We have studied the loosely bound $D^*\bar{D}^*$ system. Our results indicate that the recently observed charged charmonium-like structure $Z_c(4025)$ can be an ideal $D^*\bar{D}^*$ molecular state. We have also investigated its pionic, dipionic, and radiative decays. We stress that both the scalar isovector molecular partner $Z_{c0}\gamma_0$ and three isoscalar partners $Z_{c0\gamma\pi}$ should also exist if $Z_c(4025)$ is a $D^*\bar{D}^*$ molecular state in the framework of the one-pion-exchange model. $Z_0$ can be searched for in the channel $e^+e^-\rightarrow Y\rightarrow Z_c(4025)\ (3\pi)^{P-wave}$ where $Y$ can be $Y(4260)$ or any other excited $1^{−}\$ charmonium or charmonium-like states such as $Y(4360)$, $Y(4660)$ etc. The isoscalar $D^*\bar{D}^*$ molecular states $Z_{c0\gamma\pi}$ with $0^+(0^{++})$ and $0^+(2^{++})$ can be searched for in the three pion decay channel $e^+e^-\rightarrow Y\rightarrow Z_{c0\gamma\pi}\ (3\pi)^{P-wave}$. The isoscalar molecular state $Z_{c1}$ with $0^-(1^{−})$ can be searched for in the channel $Z_{c1}\gamma$. Experimental discovery of these partner states will firmly establish the molecular picture.

PACS numbers: 14.20.Pt, 12.40.Yx, 12.39.Hg

Since the observation of charged charmonium-like structure $Z_c(3900)$ [1], very recently the BESIII Collaboration reported a new charged structure $Z_c(4025)$ in $e^+e^-\rightarrow (D^*\bar{D}^*)^+\pi^−$ at $\sqrt{s} = 4.26\text{ GeV}$ [2]. If adopting a Breit-Wigner distribution to measure this structure, its mass and width are $(4026.3 \pm 2.6\pm 3.7)\text{ MeV}$ and $(24.8 \pm 5.6 \pm 7.7)\text{ MeV}$ [2], respectively. In addition, its obvious peculiarity is that $Z_c(4025)$ is near the $D^*\bar{D}^*$ threshold. Before the observation of $Z_c(3900)$, there are theoretical predictions of the charged charmonium-like state near the $D^*\bar{D}^*$ threshold [3-6].

This new experimental observation inspires our interest in exploring its underlying properties of $Z_c(4025)$, especially combining our former study [6-8] with new experimental information. As a charged charmonium-like structure, $Z_c(4025)$ cannot be grouped into the conventional charmonium family obviously. Since $Z_c(4025)$ is near the $D^*\bar{D}^*$ threshold, the exotic molecular state with hidden charm is a possible explanation.

As a molecular candidate, $Z_c(4025)$ is composed of the $D^*$ and $\bar{D}^*$ mesons. Since $Z_c(4025)$ appears in the $(D^*\bar{D}^*)^+(1^{++})$ invariant mass spectrum, it should be an isovector state. Due to the conservation of total angular momentum and parity in the process $e^+e^-\rightarrow Y(4260)\rightarrow Z_c(4025)^+\pi^−$, its quantum number cannot be $J^P = 0^+$. The flavor wave function of the S-wave $D^*\bar{D}^*$ molecular state can be expressed as [3]

$$
|Z_c(4025)^+\rangle = |D^*+\bar{D}^0\rangle
$$

$$
|Z_c(4025)^−\rangle = |D^*+\bar{D}^0\rangle
$$

$$
|Z_c(4025)^0\rangle = \frac{1}{\sqrt{2}}(|D^+\bar{D}^−\rangle - |D^−\bar{D}^0\rangle),
$$

where the total angular momentum $J = 0, 1, 2$. For simplicity, we denote these isovector molecular candidates as $Z_cJ$ with $J = 0, 1, 2$. For the neutral state $Z_c^0(4025)$, its C-parity is $C = (-)^J = (-)^1$ since $L = 0$ and $S = J$. Accordingly, the G-parity of its charged partner is $G = (-)^{J+1}$. Therefore, the quantum numbers of the S-wave $D^*\bar{D}^*$ molecular states with $J = 0, 1, 2$ are with $I^G(J^P) = 1^+(0^+)$, $1^+(1^+)$, $1^+(2^+)$ respectively. Since $Z_c(4025)$ was also observed in the $h_\gamma$ channel by BESIII collaboration, its G-parity is positive. In other words, the quantum number of $Z_c(4025)$ should be $I^G(J^P) = 1^+(1^+)$ while its neutral component also carries negative C-parity.

It’s very interesting to note that the $I^G(J^{PC}) = 1^+(1^+)$ hidden-charm tetraquark states were predicted to be around 4.2 GeV as shown in Table V of Ref. [6]. These tetraquark states will fall apart into the $D\bar{D}^*$, $D^*\bar{D}'$ and other hidden-charm modes very easily. Their width is expected to be around one or two hundred MeV. If $Z_c(4025)$ is a tetraquark candidate, it is very challenging to explain its narrow width and the experimental fact that such a tetraquark state does not decay into the $D\bar{D}^*$ mode. Although the tetraquark interpretation is not excluded, it seems less favorable than the molecular picture unless one can invent a special dynamical decay mechanism which forbids the $D\bar{D}^*$ mode and ensures its narrow width. In the following we focus on the molecular possibility.

Usually, the one-boson-exchange (OBE) model can be applied to obtain the effective potential of $D^*$ interacting with $\bar{D}^*$. As shown in Ref. [6], the one-pion exchange is dominant in the total effective potential of the $D^*\bar{D}^*$ system in the one-boson-exchange model. Thus, in the following we only consider the one-pion exchange contribution.

The effective Lagrangian relevant to the deduction of one pion exchange potential include [9,10]

$$
L_{H\Pi\Pi} = ig(H_b^{(Q)}\gamma_\mu A_\mu^{a\mu}H_a^{(Q)} + ig(H_a^{(Q)}\gamma_\mu A_\mu^{a\mu}H_b^{(Q)}),\ (2)
$$

where $H_b^{(Q)}$ and $H_a^{(Q)}$ are expressed as

$$
H_a^{(Q)} = \frac{1 + \gamma_5}{2}\left(P_{a\mu}^{H}Y_\mu - P_{a\gamma_5}Y_5\right), \ (3)
$$

$$
H_b^{(Q)} = \frac{1 + \gamma_5}{2}\left(P_{a\mu}^{H}Y_\mu + P_{a\gamma_5}Y_5\right)\frac{1 + \gamma_5}{2}, \ (4)
$$
where $\vec{H} = \gamma_0 H^\dagger \gamma_0$ and $\nu = (1, 0)$. The multiplet field $H^{(0)}_a$ with the heavy antiquark can be defined as

$$H^{(0)}_a = \frac{1}{2} \left[ \tilde{P}_a^\mu \gamma_\mu - \tilde{P}_a \gamma_5 \right] \frac{1 - \gamma_5}{2},$$  \hspace{1cm} (5)$$

and

$$\tilde{H}^{(0)}_a = \frac{1}{2} \left[ \tilde{P}_a^\mu \gamma_\mu + \tilde{P}_a \gamma_5 \right].$$  \hspace{1cm} (6)$$

Pseudoscalar $\mathcal{P}$ and vector $\mathcal{P}^+$ in the multiplet field $H^{(0)}_a$ are defined as $\mathcal{P}^{(*)P} = \left( D^{(*)P}, D^{(*)P+} \right)$. In the above expressions, the $\mathcal{P}(\tilde{\mathcal{P}})$ and $\mathcal{P}^+(\tilde{\mathcal{P}}^+)$ satisfy the normalization relations $\langle 0|\mathcal{P}^+(\tilde{\mathcal{P}}^+)Q(q^0)\rangle = \langle 0|\mathcal{P}Q(q^0)\rangle = \sqrt{M_P}$ and $\langle 0|\mathcal{P}^+Q(q^0)\rangle = \langle 0|\tilde{\mathcal{P}}^+Q(q^0)\rangle = -\epsilon_\mu \sqrt{M_\Pi}$. The axial current is $A^\mu = \frac{i}{2} (\xi_5 \partial_\mu \xi - \xi_\mu \xi^5) = \frac{i}{2} \partial_\mu \tilde{\pi} + \cdots$ with $\xi = \exp(i \tilde{\pi}/f_\pi)$ and $f_\pi = 132$ MeV. Here, $\tilde{\pi}$ is the two by two pseudoscalar matrix.

With the Breit approximation, the one-pion exchange potential in the momentum space can be related to the scattering amplitude by $\mathcal{V}^{D^*\bar{D}^*}_\pi(q) = -\frac{\mathcal{M}(D^*\bar{D}^* \rightarrow D^*\bar{D}^*)}{(4m_\pi^2)}.$ Thus, we obtain the potential in the coordinate space $\mathcal{V}^{D^*\bar{D}^*}_\pi(r)$ using Fourier transformation

$$\mathcal{V}^{D^*\bar{D}^*}_\pi(r) = \int \frac{dp}{(2\pi)^3} e^{ip\cdot r} \mathcal{V}^{D^*\bar{D}^*}_\pi(q) F^2(q^2, m_\tau^2),$$  \hspace{1cm} (7)$$

where the monopole form factor (FF) $F(q^2, m_\tau^2) = (\Lambda^2 - m_\tau^2)/(\Lambda^2 - q^2)$ is introduced, which reflects the structure effect of the vertex of the heavy mesons interacting with the light mesons. $m_\tau$ is the exchange $\tau$ meson mass. In addition, the introduced cutoff $\Lambda$ also plays the role of regulating the effective potential, which is around one to several GeV.

In our calculation, we consider the S-wave and D-wave mixing between $D^*\bar{D}^*$ and $\bar{D}^*\bar{D}^*$. Finally, the obtained one-pion exchange potential corresponding to the different angular momentum $J$ reads (see Ref. [3] for more details of the deduction)

$$\mathcal{V}^{D^*\bar{D}^*}_\pi(r) = \begin{cases} Y(\Lambda, m_\tau, r) + T(\Lambda, m_\tau, r), & J = 0 \\ Z(\Lambda, m_\tau, r) + T(\Lambda, m_\tau, r), & J = 1 \\ T(\Lambda, m_\tau, r), & J = 2 \end{cases},$$

where $Y(\Lambda, m_\tau, r), Z(\Lambda, m_\tau, r)$ and $T(\Lambda, m_\tau, r)$ are defined as

$$Y(\Lambda, m_\tau, r) = \frac{1}{4\pi r}(e^{-m_\tau r} - e^{-\Lambda r}) - \frac{\Lambda^2 - m_\tau^2}{8\pi \Lambda} e^{-\Lambda r},$$  \hspace{1cm} (9)$$

$$Z(\Lambda, m_\tau, r) = \sqrt{2} Y(\Lambda, m_\tau, r) = \frac{1}{r} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} Y(\Lambda, m_\tau, r),$$  \hspace{1cm} (10)$$

$$T(\Lambda, m_\tau, r) = \frac{1}{r} \frac{\partial}{\partial r} Y(\Lambda, m_\tau, r).$$  \hspace{1cm} (11)$$

Just considering the S-wave and D-wave contribution of the $D^*\bar{D}^*$ interaction, the $D^*\bar{D}^*$ states with different $I^G(JP)$ can be expressed as

$$|D^*\bar{D}^*[1^-(0^+)]\rangle = \begin{pmatrix} D^*\bar{D}^* \langle 1^- \rangle S_0 \rangle \langle 1^- \rangle D_0 \rangle \\ D^*\bar{D}^* \langle 1^- \rangle D_0 \rangle \langle 1^- \rangle D_0 \rangle \end{pmatrix},$$

$$|D^*\bar{D}^*[1^+(1^+)]\rangle = \begin{pmatrix} D^*\bar{D}^* \langle 1^+ \rangle S_1 \rangle \langle 1^+ \rangle D_1 \rangle \\ D^*\bar{D}^* \langle 1^+ \rangle D_1 \rangle \langle 1^+ \rangle D_1 \rangle \end{pmatrix},$$

$$|D^*\bar{D}^*[1^-(2^+)]\rangle = \begin{pmatrix} D^*\bar{D}^* \langle 2^- \rangle S_2 \rangle \langle 2^- \rangle D_2 \rangle \\ D^*\bar{D}^* \langle 2^- \rangle D_2 \rangle \langle 2^- \rangle D_2 \rangle \end{pmatrix},$$

which make the obtained potentials listed in Eq. \text{[3]} be of matrix form. Thus, we need to solve the coupled-channel Schrödinger equation to find bound state solutions.

The corresponding kinetic terms are

$$K_{J=0} = \text{diag} \left( -\frac{\Delta}{2m_1}, -\frac{\Delta_2}{2m_1} \right),$$  \hspace{1cm} (15)$$

$$K_{J=1} = \text{diag} \left( -\frac{\Delta}{2m_1}, -\frac{\Delta_2}{2m_1} \right),$$  \hspace{1cm} (16)$$

$$K_{J=2} = \text{diag} \left( -\frac{\Delta}{2m_1}, -\frac{\Delta_2}{2m_1}, -\frac{\Delta_2}{2m_1} \right),$$  \hspace{1cm} (17)$$

where the subscripts $J = 0, J = 1$ and $J = 2$ are introduced to distinguish the $D^*\bar{D}^*$ systems with different $J$. $\Delta = \frac{1}{2m_1} r^2 \frac{d}{dr}$, $\Delta_2 = \Delta - \frac{\Delta_1}{2}$, $m_1 = m_{D^*}/2$ are the reduced mass of the $D^*\bar{D}^*$ system, where $m_{D^*}$ denotes the mass of the $D^*$ meson.

In the one-pion exchange potential, we need the value of the coupling constant $g$ of the $D^*\bar{D}^*$ interaction in Eq. \text{[3]}. In Ref. \text{[12]}, the coupling constant $g = 0.75$ was roughly estimated by the quark model. A different set of coupling constants can be given in Ref. \text{[13]}. With our notation, $g = 0.6$. In fact, the coupling constant $g$ was calculated by using many theoretical approaches such as QCD sum rules \text{[14][17]}. Besides these theoretical estimates of the coupling constant, $g = 0.59 \pm 0.07 \pm 0.01$ \text{[18]} can be extracted by reproducing
the experimental width of $D^* [19]$. Considering the situation of the coupling constant $g$, in this work we discuss the dependence of binding energy on the $g$ value. I.e., we take some typical values of the coupling constant $g$.

With the above preparation, we perform the search for the bound state solution by solving the coupled-channel Schrödinger equation. With several typical $g$ values, we present the dependence of the binding energy $E$ on the cutoff $\Lambda$, which is listed in Fig. 1. Since we focus on the bound state with the low binding energy, we only give results of the binding energy $-20 \leq E \leq 0$ MeV.

![Graph](image)

**FIG. 1:** The variation of the obtained binding energy of the $D^* \bar{D}^*$ with $J = 0, 1, 2$ with the cutoff $\Lambda$. Here, the typical values of $g$ are taken as 0.59, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90.

If taking typical value $g = 0.59$ and requiring $\Lambda < 5$ GeV, we can find bound state solutions for the isovector $D^* \bar{D}^*$ systems with $J = 0, 1$. If we increase the $g$ value, the corresponding cutoff $\Lambda$ becomes smaller and is close to 1 GeV if obtaining the same binding energy. However, for the $D^* \bar{D}^*$ system with $J = 2$, we cannot find a bound state solution when taking $g = 0.59$ and $\Lambda < 5$ GeV. When $g = 0.8$, there exist the bound state solutions with $\Lambda \sim 5$ GeV. The corresponding $\Lambda$ value still deviates from 1 GeV even taking $g = 0.9$. Thus, our numerical results favor the explanation of $Z_c(4025)$ as the $D^* \bar{D}^*$ molecule with $J = 1$.

In addition, there also exists the $D^* \bar{D}^*$ molecular state with $I = 0$. For simplicity, we denote the isoscalar $D^* \bar{D}^*$ molecular states with $I^G(J^{PC}) = 0^+(0^{++})$, $0^-(1^{+-})$ and $0^+(2^{++})$ as $Z_{J}^I$ where $J = 0, 1, 2$. In Ref. [4], these isoscalar molecular states were studied with the coupling constant $g = 0.56$, and the bound state solutions were found at $\Lambda \approx 3$ GeV, 3.5 GeV and 1.5 GeV, respectively. If the coupling constant $g$ increases to 0.9, the solutions will appear at $\Lambda \approx 1.5$ GeV, 1.5 GeV and 1 GeV, respectively. Hence our calculation with the one-pion-exchange potential supports the existence of the isoscalar $D^* \bar{D}^*$ molecular states with $J = 0, 1, 2$.

The result shown in Fig. 1 also indicates that the obtained binding energy is strongly dependent on the cutoff, which is an inherent feature of the model itself. For the molecular states, we expect that its two constituent heavy mesons are well-separated. Thus, the long-range interaction is very important. However, the heavy mesons are not point-like particles. Therefore, in the study of the molecular states with the OBE potential model one introduces the cutoff. The role of the cutoff is to remove or suppress the contribution from the ultraviolet region of the exchanged momentum since the light pion sees the heavy mesons as a whole and does not probe their inner structure.

It is well-known that the deuteron is a loosely bound molecular state in nuclear physics. Its properties are described very well by the meson exchange model. The binding energy of the deuteron depends sensitively on the cutoff, the same as in our present study. Luckily the model-dependent cutoff parameter can be extracted through fitting to data in the deuteron case since there exist abundant experimental data. Hopefully there may accumulate more and more experimental data in heavy meson case in the future.

In the effective field theory framework, the scale dependence of the loop corrections will generally be absorbed by the short-distance counter terms in the form of low-energy constants. In the meson exchange model, one introduces the cutoff parameter in the form factor. The short-distance interaction is mimicked by the heavier meson exchange such as rho and omega etc. In the present case, the contribution from the heavier meson exchange is found to be small. We apply the OPE model to study the newly observed $Z_c(4025)$. We want to find whether similar bound state solutions exist for the $D^* \bar{D}^*$ systems with the reasonable cutoff within the framework of the OPE model.

In the following we discuss the decay behavior of the $D^* \bar{D}^*$ molecular states with $I^G(J^P) = 1^-(0^{++}), 1^+(1^{+-}), 1^-(2^{++})$, which is important to further test the inner structure of $Z_c(4025)$.

Under the heavy quark limit, the spin $S_H$ of the heavy quark of the heavy-light meson is conserved as well as the total angular momentum $J$. Thus, the spin of the light degrees of freedom, which include light quark, gluons, quark pairs and orbit angular momentum, can be defined as $\mathbf{j}_\ell \equiv \hat{J} - \hat{S}_H$. The spin structure of the heavy-light meson can be written as a form $[S_H^J \otimes S_\ell^P]_j$, where $P_H$ and $P_\ell$ are the parities of heavy quark and the remaining light degrees of freedom in heavy-light meson, which satisfy $P_H P_\ell = P$. For example, the corresponding spin structure for the $D^*$ meson is $[\frac{3}{2}^+ \otimes \frac{3}{2}^+]_1$. Thus, the S-wave $D^* \bar{D}^*$ molecular systems with $J = 0, 1, 2$ can be expressed by the spin re-coupling formula with $9-j$ symbols [20].

$$D^* \bar{D}^*[J^P] \Rightarrow \left[\frac{1}{2}_H^+ \otimes \frac{1}{2}_\ell^+\right]_1 \left[\frac{1}{2}_H^- \otimes \frac{1}{2}_\ell^-\right]_1 [S_H^J \otimes S_\ell^P]_j$$

$$= \sum_{S_H^J S_\ell^P} \frac{3}{2} \sqrt{2} S_H + 1 \sqrt{2} S_\ell + 1 \left\{ \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \right\} [S_H^J \otimes S_\ell^P]_j$$
where \( S_{\hat{H}} = \frac{1}{\sqrt{2}} [0_{\ell} \otimes 1_{\ell}]_{10} + \frac{1}{\sqrt{2}} [1_{\ell} \otimes 0_{\ell}]_{10}, \quad J = 0 \)
\( \frac{1}{\sqrt{2}} [0_{\ell} \otimes 1_{\ell}]_{11}, \quad J = 1 \),
\( \frac{1}{\sqrt{2}} [1_{\ell} \otimes 0_{\ell}]_{12}, \quad J = 2 \)

(18)

where \( S_{\hat{H}} \) denotes the angular momentum composed of the spins of \( c \) and \( \bar{c} \) quarks. Thus, \( S_{\hat{H}} = 0, 1 \). The subscript in \( S_{\hat{H}} \) denote the parity constructed by the parities of \( c \) and \( \bar{c} \) quarks directly. \( S_{\ell} \) denotes the angular momentum of the remaining light degrees of freedom in the \( D^* \bar{D}^* \) molecular state, which is similar to the definition of \( S_c \). Since the parity of the \( S \)-wave \( D^* \bar{D}^* \) molecular state is constrained to be +, thus the parity of light degrees of freedom in the \( D^* \bar{D}^* \) molecular state must be −, where we add the subscript − to \( S_{\ell} \) in Eq. (18).

In Table I the spin structures of the molecular states and the charmoniums are presented explicitly. The angular momentum and parity \( J^P \) of the emitted pion or photon can be obtained from the spin structures of the initial and final states as shown in Table I. Here, the second column is the corresponding information of \( S_{\hat{H}} \otimes S_{\hat{J}} \) of \( Z_c(4025) \) with different \( J^P \) quantum numbers as shown in Eq. (18). The final states of the pionic and radiative decays of \( Z_c(4025) \) are given in the first row. In addition, we list the spin structure of final states of the corresponding decays in the second row, where we also use the notation \( S_{\hat{H}} \otimes S_{\hat{J}} \) for charmonium, which is similar to that for the \( D^* \bar{D}^* \) molecular state. We need to specify that the definitions of \( S_{\ell} \) for charmonium and the \( D^* \bar{D}^* \) molecular state are slightly different. \( S_{\ell} \) for the \( D^* \bar{D}^* \) molecular state contains the degrees of freedom of the light valence quarks. For the pion and photon, the former definition \( S_{\hat{H}} \otimes S_{\hat{J}} \) is abbreviated as \( J^P \). In Table II we use − to mark it if the corresponding decay channel is forbidden. In addition, for those allowed decay channels, we further give the information of the \( J^P \) quantum numbers carried by the pion or photon, which is different from the intrinsic spin-parity of pion or photon. The indices \( S \) and \( P \) denote the allowed orbital angular momentum between charmonium and light meson for the corresponding decay.

Besides the angular momentum and parity conservations, the most important rule is that the decay with heavy quark spin flip should be suppressed when the heavy quark mass approaches infinity, i.e., \( 1_{\ell} \rightarrow 0_{\ell} \) is forbidden. The \( h_1(1P)\pi \) and \( J/\psi \pi \) decay modes are forbidden for the \( D^* \bar{D}^* \) molecular states with \( J^P = 0^+ \) and \( 2^+ \) due to the G-parity conservation. Besides these qualitative conclusion of the \( D^* \bar{D}^* \) molecular state decay into \( h_1(1P)\pi \) and \( J/\psi \pi \), in the following we give a quantitative discussion of these decays associated with \( Z_c(3900) \), where \( Z_c(3900) \) is another charged charmonium-like structure observed by BESIII in \( e^+e^- \rightarrow J/\psi \pi^+\pi^- \) at \( \sqrt{s} = 4.26 \text{ GeV} \). \( Z_c(3900) \) can be explained as the \( D\bar{D} \) molecular state \[22,25\], where \( Z_c(3900) \rightarrow h_1(1P)\pi \) and \( Z_c(3900) \rightarrow J/\psi \pi \) is also allowed (see Table I for its decay information).

With Eq. (18) and Table I the decay width ratio of the \( h_1(1P)\pi \) and \( J/\psi \pi \) modes of \( Z_c(4025) \) and \( Z_c(3900) \) under the heavy quark limit is

\[ \Gamma[Z_c(4025)] : \Gamma[Z_c(3900)] = 1, \]

where we have ignored the phase space difference for simplicity. The above ratio is only roughly estimated without involving any dynamics. If considering the concrete dynamics, this ratio may change. Thus, a further study on this ratio is needed with some special models, which will be another interesting research topic.

Since BESIII announced \( Z_c(3900) \rightarrow J/\psi \pi \) \[1\] and has not reported any enhancement structure near the \( D^* \bar{D}^* \) threshold up to now, one may wonder why the \( Z_c(4025) \) signal is absent in the \( J/\psi \) invariant mass spectrum of \( e^+e^- \rightarrow J/\psi \pi^+\pi^- \) process at \( \sqrt{s} = 4.26 \text{ GeV} \) if \( Z_c(4025) \) is a \( D^* \bar{D}^* \) molecular state with \( J^P = 1^+ \). One possibility is that the phase space of \( Y(4260) \rightarrow Z_c(4025) \pi \) is much smaller than that of \( Y(4260) \rightarrow Z_c(3900) \pi \). Thus, the signal of \( Z_c(4025) \) with \( J^P = 1^+ \) might be buried by that of \( Z_c(3900) \) in the \( e^+e^- \rightarrow J/\psi \pi^+\pi^- \) process.

Additionally, we also provide the radiative decay information of \( D^* \bar{D}^* \) molecular states. The decay of the \( D^* \bar{D}^* \) molecular state with \( J^P = 0^+ \) into \( \chi_{c0} \) should be suppressed because it is a \( 0 \rightarrow 0 \) process. However, for the \( D^* \bar{D}^* \) molecular state with \( J^P = 1^+ \), its radiative decays into \( \chi_{cJ} \) (\( J = 0, 1, 2 \)) are the typical \( M1 \) transition. Our calculation shows

\[ \Gamma(\chi_{c0}\gamma) : \Gamma(\chi_{c1}\gamma) : \Gamma(\chi_{c2}\gamma) = 1 : 3 : 5. \]

(20)

After considering the phase space factors proportional to the cubic of the photon energy, the above ratio becomes

\[ \Gamma(\chi_{c0}\gamma) : \Gamma(\chi_{c1}\gamma) : \Gamma(\chi_{c2}\gamma) = 1 : 1.9 : 2.4. \]

Experimental study of these radiative decays of \( Z_c(4025) \) is an interesting topic, which will be helpful to test the quantum number assignment of \( Z_c(4025) \).

We also present the hidden-charm dipion decays of the \( D^* \bar{D}^* \) molecular state as shown in Table II. The \( h_1(1P)[\pi\pi]_P \) and \( J/\psi[\pi\pi]_P \) decay modes are allowed for the \( D^* \bar{D}^* \) molecular states with \( J^P = 0^+ \) and \( 2^+ \), where the subscript \( P \) denotes the relative angular momentum between the two pions. The ratio of the decays of the \( D^* \bar{D}^* \) molecular state with \( J^P = 1^+ \) into \( \chi_{cJ}[\pi\pi]_P \) (\( J = 0, 1, 2 \)) should be similar to that of the corresponding radiative decays due to the similar quantum number and phase space factor.

Similarly for the isoscalar \( D^* \bar{D}^* \) molecular states with \( 0^+(0^+) \), \( 0^-(1^+) \) and \( 0^+(2^+) \), the possible hidden-charm decay channels are

\[ Z_{c0} \rightarrow J/\psi \omega, J/\psi \eta, \eta\eta, \]
\[ Z_{c1} \rightarrow J/\psi \omega, \eta \gamma, J/\psi \eta, \chi_{cJ} \gamma \]
\[ Z_{c2} \rightarrow J/\psi \omega, J/\psi \gamma. \]

The decay of \( Z_{c2} \rightarrow \eta \eta \) is suppressed due to the heavy quark spin flip.

In summary, stimulated by the observed \( Z_c(4025) \) by BESIII \[2\], we have investigated whether \( Z_c(4025) \) can be explained as the \( D^* \bar{D}^* \) molecular state by performing the calculation of the mass spectrum and the analysis of its pionic
TABLE I: The pionic and radiative decays of the $D^*D^*$ molecular states with different $I^c(J^P)$ in the heavy quark limit.

| $I^c(J^P)$ | $h_i(1P)$ | $\pi^\pm$ | $J/\psi$ | $\eta_c$ | $\gamma$ | $\chi^{+0}$ | $\gamma$ | $\chi^{+1}$ | $\gamma$ | $\chi^{+2}$ | $\gamma$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $D^*D^*[1^-(0^+)]$ | $\frac{\sqrt{5}}{2} |0_1^+ \otimes 1_1^6|_{10^0}$ | $1^+$ | $1^-$ | $1^-$ | $1^+$ | $1^+$ | $1^+$ | $1^+$ | $1^+$ |
| $D^*D^*[1^-(1^+)]$ | $\frac{1}{\sqrt{2}} |0_1^0 \otimes 1_1^6|_{10^0}$ | $1^+$ | $1^-$ | $1^-$ | $1^+$ | $1^+$ | $1^+$ | $1^+$ |
| $D^*D^*[1^-(2^+)]$ | $|0_1^0 \otimes 1_1^6|_{10^0}$ | $1^+$ | $1^-$ | $1^-$ | $1^+$ | $1^+$ | $1^+$ |
| $Z_c(3900)[1^-(1^+)]$ | $\frac{1}{\sqrt{2}} |0_1^0 \otimes 1_1^6|_{10^0}^*$ | $1^+$ | $1^-$ | $1^-$ | $1^+$ | $1^+$ |

TABLE II: The hidden-charm dipion decays of the $D^*D^*$ molecular states with different $I^c(J^P)$.

| $I^c(J^P)$ | $h_i(1P)$ | $[\pi\pi]_P$ | $J/\psi$ | $[\pi\pi]_P$ | $\eta_c$ | $[\pi\pi]_P$ | $\chi^{+0}$ | $[\pi\pi]_P$ | $\chi^{+1}$ | $[\pi\pi]_P$ | $\chi^{+2}$ | $[\pi\pi]_P$ |
|------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $D^*D^*[1^-(0^+)]$ | $\frac{\sqrt{5}}{2} |0_1^0 \otimes 1_1^6|_{10^0}$ | $1^+$ | $1^-$ | $1^-$ | $1^+$ | $1^+$ | $1^+$ | $1^+$ | $1^+$ |
| $D^*D^*[1^-(1^+)]$ | $\frac{1}{\sqrt{2}} |0_1^0 \otimes 1_1^6|_{10^0}$ | $1^+$ | $1^-$ | $1^-$ | $1^+$ | $1^+$ | $1^+$ | $1^+$ | $1^+$ |
| $D^*D^*[1^-(2^+)]$ | $|0_1^0 \otimes 1_1^6|_{10^0}$ | $1^+$ | $1^-$ | $1^-$ | $1^+$ | $1^+$ | $1^+$ | $1^+$ |
| $Z_c(3900)[1^-(1^+)]$ | $\frac{1}{\sqrt{2}} |0_1^0 \otimes 1_1^6|_{10^0}^*$ | $1^+$ | $1^-$ | $1^-$ | $1^+$ | $1^+$ |

and radiative decays. The effective potential of the $D^*D^*$ interaction can be obtained by the one-pion exchange model. By solving the Schrödinger equation, we find the bound state solutions with a reasonable $\Lambda$ value for the $D^*D^*$ molecular states with $J^P = 0^+, 1^+$, which indicates that $Z_c(4025)$ can be the ideal candidate of the $D^*D^*$ molecular states with $J^P = 1^+$. Under the heavy quark limit, we have also studied the decay behavior of these $D^*D^*$ molecular states. Experimental measurement of these ratios will probe the internal structure of $Z_c(4025)$.

In addition, both the scalar isovector molecular partner $Z_{00}$ and three isoscalar partners $Z_{c0,c1,c2}$ should also exist if $Z_c(4025)$ is a $D^*D^*$ molecular state within the one-pion exchange formalism. Their possible existence is strongly linked to the one-pion exchange (OPE) potential assumed in the calculation. The discovery of these additional states would give strong support of the OPE model and the molecular interpretation for $Z_c(4025)$. Experimental discovery of these partner states will establish the molecular picture.

The molecular state $Z_{c0}(4025)$ can be searched for in the channel $e^+e^- \rightarrow Y \rightarrow Z_{c0}(4025) \gamma(\pi\pi)_{p-wave}$ where $Y$ can be $Y(4260)$ or any other excited $1^-$ charmonium or charmonium-like states such as $Y(4360)$, $Y(4660)$ etc. Here the dipion acts as the quantum number filter. The isoscalar $D^*D^*$ molecular state with $0^+(0^{++})$ and $0^+(2^{++})$ can be searched for in the three pion decay channel $e^+e^- \rightarrow Y(4260, 4360, 4660) \rightarrow Z_{c0,c1,c2,3}\pi_{p-wave}^{f=0}$. The isoscalar $D^*D^*$ molecular state with $0^+(1^{++})$ can be searched for in the channel $Z_{c0}\eta$ through the same process.

More and more experimental progress on charged charmonium-like states provide us with a good platform to study exotic hadrons, which is a research field full of challenges and opportunities. More theoretical and experimental efforts are called for in order to reveal the underlying mechanism behind these novel phenomena.

Acknowledgement

This project is supported by the National Natural Science Foundation of China (Grants No. 11275235, No. 11075004, No. 11021092, No. 11035006, No. 11047606, No. 10805048), and the Ministry of Science and Technology of China (No. 2009CB825200), and the Ministry of Educa-
tion of China (FANEDD under Grants No. 200924, DPFHE under Grants No. 2009021120029, NCET under Grants No. NCET-10-0442, the Fundamental Research Funds for the Central Universities under Grants No. Iuzjby-2010-69), the Knowledge Innovation Project of the Chinese Academy of Sciences (Grant No. KJCX2-EW-N01).

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