Pad-Wafer-Slurry Interface Information from Force Data

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This paper describes a model and algorithm for converting pad-wafer shear force and normal force data measured during chemical-mechanical planarization (CMP) into information about the conditions at the lubricated interface between the pad and wafer. Insight into this interface has been notably difficult due to its inaccessibility. Results indicate that force data contain detailed information about the wafer attitude relative to the pad and information about mean solid contact and fluid pressures in the interface. Fluid pressures are particularly interesting since they are difficult to detect directly but are coupled to solid contact pressures and can therefore affect uniformity. Fluid pressures can also underlie problems like wafer slippage. As part of the analysis of the model, we derive a highly accurate analytic approximation to the solution of the Reynolds equation for the mean fluid pressure in the slurry layer during CMP.

Two algorithms are discussed for converting force data. The fastest is considerably faster than real-time force acquisition. © The Author(s) 2019. Published by ECS. This is an open access article distributed under the terms of the Creative Commons Attribution 4.0 License (CC BY, http://creativecommons.org/licenses/by/4.0), which permits unrestricted reuse of the work in any medium, provided the original work is properly cited. [DOI: 10.1149/2.0211905jss]

Because of the difficulty of making measurements within the lubricated interface between the wafer and pad during chemical-mechanical planarization (CMP), not much is known about the physical conditions there. A series of investigators at the Georgia Institute of Technology (GIT) measured and modeled the fluid pressure distribution between a perforated polishing pad and a non-rotating, planar stainless steel disk1-5 using pressure sensors embedded in the disk. They observed areas of both suction and lift with suction dominating. The distribution of pressures, which included a difficult to explain area of lift at the trailing edge, was ultimately attributed to pad tilt and to ridges in the pad radial topography caused by pad conditioning. The fluid pressure magnitudes depended mainly on sliding speed and could be comparable to the applied pressure. Using capacitance probes, Ng1 also measured changes in the orientation of the disk. The disk was found to tilt leading edge down and inside edge down during operation due to friction and the concentration of interfacial suction toward the leading edge. The separation between the pad and disk at the disk center also increased during polishing. These measurements and those of other GIT investigators provided the first basic insights into interfacial fluid dynamics and polishing head mechanics during CMP.

More recently, Zhaou et al.6,7 measured fluid pressures and wafer pitch on an instrumented 300 mm polisher with pressure and distance transducers embedded in the platen. Experiments reported in Ref. 8 were conducted using either a 10 mm thick copper disk or a silicon wafer with a deposited copper film. For the rigid copper disk, the results were similar to those from the stainless steel disk in Refs. 1 and 2: the fluid was in suction over 70% of the interface except at the trailing edge and the disk was tilted leading edge down. By contrast, the more compliant wafer flexed so that the fluid pressures were positive over 70% of the interface except near the leading edge, where there was suction. Fluid pressures were between −1.5 and +3 kPa (~0.22 to 0.44 PSI) at 80 RPM with a 3.5 kPa (0.5 PSI) load. The wafer was pitched leading edge up at an angle of about 4.3 × 10⁻⁵ degrees (7.5 × 10⁻⁶ radians). The head in Ref. 8 used a commercial membrane and a floating retainer ring.

Instrumented polishers have also been used to measure the lubricated shear force between the pad and wafer and the total normal force transmitted from the wafer to the pad.9 We shall provide two examples of such measurements. While the individual force measurements do not always show obvious evidence of fluid pressures directly, we will argue that the two measurements taken together strongly suggest that they are always present. Furthermore, we shall show that the force data can be translated directly into wafer orientation information using a physical model. Thus, force data provide a new method for understanding the origins and effects of fluid pressures during CMP.

The plan of the paper is as follows. First, we will describe the polisher on which the measurements are collected and the relevant details of the force measurements. We then show two different examples of force data and discuss what it is about the data that requires an explanation. We next introduce a physical model for shear and normal forces that is central to the paper and apply it to the two examples. The model has significant internal structure that we then develop and analyze. As part of the analysis, we derive an analytic approximation for the mean fluid pressure in the polishing interface in CMP that is very simple but highly accurate when the wafer tilt is small. Finally, we outline an algorithm for processing force data that runs faster than our real time data acquisition. The algorithm may therefore be useful for future real time problem detection and control in polishers equipped with the right metrology.

The Polisher

Force measurements in this paper were collected on an Araca APD-800 300 mm polisher and tribometer (Fig. 1; see also Ref. 10). All forces were sampled at 1,000 Hz using five load cells built into the tool. This acquisition frequency is our standard practice. It is our experience that the most important information about a run occurs at less than 150 Hz.

One of the load cells on the polisher measures the net shear force at the pad-wafer interface. On this tool, the non-oscillating, counter-clockwise wafer carrier and its drive mechanism are suspended from a sliding friction table. The shear force load cell is connected to this table; it measures the main component Fy of the shear force vector. This component runs perpendicular to the line connecting the platen and carrier centers. The radial componentFx of the shear force is not measured; however, it is known to be small (<5%) relative to the applied load from experiments on a similar 200 mm tool.11

The APD-800 polisher also measures the total normal force Fz applied to the pad. The platen and its drive mechanism sit on four load cells that collectively monitor the total force. The total force includes the applied load on the wafer, any fluid forces that might develop in the interface, and the relatively small down force from the 100 mm (4”) diameter conditioner. In this paper, the load applied to the wafer is about 1000 N (226 lb-f, or 2 PSI) while the conditioning force does not exceed 27 N (6 lb-f).

None of this information is used for polishing pressure control. Instead, the carrier uses an air chamber to apply the polishing force. An electro-pneumatic transducing regulator monitors and controls the pressure magnitudes depended mainly on sliding speed and could be comparable to the applied pressure. Using capacitance probes, Ng1 also measured changes in the orientation of the disk. The disk was found to tilt leading edge down and inside edge down during operation due to friction and the concentration of interfacial suction toward the leading edge. The separation between the pad and disk at the disk center also increased during polishing. These measurements and those of other GIT investigators provided the first basic insights into interfacial fluid dynamics and polishing head mechanics during CMP.

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None of this information is used for polishing pressure control. Instead, the carrier uses an air chamber to apply the polishing force. An electro-pneumatic transducing regulator monitors and controls the
chamber pressure. Pressure adjustments occur during polishing but are much slower than the force acquisition frequency. The load applied through the chamber is periodically checked and calibrated on a static platen.

Unlike most commercial CMP tools, the wafer carrier on the APD-800 polisher does not use a contact retaining ring. The function of such a ring is to compress the pad ahead of the wafer and squeegee off some of the surface fluid. Instead, on the APD-800, a soft wafer backing film with a non-contact ring holds the wafer. The wafer backing film is attached to a template that contains a water-filled cavity separated from the backing by a thin flexible membrane. The water in the cavity is incompressible and evens out the pressure distribution on the wafer back side during polishing. The water-filled cavity design ensures that the wafer is front-referenced; i.e., the wafer contact and deformation are determined mainly by the shape of the pad, not by the shape of the template.

Finally, an infrared sensor on the polisher measures the pad temperature in a spot a short distance in advance of the carrier. Temperature data is also acquired at 1000 Hz.

**Force Acquisition Example 1**

Fig. 2 shows a scatter-plot of \( \sim 48,000 \) (Fz, Fy) force pairs measured during a 60 sec, 300 mm blanket wafer polishing experiment run at 2.1 PSI and 1.4 m/s (59 RPM platen, 57 RPM head) on a concentrically-grooved commercial pad. The pad was dressed in situ pad conditioning was used, so there is no conditioner force included in the measured normal force. The slurry is a colloidal silica slurry with 30% solids content applied at 100 ml/min. At 20 sec and 40 sec into polishing, a 20 ml slurry pulse is suddenly applied at the left-most point of the halo. The hypothesized effect of hydrodynamic lift would be to increase the normal force and decrease the shear force. Suction would decrease the normal force while increasing the shear force.

Transducer, so that the tool is not driving the cycling. The data pose two puzzles. First, on the left side of the scatter plot, a high shear force is paired with a low normal force while on the right side a low shear force is paired with a high normal force. This is exactly the opposite of what should be the case in a classical friction force measurement and requires an explanation.

The second puzzle is that in Fig. 2, the mean normal force calculated from the data is 1161 N (2.38 PSI) while the statically calibrated applied force is 1023 N (2.1 PSI). This discrepancy also requires an explanation.

Fluid pressures can explain both puzzles. This can be appreciated intuitively. If net positive hydrodynamic pressure (lift) were to develop in the pad-wafer interface, then the fluid pressure would increase the normal force on the z load cells (Fig. 2). Simultaneously, the lift would reduce the solid contact force between the wafer and pad and reduce the shear force. This corresponds to the red arrow in Fig. 2. Similarly, negative hydrodynamic pressure, or suction, should decrease the normal force by pulling up on the platen while increasing the shear force by drawing the wafer closer to the pad. The blue arrow indicates the qualitative effect of suction. Thus, fluid suction and lift may explain the cycling in the figure. Lift, if it dominates for a significant fraction of the polishing time, would explain the discrepancy in the applied force.

**Force Acquisition Example 2**

We next present an example with a clear, intentionally induced hydrodynamic effect. The second example is a 60 sec, blanket 300 mm polishing run at 13790 Pa (2 PSI) with a 90 RPM platen and 45 RPM carrier. The pad is also a concentrically-grooved commercial pad. Ex situ pad conditioning was used, so there is no conditioner force included in the measured normal force. The slurry is a colloidal silica slurry with 30% solids content applied at 100 ml/min. At 20 sec and 40 sec into polishing, a 20 ml slurry pulse is suddenly applied at the center of the wafer track using a beaker.

Figure 3 shows the evolution of the normal force and shear force during the experiment. Both the raw data and 50 point running averages are displayed. When the slurry pulses are applied at 20 and 40 sec, there is an immediate decrease in the shear force that persists for ~2 – 3
platen rotations. There is at the same time no noticeable change in the normal force during the pulses. (The normal force has some slight upward jogs at 16 and 36 sec that are probably caused by carrier air pressure regulation). What explains the behavior of the shear force? A natural hypothesis is that the slurry pulse has temporarily increased the lubrication under the wafer and that the wafer therefore slides more easily. The mechanism of lubrication is to decrease the solid/solid contact pressure using fluid pressure, so this appears to be a clear case of the development of fluid pressures. As we shall explain below, hydrodynamic pressures induced by the additional slurry can indeed explain both the shear force and the normal force behavior during the pulses.

Figure 4 shows the force scatter plot for example 2, including a few seconds of data when the carrier is engaging with the platen at the beginning and disengaging at the end. The plot also highlights 0.1 sec of data in the dense area of the plot. The highlighted curve passes through the right-most point in the halo. Unlike example 1, in which the selected section of orbit is fairly smooth, this orbit is chaotic. We will discuss this further below.

**Figure 3.** Normal force and shear force in polishing example 2, in which two slurry pulses are applied.

**Figure 4.** Force scatter plot for example 2. The orbit sample highlighted in red is chaotic.

**Theory of Forces**

We next lay out a theory that explains what is happening in these two experiments. The basic idea in our first implementation of the theory is to convert the force measurements into wafer attitude numbers and use the latter to estimate solid contact pressures and fluid pressures. These are easier to interpret than the force data and so tell a clearer story about the interface.

We assume that the gap between the pad and wafer varies linearly. What this means is that for small variations in the pad height on a decimeter scale, the wafer is nearly conformal to the pad except for small linear perturbations imposed by the tilt of the wafer holder. In particular, the height of the wafer above the mean pad surface, whatever its shape, is assumed to have the form $h(x, y) = \delta_0 - a(x - c_x) + \beta y$. Here, the coordinate origin is at the platen center and the wafer center is at $x = c_x$. The parameters $\delta_0, a, \beta$ are respectively the altitude of the wafer center, the bank and the pitch. These define the attitude. The pitch $\beta$ is the tilt of the wafer about a diameter parallel to the x-axis, signed using the right-hand rule. The leading edge is down when $\beta > 0$. Similarly, $a$ is the tilt of the wafer about a diameter parallel to the y-axis signed using the right-hand rule. Bank is positive if the outside edge is down.

There is an alternative description of the wafer attitude that we shall also use. The attitude can be specified using the altitude $\delta_0$ along with a tilt angle $\gamma$ and a rotation angle $\theta_m$. The tilt angle is the angle between the normal vector at the wafer center and the vertical axis. The tilt is always positive. The rotation angle is the direction relative to the wafer center of the minimum point on the wafer perimeter. One attitude description can be transformed into the other using

$$\begin{align*}
\alpha &= \tan(\gamma)\cos(\theta_m) \\
\beta &= -\tan(\gamma)\sin(\theta_m)
\end{align*} \quad [1]$$

The essence of the shear and normal force theory is that given a solid contact pressure distribution $p_s$ and a fluid pressure distribution $p_f$ in the interface, the total normal force and components of the shear force are

$$\begin{align*}
F_z &= \iint_{Aw} p_s + p_fdA = \pi r_w^2 (p_s + p_f) \\
F_y &= \mu_k \iint_{Aw} p_s u_idA \\
F_x &= \mu_k \iint_{Aw} p_s u_jdA \quad [2a] \quad [2b] \quad [2c]
\end{align*}$$

In (2), $Aw$ is the wafer region, $r_w$ is the wafer radius, $\mu_k$ is the kinetic coefficient of friction (COF) for solid-solid contact and $\bar{u} = u_i \mathbf{i} + u_j \mathbf{j}$ is a local unit vector in the relative sliding direction. The relative sliding direction is described in detail below. The net shear force $F_s = F_x \mathbf{i} + F_y \mathbf{j}$ is therefore a superposition of pointwise shear forces acting at the interface in possibly different directions. We estimate the COF, in the absence of $F_x$ data, by dividing the measured component $F_y$ by the applied load and then averaging. The implications of this procedure are discussed below. It is important to note that both solid contact and fluid pressures contribute to the normal force while the shear force depends only on $p_s$. In the model, we neglect a contribution from fluid shear and a normal force contribution from the conditioning force because they are relatively small.

Given the attitude parameters, the pad-wafer solid contact pressure field $p_s$ is calculated with the Greenwood and Williamson model,

$$p_s(x, y) = 4/3E\eta_w/(1 - v^2)c_s^{1/2} \int_h^\infty (z - h)^{3/2} \phi(z)dz = G \int_h^\infty (z - h)^{3/2} \phi(z)dz. \quad [3]$$
Here, \( G \) is the cluster of coefficients in front of the integral, \( E \) is the pad Young’s modulus, \( \nu \) is the Poisson ratio, \( \eta \) is the summit density, \( \kappa \) is the mean summit curvature, and \( \phi(z) \) is the pad surface height probability density function (PDF). We measure the latter three using laser confocal microscopy.\(^{13}\) The surface height PDF is conventionally translated so that its mean is \( z = 0 \). It is a probability density function, so its integral is 1. We use the surface height PDF instead of the more correct summit height PDF because the latter is not typically produced by confocal microscopy software.

The fluid pressure field \( p_f \) is calculated using a version of the Reynolds equation,\(^{15}\)

\[
\nabla \cdot (h \nabla p_f) = 6 \mu V_R \cdot \nabla h. \tag{4}
\]

In 4, \( \mu \) is the slurry viscosity (assumed constant) and \( V_R \) is the relative sliding velocity. \( V_p \) is the difference between the pad and wafer velocities. The pad velocity in the laboratory frame is

\[
V_p = \Omega_p k \times r = \Omega_p (-yi + xj), \tag{5}
\]

where \( \Omega_p \) is the platen rotation rate and \( r \) is a position vector from the platen center to \((x,y)\). The wafer velocity in the laboratory frame is similarly

\[
V_w = \Omega_p k \times (r - c_w i) = \Omega_p (-yi + (x - c_w)j), \tag{6}
\]

from which it follows that

\[
V_R = V_p - V_w = -y(\Omega_p - \Omega_w) i + [x(\Omega_p - \Omega_w) + \Omega_w c_w] j. \tag{7}
\]

The unit vector in the sliding direction \( u = u_i + u_j \) is then \( V_R/|V_R| \).

Finally, the solution of the Reynolds equation requires a boundary condition. We require that the fluid pressure should be zero at the wafer perimeter.

Equations 2–7 comprise the model. We will discuss two solution methods. The first method starts with a measured force pair \((F_x,F_y)\) and a guess at the attitude parameters. We then use the attitude parameters to calculate \( p_f \) and \( p_s \) and the normal and shear force components. If \( F_x \) and \( F_y \) are not sufficiently close to the measurements and \( F_x \) is not sufficiently small, then a downhill simplex algorithm\(^{15}\) is used to adjust the attitude. This is repeated until the relative error measures are all within the required tolerances. This method can handle \( \sim 1000 \) points/hr on a 2.8 GHz computer and therefore requires several days of CPU time if 60,000 points need to be processed. The second solution method extracts the pressures first, much faster, and is explained below.

Application to Example 1

We first apply the analysis method to example 1 (Fig. 2) to verify that the interfacial pressures cycle as suggested. We also look in some detail at the extracted pressures and the attitude parameters.

Fig. 5a shows the calculated fluid and solid contact pressures during the highlighted section of orbit between 1.58 and 1.59 sec in Fig. 2. In the upper half of the plot, the fluid provides net lift while in the lower half there is net suction. The applied pressure target is also shown. To the right of the target, the contact force and the shear force are elevated and to the left they are reduced. We see from the figure that the orbit sample starts in suction with an elevated contact pressure and shear force and quickly cycles into lift with a reduced contact pressure and shear force, as expected. \( F_x \) and \( F_y \) from Equations 2a, 2b, 2c agree with the data with a relative error of less than \( 10^{-3} \) (Fig. 5b), so the calculated pressures can in fact account for the measured forces. \( F_x \) never exceeds a small fraction of the applied load, as required by prior experience.

Fig. 6 shows the calculated solid contact pressures and fluid pressures as a function of time for a 1 sec section of run that begins with the 0.1 ms orbit sample in Fig. 5. The fluid pressures (blue curve) oscillate between suction and lift with a period of about 22 ms. The solid contact pressures oscillate with the same period but 180° out of phase with the fluid; i.e., the contact pressure is low when the fluid pressure is high and high when the fluid pressure is low. Thus, these two pressures trade off.

We next look at the attitude parameters for the same section of run. Fig. 7 shows the attitude. The altitude oscillates between about 3.25 and 4.25 μm. It is in phase with the fluid pressure; high when there is lift, low when there is suction. The altitude has a probability distribution, which is compared with the surface height PDF in Fig. 8. The altitude stays within a narrow range in the right-hand tail of the PDF. For some runs, we have observed that the attitude PDF coincides with a secondary wear peak on the surface height PDF.

Figure 7 also shows the pitch. The pitch in the theory is a unitless slope. Since it is small, it can also be interpreted as an angle in radians. The calculations indicate that the wafer oscillates between leading edge up (negative pitch) and leading edge down (positive pitch). The

**Figure 5.** (a) Fluid and solid contact pressures extracted from Fig. 2 using the force theory in the paper. (b) Comparison of the measured and calculated forces.
pitch is out of phase with the fluid pressure, so the leading edge is up when the fluid provides lift and down when there is suction, similar to published pressure measurements. The magnitude is very small, consistent with the wafer being front-referenced and sandwiched between the pad and water-filled template. However, even this small pitch is adequate to produce the fluid pressures in Fig. 6, which come from the Reynolds equation.

While the algorithm produces numbers for the bank, we will explain below that the bank is probably unreliable and that there are circumstances in which it cannot be extracted at all from force data.

Finally, the fluid is predicted to provide lift (with some oscillation) for most of the polishing run (Fig. 9). From the full run, the mean of the calculated \( F_z \) is 1161 N (2.38 PSI), the same as the measured down force. This is a result of the good convergence of the algorithm. The mean calculated solid contact pressure, by contrast, is 2.1002 PSI, the same as the desired target contact pressure to high accuracy. We will explain this accuracy below. Thus, the theory can explain why the measured normal force is larger than the target. Fluid pressures, in the form of lift, are the explanation for the discrepancy.

Finally, there is a tentative explanation for the orbital smoothness in this example. Fig. 9 compares the pad temperature just in advance of the carrier leading edge with the calculated contact and fluid pressures. The temperature can be seen to have a shallow minimum on every platen rotation that slightly precedes the next contact pressure minimum and fluid pressure maximum. The simplest explanation for this is that there is a low area on the pad in which there is less frictional heating. The change in pad height is small, on the order of a few microns according to the altitude estimate, but the duration of each vibrational burst in Fig. 9 implies that the area is comparable to the size of the wafer. As the low area passes under the wafer, a thicker slurry film in the area induces significant hydrodynamic lift and related chatter as the wafer flexes. This is the root cause of the smooth orbit. As time goes on and the total slurry volume on the pad decreases, the burst amplitudes diminish. This mechanism will be discussed in more detail in a future paper.

Application to Example 2

We next apply the analysis method to example 2 (Figs. 3 and 4). We have already observed that during the slurry pulses, the shear force decreases while the normal force remains constant. Fig. 10 shows the extracted solid contact pressures (black) and fluid pressures (blue) starting 2 sec into the run along with the corresponding 50 ms running averages (white). Up to 4 sec, before the wafer is fully engaged with the pad, the fluid produces significant lift, consistent with excess slurry on the pad from priming. As the contact pressure stabilizes around the set point, the fluid transitions quickly into mainly suction except during the two slurry pulses, when the fluid pressure increases and again produces lift. At the end of the run, the suction deepens as the wafer is lifted from the pad. During the pulses, positive fluid pressure replaces the solid contact pressure, keeping the overall normal force constant as required by Eq. 2a. A similar transfer of force to the fluid is predicted by theories of elasto-hydrodynamic lubrication.
Figure 10. Calculated solid contact pressures (black) and fluid pressures (blue) for the slurry pulse experiment in Figs. 3 and 4 along with the corresponding 50 ms running averages (white). Periodic oscillations are visible that correspond to the carrier and platen rotation. The fluid is in suction below the horizontal red line.

Figure 11 shows the pressures in more detail during the first pulse. As in example 1, the fluid and solid pressures are 180° out of phase, but neither curve has the smooth, periodic behavior seen in Fig. 6. The irregularity is consistent with the chaotic orbital behavior visible in Fig. 4 (red curve). We believe that the pad in this case did not have any large, coherent low spots. The flow rate is also considerably lower than in example 1. The absence of low spots or of extended puddles of excess slurry would then be the root explanation of the chaotic orbits.

Turning now to the attitude parameters, prior to full engagement of the wafer and pad, the wafer center altitude is decreasing as one might expect (Fig. 12). It then stabilizes except during the two pulses and increases again at the end of the run during disengagement. During the pulses, when the fluid pressure turns positive, the altitude increases by about 0.2 μm for about 2–3 platen rotations. This is consistent with the application of excess slurry. Taking into account the change in altitude and the wafer area, during the peak of the pulse the excess slurry volume under the wafer required to explain the altitude change is only about 0.01 ml. While small, the changes in altitude and volume may be enough to significantly affect the lubrication behavior of the contacting summits, where most of the shear force must originate.

Fig. 12 also shows the calculated wafer pitch. Initially, the wafer is leading edge up (negative pitch) until it becomes fully engaged. It then stabilizes with the leading edge down except during the pulses, when the leading edge again comes up. This is consistent with what might be expected to happen when excess slurry encounters the leading edge of the wafer. At the end of the run, the leading edge briefly tips down as the wafer is raised.

Some Consequences of the Force Theory

We now derive and discuss several consequences of the theory. We have chosen to emphasize those parts of the theory that give the most insight into the above experiments and into what may be happening in CMP generally. The theory has some subtle implications and some beautiful formulas. We start with a few simple predictions, focusing primarily on the co-rotation case, in which the carrier and platen rotate at the same rate and in the same direction. The main properties of the theory can be most easily understood in this case. In Appendix B, we show that most operating conditions have behaviors that are very close to co-rotation.

If the platen and carrier co-rotate, then the mean over time of the calculated solid contact pressures always equals the applied pressure.—This result underlies the extraordinary accuracy (2.1002 PSI) with which the applied pressure was reproduced by the model in example 1.

In the theory, we estimate the COF by averaging the measured values of \( F_y \) and dividing by the applied load \( \pi r_k^2 \tilde{p} \), where \( \tilde{p} \) is the applied pressure:

\[
\mu_k = \frac{1}{\pi r_k^2} \frac{1}{N} \sum_{i \leq N} F_y^i(t_i).
\]

In 8, \( N \) is the number of points and \( F_y^i(t_i) \) is an explicit notation for the shear force measurement at time \( t_i \). If we apply the same averaging to the calculated shear force components \( F_y^i(t_i) \) using (2b), we get

\[
\frac{1}{N} \sum_{i \leq N} F_y^i(t_i) = \mu_k \frac{1}{N} \sum_{i \leq N} \iint_{Aw} p_i(x, y, t_i)\,dA
\]

where we have used the fact that \( u_i = 1 \) if the platen and carrier co-rotate. Now, if the model has converged, so that the measured and calculated shear forces are the same, then the summation in 8 may be substituted into 9 to give

\[
\pi r_k^2 \tilde{p} \mu_k = \mu_k \frac{1}{N} \sum_{i \leq N} \iint_{Aw} p_i(x, y, t_i)\,dA
\]

or

\[
\tilde{p} = \frac{1}{N} \sum_{i \leq N} \frac{1}{\pi r_k^2} \iint_{Aw} p_i(x, y, t_i)\,dA
\]
The quantity inside of the summation in 11 is the mean calculated solid contact pressure at time \( t_i \), so we conclude that

\[
\bar{p} = \frac{1}{N} \sum_{i \leq N} \bar{p}_i(t_i).
\]  

[12]

In the co-rotation case, then, the use of 8 to calculate the COF guarantees that the mean over time of the calculated solid contact pressures equals the applied pressure provided that the algorithm converges.

While this result may be somewhat comforting, the main question is whether Eq. 8 is the right way to calculate the mean kinetic solid-solid contact COF. We have rejected here the use of the measured normal force to estimate the COF for the purpose of modeling the solid contact shear force because \( F_z \) seems to include unintentionally generated fluid forces in addition to the intentional applied force. The incorporation of fluid forces in \( F_z \) is particularly unambiguous in the trade-off seen in the slurry pulse experiment, in which we deliberately induced brief hydrodynamic events. Use of \( F_z \) instead of the applied load also leads to a mysterious disagreement between the calibrated applied force and the measured normal force. The current approach provides a solution to this problem.

If the platens and wafer co-rotate, then in the absence of fluid pressures, the force theory reduces to a standard friction law in which the shear force is proportional to the normal force.—We are claiming here that the current theory builds upon the simplest sliding friction force theory. If this were not true, then the theory would be questionable. In the co-rotation case \((\Omega_p = \Omega_w)\), the y-component \( u_y \) of the unit vector in the sliding direction is always 1 and the x-component \( u_x \) is 0. This follows from Eq. 7. From Eq. 2c, it then easily follows that \( F_X = 0 \) and from (2a, 2b) that

\[
F_y = \mu_k \int_{A_w} p_u dA = \mu_k \int_{A_w} p_i dA = \mu_k F_z
\]  

[13]

The last step uses the assumption that \( p_i \) is 0. Thus, \( F_y \) is the friction force magnitude and it is proportional to the normal force. The same conclusion follows more generally if just the mean of \( p_i \) is 0.

This result means that if the mean fluid pressure is zero throughout an experiment, then all of the force measurements should fall on a straight line with slope \( \mu_k \). A scatter plot with points that do not fall on a line, as in Fig. 1, automatically implies the existence of nonzero mean fluid pressures.

In the co-rotation case, the mean fluid pressure implied by force data is—

\[
\bar{p}_f = \frac{F^m}{\pi R^2_w} - \bar{p}_i
\]  

[14a]

where

\[
\bar{p}_i = \frac{F^m}{\mu_k \pi R^2_w}
\]  

[14b]

From (2a), (2b) and \( u_y = 1 \), we have

\[
\int_{A_w} p_f dA = F^m
\]  

[15]

\[
\mu_k \int_{A_w} p_i dA = F^m
\]  

[16]

where we have again been explicit about measured forces. We can then use 15 to express the fluid pressure integral in terms of the solid contact pressures:

\[
\int_{A_w} p_f dA = F^m - \int_{A_w} p_i dA
\]  

[17]

Equation 14a follows trivially by dividing both sides of 17 by the wafer area \( \pi R^2_w \). Similarly, (14b) follows easily from 16.

While the algebra is trivial, this result is significant. Equation 14b says that the mean solid contact pressure can be obtained directly from the measured shear force if we have an estimate of the COF. In Equation 14a, we subtract the mean solid contact pressure contribution from the normal force to obtain a residual. We identify the residual as being a mean fluid pressure. We are arguing here that this residual behaves like a fluid pressure and likely is a fluid pressure. The transformations (14a) and (14b) require no modeling or theory other than the force Equations 15 and 16 and thus are very close to data.

If the wafer tilt is small compared with the altitude, then to a good approximation, the mean fluid pressure is—

\[
\bar{p}_f = \frac{-6\mu_k \bar{\Omega}_p R^2_w c^2}{8\delta^3_0}
\]  

[18]

Co-rotation is not assumed here. We derive this remarkable formula in Appendix A. We will show evidence that 18 is in fact very accurate. Equation 18 can be viewed as the main term of a series approximation, although we shall not derive it this way. The next significant term is almost six orders of magnitude smaller.

By combining 18 and (14a, 14b), we easily find that

\[
\frac{-6\mu_k \bar{\Omega}_p R^2_w c^2}{8\delta^3_0} = \left( \frac{F^m}{F^m / \mu_k} \right) / \pi R^2_w
\]  

[19]

Solving for \( \bar{p}/\delta_0^2 \), we obtain

\[
\frac{\bar{p}}{\delta_0^2} = \frac{4}{3\pi \mu_k \bar{\Omega}_p R^2_w} \left( \frac{F^m}{F^m / \mu_k} \right)
\]  

[20]

This formula expresses two unknown attitude parameters in terms of known or measured quantities and is the first of two equations for rapidly extracting wafer attitude parameters from force data.

One surprising prediction of 18 is that the mean fluid pressure depends only on the wafer attitude and pitch, not on the bank. We will explore this further next. Equation 18 also implies that the mean fluid pressure is independent of the wafer rotation rate. This is surprising and is an indication that the effects in CMP due to non-co-rotation are not of first order.

If the pitch is zero, then the mean fluid pressure is zero regardless of the bank. Furthermore, if the pitch is nonzero, then the mean fluid pressure depends only very weakly on the bank.—The essential difficulty with the bank can be seen from the Reynolds Equation 4 using a symmetry argument. In Cartesian coordinates, the equation is

\[
\frac{\partial}{\partial x} \left( \frac{\partial p_f}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial p_f}{\partial y} \right) = 6\mu_k(x)(\bar{\Omega}_p - \bar{\Omega}_w)(x - c_0)
\]  

[21]

We have explicitly worked out the right-hand side \( 6\mu_k V_R \cdot \nabla h \) and have called the resulting function \( R \). Recall that the fluid thickness is \( h(x, y) = \delta_0 - \hat{a}(x - c_0) + \beta y \). When \( \beta = 0 \), the right side reduces to \( R(x, y) = 6\mu_k(\bar{\Omega}_p - \bar{\Omega}_w) \) and the fluid thickness is unchanged, as are both terms on the left side of 21. However, the right-hand side changes sign: \( R(x, y) = -R(x, y) \). This implies that the fluid pressure must be anti-symmetric about the x-axis, \( p_f(x, y) = -p_f(x, y) \). When the pitch is zero, the mean fluid pressure under the wafer is therefore zero for any bank \( \alpha \). This means that no information whatever can be obtained about the bank from the mean fluid pressure in this case. It also provides a hint of what happens when the pitch is not zero: the mean fluid pressure depends only weakly on the bank. We will first use finite element solutions of the Reynolds equation to illustrate this and will outline an analytic argument in Appendix A.

It is easy to appreciate this result intuitively. Figure 13 shows the calculated fluid pressure field for an orientation with pure bank. The wafer is tilted slightly outside edge down. Below the x-axis, the tilt creates a converging gap between the wafer and pad in the direction of...
rotation. This configuration creates a region of positive pressure in the lower half plane as the fluid is forced into the narrowing gap. Above the x-axis, though, the fluid gap is narrowing in the direction of motion. This creates suction in the upper half plane. Since the geometry is symmetric, suction and lift exactly cancel.

By appropriately adjusting the tilt and rotation angles of the wafer using Eq. 1, it is possible to explore the effect of variable bank at constant nonzero pitch. For example, if the pitch is maintained at $-1.50 \times 10^{-7}$ radians, then over a bank angle ranging from zero to almost $10^{-7}$ radians, the mean pressure changes from 3108 Pa to 3132 Pa, or only about 0.76%. This weak dependence makes it difficult to reliably extract bank information from force data.

**If the pad surface height PDF has an exponential tail with decay length $\lambda$, then the local solid contact pressure is proportional to $\exp(-h/\lambda)$ when contact is confined to the tail.** Furthermore, to a good approximation, the mean solid contact pressure is more simply proportional to $\exp(-\delta_0/\lambda)$.—Exponential tails on the contacting side of a pad surface are very common. The PDF for example 1 is shown in Fig. 14. The PDF has an exponential tail starting at about $z = 3$ m. The decay length $\lambda$ of such a tail is defined as the horizontal distance over which the PDF decreases by a factor of e. When such a tail exists, the PDF can be approximated in the contact region by an exponential function,

$$\phi(z) = B \exp(- (z - z_c)/\lambda),$$  \hspace{1cm} [22]

where $B$ is the value of the PDF at the tail attachment point $z = z_c$. In this case, the Greenwood and Williamson contact pressure integral can be evaluated analytically, giving

$$p_r(x, y) = G \int_0^\infty (z - h)^{3/2} \phi(z)dz$$  \hspace{1cm} [23]

$$= GBe^{-h/\lambda} \int_0^\infty (z - h)^{3/2} e^{-z/\lambda} dz$$  \hspace{1cm} [24]

$$= GBe^{-h/\lambda} \sqrt{\lambda/2} \int_0^\infty u^{3/2} e^{-u} du$$  \hspace{1cm} [25]

$$= K \exp(-h/\lambda).$$  \hspace{1cm} [26]

The coefficient $K$ is

$$K = \frac{3\sqrt{\pi}}{4} Be^{-h/\lambda} \sqrt{\lambda/2} G.$$  \hspace{1cm} [27]

G is the cluster of coefficients in the Greenwood and Williamson model; see Eq. 3.

There is a very simple intuitive argument for the second claim, in which $h$ is replaced by the center altitude. When such a tail exists, the PDF can be approximated in the contact region by an exponential function,

$$\phi(z) = P \exp(- (z - z_c)/\lambda),$$  \hspace{1cm} [28]

and so

$$p_r(x, y) = K e^{-h/\lambda} \int_0^\infty e^{-u} du = K e^{-h/\lambda}. I,$$  \hspace{1cm} [29]

where we use $I$ to denote the double integral divided by the wafer area. We will show that $I$ is very close to 1. Letting $a = tcm/\lambda$, we can explicitly evaluate the inner integral in 29. The result is

$$\int_0^{tcm} e^{-u} du = \frac{1}{a} \left[ (ar_w - 1) e^{ar_w} + 1 \right].$$  \hspace{1cm} [30]

We still need to integrate 30 with respect to 0. This is easier if we apply the Taylor series expansion of the exponential first,

$$\int_0^{tcm} e^{-u} du = \frac{1}{a^2} \left[ (ar_w - 1) \sum_{n=0}^\infty \frac{1}{n!} (ar_w)^n + 1 \right]$$  \hspace{1cm} [34]

It then follows that

$$I = \frac{1}{\pi} \sum_{n=0}^\infty \frac{n+1}{(n+2)!} \int_0^\infty d^2 d\theta.$$  \hspace{1cm} [34]

Since $a = tcm/\lambda$ is proportional to $\cos(\theta - \theta_0)$, all of the odd-powered integrals in 34 are 0. The even powered terms with $n = 2m$ can also be integrated explicitly using 38

$$\int_0^\infty \cos^{2m}(\theta) d\theta = 2\pi (2m - 1)!! / (2m)!.$$  \hspace{1cm} [35]
where \((2m)!! = 2 \cdot 4 \cdot 6 \cdots (2m)\) and \((2m−1)!! = 1 \cdot 3 \cdot 5 \cdots (2m−1)\). The result is

\[
I = 1 + \sum_{m=1}^{\infty} \frac{2(2m + 1)(2m − 1)!!}{(2m + 2)!} \left( \frac{r_w}{\lambda} \right)^{2m} . \tag{36}
\]

\[
= 1 + \frac{1}{8} \left( \frac{r_w \tan(\gamma)}{\lambda} \right)^2 + \frac{1}{192} \left( \frac{r_w \tan(\gamma)}{\lambda} \right)^4 + \ldots \tag{37}
\]

For typical values of the tilt and decay length, the second term is on the order of \(10^{-4}\) or less. This shows that for small tilt to decay length ratios,

\[
\bar{p}_x = Ke^{-\delta h/\lambda} \tag{38}
\]

with only a very small error.

A fast force analysis algorithm.—Given measured shear force \(F_{x}^{m}\) we can now use (14b) and 38 to obtain an accurate estimate of the wafer attitude:

\[
\bar{p}_x = F_{x}^{m} / (\mu_k \pi r_w^2) \tag{39}
\]

\[
Ke^{-\delta h/\lambda} = F_{x}^{m} / (\mu_k \pi r_w^2) \tag{40}
\]

\[
\delta_0 = \lambda \ln(\pi r_w^2 \mu_k K / F_{x}^{m}) . \tag{41}
\]

In the co-rotation case, \(\delta_0\) can therefore be obtained very simply from the shear force. From 20, we then have that

\[
\beta = -\delta_0 \frac{4}{3 \pi \mu_k \rho c \mu_k \omega} \left( F_{x}^{m} - F_{y}^{m} \right) / \mu_k \tag{42}
\]

The two accessible parameters, the pitch and the center altitude, can therefore be obtained trivially from the force measurements using these two equations. For co-rotation in particular, two parameters are the best we can hope to extract since \(F_x\) is theoretically zero and cannot supply additional information. It is interesting and counter-intuitive that it is the shear force in CMP that determines the altitude and the normal force that determines the pitch rather than the other way around.

Equations 41 and 42 obviously lead to a very fast algorithm. On a 2.8 GHz MacBook Pro, the method described in the Theory of forces section required 1,049,400 sec (12 days) to analyze almost 60,000 force pairs in example 2. The fast algorithm processed the same data in 4.5 sec. In Fig. 15, we compare the altitude and pitch during the first slurry pulse calculated using the simplex solver (Eqs. 2–7) with the results from the fast algorithm (Eqs. 41, 42). The comparison is very good. The offsets in the results are caused by deviation of the measured tail of the surface height PDF from being exactly exponential.

Conclusions

We have presented two detailed examples of shear and normal force data collected on an instrumented CMP tool. Analysis of this data with a model that incorporates both solid contact pressures and fluid pressures indicates that many features of the resulting force scatter plots can be accounted for if significant fluid pressures are present in the interface between the pad and wafer. The fluid pressure magnitudes that we extract from force data are comparable to those measured in experiments in which pressure sensors are embedded either in the polishing head or in the platen. Using a rough surface contact model and a fluid dynamics model, we can also extract wafer attitude parameters from the force data. We find that it is possible to reliably estimate the altitude at the center of the wafer and the pitch of the wafer but not the bank. The extracted wafer attitude behavior is similar to that seen in measurements although the pitch magnitude is smaller, perhaps because of our larger applied load. Parameter extraction can be performed faster than real time, making the analysis potentially useful for in situ process monitoring to detect conditions such as hydrodynamic chatter or impending wafer slippage.

For practical applications to 10 mm and beyond, we have learned that better pad thickness uniformity control would be desirable on concentrically grooved pads in order to avoid hydrodynamically-driven vibrational bursts. Pressure-driven force excursions during such bursts may be large enough to damage delicate surface features. Alternatively, a lower slurry flow rate could be used to avoid bursts provided that the application method does not itself increase defectivity.

We have also learned several new theoretical insights about CMP. When the wafer tilt is small, as it almost certainly is in front-referenced polishing, there is a very simple and accurate formula for the mean fluid pressure in the pad-wafer interface. To our knowledge, this formula is completely new to CMP. The formula makes some interesting predictions, in particular that the mean fluid pressure does not depend on the carrier rotation rate. The implication is that one can change the carrier rate without exacerbating average fluid dynamic effects, although local fluid pressures would be affected. The formula also reinforces the conclusion that information about the bank is probably not obtainable from force data. If bank were to be monitored on a tool, it would have to be measured using a different technique than by force acquisition.

Appendix A: Approximate Solution of the Reynolds Equation

The basic idea in the analysis is to transform the Cartesian form of the Reynolds equation to the polar form with the coordinate system centered on the wafer, with \(y = r \sin(\theta)\) and \(x_w = r \cos(\theta)\). We then introduce a small parameter, expand some of the functions in the equation in terms of the parameter, and estimate the mean pressure using only the leading terms.

Preliminaries.—Starting with the Reynolds Equations 4 or 21,

\[
\nabla \cdot (\eta \nabla \sigma_{ij}) = 6 \eta \partial_t \nabla \cdot \mathbf{v} \Rightarrow R . \tag{A1}
\]

we expand the divergence operator on the left side, obtaining

\[
\eta \nabla^2 \sigma_{ij} + 3 \eta \nabla \cdot \mathbf{v} \nabla \cdot \nabla \sigma_{ij} = R . \tag{A2}
\]

In the first term, the Laplacian of the fluid pressure in polar coordinates is

\[
\nabla^2 \sigma_{ij} = \frac{1}{r^2} \partial_r \left( r^2 \partial_r \sigma_{ij} \right) + \frac{1}{r} \partial_\theta \left( \frac{r \partial_\theta \sigma_{ij}}{r} \right) . \tag{A3}
\]

The conversion of the Laplacian is straightforward but tedious. See for example16 for a detailed derivation. The gradient dot product in the second term in polar coordinates is

\[
\nabla h \cdot \nabla \sigma_{ij} = \frac{1}{r} \frac{\partial h}{\partial r} \frac{r \partial \sigma_{ij}}{r} + \frac{1}{r} h \partial_\theta \sigma_{ij} . \tag{A4}
\]

Dividing both sides of (A2) by \(h^2\) and using (A3) and (A4), we obtain the Reynolds equation in polar coordinates,

\[
c_1 \frac{1}{r^2} \partial_r \left( r^2 \partial_r \sigma_{ij} \right) + \frac{1}{r} \partial_\theta \left( \frac{r \partial_\theta \sigma_{ij}}{r} \right) + \frac{3}{R} \left( \frac{1}{r} \frac{\partial h}{\partial r} \frac{r \partial \sigma_{ij}}{r} + \frac{1}{r} h \partial_\theta \sigma_{ij} \right) = \frac{R}{R^2} \tag{A5}
\]

In (A5), we have also multiplied both sides by \(h^2\) in order to treat the potential singularity at the origin. Equation A5 also requires a boundary condition. The appropriate one is that the fluid pressure should be zero at the wafer perimeter: \(\sigma_{ij}(r_w, \theta) = 0\).
The conversion of the fluid thickness to wafer-centered polar coordinates is:

\[ h = h_0 - \alpha(x - c_x) + \beta y \]

\[ = h_0 - \alpha r \cos(\theta) + \beta r \sin(\theta) \]

\[ = h_0 - \tan(\gamma) \cos(\theta) + \tan(\gamma) \sin(\theta) \]

\[ = h_0 - r \tan(\gamma) [\cos(\theta) + \sin(\theta) \sin(\theta)] \]

\[ = h_0 - r \tan(\gamma) \cos(\theta) + \theta. \] \[ \text{[A6]} \]

In the conversion, we have used Eq. 1 to express \( h \) in terms of the tilt and rotation rather than the bank and pitch. This form of \( h \) is more natural in polar coordinates. For example, we can see immediately from (A6) that \( h \) is minimum when \( \theta = 0 \). Factoring out \( h_0 \), we have that

\[ h = h_0 \left(1 - \frac{r \tan(\gamma) \cos(\theta - \theta_0)}{h_0} \right) \]

\[ = h_0 \left(1 - r \cos(\theta - \theta_0) \right), \] \[ \text{[A7]} \]

where we have introduced the parameter \( \varepsilon = \tan(\gamma)/h_0 \). Since \( r \leq r_c \leq 0.15 \text{~mm} \) and \([\cos(\theta) - \theta_0] \leq 1\), it follows that \( r \cos(\theta - \theta_0) < \varepsilon \). Thus, if \( r \) is sufficiently small, then the pad-wafer separation at any point under the wafer is a small perturbation of the center separation.

**Mean fluid pressures.**—While the goal is to obtain the overall mean fluid pressure, we will actually do more: we will estimate the mean fluid pressure at each radius \( r \) and then derive from this the overall mean. The formulas for the two means are

\[ P_f(r) = \frac{1}{2\pi} \int_0^{2\pi} p_f(r, \theta) d\theta \]

\[ \hat{P}_f = \int_0^{r_c} \int_0^{\pi} p_f(r, \theta) rdrd\theta \]

\[ = \frac{2}{\pi^2} \int_0^{r_c} \frac{\hat{P}_f(r)dr}{r} \] \[ \text{[A8]} \]

and

\[ \hat{P}_f = \frac{6\mu B \sigma_c \text{rcm}}{k_0} (r^2 - r_c^2) \] \[ \text{[A11]} \]

In order to prove this, we start by splitting the terms in the Reynolds equation into three groups A, B, and C for separate analysis:

\[ A := r \frac{\partial p_f}{\partial r} + r \frac{\partial p_f}{\partial r} + \frac{\partial^2 p_f}{\partial \theta^2} \] \[ \text{[A12]} \]

\[ B := \frac{2}{\pi} \left( \frac{\partial p_f}{\partial r} + \frac{\partial p_f}{\partial \theta} \right) \] \[ \text{[A13]} \]

\[ C := \frac{r^2 R}{\sigma_c} \] \[ \text{[A14]} \]

The angular mean of the “A” group is

\[ \bar{A} = \frac{1}{2\pi} \int_0^{2\pi} A(r, \theta) d\theta = \int_0^{2\pi} \hat{A} dr + \int_0^{2\pi} \frac{\partial \hat{A}}{\partial \theta} d\theta \]

\[ = \frac{r^2 \hat{A}}{\sigma_c} + \frac{\partial \hat{A}}{\partial r} + \frac{\partial \hat{A}}{\partial \theta} \] \[ \text{[A15]} \]

The last step follows because \( \hat{p}_f \) and its derivatives are necessarily periodic in \( \theta \).

We next calculate the angular mean of “C”, which depends on \( 1/h \). We first introduce the shorthand notation \( c_m = \cos(\theta - \theta_0) \), so that \( h = h_0(1 - c_m) \). If \( r \) is small enough so that \( r c_m < 1 \) (this is what “small” means), then

\[ \frac{1}{h} = \frac{1}{h_0} - \frac{1}{1 - c_m} = \sum_{n=0}^{\infty} (r c_m)^n = \frac{1}{h_0} (1 + O(\epsilon)) \] \[ \text{[A18]} \]

where \( O(\epsilon) \) means that the error is a function that is bounded in absolute value by a constant times \( \epsilon \). Similarly,

\[ \frac{1}{h} = \frac{1}{h_0} \frac{1}{1 - c_m(r c_m)} = \frac{1}{h_0} \sum_{n=0}^{\infty} (n + 1)((n + 2) 2 \text{~mm} (rc_m)^n = \frac{1}{h_0} (1 + O(\epsilon)) \] \[ \text{[A19]} \]

While we have provided series expansions, the order estimates are evident without them. Using (A14) and (A19), we have that

\[ C = \frac{1}{h_0} (1 + O(\epsilon)). \] \[ \text{[A20]} \]

where from 21,

\[ R = 6\mu [\alpha (\Omega_2 - \Omega_c) y + \beta (\Omega_2 - \Omega_c) (x - c_x)] \]

\[ = 6\mu [\alpha (\Omega_2 - \Omega_c) y \sin(\theta) + \beta (\Omega_2 - \Omega_c) \cos(\theta) + \beta \Omega_2 c_x]. \] \[ \text{[A21]} \]

Since the angular mean of both \( \sin(\theta) \) and \( \cos(\theta) \) is zero,

\[ \bar{R} = 6\mu \beta \Omega_2 c_x. \] \[ \text{[A23]} \]

and therefore

\[ \bar{C} = \frac{1}{h_0} (1 + O(\epsilon)). \] \[ \text{[A24]} \]

The transition from (A22) to (A23) is where any effects due to the difference in carrier and platen rotation rates are lost due to averaging.

Turning now to the “B” terms and using the shorthand notation \( s_m = \sin(\theta - \theta_0) \), we have that

\[ B = \frac{3}{h_0} \left( \frac{\partial p_f}{\partial r} + \frac{\partial p_f}{\partial \theta} \right) \] \[ \text{[A25]} \]

\[ = \frac{3}{h_0} \frac{1}{1 + O(\epsilon)} \int_0^{r_c} \int_0^{\pi} \frac{\hat{P}_f(r)dr}{r^2} \] \[ \text{[A26]} \]

\[ = \frac{3}{h_0} \int_0^{r_c} \int_0^{\pi} \partial p_f(r)drd\theta \] \[ \text{[A27]} \]

\[ = 3\epsilon (1 + O(\epsilon)) \left( -r^2 \frac{\partial p_f}{\partial r} + r m \frac{\partial p_f}{\partial \theta} \right) \] \[ \text{[A28]} \]

Since we are assuming that both partial derivatives are bounded and since \( r_c \) and \( s_m \) are bounded, it follows that \( B = O(\epsilon) \). Thus, none of the “B” terms are of lowest order.

Combining the angular mean estimates for the three groups and neglecting the \( O(\epsilon) \) terms, to lowest order, the angular mean fluid pressure is a solution of

\[ r^2 \frac{d^2 \hat{P}_f}{dr^2} + r \frac{d \hat{P}_f}{dr} + \frac{\hat{P}_f}{\sigma_c} = \frac{R \bar{C}}{h_0} \] \[ \text{[A29]} \]

The boundary condition at the wafer perimeter requires that \( \hat{P}_f (r_c) = 0 \).

Equation A29 is an Euler equation.1 The general solution, as can be verified by direct substitution into (A29), is

\[ \hat{P}_f (r) = c_1 + c_2 \ln(r) + \frac{\bar{R}}{4h_0} r^2 \] \[ \text{[A30]} \]

where \( c_1 \) and \( c_2 \) are arbitrary constants. The first two terms in (A30) comprise the homogeneous solution (the solution of (A29) when the right side is zero) and the last term is a particular solution. The logarithmic term in (A30) is unbounded at \( r = 0 \) and has an unbounded derivative. Since we are assuming that the first derivatives of the fluid pressure are bounded, it follows that \( c_2 = 0 \). Thus

\[ \hat{P}_f (r) = c_1 + \frac{\bar{R}}{4h_0} r^2 \] \[ \text{[A31]} \]

The boundary condition at \( r = r_c \) applied to (A31) finally gives

\[ \hat{P}_f (r) = \frac{\bar{R}}{4h_0} r^2 \] \[ \text{[A32]} \]

\[ \hat{P}_f (r) = \frac{6\mu B \sigma_c \text{rcm}}{4h_0} (r^2 - r_c^2) \] \[ \text{[A33]} \]

This proves (A11).

**Result 2:** If the first partial derivatives of the fluid pressure are bounded, then

\[ \hat{P}_f = -\frac{6\mu B \sigma_c \text{rcm}}{4h_0} \] \[ \text{[A34]} \]

This is a straightforward application of formula (A10) to Eq. A11.

It is quite remarkable that the carrier rotation rate plays no role in either of the mean formulas. In Figure A1, we compare (A11) with angular averages extracted from high-resolution finite element solutions of the Reynolds equation for a 60 RPM platen and a 0, 60 or 120 RPM carrier. The finite element averages are nearly identical to each other and are indistinguishable from (A11).
Figure A1. Comparison of the angular mean fluid pressure formula (A11) with finite element solutions with 100 radial and 144 angular divisions. All calculations use a 12 μm altitude, 1.5 × 10^{-7} tilt and 45° rotation (see (A6)). The carrier rotation rate is 0, 60 or 120 RPM and the platen is at 60 RPM.

There is a more difficult method for solving the Reynolds equation using an infinite series (see Ref. 17 for an explanation of the method for ordinary differential equations). We will not present the details, but the key is to start with the right form for the series. If we assume that

\[ p_j(r, \theta) = \sum_{n=0}^{\infty} a_n(r^n - r_0^n) \]  

and substitute the series and its derivatives into the polar form of the Reynolds Equation A5, then by equating coefficients of like powers of \( r \) on the left side with those on the right, one can obtain a sequence of recurrence relations for the coefficients. Ignoring solutions of the homogeneous equation and applying the boundary condition, the final solution looks like

\[ p_j(r, \theta) = \sum_{n=0}^{\infty} a_n(r^n - r_0^n) \]  

where \( a_1 = 0 \) and

\[ a_2 = \frac{6\mu \beta \Omega_2 c_w}{8r_0^2}. \]  

The \( a_2 \) term gives (A11). The \( a_1 \) term is quite small compared with the \( a_2 \) term and is the first term showing the influence of the carrier rotation. The angular average of this term is 0. The next term is very complicated. It also includes the effect of carrier rotation but does not average to zero. Its contribution, however, is much smaller than that of the \( a_2 \) term.

The cubic coefficient (A38) can also be written in terms of the pitch and bank using the sine and cosine addition formulas and Eq. 1. After averaging, however, all of the information related to the bank zeros out, emphasizing the underlying difficulty in extracting this parameter from force data.

Appendix B: Departures from Co-rotation

When the platen and carrier do not co-rotate, any result that uses Eq. 2b with \( u_0 = 1 \) is not exact. However, the deviation is very small for most operating conditions. To illustrate this, we begin by plotting \( u_y \) in Fig. B1 for the rotation conditions in example 2 (90 RPM platen, 45 RPM carrier). We see that \( u_y \) is positive, it lies between 0.94 and 1, contours of constant \( u_y \) are straight lines, and the minimum occurs in two locations on the perimeter of the wafer. Letting \( \Delta \Omega = \Omega_p - \Omega_c \), the formula for \( u_y \) is

\[ u_y = \frac{\Delta \Omega + \Omega_c c_w}{\sqrt{(\beta \Omega_2 c_w)^2 + (\alpha \Delta \Omega + \Omega_c c_w)^2}}. \]  

Several results can be derived directly from Eq. B1. We list them without proof.

Figure B1. The y-component \( u_y \) of the shear force direction vector when the platen rotation rate is 90 RPM and the carrier rotation rate is 45 RPM.

(B2.1) If \( \Omega_p/\Omega_c < 1 + c_w/r_0 \), then \( u_y \) is positive everywhere under the wafer. For the APD-800 polisher, the number on the right side of this inequality is 2.5 for 300 mm wafers.

(B2.2) Under the above condition, \( u_y = 1 \) when \( y = 0 \).

(B2.3) \( u_y = c \), for \( c \) a constant, when \( y = \pm \sqrt{((1 - c^2)/c^2)(x + \Omega_c c_w/\Delta \Omega)} \).

(B2.4) The minimum of \( u_y \) is

\[ \min(u_y) = \frac{1}{\sqrt{1 + (c \beta \Delta \Omega/\Omega_2 c_w)^2}}. \]  

Equation B2 is used in Fig. B2 to plot the minimum of the y-component of the shear force direction vector for platen and carrier rotation rates between 30 and 120 RPM for the APD-800 polisher using 300 mm wafers. Co-rotation is indicated by the diagonal dotted line. For any platen rotation rate in this range, the carrier rotation rate can differ by between +/- 2 RPM from the platen rate for a 30 RPM platen to +/- 8 RPM for a 120 RPM platen and \( u_y \) will still exceed 0.999 everywhere under the wafer. For a band of carrier rotation rates about three times broader, \( u_y \) exceeds 0.99. The component is greater than 0.9 over most of the graph. Thus, for most polishing conditions in use, co-rotation is a practical approximation.

Figure B2. Minimum value of the y-component of the shear force direction vector as a function of platen and carrier rotation rate.
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