g-factor of a tightly bound electron

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ABSTRACT

We study the hyperfine splitting of an electron in hydrogen-like $^{209}$Bi$^{82+}$. It is found that the hfs energy splitting can be explained well by considering the g-factor reduction due to the binding effect of a bound electron. We determine for the first time the experimental value of the magnetic moment of a tightly bound electron.

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Recently, the hyperfine structure (hfs) of electronic atoms has received a re-
newed interest [1-7]. Experimentally, this situation may well be due to recent
technical progresses which enable to carry out precision measurements of the hy-
perfine splitting. Theoretically, the accurate determination of the hfs splitting
is expected to allow a novel test of QED corrections under the strong magnetic
field.

For the investigation of the hyperfine structure in electronic atoms, it is always
ideal if one makes hydrogen-like atoms. Indeed, this is done by Klaft et al.[1]
who made a very nice measurement of the hyperfine energy splitting for a tightly
bound electron state in hydrogen-like $^{209}$Bi$^{82+}$. They measured the wave length $\lambda$
of the $M1$ transition between $F = 4$ and $F = 5$ hyperfine levels of the ground state
of hydrogen-like $^{209}$Bi$^{82+}$, and obtained $\lambda_{exp} = 243.87(4)$ nm. This value should
be compared to the wave length calculated with a point Coulomb interaction. A
theoretical value is $\lambda_{point} = 212.7$ nm for hydrogen-like $^{209}$Bi$^{82+}$. (Note that this
number is obtained with the free electron g-factor.)

This difference between hfs level splitting has been discussed by Finkbeiner
et al.[2] and by Schneider et al.[3]. Finkbeiner calculated the hyperfine anomaly
due to the finite size of the nucleus. They showed that the observed value of
the hfs splitting can be understood by varying the magnetization distribution
of the nucleus. However, Schneider et al. showed that the hyperfine anomaly
of the electronic atom in $^{209}$Bi$^{82+}$ is much smaller than the observed difference
$\delta \lambda = \lambda_{exp} - \lambda_{point}$. Their calculated value of Bohr-Weisskopf effect [8] is $\delta \lambda_{BW} \approx
3.5$ nm, as compared to the observed value of $\delta \lambda = 31.17$ nm. Indeed, one can
easily confirm oneself that the hfs anomaly $\epsilon$ due to Bohr-Weisskopf effect is not
much different from 1 % if one makes use of the formula given in ref.[9]. This
is obviously much too small compared to $\delta \lambda / \lambda_{point} = 0.149$. Schneider et al.[4]
also estimate other QED corrections such as vacuum polarization. They find
δλ_{vp} = −1.6 nm, which is also small. Thus, up to now, all of the theoretical predictions are much too small to explain the observed difference δλ.

In this Letter, we show that the observed difference δλ can be explained well by considering the g-factor reduction of the bound electron in $^{209}Bi^{82+}$. This can easily be seen since the change of the g-factor of a bound electron due to the binding effect can be written for a point charge case as

$$g_{BC} = -\frac{2}{3}(1 - \sqrt{1 - \gamma^2})g_0$$

where $\gamma$ is defined as $\gamma = \frac{Z^2}{\hbar c}$. $g_0$ denotes the g-factor of a free electron. For $^{209}Bi^{82+}$, we find

$$g_{BC} = -0.136 \, g_0.$$  

Therefore, the wave length $\lambda$ should be corrected as

$$\lambda = \frac{g_0}{(g_0 + g_{BC})}\lambda_{\text{point}}.$$  

This gives

$$\lambda = 246.2 \, \text{nm}$$

which should be compared to the observed value of $\lambda_{\text{exp}} = 243.87(4) \, \text{nm}$. The agreement between theory and experiment is now remarkably good. Thus, the hfs splitting in electronic atom with high $Z$ is understood by the g-factor reduction due to the binding effect of the electron in $^{209}Bi^{82+}$.

The reduction of the g-factor in muonic atoms has been already measured by Yamazaki et al.[12]. The observed reduction of a bound muon for muonic $^{208}Pb$ atom is

$$g_{BC}^{\text{exp}}(\text{muon}) = (−0.047 \pm 0.022)g_0.$$  

This is compared to the theoretical g-factor reduction in muonic $^{208}Pb$ atom [11]

$$g_{BC}^{\text{muon}} = −0.032 \, g_0.$$
which is consistent with the observed g-factor reduction. In muonic atoms, the g-factor reduction is much smaller than that of the electronic atoms since the $1s_{\frac{1}{2}}$ muonic orbit in $^{208}$Pb atom is almost inside the nucleus and thus the Coulomb interaction is much weaker than the point Coulomb case, which is almost the case in electronic atoms.

Now, in what follows, we turn around the argument and try to extract the "experimental" g-factor of a tightly bound electron in high Z electronic atoms. For this purpose, we calculate the hfs anomaly of the electronic atoms in $^{209}$Bi$^{82+}$. All the necessary formula are given in ref.[9]. Here, we only write the results of the hfs constant $a_I$ for the $1s_{\frac{1}{2}}$ state,

$$a_I = a_I^{(0)} (1 + \epsilon)$$

where $a_I^{(0)}$ denotes the hfs constant for a point charge case and is written as

$$a_I^{(0)} = \frac{4ge\mu_N}{3I} \mu \int_0^\infty F^{(1s)}G^{(1s)}dr.$$ 

Here, $g$ denotes the g-factor of an electron and $\mu$ is the magnetic moment of the nucleus. $F^{(1s)}$ and $G^{(1s)}$ are the small and large components of the radial part of Dirac wave functions for the $1s_{\frac{1}{2}}$ state. $\epsilon$ can be expressed as

$$\epsilon = -0.62b^{(1s)} < (R/R_0)^2 > -0.38b^{(1s)} \frac{(R/R_0)^2}{\mu} \left[ \pm g_s \frac{3(I + \frac{1}{2})}{4(I + 1)} + \frac{3}{4} \frac{g_s}{g_s - g_L} (\mu - \mu_{sp}) \right]$$

for $I = \ell \pm \frac{1}{2}$. $R_0$ is a nuclear radius and can be given as $R_0 = r_0 A^{\frac{1}{3}}$ with $r_0 = 1.2$ fm. On the other hand, $b^{(1s)}$ is a constant which can be calculated in terms of relativistic electron wave functions with finite extension of the nucleus [8], and we find here $b^{(1s)} = 0.035$. $\mu_{sp}$ denotes the magnetic moment of the single particle state with $I = \ell \pm \frac{1}{2}$.

The energy splitting $\Delta E$ between $F = 5$ and $F = 4$ hyperfine states is related to $a_I$ as

$$\Delta E = 5a_I.$$
The wave length $\lambda$ of the $M1$ transition is related to $\Delta E$ as

$$\lambda = \frac{2\pi}{\Delta E}.$$  \hspace{1cm} (11)

Now, we evaluate the hfs anomaly, considering the effects of the core polarization with $\Delta \ell = 0$ transition. For $^{209}Bi$, the single particle state is $|\pi(1h_\frac{9}{2})\rangle$. The observed magnetic moment of $^{209}Bi$ nucleus is $\mu = 4.1106 \text{ n.m.}$. Also, the magnetic moment of the single particle state is $\mu_{sp} = 2.624 \text{ n.m.}$. Therefore, we find $\epsilon = -0.0083$. The calculated wave length shift due to the Bohr-Weisskopf effect becomes

$$\delta \lambda_{BW} = -\epsilon \lambda_{\text{point}} = 1.8 \text{ nm.}$$  \hspace{1cm} (12)

In addition, there is a contribution from higher order QED corrections [13,14]. This includes the vacuum polarization as well as radiative corrections, and is estimated to be

$$\delta \lambda_{QED} = -1.0 \text{ nm.}$$  \hspace{1cm} (13)

Therefore, the theoretical value of the wave length including the Bohr-Weisskopf effect and QED corrections becomes

$$\lambda_{\text{theory}} = \lambda_{\text{point}} + \delta \lambda_{BW} + \delta \lambda_{QED} = 213.5 \text{ nm.}$$  \hspace{1cm} (14)

From this value, we can extract the ”observed ” $g$-factor of the electron in $^{209}Bi^{82+}$ atom. We find

$$g_{BC}^{\text{exp}} = -0.125 \ g_0.$$  \hspace{1cm} (15)

This value should be compared to the theoretical value of $g_{BC} = -0.136 \ g_0$. The extracted experimental value is quite close to the theoretical one. Note that eq.(1) is obtained with a point charge case. The finite size effect on the $g_{BC}$ may decrease its magnitude by a few percents, which is consistent with the observed value of eq.(15).
In summary, we have shown that the hyperfine splitting of the ground state of hydrogen-like $^{209}\text{Bi}^{82+}$ is explained by the g-factor reduction of a tightly bound electron. This is the first experimental determination of the g-factor reduction of a tightly bound electron.
References

1. I. Klaft, S. Borneis, T. Engel, B. Frick, R. Grieser, G. Huber, T. Kühl, D. Marx,
   R. Neumann, S. Schröder, P. Seeling and L. Völker, Phys. Rev. Lett. 73, 2425 (1994)
2. M. Finkbeiner, B. Frick and T. Kühl, Phys. Lett. A176, 113 (1993)
3. S.M. Schneider, J. Schaffner, G. Soff and W. Greiner, J. Phys. B26, L581 (1993)
4. S.M. Schneider, W. Greiner and G. Soff, Phys. Rev. A50, 118 (1994)
5. T. Asaga, T. Fujita and K. Ito, Z. Phys. A359, 237 (1997)
6. K. Enders, O. Becker, L. Brand, J. Dembczynski, G. Marx, G. Revalde, P.B. Rao and G. Werth, Phys. Rev. A52, 4434 (1995)
7. M. Wada, K. Okada, H. Wang, K. Enders, F. Kurth, T. Nakamura, S. Fujitaka,
   J. Tanaka, H. Kawakami, S. Ohtani and I. Katayama, to appear in Nucl. Phys. A
8. A. Bohr and V.F. Weisskopf, Phys. Rev. 77, 94 (1950)
9. T. Fujita and A. Arima, Nucl. Phys. A254, 513 (1975)
10. K.W. Ford, V.W. Hughes and J.G. Wills, Phys. Rev. 129, 194 (1963)
11. T. Fujita, Ph. D Thesis, UTPN-55 (1974)
12. T. Yamazaki, S. Nagamiya, O. Hashimoto, K. Nagamine, K. Nakai,
    K. Sugimoto and K.M. Crowe, Phys. Lett. 53B, 117 (1974)
13. S.J. Brodsky and G.W. Erickson, Phys. Rev. 148, 26 (1966)
14. G.T. Bodwin and D.R. Yennie, Phys. Rev. D37, 498 (1988)