Absorption Lines in the 0.91–1.33 μm Spectra of Red Giants for Measuring Abundances of Mg, Si, Ca, Ti, Cr, and Ni

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Abstract

Red giants show a large number of absorption lines in both optical and near-infrared wavelengths. Still, the characteristics of the lines in different wavebands are not necessarily the same. We searched for lines of Mg I, Si I, Ca I, Ti I, Cr I, and Ni I in the z', Y, and J bands (0.91–1.33 μm), that are useful for precise abundance analyses, from two different compilations of lines, namely, the third release of Vienna Atomic Line Database (VALD3) and the catalog published by Meléndez & Barbuy in 1999 (MB99). We selected sufficiently strong lines that are not severely blended and ended up with 191 lines (165 and 141 lines from VALD3 and MB99, respectively), in total, for the six elements. Combining our line lists with high-resolution (λ/Δλ = 28,000) and high signal-to-noise ratio (>500) spectra taken with the WINERED spectrograph, we measured the abundances of the six elements in addition to Fe I of two prototype red giants, i.e., Arcturus and μ Leo. The resultant abundances show reasonable agreement with the values in the literature within ~0.2 dex, indicating that the available oscillator strengths are acceptable, although the abundances based on the two line lists show systematic differences by 0.1–0.2 dex. Furthermore, to improve the precision, solid estimation of the microturbulence (or the microturbulences if they are different for different elements) is necessary as far as the classical hydrostatic atmosphere models are used for the analysis.

Unified Astronomy Thesaurus concepts: Late-type stars (909); Stellar atmospheres (1584); Stellar abundances (1577); Spectrophotometry (1556)

Supporting material: machine-readable tables

1. Introduction

A list of stellar absorption lines, containing their information such as wavelengths and oscillator strengths, is essential in the analysis of chemical abundances. Compared to established lists of lines in the optical, however, the identification and characterization of absorption lines in the near-infrared range remains incomplete (see, e.g., Andreaesen et al. 2016; Matsunaga et al. 2020). The correct identification of lines is mandatory, and estimating the abundances cannot be done accurately without accurate calibration of the oscillator strengths, log gf.11

We focus on stellar absorption lines in the z'YJ bands, 0.91–1.33 μm, in this paper. In Kondo et al. (2019, hereafter referred to as Paper I), we identified 107 Fe I lines that are useful for measuring the iron abundances in the spectra of two well-studied red giants (Arcturus and μ Leo). While the iron abundance is one of the most representative parameters that indicates how stars are chemically enriched, the abundances of other elements provide us with crucial information on the evolution of the Milky Way and nearby galaxies (Freeman & Bland-Hawthorn 2002; Feltzing & Chiba 2013). Following Paper I, the purpose of the current study is to extend the identification of lines in the z'YJ bands to other elements, namely, Mg I, Si I, Ca I, Ti I, Cr I, and Ni I. These elements show ~10 or more lines, as we see below, which would enable precise chemical measurements (Adibekyan et al. 2015).

In addition to the quality of the line list, the microturbulence, ξ, is a critical ingredient for performing abundance measurements. This parameter is not required as long as one deals with weak lines within the linear region of the curve of growth. However, in practice, stronger lines are often included in the analysis to secure a sufficient number of lines. The ξ is required to reproduce the saturated region of the curve of growth with classical 1D atmospheric models with the local thermodynamic equilibrium (LTE) assumed (Gray 2005), while 3D hydrodynamical models do not require ξ given as an external parameter (Asplund 2000; Amarsi et al. 2016). The 3D models are also expected to include naturally the systematic effects caused by 3D/spherical structures in extended red giants (Dobrovolskas et al. 2013). Although the use of 3D...
hydrodynamical models has been gradually explored (Jofré et al. 2019 and references therein), it is currently more common to use 1D models for the abundance analysis because it is easier to use and allows direct comparisons with previous results based on 1D models. In a classical approach with 1D models, a depth-independent $\xi$ is estimated by demanding that the abundances estimated with individual lines show no dependency on line strengths, e.g., equivalent widths (EWs, denoted as $W$) or reduced EWs ($W/\lambda$). This method requires many lines of the same element covering a wide range of strengths, and Fe I lines are most often used. In Paper I, we performed a bootstrap analysis to determine $\xi$ and its error by using more than 50 Fe I lines in the $\zeta''YJ$ bands. Among the six elements we add in this paper, Si I and Ti I show many lines enough for doing the same analysis to determine $\xi$, and we compare the results obtained with these two elements with that obtained with Fe I.

2. Spectral Data and Line Selection

2.1. Observations and Data

We investigate the same $\zeta''YJ$-band spectra used in Paper I. The spectra of well-studied red giants (Arcturus and $\mu$ Leo) were collected with the WINERED cross-dispersed echelle spectrograph (Ikeda et al. 2016) with the wide mode, giving the resolution of $\lambda/\Delta \lambda = 28,000$. We carried out the observation on 2013 February 23 with the 1.3-m Araki Telescope at Koyama Astronomical Observatory, Kyoto Sangyo University, Japan (for more details see Table I in Paper I). The spectrum of a telluric standard, HIP 76267 (A1 IV), was used for the correction of telluric absorption lines with the method described in Sameshima et al. (2018). The wavelength ranges of the three bands in which the telluric lines can be well corrected cover 0.91–0.93, 0.96–1.115, and 1.16–1.33 $\mu$m. The continuum of the spectra was normalized to the unity after the telluric correction. The signal-to-noise ratios at around 12500 Å are $\sim$1000 before the telluric correction and are 850 and 720 in the final spectra of Arcturus and $\mu$ Leo (Figure 1), respectively. The stellar redshifts were corrected so that the absorption lines of each object are consistent between the two line lists, but the log $gf$ values in the two lists tend to be significantly different. While we considered all the lines of the six elements in MB99 as candidates, the VALD3 lines detected by Ikeda et al. were included, rather than all the VALD3 lines, in the following analysis.

We performed the line selection for the six elements (Mg I, Si I, Ca I, Ti I, Cr I, and Ni I) making use of synthetic spectra except for the final confirmation with the observed spectra. In the following analysis, we used the stellar parameters adopted from Heiter et al. (2015) as we did in Paper I; the effective temperature ($T_{\text{eff}}$), the surface gravity ($\log g$), and the global metallicity ([M/H]) are $4279 \pm 40 $ K, 1.60 $\pm$ 0.18 dex, and $-0.51 \pm 0.06$ dex for Arcturus, and 4520 $\pm$ 43 K, 2.36 $\pm$ 0.22 dex, and $+0.33 \pm 0.06$ dex for $\mu$ Leo. The spectral synthesis was done with SPTOOL developed by Y. Takeda (private communication), which utilizes the ATLAS9/ WIDTH9 codes by Kurucz (1993). For each object, we considered two synthetic spectra for which the atomic lines of VALD3 or MB99 are considered (i.e., we avoided mixing atomic lines of the two lists in our spectral analysis). In both cases, we included lines of CN, CO, C$_2$, CH, and OH molecules using the list compiled by VALD3.

As the first step of the line selection, we excluded lines in the following three ranges, as they are severely affected by telluric lines: 9300–9600 Å, 11150–11600 Å, and longer than 13300 Å. Then, we measured the depths and central wavelengths of the lines in the synthetic spectra for the two objects, Arcturus and $\mu$ Leo. If the depth of a line (the distance from the normalized continuum to the line minimum) was smaller than 0.03 in the synthetic spectra, the line was rejected. We also rejected lines with no minimum in the synthetic spectra for neither of the two objects within 5 kms$^{-1}$ around the expected wavelength. Besides, when two or more lines of the same element were detected within 30 kms$^{-1}$, we included only the strongest line if its X value was larger than those of the other neighboring line(s) by more than 0.5 dex; otherwise, we rejected all the lines in the narrow wavelength range. The X index is defined as $X = \log gf - \text{EP} \times \theta_{\text{sec}}$, where $\theta_{\text{sec}} = 5040/(0.86 T_{\text{eff}})$. It is a convenient indicator of line strength (Magain 1984; Gratton et al. 2006). These rejections, and also those in the following steps, were made independently for each combination of the line list (VALD3 or MB99) and the object (Arcturus or $\mu$ Leo).

The next step was to evaluate the blending of each target line with neighboring lines based on a few types of theoretical EWs. We used two kinds of synthetic spectra generated for each target line, i.e., the normal spectrum with all the lines included, $f_{\text{syn}}^\ast$, and the one with the target line removed, $f_{\text{syn}}^\dagger$. A normal EW around a target line ($\lambda$) is given by

$$W_1 = \int_{\lambda - \Delta \lambda/2}^{\lambda + \Delta \lambda/2} (1 - f_{\text{syn}}^\ast(\lambda)) d\lambda,$$

and we considered two different integration ranges, $\Delta_1$ and $\Delta_2$, which correspond to velocities of 30 and 60 kms$^{-1}$, respectively. In addition, we calculated the EW of contaminating lines, $W_1^\dagger$, which was estimated by Equation (1) but using $f_{\text{syn}}^\dagger$. Combining these EWs, we consider two indices,

$$\beta_1 = W_1^\dagger/W_1,$$

$$\beta_2 = (W_2^\dagger - W_1^\dagger)/W_1,$$
as the indicators of blending. The former measures the contamination to the main part of each target line, and the latter measures the contamination mainly to the continuum part around the line. We rejected lines with $\beta_1 > 0.3$ or $\beta_2 > 1$ (see Figure 3 in Paper I for some examples with different $\beta_1$ and $\beta_2$ values). Finally, we examined whether the lines selected with the synthetic spectra exist in the observed spectra. We rejected Mg I 11820.982 ($\log gf = -1.520$) selected from VALD3, though not listed in MB99, because we could not confirm its absorption in the observed spectra.

From VALD3, we selected 12 lines for Mg I, 50 for Si I, 15 for Ca I, 50 for Ti I, 25 for Cr I, and 13 for Ni I (Table 1). From MB99, the numbers are slightly smaller except for Ca I: eight for Mg I, 49 for Si I, 22 for Ca I, 34 for Ti I, 21 for Cr I, and seven for Ni I (Table 2). For Fe I, 97 and 75 lines, respectively, from VALD3 and MB99 are adopted from Paper I and included in Tables 1 and 2. Some lines were selected only for one of the two objects owing to the large difference in metallicity; some lines were too weak in the metal-poor object, Arcturus, while some other lines were severely blended with neighboring lines in the metal-rich object, $\mu$ Leo. Figure 2 shows EPs and $\log gf$ values of the selected lines. The lines of different elements tend to have different EPs; for example, Si I have high EPs ($\gtrsim 5$ eV), Ti I have low EPs ($\lesssim 3$ eV), while the EPs of the Fe I lines range

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Selected absorption lines seen in Arcturus (upper) and/or $\mu$ Leo (lower). Their spectra are presented in the scale of wavelength in the standard air. Tables 1 and 2 give details of the lines selected from VALD3 and MB99, respectively.}
\end{figure}
3. Abundance Analysis

3.1. Measurements of Individual Lines

We measured the abundance by fitting a small spectral part, within ±30 km s\(^{-1}\), around each target line using the MPFIT tool (Takeda 1995) implemented in SPTOOL as we did in Paper I, but we made some changes. This tool can search for the best match between synthetic and observational spectra by iteratively varying the abundance and some other parameters, including the line broadening width. However, unlike in Paper I, we fixed the line broadening width of each spectrum to 13.3 km s\(^{-1}\) for Arcturus and 12.0 km s\(^{-1}\) for \(\mu\) Leo, which we determined by measuring the widths of hundreds of lines of various elements. These widths correspond to the full-width at the half-maximum of each absorption line and include the broadenings intrinsic to the stellar line profile and the instrumental broadening. As a matter of fact, the instrumental broadening of the WINERED with the wide mode is 12 km s\(^{-1}\), significantly larger than the macroturbulence of Arcturus (5.59 km s\(^{-1}\); Sheminova 2015) and that of \(\mu\) Leo (2.9 km s\(^{-1}\); Smith et al. 2013). The microturbulence, \(\xi\), was fixed at each run of the MPFIT fitting, and we combined the measurements with different \(\xi\) to estimate it (Section 3.2). Besides, for the spectral synthesis with the MPFIT, we used the abundance patterns of iron and the six elements adopted from Jofré et al. (2015) and of CNO adopted from Smith et al. (2013) for each of our targets (Table 3). When we change the global metallicity, [M/H], of an atmospheric model, we increase or decrease [X/H] by the same amount and keep these patterns.
This is different from the analysis of Paper I in which we used the solar-abundance pattern taken from Anders & Grevesse (1989) for both Arcturus and $\mu$ Leo. Using the abundance pattern of each object in Table 3, instead of the solar pattern, leads to better reproduction of contaminating lines in each target’s spectrum. When we measure the abundance of a particular element X, in contrast, we change its [X/H] but keep the abundances of all the other elements.

MPFIT gives the abundances in the form of \( \log \epsilon(X) = \log N_X / \log N_H + 12 \), where \( N_X \) indicates the number density of the element X. We transformed this form to \( [X/H] = \log \epsilon(X) - \log \epsilon(X)_0 \), for which we adopted the solar compositions reported by Grevesse et al. (2007) that are given in Table 4. The compositions of Grevesse et al. (2007) were also used in Smith et al. (2013) and Jofré et al. (2015), with the results of which we compared our measurements below.

We did not measure the abundances for the lines that were expected to be too weak, depth smaller than 0.03, and for those expected to be too much blended on the basis of \( \beta_1 \) and \( \beta_2 \). The former and latter cases are indicated by the flags (w) and (b), respectively, in Tables 1 and 2. Besides, we decided not to measure the abundance for the lines whose depth is more than 0.35 in the synthetic spectra; the flag of (s) is given to these cases. In Paper I, we used the threshold of \( X = -6 \) for the Fe I lines for avoiding very strong lines. Since the X indices of different elements cannot be directly compared, we consider the depth for this selection, and 0.35 in depth roughly corresponds to \( X = -6 \) in the case of Fe I (Figure 3). Based on synthesized absorption lines of Si I and Ti I, we also
confirmed that the damping wing becomes important at the depth of 0.35 or more, which is consistent with the Fe I lines. We note that this threshold in depth depends on the spectral resolution because the resolution of the WINERED is not high enough to resolve the intrinsic line profile. We include the strong lines in our list for completeness and also because they are expected to be weaker in metal-poor stars.

While MPFIT failed in Paper I to measure the abundance of $\mu$ Leo with nine Fe I lines mainly because of unexpected blends of strong lines, we measured the abundances with those Fe I lines successfully for Arcturus and/or $\mu$ Leo in the new analysis except Fe I 11119.795, which is too strong in both objects. Some of the lines are contaminated by a neighboring line (or lines) that is not well reproduced in the synthetic spectra, but fixing the broadening widths helps to find reasonable fits to the target lines without being disturbed by the contaminated profile too much. On the other hand, we could not measure [Si/H] of $\mu$ Leo with Si I 10407.037 which looks severely disturbed with the absorption at $\sim$10406 Å that are present in the observed spectrum but not predicted by the synthetic spectrum. The flag of (*) is given to this case in Table 1.

3.2. Microturbulence and its Effects

In Paper I, we applied a bootstrap approach to Fe I lines to determine the depth-independent microturbulence ($\xi$) and the iron abundance at the same time. In this method, we measure the Fe I abundances with individual lines for a grid of $\xi$ values. Generally speaking, the estimate of [X/H] tends to decrease with increasing $\xi$ except for shallow lines whose EWs do not depend on $\xi$. The deeper, more saturated lines have the stronger dependency on $\xi$. Deep lines give higher (or lower) [X/H] than
shallow lines at a lower (higher) $\xi$. It is naively expected that [X/H] values with strong/weak lines agree at the true $\xi$. We consider the linear relation, $[X/H] = aX + b$, where $X$ is the line strength indicator (Section 2.2), to represent the dependency of [X/H] on line strength. The relation, in particular its slope, changes with varying $\xi$. A common approach for estimating $\xi$ is to find $\xi$ that gives $a = 0$ (see, e.g., Blackwell & Willis 1977; de Jager et al. 1984, for slightly different approaches). With the method in Paper I, we generate bootstrapping samples of the lines, each of which has a sequence of ($\xi$, [X/H]) values, and determine $\xi$ together with [X/H] for each bootstrapping sample repeatedly. Then, we evaluate the best estimates of the ($\xi$, [X/H]) of the star and their errors considering the distribution on the $\xi$-[X/H] plane given by the bootstrapping samples.

Since we made some changes in measuring the abundances of individual lines (Section 3.1), we performed the bootstrap analysis on the Fe I lines again. The resultant $\xi$ and [Fe/H],...
The Astrophysical Journal, 913:62 (15pp), 2021 May 20

Fukue et al.

Table 3

The Reference Abundances, $\log(\epsilon(X))$, used in the MPFIT Analysis and as the Zero-points of [X/H]

| Z | X  | Sun (AG89) | Sun (G07) | Arcturus | $\mu$ Leo |
|---|----|------------|------------|----------|-----------|
| 6 | C  | 8.56       | 8.39       | 7.96 (S13) | 8.52 (S13) |
| 7 | N  | 8.05       | 7.78       | 7.64 (S13) | 8.71 (S13) |
| 8 | O  | 8.93       | 8.66       | 8.64 (S13) | 9.05 (S13) |
| 12 | Mg | 7.58       | 7.53       | 7.56 (J15) | 8.18 (J15) |
| 14 | Si | 7.55       | 7.51       | 7.24 (J15) | 8.02 (J15) |
| 20 | Ca | 6.36       | 6.31       | 6.00 (J15) | 6.58 (J15) |
| 22 | Ti | 4.99       | 4.90       | 4.56 (J15) | 5.22 (J15) |
| 24 | Cr | 5.67       | 5.64       | 5.00 (J15) | 5.93 (J15) |
| 26 | Fe | 7.67       | 7.45       | 6.93 (J15) | 7.70 (J15) |
| 28 | Ni | 6.25       | 6.23       | 5.68 (J15) | 6.52 (J15) |

References. (AG89) Anders & Grevesse (1989), (G07) Grevesse et al. (2007), (S13) Smith et al. (2013), (J15) Jofré et al. (2015)

Table 4

Microturbulences and Abundances Obtained with the Fe, Si, and Ti Lines

| Atom | List | N | $\xi$ (kms$^{-1}$) | [X/H] (dex) |
|------|------|---|-------------------|------------|
|      |      |   |                   | Arcturus   |
| Fe I | MB99 | 53 | 1.25$^{+0.08}_{-0.08}$ | $-0.45^{+0.03}_{-0.03}$ | $-0.872$ |
| Fe I | VALD | 66 | 1.28$^{+0.14}_{-0.12}$ | $-0.65^{+0.06}_{-0.06}$ | $-0.949$ |
| Fe I | VALD NLTE | 47 | 1.43$^{+0.21}_{-0.19}$ | $-0.69^{+0.07}_{-0.07}$ | $-0.948$ |
| Si I | MB99 | 34 | 1.83$^{+0.10}_{-0.09}$ | $-0.30^{+0.02}_{-0.02}$ | $-0.663$ |
| Si I | VALD | 31 | 1.68$^{+0.29}_{-0.28}$ | $-0.40^{+0.06}_{-0.06}$ | $-0.739$ |
| Si I | VALD NLTE | 21 | 1.49$^{+0.32}_{-0.30}$ | $-0.41^{+0.09}_{-0.09}$ | $-0.835$ |
| Ti I | MB99 | 25 | 1.58$^{+0.24}_{-0.23}$ | $-0.11^{+0.04}_{-0.04}$ | $-0.788$ |
| Ti I | VALD | 34 | 1.57$^{+0.18}_{-0.17}$ | $-0.34^{+0.04}_{-0.04}$ | $-0.937$ |
| Ti I | VALD NLTE | 32 | 1.45$^{+0.13}_{-0.12}$ | $-0.22^{+0.04}_{-0.04}$ | $-0.907$ |

|      |      |   |                   | $\mu$ Leo   |
| Fe I | MB99 | 67 | 1.34$^{+0.12}_{-0.12}$ | $0.29^{+0.04}_{-0.04}$ | $-0.879$ |
| Fe I | VALD | 85 | 1.05$^{+0.17}_{-0.17}$ | $0.16^{+0.08}_{-0.08}$ | $-0.909$ |
| Fe I | VALD NLTE | 55 | 1.04$^{+0.22}_{-0.21}$ | $0.14^{+0.10}_{-0.10}$ | $-0.903$ |
| Si I | MB99 | 38 | 2.22$^{+0.21}_{-0.20}$ | $0.22^{+0.05}_{-0.05}$ | $-0.820$ |
| Si I | VALD | 33 | 1.67$^{+0.46}_{-0.45}$ | $0.21^{+0.07}_{-0.07}$ | $-0.834$ |
| Si I | VALD NLTE | 19 | 1.52$^{+0.54}_{-0.53}$ | $0.21^{+0.07}_{-0.07}$ | $-0.871$ |
| Ti I | MB99 | 29 | 2.11$^{+0.24}_{-0.23}$ | $0.26^{+0.06}_{-0.06}$ | $-0.834$ |
| Ti I | VALD | 35 | 1.81$^{+0.35}_{-0.34}$ | $0.10^{+0.07}_{-0.07}$ | $-0.921$ |
| Ti I | VALD NLTE | 30 | 1.71$^{+0.24}_{-0.24}$ | $0.16^{+0.07}_{-0.07}$ | $-0.902$ |

Note. The third column (NLTE) indicates whether the non-LTE correction was applied or not. The fourth column (N) indicates the number of lines used for estimating the microturbulence, $\xi$, and the abundance, [X/H]. The [X/H] are scaled with respect to the solar abundance in Grevesse et al. (2007). The last column (r) indicates the correlation coefficient (see the definition in Paper I) between $\xi$ and [X/H].

listed in Table 4, are consistent with the previous values in Paper I within uncertainties, but the errors get slightly smaller. In addition, we made the same analysis for Si I and Ti I. Among the elements we investigate, beside Fe I, the numbers of only the Si I and Ti I lines are large enough for the bootstrap analysis for determining $\xi$ together with the abundance. As shown in Table 4 and Figure 4, the $\xi$ obtained for Si and Ti lines tend to be higher than those for the Fe lines.

![Figure 3](http://nlte.mpia.de/gui-siuAC_secE.php)

We performed the same bootstrap analysis for subsets of the Fe I, Si I, and Ti I lines to which we could apply the non-LTE correction. Bergemann et al. (2012, 2013) carried out non-LTE line-formation calculations in the atmospheres of red supergiants for Fe I, Ti, and Si, and discussed the consequences of non-LTE effects for the J-band analysis. It was found that non-LTE effects are small for J-band Fe I lines, but significant for Ti I and Si I lines. We calculated the non-LTE corrections on the lines of Fe I, Si I, and Ti I using their website. Their online tool gives the non-LTE corrections for the abundances obtained with individual lines for a given set of stellar parameters. We added the corrections to [X/H] with individual VALD3 lines, and performed the same bootstrap analysis, but with smaller numbers of lines, to calculate $\xi$ and [X/H]. Not all the lines we selected are included in their tool, but dozens of the lines could be included in this analysis. Table 4 and Figure 4 present the resultant $\xi$ and [X/H]. In the case of Arcturus, $\xi$ gets slightly higher, but the non-LTE corrections for Fe I lines are mostly within 0.05 dex and have little impact on the $\xi$. In fact, the difference of 0.14 kms$^{-1}$ with and without the non-LTE corrections can be explained by the difference in the lines used for the calculation; using the 47 lines to which we could apply the corrections leads to $\xi \sim 1.42$ kms$^{-1}$ even if we do not apply the corrections. The non-LTE corrections are as large as 0.15 dex (or $-0.15$ dex) for some Si I and Ti I lines. In addition, there are weak correlations between the line strengths and the non-LTE correlations, leading to slightly lower $\xi$, though the impacts on $\xi$, [X/H] are not really significant (Figure 4). The same analysis can be done with the MB99 lines. However, the primary purpose of this part is to see the relative impact of

12 http://nlte.mpia.de/gui-siuAC_secE.php
the non-LTE corrections on the microturbulence, and we examined the results with the non-LTE corrections only for the VALD3 line sets.

As the contours in Figure 4 suggest, the abundance we obtain depends on the microturbulence. If we fix $\xi$ to the one obtained with the Fe I lines, $[\text{Si}/H]$ and $[\text{Ti}/H]$ get higher than the ones obtained with the bootstrap analysis (Section 3.3). If we knew the true abundances, we would be able to determine the $\xi$ that leads to accurate abundances. However, we cannot draw such a conclusion because previously reported values of $[\text{Si}/H]$ and $[\text{Ti}/H]$ have large scatters. Figure 5 compares our estimates with previous results for Arcturus (Thevenin 1998; Luck & Heiter 2005; Fullbright et al. 2007; Worley et al. 2009; Chou et al. 2010; Ramírez & Allende Prieto 2011; Britavskiy et al. 2012; Smith et al. 2013; Jofré et al. 2015) and $\mu$ Leo (McWilliam 1990; Thevenin 1998; Smith & Ruck 2000; Luck & Heiter 2007; Smith et al. 2013; Jofré et al. 2015). Here, we only refer to the reports in which the solar-abundance reference is apparent, and all the $[\text{X}/H]$ are scaled with respect to the solar compositions in Grevesse et al. (2007). Regardless of which line lists are used and which of the $\xi$ in Table 4 are used, our estimates are within the scatters of the values in the literature.

Takeda (1992) investigated the microturbulence of Arcturus and found that different groups of lines tend to give different $\xi$ showing the correlation with the depth of line forming layers (also see Gray 1981, for an earlier report on the depth-dependent $\xi$ of Arcturus). They found that the derived $\xi$ values depend on the properties of the line sets, e.g., the EPs and the ionization stage of the lines included; $\xi$ tends to be smaller with the lines formed in the inner atmosphere used in the analysis. Roughly speaking, the depth of a line forming region is well correlated with the line strength as expected, while the EP and some other parameters have additional effects (Gurtovenko & Sheminova 2015). As seen in Figure 3, the distributions of depths of the Fe I, Si I, and Ti I lines used for the analysis are similarly broad and uniform between the shallowest (0.03) and the deepest (0.35). At least, not all the Si I and Ti I lines are formed in layers higher than Fe I lines. There appears to be no simple reason to expect that both Si I and Ti I give $\xi$ higher than Fe I. Moreover, while determining $\xi$ accurately requires large numbers of lines (Mucciarelli 2011), the numbers of lines available for each group tend to be limited. The problem with...
Figure 5. Comparison of the abundances of Arcturus and μ Leo derived by using different methods and line lists on the [X/Fe]–[Fe/H] diagrams. The results based on the VALD3 lines are illustrated in blue, and those based on the MB99 lines in red. The two objects have distinctly different metallicities (Arcturus being metal-poor), and the same markers are used for both of them within each panel. Filled triangles indicate the results with the microturbulences estimated for individual elements with all the available lines used, while the VALD3 lines for which the non-LTE effects were taken into account were used for the results indicated by “×” marker (Table 4). The microturbulences were fixed to those obtained with Fe I lines to give the results indicated by open circles (Table 5). Small “±” markers indicate the literature results for the two objects (references given in text). The gray small dots indicate the abundances of red giants with the Apache Point Observatory Galactic Evolution Experiment (APOGEE) and Kepler data obtained by Hawkins et al. (2016).

Two error bars are given to each point for representing the abundances estimated for individual elements with all the available lines used, while the limited numbers of lines is particularly severe with infrared spectra; in our case, we can use less than 100 lines even for Fe I, which results in the significant statistical errors in ξ. The limited line number prevents us from more detailed discussions such as using a subset of lines, e.g., those with high or low EPs only, for the ξ estimation (see the simulation described in Appendix B concerning the number of lines required for the bootstrap analysis). Moreover, we did not apply the method for determining ξ with the line of the other elements (i.e., Mg I, Ca I, and Ni I) because each of these elements give less than ∼10 lines.

Considering the above situation, we fix ξ to the one determined for Fe I and use it for other elements, which is a standard procedure of the classical analysis involved with the depth-independent ξ. Moreover, the abundances measured with the same approach should be used together in discussions about the features on [Fe/H]–[X/Fe] diagrams, and it is more common to fix ξ to the one estimated with Fe I lines. The depth dependency of ξ is an important and interesting issue. Still, the primary purpose of this paper is the line identification, neither determining the depth-dependent ξ nor precise calibration of the oscillator strengths of individual lines.

### 3.3. Abundances of the Six Elements and Iron

In the following analysis, we used the ξ that we obtained with Fe I lines without the non-LTE effect corrected to measure the abundances of the six elements (Mg I, Si I, Ca I, Ti I, Cr I, and Ni I) as well as Fe I. For each combination of the object (Arcturus and μ Leo) and the line list (VALD3 or MB99), we made the MPFIT measurements with the ξ obtained for Fe I (Table 4). Tables 1 and 2 list obtained abundances with individual lines selected for each combination. The flags of (w) or (b) are given to the lines that were not selected because they are weak or blended (according to β1 and β2). The abundances of the lines with the (s) flag were not measured because they are selected but deeper than 0.35 in depth, while the flag of (∗) is given to Si I 10407.037 in the case of μ Leo (see Section 3.1).

Table 5 lists the number, N, of the lines with which we measured [X/H] together with the standard deviation (SD) and the interquartile range (IQR) of the N values for each combination of line list and object. Figure 6 shows box plots of derived abundances. We took the median of the N values as the best estimate of the abundance. To estimate its error, we took a bootstrap approach; we generated 10,000 bootstrapping

| Atom | List | N | SD (dex) | IQR (dex) | [X/H] (dex) |
|------|------|---|---------|----------|-------------|
| Mg I | VALD3 9 | 0.426 | 0.256 | 0.307 ± 0.122 |
| Mg I | MB99 7 | 0.384 | 0.105 | 0.166 ± 0.115 |
| Si I | VALD3 34 | 0.293 | 0.384 | 0.301 ± 0.042 |
| Si I | MB99 36 | 0.313 | 0.284 | 0.403 ± 0.044 |
| Ca I | VALD3 13 | 0.278 | 0.216 | 0.277 ± 0.054 |
| Ca I | MB99 20 | 0.239 | 0.213 | 0.341 ± 0.034 |
| Ti I | VALD3 36 | 0.433 | 0.341 | 0.330 ± 0.058 |
| Ti I | MB99 31 | 0.243 | 0.332 | 0.454 ± 0.047 |
| Cr I | VALD3 25 | 0.234 | 0.316 | 0.248 ± 0.052 |
| Cr I | MB99 21 | 0.203 | 0.236 | 0.361 ± 0.054 |
| Fe I | VALD3 84 | 0.309 | 0.355 | 0.205 ± 0.033 |
| Fe I | MB99 68 | 0.167 | 0.247 | 0.295 ± 0.027 |
| Ni I | VALD3 11 | 0.405 | 0.602 | 0.383 ± 0.182 |
| Ni I | MB99 7 | 0.391 | 0.774 | 0.067 ± 0.279 |

**Note.** For each combination of line list and object, N lines were used for the abundance measurements with the microturbulence, ξ, fixed. The abundances from N individual lines, showing the standard deviation (SD) and the interquartile range (IQR) in the table, were combined to obtain the final abundance, [X/H], and its error.

Table 5 lists the number, N, of the lines with which we measured [X/H] together with the standard deviation (SD) and the interquartile range (IQR) of the N values for each combination of line list and object. Figure 6 shows box plots of derived abundances. We took the median of the N values as the best estimate of the abundance. To estimate its error, we took a bootstrap approach; we generated 10,000 bootstrapping
samples of the N values and calculated the SD of the 10,000 median values from individual bootstrapping samples. This SD, \( \Delta_{\text{med}} \), is considered as the statistical error of the final \([X/H]\). We obtained these estimates for seven elements, including Fe I (Table 5). It is worthwhile to note that both SDs and IQRs are large for many combinations of line list and object, IQR \( \gtrsim 0.15 \) dex for Arcturus and IQR \( \gtrsim 0.25 \) dex for \( \mu \) Leo. Moreover, the ratio of IQR to SD is expected to be 1.35 for the Gaussian distribution, but the ratios in Table 5 are not normal for some combinations. These suggest that the \( \log gf \) values in the current lists are not sufficiently precise, and the errors for some lines are particularly large, leading to outliers. We will come back to this point in Section 3.4.

We also estimated how much the uncertainties in the stellar parameters \( (T_{\text{eff}}, \log g, [M/H], \xi) \) affect the estimates of the abundances. We adopt the error in each parameter from Heiter et al. (2015) as the offset, \( \sigma_p \), given in Table 6. To evaluate the effects of changing these parameters, we added positive and negative offsets to each parameter of the atmosphere models one by one. For each offset, we ran MPFIT and measured how much the abundance of each line is altered by the offset. We then took the median of the relative changes as the impact of each parameter on the abundance, \( \Delta_p \), where \( p \) is one of the stellar parameters. If the sizes of the positive and negative offsets are different from each other, we consider the root mean squares for \( \sigma_p \) and/or \( \Delta_p \); e.g., \( \sigma_\xi \) tends to be asymmetric, and \( \sigma_p \) and \( \Delta_p \) in Table 6 are the root mean squares. Figure 7 shows the \( \Delta_p \) corresponding to the positive \( \sigma_p \) offsets of each parameter for Arcturus with VALD3 used for the measurements. The size of \( \Delta_p \) varies with the element, and the sign of \( \Delta_p \) also differs from one element to another in the case of \( T_{\text{eff}} \).

The trends of \( \Delta_{\log g} \) are significant in the various elements. The trends for individual elements are described in Appendix A.

The trend of \( \Delta_{\log g} \) is noteworthy. This dependency on \( \log g \) is very small for Ca I, Ti I, and Cr I, in particular, while it is nonzero for other elements. For the three elements, the first ionization stage dominates in the entire range of line-formation layers relevant to this study, and the line absorption coefficient \( (I_\nu) \) is expected to be proportional to the electron pressure. The continuum absorption coefficient \( (\kappa_\nu) \) is also proportional to the electron pressure around \( T_{\text{eff}} \) of our targets because it is dominated by the negative hydrogen (H\(^-\)). Then, the line depths \( (\approx I_\nu/\kappa_\nu) \) are expected to be insensitive to the electron pressure and to the surface gravity as suggested by the small \( \Delta_{\log g} \) (Gray 2005; see, also, Jian et al. 2020). In case of other elements, in contrast, weak lines are formed in more ionized layers (the ionization fraction \( \sim 60 \) to almost 100%), while the ionization fraction drops significantly, down to a few percent in some cases, in the forming layers of strong lines. This results in the nonzero dependency on \( \log g \). This explains the \( \Delta_{\log g} \) presented in Figure 7 well.

Combining the \( \Delta_p \) values with the statistical error \( (\Delta_{\text{med}}) \) estimated with the bootstrap analysis, we calculated the total error, \( \Delta_{\text{tot}} \), by

\[
\Delta_{\text{tot}}^2 = \Delta_{T_{\text{eff}}}^2 + \Delta_{\log g}^2 + \Delta_{[M/H]}^2 + \Delta_\xi^2 + \Delta_{\text{med}}^2, \tag{4}
\]

where we ignored the covariant terms as we did in Paper I. The stellar parameters used for Figure 2 in Paper I show no clear correlation between any two of them \( (T_{\text{eff}}, \log g, [M/H], \xi) \). The results are given in Table 6.

### 3.4. Comparison with Previous Results

Figure 8 compares our two estimates of \([X/H]\) based on the VALD3 and MB99 for each element with those reported by Smith et al. (2013) and Jofré et al. (Jofré et al. 2014, 2015). The former study used \( H \)-band spectra, whereas the latter used optical spectra. The total errors, \( \Delta_{\text{tot}} \) in Table 6, are used for the error bars for our results. The errors we use for the results in Smith et al. (2013) and Jofré et al. also combine statistical errors and systematic errors. The definition of the statistical errors given in their works is, however, different from that of ours. Their statistical errors are the SDs of abundances from individual lines. Their SDs tend to be significantly smaller than the SDs in Table 5. This can be ascribed to the difference in the line selection and the quality of \( \log gf \) values; we have not selected the lines on the basis of the accuracy of \( \log gf \). The astrophysical calibration of the \( \log gf \) values remains to be done, desirably, for red giants with each of the abundance-analysis tools. As for the systematic errors, all of these works estimated the effects of stellar parameters and, for all the points in Figure 8, we combined \( \Delta_p \) in our notation, of the four parameters \( (T_{\text{eff}}, \log g, [M/H], \xi) \) by the square root of sum of squares without the covariant terms taken into account.

Quantitative analysis on the accuracy of \( \log gf \), or their recalibration, is outside the scope of this paper. Still, we can identify the lines that seem to have \( \log gf \) with larger errors than other lines, and give caution to them. Si I, Ti I, Cr I, and Fe I have more than 20 lines selected, and we identified outliers among their lines by considering the IQR rule. Let \( q_{25} \), \( q_{75} \), and \( q_{50} \) denote the quartiles of the \([X/H]\); \( q_{50} \) corresponds to the median, and the IQR is given by \( q_{75} - q_{25} \). We here use the \( 1.5 \times \) IQR rule for the outlier detection, i.e., the derived abundance of a line is judged as an outlier if it is smaller than \( q_{25} - 1.5 \) IQR or larger than \( q_{75} + 1.5 \) IQR. In contrast, the numbers of lines are small for Mg I, Ca I, and Ni I, and we use \( q_{50} \pm 0.5 \) dex as the thresholds of outliers. For each line in a given line list, the flag is given as follows. We combine the number of objects for which the abundance was measured \( (N_{\text{obj}}) \) and the number of measurements found to be outliers \( (N_{\text{out}}) \) and give the flag, \( N_{\text{out}}/N_{\text{obj}} \). If the line was not used for
Figure 7. The effects of changing stellar parameters on the abundance of Arcturus measured with the VALD3 lines. The effects were measured by changing each parameter at a time, and $\Delta p_\rho$ shown in this plot (also see Table 6) indicates how much [X/H] changes with the positive offset in the parameter $\rho$. $\pm 35$ K in $T_{\text{eff}}$ (red circles), $\pm 0.06$ dex in $\log g$ (green triangles), $\pm 0.08$ dex in $[\text{M}/\text{H}]$ (blue squares), and $\pm 0.14$ km s$^{-1}$ in $\zeta$ (orange crosses).

Figure 8. The final abundances of Arcturus and $\mu$ Leo based on VALD3 (blue) and MB99 (red). Their statistical errors in Table 5 and the total errors in Table 6 are drawn in the bright colors and pale colors. Black crosses and squares indicate the values reported by Smith et al. (2013) and Jofré et al. (2014) for Fe; Jofré et al. (2015) for the other elements, respectively. The definitions of the error bars are given in the text.

### Table 6

| $\Delta p_\rho$ | $\sigma_p$ | Mg I | Si I | Ca I | Ti I | Cr I | Fe I | Ni I |
|-----------------|------------|------|------|------|------|------|------|------|
| $\Delta T_{\text{eff}}$ | $\pm 35$ K | $\pm 0.001$ | $\pm 0.029$ | $\pm 0.015$ | $\pm 0.060$ | $\pm 0.038$ | $\pm 0.003$ | $\pm 0.011$ |
| $\Delta \log g$ | $\pm 0.06$ dex | $\pm 0.001$ | $\pm 0.014$ | $\pm 0.000$ | $\pm 0.002$ | $\pm 0.001$ | $\pm 0.011$ | $\pm 0.015$ |
| $\Delta [\text{M}/\text{H}]$ | $\pm 0.08$ dex | $\pm 0.021$ | $\pm 0.022$ | $\pm 0.003$ | $\pm 0.003$ | $\pm 0.001$ | $\pm 0.014$ | $\pm 0.019$ |
| $\Delta \zeta$ | $\pm 0.08$ km s$^{-1}$ | $\pm 0.005$ | $\pm 0.020$ | $\pm 0.009$ | $\pm 0.022$ | $\pm 0.017$ | $\pm 0.030$ | $\pm 0.019$ |
| $\Delta \text{med}$ | $\ldots$ | $0.074$ | $0.056$ | $0.040$ | $0.022$ | $0.032$ | $0.018$ | $0.070$ |
| $\Delta \text{tot}$ | $\ldots$ | $0.077$ | $0.071$ | $0.044$ | $0.068$ | $0.052$ | $0.039$ | $0.077$ |

#### Arcturus with VALD3

#### Arcturus with MB99

#### $\mu$ Leo with VALD3

#### $\mu$ Leo with MB99

Note. In the first four lines for each combination of the object and the line list, $\sigma_p$ indicates the error, in the parameter $p$, that was used for estimating $\Delta p_\rho$, i.e., the effect of $p$ on the abundance [X/H]. In the last two lines, $\Delta \text{med}$ indicates the error in Table 5, and $\Delta \text{tot}$ is the total error estimated with Equation (4).
the abundance measurement, with the flags such as (w) and (s), for an object, it is not included in $N_{\text{obj}}$ and the outlier detection was not performed. Thus-determined flags are given in Tables 1 and 2. The flag of “2/2” suggests that the abundances derived for both objects are judged as outliers, indicating that the log $gf$ of such a line has a large error. For example, Ca I 12816.05 was judged as an outlier in all cases, the log $gf$ of this line in both VALD3 and MB99 are considered to be rather inaccurate. We gave nonzero flags (i.e., $N_{\text{out}} \neq 0$) to 21 lines in VALD3 and 11 lines in MB99. We added the colon mark (:) to the abundances judged as outliers in Tables 1 and 2.

Concerning the systematics of the abundance scales based on the two line lists, the [X/H] based on MB99 are systematically higher than the counterparts with VALD3 by 0.1–0.2 dex for both objects except Mg I and Ni I for which the numbers of available lines are small. Roughly speaking, the best estimates in Figure 8 agree with each other allowing for scatters of $\sim 0.2$ dex. Figure 9 compares our estimates with the abundance trends on the [X/Fe]–[Fe/H] diagrams reported by Hawkins et al. (2016), who measured the abundances of disk stars. VALD3 and MB99 place Arcturus and $\mu$ Leo at slightly different positions, but the results are still consistent with the trends of disk stars as expected (e.g., Smith et al. 2013). While the difference of the metallicity scales is not negligible, $\gtrsim 0.1$ dex different in [Fe/H], we cannot select one of them based on the comparison in Figure 9.

4. Concluding Remarks

We identified absorption lines of six elements (Mg I, Si I, Ca I, Ti I, Cr I, and Ni I) that appear in the $\gamma$YJ-band spectra of red giants and are relatively isolated. From the large compilations of VALD3 and MB99, $\sim 10$ (for Ni I and Mg I) to over 30 (for Si I and Ti I) lines were selected for each element. Some detailed discussions on the selected lines for each element are given in Appendix A. These lines combined with $>50$ Fe I lines reported in Paper I are useful for the abundance analysis of red giants with $\gamma$YJ-band spectra, which will be particularly important for obtaining the precise metallicities of stars obscured by severe interstellar extinction that hampers optical spectroscopy.

With the abundance analysis making use of classical 1D LTE atmospheric models (ATLAS9 in our case), the microturbulence, $\xi$, is a crucial parameter required for obtaining accurate results. We measured $\xi$ that gives [X/H] independent of line strength for each of Si I and Ti I. They are slightly higher than those determined for Fe I, and the differences in $\xi$ are large enough to give significant impacts on the abundances, 0.1–0.2 dex. Therefore, unless it turns out that a common $\xi$ can be used for every element, finding a good $\xi$ for each element will be an essential step in order to improve the accuracy of the abundance measurements in the future. Another vital error source is the systematics of the oscillator strengths given in the two line lists, VALD3 and MB99 tend to give slightly different abundances, 0.1–0.2 dex higher if the MB99 is used. The scatters of the abundances in previous studies prevent us from judging which of VALD3 and MB99 gives more accurate results.

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Appendix A
Lines Selected for Individual Elements

1. Mg I (magnesium, Z = 12): 12 and 8 Mg I lines from VALD3 and MB99, respectively, are included in our line lists. Mg I 11820.982 in VALD3 was listed up with the synthetic spectra but not included because its absorption was not confirmed in the observed spectra. All the eight lines from MB99 are also included in the VALD3 list. The EPs of 11 lines are high, >5.9 eV, but Mg I 11828 has a low EP, ~4.35 eV, though it is too strong to be used for our abundance measurements. The abundances estimated with individual lines tend to show a large scatter, but the median values are consistent with the values in the literature within the errors. Errors in the stellar parameters have relatively small impacts on the measurements of [Mg/H].

2. Si I (silicon, Z = 14): 56 Si I lines are included in our lists in total, 50 lines from VALD3 and 49 lines from MB99 (43 lines being selected from both). All these lines have relatively high EPs, ≥5 eV. Their depths cover a wide range from shallow to deep, and several lines are too strong to be used for the abundance measurements. The abundances with individual lines show large scatters, but the resultant abundances and the errors are comparable with those in the literature. [Si/H] obtained with VALD3 are slightly lower, by ~0.1 dex, than those obtained with MB99.

3. Ca I (calcium, Z = 20): 15 lines are selected from VALD3, while 17 lines are selected from MB99. There are 10 lines in common, of which Ca I 10343 in both lists is too strong to be used for the abundance measurements. More than half of the MB99 lines are too weak in Arcturus but are measurable in μ Leo. Ca I 12816 has highly inaccurate log gf values, by more than 0.5 dex, in the two lists, but other lines lead to reasonable estimates of [Ca/H]. The abundances estimated with VALD3 are lower than those with MB99 by 0.1–0.15 dex.

4. Ti I (titanium, Z = 22): In total, 54 Ti I lines (50 lines from VALD3 and 34 from MB99) are included in our lists in total. Most of them have EPs lower than 2 eV except a few with EP ~ 4 eV. The strengths range from weak to very strong over the limit, 0.35 in depth, beyond which we did not perform the abundance measurements. The dependency of derived [Ti/H] on T eff is strong. The abundances estimated with VALD3 are lower than those with MB99 by 0.1–0.2 dex.

5. Cr I (chromium, Z = 24): 25 and 21 Cr I lines are selected from VALD3 and MB99, respectively, with 19 lines in common and 27 lines in total. There are not very strong lines, i.e., no Cr I line is deeper than 0.35 in either Arcturus or μ Leo, while several lines are too weak in Arcturus. The dependency of derived [Cr/H] on T eff is relatively strong. The abundances estimated with VALD3 are lower than those with MB99 by 0.1–0.15 dex.

6. Ni I (nickel, Z = 28): 15 Ni I lines (13 and seven from VALD3 and MB99, respectively) are included in our line lists. No line is stronger than the limit of 0.35 in depth, but a few lines are too weak in Arcturus. The line sets used for the combinations of line list and object tend to be different. The abundances from individual lines show large scatters, especially for μ Leo, leading to the large errors in the final [Ni/H], 0.1–0.3 dex, combined with the small line numbers.

Appendix B
Simulations of Bootstrap

We performed a simulation for investigating how many lines are required to derive the microturbulence, ξ, and abundance, [X/H], simultaneously with the bootstrap method. For this purpose, we used the Fe I lines in Tables 1 and 2, but excluded those with flags such as (s) and (w) in either of Arcturus or μ Leo. We found 60 VALD3 lines and 50 MB99 lines to use, and we repeat the following analysis for the four combinations of object (Arcturus or μ Leo) and line list (VALD3 or MB99). Let the set A indicate this starting line set (the number of lines

Figure 10. The simulation results on the bootstrap analysis with different numbers, N, of lines available. The parameters are described in the text. Here, presented is the case of Arcturus/ξ eff = 0.14 kms\(^{-1}\) for VALD3, 0.09 kms\(^{-1}\) for MB99, and the left three panels plots the results with VALD3 and the right panels those with MB99.
being $N_{\text{All}} = 60$ for VALD3 and 50 for MB99). We first ran the bootstrap analysis for this set A, and compared this result, $\xi_{\text{All}} \pm e_{\text{All}}$, with the results with fewer lines as follows. As simulation sets of fewer lines, we repeated the random selection $N$ times among those in the set $A$ without duplication 1000 times for each $N$ ($N$ takes 10, 15, 20, $\cdots$, 40). Each set of $N$ lines gives an estimate of $\xi(j)$ together with the 1σ error range, $[\xi(j) - e_j : \xi(j) + e_j]$, where $j$ varies from 1 to 1000. The error in $\xi(j)$ is given by $e(j) = \sqrt{(e_x^2 + e_y^2)} / 2$. We thus obtained 1000 $e(j)$ values and calculated their mean as the measure of the typical error (denoted as $e_N$) obtained with $N$ lines. In addition, the standard deviation of the 1000 $\xi(j)$, denoted as $\sigma_N$, indicates how much the selection of $N$ lines could affect the resultant microturbulence. Figure 10 shows these results for Arcturus. Both $e_N$ and $\sigma_N$ increase with decreasing $N$, as expected; the determination of $\xi$ and $[X/H]$ suffers from larger uncertainties if fewer lines are available. When the number of lines gets as small as 15–20, the statistical uncertainty ($e_N$) becomes 1.5–2 times as large as $e_{\text{All}}$. In addition, the effect of the line selection ($\sigma_N$) becomes as large as the statistical uncertainty, which means that the uncertainty significantly affected by the systematic error caused by which lines are actually available and also by the precision of oscillator strengths of the particular line set. We therefore suggest that the bootstrap method of determining $\xi$ and $[X/H]$ together requires $\sim$20 lines at least, but this limit depends on the precision of $g f$.

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The Astrophysical Journal, 913:62 (15pp), 2021 May 20

Fukue et al.