Gamma-Ray Bursts are precise distance indicators similar to Type Ia Supernovae?

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Abstract. We estimate the distance modulus to long gamma-ray bursts (LGRBs) using the Type I Fundamental Plane, a correlation between the spectral peak energy $E_p$, the peak luminosity $L_p$, and the luminosity time $T_L$ ($\equiv E_{iso}/L_p$ where $E_{iso}$ is isotropic energy) for small Absolute Deviation from Constant Luminosity (ADCL). The Type I Fundamental Plane of LGRBs is calibrated using 8 LGRBs with redshift $z < 1.4$. To avoid any assumption on the cosmological model, we use the distance modulus of 557 Type Ia supernovae (SNeIa) from the Union 2 sample. This calibrated Type I Fundamental Plane is used to measure the distance moduli to 9 high-redshift LGRBs with the mean error $\bar{\sigma}_\mu = 0.31$, which is comparable with that of SNe Ia $\bar{\sigma}_\mu = 0.26$ where $\mu$ stands for the distance modulus. The Type I Fundamental Plane is so tight that our distance moduli have very small uncertainties. From those distance moduli, we obtained the constraint $\Omega_M = 0.22 \pm 0.04$ for flat $\Lambda$CDM universe. Adding 9 LGRBs distance moduli ($z > 1.4$) to 557 SNeIa distance moduli ($z < 1.4$) significantly improves the constraint for non-flat $\Lambda$CDM universe from $(\Omega_M, \Omega_\Lambda) = (0.29 \pm 0.10, 0.76 \pm 0.13)$ for SNeIa only to $(\Omega_M, \Omega_\Lambda) = (0.23 \pm 0.06, 0.68 \pm 0.08)$ for SNeIa and 9 LGRBs.

Keywords: gamma rays: bursts — gamma rays: observations — gamma rays: cosmology

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1 Introduction

Since the discoveries of possible distance indicators of long gamma-ray bursts (LGRBs) [1–7], many attempts to measure the distance up to the high redshift universe (z > 1.7) which cannot be probed by Type Ia supernovae have been done to test cosmological models [8–20]. These attempts are very important since no other distance indicators have been known in such high redshift universe.

However the large statistical and systematic errors of the distance indicators restricted the precision of distance measurements by LGRBs. To estimate distances of LGRBs more precisely, there are two directions: to discover more precise distance indicator [6, 10, 21] and/or to take weighted average of distances derived by many distance indicators [11, 13, 16]. These studies, however, are on the verge of many difficulties, such as missing or multiple jet breaks in Swift era [22, 23] or increasing scatters of distance indicators [24–26]. To make LGRBs to be more precise and reliable standard candles, we must know the origins of the dispersion of distance indicators to control them carefully.

In our previous studies, we have investigated origins of dispersion of the spectral-brightness correlations and found possible origins of systematic errors [27, 28]: (i) spectral peak energies ($E_p$) estimated by fitting the spectra with cut-off power-law model with three free parameters tend to be overestimated compared with those obtained by using the Band model with four free parameters [29], (ii) peak luminosities ($L_p$) should be estimated for a fixed time scale in the individual rest frame rather than the observer frame, (iii) contamination of short GRBs or other unknown populations. Removing the systematic errors by (i) and (iii), and correcting (ii), we found that the Fundamental Plane of LGRBs, that is, a correlation between the spectral peak energy $E_p$, the rest frame 2.752-second peak luminosity $L_p$, and the luminosity time $T_L$ ($\equiv E_{\text{iso}}/L_p$ where $E_{\text{iso}}$ is the isotropic energy), is much tighter than the original $E_p - T_L - L_p$ correlation by R. Tsutsui et al. [17] [See also 30].

Recently, we found that dividing LGRBs into two groups according to the absolute deviation from their constant luminosity ($ADCL$) significantly improves the Fundamental Plane of LGRBs [31]. $ADCL$ is of critical importance in this work so let us summarize the main results below. $ADCL$ is a quantity which characterizes the overall shape of light curve of a GRB and defined as follows:

$$ADCL = \frac{\sum_{i=1}^{N_{\text{bin}}} |C_{i_{\text{norm}}} - 0.01 - 0.98 \times T_{i_{\text{norm}}}^\text{norm}|}{N_{\text{bin}}},$$

(1.1)
where $C_{i}^{\text{norm}}$ is normalized cumulative counts and $t_{i}^{\text{norm}}$ is the normalized time by $T_{98}$ in which the cumulative counts increase from 0.01 to 0.99. Most of small-$ADCL(< 0.17)$ events consist of many pulses with almost the same brightness, while most of large-$ADCL(>0.17)$ events consist of a bright main pulse and the weak long tail like FRED (Fast Rise and Exponential Decay). In Figure. 1, we show the cumulative light curves of LGRBs from Tsutsui, et al. [31]. The time and the cumulative counts are normalized by $T_{98}$ and the total counts, respectively. The red solid lines indicate small-$ADCL$ events while blue solid lines do large-$ADCL$ events. The black dashed line indicates the line of the virtual source with a constant luminosity. We clearly see that red lines with small ADCL ($< 0.17$) and blue solid lines with large ADCL ($> 0.17$) form different groups in this figure so that our classification scheme for subclasses of LGRB would be reasonable.

In Tsutsui, et al. [31], we found that the best-fit function for small ADCL($< 0.17$) events is given by

$$L_{p} = 10^{52.53\pm0.01} \text{ergs}^{-1} \left( \frac{E_{p}}{10^{2.71}\text{keV}} \right)^{1.84\pm0.03} \times \left( \frac{T_{L}}{10^{0.86}\text{sec}} \right)^{0.29\pm0.08},$$

with $\chi^2/d.o.f = 10.93/14$ and $\sigma_{\text{int}} = 0$, while that for large ADCL($>0.17$) events is given by

$$L_{p} = 10^{52.98\pm0.080} \text{ergs}^{-1} \left( \frac{E_{p}}{10^{2.71}\text{keV}} \right)^{1.82\pm0.093} \times \left( \frac{T_{L}}{10^{0.86}\text{sec}} \right)^{0.85\pm0.26},$$

with $\chi^2/d.o.f = 7.58/8$ and $\sigma_{\text{int}} = 0$. Since both best-fit functions separately form the plane in ($\log E_{p}$, $\log T_{L}$, $\log L_{p}$) space, we call the best fit function for small-$ADCL$ (large-$ADCL$) as Type I (Type II) Fundamental Plane, respectively. Figure. 2 shows the Type I (left) and Type II (right) Fundamental Planes for given cosmological parameters from Tsutsui, et al. [31]. In both figures, Small-$ADCL$ events are marked with red filled circles and large-$ADCL$ events are with blue squares. Orange and light blue squares indicate outliers from the Type I and Type II Fundamental Planes, respectively. (see [31] for the definition of outliers.) The left (right) of Figure. 2 is the plane perpendicular to the Type I (Type II) Fundamental Plane so that the Type I (Type II) Fundamental Plane is expressed by the solid line in each figure. In the left of Figure. 2, blue triangles which belong to the large ADCL events are above the solid line (=the Type I Fundamental Plane) while in the right of Figure. 2, red filled circles which belong to the small ADCL events are below the solid line (=the Type II Fundamental Plane) so that the existence of two Fundamental Planes can be recognized. The existence of such two subclasses is very similar to that of two Period - Luminosity relations of Cepheid variables [32].

In this paper using 21 small-$ADCL$ LGRBs from the 36 gold sample of Tsutsui et al. [31], we try to measure distance modulus up to $z = 3.57$ and determine the cosmological parameters. To avoid any assumption on cosmological models, we use the distance moduli of 557 Type Ia supernovae (SNeIa) from the Union 2 sample to calibrate the Type I Fundamental Plane of LGRBs by the 10 low-z events in $0.168 < z < 1.30$. This is a big difference from our previous paper [31] where cosmological parameters were assumed to derive the relation.
Figure 1. The cumulative light curves of LGRBs from Tsutsui, et al. [31]. The red solid lines indicate small-ADCL events while blue solid lines do large-ADCL events. The black dashed line indicates the line of the virtual source with the constant luminosity. We clearly see that red lines with small ADCL (< 0.17) and blue solid lines with large ADCL(> 0.17) form different groups.

Applying the Type I Fundamental Plane calibrated at low redshifts without cosmological parameters to high-z GRBs, we obtain the relation between the redshift and the distance modulus $\mu$ of 11 high-z events in $1.54 < z < 3.57$. From these distance moduli, we will obtain the constraint on $\Omega_M = 0.22 \pm 0.04$ for flat $\Lambda$ CDM universe. In this paper we do not use the Type II Fundamental Plane as a distance indicator to constrain the cosmological parameters due to the lack of enough number of large ADCL events. Throughout the paper, we fix the Hubble parameter as $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$.

2 Calibration

2.1 local regression

First, we calibrate the Type I Fundamental Plane of LGRBs using low-z events whose distance moduli can be obtained by those of Type Ia supernovae. Actually, to estimate the distance moduli of low-z GRBs, we use a local regression method first developed by W. S. Cleveland. [33] and W. S. Cleveland and S. J. Devlin. [34], and first applied for GRB cosmology by V. F. Cardone et al. [16]. This method does not need a global shape function of redshift-distance modulus relation and is one of interpolation procedures which use only the data near points and fit them with a low-degree polynomial. Although the number of data and the degree of polynomial are arbitrary variables, it does not depend on a shape of a global function.

Applying this method, we obtain the distance moduli of 10 low-redshift LGRBs with mean error $\sigma_\mu = 0.065$ where $\mu$ stands for the distance modulus.
Figure 2. The $E_p - T_L - L_p$ diagram both for the Type I (left) and Type II (right) Fundamental Planes for given cosmological parameters from Tsutsui, et al. [31]. Small-ADCL events are marked with red filled circles and large-ADCL events with blue squares. Except for outliers (orange triangles), most of small-ADCL events are on the Type I Fundamental Plane (=the solid line in the left figure) within their error bars without any systematic errors, while most of large-ADCL events are on the Type II Fundamental Plane (=the solid line in the right figure) except for outliers (light blue triangles). (see [31] for the definition of outliers.) In the left figure, blue squares which belong to large ADCL events are above the solid line (=the Type I Fundamental Plane) while in the right figure, red filled circles which belong to small ADCL events are below the solid line (=the Type II Fundamental Plane) so that the existence of two Fundamental Planes can be recognized. The existence of such two subclasses is very similar to that of two Period - Luminosity relations of Cepheid variables.

2.2 calibration

Having estimated the distance moduli of 10 low-redshift GRBs in a model independent way, we can calibrate the Type I Fundamental Plane of GRBs. Let us assume a linear correlation between $E_p$, $T_L$ and $L_p$ in logarithmic scale as,

$$\log L_p(E_p, T_L) = A + B \log \left( E_p / \bar{E}_p \right) + C \log \left( T_L / \bar{T}_L \right),$$

(2.1)

where $A$, $B$ and $C$ are the parameters of the model and $\bar{E}_p$ and $\bar{T}_L$ are logarithmic mean of $E_p$ and $L_p$ with $10^{2.43}$ keV and $10^{0.97}$ sec, respectively. From equation (2.1), we obtain

$$\mu(E_p, T_L, F_p) \propto 2.5 \log L_p(E_p, T_L) - 2.5 \log(4\pi F_p)$$

$$-5 \log(3.0857 \times 10^{19}),$$

(2.2)

where $F_p$ is the observed energy flux. We define a chi square function,

$$\chi^2(A, B, C) \propto \sum_{i=1}^{N} \frac{(\mu_i - \mu(E_p, T_L, F_p))^2}{\sigma^2_{\mu_{\text{local},i}} + \sigma^2_{\mu_{\text{GRB},i}}},$$

(2.3)
and
\[
\sigma_{\mu GRB,i}^2 = \frac{25}{4} \left[ (1 + 2C) \sigma_{\log E_{p,i}}^2 + B^2 \sigma_{\log E_{p,i}}^2 + C^2 \sigma_{\log T_{L,i}}^2 + \sigma_{\text{int}}^2 \right],
\]
(2.4)

where \( \sigma_{\text{int}} \) is the systematic error determined by iteration.

Before we minimize the chi square function, we use the robust statistic and outlier detection method which have been developed in Tsutsui et al. [30]. As a result, we detected two outliers (091003, 080319B), and removed them from following chi square analysis. Performing chi square regression on the remaining samples, we obtained the best fit parameters and 1-\( \sigma \) uncertainties summarized in table 1. The parameters in table 1 and outliers of the relation are consistent with those in Tsutsui et al. [31]. Figure 3 shows the \( E_p-T_L-L_p \) diagram for small-ADCL events in \( z < 1.4 \). Two outliers are marked with green triangles and the solid line indicates the best fit function of Eq.(2.1) with values of A, B and C in table 1.

**Figure 3.** The \( E_p-T_L-L_p \) diagram for small-ADCL events (red filled circles) in \( z < 1.4 \) without any assumption of cosmological parameters. The solid line is the best fit function with \( \chi^2 = 0.83 \) and \( \sigma_{\text{int}} = 0 \). The green triangles are outliers. (see [31] for the definition of outliers.)

**Table 1.** The best fit values (1-\( \sigma \) uncertainties) of the model parameters of the Type I Fundamental Plane (Eq. (2.1)).

| A     | B     | C     | \( \sigma_{\text{int}} \) | \( \chi^2 \) |
|-------|-------|-------|----------------|-----------|
| 52.06(0.02) | 1.90(0.05) | 0.25(0.13) | 0 | 0.83 |
Table 2. The redshifts, distance modulus of LGRBs estimated by local regression method and the ones estimated from the Type I Fundamental Plane. The first through the seventh column express name of the GRB, the redshift, the distance modulus determined by Type Ia supernovae, the error of $\mu_{\text{local}}$, the distance modulus determined by the best fit function of Type I Fundamental Plane (Eq. (2.1)), the error of $\mu$, the value of ADCL, respectively. Four outliers are separated in the last four rows. They are the same as the outliers for small-ADCL GRBs in Tsutsui et al. [31].

| GRB   | $z$  | $\mu_{\text{local}}$ | $\sigma_{\mu_{\text{local}}}$ | $\mu$  | $\sigma_{\mu}$ | ADCL |
|-------|------|-----------------------|-------------------------------|-------|---------------|------|
| 030329 | 0.168 | 39.48 | 0.04 | 39.61 | 0.17 | 0.13 |
| 090618 | 0.54 | 42.43 | 0.04 | 42.33 | 0.19 | 0.10 |
| 050525 | 0.606 | 42.83 | 0.07 | 42.72 | 0.13 | 0.13 |
| 970228 | 0.695 | 43.15 | 0.05 | 42.7 | 0.72 | 0.14 |
| 990705 | 0.843 | 43.75 | 0.06 | 43.86 | 0.22 | 0.10 |
| 091208B | 1.063 | 44.3 | 0.09 | 44.77 | 0.77 | 0.07 |
| 061007 | 1.261 | 44.73 | 0.06 | 44.78 | 0.16 | 0.14 |
| 990506 | 1.3 | 44.81 | 0.07 | 44.82 | 0.26 | 0.08 |
| 070125 | 1.547 | - | - | 45.39 | 0.25 | 0.10 |
| 990123 | 1.6 | - | - | 45.5 | 0.16 | 0.09 |
| 990510 | 1.619 | - | - | 45.46 | 0.34 | 0.14 |
| 090902B | 1.822 | - | - | 45.91 | 0.15 | 0.09 |
| 081121 | 2.512 | - | - | 46.9 | 0.24 | 0.07 |
| 080721 | 2.602 | - | - | 46.6 | 0.28 | 0.16 |
| 050401 | 2.9 | - | - | 46.5 | 0.42 | 0.15 |
| 971214 | 3.418 | - | - | 47.5 | 0.73 | 0.12 |
| 090323 | 3.57 | - | - | 48.05 | 0.3 | 0.12 |
| 091003 | 0.897 | 43.82 | 0.09 | 45.97 | 0.28 | 0.16 |
| 080319B | 0.937 | 43.82 | 0.08 | 45.36 | 0.17 | 0.14 |
| 110213A | 1.46 | - | - | 42.9 | 0.38 | 0.09 |
| 110422A | 1.77 | - | - | 43.51 | 0.16 | 0.10 |

3 Constraint

Applying equation (2.2) to high redshift LGRBs, we can obtain the extended Hubble diagram with the unprecedented precision. We define $\sigma_{\mu_i}$ as below,

$$
\sigma_{\mu_i}^2 = \frac{25}{4} \left[ \sigma_A^2 + (\sigma_B \log E_p) \bar{E}_p \right]^2 + \left[ (B \sigma_{\log E_p})^2 + (C \sigma_{\log T_L})^2 + (1 + 2C) \sigma_{F_p}^2 \right] + \sigma_{\text{int}}^2
$$

(3.1)

Here we neglect the effect of the gravitational lensing because M. Oguri and K. Takahashi. [35] discussed the gravitational lensing of GRBs and found that the biases are small.

Then we derive constraints on cosmological parameters. In the $\Lambda$-CDM model, the luminosity distance ($d_L$) is given as a function of the density parameters, $\Omega_M$ and $\Omega_\Lambda$.

The chi square function is defined by,

$$
\chi^2(\Omega_M, \Omega_\Lambda) = \sum_{i=1}^{N} \left[ \frac{\mu_i - \mu_i^{\text{th}}(z_i, \Omega_M, \Omega_\Lambda)}{\sigma_{\mu_i}} \right]^2
$$

(3.2)
where $\mu^{th}(z_i, \Omega_M, \Omega_\Lambda) = 5 \log(d^\text{th}_L/\text{Mpc}) + 25$. We used the robust regression and outlier detection method again. As a result, we detect two outliers (110213A, 110422A), and removed it from following chi square analysis. In table 2, we summarize the redshifts, distance modulus of LGRBs estimated by local regression method and the ones estimated from the Type I Fundamental Plane with their 1-$\sigma$ uncertainties.

![Figure 4](image_url)

**Figure 4.** Extended Hubble diagram from the Type I Fundamental Plane. *Gray squares*: the 557 SNe Ia from R. Amanullah et al.[36]. *Blue open circles*: LGRBs at $z < 1.4$ estimated from the local regression of SNe Ia data. *Red filled circles*: LGRBs estimated from the Type I Fundamental Plane. *Green triangles*: outliers. The solid line indicates a theoretical model calculated with $(\Omega_M, \Omega_\Lambda) = (0.30, 0.70)$. *Inset*: A residual Hubble diagram from the theoretical model.

Figure 4 shows an extended Hubble diagram up to $z = 3.57$. The gray squares indicate the SNe Ia from R. Amanullah et al. [36], the blue open circles indicate the low redshift LGRBs at $z < 1.4$ estimated from the local regression of SNe Ia data. Red filled circles and green triangles indicate LGRBs estimated from the Type I Fundamental Plane, and outliers, respectively. The solid line indicates a theoretical model calculated with $(\Omega_M, \Omega_\Lambda) = (0.30, 0.70)$. As Figure 4 shows, the uncertainties of distance modulus of LGRBs are as small as those of individual SNe Ia (gray squares). The mean error of distance modulus of LGRBs are $\bar{\sigma}_\mu = 0.31$, while those of SNe Ia are $\bar{\sigma}_\mu = 0.26$.

Performing the chi square analysis on the remaining 9 LGRBs, we obtained the constraint $\Omega_M = 0.22 \pm 0.04$ for flat $\Lambda$CDM universe. In Figure 5, we show the likelihood contour for non-flat $\Lambda$CDM universe from 9 LGRBs (green), with the likelihood contours from 557 SNe Ia (blue) and 9 LGRBs + 557 SNe Ia (red), and the best-fit values with 1-$\sigma$ errors are shown in table 3. Figure 4 shows that adding only 9 high redshift LGRBs ($z > 1.4$) to 557 SNe Ia ($z < 1.4$) significantly improves the constraint on $\Omega_M-\Omega_\Lambda$ plane.
Table 3. Constraints on \((\Omega_M, \Omega_\Lambda)\) for flat universe from 9 LGRBs and for non-flat universe from 557 SNe Ia and 9 LGRBs + 557 SNe Ia with their reduced chi squares. All the quoted errors are at the 1-\(\sigma\) confidence level.

|                  | \(\Omega_M\)         | \(\Omega_\Lambda\) | \(\chi^2/d.o.f\) |
|------------------|-----------------------|---------------------|-------------------|
| 9 LGRBs (flat)   | 0.22 ± 0.04           | -                   | 5.09/7            |
| 557 SNe Ia (non-flat) | 0.29 ± 0.10           | 0.76 ± 0.13         | 542.1/555         |
| Combined (non-flat) | 0.23 ± 0.06           | 0.68 ± 0.08         | 548.3/564         |

4 Discussion

In this paper we extended the Hubble diagram with the Type I Fundamental Plane of LGRBs. Because of the tightness of the relation, our distance measurements are much more precise than those of previous works. Although the number of LGRBs are much smaller than that of the previous works [11–17], our constraint is much stronger. This indicates the importance of controlling and removing systematic errors. Although we used only the Type I Fundamental Plane to measure distance modulus, in principle, we may also use the Type II Fundamental Plane as an independent distance indicator. However, because there is still a small number of long tailed events, it is difficult to calibrate the relation with our outlier rejection technique.

The constraints from 9 high redshift GRBs are consistent with those from other probes, e.g. Cosmic Microwave Background (CMB), Baryon Acoustic Oscillation (BAO)[37, 38]. We should stress here that this consistency is not trivial. Since the redshift range of GRB (1.4 < z < 3.5) is different from TypeIa supernovae (z < 1.7), BAO (z ∼ 0.35) and CMB (z = z_{rec}), they might show different cosmological parameters if the dark energy is strongly time dependent. Therefore our results suggest that time dependence of the dark energy is
not so strong even if it exists.

The purpose of this paper is to show the potential high ability of Type I Fundamental Plane of the LGRBs as a distance indicator. Combining our data with other cosmological probes is the next step. A different direction is to test more general equation of state than ΛCDM universe. We are now trying to combine our data with CMB and BAO as well as more general equation of state to constrain the property of the dark energy.

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