Cosmic-Ray Transport near the Sun

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Abstract

The strongly diverging magnetic field lines in the very inner heliosphere, through the associated magnetic focusing/mirroring forces, can, potentially, lead to highly anisotropic galactic cosmic-ray distributions close to the Sun. Using a simplified analytical approach, validated by numerical simulations, we study the behavior of the galactic cosmic-ray distribution in this newly explored region of the heliosphere and find that significant anisotropies can be expected inside 0.2 au.

Unified Astronomy Thesaurus concepts: Cosmic rays (329); Heliosphere (711); Solar wind (1534); Interplanetary turbulence (830)

1. Introduction

The Parker Solar Probe (PSP) spacecraft is currently exploring and making the first in situ Galactic cosmic-ray (GCR) measurements in the very inner heliosphere. These include particle gradient measurements (Rankin et al. 2021, 2022), potentially down to the Alfvén radius. However, the Parker (1965) transport equation, used extensively to simulate GCR transport throughout the heliosphere and valid for a nearly isotropic particle distribution, may not be applicable in this region with strongly converging (as seen from Earth) magnetic field lines. In this paper, we present a simplified model to study the (potentially anisotropic) transport of GCRs in this newly explored region of the heliosphere.

2. Field-aligned Transport

We consider only transport along the mean magnetic field, \( B = B_z \), and ignore adiabatic energy losses, as well as any 2D cross-field transport processes, including drifts and perpendicular diffusion. For the mean (or background) magnetic field, we assume a Parker (1958) heliospheric magnetic field (HMF) geometry. In the following sections, we study the field-aligned transport of GCRs using models with varying levels of complexity.

2.1. Loss-cone Distributions

In the absence of magnetic turbulence, the magnetic moment of particles is conserved during propagation and, along with the conservation of kinetic energy, any mirroring/focusing force is accompanied by an interchange between parallel and perpendicular energy: As a particle moves into a region of larger magnetic field strength, its perpendicular speed increases, with the effect that its parallel speed decreases. Ultimately, this causes the particle’s motion to be reversed and the particle is mirrored. However, not all particles entering a magnetic bottle will be mirrored. A particle starting out in a region with field strength \( B_{\text{Earth}} \) with a pitch angle \( \alpha \), and where \( \mu = \cos \alpha \), will not be able to penetrate a region of magnetic field strength \( B \), if

\[
|\mu| > \mu_c = \sqrt{1 - \frac{B_{\text{Earth}}}{B}}. \tag{1}
\]

We can now calculate this critical angle \( \mu_c \) for the case of GCRs, starting out isotropically from Earth, and propagating toward the Sun along \( B \). In Figure 1 we show the mirror ratio \( B_{\text{Earth}}/B \) and the resulting \( \mu_c = \cos^{-1} \mu_c \), as a function of radial distance. The effectiveness of the focusing/mirroring process decreases quickly away from the Sun and becomes negligible beyond Earth’s orbit as the mirror ratio approaches unity. Without any additional process that can drive anisotropic behavior, GCRs are generally observed to be nearly isotropic after propagating (diffusively) from the heliopause to Earth (see, e.g., Gil et al. 2021). In the bottom panels of the figure, we use the calculated \( \mu_c \) to estimate the associated loss-cone distributions at different distances (these distances are indicated by vertical arrows on the upper panels). These loss-cone distributions illustrate how difficult it is for GCRs to reach the inner heliosphere, an effect also studied by, e.g., Hutchinson et al. (2022).

While these results are useful for quantifying the effect of mirroring in the Parker (1958) HMF, they do not capture the full picture of particle transport: The HMF is turbulent on all scales, and these turbulent fluctuations will lead to pitch-angle scattering that tends to isotropize the GCR distribution and also disrupt the general particle drift picture (e.g., van den Berg et al. 2020, 2021). The interplay between scattering (leading to isotropic distributions) and focusing/mirroring (leading to anisotropic distributions) is therefore essential to understand. The additional effect of pitch-angle scattering is considered in the next sections.

2.2. Focused Transport

The evolution of a nearly gyrotrropic distribution function, \( f(\zeta, \mu, t) \), is given by the so-called focused transport equation,

\[
\frac{\partial f}{\partial t} + \mu \frac{\partial f}{\partial \zeta} + \frac{v}{2L} (1 - \mu^2) \frac{\partial f}{\partial \mu} = \frac{\partial}{\partial \mu} \left( D_{\mu \mu} \frac{\partial f}{\partial \mu} \right), \tag{2}
\]
Once \( f \) is obtained, an omnidirectional distribution function can be calculated as

\[
F(z, t) = \frac{\int_{-1}^{1} f(z, \mu, t) d\mu}{\int_{-1}^{1} d\mu} = \frac{1}{2} \int_{-1}^{1} f(z, \mu, t) d\mu,
\]

with an associated first-order anisotropy, given by

\[
A(z, t) = 3 \frac{\int_{-1}^{1} \mu f(z, \mu, t) d\mu}{\int_{-1}^{1} f(z, \mu, t) d\mu} = -\frac{3}{2F(z, t)} \int_{-1}^{1} \mu f(z, \mu, t) d\mu,
\]

and varying between \(-3\) (all particles propagating toward the Sun) and +3 (all particles propagating away from the Sun).

The efficiency of pitch-angle scattering is usually quantified by the parallel mean free path, calculated here following Hasselmann & Wibberenz (1968) as

\[
\lambda^0_\parallel = \frac{3}{8} v \int_{-1}^{1} (1 - \mu^2)^2 \frac{d\mu}{D_{\parallel\mu}(\mu)},
\]

which is related to the parallel diffusion coefficient through

\[
\kappa^0_\parallel = \frac{v}{3} \lambda^0_\parallel.
\]

For this work, we assume \( \lambda^0_\parallel = \lambda \) to be a constant.

### 2.3. Nearly Isotropic Formulation with Weak Focusing

Following Litvinenko & Schlickeiser (2013), we can derive an approximate transport equation for \( F(z, t) \) from Equation (2) by expanding \( f \) in terms of \( F \) and a small correction factor \( F_1(z, \mu, t) \),

\[
f(z, \mu, t) = F(z, t) + F_1(z, \mu, t),
\]

where

\[
\int F_1 d\mu = 0,
\]

and \(|F_1| \ll f\). This corresponds to the case of a nearly isotropic distribution, which is possible when \( \lambda^0_\parallel \ll L \) so that the transport is diffusion dominated in this weak-focusing/mirroring limit. Note that, from here, the explicit dependence on the quantities \( z, \mu, \) and \( t \) are not included. By furthermore assuming that \( \lambda^0_\parallel \) and \( L \) are constant, Litvinenko & Schlickeiser (2013) show that the evolution of \( F \) is governed by the following pseudodiffusion equation:

\[
\frac{\partial F}{\partial t} + \frac{\kappa^0_\parallel}{L} \frac{\partial F}{\partial z} = \kappa^0_\parallel \frac{\partial^2 F}{\partial z^2},
\]

which is in the form of a diffusion equation but with the addition of a coherent advection speed

\[
u_\parallel = \kappa^0_\parallel L.
\]

Note that the \( \kappa_\parallel \) introduced here differs from \( \kappa^0_\parallel \) as defined in Equation (7) and includes a correction to account for focusing/mirroring effects. However, in the weak-focusing/mirroring case considered here, we can approximate \( \kappa_\parallel \approx \kappa^0_\parallel \) and use the two quantities interchangeably.
The diffusive streaming flux for this scenario is given by
\[ S = \frac{v}{2} \int \mu F_i d\mu \leq L - \kappa_{||} \frac{\partial F}{\partial z}. \] (12)

In deriving the results presented here, Litvinenko & Schlickeiser (2013) assume a constant \( L \), which is definitely not the case for a Parker (1958) HMF; see the top panel of Figure 1. However, these authors also state that the analytical approximation should still be valid as long as \( L \) is constant on the scale of \( \lambda_{||} \). This is a reasonable approximation for constant (i.e., independent of radial distance) \( \lambda_{||} \), with values between 0.1 and 1 au, assumed in this work. However, modeling results of mostly solar energetic particles have shown that \( \lambda_{||} \) can vary significantly from event to event (e.g., Dröge 2000), while theoretical calculations indicate that \( \lambda_{||} \) can have a complicated radial dependence inside 1 au (e.g., Strauss & le Roux 2019).

2.4. Isotropic Formulation with No Focusing

In the case of vanishing focusing/mirroring, \( L \rightarrow \infty \), we have the evolution of the isotropic diffusion equation,
\[ \frac{\partial F}{\partial t} = \kappa_{||} \frac{\partial F}{\partial z}, \] (13)
which follows directly from Equation (10).

3. Simulation and Analytical Results

For the simulation and analytical results presented in this section, we specify an isotropic GCR distribution, with an associated (normalized) omnidirectional intensity of \( F \) (1.2 au) = 100, at the position of Earth, i.e., \( z = 1.2 \) au. At the inner boundary, located at \( z = 0.05 \) au—the approximate position of the Alfvén radius recently crossed by the PSP spacecraft (Kasper et al. 2021)—an absorbing boundary condition is assumed where \( F(0.05 \text{ au}) = 0 \). Unfortunately, we cannot extend the simulations below the Alfvén radius as the magnetic fields in this region are expected to deviate significantly from the large-scale Parker magnetic field assumed in this work.

From the omnidirectional intensity, we also compute and show the spatial gradient
\[ g(z) = \frac{1}{F} \frac{\partial F}{\partial z}, \] (14)
which is multiplied by 100% in all simulation results to be comparable to measurements, which are usually expressed in this fashion. While, usually, a radial gradient is calculated and/or measured, the gradient calculated here is directed along the mean HMF, so that we can refer to it as a magnetic-field-aligned gradient.

3.1. Isotropic Formulation with No Focusing

We start with solving Equation (13) by assuming a steady-state solution, \( \partial F/\partial t = 0 \), from which we directly obtain
\[ \frac{\partial F}{\partial z} = \text{constant} \Rightarrow F = \text{constant} \cdot z, \] (15)
with a gradient of \( g(z) = 1/z \) and, of course, an anisotropy value of \( A = 0 \).

3.2. Nearly Isotropic Formulation with Weak Focusing

A steady-state assumption for Equation (10) directly leads to a gradient of
\[ \frac{1}{F} \frac{\partial F}{\partial z} = \frac{1}{L} = g(z), \] (16)
while \( F(z) \) can be obtained by substituting the definition of \( L(z) \), given by Equation (3), and solving for \( F(z) \) in terms of the magnetic field \( B(z) \), to obtain \( d \ln F = -d \ln B \), which in turn leads to
\[ F(z) \cdot B(z) = \text{constant}. \] (17)

By using Equation (12) for the streaming flux, the anisotropy follows as
\[ A(z) = \frac{3S}{vF} = -\frac{\lambda_{||}}{L(z)}. \] (18)

Note that this expression diverges for strong focusing/mirroring, \( A \rightarrow -\infty \) as \( L \rightarrow 0 \). This is due to the assumption of weak focusing/mirroring used when driving Equation (10), i.e., \( \lambda_{||} \ll L \). This strict condition is not satisfied for all parameters assumed in this work.

3.3. Focused Transport Equation

For the simulations presented here we assume a parameterized version of \( D_{\mu\mu} \) from quasi-linear theory (Jokipii 1966),
\[ D_{\mu\mu} = D_0(1 - \mu^2)(|\mu|^q - 1 + H), \] (19)
with \( q = 5/3 \) representing the inertial range turbulence spectral index and \( H = 0.05 \) accounting for possible dynamic effects in an ad hoc fashion (e.g., Beeck & Wibberenz 1986). We do not, however, calculate \( D_0 \) from first principles (as is done by, e.g., Strauss et al. 2017), but rather specify the \( \lambda_{||} \) from which \( D_0 \) can be calculated from Equation (6). Equation (2) is solved with the open-source numerical model presented in van den Berg et al. (2020) and available online.

The resulting particle intensities, for different choices of \( \lambda_{||} \), are shown in Figure 2. For all simulations, protons with a kinetic energy of 100 MeV are assumed. However, with this specific model setup (including an energy-independent \( \lambda_{||} \), no energy losses, and the assumption of a steady-state solution), the results are independent of the particle energy considered.

The left panels of Figure 2 correspond to the case of \( \lambda_{||} = 0.1 \) au, while the right panels correspond to \( \lambda_{||} = 1 \) au. Although the mean free path is treated, in this work, as a free parameter, it is chosen to correspond to consensus values at Earth from, e.g., Bieber et al. (1994), who reported values ranging from \( \sim 0.1 \) to 1 au for protons in the rigidity range of \( \sim 10^{-4} \) MV. The top panels show the computed omnidirectional intensity, the calculated gradient, and the resulting anisotropy as a function of magnetic distance, \( z \). Results from the numerical model, solving Equation (2), are shown as red symbols, the case of weak scattering, i.e., solving Equation (10), as the solid blue line, and the case of the isotropic diffusion equation, Equation (13), as the gray dashed lines. The intensities, and hence also the gradients, as calculated by the numerical model and the analytical approach assuming weak focusing/mirroring compare relatively well.

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5 https://github.com/RDStrauss/SEP_propagator
However, the analytical approach cannot capture the radial dependence of the anisotropy as $\lambda_\parallel \ll L$ is generally violated for all assumed parameters. The diffusion equation, assuming no focusing/mirroring, gives a smaller gradient compared to the other approaches.

The bottom panels of Figure 2 show the pitch-angle distribution of the particles at different radial positions (these positions are indicated by the vertical arrows in the upper panels) as calculated from the numerical model. In these graphs, $\alpha(\mu) = 0^\circ (1)$ indicates particles streaming away from the Sun and $\alpha(\mu) = 180^\circ (-1)$ labels particles moving toward the Sun. The black shaded regions show the pitch-angle regions close to the model’s boundaries where information about the particle distribution is not available. For the assumed values of $\lambda_\parallel$, strong anisotropies, and significant anisotropic behavior, are only observed very close to the inner boundary (e.g., inside 0.1 au).

### 4. Discussion

While the loss-cone distributions associated with the Parker (1958) HMF predict highly anisotropic GCR distributions in the inner heliosphere (see Figure 1), the inclusion of scattering tends to lead to more isotropic distributions (see Figure 2), except in the very inner heliosphere, i.e., $z < 0.2$ au. In general, the cosmic-ray distribution will be isotropic if pitch-angle scattering dominates the focusing/mirroring process, i.e., when $\lambda_\parallel \ll L$. While $L$ is determined by the Parker (1958) HMF geometry, $\lambda_\parallel$ depends (in a nontrivial fashion) on the underlying turbulence structure and how particles resonate
with these fluctuations, making $\lambda_{||}$ difficult to quantify, especially in the newly explored regions closer to the Sun. However, even when adopting simplified forms of $\lambda_{||}$, the results of our numerical model suggest small but measurable first-order anisotropies in this region, increasing in magnitude toward the Sun, with a deficiency of outwardly (i.e., away the Sun) propagating GCRs due to the mirroring effect. Larger anisotropies are expected for particles with larger values of $\lambda_{||}$, such as high-rigidity cosmic rays during solar minimum conditions.

Interestingly, our results generally show that assuming an isotropic distribution governed by a diffusion equation, and therefore ignoring particle focusing/mirroring near the Sun, leads to an underestimation of the spatial gradient when compared to the numerical solution. This can potentially explain the discrepancy between measured and calculated gradients in the very inner heliosphere as shown by Rankin et al. (2021, 2022) for anomalous cosmic rays. Additionally, this suggests that traditional modulation models, based on solutions of the Parker (1965) transport equation, are not valid close to the Sun where focusing/mirroring forces have appreciable effects.

This study focused on particle transport along the mean magnetic field (essentially a 1D spatial geometry assuming scattering by quasi-wave-like parallel-propagating turbulence) and ignored any cross-field transport processes including drifts and perpendicular diffusion that will lead to 2D transport effects. At present, it is not clear whether such cross-field processes will have appreciable effects on particle anisotropies, especially given recent PSP measurements of a dominating quasi-parallel turbulent component inside 1 au (Bandyopadhyay & McComas 2021) and the fact that the higher turbulence levels in this region will significantly disrupt the drift process (van den Berg et al. 2021). Additionally, it should be kept in mind that pitch-angle scattering is primarily responsible for the isotropization of the distribution function, and given the fact that we do include this process in our modeling approach, we do not expect the results presented here to qualitatively change when additional cross-field processes are included in future work. Additionally, Ruffolo (1995) showed that the effect of adiabatic energy losses becomes negligible for solar energetic particle protons above 20 MeV. As such, we do not expect that the inclusion of adiabatic energy losses will have any appreciable effects on the simulation results presented here for higher energy GCRs.

5. Conclusion

Traditional cosmic-ray transport models ignore the magnetic mirroring effect close to the Sun. However, with the PSP spacecraft currently exploring the very inner heliosphere, it has become necessary to quantify possible mirroring effects on the anomalous cosmic-ray and GCR distributions. Such a study is presented in this work where we have included the effects of both magnetic mirroring and pitch-angle scattering. Our results predict measurable cosmic-ray anisotropies close to the Sun, i.e., inside $z < 0.2$ au, that generally increase with increasing particle rigidity (i.e., increasing $\lambda_{||}$) toward the Sun.

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