Doubly Heavy Baryon Production at $\gamma\gamma$ Collider

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Abstract

The inclusive production of doubly heavy baryons $\Xi_{cc}$ and $\Xi_{bb}$ at $\gamma\gamma$ collider is investigated. It is found that the contribution from the heavy quark pair $QQ$ in color triplet and color sextet are important.

Key Words: Doubly heavy baryon, NRQCD, Linear Collider

1 Introduction

$\gamma\gamma$ collider is one option of the International Linear Collider (ILC) in the future, where many interesting issues can be employed[1]. The doubly heavy baryon production is a potential one. It can be factorized into two stages, i.e., the production of two heavy quarks and their transformation into the baryon. The heavy quark mass sets the large scale, which enables us to calculate their creation within Perturbative QCD framework. The transformation is nonperturbative, and may be dealt with non-relativistic QCD (NRQCD) because of the small velocity of the heavy quark in the rest frame of the baryon [2]. This provides us a good opportunity to study QCD, both in the perturbative and nonperturbative aspects. For this aim, a fully inter-course between phenomenological and experimental investigations in various processes is required.

The doubly heavy baryon $\Xi_{cc}$ has been observed by SELEX collaboration [3, 4, 5], and many theoretical studies have been done, e.g., in refs. [6, 7, 8, 9, 10]. However, up to now, we can not understand its production mechanism sufficiently yet. As pointed

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out in ref. [11], the observed production rate is much larger than most of the theoretical predictions, which means that further investigation on the production mechanism as well as exploration for more experimental opportunities are still necessary.

For two identical heavy quark system, if one neglects the relative orbital angular momentum, there are only two states due to the anti-symmetry of the total wave function. One is in angular momentum $^3S_1$ and color triplet, and the other is $^1S_0$ and color sextet. In general, both of these two states can contribute to the doubly heavy baryon production, which can be described by introducing two hadronic matrix elements [8, 12]. In refs. [6, 7] the inclusive production of doubly heavy baryons at various colliders is studied, where only the contribution from quark pair in $^3S_1$ and color triplet is included. While in refs. [8, 9, 10], the contribution from both color triplet and color sextet are taken into account for $e^+e^-$ and hadron-hadron colliders. In this paper, our aim is to investigate the inclusive production of doubly heavy baryons at the future $\gamma\gamma$ collider.

This paper is organized as follows: In Sec 2 we list the basic formula for $\gamma\gamma \rightarrow H_{QQ} + X$ and give some related numerical results. The effective cross sections for $\Xi_{cc}$ and $\Xi_{bb}$ production at ILC are presented in Sec 3. Finally a short summary is given.

2 Doubly Heavy Baryon Production in $\gamma\gamma$ collisions

The doubly heavy baryon $H_{QQ}$ can be produced via the following process

$$\gamma(p_1) + \gamma(p_2) \rightarrow H_{QQ}(k) + X,$$  \hspace{1cm} (1)

where $p_1$, $p_2$ and $k$ respectively denote the four-momentum of the corresponding particles. The unobserved state $X$ can be divided into a nonperturbative part $X_N$ and a perturbative part $X_P$. At tree level, $X_P$ consists of two heavy anti-quarks $\bar{Q}\bar{Q}$. Adopting the notation in ref. [8], one can obtain the scattering amplitude for $\gamma(p_1) + \gamma(p_2) \rightarrow H_{QQ}(k) + \bar{Q}(p_3) + \bar{Q}(p_4) + X_N$ process

$$T = \frac{1}{2} \int \frac{d^4k_1}{(2\pi)^4} A_{ij}(k_1, k_2, p_3, p_4) \int d^4x_1 e^{-ik_1 \cdot x_1} \langle H_{QQ}(k) + X_N | \bar{Q}_i(x_1) \bar{Q}_j(0) | 0 \rangle,$$  \hspace{1cm} (2)

where $k_1, k_2$ denote the four-momentum of the internal heavy quarks, and $p_3, p_4$ the momentum of the anti-quarks. $i, j$ represent Dirac and color indices, $Q(x)$ is the Dirac field for the heavy quark.
The differential cross section for $\gamma(p_1) + \gamma(p_2) \rightarrow H_{QQ}(k) + \bar{Q}(p_3) + \bar{Q}(p_4) + X_N$ is
\[
d\hat{\sigma}(\hat{s}) = \frac{1}{2\hat{s}} \sum_{X_N} \frac{d^3k}{(2\pi)^3} \frac{d^3p_3}{(2\pi)^32E_3} \frac{d^3p_4}{(2\pi)^32E_4} \delta^4(p_1 + p_2 - k - p_3 - p_4 - P_{X_N})
\]
\[
\cdot \frac{1}{4} \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_3}{(2\pi)^4} \delta_{ij}(k_1, k_2, p_3, p_4) \langle \gamma^0 A^+(k_3, k_4, p_3, p_4) \rangle_{kl}
\]
\[
\cdot \int d^4x_1 d^4x_3 e^{-i(k_1 \cdot x_1 + k_3 \cdot x_3)} \langle 0\bar{Q}(k)Q_i(x_3)|H_{QQ} + X_N\rangle \langle H_{QQ} + X_N|\bar{Q}_j(x_1)\bar{Q}(0)|0 \rangle,
\]
where the average over polarization of the photons and the summation over the spin of the baryon $H_{QQ}$ and over color, spin state of two $\bar{Q}$ quarks is implied, and $\hat{s} = (p_1 + p_2)^2$. The factor $1/4$ in the bracket is induced by the identical photons and anti-quarks. The non-relativistic normalization for the heavy baryon is used here. Employing the translation invariance to eliminate the sum over $X_N$, one can obtain
\[
d\hat{\sigma}(\hat{s}) = \frac{1}{2\hat{s}} \sum_{X_N} \frac{d^3k}{(2\pi)^3} \frac{d^3p_3}{(2\pi)^32E_3} \frac{d^3p_4}{(2\pi)^32E_4} \delta_{ij}(k_1, k_2, p_3, p_4)
\]
\[
\cdot \langle \gamma^0 A^+(k_3, k_4, p_3, p_4) \rangle_{kl} \int d^4x_1 d^4x_2 d^4x_3 e^{-i(k_1 \cdot x_1 - i(k_2 \cdot x_2 + k_3 \cdot x_3)}
\]
\[
\cdot \frac{1}{16} \langle 0\bar{Q}(k)Q_i(x_3)|a^+(k)a(k)\bar{Q}_j(x_1)\bar{Q}(0)|0 \rangle,
\]
where $a^+(k)$ is the creation operator for $H_{QQ}$ with three momentum $k$. This contribution can be represented by Fig.1 where the black box represents the Fourier transformed matrix element. In the framework of NRQCD, at the zeroth order of the relative velocity between heavy quarks in the rest frame of $H_{QQ}$, the hadronic matrix element is [8]
\[
\nu^0 \int d^4x_1 d^4x_2 d^4x_3 e^{-i(k_1 \cdot x_1 - i(k_2 \cdot x_2 + k_3 \cdot x_3)} \langle 0\bar{Q}(0)Q_i(x_3)|a^+(k)a(k)\bar{Q}_j(x_1)\bar{Q}(x_2)|0 \rangle
\]
\[
= (2\pi)^4 \delta^4(k_1 - m_{Q}) (2\pi)^4 \delta^4(k_2 - m_{Q}) (2\pi)^4 \delta^4(k_3 - m_{Q})
\]
\[
\cdot [-\delta_{a_1 a_4} \delta_{a_2 a_3} + \delta_{a_1 a_3} \delta_{a_2 a_4}] \langle \tilde{P}_c \gamma^5 P_v \rangle_{ji}(P_v \gamma^5 C \tilde{P}_v)_{lk} \cdot h_1
\]
\[
+ \delta_{a_1 a_4} \delta_{a_2 a_3} - \delta_{a_1 a_3} \delta_{a_2 a_4} \langle \tilde{P}_c \gamma^5 P_v \rangle_{ji}(P_v \gamma^5 C \tilde{P}_v)_{lk} \cdot (\nu^0 \nu^0 - g_{\mu \nu} \cdot \nu^\mu)^0 \cdot h_3],
\]
where $m_{Q}$ is the mass of the heavy quark, $a_i(i = 1, 2, 3, 4)$ and $i, j, k, l$ respectively denote the color and Dirac indices, and
\[
P_v = \frac{1 + \gamma \cdot v}{2}, \quad C = i\gamma^2 \gamma^0, \quad \nu^\mu = k^\mu/M_{H_{QQ}}.
\]
$\tilde{P}_v$ is the transpose of the matrix $P_v$. $h_1$ ($h_3$) represents the probability for a $QQ$ pair in $^1S_0$ ($^3S_1$) state and in the color state of 6 ($\bar{3}$) to transform into the baryon,
\[
h_1 = \frac{1}{48} \langle 0|[\psi^a \psi^{a_2} + \psi^{a_2} \psi^a]a^+ a \psi^{a_1} \psi^a |0 \rangle,
Figure 1: Graphic representation for the contribution in eq (4), the dashed line is the cut and $k_4 = k_1 + k_2 - k_3$.

\[ h_3 = \frac{1}{72} \langle 0 | [\psi^{a_1} \varepsilon \sigma^2 \psi^{a_2} - \psi^{a_2} \varepsilon \sigma^a \psi^{a_1}] a^\dagger a \psi^{a_2 \dagger} \sigma^a \varepsilon \psi^{a_1 \dagger} | 0 \rangle, \]  

(7)

where $\sigma^j (j = 1, 2, 3)$ are Pauli matrices, $\varepsilon = i\sigma^2$ is totally anti-symmetric, $\psi$ is the NRQCD field for the heavy quark. Generally, $h_1$ and $h_3$ should be determined by nonperturbative QCD. Under NRQCD, $h_1$ and $h_3$ are at the same order. $h_3$ can be related to the non-relativistic wave function at the origin, i.e., $h_3 = |\Psi_{QQ}(0)|^2$, which can be calculated in the framework of potential models, such as in ref. [13].

With all of the above equations, one can write the cross section for $\gamma(p_1) + \gamma(p_2) \rightarrow H_{QQ}(k) + \bar{Q}(p_3) + \bar{Q}(p_4) + X_N$ process into the following form

\[ \hat{\sigma} = \frac{\alpha^2 \alpha_s^2 e_q^4}{4 m_Q^2} [f_1(\eta) \frac{h_1}{m_Q^3} + f_3(\eta) \frac{h_3}{m_Q^3}], \]  

(8)

where $\eta = \hat{s}/(16m_Q^2) - 1$, $\alpha$ is the fine structure constant, $\alpha_s$ the strong coupling, and $e_q = 2/3 (-1/3)$ for up(down)-type quark. The scaling functions $f_1(\eta)$ and $f_3(\eta)$ do not depend on $m_Q$ explicitly. The numerical results for $f_1$ and $f_3$ are displayed in fig [2]. One can find that the contribution from the color triplet $QQ$ pair is much larger than that from the color sextet one.
3 Effective cross sections at ILC

At a future linear collider, backscattered laser light may provide very high-energy photons [14]. The total effective cross section at a photon collider is

\[
d\sigma(S) = \int_0^{y_{\text{max}}} dy_1 \int_0^{y_{\text{max}}} dy_2 f_{\gamma e}^e(y_1) f_{\gamma e}^e(y_2) d\hat{\sigma}(\hat{s}),
\]

where \(\sqrt{S}\) is the \(e^+e^-\) CMS energy. The function \(f_{\gamma e}^e(y_1)\) is the normalized energy spectrum of the photons. It is given by

\[
f_{\gamma e}^e(y) = \mathcal{N}^{-1} \left[ \frac{1}{1-y} - y + \left( \frac{2y}{x(1-y)} - 1 \right)^2 \right],
\]

where \(\mathcal{N}\) is the normalization factor, and \(y\) is the fraction of the electron energy transferred to the photon in the center-of-mass frame. It takes values in the range

\[
0 \leq y \leq \frac{x}{x+1},
\]

where \(x = 4E_L E_e/m_e^2\). \(E_L\) (\(E_e\)) is the energy of the laser (electron) beam and \(m_e\) is the electron mass. In order to avoid the creation of an \(e^+e^-\) pair from the backscattered laser beam and the low energy laser beam, the maximal value for \(x\) is \(2(1 + \sqrt{2})\). For a beam
energy $E_e = 250 GeV$, this leads to an optimal laser energy $E_L = 1.26eV$. Here, we adopt these two values for $E_e, E_L$.

In our numerical calculations, we take $|\Psi_{cc}(0)|^2 = 0.039 GeV^3$ and $|\Psi_{bb}(0)|^2 = 0.152 GeV^3$ obtained in ref. [13]. For consistence, we adopt their value for quark mass, i.e., $m_c = 1.8 GeV$ and $m_b = 5.1 GeV$. We choose the scale to be $2m_Q$ for $\alpha_s$, and obtain $\alpha_s = 0.20(0.16)$ for $\Xi_{cc}(\Xi_{bb})$ production. Our predictions for the total effective cross section of $\Xi_{cc}$ ($\Xi_{bb}$) are given in Table 1 (Table 2). One can find that the contribution from the color sextet $QQ$ pair are approximately 10% of that from the color triplet one if $h_1 = h_3$. Obviously, the effective cross section decreases as the heavy quark mass increases, which is one reason for the production probability of $\Xi_{bb}$ is much smaller than that of $\Xi_{cc}$.

Additionally, taking $\Xi_{cc}$ as an example, we give the predictions for the distributions of $\cos \theta, x$ and $x_T$. Here, $\theta$ and $x$ are defined in $e^+e^-$ CMS, where $\theta$ is the angle between the moving direction of $\Xi_{cc}$ and that of the beam. $x = 2E/\sqrt{S}$ and $x_T = 2P_T/\sqrt{S}$, with $E$ and $P_T$ the energy and transverse momentum of $\Xi_{cc}$ respectively. The corresponding results are displayed in Fig.3, 4 and 5 respectively.

4 Summary

In this paper, we investigate the production of the doubly heavy baryon $H_{QQ}$ at $\gamma\gamma$ collider. We find that for $H_{QQ}$ production, the contribution from the color sextet is about 10%. This implies that the contribution from both color triplet and color sextet are important for describing the doubly heavy baryon production. Our method can be extended to study the
Figure 3: $cos\theta$-distributions, the solid line for $h_3 = h_1 = |\Psi_{cc}(0)|^2$, the dotted for $h_1 = |\Psi_{cc}(0)|^2$ and $h_3 = 0$, the dashed for $h_1 = 0$ and $h_3 = |\Psi_{cc}(0)|^2$.

Figure 4: Same as Fig. 3 but for $x$-distributions.
doubly heavy baryon $H_{QQ'}$ production. But for that case, more hadronic matrix elements have to be considered due to no restriction from Pauli principle for the heavy quark pair $QQ'$. We hope our results can be helpful for the investigation of the doubly heavy baryon production at the future linear collider.

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