Spin squeezing of the non-Hermitian one-axis twisting model

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In the absence of decay, the conditional dynamics for an open system is often describable by a non-Hermitian Hamiltonian. This study investigates spin squeezing (SS) in non-Hermitian one-axis twisting (OAT) model. Somewhat surprisingly, SS close to the limit of Hermitian two-axis counter twisting (TACT) Hamiltonian is achievable for some parameters, which significantly improves upon the optimal value realizable by Hermitian OAT model. The drawback is like with all conditional schemes, it often takes on average longer time to evolve into steady state, and the probability of no decay or success decreases as number of atoms (spins) increases. The result above for steady state SS in non-Hermitian OAT Hamiltonian is thus limited to small systems. For other parameter regimes, however, desirable SS arrives dynamically before steady state is achieved, with greatly shortened evolution time and enhanced probability of success, while still remain significantly improved over the limit of Hermitian OAT.

I. INTRODUCTION

Squeezed spin states [1–4] are symmetric states of spin-1/2 particles, whose fluctuation in one collective spin component perpendicular to the mean spin direction is smaller than the classical limit set by the summed fluctuations from independent spins pointing along the mean spin direction. This reduced fluctuation manifests quantum correlations among individual spins and has attracted considerable attention because of its potential application to improve the precision of quantum measurements [2, 3, 5–9] and to measure multipartite entanglement [10–13].

According to Kitagawa and Ueda [1], the squeezed spin states can be dynamically generated from the initial product state with all spins up (or down) under the one-axis twisting (OAT) Hamiltonian $H_{\text{OAT}} = \chi J_z^2$ and the two-axis counter twisting (TACT) Hamiltonian $H_{\text{TACT}} = \chi (J_z^2 - J_x^2)$. The collective spin $\vec{J} = \sum_{k} \hat{\sigma}_k^z/2$, with $h = 1$ hereinafter) is defined in terms of the Pauli operator of the $k$-th spin or pseudospin $\hat{\sigma}_k^z$ and $\chi$ denotes the coupling strength between two spins. When the squeezed states are applied to quantum metrology [2, 3], the property of interest is the squeezing parameter $\xi^2 = N(\Delta J_z^2)/(\langle J_z \rangle^2) < 1$, where $\Delta J_z^2 \equiv \langle J_z^2 \rangle - \langle J_z \rangle^2$ denotes the minimal fluctuation of a spin component perpendicular to the mean spin $\langle \vec{J} \rangle = \langle J_z \rangle$.

The optimal spin squeezing (SS) realizable theoretically for the OAT model is $\xi^2 \approx 1.15 N^{-2/3}$, which is reached at a time $\approx 1.2 N^{-2/3} \chi^{-1}$. A better squeezing of 4/N approaching the Heisenberg limit 1/N is achievable through the TACT at a shorter time $0.48 \ln(N)/(N \chi)$, which generally helps to mitigate accumulative influences from particle loss and dephasing. Besides, unlike the situation encountered in the OAT, the direction of optimal squeezing from the TACT remains fixed during time evolution and is independent of system size (number of spins $N$) [14]. Despite of its better performance in SS, two body interactions capable of facilitating the TACT do not occur naturally in most systems of interest. Although many proposals have been put forward to implement the TACT models [14–19], no experimental realizations have been reported so far. In contrast, the OAT-interaction induced SS has been proposed and demonstrated in several systems [5, 6, 8, 20–27], which attracted many followup studies aimed at improved SS in general OAT models [14, 19, 28–30].

A recent study [31] reports the surprising finding that the non-Hermitian TACT model can realize slightly stronger SS than its Hermitian counterpart, which is counter intuitive as damping is always viewed as causing damage to quantum coherent processes like SS. Inspired by the desire for improved understanding of SS in non-Hermitian models, we find surprisingly the non-Hermitian OAT model may be even more favorable. In addition to being more readily realizable experimentally, it provides SS approaching the limit of TACT model, which is significantly better than the optimal SS available from Hermitian OAT. This paper presents our investigation of enhanced SS in the non-Hermitian OAT model. It is organized as follows. Following this introduction section, the next two sections respectively discuss steady state SS and dynamically generated optimal squeezed states in the non-Hermitian OAT model. We compare the above results to the corresponding ones from the OAT and the TACT models before concluding in the last section.

II. STEADY STATE

Our model is built on the collective OAT interaction of an ensemble of $N$ two-state atoms (i.e., a collection of pseudospin 1/2 particles) with upper and lower states $\{| \uparrow \rangle, \downarrow \rangle \}$. Assuming a finite lifetime $1/\gamma$ for atoms in state $| \uparrow \rangle$, the atomic density matrix $\rho$ satisfies the master equation: $\partial \rho / \partial t = -i[H_{\text{OAT}}, \rho] + \mathcal{L}(\rho)$, where the super-operator

$$\mathcal{L}(\rho) = \sum_{k=1}^{N} \left[ 2 \sigma_k^+ \rho \sigma_k^- - \sigma_k^- \sigma_k^+ \rho - \rho \sigma_k^- \sigma_k^+ \right]$$

(1)

describes independent and identical single atom dissipative effects and $\sigma_k^\pm$ the Pauli operators of the $k$-th atom. As in the
non-Hermitian TACT model of Lee et al. [31], a finite and tunable value for γ can be engineered through coupling of state |↑⟩ to an unstable auxiliary state |a⟩. The Pauli operators in the above Eq. (1) are $\sigma_x^a = (\sigma_z^a)^\dagger = |\uparrow\rangle_{ka}\langle a|$, hence $\sigma_x^a\sigma_x^a = |\uparrow\rangle_{ka}\langle a|$. Conditioned on the absence of any decay event [31], whose probability is given by $P = e^{-\gamma tN}$ [32], one can remove the “real” decay terms $\sigma_x^a\rho\sigma_x^a$ in Eq. (1) and obtain $\mathcal{L}(\rho) = -\gamma(N\rho + \rho N)/2$, where $N = \sum |l\rangle\langle l|$ for the state label $l = \{\uparrow, \downarrow\}$ denotes the atom-number operator. The resulting conditional master equation becomes $\frac{d\rho}{dt} = -i[H_{\text{eff}} - \rho H_{\text{eff}}^\dagger]$, with a non-Hermitian effective Hamiltonian

$$H_{\text{eff}} = \chi J_z^2 - i\gamma N \tau/2,$$

and $N = J_z + N/2$.

The first term of the effective Hamiltonian in Eq. (2) describes the OAT interaction, which can be realized for instance in trapped ions [21, 33] or cavity QED [34], starting with a coupled Hamiltonian of the form $\mathcal{H} = \Delta a^\dagger a + g(a + a^\dagger)J_z$, where $a$ ($a^\dagger$) denotes annihilation (creation) operator of phonon in trapped ions or cavity in photon QED. For trapped ion system, this is realized through the two photon interaction of ions with two lasers of opposite detunings $\Delta = \omega_1 - \omega_2 - \omega_3$, for trapped ions system, this is realized through the EN cavitation system, photon system, this is realized through the two photon interaction of ions with two lasers of opposite detunings $\Delta = \omega_1 - \omega_2 - \omega_3$. (a) Energy level of ions with two lasers of opposite detunings $\Delta = \omega_1 - \omega_2 - \omega_3$. (b) Energy level of ions with two lasers of opposite detunings $\Delta = \omega_1 - \omega_2 - \omega_3$. (c) Energy level of ions with two lasers of opposite detunings $\Delta = \omega_1 - \omega_2 - \omega_3$.

The above results are obtained using exact numerical diagonalization. To further confirm their validity, we simulate the non-Hermitian dynamical evolution for the squeezing parameter $\xi^2$ with $\gamma/\chi$ at the value labeled by the star ‘*’ in Fig. 1(a), starting with all spins in |↓⟩ in Fig. 1(b). At short times, the non-Hermitian term has little effect and the squeezing parameter $\xi^2$ is observed to decrease according to OAT. After reaching the optimal point of minimum squeezing, which is essentially equal to limit of Hermitian OAT, its value starts to increase as the non-Hermitian term comes into play. This clearly shows up as the two curves for Hermitian and non-Hermitian OAT start to deviate from each other. The squeezing parameter $\xi^2$ for non-Hermitian OAT continues to decrease towards its steady-state value (blue dotted line) as time goes on and reaches its steady state value at a time $\sim 1/\chi$. The corresponding probability of no decay is calculated to be 20%, i.e., on average for one out five experimental runs, no decay occurs. This rate of success implies that the non-Hermitian scheme is feasible, certainly for the small size of $N = 20$ atoms evaluated here. It’s worthy pointing out that the squeezing parameter with $\gamma/\chi$ at the star ‘*’ does not correspond to the minimum. A slightly improved squeezing can be obtained at the cost of a longer evolution time and a lower success rate. The squeezing process can be intuitively illustrated by evolution of the quasi-probability distribution $Q(\theta, \psi)$ for the state $|\psi(t)\rangle$, which is determined by its projection onto the coherent spin state $|\theta, \psi\rangle = (\cos(\theta/2)|\uparrow\rangle + e^{\i\psi}\sin(\theta/2)|\downarrow\rangle)^N$, i.e., $Q(\theta, \psi) = |\langle\theta, \psi|\psi(t)\rangle|^2$, as shown in Fig. 1(c). Shaping of the initial isotropic uncertainty distribution results in reduced spin variance along one direction. Defining $\sigma_{\text{min}}$ as the angle between the optimal squeezing direction with respect to $x$-axis, it is seen that $\sigma_{\text{min}}$ tends zero under the influence of
the non-Hermitian term.

To check if the non-Hermitian scheme remains valid for large atom numbers $N$, we study scaling of the various SS quantities with respect to $N$ in Fig. 2, at $\gamma/\chi = 1/(0.03N)$, i.e., with the linewidth $\gamma$ decreasing as $N$ increases. We find squeezing parameter $\xi^2$ [Fig. 2(a) in blue circles] scales as $4/N$, which is equal to the TACT squeezing limit, and much better than the OAT squeezing limit $1.15N^{-2/3}$. In addition, the squeezing direction is almost fixed along $x$-axis or $\sigma_{\text{min}} = 0$ as shown in Fig. 2(b), representing a significant advantage over the Hermitian OAT model. The drawback is that the optimal squeezing time, which is essentially the same time for reaching steady state $\chi t_{\text{r}}$, at $\gamma/\chi = 1/(0.03N)$, is around $0.1 \sim 1$ and slightly increases as $N$ increases, as shown in Fig. 2(c). This is in contrast to the Hermitian OAT model, whose squeezing time decreases as $N$ increases. In practical implementations, this could present a serious obstacle for systems with short coherence times. Furthermore, the SS we study in the non-Hermitian OAT model is only experimentally accessible when no decay event occurs, thus the probability of success $P$ is an important consideration. A low success rate implies more measurement runs, which directly translates into longer times. As shown in Fig. 2(d), the total evolution time for success at $\chi t_{\text{r}}/P$ increases as atom number $N$ increases. For $N > 1000$, it becomes larger than $10$. In fact, at $\gamma/\chi = 1/(0.073N)$, we find the squeezing scaling becomes $3/N$, surpassing the optimal SS limit from TACT, although the relatively small $P$, on the order of $1\%$ for $N > 1000$, eventually kicks in to limit its applications.

III. OPTIMAL SQUEEZED STATE

In previous discussions, we focus on the steady state at a specific parameter of $\gamma/\chi = 1/(0.03N)$, which provides a significantly improved SS with the degree of squeezing equal to the Hermitian TACT model. The price to pay is the prolonged evolution time to settle into steady state and the low probability of success which further decreases as $N$ increases. To overcome these problems, we search for other parameter regimes. Since the steady state is not necessarily the optimal squeezed state, we investigate the optimal squeezed state obtained from non-Hermitian evolution instead of the steady state.

Figure 3(a) presents squeezing parameter $\xi^2$ obtained from non-Hermitian evolution at $\gamma/\chi = 0.1$ (blue solid line) and $\gamma/\chi = 0.5$ (blue dotted line) for $N = 1000$. For both cases, we find their $\xi^2$ reach the minimal values (marked by arrows) before settling down to the steady values. Although the optimal squeezing at these two parameters do not surpass the limit from TACT, the benefits reside in their much shortened evolution times. To understand the dependence of the optimal squeezing parameter $\xi^2_{\text{min}}$ on the parameter $\gamma/\chi$, we study their dependence in Fig. 3(b). In a wide parameter range, the degree of squeezing for our non-Hermitian OAT model again is found to surpass that of the Hermitian OAT model, which implies that the performance of our non-Hermitian scheme is insensitive to noise induced parameter fluctuations. With smaller $\gamma/\chi$, we find the degree of squeezing improves, again
obtain a degree of squeezing equal to \[ 10\log_{10} \xi \].

![Graph showing the dependence of various SS quantities on \( N \).](image)

(a) Optimal squeezing parameter \( \xi_{\text{min}}^2 \); (b) optimal squeezing time \( \chi_{\text{min}}^0 \); (c) total evolution time required for success \( \chi_{\text{min}} \). Dotted markers denote the results from non-Hermitian OAT model with \( \gamma/\chi = 0.1 \) (blue squares) and 0.5 (blue circles). The red dashed and black dashed-dotted lines denote the corresponding results from Hermitian OAT and TACT models, respectively.

at the cost of a longer evolution time, whether for a single run [Fig. 3(c)] or for the total time needed for success [Fig. 3(d)].

The results above therefore impose a trade-off between the degree of squeezing with evolution time. Reaching the optimal degree of squeezing requires a delicate balance between the two. To demonstrate this more clearly, we compare the scaling of various quantities with \( N \) at two values of parameter \( \gamma/\chi = 0.1 \) and 0.5 as presented in Fig. 4. For \( \gamma/\chi = 0.1 \) (blue squares), we find the squeezing parameter \( \xi^2 \) scales as \( 1.2/N^{1/3} \) [Fig.4(a), blue solid line], the corresponding evolution time for a single run \( \chi_{\text{min}}^0 \) scales approximately as \( 17N^{2/3} \) [Fig. 4(b), blue solid line], and the total evolution time to success \( \chi_{\text{min}}/P \) decreases as \( N \) increases [Fig.4(c)]. For the larger \( \gamma/\chi = 0.5 \) (blue circles), the degree of squeezing becomes less and scales as \( 1.2/N^{3/4} \) [Fig. 4(a), blue dashed line], the evolution time both for a single run \( 5.4N^{-2/3} \) [Fig. 4(b), blue dashed line] and for the total [Fig.4(c)] become shorter. Specifically, for \( N = 10^4 \) spins, we obtain a degree of squeezing equal to \( 10\log_{10}\xi^2 = -31.1\)dB for \( \gamma/\chi = 0.1 \) at an evolution time of \( \chi_{\text{min}} = 0.0354 \), while for the latter case of \( \gamma/\chi = 0.5 \), \(-29.0\)dB squeezing is reached at \( \chi_{\text{min}} = 0.0112 \). They can be compared to the more standard results of \(-26.2\)dB at \( \chi_{\text{min}} = 0.00254 \) for the Hermitian OAT model, and \(-34.1\)dB at \( \chi_{\text{min}} = 0.000445 \) for the Hermitian TACT model.

IV. CONCLUSION

In conclusion, we study spin squeezing in a non-Hermitian OAT model, which describes the OAT dynamics conditional on the absence of decay event. For the parameter \( \gamma/\chi = 1/(0.03N) \), we find that the steady state shows the squeezing limit \( \sim 4/N \), significant improvement over that of the Hermitian OAT model. This is achieved at the expense of prolonged evolution time and low probability of success conditional on no decay event. The former increases with \( N \), while the later decreases as \( N \) increases, combined to give an overall unfavorable scaling with \( N \). Thus while encouraging, our scheme is perhaps only applicable to systems with small \( N \). Furthermore, by investigating the non-Hermitian dynamics at other parameter regimes where optimally squeezed states arrive before steady states, we find the optimal squeezing time can be greatly shortened and success rate enhanced, at the same time impressive degrees of SS significantly beating the OAT model is maintained. Our work highlights potentially fruitful applications of non-Hermitian OAT to small samples of coupled spins.

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