FloMore: Meeting bandwidth requirements of flows

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ABSTRACT
Wide-area cloud provider networks must support the bandwidth requirements of diverse services (e.g., applications, product groups, customers) despite failures. Existing traffic engineering (TE) schemes operate at much coarser granularity than services, which we show necessitates unduly conservative decisions. To tackle this, we present FloMore, which directly considers the bandwidth needs of individual services and ensures they are met a desired percentage of time. Rather than meet the requirements for all services over the same set of failure states, FloMore exploits a key opportunity that each service could meet its bandwidth requirements over a different set of failure states. FloMore consists of an offline phase that identifies the critical failure states of each service, and on failure allocates traffic in a manner that prioritizes those services for which that failure state is critical. We present a novel decomposition scheme to handle FloMore’s offline phase in a tractable manner. Our evaluations show that FloMore outperforms state-of-the-art TE schemes including SMORE and Teavar, and also out-performs extensions of these schemes that we devise. The results also show FloMore’s decomposition approach allows it to scale well to larger network topologies.

1 INTRODUCTION
Cloud providers must ensure that their networks are designed so as to ensure business-critical applications continually operate with acceptable performance [1, 4, 16]. Networks must meet their performance objectives while coping with failure, which are the norm given the global scale and rapid evolution of networks [12, 13, 16, 26, 29, 36]. Networks support multiple services (e.g., applications, product groups, customers), each of which is associated with its own bandwidth requirement, that must typically be met a desired percentage of time. For instance, a customer of a public cloud provider may indicate that it needs “a bandwidth of at least B between a pair of sites 90% of the time”. Such bandwidth requirements may be expressed at diverse granularities ranging from individual users to an aggregate set of users [21]. Henceforth, in this paper, we use the term Flow to refer to the traffic between a pair of sites corresponding to a particular service, and assume bandwidth requirements are specified at the granularity of flows. Network architects must ensure the requirements of flows are met taking into account the likelihood the network may experience different failure states (e.g., a particular set of link failures), and the performance feasible under each failure state.

While bandwidth objectives are specified at the granularity of flows, existing traffic engineering (TE) schemes manage network resources in a much coarser fashion. First, TE schemes typically compose traffic requirements across all flows into an aggregate traffic matrix (each cell corresponding to total demand between a pair of sites across all services). Second, TE schemes optimize metrics that relate to the entire aggregate traffic matrix. For instance, many state-of-the-art TE schemes minimize the maximum traffic loss experienced across all pairs of sites [6, 19, 22].

The coarse-grained nature of TE schemes makes them unduly conservative. For instance, consider the task of ensuring that all flows see a desired bandwidth 90% of the time, given the probability the network may encounter different failure states. To achieve this goal, existing TE schemes must ensure that there exist failure states that occur 90% of the time where all flows must simultaneously meet their bandwidth demand. However, operating at the granularity of flows provides new opportunities. Specifically, each flow could meet its bandwidth requirements in different failure states. Thus, the objectives of each flow could be met even though the network may not be able to simultaneously sustain the bandwidth of all flows 90% of the time.

To tackle this, we present FloMore, a system that considers bandwidth requirements at the granularity of flows. Each flow is associated with a bandwidth demand, and a requirement that the flow’s loss must be acceptable in failure states that occur with a desired probability.

A key idea of FloMore is that not all failure states of the network are critical to meeting a particular flow’s requirement. Instead, it seeks to determine the set of critical failure states associated with each flow, i.e., those failure states where the loss associated with the flow must be acceptable so the objectives can be met. FloMore consists of (i) an offline phase,
which involves determining the critical failure states associated with each flow; and (ii) an online phase (executed when a failure occurs) which determines the bandwidth allocation of all flows recognizing that the failure state is only critical for a subset of flows identified by the offline phase.

We show that FloMore’s offline phase can be formulated as an Integer Program (IP). The IP however is large, and its complexity grows with network size, the number of failure states of the network that must be considered, and the number of flows. We devise a novel decomposition approach which solves the IP in a series of iterations, where each iteration involves solving much smaller and computationally light-weight optimization problems in a highly parallelizable manner. The decomposition need not be run to completion, and even the initial step, which does not require the solution of any IP, already delivers a better guarantee than that obtained using state-of-the-art TE schemes.

A highlight of FloMore is that it directly optimizes loss at a desired percentile. An alternative is to conservatively estimate this value using the conditional value at-risk (CVaR), i.e., the average loss encountered in failure scenarios beyond the desired percentile. This approach was recently adopted by Teavar [6], a state-of-the-art TE scheme. However, Teavar operates over coarse-grained traffic matrices, and assumes a less flexible routing strategy. To facilitate comparisons, we devise new CVaR schemes that operate at the granularity of flows, and consider adaptive routing strategies.

We evaluate FloMore over 20 topologies including many large topologies. We compare FloMore with Teavar, and SMORE (another state-of-the-art TE schemes). We show that FloMore guarantees much lower loss at a desired percentile relative to SMore and Teavar. Further, we show that while our enhanced CVaR schemes greatly improve performance over Teavar, FloMore still out-performs them. This is because CVaR is a weak approximation of the performance at a desired percentile. Our results also show the importance of our decomposition scheme and that it performs close to the optimal with acceptable computation time.

2 MOTIVATION

2.1 Background and problem context

Bandwidth objectives are defined. A network supports multiple services, where each service comprises of multiple flows, and each flow is associated with a particular ingress and egress node. A flow itself could represent traffic at different granularities – e.g., the traffic of an individual user, an application, an aggregate group of users, a product group, or a customer of a public cloud provider [21].

A flow’s bandwidth requirements must be met despite network failures. Network operators typically have empirical data which indicate the probability of the network experiencing different node/link failures [6]. We use the terms failure state and failure scenarios to refer to a state of the network where a particular set of links or nodes have failed.

Let $q$ denote a failure scenario. Each flow $f$ is associated with a bandwidth requirement $d_f$. Since the entire requirement cannot always be met under failure, let $l_q$ denote the bandwidth loss, i.e., the fraction of the bandwidth requirement $d_f$ that cannot be met in scenario $q$. Equivalently, the flow is allocated a bandwidth of $(1 - l_q)d_f$ in scenario $q$.

A flow’s requirements are typically specified as (i) a bandwidth demand $d_f$; and (ii) a requirement that despite failures, the flow must see a bandwidth loss of at most $l$ at least $\beta$ of the time. Consider Fig. 1, where each row corresponds to a flow, each column to a failure scenario, and each cell shows the corresponding loss $l_q$. To meet flow level requirements, an architect must compute the $\beta$th percentile of each row, and ensure the resulting value is at most $l$.

Abstractions and metrics of existing TE schemes. Traditionally, traffic requirements are abstracted as a set of source and destination pairs, and the traffic demand associated with each pair $i$. Effectively, the traffic requirements of all underlying services and their constituent flows are composed to obtain a traffic matrix. Given such a matrix, TE schemes allocate bandwidth to each pair $i$ so a desired performance.
metric over the entire traffic matrix aggregate is optimized, while ensuring link capacities are not exceeded.

For concreteness, consider Teavar [6], a recent and state-of-the-art TE scheme. Similar to the discussion above, let \( L_{iq} \) denote the bandwidth loss for pair \( i \) in scenario \( q \). We define the ScenLoss \(_q\) as the maximum loss across all source destination pairs in a given scenario. That is,

\[
\text{ScenLoss}_q = \max_i L_{iq}
\]

Teavar [6] seeks to allocate bandwidth to each pair so that the worst ScenLoss across a set of scenarios that occur with a probability of \( \beta \) is minimized.

Pictorially, consider Figure 2 which is similar to Figure 1, except that the rows are now source destination pairs, and each cell corresponds to the loss seen by a particular pair in a given failure scenario. Teavar effectively computes ScenLoss\(_q\) for each scenario \( q \) by aggregating along each column, and then seeks to estimate the \( \beta^{th} \) percentile of ScenLoss\(_q\) across failure scenarios. In fact, Teavar conservatively estimates the \( beta^{th} \) percentile as we will discuss in §4.

Many other TE schemes also essentially minimize ScenLoss. For instance, many TE schemes [22, 38] including the state-of-the-art SMORE [22] optimize the utilization of the most congested link (Maximum Link Utilization or MLU). Other schemes [19, 25] solve the maximum concurrent flow, and maximize the fraction \( z \) (that we also refer to as scale factor) of demand the network can handle. Minimizing MLU, or maximizing \( z \) is equivalent to minimizing ScenLoss, since \( \text{ScenLoss} = \max \{0, 1-z\} \), and \( \text{ScenLoss} = \max \{0, 1-1/\text{MLU}\} \). While most of these other TE schemes do not address percentile requirements, the natural approach to analyzing their performance under failures is also to compute the \( \beta^{th} \) percentile of ScenLoss\(_q\) (last row) across failure scenarios similar to the above.

### 2.2 Illustrating opportunity with FloMore

Unlike existing TE schemes that work with coarser traffic abstractions, and focus on optimizing an entire traffic aggregate, FloMore abstracts traffic and performance requirements at the granularity of flows. This provides opportunities not available with existing TE abstractions.

Consider the task of ensuring that no flow sees loss 99% of the time. With existing TE schemes, including Teavar, we need to compute the ScenLoss for each flow and ensure the ScenLoss is 0 for scenarios that occur 99% of the time. Effectively, this means that the network must be able to carry the traffic of all flows in failure scenarios that occur 99% of the time. While this condition is sufficient to ensure the flow requirements are met, it is not necessary. Specifically, it may be feasible to meet each flow’s requirement through a different combination of failure states. This provides significant flexibility, and an opportunity to meet a network’s bandwidth.
requirements even when state-of-the-art TE schemes cannot. This is the opportunity that FloMore exploits.

To concretely illustrate the potential opportunity, consider Fig.3, where a network must carry traffic corresponding to a flow $f_1$ from source $A$ to destination $C$, and a flow $f_2$ from source $A$ to destination $D$. Consider a requirement that each of $f_1$ and $f_2$ must support 1 unit of traffic 99% of the time. The capacities of the links are shown as in the figure. All links fail with probability 0.001, except link $AD$ which has a failure probability of 0.01.

**Can the network meet the bandwidth requirements?**

We first illustrate how the network is able to meet the requirements without any traffic loss. Consider a simple routing strategy of sending $f_1$ through $A-B-C$ and always sending $f_2$ through $A-D$. Fig. 4 shows the loss of each flow over different groups of scenarios using this strategy. For example, the first column represents the set of scenarios where link $AD$ is alive, while either link $AB$ or link $BC$ fails. In all these scenarios, we can send 1 unit of traffic for $f_2$ without any loss, but cannot send any traffic for $f_1$. Consequently, the loss of $f_1$ is 100%, while the loss of $f_2$ is 0%. The last row corresponds to ScenLoss, which is 100%, since it is the worst loss experienced across all flows in each scenario. The red ovals in each row indicate the groups of scenarios where the a particular flow can be sent without loss. Clearly, each of flows $f_1$ and $f_2$ can be sent without loss in scenarios that occur 99% of the time indicating the bandwidth requirements can be met. Notice however that each flow meets its requirements through a different combination of failure states.

**State-of-the-art TE schemes do not meet the bandwidth requirements.**

We next consider TE strategies such as Teavar [6] and SMORE [22] that minimize ScenLoss in each scenario (or equivalently, minimize MLU, or maximize demand scale factor using a maximum concurrent flow formulation). Fig. 6 shows the loss of $f_1$ and $f_2$ in different scenarios with such scenario-centric TE schemes. The figure shows that $f_1$ can only achieve 50% loss in scenarios that occur 99% of the time.

Comparing Fig. 4, and Fig. 6, a key difference is in the third column, which corresponds to the scenario, where link $AD$ fails, and all other links are alive. Here, both Teavar and SMORE route traffic as shown in Fig.3. Specifically, 0.5 units is allocated to each of flows $f_1$ and $f_2$, translating to an overall loss of 50% for each flow, and a ScenLoss of 50%. In contrast, to meet the bandwidth requirement, the network must prioritize flow $f_1$ and serves it without loss for this scenario, even though flow $f_2$ sees 100% loss (Fig. 4). This is acceptable since (i) $f_2$ already meets the requirement by performing acceptably in other scenarios; and (ii) it is necessary that $f_1$ sees no loss in this scenario to ensure its requirement is met. In doing so, the ScenLoss for this scenario is 100%, higher than the 50% achieved by the current TE schemes – yet this is desirable from the perspective of ensuring the requirements of all flows are met.

**Generalization.** A simple generalization of this example shows that both Teavar and SMORE may be arbitrarily worse than the network’s intrinsic capability. Instead, suppose $A-D$ link has a capacity of $n$, and flow $f_2$ requires $n$ unit of traffic. The requirement can still be met without any loss following the previous arguments. However, in the scenario where $A-D$ link fails, to minimize ScenLoss, $n/(n+1)$ traffic will be sent from $A$ to $D$, and $1/(n+1)$ traffic will be sent from $A$ to $C$, resulting a loss of $1 - 1/(n+1)$. As $n$ gets larger, the loss will get closer to 100%.

### 3 FLOMORE DESIGN

We now present FloMore’s design. Given a set of flows and the bandwidth demand associated with the flow, a set of failure scenarios to consider, and the probabilities associated with those scenarios, FloMore decides how to allocate bandwidth to each flow in every failure scenario so the flow sees acceptable performance over scenarios that occur with a desired probability.

A key idea of FloMore is that not all failure scenarios of the network are critical to meeting a particular flow’s requirement. Instead, it seeks to determine the set of critical failure scenarios associated with each flow, i.e., those failure scenarios where the loss associated with the flow must be acceptable so the objectives can be met. Further, each flow may be associated with a different set of critical failure scenarios. FloMore consists of (i) an offline phase, which involves determining the critical failure scenarios associated with each flow; and (ii) an online phase (executed when a failure occurs) which determines the bandwidth allocation of all flows recognizing that the failure scenario is only critical for a subset of flows identified by the offline phase.

We start by formalizing FloMore’s performance metric, and discuss FloMore’s offline phase for determining critical failure scenarios. The offline phase involves solving an Integer Program, which may require substantial computational resources with state-of-the-art solvers. Instead, we present a novel decomposition approach to tackle the problem, and heuristics to aid the offline phase. We then discuss several generalizations related to FloMore.

#### 3.1 Formalizing FloMore’s problem

Consider a network topology, represented as a graph $G = (V,E)$. Each link $e \in E$ is associated with a link capacity $c_e$. We use $P$ to represent the set of source-destination pairs. Each flow $f \in F$ is associated with traffic demand $d_f$ that must be sent along the source-destination pair $pr(f)$. $Q$ represents the set of failure scenarios. For each scenario $q \in Q$, $p_q$ represents the probability of $q$. Each pair $i$ can use
a set of tunnels $R(i)$ to route the traffic. Let $y_{tq}$ represent whether a tunnel $t$ is alive in scenario $q$. We use $x_{tq}$ to denote the bandwidth assigned to tunnel $t$ in scenario $q$, i.e., our designed routing. Table 1 summarizes notation.

For each flow $f \in F$, we define FlowLoss($f, \beta$) to be the $\beta$th percentile of loss for flow $f$. That is, there exist failure scenarios that together occur with probability $\beta$, where flow $f$ encounters a loss less than FlowLoss($f, \beta$).

We start with a formulation where FloMore determines a bandwidth allocation such that the maximum of the $\beta$th percentile loss across all flows is minimized. Specifically, we consider the following metric that we refer to as MaxFlowPctLoss (and may abbreviate as $\alpha$).

$$\text{MaxFlowPctLoss}(\alpha) = \max_{f \in F} \text{FlowLoss}(f, \beta)$$  \hspace{1cm} (1)

This formulation corresponds to a case where FloMore determines a bandwidth allocation such that all flows see a loss less than $\alpha$ over a set of scenarios that occur with probability $\beta$. It is easy to generalize FloMore to a context where bandwidth allocations must ensure each flow meets a flow-specific loss threshold (if such an allocation is feasible), as we will discuss in §3.5.

To ensure each flow’s objectives, FloMore must for each flow $f$ select scenarios that together occur with probability $\beta$ such that $f$ sees loss less than $\alpha$ in these scenarios. We denote these scenarios as critical scenarios for that flow. We use binary variable $z_{fq}$ to indicate whether scenario $q$ is critical for flow $f$. If $z_{fq} = 1$, $q$ is critical for $f$, and the loss of flow $f$ cannot exceed MaxFlowPctLoss, i.e., $l_{fq} \leq \text{MaxFlowPctLoss}$. We next present the formulation below which determines the best routing and choice of critical scenarios that can minimize $\alpha$.

\[ \begin{align*}
(I) \min_{z, x, f, \alpha} & \quad \alpha \\
\text{s.t.} & \quad \sum_{q \in Q} z_{fq}p_q \geq \beta \quad \forall f \in F \\
& \quad \alpha \geq l_{f} - 1 + z_{f} \\
& \quad \forall f \in F, q \in Q \\
& \quad \sum_{r \in R(i)} (1 - l_{fr})d_{fr} \leq \sum_{t \in R(i)} x_{tq}y_{tq} \forall i \in P, q \in Q \\
& \quad \forall i \in P, q \in Q \\
& \quad \sum_{e \in E} x_{e} \leq c_{e} \forall e \in E, q \in Q \\
& \quad x_{tq} \geq 0 \forall i \in P, r \in R, q \in Q \\
& \quad z_{f} \in \{0, 1\} \forall f \in F, q \in Q \\
& \quad 0 \leq l_{f} \leq 1 \forall f \in F, q \in Q \\
& \quad (7) \\
& \quad \text{In the above formulation, (2) ensures that for each flow, we select enough critical scenarios to cover the probability $\beta$. When $z_{fq} = 1$, (3) becomes MaxFlowPctLoss $\geq l_{fq}$ meaning we care about the loss $l_{fq}$. When $z_{fq} = 0$, (3) is satisfied.} \\
& \quad \text{no matter what MaxFlowPctLoss and $l_{fq}$ are, implying we don’t care about the loss $l_{fq}$. (4) ensures that there is enough bandwidth allocated to each pair. The LHS of (4) is the total amount of traffic required to be sent on pair $i$, and the RHS is the total allocated bandwidth on tunnels connecting pair $i$. (5) and (6) ensure the allocated bandwidth on tunnels will never exceed any link’s capacity, and the allocation is non-negative. The final two constraints indicate the $z$ variables are binary, and ensure the loss fractions are between 0 and 1.} \\
& \quad \text{3.2 Decomposing the problem} \\
& \quad \text{With $z_{fq}$ being a binary variable, (I) is an MIP formulation that simultaneously determines (i) the critical scenarios for each flow ($z_{fq}$ variables); and (ii) how the traffic should be routed in each failure scenario taking into account for which flows that scenario is critical. Solving the MIP (I) can be challenging for large topology sizes, and as the number of flows increases.} \\
& \quad \text{To tackle this, we draw on the Benders’ decomposition [30] algorithm, a systematic way to decompose an optimization problem into two stages, and then iteratively search for the optimal. In this approach, we divide the variables from the original problem into two sets for the two stages. We solve the first-stage problem (the master problem) to get values of variables in the first set. Then with the variables in the first set being fixed, we solve the second stage problem, which can learn constraints to improve the master problem. By adding the constraints learned from the second stage, the master problem can find a better solution of the first set of variables.} \\
\end{align*} \]

| notation | meaning |
|----------|---------|
| $Q$      | Set of all scenarios |
| $F$      | Set of flows |
| $P$      | Set of source-destination pairs |
| $R(i)$   | Set of tunnels for pair $i$ |
| $E$      | Set of edges |
| $\beta$  | Target probability for which the bandwidth requirement must be met |
| $pr(f)$  | Source-destination pair along which flow $f$ is being sent |
| $d_f$    | Traffic demand of flow $f$ |
| $p_q$    | Probability of scenario $q$ |
| $y_{tq}$ | 1 if tunnel $t$ is alive in scenario $q$, 0 otherwise |
| $x_{tq}$ | Allocated bandwidth on tunnel $t$ in scenario $q$ (routing variable) |
| $l_{fq}$ | Loss of flow $f$ in scenario $q$ |
| $z_{fq}$ | 1 if scenario $q$ is considered for flow $f$ to meet the target probability, 0 otherwise |

**Table 1: Notation**
By repeating these two stages iteratively, the solution gets closer to the optimal.

Fig. 7 shows how we apply the Benders’ decomposition algorithm on (I). The master problem proposes the critical scenarios for each flow, and the subproblem figures out how to route traffic when given a proposed set of critical scenarios for each flow. In our context, the subproblem itself can be further decomposed into multiple subproblems, one per failure scenario, that each determines a routing for that scenario given information regarding which flow that scenario is critical for. Each of our subproblems is extremely fast to solve, and this allows us to speed up the procedure by solving subproblems in parallel. The second stage subproblems will provide the learned constraints to the master problem so that the master problem can improve its critical scenario proposal in the next iteration.

Formally, we can rewrite the objective function of (I) as a two-step problem by separating the variables into two sets \( \{z\} \) (for the first stage) and \( \{x, l, a\} \) (for the second stage)

\[
\min_{z} \min_{x, l, a} \alpha. \quad (9)
\]

The first stage (outer) problem is to decide \( z \), which is to determine in each scenario, which flows should be taken care of. With a given \( z \), the second stage (inner) problem is to find the routing for each scenario to minimize the loss, which can be seen as a function of \( z \). And then (I) can be rewritten as

\[
(I') \min_{z} \text{MaxFlowPctLoss}(z) \quad \text{s.t.} (2), (7) \quad (10)
\]

where

\[
\text{MaxFlowPctLoss}(z) = \min_{x, l, a} \alpha \quad \text{s.t.} (3), (4), (5), (8) \quad (11)
\]

We next present some high-level intuition for the inner workings of the decomposition approach (some additional technical clarifications are presented in the Appendix). Fig. 8 shows an example \( \text{MaxFlowPctLoss}(z) \) function. Indeed, the optimal objective value for the inner problem is convex in \( z \). This is a well-known property of linear programs. The decomposition algorithm essentially searches for the minimizer of \( \text{MaxFlowPctLoss}(z) \) iteratively. Although the exact shape of \( \text{MaxFlowPctLoss}(z) \)'s is unknown at any point in the algorithm, solving (11) gives us one point on the function \( \text{MaxFlowPctLoss}(z) \). Moreover, the dual form of (11) provides a tangent of \( \text{MaxFlowPctLoss}(z) \). Thus, we derive an underestimation of \( \text{MaxFlowPctLoss}(z) \) by evaluating it at various \( z \). Each tangent is a lower bound of function \( \text{MaxFlowPctLoss}(z) \), and the pointwise maximum of these tangents is an underestimate of \( \text{MaxFlowPctLoss}(z) \). Then, we can find the current estimated minimizer of the estimated function (e.g., \( z_p \) in Fig. 8). Solving (11) at \( z^p \) gives a new tangent and a more accurate estimate of \( \text{MaxFlowPctLoss}(z) \). The process converges in finite time with an optimal solution (we discuss why in the Appendix).

The master problem shown below derives an underestimate of the optimal loss using a lower-approximation of \( \text{MaxFlowPctLoss} \) obtained via previously found tangents. These tangent constraints are learned by solving (11), i.e., the second stage problem.

\[
(M) \min_{z, \text{MaxFlowPctLoss}} \text{MaxFlowPctLoss} \quad \text{s.t.} (2), (7)
\]

\[
\text{MaxFlowPctLoss} \geq g(z) \quad \forall g \in G \quad (12)
\]

\( G \) represents the set of all tangents computed so far. So (12) ensures that all tangents are treated as a lower bound of \( \text{MaxFlowPctLoss} \), forming an estimation.

As mentioned before, (11) is to find the routing for each scenario, when provided a proposed set of flows for which each scenario is critical. Since the routing in each scenario can be derived independently of that in other scenarios, the problem in the second stage (11) decomposes by scenarios. Therefore, we solve the following subproblem for each scenario \( q \in Q \):

\[
(S_q) \min_{x, l, a} \alpha \quad \text{s.t.} \alpha \geq l_{f_q} - 1 + z_{f_q} \quad \forall f \in F \quad (13)
\]

\[
0 \leq l_{f_q} \leq 1 \quad \forall f \in F \quad (14)
\]

\[
\sum_{p \in \mathcal{F}(i)} (1 - l_{f_q}) d_f \leq \sum_{i \in \mathcal{R}(i)} x_{t_q} y_{t_q} \quad \forall i \in P \quad (15)
\]

\[
\sum_{e \in \mathcal{E}} x_{t_q} \leq c_e \quad \forall e \in E. \quad (16)
\]

Note that \( z_{f_q} \) is a parameter in (\( S_q \)). Suppose the dual variables for (13), (14), (15) and (16) are \( w_{f_q}, \theta_{f_q}, \sigma_{t_q} \) and \( u_{t_q} \) respectively. Then we will have the tangent on \( \text{MaxFlowPctLoss} \) as the following function on \( z \).

\[
g(z) = \sum_{f} (z_{f_q} - 1) w_{f_q} + \sum_{f} \theta_{f_q} + \sum_{i, \mathcal{F}(i) = i} \sigma_{t_q} d_f + \sum_{e} u_{t_q} c_e. \quad (17)
\]

Note that \( S_q \) is a small LP, and for each \( q \in Q, S_q \) can be solved independently of one another. This means that we can parallelize the solution of these LPs to speed up the solution process. Algorithm 1 provides the pseudocode (Line 13-15 can be executed in parallel). We remark that each iteration, Bender’s algorithm yields a routing strategy and the corresponding \( \text{MaxFlowPctLoss} \) can be computed easily by sorting the optimal values for \( S_q \) and computing the \( \beta \)th percentile.
Figure 7: The master problem decides which flows to consider in each scenario, and the subproblems decide how to route in each scenario, and learn constraints to improve the master problem. Note that the subproblems can be solved in parallel.

Figure 8: An example of function MaxFlowPctLoss(z). Solving (11) on $z_1$ and $z_2$ provides two tangents to estimate MaxFlowPctLoss(z). And $z_p$ is the minimal based on this estimation.

Algorithm 1 Bender’s decomposition algorithm

1. function solve_master($G$)
2. Solve ($M$) with $G$, and get variable $z$
3. return $z$
4. function solve_subproblem($z$, $q$)
5. Solve ($S_q$) and get routing variable $x$, and dual variables $w_{aq}$, $o_{aq}$, $v_{iq}$ and $u_{eq}$
6. Construct $g$ as in (17)
7. return $x$, $g$
8. function main(max_iterations)
9. $k ← 0$
10. $G ← ∅$
11. while $k < max$-iterations do
12. $z ← solve\_master(G)$
13. for $q ∈ Q$ do
14. $x_q, g ← solve\_subproblem(z, q)$
15. $G.add(g)$
16. $k ← k + 1$
17. return $x$

3.3 Heuristics to speed up decomposition

We discuss heuristics which help the decomposition algorithm converge to good solutions faster.

Identifying a good starting point. It is desirable to start with $z$ close to the real minimizer for the outer problem, so that the algorithm requires fewer iterations to converge. As shown in Algorithm 1, the master problem in first iteration will be solved with $G$ being $∅$, which means that there is no tangent to estimate. This will lead to a random $z$, which can be far away from the real minimizer. A simple heuristic is to add constraints $z_{f_q} = 1$ in ($M$) indicating all flows are critical in all scenarios. As a further optimization, we observe that a failure scenario can only be critical for a flow if the flow is connected in that scenario. Thus, we add constraints $z_{f_q} = 0$ in ($M$) if flow $f$ is disconnected in scenario $q$, and $z_{f_q} = 1$ otherwise, and use this as an even better starting point. Under either assumption, we have the proposition below (and defer a proof to the appendix).

**Proposition 1.** At the initial step of our algorithm (prior to any iteration of the master problem), the guarantee from our algorithm is already better than that from TeaVar or SMORE.

Ensure better stability. With a proper starting point, we may still end up far away from the minimal due to the current estimation being too coarse (for example, in the first few iterations, we don’t have enough tangents). Thus, we want to restrict the step we take when we update $z$. That is, in each iteration, we make sure the new estimated minimizer is still close enough to the last iterate. One way to achieve this is to add constraints in ($M$) to limit the hamming distance between current $z$ variable and $z$ variable achieved from last iteration. We remark that this constraint can hurt the convergence of the algorithm to the optimal solution. To circumvent this issue, the Hamming distance constraint can be relaxed, when no improvement is found.

Pruning scenarios. We further accelerate the decomposition strategy by recognizing that not all subproblems need to be solved each iteration. In particular, we prune out perfect scenarios where all flows can be simultaneously handled without loss. While we have not implemented, we can also prune scenarios with small losses. For details, see Appendix.
3.4 FloMore online phase
The offline phase identifies the critical failure scenarios for each flow, which may be seen as hints regarding which flows to prioritize in the online phase. When a failure occurs, a simple linear program (\(S_q\)) is solved which determines how to allocate traffic to flows so that the loss of flows for which that scenario is critical are minimized. This is a simple and fast LP, and can be easily be integrated with a system such as SMORE. The main new aspects of the LP involve leveraging "hints" regarding critical flows, and running the allocation at the granularity of flows rather than pairs.

3.5 Generalizations
Flows whose loss requirements must be simultaneously satisfied. In practice, a service may depend on multiple related flows which we call a flow set. To support a flow set, we need to send the traffic demand of all flows in it. We can define the loss of such flow set to be the max loss across its flows. Under such definition, for example, a flow set having 5% loss means its worst flow has a 5% loss. (I) can be rewritten to support flow sets so that each flow set has low loss for probability of \(\beta\). And the same decomposition method can also apply to speeding up solving flow set loss. We show the flow set formulation in Appendix.

Different loss thresholds across flows. Different services may have different criteria for meeting desirable performance. For example, service 1 may allow its flow to suffer 5% loss while service 2 requires all demand to be sent. Our formulation can be easily adapted for requirements that different flows have different loss thresholds. Suppose for any flow \(f \in F\), \(th(f)\) is the maximum loss it allows for \(\beta^{th}\) percentile loss. Then we can change (3) in (I) to

\[
\alpha \geq (l_{fq} - th(f)) - 1 + z_{fq} \quad \forall f \in F, q \in Q
\]  

(18)

With this change, this constraint will only make \(\alpha\) positive when \(z_{fq} = 1\) and \(l_{fq} > th(f)\), i.e., \(q\) is critical for flow \(f\), and \(f\)'s loss in \(q\) is exceeding its threshold. Thus, if there is a solution where \(\alpha\) (the objective) is non-positive, then we have a routing which satisfies the threshold requirements of all flows. If even in the optimal solution, \(\alpha\) is positive, then it is impossible to satisfies all requirements.

Heterogeneous percentiles across flows. The percentile targets may vary across flows. An important flow may require a guarantee 99.99% of time, while a lower priority flow may only require a guarantee 99% of the time. Since we allow critical scenarios to vary across flows, adapting for such requirements is easy in our formulation. Let \(\beta_f\) be the required percentile for flow \(f\). We can change (2) in (I) to

\[
\sum_{q \in Q} z_{fq} p_q \geq \beta_f \quad \forall f \in F
\]  

(19)

In contrast, this is not easy with current TE schemes since they operate at coarser granularities.

Shared Risk Link Groups. Note that our formulation doesn’t assume anything regarding the probability distribution of failures. Thus, FloMore can work with any kinds of failure distribution, such as each link has independent failure probability, or a group of links may fail together. As long as the set of failure scenarios is drawn from the failure probability of interest, FloMore can be applied on the set.

Capacity augmentation. The integer programming problem (I) and its decomposition strategy can be generalized to perform minimum-cost capacity augmentation on the network. In this case, we may require that MaxFlowPctLoss is constrained to be below a specified value and minimize \(\sum_e w_e \delta_e\), where \(\delta_e\) is the added capacity to link \(e\), which changes the rhs of (5) to \(c_e + \delta_e\), and \(w_e\) is the per-unit cost of adding capacity. (If there is a fixed-cost, we can include it by introducing a binary variable \(a_e\) which takes value 1 if link \(e\) is augmented, and add \(\sum_e f_e a_e\) to the cost. To ensure fixed-cost is charged with any augmentation, we add upper-bounding constraints \(0 \leq \delta_e \leq u_e a_e\), where \(u_e\) is an upper bound on the augmentation.) The decomposition strategy of §3.2 generalizes to this setting where \(c_e\) is replaced with \(c_e + \delta_e\) in (17) and this cut now describes a tangent of MaxFlowPctLoss in the \((z, \delta)\) space. Since, for a given routing, the MaxFlowPctLoss is no more than the \(\beta^{th}\) percentile of SceilLoss, it follows that this approach will lead to less costly augmentation pathways to attain performance guarantee for all flows relative to the traditional scenario-based approach.

4 FLOMORE VS. CVAR-BASED METHODS
FloMore seeks to determine bandwidth allocations that minimize the \(\beta^{th}\) percentile of flow losses. Minimizing loss at a given percentile (also referred to as Value at Risk or VaR) is a hard problem, and a standard approach in the optimization community involves approximating the same using an approach called Conditional Value at Risk (CVaR). Rather that minimize the \(\beta^{th}\) percentile, a CVaR approach involves minimizing the expected loss of the worst \((100 - \beta)\text{th}\) percentile of scenarios.

For example, consider a flow which sees a loss of 0%, 5% and 10% in three scenarios that respectively occur with probability 0.9, 0.09, and 0.01. Then, the \(90^{th}\) percentile loss (VaR) is 0%, but the CVaR is 5 * 0.09 + 10 * 0.01 = 1.45%.

Designing networks for probabilistic requirements is a challenging problem, and has only recently received attention from the networking research community. Teavar [6], a notable and representative recent work in this space draws on the notion of CVaR and applies it to traffic engineering.
There are three key differences between Teavar [6], and FloMore. First, as discussed in §2, Teavar computes the ScenLoss, and considers the $\beta^{th}$ percentile of $\text{ScenLoss}$, unlike FloMore which focuses on the $\beta^{th}$ percentile of flows. Second, Teavar further conservatively estimates the $\beta^{th}$ percentile using CVaR. In contrast, FloMore directly uses VaR. Third, Teavar assumes when a failure occurs, traffic on a source destination pair is rescaled so the same proportion is maintained on live tunnels. In contrast, FloMore like SMORE [22] allows greater flexibility in how traffic is split across tunnels.

Enhancing CVaR schemes. To appreciate the benefits of FloMore’s approach, we design two new CVaR-based TE schemes, which may be viewed generalizations of Teavar. These schemes allow us to analyze the advantages of directly considering VaR in FloMore, and decouple these benefits from other benefits of FloMore. The schemes considered are:

- Cvar-Flow-St. Here, we use CVaR to approximate the computation of MaxFlowPctLoss. Instead of directly computing $\beta^{th}$ percentile loss for flow $f$, i.e., $\text{FlowLoss}(f, \beta)$, we use CVaR of flow $f$ (denoted by $\text{CVaR}(f, \beta)$) to approximate it. Then we seek to optimize the maximum CVaR of all flows, which we denote as $\text{MaxFlowCVaR}$. Formally,

$$\text{MaxFlowCVaR} = \max_{f \in F} \text{CVaR}(f, \beta)$$

- Cvar-Flow-Ad. This is similar to Cvar-Flow-St except that we allow greater flexibility in terms of how traffic may be split across tunnels on failure.

We develop Linear Programming (LP) models for computing the routing and bandwidth allocations associated with these schemes, which we present in the appendix.

Illustrating limitations of CVaR. We next present an example to show that a scheme that optimizes for CVaR may not perform well when optimizing for the $\%ile$ loss, the measure we truly wish to optimize.

Consider Fig. 9 which shows a topology with two flows $f_1$ and $f_2$. All links have the capacity of 1 and fail with probability 0.01. Consider a requirement that each of $f_1$ and $f_2$ must support 1 unit of traffic with probability of 0.99.

First, observe that $\text{CVaR}(f_1, 0.99) = 100\%$ for any arbitrary routing scheme. This is because flow $f_1$ is disconnected in the $1\%$ of scenarios where link AB fails, and the average loss of the worst $1\%$ of scenarios for $f_1$ is thus $100\%$. Then, $\text{MaxFlowCVaR} = 100\%$ for any arbitrary routing. Thus, a scheme that seeks to optimize $\text{MaxFlowCVaR}$ could result in any arbitrary routing.

However, observe that $\text{MaxFlowPctLoss}$ can be significantly different based on the routing strategy. To see this, consider a first routing strategy that always routes $f_1$ along the link AB, and always routes $f_2$ along the link AC (red arrows). Since each link is alive $99\%$ of the time, both $f_1$ and $f_2$ achieve a $99\%ile$ loss of 0, and $\text{MaxFlowPctLoss} = 0\%$.

In contrast, consider a second routing strategy which on the failure of link AC, serves 0.5 units of each flow, and routes flow $f_2$ along the green link shown. With this strategy, the $99\%ile$ loss of $f_1$ would be $50\%$.

Thus, a scheme which optimizes $\text{MaxFlowPctLoss}$ would produce the first strategy, while a scheme that optimizes $\text{MaxFlowCVaR}$ could produce either (or a completely different routing strategy), effectively not performing well in the $\text{MaxFlowPctLoss}$ metric.

5 EVALUATIONS

We compare FloMore’s performance with state-of-the-art TE schemes. We consider the following schemes:

- FloMore and FloMore-Opt. We primarily report results using FloMore, which uses the decomposition methods and an iterative algorithm introduced in (§3.2). We set the maximum number of iterations to be 5. To understand how well our schemes perform to optimal, we also report results with FloMore-Opt which uses the routing designed by the MIP formulation (I). It is only feasible to run FloMore-Opt for smaller topologies.

- State-of-the-art TE schemes. We consider SMORE, and Teavar, two state-of-the-art TE schemes. Teavar considers the ScenLoss of each scenario (§2), and designs a proportional routing scheme that optimizes the CVaR of ScenLoss. For each failure scenario, SMORE assigns traffic to tunnels in a manner that minimizes the MLU of links, and for a given tunnel selection achieves the optimal MLU (and equivalently, the optimal ScenLoss) for each failure scenario (§2).

- Enhanced CVaR schemes. We consider two enhanced CVaR schemes (Cvar-Flow-St and Cvar-Flow-Ad) that we developed (§4). While off-the-shelf Teavar uses a proportional routing scheme, and optimizes ScenLoss, Cvar-Flow-St considers loss percentiles at the granularity of flows, and Cvar-Flow-Ad additionally considers more flexible routing (§4). The purpose of considering these schemes is to decouple FloMore’s benefits owing to its directly optimizing loss percentiles (rather than conservatively estimating using the CVaR measure), from its benefits related to considering losses at a flow granularity.
Our primary performance metric for all schemes is the $\text{MaxFlowPctLoss}$ achieved by the scheme (i.e., we consider the $\beta$th percentile of loss of each flow, and take the maximum across flows). Note that SMORE and Teavar optimize the $\text{ScenLoss}$, but for fairness, we report the $\text{MaxFlowPctLoss}$ achieved by these schemes (for any scheme, $\text{MaxFlowPctLoss}$ is no higher than $\text{ScenLoss}$). We evaluate all the schemes based on post-analysis. For each scheme, we determine the routing and bandwidth allocation in each failure scenario, compute the loss of each flow in each scenario, and then compute $\text{MaxFlowPctLoss}$.

**Topologies and traffic model.** We evaluate the models above models on 20 topologies obtained from [20] and [22] (see Table 2 in the Appendix). Our largest network contains 151 edges and 103 nodes. We remove one-degree nodes in the topologies recursively so that the networks are not disconnected with any single link failure. For each node pair in the network, we generate 3 physical tunnels so that they are as disjoint as possible, preferring shorter ones when there are multiple choices. We use the gravity model [41] to generate traffic matrices with the utilization of the most congested link (MLU) in the range $[0.5, 0.7]$ across the topologies.

Although FloMore can model settings with multiple flows per pair, we primarily report results in settings where traffic corresponding to each source-destination pair is modeled as a single flow. We note that FloMore’s benefits are guaranteed to improve (or stay the same) with more flows per pair. To stress FloMore, we evaluate its ability to scale to more flows per pair. We implement all our optimization models in Python, and use Gurobi 8.0 [17] to solve them.

**Failure scenarios.** For each topology, we use the Weibull distribution to generate the failure probability of each link, like prior work [6]. We choose the Weibull parameter so that the median failure probability is approximately 0.001, matching empirical data characterizing failures in wide-area networks [12, 26, 36]. Given a set of link failure probabilities, we sample failure scenarios based on the probability of the occurrence. Our evaluations assume independent link failures but FloMore’s approach easily generalizes to shared risk link groups (§3.5). We discard scenarios with insignificant probability ($< 10^{-6}$). Our design target is set as the most 9’s for which all flows in the network remain connected for the sampled scenarios. For example, if every flow is connected in scenarios with probability greater than 0.999, while some flow is disconnected in scenarios with probability more than 0.0001, we design for 0.999 (since the network will trivially see an $\text{MaxFlowPctLoss}$ of 1 when designing for 4 9’s).
5.1 Results

Benefits of FloMore. Fig. 10 compares the MaxFlowPctLoss of FloMore and SMORE for the CWIX topology and for 24 different traffic demands. For each demand, the MaxFlowPctLoss is computed for each scheme, and the reduction in MaxFlowPctLoss is determined. The graph shows a CDF of the reduction in MaxFlowPctLoss relative to SMORE across the traffic demands. In most cases, FloMore reduces MaxFlowPctLoss by more than 70%.

Fig. 11 shows the reduction in MaxFlowPctLoss achieved by FloMore relative to SMORE across all topologies. In most cases, FloMore reduces MaxFlowPctLoss by over 80%. In some extreme cases, FloMore achieves a reduction of 100%. To understand this, consider that there are failure scenarios where the network topology may get disconnected. In such cases, schemes such as Teavar and SMORE that optimize ScenLoss (maximum loss across all source-destination pairs in a scenario) cannot count that scenario towards meeting the requirement of any flow. However, a majority of flows may still be connected, and FloMore may still allow that scenario to count towards the requirement of some of those connected flows. In some cases, the topology was connected less than 99.9% of the time, and consequently SMORE and Teavar could only achieve a 100% loss at the 99.9%ile. In contrast, each individual flow could still be connected in scenarios that occur 99.9% of the time or higher, allowing FloMore to achieve a much lower loss at the 99.9%ile (in some extreme cases, FloMore could guarantee 0% loss). We note that FloMore provides benefits even in more richly connected topologies as we will explore later.

Comparison to CVaR-based schemes. Fig. 12 compares FloMore relative to CVaR-based schemes, including Teavar, and the new CVaR-based schemes that we designed (§4). Each curve shows a CDF of the reduction in MaxFlowPctLoss relative to Teavar for a particular scheme across all topologies. We make several points.

First, FloMore (right-most curve) achieves a significant reduction in MaxFlowPctLoss relative to Teavar. These benefits accrue owing to Teavar’s (i) focus on ScenLoss rather than flow losses; (ii) use of CVaR to approximate the percentile; and (iii) assumption of proportional routing rather than allowing traffic to be split over tunnels in more flexible fashion. Second, our enhanced scheme Cvar-Flow-Ad (which allows for flexible routing and considers flow losses) achieves a significant benefit over Teavar, but does not provide as much benefits as FloMore. This is because the benefits are limited by the use of the CVaR approximation, while FloMore directly optimizes the percentile. Finally, while Cvar-Flow-St provides the lowest reduction in MaxFlowPctLoss relative to Teavar, the benefits are still significant with a reduction of more than 50% in the median case. This indicates that the approach of considering flow losses that we advocate in this paper offers significant benefits even in the context of limited routing flexibility and a CVaR approach.

Comparison to optimal. Fig. 13 compares FloMore and FloMore-Opt on 18 topologies except the two largest ones where we could not run FloMore-Opt to completion. The curves overlap significantly indicating that FloMore performs optimally in most cases. Fig. 15 shows how MaxFlowPctLoss changes each iteration for some typical topologies. While we ran up to 5 iterations of the decomposition algorithm, our results showed the algorithm typically achieved the best performance in fewer iterations.

Benefits in richly connected topologies. As discussed above, when a topology gets disconnected in a failure scenario, SMORE, and Teavar cannot count that scenario towards any flow’s requirement (FloMore, and our enhanced CVaR schemes do not suffer from this issue). We consider FloMore’s benefits in more richly connected settings, which we create by assuming each link consists of two sub-links that fail independently. We ensure the topology remains connected in all sampled failure scenarios. Fig. 14 presents bar charts comparing the MaxFlowPctLoss achieved by SMORE, FloMore, and FloMore for multiple topologies. The results show FloMore continues to provide benefits over SMORE, and performs close to FloMore.

Benefits in computation time. Fig. 16 presents the solving time (Y-Axis) against topology size (X-Axis) for FloMore-Opt and FloMore, assuming 5 iterations for FloMore. Recall that FloMore solves multiple small LP subproblems in each iteration, and a master problem (a MIP). Even for a relatively large topology like Tinet, solving each subproblem only takes 0.02-0.06 seconds. The master problem is much smaller than the IP (I), and only takes 13-17 seconds for Tinet. The time we report for FloMore includes the solving time of the master...
Researchers have recently started exploring the design of traffic engineering mechanisms with probabilistic requirements in mind [6, 8]. Besides Teavar, Lancet [8] designs protection routing schemes with the requirement that a network should not experience congestion over failures scenarios that together occur with a certain probability. The primary metric in Lancet is MLU, and Lancet only considers a scenario acceptable if the entire traffic matrix can be handled. In contrast, FloMore focuses on losses of flows, and allows for each flow’s requirement to be met through a different combination of failure scenarios.

Another recent work NetDice [33] focuses on verifying that network configurations meet a probabilistic requirement. The focus is on distributed control planes (shortest paths, route redistribution, BGP etc.), and verifying properties such as path length for a given configuration. Earlier work [10] allows modeling probabilistic network behaviors, such as packet delivery probability on failures. In contrast, FloMore focuses on synthesizing bandwidth allocations so the loss percentile associated with every flow is acceptable, while ensuring link capacity constraints are met in each scenario.

Robust network design has received much attention [7, 19, 25, 32, 38]. These works guarantee the network remains congestion-free over all failure scenarios with flow simultaneous link failures, while allowing for different levels of flexibility in terms of how networks adapt to failures. All these works capture the performance of any given scenario using metrics such as MLU, or a demand scale factor (in the context of a max concurrent flow formulation). FloMore is distinguished by a focus on design given probability of failures, and its emphasis on metrics that capture the performance of individual flows. Recent work [34] has explored verification of distributed control planes to ensure load is not violated on failures. Earlier work has explored robust network design under single link or node failures [3, 5, 9, 14, 28, 35, 42], and robust design across traffic matrices [2, 3, 37, 40].

Centralized TE schemes [15, 18] optimally route traffic leveraging network-wide views balancing between throughput and fairness. Other schemes [11] assign bandwidth to flows so overall utility is maximized. All these works optimize metrics related to the entire traffic matrix for any given network snapshot, and do not take failure probabilities into account. In contrast, FloMore determines bandwidth allocations by looking across scenarios while considering failure probabilities, and determines critical flows that must be prioritized for each scenario. A linear program is decomposed in [11] to distribute the centralized TE controller, while FloMore involves decomposing an Integer Program. Finally, much research explores how to re-route traffic to restore connectivity on failures [23, 24, 27, 31, 39] but does not consider meeting flow loss percentiles.

7 CONCLUSIONS
In this paper, we have presented FloMore, a new approach for designing cloud provider WANs in a manner that meets the bandwidth requirements of flows over failure scenarios that occur with a desired probability. Unlike existing TE schemes that seek to meet the requirements of all flows over the same set of failure scenarios making them unduly conservative, FloMore exploits a key opportunity that each flow could meet...
its bandwidth requirements over a different set of failure scenarios. As part of FloMore, we have presented an approach to optimize the $\beta$th percentile of bandwidth losses of all flows, a hard problem, and tackled the same using a novel decomposition approach accelerated with problem-specific insights. We have extended a CVaR-based approach to our setting, a well-accepted method to approximating loss percentiles, and shown that it can be conservative. Evaluations over 20 real topologies show the benefits with FloMore are significant. FloMore (i) reduces MaxFlowPctLoss by over 80% relative to Teavar and SMORE for over 50% of topologies; and (ii) has a solving time of tens of seconds, acceptable for the offline phase, and scales well with the number of flows. Overall, the results show the promise of FloMore. This work does not raise any ethical issues.

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solution $\sum_i \lambda_i (x^i, \text{loss}^i, \alpha^i)$ is feasible when $z = \sum_i \lambda_i z^i$. This shows that, the optimal value of the inner problem at $z$ is no more than $\sum_i \lambda_i z^i$. The dual form of (11) provides a tangent of $\text{MaxFlowPctLoss}(z)$ because its feasible region does not depend on $z$.

To understand why the process converges in finite time with an optimal solution, consider that the inner problem is always feasible with $(x, \text{loss, } \alpha) = (0, 1, 1)$ and bounded between 0 and 1. If we use an extreme point of the dual feasible region to generate a tangent (as is the case using dual simplex algorithm), in finitely many iterations, a cut is developed for each extreme point, and we have an accurate representation of $\text{MaxFlowPctLoss}(\alpha)$.

Formulation for supporting related flows. In §3.5, we mention that a set of flows may require their loss requirements to be simultaneously met. Suppose $A$ represents the set of flow sets, and each flow set $a \in A$ contains multiple flows along different source destination pairs. Let $d_{ai}$ be $a$’s required demand along source destination pair $i$. We use $l_{aq}$ to denote the loss of flow set $a$ in scenario $q$, and we define $l_{aq}$ as the max loss across its flows (i.e., at least $(1 - l_{aq})d_{ai}$ can be sent along each pair $i$). Then the following formulation is a variant of (I) to minimize the max $\beta$th percentile loss across flow sets.

\[
\begin{align*}
(I') \quad & \min_{x,A,\alpha} \quad \alpha \\
\text{s.t.} \quad & \sum_{q \in Q} z_{aq}p_q \geq \beta \quad \forall a \in A \\
\alpha & \geq l_{aq} - 1 + z_{aq} \quad \forall a \in A, q \in Q \\
\sum_a (1 - l_{aq})d_{ai} & \leq \sum_{t \in R(i)} x_{iq}y_{tq} \quad \forall i \in P, q \in Q \\
\sum_{e \in t} x_{eq} & \leq c_e \quad \forall e \in E, q \in Q \\
x_{iq} & \geq 0 \quad \forall i \in P, \forall t \in R(i), q \in Q \\
z_{aq} & \in \{0, 1\} \quad \forall a \in A, q \in Q \\
0 & \leq l_{aq} \leq 1 \quad \forall a \in A, q \in Q
\end{align*}
\]

Formulations for CVaR-based schemes The following formulation, $\text{Cvar-Flow-Ad}$, minimizes the maximum conditional value at-risk across all flows. It allows the routing strategy to depend on each scenario.

\[
\begin{align*}
(I''') \quad & \min_{x,A} \quad \alpha \\
\text{s.t.} \quad & \sum_{q \in Q} z_{aq}p_q \geq \beta \quad \forall a \in A \\
\alpha & \geq l_{aq} - 1 + z_{aq} \quad \forall a \in A, q \in Q \\
\sum_a (1 - l_{aq})d_{ai} & \leq \sum_{t \in R(i)} x_{iq}y_{tq} \quad \forall i \in P, q \in Q \\
\sum_{e \in t} x_{eq} & \leq c_e \quad \forall e \in E, q \in Q \\
x_{iq} & \geq 0 \quad \forall i \in P, \forall t \in R(i), q \in Q \\
z_{aq} & \in \{0, 1\} \quad \forall a \in A, q \in Q \\
0 & \leq l_{aq} \leq 1 \quad \forall a \in A, q \in Q
\end{align*}
\]
with $\alpha$ $\zeta$

TeaVar guarantees a performance no better than $\alpha$

Therefore, for each $\chi$ $\xi$ $\zeta$

flow with a loss of $\alpha$

the maximum loss across all flows using $\alpha$

the routing strategy obtained using TeaVar and observe that $\alpha$

across all flows in scenario $\alpha$

is the same across all scenarios, i.e., $\alpha$

let $\alpha$

be the current optimal $\alpha$

More concretely, we obtain:

$$
\begin{align*}
\min_{x, f, b, s} & \quad \theta \\
\text{s.t.} & \quad \theta \geq \theta_f \quad \forall f \in F \quad (28) \\
& \quad \theta_f \geq \alpha_f + \frac{1}{1 - \beta} \sum_{q \in Q} p_q s_q f_q \quad \forall f \in F \quad (29) \\
& \quad \alpha_f + s_q f_q \geq l_q f_q \quad \forall f \in F, q \in Q \quad (30) \\
& \quad s_q f_q \geq 0 \quad \forall f \in F, q \in Q \quad (31) \\
\end{align*}
$$

(4), (5)

Here, $l_q f_q$ is the loss for flow $f$ in scenario $q$, $\theta_f$ models the conditional value-at-risk for flow $f$, and $\theta$ models max $\alpha$ $\zeta$

The following formulation, CVar-Flow-St, is derived from CVar-Flow-Ad by requiring that the routing strategy is the same across all scenarios, i.e., we add the requirement that $\chi t_q = x_q$ for all $q$. More concretely, we obtain:

$$
\begin{align*}
\min_{x, f, b, s, x'} & \quad \theta \\
\text{s.t.} & \quad \theta \geq \theta_a \quad \forall f \in F \quad (32) \\
& \quad \theta_f \geq \alpha_f + \frac{1}{1 - \beta} \sum_{q \in Q} p_q s_q f_q \quad \forall f \in F \quad (33) \\
& \quad \alpha_f + s_q f_q \geq l_q f_q \quad \forall f \in F, q \in Q \quad (34) \\
& \quad s_q f_q \geq 0 \quad \forall f \in F, q \in Q \quad (35) \\
& \quad \sum_{pr(f)=i} \left(1 - l_q f_q\right) d_f \leq \sum_{i \in R(i)} x_i y_{f q} \quad \forall i \in P, q \in Q \quad (36) \\
& \quad \sum_{e \in E} x_t \leq c_e \quad \forall e \in E \quad (37) \\
& \quad x_t \geq 0 \quad \forall i \in P \quad (38) \\
\end{align*}
$$

Proof of Proposition 1 Let $\alpha_q$ denote the maximum loss across all flows in scenario $q$, i.e., $\alpha_q$ is the optimal value of $\alpha$ $\zeta$ with $z_{a q} = 1$ for all $a$. Let $Q'$ be any minimal subset of $Q$ such that $\sum_{q \in Q'} p_q \geq \beta$ and for $q' \in Q'$ and $q \notin Q'$, $\alpha_q \geq \alpha_{q'}$. Then, we define $v = \max_{q' \in Q'} \alpha_{q'}$, which is the $\alpha$ $\zeta$ percentile of $\alpha_{q' \in Q'}$. In our first step of the algorithm, we set $z_{a q} = 1$ for all $a$ and $q', q \in Q'$. By definition, $\sum_{q' \in Q'} p_q z_{a q} \geq \beta$. In particular, for each application $a$ and $q' \in Q'$, $l_{a q} \leq v$.

Therefore, for each $a$, the $\alpha$ $\zeta$ percentile of $l_{a q} \leq v$. So, our performance guarantee, which is the maximum across all $a$ of the $\alpha$ $\zeta$ percentile of $l_{a q}$, is no more than $v$. To see that TeaVar guarantees a performance no better than $v$, let $x_t$ be the routing strategy obtained using TeaVar and observe that the maximum loss across all flows using $x_t$ for a scenario $q$ is at least $\alpha_q$. Let $r = (1 - \beta) - \sum_{q' \in Q'} p_{q'\bar{q}} \bar{q} \in Q'$ be any scenario with $\alpha_{\bar{q}} = 0$, and $s$ be the corresponding optimal $s_q$ (in TeaVar formulation). Then, observe that $r \leq p_{q\bar{q}}$ and $a + s \geq \alpha_{\bar{q}} = v$, where the inequality follows because there is at least one flow with a loss of $\alpha_q$ since $\alpha_q$ is the minimum possible loss attainable across all flows for scenario $\bar{q}$. Then, we have that TeaVar objective is no less than $\alpha + \frac{1}{1 - \beta} \sum_{q' \in Q'} p_{q'\bar{q}} s_{q'\bar{q}} + \frac{1}{1 - \beta} r v \geq \frac{1}{1 - \beta} \sum_{q' \in Q'} p_{q'\bar{q}} s_{q'\bar{q}} + \frac{1}{1 - \beta} r v \geq v$, where the first inequality is because $\sum_{q' \in Q'} p_{q'\bar{q}} + r = 1 - \beta$, $\alpha + s \geq \alpha_{q'}$, and $a + s \geq v$.

The second inequality is because $\sum_{q' \in Q'} p_{q'\bar{q}} + r = 1 - \beta$ and $\alpha_{q'} \geq v$ for $q' \in Q'$. Moreover, SMORE guarantees a loss of $v$, since the guarantee for flows in any scenario $q'$ not in $Q'$ is $\alpha_{q'}$. It follows that the guarantee from the initial step of our algorithm is at least as good as the one obtained from either SMORE or TeaVar.

Topologies summary (§5). Our evaluation is done using 20 topologies obtained from [20] and [22]. The number of nodes and the number of edges of each topology is shown in Table 2.

More advanced pruning strategies. While we have not implemented, we can further accelerate our decomposition scheme by pruning scenarios with small losses. In particular, we may begin with subproblems that yield cuts tight at the master problem solution. Then, we prune subproblems unlikely to have large loss for critical links using memoized feasible solutions for these scenarios found previously.

In more detail, the main purpose of solving the subproblem is to generate the cuts (12). Let $\tilde{z}$ be the current optimal solution of the master problem. It can be easily argued that we do not need cuts from all subproblems. In fact, a cut from a single subproblem may suffice if after introducing it to the master problem, $\tilde{z}$ is no longer optimal. One way this can be guaranteed, at an intermediate iteration, is if we find a subproblem that yields an optimal value higher than the performance guarantee from a routing strategy found in a previous iteration. In fact, this observation suggests a further acceleration strategy, that allows subproblems to be pruned. To see this, observe that the optimal value of $\alpha_q$ can be under and overestimated using previous solutions of $S_q$ for other settings of $z$. Any feasible routing strategy found in a previous iteration for $q'$ can be used to upper-bound the loss for the critical links given by $\tilde{z}$. Similarly, a cut from previous iteration, yields an underestimate $\tilde{q}(z)$. Now, we first solve scenario subproblems for which cuts in the master problem are binding at $\tilde{z}$ and assume that among these subproblems

| Topology | # nodes | # edges | Topology | # nodes | # edges |
|----------|---------|---------|----------|---------|---------|
| B4       | 12      | 19      | Janet Backbone | 29 | 45 |
| IBM      | 17      | 23      | Highwinds | 16 | 29 |
| ATT      | 25      | 56      | BTNorthAmerica | 36 | 76 |
| Quest    | 19      | 30      | CRLNetwork | 32 | 37 |
| Tinet    | 48      | 84      | Darkstrand | 28 | 31 |
| Sprint   | 10      | 17      | Integra | 23 | 32 |
| GEANT    | 32      | 50      | Xpedius | 33 | 47 |
| Xeex     | 22      | 32      | InternetMCI | 18 | 32 |
| CWIX     | 21      | 26      | Deltacom | 103 | 151 |
| Digex    | 31      | 35      | IIJ | 27 | 55 |

Table 2: Topologies used in evaluation
the largest optimal value at $z = \bar{z}$ is $\bar{\alpha}$. Then, any subproblem for which a previous overestimator (routing strategy) already yields a bound of $\bar{\alpha}$ or less does not need to solved. Since $\bar{\alpha} \geq 0$, the strategy of not solving a subproblem where all flows can be met is a special case of this more general strategy for pruning scenarios.