A Study of Holographic Dark Energy Models with Configuration Entropy

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Abstract

The holographic dark energy models provide an alternative description of dark energy. These models are motivated by the possible application of the holographic principle to the dark energy problem. In this work, we present a theoretical study of the one parameter Li holographic dark energy and the two parameter Barrow holographic dark energy models using configuration entropy of the matter distribution in the universe. The configuration entropy rate exhibits a distinct minimum at a specific scale factor that corresponds to the epoch, beyond which dark energy takes a driving role in the accelerated expansion of the universe. We find that the location of the minimum and magnitude of the entropy rate at the minimum are sensitive to the parameters of the models. We find the best fit relations between these quantities and the parameters of each model. We propose that these relations can be used to constrain the parameters of the holographic dark energy models from future observations such as the SKA. Our study suggests that the signature of a large quantum gravitational effect on the future event horizon can be detected from measurements of the configuration entropy of the matter distribution at multiple redshifts.

Key words: cosmology: theory – (cosmology:) dark energy – (cosmology:) large-scale structure of universe

1. Introduction

The current accelerated expansion of the universe remains one of the major unsolved problems in cosmology. It has been confirmed by various independent observations (Perlmutter et al. 1998; Riess et al. 1999; Komatsu et al. 2011; Ade et al. 2016) that the present universe is going through a phase of accelerating expansion which started in the recent past. The observed accelerated expansion is counter-intuitive due to the presence of matter in the universe and attractive nature of gravity. It is important to understand the driving mechanism which governs the cosmic acceleration. It is conjectured that a hypothetical component termed dark energy is responsible for this acceleration. The simplest possible candidate for dark energy is the cosmological constant, denoted as $\Lambda$ in Einstein’s equations of general relativity. The resulting model is termed the $\Lambda$CDM model. This model has been successful in explaining a large number of observations and is currently considered to be the most favored model of our universe. However, this model does not provide any insights into the physical origin of dark energy. Some recent observations point out tensions with the $\Lambda$CDM model. One of them is the famous $H_0$ tension. Some recent works (Lusso et al. 2019; Rezaei et al. 2020) concluded that the $\Lambda$CDM model is not the best fit to some data sets.

Various alternatives to $\Lambda$ such as k-essence (Armendariz-Picon et al. 2001) and rolling scalar field (Ratra & Peebles 1988; Caldwell et al. 1998) have been proposed that introduce a modification in the matter sector of Einstein’s equations. Other alternatives such as $f(R)$ gravity (Buchdahl & Lagrangians 1970) and scalar-tensor theory (Brans & Dicke 1961) modify the geometric side of the field equations of general relativity and are known as modified gravity theories. A detailed account of various alternative dark energy models can be found in Copeland et al. (2006), Amendola & Tsujikawa (2010), Nojiri & Odintsov (2011), Nojiri et al. (2017), Bamba et al. (2012). Apart from these two alternative routes, several other interesting proposals have been put forward in the literature. The backreaction (Buchert 2000), large local void (Tomita 2001; Hunt & Sarkar 2010), entropic force (Easson et al. 2011), entropy maximization (Radicella & Pavón 2012; Pavón & Radicella 2013), information storage in spacetime (Padmanabhan 2017; Padmanabhan & Padmanabhan 2017) and configuration entropy of the universe (Pandey 2017, 2019) are to name a few.

One of the most important theoretical developments in the last three decades has been the holographic principle. It was first proposed by Gerard ’t Hooft (Hooft 1993). Leonard Susskind provided a string theoretic interpretation for it (Susskind 1995) and Juan Maldacena came up with the idea of AdS/CFT correspondence (Maldacena 1999), for which many applications in different areas of physics have been found. The holographic principle states that all information contained in a volume of space can be found from the boundary of the volume. A review of the holographic principle and its connection to cosmology can be found in Bousso (2002) and Fischler & Susskind (1998), respectively. The efforts to connect the energy density of dark energy to the entropy of horizon and horizon length lead to the holographic dark energy models. The original proposal to use the holographic principle to describe dark energy came from
Cohen et al. (1999). Soon, a number of works appeared which discussed different aspects of such an effort (Hofava & Minic 2000; Thomas 2002; Hsu 2004; Li 2004). A number of works explored the possibility of generalizing the model and using holography as a potential candidate for inflation (Nojiri & Odintsov 2006; Nojiri et al. 2020; Nojiri et al. 2019; Nojiri & Odintsov 2017). The use of the Tsallis entropy formula (Tsallis & Cirto 2013) instead of the Bekenstein entropy formula (Bekenstein & Hawles 1973) lead to Tsallis holographic dark energy models (D’Agostino 2019; Chakraborty et al. 2021; Mohammadi et al. 2021). A number of works explored the interacting Ricci holographic dark energy models (Chimento et al. 2013a, 2013b; Chimento & Richarte 2013, 2011, 2012).

The recently proposed Barrow entropy formula (Barrow 2020) has led to formulation of the Barrow holographic dark energy model (Saridakis 2020) which has found many applications in cosmology (Saridakis 2020; D’Agostino et al. 2013a, 2013b; Chakraborty et al. 2021; Bhardwaj et al. 2021; Srivastava & Sharma 2021; Sheykhi 2021; Abreu et al. 2020; Pradhan et al. 2021; Sharma et al. 2021; shikha, Adhikary et al. 2021; Al Mamon et al. 2020).

Pandey (2017) showed that growth of structure in the universe starting from its initial smooth state leads to dissipation of configuration entropy. Since the rate at which structure grows depends on the cosmological model concerned, the evolution of entropy might be helpful to discern one cosmological model from another. It has been proposed that evolution of entropy might be helpful to discern one cosmological model from another. It has been proposed that evolution of configuration entropy can be used to distinguish different equations of state of dynamical dark energy (Das & Pandey 2019), to determine the mass density parameter and cosmological constant (Pandey & Das 2019), to constrain the parameters of the equation of state of dynamical dark energy (Das & Pandey 2020) and to determine the functional form of large scale linear bias of neutral hydrogen (H I) distribution (Das & Pandey 2021). In this work, we consider some holographic dark energy models and study the evolution of configuration entropy in those models. We examine how the evolution of entropy rate depends on the model and explore if the values of the model parameters can be constrained from the evolution of entropy.

2. Theory

2.1. Evolution of Configuration Entropy

Observations suggest that the universe is homogeneous and isotropic at large scales, but the universe is highly inhomogeneous and anisotropic at small scales due to formation of nonlinear structures. We choose a large enough comoving volume $V$ of the universe such that the universe is nearly homogeneous and isotropic at that length scale. We divide the volume into subvolumes $dV$. In each of these subvolumes, we denote the density of matter as $\rho(x, t)$. The density is usually defined at the center of the subvolume having comoving coordinate $x$ with respect to arbitrary origin and the density may change with time. In such a case, we can consider the matter density field as a random field and we can define the configuration entropy of the matter density field, following Shannon (1948), as Pandey (2017)

$$S_c(t) = -\int \rho(x, t) \log \rho(x, t) dV.$$  (1)

The matter distribution in the universe can be treated as an ideal fluid to a good approximation. The continuity equation of that fluid in an expanding universe is

$$\frac{\partial \rho(x, t)}{\partial t} + 3\frac{\dot{a}}{a} \rho(x, t) + \nabla \cdot (\rho(x, t) \dot{v}) = 0.$$  (2)

Here, $a$ is the cosmological scale factor and $\dot{v}$ is the peculiar velocity of the fluid element contained in $dV$. Combining Equations (1) and (2) we get the evolution equation of configuration entropy as

$$\frac{dS_c(t)}{dt} + 3\frac{\dot{a}}{a} S_c(t) - \frac{1}{a} \int \rho(x, t)(3\dot{a} + \nabla \cdot \dot{v})dV = 0.$$  (3)

To arrive at Equation (3), we note that

$$\frac{dS_c(t)}{dt} = -\int (1 + \log \rho(x, t)) \frac{\partial \rho(x, t)}{\partial t} dV.$$  (4)

We multiply Equation (2) by $(1 + \log \rho(x, t))$ and integrate over $dV$ to get

$$\int (1 + \log \rho(x, t)) \frac{\partial \rho(x, t)}{\partial t} dV + 3\frac{\dot{a}}{a} \int (1 + \log \rho(x, t)) \rho(x, t) dV + \frac{1}{a} \int \rho(x, t)(3\dot{a} + \nabla \cdot \dot{v})dV = 0.$$  (5)

The last term in Equation (5) is $\frac{1}{a} \int (1 + \log \rho(x, t)) \nabla \cdot (\rho(x, t) \dot{v})dV$. We note that

$$\nabla \cdot (\rho(x, t) \dot{v}) = (\nabla \rho(x, t)) \cdot \dot{v} + \rho(x, t) \nabla \cdot \dot{v}.$$  

So,

$$(1 + \log \rho(x, t)) \nabla \cdot (\rho(x, t) \dot{v}) = \dot{v} \cdot \nabla \rho(x, t)(1 + \log \rho(x, t)) + \rho(x, t) \nabla \cdot \dot{v} + \rho(x, t) \nabla \cdot \dot{v} + \rho(x, t) \log \rho(x, t) \nabla \cdot \dot{v}.$$  

We write, $\nabla \rho(x, t)(1 + \log \rho(x, t)) = \nabla(\rho(x, t) \log \rho(x, t))$. So,

$$(1 + \log \rho(x, t)) \nabla \cdot (\rho(x, t) \dot{v}) = \nabla(\rho(x, t) \log \rho(x, t)) \cdot \dot{v} + (\rho(x, t) \log \rho(x, t)) \nabla \cdot \dot{v} + \rho(x, t) \nabla \cdot \dot{v} + \rho(x, t) \log \rho(x, t) \nabla \cdot \dot{v}.$$  

The first term on the left-hand side of Equation (5) is $-\frac{dS_c(t)}{dt}$, while the second term gives us $3\frac{\dot{a}}{a} \int \rho(x, t) dV$ and $3\frac{\dot{a}}{a} \int \rho(x, t) \log \rho(x, t) dV = -3\frac{\dot{a}}{a} S_c(t)$. We get from the third
where $\delta(x, a)$ is the density contrast. We further simplify Equation (3) using linear perturbation theory and get

$$\frac{dS_c(a)}{da} + 3\frac{a}{2} (S_c(a) - M) + \rho \frac{f(a)D^2(a)}{a} \int \delta^2(x) dV = 0.$$  \hspace{1cm} (8)

Here, $D(a)$ is the growing mode solution of the evolution equation of density perturbation in linear approximation and $f(a) = \frac{d \log D(a)}{d \log a} = \frac{D(a)}{d(a)}$ is the dimensionless linear growth rate. We can integrate Equation (8) to get

$$\frac{S_c(a)}{S_c(a_i)} = \frac{M}{S_c(a_i)} + \left[ 1 - \frac{M}{S_c(a_i)} \right] \left( \frac{a}{a_i} \right)^3 - \rho \int_{a_i}^{a} da' a'^3 F(a'),$$ \hspace{1cm} (9)

where $F(a') = \frac{f(a')D^2(a')}{a'}$, $a_i$ is the initial scale factor and $S_c(a_i)$ is the entropy at $a_i$. We have chosen $a_i = 10^{-3}$ throughout this work.

To get the evolution of $\frac{S_c(a)}{S_c(a_i)}$ with scale factor we can either use Equation (9) or solve Equation (8) numerically. One can get the evolution of $\frac{dS_c(a)}{da}$ with scale factor by simply using Equation (8). We require the knowledge of $D(a)$ and $f(a)$ in a given cosmological model in order to study the evolution of configuration entropy. We discuss the evolution of growing mode $D(a)$ and growth rate $f(a)$ in the next section. During the initial stages of structure formation, $D(a)$ is very small, so the evolution of $\frac{S_c(a)}{S_c(a_i)}$ in Equation (9) is almost entirely determined by $S_c(a_i)$ and $M$. These quantities do not depend on the cosmological model concerned and are free parameters of the equation. If $M > S_c(a_i)$, we expect to see a sudden rise in $\frac{S_c(a)}{S_c(a_i)}$ near $a_i$, whereas $M < S_c(a_i)$ will give rise to a sudden drop. These variations are due to the choice of initial conditions and we set $M = S_c(a_i)$ to get rid of them. We also set $\rho \int \delta^2(x) dV = 1$ in Equation (9) for simplicity. An objection regarding the definition of configuration entropy is that while Shannon entropy is dimensionless, the configuration entropy has the dimension of mass. Hence, the definition of configuration entropy is wrong. However, it is easy to make the definition of configuration entropy dimensionless. We can redefine the configuration entropy as

$$S_c(t) = -\frac{1}{M} \int \rho(x, a) \log \rho(x, a) dV.$$ \hspace{1cm} (10)

Here, $M$ is the total mass inside the comoving volume $V$. If we use Equation (10) instead of Equation (1) in our formalism, we obtain the differential equation of evolution of configuration entropy as

$$\frac{dS_c(a)}{da} + 3\frac{a}{2} (S_c(a) - M) + \rho \frac{f(a)D^2(a)}{a} \int \delta^2(x) dV = 0.$$ \hspace{1cm} (11)

We can compare Equations (11) and (8) to find that they differ by some constant. Since we are interested in the temporal evolution of $S_c(a)$, we may as well set the values of these constants to 1. In that case, the evolution of entropy in Equations (8) and (11) becomes exactly equal.

### 2.2. Growth Rate of Density Perturbations

Observations of the Cosmic Microwave Background Radiation (CMBR) over the past few decades have revealed that the CMBR is very homogeneous and isotropic. However, the same observations also revealed the existence of very small inhomogeneities in the CMBR maps. It is believed that these inhomogeneities correspond to the primordial density perturbations in the matter sector which were amplified by the mechanism of gravitational instability over time leading to the present day structures. When $\delta(x, t) < < 1$, the evolution of $\delta(x, t)$ with time can be described by a differential equation as

$$\frac{\partial^2 \delta(x, t)}{\partial t^2} + 2H(a) \frac{\partial \delta(x, t)}{\partial t} - \frac{3}{2} \frac{\Omega_{m0} H_0^2}{a^3} \frac{1}{a^3} \delta(x, t) = 0.$$ \hspace{1cm} (12)

Here $H_0$ is the present value of Hubble parameter and $\Omega_{m0}$ is the present value of matter density parameter. We change the
variable of differentiation from $t$ to $a$ and introduce the deceleration parameter $q(a) = -\frac{\dot{a}}{a}^2$ to get (Linder & Jenkins 2003)

\[
\frac{\partial^2 \delta(x, a)}{a^2} + \left(2 - q(a)\right) \frac{\partial \delta(x, a)}{a} - \frac{3}{2} \frac{\Omega_{mb} \delta(x, a)}{a^2} = 0.
\]

Equation (13) can be rewritten as (Linder & Jenkins 2003)

\[
\frac{d^2 D(a)}{da^2} + \frac{3}{a} \left(1 - \frac{\omega(a)}{1 + X(a)}\right) \frac{dD(a)}{da} - \frac{3}{2} \frac{X(a) D(a)}{2 + X(a)} = 0,
\]

where we have used the fact that in linear perturbation theory $\delta(x, a) = d(a)\delta(x)$. $d(a)$ is the initial density perturbation at $x$. $D(a) = \delta(x, a), \omega(a)$ is the time dependent equation of state of dark energy and $X(a) = \frac{\Omega_{de}}{1 - \Omega_{de}}e^{\frac{3}{2} \omega(a)d\log a}$. We normalize $D(a)$ such that at present day scale factor $a_0, D(a_0) = 1$ for the $\Lambda$CDM model. Solving Equation (14) numerically, we can then find evolution of growth rate with scale factor.

To get $f(a)$ we use

\[
f(a) = \left[\Omega_{mb}a^{-3}\right]^{\gamma},
\]

where $E^2(a) = \left(\frac{\Omega_{de}a^{-3}}{1 - \Omega_{de}}\right)^{\gamma}$ (Dabrowski & Salzano 2020) ($\Omega_{de}(a)$ is the energy density of the dark energy.) and $\gamma = 0.55 + 0.05[1 + \omega(a = 0.5)]$ (Linder 2005). For simplicity, we have considered the universe to have only matter and dark energy and no interaction between them.

### 2.3. Holographic Dark Energy Models

#### 2.3.1. Li Holographic Dark Energy

If we imagine that our universe has a characteristic length scale $L$ and horizon entropy $S$, then (Cohen et al. 1999)

\[
\rho_{de} \propto SL^{-4} = 3C^2M_p^2L^{-2},
\]

where $M_p = \left(\frac{1}{8\pi G}\right)^{\frac{1}{2}}$ is the reduced Planck mass, $G$ is Newton’s constant and $S \propto L^2$ according to the Bekenstein formula (Bekenstein & Haweles 1973).

The simplest choice for $L$ is $L = \frac{1}{H(a)}$. In this case the energy density is comparable to present day dark energy density (Hofava & Minic 2000; Thomas 2002) but the equation of state is wrong (Hsu 2004). (Pavón & Zimdahl 2005 point out that interaction between dark energy and dark matter can give rise to accelerating expansion with Hubble horizon as infrared cut-off.) The particle horizon as $L$ does not produce accelerated expansion (Li 2004). The choice of future event horizon as $L$,

\[
L = a \int_0^\infty dt = a \int_a^\infty \frac{d\tilde{a}}{H(\tilde{a})},
\]

gives a model of the accelerating universe (Li 2004) with correct equation of state. The density parameter of dark energy in this model satisfies the following equation (Wang et al. 2017).

\[
\frac{d\Omega_{DE}}{da} = \frac{1}{a} \Omega_{DE}(1 - \Omega_{DE}) \left[1 + \frac{2\Omega_{DE}}{C}\right].
\]

Equation (17) can be used to find $\Omega_{DE}$ as a function of $a$ and we can use that knowledge to get $E^2(a)$. We have chosen the initial condition of Equation (17) such that $\Omega_{DE}(a_0) \sim 0.7$. The equation of state is given by Wang et al. (2017)

\[
\omega(a) = -\frac{1}{3} - \frac{2\Omega_{DE}(a)}{3C}.
\]

The model has one free parameter, $C$. We can choose different values of $C$ to get different evolution of $\Omega_{DE}(a)$ and $\omega(a)$. A number of works have constrained the value of the free parameter $C$ to be less than 1 (Huang & Gong 2004; Zhang & Wu 2005, 2007; Chang et al. 2006; Ma et al. 2009; Xu 2012, 2013; Li et al. 2013; Wang et al. 2015). In this work we have chosen four values of $C$ given by 0.6, 0.8, 1.0 and 1.2.

#### 2.3.2. Barrow Holographic Dark Energy

Recently it has been proposed that quantum gravitational effects may lead to a wrinkled horizon of a black hole instead of a smooth one. Since the Bekenstein–Hawking formula of black hole entropy is proportional to the horizon area, it is modified in case of quantum gravitational effects. The Barrow entropy formula replaces the Bekenstein–Hawking formula in this case, which is given by Barrow (2020)

\[
S_B = \left(\frac{A}{A_0}\right)^{\frac{1}{\Delta^+}}.
\]

Here $A$ is the area of the black hole horizon and $A_0$ is the Planck area. The exponent $\Delta$ encapsulates the departure from the Bekenstein–Hawking formula. $\Delta = 0$ implies no quantum effects, while $\Delta = 1$ implies maximum quantum effects.

In standard holographic dark energy, $\rho_B L^4 \leq S_B$ where $L$ is the horizon length and $S_B$ is entropy. Using the Barrow entropy formula we get (Saridakis 2020a, 2020b; Barrow et al. 2021; Anagnostopoulos et al. 2020; Dabrowski & Salzano 2020)

\[
\rho_B = 3c^2M_p^2L^{2(\Delta^+ - 1)},
\]

where $c$ is one of the free parameters. We use the future event horizon as the horizon length. (Though, Dabrowski & Salzano (2020) point out that accelerating expansion can be achieved for this model using the Hubble horizon as well, without the need to introduce interaction between dark matter and dark energy.)
where $Q = (2 - \Delta) c \frac{\Delta}{3} (H_0 \Omega_{\text{rel}}^2)^{\frac{1}{2}}$. The equation of state is given by (Saridakis 2020a, 2020b; Anagnostopoulos et al. 2020; Dabrowski & Salzano 2020)

$$\omega(a) = -\frac{1 + \Delta}{3} - \frac{Q}{3} \frac{\Delta}{\Omega_{\text{DE}}^{1/3}} (1 - \Omega_{\text{DE}}^{1/3}) a^{\frac{\Delta}{3}}. \tag{22}$$

Equation (22) reduces to the Li holographic model for $\Delta = 0$. We have used $M_p = 1$ for this model. A few works which tried to constrain the values of the free parameters of this model are Anagnostopoulos et al. (2020), Barrow et al. (2021), Dabrowski & Salzano (2020). We consider the values 0.9, 1.0 and 1.1 for $c$ and 0.1, 0.2 and 0.3 for $\Delta$.

### 3. Results and Conclusions

We show the results for the Li holographic dark energy model in Figures 1 and 2. In the top left panel of Figure 1 we depict the variation of $\Omega_{\text{DE}}(a)$ with scale factor and the top right panel displays the evolution of $\omega(a)$ with scale factor. The evolutions of $D^2(a)$ and $f(a)$ with scale factor are featured in the bottom left and right panels of Figure 1, respectively. The results for the $\Lambda$CDM model are also shown together in each panel of Figure 1 for comparison.

We depict the evolution of $\frac{dS}{da}$ with scale factor in the top left and right panels of Figure 2, respectively. Since evolution of entropy is determined by the second and third terms in the right-hand side of Equation (9) and our choice of initial conditions forces the second term to vanish, the evolution is determined by the third term which includes a product of $f(a)$ and $D^2(a)$. The top two panels of Figure 2 demonstrate that initially entropy decreases and the entropy rate $\frac{dS}{da}$ becomes more negative with increasing scale factor. The decay in the entropy rate continues up to a scale factor of $a \sim 0.5$. The rate of decrease of entropy slows down for all $C$ values after $a \sim 0.5$. The entropy rate then starts to grow while remaining negative, which signifies a slower dissipation of the configuration entropy with time. We denote the scale factor at which the entropy rate turns around as $a_{\text{min}}$ and the magnitude...
We calculate $a_{\text{min}}$ and $dS(a)/da$ for different values of the parameter $C$ in the Li model. We then determine the best fit straight lines to the numerically obtained values of these quantities in terms of the parameter $C$. The best fit relations describing these quantities in the Li model are shown in the bottom two panels of Figure 2.

The Barrow holographic dark energy model is a two parameter model and we would like to explore the evolution of configuration entropy and entropy rate for different possible combinations of the two parameters $c$ and $\Delta$. To better understand the effect of each parameter on the evolution of each quantity, we varied one of the parameters while keeping the other fixed. This resulted in two sets of plots for Barrow holographic dark energy models.

We first show $\Omega_{\text{DE}}(a)$, $\omega(a)$, $D^2(a)$ and $f(a)$ for a fixed value of $c$ but for different values of $\Delta$ in this model in Figure 3. Here we fix $c = 1.0$ and allow $\Delta$ to vary. We find that the equation of state $\omega(a)$ strongly depends on the value of $\Delta$. We also show the results for the $\Lambda$CDM model together in each panel of Figure 3.

In the left and right panels of Figure 4, we respectively show the evolution of configuration entropy and entropy rate in the Barrow model for a fixed value of $c$ and different values of $\Delta$. We find that the configuration entropy decays with scale factor in each case. The negative entropy rate turns around a specific scale factor, which is highly sensitive to the parameter $\Delta$. The value of $a_{\text{min}}$ shifts toward higher scale factors with increasing values of $\Delta$. This is a result of strong dependence of equation of state on $\Delta$. The parameter $\Delta$ represents the modifications in the area of the horizon due to quantum gravitational effects. The higher sensitivity of $a_{\text{min}}$ and $dS(a)/da$ to the parameter $\Delta$ in the Barrow model suggests that it may be possible to identify the signatures of quantum gravitational effects in the behavior of configuration entropy and entropy rate.

Since $\Delta = 0$ corresponds to no quantum effects and $\Delta = 1$ corresponds to maximum quantum effects, we separately compare the effects of a wider variation of $\Delta$ in Figure 5. The left and right panels of Figure 5 affirm that for $\Delta = 0.7$, the configuration entropy dissipates much faster than the $\Lambda$CDM model and can be easily discerned from it. The results clearly suggest that the signature...
of a large quantum gravitational effect can be identified from the study of the evolution of configuration entropy.

We then repeat the above analysis for a fixed $\Delta = 0.1$ but for different values of $c$. Different panels of Figure 6 display the variation of $\Omega_{DE}(a)$, $\omega(a)$, $D^2(a)$ and $f(a)$ with scale factor for the Barrow holographic dark energy model with $\Delta = 0.1$. The results confirm that these quantities are only mildly sensitive to $c$.

The evolutions of configuration entropy and entropy rate in these models are shown respectively in the left and right panels of Figure 7. The configuration entropy and entropy rate exhibit similar characteristics as observed in Figures 2 and 4. We note that both $a_{\text{min}}$ and $(\frac{dS}{da})_{\text{min}}$ are weakly sensitive to $c$.

We use the numerical values of $a_{\text{min}}$ for different possible combinations of the parameters $c$ and $\Delta$ in the Barrow model to...
find a best fit relation between $a_{\text{min}}$ and these parameters. Similarly, we also find the best fit relation between $(\frac{dS_c(a)}{da})_{\text{min}}$ and the parameters of the Barrow model. We show the best fit planes for $a_{\text{min}}$ and $(\frac{dS_c(a)}{da})_{\text{min}}$ in the left and right panels of Figure 8 respectively. The equations for the best fit planes in each case are expressed in the corresponding panels.

In this work we have considered two different holographic dark energy models. We obtain the evolution of growing mode and dimensionless linear growth rate by relying on knowledge of the evolution equation of dark energy density parameter and equation of state. We use these quantities to calculate the evolution of configuration entropy and entropy rate in these models. For each of the models there are one or more free
parameters. We study the dependence of configuration entropy and its time derivative on these parameters. We find from our analysis that, for all models that we considered, there is a specific scale factor up to which the entropy rate continues to decrease. The negative entropy rate turns around at a specific scale factor and thereafter the dissipation rate slows down. The scale factor at which this occurs for a particular model depends on the functional form of the equation of state as well as the values of the parameters. This particular scale factor corresponds to the era where dark energy density begins to drive the universe into a phase of accelerated expansion. We also find that at this particular scale factor, the magnitude of entropy rate is different for different values of the parameters in a particular model. We find that there exist simple approximate relations between the scale factor of the minimum and the magnitude of entropy rate and the values of the parameters. We propose that by measuring configuration entropy at different scale factors and finding the scale factor at which the minimum of the entropy rate occurs, one can constrain the values of the parameters of a particular dark energy model, provided we assume that it is the correct description of dark energy. One may ask: why use configuration entropy for this purpose? We would like to point out that although entropy is a derived quantity which depends on $D^2(a)$ and $f(a)$, $D^2(a)$, $f(a)$ and $S(a)$ are smooth functions unlike entropy rate which shows a distinct minimum. It will be easier to identify the position and magnitude of a minimum rather than finding the difference in smooth curves.

We also note that the signature of any quantum gravitational effects in the holographic dark energy models is reflected in the evolution of configuration entropy and its time derivative. The possibility of detecting any such signature using the large scale structure of the universe is certainly interesting. Currently no observational data sets are available to carry out the proposed analysis. In the future, facilities such as the Square Kilometre Array (SKA) would use the redshifted 21 cm signal to map the density of neutral hydrogen over a large redshift range.
method may then prove to be useful for the study of holographic dark energy models.

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