Glass-induced enhancement of superconducting $T_c$: Pairing via dissipative mediators

Chandan Setty∗
Department of Physics, University of Florida, Gainesville, Florida, USA

With substantial evidence of glassy behavior in the phase diagram of high $T_c$ superconductors and its co-existence with superconductivity, we attempt to answer the question: what are the properties of a superconducting state where the force driving cooper pairing becomes dissipative? We find that when the bosonic mediator is local, dissipation acts to reduce the superconducting critical temperature ($T_c$). On the other hand, contrary to naive expectations, $T_c$ behaves non-monotonically with dissipation for a non-local mediator – weakly dissipative bosons at different energy scales act coherently to give rise to an increase in $T_c$ and eventually destroy superconductivity when the dissipation exceeds a critical value. The critical value occurs when dissipative effects become comparable to the energy scale associated with the spatial stiffness of the mediator, at which point, $T_c$ acquires a maximum. We outline consequences of our results to recent proton irradiation experiments (M. Leroux et al., [1]) on the cuprate superconductor La$_{2-x}$Sr$_x$CuO$_4$ (LBCO) which observe a disorder induced increase in $T_c$ even when the transition temperature of the proximate charge density wave (CDW) is unaffected by the presence of irradiation. Our mechanism is a novel way to raise $T_c$ that does not require a ‘tug-of-war’-like scenario between two competing phases.

Introduction: In $s$-wave superconductors (SCs) where the quasiparticle excitation spectrum is fully gapped and has a constant sign of the pairing form factor across the Fermi surface, Anderson’s magic theorem keeps the critical temperature ($T_c$) robust to non-magnetic impurities. In higher angular momentum SCs ($p$-,$d$-wave etc) or SCs where the sign of the gap changes across parts of the Fermi surface (such as $s_{±}$-wave SCs), $T_c$ is drastically suppressed with the addition of impurities—magnetic or otherwise [2]. These effects hold in the independent disorder limit and in the absence of electron correlations.

At a collective level when electron correlations are taken into account, randomness can yield several interesting phases of matter [3]. Amongst these is the spin glass (SG) phase widely observed in the phase diagram of many strongly correlated systems like high $T_c$ SCs [4–20]. The SG phase exhibits a remarkable phenomenology [21] – a transition into the SG defined by a broad cusp in the specific heat, a split in the DC magnetization at the SG transition depending on whether the SG phase is field cooled (FC) or zero field cooled (ZFC), linear temperature dependence of the AC susceptibility peak, and aging. Theoretically, SGs are described by an order parameter where the spin average on each site is non-vanishing but goes to zero when averaged over the lattice [22]. Important to the discussions that follow, spin correlators at the SG critical point follow a power law of the form [22–25]

$$D(\tau) \equiv \langle S_{i\mu}(\tau)S_{i\mu}(0) \rangle \sim \frac{1}{\tau^\alpha},$$

where in frequency space reads $D(\omega) \sim |\omega|$. Here, $S_{i\mu}$ is the $\mu$-th component of the spin at site $i$, and the angular and square brackets denote thermal and site averages respectively. The linear frequency dependence of the spin correlators indicates that dissipative dynamics is a necessary – albeit not sufficient – ingredient of SGs.

In this work, we explore the robustness of $T_c$ and properties of a superconducting state where the dynamics of the pairing mediator is rendered dissipative due to collective disorder (in the aforementioned sense). To this end, we add to the Lagrangian describing the mediator a dissipative term along with a mass gap of the form [26–29]

$$\Delta L = \sum_{k,\omega_n}(\eta|\omega_n| + M^2)|\Psi(k,\omega_n)|^2.$$  

Here $k$ and $\omega_n$ are the momenta and Matsubara frequencies, $\eta$ is a measure of dissipation, the squared mass $M^2$ is proportional to the inverse correlation length, and $\Psi(k,\omega_n)$ is the bosonic field. We find that both mass and dissipative effects generally act to suppress $T_c$ when the mediator is local. This occurs because both have the effect of reducing the attractive interaction mediating cooper pairs. However, contrary to naive expectations, $T_c$ behaves non-monotonically with dissipation for a non-local mediator.

In this scenario, weakly dissipative bosons at different energy scales act coherently to give rise to an increase in $T_c$ and eventually destroy superconductivity when the dissipation exceeds a critical value. The critical value occurs when the dissipation parameter, $\eta$, becomes comparable to the energy scale associated with the velocity of the mediating bosons (or the spatial stiffness); at this crossover, $T_c$ acquires a maximum value. We also study the effects of dissipative mediator on the ratio $\frac{2\Delta(0)}{T_c}$ and the heat capacity jump at the superconducting transition and find departures from values predicted by BCS theory.

Experimental basis: We now make our case for a dissipative or ‘glassy’ mediator from experiments on a variety of high $T_c$ SCs. The SG phase has been observed extensively in the under-doped and regions proximate to superconductivity in the phase diagrams of both the cuprate [4–14] and iron based superconductors [15–20]. Existing evidence is also spread over several techniques such as DC magnetization [4, 5, 17–20], NMR/NQR [7, 11–13], µSR [9, 10] and neutron scattering [9, 16]. Given the strong evidence of a SG phase and its proximity to the superconducting dome in high $T_c$
SCs, it is already reasonable to consider its effect on the pairing problem. Additionally, there is ample experimental
evidence lending credence to a dissipative character of
fluctuations that mediate Cooper pairing. First, disorder
causes the d-electron spins (Cu spins in the cuprates and
Fe spins in the iron superconductors) to exhibit glassy be-
havior and not the dopant spins [11, 15, 19] (although in
certain iron based systems, it is the dopant spins become
glassy [30]). Second, SG and SC phases actually co-exist
in a variety of high Tc SCs [12, 15, 19]. This indicates a
strong inter-mixing of properties of the two phases, simi-
lar to what is expected in the context of other mean-field
orders (such as density waves) acquiring a glassy behav-
ior [31]. Third, neutron scattering and NMR/NQR mea-
surements in the cuprate SCs La$_2$−xSr$_x$CuO$_2$ (LSCO)
and La$_2$−xBa$_x$CuO$_4$ (LBCO) have found a direct ‘slow-
ing’ of spin fluctuations in the vicinity of glassy or-
ders [7, 9]. This has been used to estimate the dissipation of
radiation disorder [32].

Hence, the notion of a dissipative pairing mediator in
high temperature superconductors has firm foundations
in both experiment and theory. As will be argued later in
this paper, non-local dissipative mediators can help
throw light on recent proton irradiation experiments [11]
on LBCO which observe a disorder induced increase in
Tc even when the transition temperature of the proximate
charge density wave (CDW) is unaffected by the presence
of radiation disorder [31].

The mechanism we propose in this paper forms an alter-
native way to raise Tc of a supercon-
ductor that does not require a ‘tug-of-war’-like scenario
between two competing phases.

**Model for boson and gap equation:** We begin by writ-
ing the conjectured model for the bosonic propagator.
The total action consists of a free part $S_0[\Psi, \Psi^\ast]$ and a
dissipative part $S_{dis}[\Psi, \Psi^\ast]$ defined by

\[
S[\Psi, \Psi^\ast] = S_0[\Psi, \Psi^\ast] + S_{dis}[\Psi, \Psi^\ast]
\]

\[
S_0[\Psi, \Psi^\ast] = \int d^d r d\tau \left[ \kappa |\nabla \Psi(r, \tau)|^2 + |\partial_\tau \Psi(r, \tau)|^2 + M^2 |\Psi(r, \tau)|^2 \right],
\]

where $\kappa$ is the spatial stiffness or energy scale associated
with the boson velocity. As outlined in the introduction,
we take the dissipative term to be form $S_{dis}[\Psi, \Psi^\ast] = \sum_{\bm{q}, \omega_n} \left( |q| |\omega_n| + M^2 \right) |\Psi(\bm{q}, \omega_n)|^2$ in Fourier space with the
various quantities defined previously. With this total ac-
tion, the bosonic propagator, $D(\bm{q}, \omega_n - \omega_m)$, takes the
form $D(\bm{q}, \omega_n - \omega_m) = \frac{1}{\omega_n^2 + \omega_m^2 - 2q\omega_m - |\omega_n| + M^2}$. Here $q = |\bm{q}|$ and $\alpha$ is a constant with dimensions of en-
ergy that can be absorbed into an effective coupling con-
stant (similar to spin fluctuations; see for example [33]).

**Tc, non-local case ($\kappa \neq 0$):** We can make similar assump-
tions on the superconducting gap for the $\kappa \neq 0$ case.
To maintain analytical tractability and focus on the ef-
eff of dissipation parameter $\eta$, we will later set the mass
(now renormalized by the chemical potential; we use the
same symbol for ease of notation) to zero. We can now
substitute the bosonic propagator with $\kappa \neq 0$ back into
the gap equation Eq. [2]. The resulting energy integral can
be solved exactly by the method of residues and takes the
form $\int_{-\infty}^{\infty} \frac{d\eta}{(\omega^2 + \eta^2)(\omega^2 + \eta^2 + 2\eta \omega_\text{M} - \omega_\text{x}^2)} = \frac{\pi \omega_\text{x}^2}{(\omega_\text{x}^2 + \eta^2)^{3/2}}$, where $\eta^2 = \omega_\text{x}^2 - \Delta^2$
and $s = \omega_m^2 + \eta' |\omega_m| + M^2$. Performing the remaining Matsubara sum we obtain the equation for $T_c$ as

$$1 = -\lambda \left[ \frac{\psi\left(\frac{1}{2} + \frac{\eta' - ix}{2\pi T_c}\right)}{2(\eta' - ix)^2} + \frac{\psi\left(\frac{1}{2} + \frac{\eta' + ix}{2\pi T_c}\right)}{2(\eta' + ix)^2} + \frac{\pi^2 \eta'}{4\pi T_c(\eta'^2 + \kappa^2)} \right]$$

(4)

where $\eta' = 2\eta$. The solution for $T_c = T_c/\kappa$ is plotted in the right panel of Fig 2 as a function of $T_c$. As is evident, for the case of a non-local mediator, $T_c$ behaves non-monotonically with dissipation and rises up to 40% of the initial $\eta = 0$ value. This happens because weakly dissipative bosons at different energy scales act coherently to give rise to an increase in $T_c$ but eventually destroy superconductivity for large dissipation. The critical value occurs when the dissipation parameter is greater than the stiffness constant ($2\eta \sim \kappa$); at this point, $T_c$ acquires a maximum with respect to $\eta$. This physics follows from the energy integral leading to Eq. 4 above. To see this, notice that the role of the stiffness parameter $\kappa$ is to induce non-monotonicity in an ‘effective’ coupling constant as a function of $\eta$ – while $\eta$ acts only to reduce the effective coupling constant for the local case, the energy integral (leading to Eq. 4) for the non-local mediator forces the gap equation to acquire dissipative contributions that both increase and decrease the effective coupling constant. Consequently, this translates into a non-monotonic behavior in $T_c$.

**Gap variation and specific heat jump:** We now study the variation of the gap with temperature and the specific heat jump at $T_c$. In Fig 3 we plot the temperature dependence of the superconducting gap as a function of the dissipation and mass parameters for $\kappa = 0$. Both $\eta$ and $M$ reduce the zero temperature gap $\Delta(0)$ and $T_c$; however, dissipation (mass) has a greater (smaller) effect on $T_c$ compared to $\Delta(0)$. Hence, the BCS ratio $\Delta(0)/T_c$ increases (decreases) with the dissipation (mass) parameter. To get an analytical handle for the gap near $T_c$, we begin with the case of $\eta = M = 0$ where the gap equation becomes $1 = \lambda \int_{-\infty}^{\infty} d\xi \left[ \frac{\beta \xi - 2 \tanh\left(\frac{4\xi}{\pi}\right)}{4\xi^3} - 3 \beta \xi + 6 \tanh\left(\frac{\xi}{\pi}\right) + \beta \xi \tanh^2\left(\frac{\xi}{\pi}\right) \Delta^2 + \ldots \right]$

$$\approx \frac{\beta^2 a(T)}{4\pi^2} - b \Delta^2,$$

(5)

where $a(T) = \frac{1}{2} \left[ \psi\left(2, \frac{3}{2} + \frac{\beta \Lambda}{\pi^2}\right) - \frac{1}{2} \psi(2, \frac{3}{2}) \right]$ is weakly temperature dependent in the limit of $\beta \Lambda \rightarrow \infty$, $b \approx \frac{3\beta^2}{32} \frac{\pi^2}{4\pi^2}$ and $\psi(n, x)$ is the $n$-th order digamma function. Setting the gap to zero in Eq 5 we can read off the dependence of $T_c$ on the coupling as $T_c \sim \sqrt{\Lambda}$, which grows faster than the conventional BCS relation. The temperature dependence of the gap can be derived as

$$\Delta^2(T) = \frac{2a(0)\pi^2}{4\pi^2(31/32)\xi(0)} (T_c - T),$$

and therefore, the normalized specific heat jump at $T_c$ is $\gamma = 2\pi^2 N(0)/3$ is the normal state specific heat) $\Delta^2(T) = \frac{3a(0)}{32\xi(0)} \approx \frac{a(0)}{32\xi(0)} \approx 6$, which is greater than the BCS value. Similarly, in the limit where the dissipation is much larger than the temperature and mass ($\eta |\omega_m| \gg |\omega_m|^2, M^2$), we have $\frac{1}{\eta} \approx \frac{u(T)}{4\pi T \eta} \approx \frac{u(T)}{4\pi T \eta} \frac{\eta^2}{\Delta^2}$. Here $u(T) = \pi^2 - 2 \psi\left(\frac{3}{2}, \frac{3}{2} + \frac{\beta \Lambda}{\pi^2}\right)$, $v = \int_0^\infty dx \left[ H\left(\frac{1}{2} - ix\right) + c.c \right] + \frac{\psi(1, \frac{1}{2} + ix)}{x}$ and $H(z)$ is the Harmonic number. $T_c$ can be evaluated again by setting $\Delta = 0$ and we see that, in this limit, $T_c \sim \sqrt{\eta}$. The cross-over from $T_c \sim \sqrt{T_c}$ to $T_c \sim \lambda$ as a function of $\eta$ is shown in Fig 3(right). The temperature...
dependence of the gap can be evaluated from above as 
$$\Delta(T) = \frac{2\pi T_c}{\kappa} (T_c - T);$$ hence, the specific heat jump at $T_c$ takes the value $\frac{\Delta c}{\kappa T_c} = \frac{3\pi^2}{4} \sim 3.64$ which is again greater than the BCS value.

On the other hand, expanding the gap equation for a non-local mediator ($\kappa \neq 0$) in the limit of $\eta = M \to 0$, we obtain $\frac{\kappa^2}{\lambda} \sim F(\frac{\kappa}{\kappa T}) - G(\frac{\kappa}{\kappa T}) \Delta^2$. The dimensionless functions $F(x)$, $G(x)$ and $\Delta$ are defined as

$$F(x) = \frac{1}{2} \left[ H \left( -\frac{1}{2} - i \right) + c + \log 16 \right],$$

$$G(x) = \frac{-1}{x^2} \left[ 10(\gamma_E + \log 4) + 5\psi \left( 0, \frac{1}{2} - i \right) + c + c.$$ 

$$+ i x \psi \left( 1, \frac{1}{2} - i \right) + c + c - 42x^2 \xi(3) \right] ,$$

$$\Delta = \frac{\Delta_0}{T_c},$$ and $\gamma_E$ is the Euler gamma constant. In the limit $x \ll 1$, the functions $F(x)$ and $G(x)$ satisfy the property $F(x) = C_1 x^2$ and $G(x) = C_2 x^2$, where $C_1$ and $C_2$ are numerical constants. The dependence of $T_c$ on $\lambda$ goes as $T_c \sim \sqrt{\lambda}$ and the temperature dependence of the gap takes the form $\Delta(T)^2 = \frac{\Delta_0^2}{T_c} \left( 1 + T / T_c \right)$. This implies that the specific heat jump is $\frac{\Delta c}{\kappa T_c} \sim 5.6$, again larger than the BCS value. However, in the limit $\eta \omega_m \gg |\omega_m|^2, M^2$ the expansion of the gap equation gives

$$\frac{1}{\lambda} = \frac{1}{2 \pi T} \left[ \frac{\bar{\eta} \pi^2}{1 + \bar{\eta}^2} - \frac{\eta \pi^4 (3 + \bar{\eta}^2)}{12(1 + \bar{\eta}^2)} \left( \frac{\Delta}{2 \pi T_c} \right)^2 + \ldots \right] ,$$

where $\lambda = \lambda / \kappa$. Setting $\Delta = 0$, we see that $T_c \sim \lambda$ and the temperature dependence of the gap is given by $\Delta(T)^2 = \frac{24T_c(T_c - T)(1 + \bar{\eta}^2)}{(3 + \bar{\eta}^2)}$. Hence, the specific heat jump is (weakly) dependent on the dissipation parameter and is given by $\frac{\Delta c}{\kappa T_c} = \frac{36(1 + \bar{\eta}^2)}{\pi^2 (3 + \bar{\eta}^2)}$. For small $\bar{\eta}$, the normalized specific heat jump is $\approx 1.2$ and is smaller than the BCS value consistent with specific heat experiments in underdoped cuprates [32] and the pnictides [35].

**Discussions and recent experiments:** Several theoretical works have explored mechanisms that yield an enhancement of $T_c$ with disorder strength. These phenomena range from competition of superconductivity with a proximate density wave phase [36, 39], multiorbital effects [40], local inhomogeneities in the pairing interactions and mediators [41, 42] to localization [47]. In the following paragraphs we argue for the applicability of the mechanism presented in this paper to the cuprates.

We emphasize that the change in $T_c$ in our work is due to modification of the ‘effective’ coupling by dissipation, and is unrelated to pair-breaking effects originating from lowering translational symmetry (say due to magnetic/non-magnetic inhomogeneities, like those summarized in Ref. [2]). Hence, qualitative aspects of our conclusions are expected to hold for higher angular momentum pairing as well (albeit with more tedious calculations). Recent magnetization and tunnel diode (penetration depth) experiments [1] on proton irradiated LBCO at $\frac{1}{2}$ doping found up to a 50% increase in $T_c$ as a function of radiation dosage. An increased dosage above a critical value gradually suppressed $T_c$ until the eventual destruction of superconductivity. LBCO also hosts a rich phase diagram with evidence of density wave orders (CDW, SDW [48–50] as well as spin–glass behavior [6] in conjunction with superconductivity in the under-doped regime. Hence, it is natural to anticipate an influence of these phases on superconductivity and examine their implications to $T_c$ variation as a function of disorder. Of the aforementioned existing mechanisms of $T_c$ enhancement proposed in literature, a competition-based scenario between superconductivity and a density wave order seems the most promising at first sight – especially given the close proximity of the CDW phase to the superconducting dome. Indeed, this was the point of view first suggested by Leroux and co-workers in [1]. However, a closer examination of the data points to deficiencies that render this mechanism moot. First, the CDW transition temperature is unaffected by irradiation [1]. But a mechanism involving the competition between two mean field phases necessary involves a tug-of-war scenario where one phase gains stability at the expense of its competitor [38]. Second, it is unclear how non-magnetic disorder affects two different mean field phases (CDW, SDW, SC etc) asymmetrically in a parameter independent manner, except under very specific circumstances [38, 39] which do not necessarily hold in the case of LBCO and cuprates. Third, other non-magnetic impurities are well known to kill $d$-wave superconductivity monotonically [2]. Thus a consistent picture which distinguishes proton and electron irradiation with other point impurities like Zn at a microscopic level is absent. Finally, from Anderson’s theorem, one can expect that a $T_c$ enhancement that occurs through a competition based scenario must be more prevalent in $s$-wave SCs rather than higher angular momentum SCs which are far less robust to non-magnetic impurities. Experiments on the $s$-wave superconductor 2H-NbSe$_2$, however, draw conclusions that are mixed at best [51, 53]. Hence, a reasonable explanation for non-monotonic $T_c$ dependence as a function of disorder in LBCO must necessarily involve a mechanism that does not depend on the competition of two mean-field like phases. The proposed mechanism in this paper, along with the experimental evidence provided in the introduction, forms a feasible alternative that fits experiments.

In conclusion, motivated by the close proximity of glassy phases to the superconducting dome in high $T_c$ SCs, we explored the role of dissipation on superconducting properties such as $T_c$, the temperature dependence of the gap, BCS ratio and the specific heat jump at $T_c$. We found that when the mediator is local, dissipa-
sification acts to reduce the effective coupling constant and $T_c$ monotonically. On the other hand, when the mediator is non-local, two competing effects of dissipation determine the $T_c$ variation – first, the dissipative contributions of individual bosons at a given energy that act to suppress $T_c$, and second, collective contributions where dissipation acts to connect bosons at different energy scales that combine coherently to increase the effective coupling and $T_c$. The former (latter) contribution dominates when the dissipation parameter is greater (lesser) than the bosonic spatial stiffness, i.e., $\eta > \kappa$ ($\eta < \kappa$); $T_c$ peaks when these two scales are comparable to each other. We also studied the effects of a dissipative mediator on the ratio $\frac{2\Delta(0)}{T_c}$ and the heat capacity jump at $T_c$, and found departures from values predicted by BCS theory. In particular, the specific heat jump at $T_c$ acquires a value smaller than that predicted by BCS theory when the mediator is both dissipative and non-local, consistent with experiment. We pointed out consequences of our results to recent proton irradiation experiments in LBCO [1] where superconducting $T_c$ is enhanced with increased radiation disorder despite a robust CDW transition temperature, and concluded that one does not require a ‘tug-of-war’ like scenario between two competing phases to enhance superconductivity.

Acknowledgements: We thank P. J. Hirschfeld and P. W. Phillips for discussions. This work is supported by the DOE grant number DE-FG02-05ER46236.

* email for correspondence: csetty@ufl.edu

[1] M. Leroux, V. Mishra, J. P. Ruff, H. Claus, M. P. Smylie, C. Opagiste, P. Rodière, A. Kayani, G. Gu, J. M. Tranquada, et al., arXiv preprint arXiv:1808.05984 (2018).
[2] A. Balatsky, I. Vekhter, and J.-X. Zhu, Reviews of Modern Physics 78, 373 (2006).
[3] P. A. Lee and T. Ramakrishnan, Reviews of Modern Physics 35, 2204 (1995).
[4] F. Chou, N. Belk, M. Kastner, R. Birgeneau, and A. Aharony, Physical review letters 75, 2204 (1995).
[5] S. Wakimoto, S. Ueki, Y. Endoh, and K. Yamada, Physical Review B 62, 3547 (2000).
[6] F. Cordero, A. Paolone, R. Cantelli, and M. Ferretti, Physical Review B 64, 132501 (2001).
[7] A. Hunt, P. Singer, A. Cederström, and T. Imai, Physical Review B 64, 134525 (2001).
[8] M.-H. Julien, Physica B: Condensed Matter 329, 693 (2003).
[9] B. Sterlinieb, G. Luke, Y. Uemura, T. Riseman, J. Brewer, P. Gehring, K. Yamada, Y. Hidaka, T. Murakami, T. Thurston, et al., Physical Review B 41, 8866 (1990).
[10] V. Mitrovć, M.-H. Julien, C. De Vaulx, M. Horvatić, C. Berthier, T. Suzuki, and K. Yamada, Physical Review B 78, 014504 (2008).
[11] T. Imai and K. Hirota, Journal of the Physical Society of Japan 87, 025004 (2018).
[12] M.-H. Julien, F. Borsa, P. Carretta, M. Horvatić, C. Berthier, and C. Lin, Physical review letters 83, 604 (1999).
[13] S.-H. Baek, T. Loew, V. Hinkov, C. Lin, B. Keimer, B. Büchner, and H.-J. Grafe, Physical Review B 86, 220504 (2012).
[14] T. Wu, H. Mayaffre, S. Krämer, M. Horvatić, C. Berthier, C. Lin, D. Haug, T. Loew, V. Hinkov, B. Keimer, et al., Physical Review B 88, 014511 (2013).
[15] A. Dioguardi, J. Crocker, A. Shockley, C. Lin, K. Shifer, D. Nisson, M. Lawson, P. Canfield, S. Bud?ko, S. Ran, et al., Physical review letters 111, 207201 (2013).
[16] A. T. Romer, J. Chang, N. B. Christensen, B. Andersen, K. Lefmann, L. Mähler, J. Gavilano, R. Gilardi, C. Niedermayer, H. M. Remnow, et al., Physical Review B 87, 144513 (2013).
[17] H. Ryu, K. Wang, M. Opacic, N. Lazarevic, J. Warren, Z. Popovic, E. S. Bozin, C. Petrovic, et al., Physical Review B 92, 174522 (2015).
[18] H. Ryu, M. Abeykoon, K. Wang, H. Lei, N. Lazarevic, J. Warren, E. Bozin, Z. Popovic, C. Petrovic, et al., Physical Review B 91, 184503 (2015).
[19] V. Grinenko, M. Abdel-Hafiez, S. Aswartham, A. Wolter-Giraud, C. Hess, M. Kumar, S. Wurmehl, K. Nenkov, G. Fuchs, B. Holzapfel, et al., arXiv preprint arXiv:1203.1585 (2012).
[20] C. Yadav and P. Paulose, Journal of Applied Physics 107, 083908 (2010).
[21] J. Mydosh, Reports on Progress in Physics 78, 052501 (2015).
[22] N. Read, S. Sachdev, and J. Ye, Physical Review B 52, 384 (1995).
[23] A. Bray and M. Moore, Physical Review B 31, 631 (1985).
[24] J. Miller and D. A. Huse, Physical review letters 70, 3147 (1993).
[25] D. Dalidovich and P. Phillips, Physical Review B 59, 11925 (1999).
[26] A. Caldeira and A. J. Leggett, Annals of physics 149, 374 (1983).
[27] S. Chakravarty, S. Kivelson, G. T. Zimanyi, and B. I. Halperin, Physical Review B 35, 7256 (1987).
[28] S. Chakravarty, G.-L. Ingold, S. Kivelson, and G. Zimanyi, Physical Review B 37, 3283 (1988).
[29] K.-H. Wagenblast, A. van Otterlo, G. Schön, and G. T. Zimányi, Physical review letters 78, 1779 (1997).
[30] K. Nadeem, W. Zhang, D. Chen, Z. Ren, and X. Qiu, Scientific reports 5, 10700 (2015).
[31] D. F. Mross and T. Senthil, Physical Review X 5, 031008 (2015).
[32] A. Aharony, R. Birgeneau, A. Coniglio, M. Kastner, and H. Stanley, Physical review letters 60, 1330 (1988).
[33] K.-H. Bennemann and J. B. Ketterson, Superconductivity: Volume 1: Conventional and Unconventional Superconductors Volume 2: Novel Superconductors (Springer Science & Business Media, 2008).
[34] J. Tallon and J. Loram, Physica C: Superconductivity 349, 53 (2001).
[35] M. Abdel-Hafiez, S. Aswartham, S. Wurmehl, V. Grinenko, C. Hess, S.-L. Drechsler, S. Johnston, A. Wolter, B. Büchner, H. Rosner, et al., Physical Review B 85, 134533 (2012).
[36] G. Grest, K. Levin, and M. Nass, Physical Review B 25, 4562 (1982).
[37] G. C. Psaltakis, Journal of Physics C: Solid State Physics 17, 2145 (1984).
[38] R. Fernandes, M. Vavilov, and A. Chubukov, Physical Review B 85, 140512 (2012).
[39] V. Mishra and P. Hirschfeld, New Journal of Physics 18, 103001 (2016).
[40] M. N. Gastiasoro and B. M. Andersen, arXiv preprint arXiv:1712.02656 (2017).
[41] A. T. Rømer, P. Hirschfeld, and B. M. Andersen, Physical review letters 121, 027002 (2018).
[42] I. Martin, D. Podolsky, and S. A. Kivelson, Physical Review B 72, 060502 (2005).
[43] E. Arrigoni and S. Kivelson, Physical Review B 68, 180503 (2003).
[44] Y. L. Loh and E. W. Carlson, Physical Review B 75, 132506 (2007).
[45] K. Aryanpour, E. R. Dagotto, M. Mayr, T. Paiva, W. Pickett, and R. T. Scalettar, Physical Review B 73, 104518 (2006).
[46] K. Aryanpour, T. Paiva, W. E. Pickett, and R. T. Scalettar, Physical Review B 76, 184521 (2007).
[47] J. Mayoh and A. M. García-García, Physical Review B 92, 174526 (2015).
[48] Y.-J. Kim, G. Gu, T. Gog, and D. Casa, Physical Review B 77, 064520 (2008).
[49] M. Hücker, M. v. Zimmermann, G. Gu, Z. Xu, J. Wen, G. Xu, H. Kang, A. Zheludev, and J. M. Tranquada, Physical Review B 83, 104506 (2011).
[50] H. Miao, Proc. Natl. Acad. Sci. USA 114, 12430 (2017).
[51] H. Mutka, Physical Review B 28, 2855 (1983).
[52] J. Tsang, M. Shafer, and B. Crowder, Physical Review B 11, 155 (1975).
[53] K. Cho, M. Kończykowski, S. Teknowijoyo, M. A. Tanatar, J. Guss, P. Gartin, J. M. Wilde, A. Kreyssig, R. McQueeney, A. I. Goldman, et al., Nature communications 9, 2796 (2018).