Control model of Furnace Temperature Curve

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Abstract. This paper describes a measurement of the furnace temperature profile during reflow soldering by building a computational model from which we can get the furnace temperature curve under different conditions by increasing constraints with some critical parameters calibrated. Among these, the heat conduction and convection are considered as the most important heat transfer modes.

1. Introduction
Reflow soldering, which is one of the three main processes of Surface Mount Technology (SMT), can be primarily utilized to weld circuit boards that have been mounted to the components. Recently, with a great development of manufacturing process for printed circuit board assemblies, the disadvantages of conventional technology gradually emerge due to the defects of high cost. A precise control of the furnace temperature curve plays a key role during the whole process and many researchers have already invested heavily in this direction, mainly the classical 1-dimensional heat transfer models. Cai et al. studied the significant role of furnace temperature curve control of the reflow soldering [1]. Wan discussed some key points of measuring temperature curve and testing methods are expounded so as to obtain high yield and reliability[2]. Liu et al. studied the temperature zone’s setting principle[3]. Song et al. studied the main factors affecting the quality of reflow soldering during SMT involving solder paste, PCB’s speed of transmission, temperature of the heating zone and so on[4]. Farhad Sarvar et al. described a predictive model of the reflow soldering process which could be used to identify the temperature variations, effectively and accurately[5]. Atkinson et al. used one-dimensional heat flow equation and took up a mathematical heat transfer model by considering many factors such as coefficient of thermal conductivity[6]. Wang et al. established a 1D unsteady temperature distribution model with heat resource factor and the correction equation along with residual analysis[7]. Qin et al. studied a way to calculate the thermal diffusivity and conductivity and constructed the heat transfer model of infinite large plate which could be calculated with MATLAB programming software and this study provides a new way to establish the computational model[8]. Feng et al. studied the Influence of reflow soldering parameters on temperature curve with the heating model of reflow welding presented by EMPF Center of ACI[9]. Most of the above studies discussed the importance of the furnace temperature curve, but failed to provide an accurate method for measuring the furnace temperature curve. Other studies discussed the measurement and method of the furnace temperature curve in
different fields, but they could not be applied to the welding process of the reflow furnace. As an alternative, the paper outlines an establishment of a model designed for efficiency as well as accuracy. There have been lots of researchers carried extensive studies of the importance of the furnace temperature curve and given the crucial factors explaining it, but there is still no in-depth study on the specific formulation methods and solutions.

Therefore, the ideal furnace temperature curve can only be obtained by setting reasonable furnace parameters. The work of industrial soldering furnace relies on the calibration of furnace temperature curve for various parameters, so that the final emphasis will be put on the determination of standard furnace temperature curve.

Based on the present research, we put forward a reasonable method to measure the standard furnace temperature curve and put forward a model. Then, this paper introduced the background of the problem and the solution one by one.

2. Correlative information about the problem.

2.1. Restatement of the problem
As shown in figure 1, there are some small temperature zones in the reflow oven, which could be functionally divided into four large temperature zones: preheating zone (temperature zone (1~5), constant temperature zone (temperature zone (6), reflow zone (temperature zone (7~9) and cooling zone (temperature zone (10~11). And there was a gap of certain length between them. Also there was a section with certain length before and after the furnace. The circuit board was put on both sides of the conveyor into the furnace at a constant speed for soldering.

![Figure 1. The reflow furnace working principle view](image)

With the ambient temperature, the air was heated by the furnace and it would rise to a steady state in a short time after the reflow furnace was started. Each zone had a certain temperature. And the conveyor ran at a certain speed which could be adjusted within a certain range. The furnace temperature curve could be obtained through the temperature sensor testing the temperature at the centre of the welding area. And also the furnace temperature curve met certain process restrictions.

2.2. Experimental materials and selection of thermal processes
By consulting several relevant materials, this paper chose the solder paste for its fine ductility and high melting point for our research group as an intermediary material[10].

The whole heat transfer process was a classic heat conduction problem. Based on the metal characteristics of the element, thermal conductivity occurred in the vertical direction of the soldering area, with a small heating area and a fast thermal conduction speed. The whole process was under the comprehensive influence of convection and conduction between the temperature areas.
3. The Heat Conduction and the Mathematical Model

3.1. Mechanism analysis
Take the approach from easy to difficult and Variable-controlling. Firstly, the basic thermal conductivity model of temperature change in welding area was established. Assume that the conveyor runs at a certain speed and set a certain temperature for each area. According to these conditions, observe the temperature change in the center of the welding area and draw the corresponding temperature curve. After the basic model was obtained, the temperature in each temperature zone was controlled to determine the maximum belt velocity over the furnace. Considering that the ideal furnace temperature curve should have a minimum coverage area and peak temperature, the conveyor belt velocity and coverage area under this condition were determined. It was inferred that the curve is symmetric on both sides of the centerline, and the final ideal furnace temperature curve would be solved.

To solve these questions, correlated with the known conditions, the proposed model of reflow soldering process was based on the following assumptions.

- The process of the conveyor does not affect the heat transfer to the circuit board in the furnace.
- The conveyor stays stable during the transmission.
- The change of thermal conductivity of soldering paste with temperature change was ignored
- The temperature distribution had a continuous linear change within a small range of temperature difference on both sides of the boundary.

3.2. Treatments before and after the furnace
Heat conduction and convection were main ways of heat transmission in the gaps. It also took a certain amount of time for the welded plates to pass through small gaps, which would affect the temperature of PCB to some extent. For the gaps between the reflow and the cooling zone behind the furnace, 1-D heat conduction equation was constructed due to the significant temperature difference.

3.2.1. The Fundamental Thermal conductivity equation
The commonly-used theory for heat conduction was Fourier's law of heat conduction which shows that the quantity of heat per unit time and area was proportion to the temperature changing rate that perpendicular to the interface. And a ratio coefficient, “k” was introduced to the heat equation which was expressed as:

\[ q = -k \frac{\partial t}{\partial n} = -kt. \]  \hspace{1cm} (1)

Where \( q \) is heat flux density, \( k \) is thermal Conductivity, \( \frac{\partial t}{\partial n} \) is temperature alteration ratio, \( \nabla t \) is thermal gardient

It has been revealed by experimental observation that each zone reached a steady state in a short time. Therefore, without considering the specific heating process and only the heating state were studied. With a uniform inner heat source, this unsteady state heat conduction follow the law of conservation of energy was represented in equation (2).

\[ \frac{\partial t}{\partial \tau} = \alpha \nabla^2 t + \frac{\Phi}{\rho c_p}. \]  \hspace{1cm} (2)

Where \( \alpha \) is thermal diffusivity, \( \rho \) is density, \( \Phi \) is heat flow.

3.2.2. The definite condition and Heat Transfer Coefficient
Given the temperature change of the welding centre in the area in front of the furnace, the initial value conditions and boundary conditions were established respectively from the temperature of air and the zone of 1-5. A Cartesian coordinate system with the transport direction of the conveyor belt as the S-
axis was established. The initial conditions of the temperature distribution in the pre-furnace area can be expressed as:

\[ U_s(s, 0) = T^\circ C \]  \hspace{1cm} (3)

Where \( U_s \) is temperature profile before the furnace, \( s \) is variable distance before the furnace.

Considering the instance of high temperature at the boundary, make the range of the small gap was moved forward 2cm. Therefore, the outer air and the low temperature area(1~5) were under the boundary condition of constant temperature (ºC). Then the boundary condition of the area in front of the furnace can be obtained as follows:

\[
\begin{align*}
U_s(0, \tau) &= T^\circ C \\
U_s(s_1 + 2, \tau) &= T^\circ C 
\end{align*}
\]  \hspace{1cm} (4)

Where \( U_s \) is temperature profile before the furnace, \( U_s \) is temperature in preheating, \( s_1 + 2 \) is the length before the furnace, \( T \) is temperature in preheating zone.

The air thermal diffusivity could be obtained by data collection, and the heat transfer coefficient of the corresponding stage could be traversed in the range (20,100) and it was called K. The obtained heat transfer coefficient was used to inversely solve the temperature distribution of the area in front of the furnace, and the temperature distribution of the welding area at the last moment was taken as the initial value condition of the first small temperature area 1~5. One-dimensional unsteady heat conduction equations were established for each of the above zones, and the results of the solution for the area in front of the furnace were taken as the boundary conditions of the preheating zone. Then the heat transfer coefficients of each zone could be solved step by step.

3.3. The establishment of the modeling

3.3.1 outside the furnace

**A. Before the furnace.** The temperature distribution in the area in front of the furnace and the temperature distribution in the gap 1 have been solved. Taking the transmission direction of the conveyor belt as the S-axis and the front end of the area in front of the furnace as the origin, a one-dimensional heat conduction equation model changing from the distance of the area in front of the furnace was established with (3)and(4):

\[
\frac{\partial U_s}{\partial \tau} = a \frac{\partial^2 U_s}{\partial s^2}. 
\]  \hspace{1cm} (5)

**B. In and after the furnace.** On the section of the welding area, a one-dimensional thermal conductivity equation was established by taking the vertical welding direction as the axis and the welding centre as the origin. The heat conduction equations are established for the five temperature zones, the four gaps and the furnace area respectively. At the same time, in order to ensure the continuity of data, considering the large temperature difference between the cooling zone and the reflow zone, Newton's cooling formula was applied to the cooling zone and clearance 4, and the model was established as follows:
\[
\min(T_{\text{c}} - T_{\text{s}}), h \in (20,100)
\]
\[
\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2}, a = K, i = 1, ...
\]
\[
\text{initial condition}
\begin{align*}
T(x,0) &= T^0 C \\
T(x,t) &= T_{\text{in}}
\end{align*}
\]
\[
\text{boundary conditions}
\begin{align*}
\frac{\partial T}{\partial x} |_{x=a} &= 0 \\
-k \frac{\partial T}{\partial x} &= h(\alpha - T), j = 1, ...
\end{align*}
\]
\[
(6)
\]
Where \(T\) is the measured temperature, \(T^0\) is the known temperature, \(K\) is constant.

3.3.2 Modification of the thermal conductivity equation by introducing the Biot Number.

In the welding process, the most important was the measurement of furnace temperature curve. The influence of thermal conductivity, length and heat transfer coefficient of the material should be avoided as far as possible. Therefore, a set parameter could be established to represent the above three variables as a whole. The calculation of the Biot were as flows:

\[
Bi = \frac{hl}{k}
\]

(7)

Where \(h\) is heat transfer coefficient, \(l\) is the length of the object, \(k\) is thermal conductivity.

Thus, introduce the excess temperature which the deviation was less than 0.5% then we can define a new variable \(b = \frac{h}{cpl}\), using lumped parameter method to show the interaction of parameters. Then the model could be modified as follows with (3)-(6).

\[
\begin{align*}
\text{The conduction differential equation} : &\frac{dT}{d\tau} = -\frac{hA}{\rho cV}(T - U) \\
\theta &= \theta \exp(-\frac{hA}{\rho cV} \tau) = \theta_i \exp(-Bi_i Fo_i)
\end{align*}
\]

(8)

Where \(\theta = T - U\), excess temperature, \(Bi_i = \frac{h}{k A}, Fo_i = \frac{a\tau}{(V/A)^i}\) Biot and Fourier number with characteristic scale of \(V / A\).

3.3.3 Establishment of bivariate nonlinear programming model

The furnace temperature curve with the peak temperature as the center line and over a certain temperature on both sides was transformed as symmetrically as possible into the minimum temperature change rate difference on both sides at the symmetric time of the center line. And an objective function can be obtained as the minimum difference value. A two-objective nonlinear programming model was established. It could be seen from the image that when the furnace temperature curve of the part over 217 °C was symmetric with respect to the centerline with the peak. As shown in the figure 2.
Figure 2. Standard furnace temperature curve

Since \( T_a \) was in the reflow region, the temperature function could be used to solve and obtain the temperature when the temperature was equal to \( T_a \) and the temperature derivative was 0, denoted as \( t_r, t_i \).

\[
\begin{align*}
    T_r &= T_a \quad \rightarrow t_r, \\
    \frac{dT_r}{dt} &= 0 \quad \rightarrow t_i
\end{align*}
\]  \quad (9)

Take \( t_r, t_i \) as the upper and lower limits of the integral, integrate the temperature function in the temperature area of 8 and 9, and then subtract the area of the rectangle to get the area of the shaded part, denoted as \( A \), and the expression was as follows:

\[
A = \int_{t_r}^{t_i} (T_a - T_r) \cdot dt
\]  \quad (10)

It can be concluded that the two points establishing the difference of the rate of change are the two points with the same temperature on both sides of the furnace temperature curve over \( T_a \) °C, namely

\[
T_s = T_v
\]  \quad (11)

Where \( T_s, T_v \) are temperatures in 8,9 zones.

For the reflux area, the temperature change rate was positive, while for the gap between the reflux area and the cooling area and the cooling area, the temperature change rate was negative. Therefore, the difference of the temperature change rate can be directly expressed as the sum of the two:

\[
\frac{\partial T_r}{\partial t} + \frac{\partial T_i}{\partial t} = \delta
\]  \quad (12)

So the objective function was:

\[
\begin{align*}
\text{Objective function} & \quad \min A \quad A = \int_{t_r}^{t_i} (T_a - T_r) \cdot dt \\
\min \delta \quad \delta = \frac{\partial T_r}{\partial t} + \frac{\partial T_i}{\partial t} \cdot A
\end{align*}
\]  \quad (13)
4. The parameters of the simulation

4.1. Space considerations
With the ambient temperature at 25ºC, temperature set for each zone was 175ºC (T1~T5), 195ºC (T6), 235ºC (T7), 255ºC (T8~T9) and 25ºC (T10~T11) and adjustment of every zone was (-10, 10). The speed of the conveyor was 70 cm/min which could be adjusted within 65cm/min to 100cm/min and the thickness of the welding area was 0.15mm. The length of each small temperature zone was 30.5cm which could be shown in figure 3:[11]

![Figure 3. Simple graph view](image)

The temperature, speed and time limits for each stage were shown in Table 1.

| The name of the limitation                        | minimum | maximum | unit |
|--------------------------------------------------|---------|---------|------|
| The slope of temperature rise                    | 0       | 3       | ºC/s |
| The slope of temperature drop                    | -3      | 0       | ºC/s |
| The time in the process of temperature rise      | 60      | 120     | s    |
| between 150ºC~190ºC                             | 40      | 90      | s    |
| The time over 217ºC                             | 240     | 250     | ºC   |

4.2. Constraint condition
In the temperature range of 150ºC -190ºC, the boundary value of the velocity was first taken and substituted into the function to roughly observe the temperature in the welding area reaching the range of 150ºC and 190ºC. Using the temperature setting value in this temperature region, the time when the temperature reaches 150ºC and 190ºC when the temperature rises was respectively solved, denoted as (14) and the time frame was also limited.

\[
\begin{align*}
T_i &= 150ºC \rightarrow t_i \\
T_i &= 190ºC \rightarrow t_i \\
60 \leq t_i - t_i \leq 120 \\
\end{align*}
\]

(14)

When the temperature of the region reaches a certain value, the welding centre was in the reflow zone. At the same time, when the welding area reaches the cooling zone, the temperature will also reach 217ºC, and the time of the two moments can be calculated respectively, denoted as (15)

\[
\begin{align*}
T_i &= 217ºC \rightarrow t_i \\
T_i &= 217ºC \rightarrow t_i \\
40 \leq t_i - t_i \leq 90 \\
\end{align*}
\]

(15)
The peak temperature was the temperature when the derivative was 0, so the temperature when the temperature change rate was 0 could be calculated and denoted as, (or it could be the last temperature in the temperature region of 10) then the temperature constraint could be obtained as:

\[
\frac{\partial T(t)}{\partial t} = 0 \quad \rightarrow t_i
\]

\[
240 \leq t_i \leq 250
\]

(16)

The temperature variation range of the temperature in the temperature region is the given value ±10 °C of the temperature in the accessory temperature region, and the constraint conditions are established as follows:

\[
\begin{align*}
165^\circ C & \leq U_i \leq 185^\circ C \\
185^\circ C & \leq U_i \leq 205^\circ C \\
225^\circ C & \leq U_i \leq 245^\circ C \\
245^\circ C & \leq U_i \leq 265^\circ C
\end{align*}
\]

(17)

Where \( U_i \), \( i = 1, ..., 4 \) are four major temperature zones

5. Results and discussions

5.1. Calculated results

Step1 The heat conduction equation of the area in front of the furnace was mainly based on the unsteady partial differential equation. The air diffusion coefficient could be obtained from the data collection, and then the temperature distribution table of air variation with temperature could be obtained and \( \alpha = 36.15 \).

In the case of strong convection at the boundary temperature, the small gap was calculated forward to 2cm, and the distance of the area in front of the furnace was set at 27cm. We use the implicit backward Euler method to solve. The heat transfer coefficient in the furnace was 81.

Step2 In this paper, new variables \( b = \frac{h}{c_{pl}} \) were defined in the final modified model and regarded it as a whole. The heat transfer coefficient, specific heat capacity and density in each region are included in this variable of the modified model. According to the calculation results of the heat transfer coefficient, the lumped parameter problem was feasible.

The difference method was used to solve the problem, and the differential expression was substituted to solve the relationship between the temperature increase and change and the temperature in the furnace and the temperature difference in the welding area. Preliminary fitting was performed within each temperature change interval, which was expressed as a linear relationship, and the overall performance was piecewise linear function. Moreover, each temperature curve had its own constant term \( d \) and slope \( k \).

\[
y = kx + d
\]

(18)

The fitting situation is shown in the figure below:
Figure 4. Test the parameters obtained and the fitting condition

For the representation of these two parameters in the linear function, block fitting was carried out. It was concluded that the curve was almost identical with the data given in the attachment.

Step 3 The minimum area obtained was 392.5781. The 1-5 temperature zone was 184 °C, the 6 temperature zone was 205 °C, the 7 temperature zone was 235 °C, the 8-9 temperature zones was 263 °C, and the conveyor belt velocity was 95cm/min. The minimum error was 0.007444.

MATLAB was used for calculation, and the furnace temperature curve was obtained as shown in the figure below:

Figure 5. The final optimal furnace temperature curve

Combined with the above problems, the final ideal furnace temperature curve is further solved. In addition to variable-step enumeration method, simulated annealing algorithm can be used.

Figure 6. Reality and Theory Furnace Temperature Curves
Figure 7. The relationship between the model and the actual error

Figure 6 showed that the temperature difference between the simulated temperature and the measured one was small, and it was verified in the references.
Figure 7 showed that the deviation mainly distributed in the area before and after the furnace, and smaller in the key area.

6. Conclusion
When the PCB is put into the furnace for heating, without considering the thermal conductivity of the electronic components themselves, the temperature curve of the furnace can be controlled by controlling the speed of the conveyor belt and the setting of the temperature in each temperature area to achieve the best production state and improve the production efficiency. The furnace temperature curve is different for different components. In actual production, proper furnace temperature curve can be established by adjusting the range of different parameters and different solution and boundary conditions. After obtaining a reliable furnace temperature curve, only strengthening furnace temperature monitoring is needed to ensure the welding quality.

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