Scalar graviton and the modified black holes

Yu. F. Pirogov
Institute for High Energy Physics, Protvino, Russia

Abstract
Under the explicit violation of the general covariance to the unimodular one, the effect of the emerging scalar graviton on the static spherically symmetric metrics is studied. The main results are three-fold. First, there appears the two-parametric family of such metrics, instead of the one-parametric black-hole family in General Relativity (GR). Second, there may exist the one-parametric subfamily describing a pure gravitational object, the “graviball”, missing in GR. Third, in a simplifying assumption, all the metrics possess the correct Newton’s limit as in GR.

1 Introduction
In paper [1], we proposed the metric effective field theory with the explicit violation of the general covariance (GC) to the residual unimodular covariance (UC). Due to such a violation, the resulting modification of General Relativity (GR) describes the massive scalar graviton as a part of the metric, in addition to the massless tensor graviton as in GR. Associated with the scalar graviton, there appears a dimensionful parameter which is a priori arbitrary. The scalar graviton was proposed as a source of the dark matter and the dark energy of the gravitational origin. In a subsequent paper [3], this concept was applied to studying the evolution of the isotropic homogeneous Universe. It was found there that to treat the scalar graviton as the dominant source of the cold dark matter, the aforementioned dimensionful parameter should be large, of the order of the Planck mass. It follows thereof that such a parameter could strongly invalidate the Newton’s limit for the metric of the gravitating center. This question is studied in the given paper.

In Section 2, the theory of gravity with UC and the scalar graviton is briefly reviewed to be used in what follows. In Section 3, the static spherically symmetric metric of a point-like body, together with the surrounding distribution of the scalar gravitons, is investigated. The main results are three-fold. First, there exists the two-parametric family of the respective metrics instead of the one-parametric GR family, the black holes. Second, among the metrics, there may exist those for a peculiar object, the “graviball”. Third, in a simplifying assumption, all the metrics possess the correct Newton’s limit without fine tuning, making thus the proposed GR modification as robust in the Newtonian approximation as GR itself. What remains to be done is indicated in the Conclusion.

\footnote{For a brief exposition of the respective topics, see [2].}
2 Scalar graviton

Gravity action  Let us remind in brief the metric effective field theory with UC \[1\]. In addition to the massless tensor graviton, such a theory describes the massive scalar one as a part of the metric field. In the vacuum, the gravity action consists generically of two parts:

\[ S = S_g + S_s. \] (1)

The first, generally covariant part of the action, responsible for the massless tensor graviton, is as in GR:

\[ S_g = -\frac{1}{2} m_P^2 \int R(g_{\mu\nu}) \sqrt{-g} d^4 x, \] (2)

with \( m_P = (8\pi G_N)^{-1/2} \) being the Planck mass and \( G_N \) the Newton’s constant. In the above, \( x^\mu, \mu = 0, \ldots, 3, \) are the arbitrary observer’s coordinates, \( g_{\mu\nu} \) is the metric, \( g \equiv \det g_{\mu\nu}, \) \( R = g^{\mu\nu} R_{\mu\nu} \) is the Ricci scalar, with \( R_{\mu\nu} \) being the Ricci curvature tensor:

\[ R_{\mu\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\mu \Gamma^\lambda_{\lambda\nu} + \Gamma^\lambda_{\mu\nu} \Gamma^\rho_{\rho\lambda} - \Gamma^\lambda_{\mu\rho} \Gamma^\rho_{\nu\lambda}. \] (3)

Also, \( \Gamma^\lambda_{\mu\nu} \) is the Christoffel connection:

\[ \Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} (\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu}), \] (4)

so that \( \Gamma^\lambda_{\lambda\nu} = \partial_\nu \ln \sqrt{-g} \) and thus \( \partial_\mu \Gamma^\lambda_{\lambda\nu} = \partial_\mu \partial_\nu \ln \sqrt{-g}. \)

The second part of the gravity action, describing the massive scalar graviton, looks in the lowest derivative order as

\[ S_s = \int \left( \frac{1}{2} \partial_\sigma \cdot \partial_\sigma - V_s(\sigma) \right) \sqrt{-g} d^4 x, \] (5)

with \( \partial \sigma \cdot \partial_\sigma = g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma, \) etc. Here, \( \sigma \) is the field of the scalar graviton:

\[ \sigma = f_s \ln \sqrt{g/\tilde{g}}, \] (6)

with \( \tilde{g} \) being a nondynamical scalar density of the same weight as the scalar density \( g. \) Such an object is necessary to build the scalar out of \( g. \) Knowing \( \tilde{g} \) in the observer’s coordinates, one can always rescale the latter ones locally, so that \( \tilde{g} = -1. \) On the other hand, the observer’s coordinates being fixed, changing \( \tilde{g} \) by a multiplicative constant results in shifting \( \sigma \) by an additive constant. In Eq. (5), \( f_s \) is a constant with the dimension of mass. A priori, one expects \( f_s \leq O(m_P). \) \( V_s \) is a potential producing the mass for the scalar graviton. By the symmetry reasons, the potential is supposed to be suppressed.

Eq. (5) violates GC, still preserving UC. In particular, this insures that the causality is not violated. Such a modified GR is the bona fide quantum field theory, as robust theoretically as GR itself \[2\]. Nevertheless, there being no smooth restoration of GR at the quantum level, the proposed GR modification is in fact an independent theory, which is to be verified experimentally.
Gravity equations  Varying the gravity action with respect to $g^{\mu\nu}$, $\tilde{g}$ being fixed, one arrives at the modified gravity equation in the vacuum:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{m_P^2} T_{s\mu\nu},$$  \hspace{1cm} (7)

where

$$T_{s\mu\nu} = \partial_{\mu}\sigma \partial_{\nu}\sigma - \frac{1}{2} \partial\sigma \cdot \partial\sigma g_{\mu\nu} + V_s g_{\mu\nu} + f_s (\nabla \cdot \nabla \sigma + \partial V_s / \partial \sigma) g_{\mu\nu}$$ \hspace{1cm} (8)

is to be treated as the metric energy-momentum tensor of the scalar graviton. In the above

$$\nabla \cdot \nabla \sigma = \frac{1}{\sqrt{-g}} \partial_{\mu} (\sqrt{-g} g^{\mu\nu} \partial_{\nu}\sigma).$$ \hspace{1cm} (9)

Due to the Bianchi identity

$$\nabla_{\mu} \left( R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} \right) = 0,$$ \hspace{1cm} (10)

the total energy-momentum of the ordinary matter, omitted in Eq. (1), and the scalar graviton satisfies the continuity condition. For this reason, the scalar graviton can naturally be considered as a source of the dark matter and the dark energy of the gravitational origin. In the absence of the ordinary matter, the continuity condition reduces to

$$\nabla_{\mu} T_{s\mu}^{\nu} = 0.$$ \hspace{1cm} (11)

The modified pure gravity equation (7) can otherwise be written as

$$R_{\mu\nu} = \frac{1}{m_P^2} \left( T_{s\mu\nu} - \frac{1}{2} T_s g_{\mu\nu} \right),$$ \hspace{1cm} (12)

where $T_s = g_{\mu\nu} T_s^{\mu\nu}$ is as follows:

$$T_s = - \partial\sigma \cdot \partial\sigma + 4 V_s + 4 f_s (\nabla \cdot \nabla \sigma + \partial V_s / \partial \sigma),$$ \hspace{1cm} (13)

with $R = -T_s / m_P^2$ generally nonzero. Explicitly, one gets

$$R_{\mu\nu} = \frac{1}{m_P^2} \left( \partial_{\mu}\sigma \partial_{\nu}\sigma - V_s g_{\mu\nu} - f_s (\nabla \cdot \nabla \sigma + \partial V_s / \partial \sigma) g_{\mu\nu} \right).$$ \hspace{1cm} (14)

3 Modified black holes

Comoving coordinates  Under the violation of GC, to really start up the theory one should choose the particular coordinates where $\tilde{g}$ is to be defined. There is a natural choice. Namely, according to the present-day cosmological paradigm, our Universe is spatially flat, fairly homogeneous, and isotropic. Thus, given a point $P$, the observer can adjust the coordinates in the Universe so that the metric around the point at the astronomical scales, much less then the cosmological ones, may be put to the Minkowskian form, $g_{\mu\nu} = \eta_{\mu\nu}$. Call these coordinates the comoving ones. In the comoving coordinates, the nondynamical variable $\tilde{g}$, characterizing the Universe as a whole, is to vary at the cosmological scales as the Universe itself. So, $\tilde{g}$ can be treated in the given approximation as a constant, too. After the theory
starts up in the comoving coordinates, it can be rewritten in the arbitrary observer’s coordinates.

Now, let in the point \( P \) there be placed the isolated point-like body which is in the rest relative to the close ambient. The body is assumed to have no angular momentum and other physical attributes, but the mass \( m \). Such a body disturbs the Universe metric in a vicinity of the point, not violating the spherical symmetry. The interval corresponding to the static spherically symmetric metric around such a body can be chosen in the comoving coordinates most generally as follows:

\[
ds^2 = a(r)dt^2 - \left(b(r) - c(r)\right)(\mathbf{n}d\mathbf{x})^2 - c(r)d\mathbf{x}^2, \tag{15}\]

where \( r^2 = x^2 = \delta_{mn}x^mx^n, \) \( m, n = 1, 2, 3, \) etc., \( n^m = x^m/r, \) \( n_m = \delta_{ml}n^l, \) with \( \mathbf{n}^2 = 1. \) The functions \( a(r), b(r), \) and \( c(r) \) are the dynamical metric variables to be determined through the gravity equations. The latter ones are to be supplemented by the asymptotic condition: \( a(r), b(r), c(r) \rightarrow 1 \) at \( r \rightarrow \infty. \)

The respective metric looks like

\[
g_{00} = a, \quad g_{mn} = -bn_mn_n - c(\delta_{mn} - n_mn_n), \tag{16}\]

with the inverse metric

\[
g^{00} = \frac{1}{a}, \quad g^{mn} = -\frac{1}{b} n^m n^n - \frac{1}{c}(\delta^{mn} - n^m n^n). \tag{17}\]

The rest of the metric elements is zero. Rotating the spatial coordinates so that in the point \( \mathbf{x} \) there takes place \( \mathbf{n} = (1, 0, 0), \) one brings the metric in this point to the diagonal form \( (g_{\mu\nu}) = \text{diag}(a, -b, -c, -c). \) Thus, generally, the metric is anisotropic. For the isotropy, there should be fulfilled \( c = b. \) In general, one has \( g = -abc^2 \) and thus

\[
\sigma(r) = f_s \ln(\sqrt{abc}/\sqrt{-\tilde{g}}), \tag{18}\]

with \( \sigma \rightarrow f_s \ln(1/\sqrt{-\tilde{g}}) \) at \( r \rightarrow \infty. \)

One can change the original comoving coordinates \( x^m \) to the polar comoving ones \((r, \theta, \varphi)\), so that the interval becomes

\[
ds^2 = adt^2 - bdr^2 - cr^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{19}\]

with a unit of length being tacitly understood where it necessary. One has now \( g = -abc^2r^4\sin^2\theta, \) with \( \sigma \) looking nevertheless the same as in Eq. (18) due to the compensating transformation of \( \tilde{g}. \)

**Coordinate change** Having defined the theory in the comoving coordinates, one can choose, not violating the spherical symmetry, the new radial coordinate \( \hat{r} = \hat{r}(r) \), with the metric variables becoming as follows:

\[
\hat{a}(\hat{r}) = a(r(\hat{r})), \quad \hat{b}(\hat{r}) = (dr/d\hat{r})^2b(r(\hat{r})), \quad \hat{c}(\hat{r}) = (r/\hat{r})^2c(r(\hat{r})). \tag{20}\]
To preserve the asymptotic condition on the metric one should impose the restriction \( \dot{r}/r \to 1 \) at \( r \to \infty \). The scalar graviton distribution now looks like

\[
\dot{\sigma}(\dot{r}) = \sigma(r(\dot{r})) = f_s \ln \left( \sqrt{abc}/\sqrt{-\tilde{g}} \right),
\]

with

\[
\sqrt{-\tilde{g}} = (r/\dot{r})^2 (dr/d\dot{r}) \sqrt{-\tilde{g}}.
\]

A priori, all the choices of \( \dot{r} \) are equivalent under the reversibility. Due to this, by imposing a restriction on \( \dot{c} \) one can bring the metric to the form most appropriate for the particular purposes. At that, the role of the third independent dynamical variable instead of \( \dot{c} \) is played by the scalar field \( \dot{\sigma}(\dot{r}) \). The latter becomes a kind of a hidden variable in the absence of the direct interactions of the scalar graviton with the ordinary matter.

**Isotropic form**  For example, imposing the relation \( \dot{c} = \dot{b} \) by choosing \( \dot{r} \) through

\[
(r/\dot{r})^2 c = (dr/d\dot{r})^2 b,
\]

or explicitly

\[
\dot{r} = \exp \int \sqrt{b/c} \frac{dr}{r},
\]

one brings the interval to the isotropic form

\[
ds^2 = \dot{a}(\dot{r})dt^2 - \dot{c}(\dot{r})d\mathbf{x}^2,
\]

with \( \dot{r}^2 = \mathbf{x}^2 = \delta_{mn} \dot{x}^m \dot{x}^n \). The scalar graviton distribution is given by Eq. (21) at \( \dot{c} = \dot{b} \).

**Astronomic form**  Otherwise, imposing \( \dot{c} = 1 \), so that

\[
\dot{r} = r \sqrt{c(r)},
\]

one brings the interval to the form

\[
ds^2 = \dot{a}(\dot{r})dt^2 - \dot{b}(\dot{r})d\dot{r}^2 - \dot{r}^2 (d\theta^2 + \sin^2 \theta d\phi^2).
\]

Such a form is preferable to compare with observations. The scalar graviton distribution is again given by Eq. (21) at \( \dot{c} = 1 \).

**Modified Schwarzschild equations**  In what follows, we will study the static spherically symmetric metric in the polar comoving coordinates \( (r, \theta, \varphi) \). With account for Eq. (19), one gets the metric elements

\[
g_{00} = a, \ g_{rr} = -b, \ g_{\theta\theta} = -cr^2, \ g_{\varphi\varphi} = -cr^2 \sin^2 \theta,
\]

with the inverse ones

\[
g^{00} = 1/a, \ g^{rr} = -1/b, \ g^{\theta\theta} = -1/(cr^2), \ g^{\varphi\varphi} = -1/(cr^2 \sin^2 \theta),
\]

the rest of the metric elements being zero.
Designating $a' = da/dr$, etc., one gets the nonzero elements of the Christoffel connection as follows:

\[
\Gamma^0_{0r} = \frac{1}{2} \frac{a'}{a}, \quad \Gamma^\theta_{\theta r} = \Gamma^\varphi_{\varphi r} = \frac{1}{2} \frac{(cr)^2}'(cr)^2, \\
\Gamma^r_{00} = \frac{1}{2} \frac{a'}{b}, \quad \Gamma^r_{\theta \theta} = - \frac{1}{2} \frac{(cr)^2}{b}, \\
\Gamma^r_{r r} = \frac{1}{2} \frac{b'}{2b}, \quad \Gamma^r_{\varphi \varphi} = - \frac{1}{2} \frac{(cr)^2}{b} \sin^2 \theta,
\]

with

\[
\Gamma^\lambda_{\lambda r} = \frac{1}{2} \frac{a'}{a} + \frac{1}{2} \frac{b'}{2b} + \frac{(cr)^2}{cr^2}, \\
\Gamma^\lambda_{\lambda \theta} = \cot \theta.
\]

Inserting the above expressions in Eq. (3), one gets the following nonzero elements of the Ricci tensor:

\[
R_{00} = \frac{1}{2} \left( \frac{a''}{b} - \frac{1}{2} \frac{a'^2}{a} - \frac{1}{2} \frac{a'b'}{2b} \right) + \frac{1}{2} \frac{(cr)^2}'(cr)^2, \\
R_{rr} = - \frac{1}{2} \left( \frac{a''}{a} - \frac{1}{2} \frac{a'^2}{a} - \frac{1}{2} \frac{a'b'}{2ab} \right) + \frac{1}{2} \frac{(cr)^2}'(cr)^2 + \frac{1}{2} \frac{(cr)^2}{(cr)^2} - \frac{(cr)^2}{cr^2}, \\
R_{\theta \theta} = 1 - \frac{1}{2} \frac{1}{b} \left( \frac{(cr)^2}{(cr)^2} + \frac{1}{4} \frac{(cr)^2}{b} \right), \\
R_{\varphi \varphi} = \sin^2 \theta R_{\theta \theta}.
\]

Finally, with account for Eq. (14), this gives the looked for system of the three equations for the three independent variables:

\[
\frac{1}{2} \frac{a''}{a} - \frac{1}{4} \frac{a'}{a} \left( \frac{a'}{a} + \frac{b'}{b} \right) + \frac{b}{cr^2} - \frac{1}{2} \frac{(cr)^2}{cr^2} + \frac{1}{2} \frac{(cr)^2}{cr^2} + \frac{1}{4} \frac{(cr)^2}{a} + \frac{1}{4} \frac{(cr)^2}{b} = 0, \\
\frac{1}{2} \frac{(cr)^2}{cr^2} - \frac{(cr)^2}{cr^2} + \frac{1}{2} \frac{(cr)^2}{cr^2} = \frac{1}{m_p} g^2, \\
\frac{b}{cr^2} - \frac{1}{2} \frac{(cr)^2}{cr^2} - \frac{1}{4} \frac{(cr)^2}{a} - \frac{1}{4} \frac{(cr)^2}{b} = \frac{1}{m_p} b \left( f_s \nabla \cdot \nabla \sigma + \partial V_s / \partial \sigma + V_s \right),
\]

where

\[
\nabla \cdot \nabla \sigma = - \frac{1}{b} \left( \sigma'' + (\ln \sqrt{a/bcr^2})' \sigma' \right),
\]

with \( \sigma \) given by Eq. (18). At \( \tilde{g} = \text{const} \) one has

\[
\sigma' = f_s (\ln \sqrt{abc})'
\]

and

\[
\nabla \cdot \nabla \sigma = - \frac{f_s}{b} \left( (\ln \sqrt{abc})'' + \frac{2}{r} (\ln \sqrt{abc})' + (\ln \sqrt{abc})' (\ln \sqrt{abc})' \right).
\]

The system (33) supersedes the Schwarzschild equations, valid in GR (see later on). Note that the first, independent of \( \sigma \) equation of the system remains the same as in GR. Under GC, this equation holds true at any \( c \). Under UC, the equation serves to find \( c \) or, otherwise, \( \sigma \) in addition to \( a \) and \( b \).
**Schwarzschild metric** Let us recover the GR black-hole solution to be used as a reference point. Put $\sigma = 0$, restoring GC. Choose conventionally $c = 1$. Suppose also that $V_s = 0$, neglecting thus by the cosmological term. Altogether, from the last two equations of Eq. (33), one gets the Schwarzschild equations:

\[
\frac{a'}{a} + \frac{b'}{b} = 0, \\
\frac{b - 1}{r} - \frac{1}{2} \left( \frac{a'}{a} - \frac{b'}{b} \right) = 0.
\] (37)

Accounting for the asymptotic condition at $r \to \infty$, one gets $ab = 1$ and $a = 1 - r_g/r$, with $r_g$ being an integration constant. The first equation of the system (33) can be shown to be satisfied, too. This reproduces the conventional Schwarzschild interval in the astronomic form:

\[
ds^2 = (1 - r_g/r) dt^2 - \frac{1}{1 - r_g/r} dr^2 - r^2(\theta^2 + \sin^2 \theta d\phi^2).
\] (38)

To insure the Newton’s limit one should put $r_g = 2G_N m$, with $m$ being the mass of the point-like central body. Choosing the new radial variable $\hat{r}$ through

\[
r = \hat{r} \left(1 + \frac{r_g}{4\hat{r}}\right)^2,
\] (39)

and accounting for Eq. (20) at $c = 1$, one brings the Schwarzschild interval to the isotropic form:

\[
ds^2 = \left(\frac{1 - r_g/4\hat{r}}{1 + r_g/4\hat{r}}\right)^2 dt^2 - (1 + r_g/4\hat{r})^4 d\hat{x}^2,
\] (40)

or

\[
ds^2 = \left(\frac{1 - r_g/4\hat{r}}{1 + r_g/4\hat{r}}\right)^2 dt^2 - (1 + r_g/4\hat{r})^4 \left(d\hat{r}^2 + \hat{r}^2(\theta^2 + \sin^2 \theta d\phi^2)\right).
\] (41)

Otherwise, Eq. (41) can be found directly from the last two equations of Eq. (33) under $c = b$ and the missing r.h.s.

**Newtonian approximation** Now let $f_s \neq 0$, but $V_s$ can still be neglected. Decompose the solutions in the powers of $1/r$:

\[
a = 1 + \sum_{n=1} a_n r^{-n},
\]

\[
b = 1 + \sum_{n=1} b_n r^{-n},
\]

\[
c = 1 + \sum_{n=1} c_n r^{-n}.
\] (42)

Substituting this decomposition into Eq. (33) note first of all that, according to equations (35) and (36), the r.h.s. of Eq. (33) appears only in the order $O(1/r^4)$, whereas the l.h.s. is nonzero already in $O(1/r^3)$. This means that in the leading order the modified black-hole solutions possess asymptotically the same properties.
as in GR. In particular, this insures the validity of the Newton’s limit independent of $f_s$. More particularly, one gets the restriction

$$a_1 + b_1 = 0,$$  \hspace{1cm} (43)

with $c_1$ remaining arbitrary. This reflects the absence of the GC violation in the gravity equations in the given approximation. Namely, introducing the new radial coordinate

$$\hat{r} = r\left(1 + \sum_{n=1}^{\infty} \frac{\lambda_n}{r^n}\right)$$  \hspace{1cm} (44)

one gets, according to Eq. (20),

$$\hat{a}_1 = a_1,$$
$$\hat{b}_1 = b_1,$$
$$\hat{c}_1 = c_1 - 2\lambda_1,$$  \hspace{1cm} (45)

so that $c_1$ is indeed defined in the Newtonian approximation up to an additive constant as in GR.

To find the solution in the post-Newtonian approximation one should put

$$b_1 = -a_1 = r_g$$  \hspace{1cm} (46)

and fix some $c_1$. In GR, this would produce $a_2$ and $b_2$ depending on $b_1$ and $c_1$, the parameter $c_2$ being again arbitrary due to the exact GC. Repeating this procedure in GR in all the orders, one can, in principle, recover the Schwarzschild solution up to an arbitrary function $c(r)$. In particular at $c = 1$ and $c = b$, the proper coefficients $a_n$ and $b_n$ can be read off from equations (38) and (41), respectively.

Now under UC, due to the violation of GC in the r.h.s. of Eq. (33) in the higher orders, $c_2$ is not arbitrary but is determined together with $a_2$ and $b_2$. Iterating this procedure, one can built the solution depending on the two constants $b_1$ and $c_1$. Thus there should exist the two-parametric family of the modified black holes superseding the one-parametric family of the black holes in GR. Though the parameter $c_1$ gauges according to Eq. (45), it can not be disposed at all. Namely, decompose $\sigma$ as

$$\sigma = \sigma_0 + \sum_{n=1}^{\infty} \sigma_n r^n,$$  \hspace{1cm} (47)

with $\sigma_0 = 0$ without any loose of generality. Relative to the radial coordinate transformations Eq. (44) one gets

$$\hat{\sigma}_1 = \sigma_1,$$  \hspace{1cm} (48)

$\sigma_1$ being thus invariant. It is related with $c_1$ in the comoving coordinates

$$c_1 = r_s$$  \hspace{1cm} (49)

as $\sigma_1 = f_s r_s$. Similar to $r_g$, the parameter $r_s$ is a priori arbitrary and should be found through observations. The Newton’s limit for all the metrics is correctly determined by $r_g$, independent of $r_s$.

Finally note, that there are two conceivable marginal cases of the static spherically symmetric metrics. First, let $r_g \neq 0$, but $r_s = 0$. The proper object is nothing
but the GR black hole. On the contrary, let \( r_g = 0 \). This metric should describe a peculiar object which may be called the “graviball”. For the latter, \( a \) and \( b \) vary at least as \( O(1/r^2) \) at \( r \to \infty \), whereas \( c \) generally as \( O(1/r) \). Nevertheless, by the change of the radial coordinate the metric as a whole can be brought at infinity to the form varying at least as \( O(1/r^2) \), exhibiting thus the Newton’s limit for the massless body. At that, \( \sigma \) varies still as \( O(1/r) \), being though directly unobservable.

4 Conclusion

The theory of gravity with UC and the scalar graviton admits the two-parametric variety of the modified black holes, ranging from the ordinary black holes to the peculiar objects, the graviballs. The theory keeps on to be uncontradicted explicitly. The scalar graviton potential neglected, such a GR modification is as robust in the Newtonian approximation as GR itself. The post-Newtonian approximations, as well as the effect of the potential are still mandatory.

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References

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