PAPER

Effect of electric charge on anisotropic compact stars in conformally symmetric spacetime

Ksh Newton Singh¹, Piyali Bhar², Farook Rahaman³, and Neeraj Pant⁴

1 Department of Physics, National Defence Academy, Khadakwasla, Pune-411023, India
2 Department of Mathematics, Government General Degree College, Singur, Hooghly, West Bengal-712409, India
3 Department of Mathematics, Jadavpur University, Kolkata-700032, West Bengal, India
4 Department of Mathematics, National Defence Academy, Khadakwasla, Pune-411023, India

E-mail: rahaman@iucaa.ernet.in

Keywords: general relativity, compact stars, conformal symmetry

Abstract
We obtain a new class of interior solutions for charged anisotropic stars which admits non-static conformal motion. The Einstein–Maxwell field equations are solved by taking a physically reasonable choice for the $g_{rr}$ metric co-efficient and a suitable expression for charge density. From the analysis we have shown that there is a central singularity on space–time by calculating Kretschmann scalar though the central density and central pressure are finite. The emphasis are given on the causality condition and the stability analysis of the model. The masses and radii of the compact stars PSR J1614-2230, PSR J1903 + 327 and LMC X-4 obtained from the model are well match with the observed values provided by Gangopadhyay et al (2013 Mon. Not. R. Astron. Soc. 431 3216)

1. Introduction
To study the model of compact star is always an immense interest to the researchers since astronomers now believe that stars were the first large objects to form in the early universe. According to recent theoretical advances it is well known that at a very high density of the core ($\sim 10^{15}$ gm cm$^{-3}$) the pressure inside a fluid sphere shows anisotropic behavior [1], i.e., it has two components: radial pressure $p_r$ and tangential pressure $p_t$. The existence of a solid core or by the presence of type 3A superfluid [2], different kinds of phase transitions [3], pion condensation [4] may occur anisotropy. A collective works devoted to the study of anisotropic spherically symmetric general relativistic configurations are presented by Bowers and Liang [5]. Heinze and Hillebrandt [6] studied fully relativistic, anisotropic neutron star models at high densities and have shown that for arbitrary large anisotropy there is no limiting mass for neutron stars, but the maximum mass of a neutron star still lies beyond $3–4M_{\odot}$. By assuming the condition of a vanishing Weyl tensor, Hernandez and Nuñez [7] presented a general method for obtaining static anisotropic spherically symmetric solutions satisfying a nonlocal equation of state (EoS). Based on a particular form of the anisotropy factor, a class of exact solutions of Einsteins gravitational field equations describing spherically symmetric and static anisotropic stellar type configurations was obtained by Mak and Harko [8]. A variety of simple models for strange stars was constructed and compared by Avellar and Horvath [12]. Bini et al [12] studied the perturbation of white dwarf by using the ‘effective geometry’ formalism which was described as a self-gravitating fermion gas with completely degenerate relativistic EoS of barotropic type. Pagliara et al [12] studied the hydrodynamic simulation under the hypothesis that the conversion of a hadronic neutron star into a strange star is a combustion process and they also calculated the neutrino signal which is expected when a neutron star, decay into a strange quark star. Malaver [12] obtained a compact relativistic objects by considering Van der Waals modified EoS with polytropic exponent for anisotropic matter distribution. Petri [13] proposed a new class of solutions to the classical field equations of general relativity with zero cosmological constant and the proposed model has a sharp, non-continuous boundary of the matter-distribution, which is accompanied by a membrane consisting of pure tangential pressure. Numerical solutions of Einsteins field equation describing static spherically symmetric...
conglomerations of a photon star with an EoS $\rho = 3p$ was reported by Schmidt and Homann [14]. Chan et al [15] studied in detail the role of local pressure anisotropy in the onset of instabilities and they also showed that small anisotropy drastically change the stability of the system. Dev and Gleiser [16] proposed several exact solutions for anisotropic stars by taking constant density. We have extensively studied the charged and uncharged model of compact stars in some of our earlier works [17–26]. To find the exact solution, a more systematic approach is to assume conformal symmetry of the space–time. Conformal mapping is translated by the following relationship $Lg = \psi g$. Where $L$ is the Lie derivative operator and $\psi$ is the conformal factor. By assuming the supposition of spherical symmetry and the existence of a conformal Killing vector (CKV), Mak and Harko [27] found a relativistic model of strange quark star. Esculpi and Alomá [28] developed an anisotropic relativistic charged model by assuming the existence of a CKV in static spherically symmetric spacetime. Bhar et al [29] provide a new class of interior solutions for anisotropic stars admitting conformal motion in higher dimensional noncommutative spacetime by choosing a particular density distribution function of Lorentzian type as provided by Nazari and Mehdipour [30, 31] under a noncommutative geometry. Anisotropic star model admitting conformal motion have been obtained by Rahaman et al [32, 33]. A charged gravastar admitting conformal motion has been studied by Usmani et al [34] and Bhar [35] has generalized this result in higher dimension.

Inspired by these earlier work in the present paper we want to model an anisotropic charged compact star which admits a non-static conformal symmetry. Our paper is arranged as follows: in section 2, the field equations are presented; in section 3, exact general solutions are deduced using non-static conformal symmetries; in sections 4–8, the physical properties of the model are described. In the next section, we matched our interior solution to the exterior R-N line element and finally some conclusions are made in section 10.

2. Interior space–time

To describe the interior of a static and spherically symmetry object in the canonical coordinate $x^\mu = (t, r, \theta, \phi)$, we take line element

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2),$$  \hspace{1cm} (1)

where $\nu$ and $\lambda$ being the functions of the radial coordinate $r$.

The Einstein field equations with cosmological constant can be written as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi T_{\mu\nu},$$  \hspace{1cm} (2)

We have assumed the matter distribution within the star is locally anisotropic and with some net electric charge. Therefore, the energy–momentum tensor is described by,

$$T_{\mu\nu} = (\rho + p_t)\gamma_{\mu}^\lambda \gamma_{\nu}^\nu - p_t g_{\mu\nu} + (p_r - p_t)\chi_{\mu}^\lambda \chi_{\nu}^\nu + \mathcal{E}_{\mu\nu},$$  \hspace{1cm} (3)

where $\rho$, $p_t$ and $p_r$ represents the matter density, radial and transverse pressure of the fluid distribution. The quantities $v^\mu$ and $\chi^\mu$ are four-velocity and the unit spacelike vector in the radial direction that satisfies $-v^\mu v_{\mu} = \chi^\mu \chi_{\mu} = 1$. Also, $\mathcal{E}_{\mu\nu}$ represents the electromagnetic stress–energy tensor defined as

$$\mathcal{E}_{\mu\nu} = \frac{1}{4\pi} \left( -F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right).$$  \hspace{1cm} (4)

Here $F_{\mu\nu} = A_{\mu;\nu} - A_{\nu;\mu}$ represents the electromagnetic field tensor that satisfies the Maxwell’s field equations given by

$$F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} + F_{\lambda\mu;\nu} = 0,$$  \hspace{1cm} (5)

$$\left( \sqrt{-g} F^{\mu\nu} \right)_{;\nu} = 4\pi \sqrt{-g} j^\mu = 4\pi \sqrt{-g} \sigma_0 v^\mu,$$  \hspace{1cm} (6)

where $A_\mu$ is the electromagnetic four-potential, $j^\mu$ is the four-current density and $\sigma_0$ is the proper charge density.

For a static fluid configuration, the non-zero components of the four-current density is $j^0$ and function of $r$ only because of spherical symmetry. On using (6) we get

$$F^{01} = -e^{(\nu + \lambda)/2} \frac{q(r)}{r^2},$$  \hspace{1cm} (7)

where $q(r)$ is the charge enclosed within the sphere of radius $r$ and is defined as

$$q(r) = 4\pi \int_0^r e^{\nu/2} \sigma_0 \xi^2 d\xi.$$  \hspace{1cm} (8)

Assuming $G = c = 1$, for the line element (1) and the matter distribution (3) the Einstein–Maxwell field equations are written as,
8 \pi \rho = \frac{1 - e^{-\lambda}}{r^2} + \frac{e^{-\lambda} \lambda'}{r} - E^2, \quad (9)
8 \pi p_r = \frac{e^{-\lambda} - 1}{r^2} + \frac{e^{-\lambda} \nu'}{r} + E^2, \quad (10)
8 \pi p_r = -\frac{e^{-\lambda}}{2} \left( \frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'}{2r} + \frac{e^\lambda - 1}{r^2} \right) = E^2, \quad (11)

where \((\cdot)\)' represents differentiation with respect to the radial coordinate \(r\). From equations (10) and (11) the anisotropy factor \(\Delta\) is obtained as,

\[
\Delta = p_r - p_r = \frac{e^{-\lambda}}{8\pi} \left[ \frac{\nu''}{2} - \frac{\lambda' \nu'}{4} + \frac{\nu'^2}{4} - \frac{\nu'}{2r} + \frac{e^\lambda - 1}{r^2} \right] = -2E^2. \quad (12)
\]

3. Conformally symmetric charged anisotropic solution

To explore new solutions of Einstein’s field equations we are searching for symmetry using CKV. Such method was adopted by Moopanar and Maharaj [36] to find conformal Killing symmetries in spherical space–times. Recently, Radinschi et al [37] used non-static conformal symmetry for an anisotropic spherical configuration to study a classical model of electron. The non-static conformal Killing equation for (1) becomes

\[
\mathcal{L}_\xi g_{\mu\nu} = g_{\mu\nu} \delta^\xi + g_{3\mu} \mathcal{E}^\xi_{\mu} + \mathcal{E}_{\mu\nu} \mathcal{E}^\xi_{\nu} \quad \psi = \omega = \omega_0,
\]

where \(\mathcal{L}_\xi\) is the Lie derivative operator and \(\psi\) is the conformal factor. Also, the vector \(\xi\) generates the conformal symmetry and conformally mapped \(g_{\mu\nu}\) onto itself along \(\xi\). We follow Herrera et al [38, 39] assumption that \(\xi\) is non-static for a static \(\omega\) and written as

\[
\xi = \alpha(r, t) \partial_t + \beta(r, t) \partial_r, \quad \psi = \omega(r).
\]

Using (1), (13) and (14) we get

\[
\alpha = A + \frac{1}{2} k t, \quad \beta = B e^{-\lambda/2}, \quad \omega = B e^{-\lambda/2},
\]

\[
e^\nu = C^2 r^2 \exp \left[ -\frac{2k}{B} \int \frac{e^{\lambda/2}}{r} \, dr \right],
\]

where \(A, B, C, k\) are all constants.

To solve the above equations we have chosen \(A = 0\) and \(B = 1\) along with

\[
e^\lambda = \left( 1 + \frac{r^2}{R^2} \right)^4, \quad E^2 = \frac{qr^4}{R^2(r^2 + R^2)}.
\]

Now we can write (16) as

\[
e^\nu = C^2 r^2 \exp \left[ -\frac{k}{C^2} \int \frac{1 + \frac{r^2}{R^2}}{r} \, dr + C_2 \right] = r^2 C^2 \exp \left[ -\frac{k(r^4 + 4r^2R^2 + 4R^4 \log r)}{2R^4} + C_2 \right], \quad (18)
\]

where \(C_2\) is constant of integration and \(C^2 = C^2 e^{C^2}\). The behavior of \(e^\nu\) and \(e^\lambda\) are shown in figure 1.

Now using (18) and (17) we can find the expressions for density, pressure, anisotropy and charge density as

\[
8 \pi \rho(r) = -\frac{qr^4}{R^2(r^2 + R^2)} + \frac{8R^8}{(r^2 + R^2)^3} + \frac{1}{r^2} \left( 1 - \frac{R^8}{(r^2 + R^2)^3} \right), \quad (19)
\]

\[
8 \pi p_r(r) = \frac{qr^4}{R^2(r^2 + R^2)} + \frac{8R^8}{(r^2 + R^2)^3} - \frac{1}{r^2} \left( 1 - \frac{R^8}{(r^2 + R^2)^3} \right), \quad (20)
\]

\[
\Delta(r) = \frac{1}{r^2(R^2 + r^2)} \left[ R^2 \{ (k^2 + 1) r^{10} + 5(k^2 + 1) r^8 R^2 \right.
+ 10(k^2 + 1) r^6 R^4 + 10(k^2 + 1) r^4 R^6 + 5(k^2 - 1) r^2 R^8 \nonumber
+ (k^2 - 1) R^{10} - 2q r^6 (r^2 + R^2)^4 \}
\nonumber
+ \left. (k^2 - 1) R^{10} - 2q r^6 (r^2 + R^2)^4 \right], \quad (21)
\]

\[
8 \pi p_r(r) = 8 \pi p_r(r) + \Delta(r), \quad (22)
\]
Their behaviors at the interior are found to be well-behaved and free from any singularity, Figures 2–5. The EoS parameters \( p_r r \) and \( p_t t \) are also plotted in Figure 6 showing that they are \( \leq 1 \). The radial and transverse EoS are following quadratic behavior as shown in Figure 7.

We can write the density and pressure gradients as

\[
8\pi \frac{dp}{dr} = -\frac{2r}{R^2(r^2 + R^2)^5} [q^2(r^2 + 2R^2)(r^2 + R^2)^4 + R^2(r^8 + 6r^6R^2 + 15r^4R^4 + 20r^2R^6 + 50R^8)],
\]

\( q = 0.015 \) (Red)

\( q = 0 \) (Black)

\( \delta q = 0.0025 \)

\[
4\pi \sigma(r) = \frac{R^6}{q^3} \left[ \frac{3r^2 + 4R^2}{(r^2 + R^2)^3} - \frac{2}{(r^2 + R^2)^3} \right] \left( \frac{q^4}{r^2 + R^2} \right)^{1/2}.
\]

Their behaviors with respect to radial coordinates is shown in Figure 8.
The mass function, compactness parameter and surface red-shift can be found as

\[
m(r) = 4\pi \int_0^r \rho r^2 dr
\]

\[
= \frac{1}{2} \left( r - \frac{qr^2}{3} + \frac{qR^3}{5} + qR \tan^{-1} \left( \frac{r}{R} \right) - qr^2 - \frac{rR^8}{(r^2 + R^2)^4} \right),
\]

(27)
Figure 6. Variation of equation of state parameter with radial coordinates (\(R = 15, \ C_1 = 0.006, \ C_2 = 0.1, \ k = -1\) for \(q = 0\) to \(q = 0.015\) with an increment of 0.0025).

Figure 7. Relationship between \(p_r\), \(p_t\) and \(\rho\) i.e. Equation of state is plotted for \(R = 15, \ C_1 = 0.006, \ C_2 = 0.1, \ k = -1\) with \(q = 0\) to \(q = 0.015\) with an increment of 0.0025.

Figure 8. Variation of pressure and density gradients with radial coordinates (\(R = 15, \ C_1 = 0.006, \ C_2 = 0.1, \ k = -1\) for \(q = 0\) to \(q = 0.015\) with an increment of 0.0025).

\[
\frac{u(r)}{r} = 1 - \frac{q r^4}{5 R^2} + \frac{q r^2}{3} + \frac{q R^3}{r} \tan^{-1}\left(\frac{r}{R}\right) - q R^2 - \frac{R^8}{(r^2 + R^2)^4}.
\]
From the figure 9 we can observe that the maximum mass that can be obtained from the solution is about 2.5 \( M_\odot \) with compactness parameter of about 0.78. Also the the surface red-shift increases with increase in \( r_b \) with maximum value of about 1.08, figure 10.

4. Conditions for physical viability of the solutions

The following conditions are to be fulfilled by the solution in order to represent a physically viable configuration.

(i) The solution should be free from physical and geometric singularities, i.e. it should yield finite and positive values of the central pressure, central density and nonzero value of \( e_r^0 \) and \( e_1^r 0 \) = \( \rho = \rho_r \).

(ii) The causality condition should be obeyed i.e. velocity of sound should be less than that of light throughout the model. In addition to the above the velocity of sound should be decreasing towards the surface i.e. \( \frac{d \rho}{dr} \frac{d \rho}{d\rho} > 0 \) and \( \frac{d \rho}{dr} \frac{d \rho}{d\rho} > 0 \) or \( \frac{d \rho}{dr} \frac{d \rho}{d\rho} > 0 \) for \( 0 \leq r \leq \rho_b \), i.e. the velocity of sound is increasing with the increase of density and it should be decreasing outwards.

(iii) The adiabatic index, \( \gamma = \frac{\frac{\rho + \frac{\rho}{\rho}}{\frac{\rho}{\rho}}}{\rho} \) for realistic matter should be \( \gamma > \frac{4}{3} \).

(iv) The red-shift \( z \) should be positive, finite and monotonically decreasing in nature with the increase of the radial coordinate.

\[ z = e^{-\rho_r/2} - 1 = e^{\lambda_r/2} - 1 = \left(1 + \frac{r^2}{R^2}\right)^2 - 1. \] (29)
For a stable anisotropic compact star, \( \nu_0 \leq 1 \) must be satisfied [40].

(vi) Anisotropy must be zero at the center.

5. Properties of the solution

The central values of \( \rho_l, \rho_r, \rho \) and the Zeldovich’s condition can be written as

\[
8\pi p_{rc} = 8\pi p_r = \frac{4}{R^2} > 0,
\]

\[
8\pi \rho_c = \frac{12}{R^2} > 0,
\]

\[
\rho_c = \frac{1}{3} < 1.
\]

Here to get finite positive values of \( p_{rc} \) and \( \rho_c \), we must have real and finite value of \( R \).

Now the velocity of sound within the stellar object can be determined using (24) and (26) as

\[
\nu_r^2 = \frac{d\rho_l}{d\rho}, \quad \nu_t^2 = \frac{d\rho_t}{d\rho}.
\]

The solution satisfies the causality condition as the speed of sound is less than that of light, figure 11. The stability condition postulated by Herrera et al [40] i.e. \( |\nu_r^2 - \nu_t^2| \) should lie in between 0 and 1 which is again fulfilled, figure 12.

The relativistic adiabatic index is given by

\[
\Gamma_r = \frac{\rho + \rho_l}{\rho_r} \frac{d\rho}{d\rho}
\]

For a static configuration at equilibrium \( \Gamma_r \) has to be more than 4/3 [15] which is also obeyed by our solution, figure 13.

6. Conformal symmetry

Using the conformal equations, we have

\[
\psi^2 = B^2 e^{-\lambda} = B^2 \left( 1 + \frac{\nu_r^2}{R^2} \right)^{-4}.
\]

Since the value of \( B^2 > 0 \) and \( R^2 \geq 0 \), it also implies \( \psi^2 \geq 0 \). Hence the solution yields compact star configurations admitting conformal symmetry. Like wise in Farook et al [41] the conformal factor \( \psi \) is time independent, although we have used non-static conformal symmetry.
7. Kretschmann scalar and singularity

Any singularity on space–time can be confirmed by using Kretschmann scalar whether it is a true singularity or coordinate singularity. It is defined as

$$K_r = R^0_{\mu
u\alpha\beta}R_{\mu
u\alpha\beta}$$

$$= \frac{4}{r^4} \left( \frac{R^8}{(r^2 + R^2)^4} - 1 \right) + \frac{128R^8}{r^4(r^2 + R^2)^{10}} + \frac{8R^8}{r^4(r^2 + R^2)^{12}} [k(r^2 + R^2)^4$$

$$- R^4(r^2 + R^2)^2]^2 + \frac{4R^8}{r^2(r^2 + R^2)^3} [k(r^2 + R^2)^4 - R^4(r^2 + R^2)^2]^2].$$  (36)

The Kretschmann scalar divergence at $r = 0$, figure 14 showing the space–time possesses a singularity at the origin. Other than the central singularity, our solution is regular everywhere. An interesting property of the solution can be seen from the values of central density and pressure that they possess finite values even though its curvature diverges. This means that the solution has finite values of density and pressure at the singularity $r = 0$.

8. Static stability criterion

In all modeling problems, it is required to check whether the solution (numerical or exact solution) is satisfying the necessary criterion i.e. static stability criterion $|42, 43|$. In this criterion it is postulated that the total mass of the stellar structure must increase with respect to increase in central density or equivalently $\frac{\partial M}{\partial \rho_c} > 0$. However, whenever there is $\frac{\partial M}{\partial \rho_c} = 0$ we get the turning point and $\frac{\partial M}{\partial \rho_c} < 0$ for non-static region. For
the solution \( M = M(\rho_c) \) can be written as

\[
M(\rho_c) = \frac{1}{2} \left[ -\frac{1}{15} 2\pi q^2 \rho_c^5 - \frac{3q_0}{2\pi \rho_c} + \frac{3\sqrt{3} q}{2\sqrt{2} \pi^{1/2} \rho_c^{3/2}} \tan^{-1} \left( r_b \sqrt{\frac{2\pi \rho_c}{3}} \right) \right] - \frac{81r_b}{(2\pi \rho_c^2 \rho_c^3 + 3)^2} + \frac{q_0^2}{3} + r_b \right].
\]

(37)

\[
\frac{\text{d}M}{\text{d}\rho_c} = \frac{3q_0^2 (4\pi \rho_c^2 \rho_c^2 + 9)}{8\pi \rho_c^2 (2\pi \rho_c^2 \rho_c^2 + 3)} + \frac{324\pi \rho_c^2}{(2\pi \rho_c^2 \rho_c^2 + 3)^3} - \frac{1}{15} \pi q_0^2 \left( \frac{9\sqrt{3} q}{8\sqrt{2} \pi^{1/2} \rho_c^{3/2}} \tan^{-1} \left( r_b \sqrt{\frac{2\pi \rho_c}{3}} \right) \right) \geq 0.
\]

(38)

The static stability criterion is verified by the variation of mass \( M \) with \( \rho_c \) in figure 15. From the figure 15 we can conclude that the stability of stellar models is strongly depends on charge parameter \( q \). For neutral configurations \( q = 0 \), \( \text{d}M/\text{d}\rho_c = 0 \) at about \( \rho_c = 7.14 \times 10^{15} \) g cm\(^{-3}\). Therefore, \( \text{d}M/\text{d}\rho_c > 0 \) for \( \rho_c < 7.14 \times 10^{15} \) g cm\(^{-3}\) belongs to stable region and vice versa. However, for increasing \( q \), the stable range of \( \rho_c \) increases, henceforth inclusion of some electric charge improve stability of stellar configurations.
9. Boundary conditions

Assuming the exterior spacetime is the Reissner–Nodstrom solution which has to be match smoothly with the interior solution and is given by

\[
dz^2 = \left( 1 - \frac{2M}{r} + \frac{q^2}{r^2} \right) dt^2 - \left( 1 - \frac{2M}{r} + \frac{q^2}{r^2} \right)^{-1} dr^2 - r^2 d\Omega_2,
\]

where \( d\Omega_2 = d\theta^2 + \sin^2 \theta d\phi^2 \) represents the 2-sphere.

By matching the interior solution (1) and exterior solution (39) at the boundary \( r = r_b \) we get

\[
e^{\lambda_0} = 1 - \frac{2M}{r_b} + \frac{q_b^2}{r_b^2} = r_b^2 C_1^2 \exp \left[ -\frac{k(r_b^6 + 4r_b^2 R^2 + 4R^2 R^2 \log r_b)}{2R^4} \right] + C_2,
\]

\[
e^{-\lambda_b} = 1 - \frac{2M}{r_b} + \frac{q_b^2}{r_b^2} = \left( 1 + \frac{r_b^2}{R^2} \right)^{-4},
\]

\[
p_e(r_b) = 0.
\]

Using the boundary condition (40)–(42), we get

\[
q_b = \frac{R^2}{r_b^6} \left( \frac{r_b^6 + 4r_b^2 R^2 + 6r_b^2 R^4 + 4R^4}{(r_b^2 + R^2)^3} - \frac{8R^6}{(r_b^2 + R^2)^4} \right).
\]

\[
C_1^2 = \frac{1}{r_b} \left( 1 - \frac{2M}{r_b} + \frac{q_b^2}{r_b^2} \right) \exp \left[ k \left( \frac{r_b^6 + 4r_b^2 R^2 + 4R^2 R^2 \log r_b}{2R^4} \right) \right] - C_2,
\]

\[
R = r_b \left[ 1 - \frac{2M}{r_b} + \frac{q_b^2}{r_b^2} \right]^{1/8} \left( 1 - \left[ 1 - \frac{2M}{r_b} + \frac{q_b^2}{r_b^2} \right]^{1/4} \right)^{-1/2}.
\]

For physically stable static configuration, the energy condition like null energy condition, weak energy condition, strong energy condition and dominant energy condition needs to satisfy throughout the interior region i.e.

\[
\rho \geq 0; \quad \rho + p_r \geq 0; \quad \rho + p_t \geq 0; \quad \rho + p_r + 2p_t \geq 0; \quad \rho \geq \left(|p_t|, |p_i|\right).
\]

In the figure 16 it is clear that the solution satisfies all these energy conditions.

10. Results and conclusions

In the article, we have explored a new charge anisotropic solution admitting non-static conformal symmetry. We have assumed a time-dependent conformal symmetry to generate our solution, however, the obtained...
solution is a static or time-independent. All the physical parameters are well-behaved at the interior and possesses finite values at the center, however, at the center it contain a singularity with finite $p_c$ and $\rho_c$ as the Kretschmann curvature scalar diverges, figure 14. The resulting EoS and stability are strongly depends on the charge parameter $q$. For $q = 0$, the EoS behaves almost linearly, however, when $q$ increases the EoS start behaving in quadratic nature. Using the static stability criterion, the solution reveals that inclusion of more electric charge enhance the stability of the stellar configuration (figure 7). Also inclusion of more electric charge supports more masses with a bigger radius. It is also seen that inclusion of more electric charge increase the compactness parameter $2M/r_b$ within Buchdahl limit. In overall, the salient features of the solution can be summarized as: the solution holds all the energy conditions good, obeys causality condition, the relativistic adiabatic index is more than $4/3$ (and therefore static), well-behaved except at $r = 0$, stable with respect to Herrera et al [40] criterion and with respect to static stability criterion. Since the solution yields finite central density and pressure we used it to present some models of compact stars. Few models of compact stars are shown in table 1. The masses and radii of the modeled stars are well match with the observed values provided by Gangopadhyay et al [44].

The solution has distinct character as it contains a central singularity, however, possesses finite density and pressure at the center. The central singularity is in fact a geometrical or coordinate singularity. This can convince from the expression of Kretschmann scalar where one of the term contents $e^{-\nu/2}$. For this solution, the central value of $e^\nu$ is zero, hence $K = \infty$. Due to the solution possessing a central singularity, the high values of the surface red-shift may arise.

Acknowledgments

FR would like to thank the authorities of the Inter-University Centre for Astronomy and Astrophysics, Pune, India for providing the research facilities. FR is also thankful to DST-SERB for financial support. We are also grateful to the referees for their constructive suggestions.

ORCID iDs

Farook Rahaman @ https://orcid.org/0000-0003-4923-7079

References

[1] Ruderman R 1972 Annu. Rev. Astron. Astrophys. 10 427
[2] Kippenhahn R and Weigert A 1990 Stellar Structure and Evolution (Berlin: Springer)
[3] Sokolov A I 1980 Mon. Not. R. Astron. Soc. 217 361
[4] Sawyer R F 1972 Phys. Rev. Lett. 29 382–5
[5] Bowers R L and Liang E P T 1974 Astrophys. J. 188 657–65
[6] Heintzmann H and Hillebrandt W 1975 Astron. Astrophys. 38 51–5
[7] Hernandez H and Nunez L A 2004 Can. J. Phys. 82 29–51
[8] Mak M K and Harko T 2003 Proc. R. Soc. London A 459 393–408
[9] Perez-Garcia M A, Silk J and Stone J R 1995 Astrophys. J. 459 38–51
[10] Bini D, Cherubini C and Filippi S 2011 Gen. Rel. Grav. 43 239–53
[11] Petri M 2003 Gen. Rel. Grav. 35 795–803
[12] Malaver Manuel 2013 Am. J. Astron. Astrophys. 1 41–6
[13] Petri M 2003 arXiv:gr-qc/030603v3
[14] Schmidt H J and Homann F 2000 Gen. Relativ. Gravit. 32 919–31
[15] Chan R et al 1993 Mon. Not. R. Astron. Soc. 265 533
[16] Dev K and Gleiser M 2002 Gen. Rel. Grav. 34 1793–1818
[17] Singh K N et al 2016 Int. J. Mod. Phys. D 25 1650099
[18] Bhardwaj P, Singh K N and Manna T 2016 Astrophys. Space Sci. 361 284
[19] Singh K N and Pant N 2016 Astrophys. Space Sci. 361 (117)
[20] Singh K N, Pant N and Pradhan N 2016 Astrophys. Space Sci. 361 173
[21] Bhardwaj P and Ratanpal B S 2016 Astrophys. Space Sci. 361 217
[22] Singh K N, Bhardwaj P and Pant N 2016 Astrophys. Space Sci. 361 339
[23] Bhardwaj P and Govender M 2017 Int. J. Mod. Phys. D 26 1750053

Table 1. Masses and radii of few well-known compact star candidates.

| Stars               | $q$   | $R$(km) | $r_b$(km) | $M/M_\odot$ | $u$  | $\Sigma$ | $p_c$(dyne cm$^{-2}$) | $\rho_c$(g cm$^{-3}$) | Reference |
|---------------------|-------|---------|-----------|-------------|------|----------|------------------------|------------------------|-----------|
| PSRJ1614−2230       | 0.00198 | 11      | 9.69      | 1.97        | 0.406 | 2.15     | $1.59 \times 10^{36}$  | $5.31 \times 10^{35}$  | [44]      |
| PSRJ1903 + 327      | 0.0027 | 10      | 9.438     | 1.66        | 0.351 | 2.57     | $1.93 \times 10^{36}$  | $6.43 \times 10^{35}$  | [44]      |
| LMCX-4              | 0.0054 | 9       | 8.3       | 1.04        | 0.251 | 2.42     | $2.38 \times 10^{36}$  | $7.93 \times 10^{35}$  | [44]      |
[24] Bhar P, Singh K N and Manna T 2017 *Int. J. Mod. Phys.* D **26** 1750090
[25] Bhar P, Govender M and Sharma R 2017 *Eur. Phys. J.* C **77** 109
[26] Singh K N and Pant N 2016 *Eur. Phys. J.* C **76** 524
[27] Mak M K and Harko T 2004 *Int. J. Mod. Phys.* D **13** L49–56
[28] Esculpi M and Alomà E 2010 *Eur. Phys. J.* C **67** 521–32
[29] Bhar P, Rahaman F, Ray S and Chatterjee V 2015 *Eur. Phys. J.* C **75** 190
[30] Nozari K and Mehdipour S H 2009 *J. High Energy Phys.* JHEP03(2009)061
[31] Mehdipour S H 2012 *Eur. Phys. J.* Plus **127** 80
[32] Rahaman F, Jamil M, Sharma R and Chakraborty K 2010 *Astrophys. Space Sci.* **330** 249
[33] Rahaman F, Pradhan A, Ahmed N, Ray S, Saha B and Rahaman M 2015 *Int. J. Mod. Phys.* D **24** 155049
[34] Usmani A A, Rahaman F, Ray S, Nandi K K, Kuhfittig P K F, Rakib S and Hasan Z 2011 *Phys. Lett.* B **701** 388
[35] Bhar P 2014 *Astrophys. Space Sci.* **354** 457
[36] Moopanar S and Maharaj S D 2010 *Int. J. Theor. Phys.* **49** 1878
[37] Radinschi I, Rahaman F, Kalam M and Chakraborty K 2010 *Fizika* B **19** 125
[38] Herrera L et al 1984 *J. Math. Phys.* **25** 3274
[39] Herrera L and Ponce de Leon J 1985 *J. Math. Phys.* **26** 2302
[40] Herrera L and Santos N O 1997 *Phys. Rep.* **286** 53
[41] Rahaman F et al 2017 *Mod. Phys. Lett.* A **32** 1750053
[42] Harrison B K et al 1965 *Gravitational Theory and Gravitational Collapse* (Chicago: University of Chicago Press)
[43] Zeldovich Ya B and Novikov I D 1971 *Relativistic Astrophysics Vol 1: Stars and Relativity* (Chicago: University of Chicago Press)
[44] Gangopadhyay T et al 2013 *Mon. Not. R. Astron. Soc.* **431** 3216