Oscillating Taylor-Couette Flow (Azimuthal motion)

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Abstract. It has been attempted to inject the energy directly into a higher mode in a rotating Taylor-Couette system by oscillation of inner cylinder. The earlier investigation on the axial and radial velocity components revealed a dip in the excitation function which indicates a energy loss. In contrary, the azimuthal velocity component shows an increase in its excitation curve, which implies a compensation of the energy loss of other components and indicates a possible energy injection into the WVF mode which is a higher mode than TVF mode.

1. Introduction
A flow structure regarding a flow transition from laminar to turbulence has been investigated quantitatively in a rotating coaxial cylindrical system - Taylor Couette system. A series of flow instability sets in at the critical Reynolds number where spatially and spatio-temporally higher modes are induced successively, which are superposed to the main flow. Increasing Re, further higher modes are generated and superposed, while the lower mode might be weakened and disappearing. By an energy cascade concept of flow structure, the energy of each modes are fed from the main flow and cascaded subsequently to the higher mode. In a sole fluid mechanical system without heat transportation, energy cascading is considered to be continuous.

Experimentally, we succeeded to obtain an excitation function of each modes separately and confirmed the above hypothesis is valid even for very low order of modes [1]. On the platform of Taylor-Couette system with only an inner cylinder rotation, it is very persuasive to understand that the energy is given to Couette mode from the rotating wall, then flows into a Taylor vortices, wavy mode, modulated wavy mode, fast azimuthal mode and soft turbulence. This describes a sequence of flow transition in this configuration.

Based on this, we have been attempting to investigate a possibility to inject the energy directly into a higher mode [2,3]. The configuration is a circular TCF and the flow regime adopted is a wavy flow mode where only the first three modes of Couette flow, Taylor vortex and wavy vortex prevail. Excitation is made by oscillating an inner cylinder to apply a time dependent boundary condition and spatio-temporal velocity distribution was obtained by ultrasonic Doppler velocity profiler. We studied axial motion[2] and radial motion [3] and reported that the energy of the wavy mode is not affected by an oscillation frequency but it shows a dip at a certain oscillation amplitude. This finding was not well explained; to where the energy is gone. In order to see the third velocity component, we investigated the azimuthal motion of this system with oscillating inner cylinder.
2. Experimental

The apparatus is the same as reported earlier [1-3]. The test section has a radius ratio of 0.907 and water is used as a working fluid (figure 1). Reynolds number is defined using a gap distance $d$, inner radius $R_i$ and angular velocity $\Omega$ as $Re = dR_i\omega/\nu$ then reduced Reynolds number as $R^* = Re/R_c$ where $R_c$ is a critical Reynolds number for Taylor vortex flow; $R_c = 135$. The inner cylinder was oscillated as $R(t) = R_0(1 + A\sin t)$ where $R_0$ is a mean Reynolds number, $\omega$ an oscillation frequency and $A$ an excitation amplitude, which was varied from 0 to 14%.

The measurement was made using a laser PTV technique by visualizing a flow field in an $r-\theta$ plane (figure 2). Axial location of the measurement plane was set for inflow and outflow region of TVF on the averaged velocity filed.

An obtained velocity vector distribution as shown in figure 3 Left is converted to a plot on the $r-\theta$ plane as in figure 3 Right.

A space (azimuth) - temporal distribution of the azimuthal velocity component is constructed as given in figure 4, which is an example of a data at $r/d = 0.5$. Inclined color bands seem to correspond to wavy vortices which is traveling to azimuthal direction. This feature is observed clearly in the interior part of the gap but not near both walls. This feature of flow structure is same as for a change of excitation amplitude ($A$ of $Re$ oscillation) as well as an axial location of measurement on inflow and outflow region.

**Figure 1.** Experimental setup and a test section for visualization.

**Figure 2.** Visualized image of the $r-\theta$ plane.

**Figure 3.** Results of PTV analysis of the image. Left: on the Cartesian coordinate system. Right: translated into cylindrical coordinate system.
Using a data set of velocity fields obtained as a function of \((r, \theta, t)\), power spectrum was computed as \(P(r, \theta, f)\), and then space averaged power spectrum was computed for the azimuthal angle of \((0, \pi/6)\) (figure 5). A strong peak is found with its frequency to be an intrinsic frequency of wavy vortex flow. An energy of the targeted mode was evaluated as an area of this peak and plotted with respect to excitation amplitude (figure 6). It shows an increase until ca. 13\% of the excitation amplitude and then decreases.

**Figure 4.** A spatio-temporal distribution of the azimuthal velocity.

**Figure 5.** A space averaged power spectrum for \((0, \pi/6)\)

**Figure 6.** A variation of the mode energy of the WVF vs excitation amplitude. (Azimuthal component).

**Figure 7.** A variation of the mode energy of the WVF; a sum of axial and radial components.

3. Discussion

In the earlier investigation on axial and radial velocity components, the energy of the WVF mode changes with excitation amplitude showing a dip at the same amplitude. Figure 7 is a variation of energy of sum of \(u_z\) and \(u_r\). The quantity of increase for \(u\) is in the same order as of the decrease at the dip for \(u_z\) and \(u_r\). And also, the width of the increase and dip shows quite a similar tendency. This indicates that the energy of axial and radial motion is converted to that of azimuthal motion. This could be an answer to the question given in the introduction.

A radial variation of such an excitation can be compared as in figure 8 for inflow and outflow regions. An increase of energy can be observed for both regions but it is more significant in the inflow region. Observing a radial distribution of power and its change with excitation amplitude, we found that, with increasing the excitation amplitude, the energy of WVF mode increases around the intrinsic frequency and it spreads out over the whole gap region at the excitation of 13-14\%. On the other hand at the outflow region, the energy is concentrated more
around the intrinsic frequency from zero excitation amplitude and it is diffused to the whole region with increase of excitation amplitude. This would conclude that the energy as a whole increases with increasing the excitation amplitude, but the feature of the change is locally very different; the added energy is given to the WVF at the inflow region and the energy distributed in the field is concentrated to the WVF mode at the outflow region. This suggests that a direct injection of energy into the WVF is possible.

4. Conclusion
From an observation of the azimuthal motion of the WVF, it was found the energy of WVF mode is increased by oscillation of inner cylinder. The way of energy increase is different at the inflow and outflow region. Being combined with the earlier observations on axial and radial motion, it is found that the energy of axial and radial motion is converted to azimuthal motion. This indicates a possible direct energy injection to WVF mode which is the next higher order mode than TVF.

Figure 8. Excitation function as a function of radial position for inflow and outflow region.

References
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