Observational constraints on G-corrected holographic dark energy using a Markov chain Monte Carlo method

Hamzeh Alavirad and Mohammad Malekjani

1 Institute for Theoretical Physics, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany
2 Department of Physics, Faculty of Science, Bu-Ali Sina University, Hamedan 65178, Iran

We constrain holographic dark energy (HDE) with time varying gravitational coupling constant in the framework of the modified Friedmann equations using cosmological data from type Ia supernovae, baryon acoustic oscillations, cosmic microwave background radiation and X-ray gas mass fraction. Applying a Markov Chain Monte Carlo (MCMC) simulation, we obtain the best fit values of the model and cosmological parameters within 1σ confidence level (CL) in a flat universe as: \( \Omega_{b} h^2 = 0.022^{+0.0018}_{-0.0010} \), \( \Omega_{c} h^2 = 0.1121^{+0.0110}_{-0.0079} \), \( \alpha_G \equiv \dot{G}/(HG) = 0.1647^{+0.347}_{-0.2971} \) and the HDE constant \( c = 0.9322^{+0.5497}_{-0.5497} \). Using the best fit values, the equation of state of the dark component at the present time \( w_{0d} \) at 1σ CL can cross the phantom boundary \( w = -1 \).

**keywords:** Cosmology, dark energy, holographic model, gravitational constant.

I. INTRODUCTION

The astronomical data from "Type Ia supernova" [1, 2] indicate that the current universe is in an accelerating phase. These observational results have greatly inspired theorists to understand the mechanism of this accelerating expansion. In the framework of standard cosmology, an exotic energy with negative pressure, the so-called dark energy, is attributed to this cosmic acceleration.

Up to now, some theoretical models have been presented to explain the dynamics of dark energy and cosmic acceleration of the universe. The simplest but most natural candidate is the cosmological constant \( \Lambda \), with a constant equation of state (EoS) \( w = -1 \) [3, 4]. As we know, the cosmological constant confronts us with two difficulties: the fine-tuning and cosmic coincidence problems. In order to solve or alleviate these problems many dynamical dark energy models with time-varying EoS have been proposed. The quintessence [5, 6], phantom [7–9], quintom [10–12], K-essence [13, 14], tachyon [15, 16], ghost condensate [17, 18], agegraphic [19, 20] and holographic [21] are examples of dynamical models. Although many dynamical dark energy models have been suggested, the nature of dark energy is still unknown.

Models which are constructed based on fundamental principles are more preferred as they may exhibit some underlying features of dark energy. Two examples of such kind of dark energy models are the agegraphic [19, 20] and holographic [22, 23] models. In this work we focus on the holographic dark energy model. The holographic model is built on the basis of the holographic principle and some features of quantum gravity theory [21]. According to the holographic principle, the number of degrees of freedom in a bound system should be finite and is related to the area of its boundary. In holographic principle, a short distance ultra-violet (UV) cut-off is related to the vacuum energy and the IR cut-off is related to the large scale of the universe, such as Hubble horizon, particle horizon, event horizon, Ricci scalar or the generalized functions of dimensionless variables as discussed by [22, 23, 24, 25]. If we consider the Hubble length scale for \( L \), it leads to wrong equation of state for dark energy, i.e., \( w \approx 0 \) which can not result the cosmic acceleration [24, 25]. This problem can be cured by considering the interaction between dark matter and dark energy [26, 27]. In the case of particle horizon, the EoS of dark energy
is bigger than $-1/3$, hence the current accelerated expansion can not be well explained. Holographic dark energy with event horizon can provide a desired EoS to describe the cosmic acceleration. Nojiri and Odintsov investigated the HDE model by assuming IR cutoff depends on the Hubble rate, particle and future horizons. In this generalized form the phantom regime can be achieved and also the coincidence problem is demonstrated. Unification of early phantom inflation and late time acceleration of the universe is another feature of this model.

In recent years, the HDE model has been constrained by various cosmological observations. For example, Huang and Gong obtained the parameter $c$ as $c = 0.21$ by using the SNIa observations. Enqvist et al. found a connection between the holographic dark energy and low-l CMB multipoles by using CMB, LSS and supernovae data. Zhang et al. by using the OHD data constrained the parameter $c$ as $c = 0.65$.

Beside, there are some theoretical and observational evidences indicating that the gravitational coupling constant $G$ varies with cosmic time $t$. From the theoretical viewpoint one can be referred to the works of Dirac and Dyson. In Brans-Dicke theory, the variability of $G$ is also predicted. In Kaluza-Klein cosmology, time varying treatment of $G$ is related to the scalar field appearing in the metric component corresponding to the 5-th dimension. In this case, a scalar field couples with gravity by definition of a new parameter. From observational point of view, the value of the parameter $G/G$ (where an overdot represents derivative with respect to the cosmic time $t$) can be constrained by astrophysical and cosmological observations as well. For example data from SNIa observations yields $-10^{-11}yr^{-1} \leq \dot{G}/G \leq 0$.

The observations of the Binary Pulsar PSR1913 gives $-(1.10 \pm 1.07) \times 10^{-11}yr^{-1} \leq \dot{G}/G \leq 0$. The observational data from the Big Bang nuclei-synthesis results tighter constraints on this parameter as $-3.0 \times 10^{-13}yr^{-1} \leq \dot{G}/G \leq 4.0 \times 10^{-14}yr^{-1}$. This parameter can be approximated from astro-seismological data from pulsating white dwarf stars and helio-sesmiological as well.

All mentioned above motivated people to consider the holographic dark energy model with time varying gravitational coupling $G$ (G-corrected HDE model) enveloped by event horizon. In the standard Friedmann equations has been constrained by cosmological data in where for a flat universe they found $c = 0.80^{+0.16}_{-0.13}$ and $\alpha_0 \equiv \dot{G}/(HG) = -0.0016^{+0.0049}_{-0.0009}$. In this paper, by using the cosmological data of Type Ia Supernovae (SNIa), Baryon Acoustic Oscillations (BAO), Cosmic Microwave Background (CMB) radiation and X-ray gas mass fraction we will obtain the best fit values of parameters of the G-corrected HDE in the framework of the modified Friedmann equations. The evolution of EoS and deceleration parameter $q$ of the G-corrected HDE model as well as the evolution of energy density parameters. We show that within $1\sigma$ confidence level, this model can cross the phantom boundary $w = -1$.

The paper is organized as follows: In section II the G-corrected HDE is discussed briefly. Then in section III the cosmological constraining method is discussed in detail and the data fitting results are presented in section IV. The paper is concluded in section V.

II. G-CORRECTED HDE MODEL IN A FRW COSMOLOGY

The Hilbert-Einstein action with time varying gravitational coupling constant, $G(t) = G_0\phi(t)$, is described as

$$S = \frac{1}{16\pi G_0} \int \sqrt{-g} \left( R \left( \frac{\phi(t)}{G_0} \right) + L_m \right) d^4x$$

where the scalar function $\phi(t)$ is assumed for time dependency of $G(t) = \phi(t)G_0$, $G_0$ is the bare gravitational coupling constant and $L_m$ is the lagrangian of the matter fields. The first modified Friedmann equation for Robertson-Walker spacetime is obtained as

$$H^2 = \frac{8\pi G(t)}{3}(\rho_m + \rho_d) + \frac{\dot{G}}{G}$$

where an overdot represents the derivative with respect to the cosmic time $t$, $H = \dot{a}/a$ and $\rho_m$ and $\rho_d$ are matter and dark energy densities respectively. We ignore the higher time derivative of $G$ (i.e., $G/G^2$, ...) and also higher powers than one (i.e., $(G/G^2, ...)$), since the value of $G/G$ is small particularly in the late time accelerated universe. The last term on right hand side of (2.2) is due to the correction of time dependency of $G$. Equation (2.2) can also be obtained from Brans-Dicke gravity by assuming $w = 0$ and $\psi = 1/\phi(t)$ in equation (1) of [61] where $w$ is the Brans-Dicke
parameter and ψ is Brans-Dicke scalar field.

Changing the time derivative to a derivative with respect to ln a, equation (2.2) is expressed as:

\[ H^2(1 - \alpha_G) = \frac{8\pi G(t)}{3}(\rho_m + \rho_d), \]

where \( \alpha_G = \dot{G}/G \) and the prime represents derivative with respect to ln a. Putting \( \alpha_G = 0 \) and \( G(t) = G_0 \), equation (2.3) reduces to the standard Friedmann equation in flat universe.

The energy density of the G-corrected HDE model, by assuming the event horizon IR cut-off \( R_h = a \int \frac{dt}{a} = a \int \frac{H}{a} d\alpha \), is given by

\[ \rho_m + \rho_\alpha = 1 - \alpha_G. \]  

The matter (baryonic and CDM) and dark energy satisfy the following conservation equation

\[ \dot{\rho}_m + 3H\rho_m = 0, \]

\[ \dot{\rho}_d + 3H(1 + w_d)\rho_d = 0, \]

respectively, where \( w_d \) is the dark energy EoS. The Hubble parameter in the context of G-corrected HDE model in a flat geometry can be calculated from Eq. (2.2) as follows

\[ H^2(1 - \alpha_G) = H_0^2[\Omega_{m0}a^{-3} + \Omega_{d0}a^{-3(1+w_d)}], \]

where \( H_0 \) is the present value of Hubble parameter and \( \Omega_{m0} \) and \( \Omega_{d0} \) are the present values of the density parameters of matter (baryonic and CDM) and dark energy respectively. Taking the time derivative of (2.4) by using conservation equation (2.6b) as well as the relation \( \dot{R}_h = 1 + HR_h \), the equation of state for the G-corrected HDE model can be obtained as

\[ w_d = \frac{1}{3} - \frac{2}{3} \frac{\sqrt{\Omega_d}}{c} + \frac{1}{3} \alpha_G. \]

The evolutionary equation of the dark energy density parameter \( \Omega_d \) for the G-corrected HDE model can be obtained by taking derivative of \( \Omega_d = \frac{\rho_d}{\rho_{cr}} = \frac{c^2}{H^2R_c^2} \) with respect to ln a as follows

\[ \dot{\Omega}_d = -2\Omega_d\left[ \frac{c}{HR} + \frac{\dot{H}}{H^2} + 1 \right]. \]

Also, taking the time derivative of the modified Friedmann equation (2.2) yields

\[ \frac{\dot{H}}{H^2}(1 - \frac{1}{2}\alpha_G) = -\frac{3}{2}(1 + w_d\Omega_d) + 2\alpha_G. \]

Inserting (2.10) in (2.11) results

\[ \dot{\Omega}_d(1 - \alpha_G/2) = \Omega_d\left( 3(1 + w_d\Omega_d) + \frac{\sqrt{\Omega_d}}{c}(2 - \alpha_G) - 3\alpha_G - 2 \right). \]

The deceleration parameter \( q = -1 - \dot{H}/H^2 \) for determining the accelerated phase of the expansion (\( q < 0 \)) or decelerated phase (\( q > 0 \)) can be obtained for the G-corrected HDE model by using (2.10) as

\[ q(1 - \frac{1}{2}\alpha_G) = \frac{1}{2}(1 + 3w_d\Omega_d) - \frac{3}{2} \alpha_G. \]

At early times when the energy density of dark energy tends to zero and also the correction of G is negligible, one can see that \( q \rightarrow 1/2 \), representing deceleration phase in the CDM model. In the limiting case of time-independent gravitational constant G (i.e., \( \alpha_G = 0 \)) all the above relations reduce to those obtained for original holographic dark energy (OHDE) model in \[62\].
III. DATA FITTING METHOD

The constant $c$ and the quantity $\dot{G}/G$ determine evolution of the universe in the G-corrected holographic dark energy model. Therefore to study the cosmic evolution in the G-corrected HDE in the framework of the modified Friedmann equations, it is of great importance to constrain these parameters by cosmological data.

In this section we discuss the method for obtaining the best fit values of the G-corrected HDE parameters by using the cosmological data. The fitting method which we use is the maximum likelihood method. In this method the total likelihood function $L_{\text{tot}} = e^{-\chi^2_{\text{tot}}}/2$ is maximized by minimizing $\chi^2_{\text{tot}}$. To determine $\chi^2_{\text{tot}}$ we use the following observational data set: cosmic microwave background radiation (CMB) data from the seven-year WMAP [63], type Ia supernova (SNIa) data from 557 Union2 [64], baryon acoustic oscillation (BAO) data from SDSS DR7 [65], and cluster X-ray gas mass fraction data which is measured by Chandra X-ray telescope observations [66]. Therefore $\chi^2_{\text{tot}}$ is given by the relation

$$\chi^2_{\text{tot}} = \chi^2_{\text{SNIa}} + \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}} + \chi^2_{\text{gas}}. \quad (3.1)$$

In following we discuss each $\chi^2$ in detail.

The data for SNIa are 557 Union2 data [64]. In this case $\chi^2_{\text{SNIa}}$ is obtained by comparing the theoretical distance modulus $\mu_{\text{th}}(z)$ with the observed one $\mu_{\text{obs}}(z)$.

$$\chi^2_{\text{SNIa}} = \sum_i \left[ \frac{[\mu_{\text{th}}(z_i) - \mu_{\text{obs}}(z_i)]^2}{\sigma_i^2} \right], \quad (3.2)$$

with

$$\mu_{\text{th}}(z) = 5 \log_{10}[D_L(z)] + \mu_0, \quad (3.3)$$

where $\mu_0 = 5 \log_{10}(cH_0^{-1}/\text{Mpc}) + 25$ and the observational modulus distance of SNIa, $\mu_{\text{obs}}(z_i)$, at redshift $z_i$ is given by

$$\mu_{\text{obs}}(z_i) = m_{\text{obs}}(z_i) - M, \quad (3.4)$$

where $m$ and $M$ are apparent and absolute magnitudes of SNIa respectively. The Hubble-free luminosity distance $D_L$ is given by

$$D_L(z) = \frac{H_0(1+z)}{\sqrt{|\Omega_k|}} \sin\left[\sqrt{|\Omega_k|} \int_0^z dz' H(z')\right], \quad (3.5)$$

where $\sin\left[\sqrt{|\Omega_k|} x\right]$ represents respectively $\sin(\sqrt{|\Omega_k|} x)$, $\sqrt{|\Omega_k|}$ and $\sinh(\sqrt{|\Omega_k|} x)$ for $\Omega_k < 0$, $\Omega_k = 0$ and $\Omega_k > 0$. Eq. (3.2) can be written [67]

$$\chi^2_{\text{SNIa}} = A + 2B\mu_0 + C\mu_0^2, \quad (3.6)$$

where

$$A = \sum_i \left[ \frac{[\mu_{\text{th}}(z_i; \mu_0 = 0) - \mu_{\text{obs}}(z_i)]^2}{\sigma_i^2} \right] \quad (3.7a)$$

$$B = \sum_i \left[ \frac{\mu_{\text{th}}(z_i; \mu_0 = 0) - \mu_{\text{obs}}(z_i)}{\sigma_i^2} \right] \quad (3.7b)$$

$$C = \sum_i \frac{1}{\sigma_i^2} \quad (3.7c)$$

where $\mu_0 = 42.384 - 5 \log_{10} h$. The minimum of eq. (3.6) can be written as

$$\chi^2_{\text{SNIa, min}} = A - B^2/C. \quad (3.8)$$

The goodness of fit between the theoretical model and data is expressed by $\chi^2_{\text{SNIa, min}}$. 

4
For the CMB data, we use the data points \((R, l_a, z_*)\) from seven-year WMAP \[63\]. The data points parameters are as follows: \(R\) is the scaled distance to recombination \(R = \sqrt{\Omega_m / c} \int_0^{z_*} dz' / E(z')\), where \(E(z) = H(z) / H_0\) and \(z_*\) is recombination redshift \[68\]. The angular scale of the sound horizon at recombination is given by \(l_A \[69\]
\[
l_A = \frac{\pi r(z_*)}{r_s(z_*)},
\] (3.9)
where \(r(z)\) is the comoving distance \(r(z) = c / H_0 \int_0^z dz' / E(z')\) and the comoving sound horizon distance at the recombination \(r_s(z_*)\) is given by
\[
r_s(z_*) = \int_0^{a(z_*)} \frac{c_s(a)}{a^2 H(a)} da,
\] (3.10)
where the sound speed \(c_s(a)\) is defined by
\[
c_s(a) = \left[ 3(1 + 3\Omega_b h^2) \frac{d\Omega_b}{4\Omega_b} a \right]^{-1/2},
\] (3.11)
Seven-year WMAP observations give \(\Omega_m = 0.249 \times 10^{-5} h^{-2}\) and \(\Omega_b h = 0.02260 \pm 0.00053 \times 10^{-5} h^{-2} \[63\].
The recombination redshift \(z_*\) is obtained using the fitting function proposed by Hu and Sugiyama \[68\]
\[
z_* = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}][1 + g_1(\Omega_b h^2)^{0.1}] ,
\] (3.12)
where \(g_1 = (0.0783(\Omega_b h^2)^{-0.238})/(1 + 39.5(\Omega_b h^2)^{0.763})\) and \(g_2 = 0.560) / (1 + 21.1(\Omega_b h^2)^{1.81}).\) Then one can define \(\chi_{CMB}^2\) as \(\chi_{CMB}^2 = X^T C_{CMB}^{-1} X\), with \[63\]
\[
X = \begin{pmatrix}
   l_A - 302.09 \\
   R - 1.725 \\
   z_* - 1091.3
\end{pmatrix},
\] (3.13a)
\[
C_{CMB}^{-1} = \begin{pmatrix}
   2.305 & 29.698 & -1.333 \\
   293689 & 6825.270 & -113.180 \\
   -1.333 & -113.180 & 3.414
\end{pmatrix},
\] (3.13b)
where \(C_{CMB}^{-1}\) is the inverse covariant matrix.
The data from Sloan Digital Sky Survey (SDSS) Data Release 7 (DR7) \[65\] is used for the baryon acoustic oscillations (BAO) data. One can define \(\chi_{BAO}^2\) by \(\chi_{BAO}^2 = Y^T C_{BAO}^{-1} Y\), where
\[
Y = \begin{pmatrix}
   d_{0.2} - 0.1905 \\
   d_{0.35} - 0.1097
\end{pmatrix},
\] (3.14a)
\[
C_{BAO}^{-1} = \begin{pmatrix}
   30124 & -17227 \\
   -17227 & 86977
\end{pmatrix}.
\] (3.14b)
The data points \(d_{z_*} = r_s(z_*)/D_V(z_*)\), where \(r_s(z_*)\) is the comoving sound horizon distance at the drag epoch (where baryons were released from photons) and \(D_V\) is given by \[70\]
\[
D_V(z) = \left( \int_0^z \frac{dz'}{H(z')} \right)^2 \frac{cz}{H(z)}^{1/3},
\] (3.15)
The drag redshift is given by the fitting formula \[71\]
\[
z_d = \frac{1291(\Omega_m h^2)^{0.251}}{1 + 0.659(\Omega_m h^2)^{0.826}} \left[ 1 + b_1(\Omega_m h^2)^{b_2} \right] ,
\] (3.16)
where \(b_1 = 0.313(\Omega_m h^2)^{-0.419}[1 + 0.607(\Omega_m h^2)^{0.607}]\) and \(b_2 = 0.238(\Omega_m h^2)^{0.223}\).
The final data we use is X-ray gas mass fraction data from the Chandra X-ray observations \[66\]. In this case we use the definition $\chi^2_{\text{gas}}$

\[
\chi^2_{\text{gas}} = \sum \frac{(f_{\text{gas}}(z) - f_{\text{gas}}^\Lambda\text{CDM}(z))^2}{\sigma_{f_{\text{gas}}}(z)} + \frac{(s_0 - 0.16)^2}{0.0016^2} + \frac{(K - 1.0)^2}{0.01^2} + \frac{(\eta - 0.214)^2}{0.022^2},
\]

(3.17)

where $s_0 = (0.16 \pm 0.05)h_{0.5}^2$, $K = 1.0 \pm 0.1$ and $\eta = 0.214 \pm 0.022$ \[66\]. The details for the from of the mass gas fractions $f_{\text{gas}}(z)$ and $f_{\text{gas}}^\Lambda\text{CDM}$ is discussed in \[66\].

IV. DATA FITTING RESULTS

Finally we apply a Markov Chain Monte Carlo simulation on the G-corrected HDE model by modifying the publically available CosmoMC code \[72\]. The parameter space is chosen as $(\Omega_b h^2, \Omega_c h^2, \alpha_G, c)$ with the priors $\Omega_b h^2 = [0.005, 0.1], \Omega_c h^2 = [0.01, 0.99], \alpha_G = [-1, +1]$ and $c = [0, 2]$. We also consider the derived parameters $(\Omega_d, H_0, \text{age})$ as well. The results of the best fit values are presented in table I. In addition figure 1 shows the 2-dimensional constraints of the cosmological parameters contours with 1$\sigma$ and 2$\sigma$ confidence levels.

From table I one can see that all main cosmological parameters $(\Omega_b h^2, \Omega_c h^2, \Omega_d, H_0, \text{age})$ are in agreement with the results of the $\Lambda$CDM model \[73\] as one can see in the third column. The best fit value of the parameter $c$ i.e. $c = 0.9322 \pm 0.4569$ is also compatible with other works such as $c = 0.91^{+0.21}_{-0.13}$ in \[40\], $c = 0.84^{+0.14}_{-0.12}$ in \[43\] and $c = 0.68^{+0.03}_{-0.02}$ in \[42\]. Then by using the best fit values of parameters $\alpha_G$ and $H_0$ one can obtain approximately the best fit value of quantity $\dot{G}/G = +1.14 \times 10^{-11}\text{yr}^{-1}$. This results is in agreement with the results of other constraining works. For example the astroseismological data obtained from pulsating white dwarf stars result $-2.5 \times 10^{-10}\text{yr}^{-1} \leq \dot{G}/G \leq +4.5 \times 10^{-10}\text{yr}^{-1}$ \[56\] and observations of the pulsating white dwarf G117-B15A suggest $\dot{G}/G \leq +4.1 \times 10^{-11}\text{yr}^{-1}$ \[57\]. Therefore these two best values offer a self-consistency for our analysis. Lu et.al. in \[41\] constrained HDE with varying gravitational coupling constant by using SNIa, CMB, BAO and OHD (Observational Hubble Data) data in the standard Friedmann equations framework. They found the best fit values: $c = 0.80^{+0.16}_{-0.13}$ and $\alpha_G = -0.0016^{+0.0019}_{-0.0019}$. Our results in 1$\sigma$ CL are comparable with the Lu et. al. results as well.

Then we calculate the evolution of some cosmological quantities: EoS parameter of the dark energy component $w_d$, matter and dark energy density parameters, and deceleration parameter for the G-corrected HDE model based on the best fit values of cosmological parameters in table I. In the top-row of figure 2 the evolution of the EoS parameter $w_d$ (left panel) and the deceleration parameter $q$ (right panel) in terms of the redshift parameter $z$ has been plotted by solving equations (2.11) and (2.12) and using (2.8). We see that by using the best fit values in the G-corrected HDE model, within 1$\sigma$ confidence level, one obtains the present value of EoS parameter as: $-1.887 < w_d < -0.232$ which can enters to the phantom regime in lower bound. It is worthwhile to mention that in this case the phantom regime can be achieved without invoking interaction between dark matter and dark energy. In the left panel, the parameter $q$ can transit from positive values $q > 0$ to negative values ($q < 0$) which indicates the transition from early decelerated expansion to current accelerated phase of expansion. The present value of the deceleration parameter $q$ within 1$\sigma$ confidence level is obtained as: $-1.1268 < q_0 < -0.5565$. Finally, the evolution of density parameters of dark energy and pressure-less matter has been shown in the bottom row figure 2. The density parameter of the pressureless matter

### Table I
- The best fit values of the cosmological and model parameters in the G-corrected HDE model with 1$\sigma$ and 2$\sigma$ regions. Here the CMB, SNIa, BAO and X-ray gas mass fraction data together with the BBN constraints have been used.

| Parameter | Best Fit Value | $\Lambda$CDM |
|-----------|----------------|-------------|
| $\Omega_b h^2$ | 0.0222, -0.0013, 0.0046 | 0.02214 ± 0.00024 |
| $\Omega_c h^2$ | 0.1121, 0.0679, 0.0986 | 0.1187 ± 0.0017 |
| $\Omega_d$ | 0.7246, -0.0485, -0.0606 | 0.692 ± 0.010 |
| $c$ | 0.9322, -0.5447 | ... |
| $\alpha_G$ | 0.1647, 0.2971, -0.9278 | ... |
| $H_0$ | 69.8809, 1.4343, -1.457 | 67.80 ± 0.77 |
| Age (Gyr) | 13.8094, -0.3618, -0.4392 | 13.798 ± 0.45 |
V. CONCLUSION

We performed cosmological constrains on the parameters of the holographic dark energy model with time varying gravitational coupling $G$ using a Markov chain Monte Carlo simulation. We used the SNIa, CMB, BAO and X-ray mass gas fraction data for data fitting. In the framework of the modified Friedmann equations, we obtained the best fit values for the cosmological parameters as: the physical baryon matter density $\Omega_b h^2 = 0.0222^{+0.0018}_{-0.0013}$, dark matter physical density $\Omega_c h^2 = 0.1121^{+0.0110}_{-0.0079}$, Hubble parameter at the current time $H_0 = 69.8809^{+3.5339}_{-3.4423}$, and the age of the Universe $13.8094^{+0.2801}_{-0.3618}$. We constrained the G-corrected HDE parameters $c$ and $\alpha_G$ as well. The best fit value of the parameter $c = 0.9322^{+0.4569}_{-0.5447}$ is in agreement with results of the previous works $[41, 42]$. In our model the best fit value for the rate of changing the gravitational coupling constant with time is $\dot{G}/G = 1.14 \times 10^{-11}\text{yr}^{-1}$. This value is close to the value obtained by others like constraints in $[56, 57]$. Therefore the result of our analysis is compatible with observations and other analysis of the HDE model and time varying gravitational coupling constant.

The evolution of the deceleration parameter $q$, for the best fit values of cosmological parameters, indicates the transition from past decelerated to current accelerated expansion. By using the best fit values of the aforementioned parameters, within $1\sigma$ CL, the phantom regime $w < -1$ can be achieved in this model.

In summary we conclude that the holographic dark energy with a time varying gravitational coupling constant in the framework of the modified Friedmann equations, could be a candidate to describe the accelerated expansion of the universe. In addition, in future works, by using the data from Planck $[73]$ and nine-year WMAP $[74]$ projects,
one can make the constraints on the model parameters even tighter.

ACKNOWLEDGEMENTS

H. Alavirad would like to thank J. M. Weller for helpful and useful discussions and comments.

REFERENCES

[1] **Supernova Search Team** Collaboration, A. G. Riess et al., “Observational evidence from supernovae for an accelerating universe and a cosmological constant,” *Astron.J.* **116** (1998) 1009–1038, [arXiv:astro-ph/9805201 [astro-ph]].

[2] **Supernova Cosmology Project** Collaboration, S. Perlmutter et al., “Measurements of Omega and Lambda from 42 high redshift supernovae,” *Astrophys. J.* **517** (1999) 565–586, [arXiv:astro-ph/9812133 [astro-ph]].

[3] V. Sahni and A. A. Starobinsky, “The Case for a positive cosmological Lambda term,” *Int.J.Mod.Phys.* **D9** (2000) 373–444, [arXiv:astro-ph/9904398 [astro-ph]].

[4] P. Peebles and B. Ratra, “The Cosmological constant and dark energy,” *Rev.Mod.Phys.* **75** (2003) 559–606, [arXiv:astro-ph/0207347 [astro-ph]].

[5] C. Wetterich, “Cosmology and the Fate of Dilatation Symmetry,” *Nucl.Phys.* **B302** (1988) 668.

[6] B. Ratra and P. Peebles, “Cosmological Consequences of a Rolling Homogeneous Scalar Field,” *Phys.Rev.* **D37** (1988) 3406.

[7] R. Caldwell, “A Phantom menace?,” *Phys.Lett.* **B545** (2002) 23–29, [arXiv:astro-ph/9908168 [astro-ph]].
[8] S. Nojiri and S. D. Odintsov, “Quantum de Sitter cosmology and phantom matter,” *Phys.Lett.* B562 (2003) 147–152 arXiv:hep-th/0303117 [hep-th]

[9] S. Nojiri and S. D. Odintsov, “DeSitter brane universe induced by phantom and quantum effects,” *Phys.Lett.* B565 (2003) 1–9 arXiv:hep-th/0304131 [hep-th]

[10] E. Elizalde, S. Nojiri, and S. D. Odintsov, “Late-time cosmology in (phantom) scalar-tensor theory: Dark energy and the cosmic speed-up,” *Phys.Rev.* D70 (2004) 043539 arXiv:hep-th/0405034 [hep-th]

[11] S. Nojiri, S. D. Odintsov, and S. Tsujikawa, “Properties of singularities in (phantom) dark energy universe,” *Phys.Rev.* D71 (2005) 063004 arXiv:hep-th/0501025 [hep-th]

[12] A. Anisimov, E. Babichev, and A. Vikman, “B-inflation,” *JCAP* 0506 (2005) 006 arXiv:astro-ph/0504560 [astro-ph]

[13] C. Armendariz-Picon, V. F. Mukhanov, and P. J. Steinhardt, “A Dynamical solution to the problem of a small cosmological constant and late time cosmic acceleration,” *Phys.Rev.* D85 (2000) 4438–4441 arXiv:astro-ph/0004134 [astro-ph]

[14] C. Armendariz-Picon, V. F. Mukhanov, and P. J. Steinhardt, “Essentials of k essence,” *Phys.Rev.* D63 (2001) 103510 arXiv:astro-ph/0006373 [astro-ph]

[15] T. Padmanabhan, “Accelerated expansion of the universe driven by tachyonic matter,” *Phys.Rev.* D66 (2002) 021301 arXiv:hep-th/0204150 [hep-th]

[16] A. Sen, “Tachyon matter,” *JHEP* 0207 (2002) 065 arXiv:hep-th/0203265 [hep-th]

[17] N. Arkani-Hamed, H.-C. Cheng, M. A. Luty, and S. Mukohyama, “Ghost condensation and a consistent infrared modification of gravity,” *JHEP* 0405 (2004) 074 arXiv:hep-th/0312099 [hep-th]

[18] F. Piazza and S. Tsujikawa, “Dilatonic ghost condensate as dark energy,” *JCAP* 0407 (2004) 004 arXiv:hep-th/0405054 [hep-th]

[19] R.-G. Cai, “A Dark Energy Model Characterized by the Age of the Universe,” *Phys.Lett.* B657 (2007) 228–231 arXiv:0707.4049 [hep-th]

[20] H. Wei and R.-G. Cai, “A New Model of Agegraphic Dark Energy,” *Phys.Lett.* B660 (2008) 113–117 arXiv:0708.0884 [astro-ph]

[21] E. Witten, “The Cosmological constant from the viewpoint of string theory,” arXiv:hep-th/0002297 [hep-ph]

[22] S. D. Hsu, “Entropy bounds and dark energy,” *Phys.Lett.* B594 (2004) 13–16 arXiv:hep-th/0403052 [hep-th]

[23] M. Li, “A Model of holographic dark energy,” *Phys.Lett.* B603 (2004) 1 arXiv:hep-th/0403127 [hep-th]

[24] P. Horava and D. Minic, “Probable values of the cosmological constant in a holographic theory,” *Phys.Rev.* D85 (2000) 1610–1613 arXiv:hep-th/0001145 [hep-th]

[25] S. D. Thomas, “Holography stabilizes the vacuum energy,” *Phys.Rev.* D89 (2004) 081301

[26] C. Gao, X. Chen, and Y.-G. Shen, “A Holographic Dark Energy Model from Ricci Scalar Curvature,” *Phys.Rev.* D79 (2009) 043511 arXiv:0712.1394 [astro-ph]

[27] L. Xu, J. Lu, and W. Li, “Generalized Holographic and Ricci Dark Energy Models,” *Eur.Phys.J.* C64 (2009) 89–95 arXiv:0906.0210 [astro-ph.CO]

[28] D. Pavon and W. Zimdahl, “Holographic dark energy and cosmic coincidence,” *Phys.Lett.* B628 (2005) 206–210 arXiv:gr-qc/0505020 [gr-qc]

[29] W. Zimdahl and D. Pavon, “Interacting holographic dark energy,” *Class.Quant.Grav.* 24 (2007) 5461–5478

[30] Z. Zhou, B. Wang, Y. Gong, and E. Abdalla, “The Second law of thermodynamics in the accelerating universe,” *Phys.Lett.* B652 (2007) 86–91 arXiv:0705.1264 [gr-qc]

[31] V. V. Kashyap, “Thermodynamics of interacting holographic dark energy with apparent horizon as an IR cutoff,” *Class.Quant.Grav.* 27 (2010) 025007 arXiv:0910.0510 [hep-th]

[32] S. Nojiri and S. D. Odintsov, “Unifying phantom inflation with late-time acceleration: Scalar phantom-non-phantom transition model and generalized holographic dark energy,” *Gen.Rel.Grav.* 38 (2006) 1285–1304 arXiv:hep-th/0506212 [hep-th]

[33] Q.-G. Huang and Y.-G. Gong, “Supernova constraints on a holographic dark energy model,” *JCAP* 0408 (2004) 006 arXiv:astro-ph/0403590 [astro-ph]

[34] K. Enqvist, S. Hannestad, and M. S. Sloth, “Searching for a holographic connection between dark energy and the low-l CMB multipole,” *JCAP* 0502 (2005) 004 arXiv:astro-ph/0409275 [astro-ph]

[35] J.-y. Shen, B. Wang, E. Abdalla, and R.-K. Su, “Constraints on the dark energy from the holographic connection to the small 1 CMBcmb suppression,” *Phys.Lett.* B609 (2005) 200–205 arXiv:hep-th/0412227 [hep-th]

[36] X. Zhang and F.-Q. Wu, “Constraints on holographic dark energy from Type Ia supernova observations,” *Phys.Rev.* D72 (2005) 043524 arXiv:astro-ph/0506510 [astro-ph]

[37] H.-C. Kao, W.-L. Lee, and F.-L. Lin, “CMB constraints on the holographic dark energy model,” *Phys.Rev.* D71 (2005) 123518 arXiv:astro-ph/0501487 [astro-ph]

[38] Q. Wu, Y. Gong, A. Wang, and J. Alcaniz, “Current constraints on interacting holographic dark energy,” *Phys.Lett.* B659 (2008) 34–39 arXiv:0705.1096 [astro-ph]

[39] Y.-Z. Ma, Y. Gong, and X. Chen, “Features of holographic dark energy under the combined cosmological constraints,” *Eur.Phys.J.* C60 (2009) 303–315 arXiv:0711.1641 [astro-ph]

[40] X. Zhang and F.-Q. Wu, “Constraints on Holographic Dark Energy from Latest Supernovae, Galaxy Clustering, and Cosmic Microwave Background Anisotropy Observations,” *Phys.Rev.* D76 (2007) 023502 arXiv:astro-ph/0701405 [astro-ph]
[73] Planck Collaboration Collaboration, P. Ade et al., “Planck 2013 results. XVI. Cosmological parameters,” arXiv:1303.5076 [astro-ph.CO]

[74] WMAP Collaboration Collaboration, C. Bennett et al., “Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results,” arXiv:1212.5225 [astro-ph.CO]