THEORETICAL BLACK HOLE MASS DISTRIBUTIONS

CHRIS L. FRYER
Lick Observatory, University of California Observatories, Santa Cruz, CA 95064; cfryer@ucolick.org

AND

VASSILIKI KALOGERA
Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138; vkalogera@cfa.harvard.edu

Received 1999 November 12; accepted 2001 February 1

ABSTRACT

We derive the theoretical distribution function of black hole masses by studying the formation processes of black holes. We use the results of recent two-dimensional simulations of stellar core collapse to obtain the relation between remnant and progenitor masses and fold it with an initial mass function for the progenitors. Thus, we are able to derive the binary black hole mass distribution. We examine how the calculated black hole mass distributions are modified by (1) strong-wind mass loss at different evolutionary stages of the progenitors and (2) the presence of close binary companions to the black hole progenitors. The compact-remnant distribution is dominated by neutron stars in the mass range 1.2–1.6 $M_\odot$ and falls off exponentially at higher remnant masses. Our results are most sensitive to mass loss from stellar winds (particularly from Wolf-Rayet stars), and the effects of winds are even more important in close binaries. Wind mass loss leads to flatter black hole mass distributions and limits the maximum possible black hole mass ($\lesssim 10–15 M_\odot$). We also study the effects of the uncertainties in the explosion and unbinding energies for different progenitors. The distributions are continuous and extend over a broad range. We find no evidence for a gap at low values (3–5 $M_\odot$) or for a peak at higher values ($\sim 7 M_\odot$) of black hole masses, but we argue that our black hole mass distribution for binaries is consistent with the current sample of measured black hole masses in X-ray transients. We discuss possible biases against the detection or formation of X-ray transients with low-mass black holes. We also comment on the possibility of black hole kicks and their effect on binaries.

Subject headings: binaries: general — black hole physics — stars: evolution — stars: mass loss — stars: neutron — supernovae: general

1. INTRODUCTION

Soft X-ray transients provide a window into the nature of compact objects, in particular that of stellar-mass black holes. During the quiescent state of emission (low X-ray luminosity), optical/infrared observations of the compact object’s companion allow the measurement of the mass function and other binary parameters. In the case of small companion masses (late spectral type), the value of the mass function, $f(M_X)$, sets a strong lower limit to the mass of the compact object:

$$M_X = f(M_X)(1 + q)^2/\sin^3 i > f(M_X),$$

where $q \equiv M_*/M_X$ is the mass ratio and $i$ is the inclination of the binary orbit. For several systems, this lower limit exceeds 3 $M_\odot$, the optimum (independent of the equation of state at densities exceeding nuclear densities) upper limit to the gravitational mass of a neutron star (NS; Rhoades & Ruffini 1974; Kalogera & Baym 1996), strongly suggesting that these compact objects are black holes (for a recent review, see Charles 1998). Indeed, at present, there is little doubt that stellar-mass black holes (BHs) exist and that they play an important role in high-energy astrophysics (e.g., X-ray binaries and gamma-ray bursts).

Currently there are 10 X-ray transients thought to contain a BH accreting matter from a low-mass companion. They are listed in Table 1, along with the range of BH mass estimates reported in the literature. Most of the data have been taken from Bailyn et al. (1998), who presented a detailed statistical study of the BH mass measurements available at the time and the associated errors (dominated by uncertainties in the inclination angle). Recently, revised measurements of the mass function for GRO J1655–40 (Phillips, Shahbaz, & Podsiadlowski 1999) and GRO J0422+32 (Harlaftis et al. 1999) have led to a decrease in the lower mass limits. In Table 1 we also include three new systems. GRS 1009–45 (Filippenko et al. 1999) and 4U 1543–47 (Orosz et al. 1998) are included, although their mass estimates are still rather uncertain. Since the submission of this paper, an additional black hole low-mass X-ray binary has been discovered: XTE J1118+480 (McClintock et al. 2001).

Bailyn et al. (1998) studied the statistical properties of the early sample of measured BH masses (excluding 4U 1543–47, GRS 1009–45, and the revised masses for GS 1124–68 and GRO J0422+32) and concluded that almost all of the estimates are consistent with a narrow range of BH masses around 7 $M_\odot$ and that a gap between 3 and 5 $M_\odot$ is present in the distribution. In this paper we calculate the expected BH mass distribution and its dependence on a number of physical elements (stellar winds and binary evolution) based on the current theoretical understanding of BH formation.

We consider BH formation during the collapse of massive stars. This process can proceed in two different ways: either the massive star collapses directly into a BH without a supernova explosion or an explosion occurs, but its energy is too low to completely unbind the stellar envelope, and a significant part of the star falls back onto the protoneutron star, forming a BH (Colgate 1971; Woosley 1988; Fryer 1999). An alternative path, involving the collapse of an NS into a BH through hypercritical accretion (e.g., in a
TABLE 1
MEASURED BLACK HOLE MASSES

| System                  | Mass Range (M_☉) |
|-------------------------|------------------|
| GS 2023+34 (Nova Cyg 1938/1989) (V404 Cyg) | 10.3–14.2\(^b\) |
| GS 2000+25 (Nova Vul 1988)                | 5.84–18.0        |
| H1705+25 (Nova Oph 1977)                   | 4.67–8.00        |
| GRO J1655–40 (Nova Sco 1994)               | 4.1–7.9\(^c\)    |
| GS 1124–68 (Nova Mus 1991)                 | 4.56–8.17        |
| A0620–00 (Nova Mon 1975) (V616 Mon)        | 3.78–25.4        |
| GRO J0422+32 (Nova Per 1992)               | 3.16–16.9\(^d\) |
| GRS 1009–45 (Nova Vel 1993) (V1234 Oph)    | ~3.64–4.74       |
| 4U 1543–47                            | ~2.7–7.5\(^e\)  |
| XTE J1118+480                        | ~7.0–13\(^f\)   |

\(^a\) We do not include the high-mass X-ray binaries (Cyg X-1, LMC X-1, LMC X-3), whose masses are much less certain than those obtained in X-ray transients.

\(^b\) Unless otherwise stated, data are taken from Bailyn et al. 1998.

\(^c\) The lower limit for the black hole mass comes from the analysis of Phillips et al. 1999. The upper limit comes from the revised mass function of Shahbaz et al. 1999.

\(^d\) We have included the revised mass ratio and mass function from Harlaftis et al. 1999. Note that Beekman et al. 1997 have argued that the BH mass in this system may be much larger than our upper limit (~28 M_☉). They argue that the inclination angle is much lower than past estimates.

\(^e\) These system are not yet studied in detail (Filippenko et al. 1999; Orosz et al. 1998; McClintock et al. 2001), and the error bars may be revised in the future.

common envelope or in binary mergers), has often been discussed in recent years (see Fryer, Woosley, & Hartmann 1999 for a review). However, even under the assumption that BH formation occurs every time an NS goes through a common envelope phase or a merger, the event rates for NS accretion-induced collapse are lower than those for massive star collapse events by factors of 10–100 (Fryer et al. 1999). Therefore, we assume that BH formation is dominated by stellar collapse.

In this paper we use the results of up-to-date two-dimensional hydrodynamic simulations of core collapse (Fryer 1999) for explosion energies and remnant masses to obtain theoretically expected BH mass distributions. In §2, we outline the steps involved in constructing such distributions and discuss their associated uncertainties. We present our results in §3 and conclude that the BH mass distribution and particularly its slope and upper cutoff are most sensitive to the effects of stellar winds in binaries. No evidence for a gap is found at low BH masses. In the last section we compare our results to observations and discuss some possible biases and uncertainties in the observed sample.

2. BLACK HOLE MASS DISTRIBUTIONS

2.1. Outline of the Calculation

The calculation of theoretically expected BH mass distributions involves a sequence of steps. We derive the remnant mass as a function of progenitor mass\(^1\) based on an energy budget argument regarding the necessary and available energies to unbind the stellar envelope. We scale the available energy to the explosion energy calculated from core-collapse simulations. We then convolve the resulting remnant–progenitor mass relation with the initial mass function of the progenitors to obtain the BH mass distribution. Each of these steps, and hence the results, can be affected quantitatively by the evolution of the progenitor prior to collapse. Before we present our results and the associated uncertainties, we describe in detail the steps of our calculation.

2.1.1. Explosion Energy

To estimate the energy available to unbind the stellar envelope and its dependence on progenitor properties (mass and density structure), we are guided by recent two-dimensional core-collapse simulations (Fryer 1999), which provide us with the explosion energy as a function of the progenitor mass. This explosion energy is determined by the change in total energy before collapse and at the end of the ~1 s simulation of the material that is initially part of the exploding shock (for most simulations, this is the entire star beyond the 1.2 M_☉ protoneutron star). Mass loss (from winds or binary mass transfer) will not change the explosion energies unless the mass loss affects the inner ~3 M_☉ of the stellar core. Hence, the explosion energies as a function of progenitor mass from Fryer (1999) are valid for both helium and hydrogen stars. For stars in which mass loss has affected the core, we use the masses of their C/O cores at collapse calculated by Langer & Henkel (1995) to estimate the explosion energy. We find that mass loss affects the cores of stars only for a small fraction of the BH progenitors (more massive than ~40 M_☉). For example, a 40 M_☉ star with winds from Langer & Henkel (1995) will have an explosion energy closer to that of a 30 M_☉ progenitor star without winds. Such cases are discussed in more detail in §2.2.3.

Knowing the explosion energy from core-collapse models is not sufficient to estimate the final compact-remnant mass. We also need to know what fraction of the explosion energy goes into unbinding the star and what fraction is transformed into kinetic energy of the ejecta. In Figure 1 we plot (dotted lines) two different examples of the energy available to unbind the star as a function of the progenitor mass. For the top curve, we assume that 100% of the collapse energy goes into unbinding the star; this curve is a fit to the results (circles) of Fryer (1999). The bottom curve is a scaled-down version of the top curve, assuming that 10% of the explosion energy is used to unbind the star, the rest being transformed into kinetic energy of the ejecta. The simulations show that above a critical progenitor mass, no supernova explosion occurs, and the entire star collapses to form a black hole. Without winds, a progenitor mass of ~40 M_☉ appears to be the critical mass for BH formation (see Fryer 1999 for more details).

2.1.2. Stellar Binding Energy

The next step is to compare the energy available to eject material to the actual binding energy of the star, obtained from integrating the stellar mass profile. For these profiles we use the models of massive stars calculated by Woosley & Weaver (1995). In Figure 1 we plot (solid line) the energy required to unbind all but the inner 3 M_☉ for each progenitor. We assume that the maximum NS mass is 3 M_☉, and we thus calculate an upper limit to the critical initial pro-

\(^1\) Since BH progenitors can lose mass as they evolve, because of winds and binary mass transfer, the term “progenitor mass” is not uniquely defined. To avoid confusion, we use progenitor mass to refer to the initial mass of the BH progenitor unless we explicitly state otherwise (e.g., progenitor mass at collapse).
Hydrogen-rich progenitor mass dividing NS from BH formation through fallback. From Figure 1 it is evident that the critical masses are \( \sim 18 \, M_\odot \) and \( \sim 23 \, M_\odot \) for 10% and 100% of the collapse energy being available to unbind the star, respectively. If the maximum NS mass is lower, as most equations of state in the literature predict (see, e.g., Cook, Shapiro, & Teukolsky 1994), the binding energies increase and the critical progenitor masses decrease. The hydrogen envelope accounts for less than \( \sim 10\% \) of this total binding energy budget. Hence, if a star loses its hydrogen envelope before collapse, but retains its helium core, its binding energy will not change by more than 10%.

2.1.3. Remnant Mass

We can combine the estimated energies available to unbind the star and the stellar binding-energy profile (binding energy as a function of mass within the star) to calculate the remnant mass, \( M_{\text{rem}} \), as a function of progenitor mass, \( M_{\text{prog}} \):

\[
f \frac{E_{\text{explosion}}}{M_{\text{rem}}} = \int_{M_{\text{rem}}}^{M_{\text{prog}}} E(m) \, dm,
\]

where \( f \) is the fraction of the total explosion energy used to unbind the outer layers of the star (down to \( M_{\text{rem}} \)), and \( E(m) \) is the binding-energy profile. In this way, we can calculate the final remnant mass (and the amount of fallback) for a given progenitor mass. To calculate \( E(m) \), we use Woosley & Weaver (1995) massive-star models. The remnant masses as a function of progenitor mass are shown in Figure 2 for \( f = 0.1 \) and \( f = 1 \). For very low explosion energies (high masses), most of the star falls back to form a black hole, and the black hole mass is limited by the progenitor mass at collapse. In such cases, even if mass loss does not affect the explosion energy, it dictates the BH mass by limiting the total mass available to fall back and form a black hole (§2.2).

2.1.4. Initial Mass Function of Progenitors

The final step in the calculation of a theoretical BH mass distribution requires the use of an initial mass function (IMF) for the massive BH progenitors, which we transform into a remnant mass function using the derived remnant–progenitor mass relation (Fig. 2),

\[
F(M_{\text{rem}}) = F(M_{\text{prog}}) \frac{dM_{\text{rem}}}{dM_{\text{prog}}}^{-1}.
\]

There is observational evidence that the IMF in the mass ranges of interest here is very well described by a single power law, \( F(M_{\text{prog}}) \propto M_{\text{prog}}^{-\gamma} \) (for a Scalo IMF, \( \gamma = 2.7 \)). Recent studies of massive stars claim a range of values for \( \gamma \) from 1.8 to 3.1 (see Massey & Thompson 1991), with no evidence for a dependence on metallicity (for massive stars). Because of this wide spread, we investigate a range of values for \( \gamma \) between 2 and 3 (Fig. 3), while for most of the calculations we adopt \( \gamma = 2.7 \). The total number of BHs is quite sensitive to \( \gamma \) (to the critical mass dividing NS and BH formation as well). Changing its value from 2 to 3 lowers the fraction of BHs among core-collapse remnants from 30% to 12% (see rows 1 and 2 of Table 2).

\[\text{Note, however, that zero-metallicity (Population III) stars are generally believed to have a different IMF than stars with metals (see, e.g., Abel, Bryan, & Norman 2000).}\]
2.2. Results and Uncertainties

Some basic distributions of BH masses produced in core-collapse events are shown in Figures 1, 2, and 3. But how robust are these results? There are many uncertainties in the above calculation that may change our results significantly. Before we can make any firm predictions, we must study each of these uncertainties to determine how much they affect the theoretical BH mass distributions. In what follows we examine each uncertainty at its extreme to obtain a conservative view of our errors. Our goal is to identify the basic features of the mass distributions that are most robust. The range of results is summarized in Table 2.

2.2.1. Supernova Energies and Remnant Masses

The accuracy of the determined explosion energies depends on the current understanding of the supernova mechanism. Progress in this field indicates that the mechanism is sensitive to detailed physical processes and ingredients, such as the equation of state, neutrino cross sections and transport, general relativity, and multidimensional effects (see Burrows 1998 and Mezzacappa & Bruenn 1999 for reviews). Until these are worked out, no “final” answer to the question of the explosion energy as a function of progenitor mass can be given. In fact, it is likely that for a given progenitor star mass there is a range of possible supernova energies, depending on the rotation of the progenitor (Fryer & Heger 2000). It is important to note, however, that the trend—an increasing core-collapse energy up to some progenitor mass (\( \sim 15 M_{\odot} \)), followed by a decrease for more massive progenitors—is not expected to change. To assess the effect of such uncertainties, we examine our results for various slopes of this rise and decline of explosion energy with progenitor mass (Fig. 4). Note that only the rate of decline is important for determining BH mass distributions.

The core-collapse simulations we use to derive explosion energies follow just the first \( \sim 1 \) s of the explosion (Fryer 1999). In these models, the outward-moving shock is still within the oxygen layer of the star at the end of the simulation. As the shock moves out of the star, a fraction of its energy is transformed into kinetic energy of the ejecta, and another fraction goes into unbinding the core. Neutrino emission or absorption and nuclear burning and dissociation also contribute to the total energy budget, but they represent only a minor fraction of it at 1 s after bounce. In determining remnant masses, we only need an estimate of the energy fraction, \( f \), used to unbind the star. We treat this

![Diagram](image-url)
fraction as a parameter in our calculations in the range 0.1–1.0. Recent fallback models by MacFadyen, Woosley, & Heger (2000) suggest that \( f \) lies in the range 0.3–0.5. It is evident that by varying \( f \), we are able to examine the effect of the explosion energy uncertainties for a given dependence on the progenitor mass (i.e., shape of rise and decline).

The binding energy of stars at collapse is also difficult to determine. In the rotating stellar models by Heger, Langer, & Woosley (2000), the binding energies (for matter outside the inner 3 \( M_\odot \)) are nearly a factor of 2 lower than those in the Woosley & Weaver (1995) nonrotating models. The treatment of convection also introduces uncertainties in the binding energy. Currently, we are limited to the grid of precollapse stars produced by Woosley & Weaver (1995) and by the uncertainties in their assumptions for convection, rotation, and other properties. Lowering the binding energies of all stars by a factor of 2 is equivalent to raising \( f \) by a factor of 2. In this manner, varying \( f \) provides us with a way of exploring the uncertainties caused by varying the magnitude of the binding energies.

Variations of the parameter \( f \) can represent variations in the explosion energy, in the fraction of it used to unbind the envelope, or in the binding energy itself. Varying \( f \) within the range 0.1–1.0 changes the total number of BH remnants by factors smaller than 2 (Figs. 1, 2, and 3 and rows 1–10 of Table 2). In most of our models we adopt \( f = 0.5 \), in accord with the results of MacFadyen et al. (2000). Concerning the dependence of the explosion energy on the progenitor mass, we consider two cases in addition to our standard case based on the results of Fryer (1999; see Fig. 1, top curve): fast (Fig. 4, dotted line) and slow (Fig. 4, dashed line) rise and decline. The total number of black holes is not dramatically sensitive to the choice of this slope (see, e.g., rows 5, 11, and 12 of Table 2). However, the case of fast decline produces many more black holes with masses in

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**TABLE 2**

Black Hole Masses

| Row | Model Parameters | Percentage of BH Compact Remnant \( \equiv 100\% N_{\text{BH}}(N_{\text{BH}} + N_{\text{NS}}) \) |
|-----|-----------------|--------------------------------------------------|
|     | Energy\(^a\) | Winds\(^b\) | IMF\(^c\) | 3–5 \( M_\odot \) | 5–10 \( M_\odot \) | 10–15 \( M_\odot \) | >15 \( M_\odot \) |
| 1    | SD, 100%       | ... 2.0 | 29.9 | 5.0 | 8.2 | 4.7 | 12 |
| 2    | ... 3.0 | 12.1 | 3.1 | 4.1 | 1.9 | 3.0 |
| 3    | SD, 10%        | ... 2.0 | 42.1 | 8.1 | 12.2 | 6.8 | 15.0 |
| 4    | ... 3.0 | 21.3 | 6.5 | 7.6 | 3.1 | 4.1 |
| 5    | SD, 50%        | ... 2.7 | 19.1 | 4.6 | 6.3 | 3.1 | 5.1 |
| 6    | B 2.7 | 18.5 | 3.6 | 6.4 | 3.4 | 5.1 |
| 7    | W 2.7 | 16.7 | 4.3 | 9.5 | 2.9 | 3.0 |
| 8    | B + W\(^+\) 2.7 | 0.088 | 0.043 | 0.045 | 0 | 0 |
| 9    | He\(^+\) 2.7 | 0.20 | 0.048 | 0.13 | 0.024 | 0 |
| 10   | B + W 2.7 | 16.7 | 4.3 | 9.5 | 2.9 | 0 |
| 11   | Case B 2.7 | 18.5 | 3.6 | 10.4 | 2.9 | 1.5 |
| 12   | Slow, 50%      | ... 2.7 | 14.2 | 2.4 | 4.0 | 2.4 | 5.4 |
| 13   | Fast, 50%      | ... 2.7 | 18.4 | 2.7 | 2.7 | 0.30 | 10.7 |
| 14   | Slow, 50%      | B 2.7 | 14.2 | 2.4 | 4.0 | 2.6 | 5.2 |
| 15   | Fast, 50%      | B 2.7 | 18.2 | 2.8 | 3.6 | 4.6 | 2.6 |
| 16   | Slow, 50%      | W 2.7 | 12.4 | 3.1 | 7.1 | 2.2 | 0 |
| 17   | Fast, 50%      | W 2.7 | 16.5 | 3.4 | 7.6 | 2.7 | 2.8 |
| 18   | Slow, 50%      | B + W 2.7 | 12.3 | 3.2 | 9.1 | 0.0 | 0 |
| 19   | Fast, 50%      | B + W 2.7 | 16.4 | 3.6 | 11.3 | 1.5 | 0 |

\(^a\) Explosion Energy. SD \equiv standard rise/fall slope; see Fig. 1. Slow, Fast \equiv slow, fast rise/fall slope; see Fig. 4. 10%, 50%, 100%; Fraction of explosion energy available to eject material; see Fig. 1.

\(^b\) Winds, Binary Effects. B \equiv binary effects only (all binaries undergo case C mass transfer). W \equiv wind effects only. B+W\(^+\) \equiv winds plus binary effects (stellar radii from literature; most binaries undergo case B mass transfer); He\(^+\) is the same as B+W\(^+\), but assumes that the helium cores of all stars are 40% more massive than predicted by Woosley & Weaver 1995. B+W \equiv strong winds plus binary effects (all binaries undergo case C mass transfer). Case B \equiv weak winds plus binary effects (almost all binaries undergo case B mass transfer); see text.

\(^c\) Initial Mass Function power-law \( y \); see text.
excess of $10 \, M_\odot$ (Table 2; Figs. 5 and 6). For this simple case, the critical progenitor mass separating fallback and prompt BH formation decreases to $\sim 28 \, M_\odot$ (Fig. 5) from $\sim 40 \, M_\odot$ for our standard case. We note that once we include the effects of stellar winds and close binary evolution (see §§ 2.2.2 and 2.2.3), the differences caused by varying the dependence of supernova energy on progenitor mass drop to the 20%–30% level for the fraction of BH remnants in most mass ranges (Table 2).

We ignore any mass loss that might occur after black hole formation in a gamma-ray burst or similar explosion (MacFadyen & Woosley 1999) because, at this time, we are unable to determine how often (if at all) such explosions occur and do not know the amount of mass ejected even if an explosion occurs.

2.2.2. Wind Mass Loss from Single Stars

The physical processes responsible for wind mass loss from hydrogen-rich massive stars and from Wolf-Rayet stars are not well understood and may be different for these two phases of mass loss. To study the effects of mass loss on the derived BH mass distributions, we use the stellar models of Langer & Henkel (1995) and Schaller et al. (1992), who adopt different prescriptions for mass loss from hydrogen-rich and Wolf-Rayet stars.

Single stars less massive than $\sim 10 \, M_\odot$ are not affected by winds. Stars with masses up to $\sim 35 \, M_\odot$ lose a significant fraction of their hydrogen-rich envelope, most of it during the phase of core helium burning, but their inner cores ($M \leq 3 \, M_\odot$) remain largely unaffected by mass loss. As a result, the explosion energies remain unaffected by the winds and, as mentioned above, the partial or complete loss of the hydrogen-rich envelope does not have any significant effect on the stellar binding energy at the time of the explosion (less than 10%). Hence, the distribution of remnant masses from stars initially less massive than $\sim 35 \, M_\odot$ is not sensitive to winds from single stars. More massive BH progenitors, however, undergo such a dramatic decrease in their mass that they lose their entire hydrogen-rich envelope before they reach collapse. Further mass loss occurs during the strong Wolf-Rayet phase of the exposed helium cores. This strong mass loss affects the remnants of massive stars in two ways. First and foremost, the remnant mass cannot exceed the mass of the star just prior to collapse. We implement this constraint by limiting the remnant mass to the final stellar masses taken from Langer & Henkel (1995).

On a more detailed level, for massive BH progenitors ($\gtrsim 35 \, M_\odot$), stellar winds also alter the structure of the collapsing star, and this changes both the explosion energy and binding energy of the star. We account for the variation in supernova energy by assuming that the supernova energy is proportional to the C/O core mass (which we determine from the simulations of Langer & Henkel 1995). We assume that the binding energy of the outer layers of the star is similar to that of helium stars of lower mass. However, these corrections change the remnant mass by less than 10%–20%, and the primary effect of winds is simply to limit the remnant mass to the mass of the collapsing star. Combining all effects, we find that the remnant mass decreases dramatically as the progenitor mass increases beyond 35 $M_\odot$ (Fig. 7, dotted line). Understandably, these results depend sensitively upon the mass-loss rates from winds. However, such massive progenitors are so rare that they do not affect the black hole mass distribution significantly. The primary qualitative effect of single-star winds on the BH mass distribution is to impose an upper limit on the remnant mass and to flatten the slope of the BH mass distribution (Figs. 7 and 8). The value of the maximum BH mass depends on the progenitor mass above which winds can expose the helium core of the massive star. For a value of this critical progenitor mass of $35 \, M_\odot$, the maximum BH mass turns out to be $\sim 10 \, M_\odot$. As is clear from Figure 7, a lower critical progenitor mass would lead to a lower maximum BH mass. We also note that for massive stars of lower metallicity, the wind loss rates are lower and therefore the maximum BH mass is higher.

2.2.3. Black Hole Progenitors in Binaries

So far, our discussion has been limited to single-star evolution. However, stellar-mass black holes are discovered only in binaries. The short orbital periods of BH X-ray binaries (less than 1 day to a few days; see Charles 1998) imply that their formation involves a common envelope (CE) phase (Kalogera 1999). Therefore, to compare with the observations we must consider the effect of close binaries on the BH mass distribution. A common envelope phase can occur when the BH progenitor star expands and transfers mass onto its companion. In many cases, the companion is engulfed by the BH progenitor and it spirals into the hydrogen envelope of the massive star, ejecting the hydrogen layers as it spirals inward. Hence, the most important consequence of binary evolution is uncovering the helium cores of stars less massive than $\sim 35–40 \, M_\odot$. These cores can then be affected by mass loss in a Wolf-Rayet phase, unlike their single counterparts. In what follows we examine the effect of the hydrogen envelope loss on the BH mass distribution. As we will show in detail, the importance of binary effects on the BH mass distribution depends upon

**Fig. 5.—**Remnant mass as a function of progenitor mass, assuming that 50% of the explosion energy from Figs. 1 and 4 is used to unbind the star. Three different cases for the slopes of rise and decline of the explosion energy are shown: standard (solid line), fast (dotted line), and slow (dashed line).
three main factors: (1) when the common envelope phase begins with respect to the evolution of the massive star, (2) the radial evolution of highly evolved massive stars, and (3) the strength of Wolf-Rayet winds.

In the absence of stellar winds.—Binary evolution causes the hydrogen layers of the BH progenitor to be ejected during common envelope evolution, setting the mass of the star at collapse (and hence the maximum possible BH mass) equal to the mass of its helium core. However, this loss of the hydrogen envelope does not affect the inner core dramatically, and both the explosion and binding energies of these binary BH progenitors are roughly the same as the values for single stars. The core-collapse simulations of single stars less massive than $M_{\odot}$ show that some fraction of the helium core is ejected (25% for a 25 $M_{\odot}$ star, 5% for a 40 $M_{\odot}$ star). The binding energy of the ejected helium core is much greater than the binding energy of the entire hydrogen envelope, and the presence of the hydrogen envelope is not important in determining the remnant mass. For binaries that undergo mass transfer after helium ignition (case C), the remnant mass is identical to the remnants of single stars for all stars less massive than $\sim 42 M_{\odot}$ (Fig. 7, dashed line). For stars more massive than $\sim 42 M_{\odot}$, the entire star collapses to form a black hole, and the size of the hydrogen envelope (and hence binary effects) can be very important. However, these massive stars are so rare that they do not affect the remnant mass distribution significantly (Fig. 8; Table 2). If the mass transfer occurs earlier, the helium core can be as much as 10%–20% smaller and, in the most extreme circumstances (progenitors of initial mass between 35 and 40 $M_{\odot}$), the resultant remnant masses will be lower by 10%–20%.

Our conclusion—that in the absence of helium-star winds, the remnant masses do not depend on the presence or absence of a hydrogen envelope—seems to contradict what has been stated in the past, for example by Woosley (1988), Herant & Woosley (1994), and Fryer (1999). In these studies it was assumed that fallback was driven by the deceleration of the explosion shock as it enters the hydrogen envelope. The most recent simulations of fallback (MacFadyen et al. 2000), indicate that the hydrogen envelope does not influence the fallback process.

In the presence of stellar winds.—Because binary interactions eject the hydrogen envelopes, winds off hydrogen envelopes (and their uncertainties) do not affect the remnant masses of close binary systems. However, because binaries uncover the helium cores of even moderate-mass BH progenitors, the effects of Wolf-Rayet winds are even more
prominent in binaries. As we discussed in §2.2.2, Wolf-Rayet winds can significantly reduce remnant masses, but for single stars these winds only affect very massive stars (and very massive black hole remnants). In binaries, even stars less massive than 35 $M_\odot$ undergo a Wolf-Rayet wind phase, and this can affect the entire black hole mass distribution.

The amount of mass lost depends on the timing of the common envelope phase. If it occurs before helium ignition (as the star expands off the main sequence), the exposed helium star will lose mass during its entire helium-burning phase. If mass transfer does not occur before helium ignition, it will not occur until after helium exhaustion, because the star does not expand during helium burning. This is a consequence of the small radial expansion relative to the simultaneous orbital expansion caused by an aggravated wind mass loss during core helium burning. These two rates of expansion start becoming comparable only for very low metalicities ($Z = 0.002$) and are not expected to be relevant to the Galactic disk population of observed BH X-ray transients (for more details, see Kalogera & Webbink 1998).

After helium exhaustion, the star evolves so quickly that even the strong Wolf-Rayet winds currently assumed in the stellar models do not effect the mass significantly (Woosley, Langer, & Weaver 1995). We therefore have to consider separately BH progenitors that lose their hydrogen envelopes as a result of CE evolution before (case B mass transfer) or after (case C mass transfer) core helium ignition.

With the current Wolf-Rayet mass-loss rates and models for helium star evolution, BH progenitors that undergo case B mass transfer will not form black holes. In this case, the exposed helium stars lose so much mass during the core helium burning that the progenitor’s mass at collapse has decreased to $\sim 3-4 \, M_\odot$ (Woosley & Weaver 1995; Wellstein & Langer 1999). Such low-mass helium stars collapse to NSs, not BHs. Wolf-Rayet winds prevent all stars that undergo case B mass transfer from forming black holes.

However, because case C mass transfer occurs so late in a star’s life, mass transfer does not affect the mass loss from winds significantly (Wellstein & Langer 1999). The remnant masses of such binaries must satisfy the constraints of single-star wind models, as well as the limits placed by binary models (in the absence of winds). But for all progenitor masses, the constraint placed by single-star wind models is as great as (or greater than) binary constraints, and case C binaries with winds have the same remnant mass distribution as single stars with winds. Such binaries easily form massive black holes.

If black holes cannot be formed in any massive binaries that undergo case B mass transfer, then case C mass transfer is the only remaining option, and this leads to a drastic reduction of the BH formation rate (Portegies Zwart, Verbunt, & Ergma 1997). To avoid case B mass transfer, the orbit must be sufficiently wide to avoid mass transfer before helium ignition. But during core helium burning, winds cause the binary separation to widen even further, so the BH progenitor must expand significantly after core helium burning to undergo case C mass transfer. However, in most stellar models of massive stars (see, e.g., Schaller et al. 1992; Hurley, Pols, & Tout 2000), such expansion late in the evolution of the star occurs only for stars with initial masses below $\sim 25 \, M_\odot$ (see also Kalogera & Webbink 1998). This produces a narrow range of progenitor masses ($20-25 \, M_\odot$) from which black holes can actually form. Among binaries with primaries in the range 20–25 $M_\odot$, only a small fraction have orbits wide enough to avoid case B mass transfer but also tight enough to go through case C evolution. This additional constraint is satisfied typically for orbital separations in the range 1600–1800 $R_\odot$ (Kalogera & Webbink 1998). The fraction of BH progenitors in binaries which satisfy all of these constraints is less than 1%! (see row 8 of Table 2). In addition to this drastic reduction in the BH X-ray binary formation rate, the current stellar models face an even more serious problem: the black holes formed through this path are all less massive than $\sim 7 \, M_\odot$ (helium core mass of a 25 $M_\odot$ star). Such a lower limit certainly cannot explain V404 Cyg and may have trouble explaining several black hole X-ray binaries (see Table 1).

Clearly something is wrong with our set of assumptions. The current mass-loss rates from winds are too high, or the radial evolution of stars is incorrect (Kalogera 2001), or helium core masses are much higher than predicted by Woosley & Weaver (1995). Let us first consider the last possibility. Among models predicting high helium core masses are those published by Maeder (1992). Even these models predict a maximum helium core mass $\approx 10 \, M_\odot$ (from a 25 $M_\odot$ progenitor), which could only marginally account for the V404 Cyg BH mass. However, we can explore the effect of artificially increasing helium core masses relative to those predicted by Woosley & Weaver (1995). We find that the core masses have to be increased by about 40% in order to explain BH masses in excess of 10 $M_\odot$. Under this assumption, we find that one in about 500 supernovae will produce a BH, and one in about 4000 supernovae will produce a BH of such high mass (see Table 3).

3 Recall that mass transfer must occur to tighten the binary orbit and produce the observed BH X-ray binaries.
Fig. 8.—Mass distribution of compact remnants using the remnant–progenitor mass relation in Fig. 7 and assuming an IMF with $\gamma = 2.7$. Line types are the same as in Fig. 7.

2). However, we obtain this result under an assumption that is not physically motivated. Therefore, we explore in greater detail one of the other two cases, which are more physically motivated given the stellar models available in the literature: (1) the wind models for helium stars are correct, but hydrogen-rich stars more massive than $25 M_\odot$ undergo enough radial expansion after core helium exhaustion to evolve through case C mass transfer, so that we avoid the serious problem of the drastic reduction of the formation rate (“Case C + Winds” in Figs. 7 and 8); and (2) the models for the late evolution of mass-losing massive stars are correct but wind mass loss for helium stars is significantly weaker than assumed so far (“Case B + Weak Winds” in Figs. 7 and 8). Then helium stars exposed through case B evolution lose smaller amounts of mass during core helium burning and at collapse are massive enough to form black holes. In all likelihood, the problem arises from small errors in all of the above uncertainties. However, as we did with the helium core masses, let us take extreme cases (where we assume that either the winds or the stellar radii are correct) and determine the effect these uncertainties have on the remnant mass distribution.

If we assume that Wolf-Rayet wind models are correct and that the radial evolution of stars is different from that given by the current stellar evolution predictions, we can construct a scenario in which nearly all binaries undergo case C mass transfer only. As we discussed above, the constraint on the remnant masses of such binaries is identical to that of single-star winds. As a minimal test of such a case, we see that the resultant BH mass distribution can at least explain the masses of the observed black holes (Figs. 7 and 8, dotted lines). In particular, we are able to explain the high mass of V404 Cyg.

Alternatively, it is possible that the true mass-loss rates for helium stars are lower than those estimated by Langer (1989) and used by Woosley et al. (1995) and in most of the models of Wellstein & Langer (1999). In fact, recent empirical estimates of mass-loss rates for some Wolf-Rayet stars appear to be lower (Hamann & Koesterke 1998), but they still lead to significant mass reduction for helium stars (Wellstein & Langer 1999). To investigate the effect of even lower mass-loss rates, we take into account this trend of decreasing empirical rate estimates, and we calculate the BH mass distribution assuming that (1) 99% of close binaries go through case B evolution (and the remaining 1% go through case C) and that (2) the amount of mass lost from the helium star by the time it reaches collapse is half of that calculated by Woosley et al. (1995). The latter assump-
tion corresponds to mass-loss rates even lower than those indicated by Hamann & Koesterke (1998).

Hence, a 14 $M_\odot$ helium core uncovered in case B mass transfer would evolve into a 9 $M_\odot$ core at collapse (instead of $\sim 4 M_\odot$ as predicted by high mass-loss rates). In calculating the remnant masses of these binaries, we follow the procedure that we used for single stars with winds: we estimate the supernova explosion energy and binding energy from the stellar mass at collapse and, most importantly, constrain the remnant mass to that of the collapsing star (Fig. 7, thin solid line). The corresponding mass distribution (Figs. 7 and 8, thin solid lines) produces black holes with masses in excess of 10 $M_\odot$ and can easily explain massive black holes such as V404 Cyg. However, such a large cut in the total mass lost by winds may require a drastic reduction in the mass-loss rate. The mass-loss rate depends sensitively upon stellar luminosity, and it decreases as the star loses mass. Decreasing the mass-loss rate ($M_{\text{wind}}$) allows the star to retain its mass (and its high luminosity) longer and hence, it stays in a rapid mass-losing phase ($t_{\text{wind}}$) longer. Since the total mass lost by winds is $M_{\text{wind}} t_{\text{wind}}$, lowering the mass-loss rate need not significantly lower the total mass lost (Wellstein & Langer 1999).

3. DISCUSSION

In Table 2 we summarize our results for the range of theoretical BH mass distributions in terms of the percentage of BH among collapse remnants in different mass ranges. Despite the various uncertainties, some properties of the distributions are quite robust, and our goal in this paper is to identify and highlight these robust properties. In all examined cases we obtain a continuous remnant mass distribution that covers a wide range of values, from NSs up to BH masses of at least 10–15 $M_\odot$. The form of the distribution is typically well described by an exponential of varying steepness, in some cases accompanied by an upper mass cutoff. We found no cases that lead to the formation of any gap in the distribution of black hole masses below 7 $M_\odot$, although the appearance of valleys in the distribution in the range of 3–7 $M_\odot$ is possible (caused by winds and binaries; Fig. 8, dotted and thin solid lines, respectively). The fraction of BH masses in the 3–5 $M_\odot$ range varies from 15% to 30% and cannot be characterized as a gap. This fraction is mostly sensitive to the assumed power-law index $\gamma$ of the massive-star IMF. The fraction of remnants in the 5–10 $M_\odot$ range is more sensitive to the assumptions about binaries and winds. It varies from 26% to 35%, from 46% to 57%, and from 69% to 74% for single stars, single stars with winds, and binaries with winds, respectively. Finally, the fraction of remnants in the 10–15 $M_\odot$ range is most sensitive to the assumptions about stellar winds. These assumptions also determine the existence and position of an upper BH mass cutoff.

Investigations of the observed sample of compact objects (Bailyn et al. 1998; Charles 1998) conclude that there is a discontinuous jump from NS to BH masses, that is, there exists a gap in the remnant mass distribution in the range 3–5 $M_\odot$. Bailyn et al. (1998) further found that the data (excluding V404 Cyg) appear to be consistent with a narrow BH mass distribution positioned at $\sim 7 M_\odot$. Our results appear to be in disagreement with the conclusions in these studies. In what follows we address this disagreement in the light of a number of biases that may be operating, and we derive some constraints on our theoretical limits.

There are three possible ways to resolve the apparent conflict between models and observations: (1) the observed sample used by Bailyn et al. (1998) and Charles (1998) suffers from low-number statistics, and larger samples will fill in the apparent gap at low BH masses and will best fit with broader distributions; (2) there are biases operating against the identification or formation of X-ray binaries with low-mass BHs; or (3) one or more parts of the theoretical calculation presented here is incorrect.

Table 1 (and all the plots of our theoretical BH mass distributions) show the latest data on the mass estimates of the observed BH systems. Note that for seven of the nine systems the measured BH mass ranges extend to values below 5 $M_\odot$. It is also important to point out that the current sample includes two very strong candidates for low-mass BH, Nova Velorum 1993 and 4U 1543–47. These two systems have been observed only recently and were not included in the sample considered by Bailyn et al. (1998). In addition, new estimates of the mass of GRO J1655–40 predict a much lower limit for the black hole mass (Phillips et al. 1999). This result drastically changes the statistical analysis of Bailyn et al. (1998), which was dominated by the small error bars of GRO J1655–40 around a mass of $\sim 7 M_\odot$. Taking all the above into account, it is possible that the conclusions drawn from an earlier sample may no longer hold.

Another possibility is that there is some observational selection effect against the identification of systems with low-mass black holes. The transient character is one example. Dynamical measurements of masses of the accreting objects in the systems studied so far can be made only during their quiescent phase, so the source must be transient. Systems with lower-mass black holes will typically have mass ratios closer to unity (given the low-mass companions [<1 $M_\odot$ observed] than the detected binaries. This will lead to higher mass transfer rates from the companions, and it is reasonable to expect that these systems are actually persistent X-ray sources (and hence, mass measurements are hard to obtain). Such a trend has been seen in the results obtained by King, Kolb, & Szuszkiewicz (1997), who studied the transient character of BH binaries. Systems with an unevolved low-mass companion (≈1 $M_\odot$) move from the unstable to the stable regime as the mass of the accretor decreases below 5 $M_\odot$, down to 2 $M_\odot$.

In the context of biases against these systems, it is also possible that such systems have lower formation probability compared to higher-mass BH systems because of events in their formation that are not considered here, such as the effect of the BH formation (mass loss) on the binary characteristics. Using our results for the remnant mass as a function of progenitor mass (Figs. 5 and 7), we calculate the amount of mass lost during BH formation as a function of progenitor mass (Fig. 9). It is evident that the formation of lower-mass BHs (3–5 $M_\odot$) is associated with a higher fractional (relative to the progenitor or the BH) mass loss than the formation of $\sim 7 M_\odot$ black holes. Although this mass loss is not enough to unbind the postcollapse system, even for the low-mass BHs, it is enough to affect the size of the orbits after BH formation. We find that the degree of orbital expansion immediately after the collapse is higher for 3 $M_\odot$ BHs than for 7 $M_\odot$ BHs by a factor of $\sim 3$. So binaries with low-mass BHs tend to be wider than binaries with higher-mass BHs. Given the typical upper limit ($\sim 10 R_\odot$) on orbital separation for systems to reach Roche lobe overflow...
FIG. 9.—Mass ejected during BH formation as a function of progenitor mass for three different cases: our standard case from Fig. 1 (solid line) and the cases of fast (dotted line) and slow (dashed line) rise/decline of the unbinding energy from Fig. 4. We assume that 50% of the explosion energy is available to unbind the envelope and that both binary and wind effects (case C plus winds) are taken into account.

Fig. 10.—(a) Remnant mass as a function of progenitor mass; (b) distribution of remnant masses, assuming a step function for the dependence of explosion energy on progenitor mass. A gap in the range 2–5 $M_\odot$ remnant masses appears (see text).
X-1 seems to fall in this range. Our distribution is to be contrasted with that derived by Timmes, Woosley, & Weaver (1996), which has a double-peaked neutron star distribution. Their distribution may also fit the observed sample of neutron stars such as Vela X-1. However, those results were obtained without considering the effects of fallback, which would smooth out the double-peaked distribution that was reported.

3.2. Effects of Winds and Binary Evolution

Our study demonstrates the strong sensitivity of the formation of massive black holes (>10 \( M_\odot \)) to the effects of winds. In close binaries (which form the observed black hole systems) these effects become even more extreme. In the simple case of single stars, winds alter the shape of the BH mass distribution by making it flatter and decreasing the maximum BH mass to 10–15 \( M_\odot \). When the effect of a close companion is taken into account, winds drastically reduce the formation rate of BHs and cause the maximum BH mass to decrease even further (to or below 10 \( M_\odot \)). Combined with the current stellar models (and in particular, the radial evolution of stars), theory cannot explain the observed black hole systems (Kalogera 2001). The resolution of these problems lies either in significantly lowering mass-loss rates, so that helium stars lose less than half of their initial mass (see also Kalogera 1999) or in widening the range of both progenitor masses and binary orbital separations over which evolution through case C mass transfer is allowed. The latter of the two possibilities would require that current estimates of stellar radii at the end of core helium burning in massive stars are actually overestimates and that the stellar radii just prior to collapse are actually underestimates, especially for stars more massive than \( \sim 25 M_\odot \). Given the uncertainties involved in theoretically determining the radii of evolved stars, such modifications are still possible. If revised stellar radii cannot explain the black hole mass distribution, then the observed black hole masses require that mass loss from stellar winds must decrease. The downward trend of mass-loss rates from Wolf-Rayet winds (Hamann & Koesterke 1998) suggests that this may occur. However, Wolf-Rayet systems such as BR32 (Moffat, Niemela, & Marraco 1990) suggest that the mass-loss rate must be greater than what we assumed in our weak wind, case B mass transfer calculation, and it is likely that both stellar radii and winds will change in the future.

3.3. Neutron Star and Black Hole Kicks

The evidence that neutron stars receive kicks during their formation continues to grow (see Fryer et al. 1999 for a review) and it is generally accepted that neutron stars must receive kicks with a mean velocity exceeding \( \sim 100–200 \) km s\(^{-1}\), up to \( \sim 400 \) km s\(^{-1}\). The question of BH kicks is still open, however. Black hole X-ray binaries seem to have a Galactic distribution with a scale height smaller than that of NS X-ray binaries and an inferred mean velocity of only \( \sim 40 \) km s\(^{-1}\) (White & van Paradijs 1996). Tutukov & Cherepashchuk (1997) and Nelemans, Tauris, & van den Heuvel (1999) have argued that these velocities can be explained simply by mass ejection during BH formation and hence kicks are not necessary to explain the properties of BH binaries.

In contrast with the majority of BH X-ray transients, GRO J1655–40 appears to have a large space velocity. Bailyn et al. (1995) measured its radial velocity to be \( 114 \pm 19 \) km s\(^{-1}\), which provides a lower limit to the space velocity. Under the assumption of isotropocity, the measured radial velocity corresponds to a three-dimensional velocity of \( \sim 200 \) km s\(^{-1}\). Phillips et al. (1999) were able to constrain the magnitude of the space velocity in the range 143–153 km s\(^{-1}\). Nelemans et al. (1999) used the current determinations of the BH mass and a relatively high mass for the BH companion and obtained space velocities of at most 110 km s\(^{-1}\) (just compatible with the radial velocity measurement). In their study the amount of mass loss was treated as a free parameter. However, as we discussed above, our calculations link the BH mass to the amount of mass lost during BH formation. We use our results for the full set of cases (different dependence of explosion energy on the progenitor mass and different values of the parameter \( f \)) and find that space velocities in the range 100–200 km s\(^{-1}\) require precollapse orbital separations (0.1–4 \( R_\odot \)) too small to accommodate the BH companion in the binary. Imposing this latter constraint, we obtain maximum space velocities, for the different models, in the range 20–85 km s\(^{-1}\). We conclude that the measured radial velocity of GRO J1655–40 seems to require that a kick was imparted to the BH (in agreement also with Brandt, Podsiadlowski, & Sigurdsson 1995). Given that the majority of the systems appear to have low space velocities (White & van Paradijs 1996), Fryer (2000) found that a BH kick of at least \( 50 \) km s\(^{-1}\) is required to explain the velocities of the complete set of BH X-ray transients.

We have shown here that most of the black holes are formed via fallback and not promptly. Since many of the NS kick mechanisms operate before fallback occurs, it seems reasonable to expect that BHs receive kicks as well. Given the kick mechanisms, it is likely that the momentum imparted by asymmetric explosions is equal for both BHs and NSs and that therefore the kick scales with mass:
\[ \langle V_{\text{BH}} \rangle = \langle V_{\text{NS}} \rangle M_{\text{NS}}/M_{\text{BH}}. \]

If this is the case, then lower-mass \( S \) V BH T \( S \) V NS T \( M \)/M BH \( . \) BHs will receive higher kicks. This, in the context of a possible gap in the low-mass BH range, possibly provides an additional bias against the formation of low-mass BHs in binaries. We find that, for a typical NS kick magnitude of 200 km s\(^{-1}\) and for reasonable values of post-CE orbital separations, 25\%–50\% (for a fixed BH kick magnitude of 50 km s\(^{-1}\), the fraction is in the range 10\%–50\%) of the systems with 3 \( M_\odot \) BHs get disrupted, while systems with 7 \( M_\odot \) BHs remain unaffected. This effect could contribute to decreasing the formation frequency of X-ray binaries with low-mass black holes.

We thank N. Langer, S. Wellstein, and A. Heger for useful discussions. We also thank the anonymous referee, who helped in clarifying some of our main conclusions. The work of C. L. F. was supported by NASA (NAG 5-8128) and the US DOE ASCI Program (W-7405-ENG-48). V. K. acknowledges support by the Smithsonian Institution in the form of a CfA Postdoctoral Fellowship.

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