Revisiting the effect of nearby supernova remnants on local cosmic rays

Satyendra Thoudam

Astrophysical Sciences Division, Bhabha Atomic Research Centre, Trombay, Mumbai 400085, Maharashtra, India

ABSTRACT
In an earlier paper, the effect of the nearby known supernova remnants (SNRs) on the local cosmic rays (CRs) was studied, considering different possible forms of the particle injection time. The present work is a continuation of the previous work, but assumes a more realistic model of CR propagation in the Galaxy. The previous work assumed an unbounded three-dimensional diffusion region, whereas the present one considers a flat cylindrical disc bounded in both the radial and vertical directions. The study has found that the effect of the vertical halo boundary \( H \) on the local SNR contribution to the observed CR anisotropy is negligible as long as \( H \geq 2 \) kpc. Considering the values of the halo height \( H \geq 2 \) kpc obtained by different authors, the present work suggests that the study of the effect of local sources on the CR anisotropy can be carried out without having much information on \( H \), and hence using the much simpler three-dimensional unbounded solution. Finally, the present work discusses the possibility of explaining the observed anisotropy below the knee by a single dominant source with properly chosen source parameters, and claims that the source may be an undetected old SNR with a characteristic age of \( \sim 1.5 \times 10^5 \) yr located at a distance of \( \sim 0.57 \) kpc from the Sun.

Key words: cosmic rays – supernova remnants.

1 INTRODUCTION
There is a wealth of literature available on the highly isotropic nature of cosmic rays (CRs) observed at the Earth (see e.g. the references given in Thoudam 2007, hereafter Paper I). The CR anisotropy amplitude is only \( \sim 10^{-4} - 10^{-3} \) in the energy range of \( 10^{11} - 10^{15} \) eV (Guillian et al. 2007, and references therein) with the phase (direction) mainly found in the outer Galaxy, particularly in the second quadrant of the Galaxy. The possible explanations for the anisotropy are generally believed to be the global diffusion leakage of CRs from the Galaxy, the random nature of the CR sources in space–time, and the effect of the local sources. In Paper I, the effect of the known local supernova remnants (SNRs) was studied in detail by giving more emphasis to the particle release time. The study found that the observed anisotropy data favour the burst-like injection model if particles are released from the sources at an age of \( \sim (2-5) \times 10^4 \) yr.

The continuous injection model gives an anisotropy which is too large to explain the observed data. However, Paper I considered the CR diffusion zone as an unbounded three-dimensional space which is actually too far from the real geometry of the Galaxy. The present work is a continuation of the earlier work, but considers the diffusion region as a flat cylindrical disc having both radial and vertical boundaries.

In the present study, the propagation of CRs is assumed to follow the same diffusion equation as given in Paper I. The solution will be applied to local SNRs and the results will be compared with those obtained in Paper I for the burst-like model of particle injection.

2 CR SPECTRUM FROM A POINT SOURCE
In the diffusion model, neglecting convection, energy losses and particle losses due to nuclear interactions, the propagation of CR protons in the Galaxy is given by the equation

\[
\nabla \cdot (D \nabla N) + Q = \frac{\partial N}{\partial t},
\]

where \( N(r, E, t) \) is the differential number density, \( E \) is the proton kinetic energy, \( D(E) \propto E^a \) with \( a = \) constant (positive) is the diffusion coefficient which is assumed to be spatially uniform in the Galaxy, and \( Q(r, E, t) \) is the proton production rate.

The CR propagation region is assumed to be a cylindrical box bounded in both the radial and vertical directions, and our calculation takes into account the exact location of the sources with respect to the Earth. In spite of the fact that the actual spatial distribution of observed SNRs

*E-mail: satyend@barc.gov.in
extends as far as \( \sim 800 \) pc from the Galactic plane (Stupar et al. 2007), most of the CR propagation studies assume the sources to be uniformly distributed in a thin disc of half-thickness \( \sim 150-200 \) pc. Such an approximation is valid in the study of global properties of Galactic CRs since the majoriy of the sources are confined within \( \sim 200 \) pc from the plane. However, in studies like the present one where the effects of nearby discrete sources are discussed, the actual position of the sources should be considered since, for example, for the same source distance \( n(x, y, z) \) we expect to see different CR fluxes at different source heights due to the presence of the vertical halo boundary. Our calculation will also assume that the Sun is located on the Galactic plane since our Solar system is only \( \sim 15 \) pc away from the plane (Cohen 1995).

The Green’s function \( G(r, r', t, t') \) of equation (1), i.e. the solution for a \( \delta \)-function source term \( Q(r, t) = \delta (r - r') \delta (t - t') \), can be found so that the general solution can be obtained as

\[
N(r, E, t) = \int_{-\infty}^{\infty} dr' \int_{-\infty}^{\infty} dt' G(r, r', t, t') Q(r', E, t').
\]

Since the CR particles are assumed to be liberated at time \( t = t' \), the equation for \( G(r, r', t, t') \) at \( t > t' \) becomes simply

\[
\nabla \cdot (D \nabla G) = \frac{\partial G}{\partial t}.
\]

Equation (3) is solved using the proper boundary conditions and the continuity equations. While solving, we consider the origin to be located at \((x, y, z)\) from the Galactic Centre. Note that later on this point will represent the actual position of the source with respect to the observer. Then the CR density at a point \((x > 0, y > 0, z > 0)\) due to a point source [which is positioned at \((x_i, y_i, z_i)\) from the Galactic Centre] with age \( t \) is obtained using equation (2) as

\[
N(x, y, z, E, t) = \frac{q(E)}{R^2 H} \sum_{j=1}^{\infty} \left\{ \sin \left[ \frac{j\pi (R - x_j)}{2R} \right] \sin \left[ \frac{j\pi (R - x_i - |x|)}{2R} \right] \exp \left[ - \frac{j^2 \pi^2 D(t - t_j)}{4R^2} \right] \right\}
\]

\[
\times \sum_{l=1}^{\infty} \left\{ \sin \left[ \frac{k\pi (R - y_l)}{2R} \right] \sin \left[ \frac{k\pi (R - y_i - |y|)}{2R} \right] \exp \left[ - \frac{k^2 \pi^2 D(t - t_l)}{4R^2} \right] \right\}
\]

\[
\times \sum_{n=1}^{\infty} \left\{ \sin \left[ \frac{n\pi (H - z_n)}{2H} \right] \sin \left[ \frac{n\pi (H - z_i - |z|)}{2H} \right] \exp \left[ - \frac{n^2 \pi^2 D(t - t_n)}{4H^2} \right] \right\},
\]

where \( R \) and \( H \) represent the radial and the vertical boundaries of the Galaxy respectively. The solution at \((x < 0, y < 0, z < 0)\) is obtained by just replacing \((x, y, z)\) with \((-x, -y, -z)\) in equation (4). The proton flux can be calculated using \( I(E) \approx (c/4\pi a)N(E) \), where \( c \) is the velocity of light and the source spectrum \( q(E) \) is taken as

\[
q(E) = k(E^2 + 2Em_p)^{-(\Gamma + 1/2)} \times (E + m_p),
\]

in which \( m_p \) is the proton mass energy and \( k \) is the normalization constant. The source spectral index \( \Gamma \) is chosen such that \( \Gamma + a = 2.73 \), the observed proton spectral index (Haino et al. 2004).

For very large radial boundary \((R = \infty)\), the solution of equation (1) at \( z > 0 \) can be written as

\[
N(x, y, z, E, t) = \frac{q(E)}{4\pi D(t - t_0)H} \exp \left[ - \frac{(x^2 + y^2)}{4D(t - t_0)} \right] \sum_{j=1}^{\infty} \left\{ \sin \left[ \frac{n\pi (H - z_j)}{2H} \right] \sin \left[ \frac{n\pi (H - z_i - |z|)}{2H} \right] \exp \left[ - \frac{n^2 \pi^2 D(t - t_0)}{4H^2} \right] \right\}.
\]

Fig. 1 compares the proton flux at the Galactic Centre given by equation (4) with that of equation (6) for an SNR-like source located at \((1, 0, 0)\) kpc away from the Centre with an age \( t = 2 \times 10^8 \) yr. The results of equation (4) at \( R = (2, 3, 10) \) kpc are shown by the thin solid, dashed and dotted lines respectively. The thick solid lines represent the unbounded solution given by equation (6) (i.e. the solution for \( R = \infty \)). The calculations are done at \( H = 1 \) and 3 kpc assuming \( t_0 = 0 \), represented by the left- and right-hand panels respectively. The diffusion
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The coefficient is taken as \( D(E) = 2 \times 10^{28} (E/5 \text{ GeV})^{0.6} \text{ cm}^2 \text{ s}^{-1} \) for \( E > 5 \text{ GeV} \), where \( E \) is in GeV (Engelmann et al. 1990) and the injected protons are assumed to carry 10 per cent of the total explosion energy of \( \sim 10^{51} \text{ erg} \). The figures clearly show that, for sources near to the observer, the solution of equation (4) can be very well approximated by the much simpler unbounded solution for any value of \( H \) if \( R > 3 \text{ kpc} \). For example, the results at \( R = 10 \text{ kpc} \) exactly coincide with the \( R = \infty \) lines. Therefore, considering the fact that our Solar system is positioned at a distance of \( \sim 8.5 \text{ kpc} \) from the Galactic Centre and that the Galactic radius extends as far as \( \sim 20 \text{ kpc} \), the effect of the radial boundary \( R \) on the observed CRs should be negligible at least for those sources that can give appreciable density fluctuations at the Earth, i.e. for those sources located within \( \sim 1.5 \text{ kpc} \) from the Earth (see Thoudam 2006a). In the following sections where we study the effect of nearby SNRs on the observed CRs, we will therefore adopt the simpler equation (6) instead of the complicated equation (4).

3 CR ANISOTROPY

Knowing the CR density at a point \((x, y, z)\) away from a source of age \( t \), the single source anisotropy amplitude in the diffusion approximation can be calculated using (Mao & Shen 1972)

\[
\delta_i = \frac{3D}{c} \left| \nabla N_i \right|,
\]

where \( N_i \) is given by equation (6) for a point source \( i \) located at \((x_i, y_i, z_i)\) from the Earth. The total anisotropy parameter at the Earth due to a number of nearby discrete sources in the presence of an isotropic CR background is given by (Paper I)

\[
\delta = \sum_i I_i \hat{r}_i \cdot \hat{n}_m,
\]

where the summation is over the nearby discrete sources. \( \hat{r}_i \) denotes the direction of the source \( i \) giving a flux \( I_i \) and \( \hat{n}_m \) denotes the direction of maximum intensity. \( I_i = 1.37(E/\text{GeV})^{-2.77} \text{ cm}^2 \text{ s}^{-1} \text{ sr}^{-1} \text{ GeV}^{-1} \) represents the total observed flux of CR protons above \( \sim 10 \text{ GeV} \) (Haino et al. 2004). The phase of the anisotropy is taken as the direction of maximum intensity. Therefore the anisotropy \( \delta \) as well as the phase of an energy \( E \) depends on the age and distance of the nearby sources, and may be determined by different sources at different energy intervals. However, in the case of a single source dominance, the total anisotropy \( \delta \) is given by \( \delta = (I_m/I_t)\delta_m \), where ‘m’ denotes the source giving the maximum flux at the Earth.

4 COMPARISON WITH THE RESULTS OF PAPER I

In this section, we will try to investigate whether the presence of a halo boundary can affect the anisotropy at the Earth due to nearby sources. For that, we consider the 13 known SNRs located within \( 1.5 \text{ kpc} \) from the Earth as listed in table 1 of Paper I. The total anisotropy due to these SNRs is calculated using equation (8) for different \( H \) values at different values of \( h_0 \).

Fig. 2 shows the comparison of the anisotropies calculated in the present work with those obtained in Paper I for the burst-like particle injection model. The data points are taken from the compilation of various experiments given in Erlykin & Wolfendale (2006). Figs 2(a), (b), (c) and (d) are the results obtained for \( t_0 = 0, 10^3, 10^4 \) and \( 5 \times 10^4 \) yr, respectively. The thin solid lines represent the results of equation (8) for \( H = 0.5 \text{ kpc} \), the dashed lines are for \( H = 1 \text{ kpc} \), the dotted lines are for \( H = 2 \text{ kpc} \) and the dot–dashed lines are for \( H = 3 \text{ kpc} \). The thick solid lines are the results of Paper I which were obtained assuming \( H = \infty \) [equation (11) of Paper I]. In Figs 2(a)–(c), different sources determine the anisotropy at different energy ranges. These are marked by the source names along the lines. It can be seen that the results for \( H = 0.5 \text{ kpc} \) show a noticeable deviation from the \( H = \infty \) lines, while those for \( H = 1 \text{ kpc} \) show a very slight deviation. The results for other higher \( H \)-values almost overlap with the \( H = \infty \) lines and are not easily visible in the figures. This shows that, for the particle release time of \( t_0 \lesssim 10^4 \) yr, the halo height effect on the local SNR contribution to the observed CR anisotropy is almost negligible if \( H > 1 \text{ kpc} \). However, the situation is somewhat different in Fig. 2(d) where the calculations are performed at \( t_0 = 5 \times 10^4 \) yr. Note that this value of particle injection time is that at which the model calculated anisotropy values are close to the observed data (see the results of Paper I). The anisotropy here is determined solely by the Monogem SNR in the whole energy range considered here, and only those results for \( H \gtrsim 2 \text{ kpc} \) show considerable variation from the \( H = \infty \) line. The results for \( H > 2 \text{ kpc} \) show a negligible deviation. Combining all the results of Fig. 2, we can finally conclude that the effect of the halo boundary of our Galaxy on the local SNR contribution to the observed CR anisotropy is negligible as long as the boundary is greater than \( \sim 2 \text{ kpc} \). In the next section, we will combine this result along with the halo heights obtained by several authors to discuss the importance of \( H \) in the study of anisotropy due to local sources.

5 DISCUSSIONS AND CONCLUSIONS

The effect of the nearby CR sources is considered as one of the important effects that can give rise to the observed CR anisotropy at the Earth. However, the calculation of CR fluxes from any type of source in the Galaxy essentially requires the use of the proper geometry of the Galaxy as well as the actual position of the source with respect to the observer. Since our Galaxy has a cylindrical geometry with the radius much larger than the height, the radial boundary is found to have a negligible effect on the CR density and hence the geometry can be approximated by an infinite radius with a finite vertical height. Furthermore, this study has found that effect of the vertical halo boundary on the local SNR contribution to the CR anisotropy is negligible if \( H > 2 \text{ kpc} \).
vertical heights of the dominant sources above the Galactic plane are less than 60 pc which is much less than the halo heights ($H \gtrsim 0.5$ kpc) considered here.

The actual value of the halo height of our Galaxy is not exactly known. Its value is generally obtained along with other propagation parameters using observed CR data like secondary/primary ratios, CR density distribution, etc. However, the values obtained from the same experimental data are different for different CR propagation models. Webber, Lee & Gupta (1992) obtained a value of $H < 4$ kpc using a diffusion–convection model. Lukasiak et al. (1994) obtained $H = 2.8^{+1.2}_{-0.9}$ kpc using the Webber et al. (1992) model without convection. Webber & Soutoul (1998) obtained $H = 2–4$ and 2–3 kpc using the diffusion and Monte Carlo models respectively. Other studies like those of Freedman et al. (1980) and Ptuskin & Soutoul (1998) obtained $H \gtrsim 7.8$ kpc and $H = 4.9^{+4}_{-1}$ kpc respectively. A completely numerical approach using more realistic physical conditions of the Galaxy determined a value of $H > 4$ kpc for the diffusion–convection model and $H = 4–12$ kpc for the re-acceleration model (Strong & Moskalenko 1998). These results are found to be consistent with the observations of Galactic radio emission structure at 408 MHz which indicate the presence of a thick radio disc with full equivalent widths of 2.3 ± 0.2, 3.6 ± 0.4 and 6.3 ± 0.7 kpc in the Galactic radial ranges of 0–8, 8–12 and 12–20 kpc respectively (Beuermann, Kanbach & Berkhuijsen 1985), but such a wide range of values makes the Galactic halo height a very uncertain parameter in CR propagation studies. However, since most of the values obtained are found to have $H \gtrsim 2$ kpc, the conclusion given in the previous section suggests that the study of local CRs due to nearby SNRs can be carried out without having much information on $H$. This is because the effect of the nearest sources ($r \lesssim 0.5$ kpc) dominates over the influence of the other nearby sources and the CR fluxes from these sources are almost independent of the halo boundary for $H > 2$ kpc as discussed before. Hence the study of the effect of local sources on the CR anisotropy at the Earth can be done using the much simpler three-dimensional unbounded solution.

For the infinite boundary case, if a single source dominates the anisotropy in the whole energy range as in Fig. 2(d), the total anisotropy follows an energy dependence of the form $\delta \propto E^{-0.2}$ in the high-energy regime (Paper I), which for $\alpha = 0.6$ goes as $\delta \propto E^{-0.3}$. Such a decrease with energy is in fact observed in the high-energy anisotropy data somewhere above $E \sim 4 \times 10^3$ GeV up to around $3 \times 10^5$ GeV. Moreover, the increase in anisotropy from $\sim 10^{-4}$ to $10^{-1}$ in the $10^2–3 \times 10^3$ GeV energy range can also be explained by a proper choice of $(r, t, t_0)$.

Figure 2. CR anisotropy at the Earth due to nearby known SNRs assuming a burst-like injection model. Panels (a), (b), (c) and (d) are the results obtained for $t_0 = 0$, $10^3$, $10^4$ and $5 \times 10^4$ yr respectively. The thin solid lines represent the results of equation (8) for $H = 0.5$ kpc, the thin dashed lines are for $H = 1$ kpc, the dotted lines are for $H = 2$ kpc and the dot-dashed lines are for $H = 3$ kpc. The thick solid lines are the results of Paper I, i.e. for $H = \infty$. (a), (b) and (c) show that different sources determine the anisotropy at different energy intervals. These are marked by the source names along the lines. In (d), the Monogem SNR solely determines the anisotropy in the whole energy range. The thick dashed line is the best-fitting result, in the case of a single source dominance, calculated assuming infinite boundaries. Data points are taken from the compilation of various results given in Erlykin & Wolfendale (2006).
or rather \( (r, \Delta_e = t - t_0) \) for the single dominant source. We try to estimate the physical parameters of such a source that best fit the data. The best-fitting parameters are found to be \( r = 0.570 \pm 0.023 \, \text{kpc} \) and \( \Delta_e = (5.343 \pm 0.224) \times 10^5 \, \text{yr} \), and the best-fitting line is shown as the thick dashed line in Fig. 2(d). Thus for \( t_0 = 0 \) the source should have an age of \( t = \Delta_e \). However, it is possible to obtain a number of \((t, t_0)\) combinations that equally fit the data, all of them giving the same value of \( r \) and \( \Delta_e \). Therefore the present study only gives an estimate of the distance to the single dominant source; it does not give any precise information on the age and the particle release time of the source.

It should be noted that it is not the individual \((t, t_0)\) values that determine the contribution of the source, but the propagation time \( \Delta_e \) of the particles after their release from the source. We can determine the best-fitting \( t \)-value only if we know \( t_0 \), but the value of \( t_0 \) is not exactly known. It may even be that \( t_0 \) is an energy-dependent parameter, i.e. particles with different energies emitted at different times. Studies based on diffusive shock acceleration in SNRs have shown that the highest energy particles start leaving the source region already at the beginning of the Sedov phase (Berezhko, Yelshin & Ksenofontov 1996), but the major fraction of accelerated CRs remain confined for almost around \( 10^5 \, \text{yr} \) for an interstellar medium (ISM) hydrogen atom density of \( n_H = 1 \, \text{cm}^{-3} \). This implies that for the local ISM which has \( n_H \sim 1 \, \text{cm}^{-3} \) (see e.g. Thoudam 2006b, and references therein), if a single source determines the whole anisotropy, the source should have a characteristic age of \( \sim 1.5 \times 10^5 \, \text{yr} \). Unfortunately, there is no nearby known SNR with such an age located at \( r \sim 0.57 \, \text{kpc} \). However, it is quite possible that the single dominant source may be an undetected old SNR. In fact, studies assuming an adiabatic phase in SNR evolution have shown that the surface brightness of an SNR of age \( \sim 10^5 \, \text{yr} \) lies below the detection limit of radio telescopes (Leahy & Xinji 1989). The present result is further supported by the fact that almost all the nearby sources are quite young with estimated ages less than \( 10^5 \, \text{yr} \) (the generally accepted particle release time), and they might not have released the CRs into the local ISM. In addition, the possibility that some of the observed features of CRs may be due to undetected nearby sources cannot be simply ignored.

The single-source explanation of the observed CR properties can also be found in some earlier works (e.g. Erlykin & Wolfendale 2000, and references therein; Erlykin & Wolfendale 2006, etc.), but in a somewhat different context. Erlykin & Wolfendale (2000) claimed that the knee in the CR spectrum at \( E \sim 3 \, \text{PeV} \) can be attributed to the presence of a single recent supernova (as yet unidentified) in the local region. On the other hand, Erlykin & Wolfendale (2006) tried to explain the rise in the anisotropy amplitude as well as the change in its phase near the knee using a single source exploded in the direction from the Sun downward of the main CR flux, which comes predominantly from the inner Galaxy. The latter study considered the source parameters as similar to those of the Monogem SNR. Although the single source idea has not been readily accepted by the CR community, at the same time there is no reason why it should be just neglected. The present study even points out one more observed property of CRs that can possibly be explained by the single source model.

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