Parametric design of technological processes based on multidimensional geometric modelling

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Abstract. A method of parametric design of technological processes has been developed based on the theory of multidimensional geometry using elements of parametrization, multidimensional computational geometry and geometric modelling. The method allows a technological process to be described by a hypersurface defined by a set of lines of one- or two-dimensional frames in a multidimensional complex plot. Its parameters are regarded as a certain set of points and lines of a space which is defined as an intersection of the frame lines by hyperplanes of a given level. The article proposes a computer implementation of the method which provides a solution to engineering problems with required accuracy. A geometric model of a thread stitching technological process has been obtained. The model allows to set a parameter range and determine the modes of obtaining compounds of a given quality in a wide range of possible solutions depending on the package of product materials, the number of process equipment parameters and process criteria.

1. Introduction
The task of improving the design of technological processes is highly relevant at present. Because it can solve a number of problems: shorten the duration of process design and technological preparation of a process unit, provide the desired quality of production, increase productivity and ensure the competitiveness of enterprises. The main feature of technological processes is the fact that they are dependent on multiple parameters and criteria. As a result, prediction of specified modes and properties is a rather complicated and time-consuming process. Currently the majority of engineering methods of technological processes are based on the analysis of statistical information obtained from experimental studies and summing up of professional experience. However, it is difficult to select the appropriate and even more so optimal values of technological parameters without considering the multidimensional nature of arising mathematical dependencies.

The complexity of the problem also lies in the fact that the interpretation of multidimensional data by means of modern automated information processing systems is practically impossible since the attention of developers of such systems is focused on the representation of objects of only 3D physical or abstract spaces [1, 2]. Currently there are no specialized tools for automation of multidimensional data presentation. The reason for this is the lack of information technologies aimed at presenting multidimensional information on flat surfaces: monitor screens and paper documents. For the same reason methods for processing and managing multidimensional information by means of information systems are not yet developed.

At the same time research conducted by scientists of the Russian school of constructive geometric modelling makes a significant scientific contribution to the construction of models of multidimensional spaces that represent data on flat projection plots. The results of the research carried
out by this school are of a high scientific level and cannot be implemented because of the complexity of instrumental execution of multidimensional models in the form of documents by means of traditional geometric tools [3, 4]. Therefore, this richest heritage remains unused in the applied fields for the single reason of the lack of automation tools for the presentation and processing of multidimensional data which certainly became a factor hindering the development of geometric science itself.

The purpose of this study is to develop and present not only new geometrical models designed for true life technological processes, but also lay the foundation for the development of information tools that represent and process multidimensional geometric information. In the course of this research original automation tools for working with multidimensional data and for practical application in the studied technologies were developed.

2. Formulation of the problem
Suppose there are system components characterized by certain sets of parameters \(\{x_1, x_2, \ldots, x_n\}\). The state of the system is determined by the criteria \(\{y_1, y_2, \ldots, y_m\}\). The system determines the quantitative values of the criteria for each set of parameters. Each of the criteria can be given the values \(y_i^*\) and the possible deviations \(\Delta_i\) from these values \(y_i^*: |y_i - y_i^*| \leq \Delta_i\). For criteria values \(\{y_1^*, y_2^*, \ldots, y_m^*\}\) it is required to find a range of parameter values in the multidimensional space \(E^{n+m}\).

3. Theory
The proposed method is based on the following principles:
- Regardless of their physical nature technological processes can be described by a discretely defined hypersurface in the space of parameters and criteria which reflects the interrelations of parameters and criteria in the range of their allowable values.
- The hypersurface can be visualized as a set of one- or two-dimensional lines of a frame in a multidimensional complex plot.
- The parameters of the technological process working mode represent a set of points and lines of the parameter and criteria space which is defined as an intersection of the frame lines by level hyperplanes which are defined by a set of criteria values.

The theoretical basis of the method is formulated on the basis of the theory of multidimensional geometry and elements of the parametrization theory and methods of multidimensional computational geometry [5, 6]. The method includes the following steps: problem analysis; hypersurface modelling for defining the interrelations of parameters and criteria; hypersurface sections modelling; determination of the parameter values range.

3.1. Problem analysis
Problem analysis involves gathering and analyzing initial information, defining the dimension of the modeled space. This procedure is crucial for the accuracy and quality of the modelling and all subsequent results depend on it. The initial data for the construction are parameters of the system \(X = \{x_1, x_2, \ldots, x_n\}\) which influence the formation of the criteria \(Y = \{y_1, y_2, \ldots, y_m\}\). Based on the results of measurements or experimental observations the quantitative values of criteria are determined for each set of parameters in the form of a table. The selection of parameters and criteria (system components) is carried out in view of the requirements of the practical task. Their number can vary from two to several hundreds. Depending on the number of system components we differentiate cases with the number of parameters greater, equal or less than the number of criteria.

The dimension of the simulated space is determined by the number of system components \((n + m)\) needed to solve a specific parametric problem. For example, \(n + m = 2\) is a two-dimensional space, \(n + m = 3\) is a three-dimensional one, \(n + m = 4\) is a four-dimensional one and etc. If \(n + m = 3\) it is
advisable to consider the case where \( n = 2, m = 1 \). If \( n + m > 3 \) then two cases are possible \( n = m \) or \( n > m \). In particular, a space of dimension \( n + m = 4 \) may be presented by the following pairs of components: \( n = 2, m = 2 \) or \( n = 3, m = 1 \).

3.2. Hypersurface modelling

At this stage the hypersurface frame is formed either by using a found regression equation or experimentally based on approximation (interpolation) methods. In order to obtain data the simulation of the experiment should be performed using established mathematical planning methods. The selection of the experimental area of parameters and their variation intervals are determined by specific conditions of the process, such as technological capabilities of the equipment or manufacturing technology. The selection of the experiment plan is dictated by the intended model of the object of study. The results of the experiment are recorded in a statistical table.

Then a model of an \( E^{n+m} \) space is set by a rectangular coordinate system, such as \( O, x_i, x_j, f, \varphi, \chi \). Along the coordinate axes the values of the parameters \((x_i, x_j, x_k)\) and criteria \((f, \varphi, \chi)\) are plotted. The order of arrangement and designation of the coordinate axes is shown in Figure 1. The coordinate system contains the hypersurface frame which is set by a discrete set of points (frame nodes) that establish some interrelation of parameters and criteria. This is achieved by projecting the nodes onto the basic coordinate hyperplanes of projections \( x_i f, x_j f, x_i \varphi, x_j \varphi, x_i \chi, x_j \chi \). Projection points are used to construct approximating lines for the hypersurface stratification. For this purpose the parameter values along the \( x_1 \) axis in the projection plane \( x_i f, x_j \varphi, x_k \chi \) change continuously and the parameter values \( x_2 \) and \( x_3 \) change discretely. Similar constructions are performed for the planes of the projections \( x_i f, x_j \varphi, x_k \chi \). The type of the approximating (interpolating) function is based on the way the nodes (which represent experimental points) are located on the coordinate hyperplane.

3.3. Modelling of hypersurface sections

The hyperplanes of the level are perpendicular to the axes \( O f, O \varphi, O \chi \) that are projected to the level planes as level lines. Their position is defined by the values \( f^*, \varphi^* \) and \( \chi^* \), and they are located on all of the coordinate planes \( x_i f, x_j f, x_k \varphi, x_j \varphi, x_i \chi, x_j \chi \). Intersection points of the approximation lines with the given level lines are marked on the top part of Figure 1 as \( \ldots, 27, 26, 25; 24, 23, 22; 21, 20, 19; 18, 17, 16; 15, 14, 13; 12, 11, 10; 9, 8, 7; 6, 5, 4; 3, 2, 1 \). The obtained points are projected onto the planes of the projections \( x_i x_2 \) and are grouped in accordance to the values of the parameter \( x_3 \). Similar constructions are performed for the \( x_i x_3 \) and \( x_2 x_3 \) planes by grouping the points by the parameter values of the \( x_2 \) and \( x_3 \) respectively. Through these groups of points approximating curves are drawn (marked \( \ldots, 27, 26, 25; 24, 23, 22; 21, 20, 19; 18, 17, 16; 15, 14, 13; 12, 11, 10; 9, 8, 7; 6, 5, 4; 3, 2, 1 \) at the bottom part of Figure 1). The number of lines obtained is determined by the number of criteria \( f, \varphi, \chi \) and the values of the parameters \( X \).

3.4. Determination of the desired range of parameters

To determine the range of parameter values the intersection points of the frame lines are found on the \( x_i x_2 \) and \( x_2 x_3 \) planes for fixed \( x_j, x_k \) and \( x_1 \) parameters respectively \((A_1; A_2); (B_1; B_2); (C_1; C_2); (D_1; D_2); (L_1; L_2); (M_1; M_2))\). The resulting set of points forms the
ABC\( (A,B,C_1; A_2,B_2,C_2) \), DLM\( (D_1,L,M_1; D_2,L_2,M_2) \) lines that define the range of values of the parameters \( x_1, x_2, x_3 \) for given values of \( f^*, \varphi^* \) and \( \chi^* \). The coordinates of the \( N(N_1; N_2) \) point set a combination of parameter values for which \( f = f^* \), \( \chi = \chi^* \) and \( \varphi = \varphi^* \). The desired range of parameter values can be a finite number of points, curves, surfaces, hypersurfaces.

**Figure 1.** Scheme for constructing a model and sections of a hypersurface by level hyperplanes.
The process of constructing the range of parameter values is iterative. The number of iterations is determined by the dimensions of the simulated space. As a result of each iteration a \((n + m) - k\)-surface of the given hypersurface and level hyperplane intersection is constructed with a decrement in its dimension until the problem is solved in 2- or 3-planes. The coordinates of the desired area will define the combination of parameter values required for the given set of criteria values \([7, 8]\).

3.5. Computer implementation of the method

Software implementation of the engineering method is represented by the program "Hyperspace" [9] developed in Python programming language. The structure of the program library includes two classes. The main class gives functionality for working with the database, constructing a hypersurface frame, cross sections with level hyperplanes and searching for the intersection of hypersurfaces in spaces with identical dimensions. The additional class allows to process sets of points and curves on a plane. The software allows to build hypersurfaces based on existing points whose coordinates represent the values of various parameters of the processes obtained as a result of experiments. At the same time the program is capable of crossing the hypersurface by a hyperplane of a certain level (values are set by the user) and display the result on the screen. In the course of work the user can receive the necessary graphic and numerical information about previously created samples, make comparisons and analyze them according to the parameters of interest, explore the possibilities of obtaining them and choose the most rational option.

4. Approbation of the method

Approbation of the method of geometric modelling is carried out on the example of designing a technological process of sewing production. The thread stitching method of assembly remains dominant in the garments production which is due to the versatility of its use, the variety of composition parameters and properties, a comparative ease of manufacture and a wide selection of equipment for its production. In the conditions of modern manufacturing the mode of the thread stitching execution is often selected on the basis of previous experience and general recommendations. The lack of effective methods for determining the thread stitching parameters that meet the specified requirements leads to a decrease in the quality of garments as well as high financial, energy and labour costs of production [10].

4.1. Formulation of the research problem

There is a set of parameters of technological equipment:

- \(x_1\) – seam length \((L, \text{mm})\);
- \(x_2\) – needle thread tension \((F, \text{mm})\);
- \(x_3\) – linear density of sewing threads \((h, \text{tex})\).

The quality criteria for the thread seam are set:

- \(y_1\) – tearing load across the line \((P_H, \text{N})\);
- \(y_2\) – joint stiffness \((EI, \mu\text{Ncm}^2)\).

It is required to find the parameters for obtaining a thread stitching seam satisfying the following criteria: tearing load across the line \(y_1^* \geq P_H, \text{N}\); stiffness in the direction of base threads ranging from \(EI_1 \leq y_2^* \leq EI_2, \mu\text{Ncm}^2\).

4.2. Research object and methods

The quality of the thread seams is characterized by a number of indicators, the most important of which are durability and rigidity. The research object of this study are thread seams executed with a shuttle stitch (GOST 12807–2003). The stitches were performed with modern industrial equipment – a straight-line sewing machine of the shuttle stitch DDL – 8100e Juki. Polyester sewing threads of
various linear density $h = 28, 32, 37$ tex were used on a mixed cotton-polyester fabric with an insert of polyurethane threads.

Seam allowance width – 10 mm. The stitch length was changing from 2 to 4 mm with 1 mm interval. The thread needle tension was selected in accordance with established recommendations in the range from 0.2 to 0.6 daN with an interval of 0.2 daN. The pressure force of the foot on the fabric and the diameter of the needle – No. 90 were constant to provide a high-quality line for all samples of thread.

The tearing load was determined according to GOST 28073–89 by stretching perpendicular to the seam. Elementary tests were conducted on a tearing machine PT – 250M by the standard method. The stiffness of the seams was determined according to GOST 10550–93 by the console method on the PT-2 device.

4.3. Experimental indicators of the thread seam mechanical properties

Table 1 presents a portion of the experimental data for 37-tex linear density threads. During the experiment the measurement error did not exceed 3% at a confidence probability of $\alpha = 95%$.

Table 1. Durability and rigidity of the stitch.

| Linear density of sewing threads ($h$, tex) | Needle thread tension ($F$, mm) | Seam length ($L$, mm) | Tearing load across the line ($P_R$, N) | Joint stiffness ($EI$, $\mu$N·cm²) |
|-------------------------------------------|-------------------------------|----------------------|----------------------------------------|---------------------------------|
| 37                                        | 0.2                           | 2                    | 372                                    | 205630                         |
|                                           |                               | 3                    | 257                                    | 163598                         |
|                                           |                               | 4                    | 197                                    | 127123                         |
|                                           | 0.4                           | 2                    | 351                                    | 247971                         |
|                                           |                               | 3                    | 234                                    | 192614                         |
|                                           |                               | 4                    | 173                                    | 152050                         |
|                                           | 0.6                           | 2                    | 306                                    | 262618                         |
|                                           |                               | 3                    | 198                                    | 203279                         |
|                                           |                               | 4                    | 147                                    | 163152                         |

4.4. Building a geometric model of the thread stitching process

Using the experimental data and the “Hyperpass” program a geometric model of the thread stitching process was constructed. The model describes the relationship between the stiffness and durability of the seam and the technological parameters of its production. The resulting model is represented by a hypersurface frame in the form of an exploded diagram (Figure 2) and hypersurface sections with hyperplanes of a given level which define the range of process parameters (Figure 3).

5. Summary and conclusion

A method of parametric design of technological processes based on the theory of multidimensional geometry and geometric modelling has been developed. The method allows one to describe technological processes by a discretely given hypersurface of the space of parameters and criteria and to determine the range of parameter values with the required accuracy. A method of setting a hypersurface by a set of one- or two-dimensional frame lines on a multidimensional complex plot is proposed which allows localizing the studied parameter area discarding some of the equations that are not part of the search solution field and thereby reducing and simplifying the calculations and construction procedures.
Figure 2. Geometric model of the thread stitching process.

Figure 3. Determining the range of parameter values.
To implement the parametric design method the program "Hyperpass" was developed on the basis of a high-level programming language Python. The program allows building hypersurfaces using existing points, cross-sectioning the hypersurface with a certain level hyperplane and displaying the obtained results as a set of multi-dimensional model plots and parameter value range.

A multidimensional geometric model of a thread stitching technological process was created in accordance with the proposed method. Thread connections were made on a straight-line sewing machine of the shuttle stitch DDL – 8100e Juki with various parameters of the seaming process. With the help of the model a range of parameter values and modes for obtaining seams that meet the specified quality criteria were established.

References

[1] Bondarev A E and Galaktionov V A 2012 Multidimensional data analysis for multiparametric optimization problems using visualization methods Scientific Visualization (Moscow: National Research Nuclear University "MEPhI") vol 42 p 1-13
[2] Pilyugin V, Malikova E, Pasko A and Adzhiev V 2012 Scientific visualization as method of scientific data analysis Scientific Visualization vol 4(4) pp 56-70
[3] Radishchev V P 1940 About the image of multicomponent systems in the projections of regular multidimensional figures: methods for studying five-point systems (Moscow: Izv. SFHA) vol 13 rev 1 p 85
[4] Radishchev V P 1947 On the application of the geometry of four dimensions to the construction of disequilibrium physicochemical diagrams (Moscow: Izv. SFHA) vol 15 pp 129-34
[5] Volkov V Ya and Yurkov V Yu 2008 Multidimensional numerical geometry: monograph (Omsk: Publishing house OmGPU) p 244
[6] Yurkov V Yu 1998 Schubert calculus and multivalued correspondences Omsk Scientific Bulletin (Omsk: OmGTU) vol 2 pp 57-59
[7] Chizhik M A, Rasskazova M N and Starikov V I 2014 Structural approach to modelling multicomponent systems of fabric for products of clothing industry Izvestiya Vysshikh Uchebnykh Zavedenii, Seriya Teknologiya Tekstil'noi Promyshlennosti 6 (354) pp 89-94
[8] Chizhik M A, Nemirova L F and Moskovtsev M N 2018 Mathematical modelling of laser welding of textile thermoplastic fabrics Izvestiya Vysshikh Uchebnykh Zavedenii, Seriya Teknologiya Tekstil'noi Promyshlennosti 1 (373) pp 118–123
[9] Chizhik M A, Moskovtsev M N, Monastyrenko D P and Dorkin D V 2014 Certificate of state registration of computer programs No. 2014610165. "Hyperspass"
[10] Chizhik M A 2013 The graphic optimization model of the parameters of thread connections in clothes details Izvestiya Vysshikh Uchebnykh Zavedenii, Seriya Teknologiya Tekstil'noi Promyshlennosti 5 (347) p 86–90