Numerical analysis of the effect of embedment depth on the geometry of the cone failure

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Abstract. This paper presents the results of a numerical FEM analysis of the effect of embedment depth on the extent of the failure zone (cone failure) under the effect of an undercut anchor. For the establishment of the other affecting quantities, the formation of the value of the cone failure angle of the rock medium depending on the embedment depth was analysed. The problem is interesting as regards aspects of rock mass loosening during pull-out of undercut anchors. As a result of the analysis, a significant effect of embedment depth on propagation and the extent of cone failure has been found. The increasing value of embedment depth significantly decreases the extent of the failure zone measured on a free rock surface. The increasing value of cone failure angle limits the potential interaction of failure zones in multi-anchor systems.

1. Introduction

The problem of the study concerns the technology of loosening potentially large rock fragments using undercut anchors. During pulling out the anchors, lumps are obtained, the shape of which is approximated by a cone (Figure 1). In the technology of installation of infrastructure elements in concrete objects (e.g. [1–3]), such a cone is referred to as a “cone failure”, which is characterised by the angle of the forming $\alpha$ (nowadays, in papers, e.g. [4], the cone is replaced by a prism with vertex angle $\alpha$ and the length of the side of the base equal to $3h_{ef}$).

![Figure 1. Model of the cone failure under the action of the undercut anchor, $\alpha$ – the angle of cone failure, $h_{ef}$ – effective anchorage depth, $\beta$ – the angle of undercut head.](image-url)
In simplified calculation procedures (CCD method [4]) the value of the cone angle $\alpha$ is assumed to be equal to $35^0$ and the diameter of the base of the cone failure is equal to approximately $3.5h_{ef}$ (as in Figure 1).

It has been observed [5] that for smaller anchor heads the crack length at peak load is shorter than the crack length obtained for the anchors with larger anchor heads. Moreover, the crack propagation angle, measured from the loading direction, increases with the increase of both embedment depth and head size. For smaller embedment depths and small head sizes, the concrete cone is steeper than in the case of large embedment depths and larger head sizes. The two different basic modes of concrete cracking were observed in the dependence of head size and effective depth [6]. For bonded anchors, the average angle of crack propagation concerning the surface increases with increasing embedment depth from about $25^0$ ($h_{ef}=30mm$) to about $35^0$ ($h_{ef}=120mm$). This increase of the angle can be explained by fracture mechanics. Based on fracture mechanics, tension loaded anchors can be classified as a mixed-mode problem. With increasing crack length, the ratio of the stress intensity factor for Mode 1 and Mode 2 changes. As a result, the mean value of the angle changes as well [7]. According to [8], increasing the anchorage depth $h_{ef}$, results in an increase in the taper angle, as shown in Figure 2.

![Figure 2. Effect of embedment depth $h_{ef}$ on the size of the angle of the cone failure $\alpha$.](image)

In the case of anchor embedding, there is extensive literature on the subject, where the influence of anchor design parameters and embedding technology has been investigated (e.g. [9–13]and several 2D and 3D numerical models of the failure process of the medium structure under the action of anchors of different design have been built (e.g. [14]).

The extent and course of the failure zone depend on very many factors. In addition to the effective embedment depth $h_{ef}$, this zone is affected by the physical and mechanical parameters of the rock (e.g. Young’s modulus, Poisson’s ratio, tensile strength) [15], the value of the angle of the undercut head $\beta$ [16,17] and the value of the coefficient of friction in contact between the head and the rock. For sandstones, it has been found [18–20] that the extent of the failure zone is much larger than is the case for concretes. Numerical analyses have been carried out for the formation of the failure zone under the action of one anchor [21], two anchors [22], and an assembly of three anchors [23]. In the case of an anchor assembly [22,23], the interaction effect of “cone failure” that then appeared, depending on the anchor spacing, was analysed more closely. For the same embedment depth and the same value of anchor pull-out force, this effect can lead to an increase in the volume of the loosened elements (with appropriately chosen anchor spacing).
From the point of view of the proposed unconventional loosening technology, for a given embedment depth, it is important to obtain the maximum range of loosening. This translates into the volume of separated solids, which in turn affects the overall assessment of the effectiveness of the process. Particularly interesting here is the phenomenon of the interaction of cone failure, which appears for the distance of anchor holes smaller than the so-called limiting distance (Figure 3). For hole distances less than the limit \( s < s_{gr} \) and the drill hole configuration, the pull-out force on the anchor located in the next drill hole decreases significantly \( F' < F \), Fig. 3. The volume of the loosened rock also changes.

![Figure 3. Effect of cone failure interaction: F – “free” anchor pull-out force, F’ – the pull-out force of an anchor located at a distance less than the limit distance from an anchor pulled out in the “start” position, s – a distance of anchor holes, \( s_{gr} \) – a limit distance of anchor hole centres, \( h_{ef} \) – effective embedment depth.](image)

The use of numerical modelling in combination with the results obtained from experimental studies enabled a detailed understanding of the actual behaviour of engineering structures and their optimisation and allowed to reduce the number of experiments [24–29].

The subject of particular interest in the presented study, is the influence of the depth of embedment, on the formation of the cone failure angle \( \alpha \) during pull-out of anchors installed in a rock medium (sandstone). The value of this angle is important for the potential interaction of the cone failures.

### 2. FEM simulation using ABAQUS

Using ABAQUS software and the XFEM algorithm, the formation of the rock failure zone was analysed, especially the propagation angle \( \alpha \) of the cone failure at the initial stage of failure development. A constant value of the undercut anchor head angle was assumed \( \beta = 19.5^\circ \). A model with two anchors was considered, as in Figure 4.
Figure 4. Model of the interaction of two anchors on a rock.

Due to the symmetry of the model, the issue was treated as symmetrical by considering the model in one quadrant (Figure 5). Dimensions of the model $H = 500\text{mm}$, $L = H = 1000\text{ mm}$ (Figure 6a). The analysis was conducted for the value of effective embedment $h_{ef} = 50, 100, 150$ and $200\text{mm}$. Fixed anchor centre distance $s$=400mm was assumed.

**Boundary conditions:**
Restrain: on the base and sidewalls, received three translational degrees of freedom $U_1=U_2=U_3=0$. Symmetry concerning YZ plane: $U_1=U_2=U_3=0$ and XY: $U_3=U_1=U_2=0$ (as in Figure 6b).

The load was applied to the contact surface of anchor and rock (conical part of undercut head), at the anchor mesh nodes, vertically – along the $Y$-axis (as in Figure 6c). The load was applied in ascending increments up to the maximum value, with a total maximum value of 1000 N. The friction between anchor and rock was neglected in the analysis.

The method of discretising the finite element mesh model (for $h_{ef}$=50mm) is illustrated in Figure 6. In the upper part of the model, a mesh of hexagonal elements with a structured layout. In the lower part of the model, a mesh of hexagonal elements with sweep arrangement. C3D8R, eight-node, linear elements with reduced integration were used to construct the mesh.

Figure 5. The method of discretising the model with a finite element mesh.
Figure 6. Characteristic dimensions of the model quadrant (a), the method of restraining the boundary nodes (b) and the method of specifying the anchor load (c).
For the construction of the mesh, elements with a linear dimension of 25 mm were used. In the model with embedment depth \( h_{ef} = 50 \) mm, in the upper part of the model a 10 mm mesh was used.

Table 1. Characteristics of the models.

| Model | \( h_{ef} = 200 \) | \( h_{ef} = 150 \) | \( h_{ef} = 100 \) | \( h_{ef} = 50 \) |
|-------|----------------|----------------|----------------|----------------|
| Number of nodes | 32034 | 30350 | 30352 | 32903 |
| Number of elements | 28856 | 27256 | 27260 | 29706 |
| Number of equations | 96102 | 91050 | 91056 | 98709 |

Model material: Sandstone:
Young’s modulus = 14276 MPa
Poisson’s ratio = 0.247
Tensile strength \( f_t = 7.74 \) MPa.
Type of failure: Maps, maximum tensile stress in the principal direction = \( f_t = 7.74 \) MPa.
Damage evolution, Type: Energy. Fracture Energy = 0.17 N/mm. Distribution – linear.

3. Analysis of results
The simulation results obtained for each model are summarised in Figure 7.

Figure 7. Effect of embedment depth on the value of cone failure angle and failure zone interactions for \( h_{ef} \): a) 50mm, b) 100, c) 150mm, d) 200mm.

It is clear from Figure 7 that the embedment depth \( h_{ef} \) has a considerable influence on the course of destruction of the rock medium. For small values of this parameter (Figure 7a) in the initial stage of penetration, the crack develops at an angle \( \alpha = \sim 21^\circ \). As the depth increases, the value of this angle successively increases, reaching about \( 29^\circ \) for \( h_{ef} = 200 \) mm.

4. Conclusions
The analysis showed an incredibly significant effect of embedment depth \( h_{ef} \) on the extent of the rock destruction zone. As this parameter increases, the value of the angle of the cone failure \( \alpha \) increases. This
will have the effect of decreasing the failure zone interactions generated in large anchor systems or during sequential anchor pull-out. Since the value of the cone failure angle also depends on the physical and mechanical parameters of the rock and the head angle $\beta$, it is advisable to conduct further research to describe these processes more fully. Trends in changes of the cone failure angle $\alpha$ as a function of the embedment depth $h_{ef}$ are consistent with the results obtained for concrete (e.g. [6–8,30]), but the values differ.

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