Transport coefficients of hot magnetized QCD matter

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The transport coefficients such as shear viscosity, bulk viscosity, and thermal conductivity of a magnetized hot QCD matter have been estimated in the strong field limit. To model the hot QCD matter in the presence of magnetic field, a quasi-particle description of the hot QCD equation of state has been adopted. The temperature dependence of viscous coefficients (bulk and shear viscosities) and thermal conductivity have been obtained by considering, 1 → 2 processes (g → q̅q) and 2 → 2 quark-antiquark scattering processes in the presence of the strong magnetic field. All this has been done by setting up a 1 + 1-dimensional effective covariant kinetic theory for the lowest Landau level quarks in the strong field limit. This enables one to include the mean-field contributions in terms of non-trivial quasi-particle energy dispersions to the transport coefficients. Such contributions have significant impact at temperature regions which are not very far away from the QCD transition point. To realize the significance of various processes in the medium, relative behavior of the transport coefficients in the thermal medium has been investigated through their respective ratios.

Keywords: Quark-gluon-plasma, Effective kinetic theory, Strong magnetic field, Thermal relaxation time, Transport coefficients, Effective fugacity.

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I. INTRODUCTION

Relativistic heavy-ion collision (RHIC) experiments have reported the presence of strongly coupled matter-Quark-gluon plasma (QGP) as a near-ideal fluid [1, 2]. The quantitative estimation of the experimental observables such as the collective flow and transverse momentum spectra of the produced particles from the hydrodynamic simulations involve the dependence upon the transport parameters of the medium. Thus, the transport coefficients are the essential input parameters for the hydrodynamic evolution of the system.

Recent investigations show that intense magnetic field is created in the early stages of the non-central asymmetric collisions [3-6]. This magnetic field affects the thermodynamic and transport properties of the hot dense QCD matter produced in the RHIC. Ref [7] describes the extension of ECHO-QGP [8, 9] to the magnetohydrodynamic regime. The recent major developments regarding the intense magnetic field in heavy-ion collision include the chiral magnetic effect [10-12], chiral vortical effects [13-15] and very recent realization of global A-hyperon polarization in non-central RHIC [16, 17]. This sets the motivation to study the transport coefficients in presence of the strong magnetic field. The transport parameters under investigation are the viscous coefficients (shear and bulk) and the thermal conductivity of the hot magnetized QGP. Importance of the transport processes in RHIC is well studied [18] and reconfirmed by the recent ALICE results [19, 21].

Quantizing quark/antiquark field in the presence of strong magnetic field background gives the Landau levels as energy eigenvalues. The quark/antiquark degrees of freedom is governed by 1+1-dimensional Landau level kinematics whereas gluonic degrees of freedom remain intact in the presence of magnetic field [22, 23]. However, gluons can be indirectly affected by the magnetic field through the quark loops while defining the Debye mass of the system.

Shear and bulk viscosities can be estimated from Green-Kubo formulation both in the presence and absence of magnetic field [22, 24, 25]. Viscous pressure tensor quantifies the energy-momentum dissipation with the space-time evolution and is characterized by seven viscous coefficients in the strong magnetic field [27]. The seven viscous coefficients consist of two bulk viscosities (both transverse and longitudinal) and five shear viscosities. The present investigations are focused on the longitudinal component (along the direction of B̂) of shear and bulk viscosity since other components of viscosities are negligible in the strong field limit. Another key transport coefficient under investigation is thermal conductivity of the QGP medium. The temperature dependence of thermal conductivity has been studied in the absence of magnetic field in the Ref. [26]. Relative behavior of the transport coefficients will lead to physical laws and ratios such as Wiedemann-Franz law, Prandtl number etc., that can provide information about the dynamics and responses of the medium. The ratio of shear viscosity to thermal conductivity can be counted in terms Prandtl number, which is important for understanding the sound
attenuation in the fluids [29]. The first step towards the estimation of transport coefficients from the effective kinetic theory is to include proper collision integral for the processes in the strong field. This can be done within the relaxation time approximation (RTA). Microscopic processes or interactions are the inputs of the transport coefficients and are incorporated through thermal relaxation times. Note that quark-antiquark pair production \((1 \rightarrow 2\) processes) and quark-antiquark t-channel scattering \((2 \rightarrow 2\) processes) are dominant in the presence of the strong magnetic field [30].

The prime focus of the present article is to estimate the temperature behavior of the transport coefficients such as bulk viscosity, shear viscosity and thermal conductivity, incorporating the hot QCD medium effects in presence of the strong magnetic field. Estimation of the transport parameters can be done in two equivalent approaches viz., the hard thermal loop effective theory (HTL) [31,33] and the relativistic semi-classical transport theory [41–43]. The present analysis is done with the relativistic transport theory by employing the Chapman-Enskog method. Hot QCD medium effects are encoded in the quark/antiquark and gluonic degrees of freedom by adopting the effective fugacity quasiparticle model (EQPM) [23,39,10]. The transport coefficients pick up the mean field term (force term) as described in Ref [11]. The mean field term comes from the local conservations of number current and stress-energy tensor in the covariant effective kinetic theory. In the current analysis, we investigate the mean field corrections in the presence of strong magnetic field and study the temperature behavior and the relative significance of the transport coefficients. Here, the strong magnetic field restricts the calculations to 1+1-dimensional (dimensional reduction) covariant effective kinetic theory.

We organize the manuscript as follows. In section II, the mathematical formulation for the estimation of transport coefficients from the effective covariant kinetic theory is discussed along with the quasiparticle description of hot QCD medium in the strong magnetic field. Section III deals with the thermal relaxations for both \(1 \rightarrow 2\) processes and \(2 \rightarrow 2\) quark-antiquark scattering in the strong field limit. Predictions of the transport coefficients and their relative behavior are discussed in section IV. Finally, in section V the summary and outlook of the are presented.

II. FORMALISM: TRANSPORT COEFFICIENTS AT STRONG MAGNETIC FIELD

The strong magnetic field \(\vec{B} = B\hat{z}\) constraints the quarks/antiquarks motion parallel to field with the transverse density of states \(|\frac{q_{f}eB}{2\pi}\). We are working on the regime \(\alpha_s | q_{f}eB | \ll T^2 \ll | q_{f}eB |\). The first inequality means that the regime under consideration is weakly coupled and the second inequality allows us to focus on the lowest Landau state \(l = 0\) of quarks and antiquarks since the magnetic field background is considerably strong. The formalism for the estimation of transport coefficients includes the quasiparticle modeling of the system away from the equilibrium followed by the setting up of the effective kinetic theory for different processes. Quasiparticle models encode the medium effects, viz., effective fugacity or with effective mass. The later include self-consistent and single parameter quasiparticle models [42]. NJL and PNJL based quasiparticle models [43], effective mass with Polyakov loop [44] and recently proposed quasiparticle models based on the Gribov-Zwanziger (GZ) quantization [45,47].

Here, the analysis is done within the effective fugacity quasiparticle model (EQPM) where the medium interactions are encoded through temperature dependent effective quasidiquark and quark/antiquark fugacities, \(z_q\) and \(z_g\) respectively. The extended EQPM describes the hot QCD medium effects in strong magnetic field [23]. We considered the (2+1) flavor lattice QCD equation of state (EoS) (LEoS) [48,49] and the 3-loop HTLpt EOS [50,51] for the effective description of QGP in strong magnetic field [23,32].

A. Transport coefficients from effective (1+1)-D kinetic theory

We first need to define the macroscopic quantities that describe the thermodynamic state in the strong magnetic field \(\vec{B} = B\hat{z}\). The particle four flow \(N^\mu(x)\) can be defined in terms of quasiparticle (dressed) momenta \(\vec{p}_k\) within EQPM as [41],

\[
N^\mu(x) = \sum_{k=1}^{N} \nu_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3} \hat{\rho}_k f^0_k(x, \vec{p}_k) + \delta\omega \sum_{k=1}^{N} \nu_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3} \hat{\rho}_k f^1_k(x, \vec{p}_k),
\]

in which \(\nu_k\) is the degeneracy factor of the \(k\)th species. The term \((\hat{\rho}^\mu) = \Delta_{\mu\nu} \bar{\rho}^\nu\) is the irreducible tensor with \(\Delta_{\mu\nu} = g_{\mu\nu} - u^\mu u^\nu\) as the projection operator. The metric has the form \(g_{\mu\nu} = (1, -1, -1, -1)\). The quasiparticle distribution function in local rest frame with the hydrodynamic four-velocity \(u^\mu = (1,0)\) is given by

\[
f^0_q = \frac{z_q \exp[-\beta(u^\mu p_\mu)]}{1 + z_q \exp[-\beta(u^\mu p_\mu)]},
\]

with \(p^\mu = (E_p, p_\mu)\), where \(E_p = \sqrt{p_\mu^2 + m^2 + 2l | q_{f}eB |}\) is the Landau level energy eigenvalue in the strong magnetic field. Quasiparticle momenta (dressed momenta) and bare particle four-momenta can be related from the dispersion relations as,

\[
\bar{p}^\mu = p^\mu + \delta u^\mu, \quad \delta \omega_p = T^2 \partial_T \ln(z_q),
\]
which modifies the zeroth component of the four-momenta in the local rest frame. Hence, we have
\[ \rho^0 \equiv \omega_p = E_p + \delta \omega_p. \]  
(4)

The dispersion relation in Eq. (4) encodes the collective excitation of quasiparticles along with the single particle energy. In the presence of the strong magnetic field, the leading order contribution to the transport coefficients are coming from LLL \((l = 0)\) quarks and antiquarks compared to gluons. This is because the thermal density of LLL quarks and antiquarks is \(| q_{fE} B | T \) whereas for gluons is only \( T^2 \). Since we are working in the regime \( T^2 \ll | q_{fE} B | \), the thermal density of quarks and antiquarks are much larger than that of gluons. Hence, here we are considering the contribution of LLL quarks/antiquarks to the macroscopic quantities and transport coefficients. In the strong magnetic field, we have
\[
N^\mu(x) = \sum_{k=1}^{N} \left| \frac{q_{fE} B}{2\pi} \right| \nu_k \int \frac{d^3p}{(2\pi)^3} \rho_k \phi_k(\omega_p, p_z) \\
+ \sum_{k=1}^{N} \delta \omega \left| \frac{q_{fE} B}{2\pi} \right| \nu_k \int \frac{d^3p}{(2\pi)^3} \frac{\langle \rho_k \phi_k \rangle}{\omega_p} f_k^0(x, p_z),
\]
(5)

where \( \rho_k \phi_k(\omega_p, p_z) \) incorporates the longitudinal components and the integration phase factor in the strong field due to dimensional reduction [53–55] is defined as,
\[
\int \frac{d^3p}{(2\pi)^3} \left| \frac{q_{fE} B}{2\pi} \right| \int \frac{dp_z}{2\pi}.
\]
(6)

Also, \( \nu_k = 2N_c \) for SU\((N_c)\) for the quark/antiquark of each flavor.

Next, we can define energy-momentum tensor \( T^\mu{}^\nu(x) \) focusing the energy density and momentum flow in the longitudinal direction of magnetic field. In terms of dressed momenta, \( T^\mu{}^\nu(x) \) in the strong magnetic field has the following form,
\[
T^\mu{}^\nu(x) = \sum_{k=1}^{N} \left| \frac{q_{fE} B}{2\pi} \right| \nu_k \int \frac{d^3p}{(2\pi)^3} \rho_k \phi_k f_k^0(x, p_z) \\
+ \sum_{k=1}^{N} \delta \omega \left| \frac{q_{fE} B}{2\pi} \right| \nu_k \int \frac{d^3p}{(2\pi)^3} \frac{\langle \rho_k \phi_k \rangle}{\omega_p} f_k^0(x, p_z),
\]
(7)

where \( \langle \rho_k \phi_k \rangle = \frac{1}{2} (\Delta^\mu{}^\alpha \Delta^\nu{}^\beta + \Delta^\mu{}^\beta \Delta^\nu{}^\alpha) \rho_\alpha \phi_\beta \). Note that the Eq. (7) gives back the expression of energy density and pressure within the EEEQPM in the strong magnetic field as shown in [23].

Estimation of the transport coefficients requires the system away from equilibrium. We need to set-up the relativistic transport equation, which quantifies the rate of change of distribution function in terms of collision integral. The thermal relaxation time \( (\tau_{eff}) \) linearize the collision term \( (C(f_k)) \) in the following way,
\[
\frac{1}{\omega_{p_k}} \rho_k^0 \partial_\mu f_k^0(x, p_z) + F_z \frac{\partial f_k^0}{\partial p_z} = C(f_k) \equiv -\delta f_q \tau_{eff},
\]
(8)

with \( F_z = -\partial_\mu (\delta \omega \mu u_\mu) \) is the force term from the conservation of particle density and energy momentum [31,11]. The local momentum distribution function of quarks can expand as,
\[
f_q = f_q^0(p_z) + \delta f_q, \quad \delta f_q = f_q^0(1 \pm f_k^0) \phi_q.
\]
(9)

Here, \( \phi_q \) defines the deviation of the quasiparticle distribution function from its equilibrium. The Eq. (8) gives the effective kinetic theory description of the quasiparticles under EEQPM in the strong magnetic field. In order to estimate the transport coefficients, we employ the Chapman-Enskog (CE) method. Applying the definition of equilibrium quasiparticle momentum distribution function as in Eq. (2), the first term of Eq. (8) gives the number of terms with thermodynamic forces of the transport processes. The second term of Eq. (8) vanishes for a co-moving frame. Finally, we are left with,
\[
Q_k X + (\rho_k^0)(\omega_{p_k} - h_k) X_{gm} - (\langle \rho_k^0 \phi_k^0 \rangle) X_{\mu\nu} = -\frac{T \omega_{p_k}}{\tau_{eff}} \phi_k,
\]
(10)

in which the conformal factor due to dimensional reduction in the strong field limit is \( Q_k = (\bar{\rho}_k^0 - \omega_{p_k}^2 c_s^2) \) where \( c_s^2 \) is the speed of sound. Here, \( \langle \langle \rho_k^0\phi_k^0 \rangle \rangle = \{ \frac{1}{2} \Delta^\mu{}^\alpha \Delta^\nu{}^\beta + \frac{1}{2} \Delta^\mu{}^\beta \Delta^\nu{}^\alpha - \frac{1}{2} \Delta_{\alpha\beta} \Delta^\mu{}^\nu \} P^\alpha R^\beta \). The bulk viscous force, thermal force and shear viscous force are defined respectively as follows,
\[
X = \partial_u u,
\]
(11)
\[
X^\mu \equiv (\nabla^\mu T - \nabla^\mu P)/n \hbar,
\]
(12)
\[
X_{\mu\nu} = \langle (\partial_\mu u_\nu) \rangle.
\]
(13)

Note that here \( \mu = 0,3 \) describes only the longitudinal components in the strong magnetic field. Also, the deviation function \( \phi_q \) that is the linear combination of these forces can be represented as,
\[
\phi_q = A_k X + B_k^\mu X_{gm} - C_k^\nu X_{\mu\nu},
\]
(14)

where the coefficients can be defined from Eq. (10) as,
\[
A_k = \frac{Q_k}{\{ T \omega_{p_k}/\tau_{eff} \}},
\]
(15)
\[
B_k^\mu = \langle \rho_k^0(\omega_{p_k} - h_k) \rangle \{ T \omega_{p_k}/\tau_{eff} \},
\]
(16)
\[
C_k^\nu = \langle \langle \rho_k^0\phi_k^0 \rangle \rangle \{ -T \omega_{p_k}/\tau_{eff} \},
\]
(17)

with \( h_k \) as the enthalpy per particle of the system that can be defined from the basic thermodynamics and the
total enthalpy is given as, \( h = \sum_{k=0}^{N} h_k \). Following this formalism, we can estimate the viscous coefficients and thermal conductivity of the QGP medium in the strong magnetic field.

1. Shear and bulk viscosity

We can define the pressure tensor from the energy-momentum tensor as in the following way,

\[
P^{\mu\nu} = \Delta^{\mu\nu} T^{...} \Delta^{...}, \tag{18}
\]

We can decompose the \( P^{\mu\nu} \) in equilibrium and non-equilibrium components of distribution function as follows,

\[
P^{\mu\nu} = -P \Delta^{\mu\nu} + \Pi^{\mu\nu}, \tag{19}
\]

where \( \Pi^{\mu\nu} \) is the viscous pressure tensor. Following the definition of \( T^{\mu\nu} \) as in Eq. (7), \( \Pi^{\mu\nu} \) takes the form,

\[
\Pi^{\mu\nu} = \sum_{k=1}^{N} \frac{|q f_k eB|}{2\pi} \nu_k \int \frac{dp_{z_k}}{(2\pi)^2} \frac{|(\mathbf{p}_{k}^{\mu}\mathbf{p}_{k}^{\nu})|}{|\mathbf{p}_{z_k}|} \delta f_k(x, \mathbf{p}_{z_k}) + \sum_{k=1}^{N} \delta_\omega \frac{|q f_k eB|}{2\pi} \nu_k \int \frac{dp_{z_k}}{(2\pi)^2} \frac{|(\mathbf{p}_{k}^{\mu}\mathbf{p}_{k}^{\nu})|}{|\mathbf{p}_{z_k}|} \delta f_k(x, \mathbf{p}_{z_k}). \tag{20}
\]

In the very strong magnetic field, the pressure tensor has different form as compared to the case without magnetic field. This is due to the 1+1-dimensional energy eigenvalues of the quarks and antiquarks. Hence, \( \mu \) and \( \nu \) can be 0 or 3 in the strong magnetic field, describing the longitudinal components of the viscous pressure tensor. The form of viscous pressure tensor in the strong magnetic field is described in the recent works by Tuchin [27] [56]. Magnetized plasma is characterized by five shear components. Among the five coefficients, four components are negligible when the strength of the magnetic field is sufficiently higher than the square of the temperature [57]. Here, we are focusing on the non-negligible longitudinal component of shear and bulk viscous coefficients of the hot QGP medium in the strong magnetic field.

Following [29], the longitudinal shear viscous tensor has the following form,

\[
\Pi^{\mu\nu} = \Pi^{\mu\nu} - \Pi \Delta^{\mu\nu}
\]

\[
= \sum_{k=1}^{N} \frac{|q f_k eB|}{2\pi} \nu_k \int \frac{dp_{z_k}}{(2\pi)^2} \frac{|(\mathbf{p}_{k}^{\mu}\mathbf{p}_{k}^{\nu})|}{|\mathbf{p}_{z_k}|} \delta f_k(x, \mathbf{p}_{z_k}) + \sum_{k=1}^{N} \delta_\omega \frac{|q f_k eB|}{2\pi} \nu_k \int \frac{dp_{z_k}}{(2\pi)^2} \frac{|(\mathbf{p}_{k}^{\mu}\mathbf{p}_{k}^{\nu})|}{|\mathbf{p}_{z_k}|} \delta f_k(x, \mathbf{p}_{z_k}). \tag{21}
\]

Also, the bulk viscous part in the longitudinal direction comes out to be,

\[
\Pi = \sum_{k=1}^{N} \frac{|q f_k eB|}{2\pi} \nu_k \int \frac{dp_{z_k}}{(2\pi)^2} \frac{|(\mathbf{p}_{k}^{\mu}\mathbf{p}_{k}^{\nu})|}{|\mathbf{p}_{z_k}|} \delta f_k(x, \mathbf{p}_{z_k}) + \sum_{k=1}^{N} \delta_\omega \frac{|q f_k eB|}{2\pi} \nu_k \int \frac{dp_{z_k}}{(2\pi)^2} \frac{|(\mathbf{p}_{k}^{\mu}\mathbf{p}_{k}^{\nu})|}{|\mathbf{p}_{z_k}|} \delta f_k(x, \mathbf{p}_{z_k}). \tag{22}
\]

Substituting \( \phi_k \) from Eq. (14) and comparing with the macroscopic definition \( \Pi^{\mu\nu} = 2\eta(\partial \mu \partial \nu) + \zeta \delta^{\mu\nu} \partial \mu u \), we can obtain the expressions of longitudinal viscosity coefficients in the strong field limit. Note that the longitudinal component of shear viscosity, i.e., in the direction of magnetic field, is defined from \( \Pi^{33} [57] \). The longitudinal shear \( \eta \) and bulk viscosity \( \zeta \) are obtained as,

\[
\eta = \sum_{k=1}^{N} \frac{|q f_k eB|}{2\pi} \nu_k \int \frac{dp_{z_k}}{(2\pi)^2} \frac{|(\mathbf{p}_{k}^{\mu}\mathbf{p}_{k}^{\nu})|}{|\mathbf{p}_{z_k}|} \delta f_k(x, \mathbf{p}_{z_k}) + \sum_{k=1}^{N} \delta_\omega \frac{|q f_k eB|}{2\pi} \nu_k \int \frac{dp_{z_k}}{(2\pi)^2} \frac{|(\mathbf{p}_{k}^{\mu}\mathbf{p}_{k}^{\nu})|}{|\mathbf{p}_{z_k}|} \delta f_k(x, \mathbf{p}_{z_k}). \tag{23}
\]

and

\[
\zeta = \sum_{k=1}^{N} \frac{|q f_k eB|}{2\pi} \nu_k \int \frac{dp_{z_k}}{(2\pi)^2} \frac{|(\mathbf{p}_{k}^{\mu}\mathbf{p}_{k}^{\nu})|}{|\mathbf{p}_{z_k}|} \delta f_k(x, \mathbf{p}_{z_k}) + \sum_{k=1}^{N} \delta_\omega \frac{|q f_k eB|}{2\pi} \nu_k \int \frac{dp_{z_k}}{(2\pi)^2} \frac{|(\mathbf{p}_{k}^{\mu}\mathbf{p}_{k}^{\nu})|}{|\mathbf{p}_{z_k}|} \delta f_k(x, \mathbf{p}_{z_k}). \tag{24}
\]

The second term in the Eq. (23) and Eq. (24) gives correction to viscous coefficients due to the quasiparton excitations whereas the first term comes from the usual kinetic theory of bare particles.

2. Thermal conductivity

The heat flow is the difference between the energy flow and enthalpy flow by the particle,

\[
P^\mu = u_\nu T^{...} \Delta^\mu = h N^\nu \Delta^\mu. \tag{25}
\]
In terms of the modified/non-equilibrium distribution function Eq. (25) becomes,

$$\begin{align*}
I^\mu &= u_\nu \Delta^\mu \left[ \sum_{k=1}^{\infty} \frac{|q_f e B|}{2\pi} \frac{\nu_k}{T^2} \int \frac{dp_{z_k}}{(2\pi)^3} \rho^{\nu_q/p_{z_k}^2} \delta f_k(x, p_{z_k}) \right. \\
&\quad - h \Delta^\mu \left[ \sum_{k=1}^{\infty} \frac{|q_f e B|}{2\pi} \frac{\nu_k}{T^2} \int \frac{dp_{z_k}}{(2\pi)^3} \rho^{\nu_q/p_{z_k}^2} \delta f_k(x, p_{z_k}) \right. \\
&\quad + \sum_{k=1}^{\infty} \delta \omega \frac{|q_f e B|}{2\pi} \frac{\nu_k}{T^2} \int \frac{dp_{z_k}}{(2\pi)^3} \rho^{\nu_q/p_{z_k}^2} \delta f_k(x, p_{z_k}) \left. \right],
\end{align*}$$

(26)
in which heat flow retains only non-equilibrium part of the distribution function. After contracting with projection operator and hydrodynamic velocity along with the substitution of $\delta f_k$ from Eq. (8) and comparing with the macroscopic definition of heat flow, we obtain

$$I^\mu = \lambda T \chi_\rho^\mu,$$

(27)
We obtain the thermal conductivity in the strong magnetic field as,

$$\lambda = \left\{ \sum_{k=1}^{\infty} \frac{|q_f e B|}{2\pi} \frac{\nu_k}{T^2} \int \frac{dp_{z_k}}{(2\pi)^3} \rho^{\nu_q/p_{z_k}^2} \frac{(\omega_{pk} - h_k)^2}{\omega_{p_k}^2} \right. \\
- \left. \sum_{k=1}^{\infty} \delta \omega \frac{|q_f e B|}{2\pi} \frac{\nu_k}{T^2} \int \frac{dp_{z_k}}{(2\pi)^3} \rho^{\nu_q/p_{z_k}^2} \frac{h_k(\omega_{pk} - h_k)}{\omega_{p_k}^2} \right. \\
\times \left. |\vec{p}_{z_k}| f_0^2(1 - f_k^0) \right\}. \quad (28)$$

The second term with $\delta \omega$ in the heat flow comes from the $N_q$, which encodes the quasiparticle excitation in the thermal conductivity.

B. Relative momentum and thermal diffusion: The Prandtl number

The Prandtl number $P_r$ for the thermal QGP medium in the strong magnetic field is defined as follows [29],

$$P_r = \frac{\eta c_p}{\rho \lambda}, \quad (29)$$

where $c_p$ is the specific heat at constant pressure and $\rho$ is the mass density. In the strong field limit, $c_p$ can obtain from basic thermodynamics within extended EQPM [23] as,

$$c_p = \frac{16 T^3}{\pi^2} \nu_q PolyLog[4, z_q] + (T^2 \partial_T ln z_q) \frac{T^2}{\pi^2} \nu_q PolyLog[3, z_q] + T^2 (\partial_T ln z_q) \frac{T^3}{\pi^2} \nu_q PolyLog[2, z_q] + T^2 (\partial_T^2 ln z_q) \frac{T^3}{\pi^2} \nu_q PolyLog[3, z_q] - \frac{10}{3\pi^2} |qeB| \frac{T}{\nu_q} PolyLog[2, -z_q] + \frac{5}{3} |qeB| (T^2 \partial_T ln q) \frac{1}{\pi^2} \nu_q ln(1 + z_q) + |qeB| T^2 (\partial_T ln z_q) \frac{T}{\pi^2} \nu_q \frac{z_q}{1 + z_q} + |qeB| T^2 (\partial_T^2 ln z_q) \frac{T}{\pi^2} \nu_q ln(1 + z_q), \quad (30)$$

where $\nu_q = 2(N_c^2 - 1)$. The mass density $\rho$ takes the form,

$$\rho = m_D n_g + m_q (n_q + n_g), \quad (31)$$
in which $m_D$ is the Debye mass and $m_q$ is the thermal (medium) mass of quarks. The number densities $n_q, n_q, n_g$ can be obtained as,

$$n_g = \nu_q \int \frac{d^3p}{(2\pi)^3} f_0^0,$$

(32)
and

$$n_k = \frac{|eB|}{(2\pi)^3} \nu_k \int \frac{dp_{z_k}}{(2\pi)^3} f_0^0, \quad k = q, \bar{q}. \quad (33)$$

Here, the LLL quasiquark momentum distribution is defined as,

$$f_{q/\bar{q}}^{0} = z_q \exp \left( -\beta \sqrt{p_z^2 + m_q^2} \right) \frac{1}{1 + z_q \exp \left( -\beta \sqrt{p_z^2 + m_q^2} \right)}.$$

(34)
Also, the quasigluon distribution function has the form,

$$f_{g}^{0} = z_g \exp \left( -\beta |\vec{p}| \right) \frac{1}{1 + z_g \exp \left( -\beta |\vec{p}| \right)}, \quad (35)$$
in which $|\vec{p}| = E_p$ for gluons.

Medium mass of gluons and quarks

Magnetic field effects are entering into the system through the dispersion relations and the medium (thermal) mass [55]. The medium mass of gluons and quarks ($m_{g/q}$) can be obtained in terms of effective coupling constant. Being an essential input for transport processes, the effective coupling controls the behavior of transport parameters critically. There are several investigations
on the Debye masses (gluon medium mass) of the QGP in presence of magnetic field [53][61]. We recently estimated the EoS/medium dependence on the Debye screening mass and hence the effective coupling constant $\alpha_{eff}$ in the Ref. [23]. Following the same prescription, the Debye mass with finite $\mu$ can be expressed as,

$$m_g \equiv m_D^2 = 4\pi\alpha_s \left[ \frac{6T^2}{\pi^2} \text{PolyLog}[2, z_g] + \frac{3}{\pi^2} \left| \frac{|q_f eB|}{\pi^2} \left( \frac{z_q}{1+z_q} + \frac{\mu^2}{2T^2} \left( \frac{z_q - z_q^2}{1 + z_q^3} \right) \right) \right].$$

(36)

in which $\alpha_s$ is the running coupling constant at finite temperature taken from two-loop QCD gauge coupling constants [62]. Medium mass of dressed quark is defined in terms of the quark/antiquark and gluon momentum distribution function as,

$$m_q^2 = \frac{(N_c^2 - 1)}{4N_c} 4\pi\alpha_s \left( - \int \frac{d^3p}{(2\pi)^3} \partial_\mu f_0^\mu + \frac{|q_f eB|}{(2\pi)^2} \int dp_z \left( \frac{p_z f_0^z}{2} \right) \right).$$

(37)

From the quantities defined above, we estimated the Prandtl number $P_r$ for the QGP in the strong magnetic field.

### III. THERMAL RELAXATIONS IN STRONG MAGNETIC FIELD

Thermal relaxation is the essential dynamical input of the transport processes which counts for the microscopic interaction of the system. In the strong magnetic field, the $2 \rightarrow 2$ quark-antiquark t-channel scattering and the $1 \rightarrow 2$ processes (gluon to quark-antiquark pair) are dominant [30]. The $2 \rightarrow 2$ quark-gluon scattering is sub-leading to quark-antiquark scattering. This is because the thermal density of the quark/antiquark $\sim \frac{|q_f eB|}{T^3}$ dominates over the thermal density of gluons $\sim T^3$ in the regime $\alpha_s \sim eB \ll T^2 \ll |q_f eB|$. The thermal relaxation time $\tau_{eff}$, can be defined from the relativistic transport equation in terms of distribution function in the strong magnetic field $B = B\hat{z}$ as,

$$\frac{df_q}{dt} = C(f_q) = \frac{\delta f_q}{\tau_{eff}}.$$  

(38)

Here, $C(f_q)$ represents the collision integral for the process under consideration. We derived the momentum dependent thermal relaxation time $\tau_{eff}$ for $1 \rightarrow 2$ processes ($k \rightarrow p + p'$, where primed notation for antiquark) from the extended EQPM in and has the following form [52],

$$\tau_{eff}^{-1} = \frac{2\alpha_{eff} C_2 m^2}{\omega_q(1 - f_0^q)} \left( \frac{z_q}{z_q + 1} \right)(1 + f_0^q(E_{p_z})) \ln (T/m).$$

(39)

where $C_2$ is the Casimir factor of the processes and $\alpha_{eff}$ is the effective coupling constant [52] within EQPM and has the form as follows,

$$\frac{\alpha_{eff}}{\alpha_s(T)} = \frac{6T^2}{\pi^2} \text{PolyLog}[2, z_g] \left( T^2 + 3 \left| \frac{q_f eB}{2\pi^2} \right| \right) \left( T^2 + 3 \left| \frac{q_f eB}{2\pi^2} \right| \right)^2. \quad (40)$$

Here, the estimation of thermal relaxation is done with $\mu = 0$ case. The $1 \rightarrow 2$ processes are significant in the regime $p_{z'} \sim 0$ in which the dominant charge carriers have momenta in the order of $T$ [52][63]. Impact of the higher Landau levels and hot QCD medium effects on the relaxation time for $1 \rightarrow 2$ processes is explored in [52].

For $2 \rightarrow 2$ t-channel quark-antiquark scattering, the collision integral in strong magnetic field is defined in the recent work [30],

$$C(f_q) = 8\pi\alpha_{eff}\mathcal{T}_R C_2 \left| \frac{q_f eB}{2\pi} \right| m^2 \frac{\beta}{E_{p_z}} f_0^p(1 - f_0^q) \left| \frac{p_{z'} - p_z}{2} \right|,$$

$$\times \int \frac{dp_{z'}}{(2\pi)^2} \frac{1}{\omega_{q'}} \left| \frac{E_{p_{z'}} - E_{p_z}}{p_{z'} - p_z} \right| f_0^q(1 - f_0^p),$$

$$\times \frac{1}{(N_T^2 + 2(E_{p_z} E_{p_{z'}} - p_{z'}^2))} (\chi(p_{z'}) - \chi(p_z)).$$

(41)

where $\mathcal{T}_R$ is the IR cutoff and the color cover gives the factor of $T_R C_2$ in the square of scattering amplitude for the $2 \rightarrow 2$ quark-antiquark scattering processes. The response function $\chi(p_z)$ (primed notation for antiquark) is defined as

$$\delta f_q = \beta f_0^q(p_z)(1 - f_0^q(p_z)) \chi_q(p_z).$$

(42)

Note that we neglected the factor $e\frac{(p' - p)^2}{eB}$ in the strong magnetic field limit. Here,

$$|E_{p_{z'}} - E_{p_z}| = \frac{m^2}{E_{p_{z'}} + E_{p_z}} \left( p_z^2 - p_{z'}^2 \right).$$

(43)

The $2 \rightarrow 2$ quark-antiquark scattering is dominant in the regime where $p_{z'} \approx p_z$ and $N_T^2 \rightarrow \alpha_{eff} | q_f eB |$ as described in [30]. Hence $N_T^2 R$ always dominates over $(p_{z'} - p_z)^2$ in this regime within the strong field limit. Finally, we end up with,

$$C(f_q) \approx 8\pi\alpha_{eff}\mathcal{T}_R C_2 \left| \frac{q_f eB}{2\pi} \right| m^2 \beta f_0^p(1 - f_0^q) \left| \frac{p_{z'} - p_z}{2} \right|,$$

$$\times \int \frac{dp_{z'}}{(2\pi)^2} \frac{1}{\omega_{q'}} \left| \frac{E_{p_{z'}} - E_{p_z}}{p_{z'} - p_z} \right| f_0^q(1 - f_0^p)$$

$$\times \frac{1}{(\alpha_{eff} | q_f eB |)} (\chi(p_{z'}) - \chi(p_z)).$$

(44)
We have the diffusion expansion within the $p_z' \approx p_z$ approximation,

$$\langle \chi(p_z') - \chi(p_z) \rangle \approx (p_z - p_z') \partial_{p_z'} \langle \chi(p_z') \rangle.$$  \hspace{1cm} (45)

Hence, the collision integral for the quark-antiquark scattering becomes,

$$C(f_q) = \frac{\alpha_{\text{eff}} T_R C g^2 m^2 \beta f_q^0(p_z)(1 - f_q^0(p_z))}{\omega_q} \int dp_{z'} \frac{1}{\omega_{q'}} f_q^0(p_{z'})(1 - f_q^0(p_{z'})) \partial_{p_{z'}} \langle \chi(p_{z'}) \rangle.$$  \hspace{1cm} (46)

Following the Eq. (38), thermal relaxation time $\tau_{\text{eff}}$ takes the following form,

$$\tau_{\text{eff}}^2 = \frac{2}{\pi} \alpha_{\text{eff}} T_R C g^2 m^2 \beta f_q^0(1 - f_q^0)^2 \frac{1}{\omega_q}.$$  \hspace{1cm} (47)

Hot medium effects are entering through the quasiparton distribution function and the effective coupling defined in Eq. (10). Note that the momentum dependence of the relaxation time is significant in the estimation of transport properties as expressed in Eqs. (23), (24) and (28). The effective thermal relaxation time controls the behavior of transport coefficients critically.

**IV. RESULTS AND DISCUSSIONS**

We initiate the discussion with the temperature dependence on the bulk viscosity to entropy density ratio in the presence of strong magnetic field with and without the mean field corrections as shown in Fig. 1 (left panel). The bulk viscosity of the hot QCD medium for $1 \to 2$ processes has been obtained from the basic thermodynamics within the relaxation time approximation in [52]. The present analysis is done by employing the effective covariant kinetic theory using the Chapman-Enskog or Grad’s 14 method. The mean field force term which emerges from the effective theory indeed appear as the mean field corrections to the transport coefficients. The second term in the Eq. (23) describes the mean field contribution to the longitudinal bulk viscosity in the strong magnetic field whereas the first term exactly gives back the leading order contribution of $\zeta/s$ for $1 \to 2$ processes as described in [52] by the substitution of $\tau_{\text{eff}}$ as in Eq. (49). The mean term consists the term $\delta \omega$ which is the temperature gradient of the effective fugacity $\bar{z}_q/q$. At higher temperature, the region over than $T/T_c \sim 2$, the mean field effects are negligible since the effective fugacity behaves as slowly varying function in that regime. Hence, the mean field corrections due to the quasiparticle excitation are significant at temperature region near $T_c$. The longitudinal bulk viscosity to entropy ratio for $1 \to 2$ processes and $2 \to 2$ t-channel quark-antiquark scattering at $|\epsilon B| = 0.3$ GeV$^2$ is depicted in the Fig. 1 (right panel). The behavior of bulk viscosity depends on the term $\frac{1}{\omega_q}(p_{z_{k}}^2 - \omega_{p_{z_{k}}^2})^2$ and the relaxation time $\tau_{\text{eff}}$. The significance of $\frac{1}{\omega_q}(p_{z_{k}}^2 - \omega_{p_{z_{k}}^2})^2$ in longitudinal bulk viscosity is discussed in [52]. On the other hand, thermal relaxation act as the dynamical input for different processes. $\tau_{\text{eff}}$ for $1 \to 2$ processes and $2 \to 2$ t-channel processes are defined in Eq. (49) and (47) respectively. The $1 \to 2$ processes are dominant in the regime $p_{s} \sim 0$ whereas $2 \to 2$ processes are dominant when $p_{z} \sim p_{z}$ in the strong field limit. Here, we are investing the contribution of the two processes separately. The temperature dependence of the $\zeta/s$ in the strong magnetic field indi-
FIG. 2: The temperature dependence of $\eta/s$ (left panel) and thermal conductivity $\lambda$ (right panel) as a function of $T/T_c$ for different processes at $|eB| = 0.3$ GeV$^2$. Lattice data [64] and results in [28] of shear viscosity are in the absence of magnetic field. Behavior of $\lambda/T^2$ is comparing with the results in [28].

FIG. 3: Medium dependence on the transport coefficients in strong magnetic field $|eB| = 0.3$ GeV$^2$ for $1\rightarrow 2$ processes (left panel) and $2\rightarrow 2$ quark-antiquark scattering processes (right panel).

cates its rising behavior near $T_c$. As we already discussed in our previous work, the ratio $\zeta/s$ is enhanced in the presence of strong magnetic field.

The behavior of longitudinal shear viscosity along with the mean field correction for both the processes with $T/T_c$ at $|eB| = 0.3$ GeV$^2$ is plotted in Fig. 2 (left panel). Since the driving force for the longitudinal shear viscosity is in the direction of magnetic field, the Lorentz force does not interfere in the calculation. We compared the results with other parallel approach for the estimation of shear viscosity [28, 64]. Mean field correction to the thermal conductivity is explicitly shown in Eq. (28) in which thermal relaxation incorporates the microscopic interactions. We depicted the temperature behavior of $\lambda/T^2$ for different thermal relaxation time in Fig. 2 (right panel). We compared the temperature behavior of the dimensionless quantity $\lambda/T^2$ with the NJL model results [28].

EoS dependence of the transport coefficients through the effective fugacity is understood from Eqs. (23), (24), and (28). We plotted the medium dependence of the viscous coefficients and thermal conductivity in the presence of magnetic field for both for $1\rightarrow 2$ processes (left panel) and $2\rightarrow 2$ quark-antiquark scattering processes in the Fig. 3. Here, we considered the (2+1) flavor lattice
QCD EoS (LEoS) in the estimation of the transport coefficients. The EoS dependence is entering through the quasiparton distribution function and effective coupling. At very high temperature the medium interactions are negligible and hence the ratios \( \frac{\zeta}{\eta_{\text{Ideal}}} \), \( \frac{\eta}{\eta_{\text{Ideal}}} \), and \( \frac{\lambda}{\lambda_{\text{Ideal}}} \) tends to unity asymptotically.

After obtaining the expression of the longitudinal component of both shear and bulk viscosity in the strong magnetic field, their ratios have been plotted in Fig. 4 for 1 \( \rightarrow \) 2 processes and 2 \( \rightarrow \) 2 t-channel scattering (quark-antiquark). We observed similar behavior for both the processes. Note that we are focusing on the regime where \( \alpha_s eB \ll T^2 \ll eB \). At very high temperature this conditions will not hold, which may have non-negligible contributions from other components of shear viscosity other than the longitudinal one. The interplay of thermal diffusion and momentum diffusion in the longitudinal direction of the magnetic field could be understood in terms of Prandtl number. This number signifies the role of shear viscosity and the thermal conductivity in the sound attenuation in the medium. The ratio is much greater than unity as shown in Fig. 5 indicating the dominance of momentum diffusion as in the case without magnetic field that discussed in [29]. The other non-negligible components of shear viscosity at very high temperature may give corrections to \( P_r \) at high temperature.

![FIG. 4: The ratio of \( \frac{\zeta}{\eta} \) as function of \( T/T_c \) for different processes at \( |eB| = 0.3 \text{ GeV}^2 \).](image)

V. CONCLUSION AND OUTLOOK

In conclusion, we have determined the temperature behavior of the transport parameters such as longitudinal viscous coefficients (both shear and bulk viscosity components) and thermal conductivity in the strong magnetic field background while including the 1 \( \rightarrow \) 2 processes and the 2 \( \rightarrow \) 2, t-channel quark-antiquark scattering processes (dominant in the strong field limit). Thermal relaxation times of both processes are computed in the lowest Landau level approximation. Setting up an effective covariant kinetic theory within EQPM in the strong magnetic field gives the mean field contribution to the transport coefficients. We employed the Chapman-Enskog method in the effective kinetic theory for the computation of transport coefficients. The transport coefficients that have been estimated are influenced by the thermal medium. Hot QCD effects are incorporated through the quasiparton degrees of freedom along with effective coupling and the medium effects are found to be negligible at very high temperature. Furthermore, the relative behavior of these transport coefficients has been investigated in the strong field limit by evaluating the ratio \( \frac{\zeta}{\eta} \) and the Prandtl number \( P_r \). The ratio \( \frac{\zeta}{\eta} \) varies between 11\% – 7\% in the strong field limit for both processes which indicates that the longitudinal bulk viscosity might have visible effects in the hydrodynamic evolution of magnetized QGP. The dominance of momentum diffusion over thermal diffusion is indicated by the larger value of \( P_r \). We compared our results with other parallel works and found them quite consistent with other distinct approaches. Notably, the effects of the mean field term to all the transport coefficients considered here, for the temperatures not very far away from \( T_c \), are seen to be quite visible. Finally, both the mean field contributions and magnetic field effects seen to have significant impact on the transport coefficients of the hot QCD medium.

An immediate future extension of the work is to investigate the aspects of non-linear electromagnetic responses of the hot QGP with the mean field contribution along with the effective description of magnetohydrodynamic waves in the hot QGP medium. In addition, the estimation of all transport coefficients from covariant kinetic theory within the effective fugacity quasiparticle
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