Quadrupole Deformation of Barium Isotopes

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Abstract

The B(E2; 0^+_1 \rightarrow 2^+_1) values of the Ba isotopes (Z = 56) exhibit a sharp increase in deformation as the neutron numbers approach the mid-shell value of N = 66. This behavior is anomalous because the 2^+_1 level energies are very similar to those of the neighboring isotopes. By means of the axially-symmetric deformed Woods-Saxon (WS) hamiltonian plus the BCS method, we investigated the systematics of B(E2) of the Ba isotopes. We showed that 15% of the B(E2) values at N = 66 was due to the level crossing, occurring at the deformation \( \beta_{WS} \sim 0.3 \), between the proton orbits originating from the orbits \( \Omega^\pi = 1/2^- (h_{11}/2) \) and \( 9/2^+ (g_{9/2}) \) at zero deformation. The latter of these two was an intruder orbit originating from below the energy gap at \( Z = 50 \), rising higher in energy with the deformation and intruding the \( Z = 50 - 82 \) shell. These two orbits have the largest magnitude of the quadrupole moment with a different sign among the orbits near and below the Fermi surface. Occupancy and non-occupancy of these orbits by protons thus affect B(E2; 0^+_1 \rightarrow 2^+_1) significantly.

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I. INTRODUCTION

The deformation of the nuclear ground state is a fundamental quantity that can be deduced from the B(E2) value for the first $2^+$ state ($J^\pi = 2_1^+$). From the systematics of the B(E2:0$_1^+ \rightarrow 2_1^+$) values, we could infer how the deformation of the ground state changes from nucleus to nucleus. Over the past decade, the B(E2) values of the Xe-Ba-Ce nuclei have been measured extensively. See [1] for Xe, [2] for Ba, [3] for Ce. The accumulated data has enabled us to compare the data with several theoretical results to test these theories [4].

Focusing ourselves on the Ba (Z=56) isotopes, we will discuss the systematics of the B(E2:0$_1^+ \rightarrow 2_1^+$) values.

II. GRODZINS RULE

The B(E2) values of Ba [5] as well as the Xe (Z=54) isotopes [4] increase sharply as the neutron number $N$ approaches $N = 66$ [4,5] as can be seen in Fig. 1. The solid squares in Fig. 1 present the experimental data of B(E2:0$_1^+ \rightarrow 2_1^+$), which are compared in Fig. 1 with the theoretical results estimated using the Grodzins rule [6] and obtained using the proton-neutron interacting boson model (IBM-2) [7], as we will explain in detail below.

The Grodzins rule [6] expresses an empirical relation between Ex$(2_1^+)$ and B(E2):

$$B(E2 : 0^+ \rightarrow 2^+) = 1.63 \times 10^{-2} Z^2/(E \cdot A)(eb)^2$$

with $E$ being Ex$(2_1^+)$ (MeV), $Z$ the proton number, and $A$ the mass number. In deriving Eq. (1), we used Eq. (4) of Ref. [6]. The open triangles in Fig. 1 present B(E2:0$_1^+ \rightarrow 2_1^+$) derived from Eq. (1). The results obtained by this rule are in surprisingly good agreement with the data for the less deformed Ba isotopes with $N \geq 70$ but underestimate B(E2) by 20% at $N = 66$. The similar empirical relation in [4] also underestimates B(E2:0$_1^+ \rightarrow 2_1^+$) at $N = 66$. The empirical rules suggest that the sharp increase of B(E2) is anomalous because Ex$(2_1^+)$ at $N = 66$ is almost the same as those of the neighboring isotopes. We will next examine this anomaly in B(E2:0$_1^+ \rightarrow 2_1^+$) within the framework of IBM [8].

III. IBM-2

The interacting boson model (IBM) [8] has been widely used for describing the quadrupole collective states of the medium heavy nuclei. The microscopic foundation of IBM has been given within the proton-neutron IBM (IBM-2) [9]. The building blocks of IBM-2 are $s_\tau$ and $d_\tau$ bosons ($\tau = \pi, \nu$) which are considered to be approximations to the proton (neutron) pairs with spin-parity $0^+$ and $2^+$. The boson images of the fermion operators are given in terms of the OAI mapping [10].

Using IBM-2, Otsuka, Pan and Arima [7] investigated the Xe-Ba-Ce isotopes concentrating on the systematics of B(E2:0$_1^+ \rightarrow 2_1^+$) as a function of the neutron number. According to the procedures given in [7], we did the IBM-2 calculations using the code NPBOS [11] with
the parameters taken from Ref. [7] which were derived from the microscopic calculations through the method of OAI mapping [10]. The boson image of the fermion E2 operator is given by [7]

\[ T^{(E2)} = e^0_\pi Q_\pi + e^0_\nu Q_\nu, \]  

(2)

where \( e^0_\sigma (\sigma = \pi, \nu) \) are constants and

\[ Q_\sigma = \kappa_\sigma \left\{ d^\dagger_\sigma \bar{s}_\sigma + s^\dagger_\sigma \bar{d}_\sigma + \chi_\sigma \left[d^\dagger_\sigma \bar{d}_\sigma \right]^{(2)} \right\} \]  

(3)

with

\[ \kappa_\sigma = \sqrt{\frac{(\Omega_\sigma - N_\sigma)}{(\Omega_\sigma - 1)}}, \]  

(4)

\[ \chi_\sigma = \chi^0_\sigma \frac{\Omega_\sigma - 2N_\sigma}{\Omega_\sigma - 2} \cdot \kappa^{-1}_\sigma \]  

(5)

where \( \Omega_\sigma \) is the degeneracy of the major shell and \( \Omega_{\pi,\nu} = 16 \) for the Ba isotopes. We used the following parameters taken from [7]: \( \chi^0_\pi = \pm 0.672 \) (− for the particles and + for the holes), \( e^0_\pi = 0.154 \) (eb), \( e^0_\nu = 0.110 \) (eb). Using this E2 operator, we calculated B(E2:0^+_1 \rightarrow 2^+_1) for the Ba isotopes, and compared them with the experimental data [2] in Fig. 1. The solid line represents the theoretical results which were identical to the results in [7].

Quantities \( \kappa_\sigma \) and \( \chi_\sigma/\chi^0_\sigma \) represent the effects of the Pauli principle, and are referred to as the Pauli factors in [7]. These Pauli factors become unity when there is no Pauli effect, i.e., \( \Omega_\sigma \rightarrow \infty \) [7]. The dashed line in Fig. 1 is the B(E2) values calculated without the Pauli factor \( \kappa_\sigma \) in \( T^{(E2)} \) [7], i.e., with \( \kappa_\sigma = 1 \) and \( \chi_\sigma = \chi^0_\sigma (\Omega_\sigma - 2N_\sigma)/(\Omega_\sigma - 2) \).

In Fig. 1, we used the latest experimental B(E2) data, some of which [3] were measured after the publication of [7]. We should mention again that, besides the experimental data, the IBM-2 results in Fig. 1 are identical to Fig. 4 of [7] because we have taken the same parameters as was used in [7].

It is clearly seen from Fig. 1 that the IBM-2 predictions with the Pauli factor are in good agreement with the data for \( N \geq 76 \). However, for \( 70 \leq N \leq 74 \), IBM-2 without the Pauli factor gives B(E2) values closer to the data than IBM-2 with the Pauli factor. For \( N = 66, 68 \), the B(E2) values of IBM-2 with the Pauli factor saturate in the mid-shell in disagreement with the experimental trends. At \( N = 66 \), the IBM-2 calculations without the Pauli factor yield nearly the same magnitude of B(E2:0^+_1 \rightarrow 2^+_1) as the Grodzins rule. Thus, at \( N = 66 \), not only the Grodzins rule but also IBM-2 underestimate the B(E2) values.

The Xe isotopes also showed a sharp increase in the B(E2) values at the mid-shell [4]. Raman et al. pointed out in Ref. [4] that the single-shell model such as the single-shell Nilsson model failed in reproducing the B(E2) values of the Xe isotopes at \( N \sim 64-66 \), whereas several multi-shell models correctly predicted these values. They showed in [4] that the rapid rise of the \( \Omega = 9/2 \) (g9/2) oblate orbit for protons as a function of deformation and its intrusion into the 50-82 shell suggests a mechanism for increasing the B(E2:0^+_1 \rightarrow 2^+_1) value as a result of partial emptying of this orbit due to pairing. However, the detailed
manner by which the mid-shell Xenon isotopes acquire significant deformation is still unclear, as pointed out in [4].

We aim in this paper at clarifying the origin of the sharp increase of $B(E2; 0^+ \rightarrow 2^+_1)$ observed in the mid-shell Ba isotopes. To achieve this objective, we used the axially symmetric deformed Woods-Saxon (WS) hamiltonian plus the BCS method. All orbits with eigenenergies lower than the barrier height [12] were adopted in the calculations. Therefore, we can get rid of the drawbacks [4] of the single-shell models such as the single-shell Nilsson model. In contrast to the several theoretical tools examined in [4], we do not minimize a total hamiltonian to fix the intrinsic state of the ground band. Instead, as a function of deformation parameter $\beta_{WS}$ of the WS potential, we calculated intrinsic quadrupole moment $Q_0$. Then, we determined $\beta_{WS}$ directly from the experimental $Q_0$ values, by which we could discuss the variation of the intrinsic configuration of the ground band with the deformation. We will show that 15% of $B(E2; 0^+ \rightarrow 2^+_1)$ for the Ba isotope with $N = 66$ is due to the level crossing between the $\Omega = 9/2 (g9/2)$ and $\Omega = 1/2 (h11/2)$ orbits, but the effect of the level crossing on $B(E2)$ fades out rapidly as $N$ goes away from the mid-shell value of $N = 66$. This suggests that the sharp increase in $B(E2)$ at the mid-shell is due to the level crossing between these orbits.

IV. WOODS-SAXON HAMILTONIAN PLUS BCS CALCULATIONS

We assume in this paper that individual nucleons move in the axially-symmetric quadrupole deformed Woods-Saxon potential and interact with each other by the monopole pairing force. We take the same form of the Woods-Saxon (WS) potential as was used in [12]. We assume $Y_{2,0}$ deformation only, and denote the deformation parameter of the WS potential by $\beta_{WS}$ as was already mentioned.

We should mention here the $\gamma$-softness or triaxiality of the heavier Ba isotopes with $N < 82$. The Ba isotopes exhibit the $\gamma$-soft or O(6)-like level scheme for the heavier isotopes with $N < 82$ [13][14], and change gradually from the $\gamma$-soft to axially symmetric nuclei as $N$ approaches the mid-shell value of $N = 66$ [14]. Through this paper, we assumed the axially symmetric WS potential for simplicity. This assumption may not be so valid for the heavier Ba isotopes because of the $\gamma$-softness. However, our major interest lies in the rapid increase in $B(E2)$ of the Ba isotopes with $N \sim 66$. For these nuclei, the axial symmetry is a reasonable approximation.

We use the WSBETA code [12] for calculating the eigenenergies of the WS hamiltonian as well as the quadrupole moments of the eigenstates. We have several parameters in the WS potential [12] such as the depth, radius and diffuseness of the central potential and those of the spin-orbit potential. Among several parameter sets of the WS potential [12], we chose the “universal” parameter set [12]. The other WS parameter sets such as the Blomqvist-Wahlborn, Chepurnov, Rost, and “optimal” parameter sets [12] give almost the same results as the “universal” parameter set, although the details of these results are not shown in this paper. Thus, the major results of this paper on the Ba isotopes do not change among those WS parameter sets.

We assume the pairing force for the residual interaction. As our concern is the proton quadrupole moment, we consider only proton system hereafter. The pair potential acting on the individual nucleons takes up the form of
\[ V_{\text{pair}} = -\Delta \sum_{\nu > 0} (a^{\dagger}(\bar{\nu})a^{\dagger}(\nu) + a(\nu)a(\bar{\nu})) \]  

where \( a^{\dagger}(\nu) (a(\nu)) \) is the creation (annihilation) operator of a proton in the eigenstate \( \nu \) of the WS hamiltonian, \( a^{\dagger}(\bar{\nu}) (a(\bar{\nu})) \) is the corresponding time-reversed operator, and the constant \( \Delta \) represents the pairing gap whose value is fixed by \( \Delta = G \langle P^{\dagger} \rangle \) in terms of coupling constant \( G \) and pair moment \( P^{\dagger} \) defined by

\[ P^{\dagger} = \sum_{\nu > 0} a^{\dagger}(\bar{\nu})a^{\dagger}(\nu). \]  

Using the Bogoliubov-Valatin transformation \[15\], we can diagonalize the hamiltonian \( H' = H_{WS} + V_{\text{pair}} - \lambda N_p \), where \( H_{WS} \) is the WS hamiltonian, \( N_p \) the number operator for the protons, and \( \lambda \) the chemical potential. Parameters \( \Delta \) and \( \lambda \) are fixed by the self-consistency between \( \Delta \) and \( P^{\dagger} \)

\[ \Delta = G \sum_{\nu > 0} u(\nu)v(\nu), \]  

and by the constraint on the expectation value of the particle number, \( Z = \langle N_p \rangle \):

\[ Z = 2 \sum_{\nu > 0} v(\nu)^2. \]  

In Eq. (8), \( u \) and \( v \) denote the coefficients of the Bogoliubov-Valatin transformation \[15\] given by

\[ u(\nu) = 2^{-1/2} \left( 1 + \frac{e(\nu) - \lambda}{E(\nu)} \right)^{1/2}, \]  

\[ v(\nu) = 2^{-1/2} \left( 1 - \frac{e(\nu) - \lambda}{E(\nu)} \right)^{1/2} \]  

with

\[ E(\nu) = \left( (e(\nu) - \lambda)^2 + \Delta^2 \right)^{1/2}, \]

where \( e(\nu) \) stands for the single particle energy of the eigenstate \( \nu \) of the WS hamiltonian.

We determined the \( G \) value for \(^{122}\)Ba by the condition that the pairing gap for the protons should equal the empirical value given by \[14\]: \( \Delta = 12 / \sqrt{A} \sim 1.1 \text{ MeV} \) at \( \beta_{WS}=0.3 \), which \( \beta_{WS} \) value roughly corresponds to the deformation of Ba with \( N = 66 \) as will be shown below. The resultant value is \( G = 0.16 \text{ MeV} \). In the summations with respect to \( \nu \) in Eq. (8), we adopted all the WS eigenstates with their eigenenergies below the barrier height \[12\]. We note here that the value for \( G \) depends on the number of \( \nu \) in these summations. In this paper, we assume \( G \) being independent of \( N \), for simplicity. We have no free parameter except for \( \beta_{WS} \), i.e., the quadrupole deformation parameter of the WS potential. The value for \( \beta_{WS} \) will be determined later by the experimental values of the intrinsic \( Q_0 \) moment. Refer to Fig. 2.
In terms of intrinsic quadrupole moment $Q_0$ for protons, we can approximately express $B(E2; 0^+_1 \rightarrow 2^+_1)$ as

$$B(E2; 0^+_1 \rightarrow 2^+_1) = \frac{5}{16\pi} e^2 Q_0^2,$$

with $e$ being the proton charge. This is a good approximation for the axially symmetric well deformed nuclei. The quadrupole moment is given in terms of the $v$ coefficient of Eq. (9b) by

$$Q_0 = 2 \sum_{\nu>0} q(\nu) v(\nu)^2,$$

where $q(\nu)$ represents the quadrupole moment of the WS eigenstate $\nu$:

$$q(\nu) = \langle \nu | \sqrt{\frac{16\pi}{5}} r^2 Y_{2,0} | \nu \rangle.$$

We can thus compare directly the data with the results obtained by the WS hamiltonian plus the BCS method.

The intrinsic $Q_0$ moment is plotted as a function of $\beta_{\text{WS}}$ in Fig. 2. The solid curve represents the quadrupole moment calculated using Eq. (12), which we call $Q(\text{WS}+\text{BCS})$. The filled circles denote the quadrupole moment calculated without the pairing interaction, which we call $Q(\text{WS})$.

In general, the quadrupole moment $Q(\text{WS})$ increases smoothly with $\beta_{\text{WS}}$ because each occupied orbit of the ground state gains the quadrupole moment gradually with $\beta_{\text{WS}}$. It also happens that $Q(\text{WS})$ increases suddenly at a certain value of $\beta_{\text{WS}}$ owing to a change in the ground state configuration. In the plot such as the Nilsson diagram, where the level energies are plotted as a function of the potential deformation, the change in the ground state configuration can be expected to occur at level crossings of occupied and unoccupied levels at the current chemical potential.

It is clearly seen from Fig. 2 that $Q(\text{WS})$ jumps twice at $\beta_{\text{WS}} \sim 0.025$ and $\beta_{\text{WS}} \sim 0.30$. These jumps correspond to the level crossing where the wavefunction of the ground state changes its configuration. The former jump is due to the level crossing between the single particle orbits originating from the spherical sub-shells of $1d_{5/2}$ and $0g_{7/2}$. This jump is unimportant because the jump in $Q_0$ is too small to survive after the pairing force is switched on as is seen from the curve $Q(\text{WS}+\text{BCS})$. The jump at $\beta_{\text{WS}} \sim 0.3$ in $Q(\text{WS})$ is much bigger than the former one, and is due to the level crossing between the levels originating from the unique parity sub-shells, $0g_{9/2}$ and $0h_{11/2}$, which correspond to the levels with the Nilsson “asymptotic” quantum numbers (Ref. [14]) $\Omega[N_t n_z \Lambda] = \frac{9}{2}^+ [404]$ and $\frac{1}{2}^- [550]$, respectively, where $\Omega$ is the absolute value of the $z$ component of the total angular momentum, $N_t$ the total number of quanta of the deformed harmonic oscillator, $n_z$ the number of quanta in the oscillation along the symmetry axis of the same oscillator, and $\Lambda$ the component of the orbital angular momentum along the symmetry axis.

Hereafter, we denote these two orbits as $\nu_1 = \frac{9}{2}^+ [404]$ and $\nu_2 = \frac{1}{2}^- [550]$, respectively. At $\beta_{\text{WS}} = 0.3$, two protons, which occupy the $\nu_1$ and $\bar{\nu}_1$ orbits for $\beta_{\text{WS}} < 0.3$, move to the $\nu_2$ and $\bar{\nu}_2$ orbits which are unoccupied by protons for $\beta_{\text{WS}} < 0.3$. By this movement of two protons, $Q(\text{WS})$ increases sharply at $\beta_{\text{WS}} = 0.3$ by
\[ \Delta Q_0 = 2(q(\nu_2) - q(\nu_1)) = 1.12(b) \]

with \(q(\nu_1) = -0.16\) (b) and \(q(\nu_2) = 0.40\) (b). This magnitude of \(\Delta Q_0\) amounts to as much as 20\% of the experimental value \(Q_0 = 5.2 \pm 0.2\) (b) for \(^{122}\)Ba. \(\nu_1\) (\(\nu_2\)) orbit has the largest magnitude of \(q\) in Eq. (13) with a negative (positive) sign among the levels with the same sign of \(q\) near and below the Fermi surface. Therefore, emptying the \(\nu_1\) orbit and filling the \(\nu_2\) orbit strongly affect the \(Q_0\) value.

We now consider the effect of the pairing force on \(Q_0\). The current pairing force strength \(G\) yields the pairing gap \(\Delta = 1.34, 1.25, 0.99, 1.09, 1.06\) (MeV) for \(\beta_{WS} = 0.0, 0.1, 0.2, 0.3, 0.4\), respectively. Due to the level crossing between \(\nu_1\) and \(\nu_2\), \(\Delta\) increases as \(\beta_{WS}\) approaches 0.3. Owing to the pairing interaction, the sharp jumps existing in \(Q(WS)\) get smoother in \(Q(WS + BCS)\). The effect of the pairing force on \(Q_0\) depends on \(\beta_{WS}\). The force reduces \(Q_0\) for \(\beta_{WS} < 0.2\) whereas it enhances \(Q_0\) for \(0.2 < \beta_{WS} < 0.3\). The pairwise proton transfer takes place from \(\Omega = 9/2(g_{9/2})\) to \(\Omega = 1/2(h_{11/2})\), and the probability of the pairwise transfer increases as \(\beta_{WS}\) approaches the level crossing point at \(\beta_{WS} = 0.3\). The quadrupole moment \(Q(WS + BCS)\) thus gets bigger than \(Q(WS)\) for \(0.2 < \beta_{WS} < 0.3\).

Eight horizontal lines are drawn in Fig. 2, which correspond to the experimental \(Q_0\) moments for the Ba isotopes with \(N\) from 66 to 80. Namely, the height of each line is equal to the experimental value of the \(Q_0\) moment. Thus, from the intersection point between each horizontal line and the solid curve \(Q(WS + BCS)\), we can determine \(\beta_{WS}\) corresponding to the experimental \(Q_0\) moment for the individual isotope. It then turns out that roughly 15\% of B(E2:0 \(+^1\rightarrow 1^+\)) for Ba with \(N = 66\) is due to the pairwise proton transfer from \(\Omega = 9/2(g_{9/2})\) to \(\Omega = 1/2(h_{11/2})\). These results are not sensitive to choices of the WS parameter sets. When we used the WS parameter sets in [12] other than the current “universal” set, we got the values ranging from 15\% to 20\%.

The Ba isotopes can be classified into three groups by a relation between \(Q(WS + BCS)\) and \(Q(WS)\):

(a) \(Q(WS + BCS) < Q(WS)\) for \(N = 76 \sim 80\)

(b) \(Q(WS + BCS) \sim Q(WS)\) for \(N = 70 \sim 74\)

(c) \(Q(WS + BCS) > Q(WS)\) for \(N = 66 \sim 68\)

We should here note that \(\Delta\) for (b) is as big as 1 MeV although \(Q(WS + BCS) \sim Q(WS)\).

A comparison between Figs. 1 and 2 leads us to a correspondence between the results of IBM-2 and of the WS hamiltonian plus the BCS method:

(a) IBM-2 with the Pauli factor gives B(E2) closer to the experimental data than the IBM-2 without the Pauli factor.

(b) IBM-2 without the Pauli factor gives B(E2) closer to the experimental data than the IBM-2 with the Pauli factor.

(c) Both IBM-2 with and without the Pauli factor underestimate B(E2).

The OAI mapping used in the derivation of the IBM-2 E2 operator in Eqs. (2-5) with the Pauli factor is based on the seniority scheme [10]. This scheme is suitable for the case in which the single particle levels are nearly degenerate regarding energy at zero deformation. The OAI mapping thus does not take account of the intruder orbit \(\Omega = 9/2^+(g_{9/2})\) which
intrudes the $Z = 50 - 82$ shell originating at zero deformation from below the energy gap at $Z = 50$. It is thus reasonable that the IBM-2 predictions underestimate $B(E2; 0^+_1 \rightarrow 2^+_1)$ at $N = 66, 68$ while the IBM-2 predictions with the Pauli factor are in good agreement with the experimental data for the less deformed Ba isotopes with $N \geq 76$.

We should mention here the Xe ($Z = 54$) and Ce ($Z = 58$) isotopes, though the results for Xe and Ce are not shown in detail in this paper. We have applied the WS hamiltonian plus the BCS method to the Xe and Ce isotopes in a way similar to the procedures used for Ba. It then turns out that, if we chose the Chepurnov set (the “universal” set) in [12], the contribution of the level crossing between the orbits originating from $\Omega = 9/2(g_{9/2})$ and the 50-82 shell to $B(E2; 0^+_1 \rightarrow 2^+_1)$ for Xe with $N = 66$ is roughly 20% (10%). The corresponding quantity for Ba with $N = 66$ is roughly 20% (15%). For Ce with $N = 66$, the effect of the level crossings on $B(E2; 0^+_1 \rightarrow 2^+_1)$ is smaller than 5% when we ignore the error in the experimental data.

The $B(E2; 0^+_1 \rightarrow 2^+_1)$ value predicted by the Grodzins rule in Eq. (1) is only 75% of the experimental data for Xe with $N = 66$, while the prediction for Ce with $N = 66$ lies within the error bar of the experimental $B(E2; 0^+_1 \rightarrow 2^+_1)$ data [3]. The $B(E2; 0^+_1 \rightarrow 2^+_1)$ values calculated with IBM-2 without the Pauli factor are nearly the same as the experimental data for Xe with $N = 64 - 68$ [4] and Ce with $N = 66 - 74$, namely for the isotopes with relatively large $B(E2)$ values. On the other hand, for the less deformed nuclei such as Xe with $N = 70 - 80$ and Ce with $N = 76$, IBM-2 with the Pauli factor gives the $B(E2; 0^+_1 \rightarrow 2^+_1)$ values closer to the data than IBM-2 without the Pauli factor. These trends are thus consistent with the IBM-2 results for the Ba isotopes in Fig. 1. This fact suggests that some unknown mechanism cancels the effect of the Pauli factor in Eqs. (3-5), and this plays a role mainly for the well deformed nuclei. To make this mechanism clear is beyond the scope of the current paper. We should investigate this problem further in the future.

V. SUMMARY

By the use of the Woods-Saxon hamiltonian plus the BCS method, we calculated the intrinsic $Q_0$ moment as a function of the deformation parameter of the WS potential. Comparing the calculated $Q_0$ moment with the experimental data, we can determine the value of $\beta_{WS}$ for each Ba isotope. It leads us to find a detailed way how protons occupy the deformed single particle orbits. By this procedure, we found that as much as 15% of $B(E2; 0^+_1 \rightarrow 2^+_1)$ for $^{122}$Ba is due to the pairwise transfer from the proton intruder orbit originating from $\Omega = 9/2(g_{9/2})$ to the proton unique-parity orbit originating from $\Omega = 1/2^-(h_{11/2})$. We have shown that the contribution to $Q_0$ from this pairwise transfer is roughly equal to the deviation of the experimental data from the theoretical predictions by IBM-2 without the Pauli factor. Similar deviations are found for the predictions of the Grodzins rule. We also found that this contribution to $B(E2; 0^+_1 \rightarrow 2^+_1)$ decreases rapidly as $N$ goes away from the mid-shell value of $N = 66$. This fact suggests that the pairwise transfer from the orbit with $\Omega = 9/2^+(\pi g_{9/2})$ to the one with $\Omega = 1/2^- (\pi h_{11/2})$ is responsible for the sharp increase of $B(E2; 0^+_1 \rightarrow 2^+_1)$ observed in the Ba isotope with $N = 66$.

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FIG. 1. Systematics of the $B(E2;0^+_1 \rightarrow 2^+_1)$ values of the Ba isotopes. The solid squares express the experimental data. The open triangles present the predictions by the Grodzins rule. The solid (dashed) line denotes the IBM-2 results with (without) the Pauli factor.

FIG. 2. $Q_{\pi 0}$ vs. $\beta_{WS}$ plot for Ba. Proton quadrupole moment $Q_{\pi 0}$ is plotted as a function of deformation parameter $\beta_{WS}$ of the axially-symmetric deformed Woods-Saxon potential. The solid curve denotes $Q_{\pi 0}(\text{WS+BCS})$ which includes the pairing correlation. The filled circles express $Q_{\pi 0}(\text{WS})$, the quadrupole moment calculated with the vanishing pairing gap $\Delta_\pi = 0$. Eight horizontal lines in the figure correspond to the experimental data. The height of each horizontal line is equal to the experimental quadrupole moment for each Ba isotope with the neutron number $N$ ranging from $N = 66$ to 80. For the sake of clarification, we have not included the errors of the data. The experimental values for $Q_{\pi 0}$ are $5.2 \pm 0.2$, $4.6 \pm 0.1$, $4.1 \pm 0.1$, $3.8 \pm 0.1$, $3.6 \pm 0.2$, $2.9 \pm 0.1$, $2.62 \pm 0.02$, $2.00 \pm 0.01$ (b) for 66, 68, ..., 80, respectively [2].