Possible $\Lambda_c\bar{\Lambda}_c$ molecular states and their productions in nucleon-antinucleon collision

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Abstract

In this work, a study of possible molecular states from the $\Lambda_c\bar{\Lambda}_c$ interaction and their productions in nucleon-antinucleon collision is performed in a quasipotential Bethe-Salpeter equation approach. Two bound states with quantum numbers $J^{PC}=0^{-+}$ and $1^{--}$ are produced with almost the same binding energy from the $\Lambda_c\bar{\Lambda}_c$ interaction which is described by the light meson exchanges. However, the result does not support the assignment of experimentally observed $Y(4630)$ as a $\Lambda_c\bar{\Lambda}_c$ molecular state because it is hard to obtain a peak near experimental mass of the $Y(4630)$ which is far above the $\Lambda_c\bar{\Lambda}_c$ threshold. The possibility to search these states in nucleon-antinucleon collision is studied by including couplings to $NN$ and $D^0\bar{D}^{(*)}$ channels. The peaks can be found obviously near the $\Lambda_c\bar{\Lambda}_c$ threshold in the $D^*\bar{D}^*$ channel at an order of amplitude of $10 \mu$b. Too small width of state with $0^{-+}$ may lead to the difficulty to be observed in experiment. Based on the results in the current work, search for the $\Lambda_c\bar{\Lambda}_c$ molecular state with $1^{--}$ is suggested in process $N\bar{N}\to D^*\bar{D}^*$, which is accessible at PANDA.

Keywords: Molecular state, PANDA, $\Lambda_c\bar{\Lambda}_c$ interaction, Nucleon-antinucleon collision, Quasipotential Bethe-Salpeter equation

1. Introduction

The $Y(4630)$ with quantum numbers $J^{PC}=1^{--}$ was firstly observed in the exclusive process $e^+e^-\to\Lambda_c\bar{\Lambda}_c$ by Belle collaboration [1], and confirmed at BESIII after several years [53]. It carries mass and width of $M=4634^{+6}_{-7}$(stat.)$^{+3}_{-2}$(sys.) MeV and $\Gamma_{tot}=92^{+20}_{-24}$(stat.)$^{+10}_{-21}$(sys.) MeV, respectively. The internal structure of $Y(4630)$ attracts attentions from the community and many interpretations have been proposed to understand its internal exchange. It was explained as a conventional charmonium state in Refs. [3, 4, 5, 6]. The $Y(4630)$ as a tetraquark was also supported by calculation in the constituent quark model and QCD sum rule [7, 8]. Since the mass of $Y(4630)$ is close to the $\Lambda_c\bar{\Lambda}_c$ threshold, this structure was also related to the threshold effect [9] and $\Lambda_c\bar{\Lambda}_c$ baryonium [10]. Meanwhile, the $\Lambda_c\bar{\Lambda}_c$ system also attracts attentions [11, 12, 13]. Theoretical calculations suggest strong attraction between a $\Lambda_c$ baryon and a $\bar{\Lambda}_c$ baryon by $\sigma$ and $\omega$ exchanges, which favors the existence of a $\Lambda_c\bar{\Lambda}_c$ molecular states [12, 13].

It is worthwhile noting that $Y(4630)$ is near another exotic state $Y(4660)$, which was observed in the invariant mass spectrum of $\phi(2S)\pi^+\pi^-$ [14]. Within experimental uncertainties, the two states were reckoned as the same state [15, 16]. It was also proposed that the $Y(4660)$ is more likely to be a molecular state of $\phi(2S)f_0(980)$ [17]. In Ref. [18], the possibility that the $Y(4660)$ peak is of the same origin as the $Y(4630)$ was discussed. Ref. [19] questions the existence of the $Y(4630)$, and discusses that the vector $\Lambda_c\bar{\Lambda}_c$ bound state predicted could have its signal in the BESIII measurement [53].

If we put the two states together, it seems that the mass of $Y(4630)$ favors the assignment of $Y(4630)$ as a $\Lambda_c\bar{\Lambda}_c$ molecular state. But there is an obvious difficulty to fit the experimental peak of $Y(4630)$ with a $\Lambda_c\bar{\Lambda}_c$ molecular state. The $Y(4630)$ is about 60 MeV above the $\Lambda_c\bar{\Lambda}_c$ threshold while a molecular state is usually assumed as a bound state with mass below the threshold. To solve this problem, it was suggested that the data from $\Lambda_c\bar{\Lambda}_c$ threshold up to 4.7 GeV should contain signals of at least two states, a $\Lambda_c\bar{\Lambda}_c$ molecule and another one with a mass around 4.65 GeV [19].

To make clear the internal structure of the $Y(4630)$ and the structures near the $\Lambda_c\bar{\Lambda}_c$ threshold, more theoretical and experimental studies are required. Since a $\Lambda_c\bar{\Lambda}_c$ molecular state is a baryonium, it should be easy to be produced from a pair of a nucleon and an antinucleon at high energy collision by exciting a charm-anticharm quark pair. Such process is accessible in the PANDA (AntiProton annihilations at Darmstadt) detector at FAIR (Facility for Antiproton and Ion Research) Recently, some theoretical researches about $\Lambda_c\bar{\Lambda}_c$ production through proton-antiproton collision were performed in the literature, and considerably large cross section was predicted [21, 22, 23, 24, 25, 26, 27]. The charm-meson pair production from the proton-antiproton collision were also studied in the literature [21, 28, 29]. However, in these studies, the direct relation between the molecular state and the production process was not considered.

In the current work, the molecular states from the $\Lambda_c\bar{\Lambda}_c$ interaction will be studied in a quasipotential Bethe-Salpeter equation (qBSE) approach [30, 31, 32]. The $\Lambda_c\bar{\Lambda}_c$ invariant mass spectrum will be estimated to discuss the possibility of assignment of the $Y(4630)$ as a $\Lambda_c\bar{\Lambda}_c$ molecular state. As discussed above, a virtual or resonance state is required to interpret...
the mass gap between the threshold and the experimental mass. It will be seen that such requirement is difficult to be satisfied. If the \( \Lambda_c \Lambda_c \) molecular state is not the \( Y(4630) \), it is interesting to study the possibility to search the obtained molecular states in the nucleon-antinucleon collision. In this work, the coupling of the molecular states to the \( N \bar{N} \) and \( D^{(*)} \bar{D}^{(*)} \) channels will be constructed and the cross sections of processes \( N \bar{N} \rightarrow N \bar{N} \) and \( D^{(*)} \bar{D}^{(*)} \) will be calculated.

This article is organized as follows. After introduction, we present the details of theoretical frame in Section 2, which includes relevant effective Lagrangians and parameters, the potential kernel of the \( \Lambda_c \Lambda_c \) interaction and coupled-channel interaction with \( N \bar{N} \) and \( D^{(*)} \bar{D}^{(*)} \) channels, and a brief introduction about the qBSE approach. The numerical results of the molecular states from the \( \Lambda_c \Lambda_c \) interaction will be given in Section 3. The relation between such states and \( Y(4630) \) will be discussed. The coupled-channel calculation is also performed and the cross sections of the processes \( N \bar{N} \rightarrow N \bar{N}, D^{(*)} \bar{D}^{(*)} \) are also presented in this section. Finally, article ends with a summary in section 4.

2. Theoretical frame

2.1. Interaction mechanism and relevant Lagrangians

In the current work, we focus on the molecular states from the \( \Lambda_c \Lambda_c \) interaction, which mechanism is described in Fig 1 (a). Since the \( \Lambda_c \) baryon is isoscalar state, only the isoscalar \( \omega \) and \( \sigma \) mesons can be exchanged to provide the interaction.

![Figure 1: Diagrams for the interactions involved in the current work including single channels \( \Lambda_c \Lambda_c \) (a), \( N \bar{N} \) (b), and \( D^{(*)} \bar{D}^{(*)} \) (c), and coupled channels \( N \bar{N} \rightarrow \Lambda_c \Lambda_c \) (d), \( N \bar{N} \rightarrow D^{(*)} \bar{D}^{(*)} \) (e), and \( \Lambda_c \Lambda_c \rightarrow D^{(*)} \bar{D}^{(*)} \) (f).](image)

To write the potential for the \( \Lambda_c \Lambda_c \) interaction, the Lagrangians constructed with heavy quark and chiral symmetries are introduced, and presented explicitly as follows [33, 34, 35, 36, 37],

\[
L_{\Lambda_c \Lambda_c Y} = -\frac{g_{\bar{N}N} \bar{\Lambda}_c N}{m_{\Lambda_c}} \partial_\mu \bar{\Lambda}_c \gamma^\mu \partial_\mu \Lambda_c, \quad L_{\Lambda_c \Lambda_c \sigma} = ig_{\bar{N}N} \bar{\Lambda}_c \Lambda_c, \quad (1)
\]

where the \( \Lambda_c, \omega \) meson, and \( \sigma \) are for \( \Lambda_c \) baryon, \( \omega \) and \( \sigma \) meson fields, \( m_{\Lambda_c} \) is the mass of \( \Lambda_c \) baryon. The coupling constants are chosen as \( g_{\bar{N}N} = 5.9, \beta_0 = 0.87, f_b = -3.1 \) [36, 37].

To study the production of the \( \Lambda_c \Lambda_c \) molecular states in nucleon-antinucleon collision, more channels need to be introduced, that is, the \( N \bar{N} \) and \( D^{(*)} \bar{D}^{(*)} \) interactions, which are depicted with Feynman diagrams in Fig 1 (b) and Fig 1 (c). The Lagrangians for the vertices of coupling of the nucleon to pseudoscalar meson \( \pi \), vector meson \( \rho/\omega \), and \( \sigma \) meson are [38]

\[
L_{NN\pi} = -\frac{g_{N\pi}}{m_N} \bar{N} \gamma^\mu \partial_\mu N, \quad L_{N\pi\pi} = -g_{NN\pi} \bar{N} N \sigma, \\
L_{NN\rho} = -g_{NN\rho} \bar{N} \gamma^\mu \left( \gamma^\nu - i\epsilon^{\mu\nu\alpha\beta} \partial_\alpha \right) \partial_\beta N, \\
L_{NN\omega} = -g_{NN\omega} \bar{N} \gamma^\mu \left( \gamma^\nu - i\epsilon^{\mu\nu\alpha\beta} \partial_\alpha \right) \omega_\beta N, \quad (2)
\]

where \( N, \pi, \rho, \omega \) are nucleon, pion meson, \( \rho \) meson, and \( \omega \) meson fields. The coupling constants \( g_{NN\pi} = 0.989, g_{NN\rho} = -3.1, \kappa_\rho = 1.825, \kappa_\omega = 0, \) and \( g_{NN\omega} = 5 \) [38, 39, 40]. The coupling constants for the \( \omega \) meson can be related to these for the \( \rho \) meson with SU(3) symmetry as \( g_{NN\pi} = \frac{3}{2} g_{NN\rho} \). In the interaction of a baryon and an antibaryon can be easily related to the interaction with two baryons with the G-parity rule as \( V_{BBM} = (-1)^gv_{BBM} \) where \( G \) is the G-parity of exchanged meson \( M \) [31, 41]. So, we can relate the nucleon-nucleon interaction in the current work to the nucleon-nucleon interaction, which can produce the deuteron. With such interaction, the experimental mass of deuteron can be reproduced in the current model. The G-parity rule can be also applied to \( \Lambda_c \Lambda_c \) interaction.

The couplings of heavy-light charmed mesons \( D^{(*)} = (D^{(*)R}, D^{(*)I}, D^{(*)C}) \) and its antiparticle \( \bar{D}^{(*)} = (D^{(*)\bar{R}}, D^{(*)\bar{I}}, D^{(*)\bar{C}}) \) to the light mesons can be depicted by the Lagrangians with heavy quark and chiral symmetries as [33, 34, 35],

\[
L_{D^{(*)}D^{(*)}} = -\frac{2g_{MDMD}}{f_x} (-D^{(*)}_a \partial_\mu D^{(*)\mu}_a + D^{(*)\mu}_a \partial_\mu D^{(*)}_a) \partial_\nu \bar{\epsilon}_{ab} \\
+ \frac{2g_{MDMD}}{f_x} (-D^{(*)\mu}_a \partial_\mu D^{(*)\mu}_a + D^{(*)}_a \partial_\nu \bar{\epsilon}_{ab}, \quad L_{D^{(*)}D^{(*)}} = \frac{g_{MDMD}}{f_x} (D^{(*)\mu}_a \partial_\mu D^{(*)\nu}_b - \frac{g_{MDMD}}{f_x} D^{(*)\nu}_a \partial_\mu D^{(*)\mu}_b) \partial_\nu \bar{\epsilon}_{ab}, \\
L_{D^{(*)}D^{(*)}} = \sqrt{2} \frac{g_{MDMD}}{f_x} (D^{(*)\mu}_a \partial_\nu D^{(*)\nu}_b - \frac{g_{MDMD}}{f_x} D^{(*)\nu}_a \partial_\mu D^{(*)\mu}_b) \partial_\nu \bar{\epsilon}_{ab}, \quad (2)
\]

\[
L_{DDV} = -\frac{g_{D\mu}}{\sqrt{2}} D^{(*)}_a \partial_\nu D^{(*)\nu}_b + \frac{g_{D\mu}}{\sqrt{2}} \bar{D}^{(*)\mu}_a \partial_\nu \bar{D}^{(*)\mu}_b, \\
L_{D^{(*)}D^{(*)}} + \frac{i}{\sqrt{2}} g_{D\mu} (D^{(*)\mu}_a \partial_\nu D^{(*)\nu}_b - \frac{g_{D\mu}}{\sqrt{2}} D^{(*)\nu}_a \partial_\mu D^{(*)\mu}_b) \partial_\nu \bar{\epsilon}_{ab} - i \sqrt{2} \frac{g_{D\mu}}{\sqrt{2}} D^{(*)\mu}_a \partial_\nu \bar{D}^{(*)\mu}_b \partial_\nu \bar{\epsilon}_{ab}, \quad (2)
\]
\[-i2\sqrt{2}ig\varepsilon_{\mu}D_a^\mu\bar{D}_b^\sigma(\partial_\nu\gamma_\nu-\partial_\mu\gamma_\mu)_{ab},\]

\[
\mathcal{L}_{D^{0}\sigma} = -2g_\sigma m_D D_a^\sigma D_b^\sigma - 2g_\sigma m_D \bar{D}_a^\sigma \bar{D}_b^\sigma, \\
\mathcal{L}_{D^\pm \sigma} = 2g_\sigma m_D D_a^\nu D_b^\nu D^\pm \sigma + 2g_\sigma m_D \bar{D}_a^\mu \bar{D}_b^\mu \bar{D}^\pm \sigma.
\]

(3)

Here we adopt the parameters as \( f_2 = 132 \text{ MeV}, \ g = 0.59, \ \beta = 0.9, \ \lambda = 0.56 \text{ GeV}^{-1}, \ g_\nu = 5.9, \ \text{and} \ g_\sigma = 0.76 \) [33, 34, 35]. The \( \mathcal{V} \) and \( \mathcal{P} \) are the vector and pseudoscalar matrices as

\[
\mathcal{P} = \begin{pmatrix}
\frac{\lambda_{D^{0}}}{{\sqrt{6}}} & \frac{\pi^{+}}{\sqrt{6}} & K^{+} \\
-\frac{\lambda_{D^{0}}}{{\sqrt{6}}} & \frac{\pi^{-}}{\sqrt{6}} & K^{-} \\
\frac{\lambda_{D^{0}}}{{\sqrt{6}}} & \frac{\pi^{0}}{\sqrt{6}} & K^{0}
\end{pmatrix}, \\
\mathcal{V} = \begin{pmatrix}
\frac{\rho^{+}}{\sqrt{2}} & \frac{\rho^{-}}{\sqrt{2}} & K^{+} \\
\frac{\rho^{0}}{\sqrt{2}} & \frac{\rho^{-}}{\sqrt{2}} & K^{-} \\
\frac{\rho^{0}}{\sqrt{2}} & \frac{\rho^{+}}{\sqrt{2}} & K^{0}
\end{pmatrix}.
\]

The couplings between heavy-light mesons and \( J/\psi \) are also required, which are of forms [42, 43]

\[
\mathcal{L}_{D^{0}\to J/\psi} = -i\mathcal{G}^{D^{0}\to J/\psi}[\bar{D}\gamma_{\mu}D^{\mu}]\frac{i}{\sqrt{2}}(\bar{D} D^{\sigma} D^\sigma D^{\rho} \bar{D} D^{\rho}),
\]

\[
\mathcal{L}_{D^{0}\to J/\psi} = -g_{D^{0}\to J/\psi} \epsilon_{\mu\nu\rho\sigma} \left[ iD^\rho D^{\sigma} D^{\mu} \right],
\]

\[
\mathcal{L}_{D\to J/\psi} = -i\mathcal{G}^{D\to J/\psi}[\bar{D}\gamma_{\mu}D^{\mu}]\frac{i}{\sqrt{2}}(\bar{D} D^{\sigma} D^\sigma D^{\rho} \bar{D} D^{\rho}),
\]

where the coupling constants above satisfy the relation as \( g_{D^{0}\to J/\psi}/m_D = g_{D^{0}\to J/\psi}/m_D = g_{D\to J/\psi} = 2g_2 \sqrt{m_\rho} \) and \( g_2 = \sqrt{m_\rho}/2m_{f_0} \) with \( f_0 = 405 \text{ MeV} \).

Finally, the three channels considered above can be coupled to each other as shown in Fig 1(d), Fig 1(e), and Fig 1(f) by the \( D^{(*)}\Lambda\Lambda \) vertices, the Lagrangians of which are constructed under the SU(4) symmetry [44, 45, 46, 47] as follows,

\[
\mathcal{L}_{D^{0}\Lambda\Lambda} = ig_{D^{0}\Lambda\Lambda}(\bar{\Lambda}_{\mu}\Lambda_{\nu}D^\mu + \bar{\Lambda}_{\nu}\Lambda_{\mu}D^\mu), \\
\mathcal{L}_{D^{0}\Lambda\Lambda} = ig_{D^{0}\Lambda\Lambda}(\bar{\Lambda}_{\mu}\Lambda_{\nu}D^\mu + \bar{\Lambda}_{\nu}\Lambda_{\mu}D^\mu),
\]

(5)

where the coupling constants, \( g_{D^{0}\Lambda\Lambda} \), and \( g_{D^{0}\Lambda\Lambda} \) are chosen as 10.7 and \(-5.8\), respectively [27].

2.2. Potential kernel and qBSE approach

In the current work, we focus on the \( \Lambda\Lambda \) and \( \Lambda_{\mu} \) molecular states and their isoscalar states. Since baryons \( \Lambda\Lambda \) and \( \Lambda_{\mu} \) are isoscalar states, only isoscalar flavor functions are considered as [19, 48],

\[
|\Psi_{\Lambda\Lambda}\rangle = -|\Lambda_{\mu}\Lambda_{\nu}\rangle, \quad |\Psi_{NN}\rangle = \frac{1}{\sqrt{2}}(|p\bar{p}\rangle + |n\bar{n}\rangle),
\]

\[
|\Psi_{DD}\rangle = \frac{1}{\sqrt{2}}(|D^{+}\bar{D}^{-}\rangle + |D^{-}\bar{D}^{+}\rangle), \\
|\Psi_{DD}\rangle = \frac{1}{2}(|(D^{+}\bar{D}^{0}D^{-}\rangle + D^{0}\bar{D}^{+}\rangle + |D^{0}\bar{D}^{-}\rangle + |D^{0}\bar{D}^{-}\rangle)|, \\
|\Psi_{DD}\rangle = \frac{1}{\sqrt{2}}(|D^{+}\bar{D}^{0}D^{-}\rangle + |D^{-}\bar{D}^{0}\rangle)|,
\]

(6)

where \( \epsilon = \pm \) corresponds to \( C \) parity \( \epsilon = \pm \) respectively.

With the wave functions and Lagrangians, the potential can be obtained by applying the standard Feynman rules in the one-boson-exchange model as [30],

\[
\mathcal{V}_{\text{pot}} = i\hat{f}^{D^{0}}(q^2) \Gamma_2 P_{\sigma\tau} f(q^2), \\
\mathcal{V}_{\text{pot}} = i\hat{f}^{D^{0}}(q^2) \Gamma_{2\sigma} P_{\mu\nu} f(q^2),
\]

\[
\mathcal{V}_{\text{pot}} = i\hat{f}^{D^{0}}(q^2) \Gamma_2 P_{\sigma\tau} f(q^2),
\]

where the \( f^{D^{0}}(q^2) \) is flavor factors for certain meson exchange of certain interaction, which are calculated explicitly with the flavor functions and Lagrangians, and the values are listed as Table 1.

| \( \pi \) | \( \eta \) | \( \rho \) | \( \omega \) | \( \sigma \) | \( J/\psi \) | \( \Lambda_{\pi} \) |
|---|---|---|---|---|---|
| \( \Lambda_{\pi} \) | \( \Lambda_{\pi} \) | \( \Lambda_{\pi} \) | \( \Lambda_{\pi} \) | \( \Lambda_{\pi} \) | \( \Lambda_{\pi} \) | \( \Lambda_{\pi} \) |

(7)

The propagators for the exchanged pseudoscalar \( \mathcal{P} \) meson, scalar \( \sigma \) meson, vector \( \mathcal{V} \) meson, and baryon \( \mathcal{B} \) are defined as,

\[
P_{\sigma\tau} = \frac{i}{q^2-m_\sigma^2}, \quad P_{\mathcal{V}} = \frac{i}{q^2-m_\pi^2}, \quad P_{\mathcal{B}} = \frac{i}{q^2-m_\pi^2}.
\]

We introduce the \( f(q^2) \) as a form factor to compensate the off-shell effect of exchanged meson, which can be written concretely as \( f(q^2) = e^{-m_\pi^2-q^2/m_\pi^2} \) with \( m_\pi \) and \( q \) being the mass and the momentum of exchanged meson. The cutoff is parameterized as

\[
\Lambda_\pi = m_\pi + \alpha 0.22 \text{ GeV}.
\]

After inserting the potential kernel into the Bethe-Salpeter equation to obtain the scattering amplitude and applying spectator quasipotential approximation and partial-wave decomposition, a 1-dimensional integral equation with fixed spin-parity \( J^P \) can be obtained as [30, 31, 32, 48, 37],

\[
i\mathcal{M}_{\sigma\tau}(p', p) = i\mathcal{V}_{\sigma\tau}(p', p) + \sum_{J^P} \int \frac{d^2p''}{(2\pi)^2} \cdot i\mathcal{V}_{\sigma\tau}(p', p') G_0(p'' \cdot i\mathcal{M}_{\sigma\tau}(p'', p'),
\]

(9)

where the \( \mathcal{M}_{\sigma\tau}(p', p') \) is partial-wave scattering amplitude and the \( G_0(p'') \) is reduced propagator under quasipotential approximation. The partial wave potential is defined with the potential obtained in Eq. (7) as

\[
\mathcal{V}_{\sigma\tau}(p', p) = 2\pi \int d\cos \theta \mathcal{V}_{\sigma\tau}(\theta) V_{\sigma\tau}(p', p).
\]
where $\eta = PP_{p_3}^{-1} ((1 - i\epsilon_{h})/P)^{-1}$ with $P$ and $J$ being parity and spin for the $\Lambda_c\Lambda_c$ system. The initial and final relative momenta are chosen as $p = (0, 0, p)$ and $p' = (p' \sin \theta, 0, p' \cos \theta)$. The $d_{J,\pm}(\theta)$ is the Wigner d-matrix. An exponential regularization is introduced to reduced propagator $P\rightarrow G_{0}(p') \rightarrow G_{0}(p') \left[ e^{-\eta\theta_{12}^{2}/M} \right]^{2}$ with $\Lambda$ being the cutoff chosen as $\Lambda = \Lambda_{c}$ [49].

The molecular state from the $\Lambda_c\Lambda_c$ interaction corresponds to a pole of scattering amplitude $M$. With discretizing the momenta $p$, $p'$, and $p''$ to $p_i$, $p'_i$, and $p''_i$ with the weight factor $w(p''_i)$ by the Gauss quadrature the integral equation can be transformed to a matrix equation as [49],

$$M_{ik} = V_{ik} + \sum_{j=0}^{N} V_{ij} G_{M} M_{jk}.$$  
(11)

The discretized propagator is of a form

$$G_{j=0} = \frac{w(p''_j)p''_j^2}{(2\pi)^2} e^{-\eta\theta_{12}^2} G_0(p''),$$

$$G_{j>0} = -\frac{i p''_j}{2\pi^2} \left[ \frac{1}{2p''_j^2 - 2p(p''_j - p'_j)} \right].$$  
(12)

with $p''_j = \frac{1}{2\pi} \sqrt{W^2 - (M_1 + M_2)^2} [W^2 - (M_1 - M_2)^2]$. And the pole can be searched by finding a complex energy $z = E_R + i\Gamma/2$ which satisfies $|1 - V(z)G(z)| = 0$. Here $E_R$ and $\Gamma$ are mass and decay width of the molecular state, respectively.

The total cross section can be obtained as follows [50],

$$\sigma = \frac{1}{16\pi} \left[ \frac{p' \sum_{j',j \geq 0} J_{j'J} |M_{j',j}^{p'}(p',p)|^2}{4\pi} \right] .$$  
(13)

where the $s$ is the invariant mass square of the system of initial particles. $J$ and $j$ are total angular momentum of system and spins of two initial particles, respectively.

3. Numerical results

3.1. Bound states from $\Lambda_c\Lambda_c$ interaction

With the above preparation, the possible molecular states from the $\Lambda_c\Lambda_c$ interaction can be searched in the complex energy plane by adjusting the parameter $\alpha$. As usual, only the states from the S-wave interaction are considered in the current work, that is, states with quantum numbers $J^{P_{C}} = 0^{-}$ and $1^{-}$. The single-channel calculation is first performed, which produce bound state as a pole at real axis below the threshold. The results are shown in Table 2.

| $\alpha$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|----------|-----|-----|-----|-----|-----|-----|
| $0^{-}$  | 3.0 | 6.7 | 11.8| 18.6| 26.8| --- |
| $1^{-}$  | 3.0 | 6.7 | 11.8| 18.6| 26.8| --- |

As listed in Table 2, the bound states can be produced from the $\Lambda_c\Lambda_c$ interaction with both $J^{P_{C}} = 0^{-}$ and $1^{-}$. The two bound states have almost the same binding energy, which is consistent with the result in Ref. [12]. The results indicate that the attraction between $\Lambda_c$ baryon and $\Lambda_c$ baryon is considerably strong, which leads to first appearance of bound states at a value of $\alpha$ below 0. The binding energies of the bound states increase gradually with increasing of $\alpha$ value, and exceed 30 MeV at an $\alpha$ value about 0.5.

The bound state can be produced from the $\Lambda_c\Lambda_c$ interaction with $J^{P_{C}} = 1^{-}$, which are also the quantum number of the $Y(4630)$. As shown in Table 2, its binding energy is below the threshold even with an $\alpha$ value of 0. However, the experimental structure of $Y(4630)$ is about 60 MeV above the $\Lambda_c\Lambda_c$ threshold. To discuss the relation between the vector $\Lambda_c\Lambda_c$ molecular state to the experimentally observed $Y(4630)$, we make an estimation of the $\Lambda_c\Lambda_c$ invariant mass spectrum as $C p' |M_{j',j}^p(p',p)|^2$ shown in Fig. 2 with different $\alpha$ values. Here $C$ is the Gamov-Sommerfeld factor for the Coulomb enhancement effect as $C = y/(1 - e^{-\gamma})$ with $y = \pi \alpha \sqrt{1 - \beta^2}/\beta$ and $\beta = \sqrt{1 - 4m_{\Lambda_c}^2/W^2}$ [51, 52].

Figure 2: The $\Lambda_c\Lambda_c$ invariant mass spectrum estimated by $C p' |M_{j',j}^p(p',p)|^2$ with $J^{P_{C}} = 1^{-}$ at $\alpha = -0.2, -0.1, 0.0, 0.1$ and 0.2. The orange line indicates the mass of $Y(4630)$. The BESIII data for $e^+e^- \rightarrow \Lambda_c\Lambda_c$ are cited from Ref. [53].

With $\alpha$ values of 0, a peak can be observed near the $\Lambda_c\Lambda_c$ threshold, which corresponds to a bound state with a binding energy about 3 MeV. However, it is found quite difficult to produce a peak near the mass about 4630 MeV even after varying the $\alpha$ value in reasonable range. With the increase of $\alpha$ value, the pole of the bound state will leave the threshold further, which results in a smaller peak near the threshold. If smaller $\alpha$ value is chosen, the bound state will disappear, and no virtual and resonance can be produced in our model. The peak is still on the threshold. Hence, though a bound state can be produced from the $\Lambda_c\Lambda_c$ interaction with the same quantum numbers as
Y(4630), it is difficult to be used to interpret the experimental observed structure. BESIII reported the cross section of process $e^+e^- \rightarrow \Lambda_c\bar{\Lambda}_c$ near the $\Lambda_c\bar{\Lambda}_c$ threshold [53], which are also presented in Fig. 2 for reference. Here, the invariant mass spectrum is estimated by the single-channel amplitude of process $\Lambda_c\bar{\Lambda}_c \rightarrow \Lambda_c\bar{\Lambda}_c$, and can not be compared directly. However, the rapid increase can be found in both theoretical and experimental results, which suggests that the $\Lambda\bar{\Lambda}_c$ interaction plays important role near the threshold.

### 3.2. Production of $1^{−−}$ state in nucleon-antinucleon collision

Now that vector molecular states produced from the $\Lambda_c\bar{\Lambda}_c$ interaction is difficult to be explained as the Y(4630). In the following, we discuss the possibility to observe it in the nucleon-antinucleon collision. After the $NN$ and $D^{(*)}\bar{D}^{(*)}$ channels and their coupling are included, the cross section of production of $\Lambda_c\bar{\Lambda}_c$ molecular state with $J^{PC} = 1^{−−}$ in the nucleon-antinucleon collision is calculated, and the results are illustrated in Fig 3.

![Graph showing the cross section for different values of $\alpha$](image)

**Figure 3:** The pole of bound state (upper) and the cross section (lower) from the $NN$ collision with $J^{PC} = 1^{−−}$ at $\alpha = 0.1$, 0.2, 0.3 and 0.4. The color means the value of $\log(|1 - V(z)G(z)|)$ as shown in the color box.

Due to lack of experimental data, the cross section will be discussed with different $\alpha$ values. Considering the uncertainties introduced by this parameter, we will provide the results for the $NN$ collision instead of the explicit $p\bar{p}$ or $n\bar{n}$ collisions, which should have cross section at the same order of magnitude. Here, we still focus on the energy region near the $\Lambda_c\bar{\Lambda}_c$ threshold. With the coupled-channel effects are included, the pole for the molecular state leaves the real axis of the complex energy plane. With the increase of $\alpha$ value, the deviation from the real axis becomes larger, which means that the molecular state has a larger width. Generally speaking, the width of the molecular state with $J^{PC} = 1^{−−}$ is considerably small, at an order of magnitude of 0.1 MeV.

In the calculation, four channels, $NN, D\bar{D}, D\bar{D}^{*}$, and $D^{*}\bar{D}^{*}$, are considered as the final states. There are large differences between cross sections of different channels. The largest cross section can be found in the $D^{*}\bar{D}^{*}$ channel at an order of magnitude of 10 $\mu$b at $\alpha$ values from 0.1 to 0.4. Here, to show the results more explicitly, energy ranges are chosen very small in Fig 3. We would like to emphasize that the width of the state is very small, the peak should be very sharp. The cross section in the $NN$ channel is much smaller than the $D^{*}\bar{D}^{*}$ channel, at an order of magnitude of 0.1 $\mu$b. No obvious peak can be found in this channel, but with a small structure due to the interference between the contribution of molecular state and background. Similar structures can be found in the lineshapes of cross sections in other two channels, $D\bar{D}$ and $D\bar{D}^{*}$, at an order of magnitude of 0.01 $\mu$b. Hence, both the height of the peak and the lineshapes of the cross sections suggest that the $D^{*}\bar{D}^{*}$ channel is the best channel to search the $\Lambda_c\bar{\Lambda}_c$ molecular state with $J^{PC} = 1^{−−}$.

### 3.3. Production of $0^{++}$ state in nucleon-antinucleon collision

The molecular state with $0^{++}$ is also produced from the $\Lambda_c\bar{\Lambda}_c$ interaction in $S$ wave. The pole of this state with coupled-channel calculation and its production from the nucleon-antinucleon collision are shown in Fig 4. Since the decay of state with $0^{++}$ to $DD$ is forbidden, three channels, $NN, D\bar{D}^{*}$, and $D^{*}\bar{D}^{*}$, are considered as final states.

Although two bound states produced in $S$ wave are almost degenerated, the molecular state with $0^{++}$ exhibits different behaviors after the coupled-channel effects are included. A very small width about 0.1 KeV is found in our calculation, which is 3 orders of magnitude lower than state with $1^{−−}$. It reflects that the couplings of the state with $0^{++}$ to the final channels considered are very weak. The cross section in the $D^{*}\bar{D}^{*}$ channel is still at an order of magnitude of 10 $\mu$b, but the structure can not be seen obviously at small $\alpha$ values. Only a structure from interference can be found at $\alpha$ value of 0.4. A dip and peak can be found in the channels $NN$ and $D\bar{D}$, respectively, but with small magnitudes. The structures in these two channels are obvious, but should be very narrow due to the extremely small width of the molecular state. It will result in that the number of the events from these structures is very small. Hence, the existence of the possible state with $0^{++}$ may be not easy to confirmed in experiment.
D channel effects is performed to study the possibility to observe the molecular states in the nucleon-antinucleon collision. The coupled-channel calculation with \( \Lambda_\Lambda \) can couple to both quantum numbers and the \( \Lambda_\Lambda \Lambda \) pair are in S wave for both cases, the \( N\bar{N} \) collisions will produce the states with both quantum numbers simultaneously. Without the partial wave analysis, it will make the structures in this energy region more complex. However, for the process \( N\bar{N} \rightarrow D^*\bar{D}^* \), the \( 1^- \) state can stand out the background because the contribution from the \( 0^+ \) state is relatively flat. Hence, based on the results in the current work, we suggest search for the \( \Lambda_\Lambda \Lambda \) molecular state with \( 1^- \) at process \( N\bar{N} \rightarrow D^*\bar{D}^* \), which is accessible at PANDA.

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