HIGH SNR ANALYSIS OF RIS AIDED MIMO BC CHANNELS

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ABSTRACT

We analyze the influence of an reconfigurable intelligent surface (RIS) on the channel eigenvalues within a high signal-to-noise ratio (SNR) scenario. This allows to connect specific channel properties with the rank improvement capabilities of the RIS. In particular, fundamental limits due to a possible line of sight (LOS) setup between the base station (BS) and the RIS are derived. Furthermore, dirty paper coding (DPC) based schemes are compared to linear precoding in such a scenario and it is shown that under certain channel conditions, the performance gap between DPC and linear precoding can be made arbitrarily small by the RIS.

Index Terms— high SNR, eigenvalues, line of sight, waterfilling

1. INTRODUCTION

RISs draw a lot of attention currently as they are viewed as a key technology for future communication systems (e.g., [1]). An RIS is a surface consisting of many passive low-cost elements which make it possible to enhance the propagation environment. Under perfect channel state information (CSI) which is also assumed in this article, the algorithms of [5], [6], [7], [8], [9] are available which show a clear improvement when incorporating an RIS. Additionally, in [10], the rank improvement capabilities of an RIS were demonstrated.

Contributions: We show, based on a high SNR analysis, that the phase optimization performed in systems with RISs directly affects the channel eigenvalues. In particular, the optimization of the phases for DPC in the high SNR region is identified to be similar to a rank-constrained waterfilling solution. Furthermore, we discuss the limitations evolving from the often used LOS assumption for the channel between the BS and the RIS. Additionally, we motivate a multi-RIS scenario and indicate why it can circumvent these limitations. In addition to that, we show that, under certain conditions on the channel between the BS and the RIS, the gap between DPC based methods (see [7], [8]) and the earlier proposed linear scheme (see [9]) can be made arbitrarily small by increasing the number of RIS elements.

2. SYSTEM MODEL

We consider a scenario with an \( N_B \) antenna BS serving \( K \) users each having \( N_M \) antennas. Moreover, an RIS with \( N_R \) reflecting elements is available to enhance the transmission. The downlink reflecting channel from the BS to the \( k \)-th user is therefore given by

\[
H_k = H_{d,k} + H_{i,k} \Theta R \in \mathbb{C}^{N_M \times N_B}
\]

where \( H_{d,k} \in \mathbb{C}^{N_M \times N_B} \) is the direct channel from the BS to the \( k \)-th user, \( H_{i,k} \in \mathbb{C}^{N_M \times N_B} \) is the reflecting channel from the RIS to the user \( k, \Theta = \text{diag}(\theta) \in \mathbb{C}^{N_B} \) with \( \theta \in \{ z \in \mathbb{C}^{N_B} : |z_n| = 1, \forall n \} \) is the phase manipulation at the RIS and \( H_i \in \mathbb{C}^{N_R \times N_B} \) is the channel from the base station to the RIS. The stacked channel matrix, containing the channels for all the users, is defined as

\[
H = \begin{bmatrix}
H_1^H & H_2^H & \cdots & H_K^H
\end{bmatrix}^H \in \mathbb{C}^{T \times N_B}
\]

with \( T = KN_M \) where we additionally assume \( N_B \geq 1 \). Also the subchannels \( H_{d,k} \) and \( H_{i,k} \) are stacked accordingly resulting in \( H_d \in \mathbb{C}^{T \times N_B} \) and \( H_i \in \mathbb{C}^{T \times N_B} \).

3. HIGH SNR EXPRESSIONS

As performance metric, we consider the sum rate which can be written for the high SNR discussion (see e.g. [11]) as

\[
R_{\text{DPC/lin}} = r \log_2 P - r \log_2 \lambda + R_{\text{DPC/lin,offset}}
\]

with

\[
R_{\text{DPC,offset}} = - \log_2 \det \left( (H H^H)^{-1} \right),
\]

\[
R_{\text{Lin,offset}} = - \sum_{k=1}^K \log_2 \det \left( E_k^T (H H^H)^{-1} E_k \right)
\]

for DPC and linear precoding, respectively. Therefore, the slope w.r.t. the logarithmic power is the same for both schemes and they are only differing in their offsets. In [11], it has already been shown that

\[
R_{\text{DPC,offset}} \geq R_{\text{Lin,offset}},
\]

with equality for a blockdiagonal \( H H^H \) (orthogonal user channels). We will demonstrate in the following that the offsets can be also expressed by the channel eigenvalues. Defining the eigenvalue decomposition (EVD) \( H H^H = \mathbf{T} \Lambda H^H \), the offset of DPC \( R_{\text{DPC,offset}} \) reads as

\[
R_{\text{DPC,offset}} = - \log_2 \det \left( (H H^H)^{-1} \right) = r \log_2 \sqrt{\det (H H^H)} = r \log_2 \Lambda_0
\]

where \( \Lambda_0 \) is the geometric mean of the eigenvalues. The interpretation for linear precoding is less obvious. By applying two times the
inequality between the geometric and the arithmetic mean in \((a)\) and
\((b)\), it is possible to obtain

\[
-R_{\text{Lin. offset}} = \sum_{k=1}^{K} \log_2 \det \left( E_k^T (HH^H)^{-1} E_k \right) \\
= \sum_{k=1}^{K} N_M \log_2 \det \left( E_k^T (HH^H)^{-1} E_k \right) \lambda_N \\
\leq \sum_{k=1}^{K} N_M \log_2 \frac{1}{N_M} \text{tr} \left( E_k^T (HH^H)^{-1} E_k \right) \\
= N_M \log_2 \prod_{k=1}^{K} \frac{1}{N_M} \text{tr} \left( E_k^T (HH^H)^{-1} E_k \right) \\
= N_M K \log_2 \prod_{k=1}^{K} \frac{1}{N_M} \text{tr} \left( E_k^T (HH^H)^{-1} E_k \right) \\
\leq N_M K \log_2 \frac{1}{K} \sum_{k=1}^{K} \frac{1}{N_M} \text{tr} \left( E_k^T (HH^H)^{-1} E_k \right) \\
= r \log_2 \frac{1}{r} \text{tr}((HH^H)^{-1}) \\
= -\frac{r \log_2}{(HH^H)^{-1}} \\
= -r \log_2 \lambda_H \\
\text{and the offset } R_{\text{Lin. offset}} \text{ can therefore be bounded as}
\]

\[
r \log_2 \lambda_H \leq R_{\text{Lin. offset}} \leq r \log_2 \lambda_G \tag{8}
\]

where \(\lambda_H\) is the harmonic mean of the eigenvalues.

Using this interpretation, we can bound the difference of DPC and linear precoding in the high SNR region with the channel eigenvalues as

\[
\Delta R = R_{\text{DPC}} - R_{\text{Lin}} \leq r \log_2 \frac{\lambda_G}{\lambda_H} \tag{9}
\]

With the help of the RIS, we are now able to manipulate the channel and its corresponding eigenvalues. Therefore, the difference can be expressed as

\[
\Delta R(\theta_{\text{Geo}}, \theta_{\text{Har}}) = R_{\text{DPC}}(\theta_{\text{Geo}}) - R_{\text{Lin}}(\theta_{\text{Har}}) \leq r \log_2 \frac{\lambda_G(\theta_{\text{Geo}})}{\lambda_H(\theta_{\text{Har}})} \tag{10}
\]

where \(\theta_{\text{Geo}}\) are the phases that are maximizing the geometric mean and \(\theta_{\text{Har}}\) are the phases that are maximizing the harmonic mean.

4. GEOMETRIC MEAN MAXIMIZATION

Optimization problems with RISs turn out to be nonconvex due to the unimodular constraints, which the reflecting elements have to satisfy. Furthermore, they are usually hard to solve (in this case the locally optimal algorithms [5], [7], [8] are available) and it is not easy to obtain an intuition for the problem structure. We will therefore start by reformulating the high SNR DPC problem [see [4]],

\[
\max_{\theta_{\text{Lin}}, \theta_{\text{Har}}} \log_2 det(HH^H) \tag{11}
\]

in such a way, that an interpretation of the problem is available.

At first, we rewrite the matrix \(HH^H\) within our objective function as

\[
HH^H = (H_d + H_i \Theta_i H_i^H)(H_d + H_i \Theta_i H_i^H)^H \\
= H_d H_d^H + H_i H_i^H \Theta_i H_i^H + H_i \Theta_i H_i^H H_d^H \\
+ H_i \Theta_i H_i^H \Theta_i H_i^H \\
= H_d H_d^H + \sum_{l=1}^{R_s} (H_d v_l v_l^H \Theta_i H_i^H + H_i \Theta_i v_l v_l^H H_d^H) \\
+ H_i \Theta_i v_l v_l^H H_i^H \\
= H_d H_d^H + \sum_{l=1}^{R_s} (H_d v_l v_l^H \bar{H}_d^H + \bar{H}_i \theta_i H_d^H) \\
+ \bar{H}_i \theta_i H_d^H \\
= H_d H_d^H + \sum_{l=1}^{R_s} (D_l \bar{\theta}_i D_l^H - h_d v_l^H) \\
= H_d \left( I - \sum_{l=1}^{R_s} v_l v_l^H \right) H_d^H + \sum_{l=1}^{R_s} D_l \bar{\theta}_i D_l^H \\
= C + Q \tag{12}
\]

where

\[
Q = \sum_{l=1}^{R_s} D_l \bar{\theta}_i D_l^H \text{ and } C = H_d \Theta_i H_i^H \tag{13}
\]

These matrices are constructed with \(D_l = [H_l \text{ diag}(u_l) \sigma_l \ H_l v_l]\) and \(\bar{\theta}_i = [\theta_i^H 1]\) by using the singular value decomposition \(H_i = \sum_{l=1}^{R_s} \sigma_l u_l v_l^H\) and the orthogonal projector \(P_{R_s} = I - \sum_{l=1}^{R_s} v_l v_l^H\). When analyzing the expression for the matrix \(Q\) we can see that the matrix is positive semidefinite, its rank is smaller than or equal to \(R_s\), and its channel gain \(\text{tr}(Q)\) is bounded. Defining the maximum channel gain as \(G_{\text{max}}\), the geometric mean optimization can be rewritten as

\[
\max_{\theta_{\text{Geo}}, |\theta_{\text{Lin}}|=1} \log_2 det(C + Q) \tag{11}
\]

s.t. \(Q \succeq 0, \text{tr}(Q) \leq G_{\text{max}}, \text{rank}(Q) \leq R_s \tag{14}\)

where the three extra constraints were introduced to obtain an interpretation of the problem. In particular, when we could choose any positive semidefinite matrix \(Q\) under the rank and trace constraints (ignoring the last constraint), the result would be rank-constrained waterfilling. This was analyzed in [12] and is equivalent to performing waterfilling over the \(R_s\) smallest eigenvalues of the matrix \(C\).

Even though we are additionally limited by the characteristics of the RIS (the last constraint), we will see that the actual solution shows a similar behavior to the relaxed one. Moreover, we can observe that the more RIS elements we have, the more degrees of freedom we have in choosing a proper matrix \(Q\).

In comparison, the maximization of the harmonic mean results in

\[
\max_{\theta} \frac{r}{\text{tr}((HH^H)^{-1})} = \min_{\theta} \text{tr}((C + Q)^{-1}) \tag{11}
\]

Interestingly, with the discussed relaxation, the solution is also rank-constrained waterfilling (this can be shown...
by combining the results of [12] and [13]), meaning that both, DPC and linear schemes, solve the same optimization problem (with the relaxation of the RIS characteristics) at high SNR.

5. RIS OPTIMIZATION

We will now analyze how the actual problem differs from the water-filling solution discussed above. To this end, we start by a rank-one assumption for the channel \( H \), and afterwards increase the rank to an arbitrary number.

5.1. Rank-One Solution

Assume that the channel between the BS and the RIS is LOS dominated, e.g., when a millimeter wave (mmWave) scenario is discussed. We will see, however, that this assumption is quite limiting in view of the resulting degrees of freedom at the RIS. For the special case \( R_s = 1 \), the Gram channel matrix reads as

\[
HH^H = C + D_1 \bar{\theta}^H D_1^H.
\]

Introducing \( W \Phi W^H \) as the EVD (for all EVDs we assume a decreasing order of the eigenvalues) of \( C \), the solution to rank-constrained waterfilling is given by

\[
Q = w_r w_r^H G_{\text{max}}
\]

where \( w_r \) is the eigenvector corresponding to the minimum eigenvalue of \( C \). This solution already suggests that just one eigenvalue can be improved which would be clearly a limiting factor.

To see this restricting behavior, we are now considering the actual problem without the relaxation. In this case, \( D_1 \bar{\theta} \) cannot be chosen to lie perfectly within a subspace of \( W \) as in the relaxed solution above, i.e., \( W \Phi W^H D_1 \bar{\theta} \) is a vector with only non-zero elements. Therefore, the eigenvalues of \( HH^H \) interlace the ones of \( C \) (see [14], p.442, Theorem 8.5.3)) for any possible choice of \( \bar{\theta} \).

With that, the eigenvalues of the direct channel, that is, when \( \bar{\theta} = 0 \), and the eigenvalues for any choice of \( \bar{\theta} \) can be bounded by

\[
\phi_r < \lambda_r < \lambda_{r-1} < \lambda_{r-2} < \cdots < \phi_1 < \lambda_1
\]

where \( \lambda_{r} \) are the eigenvalues of the direct channel and \( \lambda_1 \) the eigenvalues with any phase configuration \( \theta \). In other words, the eigenvalue placement by the RIS is restricted by

\[
\lambda_r < \lambda_{r-1} < \lambda_{r-2} < \cdots < \lambda_2 < \lambda_1
\]

and \( \lambda_1 \) being unbounded.

From this chain of inequalities, we can infer that apart from the largest eigenvalue, we can just improve each eigenvalue to the next larger one of the direct channel with the optimization of the RIS.

To see this more clearly, we consider a scenario in which a group of \( G \) eigenvalues are very small, i.e., \( \lambda_{r} \approx \lambda_{r-1} \approx \cdots \approx \lambda_{r-G+1} \ll \lambda_{r-G} \approx \lambda_{r-G-1} \approx \cdots \approx \lambda_1 \) holds for the eigenvalues of the direct channel. In this case, the RIS could only improve one (i.e., \( \lambda_{r-G} \) of the \( G \) smaller eigenvalues (to a maximum of approximately \( \lambda_{r-G} \)). Moreover, if the RIS has strong impact such that \( \lambda_r \approx \lambda_{r-1} \approx \lambda_{r-2} \approx \cdots \approx \lambda_2 \approx \lambda_1 \) holds, it can only improve the largest eigenvalue \( \lambda_1 \) and the channel condition will get automatically worse. Note that when the influence of the RIS is very strong, the transmission would just take place via the rank one RIS channel. Additionally, if we would have a perfectly conditioned direct channel (all eigenvalues are approximately equal), the RIS could only worsen the condition by improving the largest eigenvalue.

5.2. Higher-Rank Solution

If we have a rank higher than one, the eigenvalues can be controlled better. By having a total of \( R_s \) rank-one updates available one could improve the eigenvalues up to the next \( R_s \) larger eigenvalues. Considering the example from above, we could now improve \( R_s \) of the \( G \) eigenvalues in this scenario. As in the rank-one case, if the impact of the RIS is very high, the situation changes and only the \( R_s \) largest eigenvalues can be further improved.

Regarding all cases, if we have a rank of \( R_s \geq r \) we could obtain a perfectly conditioned channel (also in the extreme case, when the transmission would effectively only take place via the RIS).

5.3. Multi RIS Scenario

To avoid the high-rank conditions for \( H \), and as furthermore, strong LOS assumptions are normally beneficial in an RIS scenario, a possible solution to the discussed rank improvement problem is to deploy multiple RISs. Consequently, the maximum number of controllable eigenvalues is given by

\[
\#\text{EVs} = \sum_{n=1}^{N_{\text{max}}} \text{rank}(H_{s,n}).
\]

6. SIMULATIONS

We consider a scenario similar to [4] and [9], where 6 single antenna (\( N_0 = 1 \)) users are uniformly distributed in a circle with radius 10m centered at (200m, 30m) and served by an \( N_B = 16 \) antenna base station located at (0m, 0m). The transmission is enhanced by an RIS located at (200m, 0m). The path losses for the channels are assumed to follow the model \( L_{db} = \alpha + \beta \log_{10}(d) \) (we assume \( \alpha = \alpha_r = \alpha_s = 30 \text{dB} \) for \( H_{s,k}, H_{r,k} \) and \( H_r \) in all simulations) where \( d \) is the distance between the receiver and the transmitter. For all plots, 1000 realizations and a noise variance of \( \sigma^2 = -100 \text{dBm} \) is assumed at each receive antenna. Furthermore, for \( H_{s,k} \) we set \( \beta_s = 3.76 \) and assume uncorrelated Rayleigh fading in all simulations.

6.1. Rank Constrained Waterfilling

At first, we will show numerically how the eigenvalues are affected by the RIS optimization. To see the impact on the eigenvalues more clearly, we introduce an extra path loss of 20dB for 3 of the 6 users. Furthermore, the channel \( H_{r,k} \) is modeled as uncorrelated Rayleigh fading with \( \beta_r = 3.76 \). To see the importance of \( \text{rank}(H_r) \), we model \( H_r \) with the Kronecker channel model as

\[
H_r = \sqrt{T_r \lambda_s} \left[ \begin{array}{cc} N_B & M \\ 0 & 0 \\ 0 & N_B - R_s \end{array} \right] S^H
\]

to analyze the behavior for different ranks. The elements of the matrix \( M \) are distributed as \( N_C(0,1) \) and the unitary matrix \( S \) is also chosen randomly by computing the QR decomposition of a matrix where all elements are distributed as \( N_C(0,1) \). The channel gain normalization \( \sqrt{T_r \lambda_s} \) is introduced as we would like to analyze the system only w.r.t. the structure (rank) of the matrix \( H_r \). The pathloss parameter for this channel is chosen as \( \beta_r = 2.2 \).

In Figure 1 the eigenvalues can be seen after different optimizations were performed for the RIS. For the geometric mean optimization (Geo-Mean) a modified version of [8] was used. The harmonic mean maximization was performed with the algorithm presented in [9]. Both algorithms were initialized with the same random phase shifts.
Increasing the rank of $H_s$ could be improved according to the rank constraint. It can be seen in Figure 1(c) that two and then all three eigenvalues can be controlled resulting in a well-conditioned channel. In the case of Rician fading (see Figure 2(b)), the channel has full rank and the smaller eigenvalues can also be improved. However, we can already see the impact of the rank-one component resulting in the increase of the largest eigenvalue.

This problem of the Rician fading assumption can also be observed in Figure 3. Here, we can see a gap between the linear scheme and DPC in the cases of only the direct channel and the random phase shifts. In case of DPC-AO and RIS-LISA, the corresponding gap can only be closed when considering Rayleigh fading (Figure 3(a)). For Rician fading (see Figure 3(b)), however, the gap remains.

Analyzing this behavior in greater depth results in Figure 4. We can see that the gap between DPC and linear precoding stays approximately constant for an increasing number of RIS elements in both cases, for random phase shifts and when only the direct channel is considered. For the DPC-AO and RIS-LISA algorithms this is different. In the Rayleigh model (see Figure 4(a)), the gap between the two schemes can be made arbitrarily small with an increasing number of elements whereas for the Rician fading model (see Figure 4(b)) the gap remains also for a higher number of elements. The gap between both algorithms is still decreasing in the Rician model as the linear scheme finds an alternative phase shift by the harmonic mean maximization (purple dash-dotted line) which results in a better conditioned channel. However, also this alternative phase shift is limited for the Rician fading model.

7. CONCLUSION

We have seen that the RIS optimization directly affects the placement of the channel eigenvalues by introducing a connection with (uniformly distributed phases) which are also presented in Figure 1 (Random).

With $H_s$, having a rank of one (see Figure 1(a)), it can be seen that the discussed limitation in $\lambda_n < \lambda_{n-1}$ holds. Therefore, only one of the $s$ smaller eigenvalues, i.e., eigenvalue 4, could be improved.

Increasing the rank of $H_s$ to two and six results in Figure 1(b) and Figure 1(c) respectively. Here, two and then all three eigenvalues could be improved according to the rank constraint. It can be seen that the channel gain is spread over the controlled eigenvalues and the similarity to the rank constrained waterfilling solution can be observed.

Furthermore, we can see in Figure 1(d) that if the RIS has enough impact (here $\beta_s = 2.2$ was set), then a rank of $\text{rank}(H_s) = r$ is enough to control all eigenvalues and with that, a well-conditioned channel can be obtained.

6.2. Performance Analysis

We will now analyze the performance of DPC and linear precoding schemes for different channel conditions. For DPC-AO we use the algorithm from [8] with the geometric mean (high SNR) initial phase values. For the linear precoding schemes, we use the RIS-LISA algorithm proposed in [9] with the harmonic mean initial phases. In all plots, we additionally show the harmonic mean (dotted lines) and the geometric mean (dashed lines) based high SNR approximations for the direct channel, for random phases, and after the geometric mean maximization.

The channel $H_{r,k}$ is assumed to follow uncorrelated Rayleigh fading with a channel gain of $\beta_r = 2.2$. Regarding $H_s$, we have already seen in the last subsection that a rank-one channel has a quite limiting influence on the possible eigenvalue locations. Instead of assuming $H_s$ to have a rank of one, we assume a more practical assumption in which $H_s$ is assumed to follow Rician fading. Here, a Rician component with a Rician factor of 6dB is added to the uncorrelated Rayleigh fading term. The Rician component is obtained by assuming a half-wavelength uniform linear array (ULA) at the BS and at the RIS where the angle of arrival (AoA) and angle of departure (AoD) are selected uniformly from the interval $[0, 2\pi)$.

This model is then compared to $H_s$ being Rayleigh (uncorrelated) distributed.

**Fig. 1.** Eigenvalues for a different rank of $H_s$ and different gains of the reflective channel $H_{r,k}$.

**Fig. 2.** Eigenvalues for the Rayleigh and Rician fading model of the matrix $H_s$.

At first, we start again by analyzing the behavior of the eigenvalues, seen in Figure 2. For Rayleigh fading (see Figure 2(a)) all eigenvalues can be controlled resulting in a well conditioned channel. In the case of Rician fading (see Figure 2(b)), the channel has full rank as well and the smaller eigenvalues can also be improved. However, we can already see the impact of the rank-one component resulting in the increase of the largest eigenvalue.

This problem of the Rician fading assumption can also be observed in Figure 3. Here, we can see a gap between the linear scheme and DPC in the cases of only the direct channel and the random phase shifts. In case of DPC-AO and RIS-LISA, the corresponding gap can only be closed when considering Rayleigh fading (Figure 3(a)). For Rician fading (see Figure 3(b)), however, the gap remains.

Analyzing this behavior in greater depth results in Figure 4. We can see that the gap between DPC and linear precoding stays approximately constant for an increasing number of RIS elements in both cases, for random phase shifts and when only the direct channel is considered. For the DPC-AO and RIS-LISA algorithms this is different. In the Rayleigh model (see Figure 4(a)), the gap between the two schemes can be made arbitrarily small with an increasing number of elements whereas for the Rician fading model (see Figure 4(b)) the gap remains also for a higher number of elements. The gap between both algorithms is still decreasing in the Rician model as the linear scheme finds an alternative phase shift by the harmonic mean maximization (purple dash-dotted line) which results in a better conditioned channel. However, also this alternative phase shift is limited for the Rician fading model.
the geometric and harmonic mean. In particular, we have seen that if the rank of the channel between the BS and the RIS is high, we can control all eigenvalues resulting in a good channel condition and the gap between linear precoding schemes and DPC vanishes. On the opposite, when the BS-RIS channel has low rank (or even only LOS) the rank improvement capabilities of the RIS are clearly limited. This can be circumvented by considering a multiple RISs.

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