Recurrence analysis of the NASDAQ crash of April 2000

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Summary. Recurrence Plot (RP) and Recurrence Quantification Analysis (RQA) are signal numerical analysis methodologies able to work with non linear dynamical systems and non stationarity. Moreover they well evidence changes in the states of a dynamical system. It is shown that RP and RQA detect the critical regime in financial indices (in analogy with phase transition) before a bubble bursts, whence allowing to estimate the bubble initial time. The analysis is made on NASDAQ daily closing price between Jan. 1998 and Nov. 2003. The NASDAQ bubble initial time has been estimated to be on Oct. 19, 1999.

Key words: Endogenous crash, Financial bubble, Recurrence Plot, Recurrence Quantification Analysis, Nonlinear Time Series Analysis, NASDAQ

1 Introduction

Recent papers have shown some analogy between crashes and phase transitions [1, 2, 3]; like in earthquakes, log periodic oscillations have been found before some crashes [4, 5], then it was proposed that an economic index \( y(t) \) increases as a complex power law, whose first order Fourier representation is

\[
y(t) = A + B \ln(t_c - t) \{1 + C \cos[\omega \ln(t_c - t) + \phi]\}
\]

(1)

where \( A, B, C, \omega, \phi \) are constants and \( t_c \) is the critical time (rupture time).

An endogenous crash is preceded by an unstable phase where any information is amplified; this critical period takes the name of 'speculative bubble'.

Recurrence Plots are graphical tools elaborated by Eckmann, Kamphorst and Ruelle in 1987 and are based on Phase Space Reconstruction [6]. In 1992, Zbilut and Webber [7] proposed a statistical quantification of RPs and gave it the name of 'Recurrence Quantification Analysis' (RQA). RP and RQA are
good in working with non stationarity and noisy data, in detecting changes in data behavior, in particular in detecting breaks, like a phase transition [8], and in informing about other dynamic properties of a time series [6]. Most of the applications of RP and RQA are at this time in the field of physiology and biology, but some authors have already applied these techniques to financial data [9, 10]. We have used RP and RQA techniques for detecting critical regimes preceding an endogenous crash seen as a phase transition and whence give an estimation of the initial bubble time.

It has been simulated a signal as in 1, the analysis is made on NASDAQ, taken over a time span of 6 years including the known crash of April 2000 [5]. The series are also divided into subseries in order to investigate changes in the evolution of the signal. Then the RPs of all time series have been observed, compared and discussed. This work is extracted by [13].

2 Recurrence Analysis

The changing state of a dynamic system can be indeed represented by sequences of ‘state vectors’ in the phase space. Each unknown point of the phase space at time \( i \) is reconstructed by the delayed vector \( y(i) = X_{i}, X_{i+d}, ..., X_{i+(m-1)d} \) in an \( m \)-dimensional space.

**Recurrence Plot**

The Recurrence Plot (RP) is a matrix of points \((i, j)\) where each point is said to be recurrent and marked with a dot if the distance between the delayed vectors \( y(i) \) and \( y(j) \) is less than a given threshold \( \epsilon \). As each coordinate \( i \) represents a point in time, RP provides information about the temporal correlation of phase space points [6].

Therefore RPs can be used to test a system deterministic behavior through the percentage of recurrent points belonging to parallel lines. In fact for a periodic or a deterministic signal patterns like parallel lines appear. In so doing, RPs are useful tools for the preprocessing of experimental time series and provide a comprehensive image of the dynamic course at a glance [8].

**Recurrence Quantification Analysis**

RQA quantifies the presence of patterns, like parallel lines of RPs, with 5 RQA variables: the percentage of recurrent points (%REC). The percentage of recurrent points forming line segments parallel to the main diagonal (%DET). The longest line segment measured parallel to the main diagonal (MAXLINE). The slope of line-of-best-fit through %REC as a function of the displacement from the main diagonal (excluding the last 10% range) (TREND). The Shannon entropy of the distribution of the length of line segments parallel to the main diagonal (ENT).
3 Analysis and Conclusions

In order to study the crash from the point of view of a phase transition with log periodic precursors, a log periodic signal, generated by equation (1), has been simulated, its RP is shown in Fig. 1(lhs). The ‘arrow’ shape is due to the trend, the not smooth border (‘color’) lines are due to the log periodicity. It has been also considered a phase transition signature. In Fig. 2(lhs) an arbitrary signal is plotted before and after a peak, taking into account the anti-bubble phenomenon after a crash [12]. The RP aspect of Fig. 2(RHS) reveals a feature far from the normal signal evolution; to be noted the well marked black bands corresponding to the crash time.

About NASDAQ after studied the whole time series the data has been divided into subseries of 200 days, overlapping each other of almost 5 months, in order to further analyze whether and how the data changes.

Fig. 3(rhs) is the RP of NASDAQ between Jan. 05, 1998 and Nov. 21, 2003. Of interest is the dark grey vertical band surrounded by a lighter grey area, delimited by horizontal coordinates $x = 452$ and $x = 690$ corresponding to Oct. 19, 1999 and Sept. 27, 2000. In correspondence of the period in which the bubble grows, RQA variables take the highest absolute values [13].

It is worth to note the same RP shape of the phase transition signature in Fig. 2. Considering that each coordinate in RP is linked with the time series, the border line of a grey or black band reveals the time when the data behavior starts to change. Noting that the dates here above fall in the same time interval as the bubble and the subsequent crash, it can be supposed that the initial bubble time occurs at $x = 452$ (Oct. 19,1999). We can thus deduce that on such a day the evolution of the system changes, i.e. the evolution passes from a normal regime to a critical regime. This is an a posteriori estimation of the initial bubble time, but through the analysis of the subseries one can argue to be able to recognize the beginning of the bubble with some delay before the bubble grows. In fact, while the RPs of the first (I), the second (II) and of the third (III) subseries do not present any remarkable pattern [13] (they are quite homogeneous except some local maximum reached by the index) the fourth subseries presents an interesting pattern: the RP in Figure 1(rhs) shows the characteristic shape typical of the strong trend of a speculative bubble as studied and pointed out in Fig. 1(lhs). The trend starts to be significant in the middle of Oct. 1999. This indicates that the RP has changed indeed when the bubble has started. Even the RQA variables, in Table 1, evidence in this period the highest values. It has to be underlined that this IV period does not include the crash time, but stops in Dec. 1999 before the bubble bursts. Even in the fifth (V) subseries RP, the bubble beginning is not so evident as it was in the fourth subseries.

In conclusion it has been shown that, with some delay as respect to the beginning but enough time before the crash (3 months in this particular case), such that a warning could be given, RP and RQA detect a difference in state and recognize the critical regime.
Fig. 1. (lhs) RP of a log periodic signal as generated by equation (1). The arrow shape on the lhs plot is the sign of a strong trend; the curve lines are due to the log periodicity. (rhs) RP of the NASDAQ subseries (IV) from Jan., 1999 to Dec., 1999. It is worth to note the similarity between these two RPs.

Fig. 2. (RHS) the RP of a simulated phase transition of a signal (LHS) following the law (1) before and after the critical event.

| Table 1. RQA of NASDAQ on 5 the subseries studied of 200 days. |
|-----------------|--------|--------|--------|--------|--------|
| Subseries periods | I      | II     | III    | IV     | V      |
| Jan.1998       | 6.075  | 9.141  | 5.513  | 17.146 | 9.246  |
| June 1998      | 35.980 | 36.119 | 29.079 | 45.018 | 54.511 |
| Oct.1998       | 158    | 83     | 179    | 166    |
| Feb.1999       | 2.522  | 3.547  | 2.585  | 4.054  | 3.301  |
| July 1999      | -105.125 | -155.824 | -97.808 | -273.775 | -138.501 |
| May2000       | 2002000 |
| TREND(units/1000points) | -105.125 | -155.824 | -97.808 | -273.775 | -138.501 |
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Fig. 3. (LHS) daily closing price of NASDAQ from Jan. 05, 1998 to Nov. 21, 2003; (RHS) RP of NASDAQ from Jan. 05, 1998 to Nov. 21, 2003. The dark grey band delimited by horizontal coordinates $x = 504$ and $x = 566$, encircled by a grey area delimited by horizontal coordinates $x = 452$ and $x = 690$, is the 'image' of the crash of April 2000. It is a 'strong event' but afterwards the normal regime is restored.

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