Electricity Industrial Organization: What About The Strategic Behavior Of Hydro And Thermal Operators?

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**Abstract**—In this paper, we develop a two-period model where we analyze and compare a hydro/thermal electrical system under different industrial organization: monopoly, Cournot competition and collusion; under storage constraint, water availability constraint and thermal turbine capacity constraint. First, we prove that the technological complementarity have an important role in the satisfaction of electricity demand in the different industrial organization. Second, by the analytical resolution, we show that inter temporal private monopoly water transfer from off-peak season to peak season is not high as the same transfer under a public monopoly and therefore this increases the power price. Under Cournot competition, an increase in the peak season implies a water transfers from off peak season to the peak season. The results of collusion depend on availability of water resource and the demand parameters. The results of collusion depend on availability of water resource and the demand parameters.

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**I. INTRODUCTION**

The market structure of the electricity sector has been characterized by natural monopolies. In the last decade, the market liberalization processes suggests that the electricity production is done by private generators which use different technologies: wind, hydroelectric, solar, thermal, etc. The main target of liberalization is to achieve more cost efficient production, lower electricity prices, better resources allocation and more supply security. The economic literature, there are little number of papers which analyze mixed hydro thermal system problems in a liberalized industry such as Scott et Read (1996) [1], Bushnell (1998) [2], Crampes and Moreaux (2001) [3] and Dakhlaoui and Moreaux (2004) [4]. Ambec and Doucet (2002) [5] compare the centralized and the decentralized industry of the hydraulic system under water turbine capacity constraint and water abundance constraint. They show that the exercise of the capacity of the market can reduce the results envisaged of the deregulation of this industry. In this paper, we use Ambec and Doucet (2002) framework to develop a two-period model where we analyze and compare a hydro/thermal electrical system under different industrial organization: monopoly, Cournot competition and collusion; under storage constraint, water availability constraint and thermal turbine capacity constraint. First, we prove that the technological complementarity have an important role in the satisfaction of electricity demand in the different industrial organization. Second, by the analytical resolution, we show that inter temporal private monopoly water transfer from off-peak season to peak season is not high as the same transfer under a public monopoly and therefore this increases the power price. Under Cournot competition, an increase in the peak season implies a water transfers from...
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The rest of the paper proceeds as follow. In the section 2, we present the model. Then, in section 3, we deal with public monopoly and private monopoly. Section 4, we analyze the production problem under Cournot competition and collusion of hydro and thermal operators. We conclude in section 5.

II. THE MODEL

We consider two-period model that analyze the production of electricity with hydro ($H$) and thermal plants ($T$). The inverse demand of electricity during period $t$ for any quantity $Q_t$: $P_t = a_t - b_t Q_t$, with $a_t > 0$ and $b_t > 0$, $t = 1, 2$. Let is $\beta$ the discount factor. The hydroelectric firm uses natural inflows of water in order to produce $Q_t^H$ units of electricity during the period $t$. $e_t$ is the exogenous volume of water supplied in the hydrographic basin during period $t$. It is assumed that one unit of water yields $\alpha$ units of electricity ($\alpha > 0$). The total production of $H$ during the two periods is:

$$Q_t^H + Q_2^H = \bar{Q}_t^H = \alpha(e_1 + e_2)$$

(1)

Water available during period 1 can be used to produce electricity during the first period, or can be stored in reservoir for use in the second period. Hence, it is able to produce at no cost during the first period any quantity $Q_1^H$ such that:

$$Q_1^H \leq \alpha e_1$$

(2)

The volume of water stored in reservoir during the period 1 is used in its entirety to produce electricity in the second period. This volume is bounded by the reservoir capacity, denoted $s$.

In terms of first period, this storage constraint is written:

$$Q_1^H \geq \alpha e_1 - s$$

(3)

A production plan is any vector, $Q^H = (Q_1^H, Q_2^H) \in \mathbb{R}_+ \times \mathbb{R}_+$, verifying the constraints.

The firm $T$ produces power gaze or fuel input. The production of $Q_t^T$ units of electricity requires an assuming quadratic cost, which is $C_t = \frac{1}{2} c_t (Q_t^T)^2$ with $c_t > 0$ for $t = 1, 2$.

During each period $t$, the $T$ plant is subject to capacity production constraint which is:

$$Q_t^T \leq \bar{Q}_t^T, \forall t = 1, 2$$

(4)

A production plan of $T$ is $Q^T = (Q_1^T, Q_2^T) \in \mathbb{R}_+ \times \mathbb{R}_+$, verifying the capacity constraint.

III. MONOPOLISTIC STRUCTURE

A. Public monopoly

The optimal monopoly production plan is a solution of the following problem:

$$\text{Max}_{Q_t^H, Q_t^T} \left\{ \mathcal{J} \left[ Q_t^H, Q_t^T \right] \right\} = \int_0^Q \left[ P_t(x) - C_t(Q_t^T) + \beta \left( \int_0^Q P_2(x) - C_2(Q_2^T) \right) \right] dx$$

With respect to (1), (2), (3) and (4).

Denoting by $\bar{\lambda}_t^H$, $\bar{\lambda}_t^T$ and $\bar{\lambda}_t^H$ the Lagrange multipliers associated respectively to the water availability constraint at the off-peak season, the storage constraint and the maximum capacity of the plant $T$.

The first order conditions yield that the monopoly equalizes the marginal benefit net of the one unity of water release for the $H$ plant at the season 1 with the actualized marginal benefit comes from the storage of water until the peak period:

$$P_1(Q_1) - \bar{\lambda}_1^H = \beta P_2(Q_2) - \bar{\lambda}_2^H$$

From the first order condition, we have:

$$C_1(Q_1^T) + \beta C_2(Q_2^T) - \bar{\lambda}_2^H = \bar{\lambda}_1^H - \bar{\lambda}_2^H$$

1) If the production maximal capacity is bounded during the peak season

Suppose that during the off-peak season, the demand is not high enough to reach the maximal capacity and that in the second period, the constraint (4). In that case, $\lambda_1^T = 0$ and $\lambda_2^T > 0$. So, the first order conditions relative to thermal plants are:

$$P_1(Q_1) = C_1(Q_1^T)$$

$$\beta P_2(Q_2) = \beta C_2(Q_2^T) + \bar{\lambda}_2^H$$

The condition (2) is reduced as such:

$$C_1(Q_1^T) = \beta C_2(Q_2^T) - \bar{\lambda}_2^H$$

The maximal capacity of the thermal plant is reached in the peak season. To satisfy the peak demand, the use of this plant in the off-peak season will intensify as will the storage of inflows to be sure to latter satisfy the peak demand. Consequently, as $\bar{\lambda}_2^H = 0$ , to balance the equation :

- $\bar{\lambda}_1^H$ increases, so more water is stored because the marginal cost of the water’s immediate exploitation increases.
- $\bar{\lambda}_2^H$ decreases, so the marginal cost of the water storage decreases.

2) If the constraints are never saturated

Proposition 1: The optimal mixed system operating at the public monopoly case is given by:

$$Q_1^H = \frac{a_1 - \beta a_2 + \beta Q^H}{b(1 + \beta)}$$

$$Q_2^H = \frac{1 + \beta}{b(1 + \beta)}$$

$$Q_1^T = \frac{a_1 + a_2 - b Q^H}{(b + c)(1 + \beta)}$$

$$Q_2^T = \frac{a_1 + a_2 - b Q^H}{(b + c)(1 + \beta)}$$
The optimal production by the T plant, in two periods depends on the water turbine capacity of the H plant, the demand characteristics and the production cost of the plant T. So, the presence of the plant H makes dynamic the problem of the thermal plant. Consequently, any increase in H turbine capacity equal to \( \Delta Q^H \), reduces the plant T production during the off-peak season by a quantity equal to \( \frac{b}{(b + c)(1 + \beta)} \Delta Q^H \). A free technology (H) supplementary exploitation of \( \frac{\beta}{1 + \beta} \Delta Q^H \), compensates for the decision to reduce expensive technology exploitation (thermal). The substitution of the expensive technology by the free one reduces the price by a quantity equal to \( \frac{\beta b c}{(1 + \beta)(b + c)} \Delta Q^H \).

To satisfy a supplementary demand during the peak season, the producer uses both technologies. The producer reduces indeed his water use during the off-peak season of \( \frac{\beta}{b(1 + \beta)} \Delta a_2 \), in order to increase his plant T production by a quantity equal to \( \frac{\beta}{(b + c)(1 + \beta)} \Delta a_2 \). The technological complementarity allows the storage of the water to face any increase of the peak season demand. In this season, the producer uses indeed all the inter temporal supplementary transfer of water (\( \frac{\beta}{b(1 + \beta)} \Delta a_2 \)) and increases the T production by a quantity equal to \( \frac{1}{(b + c)(1 + \beta)} \Delta a_2 \) in order to satisfy the peak supplementary demand. We notice that the producer increases his use of the thermal plant in different proportions (\( \Delta Q^H_{1t} < \Delta Q^T_{1t} \)). The technological complementarity allows the substitution of the expensive technology by the free one.

B. Private monopoly

The optimal production plan maximizes:

\[
\max_{Q_{1t}'H, [Q_{1t}'_L]} \Pi^{HT} = P_1(Q_{1t}'H + Q_{1t}'T) - \frac{C}{2}(Q_{1t}'T)^2 + \beta \left[ P_2(Q_{2t}'H + Q_{2t}'T) - \frac{C}{2}(Q_{2t}'T)^2 \right]
\]

with respect to (2), (3) and (4).

The first order conditions with respect to H and T plant imply: \( \Pi^{HT} - \lambda^{HT} = \beta \Pi^{HT} - \lambda^{HT} \),

\[
P_1(Q_{1t}'H + Q_{1t}'T) + P_1 - c.Q_{1t}'T = \lambda^{HT}_t \quad \text{and} \quad P_2(Q_{2t}'H + Q_{2t}'T) + P_2 - c.Q_{2t}'T = \lambda^{HT}_t \frac{\beta}{\beta}.
\]

If the constraints are never bounded, we have the following result.

**Proposition 2:** The private monopoly equilibrium is given by:

\[
Q_{1t}'H = \frac{a_1 - \beta a_2}{2b(1 + \beta)} + \frac{\beta}{1 + \beta} \Delta Q^H, \quad Q_{1t}'T = \beta \left( a_1 + a_2 - 2b\Delta Q^H \right) \frac{1}{(1 + \beta)(b + c)}
\]

\[
Q_{2t}'H = \beta a_2 - a_1 + \frac{\Delta Q^H}{1 + \beta} \quad \text{and} \quad Q_{2t}'T = \frac{a_1 + a_2 - 2b\Delta Q^H}{(2b + c)(1 + \beta)}
\]

To satisfy a supplementary peak season demand, the producer reduces his H production at the season 1 by \( \frac{\beta}{2b(1 + \beta)} \Delta a_2 \) in order to store water resources which increase the production of T by \( \frac{\beta}{(1 + \beta)(b + c)} \Delta a_2 \) units that compensates partially for the decrease of the plant H use. The water transfer face the demand growth by a supplementary production via the H plant (\( \frac{\beta}{2b(1 + \beta)} \Delta a_2 \)) and the T plant (\( \frac{\Delta a_2}{(1 + \beta)(b + c)} \)).

We find that the water transfer under a private monopoly is lower than the one under the public monopoly. This behavior guarantees a high price to the private monopoly during the peak season. Any increase of turbinage capacity equal to \( \Delta Q^H \) increases his usage during the off-peak season and the peak season by respectively, \( \frac{\beta}{1 + \beta} \Delta Q^H \) and \( \frac{1}{1 + \beta} \Delta Q^H \). On the other hand, with technological complementarity, the producer reduces his usage of T plant in the off-peak and the peak season by respectively, a quantity equal to, \( \frac{2\beta b}{(1 + \beta)(2b + c)} \Delta Q^H \) and \( \frac{2b}{(1 + \beta)(2b + c)} \Delta Q^H \).

Between the public monopoly and the private one, we notice a disproportion on the substitution of T by H during the two periods. The presence of the hydroelectric plant makes the exploitation problem of the thermal one dynamic, under the private and the public monopoly. In both cases, the technological complementarity played an important role in the efficiency of the electricity supply. The private monopoly tends to transfer less water than the public one.

IV. COMPETITION AND COLLUSION

We consider a decentralized electrical industry where first the two operators compete. Second, we analyze the collusion case.

A. Cournot competition

The H production \( q_{1t}'H = (q_{1t}'H, q_{2t}'H) \) maximizes the inter temporal profit under the H constraints (2) and (3):

\[
\max_{q_{1t}'H, q_{1t}'T, q_{2t}'H} \Pi^{H} = P_1(q_{1t}'H, q_{1t}'T)q_{1t}'H + \beta P_2(q_{2t}'H, q_{2t}'T)q_{2t}'H
\]
The first order conditions yields that the producer seeks to equalize marginal revenues to marginal costs:

$$\frac{P_1'q_1^H + P_1 - \beta(P_2'q_2^H + P_2)}{b(1 + \beta)(3b + 2c)} = \mathcal{A}^H - \mathcal{A}^H$$

The T operator production $q^T = (q_1^T, q_2^T)$ that maximizes the inter temporal profit with respect to the T capacity constraint satisfy the first order condition:

$$\mathcal{A}^T(P_2 + P_2q_2^T - C_1(q_1^2)) = \mathcal{L}^T$$

The Nash equilibrium strategies of the T and H producers, respectively $q^T$ and $q^H$. In order to resolve the equilibrium we distinguish two different cases:

Case 1: If one of the H plant’s constraints is bounded.

We distinguish two cases.

- The inflows of water are scarce: The producer exhausts the inflows of the off-peak season. The first order condition is reduced to the following equality: $\Pi^H_1 - \Pi^H_2 = \beta \Pi^H_2$ and the H production is $(\alpha_1, \alpha_2)$.

- The inflows of water are abundant: The H producer chooses to store water for a future use. The stock reaches the maximum capacity of storage s. The first order condition is reduced to: $\Pi^H_1 = \beta \Pi^H_2 - \Pi^H_1$. Besides, the inter temporal transfer of water implies the vector of production levels $(\alpha(e_1 - s), \alpha(e_2 + s))$.

Case 2: If the constraint is never bounded.

**Proposition 3:** The equilibrium Cournot game is given

$$q_1^H = \frac{(a_1 - \beta a_2)(b + c)}{b(1 + \beta)(3b + 2c)} - \frac{\beta b^H}{(1 + \beta)}$$

$$q_1^T = \frac{a_1[b(2 + 3\beta) + c(1 + 2\beta)] + a_2\beta(b + c)}{(2b + c)(1 + \beta)(3b + 2c)} - \frac{\beta b^H}{(2b + c)(1 + \beta)}$$

$$q_2^H = \frac{\beta a_2 - a_1}{b(1 + \beta)(3b + 2c)} + \frac{\beta q_1^H}{(1 + \beta)}$$

$$q_2^T = \frac{a_2[b(3 + 2\beta) + c(2 + \beta)] + a_1(b + c)}{(2b + c)(1 + \beta)(3b + 2c)} - \frac{b^H}{(2b + c)(1 + \beta)}$$

To face any increase in the peak season’s demand equal to $\Delta a_2$ units of electricity, the hydroelectric producer store during the off-peak season a quantity equal to $\frac{\beta(b+c)}{b(1 + \beta)(3b + 2c)} \Delta a_2$ of water inflows, while the T production increase in the same season by $\frac{\beta(b+c)}{(2b + c)(1 + \beta)(3b + 2c)} \Delta a_2$ units of electricity.

This transfer of water is completely used by H producer during the peak season. However, during this season the T producer increases his production by a quantity equal to $\frac{b(3 + 2\beta) + c(2 + \beta)}{(2b + c)(1 + \beta)(3b + 2c)} \Delta a_2$.

At this stage, we can assess that technological heterogeneity allows more supply security.

B. Collusion

We assume that both firms, H and T, produce quantities that maximized the joint profit. $q^H_{1m} = (q_1^{1m}, q_2^{1m})$ and $q^H_{2m} = (q_1^{2m}, q_2^{2m})$ would be the solutions of the maximization problem with regard to the variables of decision; $q^H_{1m}, q^H_{2m}, q^T_1$ and $q^T_2$:

$$\max_{q^H_{1m}, q^H_{2m}, q^T_1, q^T_2} \Pi^{joint} = \Pi^H_1(P_1(q_1^H, q_1^T), q_1^H) + \beta \Pi^H_2(P_2(q_2^H, q_2^T), q_2^H) + \Pi^T_2(P_2(q_2^T, C(q_2^T)))$$

$$\Pi^T_1(P_1(q_1^T), q^T) + \beta \Pi^T_2(P_2(q_2^T, C(q_2^T)))$$

With respect to (2), (3) and (4).

The H producer must equalize the marginal joint profit of the immediate exploitation of one unity of water in the off-peak season to actualized marginal joint profit water storage until the peak season:

$$\frac{\partial \Pi^{joint}}{\partial q_1^H} - \mathcal{A}^H = \beta \frac{\partial \Pi^{joint}}{\partial q_1^H} - \mathcal{A}^H$$

The thermal producer equalizes the marginal joint profit to zero during the off-peak season. During the peak-season, he equalizes the actualized marginal profit and the marginal cost of the capacity constraint bounding:

$$\frac{\partial \Pi^{joint}}{\partial q_1^T} = 0 \text{ and } \frac{\partial \Pi^{joint}}{\partial q_2^T} = \frac{\mathcal{L}^T}{\beta}.$$
\[
\hat{q}_1^{H} = \frac{a_1 - a_2 + \dot{q}}{2b} \quad \text{and} \quad \hat{q}_1^{T} = \frac{a_1 + a_2 - b}{2(2b + c)}
\]

\[
\hat{q}_2^{H} = \frac{\dot{q}}{2} - \frac{a_1 - a_2}{4b} \quad \text{and} \quad \hat{q}_2^{T} = \frac{a_1 + a_2 - b}{2(2b + c)}
\]

Prices are given by:
\[
\hat{P}_1 = \frac{a_1(4b + 3c) + ca_2 - 2bc\dot{q}^{H}}{4(2b + c)}
\]
\[
\hat{P}_2 = \frac{a_2(4b + 3c) + ca_1 - 2bc\dot{q}^{H}}{4(2b + c)}
\]

The collusion is possible only if the maximum capacity of the plant \(H\) belongs to the interval \([\frac{a_2 - a_1}{2b}, \frac{a_2 + a_1}{2b}]\).

The hydroelectric firm produces more during the peak season. Whereas, the thermal firm produces similar quantities during the two seasons (\(\hat{q}_2^{H} > \hat{q}_1^{H}\) and \(\hat{q}_1^{T} = \hat{q}_2^{T}\)). The maximum capacity affects (negatively) the price. An increase in the maximum capacity implies a more important decrease of the price in competition. Thus, in collusion, the price is less elastic according to the maximum capacity.

V. CONCLUSION

In this paper, we prove that the technological complementarity has an important role in the satisfaction of electricity demand in the different industrial organizations. By the analytical resolution, we show that inter temporal private monopoly water transfer from off peak to peak season is not as high as this same transfer under a public monopole and therefore this increases the power price. Under Cournot competition, an increase in the peak season implies a water transfer from off peak season to the peak season. The results of collusion depend on the availability of water resources and the demand parameters. We remark that in contrast to the competition case, collusion's quantities of \(H\) are not dependable on \(T\)'s costs. Thus, if a variation in the production cost of thermal energy has no effect on hydroelectric quantities, then and under these hypotheses, it is very likely that the competition is eased by a collusive agreement.

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