Enhanced alpha-particle optical model potential at low energies, for the mass range \( A \sim 45-209 \)

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The recent high-precision measurements of \( \alpha \)-particle induced reaction data below the Coulomb barrier make possible the understanding of actual limits and possible improvement of the \( \alpha \)-particle optical–model potentials. An updated optical potential is thus provided for \( \alpha \)-particles on nuclei within the mass number range \( 45 \leq A \leq 209 \), below the Coulomb barrier \( (B) \). The main revision concerns actually only the surface imaginary potential depth at the lowest \( \alpha \)-particle energies well below \( B \), and in fact only for the mass range above \( A \sim 130 \). A further regional point is the underestimation of reaction cross sections for the rare-earth nuclei by using the spherical optical potential unless a 7% larger values of the surface imaginary potential radius is taken into account.

I. INTRODUCTION

The recent measurements of \( \alpha \)-particle elastic–scattering and induced reaction data around the Coulomb barrier \([1,10]\) have made possible, due to their high precision, the understanding of actual limits and eventual improvement of \( \alpha \)-particle global optical model potential (OMP) parameters formerly obtained \([11,12]\). A suitable knowledge of this issue is also a condition for a reanalysis of the \( \alpha \)-particle induced reaction data around the (cold) ground–state nuclei \([13]\) and Refs. therein.

The present work on consistent description of \((\alpha,x)\) reactions follows up several earlier steps. First, we looked for the avoidance of the question marks related to (i) the rest of model parameters that are used to describe the compound–nucleus (CN) de-excitation through \( \alpha \)-particle emission (see, e.g., shaded areas in Figs. 4–9 of Ref. \([14]\)), as well as (ii) the differences between the \( \alpha \)-particles in the incoming and outgoing channels. Thus, we carried out formerly an analysis of only elastic–scattering angular distributions of \( \alpha \)-particles on \( A \sim 100 \) nuclei at energies below 35 MeV \([15]\). A semi–microscopic OMP with a double–folding model (DFM) including the explicit treatment of the exchange component was used in this respect. A dispersive correction to the microscopic DFM real potential was also considered together with a phenomenological energy–dependent imaginary part that was finally obtained. Second, a full phenomenological analysis of the same data provided a regional optical potential (ROP), to be used in further nuclear–reaction model calculations. Next, similar semi–microscopic and phenomenological analyses concerned \( A \sim 50-120 \) nuclei and energies from \( \sim 13 \) to 50 MeV, but including furthermore an ultimate statistical–model (SM) assessment of the available \((\alpha,\gamma)\), \((\alpha,n)\) and \((\alpha,p)\) reaction cross sections for target nuclei from \( ^{45}\text{Sc} \) to \( ^{118}\text{Sn} \) and incident energies below 12 MeV \([11,16]\). Third, the extension to heavy \( A=132–209 \) nuclei \([12]\), with further proof within Refs. \([17,18]\), proved the essential role of the energy dependence of surface imaginary potential depth for the understanding of the \( \alpha \)-particle interaction below the Coulomb barrier \((B)\).

Results corresponding to the OMP of Ref. \([12]\) are compared in the present work with the \((\alpha,x)\) reaction data published in the meantime. Variations of the calculated \((\alpha,x)\) reaction cross sections that may occur due to enhanced forms of SM input parameters, that have been obtained in the meantime, are discussed in Sec. II. A consequent OMP update is given on this basis in Sec. III including a particular adjustment for the deformed nuclei with \( 152<A<190 \). Conclusions are given in Sec. IV while preliminary results were presented elsewhere \([19]\).

II. \((\alpha,x)\) REACTION DATA ANALYSIS

The \((\alpha,x)\) reaction cross sections \([1,10]\) measured next to the set up of the \( \alpha \)-particle OMP \([12]\) concern mainly heavier target nuclei and incident energies well below \( B \) (Fig. 1). They are partly supporting this potential and partly pointing out the need for an update. Thus, the new measurements for the \((\alpha,n)\) reaction on the lighter target nuclei \([120]\text{Te}\) \([3]\), \([127]\text{I}\) \([2]\), and \([130,132]\text{Ba}\) \([3]\), as well as \([127]\text{I}(\alpha,\gamma)\text{Cs}\) reaction \([2]\) are rather well described by this potential. Moreover, one may also find that it provided formerly \([11,12,16]\) a better description of the \((\alpha,\gamma)\) reaction cross sections for the target nuclei \([106]\text{Cd}\), \([113]\text{In}\) and \([112]\text{Sn}\) as well as \((\alpha,n/p)\) reaction cross sections for \([106]\text{Cd}\) and \([113]\text{In}\), in comparison with later studies \([8,4]\) including lower–mass target nuclei \([10]\). On the other hand, we have met difficulties in describing even the \((\alpha,n)\) reaction data for the heavier nuclei \([141]\text{Pr}\) \([4]\) and especially the rare–earth nuclei \([165]\text{Ho}\) and \([169]\text{Er}\) \([4]\), \([169]\text{Tm}\) \([4]\), and \([168]\text{Yb}\) \([4]\), as well as the \((\alpha,\gamma)\) reaction cross sections for \([130]\text{Ba}\) \([3]\) and the
rare–earth nuclei. Consequently, further efforts had to be devoted to the OMP parameters for heavier nuclei and the distinct case of the deformed nuclei, as well as to the account of the $\gamma$-ray strength functions. On the other hand, a discussion should concern firstly the possible role of the Coulomb excitation (CE) process in the establishment of the $\alpha$-particle OMP through the ($\alpha$, $x$) reaction data analysis.

A. Coulomb excitation effects on $\alpha$-particle OMP setting up

The possible effects of the CE consideration on the so-called "$\alpha$-potential mystery" have been recently underlined by Rauscher. First, it has been pointed out that even the numerical methods employed to determine the Coulomb wave functions for low energy and high Coulomb barrier play an important role within ($\alpha$, $x$) reaction data analysis. Thus it was shown that a large difference exists between the results obtained for the $^{144}$Sm($\alpha$, $\gamma$)$^{148}$Gd reaction cross sections using either the old routine or a new one. Concerning the calculations carried on using the code SCAT2, which was involved earlier as well as within present work, their correctness in this respect is proved directly by the corresponding results shown formerly and in Fig. 1 of Ref. as being obtained with the new routine and the OMP of Ref. 20.

Second, the CE has been considered as an additional reaction channel which is competing, at the $\alpha$-particle energies well below $B$, with the CN formation while it is not present within $\alpha$-particle emission from an excited CN. Since CE was not considered in the worldwide used optical potential, which was obtained by $\alpha$-particles elastic–scattering analysis and then used in calculation of $\alpha$-particle emission data, Rauscher has adopted a decreased CN formation cross section for the $\alpha$-induced reactions. This reduction of the CN formation cross section given by the OMP has been obtained by taking into account, for each partial wave, the CE cross section that should be additionally considered for that partial wave. Next, a further reduction by a factor of 3 was found necessary in order to describe the measured $^{144}$Sm($\alpha$, $\gamma$)$^{148}$Gd reaction cross sections. Finally, it was shown that this approach is necessary for the description of the above–mentioned ($\alpha$, $\gamma$) reaction and the ($\alpha$, $n$) reaction on the target nuclei $^{141}$Pr and $^{169}$Tin, but not for the ($\alpha$, $\gamma$) reaction on $^{168}$Yb and ($\alpha$, $n$) reaction on $^{130,132}$Ba and $^{168}$Yb, and subsequently for both reactions on $^{113}$In.

While the decrease of the total reaction cross section $\sigma_R$ due to the direct–interaction channels is usually taken into account in SM calculations of reaction cross sections, the case of CE is indeed quite different. A reference paper in this respect was given, however, by Vonach et al. on $\alpha$-particle $\sigma_R$ derived from ($\alpha$, $n$) reaction cross sections through extensive SM calculations. They pointed out that, since the CE cross section becomes the dominant part of the nonelastic cross section below the Coulomb barrier, their results obtained on the basis of the measured ($\alpha$, $n$) reaction cross sections and SM calculations do not represent indeed the full nonelastic cross section but "they do, however, correctly describe the CN formation cross section needed in statistical model calculations". The use of the notation of $\sigma_R$ for these SM results, even in these conditions, may have also a theoretical support. Thus, Hussein et al. shown that the derivation of $\sigma_R$ through the use of optical theorem has the same results either paying no attention to the long–range Coulomb interaction, or including the Rutherford scattering amplitude within a generalized approach. On the other hand, a decomposition of $\sigma_R$ into a direct reaction contribution and a fusion cross section has a straightforward schematic representation in terms of partial waves, and quite distinct from the elastic–scattering and CE cross sections, only for heavy–ion interactions, and under semiclassical conditions (e.g., Fig. 1.8 of Ref.). Therefore, we should note that in the present work the approach of Vonach et al. will be followed, with the understanding that the quantity $\sigma_R$ corresponds to the CN formation.

B. Statistical model parameters

We have also used within actual ($\alpha$, $x$) reaction analysis a consistent set of nucleon and $\gamma$-ray transmission coefficients, and back–shifted Fermi gas (BSFG) nuclear level densities as before. They have been established or validated on the basis of independent experimental information for neutron total cross sections, $\gamma$-ray strength functions, and low–lying levels and resonance data, respectively. Hereafter only the points in addition to the details given formerly are mentioned, while the SM calculations were carried out us-
TABLE I: Low-lying levels number $N_d$ up to excitation energy $E_d$ used in cross-section calculations, and the levels, s-wave neutron-resonance spacings $D_0$, and average radiation widths $\Gamma_g$, in the energy range $\Delta E$ above the separation energy $S$ (with uncertainties given in parentheses, in units of the last digit), for the target–nucleus g.s. spin $I_o$, fitted to obtain the BSFG level-density parameter $a$ and g.s. shift $\Delta$ (for a spin cutoff factor calculated with a variable moment of inertia $I^2$ between half and 75% of the rigid-body value, from g.s. to $S$, and reduced radius $r_0=1.25$ fm), and the EGLO model parameters $k_o$ and $T_j$ corresponding to description of the RSF data [43, 44] and $\Gamma_{\gamma}$ values.

| Nucleus | $N_d$ | $E_d$ (MeV) | $S + \frac{\Delta}{2}$ (MeV) | $I_o$ | $D_{0}^T$ ($\text{keV}$) | $\Gamma_{\gamma}$ (MeV$^{-1}$) | $a$ | $\Delta$ | $k_o$ | $T_j$ |
|---------|-------|-------------|-----------------|-----|----------------|-----------------|-----|------|------|-----|
| $^{114}$In | 33 | 1.768 | 33 | 1.768 | 14.29 | 0.00 |
| $^{116}$Sn | 17 | 2.844 | 26 | 3.106 | 13.45 | 1.33 |
| $^{118}$Sn | 22 | 0.881 | 22 | 0.881 | 14.60 | -0.70 |
| $^{117}$Sb | 17 | 1.536 | 17 | 1.536 | 14.10 | 0.05 |
| $^{120}$Te | 20 | 2.461 | 20 | 2.461 | 14.00 | 0.87 |
| $^{121}$I | 31 | 1.453 | 31 | 1.453 | 14.00 | -0.35 |
| $^{123}$Xe | 31 | 0.876 | 31 | 0.876 | 14.50 | -0.87 |
| $^{124}$Xe | 31 | 2.832 | 29 | 2.373 | 14.20 | 0.64 |
| $^{127}$I | 33 | 1.480 | 33 | 1.480 | 14.00 | -0.35 |
| $^{130}$Xe | 26 | 2.442 | 26 | 2.442 | 14.30 | 0.86 |
| $^{130}$Cs | 11 | 0.318 | 11 | 0.318 | 14.00 | -1.32 |
| $^{131}$Cs | 20 | 1.048 | 23 | 1.212 | 13.60 | -0.54 |
| $^{136}$Ba | 19 | 2.101 | 25 | 2.280 | 14.00 | 0.57 |
| $^{136}$Ba | 32 | 2.505 | 25 | 2.374 | 13.80 | 0.63 |
| $^{136}$La | 28 | 1.319 | 28 | 1.319 | 14.00 | -0.50 |
| $^{136}$La | 17 | 1.038 | 17 | 1.038 | 13.50 | -0.60 |
| $^{137}$Ce | 121 | 1.201 | 21 | 1.201 | 14.00 | -0.45 |
| $^{134}$Ce | 16 | 2.050 | 24 | 2.304 | 13.80 | 0.56 |
| $^{135}$Ce | 13 | 1.367 | 13 | 1.367 | 13.80 | -0.10 |
| $^{136}$Ce | 14 | 2.451 | 14 | 2.451 | 13.30 | 0.88 |
| $^{141}$Yb | 23 | 1.565 | 45 | 2.190 | 13.50 | 0.50 |
| $^{144}$Nd | 52 | 2.779 | 52 | 2.779 | 0 | 0.038(2) |
| $^{144}$Nd | 11 | 0.363 | 11 | 0.363 | 15.00 | 0.84 |
| $^{144}$Pm | 28 | 1.397 | 28 | 1.397 | 15.50 | -0.21 |
| $^{144}$Sm | 28 | 2.883 | 28 | 2.883 | 15.00 | 1.34 |
| $^{144}$Sm | 33 | 2.228 | 32 | 2.214 | 14.00 | 0.64 |
| $^{144}$Sm | 22 | 0.881 | 22 | 0.881 | 14.00 | -0.44 |
| $^{144}$Eu | 28 | 1.421 | 18 | 1.244 | 13.70 | -0.03 |
| $^{146}$Gd | 15 | 1.701 | 15 | 1.701 | 17.30 | 0.49 |
| $^{146}$Gd | 23 | 2.700 | 20 | 2.633 | 17.00 | 1.30 |
| $^{150}$Gd | 25 | 1.540 | 25 | 1.540 | 18.00 | 0.20 |
| $^{150}$Gd | 50 | 0.840 | 54 | 0.888 | 18.00 | 0.076 |
| $^{150}$Gd | 25 | 1.452 | 25 | 1.452 | 18.00 | 0.90 |
| $^{156}$Dy | 39 | 1.676 | 39 | 1.676 | 17.50 | 0.13 |
| $^{156}$Dy | 32 | 0.641 | 26 | 0.568 | 18.00 | -0.86 |
| $^{156}$Dy | 27 | 1.575 | 27 | 1.575 | 17.64 | 0.17 |
| $^{156}$Dy | 34 | 0.740 | 34 | 0.766 | 17.20 | -0.80 |
| $^{156}$Dy | 18 | 1.346 | 18 | 1.346 | 16.92 | 0.02 |
| $^{158}$Ho | 24 | 0.744 | 24 | 0.744 | 18.00 | -0.62 |
| $^{160}$Er | 27 | 1.760 | 27 | 1.760 | 17.20 | 0.30 |
| $^{160}$Er | 36 | 0.813 | 39 | 0.856 | 17.91 | -0.69 |
| $^{168}$Er | 25 | 1.493 | 21 | 1.422 | 17.20 | -0.05 |
| $^{168}$Er | 29 | 0.245 | 27 | 0.366 | 18.35 | -1.12 |
| $^{168}$Er | 21 | 0.646 | 21 | 0.646 | 18.20 | -0.67 |
| $^{168}$Yb | 28 | 1.551 | 28 | 1.551 | 17.50 | 0.11 |
| $^{168}$Yb | 30 | 0.762 | 29 | 0.758 | 19.20 | -0.58 |
| $^{170}$Yb | 26 | 1.521 | 37 | 1.669 | 17.50 | 0.11 |
| $^{170}$Yb | 29 | 0.780 | 40 | 1.004 | 18.10 | -0.52 |
| $^{170}$Yb | 22 | 1.510 | 41 | 1.720 | 18.20 | 0.20 |
| $^{170}$Lu | 26 | 0.671 | 26 | 0.671 | 18.10 | -0.73 |
| $^{172}$Lu | 28 | 0.406 | 28 | 0.406 | 18.50 | -1.05 |
| $^{172}$Lu | 26 | 0.735 | 26 | 0.735 | 18.35 | -0.64 |
| $^{172}$Hf | 28 | 0.716 | 28 | 0.716 | 17.50 | -0.77 |
| $^{172}$Hf | 26 | 1.534 | 26 | 1.534 | 18.30 | 0.18 |
energies, leading to the RSF models progress.

The former Lorentzian (SLO) model for the electric dipole γ-ray strength functions, of main importance for calculation of the γ-ray transmission coefficients, has used the giant dipole resonance (GDR) line shape with the usual parameters ($\omega_0, \Gamma_0,$ and $E_0$) derived from photoabsorption data [33] and Refs. therein. Later, an energy dependence of the GDR width ($\Gamma(E_0)$), was assumed also within the energy-dependent Breit-Wigner (EDBW) model [36, 37] that was formerly involved [17, 18].

The generalized Lorentzian (GLO) model of Kopecky and Ull [38] has included in addition a further dependence on the nuclear temperature $T_f$ of the final states, to avoid the extrapolation of the SLO function in the limit of zero-γ-ray energy but a rather constant nonzero limit. Moreover, the enhanced generalized Lorentzian (EGLO) model [33, 39] assumes also an enhancement of the GLO width ($\Gamma(E_\gamma, T_f)$), going from $k_0$ at a γ-ray energy $\epsilon_0$, to $E_0$:

$$\Gamma(E_\gamma, T_f) = \left[ k_0 + \frac{E_\gamma - \epsilon_0}{E_0 - \epsilon_0} (1 - k_0) \right] \frac{\Gamma_0}{E_0^2} (E_\gamma^2 + 4\pi^2 T_f^2)$$

(1)

with the values of the two parameters $k_0$ and $\epsilon_0=4.5\text{ MeV}$ adjusted to reproduce the averaged resonance capture data. However, we found differences between the $k_0$ values given by the latest RIPL-3 form of Eqs. (143) [33] and (6.9) [40], and the related content in Fig. 6.1 of RIPL-1 [40] (e.g., 2.49 and 2.00, respectively, for a nucleus with $A=158$). At the same time we took into account the recent analysis [41] of the effects due to the assumption of the temperature $T_f$ variation from zero to the value corresponding to the BSFG model. Consequently, following also [42] and Refs. therein, we have looked for both $k_0$ and $T_f$ constant values that correspond to description of the RSF data [33, 43] and s-wave neutron–resonance average radiation widths $\Gamma_0$ [33] for the heavier nuclei of interest for the present work (Table I).

The effects of the $k_0$ and $T_f$ values on the RSF calculation using the EGOLO model, along with the corresponding results provided by the SLO and GLO models, are shown in Figs. 2, 3. The GDR as well as pigny dipole resonance parameters established within the original references [33, 44] have been used in this respect. Concerning the M1 radiation, the above–mentioned SLO model was used along with either the global parameterization [33] for the GDR energy and width, i.e. $E_0=41\cdot A^{1/3}\text{ MeV}$

FIG. 2: (Color online) Comparison of measured and calculated sum of γ-ray strength functions of the $E1$ and $M1$ radiations for the $^{148,149}$Sm nuclei, and $E1$ radiations for the $^{156–158}$Gd nuclei, using the $E1$ SLO (dotted curves), GLO (dash-dot-dotted curves), and EGLO (dash-dotted curves) models, including the effects of using the free parameters $k_0$ (dashed curves) and $T_f$ (solid curves) given in Table I. SLO strength functions are used for $M1$ and $E2$ radiations. The measured [33] and calculated s-wave neutron–resonance average radiation widths $\Gamma_0$ are given in meV.

FIG. 3: (Color online) As Fig. 2 but for the sum of γ-ray strength functions of the $E1$ and $M1$ radiations for the $^{160–164}$Dy, $^{166,167}$Er, and $^{170–172}$Yb nuclei.
reaction cross sections to the adopted $f_{E1}(E_{\gamma})$ model will be given below for the particularly questionable case of the $^{168}$Yb target nucleus.

### III. UPDATED OMP

#### A. Updated OMP

The main attribute of the recently measured cross sections of $(\alpha,x)$ reactions on heavier nuclei is the focusing at energies below $B$ while the data previously available mostly overdrawn it (Fig. 1). A first group of data consists of the $(\alpha,x)$ reaction cross sections for $^{120}$Te, $^{127}$I, and $^{130,132}$Ba target nuclei. SM calculations carried out using the global potential are compared to them in Fig. 4. The rather good agreement found for these reactions is, however, firstly due to either the target $A<130$ or the energy range above the energy limit $E_1$. A particular case is, however, the $(\alpha,n)$ reaction on $^{120}$Te within an energy range which is fully below this energy limit $E_1$. Thus, it makes possible a suitable assessment of the value of surface imaginary potential depth $W_D=3.5$ MeV for $A<130$ (see the note $b$ in Table I of Ref. 12). On the other hand, a slightly increased value $W_D=4$ MeV along with the rest of the same OMP parameters (Fig. 1) and Table II leads to a even better description, beyond the former one within the error bars of these quite accurate data.

Moreover, the new quite accurate data for the $^{141}$Pr$(\alpha,n)^{144}$Pm reaction have been more helpful in setting up the correct value of the $W_D$ parameter at the lowest energies. Thus, a significant underestimation of the data just below the energy limit $E_1$ by the OMP are entirely removed by using the value of $W_D=4$ MeV (Fig. 5). On the other hand, this value leads to an overestimation of at least the lowest-energy data points of the well-known $^{144}$Sm$(\alpha,\gamma)^{148}$Gd reaction data shown in the same figure.

Actually, the lower value $W_D\sim1.5$ MeV for $A>130$ was established through the analysis of these data characterized by incident–energy error bars as small as they are usual nowadays but unique at the end of the ’90s. Fortunately, the underestimation of these lowest–energy data remains the only question still open, while all other data analyzed formerly for heavier nuclei (Figs. 4–5 of Ref. 12) are also better described by the increased $W_D$ parameter below $E_1$. That is, however, due to the large incident–energy uncertainties of the data just then available at energies higher than $E_1$ (see, e.g., the case of $^{141}$Pr shown in Fig. 5).

#### B. Updated OMP for rare–earth nuclei

A different case is that of the recent measured data for the rare–earth nuclei $^{165}$Ho, $^{169}$Er, $^{169}$Tm, and $^{168}$Yb...
an obvious underestimation being obtained for both $(\alpha, n)$ and $(\alpha, \gamma)$ reactions using both the OMP of Ref. 12 and its updated parameter value $W_D=4$ MeV (Fig. 6).

The use of a spherical OMP 29 in the neutron-emission channel instead of a deformed optical potential, so well motivated in the rare-earth region, was the first issue deserving a careful analysis. Therefore we have replaced the former neutron transmission coefficients with the ones obtained by using the average rare-earth-actinide deformed phenomenological optical potential of Young 49 (Set A) within the Coupled-Channels (CC) model, and deformation parameters given recently 50 for Hf isotopes. The computer code EMPIRE-II 51 was used in this respect. First, we found that the measured neutron total cross sections 31, 52 are obviously described much better by the deformed OMP at energies of both tens of keV and between 1–3 MeV (Fig. 7).

Second, the calculated $^{168}$Yb$(\alpha, x)^{171,172}$Hf reaction cross sections remained however unchanged after this replacement, mainly due to the similar neutron total cross sections given by the two OMPs around the evaporation energy of $\sim 1$ MeV. This conclusion has been quite useful also for the analysis of all reactions on deformed rare-earth nuclei within this work, carried out on the basis of the spherical OMP 29.

Alternately one should take into account the fact that nuclear deformation also motivates a low-energy enhancement of the charged-particle reaction cross sections as it was proved by Lanier et al. 53 for protons on $^{151,153}$Eu and recalled recently by Grimes 54. Thus, Lanier et al. pointed out that the enhancement of $(p, n)$ reaction cross sections for $^{153}$Eu relative to $^{151}$Eu, with large difference between the corresponding ground state deformations of these nuclei, can be accounted for if spherical OMP calculations are performed with an $\sim 3\%$ larger radius for $^{153}$Eu. Since the largest sensitivity of the calculated $(\alpha, x)$ reaction data is due to the surface
imaginary potential [12], we have considered an increased radius for this potential component. Thus we found that 7% larger values of the \( r_\text{D} \) parameter may reproduce indeed the experimental data for the rare-earth target nuclei (Fig. 8) except the \((\alpha, \gamma)\) reaction on \(^{168}\text{Yb}\) [7].

Concerning the last above-mentioned \((\alpha, \gamma)\) reaction, the \((\alpha, n)\) reaction cross sections corresponding to the same target nucleus \(^{168}\text{Yb}\), which represents nearly the CN reaction cross section, is however well reproduced. Therefore, it seems that particular grounds may exist for this disagreement related to the \(\gamma\)-ray emission channel, while the 7% larger radius for the surface imaginary potential can be really considered for the present OMP (Table II) within the rare-earth deformed nuclei range (152<\(A<190\)).

### C. The \(^{168}\text{Yb}(\alpha, \gamma)\)\(^{172}\text{Hf}\) reaction

Additional calculations were carried out for the reaction \(^{168}\text{Yb}(\alpha, \gamma)\)\(^{172}\text{Hf}\) in order to understand the possible motivation of the measured data underestimation. Since an eventual agreement was reported [7] by using the \(\alpha\)-particle OMP of McFadden and Satchler [24] and SLO \(f_{E1}(E_\gamma)\) \(\gamma\)-ray strength functions, we have looked for the changes of our results that may follow the use of these particular options.

While the results of SM calculations carried out in this work using both the optical potential in Table II and the McFadden and Satchler OMP are shown in all cases (Figs. 4, 5, 6), calculations for this reaction have also considered the SLO model (Fig. 8). So thus one may see that the latter OMP has already led to larger reaction cross sections, especially at lower incident energies. Then, the replacement of the EGLO \(\gamma\)-ray strength functions by the SLO ones yields an additional increase that is however larger at higher energies. It results thus an agreement with the lower-energy measured points. Actually, the agreement between the measured and calculated data is more important at these energies where there are no effects of the level density within the corresponding \((\alpha, n)\) reaction. Such effects could easily increase or decrease an eventual agreement at higher energies. However, these effects have been avoided in the present work since the BSFG parameters used in SM calculations have also been obtained through the fit of independent experimental data, i.e., most recent low-lying discrete levels and neutron resonance data [32]. Therefore, we may conclude that a partial agreement of the calculated and measured reaction cross sections could be provided by use of less accurately established \(\alpha\)-particle OMPs and

### TABLE II: \(\alpha\)-particle OMP parameters (within the formalism of, e.g., Ref. [24]) for target nuclei with 45\(\leq A\leq 209\) at energies \(E<50\) MeV, in addition to the Coulomb potential of a uniformly charged sphere of reduced radius \(r_\text{C}=1.3\) fm. The energies and corresponding range limits are in MeV. A star used as superscript follows the parameters which were changed with respect to Ref. [12]. Particular values for nuclei involved in Refs. [11, 12] and present work, with tabular forms for the use with the TALYS code [46] and to supersede the RIPL-3 subset [12, 47], are given in [48].

| Potential depth (MeV) | Geometry parameters (fm) |
|----------------------|--------------------------|
| \(\sqrt{V}=165+0.736Z/A^{1/3}-2.64E, E\leq E_0\) | \(r_\text{R}=1.18+0.012E, E\leq E_0\) |
| \(=116.5+0.337Z/A^{1/3}-0.453E, E>E_0\) | \(=1.48, E>25\) |
| \(a_\text{R}=0.631+0.016Z/A^{1/3}-(0.001Z/A^{1/3})E_2, E\leq E_2\) | \(E\leq E_2\) |
| \(=0.631+0.016Z/A^{1/3}-(0.001Z/A^{1/3})E_3, E_2<E\leq E_4\) | \(E<25\) |
| \(=0.684-0.016Z/A^{1/3}-(0.0026-0.00026Z/A^{1/3})E, E>E_4\) | \(E>E_4\) |
| \(W_\text{V}=2.73-2.88A^{1/3}+1.11E\) | \(r_\text{V}=1.34, a_\text{V}=0.50\) |
| \(W_\text{D}=4, E\leq E_1\) | \(r_\text{D}=1.52, 152\leq A\geq 190\) |
| \(=22.2+4.57A^{1/3}-7.446E_2+6E, E_1\leq E<E_2\) | \(=1.626, 152\leq A<190\) |
| \(=22.2+4.57A^{1/3}-1.446E, E>E_2\) | \(a_\text{D}=0.729-0.074A^{1/3}\) |

\(E_1=-3.03-0.762A^{1/3}+1.24E_2, E_2=(2.59+10.4/A)Z/(2.66+1.36A^{1/3}), E_3=22.2+0.181Z/A^{1/3}, E_4=29.1-0.22Z/A^{1/3}\)
These reduction methods have been used for comparison of either the fusion or total reaction cross sections for different systems (e.g. [59] and Refs. therein), whereas the cross-section ratios are equal to those of the reduced cross sections in the present case. The quite unusual feature of the cross-section ratio in the case of the $^{168}$Yb nucleus is not only a rather constant value over $\sim 1$ MeV of incident energy but also close to unity. Neither the difference between the $Q$–values of the $(\alpha, \gamma)$ and $(\alpha, n)$ reaction channels, nor the level density of the corresponding residual nuclei, namely even–even and even–odd for the $\gamma$– and neutron–emission, respectively, can explain the particular case of the cross–section ratio for the $^{168}$Yb target nucleus. It results also from Fig. 9 that there is no particular range of the reduced energy parameter or mass range that may support this case. In fact, the calculated results of this work and Ref. [16] were not shown in Fig. 9 in spite of the suitable description of all excitation functions except the $^{168}$Yb$(\alpha, \gamma)$ reaction, in order to highlight first of all the measured data trend. Therefore the above–mentioned lack of agreement may appear due to nuclear properties that have not been yet considered.

### IV. CONCLUSIONS

The recent high–precision measurements of $\alpha$-particle induced–reaction data below the Coulomb barrier are involved in order to understand actual limits and eventually improve an $\alpha$-particle optical–model potential for nuclei with $45\leq A\leq 197$, below $B$ [12]. Statistical–model calculations of reaction cross sections have been used in this respect, while an increased attention has been paid towards enhanced forms of SM input parameters that have been obtained in the meantime. Their effects on the calculated $(\alpha, x)$ reaction cross sections are thus discussed and taken into account within analysis of differences between the measured and calculated data.

The main revision of the above–mentioned OMP [12] concerned actually only one parameter, namely the surface imaginary potential depth at the lowest $\alpha$-particle energies well below $B$, and in fact only for the mass range above $A\sim 130$. Actually the updated value corresponds to the ROP established by analysis of the well enlarged data basis available for $A\sim 50–120$ nuclei [11]. A further regional point has concerned the recent data measured for the rare–earth nuclei. The obvious underestimation of both $(\alpha, n)$ and $(\alpha, \gamma)$ reaction cross sections by using the optical potential parameters which have been found suitable for the rest of nuclei is removed if the spherical OMP calculations are performed with 7% larger values of the surface imaginary potential radius.

The only one recent data set which still has not been described concerns the $(\alpha, \gamma)$ reaction on $^{168}$Yb [7]. An additional discussion of this case has proved that a partial agreement of the calculated and measured reaction cross sections could be provided by use of less accurate either $\alpha$-particle OMPs or $\gamma$-ray strength functions.
However, the former change will be followed also by an overestimation of the related $(\alpha,n)$ reaction cross sections. The use of a consistent parameter set, established through the fit of independent experimental data, is finally leading to the marked underestimation of the measured $^{168}$Yb$(\alpha,\gamma)^{172}$Hf reaction cross sections. Therefore the lack of agreement in this case may appear due to nuclear properties that have not been yet considered. Further measurements of both $(\alpha,n)$ and $(\alpha,\gamma)$ reaction cross sections for further target nuclei, increasing the scarce actual systematics, would be most helpful in this respect. On the other hand, the updated $\alpha$-particle global OMP which provides a suitable description of the most $\alpha$-particle induced reaction data will be furthermore involved in the analysis of the significant underestimation of the $\alpha$-particle emission [13, 60].

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