Analytical Calculation of the Neutron Spectrum for Direct Measurement of $NN$ Scattering at Pulsed Reactor Yaguar*

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Abstract—Analytical calculation of a single-neutron-detector counts per YAGUAR reactor pulse is presented and comparison with coincidence scheme is given.

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1. INTRODUCTION

There is a project to measure directly $NN$ collision for checking charge symmetry of nuclear forces [1]. It is accepted that the best neutron source to perform such measurements is the Russian pulsed YAGUAR reactor. Some preliminary measurements and numerical simulations for expected experimental geometry had been performed [2]. We want to show here an analytical approach to calculations. First, we obtain analytical momentum spectrum of scattered neutrons, then the time-of-flight spectrum of neutrons detected by a single counter. After that we consider coincidence scheme, where we have two detectors, and calculate time-of-flight spectrum for one detector and delay-time spectrum for the second one. We considered coincidence scheme because from the very beginning of discussions about the project, and all the time during preparation of the experiment, many people continue to express the opinion that the coincidence scheme has an advantage comparing to the single-detector measurement. They claim that loss of intensity, which they usually estimated at the level of 20%, will be surpassed by much higher suppression of background. We show here analytically that in the coincidence scheme the effect is so much suppressed that the question about the background level becomes irrelevant.

2. ESTIMATION OF THE EFFECT

The scheme of the experiment is presented in Fig. 1 borrowed from [1]. The YAGUAR reactor 1 gives a pulse of length $t_p = 0.68$ ms, during which a huge amount of neutrons with flux density $\Phi = 0.77 \times 10^{18} n/(cm^2 s)$ is released. After a moderator at room temperature $T$ neutrons in the thermal Maxwellian spectrum arrive at the volume 2 ($V = 1.13 \text{ cm}^3$), where they collide with each other and

![Fig. 1. Scheme of the experiment on direct measurement of $nn$ scattering [1]: 1—reactor core; 2—volume of collisions; 3—neutron guide; 4—collimators; 5—detector; 6—neutron trap.](image-url)
some of them after collision fly along the neutron guide 3 with collimators 4 and arrive at detector 5, where they are registered with \( \sim 100\% \) efficiency. Collimators 4 determine the solid angle \( \Delta \Omega = 0.64 \times 10^{-4} \) sr, at which the volume \( V \) is visible by the detector. The estimated number of neutrons that can be registered at a single pulse is equal to

\[
N_e = 2n^2Vt_pv_T|b|^2d\Omega,
\]

where factor 2 takes into account that the detector can register scattered neutron or neutron-scatterer. The square of the scattering amplitude \( |b|^2 \) is defined as: \( |b|^2 = |b_0|^2/4 \), where \( b_0 \) is the singlet scattering amplitude, which is accepted to be 18 fm, and factor 1/4 is statistical weight of the singlet scattering. Therefore, \( |b|^2 = 8.1 \times 10^{-25} \) cm\(^2\). The speed \( v_T \) corresponds to the thermal speed \( v_T = 2200 \) m/s, and the factor \( v_T/|b|^2 \) determines number of collisions in the neutron gas per unit time. The factor \( n^2 \) is the square of the neutron density \( n = \Phi/v_T \) is \( \sim 3 \times 10^{12} \) cm\(^{-3}\). After substitution of all the parameters into (1) we find \( N_e \approx 170 \) neutrons per pulse. However, it is the estimated number. To find real number counted by the single detector, \( N_s \), it is necessary to calculate the scattering process. Calculation shows that \( N_s = FN_e \), where factor \( F \) is of the order of unity. Monte Carlo calculations in [1] give \( F = 0.83 \). Analytical calculation presented below give \( F = 0.705 \). The number of neutrons per pulse counted at coincidence, if the neutrons trap 6 is replaced by another detector, can be estimated as

\[
N_{sc} = N_s d\Omega\tau/t_T,
\]

where \( \tau \) is the width of the coincidence window, \( t_T = L/v_T \) is the average length of measurement time after the reactor pulse, and \( L \approx 12 \) m is the average distance between collision volume and the detectors. In the experimental scheme of Fig. 1 the time \( t_T \) is of the order of 5 ms. If we accept \( \tau/\tau_p = 0.5 \) ms, then the ratio \( \tau/t_T = 0.1 \). The factor \( d\Omega \) is included in (2) because only neutrons in this solid angle will be registered by the second detector. The total factor, which suppresses the estimated number of neutrons registered per single pulse in coincidence scheme, is of the order \( 10^{-5} \), therefore the estimated number of counts in coincidence scheme will be of \( 10^{-3} \), so the experiment becomes nonfeasible, and the level of the background, which is determined by neutron scattering on the gas atoms present at even very good vacuum conditions, becomes irrelevant. The analytical calculations, presented below, show that the real number of counted neutrons in coincidence scheme contains even additional small factor \( F_c = 0.15 \).

3. THE ANALYTICAL CALCULATION OF NEUTRON SCATTERING IN THE THERMAL NEUTRON GAS

Our calculations will be based on the standard scattering theory of neutron scattering in the atomic gas. Our main feature is that we shall make calculations directly in the laboratory reference frame without transition to the center-of-mass system. First, we remind all the definitions of the standard scattering theory and then present analytical calculations of all the required integrals.

3.1. The Standard Scattering Theory

The standard scattering theory starts with the Fermi golden rule, according to which one can write down the probability of the neutron scattering per unit time on an arbitrary system as

\[
dw(k_i \rightarrow k_f, \lambda_i \rightarrow \lambda_f) = \frac{2\pi}{\hbar} |\langle \lambda_f, k_f | U | \lambda_i, k_i \rangle|^2 \times \delta(E_{f,k} + E_f - E_i - E_f)\rho(E_{f,k}),
\]

where \(|\lambda_i\rangle, |\lambda_f\rangle\) are initial, \(|\lambda_f\rangle\) are final states of the neutron and system with energies \(E_{i,k}, E_{i,\lambda}, E_{f,k}, E_{f,\lambda}\), respectively, \(U\) is the neutron–system interaction potential, which in the neutron–atom scattering is accepted in the form of the Fermi pseudopotential

\[
U = \frac{\hbar^2}{2m} 4\pi \delta(r_1 - r_2).
\]

Here, \(r_1, r_2\) are positions of the neutron and system, \(\rho(E_{f,k})\) is the density of the neutron final states:

\[
\rho(E_k) = \left( \frac{L}{2\pi m} \right)^3 d^3k,
\]

\(E_k = \hbar^2k^2/(2m)\), \(m\) is the neutron mass, and \(L\) is the size of some arbitrary space cell.

We suppose that the system is an atom with mass \(M = m\) and momentum \(p\). The initial and final states of the neutron and atom are described with similar wave functions

\[
|k_{i,f}\rangle = \frac{1}{L^{3/2}} \exp(ik_{i,f}r),
|\lambda_{i,f}\rangle \equiv |p_{i,f}\rangle = \frac{1}{L^{3/2}} \exp(ip_{i,f}r),
\]

where \(k_{i,f}\) and \(p_{i,f}\) are initial and final neutron and atom momenta, respectively.

The flux density of the single incident neutron is

\[
j_i = \hbar k_i/(mL^3).
\]

The scattering cross section at the given initial and final states is the ratio:

\[
d\sigma(k_i \rightarrow k_f, p_i \rightarrow p_f)
\]