Simulation of the analytical solution of lasing output power for Yb\textsuperscript{3+}: YAG thin disk laser regime in a fundamental mode operation

Raed Mahdi Salih\textsuperscript{1}
Mudhir Shihab Ahmed\textsuperscript{2}

\textsuperscript{1}Al-khwarizmi Engineering College , University of Baghdad, Iraq
\textsuperscript{2}College of education for pure science Ibn Al-Haitham, University of Baghdad, Iraq
corresponding author’s e-mail : raedmahdi.rm@gmail.com

Abstract. A theoretical model is established to derivation equation of output Laser power for multi-pass pumping for a quasi-three level thin-disk laser, A numerical solution of this equation for Yb\textsuperscript{3+}:YAG thin-disk laser, which pumped at wavelength (940 nm) and lasing emission at wavelength (1030 nm), regime in fundamental transverse mode (M\textsuperscript{2}=0.95) at room temperature were performed by calculated the variation of (Nt),(d),(Mp),(R2),(T) and (Pp) on the output Laser power and find the typical value of these parameters are used to obtain high output Laser power.

Keywords: thin disk laser, fundamental mode, continuous wave, output power.

1. Introduction

A Yb\textsuperscript{3+} as an ion that extracts from a simple electronic structure. Having two manifolds separated from (10000 cm\textsuperscript{-1}), The effects like excited state absorption (ESA), cross-relaxation, and upper transformation are absent. These processes, whose exist for another rare-earth ion usually used as lasant ions, can reduce the efficiency of the laser due to an alternative pathway for energy transfer [1,2].

a Yb\textsuperscript{3+} ion contain a low quantitative defect, which leads to reduced thermal load due to the energy mismatch between the wavelengths of pumping and lasing [2,3].

By considering Y3Al5O12 (YAG) as a host material, owns several acceptable qualities for medium-power laser applications. They were very mechanically strong [1,4].One of the charming features of Yb\textsuperscript{3+}: YAG is that it can contain very high concentrations of Yb\textsuperscript{3+}. Special laser architectures, such as a thin disk Laser (TDL)[4,5].

For example, in (TDL) design, the fracture limit is measured as an inverse of the thickness of the gain medium. So, maintaining a thin gain element while still owning enough rare-earth ion density from the ground to absorb the pump can provide excellent efficiency, good pump density and enhanced thermal management [5,6,7].
2. Thin-disk laser principle:

The (TDL) concept is a common suitable way when high efficiency, high power and excellent beam quality are required at the same time. Due to the ratio of a small volume to the surface of the area, the gain may be cooled with very high efficiency [9,10].

Also, the heat flow direction is parallel to the axis of the laser resonator, which, along within a very short optical path length during an active medium, reducing the effects of the thermal lens in addition to thermally induced deviation at orders of magnitude comparing with the laser rod power[9,11].

Thus, when the density of pump power is stable above a pump spot area, there are temperature gradients available within the volume of the pump, which leading a system easily scalable in the output power by increasing the diameter of a pump spot area at a constant pump density and this keeps the benefits of a pumping more efficiency. With improved thermal management [9,11]. Figure (1) illustrate the principle of (TDL) design.

![Figure 1: The principle of thin disk lasers design](12]

Because of its low quantum defect and broad absorption band, quasi-three-level Yb$^{3+}$:YAG has been studied extensively to develop high-efficiency and output power, pumped by laser diode (InGaAs) operation at (940 nm), when the lasing wavelength is (1030), with the quantum defect is (8.7%)[13,14]. However, the (TDL) scaled to a (kW) level whereas keeping close to the fundamental mode operation ($M^2 < 1$) is unrivalled and it is commonly limited to the output power about (1KW) [15].

The beam quality ($M^2$) in thin-disk laser described as [16, 17]:

$$M^2 = \left(\frac{a_{rp}}{r_{fm}}\right)^2 \tag{1}$$

Where (rp) is the pump spot radius, (rfm) is the fundamental mode radius and (a) is the indicates of the pump beam intensity and lasing mode intensity overlap, for a thin disk lasers (a=0.85) [18].
3. Analytical model:

The typical laser material for thin disk setup is Yb\(^{3+}\):YAG, at room temperature a quasi-three-level laser material\([19,20]\). The energy level system of Yb\(^{3+}\):YAG consists of the stark split \((2F_7)\) a ground state and \((2F_5)\) an excited state.

Because of the small energy splitting of each manifold, are assumed to be very small. This justifies the use of a two-manifold model, a ground state for energy \((E_1)\), and population \((N_1)\) and excited state for energy \((E_2)\), and population \((N_2)\)\([19,20]\).

The energy levels of Yb\(^{3+}\) ion in YAG crystal at room temperature are shown in Figure (2).

\[\begin{align*}
\frac{dN_2}{dt} &= \left[\sigma_{ap}(N_t - N_2) - \sigma_{ep}N_2\right] \frac{n_{ap} \eta_{ap} \lambda_p}{\lambda_p^2 \alpha d} I_p + \left[\sigma_{al}(N_t - N_2) - \sigma_{el}N_2\right] \frac{M_L \lambda_L}{\lambda_L \alpha} I_L - \frac{N_2}{\tau} \tag{2} \\
\frac{dI_L}{dt} &= \frac{M_L \lambda_L}{2L} \left[\left(\sigma_{al} + \sigma_{el}\right)N_2 - \sigma_{al}N_t\right] I_L - \frac{c}{2L} \left[\ln(1 - T_r) - \ln(1 - \gamma)\right] I_L \tag{3} \\
N_t &= N_1 + N_2 \tag{4}
\end{align*}\]

Where \((\lambda_p)\) is the pump wavelength , \((\lambda_L)\) is the laser wavelength ,\((\tau)\) is the fluorescence lifetime of the upper manifold, \((N_t)\) is the doping concentration ,\((Tr)\) is transmission, \((\gamma)\) is the internal loss of the resonator ,\((Ip)\) is the pump intensity,\((IL)\) is the laser intensity, \((Mp)\) is the number of pump beam passes through the active medium , \((ML)\) is the number of laser beam through the resonator, \((\sigma_{ap}),(\sigma_{ep}),(\sigma_{el}),\) is absorption and emission cross section of pumping and lasing, at room temperature \((T)\) respectively, \((L)\) is the optical length of resonator , \((d)\) is the crystal thickness, \((h)\) is the Planck’s constant and \((c)\) is the velocity of an electromagnetic wave in the vacuum \([24,25]\).
When the absorption coefficient ($\alpha_p$) is given by [25]:

$$\alpha_p = \sigma_{ap}N_t - \sigma_p N2$$  \hspace{1cm} (5)

Where:

$$\sigma_p = \sigma_{ap} + \sigma_{ep}$$  \hspace{1cm} (6)

And the stokes efficiency ($\eta_s$) is defined [25]:

$$\eta_s = \frac{\lambda_p}{\lambda_L}$$  \hspace{1cm} (7)

The round trip loss ($\delta$) can be described as [24]:

$$\delta = -\ln(1-Tr) - \ln(1-\gamma)$$  \hspace{1cm} (8)

Here:

$$Tr = 1 - R2$$  \hspace{1cm} (9)

Where $R2$ is the output reflectivity.

The absorption efficiency ($\eta_a$) can be obtained [24]:

$$\eta_a = Rp[1 - \exp(1 - Mp \alpha_p d)]$$  \hspace{1cm} (10)

Where ($Rp$) is the total reflectivity of the multi-pass pumping system.

In the steady-stare conditions ($\frac{dN_2}{dt} = 0$), by solving eq.(3) to obtain the threshold population ($N_{2th}$) for laser operation:

$$N_{2th} = \frac{\sigma_{ap} N_t}{\tau L} + \frac{\delta M_p d \sigma_L}{M_L d \sigma_L}$$  \hspace{1cm} (11)

Where: $\sigma_L = \sigma_{aL} + \sigma_{eL}$  \hspace{1cm} (12)

When substituting the threshold condition ($N2 = N2th$, $Ip = Ipth$ and $IL = 0$) in eq.(2), the pump power at the threshold ($P_{th}$) can be described as:

$$P_{th} = \frac{hdcA_p}{\eta_a \tau} N_{2th}$$  \hspace{1cm} (13)

Where ($Ap$) is the pump spot area is given by [25]:

$$Ap = \pi (rp)^2$$  \hspace{1cm} (14)

Here ($rp$) is the pump spot radius.

For the steady-state condition ($\frac{dN_2}{dt} = 0$), solving eq. (2), the laser output power ($P_{out}$) can be obtained:

$$P_{out} = \frac{Tr}{\delta} \eta_a \eta_s (P_p - P_{th})$$  \hspace{1cm} (15)
4. Numerical model:

The numerical model consists of three main steps at the beam quality factor is \( M^2 = 0.95 \). In the first step, the Table (1) shows the value of all parameters used to calculate \( P_{out} \) by matlab language.

Table (1): the value of parameters for \( \gamma b_{3+}^3 \) : YAG thin disk laser

| Parameter | Value       | Unit   | Ref.       |
|-----------|-------------|--------|------------|
| \( \lambda P \) | 940x10^{-6} m |         | [22,25,26,27] |
| \( \sigma ap \) | 0.7506x10^{-24} m² |     | [22,25,26,27] |
| \( \sigma ep \) | 0.1676x10^{-24} m² |     | [22,25,26,27] |
| \( \sigma p \) | 0.9182x10^{-24} m² |     | [22,25,26,27] |
| \( \lambda L \) | 1030x10^{-6} m |        | [22,25,26,27] |
| \( \sigma nL \) | 2.1131x10^{-24} m² |     | [22,25,26,27] |
| \( \sigma eL \) | 0.1223x10^{-24} m² |     | [22,25,26,27] |
| \( \sigma L \) | 2.2354x10^{-24} m² |     | [22,25,26,27] |
| \( R_p \) | 0.77        |        | [24]       |
| \( \gamma \) | 1.7x10^{-4} |        | [28]       |
| \( a \) | 0.85        |        | [18]       |
| \( L \) | 50x10^{-3} m |        | [22,25]    |
| \( ML \) | 4            |        | [25,29]    |
| \( \text{rfm} \) | 195x10^{-6} m |      | [29]       |
| \( h \) | 6.6205x10^{-34} J.sec |    |             |
| \( c \) | 3x10^{8} m/sec |     |             |

In the second step, calculated the value of \( P_{out} \) where variation of the value of these parameters \( N_t [1,30], d [9,22,25], M_p [32], R_2 [19,22], T [24,31] \) and \( P_p [25] \) respectively.

The range of values of these coefficients used in the numerical solution in this paper was selected from the references indicated at each coefficient.

5. Numerical results:

5.1. The effects of concentration (\( N \)):

The laser output power \( (P_{out}) \) increases with increasing the concentrations, reaching its highest value at \( (N=22.26x10^{26} \ \text{m}^{-3}) \), after which it begins to decrease slightly despite the concentrations increasing, due to the indirect dependence of \( P_{out} \) on \( N \).
5.2. The effects of Disk thickness (d):

The laser output power (Pout) increases slightly by increasing the thickness of the laser thin disk and reaching its highest value at (d=80x10^{-6}m) and then decreasing slightly with increasing thickness.

5.3. The effects of number of pump beam passes through the active medium (Mp):

The laser output power (Pout) increases by increasing the number of pump beam passes through the active medium (Mp). This corresponds to the positive relationship between (Pout) and (Mp).
5.4. The effects of output reflectivity (R2):

The laser output power (Pout) is increased by increasing the reflectivity of the laser output mirror (R2), This corresponds to the positive relationship between (Pout) and (R2).

5.5. The effects of Temperature (T):

The laser output power (Pout) are slightly reduced by increasing temperature (T), due to the indirect dependence of (Pout) on (T).
5.6. The effects of input pump power (Pp):

The laser output power (Pout) in a thin disk laser increase by increasing the input pump power (Pp), this corresponds to the positive relationship between (Pout) and (Pp).

Figure 8: The laser output power (Pout) variations versus the input pump power (Pp)

In the third step, finding the typical value of these parameters are used to obtained high lasing output power shows in Table(2).
### Table (2): the typical values of parameters are used to obtained a low threshed power of laser, a high efficiency and high of output laser power

| Parameters | Value | Unit |
|------------|-------|------|
| $\eta_s$ % | 91.2621 | — |
| rf | $195 \times 10^{-6}$ | m |
| rp | $223 \times 10^{-6}$ | m |
| Nt | $26.66 \times 10^{26}$ | $\frac{1}{m^3}$ |
| $\tau$ | $970 \times 10^{-6}$ | Sec |
| d | $80 \times 10^{-6}$ | m |
| $M_p$ | 49 | — |
| $R_2$ | 0.98 | — |
| T | 300.15 | $K^0$ |
| Pp | 1000 | W |

### 6. CONCLUSIONS

In summary, a theoretical analysis is established to study the output lasing power for multi-pass pumping of Yb$^{3+}$:YAG thin-disk laser at a quasi-three level operation. In addition, we find a numerical solution of the equation of output Lasing power ($p_{out}$) at the change of ($N_t$),($d$),($M_p$),($R_2$),($T$) and ($P_p$) AT regime in a fundamental mode operation ($M^2<1$). Also we find, the ($p_{out}$) is highest value at the concentration ($26.66 \times 10^{26} \frac{1}{m^3}$) and thickness ($80 \times 10^{-6}$ m), in addition, that the output Lasing power ($p_{out}$) increases linearly by increasing the ($M_p$),($R_2$) and ($P_p$), but it is slightly reduced by increasing of the temperature, therefore we find the typical value of these parameters are used to obtain high output Laser power.
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