Multibarrier tunneling

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We study the tunneling through an arbitrary number of finite rectangular opaque barriers and generalize earlier results by showing that the total tunneling phase time depends neither on the barrier thickness nor on the inter-barrier separation. We also predict two novel peculiar features of the system considered, namely the independence of the transit time (for non resonant tunneling) and the resonant frequency on the number of barriers crossed, which can be directly tested in photonic experiments. A thorough analysis of the role played by inter-barrier multiple reflections and a physical interpretation of the results obtained is reported, showing that multibarrier tunneling is a highly non-local phenomenon.

I. INTRODUCTION

A renewed interest in a typical quantum phenomenon such as the tunnel effect has been recently achieved due to a long series of experiments aimed to measure the tunneling transit time (for reviews see, for instance, [1]). While such experiments involving electrons are usually difficult to realize (mainly due to the smallness of the electron de Broglie wavelength at usual temperatures) and even of uncertain interpretation, the observations on photonic tunneling [2]- [6] has by now provided clear data on this subject. Despite the different phenomena studied in several experiments (undersized waveguides, photonic band gap, total internal reflection) and the different frequency ranges for the light used (from the optical to the microwave region), all such experiments have shown that, in the limit of opaque barriers, the transit time to travel across a barrier of width $a$ is usually shorter than the corresponding one required for real (not evanescent) propagation through the same region of width $a$. This result can be interpreted [1] in terms of a superluminal group velocity $v_{tg} > c$ which, however, does not violate Einstein causality, since the signal velocity relevant for that [7] is never measured. Nevertheless we prefer to look at the experimental result as an observation of the simple Hartman effect [8]: for opaque barriers the tunneling phase time is independent of the barrier width. Although several definitions of the tunneling time (also related to the different experimental setups used) exist [1] and a general consensus on this is still lacking, it seems that all the experimental results can be successfully interpreted in terms of phase time [9].

Further light has been put on the problem by recent experiments involving double barrier penetration [10]. In fact, while the above effect has been confirmed in such a system too (far from the resonances of the structure), observations show that the transit time is also independent of the separation distance between the barriers (supposed to be thick). This peculiar phenomenon has been studied theoretically in Ref. [11], where the authors have provided a straightforward generalization of the Hartman effect for double barrier tunneling.

Convincing qualitative explanations of these two findings (namely that the tunneling phase time is independent of the barrier thickness as well as of the inter-barrier separation for opaque barriers) have been reported. When considering a given wavepacket entering into a potential barrier region, a reshaping phenomenon occurs in which the travelling edge of the pulse is preferentially attenuated (the leading one, thus simulating a group velocity greater than $c$ [1], [3]. In practice the Hartman effect in the tunneling through a thick barrier is explained from the fact that under the barrier no phase accumulates, and the entire phase shift comes only from the boundaries, thus being substantially independent of the thickness [12]. Furthermore, when two barriers are present, the transit time independence on the barrier separation can, instead, be understood in terms of an effective acceleration of the forward travelling waves in the inter-barrier region, which arises from a destructive interference between the two barriers [11].

Further noticeable results have been recently achieved in Ref. [12], where it has been shown that a wavepacket travels in zero time a region with $N$ arbitrary $\delta$—function barriers.

In this paper we extend all these findings by considering the case of $N$ successive opaque barriers with finite widths and heighths. While we confirm all previous results, we generalize them by showing that some peculiar tunneling properties are independent of the number...
of the barriers crossed (Sect. II). Furthermore, in order to establish a quantitative interpretation of the involved phenomena, in Sect. III we study the role of multiple reflections in double barrier tunneling and show how strongly the total tunneling phase time depends on nonlocal effects. Finally in Sect. IV we discuss the results obtained and give our conclusions.

In view of the formal analogu [13] between the Schrödinger equation and the electromagnetic Helmholtz equation, our study applies to matter particle tunneling as well as to evanescent propagation of photonic wavepackets. This is a straightforward consequence of the fact that in both cases the starting point is basically the same (in our case it is Eq. (2)) [9], on interchanging the roles of angular frequency $\omega$ and wavevector $k$ into the corresponding ones of energy $E$ and momentum $p$ through the Planck - de Broglie relations. Thus, throughout this paper, we use indifferently particle or wave terminology unless the meaning of what we are doing becomes unclear.

II. TUNNELING THROUGH $N$ SUCCESSIVE BARRIERS

Let us consider a wavepacket moving along the $x$-axis and entering at $x = 0$ into a region with a potential barrier $V(x)$ as depicted in Fig. 1:

\[
V(x) = \begin{cases} 
V_0 & (i-1) L \leq x \leq (i-1) L + a \\
0 & \text{otherwise}
\end{cases} \quad (i=1,2,\ldots,N)
\]

\[ (i=1,2,\ldots,N). \] For the sake of simplicity we choose the height $V_0$ of the potential barriers, as well as the width $a$ of each barrier, to be the same for all $N$ rectangular barriers. We further assume equally spaced barriers, $L - a$ being the inter-barrier distance.

The propagation of the wavepacket through the barriers is described by a scalar field $\psi$ representing the Schrödinger wavefunction in the particle case or some scalar component of the electric or magnetic field in the photonic case. This is the solution of the Schrödinger equation or the Helmholtz equation with potential or refractive index in Eq. (1) and, in both cases, it takes the following form *:

\[
\psi(x) = \begin{cases} 
\psi_{2i}(x) & (i-1) L \leq x \leq (i-1) L + a \\
\psi_{2i+1}(x) & \text{otherwise}
\end{cases} \quad (i=1,2,\ldots,N)
\]

with:

\[
\psi_1(x) = e^{ikx} + Re^{-ikx} \quad (3)
\]

\[
\psi_{2i}(x) = A_{2i}e^{i\chi[x-(i-1)L]} + B_{2i}e^{-i\chi[x-(i-1)L]} \quad (i=1,2,\ldots,N) \quad (4)
\]

\[
\psi_{2i+1}(x) = A_{2i+1}e^{i\chi[x-(i-1)L]} + B_{2i+1}e^{-i\chi[x-(i-1)L]} \quad (i=1,2,\ldots,N-1) \quad (5)
\]

\[
\psi_{2N+1}(x) = Te^{ik[x-(N-1)L]} . \quad (6)
\]

As noted in Ref. [9], the explicit dependence on the frequency of the (real) wavevector $k$ in the barrier-free regions and imaginary wavevector $i\chi$ in the barrier ones enters only in the final expression for the phase time. As long as possible we do not use a particular dispersion relation in order to draw general features which are common to the particle and to the wave case. The $4N$ unknown coefficients $R, T, A_i, B_i$ are obtained from the $4N$ matching conditions for the function $\psi$ and its derivative $\psi'$ at the discontinuity points $x + (i-1)L, x = (i-1)L + a$ of the potential:

\[
\psi_{2i-1}(x = (i-1)L) = \psi_{2i}(x = (i-1)L) \quad (7)
\]

\[
\psi_{2i-1}'(x = (i-1)L) = \psi_{2i}'(x = (i-1)L) \quad (7)
\]

\[
\psi_{2i}(x = (i-1)L + a) = \psi_{2i+1}(x = (i-1)L + a) \quad (7)
\]

\[
\psi_{2i}'(x = (i-1)L + a) = \psi_{2i+1}'(x = (i-1)L + a) \quad (7)
\]

\[
(i=1,2,\ldots,N). \] Note that the quantities $R$ and $T$ have the meaning of (total) reflection and transmission coefficient from the $N-$barrier system, respectively, and satisfy the unitarity condition:

\[
|R|^2 + |T|^2 = 1 . \quad (8)
\]

*Obviously, the physical field is represented by a wavepacket with a given spectrum in $\omega$: \[
\Psi(x,t) = \int d\omega \eta(\omega) \psi(x) e^{-i\omega t} ,
\]

where $\eta(\omega)$ is the envelope function. Keeping this in mind, for the sake of simplicity we deal with only stationary solutions as in Eq. (2).
We have produced a Mathematica symbolic code in order to obtain explicit analytic expressions for all the coefficients appearing in Eqs. (3)-(6). However, we here report only the interesting result obtained for the transmission coefficient $T(N)$ for an $N$-barrier system in the opaque barrier approximation $\chi a >> 1$. In this limit the quantity $T(N)$ can be factorized in the following way:

$$T(N) e^{ika} = C_0 \cdot \mathcal{E}(N) \cdot \mathcal{F}(N)$$  \hspace{1cm} (9)

$$C_0 = \frac{4i\chi k}{(k + i\chi)^2}$$

$$\mathcal{E}(N) = [e^{-\chi a}]^N$$

$$\mathcal{F}(N) = \left[ \frac{2\chi k}{2\chi k \cos(k(L-a)) - (k^2 - \chi^2) \sin k(L-a)} \right]^{N-1}$$

Note that only the real terms $\mathcal{E}$ and $\mathcal{F}$ depend on $a, L, N$, while the complex factor $C_0$ does not. As a consequence, since the tunneling phase time $\tau$ is defined as:

$$\tau = \frac{d\phi}{d\omega}$$  \hspace{1cm} (10)

and the quantity

$$\phi \equiv \arg \{T(N) e^{ika}\} = \arg \left\{ \frac{4i\chi k}{(k + i\chi)^2} \right\} = \arctan \frac{k^2 - \chi^2}{2\chi k}$$  \hspace{1cm} (11)

is independent of $a, L, N$, we arrive at the general conclusion that the tunneling phase time for a system of $N$ opaque barriers depends neither on the barrier width and inter-barrier distance nor on the number of the barriers. Let us now discuss the effects of the real terms in Eq.(9) on the tunneling probability $P_T(N) = |T(N)|^2$:

$$P_T(N) = \left[ \frac{4\chi k}{k^2 + \chi^2} \right]^2 [e^{-\chi a}]^{2N} \cdot \left[ \frac{2\chi k}{2\chi k \cos(k(L-a)) - (k^2 - \chi^2) \sin k(L-a)} \right]^{2(N-1)}$$  \hspace{1cm} (12)

We easily recognize that the last factor in Eq.(12), coming from the term $\mathcal{F}(N)$ is responsible of the resonance structure of the transmission probability. The factor $\mathcal{F}(N)$ is, of course, absent in the case of only one barrier, i.e. $N = 1$ or $N \neq 1$ but $L = a$. However, no resonance can occur even in the particular case in which the inter-barrier distance is tuned in a way that:

$$L - a = \frac{\nu\pi}{k} \hspace{1cm} (\nu = 0, 1, 2, \ldots)$$ \hspace{1cm} (13)

In this case, waves moving forward and backward in the inter-barrier regions interfere between them such that no resonance takes place.

The resonance condition for the tunneling probability is, from Eq. (12), the following:

$$\tan k(L-a) = \frac{2\chi k}{k^2 - \chi^2} \hspace{1cm} .$$  \hspace{1cm} (14)

It is worthwhile to observe that Eq.(14) does not depend on $N$, so that the resonant frequency is the same irrespective of the number of barriers to be crossed.

Finally, we point out an intriguing consequence of the resonance condition. Let us write Eq.(14) as follows:

$$\tan \phi \cdot \tan k(L-a) = 1$$  \hspace{1cm} (15)

where $\phi$ is given in Eq.(11), and take the derivative of Eq. (15) with respect to the angular frequency $\omega$. By using Eq. (10) we easily arrive at the following relation:

$$\tau + \tau_0 = 0$$  \hspace{1cm} (16)

where $\tau_0$ is the (phase) time for travelling the inter-barrier distance $L - a$ in vacuum. Keeping in mind that the total tunneling time has the same value of the tunneling time for crossing only one barrier (see above), we see that, when resonant tunneling takes place, the total time required to cover the distance $L$ (one barrier length $a$ plus one inter-barrier distance $L - a$) is zero.

### III. MULTIPLE REFLECTIONS AND NON-LOCALITY

In order to have a physical interpretation of the results obtained previously, we now consider the effect of single barriers on the propagation of the wavepacket through the entire $N$-barrier system, by invoking the superposition principle. For the sake of simplicity, we will study the case of a system of two opaque barriers.

#### A. Partial coefficients

For $N = 2$, in the barrier-free regions, Eqs.(3)-(6) reduce to the following:

$$\psi_1(x) = e^{ikx} + Re^{-ikx}$$

$$\psi_3(x) = A_3 e^{ikx} + B_3 e^{-ikx}$$

$$\psi_5(x) = T e^{ik(x-L)}$$

where the explicit expressions for the coefficients are reported in Appendix A. Let us now denote with $R_1, T_1$ and $R_2, T_2$ the (partial) reflection and transmission coefficients of the first and second barrier, respectively. In the region with $x < 0$ the reflected wave is described by the term:

$$Re^{-ikx} = R_1 e^{-ikx} + B_3 T_1 e^{-ikx}$$

while for $x > L + a$ the transmitted one is described by:

$$Te^{ik(x-L)} = A_3 T_2 e^{ik(x-L)}$$  \hspace{1cm} (17)

\[(18)\]

\[(19)\]
By taking into account multiple reflections from the two barriers in the region with \( a < x < L \), we see that the forward travelling wave is described by the term:

\[ A_3 e^{ikx} = T_1 \left[ 1 + R_1 T_2 + (R_1 T_2)^2 + \ldots \right] e^{ikx}, \tag{20} \]

while the backward one is described by:

\[ B_3 e^{-ikx} = A_3 R_2 e^{-ik(x-L)}. \tag{21} \]

Then, by introducing the quantity:

\[ S = \sum_{l=0}^{\infty} (R_1 R_2)^l = \frac{1}{1 - R_1 R_2}, \tag{22} \]

which accounts for multiple reflections, from Eqs.(18)-(21) we obtain:

\[ R = R_1 + B_3 T_1, \]
\[ T = A_3 T_2, \]
\[ A_3 = T_1 S, \]
\[ B_3 = A_3 R_2 e^{ikL}. \tag{23} \]

By solving these equations with respect to the partial reflection and transmission coefficients, we get:

\[ R_1 = \frac{R - A_3 B_3}{1 - B_3^2}, \]
\[ T_1 = \frac{A_3 - B_3 R}{1 - B_3^2}, \tag{24} \]
\[ R_2 = \frac{B_3 e^{-ikL}}{A_3}, \]
\[ T_2 = \frac{T}{A_3}. \]

In the opaque barrier limit \( \chi a \gg 1 \), for the second barrier we obtain:

\[ R_2 = R_{OB} e^{ikL}, \]
\[ T_2 = T_{OB} e^{ikL}, \tag{25} \]

while for the first barrier:

\[ R_1 = R_{OB} + R_Q + R_R, \]
\[ T_1 = T_{OB} + T_Q + T_R, \tag{26} \]

where:

\[ R_{OB} = \frac{k - i\chi}{k + i\chi} \left[ 1 - \frac{4i\chi k}{(k + i\chi)^2} e^{-2\chi a} \right], \tag{27} \]
\[ T_{OB} = \frac{4i\chi k}{(k + i\chi)^2} e^{-ika} e^{-\chi a}. \]

are the reflection and transmission coefficients corresponding to a one-barrier system \((N = 1)\) and:

\[ R_Q = - \left( \frac{k - i\chi}{k + i\chi} \right)^3 F^2 e^{2ik(L-a)} e^{-2\chi a} \]
\[ R_R = \left( \frac{k - i\chi}{k + i\chi} \right)^3 F^2 e^{ikL} e^{-2\chi a} \]
\[ T_Q = \left( \frac{k - i\chi}{k + i\chi} \right)^2 F e^{2ik(L-a)} e^{-ikL} e^{-\chi a} \]
\[ T_R = - \left( \frac{k - i\chi}{k + i\chi} \right)^2 F e^{-\chi a}. \tag{28} \]

For future reference, we also consider the partial coefficients \( R_{10}, T_{10}, R_2^0, T_2^0 \) in the approximation of no multiple reflections, as considered in Ref. [11]. These are obtained from Eqs.(23) by setting \( S = 1 \). We have:

\[ R_1 = R_{OB} + R_Q \]
\[ T_1 = T_{OB} + T_Q, \tag{29} \]

while \( R_2^0, T_2^0 \) are the same as in Eqs.(25).

**B. Unitarity conditions**

The interpretation of the quantities \( R_1, T_1 \) and \( R_2, T_2 \) as reflection and transmission coefficients of the first and second barrier is derived from the unitarity conditions satisfied by these coefficients. In fact, since \(|R|^2 + |T|^2 = 1\) and \(|R_{OB}|^2 + |T_{OB}|^2 = 1\), we find that:

\[ |R_1|^2 + |T_1|^2 = 1 \]
\[ |R_2|^2 + |T_2|^2 = 1. \tag{30} \]

It is easily recognizable as well that, assuming no multiple reflection, the total probability for scattering from the first barrier is lower than \(1\):

\[ |R_1|^2 + |T_1|^2 = 1 - F^2 e^{-2\chi a}, \tag{31} \]

\[ \frac{1}{2} \]

\[ \text{The authors of Ref. [11] have considered the case of no multiple reflections and, moreover, they also neglected the second term \( B_3 T_1 \) in the first equation in (23) corresponding to backward waves in the \( x < 0 \) region transmitted from the first barrier, reflected from the second one and again transmitted from the first barrier. While their parametrization of the wavefunction is, of course, permitted and leads to correct results, nevertheless the partial coefficients they obtained have no direct physical meaning, as we will show below.}

\[ \text{In the approximation considered in Ref. [11] the quantity \( R_1^0 \) in Eq.(29) should be replaced by the following one:}
\[ R_1^0 = R_{OB} + R_1 \]

\[ \frac{k - i\chi}{k + i\chi} \frac{d}{d} e^{ik(L-a)} e^{-2\chi a} \]

\[ \text{Instead, by using the parametrization of Ref. [11], we obtain an unphysical scattering probability greater than 1,} \]
corresponding to the double barrier crossing here consid-

\[ P_R = |R_R|^2 + |T_R|^2 = e^{-2\chi a} \]  

(32)

Thus the quantities \( R_R \) and \( T_R \), that must be added to the no multiple reflection coefficients \( R^0_1 \) and \( T^0_1 \) in order to obtain the complete ones \( R_1 \) and \( T_1 \) respectively, can be interpreted as the terms describing the phenomenon of multiple reflections between the first and second barrier.

The meaning of the picture just outlined is then, quite trivial. \( R_2 \) and \( T_2 \) corresponding to the second barrier are simply given by the one-barrier coefficients \( R_{OB}, T_{OB} \) times a phase factor which takes into account the fact that this barrier starts at \( x = L \), while the reference point in our discussion is at \( x = 0 \). Instead, \( R_1 \) and \( T_1 \) corresponding to the first barrier are given by the sum of two terms: the first one is the no multiple reflection coefficients while the second one describes the phenomenon of multiple reflections. However, it is remarkable that no multiple reflection coefficients \( R^0_2 \) and \( T^0_2 \) in Eqs.(29) do not coincide with the one-barrier coefficients \( R_{OB} \) and \( T_{OB} \). This is an obvious consequence of the fact that the scattering probability from the first barrier, neglecting multiple reflections, cannot be equal to the unity and the extra terms \( R_Q \) and \( T_Q \) in Eqs. (29) must be present in order to achieve the probability constraint in Eq.(31).

On the other hand, the scattering probability, including multiple reflections, must be equal to 1 (according to Eq. (30)), so that we can deduce that the quantities \( R_Q \) and \( T_Q \) are related to the multiple reflection coefficients \( R_R \) and \( T_R \). It is very easy to obtain from Eqs. (28) that \( R_Q \) and \( T_Q \) differ from \( R_R \) and \( T_R \) just for a phase factor (depending on \( L \) and \( a \)):

\[ \frac{R_q}{R_R} = \frac{T_Q}{T_R} = e^{i(kL-2\alpha)} \]  

(33)

Then, multibarrier tunneling is a highly non-local phenomenon driven by multiple reflections, whose influence on the determination of the reflection and transmission coefficients is (indirectly) present even in the case in which they are neglected.

C. Tunneling phase time

Let us now consider the tunneling phase time \( \tau \) in Eq.(10) corresponding to the double barrier crossing here consid-

\[ |R^0_1|^2 + |T^0_1|^2 = 1 + e^{-2\chi a} \]  

which makes impossible to give a direct physical meaning to \( R^0_1, T^0_1 \).

\[ \phi_1 = \arg \{T_1 e^{i\chi a}\} \]  

\[ \phi_2 = \arg \{T_2 e^{i\chi a}\} \]  

(34)

whose derivatives with respect to frequency give the phase times for the first barrier crossing, the second barrier crossing and the time associated to multiple reflections, respectively. Since \( T = T_1 T_2 S \) from Eqs. (23), the total tunneling phase is given by:

\[ \phi = \phi_1 + (\phi_2 - kL) + \phi_S \]  

(35)

This relation leads to the obvious conclusion that the tunneling time \( \tau \) is the sum of the partial times \( \tau_1 \) and \( \tau_2 \) spent to travel across the first and second barrier, respectively, plus the time \( \tau_S \) required by multiple reflections in the inter-barrier region of length \( L - a \). However, it is interesting to evaluate the explicit expressions for these times and, from Eqs.(34) we get:

\[ \phi_1 = \phi_0 - kL/2 + ka \]  

(36)

\[ \phi_2 - kL = \phi_0 \]  

(37)

where \( \phi_0 = \arg \{T_{OB} e^{i\chi a}\} \) is the one-barrier tunneling phase time and \( \phi_S = -\phi_0 + kL/2 - ka \).  

(38)

While the time to cross the second barrier equals exactly the one-barrier tunneling phase time (see Eq.(37)), from Eqs.(36) and (38) we see that:

\[ \phi_1 + \phi_S = 0 \]  

(39)

that is the time spent to travel from the starting edge of the first barrier to the starting edge of the second one is zero. Something similar to this statement has already been suggested in literature (see, for instance, Ref. [11]), but now we have a quantitative proof for that. Moreover, we can also deduce that, due to multiple reflections, the

**Note that the time \( \tau_2 \) corresponds to the phase \( 2kL \), since the travelling along the distance \( L \) is already taken into account in \( \phi_1 + \phi_S \) or, in other words, in the expression for the coefficient \( T_2 \) in (25) we have already considered the shift from \( x = 0 \) to \( x = L \).

\[ S = \frac{(k + i\chi)^2}{4\chi k} e^{-ikL/2} \frac{2\chi k}{2\chi k \cos kL/2 - (k^2 - \chi^2) \sin kL/2} \]  

Therefore, it is worth to observe that the probability for this phe-

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time to cross the first barrier is usually different from the one-barrier tunneling phase time since:

$$\phi_0 - \phi_1 = \frac{\phi_Q - \phi_R}{2}$$ (40)

where

$$\phi_Q = \arg\{T_Q\} = 2\phi_0 + kL - 2ka$$

$$\phi_R = \arg\{T_R\} = 2\phi_0$$

are the phase times corresponding to the terms $T_Q$ and $T_R$, the equality holding true only in the case in which the inter-barrier distance coincide with the barrier width, i.e. $L = 2a$.

**IV. CONCLUSIONS**

In this paper we have studied the tunneling of a particle or a photonic wavepacket through an arbitrary number $N$ of finite rectangular opaque barriers and obtained an analytic expression for the total transmission coefficient Eq.(9). From this we have confirmed and generalized to the present case what was found earlier for a system of one [8] or two [11] barriers: the (total) tunneling phase time is independent both of the barrier width and of inter-barrier distance. These features have been observed experimentally for single [2]- [6] and double barrier [10] tunneling using photonic setups.

Amazingly enough, we have further found that, although the tunneling probability decreases exponentially with the barrier thickness and with the number of barriers (in the opaque barrier limit), the tunneling time does not depend even on the number of barriers crossed, i.e. it is the same for one, two or more barriers. Moreover, when considering resonant tunneling, we have also shown that the position in frequency (or energy) of the resonance of the structure is independent of the number of barriers as well. These two novel predictions can be experimentally tested using, again, photonic devices.

In order to obtain a physical picture of what happens in the system considered and, especially, of the peculiar features outlined above, we have studied the role of multiple reflections between the barriers on the tunneling and found this to be a highly non-local phenomenon. In fact, as shown in Sect. III, even in the case of increasingly large separation between the barriers, the effect of multiple reflections cannot be avoided at all. In particular multiple reflections play a crucial role in the understanding of the intriguing results on the (total) tunneling time quoted above. Though in Sect. III we have dealt with a two-barrier system, the main results achieved can be easily generalized to multibarrier tunneling as follows. For $N$ barriers the partial reflection and transmission coefficients corresponding to the first $N - 1$ barriers are clearly influenced by multiple reflections occurring in the inter-barrier regions, while those associated to the last barrier are not and coincide with one-barrier coefficients up to a phase factor. In particular, as shown in Sect. III, the tunneling phase time for crossing only the last barrier equals that for a single barrier structure. Since the total tunneling time for crossing all the barriers coincides as well with the one-barrier time (see Sect. II), we immediately deduce that the time for travelling from the starting edge of the first barrier to the starting edge of the last one is zero. Note that such a result can be achieved only if we taken into account multiple reflections and, in any case, the partial times for crossing single barriers are usually different from the one-barrier tunneling time.

Finally we point out that our findings also agree with the recent results reported in [12], according to which a wavepacket travels in zero time a region with $N$ $\delta$-function barriers. In fact, as said above, the total tunneling time coincide with the transit time for the last barrier or one-barrier phase time. From [9] (see Eq.(13) of that paper) we then see that, for $\chi \to \infty$, this time tends to zero, thus recovering the result of Ref. [12]. It would then be nice, in the future, to make the connection with the recent results reported in [12], according to which a wavepacket travels in zero time a region with $N$ $\delta$-function barriers. In fact, as said above, the total tunneling time coincide with the transit time for the last barrier or one-barrier phase time. From [9] (see Eq.(13) of that paper) we then see that, for $\chi \to \infty$, this time tends to zero, thus recovering the result of Ref. [12]. It would then be nice, in the future, to make the connection between multiple reflections studied here and the tunneling interpretation in terms of superoscillations quoted in [12].

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**APPENDIX A: COEFFICIENTS FOR $N = 2$**

From Eqs.(7) we obtain the following expressions for the coefficients describing the propagation through two successive opaque barriers:

$$R \simeq \frac{k - i\chi}{k + i\chi} \left[ 1 + 2i \sin k(L - a) F e^{-2\chi a} \right]$$ (A1)

$$A_2 \simeq \frac{2k}{k - i\chi} \frac{(k - i\chi)^2}{2k} \sin k(L - a) F e^{-2\chi a}$$ (A2)

$$B_2 \simeq \frac{k - i\chi}{k + i\chi} \left\{ \frac{2k}{k - i\chi} \left[ 1 - \frac{(k - i\chi)^2}{2k} \sin k(L - a) \right] \cdot F e^{-2\chi a} \right\}$$ (A3)

$$A_3 \simeq e^{-ikL} F e^{-\chi a}$$ (A4)

$$B_3 \simeq \frac{k - i\chi}{k + i\chi} e^{ikL} F e^{-\chi a}$$ (A5)

$$A_4 \simeq 0$$ (A6)

$$B_4 \simeq \frac{2k}{k + i\chi} F e^{-\chi a}$$ (A7)

$$T \simeq \frac{4i\chi k}{(k + i\chi)^2} F e^{-2\chi a}$$ (A8)
(in all these expressions we have neglected terms of third order in $e^{\chi a}$).

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