New developments on embedding inflation in gauge theory and particle physics

A. Mazumdar 1, 2, 3

1 NORDITA, Blegdamsvej-17, Copenhagen-2100, Denmark
2 Niels Bohr Institute, Blegdamsvej-17, Copenhagen-2100, Denmark
3 Physics Department, Lancaster University, LA1 4YB, United Kingdom

Abstract: In this brief review we will discuss how a well motivated particle theory beyond the electroweak Standard Model provides ingredients and conditions for a successful inflation. We will mainly focus on a low energy supersymmetric Standard Model which provides plenty of scalars. In particular, these scalars span a multi-dimensional moduli space of gauge invariant operators which carry the Standard Model charges. The inflationary predictions which matches the current observations are robust due to the fact that inflation occurs within our own gauge sector where the couplings are well known. We further argue that based on our current understandings if there exists a string landscape of multiple vacua, then it is very natural that the last phase of inflation would be driven by one of the many supersymmetric Standard Model modulii. Only such a graceful exit from inflation would provide hot thermal Standard Model baryons, cold dark matter, conditions for baryogenesis and foremost the seed density perturbations for the cosmic microwave background radiation in just one package. Furthermore we will also discuss how some of the ingredients of inflation can be tested already by the LHC.
Contents

1. Introduction

2. What should be the inflation properties?

3. Supersymmetry as a tool
   3.1 MSSM and its potential
   3.2 F-and D-flat directions
      3.2.1 An example of F-and D-flat direction
      3.2.2 Lifting by non-renormalizable operators

4. Gauge Invariant Inflaton
   4.1 Inflation near the Saddle Point
   4.2 Slow roll
   4.3 Inflaton Properties and predictions
      4.3.1 Inflaton candidates
      4.3.2 Inflaton Predictions
   4.4 Departure from the saddle point

5. Radiative and supergravity corrections
   5.1 One-loop effective potential
   5.2 RG equations for the $LLe$ direction
   5.3 $A_6$ vs. $A_3$
   5.4 Supergravity corrections

6. Reheating and Thermalization
   6.1 Towards thermal equilibrium
   6.2 Solution to the gravitino problem

7. Cold dark matter and MSSM inflation

8. Grand unified Models and Inclusion of Right-Handed Neutrinos
   8.1 Embedding MSSM inflation in $SU(5)$ or $SO(10)$ GUT
   8.2 Including Right-Handed Majorana Neutrinos
1. Introduction

Primordial inflation as a paradigm has met with glorious successes over the past 26 years since its advent \cite{1, 2, 3, 4, 5} (for a review see \cite{6}). The virtues of inflation lies due to the fact that an accelerated expansion of the universe can give rise to a homogeneous and an isotropic universe on very large scales with a flat spatial geometry. There are very important observational consequences, such as nearly scale invariant tiny perturbations imparted to the cosmic microwave background radiation and its spectrum \cite{8} (for a review see \cite{9}) has met with an unprecedented success with the observations based on Cosmic Microwave Background (CMB) radiation, see the recent data from WMAP \cite{10, 11}.

It has been known for a while that inflation can be driven by a dynamical scalar field known as the \textit{inflaton}, an order parameter, which could either be fundamental or composite. Particularly if the inflaton rolls very slowly on a sufficiently flat potential (such that the potential energy density dominates over the kinetic term) then it is possible to show: (1) a considerably large number of e-foldings, (2) during inflation...
metric perturbations are generated with an amplitude: $P_R \sim 1.91 \times 10^{-5} k^{1.91}$, with a spectral tilt lying within: $0.92 \leq n \leq 1.0$ within $2\sigma$ (CMB+SDSS) (3) running of the spectral tilt is negligible $-0.012 \leq dn_s/d\ln k \leq 0.001$ within $2\sigma$ (CMB+SDSS for tensor to scalar ratio negligible) and, (4) the tensor to scalar ratio, $r < 0.55$ at 95% c.l. details can be found in Ref. [10].

However, inspite of all these achievements, there are some fundamental issues where we are lacking proper understanding, such as:

- What is the origin of inflation?
- What is the inflaton?
- What are the fundamental interactions of an inflaton?
- Where does the inflaton energy goes?
- How can we test the inflaton in a laboratory?

Inspite of many attempts there has been no single good candidate for an inflaton which comes naturally out of a well motivated theory of particle physics (for a review on models of inflation, see [12]). One always relies on scalar fields which are *absolute gauge singlets* possibly residing in some hidden sector or secluded sector with a small coupling to the SM gauge group. By definition an *absolute gauge singlet* does not carry any charge what so ever be the case. Therefore the masses, couplings and interactions are not generally tied to any fundamental theory or any symmetry. Such gauge singlets are used ubiquitously by model builders to obtain a desired potential and interactions at a free will in order to explain the current CMB data. In this respect the inflaton is just a phenomenological answer to inflation and its observed consequences. What has been ignored so far is the fact that if inflation explains the CMB anisotropy then it must also reheat the universe with the SM degrees of freedom $^1$. The decay of inflaton should also provide thermal conditions $^8$ to

---

$^1$An *absolute gauge singlet* inflaton can couple to all the species and sectors alike. It does not discriminate between the SM degrees of freedom to that of any other degrees of freedom. By definition an *absolute gauge singlets* can couple to any other gauge singlets, there is no symmetry which prohibits doing so. The most popular slow roll inflationary models such as chaotic inflation $^4$, hybrid inflation $^5$, new inflation $^7$, assisted inflation $^6$, $^9$, without slow roll inflation $^3$, $^10$, K-inflation $^7$, etc. all rely on absolute gauge singlets, see for a review $^2$. In this respect they all fail to explain why the couplings, masses and interactions are chosen so to explain the CMB data.
create matter-anti-matter asymmetry and thermal/non-thermal generation of cold dark matter, for a review see [19, 20].

On the other hand string theory comes up with a plethora of absolute gauge singlet modulii, which mainly arise in the gravitational sector upon various compactifications [23]. There are numerous attempts to embed inflation within string theory, for a review see [24], see also [25, 26]. However they all tend to concentrate on explaining the CMB data, but with very little to do with an aftermath of inflation [35], such as how the inflaton couples to the SM degrees of freedom ?, how the dark matter is created ?, what are the relativistic degrees of freedom after the end of inflation ?, etc. The uncertainties prevail mainly due to two facts: (a) It is not clear how to construct SM like gauge group. There are various constructions so far, but all suffer through various problems such as quantum instability, moduli de-stabilization, extra U(1)’s, etc. [27]. (b) Inflation again generically driven by absolute gauge singlets, such as the closed string modulii in the case of a race-track inflation [28], or open string moduli [29, 30], for a recent discussion on open string moduli inflation in a warped throat, see [31]. In the case of an open string moduli the inflaton need not be completely an absolute gauge singlet, but so far a realistic model of brane-anti-brane inflation lacks the SM embedding [32].

In all these cases the inflaton candidate can couple to closed string degrees of freedom and the open string degrees of freedom alike. Although there will be some preference to decay channels for an open string moduli as an inflaton, it can couple to an appropriate gauge theory with appropriate gauge couplings [32]. However there is no obvious construction which connects open string modulii as an inflaton to that of the SM sector. Moreover because of a hierarchy between the four dimensional Planck scale, string scale and the compactification scale, the 6 dimensional compactified volume is so large that a kinematical phase factor argument allows the inflaton energy density to get lost in the bulk of the volume which contains mainly the closed string degrees of freedom [30, 33]. Therefore, there is no preference why the inflaton would decay primarily into the SM baryons. One would have to make

Moreover one cannot predict definitely certain issues such as radiative corrections and stability of the potential, because the couplings and masses are all put by hand.

2The Big Bang Nucleosynthesis (BBN) demands that when the universe was nearly 1 MeV, most of the energy density of the universe must lie in the SM relativistic degrees of freedom [21]. It is therefore pertinent that the universe must have created a thermal bath of strictly speaking SM degrees of freedom before the BBN. If there were other degrees of freedom which were to couple very weakly to the SM then it would provide severe experimental constraints [22].
additional assumptions regarding various sectors the inflaton couples to or where exactly inflation occurs. The string motivated models have thus failed to explain the aftermath of inflation, particularly why the universe is filled with the SM baryons at the time of BBN. More work is required to understand this issue, until then one could at best treat stringy inflation as a nice playground to test various ideas concerning CMB data alone.

There are also some attempts to understand the CMB data without invoking inflation such as in the case of a bouncing cosmology. Some of the key ideas are stringy in origin (based on t-duality, see ), especially in the context of string gas cosmology of Brandenberger and Vafa. However, there are various caveats towards a full understanding of a thermodynamics of a gas of strings in a cosmological background. There are other attempts to obtain inflation such as in the case of a gas of D-branes, or string moduli in conjunction with string gas, or in the case of Hagedorn strings during a bounce. Moreover they all fail to address one crucial issue: how to obtain the SM degrees of freedom at the end of the day?

An important message from the above discussions is following:

*If any model of inflation or any alternatives of inflation provides the right CMB predictions then there must be a way to excite the SM baryons with a precision which can match the BBN data without making further assumptions.*

In order to facilitate this we need a guiding principle, some kind of symmetry argument, which will help us in constructing a successful model of inflation from a bottom-up approach.

Very recently an interesting construction of inflation has been suggested within a minimal extension of the SM such as in the case of Minimal Supersymmetric Standard Model (MSSM) (for a review on MSSM, see and its embedding in supergravity (SUGRA), see, for cosmological aspects of MSSM, see. Within MSSM there are many scalars, which span into a moduli

---

3In a warped throat geometry, motivated by stabilizing the dilaton, complex structure modulii and the kähler modulii, the inflaton sector and the SM sector are kept in distinctive throats. The SM throat typically addresses the hierarchy issue while the inflationary throat is situated relatively at higher energies. The two throats talk to each other through closed string sector. There are many issues which plague this geometry, such as excess Kaluza Klein graviton production, and transferring the energy density from closed string sector to the SM throat. There are various subtleties as during inflation the SM modulii are displaced away from their minimum which brings in various cosmological uncertainties which might as well hamper baryogenesis and BBN.
space of *gauge invariant* F-and D-flat directions, which carry SM charges usually baryon and/or lepton number. These modulii have an *enhanced symmetry point* near the origin (at a VEV defined by zero). Away from the origin these modulii break wholly or partly the SM gauge symmetry depending on the flat direction. But such a spontaneous breaking of charge and color in the early universe is not considered to be dangerous, provided they all settle down to their minimum before the electroweak phase transition.

In the first paper \[\cite{44}\], it was pointed out that the MSSM provides all the necessary key ingredients for a successful inflation. Out of nearly 300 flat directions \[\cite{19}\], there are only 2 directions which can support inflation with a graceful exit. Other directions do not have a graceful exit \[\cite{44, 45}\], nor do they have a slow roll phase of inflation. This is a remarkable result which puts forward the two inflaton candidates as \(LLe\) (which carries the lepton number) and, \(udd\) (which carries the baryon number). The \(L\) corresponds to the slepton (SUSY partner of SM lepton) doublet, while \(e\) corresponds to the right handed selectron (SUSY partner of electron). The \(u, d, d\) correspond to the right handed squarks (SUSY partner of quarks). These two directions can support *eternal* and *slow roll* inflation with a right amplitude of temperature anisotropy \(\mathcal{P}_R \sim \delta_H \sim 1.91 \times 10^{-5}\), an observable tilt in the power spectrum with a range \(0.92 \leq n_s \leq 1.0\), where the lower limit is saturated for a *saddle point* inflation, the upper limit corresponds to having a slow roll inflation away from the saddle point \[\cite{14, 44}\]. The running of the tilt in the spectrum is negligible and the tensor to scalar ratio is also observationally negligible. Moreover there is no production of cosmic strings, or non-MSSM degrees of freedom \[\cite{44, 45}\], and there is no large non-Gaussianity \[\cite{57}\].

Furthermore the inflaton reheating is very well understood as the inflatons carry the SM charges, they naturally decay only to the SM degrees of freedom, besides creating the lightest supersymmetric particle (LSP) as a candidate for the cold dark matter \[\cite{14, 45, 18}\]. With an R-parity LSP’s are absolutely stable and can be a SUSY cold dark matter candidate \[\cite{58, 96}\]. In Ref. \[\cite{18}\] we studied the parameter space of inflation and the neutralino type dark matter produced thermally. Remarkably we found an interesting overlap between inflationary parameters within \(m_0, m_{1/2}\) (soft masses and gaugino masses respectively) plane in (m-SUGRA) setup. This provides another hint that if the desert is filled by MSSM from the electroweak scale to the grand unified scale, then the parameters of an MSSM inflaton will provide not
only a successful inflation which matches the current observational data, but the same parameter space is also responsible for generating the observed abundance of thermally created cold dark matter [48].

Both the inflatons, \( LLe, udd \), masses are tied with a low energy SUSY required to address the hierarchy between the Planck and the electroweak scale. The mass range \( \sim \mathcal{O}(\text{TeV}) \) is adequate to explain the CMB anisotropy. An arbitrary increase in the scalar masses will ruin the CMB predictions [44, 45]. Not only this, we would also be able to put constraints on the slepton and squark masses from the LHC [48]. This will further constrain the MSSM inflation model to an unprecedented level. In this regard both \( LLe \) and \( udd \) are testable through CMB and the LHC. This is the most exciting expectation that for the first time the LHC will be able to rule out a model of inflation completely. For instance if the slepton and squark masses are beyond \( \gg 10 \text{ TeV} \), then this model of inflation is ruled out completely. For such a mass range neither \( LLe \) nor \( udd \) can generate the right amplitude of density perturbations [44, 45].

Furthermore this is the only known model of inflation where the \( N_{\text{COBE}} \) (the last number of e-foldings required to explain the CMB data) is determined fully from the fundamental interactions. The \( N_{\text{COBE}} \) is primarily determined by thermal history of the universe. Fortunately enough, in MSSM based inflationary model, the reheating and thermalization temperature can be estimated rather accurately due to known SM couplings [44, 45, 46, 47].

Another important issue is that within MSSM inflation the quantum stability of the inflationary potential can be analyzed correctly as the couplings of the inflaton are known [45]. Since inflation is driven near the saddle point, it becomes an important question to ask how stable the potential is under large radiative corrections? To our surprise what we found is that an existence of a saddle point is not ruined by radiative corrections, although the point of inflection does shift towards higher vacuum expectation values (VEVs). The predictions for CMB also does not modify at all within the current uncertainties. The model predictions are also robust as there is no supergravity (SUGRA) corrections which can spoil the cosmological flatness of the potential. Although one would expect that SUGRA to play an important role, but all such corrections are absorbed when the choice of a saddle point is made where slow roll inflation occurs. In this respect one can take a view that a saddle point inflation can address the SUGRA-eta problem [45]. Of course SUSY is the
key ingredient which helps maintaining the flatness of the potential, in this respect SUSY along with gauge symmetry not only guides us towards a more holistic model of inflation, but also with sharp predictions which can be tested at the LHC and in future CMB experiments. Note that the model does not generate observationally significant stochastic gravitational wave background radiation or the cosmic strings.

An interesting twist comes from string theory alone. As it is fairly well established by now that there exists a string theory landscape with plethora of vacua, for a review [23]. Therefore, in this landscape, only way our patch of the observable universe evolves to its present state regardless of how it began, if we only secure that the last stage of inflation were driven within an MSSM vacuum. In any case the MSSM fields (and couplings) must be in the low energy spectrum. At some stage the MSSM sector should take over. False vacuum inflation in MSSM then makes any previous stage in the history of the universe oblivious. However the observable part of the universe can still emerge since false vacuum inflation also sets the stage for a last bout of successful inflation (also happening in the MSSM sector).

In the following sections we will discuss various virtues of MSSM inflation, in section 2, we discuss general principles behind a successful inflation, in section 3, we discuss MSSM and introduce a concept behind F-and D-flat directions. In section 4, we describe MSSM inflation. In section 5, we discuss radiative and SUGRA corrections. In section 6, we discuss reheating and thermalization. In section 7, we discuss thermal neutralino cold dark matter creation. In section 8, our effort is to embed MSSM inflation with grand unified theories with an inclusion of the Majorana neutrinos. In section 9, we discuss how a small Yukawa coupling required to explain the neutrino masses could also maintain the flatness of the MSSM inflationary potential. In the same section we describe MSSM inflation within gauge mediated SUSY breaking scenarios. In the last section 10, we discuss initial conditions for an MSSM inflation, we speculate the role of string landscape in order to connect high and low scale inflations. We also point out a graceful exit from a string landscape which can happen via a slow roll MSSM inflation.

2. What should be the inflation properties?

In a minimal model, the inflaton is the only source of density perturbations and also responsible for a successful reheating. This is also true in the case of a curvaton [60]. Where the success of curvaton lies within a successful reheating of the SM degrees
of freedom during the curvaton dominance. It is possible to find an MSSM curvaton candidate which has all the properties to create an universe with the right relativistic degrees of freedom, cold dark matter and the seed for the primordial density perturbations [61]. There are also variants of curvaton in the context of inhomogeneous reheating and generating density perturbations [62]. In either cases it is important to realize that a successful reheating ensures a successful CMB predictions, as an example in the case of an MSSM curvaton, recently we have singled out an unique MSSM flat direction which would create the right CMB predictions and the SM baryons along with neutralino type cold dark matter [63].

The dream has been to embed the model of inflation in particle physics naturally. The first attempts were made by Guth [1], where inflation would occur in a false vacuum of the Higgs of the Grand Unified Theory (GUT), fell into trouble, because the end of inflation happened due to first order phase transition. The bubble of true vacuum remains cold in the sea of false vacuum and there is no way to generate thermal entropy other than colliding the bubbles (the potential energy is stored in the bubble walls). There were attempts to address this issue, but all such attempts were not so attractive, as they would all involve fields such as absolute gauge singlets from a hidden sector, see for a review [12].

A successful reheating must ensure that:

- SM Baryons are excited:
  The SM baryons are excited dominantly as light degrees of freedom to ensure a successful BBN [21]. Note that BBN puts stringent constraints on hidden light degrees of freedom [2]. At best one could accommodate one light neutrino or any relativistic species [21]. This suggests that the hidden sector particles must be heavier than few $\sim$ TeV so that they kinetically and thermally decouple from the rest of the thermal bath before they could decay through Planck suppressed interactions.

- Cold dark matter is created:
  A successful reheating must create conditions for generating thermally/non-thermally cold dark matter essential for the structure formation [10].

These two conditions do not necessarily bar the inflaton to be a part of a hidden sector. However if our main aim is to seek a model of particle physics then the hidden sector serves very little progress in our quest. Note that there is no underlying
symmetry which helps understanding the masses of the hidden sector fields and their couplings to the SM degrees of freedom.

If any model of inflation seeks to be successful, then it must possess:

- **Credibility:**
  The model parameters, such as mass, couplings are not chosen ad-hoc to match the CMB observations, rather they should arise *naturally* from an underlying theory.

- **Stability:**
  As it is well known a slow roll inflation needs a flat potential. By construction the inflaton energy density couples to gravity and the inflaton couples to at least the SM degrees of freedom, therefore, one needs to ensure that the background geometry, quantum corrections, supergravity effects do not spoil the flatness of the potential.

- **Testability:**
  It would be desirable to test the micro physical ingredients of inflation in a terrestrial laboratory. It is fair to say that we have indeed tested several ideas of inflation in CMB physics, but in order to really seek the true origin of inflation one must do more than that.

Based on the above issues, it is arguably simpler if inflation were driven solely by the SM particles. However SM spectra is full of fermions, only scalar is the SM Higgs but with a relatively small VEV to match the observational data \(^4\).

This would eventually push the inflaton candidature to the physics beyond the SM. However this is the frontier where we lack our grounds on the experimental front, hence we are forced to the speculations. This is perhaps one of the main reasons for introducing *absolute gauge singlets* as the inflaton, or modeling the inflaton in a *hidden sector*. However, here as we have reiterated, we wish to have a concrete model of inflation which has a better predictions from the CMB observations, successful reheating and of course the model should be testable in future collider.

\(^4\)Could SM Higgs be the inflaton?

The Higgs searches at LEP has pushed the Higgs mass above 114 GeV \(^6\), this means that the electroweak phase transition via the SM Higgs is of second order or perhaps crossover in nature. Inflation could potentially work out if the Higgs field rolls very slowly, however, the Higgs VEV is sufficiently low enough to generate the scalar density perturbations to match the observed temperature anisotropy.
3. Supersymmetry as a tool

Supersymmetry (SUSY) has many virtues, foremost, it is the most attractive scenario to address the hierarchy between the Planck and the electroweak scale. One particular advantage of SUSY is that the quantum corrections due to bosonic and fermionic loops cancel exactly in a SUSY limit, rendering the stability of masses, couplings and the scalar potentials. In order to address the hierarchy problem the SUSY must be broken spontaneously. At low energies, within MSSM, the SUSY breaking effects are captured by soft parameters, i.e. mass, trilinear couplings, etc. In low energy SUSY breaking scenarios, the scalar mass is \( \sim \mathcal{O}(\text{TeV}) \). Further note that SUSY allows many scalar fields (corresponding to every quarks and leptons within MSSM), therefore, it is interesting to ask what is the role of sfermions in cosmology?

In softly broken SUSY, the quantum corrections give rise to Logarithmic running to masses and couplings. This ensures that the scalar potential is at least Logarithmically stable under quantum corrections. In this respect the SUSY inspired inflationary potentials are at least stable under quantum corrections. This leads to satisfying one of the cornerstones of a successful inflationary model building.

3.1 MSSM and its potential

Let us remind the reader that the matter fields of MSSM are chiral superfields \( \Phi = \phi + \sqrt{2} \theta \bar{\psi} + \theta \bar{\theta} F \), which describe a scalar \( \phi \), a fermion \( \psi \) and a scalar auxiliary field \( F \). In addition to the usual quark and lepton superfields, MSSM has two Higgs fields, \( H_u \) and \( H_d \). Two Higgses are needed because \( H^1 \), which in the Standard Model gives masses to the u-quarks, is forbidden in the superpotential.

The superpotential for the MSSM is given by [56]

\[
W_{\text{MSSM}} = \lambda_u Q H_u u + \lambda_d Q H_d d + \lambda_e L H_d e + \mu H_u H_d,
\]

where \( H_u, H_d, Q, L, u, d, e \) in Eq. (3.1) are chiral superfields, and the dimensionless Yukawa couplings \( \lambda_u, \lambda_d, \lambda_e \) are \( 3 \times 3 \) matrices in the family space. We have suppressed the gauge and family indices. Unbarred fields are \( SU(2) \) doublets, barred fields \( SU(2) \) singlets. The last term is the \( \mu \) term, which is a supersymmetric version of the SM Higgs boson mass. Terms proportional to \( H_u^* H_u \) or \( H_d^* H_d \) are forbidden in the superpotential, since \( W_{\text{MSSM}} \) must be analytic in the chiral fields. \( H_u \) and \( H_d \) are required not only because they give masses to all the quarks and leptons, but also for the cancellation of gauge anomalies. The Yukawa matrices determine the masses.
and CKM mixing angles of the ordinary quarks and leptons through the neutral components of \( H_u = (H_u^+, H_u^0) \) and \( H_d = (H_d^0, H_d^-) \). Since the top quark, bottom quark and tau lepton are the heaviest fermions in the SM, we assume that only the \((3, 3)\) element of the matrices \( \lambda_u, \lambda_d, \lambda_e \) are important. In this limit only the third family and the Higgs fields contribute to the MSSM superpotential.

The SUSY scalar potential \( V \) is the sum of the F- and D-terms and reads

\[
V = \sum_i |F_i|^2 + \frac{1}{2} \sum_a g_a^2 D^a D^a \tag{3.2}
\]

where

\[
F_i \equiv \frac{\partial W_{MSSM}}{\partial \phi_i}, \quad D^a = \phi^\dagger T^a \phi. \tag{3.3}
\]

Here we have assumed that \( \phi_i \) transforms under a gauge group \( G \) with the generators of the Lie algebra given by \( T^a \).

### 3.2 F-and D-flat directions

For a general supersymmetric model with \( N \) chiral superfields \( X_i \), it is possible to find out the directions where the potential Eq. 3.2 vanishes identically by solving simultaneously

\[
D^a = X^\dagger T^a X = 0, \quad F_{X_i} = \frac{\partial W}{\partial X_i} = 0. \tag{3.4}
\]

Field configurations obeying Eq. (3.4) are called respectively D-flat and F-flat.

D-flat directions are parameterized by gauge invariant monomials of the chiral superfields. A powerful tool for finding the flat directions has been developed in [65, 66], where the correspondence between gauge invariance and flat directions has been employed. The configuration space of the scalar fields of the MSSM contains 49 complex dimensions (18 for \( Q_i \), 9 each for \( \bar{u}_i \) and \( \bar{d}_i \), 6 for \( L_i \), 3 for \( \bar{e}_i \), and 2 each for \( H_u \) and \( H_d \)), out of which there are 12 real D-term constraints (8 for \( SU(3)_C \), 3 for \( SU(2)_L \), and 1 for \( U(1)_Y \)), which leaves a total of 37 complex dimensions [65, 66]. The trick is to construct gauge invariant monomials forming \( SU(3)_C \) singlets and then using them as building blocks to generate \( SU(3)_C \times SU(2)_L \), and subsequently the whole \( SU(3)_C \times SU(2)_L \times U(1)_Y \) invariant polynomials [35, 60]. However these invariant monomials give only the D-flat directions. For F-flat directions, one must solve explicitly the constraint equations \( F_{X_i} = 0 \).

A single flat direction necessarily carries a global \( U(1) \) quantum number, which corresponds to an invariance of the effective Lagrangian for the order parameter \( \phi \).
under phase rotation $\phi \rightarrow e^{i\theta} \phi$. In the MSSM the global $U(1)$ symmetry is $B - L$. For example, the $LH_u$-direction (see below) has $B - L = -1$.

A flat direction can be represented by a composite gauge invariant operator, $X_m$, formed from the product of $k$ chiral superfields $\Phi_i$ making up the flat direction: $X_m = \Phi_1 \Phi_2 \cdots \Phi_m$. The scalar component of the superfield $X_m$ is related to the order parameter $\phi$ through $X_m = c \phi^m$. For a flat direction represented by polynomial the description is much more involved, see [37].

### 3.2.1 An example of F-and D-flat direction

The flat directions in the MSSM are tabulated in Table 1. An example of a D-and F-flat direction is provided by

$$
H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad L = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix},
$$

(3.5)

where $\phi$ is a complex field parameterizing the flat direction, or the order parameter, or the AD field. All the other fields are set to zero. In terms of the composite gauge invariant operators, we would write $X_m = LH_u$ ($m = 2$).

From Eq. (1.14) one clearly obtains $F_{H_u}^* = \lambda_u Q \bar{u} + \mu H_d = F_L^* = \lambda_d H_d \bar{e} \equiv 0$ for all $\phi$. However there exists a non-zero F-component given by $F_{H_u}^* = \mu H_u$. Since $\mu$ can not be much larger than the electroweak scale $M_W \sim \mathcal{O}(1)$ TeV, this contribution is of the same order as the soft supersymmetry breaking masses, which are going to

|                  | $B - L$ | $B - L$ |
|------------------|---------|---------|
| $H_u H_d$        | 0       | $LH_u$  |
| $\bar{u} \bar{d} \bar{d}$ | -1 | $QL \bar{d}$ |
| $LL \bar{e}$    | -1      | $QQ \bar{u} \bar{d}$ |
| $QQQL$          | 0       | $QL \bar{u} \bar{e}$ |
| $\bar{u} \bar{u} \bar{e}$ | 1       | $LL \bar{d} \bar{d} \bar{d}$ |
| $QQ \bar{u} \bar{e}$ | 1       | $LL \bar{d} \bar{d} \bar{d}$ |
| $QLQQLL\bar{d}$ | -2      | $\bar{u} \bar{u} \bar{d} \bar{d} \bar{d}$ |
| $QQQQLL\bar{d}$ | -1      | $QLQLQL \bar{d}$ |

Table 1: Renormalizable F and D flat directions in the MSSM
lift the degeneracy. Therefore, following [65], one may nevertheless consider $LH_u$ to correspond to a F-flat direction.

The relevant D-terms read

$$D_{SU(2)}^u = H_u^\dagger \tau_3 H_u + L^\dagger \tau_3 L = \frac{1}{2} |\phi|^2 - \frac{1}{2} |\phi|^2 \equiv 0.$$ (3.6)

Therefore the $LH_u$ direction is also D-flat.

The only other direction involving the Higgs fields and thus soft terms of the order of $\mu$ is $H_u H_d$. The rest are purely leptonic, such as $LL\bar{e}$, or baryonic, such as $\bar{u}d\bar{d}$, or mixtures of leptons and baryons, such as $QL\bar{d}$.

### 3.2.2 Lifting by non-renormalizable operators

Non-renormalizable superpotential terms in the MSSM can be viewed as effective terms that arise after one integrates out fields with very large mass scales appearing in a more fundamental (say, string) theory. Here we do not concern ourselves with the possible restrictions on the effective terms due to discrete symmetries present in the fundamental theory, but assume that all operators consistent with symmetries may arise. Thus in terms of the invariant operators $X_m$, one can have terms of the type [68, 65]

$$W = \frac{h}{dM^{d-3}} X_m^k = \frac{h}{dM^{d-3}} \phi^d,$$ (3.7)

where the dimensionality of the effective scalar operator $d = mk$, and $h$ is a coupling constant which could be complex with $|h| \sim O(1)$. Here $M$ is some large mass, typically of the order of the Planck mass or the string scale (in the heterotic case $M \sim M_{GUT}$). The lowest value of $k$ is 1 or 2, depending on whether the flat direction is even or odd under $R$-parity.

A second type of term lifting the flat direction would be of the form [68, 65]

$$W = \frac{h'}{M^{d-3}} \psi \phi^{d-1},$$ (3.8)

where $\psi$ is not contained in $X_m$. The superpotential term Eq. (3.8) spoils F-flatness through $F_\psi \neq 0$. An example is provided by the direction $\bar{u}_1 \bar{u}_2 \bar{u}_3 \bar{e}_1 \bar{e}_2$, which is lifted by the non-renormalizable term $W = (h'/M)\bar{u}_1 \bar{u}_2 \bar{d}_2 \bar{e}_1$. This superpotential term gives a non-zero contribution $F^*_{d_2} = (h'/M)\bar{u}_1 \bar{u}_2 \bar{e}_1 \sim (h'/M)\phi^3$ along the flat direction.

Assuming minimal kinetic terms, both types discussed above in Eqs. (3.7, 3.8) yield a generic non-renormalizable potential contribution that can be written as

$$V(\phi) = \frac{|\lambda|^2}{M^{2d-6}} (\phi^* \phi)^{d-1},$$ (3.9)
Figure 1: The colored curves depict the full potential, where $V(x) \equiv V(\phi)/(0.5 \, m_\phi^2 M_P^2 (m_\phi/M_P)^{1/2})$, and $x \equiv (\lambda_n M_P/m_\phi)^{1/4}(\phi/M_P)$. The black curve is the potential arising from the soft SUSY breaking mass term. The black dots on the colored potentials illustrate the gradual transition from minimum to the saddle point and to the maximum.

where we have defined the coupling $|\lambda|^2 \equiv |h|^2 + |h'|^2$. By virtue of an accidental $R$-symmetry under which $\phi$ has a charge $R = 2/d$, the potential Eq. (3.9) conserves the $U(1)$ symmetry carried by the flat direction, in spite of the fact that at the superpotential level it is violated, see Eqs. (3.7,3.8). The symmetry can be violated if there are multiple flat directions, or by higher order operator contributions. However it turns out [65] that the $B - L$ violating terms are always subdominant. This is of importance for baryogenesis considerations, where the necessary $B - L$ violation should therefore arise from other sources, e.g. such as soft supersymmetry breaking terms.

4. Gauge Invariant Inflaton

Let us recapitulate the main features of MSSM flat direction inflation [44, 45, 48]. The framework is solely based on MSSM together with gravity, so consistency dictates that all non-renormalizable terms allowed by gauge symmetry and supersymmetry should be included below the cut-off scale, which we take to be the Planck scale.
The superpotential term which lifts the $F$-flatness is given by:

$$W_{non} = \sum_{n>3} \frac{\lambda_n}{n} \frac{\Phi^n}{M_{n-3}},$$  

(4.1)

where $\Phi$ is a *gauge invariant* superfield which contains the flat direction. Within MSSM all the flat directions are lifted by non-renormalizable operators with $4 \leq n \leq 9$ [66], where $n$ depends on the flat direction. We expect that quantum gravity effects yield $M = M_P = 2.4 \times 10^{18}$ GeV and $\lambda_n \sim \mathcal{O}(1)$ [63, 65]. Note however that our results will be valid for any values of $\lambda_n$, because rescaling $\lambda_n$ simply shifts the VEV of the flat direction. Let us focus on the lowest order superpotential term in Eq. (4.1) which lifts the flat direction. Soft SUSY breaking induces a mass term for $\phi$ and an $A$-term so that the scalar potential along the flat direction reads

$$V = \frac{1}{2} m_\phi^2 \phi^2 + A \cos(n\theta + \theta_A) \frac{\lambda_n \phi^n}{n M_{n-3}} + \lambda_n^2 \frac{\phi^{2(n-1)}}{M_P^{2(n-3)}},$$  

(4.2)

Here $\phi$ and $\theta$ denote respectively the radial and the angular coordinates of the complex scalar field $\Phi = \phi \exp[i\theta]$, while $\theta_A$ is the phase of the $A$-term (thus $A$ is a positive quantity with dimension of mass). Note that the first and third terms in Eq. (4.2) are positive definite, while the $A$-term leads to a negative contribution along the directions whenever $\cos(n\theta + \theta_A) < 0$, see [1] 5. There are other attempts to embed inflation within a gauge theory [70], they however do not explain SM reheating, besides they also have some caveats related to Planckian VEVs.

### 4.1 Inflation near the Saddle Point

The maximum impact from the $A$-term is obtained when $\cos(n\theta + \theta_A) = -1$ (which occurs for $n$ values of $\theta$). Along these directions $V$ has a secondary minimum at $\phi = \phi_0 \sim (m_\phi M_{n-3}^{n-3})^{1/(n-2)} \ll M_P$ (the global minimum is at $\phi = 0$), provided that

$$A^2 \geq 8(n-1)m_\phi^2.$$  

(4.3)

At this minimum the curvature of the potential is positive both along the radial and angular directions. If the $A$ is too large, the secondary minimum will be deeper than the one in the origin, and hence becomes the true minimum. However, this

5The importance of the $A$-term was first highlighted in a successful MSSM curvaton model [63], where again the curvaton carries the SM charges. By implementing so it also leads to a successful CMB prediction, SM reheating and a detectable signature at the LHC.
is phenomenologically unacceptable as such a minimum will break charge and/or color. With a total potential: \( V \sim m_\phi^2 \phi_0^2 \sim m_\phi^2 (m_\phi M_p^{n-3})^{2/(n-2)}. \)

As discussed in [14], if the local minimum is too steep, the field will become trapped there with an ensuing inflation that has no graceful exit like in the old inflation scenario [1]. On the other hand in an opposite limit, with a point of inflection, a single flat direction cannot support inflation [39], one would require an assisted inflation with the help of many flat directions [14].

However, in the gravity mediated SUSY breaking case, the \( A \)-term and the soft SUSY breaking mass terms are expected to be the same order of magnitude as the gravitino mass, i.e.

\[ m_\phi \sim A \sim m_{3/2} \sim \mathcal{O}(1) \text{ TeV}. \] (4.4)

Therefore, as pointed out in [14], in the gravity mediated SUSY breaking it is possible that the potential barrier actually disappears and the inequality in Eq. (4.5) is saturated so that \( A \) and \( m_\phi \) are related by

\[ A^2 = 8(n-1)m_\phi^2. \] (4.5)

If the above condition is satisfied then both the first and second derivatives of \( V \) vanish at \( \phi_0 \), i.e. \( V'(\phi_0) = 0, V''(\phi_0) = 0 \). As the result, if initially \( \phi \sim \phi_0 \), a slow roll phase of inflation is driven by the third derivative of the potential.

Note that this behavior does not seem possible for other SUSY breaking scenarios such as the gauge mediated breaking [71] or split SUSY [72]. In split SUSY the \( A \)-term is protected by an \( R \)-symmetry, which also keeps the gauginos light while the sfermions are quite heavy [72].

4.2 Slow roll

The potential near the saddle point Eq. (4.3) is very flat along the real direction but not along the imaginary direction. Along the imaginary direction the curvature is determined by \( m_\phi \). Around \( \phi_0 \) the field lies in a plateau with a potential energy

\[ V(\phi_0) = \frac{(n-2)^2}{2n(n-1)} m_\phi^2 \phi_0^2 \] (4.6)

\[^6\text{In the gauge mediated case there is an inherent mismatch between } A \text{ and } m_\phi, \text{ except at very large field values where Eq. (4.4) can be satisfied. However there exists an unique possibility of a saddle point inflation within gauge mediated case which we will discuss in Section 9.2 [47].}\]
with
\[ \phi_0 = \left( \frac{m_\phi M_P^{n-3}}{\lambda_n \sqrt{2n-2}} \right)^{1/(n-2)}. \] (4.7)
This results in Hubble expansion rate during inflation which is given by
\[ H_{\text{inf}} = \frac{(n-2) m_\phi \phi_0}{\sqrt{6n(n-1)} M_P}. \] (4.8)

When \( \phi \) is very close to \( \phi_0 \), the first derivative is extremely small. The field is effectively in a de Sitter background, and we are in self-reproduction (or \textit{eternal inflation}) regime where the two point correlation function for the flat direction fluctuation grows with time. But eventually classical friction wins and slow roll begins at \( \phi \approx \phi_{\text{self}} \) \([44, 45]\)
\[ (\phi_0 - \phi_{\text{self}}) \simeq \left( \frac{m_\phi \phi_0^2}{M_P^3} \right)^{1/2} \phi_0. \] (4.9)

The regime of \textit{eternal inflation} plays an important role in addressing the initial condition problem, see section 10.

The observationally relevant perturbations are generated when \( \phi \approx \phi_{\text{COBE}} \). The number of e-foldings between \( \phi_{\text{COBE}} \) and \( \phi_{\text{end}} \), denoted by \( N_{\text{COBE}} \) follows from Eq. (10.16)
\[ N_{\text{COBE}} \simeq \frac{\phi_0^3}{2n(n-1)M_P^2(\phi_0 - \phi_{\text{COBE}})}. \] (4.10)

The amplitude of perturbations thus produced is given by \([45]\)
\[ \delta_H \equiv \frac{1}{5\pi} \frac{H_{\text{inf}}^2}{\phi} \simeq \frac{1}{5\pi} \sqrt{\frac{2}{3} n(n-1)(n-2)} \left( \frac{m_\phi M_P}{\phi_0^3} \right) N_{\text{COBE}}^2. \] (4.11)
and the spectral tilt of the power spectrum and its running are found to be \([44, 45]\)
\[ n_s = 1 + 2\eta - 6\epsilon \simeq 1 - \frac{4}{N_{\text{COBE}}}, \] (4.12)
\[ \frac{dn_s}{d\ln k} = - \frac{4}{N_{\text{COBE}}^2}. \] (4.13)

4.3 \textbf{Inflaton Properties and predictions}

4.3.1 \textbf{Inflaton candidates}

As discussed in \([44, 45]\), among nearly 300 flat directions there are two that can lead to a successful inflation along the lines discussed above.

One is \textit{udd} which, up to an overall phase factor, is parameterized by
\[ u_1^a = \frac{1}{\sqrt{3}} \phi, \quad d_1^a = \frac{1}{\sqrt{3}} \phi, \quad d_2^a = \frac{1}{\sqrt{3}} \phi. \] (4.14)
Here $1 \leq \alpha, \beta, \gamma \leq 3$ are color indices, and $1 \leq i, j, k \leq 3$ denote the quark families. The flatness constraints require that $\alpha \neq \beta \neq \gamma$ and $j \neq k$.

The other direction is $LLe$ \footnote{When the flat direction develops a VEV during inflation, it spontaneously breaks $SU(2) \times U(1)_y$, which gives masses to the corresponding gauge bosons. It is possible to obtain a seed perturbations for the primordial magnetic field in this case, see \cite{73}.}, parameterized by (again up to an overall phase factor)

\[
L_i^a = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad L_j^b = \frac{1}{\sqrt{3}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad e_k = \frac{1}{\sqrt{3}} \phi,
\]

\begin{equation}
(4.15)
\end{equation}

where $1 \leq a, b \leq 2$ are the weak isospin indices and $1 \leq i, j, k \leq 3$ denote the lepton families. The flatness constraints require that $a \neq b$ and $i \neq j \neq k$. Both these flat directions are lifted by $n = 6$ non-renormalizable operators \footnote{Since $LLe$ are $uddq$ are independently $D$- and $F$-flat, inflation could take place along any of them but also, at least in principle, simultaneously. The dynamics of multiple flat directions are however quite involved \cite{67}.},

\[
W_6 \supset \frac{1}{M_\text{P}^3}(LLe)(LLe), \quad W_6 \supset \frac{1}{M_\text{P}^3}(udd)(udd).
\]

\begin{equation}
(4.16)
\end{equation}

The reason for choosing either of these two flat directions\footnote{Since $LLe$ are $uddq$ are independently $D$- and $F$-flat, inflation could take place along any of them but also, at least in principle, simultaneously. The dynamics of multiple flat directions are however quite involved \cite{67}.} is twofold: (i) a non-trivial $A$-term arises, at the lowest order, only at $n = 6$; and (ii) we wish to obtain the correct COBE normalization of the CMB spectrum.

Those MSSM flat directions which are lifted by operators with dimension $n = 7, 9$ are such that the superpotential term contains at least two monomials, i.e. is of the type

\[
W \sim \frac{1}{M_\text{P}^{n-3}}\Psi \Phi^{n-1}.
\]

\begin{equation}
(4.17)
\end{equation}

If $\phi$ represents the flat direction, then its VEV induces a large effective mass term for $\psi$, through Yukawa couplings, so that $\langle \psi \rangle = 0$. Hence Eq. \((4.17)\) does not contribute to the $A$-term.

More importantly, as we will see, all other flat directions except those lifted by $n = 6$ fail to yield the right amplitude for the density perturbations. Indeed, as can be seen in Eq. \((3.6)\), the value of $\phi_0$, and hence also the energy density, depend on $n$.

### 4.3.2 Inflaton Predictions

According to the arguments presented above, successful MSSM flat direction inflation has the following model parameters:

\[
m_\phi \sim 1 - 10 \text{ TeV}, \quad n = 6, \quad A = \sqrt{40} m_\phi, \quad \lambda \sim \mathcal{O}(1).
\]

\begin{equation}
(4.18)
\end{equation}
Here we assume that $\lambda$ (we drop the subscript "6") is of order one, which is the most natural assumption when $M = M_P$.

The Hubble expansion rate during inflation and the VEV of the saddle point are\(^9\)

$$H_{\text{inf}} \sim 1 - 10 \text{ GeV}, \quad \phi_0 \sim (1 - 3) \times 10^{14} \text{ GeV}.$$  \hspace{1cm} (4.19)

Note that both the scales are sub-Planckian. The total energy density stored in the inflaton potential is $V_0 \sim 10^{36} - 10^{38} \text{ GeV}^4$. The fact that $\phi_0$ is sub-Planckian guarantees that the inflationary potential is free from the uncertainties about physics at super-Planckian VEVs. The total number of e-foldings during the slow roll evolution is large enough to dilute any dangerous relic away \cite{15},

$$N_{\text{tot}} \sim 10^3,$$  \hspace{1cm} (4.20)

At such low scales as in MSSM inflation the number of e-foldings, $N_{\text{COBE}}$, required for the observationally relevant perturbations, is much less than 60 \cite{74, 75}. If the inflaton decays immediately after the end of inflation, we obtain $N_{\text{COBE}} \sim 50$. Despite the low scale, the flat direction can generate adequate density perturbations as required to explain the COBE normalization. This is due to the extreme flatness of the potential (recall that $V' = 0$), which causes the velocity of the rolling flat direction to be extremely small. From Eq. (4.11) we find an amplitude of

$$\delta_H \simeq 1.91 \times 10^{-5}.$$  \hspace{1cm} (4.21)

There is a constraint on the mass of the flat direction from the amplitude of the CMB anisotropy:

$$m_\phi \simeq (100 \text{ GeV}) \times \lambda^{-1} \left( \frac{N_{\text{COBE}}}{50} \right)^{-4}.$$  \hspace{1cm} (4.22)

We get a lower limit on the mass parameter when $\lambda \leq 1$. For smaller values of $\lambda \ll 1$, the mass of the flat direction must be larger. Note that the above bound on the inflaton mass arises at high scales, i.e. $\phi = \phi_0$. However, through renormalization group flow, it is connected to the low scale mass, as will be discussed in Sect. 4.

The spectral tilt of the power spectrum is not negligible because, although the first slow roll parameter is $\epsilon \sim 1/N_{\text{COBE}}^4 \ll 1$, the other slow roll parameter is given

\(^9\)We note that $H_{\text{inf}}$ and $\phi_0$ depend very mildly on $\lambda$ as they are both $\propto \lambda^{-1/4}$.  

by $\eta = -2/N_{\text{COBE}}$ and thus, see Eq. (4.12)\textsuperscript{10}

$$n_s \sim 0.92 ,$$ \hspace{1cm} (4.23)

$$\frac{dn_s}{d \ln k} \sim -0.002 ,$$ \hspace{1cm} (4.24)

where we have taken $N_{\text{COBE}} \sim 50$ (which is the maximum value allowed for the scale of inflation in our model). In the absence of tensor modes, this agrees with the current WMAP 3-years’ data within $2\sigma$ \textsuperscript{[10]}. Note that MSSM inflation does not produce any large stochastic gravitational wave background during inflation. Gravity waves depend on the Hubble expansion rate, and in our case the energy density stored in MSSM inflation is very small.

### 4.4 Departure from the saddle point

Inflation can still happen for small deviations from the saddle point condition Eq. (4.5). To quantify this, we define a parameter $\alpha^2$ such that \textsuperscript{[45, 50]}:

$$A^2 \equiv \frac{8}{3(n-1)m_\phi^2} 1 + \left(\frac{n-2}{2}\right)^2 \alpha^2 .$$ \hspace{1cm} (4.25)

For $\alpha^2 \neq 0$, the saddle point becomes a point of inflection where $V''(\phi_0) = 0$, and

$$V'(\phi_0) = \left(\frac{n-2}{2}\right)^2 \alpha^2 m_\phi^2 \phi_0 .$$ \hspace{1cm} (4.26)

If $\alpha^2 < 0$, the potential has a local minimum and a maximum. In this case the flat direction is trapped in the local minimum. It will eventually tunnel past the maximum and a period of slow roll inflation will follow \textsuperscript{[13]}. If $\alpha^2 > 0$, the potential has no maximum or local minimum, and then slow roll inflation occurs around $\phi_0$.

For $\alpha^2 \neq 0$ the expressions for $n_s$ and $\delta_H$ are modified as \textsuperscript{[76]} (see also \textsuperscript{[50]})

$$\delta_H = \frac{1}{5\pi} \sqrt{\frac{2}{3} n(n-1)(n-2) \frac{m_\phi M_P}{\phi_0^2} \frac{1}{\Delta^2} \sin^2[N_{\text{COBE}}\sqrt{\Delta^2}]} ,$$ \hspace{1cm} (4.27)

and

$$n_s = 1 - 4\sqrt{\Delta^2} \cot[N_{\text{COBE}}\sqrt{\Delta^2}] ,$$ \hspace{1cm} (4.28)

where

$$\Delta^2 \equiv n^2(n-1)^2 \alpha^2 N_{\text{COBE}}^2 \left(\frac{M_P}{\phi_0}\right)^4 .$$ \hspace{1cm} (4.29)

\textsuperscript{10}Obtaining $n_s > 0.92$ (or $n_s < 0.92$, which is however outside the $2\sigma$ allowed region) requires deviation from the saddle point condition in Eq. (4.5); see the next subsection. For a more detailed discussion on the spectral tilt, see also Refs. \textsuperscript{[76], [14]}.\textsuperscript{10}
Figure 2: $n_s$ is plotted as a function of $\Delta^2$ for different values of $m_\phi$. $\Delta$ is defined in the text. We choose $\lambda=1$.

Note that for for $\alpha^2 = 0$, Eqs. (4.27,4.28) are reduced to (4.11,4.12) respectively. For $\alpha^2 < 0$, the spectral index will be smaller than that in Eq. (4.12), thus outside the 2$\sigma$ region from observations. The more interesting case, as pointed out in [50], happens for $\alpha^2 > 0$. We can in this case get all values within the allowed range $0.92 \leq n_s \leq 1$ [11] for

$$0 \leq \Delta^2 \leq \frac{\pi^2}{4N_{\text{COBE}}^2}. \quad (4.30)$$

The inflaton mass, $m_\phi$, is constrained by the experimental data on the spectral index $n_s$ [10, 11] and $\delta H$ [77].

We first find the solutions of $m_\phi$ by solving Eqs. (4.27,4.28). $n_s$ depends mainly on $\Delta^2$ and is mostly independent of $m_\phi$ and $\lambda$ (the coupling in Eq. (1)). The parameter $\Delta^2$ is defined in Eq. (4.30). We therefore solve $\Delta^2$ from Eq. (4.28) and apply this solution to determine the bounds on $m_\phi$ from the Eq. (4.27). In figure 3, we show $n_s$ as a function of $\Delta^2$. The range for $\Delta^2$ is determined from Eq. (4.30).

In figure 3, we show $\delta_H$ as a function of $n_s$ for different values of $m_\phi$. The blue band shows the experimentally allowed region. We find that smaller values of $m_\phi$ are preferred for smaller values of $n_s$. We also find that the allowed range of $m_\phi$ is $75 - 440$ GeV for the experimental ranges of $n_s$ and $\delta_H$. We assume $\lambda \sim 1$ for these two figures. If $\lambda$ is less than $\mathcal{O}(1)$, e.g., $\lambda \sim 0.1$ or so (which can occur in $SO(10)$ model), it will lead to an increase in $m_\phi$. We will need to study these allowed ranges of the inflaton mass in the mSUGRA model. Since the inflaton mass is related to the
Figure 3: $\delta_H$ is plotted as a function of $\Delta^2$ for different values of $m_\phi$. We used $\lambda=1$. The blue band denotes the experimentally allowed values of $\delta_H$.

parameters of the mSUGRA model, the main question is whether the allowed range of the inflaton mass is consistent with the experimentally allowed mSUGRA model or not. See section 7.

5. Radiative and supergravity corrections

Since the MSSM inflaton candidates are represented by gauge invariant combinations which are not singlets. The inflaton parameters receive corrections from gauge interactions which, unlike in models with a gauge singlet inflaton, can be computed in a straightforward way. Quantum corrections result in a logarithmic running of the soft supersymmetry breaking parameters $m_\phi$ and $A$. In this section we will discuss running of the potential with VEV-dependent values of $m_\phi(\phi)$ and $A(\phi)$ in Eq. (4.5).
Our conclusion is that the running of the gauge couplings do not spoil the existence of a saddle point. However the VEV of the saddle point is now displaced; by how much will depend precisely on the inflaton candidate. In order to discuss the situation, we derive a general expression for the one-loop effective potential for the flat directions, and then focus on the $LLe$ direction, for which the system of Renormalization Group (RG) equations can be solved analytically \footnote{add case follows similar discussion but requires numerics to solve the equations.}.

### 5.1 One-loop effective potential

The first thing to check is whether the radiative corrections remove the saddle point altogether. The object of interest is the effective potential at the phase minimum $n\theta_{\text{min}} = \pi$, for which we obtain \footnote{add case follows similar discussion but requires numerics to solve the equations.}

\[ V_{\text{eff}}(\phi, \theta_{\text{min}}) = \frac{1}{2} m_0^2 \phi^2 \left[ 1 + K_1 \log \left( \frac{\phi^2}{\mu_0^2} \right) \right] - \frac{\lambda_{n,0} A_0}{n M^{n-3}} \phi^n \left[ 1 + K_2 \log \left( \frac{\phi^2}{\mu_0^2} \right) \right] + \frac{\lambda_{n,0}^2}{M^{2(n-3)}} \phi^{2(n-1)} \left[ 1 + K_3 \log \left( \frac{\phi^2}{\mu_0^2} \right) \right]. \tag{5.1} \]

where $m_0$, $A_0$, and $\lambda_{n,0}$ are the values of $m_\phi$, $A$ and $\lambda_n$ given at a scale $\mu_0$. Here $A_0$ is chosen to be real and positive (this can always be done by re-parameterizing the phase of the complex scalar field $\phi$), and $|K_i| < 1$ are coefficients determined by the one-loop renormalization group equations.

Our aim is to find a saddle point of this effective potential, so we calculate the 1st and 2nd derivatives of the potential and set them to zero. This is a straightforward although somewhat cumbersome exercise that results in the expression \footnote{add case follows similar discussion but requires numerics to solve the equations.}

\[ \phi_0^{n-2} = \frac{M^{n-3}}{4 \lambda_n (n - 1 + K_3)} \left[ A \left( 1 + \frac{2}{n} K_2 \right) \pm \sqrt{A^2 \left( 1 + \frac{2}{n} K_2 \right)^2 - 8 m_\phi^2 (1 + K_1)(n - 1 + K_3)} \right], \tag{5.2} \]

where $m_\phi$, $A$, and $\lambda_n$ are values of the parameters at the scale $\phi_0$. Inserting this into
$V_{\phi\phi} = 0$, we can then find the condition to have a saddle point at $\phi_0$:

$$A^2 = 2m_\phi^2(n-1+K_3)F_1F_2F_3$$

$$F_1 = \left[\frac{1+K_1}{n+1+K_3}\left((n-1)(2n-3)+(4n-5)K_3\right) - 1 - 3K_1\right]^2,$$

$$F_2 = \left[(1+K_1)\left(n-1+2\frac{2n-1}{n}K_2\right) - (1+3K_1)\left(1+\frac{2}{n}K_2\right)\right]^{-1},$$

$$F_3 = \left[\frac{1+2nK_2}{n+1+K_3}\left((n-1)(2n-3)+(4n-5)K_3\right) - \left(n-1+2\frac{2n-1}{n}K_2\right)\right]^{-1}.$$  

(5.3)

In the limit when $|K_i| \ll 1$, this mercifully simplifies to

$$A^2 = 8(n-1)m_\phi^2(\phi_0) \left(1 + K_1 - \frac{4}{n}K_2 + \frac{1}{n-1}K_3\right),$$

$$\phi_0^{-2} = \frac{M^{n-3}m_\phi(\phi_0)}{\lambda_n\sqrt{2(n-1)}} \left(1 + K_1 - \frac{1}{2(n-1)}K_3\right).$$

(5.4)  

(5.5)

Note that Eqs. (5.3, 5.4) give the necessary relations between the values of $m_\phi$ and $A$ as calculated at the saddle point. The coefficients $K_i$ need to be solved from the renormalization group equations at the scale given by the saddle point $\mu = \phi_0$. Since $K_i$ are already one loop corrections, taking the tree-level saddle point value as the renormalization scale is sufficient.

The conclusion is robust, although the soft terms and the value of the saddle point are all affected by radiative corrections, they do not remove the saddle point nor shift it to unreasonable values. The existence of a saddle point is thus insensitive to radiative corrections.

5.2 RG equations for the $LLe$ direction

The form of the relevant RG equations depend on the flat direction. RG equations for $LLe$ are simpler since only the $SU(2)_W \times U(1)_Y$ gauge interactions are involved and the lepton Yukawa couplings are negligible. The case of $udd$ requires numerics if $u$ is chosen from the third family. For other choices, however, it closely resembles $LLe$. For $LLe$ the one-loop RG equations governing the running of $m_\phi^2,$ $A$, and $\lambda$
with the scale $\mu$ are given by\cite{56}

\[
\begin{align*}
\mu \frac{d m_\phi^2}{d\mu} &= -\frac{1}{6\pi^2} \left( \frac{3}{2} \tilde{m}_1^2 g_2^2 + \frac{3}{2} \tilde{m}_1^2 g_1^2 \right), \\
\mu \frac{d A}{d\mu} &= -\frac{1}{2\pi^2} \left( \frac{3}{2} \tilde{m}_1^2 g_2^2 + \frac{3}{2} \tilde{m}_1^2 g_1^2 \right), \\
\mu \frac{d \lambda}{d\mu} &= -\frac{1}{4\pi^2} \lambda \left( \frac{3}{2} g_2^2 + \frac{3}{2} g_1^2 \right),
\end{align*}
\] (5.6)

Here $\tilde{m}_1$, $\tilde{m}_2$ denote the mass of the $U(1)_Y$ and $SU(2)_W$ gauginos respectively and $g_1$, $g_2$ are the associated gauge couplings. It is a straightforward exercise to obtain the equations that govern the running of $\lambda$ and $A$ associated with the $(LLe)^2$ superpotential term (which lifts the $LLe$ flat direction). Note that $L$ has the same quantum numbers as $H_d$, and hence in this respect $LLe$ combination behaves just like $H_dLe$. One can then use the familiar RG equations that govern the Yukawa coupling and $A$-term associated with the $H_dLe$ superpotential term\cite{56}. However, as explained in\cite{78}, the coefficients of the terms on the right-hand side are proportional to the number of superfields contained in a superpotential term.\textsuperscript{12} Hence the second and third equations in (5.6) are simply obtained from those for the $H_dLe$ term after multiplying by a factor of 2. The first equation in (5.6) is also easily found by taking the electroweak charges of $L_i$, $L_j$ and $e$ superfields into account while taking into account that $m_\phi^2 = (m_{L_i}^2 + m_{L_j}^2 + m_e^2)/3$.

The running of gauge couplings and gaugino masses obey the usual equations\cite{56}:

\[
\begin{align*}
\mu \frac{d g_1}{d\mu} &= \frac{11}{16\pi^2} g_1^3, \\
\mu \frac{d g_2}{d\mu} &= \frac{1}{16\pi^2} g_2^3, \\
\frac{d}{d\mu} \left( \frac{\tilde{m}_1}{g_1^2} \right) &= \frac{d}{d\mu} \left( \frac{\tilde{m}_2}{g_2^2} \right) = 0.
\end{align*}
\] (5.7)

\textsuperscript{12}We would like to thank Manuel Drees for explaining this point to us.
The solutions of the RG equations are\(^\text{[45]}\)

\[
g_i = \frac{g_i(\mu_0)}{\sqrt{1 - b_i g_i(\mu_0)^2 \ln \frac{\mu}{\mu_0}}},
\]

\[
\tilde{m}_i = \tilde{m}_i(\mu_0) \left( \frac{g_i}{g_i(\mu_0)} \right)^2,
\]

\[
m^2_\phi = m^2_\phi(\mu_0) + \tilde{m}_2^2(\mu_0) - \tilde{m}_1^2 + \frac{1}{11} (\tilde{m}_1^2(\mu_0) - \tilde{m}_1^2),
\]

\[
A = A(\mu_0) + 6 (\tilde{m}_2(\mu_0) - \tilde{m}_1) + \frac{6}{11} (\tilde{m}_1(\mu_0) - \tilde{m}_1),
\]

\[
\lambda = \lambda(\mu_0) \left( \frac{g_2(\mu_0)}{g_2} \right)^6 \left( \frac{g_1(\mu_0)}{g_1} \right)^{\frac{\xi}{\pi}},
\]

where \(i = 1, 2\), \(b_1 = 11/8\pi^2\) and \(b_2 = 1/8\pi^2\). Ignoring the running of the gaugino masses and gauge couplings, we find that\(^{[45]}\)

\[
K_1 \approx -\frac{1}{4\pi^2} \left[ \left( \frac{\tilde{m}_2}{m_\phi} \right)^2 g_2^2 + \left( \frac{\tilde{m}_1}{m_\phi} \right)^2 g_1^2 \right],
\]

\[
K_2 \approx -\frac{3}{4\pi^2} \left[ \left( \frac{\tilde{m}_2}{A_0} \right) g_2^2 + \left( \frac{\tilde{m}_1}{A_0} \right) g_1^2 \right],
\]

\[
K_3 \approx -\frac{3}{8\pi^2} \lambda_0 \left[ g_2^2 + g_1^2 \right],
\]

where the subscript 0 denotes the values of parameters at the high scale \(\mu_0\).

For universal boundary conditions, as in minimal grand unified supergravity, the high scale is the GUT scale \(\mu_X \approx 3 \times 10^{16} \text{ GeV}\), \(\tilde{m}_1(\mu_X) = \tilde{m}_2(\mu_X) = \tilde{m}\) and \(g_1 = \sqrt{\pi/10} \approx 0.56\), \(g_2 = \sqrt{\pi/6} \approx 0.72\). Then we just use RG equations to run the coupling constants and masses to the scale of the saddle point \(\mu_0 = \phi_0 \approx 2.6 \times 10^{14} \text{ GeV}\) for \(M_P = 2.4 \times 10^{18} \text{ GeV}\), \(m_\phi = 1 \text{ TeV}\), \(\lambda_0 = 1\). With these values we obtain\(^{[45]}\)

\[
K_1 \approx -0.017 \xi^2,
\]

\[
K_2 \approx -0.0085 \xi,
\]

\[
K_3 \approx -0.029,
\]

where \(\xi = \tilde{m}/m_\phi\) is calculated at the GUT scale.

Typically the running based on gaugino loops alone results in negative values of \(K_i\). Positive values can be obtained when one includes the Yukawa couplings, practically the top Yukawa, but the order of magnitude remains the same.

Thus radiative corrections modify \(\alpha\) and we need to fine tune the potential to a few (but not all) orders in perturbation theory.
Figure 4: The running of $m_{\phi}^2$ for the $LLe$ inflaton when the saddle point is at $\phi_0 = 2.6 \times 10^{14}$GeV (corresponding to $n = 6$, $m_\phi = 1$ TeV and $\lambda = 1$). The three curves correspond to different values of the ratio of gaugino mass to flat direction mass at the GUT scale: $\xi = 2$ (dashed), $\xi = 1$ (solid) and $\xi = 0.5$ (dash-dot).

5.3 $A_6$ vs. $A_3$

One final comment is in order before closing this Section. Unlike $m_\phi$, there is no prospect of measuring the $A$ term, because it is related to the non-renormalizable interactions which are suppressed by $M_P$. However, a knowledge of supersymmetry breaking sector and its communication with the observable sector may help to link the non-renormalizable $A$-term under consideration to the renormalizable ones.

To elucidate this, let us consider the Polonyi model where a general $A$-term at a tree level is given by

$$m_{3/2}[(a-3)W + \phi(dW/d\phi)],$$

with $a = 3 - \sqrt{3}$ [56]. One then finds a relationship between $A$-terms corresponding to $n = 6$ and $n = 3$ superpotential terms, denoted by $A_6$ and $A_3$ respectively, at high scales:

$$A_6 = \frac{3 - \sqrt{3}}{6 - \sqrt{3}} A_3. \quad (5.17)$$
One can then use relevant RG equations to relate $A_6$ which is relevant for inflation, to $A_3$ at the weak scale, which can be constrained and/or measured. In principle this can also be done in general, provided that we have sufficient information about the supersymmetry breaking sector and its communication with the MSSM sector, see some related discussions in [81].

5.4 Supergravity corrections

SUGRA corrections often destroy the slow roll predictions of inflationary potentials; this is the notorious SUGRA-$\eta$ problem [80]. In general, the effective potential depends on the Kähler potential $K$ as $V \sim \left(e^{K(\phi^*,\phi)/M_P^2}V(\phi)\right)$ so that there is a generic SUGRA contribution to the flat direction potential of the type

$$V(\phi) = H^2 M_P^2 f \left(\frac{\phi}{M_P}\right),$$

where $f$ is some function (typically a polynomial). Such a contribution usually gives rise to a Hubble induced correction to the mass of the flat direction with an unknown coefficient, which depends on the nature of the Kähler potential 13.

Let us compare the non-gravitational contribution, Eq. (9.17), to that of Hubble induced contribution, Eq. (5.18). Writing $f \sim (\phi/M_P)^p$ where $p \geq 1$ is some power, we see that non-gravitational part dominates whenever

$$H_{\text{inf}}^2 M_P^2 \left(\frac{\phi}{M_P}\right)^p \ll m_\phi^2 \phi_0^2,$$

so that the SUGRA corrections are negligible as long as $\phi_0 \ll M_P$, as is the case here (note that $H_{\text{inf}} M_P \sim m_\phi \phi_0$). The absence of SUGRA corrections is a generic property of this model. Note also that although non-trivial Kähler potentials give rise to non-canonical kinetic terms of squarks and sleptons, it is a trivial exercise to show that at sufficiently low scales, $H_{\text{inf}} \ll m_\phi$, and small VEVs, they can be rotated to a canonical form without affecting the potential 14.

13If the Kähler potential has a shift symmetry, then at tree level there is no Hubble induced correction. However, at one-loop level relatively small Hubble induced corrections can be induced [82, 83].

14The same reason, i.e. $H_{\text{inf}} \ll m_\phi$ also precludes any large Trans-Planckian correction. Any such correction would generically go as $(H_{\text{inf}}/M_*)^2 \ll 1$, where $M_*$ is the scale at which one would expect Trans-Planckian effects to kick in.
6. Reheating and Thermalization

After the end of inflation, the flat direction starts rolling towards its global minimum. At this stage the dominant term in the scalar potential will be: $m_\phi \phi^2/2$. Since the frequency of oscillations is $\omega \sim m_\phi \sim 10^3 H_{\text{inf}}$, the flat direction oscillates a large number of times within the first Hubble time after the end of inflation. Hence the effect of expansion is negligible.

We recall that the curvature of the potential along the angular direction is much larger than $H_{\text{inf}}^2$. Therefore, the flat direction has settled at one of the minima along the angular direction during inflation from which it cannot be displaced by quantum fluctuations. This implies that no torque will be exerted, and hence the flat direction motion will be one dimensional, i.e. along the radial direction.

Flat direction oscillations excite those MSSM degrees of freedom which are coupled to it. The inflaton, either $LLe$ or $udd$ flat direction, is a linear combination of slepton or squark fields. Therefore inflaton has gauge couplings to the gauge/gaugino fields and Yukawa couplings to the Higgs/Higgsino fields. As we will see particles with a larger couplings are produced more copiously during inflaton oscillations. Therefore we focus on the production of gauge fields and gauginos. Keep in mind that the VEV of the MSSM flat direction breaks the gauge symmetry spontaneously, for instance $udd$ breaks $SU(3)_C \times U(1)_Y$ while $LLe$ breaks $SU(2)_W \times U(1)_Y$, therefore, induces a supersymmetry conserving mass $\sim g \langle \phi(t) \rangle$ to the gauge/gaugino fields in a similar way as the Higgs mechanism, where $g$ is a gauge coupling. When the flat direction goes to its minimum, $\langle \phi(t) \rangle = 0$, the gauge symmetry is restored. In this respect the origin is a point of enhanced symmetry \cite{84}.

There can be various phases of particle creation in this model, here we briefly summarize them below. Let us elucidate the physics, by considering the case when $LLe$ flat direction is the inflaton \footnote{Reheating happens quickly due to a flat direction motion which is \textit{strictly} one dimensional in our case. Our case is really exceptional, usually, the flat direction motion is restricted to a plane, which precludes preheating all together, for instance see \cite{85, 86, 87}.}

- Tachyonic preheating:

Right after the end of inflation, when we are close to the saddle point, the second derivative is negative. One might suspect that this would trigger tachyonic instability in the inflaton fluctuations which will then excite the inflaton couplings to matter \cite{88, 89}.  

\footnote{Reheating happens quickly due to a flat direction motion which is \textit{strictly} one dimensional in our case. Our case is really exceptional, usually, the flat direction motion is restricted to a plane, which precludes preheating all together, for instance see \cite{85, 86, 87}.}
However the situation is different in our case. As mentioned, only inflaton fluctuations with a physical momentum $k \lesssim m_{\phi}$ will have a tachyonic instability. Moreover $V'' < 0$ only at field values which are $\sim \phi_0$. Tachyonic effects are therefore expected be negligible since, unlike the case in [88], the homogeneous mode has a VEV which is hierarchically larger than $m_{\phi}$ (we remind that $\phi_0 \geq 10^{14}$ GeV) and oscillates at a frequency $\omega \sim m_{\phi}$. Further note fields which are coupled to the inflaton acquire a very large mass $\sim \hbar \phi_0$ from the homogeneous piece which suppresses non-perturbative production of their quanta at large inflaton VEVs. We conclude that tachyonic effects, although genuinely present, do not lead to significant particle production in our case.

• Instant preheating:
An efficient bout of particle creation occurs when the inflaton crosses the origin, which happens twice in every oscillation. The reason is that fields which are coupled to the inflaton are massless near the point of enhanced symmetry. Mainly electroweak gauge fields and gauginos are then created as they have the largest coupling to the flat direction. The production takes place in a short interval, $\Delta t \sim (gm_{\phi}\phi_0)^{-1/2}$, where $\phi_0 \sim 10^{14}$ GeV is the initial amplitude of the inflaton oscillation, during which quanta with a physical momentum $k \lesssim (gm_{\phi}\phi_0)^{1/2}$ are produced. The number density of gauge/gaugino degrees of freedom is given by [90], see also [91]

\[
ng \approx \frac{(gm_{\phi}\phi_0)^{3/2}}{8\pi^3}.
\]

(6.1)

As the inflaton VEV is rolling back to its maximum value $\phi_0$, the mass of the produced quanta $g\langle \phi(t) \rangle$ increases. The gauge and gaugino fields can (perturbatively) decay to the fields which are not coupled to the inflaton, for instance to (s)quarks. Note that (s)quarks are not coupled to the flat direction, hence they remain massless throughout the oscillations. The total decay rate of the gauge/gaugino fields is then given by $\Gamma = C (g^2/48\pi) g\phi$, where $C \sim \mathcal{O}(10)$ is a numerical factor counting for the multiplicity of final states.

The decay of the gauge/gauginos become efficient when [15]

\[
\langle \phi \rangle \simeq \left(\frac{48\pi m_{\phi}\phi_0}{Cg^3}\right)^{1/2}.
\]

(6.2)

Here we have used $\langle \phi(t) \rangle \approx \phi_0 m_{\phi}t$, which is valid when $m_{\phi}t \ll 1$, and $\Gamma \simeq t^{-1}$, where $t$ represents the time that has elapsed from the moment that the
inflaton crossed the origin. Note that the decay is very quick compared with the frequency of inflaton oscillations, i.e. \( \Gamma \gg m_\phi \). It produces relativistic (s)quarks with an energy \( E \):

\[
E = \frac{1}{2} g \phi(t) \simeq \left( \frac{48 \pi m_\phi \phi_0}{C g} \right)^{1/2}.
\] (6.3)

The ratio of energy density in relativistic particles thus produced \( \rho_{rel} \) with respect to the total energy density \( \rho_0 \) follows from Eqs. (6.1,6.3):

\[
\frac{\rho_{rel}}{\rho_0} \sim 10^{-2} g,
\] (6.4)

where we have used \( C \sim \mathcal{O}(10) \). This implies that a fraction \( \sim \mathcal{O}(10^{-2}) \) of the inflaton energy density is transferred into relativistic (s)quarks every time that the inflaton passes through the origin. This is so-called instant preheating mechanism \[92\] \[16\]. It is quite an efficient mechanism in our model as it can convert almost all of the energy density in the inflaton into radiation within a Hubble time (note that \( H^{-1}_{\text{inf}} \sim 10^3 m_\phi^{-1} \)).

### 6.1 Towards thermal equilibrium

A full thermal equilibrium is reached when 

1. \textit{kinetic} and
2. \textit{chemical} equilibrium

are established. The maximum (hypothetical) temperature attained by the plasma would be given by:

\[
T_{\text{max}} \sim V^{1/4} \sim (m_\phi \phi_0)^{1/2} \geq 10^9 \text{ GeV}.
\] (6.5)

This temperature may be too high and could lead to thermal overproduction of gravitinos \[96, 97\]. However the dominant source of gravitino production in a thermal bath is scattering which include an on-shell gluon or gluino leg. In the next subsection we describe a natural solution to this problem and show that the final reheat temperature is actually well below Eq. (9.37), i.e. \( T_R \ll T_{\text{max}} \).

One comment is in order before closing this subsection. The gravitinos can also be created non-perturbatively during inflaton oscillations, both of the helicity \( \pm 3/2 \) \[98\] and helicity \( \pm 1/2 \) states \[99\]. In models of high scale inflation (i.e. \( H_{\text{inf}} \gg m_{3/2} \)) helicity \( \pm 1/2 \) states can be produced very efficiently (and much more copiously than helicity \( \pm 3/2 \) states). At the time of production these states mainly consist of

\[16\] In a favorable condition the flat direction VEV coupled very weakly to the flat direction inflaton could also enhance the perturbative decay rate of the inflaton \[93\].
the inflatino (inflaton’s superpartner). However these fermions also decay in the form of inflatino, which is coupled to matter with a strength which is equal to that of the inflaton. Therefore, they inevitably decay at a similar rate as that of inflaton, and hence pose no threat to primordial nucleosynthesis [100].

In the present case \( m_\phi \sim m_{3/2} \gg H_{\text{inf}} \). Therefore low energy supersymmetry breaking is dominant during inflation, and hence helicity \( \pm 1/2 \) states of the gravitino are not related to the inflatino (which is a linear combination of leptons or quarks) at any moment of time. As a result helicity \( \pm 1/2 \) and \( \pm 3/2 \) states are excited equally, and their abundances are suppressed due to kinematical phase factor. Moreover there will be no dangerous gravitino production from perturbative decay of the inflaton quanta [94, 95, 101]. The reason is that the inflaton is not a gauge singlet and has gauge strength couplings to other MSSM fields. This makes the \( \text{inflaton} \rightarrow \text{inflatino} + \text{gravitino} \) decay mode totally irrelevant.

6.2 Solution to the gravitino problem

In order to suppress thermal gravitino production it is sufficient to make gluon and gluino fields heavy enough such that they are not kinematically accessible to the reheated plasma, even if other degrees of freedom reach full equilibrium (for a detailed discussion on thermalization in supersymmetric models and its implications, see [94, 84]). This suggests a natural solution to the thermal gravitino problem in the case of our model. Consider another flat direction with a non-zero VEV, denoted by \( \varphi \), which spontaneously breaks the \( SU(3)_C \) group. For example, if \( LLe \) is the inflaton, then \( udd \) provides a unique candidate which can simultaneously develop VEV. The induced mass for gluon/gluino fields will be:

\[
m_G \sim g \langle \varphi(t) \rangle < g \varphi_0 .
\]

The inequality arises due to the fact that the VEV of \( \varphi \) cannot exceed that of the inflaton \( \phi \) since its energy density should be subdominant to the inflaton energy density.

If \( g \varphi_0 \gg T_{\text{max}} \) the gluon/gluino fields will be too heavy and not kinematically accessible to the reheated plasma. Here \( \varphi_0 \) is the VEV of \( udd \) at the beginning of inflaton oscillations. In a radiation-dominated Universe the Hubble expansion

\[\text{To develop and maintain such a large VEV, it is not necessary that } udd \text{ potential has a saddle point as well. It can be trapped in a false minimum during inflation, which will then be lifted by thermal corrections when the inflaton decays (as discussed in the previous subsection) [13],}\]
redshifts the flat direction VEV as $\langle \varphi \rangle \propto H^{3/4}$, which is a faster rate than the change in the temperature $T \propto H^{1/2}$. Once $g\langle \varphi \rangle \simeq T$, gluon/gluino fields come into equilibrium with the thermal bath. As pointed out in Refs. [94, 84, 45], if the initial VEV of $udd$ is

$$\varphi_0 > 10^{10} \text{ GeV},$$

(6.7)

then the temperature at which gluon/gluino become kinematically accessible, i.e. $g\langle \varphi \rangle \simeq T$, is given by \[18:\]

$$T_R \leq 10^7 \text{ GeV}.$$  

(6.8)

This is the final reheat temperature at which gluons and gluinos are all in thermal equilibrium with the other degrees of freedom. The standard calculation of thermal gravitino production via scatterings can then be used for $T \leq T_R$. Note however that $T_R$ is sufficiently low to avoid thermal overproduction of gravitinos.

Finally, we also make a comment on the cosmological moduli problem. The moduli are generically displaced from their true minimum if their mass is less than the expansion rate during inflation. The moduli obtain a mass $\sim \mathcal{O}(\text{TeV})$ from supersymmetry breaking. They start oscillating with a large amplitude, possibly as big as $M_P$, when the Hubble parameter drops below their mass. Since moduli are only gravitationally coupled to other fields, their oscillations dominate the Universe while they decay very late. The resulting reheat temperature is below MeV, and is too low to yield a successful primordial nucleosynthesis.

However, in our case $H_{\text{inf}} \ll \text{TeV}$. This implies that quantum fluctuations cannot displace the moduli from their true minima during the inflationary epoch driven by MSSM flat directions. Moreover, any oscillations of the moduli will be exponentially damped during the inflationary epoch. Therefore our model is free from the infamous moduli problem [15].

7. Cold dark matter and MSSM inflation

Since $m_{\phi}$ is related to the scalar masses, sleptons ($LLe$ direction) and squarks ($udd$ direction), the bound on $m_{\phi}$ will be translated into the bounds on these scalar masses which are expressed in terms of the model parameters [15].

The models of mSUGRA depend only on four parameters and one sign. These are $m_0$ (the universal scalar soft breaking mass at the GUT scale $M_G$); $m_{1/2}$ (the

\[18\]

Note that the conditions in Eqs. (6.6, 6.7) can be simultaneously satisfied easily.
universal gaugino soft breaking mass at $M_G$; $A_0$ (the universal trilinear soft breaking mass at $M_G$)\textsuperscript{19}; $\tan \beta = \langle H_2 \rangle \langle H_1 \rangle$ at the electroweak scale (where $H_2$ gives rise to $u$ quark masses and $H_1$ to $d$ quark and lepton masses); and the sign of $\mu$, the Higgs mixing parameter in the superpotential ($W_\mu = \mu H_1 H_2$). Unification of gauge couplings within supersymmetry suggests that $M_G \simeq 2 \times 10^{16}$ GeV. The model parameters are already significantly constrained by different experimental results. Most important constraints are:

- The light Higgs mass bound of $M_{h^0} > 114.0$ GeV from LEP \textsuperscript{64}.
- The $b \to s\gamma$ branching ratio \textsuperscript{102}: $2.2 \times 10^{-4} < B(B \to X_s \gamma) < 4.5 \times 10^{-4}$.
- In mSUGRA the $\tilde{\chi}_1^0$ is the candidate for CDM. The $2\sigma$ bound from the WMAP \textsuperscript{10} gives a relic density bound for CDM to be $0.095 < \Omega_{CDM} h^2 < 0.129$.
- The bound on the lightest chargino mass of $M_{\tilde{\chi}_1^\pm} > 104$ GeV from LEP \textsuperscript{103}.
- The possible $3.3 \sigma$ deviation (using $e^+e^-$ data to calculate the leading order hadronic contribution) from the SM expectation of the anomalous muon magnetic moment from the muon $g - 2$ collaboration \textsuperscript{104}.

The allowed mSUGRA parameter space, at present, has mostly three distinct regions: (i) the stau-neutralino ($\tilde{\tau}_1 - \tilde{\chi}^1_0$), coannihilation region where $\tilde{\chi}^1_0$ is the lightest SUSY particle (LSP), (ii) the $\tilde{\chi}^0_1$ having a dominant Higgsino component (focus point) and (iii) the scalar Higgs ($A^0, H^0$) annihilation funnel ($2M_{\tilde{\chi}^0_1} \simeq M_{A^0, H^0}$). These three regions have been selected out by the CDM constraint. There stills exists a bulk region where none of these above properties is observed, but this region is now very small due to the existence of other experimental bounds. After considering all these bounds we will show that there exists an interesting overlap between the constraints from inflation and the CDM abundance \textsuperscript{115}.

We calculate $m_\phi$ at $\phi_0$ and $\phi_0$ is $10^{14}$ GeV which is two orders of magnitude below the GUT scale. From this $m_\phi$, we determine $m_0$ and $m_{1/2}$ by solving the RGEs

\textsuperscript{19}The relationship between the two $A$ terms, the trilinear, $A_0$ and the non-renormalizable $A$ term in Eq. (9.17) can be related to each other, however, that depends on the SUSY breaking sector. For a Polonyi model, they are given by: $A = (3 - \sqrt{3})/(6 - \sqrt{3})A_0$ \textsuperscript{45}. 

Figure 5: The contours for different values of $n_s$ and $\delta_H$ are shown in the $m_0 - m_{1/2}$ plane for $\tan \beta = 10$. We used $\lambda = 1$ for the contours. We show the dark matter allowed region narrow blue corridor, $(g-2)_{\mu}$ region (light blue) for $a_\mu \leq 11 \times 10^{-8}$, Higgs mass $\leq 114$ GeV (pink region) and LEPII bounds on SUSY masses (red). We also show the the dark matter detection rate by vertical blue lines.

for fixed values of $A_0$ and $\tan \beta$. The RGEs for $m_\phi$ are

$$
\frac{dm_\phi^2}{d\mu} = -\frac{1}{6\pi^2} \left( \frac{3}{2} M_2^2 g_2^2 + \frac{9}{10} M_1^2 g_1^2 \right), \quad \text{(for LLe)}
$$

$$
\frac{dm_\phi^2}{d\mu} = -\frac{1}{6\pi^2} \left( 4 M_3^2 g_3^2 + \frac{2}{5} M_1^2 g_1^2 \right), \quad \text{(for udd)}. \quad (7.1)
$$

$M_1$, $M_2$ and $M_3$ are $U(1)$, $SU(2)$ and $SU(3)$ gaugino masses respectively.

After we determine $m_0$ and $m_{1/2}$ from $m_\phi$, we can determine the allowed values of $m_\phi$ from the experimental bounds on the mSUGRA parameters space. In order to obtain the constraint on the mSUGRA parameter space, we calculate the SUSY particle masses by solving the RGEs at the weak scale using four parameters of the mSUGRA model and then use these masses to calculate Higgs mass, $BR[b \to s\gamma]$, dark matter content etc.
Figure 6: The contours for different values of $n_s$ and $\delta H$ are shown in the $m_0 - m_{1/2}$ plane for $\tan \beta = 40$. We used $\lambda = 1$ for the contours. We show the dark matter allowed region, narrow blue corridor, $(g-2)_\mu$ region (light blue) for $a_\mu \leq 11 \times 10^{-8}$, $b \rightarrow s\gamma$ allowed region (brick) and LEPII bounds on SUSY masses (red).

We show that the mSUGRA parameter space in figures 5, 6 for $\tan \beta = 10$ and 40 with the $udd$ flat direction using $\lambda = 1$ \footnote{We have a similar figure for the flat direction $LLe$ which we do not show in this paper. All the figures are for $udd$ flat direction as an inflaton.}. In the figures, we show contours correspond to $n_s = 1$ for the maximum value of $\delta_H = 2.03 \times 10^{-5}$ (at $2\sigma$ level) and $n_s = 1.0, 0.98, 0.96$ for $\delta_H = 1.91 \times 10^{-5}$. The constraints on the parameter space arising from the inflation appearing to be consistent with the constraints arising from the dark matter content of the universe and other experimental results. We find that $\tan \beta$ needs to be smaller to allow for smaller values of $n_s < 1$. It is also interesting to note that the allowed region of $m_\phi$, as required by the inflation data for $\lambda = 1$ lies in the stau-neutralino coannihilation region which requires smaller values of the SUSY particle masses. The SUSY particles in this parameter space are, therefore, within the reach of the LHC very quickly. The detection of the region at the LHC has been
Figure 7: The contours for different values of $n_s$ and $\delta_H$ are shown in the $m_0 - m_{1/2}$ plane for $\tan \beta = 10$. We used $\lambda = 0.1$ for the contours. We show the dark matter allowed region narrow blue corridor, g-2 region (light blue) for $a_\mu \leq 11 \times 10^{-10}$, Higgs mass $\leq 114$ GeV (pink region) and LEPII bounds on SUSY masses (red). The black region is not allowed by radiative electroweak symmetry breaking. We use $m_t = 172.7$ GeV for this graph.

considered in refs [105]. From the figures, one can also find that as $\tan \beta$ increases, the inflation data along with the dark matter, rare decay and Higgs mass constraint allow smaller ranges of $m_{1/2}$. For example, the allowed ranges of gluino masses are 765 GeV-2.1 TeV and 900 GeV-1.7 TeV for $\tan \beta = 10$ and 40 respectively [48].

So far we have chosen $\lambda = 1$. Now if $\lambda$ is small e.g., $\lambda \lesssim 10^{-1}$, we find that the allowed values of $m_\phi$ to be large. In this case the dark matter allowed region requires the lightest neutralino to have larger Higgsino component in the mSUGRA model. As we will see shortly, this small value of $\lambda$ is accommodated in $SO(10)$ type model. In figure 7, we show $n_s = 1, 0.98$ contours for $\delta_H = 1.91 \times 10^{-5}$ in the mSUGRA parameter space for $\tan \beta = 10$. In this figure, we find that $n_s$ can not smaller than 0.97, but if we lower $\lambda$ which will demand larger $m_\phi$ and therefore $n_s$ can be lowered.
In figure 8, we show the contours of $\lambda$ for different values of $m_\phi$ which are allowed by $n_s$ and $\delta_H = 1.91 \times 10^{-3}$. The blue bands show the dark matter allowed regions for $\tan \beta = 10$. The band on the left is due to the stau-neutralino coannihilation region allowed by other constraints and the allowed values of $\lambda$ are $0.3-1$. The first two generation squarks masses are 690 GeV and 1.9 TeV for the minimum and maximum values of $m_\phi$ allowed by the dark matter and other constraints. The gluino masses for these are 765 GeV and 2.1 TeV respectively. The band is slightly curved due to the shifting of $\phi_0$ as a function $\lambda$. (We solve for SUSY parameters from the inflaton mass at $\phi_0$). The band on the right which continues beyond the plotting range of the figure 8 is due to the Higgsino dominated dark matter. We find that $\lambda$ is mostly $\leq 0.1$ in this region and $m_\phi > 1.9$ TeV. In this case the squark masses are much larger than the gluino mass since $m_0$ is much larger than $m_{1/2}$.
8. Grand unified Models and Inclusion of Right-Handed Neutrinos

8.1 Embedding MSSM inflation in $SU(5)$ or $SO(10)$ GUT

As we have pointed out, mSUGRA makes a mild assumption that there exists a GUT physics which encompasses MSSM beyond the unification scale $M_G$. Here we wish to understand how such embedding would affect inflationary scenario, for instance, would it be possible to single out either $LLe$ or $udd$ as a candidate for the MSSM inflaton.

The lowest order non-renormalizable superpotential terms which lift $LLe$ and $udd$ are (see Eq. (9.1)):

$$\frac{(LLe)^2}{M_P^3}, \frac{(udd)^2}{M_P^3}.$$  \hspace{1cm} (8.1)

It is generically believed that gravity breaks global symmetries. Then all gauge invariant terms which are $M_P$ suppressed should appear with $\lambda \sim \mathcal{O}(1)$. Obviously the above terms in Eq. (8.1) are invariant under the SM. Once the SM is embedded within a GUT at the scale $M_G$, where gauge couplings are unified, the gauge group will be enlarged. Then the question arises whether such terms in Eq. (8.1) are invariant under the GUT gauge group or not. Note that a GUT singlet is also a singlet under the SM, however, the vice versa is not correct. To answer this question, let us consider $SU(5)$ and $SO(10)$ models separately.

• $SU(5)$:

We briefly recollect representations of matter fields in this case: $L$ and $d$ belong to $\bar{5}$, while $e$ and $u$ belong to $10$ of $SU(5)$ group. Thus under $SU(5)$ the superpotential terms in Eq. (8.1) read

$$\bar{5} \times \bar{5} \times 10 \times \bar{5} \times \bar{5} \times 10 \over M_P^3.$$ \hspace{1cm} (8.2)

This product clearly includes a $SU(5)$ singlet. Therefore in the case of $SU(5)$, we expect that $M_P$ suppressed terms as in Eq. (9.1) appear with $\lambda \sim \mathcal{O}(1)$.

\footnote{We remind the readers that inflation occurs around a flat direction VEV $\phi_0 \sim 10^{14}$ GeV. Since $\phi_0 \ll M_G$, heavy GUT degrees of freedom play no role in the dynamics of MSSM inflation, and hence they can be ignored.}

\footnote{If we were to obtain the $(LLe)^2$ term by integrating out the heavy fields of the $SU(5)$ GUT, then $\lambda = 0$. This is due to the fact that $SU(5)$ preserves $B - L$.}
SO(10):

In this case all matter fields of one generation are included in the spinorial representation 16 of SO(10). Hence the superpotential terms in Eq. (8.1) are \([16]^6\) under SO(10), which does not provide a singlet. A gauge invariant operator will be obtained by multiplying with a 126-plet Higgs. This implies that in SO(10) the lowest order gauge invariant superpotential term with 6 matter fields arises at \(n = 7\) level:

\[
\frac{16 \times 16 \times 16 \times 16 \times 16 \times 126_H}{M_P^4}. \tag{8.3}
\]

Once 126\(_H\) acquires a VEV, SO(10) can break down to a lower ranked subgroup, for instance SU(5). This will induce an effective \(n = 6\) non-renormalizable term as in Eq. (9.1) with \(\lambda \sim \langle 126_H \rangle / M_P \sim O(M_{\text{GUT}}/M_P)\). \(\tag{8.4}\)

Hence, in the case of SO(10), we can expect \(\lambda \sim O(10^{-2} - 10^{-1})\) depending on the scale where SO(10) gets broken.

We conclude that embedding MSSM in SO(10) naturally implies \(\lambda \ll 1\). Hence an experimental confirmation of the focus point region may be considered as an indication for SO(10). More precise determination of the spectral index \(n_s\) from future experiments (such as PLANCK) can in addition shed light on the scale of SO(10) breaking. Smaller values of \(n_s\) (within the range \(0.92 \leq n_s \leq 1\)) point to smaller \(\lambda\), as can be seen from figure 6. This, according to Eq. (8.4), implies a scale of SO(10) breaking, i.e. \(\langle 126_H \rangle\), which is closer to the GUT scale.

Further note that embedding the MSSM within SO(10) also provides an advantage for obtaining a right handed neutrino.

8.2 Including Right-Handed Majorana Neutrinos

Eventually one would need to supplement MSSM with additional ingredients to explain the tiny neutrino masses. Here we consider the most popular framework; the see-saw mechanism which invokes MSSM plus three RH (s)neutrinos \(N_1, N_2, N_3\) with respective Majorana masses \(M_i\). By adding new superfields to MSSM, one can write a larger number of non-renormalizable gauge-invariant terms of the form in Eq. (9.1). As a result, a given flat direction might be lifted at a different superpotential level. Then a natural question arises that whether/how adding new superfields will affect the inflaton candidates, i.e. \(LLe\) and \(udd\) flat directions.
Since, $N_i$, $1 \leq i \leq 3$, are SM singlets, we can write the following $n = 4$ superpotential terms:

$$\frac{N_i L L e}{M_P} , \quad \frac{N_i u d d}{M_P}. \quad (8.5)$$

Note that these terms are also singlet under $SU(5)$ and $SO(10)$. In the case of $SU(5)$, the terms in Eq. (8.5) read $\bar{5} \times \bar{5} \times 10 \times 1$, which includes a singlet. While in the case of $SO(10)$, since $N$ belongs to the $16$, the terms in Eq. (8.5) read $16 \times 16 \times 16 \times 16$, which includes a singlet. Hence both terms in Eq. (8.5) are allowed in $SU(5)$ or $SO(10)$ embedding of MSSM as well.

We now analyze the case for two flat directions separately.

- **LL$e$**:
  
  First let us consider the $L L e$ flat direction. Taking into account of the family indices, there are 5 independent $D$-flat directions as such $[10]$. Within MSSM, there are three directions which are $F$-flat at the $n = 3$ level, one of which survives until $n = 6$. However the term in Eq. (8.5) leads to three additional $F$-term constraints $F_{N_i} = 0$, which are more than sufficient to lift the remaining direction at the $n = 4$ superpotential level.

  Generically in this case we would expect $L L e$ to be lifted by a non-renormalizable operator $n < 6$.

- **u$dd$**:
  
  Next consider the $u d d$ direction. With family indices taken into account, there are 9 independent $D$-flat directions as such $[66]$. Within MSSM, 3 directions are lifted by $n = 4$ terms $u u d e / M_P$, while the remaining 6 will be lifted at the $n = 6$ level. Note that the superpotential term in Eq. (8.5) lead to three $F$-term constraints at the $n = 4$ level. Nevertheless, 3 directions will still survive until $n = 6$.

Based on the above analysis, if we include the RH neutrinos, we conclude that $u d d$ direction is a more promising inflaton candidate than $L L e$. The reason is that the flatness of the former will not be lifted in the presence of physically motivated right handed neutrino fields in addition to that of the MSSM fields.

---

23In the case of $SO(10)$ one can naturally obtain a right-handed neutrino.

24The gauge invariant $L L e$ direction will survive until $n = 6$ if all $M_i \gg \phi_0$. However this is not a phenomenologically viable situation.
9. Few more examples

Within MSSM there are other interesting possibilities of inflation which we will discuss in this section. In the first section we will discuss inflation with Dirac neutrinos.

9.1 Inflation with Dirac neutrinos

As we know by now that the inflaton potential has to be cosmologically flat, which is suggestive of either a symmetry or a small coupling, or both. We will see that this property of the inflaton may be related to the smallness of neutrino masses. Identifying such a connection could have important ramifications and could lead to a more fundamental theory. The model we will use as an example will contain nothing but the MSSM and the right-handed neutrinos. We will show that a viable inflation in this model favors the correct scale for the neutrino masses.

Let us consider the MSSM with three additional fields, namely the right-handed (RH) neutrino supermultiplets. The relevant part of the superpotential is

\[ W = W_{\text{MSSM}} + hN\mathcal{H}_L. \]  \hspace{1cm} (9.1)

Here N, L and \( \mathcal{H}_L \) are superfields containing the RH neutrinos, left-handed (LH) leptons and the Higgs which gives mass to the up-type quarks, respectively. For conciseness we have omitted the generation indices. We note that the RH (s)neutrinos are singlets under the standard model (SM) gauge group. However in many extensions of the SM they can transform non-trivially under the action of a larger gauge group. The simplest example is extending the SM gauge group to \( SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_{B−L} \), which is a subgroup of \( SO(10) \). Here B and L denote the baryon and lepton numbers, respectively. This is the model we consider here. In particular, the \( U(1)_{B−L} \) prohibits the RH Majorana masses\(^{25}\).

The active neutrino masses that arise from this are given by the usual seesaw relation \( h^2 \langle H_u \rangle^2 / M \) \cite{106, 107}, where \( \langle H_u \rangle \) is the Higgs vacuum expectation value (VEV). Although the seesaw mechanism allows for small neutrino masses in the presence of large Yukawa couplings, it does not require the Yukawa couplings to be of order one. It still allows one to choose between the large Yukawa couplings and large Majorana masses on the one hand, and the small Yukawas, small Majorana masses,

\(^{25}\)The monomials with \( B−L = 0 \) will be also \( D \)-flat under \( U(1)_{B−L} \), while those with \( B−L \neq 0 \) must be multiplied by an appropriate number of N superfields. In particular, \( N\mathcal{H}_L \) is now a \( D \)-flat direction.
on the other hand. Viable models for neutrino mass matrices have been constructed in both limits, including the low-scale seesaw models \cite{108, 109}, in which the Yukawa couplings are typically of the order of

\[ h \sim 10^{-12}, \]  

(9.2)
or the same order of magnitude, as it would have in the case of Dirac neutrinos in order to explain the mass scale \( \sim O(0.1 \text{ eV}) \) corresponding to the atmospheric neutrino oscillations detected by Super-Kamiokande experiment.

Let us now work in the basis where neutrino masses are diagonalized. There is a flat direction \( N_3 H_u L_3 \) spanned by the VEV of the lower and upper weak isospin components of \( H_u \) and \( L_3 \), respectively. The scalar field corresponding to the flat direction is denoted by

\[ \phi = \tilde{N}_3 + H_u^2 + \tilde{L}_3^1 / \sqrt{3}, \]  

(9.3)

where the superscripts refer to the weak isospin components. One must now include the soft SUSY breaking terms, such as the mass terms and the \( A \)-term. The \( A \)-terms are known to play an important role in Affleck-Dine baryogenesis \cite{65}, as well as in the inflation models based on supersymmetry \cite{63}.

The potential along the flat direction is found to be

\[ V(\phi) = \frac{m_\phi^2}{2} \phi^2 + \frac{h_3^2}{12} \phi^4 + \frac{A h_3}{6\sqrt{3}} \cos(\theta + \theta_h + \theta_A) \phi^3, \]  

(9.4)

where the flat direction mass is given in terms of the soft masses of \( \tilde{N}_3, \ H_u, \) and \( \tilde{L}_3 \):

\[ m_\phi^2 = \left( m_{\tilde{N}_3}^2 + m_{H_u}^2 + m_{\tilde{L}_3}^2 \right) / 3. \]  

Here we have used the radial and angular components of the flat direction \( \phi_R + i \phi_I = \sqrt{2} \phi \exp(i\theta) \), and \( \theta_h, \ \theta_A \) are the phases of the Yukawa coupling \( h_3 \) and the \( A \)-term, respectively. We note that the above potential does not contain any non-renormalizable term at all.

The last term on the right-hand side of eq. (9.4) is minimized when \( \cos(\theta + \theta_h + \theta_A) = -1 \). Along this direction, \( V(\phi) \) has the global minimum at \( \phi = 0 \) and a local minimum at \( \phi_0 \sim m_\phi / h_3 \), as long as

\[ 4m_\phi \leq A \leq 3\sqrt{2}m_\phi. \]  

(9.5)

When the inequality in eq. (9.3) is saturated, i.e., when \( A = 4m_\phi \), then both first and second derivatives of \( V \) vanish at \( \phi_0 \), \( V'(\phi_0) = V''(\phi_0) = 0 \), and the potential becomes extremely flat in the radial direction, see Fig. 9. We note that individually
Figure 9: The inflaton potential. The potential is flat near the saddle point where inflation occurs.

None of the terms in eq. (9.4) could have driven a successful inflation at VEVs lower than $M_P = 2.4 \times 10^{18}$ GeV. However the combined effect of all the terms leads to a successful inflation without the graceful exit problem.

Around $\phi_0$ the field is stuck in a plateau with potential energy

$$ V(\phi_0) = \frac{m_\phi^4}{4h_3^2}, \quad \phi_0 = \sqrt{3} \frac{m_\phi}{h_3}. $$

The first and second derivatives of the potential vanish, while the third derivative does not. Around $\phi = \phi_0$ one can expand the potential as

$$ V(\phi) = V(\phi_0) + \left(\frac{1}{3!}\right)V'''(\phi_0)(\phi - \phi_0)^3, $$

where

$$ V'''(\phi_0) = 2 \sqrt{3} h_3 m_\phi. $$

Hence, in the range $[\phi_0 - \Delta \phi, \phi_0 + \Delta \phi]$, where $\Delta \phi \sim H_{\text{inf}}^2 / V'''(\phi_0) \sim (\phi_0^3 / M_P^2) \gg H_{\text{inf}}$, the potential is flat along the real direction of the inflaton. Inflation occurs along this flat direction.

If the initial conditions are such that the flat direction starts in the vicinity of $\phi_0$ with $\dot{\phi} \approx 0$, then a sufficiently large number of e-foldings of inflation can be generated. Around the saddle point, due to the random fluctuations of the massless
field, the quantum diffusion is stronger than the classical force, $H_{\text{inf}}/2\pi > \dot{\phi}/H_{\text{inf}}$, for
\[
\frac{(\phi_0 - \phi)}{\phi_0} \lesssim \left( \frac{m_\phi \phi_0^2}{M_P^2} \right)^{1/2} = \left( \frac{3m_\phi^3}{h_3^2 M_P^2} \right)^{1/2}.
\] (9.8)

At later times, the evolution is determined by the usual slow roll. The equation of motion for the $\phi$ field in the slow-roll approximation is $3H\dot{\phi} = -(1/2)V''(\phi_0)(\phi - \phi_0)^2$.

A rough estimate of the number of e-foldings is then given by
\[
N_e(\phi) = \int \frac{H d\phi}{\dot{\phi}} \simeq \left( \frac{m_\phi}{2h_3 M_P} \right)^2 \frac{\phi_0}{(\phi_0 - \phi)},
\] (9.9)
where we have assumed $V'(\phi) \sim (\phi - \phi_0)^2 V''(\phi_0)$ (this is justified since $V'(\phi_0)$ and $V''(\phi_0)$ are both small). We note that the initial displacement from $\phi_0$ cannot be much smaller than $H_{\text{inf}}$, due to the uncertainty from quantum fluctuations.

Inflation ends when the slow roll parameters become $\sim 1$. It turns out that $|\eta| \sim 1$ gives the dominant condition
\[
\frac{(\phi_0 - \phi)}{\phi_0} \sim \frac{\sqrt{3}m_\phi}{24h_3^2 M_P^2}.
\] (9.10)

The total number of e-foldings can be computed as [44, 45]:
\[
N_e \sim \left( \frac{\phi_0^2}{m_\phi M_P} \right)^{1/2} = \left( \frac{3m_\phi}{h_3^2 M_P} \right)^{1/2},
\] (9.11)
evaluated after the end of diffusion, see eq. (9.8), when the slow-roll regime is achieved.

Let us now consider adiabatic density perturbations. As in Ref. [44, 45, 46], one finds
\[
\delta_H \simeq \frac{1}{5\pi} \frac{H_{\text{inf}}^2}{\dot{\phi}} \sim \frac{h_3^2 M_P}{3m_\phi} N_{\text{COBE}}^2.
\] (9.12)

In the above expression we have used the slow roll approximation $\dot{\phi} \simeq -V'''(\phi_0)(\phi_0 - \phi)^2/3H_{\text{inf}}$, and eq. (10.16). The exact number depends on the scale of inflation and on when the Universe becomes radiation dominated (we note that full thermalization is not necessary as it is the relativistic equation of state which matters). In our case $N_{\text{COBE}} < 60$ as we shall see below.

The spectral tilt of the power spectrum and its running are
\[
n_s = 1 + 2\eta - 6\epsilon \simeq 1 - \frac{4}{N_{\text{COBE}}},
\] (9.13)
\[
\frac{d n_s}{d \ln k} = -\frac{4}{N_{\text{COBE}}^2},
\] (9.14)
cf. \cite{14}. (We note that $\epsilon \ll 1$ while $\eta = -2/\mathcal{N}_{\text{COBE}}$.)

It is a remarkable feature of the model that for the weak-scale supersymmetry and for the correct value of the Yukawa coupling, namely,

$$
m_\phi \simeq 100 \text{ GeV} - 10 \text{ TeV}, \quad h_3 \sim 10^{-12},
$$  \hspace{1cm} (9.15)

the flat direction $N_3H_uL_3$ leads to a successful low scale inflation near $\phi_0 \sim (10^{14} - 10^{15}) \text{ GeV} \ll M_P$, with

$$
V \sim 10^{32} - 10^{36} \text{ GeV}^4, \quad H_{\text{inf}} \sim 10 \text{ MeV} - 1 \text{ GeV},
$$

$$
\mathcal{N}_e \sim 10^3, \quad T_{\text{max}} \sim 10^8 - 10^9 \text{ GeV}.
$$  \hspace{1cm} (9.16)

The total number of e-foldings driven by the slow roll inflation, $\mathcal{N}_e \sim 10^3$, is more than sufficient to produce a patch of the Universe with no dangerous relics. Those domains that are initially closer to $\phi_0$ enter self-reproduction in eternal inflation. Since the inflaton, $N_3H_uL_3$, couples directly to MSSM particles, after inflation the field oscillates and decays to relativistic MSSM degrees of freedom. The highest temperature during reheating is $T_{\text{max}} \sim V^{1/4}$. This temperature determines the total number of e-foldings required for the relevant perturbations to leave the Hubble radius during inflation; in our case it is roughly $\mathcal{N}_{\text{COBE}} \sim 50$.

Despite the low scale of inflation, the flat direction can generate density perturbations of the correct size for the parameters listed above. Indeed, from eqs. (9.26), (9.29), and (9.15), we obtain: $\delta_H \sim 10^{-5}$. Following the discussion of Section 4.4, we obtain $0.91 \leq n_s \leq 1.0$ with a negligible running. The spectral tilt agrees with the current WMAP 3-years’ data within $2\sigma$ \cite{10}. The tensor modes are negligible because of the low scale of inflation.

We emphasize that the VEV of the flat direction is related to the Yukawa coupling that can generate the Dirac neutrino mass $\sim 0.1$ eV. The scale of the neutrino mass appears to be just right to get the correct amplitude in the CMB perturbations.

The inflaton has gauge couplings to the electroweak and $U(1)_{B-L}$ gauge/gaugino fields. It therefore induces a VEV-dependent mass $\sim g\langle \phi \rangle$ for these fields ($g$ denotes a typical gauge coupling). After the end of inflation, $\phi$ starts oscillating around the global minimum at the origin with a frequency $m_\phi \sim 10^3 H_{\text{inf}}$, see eq. (9.16). When the inflaton passes through the minimum, $\langle \phi \rangle = 0$, the induced mass undergoes non-adiabatic time variation. This results in non-perturbative particle production \cite{30}. 

---
9.2 Inflation in gauge mediated scenarios

In a Gauge Mediated Supersymmetry Breaking (GMSB) scenario the two-loop correction to the flat direction potential results in a logarithmic term above the messenger scale, i.e. $\phi > M_S$ [10]. Together with the $A$-term this leads to the scalar potential

$$V = M_F^4 \ln \left( \frac{\phi^2}{M_S^2} \right) + A \cos(n\theta + \theta_A) \frac{\lambda_n \phi^n}{n M_{\text{GUT}}^{n-3}} + \lambda_n^2 \frac{\phi^{2(n-1)}}{M_{\text{GUT}}^{2(n-3)}},$$

(9.17)

where $M_F \sim (m_{\text{SUSY}} \times M_S)^{1/2}$ and $m_{\text{SUSY}} \sim 1$ TeV is the soft SUSY breaking mass at the weak scale. For $\phi > M_F^2/m_{3/2}$, usually the gravity mediated contribution, $m_{3/2}^2 \phi^2$, dominates the potential where $m_{3/2}$ is the gravitino mass. Here we will concentrate on the VEVs $M_s \ll \phi \ll M_F^2/m_{3/2}$.

Although individual terms are unable to support a sub-Planckian VEV inflation, but as shown in Refs. [44, 46, 45, 50], a successful inflation can be obtained near the saddle point, which we find by solving, $V'(\phi_0) = V''(\phi_0) = 0$ (where derivative is w.r.t $\phi$).

$$\phi_0 = \left( \frac{M_{\text{GUT}}^{n-3} M_F^2}{\lambda_n} \right)^{1/(n-1)} \sqrt{\frac{n}{(n-1)(n-2)}},$$

(9.18)

$$A = \frac{4(n-1)^2 \lambda_n n M_{\text{GUT}}^{n-3}}{M_{\text{GUT}}^{n-3}} \phi_0^{n-2}.$$  

(9.19)

In the vicinity of the saddle point, we obtain the total energy density and the third derivative of the potential to be [47]:

$$V(\phi_0) = M_F^4 \left[ \ln \left( \frac{\phi_0^2}{M_S^2} \right) - \frac{3n - 2}{n(n-1)} \right],$$

(9.20)

$$V'''(\phi_0) = 4n(n-1)M_F^4 \phi_0^{-3}.$$  

(9.21)

There are couple of interesting points, first of all note that the scale of inflation is extremely low in our case, barring some small coefficients of order one, the Hubble scale during inflation is given by:

$$H_{\text{inf}} \sim M_F^2 / M_P \sim 10^{-3} - 10^{-1} \text{ eV},$$

(9.22)

for $M_F \sim 1 - 10$ TeV. For such a low scale inflation usually it is extremely hard to obtain the right phenomenology. But there are obvious advantages of having a low scale inflation, $M_F \gg H_{\text{inf}}$. The supergravity corrections and the Trans-Planckian corrections are all negligible [47], therefore the model predictions are trustworthy.
Perturbations which are relevant for the COBE normalization are generated a number \( N_{\text{COBE}} \) e-foldings before the end of inflation. The value of \( N_{\text{COBE}} \) depends on thermal history of the universe and the total energy density stored in the inflaton, which in our case is bounded by \( V_0 \leq 10^{16} \) (GeV)\(^4\). The required number of e-foldings yields in our case, \( N_{\text{COBE}} \sim 40 \) [75], provided the universe thermalizes within one Hubble time. Although within SUSY thermalization time scale is typically very long [84], however, in this particular case it is possible to obtain a rapid thermalization.

Near the vicinity of the saddle point, \( \phi_0 \), the potential is extremely flat and one enters a regime of self-reproduction [6]. The self-reproduction regime lasts as long as the quantum diffusion is stronger than the classical drag; \( H_{\text{inf}}/2\pi > \dot{\phi}/H_{\text{inf}} \), for \( \phi_s \leq \phi \leq \phi_0 \), where \( \phi_0 - \phi_s \simeq M_F (\phi_0/M_P)^{3/2} \). From then on, the evolution is governed by the classical slow roll. Inflation ends when \( \epsilon \sim 1 \), which happens at \( \phi \simeq \phi_e \), where [47]

\[
\phi_e - \phi_0 = -\sqrt{\frac{V_0 \phi_0^3}{2n(n-1)M_F^4 M_P^4}}. \tag{9.23}
\]

Assuming that the classical motion is due to the third derivative of the potential, \( V' \simeq (1/2) V''(\phi_0)(\phi - \phi_0)^2 \), the total number of e-foldings during the slow roll period is found to be [17]:

\[
N_{\text{tot}} = \int \frac{H d\phi}{\dot{\phi}} \simeq \frac{2V_0 \phi_0^3}{4n(n-1)M_F^4 M_P^2} \left( \frac{1}{\phi_0 - \phi_s} \right). \tag{9.24}
\]

This simplifies to [17]

\[
N_{\text{tot}} \simeq \frac{\phi_0^{3/2}}{(M_P^{1/2} M_F)}. \tag{9.25}
\]

Let us now consider the adiabatic density perturbations. Despite \( H_{\text{inf}} \ll 1 \) eV, the flat direction can generate adequate density perturbations as required to explain to match the observations. Recall that inflation is driven by \( V'' \neq 0 \), we obtain [47]

\[
\delta_H \simeq (1/5\pi)(H_{\text{inf}}^2/\dot{\phi}) \sim M_F^2 M_P N_e^2 \phi_0^{-3} \sim 10^{-5}. \tag{9.26}
\]

Note that for \( M_F \sim 10 \) TeV, and \( N_{\text{COBE}} \sim 40 \), we match the current observations [10], when \( \phi_0 \sim 10^{11} \) GeV. The validity of Eq. (9.17) for such a large VEV requires that \( M_F^2 > (10^{11} \text{ GeV}) \times m_{3/2} \). For \( M_F \sim 10 \) TeV this yields the bound on the gravitino mass, \( m_{3/2} < 1 \) MeV, which is compatible with the dark matter constraints as we will see.
We can naturally satisfy Eq. (9.26) provided, \( n = 6 \). The non-renormalizable operator, \( n = 6 \), points towards two MSSM flat directions out of many, \( LLe \) and \( udd \).

\[ (9.27) \]

As we discussed before in [44], these are the only directions which are suitable for inflation as they give rise to a non-vanishing \( A \)-term. Note that the inflatons are now the gauge invariant objects. The total number of e-foldings, during the slow roll inflation, after using Eq. (9.25) yields,

\[ N_{\text{tot}} \sim 10^3. \]  

(9.28)

While the spectral tilt and the running of the power spectrum are determined by \( N_{\text{COBE}} \sim 40 \ll N_{\text{tot}} \).

\[ n_s = 1 + 2\eta - 6\epsilon \simeq 1 - \frac{4}{N_{\text{COBE}}} \sim 0.90, \]  

(9.29)

\[ \frac{d n_s}{d \ln k} = 16\eta - 24\epsilon^2 - 2\xi^2 \simeq -\frac{4}{N_{\text{COBE}}^2} \sim -10^{-3}, \]  

(9.30)

where \( \xi^2 = M_F^4 V'V'\prime'V''/V^2 \). Note that the spectral tilt is slightly away from the 2\( \sigma \) result of the current WMAP 3 years data, on the other hand running of the spectrum is well inside the current bounds [10].

At first instance one would discard the model just from the slight mismatch in the spectral tilt from the current observations. However note that our analysis strictly assumes that the slow roll inflation is driven by \( V''(\phi_0) \). This is particularly correct if \( V'(\phi_0) = 0 \) and \( V''(\phi_0) = 0 \). Let us then study the case when \( V'(\phi_0) \neq 0 \), as discussed in [43, 50].

The latter case can be studied by parameterizing a small deviation from the exact saddle point condition by solving near the point of inflection, where we wish to solve \( V''(\phi_0) = 0 \) and we get up-to 1st order in the deviation, \( \delta < 1 \),

\[ \tilde{A} = A(1 - \delta), \quad \tilde{\phi}_0 = \phi_0 \left( 1 - \frac{n-1}{n(n-2)}\delta \right), \]  

(9.31)

with \( A \) and \( \phi_0 \) are the saddle point solutions. Then the 1st derivative is given by

\[ V'(\phi_0) = 4\frac{n-1}{n-2} M_F^4 \phi_0^{-1} \delta. \]  

(9.32)

Therefore the slope of the potential is determined by, \( V''(\phi) \simeq V''(\phi_0) + (1/2)V''''(\phi_0)(\phi - \phi_0)^2 \).
Note that both the terms on the right-hand side are positive. The fact that $V'(\phi_0) \neq 0$ can lead to an interesting changes from the saddle point behavior, for instance the total number of e-foldings is now given by

$$N_{\text{tot}} = \frac{V(\phi_0)}{M_P^2} \int_{\phi_{\text{end}}}^{\phi_0} \frac{d\phi}{V'(\phi_0) + \frac{1}{2} V''(\phi_0)(\phi - \phi_0)^2}. \quad (9.33)$$

First of all note that by including $V'$, we are slightly away from the saddle point and rather close to the point of inflection. This affects the total number of e-foldings during the slow roll. It is now much less than that of $N_{\text{tot}}$, i.e. $N_{\text{tot}} \ll 10^3$, see Eq. (9.28).

When both the terms in the denominator of the integrand contributes equally then there exists an interesting window.

$$\frac{\kappa}{8} \leq \delta \leq \frac{\kappa}{2}. \quad (9.34)$$

where

$$\kappa \equiv \frac{n - 2}{n(n - 1)^2} \left[\ln\left(\frac{\phi_0^2}{M_S^2}\right) - \frac{3n - 2}{n(n - 1)}\right]^2 \frac{\phi_0^4}{M_P^4 N_{\text{COBE}}^2}. \quad (9.35)$$

The lower limit in Eq. (9.34) is saturated when $V'(\phi_0) = 0$, while the upper limit is saturated when $N_{\text{tot}} \simeq N_{\text{COBE}} \simeq 40$. It is also easy to check that there will be no self-reproduction regime for the field values determined by $\delta$.

It is a straightforward but a tedious exercise to demonstrate that when the upper limit of Eq. (9.34) is saturated the spectral tilt becomes $n_s \simeq 1$, when the lower limit is satisfied we recover the previous result with $n_s = 0.9$. This value, $n_s \rightarrow 1$, can be easily understood as $\phi_{\text{COBE}} \rightarrow \phi_0$ (where $\phi_{\text{COBE}}$ corresponds to the VeV where the end of inflation corresponds to $N_{\text{COBE}} \sim 40$), in which case, $\eta \rightarrow 0$. Therefore the spectral tilt becomes nearly scale invariant. We therefore find a range $\left[15, 50, 47\right]$, $0.90 \leq n_s \leq 1$, \(9.36\)

whose width is within the $2\sigma$ error of the central limit \[10\]. Similarly the running of the spectral tilt gets modified too but remains within the observable limit $^{26}$, while the amplitude of the power spectrum is least affected \[15, 50, 47, 48\].

Let us now discuss the issue of reheating and thermalization. Important point is to realize that the inflaton belongs to the MSSM, i.e. $LLe$ and $udd$, both carry

$^{26}$A similar exercise can be done for the running of the spectral tilt and the running lies between $-16/N_{\text{COBE}}^2 \leq dn_s/d\ln k \leq -4/N_{\text{COBE}}^2 \ [13, 54]$. 


MSSM charges and both have gauge couplings to gauge bosons and gauginos. After inflation the condensate starts oscillating. The effective frequency of the inflaton oscillations in the Logarithmic potential, Eq. (9.17), is of the order of $M_F^2/\phi_0$, while the expansion rate is given by $H_{\text{inf}} \sim M_F^2/\phi_0$. This means that within one Hubble time the inflaton oscillates nearly $M_F/\phi_0 \sim 10^7$ times. The motion of the inflaton is strictly one dimensional from the very beginning. During inflation, the imaginary direction is very heavy and settles down in the minimum of the potential. This also prohibits fragmentation of the flat direction to form Q-balls [19, 45]. Although one could still argue positive and negative charged Q-ball formation [101], but they do not form adequately to alter reheating and thermalization in any significant way.

An efficient bout of particle creation occurs when the inflaton crosses the origin, which happens twice in every oscillation. The reason is that the fields which are coupled to the inflaton are massless near the point of enhanced symmetry. Mainly electroweak gauge fields and gauginos are then created as they have the largest coupling to the flat direction. The production takes place in a short interval. Once the inflaton has passed by the origin, the gauge bosons/gauginos become heavy by virtue of VeV dependent masses and they eventually decay into particles sparticles, which creates the relativistic thermal bath. This is so-called instant preheating mechanism [92]. In a favorable condition, the flat direction VeV coupled very weakly to the flat direction inflaton could also enhance the perturbative decay rate of the inflaton [98]. In any case there is no non-thermal gravitino production [98] as the energy density stored in the inflaton oscillations is too low.

A full thermal equilibrium is reached when a) kinetic and b) chemical equilibrium are established [94]. The maximum temperature of the plasma is given by

$$T_R \sim \left[ V(\phi_0) \right]^{1/4} \sim M_F \lesssim 10\text{TeV},$$

(9.37)

when the flat direction, either $LLe$ or $udd$ evaporates completely. This naturally happens at the weak scale. There are two very important consequences which we summarize below.

### 9.2.1 Cold electroweak Baryogenesis

The model strongly points towards electroweak baryogenesis within MSSM. Note that the reheat temperature is sufficient enough for a thermal electroweak baryogenesis [111].
However, if the thermal electroweak baryogenesis is not triggered, then cold electroweak baryogenesis is still an option. During the cold electroweak baryogenesis, the large gauge field fluctuations give rise to a non-thermal sphaleron transition. In our case it is possible to excite the gauge fields of $SU(2)_L \times U(1)_Y$ during instant preheating provided the inflaton is $LLe$. The $LLe$ as an inflaton carries the same quantum number which has a $B-L$ anomaly and large gauge field excitations can lead to non-thermal sphaleron transition to facilitate baryogenesis within MSSM.

9.2.2 Gravitino dark matter

Within GMSB gravitinos are the LSP and if the $R$-parity is conserved then they are an excellent candidate for the dark matter. There are various sources of gravitino production in the early universe. However in our case the thermal production is the dominant one and mainly helicity $\pm 1/2$ gravitinos are created. Gravitinos thus produced have the correct dark matter abundance for

$$m_{3/2}/100 \text{ KeV} \simeq \frac{1}{\text{few}} \left( \frac{T_R}{1 \text{ TeV}} \right) \left( \frac{M_{\tilde{g}}}{1 \text{ TeV}} \right)^2,$$

where $M_{\tilde{g}}$ is the gluino mass. For $m_{3/2} \gtrsim 100$ KeV, Eq. (9.38) is easily satisfied for $M_{\tilde{g}} \sim 1$ TeV and $T_R \lesssim 10$ TeV. We remind that for $\mathcal{O}(\text{keV}) \lesssim m_{3/2} < 100$ KeV gravitinos produced from the sfermion decays overclose the universe.

10. Quantum initial conditions, cosmological constant and a string landscape

One important result is that the MSSM inflation produces many e-foldings of slow roll inflation, $N_e \sim 10^3$, with a preceding self-reproduction regime. One difficulty of MSSM inflation, though, is that it requires a fine-tuning of the initial conditions for slow-roll.

Our aim is to address this initial condition issue by studying the quantum fluctuations of the MSSM inflaton during a false vacuum inflation which naturally fine tunes the initial conditions for MSSM inflation, i.e. why the MSSM inflation occurs near the saddle point of the potential. The idea is to have an early bout of false vacuum during which the MSSM inflaton will obtain false vacuum induced quantum fluctuations from the point of enhanced symmetry to the saddle point. Once the false vacuum tunnels to a lower cosmological constant or the vacuum energy density
comparable to that of the MSSM inflation then the latter potential takes over for a last stage of MSSM driven inflation.

Without a prior phase of inflation it would be difficult to explain the initial condition otherwise. For example, one may not invoke thermal effects to trap the flat direction near the saddle point as it does not correspond to a false minimum, or a point of enhanced symmetry. As we shall see, old inflation and MSSM inflation complement each other nicely.

Also, there is some evidence that string theory has a “landscape” of metastable vacua with varying cosmological constant, moduli VEVs (and therefore couplings), supersymmetry (SUSY) breaking scale, and so on, which can be studied using statistical arguments (see [113, 116, 23] for reviews). Since many of these metastable vacua have large energy densities (as we describe below), it seems likely that the early universe could have existed as a de Sitter spacetime with a large cosmological constant, which then decayed by tunnelling, as in old inflation [1]. In fact, such a cosmology with multiple stages of inflation [75] provides a mechanism by which the full landscape of vacua is populated, as in [117, 118, 119]. With some caveats, this mechanism can also relax the cosmological constant quickly [117, 118, 119, 120, 121]. In particular, the cosmological constant must be able to decay even though other sectors will dominate once the energy density reaches \( \sim 1 \, (\text{TeV})^4 \).

One obvious worry about this picture is that, if the universe were to tunnel out of the false vacuum, then the universe would be devoid of any entropy as the nucleated bubble would keep expanding forever with a negative spatial curvature. Such a universe would have no place in a real world, so it is important that the last stage of inflation be driven in an observable sector, such as in the case of MSSM inflation which not only produces the right amplitude of density spectrum but also provides us with a desired thermal entropy, and cold dark matter abundance.

### 10.1 Old Inflation on the Landscape

The most basic fact about the landscape (which is usually not discussed, being of little interest for present-day physics) is that the cosmological constant is generically large. A simple way to see that follows [118], which describes the landscape contribution to the cosmological constant as arising from string theory flux. In this picture, the vacuum energy

\[
V = M_P^2 \Lambda = M_P^2 \Lambda_0 + \alpha'^{-2} \sum_i c_i n_i^2 ,
\]  

(10.1)
where \( c_i \lesssim 1 \) are constants and \( n_i \) are flux quantum numbers (note that \( \Lambda \) has dimension mass\(^2\) in our notation as it enters the Einstein equation as \( \Lambda g_{\mu\nu} \)). It is clear that large \( \Lambda \) corresponds to a large radius shell in the space of flux quanta, so larger \( \Lambda \) will have more possible states. All in all, string theory (from the landscape point of view) could have from \( 10^{500} \) to even \( 10^{1000} \) vacua \([115, 116, 23, 118]\), with the vast majority of those having large cosmological constants.

### 10.2 Decay time scales and inflation

The decay rate per volume (by tunnelling) of a metastable vacuum to the nearest neighboring vacuum\(^{27}\) takes the form

\[
\Gamma/V = C \exp \left( -\Delta S_E \right), \tag{10.2}
\]

where \( C \) is a one-loop determinant and \( \Delta S_E \) is the difference in Euclidean actions between the instanton and the background with larger cosmological constant. The determinant \( C \) can at most be \( C \lesssim M_P^4 \), simply because \( M_P \) is the largest scale available, and estimates (ignoring metric fluctuations) give a value as small as \( C \sim r^{-4} \), with \( r \) the instanton bubble radius \([124]\). If we therefore look at a decay rate in a (comoving) Hubble volume, we find

\[
\Gamma \lesssim \frac{M_P^4}{H^3} \exp \left( -\Delta S_E \right). \tag{10.3}
\]

Especially with a large Hubble scale, the associated decay time is much longer than \( 1/H \), given that typically \( \Delta S_E \gg 1 \).

In fact, as given in \([125]\) following \([126, 127, 128]\), the Euclidean action takes the form

\[
\Delta S_E = 2\pi^2 r^3 \tau_e, \tag{10.4}
\]

where the bubble radius and effective tension \( \tau_e \) are given in terms of dimensionless cosmological constants \( \lambda = M_P^4 \Lambda / \tau^2 \), and the actual tension of the bubble wall \( \tau \). The full formulae are listed in the appendix.

Unfortunately, a given tunnelling process in the landscape probably has \( \lambda_+ \sim 1 \) and \( \lambda_- \lesssim 1 \). However, it may be instructive to look at three limits, \( \lambda_+ \to 0, \infty \). In

\(^{27}\)To avoid subtleties, we require that both states have nonnegative energy density. See, for example, \([122]\) for issues in the negative \( \Lambda \) case. Also, \([123]\) has discussed the importance of negative \( \Lambda \) vacua in possibly separating parts of the landscape from each other. We adopt the view, as discussed in that paper, that the landscape is sufficiently complicated that there are no isolated regions.
the latter case, we simultaneously take $\lambda_- \to 0$ or $\lambda_- \to \lambda_+$ (these are the same for $\lambda_+ \to 0$). As $\lambda_+$ vanishes, we find
\[
\Delta S_E \rightarrow 24\pi^2 \frac{M_P^6}{\tau^2 \lambda_+} = 24\pi^2 \frac{M_P^2}{\Lambda_+}. \tag{10.5}
\]
(The tension surprisingly drops out!) For $\lambda_+ \to \infty$, we find the limits
\[
\Delta S_E \rightarrow 12\pi^2 \frac{M_P^6}{\tau^2 \lambda_+} = 12\pi^2 \frac{M_P^2}{\Lambda_+} \left( \lambda_- \to 0 \right), \tag{10.6}
\]
\[
\Delta S_E \rightarrow 6\sqrt{3}\pi^2 \frac{M_P^6}{\tau^2 \lambda_+^{3/2}} = 6\sqrt{3}\pi^2 \frac{\tau}{\Lambda_+^{3/2}} \left( \lambda_- \to \lambda_+ \right). \tag{10.7}
\]

The only parametrically different behavior is the last limit as $\lambda_+ \to \infty$, which means either that the tension is very small or that the cosmological constants are both very large (or that $M_P \to \infty$, corresponding to field theory without gravity). Technically, the instanton approximation can break down in the limit (10.7), if the Euclidean action becomes much smaller than one (as a full quantum mechanical treatment would be necessary), but the important point is that $\Gamma/H \gtrsim 1$ if that is the case.

From (10.3), we might expect that $\Gamma/H \gg 1$, but we should remember that the coefficient of the exponential in (10.3) is an upper limit and that quantum mechanical effects could limit the decay rate to $\Gamma \sim H$. In section 10.3, we give an improved form of the argument of [118] that shows how old inflation on the landscape can source MSSM inflation without inflaton trapping and with only small jumps in the cosmological constant.

Note that, when MSSM inflation starts, the “bare” cosmological constant (that not associated with the MSSM inflaton) might still be considerably larger than the present value. This means that further instanton decays should take place to reduce the bare cosmological constant, and these decays should occur during MSSM inflation in order for the bubble regions to grow long enough. Since it seems likely that the instanton bubble tension will be large compared to the scale of MSSM inflation, the decay rate will be given by (10.7), which is highly suppressed. This would then require MSSM inflation to last for extremely many e-foldings. Fortunately, MSSM inflation naturally includes a self-reproduction (eternal inflation) regime prior to slow-roll [44, 45]. In other words, it seems that a low-scale eternal inflation is a necessary part of using decays to solve the cosmological constant problem, and MSSM inflation includes it naturally. Eternal inflation solves several of problems relating false vacuum inflation to observations, and it is particularly viable at the TeV scale of MSSM inflation.
10.3 Initial quantum kicks for an MSSM fields

Let us now discuss what happens to the observable sector in the background of inflation spacetime. As we have noticed, the universe cascades from large cosmological constant to another somewhat smaller as time progresses. Given the decay time, it is fairly evident that huge number of e-foldings can be generated.

During the false vacuum inflation the energy density of the universe, and hence the expansion rate $H_{\text{false}}$, remains constant for a given $\Lambda$.

The flat direction potential receives corrections from soft SUSY breaking mass term, the non-renormalizable superpotential correction, and corrections due to the large Hubble expansion for a minimal choice of Kähler potential. The Hubble correction is relevant only when $m_\phi \ll H(t)$, which is parameterized by two constants $c \sim \mathcal{O}(1)$ and $a \sim \mathcal{O}(1)$, result in [68, 65, 19, 20]

$$V = \frac{1}{2} (m_\phi^2 + c H_{\text{false}}^2) |\phi|^2 + \left[ (A + a H_{\text{false}}) \lambda_n \frac{\phi^n}{n M_P^{n-3}} + \text{h.c.} \right] + \lambda_n^2 \frac{|\phi|^{2(n-1)}}{M_P^{2(n-3)}}, \quad (10.8)$$

where the soft SUSY breaking mass term is generically small compared to the Hubble expansion rate of the false vacuum, $m_\phi \sim 1 \text{ TeV} \ll H_{\text{false}}$. We define $\Phi \equiv \phi e^{i\theta}$, and the dynamics of $\phi$ in an inflationary background depends on $c$ and $a$. Therefore we consider different cases separately.

It turns out that the dynamics of the MSSM flat direction largely follow the physics discussed in [118]. We review and elaborate on their discussion in the context of the MSSM and give an important improvement on their result in the context of negligible Hubble corrections.

10.3.1 Positive Hubble induced corrections

A positive Hubble induced correction provides $c \sim + \mathcal{O}(1), a \sim \mathcal{O}(1)$. This is a typical scenario when the Kähler potential for the string modulus comes with a canonical kinetic term. Although this could be treated as a special point on the Kähler manifold, it is nevertheless important to discuss this situation. This is also

\[28\]

The origin of the Hubble induced terms is due to couplings between the modulus which drives the false vacuum inflation and the MSSM sector. Apriori, even in string theory, the Kähler potential for the modulus is not well known at all points of the parameter space. Similarly, within the MSSM, the Kähler potential is unknown. Therefore, the coefficients $c, a$ are not fixed. For a no-scale type model, the Hubble induced corrections are vanishing at tree level; however, they do appear at one loop $|c| \sim 10^{-2}$ [82, 83].
the simplest scenario out of all possibilities. A generic flat direction gets a large mass of the order of Hubble expansion rate; therefore, its fluctuations are unable to displace the flat direction from its global minimum.

On phenomenological grounds, this is an uninteresting and undesirable case, since the bubble does not excite any of the MSSM fields. Therefore, the bubble remains empty and devoid of energy with no graceful exit of inflation from the false vacuum. The universe continues cascading to smaller $\Lambda$ with smaller $H_{\text{false}}$. Eventually, when $H_{\text{false}} \leq m_\phi$, the MSSM flat directions would be free to move. However, through this time the fields were never displaced from their minimum and therefore the dynamics of the flat directions would remain frozen. The universe would be cold, as it was before, and the spatial curvature would remain negative. It is fair to say that, on phenomenological grounds, such a universe is already ruled out. This paves the way for more interesting scenarios, which we discuss next.

10.3.2 Negligible Hubble induced corrections

In this case, the potential is not affected by the false vacuum inflation at all, namely $|c|, |a| \ll 1$. So long as $V''(\phi) \ll H^2_{\text{false}}$, the flat direction field $\phi$ makes a quantum jump of length $H_{\text{false}}/2\pi$ within each Hubble time.\(^{29}\) These jumps superpose in random walk fashion resulting in

$$\left( \frac{d\langle \phi^2 \rangle}{dt} \right)_{\text{fluctuations}} = \frac{H_{\text{false}}^4}{4\pi^2}. \quad (10.9)$$

On the other hand, the classical slow roll due to the potential leads to

$$\left( \frac{d\langle \phi^2 \rangle}{dt} \right)_{\text{slow roll}} = -\frac{2\langle V'(\phi)\phi \rangle}{3H_{\text{false}}}. \quad (10.10)$$

For a massive scalar field $V(\phi) \sim m_\phi^2 \phi^2/2$, the combined effects yield

$$\langle \phi^2 \rangle = \frac{3H_{\text{false}}^4}{8\pi^2 m_\phi^2} \left[ 1 - \exp \left( -\frac{2m_\phi^2}{3H_{\text{false}}} t \right) \right]. \quad (10.11)$$

The maximum field value

$$\phi_{\text{r.m.s.}} = \sqrt{\frac{3H_{\text{false}}^2}{8\pi^2 m_\phi}}, \quad (10.12)$$

at which the slow roll motion (10.10) counterbalances the random walk motion (10.9), is reached for $\Delta t \gg 3H_{\text{false}}/2m_\phi^2$. This amounts to a number

$$N_{\text{false}} \gg \frac{3}{2} \left( \frac{H_{\text{false}}}{m_\phi} \right)^2. \quad (10.13)$$

\(^{29}\)To be more precise, the quantum fluctuations of $\phi$ have a Gaussian distribution, and the r.m.s. of modes which exit the horizon within one Hubble time is $H_{\text{false}}/2\pi$. 


of e-foldings of inflation in the false vacuum.

In the absence of Hubble induced corrections, the potential in (10.8) has a saddle point at

$$\phi_0 = \left( \frac{m_\phi M_p^{n-3}}{\lambda_n \sqrt{2n - 2}} \right)^{1/(n-2)}, \quad (10.14)$$

where $V'(\phi_0) = V''(\phi_0) = 0$ while $V'''(\phi_0) \neq 0$, provided that $A^2 = 8(n-1)m_\phi^2$.$^{30}$ For $m_\phi \sim 100 \text{ GeV} - 10 \text{ TeV}$ and $\lambda_n \sim O(1)$, and for $n = 6$, the VEV is $\phi_0 \sim 10^{14} - 10^{15} \text{ GeV}$. The suitable flat directions are $LLe$ and $udd$, which are lifted by $n = 6$ superpotential terms and also have a non-zero $A$-term as required by the condition for a saddle point.$^{31}$

For $\phi < \phi_0$ the mass term dominates the flat direction potential. Then quantum fluctuations can push $\phi$ to the vicinity of $\phi_0$ if $\phi_{r.m.s} \geq \phi_0$.$^{32}$ This, according to (10.12), requires that

$$H_{\text{false}} \geq \left( \frac{8\pi^2}{3} \right)^{1/4} (m_\phi \phi_0)^{1/2} \simeq 10^9 \text{ GeV}. \quad (10.15)$$

The number of e-foldings needed for this to happen is

$$\tilde{N}_{\text{false}} \leq \left( \frac{H_{\text{false}}}{10^9 \text{ GeV}} \right)^2 10^{12}. \quad (10.16)$$

Indeed for $H_{\text{false}} \geq \phi_0$ the inherent uncertainty due to quantum fluctuations implies that $\phi > \phi_0$ within one Hubble time.

A last stage of MSSM inflation with an expansion rate

$$H_{\text{MSSM}} = \frac{n - 2}{\sqrt{6n(n - 1)}} \frac{m_\phi \phi_0}{M_P} \sim O(1 \text{ GeV}) \quad (10.17)$$

$^{30}$This can happen in the gravity mediated case, where $A \sim m_\phi \sim m_{3/2}$, where $m_{3/2} \sim O(1 \text{ TeV})$ is the gravitino mass. The situation is quite different in the case of a gauge mediated SUSY breaking scenario.$^{47}$

$^{31}$In order to have successful inflation, we require the condition $A^2 = 8(n-1)m_\phi^2$ to be satisfied to one part in $10^9$. Although this requires a fine tuning, SUSY can allow it to be maintained order by order if $A/m_\phi$ acts as an infrared fixed point of the renormalization group flow.$^{83}$ Also, as noted in Ref. $^{83}$, this tuning can be explained naturally by the landscape picture. For larger deviations there is a point of inflection with large $V'(\phi_0)$ (or a negligible $A$-term discussed in $^{69}$), or a pocket of false minimum.$^{83}$ Neither case leads to a slow roll inflation within the MSSM.

$^{32}$Due to the Gaussian distribution of fluctuations, the probability of having $\phi > \phi_{r.m.s}$ is exponentially suppressed.
starts if $V(\phi)$ dominates the energy density of the universe, i.e. $V(\phi) > 3H^2_{\text{false}}M^2_P$.

An observationally consistent inflation in the slow roll regime requires that the displacement from the saddle point satisfy $|\phi - \phi_0| < \Delta \phi$, where \[ \Delta \phi = \frac{\phi_0^3}{4n(n-1)M^2_P} \simeq 10^6 \text{ GeV}. \] (10.18)

In the landscape, the universe can begin in a false vacuum with arbitrarily large $H_{\text{false}}$ (as long as $H_{\text{false}} \ll M_P$). Therefore, we generically expect that $\phi$ is quickly pushed to field values $\phi \gg \phi_0$. However, $H_{\text{false}}$ slowly decreases as a result of tunnelling to vacua with smaller cosmological constant \[ 33 \] For a massive scalar field with the potential $V(\phi) \sim m^2\phi^2/2$, this implies a gradual decrease of $\phi_{r.m.s}$, see (10.12). Since quantum fluctuations can at most push $\phi$ to $\phi_{r.m.s}$, this also implies that $\phi$ is slowly decreasing in time. Indeed for $H_{\text{false}} < 10^9 \text{ GeV}$, we find that $\phi < \phi_0$, irrespective of how large $\phi$ initially was. This is the case in the discussion of [118], which is why [118] requires a jump from large cosmological constant directly to the slow-roll inflationary stage.

Note, however, that the potential becomes very flat around $\phi_0$ as a result of the interplay among different terms in (10.8). In fact, for $|\phi - \phi_0| \ll \phi_0$ we have \[ V(\phi) \approx V(\phi_0) + \frac{1}{3(n-2)^2 \phi_0^2} m^2_{\phi}(\phi - \phi_0)^3. \] (10.19)

Once $H_{\text{false}} \sim 10^9 \text{ GeV}$, $\phi$ reaches this plateau (from above). Within the plateau, $V'(\phi)$ becomes increasingly negligible, and so does the classical slow roll; they exactly vanish at $\phi = \phi_0$. In consequence, quantum jumps dominate the dynamics and freely move $\phi$ throughout the plateau. Hence the flatness of potential, which is required for a successful MSSM inflation, guarantees that $\phi$ will remain within the plateau during the landscape evolutionary phase. It is possible to generalize this argument to other models of low-scale inflation; we see that a sufficiently flat inflaton potential can trap inflaton fluctuations in the slow-roll region.

The flat direction eventually dominates the energy density of the universe when $H_{\text{false}} < 1 \text{ GeV}$, see (10.17). MSSM inflation then starts in those parts of the universe

---

\textsuperscript{33}Note that we need to stay in an MSSM-like vacuum all the way until MSSM inflation begins. Given the scarcity of MSSM-like vacua in the landscape, the probability of tunnelling from one such vacuum to another is $\sim 10^{-9}$. However, due to eternal inflation in the false vacuum, the physical volume of the universe increases by a factor of $\exp(3H_{\text{false}}/\Gamma)$ within a typical time scale for bubble nucleation ($\Gamma$ is the false vacuum decay rate, see subsection 2.2). This easily wins over the suppression factor $10^{-9}$ for $\Gamma < 9H_{\text{false}}$. 

which obey (10.18). Having \( \phi \) so close to \( \phi_0 \) is possible since the uncertainty due to quantum fluctuations is \( \sim 1 \) GeV at this time.

The false vacuum inflation paves the way for an MSSM inflation inside the nucleated bubble. The modulus which was responsible for the false vacuum inflation continues tunnelling to minima with smaller (eventually the currently observed) cosmological constant inside the MSSM inflating bubble. The modulus will oscillate around the minimum of its potential as the curvature of its potential dominates over the Hubble expansion rate. The fate of oscillations would depend on the coupling of the modulus to the MSSM fields. For a coupling of gravitational strength, oscillations are long-lived. Moreover, the decay will be kinematically forbidden if the decay products are coupled to the flat direction \( \phi \) (which has obtained a large VEV \( \simeq \phi_0 \)). However, such details are largely irrelevant as the flat direction dominates the energy density of the universe and drives inflation, diluting the energy density in oscillations.

In particular, we note that MSSM inflation has a self-reproducing regime [44, 45] because \( V'(\phi) \) is extremely small and the potential becomes very flat close to \( \phi_0 \). The observable part of our universe can spend an arbitrarily long time in the self-reproducing regime before moving into the standard slow-roll inflation. During this period, the cosmological constant can continue to decay and eventually settle at an observationally acceptable value.

Before we conclude this discussion, let us remind the readers that obtaining the flat direction VEV, \( \phi_0 \), depends on the false vacuum inflation, which requires \( H_{\text{false}} \geq 10^9 \) GeV at some time (see (10.15)). This condition, although very probable in the landscape picture, need not be satisfied always. Then the quantum fluctuations would not be large enough to push the flat direction to the vicinity of \( \phi_0 \) as required for a final stage of MSSM inflation. This would therefore ruin the inflationary and phenomenological predictions. This can be avoided if the flat direction is trapped in a false minimum which evolves with time, as discussed below.

### 10.3.3 Negative Hubble induced corrections

The case with \( c \sim -O(1) \) may arise naturally for non-minimal Kähler potential [68, 63]. For \( H_{\text{false}} \gg m_\phi \) the potential in (10.8) becomes tachyonic, and its true minimum is located at

\[
\phi_{\text{min}} \simeq \left( \frac{\sqrt{|c|}}{\lambda_n \sqrt{2n-2} H_{\text{false}} M_P^{n-3}} \right)^{1/(n-2)}.
\]  

(10.20)
Note that $\phi_{\text{min}}$ is initially larger than $\phi_0$, see (10.14).

The curvature of the potential at the minimum is $V''(\phi_{\text{min}}) = (4n - 7)|c|H^2_{\text{false}} \gg H^2_{\text{false}}$. This implies that $\phi$ is driven away from the origin, due to quantum fluctuations, and quickly settles down at $\phi_{\text{min}}$. The MSSM flat direction is trapped inside the minimum, held due to false vacuum inflation, and gradually tracks the instantaneous value of $\phi_{\text{min}}$ as $H_{\text{false}}$ decreases.\(^{34}\)

The minima at $\phi = 0$ and $\phi = \phi_{\text{min}}$ become degenerate when $H_{\text{false}} \sim m_\phi$. For $H_{\text{false}} \ll m_\phi$ the true minimum is at $\phi = 0$ and the one at $\phi_{\text{min}}$ will be false. The Hubble induced corrections are subdominant in this case. We then find \(^{35}\)

$$\phi_{\text{min}} \simeq \phi_0 \left(1 + \frac{1}{n - 2} \frac{\sqrt{|c|H_{\text{false}}}}{m_\phi}\right) \quad (10.21)$$

$$V''(\phi_{\text{min}}) \simeq 2(n - 2)\sqrt{|c|H_{\text{false}}m_\phi}. \quad (10.22)$$

The $\phi$ field can track down the instantaneous value of $\phi_{\text{min}}$ provided that $\sqrt{V''(\phi_{\text{min}})}$ is greater than the Hubble expansion rate. In fact this is the case so long as $H_{\text{false}} > H_{\text{MSSM}} \sim 1$ GeV, see (10.22). Once $H_{\text{false}} \simeq H_{\text{MSSM}}$, the flat direction potential dominates the energy density of the universe, and MSSM inflation begins at a Hubble expansion rate $H_{\text{MSSM}}$. In the meantime, landscape tunnelling to vacua with smaller cosmological constant continues, and the location of the false minimum $\phi_{\text{min}}$ continuously changes.\(^{35}\) Eventually $V''(\phi_{\text{min}}) < H^2_{\text{MSSM}}$ when

$$H_{\text{false}} \simeq \frac{1}{2(n - 2)^2} \frac{H^2_{\text{MSSM}}}{m_\phi}, \quad (10.23)$$

at which time

$$\phi_{\text{min}} - \phi_0 \simeq \frac{\phi_0}{2(n - 2)^2} \left(\frac{H_{\text{MSSM}}}{m_\phi}\right)^2. \quad (10.24)$$

It turns out from (10.17,10.18) that $\phi_{\text{min}} - \phi_0 \ll \Delta \phi$. Therefore $\phi$ is already inside the interval required for a successful MSSM inflation. At this point the Hubble induced corrections become largely unimportant, and all the successes of MSSM inflation are retained, as discussed in the previous subsection. The fate of the string moduli inside the MSSM bubble remains the same as in the previous subsection. In particular, it

\(^{34}\)Here we are assuming that the difference between the energy densities of the two false vacua is small compared to their average energy density.

\(^{35}\)Note that tunnellings do not affect the Hubble expansion rate anymore since the flat direction dominates the energy density of the universe now.
does not matter whether the universe tunnels right away to the currently observed value of \( \Lambda \) or not. Inflation dilutes any excitations of the modulus oscillations during inflation. Our Hubble patch reheats when the MSSM flat direction rolls down to its minimum and starts creating MSSM quanta as discussed in the previous subsection.

Note that all needed for the success of this scenario is to start in a false vacuum in the landscape where \( H_{\text{false}} > m_\phi \sim 1 \text{ TeV} \). This ensures that the flat direction will roll way from the origin and settle at \( \phi_{\text{min}} \), which is the true minimum of \( V(\phi) \) at that time. It will then track \( \phi_{\text{min}} \) as \( H_{\text{false}} \) slowly decreases, and will eventually land inside the appropriate interval around \( \phi_0 \). This is a much milder condition than that in the case of negligible Hubble induced corrections \( H_{\text{false}} \geq 10^9 \text{ GeV} \) (see the previous subsection).

One comment is in order. This scenario has some similarities to the Affleck-Dine baryogenesis with negative Hubble induced corrections \([68, 65]\). It can be seen from (10.8) that the equation of motion of the flat direction has a fixed point so long as Hubble induced corrections are dominant. The flat direction tracks the fixed point in a radiation or matter dominated universe. However, a non-adiabatic change in the potential occurs when \( H \sim m_\phi \), and the flat direction starts oscillating around the origin. However, in our case the universe is in a de Sitter phase with a slowly varying \( H_{\text{false}} \), and the flat direction tracks the false minimum of its potential until it dominates the universe. As explained, this is necessary for having a successful MSSM inflation.

11. Acknowledgments

The author would like to thank all his collaborators: Rouzbeh Allahverdi, Bhaskar Dutta, Kari Enqvist, Andrew Frey, Juan Garcia-Bellido, Askö Jokinen and Alex Kusenko for successful collaborations, invigorating, delightful and extremely helpful discussions on various aspects of the physics involved in understanding inflation and its consequences.

We also wish to thank Cliff Burgess, Manuel Drees, John Ellis, Jaume Garriga, and Tony Riotto for valuable discussions and various suggestions they have made. We also benefited from the discussions with Shanta de Alwis, Steve Abel, Mar Bastero-Gil, Micha Berkooz, Zurab Berezhiani, Robert Brandenberger, Ramy Brustein, Kostas Dimopoulos, Damien Easson, Renata Kallosh, Gordy Kane, Justin Khoury, George Lazarides, Andrei Linde, Andrew Liddle, David Lyth, Hans Peter Nilles,
Pavel Naselsky, Lyman Page, Maxim Pospelov, Subir Sarkar, Qaisar Shafi, Misha Shaposhnikov, Paul Steinhardt, Scott Thomas and Igor Tkachev.

The research of A.M. is partly supported by the European Union through Marie Curie Research and Training Network “UNIVERSENET” (MRTN-CT-2006-035863).

References

[1] A. H. Guth, Phys. Rev. D 23, 347 (1981).

[2] A. A. Starobinsky, Phys. Lett. B 91 (1980) 99. A. A. Starobinsky, Phys. Lett. B 117 (1982) 175.

[3] A. D. Linde, Phys. Lett. B 108, 389 (1982).

[4] A. D. Linde, Phys. Lett. B 129 (1983) 177.

[5] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).

[6] A. D. Linde, Contemp. Concepts Phys. 5, 1 (2005). [arXiv:hep-th/0503203].

[7] A. D. Linde, D. A. Linde and A. Mezhlumian, Phys. Rev. D 49, 1783 (1994) [arXiv:gr-qc/9306035].

[8] V. F. Mukhanov and G. V. Chibisov, JETP Lett. 33 (1981) 532 [Pisma Zh. Eksp. Teor. Fiz. 33 (1981) 549]. V. F. Mukhanov and G. V. Chibisov, Sov. Phys. JETP 56 (1982) 258 [Zh. Eksp. Teor. Fiz. 83 (1982) 475]. G. V. Chibisov and V. F. Mukhanov, Mon. Not. Roy. Astron. Soc. 200 (1982) 535. J. M. Bardeen, P. J. Steinhardt and M. S. Turner, Phys. Rev. D 28, 679 (1983). J. M. Bardeen, Phys. Rev. D 22 (1980) 1882. A. H. Guth and S. Y. Pi, Phys. Rev. Lett. 49 (1982) 1110.

[9] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rept. 215, 203 (1992).

[10] D.N. Spergel, et.al., astro-ph/0603449.

[11] W. H. Kinney, E. W. Kolb, A. Melchiorri and A. Riotto, Phys. Rev. D 74, 023502 (2006).

[12] D. H. Lyth and A. Riotto, Phys. Rept. 314, 1 (1999) [arXiv:hep-ph/9807278].

[13] A. D. Linde, Phys. Rev. D 49, 748 (1994) [arXiv:astro-ph/9307002].
[14] A. R. Liddle, A. Mazumdar and F. E. Schunck, Phys. Rev. D 58, 061301 (1998) [arXiv:astro-ph/9804177]. E. J. Copeland, A. Mazumdar and N. J. Nunes, Phys. Rev. D 60, 083506 (1999) [arXiv:astro-ph/9904309].

[15] S. Dimopoulos, S. Kachru, J. McGreevy and J. G. Wacker, arXiv:hep-th/0507205.

[16] T. Damour and V. F. Mukhanov, Phys. Rev. Lett. 80, 3440 (1998) [arXiv:gr-qc/9712061]. A. R. Liddle and A. Mazumdar, Phys. Rev. D 58, 083508 (1998) [arXiv:astro-ph/9806127]. A. Linde, JHEP 0111, 052 (2001) [arXiv:hep-th/0110195].

[17] C. Armendariz-Picon, T. Damour and V. F. Mukhanov, Phys. Lett. B 458, 209 (1999) [arXiv:hep-th/9904075].

[18] A. Albrecht, P. J. Steinhardt, M. S. Turner and F. Wilczek, Phys. Rev. Lett. 48, 1437 (1982).

[19] K. Enqvist and A. Mazumdar, Phys. Rept. 380, 99 (2003) [arXiv:hep-ph/0209244].

[20] M. Dine and A. Kusenko, Rev. Mod. Phys. 76, 1 (2004) [arXiv:hep-ph/0303065].

[21] G. Steigman, Int. J. Mod. Phys. E 15, 1 (2006) [arXiv:astro-ph/0511534].

[22] V. Barger, P. Langacker and H. S. Lee, Phys. Rev. D 67, 075009 (2003) [arXiv:hep-ph/0302066].

[23] M. R. Douglas and S. Kachru, Rev. Mod. Phys. 79, 733 (2007) [arXiv:hep-th/0610102].

[24] R. Kallosh, arXiv:hep-th/0702059.

[25] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D 68, 046005 (2003) [arXiv:hep-th/0301240].

[26] S. Kachru, R. Kallosh, A. Linde, J. M. Maldacena, L. McAllister and S. P. Trivedi, JCAP 0310, 013 (2003) [arXiv:hep-th/0308055].

[27] R. Blumenhagen, B. Kors, D. Lust and S. Stieberger, Orientifolds and HEP-TH/0610327;

[28] J. J. Blanco-Pillado et al., JHEP 0609, 002 (2006) [arXiv:hep-th/0603129].

[29] G. R. Dvali and S. H. H. Tye, Phys. Lett. B 450, 72 (1999) [arXiv:hep-ph/9812483]. C. P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh and R. J. Zhang, JHEP 0107, 047 (2001) [arXiv:hep-th/0105204].
[30] A. Mazumdar, S. Panda and A. Perez-Lorenzana, Nucl. Phys. B 614, 101 (2001) [arXiv:hep-ph/0107058].

[31] D. Baumann, A. Dymarsky, I. R. Klebanov, L. McAllister and P. J. Steinhardt, arXiv:0705.3837 [hep-th]. D. Baumann, A. Dymarsky, I. R. Klebanov and L. McAllister, arXiv:0706.0360 [hep-th]. S. Panda, M. Sami and S. Tsujikawa, arXiv:0707.2848 [hep-th].

[32] N. T. Jones, H. Stoica and S. H. H. Tye, JHEP 0207, 051 (2002) [arXiv:hep-th/0203163].

[33] N. Barnaby, C. P. Burgess and J. M. Cline, JCAP 0504, 007 (2005) [arXiv:hep-th/0412040].

[34] L. Kofman and P. Yi, Phys. Rev. D 72, 106001 (2005) [arXiv:hep-th/0507257].

[35] A. R. Frey, A. Mazumdar and R. Myers, Phys. Rev. D 73, 026003 (2006) [arXiv:hep-th/0508139].

[36] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D 64, 123522 (2001) [arXiv:hep-th/0103239]. R. Y. Donagi, J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, JHEP 0111, 041 (2001) [arXiv:hep-th/0105199]. R. Kallosh, L. Kofman and A. D. Linde, Phys. Rev. D 64, 123523 (2001) [arXiv:hep-th/0104073].

[37] T. Biswas, A. Mazumdar and W. Siegel, JCAP 0603, 009 (2006) [arXiv:hep-th/0508194].

[38] A. Nayeri, R. H. Brandenberger and C. Vafa, Phys. Rev. Lett. 97, 021302 (2006) [arXiv:hep-th/0511140]. N. Kaloper, L. Kofman, A. Linde and V. Mukhanov, JCAP 0610, 006 (2006) [arXiv:hep-th/0608200].

[39] R. H. Brandenberger and C. Vafa, Nucl. Phys. B 316, 391 (1989).

[40] R. Danos, A. R. Frey and A. Mazumdar, Phys. Rev. D 70, 106010 (2004) [arXiv:hep-th/0409162]. R. Easther, B. R. Greene and M. G. Jackson, Phys. Rev. D 66, 023502 (2002) [arXiv:hep-th/0204099].

[41] R. Brandenberger, D. A. Easson and A. Mazumdar, Phys. Rev. D 69, 083502 (2004) [arXiv:hep-th/0307043].

[42] T. Biswas, R. Brandenberger, D. A. Easson and A. Mazumdar, Phys. Rev. D 71, 083514 (2005) [arXiv:hep-th/0501194].
[43] T. Biswas, R. Brandenberger, A. Mazumdar and W. Siegel, arXiv:hep-th/0610274.

[44] R. Allahverdi, K. Enqvist, J. Garcia-Bellido and A. Mazumdar, Phys. Rev. Lett. 97, 191304 (2006) [arXiv:hep-ph/0605035].

[45] R. Allahverdi, K. Enqvist, J. Garcia-Bellido, A. Jokinen and A. Mazumdar, JCAP 0706, 019 (2007) [arXiv:hep-ph/0610134].

[46] R. Allahverdi, A. Kusenko and A. Mazumdar, arXiv:hep-ph/0608138. To be published in JCAP.

[47] R. Allahverdi, A. Jokinen and A. Mazumdar, arXiv:hep-ph/0610243.

[48] R. Allahverdi, B. Dutta and A. Mazumdar, Phys. Rev. D 75, 075018 (2007) [arXiv:hep-ph/0702112].

[49] R. Allahverdi, A. R. Frey and A. Mazumdar, Phys. Rev. D 76, 026001 (2007) [arXiv:hep-th/0701233].

[50] R. Allahverdi and A. Mazumdar, arXiv:hep-ph/0610069.

[51] D. H. Lyth, hep-ph/0605283.

[52] D. J. H. Chung, L. L. Everett, G. L. Kane, S. F. King, J. D. Lykken and L. T. Wang, Phys. Rept. 407, 1 (2005) [arXiv:hep-ph/0312378].

[53] H. E. Haber and G. L. Kane, Phys. Rept. 117, 75 (1985).

[54] D. Z. Freedman, P. Van Nieuwenhuizen and S. Ferrara, Phys. Rev. D 13, 3214 (1976); S. Deser and B. Zumino, Phys. Lett. B 62, 335 (1976); A. H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49, 970 (1982).

[55] R. Barbieri, S. Ferrara and C. A. Savoy, Phys. Lett. B 119, 343 (1982); L. J. Hall, J. D. Lykken and S. Weinberg, Phys. Rev. D 27, 2359 (1983); P. Nath, R. Arnowitt and A. H. Chamseddine, Nucl. Phys. B 227, 121 (1983);

[56] H. P. Nilles, Phys. Rept. 110, 1 (1984).

[57] K. Enqvist, A. Jokinen, A. Mazumdar, T. Multamaki and A. Vaihkonen, Phys. Rev. Lett. 94, 161301 (2005) [arXiv:astro-ph/0411394]. K. Enqvist, A. Jokinen, A. Mazumdar, T. Multamaki and A. Vaihkonen, JCAP 0503, 010 (2005) [arXiv:hep-ph/0501076]. A. Jokinen and A. Mazumdar, JCAP 0604, 003 (2006) [arXiv:astro-ph/0512368].

[58] G. Bertone, D. Hooper and J. Silk, Phys. Rept. 405, 279 (2005).
[59] J. Ellis, K. Olive, Y. Santoso, and V. Spanos, Phys. Lett. B 565, 176 (2003); R. Arnowitt, B. Dutta, and B. Hu, arXiv:hep-ph/0310103; H. Baer, C. Balazs, A. Belyaev, T. Krupovnickas, and X. Tata, JHEP 0306, 054 (2003); B. Lahanas and D.V. Nanopoulos, Phys. Lett. B 568, 55 (2003); U. Chattopadhyay, A. Corsetti, and P. Nath, Phys. Rev. Lett 68, 035005 (2003); E. Baltz and P. Gondolo, JHEP 0410, 052 (2004) 052; A. Djouadi, M. Drees and J. L. Kneur, JHEP 0603, 033 (2006); J. L. Feng and K. T. Matchev, Phys. Rev. Lett. 86, 3480 (2001).

[60] A. D. Linde and V. F. Mukhanov, Phys. Rev. D 56, 535 (1997) [arXiv:astro-ph/9610219]. K. Enqvist and M. S. Sloth, Nucl. Phys. B 626, 395 (2002) [arXiv:hep-ph/0109214]. T. Moroi and T. Takahashi, Phys. Lett. B 522, 215 (2001) [Erratum-ibid. B 539, 303 (2002)] [arXiv:hep-ph/0110096]. D. H. Lyth and D. Wands, Phys. Lett. B 524, 5 (2002) arXiv:hep-ph/0110002.

[61] K. Enqvist, S. Kasuya and A. Mazumdar, Phys. Rev. Lett. 90, 091302 (2003) [arXiv:hep-ph/0211147]. M. Postma, Phys. Rev. D 67, 063518 (2003) [arXiv:hep-ph/0212005]. K. Enqvist, A. Jokinen, S. Kasuya and A. Mazumdar, Phys. Rev. D 68, 103507 (2003) [arXiv:hep-ph/0303165]. A. Mazumdar and A. Perez-Lorenzana, Phys. Rev. Lett. 92, 251301 (2004) [arXiv:hep-ph/0311106]. K. Enqvist, S. Kasuya and A. Mazumdar, Phys. Rev. Lett. 93, 061301 (2004) [arXiv:hep-ph/0311224]. A. Mazumdar and A. Perez-Lorenzana, Phys. Rev. D 70, 083526 (2004) [arXiv:hep-ph/0406154]. K. Enqvist, A. Mazumdar and A. Perez-Lorenzana, Phys. Rev. D 70, 103508 (2004) [arXiv:hep-th/0403044].

[62] G. Dvali, A. Gruzinov and M. Zaldarriaga, Phys. Rev. D 69, 023505 (2004) [arXiv:astro-ph/0303591]. G. Dvali, A. Gruzinov and M. Zaldarriaga, Phys. Rev. D 69, 083505 (2004) [arXiv:astro-ph/0305548]. K. Enqvist, A. Mazumdar and M. Postma, Phys. Rev. D 67, 121303 (2003) [arXiv:astro-ph/0304187]. A. Mazumdar, Phys. Rev. Lett. 92, 241301 (2004) [arXiv:hep-ph/0306026]. R. Allahverdi, Phys. Rev. D 70, 043507 (2004) [arXiv:astro-ph/0403351]. A. Mazumdar and M. Postma, Phys. Lett. B 573, 5 (2003) [Erratum-ibid. B 585, 295 (2004)] [arXiv:astro-ph/0306509].

[63] R. Allahverdi, K. Enqvist, A. Jokinen and A. Mazumdar, JCAP 0610, 007 (2006) [arXiv:hep-ph/0603255].

[64] ALEPH, DELPHI, L3, OPAL Collaborations, G. Abbiendi, et al. (The LEP Working Group for Higgs Boson Searches), Phys. Lett. B 565, 61 (2003).
[65] M. Dine, L. Randall and S. Thomas, Nucl. Phys. B 458, 291 (1996) [arXiv:hep-ph/9507453].

[66] T. Gherghetta, C. F. Kolda and S. P. Martin, Nucl. Phys. B 468, 37 (1996) [arXiv:hep-ph/9510370].

[67] K. Enqvist, A. Jokinen and A. Mazumdar, JCAP 0401, 008 (2004) [arXiv:hep-ph/0311336].

[68] M. Dine, L. Randall and S. Thomas, Phys. Rev. Lett. 75, 398 (1995) [arXiv:hep-ph/9503303].

[69] A. Jokinen and A. Mazumdar, Phys. Lett. B 597, 222 (2004) [arXiv:hep-th/0406074].

[70] G. Lazarides and Q. Shafi, Phys. Lett. B 308, 17 (1993) [arXiv:hep-ph/9304247].
S. Kasuya, T. Moroi and F. Takahashi, Phys. Lett. B 593, 33 (2004) [arXiv:hep-ph/0312094].
R. Brandenberger, P. M. Ho and H. c. Kao, JCAP 0411, 011 (2004) [arXiv:hep-th/0312288].
A. Perez-Lorenzana, M. Montesinos and T. Matos, arXiv:0707.1678 [astro-ph].

[71] G. F. Giudice and R. Rattazzi, Phys. Rept. 322, 419 (1999) [arXiv:hep-ph/9801271].

[72] N. Arkani-Hamed, S. Dimopoulos, G. F. Giudice and A. Romanino, Nucl. Phys. B 709, 3 (2005) [arXiv:hep-ph/0409232].

[73] K. Enqvist, A. Jokinen and A. Mazumdar, JCAP 0411, 001 (2004) [arXiv:hep-ph/0404269].

[74] A. R. Liddle and S. M. Leach, Phys. Rev. D 68, 103503 (2003).

[75] C. P. Burgess, R. Easther, A. Mazumdar, D. F. Mota and T. Multamaki, JHEP 0505, 067 (2005) [arXiv:hep-th/0501125].

[76] J. C. B. Sanchez, K. Dimopoulos and D. H. Lyth, arXiv:hep-ph/0608299.

[77] A. R. Liddle, D. Parkinson, S. M. Leach and P. Mukherjee, Phys. Rev. D 74, 083512 (2006).

[78] Y. Yamada, Phys. Rev. D 50, 3537 (1995) [arXiv:hep-ph/9401241].

[79] K. Enqvist, A. Jokinen and J. McDonald, Phys. Lett. B 483, 191 (2000) [arXiv:hep-ph/0004050].
[80] M. Dine, W. Fischler, and D. Nemeschansky, Phys. Lett. B 136, 169 (1984); G. D. Coughlan, R. Holman, P. Ramond, and G. G. Ross, Phys. Lett. B 140, 44 (1984); A. S. Goncharov, A. D. Linde, and M. I. Vysotsky, Phys. Lett. B 147, 279 (1984); O. Bertolami, and G. G. Ross, Phys. Lett. B 183, 163 (1987); E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart, and D. Wands, Phys. Rev. D 49, 6410 (1994) [arXiv:astro-ph/9401011].

[81] K. Enqvist, L. Mether and S. Nurmi, arXiv:0706.2355 [hep-th].

[82] M. K. Gaillard, H. Murayama and K. A. Olive, Phys. Lett. B 355, 71 (1995) [arXiv:hep-ph/9504307].

[83] R. Allahverdi, M. Drees and A. Mazumdar, Phys. Rev. D 65, 065010 (2002) [arXiv:hep-ph/0110136].

[84] R. Allahverdi and A. Mazumdar, JCAP 0610, 008 (2006) [arXiv:hep-ph/0512227].

[85] R. Allahverdi, B. A. Campbell and J. R. Ellis, Nucl. Phys. B 579, 355 (2000) [arXiv:hep-ph/0001122].

[86] M. Postma and A. Mazumdar, JCAP 0401, 005 (2004) [arXiv:hep-ph/0304246].

[87] R. Allahverdi and A. Mazumdar, arXiv:hep-ph/0608296.

[88] G. N. Felder, J. Garcia-Bellido, P. B. Greene, L. Kofman, A. D. Linde and I. Tkachev, Phys. Rev. Lett. 87, 011601 (2001) [arXiv:hep-ph/0012142].

[89] J. Garcia-Bellido and E. Ruiz Morales, Phys. Lett. B 536, 193 (2002) [arXiv:hep-ph/0109230].

[90] J. H. Traschen and R. H. Brandenberger, Phys. Rev. D 42, 2491 (1990). L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. 73, 3195 (1994) [arXiv:hep-th/9405187]; Y. Shtanov, J. H. Traschen and R. H. Brandenberger, Phys. Rev. D 51, 5438 (1995) [arXiv:hep-ph/9407247]. L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D 56, 3258 (1997) [arXiv:hep-ph/9704452].

[91] D. Cormier, K. Heitmann and A. Mazumdar, Phys. Rev. D 65, 083521 (2002) [arXiv:hep-ph/0105236].

[92] G. N. Felder, L. Kofman and A. D. Linde, Phys. Rev. D 59, 123523 (1999) [arXiv:hep-ph/9812289].

[93] R. Allahverdi, R. Brandenberger and A. Mazumdar, Phys. Rev. D 70, 083535 (2004) [arXiv:hep-ph/0407230].
[94] R. Allahverdi and A. Mazumdar, arXiv:hep-ph/0505050.

[95] R. Allahverdi and A. Mazumdar, arXiv:hep-ph/0603244. R. Allahverdi, Phys. Rev. D 62, 063509 (2000) [arXiv:hep-ph/0004035]. S. Davidson and S. Sarkar, JHEP 0011, 012 (2000) [arXiv:hep-ph/0009078]. P. Jaikumar and A. Mazumdar, Nucl. Phys. B 683, 264 (2004) [arXiv:hep-ph/0212265]. R. Allahverdi and M. Drees, Phys. Rev. D 66, 063513 (2002) [arXiv:hep-ph/0205246].

[96] J. R. Ellis, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B 145, 181 (1984).

[97] M. Bolz, A. Brandenburg and W. Buchmüller, Nucl. Phys. B 606, 518 (2001) [arXiv:hep-ph/0012052].

[98] A. L. Maroto and A. Mazumdar, Phys. Rev. Lett. 84, 1655 (2000) [arXiv:hep-ph/9904206].

[99] G. F. Giudice, A. Riotto and I. Tkachev, JHEP 9911, 036 (1999) [arXiv:hep-ph/9911302]. G. F. Giudice, I. Tkachev and A. Riotto, JHEP 9908, 009 (1999) [arXiv:hep-ph/9907510]. R. Kallosh, L. Kofman, A. D. Linde and A. Van Proeyen, Phys. Rev. D 61, 103503 (2000) [arXiv:hep-th/9907124]. M. Bastero-Gil and A. Mazumdar, Phys. Rev. D 62, 083510 (2000) [arXiv:hep-ph/0002004].

[100] R. Allahverdi, M. Bastero-Gil and A. Mazumdar, Phys. Rev. D 64, 023516 (2001) [arXiv:hep-ph/0012057]. H. P. Nilles, M. Peloso and L. Sorbo, Phys. Rev. Lett. 87, 051302 (2001) [arXiv:hep-ph/0102264]. H. P. Nilles, M. Peloso and L. Sorbo, Phys. Rev. Lett. 87, 051302 (2001) [arXiv:hep-ph/0102264].

[101] K. Enqvist, S. Kasuya and A. Mazumdar, Phys. Rev. Lett. 89, 091301 (2002) [arXiv:hep-ph/0204270]. K. Enqvist, S. Kasuya and A. Mazumdar, Phys. Rev. D 66, 043505 (2002) [arXiv:hep-ph/0206272].

[102] M. Alam et al., Phys. Rev. Lett 74, 2885 (1995).

[103] Particle Data Group, S. Eidelman et al., Phys. Lett. B 592, 1(2004).

[104] Muon $g-2$ Collaboration, G. Bennett et al., Phys. Rev. Lett. 92, 161802 (2004); S. Eidelman, Talk at ICHEP 2006, Moscow, Russia.

[105] R. Arnowitt et al., arXiv:hep-ph/0608193; R. Arnowitt, B. Dutta, T. Kamon, N. Kolev and D. Toback, Phys. Lett. B 639, 46 (2006).

[106] P. Minkowski, Phys. lett. B367 , 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, Supergravity (P. van Nieuwenhuizen et al. eds.), North Holland,
Amsterdam, 1980, p. 315; T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe* (O. Sawada and A. Sugamoto, eds.), KEK, Tsukuba, Japan, 1979, p. 95; S. L. Glashow, *The future of elementary particle physics*, in *Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons* (M. Lévy et al. eds.), Plenum Press, New York, 1980, pp. 687; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[107] For review, see, e.g., R. N. Mohapatra *et al.*, arXiv:hep-ph/0510213.

[108] A. de Gouvea, Phys. Rev. D **72**, 033005 (2005).

[109] T. Asaka, S. Blanchet and M. Shaposhnikov, Phys. Lett. B **631**, 151 (2005);

[110] A. de Gouvea, T. Moroi and H. Murayama, Phys. Rev. D **56**, 1281 (1997).

[111] V. A. Rubakov and M. E. Shaposhnikov, Usp. Fiz. Nauk **166**, 493 (1996) [Phys. Usp. **39**, 461 (1996)];

[112] J. Garcia-Bellido, D. Y. Grigoriev, A. Kusenko and M. E. Shaposhnikov, Phys. Rev. D **60** (1999) 123504.

[113] F. D. Steffen, arXiv:hep-ph/0605306.

[114] R. Allahverdi, S. Hannestad, A. Jokinen, A. Mazumdar and S. Pascoli, arXiv:hep-ph/0504102.

[115] M. R. Douglas, Comptes Rendus Physique **5**, 965 (2004) [arXiv:hep-th/0409207].

[116] J. Kumar, Int. J. Mod. Phys. A **21**, 3441 (2006) [arXiv:hep-th/0601053].

[117] L. F. Abbott, Phys. Lett. B **150**, 427 (1985).

[118] R. Bousso and J. Polchinski, JHEP **0006**, 006 (2000) [arXiv:hep-th/0004134].

[119] B. Freivogel and L. Susskind, Phys. Rev. D **70**, 126007 (2004) [arXiv:hep-th/0408133].

[120] S. H. Henry Tye, arXiv:hep-th/0611148.

[121] K. Freese, J. T. Liu and D. Spolyar, arXiv:hep-th/0612056.

[122] T. Banks, arXiv:hep-th/0211160.

[123] T. Clifton, A. Linde and N. Sivanandam, JHEP **0702**, 024 (2007) [arXiv:hep-th/0701083].
[124] J. Garriga, Phys. Rev. D 49, 6327 (1994) [arXiv:hep-ph/9308280].

[125] A. R. Frey, M. Lippert and B. Williams, Phys. Rev. D 68, 046008 (2003) [arXiv:hep-th/0305018].

[126] S. R. Coleman and F. De Luccia, Phys. Rev. D 21, 3305 (1980).

[127] J. D. Brown and C. Teitelboim, Phys. Lett. B 195 (1987) 177.

[128] J. D. Brown and C. Teitelboim, Nucl. Phys. B 297, 787 (1988).