Profile of the Electroweak Bubble Wall\textsuperscript{a}

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We study the CP-violating bubble wall by solving the coupled equations of motion derived from the two-Higgs-doublet model for the moduli $\rho_i$ and phases $\theta_i$ of the two scalars $(i = 1, 2)$ at the transition temperature. We find a solution that the CP-violating phase $\theta \equiv \theta_1 - \theta_2$ connecting the CP-conserving vacua strongly violates CP in the intermediate region while the moduli largely deviate from the kink shape there. We estimate the chiral charge flux through the wall surface, which may be efficient enough to produce the baryon asymmetry by the sphaleron process.

1 Introduction

It is essential in any scenario of the electroweak baryogenesis\textsuperscript{b} to know the spacetime-varying profile of the CP violation created at the electroweak phase transition. In literatures, some functional forms of the profile were assumed\textsuperscript{c, d}. They should be determined, however, by the dynamics of the gauge-Higgs system, in which the modulus and phase of the profile would be governed by some classical equations of motion derived from the effective Higgs potential at the transition temperature.

In a previous paper\textsuperscript{4}, we presented a solution in the two-Higgs-doublet model such that the CP-violating phase $\theta$ spontaneously generated becomes as large as $O(1)$ around the wall while it completely vanishes in the broken and symmetric phase limits. In the solution, the moduli $\rho_i$’s of the scalars were fixed to be the kink shape. This solution, which we call as ‘solution with kink ansatz’, is interesting since it does yield an efficient amount of the chiral charge flux through the wall surface, which will be turned into the baryon density in the symmetric phase region by the sphaleron transition. From the dynamical point of view, however, such a large $\theta$ would affect $\rho_i$’s in turn.

In this article, we solve coupled equations of motion for $(\rho_i, \theta_i)$ derived from the two-Higgs-doublet model by imposing the discrete symmetry $(\rho_1, \theta_1) \leftrightarrow (\rho_2, \theta_2)$. In this solution, which we call as ‘solution without kink ansatz’, $\rho_i$’s deviate largely from the kink shape while $\theta$ remains to be $O(1)$. We also estimate the chiral charge flux due to this solution, which is found also to be sufficiently large.

2 Equations of Motion

At the temperature $T_C$ of the phase transition, the neutral components of the Higgs fields develop VEV’s as the profile of the bubble wall:

$$\langle \Phi_i(x) \rangle = \left( \frac{0}{1+2\rho_i(x)e^{i\theta_i(x)}} \right), (i = 1, 2).$$

Let the effective Higgs potential at $T_C$ be $V_{eff}(\rho_i, \theta_i)$. Regarding the bubble wall as a static planar object, the equations of motion are given by

$$\frac{d^2 \rho_i(z)}{dz^2} - \rho_i(z) \left( \frac{d\theta_i(z)}{dz} \right)^2 - \frac{\partial V_{eff}}{\partial \rho_i} = 0, (2)$$

$$\frac{d}{dz} \left( \rho_i^2(z) \frac{d\theta_i(z)}{dz} \right) - \frac{\partial V_{eff}}{\partial \theta_i} = 0, (3)$$

where $z$ is the coordinate perpendicular to the wall. From the requirement that the gauge

\textsuperscript{a}Talk presented by F. Toyoda at 28th International Conference on High-energy Physics (ICHEP 96), Warsaw 1996. To be published in the Proceedings.

\textsuperscript{b}To be published in the Proceedings.
fields are pure-gauge type, a constraint equation, “sourcelessness condition”, is added to them:
\[ \rho_1^2(z) \frac{d \theta_1(z)}{dz} + \rho_2^2(z) \frac{d \theta_2(z)}{dz} = 0. \] (4)

As \( V_{eff} \) at \( T_C \), we postulate the same one examined in the previous paper:
\[ V_{eff}(\rho_i, \theta_i) = \frac{1}{2} m_1^2 \rho_1^2 + \frac{1}{2} m_2^2 \rho_2^2 + m_3^2 \rho_1 \rho_2 \cos \theta \\
+ \frac{\lambda_1}{8} \rho_1^4 + \frac{\lambda_2}{8} \rho_2^4 \\
+ \frac{\lambda_3 - \lambda_4}{4} \rho_1^2 \rho_2^2 - \frac{\lambda_5}{4} \rho_1^2 \rho_2^2 \cos 2\theta \\
- \frac{1}{2} (\lambda_6 \rho_1^2 + \lambda_7 \rho_2^2) \rho_1 \rho_2 \cos \theta \\
- (A \rho_1^2 + B \rho_1 \rho_2 \cos \theta \\
+ C \rho_1 \rho_2^2 \cos \theta + D \rho_2^4), \] (5)

where \( \theta \equiv \theta_1 - \theta_2 \). Here all the coefficients are assumed to include the radiative and the finite-temperature corrections, and are assumed to be real so that CP is violated spontaneously. The \( \rho^2 \) terms are expected to arise at finite temperatures so that the moduli of VEV’s in [3] take the kink shape for \( \theta = 0 \) as below.

2.1 Kink ansatz
In order for the kink shape moduli,
\[ \rho_1(z) = \frac{v}{2} \cos \beta (1 + \tanh(az)), \]
\[ \rho_2(z) = \frac{v}{2} \sin \beta (1 + \tanh(az)), \] (6)
to be dynamically realized for \( \theta(z) = 0 \), the parameters in [3] have to satisfy certain set of relations. Here \( v \cos \beta \) and \( v \sin \beta \) are VEV’s of \( \Phi_1 \) and \( \Phi_2 \) respectively, and \( 1/a \) is the common wall width. As in the previous paper, it is convenient to use the following parameters:
\[ b \equiv -\frac{m_3^2}{4a^2 \sin \beta \cos \beta}, \]
\[ c \equiv \frac{v^2}{32a^2} (\lambda_1 \cot^2 \beta + \lambda_2 \tan^2 \beta \\
+ 2(\lambda_3 - \lambda_5) - \frac{1}{2} \sin^2 \beta \cos^2 \beta) \\
= \frac{v^2}{8a^2} (\lambda_6 \cot \beta + \lambda_7 \tan \beta), \]
\[ d \equiv \frac{\lambda_5 v^2}{4a^2}. \]

\[ e \equiv \frac{v}{4a^2 \sin^2 \beta \cos^2 \beta} \left( A \cos^3 \beta \\
+ D \sin^3 \beta - \frac{4a^2}{v} \right) \\
= -\frac{v}{4a^2} \left( B \sin \beta + C \cos \beta \right) \]

where \( \lambda_3 \equiv \lambda_3 - \lambda_4 \), and we have used the relations among parameters to rewrite \( c \) and \( e \). The condition that \( (\rho_1, \theta_1) = (0, 0) \) and \( (\rho_1, \theta_1) = (v \cos \beta, v \sin \beta) \) are the local minima of \( V_{eff} \) for \( \theta(z) = 0 \) leads to inequalities among the parameters:
\[ b > -1, \quad b - 2c + 3c > -1 + (\lambda_3 - \lambda_3) v^2 / 4a^2. \] (8)

The equations of motion together with the “sourcelessness condition” are reduced to a single equation for \( \theta \) with the kink ansatz as examined in the previous paper.

2.2 Discrete symmetry without kink ansatz
When we do not impose the kink ansatz, we have many free parameters. To reduce the number of them, we require that \( V_{eff} \) is symmetric under \((\rho_1, \theta_1) \leftrightarrow (\rho_2, \theta_2)\) and that this discrete symmetry is not spontaneously broken. This means that the parameters in [3] should satisfy
\[ m_1^2 = m_2^2 = m^2, \]
\[ \lambda_1 = \lambda_2, \quad \lambda_6 = \lambda_7, \quad A = D, \quad B = C, \] (9)
and that tan \( \beta = 1 \). For
\[ \rho_1(z) = \rho_2(z) \equiv \rho(z) / \sqrt{2}, \]
\[ \theta_1(z) = -\theta_2(z) \equiv \theta(z) / 2, \] (10)
we have two coupled equations:
\[ \frac{d^2 \rho(z)}{dz^2} - \frac{1}{4} \rho(z) \left( \frac{d \theta(z)}{dz} \right)^2 - \frac{\partial V_{eff}}{\partial \rho} = 0 \] (11)
\[ \frac{1}{4} \frac{d}{dz} \left( \rho^2(z) \frac{d \theta(z)}{dz} \right) - \frac{\partial V_{eff}}{\partial \theta} = 0 \] (12)
while “sourcelessness condition” is automatically satisfied. Under the discrete symmetry, there remains only one free parameter in \( V_{eff} \) once \( b, c, d \) and \( e \) are given. We take \( \lambda_1 \) as the free parameter. Since tan \( \beta = 1 \), the parameters are given by
\[ m_3^2 = -2a^2 b, \quad m^2 = 4a^2 - m_3^2, \quad \lambda_5 = 4a^2 v^{-3} d, \]
\[ \lambda_3 = \lambda_5 + \frac{16a^2}{v^2}(c + 2) - \lambda_1, \]
\[ \lambda_6 = \frac{\lambda_1 + \lambda_3 - \lambda_5}{4} - \frac{8a^2}{v^2}, \]
\[ A = \frac{\sqrt{2}a^2}{v}(e + 4), \quad B = -A + \frac{4\sqrt{2}a^2}{v}. \]

3 Solutions with and without Kink Ansatz

3.1 Solution with kink ansatz

The parameter set for this type of solution is constrained from the asymptotic behaviors of \( \phi \) in the background of the kink-shape \( \rho \) in (2.6). One of the solutions presented in the previous paper \[ \text{is obtained for the set} (b, c, d, e) = (3, 12.2, -2, 12.2), \] which is an example satisfying the constraints \[ \text{The boundary conditions for this solution is} \]
\[ \theta_b \equiv \theta(+\infty) = 0, \quad \theta_s \equiv \theta(-\infty) = 0 \] in the broken(symmetric) phase limit. The profile of the bubble wall of this solution is given in Fig.1. Note that the CP-violating imaginary part is large just around the wall.

3.2 Solution without kink ansatz

We impose the boundary conditions \( \rho_b \equiv \rho(+\infty) = v \) and \( \rho_s \equiv \rho(-\infty) = 0 \) together with \( \theta_b = 0, \theta_s = 0 \) in the broken(symmetric) phase limit. We take the same values of \( b, c, d, e \) for comparison, though they do not need to be constrained. For definiteness, we set \( (v, a) = (246, 10) \) in the unit of GeV. The profile of the bubble wall is shown in Fig.2. The real part of the wall is drastically altered from the kink shape. The imaginary part of the wall is much larger than that of the solution with the kink ansatz.

4 Concluding Remarks

We have investigated the CP-violating profile of the wall by solving the equations of motion from the two-Higgs-doublet model. The most remarkable feature may be that, while CP is conserved in the broken and symmetric phase limits, the CP-violating phase \( \theta \) becomes \( O(1) \) in the intermediate region. The model could avoid the light scalars due to the Georgi-Pais theorem \( ^b \). We give some comments.

(1) The baryon asymmetry of the universe through the sphaleron process along the charge transport scenario is roughly given by
\[ \frac{\rho_B}{s} \simeq \frac{10^{-7} F_Q}{u} \tau, \]
where the entropy density is given by \( s = 2\pi^2 g_* T^3/45 \) with \( g_* \simeq 100 \), \( u \) is the wall velocity, \( \tau \) is the transport time within which the scattered fermions are captured by the wall and \( F_Q \) is the chiral charge flux through the CP-violating wall. Fig.3 gives the contour plot of \( F_Q \) due to the solution without kink ansatz for various choices of \( m/T \) and \( a/T \), where \( m \) is the relevant fermion mass \( ^b \). This should be compared with the corresponding one due to the solution with kink ansatz given in the previous paper. For the both solutions, \( F_Q \) may be said to be large enough because of the large \( \theta \)'s around the wall surface. This would be important since there are many possible mechanisms to diminish the net baryon asymmetry in the baryogenesis scenarios, whether spontaneous or charge transport.

(2) It may be interesting to see how the deviation of \( \rho \) from the kink shape affects the energy density of the wall per unit area, which is given as
\[ \mathcal{E} = \int_{-\infty}^{\infty} dz \left\{ \frac{1}{2} \sum_{i=1,2} \left[ \frac{d\rho_i}{dz} \right]^2 + \rho_i^2 \left( \frac{d\theta_i}{dz} \right)^2 \right\} + V_{\text{eff}}(\rho_1, \rho_2, \theta). \]

For the trivial solution \( \theta \equiv 0 \) with the kink ansatz, \( \mathcal{E}|_{\theta=0} = av^2/3 \). The energy densities of the above solutions are
\[ \Delta \mathcal{E} \equiv \mathcal{E} - \mathcal{E}|_{\theta=0} = \begin{cases} 2.056 \times 10^{-3} av^2 \sin^2 \beta \cos^2 \beta, \\ -3.473 \times 10^{-2} av^2 \sin^2 \beta \cos^2 \beta, \end{cases} \]
where the upper value is obtained with kink ansatz and the lower one without kink ansatz, and we fix \( \sin^2 \beta \cos^2 \beta = 1/4 \) because it is required for the latter case by the discrete symmetry. The enhancement factor \( \exp\left( -4\pi R_C^2 \Delta \mathcal{E}/T_C \right) \) to form the respective bubble over that with the trivial solution is
\[ \exp\left( -\frac{4\pi R_C^2 \Delta \mathcal{E}}{T_C} \right) = \begin{cases} 1.25 \quad \text{(kink)}, \\ 43.7 \quad \text{(nonkink)}, \end{cases} \]
where the radius of the critical bubble \( R_C \) is approximately given by \( \sqrt{3F_C/(4\pi av^2)} \) with \( F_C \) being the free energy of the critical bubble, and

\( ^b \) For a numerical method how to obtain the chiral transmission and reflection coefficients for such solutions, see.
$F_C = 145T$ is taken as various authors estimate $F_C \simeq (145 \sim 160)T$.

(3) Since $V_{eff}$ is symmetric under the exchange of $\theta \leftrightarrow -\theta$, it is expected that the bubble with $\theta > 0$ and that with $\theta < 0$ are created with the equal probability, so that the chiral charge flux or the net baryon number density would be averaged to vanish. An explicit CP breaking to violate the symmetry could avoid this difficulty as pointed out by Comelli et al. In another article we give an estimate that, even if the explicit breaking at the phase transition is as small as the Kobayashi-Maskawa scheme, the relative enhancement factor between the two kinds of bubbles is able to amount to $O(10)$ when the explicit breaking is taken into account with kink ansatz.

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