Critical behavior of long straight rigid rods on two-dimensional lattices:

Theory and Monte Carlo simulations

D. A. Matoz-Fernandez, D. H. Linares, and A.J. Ramirez-Pastor†

Departamento de Física, Instituto de Física Aplicada,
Universidad Nacional de San Luis-CONICET,
Chacabuco 917, D5700BWS San Luis, Argentina

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The critical behavior of long straight rigid rods of length $k$ ($k$-mers) on square and triangular lattices at intermediate density has been studied. A nematic phase, characterized by a big domain of parallel $k$-mers, was found. This ordered phase is separated from the isotropic state by a continuous transition occurring at a intermediate density $\theta_c$. Two analytical techniques were combined with Monte Carlo simulations to predict the dependence of $\theta_c$ on $k$, being $\theta_c(k) \propto k^{-1}$. The first involves simple geometrical arguments, while the second is based on entropy considerations. Our analysis allowed us also to determine the minimum value of $k$ ($k_{\text{min}} = 7$), which allows the formation of a nematic phase on a triangular lattice.

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† To whom all correspondence should be addressed.
I. INTRODUCTION

The isotropic-nematic (I-N) phase transition has been a topic of active theoretical and experimental studies over the past few decades. An early seminal contribution to this subject was made by Onsager\textsuperscript{1} with his work on the I-N phase transition of infinitely thin rods. This theory predicted that excluded volume interactions alone can lead to long-range orientational (nematic) order. Later, several papers contributed greatly to the understanding of the statistics of rigid rods\textsuperscript{2,3,4,5,6,7}. Successive works have established detailed phase diagrams for several hard-body shapes\textsuperscript{8,9,10,11}. For the continuum problem, there is general agreement that in three dimensions, infinitely thin rods undergo a first-order I-N transition, as was pointed out by Onsager\textsuperscript{1}. In two dimensions, the nature of the isotropic-nematic (I-N) phase transition depends crucially on the particle interactions and a rich variety of behaviors is observed\textsuperscript{12,13}.

A notable feature is that nematic order is only stable for sufficiently large aspect ratios while isotropic systems of short rods not show nematic order at all. The long-range orientational order also disappears in the case of irreversible adsorption (no desorption)\textsuperscript{14,15}, where the distribution of adsorbed objects is different from that obtained at equilibrium\textsuperscript{16,17}. Thus, at high coverage, the equilibrium state corresponds to a nematic phase with long-range correlations, whereas the final state generated by irreversible adsorption has infinite memory of the process and orientational order is purely local.

In the case of lattice models, two previous articles\textsuperscript{18,19}, referred to as papers I and II, respectively, were devoted to the study of the I-N phase transition in a system of long straight rigid rods of length $k$ ($k$-mers) on two-dimensional lattices with discrete allowed orientations. The model of a two-dimensional gas of rigid $k$-mers is the simplest representation of a strongly adsorbed film of linear molecules in submonolayer or monolayer
regime. Examples of this kind of systems are monolayer films of $n$-alkanes adsorbed on monocrystalline surfaces of metals, such as Pt(111)\textsuperscript{20} and Au(111)\textsuperscript{21,22}.

Papers I and II were inspired in the excellent paper by Ghosh and Dhar\textsuperscript{23}. In Ref. \textsuperscript{23}, the authors presented strong numerical evidence that a system of square geometry, with two allowed orientations, shows nematic order at intermediate densities for $k \geq 7$ and provided a qualitative description of a second phase transition (from a nematic order to a non-nematic state) occurring at a density close to 1. However, the authors were not able to determine the critical quantities (critical point and critical exponents) characterizing the I-N phase transition occurring in the system.

In this context, extensive Monte Carlo (MC) simulations were used in paper I to resolve the universality class of the first phase transition occurring in the system of Ref. \textsuperscript{23}. Lattices of various sizes were considered and finite-size scaling theory was utilized. As it was evident from the calculation of the critical exponents and the behavior of Binder cumulants, the universality class was shown to be that of the 2D Ising model for square lattices with two allowed orientations, and the three-state Potts model for triangular lattices with three allowed orientations.

Paper II was a step further, analyzing the I-N phase transition in terms of entropy. For this purpose, the configurational entropy of a system of rigid rods deposited on a square lattice was calculated by Monte Carlo (MC) simulations and thermodynamic integration method\textsuperscript{24}. The numerical data were compared with the corresponding ones obtained from a fully aligned system (nematic phase), whose calculation reduces to the one-dimensional case\textsuperscript{25}. The study allowed us 1) to confirm previous results in the literature\textsuperscript{23}, namely, the existence of i) a I-N phase transition at intermediate densities for $k \geq 7$ and ii) a second phase transition from a nematic order to a non-nematic state at high density. In
the second case, Ref. [19] represents the first numerical evidence existing in the literature about this important point; 2) to provide an interpretation on the underlying physics of the observed $k$ dependence of the isotropic-nematic phase transition; and 3) to test the predictions of the main theoretical models developed to study adsorption with multisite-occupancy.

Even though many aspects of the problem have been treated in papers I, II and Ref. [23], other points remain still open. Among them, there exists no studies on the dependence of the critical density characterizing the I-N phase transition, $\theta_c$, on the size $k$ of the rod. In this work we attempt to remedy this situation. For this purpose, extensive MC simulations complemented by finite-size scaling techniques and theoretical modeling have been applied. We restrict the study to the first phase transition occurring in the system (or I-N phase transition at intermediate density). Our analysis allowed us (1) to obtain $\theta_c$ as a function of $k$ for square and triangular lattices, being $\theta_c(k) \propto k^{-1}$; and (2) to determine the minimum value of $k$ ($k_{min} = 7$), which allows the formation of a nematic phase on triangular lattices.

The outline of the paper is as follows. In Sec. II we describe the lattice-gas model, the simulation scheme, and we present the behavior of $\theta_c(k)$, obtained by using the MC method. In Sec. III we present the analytical approximations and compare the MC results with the theoretical calculations. Finally, the general conclusions are given in Sec. IV.

II. LATTICE-GAS MODEL AND MONTE CARLO SIMULATION SCHEME

A. Model and Monte Carlo method

We address the general case of adsorbates assumed to be linear rigid particles containing $k$ identical units ($k$-mers), with each one occupying a lattice site. Small adsorbates would
correspond to the monomer limit ($k = 1$). The distance between $k$-mer units is assumed to be equal to the lattice constant; hence exactly $k$ sites are occupied by a $k$-mer when adsorbed. The only interaction between different rods is hard-core exclusion: no site can be occupied by more than one $k$-mer unit. The surface is represented as an array of $M = L \times L$ adsorptive sites in a square or triangular lattice arrangement, where $L$ denotes the linear size of the array.

The degree of order in the adsorbed phase is calculated for each configuration according to the standard method used for the Potts model\textsuperscript{26}. To this end, we first build a set of vectors $\{\vec{n}_1, \vec{n}_2, \cdots, \vec{n}_m\}$ with the following properties: (i) each vector is associated to one of the $m$ possible orientations (or directions) for a $k$-mer on the lattice; (ii) the $\vec{n}_i$’s lie in the same plane (or are co-planar) and point radially outward from a given point $P$ which is defined as coordinate origin; (iii) the angle between two consecutive vectors, $\vec{n}_i$ and $\vec{n}_{i+1}$, is equal to $2\pi/m$; and (iv) the magnitude of $\vec{n}_i$ is equal to the number of $k$-mers aligned along the $i$-direction. Note that the $\vec{n}_i$’s have the same directions as the $q$ vectors in Ref.\textsuperscript{26}. These directions are not coincident with the allowed directions for the $k$-mers on the real lattice. Then the order parameter $\delta$ of the system is given by

$$\delta = \frac{\sum_{i=1}^{m} \vec{n}_i}{\sum_{i=1}^{m} |\vec{n}_i|}$$

(1)

$\delta$ represents a general order parameter measuring the orientation of the $k$-mers on a lattice with $m$ directions. In the case of square lattices, $m = 2$ and the angle between $\vec{n}_1$ and $\vec{n}_2$ is $\pi$. Accordingly, the order parameter reduces to $\delta = |n_1 - n_2| / (n_1 + n_2)$, $n_1$ ($n_2$) being the number of $k$-mers aligned along the horizontal (vertical) direction. This expression coincides with the order parameter $Q$ defined in Ref.\textsuperscript{23}. On the other hand, $m = 3$ for triangular lattices and $\vec{n}_1$, $\vec{n}_2$ and $\vec{n}_3$ form angles of $2\pi/3$ between them.

When the system is disordered ($\theta < \theta_c$), all orientations are equivalents and $\delta$ is zero.
As the density is increased above $\theta_c$, the $k$-mers align along one direction and $\delta$ is different from zero. Thus, $\delta$ appears as a proper order parameter to elucidate the phase transition.

The problem has been studied by grand canonical MC simulations using a typical adsorption-desorption algorithm\textsuperscript{27,28,29}. The procedure is as follows. Once the value of the chemical potential $\mu$ is set, a linear $k$-uple of nearest-neighbor sites is chosen at random. Then, if the $k$ sites are empty, an attempt is made to deposit a rod with probability $W = \min \{1, \exp (\mu/k_B T)\}$, where $k_B$ is the Boltzmann constant and $T$ is the temperature; if the $k$ sites are occupied by units belonging to the same $k$-mer, an attempt is made to desorb this $k$-mer with probability $W = \min \{1, \exp (-\mu/k_B T)\}$; and otherwise, the attempt is rejected. In addition, displacement (diffusional relaxation) of adparticles to nearest-neighbor positions, by either jumps along the $k$-mer axis or reptation by rotation around the $k$-mer end, must be allowed in order to reach equilibrium in a reasonable time. A MC step (MCs) is achieved when $M$ $k$-uples of sites have been tested to change its occupancy state. Typically, the equilibrium state can be well reproduced after discarding the first $r' = 10^6$ MCs. Then, the next $r = 2 \times 10^6$ MCs are used to compute averages.

In our Monte Carlo simulations, we varied the chemical potential $\mu$ and monitored the density $\theta$ and the order parameter $\delta$, which can be calculated as simple averages. The reduced fourth-order cumulant $U_L$ introduced by Binder\textsuperscript{30} was calculated as:

$$U_L = 1 - \frac{\langle \delta^4 \rangle}{3\langle \delta^2 \rangle^2},$$  \hspace{1cm} (2)

where $\langle \cdots \rangle$ means the average over the $r$ MC simulation runs.

Finally, the configurational entropy of the system $S$, was calculated by using thermodynamic integration method\textsuperscript{31,32,33,34}. The method in the grand canonical ensemble relies upon integration of the chemical potential $\mu$ on coverage along a reversible path between an arbitrary reference state and the desired state of the system. This calculation also
requires the knowledge of the total energy $U$ for each obtained coverage. Thus, for a system made of $N$ particles on $M$ lattice sites, we have:

$$S(N, M, T) = S(N_0, M, T) + \frac{U(N, M, T) - U(N_0, M, T)}{T} - \frac{1}{T} \int_{N_0}^{N} \mu dN'$$

(3)

In our case $U(N, M, T) = 0$ and the determination of the entropy in the reference state, $S(N_0, M, T)$, is trivial [$S(N_0, M, T) = 0$ for $N_0 = 0$]. Note that the reference state, $N \to 0$, is obtained for $\mu/k_B T \to -\infty$. Then,

$$\frac{s(\theta, T)}{k_B} = -\frac{1}{k_B T} \int_0^\theta \frac{\mu}{k} d\theta'$$

(4)

where $s(= S/M)$ is the configurational entropy per site and $\theta(= k N/M)$ is the surface coverage (or density).

**B. Computational results**

Computational simulations have been developed for a system of straight rigid rods of length $k$ ($k = 2 - 14$) on a lattice. The surface was represented as an array of adsorptive sites in a square or triangular lattice arrangement. In addition, conventional periodic boundary conditions were considered. The effect of finite size was investigated by examining square lattices with $L/k = 5, 10, 15, 20$ and triangular lattices with $L/k = 10, 15, 20, 25$.

As it was established in Ref. [23], the minimum value of $k$, which allows the formation of a nematic phase on a square lattice at intermediate densities, is $k_{min} = 7$. This critical quantity has not been calculated yet for other geometries. Then, our first objective is to obtain $k_{min}$ for triangular lattices. For this purpose, two criteria have been applied: 1) the comparison between the configurational entropy of the system, obtained by MC
simulations, and the corresponding to a fully aligned system (nematic phase), whose
calculation reduces to the 1D case\textsuperscript{19}, and 2) the behavior of the nematic order parameter
\( \delta \) as a function of coverage.

The results obtained in the first case are shown in Fig. 1. Dotted line and symbols
represent MC data for triangular lattices, \( k = 6, 7 \) and 8 and \( L/k = 20 \). The calculation of
\( s(\theta)/k_B \) through eq. (4) is straightforward and computationally simple, since the coverage
dependence of \( \mu/k_B T \) is evaluated following the standard procedure of MC simulation
described in previous section. Then, \( \mu(\theta)/k_B T \) is spline-fitted and numerically integrated.

On the other hand, when the nematic phase is formed, the system is characterized by
a big domain of parallel \( k \)-mers. The calculation of the entropy of this fully aligned state
having density \( \theta \) reduces to the calculation of a one-dimensional problem\textsuperscript{25}

\[
\frac{s(\theta)}{k_B} = \left[ 1 - \frac{(k-1)}{k} \theta \right] \ln \left[ 1 - \frac{(k-1)}{k} \theta \right] \\
- \frac{\theta}{k} \ln \frac{\theta}{k} - (1 - \theta) \ln (1 - \theta). \tag{5}
\]

Results from eq. (5) for \( k = 6, 7 \) and 8 are shown in Fig. 1 (solid lines). As it can be
observed, for \( k \leq 6 \), the 1D results present a smaller \( s/k_B \) than the 2D simulation data
over all the range of \( \theta \). For \( k \geq 7 \), there exists a range of coverage for which the difference
between the 1D value and the true 2D value is very small. In other words, for \( k \geq 7 \) and
intermediate densities, it is more favorable for the rods to align spontaneously, since the
resulting loss of orientational entropy is by far compensated by the gain of translational
entropy. This finding is a clear indication that \( k_{\text{min}} = 7 \) for triangular lattices. In addition, \( \theta_c \) can be calculated from the minimum value of \( \theta \) for which the near superposition
of the 1D and 2D results occurs. Thus, the technique provides an alternative method
of determining the critical coverage characterizing the I-N phase transition without any
special requirement and time consuming computation. However, it is important to em-
phasize that the calculation of the entropy of the nematic phase from the 1D model is an approximation (especially at the moderate densities, where the phase is not completely aligned). Consequently, a precise determination of $\theta_c$ should require an extensive work of MC simulation and finite-size scaling techniques.

As an independent corroboration of the results previously obtained, the inset in Fig. 1 presents the nematic order parameter $\delta$ as a function of coverage for $k = 6, 7$ and $8$ and $L/k = 20$. The behavior of $\delta$ confirms the existence of nematic order for $k \geq 7$. This value of $k_{\text{min}}$ coincides with that obtained for square lattices.

Once $k_{\text{min}}$ has been established, we now discuss the behavior of the critical density as a function of the size $k$. In the case of the standard theory of finite-size scaling, when the phase transition is temperature driven, the technique allows for various efficient routes to estimate $T_c$ from MC data. One of these methods, which will be used in this case, is from the temperature dependence of $U_L(T)$, which is independent of the system size for $T = T_c$. In other words, $T_c$ is found from the intersection of the curve $U_L(T)$ for different values of $L$, since $U_L(T_c) = \text{const}$. In our study, we modified the conventional finite-size scaling analysis by replacing temperature by density. Under this condition, the critical density has been estimated from the plots of the reduced four-order cumulants $U_L(\theta)$ plotted versus $\theta$ for several lattice sizes. As an example, Fig. 2 shows the results for rods of size $k = 10$ on square [Fig. 2(a)] and triangular [Fig. 2(b)] lattices. In the cases of the figure, the values obtained were $\theta_c = 0.502(1)$ and $\theta_c = 0.530(1)$, for square and triangular lattices, respectively. The curves of the order parameter, which were used to obtain $U_L(\theta)$, are shown in the insets of the figure.

The procedure of Fig. 2 was repeated for $7 \leq k \leq 14$ and the results are collected in Fig. 3. The log-log plots show that the critical density follows a power law as $\theta_c(k) \propto k^{-1}$. 
The understanding of this dependence of $\theta_c(k)$ on $k$ can be developed by using simple geometric arguments. An example follows in order to make this point clear. Fig. 4(a) shows a snapshot corresponding to a “small window” of side $l$, which is embedded in an infinite square lattice. Open squares correspond to empty sites and black bars represent adsorbed $k$-mers (in the case of the figure, $k = 4$ and $l = 8$). Then, the local coverage or fraction of occupied sites belonging to the window is $\theta = kN/(l \times l)$, being $N$ the number of $k$-mers into the window. We can now think of a transformation $L \rightarrow L'$ from the original lattice $L$ to a new lattice $L'$, where each characteristic length of $L'$ is $s$ times the corresponding one of $L$ ($s = 1, 2, 3, 4, \cdots$). Then, $l' = sl$, $k' = sk$ and $\theta' = k'N/(l' \times l')$. By following this procedure, Fig. 4(b) was obtained from Fig. 4(a). In this case, $s = 3$, and consequently, $l' = 3l = 24$ and $k' = 3k = 12$. As it can be observed, $L$ and $L'$ represent the “same situation” for two different values of $k$.

In general,

$$\frac{\theta'}{\theta} = \frac{k'N(l \times l)}{kN(l' \times l')}$$

and $l/k = l'/k'$. Then, the relationship between a coverage in $L$ and the corresponding one in $L'$ results

$$\frac{\theta}{\theta'} = \frac{k'}{k}.$$  

By taking the system $L'$ as reference, it is possible to write,

$$\theta = (k'\theta')k^{-1}$$

$$\propto k^{-1},$$

as it was found from the MC data. A more rigorous justification of the observed $k$ dependence of the I-N phase transition will be presented in the next section.
It is worth pointing out that we did not assume any particular universality class for the transitions analyzed here in order to calculate their critical temperatures, since the analysis relied on the order parameter cumulant’s properties. However, a systematic analysis of critical exponents was carried out in Ref. [18], where the universality class was shown to be that of the 2D Ising model for square lattices and the two-dimensional Potts model with \( q = 3 \) for triangular lattices.

### III. ANALYTICAL APPROXIMATIONS AND COMPARISON BETWEEN SIMULATED AND THEORETICAL RESULTS

Next, the transition is analyzed from the main theoretical models developed to treat the polymers adsorption problem. The study allows us to calculate the dependence of \( \theta_c(k) \) on \( k \) from three well-known multisite-adsorption theories: the Flory-Huggins (FH) approximation\(^{2,3}\), the Guggenheim-DiMarzio (GD) approach\(^{4,5}\) and the recently developed Semiempirical Model for Adsorption of Polyatomics (SE)\(^{36,37}\). The corresponding expressions for the configurational entropy per site are:

\[
\frac{s(\theta)}{k_B} = -\frac{\theta}{k} \ln \frac{\theta}{k} - (1 - \theta) \ln (1 - \theta) - \frac{\theta}{k} \left[ k - 1 - \ln \left( \frac{c}{2} \right) \right] \tag{FH} \]

\[
\frac{s(\theta)}{k_B} = -\frac{\theta}{k} \ln \frac{\theta}{k} - (1 - \theta) \ln (1 - \theta) + \left( \theta - \frac{c}{2} \right) \ln \left( \frac{c}{2} \right) + \left[ \frac{c}{2} - \frac{(k - 1)}{k} \theta \right] \ln \left[ \frac{c}{2} - \frac{(k - 1)}{k} \theta \right] \tag{GD} \]

and

\[
\frac{s(\theta)}{k_B} = -\frac{\theta}{k} \ln \frac{\theta}{k} - (1 - \theta) \ln (1 - \theta) + \theta \left[ \frac{1}{2} - \frac{c}{4} + \frac{1}{k} \ln \left( \frac{c}{2} \right) \right] + \frac{1}{2} \frac{k}{(k - 1)} \left[ 1 - \frac{(k - 1)^2}{k^2} \theta^2 \right] \ln \left[ 1 - \frac{(k - 1)}{k} \theta \right] - \frac{c}{4} \left[ \theta + \frac{k(c - 4) + 4}{2(k - 1)} \right] \left[ 1 - \frac{2(k - 1)}{ck} \theta \right] \ln \left[ 1 - \frac{2(k - 1)}{ck} \theta \right] \tag{SE} \]
From a given value of $k$ (which depends on the approximation considered), the 2D \[\text{eqs. (9-11)}\] and 1D \[\text{eq. (5)}\] curves cross at intermediate densities and two well differentiated regimes can be observed. In the first regime, which occurs at low densities, the 2D approaches predict a larger entropy than the 1D data. In the second regime (at high densities) the behavior is inverted and the 2D data present a smaller $s/k_B$ than the 1D results. Given that the theoretical results in 2D assume isotropy in the adlayer (interested readers are referred to Ref. \cite{5}, where this point is explicitly considered), the crossing of the curves shows that, in the second regime, the contribution to the 2D entropy from the isotropic configurations is lower than the contribution from the aligned states. Then, the existence of an intersection point is indicative of a I-N transition and allows us to estimate $k_{\text{min}}$ and $\theta_c$ from the different approximations studied. This intersection point can be easily calculated through a standard computing procedure; in our case, we used Mathematica software. The results are shown in Fig. 5 for the different studied cases. The symbology is indicated in the figure.

In the case of square lattices, FH and GD predict values of $k_{\text{min}} = 3$ and $k_{\text{min}} = 4$, respectively. On the other hand, SE performs significantly better than the other approaches, predicting the “exact” value of $k_{\text{min}} = 7^{23}$. With respect to triangular lattices, the values obtained for the critical $k$ are $k_{\text{min}} = 4$ (FH), $k_{\text{min}} = 4$ (GD) and $k_{\text{min}} = 10$ (SE). Finally, the behavior of $\theta_c$ as a function of $k$ for the analytical approximations follows qualitatively the Monte Carlo simulation results, which reinforces the robustness of the analysis introduced here.
IV. CONCLUSIONS

In the present work, we have addressed the critical properties of long straight rigid rods on square and triangular lattices at intermediate density. The results were obtained by combining Monte Carlo simulations, finite-size scaling techniques and three well-known multisite-adsorption theories: the Flory-Huggins approximation, the Guggenheim-DiMarzio approach and the Semiempirical Model for Adsorption of Polyatomics.

Two main conclusions can be drawn from the present work. On one hand, the critical density dependence on the particle size $k$ has been reported. We found that $\theta_c(k)$ follows a power law as $\theta_c(k) \propto k^{-1}$. On the other hand, our analysis allowed us also to determine the minimum value of $k$ ($k_{min} = 7$), which allows the formation of a nematic phase on a triangular lattice.

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Figure Captions

Fig. 1: Configurational entropy per site (in units of $k_B$) versus surface coverage for different $k$ as indicated and $L/k = 20$. Dotted line and symbols represent MC results for triangular lattices, and solid lines correspond to exact results for one-dimensional systems. Inset: Surface coverage dependence of the nematic order parameter for different $k$ as indicated and $L/k = 20$.

Fig. 2: (a) Curves of $U_L$ vs $\theta$ for square lattices, $k = 10$ and different lattice sizes as indicated. From their intersections one obtains $\theta_c$. Inset: Size dependence of the order parameter, $\delta$, as a function of coverage. (b) Same as part (a) for triangular lattices.

Fig. 3: Simulated results for critical density $\theta_c$ as a function of $k$. The symbology is indicated in the figure.

Fig. 4: (a) An example of a “small window” of side $l = 8$, which is embedded in an infinite lattice. Open squares correspond to empty sites and black bars represent adsorbed $k$-mers with $k = 4$. (b) Same as part (a) for rescaled values of $l$ and $k$. In this case, $l = 24$ and $k = 12$.

Fig. 5: Theoretical results for critical density $\theta_c$ as a function of $k$. The symbology is indicated in the figure.
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38 Note that the expression $\theta = kN/M$ is valid in the thermodynamic limit, where the contribution to the local coverage due to $k$-mers adsorbed on the window boundaries is negligible.
Fig. 1: Graph showing the relationship between $s/k_B$ and $\theta$ for different values of $k$. The inset highlights the data for $k=7$ and $k=8$. The main graph includes data points for $k=8$.
Square lattice

$U^* = 0.615(5)$

$\theta_c = 0.502(1)$

Fig. 2(a): Matoz-Fernandez et al.
Triangular lattice

\(U^* = 0.613(5)\)
\(\theta_c = 0.530(1)\)

Fig. 2(b): Matoz-Fernandez et al.
Fig. 3: Matoz-Fernandez et al.

![Graph showing the relationship between $\theta_c$ and $k$ for triangular and square lattices. The graph includes data points and a line indicating a slope of approximately -1.]

- Triangular lattice
- Square lattice

slope $\approx -1$
Fig. 4(a): Matoz-Fernandez et al.
Fig. 4(b): Matoz-Fernandez et al.
Fig. 5: Matoz-Fernandez et al.

$\theta_c$ vs. $k$

- Dotted line: SE, square lattice
- Dot-dashed line: GD, square lattice
- Dashed line: FH, square lattice
- Solid line: SE, triangular lattice
- Triangle: GD, triangular lattice
- Triangle with a dot: FH, triangular lattice

The data follows a power law with $\theta_c \sim k^{-1}$. 

Fig. 5: Matoz-Fernandez et al.