Gauge-Higgs Unification on Flat Space Revised

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Abstract. Models with gauge-Higgs unification on a flat space are typically affected by common problems, the main of which are the prediction of a too small top and Higgs mass and a too low compactification scale. We show how, by breaking the SO(4,1) Lorentz symmetry in the bulk and introducing a $\mathbb{Z}_2$ “mirror” symmetry, a potentially realistic model arises, in which all these problems are solved.

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INTRODUCTION

The idea of identifying the Higgs field with the internal components of a gauge field in TeV–sized extra dimensions (Gauge-Higgs Unification (GHU)) has been realized to offer a possible solution to the SM instability of the electroweak scale. However GHU models compactified on a flat $S^4/\mathbb{Z}_2$ orbifold usually present some common drawbacks: too small Higgs and top masses and a too low compactification scale [1].

To solve these problems we propose to break the Lorentz symmetry in the bulk along the fifth direction [2]. In this way, Yukawa couplings are no longer constrained by the 5D Lorentz invariance and one can obtain order one couplings which are needed to get a realistic top mass. This leads also to an enhancement of the Higgs quartic coupling which pushes the Higgs mass above the current experimental bounds ($m_H > 115$ GeV).

Moreover, the introduction of a $\mathbb{Z}_2$ “mirror” symmetry, which doubles a subset of the bulk degrees of freedom, allows for a partial cancellation of the mass term in the effective Higgs potential, thus generating a substantial gap between the electroweak and compactification scales.

THE MODEL

We consider a 5D gauge theory compactified on $S^4/\mathbb{Z}_2$. The gauge group is taken to be $SU(3)_w \times G_1 \times G_2$, where $G_i = U(1)_i \times SU(3)_{i,s}$, $i = 1,2$, with the requirement of having a Lagrangian invariant under the $\mathbb{Z}_2$ “mirror” symmetry $1 \leftrightarrow 2$. The $\mathbb{Z}_2$ orbifold projection is embedded non-trivially in the $SU(3)_w$ group, by means of the diagonal matrix $P = \text{diag}(-1,-1,1)$; this choice of parities leads to the breaking

1 In the following we consider an explicit breaking of the Lorentz symmetry. However, in a purely 5D context, it can be generated by an axion–like field with twisted boundary conditions, as shown in [2].
SU(3)_w → SU(2)_L × U(1)_w. For the Abelian U(1)_i and non-Abelian SU(3)_1,s fields, we take \(A_1(y ± 2\pi R) = A_2(y)\) and \(A_1(-y) = \eta A_2(y)\), where \(\eta_\mu = 1, \eta_5 = -1\). The unbroken gauge group at \(y = 0\) is \(SU(2) × U(1) × G_+\), whereas at \(y = \pi R\) we have \(SU(2) × U(1) × G_1 × G_2\), where \(G_+\) is the diagonal subgroup of \(G_1\) and \(G_2\). The \(Z_2\) mirror symmetry also survives the compactification and remains as an exact symmetry. The boundary conditions can be diagonalized by a redefinition of the bulk fields \(A_± = (A_1 ± A_2)/\sqrt{2}\); these combination are respectively periodic and antiperiodic on \(S^1\) and have a multiplicative charge +1 for \(A_+\) and −1 for \(A_-\) under the mirror symmetry. The 4D zero-modes are the gauge fields in the adjoint of \(SU(2) × U(1) \in SU(3)_w\), the \(U(1)_+\) and glue gauge bosons \(A_+\) and a charged scalar doublet identified with the Higgs field, which arises from the even internal components of the \(SU(3)_w\) 5D gauge fields (namely \(A^2_5, 5, 6, 7\)). The \(SU(3)_+, s\) and \(SU(2)\) gauge groups are identified respectively with the SM \(SU(3)_QCD\) and \(SU(2)_L\) ones, while the hypercharge \(U(1)_Y\) is the diagonal subgroup of \(U(1)_w\) and \(U(1)_+\). The extra \(U(1)_X\) gauge symmetry which survives the orbifold projection is anomalous (see [1, 2, 3]) and its corresponding gauge boson decouples from the low-energy effective theory. This mechanism is required in order to obtain the correct value of the weak mixing angle. The most general gauge Lagrangian compatible with the mirror symmetry and the 4D Lorentz symmetry is

\[
\mathcal{L}_g = \sum_{i=1,2} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\rho^2}{2} F_{\mu5} F^{\mu5} \right] - \frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} - \rho_w^2 \text{Tr} F_{\mu5} F^{\mu5},
\]

where we omitted for simplicity the gluon fields, the ghost Lagrangian and the gauge-fixing terms.

An additional spontaneous symmetry breaking to \(U(1)_{EM}\) is induced by an Higgs VEV \(\langle A_{w5}\rangle = \frac{2\alpha}{g_5 R^s t_a}\), where \(g_5\) is the 5D charge of the \(SU(3)_w\) group and \(t_a\) its generators. The Higgs VEV is associated to a Wilson line \(W = e^{i\alpha t a}\), thus the electroweak symmetry breaking (EWSB) is equivalent to a Wilson line symmetry breaking [4].

In the bulk we introduce couples of fermions \((\Psi, \bar{\Psi})\), with identical quantum numbers and opposite orbifold parities. There are couples \((\Psi_1, \bar{\Psi}_1)\) which are charged under \(G_1\) and neutral under \(G_2\) and, by mirror symmetry, an equal number of couples \((\Psi_2, \bar{\Psi}_2)\) charged under \(G_2\) and neutral under \(G_1\). For each quark family we include one pair of couples \((\Psi^i_{1,2}, \bar{\Psi}^i_{1,2})\) in the 6 representation of \(SU(3)_w\) and one pair of couples \((\Psi^b_{1,2}, \bar{\Psi}^b_{1,2})\) in the 3 representation of \(SU(3)_w\). Both pairs have \(U(1)_{1,2}\) charge +1/3 and are in the 3 representation of \(SU(3)_1, 2, s\). These fields satisfy twisted boundary condition similar to the gauge field ones, in particular the combinations \(\Psi_{±} = (\Psi_1 ± \Psi_2)/\sqrt{2}\) are respectively periodic and antiperiodic on \(S^1\). The bulk fermion Lagrangian for a quark family has the form

\[
\mathcal{L}_\Psi = \sum_{i=1,2} \sum_{a=1,b} \left[ \bar{\Psi}^i_a \left[ i D_4 - k_a D_5 \gamma^5 \right] \Psi^i_a + \bar{\Psi}^i_{\bar{a}} \left[ i D_4 - \bar{k}_a D_5 \gamma^5 \right] \Psi^i_{\bar{a}} - M_a \left( \bar{\Psi}^i_a \Psi^i_a + \bar{\Psi}^i_{\bar{a}} \Psi^i_{\bar{a}} \right) \right].
\]

We also introduce massless fermions localized at the \(y = 0\) fixed point with mirror charge +1. These fields can couple only with periodic even bulk fields through localized
mass terms (see [2, 3] for a detailed description of the couplings). In particular we put an \( SU(2)_L \) doublet \( Q_L \) and two singlets \( t_R \) and \( b_R \), all in the fundamental representation of \( SU(3)_{+,s} \) and with \( U(1)_{+} \) charge +1/3.

**THE ELECTROWEAK SYMMETRY BREAKING**

The 5D gauge symmetry, which remains unbroken, forbids the appearance of any local Higgs potential in the bulk, moreover a localized Higgs potential is also forbidden. The Higgs potential is radiatively generated and induced by non-local operators, thus it is finite. The contributions of the various fermionic and gauge fields to the effective potential are fully reported in [2, 3]. The whole effective potential is dominated by the contributions of the top and bottom quark, which provide a negative contribution for the Higgs mass squared needed to generate a spontaneous EWSB. The presence of an unbroken mirror symmetry forces bulk fermions to come in pairs of periodic and antiperiodic fields with the same quantum numbers and the same Lorentz-breaking couplings. This allows for a natural cancellation of the radiatively induced Higgs mass, lowering the position of the minimum of the effective potential and reducing the amount of fine-tuning needed to satisfy the experimental bounds [3].

After EWSB the W boson zero-mode acquires a mass \( m_W = \frac{\alpha}{R} \). The whole spectrum of the fermion system is described in [2, 3]. In particular the top mass, in the limit of large bulk-boundary mixing and small \( \alpha \), is given by

\[
m_t \simeq k_t m_W \frac{2\lambda_t/k_t}{\sinh(2\lambda_t/k_t)},
\]

where \( \lambda_t = \pi R M_t \). From this relation we deduce that \( k_t \sim 2 - 3 \) is needed to get the correct top mass. The bulk-boundary fermion system provides also the lightest non-SM states. These states are coloured fermions with a mass of order 1–2 TeV, and in particular they are given by an \( SU(2)_w \) triplet (with \( Y = 2/3 \)), a doublet (with \( Y = -1/6 \)) and a singlet (with \( Y = -1/3 \)).

**PHENOMENOLOGICAL BOUNDS**

Flavour and CP conserving new physics effects can be parameterized by the universal parameters \( \hat{S}, \hat{T}, \hat{U}, V, X, W \) and \( Y \), and by 3 parameters which describe the distortions of the \( \hat{Z} \) boson couplings with the quarks [5].

The light quarks are nearly exactly localized, thus their couplings with the \( \hat{Z} \) are undistorted. The third quark family has a stronger mixing with the bulk fields. Available experimental data put tight bounds on the \( \hat{Z}b_Lb_L \) vertex, which, in our model, gets corrections due to two effects. One effect is the mixing of \( b_L \) with the KK tower of an \( SU(2)_w \) triplet and an \( SU(2)_w \) singlet bulk fermions. However this distortion is much suppressed due to the choice of the bulk fermion quantum numbers and vanishes in the limit of zero \( b \) mass [3]. The other distortion comes from the couplings of the \( b_L \) field with the KK tower of the anomalous \( U(1)_X \) gauge subgroup. The correction turns out to be \( -\delta g_b \lesssim 3\alpha^2 \), and can be neglected for \( \alpha \lesssim 3 \times 10^{-2} \).
FIGURE 1. Constraints coming from a $\chi^2$ fit on the electroweak precision tests. The shaded band shows the experimentally excluded values for the Higgs mass ($m_H < 115$ GeV). The dots represent the predictions of our model for different values of the microscopic parameters.

Out of the 7 universal parameters, only $\hat{S}$, $\hat{T}$, $Y$ and $W$ arise at leading order in $\alpha$:

\[
\begin{align*}
\hat{S} &= \frac{2}{3} \alpha^2 \pi^2, \\
\hat{T} &= \alpha^2 \pi^2, \\
Y &\simeq \frac{1}{3} \alpha^2 \pi^2, \\
W &= \frac{1}{3} \alpha^2 \pi^2,
\end{align*}
\]

(4)

the others are suppressed by higher powers of $\alpha$ and can be neglected in the fits.

In fig. 1, we report the constraints on the Higgs mass and the compactification scale obtained by a $\chi^2$ fit using the values in eq. (4). From the fit, one can extract a lower bound on the compactification scale $1/R \gtrsim 4 - 5$ TeV (which corresponds to $\alpha \lesssim 0.016 - 0.02$) and an upper bound on the Higgs mass which varies from $m_H \lesssim 600$ GeV at $1/R \sim 4$ TeV to $m_H \lesssim 250$ GeV for $1/R \gtrsim 10$ TeV. Notice that the values for $\hat{S}$, $W$ and $Y$ found in our model are essentially those expected in a generic 5D theory with gauge bosons and Higgs in the bulk [6]. On the contrary, the $\hat{T}$ parameter is much bigger and is essentially due to the anomalous gauge field $A_X$, which generates a distortion of the SM $\rho$ parameter. The high value of the $\hat{T}$ parameter compensates the effects of a heavy Higgs.

Performing a random scan of the microscopic parameters (see fig. 1), we found that the bound on the compactification scale can be easily fulfilled if a certain amount of fine-tuning is allowed. We estimated the fine-tuning to be of order a few % [3].

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