Rotation Curves and Constraints on Dark Matter Annihilation

Maneenate Wechakama
Department of Physics, Faculty of Science, Kasetsart University, Bangkok, Thailand
E-mail: mwechakama@gmail.com

Abstract. The rotation curves of gas-rich dwarf and low surface brightness (LSB) spiral galaxies have provided the most important pieces of evidence for the existence of dark matter. However, their shape in the inner regions is one of the outstanding issues in modern cosmology. In order to explain the so-called “cusp-core” problem, we have applied the effect of the gas pressure from dark matter annihilation into electron-positron pairs to the rotation curves of LSB galaxies. The propagation of the electrons and positrons has been determined by the diffusion-loss equation. We have assumed a uniform diffusion coefficient and considered energy loss mechanisms. By fitting rotation curves of LSB galaxies, we are able to find the constraints on dark matter particles.

1. Introduction
One of the first and most important evidence for the existence of dark matter is the measurements of galactic “rotation curves” of spiral galaxies [1]. However, observed rotation curves are found to rise approximately linearly with radius, consistent with a constant density core in the dark matter distribution rather than the steep power law predicted by cosmological N-body simulations. Many strategies have been proposed in order to explain the so-called “cusp-core” problem [2]. Recently, we have shown that dark matter annihilation into electron-positron pairs may affect the observed rotation curve of spiral galaxies by a significant amount [3]. The results indicate that the effect of dark matter pressure on the rotation curves of galaxies is significant enough to be used as a tool to probe the properties of dark matter particles and explore their potential role in the “cusp-core” problem. Therefore, in the present work we will extend the previous study to constrain the properties of dark matter particles.

2. Method
2.1. Rotation curves and dark matter pressure
We focus on dark matter annihilations into electrons and positrons. We adopt a model-independent approach, in which all particles are created with the same initial energy $E_0 \sim m_{dm}c^2$. Electrons and positrons will be relativistic at the moment of their creation. They can efficiently lose their energy through different processes, such as inverse Compton scattering, synchrotron radiation, Coulomb collisions, bremsstrahlung, and ionization. We will often use the Lorentz factor $\gamma$ to express the energy $E = \gamma m_e c^2$, where $m_e$ denotes the rest mass of electron, and $c$ is the speed of light.
The pressure associated to these particles, hereafter referred to as “dark matter pressure”, is given by
\[
P_{\text{dm}}(r) = \frac{m_e c^2}{3} \int_{1}^{\infty} \frac{\mathrm{d}n(r, \gamma)}{\mathrm{d}\gamma} \left( \frac{\gamma^2 - 1}{\gamma} \right) \mathrm{d}\gamma,
\]
where the electron-positron spectrum \( \frac{\mathrm{d}n}{\mathrm{d}\gamma}(r, \gamma) \) is the number density of particles with Lorentz factor \( \gamma \) at a radius \( r \) from the centre of the dark matter halo. The pressure gradient induces an acceleration
\[
a_{\text{dm}}(r) = -\frac{1}{\rho_g(r)} \frac{\mathrm{d}P_{\text{dm}}(r)}{\mathrm{d}r},
\]
where \( \rho_g(r) \) is the gas density at radius \( r \), that opposes the gravitational pull towards the centre, affecting observable quantities such as the circular velocity
\[
v_c(r) = \sqrt{\frac{G M(r)}{r} + \frac{r}{\rho_g(r)} \frac{\mathrm{d}P_{\text{dm}}(r)}{\mathrm{d}r}}.
\]

2.2. Propagation
The propagation of electrons and positrons through the interstellar medium (ISM) is determined by the diffusion-loss equation
\[
\frac{\partial}{\partial t} \frac{\mathrm{d}n}{\mathrm{d}\gamma}(\mathbf{x}, \gamma) = \nabla \left[ K(\mathbf{x}, \gamma) \nabla \frac{\mathrm{d}n}{\mathrm{d}\gamma}(\mathbf{x}, \gamma) \right] + \frac{\partial}{\partial \gamma} \left[ b(\mathbf{x}, \gamma) \frac{\mathrm{d}n}{\mathrm{d}\gamma}(\mathbf{x}, \gamma) \right] + Q(\mathbf{x}, \gamma).
\]
We assume a diffusion coefficient of the form
\[
K(\gamma) = K_0 \gamma^\delta.
\]
The energy loss rate
\[
b(\mathbf{x}, \gamma) \equiv -\frac{\mathrm{d}\gamma}{\mathrm{d}t}(\mathbf{x}, \gamma) = \sum_i b_i(\mathbf{x}, \gamma)
\]
is a sum over the relevant physical processes, i.e. inverse Compton scattering, synchrotron radiation, Coulomb collisions, bremsstrahlung, and ionization.

The source term \( Q(\mathbf{x}, \gamma) \) represents the instantaneous electron-positron injection rate. Since the electrons and positrons in our model originate from the annihilation of dark matter particles and we consider a perfectly spherically-symmetric halo, the production rate is dictated by the dark matter number density and the annihilation rate into electron-positron pairs,
\[
Q(r, \gamma) = n_{\text{dm}}(r) \; n_{\text{dm}*}(r) \; \langle \sigma v \rangle_{e^\pm} \frac{\mathrm{d}N_{e^\pm}}{\mathrm{d}\gamma},
\]
where \( n_{\text{dm}} \) and \( n_{\text{dm}*} \) denote the number densities of dark matter particles and anti-particles, respectively, \( \langle \sigma v \rangle_{e^\pm} \) is the thermal average of the annihilation cross-section times the dark matter relative velocity, and \( \frac{\mathrm{d}N_{e^\pm}}{\mathrm{d}\gamma} \) is the injection spectrum of electrons and positrons in the final state. Assuming all electrons and positrons are injected with the same energy \( \gamma_0 \sim m_{\text{dm}}/m_e \),
\[
Q(r, \gamma) = Q_0(r) \; \delta(\gamma - \gamma_0)
\]
where
\[ Q_0(r) = 2 \left[ \frac{\rho_{dm}(r)}{m_{dm}} \right]^2 \langle \sigma v \rangle_{e^\pm} \]  
(9)
is the local production rate per unit volume per unit time and \( \delta(\gamma - \gamma_0) \) denotes a Dirac delta function.

For the dark matter density \( \rho_{dm}(r) \), we consider a perfectly spherically-symmetric halo described by a density profile of the form
\[ \rho_{dm}(r) = \frac{\rho_s}{\left( \frac{r}{r_s} \right)^\alpha \left( 1 + \frac{r}{r_s} \right)^{3-\alpha}}, \]  
(10)
where \( r_s \) and \( \rho_s \) denote a characteristic density and radius of the halo, respectively, and the \( \alpha \) is the inner logarithmic slope of the density profile [4].

Given enough time, the electron-positron population will approach a steady-state distribution and the electron-positron spectrum is given by
\[ \frac{d^2n}{d\gamma d^3x} = \frac{1}{b(x, \gamma)} \int_0^\infty d\gamma_s \int_0^\infty d^3x_s \exp \left( -\frac{|x - x_s|^2}{2\Delta \lambda^2} \right) \frac{(2\pi \Delta \lambda^2)^{3/2}}{Q(x, \gamma_s)} \]  
(11)
where the quantity
\[ \Delta \lambda^2 = \lambda^2(\gamma) - \lambda^2(\gamma_s) \]  
(12)
is related to the characteristic diffusion length of the electrons and positrons, and \( \gamma_s \) denotes their initial energy. The variable \( \lambda \) is defined as
\[ \lambda^2(\gamma) = \int_\gamma^\infty \frac{2K(\gamma)}{b(\gamma)} d\gamma. \]  
(13)

Considering the dark matter halo as a spherically-symmetric source, the spatial integral can be reduced to one dimension, and the electron-positron spectrum is finally given by the expression
\[ \frac{d^2n}{d\gamma d^3x}(r, \gamma) = \frac{1}{b(\gamma)} \exp \left( -\frac{r^2}{2\Delta \lambda^2} \right) \times \left\{ \int_\gamma^\infty d\gamma_s \int_0^\infty dr_s \exp \left( -\frac{r_s^2}{2\Delta \lambda^2} \right) \right\} \]  
(14)

2.3. Constraints on the dark matter mass
In order to constrain the mass of dark matter particles, we compare our predicted rotation curves with dark matter pressure (3) with observations of dwarf and low surface brightness galaxies compiled by [5]. We consider their constant mass-to-light ratio and maximum disk models, adopting the corresponding best-fitting values of \( V_{200} \) and \( c_{200} \) in [5].

For the preliminary results we have defined our canonical model with \( \langle \sigma v \rangle_{e^\pm} = 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \), and a dark matter density profile with \( \alpha = 1 \) [4]. We assume that the ISM is mainly composed of neutral hydrogen atoms (\( X_{\text{ion}} = 0 \)) with number density \( \rho_s/m_p \sim 1 \text{ cm}^{-3} \) [6], and it is permeated by a tangled magnetic field whose intensity is \( B \sim 6 \mu \text{G} \) throughout the galaxies [7]. We consider the injection energy (i.e. the mass of the dark matter particle) as a free parameter and investigate values of the initial Lorentz factor \( \gamma_0 \) between \( 2 \times 10^3 \) and \( 2 \times 10^7 \), corresponding to injection energies \( E_0 = \gamma_0 m_e c^2 \) from 1 GeV to 10 TeV. Our constraints are derived by the best fit of least chi-square of each galaxies.
3. Results and Discussion

By fitting the 26 rotation curves of low surface brightness (LSB) galaxies, we are able to find the constraints on dark matter mass. We have derived limits on the mass of dark matter particles by applying the effect of the pressure from dark matter annihilation into electron-positron pairs to the rotation curves of LSB galaxies. We have found that the limits of the mass of dark matter particles are between 1 GeV to 30 GeV.

To improve to previous results, realistic models are needed. We aim to extend the previous study by using realistic models of the dark matter halo, the structure of the interstellar medium, magnetic field, and the propagation of relativistic particles to investigate the properties of dark matter and its distribution in the Milky Way and gas-rich dwarf and low surface brightness (LSB) galaxies. This research may bring the better understanding of dark matter in the Universe.

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References

[1] Rubin V C, Thonnard N, and Ford Jr. W K 1978 ApJL. 225 107
[2] de Blok W J G 2010 Advances in Astronomy 2010
[3] Wechakama M, and Ascasibar Y 2011 MNRAS 413 1991
[4] Navarro J F, Frenk C S, and White S D M 1996 ApJ 462 563
[5] de Blok W J G, and Bosma A 2002 A&A 385 816
[6] Ferrière K M 2001, Rev. Mod. Phys. 73 1031
[7] Beck R 2001, Space Science Reviews 99 243