Electroweak Baryogenesis and the Phase Transition Dynamics

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Abstract

The baryogenesis is reanalyzed based on the model by A.G.Cohen et al., in which the lepton number, generated by the neutrinos' scattering from the bubble walls appearing in the development of the electroweak phase transition, is converted to the baryon number excess through the sphaleron transition. A formula obtained in this paper on the lepton number production rate is correct for both thin and thick walls within the linear approximation.

Investigation on the time-development of the first order phase transition is simulated, including the temporal change of the wall velocity as well as the fusion effect of the bubbles. The details of such phase transition dynamics are found to affect considerably the final value of the baryon number excess.
Baryogenesis in the universe was studied originally in the grand unified theories in the late 70’s. After the discovery of the baryon number and lepton number violation mechanism through the sphaleron transition, we have another possibility of generating baryons at the electroweak (EW) scale.

Among the various model of baryogenesis at the EW scale, we take up the model by A.G.Cohen et al., in which the lepton number, generated by the neutrino’s scattering from the bubble walls appearing in the development of the EW phase transition, is converted to the baryon number excess through the sphaleron transition.

The purpose of his paper is to find a formula of the lepton number production rate which is correct for both thin and thick walls when the vacuum expectation value of the Higgs scaler changes linearly within the wall (which may be called as the linear approximation).

The other aim of ours is to elucidate the time development of the phase transition in detail including the temporal change of the wall velocity as well as the fusion effects of the nucleated bubbles.

A Estimation of the Lepton Number Production Rate from the Bubble Wall

Our starting Lagragian is that of Cohen et al., namely

\[
\mathcal{L} = -\mathcal{L}(\text{standard model}) + \Delta \mathcal{L},
\]

where

\[
\Delta \mathcal{L} = \frac{1}{2} N_L i\gamma^\mu \partial_\mu N_L - \frac{1}{2} \phi N^C_L \lambda^1_M N_L - H \overline{\psi}_L \lambda_D N^C_L + (h.c.).
\]

Here \( \psi_L \) and \( N^C_L \) represent left-handed lepton doublets and right-handed neutrinos of \( G \) generations, respectively, \( \phi \) stands for the additional singlet
Higgs-scalar, and $H$ is the usual doublet one. The Majorana and Dirac mass terms of the neutrinos are given by the $G \times G$ mixing-matrices $\lambda_M^\text{I}$ and $\lambda_D$, respectively. ($C$ denotes the charge conjugation as usual.)

Since the electroweak phase-transition is of first order, bubbles of the broken phase are nucleated within the unbroken vacuum when the phase transition begins and the system becomes supercooling. These nucleated bubbles grow and fuse with other bubbles; finally the whole space is filled up with the broken phase. Therefore we have the thermal non-equilibrium, one of the necessary conditions of the baryogenesis, during the development of this first order phase transition.

We have, however, two kinds of Higgs scalars, $H$ and $\phi$.

If they acquire vacuum expectation values at different phases, then we have a complex configuration of the admixture of various kinds of bubbles. In order to avoid the complexity, we have assumed that there is only one kind of bubbles. For this purpose, the position-dependency of the vacuum expectation values $\langle \phi(x) \rangle$ and $\langle H^0(x) \rangle$ near the interface of bubbles is assumed to be identical $\langle \phi(x) \rangle \propto \langle H^0(x) \rangle$. This rather unrealistic assumption is not too bad, if the lepton number production rate $f_L$ comes mainly from the lighter components of neutrinos and not from the heavier ones the latter of which feel the configuration $\langle \phi(x) \rangle$ sensitively.

The Dirac equation of neutrinos for a given configuration of $\langle \phi(x) \rangle$ and $\langle H^0(x) \rangle$ reads from Eq.(2)

\[ i(\partial_0 - \sigma \cdot \nabla)\Psi(x) - i\sigma_2 M(x)\Psi(x)^* = 0, \]

where

\[ \Psi(x) = [\nu_1, \nu_2, \cdots, \nu_G, N_1, N_2, \cdots, N_G]^T, \]

and $\sigma_i(i = 1, 2, 3)$ are the usual Pauli matrices.

In Eq.(3) the mass matrix $M(x)$ is given by

\[ M(x) = \begin{pmatrix} 0, & \lambda_D \varphi(x), \\ \lambda_D^T \varphi(x), & \lambda_M \varphi(x) \end{pmatrix}, \]

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where $\varphi(x)$ stands for the common configuration of $\langle \phi(x) \rangle$ and $\langle H^0(x) \rangle$ near the bubble wall, taking 0 outside the bubble (unbroken phase), 1 inside the bubble (broken phase), and the value in between within the wall of the bubble (interface of the two phases). Under the normalization adopted here,

$$|\langle \lambda_D \rangle_{ij}| \ll |\langle \lambda_M \rangle_{kl}|$$

(6)

is understood for the see-saw mechanism to work.

For a sufficiently large bubble of the radius $R \gg T^{-1}$ with the typical temperature $T \sim O(100 GeV)$, we can reduce the three spatial variables to one, say $z$, representing the radial direction of the bubble. The decomposition of $\Psi(z)$ in terms of the spin $S_z$,

$$\Psi(z) = \begin{bmatrix} \psi_1^*(z)e^{iEt} \\ \psi_1(z)e^{-iEt} \end{bmatrix} + \begin{bmatrix} \psi_3(z)e^{-iEt} \\ \psi_2^*(z)e^{iEt} \end{bmatrix}$$

(7)

indicates that the particle $\psi_1$ with $S_z = -1/2$ is coming from the left, a part of which is reflected to the left as the anti-particle $\psi_4$ with the same spin of $S_z = -1/2$; Similarly the anti-particle $\psi_2$ with $S_z = +1/2$ is coming from the left, a part of which is reflected as the particle $\psi_3$ with $S_z = +1/2$.

We consider that the regions $z < 0$ and $z > \delta_w$ belong to the unbroken and broken phases, respectively, while the region $0 < z < \delta_w$ is occupied by the bubble wall.

The reflection coefficients $R$ and $\bar{R}$ for the particle $(\nu, N)$ and its anti-particle $(\bar{\nu}, \bar{N})$ are defined by

$$\psi_4(0) = R\psi_1(0) \quad \text{and} \quad \psi_3(0) = \bar{R}\psi_2(0)$$

(8)

under the boundary conditions of $\psi_1(\infty) = 0$ and $\psi_2(\infty) = 0$, respectively.

In the derivation of $R$ and $\bar{R}$, we need to estimate the path ordered integral.

$$I \equiv P \exp \left( i \int_0^{\delta_w} dz A(\varphi(z)) \right)$$

(9)
with $2G \times 2G$ constant matrix $A$.

We approximate it to

$$I_A \cong \exp \left( i \int_0^{\delta_w} d\varphi(z)A \right) = e^{i\eta A},$$

(10)

where $\eta$ is a parameter representing the shape of the wall ($0 < \eta < 1$);

$$\eta \equiv \frac{1}{\delta_w} \int_0^{\delta_w} d\varphi(z)$$

(11)

It should be noted that the approximation (10) is exact when $\varphi(z)$ changes linearly within the wall, $0 < z < \delta_w$, and for $\eta = 1/2$. Therefore, the approximation (called linear one) is a rather good approximation.

Now the reflection coefficients of particle $[(\nu, N) \to (\bar{\nu}, \bar{N})]$ and anti-particle $[(\bar{\nu}, \bar{N}) \to (\nu, N)]$ read

$$R \cong -U^T DU \quad \text{and} \quad \bar{R} \cong U^\dagger DU^*,$$

(12)

where the diagonal $2G \times 2G$ matrix $D$ in the generation space denotes

$$D = D(E) = \frac{s + i(sE - c\eta M_b^{(\text{diagonal})}) \times P_{\eta}^{-1} \tan(\delta_u P_{\eta})}{c - i(cE - s\eta M_b^{(\text{diagonal})}) \times P_{\eta}^{-1} \tan(\delta_u P_{\eta})}.$$  

(13)

Here the diagonal matrices $c_{\eta}$, $s_{\eta}$ and $P_{\eta}$ are defined as

$$c_{\eta} = \cosh \theta_{\eta}, \quad s_{\eta} = \sinh \theta_{\eta},$$

(14)

with

$$\tanh 2\theta_{\eta} = \eta (M_b^{(\text{diagonal})}/E),$$

(15)

and

$$P_{\eta} = \sqrt{E^2 - (\eta M_b^{(\text{diagonal})})^2},$$

(16)
where \( c = c_1 \) and \( s = s_1 \). The detail of the derivation of (13) will be written elsewhere [0].

Viewing the expression Eq.(13), we can understand that \( D \) is the reflection coefficient for each mass eigen-states \((\nu, N)\) or \((\bar{\nu}, \bar{N})\), and it is sandwiched by the matrices \( U' \)'s which transform the weak eigen-states to the mass eigen-states, namely,

\[
UM(z \text{ in the broken phase})U^T = M_b^{\text{diagonal}}. \tag{17}
\]

Cooperation of the phase shift including in \( D \) and of the complex phases in the mass matrix (or in \( U \)) makes the difference between particle \((\nu, N)\) and anti-particles \((\bar{\nu}, \bar{N})\). The number of the complex phases in \( U \) is \( G(G-2) + 1 \), so that the minimum number of the generation is 2 for our purpose of the baryogenesis.

We can read easily the following properties from our results Eq.(13):

i. \(|D(E)|^2 \to \frac{1}{4}(m_\nu/E)^2 \text{ for } m_\nu/E \to 0\), which means that for a massless neutrino the barrier or the wall disappears so that it perfectly transmit the barrier and does not be reflected.

ii. \(|D(E)|^2 \to 1 \text{ for } E \leq m_\nu\), that is, such neutrino is completely reflected by the "potential" barrier.

iii. Near the threshold \( E = m_\nu + \Delta E (\Delta E > 0) \), we have

\[
|D(E)|^2 \sim 1 - 2\sqrt{\frac{2\Delta E}{m_\nu}} \frac{1 + \eta}{1 + \eta \cos 2(2\sqrt{1 - \eta^2}\delta w m_\nu)} \tag{18}
\]

giving the cusp behavior \( \sqrt{E - m_\nu} \), and damping rapidly for \( E \geq m_\nu \). This is the origin of "resonance behavior" in the baryon number production rate so called by Nelson et al. [0].

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iv. Asymptotically for \( E \gg m_\nu \),

\[
|D(E)|^2 \sim \left( \frac{m_\nu}{E} \right)^2 \left\{ \frac{1}{4} \cos^2 (\delta_w E) + \left( \frac{1}{2} - \eta \right)^2 \sin^2 (\delta_w E) \right\}, \tag{19}
\]

where a series of ”resonances” appear for \( \delta_w E \neq 0 \).

In general the phase transition occurs at \( T = O(100 \text{GeV}) \), which is approximately the physical Higgs mass \( m_H \), and wall width \( \delta_w \) is roughly \( m_H^{-1} \). Therefore, \( \delta_w E \) and \( \delta_w P_\eta \) are \( O(1) \) and are not negligible, so that we will be troubled with the summing up a series of ”resonances”.

Difference of the reflection rates between the two processes \( \nu_i \rightarrow \bar{\nu}_j \) and \( \bar{\nu}_i \rightarrow \nu_j \) triggers the lepton number production

\[
\Delta_{ji} = |R_{ji}|^2 - |\bar{R}_{ji}|^2 = \sum_{k \neq l} \text{Im}(D_k D_l^*) \times J_{ji}^{lk}, \tag{20}
\]

where

\[
J_{ji}^{lk} = \text{Im}(U_{kj} U_{kl}^* U_{ij}^* U_{li}) \tag{21}
\]

is the so-called Jarlskog parameter \( \mathcal{J} \), typically representing the magnitude of \( \mathcal{CP} \) violation arising from the mass matrix. The expression eq.(20) shows that in order for the \( \mathcal{CP} \) violation to manifest itself, the other dynamical phase, \( \text{Im}(D_k D_l^*) \) is required to take part in the problem.

Here we will meet with a difficult problem: The initial states \( i \) and the final states \( j \) are thermally averaged, so that if the thermal distribution is common for all \( i \) or for all \( j \), then we have no lepton number production, \( \sum_i \Delta_{ji} = \sum_j \Delta_{ji} = 0 \). This reflects the \( \mathcal{CP} \mathcal{T} \) invariance and the GIM cancellation. The way to overcome this difficulty has been proposed by Farrar and Shaposhnikov \( \mathcal{J} \). They have considered that the initial and final particles have masses due to the finite temperature effects that lifts the common distribution for initial and final particles.

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Here we restrict ourselves only to the process in which the neutrinos are coming from the unbroken phase and are reflected by the bubble wall. Then, initial and final neutrinos are massless at $T = 0$, but acquire the following finite temperature masses for $\nu_L$ and $N_L$,

\[
M_{\nu_L}(T^2) = \frac{T^2}{16} \lambda_D \lambda_D^\dagger
\]

\[
M_{N_L}(T^2) = \frac{T^2}{16} (\lambda_M^\dagger \lambda_M)^T,
\]

without breaking the chiral invariance $[0]$. Then, we have obtained the lepton number production rate $f_L$ per unit time generated from the unit area of the bubble wall, under the situation that the wall is moving toward the incoming neutrinos with the velocity $v_\omega$:

\[
f_L = \frac{1}{\gamma_\omega} \int \frac{d^3 k_W}{(2\pi)^3} \sum_{mn} (1 - f_n(E_n^\dagger)) \Delta L_{nm}(|k^w_\perp|) \times \frac{k^w_\perp}{E_m} f_m(E_m),
\]

with

\[
\Delta L_{nm} = \sum_{i,j} (L_j - L_i) \{ |a^\dagger_{nj} R_{ji} a_{im}|^2 - |a^\dagger_{nj} \bar{R}_{ji} a_{im}|^2 \}
\]

Here, $L_{i(j)}$ denotes the lepton number of the neutrino $\nu_{i(j)}$ (or $N_{i(j)}$) in the flavor eigenstates, $m(n)$ represent the mass eigenstates diagonalized by the matrix $\{a_{im}\}$, $f(E)$ is the Fermi-Dirac distribution in the thermal frame, the energy and momentum with the affix $W$ stand for the variables in the wall rest frame, and $\gamma_\omega = (1 - v^2_\omega/c^2)^{-1/2}$.

Approximately we have the following expression

\[
f_L \approx T^3 \sum_{m,n} \frac{M^2_m(T) - M^2_n(T)}{2 T^2} \int_0^\infty \frac{d(k^w_\perp/T)}{(2\pi)^2} \frac{k^w_\perp/T}{\Delta L_{nm}(|k^w_\perp|)} \times \left\{ \ln \gamma_\omega + \ln(\sqrt{(k^w_\perp)^2 + M^2_m(T)/T}) + \gamma_E + \ln 2 - 2 \gamma_\omega \sqrt{(k^w_\perp)^2 + M^2_m(T)/T} \right\}
\]

\[\text{(27)}\]
where $\gamma_E$ is the Euler’s constant ($= -0.577 \ldots$).

Then, $f_L$ is the sum of 3 terms, having different dependence on the wall velocity $v_\omega$ (or $\gamma_\omega$);

$$f_L/T^3 \approx (A \ln \gamma_\omega + B - C\gamma_\omega) \times J,$$

(28)

where the constants $A, B$ and $C$ depend on the model and the phase transition temperature $T$.

As an example, we take the 2 generation model, where only two lighter neutrinos $\nu_1$ and $\nu_2$ are assumed to contribute to the lepton number generation. Using the parameters of $m_1 = M_1(T) = 100\text{GeV}$, $m_2 = M_2(T) = 50\text{GeV}$ and $T = 100\text{GeV}$, we have

$$A = 0.625 \times 10^{-3}$$
$$B = 0.110 \times 10^{-2}$$
$$C = 0.185 \times 10^{-2}.$$

(29)

The same result is obtained in case of doubling the energy scale as $m_1 = M_1(T) = 200\text{GeV}$, $m_2 = M_2(T) = 100\text{GeV}$ and $T = 200\text{GeV}$. Difficulty existing in the calculation of (27) is the integration over an infinite tower of small ”resonances”, so that the obtained values are preliminary. Using the thin wall approximation, there is no such trouble. Final lepton number production rate depends on $J$, the magnitude of the $CP$ violation.

\section*{B Temporal Development of the Phase Transition Dynamics}

Next theme which we are going to study in this paper is the phase transition dynamics. During the development of the EW phase transition, the bubbles of
the broken phase are nucleated at a rate of \( I \) per unit time and unit volume. They grow and fuse with each other. Finally the whole space is covered by the broken phase. The lepton number production occurs at the interfaces between the broken and unbroken phases, or the bubble walls. Therefore we need to know the temporal change of the total area \( A \) of these bubble walls, that is \( A = A(t) \).

If the wall velocity \( v_\omega \) is constant at any situation, then the total number of the lepton number production can be obtained without the knowledge of \( A(t) \):

\[
N_L = \int v_\omega^{-1} f_L(v_\omega) \cdot v_\omega A(t) dt = v_\omega^{-1} f_L(v_\omega)V_{total},
\]

(30)
giving the lepton number density \( n_L \) of the universe as

\[
n_L^{(0)} = v_\omega^{-1} f_L(v_\omega).
\]

(31)

The wall velocity \( v_\omega \) is, however, by no means a constant, but is time-dependent \( t \):

\[
v_\omega(t) = \frac{dR(t)}{dt} = 2\Gamma \left( \frac{1}{R_c} - \frac{1}{R(t)} \right)
\]

(32)

where \( R(t) \) is the radius of the bubble nucleated at time \( t \), and \( R_c \) is the critical radius with which the bubble is nucleated. The \( \Gamma^{-1} \) is the friction coefficient which the global bubble feels when it grows in the heat bath;

\[
\frac{d\phi}{dt} = -\Gamma \frac{\delta F[\phi]}{\delta \phi},
\]

(33)

where \( F[\phi] \) is the free energy of the Higgs field \( \phi \) making the bubble. Therefore \( v_\omega(t) \) increases exponentially \( \exp(2\Gamma T/R_c^2) \) and approaches to the constant velocity of \( v_\omega(\infty) = 2\Gamma/R_c \). Then the typical length scale and time scale in this problem are \( R_c \) and \( t_0 = R_c^2/2\Gamma \). The \( \Gamma \) may be estimated using the linear response theory, and giving \( O(T^{-1}) \sim O(100GeV)^{-1}) \).
The other important quantity is the nucleation rate \( I \) of the bubbles. We have the expression

\[
I = T^4 \left( \frac{F_c(T)}{2\pi T} \right)^{3/2} e^{-F_c(T)/T}
\]  

(34)

with \( F_c(T) \), the free energy of the bubble with the critical radius. Using the finite temperature effective potential estimated at 1-loop level reads

\[
V = \frac{\lambda T}{4} \phi^4 - ET\phi^3 + D(T^2 - T_0^2)\phi^2
\]  

(35)

with

\[
D = \frac{1}{4v^2} (2m_W^2 + m_Z^2 + 2m_t^2)
\]  

(36)

\[
E = \frac{1}{\sqrt{2\pi}v^3} (2m_W^3 + 3m_Z^3)
\]  

(37)

\[
T_0 \sim \frac{1}{2\sqrt{D}} m_H
\]  

(38)

and

\[
\lambda_T \sim \lambda \sim \frac{1}{2} (m_H/v)^2.
\]  

(39)

For the choice of values \( m_W = 80\) GeV, \( m_Z = 90\) GeV, \( m_t = 150(170)\) GeV, \( v = 246\) GeV and the unknown parameter \( m_H = 100\) GeV, we have \( D \sim 0.27(0.33), E \sim 0.05, T_0 \sim 100\) GeV and \( \lambda \sim 0.08 \).

Correspondingly, we have roughly

\[
\delta_\omega \sim 2\sqrt{\lambda}/ET \sim (9\text{GeV})^{-1}
\]  

(40)

\[
R_c \sim 2(DT)^3/\lambda^{3/2} \cdot \epsilon
\]  

(41)

\[
(42)
\]
with the latent heat

$$\epsilon \sim (T_c^2 - T^2)\phi_c^2 D$$

(43)

where $T_c$ is the temperature below which the broken phase appears with $\langle \phi \rangle = \phi_c$. During the temperature $T_c > T > T_0$, the EW phase transition develops. The free energy of the critical bubble

$$F_c(T) \sim \frac{2\sqrt{2\pi}E^5T^5}{3^{7/2}D^2(T_c^2 - T^2)}$$

(44)

gives the nucleation rate $I$ as a function of $x = T_c - T$; Fig 1 depicts the function $I = I(x)$, where the vertical scale is normalized by $R_{ct_0}^2$ and $x$ is in the unit of $GeV$. It is a problem of determining a fixed value of the temperature at which the first order phase transition develops; the problem should be answered by coupling the phase transition dynamics with the expansion of the universe. Here we try two typical value $T_{c1} = 100GeV$ and $T_{c2} = 200GeV$ for $T_c$. For these values, we have fixed the temperature at $T_1 = T_{c1} - 0.90GeV$ and $T_2 = T_{c2} - 0.29GeV$, respectively, during the phase transition.

We performed the computer simulation at $T_1$ or $T_2$. In the simulations, we generated the critical bubbles at the rate of $I(T_1)$ or $I(T_2)$, these bubbles grow by changing their wall velocities according to (32), and the fusion effect of the bubbles is taken into account.

The results of these simulation are given in Fig. 2 and 3. The Fig 2 and 3 give the temporal evolution of the area of the wall $A(t)$ and the volume $V(t)$ of the broken phase, respectively. The value $T_1$ and $T_2$ are so chosen that the both simulations of $A(t)$ and $V(t)$ at $T_1$ and $T_2$ become identical.

There is the exactly solvable model of Kolmogorov and Avrami [1], in which the $v_\omega$ and $I$ are kept to be constant. This extremely attractive theory predicts for $D = 3$ as

$$A(t)/V_{total} = \frac{4\pi}{3}v_\omega^3 I_0 t^3 e^{-\frac{4}{3}v_\omega^2 I_0 t^4}.$$  

(45)
Our simulation differs considerably from the simple theory of the phase transition dynamics \([13]\). In our simulation, each portion \(i\) of the bubble walls gives different time-development, so that the finally produced lepton number density \(n_L\) of our simulation becomes

\[
n_L = \sum_i \int f_L(v_i^\omega(t)A^i(t))dt/V_{\text{total}}. \tag{46}
\]

Then, we have

\[
\frac{n_L}{T^3} = \begin{cases} 
-0.299 \times 10^{-2} \cdot J & \text{for the case 1} \\
-0.303 \times 10^{-2} \cdot J & \text{for the case 2}
\end{cases}, \tag{47}
\]

where the parameters used in the case 1 are \(T_{c1} = 100\text{GeV}, T_1 = T_{c1} - 0.90\text{GeV}, m_1 = M_1(T) = 100\text{GeV}, m_2 = M_2(T) = 50\text{GeV},\) and \(v_\omega(\infty) = 0.482,\) whereas those used in the case 2 are \(T_{c1} = 200\text{GeV}, T_2 = T_{c2} - 0.29\text{GeV} , m_1 = M_1(T) = 200\text{GeV} , m_2 = M_2(T) = 100\text{GeV} ,\) and \(v_\omega(\infty) = 0.311.\)

The corresponding lepton number densities \(n_L^0\) obtained in the Kolmogorov-Avrami model of the phase transition, read

\[
n_L^0/T^3 = \begin{cases} 
-0.108 \times 10^{-2} \cdot J & \text{for the case 1} \\
-0.209 \times 10^{-2} \cdot J & \text{for the case 2}
\end{cases}. \tag{48}
\]

Therefore, we have

\[
\frac{n_L}{n_L^0} = \begin{cases} 
2.769 & \text{for the case 1} \\
1.450 & \text{for the case 2}
\end{cases}, \tag{49}
\]

which is the most important results of our paper; The phase transition dynamics with and without including the change of the wall velocity are found to affect considerably the final value of the lepton number (as well as the baryon number) production.

The detailed analysis will be given elsewhere \([4]\).
C Baryogenesis from Leptogenesis

One of the difficulty in considering the baryogenesis at EW scale is the constraint of $m_H \leq 45\text{GeV}$ which guarantees that the sphaleron transition is suppressed by the expansion of the universe and that the produced baryon number may not be washed away by the spharelon.

In our case the sphaleron transition should be rapid enough to make the chemical equilibrium between the lepton number $L$ and the baryon number $B$. Thanks to the $B - L$ conservation, we have the non-vanishing equilibrium value for $B$.

$$\langle B \rangle = \frac{1}{2}(1 + x)\langle B - L \rangle$$

$$= -\frac{1}{2}(1 + x)\langle \text{produced } L \rangle,$$

(50)

where $\langle J_3 \rangle = \langle Y \rangle = 0$ is assumed with $x = O(1); x = -18/15$ for the light Higgs doublet $H$ and the light singlet scaler $\phi$ with three fermions of quarks and leptons and, $x = -1/2$ without this $\phi$. Anyway we have $n_B$ at the same order of magnitude of the produced $n_L$ during the EW phase transition, without the unrealistic constraint of $m_H \leq 45\text{GeV}$. This is the good point of this baryogenesis model induced by the neutrinos. To obtain a realistic value for $n_L/n_\gamma$, we need to have CP violation $J$ of $O(10^{-5} \sim 10^{-7})$.

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Figure 1: nucleation rate $I$ as a function of $x = T_c - T$ [GeV]. The unit of the vertical axis is $V = T^4$. 
Figure 2: Temporal development of the total area of the wall for our simulation and for the Kolmgorov-Avrami theory (solid line: our simulation, dotted line: Kolmgorov-Avrami)
Figure 3: Temporal development of the volume fraction of the broken phase for our simulation and for the Kolmogorov-Avrami theory (solid line: simulation, dotted line: Kolmogorov-Avrami)
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