Constraint on Vector Coherent Oscillation
Dark Matter with Kinetic Function

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Abstract

A spatially uniform vector condensate can be formed during inflation if the vector boson is coupled to the inflaton through nontrivial kinetic function. The coherent oscillation of such a massive vector boson is a dark matter candidate. In this paper we consider the case where the vector boson energy density increases during inflation and show that the curvature/isocurvature perturbation gives stringent constraint on this scenario.
1 Introduction

Very light bosonic dark matter (DM) scenarios recently draw lots of attention [1, 2]. Axion-like particle is the most widely studied scenario in this class of models, but a massive vector boson is also a plausible DM candidate. One caveat of the vector DM model is that it is a bit nontrivial to obtain a correct abundance of the present DM compared with the case of scalar field.

So far several mechanisms to produce a correct amount of vector DM have been proposed: vector boson production through the (pseudo-)scalar coupling [3–6], inflationary fluctuation [7], gravitational particle production [8], production through the cosmic string dynamics [9] and coherent oscillation of the vector boson [2, 10, 11].

In this paper we focus on the coherent oscillation scenario. In contrast to a scalar field, a minimal massive vector field cannot develop a condensate during inflation. A possible extension is to introduce a kinetic coupling to the inflaton $\phi$ through the form of $L \sim f^2(\phi)F_{\mu\nu}F^{\mu\nu}$. If the kinetic function $f(\phi)$ has a particular time dependence, the vector boson condensate does not decay during inflation and one can sustain a homogeneous vector field. Later it begins a coherent oscillation and it behaves as non-relativistic matter. Such a scenario was extensively studied in the context of vector curvaton [12–15] and a similar idea has been applied to the vector coherent oscillation DM scenario [11]. This model has an advantage that it does not suffer from the ghost instability [16–19]. In Ref. [11] the case of $f^2(\phi) \propto a(t)^\alpha$ with $\alpha = -4$ or $2$, where $a(t)$ denotes the cosmic scale factor, was considered.
as an illustration and it was shown that the physical vector field value\(^\#1\) can remain constant during inflation for this particular choice. However, there is a priori no reason to choose this parameter and parameter tuning is required for this scenario to realize.

In this paper we mainly study the case of \(\alpha < -4\) or \(\alpha > 2\). In this case the physical vector boson field is amplified during inflation. The vector boson energy density grows and eventually the backreaction of the vector field to the inflaton dynamics becomes important. It is known that in such a case the background vector boson can support another inflation regime, called the anisotropic inflation [20–22]. The vector boson energy density is saturated at some value during the anisotropic inflation and we will consider a possibility that this vector boson will later become a coherent oscillation DM.

In Sec. 2 we study the dynamics of the inflaton and vector boson of the homogeneous mode. It is found that the vector boson in this scenario can indeed have a correct abundance as total DM. In Sec. 3 we discuss constraints on this scenario through the properties of the curvature and isocurvature perturbation. Actually they give very stringent constraint and we are left with only a limited possibility as a consistent DM scenario. Sec. 4 is devoted to conclusions and discussion.

2 Dynamics of vector condensate

We consider the following action for a massive vector boson \(A_M\) and inflaton \(\phi\):

\[
S = \int d^3 x dt \sqrt{-g} \left[ -f^2(\phi) \frac{1}{4} F_{MN} F^{MN} - \frac{h^2(\phi)}{2} m_A^2 A_M A^M - \frac{1}{2} \partial_M \phi \partial^M \phi - V(\phi) \right]. \tag{1}
\]

We have introduced kinetic function \(f(\phi)\) and mass function \(h(\phi)\), whose functional form will be given later. Taking account of the effect of the background homogeneous vector field, whose direction is taken to be \(x\) direction without loss of generality, i.e. \(A_i = (A, 0, 0)\), the metric is taken to be the so-called Bianchi type-I form:

\[
d s^2 = -dt^2 + a^2(t) \left[ e^{-4\sigma(t)} dx^2 + e^{2\sigma(t)} (dy^2 + dz^2) \right], \tag{2}
\]

where \(a(t)\) is the cosmic scale factor and \(\sigma(t)\) represents the anisotropic expansion. For a while we neglect the anisotropy, i.e. \(\sigma = 0\), by assuming that the energy density of the vector field is much smaller than the inflaton. This will be justified later.

It is often useful to rescale the vector field as \(A_\mu = (a A_0, A_i)\) and use the conformal time \(d\tau = dt/a\) to obtain the action

\[
S = \int d\tau d^3 x \left[ -f^2 \frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{a^2 h^2}{2} m_A^2 \eta^{\mu\nu} A_\mu A_\nu - \frac{V}{\phi} d\phi \right]. \tag{3}
\]

Below we consider the special form of the kinetic function:

\[
f(\phi) = \exp \left( -\frac{\gamma}{2 M_P^2} \int \frac{V}{\phi} d\phi \right), \tag{4}
\]

\(^\#1\) The “physical” vector field is defined later below Eq. (10).
where $V_\phi \equiv \partial V / \partial \phi$. A particular example is the chaotic inflation:

$$f(\phi) = \exp \left( - \frac{\gamma}{4n} \frac{\phi^2}{M_P^2} \right), \quad V(\phi) = \frac{\lambda \phi^n}{n}. \quad (5)$$

For the new (hilltop) inflation model, it is given by

$$f(\phi) = \exp \left( - \frac{\gamma}{4n(n-2)} \frac{v^n}{M_P^2 \phi^{n-2}} \right), \quad V(\phi) = \Lambda^4 \left[ 1 - \left( \frac{\phi}{v} \right)^n \right]^2. \quad (6)$$

They lead to the scaling of $f^2 \propto a^\gamma$ during the standard slow-roll inflation when the effect of backreaction of the vector field to the inflaton dynamics is negligible. Note also that $f \simeq 1$ soon after inflation ends. The case of $\gamma = -4$ and 2 have been discussed in Ref. [11]. We will consider the case of $\gamma < -4$ and $\gamma > 2$. For the moment we do not assume any specific functional form of $h(\phi)$ except that it soon approaches to $h(\phi) \to 1$ after inflation ends.

### 2.1 Dynamics during inflation

Let us describe the vector and inflaton dynamics during inflation neglecting the spatial fluctuation. We follow the analysis given in Refs. [20–22]. The equation of motion is given by

$$a \frac{\partial}{\partial t} \left( a f^2 \dot{A} \right) + a^2 h^2 m_A^2 A = 0, \quad (7)$$

$$\ddot{\phi} + 3H \dot{\phi} + V_\phi - \frac{1}{a^2 f f_\phi} \dot{A}^2 = 0, \quad (8)$$

where $f_\phi \equiv \partial f / \partial \phi$. The Hubble parameter $H = \dot{a} / a$ is given by

$$3M_P^2 H^2 = \rho_\phi + \rho_A, \quad (9)$$

with $M_P$ being the reduced Planck scale and $\rho_\phi$ the inflaton energy density. The vector boson energy density $\rho_A$ is given by

$$\rho_A = \frac{1}{2a^2} \left( f^2 \dot{A}^2 + h^2 m_A^2 A^2 \right) = \frac{1}{2} \left\{ \frac{\dot{A}}{A} + \left( H - \frac{f}{f_\phi} \right) \frac{A}{f} \right\}^2 + \frac{h^2 m_A^2}{f^2 A^2}, \quad (10)$$

where we have defined the “physical” vector field as $\overline{A} \equiv f A / a$ [11].

Let us consider the case where the vector boson mass term can be safely ignored. Then we immediately obtain

$$a f^2 \dot{A} = \text{const.} \quad (11)$$

Supposing the scaling $f^2 \propto a^\alpha$ with $\alpha$ being a numerical constant, we schematically obtain

$$A = C_1 + C_2 a^{-(1+\alpha)}, \quad (12)$$
during inflation where \( C_1 \) and \( C_2 \) are constants. As soon explained below, \( \alpha = \gamma \) when the vector energy density is negligible but \( \alpha \) can take different value from \( \gamma \) when the backreaction is important. The physical field roughly behaves as

\[
\overline{A} \propto a^{(|1+\alpha|-3)/2} = \begin{cases} 
  a^{\alpha/2-1} & \text{for } 1 + \alpha > 0 \\
  a^{-\alpha/2-2} & \text{for } 1 + \alpha \leq 0
\end{cases}
\]  

(13)

Note that, although \( A \) is increasing for \( \alpha < -4 \) and \( \alpha > 2 \), the \( C_1 \) term does not contribute to the kinetic energy in (10). Since the \( C_1 \) term is dominant for \( 1 + \alpha > 0 \), actually the energy density is actually decreasing for \( \alpha > 2 \). In both cases the vector energy density scales as \( \rho_A \propto f^{-2}a^{-4} \propto a^{-\alpha-4} \). Below we consider the case of \( \gamma < -4 \) and \( \gamma > 2 \) separately.

### 2.1.1 \( \gamma < -4 \)

As studied in Ref. [11], \( \rho_A \) remains constant for \( \gamma = -4 \) that ensures the establishment of the vector boson homogeneous condensate during inflation. On the other hand, it increases during inflation for \( \gamma < -4 \) and hence eventually the backreaction will become important. The inflaton equation of motion is written as

\[
\ddot{\phi} + 3H\dot{\phi} + V_{\phi} \left( 1 + \frac{\gamma}{2\epsilon_V} \frac{\rho_A}{V} \right) = 0,
\]

(14)

where \( \epsilon_V \equiv M_p^2 (V_{\phi}/V)^2/2 \) is the slow-roll parameter.\(^\#2\) Thus it is seen that if the vector boson energy density satisfies \( \rho_A \ll (2\epsilon_V/|\gamma|)V \), the effect of the vector boson on the inflaton dynamics is safely neglected. For \( \gamma < -4 \), however, \( \rho_A \) increases during inflation and \( \rho_A \) will become comparable to \( (2\epsilon_V/|\gamma|)V \). It is expected that \( \dot{\phi} \) will slow down at this stage since the parenthesis in the last term of Eq. (14) will approach to zero, which effectively “flattens” the inflaton potential. Correspondingly the time evolution of the function \( f(\phi) \) also changes so that \( \rho_A \) approximately remains constant: \( f^{-2}a^{-4} \sim \text{const} \). This requires the following relation:

\[
\dot{\phi} \simeq \frac{4HM_p^2 \phi}{\gamma V} = \frac{4}{3\gamma H} V_{\phi}.
\]

(15)

In order for this solution to be consistent with slow-roll equation of (14), the energy density should satisfy

\[
\frac{\rho_A}{\rho_{\phi}} = -2\epsilon_V \frac{\gamma + 4}{\gamma^2} \equiv R_A.
\]

(16)

Here \( \rho_\phi \simeq V \) is the inflaton energy density. To summarize, the slow-roll inflaton dynamics is described by

\[
3H\dot{\phi} \simeq \begin{cases} 
  -V_{\phi} & \text{for } \frac{\rho_A}{\rho_{\phi}} \ll R_A, \\
  \frac{4}{\gamma}V_{\phi} & \text{for } \frac{\rho_A}{\rho_{\phi}} \simeq R_A
\end{cases}
\]

(17)

\(^\#2\) Note that \( \epsilon_H \equiv -\dot{H}/H^2 = -(4/\gamma)\epsilon_V. \)
One can see that the potential is effectively flattened by a factor $-4/\gamma$ due to the vector backreaction. This second case is a slow-roll inflation supported by the vector field and it is called the anisotropic inflation because the vector field condensate implies a preferred direction. Even if the initial vector energy density is negligibly small, it will be exponentially amplified during inflation and it enters the regime of anisotropic inflation, although still the vector energy density is much smaller than the inflaton itself at this stage.

Fig. 1 shows the result of numerical solution of the equation of motion of the inflaton (8) and vector boson (7) for the inflaton potential $V = m_\phi^2 \phi^2 / 2$ and $\gamma = -5$. Time evolution of the energy density of the inflaton ($\rho_\phi$) and vector boson ($\rho_A$) normalized by $m_\phi^2 M_P^2$ are shown in the left panel. We have taken $\phi = 20 M_P$ and $\vec{A} = 10^{-4} m_\phi M_P$ for (a) and $10^{-6} m_\phi M_P$ for (b) as initial conditions and the massless limit $m_A \to 0$. Time evolution of the ratio $\rho_A/\rho_\phi$ compared with $R_A$ (16) is shown in the right panel. Parameters are the same as the left panel. Similarly, Fig. 2 shows the result of numerical calculation for the new inflation model (6) with $n = 6$. We have taken $\gamma = -5$, $v = M_P$, $\phi = 0.3 \phi_{\text{end}}$ and $\vec{A} = 10^{-6} M_P$ for (a) and $10^{-8} M_P$ for (b) as initial condition. Here $\phi_{\text{end}}$ denotes the inflaton field value at which inflation ends: $2 n (n-1) (\phi_{\text{end}} / v)^{n-2} = v^2 / M_P^2$. In both cases it is clearly seen that the ratio $\rho_A/\rho_\phi$ approaches to the value given by $R_A$ (16) independently of the initial condition.

Note that in the calculation performed for these figures the anisotropic inflation regime does not last for very long time, but it is an artifact of the choice of the initial condition. If, for example, the calculation starts from much larger (smaller) inflaton field value for the chaotic (new) inflation model, the vector boson density is saturated at much earlier time and the anisotropic inflation lasts for much longer time (say, much longer than 60 e-folds). We do not go into details of the problem of initial condition since it is related with the dynamics before the “observable” inflation happens and just treat the initial condition as free parameters.\#3

So far we have ignored the anisotropic expansion. The equation for $\Sigma \equiv \dot{\sigma}$ is given by

$$\dot{\Sigma} + 3 H \Sigma = \frac{2 \rho_A}{3 M_P^2}. \quad (18)$$

It is expected that $\Sigma$ converges to a nearly constant value

$$\frac{\Sigma}{H} \sim \frac{2 \rho_A}{3 \rho_\phi} = - \frac{4 \epsilon_V \gamma + 4}{3 \gamma^2} = \frac{\epsilon_H \gamma + 4}{3 \gamma}. \quad (19)$$

It is suppressed by the slow-roll parameter $\epsilon_H$. Therefore the homogeneous dynamics is not much affected by the inclusion of the anisotropic expansion.

2.1.2 $\gamma > 2$

In this case, as far as the vector boson mass $m_A$ is negligible, the vector energy density $\rho_A$ (or its kinetic part) decreases as $\rho_A \propto a^{-\gamma-2}$ during inflation and hence it rapidly approaches

\#3 It is possible that the long wavelength vector perturbation accumulates to constitute a “homogeneous” mode if the total duration of inflation is long enough [24, 25].

5
Figure 1: Numerical results for chaotic inflation model (5) with $n = 2$. (Left) Time evolution of the energy density of the inflaton ($\rho_\phi$) and vector boson ($\rho_A$) normalized by $m_\phi^2 M_P^2$. We have taken $\gamma = -5$ and $A = 10^{-4} m_\phi M_P$ for (a) and $10^{-6} m_\phi M_P$ for (b) as initial condition. (Right) Time evolution of the ratio $\rho_A/\rho_\phi$ compared with $R_A$. Parameters are the same as the left panel.

Figure 2: Numerical results for new inflation model (6) with $n = 6$. (Left) Time evolution of the energy density of the inflaton ($\rho_\phi$) and vector boson ($\rho_A$) normalized by $\Lambda^4$. We have taken $\gamma = -5$ and $A = 10^{-6} M_P$ for (a) and $10^{-8} M_P$ for (b) as initial condition. (Right) Time evolution of the ratio $\rho_A/\rho_\phi$ compared with $R_A$. Parameters are the same as the left panel.
to zero, while the $\mathcal{A}$ increases as $\mathcal{A} \propto a^{\gamma/2-1}$. Since $\rho_A$ is negligible, there is no backreaction of the vector field to the inflaton and the anisotropic inflation does not occur.

On the other hand, the condition that the vector boson mass is negligible is written as $hm_A/f \ll H_{\text{inf}}$ at least during the last 60 e-foldings of inflation. For $h = 1$ for example, it gives a constraint on the vector boson mass as

$$m_A \ll e^{-30\gamma} H_{\text{inf}} \sim 10^{-13\gamma} H_{\text{inf}}. \quad (20)$$

Then the vector boson energy density at the end of inflation is bounded as

$$\frac{|\rho_A|}{\rho_\phi} \bigg|_{\tau_{\text{end}}} \ll 10^{-26\gamma}. \quad (21)$$

If the total duration of inflation is much longer than 60 e-foldings, the constraint becomes much more stringent. If this condition is violated, the vector boson mass would make rapid decay of the amplitude $\mathcal{A}$ during inflation.

Another choice is $h(\phi) = f(\phi)$. In this case, the only requirement is $m_A \ll H_{\text{inf}}$. Thus the upper bound on the vector energy density at the end of inflation is just

$$\frac{|\rho_A|}{\rho_\phi} \bigg|_{\tau_{\text{end}}} \approx \frac{m_A^2 \mathcal{A}^2}{6H_{\text{inf}}^2 M_P^2} \ll \left( \frac{\mathcal{A}}{M_P} \right)^2. \quad (22)$$

Fig. 3 shows the time evolution of the energy density of the inflaton ($\rho_\phi$) and vector boson ($\rho_A$) for $\gamma = 5/2$ and $h(\phi) = f(\phi)$ for chaotic inflation model (5) with $n = 2$ (left) and new inflation model (6) with $n = 6$ (right). The vector boson mass is taken to be $m_A = 10^{-5} H_{\text{inf}}$ (left) and $m_A = 10^{-3} H_{\text{inf}}$ (right). As initial condition, we have taken $\mathcal{A} = 10^{-4} M_P$, $\phi = 15 M_P$ (left) and $\phi = 0.4 \phi_{\text{end}}$ (right). It is seen that first $\rho_A$ decreases exponentially but later the mass term begins to dominate and it increases until the end of inflation.

### 2.2 Dynamics after inflation

After inflation ends, the kinetic function and mass function is taken to be $f \simeq h \simeq 1$. The inflaton coherent oscillation behaves as non-relativistic matter until the reheating is completed at $H = \Gamma_{\phi}$ where $\Gamma_{\phi}$ denotes the inflaton decay width. After the completion of reheating, the radiation-dominated universe begins. This thermal history is described by the equation of state parameter $w$, which takes $w = 0$ ($1/3$) for the matter (radiation)-dominated era.

The equation of motion of the vector field $\mathcal{A} \equiv fA/a$ is given by

$$\ddot{\mathcal{A}} + 3H\dot{\mathcal{A}} + \left( m_A^2 + \frac{1-3w}{2} H^2 \right) \mathcal{A} = 0. \quad (23)$$
Figure 3: Time evolution of the energy density of the inflaton ($\rho_\phi$) and vector boson ($\rho_A$) for $\gamma = 5/2$ and $h(\phi) = f(\phi)$ for chaotic inflation model (5) with $n = 2$ (left) and new inflation model (6) with $n = 6$ (right). The vector boson mass is taken to be $m_A = 10^{-5}H_{\text{inf}}$ (left) and $m_A = 10^{-3}H_{\text{inf}}$ (right). As initial condition, we have taken $\lambda = 10^{-4}M_P$, $\phi = 15M_P$ (left) and $\phi = 0.4\phi_{\text{end}}$ (right).

For $H \gg m_A$, we find the solution to this equation as

$$\lambda = d_1a^{-1} + d_2a^{(3w-1)/2},$$

with $d_1$ and $d_2$ being some constants. Notice that the $d_1$ term corresponds to the solution $A = \text{const}$. It is consistent with the solution during inflation for $\gamma > 2$, which corresponds to the $C_1$ term in (12) during inflation. For $\gamma < -4$, on the other hand, the $d_2$ term solution applies. Below we consider the case of $\gamma < -4$ and $\gamma > 2$.

2.2.1 $\gamma < -4$

For $\gamma < -4$, taking the $d_2$ term in (24), we obtain $\lambda \propto a^{-1/2}$ for $w = 0$ and $\lambda \propto a^0$ for $w = 1/3$, which means $\rho_A \propto a^{-3}$ for both cases. This behavior is seen in Figs. 1 and 2. On the other hand, for $H \ll m_A$, the equation is the same as the minimal scalar field: it begins coherent oscillation at $H \sim m_A$ and behaves as non-relativistic matter thereafter. Hence we have $\rho_A \propto a^{-3}$ for $H \ll m_A$ and it is a candidate of DM.

Keeping this in mind, we can now evaluate the vector DM abundance. First we consider the case of $\Gamma_\phi > m_A$. In this case the final energy density to the entropy density ($s$) ratio is evaluated as

$$\rho_A/s = \left(\frac{\rho_A}{\rho_\phi}\right)_{H=\Gamma_\phi} \left(\frac{\rho_\phi}{s}\right)_{H=m_A} = \frac{3R_A}{4} \left(\frac{90}{\pi^2g_*}\right)^{1/4} \left(\frac{\Gamma_\phi}{H_{\text{inf}}}\right)^{2/3} \sqrt{m_A M_P},$$

(25)
where $\rho_\phi$ collectively denotes the inflaton energy density or the radiation energy density produced by the inflaton decay. For the other case $\Gamma_\phi < m_A$, we have

$$\frac{\rho_A}{s} = \left(\frac{\rho_A}{\rho_\phi}\right)_{H=m_A} \left(\frac{\rho_\phi}{s}\right)_{H=\Gamma_\phi} = \frac{3R_A}{4} \left(\frac{m_A}{H_{\text{inf}}}\right)^{2/3} T_R,$$  

with $T_R$ being the reheating temperature. Numerically they are summarized as

$$\frac{\rho_A}{s} \simeq \begin{cases} 
3.7 \times 10^{-10} \text{GeV} & \left(\frac{R_A}{0.1}\right) \left(\frac{m_A}{10^{-8} \text{GeV}}\right)^{1/2} \left(\frac{10^{14} \text{GeV}}{H_{\text{inf}}}\right)^{2/3} \left(\frac{T_R}{10^6 \text{GeV}}\right)^{4/3} & \text{for } m_A < \Gamma_\phi, \\
3.5 \times 10^{-10} \text{GeV} & \left(\frac{R_A}{0.1}\right) \left(\frac{m_A}{1 \text{GeV}}\right)^{2/3} \left(\frac{10^{14} \text{GeV}}{H_{\text{inf}}}\right)^{2/3} \left(\frac{T_R}{10 \text{GeV}}\right) & \text{for } m_A > \Gamma_\phi. 
\end{cases}$$  

(26)

(27)

(27)

It is consistent with the observed DM abundance ($\simeq 4 \times 10^{-10} \text{GeV}$ in terms of the energy to entropy density ratio) for wide parameter ranges.

### 2.2.2 $\gamma > 2$

For $\gamma > 2$, the vector energy density at the end of inflation is bounded as (21). After inflation, the vector energy density scales as $\rho_A \simeq m_A^2 A_t^2 \propto a^{-2}$. For $h = 1$, for example, assuming the limit of instant reheating, i.e., the radiation dominated universe starts just after inflation, the final vector boson abundance is evaluated as

$$\rho_A \ll 10^{-26} \gamma \frac{H_{\text{inf}} M_P^{1/2}}{m_A^{1/2}} \lesssim 10^{-13} \text{GeV} \left(\frac{10^{-22} \text{eV}}{m_A}\right)^{1/2} \left(\frac{H_{\text{inf}}}{10^{14} \text{GeV}}\right),$$  

(28)

where we have taken $\gamma = 2$ when evaluating the most right hand side. If the reheating is delayed, there is a further suppression factor of $(\Gamma_\phi / H_{\text{inf}})^{1/3}$. Since the DM should be heavier than $\sim 10^{-22} \text{eV}$ from the galactic structure [23], we conclude that it cannot explain total DM abundance as far as the standard thermal history is assumed in the early universe. It may be possible that the universe enters the kination regime before the completion of the reheating. In such a case the DM abundance can be enhanced, but we do not pursue this possibility in this paper.

For another choice $h(\phi) = f(\phi)$, as mentioned in Sec. 2.1.2, there is no strong suppression for the vector energy density at the end of inflation. The physical field $A$ increases as $A \propto a^{(\gamma-2)/2}$ during inflation and one can take $A_{\text{end}} = \overline{A}(\tau_{\text{end}})$ as a free parameter in this case. Taking the scaling $\rho_A \propto a^{-2}$ after inflation until $H \sim m_A$, as mentioned above and actually seen in Fig. 3, the final abundance is

$$\frac{\rho_A}{s} \simeq \frac{1}{8} \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \left(\frac{A_{\text{end}}}{M_P}\right)^2 \left(\frac{\Gamma_\phi}{H_{\text{inf}}}\right)^{4/3} \left(\frac{m_A}{\Gamma_\phi}\right) \sqrt{m_AM_P},$$  

(29)
for $m_A < \Gamma_\phi$, and

$$\rho_A \approx \frac{T_R}{8} \left( \frac{\bar{A}_{\text{end}}}{M_P} \right)^2 \left( \frac{m_A}{H_{\text{inf}}} \right) \frac{4/3}{},$$

(30)

for $m_A > \Gamma_\phi$. Numerically we have

$$\rho_A \approx \begin{cases} 
2.5 \times 10^{-11} \text{GeV} & \left( \frac{T_R}{10^9 \text{GeV}} \right)^{2/3} \left( \frac{m_A}{1 \text{ GeV}} \right)^{3/2} \left( \frac{10^{14} \text{ GeV}}{H_{\text{inf}}} \right)^{4/3} \left( \frac{\bar{A}_{\text{end}}}{M_P} \right)^2 \\
1.3 \times 10^{-10} \text{GeV} & \left( \frac{T_R}{10^7 \text{GeV}} \right)^{4/3} \left( \frac{m_A}{10^2 \text{ GeV}} \right)^{4/3} \left( \frac{10^{14} \text{ GeV}}{H_{\text{inf}}} \right)^{4/3} \left( \frac{\bar{A}_{\text{end}}}{M_P} \right)^2 
\end{cases}$$

(31)

for $m_A < \Gamma_\phi$. For $m_A > \Gamma_\phi$, we have

$$\rho_A \approx \begin{cases} 
2.5 \times 10^{-11} \text{GeV} & \left( \frac{T_R}{10^9 \text{GeV}} \right)^{2/3} \left( \frac{m_A}{1 \text{ GeV}} \right)^{3/2} \left( \frac{10^{14} \text{ GeV}}{H_{\text{inf}}} \right)^{4/3} \left( \frac{\bar{A}_{\text{end}}}{M_P} \right)^2 \\
1.3 \times 10^{-10} \text{GeV} & \left( \frac{T_R}{10^7 \text{GeV}} \right)^{4/3} \left( \frac{m_A}{10^2 \text{ GeV}} \right)^{4/3} \left( \frac{10^{14} \text{ GeV}}{H_{\text{inf}}} \right)^{4/3} \left( \frac{\bar{A}_{\text{end}}}{M_P} \right)^2 
\end{cases}$$

(32)

It is possible to have a correct vector DM abundance in this case.

### 3 Constraint from curvature and isocurvature perturbation

We have considered the dynamics of homogeneous mode of the vector boson in the previous section. Let us consider fluctuation of the inflaton and vector boson generated during inflation and its observational consequences.

As shown in the previous section, when the vector energy density is negligible, the standard slow-roll inflation driven by just an inflaton field happens. We call this as “isotropic” regime. Then the vector boson backreaction to the inflaton becomes important if $\gamma < -4$ and the anisotropic inflation regime follows. The e-folding number of the anisotropic regime, $N_{\text{ani}}$, depends on the initial condition. If $N_{\text{ani}} \gtrsim 60$ fluctuations of all the observable scale must arise during the anisotropic inflation, while if $N_{\text{ani}} \lesssim 60$ the large scale fluctuations in the present universe may arise from the isotropic regime and only the small scale fluctuations may be affected by the anisotropic inflation. Thus we consider three cases for $\gamma < -4$:

- (i) There is no anisotropic inflation regime ($N_{\text{ani}} = 0$).
- (ii) Anisotropic inflation regime is not long enough ($0 < N_{\text{ani}} \lesssim 60$).
- (iii) Anisotropic inflation regime lasts long enough ($N_{\text{ani}} \gtrsim 60$).

We note that there is only the case (i) for $\gamma > 2$.

#### 3.1 Isocurvature perturbation

It is convenient to move to the Fourier space. The vector boson can be decomposed into the transverse and longitudinal mode as $\vec{A}(k) = \vec{A}_T/f + \kappa A_L/g$, where the transverse mode
satisfies $\vec{k} \cdot \vec{A}_T = 0$ and $g \equiv f \sqrt{a^2 m_A^2 / (a^2 m_A^2 + k^2)}$. The action is written as

$$S = S_T + S_L,$$

$$S_T = \int \frac{d^3 k d\tau}{(2\pi)^3} \frac{1}{2} \left[ |\vec{A}_T(k)|^2 - \left( k^2 + \frac{a^2 h^2 m_A^2}{f^2} - \frac{f''}{f} \right)|\vec{A}_T(k)|^2 \right],$$

$$S_L = \int \frac{d^3 k d\tau}{(2\pi)^3} \frac{1}{2} \left[ |A_L(k)|^2 - \left( k^2 + \frac{a^2 h^2 m_A^2}{f^2} - \frac{g''}{g} \right)|A_L(k)|^2 \right].$$

Below we consider the isocurvature fluctuation of the vector boson and derive constraint on the vector DM scenario. For this purpose, it is enough to consider only the transverse mode because the magnitude of longitudinal fluctuation is at most the same order of the transverse one. As studied in Ref. [11], a peculiar evolution of the longitudinal mode for $k/a \ll h m_A / f$ makes suppression for the longitudinal power at large scale. More concretely, if the present cosmological horizon scale, $k_0^{-1}$, is longer than $(a_{end} m_A)^{-1}$, the large scale fluctuation is dominated by the transverse one while the transverse and longitudinal power are comparable if $k_0^{-1} < (a_{end} m_A)^{-1}$. The condition $k_0^{-1} < (a_{end} m_A)^{-1}$ is roughly rewritten as

$$m_A \lesssim 0.1 \text{ eV} \left( \frac{H_{\text{inf}}}{10^{14} \text{ GeV}} \right)^{2/3} \left( \frac{10^9 \text{ GeV}}{T_R} \right)^{1/3}. \quad (35)$$

If the condition (35) is satisfied, the transverse and longitudinal modes have the same power at the observable scale. #4 On the other hand, if the condition (35) is violated, the CDM isocurvature perturbation on the present cosmological scale is (maximally) statistically anisotropic. #5 In any case, the longitudinal power is at most the same order of the transverse one and hence below we only consider the transverse fluctuation.

The equation of motion of the transverse mode during inflation is given by

$$\vec{A}_T''(k) + \left( k^2 + \frac{a^2 h^2 m_A^2}{f^2} - \frac{\alpha(2 + \alpha)}{4} H^2 \right) \vec{A}_T(k) = 0. \quad (36)$$

where we substituted $f^2 \propto a^\alpha$. As explained in Sec. 2.1, for $\gamma < -4$, the standard slow-roll inflation happens when the vector boson energy density is negligible and in this regime we have $\alpha = \gamma$. However, the inflationary universe will eventually enter the regime of anisotropic inflation supported by the vector condensate and in this regime we have $\alpha = -4$ independently of the value of $\alpha$. Let us define $\tau_{ani}$ as the conformal time when the anisotropic inflation regime starts. We have

$$\alpha = \begin{cases} \gamma & \text{for } \tau < \tau_{ani} \\ -4 & \text{for } \tau > \tau_{ani} \end{cases}. \quad (37)$$

#4 They cancel with each other for the statistically anisotropic component of the CDM isocurvature perturbation.

#5 In Ref. [11] it was implicitly assumed that the condition (35) is violated. Actually, however, for very light vector DM this condition is naturally satisfied.
Thus the property of fluctuations of the observable scale depends on whether the present cosmological scale, \( k_0^{-1} \), is longer or shorter than \( |\tau_{\text{ani}}| \). On the other hand, for \( \gamma > 2 \), the vector backreaction never becomes effective and there is no such anisotropic inflation regime, and hence \( \alpha = \gamma \) throughout the inflation regime.

Neglecting the mass term, i.e., assuming \( m_A/f \ll H_{\text{inf}} \), the solution to this equation is given by

\[
A_{T,\lambda}(k, \tau) = e^{i(2\nu+1)\pi/4} \frac{1}{\sqrt{2k}} \sqrt{-\pi \kappa \tau} H^{(1)}_{\nu}(-k\tau), \quad \nu \equiv \frac{|1+\alpha|}{2},
\]

where \( \lambda = \pm \) denotes the two transverse helicity states and \( H^{(1)}_{\nu}(x) \) is the Hankel function of the first kind. The limiting form in the subhorizon and superhorizon limit are given by

\[
A_{T,\lambda}(k, \tau) \simeq \begin{cases} 
\frac{1}{\sqrt{2k}} e^{-ik\tau} & \text{for } k/a \gg H_{\text{inf}} \\
\frac{1}{\sqrt{2k}} \frac{\Gamma(\nu)}{\Gamma(3/2)} \left( \frac{2}{-k\tau} \right)^{\nu-1/2} & \text{for } k/a \ll H_{\text{inf}}.
\end{cases}
\]

Actually we have chosen the overall coefficient so that the mode function coincides with the Minkowski form in the short wavelength limit. Thus it evolves as \( a^{\nu-1/2} \) after the horizon exit. The “physical” field \( \tilde{A}_{T,\lambda} = A_{T,\lambda}/a \) evolves as \( a^{\nu-3/2} \). It is the same scaling as that of the homogeneous mode \( \tilde{A} (13) \), as expected.

The transverse power spectrum is defined as

\[
\langle \tilde{A}_T(k) \tilde{A}^*_T(k') \rangle = \frac{4\pi^2 a^2}{k^3} P_T(k)(2\pi)^3 \delta(k - k').
\]

The shape of the spectrum depends on the case (i)–(iii). First, for the case (i) all the cosmologically relevant scales correspond to the modes that exit the horizon during the standard slow-roll inflation. Thus

\[
P_T(k) = \left( \frac{H_{\text{inf}}}{2\pi} \right)^2 \left( \frac{\Gamma(\nu)}{\Gamma(3/2)} \right)^2 \left( \frac{2a H_{\text{inf}}}{k} \right)^{2\nu-3}.
\]

Therefore, for \( \nu > 3/2 \ (\alpha < -4) \), the spectrum is red tilted. For the case (ii), large scale fluctuations \( k < k_{\text{ani}} \) experience both the standard slow-roll inflation and anisotropic inflation regime. Thus

\[
P_T(k) = \begin{cases} 
\left( \frac{H_{\text{inf}}}{2\pi} \right)^2 \left( \frac{\Gamma(\nu)}{\Gamma(3/2)} \right)^2 \left( \frac{2a_{\text{ani}} H_{\text{inf}}}{k} \right)^{2\nu-3} & \text{for } k < k_{\text{ani}} \\
\left( \frac{H_{\text{inf}}}{2\pi} \right)^2 \left( \frac{2a_{\text{ani}} H_{\text{inf}}}{k} \right)^{2\nu-3} & \text{for } k > k_{\text{ani}}.
\end{cases}
\]

For the case (iii), all the cosmologically relevant scales correspond to the modes that exit the horizon during the anisotropic inflation at which \( \nu = 3/2 \), hence

\[
P_T(k) = \left( \frac{H_{\text{inf}}}{2\pi} \right)^2.
\]
The typical magnitude of the fluctuation with a comoving wavenumber $k$ is given by $\sqrt{\mathcal{P}_T(k)}$. The isocurvature perturbation of the vector field is then given by

$$S_A(k) = \frac{\delta \rho_A(k)}{\rho_A} \bigg|_{H=m_A} \frac{2\sqrt{\mathcal{P}_T(k)}}{A} \bigg|_{a_{\text{end}}},$$

where the most right hand side is evaluated at the end of inflation $a = a_{\text{end}}$. The observational constraint is $S_A \lesssim 9 \times 10^{-6}$ at the present cosmological scale [26].

### 3.2 Curvature perturbation

Here we briefly describe the statistical anisotropy in the curvature perturbation. Since there is a vector background during inflation, it can affect the statistical properties of the curvature perturbation. In particular, the power spectrum of the curvature perturbation may have the following quadrupolar asymmetric form:

$$P_{\zeta}(\vec{k}) = P_0^0(\zeta)(1 + g^* \sin^2 \theta_k),$$

where $P_0^0(\zeta)$ is the isotropic part of the dimensionless curvature perturbation power spectrum, normalized as $P_0^0(\zeta) \simeq 2.1 \times 10^{-9}$ at the present horizon scale [26], $\theta_k$ is the angle between the wave vector $\vec{k}$ and the preferred direction and $g^*$ represents the magnitude of the statistical anisotropy. The observational constraint on this type of quadrupolar asymmetry is $|g^*| \lesssim 10^{-2}$ [26].

There are several effects that generates nonzero $g^*$ as extensively studied in e.g. Refs. [22, 24]. The dominant effect comes from the inflaton-vector boson interaction in the Lagrangian after expanding $f \simeq f_0 \delta \phi$ and $\vec{A} = \vec{A}_0 + \delta \vec{A}$ around the homogeneous background, which gives the additional contribution to the inflaton 2-point function. The result is [22, 24]

$$g^* = \frac{48 \rho_A A^2 N^2(k)}{\epsilon_H \rho_\phi},$$

where $N(k)$ denotes the e-folding number from the horizon exit for the wave number $k$ to the end of inflation. In the regime of anisotropic inflation where the vector boson is saturated as (16), it becomes

$$g^* = \frac{24(\gamma + 4)}{\gamma} N^2(k).$$

Thus the constraint $|g^*| \lesssim 10^{-2}$ severely restricts the vector boson energy density when the observable scales exit the horizon.
3.3 Constraint

3.3.1 $\gamma < -4$

Now we are going to discuss constraint on the vector DM scenario from the isocurvature perturbation. Here we consider $\gamma < -4$. First let us focus on the case (i). In this case the fluctuation is enhanced after the horizon exit. For the present horizon scale $k_0$, by using (41), the isocurvature perturbation is estimated as

$$\sqrt{P_T(k_0)} \sim \frac{H_{\text{inf}}}{2\pi} \left( e^{60} \right)^{\nu - 3/2},$$

Therefore, unless $2\nu - 3$ is very close to zero, the enhancement factor is extremely large. The situation is analogous to the development of tachyonic instability of a scalar field during inflation and the separation of the vector field into homogeneous mode plus small fluctuation may not make sense. Note that the homogeneous mode $\overline{A}$ is also exponentially enhanced during inflation by the same factor. It is unlikely that the backreaction of the vector field to the inflaton dynamics is negligible during the last 60 e-foldings. Thus the case (i) is not likely to be consistent unless the tuning of $2\nu - 3 \simeq 0$ is done.

Next consider the case (iii) where the last 60 e-foldings is spent by the anisotropic inflation. In this case the isocurvature perturbation is given by

$$S_A \sim \frac{H_{\text{inf}}}{\pi \overline{A}_{\text{end}}} \sim \sqrt{\frac{3}{2}} \frac{H_{\text{inf}}}{R_A \pi M_P},$$

where $\overline{A}_{\text{end}} = \overline{A}(a = a_{\text{end}})$ and used the fact that the vector zero mode is saturated at which $\rho_A/\rho_\phi = R_A$ according to the discussion in Sec. 2.1. Noting that $\sqrt{R_A} \propto \sqrt{\epsilon_V} \propto H_{\text{inf}}$, simply assuming small $H_{\text{inf}}$ may not reduce the isocurvature perturbation\(^\#6\) and hence the isocurvature constraint is severe. Moreover, in this case, the statistical anisotropy of the curvature perturbation (47) is too large to be consistent with observation.

Finally we consider the case (ii). In this case the last 60 e-foldings is divided into $60 \sim N_{\text{st}} + N_{\text{ani}}$ where $N_{\text{st}}$ and $N_{\text{ani}}$ represent the e-folding number of the standard slow-roll inflation and anisotropic inflation, respectively. By using (42), we have

$$\sqrt{P_T(k_0)} \sim \frac{H_{\text{inf}}}{2\pi} \left( e^{N_{\text{st}}} \right)^{\nu - 3/2},$$

at the end of inflation. For example, for $N_{\text{st}} \simeq 10$ and $\nu - 3/2 \sim O(1)$, the enhancement factor may not be too large and it can lie in the reasonable value for low scale inflation such as the new inflation given in (6). Noting that the homogeneous field value $\overline{A}$ also receives the same enhancement factor, the isocurvature perturbation may be evaluated as

$$S_A \sim \frac{H_{\text{inf}}}{\pi A_{\text{be}}},$$

\(^\#6\) Precisely speaking, the relation $\sqrt{\epsilon_H} \propto H_{\text{inf}}$ should hold when the observable scale exits the horizon and $\epsilon_V$ differs from $\epsilon_H$. In addition, $R_A$ appearing in (49) may be evaluated at later epoch (see Fig. 2).
where $\overline{A}_{be}$ denotes the field value evaluated when the cosmological scales exit the horizon: $\overline{A}_{be} = e^{-N_{st}(v-3/2)}A_{\text{end}} \sim e^{-N_{st}(v-3/2)}\sqrt{2}$ $\overline{R}_A/3M_P$. It is in principle possible that this is below the observational bound. Furthermore, in this scenario, the vector energy density when the observable scales exit the horizon is much smaller than the saturation value: $\rho_A \sim e^{-N_{st}(2v-3)}R_A \rho_\phi \ll R_A \rho_\phi$. From Eq. (46) we thus obtain

$$|g_*| \sim \frac{36|\gamma + 4|}{\gamma} \frac{N^2(k)}{\pi^2 S_A^2 R_A} \left(\frac{H_{\text{inf}}}{M_P}\right)^2 \sim 576N^2(k)\frac{P_0^0}{S_A^2}.$$  (52)

Clearly this is too large. Note that it is independent of the inflation scale.

Let us also comment on the case of $\gamma + 4 \simeq 0$. In this case one can practically regard $\overline{A}$ as a constant during inflation and the backreaction to the inflaton dynamics never becomes effective. Treating $\overline{A} = A_{\text{end}}$ just as a free parameter, the isocurvature perturbation is given by

$$S_A \simeq \frac{H_{\text{inf}}}{\pi \overline{A}},$$  (53)

hence the observational constraint is satisfied when $H_{\text{inf}} \lesssim 3 \times 10^{-5} \overline{A}$. However, the statistical anisotropy of the curvature perturbation (46) is given by

$$|g_*| \simeq 576N^2(k)\frac{P_0^0}{S_A^2},$$  (54)

which is the same expression as (52) and it is clearly too large. To summarize, for all the cases we have considered here the vector condensate DM scenario is excluded observationally.

A loophole is that the inflaton may not responsible for the curvature perturbation. In the curvaton scenario, the observed curvature perturbation is originated from the other scalar field fluctuation than the inflaton, called the curvaton [27–29]. In this case $P_0^0$ appearing e.g. in Eq. (52) should be interpreted as the sub-dominant inflaton contribution to the curvature perturbation, and it can take much smaller value than the observed value. In such a case the constraint from the statistical anisotropy can be avoided if $P_0^0 \lesssim 5 \times 10^{-19}$.

### 3.3.2 $\gamma > 2$

For $\gamma > 2$, an important difference from the $\gamma < -4$ case is that the energy density of the vector homogeneous background can be extremely small: $\rho_A \simeq m_A^3/2$ for $h = f$, since the kinetic energy vanishes for the $C_1$ solution in (12). Therefore, the statistical anisotropy (46) is negligibly small. The constraint on this scenario comes from the isocurvature perturbation. The typical magnitude of the long wave fluctuation is given by Eq. (48). Thus we need to

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#7 We used the standard slow-roll inflation relation: $P_0^0 \simeq V/(24\pi^2 M_P^4 \epsilon_V)$.

#8 In Ref. [11] constraint from the statistical anisotropy of the curvature perturbation was not taken into account.
Figure 4: Constraints on the model with \( h(\phi) = f(\phi) \). We have taken \( T_R = 10^{10} \) GeV and determined \( A_{\text{end}} \) so that it reproduces the observed DM abundance at each value of \((m_A, H_{\text{inf}})\). The region above the black line is excluded for \( \gamma = 2 \) (upper line) and \( \gamma = 2.5 \) (lower line) from the isocurvature constraint. In the region above the red dashed line, \( A_{\text{end}} \) becomes larger than \( M_P \) in order to explain the DM abundance.

Assume \( \nu - 3/2 \) close to zero, or \( \gamma - 2 \simeq 0 \), in order to avoid too large fluctuation amplitude itself. For \( \gamma = 5/2 \), for example, the enhancement factor is about \( e^{15} \simeq 3 \times 10^6 \) and not very unreasonable. The isocurvature perturbation is then given by

\[
S_A \sim \frac{H_{\text{inf}}}{\pi A_{\text{be}}},
\]

where \( A_{\text{be}} \simeq e^{-30(\gamma-2)} A_{\text{end}} \) is the vector field value when the observable scales exit the horizon. Thus the constraint from the isocurvature perturbation can be satisfied if \( H_{\text{inf}} \lesssim 10^{-5} A_{\text{be}} \). However, one should note that this is a minimum requirement. If inflation lasts much longer than 60 e-foldings, the constraint becomes much more stringent and accordingly \( \gamma \) must become much closer to 2.

Fig. 4 shows constraints on the model with \( h(\phi) = f(\phi) \). We have taken \( T_R = 10^{10} \) GeV and determined \( A_{\text{end}} \) so that it reproduces the observed DM abundance at each value of \((m_A, H_{\text{inf}})\). The region above the black line is excluded for \( \gamma = 2 \) (upper line) and \( \gamma = 2.5 \) (lower line) from the isocurvature constraint. In the region above the red dashed line, \( A_{\text{end}} \) becomes larger than \( M_P \) in order to explain the DM abundance.
4 Conclusions and discussion

In this paper we studied scenario for vector coherent oscillation DM with the action given by (1). The homogeneous vector condensate can be formed for $\gamma \leq -4$ or $\gamma \geq 2$. The particular case of $\gamma = -4$ and 2 was studied in Ref. [11] and in this paper we mainly considered $\gamma < -4$ and $\gamma > 2$.

For $\gamma < -4$, the vector condensate energy density increases during inflation and eventually the backreaction becomes important and the so-called anisotropic inflation occurs [20–22]. It is indeed possible that the vector condensate will become a coherent oscillation and its abundance is consistent with the observed DM abundance. However, it is found that the combination of constraints from DM isocurvature fluctuation and also the statistical anisotropy of the curvature perturbation almost exclude vector coherent oscillation DM scenario. A possible loophole is that the inflaton is not responsible for the observed curvature perturbation and the curvaton explains the curvature perturbation, or thermal history after inflation has an epoch of non-standard equation of state such as kination regime.

For $\gamma > 2$, the vector abundance crucially depends on the form of the mass function $h$ in (1). For the simplest case $h = 1$ the vector coherent oscillation abundance is too low to explain total DM. For $h = f$, the vector coherent oscillation can be total DM. In this case there is no constraint from the statistical anisotropy of the curvature perturbation and constraint from the isocurvature perturbation can be satisfied if $\gamma$ is not far from 2, for example, $2 < \gamma \lesssim 2.5$. The constraint is summarized in Fig. 4.

To summarize, the vector coherent oscillation DM is severely restricted from cosmological observation and only the limited model remains viable. If it constitutes the present DM, it may be detectable by experiments proposed so far [30–41] through the (small) kinetic mixing between the vector boson and the Standard Model photon.

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