Electric-field-coupled oscillators for collective electrochemical perception in biohybrid robotics

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Abstract
This work explores the application of nonlinear oscillators coupled by an electric field in water, inspired by weakly electric fish. Such coupled oscillators operate in clear and colloidal (mud, bottom silt) water and represent a collective electrochemical sensor that is sensitive to global environmental parameters, the geometry of the common electric field and spatial dynamics of autonomous underwater vehicles (AUVs). Implemented in hardware and software, this approach can be used to create global awareness in a group of robots, which possess limited sensing and communication capabilities. Using oscillators from different AUVs enables extension of the range limitations related to the electric dipole of a single AUV. Applications of this technique are demonstrated for detecting the number of AUVs, distances between them, perception of dielectric objects and synchronization of behavior. Recognizing self-/nonself-generated signals by electric fish is re-embodied in a technological way through an 'electrical mirror' for discrimination between 'collective self' and 'collective nonself'. These approaches have been implemented in several research projects with bioinspired/biohybrid systems in fresh and salt water, and electrochemical sensing in fluidic media.

1. Introduction
Underwater exploration has recently become an important economic, ecological and social field. Possible tasks for autonomous underwater vehicles (AUVs) include monitoring ecologically sensitive areas, exploration of the seabed, safety inspections and finding objects of interest [1, 2]. The challenges of underwater robotics are related, among other things, to the high damping factor of water, which limits the sensing and communication capabilities of AUVs [3, 4].

To overcome these limitations the state of the art approach is to use multiple AUVs [5–7]. These are small robot platforms designed to operate in a collective way. This approach has several advantages, such as exploration of large areas, increased functional operability through heterogeneity, enhanced reliability and a lower platform cost. Research and technological developments concern collective cognitive capabilities, coordination of the whole robot group, platform design and other issues [8].

An interesting bio-inspired solution observed in weakly electric fish [9, 10] consists of using underwater electric fields for collective sensing and communication. This approach, implemented for different purposes in potential and current modes [11–13], demonstrated strong and weak aspects of such technology. For example, some difficulties are related to the electric dipole of a single AUV being limited by the body length of the AUV [14]. Due to the small dipole size, the effective range of electric-field-based sensing is also limited.

Improvement of this technology can be achieved by using multiple oscillating dipoles coupled by an electric field. Such coupled oscillators are sensitive to the spatial distribution of dipoles, their orientation and the presence of obstacles in a common field. Variation of these parameters influences common synchronization patterns and the amplitude/phase of signals. Specific frequency–current dynamics of oscillating dipoles can be used in a type of electrochemical impedance spectroscopy [15] with different oscillating frequencies. Such a collective electrochemical sensor is used for environmental sensing [16],
detection of conducting/dielectric objects and sensing in colloidal (mud, bottom silt) solutions. Different interference patterns enable identification of AUVs and underwater beacons. Specific research and technological challenges are related to the detection of physical and chemical properties of fluidic media [17], collective sensing by electric fields on micro and macro scales, achieving awareness of a global environmental state by measuring parameters of local oscillations and other issues.

In this paper, nonlinear effects of electric-field-coupled oscillators are explored for application to collective mechanisms of perception and coordination [18, 19] in bioinspired AUVs [7, 20]. It is shown that changes in the amplitude, phase and temporal patterns of signals can be attributed to various group–internal/group–external events. Global synchronization effects can also be used for behavioral coordination. In particular, we explore the capacity to discriminate between collectively self- and nonself-generated signals by a so-called ‘electrical mirror’ [21]. Similarly to an optical mirror, an electrical mirror reflects received electrical signals and allows a robot to receive its own electrical reflection. The correlation between emitted and received signals enables robots to determine group affiliation or to discriminate between ‘collective self’ and ‘collective nonself’. Electric fish recognize self-/nonself-generated signals in a similar way [22].

These techniques have been demonstrated in five experiments implemented in several past projects with biohybrid (robot–fish interactions [23]) and
bioinspired underwater robots [7, 20, 21, 24] operating in fresh and salt water. They represent one of the lines of development for designing collective cognitive capabilities in a group of AUVs. New applications of this approach are primarily related to collective electrochemical sensing at macro (e.g. collective AUVs in colloidal water) and micro (e.g. ionic interfaces to living systems) scales, as demonstrated in recent projects on biohybrid systems [25–27].

2. Biological oscillators with embodied coupling

Coupled oscillators are well known in biology [28], theoretical physics [29] and nonlinear control [30], where different spatio-temporal effects are observed [31]. Both low-dimensional central pattern generators (CPGs) [32] and high-dimensional coupled map lattices (CMLs) [33] have applications in functional control of robotic systems. CPGs have been investigated, for example, in neuroscience, biology [34] and locomotive control [35], where models at collective and individual levels are distinguished. The state variables of CPGs are embodied (e.g. the amplitude of a biped movement); the disembodied ‘virtual channels’ are used for information exchange between oscillators. Due to embodiment, CPG-based coupled oscillators are widely used in robot controllers [36, 37]. The CPG approach represents a balance between the use of embodied processes as state variables on the individual level and the possibility of analytical treatments for complex biological or technological processes on the collective level. Since there is no direct transition from desired collective behavior to individual rules that generate that behavior, there are different bioinspired, stimulative or synergetic [38] strategies for resolving this problem.

The CML approach represents a useful modeling tool for studying the spatio-temporal dynamics of complex nonlinear systems and is widely used in complexity science [39] (e.g. for exploring chaotic modes and emergent behavior). Examples are the concepts of algorithmic and analytic agents [40, 41] decision-making processes [42, 43], homeostatic regulation [44] or underwater swarm algorithms [45] on a collective level. Due to disembodiment of state variables and couplings, the dynamics of CMLs can be investigated analytically as well as numerically. However, such a disembodiment imposes several limitations on applications of this technique in robotics [46].

This work is inspired by electric fish and combines CML- and CPG-based methodologies. The idea is to involve physical embodiment for coupling instead of state variables. Such an approach is more natural for operations in liquid media with a high damping factor. For instance, interference between electric fields emitted by oscillating dipoles can represent an embodied coupling mechanism (see figure 1(a)). This coupling depends on the parameters of water (e.g. salinity, geometry of sensing area, presence of obstacles), the frequency of individual oscillations, the number of dipoles and the distances between them and the relative positions and orientations of dipoles in three-dimensional space. All these parameters reflect the global state of the system. The perception of this information is strongly local. By analyzing perturbations of oscillations [47], each robot can become aware of the global situation and can undertake corresponding activities.

Structures of oscillating devices are shown in figure 2. For modelling we use time-discrete dynamical systems [48, 49] because they are suitable for implementation in a microcontroller with a specific digital-to-analog converter (DAC)/analog-to-digital converter (ADC) processing time. The signal from oscillator $\xi_n$ at each time step $n$ is converted into an analog signal by the DAC, which is connected to the positive $E^+$-emitting and negative $E^-$-emitting electrodes. In potential mode, the positive $R^+$-receiving
and negative $R^-$-receiving electrodes are connected to a high-pass filter, amplifier and ADC and represent the signal $\xi_{wn}$ from water at the time step $n$ (see figure 2(a)). In current mode, a single pair of electrodes is used to produce the potential and to sense a current between electrodes (see figure 2(b)). Analytical considerations, represented below, can be applied to both modes of operation.

Let $x_n$ be a state variable and $f(x_n, \alpha)$ a function of $x_n$ and the control parameter $\alpha$. The dynamical system can be written in the form

$$\xi_n = f(x_n, \alpha), \quad (1)$$

$$x_{n+1} = f(x_n, \alpha) + k_2 \xi_n \quad (2)$$

where $k_2$ is a coefficient. The variable $\xi_n$ contains its own signal sent by the emitting electrodes as well as signals from other emitting devices. Since its own signal has an essential impact on the dynamics of the system (2), we can follow two strategies to remove it from $\xi_n$. First, we can set $k_2 = 0$ and investigate the dynamics of $\xi_n$ without modifying the dynamics of $x_n$. Since the coupling is embodied, the dynamics of $\xi_n$ can provide enough information about the global state of the system. In the second strategy, the signal is subtracted from equation (2). The term $(\xi_n - k_1 \xi_{on})$ represents the coupling part from other oscillating devices, where $k_1$ is the coefficient reflecting the damping factor of water as well as the amplification in electronics. The following condition

$$\xi_n - k_1 \xi_{on} = 0 \quad (3)$$

should be satisfied when no other oscillating devices are in the water. This is achieved by calibrating the device and setting the coefficient $k_1$. The following system

$$\xi_n = f(x_n, \alpha), \quad (4)$$

$$x_{n+1} = f(x_n, \alpha) + k_2(\xi_n - k_1 \xi_{on}) \quad (5)$$

represents the model of an oscillating device. In the following we use only equation (5), bearing in mind $\xi_{on} = f(x_n, \alpha)$.

Weakly electric fish uses spike-based oscillators for generating electric impulses [9]. Such dynamics can be produced by the well-known logistic map $x_{n+1} = \alpha x_n (1 - x_n)$ in period-two dynamics, where $x \in R$ is
the state variable and $\alpha$ is the control parameter. Since the input from water does not include a constant potential (which is removed by a high-pass filter), the logistic map must also be transformed to remove such a potential. This is achieved by eliminating the non-periodic stationary states. Solving $x_n = \alpha x_n (1 - x_n)$, we obtain $x_{n1,2} = \{0, \frac{\alpha - 1}{\alpha}\}$. From the linear stability analysis we know that $x_{n1} = 0$ is stable in the region $\alpha = (-1 \ldots 1)$ and $x_{n2} = \frac{\alpha - 1}{\alpha}$ is stable in the region $\alpha = (1 \ldots 3)$. By inserting a new variable $y_n = x_n + \frac{\alpha - 1}{\alpha}$ and $y_{n+1} = x_{n+1} + \frac{\alpha - 1}{\alpha}$ and rewriting the systems in terms of the variables $x_n$ and $x_{n+1}$, we obtain

$$x_{n+1} = x_n (2 - \alpha x_n - \alpha).$$

(6)

The stationary state $x_{n1} = -\frac{\alpha - 1}{\alpha}$ is stable in the region $\alpha = (-1 \ldots 1)$ and $x_{n2} = 0$ in the region $\alpha = (1 \ldots 3)$.

For further tests in section 3 we use equation (6) in the form of equation (5); thus

$$x_{n+1} = x_n (2 - \alpha x_n - \alpha) + k_2 (\xi_n^\omega - k_1 \xi_n^\phi).$$

(7)

is used in the microcontroller for generating oscillations. Coupling through an electric field in water for low frequencies can be approximated by $\sum_{j=1}^{m} g_{ij} x_j^n$, where $g$ represents the distance from device $j$ to a mutual center of the system $[50]$. For the case shown in figure 1(a), we assume that the coefficient $k_1$ is calibrated so that its signal is filtered from the received signal. Since potential fields have properties of superposition, the system (7) for several devices can be represented as

$$x_{n+1}^{i'} = x_n^{i'} (2 - \alpha x_n^{i'} - \alpha) + \sum_{j=1}^{m} g_{ij} x_j^n, \quad i = 1, \ldots, m,$$

(8)

where $m$ is a dimension of the system (8) and the coefficients $g_{ij}$ reflect factors such as amplifications of $\xi_n^\omega$, losses in the high-pass filters, water damping and spatial conditions between emitting devices. Since only periodic signals are taken into consideration (see section 3), we can use only a period-two motion of (8) and corresponding global synchronization effects as reported in $[51]$. 

Figure 5. Strategies for the coupled system (11) at $m > 2$ (initial conditions randomly selected in the range $[0 \ldots 0.1]$, $\alpha = 3.1$). At each $N$ 100 attempts are performed. (a) Dependence between $\frac{x_{n-1}}{m}$ and $m$, $k_2$ is selected as (10), $e = -0.01$. (b) Dependence between $\frac{x_{n-1}}{m}$ and $e$ from (10) for $m = 5, 10, 15$.

Figure 6. Bifurcation diagrams with different delayed feedback: (a) $0.45 x_{n-1} - 0 x_{n-2}$ and (b) $-0.45 x_{n-1} - 0.45 x_{n-2}$.
Figure 7. Bifurcation diagrams with different delayed feedback at $\alpha = 3.1$: (a) $k_3 = 0$ (initial conditions are random between the intervals $[0 \ldots 0.1]$) and (b) $k_3 = 0$ (initial conditions are random between the intervals $[0 \ldots 0.1]$).

Figure 8. Eigenvalues of (14) evaluated on non-periodical stationary states at $\alpha = 3.1$ for (a) $k_4 = 0$ and (b) $k_3 = 0$.

2.1. Coupled oscillator mode with $k_2 = 0$

The coupled oscillator mode corresponds to the case in which each of the devices implements equation (7), emits $\uparrow \xi_n$ and receives $\downarrow \xi_n$ from the water. Since coupling is performed using an electric field with additive properties, the term $\downarrow \xi_n$ contains all signals from the other oscillators. In the case of $k_2 = 0$, the term $\downarrow \xi_n$ does not influence the dynamics of equation (7).

The dynamics of $\downarrow \xi_n$ is determined by two factors: different $g_j$ caused by the distance between devices and phase desynchronization of oscillators (see more in section 1).

To exemplify the effect of phase desynchronization, in figure 3(a) we plot the behavior of two versins of equation (7) with slightly different resolutions of step $n$ (this is performed in numerical simulation) and, in figure 3(b), the term $\sum_{j=1}^{m} g_j \xi_n$. The resolution of step $n$ in figure 3(b) is ten times higher than in figure 3(a), i.e. one step $n$ is represented by 10 points.

We can see that the maximum and minimum values are generated by $x_n(g^1 + g^2)$ and $x_n(g^1 - g^2)$. Since $g^j$ is primarily related to the distances between oscillators, the maximum value of the signal provides information about $\sum_{j=1}^{m} g^j$, that is, the sum of all distances between devices.

2.2. Coupled oscillator mode with $k_2 \neq 0$

For the case $k_2 \neq 0$ it is assumed that $k_1$ is calibrated so that the influence of its own variables in $\downarrow \xi_n$ is neglected. Thus, the system (8) has only one-way coupling with another variable, denoted as $x_{n+1}$. To summarize the changes caused by $g^j$ and by $k_2$, we consider only the coefficient $k_2$. For simplification, equal distances between devices are assumed. Non-linear effects related to asynchronous updates [52] and desynchronization are neglected. To analyze the periodical behavior, we consider the second iteration of the map, i.e. $f(f(x))$, which takes the following form for the system (8) with $m = 2$:

$$x_{n+1} = (x_n(2 - \alpha x_n - \alpha) + k_2 x_{n+1}) \times (2 - \alpha(x_n(2 - \alpha x_n - \alpha) + k_2 x_{n+1}) - \alpha) + k_2 (x_{n+1}(2 - \alpha x_{n+1} - \alpha) + k_2 x_{n+1}),$$

(9)

considering boundary conditions for $x_{n+1}$. This is a fourth-order system with 16 stationary states $x_{ni}$. 

6
Here, period-two stationary states from (8) become non-periodic and can be analyzed. The Jacobian of system (9) has two eigenvalues $\lambda_{1,2}$. From the stability conditions $|\lambda_{1,2}| \leq 1$ evaluated on each $x_n$, we can estimate which stationary states are stable. This step is done by combining analytical and numerical approaches. The stable distances between AUVs and without calibration of $k_2$ are also stable in this region of the parameter $k_2$, and points to spatial configuration as the sum of all distances between the AUVs. We see that the local value $\frac{\sum_{j=1}^{m} g_j x_j}{x_n}$ can be used to estimate these global values.

This curve demonstrates a clear dependence on $m$, even when the distances between robots are different. Since the synchronous update strategy is used, the term $\sum_{j=1}^{m} g_j x_j$ is calculated before evaluation of all $x_{n+1}$. As an alternative strategy, the number of devices can be fixed and the spatial distribution of the group can be explored. In figure 5(b) we plot the values of $\frac{\sum_{j=1}^{m} g_j x_j}{x_n}$ independent of $e$ from (10) at fixed $m = 5, 10, 15$. The value $e$ determines all $g_j$ from $\sum_{j=1}^{m} g_j x_j$ and points to spatial configuration as the sum of all distances between the AUVs. We see that the local value $\frac{\sum_{j=1}^{m} g_j x_j}{x_n}$ can be used to estimate these global values.

### 2.3. Electrical mirror mode

A swarm of AUVs in coupled oscillator mode with equation (7) can interact either with another swarm of AUVs or with the electrical mirror, as shown in figure 1. In the second case shown in figure 1(b), the AUV’s individual oscillations will be reflected by the mirror. Since the correlation between self- and reflected signals dramatically changes the dynamics, the swarm can collectively recognize this global situation.

The mirror device stores the common signal (i.e. the term $\sum_{j=1}^{m} g_j x_j$) at each time step and sends it back with delay $\gamma$. This delayed signal creates

$$x_{n+1} = x_n(2 - \alpha x_n - \alpha) + \sum_{j=1}^{m} g_j x_j + \sum_{j=1}^{m} \tilde{g}_j x_{n-\gamma},$$

(11)

where $\tilde{g}_j$ at the delayed terms represents the previous state of the system. For a system with no spatial changes between emitting devices we can set $\tilde{g}_j = g_j$.

The electrical mirror corresponds to the delayed feedback produced when the same signal with delayed $\gamma$ steps $x_{n-\gamma}$ is added to the system (for $\gamma = 0$ we add a non-delayed signal). Assuming the coefficient $k_2$ is calibrated and the self-signal is filtered out, we rewrite

![Figure 9](Image)

**Figure 9.** Temporal behavior of the system (11), $m = 10$, $g_j$ is defined by (10), $\epsilon = -0.01$. (a) Period-two motion without an electrical mirror; we observe global synchronization within the first ten steps. (b) Period-four motion with an electrical mirror $2 \sum_{j=1}^{m} \tilde{g}_j x_{n-\gamma}$; we observe global synchronization within the first 80 steps.
Figure 10. Experimental set-up with the potential mode electrodes (four electrodes) (a, b) and current mode electrodes (c) (electrodes are slightly removed from the water for demonstration purposes). Since the current mode is sensitive to optical and EMF excitations, this setup contains embedded 490 nm LED and EMV actuators for conducting corresponding experiments.

Figure 11. Examples of AUVs that use the development with electrical sensors: (a) the potential mode in AUVs from the CoCoRo project [21] and (b) the current mode in AUVs from the SubCULTron project [7, 53].

equation (6) in the following form:

\[ x_{n+1} = \alpha x_n (2 - \alpha x_n - \alpha) + k_3 x_{n-1} + k_4 x_{n-2}. \]  

Equation (12) can be considered as a model of time-delayed feedback for two emitting devices (one oscillator and one electrical mirror). Each of the delayed terms has a specific influence on the dynamics: they shift the corresponding bifurcation and to some extent ‘stretch’ or ‘squeeze’ the whole dynamics. In figure 6(a) we demonstrate the term \(0.45x_{n-1} - 0.45x_{n-2}\), which shifts the first period-doubling bifurcation backwards with regard to parameter \(\alpha\). Figure 6(b) shows the term \(-0.45x_{n-1} - 0.45x_{n-2}\).
which shifts the first bifurcation forwards and the second bifurcation backwards.

Since the delayed feedback produces behavior that is qualitatively different from the coupled mode, a robot can recognize whether another emitting device is a ‘robot’ or a ‘mirror’. The approach considered in section 2.2 uses different amplitudes of the period-two oscillation; here a transition to non-periodic or to period-four oscillation by means of delayed feedback is implemented. These two possibilities are shown in figure 7(a), where the values of $k_3$ in the range $[-1 \ldots -0.0333]$ at $\alpha = 3.1$ lead to non-periodic behavior and in figure 7(b) the values of $k_4$ in the range $[0 \ldots 0.3]$ at $k_2 = 0.138$ and $\alpha = 3.1$ lead to period-four behaviour (this case is sensitive to the choice of initial conditions).

To demonstrate the analysis of equation (12), we rewrite it as

$$x_{n+1} = x_n(2 - \alpha x_n - \alpha) + k_3 y_n + k_4 z_n,$$

$$y_{n+1} = x_n,$n+1 = y_n.$$

The non-periodical stationary states are $x_{st} = y_{st} = z_{st} = \{0, \frac{2 - 1 - k_3 + k_4}{\alpha}\}$. The Jacobian of the system (13) is given by

$$\begin{bmatrix}
2 - 2\alpha x_n - \alpha & k_3 & k_4 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}.$$  

We plot eigenvalues $|\lambda_{1,2,3}|$ for two cases of $k_4 = 0$ and $k_3 = 0$ in figure 8.

As we can see, the terms $k_3x_{n-1}$ and $k_4x_{n-2}$ both influence the stability of eigenvalues and thus can shift the first period-doubling bifurcation forwards and backwards. Similarly, we can investigate the linear stability of the second period-doubling bifurcation by using the second-iterated map obtained from (13). By varying $k_3$ and $k_4$, the first and second bifurcations can be simultaneously shifted, as shown in figure 6(b). Thus, the electrical mirror will produce period-four motions whereas coupled oscillators without an electrical mirror will produce only period-two motions. Based on this qualitatively different behavior, all AUVs can locally recognize a self-reflection.

In the case of several emitting devices (i.e. $m > 2$), we perform a numerical simulation of the system (11). Figure 9 demonstrates the temporal behavior of this system with and without the delayed term.

The numerical value of non-delayed and delayed terms $\sum_{j=1}^{m} g^j x_0$ is small; thus an amplification of the delayed signal is needed to shift the dynamics into the period-four region. In both cases global synchronization is observed.

3. Experimental setup

The experimental setup includes both the potential mode operation, shown in figures 2(a) and 11, and the current mode operation, shown in figure 2(b). Experiments were performed with two to eight electric sensors. Sending and receiving electrodes are placed in plastic tubes on a movable platform in an aquarium (see figure 10(b) for the potential mode and figure 10(c) for the current mode). Since the current mode is sensitive to optical and electromagnetic field (EMF) excitations [17], this setup includes embedded 490 nm LEDs and EMV drivers for conducting corresponding experiments. Electrodes are connected to electronic boards with analog filters, amplifiers and DAC/ADC. During all experiments, the water in the aquarium was grounded and all electronic devices, for example the water pump, were switched off. The laptops used for data collection ran on batteries for each of the emitting/receiving devices separately. Both potential and current mode systems have been implemented in AUV’s from CoCoRo [21] and subCULTron [7, 53] projects (shown in figures 11(a) and (b)) and several other developments [20, 24], such as active electric mirrors (see figure 26).

Two receiving electrodes in potential mode are connected to a high-pass filter with a cut-off frequency of 16 Hz (figure 12).

The high-pass filter is needed to filter any unwanted constant electrical fields that may be present in the experimental environment. The output of these filters is connected to the differential inputs of an instrumental OpAmp. With a supply voltage of $\pm 5 \text{ V}$, the amplifier is able to process input voltages between $\pm 10 \text{ V}$ without saturation. The emitting electrodes are connected to the outputs of a two-channel DAC. This DAC can generate an analog voltage between $0 \text{ V}$ and $+5 \text{ V}$ on both outputs independently, which leads to a relative voltage of $\pm 5 \text{ V}$ between the two electrodes. Additionally the outputs can be deactivated (high impedance) to reduce unwanted influence on the ADC.

For mapping between the values of $x_n$ and the output voltage $\pm 5 \text{ V}$ we use the coefficient $k_{DAC}$, calculated from the condition $|x_n|k_{DAC} < 2048$ for the maximum positive and negative values of $x_n$. In contrast to numerical analysis of coupled maps, where all maps are iterated sequentially, real experimental hardware iterates all maps in parallel. This imposes several conditions on the timing of $x, \xi_0$, and $\xi_n$. Since all variables of the right-hand side of
equation (7) are of time $n$, we first write $x$ into the DAC, read ↓ $\xi$ from the ADC and then iterate the map. The measured output amplitudes of the signals for the period-two ($x_0 = 0.08714, -0.11940$) and chaotic ($x_0 = [0.25, ..., -0.75]$) modes are shown in figures 13(a) and (b). We measured the time needed to raise the level of voltage, as shown in figure 13(c). Switching between ±0.7 V takes about 50 µs. The minimum delay from the software and serial peripheral interface communication for switching between two levels of voltage is about 20 µs (see figure 13(d)).

4. Experiments

In this section we demonstrate five experiments that target different aspects of the proposed approach: estimation of interaction ranges, collective coordination and sensing, the electrical mirror and the current mode. All of them allows better understanding of the system in different applications.

4.1. Part 1: dependency between emitted and received signals

In the first project we investigated the dependency between ↑$\xi_i$ and ↓$\xi_i$ in potential mode by varying the distance $d$ between two oscillating devices (see figure 14). The first device sends the generated signal ↑$\xi_i$ using equation (7) without receiving ↓$\xi_i$; the second device receives ↓$\xi_i$ without sending ↑$\xi_i$.

We expect to see two different signal values depending on obstacles [14]. To explore this effect, two experiments are performed: one with the electrodes installed on the glass wand of the aquarium (figure 10(b)) and one with the electrodes placed in the middle of the aquarium (figure 14), where the distance to the closest obstacle is twice the maximum distance between electrodes. In both experiments the electrodes are placed at the same depth. We used a period-two behavior at $\alpha = 3.1$, mapped to the output voltage of $-4.7 \text{ V} \ldots +4.4 \text{ V}$. The measured level of noise is shown in figure 16(a). The maximum amplitude of noise varied between $+0.03 \text{ V}$ and $-0.05 \text{ V}$; we observed a smaller level of noise.
(+0.01 V and −0.02 V) in the middle of the aquarium and a bias of the average value to negative (around −0.007 V). This bias arises from the non-symmetrical amplification of the signal from the electrodes.

Figure 15 shows a plot of experimental data for the first position of the electrodes. Due to greater noise, we considered this as the worst case for communication. The distance between electrodes was increased from 5 cm to 32.5 cm in steps of 2.5 cm. At d < 5 cm, the input voltage is around the saturation level of the amplifier. Values up to 25 cm are above the noise level. The voltage values for d > 25 cm are within the noise
areas and need statistical processing. Thus, $d = 5–25$ cm can be considered as the working range of the system. At a distance between emitting electrodes of $7.5$ cm, the communication distance is about $3.3$ times the body length.

Figure 16 shows the decay of periodic signals for all measured values of $d$ for positions 1 and 2. We observe a larger variation in the signal for position 1; moreover signal decay is higher in the middle of the aquarium.

4.2. Part 2: calibration of the coefficient $k_1$

The need for calibration is explained by expression (3). The self-signal in the term $\xi_w^o - k_1 \xi_w^o$ should be compensated, so that $\xi_w^o - k_1 \xi_w^o = 0$ in the absence of any other emitting devices. In this experiment the position of electrodes is first fixed in relation to the level of water and $k_1$ is calculated for this setup. For later experiments, the device autocalibrates.

In figure 17(a) we plot $\xi_w^o$, which is calculated directly in the microcontroller. We observe different values of $k_1$ in figure 17(a) for each type of behavior. For period-two motion of the system (7) at $\alpha = 3.1$ we find an oscillating behavior of $\xi_w^o$ between $k_1^p = 0.13$ for positive and $k_1^n = 0.03$ for negative values.
Figure 20. Different stages of phase desynchronization between emitting devices.

Figure 21. Measuring the intensity of the electrical field; both emitting devices $D_1$ and $D_3$ run equation (7) ($\alpha = 3.1, k_2 = 0$, $d_{13} = 15$ cm). (a) 'Position: left'—the measuring device is close to the left-hand emitting device (0.4 is added to all values); 'position: center'—the measuring device is between the two emitting devices (0.2 is added to all values); 'position: right'—the measuring device is close to the right-hand emitting device. The sampling frequency of the measuring device is 0.5 kHz. (b) Region 'A' with a higher sampling frequency of 20 kHz.

Figure 22. Desynchronization effects at $\alpha = 3.1$ ($k_2 = -0.1, x_i = 0.1, k_1$ is autocalibrated before experiments, $d = 15$ cm, duration of pulses 10 ms): (a) device $D_1$; (b) device $D_2$. 
Figure 23. Measuring distances between emitting devices by measuring maximum and minimum amplitudes ($\alpha = 3.1, k_2 = 0$): (a) 10 cm between devices; (b) 5 cm between devices.

Figure 24. Global synchronization effects ($\alpha = 3.1, k_2 = 0.7, x_1 = 0.1, k_1$ is autocalibrated before each experiment; 0.5 is added to values from $D_2$, 1.0 is added to values from $D_1$): (a) behavior of coupled oscillators for $d_{12} = d_{23} = d_{13} = 15$; (b) behavior of coupled oscillators for $d_{12} = d_{23} = d_{13} = 10$.

of $x_n$. This difference can be explained by nonsymmetrical gain in the operational amplifier. Thus, in expression (3) we must consider the sign of $x_n$. Iteration from such initial conditions, close to the value of the long-term dynamics, is recommended. Moreover, it also depends on the delay between sending a value to the DAC and reading the voltage from the ADC. Figure 17(b) shows the values of $\xi_n - k_1 \xi_n$ with the calculated coefficient $k_1$ for $\alpha = 3.1$. We observe the appearance of structures for $\alpha > 3.4$, which will change the dynamics of original system in this area. Figure 18 demonstrates two bifurcation diagrams with the term $\pm 0.5(\xi_n - k_1 \xi_n)$ and changes in the high-periodical and chaotic regions of the parameter $\alpha$.

However, for the calculated values of $\alpha = 3.1$, we observe a compensation of the self-signal up to the level of noise (the maximum amplitude of this term depends on calibration, it is about $\pm 5 \times 10^{-3}$) (see figure 17(b)). This allows working with even larger values of $k_2$ in further experiments.

There are two strategies for using the coefficient $k_1$. Since it is calculated only for some regions of $x_n$, values outside this region will increase noise and can change the dynamics of the system (see figure 18(b)). Thus, the first strategy is to limit the working region of $x_n$. The second strategy is to calibrate $k_1$ each time with values taken from, for example, a look-up table. In further experiments the coefficient $k_1$ is autocalibrated each time the device is switched on.

4.3. Part 3: coordination and sensing in coupled oscillator mode

For tests of the coupled oscillator mode, all devices are programmed with the same program and $x_n$ values are read from them. We performed three experiments: (a) synchronization and desynchronization of oscillations, (b) different distances between oscillating devices, (c) different numbers of oscillating devices. The experimental setup for these experiments is shown in figure 19. All three devices $D_1$–$D_3$ were placed either in a triangle or in a line at distance of $d_{12}, d_{23}, d_{13}$ from each other.
Before each experiment, all devices self-calibrate to estimate values of the coefficient $k_1$.

(a) Synchronization and desynchronization of oscillations. A desynchronization effect appears for two reasons: first, not all the main clocks of the devices have exactly the same frequency; second, the positive and negative pulses initially overlap, since the oscillators start asynchronously. Potentially, desynchronization of coupled oscillators is undesirable; however, it can lead to several interesting effects. In figure 20 we show two oscillograms of three devices, running equation (7) at $\alpha = 3.1$ and a pulse of duration 2 ms.

We changed the clock divider of each device to exhibit an approximately 0.2 Hz frequency shift. Thus, the periods of oscillations in different devices can exactly overlap, can have inverse phases (see figure 20(a)) and can be shifted differently from each other (see figure 20(b)).

To confirm model (8) and our ideas regarding adding and subtracting amplitudes in asynchronous mode, two devices ran equation (7) with $k_2 = 0$ (that is, without reading values from the water). The third device reads the ADC and measures the intensity of the electric field without emitting functionality. We performed three measurements, shown in figure 21(a).

As expected, a spatially dependent addition and subtraction of phases is observed. Due to a low sampling frequency in figure 21(a), we see a step-wise phase exchange. Figure 21(b) shows region ‘A’ with a higher sampling rate, where addition and subtraction of phases is readily visible. The decrease of potential (region ‘B’) in figure 21(b) is explained by the high-pass filter. Another interesting effect is also visible in figure 21(a); the left and right emitting devices have slightly different waveforms, resulting in symmetry breaking when approaching the left or right sides of aquarium. Based on this effect, a robot can distinguish between emitting devices and thus navigate.

For $k_2 \neq 0$ phase desynchronization appears as a switch between two amplitudes of period-two motion. To make this effect more visible, we increase the duration of pulses to 10 ms and plot the values $\xi_n$ from the water for two oscillating devices, as shown in figure 22.

Increasing amplitude in the first device and, at the same time, decreasing amplitude in the second device are clearly visible.

(b) Different distances between oscillating devices. To measure distances between emitting devices, we can apply two approaches: first, one based on a...
phase desynchronization for \( k_2 = 0 \) and, second, one based on measuring the amplitude of \( x_n \).

Both approaches worked well in the test environment. Figure 23 shows the difference between the maximum and minimum of the amplitudes for \( k_2 = 0 \).

In contrast to the measurement shown in figure 21, in this experiment we measured the intensity of the field simultaneously with the self-emitted signals. The resolution of this approach is primarily defined by measurement accuracy and the amplification factor of input electronics.

To demonstrate the approach with \( k_2 \neq 0 \), we used a strong positive coupling coefficient \( k_2 \), which leads to global synchronization and changes in qualitative behavior. Three experiments for \( d_{12} = d_{23} = d_{13} = 25 \) cm (period-two behavior), for \( d_{12} = d_{23} = d_{13} = 15 \) and for \( d_{12} = d_{23} = d_{13} = 10 \) cm were performed (see figure 24).

We clearly see a locking of phases and amplitudes for all three oscillators. The distance between devices can be obtained by measuring the pulse frequency.

(c) Different numbers of oscillating devices. As indicated in section 2.2, either the number of devices or the distances between them can be measured. In this case we were interested in the number of devices and repeated the experiment with a strong positive coupling coefficient \( k_2 \). However, in this experiment all oscillating devices were placed in a line, at distances of \( d_{13} = 22 \) cm and \( d_{12} = d_{23} = 11 \) cm. Moreover, we slightly unbalanced oscillators 1 and 2 by varying the coefficient \( k_1 \). First two close oscillators 1 and 2 were turned on and became synchronized (see figure 25(a)). Oscillators 1 and 3, at a distance of 22 cm, were only slightly perturbed in the period-two behavior. After this, we turned on oscillator 3. The behavior of all devices changed
to a mode where synchronization and desynchronization phases exchange (see figure 25(b)).

Generally, when all devices have different oscillation parameters, caused for example by self-calibration of $k_1$ or by different distances between devices, we observe a different period of oscillation, as shown in figure 25(a). This leads to an appearance of temporal desynchronization, after which all devices resynchronize. This mode can be used for detection of a group of underwater devices, even at the furthest range of sensitivity. Moreover, by measuring the degree of desynchronization, it is possible to draw conclusions about the spatial distribution of devices.

4.4. Part 4: electrical mirror mode

For experiments in electrical mirror mode, one or two devices oscillated and one device represented the electrical mirror implemented as an independent system (see figure 26). All devices had the same time interval (2 ms) as in other experiments. The mirror device first sensed the electrical field without sending its own signal and stored the received values in an internal array. After this, the value received from the previous step (generally the value $x_{n-\gamma}$, where $\gamma$ is the time delay) was sent to the emitting electrodes. After 2 ms, the emitting electrodes were switched off, the value of the electrical field read by the ADC and the cycle repeated. For this experiment it is necessary to calibrate: (a) a proportional coefficient between the received and emitted values, first in order not to saturate the amplifiers and second to provide a possible large interaction radius; (b) the time interval when the emitting electrodes are switched on and off.

In the first experiment we intended to confirm the model (12) from section 2.3 regarding time-delayed feedback with two devices (one oscillator and one electrical mirror). Figure 27 shows the values received from the oscillating device.

The oscillating device implements equation (7). The mirroring device returns $-x_{n-1}$ each time, whereas we change $k_2 = 0.5$ and $k_2 = -0.5$ in the feedback term of the oscillating device. We observed a characteristic behavior in the delayed feedback (see also figure 8(a)) when the signal was compensated to zero (as shown in figure 27(a)) or when the amplitude of the period-two motion was increased (as shown in figure 27(b)) for positive and negative values of the
feedback signal, respectively. The values of the coefficient $k_3$ in equation (12) differ from those shown here, due to amplifications, losses in the water/mirror and other factors; they need to be experimentally calibrated.

In figure 28 two devices $D_2$ and $D_3$ oscillate and $D_1$ acts as an electrical mirror. First, $D_2$ and $D_3$ are synchronized, as shown in figure 28(a), then the mirror device $D_1$ is turned on (see figure 28(b)).

We observed a crash of global synchronization and a change in the oscillating pattern. Comparing the results with figure 24, we see that the global synchronization pattern enables robots to recognize whether the signal is emitted by another swarm or their own signal is reflected by the mirror.

The system is also sensitive to a non-delayed signal. In figure 29 we demonstrate the case where the mirroring device returns $-x_n$ and the oscillating device has the coefficient $k_2 = 0.5$. Oscillations are sensitive to the presence of this feedback signal even at the boundary of the sensing range.

4.5. Part 5: current mode sensor
Experiments in the current mode are aimed at demonstrating coupling by physical means (by interference of electric fields in water) and sensitivity of common electric fields to placement and movement of dipoles and dielectric objects in such a common field. The general setup is shown in figure 30 and includes eight electric dipoles combined in two

![Graph](image1)

**Figure 31.** Electrochemical dynamics of a single dipole (the last right dipole from figure 30) for (a) movement of dielectric objects and other dipoles in a sensing area of the group and (b) approaching other dipoles with different oscillating frequencies and appearance of interference patterns.
groups on movable holders. Each dipole has its own fixed oscillating frequency between 70 and 450 Hz. Since the coupling part of equation (5) is implemented in a ‘physical way’ (similar to the case $k_2 = 0$), the current mode is simpler than the potential mode; however, it is also less controllable. The current between electrodes at the applied potential is calculated as impedance (in RMS form for AC signals). Sensing is performed by the last pair of dipoles, as shown in figure 31. The size of dipole is related to the dipole–dipole distance as 1:8, to the size of the whole group as 1:24 and to size of the sensing area as 1:40. Due to its larger sensing area, this setup has a smaller dipole size and is also used to study electrochemical and optical effects in fluidic media [16, 17] and biological organisms [26], and includes LED and EMF actuators.

Since the configuration of a common electric field is sensitive to the positions of dipoles and the geometry of field with included obstacles, this can be used for sensing purposes. To demonstrate this, a plexiglass object of about twice the dipole size in figure 30 is placed in random positions (the time in each position is about 3 min to demonstrate stable transient dynamics). As shown in figure 31(a), changes of position are well detectable in a common electric field, sensed even by the most distant dipoles of this coupled system. A localization task was not pursued in this experiment; however, it can be targeted if oscillating frequencies of all dipoles are different and the system is calibrated (similar to interference patterns in figure 31(b)). Figure 31(a) demonstrates electrochemical dynamics measured in cases where dipoles have different positions in the sensing area. The influence of dipole displacements is larger than that of dielectric objects; this allows separation of signal levels for group-external objects and for group-internal dipoles.

All dipoles oscillate at different frequencies and the interference pattern of the electric field is specific to each of these dipoles. Figure 31(b) demonstrates examples of such patterns, detectable on the level of a single dipole. This approach can be used for different purposes, for example for identification of beacons and AUVs.

5. Discussion

This work has demonstrated five applications of coupling by electric field oscillators in potential and current modes of operation in fluidic media. The original inspiration for this work came from weakly electric fish, while further development of the current mode sensor led to the discovery of multiple parallels with electrochemical impedance spectroscopy, used in physical chemistry. Therefore, this approach combines typical elements of both collective (swarm) and ionic (molecular) systems. Such parallels explain the wide application field from macro biohybrid and bioinspired systems, such as robot–robot [20], robot–fish [23] and robot–plant [26] interactions, or micro biohybrid systems such as ionic interfaces with organic tissues [25], molecular species [17] or microorganisms [16]. This work mostly focuses on macro applications from the domain of AUV swarm systems and targets collective mechanisms of perception and coordination [18, 19].

The common electric field in water presents a spatially excited medium that is sensitive to the distribution of oscillating dipoles, their orientation, the presence of obstacles and individual dynamics of dipoles. These global (collective) parameters can be sensed on a local (individual) level in the form of synchronization patterns or changes in amplitude/phase of signals and used for collective sensing and coordination purposes. Challenges of transition from collective to individual levels are resolved by using analytical methods from nonlinear dynamics and complexity science, which remains the same for micro and macro applications. Implemented hardware and software solutions have demonstrated the effectiveness of such an approach.

The intended increase in individual sensing range is achieved due to using a larger interaction range for sensing purposes. For instance, sensing is increased from one to two times body length [14] (the size of electric dipole to the distance between electrodes) to three to four times body length with ±5 V driven hardware in potential mode in fresh water. Using step-up converters for, for example, ±15 V output waveforms enables further extension up to five times body length in potential mode in a large-scale setup. The current mode sensor is more sensitive, we observed an interaction range of more than ten times body length with high-resolution 24 bit ADC sampling in fresh water in a small-scale setup. Obviously, practical applications will require a trade-off between the properties of water (fresh, with specific salinity), excitation potential (e.g. ±5 V, ±15 V), operational mode (potential, current, passive), setup scales (small, large) and sensing/interaction ranges. Various experiments in different projects have demonstrated a nonlinear dependency between these parameters and required several test runs.

6. Conclusion

This work has explored the behavior of nonlinear oscillators, coupled through an electric field in water. We have demonstrated different schemes for potential and current sensing, where two effects have an impact on the behavior of nonlinear oscillators: phase desynchronization of pulses and global synchronization of amplitudes. A combination of both effects enables sensing of either distances or the number of emitting devices. Analysis of temporal patterns allows the devices to draw more complex conclusions about the
spatial configurations of AUVs or dielectric objects in the sensing area. Moreover, temporal synchronization of amplitudes makes it possible to coordinate activities of AUVs without global communication.

Using one device as an electric mirror to reflect the received signals can change the qualitative behavior of the coupled system. The main argument for these experiments is that by comparing the self- and received signals, each AUV can recognize its membership in the group. This approach underlies more complex cognitive strategies of group-based identification and collective recognition of the ‘self’ and ‘nonself’. Experiments demonstrated a good correlation between numerical analysis of corresponding equations with their dynamics executed on micro-controllers of real AUVs. Expected analytical results have been confirmed in experiments; the results can be used for designing collective capabilities of larger underwater systems.

Further research in biohybrid systems clearly trends toward micro applications on molecular species and ionic interfaces to bio-objects. Here, current mode sensing has advantages from electrochemistry and impedance spectroscopy and, in addition to the design of collective properties of swarm-like systems, enables specific ionic analysis of fluidic media.

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Data availability statement

No new data were created or analysed in this study.

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