Z-scaling from tens of GeV to TeV

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Abstract

The concept of $z$-scaling reflecting the general features of internal particle substructure, constituent interaction and mechanism of particle formation at high-$p_T$ is reviewed. Experimental data on inclusive cross sections obtained at U70, ISR, SpS, RHIC and Tevatron are used in the analysis. The properties of data $z$-presentation such as the energy independence, power law and $A$-dependence are discussed. The properties of $z$-scaling are argued to be connected with the fundamental symmetries such as self-similarity, locality and fractality. The use of $z$-scaling to search for new physics phenomena in hadron-hadron and hadron-nucleus collisions is suggested. RHIC data used in our new analysis confirm $z$-scaling in $pp$ collisions. High-$p_T$ particle spectra at LHC energies are predicted. Violation of $z$-scaling characterized by the change of anomalous fractal dimension is considered as a new and complementary signature of new physics phenomena.

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1 Introduction

More prominent properties of particle are observed at high energy and transverse momentum [1]-[4]. The kinematic regime is used to perform calculations of physical quantities in the framework of perturbative Quantum Chromodynamics (QCD). Deviations of theoretical results obtained in a high-$p_T$ region using available experimental data are often considered as manifestations of new physics phenomena. At the same time we should note that nonperturbative effects are not well controlled by the theory. Therefore search for new scaling features of particle interaction in the region is of interest for development of the theory.

The fundamental problem of high energy physics is the origin of mass, spin and charge of particles. The study of particle interactions over a wide kinematic range and especially at small scales is necessary to resolve the problem and understand underlying physics phenomena. One assumes that all types of interactions are unified at small scales. New ideas and theories such as extra dimensions, anisotropy and fractality of space-time, quark compositeness, theories of Grand Unification, Super Symmetry and Super Gravity are intensively developed.

New feature ($z$-scaling) of high-$p_T$ particle production in hadron-hadron and hadron-nucleus collisions at high energies was established in [5]-[11]. The scaling function $\psi$ and scaling variable $z$ are expressed via experimental quantities such as the inclusive cross section $E d^3\sigma/dp^3$ and the multiplicity density of charged particles $dN/d\eta$. Data $z$-presentation is found to reveal symmetry properties (energy independence, $A$-dependence, power law). The properties of $\psi$ at high $z$ are assumed to be relevant to the structure of space-time at small scales [12, 13]. The function $\psi(z)$ is interpreted as the probability density to produce a particle with a formation length $z$. The concept of $z$-scaling and the method of data analysis are developed for description of different particles (charged [6, 7, 14] and neutral [10, 11] hadrons, direct photons [8, 15], jets [9]) produced in high energy hadron-hadron and hadron-nucleus interactions. The proposed method is complementary to a method of direct calculations developed in the framework of QCD [16] and methods based on Monte Carlo generators [17]-[24]. Therefore we consider that the use of the method allow to reduce some theoretical uncertainties which are ambiguously estimated by theory.

In the report the general concept of $z$-scaling, the properties of data $z$-presentation and some new results of data analysis are presented. The fundamental principles such as self-similarity, locality, fractality and scale-relativity are formulated and discussed in the framework of the $z$-scaling concept. Verification $z$-scaling validity at RHIC and LHC is suggested. Violation $z$-scaling is considered to be indication of new physics phenomena.

2 Z-scaling

In the section we discuss underlying ideas of $z$-scaling. A general scheme of data $z$-presentation is described. Physical meaning of the introduced quantities is explained.

2.1 Locality

The idea of $z$-scaling is based on the assumptions [25] that gross feature of inclusive particle distribution of the process (1) at high energies can be described in terms of
the corresponding kinematic characteristics

\[ M_1 + M_2 \rightarrow m_1 + X \]  

(1)

of the constituent subprocess written in the symbolic form (2)

\[ (x_1M_1) + (x_2M_2) \rightarrow m_1 + (x_1M_1 + x_2M_2 + m_2) \]  

(2)

satisfying the condition

\[ (x_1P_1 + x_2P_2 - p)^2 = (x_1M_1 + x_2M_2 + m_2)^2. \]  

(3)

The equation is the expression of locality of hadron interaction at constituent level. The \( x_1 \) and \( x_2 \) are fractions of the incoming momenta \( P_1 \) and \( P_2 \) of the colliding objects with the masses \( M_1 \) and \( M_2 \). They determine the minimum energy, which is necessary for production of the secondary particle with the mass \( m_1 \) and the four-momentum \( p \). The parameter \( m_2 \) is introduced to satisfy the internal conservation laws (for baryon number, isospin, strangeness, and so on).

The equation (3) reflects minimum recoil mass hypothesis in the elementary subprocess. To connect kinematic and structural characteristics of the interaction, the quantity \( \Omega \) is introduced. It is chosen in the form

\[ \Omega(x_1, x_2) = m(1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}, \]  

(4)

where \( m \) is a mass constant and \( \delta_1 \) and \( \delta_2 \) are factors relating to the anomalous fractal dimensions of the colliding objects. The fractions \( x_1 \) and \( x_2 \) are determined to maximize the value of \( \Omega(x_1, x_2) \), simultaneously fulfilling the condition (3)

\[ d\Omega(x_1, x_2)/dx_1|_{x_2=x_2(x_1)} = 0. \]  

(5)

The fractions \( x_1 \) and \( x_2 \) are equal to unity along the phase space limit and cover the full phase space accessible at any energy.

2.2 Self-similarity

Self-similarity is a scale-invariant property connected with dropping of certain dimensional quantities out of physical picture of the interactions. It means that dimensionless quantities for the description of physical processes are used. The scaling function \( \psi(z) \) depends in a self-similar manner on a single dimensionless variable \( z \). It is expressed via the invariant cross section \( E\sigma^3/d\sigma^3 \) as follows

\[ \psi(z) = -\frac{\pi s}{(dN/d\eta)\sigma_m} J^{-1} E\frac{d^3\sigma}{dp^3} \]  

(6)

Here, \( s \) is the center-of-mass collision energy squared, \( \sigma_m \) is the inelastic cross section, \( J \) is the corresponding Jacobian. The factor \( J \) is the known function of the kinematic variables, the momenta and masses of the colliding and produced particles.

The function \( \psi(z) \) is normalized as follows

\[ \int_0^\infty \psi(z)dz = 1. \]  

(7)
The relation allows us to interpret the function $\psi(z)$ as a probability density to produce a particle with the corresponding value of the variable $z$.

We would like to emphasize that the existence of the function $\psi(z)$ depending on a single dimensionless variable $z$ and revealing scaling properties is not evident in advance. Therefore the method proposed to construct $\psi(z)$ and $z$ could be only proved a posteriori.

### 2.3 Fractality

Principle of fractality states that variables used in the description of the process diverge in terms of the resolution. This property is characteristic for the scaling variable

$$z = z_0 \Omega^{-1}, \quad (8)$$

where

$$z_0 = \sqrt{\hat{s}_\perp / (dN/d\eta)}. \quad (9)$$

The variable $z$ has character of a fractal measure. For the given production process (1), its finite part $z_0$ is the ratio of the transverse energy released in the binary collision of constituents (2) and the average multiplicity density $dN/d\eta |_{\eta=0}$. The divergent part $\Omega^{-1}$ describes the resolution at which the collision of the constituents can be singled out of this process. The $\Omega(x_1, x_2)$ represents relative number of all initial configurations containing the constituents which carry fractions $x_1$ and $x_2$ of the incoming momenta. The $\delta_1$ and $\delta_2$ are the anomalous fractal dimensions of the colliding objects (hadrons or nuclei). The momentum fractions $x_1$ and $x_2$ are determined in a way to minimize the resolution $\Omega^{-1}(x_1, x_2)$ of the fractal measure $z$ with respect to all possible subprocesses (2) subjected to the condition (3). The variable $z$ was interpreted as a particle formation length.

As we will show later the scaling function of high-$p_T$ particle production is described by the power law, $\psi(z) \sim z^{-\beta}$. Both quantities, $\psi$ and $z$, are scale dependent. Therefore we consider the high energy hadron-hadron interactions as interactions of fractals. In the asymptotic region the internal structure of particles, interactions of their constituents and mechanism of real particle formation manifest self-similarity over a wide scale range.

### 2.4 Scale-relativity

The properties of particle interactions in the space-time reflect symmetries of Nature. The principle of motion relativity has been used to formulate non-relativistic and relativistic theory. The special theory deals with only inertial coordinate systems while the expression of physical laws in the general theory of relativity should be written into any curvilinear coordinate system. Principle of general relativity states that "the laws of physics must be of such a nature that they apply to systems of references in any kind of motion". Application of relativity principle can be extended to state of scale of reference system [13]. There are convincing evidence to consider that scale as well as other quantities characterizing a reference frame should be used to describe a particle state in a high-$p_T$ range. In this range elementary probes such as direct photons, high-$p_T$ hadrons and jets are not point-like objects. They have a complicated structure. The last is resulted from interactions of quarks, gluons and heavy bosons which are fundamental objects of theory. Therefore experimentally measurable quantities should
depend on a ratio between scales of a studied object and a probe. In other words the variables used in the description of the process depends on the resolution.

A generalization of the motion-relativity principle to the scale-relativity principle requires that “the laws of physics must be of such a nature that they apply to systems of references in any kind of motion and whatever its state of scale”.

In the framework of data $z$-presentation the mechanism of particle formation is described by the scaling function $\psi(z)$. Both quantities, $\psi$ and $z$, depend on the resolution $\Omega^{-1}$ while the anomalous fractal dimension $\delta$ as found from our analysis of numerous experimental data to be resolution independent. We consider that the study of $z$-scaling of high-$p_T$ particle production over a wide kinematic range of $p_T$ and $\sqrt{s}$ and determination of $\delta$ could give new insight in theory of scale-relativity [12, 13].

3 Properties of data $z$-presentation

In the section we present and discuss some properties of data $p_T$- and $z$-presentation of high-$p_T$ particle production in $pp$, $\bar{p}p$ and $pA$ collisions. We show that the scaling functions for different processes reveal the same properties. They are the energy independence of $\psi$, the power behavior of $\psi$ at high-$z$ and $A$-dependence.

3.1 Energy independence of $\psi(z)$

It is well known that numerous experimental data on high-$p_T$ particle spectra manifest the strong dependence on collision energy $\sqrt{s}$. The effect enhances as the transverse momentum of produced particle increases.

3.1.1 $\pi^+$-mesons

Figures 1(a) and 2(a) show the dependence of the inclusive cross sections of $\pi^+$ and $K^+$ meson production in $pp$ collisions on the transverse momentum $p_T$ at incident proton momentum $p_L = 70, 200, 300, 400$ and $800$ GeV/c and an angle $\theta_{cm} \simeq 90^\circ$. The data were obtained at Protvino [26] and Batavia [27, 28]. Transverse momenta of produced particles shown in Figures 1(a) and 2(a) change from 1 to 10 GeV/c. We would note that data [27] and [28] corresponding to the momentum $p_L = 400$ GeV/c are complementary and are in a good agreement each other. As seen from Figures 1(a) and 2(a) particle spectra have power behavior at $p_L = (200 - 800)$ GeV/c. The effect of kinematic boundary is visible at the end of spectrum at $p_L = 70$ GeV/c. The energy independence of data $z$-presentation means that the scaling function $\psi(z)$ has the same shape for different $\sqrt{s}$ over a wide $p_T$ range.

As seen from Figures 1(b) and 2(b) $z$-presentation of the same data sets demonstrates the energy independence of $\psi(z)$ over a wide collision energy and transverse momentum range. We would like to emphasize that the data [28] used in our new analysis confirm our earlier results [5, 29].

3.1.2 $\pi^0$-mesons

The PHENIX Collaboration published the new data [30] on inclusive spectrum of $\pi^0$-mesons produced in $pp$ collisions in central rapidity range at RHIC energy $\sqrt{s} = 200$ GeV. The transverse momenta of $\pi^0$-mesons were measured up to 13 GeV/c.
The data $p_T$- and $z$-presentations for $\pi^0$-meson spectra obtained at ISR [31]-[36] and RHIC [30] are shown in Figures 3(a) and 3(b). One can see that $p_T$-spectra of $\pi^0$-meson production reveal the properties similar to that found for charged hadrons. The new data [30] on $\pi^0$-meson inclusive cross sections obtained at RHIC as seen from Figure 3(b) are in a good agreement with our earlier results [10]. Thus we can conclude that available experimental data on high-$p_T$ $\pi^0$-meson production in $pp$ collisions confirm the property of the energy independence of $\psi(z)$ in $z$-presentation.

### 3.1.3 Charged hadrons

The STAR Collaboration published new data [37] on inclusive cross sections of charged hadrons produced in $pp$ collisions at RHIC energy $\sqrt{s} = 200$ GeV. The RHIC data and other ones obtained at U70 [26], ISR [38] and Tevatron [27, 28] are shown in Figure 4(a). Charged hadron spectra were measured over a wide kinematic range $\sqrt{s} = (11.5 - 200)$ GeV and $p_T = (0.5 - 9.5)$ GeV/$c$. The strong energy dependence and the power behavior of particle $p_T$-spectrum are found to be clearly. The energy independence of data $z$-presentation shown in Figure 4(b) is confirmed. It is of interest to verify the asymptotic behavior of $\psi$ at $\sqrt{s} = 200$ GeV and reach value of $z$ up to 30 and more.

### 3.1.4 Direct $\gamma$

Direct photons are considered as the best probes of constituent interactions at high-$p_T$. The calculations of direct photon cross sections in the framework of QCD are developed in next-to-leading order [39]. The basic mechanisms of direct photon production are considered to be Compton scattering ($gq \rightarrow \gamma q$), and annihilation process ($\bar{q}q \rightarrow \gamma g$). These are direct mechanisms of photon production. In high-$p_T$ range direct photons are also produced indirectly via bremsstrahlung of quarks ($qq \rightarrow qq\gamma$, $qg \rightarrow qg\gamma$). The contribution of indirect mechanisms of high-$p_T$ photon production can be significant. However there are significant theoretical uncertainties due to the choice of structure and fragmentation functions and renormalization, factorization and fragmentation scales. Therefore any reliable estimates of direct photon cross sections allowing to reduce some of theoretical uncertainties is of interest.

The results of data analysis for direct photon production in $\bar{p}p$ collisions are presented in Figure 5. Inclusive cross sections versus the transverse momentum at $\sqrt{s} = (24 - 1800)$ GeV over the range $p_T = (4 - 110)$ GeV/$c$ are shown in Figure 5(a). Data used in the analysis are obtained by UA1 [40], UA2 [41], UA6 [42], CDF [43] and D0 [44] Collaborations. Data $z$-presentation (see Figure 5(b)) demonstrates the energy independence of the scaling function $\psi$ of high-$p_T$ direct photon production over a wide kinematic range.

### 3.1.5 Jets

First observation of jets in $\bar{p}p$ collisions at SpS was considered as compelling confirmation of parton structure of hadrons. In high energy collisions of hadrons copiously jet production due to hard parton scattering is observed. A jet represents a group of moving collimated particles. In the framework of QCD jets are distinguished to can be quark and gluon. Quark and gluon jets are initiated by fastest quark and gluon, respectively. It should be noted that the mechanism of jet formation is insufficient
explored and not clearly understood till now. Therefore we hope that scaling features of jet production could be useful to obtain additional constraints for models of jet formation.

In Figures 6(a) we show the invariant cross sections of inclusive jet production in \( \bar{p}p \) collisions at \( \sqrt{s} = 630 \) and 1800 GeV. These experimental data are obtained by CDF [45] and D0 [46] Collaborations. A clear energy dependence of the cross section is observed to be. Difference between cross sections at \( \sqrt{s} = 630 \) and 1800 GeV increases with transverse energy of jet. Data \( z \)-presentation shown in Figure 6(b) demonstrates independence on collision energy \( \sqrt{s} \). The anomalous fractal dimensions for jet production in \( pp \) and \( \bar{p}p \) collisions was found to be constant and equal to 1 [9].

3.2 A-dependence of \( \psi(z) \)

A comparison of particle yields in hadron-hadron and hadron-nucleus collisions is a basic method to study the nuclear matter influence on particle production. The elementary process is considered as probe of more complex system like nucleus. The difference between cross sections of particle production on free and bound nucleons was considered as an indication of unusual physics phenomena like EMC-effect, \( J/\psi \)-suppression and Cronin effect.

A change of the shape of \( p_T \) spectra is considered to be evidence that the mechanism of particle formation in nuclear matter is modified. Therefore it is convenient to compare scaling functions corresponding to different \( pA \) processes over a wide range of \( \sqrt{s} \) and \( p_T \).

The search for scaling features of particle formation in \( pA \) as well as in \( pp \) collisions and the study of their dependence on the atomic weight \( A \) is of interest for development of theory.

\( A \)-dependence of \( z \)-scaling of hadron production in \( pA \) collisions was studied in [7, 11]. It was established \( z \)-scaling for different nuclei (\( A = D - Pb \)) and types of produced particles (\( \pi^{\pm,0}, K^{\pm}, \bar{p} \)). To compare the scaling functions for different nuclei the symmetry transformation

\[
z \rightarrow \alpha_A z, \quad \psi \rightarrow \alpha_A^{-1} \psi
\]

has been used. The parameter \( \alpha \) of the scale transformation (10) depends on the atomic weight \( A \). It was parameterized by the formula \( \alpha(A) = 0.9A^{0.15} \).

Figures 7(a) demonstrates the spectra of \( \pi^+ \)-mesons produced in proton-nucleus collisions at \( \sqrt{s} = 11.5 \) and 27.4 GeV and \( \theta_{cm}^{NN} \simeq 90^0 \). Our new data analysis includes experimental data obtained at Protvino [26] and Batavia [27, 28]. The data [28] extend transverse momentum range up to \( p_T = 8.5 \) GeV/c. A good compatibility of [27] and [28] data sets in the overlapping region was observed. Solid and dashed lines are obtained by fitting of the data for \( W, Pb \) and \( D \), respectively. They demonstrate the strong dependence of \( p_T \)-spectra on collision energy \( \sqrt{s} \).

Figure 7(b) shows the \( z \)-presentations of the same data. The obtained results is the new confirmation of \( z \)-scaling of high-\( p_T \) hadron production in \( pA \) collisions. The universality of the scaling function \( \psi \) for different nuclei means that mechanism of high-\( p_T \) particle formation in nuclear matter reveals property of self-similarity.

3.3 Power law

One of the general properties of data \( z \)-presentation is the power law of scaling
Such behavior of $\psi$ as seen from Figures 1(b)-7(b) is observed for different particles (hadrons, direct photons and jets) produced at $z > 4$. The data sets demonstrate a linear $z$-dependence of $\psi(z)$ on the log-log scale at high $z$. The quantity $\beta$ is the slope parameter. The value of the slope parameter $\beta$ is found to be constant with high accuracy and it is independent of energy $\sqrt{s}$ over a wide high transverse momentum range. Some indications (for $\pi^0$ mesons, charged hadrons, direct photons, jets) are obtained that the value of slope parameter for $pp$ collisions, $\beta_{pp} > \beta_{\bar{p}p}$.

The existence of the power law, means, from our point of view, that the mechanism of particle formation reveals fractal behavior.

4 Multiplicity charged particle density

The important ingredient of $z$-scaling concept is the multiplicity density of charged particles, $dN/d\eta(s, \eta)$. The scaling function $\psi$ and the scaling variable $z$ is proportional to $[dN/d\eta]^{-1}$. In the first case the quantity is included in the expression (5) to normalize the function $\psi$ and to give the physical meaning for it as a probability density to produce a particle with formation length $z$. In the second one (7) the multiplicity density is taken at $\eta = 0$. Therefore $z$ is proportional to energy of elementary subprocess per one particle produced in the initial hadron collision.

The energy dependence of the multiplicity charged particle density for inelastic and non-single diffractive $pp$ and $\bar{p}p$ collisions is shown in Figure 8(a) [47]. The collision energy $\sqrt{s}$ changes from 14 to 1800 GeV. New data for $dN/d\eta(s, \eta)$ as well as for inclusive cross section $Ed^3\sigma/dp^3$ for $pp$ collisions at RHIC energies are of interest for verification of $z$-scaling.

5 $\gamma/\pi^0$ ratio for $pp$ and $\bar{p}p$

The properties of the scaling function for direct $\gamma$ and $\pi^0$-meson can be used to estimate the dependence of the $\gamma/\pi^0$ ratio of inclusive cross sections on transverse momentum at LHC energies.

The asymptotic behavior of $\psi(z)$ was found to be described by the power law for $\pi^0$-meson and direct photon production in $pp$ and $\bar{p}p$ collisions. The slope parameters are satisfied to the relations $\beta_{\gamma pp} > \beta_{\gamma pp}^{\pi^0}$, $\beta_{\gamma pp}^{\pi^0} > \beta_{\pi^0 pp}$, $\beta_{\pi^0 pp} > \beta_{\gamma pp}^\pi$ and $\beta_{\pi^0 pp}^\pi > \beta_{\gamma pp}$. As seen from Figure 3(b) the cross section data [30] of $\pi^0$-meson production in $pp$ collisions obtained by the PHENIX collaboration at RHIC are in a good agreement with the asymptotic behavior of $\psi(z)$.

Figure 8(b) shows the $\gamma/\pi^0$ ratio of inclusive cross sections as a function of the transverse momentum $p_T$ at $\sqrt{s} = 5.5$ and 14 TeV. The ratio was found to be different for $pp$ and $\bar{p}p$ collisions. It increases with $p_T$. The ratio has the cross-over point at $p_T \simeq (60-70)$ GeV/c for $pp$ and $\bar{p}p$ collisions, respectively.

6 $z - p_T$ plot

The $z - p_T$ plot is the dependence of the variable $z$ on the transverse momentum
$p_T$ of produced particle for a given process. The plot allows us to determine the high transverse momentum range interesting for verification of $z$-scaling and experimentally inaccessible up to now.

Figure 9(a) shows the $z - p_T$ plot for the $pp \to \pi^+ X$ process at $\sqrt{s} = (24 - 14000)$ GeV. As seen from Figure 1(b) the scaling function $\psi(z)$ was measured up to $z \simeq 30$. The function $\psi(z)$ demonstrates the power behavior at $z > 4$. Therefore the kinematic range $z > 30$ is of more preferable for experimental investigations of $z$-scaling violation. The boundary $z = 30$ corresponds to the different values of the transfers momentum $p_T$ depending on collision energy $\sqrt{s}$.

Figure 9 (b) shows our predictions of the dependence of the inclusive cross section $E d^3\sigma/dp^3$ on the transverse momentum $p_T$ for $\pi^+$-mesons produced in $pp$ collisions at the ISR, RHIC and LHC energies and an angle $\theta_{cm} = 90^0$. The verification of the predictions is of interest to determine the region of the scaling validity and search for new physics phenomena.

7 "$\delta$-jump"

Mechanism of particle formation at high transverse momenta in $z$-presentation is described by the power law (11). Such behavior depends on the values of the anomalous fractal dimension of colliding particles, $\delta_1$ and $\delta_2$. The dimensions for hadrons, direct photons and jets produced in $pp$ collisions were found to satisfy the relation $\delta_h < \delta_\gamma < \delta_{jet}$ and to be independent of $\sqrt{s}$ and $p_T$. The anomalous fractal dimension for nucleus $\delta_A$ is expressed via the dimension for nucleon $\delta_N$ as follows $\delta_A = A \cdot \delta_N$.

Figure 10(a) shows the dependence of the anomalous fractal dimension $\delta$ for $\pi^0$-meson production in $pp$ and $\bar{p}p$ [48] collisions on energy $\sqrt{s}$. The value of $\delta_h = 0.5$ used in our previous data analysis is confirmed by the new data [30] on inclusive cross section (see Figure 3(b)) obtained by the PHENIX Collaboration at RHIC. Figure 10(b) gives evidence that values of the slope parameter $\beta$ of the scaling function for $pp$ and $\bar{p}p$ collisions differ each other at $z > 6$.

The change of the fractal dimension $\delta$ or "$\delta$-jump" is considered as an indication on new mechanism of particle formation. It is assumed that the energy dependence of the quantity is especially sensitive in the high-$p_T$ range. Therefore the study of $z$-scaling at higher $\sqrt{s}$ and $p_T$ is of interest for search for new physics phenomena.

8 Direct $\gamma$ and $\eta^0$-meson yields in $pp$ and $pPb$ collisions at RHIC and LHC

The scaling properties of data $z$-presentation for $pp$ and $pA$ collisions can be used to estimate particle yields in the kinematic region experimentally inaccessible at present time and to compare with other model predictions.

Figures 11 and 12 shows our predictions of the dependence of the inclusive cross section $E d^3\sigma/dp^3$ on the transverse momentum $p_T$ for direct photon (a) and $\eta^0$-mesons (b) produced in $pp$ and $pPb$ collisions at RHIC and LHC energies and an angle $\theta_{cm}^{NN} = 90^0$. The data on the cross sections [33, 42, 49] obtained at ISR energy $\sqrt{s} = (24 - 63)$ GeV are also shown for comparison.
9 Conclusions

The general concept of $z$-scaling for particle production in hadron-hadron and hadron-nucleus collisions with high transverse momenta was reviewed. The development of new method of data analysis based on data $z$-presentation was presented. The scaling function $\psi(z)$ and scaling variable $z$ were shown to be expressed via the experimental quantities, the invariant inclusive cross section $E d^3\sigma/dp^3$ and the multiplicity charged particles density $dN/d\eta(s, \eta)$.

Physical interpretation of the scaling function $\psi(z)$ and variable $z$ as a probability density to produce a particle with formation length $z$ was argued. The quantity $z$ was shown to reveal the property of the fractal measure and $\delta_{1,2}$ are the anomalous fractal dimensions of colliding particles. It was argued that $z$-scaling reflects the fundamental symmetries such as locality, self-similarity, fractality and scale relativity.

Results of new analysis of experimental data on the inclusive cross sections obtained at U70, ISR, SpS, Tevatron and RHIC were presented. The scaling properties of data $z$-presentation such as the energy independence, $A$-dependence and the power law were discussed. A complementary confirmation of $z$-scaling for $\pi^0$-meson and charged hadron production in $pp$ collisions at RHIC was obtained.

New measurements of the multiplicity charged particle density and the inclusive cross sections of particle production in the experimentally inaccessible kinematic region for the study of $z$-scaling were suggested. The change of the anomalous fractal dimension ("$\delta$-jump") was suggested to consider as a new and complementary signature of new physics phenomena of high-$p_T$ particle production in hadron-hadron and hadron-nucleus collisions at high energies. The $z-p_T$ plot was used to determine the regions that are of more preferable for experimental search for $z$-scaling violation. The properties of data $z$-presentation were used to predict high-$p_T$ particle spectra at RHIC and LHC energies.

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Figure 1. (a) The inclusive differential cross sections for $\pi^+$-mesons produced in $pp$ collisions at $p_{lab} = 70, 200, 300, 400$ and $800$ GeV/c and $\theta_{cm} \simeq 90^0$ as functions of the transverse momentum $p_T$. Experimental data are taken from [26, 27, 28]. (b) The corresponding scaling function $\psi(z)$.

Figure 2. (a) The inclusive differential cross sections for $K^+$-mesons produced in $pp$ collisions at $p_{lab} = 70, 200, 300, 400$ and $800$ GeV/c and $\theta_{cm} = 90^0$ as functions of the transverse momentum $p_T$. Experimental data are taken from [26, 27, 28]. (b) The corresponding scaling function $\psi(z)$.
Figure 3. (a) The dependence of the inclusive cross section of $\pi^0$-meson production on the transverse momentum $p_{\perp}$ in $pp$ collisions at $\sqrt{s} = 30, 53, 62$ and 200 GeV and an angle $\theta_{cm}$ of 90°. The experimental data are taken from [30]-[36]. (b) The corresponding scaling function $\psi(z)$.

Figure 4. (a) Inclusive cross sections of charged hadron production in $pp$ collisions versus transverse momentum at U70, ISR, Tevatron and RHIC energies $\sqrt{s} = (11.5 - 200)$ GeV and an angle $\theta_{cm}$ of 90°. Experimental data are taken from [26, 27, 28, 37, 38]. (b) The corresponding scaling function $\psi(z)$.
Figure 5. (a) The dependence of the inclusive cross section of direct photon production on the transverse momentum $p_T$ in $p\bar{p}$ collisions at $\sqrt{s} = (24 - 1800)$ GeV. Experimental data obtained by the UA1, UA2, UA6, CDF and D0 Collaborations are taken from [40]-[44]. (b) The corresponding scaling functions.

Figure 6. (a) Inclusive cross sections of jet production in $p\bar{p}$ collisions versus transverse momentum at Tevatron energies $\sqrt{s} = 630$ and 1800 GeV and $\theta_{cm} \simeq 90^0$ obtained by CDF and D0 Collaborations. Experimental data are taken from [45, 46]. (b) The corresponding scaling function $\psi(z)$. 
Figure 7. (a) Inclusive differential cross section for $\pi^+$-mesons produced in $pA$ collisions at $\sqrt{s} = 11.5$ and 27.4 GeV/c and $\theta_{NN} \approx 90^\circ$ as a function of the transverse momentum $p_T$. Solid and dashed lines are obtained by fitting of the data for $W, Pb$ and $D$, respectively. Experimental data are taken from [26, 27, 28]. (b) The corresponding scaling function $\psi(z)$.

Figure 8. (a) The multiplicity charged particle density $dN/d\eta$ as a function of collision energy $\sqrt{s}$ at $\eta = 0$ for $pp$ and $\bar{p}p$ collisions. Experimental data are taken from [47]. (b) The $\gamma/\pi^0$ ratio of inclusive cross sections versus the transverse momentum $p_T$ in $pp$ and $\bar{p}p$ collisions at $\sqrt{s} = 5.5$ and 14. TeV.
Figure 9. (a) The $z - p_T$ plot and (b) the dependence of the inclusive cross section of $\pi^+$-meson production in $pp$ collisions on the transverse momentum $p_T$ at $\sqrt{s} = (24 - 14000)$ GeV and an angle $\theta_{cm}$ of 90°.

Figure 10. (a) The dependence of the anomalous fractal dimension $\delta(s)$ on collision energy $\sqrt{s}$. (b) The scaling function $\psi(z)$ of $\pi^0$-meson production in $pp$ and $\bar{p}p$ collisions on the transverse momentum $p_T$ at energy $\sqrt{s} = 30 - 200$ and 540 GeV and an angle $\theta_{cm}$ of 90°, respectively. The experimental data are taken from [30]-[36] and [48].
Figure 11. The dependence of inclusive cross sections of direct photon (a) and $\eta^0$-meson (b) production on the transverse momentum $p_T$ in $pp$ collisions at $\sqrt{s} = (24 - 14000)$ GeV. Experimental data are taken from [33, 42, 49]. Solid lines and points ($\ast, \triangle, \ast, \times$) are the calculated results.

Figure 12. (a) The dependence of inclusive cross sections of direct photon (a) and $\eta^0$-meson (b) production on the transverse momentum $p_T$ in $pPb$ collisions at $\sqrt{s} = (31 - 8800)$ GeV. Points ($\triangle, \circ, \ast, \ast, +$) are the calculated results.