Relational Generalized Few-Shot Learning

Xiahan Shi  
Bosch Center for Artificial Intelligence  
xiahan.shi@de.bosch.com

Leonard Salewski  
Bosch Center for Artificial Intelligence  
leonard.salewski@gmail.com

Martin Schiegg  
Bosch Center for Artificial Intelligence  
martin.schiegg@de.bosch.com

Zeynep Akata  
UvA Bosch DELTA Lab  
z.akata@uva.nl

Max Welling  
University of Amsterdam  
m.welling@uva.nl

Abstract

Transferring learned models to novel tasks is a challenging problem, particularly if only very few labeled examples are available. Although this few-shot learning setup has received a lot of attention recently, most proposed methods focus on discriminating novel classes only. Instead, we consider the extended setup of generalized few-shot learning (GFSL), where the model is required to perform classification on the joint label space consisting of both previously seen and novel classes. We propose a graph-based framework that explicitly models relationships between all seen and novel classes in the joint label space. Our model Graph-convolutional Global Prototypical Networks (GcGPN) incorporates these inter-class relations using graph-convolution in order to embed novel class representations into the existing space of previously seen classes in a globally consistent manner. Our approach ensures both fast adaptation and global discrimination, which is the major challenge in GFSL. We demonstrate the benefits of our model on two challenging benchmark datasets.

1 Introduction

Few-shot learning (FSL) \cite{1, 2, 3, 4} is inspired by the human ability to learn new concepts from very few or even only one example. This extreme low-data setup is particularly challenging for deep neural networks, which require large amounts of data to ensure generalization. Recently, FSL has mostly been approached from the meta-learning perspective, focusing on the problem setup of \textit{N-way K-shot classification}. For each \textit{N-way K-shot} task, the model has to discriminate \textit{N} novel few-shot classes with only \textit{K} labeled examples available per class. Unlike in standard transfer learning, meta-learning requires the model to adapt well across a range of \textit{various} previously unknown tasks instead of a fixed, \textit{specific} target task. Therefore, efficient on-the-fly model adaptation based on very few examples is at the core of most FSL models \cite{5, 6, 7, 8, 9, 10}.

However, this FSL problem setup focuses only on discriminating novel classes from each other and offers no incentive for the model to remember previously seen classes or to maintain a globally consistent label space. From a practical point of view, however, we would like the model to incorporate few-shot novel classes into the label space of previously seen classes. This leads us to an extended scenario called \textit{generalized few-shot learning} (GFSL), where the model has to discriminate the joint label space consisting of both previously seen and novel classes. This terminology is adopted

Preprint. Under review.
Figure 1: **Relational generalized few-shot learning**: The task is to discriminate the joint label space $\mathcal{Y}_{\text{joint}} = \mathcal{Y}_{\text{seen}} \cup \mathcal{Y}_{\text{novel}}$, where $\mathcal{Y}_{\text{novel}}$ consists of $N$ previously unseen classes with only $K$ labeled examples per class (support set). We propose GcGPN, which maps all support set and query images to a feature space, where classes are represented by prototypes (circles for novel and blue symbols for seen classes). The core idea of GcGPN is to explicitly model inter-class relations by a weighted graph (edge in yellow, edge weight indicated by width), which can be obtained in different ways such as similarity learning, attention modeling or side information. GcGPN employs graph-convolutions (GCN) to propagate information among classes according to the relational graph, resulting in jointly updated prototypes. The query $x$ (gray rectangle) is classified according to its cosine similarity to the updated prototypes. The model is trained end-to-end in a meta-learning setup.

from zero-shot learning (ZSL) and generalized zero-shot learning (GZSL), where novel classes come with no labeled examples at all and classification relies on side information such as attributes or semantic label embeddings [11][12]. It is a well-known observation that many ZSL models fail dramatically at discriminating the joint label space (GZSL) despite good performance on novel label space (ZSL) [13][12]. Similarly, GFSL is a much more challenging task compared to FSL due to the trade-off between fast adaptation to novel classes and maintaining a global consistency across the joint label space.

We address the GFSL problem setup by explicitly modeling inter-class relationships as a weighted graph. We propose the **Graph-convolutional Global Prototypical Network** (GcGPN) which models representative prototypes for all novel and seen classes jointly. In particular, the prototypes are updated by graph convolutional operations [14] according to the relationship graph. Fig. 1 provides an illustration of our approach.

To summarize, our main contributions are: We propose the first flexible framework for relational GFSL that (i) considers an arbitrary weighted graph describing relations between classes (from any source of side information, attention mechanism or other similarity measures), (ii) learns consistent class embeddings by taking all class relations into account to allow for better generalization to novel tasks, (iii) is the first model which applies graph-convolution for prototype learning in GFSL and allows for end-to-end learning, (iv) allows to derive previous (G)FSL methods [6][10] as special cases of our model, and (v) achieves state-of-the-art performance on GFSL tasks.

## 2 (Generalized) Few-Shot learning

In this section, we recap the task definition of few-shot learning and generalized few-shot learning.

**Few-shot learning (FSL)**: We consider $N$-way $K$-shot classification, which is the most widely studied problem setup for FSL. The classifier has to perform a series of $N$-way $K$-shot tasks, where each task consists of $N$ previously unseen, novel classes with $K$ labeled examples each (often $K \leq 5$). More precisely, let $\mathcal{Y}_{\text{novel}}$ denote the novel class label space with $|\mathcal{Y}_{\text{novel}}| = N$, and let $\mathcal{D}_{\text{novel}} = \bigcup_{n=1}^{N} \{ (x_{n,k}, y_{n}) \}_{k=1}^{K}$ denote the support set, where $x_{n,k}$ is the $k$-th labeled example of the class with label $y_{n}$. For a new query image $x$, the FSL prediction is given by

$$\hat{y} = \arg \max_{y \in \mathcal{Y}_{\text{novel}}} p_{\psi}(y|x, \mathcal{D}_{\text{novel}}).$$

(1)
\(N\)-way \(K\)-shot classification considers FSL from a meta-learning perspective. Unlike in standard transfer learning, the goal is not to generalize to a specific novel label space but to adapt and perform well across a series of various novel label spaces presented at test time. Therefore, most FSL methods adopt episodic training \([5]\), where a new \(N\)-way \(K\)-shot task gets randomly sampled from a larger training set in every episode. This involves randomly selecting \(N\) “virtual” novel classes and sampling \(K\) support set images per class along with a batch \(B\) of query images. The loss on this batch is then given by \(\frac{1}{|B|} \sum_{(x,y)\in B} - \log p_{\psi}(y|x, D_{\text{novel}})\), which gets back-propagated to update the model parameters \(\psi\).

An FSL model is only concerned with discriminating the novel label space since all test time queries, by design of the task setup, come from one of the novel classes. Hence the arg max in eq. (1) is only over \(Y_{\text{novel}}\). Classes seen during training no longer play a role at test time. This setup emphasizes fast adaptation to varying new tasks but does not encourage the model to accumulate knowledge, which may not always be very practical. Many real-world applications require the model to incorporate novel few-shot classes into the existing space of seen classes while maintaining global discrimination. Therefore, we consider the extended setup of generalized few-shot learning (GFSL) with test time queries that may come from both novel and seen classes.

### Generalized few-shot learning (GFSL)

In generalized \(N+1\)-way \(K\)-shot classification, the model has to discriminate the joint label space \(Y_{\text{joint}} = Y_{\text{seen}} \cup Y_{\text{novel}}\) consisting of all novel few-shot classes and previously seen training classes. We denote the training set by \(D_{\text{seen}} = \cup_{i=1}^{N_{\text{seen}}} \{(x_{n,k}, y_{n,k})\}_{k=1}^{K_{n}}\), where \(N_{\text{seen}}\) is the number of training classes and \(K_{n}\) is the number of labeled examples available for class \(y_{n} \in Y_{\text{seen}}\). In general, \(N_{\text{seen}} \gg N\) and \(K_{n} \gg K\). For a new query \(x\), a GFSL model performs

\[
\hat{y} = \arg\max_{y \in Y_{\text{seen}}} p_{\psi}(y|x, D_{\text{novel}}) .
\] (2)

In contrast to eq. (1), the arg max is over \(Y_{\text{joint}}\) instead of \(Y_{\text{novel}}\) since \(x\) may come from any of the seen and novel classes. In particular, GFSL requires discrimination of a much larger label space than FSL (\(N_{\text{seen}} + N\) instead of \(N\)). In addition, the model has to maintain a globally consistent joint label space while, at the same time, achieve fast adaptation to novel classes based on very few examples. In general, we cannot expect FSL models to perform well in GFSL since there is no explicit reward for remembering the training classes and learning a well-separated joint label space.

### 3 Graph-convolutional Global Prototypical Networks

We approach GFSL by explicitly modeling the relationships among classes through a weighted graph where nodes represent classes and edges represent inter-class dependencies as depicted in Figure 1. We propose Graph-convolutional Global Prototypical Networks (GcGPN) for relational GFSL. The main idea is to represent seen and novel classes by prototypes and learn these node representations jointly through a graph-convolutional network \([14]\) while considering inter-class relationships.

#### 3.1 GcGPN: Model Overview

Figure 1 visualizes how our model operates on a GFSL classification task. First, GcGPN maps all support set and query images into a \(d\)-dimensional feature space by a feature extractor \(f_{\psi}(\cdot)\). Next, initial prototypes \(c_{n} \in \mathbb{R}^{d}\), \(n = 1, \ldots, N_{\text{seen}} + N\), are computed for all classes. While seen class prototypes are learned as model parameters, novel class prototypes are initialized on-the-fly since the novel label space varies at test time. The novel initial prototypes are given by the per-class average \(c_{n} = \frac{1}{K} \sum_{k=1}^{K} \tilde{z}_{n,k}\) of the normalized support set examples \(\tilde{z}_{n,k} = \frac{f_{\psi}(x_{n,k})}{\|f_{\psi}(x_{n,k})\|}\), \(n = N_{\text{seen}} + 1, \ldots, N_{\text{seen}} + N\), as in \([6]\) \([10]\). Then, a graph-convolution block \(\hat{g}(\cdot)\) as defined in section 3.2 updates these graph node initializations jointly according to the inter-class relationships specified in the edge weights. The updated prototypes \(c'_{n}\), \(n = 1, \ldots, N_{\text{seen}} + N\), are then adapted representations of the joint label space of the \(N+1\)-way \(K\)-shot task at hand. Finally, the predicted class probabilities for a query \(x\) are obtained from its cosine similarities between its feature representation and the updated prototypes using \([10]\):

\[
p(y = n|x, c'_{1}, \ldots, c'_{N_{\text{seen}}+N}) = \frac{\exp (\tau \cos(f_{\psi}(x), c'_{n}))}{\sum_{m=1}^{N_{\text{seen}}+N} \exp (\tau \cos(f_{\psi}(x), c'_{m}))} .
\] (3)
where $\tau$ is a learnable temperature. We adopt the cosine classifier since it was found to be preferable over the originally proposed L2 distance \cite{Li19} when combining existing and novel classes \cite{Sun20, Herbrich98}. To train the model, we use the cross-entropy loss on eq. (3). Note that the sum in eq. (3) is over all class prototypes in the joint label space, which is in accordance with eq. (2) and differs from the FSL objective. Further, we apply episodic training for GFSL: For each episode, $N$ out of $N_{\text{seen}}$ training classes are sampled to act as “virtual” novel classes and the remaining $N_{\text{seen}} - N$ are treated as the label space of seen classes. Contrary to an FSL episode, the GFSL query set $Q$ must also contain images from the seen classes, thus rewarding the model for maintaining global discrimination instead of focusing only on the novel classes. In every episode, the loss is back-propagated to update all learnable parts of the model including the parameters $\psi$ of the feature extractor, the initial prototypes $c_n$ of seen classes, trainable components of the graph-convolutional block $\tilde{g}()$ and the classifier temperature $\tau$. Unlike previous work \cite{Herbrich98}, our model does not require multi-stage training but trains all parts of GcGPN jointly.

### 3.2 The Graph-Convolutional Block

The graph-convolutional block $\tilde{g}(\cdot)$ is at the core of GcGPN. To recap, a graph-convolutional block \cite{Kipf17} consists of $L$ graph-convolutional layers $g(\cdot) = g^L(g^{L-1}(...(g^1(\cdot))))$ on a graph of $V$ nodes, which is given in its general form by

$$X^{(l+1)} = g^l(X^{(l)}) = \rho\left(\sum_{B \in \mathcal{A}} BX^{(l)}\theta_B^{(l)}\right), \ l = 1, \ldots, L,$$

where $l$ indexes the layer of the block, $X^{(l)}$ is the $(V \times d_l)$-dimensional input matrix containing $d_l$-dimensional node features in its rows, $\mathcal{A}$ denotes a set of $(V \times V)$-dimensional linear operators such as the adjacency or weight matrix of the graph, $\theta_B^{(l)}$ with $B \in \mathcal{A}$ denotes a $(d_l \times d_{l+1})$-dimensional matrix containing learnable parameters of the $l$-th layer and $\rho(\cdot)$ is a non-linearity. For example, if $B$ is the adjacency matrix of the graph, the local convolutional operation $BX^{(l)}$ computes for each node the sum of its neighbors.

In our GFSL model, eq. (4) is applied to the class prototypes to model relations between them. More precisely, let $C$ denote the $((N_{\text{seen}} + N) \times d)$-dimensional matrix, where the $n$-th row contains the initial prototype $c_n$ of the $n$-th class. Further, let the operator entries $B_{m,n}$ encode a similarity score between classes represented by $c_m$ and $c_n$. Then, the a $L$-layered graph-convolutional network computes the class prototype updates as $C' = \tilde{g}(C) = g^L(...g^1(C))$.

Note that graph-convolution can be interpreted as performing two steps to update a class prototype: First, a weighted average of similar prototypes is computed with weights given in the convolution operator $B$ and second, a non-linear post-convolution transform is applied given by $\theta_B^{(l)}$ and $\rho(\cdot)$. The first part models interactions among classes and operates only on node-level, while the second part operates only on feature-level by applying the same transform to all classes.

The general graph-convolution definition from eq. (4) operates in Euclidean spaces. We adopt the graph-convolutional block to be consistent with cosine similarities as used in eq. (3) by intermediate normalizations $\tilde{x} = \frac{x}{\|x\|}$ to keep the prototypes at unit length. Our graph-convolutional block is thus defined by $C' = \tilde{g}(C) = \tilde{g}^L(...\tilde{g}^1(C))$ with

$$\tilde{g}^l(C^{(l)}) = \rho\left(\sum_{B \in \mathcal{A}} s_B \overline{BC^{(l)}\theta_B^{(l)}}\right), \ l = 1, \ldots, L,$$

where scalars $s_B$ are introduced to trade-off between different operators in $\mathcal{A}$. We call the operators $B \in \mathcal{A}$ semantic operators.

In the following, we discuss the building blocks of the graph-convolutional block essential for GFSL.

**Convolution operators:** The relational information between the class prototypes is modeled by the operators $B \in \mathcal{A}$. For big graph-structured data such as recommender systems or social network analysis, the adjacency matrix and its higher-order versions are a popular choice \cite{Kipf17}. In our case, where the data may not be inherently graph-structured, edge weights are used to express the degree of similarity. In general, there are several possible strategies to define the operator entries $B_{n,m}$:

1. Any standard distance or similarity measure on the prototype space such as L2 distance or cosine similarity.
2. Learned similarities, either using a standard measure in a learned transformed space or learning a flexible transform of the element-wise absolute differences as done in [9].
3. Similarities within a key space by learning a key $k_n$ for each class as proposed in [10].
4. Side information, either given directly as relational scores (e.g., shortest-path distance between two class names in an ontology [16]), or computed as similarity between per-class embeddings such as word vectors or attributes [11].

Due to the multi-operator design, our model can naturally combine multiple of the above strategies, resulting in a general and flexible framework for relational GFSL.

**Post-Convolutional Transforms:** The parameters $\theta_B$ are another crucial component of the model. Since the output of $\hat{g}(\cdot)$ are updated prototypes, we choose $\theta_B$ to preserve the dimensionality. Using a learnable quadratic weight matrix is the most straight-forward approach, although restricting $\theta_B$ similar to [10] is also a competitive option.

### 3.3 Generalization of existing approaches

There is a naive extension of the state-of-the-art FSL method Prototypical Networks [6] (PN) to the GFSL setup. For a readily trained PN, seen class prototypes can be obtained as feature averages over all available training examples for that class, which can then be used to perform GFSL tasks. This extension referred to as PN$^*$ corresponds to the assumption that there are no inter-class dependencies at all, which is equivalent to setting all $B \in \mathcal{A}$, $\rho(\cdot)$, $s_B$, $\theta_B$ to identity matrices or functions in our GcGPN framework.

The model in [10] addresses GFSL successfully by an attention-based weight generator that computes classifier weights $w^*$ for novel classes based on their support sets and their similarities to seen classes. Both our model and theirs utilize a cosine classifier. However, while the cosine classifier in our model operates on representative prototypes in the feature space, theirs operates in the weight space and computes cosine similarities between seen class weights and transformed support set image features. The Average Weight Generator variant in [10] can be recovered in our framework by using a GcGPN with prototype initialization as in sec. 3.1 and one graph-convolution layer ($L = 1$) with $\mathcal{A} = \{\hat{B}_1, \hat{B}_2\}$ containing two block-structured operators

$$
\hat{B}_1 = \begin{pmatrix} I_{N_{seen} \times N_{seen}} & 0_{N_{seen} \times N} \\ 0_{N \times N_{seen}} & 0_{N \times N} \end{pmatrix}, \quad \hat{B}_2 = \begin{pmatrix} 0_{N_{seen} \times N_{seen}} & 0_{N_{seen} \times N} \\ 0_{N \times N_{seen}} & I_{N \times N} \end{pmatrix}
$$

(6)

with identity matrix on the seen- and novel-class blocks respectively and zeros elsewhere. This corresponds to not modeling inter-class relations at all. $\theta_{B_1}$ is the identity matrix and $\theta_{B_2}$ is a learnable diagonal matrix. The Attention Weight Generator variant in [10] can be recovered by adding one more operator to $\mathcal{A}$, whose lower-left block contains scores from an attention mechanism in a learned key space. This corresponds to an underlying relational graph with weighted directed edges from seen to novel classes, such that novel prototypes do not only depend on the support set but also on similar seen classes. To summarize, GcGPN generalizes over [10] in several respects: (i) We use a fully connected graph, allowing not only relations from seen to novel classes but among all classes (i.e., operators in $\mathcal{A}$ may be full matrices); (ii) our framework accommodates any kind of similarity modeling (not only attention matching) and offers a natural way to combine multiple strategies discussed in sec. 3.2; (iii) more general post-convolution transforms and layer stacking ($L \geq 1$) result in a more flexible joint model for class prototypes; (iv) all parameters can be trained end-to-end through a GFSL objective, thus does not require the 2-stage training procedure from [10].

### 4 Related Work

**Few-Shot Learning (FSL)** has been approached from different perspectives. FSL modeling alternatives include to mimic the human learning behavior by modeling high-level concepts [17], to learn similarity measures to one-shot learning [18], and to extend deep neural networks by an external memory module to allow for direct incorporation of few-shot examples [19] [20]. Moreover,

---

1 This is in contrast to our method, where prototypes are learnable parameters, initialized with an average over the support points.
recent meta-learning approaches that focus on $N$-way $K$-shot classification can be divided in two categories: Optimization-based methods [7, 8, 21] rely on a meta model that learns an optimal strategy which is carried out by an inner model in order to efficiently adapt this model to new tasks. Distance-based methods such as Matching Networks [5] and Prototypical Networks [6] perform nearest-neighbor-based classification with a learned distance measure, either by selecting the closest support set example [5], or by choosing the class of the closest prototype obtained by averaging the respective support set examples on the feature space [6]. Despite its simplicity, the method in [6] achieves state-of-the-art performance and has inspired extensions which parameterize the distance measure or the prototype mechanism in a more flexible way [22].

Generalized Few-Shot Learning (GFSL) in the context of meta-learning is not yet a well-studied task. Alternatives to meta-learning include using pre-trained image features [23, 24] or learning a generative model that models the global class structure in the joint label space [25]. The most relevant work to our setup is Dynamic FSL without Forgetting [10], which utilizes an attention-based weight generator for novel classes to extend the classifier from seen classes to the joint label space. Its connections to our work is discussed in detail in 3.3.

Side Information plays a crucial role in zero-shot learning (ZSL), where no labeled examples are available for novel classes at all. In particular, ZSL methods typically build on side information from knowledge graphs [26], semantic word embeddings [27, 28] or visual attributes [29]. Generalization can be achieved by relating the feature space to the side information either through a learned mapping or a joint embedding space [50] [31] [11] [24]. Furthermore, graph-convolutions can be applied to distill information from relational knowledge graphs and class embeddings to predict the classifier weights for novel classes [16]. Apart from ZSL, a range of FSL methods exist that incorporate side information, e.g., to regularize the feature space with textual embeddings for alignment on a distributional level [32], or to use attention mechanisms for synthesizing additional training examples for few-shot classes [33].

Graph-Convolutional Networks (GCN) [34, 35, 14] are a powerful tool to jointly learn node representations for inherently graph-structured data such as items in recommender systems or users of social networks [36]. GCNs have been applied to FSL [9] by representing image instances with graph nodes in an $N$-way $K$-shot classification setup. In contrast, we represent classes by graph nodes with a GFSL setup. Class-level graph-convolution has been used in a similar way for ZSL [16]. Alternatively, GCNs may exploit the manifold structure in the data to propagate labels from labeled to unlabeled images by using edge weights that depend on learned distances in the feature space [37].

5 Experimental Results

We evaluate our method on two widely used benchmark datasets. First, we use the FSL benchmark dataset miniImageNet [5], which is a subset taken from ImageNet [38] consisting of 100 classes and 600 images per class. We adopt the split specified in [8] to obtain 64 seen, 16 novel validation and 20 novel test classes. To obtain training, validation and test sets for the seen class label space, we further follow the approach in [10]. We enrich this dataset with side information based on conceptual semantics and lexical relations by mapping class names into the ontology WordNet [26]. In particular, we use WordNet path similarities [39] between class labels, which are scores based on the shortest path distances between words in the taxonomy. Second, we evaluate our method on Caltech-UCSD Birds-200-2011 (CUB) [40], which is widely used for ZSL. This dataset contains 11,788 images across 200 classes of different bird species. Each class has 312 annotated continuous attributes describing visual characteristics of the respective bird species. We follow the standard split used in ZSL [41] to obtain 150 seen and 50 novel test classes. Further, we randomly select 20 from the 150 seen classes for validation. For each seen class, 25% of the images are hold out as seen class test set and 10% as seen class validation set. In this dataset, we obtain edge weights by computing pairwise cosine similarities between class attributes. Fig. shows the semantic operators $B$ when class similarities are used as edge weights; brighter color indicates higher similarity.

Experimental Setup Our goal is to evaluate the ability of GcGPN to leverage side information for relational GFSL. To this end, we explore multiple variants of GcGPN with different specifications for the graph-convolution block. At the core of all model variants is the semantic operator $B$ containing
all graph edge weights, i.e., similarities among all $N_{seen} + N$ classes. Since graph-convolution operators usually require normalization, we apply row-wise softmax with a learnable temperature such that each row sums up to 1. We use one graph-convolution layer and diagonal post-convolution transform $\theta_B$ with learnable entries.

We exploit the model’s flexibility to combine multiple operators and include variants where the operator set $A$ is augmented by the two auxiliary operators $\hat{B}_1$ and $\hat{B}_2$ defined in eq. (6) (variant indicated by -aux). This allows the model to trade-off between self-connection and the effect of similar prototypes. Further, note that the operators have an inherent four-block structure with relations between seen-seen, seen-novel, novel-seen and novel-novel classes in the upper-left, lower-left, upper-right and lower-right block, respectively (similar to eq. (6)). We explore the effect of either utilizing only one semantic operators $A = \{B\}$ with fully connected class similarities or splitting $B$ into four individual operator $A = \{B_{ss}, B_{sn}, B_{ns}, B_{nn}\}$ with one activated block each. The latter variant, indicated by -split, allows the model to learn specialized post-convolution transforms for each block.

To further study the effect of the semantic operator and the post-convolution transform, we conduct two more experiments on CUB: Variant GcGPN-aux-sn has reduced capacity in the operator by deactivating all inter-class relations other than the seen-novel block, whereas GcGPN-aux-fc$\theta_B$ has increased capacity in the post-convolution transform by using fully connected instead of diagonal $\theta_B$.

For comparability, we adopt the same feature extractor architecture as in [6] and [10] with 4 convolutional blocks and 128 output feature maps, where each block consists of a 3x3 convolution layer followed by batch normalization, ReLU and 2x2 max-pooling. We train all models from scratch, which is in contrast to the two-phase training used in [10]. All models are trained for 75 epochs on mIN and 45 epochs on CUB using an SGD optimizer with a momentum of 0.9, a weight decay parameter of 0.0005 and an initial learning rate of 0.1 that was reduced after 20, 40, 50 and 60 epochs. Performance is monitored on the validation set to allow for early stopping.

**Results and Discussion**

Tables 1 and 2 show results for generalized $5^+$-way $K$-shot classification on miniImageNet and CUB datasets. In addition to the evaluation measures in [10], we follow the convention in GFZL [12] and report Seen-Joint and Novel-Joint accuracies together with their harmonic mean, which capture the joint label space classification performance separately for seen and novel class queries. We compare to the GFSL extension of Prototypical Networks [6], PN* (see sec. 3.3), and to DFSLwoF [10], which are the most related approaches to ours. All models are evaluated across 600 episodes randomly sampled from the test set and the average accuracy with 95% confidence intervals is reported.

The first observation is the drastic performance drop of PN* on novel class queries when changing from the novel to the joint label space (comparing Novel-Novel and Novel-Joint). The novel classes are well-separated from each other but not consistently embedded into the seen label space, which is what we anticipate since PN* is only trained for FSL.
We do observe improvements from using auxiliary operators, however, the simplest GcGPN already, which is suggested by the significant increase in Novel-Joint accuracy while the Seen-Joint accuracy while being competitive with the baseline DFSLwoF [10] on the 5-shot task. On CUB, our model provides

Table 1: Test set results for 5+-way 1-shot and 5+-way 5-shot classification on miniImageNet.

|       | FSL          | GFSL         |
|-------|--------------|--------------|
|       | 1-shot       | 5-shot       |
|       | Seen-Seen    | Novel-Novel  | Seen-Joint    | Novel-Joint    | Joint-Joint  | Seen-Joint    | Novel-Joint    | Joint-Joint  | H-Mean       |
| PN    | 54.02        | 53.88±0.78   | 27.02±0.23   | 54.02         | 0.02±0.01     | 0.04±0.03    |
| DFSLwoF [10] | 69.93        | 55.80±0.78   | 49.42±0.41   | 58.54         | 40.30±0.74    | 46.95±0.55   |
| GcGPN-aux | 63.68        | 55.67±0.73   | 46.82±0.41   | 51.08         | 42.57±0.72    | 43.68±0.48   |
| GcGPN-split | 62.26        | 55.68±0.76   | 49.60±0.41   | 55.22         | 43.98±0.76    | 48.13±0.49   |
| GcGPN-aux-split | 68.13       | 60.40±0.71   | 51.63±0.41   | 54.68         | 48.59±0.72    | 50.83±0.45   |

Table 2: Test set results for 5+-way 1-shot and 5+-way 5-shot classification on CUB.

|       | FSL          | GFSL         |
|-------|--------------|--------------|
|       | 1-shot       | 5-shot       |
|       | Seen-Seen    | Novel-Novel  | Seen-Joint    | Novel-Joint    | Joint-Joint  | Seen-Joint    | Novel-Joint    | Joint-Joint  | H-Mean       |
| PN    | 35.16        | 58.87±0.91   | 17.61±0.21   | 35.16         | 0.05±0.02     | 0.09±0.04    |
| DFSLwoF [10] | 47.02        | 59.87±0.93   | 37.87±0.48   | 41.55         | 34.19±0.82    | 36.34±0.56   |
| GcGPN-aux | 43.96        | 70.49±0.81   | 45.46±0.48   | 34.92         | 56.00±0.84    | 42.21±0.47   |
| GcGPN-split | 46.26        | 71.17±0.79   | 47.61±0.47   | 36.35         | 58.88±0.78    | 44.21±0.48   |
| GcGPN-aux-split | 40.60        | 71.77±0.81   | 46.09±0.48   | 30.49         | 61.68±0.80    | 40.15±0.50   |
| Ablations | GcGPN-aux-fcθB | 51.88        | 72.72±0.80   | 47.49±0.46   | 47.33         | 47.66±0.74    | 46.77±0.48   |
| GcGPN-aux-sn | 38.71        | 70.25±0.84   | 44.67±0.48   | 29.26         | 60.09±0.81    | 38.61±0.52   |

|       | 5-shot       | 5-shot       |
|       | Seen-Seen    | Novel-Novel  | Seen-Joint    | Novel-Joint    | Joint-Joint  | Seen-Joint    | Novel-Joint    | Joint-Joint  | H-Mean       |
| PN    | 43.04        | 75.81±0.67   | 25.26±0.26   | 42.90         | 7.62±0.32     | 12.45±0.44   |
| DFSLwoF [10] | 48.37        | 74.73±0.79   | 44.97±0.51   | 45.09         | 44.85±0.82    | 44.19±0.54   |
| GcGPN-aux | 44.33        | 76.98±0.75   | 50.35±0.46   | 36.44         | 64.26±0.75    | 45.92±0.48   |
| GcGPN-split | 50.73        | 75.87±0.74   | 50.62±0.49   | 45.92         | 55.33±0.79    | 49.53±0.48   |
| GcGPN-aux-split | 52.31        | 76.49±0.74   | 49.16±0.48   | 48.37         | 49.95±0.78    | 48.42±0.49   |
| Ablations | GcGPN-aux-fcθB | 51.39        | 76.63±0.75   | 48.87±0.50   | 47.79         | 49.95±0.80    | 48.81±0.52   |

On miniImageNet, GcGPN benefits from auxiliary operators and splitting on both tasks. Our best variant achieves state-of-the-art Joint-Joint accuracy and harmonic mean on the 1-shot task while being competitive with the baseline DFSLwoF [10] on the 5-shot task. On CUB, our model outperforms state-of-the-art by a wide margin on both 1-shot and 5-shot tasks and in terms of both Joint-Joint accuracy and harmonic mean performance. This improvements mainly stem from the model’s enhanced ability to incorporate novel classes consistently into the seen class label space, which is suggested by the significant increase in Novel-Joint accuracy while the Seen-Joint accuracy remains comparable with the baseline [10]. Unlike on miniImageNet, splitting was not beneficial. We do observe improvements from using auxiliary operators, however, the simplest GcGPN already outperforms the baselines significantly. Note that our model variants do not require learning an additional key space and an attention module as in DFSLwoF, but instead relies on side information only. Thus, the quality of the side information becomes crucial. The attributes on CUB provides fine-grained visual information which, according to our empirical results, proves to be a richer source of relational information compared to the taxonomy-based similarity for miniImageNet.

The ablation study on CUB suggests that increasing the post-convolution transformation capacity (GcGPN-aux-fcθB) improves the model’s discriminative power. Restricting the relational graph to novel-seen dependencies turns out to harm the performance, which is in line with our key intuition that learning prototypes jointly by incorporating all inter-class relationships results helps to handle the challenging trade-off in GFSL.
6 Conclusion

We propose Graph-convolutional Global Prototypical Network for GFSL, which takes inter-class relationships defined by an arbitrary weighted graph into account to consistently embed previously seen and novel classes into a joint prototype space. This allows for better generalization to novel tasks while at the same time maintaining discriminative power over not only novel but also seen classes. Our model generalizes existing approaches in FSL and GFSL and achieves strong state-of-the-art results on 3 out of 4 benchmark tasks by leveraging side information.

References

[1] Sebastian Thrun. Is learning the n-th thing any easier than learning the first? In NIPS, 1996.
[2] Erik G. Miller, Nicholas E Matsakis, and Paul A. Viola. Learning from one example through shared densities on transforms. In CVPR, 2000.
[3] Evgeniy Bart and Shimon Ullman. Cross-generalization: Learning novel classes from a single example by feature replacement. In CVPR, 2005.
[4] Li Fei-Fei, Rob Fergus, and Pietro Perona. One-shot learning of object categories. TPAMI, 28(4), 2006.
[5] Oriol Vinyals, Charles Blundell, Timothy Lillicrap, Koray Kavukcuoglu, and Daan Wierstra. Matching networks for one shot learning. In NIPS, 2016.
[6] Jake Snell, Kevin Swersky, and Richard Zemel. Prototypical networks for few-shot learning. In NIPS, 2017.
[7] Chelsea Finn, Pieter Abbeel, and Sergey Levine. Model-agnostic meta-learning for fast adaptation of deep networks. In ICML, 2017.
[8] Sachin Ravi and Hugo Larochelle. Optimization as a model for few-shot learning. In ICLR, 2017.
[9] Victor Garcia and Joan Bruna. Few-shot learning with graph neural networks. In ICLR, 2018.
[10] Spyro Gidaris and Nikos Komodakis. Dynamic few-shot visual learning without forgetting. In CVPR, 2018.
[11] Zeynep Akata, Florent Perronnin, Zaid Harchaoui, and Cordelia Schmid. Label-embedding for image classification. TPAMI, 38(7), 2016.
[12] Yongqin Xian, Christoph H. Lampert, Bernt Schiele, and Zeynep Akata. Zero-shot learning - a comprehensive evaluation of the good, the bad and the ugly. TPAMI, 2018.
[13] Wei-Lun Chao, Soravit Changpinyo, Boqing Gong, and Fei Sha. An empirical study and analysis of generalized zero-shot learning for object recognition in the wild. In ECCV, 2016.
[14] Thomas N. Kipf and Max Welling. Semi-supervised classification with graph convolutional networks. In ICLR, 2017.
[15] Matthias Bauer, Mateo Rojas-Carulla, Jakub B. Świątkowski, Bernhard Schölkopf, and Richard E. Turner. Discriminative k-shot learning using probabilistic models. In Second Workshop on Bayesian Deep Learning at NIPS, 2017.
[16] Xiaolong Wang, Yufei Ye, and Abhinav Gupta. Zero-shot recognition via semantic embeddings and knowledge graphs. In CVPR, 2018.
[17] Brenden M. Lake, Ruslan Salakhutdinov, and Joshua B. Tenenbaum. Human-level concept learning through probabilistic program induction. Science, 350(6266), 2015.
[18] Gregory Koch, Richard Zemel, and Ruslan Salakhutdinov. Siamese neural networks for one-shot image recognition. In ICML deep learning workshop, 2015.
[19] Adam Santoro, Sergey Bartunov, Matthew Botvinick, Daan Wierstra, and Timothy Lillicrap. Meta-learning with memory-augmented neural networks. In ICML, 2016.
[20] Łukasz Kaiser, Ofir Nachum, Aurko Roy, and Samy Bengio. Learning to remember rare events. In ICLR, 2017.
[21] Andreas Antoniou, Harrison Edwards, and Amos Storkey. How to train your MAML. In ICLR, 2019.
[22] Flood Sung, Yongxin Yang, Li Zhang, Tao Xiang, Philip H. S. Torr, and Timothy M. Hospedales. Learning to compare: Relation network for few-shot learning. In CVPR, 2018.
[23] Vinay K. Verma and Piyush Rai. A simple exponential family framework for zero-shot learning. In ECML-PKDD, 2017.
[24] Edgar Schönfeld, Sayna Ebrahimi, Samarth Sinha, Trevor Darrell, and Zeynep Akata. Generalized zero-and few-shot learning via aligned variational autoencoders. CVPR, 2019.
[25] Xuechen Li, Will Grathwohl, Eleni Triantafillou, David Duvenaud, and Richard Zemel. Few-shot learning for free by modelling global class structure. *2nd Workshop on Meta-Learning at NeurIPS*, 2018.

[26] George A Miller. WordNet: a lexical database for English. *Communications of the ACM*, 38(11):39–41, 1995.

[27] Tomas Mikolov, Ilya Sutskever, Kai Chen, Greg S Corrado, and Jeff Dean. Distributed representations of words and phrases and their compositionality. In *NIPS*, 2013.

[28] Jeffrey Pennington, Richard Socher, and Christopher Manning. Glove: Global vectors for word representation. In *EMNLP*, 2014.

[29] Kun Duan, Devi Parikh, David Crandall, and Kristen Grauman. Discovering localized attributes for fine-grained recognition. In *CVPR*, 2012.

[30] Andrea Frome, Greg S Corrado, Jon Shlens, Samy Bengio, Jeff Dean, Tomas Mikolov, et al. Devise: A deep visual-semantic embedding model. In *NIPS*, 2013.

[31] Mohammad Norouzi, Tomas Mikolov, Samy Bengio, Yoram Singer, Jonathon Shlens, Andrea Frome, Greg S Corrado, and Jeffrey Dean. Zero-shot learning by convex combination of semantic embeddings. *ICLR*, 2014.

[32] Yao-Hung H. Tsai, Liang-Kang Huang, and Ruslan Salakhutdinov. Learning robust visual-semantic embeddings. In *ICCV*, 2017.

[33] Yao-Hung H. Tsai and Ruslan Salakhutdinov. Improving one-shot learning through fusing side information. *Learning with Limited Data Workshop at NIPS*, 2017.

[34] David K Duvenaud, Dougal Maclaurin, Jorge Ibarra, Rafael Bombarell, Timothy Hirzel, Alán Aspuru-Guzik, and Ryan P Adams. Convolutional networks on graphs for learning molecular fingerprints. In *NIPS*, 2015.

[35] Michaël Defferrard, Xavier Bresson, and Pierre Vandergheynst. Convolutional neural networks on graphs with fast localized spectral filtering. In *NIPS*, 2016.

[36] Rianne van den Berg, Thomas N. Kipf, and Max Welling. Graph convolutional matrix completion. *arXiv preprint arXiv:1706.02263*, 2017.

[37] Yanbin Liu, Juho Lee, Minseop Park, Saehoon Kim, Eunho Yang, Sungju Hwang, and Yi Yang. Learning to propagate labels: transductive propagation network for few-shot learning. In *ICLR*, 2019.

[38] Jia Deng, Wei Dong, Richard Socher, Li-Jia Li, Kai Li, and Li Fei-Fei. ImageNet: A large-scale hierarchical image database. In *CVPR*, 2009.

[39] Ted Pedersen, Siddharth Patwardhan, and Jason Michelizzi. WordNet: Similarity: measuring the relatedness of concepts. In *HLT-NAACL*, 2004.

[40] Cathrine Wah, Steve Branson, Peter Welinder, Pietro Perona, and Serge Belongie. The Caltech-UCSD Birds-200-2011 Dataset. Technical Report CNS-TR-2011-001, California Institute of Technology, 2011.

[41] Pedro Morgado and Nuno Vasconcelos. Semantically consistent regularization for zero-shot recognition. In *CVPR*, 2017.
A Supplementary Material

A.1 Semantic Operators

In this section we show the semantic operators for the miniImageNet \cite{Vinyals2016} dataset and for the Caltech-UCSD Birds-200-2011 (CUB) \cite{WahCUB_200_2011} dataset.

Figure 3: Softmax normalized semantic operators for the miniImageNet \cite{Vinyals2016} dataset (with temperature $\tau = 1$) for a typical GFSL $5^+$-way $K$-shot task. Brighter color indicates higher similarity and block structures arise when similar classes are listed next to each other. The two largest blocks for example are animate and inanimate things. The blue lines indicate the four quadrants Seen-Seen, Seen-Novel, Novel-Seen and Novel-Novel. Note that the colormap clips the largest values on the diagonal to visualize the off-diagonal structure of the side information. (Best viewed in color.)
Figure 4: Softmax normalized semantic operators for the Caltech-UCSD Birds-200-2011 (CUB) [40] dataset (with temperature $\tau = 1$) for a typical GFSL $5^+$-way $K$-shot task. Brighter color indicates higher similarity and block structures arise when similar classes are listed next to each other. The largest continuous block for example are several different sparrow species. The blue lines indicate the four quadrants Seen-Seen, Seen-Novel, Novel-Seen and Novel-Novel. Note that the colormap clips the largest values on the diagonal to visualize the off-diagonal structure of the side information. (Best viewed in color.)