Research on energy conversion mechanism of rotodynamic pump and design of non-overload centrifugal pump

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Abstract. In this paper, an attempt has been made for the calculation of an expression for the intrinsic law of input power which has not yet been given by current theory of Rotodynamic pump. By adequate recognition of the characteristics of non-inertial system within the rotating impeller, it is concluded that the input power consists of two power components, the first power component, whose magnitude increases with the increase of the flow rate, corresponds to radial velocity component, and the second power component, whose magnitude decreases with the increase of the flow rate, corresponds to tangential velocity component, therefore, the law of rise, basic levelness and drop of input power curves of centrifugal pump, mixed-flow pump and axial-flow pump can be explained reasonably. Through further analysis, the main ways for realizing non-overload of centrifugal pump are obtained, and its equivalent design factor is found out, the factor correlates with the outlet angle of leading face and back face of the blade, wrap angle, number of blades, outlet width, area ratio, and the ratio of operating flow rate to specified flow rate and so on. These are verified with actual example.

1. Introduction
Rotodynamic pump (including centrifugal pump, mixed-flow pump and axial-flow pump) is a machinery for realizing energy conversion, which converts the mechanical energy of the prime mover into the liquid energy of the conveyed medium (sum of pressure energy, velocity energy and elevation energy), this involves the energy conversion mechanism, i.e. the relationship of conversion between input power and output power of the pump. Correct understanding of the intrinsic character of input power and obtaining the calculation expressions for the intrinsic law of input power is of important significance for knowing energy conversion mechanism and pump design, etc., for example, effectively controlling the change law of the input power curve so as to make the power curve of centrifugal pump to have maximum value, and thus achieve non-overload design of centrifugal pump; reducing input power as far as possible so as to find out key design factors for improving pump efficiency; realizing prediction of power curve, numerical simulation, etc.

The pump theory and design are still not perfect, many mechanism problems need to be solved, thus it is of important significance to research on input power of pump and relevant problems.

2. Discussion on input power law of pump theories at present
The actual shape of input power (shaft power) curve of rotodynamic pump is shown in Fig. 1(a)[1-7], it
is characterized that the input power curve of centrifugal pump, mixed-flow pump and axial-flow pump presents ascending, basically level and descending tendency, respectively.

However, the calculation formula \( N = \rho g Q_{\text{in}} H_{\text{sh}} \) for input power is given in literature [1,3-5] (neglecting mechanical losses of bearing, etc), their characteristics are: all the input power curves of rotodynamic pump present ascending law, as shown in Fig. 1(b). This viewpoint cannot reasonably explain the actual phenomenon that the shaft power of mixed-flow pump at zero flow rate is approximately equal to rated power and the shaft power of axial-flow pump at zero flow rate is significantly higher than the rated power, etc. Literature [7] points out that the difference between theories and actual value of input power in the small-capacity zone is very obvious, its cause is still not clear.

3. New knowledge on input power and energy conversion mechanism

Input power is the mechanical energy of the prime mover corresponding to the impelling force of the blades on the medium (neglecting mechanical losses as of bearing, etc.). If the impelling force (unit mass impelling force) of blades on the medium is obtained, the torque of blades on the medium can be obtained, further the input power can be obtained. Therefore, derivation of the impelling force of the blades on the medium is the key to the study of input power and relevant problems.

On the other hand, the head is the energy which is acquired by the medium of unit mass. Therefore, there certainly exists a corresponding relationship between the head and input power, that is, there certainly exists a corresponding relationship between the head and the impelling force of the blades on the medium.

Then, according to the principle that the force can be decomposed and recomposed in the theoretical mechanics, in order to obtain the impelling force of the blades, first the total head needs to be decomposed, and the impelling force of the blades to which each head component corresponds on the medium is to be obtained. Thus the input power component to which each head component corresponds is obtained. Further it is not difficult to obtain the corresponding relationship between input power component and output power component, that is, the relationship between energy conversion mechanisms of input power and output power.

What is worth pointing out is that only the total head is decomposed by the intrinsic characteristics correctly, can the nature of each head component be recognized, and can the correct solution for the impelling force of the blades to which each head component corresponds be found out, thus the action of blades to which each head component corresponds can be obtained correctly, the calculation expression for intrinsic law of input power and the relationship of conversion between input power and output power can be obtained.

3.1 Decomposition and division of the head
In literature [1, 3, 5-7], one of the equations currently used to calculate theoretical head $H_{th}$ is as follows:

$$H_{th} = \frac{u_2^2 - u_1^2}{2g} + \frac{(w_1^2 - w_2^2)}{2g} + \frac{(c_2^2 - c_1^2)}{2g}.$$  \hspace{1cm} (1)

In literature [1, 3, 5-7], $(w_1^2 - w_2^2)/2g$ is classified into potential head, this is evidently inappropriate. If it (strictly, it should be $(w_1^2 - w_2^2)/2g$) is classified into kinetic head, then head classification is very appropriate. The reasons are:

1. $w$ is a velocity of movement in a relative coordinates;
2. within the impeller there indeed exists friction losses $h$ due to $w$, and $h \propto w^2$. This is one of the important characteristics of kinetic head;
3. the characteristic of $w_u$ is the same as $c_u$, but it is different from $u$ (refer to the description below);  
4. because $(w_1^2 - w_2^2)/2g$ is converted into pressure, pump theory at present classifies it into the potential head. However, $(c_2^2 - c_1^2)/2g$ is finally converted into pressure through pump output, but it undoubtedly belongs to the kinetic head.

Because of $c = c_u + c_m, w = w_u + w_m, c_m = w_m$, as shown in Fig. 2, by slight transformation, Equation (1) becomes:

$$H_{th} = \frac{u_2^2 - u_1^2}{2g} + \frac{(c_{u2}^2 - c_{u1}^2)}{2g} + \frac{(w_{u1}^2 - w_{u2}^2)}{2g}$$ \hspace{1cm} (2)

Equation (2) more clearly reflects the essential law of head because it does not contain meridional velocity $c_m$ and $w_m$. Therefore, we can classify head as follows:

The right-hand side term of Equation (2) $(u_2^2 - u_1^2)/2g$ is potential head $H_p$, i.e.

$$H_p = \frac{u_2^2 - u_1^2}{2g}$$ \hspace{1cm} (3)

The right-hand side term of Equation (2) $(c_{u2}^2 - c_{u1}^2)/2g + (w_{u1}^2 - w_{u2}^2)/2g$ is kinetic head $H_k$, i.e.

$$H_k = \frac{(c_{u2}^2 - c_{u1}^2)}{2g} + \frac{(w_{u1}^2 - w_{u2}^2)}{2g}$$ \hspace{1cm} (4)

**Figure 2.** Motion analysis of medium within the impeller, i.e. space velocity triangles

It is not difficult to know that potential head $H_p$ is constant and kinetic head $H_k$ decreases with the increase of the flow rate. The inlet diameter and outlet diameter of axial-flow pump is the same ($D_1 = D_2$), the medium flows along the direction of axis, there is no radial velocity component ($v_r = 0$), therefore, the potential head $H_p$ of axial-flow pump is zero. The law of potential head curve and kinetic head curve is shown in Fig. 3.
3.2 Action of unit mass of the blades on the medium

3.2.1 Action of blades to which potential head corresponds

If making further analysis on Coriolis force, we will get the calculation formula for the impelling force to which potential head corresponds $f_p$.

Because of $a_{cor} = 2\omega \times w$ and $w = w_u + w_m = w_u + w_r + w_z$, it is not difficult to know that only $w_u$ and $w_r$ generates the Coriolis acceleration, that is, $a_{cor} = 2\omega \times w = 2\omega \times w_u + 2\omega \times w_r = 2\omega \times w_u + 2\omega \times c_r$ (due to $w_c = c_c$). Therefore, only $w_u$ and $c_r$ generates the Coriolis force, their direction is radial and tangential, and their magnitude is $2\omega w_u$ and $2\omega c_r$, respectively. The radial Coriolis force $2\omega w_u$ has no direct relation with torque, whereas the tangential Coriolis force $2\omega c_r$ has direct relation with torque.

The constraining force (also called constraining counterforce) of the tangential Coriolis force $2\omega c_r$ is the potential impelling force $f_p$, i.e.

$$f_p = 2\omega c_r$$

The potential impelling force $f_p$ corresponds to potential head $(u_2^2 - u_1^2)/2g$. The reason is that from blade entrance to exit edges $f_p$ produces increment of moment of momentum, and from moment of momentum theorem (integral formulation), the increment is

$$\int_{u_1}^{u_2} mf_u r dt = m\omega \int_{r_1}^{r_2} 2c_r \cdot r dt = m\omega \int_{r_1}^{r_2} 2r dr = m(u_2 r_2 - u_1 r_1)$$

It's easy to know that the increment of the moment of momentum is the change of moment of momentum corresponding to $u_2$ (at radius $r_2$) and $u_1$ (at radius $r_1$), and $u_2$ (at radius $r_2$) and $u_1$ (at radius $r_1$) correspond to potential head $(u_2^2 - u_1^2)/2g$.

Note: ①the study object here is free body, as shown in Fig. 4(a); ②$\omega \cdot dt = dr$; ③ $u_2 = \omega r_2$, $u_1 = \omega r_1$; ④where $m$ is the mass within the free body.

When the medium has no displacement in radial direction, i.e. $c_r = 0$, then $f_p = 0$, and its corresponding energy input is also zero, only when the medium has radial displacement, $c_r \neq 0$, then $f_p \neq 0$, and it have corresponding energy input. Again, in axial flow pump, $(u_2^2 - u_1^2)/2g = 0$, it happens that $c_r = 0$, i.e. $f_p = 0$.

3.2.2 Impelling force to which kinetic head corresponds

From Equation (4), the kinetic impelling force
\( f_k \) corresponds to kinetic head \( H_k \) includes \( f_{cu} \) (corresponds to \( (c_{u2}^2 - c_{u1}^2)/2g \)) and \( f_{wu} \) (corresponds to \( (w_{u1}^2 - w_{u2}^2)/2g \)), i.e.

\[
f_k = f_{cu} + f_{wu}
\]

(6)

According to the characteristics of kinetic impelling force, when applying the moment of momentum theorem to obtain \( f_{cu} \), corresponding process is needed. ①Take the control body of the infinite thin annulus as the study object, as shown in Fig. 4(b); ②conventionally, take \( c_{u1} = 0 \). ③In the classical mechanics, the change in momentum is \( m(v_2 - v_1) \) or \( m\Delta v \), that is, \( m \) is constant, while velocity \( v \) changes. But in the derivation herein, the change of momentum can be considered as \( e_u\Delta m_t \), that is, \( e_u \) is constant (\( e_u \) is regarded as constant when the annulus is developed, similar to the processing on axial flow impeller developed), while \( \Delta m_t \) refers to accumulative mass of medium when it has \( e_u \) within the time of \( dt \). From the moment of momentum theorem, then

\[
mf_{cu} \cdot r dt = c_u \Delta m_t \cdot r \cdot c_u
\]

where \( r \) is infinite thin annulus (the control body) radius, \( e_u \) is tangential component of absolute velocity \( c \) (at radius \( r \)), \( m \) is actual mass within the control body, and \( \Delta m_t \) is accumulative mass, means the product of number of revolutions of the control body within the time of \( dt \) and the actual mass \( m \), i.e. \( \Delta m_t = m \cdot \frac{\omega}{2\pi} \cdot dt \). Therefore, it is obtained:

\[
f_{cu} = \frac{1}{2\pi} \omega c_u
\]

(7)

Similarly, from the moment of momentum theorem, the calculation formula for \( f_{wu} \) can be obtained as follows:

\[
f_{wu} \propto -\frac{1}{2\pi} \omega w_u
\]

(8)

If comparing the calculation formulas of \( f_{cu} \) and \( f_p \), an interesting phenomenon can be found: When the centrifugal pump and the mixed flow pump run at the rated flow rate, the value of \( e_u \) in the impeller discharge is approximately 10 times of that of \( e_r \) (\( e_r \) is different from \( e_u \)), whereas the ratio of coefficients in the formula of \( f_{cu} \) and \( f_p \) is \( 1/(4\pi) \), that is, In the impeller discharge, the value of \( f_{cu} \) is close to that of \( f_p \). This indicates the value of \( f_{cu} \) is not divorced from reality.
3.2.3 Discussion and analysis

The two typical kinds of motion in fluid mechanics are shown as in Fig. 5(a) and 5(b).

Due to the impelling force on the medium, the flow within the impeller is a complex non-inertial motion.

The motion to which potential head corresponds within the impeller is analogous to rotary motion as shown in Fig. 5(a).

When the impeller is developed (similar to the processing on axial flow impeller developed), the motion to which kinetic head corresponds within impeller is analogous to accelerating linear motion as shown in Fig. 5(b), the pressure difference between leading face and back face of blades is similar to the pressure difference between two wall faces of the vessel as shown in Fig. 5(b). Note: ① according to the cascade theory, the developed shape of blades of axial-flow pump is cascade, in fact, the motion of cascade after development is analogous to accelerating linear motion; ② Annular cascade of centrifugal pump is changed to cascade by conformal mapping in literature [8].

The non-inertial motion within the impeller contains important scientific problems in the aspects of fluid mechanics, rotodynamic pump, etc.
3.3 Input power and energy conversion mechanism

3.3.1 Input power and output power to which potential head corresponds

Because the impelling force of blades \( f_p \) to which the potential head corresponds is \( 2\omega c_r \), and \( v_r \) is proportional to flow rate \( Q \), therefore, it is not difficult to conclude that the input power to which potential head corresponds is proportional to flow rate. Through further derivation, the calculation formula for input power to which potential head corresponds is obtained, as follows:

\[
N_p = K_p Q \tag{9}
\]

Where \( K_p > 0 \), and relates to factors such as geometric parameters of impeller, pump speed, etc.

On the other hand, the output power to which potential head corresponds is apparently proportional to flow rate. The law of input power and output power to which potential head corresponds is shown in Fig. 6(a).

3.3.2 Input power and output power to which kinetic head corresponds

According to Equation (7) and Equation (8), the impelling force of blades \( f_k \) to which kinetic head corresponds is proportional to \( C_u \) and is inversely proportional to \( W_u \). On the other hand, it is easy to know: ① \( C_u \) is inversely proportional to flow rate \( Q \), and \( W_u \) is proportional to flow rate \( Q \); ② when \( Q = 0 \), \( C_u \) rises to maximum value \( (= u) \), and \( W_u \) drops to minimum value \( (= 0) \). Therefore, input power to which kinetic head corresponds is inversely proportional to flow rate \( Q \). Through further derivation, the calculation formula for input power to which kinetic head corresponds is obtained, as follows:

\[
N_k = \text{Const} - K_k Q \tag{10}
\]

Where \( K_k > 0 \), and relates to factors such as geometric parameter of impeller, pump speed, ratio
of actual flow rate to rated flow rate, area ratio, etc.

The output power to which kinetic head corresponds is apparently proportional to flow rate $Q$. The law of input power and output power to which kinetic head corresponds is shown in Fig. 6(b).

![Diagram](image)

(a) Law of input power and output power component corresponding to potential head $H_p$

(b) Law of input power and output power component corresponding to kinetic head $H_k$

**Figure 6.** Component of input power and component of output power

4. Analysis of shaft power curve of rotodynamic pump

Shaft power of rotodynamic pump is composed of two kinds of power components to which potential head and kinetic head correspond, and the power component to which potential head corresponds increases with the increase of flow rate, the increasing amplitude decreases with the increase of specific speed $n_s$ (because the $\frac{D_2}{D_1}$ decreases with the increase specific speed $n_s$).

The power component of this term in axial-flow pump decreases to zero. The power component to which kinetic head corresponds decreases with the increase of flow rate. Thus it can reasonably be explained that the law which the input power curves of centrifugal pump, mixed-flow pump and axial-flow pump present the law of ascending, approaching to level and descending, respectively. As shown in Fig. 7(a), (b) and (c).

![Diagram](image)

(a) Centrifugal pump

(b) Mixed-flow pump

Flow rate $Q$ (m$^3$/h)
5. Non-overload design of centrifugal pump

In normal case, the shaft power curve of centrifugal pump rises continuously with the increase of flow rate, therefore, it is easy to generate overload in large-capacity zone, and even burn out the prime mover. In some cases, in order to guarantee safe and reliable running of centrifugal pump, the centrifugal pump must be provided with non-overload feature, i.e., the input power curve of centrifugal pump has maximum value, thus no overload problem will be generated over the entire flow rate range.

5.1 Equivalent design factor of non-overload centrifugal pump

From Equation (9) and (10), the shaft power $N$ is $N_p + N_k = K_p Q + (Const - K_k Q)$, i.e.

$$N = Const + (K_p - K_k)Q$$  \hspace{1cm} (11)

In general case, $K_p > K_k$ for centrifugal pump, hence, it cannot realize non-overload feature. However, $K_p$ and $K_k$ are the design factors which depend on multiple aspects of hydraulic model, by some pertinent designs, the change law of shaft power curve can be controlled over a certain flow rate, the rise amplitude of power component of shaft power to which potential head corresponds ($K_p$) is less than the drop amplitude of power component of shaft power to which kinetic head corresponds ($K_k$), so as to achieve maximum value of shaft power at a certain flow rate. Thereby, the equivalent design factor for non-overload centrifugal pump obtained is

$$K_p < K_k$$  \hspace{1cm} (12)

5.2 Relevant design factors of non-overload centrifugal pump

The hydraulic model of centrifugal pump involves multiple design factors, likewise, the equivalent design factor of non-overload centrifugal pump $K_p < K_k$ also relates to multiple design factors, e.g., the outlet angles of leading face and back face of blades, wrap angle, number of blades, outlet width, the area ratio, etc. Moreover, for a certain centrifugal pump, $K_k$ has a slight increase as the flow rate increases, therefore, another feature of non-overload centrifugal pump is that maximum value of input...
power can only appear at a relatively large flow rate point.

5.3 Design example

In a recent subject research, the authors have designed high-efficiency non-overload low specific speed centrifugal pump. By optimizing multiple design factors of hydraulic model (the outlet angles of leading face is 11.5°, the outlet angles of back face is 15.5°, wrap angle is 198°, number of blades is 4, outlet width is 9.5mm, the area ratio is 1.1), the equivalent design factor of non-overload centrifugal pump is satisfied, the shaft power has maximum value, and ideal result is obtained. The centrifugal pump with low specific speed $n_s = 62$, various performance indexes of its prototype (non-overload feature, efficiency, cavitation performance, etc) reach and even are better than the target requirements, among them, the non-overload feature is good, the maximum value of shaft power is 1.1 times of rated shaft power, and the flow rate at maximum shaft power is 1.5 times of rated flow rate, as shown in Fig. 8.

![Non-overload power curve of low specific speed centrifugal pump](image)

**Figure 8.** Non-overload power curve of low specific speed centrifugal pump

6. Conclusion

(1) The calculation expression for the intrinsic law of input power which has not yet been given by current theory of Rotodynamic pump;

(2) It is very appropriate that $(w_{u1}^2 - w_{u2}^2)/2g$ is classified into kinetic head, then potential head $H_p = (u_2^2 - u_1^2)/2g$, and kinetic head $H_k = (c_{u2}^2 - c_{u1}^2)/2g + (w_{u1}^2 - w_{u2}^2)/2g$;

(3) The potential impelling force $f_p$ corresponds to potential head $H_p$, and $f_p = 2\omega c$, i.e. the constraining force (also called constraining counterforce) of the tangential Coriolis force;

(4) The kinetic impelling force $f_k$ corresponds to kinetic head $H_k$ includes $f_{cu}$ (corresponds to $(c_{u2}^2 - c_{u1}^2)/2g$) and $f_{wu}$ (corresponds to $(w_{u1}^2 - w_{u2}^2)/2g$), i.e. $f_k = f_{cu} + f_{wu}$, and $f_{cu} = \frac{1}{2\pi}\omega c\cdot$, $f_{wu} = -\frac{1}{2\pi}\omega w$;

(5) The input power component to which potential head corresponds is proportional to flow rate $Q$, and $N_p = K_pQ$;

(6) The input power component to which kinetic head corresponds is inversely proportional to flow
rate $Q$, and $N_k = Const - K_k Q$;

(7) The shaft power of rotodynamic pump is composed of two kinds of power components to which potential head and kinetic head correspond, i.e. shaft power $N = Const + (K_p - K_k)Q$, and the power component to which potential head corresponds increases with the increase of flow rate, the increasing amplitude decreases with the increase of specific speed $n_s$ (the power component of this term in axial-flow pump decreases to zero), the power component to which kinetic head corresponds decreases with the increase of flow rate. Thus it can reasonably be explained that the law which the input power curves of centrifugal pump, mixed-flow pump and axial-flow pump present the law of ascending, approaching to level and descending, respectively;

(8) The equivalent design factor for non-overload centrifugal pump obtained is $K_p < K_k$, and the equivalent design factor correlates with the outlet angle of leading face and back face of the blade, wrap angle, number of blades, outlet width, area ratio, and the ratio of operating flow rate to specified flow rate and so on;

(9) The authors have designed high-efficiency non-overload low specific speed centrifugal pump, and the non-overload feature is good, the maximum value of shaft power is 1.1 times of rated shaft power.
Nomenclature

\( a_{cor} \) = Coriolis acceleration, \( a_{cor} = 2\omega \times w = 2\omega \times w_u + 2\omega \times w_r = 2\omega \times w_u + 2\omega \times c_r \)

\( c, c \) = absolute velocity of flow, \( c = u + w, \ c = c_u + c_m = c_u + c_r + c_z \)

\( D \) = impeller diameter

\( f \) = impelling force of blade (per unit mass), \( f = f_p + (f_{cu} + f_{wu}) \)

\( f_{cu} + f_{wu}\) corresponds to \( (w_{u2}^2 - w_{u1}^2)/2g \)

\( f_p = (u_2^2 - u_1^2)/2g \)

\( H \) = head

\( m \) = (actual) mass within the free body or the control body of the infinite thin annulus

\( \Delta m_i \) = accumulative mass, means the product of number of revolutions of the control body within the time of \( dt \) and the actual mass \( m \), i.e. \( \Delta m_i = m \cdot \frac{\omega}{2\pi} \cdot dt \)

\( N \) = input power, \( N = N_p + (N_{cu} + N_{wu}) \)

\( N_{cu} + N_{wu} \) = \( N_{cu} \) corresponds to \( f_{cu} \), \( N_{wu} \) corresponds to \( f_{wu} \)

\( N_p \) = corresponding to \( f_p \)

\( Q \) = flow rate

\( r \) = radius

\( u \) = peripheral velocity of impeller, \( u = \omega r \)

\( w \) = relative velocity of flow, \( w = w_u + w_m = w_u + w_r + w_z, \ w_r = c_r, \ w_z = c_z \)

\( \omega, \omega \) = angular velocity of impeller

Subscript

1 = blade entrance
2 = blade exit edge
\( k \) = corresponding to kinetic head
\( p \) = corresponding to potential head
\( m \) = meridional direction (component of velocity)
\( r \) = radial direction (component of velocity)
\( \theta \) = theoretical
\( u \) = tangential direction (component of velocity)
\( z \) = axial direction (component of velocity)
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