Construction of multiple dependent state sampling plan for variables based on logistic distribution

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Abstract
Acceptance sampling plans are specific rules used in statistical quality control to reach a decision on acceptance or rejection of lots of manufactured products submitted for inspection subject to the quality requirements of the sampled units. In order to reduce the inspection time and cost, the variable sampling plans are used as it needs comparatively small sample size. The multiple dependent state sampling plan is one of the conditional sampling plans which acquires the sample information from present lot as well as the prior lots for disposition of a lot. The MDS plan is applicable in the situations where lots are submitted for inspection in sequence of production from a process having a constant proportion non-conforming. MDS plan under the assumption of normality are found in literature. In this article, a multiple dependent state sampling plan is developed for sentencing lots of products whose quality characteristics follows a Logistic distribution. The proposed plan is designed by considering the two points on the OC curve, by formulating it as a non-linear optimization problem, when standard deviation is known and unknown. The tables are constructed for the selection of the proposed plan parameters. The proposed plan is compared with the single variable sampling plan based on Logistic distribution.

Keywords: Multiple dependent sampling plan, logistic distribution, operating characteristic curve, acceptable quality level, limiting quality level

Introduction
The concept of multiple dependent (or deferred) state sampling plan for attribute, developed by Wortham and Baker (1976) \(^{(10)}\), is one of the conditional sampling procedure. In this plan, the decision on accepting or rejecting of a lot is based on the sample information from the current lot together with consecutive preceding lots (in the case of dependent state sampling) or upcoming lots (in the case of deferred state sampling). Vaerst (1982) \(^{(11)}\) further modified the operating procedure and characteristic function of multiple dependent (or deferred) state sampling plan for attributes by Wortham and Baker to make it on a par with chain sampling plan (ChSP-1). The Multiple Dependent State Sampling plan is applicable in the case of Type B situations (i.e., sampling from a continuous process) where lots are submitted for inspection serially in the order of production. Govindaraju and Subramani (1990) \(^{(16)}\) have developed a table and procedure for finding the multiple deferred state sampling plan of the type MDS-1 involving a minimum sum of risks for specified acceptable quality level and limiting quality level. Soundararajan and Vijayaraghavan (1990) \(^{(16)}\) have designed multiple deferred state MDS-1 (0,2) sampling plan indexed by acceptable quality level and limiting quality level. Balamurali and Jun (2007) \(^{(2)}\) developed MDS Sampling plan for variables, when the quality characteristics is normally distributed. The decision upon the acceptance of the lot is based on the states of the preceding lots or on the states of the forthcoming lots. Aijun Yan, Sanyang Liu and Xiaojuan Dong (2016) \(^{(1)}\) developed a multiple dependent state (MDS) sampling plan based on the coefficient of variation when the quality characteristic follows a normal distribution with unknown mean and variance. Muhammad Aslam, Balamurali and Chi-Hyuck Jun (2019) \(^{(12)}\) proposed a new multiple dependent state sampling plan which comprises the features of existing multiple dependent state sampling plan and repetitive group sampling plan based on the process capability index where the quality characteristic under investigation follows the normal distribution with unknown mean and unknown variance.
2. Logistic Distribution

Let $X$ be a random variable distributed according to a Logistic distribution with two parameters $\alpha$ (location) and $b$ (scale). The probability density function is given by

$$f(x; a, b) = \frac{e^{-(x-a)/b}}{b(1+e^{-(x-a)/b})^2}$$  \hspace{1cm} (1)$$

Where $\alpha$ is the mean and $b$ is a scale parameter proportional to the standard deviation. Thus, the mean and variance are respectively given by $\alpha$ and $b^2\pi^2/3$. Taking transformation as $\alpha = 0$ and $b = 1$, logistic distribution becomes standard logistic distribution with pdf and cdf respectively given by

$$f(x) = \frac{e^{-x}}{(1+e^{-x})^2}$$  \hspace{1cm} (2)$$

and

$$F(x) = \frac{1}{1+e^{-x}}$$  \hspace{1cm} (3)$$

3. Variable MDS Plan for Unknown Sigma

In multiple dependent state sampling plan, the past lots will be considered for acceptance of the current lot, consequently the accepting or rejecting decision is effectively postponed. Although the MDS plan is reported to be well-organized in terms of the sample size required. The following assumptions should be considered for the application of MDS plan.

1. Lots are submitted for inspection sequentially in the order of production from a process having a constant proportion non-conforming.
2. The consumer has assurance in the supplier and there should be no reason to believe that a particular lot is poorer than the preceding lots.
3. The quality characteristic follows a Logistic distribution.

3.1 Operating Procedure

Suppose the quality characteristic follows Logistic Distribution with mean $\mu$ and standard deviation $\sigma\sqrt{\pi^2/3}$. Let the upper specification limit be $U$. The proposed plan operates as follows:

Step1: From each submitted lot take a random sample of size $n$, $(x_1, x_2, ..., x_n)$ and compute

$$\bar{x} = \frac{1}{n}\Sigma_{i=1}^{n}x_i$$

and $S = \sqrt{\frac{1}{n-1}\Sigma_{i=1}^{n}(x_i - \bar{x})^2}$ is an unbiased estimate of $\sigma^2$.

Step2: Accept the lot if $v \geq k_a$, and reject the lot if $v < k_r$. If $k_r \leq v < k_a$, then accept the current lot provided that the foregoing $m$ lots were accepted on the condition that $v \geq k_a$.

Thus the proposed unknown sigma variables MDS plan is characterized by four parameters, namely $n$ – sample size, $m$ – preceding lots, $k_a$ – acceptance constant and $k_r$ – acceptance constant.

It is known that $\bar{X} \pm k_aS$ for large sample size approximately follows (Duncan, 1986) [6]:

$$\bar{X} \pm k_aS \sim N\left(\mu \pm k_a\sigma, \frac{\sigma^2}{n} + \frac{k_a^2\sigma^2}{2m}\right)$$

The lot quality is defined by

$$p = P(X > U|\mu) = 1 - L\left(\frac{U - \mu}{S}\right)$$  \hspace{1cm} (4)$$

Where $L(.)$ is the cumulative distribution function of the standard Logistic Variate. Now, the OC function of the variable MDS sampling plan, which gives the proportion of lots that are expected to be accepted for a lot quality $p$, is defined by

$$P_a(p) = P(v \geq k_a) + P(k_r \leq v < k_a)P(v \geq k_a)^m$$  \hspace{1cm} (5)$$

Where the first term in the right hand side represents the probability of accepting a lot based on a current sample and the second term is the probability of accepting a lot based on the states of the preceding lots.

The acceptable quality level and limiting quality level respectively defined by $p_1 = 1 - F(x_{p_1})$ and $p_2 = 1 - F(x_{p_2})$ where $x_{p_1}$ and $x_{p_2}$ are the values corresponding to standardized logistic distribution at $p_1$ and $p_2$.

Analogously, the lot acceptance probability for sigma unknown plan is rewritten as

$$P_a(p) = \Phi(\omega_2) + [\Phi(\omega_1) - \Phi(\omega_2)][\Phi(\omega_2)]^m$$  \hspace{1cm} (6)$$
Where

\[ \omega_1 = (x_{p_1} - k_r) \sqrt{\frac{n}{1+k_r^2}} \quad \text{and} \quad \omega_2 = (x_{p_2} - k_a) \sqrt{\frac{n}{1+k_a^2}} \]

The proposed sampling plan parameters are obtained by considering two points on the OC curve namely, \((p_1, 1-\alpha)\) and \((p_2, \beta)\). where \(\alpha\) and \(\beta\) are respectively producer’s risk and consumer’s risk associated with \(p_1\) and \(p_2\).

For specified \((p_1, 1-\alpha)\) and \((p_2, \beta)\) the parameters of the proposed plan should satisfy the following conditions

\[
P_a(p_1) = \Phi(\omega_{21}) + [\Phi(\omega_{11}) - \Phi(\omega_{21})][\Phi(\omega_{21})]^m \geq 1 - \alpha
\]

\[
P_a(p_2) = \Phi(\omega_{22}) + [\Phi(\omega_{12}) - \Phi(\omega_{22})][\Phi(\omega_{22})]^m \leq \beta
\]

where \(\omega_{11}\) be the value of \(\omega_1\) at \(p = p_1\), \(\omega_{21}\) be the value of \(\omega_2\) at \(p = p_1\), \(\omega_{12}\) be the value of \(\omega_1\) at \(p = p_2\) and \(\omega_{22}\) be the value of \(\omega_2\) at \(p = p_2\).

In order to obtain the optimum plan parameters, the problem is formulated as an optimization problem as follows

Minimize \(n\) Subject to

\[ P_a(p_1) \geq 1 - \alpha \]

\[ P_a(p_2) \leq \beta \]

\[ n \geq 1, k_a > k_r > 0 \]

To solve the non-linear optimization problem, the routine “fmincon” in the Matlab software was used which is based on the sequential quadratic programming algorithm proposed by Nocedal and Wright (1999) \(^{[13]}\).

4. Variable MDS Plan for Known Sigma

Suppose the quality characteristic follows a Logistic distribution, with upper specification limit \(U\), specified. The operating procedure for a proposed variable MDS sampling plan is as follows:

Step 1: Draw a sample of \(n'\) units from a lot and observe the measurements, \(x_1, x_2, \ldots, x_{n'}\) of quality characteristic, \(X\) and compute

\[ v = \frac{\bar{X} - \mu}{\sigma} \]

Step 2: Accept the lot if \(v \geq k_{a'}\) and reject the lot if \(v < k_r'\). If \(k_{a'} \leq v < k_r'\) then accept the current lot provided that the preceding \(m'\) lots were accepted on the condition that \(v \geq k_{a'}\). Reject the lot otherwise. The proposed variable MDS plan is designated by four parameters namely \(n', m', k_{a'}, k_r'\). The fraction non-conforming in a lot will be expressed as

\[ p = P[X > U | \mu] = 1 - L \left( \frac{\bar{X} - \mu}{\sigma} \right) \]

where \(L(.)\) is the cumulative distribution function of the standard Logistic Variate. Thus, the known mean is related to the fraction non-conforming \(p\) through \((9)\). Thus, the probability of acceptance in the lot is defined by

\[ P_a(p) = \Phi(\omega_2) + [\Phi(\omega_1) - \Phi(\omega_2)][\Phi(\omega_2)]^{n'} \]

Where

\[ \omega_1 = (k_{p_1} - k_r')\sqrt{n'} \quad \text{and} \quad \omega_2 = (k_{p_2} - k_{a'})\sqrt{n'} \]

The sampling plan parameters are obtained by considering two points on the OC curve, namely, \((p_1, 1-\alpha)\) and \((p_2, \beta)\).

Values of the plan parameters for \(m\) or \(m'\) = 1, 2 & 3 for several combinations of \((p_1, p_2)\) under \(\alpha = 0.05\) and \(\beta = 0.1\) are tabulated.

It is observed that the proposed plan becomes single variable sampling plan with parameter \(n, k\) \((or\ n', k_{a'})\) when \(m\) or \(m'\) goes to infinity. Also when \(k_{a} = k_r\), \((k_{a'} = k_r')\) the proposed becomes variable single sampling plan.

5. Numerical Example

Suppose that it is preferred to set up a MDS sampling plan by giving protection to the consumer and the producer in terms of quality levels \(p_1 = 0.01\) and \(p_2 = 0.05\) associated with \(\alpha = 0.05\) and \(\beta = 0.1\), when the quality characteristics is distributed according to Logistic distribution. The plan parameters are \(n = 21\), \(k_a = 2.1436\), \(k_r = 1.5482\), when \(m = 2\). For each submitted lot, take a random sample of size 21, and compute \(\bar{X}, S\) and evaluate \(v = \frac{\bar{X} - \mu}{S}\). Accept the lot if \(v \geq 2.1436\) and reject the lot if \(v \leq 1.5482\). If \(1.5482 \leq v < 2.1436\), then accept the current lot provided that the preceding two lots were accepted on the condition that \(v \geq 2.1439\), otherwise reject the lot.

6. Comparison of Sampling Plans

When sigma is unknown, the optimum plan parameters for \(p_1 = 1\%, \alpha = 5\%, p_2 = 5\%\) and \(\beta = 10\%\) are
For the same protection, the optimum parameters of single variable sampling plan (S. Geetha and R. Vijayaraghavan, 2011)\(^{17}\) are \(n = 32, k = 2.022\)

Fig 1: OC Curve for Variable MDS Plan for \(m = 1, 2 & 3\) and Variable single sampling Plan

It can be observed from the OC curve that the proposed plan provides almost same protection for consumer and producer as the variable single sampling plan, but with smaller sample size. The proposed MDS plan is compared with single sampling plan for variables under Logistic distribution with respect to sample size. Table 1 is constructed for comparing the MDS and SSP, and can be inferred that the sample size required for proposed plan is comparably less.

Table 1: Comparison of MDS plan with SSP for Variables

| \(P_1\) | \(P_2\) | Unknown Sigma | Known Sigma |
|--------|--------|---------------|-------------|
|        |        | MDS | SSP | MDS | SSP |
| 0.001  | 0.002  | 276 | 435 | 39  | 59  |
| 0.0025 | 0.005  | 213 | 335 | 21  | 59  |
| 0.005  | 0.020  | 38  | 59  | 9   | 15  |
| 0.010  | 0.050  | 21  | 32  | 6   | 11  |
| 0.020  | 0.100  | 6   | 23  | 3   | 10  |

7. Conclusion
The intention of this article is to construct a multiple dependent state (MDS) sampling inspection plans for variables, in which the quality characteristics of interest follows logistic distribution. The sample size of the proposed MDS sampling plan for variables is comparably lesser than the conventional variable single sampling plan for logistic distribution. The proposed plan assures nearly the same preservation for producer and consumer as the single sampling plan for logistic distribution, but with a smaller sample size. Hence the plan can be implemented when the testing is destructive and costly. Tables are designed at various values of AQL and LQL for the selection of proposed plan.

Table 2: Variable MDS Sampling plans for \(m \) (or \(m'\)) = 1 indexed by AQL and LQL for \(\alpha = 5\%\) and \(\beta = 10\%\)

| \(P_1\) | \(P_2\) | Unknown Sigma | Known Sigma |
|--------|--------|---------------|-------------|
|        |        | \(n\) | \(k_a\) | \(k_r\) | \(n'\) | \(k_a'\) | \(k_r'\) |
| 0.001  | 0.002  | 276  | 3.6717 | 3.4301 | 39  | 3.6863 | 1.8432 |
|        | 0.004  | 63   | 3.5324 | 3.0905 | 9   | 3.5386 | 3.0053 |
|        | 0.005  | 46   | 3.4869 | 2.9877 | 7   | 3.4951 | 2.8758 |
|        | 0.010  | 20   | 3.3434 | 2.6852 | 3   | 3.3598 | 2.4724 |
|        | 0.020  | 11   | 3.1965 | 2.4064 | 2   | 3.2235 | 2.0661 |
|        | 0.040  | 6    | 3.0449 | 2.1370 | 1   | 3.0851 | 1.6537 |
|        | 0.050  | 5    | 2.9948 | 2.0537 | 1   | 3.0398 | 1.5187 |
| 0.0025 | 0.005  | 213  | 3.1653 | 2.9256 | 21  | 3.2470 | 2.8941 |
|        | 0.006  | 130  | 3.1287 | 2.8338 | 14  | 3.2115 | 2.7882 |
|        | 0.010  | 48   | 3.0249 | 2.5897 | 7   | 3.1117 | 2.4907 |
Table 3: Variable MDS Sampling plans for \( m \) or \( m' \) = 2 indexed by AQL and LQL for \( \alpha = 5\% \) and \( \beta = 10\% \)

| \( P_1 \) | \( P_2 \) | Unknown Sigma | Known Sigma |
|-----------|-----------|---------------|-------------|
|           |           | \( n \)       | \( k_a \)    | \( k_r \)    | \( n' \)    | \( k_a' \)   | \( k_r' \)   |
| 0.005     | 0.005     | 46            | 3.4274      | 2.8023      | 7           | 3.4292      | 2.5770      |
| 0.010     | 0.010     | 20            | 3.2617      | 2.4593      | 3           | 3.2653      | 2.0443      |
| 0.020     | 0.020     | 11            | 3.0943      | 2.1484      | 2           | 3.1002      | 1.5078      |
| 0.050     | 0.050     | 6             | 2.8682      | 1.7703      | 1           | 2.87789     | 0.7850      |
| 0.005     | 0.010     | 170           | 2.7541      | 2.4436      | 36          | 2.7545      | 2.3857      |
| 0.020     | 0.020     | 38            | 2.5878      | 2.0485      | 9           | 2.5894      | 1.8492      |
| 0.040     | 0.040     | 15            | 2.4183      | 1.7010      | 4           | 2.4218      | 1.3046      |
| 0.050     | 0.050     | 12            | 2.3628      | 1.5957      | 3           | 2.3670      | 1.1264      |
| 0.001     | 0.001     | 100           | 1.9776      | 1.6685      | 34          | 1.9781      | 1.6011      |
| 0.020     | 0.020     | 15            | 1.7453      | 1.1458      | 6           | 1.7479      | 0.8529      |
| 0.040     | 0.040     | 6             | 1.5523      | 0.7686      | 3           | 1.5575      | 0.2342      |
| 0.050     | 0.050     | 39            | 1.5210      | 1.1235      | 18          | 1.5219      | 1.0039      |
| 0.001     | 0.010     | 100           | 1.3287      | 0.6989      | 5           | 1.3316      | 0.3852      |
| 0.020     | 0.020     | 10            | 1.2005      | 0.4308      | 2           | 1.2047      | 0.0000      |
| 0.040     | 0.040     | 63            | 1.4474      | 1.1281      | 31          | 1.4480      | 1.0533      |
| 0.050     | 0.050     | 13            | 1.2554      | 0.6836      | 7           | 1.2576      | 0.4346      |
| 0.500     | 0.500     | 3             | 0.9228      | 0.0000      | 2           | 0.9081      | 0.0000      |

Table 4: Variable MDS Sampling plans for \( m \) or \( m' \) = 3 indexed by AQL and LQL for \( \alpha = 5\% \) and \( \beta = 10\% \)

| \( P_1 \) | \( P_2 \) | Unknown Sigma | Known Sigma |
|-----------|-----------|---------------|-------------|
|           |           | \( n \)       | \( k_a \)    | \( k_r \)    | \( n' \)    | \( k_a' \)   | \( k_r' \)   |
| 0.001     | 0.010     | 21            | 3.2238      | 2.9098      | 3           | 3.2242      | 1.6603      |
| 0.020     | 0.020     | 11            | 3.0459      | 1.9625      | 2           | 3.0466      | 1.0069      |
| 0.050     | 0.050     | 6             | 2.8062      | 1.5667      | 1           | 2.8074      | 0.1271      |
| 0.005     | 0.010     | 181           | 2.7421      | 1.3710      | 38          | 2.7421      | 1.3710      |
| 0.020     | 0.020     | 40            | 2.5643      | 1.9247      | 9           | 2.5645      | 1.6163      |
| 0.050     | 0.050     | 12            | 2.3248      | 1.4370      | 3           | 2.3253      | 0.7362      |
| 0.010     | 0.040     | 31            | 2.1754      | 1.5431      | 9           | 2.1756      | 1.2169      |
| 0.050     | 0.050     | 22            | 2.1164      | 1.4162      | 7           | 2.1166      | 0.9999      |
| 0.100     | 0.100     | 9             | 1.9274      | 1.0415      | 3           | 1.9280      | 0.3057      |
| 0.050     | 0.100     | 58            | 1.9064      | 1.4388      | 21          | 1.9065      | 1.2655      |
| 0.010     | 0.080     | 16            | 1.7175      | 1.0107      | 6           | 1.7178      | 0.5713      |
| 0.200     | 0.200     | 6             | 1.5124      | 0.5893      | 3           | 1.5129      | 0.0000      |
| 0.040     | 0.080     | 76            | 1.5663      | 1.1834      | 34          | 1.5664      | 1.0686      |
| 0.200     | 0.200     | 11            | 1.2994      | 0.5462      | 6           | 1.2997      | 0.0875      |

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|    | 0.400 | 4 | 1.0511 | 0.0000 | 2 | 1.0468 | 0.0000 |
|----|-------|---|--------|--------|---|--------|--------|
| 0.050 | 0.100 | 67 | 1.4346 | 1.0464 | 33 | 1.4347 | 0.9292 |
|      | 0.200 | 13 | 1.2297 | 0.5423 | 8  | 1.2299 | 0.1758 |
|      | 0.500 | 3  | 0.8750 | 0.0000 | 2  | 0.8656 | 0.0000 |