GRAIN DYNAMICS IN DEBRIS DISKS: CONTINUOUS OUTWARD FLOWS AND EMBEDDED PLANETS

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ABSTRACT

This study employs grain dynamic models to examine the density distribution of debris disks, and discusses the effects of the collisional time intervals of asteroidal bodies, the maximum grain sizes, and the chemical compositions of the dust grains of the models, in order to find out whether a steady outmoving flow with an $l/R$ profile could be formed. The results showed that a model with new grains every 100 years, a smaller maximum grain size, and a composition C400 has the best fit to the $l/R$ profile because: (1) the grains have larger values of $\beta$ on average, therefore, they can be blown out easily and (2) the new grains are generated frequently enough to replace those that have been blown out. With the above two conditions, some other models can have a steady out-moving flow with an approximate $l/R$ profile. However, those models in which new grains are generated every 1000 years have density distributions far from the profile of a continuous out-moving flow. Moreover, the analysis on the signatures of planets in debris disks showed that there are no indications when a planet is in a continuous out-moving flow, however, the signatures are obvious in a debris disk with long-lived grains.

Key words: circumstellar matter – planetary systems – stellar dynamics

1. INTRODUCTION

Vega, one of the brightest stars in the Solar neighborhood, has become a typical example of stars having disks of dust due to large infrared excess, as attributed to thermal dust emissions, discovered by the Infrared Astronomical Satellite (Aumann et al. 1984). After that, many other main-sequence stars observed from optical to submillimeter wavelengths, and revealed to have dusty disk-like structures, thus, were named “Vega-like stars.”

Debris disks are the dust disks that surround these “Vega-like stars.” It is still unclear how these debris disks form. Naturally, one would expect them to be a product of the processes of star formation. The stars are formed through the collapse of a molecular cloud, which is a mixture of dust and gas, with a mass ratio of about 0.01, as implied by the compositions of the interstellar medium. The dust grains embedded in the collapsing cloud are the seeds that grow into larger grains and planetesimals. In the standard scenario, debris disks are constructed at the time when planetesimals are frequently forming and colliding. Thus, debris disks can be generated only when there are km-sized planetesimals colliding and producing a huge amount of new dust grains. This would take place at the stellar age of a million years, when the original seed grains grow to become km-sized planetesimals (Cuzzi et al. 1993). Moreover, in addition to creating new dust grains, the planetesimals would further grow into asteroids and trigger the formation of planets. In the end, the gaseous parts are gradually depleted by stellar winds, and the debris disks are constructed.

High-resolution images of some debris disks show the presence of asymmetric density structures or clumps, and before extra-solar planets were discovered by the Doppler Effect, these clumpy structures gave indirect evidences of the existence of planets. If there were no planets around Vega-like stars, it would be much more difficult to explain the asymmetrical structures of debris disks.

Astronomers’ observational efforts have led to rapid progress on the discovery of planets, and there are now more than 200 detected extra-solar planetary systems. Many theoretical works on their dynamic structures have been written (Laughlin & Chambers 2001; Kinoshita & Nakai 2001; Gozdziewski & Maciejewski 2001; Jiang et al. 2003; Ji et al. 2002; Zakamska & Tremaine 2004; Ji et al. 2007). The possible effects of disks on the evolution of planetary systems also have been investigated (Jiang & Yeh 2004a, 2004b, 2004c), and in fact, some of these systems are associated with the disks of dust. For example, using a submillimeter camera, Greaves et al. (1998) detected dust emissions around the nearby star Epsilon Eridani. This ring of dust is at least 0.01 Earth Mass and the peak is at 60 AU, and it is thus claimed to be a young analog to the Kuiper Belt in our Solar System. Furthermore, Hatzes et al. (2000) discovered a planet orbiting Epsilon Eridani by radial velocity measurements, making the claim by Greaves et al. (1998) even more impressive.

Therefore, the existence of debris disks implies the existence of planetesimals, and probably planets. The study of debris disks is very interesting and important because the density structures and evolutionary histories of debris disks actually provide hints to the evolution of planetesimals and the formation of planets. Since the Vega system has one of the closest and brightest debris disks, many observations of it have been performed, which reveal detailed information (see Harvey et al. 1984; Zuckerman & Becklin 1993; Van der Bieke et al. 1994; Heinrichsen et al. 1998; Mauro & Dole 1998; Holland et al. 1998; Koerner et al. 2001; Wilner et al. 2002). Moreover, Wilner et al. (2002) showed that the two clumps within Vega’s inner disk could be theoretically explained by the resonance with a Jupiter-mass planet in an eccentric orbit.

In addition to the Vega system, Artymowicz (1997) and Artymowicz & Clampin (1997) discussed the dust disks around $\beta$ Pic, Fomalhaut, and $\alpha$ Lyr. Grigorieva et al. (2007) simulated collisional dust avalanches of debris disks. Takeuchi & Lin (2002) employed a simplified model to study the dynamics of dust grains in gaseous proto-stellar disks. Using a disk model analogous to the primordial solar nebula, they examined the effect of a dust grain’s size on the dust’s radial migrations. In principle, the particles at high altitudes move outward, and those at lower altitudes move inward. In fact, Takeuchi & Artymowicz (2001) also investigated the same problems.

Interestingly, Su et al. (2005) showed the images of Vega, as observed by the Spitzer Space Telescope, and confirmed that the size of a Vega debris disk is much larger than previously thought.
Furthermore, from the radial profiles of surface brightness, they suggested several model fits, with different combinations of grain sizes, and all models require an inverse radial \((l/R)\) surface number density profile. Asteroidal bodies between 86 and 200 AU continue to produce new grains, which migrate outward and form an \(l/R\) density profile of the outer disk. Most grains are blown outward, and their lifetime on the debris disk is relatively short, i.e., less than 1000 years.

However, the above configuration derived from the observations raises a few important questions on debris disks in general. What is the necessary condition to produce the \(l/R\) dust density profile? How important are the effects of chemical composition? How does the grain size affect the dust density profile? How frequently must collisions occur in order to produce enough new dust grains to maintain the \(l/R\) profile?

In order to clarify the above issues, this study aimed at finding a self-consistent dynamic models for debris disks, and assumed that the asteroidal bodies within the inner disk continue generating new grains through their collisions. These grains are then added into the system and move to where it should be according to the equations of motion. The distributions of these grains are examined to see whether they follow \(l/R\) profiles. There are many physical processes and parameters to be explored for the above models. However, this paper particularly addresses the effects of collisional time intervals of asteroidal bodies, the effects of maximum grain sizes, and the influences of the chemical compositions of dust grains.

The remainder of this paper is organized as follows. Section 2 presents the model and initial conditions, Section 3 describes the simulations of a continuous flow, Section 4 discusses the possible signatures of planets, and Section 5 gives the conclusions.

2. THE MODEL CONSTRUCTION

This study aimed at investigating the possible self-consistent dynamic models of dust distribution on debris disks. The mass of the dust grains on a debris disk could be \(3 \times 10^{-3} M_\oplus\) (Su et al. 2005). Thus, if the density of each dust grain is 3.5 g cm\(^{-3}\) and the size is 2 \(\mu\)m, the total number of grains is in the order of \(10^{15}\). This estimated number of dust particles is too large for any possible numerical simulations. Therefore this paper uses 10,000 and 30,000 dust grains to represent the outer debris disk in models. This number is large enough to make the spatial resolution of density distributions sufficiently high for the purpose in this paper. However, we do not mean that each particle represents a body consisting of a huge number of dust grains. In the simulations, each particle only represents one single dust grain. Because the dust grains do not influence each other in the models, we could use a certain number of them as tracers for the system. In other words, in our simulations, only density distribution is important, and the total mass of grains is irrelevant.

This paper focuses on those effects that influence orbital evolution and density distributions of dust grains. We plan to study; (1) the time intervals between successive collisional events of asteroidal bodies, i.e., the frequency of the generation of new grains; (2) the effect of maximum grain sizes; and (3) the influence of chemical compositions. To complete the above three studies, this paper chooses 2 chemical compositions (C400 and MgFeSiO\(_4\)), 2 maximum grain sizes \((a_{\text{max}} = 9.57 \ \mu\text{m}, \ \text{and} \ 14.04 \ \mu\text{m})\), and 2 time intervals (100 and 1000 years) between successive collisions of asteroidal bodies in simulations.

### Table 1: The Ingredients of Models

| Model | Composition | Grain Density | Time Interval | \(a_{\text{max}}\) (\(\mu\text{m}\)) | \(\beta_{\text{min}}\) |
|-------|-------------|---------------|--------------|----------------|----------------|
| C2S   | C400        | 2.26 (g cm\(^{-3}\)) | 100 (years) | 9.57 | 0.62 |
| C2L   | C400        | 2.26 (g cm\(^{-3}\)) | 100 (years) | 14.04 | 0.42 |
| C3S   | C400        | 2.26 (g cm\(^{-3}\)) | 100 (years) | 9.57 | 0.62 |
| C3L   | C400        | 2.26 (g cm\(^{-3}\)) | 100 (years) | 14.04 | 0.42 |
| Mg2S  | MgFeSiO\(_4\) | 3.3 (g cm\(^{-3}\)) | 100 (years) | 9.57 | 0.43 |
| Mg2L  | MgFeSiO\(_4\) | 3.3 (g cm\(^{-3}\)) | 100 (years) | 14.04 | 0.29 |
| Mg3S  | MgFeSiO\(_4\) | 3.3 (g cm\(^{-3}\)) | 1000 (years) | 9.57 | 0.43 |
| Mg3L  | MgFeSiO\(_4\) | 3.3 (g cm\(^{-3}\)) | 1000 (years) | 14.04 | 0.29 |

Thus, there will be eight models, and their results would provide opportunities to connect the grains’ orbits and the density distribution of debris disks with the three physical ingredients. In general, whether the grains form a continuous out-moving flow and approach a steady \(l/R\) density profile will be examined. For convenience, a simulation with C400, a time interval of \(10^3\) years, and a smaller value of maximum gain sizes are presented in Model C2S. Similarly, Model Mg3L stands for a simulation with MgFeSiO\(_4\), a time interval of \(10^3\) years, and a larger value of maximum gain sizes. Table 1 lists these eight models.

In our models, the dust grains’ motion is governed by gravity and radiation pressure from the central star. Through the calculations of the orbital evolution of these dust grains, the density distribution of the debris disk at any particular time could be determined.

2.1. The Units

For the equations of motion, the unit of mass is \(M_\odot\), the unit of length is AU, and the unit of time is a year. Thus, the gravitational constant \(G = 6.672 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^2 = 38.925 \text{AU}^3 \text{M}^{-1}_\odot \text{yr}^{-2}\), and the speed of light \(c = 3 \times 10^8 \text{m s}^{-1} = 6.3 \times 10^4 \text{AU} \text{yr}^{-1}\).

2.2. The Equations of Motion

All dust grains are assumed to be in a two-dimensional plane, governed by gravity and radiation pressure from the central star. For any given time, the central star is fixed at the origin, and the dust grain’s equations of motion are, as in Moro-Martin & Malhotra (2002):

\[
\frac{d^2x}{dt^2} = -\frac{Gm_0(1-\beta)}{R^3} x - \frac{\beta_{\text{sw}} Gm_0}{c R^2} \left[ \left( \frac{\dot{R}}{R} \right) x + \frac{dx}{dt} \right],
\]

\[
\frac{d^2y}{dt^2} = -\frac{Gm_0(1-\beta)}{R^3} y - \frac{\beta_{\text{sw}} Gm_0}{c R^2} \left[ \left( \frac{\dot{R}}{R} \right) y + \frac{dy}{dt} \right],
\]

where,

\[
\begin{align*}
R &= \sqrt{x^2 + y^2}, \\
\dot{R} &= \frac{x dx}{R dt} + \frac{y dy}{R dt}, \\
\beta_{\text{sw}} &= (1 + \text{sw})\beta,
\end{align*}
\]

further, \((x, y)\) is the coordinate of a particular grain, \(G\) is the gravitational constant, \(m_0\) is the central star’s mass (2.5 \(M_\odot\)), \(c\) is the speed of light, \(\beta\) is the ratio between the radiation pressure force and the gravitational force, and “\(\text{sw}\)” is the ratio of the solar wind drag to the \(P-R\) drag. In this paper, \(\text{sw}\) is taken to be zero.
Figure 1. (a) Radial velocities, $v_r$, as functions of time for grains with different values of $\beta$. The grains are at 100 AU initially. The bottom solid curve is for $\beta = 0.35$. The dotted curve is for $\beta = 0.45$. The short dashed curve is for $\beta = 0.55$. The long dashed curve is for $\beta = 0.65$. The short dashed–dotted curve is for $\beta = 1.0$. The long dashed–dotted curve is for $\beta = 1.5$. The top two solid curves are for $\beta = 2.0$ and $\beta = 2.5$. (b)–(d) The radial distances, $R$, as functions of time for grains with different values of $\beta$. The grains are initially at 100, 200, and 300 AU for Panels (b)–(d). The solid curves are for $\beta = 0.2$, dotted curves are for $\beta = 0.3$, dashed curves are for $\beta = 0.4$, and long dashed curves are for $\beta = 0.5$.

Figure 2. (a-1) Grain size distribution for the models with $a_{\text{max}} = 9.57 \mu m$; (a-2) The histograms of the values $\beta$ of models with $a_{\text{max}} = 9.57 \mu m$, where triangles are for those models with C400 and circles are for the models with MgFeSiO$_4$; (b-1) The grain size distribution for the models with $a_{\text{max}} = 14.04 \mu m$; (b-2) The histograms of the values $\beta$ of models with $a_{\text{max}} = 14.04 \mu m$, where triangles are for those models with C400 and circles are for the models with MgFeSiO$_4$.
2.3. The Optical Parameters of Dust Grains

The equations of motion show that, when a parameter is $\beta < 0.5$, the grain's orbit is likely to be bounded; when $\beta > 0.5$, the grain would have an unbound orbit. To demonstrate the effect of $\beta$ on the orbital evolution, the evolution of radial velocities and distances are presented in Figure 1. Figure 1(a) shows the radial velocities of grains with given $\beta$ as functions of time when they are initially located at $R = 100$ AU. For a grain with $\beta = 0.35$, the radial velocity can be positive or negative, and oscillate around zero. It moves on an elliptical orbit because it has a negative total energy. A grain with $\beta = 0.45$ has similar behavior, however, the orbital period is much longer. For a grain with $\beta = 0.55$, the orbit becomes unbound and the radial velocity approaches a constant value, i.e., terminal velocity. Similarly, grains with $\beta = 0.65, 1.0, 1.5, 2.0,$ and $2.5$ also have unbound orbits and approach terminal velocities. The curves in Figure 1(a) show that grains with larger $\beta$ have larger terminal velocities. Figures 1(b)–(d) show the radial distances of grains with given $\beta$ as functions of time when they are initially located at $R = 100, 200, \text{and } 300$ AU. The solid curves are for $\beta = 0.2$, dotted curves are for $\beta = 0.3$, dashed curves are for $\beta = 0.4$, and the long dashed curves are for $\beta = 0.5$. The grains with $\beta = 0.5$ all escape from the systems. For the grains with $\beta = 0.4$, they reach 500 AU if initially located at 100 AU, reach 1000 AU if initially set at 200 AU, and reach 1500 AU when initially put at 300 AU. The parameter $\beta$ in the equations of motion determines how important the radiation pressure is and can be calculated as (Burns et al. 1979)

$$\beta = \frac{3L}{16\pi G m_0 c \rho a} \frac{\int Q_{pr}(a, \lambda) F_\lambda d\lambda}{\int F_\lambda d\lambda},$$  

(3)

where, $G, m_0$, and $c$ are as previously defined. $L$ is the central star’s luminosity, $\rho$ is the dust grain’s density, $a$ is the radius of the dust particle, $F_\lambda$ is the central star’s spectrum. The radiation pressure factor of optical parameters, $Q_{pr}(a, \lambda)$, is a function of grain’s radius $a$ and the incident electromagnetic wave’s
wavelength $\lambda$, and depends on the chemical composition of the considered grain. In general, the smaller grains would have larger $\beta$. In this paper, we choose C400 and MgFeSiO$_4$ as the compositions of the dust grains to calculate their corresponding $\beta$. The reason why we choose these two is that C400’s values of $\beta$ are among one of the largest, and MgFeSiO$_4$’s values of $\beta$ are around one of the smallest (as seen in Figure 5 of Moro-Martín et al. 2005). According to Laor & Draine (1993), the density of C400 grains is 2.26 g cm$^{-3}$, and that of MgFeSiO$_4$ grains is 3.3 g cm$^{-3}$. Then, Mie Scattering Theory and Vega’s spectrum are used to determine $\beta$ of a particular grain (please see the Appendix for details).

2.4. The Grain Size Distributions

As shown in Figure 1, the nongravitational influence on the grains’ orbital evolution is completely determined by the parameter $\beta$. When the chemical compositions of dust grains are chosen, the values of $\beta$ mainly depend on the grain sizes. For the size distributions in this study, the classic standard power law, with an index $-3.5$ is applied, i.e.,

$$\frac{dN}{da} = Ca^{-3.5},$$

where $N$ is the grain number, $a$ is the grain radius, $C$ is a constant. Numerically, it can be written as:

$$\Delta N = Ca^{-3.5}\Delta a,$$

where $\Delta a$ is the chosen bin size and $\Delta N$ is the expected grain number in the bin with a grain size around $a$. We set $a_{\text{bottom}} = 1 \mu m$ and $a_{\text{top}} = 46 \mu m$ and choose a uniform bin size in the logarithmic space as $(\ln a_{\text{top}} - \ln a_{\text{bottom}})/100 = 0.038286$. We then have $a_i = \exp(0.038286 \times (i-1))$ for $i = 1, 2, \ldots, 100$ and define $\bar{a}_i = (a_{i+1} + a_i)/2$ for $i = 1, 2, \ldots, 99$ as the possible grain sizes. By Equation (5),

$$\Delta N_i(\bar{a}_i) = C\bar{a}_i^{-3.5}(a_{i+1} - a_i) \quad \text{for} \quad i = 1, 2, \ldots, 99.$$
From the above, we have

\[ N_{\text{tot}} \equiv \sum_{i=1}^{99} \Delta N_i(\bar{a}_i) = C \sum_{i=1}^{99} \bar{a}_i^{-3.5}(a_{i+1} - a_i). \]  

(6)

Once the total particle number \( N_{\text{tot}} \) is given, the parameter \( C \) can be determined from Equation (6). Thus, we set the first grain size as \( \bar{a}_1 \), and the number of this size of grains to be \( \text{INT}[\Delta N_1(\bar{a}_1)] + 1 \) = \( \text{INT}[C\bar{a}_1^{-3.5}(a_2 - a_1)] + 1 \), where \( \text{INT} \) is an operator to take the integer part of a real number. The second grain size is \( \bar{a}_2 \) and the number of this size is similarly determined. We continue this process until the total number of grains approaches \( N_{\text{tot}} \) as possible as it can be. For example, in this paper, when \( N_{\text{tot}} = 10,000 \), we start from the first grain size \( \bar{a}_1 \) until \( \bar{a}_{59} \). We find \( \Delta N_{59}(\bar{a}_{59}) = 3.54 \), so the number of grains with size \( \bar{a}_{59} \) is four. At this stage, the total number of grains is 9996. Luckily, \( \Delta N_{69}(\bar{a}_{69}) = 3.21 \), therefore, we set the number of grains with size \( \bar{a}_{69} \) as four and the total number of grains is 10,000. Please note that \( \bar{a}_{60} = 9.57 \, \mu m \), which is smaller than \( a_{\text{up}} \). Thus, in our simulations, when the total grain number is 10,000, the maximum grain size \( a_{\text{max}} \) is 9.57 \( \mu m \). When \( N_{\text{tot}} = 30,000 \), we proceed similarly and find \( \Delta N_{60}(\bar{a}_{60}) = 4.08 \). The number of grains with size \( \bar{a}_{60} \) is five and the total number is now 29,997. Although \( \Delta N_{70}(\bar{a}_{70}) = 3.7 \), we still set the number of grains with size \( \bar{a}_{70} \) to be three only, in order to make the total number of grains be 30,000. Thus, the maximum grain size \( a_{\text{max}} \) is \( \bar{a}_{70} = 14.04 \, \mu m \) when \( N_{\text{tot}} = 30,000 \). Figure 2(a-1) shows the number of grains as a function of grain size of models with \( a_{\text{max}} = 9.57 \, \mu m \) and Figure 2(b-1) shows the one of models with \( a_{\text{max}} = 14.04 \, \mu m \). Figures 2(a-2) and 2(b-2) show the histograms of the grains’ corresponding \( \beta \) values. One can see that the \( \beta \) values of C400 grains, triangles, are larger than those of MgFeSiO4 grains, circles, in both Figures 2(a-2) and 2(b-2).

2.5. The Initial Distributions

The dust particles are supposed as produced through the collisions of asteroidal bodies in the ring region, between 86 and
Figure 6. Surface mass densities of grains as functions of radii of Model C3L. The ith panel (i = 1, 2, ..., 20) is at the time t = 200 × i, where the solid curve is the best l/R fitting function. In all panels, the triangles are for grains with β ≥ 0.5, the crosses are for grains with β < 0.5, and the circles are the total. The unit of R is AU and the unit of S is 10^{-12} g AU^{-2}.

200 AU. The initial positions of the dust grains in all models are therefore placed in this region. From 86 to 100 AU, the surface number density is set to be a constant. At $R = 100$ AU the surface number density starts to decrease as $l/R^2$ until 200 AU. Thus, the surface number density is

$$\Sigma_l(R) = \begin{cases} \Sigma_0/100^2 & \text{when } 86 \leq R \leq 100, \\ \Sigma_0/R^2 & \text{when } 100 < R \leq 200, \end{cases}$$  \hspace{1cm} (7)$$

where $\Sigma_0$ is a constant. At the beginning of the simulations, i.e., $t = 0$, the initial particle number is $N_{ini} = 10,000$ for those models with $a_{max} = 9.57$ μm, and $N_{ini} = 30,000$ for models with $a_{max} = 14.04$ μm. Thus, the constant $\Sigma_0$ can be determined by

$$N_{ini} = \int_{86}^{100} 2\pi R \frac{\Sigma_0}{100^2} dR + \int_{100}^{200} 2\pi \frac{\Sigma_0}{R} dR.$$  \hspace{1cm} (8)$$

After that, we add $N_{ini}$ grains into the system with the above distribution at $t = 100 \times i$, $i = 1, 2, \ldots, 40$ (for Models C2S, C2L, Mg2S, Mg2L) or at $t = 1000 \times i$, $i = 1, 2, \ldots, 4$ (for Models C3S, C3L, Mg3S, Mg3L). Thus, the total simulation time would be 4000 years. The basic ingredients of all models are summarized in Table 1.

2.6. The Initial Velocities

The asteroidal bodies in the ring region could move on any orbits, but their average velocities should be close to the velocities of circular motions. To simplify the models, we assume all dust grains that are supposed as generated from the larger asteroidal bodies, move on circular orbits initially.

2.7. Fitting Functions and Scaling Factors

In order to determine whether the surface mass density of the simulation result follows $l/R$ distributions, we must first
determine the best $l/R$ fitting functions. Following the least-squares method of Cheney & Kincaid (1994), the function $\phi$ is defined by

$$\phi(c) = \sum_{i=1}^{n} \left( \frac{c}{R_i} - S_i \right)^2,$$

where $S_i$ is the surface mass density at the radius $R_i$ (The disk is now separated into $n$ annulus,) and the surface mass density is considered beyond 200 AU for this fitting. The best value of $c$ can be determined through $d\phi/dc = 0$, and this value $c_s$ divided by $R$ is the best $l/R$ fitting function, $c_s/R$.

2.8. Low Collisional Probability between Dust Grains

The estimation of possible collision rates between dust grains could be complicated as it is related to the grain sizes, the spatial distributions, and the total mass. Thebault & Augereau (2007) conducted simulations to address this important issue, and defined a collisional lifetime $t_{\text{coll}}$, which is the average time it takes for an object to lose 100% of its mass by collisions.

The results of the collisional lifetimes are presented in their Figure 4, where a debris disk with total masses $0.1 \, M_\oplus$ (left panel) and $0.001 \, M_\oplus$ (right panel) is considered. The total mass of the debris disk considered in this paper is about $3 \times 10^{-3} \, M_\oplus$, therefore, the right panel of their Figure 4 fits our case. Because the grain size in our models is around $10 \, \mu$m, the $t_{\text{coll}}$ is from $2 \times 10^4$ years to $2 \times 10^6$ years. As the simulation time of our models will be only 4000 years, the collisional lifetime $t_{\text{coll}}$ is much longer than the timescale we consider in our simulations. Therefore, the possible collisions between dust grains are ignored in this paper.

3. SIMULATIONS OF A CONTINUOUS OUT-MOVING FLOW

This section will discuss the grains’ distribution on the disk for all simulation models. The simulations started at $t = t_0 \equiv 0$, and terminated at $t = t_{\text{end}} \equiv 4000$ years. To clearly present the evolution during the simulation, the dust distribution is plotted every 200 years for 20 panels. In order to understand the
evolution of both long-lived and short-lived grains, the grains are divided into two groups according to their $\beta$, i.e., the larger grains are those with $\beta < 0.5$ and the smaller grains are those with $\beta \geq 0.5$. Table 2 gives the number percentages of the smaller and larger grains in our models.

Accordingly, in all the plots of grains' density distributions, the crosses are for the larger grains, the triangles are for the smaller grains, and the circles are the total surface mass density. Figure 3 shows the surface mass density of dust grains distributed between 200 and 1400 AU at $t = 200 \times i$ years, where $i = 1, 2, 3, \ldots, 20$, for the Model C2S. Since all grains are located between 86 and 200 AU initially, there is no grain in the region beyond 200 AU at $t = 0$. To save the space, the grain distribution at time $t = 0$ is not plotted. As shown in Table 2, there are no larger grains (i.e., those with $\beta < 0.5$) in this model, so the crosses are always at the value of zero. Panel 1 shows that at $t = 200$, the smaller grains spread over the region between 200 and 500 AU. Since new grains are added into the system every 100 years, even more grains appear in the region between 200 and 500 AU and some grains migrate even furthermore, as shown in Panel 2, 3, and 4. While more and more grains move outward, the whole outer disk beyond 200 AU can be well fitted by an $I/R$ profile by time $t = 2000$ (Panel 10), and after, the outer disk becomes a steady continuous flow and always follows an $I/R$ profile. In order to understand disk evolution when collisions between asteroid bodies occur much less often, we
Figure 9. Surface mass densities of grains as functions of radii of Model Mg2L. The $i$th panel ($i = 1, 2, \ldots, 20$) is at the time $t = 200 \times i$, where the solid curve is the best $l/R$ fitting function. In all panels, the triangles are for grains with $\beta \geq 0.5$, the crosses are for grains with $\beta < 0.5$, and the circles are the total. The unit of $R$ is AU and the unit of $S$ is $10^{-12}$ g AU$^{-2}$.

also do simulations as new grains are added every 1000 years. Model C3S is for this purpose, and the results are presented in Figure 4. At $t = 200$, as shown in Panel 1, some initial grains move to the region between 200 and 500 AU. These grains move outward even further in the following panels, however, the surface mass density decays as there are no new grains to maintain the profile between 200 and 500 AU. At $t = 1000$ (Panel 5), the density profile decays to nearly flat, and 10,000 new grains are added at that time. These new grains migrate outward, thus, the surface mass density between 200 and 500 AU raises in Panel 6. In general, Panels 6 to 10 repeat the decaying seen in Panels 1 to 5. Similar processes happen from Panels 11 to 15 and Panels 16 to 20. The $l/R$ profile does not have good fitting function for the surface mass density at any time in this model. Figure 5 shows the evolution of the disk’s surface mass density of Model C2L, which is a model with a small fraction of larger grains (0.07%) and adding new grains every 100 years. At $t = 200$, some smaller grains migrate up to about 500 AU but there are almost no larger grains appearing in this region. At $t = 400$ (Panel 2), the surface mass density of smaller grains increases and some larger grains migrate up to around 200 and 300 AU. Both the smaller and larger grains continue moving outward, and the overall surface mass density approaches a decaying function with some fluctuations. Because most of the larger grains are bounded within the system, only the smaller grains form a steady out-moving flow. The existence of larger grains makes it more difficult for the total surface mass density to become an $l/R$ profile. It cannot be fitted by an $l/R$ function until $t = 3200$ (Panel 16), and density fluctuation deviation from the $l/R$ curve is larger than in Model C2S. Figure 6 presents the evolution of the grain’s surface mass density of Model C3L. As in Model C3S, the profile cannot be maintained due to new grains not being added with enough frequency. The main difference is that the larger grains exist and stay in the region between 200 AU and 600 AU. Their persistence makes the total surface mass density closer to the $l/R$ profile at some particular time. For example, the distribution in Panels 7 and 12 are closer to the $l/R$ profile, however, there is no steady outgoing flow in this model. In order to investigate the effects of chemical compositions, this paper offers another set of
Figure 10. Surface mass densities of grains as functions of radii of Model Mg3L. The $i$th panel ($i = 1, 2, \ldots, 20$) is at the time $t = 200 \times i$, where the solid curve is the best $l/R$ fitting function. In all panels, the triangles are for grains with $\beta \geq 0.5$, the crosses are for grains with $\beta < 0.5$, and the circles are the total. The unit of $R$ is AU and the unit of $S$ is $10^{-12}$ g AU$^{-2}$.

four models Mg2S, Mg3S, Mg2L, and Mg3L (please see details in Table 1 and Table 2), and their results are shown in Figures 7–10. Table 2 shows that the fractions of long-lived grains of these four models (i.e., those with $\beta < 0.5$) are larger than those models with C400. In general, these larger grains would remain around the system for a timescale much longer than the smaller grains, and their persistence cause the density deviations from the $l/R$ profile. For example, Figure 7 shows that, in Model Mg2S, the smaller grains form a continuous out-moving flow starting from $t = 3000$ (Panel 15), however, the larger grains continue moving out slowly. The overall density distribution still approaches an $l/R$ profile, though much slower and with larger fluctuations. Figure 8 shows that, in Model Mg3S, the new grains are not generated frequently enough to form a steady density profile. The surface mass density is often very small in all areas of the disk. Figure 9 presents the grains’ distributions of Model Mg2L, which is a model similar to Model Mg2S, but with a greater fraction of larger grains. The contribution of the persistent larger grains make it very difficult to have an $l/R$ density profile, though the profile approximately becomes a steady state after $t = 2400$ (Panel 12). Finally, the results of Model Mg3L are shown in Figure 10, because new grains are added every 1000 years, the density goes up and down randomly. There are some larger grains remaining, however, the density cannot be fitted by an $l/R$ profile.

4. SIGNATURES OF PLANETS

As discussed in Section 1, planets could exist in debris disks. For example, two clumps of Vega’s inner disk, as studied by Wilner et al. (2002), could be due to the resonant capture of dust grains by a Jupiter-mass planet. Thus, the nonaxisymmetric structures of debris disks give obvious signatures of planets and the resonant trapping is a natural explanation for these clumpy structures. In addition to the resonant capture, gravitational scattering by the planet also influences grain distribution of debris disks. It is therefore interesting to see what could happen if there is a planet moving around the debris disk, with the continuous out-moving grain flow produced in the previous section. First, this study examined the disk’s structure in one
of the previous models, Model C2S, in the case when a five-Jupiter-mass planet is added. In Run 1, the planet is initially located at \((x, y) = (100 \text{ AU}, 0)\), and follows simple circular motions. However, in Run 2, the planet is initially located at \((x, y) = (250 \text{ AU}, 0)\), and the remaining details of the above two runs are the same as in Model C2S. Figure 11 shows the disk’s surface mass density in Model C2S, Run 1, and Run 2 from \(R = 0\) to \(R = 1000 \text{ AU}\). The circles represent Model C2S, the crosses represent Run 1, and the triangles represent Run 2. It is clear that the disks’ profiles in these three models are the same at any point during the simulations. When the planet orbits at \(R = 100 \text{ AU}\), as in Run 1, it would not greatly affect the grains, as almost all grains leave their birth places (86 to 200 AU). The density peaks were around 250 AU, therefore, we choose the planet to move in a circular orbit, with a radius of \(R = 250 \text{ AU}\), in Run 2. However, it is found that the planet does not affect the disk’s density profile. In order to observe the disk’s evolution from another view, the distribution of dust grains on the \(x-y\) plane in Run 2 is presented in Figure 12. The small dots represent the dust grains, and the full circle represents the planet. It is obvious that the grain distribution in every panel looks almost identical, and there is no signature for the planet. The corresponding plots for Model C2S and Run 1 are the same as in Figure 12, and thus, are not shown here. The planet is hidden among the debris disk with a continuous out-moving flow. This is due to the scattering probability between the planet and dust grains, which is very small in a continuous out-moving flow. The dust grains pass by and move outward quickly in a short time. Secondly, in order to demonstrate the outcome when the dust grains have a much longer lifetime, in Run 3, a simulation was performed, where 10,000 dust grains were placed in the region between 86 and 200 AU, following Equation (7), as in Model C2S. However, all dust grains had \(\beta = 0.1\), which corresponds to grain radius \(a = 58.53 \mu m\) for C400, and \(a = 40.62 \mu m\) for MgFeSiO4. These represent larger dust grains in the ring region, which would only have tiny migrations and not be blown out. In Run 3, a five-Jupiter-mass planet is initially located at \((x, y) = (100 \text{ AU}, 0)\), and move in a circular orbit. The evolution of grain distributions on the \(x-y\) plane for Run 3 is shown in Figure 13. The locations of dust grains are...
Figure 12. Grain distributions on the $x - y$ plane of Run 2. The $i$th panel ($i = 1, 2, \ldots, 20$) is at the time $t = 200 \times i$. In all panels, the dots show the locations of grains, and the full circle represents the planet. The unit of both axes is AU.

shown by dots, and the full circle represents the planet. The first panel, which is the situation at $t = 200$, shows that dust grains are remaining in their birthplaces. However, in Panel 2, some grains are scattered by the planet and move on elongated orbits. In Panels 3 and 4, the spiral-shape gaps are formed gradually. Later, a ring-like structure becomes obvious in the 5th and 6th panels. In Panels 7, 8, and 9, the ring becomes larger and another arc-like structure forms inside the ring. Due to the influence of the planet, these nonaxisymmetric structures are developed continuously until the end of the simulation. For purposes of comparison, another simulation was performed as Run 4, which is the same as Run 3, except there is no planet in this run. The evolution of dust grains on the $x - y$ plane for this simulation is shown in Figure 14. Due to the dust grains being subjected to radiation pressure, the orbit expands slightly, and the radial oscillations cause rings to form at different places. To carefully examine particle distribution, the color contours of the final panel of Figures 12–14 (i.e., Runs 2, 3, 4) are shown in the top right, bottom left, and bottom right of the Figures 15. In addition, the color contour of the final grain distribution of Model C2S is shown in the top-left panel of Figure 15, for comparison. Thus, those models with a continuous out-moving flows could have a larger disk with a density peak at $R = 250$ AU. Their contours look the same no matter whether there is a planet
or not. The models with long-lived grains have smaller disks, and the one with a planet added could have nonaxisymmetric structures. To summarize, from the first two panels of Run 3, it is found that, to make the planet-grain scattering strong enough to produce signatures in debris disks, the dust grains have to stay in a close-by orbit for at least 400 years. When dust grains of the debris disk are short-lived, they do not stay in the same orbital radius for many periods, and simply keep moving, and continuously approach 1000 AU. This is the reason there is no signature in a debris disk, when a planet is added in an out-moving flow, and it looks as though the planet is hidden in a continuous flow. On the other hand, when dust grains are long-lived, the nonaxisymmetric structures are easily formed due to the scattering between the planet and dust grains.

5. CONCLUDING REMARKS

This study investigated the possibilities of constructing general self-consistent dynamic models of debris disks, and particularly, examined the effects of collisional time intervals of asteroidal bodies, the effects of maximum grain sizes, and the influence of chemical compositions of dust grains. In all the simulations, the grains' orbits are calculated, the density distributions of debris disks are then determined, and compared with the $l/R$ functions. The results showed the grains in the
Model C2S give the best fit to the $l/R$ profile because: (1) the grains have larger values of $\beta$ in average, and thus, they can be blown out easily; and (2) the new grains are generated frequently enough to replace those have been blown out. The above two conditions make it easier to form a continuous out-moving flow and thus approach an $l/R$ profile. It is worth noting that it only takes about 2000 years to become an $l/R$ profile. When there are not enough new grains, the profile cannot be maintained, as shown in Model C3S and Model C3L. The persistence of some larger grains in Model C2L make the density profile slightly more complicated. The models with MgFeSiO$_4$ have more bound grains, therefore, the deviations from the $l/R$ profile are larger than with C400. To conclude, those models in which new grains are generated every 1000 years have density distributions far from the profile of a continuous out-moving flow. In the study of the signatures of planets in debris disks, the results showed that there is no sign at all when the planet is in a continuous out-moving flow, however, the signatures are obvious in a debris disk with long-lived grains.

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APPENDIX

MIE SCATTERING THEORY

The exact solution of the scattering of an electromagnetic wave by a dielectric sphere of arbitrary size is referred to as the Mie Scattering Theory. Using the radial components of the electric and magnetic Hertz vectors ($\pi_1$, $\pi_2$), the Hertz potential can be found and the general expressions for the scattered fields can be obtained (Ishimaru 1991). Applying the boundary conditions on $\pi_1$ and $\pi_2$, the constant $a_n$ and $b_n$ in the general expressions can be determined. It is found that both $a_n$ and $b_n$ are related with spherical Bessel functions $j_n(x)$ and $h_n^{(2)}(x)$. The independent variable $x$ is usually called the size parameter and can be defined as

$$x = \frac{2\pi a}{\lambda},$$  \hspace{1cm} (A1)

where $\lambda$ is the wavelength of the incident wave, and $a$ is the radius of the dust grain. We also need to define $\psi_n(x)$ and $\zeta_n(x)$ as

$$\psi_n(x) = x j_n(x)$$ \hspace{1cm} (A2)

$$\zeta_n(x) = x h_n^{(2)}.$$ \hspace{1cm} (A3)

The complex index of refraction is $m$, and $y$ is defined as

$$y = mx.$$ \hspace{1cm} (A4)

Then, we can explicitly express $a_n$ and $b_n$ as

$$a_n = \frac{\psi_n(y)\psi_n(x) - m\psi_n(y)\psi'_n(x)}{\psi'_n(y)\zeta_n(x) - m\psi_n(y)\zeta'_n(x)}$$

$$b_n = \frac{m\psi'_n(y)\psi_n(x) - \psi'_n(y)\psi'_n(x)}{\psi'_n(y)\zeta_n(x) - m\psi_n(y)\zeta'_n(x)}$$

Figure 15. Color contour of the grain particle distribution at $t = 4000$, where the top-left, top-right, bottom-left, and bottom-right panels are for Model C2S, Run 2, Run 3, and Run 4, respectively.
The extinction factor $Q_{\text{ext}}$, defined as the total cross section ($\sigma_t$) divided by the geometric cross section ($\pi a^2$), can be calculated as (Ishimaru 1991)

$$Q_{\text{ext}} = \frac{2}{x^2} \Re \left[ \sum_{n=1}^{\infty} (2n+1)(a_n + b_n) \right],$$

(A6)

where $\Re$ means taking the real part only. Similarly, the scattering factor $Q_{\text{sca}}$, defined as the scattering cross section ($\sigma_s$) divided by the geometric cross section ($\pi a^2$), can be calculated as

$$Q_{\text{sca}} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2).$$

(A7)

The absorption factor $Q_{\text{abs}}$ is defined as

$$Q_{\text{abs}} = Q_{\text{ext}} - Q_{\text{sca}}.$$  

(A8)

Please note that Van de Hulst (1957) also derived the above results similarly and further define the radiation pressure factor $Q_{\text{pr}}$ as

$$Q_{\text{pr}} = Q_{\text{ext}} - \frac{4}{x^2} \sum_{n=1}^{\infty} \frac{n(n+2)}{n+1} \Re(a_n a_{n+1}^* + b_n b_{n+1}^*)$$

$$- \frac{4}{x^2} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \Re(a_n b_n^*).$$

(A9)

The relation between a wavelength and the complex index of refraction $m$ for a grain with a particular chemical composition can be obtained from the website, http://www.astro.uni-jena.de/Laboratory/Database/. We get that for C400 and MgFeSiO$_4$ and calculate the $Q_{\text{pr}}$ and $Q_{\text{abs}}$ numerically through the above equations. The spectrum of Vega is taken from the site, ftp://ftp.stsci.edu/cdbs/cdbs2/grid/k93models/standards/.

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