Optimization Methods to Reduce Capacitor Stress in Modular Multilevel Converters

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The work of Nikola Petranovic was supported in part by a Samaha Research Scholarship, and in part by an Australian Government Research Training Program Scholarship at The University of Western Australia.

ABSTRACT Modular multilevel converters consist of many capacitor half-bridge or full-bridge converter cells. Stresses on the capacitors result from peak capacitor voltage, capacitor voltage ripple and current ripple. Three new tractable convex optimisation problems are presented that reduce peak capacitor voltage and limit rms arm current using injected zero-sequence voltages, circulating currents and selecting the initial stored capacitor energy. These optimisation problems are compared to assess their performance in terms of key indicators which include factors related to capacitor stress, capacitor and semiconductor ratings, and power loss: peak capacitor voltage, capacitor voltage ripple, capacitor current ripple, average and rms arm current. The optimisation methods permit designers to understand the trade-offs in achieving the limits imposed by device specifications and power losses.

INDEX TERMS Design optimisation, equivalent circuits, modelling, modular multilevel converters.

I. INTRODUCTION A high efficiency converter with medium and high power applications is the modular multilevel converter (MMC) [1]–[4]. A circuit schematic of a three-phase MMC is shown in Fig. 1. Each MMC leg (a,b,c) contains two arms (upper u and lower l), each of which consists of serially connected submodules (SMs) which are full-bridge capacitor cells (or alternatively half-bridge cells [5]).

An important consideration in the design of MMCs is to limit the stress on the components. Current ratings of semiconductors and capacitors require limits on the maximum arm current. Power losses and thermal stresses on semiconductors are related to rms currents [6], [7]. The stress on capacitors depend upon the peak capacitor voltage [8], [9]. Selection of semiconductor voltage specification is based upon the maximum peak capacitor voltage [10]. Other stresses on capacitors are caused by ripple voltage and ripple current [9], [11].

To decrease stress on capacitors, papers in the literature proposed methods that determine the zero-sequence voltages and/or circulating currents to reduce peak-to-peak voltage ripple [12]–[21] or peak capacitor voltage [22]. Brief summaries of some of these papers are given below:

Exhaustive search methods exist that minimise peak-to-peak capacitor voltage ripple using circulating currents [12], [13]. The disadvantages of exhaustive search techniques relative to optimisation-based solvers are time-inefficiency and the solution’s dependence on quantisation of optimisation parameters. In [12], the magnitude and phase angle of a second-order and a fourth-order circulating current harmonic are exhaustively sought to find optimal solutions for three optimisation problems: the minimisation of capacitor voltage ripple; the minimisation of a weighted sum of capacitor voltage ripple and rms current, and; the minimisation of capacitor voltage ripple subject to a constraint on rms current. The authors of [13] present an exhaustive search of the same circulating current harmonic parameters as those considered in [12]. The method presented in [13] also minimises capacitor voltage ripple, however the authors utilise a different model for capacitor voltage that is proposed in [23]. The technique studied in [13] was designed to overcome the drawbacks of another method to reduce capacitor voltage ripple, shown in [14], in which an approximate expression for capacitor voltage ripple is minimised using circulating currents. Neither
The key contributions of this paper are: 1. A detailed formulation of a simplified MMC model with all assumptions clearly described. 2. Three new and tractable two-step convex optimisation problems that reduce peak capacitor voltage using injected zero-sequence voltages, circulating current and initial capacitor energy. 3. The introduction of an optimisation constraint which imposes an upper limit on rms arm current. 4. The solutions to each of the three two-step optimisation problems are compared in terms of key performance indicators related to capacitor stress and power losses.

In this section, the MMC arm voltages, current and capacitor energy are modelled in terms of the nominal (operational) condition parameters, injected zero-sequence voltage...
and circulating currents parameters, and MMC components parameters. Tellegen’s theorem is used to derive a model for the stored capacitor energy in an arm as a function of the arm voltage and arm current. Similar stored capacitor energy relationships have been reported in [9], [12], [18], [20], [22], however alternative assumptions are used and it is not clear how these models are related to the one proposed in this paper. Experimental validation of the model is presented in Section III-C. The optimisation criteria and constraints in the optimisation problems, which are introduced in Section III, are defined in terms of this model’s voltages, currents and energies.

### A. ARM VOLTAGES AND CURRENTS

With reference to Fig. 1, let \( v_{pq}(t) \) and \( i_{pq}(t) \) denote voltages and currents in an arm, where \( p \in \{a, b, c\} \) and \( q \in \{u, l\} \). The voltages and currents in the upper and lower arms of the MMC, \( v_{pu}(t) \), \( i_{pu}(t) \), \( v_{pl}(t) \) and \( i_{pl}(t) \), are defined in (1)-(4).

\[
v_{pu}(t) \triangleq \frac{V_{dc}}{2} - V_{1g} \cos(\omega_0(t - T_p)) - V_{1h} \sin(\omega_0(t - T_p)) - \sum_{m \in \mathcal{V}} \left[ V_{mg} \cos(m\omega_0(t - T_p)) + V_{mh} \sin(m\omega_0(t - T_p)) \right] \frac{i_{Vpu}(t)}{v_{Vpu}(t)} \quad (1)
\]

\[
i_{pu}(t) \triangleq -\frac{I_{dc}}{3} + I_{1g} \cos(\omega_0(t - T_p)) + I_{1h} \sin(\omega_0(t - T_p)) - \sum_{m \in \mathcal{I}} \left[ I_{mg} \cos(m\omega_0(t - T_p)) + I_{mh} \sin(m\omega_0(t - T_p)) \right] \frac{i_{Cpu}(t)}{i_{Vpu}(t)} \quad (2)
\]

\[
v_{pl}(t) \triangleq \frac{V_{dc}}{2} + V_{1g} \cos(\omega_0(t - T_p)) + V_{1h} \sin(\omega_0(t - T_p)) + \sum_{m \in \mathcal{V}} \left[ V_{mg} \cos(m\omega_0(t - T_p)) + V_{mh} \sin(m\omega_0(t - T_p)) \right] \frac{i_{Vpl}(t)}{v_{Vpl}(t)} \quad (3)
\]

\[
i_{pl}(t) \triangleq -\frac{I_{dc}}{3} - I_{1g} \cos(\omega_0(t - T_p)) - I_{1h} \sin(\omega_0(t - T_p)) - \sum_{m \in \mathcal{I}} \left[ I_{mg} \cos(m\omega_0(t - T_p)) + I_{mh} \sin(m\omega_0(t - T_p)) \right] \frac{i_{Cpl}(t)}{i_{Vpl}(t)} \quad (4)
\]

where \( \omega_0 \) is the fundamental frequency, \( T_0 = 2\pi/\omega_0 \) and

\[
T_p = \begin{cases} 0, & p = a \\ T_0/3, & p = b \\ 2T_0/3, & p = c \end{cases}
\]

The arm voltages and currents have been formulated generically to include nominal voltage \( v_{Vpq}(t) \) and current \( i_{Vpq}(t) \), injected voltages, \( v_{Vpu}(t) \), \( v_{Vpl}(t) \), and circulating currents, \( i_{Cpq}(t) \); which can be created in an MMC [12]–[21], [26], [27]. The \( T_0 \)-periodic arm voltages and currents are formulated to be three-phase time offset by \( T_0/3 \). In (1)-(4) the harmonics due to PWM switching in the SMs are assumed to be negligible and the arm inductances are assumed to be small so that the arm voltages required to create the arm currents are insignificant. In the general case the harmonics injected in arm voltages and currents in (1)-(4) are assumed to be harmonics of the fundamental \( \omega_0 \) up to some maximum \( N_H \) as defined by the sets \( \mathcal{V} \) and \( \mathcal{I} \).

\[
\mathcal{V} \triangleq \{ 2, \ldots, N_H \} \quad (6)
\]

\[
\mathcal{I} \triangleq \{ 2, \ldots, N_H \} \quad (7)
\]

The arm voltage and currents are re-written in vector form in (8)-(11), where \( \mathbf{f}_p(t) \), \( \mathbf{x}_u \), and \( \mathbf{x}_l \) are defined in the Appendix A.

\[
v_{pu}(t, \mathbf{x}_u) = v_{Vpu}(t) - \mathbf{f}_p(t) \mathbf{x}_u \quad (8)
\]

\[
i_{pu}(t, \mathbf{x}_u) = i_{Vpu}(t) - \mathbf{f}_p(t) \mathbf{x}_i \quad (9)
\]

\[
v_{pl}(t, \mathbf{x}_l) = v_{Vpl}(t) + \mathbf{f}_p(t) \mathbf{x}_u \quad (10)
\]

\[
i_{pl}(t, \mathbf{x}_l) = i_{Vpl}(t) + \mathbf{f}_p(t) \mathbf{x}_i \quad (11)
\]

### B. STORED CAPACITOR ENERGY

The circuit in Fig. 2 shows the serially connected submodules in any arm \( pq \). In Fig. 2, the rest of the MMC circuit is represented by the one-port element characterised by the relationship of arm voltage to arm current, \( v_{pq}(t, \mathbf{x}_u) \) and \( i_{pq}(t, \mathbf{x}_l) \) as defined in (8) to (11).

\[
v_{Vpq}(t) \text{ and } i_{Vpq}(t) \text{ represent the voltage and current at } k \text{th switch on the } j \text{th submodule where } k = 1, \ldots, 4 \text{ and } j = 1, \ldots, J. C_{SMj} \text{, } v_{Cpq}(t) \text{ and } i_{Cpq}(t) \text{ represents the capacitance, voltage and current of the } j \text{th submodule capacitor. Using Tellegen’s theorem [28] for the circuit in Fig. 2:}
\]

\[
\sum_{j=1}^{J} \sum_{k=1}^{4} \left[ v_{Vpq,j,k}(t) i_{Vpq,j,k}(t) \right] - v_{pq}(t, \mathbf{x}_u) i_{pq}(t, \mathbf{x}_l) + \sum_{j=1}^{J} v_{Cpq}(t) i_{Cpq}(t) = 0 \quad (12)
\]

Assuming ideal lossless switches, then the first term in (12) is zero. Under this assumption, and considering the definite
\[
\sum_{j=1}^{J} \int_{t_0}^{t_0+T_{pq}} v_{pq}(\tau, x_i) i_{pq}(\tau, x_i) d\tau = 0 \quad (13)
\]

where

\[
T_{pq} = T_p + T_q \quad (14)
\]

\[
T_q = \begin{cases} 
0, & q = u \\
\alpha, & q = l
\end{cases} \quad (15)
\]

The parameter \(\alpha\) is introduced to allow for a time shift that exists, for the cases of interest in the paper, between the upper and lower arm voltage and current functions in the same phase, as defined in (16).

\[
v_{pq}(t, x_i) = v_{pq}(t - \alpha, x_i), \quad \forall p
\]

\[
i_{pq}(t, x_i) = i_{pq}(t - \alpha, x_i), \quad \forall p
\]

The value of \(\alpha\) is determined in Section III-B7 for a special set of \(V\) and \(I\) that are of interest in this paper.

From (13) and the voltage-current relationship in a capacitor, it follows that:

\[
\sum_{j=1}^{J} \int_{t_0}^{t_0+T_{pq}} C_{SM,pq} \left[ v_{pq}^2(t + T_{pq}) - v_{pq}^2(t_0 + T_{pq}) \right] d\tau = \int_{t_0}^{t_0+T_{pq}} v_{pq}(\tau, x_i) i_{pq}(\tau, x_i) d\tau \quad (17)
\]

It is assumed that there are balanced capacitor voltages and equal submodule capacitor currents in an arm, i.e.

\[
v_{pq}(t) = v_{CPq}(t), \quad i_{pq}(t) = i_{CPq}(t), \quad j = 1, \ldots, J \quad (18)
\]

Control strategies for achieving the balance described in (18) have previously been proposed in the literature [23].

Given every SM capacitor has the same capacitance, \(C_{SM}\), the subscript \(j\) is dropped from (17) to obtain:

\[
\int_{t_0}^{t_0+T_{pq}} v_{pq}(\tau, x_i) i_{pq}(\tau, x_i) d\tau = \frac{JC_{SM}}{2} \left( v_{pq}^2(t + T_{pq}) - v_{pq}^2(t_0 + T_{pq}) \right) \quad (19)
\]

Consider the definitions in (20)-(22).

\[
e_{C_{pq}}(t + T_{pq}) \leq \frac{C_{SM}}{J} \quad (20)
\]

\[
x_{C_{pq}}(t + T_{pq}) \leq \frac{C_{sm}}{J} (J v_{pq}(t + T_{pq}))^2 \quad (21)
\]

Equation (19) is then re-written in terms of the variables \(C_{sm}\), \(e_{C_{pq}}(t)\) and \(x_{C_{pq}}(t)\) in (23).

\[
e_{C_{pq}}(t + T_{pq}) = \int_{t_0}^{t_0+T_{pq}} v_{pq}(\tau, x_i) i_{pq}(\tau, x_i) d\tau + x_{C_{pq}}(t_0 + T_{pq}) \quad (23)
\]

The time-average, defined in (24), of the function \(e_{C_{pq}}(t + T_{pq})\) is assumed to be equal in all the arms. This assumption is stated in (25). An experimental method to enforce (25) is described in [26].

\[
\text{ave} [e_{C_{pq}}(t + T_{pq})] = \text{ave} [e_{C_{pq}}(t_0 + T_{pq})] \quad (24)
\]

Using the arm voltage and current relationships defined in (1)-(4) and (16), it can be shown that (25) implies (26). The derivation is detailed in Appendix C.

\[
x_{C_{pq}}(t_0 + T_{pq}) = x_{C_{pq}}(t_0) \quad (26)
\]

It follows from (26), that the \(e_{C_{pq}}(t + T_{pq})\) in the MMC arms are time-offset in the same manner as the voltages and currents as defined in (1)-(4), (16), as described in (27). The derivation of (27) is detailed in Appendix C.

\[
e_{C_{pq}}(t + T_{pq}) = e_{C_{pq}}(t) \Leftrightarrow e_{C_{pq}}(t) = e_{C_{pq}}(t - T_{pq}) \quad (27)
\]

An interpretation of (27) is: the behaviour of \(e_{C_{pq}}(t + T_{pq})\) for all \(pq\) is known if \(e_{C_{pq}}(t + T_{pq})\) for any \(pq\) is known.

As explained in Section III-B7, for a special subset of voltage and current harmonics, only one of the six arm’s voltage, current and energy functions needs to be considered when solving the optimisation problems presented in this paper.

The upper arm of leg \(a, p = a\) and \(q = u\), is arbitrarily selected hereafter and for simplicity, the subscript \(au\) is dropped from the notation. Substituting \(T_{pq} = T_{au} = 0\), (8) and (9) into (23), it can be shown that \(e_{C}(t)\) in arm \(au\) can be...
written as in (28). Furthermore, it is made explicit that $e_C(t)$ is a function of the $x_v$, $x_i$, and $x_{ec0}$ in (28).

$$e_C(t, x_v, x_i, x_{ec0}) = x_{ec0} + \int_{t_0}^t v_N(\tau) i_N(\tau) d\tau - x_i^T \phi_i(t) - x_v^T \phi_v(t) + x_{ec0}^T P(t)x_i$$

where $\phi_i(t)$, $\phi_v(t)$ and $P(t)$ are defined in the Appendix A1.

III. OPTIMISATION PROBLEM FORMULATION

The optimisation problems proposed in this paper minimize the peak capacitor voltage subject to constraints, using the optimisation parameters that are controllable in the MMC: $x_v$, $x_i$ and $x_{ec0}$. The objective function, constraints and the proposed two-step optimisation problems are detailed in this section.

A. OBJECTIVE

Stresses on a capacitor arise from large peak voltages. In this paper, the objective function, $f_0(t, x_v, x_i, x_{ec0})$, of the optimisation problems is devised such that peak capacitor voltages are minimized, thereby decreasing stress on capacitors. Thus the objective function given by (29) is proposed.

$$f_0(t, x_v, x_i, x_{ec0}) \triangleq \max_t v_C(t, x_v, x_i, x_{ec0}) \quad (29)$$

For correct operation of the diode/switch assembly, the capacitor voltages need to be always positive, i.e. $v_C(t) \ge 0$. It follows from (29) and $v_C(t) \ge 0$ that the objective function defined by (30) is equivalent to that defined by (29).

$$\max_t e_C(t, x_v, x_i, x_{ec0}) \quad (30)$$

The objective function (30) avoids any issues associated with the square root operator which would arise in (29).

B. CONSTRAINTS

This section introduces the constraints that are utilized in the optimisation problems.

1) PEAK ARM CURRENT CONSTRAINT

The allowable peak arm current is dependent on the rating of the MMC IGBTs/semiconductors. Thus the magnitude of the current needs to be constrained below some specified maximum, as defined in (31).

$$i(t, x_i) \le I_{MAX}, \quad \forall t \quad (31)$$

The two linear inequality constraints on the current in (32) are equivalent to the required constraint defined by (31).

$$i_N(t) - \bar{I}^F(t)x_i \le I_{MAX} \quad \forall t \quad (32)$$

2) ROOT-MEAN-SQUARE CURRENT CONSTRAINT

Thermal limits of semiconductors and power loss considerations are addressed by imposing an upper bound on arm rms current. It can be shown that the rms current in an arm, $i_{rms}(x_i)$

$$i_{rms}(x_i) = \sqrt{\frac{I_{dc}^2}{9} + \frac{I_{ls}^2}{2} + \frac{I_{th}^2}{2} + \frac{1}{2} x_i^T x_i} \quad (33)$$

where

$$i_{rms}(x_i) \triangleq \sqrt{\frac{1}{T_0} \int_{t_0}^t i^2(t, x_i) dt} \quad (34)$$

It is clear from (33) that the injection of circulating currents increases the arm rms current relative to nominal conditions, that is

$$i_{rms}(x_i = 0) \le i_{rms}(x_i) \quad (35)$$

Thus, as defined in (36) a parameter $I_{RMSF} \ge 1$, is introduced to define an upper bound on the rms current. Note that the bound imposes a maximum relative to the nominal rms current.

$$i_{rms}(x_i) \le I_{RMSF}i_{rms}(x_i = 0) \quad (36)$$

Root-mean-square quantities are positive, and therefore deriving the root-mean-square current from the mean-square quantity involves a monotone transformation. Thus the constraint defined by (37) is equivalent to the constraint defined by (36).

$$\frac{I_{dc}^2}{9} + \frac{I_{ls}^2}{2} + \frac{I_{th}^2}{2} + \frac{1}{2} x_i^T x_i \le (I_{RMSF}i_{rms}(x_i = 0))^2 \quad (37)$$

Equation (37) is a direct way of placing an upper bound on the arm rms current as opposed to adding a term to the objective function [12], [19], which would require a decision on the weight to be placed on this term.

3) MINIMUM ARM VOLTAGE CONSTRAINT

For correct operation of MMC submodules implemented as unipolar (half-bridge) converter submodules, the arm voltages need to be greater than or equal to zero. Thus, the constraint (38) is proposed.

$$v(t, x_i) \ge 0, \quad \forall t \quad (38)$$

It is not difficult to show that the more convenient form (39) is equivalent to (38).

$$v_N(t) - \bar{F}(t)x_i \ge 0, \quad \forall t \quad (39)$$

Note that bipolar (full-bridge) converter submodules can function as unipolar submodules, but the reverse is not true.

4) PEAK ARM VOLTAGE CONSTRAINT

Similar to peak currents, another constraint is introduced to place an upper limit on peak arm voltage as defined in (40).

$$|v(t, x_i)| \le V_{MAX}, \quad \forall t \quad (40)$$

Given the inclusion of the constraint in (38), it is clear that (40) is equivalently achieved using (41).

$$v(t, x_i) \le V_{MAX}, \quad \forall t \quad (41)$$
Furthermore, the linear inequality constraint on the arm voltage in (42) is equivalent to the required constraint defined by (41).

\[ v_N(t) - f(t)x_v \leq V_{MAX} \quad \forall t \]  

(42)

5) MINIMUM CAPACITOR STORED ENERGY REQUIREMENT

Due to the half-bridge structure of submodules, (43) needs to hold during the operation of an MMC [8, 9].

\[ J_{vc}(t, x_v, x_i, x_{ec}) \geq v(t, x_v), \quad \forall t \]  

(43)

It can be shown that (44) is equivalent to the desired constraint in (43).

\[ e_C(t, x_v, x_i, x_{ec}) \geq E_{MIN}(t, x_v), \quad \forall t \]  

(44)

where

\[ E_{MIN}(t, x_v) = \frac{C_{eff}}{2}v^2(t, x_v) \]  

(45)

The definition of \( E_{MINpq}(t, x_v) \) for any arm is given in (106). Note that \( E_{MIN}(t, x_v) \) is the lower bound of \( e_C(t, x_v, x_i, x_{ec}) \). To simplify notation, the inequality (44) can be replaced by the equivalent inequality in (46).

\[ e_C(t, x_v, x_i, x_{ec}) \geq 0, \quad \forall t \]  

(46)

where

\[ e_C(t, x_v, x_i, x_{ec}) = e_C(t, x_v, x_i, x_{ec}) - E_{MIN}(t, x_v) \]  

(47)

The definition of \( e_{CPq}(t, x_v, x_i, x_{ec}) \) for any arm is given in (110). Note that the constraint defined by (46) can be revised to include a margin by introducing a positive parameter on the right-hand side of the inequality.

6) MINIMUM INITIAL CAPACITOR ENERGY

Given the definition of \( x_{ec} \) in (22), it is clear that

\[ x_{ec} \geq 0 \]  

(48)

It can be shown the constraint in (48) is met upon enforcing the constraint in (46). Therefore (48) is met without explicitly including it as a constraint in the formulation of the optimisation problems.

7) HARMONIC SELECTION

The notation used in this paper considers injection of all integer harmonics from 2 to \( N_H \) into arm voltages and currents as shown in (76). Generally, MMCs are operated such that a special subsets of voltage and current harmonics, \( V_s \subseteq \mathcal{V} \) and \( I_s \subseteq \mathcal{I} \), are injected.

\( a: \) EQUALITY CONSTRAINTS

A method using equality constraints is defined to enforce the selection of harmonic numbers in \( V_s \) and \( I_s \) in the optimisation problems: Square matrices \( B_a \) and \( B_b \) can be defined such that the equality constraints defined by (49) and (50) ensure that any harmonic in \( x_v \) and \( x_i \) that is an element of \( V_s^c \cap \mathcal{V} \) or \( I_s^c \cap \mathcal{I} \), respectively, is set to zero and the desired harmonic selection is achieved. The equality constraints defined by (49) and (50) appear in the optimisation problems as required.

\[ B_a x_v = 0 \]  

(49)

\[ B_b x_i = 0 \]  

(50)

b: SELECTION OF VOLTAGE AND CURRENT HARMONICS

Special subsets of the injected voltages and currents defined by (6) and (7) are identified. Let \( Z_{++} \) be the positive integers and \( N_V, N_I \in Z_{++} \). \( N_V, N_I \leq N_H \). Let the special subsets of \( V \) and \( I \), \( V_s \) and \( I_s \), be defined as:

\[ V_s \triangleq \{ a_i | a_i \in Z_{++}, \frac{a_i}{2} \notin Z_{++}, i = 1, \ldots N_V \} \]  

(51)

\[ I_s \triangleq \{ b_i | b_i \in Z_{++}, \frac{b_i}{2} \notin Z_{++}, i = 1, \ldots N_I \} \]  

(52)

For voltage harmonics in \( V_s \), the injected voltages \( v_{NPQ}(t) \) are zero-sequence voltages, which are triplen harmonics utilised since they do not appear in the line-to-line output voltages [18]. Further, for current harmonics in \( I_s \), which are even non-triplen harmonics, it can be shown that the sum of arm currents add to \( I_{dc} \), that is:

\[ \sum_{p \in \{a,b,c\}} i_{pq}(t, x_i) = I_{dc}, \quad q \in \{u,l\} \]  

(53)

and therefore the dc-link current, \( I_{dc} \) shown in Fig. 1 is not altered by injection of circulating currents.

For the special subsets \( V_s \) and \( I_s \), it can be shown in Appendix B that \( \alpha = T_0/2 \). Substituting \( \alpha = T_0/2 \) into (16) and (27) leads to:

\[ v_{pl}(t, x_v) = v_{pdu}(t - \frac{T_0}{2}, x_v), p \in \{a,b,c\} \]  

(54)

\[ i_{pl}(t, x_i) = i_{pdu}(t - \frac{T_0}{2}, x_i), p \in \{a,b,c\} \]  

(55)

\[ e_{CPq}(t, x_v, x_i) = e_{CPq}(t - \frac{T_0}{2}, x_v, x_i, x_{ec}), p \in \{a,b,c\} \]  

(56)

Given the result in (54) and the definition of \( E_{MINpq}(t, x_v) \) in (106), leads to (57).

\[ E_{MINpq}(t) = E_{MINpq}(t - \frac{T_0}{2}), p \in \{a,b,c\} \]  

(57)

The relationships in (54)-(57) are termed here as the symmetric properties of the circuit functions.

For voltage and current harmonics in \( V_s \) and \( I_s \), it can be shown that \( e_{CPq}(t, x_v, x_i, x_{ec}) \) is \( T_0 \)-periodic if and only if (58) is satisfied.

\[ I_{dc}V_{dc} + I_{lg}V_{lg} + I_{lh}V_{lh} = 0 \]  

(58)

It can be shown that \( e_{CPq}(t, x_v, x_i, x_{ec}), E_{MINpq}(t, x_v) \) and \( e_{CPq}(t, x_v, x_i, x_{ec}) \) are \( T_0 \)-periodic, three-phase time-offset and symmetric. The derivation is detailed in the Appendix C, Appendix D and Appendix E.

In view of the properties established for the circuit functions in the arms, the circuit function \( v_{NPQ}(t, x_v), i_{pq}(t, x_i), \)
$e_{Cpq}(t, x_q, x_t, x_{CC})$ and $E_{MNpq}(t, x_t)$ for any arm can be determined from the corresponding circuit functions in arm \textit{au}. This important result implies only the circuit parameters in one arm needs to be treated in the optimisation problem. For this reason, only the \textit{upper} arm of phase \textit{a} is considered.

\section*{C. MODEL VALIDATION}

The MMC model presented in Section II contains a number of assumptions that permits the formulation of tractable optimisation problems. It is important to check that this idealised model is a good approximation of the a real MMC system. The voltage, current and energy waveforms that are derived from the model are computed in Matlab. These waveforms are compared to those generated in an MMC experimental setting to establish the validity of the model. In this comparison, the nominal and injected voltage and current harmonics are the same in the model and three-phase MMC experimental setup. The $C_{\text{eff}}$ used in the model is set to the value used in the experimental setup (see Table 1), that is:

$$C_{\text{eff}} = \frac{5.93 \text{mF}}{8} = 0.74 \text{mF}$$

\textbf{TABLE 1. Experimental system parameters.}

| Parameter               | Value |
|-------------------------|-------|
| $L_{\text{arm}}$        | 6 mH  |
| $C_{SM}$                | 5.93 mF |
| No. submodules/arm, J   | 8     |
| Load resistance         | 10 \Omega |
| Control frequency       | 5 kHz |

The fundamental frequency used in the Matlab-computed model and experimental setup is $10\pi$ rad/s (5Hz) which is a typical low speed operational condition in drive applications, where injection of harmonic components is often utilised to prevent excessive capacitor voltage peaks [4]. The nominal conditions and injected voltage and current parameters are presented in Table 2.

\textbf{TABLE 2. Nominal and injected harmonics voltages and currents.}

| Parameter               | Value |
|-------------------------|-------|
| $I_{DC}$                | 1.5818 A |
| $I_{1g}, I_{1h}$        | -2.9 A, 0 A |
| $V_{DC}$                | 110 V |
| $V_{1g}, V_{1h}$        | 60V, 0V |
| $\varpi$                | 10\pi rad/s |
| $I_{2g}, I_{2h}$        | 1.37 A, -0.34 A |
| $I_{4g}, I_{4h}$        | -0.15 A, 0.22 A |
| $I_{8g}, I_{8h}$        | -0.08 A, 0.03 A |
| $V_{10g}, V_{10h}$      | -0.02 A, -0.05 A |
| $v_{C, ave}, jv_{C, ave}$ | 13.5 V, 108 V |
| $V_{1g}, V_{3h}$        | -12.95 V, -0.12 V |
| $V_{2g}, V_{6h}$        | -1.00 V, -0.12 V |

Experimental results were obtained using the setup described in [30], with system parameters shown in Table 1. Details of the signal processor and control structure are provided in [30]. The experimental setup controls the currents and average submodule capacitor voltage, $v_{C, ave}$ at

The arm experimental energy waveforms are presented in Fig. 3. The waveforms in Fig. 3 are periodic and three-phase time offset, thus adhering to the properties shown earlier in Section III-B7. The arm \textit{au} experimental and model energy waveforms are shown in the top-left of Fig. 4. The resultant model waveforms are computed for arm \textit{au} only since the waveforms of other arms are simply time-offset with respect to arm \textit{au}. The superscript \textit{ex} describes the result obtained from the experimental setup. The resultant $E_{MN}(t)$ and $E_C(t)$ align closely from the model and experimental results. Note that the model waveforms have been time-shifted to adjust for sampling offset with the experimental waveforms. There is close agreement between the \textit{mo} and \textit{ex} results. The small discrepancies arise from the voltage drops across the semiconductors, arm inductances and resistances that were not modelled.

The resultant arm voltages and currents for the upper arms across the three phases are presented in the right two subplots of Fig. 4. The results of the model are superimposed on these experimental results. Again, close alignment of the arm voltage and current of the modelling and experimental results occurs. As expected, voltages and currents are periodic and three-phase time-offset. The nonnegative constraint (39) ensures nonnegative arm voltages and this means that bipolar (full-bridge) converters submodules are not required in the proposed approach. Rather, unipolar (half-bridge) converters can be used to implement these optimisation results if desired. It is clear from the bottom-left of Fig. 4 that each submodule capacitor voltage in an arm is are closely balanced throughout the experiment. These preliminary results provide evidence that the modelling assumptions are acceptable.

\section*{D. PROPOSED OPTIMISATION PROBLEMS STEPS}

This subsection presents the “steps” which form part of the two-step optimisation methods proposed in this paper.

It follows from (28) that objective function in (30) is a bilinear function of injected voltage and current harmonics, and is in general nonconvex. A drawback of nonconvex objective functions is that finding a minimum does not guarantee that it is the global minimum. A benefit of convex optimisation problems is that widely-available solving techniques can consistently find optimal values to such problems [31]. Further, the minimum of convex optimisation is the global minimum.

Two-step optimisation problems that involve subsets of the optimisation variables ($x_t$, $x_i$ and $x_{CC}$) are proposed such that each optimisation step involves a convex optimisation problem. The optimisation problem in each step is tractable and can be solved using known methods. Four optimisation steps are proposed: LPv, QPv, LPi and QPi. Their details are discussed subsequently. As will be evident, each of the proposed optimisation steps involves either the voltage...
QPv is the optimisation problem in (60). The optimisation problem.

1) QPv

QPv is the optimisation problem in (60). The optimisation problem minimizes $e_C(t, x_v, \bar{x}_i, x_{ECO})$ over the optimisation variables $x_v$ and $x_{ECO}$ for a fixed value for $x_i$ that is denoted $\bar{x}_i$. From (28) it is clear that for a given $\bar{x}_i$, that $e_C(t, x_v, \bar{x}_i, x_{ECO})$ is a linear function of $x_v$ and $x_{ECO}$.

$$\begin{align*}
\text{minimize} & \quad \max_{x_v, x_{ECO}} e_C(t, x_v, \bar{x}_i, x_{ECO}) \\
\text{subject to} & \quad e_C(t, x_v, \bar{x}_i, x_{ECO}) \geq 0 \\
& \quad v(t, x_v) \geq 0 \\
& \quad v(t, x_v) \leq V_{MAX} \\
& \quad B_v x_v = 0
\end{align*}$$

(60)

From (85) it obvious that $e_C(t, x_v, \bar{x}_i, x_{ECO})$ in (60) is a quadratic function of $x_v$. Note that QP is an abbreviation given to the names of problems that can be formulated as a quadratic program.

2) LPv

Another optimisation problem step which is solved over voltage harmonics, $x_v$, is proposed. An approach to reduce (30) is to minimize the maximum of $E_{MIN}(t, x_v)$, which is a function of only the optimisation variables, $x_v$. The optimisation problem, LPv, defined by (61) is introduced.

$$\begin{align*}
\text{minimize} & \quad \max_{x_v} E_{MIN}(t, x_v) \\
\text{subject to} & \quad v(t, x_v) \geq 0 \\
& \quad B_v x_v = 0
\end{align*}$$

(61)

$E_{MIN}(t, x_v)$ is a quadratic function of $x_v$.

It can be shown that the optimisation problem in (62) is equivalent to the optimisation problem in (61) because the objective function in (62) is related to the objective function in (61) by a monotone transformation, $(\cdot)^2$, over the domain of feasible solutions [32].

$$\begin{align*}
\text{minimize} & \quad \max_{x_v} v(t, x_v) \\
\text{subject to} & \quad v(t, x_v) \geq 0 \\
& \quad B_v x_v = 0
\end{align*}$$

(62)

Note that LP is an abbreviation given to the names of problems that can be formulated as a linear program.

3) LPi

The optimisation problem, LPi, is defined by (63). In this problem $e_C(t, x_v, \bar{x}_i, x_{ECO})$ is minimized with respect to the

parameters (LPv and QPv) or the current parameters (LPi and QPv) but not both.

All functions considered in the objective and constraints are $T_0$-periodic and thus every optimisation problem is solved over the interval $t \in [t_0, t_0 + T_0)$.

It is noted that if the injection of voltage harmonics is considered undesirable, then the LPv or QPv steps can be omitted, and the voltage harmonic coefficients can be set to zero.

FIGURE 3. $E_{MIN}^a(t)$ and $e_C(t)$ in each upper (left) and lower (right) arm resulting from the experiment, using parameters in Tables 1 and 2. Solid lines for leg a, dashed lines for leg b, and dotted lines for leg c.

FIGURE 4. $E_{MIN}(t)$ and $e_C(t)$ in arm au (top-left), arm voltages (top-right) and currents (bottom-right) in the upper arms, which result from Matlab-computed model, superscript mo, and the experimental setup, superscript ex. The experimental submodule capacitor voltages (bottom-left) in an arm.
optimisation variables $x_i$ and $x_{eCO}$ for a given $\tilde{x}_i$.

\[
\begin{align*}
\text{minimise} & \quad \max_{x,f(t, \tilde{x}_i, x_i, x_{eCO})} \quad e_C(t, \tilde{x}_i, x_i, x_{eCO}) \\
\text{subject to} & \quad e_C(t, \tilde{x}_i, x_i, x_{eCO}) \geq 0 \\
& \quad |i(t, x_i)| \leq I_{\text{MAX}} \\
& \quad B_i x_i = 0
\end{align*}
\]

(63)

4) QPi

Finally, the optimisation problem QPi is proposed. Compared to LPi the key addition in the optimisation problem QPi defined in (64) is the inclusion of a constraint on the rms current. From (37) it is clear that this is a quadratic constraint.

\[
\begin{align*}
\text{minimise} & \quad \max_{x,f(t, \tilde{x}_i, x_i, x_{eCO})} \quad e_C(t, \tilde{x}_i, x_i, x_{eCO}) \\
\text{subject to} & \quad e_C(t, \tilde{x}_i, x_i, x_{eCO}) \geq 0 \\
& \quad |i(t, x_i)| \leq I_{\text{MAX}} \\
& \quad i_{\text{rms}}(t, x_i) \leq (I_{\text{RMS}} i_{\text{rms}}(x_i = 0))^2 \\
& \quad B_i x_i = 0
\end{align*}
\]

(64)

E. PROPOSED TWO-STEP OPTIMISATION PROBLEMS

The names given to each two-step optimisation problem indicates which optimisation problems steps are solved and their sequence. For example LPvLPi solves LPv first, followed by LPi second.

The three two-step optimisation problems proposed in this paper are: 1. LPvLPi. 2. LPvQPv. 3. QPvQPi.

The selection of fixed value parameters in each step of the two-step optimisation problems are described as follows. For each two-step problem, the first step selects the fixed values to be zero, whereas the second step selects the fixed values to be the optimal values of the first step. That is, for LPv, the fixed values, $\tilde{x}_{eCO}$ and $\tilde{x}_i$, are selected to be zero. For QPv $\tilde{x}_i$ is selected to be zero. For LPi and QPi, $\tilde{x}_i$ is selected to be the optimal $x_i$ that results from LPv or QPv.

IV. OPTIMISATION IMPLEMENTATION AND RESULTS

In this section, methods are presented for the simplification, computation and comparison of the optimisation problems proposed in Section III-D.

A. DIMENSION REDUCTION

The linear equality constraints defined in (49) and (50) in each of the defined optimisation problems have a special structure which sets undesired voltage and current harmonics to zero. This structure can be exploited to reduce the dimension of the parameter vector in each of the optimisation problems. It can be shown, that there exists matrices $P_{s,v}$ and $P_{s,i}$ such that:

\[
\begin{align*}
\mathbf{z}_v &\equiv P_{s,v} \mathbf{x}_v \\
\mathbf{z}_i &\equiv P_{s,i} \mathbf{x}_i
\end{align*}
\]

where the resulting parameter vectors of $\mathbf{z}_v$ and $\mathbf{z}_i$ contain only the injected voltages and currents of the desired sets $V_s$ and $I_s$, respectively. The dimensions of the vectors in (65) are reduced with respect to $x_v \in \mathbb{R}^{2NH}$ and $x_i \in \mathbb{R}^{2NH}$ in (76) because $N_v, N_f \leq N_H$. In this paper two voltage and four current harmonics are injected, $V_{3g}, V_{3h}, V_{9g}, V_{9h}$ and $I_{2g}, I_{2h}, I_{4g}, I_{4h}, I_{8g}, I_{8h}, I_{10g}, I_{10h}$, and it is found that $\mathbf{z}_v \in \mathbb{R}^{4}$ and $\mathbf{z}_i \in \mathbb{R}^{8}$.

It can be shown that the range-space of $P_{s,v}$ and $P_{s,i}$ is equal to the nullspace of $B_v$ and $B_i$, respectively. Because of this property, the equality constraints defined by matrices $B_v$ and $B_i$ are removed without affecting the optimal solution [31] when considering the optimisation problems in terms of the reduced dimension parameter vectors, $\mathbf{z}_v$ and $\mathbf{z}_i$.

1) PROPOSED OPTIMISATION STEPS OF REDUCED DIMENSION

Each proposed optimisation step is re-written in terms of the reduced dimension parameter vectors, $\mathbf{z}_v$ and $\mathbf{z}_i$. This reduced dimension vector form is considered hereafter and is used to generate the results in Section IV-H.

a: LPv

\[
\begin{align*}
\text{minimize} & \quad \max_{t, \mathbf{z}_v} \quad v(t, \mathbf{z}_v) \\
\text{subject to} & \quad v(t, \mathbf{z}_v) \geq 0
\end{align*}
\]

(66)

b: QPv

\[
\begin{align*}
\text{minimize} & \quad \max_{t, \mathbf{z}_v, \mathbf{z}_i} \quad e_C(t, \mathbf{z}_v, \mathbf{z}_i, x_{eCO}) \\
\text{subject to} & \quad e_C(t, \mathbf{z}_v, \mathbf{z}_i, x_{eCO}) \geq 0 \\
& \quad v(t, \mathbf{z}_v) \geq 0 \\
& \quad v(t, \mathbf{z}_v) \leq V_{\text{MAX}}
\end{align*}
\]

(67)

c: LPi

\[
\begin{align*}
\text{minimize} & \quad \max_{t, \mathbf{z}_v, \mathbf{z}_i} \quad e_C(t, \mathbf{z}_v, \mathbf{z}_i, x_{eCO}) \\
\text{subject to} & \quad e_C(t, \mathbf{z}_v, \mathbf{z}_i, x_{eCO}) \geq 0 \\
& \quad |i(t, \mathbf{z}_i)| \leq I_{\text{MAX}}
\end{align*}
\]

(68)

d: QPi

\[
\begin{align*}
\text{minimize} & \quad \max_{t, \mathbf{z}_v, \mathbf{z}_i} \quad e_C(t, \mathbf{z}_v, \mathbf{z}_i, x_{eCO}) \\
\text{subject to} & \quad e_C(t, \mathbf{z}_v, \mathbf{z}_i, x_{eCO}) \geq 0
\end{align*}
\]
\[ |i(t, z_i)| \leq I_{\text{MAX}} \]
\[ i_{\text{rms}}^2(t, z_i) \leq (I_{\text{RMSF}}i_{\text{rms}}(z_i = 0))^2 \]  

(69)

\section*{B. SAMPLING}

The optimisation problems defined so far involve objective functions and constraints over continuous time and hence are in general infinite dimensional problems. Discretisation of the functions in the time domain is one approach to generating finite dimensional optimisation problems that approximate the infinite dimensional problems. The discretisation approach adopted in this paper is to consider uniformly sampled values over one period. The selection of appropriate sampling interval and the effect of this simple discretisation approach is an important area that requires in-depth investigation but is beyond the scope of this paper. For simplicity, the continuous time notation is retained with the understanding that \( t \) belongs to a finite set of values \( \{t_0, t_0 + 1/f_s, t_0 + 2/f_s, \ldots, t_0 + T_0 - 1/f_s\} \), where \( f_s \) is the sampling frequency. As such, the optimisation problems in (66)-(69) can be solved using known numerical algorithms.

\section*{C. ALGORITHM}

There exists multiple Matlab functions to solve the optimisation problems LPv and LPi. Examples include, \texttt{fmincon}, \texttt{fminimax} and \texttt{linprog}.

Matlab functions which can be used to solve QPv and QPi include \texttt{fmincon} and \texttt{fminimax}.

The \texttt{fmincon} Matlab function and interior-point algorithm [33]–[35] are selected to study the solutions of the three two-step optimisation problems. The optimisation tolerances on optimality and constraints are the same for each two-step optimisation problem considered. By using the same Matlab function and algorithm for each problem ensures the stopping criteria is consistent, which gives an effective means for comparative studies.

To solve using \texttt{fmincon}, the optimisation problems need to be transformed into the equivalent epigraph form, which is possible given the optimisation problems are convex [31]. This is achieved by introducing an auxiliary variable to the optimisation problems [31].

\section*{D. OPTIMISATION OVER A RANGE OF POWER FACTORS}

The optimisation problems are solved over a range of power factors by considering the nominal operational parameters in (70).

\[ V_{1g} = -\sin(\theta_V) \quad V_{1h} = -\cos(\theta_V) \]
\[ I_{1g} = 0 \quad I_{1h} = 1 \]  

(70)

where \( \theta_V \in [-\pi, \pi] \). Using (70), the ac components (subscript ~) of the nominal voltages and currents are

\[ v_{N-\text{rms}}(t) = \sin(\omega_0 t + \theta_V) \ V \]  

(71)

\[ i_{N-\text{rms}}(t) = \sin(\omega_0 t) \ A \]  

(72)

Therefore, the optimisation problems can be solved over a range of power factors. Hereafter, the functions which depend on \( \theta_V \) are made explicit in the arguments.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Parameter & Value \\
\hline
\( f_0 \) (Hz) & 50 \\
\hline
\( f_s \) (Hz) & 9000 \\
\hline
\( \theta_V \) (rad) & [-\pi/2, \pi/2] \\
\hline
\( t \) & [0, 1/9000, 1/50] \\
\hline
\( t_0 \) & 0 \\
\hline
\( I_{\text{RMSF}} \) & [1.03:0.04:1.63] \\
\hline
\( I_{\text{MAX}} \) (A) & 1.875 \\
\hline
\( V_{\text{MAX}} \) (V) & 244 \\
\hline
\( V_{dc}, V_{1g}, V_{1h} \) (V) & 2, -\sin(\theta_V), -\cos(\theta_V) \\
\hline
\( I_{dc}, I_{1g}, I_{1h} \) (A) & 1.5 \cos(\theta_V), 0, 1 \\
\hline
OptimalityTolerance & 10^{-6} \\
\hline
ConstraintTolerance & 10^{-7} \\
\hline
\end{tabular}
\caption{Numerical optimisation parameter values.}
\end{table}

\section*{E. PARAMETERS FOR NUMERICAL OPTIMISATION}

The peak arm current upper bound, \( I_{\text{MAX}} \), is selected to be:

\[ I_{\text{MAX}} = 1.25 \max_{\theta_V \in [-\pi, \pi]} \left( \max_{t \in [t_0, t_0 + T_0]} (i_n(t, \theta_V)) \right) \]  

(73)

The upper bound, \( V_{\text{MAX}} \), is set to be large to keep the peak arm voltage constraint inactive and therefore exclude the effects of \( V_{\text{MAX}} \) as a factor upon comparing the different optimisation problems.

\section*{F. NOMINAL CONDITION PARAMETERS}

In the nominal conditions, there are no injected voltages or currents, \( z_i = 0 \). The nominal parameter \( x_{\text{eoN}} \) needs to be selected to satisfy the constraint in (46). In order to compare the results of the nominal conditions to those generated from the optimisation problems, a selection process is defined for the nominal \( x_{\text{eoN}} \). Because the focus on this paper is on peak capacitor voltage reduction, it follows that the lowest possible \( x_{\text{eoN}} \) that satisfies the condition in (46) is defined by (74).

\[ x_{\text{eoN}} = \max_{t \in [t_0, t_0 + T_0]} (y(t, \theta_V)) \]  

(74)

This is because the definition of \( x_{\text{eoN}} \) in (74) implies that the minimum of the nominal \( \varepsilon_C \), \( \varepsilon_C(t, z_i = 0, z_i = 0, x_{\text{eoN}}) \), is zero, that is:

\[ \min_{t \in [t_0, t_0 + T_0]} (\varepsilon_C(t, \theta_V, z_i = 0, z_i = 0, x_{\text{eoN}})) = 0 \]  

(75)

The result in (75) means that using (74) for nominal conditions creates a capacitor energy waveform which touches \( E_{\text{MIN}}(t, z_i = 0) \).

\section*{G. PERFORMANCE INDICATORS}

The performance indicators are defined in Table 4. Each of the performance indicators are key sources of capacitor stress [8], [9], [11] and MMC power losses [7] which are considered...
to select component ratings for MMCs. Trends in these performance indicators give designers an overview of the trade-offs among power losses and stresses on components. The $\hat{\cdot}$ notation denotes a variable that is evaluated at the optimal parameters $\hat{z}_v$, $\hat{z}_i$, and $\hat{x}_{C0}$ that result from numerically solving the relevant two-step optimisation problems.

### H. RESULTS COMPARISON

All three two-step optimisation problems are solved numerically for each $\theta_V$ and $I_{\text{RMSF}}$ defined in Table 3. The resulting performance indicators for each optimisation problems at every $\theta_V$ and $I_{\text{RMSF}}$ are termed here as the optimisation results.

Examples of optimisation results generated using QPvQPi are illustrated in 3D plots in Fig. 5 and Fig. 6. Fig. 5 illustrates the rms current of QPvQPi, which approaches to the nominal rms current for all $\theta_V$ as the upper bound on rms current is tightened (decreased). This is the result of defining the rms current constraint limit relative to the rms of the nominal current. It is clear from Fig. 6 that the cost incurred for reducing rms currents is the increase in peak capacitor voltages.

Examples of optimal injected voltage and current harmonic parameters, $\hat{V}_{3g} \in \hat{z}_v$ and $\hat{I}_{2h} \in \hat{z}_i$ are shown in Fig. 7 and Fig. 8, respectively. It is clear that as rms current constraint is loosened (increased), the optimal parameter $\hat{I}_{2h}$ increases. Together, Fig. 6 to Fig. 8 illustrate the variation of performance indicators and optimisation parameters with varying constraint, $I_{\text{RMSF}}$, and power factor, $\theta_V$.

The optimisation results, which can be shown as 3D plots in Fig. 5 and Fig. 6, are condensed as 2D plots by applying the operator $\max_{\theta_V}(\cdot)$. This is for convenience of assessing overall performance in terms of the maximum, which is an important indicator for design purposes.

All 2D plots show that the LPvQPi optimisation results approach those of LPvLPi as $I_{\text{RMSF}}$ is loosened. This occurs because the only difference between LPi and QPi is the rms current constraint. The removal of the rms current constraint from QPi, which is effectively achieved by loosening $I_{\text{RMSF}}$, gives the same results as LPi. LPi does not have a constraint on rms current, and therefore it is independent of $I_{\text{RMSF}}$.

### TABLE 4. Performance indicators.

| Performance indicator | Description |
|-----------------------|-------------|
| $\max_{\theta_V}(J\hat{V}_C)$ | Peak capacitor voltage |
| $J_{\text{V}_{\text{C}p}} \triangleq \max_{\theta_V} J_{\text{V}_C} - \min_{\theta_V} J_{\text{V}_C}$ | Capacitor voltage ripple |
| $t_{\text{C}_{\text{ip}}} \triangleq \max_{\theta_V} t_{\text{C}} - \min_{\theta_V} t_{\text{C}}$ | Capacitor current ripple |
| $I_{\text{rms}}$ | Arm rms current |
| $i_{\text{ave}} \triangleq \text{ave}_{t}(i(t))$ | Arm average current |

### FIGURE 5. The optimal rms current at each $I_{\text{RMSF}}$ and $\theta_V$ which results from QPvQPi using parameters in Table 3 at $C_{\text{eff}} = 3$ mF. The nominal rms current is the black line.

### FIGURE 6. The optimal peak capacitor voltage at each $I_{\text{RMSF}}$ and $\theta_V$ which results from QPvQPi using parameters in Table 3 at $C_{\text{eff}} = 3$ mF.

### FIGURE 7. The optimal third harmonic voltage coefficient of Cosine for each $I_{\text{RMSF}}$ and $\theta_V$ which results from QPvQPi using parameters in Table 3 at $C_{\text{eff}} = 3$ mF.

### FIGURE 8. The optimal second harmonic voltage coefficient of Sine for each $I_{\text{RMSF}}$ and $\theta_V$ which results from QPvQPi using parameters in Table 3 at $C_{\text{eff}} = 3$ mF.
as shown by constant LPvLPi lines in the 2D plots. For the same reason, the nominal results are constant in the 2D plots.

Fig. 9 to Fig. 13 present the optimisation results for $C_{\text{eff}} = 3\, \text{mF}$. Fig. 9 shows the peak capacitor voltage performance indicator as a function of $I_{\text{RMSF}}$. These results reflect the optimal objective criteria as the upper bound of the rms current constraint is varied. All optimisation problems give significantly lower peak capacitor voltage compared to the nominal case. This demonstrates the effectiveness of the optimisation problems, in particular the objective function, in decreasing capacitor stress caused by peak voltage.

The peak capacitor voltage ripple, arm rms and average current, are plotted against $I_{\text{RMSF}}$ in Fig. 10, Fig. 11 and Fig. 12, respectively. The cost of reducing arm rms current are an increase in peak capacitor voltage and peak capacitor voltage ripple. However, the benefit is a decrease in average arm current and, obviously, rms current. Lower rms and average currents give lower power losses [6], [7], [36]. These figures illustrate the trade-offs in terms of increases in the capacitor stresses and lower power losses. Further, it is clear that the $I_{\text{RMSF}}$ is effective in limiting the rms currents which increase with circulating current injection.

The capacitor current ripple performance indicator for $C_{\text{eff}} = 3\, \text{mF}$ is presented in Fig. 13. All optimisation problems result in lower capacitor current ripple with respect to the nominal, which is favourable for reducing capacitor stress. The lower capacitor stress from a combination of lower peak capacitor voltage and current ripple, along with limits on rms and average arm currents, exhibit the benefits of LPvQPi and QPvQPi. Further, QPvQPi is particularly useful when considering power losses and reducing peak capacitor voltage for all $I_{\text{RMSF}}$.

The optimisation results for a larger $C_{\text{eff}}$ (6 mF) are presented in Fig. 14 to Fig. 16. Both peak capacitor voltage and capacitor voltage ripple decrease with larger capacitance when compared to Fig. 9 and Fig. 10.

The capacitor current ripple is increased with respect to those presented with $C_{\text{eff}} = 3\, \text{mF}$. In addition, for $C_{\text{eff}} = 6\, \text{mF}$, the nominal capacitor current ripple is lower than those resulting from the optimisation problems.
1) KEY OUTCOMES
The key outcomes of the applying the two-step optimisations are summarized as follows:

1) The peak capacitor voltages are significantly reduced with respect to the nominal for each two-step optimisation problem presented.
2) The QPi optimisation step has demonstrated its ability to constrain arm rms current and lower average current, which are important for power loss and thermal considerations.
3) For a lower capacitance value, lowering rms current does not come at a cost of capacitor voltage and current ripple that are larger than the nominal.
4) Loosening rms upper bound is beneficial in reducing capacitor stress in terms of peak and ripple capacitor voltages. However there is a trade-off since rms and average arm currents increase.
5) Holistically, LPvQPi and QPvQPi are good candidates for reducing peak capacitor voltages.

These results highlight the trade-offs which occur when optimising for lower capacitor stress and power losses. The key outcomes give designers a perspective on how the circuit parameters behave when adjusting constraints over a range of operating points.

V. CONCLUSION
This paper proposed three two-step convex optimisation problems which serve as tools for designers to reduce peak capacitor voltage and limiting rms currents in MMCs based on a simplified circuit model of an MMC. Each has been compared in terms of performance indicators which reflect capacitor stresses and power losses. The proposed methods have been shown to clearly identify that a trade-off that exists between capacitor stress and power losses.

Note that it is possible to consider variants of the two-step optimisation problems considered. For example, the order of QPvQPi can be changed to QPiQPv, or even alternation optimisation can be considered for the former or latter.
APPENDICES

A. VECTOR FORM OF ARM VOLTAGE AND CURRENT

The vector forms of arm voltages and currents are defined. Let the vectors containing the harmonic coefficients for voltages, \( x_v \), and currents, \( x_i \), be defined in \( (76) \). The vectors each contain coefficients for both Sine and Cosine terms for all harmonics from 2 to some integer \( N_H \), where \( N_H \geq 2 \).

\[
x_v \triangleq \begin{bmatrix} V_{2g} \\ \vdots \\ V_{N_{H}g} \\ V_{2h} \\ \vdots \\ V_{N_{H}h} \\ N_{H} \end{bmatrix}, \quad x_i \triangleq \begin{bmatrix} I_{2g} \\ \vdots \\ I_{N_{H}g} \\ I_{2h} \\ \vdots \\ I_{N_{H}h} \\ N_{H} \end{bmatrix}
\]

(76)

\[
f_p(t) \triangleq \begin{bmatrix} \cos(2\omega_0(t - T_p)) \\ \vdots \\ \cos(N_{H}\omega_0(t - T_p)) \\ \sin(2\omega_0(t - T_p)) \\ \vdots \\ \sin(N_{H}\omega_0(t - T_p)) \end{bmatrix}_{2N_{H}}
\]

(77)

\[v(t, x_v) = v_N(t) - f^T(t)x_v\]

(78)

\[i(t, x_i) = i_N(t) - f^T(t)x_i\]

(79)

where \( T_p = T_g = T_{pq} = 0 \) and \( f(t) \triangleq f_p(t) \). Other key functions are defined in \( (80)-(83) \).

\[
\varphi_v(t) \triangleq \int_{t_0}^t f(\tau)v_N(\tau)d\tau \in \mathbb{R}^{2n},
\]

(80)

\[
\varphi_i(t) \triangleq \int_{t_0}^t f(\tau)i_N(\tau)d\tau \in \mathbb{R}^{2n},
\]

(81)

\[
P(t) \triangleq \int_{t_0}^t f(\tau)f^T(\tau)d\tau \in \mathbb{R}^{2n \times 2n},
\]

(82)

\[
y(t) \triangleq \int_{t_0}^t v_N(\tau)i_N(\tau)d\tau - C_{eff}V_c^2(t)
\]

(83)

Using these defined functions, it can be shown that the vector form of key circuit functions \( e_C(t, x_v, x_i, x_{ec0}) \) and \( \varepsilon_C(t, x_v, x_i, x_{ec0}) \) are given by \( (84) \) and \( (85) \).

\[
e_C(t, x_v, x_i, x_{ec0}) = x_{ec0} + \int_{t_0}^t v_N(\tau)i_N(\tau)d\tau - x_v^T\varphi_v(t) - x_i^T\varphi_i(t) + x_v^Tf(t)x_i
\]

(84)

\[
\varepsilon_C(t, x_v, x_i, x_{ec0}) = y(t) + x_{ec0} - \frac{C_{eff}}{2}x_v^Tf(t)f^T(t)x_v + C_{eff}v_Nf^T(t)x_v - q^T(t)x_v - q^T(t)\varphi_v(t) + x_v^Tf(t)x_i
\]

(85)

B. VOLTAGE AND CURRENT SYMMETRY FOR \( V_s \subseteq V \) AND \( I_s \subseteq I \)

The \( T_0/2 \)-delayed voltage across any of the upper arms is

\[
v_{pu}(t - \frac{T_0}{2}) = \frac{V_{dc}}{2} - V_{1g}\cos(\omega_0(t - \frac{T_0}{2} - T_p))
\]

\[
- V_{1h}\sin(\omega_0(t - \frac{T_0}{2} - T_p))
\]

\[
- \sum_{m \in V_s} V_{mg}\cos(m\omega_0(t - \frac{T_0}{2} - T_p))
\]

\[
- \sum_{m \in V_s} V_{mh}\sin(m\omega_0(t - \frac{T_0}{2} - T_p))
\]

\[
= \frac{V_{dc}}{2} - V_{1g}\cos(\omega_0(t - T_p) - \pi)
\]

\[
- V_{1h}\sin(\omega_0(t - T_p) - \pi)
\]

\[
- \sum_{m \in V_s} V_{mg}\cos(m\omega_0(t - T_p) - m\pi)
\]

\[
- \sum_{m \in V_s} V_{mh}\sin(m\omega_0(t - T_p) - m\pi)
\]

(86)

The \( T_0/2 \)-delayed current in any of the upper arms is

\[
i_{pu}(t - \frac{T_0}{2}) = \frac{-I_{dc}}{3} + I_{1g}\cos(\omega_0(t - \frac{T_0}{2} - T_p))
\]

\[
+ I_{1h}\sin(\omega_0(t - \frac{T_0}{2} - T_p))
\]

\[
- \sum_{m \in I_s} I_{mg}\cos(m\omega_0(t - \frac{T_0}{2} - T_p))
\]

\[
- \sum_{m \in I_s} I_{mh}\sin(m\omega_0(t - \frac{T_0}{2} - T_p))
\]

\[
= \frac{-I_{dc}}{3} + I_{1g}\cos(\omega_0(t - T_p) - \pi)
\]

\[
+ I_{1h}\sin(\omega_0(t - T_p) - \pi)
\]

\[
- \sum_{m \in I_s} I_{mg}\cos(m\omega_0(t - T_p) - m\pi)
\]

\[
- \sum_{m \in I_s} I_{mh}\sin(m\omega_0(t - T_p) - m\pi)
\]

(87)
The harmonic numbers in $I_s$, $m$, are even, it follows that

$$I_{p}(t - T_0) = \frac{I_{dc}}{3} - I_{th} \cos(\omega_0(t - T))$$

$$- I_{th} \sin(\omega_0(t - T))$$

$$- \sum_{m \in I_s^2} I_{mh} \cos(m \omega_0(t - T))$$

$$- \sum_{m \in I_s^2} I_{mh} \sin(m \omega_0(t - T)) = I_{p}(t) \quad (89)$$

From (87) and (89), it is clear that $\alpha = T_0/2$.

**C. PROPERTIES OF $e_{pq}(t + T, x, x, x, x_{e_{eqpq}})$**

At times the arguments $x, x, x_{e_{eqpq}}$ are dropped from $e_{pq}(t + T, x, x, x_{e_{eqpq}})$ to simplify notation. In this subsection, the relationship among $e_{pq}(t + T, x, x, x_{e_{eqpq}})$ for all $p, q$, are derived under the assumption that the time average of the $e_{pq}(t + T, x, x, x_{e_{eqpq}})$ is equal for all $p, q$. Consider the time-average of $e_{pq}(t + T, x, x, x_{e_{eqpq}})$:

$$\int_t e_{pq}(t + T, x, x, x_{e_{eqpq}})$$

$$= \frac{1}{T_0} \int_t e_{pq}(t + T, x, x, x_{e_{eqpq}})dt$$

$$= \frac{1}{T_0} \int_t \left( \int_{t_0 + T}^{t + T} v_{pq}(\tau, x, x_{e_{eqpq}})(\tau, x, x_{e_{eqpq}})d\tau \right)$$

$$+ x_{e_{eqpq}}(t_0 + T, x, x_{e_{eqpq}})d\tau$$

$$= \frac{1}{T_0} \int_t \left( \int_{t_0 + T}^{t + T} v_{pq}(\tau, x, x, x_{e_{eqpq}})(\tau, x, x_{e_{eqpq}})d\tau \right)$$

$$+ x_{e_{eqpq}}(t_0 + T, x, x_{e_{eqpq}})d\tau \quad (90)$$

Let $\mu = T - T, x, x, x_{e_{eqpq}}$, then:

$$\int_t e_{pq}(t + T, x, x, x_{e_{eqpq}})$$

$$= \frac{1}{T_0} \int_t \left( \int_{t_0 + T}^{t + T} v_{pq}(\mu, x, x, x_{e_{eqpq}})(\mu, x, x_{e_{eqpq}})d\mu \right)$$

$$+ x_{e_{eqpq}}(t_0 + T, x, x_{e_{eqpq}})d\tau \quad (91)$$

Consider now the time average energy in arm $au$:

$$\int_t e_{au}(t) = \frac{1}{T_0} \int_t e_{au}(t)dt$$

$$= \frac{1}{T_0} \int_t \left( \int_{t_0 + T}^{t + T} v_{au}(\tau, x, x, x_{e_{eqpq}})(\tau, x, x_{e_{eqpq}})d\tau \right)$$

$$+ x_{e_{au}}(t_0 + T, x, x_{e_{eqpq}})d\tau \quad (92)$$

Since it is assumed that the time-average of $e_{pq}(t + T, x, x, x_{e_{eqpq}})$ and $e_{au}(t)$ are equal, that is:

$$\int_t e_{au}(t) = \int_t e_{pq}(t + T, x, x, x_{e_{eqpq}})$$

From (91), (92) and (93) it follows that:

$$x_{e_{au}}(t_0 + T, x, x_{e_{eqpq}}) = x_{e_{eqpq}}(t_0 + T, x, x_{e_{eqpq}}) \quad (94)$$

Substitution of (94) in the equation for $e_{pq}(t + T, x, x, x_{e_{eqpq}})$ yields:

$$e_{pq}(t + T, x, x, x_{e_{eqpq}}) = \int_{t_0 + T}^{t + T} v_{pq}(\tau, x, x, x_{e_{eqpq}})(\tau, x, x_{e_{eqpq}})d\tau + x_{e_{au}}(t_0 + T, x, x_{e_{eqpq}})$$

$$e_{au}(t) = \int_{t_0 + T}^{t + T} v_{au}(\tau, x, x, x_{e_{eqpq}})(\tau, x, x_{e_{eqpq}})d\tau + x_{e_{au}}(t_0 + T, x, x_{e_{eqpq}})$$

$$\quad (95)$$

Now consider the function $e_{au}(t)$, where $T, x, x, x_{e_{au}} = 0$:

$$e_{au}(t) = \int_{t_0}^{t} v_{au}(\tau, x, x, x_{e_{eqpq}})(\tau, x, x_{e_{eqpq}})d\tau + x_{e_{au}}(t_0)$$

$$\quad (96)$$

Substitution of (96) into (95) yields:

$$e_{pq}(t + T, x, x, x_{e_{eqpq}}) = e_{au}(t) = e_{pq}(t + T, x, x, x_{e_{eqpq}})$$

$$\quad (97)$$

1) THREE-PHASE TIME-OFFSET

From (97), it can be shown that the relationships of $e_{pq}(t + T, x, x, x_{e_{eqpq}})$ and $e_{au}(t)$ with respect to $e_{au}(t)$ are:

$$e_{pq}(t + T, x, x, x_{e_{eqpq}}) = e_{au}(t)$$

$$e_{pq}(t + T, x, x, x_{e_{eqpq}}) = e_{au}(t)$$

$$\quad (98)$$

Substituting $T, x, x, x_{au} = 0$ for each $p \in \{a, b, c\}$, it is clear that the stored capacitor energy functions are three-phase time-offset in the upper arms as well as the lower arms.

2) SYMMETRIC PROPERTY OF $e_{pq}(t + T, x, x, x_{e_{eqpq}})$ FOR $V_s \subseteq V$ AND $I_s \subseteq I$

Substituting $T, x, x, x_{au} = 0$ into (98) gives (99).

$$e_{pq}(t + T, x, x, x_{e_{eqpq}}) = e_{au}(t)$$

$$e_{pq}(t + T, x, x, x_{e_{eqpq}}) = e_{au}(t)$$

$$\quad (99)$$

Equation (99) implies (100).

$$e_{pq}(t) = e_{pq}(t + T, x, x, x_{e_{eqpq}})$$

$$\quad (100)$$

Equation (100) shows the symmetric property of $e_{pq}(t + T, x, x, x_{e_{eqpq}})$ for $V_s \subseteq V$ and $I_s \subseteq I$.

3) PERIODICITY FOR $V_s \subseteq V$ AND $I_s \subseteq I$

The circuit function $e_{pq}(t + T, x, x, x_{e_{eqpq}})$ is $T_0$-periodic if and only if:

$$e_{pq}(t + T, x, x, x_{e_{eqpq}}) = e_{pq}(t + T, x, x, x_{e_{eqpq}}), \forall t$$

Equation (101) yields:

$$\int_{t_0 + T}^{t + T} v_{pq}(\tau, x, x, x_{e_{eqpq}})(\tau, x, x_{e_{eqpq}})d\tau + x_{e_{eqpq}}(t_0 + T, x, x_{e_{eqpq}})$$

$$\quad (t_0 + T, x, x, x_{e_{eqpq}})$$

$$\quad (102)$$

Equation (102) leads to:

$$\int_{t_0 + T}^{t + T} v_{pq}(\tau, x, x, x_{e_{eqpq}})(\tau, x, x_{e_{eqpq}})d\tau$$

$$\quad (103)$$
\[ E \triangleq \int_{t_0+T_{pq}}^{t+T_{pq}+T_0} v_{pq}(\tau;x_i) i_{pq}(\tau;x_i) d\tau, \quad \forall t \] (103)

Equation (103) implies (104).
\[ \int_{t+T_{pq}}^{t+T_{pq}+T_0} v_{pq}(\tau;x_i) i_{pq}(\tau;x_i) d\tau = 0, \quad \forall t \] (104)

It can be shown that \( v_{pq}(\tau;x_i) i_{pq}(\tau;x_i) \) contains dc components, and sinusoids with frequencies of \( \omega_0 \) and integer multiples of \( \omega_0 \). Therefore, (104) holds if and only if the dc component of \( v_{pq}(\tau;x_i) i_{pq}(\tau;x_i) \) is zero. It can be shown that the dc component of \( v_{pq}(\tau;x_i) i_{pq}(\tau;x_i) \) is equal to: \( V_{dc} I_{dc}/6 + V_{1g} I_{1g}/2 + V_{1h} I_{1h}/2 \), and thus \( \varepsilon_{Cpq}(t + T_{pq}) \) is \( T_0 \)-periodic if and only if:
\[ \frac{V_{dc} I_{dc}}{6} + \frac{V_{1g} I_{1g}}{2} + \frac{V_{1h} I_{1h}}{2} = 0 \] (105)

### D. PROPERTIES OF \( E_{MINpq}(t,x_i) \)

The definition of \( E_{MINpq}(t,x_i) \) is:
\[ E_{MINpq}(t,x_i) \triangleq \frac{C_{eff}}{2} v_{pq}^2(t,x_i) \] (106)

1) THREE-PHASE TIME-OFFSET

It can be shown that the relationships between \( E_{MINpq}(t,x_i) \) and \( E_{MINpq}(t,x_i) \), with respect to \( E_{MINpq}(t,x_i) \), are given by:
\[ E_{MINpq}(t + T_p,x_i) = E_{MINpq}(t,x_i) \]
\[ E_{MINpq}(t + T_p + \alpha,x_i) = E_{MINpq}(t,x_i) \] (107)

Substituting \( T_p \) for each \( p \in \{a,b,c\} \), it is clear that \( E_{MINpq}(t,x_i) \) are three-phase time-offset in the upper arms as well as the lower arms.

2) \( T_0 \)-PERIODICITY

It can be shown that \( E_{MINpq}(t,x_i) \) can be described as a constant plus a sum of sinusoids. The greatest common frequency divisor of the sinusoids in \( E_{MINpq}(t,x_i) \) is the fundamental frequency, \( \omega_0 \), and therefore \( E_{MINpq}(t,x_i) \) is \( T_0 \)-periodic.

3) SYMMETRIC PROPERTY OF \( E_{MINpq} \) FOR \( V_s \subseteq \mathcal{V} \) AND \( \mathcal{I}_s \subseteq \mathcal{I} \)

Substituting \( \alpha = T_0/2 \) into (107) gives (108).
\[ E_{MINpq}(t + T_p,x_i) = E_{MINpq}(t,x_i) \]
\[ E_{MINpq}(t + T_p + T_0/2,x_i) = E_{MINpq}(t,x_i) \] (108)

The equations (108) imply (109).
\[ E_{MINpq}(t,x_i) = E_{MINpq}(t + T_0/2,x_i) \] (109)

### E. PROPERTIES OF \( \varepsilon_{Cpq}(t,x_i,x_{e_0}) \)

At times the arguments \( x_i, x_{e_0} \) are dropped from \( \varepsilon_{Cpq}(t,x_i,x_{e_0}) \) to simplify notation. The definition of \( \varepsilon_{Cpq}(t) \) is given below:
\[ \varepsilon_{Cpq}(t) \triangleq \varepsilon_{Cpq}(t + T_{pq}) - E_{MINpq}(t,x_i) \] (110)

1) THREE-PHASE TIME-OFFSET

The equations in (98) and (107) imply that \( \varepsilon_{Cpq}(t) \) has the following relationships:
\[ \varepsilon_{Cpq}(t + T_p) = \varepsilon_{Cas}(t) \]
\[ \varepsilon_{Cpq}(t + T_p + \alpha) = \varepsilon_{Cas}(t) \] (111)

and therefore \( \varepsilon_{Cpq}(t) \) is three-phase time-offset in the upper and lower arms.

2) PERIODICITY FOR \( V_s \subseteq \mathcal{V} \) AND \( \mathcal{I}_s \subseteq \mathcal{I} \)

Under the condition in (105), and because \( E_{MINpq}(t,x_i) \) it \( T_0 \)-periodic, it follows from (110) that \( \varepsilon_{Cpq}(t) \) is \( T_0 \)-periodic.

3) SYMMETRIC PROPERTY OF \( \varepsilon_{Cpq}(t) \) FOR \( V_s \subseteq \mathcal{V} \) AND \( \mathcal{I}_s \subseteq \mathcal{I} \)

Substituting \( \alpha = T_0/2 \) into (111) gives (112).
\[ \varepsilon_{Cpq}(t) = \varepsilon_{Cpq}(t + T_0/2) \] (112)

### F. ARM VOLTAGE, CURRENT AND ENERGY

### RELATIONSHIPS FOR \( V_s \subseteq \mathcal{V} \) AND \( \mathcal{I}_s \subseteq \mathcal{I} \)

The key arm voltage, current and energy relationships when considering the specific voltage and current harmonics defined for \( V_s \) and \( \mathcal{I}_s \) are summarised below. All relationships are defined with respect to the function of arm \( au \).
\[ T_{pq} = T_p + \frac{T_0}{2} \]
\[ v_{Cpq}(t) = v_{Cas}(t - T_{pq}) \]
\[ i_{Cpq}(t) = i_{Cas}(t - T_{pq}) \]
\[ E_{MINpq}(t) = E_{MINpq}(t - T_{pq}) \]
\[ \varepsilon_{Cpq}(t) = \varepsilon_{Cas}(t - T_{pq}) \]
\[ \varepsilon_{Cpq}(t) = \varepsilon_{Cas}(t - T_{pq}) \] (114)

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