Local Realism for $K^0\bar{K}^0$ pairs

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Abstract

In this talk we discuss the predictions of local realistic theories for the evolution of a $K^0\bar{K}^0$ quantum entangled pair created in the decay of the $\phi$–meson. It is shown, in agreement with Bell’s theorem, that the most general local hidden–variable model fails in reproducing the whole set of quantum–mechanical observables. We achieve this conclusion by employing two different approaches. In the first approach, the local realistic observables are deduced from the most general premises concerning locality and realism, and Bell–like inequalities are not employed. The other approach makes use of Bell’s inequalities. Under particular conditions for the detection times, within the first approach the discrepancy between quantum mechanics and local realism for the asymmetry parameter turns out to be not less than 20%. A similar incompatibility can be made evident by means of a Bell–type test, by employing a Clauser–Horne–Shimony–Holt’s inequality written in terms of properly normalized observables. Because of its relatively low experimental accuracy, the data obtained by the CPLEAR collaboration do not yet allow a decisive test of local realism. Such a test, both with and without the use of Bell’s inequalities, should be feasible in the future at the Frascati $\Phi$–factory.

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In 1935 Einstein Podolsky and Rosen (EPR in the following) [1] advanced a strong criticism concerning the interpretation of quantum theory. Starting from a few premises concerning completeness, physical reality and locality and considering the behaviour of a correlated and non–interacting system composed by two separated entities, EPR arrived at the conclusion that the description of reality given by Copenhagen’s interpretation is incomplete. At the very heart of their logical conclusion is the following fact: their assumption, that a quantum system has real and well defined properties also when does not interact with other systems (including a measuring apparatus), is contradicted by quantum mechanics.

This was the point attacked by Bohr in his famous response [2] to EPR’s paper. He noticed that EPR’s criterion of reality contained an ambiguity if applied to quantum phenomena. Starting from the complementarity point of view, Bohr stressed again that quantum mechanics within its scope [namely, in its form restricted to human knowledge] would appear as a completely rational description of the physical phenomena.

The first hypothesis for the solution of the paradoxical conclusion of EPR was proposed by Furry [3] in 1935. He assumed that in presence of EPR correlations between two quantum subsystems which are very far away one from each other, the state of the global system is no longer given by a superposition of tensorial products of states but it is simply represented by a statistical mixture of products of states. However, different experiments excluded a possible separability of the many–body wave function even in the case of space–like separated particles.

EPR’s paradox was interpreted as the need for the introduction, in quantum theory, of additional variables, in order to restore completeness, relativistic causality (namely locality) and realism. In 1952 Bohm [4] suggested an interpretation of quantum theory in terms of hidden–variables, in which the general mathematical formulation and the empirical results of the theory remained unchanged. In Bohm’s interpretation the paradoxical behaviour of correlated and non–interacting systems revealed by EPR find an explanation. However, for such systems Bohm’s theory exhibits a non–local character.

This result is consistent with what Bell obtained in 1964 [5]. He proved that any deterministic local hidden–variable theory is incompatible with some statistical prediction of quantum mechanics. This is the content of Bell’s theorem in its original form, which has been generalized in [6] to include non–deterministic theories. From then, Bell and other authors [7–10] derived different inequalities suitable for testing what has been called local realism.

Once established the particularity of Bell’s local realism, different experiments have been carried out to test these theories. The oldest ones [11,12] measured the linear polarization correlations of photon pairs created in radiative atomic cascade reactions or in electron–positron annihilations, whereas, more recently, parametric down–conversion photon sources have been employed [13–15]. Essentially all the experiments performed until now (in optics and atomic physics) have proved that the class of theories governed by Bell’s theorem are
unphysical. Actually, to be precise, because of apparatus non-idealities and other technical problems, supplementary assumptions are needed in the interpretation of the experiments, and, consequently, no test employed to refute local realism has been completely loophole free [10,13].

It is then important to continue performing experiments on correlation properties of many particle systems, possibly in new sectors, especially in particle physics, where entangled $K^0\bar{K}^0$ and $B^0\bar{B}^0$ pairs are considerable examples. If future investigations will confirm the violation of Bell’s inequalities, it is clear that, under the philosophy of realism, the locality assumption would be incompatible with experimental evidence. This fact is not in conflict with the theory of relativity: actually, there is no way to use quantum non-locality for faster-than-light communication.

In this talk we discuss the predictions of local realistic schemes for a pair of correlated neutral kaons created in the decay of the $\phi$–meson. Unlike photons, kaons are detectable with high efficiency. Moreover, for $K^0\bar{K}^0$ pairs, which can be copiously produced at a high luminosity $\Phi$-factory, additional assumptions regarding detection not implicit in local realism (always implemented in the interpretation of experiments with photon pairs [10]) are not necessary to derive Bell’s inequalities [14]. A correlation experiment discriminating between local realism and quantum mechanics could be performed at the Frascati $\Phi$-factory [16] in the future.

II. EPR’S ARGUMENT AND LOCAL REALISM FOR $\phi \to K^0\bar{K}^0$

The starting point of EPR’s argumentation was the following condition for a complete theory: every element of physical reality must have a counterpart in the physical theory. They defined the physical reality by means of the following sufficient criterion: if, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this quantity. In addition, for a system made of two correlated, spatially separated and non-interacting entities, EPR introduced the following locality assumption: since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system. The previous criterion of reality supports the anthropocentric point of view nowadays called realism: it asserts that quantum systems have intrinsic and well defined properties even when they are not subject to measurements.

To exemplify EPR’s argumentation, consider the case (EPR–Bohm’s gedanken experiment [17]) of a particle with total angular momentum zero which decays, at rest, into two spin 1/2 particles, 1 and 2, which fly apart with opposite momenta. After a certain time, when they do not interact any more, the normalized spin wave function of the global system, which does not depend on the quantization direction of the spin, is:
\[
|S = 0, S_z = 0\rangle = \frac{1}{\sqrt{2}} \left[ |+\rangle_1 |_2 - |−\rangle_1 |+\rangle_2 \right].
\] (2.1)

For particles 1 and 2, \(|+\rangle\) and \(|−\rangle\) represent spin–up and spin–down states, respectively, along a direction chosen as \(z\)–axis. A measurement of the spin component of particle 1 along \(z\) produces a given outcome [which is not predetermined by the quantum state (2.1)] and forces, immediately, the spin of particle 2 along the opposite direction. Following EPR, the spin component of particle 2 is an element of physical reality, since it can be predicted with certainty and without in any way disturbing particle 2. Moreover, in order to fulfil the locality assumption (no action–at–a–distance), EPR assumed that such an element of reality existed independently of any measurement performed on particle 1.

Following EPR’s argumentation, the interpretation of the above experiment by means of quantum mechanics leads to a difficulty. In fact, if we had performed a measurement of the spin component of particle 1 along another direction, say along the \(x\)-axis, this would have defined the \(x\) component of the spin of particle 2 as another element of reality, again independent of measurement. Obviously, this is also valid for any spin component; then it should be possible, in the supposed complete theory, to assign different spin wave functions to the same physical reality. Therefore, one arrives at the conclusion that two or more physical quantities which correspond to non–commuting quantum operators, can have simultaneous reality. However, this is not possible in quantum mechanics. Therefore, there exist elements of physical reality for which quantum mechanics has no counterpart, and, according to EPR’s completeness definition, quantum theory cannot give a complete description of reality.

Actually, one could object, with Bohr [2], that in connection with a correlated system of non–interacting subsystem, EPR’s reality criterion reveals the following weak point: it is not correct to assert that the measurement on subsystem 1 does not disturb system 2; in fact, in quantum mechanics the measurement do separate systems 1 and 2, which are not separated entities before the reduction of the wave packet. Then, from the point of view of orthodox quantum mechanics, EPR’s argumentation ceases to be a paradox: EPR’s proof of incompleteness is mathematically correct but is founded on premises which are inapplicable to microphenomena.

Now we come to the neutral–kaon system. In the following discussion we shall neglect the (small) effects of \(CP\) violation. Then, the \(CP\) eigenstates are identified with the short and long living kaons (mass eigenstates): \(|K_+\rangle \equiv |K_S\rangle \ (CP = +1)\), \(|K_−\rangle \equiv |K_L\rangle \ (CP = −1)\). In this approximation the strong interaction eigenstates \(|K^0\rangle\) and \(|\bar{K}^0\rangle\) are given by:

\[
|K^0\rangle = \frac{1}{\sqrt{2}} \left[ |K_S\rangle + |K_L\rangle \right], \quad |\bar{K}^0\rangle = \frac{1}{\sqrt{2}} \left[ |K_S\rangle - |K_L\rangle \right].
\] (2.2)

The time evolution of the mass eigenstates is:

\[
|K_{S,L}(\tau)\rangle = e^{-i\lambda_{S,L}\tau} |K_{S,L}\rangle,
\] (2.3)

where \(|K_{S,L}\rangle \equiv |K_{S,L}(0)\rangle\), \(\tau = t\sqrt{1 - v^2}\) is the kaon proper time \([t \ (v)\) being the time (kaon velocity) measured in the laboratory frame] and:
\[ \lambda_{S,L} = m_{S,L} - \frac{i}{2} \Gamma_{S,L}, \quad (2.4) \]

\( m_{S,L} \) denoting the \( K_S \) and \( K_L \) masses and \( \Gamma_{S,L} \) the corresponding decay widths: \( \Gamma_{S,L} \equiv 1/\tau_{S,L} \) (we use natural units: \( \hbar = c = 1 \)).

Consider now the strong decay of the \( J^{PC} = 1^{--} \)–meson into \( K^0\bar{K}^0 \). Just after the decay, at proper time \( \tau = 0 \), the quantum–mechanical state is given by the following superpositions:

\[ |\phi(0)\rangle = \frac{1}{\sqrt{2}} \left[ |K^0\rangle_1 |\bar{K}^0\rangle_2 - |\bar{K}^0\rangle_1 |K^0\rangle_2 \right] = \frac{1}{\sqrt{2}} \left[ |K_L\rangle_1 |K_S\rangle_2 - |K_S\rangle_1 |K_L\rangle_2 \right]. \quad (2.5) \]

From eqs. (2.2) and (2.3) the time evolution of state (2.3) is obtained in the following form:

\[ |\phi(\tau_1, \tau_2)\rangle = \frac{1}{\sqrt{2}} \left\{ e^{-i(\lambda_L \tau_1 + \lambda_S \tau_2)} |K_L\rangle_1 |K_S\rangle_2 - e^{-i(\lambda_S \tau_1 + \lambda_L \tau_2)} |K_S\rangle_1 |K_L\rangle_2 \right\} \]
\[ = \frac{1}{2\sqrt{2}} \left\{ \left[ e^{-i(\lambda_L \tau_1 + \lambda_S \tau_2)} + e^{-i(\lambda_S \tau_1 + \lambda_L \tau_2)} \right] \left[ |K^0\rangle_1 |\bar{K}^0\rangle_2 - |\bar{K}^0\rangle_1 |K^0\rangle_2 \right] \]
\[ + \left[ e^{-i(\lambda_L \tau_1 + \lambda_S \tau_2)} - e^{-i(\lambda_S \tau_1 + \lambda_L \tau_2)} \right] \left[ |K^0\rangle_1 |\bar{K}^0\rangle_2 - |\bar{K}^0\rangle_1 |K^0\rangle_2 \right] \right\}. \quad (2.6) \]

Therefore, quantum mechanics predicts (and we know it is a well tested property) a perfect anti–correlation in strangeness and \( CP \) values when both kaons are considered at the same time. In the case in which both kaons are undecayed, if an experimenter observes, say along direction 1, a \( K^0 \) (\( K_L \)), at the same time \( \tau_1 \), along direction 2, because of the instantaneous collapse of the two–kaon wave function, one can predict the presence of a \( \bar{K}^0 \) (\( K_S \)). Thus, at time \( \tau_1 \) to the kaon moving along direction 2 we assign an element of reality (since, following EPR’s reality criterion, the value of the corresponding physical quantity is predicted with certainty and without in any way disturbing the system), the value –1 (+1) of strangeness (\( CP \)). The same discussion is valid when the state observed along direction 1 is \( \bar{K}^0 \) (or \( K_S \)) as well as when one exchanges the kaon directions: \( 1 \leftrightarrow 2 \). For times \( \tau_2 \) following the observation at time \( \tau_1 \) along direction 1 of a \( K_L \) (\( K_S \)), a \( CP \) measurement on the other kaon will give with certainty the same result \( CP = +1 \) (\( CP = -1 \)) one expects at time \( \tau_1 \). This expresses \( CP \) conservation. In the case in which neither kaon has decayed, when the kaon detected at time \( \tau_1 \) is \( K^0 \) (\( \bar{K}^0 \)), at times \( \tau_2 > \tau_1 \) along direction 2 quantum mechanics predicts the possibility to observe a \( \bar{K}^0 \) (\( K^0 \)) as well as a \( \bar{K}^0 \) (\( K^0 \)): since strangeness is not conserved during the evolution of the system, perfect anti–correlation on strangeness only exists when both particles are considered at the same time.

Following EPR’s argument, in a local realistic approach one then associates to both kaons of the pair, at any time, two elements of reality, which are not created by measurements eventually performed on the partner when the particles are space–like separated (locality): one determines the kaon \( CP \) value, the other one supplies the kaon strangeness \( S \). They are both well defined also when the meson is not observed (realism) and can take two values, ±1, which appear at random with the same frequency in a statistical ensemble of kaons.
Therefore, four different single kaon states can appear just after the $\phi$ decay, with the same frequency (25%). They are quoted in table I. It is clear that this classification is incompatible with quantum mechanics, where strangeness and $CP$ cannot be measured simultaneously.

### III. QUANTUM–MECHANICAL EVOLUTION

By introducing the shorthand notation:

$$E_{S,L}(\tau) = e^{-\Gamma_{S,L} \tau}, \quad \Delta m = m_L - m_S,$$

from eq. (2.6) the quantum–mechanical (QM) probability $P_QM[K^0(\tau_1), \bar{K}^0(\tau_2)] \equiv |\langle K^0|\rho(\tau)|\phi(\tau_1, \tau_2)\rangle|^2$ that a measurement detects a $K^0$ at time $\tau_1$ along direction 1 and a $\bar{K}^0$ at time $\tau_2$ along direction 2 is:

$$P_QM[K^0(\tau_1), \bar{K}^0(\tau_2)] = P_QM[\bar{K}^0(\tau_1), K^0(\tau_2)]\quad (3.2)$$

$$= \frac{1}{8} [E_L(\tau_1)E_S(\tau_2) + E_S(\tau_1)E_L(\tau_2)] [1 + A_QM(\tau_1, \tau_2)].$$

The other probabilities relevant for our discussion are the following ones:

$$P_QM[K^0(\tau_1), K^0(\tau_2)] = P_QM[K^0(\tau_1), \bar{K}^0(\tau_2)]$$

$$= \frac{1}{8} [E_L(\tau_1)E_S(\tau_2) + E_S(\tau_1)E_L(\tau_2)] [1 - A_QM(\tau_1, \tau_2)], \quad (3.3)$$

$$P_QM[K_L(\tau_1), K_S(\tau_2)] = \frac{1}{2} E_L(\tau_1)E_S(\tau_2), \quad (3.4)$$

$$P_QM[K_S(\tau_1), K_L(\tau_2)] = \frac{1}{2} E_S(\tau_1)E_L(\tau_2). \quad (3.5)$$

In eqs. (3.2) and (3.3):

$$A_QM(\tau_1, \tau_2) \equiv \frac{P_QM[K^0(\tau_1), \bar{K}^0(\tau_2)] - P_QM[K^0(\tau_1), K^0(\tau_2)]}{P_QM[K^0(\tau_1), K^0(\tau_2)] + P_QM[K^0(\tau_1), \bar{K}^0(\tau_2)]} \quad (3.6)$$

$$= 2 \frac{\sqrt{E_L(\tau_2 - \tau_1)E_S(\tau_2 - \tau_1)}}{E_L(\tau_2 - \tau_1) + E_S(\tau_2 - \tau_1)} \cos \Delta m(\tau_2 - \tau_1),$$

is the quantum–mechanical asymmetry parameter.
TABLE II. Realistic states for the kaon pair at initial time $\tau = 0$.  

| Direction 1 | Direction 2 |
|-------------|-------------|
| $K_1 \equiv K_S^0$ ($S = +1, CP = +1$) | $K_4 \equiv K_L^0$ ($S = -1, CP = -1$) |
| $K_2 \equiv \bar{K}_S^0$ ($S = -1, CP = +1$) | $K_3 \equiv K_L^0$ ($S = +1, CP = -1$) |
| $K_3 \equiv K_L^0$ ($S = +1, CP = -1$) | $K_2 \equiv \bar{K}_S^0$ ($S = -1, CP = +1$) |
| $K_4 \equiv \bar{K}_L^0$ ($S = -1, CP = -1$) | $K_1 \equiv K_S^0$ ($S = +1, CP = +1$) |

TABLE III. Local realistic states for the kaon pair at times $\tau_2 \geq \tau_1$.  

| Probabilities | Direction 1 (Left) Time $\tau_1$ | Direction 2 (Right) Time $\tau_2$ |
|---------------|----------------------------------|----------------------------------|
| $P_{1}(\tau_1, \tau_2; \lambda)$ | $K_1 \equiv K_S^0$ | $K_4 \equiv K_L^0$ |
| $P_{2}(\tau_1, \tau_2; \lambda)$ | $K_1 \equiv K_S^0$ | CP = -1 DP |
| $P_{3}(\tau_1, \tau_2; \lambda)$ | CP = +1 DP | $K_4 \equiv \bar{K}_L^0$ |
| $P_{4}(\tau_1, \tau_2; \lambda)$ | $K_1 \equiv K_S^0$ | $K_3 \equiv K_S^0$ |
| . . . . . . . . | . . . . . . . . | . . . . . . . . |

IV. LOCAL REALISTIC EVOLUTION

In this section we discuss the predictions of local hidden–variable models for the kaon–pair observables. More details can be found in ref. [18].

The quantum–mechanical expectation values for the evolution of a single kaon can be reproduced by a realistic approach [19]. Consider now the time evolution of a kaon pair in $\phi \rightarrow K^0\bar{K}^0$. At time $\tau = 0$, immediately after the $\phi$ decay, in the realistic picture there are four possible states for the pair, each appearing with a probability equal to $1/4$: they are listed in table II. We assume, as in quantum mechanics, a perfect anti–correlation in strangeness and $CP$ when both kaons are considered at equal times.

When the system evolves, the kaons fly apart from each other, and at two generic times $\tau_1$ and $\tau_2$ (corresponding to opposite directions of propagation labeled 1 and 2, respectively) the kaon pair is in one of the states reported in table II. The first row refers to the state with a $K_1$ at time $\tau_1$ along direction 1 (left) and a $K_4$ at time $\tau_2$ along direction 2 (right). Given the classification of the table, in our discussion we consider $\tau_2 \geq \tau_1$: the isotropy of space guarantees the invariance of the two–kaon states by exchanging the directions 1 and 2. In the second row the state corresponds to a left going $K_1$ at time $\tau_1$ and $CP = -1$ decay products (DP) at time $\tau_2$ on the right. These decay products originate from the instability of the $K_3$ and $K_4$ pure states, which are both long living kaons, namely $CP = -1$ states. At time $\tau_1$ the state correlated with a left going $K_1$ is necessarily either a $K_4$ or a state containing $CP = -1$ decay products, $K_3^{DP}$ or $K_4^{DP}$. Then, at time $\tau_2$ ($> \tau_1$) on the right we can have: i) a $K_4$ (state in the first row), ii) $CP = -1$ decay products (state in the
second row) or iii) a $K_3$ (state in the fourth row). The former case refers to the transition $K_4(\tau_1) \rightarrow K_4(\tau_2)$, the latter to $K_4(\tau_1) \rightarrow K_3(\tau_2)$, both along direction 2. Occurrence ii) takes contributions from the following transitions: $K_3^{DP}(\tau_1) \rightarrow K_3^{DP}(\tau_2)$, $K_4^{DP}(\tau_1) \rightarrow K_4^{DP}(\tau_2)$, $K_4(\tau_1) \rightarrow K_4^{DP}(\tau_2)$ and $K_4(\tau_1) \rightarrow K_3(\tau_1 < \tau < \tau_2) \rightarrow K_3^{DP}(\tau_2)$. The other 14 local realistic states not quoted in table III can be obtained in the same way [18].

It is important to stress that the states listed in table III are assumed to be well defined for all times $\tau_1$ and $\tau_2$ with $\tau_1 \leq \tau_2$: this is the main requirement of the realistic approach. For a given kaon pair, in a deterministic theory only one of the possibilities of table III really occurs for fixed $\tau_1$ and $\tau_2$. This means that we are making the hypothesis (realism) that there exist additional variables $\lambda$, called hidden–variables, that provide a complete description of the pair, which is viewed as really existing and with well defined properties independently of any observation. The state representing the meson pair for given times $(\tau_1, \tau_2)$ is completely defined by these hidden–variables: they are supposed to determine in advance (say when the two kaons are created) the future behaviour of the pair. Thus, the times in correspondence of which the instantaneous $|\Delta S| = 2$ jumps and the decay occur for a given kaon are predetermined by its hidden–variables. Under this hypotheses there is no problem concerning a possible causal influence acting among the different entities of entangled systems when a measurement takes place on one subsystem. However, the new variables are unobservable since they are averaged out in the measuring processes, and unobservable are the states of table III. Besides, 1) also the measuring apparatus could be described by means of hidden–variables, which influence the results of measurement, and 2) hidden–variables associated to the kaon pair could show a non–deterministic behaviour. For further details concerning the hidden–variable interpretation of the states in table III see ref. [18].

In ref. [18] we have studied the range of variability of the meson pair observables compatible with the most general local realistic model, obtaining the following inequality on the asymmetry parameter:

$$2|Q_+(\tau_2) - Q_-(\tau_1)| - 1 \leq A_{LR}(\tau_1, \tau_2) \leq 1 - 2|Q_+(\tau_2) - Q_-(\tau_1)|, \quad (4.1)$$

where:

$$Q_\pm(\tau) = \frac{1}{2} \left[ 1 \pm 2\sqrt{E_L(\tau)E_S(\tau)} \cos \Delta m \tau \right]. \quad (4.2)$$

V. COMPATIBILITY BETWEEN LOCAL REALISM AND QUANTUM MECHANICS

Local realism reproduces the quantum–mechanical predictions for the single kaon observables and the joint probabilities (3.4), (3.5). The same conclusion would be true for the
observables (3.2) and (3.3) involving $K_S-K_L$ mixing if the time–dependent local realistic asymmetry parameter had the same expression it has in quantum mechanics:

Local Realism equivalent to Quantum Mechanics \iff $A_{LR}(\tau_1, \tau_2) \equiv A_{QM}(\tau_1, \tau_2)$. (5.1)

From eq. (5.1) it follows that for the special case of $\tau_2 = \tau_1 \equiv \tau$, local realism is compatible with quantum mechanics. This also occurs when $\tau_1 \equiv 0$. Another special case is when, for instance, $\tau_2 = 1.5\tau_1$: in this situation, the local realistic asymmetry does not satisfy the compatibility requirement (5.1). This is depicted in figure [1]. There is an

![Image of diagram](attachment:image.png)

FIG. 1. Local realistic and quantum–mechanical asymmetry parameters for $\tau_2 = 1.5\tau_1$ plotted vs $\tau_1/\tau_S$. There is an evident discrepancy when $0 < \tau_1 < 2.3\tau_S$, the largest incompatibility corresponding to $\tau_1 \approx 1.5\tau_S$, where $[A_{QM} - A_{LR}^{\text{Max}}]/A_{QM} \approx 20\%$. In general, local realism and quantum mechanics are incompatible when $\tau_2 = \alpha\tau_1$ with $\alpha > 1$. The degree of incompatibility increases for increasing $\alpha$. For instance, when $\tau_2 = 2\tau_1 \approx 2.4\tau_S$, $A_{QM}$ is 27 % larger than $A_{LR}^{\text{Max}}$.

However, it is important to stress the following restriction concerning the choice of the detection times $\tau_1$ and $\tau_2$. In order to satisfy the locality condition, namely to make sure that the measurement on the right is causally disconnected from that on the left, these events must be space–like separated. In the center of mass of the process $\phi \rightarrow K^0\bar{K}^0$, this requirement corresponds to choose detection times in the interval: $1 \leq \tau_2/\tau_1 < 1.55$. 

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An experiment that measured the asymmetry parameter was performed by the CPLEAR collaboration at CERN [20]. The $K^0\bar{K}^0$ pairs were produced by proton–antiproton annihilation at rest. Unfortunately, because of their large error bars, the CPLEAR data are in agreement, within one standard deviation, with both quantum mechanics and local realism.

VI. BELL’S INEQUALITIES

Because of the particular values of the kaon lifetimes ($\Gamma_S$ and $\Gamma_L$) and of the quantity $\Delta m \equiv m_L - m_S$, it is impossible to show a violation, by quantum mechanics, of Bell’s inequalities exploiting strangeness measurements at different times. This was the conclusion of ref. [21]. In this section we consider again this question in order to show how a Bell–type test is actually feasible. The reason of the difficulty in designing a Bell test with kaons lies in the very short $K_S$ lifetime ($\tau_S$) compared with the typical time ($2\pi/\Delta m \simeq 13\tau_S$) of the strangeness oscillations.

Consider joint probabilities normalized to undecayed kaon pairs:

$$P[\bar{K}^0(\tau), \bar{K}^0(\tau')] \to P_{\text{ren}}[\bar{K}^0(\tau), \bar{K}^0(\tau')] \equiv \frac{P[\bar{K}^0(\tau), \bar{K}^0(\tau')] - P[-(\tau), -(\tau')]}{4[1 - A(\tau, \tau')]} = 1,$$

where

$$P[-(\tau), -(\tau')] = \frac{1}{2}[E_S(\tau)E_L(\tau') + E_L(\tau)E_S(\tau')],$$

both in the local realistic description and in quantum mechanics. The renormalized observables are less damped than the original ones, and, as a consequence, a Bell–type test can be performed.

The same derivation that supplies Clauser–Horne–Shimony–Holt’s (CHSH’s) inequality [7,9] in the standard (namely unrenormalized) case can be applied to the renormalized observables of eq. (6.1). By introducing four detection times ($\tau_1$ and $\tau_2$ for the left going meson, $\tau_3$ and $\tau_4$ for the right going meson), CHSH’s inequality for strangeness $-1$ detection is then:

$$-1 \leq S_{LR}(\tau_1, \tau_2, \tau_3, \tau_4) \leq 0,$$

with:

$$S_{LR}(\tau_1, \tau_2, \tau_3, \tau_4) \equiv P_{LR}[\bar{K}^0(\tau_1), \bar{K}^0(\tau_3)] - P_{LR}[\bar{K}^0(\tau_1), \bar{K}^0(\tau_4)] + P_{LR}[\bar{K}^0(\tau_2), \bar{K}^0(\tau_3)] - P_{LR}[\bar{K}^0(\tau_2), \bar{K}^0(\tau_4)],$$

where

$$P_{LR}[\bar{K}^0(\tau)] \equiv P_{LR}[\bar{K}^0(\tau)]/P_{LR}[-(\tau)] = 1/2.$$
FIG. 2. Violation of CHSH’s inequality (6.3) for \( \tau_1/p = \tau_2/(p+2) = \tau_3/(p+1) = \tau_4/(p+3) \equiv \tau \). The function \( S_{QM} \) of eq. (6.6) is plotted versus \( \tau \). See text for further details.

\[
\tau_3 - \tau_1 = \tau_2 - \tau_3 = \tau_4 - \tau_2 = \frac{1}{3}(\tau_4 - \tau_1) \equiv \tau.
\]

Thus, in quantum mechanics eq. (6.4) reduces to:

\[
S_{QM}(\tau) = \frac{1}{4} [2 - 3A_{QM}(\tau) + A_{QM}(3\tau)] - 1.
\]

If we choose \( \tau_1 \equiv \tau \), the other times become: \( \tau_2 = 3\tau \), \( \tau_3 = 2\tau \) and \( \tau_4 = 4\tau \), and, in the limit of stable kaons \( (\Gamma_S = \Gamma_L = 0) \), both side of inequality (6.3) are violated by quantum mechanics in periodical intervals of \( \tau \) (see curve marked \textit{spin} in figure 2): this situation correspond to the case of the spin–singlet system system (2.1).

As far as the real case for kaons is considered, quantum mechanics does not violate inequality (6.3) when unrenormalized expectation values are used (see curve \textit{unren} in figure 2). The conclusion is different once one employs probabilities normalized to undecayed kaon pairs: as it is shown in figure 2 (curve \textit{ren}), for \( 0 < \tau < 1.4\tau_S \) quantum–mechanical expectation values are incompatible with the left hand side of inequality (6.3).

The largest violation of the inequality \((-1.087 < -1\)) corresponds to \( \tau \simeq 0.81\tau_S \) and \( P_{QM}[\bar{K}^0(\tau_1), \bar{K}^0(\tau_3)] \simeq 0.036 \), \( P_{QM}[\bar{K}^0(\tau_1), \bar{K}^0(\tau_4)] \simeq 0.195 \).
With the previous choice of the four detection times the locality condition $1 \leq \tau_2/\tau_1 < 1.55$ is not satisfied, since: $\tau_4/\tau_1 = 4 > 1.55$. In order to fulfil this requirement when relation (6.5) is used, one can introduce times $\tau_1 = p\tau$, $\tau_2 = (p + 2)\tau$, $\tau_3 = (p + 1)\tau$ and $\tau_4 = (p + 3)\tau$ ($p \geq 0$) and require $\tau_4/\tau_1 = (p + 3)/p < 1.55$, thus $p > 5.45$. However, since the renormalized quantum–mechanical probabilities only depend on the difference between the observation times [see eqs. (6.1), (3.6)], the result ren of figure 2 is independent of $p$, and the locality condition is satisfied. Thus, experimentally one could choose to use, for instance, $p = 6$, namely $\tau_1 = 6\tau$, $\tau_2 = 8\tau$, $\tau_1 = 7\tau$, $\tau_4 = 9\tau$, and the largest violation of the inequality would be again for $\tau \simeq 0.81\tau_S$. However, as $p$ increases, even if the renormalized probabilities are unchanged, the strangeness detection becomes more and more difficult, because of the kaon decays, thus small $p$ are preferable. Also the curve corresponding to the limit $\Gamma_S = \Gamma_L = 0$ is the same for any $p$. The curve corresponding to the inequality that makes use of unrenormalized probabilities depends on $p$, but this case is not interesting since it cannot be used for a discriminating test whatever the choice of $p$ is.

VII. CONCLUSIONS

In agreement with Bell’s theorem, in this talk we have shown that quantum mechanics for the two–neutral–kaon system cannot be completed by a theory which is both local and realistic: the separability assumed in Bell’s local realistic theories for the joint probabilities contradicts the non–separability of quantum entangled states. Any local realistic approach is only able to reproduce the non–paradoxical predictions of quantum mechanics like the perfect anti–correlations in strangeness and $CP$ and the single particle observables.

The incompatibility proof among quantum mechanics and local realistic models has been carried out by employing two different approaches. We started discussing the variability of the $K^0\bar{K}^0$ expectation values deduced from the general premises concerning locality and realism. The realistic states have been interpreted within the widest class of hidden–variable models. Under particular conditions for the experimental parameters (the detection times), the discrepancies among quantum mechanics and local realistic models for the time–dependent asymmetry parameter are not less than 20%. The data collected by the CPLEAR collaboration for the asymmetry do not allow for conclusive answers concerning a refutation of local realism.

The other approach that we followed makes use of Bell–like inequalities involving $K_S–K_L$ mixing. Contrary to what is generally believed in the literature, we have shown that a Bell–type test is feasible at a $\Phi$–factory, using CHSH’s inequalities.

Concluding, by employing an experimental accuracy for joint kaon detection considerably higher than that corresponding to the CPLEAR measurement, a decisive test of local realism vs quantum mechanics both with and without the use of Bell’s inequalities will be feasible in the future at the Frascati $\Phi$–factory.
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