Parton Content of Polarized Photons:  
Theoretical Status and Experimental Prospects

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Parton Content of Polarized Photons: Theoretical Status and Experimental Prospects

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Abstract

The theoretical framework for a next-to-leading order QCD analysis of the spin-dependent parton densities \( \Delta f^\gamma(x, Q^2) \) of the longitudinally polarized photon and of its structure function \( g_1^\gamma(x, Q^2) \) is reviewed in some detail. Special emphasis is given to the specific features of different factorization schemes. Two conceivable scenarios for the polarized parton densities \( \Delta f^\gamma(x, Q^2) \) are introduced, which are suitable to study the sensitivity of future experiments to the so far unmeasured \( \Delta f^\gamma \). The experimental prospects of determining \( \Delta f^\gamma \) at a polarized collider mode of HERA or at a linear \( e^+e^− \) collider are outlined. Finally, the \( Q^2 \)-evolution of the parton content of linearly polarized photons, which shows some remarkable differences, is briefly discussed.

1 Introduction

The past months have seen a substantial amount of new experimental results on unpolarized deep-inelastic electron-photon scattering from LEP and LEP2 runs \[1\]. In these measurements of the photon structure function \( F_2^\gamma(x, Q^2) \) the kinematical coverage in \( x \) and \( Q^2 \) has been considerably extended as compared to all previous results since PEP and PETRA. Complementary information on the partonic structure of photons is provided by increasingly precise photoproduction measurements at HERA, in particular from (di-)jet production data \[2\]. The combination of the available \( e^+e^− \) and \( ep \) results should considerably improve our knowledge of the photon structure and should seriously challenge the various, presently available theoretical models.

A similar analysis in longitudinally polarized \( e^+e^− \) and \( ep \) collisions would be desirable. By measuring the difference between the two independent helicity combinations of the initial particles, i.e.,

\[
\Delta \sigma = \frac{1}{2} [\sigma(++) - \sigma(+-)]
\]

instead of the sum, as in unpolarized (helicity-averaged) experiments, one would gain access to the parton structure of longitudinally (more precisely, circularly) polarized photons, which is completely unmeasured so far. These densities are defined by

\[
\Delta f^\gamma(x, Q^2) = f_{\gamma}^{+-}(x, Q^2) - f_{\gamma}^{--}(x, Q^2),
\]
where \( f_{+}^{\gamma} \) (\( f_{-}^{\gamma} \)) denotes the density of a parton \( f \) with helicity ‘+’ (‘−’) in a photon with helicity ‘+’ (‘−’). The densities \( \Delta f \) contain information different from that contained in the more familiar unpolarized ones (defined by taking the sum in (2)) and their measurement would complete our understanding of the partonic structure of photons.

The complete NLO QCD framework for the \( Q^2 \)-evolution of the densities \( \Delta f^{\gamma}(x, Q^2) \) and the calculation of the polarized photon structure function \( g_{1}^{\gamma}(x, Q^2) \) has become available recently with the calculation of the required spin-dependent two-loop parton-parton and photon-parton splitting functions \( [3] \) and will be reviewed here. Although such a study seems to be somewhat premature at the first sight in view of the lack of any experimental information on \( \Delta f^{\gamma} \) up to now, interesting theoretical questions arise when going beyond the leading order. Apart from getting a feeling for the typical size of the NLO corrections, it is moreover important to analyze the necessity (and feasibility) to introduce a suitable factorization scheme which overcomes expected problems with perturbative instabilities arising in the \( \overline{\text{MS}} \) scheme, in particular for large values of \( x \) (already known from the unpolarized case, see, e.g., \( [5] \)).

Furthermore, it is no longer inconceivable to longitudinally polarize also the proton beam at HERA \( [3] \), i.e., to run HERA in a polarized collider mode. Measurements of spin asymmetries in, e.g., the photoproduction of large-\( p_T \) (di-)jets can then in principle reveal information on the \( \Delta f^{\gamma} \) through the presence of ‘resolved’ photon processes (similar to the already extensively studied case with unpolarized beams). Future polarized linear \( e^+e^- \) colliders could provide additional information on the \( \Delta f^{\gamma} \) by measuring the spin-dependent photon structure function \( g_{1}^{\gamma}(x, Q^2) \) or spin asymmetries in ‘resolved’ two-photon reactions. To estimate the feasibility to pin down the so far unknown \( \Delta f^{\gamma} \) in such experiments, one has to invoke some theoretical models for \( \Delta f^{\gamma} \). Moreover, recent progress in calculating spin-dependent cross sections up to NLO QCD \( [4, 8] \) demonstrates the demand for some model distributions even in NLO QCD in order to study the impact of the NLO corrections or the remaining scale dependence in a consistent manner.

A brief survey of the required theoretical framework for the \( Q^2 \)-evolution of the densities \( \Delta f^{\gamma} \) in NLO QCD is given in Sec. 2, including a discussion of the photonic structure function \( g_{1}^{\gamma} \), different factorization schemes, and the so-called ‘asymptotic’ solution for \( \Delta f^{\gamma} \). Two different scenarios for the \( \Delta f^{\gamma} \) are presented in Sec. 3 alongside a discussion of theoretical constraints on the \( \Delta f^{\gamma} \). In Sec. 4 experimental prospects for measuring the \( \Delta f^{\gamma} \) are discussed, with special emphasis on the photoproduction of (di-)jets at a polarized HERA. Finally, Sec. 5 is devoted to a brief discussion of the theoretical framework for linearly polarized photons, where some interesting new features arise.

### 2 \( Q^2 \)-evolution, factorization schemes and all that

Since the photon is a genuine elementary particle, it can directly interact in hard scattering processes, in addition to its partonic quark and gluon content \( \Delta q^{\gamma} \) and \( \Delta g^{\gamma} \), respectively. Therefore the latter distributions obey the well-known \textit{inhomogeneous} evolution equations schematically given by\( ^{1} \)

\[
\frac{d\Delta q_{i}^{\gamma}(x, Q^2)}{d\ln Q^2} = \Delta k_{i}(x, Q^2) + (\Delta P_{i} \ast \Delta q_{i}^{\gamma})(x, Q^2),
\]

where \( i \) stands for the flavor non-singlet (NS) quark combinations or the singlet (S) vector \( \Delta q_{S}^{\gamma} \equiv \left( \Delta \Sigma^{\gamma}\Delta g^{\gamma} \right) \), where \( \Delta \Sigma^{\gamma} \equiv \sum_{f}(\Delta f^{\gamma} + \Delta \bar{f}^{\gamma}) \) with \( f \) running over all active quark

\( ^{1} \) We follow closely the notation adopted in the unpolarized case as presented in Refs. \( [4] \) and \( [5] \).
flavors ($f = u, d, s$). The symbol $\ast$ denotes the usual convolution in Bjorken-$x$ space. The polarized photon-to-parton and parton-to-parton splitting functions, $\Delta k_i(x, Q^2)$ and $\Delta P_i(x, Q^2)$, respectively, in Eq. (3) receive the following 1-loop (LO) and 2-loop (NLO) contributions:

$$
\Delta k_i(x, Q^2) = \frac{\alpha}{2\pi} \Delta k_i^{(0)}(x) + \frac{\alpha \alpha_s(Q^2)}{(2\pi)^2} \Delta k_i^{(1)}(x)
$$
$$
\Delta P_i(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \Delta P_i^{(0)}(x) + \left( \frac{\alpha_s(Q^2)}{2\pi} \right)^2 \Delta P_i^{(1)}(x). \tag{4}
$$

In the singlet (S) case Eq. (3) becomes, of course, a coupled 2 matrix equation. The $\Delta P_f^{(0,1)}$ can be found in [3] and, apart from obvious NS and S charge factors, the spin-dependent photon-to-parton splitting function $\Delta k_q^{(0)}$ can be obtained from $\Delta P_q^{(0)}$ by multiplying it with $N_f N_C / T_F$, where $N_f = 3$, $T_F = N_f/2$ and $N_f$ being the number of active flavors; similarly the NLO quantities $\Delta k_q^{(1)}$ and $\Delta k_q^{(1)}$ correspond to the $C_F T_F$ terms of $\Delta P_q^{(1)}$ and $\Delta P_q^{(1)}$, respectively, multiplied by $N_f N_C / T_F$ and are listed in [4].

The evolution equations (3) are most conveniently solved directly in Mellin-$n$ space, where the solutions can be given analytically. Taking the $n$th moment of Eq. (3), the various convolutions simply factorize. The required moments of the LO and NLO $\Delta k^{(j)}$ and $\Delta P_f^{(j)}$ ($j = 0, 1$) can be found in [4] and [10], respectively, along with the prescriptions for an analytic continuation in $n$, which is required for a numerical Mellin inversion back into $x$ space. The solution of Eq. (3) can be decomposed into a ‘pointlike’ (inhomogeneous) and a ‘hadronic’ (homogeneous) part, i.e.,

$$
\Delta q_i^{\gamma,n}(Q^2) = \Delta q_i^{\gamma,n,PL(Q^2)} + \Delta q_i^{\gamma,n,had}(Q^2) \tag{5}
$$

($i = \text{NS}, \text{S}$) and can be found in [4] (with the obvious replacements of all unpolarized quantities by the corresponding polarized ones, e.g., $k_i^{(1)n} \rightarrow \Delta k_i^{(1)n}$). Having solved the evolution equations (3) for $\Delta q_{NS}^{\gamma,n}(Q^2)$, $\Delta \Sigma^{\gamma,n}(Q^2)$, and $\Delta g^{\gamma,n}(Q^2)$, one finally obtains the desired $\Delta f^{\gamma,n}(Q^2)$ ($f = u, d, s, g$) by a straightforward flavor decomposition.

Turning now to spin-dependent deep-inelastic electron-photon scattering, which can be parametrized in terms of the polarized structure function $g_1^{\gamma}(x, Q^2)$ (in analogy to the helicity-averaged case with $F_2^\gamma$ and $F_L^\gamma$). In moment-$n$ space the NLO expression for $g_1^{\gamma}$ is given by [4] (note that $\Delta f^{\gamma} = \Delta f^{\gamma}$)

$$
g_1^{\gamma,n}(Q^2) = \frac{1}{2} \sum_{f=u,d,s} e_f^2 \left\{ 2 \Delta f^{\gamma,n}(Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \left[ 2 \Delta C_q^m \Delta f^{\gamma,n}(Q^2) + \frac{1}{N_f} \Delta C_g^m \Delta g^{\gamma,n}(Q^2) \right] \right\} + \frac{1}{2} N_f N_C (e^4 \frac{\alpha}{2\pi}) \Delta C_\gamma^m \tag{6}
$$

with the usual hadronic spin-dependent Wilson coefficients $\Delta C_q^m$ and $\Delta C_g^m$, which in the conventional \underline{$A\overline{MS}$} scheme can be found, e.g., in [11, 4]. The photonic coefficient $\Delta C_\gamma^m$ can be easily derived from $\Delta C_q^m$ and is given in [4], but for the discussions below it might be useful to quote here its explicit $x$-space expression

$$
\Delta C_\gamma(x) = 2 \left( 2x - 1 \right) \left( \ln \frac{1-x}{x} - 1 \right) + 2(1-x) \tag{7}
$$

\footnotetext{2}{Note that $\Delta k_q^{(0)} = 0$ due to the missing photon-gluon coupling in lowest order.}

\footnotetext{3}{By definition, the pointlike part satisfies the boundary condition $\Delta q_i^{\gamma,n,PL(\mu^2)} = 0$ at the input scale $\mu$.}
The LO expression for $g_1^\gamma$ is entailed in the above formula (3) by simply dropping all NLO terms. Only the contribution of the light flavors has been written out in (3). Heavy quark contributions to $g_1^\gamma$ should be included via the massive polarized direct and resolved fusion subprocesses (see, e.g., [12]). These expressions are only available in LO so far.

Let us now turn to the specific features of different factorization schemes in NLO. It is convenient to introduce a decomposition of $g_1^\gamma(Q^2)$ analogously to Eq. (3):

$$g_1^\gamma(Q^2) \equiv g_{1,PL}^\gamma(Q^2) + g_{1,had}^\gamma(Q^2),$$

where $g_{1,PL}^\gamma(Q^2)$ is obtained from Eq. (3) by taking only $\Delta f_{1,PL}^\gamma(Q^2)$ with $\Delta f_{1,PL}^\gamma(Q^2)$ as defined in [3]. Conversely, for $g_{1,had}^\gamma(Q^2)$ one uses the $\Delta f_{had}^\gamma(Q^2)$ of (3), and one obviously has to omit the $\Delta C_\gamma^\gamma$ term in (3) in this case.

The solutions for $\Delta f_{1,PL}^\gamma(Q^2)$ ($\Delta f_{PL}^\gamma(x, Q^2)$) depend on the up to now unspecified hadronic input distributions at the input scale $Q^2 = \mu^2$, i.e., on the boundary conditions for the hadronic pieces $\Delta f_{had}^\gamma$ in (3), which one would intuitively relate to some model inspired by vector meson dominance (VMD). On the other hand, beyond LO both the pointlike as well as the hadronic pieces in (3) depend on the factorization scheme chosen, and it is a priori not clear in which type of factorization schemes it actually makes sense to impose a pure VMD hadronic input. Indeed, in the unpolarized case it was observed that [5] the ln$(1-x)$ term in the photonic coefficient function $C_{2,\gamma}(x)$ for $F_2^\gamma$, which becomes negative and divergent for $x \to 1$, drives the pointlike part of $F_2^\gamma(x, Q^2)$ in the $\overline{\text{MS}}$ scheme to large negative values as $x \to 1$, leading to a strong difference between the LO and the NLO results for $F_2^\gamma, PL$ in the large-$x$ region. As illustrated in Fig. 1, a very similar thing happens in the polarized case: here it is the ln$(1-x)$ term in the polarized photonic coefficient function $\Delta C_\gamma(x)$ (see Eq. (4)) for $g_1^\gamma$ that causes large negative values of the pointlike part of $g_1^\gamma(x, Q^2)$ in the $\overline{\text{MS}}$ scheme as $x \to 1$, strongly differing from the corresponding LO result also shown in Fig. 1. Clearly, the addition of a VMD-inspired hadronic part $\Delta f_{had}^\gamma(Q^2)$ cannot be sufficient to cure this observed instability of $g_{1,PL}$ in the large-$x$ region since any VMD input vanishes as $x \to 1$. Instead, as in the unpolarized case, an appropriately adjusted non-VMD hadronic NLO input would be required in the $\overline{\text{MS}}$ scheme, substantially differing from the LO one, as the only means of avoiding physically not acceptable perturbative instabilities for physical quantities like $g_1^\gamma(x, Q^2)$.

In the unpolarized case the so-called DIS$_\gamma$ scheme [7] was introduced to avoid such ‘inconsistencies’ by absorbing the photonic Wilson coefficient for $F_2^\gamma$ into the photonic quark distributions. Analogously, one expects that a similar procedure for the coefficient $\Delta C_\gamma$ for $g_1^\gamma$ cures the problem observed for $g_{1,PL}$ in the $\overline{\text{MS}}$ scheme. This redefinition of the polarized photonic quark distributions implies, of course, also a transformation of the NLO photon-to-parton splitting functions $\Delta k_{1i}^{(1)}$ due to the requirement that the physical quantity $g_1^\gamma$ has to be scheme independent (see [7] for details). The result for $g_{1,PL}$ after the transformation to the DIS$_\gamma$ scheme is also shown in Fig. 1. The similarity between the NLO (DIS$_\gamma$) and the LO curves strongly suggests that it is indeed recommendable also in the polarized case to work in the DIS$_\gamma$ scheme. Moreover, the DIS$_\gamma$ scheme, also eliminates all terms $\sim \ln^2 x$ from the polarized NLO $\Delta k_{1i}^{(1)}(x)$, i.e., removes the $\overline{\text{MS}}$ terms leading for $x \to 0$ (for corresponding observations in the unpolarized case see [12]).

The $\Delta f_{\overline{\text{MS}}}^\gamma$ in the $\overline{\text{MS}}$ and the DIS$_\gamma$ scheme are simply related by [7]

$$\Delta f_{\overline{\text{MS}}}^\gamma(x, Q^2) = \Delta f_{\text{DIS}_\gamma}^\gamma(x, Q^2) + \delta \Delta f^\gamma(x, Q^2)$$

(9)

with

$$\delta \Delta q^\gamma(x, Q^2) = \delta \Delta q^\gamma(x, Q^2) = -N_C e_q^2 \frac{\alpha}{4\pi} \Delta C_\gamma(x), \quad \delta \Delta g^\gamma(x, Q^2) = 0 ,$$

(10)
Figure 1: The ‘pointlike’ part of $xg_{1,PL}^\gamma/\alpha$ (see Eq. (8)) in LO and NLO for the $\overline{\text{MS}}$ and the DIS\text{$_\gamma$} factorization schemes. Also shown is the result obtained when extending the factorization scheme of [13] to the polarized case (‘AFG’, see [4] for details). The toy input scale $\mu = 1$ GeV, the QCD scale parameter $\Lambda = 200$ MeV and $N_f = 3$ flavors have been used. For illustration the NLO ‘asymptotic’ solution (see text and [4]) is included for $Q^2 = 20$ GeV$^2$.

where $\Delta C_\gamma(x)$ is given in Eq. (7).

Let us finish this technical section with a short comment on the so-called ‘asymptotic’ solution for the $\Delta f^\gamma$, which is obtained by dropping all terms in the full solution which decrease with increasing values of $Q^2$. In this way all dependence on the input scale and the boundary conditions is eliminated, and one ends up with the unique QCD prediction (see [15, 9, 5] for a discussion of the asymptotic solution in the unpolarized case)

$$
\Delta \mathbf{q}_{\gamma,n}^{P_L}(Q^2) = \frac{4\pi}{\alpha_s(Q^2)} \Delta \mathbf{a}^n + \Delta \mathbf{b}^n,
$$

(11)

where $\Delta \mathbf{a}^n$ and $\Delta \mathbf{b}^n$ depend on the splitting functions (see [4]). However, the practical utility of the asymptotic solution is very limited since it only applies at very large $Q^2$ and $x$: the determinants of the denominators in $\Delta \mathbf{a}^n$ and $\Delta \mathbf{b}^n$ can vanish, causing completely unphysical poles of the asymptotic solution which are not present in the full solution where subleading (‘non-asymptotic’) terms regulate such pole terms. This implies, for instance, that the NLO singlet asymptotic solution will rise as $\approx x^{-1.57}$ as $x \to 0$, i.e., the asymptotic result for $g_1^\gamma$ will not be integrable anymore [4], whereas for the full solution the first moment of $g_1^{\gamma,P_L}$ is conserved

$$
\int_0^1 g_1^{\gamma,P_L}(x, Q^2) dx = 0.
$$

(12)

This clearly underlines that the asymptotic solution as considered in LO in [16] can in general not be regarded as a reliable or realistic estimate for the polarized photon structure.
3 Available models and theoretical constraints

As already mentioned one has to fully rely on theoretical models for the polarized photon densities $\Delta f^\gamma$ for the time being. However, certain theoretical constraints on $\Delta f^\gamma$ might have to be taken into account when constructing such models:

- **‘Positivity’**: Positivity of the helicity dependent cross sections on the r.h.s. of Eq. (1) demands that $|\Delta \sigma| \leq \sigma$, which can be directly translated into a useful constraint on the densities:

  \[ |\Delta f^\gamma(x, Q^2)| \leq f^\gamma(x, Q^2). \tag{13} \]

- **‘Current conservation’**: In [17] it was shown that the first moment of $g_1^\gamma$ vanishes irrespective of $Q^2$. This result is non-perturbative: it holds to all orders in perturbation theory and at every twist provided that the fermions in the theory have non-vanishing mass [17]. Due to Eq. (12) this sum rule can be realized in LO and NLO (MS or DIS) by demanding

  \[ \Delta q^\gamma,n=1_{\text{had}} = 0, \tag{14} \]

  i.e., a vanishing first moment of the photonic quark densities at the input scale (the gluon input is not constrained since $\Delta C^m=1_g = 0$ in (6)).

To obtain a realistic estimate for the theoretical uncertainties in the polarized photon structure functions coming from the unknown hadronic input, we consider two very different scenarios in LO [18, 12] and NLO (DIS) [4] based on the positivity bound (13): for the first (‘maximal scenario’) we saturate (13) using the phenomenologically successful unpolarized GRV photon densities [19]

\[ \Delta f^\gamma_{\text{had}}(x, \mu^2) = f^\gamma_{\text{had}}(x, \mu^2), \tag{15} \]

whereas the other extreme input (‘minimal scenario’) is defined by

\[ \Delta f^\gamma_{\text{had}}(x, \mu^2) = 0 \tag{16} \]

with $\mu = \mu_{\text{LO,NLO}} \approx 0.6$ GeV [19]. Of the two extreme hadronic inputs only the ‘minimal’ one (Eq. (16)) satisfies (14). However, we are interested only in the region of, say, $x > 0.01$ here, such that for the ‘maximal’ scenario (13) the constraint (14) could well be implemented by contributions from smaller $x$ which do not affect the evolutions at larger $x$. In addition the sum rule is not expected to hold in massless QCD. Rather than artificially enforcing the vanishing of the first moment of the $\Delta q^\gamma_{\text{had}}(x, \mu^2)$ in the ‘maximal’ scenario, we therefore stick to the two extreme scenarios as introduced above.

In Fig. 2 we compare our LO and NLO (DIS) distributions $x\Delta u^\gamma/\alpha$, $x\Delta g^\gamma/\alpha$ for the two extreme scenarios at $Q^2 = 10$ GeV$^2$. These two extreme sets should be useful and sufficient in studies of the prospects of future spin experiments.

\[ ^4 \text{Strictly speaking positivity applies only to physical quantities like cross sections and not to parton densities beyond the LO where they become scheme-dependent (‘unphysical’) objects. Of course, (13) still serves as a reasonable ‘starting point’ for the NLO densities as (13) is preserved by the NLO evolution kernels.} \]
Figure 2: The LO and NLO (DIS_\gamma) polarized photonic parton densities according to the ‘maximal’ and ‘minimal’ inputs of Eqs. (15) and (16), respectively, evolved to Q^2 = 10 GeV^2.

4 Experimental prospects

Due to the lack of space, we concentrate here mainly on jet photoproduction at a polarized HERA, which seems to be the most promising tool to gain some information about the \( \Delta f^\gamma \) (similar unpolarized measurements at HERA have already successfully reduced our ignorance of the helicity-averaged densities \( f^\gamma \)). A detailed study of the physics case of the polarized collider mode option for HERA can be found in [6].

Schematically, any photoproduction cross section \((\Delta)\sigma\) is a sum of a so-called ‘direct’ and a ‘resolved’ photon contribution:

\[
(\Delta)\sigma = (\Delta)f^p \ast (\Delta)\hat{\sigma}_f + (\Delta)f^p \ast (\Delta)f^\gamma \ast (\Delta)\hat{\sigma}_{f'} \, .
\]

Apart from the \( \Delta f^\gamma \), the polarized parton densities \( \Delta f^p \) of the proton enter in (17) as well, which complicates a determination of the \( \Delta f^\gamma \) since the \( \Delta f^p \), especially \( \Delta g^p \), are not very well constrained by presently available DIS data (see, e.g., [10]). However, at the time polarized HERA could be operational, new information on the \( \Delta f^p \), in particular on \( \Delta g^p \), will be available from the upcoming experiments COMPASS at CERN and BNL-RHIC.

Studying laboratory frame rapidity distributions at HERA\footnote{As conventional for HERA, \( \eta_{LAB} \) is defined to be positive in the proton forward direction.} is a particularly suited way of ‘separating’ off the direct from the resolved contributions in (17) in a single-inclusive measurement of jets or hadrons [20]: for negative \( \eta_{LAB} \) the main contribution is expected to come from the region of \( x_\gamma \to 1 \) and thus mostly from the direct part. The situation is reversed at positive \( \eta_{LAB} \), where one should become sensitive to the unknown \( \Delta f^\gamma \). To investigate this conjecture, Fig. 3 shows our results for the polarized single-inclusive jet cross section and its asymmetry \( A^{1-jet} \equiv \Delta\sigma/\sigma \), which is the experimentally relevant quantity, vs. \( \eta_{LAB} \) for four different sets of the \( \Delta f^p \) (for more details see [20]). For Figs. 3a,b we have used the ‘maximally’ saturated set of \( \Delta f^\gamma \), whereas Figs. 3c,d correspond to the ‘minimally’ saturated one. A comparison of these results shows indeed a sensitivity to the different \( \Delta f^\gamma \) only in the forward direction for \( \eta_{LAB} \geq 1 \), where the resolved contribution dominates. Fig. 3 also includes the expected statistical errors for such a measurement at HERA [20] assuming an integrated luminosity \( \mathcal{L} = 100 \text{ pb}^{-1} \).
Figure 3: a: $\eta_{\text{LAB}}$ dependence of the polarized single-jet inclusive photoproduction cross section at HERA, integrated over $p_T > 8$ GeV (see [20] for details). The resolved contribution to the cross section has been calculated with the 'maximally' saturated set of polarized photonic densities. b: Asymmetry corresponding to a. Also shown are the expected statistical errors, see [20]. c,d: Same as a, b, but for the 'minimal' scenario of the $\Delta f^\gamma$.

Clearly, one can really learn something about the completely unknown polarized photon structure, provided the proton densities $\Delta f_p$ are pinned down more precisely in the future. Similar results can be obtained from single-inclusive charged hadron production [20, 21]. It should be also noted that the shown LO asymmetries in Fig. 3 are rather stable under variations of the factorization scale in contrast to the polarized LO cross sections $\Delta \sigma$.

Even more promising is dijet production. The important point here is that such a measurement allows for fully reconstructing the kinematics of the underlying hard process, i.e., experimentally determining the momentum fraction $x_\gamma$ for each event. Thus it becomes possible to experimentally select a resolved sample by taking only events with, say, $x_\gamma \leq 0.75$. Again with $\mathcal{L} = 100$ pb$^{-1}$ a decent measurement should be possible [20, 21]. In [21] we have furthermore shown that the LO QCD parton level calculations do not differ too much from MC results including QCD radiation, hadronization, etc.

Fig. 4 shows the LO 2-jet asymmetry in three different $x_\gamma$-bins for the two extreme $\Delta f^\gamma$ sets. For the $\Delta f^p$ the GRSV LO 'standard' distributions [10] have been used (of course, as for Fig. 3, there is a potential ambiguity due to our present ignorance of $\Delta g^p$, which has to be reduced before any information on the $\Delta f^\gamma$ can be obtained). Based on a rather old idea [23], the various different resolved subprocess cross sections and parton

6Very recently, the complete NLO corrections were calculated [7], but no phenomenological studies have been performed yet.
Figure 4: The LO di-jet spin asymmetry in different $x_\gamma$ bins as a function of the jet transverse energy $E_{\text{jet}}^T$ for the two extreme set of $\Delta f^\gamma$. Cuts on the jet rapidities are as in the recent unpolarized measurement by H1 [22]. The dotted lines correspond to the approximation as described in the text.

combinations entering the analysis can be approximated by an effective cross section which depends only on a certain combination of the proton and photon densities

$$ (\Delta) f_{\text{eff}}^{\gamma\rho} = \sum_q [(\Delta) q + (\Delta) \bar{q}] + (\Delta) a (\Delta) g, \tag{18} $$

where $a = 9/4$ [23] and $\Delta a = 11/4$ [24]. The result of this rather accurate approximation is also shown in Fig. 4 (dotted lines). The unpolarized combination $f_{\text{eff}}^{\gamma\rho}$ was recently measured by H1 at HERA using this method [22], and the prospects of unfolding $\Delta f_{\text{eff}}^{\gamma\rho}$ in a similar way are currently under investigation [24].

It should be mentioned here that heavy flavor production at a polarized HERA is not suited to determine either $\Delta g^p$ or the $\Delta f^\gamma$. A similar remark applies to prompt photon production. In both cases the statistical accuracy would be far too limited [21].

The helicity transfer $\Lambda$-baryon photoproduction process $\bar{e} p \rightarrow \Lambda X$, which can be studied without polarized protons at HERA, is, unfortunately, mainly sensitive to the polarized $\Lambda$ fragmentation functions $\Delta D_\Lambda^\gamma$ and only to a much lesser extent to the $\Delta f^\gamma$ [25]. However, such a measurement can shed some light on the also poorly understood spin-dependent fragmentation functions, which is an issue interesting in its own.
Finally, some brief remarks about the prospects of a future polarized linear $e^+e^-$ collider. This would be a unique place to study for the first time the structure function $g_{1\gamma}^\gamma$ in spin-dependent deep-inelastic electron-photon scattering. Moreover, di-jets, etc. can be studied in single and double-resolved photon processes. However, compared to similar $ep$ cross sections, which are $\mathcal{O}(\alpha^2\alpha_s)$, $\mathcal{O}(\alpha^4)$ $e^+e^-$ reactions are further suppressed and much higher luminosities are required to keep the statistical errors small enough to discriminate between different $\Delta f^\gamma$ scenarios. A possible improvement would be the use of polarized backscattered laser photons [26], i.e., a dedicated polarized $\gamma\gamma$ collider. The spectrum of backscattered laser photons can lead to considerably larger cross sections as compared to the ‘usual’ equivalent photon approximation and detailed quantitative analyses are currently under way.

5 Linearly polarized photons

Finally, let us turn to the parton content of linearly polarized photons. There are no quark densities for this kind of polarization [27] and the linearly polarized gluon density of the photon is defined by

$$\Delta_L g_{\gamma}^\gamma \equiv g_{\hat{x}}^{\gamma z} - g_{\hat{y}}^{\gamma z},$$

(19)

where $\hat{x}$ ($\hat{y}$) denotes linear polarization along the $x$ ($y$) axis. The $\Delta_L g_{\gamma}^\gamma$ obeys a very simple, non-singlet type, inhomogeneous evolution equation

$$\frac{d\Delta_L g_{\gamma}^\gamma}{d\ln Q^2} = \Delta_L k_g + \Delta_L P_{gg} * \Delta_L g_{\gamma}^\gamma,$$

(20)

where the relevant linearly polarized splitting functions $\Delta_L k_g$ and $\Delta_L P_{gg}$ are taken to have a perturbative expansion as in (4). $\Delta_L P_{gg}$ was recently calculated in NLO and can be found in [29]. The $C_T F$ part of $\Delta_L P_{gg}^{(1)}$ also determines the first non-vanishing, lowest order photon-gluon splitting function $\Delta_L k_g^{(1)}$ along the same lines as described below Eq. (4).

When solving (20), this $\mathcal{O}(\alpha\alpha_s) \Delta_L k_g^{(1)}$ has to be combined with the lowest order $\mathcal{O}(\alpha_s)$ gluon-gluon splitting function $\Delta_L P_{gg}^{(0)}$ in LO QCD. The bottom line is that the partonic structure of linearly polarized photons in LO QCD is not of $\mathcal{O}(\alpha/\alpha_s)$ anymore, as in the case of unpolarized or longitudinally polarized photons (see, e.g., (11)), but $\mathcal{O}(\alpha)$, due to the lack of a $\mathcal{O}(\alpha)$ quark ‘driving term’ $\sim (\Delta) k_q^{(0)}$ in (20). Both terms on r.h.s. of Eq. (20) are then $\mathcal{O}(\alpha\alpha_s)$, and a consistent NLO evolution of $\Delta_L g_{\gamma}^\gamma$ would require the knowledge of $\Delta_L k_g^{(2)}$. In addition, all resolved processes are $\alpha_s$-suppressed compared to the direct photon contribution, i.e., are formally NLO effects. More details will be presented in [30].

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\footnote{It should be noted that the structure function of linearly polarized photons is sometimes denoted also by $F_3^\gamma$ in the literature [28].}
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