Indirect limits on the CPT violating background in the neutrino sector

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Abstract

CPT violation in the neutrino sector, suggested as a new way to reconcile different neutrino anomalies, induces at the radiative level observable effects among charged leptons, where high-precision tests of the CPT symmetry are available. We show that, in the models with heavy right-handed Majorana neutrinos, constraints imposed by these experiments require CPT violation in neutrino spectrum be suppressed to a level undetectable for any conceivable neutrino experiment. We find that the CPT violation in the neutrino sector may evade indirect constraints only at the expense of light right-handed neutrinos with small Yukawa couplings to the Standard Model sector or by allowing non-locality well below the electroweak scale.
1 Introduction

In Quantum Field Theory, any local, hermitian, Poincaré invariant action is CPT invariant. CPT invariance is thus considered to be a fundamental symmetry of particle physics. This does not imply, however, that the CPT invariance cannot be broken: field theory can be just a low energy limit of another fundamental theory (e.g., string theory), where one or more of the conditions for the CPT theorem are violated (e.g., locality). At low energies, field theory still presents a perfect description and therefore all effects of CPT violation must be described in the form of effective CPT-odd operators. Invariably, such operators would break Lorentz invariance, so that the breaking of CPT can be ascribed to some background (vector or axial-vector) fields that define preferred directions. A classification of such backgrounds, coupled to the operators of lowest dimensions built from the Standard Model fields was given in Ref. [1]. Specific realizations of the CPT breaking were discussed in the context of string theory [2, 3], in chiral gauge theories on spacetimes with a particular topological structure [4] and in non-commutative geometries [5, 6]. Phenomenologically, CPT violation can be motivated by an interesting, although quite speculative idea of equilibrium baryogenesis [7].

On the experimental side, there have been diverse efforts to detect possible signatures of CPT/Lorentz violating backgrounds that resulted in a collection of extremely tight bounds on the CPT-odd physics [8]-[12]. It is clear that, up to date, the most precise experiments that check Lorentz symmetry and CPT where performed with “friendly” matter: electrons, nuclei, relatively long-lived mesons. Direct constraints on the CPT violation in the neutrino sector are by far more modest.

The absence of direct constraints on the CPT violation in neutrino sector has sprung a number of interesting speculations that CPT breaking may be comparable to neutrino masses and splittings between different generations [13, 14]. The presence of CPT violation creates a possibility to accommodate all neutrino data: atmospheric and solar neutrino anomalies, as well as the LSND effect, without introducing new light degrees of freedom such as sterile neutrinos. To account for the LSND anomaly, the authors of Ref. [14] split the “masses” of electron neutrino and antineutrino by \( O(1\,\text{eV}) \). In a more adequate language, this amounts to asymmetrically modifying the dispersion relations for neutrino and antineutrino by an eV-size term. It is easy to see
that, quite generically, these terms can be combined to form an effective
operator of the form $\bar{\nu}b_{\mu}\gamma_{\mu}(\gamma_{5})\nu$. Another equally speculative motivation for
CPT/Lorentz violation in the neutrino sector are the hints on the “negative”
$m_{\nu}^{2}$ in the experiments searching for neutrino masses using the end-points of
beta decays and the seasonal variations of this effect [13].

Considering as an example the models in [6], we would like to argue that
CPT violation would naturally arise in the sector of singlet neutrinos. In
the higher dimensional set up of Ref. [6], with non-commutativity between
usual 4d and extra-dimensional coordinates, the CPT violation is induced for
a fermionic Kaluza-Klein zero mode if this fermion is allowed to propagate
in the bulk. Charged matter needs to be chiral, and the projecting out of
unwanted states leads to CPT even physics [6]. On the other hand, singlet
neutrino is allowed to live in higher dimensions and therefore is susceptible
to the CPT violating effects.

But even though the neutrino sector is the most likely recipient of CPT vi-
olation induced by some fundamental physics (string theory, non-commutativity,
etc.), it is easy to argue from general principles that CPT violation cannot
reside solely in the neutrino sector. The charged leptons and neutrinos are
connected by electroweak and Yukawa interactions, and therefore CPT viola-
tion in the neutrino sector will induce CPT violation among charged leptons.
The coupling of the axial vector background to the electron current $\bar{e}\gamma_{\mu}\gamma_{5}e$
is limited to an impressive accuracy [12]

$$|\vec{\theta}_{\text{electron}}| \lesssim 10^{-28} \text{ GeV}$$

with slightly milder limits coming from the clock comparison experiments
that involve paramagnetic atoms [8].

The purpose of this letter is to study the mechanisms that import CPT viola-
tion from neutrinos to the charged lepton sector and make a full use of
the constraint (1). This allows to place strong bounds on the CPT-violating
neutrino phenomenology and narrow down certain theoretical constructions
that would evade these constraints.

2 CPT-violation with minimal field content

We start from the models which are the most economical from the point of
view of the number of light degrees of freedom. We assume that at low ener-
gories ($E < M_W$) the neutrino spectrum consists of only three generations of the Standard Model neutrinos. To have a non-trivial neutrino phenomenology, we extend the Standard Model by a set of heavy Majorana neutrinos $N_i$. By heavy we mean not necessarily the GUT scale neutrino: for the purposes of the present discussion it is sufficient to assume that $M_i \gtrsim M_W$. Now, we introduce the CPT-violating background in the singlet neutrino sector, that would couple to bilinear combinations of $N$.

For the three generation case, two types of backgrounds are possible:

$$\mathcal{L}_N = \frac{1}{2} N^T C M N + \bar{\tilde{N}} A_{\mu} \gamma_{\mu} N + \bar{\tilde{N}} B_{\mu} \gamma_{\mu} \gamma_{5} N,$$

(2)

The matrix $A_{\mu}$ is antisymmetric and $B_{\mu}$ is symmetric in generation space. For a single generation case, only the $B_{\mu}$ term is allowed. For the clarity of the discussion, we shall work with the first generation and extend it to the multi-generation case when needed. We further assume that the CPT violating terms are generated at some scale $\Lambda_{UV}$. It is natural to think that this scale is also large, $\Lambda_{UV} \gtrsim M_W$.

Integrating out heavy degrees of freedom, at the tree level, we get the standard see-saw mechanism for the left handed neutrinos, i.e. dimension five $\Delta L = 2$ effective operators, corrected for the presence of the CPT-violating terms. The relevant tree level diagrams are given in Figure 1.

$$\mathcal{L}_\nu = \nu^T C \frac{(y v)^2}{2 M} \nu + \nu \frac{(y v)^2 B_{\mu} \gamma_{\mu} \gamma_{5} N}{2(B^2 + M^2)} \nu.$$  

(3)

Here $y$ is the Yukawa coupling and $v = 246$ GeV is the electroweak v.e.v.
For future convenience, we also introduce a notation

\[ b_{\mu}^{\text{tree}} = \frac{(y v)^2 B_\mu}{2(B^2 + M^2)} \]  

(4)

Now we take into account the loop correction coming from the \( H - N \) exchange diagram, Figure 2. We choose to work in the explicitly renormalizable gauge and by \( H \) we mean all four scalars that belong to the Standard Model Higgs doublet \( H \). In the unitary gauge, the exchange by charged scalars and a pseudoscalar will be equivalent to taking into account longitudinal components of gauge bosons in \( W - N \) and \( Z - N \) exchange diagrams. In the CPT-even channel this loop is not important, as it brings a small correction to the kinetic term of the left-handed doublet \( L \). In the CPT-odd channel, the corrections are generated for both the neutral and charged components of \( L \).

\[ \mathcal{L}_L = \bar{L} \gamma_\mu \gamma_5 L b_{\mu}^{\text{loop}} \]  

(5)

Thus, at the loop level, \( b_{\mu}^{\text{loop}}(\text{electron}) \approx b_{\mu}^{\text{loop}}(\text{neutrino}) \) and both quantities are subject to the constraint (1). Therefore, the only way of having the CPT-odd neutrino phenomenology at \( O(1 \text{eV}) \)-level is to have \( b_{\mu}^{\text{loop}}/b_{\mu}^{\text{tree}} < 10^{-19} \)!

We now turn to the explicit result for \( b_{\mu}^{\text{loop}} \):

\[ b_{\mu}^{\text{loop}} = \frac{y^2 B_\mu}{64 \pi^2} \begin{cases} \ln \frac{\Lambda_{UV}^2}{B^2 + M^2} & \text{if } \Lambda_{UV}^2 > B^2 + M^2 \\ c_1 \frac{\Lambda_{UV}^2}{B^2 + M^2} & \text{if } B^2 + M^2 > \Lambda_{UV}^2 > M_W^2 \\ c_2 \frac{\Lambda_{UV}^2}{M_W(B^2 + M^2)} & \text{if } M_W^2 > \Lambda_{UV}^2 \end{cases} \]  

(6)

where the loop integrals were cutoff at \( \Lambda_{UV} \), which was kept arbitrary and \( c_1 \) and \( c_2 \) are order one coefficients.
It is convenient to rewrite this result in the following form:

\[
\frac{b^{\text{loop}}}{b^{\text{tree}}} = \frac{1}{32\pi^2} \begin{cases} 
\frac{B^2 + M^2}{v^2} \ln \frac{\Lambda_{\text{UV}}^2}{B^2 + M^2} & \text{if } \Lambda_{\text{UV}}^2 > B^2 + M^2 \\
\frac{\Lambda_{\text{UV}}^2}{B^2 + M^2} & \text{if } B^2 + M^2 > \Lambda_{\text{UV}}^2 > M_W^2 \\
c_1 \frac{\Lambda_{\text{UV}}^2}{B^2 + M^2} & \text{if } M_W^2 > \Lambda_{\text{UV}}^2
\end{cases}
\]  

(7)

which is valid for every component of \(b_\mu\).

Examining the expression (7), we observe that the first two lines are larger than one, so that \(b^{\text{loop}} > \sim 10^{-2} b^{\text{tree}}\) according to our assumptions. This, together with the experimental bound (1) immediately renders the following constraint on the CPT-odd terms in the sector of the left-handed neutrinos:

\[
b^{\text{loop}}_{\text{neutrino}} \text{ and } b^{\text{tree}}_{\text{neutrino}} < 10^{-17} \text{eV for } \Lambda_{\text{UV}}, M > M_W, \tag{8}
\]

which is applicable to the spatial components of the CPT-background. One may wonder if the bound (8) could be relaxed if the four-vector \(b\) is chosen to be exactly time-like. If \(b\) is time-like in the galactic frame, the motion of the solar system relative to galactic halo translates into \(b_i \sim 10^{-3} b_0\). If this effect is also fine-tuned, the motion of the Earth around the Sun breaks it at the level of \(O(10^{-4})\). In other words, a careful choice of frame for \(b_\mu\) may relax the bound (8) to the level of \(10^{-13}\)eV, which is still dramatically lower than the detection possibilities for any direct experiments with neutrinos [16]. Would it be possible to have eV CPT-odd terms for other flavours, as the CPT violation is by far less restricted for muons and tau-leptons? The strength of the bound in this case would depend, of course, on the mixing angle with the electron neutrinos. However, suppressing \(\theta_{1i}\) to a \(O(10^{-8})\) level would also make these neutrinos useless for “normal” CPT-even oscillations, making the consistency of all neutrino anomalies even more problematic. In any event, this possibility will appear again as a severe fine-tuning. We conclude that the minimal scenario - three light neutrinos below the electroweak scale and \(O(\text{eV})\)-size CPT violation coming from short distances - is highly unnatural on account of the strong bounds coming from the tests of CPT and Lorentz invariance for electrons.

\section{Small \(\Lambda_{\text{UV}}\) and/or Dirac neutrinos}

While excluding the minimal and the most natural possibilities for the CPT-odd neutrino phenomenology, Eqs. (7-8) also suggest how to suppress the
size of effects in the charged lepton sector.

**Option 1. Dirac neutrinos: \( M \equiv 0, \text{ small} \)**

One could consider adding right handed fields with very small Yukawa couplings to the Standard Model, so that neutrinos would get small Dirac masses

\[
m_\nu = \frac{y v}{\sqrt{2}} \sim eV
\]

In order to obtain the desired CPT-violating neutrino phenomenology, \( B_\mu \) should also be of the order of 1eV. The effect of CPT violation for electrons is obtained by the same radiative mechanism as before, but now it is strongly suppressed by the small Yukawa couplings.

\[
b_\mu^{\text{loop}} = \frac{y^2 B_\mu}{64 \pi^2} \ln \frac{\Lambda_{UV}^2}{M_W^2} = \left( \frac{m_\nu}{v} \right)^2 \frac{B_\mu}{32 \pi^2} \ln \frac{\Lambda_{UV}^2}{M_W^2} \sim 10^{-23} B_\mu \frac{\Lambda_{UV}^2}{M_W^2}
\]

For a cut-off \( \Lambda_{UV} \) of the order of the GUT scale, \( \Lambda_{UV} \sim 2 \times 10^{16}\text{GeV} \), one gets \( b_\mu^{\text{loop}} \sim 3 \times 10^{-24} B_\mu \sim 3 \times 10^{-33}\text{GeV} \). This is about four orders of magnitude below the present limit (\( b_\mu \)) but might be within the reach of the new generation of clock comparison experiments (\[17\]). This scenario avoids the constraints from the charged lepton sector, but introduces new degrees of freedom far below the electroweak scale and does not have a natural understanding of the smallness of neutrino masses.

**Option 2. Small values of \( \Lambda_{UV} \)**

Another, more exotic possibility, is that the scale at which CPT-odd physics is generated could be much lower than the electroweak scale and the loop effects are suppressed as \( \Lambda_{UV}^4/v^4 \) (Eqs. (6) and (7)). In order to have loop effects under control and the eV-size CPT violation in the neutrino sector, one should have \( \Lambda_{UV} \) in the 100 KeV range or smaller. Needless to say that this corresponds to a drastically different physics for the right-handed neutrino. Without having any convincing model for such a possibility, one can imagine an effective CPT-odd non-local interaction

\[
\bar{\nu}_{\gamma_\mu} \gamma_5 B_\mu \left( \frac{\Lambda_{UV}^2}{\partial^2 + \Lambda_{UV}^2} \right)^n \nu
\]

with some positive power \( n \). Such form of the CPT-odd interaction will be seen as a usual axial-vector background at energies smaller than \( \Lambda_{UV} \), at the same time providing a form factor/cutoff for loop integral at the energies larger than \( \Lambda_{UV} \).
This possibility is hard to realize unless one very specific phenomenon happens in the right-handed neutrino sector. If the right-handed neutrinos are light and very strongly-interacting due to a force of yet unknown origin, they might develop a fermion condensate with an open fermion number. At low energies a cubic root from the value of the corresponding condensate will determine $B_\mu$ and at high energies all CPT-odd pieces of propagators will be power-like suppressed (similarly to non-perturbative pieces in high-energy quark and gluon propagators in QCD).

4 Conclusions

The main finding of our paper is that the CPT violating effects are very efficiently transmitted from neutrinos to the charged lepton sector if the Majorana right-handed neutrinos and the scale of CPT breaking are heavier than the electroweak scale. Using the results of [12], we have shown that the CPT-odd energy splitting in the neutrino sector should be suppressed down to at least the $10^{-13}$ eV level, which makes it useless for reconciling different neutrino anomalies within the minimal model of three light SM neutrinos.

We see two potential ways of avoiding strong constraints coming from the charged lepton sector. Both of them require drastic modifications of the theory at the scales lower than the electroweak scale. One possibility is to make right-handed neutrinos light, so that corresponding Yukawa couplings are small, and the radiative mechanism of transmitting CPT violation to the charged lepton sector will be suppressed by an additional factor of $(m_\nu/M_W)^2$. But this “solution” introduces new degrees of freedom far below the electroweak scale and thus defies the purpose of having CPT-odd neutrino phenomenology, which was invented as an alternative to sterile neutrino models.

Another possibility is even more exotic. The CPT violating terms should have a “form factor”, i.e. the theory becomes effectively non-local at the scale of 1 GeV or less. Finally, both possibilities can be merged together if the right-handed neutrinos are light and for some reasons develop a condensate with a non-zero fermion number. It will be a serious model-building challenge to find a model for right-handed neutrinos that would furnish this property.

As a concluding remark, we would like to mention that if one of this possibilities is realized and CPT violation in neutrino sector escapes indirect
constraints (however unlikely it looks at the moment), the CPT-odd neutrino phenomenology can be highly anisotropic and oscillation patterns can be different from what was discussed in [13, 14, 16]. Previous discussions of CPT odd effects effectively concentrated on the time-like vectors $b_\mu \sim (b_0, 0, 0, 0)$. At the moment there are no solid reasons to argue that the space components of $b$-vector should be zero. This brings a question of directional dependence and sidereal variations in CPT-odd neutrino phenomenology that come atop of already known anisotropies in CPT-even neutrino physics such as azimuthal dependence of atmospheric neutrino fluxes, day/night effects, etc.

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