Variable Speed of Light Cosmology: An Alternative to Inflation

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Abstract

It is generally believed that inflationary cosmology explains the isotropy, large scale homogeneity and flatness as well as predicting the deviations from homogeneity of our universe. We show that this is not the only cosmology which can explain successfully these features of the universe. We consider anew and modify a model in which local Lorentz invariance is spontaneously broken in the very early universe, and in this epoch the speed of light undergoes a first or second order phase transition to a value \( \sim 30 \) orders of magnitude smaller, corresponding to the presently measured speed of light. Before the phase transition at a time \( t \sim t_c \), the entropy of the universe is reduced by many orders of magnitude, allowing for a semiclassical quantum field theory calculation of a scale invariant fluctuation spectrum. After the phase transition has occurred, the radiation density and the entropy of the universe increase hugely and the increase in entropy follows the arrow of time determined by the spontaneously broken direction of the vev \( \langle \phi^a \rangle_0 \). This solves the enigma of the arrow of time and the second law of thermodynamics. A new calculation of the primordial Gaussian and adiabatic fluctuation spectrum is carried out, leading to a scale invariant scalar component of the power spectrum. We argue that there are several attractive features of VSL theory compared to standard inflationary theory, and that it provides an alternative cosmology with potentially different predictions.

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1 Introduction

It is ten years ago that an alternative solution to the initial value problems of cosmology based on a variable speed of light (VSL) was published [1]. The model was based on the idea that in the very early universe at a time $t \sim t_P \sim 10^{-43}$ sec., where $t_P$ denotes the Planck time, the local Lorentz invariance of the ground state of the universe was spontaneously broken by means of a non-zero vacuum expectation value (vev) of a vector field, $\langle \phi^a \rangle_0 \neq 0$, where $a$ labels the flat tangent space coordinates of four-dimensional spacetime. At a temperature $T < T_c$, the local Lorentz symmetry of the vacuum was restored corresponding to an “anti-restoration” of the symmetry group $SO(3, 1)$. Above $T_c$ the symmetry of the ground state of the universe was broken from $SO(3, 1)$ down to $O(3)$, and the domain formed by the direction of $\langle \phi^a \rangle_0$ produced an arrow of time pointing in the direction of increasing entropy and the expansion of the universe.

The notion that as the temperature of the universe increases, a larger symmetry group $SO(3, 1)$ can spontaneously break to a smaller group $O(3)$ seems counter-intuitive. Heating a superconductor restores gauge invariance, and heating a ferromagnet restores rotational invariance. Anti-restoration would appear to violate the second law of thermodynamics. This, however, is not the case, for certain ferroelectric crystals such as Rochelle or Seignette salt, possess a smaller invariance group above a critical temperature, $T = T_c$, than below it [2] Explicit models of 4-D field theories have been constructed in which the symmetry non-restoration of symmetries occurs at high temperatures [3].

Recently, Hollands and Wald [4] investigated the issue of whether an alternative to inflation theory could exist and predict a scale invariant fluctuation spectrum in the early universe. They also considered the issue of fine-tuning in inflationary cosmology, and argued that the present models of inflation do not avoid fine-tuning of the initial conditions of the universe, although they do improve considerably the extreme fine-tuning that occurs in the standard Friedmann, Robertson and Walker (FRW) big bang model. They stressed that an important issue in the initial value problem in cosmology is to explain how the second law of thermodynamics came into being.

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2 Rochelle salt possesses a lower Curie point at $-18^\circ C$, below which the Rochelle crystal is orthorhombic and above which it is monoclinic.

3 Temperature dependent models of spontaneous symmetry breaking of $SO(3, 1) \rightarrow O(3)$ with increasing temperature will be investigated in a separate article.
Kofman, Linde and Mukhanov \cite{5} argued that inflation theory is not as fine-tuned in the initial universe as claimed by Hollands and Wald, although the avoidance of such a fine-tuning relies to some extent on the use of the “anthropic principle”, in that chaotic inflation postulates enough initial patches of potential inflation such that one of them can develop enough e-folds of inflation to solve the horizon and flatness problems in our universe \cite{6}. They also stressed that without a scenario such as inflation in the early universe, it would not be possible to dilute the initial density of radiation and matter to allow a sensible semi-classical quantum field theory calculation of quantum fluctuations. Indeed, the fluctuation calculation of Hollands and Wald would have to be performed at a density, $\rho \sim 10^{95} \rho_P$, where $\rho_P = c^5/\hbar G^2$ is the Planck density, a density so large that it would not allow any standard quantum field theory calculations to be carried out. Inflation models do significantly dilute the radiation and matter density of the early universe and they also exponentially reduce the entropy of the universe, leading to a resolution of the flatness problem. However, the matter density and entropy have to be re-instated by a period of re-heating in which the inflaton field and the large vacuum energy undergo decay.

This then leads us inevitably to the question: Does there exist an alternative to inflation, which can successfully allow a quantum field theory calculation of a scale invariant primordial spectrum? In spite of the successes of inflation theory, it is important to seek alternatives to it to see whether a different scenario could overcome some of the shortcomings of inflation, such as the problem of vacuum energy, the fine-tuning of the coupling constant to give the correct density profile in the present universe, and the unnaturally flat potentials needed to solve the initial value problems. In the following, we shall consider anew the VSL cosmology associated with a spontaneous symmetry breaking of Lorentz invariance and a phase transition in the speed of light in the very early universe.

Alternative VSL models, such as those considered by Albrecht and Maguiejo, and Barrow \cite{7} were based on a “hard” breaking of Lorentz invariance. We, instead, attempt to emulate the successes of the standard model of particle physics \cite{8}, in which “soft”, spontaneous breaking of the internal symmetries by a Higgs mechanism plays a crucial role. In our scenario, local Lorentz symmetry is simply an accident of nature, i.e. the ground state of our current universe happens to be found in a particular false vacuum, and transitions away from this ground state may well have happened in the early universe.

Another alternative model of VSL theory has been based on a bimetric
theory of gravity [9, 10, 11]. In this model there are two metrics in which the light cones are associated, respectively, with the speed of light and the speed of gravitational waves, and they are linked by the gradient of a scalar field. These models maintain local relativistic invariance and diffeomorphism invariance, and do resolve the early universe initial value problems. The basic parameter in these models is the dimensionless ratio of the speed of gravitational waves (graviton) to the speed of light (photon). However, a calculation of the primordial scale invariant fluctuation spectrum has, so far, only been performed by using a “slow roll” approximation for the potential of the scalar field that links both metrics [10]. Such a scheme falls into the category of inflationary cosmology. Magueijo [12] has also published a version of a relativistic VSL model, but this model has not yet succeeded in producing a viable scale invariant fluctuation spectrum, which must play a crucial role in confirming models of the early universe.

A vierbein $e^a_\mu$ is used to convert $\phi^a$ into a 4-vector in coordinate space: $\phi^\mu = e^a_\mu \phi^a$ and it satisfies:

$$e^a_\mu e^\mu_b = \delta^a_b, \quad e^\mu_a e_\nu = \delta^\mu_\nu. \tag{1}$$

The vierbein obeys the Lorentz transformation rule

$$e^a_\mu(x) = L^b_a(x) e^b_\mu(x). \tag{2}$$

The metric tensor is obtained from the vierbeins by the formula

$$g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu. \tag{3}$$

The covariant derivative operator acting on $\phi^a$ is defined by

$$D_\mu \phi^a = [\partial_\mu \delta^a_b + (\Omega_\mu)_b^a] \phi^b, \tag{4}$$

where $(\Omega_\mu)_b^a$ denotes the spin, gauge connection:

$$\Omega_\mu = \frac{1}{2} \sigma^{ab} e^a_\mu \nabla^b \epsilon \epsilon^b, \tag{5}$$

and $\nabla_\mu$ denotes covariant differentiation with respect to the Christoffel symbol $\Gamma^\lambda_{\mu\nu}$:

$$\Gamma^\lambda_{\mu\nu} = g^{\lambda\rho} \eta_{ab} (D_\mu e^a_\nu) e^b_\rho. \tag{6}$$

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4 The indices $\mu, \nu...$ and $a, b...$ run from 0, ..., 3 and the Minkowski metric signature is $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. 

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Moreover, the $\sigma_{ab}$ are the six generators of the Lorentz group.

We shall describe the dynamical behavior of the speed of light $c(x)$ by a scalar field: $c(x) = \bar{c}\chi(x)$ where $\bar{c}$ is a constant with the dimensions of velocity. The total action of the theory is

$$S = S_G + S_M + S_\phi + S_\chi,$$

where

$$S_G = -\frac{c^4}{16\pi G} \int d^4 x e (R + 2\Lambda),$$

and $e = \sqrt{-g} = \det(e^a_\mu e^a_\nu)$, $\Lambda$ is the cosmological constant and $S_M$ is the matter action. Moreover,

$$S_\phi = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} D_\mu \phi_a D^\mu \phi_a - V(\phi) \right].$$

We demand that $\phi^a$ (or $\phi^\mu$) be a timelike vector, which ensures that the kinetic energy term $D_\mu \phi_a D^\mu \phi^a > 0$ for all events in the past and future light cones of the flat tangent space, which avoids the occurrence of negative energy modes in the Hamiltonian. We could add a Lagrange multiplier term to the action to guarantee the timelike nature of the vector $\phi^a$.

We also have

$$S_\chi = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} D_\mu \chi D^\mu \chi - V(\chi) - V(\chi \phi) \right],$$

where $V(\phi \chi)$ denotes a potential energy contribution coupling the fields $\phi^\mu$ and $\chi$ (e.g. a Yukawa coupling contribution $\partial_\mu \phi^\mu \chi$).

We choose the potential $V(\phi)$ to be of the form

$$V(\phi) = -\frac{1}{2} \mu^2 \phi_a \phi^a + \lambda (\phi_a \phi^a)^2,$$

where $\phi_a \phi^a > 0$ and the coupling constant $\lambda > 0$, so that the potential is bounded from below. If $V$ has a minimum at $\phi_a = v_a$, then the spontaneously broken solution is given by $v_a^2 = \mu^2 / 4\lambda$. We can choose $\phi_a$ to be

$$\phi_a = \delta_{a0} v = \delta_{a0} (\mu^2 / 4\lambda)^{1/2}.$$
All the other solutions of $\phi_a$ are related to this one by a Lorentz transformation. Then, the homogeneous Lorentz group $SO(3,1)$ is broken down to the spatial rotation group $O(3)$. The three rotation generators $J_i$ ($i = 1, 2, 3$) leave the vacuum invariant, $J_i v_i = 0$, while the three Lorentz-boost generators $K_i$ break the vacuum symmetry, $K_i v_i \neq 0$.

Let us consider small oscillations about the true minimum and define a shifted field $\phi'_a = \phi_a - v_a$. By performing a Lorentz transformation, we obtain

$$
\phi^0 = \psi, \quad \phi^1 = \phi^2 = \phi^3 = 0.
$$

In this special coordinate frame, the remaining component $\psi$ is the scalar physical particle that survives after the three Goldstone modes have been removed. This corresponds to the “unitary gauge” in the standard electroweak theory. In the broken phase, the Einstein tensor $G^{\mu \nu} = R^{\mu \nu} - \frac{1}{2} g^{\mu \nu} R$ still satisfies the Bianchi identities: $\nabla_\nu G^{\mu \nu} \equiv 0$, but the conservation law for the energy momentum tensor is modified to be

$$
\nabla_\nu T^{\mu \nu} = -\nabla_\nu (K^{\mu \nu} + H^{\mu \nu}),
$$

where $K^{\mu \nu}$ and $H^{\mu \nu}$ are non-vanishing contributions that arise in the spontaneously broken phase due to the “Higgs mechanism” for the spin gauge field $\Omega_\mu$, and the energy momentum tensor for the physical fields $\psi$ and $\chi$, respectively. In the unbroken phase, we regain the standard energy momentum conservation law $\nabla_\nu (T^{\mu \nu} + H^{\mu \nu}) = 0$, since $K^{\mu \nu} = 0$ and the spin connection becomes that of a massless graviton gauge field.

### 2 Variable Speed of Light and Solutions to the Horizon and Flatness Problems

In the spontaneously broken phase of the evolution of the universe, the space-time manifold has been broken down to $R \times O(3)$. The three-dimensional space with $O(3)$ symmetry is assumed to be the homogeneous and isotropic FRW solution:

$$
d\sigma^2 = R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right].
$$

where $k = 0, +1, -1$ corresponding to a flat, closed and open universe, respectively, and $t$ is the external time variable. This describes the space of our
ordered ground state in the symmetry broken phase and it has the correct
subspace structure for our FRW universe with the metric
\[ ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = dt^2c^2(t) - R^2(t)\left[\frac{dr^2}{1-k_r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \tag{16} \]

The Newtonian “time” \( t \) is the absolute time measured by standard clocks.

In the spontaneously broken Lorentz symmetry phase, we can now have
the speed of light \( c \) undergo a phase transition, since we are no longer required
to satisfy Einstein’s second postulate of special relativity: The speed of light \( c \)
is the same constant with respect to all observers irrespective of their motion
and the motion of the source.

Close to the phase transition at the time \( t \approx t_c \), we assume that
\[ c(t) = c_0\theta(t_c - t) + c_m\theta(t - t_c), \tag{17} \]
where \( \theta(t) \) is the Heaviside step function which satisfies \( \theta(t) = 1 \) for \( t > 0 \)
and \( \theta = 0 \) for \( t < 0 \). Moreover, \( c_0 \) and \( c_m \) denote the values of \( c \) before and
after the phase transition, respectively, where \( c_m = 299792458 \text{ m s}^{-1} \) is the
presently measured value of \( c \) and \( c_0 \gg c_m \).

The metric \( g_{\mu\nu} \) now has the bimetric form:
\[ g_{\mu\nu} = g_{0\mu\nu} + g_{m\mu\nu}, \tag{18} \]
where
\[ ds_0^2 \equiv g_{0\mu\nu}dx^\mu dx^\nu = dt^2c_0^2\theta(t_c - t) - R^2\left[\frac{dr^2}{1-k_r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{19} \]
and
\[ ds_m^2 \equiv g_{m\mu\nu}dx^\mu dx^\nu = dt^2c_m^2\theta(t - t_c) - R^2\left[\frac{dr^2}{1-k_r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \tag{20} \]

The phase transition in \( c \) produces two light cones: \( ds_0^2 = 0 \) and \( ds_m^2 = 0 \)
and their relative sizes are determined by the dimensionless ratio \( \gamma = c_0/c_m \). When \( \gamma = 1 \) and there is no phase transition the model becomes
the same as local special relativity with one light cone. When \( \gamma \) becomes
very large in the spontaneously broken phase, the Minkowski light cone,
determined by the metric (21), is contained within the much larger light
cone determined by the metric (19). As in alternative bimetric theories,
a diffeomorphism transformation in time cannot remove the speed of light dependence from both metrics $g_{0\mu\nu}$ and $g_{m\mu\nu}$ simultaneously. Only when $\gamma = 1$ can a diffeomorphism time transformation $dt' = dt c(t)$ remove $c(t)$ completely, for then we have only one light cone and one speed of light corresponding to local special relativity.

The proper horizon scale is given by

$$d_H(t) = R(t) \int_0^t \frac{dt' c(t')}{R(t')}.$$  \hspace{1cm} (21)

We obtain for $t > t_c$ the usual result, $d_H \sim 2c_m t$, since for a radiation dominated universe $R(t) \propto t^{1/2}$ and $c = c_m$. On the other hand, we have for a radiation dominated universe in the spontaneously broken phase before the phase transition in $c(t)$, $d_H \sim 2c_0 t$, and for $\gamma \to \infty$ the proper horizon size is stretched and this means that all observers in the spontaneously broken phase were in causal contact. The forward light cone beginning at the time of the big bang is considerably expanded for $\gamma \to \infty$ and is made larger than the region from which microwave photons are reaching us today and this solves the isotropy problem.

To see how the flatness problem is resolved, we write the Friedmann equation in the spontaneously broken phase:

$$H^2 + \frac{c^2 k}{R^2} = \frac{8\pi G \rho}{3} + \frac{c^2 \Lambda}{3},$$  \hspace{1cm} (22)

where $H = \dot{R}/R$. We set the cosmological constant $\Lambda = 0$, and obtain

$$\epsilon \equiv |\Omega - 1| = \frac{c^2 |k|}{R^2},$$  \hspace{1cm} (23)

where $\Omega = 8\pi G \rho / 3H^2$. We now find that

$$\dot{\epsilon} = -\frac{2c^2 |k| \dot{R}}{R^3} + 2 \left( \frac{\dot{c}}{c} \right) \left( \frac{c^2 |k|}{R^2} \right).$$  \hspace{1cm} (24)

For a radiation dominated universe $\ddot{R} < 0$ and for a speed of light $c$ that decreases in a phase transition to the small value $c_m$, we have $\dot{c}/c < 0$ and $\dot{\epsilon} < 0$ corresponding to an attractor solution with $\epsilon \sim 0$ and an approximately spatially flat universe.
An alternative way of seeing how the flatness problem is resolved is to write

$$\Omega(t) = 1 + x(t),$$

(25)

where

$$x(t) = \frac{c^2 k}{R^2 H^2} \sim \frac{c^2 k/R^2}{8\pi G \rho_r/3},$$

(26)

where $\rho_r$ is the radiation density, $\rho_r = \rho_{0r} \left( R_0/R \right)^4$ with $\rho_{0r}$ and $R_0$ denoting the present values of the radiation density and $R$, respectively. Then, we have $x \sim c^2 k R^2/R^*$ where $R^* = 8\pi G \rho_{0r} R_0^4/3$. This yields close to the phase transition with $\gamma = c_0/c_m$:

$$|\Omega(10^{-43}\text{ sec}) - 1| \sim \mathcal{O}(\gamma^2 10^{-60}).$$

(27)

Thus, in the time the universe is in the broken phase before the phase transition in the speed of light, we obtain for $\gamma \sim 5 \times 10^{29}$:

$$|\Omega(10^{-43}\text{ sec}) - 1| \sim \mathcal{O}(1),$$

(28)

which implies much less fine-tuning than the standard FRW model.

## 3 The Entropy Problem and the Arrow of Time

A calculation of the energy density of photons gives

$$\mathcal{E}_\gamma = \sigma_B T^4,$$

(29)

where $\sigma_B$ is the Stefan-Boltzmann constant, $\sigma_B = \pi^2 k_B^4/15c_m^3 h^3 = 7.5641 \times 10^{-15} \text{ erg cm}^{-3} K^{-4}$ ($k_B$ is Boltzmann’s constant). For $\log_{10} \gamma \sim 30$, we see that the Stefan-Boltzmann constant is significantly reduced to: $\sigma_B = \pi^2 k_B^4/15c_0^3 h^3 \sim 7.56 \times 10^{-104} \text{ erg cm}^{-3} K^{-4}$. Thus, in the early universe in the symmetry restored phase for a temperature $T \sim 10^{12} K$, we have $\mathcal{E}_\gamma \sim 7.6 \times 10^{33} \text{ erg cm}^{-3}$ whereas in the spontaneously broken phase the energy density of photons is significantly diluted, $\mathcal{E}_\gamma \sim 400 \text{ erg cm}^{-3}$.

The entropy of relativistic particles is given by [15]:

$$S = \frac{R^3}{T} (\rho c^2 + p) = \frac{4\sigma_B}{3} (RT)^3 f,$$

(30)
where $f$ is a numerical factor of order unity. For $\gamma = 1$, $T \sim 10^{12} K$ and $R \sim 10^{28} \text{cm}$ we get $S \sim 10^{118} \text{erg} K^{-1}$. In the broken symmetry phase, near the phase transition in $c$, and for $\log_{10} \gamma \sim 30$ this will be reduced to the small entropy, $\sim 300 \text{erg} K^{-1}$. The phase transition in the speed of light with $\gamma \to 1$, results in an enormous increase in the entropy of the universe.

A similar situation occurs in inflationary models in which the exponential expansion of the universe decreases considerably the entropy and results in a re-heating phase when inflation ceases.

In our VSL scenario, the large increase in entropy at the phase transition time $t \sim t_c$ is in the direction of the spontaneous symmetry breaking domain $\langle \phi^a \rangle_0 \neq 0$, which corresponds to the direction of the arrow of time in the expanding universe. To solve the problem of the arrow of time and the second law of thermodynamics, we should expect that the entropy of the universe at or near the big bang should be small [16]. The sudden large increase in entropy in our VSL scenario also leads to a solution of the flatness problem, as in the case of inflationary models.

In the spontaneously broken phase near the phase transition $c_0 \to \infty$, the Planck length $L_P = \sqrt{\hbar G/c_0^3} \to 0$, and the Planck density $\rho_P = c_0^3/\hbar G^2 \to \infty$. Thus, the super-Planck density is far removed from the region in the spontaneously broken phase, in which the radiation energy $E$, and the entropy $S$ are diluted as $c_0 \to \infty$, and we do not have to concern ourselves with ultra-Planck energy corrections to the primordial fluctuation spectrum, which is calculated in the spontaneously broken phase.

4 Calculation of Scale Invariant Fluctuation Spectrum

Inflationary models provide a simple and successful answer to how the departures from inhomogeneity arise from quantum fluctuations in the early universe. This prediction of a scale invariant spectrum has been confirmed during the past two years by high precision measurements of the cosmic microwave background (CMB) [17]. We shall now show how our VSL model can predict equally well a scalar, adiabatic Gaussian scale-free perturbation spectrum. We shall use a simple method for calculating the spectrum, avoiding many of the technical details, so that we can see how the mechanism works at an intuitive level [4].
We shall consider a simple model of a free, minimally coupled scalar field \( \psi \), which we identify with our physical field \( \psi \) in the “unitary gauge” after the three Goldstone modes have been removed in our model of spontaneous symmetry breaking of Lorentz invariance. We choose for simplicity the flat spacetime with \( k = 0 \). The scalar field \( \psi \) is pictured as a plane wave mode with coordinate wave vector \( \vec{k} \):

\[
\psi(\vec{x}, t) = \psi_k(t) \exp(i\vec{k} \cdot \vec{x}),
\]

(31)

which satisfies

\[
\ddot{\psi}_k + 3H \dot{\psi}_k + \frac{c^2 k^2}{R^2} \psi_k = 0,
\]

(32)

and we have defined

\[
\psi_k = \frac{1}{(2\pi)^{3/2}} \int d^3 x \psi(\vec{x}) \exp(-i\vec{k} \cdot \vec{x}).
\]

(33)

Here, we consider that the quantum fluctuation modes are created in the spontaneously broken ground state and that their proper wavelength \( \lambda_p \) is tiny compared to the Hubble radius, \( R_H = c/H \). The equation of motion for the dynamical field \( \chi(t) \) in the preferred frame gauge, \( \phi^0 = \psi \), is of the form:

\[
\ddot{\chi} + 3H \dot{\chi} + \frac{dV(\chi)}{d\chi} + \frac{dV(\phi \chi)}{d\chi} + I(\chi, g) = 0,
\]

(34)

where \( I(\chi, g) \) denotes the contribution coming from the variation of \( \chi \) in the Einstein-Hilbert action \( S_G \) in (8). A possible solution for \( c(t) = \bar{c}\chi(t) \) given the potentials \( V(\chi), V(\phi \chi) \) and \( I(\chi, g) \) is

\[
c(t) = \frac{a}{t^b} + c_0 \theta(t_c - t) + c_m \theta(t - t_c),
\]

(35)

where \( c(t) \to c_0 \) from above as \( t \to t_c \).

Eq. (32) has the same form as the harmonic oscillator equation with a unit mass, a variable spring constant \( c^2 k^2 / R^2 \), and a variable friction damping coefficient \( 3H \). The Lagrangian for our harmonic oscillator has the form

\[
L_k = \frac{R^3}{2} \left( \dot{\psi}_k^2 - \frac{c^2 k^2}{R^2} \psi_k^2 \right).
\]

(36)

The ground state of the oscillator at some fixed time \( t \) has the form of a Gaussian wave function, with a spread given by

\[
(\Delta \psi_k)^2 = \frac{1}{2R^2ck}.
\]

(37)
In the case of generic inflationary models, we have $R(t) \sim \exp(HT)$ and $H$ is constant in time. When the proper wavelength, $\lambda_p = R/k$, of the normal mode is much smaller than the Hubble radius $R_H = c_m/H$, the mode oscillates like an ordinary harmonic oscillator with small damping, and the adiabatic vacuum corresponds to that of a flat Minkowski spacetime. However, when $\lambda_p$ is much greater than $R_H$ the mode enters an overdamped phase with $\dot{\psi}_k \sim 0$ and the mode “freezes”. Indeed, we have that $\lambda_p = R/k \sim \exp(HT)$, while the Hubble radius $R_H$ remains constant during the inflationary period, so that the proper wavelengths of the normal modes quickly overtake the horizon and make a frozen imprint on the spacetime metric. On the other hand, in the standard FRW model for the radiation equation of state, $p = \frac{1}{3} \rho$, we obtain $\lambda_p \propto t^{1/2}$ while $R_H \sim 2c_m t$, so that the proper wavelengths of the initial tiny wave modes never catch up to the Hubble horizon and cross it to produce a scale-free fluctuation spectrum.

For our spontaneously broken VSL model, we have from (32) and (35) for times $t < t_c$ and for a radiation dominated background universe with $R \sim A t^{1/2}$ and $H \sim 1/2t$:

$$\ddot{\psi}_k + \frac{3}{4t^2} \dot{\psi}_k + \left(\frac{a}{A}\right)^2 \frac{k^2}{t^{2b+1}} \psi_k = 0,$$

(38)

where we have chosen $\dot{\psi}_k \sim H \psi_k \sim (1/2t) \psi_k$. We observe that as $t \to 0$ for $b \geq 1$, there will be a period in the spontaneously broken phase in which (38) produces oscillating modes, for the wavelengths $\lambda_p \sim t^{1/2}$ are much smaller than the Hubble radius, $R_H \sim c/H \sim a/t^{b-1}$. At this time, the universe evolves adiabatically in a Minkowski flat spacetime vacuum and the ground state remains more or less as in (37). However, as the universe expands and $t$ increases, the wavelengths $\lambda_p \sim t^{1/2}$ will overtake the Hubble radius $R_H \sim a/t^{b-1}$ and cross it as $t$ approaches the phase transition time $t_c$. The overdamped modes cease to oscillate and $\Delta \psi_k$ will become constant at the fixed time $t = t_h$ when the wavelengths cross the horizon.

After the comoving wavelengths pass through the horizon, they freeze and the spectrum spread is given by

$$\left(\Delta \psi_k\right)_h^2 = \frac{1}{2R_h^2 c_h k},$$

(39)

where $c_h$ and $R_h$ are the values of $c$ and $R$ at the time the modes cross the
Hubble radius, \( R_h = c_h/H_h \), i.e. when

\[
\frac{R_h}{k} = \frac{c_h}{H_h}.
\]

(40)

Therefore, the fluctuation modes at later times have the spectrum

\[
(\Delta \psi_k)^2_h \sim \frac{H_h^2}{c_h^3 k^3}.
\]

(41)

The horizon radius \( R_h \) can be made to coincide with the phase transition with \( R_h \sim c_{ph}/H_{ph} \) when \( H_{ph} \) is expected to be approximately constant.

This constitutes the prediction of a scale invariant spectrum with

\[
k^3|\delta_k| \sim \text{constant},
\]

(42)

where \( \delta_k \) is the fractional energy density fluctuation in momentum space. We observe that the difference between (37) and (39) at later times is given by

\[
\left( \frac{R}{R_h} \right)^2 \left( \frac{c_0}{c_h} \right) (\Delta \psi_k)_h^2 = (\Delta \psi_k)^2_h.
\]

(43)

We see that for \( c_h = c_m \), the spread \( (\Delta \psi_k)^2 \) is magnified by the huge factor \( \log_{10} \gamma \sim 30 \), so that the late time quantum fluctuations have macroscopically relevant cosmological interest. In inflation theory, it is the factor \( (R/R_h)^2 \) that is exponentially enhanced and also produces macroscopically large fluctuation effects.

5 Comparison of VSL and Inflationary Cosmologies

Let us compare the VSL and inflationary models. Regarding the problem of fine-tuning of the initial conditions after the big bang, the argument given by Hollands and Wald [4] that the initial conditions in inflation cannot be natural, depends on whether inflation models can be considered time reversible, so that the probability that a universe would get large by undergoing an era of inflation is equal to the probability that a universe will undergo an era of “deflation” when it recollapses. It is argued that the probability that a universe dominated by ordinary matter will deflate is very small, so that by time invariance the probability of inflation must be small too.
It is argued by Kofman, Linde and Mukhanov [5] that the dynamical evolution of the universe does not preserve the measure of probability, or the number of degrees of freedom. This is mainly due to the circumstance that in inflationary models the total energy of the scalar inflaton field and the particles created by its decay is not conserved. Since all the $\sim 10^{88}$ particles we see now within our horizon were created by the scalar inflaton, then inflation had removed them at the beginning of the universe and thereby guaranteed the absence of adiabaticity. Thus, inflationary evolution can never produce the same initial conditions at the universe’s beginning. This circumstance should be considered in contrast to the fact that the equations of Einstein’s general relativity are time reversible invariant, so that there is an equal number of decreasing and growing entropy universes.

What can we say about the likelihood of a spontaneous symmetry breaking of Lorentz invariance of the ground state occurring in the early universe, and a sudden phase transition happening in the speed of light? The Lorentz symmetry of the ground state of the universe is just an accident, for the symmetry occurs in a false vacuum state and could occur at any time. It is, of course, difficult to measure the probability of such an event, as it would be for the spontaneous breaking of the internal symmetries of the standard model.

A fundamental difference between VSL cosmology and inflation is that we can choose the cosmological constant $\Lambda$ to be small or zero from the beginning of the universe. If the data supporting an accelerating expansion of the universe continues to be affirmed by more observations [18], then we can have a small positive cosmological constant in the present universe. In inflationary models, the initial vacuum energy coming from the inflaton potential is huge, so that enough e-folds of inflation can be sustained. How do we know that the long-sought mechanism for the explanation of the smallness of the effective cosmological constant will not cancel out the large vacuum energy needed in the inflationary era? [19]. Moreover, the decaying vacuum energy has to be fine-tuned to fit the present observational data supporting a small positive cosmological constant.

In our VSL model, the vacuum energy does not play a crucial role in solving the initial value problems. The pressure in our perfect fluid model can always be positive, i.e. for the equation of state for radiation and matter, $p = w \rho$, we can have $0 \leq w \leq 1/3$ in the early universe. Therefore, for dark matter and radiation there is no violation of the positive energy conditions. However, if there is dark energy causing an acceleration of the present uni-
verse, then the equation of state for dark energy would be $p_{\text{DE}} = w_{\text{DE}}\rho_{\text{DE}}$ and $-1 \leq w_{\text{DE}} < -2/3$.

Another feature associated with generic inflationary models is the extreme flatness of the inflaton potential required to permit sufficient inflation to occur. Apart from the necessary “Mexican hat” potential required to allow for the spontaneous symmetry breaking of local Lorentz invariance of the vacuum, the VSL model does not require a fine-tuning of potentials of the kind needed by inflationary models. Of course, we do not yet possess a microscopic model of the phase transition in the speed of light, but we do possess an effective theory that can be modelled in analogy with a semi-classical description of phase transitions in critical phenomena.

6 Conclusions

We have investigated anew and modified a model of VSL cosmology, first published a decade ago, and compared it with standard inflationary cosmology. A new calculation of the scale invariant fluctuation spectrum agrees well with the data [20], in the same way as the equivalent calculation of the spectrum in inflation models. However, a more technical calculation of the spectrum, including a derivation of the tensor component needs to be performed [21]. It is expected that the tensor and gravitational wave component of the fluctuation spectrum will not necessarily agree with that predicted by inflation, providing a new competitive prediction to be tested by observations.

We cannot, of course, go back to the beginning of the universe to observe whether it actually went through an era of exponential or power law inflation, or an epoch in which the Lorentz invariance of the ground state of the universe was spontaneously broken, accompanied by a phase transition in the speed of light. Therefore, we must rely on the self-consistent results of calculations of the primordial power spectrum and observations of CMB anisotropies to guide us in our understanding of early universe cosmology.

We believe that the VSL cosmology considered here is a viable alternative to standard inflationary models, and that it may overcome certain shortcomings in the latter models, and produce new predictions that could be tested and compared with inflationary scenarios.

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