Calibration Error Analysis of Shadowgraph Station for Large Caliber Projectile Motion Attitude Measurement

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Abstract. The calibration principle and working process of shadowgraph station for large caliber projectile motion attitude measurement is studied. The main error factors affecting the calibration accuracy of the system are analysed, and the various errors in the calibration process are statistically analysed. The calibration error of the measurement system is calculated to be 0.062 mm. The error analysis process shows that the coincidence error between the laser optical axis and the central axis of double square nets is the main error of calibration. Among these errors, the installation error of optical window and the centering error of filtering target (Angle deviation between parallel light source beam axis and double square nets axis caused by coincidence error of reflected facula and incident facula) are the main sources of errors.

1. Introduction

The calibration method of multiphase cameras mainly depends on the calibrator. ZhengYou Zhang et al [1], proposed a calibration method based on 1D rigid body. The collinear feature points and the distance between the feature points on the calibration object were taken as constraints to establish the relationship between the multi-camera image and the external space. Daucher et al [2][3]. used sphere center instead of 2D to calibrate corner information in checkerboard. However, these calibration methods require continuous public field of view between multiple cameras. And these methods have not been applied to multi-station calibration of target range for the time being. At present, the calibration system of ballistic trajectory can be divided into two types: catenary and carrier [4].

Different from the previous shadowgraph station, the measurement of large caliber projectile motion attitude by shadowgraph method should be carried out in the field. Besides the conventional error factors, the calibration measurement errors may also be affected by natural wind, stray light, etc. The calibration principle and working process of shadowgraph station for large caliber projectile attitude measurement are studied. The main error factors affecting the calibration accuracy of the system are analyzed, and the errors in the calibration process are analyzed. Calculations show that the calibration error of the measurement system is 0.062 mm, and the main source of the error is clarified, which points out the direction for the improvement of the calibration accuracy and the control of the error.
2. Large Caliber Projectile Motion Attitude Measurement System

The overall structure of the calibration system for outfield projectile motion attitude measurement is shown in Fig. 1. It consists of a collimating light source module, a double square nets pose detection module, a hollow double square nets adjustment module and an inter-station handling module. The collimating light source module is composed of parallel light source, light source installation adjustment mechanism and vibration isolation belt. Double square nets pose detection module is composed of filtering target, optical window and PSD. The adjusting module of hollow double square nets consists of rigid base, movable adjusting platform, six-degree-of-freedom(6-DOF) platform, base and double square nets. The inter-station handling module is composed of wheeled mobile car and stacking truck.

![Figure 1. Sketch of calibration system for outfield projectile motion attitude measurement.](image)

The parallel laser tube is mounted at the muzzle through a tripod and a three-dimensional pan-tilt, and the three-dimensional pan-tilt is adjusted to make the beam level of the parallel laser tube point to the shooting direction. The mobile lifting platform vehicle is placed on the track, the 6-DOF parallel robot is fixed on the mobile lifting platform vehicle, and double square nets is installed on the 6-DOF parallel robot through the base. Pushing the mobile lifting platform vehicle, double square nets is located in the middle of the shadowgraph station, and the optical target is erected near the laser side of double square nets. The optical target is adjusted to make the laser beam pass through the center of the target. The 6-DOF parallel robot is used to adjust the position of double square nets, so that the reflected facula on the optical target coincides with the incident facula, and the position coordinate of the transmitted beam facula on the PSD is (0,0). At this time, double square nets is in the calibration posture. The current shadowgraph station can be calibrated by using double square nets as the reference material. The calibration instrument can be moved to the next camera station and calibrated until the calibration of all camera stations in the target track is completed [5].
3. Analysis of error factors affecting calibration

Figure 2. Influencing factors of error

The calibration error of measurement system includes many factors. According to the calibration device and method, it can be summarized as shown in Fig. 2. It mainly consists of the coincidence error between the optical axis of the parallel light source and the central axis of double square nets, the coincidence error between the optical axis of the camera and the central line of the grid plane before and after double square nets, and the vertical error between the reflecting screen and the optical axis of the camera.

3.1. Coincidence error between optical axis of parallel light source and central axis of double square nets

3.1.1. Horizontal Error of Parallel Light Source. In the calibration experiment, the target with Leica reflective patch is set in front of the parallel light source, and the facula of the parallel light source falls on the center of the reflective patch. The angle between the center of the parallel light source and the center of the reflective patch is measured by total station. When the angle is 0, the parallel light source is horizontal. Leica TS 60 total station used in calibration, and the accuracy of angle measurement is 0.5". Due to the limited resolution of the adjusting mechanism, the slope of the parallel light source is 2°. Because of the long calibration time, it is assumed that the height of parallel light source varies by 2 mm during the calibration process. When the angle is changed by 2.73°, the horizontal adjustment error of parallel light source is

\[ \delta_{h} = \sqrt{2^2 + 0.5^2 + 2.73^2} = 3.42". \]

3.1.2. Installation error of optical window. When the axis of double square nets is parallel to that of laser based on the principle of optical lever, it is required that the optical window be perpendicular to the axis of double square nets. Due to the influence of adjustment and processing errors, the optical window will deviate from the axis of double square nets at an angle of \( \theta \), as shown in Fig. 3.
When installing and adjusting the optical window, a parallel light source is set up at a distance of 5 m, and the light source is adjusted to the horizontal position. Installation is completed in a short time, without considering the angle change of parallel light source, the error is $2.06^\circ$. Double square nets is adjusted to the horizontal position, and the error is $4^\circ$ by using horizontal bubble strip detection. Adjust the position of the optical window until the facula reflected by the optical window coincides with the parallel light source, and the coincidence reading error is 0.5 mm. The final optical window installation error is $\delta = \sqrt{4^2 + 2.06^2 + \left(\frac{1}{2} \cdot \arctan(0.5/5000)\right)^2} = 11.25^\circ$.

3.1.3. Centering error of filtering target. The angle deviation between parallel light source beam axis and double square nets axis caused by the coincidence error of reflection facula and incident facula is $\delta_{13} = \frac{1}{2} \cdot \arctan(0.5/5000) = 10.313^\circ$.

3.1.4. Coaxality error of double square nets. Due to the limitation of the processing and assembly technology and the accuracy of the machine tool, the coaxiality of the central axis of double square nets will produce errors in the processing process. According to the existing processing level, the coaxiality error of the center axis of double square nets is 0.01 mm, or $\delta_c = \arctan(0.01/300) = 6.88^\circ$.

The angle error between the optical axis of the reference light source and the axis of double square nets is $\varphi = \sqrt{\delta_{11}^2 + \delta_{12}^2 + \delta_{13}^2 + \delta_{14}^2} = 17.09^\circ$. The tilt of double square nets will cause the camera to rotate and make the actual imaging point coordinates deviate from the ideal imaging point coordinates.

As shown in Fig. 4, when $OXY$ rotates $\varphi$ around the origin to $O'X'Y'$, there are

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

Let the coordinates of $P'$ in $O'X'Y'$ be $(m,n)$, then the coordinates of $P'$ in $OXY$ coordinates will be $\left(m\cos\varphi + n\sin\varphi, -m\sin\varphi + n\cos\varphi\right)$, and $P$ in $OXY$ coordinates will be $(m,n)$. 

![Figure 3. Optical Window Installation Diagram](image-url)
According to the function of the distance between two points, the deviation $\Delta d$ of $P$ between the actual imaging plane $OX'Y'$ and the ideal imaging plane $OXY$ is

$$\Delta d = pp = \sqrt{(m(1-\cos \phi) - n\sin \phi)^2 + (m\sin \phi + n(1-\cos \phi))^2}$$

When the projectile is located in the range of abscissa [-1000,1000] and ordinate [-3200,3200], the angle $\phi$ between the axis of the reference light source and the axis of double square nets is $17.09^\circ$ which causes the displacement of the imaging point as shown in Fig. 5. The maximum offset is 0.28 pixel.

According to the pinhole imaging model, the measurement error caused by the coincidence error between the optical axis of parallel light source and the central axis of double square nets is

$$\delta_i = \left( \frac{1700 \times 5.5}{50} \right) \times 0.28 = 52.36 \mu m$$

3.2. Coincidence error between camera optical axis and the central line of the grid plane before and after double square nets.

After the parallel light source beam axis coincides with the axis of double square nets, the position of the camera relative to double square nets is adjusted based on double square nets, so that the camera optical axis coincides with the central line of double square nets before and after double square nets.
3.2.1. **Horizontal error of Z-axis in double square nets.** Adjust the six-axis parallel robot to make double square nets in the horizontal position. The accuracy of strip horizontal bubbles is as follows: $4''$ when the grid position is detected by strip horizontal bubbles. The height difference between the latitudes of the grid lines is $\delta_{z1} = 0.0058 \text{mm}$, and the flatness of double square nets frame is $\delta_{z2} = 0.01 \text{mm}$.

3.2.2. **Parallelism Error of Center Line Axis of Mesh Surface.** The grid line positioning of double square nets relies on line hole positioning, and the machining error of line hole directly affects the parallelism of grid line. Line hole processing is based on the frame plane of double square nets. When the verticality of the three datum planes of double square nets is $0.03 \text{ mm}$, the maximum geometric tolerances of mesh lines is $t = \sqrt{3 \times 0.03} = 0.05 \text{mm}$.

3.2.3. **Mesh line error.** When the diameter of the thin line is $0.5 \text{mm} \pm 0.1 \text{mm}$, the coincidence error of the front and back grids is $\delta_{c5} = 0.2 \text{mm}$.

3.2.4. **Parallax error of grid line coincidence.** In calibration, the center cross line of the camera image before and after double square nets is reconstructed as the criterion for the coincidence of the camera optical axis and the center axis of double square nets plane. The accuracy of coincidence interpretation is $0.5 \text{ pixel}$. Therefore, the error of grid line coincidence interpretation is $\delta_{c6} = 0.09 \text{mm}$.

3.2.5. **Effect of wind force on double square nets.** In the field environment, the field grid will be affected by wind force, which will cause frame deformation. The deformation results are shown in Fig. 6 by finite element simulation. Considering the change of wind direction, the deformation of double square nets is $\delta_{w} = 0.012 \text{mm}$.

![Figure 6. Effect of wind force on double square nets.](image)

To sum up, the deviation of the thin line axis is $\Delta l = \sqrt{\delta_{z1}^2 + \delta_{z2}^2 + \delta_{c5}^2 + \delta_{c6}^2 + \delta_{w}^2 + \delta_{c5}^2} = 0.231 \text{mm}$. As shown in Fig. 7, the center deviation angle between the camera optical axis and the ideal front and rear mesh surface is $\Delta l = \sqrt{\delta_{z1}^2 + \delta_{c5}^2 + \delta_{c6}^2 + \delta_{w}^2 + \delta_{c5}^2} = 0.231 \text{mm}$. 


Figure 7. Schematic diagram of grid line deviation before and after.

The deflection angle between the optical axis of the camera and the center of the ideal front and rear mesh surface will lead to the tilt of the camera, which will lead to the change of the angle between the camera and the scene. As shown in Fig. 8.

Figure 8. Image point migration caused by camera tilt.

From geometric relation, the offset of image point can be calculated as

\[ \Delta d = \frac{hf - hf \cos \theta}{L - h \sin \theta} \]

3.3. Vertical error between reflecting screen and camera optical axis

3.3.1. Surface roughness error of reflecting screen. The current imaging system can only image objects within a certain distance clearly. The camera needs to be focused before shooting in order to get a clear image. Due to the limitation of manufacturing technology, the surface of the reflecting screen will be uneven, which makes the distance between the reflecting screen and the camera change.
As shown in Fig. 9, when the distance between the target and the lens changes, the image formed by the point on the non-focus plane on the image plane will no longer be the point image, but the cross section of the corresponding beam, forming a dispersion circle of diameter \(\delta\). If the diffuse circle size is small enough, it can be considered that the image composed of diffuse speckles is "point" image, which is "clear" and can obtain clear image on the CCD image plane is called depth of field\(^6\).

The relation between depth of field and dispersion circle is

\[
\Delta L = \frac{2f^2F\delta(u - f)}{f^2 - F^2\delta^2(u - f)^2}
\]

The reflective screen is composed of frame and 3M directional reflective film. The frame is made of aluminium plate. The backside is supported by reinforcing tendons. The 3M directional reflective film is posted on the surface of aluminium plate. The size of aluminium plate is 3\(\times\)4 m\(^2\). According to machining tolerance standard, the flatness of aluminium sheet is 1 mm. When \(\Delta L\) equals 1 mm, the aperture number \(F\) equals 5.6, the diameter of the dispersion circle \(\delta\) equals 0.0035 pixel, and the space length of the characterization is 0.6545 \(\mu\)m, the CCD camera can still obtain a clear image. Therefore, the depth-of-field error caused by the irregularity of the reflecting screen can be neglected.

3.3.2. The position error of the reflecting screen. The reflecting screen is mounted on the precision mechanical support with double square nets as the reference. The reflecting screen is fine-tuned to make the reflecting screen parallel to the grid plane. Because of the adjustment error and interpretation error, the reflecting screen will inevitably deviate from its ideal position at a small angle after adjustment. The setting error of the reflecting screen is \(\theta=1^\circ\). In the experiment, the original reflecting screen is selected. When the setting error \(\theta\) of the reflecting screen is \(1^\circ\), the imaging light path will not change. As shown in Fig. 10.
According to the pinhole imaging model, the projection of the scene on the reflecting screen changes from point $A$ to point $A'$, resulting in the change of the shadow to the camera imaging distance is $\Delta d = 26 \, \text{mm}$. The diameter of the dispersion circle is 0.092 pixel. From the pinhole model:

$$\delta = \frac{1700 \times 5.5}{50} \times 0.092 = 17.2 \, \mu\text{m}$$

### 4. Calibration error synthesis

The error distribution of single station calibration is shown in Table 1.

| Number | Error source                                                                 | Error value/μm |
|--------|-------------------------------------------------------------------------------|----------------|
| 1      | The coincidence error between the laser optical axis and the central axis of double square nets $\delta_1$ | 456            |
| 2      | Coincidence error between camera optical axis and the central line of the grid plane before and after double square nets $\delta_2$ | 213            |
| 3      | Vertical error between reflecting screen and camera optical axis $\delta_3$     | 654            |

### 5. Conclusion

The calibration principle and working process are studied. The main error factors affecting the calibration accuracy of the system are analyzed, and the various errors in the calibration process are statistically distilled. The calibration error of the measurement system is calculated to be 0.062 mm. Among many error factors affecting the calibration accuracy, the coincidence error $\delta_1$ between the laser optical axis and the central axis of double square nets is the main error of calibration. Among these errors, the installation error of optical window and the centering error of filtering target (Angle deviation between parallel light source beam axis and double square nets axis caused by coincidence error of reflected facula and incident facula) are the main sources of errors.
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