Static Strength Assurance Factor and Ensuring the Required Level of Structure Safety

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Abstract. The relation of the static strength assurance factor with the probability of non-destruction of the structure is considered in this article. As a result, it becomes possible to assign the minimum allowed assurance factors, based on the need to ensure the given level of structure safety during operation.

1. Introduction

Components and structures which are under constant or monotonously increasing loads are calculated for static strength. The condition of static strength in design calculations [1] is:

\[ X \leq [X], \]  

(1)

where \( X \) is, as a rule, the numerical value of the mechanical stresses, which is determined by the balanced deformed state of the object being calculated, and \([X]\) is the limit value of this variable, taking into account the material, special aspects of the manufacturing technology and operation.

Depending on the accepted theory of strength, within which the calculation results are estimated, various combinations of the stress tensor components or strains can be used as \( X \). Most often, the stress-strain state is multiaxial. Then some equivalent stresses, calculated using known formulas [2, 3], or using finite element simulation, is used as \( X \), and either a fraction of the yield strength for plastic, or a fraction of the tensile strength for brittle materials [1, 2] is used as \([X]\). This fraction defines the concept of the static strength assurance factor, for example, \( k_T \) [2]:

\[ \frac{\sigma_r}{\sigma_i} = k_T, \]  

(2)

where \( \sigma_r \) – yield strength, and \( \sigma_i \) – value of the equivalent stress in the local zone of concentration arising from possible design loads.

The static strength condition is satisfied if the design value of \( k \) exceeds the min permitted value \([k]\), specified in the regulatory documents governing the rules for conducting calculations [4–7].

Assurance factors, among other things, determine the probability of destruction during loading or the safety level of a structure [8]. Unreasonable overstatement of the accepted assurance factor leads to the creation of an uneconomical design, and understatement leads to an insufficient safe design.
2. Statement of the problem
In full-scale parts with the same manufacturing technology, different stress values may arise due to tolerances during manufacturing or a random deviation of the level of the received load, that is, the loading can be described by a random value $X$, which has a density function $f(X)$, its mathematical expectation $\bar{X}$ and its variance $\sigma_X^2$.

For each material, the strength (endurance limit, yield strength or tensile strength) can also be described by a random value $Y$, which has its own density function $f(Y)$, mathematical expectation $\bar{Y}$ and variance $\sigma_Y^2$. In Figure 1, for example, two bell-shaped distributions are shown: $f(X)$ and $f(Y)$.

![Figure 1. Probability-density function: dashed line – loading; solid line – strength.](image)

When designing a product, ensuring the necessary reliability as the ability to perform its functions during the period of operation under the established conditions [9] is carried out by spacing the average values of these distributions taking into account the variance. At the same time, this means choosing the necessary assurance factor.

If $n$ of $N$ parts fail under loading, then we can speak about the statistical probability of the occurrence of a destruction event [10, 11], determined by the formula

$$W_i = \frac{n}{N},$$  \hspace{1cm} (3)

and the probability of non-destruction, determined by the formula

$$W_0 = \frac{N-n}{N}.$$  \hspace{1cm} (4)

Since failure-free operation and failure are mutually antithetical events, the sum of their probabilities is equal to one.

3. Theoretical part
The probabilities determined by formulas (3) and (4) can be calculated based on the probability-density functions, which are shown schematically in Figure 2.
Figure 2 shows the state of the structure depending on the relationship “strength – loading”: “1” – the state of the destruction; “0” – the state of non-destruction; “н” – the state of uncertainty. These states form areas of destruction and non-destruction, respectively, as well as a band of uncertainty.

The event, consisting in that the structure under load will be in the \( ij \)th state, is actually determined by the simultaneous occurrence of two statistically independent events: the loading is described by \( X_i \) value, and the strength – by \( Y_j \) value. Therefore, the probability of such a complex event [10] is equal to the product:

\[
\Delta W(i, j) = f(X_i) \cdot f(Y_j) \cdot \Delta X \cdot \Delta Y .
\]

Then the probability that destruction does not occur will be determined by summing the probabilities of all the states corresponding to “0” or by the integral of the form:

\[
W_0 = \int_0 \int f(X) \cdot f(Y) \cdot dX \cdot dY .
\]

The probability of destruction will be determined by summing the probabilities of all the states corresponding to “1”. The states of “н” should be added to \( W_i \), as a margin:

\[
W_i = \int \int f(X) \cdot f(Y) \cdot dX \cdot dY .
\]

The result of calculations by the formulas (6), (7) is schematically presented in Figure 3.

The cutting plane passing through the band of uncertainty “н” perpendicular to the “strength – loading” plane divides the “bell” of the probability density into two volumes. The greater the proportion of the “bell” volume lies in the destruction area, the higher the probability of destruction, respectively. In Figure 3, almost the entire volume lies in the non-destruction area.

Figure 4 shows schematically the dependence of the probability of non-destruction on the assurance factor.
4. Practical application

According to the requirements of technical regulations, elements of railway rolling stock “...must have static strength assurance <...> within the period of their full examination or service life specified in the design documentation” [12, 13]. This means that, for example, the axle of a rolling stock wheelset must have the necessary static strength assurance factor, providing a given probability of non-destruction under quasistatic loading during the entire service life. Figure 5 shows the result of the finite element calculation of the stress-strain state of the axle of the rolling stock wheelset made according to the procedures [14–16].

The maximum equivalent stresses in the axle, calculated according to [14–16], reach $\sigma_i = 253$ MPa in the section after the wheel seat. The stress variance can be taken as $0.1 \cdot \sigma_i = 25.3$ MPa [15, 16]. Therefore, the probability density of the equivalent stresses distribution can be described by the expression:

$$f(\sigma) = C_i \cdot \exp\left(-0.5 \cdot (\sigma_i - 253)^2 / 25.3^2 \right),$$  \hspace{1cm} (8)

where $C_i$ – normalization factor.

According to [17], the yield strength of OC steel grade is in the range from 300 to 330 MPa, therefore, similarly to expression (8), it is possible to make the distribution density function for the fatigue limit:

$$f(\sigma_f) = C_f \cdot \exp\left(-0.5 \cdot (\sigma_f - 315)^2 / 5^2 \right),$$  \hspace{1cm} (9)

where $C_f$ – normalization factor.
Thus, the static strength assurance factor is equal $n = 315: 253 = 1.245$, what satisfies regulatory requirements [15, 16, 18, 19]. Integration and normalization of expressions (8) and (9) on the interval $[-200; 800]$ MPa with a step of 0.1 MPa give a value for the non-destruction probability $W_n = 0.99185$ with uncertainty bandwidth of not more than 0.00009. Variations of the integration limits and the unit cell size contribute to the calculation error and should be chosen depending on the specified accuracy of the probability determination. At the same time, as the cell size decreases, the width of the “$n$” uncertainty band decreases and, in the limit, tends to zero, without affecting the result.

5. Conclusions

5.1. While at the axle design stage and having been given the selected loading and strength distributions, the designer can expect during the specified service life less than 9 axle destructions (construction of Figure 5) out of 1,000. For a specific full-scale axle of the wheelset, metal properties are determined by standardized methods. So the yield strength variance for a particular axle in (9) will be lower than it is established by [17]. Clarifying operation conditions will reduce the stress variance in (8). Thus taking static strength assurance factor equal $n = 1.245$, for certain axles and regions destruction probability will be substantially lower, then 0.009.

5.2. The absence of facts of this type axle destructions in operation due to insufficient static strength means that the destruction probability is less than $1/N$, where $N$ is the total number of axles of this type in operation.

5.3. The stated method of probabilities calculating cannot be applied to a single axle subject to cyclic loads, since it does not take into account the fatigue strength and the degradation of metal physical and mechanical properties over time, and the definition of the concept of probability in this case cannot be carried out using formulas (3), (4).

5.4. The probability of destruction, taking into account the actual operational loading and the adopted repair structure, is of particular interest, as it directly determines the risks of adverse emergencies and the associated financial losses [20].

6. References

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