Application of geometric flow and bandelet coefficient combining method to image fusion

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Abstract. Based on the theory of Bandelet transformation, we use Beamlab200 toolbox and Matlab software, adopt new construction of geometric flow and Bandelet coefficient combination method, apply new introduction of smaller absolute value rules, and take advantage of usage of computer simulation to experiment image fusion to explore the coefficient combination method and new introduction of smaller absolute value rules for geometric flow and Bandelet and test effect of their application to image fusion. The result after the experiment proves that this method is superior than the Beamlab 200 toolbox method in terms of running time, convergence speed, standard deviation, signal-to-noise ratio, gradient and image effect. The method can be applied to repair damaged images and others according to the results.

1. Introduction
The domestic and traditional sampled tensor product wavelet does not have translation invariance, which is only representing singular features of isotropic. Taking a digital image as an example, when the outline and edge of an image are represented, the points on the boundary are isolated, in which continuity or smoothness along the edge direction is not utilized. In order to overcome the shortcomings of wavelet analysis, foreign scholars Pennec and Mallat proposed the Bandelet transformation in 2000. Bandelet transformation is an edge-based image fusion method, and is also an image fusion method based on the maximum absolute value fusion rule, which can adaptively track the geometric regular direction of the image. The two scholars believe that during the image processing task, if the geometric regularity of the image can be known in advance and fully utilized, the approximation performance of the image transformation method will be improved. In this study, we improved the image fusion method based on the maximum absolute value fusion rule, by introducing smaller absolute value rules, constructing a geometrical flow and by using Bandelet coefficient combining method for image fusion experiments, then obtained a good result [1].

2. Bandelet transformation theory

2.1. Geometric regular images and geometric flow
Let the support area of image \( f(x, y) \) be \([0,1]^2\), and the only continuous uninterrupted edge in \([0,1]^2\) is denoted by \( \Gamma \). The continuous edge \( \Gamma \) in the square region has a finite geometric length. For convenience of representation, let the continuous edge be \([a, b]\) and \([a, b]\) be the projection interval of \( \Gamma \) on the \( x \) axis, and \( \Gamma \) can have the form like (1). [3]
\[
\Gamma = \{(x, y(x)) | a \leq x \leq b \}\tag{1}
\]

Let \(\Omega = [0,1]^2 - \Gamma\), \(C^\alpha(\bar{\Omega})\) represents the \(\alpha\)-order Hölder space. The image \(f(x, y)\) is called \(\alpha\)-order geometric regular image if \(f(x, y) \in C^\alpha(\bar{\Omega})\), \(\gamma(x) \in C^\alpha[a, b]\). \(f(x, y)\) is smooth in other areas except for a segmented smooth edge line. Considering that the edges of any natural image have a certain width, so we can correct \(f(x, y)\)

\[
f = \tilde{f} \star h\tag{2}
\]

Where \(\tilde{f} \in C^\alpha[a, b]\), \(\Omega = [0,1]^2 - \Gamma\) is a fuzzy kernel in a region, and it is a tightly supported one that has not yet been determined \[3\].

2.2. Optimal geometric flow

For the needs of the experiment, the coding of several geometric flow images should be considered in two directions: bit rate and distortion. Inside the square area \(S\), it should be the best geometric flow direction, that is, we should take the smallest Lagrangian function as the formula (3).

\[
(f_\theta, R) = \left| f_\theta - \tilde{f}_\theta \right| + \lambda T^2 (R_g + R_b)\tag{3}
\]

Where \(\tilde{f}_\theta\) is a one-dimensional reconstructed signal, and it is also a Bandelet coefficient that must be uniformly quantized; \(T\) is a uniform quantization threshold, \(R_g\) is the number of bits necessary to encode the geometry stream image, \(R_b\) is the number of bits required for the Bandelet coefficient, and it is the geometrically streamed image that is encoded and consistently quantized. \(\lambda\) is the Lagrange multiplier factor. It is concluded that \(\lambda = 5/28\) is taken in the computer simulation experiment according to Pennec's conclusion.

We can divide the rounded corners \([0, \pi]\) into \(L^2 - 1\) at equal angles for the small squares of \(L \times L\), that is, the possible value of \(\theta\) is as illustrated in the formula (4).

\[
\theta = \frac{k\pi}{L^2 - 1}, k = 0, 1, 2, \ldots, L^2 - 2\tag{4}
\]

However, if the experiment encounters a situation where there is no geometric flow, use \(\theta = \text{Inf}\) as a feature tag to indicate that Bandeletization is not implemented here.

Consistent uniform quantization is required for Bandelet coefficients. Here we can use \(X\) to represent Bandelet coefficients, \(T\) is a uniform uniform quantization threshold, so the quantized value \(Q(X)\) is calculated by the formula (5).

\[
Q(X) = \begin{cases} 
0, & |X| \leq T \\
\text{sgn}(X) \left( q + \frac{1}{2} \right) T, & q T \leq |X| \leq (q + 1) T 
\end{cases}\tag{5}
\]

3. Geometric flow and Bandelet coefficient combining and smaller absolute value rules

Image fusion is as following after geometric flow and Bandelet coefficient combining rules are given.

(1) The two original images are geometrically registered.

(2) In region \(\Omega_s\), re-sampling the sample of the calculated image.
(3) Calculate the geometric flow \( G_j(i), (j = 1, 2, \cdots, N) \) of each region and its corresponding Bandelet coefficient. \( N \) represents the total number of original images. \( G_j(x, y, i) \) is the Bandelet coefficient of pixel \((x, y)\), and it is for the \( j \)th image of original image.

(4) Fusion rules are as follows.

1. Combine geometric flow images with smaller absolute value fusion rules

\[
G_F(i) = \begin{cases} 
G_1(i), & G_1(i) \leq G_2(i) \\
G_2(i), & G_1(i) > G_2(i) 
\end{cases}
\]  

(6)

2. For Bandelet coefficients, use smaller absolute value fusion rules

\[
G_F(x, y, i) = \begin{cases} 
G_1(x, y, i), & |G_1(x, y, i)| \leq |G_2(x, y, i)| \\
G_2(x, y, i), & |G_1(x, y, i)| > |G_2(x, y, i)| 
\end{cases}
\]  

(7)

In formulas (6) and (7), let \( G_F(i) \) be the geometric flow of the region \( \Omega \) after fusing the two images, and use \( G_F(x, y, i) \) to represent the Bandelet coefficient, which is the coefficient at the pixel point \((x, y)\) of the fused image after fusing the two images.

(5) The fused image is obtained by the geometric flow \( G_F(i) \) and the Bandelet coefficient \( G_F(x, y, i) \), through reconstruction by the inverse of the Bandelet transformation.

4. The simulation experiment

4.1. Image effect comparison

In order to compare the effects of image fusion using Beamlab200 toolbox method, geometric flow and Bandelet coefficient combining method, the simulation experiment was carried out with aid of Matlab7.01 software. The images of experimental results are shown below in Figure 1, Figure 2, Figure 3. Figure 4.

\[\text{Figure 1. Beamlab200 toolbox fusion}
\]

Different image fusion based on Beamlab200 toolbox
Figure 2. Different image fusion based on Geometric flow and Bandelet coefficient combining method

Figure 3. Half-images fusion based on Beamlab200 toolbox

Figure 4. Half-images fusion based on Geometric flow and Bandelet coefficient combining method

4.2. Convergence speed comparison
In order to compare the convergence speeds of Beamlab200 toolbox method, geometry flow and Bandelet coefficient combination method respectively, the simulation experiment was carried out with aid of Matlab7.01 software. The images of experimental results are shown below in Figure 5.
4.3. Parameter comparison

In order to compare the performance parameters of Beamlab200 toolbox method, geometric flow and Bandelet coefficient combining method respectively, the simulation experiment was carried out with aid of Matlab7.01 software. The images of experimental results are shown below in Table 1.

| Image fusion method name | Standard deviation | Signal to noise ratio (db) | The number of iterations | The average gradient | Entropy | Running time(s) |
|--------------------------|--------------------|---------------------------|-------------------------|---------------------|--------|-----------------|
| Beamlab200 toolbox method | 71.4153            | 38.6529                   | 131                     | 19.6584             | 7.7583 | 593             |
| Geometric flow and Bandelet coefficient combing method | 69.2548            | 40.0058                   | 96                      | 18.5389             | 7.7695 | 386             |

5. Conclusion

Through the computer numerical simulation experiment, the results are as follows:

First, the simulation results prove the effectiveness of the geometric flow and Bandelet coefficients combining method, and the new introduction of new smaller absolute value rules. Through comparison between Figure 1 and Figure 2 as well as between Figure 3 and Figure 4, it proves that we can process the edge texture more clearly, display the stereo features of the face more realistically, and make it more close to the original image color by using this method. Second, the simulation results prove the convergence of the geometric flow and Bandelet coefficients combining method and the new introduction of new smaller absolute value rules. According to the comparison presented in Figure 5, we converged more than 30 steps in advance by using this method, validating convergence effect of adopting this method. Third, the simulation results prove the advantage of the geometric flow and Bandelet coefficients combing method, and the new introduction of smaller absolute value rules, According to the comparison presented in Table 1, the standard deviation after using the method is slightly smaller, the signal-to-noise ratio is slightly larger, the average gradient becomes smaller, the entropy value is slightly smaller, and 200 seconds of running time is saved, which shows advantage.

We met the following conditions when perform the computer simulation experiments. First, the value of threshold T with uniform, quantization, and even nature is the key. T=50 in the experiment, and the general value is T=40-60. If it exceeds this range, the fused image will not work properly, and the convergence speed will slow down more than 100 steps.
Second, the value of \( N \) is also very important. The value of \( N \) in the experiment is \( N=215-335 \). If the value exceeds this range, the convergence speed will be slower by 50 steps or more and the effect of fused image is not good.

Third, value of \( \lambda \) and other coefficients can make emergence of better image fusion effect and ideal convergence speed after many experiments.

Fourth, the image pixel values are linear before and after the fusion, and the pixel values are in corresponding to position. If we use non-linear values like quadratic function, the fusion effect is not good, and the convergence speed is slower more than 100 steps.

**References**

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