Approximate quantum gates compilation for superconducting transmon qubits with self-navigation algorithm

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Abstract
Precise and fast qubit control is crucial when compiling quantum gates for successful implementation of quantum algorithms. However, the presence of environmental noise and the nonzero bandwidth of control pulses pose challenges for the effective control, particularly for weakly anharmonic systems such as superconducting transmon qubits. To address these problems, in this work we propose a self-navigation algorithm to approximately compile single-qubit gates with high accuracy in the context of transmon qubit. By utilizing this algorithm, the overall rotation distance for the target gate operation is significantly shorter than that of the commonly used $U_3$ gate technique. As a result, a shorter gate time can be obtained. The necessary number of pulses and the runtime of scheme designing scale up as $O(\log(1/\epsilon))$ with a small pre-factor, indicating a low overhead cost. Moreover, we investigate the trade-off between effectiveness and cost, and a balance point is identified. Our results demonstrate a reduction in both gate time and noise effects, but without an increase in leakage. Our work opens up a new avenue for efficient quantum algorithm implementations with contemporary superconducting quantum technology.

Keywords Quantum gate · Approximate compilation · Superconducting transmon qubits

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1 Introduction

Quantum algorithms have shown superpolynomial or exponential speedup compared to their classical counterparts [1–3] and their advantages have been demonstrated in different physical systems during the past few years [4–9]. Quantum logical operation is always required for quantum computing and it can be constructed by single- and two-qubit elementary gates [1, 10–12]. Then a given unitary can be decomposed into a sequence of elementary gates. A lot of works have focused on the optimal decomposition, including the Cartan’s KAK [13, 14], Gray Code [15], Revised Greedy [16], Cosine-Sine [17], etc. In addition, gate synthesis has been proposed to reduce the number of gates that must be executed in experiments [18]. Recently, nascent machine learning has also shown its potential in this field [19–24].

To enact a quantum program, the abstract unitary operators (such as the Hadamard or CNOT gate) have to be transformed into executable forms depending on the experimental platform, such as microwave pulse sequences in superconducting circuit systems. This translation step, also known as gate compilation, presents new opportunities for performance improvement [25–27]. The Solovay-Kitaev algorithm and its variants [28, 29] are effective methods for fault-tolerant quantum computation. It can approximate unitaries to arbitrary accuracy by using discrete hardware-supported gates. However the Solovay-Kitaev algorithm requires extravagant hardware resources that can only be realized by future fault-tolerant quantum devices and it may take years or even decades in the era of noisy intermediate-scale quantum (NISQ) devices [30]. For the long evolution time, the qubits are susceptible to noise, and the size of quantum processors that can be reliably controlled is also relatively limited. Normally the noise will impose deleterious effects on the qubits, leading to reduced fidelity [31]. Typically, the detrimental effects of noise on the devices become serious over time [32].

Circuit-approximation schemes [32, 33] have been proposed to improve the performance in NISQ devices. These schemes do not seek to faithfully execute a given algorithm circuit, but explore approximated versions with fewer multi-qubit gates and shorter circuit depths. Approximate circuits have shown their potential to outperform theoretically ideal but deeper circuits. A lot of research have been focused on circuit optimization, while few on the approximating quantum gates for practical implementation. The latter presents a promising approach for harnessing the potential of NISQ devices, particularly in controlling qubits made from weakly anharmonic oscillators. In this case, unwanted leakage poses another challenge to realize fast and precise quantum gates. For instance, in superconducting transmon qubits, nonzero overlaps between the drive and the leakage transition frequencies caused by finite pulse duration can drive the qubits out of their subspace. The derivative reduction by adiabatic gate (DRAG) scheme has been proposed to mitigate this population leakage by modifying the quadrature amplitudes of the microwave drives [34, 35]. However, the optimal value of the scaling parameter in DRAG is sensitive to pulse distortions and requires repeated identification and calibration in the experiments [36, 37].

The spectral content of leakage frequency will decay exponentially as a function of pulse duration, then an alternative approach to prevent leakage is to use slower operations with time-domain broadened control pulses. This results in a control bandwidth significantly smaller than the anharmonicity. However, this strategy generally extends
gate execution time, which limits the reliable circuit depth due to the decoherence. If the unitary is compiled optimally, it is possible to reduce the overall gate time by finding a short path to compensate for the extended time spent on unit rotation distance. In this paper, we propose a feasible method (SN algorithm) for approximating single-qubit gates compilation. We demonstrate that any desired accuracy can be achieved compared to the precise gate, and the designed overall rotation distance is much shorter than that of the commonly used U3-gate technique \([26, 38, 39]\). The SN algorithm provides both the number of elementary rotations and runtime in \(O[\log(1/\epsilon)]\) with a small prefactor, resulting in minimal cost. Evaluation results indicate that the control scheme designed by the SN algorithm significantly reduces the gate time and effectively suppresses the leakage. Furthermore, unlike methods based on machine learning \([19–22]\), our approach does not require time-consuming training processes and provides high accessibility.

2 Model

The transmon qubit \([40]\) is a widely used type of qubit that is constructed from weakly anharmonic oscillators. In the rotating frame with respect to the driving frequency and under the rotating wave approximation, its effective Hamiltonian can be written as \([31, 41]\)

\[
H_q = -\frac{\hbar}{2} \delta \sigma_z + \frac{\hbar}{2} \Omega (\cos \phi \sigma_x + \sin \phi \sigma_y),
\]

where \(\delta\) denotes the qubit detuning from the microwave drive frequency. When the qubit resonates with the microwave \((\delta = 0)\), the rotation around the axis in the \(XY\)-plane can be realized by microwave drive for certain time. \(\Omega\) refers to the drive amplitude, and \(\sigma_i\) \((i = x, y, z)\) are the Pauli matrices. The phase of the microwave \(\phi\) determines the rotation axis. For simplicity, we set \(\hbar = 1\) and take \(1/\hbar\) as the time-scale throughout.

The shift of the qubit frequency \(\delta\) in Eq. (1) leads to a rotation around \(z\)-axis and can be tailored by modulating the flux bias of the SQUID (superconducting quantum interference device) \([42]\). However, this is not necessary in the experiments. It can be accessed alternatively by the so-called “virtual \(Z\) gate” technique \([38]\), which can be completed by applying a specific phase offset \(\phi\) to the microwave signals for subsequent rotation around the axis in the \(X-Y\) plane. For example,

\[
\exp(-i \frac{\theta}{2} [\cos \phi \sigma_x + \sin \phi \sigma_y]) = Z_{-\phi} X_{\theta} Z_{\phi},
\]

\[
\exp(-i \frac{\theta}{2} \cos(\frac{\pi}{2} + \phi) \sigma_x + \sin(\frac{\pi}{2} + \phi) \sigma_y) = Z_{-\phi} Y_{\theta} Z_{\phi}.
\]

Note that they leave an extra \(Z_{-\phi}\) which does not change the outcome of the measurement along \(z\). The implemented virtual \(Z\) gate is “perfect”, because it requires no additional control pulse and therefore taking zero-time and having unity gate fidelity nominally.
A generic single-qubit gate (up to a global phase) can be realized by three successive rotations around the $z$, $x$- and $z$-axes. These are both native operations in the superconducting circuits model [26, 38, 43]

$$U(\theta, \phi, \lambda) = Z\phi X\theta Z\lambda = \begin{bmatrix} \cos(\theta/2) & -e^{i\lambda} \sin(\theta/2) \\ e^{i\phi} \sin(\theta/2) & e^{i(\lambda+\phi)} \cos(\theta/2) \end{bmatrix},$$  

(4)

where $\theta$, $\phi$ and $\lambda$ represent 3 Euler angles. Combined with the identity

$$X\theta = Z_{-\pi/2} X_{\pi/2} Z_{-\theta} X_{\pi/2} Z_{-\pi/2},$$  

(5)

Eq. (4) can be reexpressed as

$$U(\theta, \phi, \lambda) = Z_{-\pi} X_{\pi} Z_{-\theta} X_{\pi} Z_{-\pi/2}.$$  

(6)

The above is the so-called $U3$ gate technique, a commonly used strategy to compile single-qubit logical operations in the experiments [26, 39, 43]. As mentioned before, the rotations around $z$-axis can be included into the microwaves used for the $X_{\pi/2}$ as an additional phase. That is to say, the requisite overall rotation distance is always that used to implement two $X_{\pi/2}$ operations, i.e., $\pi$. Can we find a shorter rotation path, thereby reduce the gate time and hence reduce the effects of noise when using prolonged driving pulses? In this paper, we obtain such a shorter rotation distance compared to the $U3$ gate by using the SN algorithm.

3 Algorithm

An arbitrary single-qubit gate can be represented by a rotation around an axis $\vec{n}$ with an angle $\theta$:

$$R_{\vec{n}}(\theta) = \cos \left( \frac{\theta}{2} \right) - i \sin \left( \frac{\theta}{2} \right) (n_x \sigma_x + n_y \sigma_y + n_z \sigma_z)$$  

(7)

We firstly consider the rotations around a fixed axis $\vec{n}$. Assume the target rotation angle is $\theta_T$ and the current angle is $\theta_I$. Here the rotation speed and duration have been included in the rotation angle. To determine the appropriate next rotation angle, by defining the fidelity $F$ between the two unitaries as the Hilbert-Schmidt distance [44, 45], we observe that

$$F = \left| \frac{\text{Tr} \left[ R_{\vec{n}}^\dagger (\theta_T) R_{\vec{n}} (\theta_I) \right] }{2} \right|^2 = |\cos \left( \frac{\Delta \theta}{2} \right) |^2.$$  

(8)

Here $\Delta \theta = \theta_T - \theta_I$. Then we have

$$\Delta \theta = 2\arccos(\sqrt{F}).$$  

(9)
In other words, we can obtain the required subsequent rotation angle $\Delta \theta$ via the current fidelity in the coaxial case.

However, the presumed rotations around the axis $\vec{n}$ in Eq. (7) may not be experimentally realizable. Our strategy to address this issue is to use an experimentally accessible approximate axis to replace the theoretical axis. This approximation axis performs best among all experimentally accessible axes when the rotation angle equals $\Delta \theta$. Then with the corresponding unitary obtained, another rotation angle and approximate axis can be performed iteratively in a similar manner, until the compilation error $\epsilon = 1 - F$ is less than a desired target value $\epsilon_T$. All of these approximate rotation operations eventually form a sequence in order. The adjacent and commutable rotations in the sequence can be merged together to reduce the redundant pulses. At last, the $z$-rotations are absorbed into the subsequent $X$-$Y$ operations by taking advantage of the virtual $Z$ gate technique. The workflow of this algorithm is shown in Fig. 1. We point out that the appropriate rotation axes and angles can be determined by this algorithm, then an arbitrary single-qubit unitary operator is compiled dynamically and automatically. From this perspective, we name it the self-navigation algorithm.

4 Results and discussions

To evaluate the performance of our $SN$ algorithm in the context of superconducting transmon qubit, we exemplarily take $z$-axis and several axes located in the $XY$-plane as the approximate rotation axes. Note that these axes are accessible on this superconducting transmon qubit platform, which can be parameterized by the phase offset $\phi$ in Eq. (1). Now we study the performance of the $SN$ algorithm with different numbers of allowed rotation axes $N_{\text{axes}}$. Specifically, we take $N_{\text{axes}} = 3, 5, 9$ and 17 as examples. In each case, $N_{\text{axes}} - 1$ axes are distributed uniformly on the $XY$-plane. For example, when $N_{\text{axes}} = 5$, 4 axes are represented by $\phi = 0, \pi/4, \pi/2, 3\pi/4$, respectively. Arbitrary single-qubit gates can be realized by sequentially rotating around the $z$-, $x$-
Fig. 2. The averaged achieved accuracies $\bar{\epsilon}$ (a), overall rotation distances $\bar{d}$ (b), numbers of pulses $\bar{n}$, and the time $\bar{t}$ (d) versus the desired target accuracy $\epsilon_T$ for different number of the allowed rotation axes $N_{\text{axes}}$. The dashed line in (a) corresponds to the case $\bar{\epsilon} = \epsilon_T$. The solid curves in (b), (c) and (d) are the fitting functions based on the corresponding actual data. The $SN$ algorithm is used to compile arbitrary single-qubit gates and $z$-axes with randomly sampled angles $\theta_0$, $\theta_1$ and $\theta_2$, respectively. In this paper, we take 128 unitaries as the evaluation dataset.

The critical metrics for evaluating a gate-compilation algorithm include the accuracy and efficiency. In this paper, the accuracy is characterized by the compiling error ($\epsilon = 1 - F$) between the achieved gate and the target gate. The efficiency is determined by the rotation distance and the number of required pulses. A shorter overall rotation distance indicates a shortcut, which can reduce the gate time. Additionally, the runtime of the algorithm is also an important criterion, representing the accessibility of the algorithm.

Now we use our $SN$ algorithm to approximately compile quantum gates and evaluate its performance by the accuracy and efficiency. In Fig. 2a, we plot the averaged achieved accuracies $\bar{\epsilon}$ as functions of the target accuracy $\epsilon_T$ for different allowed axes. The dashed line in Fig. 2a represents the threshold $\bar{\epsilon} = \epsilon_T$. Figure 2a shows that any given approximate accuracy can be achieved with the $SN$ algorithm for different axes. The value of $\bar{\epsilon}$ is very close to $\epsilon_T$.

Figure 2b exhibits the averaged overall rotation distances $\bar{d}$ as functions of $\epsilon_T$ with various allowed axes. From Fig. 2b we can see that for certain $N_{\text{axes}}$, at first $\bar{d}$ increases
with decreasing $\epsilon_T$ then tends to converge to a fixed value, e.g., 1.88 for $N_{\text{axes}} = 9$ and $\epsilon_T = 1 \times 10^{-7}$. Note that this value is only about 60% of the fixed $\pi$ distance obtained by the $U3$ gate technique [26, 38, 39]. A shortcut can be found to significantly shorten the actual gate-time by our $SN$ algorithm. In addition, we observe that $\bar{d}$ decreases with increasing $N_{\text{axes}}$ for certain $\epsilon_T$. However, this trend gradually weakens and nearly disappears when $N_{\text{axes}} = 9$. Therefore, $9$ allowed axes is sufficient for this compiling task, as additional axes only yield negligible improvements.

Figure 2c illustrates the averaged numbers of required pulses $\bar{n}$ as functions of $\epsilon_T$. For $N_{\text{axes}} = 3, 5, 9$ and 17, the values of $\bar{n}$ are 5.31, 3.68, 3.75 and 3.92 respectively, with $\epsilon_T = 1 \times 10^{-4}$. And they are 9.90, 5.82, 5.97 and 5.95 respectively, with $\epsilon_T = 1 \times 10^{-7}$. In Fig. 2c, it is evident that $\bar{n}$ scales up with $\mathcal{O}[\log(1/\epsilon)]$ as $\epsilon_T$ decreases. Notably, $N_{\text{axes}} = 9$ is sufficient for the implementation.

The execution time of the algorithm to determine the appropriate pulse sequence is also an important metric. Figure 2d plots the averaged runtime $\bar{t}$ versus $\epsilon_T$ for different number of allowed axes. Figure 2d shows that bigger $N_{\text{axes}}$ corresponds to longer $\bar{t}$ as expected. $\bar{t}$ roughly scales up as $\mathcal{O}[\log(1/\epsilon)]$ with small prefactors (about $10^{-3} \sim 10^{-2}$) for all cases.

Next, we consider the noise on the quantum gates. In practical quantum computing, the physical transmon qubits will inevitably suffer from a nonzero bandwidth of the control pulses and external noise, causing population leakage and limiting the coherence time. For the population leakage, the third energy level spectroscopically contributes the greatest threat to the manipulation on the qubit. In this case the Hamiltonian (1) can be rewritten as [35, 36]

\[
H(t) = \delta(t)|1\rangle\langle 1| + (2\delta(t) + \Delta)|2\rangle\langle 2| + \left[ \frac{\Omega_x(t)}{2} \sigma_{0,1}^x + \frac{\Omega_y(t)}{2} \sigma_{0,1}^y \right] + \lambda \left[ \frac{\Omega_x(t)}{2} \sigma_{1,2}^x + \frac{\Omega_y(t)}{2} \sigma_{1,2}^y \right],
\]

where $\sigma_{j,k}^x = |j\rangle\langle k| + |k\rangle\langle j|$, and $\sigma_{j,k}^y = -i|j\rangle\langle k| + i|k\rangle\langle j|$ for $k > j$. $\lambda$ is a dimensionless parameter which represents the relative strength between transitions $|1\rangle \rightarrow |2\rangle$ and $|0\rangle \rightarrow |1\rangle$. Here we let $\lambda = 1$ as in Ref. [35]. $\Omega_{x,y}(t)$ are the amplitudes of microwave pulses and $\Delta$ is the anharmonicity.

To suppress the leakage to the third energy level, we use the pulses with a Gaussian envelope $\Omega_G$ as in Refs [35, 36] to drive the qubit, i.e., $\Omega_{x,y}(t) = \Omega_G(t)$. It takes the form

\[
\Omega_G(t) = \theta \frac{\exp\left[-\left(t-t_g/2\sigma^2\right)^2\right] - \exp\left[-t_g^2/8\sigma^2\right]}{\sqrt{2\pi}\sigma \text{erf}\left[t_g/\sqrt{8}\sigma\right] - t_g \exp\left[-t_g^2/8\sigma^2\right]},
\]

where $\sigma$ refers to the standard deviation, $t \in [0, t_g]$ and $t_g = 4\sigma$ refers to the pulse duration. The parameter $\theta$ specifies the rotation angle that will be implemented. This form ensures that the pulse starts and ends with zero intensity.

Now we compare the performance of gate implementation designed by our scheme with $U3$ technique in the presence of noise. We choose the $H^{1/4}$ gate as an example, which corresponds to a rotation of $\pi/4$ radians around the $x+z$ axis of the Bloch...
sphere. As mentioned before, with the virtual \( Z \) gate and \( U3 \) technique, the required rotation angles to implement this \( H^{1/4} \) gate are two \( \pi/2 \) around the \( x \)-axis. Using our \( SN \) scheme with 9 allowed axes these angles are 0.3927 and 0.1575 around the \( x \) axis and the axis corresponding to \( \phi = 7\pi/8 \) in Eq. (1), respectively. We set the standard deviation of the Gaussian pulses as \( \sigma = \theta \pi/\Delta \). Compared to the \( U3 \) technique, our scheme saves 82.5\% of the overall rotation angle and gate time.

The leakage noise \( L \) can be quantified by the population escaping from the qubit subspace during the gate time. The population leakage \( L \) versus evolution time during the \( H^{1/4} \) gate time is plotted in Fig. 3a. We can see that the execution time based on the \( SN \) scheme (6.91) is only 17.5\% of the time based on the \( U3 \) scheme (39.48), while the final leakages are both well suppressed (1.2095 \( \times 10^{-4} \) and 1.196 \( \times 10^{-4} \), respectively).

We only consider the leakage noise in the above discussions. Next we consider the amplitude damping channel, which describes the energy dissipation of the quantum system due to the interaction with the environment [1]

\[
\epsilon(\rho) = E_0 \rho E_0^\dagger + E_1 \rho E_1^\dagger,
\]

with Kraus operators

\[
E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix},
\]

where \( \rho \) and \( \epsilon(\rho) \) are the initial and final density matrices of the quantum states, respectively. \( \gamma \) refers to the dissipation factor. We assume that the dissipation factor of the channel is proportional to the gate time and we use Monte Carlo method to obtain the simulation results by multiple sampling. Figure 3b displays the averaged final fidelity of quantum states generated by the \( H^{1/4} \) gate based on \( SN \) and \( U3 \) schemes as functions of dissipation factor \( \gamma \). Higher fidelities can be obtained by using our \( SN \) scheme due to the shorter gate time.
Given the limitations of quantum computing hardware presently accessible, we simulate quantum computing on a classical computer and generate the corresponding data. Our algorithms are implemented with PYTHON 3.7.9, MindQuantum 0.8.0 and run on a computer with a four-core 1.80 GHz CPU and 8 GB memory. The source code supporting this work can be found in the Sec. Data and code availability.

5 Conclusion

In this work, we propose an $SN$ algorithm to approximately compile arbitrary single-qubit gates with rotations all natively available in the superconducting transmon qubits. The evaluation results show that the overall rotation distance and gate time of the control scheme designed by our $SN$ algorithm are significantly shorted compared to those of the commonly used $U3$ gate technique. In addition, any desired accuracy for the target gate can be achieved at the expense of a reasonable increase in the pulse number and pulse designing time. Combined with the virtual $Z$ gate technique, we observe that both of these values are modest and only logarithmic in $1/\epsilon$ with a small prefactor, indicating a low overhead cost. At last, we consider the leakage and amplitude damping noise during gate time. We use Gaussian pulses to suppress the leakage and the simulation results indicate that both gate time and the effects of the noise with amplitude damping have been significantly reduced.

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Data and code availability The code, running environment of algorithm and all data presented in this paper are available from the corresponding author upon reasonable request or from Gitee in (https://gitee.com/mindspore/mindquantum/tree/research/paper_with_code).

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