Improved inference for the panel data model with unknown unit-specific heteroscedasticity: A Monte Carlo evidence

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Abstract: For a panel data model (PDM), it is common that the error terms of panel regression model are heteroscedastic. In the available literature, the heteroscedastic consistent covariance matrix estimators (HCCMEs) have been used for adequate testing of the coefficients of PDM. Usually, these HCCMEs are based on the residuals derived from ordinary least square (OLS) estimator which is considerably inefficient in the presence of heteroscedasticity. To get efficient estimation, the existing literature proposes some adaptive estimators for the PDM. This paper presents the HCCMEs, derived from some adaptive estimator, while considering the panel dataset with unit-specific heteroscedasticity. Through the Monte Carlo simulations, we present the numerical evaluation and attractive findings.

Subjects: Science; Mathematics & Statistics; Statistics & Probability; Statistics; Statistics & Computing; Statistical Theory & Methods

Keywords: adaptive estimator; HCCME; leverage points; panel data model; power of test; size distortion

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PUBLIC INTEREST STATEMENT
Panel data are multi-dimensional data consisting of measurements over time. The observations of multiple phenomena, obtained over multiple time periods for the same companies, firms, countries or individuals etc., constitute the panel data. Panel data have several advantages over purely cross-sectional or purely time series data. To model such data, a panel data model (PDM) is used that provides information on individual behaviour, both across individuals and over time. In spite of its many advantages, a PDM may pose several estimation and inference problems due to several reasons and heteroscedasticity in cross-sectional units at the same point in time (i.e. unit-specific heteroscedasticity) is one of them. The present article addresses the same issue and suggests how one can improve in the inferential issue in the presence of unit-specific heteroscedasticity.
1. Introduction

In econometrics, one important type of data is known as the panel data. Panel data are based on various observations, collected from same individuals over several time periods. A regression model that fits panel data is known as the panel data model (PDM). In econometrics, analysis of PDM is the most dynamic assortment somewhat in light of the fact that panel data-sets give a rich domain to advancement of estimation methods and hypothetical results. In more useful terms, researchers have possessed the capacity to utilize time-series and cross-sectional data to inspect issues which could not be handled in either setting alone.

An important assumption of classical linear regression model (CLRM) is homoscedasticity that the variance of error term remains constant and thus, the error term is identically distributed. If this assumption is not met, there exists the issue of heteroscedasticity and the OLS results are inadequate in this case. Heteroscedasticity is a common problem in the PDM and it is desirable to concentrate on it for making robust inference. The ordinary technique used for estimation of PDM like the OLS does not lead to efficient estimation and correct inference in the presence of heteroscedasticity. The OLS estimator is not biased and inconsistent but does not remain linear unbiased estimator (BLUE) when the assumption of homoscedasticity is violated. Furthermore, usual t and F statistic are unable to construct precise confidence interval and to perform correct testing of hypothesis. Moreover, presence of the high leverage points in given data-set may also lead to incorrect inference. Therefore, focus of this study is to bring improvement in inference of linear PDM suffering from heteroscedasticity, namely the unit-specific heteroscedasticity (USH).

Mazodier and Trognon (1978) were the first who studied the problem of heteroscedasticity in the PDM, and later, Baltagi and Griffin (1988) and Randolph (1988) considered it. For the efficient estimation of the PDM under heteroscedasticity, some adaptive estimators are available in the literature. Li and Stengos (1994) developed an adaptive estimator for the unit time-varying heteroscedasticity (UTVH) and Roy (2002) proposed an adaptive estimator for the USH. Baltagi, Bresson, and Pirotte (2004) studied performance of both of these estimators and found that Roy’s estimator performed well in terms of relative MSE and was not dependent on selection of bandwidth. However, the estimator proposed by Li and Stengos for UTVH showed loss in efficiency for smaller bandwidth but performed well under higher bandwidth.

To tackle the problem of heteroscedasticity, Eicker (1963) and White (1980) proposed HCCME for non-panel data which made it conceivable to draw asymptotically robust inference. In the existing literature, it can be seen that Arellano (1987) built White’s estimator for the PDM. For common regression models, Ahmed, Aslam, and Pasha (2011) and Aslam, Riaz, and Altaf (2013) used the adaptive HCCME (AHCCME). Cribari-Neto (2004) proposed a variant of the HCCME for common regression models to take into account the effect of leverage points. This estimator is known as HC4. Cribari-Neto, Souza, and Vasconcellos (2007) proposed another version of the HCCME for linear regression models to study the effect of maximal leverage on associated inference. It is termed as the HC5. Adaptive versions of the HC4 and HC5 have been used for common regression models by Aslam et al. (2013). It has been noticed in the available literature that the AHCCMEs are not used for the PDM. Therefore, this is the main concern of the current study.

This article unfolds as follows. Section 2 describes the PDM with USH and adaptive estimator. Section 3 describes the AHCCME. In Section 4, the quasi-t test statistic and computation of confidence interval and power of test are discussed. Empirical results are presented in Section 5. An illustrative example has been given in Section 6 and, finally, Section 7 concludes the said work.

2. Adaptive estimator for USH

Following Li and Stengos (1994) and Roy (2002), consider the standard one-way error component model

\[ y_{it} = x_{it}' \beta + u_{it}; \quad i = 1, 2, \ldots, N; \quad t = 1, 2, \ldots, T \]  

with \( u_{it} = \mu_i + \nu_{it} \),

(1)
where $x_i$ is a unit-time varying error component (UTVEC) and $\mu_i$ is the unit-specific error (USE) component assumed to be i.i.d. with that $E(\mu_i|x_i) = 0$, $\text{var}(\mu_i|x_i) = \omega_0$, where $\tilde{x}_i = \frac{1}{T} \sum_{t=1}^{T} x_{it}$. In other words, the conditional variance of USE is suffering from heteroscedasticity of unknown form. Throughout the paper, we assume $T$ is small and $N$ is large.

Matrix form of Model (1) is presented by Li and Stengos (1994) as

$$
y = X\beta + Z\mu + v,
$$

(2)

where $Z = I_t \otimes e_t$, $e_t$ is a T-dimensional column vector of ones, $\otimes$ denotes the Kronecker product, $\mu = [\mu_1, \mu_2, \ldots, \mu_n]'$, $y$, and $v$ are the $NT \times 1$ column vectors of dependent variable and UTVEC, respectively, while $x$ is an $NT \times q$ matrix of regressors.

Following the work of Baltagi and Griffin (1988), Roy (2002) presented inverse of the conditional variance–covariance matrix of error term $(Z\mu + v)$ in (2), which is denoted by $W^{-1}$,

$$
W^{-1} = \text{diag} \left( \frac{1}{\sigma^2_i} \right) \otimes \left( \frac{I_T}{T} \right) + \text{diag} \left( \frac{1}{\sigma^2_v} \right) \otimes \left( I_T - \frac{J_T}{T} \right),
$$

(3)

where $\sigma^2_i = T \omega_i + \sigma^2_v i$ and $J_T$ is $T \times T$ matrix of ones. For Model (2), the true generalized least square (TGLS) estimator of $\beta$ is

$$
\hat{\beta} = (X'W^{-1}X)^{-1}X'W^{-1}y,
$$

(4)

The estimation of (4) involves covariance matrix of order $NT \times NT$. So for large data-set, Roy proposed following version of (4)

$$
\hat{\beta} = \left( \sum_{i=1}^{N} x_i'A_i^{-1}x_i \right)^{-1} \sum_{i=1}^{N} x_i'A_i^{-1}y_i,
$$

(5)

where $x_i$ is a $T \times q$ matrix of regressors for the $i$th individual, $y_i$ is $T \times 1$ and $A_i^{-1}$ is $T \times T$ covariance matrix

$$
A_i^{-1} = \frac{1}{\gamma_i (1 - \rho_i)} \left[ I_T - e_i e_i' \rho_i \right],
$$

(6)

where $\rho_i = \frac{\omega_i}{T}$ and $\gamma_i = \omega_i + \sigma^2_v$. To find estimates of $\omega_i$, $\sigma^2_v$, and $\gamma_i$ need to be estimated which are unknown parameters in (6). Roy (2002) estimated $\sigma^2_v$ as

$$
\hat{\sigma}^2_v = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} \left( (y_{it} - \tilde{y}_i) - \hat{\beta}_w (x_{it} - \tilde{x}_i) \right)^2}{N(T - 1) - q},
$$

where $\tilde{y}_i$ is similarly defined as $\tilde{x}_i$, and $\hat{\beta}_w$ is within group estimator (WGE) (for more details; see Greene, 1997).

Roy (2002) defined $\gamma_i = E(\mu_i^2|x_i) = \omega_i + \sigma^2_v$ and proposed the following kernel estimator for $\gamma_i$

$$
\hat{\gamma}_i = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\sigma}^2_v K \left\{ \frac{(x_{it} - \bar{x})}{d} \right\}}{\sum_{i=1}^{N} \sum_{t=1}^{T} K \left\{ \frac{(x_{it} - \bar{x})}{d} \right\}}; \quad i = 1, 2, \ldots, N,
$$

(7)

where $\hat{\sigma}^2_v$ is the OLS residual from the regression of $y_i$ on $x_i$, $K(\cdot)$ is the kernel function with $d$ as the smoothing parameter. Using (7), the estimates of $\omega_i$ can be found as $\hat{\omega}_i = \hat{\gamma}_i - \hat{\sigma}^2_v$ and hence an estimator of $A_i^{-1}$ can be obtained as
\[ \hat{\beta} = \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} x_i \right)^{-1} \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} y_i. \]

3. The AHCCME

For common regression models (i.e., models for cross-sectional data), Ahmed et al. (2011) used the AHCCME (AHCC0-AHCC3). For common regression models, Cribari-Neto (2004) proposed HC4 and Cribari-Neto et al. (2007) proposed HC5. The AHCC4 and AHCC5 have been used for common regression models by Aslam et al. (2013). However, such covariance estimators have not been studied yet by any author for the PDM. According to the above cited studies, the HC3, HC4, and HC5 give attractive performance to improve testing. Therefore, in the present study, we skip HC1 and HC2 but HC0 is included as a standard estimator.

The usual covariance matrix of \( \hat{\beta} \) is

\[ \Psi = \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} x_i \right)^{-1} \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} E(u_i u_i') \hat{A}_i^{-1} x_i \right) \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} x_i \right)^{-1}. \]

Following White (1980), Ahmed et al. (2011), and Aslam et al. (2013), we define a consistent estimator of the PDM as follows:

\[ \hat{\Psi}(0) = \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} x_i \right)^{-1} \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} \phi_i^{(0)} \hat{A}_i^{-1} x_i \right) \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} x_i \right)^{-1}, \]

where \( \phi_i^{(0)} = (\hat{\omega}, \hat{\omega}') \) and \( \hat{\omega}_i = (\hat{\omega}_{1i}, \hat{\omega}_{2i}, \ldots, \hat{\omega}_{ni})' \) is the AGLS residual given as \( \hat{\omega} = y - \hat{x} \hat{\beta} \).

The estimator in (8) is termed as AHCC0.

Consider \( h_i = (h_{1i}, h_{2i}, \ldots, h_{ni})' \) and \( h_{\pi} \) as the \( i \)th diagonal element of hat matrix \( H = \sum_{i=1}^{N} x_i \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} x_i \right)^{-1} \hat{A}_i^{-1} x_i \), then the AHCC3 can be defined as

\[ \text{AHCC3} = \hat{\Psi}^{(3)} = \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} x_i \right)^{-1} \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} \hat{\phi}_i^{(3)} \hat{A}_i^{-1} x_i \right) \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} x_i \right)^{-1}, \]

where \( \hat{\phi}_i^{(3)} = (\hat{\omega}, \hat{\omega}') \text{diag} (1 - h_i)^{-2} \) (see similar construction of the HCCME in Uchoa, Cribari-Neto, and Menezes (2014)). An observation with \( h_\pi \geq \frac{2\pi}{NT} \) is declared as a high leverage point by Hoaglin and Welsch (1978). A general rule-of-thumb, cited in Cribari-Neto (2004), is that the values of \( h_\pi \) in excess of two or three times the average (i.e., \( \frac{2\pi}{NT} \) and \( \frac{3\pi}{NT} \)) are regarded as influential. The adaptive versions of Cribari-Neto (2004) and Cribari-Neto et al. (2007) estimator have not been studied in context of the PDM yet by any author. Therefore, we propose to use AHCC4 and AHCC5 for the PDM. The AHCC4 and AHCC5 are

\[ \text{AHCC4} = \hat{\Psi}^{(4)} = \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} x_i \right)^{-1} \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} \hat{\phi}_i^{(4)} \hat{A}_i^{-1} x_i \right) \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} x_i \right)^{-1}, \]

where \( \hat{\phi}_i^{(4)} = (\hat{\omega}, \hat{\omega}') \text{diag} (1 - h_i)^{-1} \), \( \pi_i = \min \left( \frac{4, \hat{\pi}_i}{h_i}, \ldots, \min \frac{4, \hat{\pi}_i}{h_i} \right) \), \( h_{\pi} \) being the average leverage (i.e., the average value of all leverages). Since \( 0 < h_i < 1 \) and \( \pi_i > 0 \), hence \( 0 < (1 - h_i)^{\pi_i} < 1 \).

\[ \text{AHCC5} = \hat{\Psi}^{(5)} = \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} x_i \right)^{-1} \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} \hat{\phi}_i^{(5)} \hat{A}_i^{-1} x_i \right) \left( \sum_{i=1}^{N} x_i' \hat{A}_i^{-1} x_i \right)^{-1}, \]
where \( \hat{\phi}_i^{(s)} = (\hat{u}_i, \hat{u}_i') \text{diag}(1 - h_i)^{-\delta_i}, \delta_i = \min \left( \left\{ \frac{h_i}{h_{i1}}, \ldots, \frac{h_i}{h_{i1}} \right\}, \max \left( \left\{ 4, \frac{c_{i1}}{h_{i1}} \right\}, \ldots, \left\{ 4, \frac{c_{i1}}{h_{i1}} \right\} \right) \right) \),
\[ 0 < c < 1, \ h_{max} \] is the maximum value of leverages. Since \( 0 < h_i < 1 \) and \( \delta_i > 0 \), hence \( 0 < (1 - h_i)^{\delta_i} < 1 \).

Generally, the estimators presented above can be written in unified fashion as
\[
\hat{\Psi}^{(s)} = \left( \sum_{i=1}^{N} x_i' A_i^{-1} x_i \right)^{-1} \left( \sum_{i=1}^{N} x_i' A_i^{-1} \phi_i^{(s)} A_i^{-1} x_i \right) \left( \sum_{i=1}^{N} x_i' A_i^{-1} x_i \right)^{-1}, \ s = 0, 3, 4, 5. \tag{9}
\]

4. Adaptive heteroscedasticity consistent interval estimators (AHCIE), test statistic, and power of test

Cribari-Neto and Lima (2009) considered heteroscedasticity consistent interval estimators (HCIE) based on \( \hat{\beta} \) and HCCMEs for common regression models. Aslam et al. (2013) used the AHCIE in their work for non-panel regression models. But, we are going to consider the AHCIE for the PDM.

Let \( \theta = h(\beta): R^k \rightarrow R \) be a function of parameter of interest, \( \hat{\theta} \) is its estimate and \( s(\hat{\theta}) \) is the asymptotic standard error. Consider the studentized statistic, \( t_n = \frac{\hat{\theta} - \theta}{s(\hat{\theta})} \). It is quite easy to show that \( t_n \rightarrow^d N(0, 1) \).

Consider the hypothesis, \( H_0: \beta_r = \beta_r^0 \) against \( H_1: \beta_r = \beta_r^1 \), where \( \beta_r^0 \) is a hypothesized value of \( \beta_r \) under \( H_0 \).

Under homoscedasticity, the test statistic for above given hypothesis is
\[
\hat{t} = \frac{(\hat{\beta}_r - \beta_r^0)}{\sqrt{\hat{\Psi}_{rr}(n)}}, \tag{10}
\]
where \( \hat{\Psi}_{rr} \) is the \( r \)th diagonal element of \( \hat{\Psi} \) and \( r = 0, 1, 2, \ldots, q-1 \). Then \( \hat{t} \) is likely to follow a Student’s \( t \) distribution with degree of freedom \( (NT - \text{tr}(H)) \), such that \( \hat{t} > t_{1-\alpha} \text{tr}(H) \). For large sample size, the quantity above converges in distribution to the standard normal distribution (for more details; see (Cribari-Neto & Lima, 2009)). Thus, a test of asymptotic significance \( \alpha \) rejects \( H_0 \) if \( |\hat{t}| > Z_p \), where \( Z_p \) is the \( \left( 1 - \frac{\alpha}{2} \right) \) quantile of standard normal distribution. Thus, the true size of test can be computed as
\[
P(\text{reject } H_0 | H_0) = P\left( |\hat{t}| > Z_{1-\frac{\alpha}{2}} | \beta_r = \beta_r^0 \right). \tag{11}
\]

Similar, the power of test can be measured as
\[
P(\text{reject } H_0 | H_1) = P\left( |\hat{t}| > Z_{1-\frac{\alpha}{2}} | \beta_r = \beta_r^1 \right). \tag{12}
\]

when the errors are heteroscedastic, the statistic in (10) can be re-defined as follows:
\[
\hat{t} = \frac{(\hat{\beta}_r - \beta_r^0)}{\sqrt{\hat{\Psi}_{rr}(n)}}, \ s = 0, 3, 4 and 5.
\]

In a similar manner, the confidence interval can be constructed. A \((1-\alpha)\times100%\) (two tailed) confidence interval based on the AHCCME is
\[
\hat{\beta}_r \pm Z_{1-\frac{\alpha}{2}} \sqrt{\hat{\Psi}_{rr}(n)} s = 0, 3, 4, 5. \tag{13}
\]
5. Empirical results

For the empirical results, we use the same Monte Carlo scheme as used in some previous studies like Li and Stengos (1994), Roy (2002), and Rilstone (1991)

The considered model is

\[ y_{it} = \beta_0 + \beta_1 x_{it} + \mu_i + v_{it}, \quad i = 1, 2, \ldots, N; t = 1, 2, \ldots, T, \]

where \( x_{it} = 0.5 w_{i,t+1} + w_i \) and \( w_i \sim \text{i.i.d.} \text{Exp}(N(0, 0.4^2)) \), i.e. \( w_i \) is generated from lognormal distribution. The values assigned to \( \beta_0 \) and \( \beta_1 \) are 5 and 0.5, respectively. The \( v_i \) and \( \mu_i \) can be generated as 

\[ v_i \sim \text{i.i.d.} N(0, \sigma_v^2), \quad \mu_i \sim \text{i.i.d.} N(0, \omega_i), \]

with \( \omega_i = \omega(\bar{x}_i) = \sigma_i^2 (1 + \bar{x}_i)^2 \). It is supposed that heteroscedasticity is of additive form. Let the total variance \( \gamma_i \) and the expected variance of \( \mu_i \) is

\[ \gamma_i = \omega_i + \sigma_v^2, \]

The values of \( \lambda \) are 0, 1, 2, and 3, where 0 indicates the homoscedastic USE and other shows different levels of heteroscedasticity for the fixed value of \( \sigma_v^2 \) and the values assigned to \( \omega_i \) are 2, 4, and 6. Increase in \( \lambda \) cause increase in degree of heteroscedasticity.

### Table 1. Mean and MSE \((N = 50, T = 3)\)

| Estimator   | \( \lambda = 0 \) | \( \sigma_v^2 = 2 \) | Mean | MSE | Mean | MSE | Mean | MSE |
|-------------|-------------------|----------------------|------|-----|------|-----|------|-----|
| OLS (for \( \beta_0 = 5 \)) | 5.00 | 0.43 | 4.99 | 0.43 | 4.99 | 0.44 |
| AGLS (for \( \beta_0 = 5 \)) | 5.00 | 0.27 | 5.00 | 0.35 | 4.99 | 0.42 |
| OLS (for \( \beta_1 = 0.5 \)) | 0.50 | 0.19 | 0.51 | 0.19 | 0.50 | 0.19 |
| AGLS (for \( \beta_1 = 0.5 \)) | 0.50 | 0.07 | 0.50 | 0.13 | 0.50 | 0.17 |
| WG (for \( \beta_1 = 0.5 \)) | 0.50 | 0.07 | 0.50 | 0.14 | 0.50 | 0.21 |
| \( \lambda = 1 \) | | | |
| OLS (for \( \beta_0 = 5 \)) | 5.00 | 0.44 | 5.02 | 0.44 | 4.99 | 0.44 |
| AGLS (for \( \beta_0 = 5 \)) | 5.00 | 0.26 | 5.01 | 0.35 | 4.99 | 0.41 |
| OLS (for \( \beta_1 = 0.5 \)) | 0.50 | 0.19 | 0.51 | 0.19 | 0.50 | 0.19 |
| AGLS (for \( \beta_1 = 0.5 \)) | 0.50 | 0.07 | 0.49 | 0.13 | 0.51 | 0.17 |
| WG (for \( \beta_1 = 0.5 \)) | 0.50 | 0.07 | 0.49 | 0.14 | 0.50 | 0.21 |
| \( \lambda = 2 \) | | | |
| OLS (for \( \beta_0 = 5 \)) | 5.01 | 0.44 | 4.99 | 0.44 | 4.99 | 0.44 |
| AGLS (for \( \beta_0 = 5 \)) | 5.01 | 0.26 | 4.99 | 0.35 | 4.99 | 0.41 |
| OLS (for \( \beta_1 = 0.5 \)) | 0.50 | 0.19 | 0.51 | 0.19 | 0.51 | 0.19 |
| AGLS (for \( \beta_1 = 0.5 \)) | 0.50 | 0.07 | 0.51 | 0.13 | 0.51 | 0.17 |
| WG (for \( \beta_1 = 0.5 \)) | 0.50 | 0.07 | 0.51 | 0.14 | 0.51 | 0.22 |
| \( \lambda = 3 \) | | | |
| OLS (for \( \beta_0 = 5 \)) | 5.01 | 0.44 | 4.99 | 0.44 | 4.99 | 0.44 |
| AGLS (for \( \beta_0 = 5 \)) | 5.01 | 0.26 | 4.99 | 0.35 | 4.99 | 0.41 |
| OLS (for \( \beta_1 = 0.5 \)) | 0.50 | 0.19 | 0.51 | 0.19 | 0.51 | 0.19 |
| AGLS (for \( \beta_1 = 0.5 \)) | 0.50 | 0.07 | 0.50 | 0.13 | 0.50 | 0.17 |
| WG (for \( \beta_1 = 0.5 \)) | 0.50 | 0.07 | 0.51 | 0.14 | 0.51 | 0.21 |
Table 2. Mean and MSE ($N = 100, T = 3$)

| Estimator          | $\lambda = 0$ | $\sigma^2 = 2$ | $\sigma^2 = 4$ | $\sigma^2 = 6$ |
|--------------------|---------------|----------------|----------------|----------------|
|                    | Mean | MSE | Mean | MSE | Mean | MSE | Mean | MSE |
| OLS (for $\beta_0 = 5$) | 5.00 | 0.18 | 5.00 | 0.18 | 5.00 | 0.18 |
| AGLS (for $\beta_0 = 5$) | 5.00 | 0.13 | 5.00 | 0.16 | 5.00 | 0.18 |
| OLS (for $\beta_1 = 0.5$) | 0.50 | 0.10 | 0.50 | 0.10 | 0.50 | 0.10 |
| AGLS (for $\beta_1 = 0.5$) | 0.50 | 0.03 | 0.50 | 0.07 | 0.50 | 0.09 |
| WG (for $\beta_1 = 0.5$) | 0.50 | 0.04 | 0.50 | 0.08 | 0.50 | 0.12 |

$\lambda = 1$

| OLS (for $\beta_0 = 5$) | 5.01 | 0.19 | 5.01 | 0.18 | 5.02 | 0.18 |
| AGLS (for $\beta_0 = 5$) | 5.01 | 0.12 | 5.00 | 0.16 | 5.02 | 0.18 |
| OLS (for $\beta_1 = 0.5$) | 0.49 | 0.10 | 0.50 | 0.10 | 0.49 | 0.10 |
| AGLS (for $\beta_1 = 0.5$) | 0.49 | 0.04 | 0.50 | 0.07 | 0.49 | 0.07 |
| WG (for $\beta_1 = 0.5$) | 0.49 | 0.04 | 0.50 | 0.08 | 0.49 | 0.08 |

$\lambda = 2$

| OLS (for $\beta_0 = 5$) | 4.99 | 0.19 | 5.01 | 0.19 | 5.00 | 0.19 |
| AGLS (for $\beta_0 = 5$) | 5.00 | 0.12 | 5.00 | 0.16 | 5.00 | 0.16 |
| OLS (for $\beta_1 = 0.5$) | 0.50 | 0.10 | 0.50 | 0.10 | 0.50 | 0.10 |
| AGLS (for $\beta_1 = 0.5$) | 0.50 | 0.04 | 0.50 | 0.07 | 0.51 | 0.07 |
| WG (for $\beta_1 = 0.5$) | 0.50 | 0.04 | 0.50 | 0.08 | 0.50 | 0.08 |

$\lambda = 3$

| OLS (for $\beta_0 = 5$) | 5.00 | 0.19 | 5.01 | 0.19 | 5.00 | 0.19 |
| AGLS (for $\beta_0 = 5$) | 5.00 | 0.12 | 5.01 | 0.16 | 5.00 | 0.16 |
| OLS (for $\beta_1 = 0.5$) | 0.50 | 0.10 | 0.50 | 0.10 | 0.50 | 0.10 |
| AGLS (for $\beta_1 = 0.5$) | 0.50 | 0.04 | 0.50 | 0.07 | 0.51 | 0.07 |
| WG (for $\beta_1 = 0.5$) | 0.50 | 0.04 | 0.50 | 0.08 | 0.50 | 0.08 |

Figure 1. Empirical size (%) for $\beta$. 

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Moreover, the value of \( \bar{\omega} \) can be obtained using different values of \( \lambda \) for each value of \( \sigma_v^2 \) and \( \alpha \) is obtained using the additive heteroscedastic design specified above for given \( \lambda \). Thus, the values of \( \bar{\omega} \) for each \( \sigma_v^2 \) under the four different values of \( \lambda \) are obtained. The Gaussian kernel cited in Roy (Roy, 2002) is used and defined as 

\[ k(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}. \]

Roy (2002) used 0.5, 1, and 1.5 as bandwidth. In the present work, we used 0.5 as bandwidth.

The simulations are 5000 with two schemes for fixed small \( T \) but large \( N \):

1. Scheme I: \( N = 50; T = 3; NT = 150 \)
2. Scheme II: \( N = 100; T = 3; NT = 300 \)

For empirical investigation, the concerned estimators are given below:

1. The pooled OLSE
2. The WGE
3. The AGLS estimator (AGLSE)
4. The AHCCME

The numerical evaluation in this section has been divided into four parts; the first part compares the efficiency of the estimators in terms of mean squares error (MSE), the second part presents evaluation of the covariance estimators for their performance in hypothesis testing in terms of null rejection rate (NRR), the third part is reserved for performance of the AHCCMEs, and the fourth part compares performance of the estimators in terms of power of test. Empirical size and coverage is given in percentage form. Empirical size is studied at 1, 5, and 10% nominal level of significance (LOS) and nominal coverage is taken to be 95%. The estimators have been studied under different degrees of heteroscedasticity, namely \( \lambda = 0, 1, 2, \) and \( 3 \) as used by Roy (2002).

Tables 1 and 2 show mean and MSE for Schemes I and II, respectively. Intercept is excluded in the WG estimation, therefore it does not appear in these tables and discussion is concentrated only on the slope estimates. Table 1 shows that all the estimators remain almost unbiased and there is no issue of bias under heteroscedasticity. But the OLSE is inefficient for smaller UTVH (\( \sigma_v^2 = 2 \)) as it yields higher MSEs than the AGLSE. For \( \sigma_v^2 = 2 \), the MSE of OLSE is more than twice of AGLSE and WGE. The WGE performs better than OLSE in terms of MSE but outperformed by AGLSE for \( \sigma_v^2 = 4 \). Due to gain in efficiency, the AGLSE remains an attractive choice. Such results are actually due to Roy (2002). Because of the OLSE improves for larger UTVH (\( \sigma_v^2 = 6 \)). For \( \sigma_v^2 = 6 \) and \( \lambda = 1 \), the MSE of OLSE is identical to AGLSE. The similar behavior of all the estimators is observed in Table 2 as noticed in Table 1. The MSE of OLSE decreases with the increase of sample size but it is still less efficient than the AGLSE and WGE for smaller UTVH (\( \sigma_v^2 = 2 \)).

Empirical sizes are displayed in Figure 1 at 5% LOS and \( \sigma_v^2 = 2 \). The OLSE curve shows high over-rejection. While the curves produced by the AGLSE and WGE are closer to nominal LOS (5%). The curve of AHCO shows deviation from nominal level (5%) under mild and moderate heteroscedasticity but becomes closer to 5% under severe heteroscedasticity for small sample. However, the AHCO gets improvement in performance for large sample. Similar results have been reported by Long and Ervin (2000) for cross-sectional data. The AHCS4 and AHCS5 curves are closer to the nominal LOS (5%).

Tables 3 and 4 display empirical sizes for Schemes I and II, respectively. In Table 3, the test based on the OLS variance estimator is largely liberal under smaller UTVH (\( \sigma_v^2 = 2 \)). It expresses high size distortion for the cases of heteroscedasticity. Under severe heteroscedasticity (\( \lambda = 3 \)), the NRR produced by the OLS variance estimator based quasi-t test is 8.30% at 5% LOS for smaller UTVH (\( \sigma_v^2 = 2 \)). However, the
quasi-t test, based on the OLS variance estimator, gives better NRR for the larger UTVH ($\sigma^2_v = 6$). The quasi-t test, based on the AGLS variance estimator, performs better than the test based on the OLS variance estimator. For instance, for $\lambda = 3$, the NRR yields by AGLS variance estimator based quasi-t test is 5.14% at 5% LOS for smaller UTVH ($\sigma^2_v = 2$). It verifies the reported results of Roy. The quasi-t tests that employ AHCCMEs yield good NRR from smaller UTVH ($\sigma^2_v = 2$) to the larger UTVH ($\sigma^2_v = 6$). The best NRR among the AHCCMEs, is observed by the tests based on AHC4 and AHC5. In case of severe heteroscedasticity ($\lambda = 3$), the AHC4 yields exact NRR for $\sigma^2_v = 6$ at 5% LOS. The results given by the AHC4 and AHC5 confirm the findings made by Cribari-Neto (2004) and Cribari-Neto et al. (2007) for the non-panel data and also justify their formulation for the PDM.

| $\sigma^2_v$ | $\alpha$ | 2 | 4 | 6 |
|--------------|----------|---|---|---|
| $1\%$ | $5\%$ | $10\%$ | $1\%$ | $5\%$ | $10\%$ | $1\%$ | $5\%$ | $10\%$ |
| OLS | 1.04 | 5.40 | 11.48 | 1.26 | 5.40 | 10.28 | 0.86 | 5.08 | 10.26 |
| AGLS | 1.20 | 5.42 | 10.46 | 1.42 | 5.44 | 10.76 | 1.34 | 5.58 | 10.60 |
| WG | 1.32 | 5.24 | 10.34 | 1.26 | 5.22 | 10.06 | 1.24 | 5.48 | 10.96 |
| AHC0 | 1.78 | 6.52 | 11.30 | 1.90 | 6.04 | 12.16 | 1.66 | 6.48 | 11.66 |
| AHC3 | 1.38 | 5.62 | 10.28 | 1.62 | 5.24 | 10.68 | 1.38 | 5.68 | 10.60 |
| AHC4 | 1.26 | 5.10 | 9.62 | 1.46 | 4.92 | 9.88 | 1.22 | 5.24 | 9.96 |
| AHC5 | 0.92 | 4.54 | 8.52 | 1.30 | 4.28 | 8.60 | 1.02 | 4.62 | 9.20 |

Table 3. NRR of quasi-t test for $N = 50$, $T = 3$

The best NRR among the AHCCMEs, is observed by the tests based on AHC4 and AHC5.
In Table 4, behavior of all the estimators is similar as presented in Table 3. The tests, based on the AGLSE variance estimator, perform well in terms of NRR as reported by Roy (Roy, 2002). Performance of the AHC4 and AHC5 remains attractive and justifies our proposal for the PDM.

Table 4. NRR of quasi-t test for $N = 100, T = 3$

| $\sigma^2$ | 2      | 4      | 6      |
|------------|--------|--------|--------|
| $\alpha$   | 1%     | 5%     | 10%    | 1%     | 5%     | 10%    | 1%     | 5%     | 10% |
| $\lambda = 0$ |       |        |        |        |        |        |        |        |      |
| OLS        | 1.78   | 6.34   | 11.88  | 1.50   | 6.88   | 12.36  | 1.06   | 5.94   | 11.30|
| AGLS       | 1.22   | 5.22   | 10.24  | 1.56   | 6.46   | 11.60  | 1.30   | 5.74   | 10.88|
| WG         | 1.08   | 5.20   | 10.38  | 1.36   | 5.76   | 11.68  | 1.26   | 5.50   | 10.74|
| AHC0       | 1.16   | 4.94   | 9.90   | 1.74   | 6.22   | 11.46  | 1.30   | 5.50   | 10.36|
| AHC3       | 1.02   | 4.60   | 9.26   | 1.64   | 5.86   | 11.04  | 1.16   | 5.06   | 9.82 |
| AHC4       | 0.84   | 4.38   | 8.76   | 1.42   | 5.44   | 10.60  | 1.08   | 4.82   | 9.42 |
| AHC5       | 0.76   | 3.88   | 7.96   | 1.16   | 4.80   | 9.52   | 0.78   | 4.44   | 8.74 |

| $\lambda = 1$ |       |        |        |        |        |        |        |        |      |
| OLS        | 2.52   | 9.80   | 16.66  | 2.34   | 8.46   | 14.98  | 1.70   | 7.00   | 13.26|
| AGLS       | 1.04   | 5.10   | 10.62  | 1.36   | 5.68   | 10.98  | 1.16   | 5.96   | 11.42|
| WG         | 0.94   | 5.42   | 10.26  | 1.16   | 5.42   | 10.26  | 0.98   | 4.62   | 10.06|
| AHC0       | 1.02   | 4.78   | 9.70   | 1.18   | 5.42   | 10.60  | 1.14   | 5.86   | 11.20|
| AHC3       | 0.86   | 4.44   | 8.80   | 1.04   | 4.98   | 9.86   | 1.02   | 5.52   | 10.48|
| AHC4       | 0.84   | 4.14   | 8.60   | 1.02   | 4.72   | 9.58   | 0.98   | 5.18   | 9.98 |
| AHC5       | 0.70   | 3.72   | 7.64   | 0.86   | 4.10   | 8.90   | 0.80   | 4.54   | 9.14 |

| $\lambda = 2$ |       |        |        |        |        |        |        |        |      |
| OLS        | 2.80   | 9.66   | 16.62  | 2.26   | 8.22   | 14.28  | 1.58   | 6.40   | 12.20|
| AGLS       | 1.02   | 5.00   | 10.28  | 1.24   | 5.60   | 10.34  | 1.30   | 5.18   | 10.64|
| WG         | 1.14   | 4.92   | 9.62   | 1.08   | 5.34   | 10.02  | 1.08   | 5.36   | 10.04|
| AHC0       | 1.02   | 4.58   | 9.46   | 1.30   | 5.28   | 9.86   | 1.22   | 5.02   | 10.58|
| AHC3       | 0.96   | 4.14   | 8.80   | 1.14   | 4.68   | 9.24   | 1.08   | 4.64   | 9.60 |
| AHC4       | 0.90   | 3.96   | 8.42   | 1.06   | 4.56   | 8.96   | 0.98   | 4.44   | 9.34 |
| AHC5       | 0.72   | 3.52   | 7.54   | 0.98   | 4.04   | 8.28   | 0.90   | 4.16   | 8.50 |

| $\lambda = 3$ |       |        |        |        |        |        |        |        |      |
| OLS        | 3.38   | 10.36  | 17.04  | 2.88   | 8.64   | 15.12  | 1.72   | 7.08   | 12.74|
| AGLS       | 1.02   | 5.52   | 10.40  | 1.26   | 5.00   | 9.72   | 1.24   | 5.30   | 10.92|
| WG         | 0.88   | 5.10   | 10.16  | 1.12   | 4.68   | 9.48   | 0.96   | 4.98   | 10.16|
| AHC0       | 0.90   | 4.68   | 9.92   | 1.12   | 4.76   | 8.90   | 1.18   | 5.30   | 10.60|
| AHC3       | 0.78   | 4.22   | 9.16   | 1.06   | 4.32   | 8.24   | 1.02   | 4.68   | 10.06|
| AHC4       | 0.70   | 3.96   | 8.70   | 1.02   | 4.12   | 7.84   | 1.00   | 4.44   | 9.70 |
| AHC5       | 0.64   | 3.50   | 7.80   | 0.90   | 3.74   | 7.30   | 0.90   | 3.94   | 8.98 |
Estimation of confidence interval is done as illustrated in Equation (13). For $\sigma^2 = 2$, empirical coverage is presented in Figure 2. The OLSE curve exhibits under-coverage, while the curve of AGLSE is closer to the nominal coverage (95%). The curve of AHC0 shows under-coverage for small sample but coverage rate is closer to the nominal coverage (95%) for the large samples. On the other side, the curves of AHC4 and AHC5 are closer to the nominal coverage (95%).

For the above-mentioned estimators, Tables 5 and 6 carry empirical coverage and average length for Schemes I and II, respectively. Performance of the OLSE is not satisfactory in for smaller UTVH as it shows under-coverage. However, the OLSE gets improvement in performance from smaller ($\sigma^2 = 2$) to larger UTVH ($\sigma^2 = 6$). While the AGLSE shows remarkable performance for homoscedastic as well as for all types of heteroscedastic cases. The empirical coverage of the WGE is closer to nominal coverage (95%) for all degrees of heteroscedasticity and it outperforms the OLSE. It is noticed that the best empirical coverage among AHCCMEs are produced by our AHC4 and AHC5. The AHC4 exhibits exact coverage for $\sigma^2 = 2$ in case of mild heteroscedasticity ($\lambda = 1$) and also for larger UTVH ($\sigma^2 = 6$) when $\lambda = 3$. The AHC5-based confidence intervals display coverage that is close to the nominal coverage (95%).

Performance of the estimators in Table 6 is similar to that observed in Table 5. For the large samples, Roy’s estimator outperforms the OLSE. Among the AHCCME, AHC4 and AHC5 express very good coverage and average interval length and they remain attractive choice. 
Figures 3–5 show empirical power curves, built upon all the above mentioned estimators for Scheme I. For \( \sigma^2 = 2 \), Figure 3 gives indication that for homoscedastic (\( \lambda = 0 \)) and heteroscedastic situations (\( \lambda = 1, 2 \) and 3), all the estimators show identical power of test to that of the AGLSE except OLSE. However, as \( \sigma^2 \) increases, the OLSE gets improvement in such a way that for \( \sigma^2 = 6 \), all the estimators become near to identical in power of test. Table 7 gives numerical values of the empirical power for a specific case i.e. \( \sigma^2 = 2, N = 50, T = 3 \). Aslam (2006) presented power curve analysis of the above mentioned estimators in context of the PDM and our results verify his findings.

### Table 5. 95% Confidence interval: coverage (%) and length (\( N = 50, T = 3 \))

| \( \sigma^2 \) | 2 | 4 | 6 |
|----------------|---|---|---|
| \( \lambda = 0 \) |   |   |   |
| OLS            | 94.60 | 1.69 | 94.60 | 1.69 | 94.92 | 1.70 |
| AGLS           | 94.58 | 1.02 | 94.56 | 1.38 | 94.42 | 1.61 |
| WG             | 94.76 | 1.09 | 94.78 | 1.47 | 94.52 | 1.96 |
| AHC0           | 93.48 | 1.00 | 93.96 | 1.36 | 93.52 | 1.59 |
| AHC3           | 94.34 | 1.04 | 94.76 | 1.42 | 94.32 | 1.65 |
| AHC4           | 94.90 | 1.07 | 95.08 | 1.46 | 94.76 | 1.69 |
| AHC5           | 95.46 | 1.12 | 95.72 | 1.53 | 95.38 | 1.76 |
| \( \lambda = 1 \) |   |   |   |
| OLS            | 93.30 | 1.70 | 92.98 | 1.70 | 94.42 | 1.70 |
| AGLS           | 94.48 | 1.02 | 94.22 | 1.39 | 94.24 | 1.61 |
| WG             | 94.58 | 1.10 | 94.58 | 1.48 | 94.56 | 1.96 |
| AHC0           | 93.88 | 1.01 | 93.48 | 1.38 | 93.58 | 1.59 |
| AHC3           | 94.62 | 1.05 | 94.42 | 1.43 | 94.72 | 1.65 |
| AHC4           | 95.00 | 1.08 | 94.74 | 1.47 | 95.18 | 1.70 |
| AHC5           | 95.68 | 1.12 | 95.26 | 1.54 | 95.76 | 1.76 |
| \( \lambda = 2 \) |   |   |   |
| OLS            | 92.54 | 1.70 | 92.68 | 1.70 | 93.80 | 1.70 |
| AGLS           | 94.80 | 1.02 | 94.50 | 1.39 | 94.08 | 1.62 |
| WG             | 94.72 | 1.10 | 94.70 | 1.48 | 94.32 | 1.96 |
| AHC0           | 93.82 | 1.01 | 93.54 | 1.38 | 93.04 | 1.60 |
| AHC3           | 94.52 | 1.05 | 94.18 | 1.44 | 93.86 | 1.66 |
| AHC4           | 94.98 | 1.08 | 94.92 | 1.48 | 94.34 | 1.70 |
| AHC5           | 95.44 | 1.12 | 95.52 | 1.54 | 94.88 | 1.77 |
| \( \lambda = 3 \) |   |   |   |
| OLS            | 91.70 | 1.70 | 92.86 | 1.71 | 93.50 | 1.71 |
| AGLS           | 94.86 | 1.02 | 94.88 | 1.39 | 94.04 | 1.62 |
| WG             | 95.02 | 1.10 | 94.94 | 1.48 | 94.12 | 1.96 |
| AHC0           | 94.54 | 1.01 | 93.74 | 1.38 | 93.52 | 1.61 |
| AHC3           | 95.36 | 1.05 | 94.50 | 1.44 | 94.24 | 1.68 |
| AHC4           | 95.84 | 1.08 | 95.02 | 1.47 | 95.00 | 1.72 |
| AHC5           | 96.16 | 1.13 | 95.86 | 1.54 | 95.58 | 1.78 |
For all the estimators under consideration, empirical power curves for Scheme II are displayed in Figures 6–8. In case of larger sample, it is noticed that power curves of all the estimators get slumber. Performance of the OLSE is not good for smaller UTVH ($\sigma_v^2 = 2$) but it becomes closer to the AGLSE for larger UTVH ($\sigma_v^2 = 6$).

| $\sigma_v^2$ | 2 | 4 | 6 |
|--------------|---|---|---|
|               | Coverage | Length | Coverage | Length | Coverage | Length |
| $\lambda = 0$ | OLS   | 93.66 | 1.24 | 93.12 | 1.24 | 94.06 | 1.24 |
|               | AGLS  | 94.78 | 0.77 | 93.54 | 1.04 | 94.26 | 1.19 |
|               | WG    | 94.80 | 0.83 | 94.24 | 1.12 | 94.50 | 1.37 |
|               | AHC0  | 95.06 | 0.79 | 93.78 | 1.06 | 94.50 | 1.20 |
|               | AHC3  | 95.40 | 0.80 | 94.14 | 1.08 | 94.94 | 1.23 |
|               | AHC4  | 95.62 | 0.82 | 94.56 | 1.09 | 95.18 | 1.24 |
|               | AHC5  | 96.12 | 0.84 | 95.20 | 1.13 | 95.56 | 1.28 |
| $\lambda = 1$| OLS   | 90.20 | 1.25 | 91.54 | 1.25 | 93.00 | 1.25 |
|               | AGLS  | 94.90 | 0.77 | 94.32 | 1.05 | 94.04 | 1.05 |
|               | WG    | 94.58 | 0.83 | 94.58 | 1.12 | 95.38 | 1.12 |
|               | AHC0  | 95.22 | 0.80 | 94.58 | 1.07 | 94.14 | 1.07 |
|               | AHC3  | 95.56 | 0.81 | 95.02 | 1.09 | 94.48 | 1.09 |
|               | AHC4  | 95.86 | 0.83 | 95.28 | 1.11 | 94.82 | 1.11 |
|               | AHC5  | 96.28 | 0.86 | 95.90 | 1.14 | 95.46 | 1.14 |
| $\lambda = 2$| OLS   | 90.34 | 1.25 | 91.78 | 1.25 | 93.60 | 1.25 |
|               | AGLS  | 95.00 | 0.77 | 94.40 | 1.05 | 94.82 | 1.05 |
|               | WG    | 95.08 | 0.84 | 94.66 | 1.12 | 94.64 | 1.12 |
|               | AHC0  | 95.42 | 0.80 | 94.72 | 1.07 | 94.98 | 1.07 |
|               | AHC3  | 95.86 | 0.98 | 95.32 | 1.10 | 95.36 | 1.10 |
|               | AHC4  | 96.04 | 0.83 | 95.44 | 1.11 | 95.56 | 1.11 |
|               | AHC5  | 96.48 | 0.85 | 95.96 | 1.15 | 95.84 | 1.15 |
| $\lambda = 3$| OLS   | 89.64 | 1.25 | 91.36 | 1.25 | 92.92 | 1.25 |
|               | AGLS  | 94.48 | 0.77 | 95.00 | 1.05 | 94.70 | 1.05 |
|               | WG    | 94.90 | 0.84 | 95.32 | 1.12 | 95.02 | 1.12 |
|               | AHC0  | 95.32 | 0.80 | 95.24 | 1.08 | 94.70 | 1.08 |
|               | AHC3  | 95.78 | 0.82 | 95.68 | 1.10 | 95.32 | 1.10 |
|               | AHC4  | 96.04 | 0.83 | 95.88 | 1.11 | 95.56 | 1.11 |
|               | AHC5  | 96.50 | 0.85 | 96.26 | 1.15 | 96.06 | 1.15 |
Figure 3. Empirical power at 5% LOS ($N = 50$, $T = 3$; $\sigma_2 = 2$).
Figure 4. Empirical power at 5% LOS ($N = 50, T = 3; \sigma_v^2 = 4$).
Figure 5. Empirical power at 5% LOS ($N = 50$, $T = 3$; $\sigma^2 = 6$).
6. Illustrative example

We take an example of panel data of productivity of USA (Munnell, 1990) with \( T = 17 \) and \( N = 48 \). The model of interest is

\[
y_{it} = \beta_0 + \beta_1 x_{1it} + \beta_2 x_{2it} + \beta_3 x_{3it} + \beta_4 x_{4it} + \beta_5 x_{5it} + \beta_6 x_{6it} + u_{it}; \quad (i = 1, 2, \ldots, 48, t = 1, 2, \ldots, 17),
\]

where \( y \) denotes gross production, \( x_1 \) is high way capital, \( x_2 \) is water utility capital, \( x_3 \) is utility capital, \( x_4 \) is private capital, \( x_5 \) is employed capital, and \( x_6 \) is unemployed capital.

### Table 7. Power results (%) for \( \sigma^2 = 2, N = 50, T = 3 \)

| OLS | AGLS | AHC0 | AHC3 | AHC4 | AHC5 |
|-----|------|------|------|------|------|
| \( \lambda = 0 \) |      |      |      |      |      |
| 5.40 | 5.42 | 6.52 | 5.66 | 5.10 | 4.54 |
| 12.14 | 21.76 | 22.92 | 20.86 | 19.46 | 17.34 |
| 30.16 | 63.66 | 63.72 | 61.16 | 58.98 | 55.26 |
| 55.20 | 92.88 | 93.06 | 91.76 | 90.74 | 88.72 |
| 78.78 | 99.60 | 99.56 | 99.38 | 99.26 | 98.86 |
| 92.76 | 99.99 | 99.99 | 99.99 | 99.98 | 99.98 |
| \( \lambda = 1 \) |      |      |      |      |      |
| 6.70 | 5.42 | 5.52 | 6.12 | 5.38 | 4.32 |
| 13.00 | 20.20 | 20.88 | 22.10 | 20.02 | 17.28 |
| 30.54 | 60.98 | 63.02 | 63.64 | 60.34 | 54.84 |
| 53.84 | 91.98 | 92.80 | 92.80 | 91.68 | 88.42 |
| 76.44 | 99.28 | 99.60 | 99.50 | 99.34 | 98.64 |
| 90.98 | 99.99 | 99.98 | 99.98 | 99.98 | 99.98 |
| \( \lambda = 2 \) |      |      |      |      |      |
| 7.46 | 5.28 | 5.20 | 6.18 | 5.48 | 4.56 |
| 13.96 | 20.80 | 21.64 | 22.74 | 20.48 | 17.52 |
| 31.44 | 62.42 | 64.00 | 64.50 | 61.72 | 56.00 |
| 54.52 | 91.98 | 93.24 | 93.22 | 91.76 | 88.48 |
| 76.48 | 99.32 | 99.52 | 99.56 | 99.46 | 99.00 |
| 91.36 | 99.99 | 99.99 | 99.99 | 99.98 | 99.92 |
| \( \lambda = 3 \) |      |      |      |      |      |
| 8.30 | 4.98 | 5.14 | 5.46 | 4.16 | 3.84 |
| 14.12 | 20.62 | 21.84 | 23.32 | 20.26 | 18.52 |
| 31.60 | 61.70 | 63.60 | 63.96 | 59.12 | 56.12 |
| 55.50 | 92.04 | 93.20 | 93.16 | 90.56 | 88.42 |
| 75.70 | 99.52 | 99.60 | 99.44 | 99.22 | 98.72 |
| 90.30 | 99.99 | 99.99 | 99.99 | 99.99 | 99.96 |
In order to evaluate the testing performance of all the stated estimators, following Cribari-Neto (2004), an extra variable (e.g. $X^2$) is being added as an explanatory variable. Thus, Model (14) is reformulated as

Figure 6. Empirical power at 5%
LOS ($N = 100, T = 3; \sigma^2 = 2$).
The USH is found after the Wald test (with $p$-value < 0.01). Table 8 displays the comparative statistics obtained from Model (14). The results obtained from fitting of Model (15) are presented in Table 8. All the regression coefficients are found to be statistically significant while referring to Table 7.

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + \beta_5 x_{5t} + \beta_6 x_{6t}^2 + u_t.$$  

(15)
In Model (15), we include square of employed capital as an extra explanatory variable with the expectedly no impact on determining the gross production. Thus, it should be statistically non-significant. We perform the inference again. In this situation, the attractive estimator would be one that
does not reject the null hypothesis of $\beta_7 = 0$. Table 9 shows that the tests based on only AHC4 and AHC5 do not reject the hypothesis of $\beta_7 = 0$ at 1% LOS.

### Table 8. Comparative statistic of model (14)

|       | OLS | AGLS | WG  | AHC0 | AHC3 | AHC4 | AHC5 |
|-------|-----|------|-----|------|------|------|------|
| $\hat{\beta}_0$ | 1.926 | 2.148 | 2.192 | -    | -    | -    | -    |
| $\hat{\beta}_1$ | 0.058 | 0.064 | 0.077 | -    | -    | -    | -    |
| $\hat{\beta}_2$ | 0.199 | 0.077 | 0.079 | -    | -    | -    | -    |
| $\hat{\beta}_3$ | 0.009 | -0.096 | -0.115 | -    | -    | -    | -    |
| $\hat{\beta}_4$ | 0.312 | 0.273 | 0.235 | -    | -    | -    | -    |
| $\hat{\beta}_5$ | 0.550 | 0.746 | 0.801 | -    | -    | -    | -    |
| $\hat{\beta}_6$ | -0.007 | -0.006 | -0.005 | -    | -    | -    | -    |
| $se(\hat{\beta}_0)$ | 0.053 | 0.138 | 0.216 | 1.005 | 1.024 | 1.034 | 1.038 |
| $se(\hat{\beta}_1)$ | 0.015 | 0.022 | 0.031 | 0.113 | 0.115 | 0.116 | 0.116 |
| $se(\hat{\beta}_2)$ | 0.012 | 0.014 | 0.015 | 0.038 | 0.039 | 0.039 | 0.039 |
| $se(\hat{\beta}_3)$ | 0.012 | 0.017 | 0.018 | 0.054 | 0.056 | 0.056 | 0.056 |
| $se(\hat{\beta}_4)$ | 0.011 | 0.020 | 0.026 | 0.146 | 0.151 | 0.155 | 0.157 |
| $se(\hat{\beta}_5)$ | 0.016 | 0.025 | 0.030 | 0.162 | 0.167 | 0.172 | 0.174 |
| $se(\hat{\beta}_6)$ | 0.014 | 0.001 | 0.001 | 0.003 | 0.003 | 0.004 | 0.004 |
| $t(\hat{\beta}_0)$ | 36.684 | 15.576 | 10.132 | 2.138 | 2.099 | 2.076 | 2.070 |
| $t(\hat{\beta}_1)$ | 3.821 | 2.968 | 2.457 | 0.565 | 0.556 | 0.552 | 0.552 |
| $t(\hat{\beta}_2)$ | 9.597 | 5.556 | 5.245 | 2.001 | 1.981 | 1.969 | 1.967 |
| $t(\hat{\beta}_3)$ | 0.692 | -5.712 | -6.325 | -1.771 | -1.739 | -1.725 | -1.723 |
| $t(\hat{\beta}_4)$ | 28.142 | 13.786 | 8.964 | 1.857 | 1.805 | 1.756 | 1.739 |
| $t(\hat{\beta}_5)$ | 35.380 | 29.948 | 26.924 | 4.597 | 4.460 | 4.346 | 4.291 |
| $t(\hat{\beta}_6)$ | -5.255 | -6.766 | -5.288 | -1.753 | -1.709 | -1.674 | -1.659 |
| $p(\hat{\beta}_0)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $p(\hat{\beta}_1)$ | 0.000 | 0.000 | 0.000 | 0.007 | 0.008 | 0.008 | 0.008 |
| $p(\hat{\beta}_2)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $p(\hat{\beta}_3)$ | 0.003 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $p(\hat{\beta}_4)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $p(\hat{\beta}_5)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $p(\hat{\beta}_6)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
7. Conclusion
To improve the testing of coefficients of the PDM with the problem of USH, we have used the HCCMEs, based on Roy’s (2002) adaptive estimator. It is found that the AHC4 and AHC5 perform better than all the competing estimators in terms of NRR, power of tests and empirical coverage of interval estimators. On the basis of our findings, the adaptive versions of HCCME are found to be as attractive choice for the testing of PDM as they are for the linear regression models with heteroscedastic errors.

Table 9. Comparative statistic of model (15)

|       | OLS  | AGLS | WG   | AHC0 | AHC3 | AHC4 | AHC5 |
|-------|------|------|------|------|------|------|------|
| \( \hat{\beta}_0 \) | 3.368 | 2.604 | 2.489 | -    | -    | -    | -    |
| \( \hat{\beta}_1 \) | 0.003 | 0.052 | 0.071 | -    | -    | -    | -    |
| \( \hat{\beta}_2 \) | 0.128 | 0.076 | 0.079 | -    | -    | -    | -    |
| \( \hat{\beta}_3 \) | 0.010 | -0.089 | -0.112 | -    | -    | -    | -    |
| \( \hat{\beta}_4 \) | 0.323 | 0.270 | 0.228 | -    | -    | -    | -    |
| \( \hat{\beta}_5 \) | 0.190 | 0.631 | 0.736 | -    | -    | -    | -    |
| \( \hat{\beta}_6 \) | -0.007 | -0.006 | -0.005 | -    | -    | -    | -    |
| \( \hat{\beta}_7 \) | 0.027 | 0.009 | 0.006 | -    | -    | -    | -    |
| se(\( \hat{\beta}_0 \)) | 0.146 | 0.260 | 0.329 | 1.465 | 1.497 | 1.518 | 1.531 |
| se(\( \hat{\beta}_1 \)) | 0.015 | 0.021 | 0.032 | 0.111 | 0.113 | 0.114 | 0.114 |
| se(\( \hat{\beta}_2 \)) | 0.012 | 0.014 | 0.015 | 0.037 | 0.037 | 0.037 | 0.037 |
| se(\( \hat{\beta}_3 \)) | 0.012 | 0.017 | 0.018 | 0.054 | 0.055 | 0.053 | 0.055 |
| se(\( \hat{\beta}_4 \)) | 0.010 | 0.019 | 0.027 | 0.137 | 0.142 | 0.146 | 0.152 |
| se(\( \hat{\beta}_5 \)) | 0.037 | 0.057 | 0.064 | 0.231 | 0.242 | 0.253 | 0.276 |
| se(\( \hat{\beta}_6 \)) | 0.001 | 0.001 | 0.001 | 0.003 | 0.003 | 0.003 | 0.004 |
| se(\( \hat{\beta}_7 \)) | 0.003 | 0.004 | 0.005 | 0.016 | 0.016 | 0.017 | 0.017 |

\[
\begin{align*}
\text{t}(\hat{\beta}_0) & = 23.132, \\
\text{t}(\hat{\beta}_1) & = 0.193, \\
\text{t}(\hat{\beta}_2) & = 10.982, \\
\text{t}(\hat{\beta}_3) & = 0.823, \\
\text{t}(\hat{\beta}_4) & = 30.955, \\
\text{t}(\hat{\beta}_5) & = 5.130, \\
\text{t}(\hat{\beta}_6) & = -5.492, \\
\text{t}(\hat{\beta}_7) & = 10.524
\end{align*}
\]

\[
\begin{align*}
\text{p}(\hat{\beta}_0) & = 0.000, \\
\text{p}(\hat{\beta}_1) & = 0.104, \\
\text{p}(\hat{\beta}_2) & = 0.000, \\
\text{p}(\hat{\beta}_3) & = 0.000, \\
\text{p}(\hat{\beta}_4) & = 0.001, \\
\text{p}(\hat{\beta}_5) & = 0.000, \\
\text{p}(\hat{\beta}_6) & = 0.000, \\
\text{p}(\hat{\beta}_7) & = 0.000
\end{align*}
\]
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