Simulation of fluid outflow from a channel with complex geometry

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Abstract. Topic of the paper is the simulation of fluid outflow from a channel with complex geometry. Subject of the study is the outflow of fluid from a channel with an arbitrary section, in particular, consisting of a parabolic inlet, cylindrical middle and hyperbolic outlet sections. The paper considers the integral form of determining the pressure and the average flow rate of the fluid in a channel with an arbitrary cross section. From the obtained expression for the pressure and from the equation of conservation of mass, given in integral form, it is necessary to determine the pressure and the average flow rate in successively located channels with a parabolic, cylindrical and hyperbolic section. Research methods are based on: Newton's rheological law; the continuity equation and the Navier-Stokes equation, which are the basic equations of fluid motion; the method of mathematical modeling and the analytical method for their solution. The paper contains integral expressions for the hydrodynamic characteristics of a fluid flow in a channel with an arbitrary section. The pressure and average flow rate in the channel of successively located parabolic, cylindrical, hyperbolic segments are determined. Analytical expressions are obtained for the pressure and average flow rate of the liquid in a channel with an arbitrary cross section. As an example, the pressures and the average flow rate in the channel from the parabolic inlet, cylindrical middle and hyperbolic outlet sections are determined. In perspective, by substituting in them an arbitrary arithmetic expression of the channel cross-section, it is possible to determine the hydrodynamic characteristics of the flow with any geometry of the flow channel.

1 Introduction

Anti-vibration baffles in pipeline transport improve flow characteristics and are the reason for flow establishment. In this case, the determination of the velocity and pressure of the liquid in this area plays an important role.

Analyzing numerous studies aimed at improving the hydrodynamic characteristics by changing the geometry of a segment in a pipeline, segments with a circular and elliptical section were proposed [1]. Comparing spiral baffles with cylindrical tubes and conventional segmented baffles with cylindrical tubes, square twisted tubes were recommended [2].

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The determination of the flow parameters depending on the geometry [1-5], external factors [6], internal factors [7-9] are solved using numerical [2,5,8] and analytical [9-10] methods. In many works, such studies are carried out by numerical methods using modern information and communication systems. For this purpose, new methods of numerical modeling and numerical calculations are being developed. Attracting these new mathematical and numerical models for a variety of problems is time consuming work.

In order to avoid vibrations caused by the flow and eliminate stagnant recirculation zones, a type of anti-vibration baffle with parabolic inlet, cylindrical middle and hyperbolic outlet sections is proposed below. The fluid motion is described by the Navier-Stokes equation. Assuming that the motion is a Poiseuille flow, analytical expressions for the pressure and average flow rate for each segment are obtained by the “stitching” method.

The problem relates to a segmental section of pipeline transport and is solved by the analytical method.

2 Object

The object of the study is the outflow of fluid from a channel of arbitrary shape, in particular, from a channel with a parabolic inlet, cylindrical middle and hyperbolic outlet sections. It is necessary to:

• obtain analytical expressions for the pressure and the average flow rate of the fluid in the flow channel with any cross-section, so that by substituting any arithmetic expression of the channel cross-section into them, one can determine the hydrodynamic characteristics of the flow;
• determine the pressure and the average flow rate of the fluid in the channel with parabolic inlet, cylindrical middle and hyperbolic outlet sections by analytical method;
• calculate the hydrodynamic parameters of the flow during the outflow of fluid from successively located parabolic, cylindrical and hyperbolic channels according to the obtained analytical solutions.

3 Methods

Research methods are based on: Newton’s rheological law; continuity equations and Navier-Stokes equations, which are the basic equations of fluid motion; methods of mathematical modeling and analytical method for their solution.

4 Materials

The movement of the fluid is considered in the channel of the anti-vibration segment consisting of a parabolic inlet, cylindrical middle and hyperbolic outlet sections, determined by the dependence:

$$R(x) = \begin{cases} \frac{b}{a} \sqrt{x^2 + a^2} & \text{when } x > x_l \\ \frac{R_l}{x} & \text{when } -x \leq x \leq x_l \\ ax^2 + c & \text{when } x \leq -x_l \end{cases}$$ (1)

It is required to determine the flow parameters in this channel.
Dynamic changes in hydrostatic pressure and average flow rate in an elementary section of an anti-vibration segment of a horizontal pipeline were described by the Navier-Stokes fluid motion equation [11]. Many works [12-20] are devoted to the solution of various problems by this equation and the analysis of this equation itself. So, we take the Navier-Stokes equation in divergent form:

\[
\frac{\partial u}{\partial t} + \frac{1}{r} \frac{\partial u r v r}{\partial r} + \frac{\partial u^2}{\partial x} = -\frac{1}{\rho} + \nu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{\partial^2 u}{\partial x^2} \right]
\]  
(2)

To directly solve the problem in order to determine the change in pressure and average flow rate in three segments, multiplying (2) by \(2\pi \rho r\) and integrating over \(r\) from 0 to \(R(x)\), one can obtain [11]:

\[
2\pi \frac{\partial}{\partial x} \int_0^R (\rho u^2 + P) r dr = \frac{\partial Q}{\partial t} + \nu \frac{\partial^2 Q}{\partial x^2} + 2\pi R \tau_R
\]  
(3)

Here \(u\) is the longitudinal component of the velocity; \(P\) is the pressure; \(\rho, \nu\) are the density and kinematic viscosity of the liquid; \(Q\) is the flow rate of the liquid; \(\tau_R\) is the shear stress. Replacing the velocity \(u\) with its average consumption value:

\[
Q = 2\pi \rho \int_0^R u r dr = \pi R^2 u \rho
\]

from (3) we obtain:

\[
\frac{\partial}{\partial x} [(\rho U^2 + P) R^2] = -\rho R^2 \frac{\partial U}{\partial t} + \mu \frac{\partial^2 R^2 U}{\partial x^2} + 2R \tau_R
\]  
(4)

Assuming the motion corresponds to the Poiseuille flow

\[
u(r) = \frac{\Delta P}{4 \mu l} (R^2 - r^2), Q = \frac{\pi \Delta P}{8 v l} R^4
\]  

we obtain:

\[
u = \frac{2Q}{\pi \rho R^2} (R^2 - r^2)
\]  
(6)

From here it follows:

\[
\tau_R = -\frac{4vQ}{\pi R^3}
\]  
(7)

Taking into account that \(Q = \pi \rho R^2 U = \pi \rho R^2 U\), from (1.5) one can obtain:

\[
\frac{\partial P R^2}{dx} = -\frac{Q}{\pi} \frac{\partial U}{\partial x} - \frac{8vQ}{\pi R^2}
\]  
(8)
Integrating (8) over \( x \) from \( x_\) to \( x \), we determine the pressure for the channel \( R \):

\[
P(x) = \rho U (U - U_\) + 8\nu \int_{x_\)}^{x} \frac{dx}{R^2}
\] (9)

From the equation of conservation of mass, given in integral form \( R^2 U = R^2 U_\) , follows the formula for determining the average flow rate in the inlet parabolic section of the channel:

\[
U(x) = \left( \frac{R}{ax^2 + c} \right)^2 U_\)
\] (10)

The pressure change in this section of the channel is determined from (9) by integrating for \( R = ax^2 + c \) and has the form:

\[
\frac{P(x)}{\rho U_\}^2} = \left( \frac{R}{ax^2 + c} \right)^2 \left[ 1 - \left( \frac{R}{ax^2 + c} \right)^2 \right] - \frac{4R}{e\Re} \left( - \frac{x}{ax^2 + c} + \frac{1}{\sqrt{ab} \arctg \sqrt{a/bx}} \right) \bigg|_{x_\} }^{x}
\] (11)

Here \( \Re = \frac{R - U_\) }{\nu} \) – Reynolds criterion at the channel inlet.

Next comes a section of a cylindrical channel with a constant radius \( R_i \) in the inlet section of which we have the conditions:

\[
U(-x_i) = \left( \frac{R}{R_i} \right)^2 U_-
\]

Since the free cross-section of the channel does not change in the section \( -x_i \leq x \leq x_i \), then, according to the law of conservation of the mass of the liquid, the average flow rate remains constant:

\[
U(x) = \left( \frac{R}{R_i} \right)^2 U_-
\] (12)

In this section, the intermediate integral of the equation of motion is simplified due to \( R = const. \) \( \frac{\partial U}{\partial x} = 0 \) and \( Q = \rho U \pi R^2 \), takes the form:

\[
\frac{\partial P}{dx} = - \frac{8\nu Q}{\pi R^4} = - \frac{8\nu \rho U}{R^2}
\] (13)

whence the dependence follows:

\[
P(x) = P(-x_i) - \frac{8\nu \rho U}{R^2} \bigg|_{x_i}^{x}
\] (14)

Thus, at the end of the cylindrical channel we have:
These values of the hydrodynamic parameters serve as the flow input data for the third hyperbolic section of the channel.

To determine the pressure in the hyperbolic section of the channel, we will use equation (9). After integrating $R = \frac{b}{a} \sqrt{ab}$ from $x_1$ to $x$, we have:

$$P(x) - P(x_-) = \rho U^2 \left[ \frac{x^2L^2}{x^2 + a^2} - 8v \frac{a}{b^2} (\arctg \frac{x}{a} - \arctg \frac{x_-}{a}) \right]$$

Thus, analytical expressions have been obtained for the hydrodynamic parameters of fluid outflow from a channel with an arbitrary cross section, in particular, one consisting of a parabolic inlet, cylindrical middle and hyperbolic outlet sections.

5 Results and discussion

Let us present the results of computational experiments on the obtained analytical solution in the form of graphs.
Fig. 1. Velocity profiles at 1) $-0.5 \leq \bar{x} \leq -0.3$ and 2) $0.3 \leq \bar{x} \leq 0.5$.

Fig. 2. Velocity profiles at 1) $-0.2 \leq \bar{x} \leq 0.6$ and 2) $-0.6 \leq \bar{x} \leq 0.2$.

Figures 1 and 2 show the velocity distributions in the channels where the cylindrical section has the length $0.2\bar{x}$. In Fig. 1, this section is located in the intervals $-0.5 \leq \bar{x} \leq -0.3$ (1) and $0.3 \leq \bar{x} \leq 0.5$ (2). In Fig. 2, the cylindrical part of the channel is located in the intervals $-0.2 \leq \bar{x} \leq 0.6$ (1) and $-0.6 \leq \bar{x} \leq 0.2$. In other variants of the location of the cylindrical section, the same flow patterns are also observed, i.e. if the cylindrical section is closer to the beginning of the channel, then the velocity increases monotonically in the first section, and in the third section it decreases gradually. With increasing distance from the beginning of the channel to the cylindrical section, the opposite picture is observed, i.e. before the cylindrical section, the velocity increases moderately, after this section, it decreases monotonically. And in the middle section, it remains constant, reaching its maximum value.

Based on the study, the following results (conclusions) were obtained:

1. Integral forms for determining the pressure and average flow rate of a liquid in a flow channel with an arbitrary cross section have been obtained.
2. Substituting in them an arbitrary arithmetic expression of the channel cross-section, one can determine the hydrodynamic characteristics of the flow in this channel.
3. Analytical expressions are obtained for the hydrodynamic parameters in the channel of successively located parabolic, cylindrical, and hyperbolic segments.
4. Calculations based on the obtained solutions have been carried out. The results of the computational experiments carried out correspond to the physical processes that actually occur in a fluid flow.
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