Non-spectator Contributions To The Lifetime of $\Lambda_b$

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Abstract

In this work, we evaluate the contributions of non-spectator effects to the lifetimes of $\Lambda_b$ and B-mesons. Based on the well-established models and within a reasonable range of the concerned parameters, the contributions can reduce the lifetime of $\Lambda_b$ by $7 \sim 8\%$ compared to that of B-mesons which are not significantly affected. This might partly explain the measured ratio $	au(\Lambda_b)/\tau(B^0) = 0.79$ [1], which has been a long-standing discrepancy between theory and experimental data.

1 Introduction

The heavy-flavor world provides us an opportunity to get insight into the fundamental physics. The most intriguing problem at the present stage of the theoretical studies on hadron properties and the processes of relatively low energies is lack of solid knowledge on non-perturbative QCD. The non-perturbative QCD effects which govern the hadronic transition matrix elements, are entangled with the hard subprocesses.

For the processes where some heavy flavors (b and/or c) are involved, by the heavy quark effective theory (HQET) [2, 3], the short and long-distance QCD effects are separated in a systematic way. Beneke et al. [4] demonstrated how to correctly apply the factorization procedure to the processes where heavy mesons transit into light ones.

On other side, there are still some unsolved problems in the heavy flavor physics. A protru- dent problem is the lifetime of $\Lambda_b$. The present data for the ratios are $^{[1]}$

\[
\frac{\tau(B^-)}{\tau(B_d)} = 1.06 \pm 0.04, \quad (1)
\]
\[
\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.79 \pm 0.06. \quad (2)
\]

In the traditional spectator scenario, if there are no strong decay channels the weak decays of b-hadrons as well as their lifetimes are fully determined by the weak decays of b-quark, thus the above ratios must be very close to unity. For D-meson, the lifetime of $D^\pm$ is almost 2.55 times larger than that of $D^0$. This difference is perfectly explained in the QCD framework by many authors $^[5]$, where the non-spectator effects play a crucial role as the Pauli-Interference (PI) strongly suppresses the total width of $D^\pm$ compared to $D^0$, but the effects are not so obvious
for $B^\pm$ and $B^0$ due to the $\Lambda_{QCD}/m_b$ suppression. We would think that the lifetimes of $D^\pm$, $D^0$ and $B^\pm$, $B^0$ are well understood, but for the $\tau(\Lambda_b)$ so far there is not a satisfactory answer to the puzzle yet, it is also worth noticing that this problem is even more serious in the $\Lambda_c$ lifetime which we will discuss in our coming work.

As one attempts to explain the puzzle of the $\Lambda_b$ lifetime, it is natural to consider the non-spectator contributions. Even though the study of the lifetime difference of $B^\pm$ and $B^0$ indicates that the non-spectator contribution can only result in a small effect for B-meson cases because of the $\Lambda_{QCD}/m_b$ suppression, it still is necessary to investigate how large such effects can be in the b-baryon case. Namely before one can claim a new physics or mechanism which result in the difference, he has to thoroughly explore possible explanations in the standard QCD framework.

To calculate the lifetime, one only needs to deal with the inclusive processes which are relatively simple compared to the exclusive processes, therefore the results are more reliable. In this work we consider the contributions from the non-spectator effects to the lifetime of $\Lambda_b$. At the quark level, the foundation is the weak effective Lagrangian [6]. We not only consider the W-exchange(WE) diagrams and the Pauli interference(PI) diagrams ¹ where only two quarks take part in the reaction with the other one being a spectator, but also account for the corresponding diagram where all the three valence quarks are involved. The last mechanism does not exist in the meson case, and seems to result in a difference between $\tau(\Lambda_b)$ and $\tau(B^0(B^\pm))$.

Moreover we also consider the diquark structure where the two light quarks constitute a boson-like diquark of color anti-triplet and it participates in the reaction of WE or PI as a whole subject. Then we compare the results obtained in the three-valence-quark picture and the heavy-quark-light-diquark picture. It could also be a test of the supposed diquark structure of baryons.

The most difficult task is to evaluate the hadronic transition matrix elements which are fully governed by the non-perturbative QCD. Even though we are able to separate the long-distance effects from the short distance subprocess, we still cannot evaluate the hadronic matrix elements in a well-established way, but need to carry out a model-dependent calculation. In this work we adopt the simplest non-relativistic harmonic oscillator model [10] for the hadron wavefunctions. To obtain the concerned parameters in the model, we first evaluate the semi-leptonic decays of $\Lambda_b \rightarrow \Lambda_c + e^- + \bar{\nu}$ and $B \rightarrow D + e + \bar{\nu}$ which are not contaminated by the final state interactions.

Our results are comparable with the works on this aspect in the literatures [8] and a recent work by Chakraverty et al [9]. Then with the parameters, we calculate the lifetimes of B-mesons and $\Lambda_b$.

To evaluate the ratios $\tau(\Lambda_b)/\tau(B^0)$ and $\tau(B^\pm)/\tau(B^0)$, we calculate the non-spectator contributions to the lifetimes of $B^0$ and $B^\pm$ in the same model, namely we achieve the wavefunctions of $B^0$ and $B^\pm$ in the two-body harmonic oscillator model. Evaluating the hadronic matrix elements of $< B | O | B >$ which appears in the semi-leptonic decay rate of B-mesons, and then by fitting the data, we gain the model parameters. We hope that since we evaluate lifetimes of both $\Lambda_b$ and B-meson in the same model, the model-dependence of the theoretically evaluated ratios can be partly cancelled, and the results would make more sense.

Our paper is organized as follows. After this introduction we briefly introduce the non-relativistic harmonic oscillator model for $\Lambda_b$ in the picture with three valence quarks and as

¹Here PI refers to the mechanism where a constituent of the final state joins the b-quark of the initial state at the effective vertex and vice versa. Namely there exists a crossing of constituent lines (see Figs.1 and 2 for illustration). In the B-meson case, it is indeed a Pauli interference, but for the $\Lambda_b$ it is only a manifestation of such mechanisms.
well as that with one heavy quark and one light diquark, then we discuss B-mesons in the same model. In section 3, we calculate the semi-leptonic decays of B-mesons and \( \Lambda_b \) where we also use the two pictures of 3-quark and quark-diquark structures respectively. In sect. 4, we use the model to evaluate the lifetimes of B-mesons and \( \Lambda_b \), then in sect. 5, we present our numerical results. The last section is devoted to our conclusion and discussion.

2 The wavefunctions of b-hadrons in the non-relativistic harmonic oscillator quark model

2.1 For the B-mesons

In the valence quark model, B-mesons contain a heavy quark b and one light flavor. The spatial coordinates of the two constituents in B-mesons are denoted by \( r_b \) and \( r_q \). It is convenient to introduce the Jacobi coordinates \( R \) and \( \rho \) which are defined as

\[
\begin{align*}
R &= \frac{m_q r_q + M_b r_b}{M_b + m_q}, \\
\rho &= r_b - r_q.
\end{align*}
\]

The corresponding canonical conjugate momenta \( P \) and \( p_\rho \) are respectively,

\[
\begin{align*}
P &= p_b + p_q, \\
p_\rho &= \frac{m_q p_b - M_b p_q}{M_b + m_q}.
\end{align*}
\]

In the non-relativistic harmonic oscillator model, the normalized state vector is written as,

\[
| B_M(P, s) > = A_M \sum_{color, spin} C_{s_q, s_b}^s \chi_{s_q, s_b} \varphi_{color} \int d^3 p_\rho \psi_{B_M}(p_\rho) \left| q(p_q, s_q); \bar{b}(p_b, s_b) \right>,
\]

where \( \chi \), \( \varphi \) and \( \psi \) are the flavor-spin, color and spatial wavefunctions in the momentum space respectively. \( A_M \) is a normalization factor, thus \( | B_M(P, s) > \) satisfies the normalization condition

\[
< B_M(P, s) | B_M(P', s') > = (2\pi)^3 2\omega_P \delta^3(P - P') \delta_{s,s'}.
\]

\( C_{s_q, s_b}^s \) are the Clebsch-Gordon coefficients for combining two spin-\( \frac{1}{2} \) constituent quarks into a spin-0 meson. The wave function \( \psi_{B_M}(p_\rho) \) is the ground state eigenfunction of the harmonic oscillator hamiltonian and has the form

\[
\psi_{B_M}(p_\rho) = \exp(-\frac{P_\rho^2}{2\alpha_M^2}),
\]

where the parameter \( \alpha_M \) is a free parameter of the model and to be determined by fitting data of semileptonic decays [8].
2.2 For $\Lambda_b$

(a) 3-valence-quark model

In a non-relativistic quark model (NRQ), $\Lambda_b$ contains a heavy quark $b$ and two light quarks ($u$ and $d$). The spatial coordinates of the three quarks in $\Lambda_b$ are denoted by $r_b$, $r_{q_1}$, and $r_{q_2}$. Similar to the meson case, we also introduce the Jacobi coordinates $R$, $\rho$ and $\lambda$ defined as

\[
\begin{align*}
R &= \frac{m_{q_1} + m_{q_2} + M_b r_b}{2m + M_b}, \\
\rho &= \frac{1}{\sqrt{2}} (r_{q_1} - r_{q_2}), \\
\lambda &= \frac{1}{\sqrt{2(2m + M_b)}} (r_{q_1} + r_{q_2} - 2r_b).
\end{align*}
\]

In this model, the normalized wavefunction of $\Lambda_b$ has the form

\[
|\Lambda_b(P,s)\rangle = A_B \sum_{\text{color,spin}} C^s_{s_{q_1},s_{q_2},s_b} \chi_{\text{spin}} \chi_{\text{color}} \int d^3p_\rho d^3p_\lambda \psi_{B_B}(p_\rho, p_\lambda) |
\]

\[
q_1(p_{q_1}, s_{q_1}), q_2(p_{q_2}, s_{q_2}), b(p_b, s_b),
\]

\[. \]

thus the $|\Lambda_b(P,s)\rangle$ satisfies the normalization condition

\[
< \Lambda_b(P,s) | \Lambda_b(P',s') > = (2\pi)^3 \frac{M_{B_B}}{\omega_P} \delta^3(P - P') \delta_{s,s'}.
\]

Here $C^s_{s_{q_1},s_{q_2},s_b}$ are the Clebsch-Gordon coefficients for combining the three spin-$\frac{1}{2}$ constituent quarks into a spin-$\frac{1}{2}$ baryon and $A_B$ is a normalization factor. The spatial part of the wave function $\psi_{B_B}(p_\rho, p_\lambda)$ is the eigenfunction of the three-body harmonic oscillator Hamiltonian and has the form

\[
\psi_{B_B}(p_\rho, p_\lambda) = \exp(-\frac{P_\rho^2}{2\alpha^2} - \frac{P_\lambda^2}{2\alpha^2}),
\]

where the parameters $\alpha_\rho$ and $\alpha_\lambda$ are free parameters and to be determined by fitting the data of $\Lambda_b \rightarrow \Lambda_c + e^- + \bar{\nu}$.

(b) For the diquark picture.

In the diquark picture, the two light quarks constitute a boson-like subject in the color-anti-triplet so called as diquark and $\Lambda_b$ is supposed to be composed of the heavy quark $b$ and one light scalar diquark. In analog to the meson quark model, the spatial coordinates of the two constituents in $\Lambda_b$ are denoted by $r_b$ and $r_D$, and we can also introduce the Jacobi coordinates $R$ and $\rho$:

\[
\begin{align*}
R &= \frac{m_D r_b + M_b r_b}{m_D + M_b}, \\
\rho &= r_b - r_D,
\end{align*}
\]

where $m_D$ is the diquark mass. In this model, the normalized state vector is written as

\[
|\Lambda_b^{(D)}(P,s)\rangle = A_D \sum_{\text{color,spin}} C^s_{s_D,s_b} \chi_{s_D,s_b} \chi_{\text{color}} \int d\rho \psi_{B_D}(\rho) | D(p_D,s_D), b(p_b,s_b) >,
\]
where the superscript D denotes the diquark picture and \( \Lambda_b^{(D)}(P, s) \) satisfies the normalization condition
\[
< \Lambda_b^{(D)}(P, s) | \Lambda_b^{(D)}(P', s') > = (2\pi)^3 \frac{M_{BD}}{\omega_P} \delta^3(P - P') \delta_{s,s'}.
\] (13)

Here \( C_{sD,sb}^s \) are the Clebsch-Gordon coefficients for combining one spin-\( \frac{1}{2} \) constituent quark \( b \) and one spin-0 constituent diquark into a spin-\( \frac{1}{2} \) baryon. The corresponding wave function \( \psi_{BD}(p) \) is
\[
\psi_{BD}(p) = \exp(-\frac{p_b^2}{2\alpha_D}),
\] (14)

where the parameters \( \alpha_D \) is also a free parameter and should be determined by fitting the data of \( \Lambda_b \rightarrow \Lambda_c + e^- + \bar{\nu} \) in the heavy-quark-light-diquark structure.

### 3 The Semi-leptonic decays of b-hadrons

The matrix elements for the processes \( B^- (\bar{B}^0) \rightarrow D^0(D^+)e^−\bar{\nu}_e \) and \( \Lambda_b \rightarrow \Lambda_c e^−\bar{\nu}_e \) are expressed as
\[
\mathcal{M}(s', s) = \frac{G_F}{\sqrt{2}} V_{cb} L^\mu H_\mu(s', s)
\] (15)

with the leptonic and hadronic parts being
\[
L^\mu = \bar{u}(p_e)\gamma^\mu(1 - \gamma_5)\nu(p_\nu), \\
H_\mu(s, s') = < h_c(P, s) | \bar{c}(p_c, s_c)\gamma_\mu(1 - \gamma_5)b(p_b, s_b) | h_b(P', s') >,
\] (16) (17)

where \( h_b \) and \( h_c \) are corresponding b and c hadrons and \( P' \), \( P \) are the four-momenta of \( h_b \) and \( h_c \) respectively. Defining
\[
\vec{K} \equiv P' - P,
\] (18)

we obtain
\[
\frac{d\Gamma}{dP} = F_{Had} \frac{G_F^2 V_{cb}^2}{12} (\vec{K}_\mu \vec{K}_\nu - \vec{K}^2 g_{\mu\nu}) \sum_{spin} T^{\mu\nu}_{\text{had}},
\] (19)

with
\[
T^{\text{had}}_{\mu\nu} = H_\mu^T H_{\nu}. 
\] (20)

#### 3.1 Semi-leptonic decays of B-mesons

In the rest frame of the B-meson \( (P' = (M_B, 0)) \), we have
\[
\begin{align*}
\{ p_b = p_b^b, & \quad p_c = \frac{m_c}{m_q + m_c} P + p_c^c, \\
\{ p_b^b = -p_b^b, & \quad p_c^c = \frac{m_q}{m_q + m_c} P - p_c^c. \\
\end{align*}
\] (21)

In the semileptonic decays, the light quark is indeed a spectator, so we have
\[
\begin{align*}
\{ p_b^b = p_c^c, & \quad \mathbf{P} = p_c - p_b. \\
\end{align*}
\] (22)

5
Thus \(H_\mu = A_M^B A_M^D \int d^3p^b d^3p^c \psi_M^b \psi_M^c < c_{pc,sc}, \bar{q}_{pq,qs} | \bar{c}(p_c, s_c) \gamma_\mu b(p_b, s_b) | b_{pq,qs}, \bar{q}_{pq,qs} >,\)

\[\delta^3(p^b_\rho - p^c_\rho - \frac{m_q}{m_q + m_c} P) \bar{c}(p_c, s_c) \gamma_\mu b(p_b, s_b). \tag{23}\]

Defining

\[p_\rho \equiv p^b_\rho + \frac{m_q \alpha_B^2}{(m_q + m_c)(\alpha_B^2 + \alpha_C^2)} P, \tag{24}\]

we obtain

\[H_\mu = (2\pi)^3 A_M^B A_M^D \exp[-\frac{m_q^2 p^2}{2(m_q + m_c)^2(\alpha_B^2 + \alpha_C^2)}] \int d^3p_\rho E_\rho m_q \exp[-\frac{\alpha_B^2 + \alpha_C^2}{2\alpha_B^2 \alpha_C^2} - \frac{m_q^2 p^2}{2(m_q + m_c)^2(\alpha_B^2 + \alpha_C^2)}] \bar{c}(s_c, \gamma_\mu b_{s_b}). \tag{25}\]

In the non-relativistic limit, we finally reach an expression as

\[\frac{d\Gamma}{dP} = \frac{G_F^2 V_{cb}^2}{2 \times 192(2\pi)^4 M_B M_D} (\bar{K}^\mu \bar{K}^\nu - \bar{K}^2 g^{\mu\nu}) \sum_{spin} H_\mu^\dagger H_\nu. \tag{26}\]

### 3.2 Semi-leptonic decay of \(\Lambda_b \to \Lambda_c + e^- + \bar{\nu}\)

(a) For the 3-valence-quark model

In the rest frame of \(\Lambda_b (P' = (M_B, 0))\), we have

\[
\begin{align*}
\{ p_b &= -\frac{2m_b}{2m_b + m_c} P^\lambda, \\
p^{q_1}_c &= \frac{m_c}{2(2m_b + m_c)} P^\lambda + \frac{1}{\sqrt{2}} p^b_\rho, \\
p^{q_2}_c &= \frac{m_c}{2(2m_b + m_c)} P^\lambda - \frac{1}{\sqrt{2}} p^b_\rho,
\end{align*}
\]

\[
\begin{align*}
\{ p^c_\rho &= p^{q_1}_c, \\
p^{q_2}_c &= p^{q_2}_c, \\
\bar{P} &= p_c - p_b
\end{align*}
\]

and

\[
\begin{align*}
\{ p^{q_1}_b &= p^{q_1}_c, \\
p^{q_2}_b &= p^{q_2}_c,
\end{align*}
\]

Thus

\[H_\mu = (2\pi)^6 A_{\Lambda_b} A_{\Lambda_c} \frac{(A(2m_b + m_c)) \alpha_B^2 \alpha_C^2}{m_c^2} \int d^3p^b d^3p^c d^3p_\rho d^3p_\lambda E^{q_1} E^{q_2} \exp[-\frac{p^b_\rho (p^b_\rho)^2}{2\alpha_\rho^2} - \frac{p^c_\rho (p^c_\rho)^2}{2\alpha_\rho^2} - \frac{p^\lambda_\rho (p^\lambda_\rho)^2}{2\alpha_\lambda^2} - \frac{p^\lambda_\rho (p^\lambda_\rho)^2}{2\alpha_\lambda^2}]
\]

\[\delta^3(p^c_\rho - p^b_\rho) \delta^3(p^\lambda_\rho - a_1 p^\lambda_\rho + a_2 \bar{P}) \bar{c}(p_c, s_c) \gamma_\mu (1 - \gamma_5) b(p_b, s_b). \tag{29}\]
Defining

\[
\begin{aligned}
\{ & P_\rho \equiv P_\rho^b, \\
& P_\lambda \equiv P_\lambda^b - \frac{m\sqrt{2m_b(2m+m_b)}}{m_c(2m+m_b)\alpha^b_\lambda^2+m_b(2m+m_c)\alpha^b_\lambda^2} P,
\end{aligned}
\]

we have

\[
H_\mu = (2\pi)^6 A_{\Lambda c} A_{\Lambda c} \frac{4(2m+m_b)}{m_c} \frac{4}{4} \exp \left[-a_3 P^2\right] \int d^3p_\lambda d^3p_\rho \frac{E_{q_1} E_{q_2}}{m^2} \exp[-\frac{a_2^2}{2\alpha^2_\rho} P^2] \sqrt{c(p_c, s_c)\gamma_\mu(1 - \gamma_5)b(p, s_b)}
\]

and

\[
\begin{aligned}
\{ & a_1 = \frac{m_b(2m+m_b)}{m_c(2m+m_b)^{\frac{1}{2}}}, \\
& a_2 = m \left(\frac{2}{m_c(2m+m_b)^{\frac{1}{2}}}\right)^2, \\
& a_3 = \frac{(2m+m_b)m_c(2m+m_b)\alpha^2_\lambda + m_b(2m+m_c)\alpha^2_\lambda}{2m_c(2m+m_b)\alpha^2_\rho\alpha^2_\lambda}, \\
& a_4 = \frac{m_c(2m+m_b)\alpha^2_\lambda + m_b(2m+m_c)\alpha^2_\lambda}{2m_c(2m+m_b)\alpha^2_\rho\alpha^2_\lambda},
\end{aligned}
\]

then the final expression is reached

\[
\frac{d\Gamma}{dP} = \frac{(2\pi)^2 G_F^2 V_{cb}^2}{192} (\bar{K}^\mu K^\nu - \bar{K}^2 g^{\mu\nu}) \sum_{\text{spin}} H_\mu H_\nu.
\]

(b) For the diquark Model

Now let us turn to the model for the quark-diquark structure. In analog with the B-meson case, the semileptonic decay can be evaluated with the two-body wavefunction and then \( H_\mu \) is

\[
H_\mu = 2(2\pi)^3 A_D^b A_D^c \exp\left[-\frac{m^2_D P^2}{2(m_D+m_c)^2(\alpha^2_D + \alpha^2_D)}\right] \int d^3p_\omega \frac{\omega}{m_D} \exp\left[-\frac{\omega^2}{2\alpha^2_D \alpha^2_D} P^2\right] \bar{c}_c \gamma_\mu(1 - \gamma_5)b_{s_b}
\]

and

\[
\frac{d\Gamma}{dP} = \frac{4G_F^2 V_{cb}^2}{192(2\pi)^4} (\bar{K}^\mu K^\nu - \bar{K}^2 g^{\mu\nu}) \sum_{\text{spin}} H_\mu H_\nu.
\]

4 Inclusive decays of b-hadrons

(a) The spectator contributions to the lifetimes of \( \Lambda_b, B^\pm \) and \( B^0 \)

With quark-hadron duality and the optical theorem, the full inclusive decay width corresponding to the lifetime of a heavy hadron \( h_b \) (containing a heavy quark \( b \)) is related to the
absorptive part of the forward scattering matrix element

\[
\Gamma(h_b \to X) = \frac{1}{\rho N_i} \sum_{\text{spin},f} \int d\prod_f (2\pi)^4 \delta^4(p_i - p_f) \ | \mathcal{M}(H_Q \to X) |^2
\]

\[
= \frac{1}{\rho N_i} \sum_{\text{spin}} 2\text{Im}\mathcal{M}(H_Q \to H_Q)
\]

\[
= \frac{2}{\rho N_i} \text{Im} \int d^4x <H_Q | \hat{T} | H_Q > = \frac{2}{\rho N_i} <H_Q | \hat{\Gamma} | H_Q >
\]

(36)

where

\[
\hat{T} = T\{i\mathcal{L}_{eff}(x), \mathcal{L}_{eff}(0)\},
\]

(37)

and \(\rho, N_i\) are the state density factors and \(L_{eff}\) is the relevant effective weak Lagrangian which is responsible for the decay. For the concerned final state \(X\) with the designated quark-antiquark combination, up-to order \(1/m^3_Q\) one finds [5, 7]:

\[
\Gamma(H_Q \to X) = \frac{G_F^2 m_Q^2}{192\pi^3} |V_{CKM}|^2 \{c_X^3 <H_Q | \bar{Q}Q | H_Q > + c_5^X <H_Q | \bar{Q}_i \sigma \cdot GQ | H_Q >
\]

\[
+ \sum_i c_{6,a}^X <H_Q | (\bar{Q}\Gamma_i q)(\bar{q}\Gamma_i Q) | H_Q > + O(1/m^4_Q)\}.
\]

(38)

Here only the heavy quark (b,c quark) decays are concerned. In the spectator scenario, the light flavor(s) in the heavy meson (baryon) remains as a spectator. We can write the spectator contribution as

\[
\Gamma_{\text{spec}} = \sum_{l=e,\mu,\tau} \Gamma_{b \to c\bar{l}l} + \sum_{q=u,d,s,c} \Gamma_{b \to c\bar{q}q}.
\]

(39)

The pure b-quark decay rates of \(b \to c\bar{u}s, c\bar{u}d + ce\bar{\nu}_e, cm\bar{\nu}_\mu\) and \(ct\bar{\nu}_\tau\) have been carefully evaluated by Bagan et al. [22] as

\[
\Gamma(b \to c\bar{u}s + c\bar{u}d) = (4.0 \pm 0.4)\Gamma_{b \to ce\bar{\nu}_e},
\]

\[
\Gamma(b \to ct\bar{\nu}_\tau) = 0.25\Gamma_{b \to ce\bar{\nu}_e},
\]

(40)

and the measured semileptonic branching ratio is given in ref.[22] as \(B(b \to ce\bar{\nu}_e) = (11.6 \pm 1.8)\%\). The theoretically formula at the tree level reads

\[
\Gamma(b \to ce\bar{\nu}_e) = |V_{cb}|^2 \frac{G_F^2 m_b^5}{192\pi^3} \left(1 - 8\left(\frac{m_c}{m_b}\right)^2 - 12\left(\frac{m_c}{m_b}\right)^4 \ln\frac{m_c^2}{m_b^2} + 8\left(\frac{m_c}{m_b}\right)^6 - \left(\frac{m_c}{m_b}\right)^8\right).
\]

(41)

The semi-leptonic and non-leptonic decay rates of b-quark up-to order \(1/m^2_b\) have been evaluated by many authors [5]. In our numerical computations we need to use their formulas, for the readers’ convenience we collect them in Appendix A.

(b) The non-spectator contributions to the inclusive decays of b-hadrons.

The total width of a b-hadron decay (the lifetime) includes two contributions

\[
\Gamma(H_Q \to X) = \Gamma_b^{\text{spec}} + \Gamma_b^{\text{non}},
\]

(42)
where the superscripts spec and non refer to the spectator and non-spectator contributions respectively. To estimate the non-spectator contribution to b-hadron decays, the basis is the weak effective Lagrangian [13],

\[
\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \left\{ c_1(m_b) \left[ d' \gamma^\mu L \bar{c} \gamma_\mu L b + s' \gamma^\mu L \bar{c} \gamma_\mu L b \right] + c_2(m_b) \left[ \bar{c} \gamma^\mu L u d' \gamma_\mu L b + \bar{c} \gamma^\mu L cs' \gamma_\mu L b \right] + \sum_{l=e,\mu,\tau} \bar{l} \gamma^\mu L \nu_l \bar{c} \gamma_\mu L b \right\},
\]

(43)

where \( d' = d \cos \theta_c + s \sin \theta_c \), \( s' = s \cos \theta_c - d \sin \theta_c \) and \( \theta_c \) is the Cabibbo angle. \( \mathcal{H}_{\text{eff}} \) can be further written as

\[
\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \left[ V_{ub}^* V_{cq}(c_{1u}(\mu) O_1^{u} + c_{2u}(\mu) O_2^{u}) + V_{cb}^* V_{cq}(c_{1c}(\mu) O_1^{c} + c_{2c}(\mu) O_2^{c}) - V_{tb}^* V_{ti} \sum_{i=3}^{6} c_i(\mu) O_i \right],
\]

(44)

where the operations are

\[
O_1^{Q} = \bar{Q} \gamma^\mu L b \bar{q} \gamma_\mu L u,
\]

\[
O_2^{Q} = \bar{Q} \gamma^\mu L b_j \bar{q}_j \gamma_\mu L u_i,
\]

\[
O_3,5 = \bar{q} \gamma^\mu L \sum q' \gamma_\mu L(R) q',
\]

\[
O_{4,6} = \bar{q}_i \gamma^\mu L b_j \sum q' \gamma_\mu L(R) q_i,
\]

(45)

and \( L(R) = \frac{1 + \gamma_5}{2} \), \( q' \) is summed over \( u, d, s, c, b \) and \( t, q \) can be \( s \) or \( d \), \( i \) and \( j \) are the color indices.

(c) Inclusive decays of B-mesons

The non-spectator operators for B-meson decays have been given [13], because of the Cabibbo suppression the W-exchange (WE) process only exists for \( B^0 \) and the PI-process applies uniquely to \( B^{\pm} \). One can have [5]

\[
\hat{\Gamma}_{P_{\text{f}}}^{B^{\pm}} = \frac{2G_F^2 m_b^2}{\pi} |V_{cb}|^2 (1 - z)^2 \left\{ (2c_1 c_2 + \frac{1}{N_c}(c_1^2 + c_2^2))O_{V-A}^n + 2(c_1^2 + c_2^2)T_{V-A}^n \right\},
\]

(46)

\[
\hat{\Gamma}_{W_{\text{E}}}^{B^0} = -\frac{2G_F^2 m_b^2}{3\pi} |V_{cb}|^2 (1 - z)^2 \left\{ (2c_1 c_2 + \frac{1}{N_c}c_1^2 + N_c c_2^2) [(1 + \frac{z}{2})O_{V-A}^d - (1 + 2z)O_{S-P}^d] + 2c_1^2[(1 - z)T_{V-A}' - (1 + 2z)T_{S-P}'] \right\},
\]

(47)

where \( z = \frac{m_c^2}{m_b^2} \) and \( N_c = 3 \). The local four-quark operators appearing in this expression are

\[
O_{V-A}^n = \bar{b} \gamma^\mu L q q \gamma_\mu L b,
\]

\[
O_{S-P}^d = \bar{b} \Lambda q \bar{q} R b,
\]

\[
T_{V-A}^n = \bar{b} \gamma^\mu L T^a q q \gamma_\mu L T^a b,
\]

\[
T_{S-P}^d = \bar{b} \Lambda L T^a q \bar{q} R T^a b,
\]

(48)
where $T^a = \frac{\lambda^a}{2}$ are the generators of the color SU(3). The hadronic matrix element is

$$\Gamma_{\text{non}}^{B_q} = \frac{1}{M_{B_q}} \langle B_q | \hat{\Gamma}_{\text{non}} | B_q \rangle = \frac{1}{M_{B_q}} | A_M |^2 \sum_{\text{color,spin}} \int d^4 p' d^4 p \psi^*_B(p') \psi_B(p) \langle q(p_q, s_q), b(p_b, s_b) | \hat{\Gamma}_{\text{non}} | q(p_q, s_q), b(p_b, s_b) \rangle. $$ (49)

It is reasonable to assume that the quarks in the bound states are only slightly off-shell, we can carry out the calculation of the matrix elements in our model described in last section.

$$\Gamma_{\text{F1}}^{B_{\pm}} = \frac{3G_F^2 m_b^2}{\pi M_{B^0}} | V_{cb} |^2 \left( 2c_1 c_2 + \frac{1}{N_c} (c_1^2 + c_2^2) \right) (1 - z)^2 | A_{B_{\pm}} |^2 \sum_{\text{spin}} \int d^3 p' d^3 p \exp\left[ -\frac{p_{\rho}'^2 + p_{\rho}^2}{2\alpha_{B_{\pm}}^2} \right] b_\gamma \mu L u \bar{u}_\gamma \mu L b, $$ (50)

$$\Gamma_{\text{F2}}^{B_{0}} = -\frac{G_F^2 m_b^2}{\pi M_{B^0}} | V_{cb} \cos \theta_c |^2 \left( 2c_1 c_2 + \frac{c_1^2}{N_c} + N_c c_2^2 \right) (1 - z)^2 | A_{B_{0}} |^2 \sum_{\text{spin}} \int d^3 p' d^3 p \exp\left[ -\frac{p_{\rho}'^2 + p_{\rho}^2}{2\alpha_{B_{0}}^2} \right] \left( \frac{1}{2} + z \right) b_\gamma \mu L d \bar{d}_\gamma \mu L b + (1 + 2z) b \bar{L} d \bar{d} \bar{L} b. $$ (51)

(d) The inclusive Decays of $\Lambda_b$

(1) For the 3-valence-quark model

The concerned non-spectator Feynman diagrams are shown in Fig.1 (a), (b) and (c). One of the light valence quarks and the b-quark exchange a W-boson while the other light quark stands by as a spectator. Fig.1 (c) is a PI diagram where three quarks take part in the reaction. Due to the CKM entries, other diagrams are much suppressed compared to that in Fig.1.(a),(b) and (c), thus can be neglected.

$$\hat{\Gamma}_{\text{F1}}^{\Lambda_{b}} = \frac{G_F^2 M_{\pi}^2}{\pi} | V_{cb} V_{ud} |^2 \left( 1 - z_+ \right) \left( 1 - 8 z_+ \right) \left[ \left( c_1^2 + c_2^2 \right) + \frac{2 c_1 c_2}{N_c} \right] | O_{V-A}^c + 4 c_1 c_2 T_{V-A}^c \right] \right),
$$

$$\hat{\Gamma}_{\text{F2}}^{\Lambda_{b}} = \frac{G_F^2 | V_{cb} V_{ud} |^2}{4 m_d \Gamma_{c}} \left\{ c_1^2 \bar{O}_{V-A}(b_i, b_j, u_j, d_j, d_k, u_k) + c_2^2 \bar{O}_{V-A}(b_i, b_j, u_k, d_i, d_j, u_k) + 2 c_1 c_2 \bar{O}_{V-A}(b_i, b_j, u_k, d_k, d_j, u_i) \right\} \left( g^{\mu \lambda} p_c^\mu - g^{\mu \lambda} p_c^\mu - i e^{\mu \lambda \rho} p_c^\rho \right), $$ (52)

As discussed in the previous footnote, in fact, this PI diagram is not exactly the Pauli Interference diagrams discussed in the literature. In the diagram a quark from the initial state crosses with one from the final state. One can see that a quark from the "right" state joins the "left" vertex and vice versa, thus for such a situation, we just keep the terminology as "PI" diagrams.
We finally achieve

\[
\begin{align*}
O_{V-A}^c &= \bar{b}\gamma^\mu Lb\bar{u}\gamma_\mu Lu, \\
T_{V-A}^c &= \bar{b}\gamma^\mu LT^a b\bar{u}\gamma_\mu LT^a u, \\
\bar{O}_{V-A} &= \bar{b}\gamma_\mu Lb\bar{u}\gamma_\mu Ld\bar{d}\gamma_\lambda Lu. 
\end{align*}
\]

The hadronic matrix elements are evaluated in the non-relativistic harmonic oscillator model as

\[
\Gamma_{WE} = < \Lambda_b(P = 0) | \hat{\Gamma}_{WE} | \Lambda_b(P' = 0) > \equiv (2\pi)^3 | A_{\Lambda_b} |^2 \sum_{\text{color,spin}} \int d^3\tilde{p}_\rho d^3\tilde{p}_\lambda d^3k \frac{E_d}{m} \delta^3(p'-p_d) \exp[-\frac{\tilde{p}_\rho^2 + \tilde{p}_\lambda^2}{2\alpha_\rho^2} - \frac{\tilde{p}_\rho^2 + \tilde{p}_\lambda^2}{2\alpha_\lambda^2}] < u(p'_u, s'_u), b(p'_b, s'_b) | \hat{\Gamma}_{WE} | (p_u, s_u), b(p_b, s_b) >. 
\]

Setting

\[
k = p_b - p'_b, \tag{55}
\]

we have

\[
\begin{cases}
p'_\rho = p_\rho + \frac{1}{\sqrt{2}}k, \\
p'_\lambda = p_\lambda + \sqrt{\frac{2m+mb}{2mb}}k,
\end{cases}
\]

thus

\[
\begin{cases}
\bar{p}_\rho = \sqrt{2}p_\rho + \frac{1}{2}k, \\
\bar{p}_\lambda = \sqrt{2}p_\lambda + \frac{1}{2}\sqrt{\frac{2m+mb}{mb}}k.
\end{cases}
\]

We finally achieve

\[
\Gamma_{WE} = (2\pi)^3 \frac{G_F^2 M_b^2}{4} | V_{cb} V_{ub} |^2 (c_{1c} - c_{2c})^2 (1 - z_+(1 - 8z_+)) \frac{9(2m + mb)}{8mb} \frac{\frac{9}{8m_b}}{8} | A_{\Lambda_b} |^2 \sum_{\text{spin}} \int d^3\tilde{p}_\rho d^3\tilde{p}_\lambda d^3k \frac{E_d}{m} \exp[-\frac{\tilde{p}_\rho^2 + \tilde{p}_\lambda^2}{2\alpha_\rho^2} - \frac{\tilde{p}_\rho^2 + \tilde{p}_\lambda^2}{2\alpha_\lambda^2} - \frac{m_b \alpha_\lambda^2 + (2m + mb) \alpha_\rho^2 k^2}{8mb \alpha_\rho^2 \alpha_\lambda^2}] \bar{b}\gamma^\mu (1 - \gamma_5)b\bar{u}\gamma_\mu (1 - \gamma_5)u, \tag{56}
\]

and

\[
\Gamma_{PI} = < \Lambda_b(P = 0) | \hat{\Gamma}_{PI} | \Lambda_b(P' = 0) > = \frac{G_F^2}{4} | V_{cb} V_{ud} |^2 (c_{1c} - c_{2c})^2 | A_{\Lambda_b} |^2 \sum_{\text{spin}} \int d^3\tilde{p}_\rho d^3\tilde{p}_\lambda d^3k \frac{E_d}{m} \exp[-\frac{\tilde{p}_\rho^2 + \tilde{p}_\lambda^2}{2\alpha_\rho^2} - \frac{\tilde{p}_\rho^2 + \tilde{p}_\lambda^2}{2\alpha_\lambda^2}] \bar{b}\gamma^\mu (1 - \gamma_5)b\bar{u}\gamma_\mu (1 - \gamma_5)d\bar{d}\gamma_\lambda (1 - \gamma_5)u(g^{\mu\lambda} p_\nu - g^{\nu\lambda} p_\mu + g^{\mu\nu} p_\lambda - i\varepsilon^{\mu\nu\lambda\rho} p_{\epsilon\rho}). \tag{57}
\]
(2) The quark-diquark structure

(i) Now we consider the non-spectator effects in the quark-diquark scenario. In this picture, b-quark and the light diquark exchange a W-boson as

\[ b + D \rightarrow c + D', \]

where \( D \) and \( D' \) are scalar or vector diquarks. Supposing that the diquarks of \( \Lambda_b \) and \( \Lambda_c \) are in ground states, i.e. \( D \) and \( D' \) are scalars only. The corresponding Feynman diagrams for the non-spectator contributions to the inclusive decay rate of \( \Lambda_b \) are shown in Fig.2. The effective diquark-W-boson interaction vertex was given \[15\]

\[ V_s = -iG_s(q_1 + q_2)\mu W_\mu, \quad \text{for } DWD' \]

where \( D \) and \( D' \) stand for scalar diquarks, \( q_1 \) and \( q_2 \) are the four-momenta of \( D \) and \( D' \), \( G_s \) is a form factor which is determined by fitting data \[15\] and generally one can recast it as

\[ G_s(Q^2) = \frac{\tilde{\alpha}_s(Q^2)Q_0^2}{Q_0^2 + Q^2}, \]

with \( \tilde{\alpha}_s(Q^2) \) the effective non-perturbative QCD coupling constant and takes a reasonable value similar to that used in the potential model. The transition amplitude for \( b(p_1) + D(q_1) \rightarrow c(p_2) + D'(q_2) \) via the W-boson exchange reads as

\[ T_{eff}^s = \frac{G_F}{\sqrt{2}}(V_{cb}V_{ts}^*)\bar{c}\gamma_\mu(1 - \gamma_5)b(q_1 + q_2)\mu F_s(Q^2). \]

The same process can occur via the penguin-induced effective vertex. The transition \( b(p_1) + D(q_1) \rightarrow c(p_2) + D'(q_2) \) via the penguin, where a virtual gluon is exchanged between the quark and diquark, and the formulation is similar to that for W-boson exchange. The effective vertex for \( b \rightarrow s + g \) is given \[14\] as

\[ V_\mu = \frac{G_F}{\sqrt{2}}\frac{g_s}{4\pi^2}(V_{tb}V_{ts}^*)\bar{s}\gamma_\mu\gamma_\nu\sigma_{\mu\nu}\bar{q}m_bRb, \]

with \( \Delta F_1 = F_1^t - F_1^c \), \( F_1^t \approx 0.25 \), \( F_1^c = -\frac{2}{3}\ln\frac{m_b^2}{M_W^2} \approx 5.3 \), \( F_2 \approx 0.2 \).

Thus we can ignore the \( F_2 \) part and the transition amplitude is

\[ T_{eff}^s = \frac{G_F}{\sqrt{2}}\frac{\alpha_s}{\pi}(V_{tb}V_{ts}^*)\bar{s}\gamma_\mu\gamma_\nu\sigma_{\mu\nu}\bar{q}m_bRb, \]

(ii) The effective operators for the inclusive processes of \( \Lambda_b \)
The weak W-exchange (WE) operator is
\[ \hat{\Gamma}_{\text{tree}}^{t_{\text{W}E}} = \frac{G_F^2 M^2}{8\pi} |V_{cb}V_{ud}|^2 (1 - \bar{z}_+ ) \bar{b}(\frac{\bar{M}_+^2}{2} - m_c^2)(\bar{\psi}_D + \bar{\psi}_D') + (\frac{3}{4} - 3m_c^2)\bar{\psi}_+ + \frac{1}{2} \bar{\psi}_D \bar{\psi}_+ \bar{\psi}_D |LbF_s^2, \] (65)

where \( p_b, p'_b, p_D \) and \( p'_D \) are the four momenta of the initial and final b-quarks and diquarks, 
\( p_+ = p_b + p_D, \bar{M}_+^2 = (p_b + p_D)^2 \approx M_b^2, \) and \( \bar{z}_+ = \frac{m_c^2}{M_b^2}. \)

For the Pauli-interference operators (PI), it is noted that as we indicated in the previous footnote that Pauli interference here is only a terminology for such classes of Feynman diagrams, because the duquark is a boson-like subject, so by no means needs to obey the Pauli principle.

There are contributions from both the tree and penguin mechanisms to the Pauli interference (PI) operators. In non-relativistic limit, \( Q_i^2 \sim Q_f^2 \sim 0, \) and \( F_s \sim \bar{\alpha}_s(Q^2) \), they are
\[ \hat{\Gamma}_{\text{tree}}^{t_{\text{PI}}} = \frac{G_F^2 M^2}{8\pi} |V_{cb}V_{ud}|^2 (1 - \bar{z}_- ) \bar{b}(\frac{3}{4} - 3m_c^2)\bar{\psi}_- + (\frac{3}{2} - m_c^2)(\bar{\psi}_D + \bar{\psi}_D') + \frac{1}{2} \bar{\psi}_D \bar{\psi}_+ \bar{\psi}_D |LbF_s^2, \] (66)

\[ \hat{\Gamma}_{\text{penguin}}^{t_{\text{PI}}} = \frac{G_F^2}{(4\pi)^3} c_s^2 |V_{tb}V_{ts}|^2 \bar{b}_b[\bar{M}_-^2 \frac{3}{2} \bar{\psi}_- - \bar{\psi}_D' - \bar{\psi}_D] + \bar{\psi}_D \bar{\psi}_D' \bar{b}_b \frac{1}{2} \bar{\psi}_D \bar{\psi}_+ \bar{\psi}_D |LbF_s^2, \] (67)

where \( p_- = p_b - p'_D, \bar{M}_-^2 = (p_b - p_D)^2 \approx (m_b - M_b)^2 \) and \( \bar{z}_- = \frac{m_c^2}{M_b^2}. \) In the operators, we set the current quark mass of s-quark to be zero. Sandwiching the operators between initial and final \( \Lambda_b \) states of the forward scattering, we obtain the hadron matrix elements as

\[ \Gamma_{t_{\text{WE}}} = < \Lambda_b(\mathbf{P} = 0, s) | \hat{\Gamma}_{t_{\text{WE}}}^{t_{\text{WE}}} | \Lambda_b(\mathbf{P} = 0, s) > \]

\[ = \frac{G_F^2 M^2}{8\pi} |V_{cb}V_{ud}|^2 (1 - \bar{z}_+) \frac{2}{\sum_{\text{spin}}} \int d^3p_\rho d^3p_\rho \exp[-\frac{p_\rho^2 + p_\rho'^2}{2\alpha_D^2}] \]

\[ \bar{b}(\frac{\bar{M}_-^2}{2} - m_c^2)(\bar{\psi}_D + \bar{\psi}_D') + (\frac{3}{4} - 3m_c^2)\bar{\psi}_+ + \frac{1}{2} \bar{\psi}_D \bar{\psi}_+ \bar{\psi}_D |LbF_s^2, \] (68)

\[ \Gamma_{t_{\text{PI}}} = < \Lambda_b(\mathbf{P} = 0, s) | \hat{\Gamma}_{t_{\text{PI}}}^{t_{\text{PI}}} | \Lambda_b(\mathbf{P} = 0, s) > \]

\[ = \frac{G_F^2 M^2}{8\pi} |V_{cb}V_{ud}|^2 (1 - \bar{z}_-) \frac{2}{\sum_{\text{spin}}} \int d^3p_\rho d^3p_\rho \exp[-\frac{p_\rho^2 + p_\rho'^2}{2\alpha_D^2}] \]

\[ \bar{b}(\frac{3}{4} - 3m_c^2)\bar{\psi}_- + (\frac{3}{2} - m_c^2)(\bar{\psi}_D + \bar{\psi}_D') + \frac{1}{2} \bar{\psi}_D \bar{\psi}_+ \bar{\psi}_D |LbF_s^2, \] (69)

\[ \Gamma_{p_{\text{penguin}}}^{t_{\text{PI}}} = < \Lambda_b(\mathbf{P} = 0, s) | \hat{\Gamma}_{p_{\text{penguin}}}^{t_{\text{PI}}} | \Lambda_b(\mathbf{P} = 0, s) > \]

\[ = \frac{11G_F^2}{18(4\pi)^3} c_s^2 |V_{tb}V_{ts}|^2 |A_{\Lambda_b}|^2 \frac{2}{\sum_{\text{spin}}} \int d^3p_\rho d^3p_\rho \exp[-\frac{p_\rho^2 + p_\rho'^2}{2\alpha_D^2}] \]

\[ \bar{b}[\bar{M}_2^2 (\bar{\psi}_D + \bar{\psi}_D') + (\frac{3}{2} \bar{\psi}_-) + \bar{\psi}_D \bar{\psi}_- \bar{\psi}_D |b(\Delta F_1)|^2 F_s^2. \] (70)
5 The Numerical Results

To evaluate the lifetimes of $B^0$, $B^\pm$ and $\Lambda_b$, the input parameters are taken as follows

\[
\begin{align*}
V_{ud} &= 0.9742, \quad V_{us} = 0.219, \quad V_{tb} = 0.9993, \quad V_{ts} = 0.035, \quad V_{cb} = 0.037, \\
N_c &= 3.0, \quad m_b = 4.79\text{GeV}, \quad m_c = 1.25\text{GeV}, \quad m_u = m_d = 0.40\text{GeV}, \\
m_D &= 0.6\text{GeV}, \quad m_{B^0} = 5.28\text{GeV}, \quad m_{B^\pm} = 5.28\text{GeV}, \quad m_{\Lambda_b} = 5.6\text{GeV},
\end{align*}
\]

and the current quark masses $\bar{m}_u = \bar{m}_d = \bar{m}_s = 0$. In addition, in the diquark model, we take the values given in ref.\[15\],

\[
\begin{align*}
\Delta F_1 &= -5.05; \quad Q_0^2 = 3.22\text{GeV}^2; \quad \alpha_s(m_b^2) = 0.246; \quad \alpha_s = 0.87.
\end{align*}
\]

Meanwhile the pure b-quark decay rate has been evaluated by Bagan et al \[12\] as

\[
\Gamma_b = 4.13 \times 10^{-13}\text{GeV}. \quad (71)
\]

(a) For the heavy B-mesons

The experimental data of $B^0$ and $B^-$ are \[11\]

\[
\begin{align*}
\begin{cases}
\tau_{B^0} = (1.548 \pm 0.032)\text{ps}, \\
BR(B^0 \to D^+ e \bar{\nu}_e) = (2.10 \pm 0.19)\%, \\
\Gamma_{SL}(B^0 \to D^+ e \bar{\nu}_e) = (8.121 \sim 9.737) \times 10^{-15}\text{GeV}.
\end{cases} \quad (72)
\end{align*}
\]

and

\[
\begin{align*}
\begin{cases}
\tau_{B^\pm} = (1.653 \pm 0.028)\text{ps}, \\
BR(B^- \to D^0 e \bar{\nu}_e) = (2.15 \pm 0.22)\%, \\
\Gamma_{SL}(B^- \to D^0 e \bar{\nu}_e) = (7.685 \sim 9.347) \times 10^{-15}\text{GeV}.
\end{cases} \quad (73)
\end{align*}
\]

Fitting the data of the semi-leptonic decays of $B$, we obtain the corresponding parameters $\alpha_{B^0}$ and $\alpha_{B^\pm}$ in the wavefunctions as

\[
\alpha_{B^0} = 0.573; \quad \alpha_{B^\pm} = 0.541; \quad (74)
\]

With the Wilson coefficients evaluated in ref.\[13\]

\[
c_1 = 1.105; \quad c_2 = -0.245.
\]

we get the theoretical values of the lifetimes of B-mesons which are shown in Table.1:

|       | $\tau_{B^0}(\text{ps})$ | $\Gamma_{B^0} \times 10^{-13}\text{GeV}$ | $\Gamma_{WE}/\Gamma_b$ | $\Gamma_{PL}/\Gamma_b$ |
|-------|------------------------|------------------------------------------|------------------------|------------------------|
| $B^0$ | 1.56                   | 4.17                                    | 1.0%                   | -1.9%                  |
| $B^\pm$ | 1.63               | 4.05                                    | -1.9%                  | -1.9%                  |
Table 1: The lifetimes of $B^0$ and $B^\pm$ evaluated in our model with the parameters obtained by fitting data of the semi-leptonic decays, where $\Gamma_{B_q} = \Gamma_b + \Gamma_{non}$.

Thus theoretical result is $\tau(B^-)/\tau(B_d) = 1.03$. which is in general consistent with the data.

(b) For the heavy baryon $\Lambda_b$

The experiment data of $\Lambda_b$ are given \[1\]

\[
\begin{align*}
\tau_{\Lambda_b} &= (1.229 \pm 0.080)\text{ps}, \\
BR(\Lambda_b \to \Lambda_c e\bar{\nu}_e) &= (7.9 \pm 1.9)\%, \\
\Gamma_{SL}(\Lambda_b \to \Lambda_c e\bar{\nu}_e) &= (3.213 \sim 5.249) \times 10^{-14}\text{GeV}.
\end{align*}
\]

By fitting the data of the semi-leptonic decays of $\Lambda_b$, we get the corresponding parameters which are presented in Table 2. Since the measured $\Gamma_{SL}$ which is the input parameter for our model, has a relatively large tolerance range, the model parameters can accordingly take various values, here we adopt four typical values $\Gamma_{SL} - \sigma$, $\Gamma_{SL}$, $\Gamma_{SL} + \sigma$ and $\Gamma_{SL} + 2\sigma$, as input. $\Gamma_{SL}$ and $\sigma$ are the central value and standard deviation of the measured decay rate\[1\].

| $\Gamma(\Lambda_b \to \Lambda_c e\bar{\nu}_e) \times 10^{-14}\text{GeV}$ | $\alpha_{\Lambda_b}^{\rho}$ | $\alpha_{\Lambda_b}^{\Lambda}$ | $\alpha_{\Lambda_c}^{\rho}$ | $\alpha_{\Lambda_c}^{\Lambda}$ | $\alpha_{D_i}^{\Lambda_b}$ | $\alpha_{D_i}^{\Lambda_c}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 3.213 (a)       | 0.215           | 0.601           | 0.101           | 0.364           | 0.669           | 0.435           |
| 4.231 (b)       | 0.221           | 0.627           | 0.114           | 0.377           | 0.712           | 0.463           |
| 5.294 (c)       | 0.234           | 0.657           | 0.127           | 0.394           | 0.745           | 0.486           |
| 6.267 (d)       | 0.255           | 0.674           | 0.142           | 0.411           | 0.788           | 0.502           |

Table 2: These four sets of parameters which are obtained by fitting data of semi-leptonic decay $\Lambda_b \to \Lambda_c e\bar{\nu}_e$ and corresponding to (a) ($\Gamma_{SL} - \sigma$), (b) $\Gamma_{SL}$, (c) ($\Gamma_{SL} + \sigma$) and (d) ($\Gamma_{SL} + 2\sigma$) respectively.

With these parameters we can numerically calculate the lifetime of $\Lambda_b$ similarly to what we have done for $B^0$ and $B^\pm$. The results are shown in Table 3.
Table 3: The lifetime of $\Lambda_b$ evaluated in our model with different sets of the parameters obtained by fitting data of the semi-leptonic decays, and $\Gamma_{\Lambda_b} = \Gamma_b + \Gamma_{\text{non}}$.

The theoretical results indicate that $\tau(\Lambda_b)/\tau(B_d) \sim 0.91 \sim 0.95$ in the 3-valence quark model and $\tau(\Lambda_b)/\tau(B_d) \sim 0.93 \sim 0.96$ in the diquark model while the measured value of the ratio is 0.79.

6 Conclusion and Discussion

The Standard Model (SM) is definitely responsible for the weak transition and the lifetimes of B-mesons, as well as $\Lambda_b$ because they do not have strong decay channels. By the common understanding, such inclusive decay modes are dominated by the spectator mechanism, namely the decay rate is almost fully determined by the decays of the heavy flavor in the hadron. However, some puzzles in b-physics emerge as indicated by many authors [17], such as the lifetime of $\Lambda_b$, charm number missing in B decays etc. Among them the lifetime of $\Lambda_b$ raises the most challenging problem.

Bigi et al. discussed the lifetime difference of $D^\pm$ and $D^0$ [5], they showed that the non-spectator effects play a crucial role to the lifetimes. In fact, the Pauli interference(PI) mechanism greatly suppresses the decay rate of $D^\pm$ compared to $D^0$, and QCD can almost perfectly explain why $\tau(D^\pm) \sim 2.55\tau(D^0)$, but $\tau(B^\pm) \sim \tau(B^0)$. Therefore it is natural to consider to take into account the non-spectator effects for evaluating the lifetime of $\Lambda_b$. Even though as Bigi et al. indicated, such effects are not so important for b-physics, one may still think that the contributions may be not negligible, because the baryon structure is different from that of mesons, at least there are two light valence quarks and each of them can join the b-quark to make a non-spectator contribution. Moreover, there is a diagram where three valence quarks(b u d) participate in the reaction (see Fig1(c)), and it does not appear for the meson case. Unfortunately our numerical results indicate that this contribution is too small to result in any substantial change to the lifetime of $\Lambda_b$.

The derivation at quark level is standard, the main difficult part comes from the evaluation of the hadronic matrix elements. We employ the simplest non-relativistic harmonic oscillator model which has been proved to be successful in phenomenology, but definitely is not accurate. We determine the concerned parameters by fitting the data of semi-leptonic decays and then
use them to evaluate the lifetimes. In the process we can reduce the errors and uncertainties in the model-dependent calculation. Then we repeat the procedure with the same model to deal with the B mesons, thus when we compute the ratios of $\tau(B^+)/\tau(B^0)$ and $\tau(\Lambda_b)/\tau(B^0)$, the model-dependence is further reduced.

For $\Lambda_b$, we employ two pictures, the three-valence-quark picture and one-heavy-quark-one-light-diquark picture to evaluate the lifetime of $\Lambda_b$, the result obtained in the two pictures are qualitatively consistent.

Our numerical results indicate that the non-spectator effects can only result in about at most 7% reduction of the lifetime of $\Lambda_b$ whereas the experimental data demand a 21% reduction. Therefore we can claim that the non-spectator contribution is sizable and cannot be neglected, taking into account the effects can remarkably alleviate the discrepancy which was not improved by just changing the matrix element parameters [16]. However, on the other side, the discrepancy still stands and demands a more satisfactory explanation. Because the large fraction of 21% deviation in the ratios cannot be compensated by only the non-spectator effects, there must be some unknown mechanisms which induce the difference between baryon and meson. For example, the proposed three-body force in baryons [2] might lead to a dramatic difference between baryon and meson, all these mechanisms are worth careful studies and may provide us with better understanding of the hadron structure and fundamental interactions.

As the verbal work of the paper is near completion, we notice Gabbiani et al’s work [18], they also evaluate the non-spectator effects on the lifetime of $\Lambda_b$, even though in a different approach, our results are consistent with theirs.

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References

[1] The Data Group, Phys. Rev. D66(2002)010001;

[2] N. Isgur and M. Wise, Phys. Lett. B232(1989)113; B237(1990)527;

[3] H. Georgi, Nucl. Phys. B361(1991)339; M. Neubert, Phys. Rep. 245(1994)259;

[4] M. Beneke, G. Buchulla, M. Neubert and C. Sachrajda, Phys. Rev. Lett. 83(1999)1914; Nucl. Phys. B591(2000)313;

[5] I. Bigi, N. Uraltsiev Phys. Lett. B280(1992)120; I. Bigi, N. Uraltsiev and A. Vainshtein, Phys. Lett. B293(1992)430; (E) B297(1993)477; B. Blok, M. Shifman, Nucl. Phys. B399(1993)441; 459; I. Bigi, B. Blok, M. Shifman, N. Uraltsiev and A. Vainshtein, In B Decays, edited by S. Stone, World Scientific Pub.Co.,Singapore 1994; G. Belliui et al, Phys. Rep. 289(1997)1; M. Neubert and C. Sachrajda, [hep-ph/9603202] W. Dai et al, Phys. Rev. D60(1999)034005;

[6] N. Deshpande and X. He, Phys. Lett. B336(1994)471;
Appendix A

The semileptonic and non-leptonic decay rates of b-quark up to order $1/m_Q^2$ are given as
the expression of \( \eta \) to the semileptonic decay rate and its general analytic expression is given in \([20]\). In a special case the expression of \( \eta(x, 0, 0) \) describing the \( b \to c\bar{c}s \) transitions. \( \eta(x_c, x_l, 0) \) is the QCD radiative correction to the semileptonic decay rate and its general analytic expression is given in \([20]\). In a special case the expression of \( \eta(x, 0, 0) \) is given in \([21]\) and numerically it can be approximated as \([22, 23]\).

\[
\eta(x_c, x_l, 0) = 1 - \frac{2\alpha_s}{3\pi}[(\pi^2 - \frac{31}{4})(1 - \sqrt{x})^2 + \frac{3}{2}].
\]  

(79)

For the decay \( b \to c\tau\bar{\nu} \), according to ref.\([12]\) we roughly have

\[
\Gamma(b \to c\tau\bar{\nu}) \sim 0.25\Gamma_{b \to e\bar{\nu}c}.
\]  

(80)

For the dimension-three operator \( \bar{Q}Q \), the expectation value can be expressed as

\[
< H_Q | \bar{Q}Q | H_Q > = 1 - \frac{\langle p_Q^2 \rangle^2 > H_Q}{2m_Q^2} + \frac{\mu_G^2 > H_Q}{2m_Q^2} + O(1/m_Q^2),
\]  

(81)
where \( < (p_Q)^2 > \equiv < H_Q \mid \bar{Q}(iD)^2Q \mid H_Q > \) denotes the average kinetic energy of the quark Q moving inside the hadron and \( < \mu_Q^2 > \equiv < H_Q \mid \bar{Q}\frac{1}{2}\sigma \cdot GQ \mid H_Q > \).

According to ref. [19] the kinetic energies of b and c quarks are respectively

\[
\frac{< (p_b)^2 >_B}{2m_b^2} \simeq 0.016. \tag{82}
\]

For the QCD operator, one finds \( < \mu_Q^2 >_{PQ} = \frac{3}{2}m_Q(M_{VQ} - M_{PQ}), \) where \( P_Q \) and \( V_Q \) denote the pseudoscalar and vector mesons, respectively.
Figure 1: The Feynman diagrams for the non-spectator effects in the 3-valence-quark-picture picture, (a) is the WE and (b) is the PI where only one of the light quarks exchanges W-boson with the b-quark while the rest one stands by as a spectator. The Pauli interference is only a terminology and its real meaning is explained in the footnote of the text. Fig.1 (c) refers to a "PI" and "WE" mixed Feynman diagram where all the three quarks take part in the reaction.

Figure 2: The Feynman diagrams for the non-spectator effects in the one-heavy-quark-one-light-diquark picture, (a) is the WE, and (b) is PI where the light diquark exchanges W-boson with the b-quark. The Pauli interference is only a terminology.