BRST FORMALISM FOR SYSTEMS WITH HIGHER ORDER DERIVATIVES OF GAUGE PARAMETERS

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Abstract
For a wide class of mechanical systems, invariant under gauge transformations with higher (arbitrary) order time derivatives of gauge parameters, the equivalence of Lagrangian and Hamiltonian BRST formalisms is proved. It is shown that the Ostrogradsky formalism establishes the natural rules to relate the BFV ghost canonical pairs with the ghosts and antighosts introduced by the Lagrangian approach. Explicit relation between corresponding gauge-fixing terms is obtained.

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1 Introduction

Gauge invariant systems are described by singular Lagrangians. Quantizing such a theory it is desirable to keep an initial covariance of the system. Hence, to have a consistent state space in corresponding quantum theories, it is necessary to modify an initial velocity phase space of such systems on the classical level. The most popular way to do it – BRST formalism – is based on the notion of the BRST-symmetry [1], discovered first for Yang-Mills field theory.

At present there have been elaborated two different approaches to construct an effective BRST invariant theory from the initial singular system. In the first approach, called Lagrangian BRST formalism [2], one starts from a gauge invariant system and, using consequences of the gauge invariance, constructs a nonsingular BRST invariant Lagrangian. Another method (Hamiltonian BRST (BFV) formalism [3, 4]) is based on Hamiltonian description of the constrained system. The question of correspondence between Lagrangian and Hamiltonian BRST formalisms was considered in many papers [5, 6, 7, 8] by the use of various approaches.

As we know, the most interesting from the physical point of view systems have a gauge symmetry under transformations depending on time derivatives of gauge parameters only up to the first order. The equivalence of Lagrangian and Hamiltonian BRST formalisms for such systems was proven in Refs. [3, 7, 8] in such a way, that relations between BRST charges, BRST invariant Hamiltonians and gauge-fixing terms of two approaches were explicitly established. In these works there was also obtained the form of the constraints and structure functions which appear when one rewrites the corresponding Lagrangian functions through Hamiltonian variables. Now it would be interesting to perform the same analysis for gauge invariant systems, whose symmetry transformations depend arbitrarily on gauge parameters (we mean an arbitrary order of time derivatives of infinitesimal gauge parameters). Actually, within the framework of Lagrangian BRST formalism infinitesimal parameters of gauge transformations are replaced by ghost variables. Then, we shall get an effective BRST invariant system with higher order derivatives because of nonsingular Lagrangian is constructed with the help of terms, including BRST transformations of the velocity phase space coordinates. But the BFV prescription introduces the ghosts as additional canonical variables, simply associating them with the constraints. Thus, there arises one more question: what rules relate the ghosts of Lagrangian and Hamiltonian BRST formalisms? Remember that the same question had also appeared in the paper by I.V. Tyutin with collaborators (see Ref. [5]). We will be convinced that within the framework of our analysis the corresponding problem is consistently solved.

In Ref. [9] Hamiltonian description for gauge invariant systems, having the symmetry transformations with arbitrary dependence on gauge parameters, has been constructed. In the same paper the explicit form of the corresponding constraint algebra was obtained. We shall use the results of Ref. [9].

This paper is organized in the spirit of [7]. In Section 1 we construct Lagrangian BRST formalism for gauge invariant systems with arbitrary dependence on gauge parameters. By this we consider the gauge transformations, depending only on the velocity phase space coordinates. Section 2 is devoted to Hamiltonian (BFV) BRST formalism for considered mechanical systems. In Section 3 we prove the equivalence between Lagrangian and Hamiltonian BRST formalisms and present explicit connection of gauge-fixing terms of the two
approaches.

In this paper we restrict us to the initial gauge invariant systems being bosonic, but the generalization of our results to the case of mechanical systems, described by both even and odd variables, is not a difficult problem [13, 14].

We assume the summation over repeated indexes, and all the partial derivatives to be the left partial derivatives.

2 Lagrangian BRST formalism

Let the velocity phase space [10] be described by the set of generalized coordinates \( q^r \) and generalized velocities \( \dot{q}^r \), \( r = 1, \ldots, R \). Consider on this space the mechanical system given by Lagrangian \( L(q, \dot{q}) \) having the symmetry under the gauge transformations of the form

\[
\delta_\varepsilon q^r = \sum_{k=0}^{N} (\varepsilon^k \psi_{\alpha}^r(q, \dot{q})), \quad \alpha = 1, \ldots, A,
\]

(2.1)

where \( \varepsilon^\alpha \) are arbitrary infinitesimal functions of time, and \( N > 1 \). Hence, we have

\[
\delta_\varepsilon L = \frac{d}{dt} \Sigma_\varepsilon.
\]

(2.2)

As well as in Ref.[9], the following notations are used in this paper: integers within ordinary brackets (parentheses) over characters display an order of time derivative of corresponding functions, and all the integers within square brackets (both subscripts and superscripts of characters) just mark the functions, simply giving them numbering.

Equations of motion (Lagrange equations) of the system are differential equations of the form

\[
L_r(q, \dot{q}, \ddot{q}) \equiv W_{rs}(q, \dot{q})\dot{q}^s - R_r(q, \dot{q}) = 0,
\]

(2.3)

where

\[
R_r(q, \dot{q}) = \frac{\partial L(q, \dot{q})}{\partial q^r} - \dot{q}^s \frac{\partial^2 L(q, \dot{q})}{\partial q^s \partial \dot{q}^r},
\]

(2.4)

\[
W_{rs}(q, \dot{q}) = \frac{\partial^2 L(q, \dot{q})}{\partial \dot{q}^r \dot{q}^s}.
\]

(2.5)

The matrix \( W_{rs} \) is called the Hessian of the system.

From the symmetry equations (2.1),(2.2) we get the Noether identities

\[
\sum_{k=0}^{N} (-1)^k q^k \frac{d}{dt} \left( \psi_{\alpha}^r L_r \right) = 0,
\]

(2.6)

which express the functional dependence of the Lagrange equations. Using the Noether identities written in a more appropriate form

\[
\Lambda_{\alpha} = \psi_{\alpha}^r R_r - \dot{q}^s \frac{\partial \Lambda_{\alpha}}{\partial q^s},
\]

(2.7)

\[
\psi_{\alpha}^r W_{rs} = -\frac{\partial \Lambda_{\alpha}}{\partial \dot{q}^r},
\]

(2.8)
where $k = 0, 1, \ldots, N$ and $\Lambda_\alpha = \Lambda^{[N+1]}_\alpha \equiv 0$ by definition, we get for $\Sigma_\varepsilon$ the expression

$$\Sigma_\varepsilon = \delta_\varepsilon q^r \frac{\partial L}{\partial \dot{q}^r} + \sum_{k=0}^{N-1} \varepsilon^{[k]} \Lambda^{[N-k]}_\alpha.$$  \hspace{1cm} (2.9)

It follows from Noether identities (2.8) that the Hessian of the system is singular, and the Lagrange equations have no unique solution for any initial values of $q^r(t)$ and $\dot{q}^r(t)$. We suppose the null-vectors $\psi^r_\alpha(q, \dot{q})$, $\alpha = 1, \ldots, A$, of the Hessian to be linearly independent, and any null-vectors of the matrix $W_{rs}(q, \dot{q})$ to be linear combinations of the vectors $\psi^r_\alpha$. Hence, we have

$$\text{rank } W_{rs}(q, \dot{q}) = R - A, \quad \text{rank } \psi^r_\alpha(q, \dot{q}) = A$$  \hspace{1cm} (2.10)

for any values of $q^r$ and $\dot{q}^r$. Besides, let the gauge transformations (2.1) be nontrivial for any choice of arbitrary functions $\varepsilon^r(t)$ and any trajectory of the system. It can be shown that this condition is equivalent to the linear independence of the set formed by the vectors $\psi^r_\alpha$, $k = 0, 1, \ldots, N$.

The Lagrange equations are solvable with respect to only $R - A$ accelerations $\ddot{q}^r$, and, to determine the evolution of the system, the Lagrangian constraints

$$\psi^r_\alpha R_r = 0,$$  \hspace{1cm} (2.11)

must be kept. The latter follow directly from Eqs.(2.8),(2.3). Such relations restrict the possible values of $q^r$ and $\dot{q}^r$ and are called the primary Lagrangian constraints.

The stability condition for the primary Lagrangian constraints $\Lambda_\alpha = \psi^r_\alpha R_r$ gives rise to other Lagrangian constraints of the system. From the Noether identities we get that the complete set of the Lagrangian constraints of the system is given by the relations

$$\Lambda^k_\alpha(q, \dot{q}) = 0, \quad k = 1, \ldots, N.$$  \hspace{1cm} (2.12)

Suppose now that gauge transformations (2.1) form a closed gauge algebra. So, for any two sets of infinitesimal functions $\varepsilon_{1}^r(t)$ and $\varepsilon_{2}^r(t)$ we have the commutator of corresponding gauge transformations of type (2.1) to be of the same type

$$[\delta_{\varepsilon_{1}}, \delta_{\varepsilon_{2}}] q^r = \delta_{\varepsilon} q^r,$$  \hspace{1cm} (2.13)

where $\varepsilon^r$ are, in general, some functions of $\varepsilon_{1}^r$, $\varepsilon_{2}^r$ and the trajectory of the system.

Taking into account the linear independence of the vectors $\psi^r_\alpha$, $k = 0, 1, \ldots, N$, from Eqs.(2.13) and (2.7) we obtain the relations of the gauge algebra of the system

$$\frac{\partial}{\partial q^s} \psi^r_\alpha + \frac{\partial}{\partial \dot{q}^s} \psi^r_\alpha = 0.$$  \hspace{1cm} (3)
\[
- \psi^\beta_\alpha \frac{\partial}{\partial q^\alpha} \psi^\beta_\alpha \frac{\partial}{\partial q^\alpha} - \left( \frac{[N-n]}{[N-m+n]} \psi^\beta_\alpha \frac{\partial}{\partial q^\alpha} + \frac{[N-n]}{[N-m+n]} \psi^\beta_\alpha \frac{\partial}{\partial q^\alpha} \right) \frac{[N-n]}{[N-m+n]} \psi^\beta_\alpha \frac{\partial}{\partial q^\alpha} = 0
\]

\[
= \sum_{i=0}^{N} \sum_{j=0}^{[i]} \left( \frac{[n-j]}{[m-i]} \psi^\gamma_\alpha \frac{\partial}{\partial q^\alpha} \right)
\]

\[
+ \left[ [n] \frac{[1]}{[m]} \psi^\gamma_\alpha \right] \frac{\partial}{\partial q^\alpha} \left( [n] \frac{[1]}{[m]} \psi^\gamma_\alpha + 2 [n] \frac{[1]}{[m]} \psi^\gamma_\alpha + 2 [n] \frac{[1]}{[m]} \psi^\gamma_\alpha \right) \frac{[0]}{[k]} \psi^\gamma_\alpha \right],
\]

where \( n = 0, 1, \ldots, N + 1; m = 0, 1, \ldots, 2N + 1 \). Here \( \frac{A^\gamma_\alpha}{[k]} \) are some functions of the generalized coordinates \( q^r \), called the structure functions of the gauge algebras (remember that for the case of \( N = 1 \) [11, 12] the structure functions of the corresponding gauge algebras, in general, depend on both generalized coordinates and generalized velocities).

These functions satisfy the symmetry property

\[
\frac{A^\gamma_\alpha}{[k]} = - \frac{A^\gamma_\alpha}{[k]}, \quad l \leq k \leq l + 1,
\]

and connect the infinitesimal parameters of gauge transformations in (2.13) by the relation

\[
\varepsilon^\gamma = \sum_{k=0}^{N+1} \sum_{l=0}^{k} (k-l)^{\alpha} \frac{[0]}{1} \varepsilon^\gamma \frac{[l]}{2} \frac{A^\gamma_\alpha}{[k]}.
\]

From (2.13) we have that the only nonzero structure functions are \( \frac{A^\gamma_\alpha}{[k]} \), \( \frac{A^\gamma_\alpha}{[k]} \), \( \frac{A^\gamma_\alpha}{[k]} \), \( \frac{A^\gamma_\alpha}{[k]} \). Besides, as follows from Eq.(2.14), for the case of \( N > 1 \) all the structure functions are turned out to be constant, thus the terms within the square brackets in the r.h.s. of (2.14) are equal to zero.

Finally, let us single out the following relations from the gauge algebra

\[
\frac{[0]}{[k]} \psi^\gamma_\alpha \frac{\partial}{\partial q^\alpha} + \frac{[N-n]}{[N-m]} \psi^\gamma_\alpha \frac{\partial}{\partial q^\alpha} = 0
\]

and note that for \( k < N - 1 \) the r.h.s. of Eq.(2.17) is equal to zero because of the above properties of the structure functions.

From the Jacobi identities for the gauge transformations (2.1)

\[
\left( [\delta_{e_1} , [\delta_{e_2} , \delta_{e_3} ]] + [\delta_{e_2} , [\delta_{e_1} , \delta_{e_3} ]] + [\delta_{e_3} , [\delta_{e_1} , \delta_{e_2} ]] \right) q^r = 0,
\]

we get the relations

\[
\frac{[N-n]}{[N-m]} \frac{\partial}{\partial q^\alpha} A^\gamma_\alpha + \frac{[N-m]}{[N-n]} \frac{\partial}{\partial q^\alpha} A^\gamma_\alpha + \frac{[N-n]}{[N-m]} \varepsilon \frac{[l]}{[l+m]} \frac{\partial}{\partial q^\alpha} = 0.
\]
These relations, called the generalized Jacobi identities, give the necessary and sufficient conditions for existence of the structure functions of the gauge algebra.

To construct Lagrangian BRST formalism \[2\], enlarge the configuration space of the system, adding to (even) initial generalized coordinates \(q^r\) the set of odd variables \(c^\alpha, \alpha = 1, \ldots, A\), called the ghost variables, or simply the ghosts, and odd variables \(\bar{c}_\alpha\), called the antighosts. The ghosts and antighosts are endowed with the ghost numbers, respectively equal to 1 and \(-1\). Define the infinitesimal BRST transformations for initial coordinates as follows

\[
\delta_\lambda q^r = \sum_{k=0}^{N} \lambda^{(k)} c^{\alpha \{N-k\}} \psi^r_{\alpha}(q, \dot{q}),
\]  

(2.20)

where \(\lambda\) is the infinitesimal odd parameter.

Let \(s\) be an odd vector field, connected with BRST transformations (2.20) by the relation

\[
s(q^r) = \sum_{k=0}^{N} \lambda^{(k)} c^{\alpha \{N-k\}} \psi^r_{\alpha}(q, \dot{q}).
\]  

(2.21)

From the nilpotency condition for BRST transformations we get the commutator of the vector field \(s\) with itself has to be equal to zero

\[
[s, s] = 2s^2 = 0,
\]  

(2.22)

where the symbol \([, ,]\) means the generalized commutator of vector fields \([13, 14]\).

Using the relations of the gauge algebra we obtain from condition \(s^2(q^r) = 0\) the expression for BRST transformation of the ghosts

\[
s(c^\alpha) = -\frac{1}{2} \sum_{k=0}^{2} \sum_{\substack{l=0,1 \ k-l}} (c^{\alpha \{l\}} c^{\beta \{l\}} c^{\gamma \{l\}} A^{\beta \gamma \{l\}} b_{\alpha \{l\}})
\]  

(2.23)

From the Jacobi identities (2.19) we get \(s^2(c^\alpha) = 0\).

Introduce even auxiliary variables \(b_\alpha, \alpha = 1, \ldots, A\), and define BRST transformations for the antighosts as follows

\[
s(\bar{c}_\alpha) = b_\alpha.
\]  

(2.24)

Supposing that

\[
s(b_\alpha) = 0,
\]  

(2.25)

we directly get \(s^2(\bar{c}_\alpha) = 0\) and \(s^2(b_\alpha) = 0\).

To remove the degeneracy of the initial gauge invariant Lagrangian, one performs the ordinary BRST gauge-fixing procedure \([13]\). To this end, introduce on the enlarged velocity phase space odd function \(F\), having the ghost number \(-1\), and define a new Lagrangian by the relation

\[
L' = L + s(F).
\]  

(2.26)

Choose \(F\) in the most convenient form

\[
F = \bar{c}_\alpha \chi^\alpha(q, \dot{q}) + \frac{1}{2} \bar{c}_\alpha b_\gamma \gamma^\alpha \beta,
\]  

(2.27)
where $\gamma^{\alpha\beta}$ is some non–singular constant matrix, and

$$\chi^\alpha(q, \dot{q}) = \dot{q}^r \chi^\alpha_r(q) + \nu^\alpha(q).$$  \hfill (2.28)

Using BRST transformations of both even and odd variables, we get from (2.26) and (2.27) the expression

$$L' = L + b_\alpha \chi^\alpha + \frac{1}{2} b_\alpha b_\beta \gamma^{\alpha\beta} - \bar{c}_\alpha s(q^r) \frac{\partial \chi^\alpha}{\partial q^r} - \bar{c}_\alpha s(q^r) \frac{\partial \chi^\alpha}{\partial q^r}. \hfill (2.29)$$

Taking into account the equations of motion for the auxiliary variable $b_\alpha$, that are simple algebraic relations

$$b_\alpha = -\gamma_{\alpha\beta} \chi^\beta,$$  \hfill (2.30)

where $\gamma_{\alpha\delta} \gamma^{\delta\beta} = \delta^\beta_{\alpha}$, we may rewrite the expression for $L'$ in the form

$$L'' = L - \frac{1}{2} \chi^\alpha \gamma_{\alpha\beta} \chi^\beta - \bar{c}_\alpha s(q^r) \chi^\alpha_r + \bar{c}_\alpha s(q^r) \frac{\partial \chi^\alpha}{\partial q^r} - \frac{d}{dt} \left( \bar{c}_\alpha s(q^r) \frac{\partial \chi^\alpha}{\partial q^r} \right), \hfill (2.31)$$

where we use the notation $\chi^\alpha_r$ for the variational derivative $\frac{\partial \chi^\alpha}{\partial q^r} - \frac{d}{dt} \left( \frac{\partial \chi^\alpha_r}{\partial \dot{q}^r} \right)$.

Finally, removing from $L''$ the last term, which is a full time derivative, we obtain the BRST invariant Lagrangian of the form

$$L_B = L - \frac{1}{2} \chi^\alpha \gamma_{\alpha\beta} \chi^\beta - \bar{c}_\alpha s(q^r) \chi^\alpha_r + \bar{c}_\alpha s(q^r) \frac{\partial \chi^\alpha}{\partial q^r}. \hfill (2.32)$$

To obtain nondegenerate effective system, one needs to investigate the super-Hessian, corresponding to the final Lagrangian $L_B$. One can easily verify that the Lagrangian $L_B$ is nonsingular if and only if the matrix

$$v^\beta_\alpha = \psi^r_\beta \frac{\partial \chi^\alpha}{\partial q^r} = \psi^r_\beta \chi^\alpha_r$$  \hfill (2.33)

is nonsingular. We suppose that this is the case.

Let us obtain the BRST charge, corresponding to BRST symmetry of $L_B$. The initial gauge invariant Lagrangian is BRST invariant, hence, from the nilpotency of BRST transformations we get that $L'$ in Eq.(2.29) should be also BRST invariant. Note, that

$$s(L'') = s(L')|_{b_\alpha = -\gamma_{\alpha\beta} \chi^\beta}, \hfill (2.34)$$

and

$$s(L_B) = s(L'') + \frac{d}{dt} \left( s(\bar{c}_\alpha s(q^r) \frac{\partial \chi^\alpha}{\partial q^r}) \right). \hfill (2.35)$$

Hence, we have

$$s(L_B) = \frac{d}{dt} \Sigma_B, \hfill (2.36)$$

where

$$\Sigma_B = s(q^r) \frac{\partial L}{\partial q^r} + \sum_{k=0}^{N-1} c^{(k)} \Lambda_\alpha \gamma_{\alpha\beta} \chi^\beta + s(q^r) s(q^r) \frac{\partial^2 \chi^\alpha}{\partial q^r \partial q^r}. \hfill (2.37)$$
From Eqs. (2.36), (2.37) we get the expression for the BRST charge

$$q_B = \Sigma_B - s(q^r) \frac{\partial L_B}{\partial \dot{q}^r} - s(\bar{c}_a) \frac{\partial L_B}{\partial \dot{\bar{c}}_a} - \sum_{k=0}^{N-1} \left( s^{(k)} \sum_{l=k}^{N-1} (-1)^{l-k} \frac{d^{l-k}}{dt^{l-k}} \frac{\partial L_B}{\partial (l+)_{\alpha}} \right).$$

(2.38)

So, starting from the singular (gauge invariant) system, we have constructed the effective nonsingular BRST invariant Lagrangian (2.32) and obtained the expression for corresponding BRST charge (2.38). Note that, unlike the case of $N = 1 [6, 7, 8]$, we have now the system with higher order derivatives, since the Lagrangian $L_B$ contains time derivatives of the ghosts up to $N$-th order. This circumstance forces us to use further the Ostrogradsky formalism [7].

### 3 Hamiltonian BRST (BFV) formalism

Consider a mechanical system, defined by the Hamiltonian $h$ and the set of irreducible constraints of the first class $\varphi_a$ [13]. We have the constraint algebra with respect to the Poisson brackets of the form

$$\{h, \varphi_a\} = h^b_a \varphi_b,$$

(3.1)

$$\{\phi_a, \varphi_b\} = f^b_{ab} \varphi_c,$$

(3.2)

where $h^b_a$ and $f^c_{ab}$ are some functions of the phase space coordinates of the system. These functions called the structure functions of the constraint algebra.

To construct a Hamiltonian BRST formalism [3, 4] for the constrained system under consideration, let us enlarge the phase space, adding to (even) initial coordinates, describing its points, the set of odd variables $\theta^a, \pi_a$, associated with the constraints $\varphi_a$. We put $\theta^a$ to be the ghost variables with the ghost number 1, whereas $\pi_a$ – are canonically conjugate to them generalized ghost momenta, having the ghost number $-1$. The initial phase space coordinates have the ghost number equal to zero. We set the Poisson brackets of the odd variables to be of the form

$$\{\theta^a, \theta^b\} = 0, \quad \{\pi_a, \pi_b\} = 0,$$

(3.3)

$$\{\pi_a, \theta^b\} = -\delta^b_a.$$  

(3.4)

The principal ingredients of BFV formalism are the nilpotent (odd) BRST charge with the ghost number equal to 1, and the BRST invariant Hamiltonian, which is an even function, having the ghost number equal to zero. These functions are given on the extended phase space of even and odd canonical variables, the general form of BRST charge being represented by the following series

$$\Omega_B = \sum_{n \geq 0} [n] \Omega_B = \sum_{n \geq 0} [n] \Omega_{a_1 \ldots a_{n+1}} \theta^{a_{n+1}} \ldots \theta^{a_2} \pi_b \ldots \pi_{b_1},$$

(3.5)

where

$$[0] \Omega_{a_1} = \varphi_{a_1},$$

(3.6)
and the quantities \( \Omega_{a_1 \ldots a_{n+1}}^{b_1 \ldots b_n} \) \((n > 0)\) are determined by the nilpotency condition
\[
\{ \Omega_B, \Omega_B \} = 0. \tag{3.7}
\]
The BRST invariant Hamiltonian may be written in the form
\[
H_A = \sum_{n \geq 0} [n] H_A = \sum_{n \geq 0} [n] H_{a_1 \ldots a_n}^{b_1 \ldots b_n} \theta^{a_1} \ldots \theta^{a_n} \pi_{b_1} \ldots \pi_{b_n}. \tag{3.8}
\]
Assuming, that
\[
H = h, \tag{3.9}
\]
we can find the quantities \( [n] H_{a_1 \ldots a_n}^{b_1 \ldots b_n} \) \((n > 0)\) from the BRST invariance condition for \( H_A \)
\[
\{ \Omega_B, H_A \} = 0. \tag{3.10}
\]
The general theorem of existence of the higher order structure functions \( \Omega_{a_1 \ldots a_{n+1}}^{b_1 \ldots b_n} \) and \( [n] H_{a_1 \ldots a_n}^{b_1 \ldots b_n} \) of BFV formalism has been proved in Ref.[4]. In particular, we have for \( n = 1 \)
\[
\Omega_B = -\frac{1}{2} f_{abc} \theta^b \theta^a \pi_c, \quad [1] H_A = h_\theta \theta \pi_{\theta b}. \tag{3.11}
\]
From the nilpotency of the BRST charge we get that Eq.(3.10) defines \( H_A \) only up to BRST exact term. Hence, the general form of the BRST invariant Hamiltonian is given by the expression
\[
H_B = H_A - \{ \Omega_B, \Psi \}, \tag{3.12}
\]
where \( \Psi \) is an odd function, having the ghost number equal to \(-1\). Thus, the gauge–fixing procedure within the framework of Hamiltonian BRST formalism consists of the choice of \( \Psi \)-function.

Hamiltonian formalism for the gauge invariant system, considered in the previous Section, has been constructed in Ref.[9]. In the same paper the explicit relations of the constraint algebra were obtained. Let us briefly recall the results of [9], that are necessary for the further consideration.

Introduce \( 2R \)-dimensional phase, the points of which are described by the generalized coordinates \( q^r \) and generalized momenta \( p_r, r = 1, \ldots, R \). Suppose the latter to be canonically conjugate pairs
\[
\{ q^r, p_s \} = \delta^r_s, \tag{3.13}
\]
and define a usual mapping of the velocity phase space to the phase space as follows
\[
p_r(q, \dot{q}) = \frac{\partial L(q, \dot{q})}{\partial \dot{q}^r}. \tag{3.14}
\]
From the gauge invariance of the system we get that this mapping is singular. The image of the velocity phase space under the mapping, given by (3.14), is a \((2R - A)\)-dimensional surface in the phase space, the points of which may be described by the following relations
\[
\Phi_\alpha(q, p) = 0, \quad \alpha = 1, \ldots, A, \tag{3.15}
\]
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where the functions \( \Phi_\alpha \) are functionally independent. Hence, we have introduced by Eq.\((3.15)\) the primary constraints of the system \([15]\) and, respectively, the primary constraint surface \([16, 11, 7, 9]\).

It can be shown that
\[
\frac{\partial [0] \Phi_\alpha}{\partial p_r}(q, p(q, \dot{q})) = -[0] u_\alpha^\beta(q, \dot{q}) \psi_\beta^r(q, \dot{q}), \tag{3.16}
\]
where \( u_\alpha^\beta \) is nonsingular matrix. We choose this matrix to be equal to the inverse one of the matrix \( v_\alpha^\beta(q, \dot{q}) \), given by Eq.\((2.33)\):
\[
v_\alpha^\delta u_\delta^\beta = \delta_\alpha^\beta, \quad v_\alpha^\beta = \chi_r^r \psi_\alpha^r. \tag{3.17}
\]

Let \( F(q, p) \) be a function defined on the phase space. There exists a function \( f(q, \dot{q}) \) on the velocity phase space, such that
\[
f(q, \dot{q}) = F(q, p(q, \dot{q})). \tag{3.18}
\]
In this, the function \( f \) takes constant values at points of the surfaces, given parametrically in the form \([16]\)
\[
q^r(\tau) = q^r, \tag{3.19}
\]
\[
\dot{q}^r(\tau) = \dot{q}^r + \tau^\alpha \psi_\alpha^r(q, \dot{q}). \tag{3.20}
\]
This fact may be easily expressed by the differential equations of the form
\[
\psi_\alpha^r \frac{\partial f}{\partial \dot{q}^r} = 0, \quad \alpha = 1, \ldots, A. \tag{3.21}
\]

But for a given function \( f(q, \dot{q}) \) on the velocity phase space, there not always exists a corresponding function \( F(q, p) \) on the phase space, which is related to \( f \) as follows
\[
F(q, p(q, \dot{q})) = f(q, \dot{q}). \tag{3.22}
\]
Eq.\((3.21)\) gives the necessary conditions for the existence of the function \( F(q, p) \). We assume, following Ref.\([3]\), that these relations are also sufficient conditions for Eq.\((3.22)\) to be valid. It means that we restrict us to the systems for which any point of the primary constraint surface \((3.15)\) is the image of an unique surface of the form \((3.19), (3.20), (16)\).

In this case, for any function \( f(q, \dot{q}) \), which satisfies the equalities \((3.21)\) one can find a function \( F(q, p) \), connected with \( f \) by Eq.\((3.22)\). We shall call such a function \( f \) the projectable to the primary constraint surface, or simply projectable, and write
\[
F = f. \tag{3.23}
\]
We have by definition
\[
[0] \Phi_\alpha = 0, \tag{3.24}
\]
hence, any function $F$ of the form

$$F = F_0 + F^\alpha \Phi_\alpha,$$  \hspace{1cm} (3.25)

where $F_0$ satisfies (3.23) and $F^\alpha$ are some arbitrary functions, fulfils the same equality (3.23) as well. Indeed, the relation (3.23) determines the function $F$ only on the primary constraint surface and the solution of this equation is defined up to a linear combination of the primary constraints. Hence, the expression (3.25) gives the general solution of Eq.(3.23). Note that the standard extension method [11, 8, 9], we shall use here, will consist in the way to fix the specific form of functions $F_0$ and $F^\alpha$.

Introduce the energy function $E(q, \dot{q})$:

$$E = \dot{q}^r \frac{\partial L}{\partial \dot{q}^r} - L.$$  \hspace{1cm} (3.26)

This function is projectable to the primary constraint surface, hence we can define the Hamiltonian of the system by the relation

$$H = E.$$  \hspace{1cm} (3.27)

Note that the Lagrangian constraints of the system, given by Eq.(2.12), satisfy the conditions (3.21). This fact is a direct consequence of the Noether identities. Hence, we may define the set of functions on the phase space, corresponding to the Lagrangian constraints as follows

$$[k] \Phi_\alpha = [k] \Lambda_\alpha, \quad k = 1, \ldots, N.$$  \hspace{1cm} (3.28)

In Ref.[9] it has been shown that the functions $[k] \Phi_\alpha$ are the secondary Hamiltonian constraints of $k$-th stage. Note again that the Eqs.(3.27) and (3.28) determine, respectively, the Hamiltonian and the constraints of the system only on the primary constraint surface. To get these functions and corresponding constraint algebra (with respect to the Poisson brackets), it is necessary to define the way of extension of functions from the primary constraint surface to the total phase space. It may be performed e. g. within the framework of the standard extension [11, 9]. Remember that various extensions differ from each other by a linear combination of the primary constraints. Here we do not discuss the conditions for the existence of the standard extension, assuming that they are valid, but refer to Ref.[11, 9].

Function $F(q, p)$ is called the standard if it satisfies the relations

$$\chi_\alpha^r \frac{\partial F}{\partial p_r} = 0,$$  \hspace{1cm} (3.29)

where the vectors $\chi_\alpha^r(q)$ are dual to the vectors $u^\alpha_\beta \psi_\beta^r(q, \dot{q})$. Let the Hamiltonian $H$ and constraints $[k] \Phi_\alpha$ be the standard functions. Besides, we choose the so-called standard primary constraints, satisfying the equalities

$$[0] \Phi_\alpha = 0, \quad \frac{\partial [0] \Phi_\alpha}{\partial p_r} = -([0] \psi_\alpha^r) \psi_\beta^r.$$  \hspace{1cm} (3.30)
where, and hereafter, the symbol \((f)^0\) denotes the standard Hamiltonian analog for corresponding function \(f\). Then, the constraint algebra of the system under consideration in the standard extension is given by the relations \[3.31\]

\[\{\Phi^0_\alpha, \Phi^0_\beta\} = \frac{\partial \Phi^0_\alpha}{\partial p_r} \chi^r_{rs} \frac{\partial \Phi^0_\beta}{\partial p_s} \Phi^0_\gamma,\]

\[3.32\]

\[\{\Phi^k_\alpha, \Phi^0_\beta\} = \left(\frac{\partial \Phi^0_\beta}{\partial q^{k}}\right)^0 \frac{\partial \Phi^0_\beta}{\partial p_s} \Phi^0_\gamma,\]

\[3.33\]

\[\{\Phi^k_\alpha, \Phi^k_\beta\} = \left(\frac{\partial \Phi^0_\beta}{\partial q^{k}}\right)^0 \frac{\partial \Phi^0_\beta}{\partial p_s} \Phi^0_\gamma,\]

\[\{H, \Phi^0_\alpha\} = (\Phi^0_\alpha)^0 \frac{\partial \Phi^0_\beta}{\partial p_s} \frac{\partial \Phi^0_\beta}{\partial p_s} \Phi^0_\gamma,\]

\[\{H, \Phi^k_\alpha\} = (\Phi^k_\alpha)^0 \frac{\partial \Phi^0_\beta}{\partial p_s} \frac{\partial \Phi^0_\beta}{\partial p_s} \Phi^0_\gamma,\]

where \(k, l = 1, \ldots, N > 1, i > N - k - l\), and we use the same notations for \(\mu^\alpha(q, \dot{q}), u^k(q, \dot{q})\) and \(\chi^\alpha_{rs}\) as in Ref.\[3\]:

\[\mu^\alpha = \dot{q}^r \chi^\beta_{rs} u^\alpha_{\beta}, \quad u^\alpha_{\beta} = \psi^r \chi^\gamma \psi^\alpha_{\gamma}, \quad \chi^\alpha_{rs} = \frac{\partial \chi^\alpha_{rs}}{\partial q^s} - \frac{\partial \chi^\alpha_{rs}}{\partial q^r}.\]

Thus, we have the constraint system of the first class, and one can apply to it the general BFV formalism, given in the beginning of this Section. To this end, let us extend the initial phase space by adding to the canonical pairs \(q^r, p_r\) the set of odd ghost coordinates \(\eta^\alpha\) and ghost momenta \(\pi^\alpha\), \(k = 0, \ldots, N, \alpha = 1, \ldots, A\), having, the ghost numbers, respectively, 1 and \(-1\). We suppose the non-zero Poisson brackets for the ghost variables to be of the form

\[\{\pi^\alpha, \eta^\beta\} = -\delta^{kl} \delta^\beta_{\alpha}, \quad k, l = 0, \ldots, N.\]

The BRST charge \(\Omega_B\) and the BRST invariant Hamiltonian \(H_A\), corresponding to the system with standard constraints \(\Phi^0_\alpha\) and the standard Hamiltonian \(H\), may be written in the form

\[\Omega_B = \sum_{k=0}^{N} \eta^\alpha \Phi^k_\alpha + \Delta \Omega_B,\]

\[H_A = H + \Delta H_A,\]

where \(\Delta \Omega_B\) and \(\Delta H_A\) consist of the terms of \(n \geq 1\) according to Eqs.\[3.35\), \(3.36\).
Note that the BRST structure functions for \( n = 1 \) are given directly by the structure functions of the constraint algebra (3.31)–(3.36) according to Eq. (3.11). The BRST structure functions of order \( n = 2 \) are constructed by using of the Poisson brackets of the constraints and Hamiltonian with arbitrary standard functions. The corresponding expressions for these Poisson brackets has been calculated for the considered class of systems \((N > 1)\) in Ref. [9] (for the case of \( N = 1 \) see [6, 7, 8]).

In this paper we shall not perform calculations of the higher order BRST structure functions, but only prove in the next Section the equivalence between Lagrangian and Hamiltonian BRST formalisms, presented in the two previous Sections. Note that we will do it in the spirit of Ref. [7].

4 Relationship between Lagrangian and Hamiltonian approaches

To compare the above formalisms, let us construct Hamiltonian description for the system, given by the effective nonsingular BRST invariant Lagrangian \( L_B \) (4.32). Recall that it corresponds to the mechanical system with higher order derivatives. Hence one should use the Ostrogradsky approach (see e.g. [17]). To this end, let us introduce the set of odd variables, putting

\[
[k] \theta^\alpha = (N-k) c^\alpha, \quad k = 1, \ldots, N. \tag{4.1}
\]

Define the generalized momenta, corresponding to the system with the Lagrangian \( L_B \), as follows

\[
p_r = \frac{\partial L_B}{\partial \dot{q}^r} = \frac{\partial L}{\partial \dot{q}^r} - \chi^r_{\gamma} \partial_s (q^t) \chi_{r}^\alpha - \bar{c}^\alpha \partial_s (q^t) \chi_{r}^\alpha, \quad \chi^r_{\gamma}; \tag{4.2}
\]

\[
p^\alpha = \frac{\partial L_B}{\partial \bar{c}^\alpha} = \sum_{k=0}^{N} \chi^r_{\gamma} \psi_{\beta}^r \chi_{r}^\alpha, \tag{4.3}
\]

\[
[\beta] p_{\alpha} = \sum_{l=1}^{k} (-1)^{k-l} \frac{d^{k-l}}{dt^{k-l}} \left( \frac{\partial L_B}{\partial \bar{c}^{(N-l+1)}} \right) \chi_{r}^\alpha, \quad k = 1, \ldots, N. \tag{4.4}
\]

Remember that all the partial derivatives are understood as the left partial derivatives [13, 14].

From (4.3), (4.4) using Eqs. (3.17), (3.36) and taking into account definition (4.1), we get

\[
\dot{c}^\alpha = - \bar{p}^{\beta} \chi_{\gamma}^r \psi_{\beta}^r u_{\alpha}^{[\beta]}, \tag{4.5}
\]

\[
(N) c^\alpha = p^\beta u_{\alpha}^{[\beta]} - \sum_{k=1}^{N} \theta^\beta u_{\beta}^{[k]}. \tag{4.6}
\]

Let us introduce the projector

\[
\Pi^r_s = \delta^r_s - \chi^r_{\gamma} u_{\alpha}^{[\beta]} \psi_{\beta}^r, \quad \Pi^r_s \Pi^r_s = \Pi^r_s, \tag{4.7}
\]
and define for the singular Hessian \( W_{rs} \) the corresponding pseudo-inverse matrix \( W^{rs} \), which is uniquely determined by the relations \([8]\)

\[
W^{rt}W_{ts} = \Pi^r_s, \quad W^{rs}\chi^\alpha_s = 0.
\]

(4.8)

Representing the BRST transformations of the generalized coordinates \( q^r \) in the form

\[
s(q^r) = p^\beta u^\alpha_\beta q^r_\alpha + \sum_{k=1}^N \theta^\alpha \psi^s_\alpha \Pi^r_s,
\]

(4.9)

we may rewrite the expression for the generalized momenta \( p_r \) as follows

\[
p_r = \bar{M}_r + \{1\}_r,
\]

(4.10)

where we use the notations

\[
\{0\}_r = \frac{\partial L}{\partial \dot{q}^r} - \chi^\alpha_r \gamma^\alpha_\beta \chi^\beta,
\]

(4.11)

\[
\{1\}_r = \left( -p^\alpha \frac{\partial u^\alpha_\alpha}{\partial \dot{q}^r} + \sum_{k=1}^N \theta^\alpha \frac{\partial u^\alpha_\alpha}{\partial \dot{q}^r} \right) \bar{p}_\beta + \sum_{k=1}^N \theta^\alpha \frac{\partial (\psi^s_\alpha \Pi^t_s)}{\partial \dot{q}^r} \lambda_{\beta t} \bar{c}_\beta
\]

\[
+ \left( p^\alpha \frac{\partial u^\alpha_\alpha}{\partial \dot{q}^r} + \sum_{k=1}^N \theta^\alpha \psi^t_\alpha \Pi^r_s \right) \chi^\gamma_{\rho t} \bar{c}_\gamma.
\]

(4.12)

From Eqs. (2.37), (2.38) using Eqs. (4.5), (4.6) and (4.10)–(4.12) we get the following expression for the BRST charge

\[
q^r_\alpha = \dot{q}^r_\alpha \chi^\beta + \sum_{k=1}^N \theta^\alpha \Lambda^\gamma_\alpha - s(q^r) \bar{M}_r - \sum_{k=1}^N s(\theta^\alpha) \bar{p}_\alpha + \frac{1}{2} s(q^r)s(q^r) \chi^\alpha \bar{c}_\alpha, \]

(4.13)

where \( s(q^r) \) is given by Eq. (4.13). Hence, we have expressed the BRST charge via generalized ghost coordinates and ghost momenta, and initial even variables \( q^r, \dot{q}^r \).

Now according to the Ostrogradsky formalism we introduce the energy function which corresponds to the effective Lagrangian \( L_B \) as follows

\[
E_B = q^r \frac{\partial L_B}{\partial \dot{q}^r} + \hat{c}_\alpha \frac{\partial L_B}{\partial \dot{c}_\alpha} + \sum_{k=0}^{N-1} (N-k) \chi^\alpha_{k+1} \sum_{l=1}^{k+1-l} (-1)^{k+1-l} \frac{d^{k+1-l}}{dt^{k+1-l}} \left( \frac{\partial L_B}{\partial \chi^\alpha_{N-l}} \right) - L_B.
\]

(4.14)

Using the gauge algebra relations (2.17) and properties of the projector \( \Pi^r_s \), from Eqs. (2.32) and (4.14) we see that the energy function \( E_B \), expressed through the Hamiltonian ghost variables (4.5), (4.6) and even coordinates of the initial velocity phase space, has the form

\[
E_B' = E + \hat{q}^r \Pi^r_s \{0\}_r + p^\alpha u^\alpha_\beta \{1\}_r + \sum_{k=1}^N \theta^\alpha \left( \bar{p}_\beta - \left( u^\beta_\alpha + \mu^\beta_\alpha \right) A^\gamma_\alpha \right) \bar{p}_\beta
\]

\[
+ \left( \chi^\alpha_\beta - \frac{1}{2} \gamma^\alpha_\beta \chi^\beta \right) \chi^\beta_\gamma \bar{c}_\gamma,
\]

(4.15)
where the functions $\hat{u}_\beta$ and $\mu$ are given by Eq. (3.36), and $E$ is the energy function, corresponding to the initial gauge invariant Lagrangian $L$.

Making use of the method, presented in Ref. [7], we may express the generalized velocities $\dot{q}^r$ in the form of expansion over even powers of the Hamiltonian ghost variables (the coefficients of such an expansion will be functions of even canonical variables $q^r, p_r$). In order to do this, denote the functions entering right-hand side of (4.10)-(4.12) by

$$M_r = \sum_{n=0,1} [n] M_r,$$

(4.16)

where the functions $[0] M_r$ do not depend on ghost variables $\theta^a, \pi_a$, while the functions $[1] M_r$ are quadratic in these variables. Let us denote the functions, expressing the variables $\dot{q}^r$ via $q^r, p_r$ and $\theta^a, \pi_a$ by $N^r(q, p, \theta, \pi)$. Thus, we have the equality

$$M_r(q, N(q, p, \theta, \pi), \theta, \pi) = p_r,$$

(4.17)

that may be shortly written as

$$M_r(N) = p_r.$$ 

(4.18)

Represent the functions $N^r$ in the form

$$N^r = \sum_{n \geq 0} [n] N^r,$$

(4.19)

where the functions $[n] N^r$ have the degree $2n$ in ghost variables $\theta^a$ and $\pi_a$. Expanding (4.18) over degrees of the ghost variables, we get

$$\sum_{n=0,1, k \geq 0} \left( \frac{1}{k!} \frac{\partial^k [n] M_r}{\partial \dot{q}^{s_1} \cdots \partial \dot{q}^{s_k}} \sum \frac{[l_1] N^{s_1} \cdots [l_k] N^{s_k}}{l_1 \cdots l_k} \right) = p_r.$$ 

(4.20)

In particular, we have

$$[0] M_r(N) = p_r,$$

(4.21)

$$\frac{[0] M_r(N)}{\partial \dot{q}^{s_1}} [1] N^{s_1} + [1] M_r(N) = 0.$$ 

(4.22)

Recall that the functions $[0] N^r$ have the following important property [7]. Let a function $f(q, \dot{q})$ be projectable, so that

$$\psi^r_\beta \frac{\partial f}{\partial \dot{q}^r} = 0.$$ 

(4.23)

Consider the function $F$ connected with $f$ by the relation

$$F = f[N].$$

(4.24)
From (4.21) it follows that
\[ f = F(M). \]  
(4.25)

Using this equality, it is easy to show that
\[ \chi^r_\alpha \frac{\partial F}{\partial p_r} = 0. \]  
(4.26)

Hence, the function \( F \) is a standard function. For any standard function \( F \) we have
\[ F(M) = F(\partial L/\partial \dot{q}^r). \]  
(4.27)

Therefore, if the function \( f \) satisfies the conditions (4.23), then
\[ f(N) = f^0. \]  
(4.28)

Introduce the notation
\[ G_{rs} = \frac{\partial M_r}{\partial q^s}. \]  
(4.29)

From (4.11) it follows that
\[ G_{rs} = W_{rs} - \chi^r_\alpha \gamma_{\alpha \beta} \chi^s_\gamma. \]  
(4.30)

It is easy to check that the matrix \( G_{rs} \) is nonsingular. Actually, we have
\[ G_{rt} G_{ts} = \delta^t_r, \quad G_{rs} = W_{rs} - \psi^r_\gamma u^0_\gamma \gamma_{\alpha \beta} u^0_\beta \psi^s_\gamma. \]  
(4.31)

From (4.22) we now get the equality
\[ N^r(M) = -G^{rs} M_s. \]  
(4.32)

Finally, using Eqs. (4.28), (4.32) from (4.13) we get the following expression for the BRST charge
\[ Q_B = p^a \Phi^a + \sum_{k=1}^N \theta^a \Phi^a + \sum_{k=1}^N \left( u^0_\beta \left[ A_{N-k+1} \right]_{\gamma} \right) \left[ A_{N-l+1} \right]_{\alpha} p^\beta \theta^\alpha \bar{p}^\gamma \]  
\[ + \frac{1}{2} \sum_{k,l=1}^N \left( \frac{\partial \Phi^a}{\partial p_r} \chi^r_{\alpha \beta} \frac{\partial \Phi^b}{\partial p_s} \right) p^a p^b \bar{c}_\gamma + \sum_{k=1}^N \left( \frac{\partial \Phi^a}{\partial p_r} \chi^r_{\alpha \beta} \frac{\partial \Phi^b}{\partial p_s} \right) \theta^\alpha \theta^\beta \bar{p} \]  
\[ + \frac{1}{2} \sum_{k,l=1}^N \left( \frac{\partial \Phi^a}{\partial p_r} \chi^r_{\alpha \beta} \frac{\partial \Phi^b}{\partial p_s} \right) \theta^\alpha \theta^\beta \bar{c}_\gamma + \Delta Q_B, \]  
(4.33)
where $\Delta Q_B$ contains the terms having more than cubic powers in the ghost variables. Note that all the constraints and higher order structure functions in Eq. (4.33) turn out to be the standard functions. Besides, we have

$$\{Q_B, Q_B\} = 0.$$  \hfill(4.34)

Indeed, the vector field $s$, which defines the BRST transformations, satisfies the nilpotency condition (2.22), at least on the equations of motion, following from $L_B$. BRST charge $Q_B$ is nothing but the Hamiltonian analog of the vector field $s$. Hence, the Poisson brackets $\{Q_B, Q_B\}$ have to be constant. Since the ghost number of this constant is equal to 2, we get it to be zero.

Now it is quite natural to identify the ghost variables of the previous Section with those we have in this one by the following rules

$$p^\alpha = [0] \eta^\alpha, \quad q^\alpha = [k] \eta^\alpha,$$

$$\bar{c}_\alpha = [0] \pi^\alpha, \quad \bar{p}_\alpha = [k] \pi^\alpha,$$ \hfill (4.35, 4.36)

for any $k = 1, \ldots, N$. From Eqs. (3.38), (4.33) we now see that, using the arbitrariness in definition of the constraints, one can always get

$$Q_B = \Omega_B.$$  \hfill (4.37)

To proceed to the Hamiltonian, consider the following odd function, having the ghost number equal to $-1$,

$$\psi = \bar{c}_\alpha \left( \nu^\alpha - \frac{1}{2} \chi^\alpha \right).$$  \hfill (4.38)

Taking into account the equations of motion, following from $L_B$, we obtain the expression for BRST transformation of this function

$$s(\psi) = - \left( \nu^\alpha - \frac{1}{2} \chi^\alpha \right) \gamma_{\alpha \beta} \chi^\beta + s(q^r) \frac{\partial \nu^\alpha}{\partial q^r} \bar{c}_\alpha.$$  \hfill (4.39)

Using the above reasoning, we get the relation

$$\{\Psi, Q_B\}(M) = s(\psi),$$  \hfill (4.40)

where $\Psi$ is the Hamiltonian analog of odd function $\psi$. Explicitly, $\Psi$-function is given by the expression

$$\Psi = \bar{c}_\alpha \left( \nu^\alpha - \frac{1}{2} \gamma_{\alpha \beta} \left( [0] \Phi^\beta - \sum_{k=1}^N [k] \Phi^\beta \left( [0] A_{[N-k+1]} [1] \delta_{\gamma} \right) [1] \bar{p}_{\delta} \right) \right)$$

$$+ \frac{1}{2} \bar{c}_\gamma [\gamma_{\alpha \beta} \left( [0] \nu^\delta \left( [0] \psi^r_{\alpha \beta} \psi^s_{\gamma} \right) + \sum_{k=1}^N [k] \nu^\delta \left( [k] \psi^r_{\alpha \beta} \Pi^s_{B_s} [0] \psi^r_{\gamma} \right) \right) \chi_{rt} \bar{c}_\gamma \right) + \Delta \Psi,$$  \hfill (4.41)

where $\Delta \Psi$ consists of the terms of more than cubic powers in the ghost variables. Note that for the primary constraints, which are linear in generalized momenta $p_r$, this term $\Delta \Psi$ is equal to zero.

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Denote the Hamiltonian, corresponding to the effective Lagrangian $L_B$, by $H_B$, and consider the function $H_A$, connected with $H_B$ by the relation

$$H_B = H_A - \{\Psi, Q_B\}.$$  \hspace{1cm} (4.42)

Since the BRST transformations are the symmetry transformations for the Lagrangian $L_B$, we see that the relation

$$\{Q_B, H_B\} = 0$$  \hspace{1cm} (4.43)

is indeed valid. Hence, because of nilpotency of the BRST charge $Q_B$, the Hamiltonian $H_A$ is also BRST invariant

$$\{Q_B, H_A\} = 0.$$  \hspace{1cm} (4.44)

Now, using the technique of Ref.[7] for the BRST charge (4.13), (4.33) from Eq.(4.15) we obtain that

$$H_A = H + p^\alpha \left( u^\beta_{\alpha} \right) [0]_\beta \bar{p} + \sum_{k=1}^{N} \theta^\alpha \left( p^{k+1} \bar{p} \alpha - \left( u^\beta_{\alpha} + \mu^\delta \left( A \right) [1]_\beta \bar{p} \right) \right)$$

$$+ p^\alpha \left( \frac{\partial H}{\partial p_r} \chi_{rs} \frac{\partial \Phi_{\alpha}}{\partial p_s} \right) \bar{c}_{\beta} + \sum_{k=1}^{N} \theta^\alpha \left( \frac{\partial H}{\partial p_r} \chi_{rs} \frac{\partial \Phi_{\alpha}}{\partial p_s} \right) \bar{c}_{\beta} + \Delta H_A,$$  \hspace{1cm} (4.45)

where $\Delta H_A$ contains the terms having more than quadratic powers in the ghost variables. In this, the Hamiltonian $H$ and all the structure functions in Eq.(4.45) are the standard functions. Comparing Eqs.(3.39) and (4.45), using the identification rules (4.35), (4.36) and taking into account the arbitrariness in definition of the Hamiltonian and constraints we see that one can always construct the BRST invariant Hamiltonian within the framework of BFV formalism in such a way, that it will coincide with the Hamiltonian, obtained from the effective BRST invariant Lagrangian.

5 Conclusion

We have constructed both Lagrangian and Hamiltonian BRST formalisms for the systems having the gauge symmetry under transformations (2.1), forming closed gauge algebra, and proved the equivalence of these two approaches. In this way, we have also obtained an explicit relation between the so-called gauge fermions, which remove the degeneracy of the system within the framework of Lagrangian and Hamiltonian BRST formalisms. Besides, having used the Ostrogradsky formalism we shown the correspondence between the ghosts with higher order time derivatives of the Lagrangian approach and the canonical ghosts of the BFV formalism. Note that the Lagrangian BRST charge written in the terms of the Hamiltonian variables has been expressed through the standard constraints. We observed the same appearance of the standard extension in Refs.[3, 7, 8], where the case of $N = 1$ with both closed and open gauge algebras had been considered.

Obviously, to simplify the calculations it is desirable to suppose the quantities $\chi_{rs}$ from Eq.(3.36) to be equal to zero for any values of the velocity phase space coordinates $q_r, \dot{q}_r$ (i.e. globally). It can be shown [3] that there exists the corresponding choice of the vectors
$\chi^\alpha_r$ in general if, and only if, the vector fields $\frac{\partial}{\partial q^r} \psi^\alpha_r \partial / \partial q^r$ form an abelian Lie subalgebra of the gauge algebra.

Note finally that our consideration, done on the classical level, allows us to prove the equivalence between Lagrangian and Hamiltonian BRST formalisms on quantum level as well.

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