Robust Fault Diagnosis for Pipelines

Lizeth Torres

Abstract—This paper presents the design of a diagnosis system for the detection, identification and reconstruction of faults in pipelines. The design of such diagnosis system is based on redundant relations and nonlinear observers, taking into account faults in sensors, damages in pumps, and unknown extractions. The proposed algorithm is developed based on a model described by nonlinear equations of the fluid behavior in a pipeline, considering the principles of conservation of mass and momentum. In order to distinguish among different types of faults and to reconstruct their behavior, the diagnosis system operates in stages. The first stage called detection & fault isolation aims to isolate a fault symptom with a set of redundant relations deduced from the analysis of the model in nominal conditions and assuming measurements of the standard variables at the extremes of the pipeline. In the second stage named fault reconstruction, nonlinear observation algorithms estimate the temporal evolution of the isolated fault. The complete diagnosis system is validated through a series of experiments in a hydraulic pilot pipeline of 200 [m].

I. INTRODUCTION

The principal purpose of a monitoring automatic system in real time is to locate, as quickly as possible, the presence of faults with the minimal instrumentation and cost. Different physical principles and tools have been considered to design automated systems of aqueducts, pipelines, pumping systems, etc. [1]. Usually in pipeline networks, the anomalies to locate in real time are: faults in valves and pumps, leaks in the pipeline body, obstructions, air bubbles, cracks, corrosion, etc. If the pipelines are underground or underwater, in addition to the inability to suspend the process operation, the fault diagnosis becomes an arduous task. Therefore, the diagnosis should be realized considering the dynamical phenomena associated with the fluid. For the detection and location of leaks in pipelines, various models of the static behavior of the fluid have been used [2]. Recently, these models have been successfully extended to the case of pipelines with nonuniform topographic profiles [3]. Despite their simplicity, the main disadvantage of using these models is that they can not be used when the fluid does not operate around static conditions. For distribution networks, leak detection analysis using time-frequency signals as wavelets have been reported [4], as well as advanced genetic algorithms [5]. For the automatic monitoring of pipelines is mandatory the continuous improvement of issues related to the hardware and software. Sensors should be faster and more accurate, and the localization algorithms must be able to discern among a great number of fault scenarios, i.e., the monitoring systems should include fault alarms for sensors and pump systems to become intelligent monitoring systems for pipeline networks [6]. This fact has motivated the development of this work where the main contribution is a diagnosis system, which considers faults in measuring instruments, damages in pumps and pipeline leaks. The system uses an algorithm with the ability to distinguish among different faults in real time by means of five residuals with faults signatures for each event, and a bank of high gain observers to reconstruct the evolution of the identified fault.

This paper is organized as follows: Section 2 exposes the fluid model in nominal and fault conditions used for the design of the fault diagnosis system; Section 3 contains the overall description of the proposed diagnosis system for five diverse faults. Finally, Sections 4 and 5 show the experimental validation of the proposed approach and some conclusions about it.

II. DYNAMICAL MODEL OF THE FLUID

Assuming convective changes in velocity to be negligible, constant liquid density and constant pipe cross-sectional area, the momentum and continuity equations governing the dynamics of the fluid in a horizontal pipeline can be expressed as [7].

\[
\begin{align*}
\frac{\partial Q(z,t)}{\partial t} + a_1 \frac{\partial H(z,t)}{\partial z} + \mu Q(z,t)|Q(z,t)| &= 0 \\
\frac{\partial H(z,t)}{\partial t} + a_2 \frac{\partial Q(z,t)}{\partial z} &= 0 
\end{align*}
\]

where \( (z,t) \in (0, L) \times (0, \infty) \) are the time \([s]\) and space \([m]\) coordinates respectively, \( L \) is the length of the pipe, \( H(z,t) \) is the pressure head \([m]\) and \( Q(z,t) \) is the flow rate \([m^3/s]\). The physical parameters of the pipeline are

\[ a_1 = gA_r, a_2 = \frac{b^2}{gA_r}, \mu = \frac{f}{2\phi A_r} \]

where \( b \) is the wave speed in the fluid \([m/s]\), \( g \) is the gravitational acceleration \([m/s^2]\), \( A_r \) is the cross-sectional area of the pipe \([m^2]\), \( \phi \) is the inside diameter of the pipe \([m]\), and \( f \) is the Darcy-Weichbach friction factor.

The presence of a leak in a given position \( z_{f_i} \) must be handled as a boundary condition for the system (1) with a flow loss:

\[ Q_{f_i}(t) = \sigma_i \sqrt{H_{f_i}(z_{f_i}, t)} \]

where \( \sigma_i = \sqrt{2g A_{f_i} C_{f_i}} > 0, A_{f_i} \) is the sectional area of the leak, \( C_{f_i} \) the discharge coefficient and \( H_{f_i} \) is the pressure at the leak position \( z_{f_i} \).
A. Nonlinear finite model

To obtain a model of finite dimension from the set of equations (1) is necessary to define their boundary and initial conditions. The boundary conditions express the temporal profiles of $Q(z, t)$ and $H(z, t)$ in the spatial coordinates $z = 0$ and $z = L$. Apart from, the initial conditions express the spatial profiles of $Q(z, t)$ and $H(z, t)$ at the instant $(t = 0)$ and they are denoted here as: $H(z, 0) = H^0(z), \ Q(z, 0) = Q^0(z)$. As boundary conditions, the following Dirichlet conditions can be used:

- Upstream pressure head: $H(0, t) = H_{in}(t)$
- Downstream pressure head: $H(L, t) = H_{out}(t)$
- Upstream flow rate: $Q(0, t) = Q_{in}(t)$
- Downstream pressure head: $Q(L, t) = Q_{out}(t)$

System (1) with included leaks can be discretized using the Finite Difference Method in $n_x$ sections as follows [7]:

$$
\dot{Q}_i = \frac{a_1}{\Delta x_i} (Q_i - Q_{i+1}) - \mu Q_i |Q_i|; \quad (3)
$$

$$
\dot{H}_{i+1} = \frac{a_2}{\Delta x_i} (Q_i - Q_{i+1} - \sigma_i \sqrt{H_{i+1}}); \quad (4)
$$

with $i = 1, \ldots, n_x$.

III. DIAGNOSIS SYSTEM FOR A SET OF FAULTS IN A PIPELINE

A Fault Detection and Isolation (FDI) procedure involve three tasks [8]: (i) Fault detection: to take a binary decision on the status of the system according to the nominal operating conditions. (ii) Fault isolation: to determine the type of fault from a symptom. (iii) Fault identification: to estimate the size and type of fault, including its temporal evolution. For the three tasks in this work the generic representation of concentrated parameters

$$
\dot{x}(t) = f(x(t), u(t), \theta(t)) \quad (5)
$$

$$
y = h(x(t))
$$

is assumed, where $x(t) \in \mathbb{R}^n$ are the states associated to the flows and pressures of the fluid along the pipeline, $u(t) \in \mathbb{R}^m$ are the exogenous inputs or control signals, $y(t) \in \mathbb{R}^p$ represents known variables of the fluid and $\theta(t) \in \mathbb{R}^{n_\theta}$ represents the physical parameters of the pipeline.

By means of the data of the process and model (5) the detection task consists to generate a residual $r(t)$ with the following property [9]:

$$
r(t) = \begin{cases} 0 & \text{normal condition} \\ \neq 0 & \text{abnormal condition} \end{cases} \quad (6)
$$

In particular for the residual generation task, let consider the analytical model of the fluid (3)-(4), as well as the available measurements of flows and pressures at the ends of the pipeline $(H_1 = H_{in}, \ H_{n+1} = H_{out}, \ Q_1 = Q_{in}$ and $Q_n = Q_{out})$. For the design of the robust fault diagnosis the following fault scenarios are considered:

- E1: Fault in the upstream flow rate sensor, $\delta Q_{in}$.
- E2: Fault in the downstream flow rate sensor $\delta Q_{out}$.
- E3: Fault in the pumping system corresponding to an actuator fault $\delta H_{in}$.
- E4: Abnormal condition in the downstream storage system associated with additive faults $\delta H_{out}$ in the model.
- E5: Leak in the pipeline with unknown position and outflow $Q_f$.

Thus, the diagnosis system presented here is composed of two tasks, the detection & isolation step followed by a specific reconstruction stage (which is activated according to the isolated fault). The first one counts on five analytical redundancy relations to detect and isolate the faults with a different signature for each scenario. This stage simplifies the reconstruction of the faults by a set of independent nonlinear observers of dimension lower than the global model with each fault.

A. Fault detection and isolation system

To obtain residuals that fulfill the property (6) for the fault scenarios (E1-E5), the model (3)-(4) is considered. Since there is not a solution to the design of residuals sensitive only to one fault scenario and robust to the rest, residual patterns assuming diverse subsets of faults and disturbances must be generated.

The simplest and basic residual is based on the mass balance and is generated from the difference between the flow rates

$$
r_1 = Q_{in} - Q_{out} \quad (7)
$$

which is zero when there is not an unknown fluid extraction or the sensors of the variables $Q_{in}$ and $Q_{out}$ operates in normal conditions and becomes nonzero when there are leaks in the system or any flow rate measurement is erroneous, i.e. $r_1$ deviates from zero for the scenarios E1, E2 and E5 and remain around zero for E2 and E4.

Considering that a pipeline of length $L$ operates in steady state, from (3)-(4), the analytical redundant relation

$$
r_2 = -\mu Q_{in} |Q_{in}| + \frac{a_1}{L}(H_{in} - H_{out}) \quad (8)
$$

is zero in normal conditions and becomes nonzero for four fault scenarios: E1, E3, E4 and E5. Since (8) does not depend on the measurement of the variable $Q_{out}$, this relation is robust to the fault in the measurement of the downstream flow. By symmetry of the model with respect to the flows the relation

$$
r_3 = -\mu Q_{out} |Q_{out}| + \frac{a_1}{L}(H_{in} - H_{out}) \quad (9)
$$

is zero when the pipeline is free of leaks, and the pumping system, pressure data and downstream flow are correct. On the contrary, the residual deviates from zero when there is a leak or scenarios E2, E3 and E4 occur. Since $r_3$ is independent from $Q_{in}$, this is only insensible to the upstream flow rate.
With these three residuals (7-9), the following Fault Signature Matrix (FSM) is obtained

|    | E1 | E2 | E3 | E4 | E5 |
|----|----|----|----|----|----|
| r1 | ●  | ●  | ●  | ●  | ●  |
| r2 | ●  | ●  | ●  | ●  | ●  |
| r3 | ●  | ●  | ●  | ●  | ●  |

where ● means a residual different from zero for the abnormality indicated at the top of each column. Thus, only the presence of leaks and faults in the flowmeters can be isolated assuming the presence of a unique fault scenario. Faults in the pumping system or pressure sensors (δH_{in} and δH_{out}) can be detected with (8) and (9) but not isolate, because both residuals r_{2} and r_{3} are sensitive to both faults.

By adding the new residuals to the FSM (10), this becomes

\[
\begin{align*}
\dot{Q}_{in} &= \mu Q_{in} |Q_{in}| + \frac{a_{1}}{0.5L}(H_{in} - H_{m}) \\
\dot{H}_{m} &= \frac{a_{2}}{0.5L}(Q_{in} - Q_{out}) \\
\dot{Q}_{out} &= \mu Q_{out} |Q_{2}| + \frac{a_{1}}{0.5L}(H_{m} - H_{out})
\end{align*}
\]

and eliminating the unknown pressure at the middle of the line H_{m}, the following analytical relation is obtained

\[
r_{4} = Q_{in} + \mu Q_{in} |Q_{in}| + \frac{\alpha_{1}}{\alpha_{2}} \int (Q_{in} - Q_{out})dt - \alpha_{2} H_{in}
\]

with \(\alpha_{1} = \frac{\alpha_{1}}{0.25L}\) and \(\alpha_{2} = \frac{\alpha_{1}}{0.5L}\). This relation is zero if there are deviations in the downstream pressure head H_{out} and then allows to isolate faults in the actuator and pressure sensors. It is to note that the implementation of the residual require to estimate on-line the initial condition of the integral or to generate the residual as a second order differential equation. By symmetry of the structure, the following analytical relation is obtained

\[
r_{5} = Q_{out} + \mu Q_{out} |Q_{out}| - \frac{\alpha_{1}}{\alpha_{2}} \int (Q_{in} - Q_{out})dt + \alpha_{2} H_{out}
\]

which does not depend on the upstream pressure head H_{in}.

By adding the new residuals to the FSM (10), this becomes

|    | E1 | E2 | E3 | E4 | E5 |
|----|----|----|----|----|----|
| r1 | ●  | ●  | ●  | ●  | ●  |
| r2 | ●  | ●  | ●  | ●  | ●  |
| r3 | ●  | ●  | ●  | ●  | ●  |
| r4 | ●  | ●  | ●  | ●  | ●  |
| r5 | ●  | ●  | ●  | ●  | ●  |

From this matrix, one can observe that a different pattern of the residuals exists for each scenario, such that all faults are isolable.

B. Fault reconstruction

The second stage of the diagnosis system is the reconstruction of the five concerned faults. To design this stage, specific models of the form (5) with the corresponding fault are considered, where the parameters \(\theta\) associated to the faults are additional states according to the extended vector

\[
x_{e}(t) \rightarrow \begin{bmatrix} x(t) \\ \theta(t) \end{bmatrix}
\]

and they can be estimated via nonlinear observers. In this work, high gain observers are designed for the aim. The start point of the algorithm design consists on transforming the finite model (3)-(4), into the form

\[
f(x(t), u(t), \theta(t)) = F(x_{e}(t), u(t)) + Gy(t),
\]

to obtain the following system

\[
\dot{x}_{e}(t) = F(x_{e}(t), u(t)) + G(y(t))
\]

where the input vector is given by \(u(t) = [u_{1}(t), u_{2}(t)]^{T} = [H_{in}(t), H_{out}(t)]^{T}\), composed by the pressure heads at the ends of the pipeline. In case of constant pressure heads, \(u(t) = u_{0}\) is assumed. For the design of the high gain observers, the flow rates at the ends of the pipeline are considered to be measurable, i.e.:

\[
y = h(x_{e}) = \begin{bmatrix} Q_{in} \\ Q_{out} \end{bmatrix}^{T}
\]

It is to remark that both measurements are not required in all the reconstruction cases. Thus, the used measurements are defined for each fault scenario.

1) Reconstruction of a leak: The aim of this task is the reconstruction of the leak position and its coefficient. For the conception of the observer, the model (3)-(4) must be fixed (at least) with \(n_{s} = 2\) to represent two sections with different flow rates: the flow before the leak point, and the flow rate after it. Therefore, the following equations set is proposed:

\[
\begin{align*}
\dot{Q}_{in} &= \mu Q_{in} |Q_{in}| + \frac{a_{1}}{\Delta z_{1}} (H_{in} - H_{f}) \\
\dot{H}_{f} &= \frac{a_{2}}{\Delta z_{1}} (Q_{in} - Q_{out} - \sigma_{f} \sqrt{H_{f}}) \\
\dot{Q}_{out} &= \mu Q_{out} |Q_{out}| + \frac{a_{1}}{\Delta z_{2}} (H_{f} - H_{out})
\end{align*}
\]

where the leak position is associated with the size of the first section \(\Delta z_{1} \in [0, L]\), whereas the second section has a size of \(\Delta z_{2} = L - \Delta z_{1}\).

To complete the model for parameter estimation purposes, the state vector of system (17), \(x = \begin{bmatrix} Q_{in} \\ H_{f} \\ Q_{out} \end{bmatrix}^{T}\), must be extended with two additional states corresponding to the parameters associated (position and leak coefficient) to the unknown extraction

\[
\theta(t) = \begin{bmatrix} \Delta z_{1} \\ \sigma_{f} \end{bmatrix}^{T}
\]

setting up the augmented vector (14). Taking into account that the parameters are constant, their derivatives are equalised to zero, thus

\[
\dot{\Delta z}_{1} = 0 \\
\dot{\sigma}_{f} = 0
\]

Following the procedure design of a high gain observer (described in [10], [11], [12]) for the system with the state vector

\[
x_{e} = \begin{bmatrix} Q_{in} \\ H_{f} \\ Q_{out} \\ \Delta z_{1} \\ \sigma_{f} \end{bmatrix}^{T}
\]
the following state observer is obtained:

\[
\begin{bmatrix}
\dot{\hat{Q}}_{in} \\
\dot{\hat{H}}_f \\
\dot{\hat{Q}}_{out} \\
\dot{\delta}_{f_1}
\end{bmatrix} =
\begin{bmatrix}
\mu \hat{Q}_{in} | \dot{\hat{Q}}_{in} | + \frac{a_1}{\Delta z_1} (H_{in} - \hat{H}_f) \\
0 \\
\frac{a_2}{\Delta z_1} (\hat{Q}_{in} - \hat{Q}_{out} - \dot{\delta}_{f_1} \sqrt{\hat{H}_f}) \\
0
\end{bmatrix}
\]

\[\text{with } K (\cdot) = \begin{bmatrix} 2\lambda & 0 & 0 & \lambda \Delta z_1 \hat{H}_f \\
0 & 0 & 3\lambda & \lambda \Delta z_1 \\
0 & 3\lambda (L - \Delta z_1) & 0 & \lambda \Delta z_1 \\
0 & 0 & \lambda \Delta z_1 & 0 \end{bmatrix} \]

\[\text{In case of a fault in the downstream pressure system, the observer uses only the flow rate measurement at the upstream, i.e., } y = h(x_e) = Q_{in}. \text{ Finally, assuming } (\hat{Q}_{in} - \hat{Q}_{in}) > 0, \text{ the gain is reduced to:}
\]

\[K = \begin{bmatrix}
-\frac{2\lambda}{\gamma} \\
\gamma - \frac{2\lambda}{\gamma} + 2\lambda
\end{bmatrix} \]

A similar observer can be designed for the reconstruction of the downstream flow sensor.

3) Reconstruction of faults in pressure systems: In case of faults in the pumping or storage systems at the end of the pipeline, the fault models to be considered are:

\[\hat{H}_{in} = H_{in} + \delta H_{in}, \quad \hat{H}_{out} = H_{out} + \delta H_{out} \]

To conceive the reconstruction observer, a finite model with at least two spatial sections is required. Therefore, for the upstream case, the considered model is

\[\dot{\hat{Q}}_{in} = \mu (\hat{Q}_{in} - \hat{\delta}_{Q_{in}}) | \hat{Q}_{in} - \hat{\delta}_{Q_{in}} | + \frac{a_1}{\Delta z_1} (H_{in} - H_{out}) \]

\[\dot{\delta}_{Q_{in}} = 0 \]

with \(x_e = [\hat{Q}_{in} \quad \hat{\delta}_{Q_{in}}].\) and

(ii) Fault in the downstream flow sensor:

\[\dot{\hat{Q}}_{out} = \mu (\hat{Q}_{out} - \hat{\delta}_{Q_{out}}) | \hat{Q}_{out} - \hat{\delta}_{Q_{out}} | + \frac{a_1}{\Delta z_1} (H_{in} - H_{out}) \]

\[\dot{\delta}_{Q_{out}} = 0 \]

with \(x_e = [\hat{Q}_{out} \quad \hat{\delta}_{Q_{out}}].\)
this fault scenario (23) is given by

\[
\begin{bmatrix}
\dot{Q}_{in} \\
\dot{H}_f \\
\dot{Q}_{out} \\
\dot{\delta H}_{in}
\end{bmatrix}
= \begin{bmatrix}
\mu Q_{in} + \frac{a_1}{\Delta z_1} \left( H_{in} - \dot{H}_f - \delta H_{in} \right) \\
\frac{a_2}{\Delta z_1} (Q_{in} - Q_{out}) \\
\mu Q_{out} (Q_{out}) + \frac{a_1}{\Delta z_2} \left( \dot{H}_f - H_{out} \right) \\
- K \begin{bmatrix}
\dot{Q}_{in} - Q_{in} \\
\dot{Q}_{out} - Q_{out}
\end{bmatrix}
\end{bmatrix}
\]

which have been conceived considering as measurement outputs, the flows at the extremities, i.e., using Eq. (16). Assuming \( Q_{in} > 0 \), the reduced gain is expressed as follows:

\[
K = \begin{bmatrix}
2\lambda & 0 & \Delta z_2 \lambda^2 & 0 \\
0 & \frac{\Delta z_2 \lambda^2}{a_1} & - 4\mu \dot{Q}_{out} \Delta z_2 \lambda & 2\lambda \\
- \frac{\Delta z_1 \lambda^2}{a_1} + 4\mu \dot{Q}_{in} \Delta z_1 \lambda & - \frac{\Delta z_2 \lambda^2}{a_1} & - 4\mu \dot{Q}_{out} \Delta z_2 \lambda & 4\mu \dot{Q}_{out} \Delta z_2 \lambda
\end{bmatrix}
\]

Similar observer can be designed for \( \delta H_{out} \) using Eq. (24).

IV. EXPERIMENTAL RESULTS

The hydraulic pilot pipeline counts on six valves (V1, V2, V3, V4, V5, V6) used to emulate leaks at 11.535 [m], 49.825 [m], 80.355 [m], 118.365 [m], 148.925 [m], 186.945 [m], respectively from the upstream. The physical parameters values of this pipeline are \( b = 1497 \ [m/s] \), \( \phi = 0.1047 \ [m], \ L = 200.16 \ [m], \ \dot{f} = 2.785 \times 10^{-2}. \)

With the purpose to evaluate the performance of the fault diagnosis system for pipelines, several experiments were realized provoking a extraction (leak emulation) and altering the recorded measurements supplied by the flow and pressure sensors. The leak emulation was originated opening the valve V4 at 100 [s]. The effect of the leak can be observed in the behavior of the flows plotted in Fig. 1 clearly these are deviated from the operation point. In the same figure, the measurement signals corresponding to the pressure heads at the end points are also plotted. For the validation of the system robustness, experiments are carry out considering the following three scenarios:

- E5: Reconstruction of a leak with a position \( \Delta z_1 = 118.365 \ [m] \) via the algorithm given by the non linear observer (18) and full described in (13).
- E1: Reconstruction of the offset \( \delta Q_{in} = 11.11 - 22.22\% \) in the upstream flowmeter with the algorithm (20).
- E3: Reconstruction of the fault \( \delta H_{in} = 20\% \) in the sensor of the pumping system with algorithm (25).

A. Fault detection and isolation

The evolution of the set of residuals (7, 8, 9, 11 and 12) for scenarios E1, E3 and E5 are shown in Fig. 2. One can see that the signature for each scenario corresponds with the FSM given by (13), both the magnitude of each residual is disperse with respect to faults. A normalized procedure for the residual is suggested considering maximum physical parameters of the faults. It is to remark that to detect leaks \( r_1 \) is the standard residual; however the loss volume involved in the integral term of residuals \( r_4 \) or \( r_5 \), seems to be more appropriated from sensitivity point of view.

B. Fault reconstruction

In accordance to the fault nature, once a fault has been detected and isolated, the reconstruction of the fault along the time is determined using a corresponding estimation algorithm. Following, some experimental results concerning the reconstruction performed by the observers are presented.

1) Reconstruction of a leak: The observer given in (18) was initialized at \( t = 160 \ [s] \), once the leak has been isolated from the residuals behavior. In Fig. 3 the estimation of the leak position is presented for different values of \( \lambda \). The estimated values are approximated to the real value of the leak position with a decreasing and increasing of noise depending on the choice of \( \lambda \).

2) Reconstruction of faults in flow sensors: In order to evaluate the observer for the reconstruction of a fault in the upstream flowmeter (20), a signal has been added to the upstream measurement. During the experiment shown in Fig. 4 the observer was initialized at \( t = 0 \ [s], \ i.e., \), when there

Fig. 3. Position estimation

Fig. 4. Position estimation.
The estimations results are exposed in Fig. 4 (top).

**Estimation of a damage in the pumping system**

Fig. 4. (top) Estimation of the offset in the upstream flowmeter, (bottom) chosen with different values with respect to the real initial conditions of the observer were appreciated.

In this work, a robust algorithm was presented to diagnose faults in pipelines, principal contribution of this paper. The originality of such system is its two-stages composition with specific tasks. In the first step the aim is to distinguish among five fault scenarios including a possible leak. This task is performed by the use of redundant relations obtained from the fluid transport model of the pipeline. In the second stage, the task is the reconstruction of the temporal evolution of the fault, once this has been previously identified. The evaluation results of the diagnosis system with experimental data were satisfactory, making it feasible for implementation in industrial pipelines.

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