Weak measurement amplification in optomechanics via a squeezed coherent state pointer

Gang Li¹, Li-Bo Chen², Xiu-Min Lin³ and He-Shan Song¹

¹School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian 116024, People’s Republic of China
²School of Science, Qingdao Technological University, Qingdao 266033, People’s Republic of China
³Fujian Provincial Key Laboratory of Quantum Manipulation and New Energy Materials, College of Physics and Energy, Fujian Normal University, Fuzhou 350007, People’s Republic of China

E-mail: ligang0311@sina.cn and hssong@dlut.edu.cn

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Abstract
We present a scheme for achieving amplification of the displacement of a mirror in an optomechanical cavity using single-photon postselection where the mirror is initially prepared in a squeezed coherent state. The amplification depends on the enhanced fluctuations of the squeezed coherent state and is caused by the noncommutativity of quantum mechanics relying on the squeezed coherent state, which cannot be explained by the standard weak measurement theory [1, 25].

Keywords: amplification, squeezed coherent state, optomechanics

1. Introduction

Weak measurement theory, first introduced by Aharonov, Albert and Vaidman [1], describes a measurement situation where the measured system is weakly coupled to the measuring device. Such weak measurement has been used to solve basic problems in quantum mechanics [2, 3] and explain certain paradoxes [4].

Weak measurement has been realized [5] and can be applied to amplifying tiny physical effects, such as the spin Hall effect [6] and ultrasensitive beam direction [7], and measuring physical quantities, such as the direct measurement of wave functions [8]. More experimental protocols have been proposed [9–17]. Recently, a weak measurement protocol combined with cavity optomechanics [18, 19] was provided in [20, 21] and further applications of weak measurement are reviewed in [22, 23].

In most discussions about weak measurement, the initial state of the pointer considered is a Gaussian state (classical), such as in [1, 6, 7, 20, 21], but here we are interested in whether a squeezed coherent state (quantum) pointer can further improve the amplification effect of weak measurement compared to a Gaussian pointer. In this paper we use the same optomechanical model as in [20, 24], but with the mirror initially prepared in a squeezed coherent state, and find that the amplification of the displacement of the mirror depends on the enhanced fluctuations of the squeezed coherent state, which are larger than those with a Gaussian pointer [20, 21]. This result is counter-intuitive since the enhanced fluctuations in a quadrature of motion are considered to be of no use. To the best of our knowledge, this is the first positive example for the use of the enhanced fluctuations of the squeezed coherent state. The amplification effect is caused by the noncommutativity of quantum mechanics relying on the squeezed coherent state, which cannot be explained by the standard weak measurement theory [1, 25] (see appendix).

The structure of our paper is as follows. In section 2, we state the main result of this work, including the amplification effect with squeezed vacuum state and squeezed coherent state pointers, and in section 3, we present the conclusions of this work.

2. Amplification in optomechanics

2.1. The optomechanical model

In figure 1 the optomechanical cavity A is embedded in one arm of the March–Zehnder interferometer and a stationary
2. Amplification about position variable $\hat{q}$ with a squeezed vacuum state pointer

Suppose that one photon is input into the interferometer, the state of the photon after the first beam splitter becomes

$$|\psi_i\rangle = \frac{1}{\sqrt{2}} (|1\rangle_\lambda |0\rangle_m + |0\rangle_\lambda |1\rangle_m).$$

and after interacting weakly with the mirror prepared at the squeezing vacuum state $S(\epsilon)|0\rangle$, the state of the total system is

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} (|1\rangle_\lambda |0\rangle_m |\psi(\xi, \eta, 0)\rangle_m + |0\rangle_\lambda |1\rangle_m \times S(\eta) |0\rangle_m).$$

After the second beam splitter and detecting at the dark port, that is, postselecting for an single-photon state

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|1\rangle_\lambda |0\rangle_m - |0\rangle_\lambda |1\rangle_m),$$

which satisfies this case of the orthogonal postselection, i.e., $\langle\psi|\psi\rangle = 0$. Then the final state of the mirror becomes

$$|\Psi_m(t)\rangle = \frac{1}{2} (|\psi(\xi, \eta, 0)\rangle_m - S(\eta) |0\rangle_m).$$

The average displacement of pointer variable $\hat{q}$ of the mirror is

$$\langle \hat{q} \rangle = \frac{\text{Tr}[|\Psi_m(t)\rangle \langle \Psi_m(t) | \hat{q} \rangle]}{\text{Tr}[|\Psi_m(t)\rangle \langle \Psi_m(t) |]} = \text{Tr} \left( S(\eta) |0\rangle_m \langle 0 | m \right) \times S(\eta) |\hat{q}\rangle.$$ (7)

Here we use the position operator $\hat{q} = (c + c^\dagger)\sigma$. Substituting (6) into (7), as a result, we have

$$\langle \hat{q}(t) \rangle = \sigma \left[ \xi(t) + \xi^*(t) - e^{i\theta} \left( e^{\phi(t)} v(t) \mu^*(t) + e^{-i\phi(t)} \mu(t) v^*(t) \right) \right] / \left( 2 - e^{i\phi(t)} e^{-i\phi(t)} \right).$$

(8)

where $\mu(t) = \cosh r - e^{i(\theta - 2\omega_m t)} \sinh r$ and $v(t) = \xi(t) \cosh r + e^{i(\theta - 2\omega_m t)} \sinh r$.

The average displacement $\langle \hat{q}(t) / \sigma \rangle$ of the mirror is shown in figure 2 as a function of $\omega_m t$ with $k = 0.005$, $r = 2$ and $\theta = \pi$. We can see that there are two amplification zones. One is that there exist amplification effects around time $\omega_m t = (2n + 1)\pi$ ($n = 0, 1, 2, \cdots$). The other amplifications occur around the vibration periods of the mirror $\omega_m t = 2n\pi$.
(n = 1, 2, …). In particular, the maximal positive and negative amplifications around time $\omega_m t = 2n\pi$ (n = 1, 2, …) are more prominent and they can also reach the maximal value [28] \( \langle q(t) \rangle = \pm \varepsilon^2 \sigma \), i.e., the level of the squeezing vacuum-state fluctuation \( \pm \sqrt{\frac{e^{2r} \sin^2 \frac{\theta}{2} + e^{-2r} \cos^2 \frac{\theta}{2}}{2} \sigma} \) when \( r = 2 \) and \( \theta = \pi \) (\( \sigma \) is the level of the vacuum-state fluctuation). Note that the maximal displacement of the mirror caused by one photon in cavity A (see figure 1) is 4\( \delta \), therefore the amplification factor is \( Q = \pm \varepsilon^2 / 4k \) which is \( \approx 369.5 \) when \( k = 0.005 \). Compared with [20], not only are the maximal amplifications around time \( \omega_m t = 2n\pi \) (n = 1, 2, …) generated, but the maximal amplification can also be much larger since the squeezing vacuum state of the mirror is prepared.

In order to understand the amplification effects appearing around time \( T = 2n\pi \) (n = 1, 2, …), we can perform a small quantity expansion about time \( T \) up to the second order when \( \theta = \pi \). Suppose that \( |\omega_m t - T| \ll 1 \), \( k \ll 1 \) and \( k^2 T \ll 1 \), then

\[
|\Psi(\omega_m t - T)\rangle \approx \frac{1}{2} \left[ i k^2 T S(\eta) \right] |0\rangle_m + i k (\omega_m t - T) \times e^i S(\eta) |1\rangle_m \]  

(9)

where \( \eta = r e^{i(\pi - 2(\omega_m t - T))} \). It should be noted that we use the approximation \( e^{i\phi(t)} \approx 1 + i k^2 T \). It can be easily seen that the squeezed vacuum state \( S(\eta)|0\rangle \) of (9) is generated due to the Kerr phase. Substituting (9) into (7), the average value of the displacement operator \( \hat{q} \) is given by

\[
\langle q(t) \rangle = \frac{1}{2} \left( k^2 T^2 + k^2 (\omega_m t - T) \right) e^{i T} \]

(10)

which obtains its maximal value \( \varepsilon \sigma \) when \( k^2 T = k (\omega_m t - T) e^i \), and obtains its minimal value \( -\varepsilon \sigma \) when \( k^2 T = -k (\omega_m t - T) e^i \). Therefore, the mirror state achieving the positive amplification is \( S(\eta)|1\rangle \), \( \frac{1}{\sqrt{2}} (|0\rangle_m + |1\rangle_m) \) and the state achieving the negative amplification is \( S(\eta)|\rangle \), \( \frac{1}{\sqrt{2}} (|0\rangle_m - |1\rangle_m) \). It is obvious that the key to understanding the amplification is the superposition of the squeezing vacuum state and the squeezing one-phonon state of the mirror, which is due to the Kerr phase. It is well-known that the squeezed state has many applications in the diverse fields of quantum physics since it has fewer fluctuations in one quadrature component at the expense of increased fluctuations in another quadrature component, which is a key characteristic of the squeezed state on the application side. In contrast, the application of its quadrature component with increased fluctuations is seldom mentioned. Here we show that the amplification achieved here is associated with the quadrature component with increased fluctuations in the squeezed state, i.e., the level of the squeezing vacuum-state fluctuation \( \pm \varepsilon \sigma \). We can see that the amplification will become larger if the level of the increased fluctuations of the squeezed coherent state in another quadrature direction become larger and it becomes more easily measurable. It will be shown that the maximal amplification due to the Kerr phase obtained here is larger than the level of vacuum-state fluctuation \( \sigma \) which cannot be achieved if the mirror is initially prepared in the ground state [20] or the coherent state [21].

Also, in this case of orthogonal postselection, the amplifications around time \( \omega_m t = 2n\pi \) (n = 1, 2, …) are not explained by the weak measurement theory [1, 25] (see appendix) which is applicable to the case that is a near-orthogonal postselection.

2.3. Amplification about position variable \( \hat{q} \) with a squeezed coherent state pointer

Suppose that the mirror is initially prepared at the squeezed coherent state \( S(\xi)|\alpha\rangle \) with \( \alpha = |\alpha| e^{i\beta} \), where \( |\alpha| \) and \( \beta \) are real numbers called the amplitude and phase of the state, respectively. If steps corresponding to those of (3)–(5) are carefully carried out in this case, then the final state of the mirror becomes

\[
|\Psi(\omega_m t)\rangle = \frac{1}{2} \left[ \left( \psi(\xi, \eta, \phi) \right) \right] |0\rangle_m - S(\eta)|0\rangle_m \]  

(11)

For the sake of making the analysis simple, we can displace the state of (11) to the origin point in phase space, defining

\[
|\xi(\omega_m t)\rangle = D\left( e^{-i\omega_m t} \alpha \cosh \tau - \alpha^* \sinh \tau \right) |\Psi(\omega_m t)\rangle \]

(12)

where the phase \( e^{i\xi} \) with \( \tau(t) = -i \left[ \alpha^* \omega_m t - \omega \alpha \right] \) is a relative phase between the states \( D(\xi(t)) S(\eta)|0\rangle_m \) and \( S(\eta)|0\rangle_m \) and it is obtained when we use the property of the displacement operators \( D(\alpha)D(\beta) = \exp[i(\alpha^* \beta - \alpha \beta^*)] \) \( D(\beta)D(\alpha) \), i.e., the noncommutativity of quantum mechanics relying on the squeezed coherent state, which is similar to the derivation of the relative phase caused by the coherent state in [21].
Figure 3. The average displacement $\langle q(t)\rangle/\sigma$ as a function of $\omega_m t$ with $k=0.005$, $r = 2$ and $\theta = \pi$ for different amplitudes and the phases, $|\alpha| = 1/2$, $\beta = 2\pi$ (solid line) and $|\alpha| = 400$, $\beta = \pi/2$ (dashed line).

Substituting (12) into (7), as a result, we have

$$\langle q(t)\rangle = \sigma \left[ \xi(t) + \xi^*(t) + e^{-i\theta} \left( e^{i\phi(t)+i\tau(t)} b(t) \mu^*(t) \right. \right. $$

$$\left. \left. + e^{-i\phi(t)+i\tau(t)} b^*(t) \mu(t) \right) \right] \left( 2 - e^{-2i\phi(t)} \right) \left( e^{i\phi(t)} \right. $$

$$\times \left. e^{i\tau(t)} + e^{-i\phi(t)+i\tau(t)} \right].$$

(13)

Figure 3 shows the average displacement $\langle q(t)\rangle/\sigma$ of the mirror versus time $\omega_m t$ with $k = 0.005$, $r = 2$ and $\theta = \pi$ for different amplitudes and the phases, $|\alpha| = 1/2$, $\beta = 0$ (solid line) and $|\alpha| = 400$, $\beta = \pi/2$ (dashed line). It can be seen clearly that the maximal amplification cannot only occur around $\omega_m t = (2n+1)\pi$ $(n = 0, 1, \cdots)$ and the vibration periods of the mirror $\omega_m t = 2n\pi$ $(n = 1, 2, \cdots)$ but also occur near the initial time where their maximal values are $e^{i\phi} \sigma$, i.e., the level of the increased fluctuations of the squeezed coherent state.

Similar to (9), we can also perform a small quantity expansion about time $T = 0$ up to the second order when $\theta = \pi$. Suppose that $|\omega_m t - T| \ll 1$, i.e., $\omega_m t \ll 1$, and $k \ll 1$, then

$$\left| \chi_{\omega_m} (t) \right| \approx \frac{1}{2} \left[ \left| 2k \right| \left| \zeta \right| \left| S(\eta_2) \right| 0 \right|_m + \left| i k \omega_m t e^\zeta S(\eta_2) \right| 1 \right|_m \right].$$

(14)

where

$$\eta_2 = r e^{i(\pi - 2\omega_m t)}$$

and

$$\zeta = e^{i\omega_m t} \cos \beta + e^{-i\omega_m t} \frac{\sin \beta}{2}.$$ Note that we use the approximation $e^{it} \approx 1 + 2ik \left| \alpha \right| \zeta$. It can be easily seen that the squeezed vacuum state $S(\eta_2)|0\rangle$ of (14) is generated due to the relative phase $e^{i\phi}$. Substituting (14) into (7), the average value of displacement operator $\hat{q}$ is given by

$$\langle q(t)\rangle = \sigma 4 k^2 \left| \alpha \right| \zeta e^{i\omega_m t} \left( \zeta e^{2i\omega_m t} + k^2 (\omega_m t)^2 e^{2\eta_2} \right).$$

(15)

which obtains its maximal value $e^{i\phi} \sigma$ or minimal value $-e^{i\phi} \sigma$ when $2k \left| \alpha \right| \zeta = \pm k e^{i\omega_m t}$, respectively. The results clearly indicate that the maximal amplifications appearing near the initial time arise from the equal superposition of the squeezing vacuum state and the squeezing one-phonon state of the mirror, i.e., $S(\eta_2) \frac{1}{\sqrt{2}} (|0\rangle_m \pm |1\rangle_m)$, which is due to the relative phase $e^{i\phi}$ caused by the noncommutativity of quantum mechanics.

As a result, the amplifications due to the phase $e^{i\phi}$ occurring near the initial time are different from the amplifications due to the Kerr phase since the reasons for the two amplifications are essentially different. The relative phase $e^{i\phi}$ after an orthogonal postselection is caused by the noncommutativity of quantum mechanics. In the standard weak measurement [1, 25] the relative phase results from near-orthogonal postselection (see appendix). Therefore, the amplification using the squeezed coherent state cannot be explained by the weak measurement theory [1, 25]. Remarkably, the amplifications using the squeezed coherent state not only depend on the quadrature component with increased fluctuations but can also appear near the initial time, i.e., one photon is successfully detected at the dark port within a very short time. The maximal amplification using the squeezed coherent state can reach the level of the increased fluctuations $e^{i\phi} \sigma$ and it can be detected more easily than the maximal amplification value (the level of the vacuum-state fluctuation $\sigma$) with the ground state pointer [20] or the coherent state pointer [21] since $e^{i\phi} \sigma$ exceed the strong coupling limiting $\sigma$ [28]. Therefore, the amplification result provided here is a surprising one in the weak coupling regime.

Taking into account dissipation, the master equation of the mechanical system [26] is given by

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar} [H, \rho(t)] + \gamma_m \frac{2c^\dagger c \rho(t) - \rho(t) c^\dagger c]}{2},$$

(16)

where $\gamma_m$ is the damping constant.

Similar to the result of the dissipation in [20], because the actual $\gamma \approx \gamma_m \rho_{\omega_m}$ can be very small ($\gamma = 5 \times 10^{-7}$ in proposed device no. 2 [24]) and is almost close to $\gamma = 0$, all the amplification values in the presence of the damping are almost unscathed.

2.4. Discussion

Considering the feasibility of the proposed scheme, we discuss the experimental requirements. First, the mechanical oscillator (mirror) of our device is initially prepared in a squeezed coherent state. The coherent state of the mechanical oscillator is prepared using iteranent microwave fields [29]. Achieving squeezed states in mechanical oscillators has not been realized experimentally but several schemes have been proposed, such as resolved sideband cooling using squeezed [30] or modulated input light [31]. It is difficult to observe the amplification effect using the squeezed vacuum state pointer because of the very small
postselected probability of success and the very limited time zones for the appearance of the maximal displaced state. Nevertheless, the amplification using the squeezed coherent state pointer is not the case. The probability density of a photon released from an optomechanical cavity after time $\omega_b t$ is $\kappa \exp(-\kappa t)$, where $\kappa$ is the decay rate of the cavity. The probability of a successful postselection released after $\omega_b t$ is

$$
\frac{1}{4} \left[ 2 - e^{-\frac{1}{r^2}} (e^{\phi(t)} + e^{-i\phi(t)}) \right].
$$

For $k \ll 1$, this is approximately $\frac{1}{4} \left[ |\nu(t)|^2 + \tau(t)^2 \right]$. Multiplying these results, the photon arrival rate density in an optomechanical cavity will be given by

$$
\frac{\kappa}{4P} \exp(-\kappa t) \left( |\nu(t)|^2 + \tau(t)^2 \right),
$$

(17)

where $P$ is the overall single photon probability of the state in (12):

$$
P = \frac{1}{4} \int_0^\infty \kappa \exp(-\kappa t) \left( |\nu(t)|^2 + \tau(t)^2 \right) dt
$$

$$
= \frac{k^2 \kappa \left( 3 \omega_m^4 + 2 e^6 \omega_m^4 + e^8 \omega_m^2 \kappa^2 \right)}{2e^4 \left( \kappa^5 + 5 e^3 \omega_m^4 + 4 e^6 \omega_m^4 \kappa \right)},
$$

(18)

where $|\alpha| = \frac{1}{2}$, $\beta = 2\pi$, $r = 2$ and $\theta = \pi$. The photon arrival rate density is shown in Figure 4. It can be seen clearly that in the bad-cavity limit $\kappa > \omega_m$, i.e., the non-sideband resolved regime, and as the decay rate of the cavity $\kappa$ increases, such as $\kappa = 10\omega_m$, the photon arrival rate density increasingly distributes mainly at time $t$ near 0 where it is very narrow. Because of the photon arrival rate concentrating near the zero time (blue line) in Figure 4 and the maximal amplification occurring at time $t$ near 0 (solid line) in Figure 3, for a repeated experimental set-up with identical conditions, the ‘average’ position displacement of the pointer is given by

$$
\langle q(t) \rangle = \frac{\kappa}{4P} \int_0^\infty \exp(-\kappa t) \left( |\nu(t)|^2 + \tau(t)^2 \right) \langle q(t) \rangle dt
$$

$$
= 6.98\sigma,
$$

(19)

where $\langle q(t) \rangle$ is the same as $\langle q(t) \rangle$ in (13). Note that the result can be completely detected in an experiment since it is beyond the strong coupling limit [28], i.e., the level of vacuum-state fluctuation $\sigma$.

Second, we discuss the experimental requirements for the optomechanical device. The successful postselection probability of our required device is common, although its precise value depends on the dark count rate of the detector and the stability of the set-up. As shown in (18), the probability of successful postselection in an optomechanical device with $k = 10\omega_m$ is approximately 0.525$k^2$. The window in which the detectors will need to be open for photons is approximately 1/$\kappa$, leading to a requirement that the dark count rate be lower than 0.525$k^2$/-$\kappa$. Because the best silicon avalanche photodiodes have a dark count rate of ~2 Hz, we obtain $k \geq 0.0036$ for a 4.5 kHz device [24], i.e., the proposed device no. 2, but $k = 10\omega_m$. In other words, the optical finesse $F$ in proposed device no. 2 is reduced to $3.33 \times 10^4$. Such an optomechanical cavity is easy to prepare. Therefore, the implementation of the scheme provided here is feasible in experiments.

3. Conclusion

In summary, when combined with weak measurement, we use the squeezed coherent state to enhance the displacement of the mirror in an optomechanical system. The amplification of the displacement of the mirror depends on the enhanced fluctuations of the squeezed coherent state, which is larger than that with a Gaussian state [20, 21]. Such a result is due to the noncommutativity of quantum mechanics relying on the squeezed coherent state, which cannot be explained by the standard weak measurement [1, 25]. Moreover, the amplification occurring at a time near $t = 0$, which is important for a bad optomechanical cavity with a non-sideband resolved regime, makes our proposed scheme feasible in experiments. These results not only extend the application of weak measurement to optomechanical systems, but also deepen our understanding of the weak measurement.

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Appendix. Weak measurement with a ground state pointer

In [25], they consider the standard weak measurement model but the initial state of the pointer is assumed to be the ground state $|0\rangle_m$. Suppose the state $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is the initial state of the system to be measured, where $|0\rangle$ and $|1\rangle$ are eigenstates of $\hat{\sigma}$. The Hamiltonian between the pointer and the system is given in general as

$$\hat{H} = h\chi(t)\hat{\sigma} \otimes \hat{\rho}, \quad (A1)$$

where $\sigma$ is an observable of the system to be measured, $\hat{\rho}$ is the momentum operator of the pointer and $\chi(t)$ is a narrow pulse function with integration $\chi$. Suppose that $\hat{\xi}$ is the position operator of the pointer that is conjugates to $\hat{\rho}$. As in [25], if defining an annihilation operator $\hat{c} = \frac{1}{\sqrt{2}}\hat{\xi} + \frac{1}{\sqrt{2}}\hat{\rho}$, where $\sigma$ is the zero-point fluctuation of the pointer ground state, the Hamiltonian of (A1) can be rewritten as

$$\hat{H} = -\frac{\hbar^2\chi(t)}{2\sigma}\hat{\sigma} \cdot (\hat{c} - \hat{c}^\dagger). \quad (A2)$$

Then the time evolution of the total system is given by

$$e^{-i\hat{H}dt} |+\rangle |0\rangle_m = \exp\left[-i\eta\hat{\sigma} \cdot (\hat{c} - \hat{c}^\dagger)\right] |+\rangle |0\rangle_m$$

$$= \frac{1}{\sqrt{2}} \left[ |0\rangle_m D(\eta) |0\rangle_m + |1\rangle_m D(-\eta) |0\rangle_m \right]. \quad (A3)$$

where $D(\eta) = \exp[\eta\hat{c}^\dagger - \eta^*\hat{c}]$ with $\eta = \frac{\hbar}{2\sigma}$ is a displacement operator and $\eta \ll 1$. In the weak measurement regime [1] the postselected state of the system is closely orthogonal to the initial state of the system which is usually chosen as $|\epsilon\rangle = |+\rangle + |-\rangle$, where $|\epsilon\rangle \ll 1$. After postselection the final state of the pointer becomes

$$|\psi\rangle_m = \frac{1}{\sqrt{2}} \left[ (1 + \epsilon)D(\eta) |0\rangle_m - (1 - \epsilon)D(-\eta) |0\rangle_m \right]. \quad (A4)$$

When $|\epsilon| \ll 1$ and $\eta \ll 1$, there is

$$|\psi\rangle_m \approx \frac{1}{2} \left[ (1 + \epsilon) \left( 1 - \eta^*\hat{\sigma} \cdot (\hat{c} - \hat{c}^\dagger) \right) |0\rangle_m - (1 - \epsilon) \left( 1 - \eta\hat{\sigma} \cdot (\hat{c} - \hat{c}^\dagger) \right) |0\rangle_m \right]$$

$$\approx \epsilon |0\rangle + \eta |1\rangle. \quad (A5)$$

Note that the tiny relative phase $\epsilon$ arises from a near-orthogonal postselection on the system. Using the expression of the pointer's displacement

$$\langle \hat{q} \rangle = \frac{\langle \psi | \hat{q} | \psi \rangle_m}{\langle \psi | \psi \rangle_m} - \langle 0 | \hat{q} | 0 \rangle_m, \quad (A6)$$

and

$$\langle \hat{\rho} \rangle = \frac{\langle \psi | \hat{\rho} | \psi \rangle_m}{\langle \psi | \psi \rangle_m} - \langle 0 | \hat{\rho} | 0 \rangle_m. \quad (A7)$$

Hence in this case of near-orthogonal postselection, i.e., $\langle -|+\rangle \neq 0$, we can find that

$$\langle \hat{q} \rangle = \frac{2\epsilon^2}{\epsilon^2 + \eta^2} \sigma \quad (A8)$$

and

$$\langle \hat{\rho} \rangle = 0. \quad (A9)$$

When $\epsilon = \pm \eta$ we will have the largest displacement $\pm \sigma$ in position space and when $\epsilon = 0$, indicating that the postselected state of the system is absolutely orthogonal to the initial state of the system, i.e., $\langle -|+\rangle = 0$, the displacement of the pointer position is zero. However, the displacement of the pointer is always zero in momentum space.

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