Research about the Sensorless Vector Control of Permanent Magnet Synchronous Motor Based on Two-stage Filter Sliding Mode Observer

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Abstract. In order to overcome the disadvantages of installing mechanical sensors and the problems of chattering and low observation accuracy in traditional sliding mode observer sensorless control, a two-stage filter Sliding Mode Observer (SMO) is proposed in this paper. By collecting the current and voltage of the Permanent Magnet Synchronous Motor (PMSM), the SMO algorithm is realized by using the state equation of the motor in the synchronous stationary coordinate system; the position of the rotor is estimated by the arc tangent function, the observation accuracy of the rotor position is improved by increasing phase compensation; the variable cut-off frequency filter is introduced to make the Low Pass Filter (LPF) cut-off frequency can be self-adjusted with the change of rotational speed, which improves the estimation accuracy of rotor position at different rotational speeds. Kalman filter is introduced to form a two-stage filter with variable cut-off frequency LPF, which greatly weakens the chattering of the motor and reduces the observation error. Finally, the simulation is carried out under MATLAB/Simulink. The simulation results show that the PMSM vector control system with two-stage filter sliding mode observer has high estimation accuracy, good dynamic and steady-state performance, strong anti-jamming ability and robustness.

1. Introduction
In recent years, with the development of power electronics technology and control technology, Permanent Magnet Synchronous Motor (PMSM) is widely used in fan, electric vehicle and other fields because of its high power density, good reliability [1], simple structure and fast response.

In order to obtain the efficient control performance of PMSM, it is often necessary to obtain accurate rotor position and speed information of PMSM. Nowadays, the position and speed of PMSM are mainly obtained by installing encoder or resolver sensors in PMSM. However, the installation of mechanical sensors increases the size, weight and cost of PMSM, reduces the stability performance and limits its application in harsh environments. PMSM control technology without position sensor arises at the historic moment.

Sensorless control technology estimates the position and speed of the rotor by detecting the easily measured electrical signals in the motor windings and adopting a certain control algorithm. At present, PMSM sensorless control methods mainly include model reference adaptive algorithm, high frequency signal injection method, sliding mode observer algorithm and extended Kalman filter algorithm [2-5]. The high frequency signal injection method uses the salient pole effect of the motor rotor to inject...
pulsating or rotating high frequency signals. Through this method, the rotor position information of the motor at very low speed can be obtained. This method is complex and not suitable for medium and high speed operation, nor for hidden pole permanent magnet synchronous motor. Sliding Mode Control (SMC) is a discontinuous non-linear control system, that is, a switching characteristic that makes the structure of the system change at any time. The key to this method lies in the selection of sliding mode surface function and sliding mode gain, which not only ensures the convergence speed, but also avoids the problem of excessive chattering caused by excessive gain. Sliding Mode Control is a robust control method because it does not require high accuracy of the system model and is insensitive to parameter changes and external disturbances. In this paper, a PMSM model of sliding-mode observer based on two-stage filter is proposed. This model adds variable cut-off frequency filter and Kalman filter to the traditional sliding-mode observer. After two-stage filter, noise is eliminated, chattering is reduced, and observation accuracy is further improved, so the performance of the system is improved. Finally, a simulation model is built in Simulink to verify the superiority of the algorithm.

2. Design of SMO algorithm for permanent magnet synchronous motor

2.1. Mathematical model of permanent magnet synchronous motor

In this paper, three-phase surface mounted permanent magnet synchronous motor (PMSM) is selected. Its mathematical model in two-phase synchronous static coordinate system is as follows:

\[
\begin{bmatrix}
    u_α \\
    u_β
\end{bmatrix} =
\begin{bmatrix}
    R_s + L_s \frac{di_α}{dt} & 0 \\
    0 & R_s + L_s \frac{di_β}{dt}
\end{bmatrix}
\begin{bmatrix}
    i_α \\
    i_β
\end{bmatrix} + \begin{bmatrix}
    e_α \\
    e_β
\end{bmatrix}
\]

Type \([e_α, e_β]^T = [−ω_r ψ_f \sin θ_r, ω_r ψ_f \cos θ_r]\) is the extended back Electromotive Force(EMF) of the motor.

\(L_s\) is the stator inductance, \(R_s\) is the stator resistance, \([u_α, u_β]^T\) is the stator voltage, \([i_α, i_β]^T\) is the stator current, \(ω_r\) is the electrical angular velocity of the motor rotor, \(ψ_f\) is the rotor flux linkage of the motor [6].

2.2. Design of sliding mode observer

It can be seen from equation (1) that the extended back electromotive force of the motor is a sine wave and contains the rotor position \(θ_r\) information of the motor and the rotational speed \(ω_r\) information. When the rotational speed \(ω_r\) is faster, the back electromotive force is larger and vice versa. Therefore, the rotor position information of the motor can be obtained by observing the back electromotive force information.

The extended back-EMF of PMSM contains all the information of the rotor position and speed, so only when the extended back-EMF is obtained accurately can the information of the rotor position and speed be calculated. The principle of the sliding mode observer is to measure current and voltage, and estimate the back electromotive force according to the sliding mode algorithm, so that the rotor position and speed can be calculated. In order to observe the extended back-EMF of the motor with SMO, the voltage equation of equation (1) is rewritten to the state equation of current.

\[
\begin{bmatrix}
    \frac{di_α}{dt} \\
    \frac{di_β}{dt}
\end{bmatrix} = \begin{bmatrix}
    -R_s & 0 \\
    0 & -R_s
\end{bmatrix} \begin{bmatrix}
    i_α \\
    i_β
\end{bmatrix} + \begin{bmatrix}
    u_α \\
    u_β
\end{bmatrix} - \begin{bmatrix}
    e_α \\
    e_β
\end{bmatrix}
\]

(2)

In order to obtain the estimated value of extended back-EMF, we can construct the SMO equation as follows:

\[
\frac{d}{dt} \begin{bmatrix}
    \hat{i}_α \\
    \hat{i}_β
\end{bmatrix} = \begin{bmatrix}
    -R_s & 0 \\
    0 & -R_s
\end{bmatrix} \begin{bmatrix}
    \hat{i}_α \\
    \hat{i}_β
\end{bmatrix} + \begin{bmatrix}
    u_α \\
    u_β
\end{bmatrix} - \begin{bmatrix}
    \nu_α \\
    \nu_β
\end{bmatrix}
\]

(3)

Among them, \(i_α\) and \(i_β\) are the observed values of stator current, \(\hat{i}_α\) and \(\hat{i}_β\) are the stator current, \(u_α\) and \(u_β\) are the stator voltage, which is the control input of the observer, \(\nu_α\) and \(\nu_β\) are the preliminary estimates of back-EMF.
In order to further increase the estimation accuracy, the observed back-EMF value is sent to the SMO observer again, and then the difference between equation (2) is obtained to obtain the motor stator current estimation error equation:

\[
\frac{d}{dt}[\hat{i}_a] = \frac{1}{L_s} \begin{bmatrix} -R_s & 0 \\ 0 & -R_s \end{bmatrix} [\hat{i}_a] - \frac{1}{L_s} [\hat{e}_a] - \frac{1}{L_s} [v_{pa}]
\]

(4)

In the formula: \(i_a = i_a - i_a\), \(\hat{i}_a = i_a - i_a\) is the error of current observation; \(\hat{e}_a = \hat{e}_a - e_a\), \(\hat{e}_b = \hat{e}_b - e_b\) is the error of back electromotive force observation.

According to the sliding mode variable structure theory, the sliding mode surface function \(s(x) = [\hat{i}_a \quad \hat{i}_b]^T\), \(sgn(x)\) is a symbolic function. When \(\hat{i}_a = i_a - i_a = 0\) and \(\hat{i}_b = i_b - i_b = 0\), the state of the SMO observer remains on the sliding surface, and the equation (4) is simplified as follows:

\[
\begin{bmatrix} \dot{\hat{e}}_a \\ \dot{\hat{e}}_b \end{bmatrix} = - \begin{bmatrix} \nu_a \\ \nu_b \end{bmatrix} = - \begin{bmatrix} ksgn(i_a - i_a) \\ ksgn(i_b - i_b) \end{bmatrix}
\]

(5)

It can be seen from formula (5) that the value of extended back EMF is related to the sign function, so the SMC law is designed as follows:

\[
\begin{bmatrix} \nu_a \\ \nu_b \end{bmatrix} = \begin{bmatrix} ksgn(i_a - i_a) \\ ksgn(i_b - i_b) \end{bmatrix}
\]

(6)

In formula: k represents switching gain. The initial estimates of back EMF \(\nu_a\) and \(\nu_b\) can be transformed into continuous smooth estimates after filtering, so that the rotor position and speed can be estimated.

2.3. Rotor Position Estimation Based on Arbitrary Tangent Function

Formula (6) shows that the switching function contains back-EMF information, but because the actual control quantity is a discontinuous high-frequency switching signal, the estimated back-EMF value will be distorted to some extent, showing high-frequency discontinuity, containing large interference, and then the estimation accuracy will be greatly reduced. Therefore, in order to obtain continuous extended back-EMF estimates, we usually need a low-pass filter to filter the estimated back-EMF values and then observe the position and velocity information, so the estimated back-EMF is:

\[
\begin{bmatrix} \hat{e}_a \\ \hat{e}_b \end{bmatrix} = \frac{\omega_c}{s + \omega_c} \begin{bmatrix} ksgn(i_a - i_a) \\ ksgn(i_b - i_b) \end{bmatrix}
\]

(7)

However, when the equivalent control variable is filtered by low-pass filter, the estimated value of extended back-EMF will change in amplitude and phase [7-8], while the high-frequency switching signal is filtered out. Usually, we can obtain the position information of the rotor by the method of arctangent function, which can be obtained by formula (1):

\[
\hat{\theta}_{eq} = -\arctan \left( \frac{\hat{e}_a}{\hat{e}_b} \right)
\]

(8)

The estimated value of back-EMF obtained by low-pass filter filtering will cause phase delay, which will directly affect the estimation of rotor position. Therefore, in practical applications, we usually need to increase phase compensation to reduce the estimation error of rotor position caused by the delay of low pass filter, so as to improve the observation accuracy of SMO. Finally, the rotor position is obtained as follows:

\[
\hat{\theta}_r = \hat{\theta}_{eq} + \arctan \left( \frac{\hat{e}_a}{\hat{e}_b} \right)
\]

(9)

The estimated speed of the motor can be obtained from the extended back-EMF in Formula (1):

\[
\omega_e = \sqrt{\frac{\hat{e}_a^2 + \hat{e}_b^2}{\psi_f}}
\]

(10)

Through the above analysis, we can obtain the structure diagram of the traditional sliding mode observer shown in Figure 1.
3. Design of Sliding mode observer with two-stage filtering

3.1. Design of variable cut-off frequency filter

The high frequency component of the motor back EMF observation changes with the change of speed.

The cut-off frequency of LPF designed in Section 2.3 is fixed and can not adapt to the change of speed. When the speed of the motor is different, the filtering effect will be different. It is difficult to get the ideal filtering effect in each speed area of the motor, and the accuracy of rotor position estimation will be greatly reduced. Therefore, this section designs an LPF with a cut-off frequency to improve the filtering effect and the estimation accuracy of the rotor position.

The mathematical model of LPF is as follows:

\[
\begin{align*}
\frac{d \hat{e}_\alpha}{dt} &= \omega_c (v_\alpha - \hat{e}_\alpha) \\
\frac{d \hat{e}_\beta}{dt} &= \omega_c (v_\beta - \hat{e}_\beta)
\end{align*}
\]  

(11)

The transfer function form of the back EMF observation can then be obtained:

\[
\begin{align*}
\hat{e}_\alpha &= \frac{\omega_c}{s} (v_\alpha - \hat{e}_\alpha) \\
\hat{e}_\beta &= \frac{\omega_c}{s} (v_\beta - \hat{e}_\beta)
\end{align*}
\]  

(12)

The relationship between cut-off frequency of LPF with variable cut-off frequency and rotor speed is as follows:

\[
\hat{\omega}_c = k_f \omega_r + k_e
\]  

(13)

Among them, \(k_f\) is positive and \(k_e\) is positive constant. In order to make the system work properly at low speed, we can usually set \(k_e\) to a smaller positive number.

Variable cut-off frequency LPF is designed as follows:

\[
H(j\omega_r) = \frac{\hat{\omega}_c}{j\omega_r + \hat{\omega}_c}
\]  

(14)

It can be seen that the cut-off frequency \(\hat{\omega}_c\) will adjust itself with the change of speed.

3.2. Design of two-stage filter based on kalman

Because the back-EMF observation values \(\hat{e}_\alpha\) and \(\hat{e}_\beta\) obtained after LPF with variable cut-off frequency still contain some measurement noise, if \(\hat{e}_\alpha\) and \(\hat{e}_\beta\) are directly estimated, the estimation information of the final rotor position and speed will be inaccurate, so it can not be directly applied to high-performance motor driving occasions. In order to further improve the observation effect of SMO, improve the estimation accuracy of rotor position, minimize chattering and reduce observation error, this section designs a Kalman filter and LPF to form cascaded secondary filtering.

The Kalman filter can better reduce the deviation of the estimation result caused by the motor parameter error, and effectively reduce the measurement noise, measurement error and random interference. The back electromotive force first passes through the variable cutoff frequency LPF and then undergoes Kalman secondary filtering. It can filter out the high frequency ripple components in \(\hat{e}_\alpha\) and \(\hat{e}_\beta\). Finally, we can obtain more continuous smooth optimal observation signal values \(\hat{E}_\alpha\) and \(\hat{E}_\beta\). Bipolar filtering effectively reduces the estimation error and suppresses the system's...
Chattering makes the PMSM control system have better steady-state effects, dynamic response, and better working performance.

Deriving the back EMF equation in equation (1):

\[
\begin{align*}
\frac{d\hat{e}_a}{dt} &= -\omega_r^2\psi_f \cos \theta_r - \frac{d\omega_r}{dt}\psi_f \sin \theta_r \\
\frac{d\hat{e}_\beta}{dt} &= -\omega_r^2\psi_f \sin \theta_r + \frac{d\omega_r}{dt}\psi_f \cos \theta_r
\end{align*}
\]  

(15)

In the PMSM sensorless control mode, the sampling frequency is much smaller than the differential value of the actual motor speed. Therefore, it can be assumed that the motor speed change is 0 in a single sampling frequency, \(d\omega_r/dt = 0\).

In the PMSM sensorless control mode, during operation, the sampling frequency is much smaller than the differential change of the actual speed of the motor. Therefore, it can be assumed that the speed change of the motor is 0 in a single sampling frequency, that is \(d\omega_r/dt = 0\), substituting the back-EMF equation in equation (1), we get:

\[
\begin{align*}
\frac{d\hat{e}_a}{dt} &= -\omega_re_\beta \\
\frac{d\hat{e}_\beta}{dt} &= -\omega_re_a
\end{align*}
\]  

(16)

Thus, the Kalman filter can be designed as:

\[
\begin{align*}
\frac{d\hat{E}_a}{dt} &= -K_1(\hat{E}_a - \hat{e}_a) - \hat{\omega}_e\hat{E}_\beta \\
\frac{d\hat{E}_\beta}{dt} &= -K_1(\hat{E}_\beta - \hat{e}_\beta) + \hat{\omega}_e\hat{E}_a \\
\frac{d\hat{\omega}_r}{dt} &= (\hat{E}_a - \hat{e}_a)\hat{E}_\beta - (\hat{E}_\beta - \hat{e}_\beta)\hat{E}_a
\end{align*}
\]  

(17)

In the formula, \(K_1\) is Kalman filter gain, \(\hat{E}_a\) and \(\hat{E}_\beta\) are the final values of back-EMF observation. The value of \(K_1\) in the above equation affects the stability of the system operation. When \(K_1\) is too large, the system may oscillate. When the value of \(K_1\) is too small, the response speed of the system will be slower, which will reduce the high-speed observation effect of the sliding mode observer. In severe cases, the actual speed may not meet the set speed. Therefore, the Kalman filter gain \(K_1\) in this paper is designed to be able to adaptively adjust the change with the speed \(\omega_r\) to ensure the stability of the system when the motor speed changes.

After introducing variable cut-off LPF and Kalman filter, the structure of SMO is shown in Figure 2.

![Figure 2. Structural block diagram of two-stage filter SMO](image)

4. Simulation results and analysis
In order to verify the effectiveness of PMSM vector control based on two-stage filter proposed in this paper, a simulation model of sensorless sliding mode observer for three-phase surface-mounted PMSM under vector control mode of \(i_d = 0\) had been built in MATLAB/Simulink.

Firstly, in order to verify the superiority of PMSM vector control based on two-stage filter over traditional sliding mode observer, two simulation models had been built. The simulation conditions were the same: simulation time was 0.1 s, speed was 1200 r/min, no-load started. The simulation
results were shown in Figure 3-4.

**Figure 3.** (a) Traditional SMO

![Figure 3. (a) Traditional SMO](image)

**Figure 3.** (b) Two-stage filter SMO

![Figure 3. (b) Two-stage filter SMO](image)

**Figure 3.** Speed change curve

**Figure 4.** (a) Traditional SMO

![Figure 4. (a) Traditional SMO](image)

**Figure 4.** (b) Two-stage filter SMO

![Figure 4. (b) Two-stage filter SMO](image)

**Figure 4.** Variation curve of speed estimation error

It can be seen from the figure that the two SMOs can reach the set speed, but the speed of the two-stage filtering SMO is much smoother than the traditional SMO, and the jitter is small, the speed estimation error is small, and there are obvious high frequency and Chatter suppression effect.
In order to verify the observing ability of the designed two-stage filter SMO for rotor position and speed and the dynamic performance of the system, the initial speed was set to 500r/min and no-load operation was set. At t=0.05s, the speed was set to 1200r/min, at t=0.1s, the load torque was 5N.m, and the simulation time was 0.15. The dynamic simulation results were shown in Figure 5.

![Figure 5. (a) Speed change curve](image1)

![Figure 5. (b) Rotor position change curve](image2)

![Figure 5. (c) Torque Curve of Motor](image3)

![Figure 5. (d) Current curve of motor](image4)

Figure 5. Dynamic simulation results

It can be seen from the figure that in the process of starting and accelerating the motor, the acceleration of the motor is faster, the torque and flux linkage fluctuate greatly, and the current is larger, which causes the SMO to reciprocate around the sliding mode plane. The place that tends to be
flat will soon reach stability and the error will decrease. At this time, the torque of the motor is 0, and the current is small. In the process of starting and speed-up, in order to achieve the set speed, the required stator current is large, so the torque is also large; when the motor is stable, the output torque of the motor is 0; when the load changes, the output torque of the motor quickly reaches the load torque value. In the process of sudden load, the motor's speed has just begun to fluctuate a little, and soon reaches the actual speed. The fluctuation range of speed estimation error is small. The rotor position estimation value can track the actual value well and the estimation error is small. The current output is smooth and the noise harmonics are small. It shows good anti-interference ability and robustness. It satisfies the requirement of actual control.

5. Conclusions
In this paper, a PMSM control strategy based on two-stage filter SMO is proposed, and the strategy is analyzed in detail. At the same time, the simulation is carried out in MATLAB/Simulink. The simulation results show that by introducing variable cut-off frequency filter and Kalman filter, the observation error is reduced, the chattering matrix of the system is reduced, and the observation accuracy is greatly improved. Compared with traditional SMO, the back-EMF information obtained by the designed two-stage filter SMO is smoother; With the sudden change of speed, the response speed of the motor is faster, and the estimation error of the position and speed of the motor is smaller, the stability is higher, so it has good feasibility and robustness. With the sudden increase of load, the speed fluctuation of the motor is small, which is in accordance with the set speed very quickly, the current curve is smooth and the harmonic noise is less, which shows that the designed control system has better dynamic performance and anti-disturbance ability. In the future, the feasibility and effect of this method can be verified in actual control systems.

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