Studying the dynamics of contact interactions during machining based on a system of nonlinear piecewise linear differential equations

D V Vasilkov¹, A S Alexandrov¹, V V Golikova¹, and T B Kochina²

¹ Baltic State Technical University VOENMEH, 1, 1st Krasnoarmejskaja street, St. Petersburg, 190005, Russia
² Nizhnevartovsk State University, 56, Lenina street, Nizhnevartovsk, Khanty-Mansiiskii avtonomous okrug - Iugra, 630073, Russia

E-mail: vasilkovdv@mail.ru

Abstract. When manufacturing parts for thermal control systems for operation in especially difficult conditions of the North and in spacecraft, titanium and complex alloyed alloys are used. They are subject to increased quality requirements. They are achieved through simulation modeling. The solution is based on a dynamic model in which the contact interaction between the tool flank and the workpiece is differentiated. The process of contact interaction is considered as two-phase in the form of a sequence of states of adhesion and sliding. In the space of state variables, a system of nonlinear differential equations of piecewise linear type is constructed based on a set of rheological models. Based on this model, simulation modeling was carried out in the form of a computational experiment. A periodic solution is obtained which is formed as a result of switching the sliding and adhesion phases. In the graphs presented in the paper, the transition from the adhesion phase to the sliding phase is accompanied by a surge in displacement. In this case, in the phase portrait, the phase trajectory reaches the limit cycle with a limited amplitude. At the moment of transition from the adhesion phase to the sliding phase, a characteristic deviation of the phase trajectory from the limit cycle with an increased amplitude is observed, followed by a return to the limit cycle. With a wide variation in cutting speed, the conditions for technical stability are determined. During the simulation, a dynamic manifestation of the formation of chip elements was found, which did not have a serious impact on the nature of dynamic processes. The results obtained make it possible to study the dynamics of contact interactions during cutting in the framework of nonlinear dynamic models. This makes it possible to assess the level of vibration amplitudes and determine the regulated parameters of processing accuracy and surface roughness of manufactured products.

1. Introduction

Devices and systems operating in the harsh conditions of the north, as well as in spacecraft serving the northern territories, operate using thermal control systems that ensure their normal operation [1]. In the manufacture of the drive part of these systems, the big problem is thin-walled parts such as bodies of revolution from complex alloyed and titanium alloys, such as shells, bushings, flanges, housings, etc. Some thin-walled elements have a thickness of 0.3 ... 0.9 mm [2].

In the manufacture and operational loads, the accuracy of the size, shape and relative position of
the surfaces of these parts must be maintained in accordance with the requirements of the drawing and design documentation. Prediction of output characteristics during manufacturing is carried out by simulation based on nonlinear dynamic systems. The study of the dynamics of contact interactions during cutting has become widespread in domestic and foreign developments [3-11]. An analysis of the processes occurring during the contact interaction of the cutting tool and the workpiece made it possible to present this process as two phases: seizure and slide [12-14].

The molecular-mechanical processes that occur during the interaction of the cutting tool and the workpiece in the space of state variables can be described on the basis of a system of differential equations

\[
\dot{u} = Du + S(u),
\]

where \( u \) is the vector of state variables; \( D \) is transformation matrix with constant elements; \( S(u) \) is a vector function with piecewise linear components.

Model solutions as applied to the system of differential equations (1) are considered in detail in [12, 14, 15]. Its feature is that the vector function \( S(u) \) is a piecewise linear function.

2. Materials and methods

Complex alloyed, heat-resistant and aluminum alloys are prone to the formation of adhesive bonds in a wide range of cutting speeds. In [5], it was confirmed that in the zone of contact interaction between the cutting tool and the workpiece, sequential formation (seizure) and destruction (sliding) of adhesive bonds continuously occur. Each of these states is described by a simple dynamic model. The transitions between the slip and seizure phases are determined by the kinematic and force characteristics embedded in the model and are monitored by the switching function \( Sg1 \), which can be written in the form of the relation [12]

\[
Sg1 = \begin{cases} 
1, & \text{seizure phase} \\
0, & \text{sliding phase} 
\end{cases}
\]

The considered dynamic system (1) in the space of state variables is nonlinear piecewise linear system. Its solution is represented by expressions described in detail in [12-14]. Based on this dynamic system, it is possible to study the frequency composition and level of the amplitudes of oscillations with a wide variation of the model parameters.

3. Results

To assess the nature of the movements, we formulate the conditions of a computational experiment.

Workpiece parameters: diameter \( d = 90 \) mm, offset is 135 mm, material is XH65BMTJu (CrNi65BCuTeAl). Tool parameters: trailing angle \( \alpha = 8^\circ \); rake angle \( \gamma = -8^\circ \); the main entering angle \( \varphi = 70^\circ \); auxiliary entering angle \( \varphi_1 = 20^\circ \); material of the cutting part of the tool: at \( V = 30-70 \) m/min GC4235 “Sandvik” (Sweden); at \( V = 80-200 \) m/min M101S “Tungaloy” (Japan); section of the holder is 40x20; holder offset is 95 mm. Cutting modes: feed \( S = 0.19 \) mm/rev, cutting depth \( t = 1.0 \) mm; cutting speed \( V = 30 \ldots 220 \) m/min. Processing without cooling.

In all cases considered, the movement in the model begins with the seizure phase. This is evidenced by the graph of the switching function (time diagram 1 in Fig. 1a). The relative displacement along the normal to the forming point of the tool in the y direction (curve 2 in Fig. 1a) was obtained as a result of calculation according to model (1).

The periodic solution, which is formed as a result of switching the sliding and seizure phases, is clearly visible on it. In the graphs, the transition from the seizure phase to the sliding phase is accompanied by a surge in the displacement y (curve 2 in Fig. 1a). In this case, the phase trajectory exits the phase trajectory to the limit cycle with an amplitude of 2–5 \( \mu m \) (curve 3 in Fig. 1b). At the moment of transition from the seizure phase to the slip phase, a characteristic deviation of the phase trajectory from the limit cycle with an amplitude of 3-12 \( \mu m \) (curve 4 in Fig. 1b) is observed, followed by return to the limit cycle. This characteristic manifestation of a dynamic system is observed in the
range of cutting speeds \( V = 30 \ldots 50 \) m/min (pos. 1-3, Table 1) and in the range of cutting speeds \( V = 70 \ldots 200 \) m/min (pos. 5-12, Table 1). In the indicated modes, the duration of the seizure phase was on average \( 10^{-4} \) s, and the sliding phases were an order of magnitude longer.

At a cutting speed of \( V = 60 \) m/min, there is a sharp increase in the level of vibration with a transition to the second limit cycle with an amplitude of 30 \( \mu \)m (pos. 5, Table 1). This mode is unacceptable in all cases and excluded from the list for possible implementation.

A comparison of the quantitative results of the calculation from the standpoint of technical stability is presented in Table 1. The range of acceptable modes is defined for tools made of hard alloys and mineral ceramic. Moreover, cermets are most effective at a temperature in the cutting zone, when there is a drop in the strength and yield strengths for a given processed material. This is evidenced by a decrease in the level of vibration amplitudes of less than 5 \( \mu \)m.

In the system of differential equations (1), the main mechanism of the adhesive-deformation interaction in the contact connection between the tool and the workpiece in the cutting zone, formed by a sequence of slip and seizure phases, is laid down. In this case, the phase switching function is formed synergistically by the system itself in accordance with expression (2).

**Figure 1.** The calculated vibration displacements normal to the shaping point of the tool at \( V = 40 \) m/min: a - graph of displacements in the \( y \) direction, \( \mu \)m; time diagram of the function of switching dynamic modes; b - phase portrait of a dynamic system.
In addition to this self-adjusting mechanism of a dynamic system, the mechanism for the formation of chip elements integrated in expressions (1) can be considered and dynamically modeled. In accordance with the accepted initial assumption, this mechanism is insignificant in terms of impact in comparison with the mechanism of adhesive-deformation interaction. It was assumed that the complication of the model by taking into account this mechanism will lead to unreasonable complication of the calculation results.

Omitting the mathematical calculations, we present the results of modeling a dynamic piecewise linear system, in which only switching, which reflects the formation of chip elements, is taken into account. Figure 2 presents the simulation result, taking into account only the formation of chip elements at \( V = 200 \text{ m/min} \). The graph is built at high magnification. In this case, the switching function displays the formation of chip elements. This result has an important theoretical meaning, since it allows identifying the frequency of formation of chip elements in the model solution and determine the conditions of elemental and adiabatic chip formation.

### Table 1. Calculated data on the stability of a dynamic system

| Pos. No. | Cutting speed [m/min] | Technical stability | max amplitude of vibrations [\( \mu \text{m} \)] | Tool material (+/-) | Hard alloy | Mineral ceramics |
|----------|-----------------------|---------------------|-----------------------------------------------|---------------------|-----------|-----------------|
| 1        | 30                    | SLC, \( a = 3 \mu \text{m} \) | 12 \( \mu \text{m} \) | +                   | SLC       | Mineral ceramics |
| 2        | 40                    | SLC, \( a = 4 \mu \text{m} \) | 9 \( \mu \text{m} \)  | +                   | SLC       | Mineral ceramics |
| 3        | 50                    | SLC, \( a = 5 \mu \text{m} \) | 10 \( \mu \text{m} \) | +                   | SLC       | Mineral ceramics |
| 4        | 60                    | NSLC, \( a = 30 \mu \text{m} \) | 30 \( \mu \text{m} \) | +                   | SLC       | Mineral ceramics |
| 5        | 70                    | SLC, \( a = 6 \mu \text{m} \) | 12 \( \mu \text{m} \) | +                   | SLC       | Mineral ceramics |
| 6        | 80                    | SLC, \( a = 4 \mu \text{m} \) | 9 \( \mu \text{m} \) | -                   | SLC       | Mineral ceramics |
| 7        | 120                   | SLC, \( a = 3.5 \mu \text{m} \) | 8 \( \mu \text{m} \) | -                   | SLC       | Mineral ceramics |
| 8        | 140                   | SLC, \( a = 3 \mu \text{m} \) | 8 \( \mu \text{m} \) | +                   | SLC       | Mineral ceramics |
| 9        | 160                   | SLC, \( a = 3 \mu \text{m} \) | 7 \( \mu \text{m} \) | -                   | SLC       | Mineral ceramics |
| 10       | 180                   | SLC, \( a = 2 \mu \text{m} \) | 3 \( \mu \text{m} \) | +                   | SLC       | Mineral ceramics |
| 11       | 190                   | SLC, \( a = 2 \mu \text{m} \) | 3 \( \mu \text{m} \) | -                   | SLC       | Mineral ceramics |
| 12       | 200                   | SLC, \( a = 2 \mu \text{m} \) | 3 \( \mu \text{m} \) | +                   | SLC       | Mineral ceramics |

SLC – stable limit cycle; NSLC – non-stable limit cycle; \( a \) – limit cycle amplitude.

**Figure 2.** Calculated vibration displacements normal to the forming point of the instrument at high resolution \( (V = 200 \text{ m/min}) \): 1 - graph of displacements in the \( y \) direction, \( \mu \text{m} \); 2 is a time diagram of the dynamic mode switching function.
5

The graph clearly shows small deviations (Fig. 2) associated with the formation of chip elements. They indicate the formation of chip elements, but do not have a serious impact on the nature of dynamic processes. This is also confirmed by the fact that the frequency of formation of chip elements is 11.4 kHz.

4. Conclusion
The results obtained enable the study of the dynamics of contact interactions during machining in the framework of nonlinear dynamic models. This allows assessing the level of oscillation amplitudes and determine the regulated parameters of processing accuracy and surface roughness of manufactured products.

Acknowledgments
The work was carried out at BSTU «VOENMEH» as part of the project «The creation of a high-tech import-substituting production of the elements of executive device systems with high life for transport and aerospace equipment, ensuring the development and use of the World Ocean, Arctic and Antarctica» with financial support from the Ministry of Science and Higher Education of the Russian Federation (agreement No. 075-11-2019-077 as of 13.12.2019) in accordance with the government decree of the Russian Federation No. 218 as of 09.04.2010.

References
[1] Bobkov A V and Tsvetkov E O 2011 Features of the balance of power losses in electric pump units of spacecraft thermal control systems Bull. of the Samara Scientific Center of the Russian Academy of Sciences 13(1(2)) 290-292
[2] Vasilkov D V, Kuznetsova Z A and Nikitin A V 2020 Technological quality support when turning thin-walled parts made of titanium alloys of electric pump units of the spacecraft temperature control system Metalworking 1(115) 3-9
[3] Kudinov V A 1967 Dynamics of machines (Moscow: Mechanical Engineering) 359 p.
[4] Elyasberg M E 1993 Self-oscillations of metal-cutting machines: Theory and practice (St. Petersburg: OKBS) 180 p.
[5] Vasilkov D V, Weitz B L and Shevchenko V S 1997 Dynamics of the technological system of mechanical processing (Saint-Petersburg: Publishing Tool) 230 p.
[6] Maksarov V 2015 Improving the accuracy of manufacturing of hydraulic power cylinders using vibration-proof cutting tool Agronomy Res. 13(3) 671-679
[7] Olt J, Liivapuu O, Maksarov V, Liyvapuu A and Tärgla T 2016 Mathematical modelling of cutting process system Springer Proc. in Mathematics and Statistics 178 173-186
[8] Maksarov V V and Efimov A E 2018 Simulation modeling of dynamic characteristics of machining in NI LabView software environment to improve processing technique of a rod component IOP Conf. Ser.: Earth Env. Sci. 194(2) 022021
[9] Olt J, Liyvapuu A, Mabisoo M and Maksarov V 2016 Dynamic simulation of chip formation in the process of cutting Int. J. of Materials and Product Technology 53(1) 1-14
[10] Maksarov V V, Khalimonenko A D and Matrenichev K G 2017 Stability analysis of multipoint tool equipped with metal cutting ceramics IOP Conf. Ser.: Earth Environ. Sci. 7(8) 082030
[11] Olt J and Maksarov V V 2015 Cutting process simulation on the basis of rheological properties of metals Annals of DAAAM and Proc. of the Int. DAAAM Symp., pp 229-237
[12] Vasilkov D V and Kochina T B 2013 Modelling of contact interactions taking into account the rheology in the cutting zone at high-speed processing of products from heat resisting alloys Metalworking 4(76) 2-10
[13] Vasilkov D V and Kochina T B Dynamic features of formation of conditions of contact interaction in a cutting zone on the basis of piecewise and linear approximation Metalworking 5-6(77-78) 2-3
[14] Nikitin A V, Tarikov I Ya and Vasilkov D V 2019 Determination of technological residual
stresses in the surface layer of parts with thin-walled elements during turning \textit{J. of Advanced Res. in Dynamical and Control Systems} \textbf{11(08)} 2926-2932

[15] Maksarov V, Efimov A and Golikov T 2020 Treatment of Titanium Alloys Based on Preliminary Plastic Impact \textit{Key Engineering Materials Submitted} \textbf{836} 63-70