Rectangular Plates of a Trapezoidal Cross-Section Subjected to Thermal Load

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Abstract. The method to calculate rectangular tanks as a system of bi-directionally bent plates with the use of separated plates methodology is a widely known and currently most often used approach to verify numerical calculations obtained from computer software supporting the design process, in which spatial operation of tanks is taken into account. In these calculations, due to their static scheme, it is possible to distinguish supported plates, plates fixed on four edges and plates with one edge free and three fixed. The subject literature contains publications on plates or tanks with walls of a constant thickness, however, there are very few references on plates or tanks with walls of linearly variable thickness. The wall plate of a tank is subject to hydrostatic load or soil pressure and might be exposed to thermal load in the case of, i.e. filling it with hot liquid or during climate action. The article presents the results of static calculations for rectangular plates with a linearly variable thickness, a trapezoidal cross-section, three fixed edges and one edge free, subjected to permanent and thermal loads. Trapezoidal cross-section walls are optimal when used in structures where load distribution is triangular in shape (hydrostatic load). For tanks recessed in the ground, the load on walls increases along with the depth of foundations and obtains the highest value in the bottom part of the wall. Trapezoidal or triangular load distribution causes that the highest values of bending moments in the vertical cross-section occur at the point where a wall connects to the bottom, while the upper free edge of the tank takes zero value. The above statements lead to the conclusion that structural and economic considerations should determine the choice of walls with a thickness increasing along with the tank depth, since it is more economical in terms of material usage. The impact of thermal load is often neglected in the design process, which may cause operational problems and even pose a threat to the safety of use. In addition to the numerical analysis, the article presents the results of model tests for a plate with a linearly variable thickness made of resin, subjected to thermal load. The convergence of the obtained results proves the correctness of calculations and tests performed. This also contributes to the recognition of statics in rectangular plates of a trapezoidal cross-section.

1. Introduction
The increase in ecological awareness among societies, economic development and higher expenditures on generally understood environmental protection have given rise to a growing demand for implementing engineering facilities such as tanks in sewage treatment plants or on large farms [1, 2, 3, 4]. The second reason for the increasing interest in engineering structures is a gradual social enrichment and the corresponding demand for, e.g. recreational swimming pools or floating platforms [5].

Despite access to software supporting the design process, it is still difficult to calculate rectangular tanks as spatially working structures. One of the analytical procedures used for calculation is the method...
of separated plates that considers a tank as a system of component plates, which are: walls, bottom plate and cover plate (for closed tanks). The first stage of calculations examines all plates as operating independently, specifying their static schemes. The wall plate referred to in the article had its side edges fixed in adjacent walls, while its bottom was fixed in the bottom plate. Calculations carried out for each plate separately may lead to obtaining varied results for supporting moments for two plates converging in the corner. In fact, there is only one supporting moment value. Bearing in mind the correctness of results, if the difference in bending moments is not large, i.e. up to 10%, a higher value is assumed to be reliable. However, if the difference in results is greater, researchers apply the balancing of moments, i.e. by the Hardy Cross method, which consists in separating the difference in supporting moments of plates converging at the edge of the tank proportionally to their rigidity [1]. The method of calculating rectangular plates with a constant thickness has been quite satisfactorily recognised in the literature [3, 6, 7, 8, 9, 10], while the topic of calculating plates with a variable thickness has been discussed in a significantly fewer number of publications [11, 12]. The wall plate of a tank is subject to hydrostatic load or soil pressure and might be exposed to thermal load. Temperature can work on building objects in two ways: by uniform heating or cooling of the entire cross-section of the element or by occurrence of the temperature difference between planes of the element [2, 3, 11]. The plate analysed in the article had the following static scheme: three fixed edges, one edge free and a linearly variable thickness. Trapezoidal cross-section walls are optimal when used in structures where load distribution is triangular in shape (hydrostatic load). For tanks recessed in the ground, the load on the walls increases with the depth of foundations and obtains the highest value in the bottom part of the wall. Trapezoidal or triangular load distribution causes that the highest values of bending moments in the vertical cross section occur at the point where a wall connects to the bottom, and while the upper free edge of the tank takes zero value. Insufficiency of publications might be one of the reasons for constant occurrence of difficulties in the design and construction of plates or tanks with walls of a linearly variable thickness [13, 14]. Presumably, as a consequence, a small number of such structures have been put into use.

2. Static calculations

Static calculations for the plate of trapezoidal cross-section were made with the finite difference method in the variation approach. The function describing the energy of elastic deflection and the potential energy resulting from load action on plates is as follows [6, 8, 12, 15]:

\[
V = \frac{D}{2} \int \left\{ \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + 2 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 w}{\partial y^2} \right)^2 \right\} + 2 \nu \left[ \frac{\partial^4 w}{\partial x^4} + \frac{\alpha_{\Delta T} \partial^3 w}{\partial x \partial y^2} - \frac{\partial^3 w}{\partial y^2} \right] \right\} \int q w dA - \int q w dA
\]

\[
D = \frac{E h^3}{12(1 - \nu^2)} \quad \text{flexural rigidity},
\]

E – elasticity modulus,
\( \nu \) – Poisson’s ratio,
h – plate thickness,
w – plate deflection,
q – load perpendicular to the central surface of the plate,
A – plate area,
\( \Delta T \) – difference in temperature between the lower plate and the upper plate determined by correlation: \( \Delta T = T_u - T_g \)
\( \alpha_t \) – coefficient of thermal expansion
In order to obtain a solution with the finite difference method, the analysed plate was divided into elementary subareas with a discretisation grid. During calculations the plate of trapezoidal cross-section was changed into a plate with a discrete variable thickness, where the thickness step value was at half the height of each grid mesh. Using Functional (1), the system of linear algebraic equations was completed with appropriate differential quotients instead of derivatives. The authors also complied with the condition that for the system in a stable equilibrium, its energy reaches a minimum. At the same time, summation after elementary sub-areas replaced integration by surface. In order to verify the correctness of calculations concerning the temperature effect performed with the finite difference method, there were made calculations of the plate taking into account acting constant load. The results obtained were compared with generally available tables for plates with a variable thickness [15]. The plate used for calculations is shown in figure 1.

**Figure 1.** Analysed plate with the assumed discretisation grid and numbering of nodes

Calculations were made for the Poisson's ratio $\nu=0$ and for two types of loads: uniformly distributed on the whole plate and the temperature difference identical for the whole plate. The analysis included a plate with the following thickness values: $h_0$ in the upper part and $h_8$ in the lower part. For the assumed discretisation grid, being eight meshes after the height of the plate, there were calculated thickness values in individual grid meshes with the use of general formulas. For this purpose, the constant $\lambda$ was taken, which specifies the rigidity ratio of the bottom part to the upper part of the plate. Below are presented the dependencies of individual thickness values from $h_0$, which was taken as the starting value to create a calculation algorithm for plates with different thicknesses.

The constant $\lambda$ defines dependencies:

$$\frac{D_8}{D_0} = \lambda, \quad \frac{h_8^3}{h_0^3} = \lambda \quad \text{and} \quad \frac{h_8}{h_0} = 2$$

(2)

where:

- $h_8$ – maximum plate thickness,
- $h_0$ – minimum plate thickness,
- $D_8$ – plate rigidity with $h_8$,
- $D_0$ – plate rigidity with $h_0$. 

After transforming (2), the result was:

$$h_k = h_0 \sqrt[3]{\lambda} \text{ and } D_k = 2D_0 \tag{3}$$

The following results were obtained for the given plate thicknesses:

$$h_7 = h_0 + \frac{7}{8}(h_k - h_0) = h_0 \left[1 + \frac{7}{8}(\sqrt[3]{\lambda} - 1)\right] \text{ and } D_7 = D_0 \frac{h_7^3}{h_0^3} \tag{4}$$

$$h_6 = h_0 + \frac{6}{8}(h_k - h_0) = h_0 \left[1 + \frac{6}{8}(\sqrt[3]{\lambda} - 1)\right] \text{ and } D_6 = D_0 \frac{h_6^3}{h_0^3} \tag{5}$$

$$h_5 = h_0 + \frac{5}{8}(h_k - h_0) = h_0 \left[1 + \frac{5}{8}(\sqrt[3]{\lambda} - 1)\right] \text{ and } D_5 = D_0 \frac{h_5^3}{h_0^3} \tag{6}$$

$$h_4 = h_0 + \frac{4}{8}(h_k - h_0) = h_0 \left[1 + \frac{4}{8}(\sqrt[3]{\lambda} - 1)\right] \text{ and } D_4 = D_0 \frac{h_4^3}{h_0^3} \tag{7}$$

$$h_3 = h_0 + \frac{3}{8}(h_k - h_0) = h_0 \left[1 + \frac{3}{8}(\sqrt[3]{\lambda} - 1)\right] \text{ and } D_3 = D_0 \frac{h_3^3}{h_0^3} \tag{8}$$

$$h_2 = h_0 + \frac{2}{8}(h_k - h_0) = h_0 \left[1 + \frac{2}{8}(\sqrt[3]{\lambda} - 1)\right] \text{ and } D_2 = D_0 \frac{h_2^3}{h_0^3} \tag{9}$$

$$h_1 = h_0 + \frac{1}{8}(h_k - h_0) = h_0 \left[1 + \frac{1}{8}(\sqrt[3]{\lambda} - 1)\right] \text{ and } D_1 = D_0 \frac{h_1^3}{h_0^3} \tag{10}$$

After taking into account $\nu = 0$, the result was:

$$V = \frac{D}{2} \iint_A \left(\frac{\partial^2 w}{\partial x^2} + 2\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\frac{\alpha \Delta T}{h} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\alpha \Delta T}{h}\right)\right) dA - \iint_A qwdA \tag{11}$$

The plate thickness step was characterised by the assumed rigidity $D_s$ adopted as the arithmetic mean, i.e. $D_{so1} = 0.5(D_0 + D_1)$. Further calculations planned to use the rigidity as the harmonic mean [16].

2.1. Calculation example

The analysis included a rectangular plate with three edges fixed and one edge free. The plate used for calculations is shown in figure 1. Calculations were made for the Poisson's ratio $\nu = 0$ and for the load uniformly distributed on the whole plate and the temperature load. The analysed case concerned the plate with $\lambda = 8$, which means that the lower part of the plate was eight times more rigid in relation to its upper part. When the thickness of plate is considered, it corresponds to the case when the bottom part of the plate is twice as thick as its upper part. According to formulas from (4) to (10), it was assumed:

$$h_0 = 1 \quad \text{and } D_0 = 1,$$

$$h_1 = 1.125 \ h_0 \quad \text{and } D_1 = 1.4238 \ D_0,$$

$$h_2 = 1.25 \ h_0 \quad \text{and } D_2 = 1.9530 \ D_0,$$

$$h_3 = 1.375 \ h_0 \quad \text{and } D_3 = 2.5996 \ D_0,$$

$$h_4 = 1.5 \ h_0 \quad \text{and } D_4 = 3.3750 \ D_0,$$

$$h_5 = 1.625 \ h_0 \quad \text{and } D_5 = 4.2910 \ D_0.$$
\[ \begin{align*}
\theta_0 &= 1.75 \ h_0 \quad \text{and} \quad D_0 = 5.3594 \ D_0, \\
\theta_1 &= 1.875 \ h_0 \quad \text{and} \quad D_1 = 6.5918 \ D_0, \\
\theta_2 &= 2 \ h_0 \quad \text{and} \quad D_2 = 8 \ D_0,
\end{align*} \]

Similarly, the average values of rigidity at the plate thickness step were calculated as the arithmetic means of rigidity corresponding to subsequent plate thickness values.

After solving the system of displacement equations, there were obtained the values of bending moments in all nodes of the assumed discretisation grid. Based on deflections, there were calculated the values of bending moments according to the following formulas (12, 13):

\[ \begin{align*}
M_x &= -D \left( \frac{\partial^2 W}{\partial x^2} + \frac{\alpha \Delta T}{h} \right) \\
M_y &= -D \left( \frac{\partial^2 W}{\partial y^2} + \frac{\alpha \Delta T}{h} \right)
\end{align*} \] (12)

\[ \begin{align*}
M_x &= -D \left( \frac{\partial^2 W}{\partial x^2} + \frac{\alpha \Delta T}{h} \right) \\
M_y &= -D \left( \frac{\partial^2 W}{\partial y^2} + \frac{\alpha \Delta T}{h} \right)
\end{align*} \] (13)

In order to verify the correctness of the displacement matrix, equation system and results, there were calculated the values of bending moments taking into account acting constant loads for a plate of a linearly variable thickness. The results were compared with generally available tables [15]. In addition, table 1 includes the values of bending moments for a plate with a constant thickness in order to show the difference between the values of bending moments for a plate with variable and constant thicknesses. Table 1 summarises the values of bending moments.

**Table 1. Comparison of bending moments for the plate with a linearly variable thickness**

| Compared values | Plate with a linearly variable thickness MRS – work | Plate with a linearly variable thickness – acc. [15] | Plate with a constant thickness – acc. [15] |
|-----------------|--------------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Constant load   | Constant load                                    | Constant load                                 | Constant load                                 |
| \( M_x^8 \)    | 1.9026 \( qs^2 \)                               | 2.3296 \( qs^2 \)                             | 6.2976 \( qs^2 \)                             |
| \( M_x^{40} \) | 2.2970 \( qs^2 \)                               | 2.5344 \( qs^2 \)                             | 2.7648 \( qs^2 \)                             |
| \( M_x^C \)    | -7.7168 \( qs^2 \)                              | -6.6560 \( qs^2 \)                            | -18.9952 \( qs^2 \)                           |
| \( M_x^B \)    | -8.0237 \( qs^2 \)                              | -8.5248 \( qs^2 \)                            | -8.1664 \( qs^2 \)                            |
| \( M_x^{40} \) | -2.4853 \( qs^2 \)                              | -2.9312 \( qs^2 \)                            | 0.7424 \( qs^2 \)                             |
| \( M_x^A \)    | -18.6000 \( qs^2 \)                             | -18.8288 \( qs^2 \)                           | -13.0816 \( qs^2 \)                           |

where: \( s = \frac{l_x}{16} = \frac{l_y}{8} \)

The analysis of data provided in table 1 confirmed a satisfactory compliance between the results obtained with the finite difference method according to tables [15] and the results of calculations obtained with the finite difference method in the variation approach for the same example, however with an innovative procedure. It led, as a result, to the conclusion that the matrix of displacement equations – used for calculating the values of bending moments for a plate of trapezoidal cross-section loaded with temperature – was developed correctly. Since there are no publications in the literature that could serve as a scientific reference in term of plates loaded with temperature, the analytical calculations were subjected to model-based verification.
3. Model tests

The test-plate was cast from resin and fixed in a steel frame. The upper plate thickness was 8 mm and the lower plate thickness – 16 mm, which corresponded to the assumptions made in the analysis, i.e. \( h_0 = 2h_0 \) and \( \lambda = 8 \). The static scheme of the plate was assumed as three edges fixed in a steel frame and one edge free. The test-plate dimensions were \( L_x = 360 \) mm and \( L_y = 180 \) mm, whereas the assumed grid size was \( s_x = 22.5 \) mm and \( s_y = 22.5 \) mm. In order to determine the elasticity modulus used for making this resin plate, preliminary tests were carried out on a bar with the following dimensions: width 68.56 mm, thickness 11.00 mm and length 500 mm. The deflection tests for the bar loaded with force concentrated in the middle of its span were carried out under laboratory conditions at temperature 24°C. Deflection was measured in the middle of its span, and then using formula (14) and after its transformation according to formula (15), the elastic modulus of resin was calculated.

\[
f = \frac{P l^3}{48EJ}
\]

\[
E = \frac{P l^3}{48fJ}
\]  

(14)  

(15)

Material constants \( E \) and \( \alpha_t \) (coefficient of linear thermal expansion) should be determined for the temperature occurring during the tests. The elasticity modulus \( E \) at temperature 24°C was 432.65 kNcm\(^{-2}\), whereas its value at temperature 45°C was 304.19 kNcm\(^{-2}\). To determine the coefficient \( \alpha_t \), the authors used a bar, which was 500 mm long at 24°C. Then, it was placed in a trough with water at 50°C. The expansion value of the bar for this temperature was 1.078 mm. Each of the temperature value measured was completed by the measurements of its expansion value. Then, the coefficient \( \alpha_t \) was determined using formula (16) and after being transformed into formula (17).

\[
\Delta l = l \alpha_t \Delta T
\]

\[
\alpha_t = \frac{\Delta l}{l \Delta T}
\]

(16)  

(17)

Based on the research conducted, there was determined the coefficient of linear thermal expansion as \( \alpha_t = 8.3 \times 10^{-5} \)°C\(^{-1}\). The test-plate was subjected to a temperature load, which was modelled by blowing warm air in the tunnel. The temperature was measured using contact thermometers attached to the test-plate. It was assumed that the temperature distribution was linear on the thickness of the plate. Plate deflections were measured using four sensors, two devices with measurement accuracy of 0.001 mm were located on the free edge, another two with measurement accuracy of 0.01 mm at half the height of the test-plate. Sensor readings were made when the temperature on both sides of the test-plate stabilised. The numbering of measuring points in which the deflections of test-plate were measured was taken according to figure 1. Figure 2 shows the view of the plate during tests.

Figure 2. View of the plate during tests
Measurements of deflections were made when the temperature from the heated side in the tunnel was $T = 50^\circ C$, while the temperature of the unheated side in the upper part, on the free edge was $T_g = 32.4^\circ C$, and at half the height of the test plate was $T_d = 30.5^\circ C$. Thus, the adopted temperature differences were:

- for the free edge of the plate: $\Delta T_g = T - T_g = 17.6^\circ C$,
- at half the height of the plate: $\Delta T_d = T - T_d = 19.5^\circ C$.

Table 2 summarises the values of deflections measured on the model and calculated with the finite difference method. Numerical calculations were made for the following data: $\alpha_t = 8.3 \times 10^{-5} \, ^\circ C^{-1}$, $\Delta T_g = 17.6^\circ C$, $\Delta T_d = 19.5^\circ C$. The values of deflections obtained traditionally with the finite difference method were multiplied by the calculated multiplier value for temperature loads.

| Deflections in points | Values measured on the model $\text{[mm]}$ | Values calculated traditionally with the finite difference method $\text{[mm]}$ |
|----------------------|------------------------------------------|--------------------------------------------------|
| 4                    | 0.872                                    | 0.806                                            |
| 8                    | 1.135                                    | 1.145                                            |
| 36                   | 0.160                                    | 0.159                                            |
| 40                   | 0.250                                    | 0.259                                            |

By analysing the values summarised in table 2, it can be concluded that the consistency of results is satisfactory. In the next stage of numerical calculations, it is planned to adopt the harmonic mean instead of the arithmetic one at the plate thickness step as it was done in the case of calculations for bars with a variable rigidity provided in the work [16]. The obtained results show the correct course of tests and the designation of material data for the resin from which the plate was cast.

4. Summary

The literature of the subject provides few publications concerning plates of a trapezoid cross-section subjected to thermal load. The plate presented in the article was calculated traditionally with the use of the finite difference method. The consistency of calculation results and model tests confirms the correctness of the determined matrix and equation systems. The temperature effect is often neglected in calculating plate structures, and yet it gives higher values of bending moments than other loads. Bending moments due to acting thermal load increase proportionally to the second power of the wall thickness. By using the formula for bending moment caused by the temperature load (18) [17, 18, 19].

$$M_t = \frac{Eh_0^2}{12} \alpha_t \Delta T$$

and referring to the plate analysed in the work, it can be stated that the value of bending moment for the upper part of the plate with $h_0$ is $M_t = \frac{Eh_0^2}{12} \alpha_t \Delta T$, while for the lower part of the plate, where $h_0=2h_0$ it equals $M_t = 4 \frac{Eh_0^2}{12} \alpha_t \Delta T$. In engineering practice, the most common are tanks designed and constructed with walls of a constant thickness. However, the most desirable solution for tanks are walls of a linearly variable thickness, adapted to the bending moments that reach their highest value in the lower part, and the upper one on the free edge takes the zero value. The values of bending moments summarised in Table 1 due to constant load for plates with fixed and linearly variable thicknesses demonstrate how important in terms of a design-optimised solution is to construct walls with a variable thickness. The above statements lead to the conclusion that structural and economic considerations should determine the selection of walls with a thickness increasing accordingly to the depth of the tank since the usage of material in such walls is more cost-effective.
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