Model independent investigation of rare semileptonic $b \to ul\bar{\nu}_l$

decay processes

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Abstract

Motivated by the recent observation of lepton universality violation in the flavour changing charged current transitions $b \to cl\bar{\nu}_l$, we intend to scrutinize the lepton non-universality effects in rare semileptonic $B$ meson decays involving the quark level transitions $b \to ul\bar{\nu}_l$. In this regard, we envisage the model-independent approach and consider the generalized effective Lagrangian in the presence of new physics and constrain the new parameters by using the experimental branching fractions of $B^+_u \to l^+\nu_l$ and $B^- \to \pi^0\mu^-\bar{\nu}_\mu$ processes, where $l = e, \mu, \tau$. We then estimate the branching ratios and forward-backward asymmetries of $B_{(s)} \to P(V)l\bar{\nu}_l$ processes, where $P(= K, \pi, \eta^{(')} )$ denotes the pseudoscalar meson and $V(= K^*, \rho)$ is the vector meson. We also find out various lepton non-universality parameters in these processes in the presence of new physics.

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I. INTRODUCTION

In recent times, flavour physics has become quite interesting as several deviations at the level of \((2 - 4)\sigma\) have persistently been observed in semileptonic \(B\) decays. Specifically, the LHCb experiment has observed several anomalies in the rare semileptonic \(B\) decays driven by the flavour changing neutral current (FCNC) \(b \to s\) transitions. The most leading ones are the observation of \(3.7\sigma\) deviation in the angular observables \(P_5'\) \([1,2]\), the decay rate of \(\bar{B} \to \bar{K}^{(*)}\mu^+\mu^-\) mode \([3]\) and also the \(3\sigma\) \([4]\) discrepancy in the decay rate of \(B_s \to \phi\mu^+\mu^-\) process in the low \(q^2\) region. Besides these anomalies, recently LHCb and \(B\) factories have observed the violation of lepton flavour universality in \(B \to D^{(*)}\bar{\nu}_\ell\) and \(B \to K^{(*)}\bar{\ell}\bar{\nu}_\ell\) processes, which comprises some additional tension. The lepton non-universality (LNU) parameter \((R_K)\), defined as the ratio of the branching fractions of \(B^+ \to K^+\mu^+\mu^-\) over \(B^+ \to K^+e^+e^-\) and its measured value in the low \(q^2 \in [1,6]\) region \([5]\)

\[
R_{K}^{\text{Expt}} = \frac{\text{BR}(B^+ \to K^+\mu^+\mu^-)}{\text{BR}(B^+ \to K^+e^+e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036, \tag{1}
\]

has \(2.6\sigma\) deviation from the corresponding SM result \(R_{K}^{\text{SM}} = 1.0003 \pm 0.0001\) \([6]\). In addition, very recently LHCb Collaboration has also reported discrepancy of \(2.2\sigma\) in \(R_{K^*}\) \([7]\)

\[
R_{K^*}^{\text{Expt}} = \frac{\text{BR}(B \to K^*\mu^+\mu^-)}{\text{BR}(B \to K^*e^+e^-)} = 0.660^{+0.110}_{-0.070} \pm 0.024, \tag{2}
\]

from the corresponding SM prediction \(R_{K^*}^{\text{SM}} = 0.92 \pm 0.02\) \([8]\) in the \(q^2 \in [0.045,1.1]\) GeV\(^2\) bin and \(2.4\sigma\) discrepancy \([7]\)

\[
R_{K^*}^{\text{Expt}} = 0.685^{+0.113}_{-0.069} \pm 0.047, \tag{3}
\]

has been found in \(q^2 \in [1.1,6]\) GeV\(^2\) region from its SM predicted value \(R_{K^*}^{\text{SM}} = 1.00 \pm 0.01\) \([8]\).

Analogously, in the charged current transition processes mediated through \(b \to c\tau\bar{\nu}_\tau\), LHCb as well as both the \(B\) factories Belle and BaBar have measured the LNU parameter \(R_{D^{(*)}}\) in \(B \to D^{(*)}\bar{\nu}_l\) decay processes and the measured values \([9,11]\)

\[
R_{D}^{\text{Expt}} = \frac{\text{BR}(B \to D\tau\bar{\nu}_l)}{\text{BR}(B \to Dl\bar{\nu}_l)} = 0.397 \pm 0.040 \pm 0.028, \tag{4}
\]

\[
R_{D^*}^{\text{Expt}} = \frac{\text{BR}(B \to D^*\tau\bar{\nu}_l)}{\text{BR}(B \to D^*l\bar{\nu}_l)} = 0.316 \pm 0.016 \pm 0.010, \tag{5}
\]
have respectively $1.9\sigma$ and $3.3\sigma$ deviation from the corresponding SM predictions [12,13]

$$R_{D}^{SM} = 0.300 \pm 0.008, \quad R_{D^*}^{SM} = 0.252 \pm 0.003. \quad (6)$$

In this context, we wish to explore the possibility of observing LNU parameters and other asymmetries in the rare semileptonic $b \rightarrow ul\bar{\nu}_l$ decay processes, in order to corroborate the observed results on lepton non-universality.

In the SM, the $V-A$ current structure of the weak interactions describes various charged current interactions for all three generation of quarks and leptons to a high precision. However, the recent experimental data indicates that among all the leptonic and semileptonic decays of $B$ mesons, the decay processes involving third generation of fermions in the final state are comparatively less precise than the first two generations. The coupling of the third generation fermions to the electroweak gauge sector is relatively stronger due to the heavier mass of the tau lepton and thus, more sensitive to new physics (NP) which could modify the $V-A$ structure of the SM. The decays with third generation fermions in the final state are sensitive to non-SM contributions arising from the violation of LFU, hence, these processes could be ideally suited for probing the NP signature. In this respect, the study of $B \rightarrow (\pi, \rho, \eta^{(i)})l\bar{\nu}_l$ and $B_s \rightarrow K^{(*)}l\bar{\nu}_l$ charged current processes, involving the quark level transitions $b \rightarrow u$ would be quite interesting to test the lepton flavour non-universality. In this paper, we adopt the model-independent approach to analyze the effect of NP in the rare semileptonic $b \rightarrow ul\bar{\nu}_l$ decay processes. For this purpose, we consider the generalized effective Lagrangian, including the possible new parameters allowed by Lorentz invariance. We constrain the new coefficients by using the experimental data on the branching fractions of $B_u^+ \rightarrow l^+\nu_l$ processes. We then compute the branching ratios, forward-backward asymmetries and various LNU parameters of semileptonic $B \rightarrow (\pi, \rho, \eta^{(i)})l\nu_l$ and $B_s \rightarrow K^{(*)}l\nu_l$ processes. Although these processes have been extensively studied in the literature [14–24], in the context of various new physics models and also in model-independent way, but the search for lepton nonuniversality parameters are not being explored.

The outline of the paper is as follows. In section II, we describe the most general effective Lagrangian responsible for the $b \rightarrow ul\bar{\nu}_l$ processes. We also show the constraints on the new parameters by using the branching ratios of $B_u^+ \rightarrow l^+\nu_l$ processes. The constraint on new physics couplings from the $B^- \rightarrow \pi^0\mu^-\bar{\nu}_\mu$ process is presented in section III. We also estimate the branching ratios, forward-backward asymmetries and the LNU parameters of
the $B \to P l \bar{\nu}_l$ processes, where $P(= K, \pi, \eta(0))$ represents the pseudoscalar meson, in section III. In section IV, we study the rare semileptonic $B \to V l \bar{\nu}_l$ processes, where $V(= K^*, \rho)$ denotes the vector meson. Our findings are summarized in section V.

II. GENERAL EFFECTIVE LAGRANGIAN FOR $b \to ul \bar{\nu}_l$ TRANSITIONS

The most general effective Lagrangian for $b \to ul \bar{\nu}_l$ process is given by [25]

$$L_{\text{eff}} = -\frac{4 G_F}{\sqrt{2}} V_{ub} \left\{ (1 + V_L) \bar{L} \gamma_\mu \nu_L \bar{u} \gamma^\mu b_L + V_R \bar{L} \gamma_\mu \nu_L \bar{u} \gamma^\mu b_R \\
+ S_L \bar{R} \gamma_\mu \nu_L \bar{u} \gamma^\mu b_R + S_R \bar{L} \gamma_\mu \nu_L \bar{u} \gamma^\mu b_R + T_L \bar{L} \sigma_\mu\nu \nu_L \bar{u} \sigma^{\mu\nu} b_L \right\} + \text{h.c.}, \quad (7)$$

where $G_F$ is the Fermi constant, $V_{ub}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element and $q_{L(R)} = L(R)q$ are the chiral quark fields with $L(R) = (1 \mp \gamma_5)/2$ as the projection operator. Here $V_{L,R}, S_{L,R}$ and $T_L$ are the vector, scalar and tensor new physics couplings associated with the left-handed neutrinos, which are zero in the SM. The constraint on the new coefficients obtained from the leptonic $B_u^+ \to l^+ \nu_l$ processes are discussed in the subsection below.

A. Constraints on new couplings from rare leptonic $B_u^+ \to l^+ \nu_l$ processes

The rare leptonic $B_u^+ \to l^+ \nu_l$ processes are mediated by the quark-level transitions $b \to u$ and are theoretically very clean. The only non-perturbative quantity involved in these processes is the decay constant of $B_u$ meson. Including the new coefficients from Eqn. [7], the branching ratios of $B_u^+ \to l^+ \nu_l$ processes in the presence of NP are given by [26]

$$\text{BR}(B_u^+ \to l^+ \nu_l) = \frac{G_F^2 M_{B_u} m_l^2}{8 \pi} \left( 1 - \frac{m_l^2}{M_{B_u}^2} \right)^2 \left| f_{B_u} V_{ub} \right|^2 \tau_{B_u}$$

$$\times \left| (1 + V_L - V_R) - \frac{M_{B_u}^2}{m_l (m_b + m_u)} (S_L - S_R) \right|^2,$$

where $M_{B_u} \ (f_{B_u})$ is the mass (decay constant) of $B_u$ meson and $m_l$ is the lepton mass. In our analysis, all the particle masses and the life time of $B_u^+$ meson are taken from [27]. The decay constant of $B_u$ meson is taken as $f_{B_u} = 190.5 \ (4.2) \ \text{MeV}$ [28], and for the CKM matrix element, we use the Wolfenstein parametrization with the values $A = 0.811 \pm 0.026,$
\( \lambda = 0.22506 \pm 0.00050, \bar{\rho} = 0.124^{+0.019}_{-0.018} \) and \( \bar{\eta} = 0.356 \pm 0.011 \) \[27\]. Using these values, the obtained branching fractions of \( B_u^+ \rightarrow l^+\nu_l \) processes in the SM are given as

\[
\begin{align*}
\text{BR}(B_u^+ \rightarrow e^+\nu_e)|^\text{SM} &= (8.9 \pm 0.23) \times 10^{-12}, \\
\text{BR}(B_u^+ \rightarrow \mu^+\nu_\mu)|^\text{SM} &= (3.83 \pm 0.1) \times 10^{-7}, \\
\text{BR}(B_u^+ \rightarrow \tau^+\nu_\tau)|^\text{SM} &= (8.48 \pm 0.28) \times 10^{-5},
\end{align*}
\]

and the corresponding experimental values are \[27\]

\[
\begin{align*}
\text{BR}(B_u^+ \rightarrow e^+\nu_e)|^\text{Expt} &< 9.8 \times 10^{-7}, \\
\text{BR}(B_u^+ \rightarrow \mu^+\nu_\mu)|^\text{Expt} &< 1.0 \times 10^{-6}, \\
\text{BR}(B_u^+ \rightarrow \tau^+\nu_\tau)|^\text{Expt} &= (1.09 \pm 0.24) \times 10^{-4}.
\end{align*}
\]

Since \( B_u^+ \rightarrow l^+\nu_l \) processes do not receive any contribution from tensor coupling, we ignore the effect of tensor operator in this work. In our analysis, we consider the new coefficients \( V_{L,R}, S_{L,R} \) as complex. For simplicity, we consider the presence of only one coefficient at a time and constrain its real and imaginary parts by comparing the predicted SM branching fractions of \( B_u^+ \rightarrow l^+\nu_l \) processes with the corresponding experimental results. For \( B_u^+ \rightarrow \tau^+\nu_\tau \), we compare with the 1\( \sigma \) range of observed data. In Fig. 1, we show the constraints on the real and imaginary parts of the \( V_L \) coefficient obtained from the \( B_u^+ \rightarrow e^+\nu_e \) (top-left panel), \( B_u^+ \rightarrow \mu^+\nu_\mu \) (top-right panel) and \( B_u^+ \rightarrow \tau^+\nu_\tau \) (bottom panel) processes. Analogously, the allowed ranges of the real and imaginary parts of \( S_L \) coefficient derived from the \( B_u^+ \rightarrow e^+\nu_e \) (top-left panel), \( B_u^+ \rightarrow \mu^+\nu_\mu \) (top-right panel) and \( B_u^+ \rightarrow \tau^+\nu_\tau \) (bottom panel) processes are shown in Fig. 2. The constraint on the imaginary part of the \( V_R \) (\( S_R \)) coefficient is same as \( V_L \) (\( S_L \)) coefficient and the corresponding real part is related by \( \text{Re}[V_R] (\text{Re}[S_R]) = -\text{Re}[V_L] (\text{Re}[S_L]) \). It should be noted that the bounds obtained from \( B_u^+ \rightarrow e^+\nu_e(\mu^+\nu_\mu) \) process are comparatively weak as only the upper limits on the branching ratios of these processes exist. Furthermore, the bounds on new coefficients obtained from \( B_u^+ \rightarrow e^+\nu_e \) process are too weak to make reasonable predictions for the observables associated with \( b \rightarrow ue^+\nu_e \) decay modes. Therefore, we only present the results for semileptonic \( B \) decays with \( \mu(\tau) \) in the final state.
FIG. 1: Constraint on the real and imaginary parts of $V_L$ parameter obtained from $B^+_u \rightarrow e^+\nu_e$ (top-left panel), $B^+_u \rightarrow \mu^+\nu_\mu$ (top-right panel) and $B^+_u \rightarrow \tau^+\nu_\tau$ (bottom panel).

III. $B \rightarrow P l \bar{\nu}_l$ PROCESSES

In this section, we discuss the rare $B \rightarrow P l \bar{\nu}_l$ processes, where $P = \pi, K, \eta$. The matrix elements of various hadronic currents between the initial $B$ meson and the final pseudoscalar meson $P$, can be parametrized in terms of two form factors $F_0, F_1$ as

$$\langle P(k)|\bar{u}\gamma_\mu b|B(p_B)\rangle = F_1(q^2) \left[(p_B + k)_\mu - \frac{M_B^2 - M_P^2}{q^2} q_\mu\right] + F_0(q^2) \frac{M_B^2 - M_P^2}{q^2} q_\mu , \quad (11)$$

where $p_B$ and $k$ are respectively the four momenta of the $B$ and $P$ mesons and $q = p_B - k$ is the momentum transfer. Now using the above form factors, the double differential decay distribution of $B \rightarrow P l \nu_\ell$ processes in terms of the helicity amplitudes $H_0, H_t$ and $H_S$ are
FIG. 2: Constraint on the real and imaginary parts of the $S_L$ parameter obtained from $B^+ \to e^+ \nu_e$ (top-left panel), $B^+ \to \mu^+ \nu_\mu$ (top-right panel) and $B^+ \to \tau^+ \nu_\tau$ (bottom panel).

Given by [30]

\[
\frac{d\Gamma(B \to Pl\nu_l)}{dq^2} = \frac{G_F^2 |V_{ub}|^2 q^2 \sqrt{\lambda_P(q^2)}}{192\pi^3 M_B^3} \left(1 - \frac{m_l^2}{q^2}\right)^2 \times \left\{1 + V_L + V_R\right\}^2 \left[\left(1 + \frac{m_l^2}{2q^2}\right) H_0^2 + \frac{3 m_l^2}{2 q^2} H_t^2\right] + \frac{3}{2} |S_L + S_R|^2 H_S^2 + 3 \Re [(1 + V_L + V_R)(S_L^* + S_R^*)] \frac{m_l}{\sqrt{q^2}} H_S H_t, \tag{12}
\]

where

\[
\lambda_P = \lambda(M_B^2, M_P^2, q^2) = M_B^4 + M_P^4 + q^4 - 2(M_B^2 M_P^2 + M_P^2 q^2 + M_B^2 q^2), \tag{13}
\]
and the helicity amplitudes \((H_{0,t,s})\) in terms of the form factors \((F_{0,1})\) are given as

\[
H_0(q^2) = \sqrt{\frac{\lambda_P(q^2)}{q^2}} F_1(q^2),
\]
\[
H_t(q^2) = \frac{M_B^2 - M_P^2}{\sqrt{q^2}} F_0(q^2),
\]
\[
H_S(q^2) = \frac{M_B^2 - M_P^2}{m_b - m_u} F_0(q^2).
\]

(14)

Here \(M_P\) is the mass of the \(P\) meson and \(m_b\) (\(m_u\)) is the mass of the \(b\) (\(u\)) quark.

The lepton forward-backward asymmetry, which is an interesting observable to look for NP, defined as

\[
A_{FB}(q^2) = \frac{\int_0^1 \frac{d\tau}{dq^2} d\cos \theta d\cos \theta - \int_{-1}^0 \frac{d\tau}{dq^2} d\cos \theta d\cos \theta}{d\Gamma/dq^2}.
\]

(15)

Besides the branching ratio and forward-backward asymmetry, another important observable is the LNU ratio. Similar to \(R_{D^{(*)}}\) observables, we define the LNU parameter for \(B \to P l \nu_l\) processes as

\[
R_{P}^{l\mu} = \frac{\text{BR}(B \to P \tau \bar{\nu}_\tau)}{\text{BR}(B \to P \mu \bar{\nu}_\mu)}.
\]

(16)

in order to scrutinize the violation of lepton universality effect in \(b \to ul\nu_l\) decays. In Ref. [13], the authors have studied the lepton universality violating ratio \(\text{BR}(B \to P \tau \bar{\nu}_\tau)/\text{BR}(B \to P l \nu_l)\), where \(l = e, \mu\). Since the constraints on new coefficients obtained from \(B_u^+ \to e^+ \nu_e\) process are too weak, it would not be possible to predict reasonably constrained result for the \(\text{BR}(B \to P \tau \bar{\nu}_\tau)/\text{BR}(B \to P e \nu_e)\) ratio. Therefore, we only consider the \(\text{BR}(B \to P \tau \bar{\nu}_\tau)/\text{BR}(B \to P \mu \bar{\nu}_\mu)\) parameter in our analysis.

In order to explore few other observables which are sensitive to NP in the \(b \to ul\nu_l\) processes, we define the parameter \(R_{PP'}^l\) as ratio of branching fractions of \(B \to Pl^{-}\bar{\nu}_l\) to \(B \to P' l^{-}\bar{\nu}_l\) processes

\[
R_{PP'}^l = \frac{\text{BR}(B \to Pl^{-}\bar{\nu}_l)}{\text{BR}(B \to P' l^{-}\bar{\nu}_l)}.
\]

(17)

These processes differ only in the spectator quark content and hence, any deviation from SM prediction, if observed would hint towards the existence of NP.

After setting the stage, we now proceed for numerical analysis. We consider all the particle masses and the life time of \(B\) meson from the Ref. [27]. To make predictions for the
various observables or to extract information about potentially new short distance physics, one should have sufficient knowledge on the associated hadronic form factors. For the form factors of $\bar{B}_s \to K^+ l^- \bar{\nu}_l$ processes, we consider the perturbative QCD (PQCD) calculation \[17, 18\] based on the $k_T$ factorization \[31\] at next-to-leading order (NLO) in $\alpha_s$ \[32\], which gives

\[
F_{B_s \to K^+}^{1}(q^2) = F_{B_s \to K^+}^{1}(0) \left( \frac{1}{1 - q^2/M_{B_s}^2} + \frac{a_1 q^2/M_{B_s}^2}{(1 - q^2/M_{B_s}^2)(1 - b_1 q^2/M_{B_s}^2)} \right),
\]

\[
F_{B_s \to K^0}^{0}(q^2) = \frac{F_{B_s \to K^0}^{0}(0)}{(1 - a_0 q^2/M_{B_s}^2 + b_0 q^4/M_{B_s}^4)},
\]

(18)

where $M_{B_s}$ is the mass of $B_s$ meson and the values of the parameters $a_{0,1}, b_{0,1}$ and $F_{0,1}^{B_s \to K}$ are listed in Table I.

**TABLE I: Numerical values of the $B_s \to K$ form factors in the PQCD approach \[17\].**

| Parameters | PQCD |
|------------|------|
| $F_0(0)$   | $0.26^{+0.04}_{-0.03} \pm 0.02$ |
| $a_0$      | $0.54 \pm 0.00 \pm 0.05$ |
| $b_0$      | $-0.15 \pm 0.00 \pm 0.00$ |
| $F_1(0)$   | $0.26 \pm 0.035 \pm 0.02$ |
| $a_1$      | $0.57 \pm 0.01 \pm 0.02$ |
| $b_1$      | $0.50 \pm 0.01 \pm 0.05$ |

For $B \to \pi$ form factors, we use the light cone sum rule (LCSR) results as input for a $z$-series parametrization which yield the $q^2$ shape in the whole semileptonic region of $B \to \pi l \nu_l$ processes. The $q^2$ dependence of the form factors is parametrized as \[33\]

\[
F_1(q^2) = \frac{F_1(0)}{1 - \frac{q^2}{M_{B_s}^2}} \left\{ 1 + \sum_{k=1}^{N-1} b_k \left( z(q^2, t_0)^k - z(0, t_0)^k - (-1)^{N-k} \frac{k}{N} \left[ z(q^2, t_0)^N - z(0, t_0)^N \right] \right) \right\},
\]

\[
F_0(q^2) = F_0(0) \left\{ 1 + \sum_{k=1}^{N} b_0^k \left( z(q^2, t_0)^k - z(0, t_0)^k \right) \right\},
\]

(19)

where $N = 2$ for $F_1(q^2)$ form factor and for $F_0(q^2)$ form factor, $N = 1$. Here the function $z(q^2, t_0)$ is defined as \[34\]

\[
z(q^2, t_0) = \frac{\sqrt{(M_B + M_{\pi})^2 - q^2} - \sqrt{(M_B + M_{\pi})^2 - t_0}}{\sqrt{(M_B + M_{\pi})^2 - q^2} + \sqrt{(M_B + M_{\pi})^2 - t_0}},
\]

(20)
where \( t_0 = (M_B + M_\pi)^2 - 2\sqrt{M_B M_\pi} \sqrt{(M_B + M_\pi)^2 - q_{min}^2} \) is the auxiliary parameter. Here the values of various parameters involved are \( F_1(0) = F_0(0) = 0.281 \pm 0.028, b_1 = -1.62 \pm 0.70 \) and \( b_0^1 = -3.98 \pm 0.97 \).  

The \( B^- \to \eta^{(0)} l^- \bar{\nu}_l \) processes are also mediated by the flavour changing charged current (FCCC) transitions \( b \to u \). For the study of these processes, we use \( SU(3)_F \) flavour symmetry to relate the form factors of \( F_1^{B\to\eta^{(0)}} \) to \( F_1^{B\to\pi} \). We choose the scheme as discussed in Refs. [35, 36], and consider

\[
|\eta\rangle = \cos \phi |\eta_q\rangle - \sin \phi |\eta_s\rangle, \\
|\eta'\rangle = \sin \phi |\eta_q\rangle + \cos \phi |\eta_s\rangle, \tag{21}
\]

for the \( \eta - \eta' \) mixing, where \( |\eta_q\rangle = (u\bar{u} + d\bar{d})/\sqrt{2}, \eta_s = s\bar{s} \) and \( \phi \) is the fitted mixing angle \( (\phi = 39.3^\circ) \). With these input parameters in hand, we now proceed to discuss four different new physics scenarios and their effect on \( b \to ul\nu_l \) processes.

**FIG. 3:** Constraint on the real and imaginary parts of the \( V_L \) (left panel) and \( S_L \) (right panel) parameters obtained from \( B_u^- \to \pi^0 \mu \bar{\nu}_\mu \) process.

### A. Case A: Effect of \( V_L \) only

In this case, we assume that only the new \( V_L \) coefficient is present in addition to SM contribution, in the effective Lagrangian (7). From Eqn. (12) it should be noted that as the NP has the same structure as the SM, the SM decay rate gets modified by the factor \( |1+V_L|^2 \). The constraints on the real and imaginary parts of \( V_L \) coefficient for \( b \to u\tau \bar{\nu}_\tau \) are obtained from the branching ratio of \( B_u^+ \to \pi^+ \nu_\tau \) process as discussed in section II. From the bottom
one can notice that the constraint on $V_L$ is $|V_L| \leq 2.5$, obtained from $B_u \rightarrow \tau \bar{\nu}_\tau$ process. In our analysis, we consider the values for real and imaginary parts of $V_L$, which give the maximum and minimum values of the branching ratio within the 1σ limit. Thus, imposing the extrema conditions, the allowed parameters are found as $(\text{Re}[V_L], \text{Im}[V_L])^{\text{max}} = (0.130, 0.761)$ and $(\text{Re}[V_L], \text{Im}[V_L])^{\text{min}} = (-0.929, 0.841)$. For $b \rightarrow u \mu \bar{\nu}_\mu$ transition as only the upper limit of $B_u \rightarrow \mu \bar{\nu}_\mu$ is known, it will not provide any strict bound on the NP coefficient $V_L$. Therefore, to avoid overestimation of the predicted values of various physical observables, we consider the branching ratio of $B^- \rightarrow \pi^0 \mu^- \bar{\nu}_\mu$ process. Comparing the SM predicted value $\text{BR}(B^- \rightarrow \pi^0 \mu^- \bar{\nu}_\mu)^{\text{SM}} = (7.15 \pm 0.55) \times 10^{-5}$ with the 1σ range of corresponding measured value $\text{BR}(B^- \rightarrow \pi^0 \mu^- \bar{\nu}_\mu)^{\text{Expt}} = (7.80 \pm 0.27) \times 10^{-5}$, we obtain the maximum and minimum values of the $V_L$ parameter as $(\text{Re}[V_L], \text{Im}[V_L])^{\text{max}} = (-0.233, 0.769)$ and $(\text{Re}[V_L], \text{Im}[V_L])^{\text{min}} = (-0.833, 0.968)$. The corresponding allowed parameter space is shown in the left panel of Fig. 3.

Using the allowed constrained values, we show the plots for the variation of branching fractions of various $B \rightarrow P \mu^- \bar{\nu}_\mu$ processes with respect to $q^2$ in Fig. 4 both in the SM and in NP scenario. Here the plot for $\bar{B}_s \rightarrow K^+ \mu^- \bar{\nu}_\mu$ process is represented in the top-left panel, the top-right panel is for the branching ratio of $\bar{B}^0 \rightarrow \pi^+ \mu^- \bar{\nu}_\mu$, the bottom-left plot is for $B^- \rightarrow \eta \mu^- \bar{\nu}_\mu$ process and the branching ratio of $B^- \rightarrow \eta' \mu^- \bar{\nu}_\mu$ process is presented in the bottom-right panel. In these figures, the red bands are due to the contribution coming from $V_L$ new physics parameter in addition to SM and the blue dashed lines are due to SM. The green bands are the corresponding SM theoretical uncertainties, which arise due to the uncertainties in the SM input parameters such as CKM elements and form factors. Analogous plots for the variation of the branching ratios of $\bar{B}_s \rightarrow K^+ \tau^- \bar{\nu}_\tau$ (top-left panel), $\bar{B}^0 \rightarrow \pi^+ \tau^- \bar{\nu}_\tau$ (top-right panel), $B^- \rightarrow \eta \tau^- \bar{\nu}_\tau$ (bottom-left panel) and $B^- \rightarrow \eta' \tau^- \bar{\nu}_\tau$ (bottom-right panel) processes are shown in Fig. 5. The integrated values of the branching ratios for these processes are given in Table II. Due to the inclusion of new $V_L$ coefficient, we found certain deviation in the branching ratios of $B \rightarrow P \tau \bar{\nu}_\tau$ processes from the SM values, whereas the deviation in the branching ratios of $B \rightarrow P \mu \bar{\nu}_\mu$ processes are relatively small. Our predicted results for $B \rightarrow (\pi, \eta, \eta') \nu \bar{\nu}_\tau$ processes are consistent with the existing
FIG. 4: The plots for the $q^2$ variation of the branching ratios of $\bar{B}_s \to K^+ \mu^- \bar{\nu}_\mu$ (top-left panel), $\bar{B}^0 \to \pi^+ \mu^- \nu_\mu$ (top-right panel), $B^- \to \eta \mu^- \bar{\nu}_\mu$ (bottom-left panel) and $B^- \to \eta' \mu^- \bar{\nu}_\mu$ (bottom-right panel) processes for the NP contribution coming from only $V_L$ coupling. Here the red bands represent the contributions due to the $V_L$ coupling. The blue dashed lines are for the SM contribution and the green bands are due to the contributions coming from the theoretical uncertainties.

experimental data [27]

$$\text{BR}(B^+ \to \eta l^+ \nu_l)^{\text{Expt}} = (3.8 \pm 0.6) \times 10^{-5}, \quad \text{BR}(B^0 \to \pi^- l^+ \nu_l)^{\text{Expt}} = (1.45 \pm 0.05) \times 10^{-4}, \quad \text{BR}(B^+ \to \eta' l^+ \nu_l)^{\text{Expt}} = (2.3 \pm 0.8) \times 10^{-5}, \quad \text{BR}(B^0 \to \pi^- \tau^+ \nu_\tau)^{\text{Expt}} < 2.5 \times 10^{-4}. \quad (22)$$

Since the $V_L$ contribution has the same structure as SM, the forward-backward asymmetry parameter of $B \to P \mu^- \bar{\nu}_\mu$ ($\tau^- \bar{\nu}_\tau$) processes do not deviate from their SM values, and the corresponding integrated values (integrated over the whole $q^2$ range) are presented in Table II. In Fig. 6 we show the plots for the LNU parameters of $\bar{B}_{(s)} \to P l \bar{\nu}_l$ processes, $R^\mu_K$ (top-left panel), $R^\mu_\pi$ (top-right panel), $R^\mu_\eta$ (bottom-left panel) and $R^\mu_\eta'$ (bottom-right panel). Including only $V_L$ coupling, we also compute the $R^l_\pi K$, $R^l_\pi \eta$ and $R^l_\pi \eta'$ parameters, however, no deviation has been found from their corresponding SM result. The numerical values of these parameters are listed in Table III.
FIG. 5: The plots for the $q^2$ variation of the branching ratios of $\bar{B}_s \to K^+\tau^-\bar{\nu}_\tau$ (top-left panel), $\bar{B}^0 \to \pi^+\tau^-\nu_\tau$ (top-right panel), $B^- \to \eta\tau^-\bar{\nu}_\tau$ (bottom-left panel) and $B^- \to \eta'\tau^-\bar{\nu}_\tau$ (bottom-right panel) processes for the NP contribution due to $V_L$ coupling.

B. Case B: Effect of $V_R$ only

Here we consider the effect of only $V_R$ coefficient in addition to the SM contribution. The constraints obtained on real and imaginary parts of $V_R$ coupling from $B_u \to \tau\nu$ process are related to that of $V_L$ as $\text{Re}[V_R] = -\text{Re}[V_L]$ and $\text{Im}[V_R] = \text{Im}[V_L]$, and thus, allowed parameter space for $V_R$ is same as that of $V_L$ with a sign flip for the real parts. The minimum and maximum values of the $V_R$ parameters are obtained using the extrema conditions as $(\text{Re}[V_R], \text{Im}[V_R])^{\text{max}} = (-0.242, -0.561)$ and $(\text{Re}[V_R], \text{Im}[V_R])^{\text{min}} = (0.259, -0.406)$. However, the constraints on $V_R$ obtained from $B^- \to \pi^0\mu^-\bar{\nu}_\mu$ for $b \to u\mu\bar{\nu}_\mu$ transition are same as $V_L$. Thus, the predicted branching ratios for $B \to P\mu\bar{\nu}_\mu$ processes in the presence of $V_R$ coupling are same as those with $V_L$ coupling. Using the allowed values of the couplings, the plots for the branching ratios of $\bar{B}_s \to K^+\tau^-\bar{\nu}_\tau$ (top-left panel), $\bar{B}^0 \to \pi^+\tau^-\nu_\tau$ (top-right panel), $B^- \to \eta\tau^-\bar{\nu}_\tau$ (bottom-left panel) and $B^- \to \eta'\tau^-\bar{\nu}_\tau$ (bottom-right panel) processes in the presence of $V_R$ coupling are shown in Fig. 7. In these plots, the cyan bands are obtained by using the allowed parameter space of $V_R$. The predicted integrated values of branching ratios...
FIG. 6: The plots for the LNU parameters $R_{K}^{T}(q^{2})$ (top-left panel), $R_{\pi}^{T}(q^{2})$ (top-right panel), $R_{\eta}^{T}(q^{2})$ (bottom-left panel) and $R_{\eta'}^{T}(q^{2})$ (bottom-right panel) for the NP contribution due to $V_{L}$ coupling.

ratios of these processes are listed in Table II. Like the previous case, the forward-backward asymmetry parameters are also not affected due to $V_{R}$ coupling. In Fig. 8, we present the plots for the LNU parameters $R_{K}^{T}(q^{2})$ (top-left panel), $R_{\pi}^{T}(q^{2})$ (top-right panel), $R_{\eta}^{T}(q^{2})$ (bottom-left panel) and $R_{\eta'}^{T}(q^{2})$ (bottom-right panel). In the presence of $V_{R}$ coupling, the parameters $R_{l}^{lK}$, $R_{l}^{l(\pi\eta')}$ don’t have any deviation from their corresponding SM predictions. In Table III, we present the numerical values of these parameters.

C. Case C: Effect of $S_{L}$ only

In this subsection, we wish to see the effect of only $S_{L}$ coupling on various observables associated with $B \to PL\bar{\nu}$ processes. For $b \to u\tau\nu$ transition, using the extrema conditions, we obtain the maxima and minima of $S_{L}$ parameter as $(\text{Re}[S_{L}], \text{Im}[S_{L}])^{\text{max}} = (0.0106, -0.0063)$ and $(\text{Re}[S_{L}], \text{Im}[S_{L}])^{\text{min}} = (-0.5397, 0.0244)$, from the allowed parameter space in the bottom panel of Fig. 2. Analogously, for $b \to u\mu\bar{\nu}_{\mu}$, the extrema values of $S_{L}$ are
FIG. 7: The plots for the branching ratios of $B_s \rightarrow K^+ \tau^- \bar{\nu}_\tau$ (top-left panel), $\bar{B}^0 \rightarrow \pi^+ \tau^- \bar{\nu}_\tau$ (top-right panel), $B^- \rightarrow \eta \tau^- \bar{\nu}_\tau$ (bottom-left panel) and $B^- \rightarrow \eta' \tau^- \bar{\nu}_\tau$ (bottom-right panel) processes for the NP contribution of only $V_R$ coupling. Here the cyan bands are for the $V_R$ NP coupling contributions. The corresponding plots for $\bar{B}_s \rightarrow P^{+} \tau^- \bar{\nu}_\tau$ processes are given in Fig. [7]. Including the additional contributions from $S_L$ coupling, the obtained branching ratios for various processes are listed in Table IV. It is observed that the branching ratios of $\bar{B}_s \rightarrow P^{+} \tau^- \bar{\nu}_\tau$ processes comparatively deviate more than the corresponding processes with muon in the final state.

Fig. [9] represents the $q^2$ variation of the forward-backward asymmetry of $\bar{B}_s \rightarrow K^+ \mu^- \bar{\nu}_\mu$ (top-left panel), $\bar{B}^0 \rightarrow \pi^+ \mu^- \bar{\nu}_\mu$ (top-right panel), $B^- \rightarrow \eta \mu^- \bar{\nu}_\mu$ (bottom-left panel) and $B^- \rightarrow \eta' \mu^- \bar{\nu}_\mu$ (bottom-right panel) processes for only $S_L$ coupling. The corresponding plots for $\bar{B}_s \rightarrow P \tau \bar{\nu}_\tau$ processes are given in Fig. [10]. Due to the additional $S_L$ contribution, the forward-backward asymmetry parameters of these processes deviate significantly from SM. The corresponding integrated values are presented in Table IV. Fig. [11] represents the plots for the LNU parameters $R^K_{\tau \mu}(q^2)$ (top-left panel), $R^\pi_{\tau \mu}(q^2)$ (top-right panel), $R^\eta_{\tau \mu}(q^2)$
FIG. 8: The plots for the LNU parameters $R_{\pi K}^\tau(q^2)$ (top-left panel), $R_{\pi\pi}^\tau(q^2)$ (top-right panel), $R_{\eta}^\tau(q^2)$ (bottom-left panel) and $R_{\eta'}^\tau(q^2)$ (bottom-right panel). The variation of $R_{\tau\pi K}, R_{\tau\pi\eta}$ parameters with respect to $q^2$ are shown in Fig. 12. In Table V, we give the numerical values of these parameters.

D. Case D: Effect of $S_R$ only

Here we perform an analysis of $B \to P l^{-}\bar{\nu}_l$ processes with the additional $S_R$ coupling. As discussed in section II, the real part of $S_R$ coupling differs from the real part of $S_L$ by a negative sign while their imaginary parts are same. The minimum and maximum values of $S_R$ parameter are found as $(\text{Re}[S_R], \text{Im}[S_R])^{\text{max}} = (0.003, 0.268)$ and $(\text{Re}[S_R], \text{Im}[S_R])^{\text{min}} = (-0.54, -0.03)$ for $b \to u\tau\bar{\nu}_\tau$ process. For $b \to u\mu\nu$ the constraints on $S_R$ couplings are same as $S_L$. Using these value the $q^2$ variation of the forward-backward asymmetries for $B^- \to P^0\tau^{-}\bar{\nu}_\tau$ processes are shown in Fig. 13. The branching ratios and forward-backward asymmetries of these processes are presented in Table IV. Fig. 14 represents the variation of the LNU parameters $(R_{K,\pi,\eta,\eta'}^\tau)$ due to only $S_R$ coupling. The variation of $R_{\eta,\eta'}^{\tau\mu}$ parameters
TABLE II: The predicted branching ratios and forward-backward asymmetries of $B_{(s)} \rightarrow P l \bar{\nu}_l$ processes, where $P = K, \pi, \eta^{(l)}$ and $l = \mu, \tau$ in the SM and in the presence of $V_{L,R}$ NP couplings.

| Observables | Values in the SM | Values for $V_L$ coupling | Values for $V_R$ coupling |
|-------------|------------------|---------------------------|---------------------------|
| $\text{BR}(B_s \rightarrow K^+ \mu^{-} \bar{\nu}_\mu)$ | $(1.03 \pm 0.082) \times 10^{-4}$ | $(1.03 - 1.22) \times 10^{-4}$ | $(1.03 - 1.22) \times 10^{-4}$ |
| $\langle A_{FB}^\mu \rangle$ | $(2.98 \pm 0.238) \times 10^{-3}$ | $2.98 \times 10^{-3}$ | $2.98 \times 10^{-3}$ |
| $\langle A_{FB}^\tau \rangle$ | $0.275 \pm 0.022$ | $0.275$ | $0.275$ |
| $\text{BR}(B \rightarrow \pi^+ \mu^{-} \bar{\nu}_\mu)$ | $(1.35 \pm 0.1) \times 10^{-4}$ | $(1.35 - 1.59) \times 10^{-4}$ | $(1.35 - 1.59) \times 10^{-4}$ |
| $\langle A_{FB}^\mu \rangle$ | $(2.94 \pm 0.235) \times 10^{-3}$ | $2.94 \times 10^{-3}$ | $2.94 \times 10^{-3}$ |
| $\langle A_{FB}^\tau \rangle$ | $(0.27 \pm 0.021)$ | $0.27$ | $0.27$ |
| $\text{BR}(B^- \rightarrow \eta \mu^{-} \bar{\nu}_\mu)$ | $(3.143 \pm 0.25) \times 10^{-5}$ | $(3.143 - 3.7) \times 10^{-5}$ | $(3.143 - 3.7) \times 10^{-5}$ |
| $\langle A_{FB}^\mu \rangle$ | $(3.45 \pm 0.276) \times 10^{-3}$ | $3.45 \times 10^{-3}$ | $3.45 \times 10^{-3}$ |
| $\langle A_{FB}^\tau \rangle$ | $(0.292 \pm 0.023)$ | $0.292$ | $0.292$ |
| $\text{BR}(B^- \rightarrow \eta' \mu^{-} \bar{\nu}_\mu)$ | $(1.45 \pm 0.116) \times 10^{-5}$ | $(1.45 - 1.7) \times 10^{-5}$ | $(1.45 - 1.7) \times 10^{-5}$ |
| $\langle A_{FB}^\mu \rangle$ | $(4.1 \pm 0.328) \times 10^{-3}$ | $4.1 \times 10^{-3}$ | $4.1 \times 10^{-3}$ |
| $\langle A_{FB}^\tau \rangle$ | $(0.317 \pm 0.026)$ | $0.317$ | $0.317$ |

are similar to those with $S_L$ coupling. Table V contains the numerical values of these parameters.

The rare semileptonic $B_s \rightarrow K l \bar{\nu}_l$ and $B \rightarrow \pi l \bar{\nu}_l$ processes are investigated in Refs. [16, 17]. The analysis of $B \rightarrow \pi l \bar{\nu}_l$ processes using the light cone QCD sume rule approach [24] and 2HDM [20] are also studied in the literature. In Ref. [21–23], $B \rightarrow \eta^{(l)} l \bar{\nu}_l$ processes are studied by using various model-dependent approaches. The model independent analysis of $b \rightarrow u l \bar{\nu}_l$ processes can be found in [13]. Our predicted SM values of the branching ratios of $\bar{B}_{(s)} \rightarrow P^+ l^- \bar{\nu}_l$ processes are found to be consistent with the predicted results in the literature, though due to updated input parameters, the central values of the branching ratios of these processes have slight deviations.
TABLE III: The predicted values of various parameters ($R^{\mu}_{p'}$ and $R^l_{P'P''}$) of $\bar{B}_{(s)} \to P l \bar{\nu}_l$ processes in the SM and in the presence of $V_{L,R}$ NP couplings.

| Observables | Values in the SM | Values for $V_L$ coupling | Values for $V_R$ coupling |
|-------------|------------------|---------------------------|---------------------------|
| $R^{\tau \mu}_K$ | 0.649 | 0.46 − 1.02 | 0.489 − 1.13 |
| $R^{\tau \mu}_\pi$ | 0.7 | 0.497 − 1.1 | 0.528 − 1.22 |
| $R^{\tau \mu}_\eta$ | 0.624 | 0.45 − 0.982 | 0.47 − 1.09 |
| $R^{\tau \mu}_\eta'$ | 0.54 | 0.385 − 0.85 | 0.408 − 0.946 |
| $R^{\mu}_{\pi K}$ | 1.31 | 1.3 − 1.31 | 1.3 − 1.31 |
| $R^{\mu}_{\pi \eta}$ | 4.3 | 4.3 | 4.3 |
| $R^{\mu}_{\pi \eta'}$ | 9.3 | 9.3 − 9.35 | 9.3 − 9.35 |
| $R^{T}_{\pi K}$ | 1.4 | 1.4 − 1.41 | 1.373 − 1.39 |
| $R^{T}_{\pi \eta}$ | 4.8 | 4.785 − 4.808 | 4.709 − 4.723 |
| $R^{T}_{\pi \eta'}$ | 12.0 | 11.96 − 12.1 | 11.82 − 11.86 |

IV. $B \to Vl\bar{\nu}_l$ PROCESSES

In this section, we study the $B \to Vl\bar{\nu}_l$ processes, where $V = K^*, \rho$. The hadronic matrix element of the $B \to Vl\bar{\nu}_l$ processes can be parametrized as

$$\langle V(k, \varepsilon) | \bar{u} \gamma_\mu b | B(p_B) \rangle = -i \epsilon_{\mu \nu \rho \sigma} \varepsilon^{\nu \sigma} p_B^\rho k^\sigma \frac{2V(q^2)}{M_B + M_V},$$

$$\langle V(k, \varepsilon) | \bar{u} \gamma_\mu \gamma_5 b | B(p_B) \rangle = \varepsilon^{\mu \ast} (M_B + M_V) A_1(q^2) - (p_B + k)_\mu (\varepsilon^\ast \cdot q) \frac{A_2(q^2)}{M_B + M_V}$$

$$- q_\mu (\varepsilon^\ast \cdot q) \frac{2M_V}{q^2} [A_3(q^2) - A_0(q^2)],$$

(23)

where

$$A_3(q^2) = \frac{M_B + M_V}{2M_V} A_1(q^2) - \frac{M_B + M_V}{2M_V} A_2(q^2).$$

(24)
FIG. 9: The plots for the $q^2$ variation of forward-backward asymmetry of $\bar{B}_s \to K^+ \mu^- \bar{\nu}_\mu$ (top-left panel), $\bar{B}^0 \to \pi^+ \mu^- \bar{\nu}_\mu$ (top-right panel), $B^- \to \eta \mu^- \bar{\nu}_\mu$ (bottom-left panel) and $B^- \to \eta' \mu^- \bar{\nu}_\mu$ (bottom-right panel) processes.

The differential decay rate of $B \to Vl\bar{\nu}_l$ processes with respect to $q^2$ is given by

$$
\frac{d\Gamma(B \to Vl\bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 M_B^3} q^2 \sqrt{\lambda_V(q^2)} \left(1 - \frac{m_l^2}{q^2}\right)^2 \left\{ |1 + V_L|^2 + |V_R|^2 \right\} \\
\times \left[ \left(1 + \frac{m_l^2}{2q^2}\right) \left(H_{V,+}^0 + 2H_{V,-}^0 + 2H_{V,0}^0 + \frac{3}{2} \frac{m_l^2}{q^2} H_{V,t}^2 \right) \right] \\
- \left[ \left(1 + \frac{m_l^2}{2q^2}\right) \left(H_{V,0}^0 + 2H_{V,+}^0 + 2H_{V,-}^0 \right) + \frac{3}{2} \frac{m_l^2}{q^2} H_{V,t}^2 \right] \\
+ \frac{3}{2} |S_L - S_R|^2 H_{S}^2 + 3 \text{Re}[(1 + V_L - V_R)(S_L^* - S_R^*)] \frac{m_l}{\sqrt{q^2}} H_S H_{V,t}, \quad (25)
$$
FIG. 10: The plots for the $q^2$ variation of forward-backward asymmetry of $\bar{B}_s \to K^+\tau^-\bar{\nu}_\tau$ (top-left panel), $\bar{B}_0 \to \pi^+\tau^-\bar{\nu}_\tau$ (top-right panel), $B^- \to \eta\tau^-\bar{\nu}_\tau$ (bottom-left panel) and $B^- \to \eta'\tau^-\bar{\nu}_\tau$ (bottom-right panel) processes.

where $\lambda_V = \lambda(M_B^2, M_V^2, q^2)$ and the hadronic amplitudes in terms of the form factors are given as

$$H_{V,\pm}(q^2) = (M_B + M_V) A_1(q^2) \mp \frac{\sqrt{\lambda_V(q^2)}}{M_B + M_V} V(q^2),$$

$$H_{V,0}(q^2) = \frac{M_B + M_V}{2M_V \sqrt{q^2}} \left[ - (M_B^2 - M_V^2 - q^2) A_1(q^2) + \frac{\lambda_V(q^2)}{(M_B + M_V)^2} A_2(q^2) \right],$$

$$H_{V,t}(q^2) = -\sqrt{\frac{\lambda_V(q^2)}{q^2}} A_0(q^2),$$

$$H_S(q^2) = -H_{S_2}(q^2) \simeq -\frac{\sqrt{\lambda_V(q^2)}}{m_b + m_u} A_0(q^2).$$

(26)

For the momentum transfer dependence of the form factors, we consider the most intuitive and the simplest parametrization of the $B_{(s)} \to (K^*)\rho$ form factors, $(V(q^2), A_{0,1,2}(q^2))$ from Ref. 37. The masses of all the particles are taken from 27. Using these input values and the bounds on $V_L$ coupling obtained from $B_u^+ \to \tau^+\nu_\tau$ and $B^- \to \pi^0\mu^-\bar{\nu}_\mu$ processes (discussed in sections II and III), we show the plots for the $q^2$ variation of branching ratios.
FIG. 11: The plots for the LNU parameters $R_{K}^{\tau\mu}(q^2)$ (top-left panel), $R_{\pi}^{\tau\mu}(q^2)$ (top-right panel), $R_{\eta}^{\tau\mu}(q^2)$ (bottom-left panel) and $R_{\eta'}^{\tau\mu}(q^2)$ (bottom-right panel) due to $S_L$ coupling.

for $\bar{B}_s \to K^{*+}\mu^-\bar{\nu}_\mu$ (top-left panel) and $\bar{B}_s \to K^{*+}\tau^-\bar{\nu}_\tau$ (top-right panel) processes in the presence of $V_L$ in Fig. 15. The corresponding plots in the bottom panel of this figure are for $V_R$ coupling. In the presence of $V_R$ coupling, we found reasonable deviation of the branching ratios from the SM predictions, whereas $V_L$ affects mainly $\bar{B}_s \to K^{*+}\tau^-\bar{\nu}_\tau$ process. In the top-left panel of Fig. 16 we show the $q^2$ variation of forward-backward asymmetries of $\bar{B}_s \to K^{*+}\mu^-\bar{\nu}_\mu$ processes for $V_R$ coupling. The forward-backward asymmetry of $\bar{B}_s \to K^{*+}\tau^-\bar{\nu}_\tau$ processes for $V_R$ (top-right panel), $S_L$ (bottom-left panel) and $S_R$ (bottom-right panel) couplings are presented in Fig. 16. We found significant deviation in the forward-backward asymmetry parameters from SM values due to the additional $V_R$ and $S_{L,R}$ couplings. The presence of $V_L$ coupling does not affect the forward-backward asymmetry parameters. As seen from the figure, due to $S_{L,R}$ couplings, the forward-backward asymmetry of $\bar{B}_s \to K^{*+}\tau^-\bar{\nu}_\tau$ process receives significant deviation from its SM values, whereas the deviation is negligible for $\bar{B}_s \to K^{*+}\mu^-\bar{\nu}_\mu$ process. The integrated values of the branching ratios and the forward-backward asymmetries for $V_{L,R}$ and $S_{L,R}$ couplings are presented in Table VI and VII respectively. In Fig. 17 we present the plots for the $R_{K}^{\tau\mu}(q^2)$ parameters for $V_L$
FIG. 12: The plots for $R_{\pi K}^\tau (q^2)$ (top-left panel), $R_{\pi\eta}^\tau (q^2)$ (top-right panel) and $R_{\pi\eta'}^\tau (q^2)$ (bottom panel) parameters.

(top-left panel), $V_R$ (top-right panel), $S_L$ (bottom-left panel) and $S_R$ (bottom-right panel) couplings and the corresponding integrated values are presented in Table VIII.

The $q^2$ variation of the branching ratios of $\bar{B} \to \rho^+ l^- \bar{\nu}_l$ processes for $V_{L,R}$ couplings are presented in Fig. 18. In the presence of $S_{L,R}$ couplings, the branching ratios of $\bar{B} \to \rho^+ l^- \bar{\nu}_l$ processes have negligible deviation from the SM predictions. The predicted values of the branching ratios of these processes are given in Table VI and VII respectively. The experimental branching ratio of $B^+ \to \rho^0 l^+ \nu_l$ process is [27]

$$\text{BR}(B^+ \to \rho^0 l^+ \nu_l)_{\text{Expt}} = (1.58 \pm 0.11) \times 10^{-4}. \quad (27)$$

Our predicted results for $B^- \to \rho^0 \mu^- \bar{\nu}_\mu$ process is consistent with the above experimental data (though a part of the allowed parameter space of $V_{L,R}$ and $S_{L,R}$ give values on the higher side of the observed central value). The forward-backward asymmetry plots for $\bar{B} \to \rho^+ l^- \bar{\nu}_l$ are presented in Fig. 19 and the corresponding numerical values are given in Table VI and VII. Fig. 20 represents the plots of LNU parameter $R_{\rho K}^\tau (q^2)$ for $V_L$ (top-left panel), $V_R$ (top-right panel), $S_L$ (bottom-left panel) and $S_R$ (bottom-right panel) couplings. In Fig. 21 we show the variation of the parameter $R_{\rho K}^\tau (q^2)$ with respect to $q^2$ for only $S_L$ (left
V. CONCLUSION

Inspired by the recent measurement of $R_{K^{(*)}}$ parameter at LHCb and the observed $R_{D^{(*)}}$ anomalies in $b \to s l^+ l^-$ and $b \to c l \bar{v}_l$ processes, we performed a model independent analysis of the rare semileptonic $b \to u l \bar{v}_l$ processes in this paper. We considered the generalized

| Observables | Values for $S_L$ coupling | Values for $S_R$ coupling |
|-------------|--------------------------|--------------------------|
| $\text{BR}(B_s \to K^+ \mu^- \bar{\nu}_\mu)$ | $(1.1 - 1.15) \times 10^{-4}$ | $(1.1 - 1.15) \times 10^{-4}$ |
| $\text{BR}(B_s \to K^+ \tau^- \bar{\nu}_\tau)$ | $(0.62 - 1.29) \times 10^{-4}$ | $(4.97 - 7.4) \times 10^{-5}$ |
| $\langle A_{FB}^L \rangle$ | $(-3.32 \to 3.52) \times 10^{-3}$ | $(-3.32 \to 3.52) \times 10^{-3}$ |
| $\langle A_{FB}^R \rangle$ | $0.255 - 0.272$ | $0.058 - 0.26$ |
| $\text{BR}(\bar{B} \to \pi^+ \mu^- \bar{\nu}_\mu)$ | $(1.39 - 1.49) \times 10^{-4}$ | $(1.39 - 1.49) \times 10^{-4}$ |
| $\text{BR}(\bar{B} \to \pi^+ \tau^- \bar{\nu}_\tau)$ | $(0.82 - 1.93) \times 10^{-4}$ | $(0.66 - 1.02) \times 10^{-4}$ |
| $\langle A_{FB}^L \rangle$ | $(-3.86 \to 3.51) \times 10^{-3}$ | $(-3.86 \to 3.51) \times 10^{-3}$ |
| $\langle A_{FB}^R \rangle$ | $0.25 - 0.27$ | $0.0264 - 0.2468$ |
| $\text{BR}(B^- \to \eta^0 \mu^- \bar{\nu}_\mu)$ | $(3.28 - 3.44) \times 10^{-5}$ | $(3.28 - 3.44) \times 10^{-5}$ |
| $\text{BR}(B^- \to \eta^0 \tau^- \bar{\nu}_\tau)$ | $(1.74 - 3.82) \times 10^{-5}$ | $(1.32 - 2.12) \times 10^{-5}$ |
| $\langle A_{FB}^L \rangle$ | $(-3.39 \to 4.0) \times 10^{-3}$ | $(-3.39 \to 4.0) \times 10^{-3}$ |
| $\langle A_{FB}^R \rangle$ | $0.27 - 0.277$ | $0.085 - 0.272$ |
| $\text{BR}(B^- \to \eta^0 \mu^- \bar{\nu}_\mu)$ | $(1.49 - 1.55) \times 10^{-5}$ | $(1.49 - 1.55) \times 10^{-5}$ |
| $\text{BR}(B^- \to \eta^0 \tau^- \bar{\nu}_\tau)$ | $(0.7 - 1.46) \times 10^{-5}$ | $(5.0 - 8.33) \times 10^{-6}$ |
| $\langle A_{FB}^L \rangle$ | $(-2.82 \to 4.68) \times 10^{-3}$ | $(-2.92 \to 4.68) \times 10^{-3}$ |
| $\langle A_{FB}^R \rangle$ | $0.287 - 0.31$ | $0.153 - 0.298$ |

panel) and $S_R$ (right panel) couplings. The integrated values of these parameters are given in Table VIII. The additional $V_{L,R}$ couplings don’t affect the $R_{\rho K^*}$ parameters.

In the literature, the $B \to V \ell \nu_\ell$ processes are investigated in both model-dependent and independent ways [15, 19]. Our findings on these processes are consistent with these predictions.
TABLE VI: The predicted branching ratios, forward-backward asymmetries of $\bar{B}(s) \to V^+ l^− \bar{\nu}_l$ processes, where $V = K^*, \rho$ and $l = \mu, \tau$ in the SM and for the case of $V_{L,R}$ NP couplings.

| Observables | Values in the SM | Values for $V_L$ coupling | Values for $V_R$ coupling |
|-------------|------------------|---------------------------|---------------------------|
| $\text{BR}(B_s \to K^{*+}\mu^-\bar{\nu}_\mu)$ | $(3.97 \pm 0.32) \times 10^{-4}$ | $(3.97 - 4.68) \times 10^{-4}$ | $(3.97 - 8.05) \times 10^{-4}$ |
| $\text{BR}(B_s \to K^{*+}\tau^-\bar{\nu}_\tau)$ | $(2.16 \pm 0.173) \times 10^{-4}$ | $(1.54 - 4.0) \times 10^{-4}$ | $(1.92 - 3.8) \times 10^{-4}$ |
| $\langle A_{FB}^\mu \rangle$ | $-0.293 \pm 0.023$ | $-0.293$ | $-0.293 \to -0.052$ |
| $\langle A_{FB}^\tau \rangle$ | $-0.146 \pm 0.012$ | $-0.146$ | $-0.138 \to 0.037$ |
| $\text{BR}(B^- \to \rho^0\mu^-\bar{\nu}_\mu)$ | $(1.56 \pm 0.124) \times 10^{-4}$ | $(1.56 - 1.85) \times 10^{-4}$ | $(1.56 - 3.0) \times 10^{-4}$ |
| $\text{BR}(B^- \to \rho^0\tau^-\bar{\nu}_\tau)$ | $(8.97 \pm 0.71) \times 10^{-5}$ | $(0.64 - 1.67) \times 10^{-4}$ | $(0.8 - 1.52) \times 10^{-4}$ |
| $\langle A_{FB}^\mu \rangle$ | $-0.362 \pm 0.028$ | $-0.362$ | $-0.362 \to -0.065$ |
| $\langle A_{FB}^\tau \rangle$ | $-0.184 \pm 0.015$ | $-0.184$ | $-0.168 \to 0.024$ |

effective Lagrangian in the presence of new physics, which contributes additional coefficients to the SM. In our work the new coefficients are considered to be complex and we have taken into account the effect of one Wilson coefficient at a time to compute the allowed parameter space of these new coefficients. Using the experimental branching ratios of $B_s^+ \to \tau^+\nu_\tau$ and
FIG. 13: The plots for the $q^2$ variation of forward-backward asymmetry of $\bar{B}_s \to K^+\tau^-\bar{\nu}_\tau$ (top-left panel), $\bar{B}^0 \to \pi^+\tau^-\bar{\nu}_\tau$ (top-right panel), $B^- \to \eta\tau^-\bar{\nu}_\tau$ (bottom-left panel) and $B^- \to \eta'\tau^-\bar{\nu}_\tau$ (bottom-right panel) processes.

$B^- \to \pi^0\mu^-\bar{\nu}_\mu$ processes, we have constrained the new couplings. We then calculated the branching ratios, forward-backward asymmetries of $B \to P l\bar{\nu}_l$ processes, where $P = K, \pi, \eta^{(l)}$ for all possible cases of new couplings. In the presence of $V_{L,R}$ couplings, we found reasonable deviation in the branching ratios of these processes from the corresponding SM predictions, but the corresponding forward-backward asymmetry parameters don’t show any deviation. In the case of $S_{L,R}$ couplings, the branching ratios have slight deviation from the SM predictions. However, the forward-backward asymmetry parameters have comparatively large deviations from the SM values. We then computed the lepton non-universality parameters, in order to test the presence of the violation of lepton universality in $b \to ul\bar{\nu}_l$ processes.

Besides the semileptonic decays of $B$ meson to a pseudoscalar meson, we also studied the $B \to V l\bar{\nu}_l$ processes, where $V$ is a vector meson and $V = K^*, \rho$. We calculated the branching ratios, forward-backward asymmetries and the lepton non-universality parameters for these processes. The presence of additional $V_{L,R}$ Wilson coefficients result larger deviation in the branching ratios and other observables in the $B \to V l\bar{\nu}_l$ processes. The effect of $S_{L,R}$
FIG. 14: The plots for the LNU parameters $R_{\tau\mu}^K(q^2)$ (top-left panel), $R_{\tau\mu}^\pi(q^2)$ (top-right panel), $R_{\eta}^{\tau\mu}(q^2)$ (bottom-left panel) and $R_{\eta'}^{\tau\mu}(q^2)$ (bottom-right panel) due to $S_R$ coupling.

couplings on branching ratios of these processes is almost negligible. However, the forward-backward asymmetry of $B \to V\tau\bar{\nu}_\tau$ process deviates significantly from SM. We also observe that, the rare semileptonic $b \to ul\bar{\nu}_l$ processes also violate the lepton flavour universality. Thus, the study of $b \to ul\bar{\nu}_l$ processes are necessary in both theoretical and experimental point of view in order to search new physics.

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FIG. 15: The plots in the top panel represent the $q^2$ variation of the branching ratios of $\bar{B}_s \to K^{*+}\mu^−\bar{\nu}_\mu$ (top-left panel) and $\bar{B}_s \to K^{*+}\tau^−\bar{\nu}_\tau$ (top-right panel) processes for only $V_L$ coupling. The corresponding plots for only $V_R$ coupling are shown in the bottom panel.

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FIG. 16: The plots for the $q^2$ variations of the forward-backward asymmetry of $\bar{B}_s \to K^{*+}\tau^-\bar{\nu}_\tau$ processes for only $V_R$ (top-right panel), $S_L$ (bottom-left panel) and $S_R$ (bottom-right panel) couplings. The top-left panel represents the plots for the forward-backward asymmetry of $\bar{B}_s \to K^{*+}\mu^-\bar{\nu}_\mu$ processes for only $V_R$ coupling.

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FIG. 17: The plots for $R_{K^*}(q^2)$ parameters verses $q^2$ for only $V_L$ (top-left panel), $V_R$ (top-right panel), $S_L$ (bottom-left panel) and $S_R$ (bottom-right panel) couplings.

TABLE VII: Same as Table VI in the presence of $S_{L,R}$ couplings.

| Observables                             | Values for $S_L$ coupling | Values for $S_R$ coupling |
|-----------------------------------------|---------------------------|---------------------------|
| $BR(B_s \to K^{*+} \mu^- \bar{\nu}_\mu)$ | $(3.97 - 4.0) \times 10^{-4}$ | $(3.97 - 4.0) \times 10^{-4}$ |
| $BR(B_s \to K^{*+} \tau^- \bar{\nu}_\tau)$ | $(2.1 - 2.58) \times 10^{-4}$ | $(1.99 - 2.2) \times 10^{-4}$ |
| $\langle A^\mu_{FB} \rangle$              | $-0.293 \to -0.291$       | $-0.293 \to -0.286$       |
| $\langle A^\tau_{FB} \rangle$             | $-0.169 \to -0.043$       | $-0.144 \to -0.056$       |
| $BR(B^- \to \rho^0 \mu^- \bar{\nu}_\mu)$ | $(1.57 - 1.6) \times 10^{-4}$ | $(1.57 - 1.6) \times 10^{-4}$ |
| $BR(B^- \to \rho^0 \tau^- \bar{\nu}_\tau)$ | $(0.87 - 1.12) \times 10^{-4}$ | $(8.9 - 9.2) \times 10^{-5}$ |
| $\langle A^\mu_{FB} \rangle$              | $-0.36 \to -0.35$         | $-0.36 \to -0.35$         |
| $\langle A^\tau_{FB} \rangle$             | $-0.21 \to -0.07$         | $-0.32 \to -0.18$         |

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FIG. 18: Same as Fig. 15 for $B^{-} \rightarrow \rho^{0} l^{-}\bar{\nu}_{l}$ processes.

TABLE VIII: Values of $R_{K^{*}}^{\tau\mu}$, $R_{\rho}^{\tau\mu}$, $R_{\rho K^{*}}^{\mu}$, and $R_{\rho K^{*}}^{\tau}$ parameters for different cases of NP couplings.

| Model | $R_{K^{*}}^{\tau\mu}$ | $R_{\rho}^{\tau\mu}$ | $R_{\rho K^{*}}^{\mu}$ | $R_{\rho K^{*}}^{\tau}$ |
|-------|----------------------|---------------------|-----------------------|----------------------|
| SM    | 0.544                | 0.573               | 0.393                 | 0.415                |
| $V_{L}$ | 0.388 – 0.856         | 0.41 – 0.9          | 0.393 – 0.395         | 0.415 – 0.42         |
| $V_{R}$ | 0.47 – 0.474         | 0.5 – 0.51          | 0.373 – 0.393         | 0.4 – 0.42           |
| $S_{L}$ | 0.522 – 0.646        | 0.542 – 0.712       | 0.393 – 0.4           | 0.414 – 0.434        |
| $S_{R}$ | 0.497 – 0.544        | 0.5 – 0.573         | 0.393 – 0.4           | 0.4 – 0.42           |

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FIG. 19: Same as Fig. 16 for $B^- \to \rho^0 l^- \bar{\nu}_l$ processes.
FIG. 20: Same as Fig. 17 for $B^- \to \rho^0 l^- \bar{\nu}_l$ processes.

FIG. 21: The plots for $R^\tau_{\rho K^*}(q^2)$ parameters verses $q^2$ for only $S_L$ (left panel) and $S_R$ (right panel) couplings.

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