QUASAR ABUNDANCE AT HIGH REDSHIFTS AS A PROBE OF INITIAL POWER SPECTRUM ON SMALL SCALE

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ABSTRACT

The dependence of the number density of the bright QSOs at different redshifts \( n_{QSO}(z) \) on initial power spectrum is studied. It is assumed that QSO phenomenon is an early short term stage of evolution of massive galaxies with \( M \geq 2 \times 10^{11}h^{-1}M_{\odot} \). The duration of such QSO stage which is passed through by fraction \( \alpha \) of galaxies is determined by means of minimization of the divergence of the theoretical number density of QSOs at different redshifts for specified initial spectrum from observable one [1]. It is shown that the nearest number densities of QSOs at \( 0.7 \leq z \leq 3.5 \) to observable ones are obtained for the tilted CDM model (\( \Omega_b = 0.1, n = 0.7 \)). The QSO stage lasts \( \sim 7 \times 10^7/\alpha \) years and begins soon after the moment of rise of the first counterflow in collisionless component and shock wave in gas.

The possibility of the reconstruction of initial power spectrum on small scale on the base of the observable data on number density of QSOs at different \( z \) is considered too. Such reconstructed spectrum in comparison with standard CDM has steep reducing of power at \( k \geq 0.5h\text{Mpc}^{-1} \).

Subject headings: cosmology:initial power spectrum - dark matter - quasars
1. Introduction

The problem of formation of galaxies and large scale structure of the Universe has an astrophysical aspect consisting in search of connection between properties of luminous objects and dark matter halo, as well as cosmological aspect consisting in determination of the initial power spectrum of density fluctuations. Obtaining such spectrum from first principles would essentially simplify the solving of problem on the whole. Unfortunately, today we can not surely say which the postinflation spectrum is: scale invariant \( P_{\text{Pl}} = A k^n \) with \( n = 1 \), tilted with \( n \neq 1 \), or more complicated. The nature of dark matter (baryonic, collisionless cold dark matter, further CDM, hot dark matter, further HDM, or their mixture H+CDM, or other), and values of cosmological parameters \( h = \frac{H_0}{100 \text{ km/s Mpc}} \) (Hubble constant), \( \Omega_1 = \frac{8 \pi G}{c^2} \rho_1 \) (total mean density), \( \Omega_5 = \frac{8 \pi G}{c^2} \rho_b \) (mean density of baryons) and cosmological constant \( \Lambda \) remain uncertain. More preferable from theoretical point of view spectra (standard CDM model, tilted CDM, CDM+\( \Lambda \) or hybrid H+CDM) give predictions consistent with the observable data marginally only \( \Delta \). Furthermore, the different spectra normalised to the COBE r.m.s. cosmic microwave background (CMB) anisotropy predict close characteristics of the large scale structure of the Universe, e.g. \( \Delta T/T \) at degree angular scale, large scale peculiar velocity field etc. \( \Delta \) so that it is impossible to distinguish them. Therefore, it is very important to build up the new cosmological tests based on extension of a class of objects with predicted observational manifestations sensitive to initial power spectrum on small scale. Here we shall investigate the dependence of the number density of quasars (QSOs) at different redshifts against initial power spectra. To make it possible we suppose that a QSO is the early short term stage of evolution of a massive galaxy and appears in the corresponding peaks of initial random Gaussian field of density fluctuations. The number density of already collapsed peaks of certain scale at different redshifts \( z \) strongly depends on an amplitude and slope of the power spectrum of such field at small scales \( (k \sim 0.1 - 10 h \text{ Mpc}^{-1}) \).

Such approach was applied independently by some authors. For example, Cen et al. \( \Delta \) have calculated the comoving number density of nonlinear objects as a function of redshift \( z \) using Press-Schechter approximation \( \Delta \). Their results show that observ-
phenomenological spectrum on small scale.

2. The main assumptions and method

We explore the distribution of quasar abundance over redshift $n_{QSO}(z)$ in the framework of the theory of formation of the large scale structure of the Universe in the peaks of random Gaussian field of scalar density fluctuations as consequence of gravitational instability on the Einstein-de Sitter cosmological background. In this approach galaxies and objects of other scales are formed in the peaks of density fluctuations which have an amplitude $\delta = \delta_p \geq \nu_{th} \sigma_0(R_f)$, where $\sigma_0$ - r.m.s. amplitude of the fluctuations of certain scale $R_f$, $\nu_{th}$ - some threshold height which corresponds to minimal amplitude when objects of this scale forms still.

In the theory of random Gaussian fields \[ n_{th}^0 \approx (4.6 h^{-1} \text{Mpc})^{-3}. \] Using eq.(1) for massive galaxies

\[ n(\nu_{th}, q) = n_{th}^0 \] one parameter of threshold function (e.g. $\nu_{th}^0$) can be found.

As follows from Tolmen model, a homogeneous spherical-symmetrical adiabatic fluctuation on the Einstein-de Sitter cosmological background collapses at

\[ z_c \equiv \left( \frac{t_0}{t_c} \right)^{\frac{2}{3}} - 1 = 0.59 \sigma_0 \nu_c - 1, \] where $t_0 \simeq 1.3 \times 10^{10}$ years is present cosmological time (here and further we assume $H_0 = 50 km/Mpc$), $t_c$ - the moment of collapse, $\nu_c$ - corresponding height of peak. At this moment $t_c$ appearance of the first counterflows in dark matter and generation of shock wave in baryon component take place. Massive black hole in central region of such peak may be formed during $\sim 10^8$ years as it follows from the physical models of quasar mechanism (see, for example, \[ \text{[21]} \]). Therefore, we suppose that QSO stage of galaxy evolution begins soon after collapse of central region of peak $t_c$ or later by certain time interval $\Delta t$ called a time delay. Duration of the QSO stage or quasar lifetime is $\tau_{QSO}$.

In such approach the number density of the QSOs at redshift $z$ is the number density of corresponding peaks of density fluctuations (precursors of massive galaxies) which had collapsed between $t_c - \tau_{QSO}$ and $t_c$. These peaks we can see as QSOs at redshift $z$ corresponding to time $t(z) = t_c(z) + \Delta t$:

\[ z = \left( \frac{t_0}{t_c(z_c) + \Delta t} \right)^{\frac{2}{3}} - 1. \] The height of such peaks $\nu$ is in the range of $(\nu_c, \nu_c + \Delta \nu)$, where

\[ \nu_c = 1.69 \sigma_0^{-1}(z_c + 1), \] \[ \Delta \nu = 1.13(z_c + 1) \frac{2.5 \tau_{QSO}}{\sigma_0 t_0}, \] as it follows from eq. (3). Then the number density of QSOs is

\[ n_{QSO}(z) = \alpha \int_{\nu_c}^{\nu_c + \Delta \nu} p(\nu; \nu_{th}, q) N_{pk}(\nu) d\nu \]
where $z$, $\nu_c$, and $\Delta \nu$ are defined by eq. 4, 5, 6 respectively. As we can see, the number density of QSOs $n_{QSO}(z)$ depends upon the duration of the quasar stage $\tau_{QSO}$, the time delay $\Delta t$, parameters of the threshold function $\nu_i$ and $q$, momenta of spectrum smoothed by galactic filter $\sigma_0$, $\sigma_1$, $\sigma_2$ and fraction $\alpha$ of peaks-precursors passing quasar stage and visible by Earth observer. If quasar stage is long-term ($\tau_{QSO} \approx t_0$) then the number density of QSOs at all $z$ is equal to fraction of already collapsed peaks-precursors of massive galaxies visible from Earth at quasar stage of their evolution: $n_{QSO}(z) = \alpha n_g(z)$. If quasar stage is short-term ($\tau_{QSO} \ll t_c$), then it follows from (7) and (6) that

$$n_{QSO}(z) = 1.13\alpha \frac{(z_c + 1)^{2.5}}{\sigma_0 t_0} p(\nu; \nu_i, q) N_{pk}(\nu) \tau_{QSO}. \quad (8)$$

It is valid for above mentioned estimation of $\tau_{QSO} \sim 10^7 - 10^8$ years.

We suppose that the duration of the quasar stage $\tau_{QSO}$, the time delay $\Delta t$ and parameters of threshold function $\nu_i$ and $q$ do not depend on the $z$. Also we assume that the sample of bright quasars given by Schmidt et al. \cite{1} and Boyle et al. \cite{7} is complete for all $z$ in the range 0.7-4.7. Here we do not discuss the problem concerned with the observational data but pay particular attention to the theoretical approach of their using for cosmological problems.

Now we can calculate the number density of QSOs for set of redshifts and given values of the duration of QSO stage $\tau_{QSO}$, the time delay $\Delta t$, the fraction $\alpha$ of massive galaxies passing quasar stage, parameters $\nu_i$ and $q$ of threshold function and momenta of spectrum $\sigma_0$, $\sigma_1$, $\sigma_2$. For present spectrum normalised in a certain way the momenta $\sigma_0$, $\sigma_1$, $\sigma_2$ are calculated independently. The time delay $\Delta t$ for given spectrum we find in following manner. Using eq.(8) we calculate the redshift of maximum $z_{max}$ of dependence $n_{QSO}(z)$. If $z_{max}$ is higher than one of observable $n_{QSO}^{obs}(z) \equiv n_{QSO}^{max} \approx 2.2$ then we find the time delay $\Delta t$ from equation

$$\frac{dn_{QSO}(z)}{dz}_{z_{max} = 2.2} = 0. \quad (9)$$

In those models where $z_{max}$ is less than or equal $z_{max} \approx 2.2$ we assume $\Delta t = 0$. Rest four values $\alpha$, $\tau_{QSO}$, $\nu_i$ and $q$ are unknown. Finding of them by method of minimization of divergence vector is a matter of testing of known spectra.

We try also to reconstruct the initial power spectrum from observable data on number density of QSOs in wide range of redshifts and massive galaxies on $z \approx 0$. For this we consider momenta of spectrum $\sigma_0$, $\sigma_1$, $\sigma_2$ as unknown values and find them in the same way.

In all cases we write eq.(7) for 6 points of data by Schmidt et al. \cite{1} on the $z_1 = 4.7$, $z_2 = 4.05$, $z_3 = 3.7$, $z_4 = 3$, $z_5 = 2.8$, $z_6 = 2.2$, for 3 points on curve by Boyle et al. \cite{7} on $z_7 = 2.0$, $z_8 = 1.5$, $z_9 = 0.7$ and the eq. (2) for number density of massive galaxies on $z_{10} = 0:

$$\alpha \int_{\nu_i}^{\nu_i + \Delta \nu_i} p(\nu; \nu_i, q) \, N_{pk}(\nu) \, d\nu = n_{QSO}^{obs}(z_i), \quad (i = 1, 2, ..., 9),$$

$$\int_0^\infty p(\nu; \nu_i, q) \, N_{pk}(\nu) \, d\nu = n_{QSO}^{obs}, \quad (10)$$

where $\nu_i$, $\Delta \nu_i$ are calculated for $z_i$ accordingly to (5) and (6) respectively. So, we solve the overdetermined system of equations (10), (11) (or (10) only) by method of minimization of the divergence vector using MathCad \cite{22} software.

3. Results

3.1. Testing of spectra

We shall test the preferable initial power spectra of density fluctuations on the flat Friedman background expected in standard CDM model ($\Omega_{CDM} = 0.9$, $\Omega_b = 0.1$), tilted CDM ones and hybrid H+CDM one ($\Omega_{CDM} = 0.6$, $\Omega_{CDM} = 0.3$, $\Omega_b = 0.1$), which marginally match other observational data on large scale structure of the Universe \cite{2, 3}. The transfer function $T(k)$ in the spectra $P(k) = A k^n T^2(k)$, where $A$ is constant of normalisation $\Omega$, is taken from the paper by Holtzman \cite{24}. All spectra we normalised to produce the COBE data on CMB temperature anisotropy at angular scales $10^\circ$ \cite{24, 25}: $< (\Delta T/T)^2 > = (1.1 \pm 0.1) \times 10^{-5}$. We calculate $< (\Delta T/T)^2 > = C(0; 10^\circ)$ using the approximation formula by Wilson and Silk \cite{26} for correlation function of CMB temperature anisotropy measured by receiver with Gaussian response function and exact formula \cite{27, 28} for correlation function $C(\alpha) \equiv < \Delta T(0) \Delta T(\alpha) >$. Both Sachs-Wolfe and Doppler effects are taken into account (see also \cite{28}). The con-

\footnote{The parameter $t_1$ of transfer function fitting formula by Holtzman \cite{24} are accepted equal to 1.}
tribution of tensor mode to $\Delta T/T$ is different in different models of inflation. In the most of them it is not dominating at the COBE angular scale (see review by [29] and references cited therein), in the some ones it is negligible even if $n < 1$ (for example ”natural” inflation, [30]). In this work, like ones cited here, we consider only models of inflation which give no significant tensor mode. But one can easily cross over to models of inflation with significant tensor mode when ratio $\alpha_T \equiv (\Delta T/T)_T / (\Delta T/T)_S$ is known (here $(\Delta T/T)_T$ and $(\Delta T/T)_S$ is contribution of tensor and scalar modes correspondingly). The constant of normalization then will be $A' = A/(1+\alpha_T^2)$, the momenta of spectrum $\sigma'_j = \sigma_j/\sqrt{1+\alpha_T^2}$, peaks-precursors of QSOs will be collapsed at $z'_j = (z_j + 1)/\sqrt{1+\alpha_T^2} - 1$, where values without (‘) are ones calculated without tensor mode. Parameters of cosmological models, constants of normalisation $A$ of power spectra, their momenta $\sigma_0$, $\sigma_1$, $\sigma_2$ and biasing parameters $b \equiv b/\sigma_8$, where $\sigma_8$ is the r.m.s. mass fluctuations in the top-hat sphere with radius $8h^{-1}\text{Mpc}$, are presented in Tabl.1. (For comparison, our normalization constant $A$ for paper [31] reduced to $h=0.5$ and $\sigma(10^\circ) = 30.5 \mu K$ with accuracy better then 0.1%).

For beginning we analyse the dependence of the $n_{QSO}(z)$ on $\tau_{QSO}$, $\Delta t$, $\nu_h$ and $q$ for different spectra accepting $\alpha$ equal to 1. Results are presented in Fig.1-3.

As we can see, the number density of QSOs is proportional to $\tau_{QSO}$, meanwhile the time delay displaces the maximum of $n_{QSO}$ to lower $z$ (Fig.1). Reducing of power spectrum at small scale results in steeper declination of $n_{QSO}(z)$ at high $z$ and displacing of its maximum to lower $z$ (Fig.1-2). Increasing of the value of the threshold parameter $q$ ($\tau_{QSO}$, $\nu_h$ are constants) reduces the number density of QSOs at low redshift and displaces its maximum to high $z$ (Fig. 3a). Dependence $n_{QSO}(z)$ on $\nu_h$ ( $\tau_{QSO}$ and $q$ are constants) is similar (Fig. 3b). Increasing of $q$ with finding of corresponding $\nu_h$, which ensures $n_g = n_{oha}$, results in steeper slope of $n_{QSO}(z)$ at low redshifts $z \lesssim 2$ (Fig. 3c). The curve labeled by 5 in Fig. 3 is the number density of QSOs calculated without threshold function ($\nu_h = 0$). For comparison we have calculated also the $n_{QSO}(z)$ without threshold function for another minimal linear overdensity corresponding to a bound object at $z = 0 \delta_0 = 1.33$ adopted by Nusser and Silk [4] (line labeled by 6 in Fig. 3a).

Now let us find for each model such values of $\alpha$, $\tau_{QSO}$, $\nu_h$ and $q$ which give minimal divergence $n_{QSO}(z)$ from observable one and ensure the number density of massive galaxies at $z \approx 0$. In all cases analyzed here the amplitude of theoretical $n_{QSO}(z)$ is roughly equal to observable $n_{QSO}^{th}(z) \approx 3 \times 10^7 h^3 Mpc^{-3}$ when $\tau_{QSO} \sim 10^6$ years that is $\ll t_0$. It means that (8) is good approximation of (10) in the range $10^{-3} \leq \alpha \leq 1$ and that $n_{QSO} \propto \alpha \tau_{QSO}$. As the result, we can not find both values $\alpha$ and $\tau_{QSO}$ simultaneously but their product $\alpha \tau_{QSO}$ only. Approximate solutions of system of equations (10)-(11) are obtained by using of MathCad software package and presented in the Table 2. The quasar abundances $n_{QSO}(z)$ for them are shown in the Fig.4.

As we can see any model does not explain the observable $n_{QSO}^{obs}(z)$ at high $z$. Therefore we have repeated the same procedure without the equation for concentration of bright galaxies (11). Results are presented in the Table 3 and Fig.5.

In this case the parameter $\alpha$ has other interpretation: it is the fraction of peaks which are selected by threshold function $p(\nu; \nu_{th}^{QSO}, Q_{QSO})$ and which pass through quasar stage and are visible by Earth observer. Let $\alpha'$ denote it. The ratio of the number density of all peaks selected in a such way to number density of bright galaxies

$$K = \frac{\int_0^\infty p(\nu; \nu_{th}^{QSO}, Q_{QSO}) N_{pb}(\nu) \, d\nu}{\int_0^\infty p(\nu; \nu_{th}, q^b) N_{pb}(\nu) \, d\nu}$$

is calculated for each model and presented in Table 3. In all models, with the exception of standard CDM, $K \ll 1$ and can be interpreted as $\alpha_1$ above mentioned, that is fraction of the peaks-precursors of bright galaxies passed through QSO stage. Thus, in these models the peaks resulting in bright galaxies through QSO stage are higher at average than main part of all peaks. The picture is inverse in standard CDM, where $K \approx 7$, and may mean that peaks of galactic mass $M \geq 2 \times 10^{11} h^{-1} M_\odot$ survive after QSO stage and become bright galaxies if their gravitational potential holes are sufficiently deep. Whether it is valid or not can be proved only by means of elaborate numerical simulations of the evolution of such peaks in different cosmological models. As it follows from Fig. 5, the theoretical number density of QSOs at low $z$ is satisfactory in all models. At high $z$ only CDM model with $n = 0.7$ gives the number density of QSOs sufficiently close to observable one. But even this model does not explain the observable number
Table 1: The constant of normalisation of the spectra, theirs momenta $\sigma_0$, $\sigma_1$, $\sigma_2$ on galactic scale ($R_f = 0.35h^{-1}Mpc$) and biasing factor $b$, defined by $b = \sigma_8^{-1}$, where $\sigma_8$ is r.m.s. mass fluctuation on a scale of $8h^{-1}Mpc$.

|   | CDM | $H + CDM$ |
|---|-----|-----------|
| $n$ | $7.33 \times 10^6$ | $7.57 \times 10^6$ | $7.51 \times 10^6$ |
| $A$ | $7.33 \times 10^6$ | $7.57 \times 10^6$ | $7.51 \times 10^6$ |
| $\sigma_0$ | 4.78 | 2.37 | 1.67 | 1.39 |
| $\sigma_1$ | 4.77 | 2.18 | 1.47 | 0.91 |
| $\sigma_2$ | 7.93 | 3.50 | 2.31 | 1.32 |
| $b$ | 1.05 | 1.77 | 2.30 | 1.66 |

Fig. 1.— Quasar abundance against redshift for different values of the $t_{QSO} = 10^4$, $10^5$, $10^6$, $10^8$, $10^{10}$ years (dashed lines 1, 2, 3, 4, 5 respectively in each panel) and time delay $\Delta t = 0$ (a), $\Delta t = 10^9$ years (b), $\Delta t = 1.61 \times 10^9$ years (c) for CDM spectrum with $n = 1$, $\Omega_{CDM} = 0.9$ and $\Omega_b = 0.1$. The solid line with error bars is the comoving space density of quasars from Schmidt et al. (1991). The upper dashed line (5) is the number density of already collapsed peaks-precursors of bright galaxies. In all cases $\alpha = 1$.

density of QSOs at $z \geq 4$. Is it possible to explain the whole observable redshift distribution of quasar abundance given by Schmidt et al. [1] in such approach, in principle?

3.2. Finding of phenomenological spectrum

For answer to this question we attempted to solve the system of equations (10)-(11) with respect to unknown values $\sigma_0$, $\sigma_1$, $\sigma_2$, $\alpha t_{QSO}$, $n_{th}$ and $q$. The time delay $\Delta t$ is assumed to be equal to zero. Solutions are obtained in the same way and are presented in the Table 4. Quasar abundances $n_{QSO}(z)$ for them are shown in the Fig.6. and match the observable data quite well.

As we can see, the main feature of all solutions is the relation between momenta: $\sigma_1 < \sigma_2 < \sigma_0$. What kind of spectrum have such relations between its momenta on the galaxy scale? We attempted to modify the standard CDM spectrum on small scale by force of reducing and enhancing of its power in order to obtain momenta of spectra from Table 4. Locations and amplitudes of such modifications were unknown parameters and were found from 3 equations for given momenta $\sigma_1$, $\sigma_2$, $\sigma_0$ using MathCad software package. It was successful for momenta from the column 1 in Table 4 only, for which parameter $\gamma \equiv \sigma_1^2/\sigma_0 \sigma_2 = 0.3$. The phenomenological power spectrum of density
fluctuations obtained in such way is

\[ P_{ph}(k) = A k T_{CDM}^2 \begin{cases} 
    1, & k < k_0, \\
    e^{(-k/k_0)^p}, & k_0 \leq k \leq k_1, \\
    e^{(-k_1/k_0)^p}, & k > k_1,
\end{cases} \]

where \( k_0 = 0.48 h \text{Mpc}^{-1}, k_1 = 0.82 h \text{Mpc}^{-1}, p = 2.3, \)
\( A \) is the same as in standard CDM. The spectrum has shelf-like reducing of power at \( k \sim 0.7 h \text{Mpc}^{-1} \) in comparison with standard CDM normalised to the COBE 10° angular scale temperature anisotropy of CMB (\( P_{ph}(k) \) spectrum in Fig. 7).

The correlation function of galaxies for it does not contradict observable data. Other cosmological consequences of such spectrum are being investigated yet.

The parameter \( \gamma \) for momenta from columns 2-5 in Table 4 is extremely small (0.04-0.0004), so that phenomenological power spectrum for them, obviously, must be extraordinary too. Indeed, spectrum with momenta from column 2 (\( P_{2, ph}(k) \) spectrum in Fig. 7) has two narrow bumps at \( k \approx 0.02 \) and \( \approx 10 h \text{Mpc}^{-1} \) and shelf-like reducing of power at \( k \sim 0.03 h \text{Mpc}^{-1} \) in comparison with standard CDM. Since its correlation function of bright galaxies strongly contradicts observable one, the solutions from columns 2-5 in Table 4 can be interpreted as meaningless from physical point of view. So, only spectra with momenta from column 1 (\( \gamma \approx 0.3 \)), or close to them, are cosmologically significant.

### 3.3. Estimation of fraction of host galaxies

Masses of galaxies which pass through quasar stage (host galaxies) are \( M_g \geq 2 \times 10^{11} h^{-1} M_\odot \), so that it is coordinated with the black hole accretion models of quasar phenomenon [4, 21, 82, 85]. An optical lu-
Fig. 3.— Quasar abundance against redshift in tilted CDM model (n = 0.7) for different parameters of threshold function: (a) - q = 16 (line 1), q = 12 (2), q = 8 (3), q = 4 (4), without threshold function (5), \( \tau_{QSO} = 6.9 \times 10^7 \) years, \( \nu_{th} = 5.89 \); (b) - \( \nu_{th} = 10 \) (line 1), \( \nu_{th} = 8 \) (2), \( \nu_{th} = 5.89 \) (3), \( \nu_{th} = 3 \) (4), without threshold function (5), \( \tau_{QSO} = 6.9 \times 10^7 \) years, \( q = 8.08 \); (c) - \( q = 16 \), \( \nu_{th} = 2.5 \) (line 1), \( q = 12.4 \), \( \nu_{th} = 2.52 \) (2), \( q = 8 \), \( \nu_{th} = 2.6 \) (3), \( q = 4 \), \( \nu_{th} = 2.95 \) (4), without threshold function (5), \( \alpha \tau_{QSO} = 2.4 \times 10^8 \) years, \( \nu_{th} \) for given \( q \) was obtained from eq. (11). The rest is the same as in Fig. 1.

Minority of the bright quasars is \( L \geq 10^{47} \) \( h^{-2} \) erg/s. Their lifetime multiplied by \( \alpha \) in all cases is \( \alpha \tau_{QSO} \geq 10^5 \) years, because we denote \( \tau_{QSO} = 10^5 \) \( \tau_5/\alpha \), where \( \tau_5 \) is \( \alpha \tau_{QSO} \) in the units of \( 10^5 \). If the fraction of the black hole mass converted into optical radiation is \( \epsilon \), the ratio of the quasar’s mass to that of the host galaxy is \( F \), then

\[
M_g \approx 2 \times 10^5 L_{47} \tau_5/(\alpha \epsilon F) \, h^{-2},
\]

that gives

\[
\alpha \approx 10^{-3} L_{47} \tau_5/(\epsilon_0.1 F_{0.01}) \, h^{-1},
\]

where \( \epsilon_0.1 \equiv \epsilon/0.1 \), \( F_{0.01} \equiv F/0.01 \), \( L_{47} \equiv L/10^{47} \).

Thus, in the models with phenomenological power spectrum and spectra tested here only less than 1% of massive galaxies pass through quasar stage and are visible by Earth observer.

4. Discussion

Recently we \cite{14, 33} carried out the similar calculations of evolution of quasar number density for the same spectra (CDM with \( n=1, 0.8, 0.7 \) and H+CDM) normalised to \( \sigma_8 = b_g^{-1} \), where galactic biasing parameter \( b_g \) was calculated in the framework of Gaussian statistics of density peaks \cite{17, 30} and was equal to 1.40, 1.47, 1.53 and 1.76 respectively. From comparison of results of this work with previous ones it follows that the dependence of quasar number density on \( z \) is very sensitive to the normalisation of spectrum in all models. In our previous papers the modified on small scale standard CDM spectrum for reproducing of the observable data on number density of QSOs at different redshifts has reducing of power on scale \( 1 \leq k \leq 10 h Mpc^{-1} \) and bump on scale \( k \sim 10 h Mpc^{-1} \) (dotted line in Fig.7). Its difference from spectrum shown in Fig.7 is caused by different power of initial CDM spectrum on \( k \leq 1 h Mpc^{-1} \) as a result of different normalisation.

It should be remarked here, though formally this method is sensitive to spectrum on small scale up to \( k \sim 10 h Mpc^{-1} \) when it is smoothed by Gaussian filter function with \( R_f = 0.35 h^{-1} Mpc \) (or top-hat \( R_{TH} = 0.6 h^{-1} Mpc \) nevertheless we are not sure that calculation of peak number density of galaxy scale is exact for cases of spectrum like standard CDM and, especially, CDM + short wave bump models. This is concerned mainly with the well known problem ‘cloud-in-cloud’ unresolved by now in the frame of peak formalism used here.

Now let us compare our results with those by Nusser and Silk \cite{14}. The curve \( n_{QSO}(z) \) for \( \delta_c^2 = 1.33 \)
Table 3: Solutions \( \alpha \tau_{QSO}, \nu_{th}, q \) of system of equations (10) and ratio \( K \) of number density of peaks which has passed through quasar stage to one of bright galaxies.

| \( n \) | CDM | \( H + CDM \) |
|--------|------|---------------|
| \( \alpha \tau_{QSO} \) (years) | \( 5.73 \times 10^4 \) | \( 1.83 \times 10^7 \) | \( 6.91 \times 10^7 \) | \( 1.64 \times 10^8 \) |
| \( \Delta t \) (years) | \( 1.3 \times 10^9 \) | \( 5.19 \times 10^8 \) | \( 0.00 \) | \( 0.00 \) |
| \( \nu_{th} \) | 1.09 | 7.26 | 5.89 | 4.95 |
| \( q \) | 4.96 | 4.85 | 8.08 | 12.32 |
| \( K \) | 7.2 | 0.02 | 0.006 | 0.003 |

Table 4: Partial solutions \( \alpha \tau_{QSO}, \nu_{th}, q, \sigma_0, \sigma_1 \) and \( \sigma_2 \) of system of equations (10)-(11).

| \( \alpha \tau_{QSO} \) (years) | \( 3.20 \times 10^5 \) | \( 3.51 \times 10^6 \) | \( 3.53 \times 10^6 \) | \( 4.41 \times 10^8 \) | \( 4.0 \times 10^4 \) |
| \( \Delta t \) (years) | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| \( \nu_{th} \) | 2.06 | 1.69 | 1.69 | 1.67 | 1.67 |
| \( q \) | 8.71 | 9.28 | 9.28 | 9.31 | 9.31 |
| \( \sigma_0 \) | 2.38 | 2.77 | 2.78 | 2.81 | 2.82 |
| \( \sigma_1 \) | 1.48 | 0.22 | 0.17 | 0.02 | 0.002 |
| \( \sigma_2 \) | 2.37 | 0.43 | 0.34 | 0.03 | 0.004 |

(line 6 in Fig.3) without threshold function, recalculated to \( \alpha \tau_{QSO} = 10^8 \) years, coordinates with the corresponding curve in Fig.2 of work by Nusser & Silk [14] quite satisfactory at high \( z \). The some divergence of these curves at low \( z \) are caused mainly by diferent ways of definition the mass of peaks.

In our work the value \( \delta_c^* = 1.68 \) corresponding to collapse of spherical-symmetrical peaks, is adopted. Therefore, in our approach the QSOs are formed in spheroidal peaks only. The fraction of spheroidal peaks is \( \leq 4\% \) [18], the fraction of massive galaxies with \( M \geq 2 \times 10^{11} h^{-1} M_\odot \) passed through quasar stage in all cases does not exceed 1%. So, the assumption that QSOs appear only in spherical peaks of density fluctuations is not contradictory in our approach. But its conformity to reality should be proved or objected by elaborated numerical simulations of the evolution of QSOs which start from initial conditions determined at the linear stage of the evolution of peaks. Substantiations of the value \( \delta_c^* \) as well as dependence or independence of both quasar lifetime \( \tau_{QSO} \) and time delay \( \Delta t \) on redshift \( z \) will be the subject of the future works. c We put constraints on the cosmological models based on their ability to reproduce the rise of quasar abundance up to redshifts \( \approx 2.2 \) and following fall up to \( \approx 4.7 \) assuming that quasar nature is the same at different redshifts and \( \tau_{QSO} \). \( \Delta t \) and \( \alpha \) do not depend on \( z \). Moreover, if we have only 3 unknown values \( \alpha \tau_{QSO}, \nu_{th} \) and \( q \) in equation system (10-11) then we can not reproduce the \( n_{QSO}^{obs}(z) \) given by Boyle et al. [17] and Schmidt et al. [3] with sufficient accuracy in the models tested here. But when \( \sigma_0, \sigma_1 \) and \( \sigma_3 \) are supposed to be unknown values too, such reproducing is possible. Apparently, for other models of quasars, which explain the rise and fall of the \( n_{QSO}^{obs}(z) \) by astrophysics of quasars (e.g. [37]), the constraints on the cosmological models may be other. But it is the subject of separate work. Here we only confirm that such dependence of quasar abundance on \( z \) can be explained in the framework of most simple astrophysical models of quasars, which look rather plausible and motivated in order to be put in the base of similar research.

The phenomenological power spectrum obtained here (§3.2) has shelf-like reducing of power at \( k \sim 0.7 h Mpc^{-1} \). Such spectra with broken scale invari-
Fig. 4.— Quasar abundance against redshift obtained by minimization of divergence vector including also number density of bright galaxies at \(z \approx 0\): 1 - CDM \(n = 1\), 2 - CDM \(n = 0.8\), 3 - CDM \(n = 0.7\), 4 - H+CDM \(n = 1\).

ance is generated in some inflationary scenarios (e.g. [38, 39, 40, 41]). The typical scale and the height of the shelf-like reducing determined here can be connected with some constants, which characterise the underlying inflationary models.

If we treat the used observational data more critically and suppose that \(n_{\text{obs}}^{QSO}(z)\) given by Schmidt et al. [1] and Boyle et al. [17] is only lower limit for number density of QSOs at all \(z\) then the next constraints follow from the presented results: on the small scale the initial power spectrum of density fluctuations is nearly CDM with \(n \geq 0.7\), \(b_g \leq 2.3\) and \(\alpha_{\tau_{QSO}} \geq 5 \times 10^5\) years, \(\Delta t \leq 5 \times 10^8\) years.

5. Conclusions

Thus, the main assumption that quasar phenomenon is active short term stage of evolution of some small fraction of massive galaxies with \(M \geq 2 \times 10^{11} h^{-1} M_\odot\) and appears in peaks of random Gaussian density fluctuation field allows to explain the general feature of \(n_{\text{obs}}^{QSO}(z)\) given by Boyle et al. [7] and Schmidt et al. [1]: fast increasing of number density from \(z \approx 0\) to \(z \approx 2.2\) up to value \(\approx 3 \times 10^{-7}\) \(h^3 Mpc^{-3}\) and following monotonous decreasing on \(z > 2.5\). Apparently, a constant quasar abundance from \(z \approx 2\) to 4

Fig. 5.— Quasar abundance against redshift obtained by minimization of divergence vector without abundance of bright galaxies: 1 - CDM \(n = 1\), 2 - CDM \(n = 0.8\), 3 - CDM \(n = 0.7\), 4 - H+CDM \(n = 1\).

If physically motivated quasar lifetime is \(\sim 10^7 - 10^8\) years, then \(\alpha \approx 0.01 - 0.001\) that matches the estimations of \(\alpha\) done above.

Thus, redshift distribution of quasar abundance is very sensitive to the amplitude and slope of the initial power spectrum, and the number density of QSOs is determined by the underlying inflationary model. The constraints obtained here can be used for testing of such models.
power spectrum of density perturbations on galaxy scale. But it can be effective test when complete number density of QSOs with mass larger than fixed value will be known with confidence for all redshifts and physical parameters connected with nature of quasar phenomenon (the quasar lifetime \(\tau_{QSO}\), time delay \(\Delta t\), fraction of galaxies which pass through quasar stage \(\alpha\), linear amplitude of peaks \(\delta_o\) collapsing just now and threshold function) will be substantiated. The last needs the detailed investigation of connection of quasar phenomenon with initial configuration of density fluctuation by the help of numerical simulations of its evolution.

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REFERENCES

Schmidt M., Schneider D. P., Gunn J. E., 1991, in The Space Density of Quasars, ed.D. Crampton (SF: ASP), 1991, p.109

Davis M. J., Summers F.J. & Schlegel D., 1992, Nature, 359, 393

Taylor A.N. & Rowan-Robinson M., 1992, Nature, 359, 396

van Dalen A. & Schaefer R.K., 1992, ApJ, 398, 33

Cen R., Gnedin N.Yu., Kofman L.A. & Ostriker J.P. 1992, ApJ, 399, L11

Pogosyan D.Yu. & Starobinsky A.A., 1993, MNRAS, 265, 507

Kofman L.A., Gnedin N.Y. & Bahcall N.A., 1993, ApJ, 413, 1

Novosyadlyj B.S., 1994, Kinematica i fizika nebesnyck tel, 10, 13

Hnatyk B.I., Lukash V.N. & Novosyadlyj B.S., 1995, A&A, 300, 1

Press W. H. & Schechter P., 1974, ApJ, 187, 425

Blanchard A., Buchert T., Klauff R., 1993, A&A, 267, 1

Smoot G.F., Bennett C.L., Kogut A. et al., 1992, ApJ, 396, L1

Kashlinsky A., 1993, ApJ, 406, L1
Nusser A. & Silk J., ApJ, 411, L1
Bardeen J.M., Bond J.R., Kaiser N. & Szalay A.S., 1986, ApJ, 304, 15
Ma C., Bertschinger E., 1994, ApJ, 434, L5
Boyle B.J., Shanks T. & Peterson B.A., 1988, MNRAS, 235, 935
Doroshkevich A.G., 1970, Astrophysics, 6, 581
Turner E.L., 1991, AJ, 101, 5
Loeb A., Rasio F.A., 1994, ApJ, 432, 52
Davis M. J., Huchra J., 1982, ApJ, 254, 437
MathCAD. Version 2.5. User Guide. MathSoft Ins., Cambridge, 1989, 262p.
Holtzman J.A., 1989, ApJS, 71, 1
Bennett C.L. et al. 1994, ApJ, 436, 423
Wright E.L., Smoot G.F., Bennett C.L. & Lubin P.M. 1994, ApJ, 436, 443
Wilson M.L. & Silk J., 1981, ApJ, 243, 14
Martinez-Gonzales E. & Sanz J., 1989, ApJ, 347, 11
Novosyadlyj B.S., 1996, Astron. and Astroph. Transactions, 10, 85
White M., Scott D. & Silk J. 1994, Annu. Rev. Astron. Astrophys., 47, 426
Adams F.C. et al. 1992, Phys. Rev., D47, 426
Bunn E.F., Scott D. & White M. 1995, ApJ, 441, L9
Efstathiou G. & Rees M.J., 1988, MNRAS, 230, 5
Kashlinsky A. & Jones B.J.T., 1991, Nature, 349, L1
Chornij Yu.B. & Novosyadlyj B.S., Astron. and Astroph. Transactions, 1996, 10, 77
Novosyadlyj B.S. & Chornij Yu.B., 1996, Kinematica i fizika nebesnyck tel, 12, 30
Hnatyk B.I., Lukash V.N., Novosyadlyj B.S., Kinematica i fizika nebesnyck tel, 7, 48
Haehnelt M. & Rees M.J. 1993, MNRAS, 263, 168
Amendola A., Occhionero F. & Saez D. 1990, ApJ, 349, 399
Gottlöber S., Müller V. & Starobinsky A.A. 1991, Phys. Rev. D43, 2510
Starobinsky A.A. 1992, Pis’tma Zh. Eksp. Teor. Fiz., 55, 477
Peter P., Polarski D. & Starobinsky A.A., 1994, Phys. Rev., D50, 4827
Irwin M., McMahon R.G & Hazard C., 1991, in The Space Density of Quasars, ed.D. Crampton (SF: ASP), 1991, p.117

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