Rotating String

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Abstract

The qualitative results for string rotation in the frame of Relativistic Flux Tube Model and the quantitative and qualitative results for decay of massive open string states of string theory are used as the basic principles in the Monte Carlo implementation for decay of massive open string. The presented model can be used as an ingredient into any Monte Carlo model of multiparticle production for different types of colliding particles. In the interval of total c.m. energy from 3 GeV ($e^+e^-$ annihilation) to 1800 GeV (proton-antiproton interaction) the presented "soft" (i.e. without $qq, gg, qg$ scattering) model has agreement with experimental data on transverse momentum distribution of secondary particles up to 4 GeV. According to widely known theoretical hypothesis, the secondary particles from interval $p_T > 1$ GeV are results of hard scattering QCD states. Therefore (maybe) one can recover a question about relation between the classical string solutions of string theory and solutions of QCD equations.
1 Introduction

If we remove the quarks from QCD, there would remain a nontrivial $SU(3)$ Yang-Mills theory which must have its own spectrum of states. These states will become the "glueball" states of QCD (See Ref.[1]). The simple quark model can be extended to incorporate such gluonic degree of freedom. In the bag [2] model, such approach is to proceed by analogy with the "constituent quark" to posit the existence of a "constituent gluon" with the quantum numbers $(J^{PC} = 1^{-+})$ of a gluon of weak-coupling perturbation theory [3]. In the non-relativistic Flux Tube model [1] by Isgur and Paton there is the long-range strong-coupling limit in which the gluonic degree of freedom have condensed into collective string like flux tubes. A flux tube is a directed element (or "string"), and quark(antiquark) acts as a unit source of the three-dimentional flux tube. Therefore a "junction" can annihilate (be created) there, i.e. there are flux tube breaking and fusion. Such theoretical picture leads (see [1]) to an understanding of the linear confinement (quark-antiquark) potential, to linearly rising Regge trajectories [1]. Hence, in the potential flux tube model, a simple QCD-motivated potential [1], [3] $V(r) = -\frac{k}{r} + a \cdot r$ (here $k \simeq 0.5$ and $a \simeq 0.2 GeV^2$) is used for calculation of interaction along to string (flux tube) lenght.

In the frame of classical analysis [3] Olson, Olsson and Williams have demonstrated that for large angular momenta the leading relativistic QCD corrections can be interpreted as the angular momentum and angular energy of a rotating flux tube. Therefore, the simplest interpretation of these relativistic corrections is to consider the momentum as well as the energy of the interacting field. So, the relativistic flux tube model [4], [7] arises as the successor to the potential model.

In the Relativistic Flux Tube (RFT) model [4] (see Fig.1) the energy of motion of the $i$-th quark (fermionic) is $H_i = \alpha \cdot p^\mu_{qi} + \beta \cdot m_i$. The tube is inserted by the four momentum substitution [8] $p^\mu_q \rightarrow p^\mu - p^\mu_t$, where $p^\mu$ is the new canonical four momentum and $p^\mu_t$ is the tube four momentum computed by integration along the tube [3]

$$(p_{ti})_r = 0 ,$$

$$(p_{ti})_\theta = L_{ti} = \frac{r_i}{2 \cdot \vartheta_{\perp i}} (H_{ti} - a \cdot r_i (1 - \vartheta_{\perp i}^2)^{0.5}) , \quad (1)$$

$$H_{ti} = a \cdot r_i (\vartheta_{\perp i} \cdot r_i \sin \vartheta_{\perp i})^{-1} ,$$

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where $\vartheta_{\perp i}$ is transverse velocity of $i$-th quark. As one can see, the tube substitution is analogous to the introduction of the four potential in QED. For small quark masses $m_i \ll ar_i$, it can be shown [8] that $\vartheta_{\perp i} \to 1$ and the quarks are dynamically unimportant for large angular momenta. In this limit we recover (see [6]) the Nambu-Goto string with Regge slope $\frac{L}{M^2} = \frac{1}{2\pi a}$. 

The basic assumption of the RFT model (see [9]) is that the QCD dynamical ground state for large quark separation consists of a rigid straight tubelike color flux configuration connecting the quarks. The similar physical picture is for classical string solutions in Ref. [10] where Mitchell, Sundborg and Turok had considered the decay of massive open string states on the leading Regge trajectory. These correspond to classical string solutions in the form of rigidly rotating rods whose ends move at the speed of light (see above limit $\vartheta_{\perp i} \to 1$). For these states the length $L$ of the string is proportional to the mass ($L \propto M$). In [10] Mitchell, Sundborg and Turok had found that the decay rate for a string of a "classical" length $L$ is proportional to $L^{(d-14)/12} \propto M^{(d-14)/12}$, where $d$ is a space dimension. For example, for our case $d=4$ and the decay rate $\Gamma \propto L^{-0.83} \propto M^{-0.83}$.

Hence, rotation of string plays the fundamental role in both the flux tube model (which has correct theoretical basis for low and middle mass calculations) and the string model which has theoretical description for decay up to high mass of string (see [10]). The question about rotation of string has deep physical connection with a question about relation between radial and orbital excitations, i.e. relation between momenta $P_r$ and $P_\theta$ in Fig.1.

As it had described in Refs. [11], [12] for the case of large orbital excitations $L \gg n_r$ (or $L_z \gg n_r$, where $n_r$ is the radial quantum number) the system behaves as the transverse rotating string (i.e. for the quasiclassical long distance approximation it is $P_\theta > P_r$ in Fig.1). It is an analogy with so called dominantly orbital excited states [14]. In the opposite case $L \ll n_r$ (i.e. $P_\theta < P_r$ in Fig.1) the string is nearly pure inert and almost doesn’t contribute into the kinetic part of the Hamiltonian [11].

For example, in the Lund fragmentation scheme [3], for the force field between quark and antiquark, an approximation of quasiclassical relativistic massless string is realized[4]. Hence this string does not lie on the Regge trajectory, has not orbital momentum and rotation (i.e. the momentum $P_\theta$ in Fig.1 is neglected). As it is emphasized in [13] the surface spanned by the

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1 The string model becomes semiclassical when we account for its breaking [16].
massless relativistic string is always a minimal surface. This means that (see \[13\]), in the Lund fragmentation scheme, all interior properties of massless relativistic string is determined by the boundary. Therefore in this approximation the string is nearly pure inert and only (anti)quark as the excited end-point of string has energy-momentum. Therefore, the momentum \(P_\theta\) is neglected and the process is one-dimensional. For high radial excitation of end-point, this approximation is correct to a high degree of accuracy.

At the same time, after Lund string fragmentational procedure, the final state consists of a set of string pieces [13], which have the energies, momenta and masses, i.e. ones lie on the linear Regge trajectories and have rotation at rest. Hence, in Fig.1, the value \(P_\theta\) can’t be neglected. Therefore in the Lund fragmentation model there are the classical massive (rotating) string solutions only for small string masses (meson and baryon hadron states).

The Lund approximation is starting point for our model. According to results of Ref. [10] in our string fragmentation scheme we suppose existence of the classical massive (rotating) string solutions not only for small mass string states, but for the mean and large mass string states too.

In the Relativistic Flux Tube model [1,9] and for classical string solutions of string theory [10], for large quark separation a rigid straight tube like color flux configuration connects the quarks. At the same time, in the soft (i.e. without qq, gg, qg scattering) proton-(anti)proton interaction, and in the proton-electron interaction, and in electron-positron annihilation for high c.m. energy, the above both quark and antiquark have long distance strong interaction, ultrarelativistic opposite momenta (in their center of mass system) and, due to orbital momentum conservation, they move with non-zero impact parameter. Therefore for the short time period between formation and decay of string, there are physical conditions (for formation and decay) which are the same as in the flux tube model or in the classical string solutions of string theory [10]. Hence, at high energy interaction, the cascade decay of massive rotating open string can be serious realistic candidate for description of multiparticle production by the use of cascade multistring production.

The primary string (arising in electron-positron annihilation, proton-electron or proton-(anti)proton interaction) has radial direction which is directed along

\[\text{In the Relativistic Flux Tube model the process is three-dimensional, because the quark acts as a unit source of the three-dimentional flux tube (string) which has the mass, momentum-energy and angular momentum and lies on the Regge trajectory.}\]
To summary momentum vector of both the quark and antiquark which act as units sources of the three-dimensional primary string (flux tube). Therefore there are correspondingly two degree of freedom for excitation of meson (baryon) system \([17]\) in its rest frame: the values \(P_\theta\) and \(P_r\) (in Fig.1). In Refs. \([17],[18]\) it is shown that for conventional meson states, where the flux tube is in its ground state, the transverse confinement leads to Gauss law distribution for transverse degree of freedom and the string transverse size is about a few hundred MeV (i.e. \(\sim 0.3\) GeV). We use this theoretical result in our model, where the longitudinal momentum \(P_r\) grows with the mass of string. Therefore in our model there is the average summary physical picture:

- the string is breaking along to direction which is determined by angle \(\theta\);
- the angle \(\theta\) is determined by relation between the momenta \(P_\theta\) and \(P_r\) from the previous decays; i.e. rotation of daughter string has direct connection with rotation of mother string
- the momentum \(P_\theta\) has ”restricted” value (which is determined by Gauss law), whereas \(P_r\) grows with the string mass. Therefore the string of very large mass is breaking longitudinally (i.e. the angle \(\theta\) is about zero), and the string of small mass is breaking approximately isotropically;
- the average value of angle \(\theta\) grows with number of decays of string. This is natural result, which corresponds to classical (rotating) string solutions \([10]\) and to the Relativistic Flux Tube model \([4]\) where the string rotation is one in a few basic principles.

As it is noted above, in Ref. \([10]\), Mitchell, Sundborg and Turok considered the decay of massive open string states and found that the decay rate for a string of ”classical” length \(L\) (and of mass \(M\)) is proportional to \(L^{-0.83}\), i.e. \(\Gamma \propto M^{-0.83}\) for the space-time dimension \(d=4\). The branching ratios to differ mass level states illustrate this behaviour graphically in Fig.1 (in Ref.\([10]\)), where the dominant decay modes are into one small string and one large string. The same physical picture can be arise in our model, if we for simplicity suppose\(^3\) that the partial decay rate \(\Gamma_i\) in the differential kinematical interval of daughter strings is proportional to the kinematical characteristics

\(^3\)Of course, it is rough additional supposition which (in future MC implementation) should be changed over correct theoretical result.
of daughter strings from this interval, i.e. \( \Gamma_i \propto (\varepsilon_i + p_{zi})^{-0.83} \), where \( \varepsilon_i \) and \( p_{zi} \) are energy and \( z \)-projection (i.e. along to \( P_r \) momentum in Fig.1) of momentum of \( i \)-th daughter string in rest frame of decaying string. Therefore the (density of) probability for decay of massive open string with mass \( M \) in to daughter strings with above kinematical characteristics from differential interval is \( w_i = \frac{\Gamma_i}{\Gamma} \propto \frac{(\varepsilon_i + p_{zi})^{-0.83}}{M^{-0.83}} = (\frac{\varepsilon_i + p_{zi}}{M})^{-0.83} = z_i^{-0.83} \), where convenient variable \( z_i = \frac{\varepsilon_i + p_{zi}}{M} \) is used. Both the \( z_1 \)- and \( z_2 \)-distribution functions and Gauss law for projections of transverse momentum (see above) of daughter string in rest frame of decaying string allow to us to determine the masses, energies and momentum projections of two daughter strings in the rest frame of decaying string (see Fig.1)[4].

The above theoretical principles and results we use in the algorithm (see Sect.2) which is an attempt to construct the Monte Carlo implementation for breaking of massive open string according to quantitative results for decay of classical string solutions of string theory and to the results of RFT model also.

The results of Monte Carlo calculations is discussed in Sects. 3 and 4.

2 Decay of string

Monte Carlo (MC) algorithm of decay of string is described in this Section. The diagram of cascade breaking of string is shown in Fig.2. The double line marks the diquark. The cascade breaking of string is described by the iteration of an elementary process presented in Fig.3: the string is breaking into two non-interacting colourless daughter strings. Three schemes of decay of the string into two strings (Fig.3) are taking into consideration in our model. According to the Flux Tube model (see Fig.1) and in the frame of cascade logic we generate the break of each string in its rest frame. By every break, for example of secondary string of mass \( M \) (see Fig.2), we know following its characteristics (in the \( K_0 \) c.m.s)[5]: the scheme of the decay of secondary

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4According to Relativistic Flux Tube model, and classical string solutions of string theory[11], for large quark separation a rigid straight tube like color flux configuration connects the quarks. Therefore it is necessary to stress, at Monte Carlo simulation of decay of above rigid straight tube (string), the \( z_1 \) and \( z_2 \) can’t be generated independently.

5It can be, for example, c.m.s. of \( pp \)-collisions, or \( e^+e^- \) annihilation c.m. system with \( z \) axis along to jet axis.
string, the flavours of the partons participating in this decay, the mass $M$, the energy $E_0$ and the momentum vector $\vec{P}_0$ of the decaying string.

We generate the break of each string in its rest frame. For transition from $K_0$ to string rest frame, let us define new $K_L$ frame, where longitudinal momentum of decaying string is equal to zero. This $K_L$ frame moves with $\beta_{z0} = P_{z0}/E_0$ velocity along to $z_0$ axis in the $K_0$ frame and with Lorentz-factor $\gamma_0 = E_0/M_\perp = E_0/(E_0^2 - P_{z0}^2)^{0.5}$, where $M_\perp = (M^2 + P_{\perp 0}^2)^{0.5}$ is transverse mass of $i$-th string.

In the $K_L$ frame, the decaying $i-th$ string has transverse momentum $\vec{P}_{\perp 0}$ with projections $P_{x0}$ and $P_{y0}$. We turn the $K_L$ frame (in the transverse $xy$ plane) to obtain the $x_L$ axis along to the vector $\vec{P}_{\perp 0}$. The parameters of this transformation are $\cos \varphi_\perp = P_{x0}/P_{\perp 0}$, $\sin \varphi_\perp = P_{y0}/P_{\perp 0}$. Let us define this new system as $K_\perp$ one, and let the $K_L$ frame move with $\vec{\beta}_{\perp 0} = \vec{P}_{\perp 0}/M_\perp$ velocity and with Lorentz-factor $\gamma_{\perp 0} = E_{\perp 0}/M = M_\perp/M$, where $E_{\perp 0}$ is energy of decaying string in $K_L$ frame. Thereby we obtain the $K'$ rest frame of the string.

The longitudinal $z'$ axis of $K'$ frame is parallel to the longitudinal $z_0$ axis of $K_0$ frame. But according to the Flux Tube model (see Fig.1) the breaking string has rotation, i.e. there is nonzero angle $\theta$ between the flux tube and the $z'$ axis in our $K'$ rest frame (it is C.M. frame in Fig.1). At present we can’t calculate $\theta$ angle (according to theory and simultaneously) in frame of our MC model (it is interesting problem which should be overcome for future MC implementation), and so for continuity of rotation (of rigid straight tube) from the mother to daughter string we determine the polar angle $\theta$ by the formula $\sin \theta = P_{\perp 0}/P_{1(2)}$, where $P_{1(2)}$ is the modulus of the momentum vector of first(second) daughter string in the $K'$ rest frame, and the azimuthal angle $\varphi$ is generated uniformly in the interval $(0, 2\pi)$. The above formula takes into account the $\theta$ fluctuations at fixed $P_{\perp 0}$ and the average $\theta$ angle grows with $P_{\perp 0}$ of string. In the string rest frame $K'$ the angles $\theta, \varphi$ determine the line of string stretch. Along this line the string break is generated, and so we have to determine the new $K$ rest system which is given by the $\theta, \varphi$ angles in $K'$.

$^6$The $P_{1(2)}$ value is given by eqs.(3),(20), and so if $P_{\perp 0} > P_{1(2)}$, we can use an approximation $\sin \theta = 1$.

$^7$As it is emphasized in Sect.1, in the string breaking models [19], [20] the angle $\theta$ (in Fig.1) equals to zero.

$^8$In future MC implementation it is necessary to take into account that the azimuthal isotropy can be broken by angular momentum conservation.
system and with the $y$ axis along to the vector product $\vec{z} \times \vec{z}'$.

A simple model for string breaking involves quark pair creation by the strong chromoelectric field inside the string (see [16]). The flavours of quarks, produced from vacuum at the decay of string, are generated according to the relation [19]

$$u : d : s = 3 : 3 : 1 .$$  \hfill (2)

In the RFT model [17],[18] the transverse confinement leads to Gauss law distribution for transverse degree of freedom and the string transverse size is about a few hundred MeV. We use this theoretical result in our model, where in the $K$ frame $P_{x1}$ and $P_{y1}$ components of the transverse momentum $P_\perp$ of first daughter string are generated according to Gauss law (see Sect.1)

$$r_1(r_2) = \sigma^{-1}(2\pi)^{-0.5} \int_{-\infty}^{P_{x1}(P_{y1})} \exp(-x^2/2\sigma^2) \ dx ,$$  \hfill (3)

where $r_i$ is uniformly distributed random number in the range (0,1) and $\sigma$ is parameter, which is $\sigma_{qq} = 0.35$ GeV for the string decays in Figs.3b and 3c. For the string decay in Figs.3a the model parameter $\sigma$ is $\sigma_{u(d)}=0.25$ GeV (if valence quarks are u(d)-quarks), and the model parameter $\sigma$ is $\sigma_s=0.30$ GeV (if only one valence quark is s-quark).

In the $K$ frame we determine (see Sect.1) the following variables:

$$z_1^+ = (E_1 + P_{z1})/M, \quad z_2^+ = (E_2 + P_{z2})/M, \quad z_1^- = (E_1 - P_{z1})/M,$$
$$z_2^- = (E_2 - P_{z2})/M,$$  \hfill (4)

where $E_1$ and $E_2$ ($P_{z1}$ and $P_{z2}$) are energies (the momentum projections (on $z$ axis)) of first and second daughter strings. Generally speaking, the $z$-distribution can be singular one. In this case the $z$-variable should be generated from interval ($z_{min}, z_{max}$). The possible choice for $z_{min}$ and $z_{max}$ is the following. According to law of the conservation of energy, at the production of first daughter string we generate the variable $z_1^+$ in the interval

$$z_{1min}^+ < z_1^+ < z_{1max}^+ ,$$  \hfill (5)

where

$$z_{1min}^+ = \frac{(E_1 - |P_{z1}^{max}|)}{M} ,$$  \hfill (6)

\footnote{There fore third nonphysical Eulerian angle $\psi$ is absent in our calculations.}
\[ z_{1 \text{max}}^+ = \left( E_1 + |P_{z1}^\text{max}| \right) / M , \]

where
\[ E_1 = (M^2 + (M_1^{\text{min}})^2 - (M_2^{\text{min}})^2) / (2M) , \]
\[ |P_{z1}^\text{max}| = ((E_1)^2 - (M_1^{\text{min}})^2)^{0.5} , \]

where \( M_1^{\text{min}} \) and \( M_2^{\text{min}} \) are the minimal masses of first and second daughter strings (see Fig.3). Any daughter string is converted into a hadron if it cannot be broken into two lightest hadrons, i.e. the minimal mass of first (second) daughter string equals to the sum of the masses of two lightest hadrons, which have the flavours depending on the quark contents of first (second) daughter string. Therefore (for simplification) the minimal mass of first (second) daughter string can be determined as the double mass of lightest hadron with quark composition of first (second) daughter string (see Appendix 1).

For production of first daughter string, the distribution density of \( z_1^+ \) variable (for all schemes of the string breaking in Fig.3) is parameterized by the form (see Sect.1)
\[ f(z) = z^{-0.83} . \]

The same scaling\(^\text{10}\) function (10) can be used for production of second daughter string if we generate the variable \( z_2^- \) (see eq.(3)). In the \( K \) frame the equalities (4) lead to ones:
\[ z_1^+ + z_2^+ = 1 , \]
\[ z_1^+ . z_1^- = ((M_1)^2 + (P_\perp)^2) / M^2 , \]
\[ z_2^+ . z_2^- = ((M_2)^2 + (P_\perp)^2) / M^2 , \]

where \( M_1 \) and \( M_2 \) are the masses of first and second daughter strings (see Fig.2). Therefore (after generation of the variable \( z_1^+ \)) the variable \( z_2^- \) is generated in the \( K \) frame (according to the scaling function (10)) in the interval
\[ z_{2\text{min}}^- < z_2^- < z_{2\text{max}}^- , \]

where
\[ z_{2\text{min}}^- = \left( (M_2^{\text{min}})^2 + (P_\perp)^2 \right) / \left( M^2(1 - z_1^+) \right) , \]

\(^\text{10}\)According to theory of decay of massive open string [10], in our Monte Carlo implementation the distribution of \( z \) does not depend on the mass of mother (decaying) string, i.e. this distribution is scaling one.
\[ z_{2\text{max}} = 1 - \frac{((M_{1 \text{min}})^2 + (P_{\perp})^2)}{(M^2 z^+_1)} \]  

are determined according to law of the conservation of energy. In the \( K \) frame from the conservation of energy and momentum after generation of \( z_1^+ \) and \( z_2^- \) we can find (see (4)) the variables

\[ z_1^- = 1 - z_2^- , \]  

\[ z_2^+ = 1 - z_1^+ . \]  

The energies and longitudinal momenta of daughter strings are given by formulas (see Appendix 2)

\[ E_i = M(z_i^+ + z_i^-)/2 , \]  

\[ P_{z_i} = M(z_i^+ - z_i^-)/2 , \quad i = 1, 2. \]  

Because of an equality \( z_1^+ = (1 - z_2^+) \) and from eq.(10) one can see that the variable \( z^+ \) has \( z^{-0.83} \) distribution for first daughter string and (according to energy conservation) \( (1 - z)^{-0.83} \) distribution for second daughter string. Therefore (in the frame of presented model) for the fitted values of the free parameters, for small mass of mother string and for the random numeration of daughter strings, at the decay of mother string into two daughter strings the distributions of first and second daughter strings on the part of energy (and momentum) of mother string are similar to momentum distributions for valence quark and antiquark in meson\(^{11} \), i.e. similar to \( f(z) = z^{-0.5}(1 - z)^{-0.5} \) (see Ref. [15]).

In the models [19], [20] there is no such result, because for the force field between quark and antiquark, an approximation of quasiclassical relativistic massless string is realized (see [13] and Sect.1), therefore the scaling functions are used for the other process (see Fig.4a and Sect.1). For example, in Ref.[21] the scaling function for conversion of the quark into meson+quark is equal to \( f(z) = 1-a+3a(1-z)^2 \), with \( a=0.88 \), in Ref.[19] \( f(z) = (1+c)(1-z)^c \), with \( c \approx 0.3 \div 0.5 \).

After determination of the momenta of daughter strings, we control the inequalities

\[ E_i > P_i , \quad i = 1, 2 \]  

\(^{11}\) For \( z^- \) variable there is the same result because the \( z_1^+ \) and \( z_2^- \) variables have the same parametrization (10).
If inequalities (21) are fulfilled for the i-th string, we determine the string mass 
\[ M_i = (E_i^2 - (P_i)^2)^{0.5} \]
and the decay scheme of i-th string and flavours of quarks, produced in this decay from vacuum, are generated. Then we control the inequality 
\[ M_i > M_{i1}^{\text{min}} + M_{i2}^{\text{min}}, \] (22)
where the \( M_{i j}^{\text{min}} \) is minimum mass of the j-th hadron with the fixed quark composition, which can be produced by decay of i-th string. The decay scheme (see Fig.3) is generated according to the relation [19]
\[ a : b = 0.95 : 0.05. \] (23)
A model has six parameters ( see eqs. (2),(3),(10),(23) ), which have correct theoretical validity.

If the inequalities (21),(22) are fulfilled for both daughter strings, their decays are simulated. The algorithm for decay of daughter string of the mass \( M_1(M_2) \) is similar to algorithm for decay of secondary string of the mass \( M \).

If the inequalities (21),(22) are fulfilled for only one daughter string (for definiteness we take the first string), the decay of the mother string into first daughter string and hadron is generated. If \( E_2 < P_2 \) we select the lightest hadron from hadrons with the given quark composition. The early generated \( z_1^+, P_{x1} \) and \( P_{y1} \) values are not changed, but we generate the hadron mass \( M_2 \) instead of \( z_2^- \), if the hadron is resonance. After determination of \( z_2^- \) from (18), we determine \( z_2^- \) according to (see (13))
\[ z_2^- = \frac{(M_2^2 + (P_{\perp})^2)}{M_2 z_2^+}, \] (24)
and \( z_1^- \) from (17). The energies and longitudinal momenta of the daughter string and hadron are calculated from (19),(20) (see Appendix 2). If the inequalities (21),(22) are fulfilled for the daughter string, the algorithm of it decay is similar to algorithm for decay of secondary string of the mass \( M \).

The decay of mother string into two hadrons is generated, if at the decay of the mother string into two strings the inequalities (21),(22) are not fulfilled for the both strings, or if at the decay of mother string into a string and a hadron the inequalities (21),(22) are not fulfilled for the daughter string. There are some decay modes for the given quark content of these hadrons. We attribute a weight to each decay mode. This weight is equal to the product of three factors. The spin factor is equal to \( (2J_1 + 1)(2J_2 + 1) \), where \( J_1 \) and \( J_2 \) are
spins of the hadrons. The kinematic factor is equal to the two body phase space volume or to zero, if the string mass $M$ is smaller than the sum of the masses of daughter hadrons. The $SU_3$-factor is taken into consideration, if there are several hadrons with the same quark content, spin and parity. For example, $SU_3$-factor of $\eta$-meson which is formed from $\pi u$-pair is equal to $1/6$, and the same $SU_3$-factor of $u$-meson is equal to $1/2$.

The $M_1$ and $M_2$ masses of resonances are generated after the generation of the decay mode. For example, if string decays into two resonances and the mass of first resonance must be generated at first, the $M_1$ and $M_2$ masses can be generated according to the Breit-Wigner distributions in intervals

$$M_{1_{\text{min}}} < M_1 < M - M_{2_{\text{min}}} ,$$

$$M_{2_{\text{min}}} < M_2 < M - M_1 ,$$

where $M_{i_{\text{min}}}$ is maximum sum of masses of particles produced by the decay of $i$-th resonance. In the $K$ frame the square of transverse momentum of resonance $P_{\perp} = P_{x_1} + P_{y_1}$ is generated in the interval $(0, P^2)$, and the azimuthal angle is generated uniformly in the interval $(0, 2\pi)$. The momentum of first resonance is supposed to have the sharp angle with $z$ axis. The decays of unstable hadrons into stable and quasistable particles are generated in the $K$ frame, for example, according to algorithm [26].

The momenta of stable and quasistable particles are transformed from the $K$ frame to the $K'$ frame according to formulas

$$p'_{x_i} = -p_{x_i} \cos \theta \cos \varphi - p_{y_i} \sin \varphi - p_{z_i} \sin \theta \cos \varphi ,$$

$$p'_{z_i} = p_{x_i} \sin \theta - p_{z_i} \cos \theta ,$$

$$p'_{y_i} = -p_{x_i} \cos \theta \sin \varphi + p_{y_i} \cos \varphi - p_{z_i} \sin \theta \sin \varphi ,$$

where $p_{x_i} \ , p_{y_i} \ , p_{z_i}$ are the momentum projections of $i$-th particle (in the $K$ frame), and $\theta, \varphi$ angles are determined above. After Lorentz transformation of the energies and momentum vectors of particles from the $K'$ frame to the $K_{\perp}$ frame, momentum vectors of particles are transformed from the $K_{\perp}$ to $K_L$ frame according to the parameters of transformation $\cos \varphi_{\perp} \ , \sin \varphi_{\perp}$ (see above) in the transverse $xy$ plane. After Lorentz transformation of the energies and momentum vectors of stable and quasistable particles from the $K_L$ frame to the $K_0$ frame we know all characteristics of secondary particles in the $K_0$ c.m.s.
3 $e^+e^-$ annihilation

3.1 $e^+e^-$ annihilation at low energy

In Sect.2 a cascade model of string breaking (see Figs.2,3) had been constructed as a new model for hadronization. In $e^+e^-$ annihilation with center of mass energy $E_{c.m.}$ up to a few GeV the process of gluon emission \cite{[XX],[YY]} practically is absent \cite{[Z][ZZ]}. Therefore we deal with only one primary quark-antiquark string (see Fig.5). Therefore in this energy interval there is unique possibility to check MC cascade model of string breaking (Sect.2). However, our model incorporates the production and decay the hadrons which are constructed by $u$, $d$, $s$ quarks only. Therefore there is a possibility to check this model at center of mass energy only which is under threshold of charm quark pair production, i.e. at c.m.energy equals to 3 GeV \cite{[AA],[BB]}.

3.2 The formation of the primary string and its decay

The $e^+$ and $e^-$ annihilate to form a virtual photon which produces a quark-antiquark pair (Fig.5). In the presented model the flavour of the quark $q$ (and antiquark $\overline{q}$ ) (Fig.5) is generated according to relation (between the probabilities for $u\overline{u}$, $d\overline{d}$, $s\overline{s}$ quark-antiquark pair production)

$$u : d : s = 4 : 1 : 1 \ .$$  \hfill (30)

In the process $e^+e^- \rightarrow q\overline{q}$ (in the $e^+e^-$ center mass system) the angle distribution for the quark is given by the form

$$N^{-1}dN/d\Omega = (1 + \cos^2\theta) \ 3/16\pi \ .$$  \hfill (31)

where $\theta$ is polar angle between the quark momentum vector and electron momentum vector, $d\Omega = 2\pi sin\theta d\theta$.

A quark $q$ and antiquark $\overline{q}$ (Fig.5) stretch the primary string $A$ which decays into secondary hadrons according to the algorithm of cascade of string breaking which is described in details in Sect.2. In $e^+e^-$ c.m.s. the quark momentum vector determinates the direction of the string breaking (see Sects. 1,2).

3.3 Comparison with experimental data for $e^+e^-$ annihilation
In $e^+e^-$ center of mass system (see Fig.5) the quark momentum vector is opposite to momentum vector of antiquark. Therefore after hadronization there are two jets of secondary hadrons. A direction for jet axis (see Fig.6) is determined by Monte Carlo method (\cite{31}, \cite{32}).

At $E_{c.m.}=3$ GeV the time for Monte Carlo simulation (see above) of one $e^+e^-$ annihilation is 0.40 second. Illustration of a hadronic event from $e^+e^-$ annihilation is in Fig.6. The results of calculations according to the model are the curves in Fig.7 (the multiplicity distribution), Fig.8 (rapidity distribution), Figs.9 ($p_\perp$-distribution). The variables $x$ and $x_\parallel$ (see Figs.8,9) are defined by $x = 2P/E_{c.m.}$ and $x_\parallel = 2P_\parallel/E_{c.m.}$ (see Fig.6 and Refs. \cite{31}, \cite{32}).

4 Conclusion

The last few years have witnessed rapid progress in the description of string-string interactions \cite{23} and semiclassical decays of strings (flux tubes). For example, in Ref.\cite{16} Gupta and Rosenzweig explored the implications of string breaking (flux tube fission) for hadron decays; and in Ref.\cite{34} Nussinov shown that the flux tube model can be successfully applied to analytical calculations for various stages of chromoelectric flux tube intersection, rearrangement, and eventual fragmentation (“hadronization”) of the resulting highly excited flux tubes; in the Ref.\cite{10} Mitchell et. all calculated the decay rates for arbitrarily massive states on the leading Regge trajectory for open strings.

In our Monte Carlo (MC) model \cite{35} we attempt to construct the string (and its decay) which has the physical properties the same as the string in the Relativistic Flux Tube model \cite{4}, \cite{7}, \cite{6} and simultaneously the same as the classical massive open string solutions \cite{10} of string theory. Therefore in our MC model the string decay is computed taking the effect of string rotation into account, it is the first. The second, in the Ref.\cite{10} Mitchell, Sundborg and Turok have calculated the decay of the fundamental string in an arbitrary number of dimensions. For calculation of decay probability we use decay rates \cite{10} for the space-time dimension $d=4$.

We can make a comparison with some results on fundamental (Nambu-Goto) strings and RFT model. In the decay of a fundamental string \cite{10} in the critical dimension $d=26$ Mitchell, Sundborg and Turok find that the string is likely to break symmetrically (in contrast to the noncritical dimension) into two states, each with roughly half the (mass) excitation number of the original
state. The same result is for semiclassical decay of excited string states in the flux tube model [16], where the space-time dimension \( d=4 \). At the same time at the decay of a fundamental string [10] in noncritical dimension the string decays by emitting a state close to the ground state and a highly excited state. In our Monte Carlo model we use decay rates (from Ref. [10]) for the space-time dimension \( d=4 \). However, because of momentum-energy conservation we generate the kinematical characteristics of the both daughter strings from intervals which are determined by the conservation laws, the first. The second, because of isotropy of time-space, and because of \( P \) (space inversion) invariance of strong interaction, we can suppose that in the statistical set of decays of mother string (with fixed mass) the distributions for kinematical characteristics for first daughter string are the same as ones for second daughter string inside of the above kinematical intervals. Therefore our MC calculations yield result that the share of string decays into the state close to the hadron state and a highly excited state is about 25\% only.

The some other properties of our model one can see by comparison with electron-positron experimental data at the total center of mass energy equal to 3 \( \text{GeV} \), where the process of gluon emission practically is absent and we can check model at center of mass energy which is under threshold of charm quark pair production.\(^{14}\)

The most interesting quantitative results for proton-(anti)proton interactions will be given elsewhere\(^{15}\), but now it is necessary to stress one interesting property which has direct connection with massive open string dynamics [10].

Because of the rotation (Fig.1) of decaying string (Fig.3), the average transverse momentum of secondary strings (Fig.2) grows with the mass of primary string (the A string in Figs.2,5). A quantitatively illustration of this is in Fig.10. In other side the average mass of primary string grows with the total energy of collising particles \( (e^+e^-, \not p p, pp) \). Therefore the average transverse momentum of secondary strings grows with the total energy of collising particles, and so for the interval of total energy (in the center of mass system of collising particles) from 3 \( \text{GeV} \) \( (e^+e^- \ \text{annihilation}) \) to 1800

\(^{12}\)At the random numeration of both strings.

\(^{13}\)See choice for intervals and distributions for simulation of \( z_1 \) and \( z_2 \) variables from Sect.2.

\(^{14}\)In our hadronization scheme only lightest \( u-, d-, s- \) quarks is used.

\(^{15}\)The preliminary results see, for example, in [21].
GeV (proton-(anti)proton interactions\textsuperscript{16}) there is agreement between experimental data and our theoretical MC calculations on transverse momentum $p_{\perp}$ distribution of secondary particles up to $p_T = 4\text{GeV}$ (see Figs.11-13). It is interesting because there is widely known theoretical hypothesis that in the $p_{\perp}$ interval (approximately) from 1GeV to 4GeV the particle production is a result of QCD hard ($qq, gg, qg$) scattering states (see \textsuperscript{23}). But in our model, all secondary particles are result only of decay \textsuperscript{10} of states of massive open strings (classical string solutions of string theory \textsuperscript{10}) and with the same behaviour as for particles which are result of QCD hard scattering states\textsuperscript{17} (at least in the $p_{\perp}$ scale). Therefore (maybe) one can recover a question about relation between the classical string solutions of string theory and solutions of QCD equations.

Our results are a hint that it would be very interesting to construct the Monte Carlo implementation for the decay of massive open string most closely to results of string theory\textsuperscript{18}. We hope that our model is first step to this aim. But already the presented string breaking model can be used as an ingredient into any Monte Carlo model of multiparticle production for different types of collising particles ($pp, e^+e^-, e^-p$, heavy-ion reactions etc).

For heavy-ion reactions there is widely known theoretical prediction that hard parton-parton interaction and formation of quark-gluon plasma are two sources of secondary particles with high transverse momenta. We hope that our calculations shown that the string rotation is also the source of high transverse momenta, and so this result can’t be ignored in the realistic calculations of high energy reactions.

\textsuperscript{16}In proton-(anti)proton interactions for formation of primary strings, the well-known soft version of dual parton model \textsuperscript{22} is used, i.e. hard interactions ($qq, gg, qg$) between partons of collising hadrons is not calculated.

\textsuperscript{17}The details for production of secondary particles which are the result of QCD hard ($qq, gg, qg$) scattering states, and the detail discussion about $p_{\perp}$ distribution are in Refs. \textsuperscript{24}, \textsuperscript{25}, \textsuperscript{33}.

\textsuperscript{18}For example, for MC implementation it is best to have the strict (without our additional suppositions) theoretical partial decay rate, which depends on the kinematical characteristics of daughter string, it is the first. The second, for MC scheme for long-distance interaction between quark and antiquark, it it necessary to have a theoretical (share or probability for) relation between the states with angular (orbital) momentum $L = 0$ (here the quark degrees of freedom dominate) and the states with angular (orbital) momentum $L \gg 1$ (here the string degrees of freedom dominate).
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Appendix 1

According to experimental data [36] for high energy hadron-hadron collisions, for the produced primary hadron from every hadron multiplet the share of the members with big mass grows with total energy of collision. Therefore there is second possibility for determination of the minimal mass (of first (second) daughter string), i.e. in second case it is the product of two factors. The first factor equals to 2. The second factor grows with energy. For example, at $\sqrt{s} < 100$ GeV the second factor is the mass of lightest hadron with quark composition of first (second) daughter string; at $100$ GeV $< \sqrt{s} < 1000$ GeV the second factor is (with equal probability) the mass of any hadron with quark composition of first (second) daughter string; at $\sqrt{s} > 1000$ GeV the second factor is the biggest mass of the hadron with quark composition of first (second) daughter string. This determination of the minimal mass of the daughter strings leads to the better agreement with the experimental data, but it is not a decisive factor.

Appendix 2

At decay of string of the big mass the sharp angle $\theta$ (in Fig.1) leads to the sharp angle between the $z$ axis and the momentum vector of the daughter string. Therefore it is necessary to rewrite the formulas (19) as the following

$$P_{z1} = M \left| z_1^+ - z_1^- \right| / 2$$
$$P_{z2} = -M \left| z_2^+ - z_2^- \right| / 2$$
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**Figure Captions**

**Fig.1** In the Flux Tube model [4]: the portion of a meson consisting of a segment of flux tube from the center of momentum to the $i^{th}$ quark.

**Fig.2** The diagram of our model for the cascade break of the primary string $A$. 1 and 2 are the partons stretching the secondary string of mass $M$. $M_1$ and $M_2$ are the masses of daughter strings produced at the break of the mother string. $\bar{1}$ is antiquark, 2 is quark. Here and below the double line is diquark.

**Fig.3** Shemes of the string breaking in our model.

**Fig.4** a) An elementary Monte Carlo process of the Models [19], [20]: $q \rightarrow q' + h$. b) The diagram of the Lund model [19] (and model [20]) of breaking of the colour singlet $q'\bar{q}'$ string into hadrons $h$ and secondary small mass $q''\bar{q}''$ string, which is been brouken into two hadrons.

**Fig.5** At the low energy $E_{c.m.}$ by $e^+e^-$ annihilation a single virtual photon $\gamma$ produces a quark-antiquark ($q\bar{q}$) pair, tensing the primary string $A$, which decays (see Fig.2) into secondary hadrons according to algorithm of cascade model of string breaking.

**Fig.6** Illustration [32] of a hadronic event from $e^+e^-$ annihilation showing the jet axis and the components of the momentum $\vec{p}$ of a particle parallel to ($p_\parallel$) and perpendicular to ($p_\perp$) the jet axis.

**Fig.7** Charged particle multiplicity distribution. $K_s^0 \rightarrow \pi^+\pi^-$ decays are included [31], [32]. Here and below the line is the calculation according to the model.

**Fig.8** Particle-density distribution $\sigma^{-1}d\sigma/dy$ vs $y$ for jets (see Fig.6) with
$x_{\text{max}} > 0.3$. $x_{\text{max}}$ is the highest-$x$ particle on one side of the event. The jet direction is oriented so that $x_{\text{max}}$ is at positive $y$. $y$ is the rapidity of the particle relative to the jet direction assuming a pion mass. $x_{\text{max}}$ is at positive $y$ and is not plotted. The distributions are normalized to the cross sections for jets with $x_{\text{max}} > 0.3$. Experimental data are from [31], [32].

**Fig.9** Particle-density distribution $\sigma^{-1}d\sigma/dp_T$ vs $p_T$ for particles opposite (negative $x_\parallel$) jets with $x_{\text{max}} > 0.3$. $p_T$ is the component of particle momentum perpendicular to the jet direction (see Fig.6). Experimental data are from [31], [32].

**Fig.10.** The theoretical MC dependence of the average transverse momentum of secondary strings on the mass of primary string A (see Figs.2,5).

**Fig.11.** The dependence of the inclusive production cross section of charged particles on the transverse momentum $p_T$. Experimental data are from [37].

**Fig.12.** The same as in Fig.11. Experimental data are from [37].

**Fig.13.** The same as in Fig.11. Experimental data are from [38].
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