Some comments on $\bar{n}p$-annihilation branching ratios into $\pi\pi$-, $\bar{K}K$- and $\pi\eta$-channels

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Abstract

We give some remarks on the $\bar{n}p$-partial branching ratios in flight at low momenta of antineutron, measured by OBELIX collaboration. The comparison is made to the known branching ratios from the $p\bar{p}$-atomic states. The branching ratio for the reaction $\bar{n}p \rightarrow \pi^+\pi^0$ is found to be suppressed in comparison to what follows from the $p\bar{p}$-data. It is also shown, that there is no so called dynamic $I = 0$-amplitude suppression for the process $N\bar{N} \rightarrow K\bar{K}$.

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1 Some useful definitions

Let us consider first the $N\bar{N}$-system. By definition $|I, I_3>$ is the isospin wave function of the $N\bar{N}$-system with isospin $I$ and its projection $I_3$. Using notations of ref. [1], we write the following relations between the physical states $|N\bar{N}>$ and states of definite isospin $|I, I_3>$:

$$|p\bar{p}> = \frac{1}{\sqrt{2}} [ |1, 0> + |0, 0>], \quad |n\bar{n}> = \frac{1}{\sqrt{2}} [ |1, 0> - |0, 0>].$$

(1)

On the contrary in terms of physical states the wave function $|I, I_3>$ looks for isosinglet state as

$$|0, 0> = -\frac{1}{\sqrt{2}} [ |p\bar{p}> + |n\bar{n}>],$$

(2)

and for isotriplet as

$$|1, -1> = |n\bar{p}>, \quad |1, 0> = \frac{1}{\sqrt{2}} [ |p\bar{p}> - |n\bar{n}>], \quad |1, 1> = |n\bar{p}>.$$  

(3)

Each wave function is normalized as:

$$<N\bar{N} | N\bar{N}> = 1, \quad <I, I_3 | I, I_3> = 1.$$

Let us also define wave function for the hadron final state $|a>$ with definite isospin $I$: $|a>I$. We shall use the notations $\hat{V}_a^I$ for transition operator from initial $|I, I_3>N\bar{N}$-state to $|a>I$ and

$$V_a^I = \langle a | \hat{V}_a^I | I, I_3 >_{N\bar{N}},$$

(4)

is matrix element for this operator. It doesn’t depend on $I_3$. Evidently that

$$\hat{V}_a^I | J, J_3 >_{N\bar{N}} = 0$$

in the case $I \neq J$.

2 Matrix elements for the transitions $N\bar{N} \rightarrow \pi\pi$ and $N\bar{N} \rightarrow K\bar{K}$.

Consider only the transitions to the final $\pi\pi$-states from the initial $N\bar{N} S$-wave ($^3S_1$). In this case the $\pi\pi$-system is produced in $I = 1$ isospin state. So there is only one operator $\hat{V}_\pi^1$. The expansion of the $|\pi\pi>$-wave function in terms of the states with definite isospin has the form:

$$|\pi^+\pi^-> = \frac{1}{\sqrt{3}} |0, 0> + \frac{1}{\sqrt{2}} |1, 0> + \frac{1}{\sqrt{6}} |2, 0>,$$

(5)

$$|\pi^+\pi^0> = \frac{1}{\sqrt{2}} |1, 1> - \frac{1}{\sqrt{2}} |2, 1>.$$
Thus using definitions (1), (3) and (4), we get
\[ < \pi^+ \pi^0 | \hat{V}_\pi^1 | \bar{n}p > = \frac{1}{\sqrt{2}} V_\pi^1, \]
\[ < \pi^+ \pi^- | \hat{V}_\pi^1 | p\bar{p} > = \frac{1}{2} V_\pi^1. \]

It means, that the processes \( p\bar{p} \rightarrow \pi^+\pi^- \) is to be at least by factor two suppressed in comparison to \( \bar{n}p \rightarrow \pi^+\pi^0 \).

Let us now consider the transitions into \( K\bar{K} \)-final states. Isospin wave functions for \( K\bar{K} \)-states have the following form:
\[ | K^+ K^- > = \frac{1}{\sqrt{2}} [ | 1, 0 > - | 0, 0 > ], \]
\[ | K^0 \bar{K}^0 > = - \frac{1}{\sqrt{2}} [ | 1, 0 > + | 0, 0 > ], \]
\[ | K^+ \bar{K}^0 > = | 1, 1 >, \hspace{1cm} | K^0 K^- > = - | 1, -1 >. \]

In this case \( | K\bar{K} > \) final state is indeed a mixture of both \( I = 0 \) and \( I = 1 \) isospin states \( (I_3 = 0) \). Hence both operators \( \hat{V}_K^1 \) and \( \hat{V}_K^0 \) give contribution to this reaction, and
\[ K < 1, I_3 | \hat{V}_K^1 | 1, I_3 >_{NN} = V_K^1, \hspace{1cm} K < 0, 0 | \hat{V}_K^0 | 0, 0 >_{NN} = V_K^0. \]

In terms of \( V_K^1 \) and \( V_K^0 \) we may calculate matrix elements between the physical states:
\[ < K^+ K^- | \hat{V}_K | p\bar{p} > = \frac{V_K^0 + V_K^1}{2}, \hspace{1cm} < K^0 \bar{K}^0 | \hat{V}_K | p\bar{p} > = \frac{V_K^0 - V_K^1}{2}, \]
\[ < K^+ \bar{K}^0 | \hat{V}_K | \bar{n}p > = V_K^1. \]

The matrix elements (9),(10) are related to the corresponding partial cross-sections:
\[ \sigma = 4\pi \frac{q}{k} | < f | V | i > |^2, \]
where \( q \) and \( k \) are final and initial c.m. momenta. We get the agreement for the expression (10) with what is given in the ref.[1], but expressions (9) differ from that of ref. [1]. Namely, redefining the operators according to equation (32) of ref. [1], we get:
\[ \sigma(p\bar{p} \rightarrow K^+ K^-) + \sigma(p\bar{p} \rightarrow K^0 \bar{K}^0) = | A_0 |^2 + | A_1 |^2 \]
(11)
and
\[ \sigma(\bar{n}p \rightarrow K^+ \bar{K}^0) = 2 | A_1 |^2. \]
(12)
Notice, that factor 2 in the right-hand side of the equation (12) is not present in equation (35) of the paper [1]. Historically this factor was also lost in the papers [2, 3], and this error was
reproduced later in some review papers, see, e.g. [1, 2]. That is why the conclusion of the papers [1, 2, 3] on \( I = 0 \)-amplitude suppression seems to be incorrect and is to be revised. We shall discuss this problem in Section 4.

3 Some relations between branching ratios in \( p\bar{p} \)- and \( \bar{n}p \)-annihilation processes

Let us first consider the \( \pi\pi \)-case. By definition of the branching ratio we have:

\[
\text{Br}_{\pi^+\pi^0}(\bar{n}p) = \frac{\sigma(\bar{n}p \to \pi^+\pi^-)}{\sigma(\bar{n}p \to \text{all})}
\]

and similar expression for the \( p\bar{p} \)-case. So the ratio of branching ratios is:

\[
\frac{\text{Br}_{\pi^+\pi^0}(\bar{n}p)}{\text{Br}_{\pi^+\pi^-}(p\bar{p})} = \frac{\sigma(\bar{n}p \to \pi^+\pi^0)}{\sigma(\bar{n}p \to \text{all})} : \frac{\sigma(p\bar{p} \to \pi^+\pi^-)}{\sigma(p\bar{p} \to \text{all})}.
\] (13)

Notice, that at low energies, if only \( S \)-wave contribute, we have:

\[
\sigma(p\bar{n} \to \pi^+\pi^0) = 4\pi \frac{3}{4} |<\pi\pi | \hat{V}_1 \pi | p\bar{n}>|^2 \frac{q}{k}
\] (14)

and

\[
\sigma(p\bar{p} \to \pi^+\pi^-) = 4\pi \frac{3}{4} C^2(k) |<\pi\pi | \hat{V}_1 \pi | p\bar{p}>|^2 \frac{q}{k}.
\] (15)

Here \( C^2(k) \) is the Gamov factor,

\[
C^2(k) = \frac{2\pi}{ka_B}/[1 - \exp(-\frac{2\pi}{ka_B})],
\]

and \( a_B = 57.6 \text{fm} \) is the \( p\bar{p} \)-Bohr radius. Taking into account (13)-(15), we get:

\[
\frac{\text{Br}_{\pi^+\pi^0}(\bar{n}p)}{\text{Br}_{\pi^+\pi^-}(p\bar{p})} \approx 2R,
\] (16)

where \( R \) is now a well defined and finite quantity:

\[
R = \frac{\lim_{k \to 0}[\beta C^{-2}(k)\sigma^{ann}(p\bar{p} \to \text{all})]}{\lim_{k \to 0}[\beta\sigma^{ann}(\bar{n}p \to \text{all})]}.
\] (17)

From the experimental data of refs. [6] and [8] we get the value of \( R \) at low momenta of incident antiproton \( (p_{lab} = 50 - 70 \text{MeV}/c) \):

\[
R = \frac{32 \pm 2}{25.3 \pm 1.0} \approx 1.26 \pm 0.10.
\] (18)

Notice, that this value coincides with what follows from the experimental data on annihilation of antiprotons off deuteron [7]. So we conclude, that the data [8] on total annihilation
\( \tilde{n}p \)-cross section are in agreement with the results of quite independent experiment for the annihilation of antiproton on deuteron [7]. One may find the more detailed discussion of value \( R \) extracted from the different data on deuteron and some heavier nuclei in the review paper [9].

A case of kaons looks very similar. Using eqs. (9)-(10) as well as the definition of the ratio \( R \) (17), one gets the following relation between branchings for the reactions \( p\bar{p} \rightarrow K^+K^- \), \( p\bar{p} \rightarrow K^0\bar{K}^0 \) and \( \tilde{n}p \rightarrow K^+\bar{K}^0 \):

\[
\frac{|V_K^1|^2 + |V_K^0|^2}{2|V_K^1|^2} = R \frac{Br(p\bar{p} \rightarrow K^+K^-) + Br(p\bar{p} \rightarrow K^0\bar{K}^0)}{Br(\tilde{n}p \rightarrow K^+\bar{K}^0)}. \tag{19}
\]

4 The analysis of the experimental situation

In Ref. [8] the branching ratio for the reaction \( \tilde{n}p \rightarrow \pi^+\pi^0 \) in the momentum interval 50-150 MeV/c (S-wave) was found to be equal:

\[
Br(\tilde{n}p \rightarrow \pi^+\pi^0) = (2.3 \pm 0.4) \times 10^{-3}. \tag{20}
\]

This value is to be compared with what follows from the \((p\bar{p})\)-atomic experiment for the reaction \( p\bar{p} \rightarrow \pi^+\pi^- \). The separation of the S- and P-wave contribution to last reaction was provided in the Refs. [10, 11]. So we get for the branching ratio into the \( \pi^+\pi^- \)-channel from atomic S-state:

a) \((2.37 \pm 0.23) \times 10^{-3} \) [10]; b) \((2.04 \pm 0.17) \times 10^{-3} \) [11].

Substituting these numbers into eq. (16), we get the evident contradiction. It means, that something is wrong with the branchings. If one believes in the experimental branchings for both \( \tilde{n}p \)- and \( \bar{p}p \)-channels, the only possible way to solve the problem is to suggest, that the \( p\bar{p} \)-atomic wave function at small distances has an abnormal admixture of the \( \tilde{n}n \)-component. We shall discuss this hypothesis in the next Section.

Now let us discuss a case of kaons. The only information on branching ratio \( N\bar{N} \rightarrow KK \) for isospin \( I = 1 \) channel for long time was available from the old data for absorption of antiproton on deuteron [12],

\[
Br(\bar{p}n \rightarrow K^0K^-) = (1.47 \pm 0.21) \times 10^{-3}.
\]

Nowadays the OBELIX collaboration gives [1] (S-wave):

\[
Br(\bar{n}p \rightarrow K^+K^-) = (0.92 \pm 0.23) \times 10^{-3}.
\]

It means, that the branching into \( K^+\bar{K}^0 \) is:

\[
Br(\bar{n}p \rightarrow K^+\bar{K}^0) = 2Br(\bar{n}p \rightarrow K^+K^-) = (1.84 \pm 0.46) \times 10^{-3}.
\]

It is seen, that this last number for branching does not contradict the old data by Bettini et al. [12].
At the same time from the ASTERIX experiments [3,13] we have:

\[
Br(p\bar{p} \rightarrow K^+K^-) = (1.08 \pm 0.05)10^{-3},
\]

\[
Br(p\bar{p} \rightarrow K^0\bar{K}^0) = (0.83 \pm 0.05)10^{-3}.
\]

Using these values and taking into account equation (19), we get

\[
| V_K^0 | \approx 1.3 | V_K^1 |.
\]

So we conclude, that there is no evidence for any suppression of \( I = 0 \)-amplitude for the reaction \( N\bar{N} \rightarrow K\bar{K} \) in the S-wave. The dynamic selection rule for this process, declared in the Refs.[1-5] is the consequence of incorrect formulae for branchings used in refs.[1,2].

Let us also discuss a case of \( \pi\eta \)-channel. From the experiment [8] it follows, that in the momentum interval 150-250 MeV/c (P-wave)

\[
Br(\bar{n}p \rightarrow \pi^+\eta) = (0.99 \pm 0.22)10^{-3}.
\]

At the same time from the paper [10] we have:

\[
Br(p\bar{p} \rightarrow \pi^0\eta) = (7.7 \pm 1.13)10^{-4}.
\]

So again we come to the conclusion that the ratio

\[
\frac{Br(\bar{n}p \rightarrow \pi^+\eta)}{Br(p\bar{p} \rightarrow \pi^0\eta)}
\]

is significantly less than \( 2R \) (see eq.(16)).

## 5 Possible solution of the problem for the \( N\bar{N} \rightarrow \pi\pi \) branchings

In line with the papers [1,14,15] let us suppose, that the wave function for \( p\bar{p} \)-atom at small distances is a superposition of \( | p\bar{p} > \) and \( | n\bar{n} > \) configurations, i.e.:

\[
| \psi_{at} > = \frac{1}{\sqrt{1 + \epsilon^2}}[| p\bar{p} > + \epsilon | n\bar{n} >].
\]

In terms of the states of definite isospin it means, that

\[
| \psi_{at} > = \frac{1}{\sqrt{2(1 + \epsilon^2)}}[(1 - \epsilon) | 1, 0 > -(1 + \epsilon) | 0, 0 >].
\]

So it follows immediately, that:

\[
\frac{Br(\psi_{at} \rightarrow \pi^+\pi^-)}{Br(\bar{n}p \rightarrow \pi^+\pi^0)} = \frac{(1 - \epsilon)^2}{2(1 + \epsilon^2)R}
\]
A case $\epsilon = 0$ corresponds to the usual suggestion of the absence of the $n\bar{n}$-component in the $p\bar{p}$-atom. In the limit $\epsilon = -1$ the atomic state is that of definite isospin $I = 1$. Substituting the experimental numbers for the $\pi\pi$-branchings (see Section 4), we conclude, that it is possible to fit the parameter $\epsilon$ so the equation (23) is justified. For example, taking $\text{Br}(p\bar{p} \to \pi^+\pi^-) = 1.87$ (lower limit) and $\text{Br}(\bar{n}p \to \pi^+\pi^0) = 2.7$ (upper limit), we get $\epsilon = -2.24$, that corresponds to the value of mixing angle $\cos \alpha = 1/\sqrt{1 + \epsilon^2}; \quad \alpha \approx 66^\circ$. It means, that the admixture of the $\bar{n}n$-component should be large to fit the experimental data.

6 Conclusion

a) The data on the $\bar{n}p$-total annihilation cross section, presented by OBELIX Collaboration [8], are in agreement with the data on the value of the ratio $R$, determined from the absorption of antiprotons on deuteron (see [7] and references in [9]).

b) The branching ratios for the reactions $\bar{n}p \to \pi^+\pi^0$ and $\bar{n}p \to \pi^+\eta$ at low energies [8] seem to be too large in comparison to what follows from the analysis of the known branching ratios for the $p\bar{p}$-atom.

c) The branching for the reaction $\bar{n}p \to K^+K_S$ is in agreement to the known branching for the reaction $\bar{p}n \to K^0K^-$ from the deuteron data [12]. There is no suppression for the $I = 0$ $N\bar{N} \to K\bar{K}$-reaction amplitude in S-wave (no specific dynamic selection rule).

d) Some admixture of the $|n\bar{n}>$-component in the $p\bar{p}$-atomic wave function may help in solving the problems with the branching into two pions and $\pi\eta$. However to solve this problem, the admixture should be large enough.

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