Quantum Noise from a Bohmian perspective: fundamental understanding and practical computation

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Abstract  The study of electron transport in quantum devices is mainly devoted to DC properties, while the fluctuations of the electrical current (or voltage) around these DC values, the so-called quantum noise, are much less analyzed. The computation of quantum noise is intrinsically linked (by temporal correlations) to our ability to understand/compute the time-evolution of a quantum system that is measured several times. Therefore, quantum noise requires a salutary understanding of the perturbation (collapse) of the wave function while being measured. There are several quantum theories in the literature that provide different (but empirically equivalent) ways of understanding/computing it. In this work, the quantum noise (and the collapse) associated to an electron impinging upon a semitransparent barrier are explained using Bohmian mechanics. The ability of this theory (which deals with wave and point-like particles) in explaining the collapse in a natural way, from the conditional wave function, are emphasized. From this result, the fundamental understanding and practical computation of quantum noise with Bohmian trajectories are discussed. Numerical simulations of low and high frequency features of quantum shot noise in a resonant tunneling diode are presented (through the BITLLES simulator), showing the usefulness of the Bohmian approach.

Keywords  Quantum Noise · Bohmian trajectories · Multi-time measurement · Collapse wave function

1 Introduction

Historically, the definition of noise was related to the sound: A noise is an unwanted, unpleasant and confusing type of sound. However, such definition is ambiguous. What does it mean unwanted, unpleasant or confusing? An attempt to provide a more academic definition comes from music: Noise is a non-harmonious or discordant group of sounds. Again, however, the definition is not free from ambiguities because one man’s noise is another man’s music.

A more scientific definition closer to our interest in electrical devices comes from communications: A noise is an electric disturbance that interferes with or prevents reception of a signal or of information. For example, the buzz in a telephone call. Thus, we realize that once we have a precise definition of what is a signal, the meaning of what is noise becomes perfectly clear: It is the difference between the measured value and the signal.

1.1 Quantum noise in electrical devices from an experimental point of view

As discussed above, the answer on what is noise in electrical devices depends on our definitions of the electrical signal. For most DC applications, the signal is just the time-averaged value of the current. For frequency applications, the signal is equivalently defined as a time average value, but using a shorter time interval (related to the inverse of the operating frequency). In other applications, mainly digital applications, the signal is related
to the time-averaged value of the voltage in a capacitor. Hereafter, we will assume that the electrical signal is the DC value of the current, referenced by the symbol $\langle I \rangle$. To simplify the discussion, we assume the temporal average equal to the ensemble average. All fundamental and practical issues discussed here for the DC signal (and its noise) can be easily and straightforwardly extended to those other types of electrical signals.

Once we have defined the signal $\langle I \rangle$ as the DC value, the noise can be simply quantified, by time-averaging the difference between the measured value of the current $I(t)$ and the signal in a unique device during a large period of time $\tau$:

$$\Delta I^2 = \lim_{t \to \infty} \frac{1}{\tau} \int_0^\tau (I(t) - \langle I \rangle)^2 dt.$$  \hfill (1)

The square of the difference is considered to avoid positive and negative cancellations. Due to the ergodic assumption, the noise in Eq. (1) can be alternatively calculated from an ensemble-average of all possible values of the current $I_i$ measured at one time $t$ over an ensemble of (identical) devices:

$$\Delta I^2 = \sum_i (I_i - \langle I \rangle)^2 P(I_i).$$  \hfill (2)

The index $i$ indicates different values of the measured current. For simplicity, here, we assume a discrete range of current values. The probability $P(I_i)$ of measuring the current $I_i$ is defined as the ratio of the number of devices $N_i$ giving $I_i$ over the total number $N$ of devices, this means $P(I_i) = N_i/N$. In the ideal case, $N$ tends to infinite.

However, electronic systems cannot measure all the noise appearing in Eq. (1) and Eq. (2). Most of electronic apparatuses, and the ammeter itself, have to be interpreted as low-pass filters. Therefore, they are not able to measure all noise of the spectrum, but only up to a maximum cut-off frequency. In practice, what can be measured is the power spectral density of the noise below the cut-off frequency. Such power spectral density is related (by a Fourier transform) to the correlation function. The correlation function is the ensemble-average of an event defined as measuring the current $I_i(t_1)$ at time $t_1$ and the current $I_j(t_2)$ at time $t_2$:

$$\langle I(t_2)I(t_1) \rangle = \sum_i \sum_j I_j(t_2)I_i(t_1)P(I_j(t_2), I_i(t_1)).$$  \hfill (3)

It is important to emphasize (for a posterior discussion) that the probability $P(I_j(t_2), I_i(t_1))$ implies a unique system whose current is measured at time $t_1$ giving the value $I_i$, evolving without measurement until time $t_2$, when the system is measured again giving $I_j$.

Let us emphasize that the previous discussion is valid for either classical or quantum devices. We repeat. Once the signal is defined, the (quantum) noise is the result of subtracting the (quantum) signal $\langle I \rangle$ to the (quantum) experimental value $I(t)$. Expressions (1), (2) and (3) are equally valid for classical or quantum electrical devices. The adjective quantum emphasizes that the signal and the noise are computed or measured in an electrical device governed by quantum laws [2,3,4,5].

Up to here, we realize that the definition of quantum noise seems very trivial. Then, why does the concept of quantum noise have a halo of mystery around it?

### 1.2 Quantum noise in electrical devices from a computational point of view

Our previous definition on what is quantum noise do not answer the question on how we compute it. If we want to predict the values $I(t)$ in Eq. (1) or the probabilities $P(I)$ and $P(I_j(t_2), I_i(t_1))$ in Eq. (2) and Eq. (3), we require a quantum theory.

There are several quantum theories available in the literature that, by construction, are empirically equivalent when explaining all quantum phenomena. Among others, the so-called Copenhagen or orthodox interpretation [6,7], Bohmian mechanics [8,9,10] or the many-worlds theory [11]. Any theory has usually two different planes. First the formalism, which is a set of mathematical rules (using elements such as wave functions, operators, trajectories) that allows us to make practical computations that reproduce experimental results. The formalism of a theory provides an answer to the question: How do we compute quantum noise? The second plane of a theory is its interpretation. It tries to provide a deep connection on how the mathematical rules and its elements (wave functions, operators, trajectories, etc.) explain the ultimate reasons on how nature works. The interpretation of the theory provides answers to the question: Which is the physical origin of quantum noise? Each quantum theory will provide its own answers to both questions.

Many people argue that the only important part of a quantum theory (once we know it is empirically valid) is its formalism because it is the only part we need to make computations. Certainly, one can make noise computations using any of the available formalisms without worrying about its interpretation. At the end of the day, by construction, each theory should give the same predictions. Other people argue that even when one is only
interested in computations, a correct understanding on the interpretational issues of each theory is fruitful because it provides an enlarged vision on how correctly apply the theory in unsolved problems (abandoning the *shut up and calculate* [12] attitude when facing quantum problems). We will return to this very point later, in the conclusions in Sec. 5.

At this point, we want to clarify why quantum noise is specially sensible to fundamental quantum mechanical issues. Any electrical device (or any experiment) is connected to a measuring apparatus. In our case, an ammeter to get the electrical current. Quantum noise is sensible to the (ammeter) measuring process. As stated in Eq. (3), in order to obtain the noise, the quantum system has to be measured twice. This two-time measurement faces directly with one of the more complex issues in quantum mechanics, namely, which is the perturbation of the quantum wave function when a measurement is performed. Historically, this perturbation is known with the somehow scary name of the *collapse of the wave function*. Can we ignore it? Definitely not if temporal correlations need to be correctly predicted. In the literature on quantum noise in mesoscopic systems, the need for a proper understanding of the measurement process for computing quantum noise is obviously acknowledged. See for example, Ref. [13]: “The fluctuations ... are a consequence of a probabilistic reflection and transmission probability (a wave phenomena) and are a consequence of the fact that detectors register either a transmitted or a reflected particle (a particle phenomena)”. The measurement process is hidden in the word *detectors*.

A quantum theory is more than just the collapse of the wave function discussed above. Many other complex phenomena are present. For example, the charge of electrons (that implies Coulomb interaction among different electrons) provides enormous computational difficulties in the quantum regime. Equivalently, the spin of electrons (with the exchange interaction, also known as the Pauli principle) plays a very relevant role in determining the noise in a quantum device. In any case, in this letter, we will mainly focus our discussion on a very simple quantum system: a flux of (single-particle) electrons tunneling through a barrier. All conceptual issues discussed in our simplified system can be straightforwardly extended to more complex and real systems (as shown in Sec. 4). In any case, we believe that the origin of the difficulties accompanying the concept of quantum noise is the fundamental role the collapse of the wave function plays in its understanding.

As a last comment of this introduction, we mention that the fundamental understanding/computing of the measurement process can be largely relaxed when dealing with DC predictions. They can be computed from an ensemble of devices with only one measurement in each device, so we can ignore the evolution of the quantum system after the measurement. See Appendix A to enlarge this point.

In this paper, we will provide an explanation on the role of collapse in the quantum noise from a Bohmian perspective. We emphasize that we are not saying that the Bohmian answer is the best one. Answers from other theories are equally satisfactory, and provide the same predictions. We are just defending that it is a consistent answer that in the authors’ opinion provides a quite intuitive and understandable explanation of the quantum noise and also a numerically accessible formalism. In Sec. 2 we explain how Copenhagen interpretation explains the multi-time measurement process in a experiment with a flux of electrons impinging upon a tunneling barrier, by introducing the notion of operators. Then, in Sec. 3 we illustrate how the formalism of Bohmian mechanics exposed in Sec. 3 can be applied in practical problems to calculate the quantum noise in electrical devices, including Coulomb and exchange interactions. Finally, we conclude in Sec. 5 explaining how the different theories explain the origin of quantum noise.

### 2 Multi-time measurement with operators

A typical scenario when discussing quantum noise in electrical devices is a flux of electrons impinging upon a partially transparent barrier (located in the middle of the active region). Electron transport through the barrier takes place by tunneling. Electron is either transmitted or reflected, but not both! [5,13]15 We get a transmitted electron with a probability $T$, while a reflected one with probability $R = 1 − T$. To simplify the discussion, we consider a constant injection of electrons (at zero temperature), one by one. Each electron, after measurement at time $t$, will appear randomly at the left or the right of the barrier. The time averaged number of transmitted electrons will be proportional to $T$, but the number of transmitted electrons fluctuates instantaneously because of the randomness of the transmission. These fluctuations on the number of transmitted electrons (when compared with the DC signal) are named partition noise [4,13,15].

There are many other sources of noise in electrical devices, for example, the $1/f$ noise which become very relevant at low frequencies [5,13]. In this paper, we will only deal with partition noise due to a tunneling barrier. In Sec. 4 we will discuss partition noise considering...
also the injection of electrons at finite temperature (the so-called thermal noise). The fluctuations due to both processes simultaneously are known in the literature as shot noise [13,16].

In this section we discuss how the partition noise is understood within the orthodox interpretation of quantum mechanics, also known as Copenhagen interpretation [6,7]. Let us specify that most available formulations of shot noise are developed within this orthodox interpretation [13,14,15]. In this section, we consider a very simple example, but with a detailed discussion of the role played by the measuring apparatus (the ammeter). The Copenhagen interpretation associates a wave function $\Psi(x_N, t)$ to a system of $N$ particles. In principle, such wave function lives in a $3N + 1$ dimensional configuration space. Within the first non-relativistic quantization language, the evolution of this wave function is defined by two laws [17]. The first law, known as Schrödinger equation, states that (when the system is not measured) the wave function evolves unitarily and deterministically according to the following equation

$$i\hbar \frac{\partial \Psi(x_N, t)}{\partial t} = H\Psi(x_N, t),$$

where $H = \left[ \sum_i -\frac{\hbar^2}{2m_i} \nabla_i^2 + U(x_N, t) \right]$. With $U(x_N, t)$ we denote a generic interaction potential in the position representation, with $m_i$ the mass of the $i$-th particle and with $x_N = (x_1, x_2, ..., x_N)$ the multidimensional vector in the configuration space. To provide a simple discussion of the partition noise in a tunneling barrier, let us assume that each electron in our experiment can be described by a single-particle wave function (we neglect the exchange and the Coulomb interaction among electrons). In Fig. 1 we plot the (unitary) evolution of such wave function solution of Eq. (4). Is the (unitary) Schrödinger equation alone depicted in Fig. 1 enough to understand quantum noise? No (we will clarify this answer later). The orthodox theory has a second law, known as the collapse of the wave function, that takes into account the effects of the interaction of a measuring apparatus with the quantum (sub)system. This second law can be found in many textbooks [18]. It requires a new non-unitary operator $A$ (different from the Hamiltonian which is present in the Schrödinger equation) able to encapsulate all the interactions of the quantum systems with the rest of the particles (including the ammeter, the cables, the environment, etc.). This operator is the only tool provided by the theory to determine the possible results of a measurement. In principle we do not know anything about this operator except that it is a (hermitian $A^\dagger A = I$) function whose (real) eigenvalues $\lambda_n$ of its spectral decomposition are the possible results of the measurement. Once the system in Fig. 2 is measured (and not before), the wave function is projected to one of the eigenstates of the mentioned operator in a non-unitary evolution [6]. After the collapse, the new wave packet evolves again according to the time-dependent Schrödinger equation until a new measurement is done.

For simplicity, in our present conceptual discussion let us assume a reasonable (but ad-hoc) operator (why this operator is reasonable will be clarified in Sec. 5). Such operator provides the following perturbation of the wave function. If the electron is randomly measured as a reflected electron at $t_1$, the transmitted part of the wave function is eliminated. This measuring process corresponds to Figs. 2 (c) and (d) where only the reflected wave function survives after $t_1$. Equivalently, the measurement process associated to randomly getting a transmitted electron corresponds to eliminating the reflected part, as seen in Figs. 2 (g) and (h).

The measurement described in most textbooks is called “projective” (strong) measurement. There exists, for example, another type of measurement known as weak measurement, which is useful to describe situations where the effects of the apparatus on the measured system is just a small perturbation.
wave function for a reflected electron detected at time (a), (b), (c) and (d) Non unitary evolution of the
Fig. 2
Quantum Noise from a Bohmian perspective: fundamental understanding and practical computation 5
be found in a later time
electron found at time (e), (f), (g) and (h) Non unitary evolution of
By looking at Fig. 1, it could be the case that an elec-
tion Eq. (3) where a two time measurement is required.
Now, by comparing the evolutions of the wave func-
tions in Fig. 1 and Fig. 2, it is obvious that the former
is wrong. Imaging that we require the use of expres-
sions in Fig. 1 and Fig. 2, it is obvious that the former
figure online)
Squared Modulus of Wave Function (a.u.)
Electron Position x (nm)
(d) t = t₁
(b) Interaction with the barrier
(a) t = 0
(h) t = t₁
(c) t = t₂
(g) t = t₁
(f) Interaction with the barrier
(e) t = 0
Fig. 2 (a), (b), (c) and (d) Non unitary evolution of the wave function for a reflected electron detected at time t₁ on the left side. (e), (f), (g) and (h) Non unitary evolution of the wave function for a transmitted electron detected at time t₁ on the right side. Symbols are the same of Fig. 1 (Color figure online)

3 Multi-time measurement without operators
In the previous section, we discussed how the Copenhagen formalisms can be successfully used to understand quantum noise in electrical devices. One technical difficulty with this formalism is the proper definition of the right operator that determines the collapse of the wave function when, for example, the total (conduction plus displacement) current is measured.

There are alternative theories which account for the perturbation of the wave function during a measurement process in a different way, without operators. The one that we will develop here is Bohmian mechanics. Let us emphasize again that both (Copenhagen and Bohmian) theories are empirically equivalent so that the preferences of one in front of the other is related only to non-fundamental issues like computational abilities, intuitive result, etc. [9]

In the Bohmian theory, the complete description of a quantum system of N particles is given by the (same) wave function, Ψ(¯X_N, t) mentioned in Sec. 2 and by the actual positions of the point-like particles, X_N(t) = (X_1(t), X_2(t), ..., X_N(t)) (see Appendix B and Refs. 9,10 for a more detailed discussion of this theory). We emphasize that the evolution law for the wave function Ψ(¯X_N, t) is the same as in standard quantum mechanics: the Schrödinger equation (Eq. 1). The wave function guides the movement of the actual positions of the particles in time, according to the so-called guidance equation, which defines the velocity of the i-th particle as

3 We denote with capital letter X the actual position of the particle, while lower case letter x are used to indicate generic position. With barred letter we refer to a multidimensional vector in the configuration space, while letter without bar denotes the 3-dimensional vector in physical space.
associated to particle \( \text{Im} \) denote the imaginary part. We time the velocity field for the \( i \)-particle and depends on the actual position of all the particles of the system \( \hat{X}_N(t) \). Each particle follows a definite trajectory which can be obtained integrating in time the velocity field

\[
X_i(t) = X_i(0) + \int_0^t v_i(t')dt',
\]

where \( X_i(0) \) is the initial position of particle \( i \).

A proper ensemble of these trajectories (proper means that the initial position of each trajectory of the ensemble is selected according to the initial squared modulus of the wave function, see Eq. \( \text{[23]} \) in Appendix \( \text{[B.2]} \) reproduces the time-evolution of the many particle wave function, at any later time. In Sec. \( \text{[2]} \) we saw that in order to reproduce the experimental results, we have used the notion of operators to describe how the wave function of a measured system is modified under the measurement process. In the Bohmian theory, we simply consider the apparatus as another (big and complex) quantum system interacting with our measured system. The interaction among the system is an electron labelled as \( x_1 \) and depends on the actual position of all the particles excluding that of the pointer of the apparatus. The interaction between the electron and the pointer can be modeled as:

\[ H_{\text{int}} = \lambda Q(x_1)P_{x_2} = -i\hbar \lambda Q(x_1) \frac{\partial}{\partial x_2}, \tag{7} \]

where \( P_{x_2} = -i\hbar \frac{\partial}{\partial x_2} \) is the momentum operator of the detector and \( \lambda = 50 \text{ nm/ps} \) is the interaction constant. \( Q(x_1) \) is a function that is equal to zero when the electron is outside the detector, \((x_1) < 75 \text{ nm} \) in Fig. \( \text{[3]} \), and is equal to one when the particle is inside the detector \((x_1) > 75 \text{ nm} \). In Fig. \( \text{[3]} \) the region in the configuration space in which this function is different from zero is represented by a rectangle. The many-particle Schrödinger equation reads

\[
\begin{align*}
\frac{i\hbar}{2m} \frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2M} \frac{\partial^2}{\partial x_2^2} + U(x_1) - i\hbar \lambda Q(x_1) \frac{\partial}{\partial x_2} \end{align*}
\]

\[ \Psi(x_1, x_2, t) = \left( \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x_1^2} + \frac{\hbar^2}{2M} \frac{\partial^2}{\partial x_2^2} + U(x_1) - i\hbar \lambda Q(x_1) \frac{\partial}{\partial x_2} \right) \Psi(x_1, x_2, t), \tag{8} \]

where \( m \) is the effective mass of the electron, \( M \) is the mass of the apparatus pointer, and \( U(x_1) \) is the external potential energy barrier.

The main feature of a transmitted charge detector is that the center of mass of the wave function in the \( x_2 \) direction has to move if the electron is transmitted and it has to be at rest if the electron is reflected. We solve Eq. \( \text{[3]} \) numerically considering as initial wave function the products of two gaussian wave packets, i.e. \( \Psi(x_1, x_2, 0) = \psi(x_1, 0)\phi(x_2, 0) \). All details of this simulation can be found in \( \text{[19]} \). In particular we are considering \( M = 75000 \text{ m} \). In Fig. \( \text{[3]} \) the numerical solution of the squared modulus of \( \Psi(x_1, x_2, t) \) is plotted at four different times. At the initial time \( t = 0 \), Fig. \( \text{[3]} \) (a), the entire wave function is at the left of the barrier. At a later time \( t_0 \) the wave function has split up into reflected and transmitted parts due to the barrier, see Fig. \( \text{[3]} \) (b). At this time, because the electron has not been moved by the reflection, the transmitted charge detector can detect the successful transmission of an electron.
the wave function appears. For time interaction of the detector with the transmitted part of option with the electron. Then, Figs. 3 (c) and (d), the transmitted part of the wave function is shifted up in the \( x_2 \) direction while the reflected part does not move. The interaction with the apparatus thus produces two channels in the configuration space, one corresponding to the electron being transmitted and the other corresponding to the electron being reflected.

In Fig. 3 we also plot the actual positions of the system and detector \( \{ X_1(t), X_2(t) \} \) for four different possible initial positions \( \{ X_1(0), X_2(0) \} \), corresponding (say) to four distinct runs of the experiment (labelled by \( \alpha = 1, ..., 4 \)). Of the four possible evolutions shown, three have the electron transmitting (\( \alpha = 2, 3, 4 \)) and one has it reflecting (\( \alpha = 1 \)). While the pointer position \( X_2(t) \) does not move for the reflected particle, its evolution for the transmitted ones clearly shows a movement. In conclusion, looking at the detector position we can perfectly certify if the particle has been reflected (\( X_1(t) < -50 \) nm and \( X_2(t) = 0 \) nm) or transmitted (\( X_1(t) > -50 \) nm and \( X_2(t) \approx 15 \) nm). We hope the reader will realize how trivially we have been able to explain the measurement, using only a \textit{channellized} (unitary) time-evolution of 2D wave function plus two Bohmian trajectories, one for the system and another for the measuring apparatus.

Once we have solved the complete problem of the measurement in (2D) configuration space, we can do-

\[
\Psi(x_1, x_2, t_0) = |\psi_T(x_1, t_0) + \psi_R(x_1, t_0)| \phi(x_2, t_0), \quad (9)
\]

in which the detector is in an entangled superposition with the electron. Then, Figs. 3 (c) and (d), the interaction of the detector with the transmitted part of the wave function appears. For time \( t > t_0 \) the transmitted part of the wave function is shifted up in the \( x_2 \) direction while the reflected part does not move. The interaction with the apparatus thus produces two channels in the configuration space, one corresponding to the
scribe the same measurement in (1D) physical space with the help of the conditional wave function. The key point illustrated here is that the collapse of the one-particle wave function for the electron, which collapse is of course postulated through the second law seen in Sec. 2 in ordinary quantum theory, instead arises naturally and automatically in Bohmian mechanics. It is simply a consequence of slicing the unitary-evolving (2D) wave function $\Psi$ along the (moving) line $x_2 = X_2(t)$, resulting $\psi_1(x_1, t) = \Psi(x_1, X_2(t), t)$. In Fig. 3 we have plotted two solid horizontal lines corresponding to a slice of the wave function at two different values of $X_2(t)$. In Fig. 4 we report the evolution of these (time-dependent) slices of the many-particle wave function, the conditional wave function for the electron, for the trajectories $\alpha = 1$ and $\alpha = 3$ from Fig. 3. We clearly see that if the particle is reflected, as is the case for $\alpha = 1$, the position of the pointer does not change with time and, after the interaction with the detector has been performed, the electron’s conditional wave function includes only a reflected part. See Figs. 4 (c) and (d). On the other hand, when the particle is transmitted (e.g., $\alpha = 3$), it is the reflected part of the conditional wave function which collapses away, leaving only the transmitted packet. See Figs. 4 (g) and (h). Note in particular that the evolution of $\psi_1(x_1, t)$ (the electron’s conditional wave function) is not unitary, even though the evolution of $\Psi$ is.

While the wave function provides only statistical information about the experimental results, with the help of the Bohmian trajectories, we have been able to recover the individual result of each experiment. In fact for each experiment the pointer of the detector is either moving (corresponding to a transmitted electron) or not (reflected electron), while an ensemble of repeated experiments (where the initial positions of the particles, both the electron $X_1(0)$ and the detector $X_2(0)$, are selected according to the squared modulus of the wave function at the initial time $|\Psi(x_1, x_2, 0)|^2$) reproduce the same statistical results.

Thus with the previous numerical example we have reproduced the collapse-behaviour of the wave function of a transmitted (or reflected) electron. This allows us to conclude that the same results of standard formalism (we explained in Sec. 2) are obtained within Bohmian mechanics (see [920] for a formal derivation of the empirical equivalence of the two theories). Apart from irrelevant technicalities (related on how we define the measuring apparatus) the results in Fig. 2 and Fig. 3 are conceptually identical. We emphasize that, the collapse in Bohmian theory is naturally derived. Such a natural derivation of the collapse behavior demystifies the measurement process (and the quantum noise). We underline that we achieve the non-unitary evolution of the wave function of a measured system simply slicing the enlarged wave function (which includes the apparatus) in the configuration space. Even more, the fact that the apparatus, what we have called transmitted charge detector, is directly treated into the Hamiltonian of the Schrödinger equation allows to study such a situation where it is not completely clear which is its actual effect on the measured system and where it is difficult to find the right operator able to reproduce the experimental results.

Let us return to the questions posted at the end of Sec. 2 about the measurement of the total (conduction or displacement) current. Is this measurement process continuous or instantaneous? Does it provide a strong or a weak perturbation of the wave function? The Bohmian theory does not provide simple answers to these questions, but it clearly indicates the path. We need to include (somehow) the measuring apparatus in the Hamiltonian. Here, the electrostatic interaction between the electrons in the system and those in the contacts, cables, etc. See a preliminary work on this direction in Ref. [21].

A powerful simulator which uses quantum Bohmian mechanics to compute DC and quantum noise is the BITLLES simulator [2223]. This simulator allows us to work with a lot of flexibility, being able of introducing any sort of potential, including Coulomb correlations and exchange interaction. The next Section is dedicated to expose the fundamental ideas of the simulator and an example of the calculation of noise with it.

### 4 Practical application

We have previously exposed the main features of Bohmian mechanics in explaining in a quite trivial way the partition noise in a tunneling barrier. However, as it occurs for all theories, there is a huge step between its general formulation and its practical application. In fact, it happens many times that the problem of interest is unsolvable both analytically and numerically. Some kind of approximations are always required. The paradigmatic example of the need of approximations in quantum theories is the well-known many-body problem, that reminds us that the celebrated Schrödinger equation in Eq. (4) (or any alternative formulation) can only be solved exactly for very few (one, two, three,..) degrees of freedom [2425].

In principle we have seen in Sec. 3 that to reproduce the collapse of the wave function in Bohmian mechanics we have to include a suitable interaction with an external apparatus. Then we can write down the Schrödinger
equation for our complete system including all the electrons in the active region of the device plus all the particles composing the detector. But solving numerically this problem is obviously impossible. Again, the many-body problem appears. Then we should look for suitable approximations able to reduce the complexity of our problem. Let us emphasize that the (technical) approximations that we will show, do not alter the general framework we have previously presented.

4.1 An approximation for the interaction between the electron and the measuring apparatus

The first kind of approximation regards the inclusion of the apparatus in our simulations. It seems that its inclusion is unavoidable in order to provide the collapse of the wave function. And this is true, but in the particular case of quantum noise in electrical devices the fact of playing with (Bohmian) trajectories will greatly simplify the problem. In Appendix B.2, we have reported how any experimental value is calculated in Bohmian mechanics. The important thing is that any expectation value of a given operator is simply calculated as a function of the actual particles positions over an ensemble of repeated experiments (see Eq. (21) in Appendix B.2). Thus what really matters in the computation of a property of the quantum system are only the trajectories of the Bohmian particles (not the wave functions). Therefore, if they are enough accurate (this means if the error due to neglecting the apparatus is reasonably small compared to the exact solution) we can get accurate results with a minimal computational effort. In the case of the transmitted charge detector of Sec. 3 it has been demonstrated [19] that the error due to the exclusion of the apparatus from the simulations is almost negligible for the computation of the trajectories. In this way we can decrease enormously the computational burden, removing all the degrees of freedom related to the apparatus from our computations.

We can provide a more didactic discussion on why the previous technical approximation for the measuring apparatus works quite well when using Bohmian trajectories. In Sec. 2 we conclude that the reason why the wave function evolution in Fig. 1 where wrong is because the wrong possibility that an electron that is transmitted at time \( t_1 \) is later reflected at time \( t_2 \). This unphysical result simply disappears when using Bohmian trajectories: the dynamic of a transmitted electron at time \( t_1 \) will be determine locally by the guidance law Eq. (4) that only takes into account the transmitted part of the wave function. We can, for all practical purposes, completely ignore the reflected part of the wave function. Therefore, at time \( t_2 \), this electron will remain as a transmitted electron with full certainty. Thus if the trajectory without detector is enough accurate (as explained above) it does not matter if the collapse of the wave function is taken into account or not.

Finally, let us emphasize that, in principle, the measuring apparatus has also a role in the classical simulation of electronic devices. Such interaction with the apparatus is included at a classical level, at best, by a proper boundary conditions for the scalar potential of the Hamiltonian (i.e. the Poisson equation) ensuring overall charge neutrality. Obviously, this kind of approximations can also be included here.

4.2 An approximation for the Coulomb and exchange interaction among electrons

Once we have practically eliminated the apparatus from our computations, a second kind of approximation regards the interactions among the electrons of our device. The active region of the electronic device can contain hundreds of electrons. Also in this case, as we mentioned, the many-particle Schrödinger equation can be solved only for very few degrees of freedom. A standard way to proceed consists then on reducing the complexity of the problem by tracing out certain degrees of freedom. This process ends up with what is called the reduced density matrix. When the reduced density matrix is used, its equation of motion is no longer described by the Schrödinger equation but in general by a non-unitary operator. The reduced density matrix is no longer a pure state, but a mixture of states and their evolution is in general irreversible [20]. Now we discuss how Bohmian mechanics allows us to proceed in a very different way. As it will be seen below, the concept of conditional wave function [20] provides an original tool to deal with many-body open quantum systems [21, 22, 23]. While in the first approximation we have excluded some degrees of freedom (the ones of the apparatus) because its interaction with the system was small enough to certify that the perturbation of the (system) trajectories was negligible, this is not the case for this second approximation. In fact the interactions among the electrons cannot be avoided to go beyond the mean field approximations.

As said, once again the key instrument is the conditional wave function. In order to use the conditional wave function to reduce the degrees of freedom of a system we must know how it evolves in time. It can be demonstrated [21] that the single-particle conditional wave function of particle 1, \( \psi_1(x_1, t) \), for a system of \( N \)
interacting particles, obeys the following wave equation:
\[
\frac{i\hbar}{\partial t} \psi_1(x_1,t) = \left\{ -\frac{\hbar^2}{2m} \nabla^2_1 + U_1(x_1,\bar{X}_{N-1}(t),t) + G_1(x_1,\bar{X}_{N-1}(t),t) + iJ_1(x_1,\bar{X}_{N-1}(t),t) \right\} \psi_1(x_1,t). \tag{10}
\]

The explicit expression of the potentials \(G_1(x_1,\bar{X}_{N-1}(t),t)\) and \(J_1(x_1,\bar{X}_{N-1}(t),t)\) that appears in Eq. (10) can be found in reference [27]. However, their numerical values are in principle unknown and need some educated guesses. On the other hand, the total electrostatic potential energy among the \(N\) electrons that appears in Eq. (4), has been divided into two parts:
\[
U(x_1,\bar{X}_{N-1}(t),t) = U_1(x_1,\bar{X}_{N-1}(t),t) + U_{N-1}(\bar{X}_{N-1}(t),t). \tag{11}
\]

The term \(U_1(x_1,\bar{X}_{N-1}(t),t)\) can be any type of many-particle potential defined in the position-representation, in particular it can include short-range and long-range Coulomb interactions. The other term \(U_{N-1}(\bar{X}_{N-1}(t),t)\) in Eq. (11) is contained in the coupling potential \(G_1\) in Eq. (10). The same procedure can be done for the rest of the \(N-1\) particles, for example for particle 2 we fix the positions of particle 1, 3, ..., \(N\) obtaining the analogous of Eq. (11) for \(\psi_2(x_2,t)\). From our practical point of view, all quantum trajectories \(\bar{X}_N(t)\) have to be computed simultaneously. In order to gather all the above concepts, let us discuss a practical computation with conditional wave functions by detailing a sequential procedure:

1. At the initial time \(t = 0\), we fix the initial position of all \(i\)-particles, \(X_i(0)\), according to the initial probability distribution \(|\Psi(E_N,0)|^2\), and their associated single-particle wave function \(\psi_i(x_i,0)\).

2. From all particle positions, we compute the exact value of the potential \(U_1(x_i,\bar{X}_{N-1}(0),0)\) for each particle. An approximation for the terms \(G_i\) and \(J_i\) is required at this point.

3. We then solve each single-particle Schrödinger equation, Eq. (10), from \(t = 0\) till \(t = dt\).

4. From the knowledge of the single-particle wave function \(\psi_i(x_i,dt)\), we can compute the new velocities \(v_i(dt)\) for each \(i\)-particle (see Eq. (22) in Appendix B.1).

5. With the previous velocity, we compute the new position of each \(i\)-particle as \(X_i(dt) = X_i(0) + v_i(dt)dt\).

6. Finally, with the set of new positions and wave functions, we repeat the whole procedure (steps 1 till 5) for another infinitesimal time \(dt\) till the total simulation time is finished.

The advantage of the above algorithm using Eq. (10) instead of the many-particle Schrödinger equation (Eq. (1)) is that, in order to find approximate trajectories, \(X_i(t)\), we do not need to evaluate the wave function and potential energies in the whole configuration space, but only over a smaller number of configuration points, \(\{x_i,\bar{X}_{N-1}(t)\}\), associated with those trajectories defining the highest probabilities according to \(|\psi(\bar{x}_N,t)|^2\).

For spinless electrons, the exchange interaction is naturally included in Eq. (10) through the terms \(G_i\) and \(J_i\). Due to the Pauli exclusion principle, the modulus of the wave function tends to zero, \(|\psi(x_i,\bar{X}_{N-1}(t),t)| \to 0\), in any neighborhood of \(x_i\) such that \(|x_i - \bar{X}_{N-1}(t)| \to 0\). Thus, both terms, \(G_i(x_i,\bar{X}_{N-1}(t),t)\) and \(J_i(x_i,\bar{X}_{N-1}(t),t)\), have asymptotes at \(x_i \to \bar{X}_{N-1}(t)\) that repel the \(i\)-particle from other electrons. However, in order to exactly compute the terms \(G_i\) and \(J_i\) we must know the total wave function, which is in principle unknown. There are however a few ways to introduce the symmetry of the wave function without dealing directly with these two coupling terms [27,29,30]. Clearly, the complexity of the algorithm increases as we go beyond the single-particle quantum transport scenario mentioned in Sec. 2 and Sec. 3.

4.3 Calculating noise with trajectories

In a real device, apart from the uncertainty in the initial position in the quantum trajectories (the \(\alpha\) distribution explained in Appendix B), we have also to take into account the uncertainty on the properties of the injected electrons which we refer to the parameter \(b = \{1, ..., M_b\}\), that arises because the active region is an open system. This properties are the initial energies, momentums, etc. At finite temperature, the thermal noise introduces fluctuations on the energies of the electrons entering inside the device. As discussed at the introduction of Sec. 2, the study of the noise in electrical devices due to the partition noise of the barrier plus the thermal noise of the injection are traditionally known as quantum shot noise [1,3,13]. This is the noise studied in this Sec. 4.

There are many other sources of noise in electrical devices, for example, the \(1/f\) noise which become very relevant at low frequencies [5,4] that we do not discuss here explicitly. Finally, once we have the trajectories with both sources of uncertainties, this means having the actual positions of particles at each time, it is straightforward to compute the current value and any higher moment of the current distribution because as said before any expectation value depends only on the particles positions.

Within the approximation mentioned in Sec. 4.1 and Sec. 4.2 the total current value can be calculated as
the sum of the particle or conduction current plus the displacement current:

\[ I^{\alpha,h}(t) = I^{\alpha,h}_1(t) + I^{\alpha,h}_2(t) = \int \sum_{i=1}^{N} q_i v_i(X_i^{\alpha,h}(t)) \delta(x_S - X_i^{\alpha,h}(t)) \cdot ds + \int \sum_{i=1}^{N} \epsilon(x_S) \frac{dE(x_S; X_i^{\alpha,h}(t), t)}{dt} \cdot ds, \tag{12} \]

where \( S \) is the surface where we want to calculate the current, \( x_S \) are the points of the chosen surface, \( \epsilon(x_S) \) is the dielectric constant in the same surface and \( E(x_S; X_i^{\alpha,h}(t), t) \) is the electric field in the surface \( S \) which depends on the actual position of all the electrons. In the second and third row of Eq. (12) the products involving \( ds \) are clearly scalar products.

This discussion can be familiar for those people who works in semi-classical approaches. In fact, the Bohmian procedure is very similar to that of, for instance, the Monte-Carlo simulations of the Boltzmann equation. But instead of being the electric-field the one who guides the electrons, it is the wave function, through the guiding velocity field in Eq. (5). In fact, the way to proceed is similar: many experiments are reproduced, following some trajectories (which are determined in one case by the electric field and in the other by the wave function), and statistics of the studied system are extracted from them.

Once we know \( I^{\alpha,h}(t) \) at any time, the algorithm to compute the current fluctuations is quite simple following Eq. (1). The fluctuating signal of the current can be defined from \( \Delta I^{\alpha,h}(t) = I^{\alpha,h}(t) - \langle I^{\alpha,h}(t) \rangle \) and the variance (or the mean square or the second moment) defined as \( \langle \Delta I(t)^2 \rangle = \langle I(t)^2 \rangle - \langle I(t) \rangle^2 \). However, as mentioned in Sec. 1 experimentalists are interested in having information on how the noise is distributed along different frequencies. Then, the fluctuations of the current are computed from the covariance:

\[ \langle \Delta I(t_1) \Delta I(t_2) \rangle = \lim_{M_b \to \infty} \frac{1}{M_b M_h} \sum_{\alpha=1}^{M_a} \sum_{\beta=1}^{M_b} \Delta I^{\alpha,h}(t_1) \Delta I^{\beta,h}(t_2). \tag{13} \]

If the process is ergodic, i.e. \( \langle \Delta I^{\alpha,h}(t) \Delta I^{\alpha,h}(t + \tau) \rangle = \Delta I(t) \Delta I(t + \tau) \), we can compute the noise equivalently from the autocorrelations function:

\[ \Delta I(t) \Delta I(t + \tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \Delta I^{\alpha,h}(t) \Delta I^{\alpha,h}(t + \tau) dt. \tag{14} \]

These expressions are directly related to Eq. (13) which has played a central role in all our argumentation. A process is called wide-sense (or weakly) stationary if its mean value is constant and its autocorrelation function depends only on \( \tau = t_2 - t_1 \). Then, we define the autocorrelation function \( R(\tau) \) as:

\[ R(\tau) = \Delta I(t) \Delta I(t + \tau), \tag{15} \]

because depends only on \( \tau = t_2 - t_1 \). Wide-sense stationary processes are important because the autocorrelation function in Eq. (13) and the power spectral density function \( S(f) \) (measured by experimentalists) form a Fourier transform pair:

\[ S(f) = \int_{-\infty}^{\infty} R(\tau)e^{-j2\pi f \tau} d\tau. \tag{16} \]

This is known as the Wiener-Khinchine theorem. In many systems, one obtains the well known Schottky’s result \( S_{II*}(0) = 2q \langle I \rangle \).

4.4 Practical example

As a practical example of the computation of the fluctuations, we show here the current response to a step input voltage in the Negative Differential Conductance region of a RTD. The input signal is the step voltage \( V(t) = V_1 u(t) + V_2 [1 - u(t)] \) where \( u(t) \) is the Heaviside (step) function. The voltages \( V_1 \) and \( V_2 \) are constant. Then the current response can be expressed as \( I(t) = I_{tran}(t) + I_1 u(t) + I_2 [1 - u(t)] \) where \( I_1 \) and \( I_2 \) are the stationary currents corresponding to \( V_1 \) and \( V_2 \) respectively, and \( I_{tran} \) is the intrinsic transient current.

The results are reported in Fig. 5 where \( I_{tran} \) manifests a delay with respect to the step input voltage, due to the dynamical adjustment of the electric field in the leads. After this delay, the current response becomes a RLC-like response (dot-dashed line RLC response 2).
i.e. purely exponential. Performing the Fourier transform of $I_{\text{tran}}(t)$ in Fig. 6 and comparing it with the single pole spectra (Fourier transform of the RLC-like responses, dashed and dashed dotted lines), we are able to estimate the cut-off frequency and the frequency offset due to the delay [22].

In order to understand how the many-body Coulomb interaction affects the noise in RTDs, we also investigate the correlation between an electron trapped in the resonant state during a dwell time $\tau_d$ and those remaining in the left reservoir. This correlation occurs essentially because the trapped electron perturbs the potential energy felt by the electrons in the reservoir. In the limit of non-interacting electrons, the Fano factor will be essentially proportional to the partition noise, however, if the dynamical Coulomb correlations are included in the simulations (see Fig. 7) this result is no longer reached, it shows super-poisonian values. Finally, we are also interested in the high frequency spectrum $S(f)$ given by Eq. (16) revealing information about the internal energy scales of the RTD that is not available from DC transport (see Fig. 8).

5 Conclusions

In the present paper we discuss the understanding and the computation of quantum noise in electrical devices from a Bohmian perspective. Computations of quantum noise are quite complex because noise is generally quantified in terms of temporal correlations. Such correlations must include the time-evolution of a quantum system during and after one measurements. Usually, many quantum computations do only require a final measurement, so that their time-evolution from the initial until the final times is uniquely determined by the unitary (Schrödinger like) evolution. As discussed in Fig. 2 this unitary evolution is not enough to compute time correlations which require mixing unitary and non-unitary (the so-called collapse of the wave function) time evolutions.

There are several (empirically equivalent) quantum theories. Each quantum theory has its own formalisms that is able to connect the experimental values with some abstracts elements such as wave functions, operators, trajectories, etc. that are able to satisfactorily reproduce (or predict) experimental results. We discuss how the Copenhagen interpretation and Bohmian mechanics give explanation to the partition noise. For a flux of electrons impinging upon a tunneling barrier, we analyze how a measurement process affects partition noise in a quantum device. For simplicity, to focus on the importance of the measurement process, we
consider spinless electrons without Coulomb and exchange interaction. In Sec. 2 we explain how the standard quantum theory provides an answer for the measurement problem by means of the introduction of the notion of operators. We see that this notion is not always satisfactory, because the definition of which is the right operator is left undefined in many cases. Then in Sec. 3 we discuss an alternative way to deal with the collapse without introducing the idea of operator. In fact within Bohmian mechanics, a theory of wave and particles, the collapse is derived trivially by means of the introduction of the conditional wave function (the wave function of a subsystem), a tool exclusively belonging to the Bohm’s theory. Obviously, each theory gives a different formalism to compute quantum noise and different interpretation on its origin. In any case, at the end of the day, the same empirical predictions are achieved by using the orthodox quantum theory or Bohmian mechanics.

In Sec. 1 because the Bohmian formulation uses trajectories (not the wave function) to compute experimental results, we see that a very reasonable approximation to include collapse can be achieved with a very small computational effort. Finally, details of the simulator named BITLLES based on Bohmian mechanics and numerical result for low and high frequency noise of the current in a resonant tunneling diode are presented. We emphasize that the presented formalism and the procedure for computing the properties of a system (in our case current, noise, etc.) has many similarities with the one used in semi-classical simulations (for example Monte-Carlo of the Boltzmann equation \cite{32}). In any case, the Bohmian formalism is not at all a semi-classical approach but a complete quantum theory that can be applied to study any non-relativistic quantum phenomena, quantum noise and collapse among them.

Now, we can provide an answer to the question What is the origin of quantum noise in the orthodox interpretation? The wave function alone provides only a probabilistic information about the location of the electron. A crucial role in our understanding of the quantum noise is played by the operator. The operator selects the outcome of an experiment randomly. The partition noise in the tunneling barrier discussed in Fig. 2 is due to this stochastic evolution of the wave function when measured (the collapse). Alternatively, we can also answer what is the origin of quantum noise according to Bohmian mechanics? Each experiment has a lot of degrees of freedom (the position of electrons in the device, the particles composing the cable, the ammeter, etc.) being impossible to control all of them. The randomness in each experiment, i.e. the particle being transmitted or reflected, is then a consequence of the absolute uncertainty that is due to a global quantum equilibrium of the entire system (for more details see \cite{20}). The measurement, which is treated as any other interaction among particles, has no special role. In summary, according to the orthodox interpretation, the partition noise has its origin in the inherent stochasticity of the orthodox measurement process. On the contrary, Bohmian mechanics says that the origin of noise is the absolute uncertainty of the initial position of the trajectory in each realization of the experiment. Although both theories gives the same predictions, in the authors’ opinion, the latter has a more natural and understandable explanation of the origin of quantum noise. The collapse in Bohmian theory is so naturally derived that quantum measurement problem is somehow demystified. Then, the halo of mystery around the concept of quantum noise disappears. We underline that Bohmian mechanics achieve the non-unitary evolution of the wave function of a measured system simply slicing the enlarged wave function (which includes the apparatus) in the configuration space.

We accept that preferences between the explanation of the origin of quantum noise in terms of the orthodox or Bohmian interpretations are subjective. Therefore, in this paper we have also developed objective arguments about the computational advantages of the Bohmian formalism. The facts that the measuring apparatus, what we call the ammeter, is directly treated into the Hamiltonian of the Schrödinger equation and that the currents values are computed from trajectories (not from the wave functions) allows us to study system+apparatus scenarios (or look for reasonable approximations). This ability is very relevant, for example, in the computation of high frequency currents where it is difficult to find the right operator. It is for all these reasons, that we conclude that quantum noise is easily understood and computed from a Bohmian perspective.

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Appendix A The process of measurement in DC current

When the ergodic theorem is applicable, as we have assumed in all this paper, the prediction of the DC
current, $\langle I \rangle$, measured in a laboratory can be computed using two different protocols. First, we can compute $\langle I \rangle$ by time-averaging the measured value of the total current $I(t)$ from a unique device during a large (ideally infinite) period of time $\tau$:

$$\langle I \rangle = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{0}^{\tau} I(t) dt. \quad (17)$$

Second, we can compute $\langle I \rangle$ from an ensemble-average of all possible values of the current $I_i$ measured, at one particular time $t$, over an ensemble of (identical) devices:

$$\langle I \rangle = \sum_{i=1}^{M} I_i P(I_i). \quad (18)$$

The meaning of the index $i$ and the probability $P(I_i)$ is the same as the one given in Sec. 12.

For DC quantum transport computations, Eq. (18) is greatly preferred because it deals directly with the probabilistic interpretation of the wave function. It is important to realize that while Eq. (17) implies measuring the quantum current many times, Eq. (18) involves only one measurement. We do not need to worry about the evolution of the wave function after the measurement when using Eq. (18). Let us discuss this point in more detail. We can define the eigenstates $|\psi_i\rangle$ of a particular operator $I$, as those vectors that satisfy the equation $I|\psi_i\rangle = I_i|\psi_i\rangle$. The eigenvalue $I_i$ is one of the possible measured values in Eq. (18). Since the entire set of eigenstate form a basis for the Hilbert space, the wave function can be decomposed as $|\psi(t)\rangle = \sum_{i=1}^{M} c_i(t)|\psi_i\rangle$, with $c_i(t) = \langle \psi_i|\psi(t)\rangle$. Then, we can rediscover Eq. (18) as follow:

$$\langle I \rangle = \langle \psi(t)|I|\psi(t)\rangle = \sum_{j=1}^{M} c_j(t)\langle \psi_j|I_i|\psi_i\rangle = \sum_{i=1}^{M} I_i P(I_i), \quad (19)$$

where we have used the orthonormal property of the eigenstates $\langle \psi_j|\psi_i\rangle = \delta_{ij}$ and the definition of the (Born) probability $P(I_i) = |c_i(t)|^2$.

At this point, it is mandatory to provide some discussion about the use of the ergodic theorem. Strictly speaking, no ergodic theorem exists for an out of equilibrium system. Indeed, the out of equilibrium system is represented by a distribution function, or probability function, that is different from that in equilibrium and arises from a balance between the driving forces and the dissipative forces. The applied bias used to measure the DC current of any device implies that the device is quite likely in a far from equilibrium state. Therefore, the ergodic connection between (17) and (18) has to be considered as only a very reasonable approximation for DC transport, but not as an exact result. In this approximation is then legitimated to use Eq. (18) instead of Eq. (17) when computing DC currents. Eq. (18) avoids the complex discussions of collapse, because only one-time measurement is required (while Eq. (17) needs a multi-time measurement). For the low and high frequency noises discussed in Sec. 4 a multi-time measurement is unavoidable. Let us say that the multi-time measurement problem does not only appear for noise, but also in transients or high frequency scenarios where the knowledge of the instantaneous current is required, implying different measurements at different times.

**Appendix B ** Bohmian mechanics

Bohmian mechanics is a version of quantum theory whose basic elements are waves and point-like particles. The many-particle wave function evolves according to the Schrödinger equation (4) while particles have definite position at any time with a law given by Eq. (5), therefore being a fully deterministic theory. The configuration of the particles, say at time $t = 0$, is chosen randomly according to $|\Psi_0|^2$, known as quantum equilibrium hypothesis [20]. Thanks to the continuity equation

$$\frac{\partial \rho}{\partial t} = -\nabla (\rho v), \quad (20)$$

where $\rho = |\Psi|^2$ and $v$ the Bohmian vector field, the configuration of the particle at any time $t$ is random with probability $|\Psi|^2$, known as equivariance [20]. An important consequence of the quantum equilibrium hypothesis and equivariance is the empirical equivalence between Bohmian mechanics and quantum theory: for any kind of experiment in quantum mechanics, Bohmian mechanics makes the same predictions.

**B.1 The Conditional Wave Function**

Consider a quantum system of $N$ particles and a partition of it in such a way that its spatial coordinates can be split as $\hat{x}_N = \{x_1, x_{N-1}\}$. Where we denote with $x_1$ the position in $\mathbb{R}^3$ space of the electron 1, while with $x_{N-1}$ the positions of the rest of the electrons in a $\mathbb{R}^{3(N-1)}$ space. The actual particle trajectories are accordingly denoted by $\hat{X}_N(t) = \{X_1(t), X_{N-1}(t)\}$. How can one assign a wave function to the electron 1? In general this is not possible if the two subsystems are entangled, i.e. the total wave function cannot be written as a product $\Psi(\hat{x}_N) = \psi_1(x_1)\psi_{N-1}(\hat{x}_{N-1})$. However,
we can modify our question and ask what is the wave function of the electron that provides the exact velocity $v_1$ given a particular configuration $X_{N-1}(t)$ for the rest of the particles. The answer given by Bohmian mechanics is the so called conditional wave function \[ \psi_1(x_1, t) = \Psi(x_1, X_{N-1}(t), t), \] (21)
which constitutes a slice of the whole multi-dimensional wave function. The wave function constructed in such a way gives exactly the same Bohmian velocity
\[ v_1(t) = \frac{\hbar}{m_1} \text{Im} \frac{\nabla \psi}{\psi} |_{x=N} = \frac{\hbar}{m_1} \text{Im} \frac{\nabla \psi_1}{\psi_1} |_{x_1=X_1(t)}. \] (22)

\section*{B.2 Computation of mean value of an operator}

If needed, Bohmian mechanics can make use of operators, but only as a mathematical trick. Without any physical or fundamental role in the operator. We briefly explain how it is possible to calculate the mean value of a general hermitian operator with Bohmian trajectories. The quantum equilibrium hypothesis at the initial time $t=0$ can be expressed in terms of the trajectories as follows
\[ |\psi(x_N, 0)|^2 = \lim_{M_\alpha \to \infty} \frac{1}{M_\alpha} \sum_{\alpha=1}^{M_\alpha} \prod_{i=1}^{N} \delta(x_i - X_i^\alpha(0)), \] (23)

where the superindex $\alpha$ takes into account the uncertainty in the initial position of the particles. It can be easily demonstrated that the evolution of the above finite set of quantum trajectories $\alpha = 1, 2, ..., M_\alpha$ reproduce at any time $t$ the probability distribution, $|\psi(x_N, t)|^2$. For computing the mean value of an operator $A$ it can be demonstrated [9] that
\[ \langle A \rangle_\psi = \lim_{M_\alpha \to \infty} \frac{1}{M_\alpha} \sum_{\alpha=1}^{M_\alpha} A_B(\bar{X}_N^\alpha(t)), \] (24)

where $A_B(\bar{x}_N)$ is the “local” mean value of $A$ [9].

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