Factor-Graph-Based Soft Self-Iterative Equalizer for Multipath Channels

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We consider factor-graph-based soft self-iterative equalization in wireless multipath channels. Since factor graphs are able to characterize multipath channels to per-path level, the corresponding soft self-iterative equalizer possesses reduced computational complexity in sparse multipath channels. The performance of the considered self-iterative equalizer is analyzed in both single-antenna and multiple-antenna multipath channels. When factor graphs of multipath channels have no cycles or mild cycle conditions, the considered self-iterative equalizer can converge to optimum performance after a few iterations; but it may suffer local convergence in channels with severe cycle conditions.

Keywords and phrases: factor graph, equalizer, iterative processing, multipath fading, MIMO.

1. INTRODUCTION

A multipath fading channel, which can be mathematically described by a convolution of transmitted signals and linear channel response, is one of many typical channel models occurring in digital communications. In general, an equalizer that makes detection based on a number of adjacent received symbols is necessary to achieve optimal or near-optimal performance in multipath channels. In classical communication theory, different representations of multipath channels have led to equalizers with different designs. By representing multipath channels as trellis structures, the optimum sequence detector can be computed by the Viterbi algorithm [1], and the optimum symbol detector can be computed by BCJR algorithm [2]. Starting from the transfer function representation of linear multipath systems, people proposed various low-complexity designs such as linear zero-forcing (ZF) equalizer, linear minimum mean-square-error (MMSE) equalizer, nonlinear zero-forcing decision feedback equalizer (ZF-DFE), non-linear MMSE-DFE, and so forth. [3]. In this work, the multipath channels are represented by factor graphs, and soft self-iterative equalizers that execute belief propagation algorithm on factor graphs are studied. (Please refer to [4] for an excellent tutorial on factor graph and its applications.)

One question might rise regarding the motivation of this work, since we have already had both Viterbi algorithm and BCJR algorithm as exact optimum equalizers. The answer to this question lies in the flexibility of factor graph in characterizing multipath channels to per-path level. As a well-known fact, the computational complexity of Viterbi and BCJR algorithms are exponential in the total number of multipaths $L$. In practice, there exist cases when only $L'$ out of $L$ paths (with $L' < L$) have significant channel gains and
moreover, the location of these significant $L'$ paths can be slowly changing in time, for example, rural wireless channels.

Then, a reduced-complexity equalizer that avoids or reduces the computations spent on those zero multipath taps is desirable. Some efforts along this direction have been made in earlier works, for example, parallel Viterbi and parallel BCJR algorithms in [5, 6], which however may require specifically designed control logic for a different multipath scenario. In the considered factor-graph-based soft iterative equalizer, the log-likelihood probabilities are passed as messages in factor graphs between channel nodes and information nodes only along the edges that correspond to paths with significant gain, thus it inherently results in a complexity reduction owing to the sparseness of multipath channels. In particular, we consider three schemes to compute the messages passed from channel nodes to information nodes, namely the scheme based on the a posteriori probability (APP) algorithm, the one based on the linear-MMSE-soft-interference-cancelation (LMMSE-SIC), and the one based on match-filter-soft-interference-cancelation (MF-SIC); and we analyze their performance and applicabilities in practical multipath channels.

One main focus of this paper is the effect of cycles that existed in factor graph on the equalization performance. As compared to the Viterbi and BCJR algorithms which themselves are belief propagation algorithms operating in trellis trees of multipath channels and guarantee the optimum performance, the belief propagation algorithm operating in factor graphs guarantee global optimality only if the underlying factor graph is a tree. Although the condition of factor graph being a tree (i.e., without cycles) is not always met in practice, the factor-graph-based belief propagation algorithm has achieved great success in decoding cycle-contained linear turbo codes and low-density parity-check (LDPC) codes. For the considered self-iterative equalizer, we quantitatively analyze the cycle effect in single-input single-output (SISO), multiple-input single-output (MISO), and multiple-input multiple-output (MIMO) wireless systems; and discuss an alternative representation of factor graphs that ameliorates the performance degradation due to cycle effects.

While it bears similarities to various iterative receivers developed earlier, for example, [7, 8, 9, 10], we highlight that the soft self-iterative equalizer is a self-iterative device which successfully improves the equalization performance by taking advantage of the constraints in received signals due to multipaths, instead of other constraints for instance imposed by error-control coding. Moreover, since the considered equalizer inputs prior and outputs a posteriori probabilities of information symbols, it can easily concatenate with other receiver modules to achieve the turbo receiver processing gains [8].

The rest of this paper is organized as follows. In Section 2, the system model and factor graph representation of multipath channels are described. In Section 3, the factor-graph-based soft iterative equalizer is derived. In Section 4, the performance of the soft self-iterative equalizer is analyzed by numerical simulations for both single-antenna and multiple-antenna systems. Finally, Section 5 contains the conclusions.

2. SYSTEM MODEL AND FACTOR GRAPH REPRESENTATION

Assume match-filtering and symbol-rate sampling, the received signals of multipath channels are normally described by the following time-domain equation [11]:

$$y_t = \sum_{i=0}^{L-1} h_{it} x_{t-i} + n_t, \quad t = 1, 2, \ldots, T,$$

where $y_t \in \mathcal{C}$ and $x_t \in \Omega$ are the receive and transmit signals at time $t$, respectively; $\Omega$ is the modulation set; $h_{ij} \in \mathcal{C}$ is the channel impulse response with delay of $i$ times the symbol rate at time $t$; $n_t \in \mathcal{C} \sim \mathcal{N}(0, \sigma^2)$ is the zero-mean $\sigma$-variance circularly symmetrical Gaussian ambient noise that has been properly whitened and is independent of data; $L$ is the total number of multipaths; $T$ is the frame length. In this paper, we are concerned with block signal processing, and assume that zero prefix is inserted in each signal frame, that is, $x_t = 0, \quad t = -L + 1, \ldots, -1$. For ease of comparison, we also assume that channel gain is properly normalized: in static channels, $\sum_{i=0}^{L-1} |h_{it}|^2 = 1$; and in fading channels, $\sum_{i=0}^{L-1} \mathbb{E}(|h_{it}|^2) = 1$, where $\mathbb{E}(\cdot)$ denotes the expectation over random variables $h_{it}$, for all $t$. As mentioned earlier, we only consider uncoded systems in this work, thus $x_t$ have equal prior probabilities and are assumed to be independent for different $t$.

Equivalently, (1) can be written in a matrix form as

$$\begin{bmatrix} y_1 \\
\vdots \\
y_T \end{bmatrix} = \begin{bmatrix} h_{1,L-1} & h_{1,L-2} & \cdots & h_{1,0} \\
\vdots & & & \vdots \\
h_{T,L-1} & h_{T,L-2} & \cdots & h_{T,0} \\
\end{bmatrix} \begin{bmatrix} x_{t-L+2} \\
\vdots \\
x_{t-T+2} \end{bmatrix} + \begin{bmatrix} n_1 \\
\vdots \\
n_T \end{bmatrix},$$

$$\begin{bmatrix} x_{t-L+2} \\
\vdots \\
x_{t-T+2} \end{bmatrix} \times \begin{bmatrix} h_{1,L-1} & h_{1,L-2} & \cdots & h_{1,0} \\
\vdots & & & \vdots \\
h_{T,L-1} & h_{T,L-2} & \cdots & h_{T,0} \\
\end{bmatrix} = \begin{bmatrix} y_1 \\
\vdots \\
y_T \end{bmatrix}.$$
and (2) as well as $\mathbf{I}_H$ can be extended to multiple-input single-output (MISO) and multiple-output multiple-output (MIMO) cases, by simply replacing $y_t$, $h_{t,i}$, $x_t$, $n_t$ with their matrix/vector counterparts $\mathbf{y}_t$, $\mathbf{h}_{t,i}$, $\mathbf{x}_t$, $\mathbf{n}_t$. As a result, $\mathbf{H}$ and $\mathbf{I}_H$ now become $N_rT \times N_t \cdot (T + L - 1)$ matrices, where $N_r$ and $N_t$ are the number of receive and transmit antennas, respectively.

The above multipath channels in (1) and (2) can also be depicted by factor graphs. The example of the factor graph representations of SISO multipath and MIMO single-path channels are given in Figures 1 and 2. There are two types of nodes in the factor graph: the channel nodes for $y_t$, for all $t$, and the information nodes $x_t$, for all $t$. An edge connects channel node $t$ and information node $t'$, only if the channel gain is significant, that is, $|h_{t,t'}|^2 > 0$. We remark that by no means the factor graphs shown in the figures are unique representation of the corresponding multipath channels; indeed, different representations of the same multipath channel lead to different designs of the factor-graph-based soft iterative equalizer, which we will discuss in Section 4.3.

3. SOFT SELF-ITERATIVE EQUALIZER BASED ON FACTOR GRAPH

The considered soft self-iterative equalizer computes the marginal probabilities of information symbol $\{x_t\}_{t=0}^T$ based on prior probabilities of the receive signals $\{y_t\}_{t=0}^T$, and $\{x_t\}_{t=0}^T$, by executing belief propagations in factor graphs. (As a comparison, both Viterbi algorithm and BCJR algorithm execute belief propagation in trellis trees.)
The messages, defined as the log-likelihood ratio (LLR) of information symbols, are iteratively passed among the nodes in factor graphs, such as to compute the marginal probabilities of information symbols. For BPSK modulation, the message is 1-tuple. In this paper, we will mainly study the complex modulation schemes such as MPSK and MQAM for which the message is \( \log_2 |\Omega| \)-tuple. Let \( m_{c,i}^{(p)} \) be the message passed from the channel node \( c \) to the information node \( i \) at the \( p \)th iteration, \( m_{c,i}^{(p)} \triangleq (m_{c,i}, m_{c,i,1}^{(p)}, \ldots, m_{c,i,|\Omega|-1}^{(p)}) \); and it is updated as

\[
m_{c,i}^{(p)} \triangleq F_{c,i,k}(y_c, m_{c,i}^{(p)}, \forall i' \in U_c)
\]

\[
= \log \frac{\Pr[b_{i,k} = 0 | y_c, m_{c,i}^{(p-1)}, \forall i' \in U_c \setminus \{i\}, m_{c,i,k'}^{(p-1)}, \forall k' \neq k]}{\Pr[b_{i,k} = 1 | y_c, m_{c,i}^{(p-1)}, \forall i' \in U_c \setminus \{i\}, m_{c,i,k'}^{(p-1)}, \forall k' \neq k]}, \quad \forall k,
\]

(3)

where the mapping function from \( \log_2 |\Omega| \)-tuple \( (b_{i,0}, \ldots, b_{i,\log_2 |\Omega|-1}) \) to complex symbol \( x_i \) is usually referred to as modulation format; \( m_{c,i} \) is the message sent from information node \( i' \) to channel node \( c \), as explained next; \( U_c \) denotes the set of all information nodes incident to channel node \( c \), \( U_c \setminus \{i\} \) denotes \( U_c \) excluding information node \( i \); and \( y_c \) is the received signal at time \( c \). The message update rule in (3) follows the general principle of a belief propagation algorithm, that is, the component message \( m_{c,i}^{(p)} \) sent from channel node \( c \) to information node \( i \) is updated based on received signal \( y_c \) and all incident messages to channel node \( c \) except for the same incident component message \( m_{c,i}^{(p-1)} \). Similarly, we let \( m_{i,c}^{(p)} \) be the message passed from the information node \( i \) to the channel node \( c \) at the \( p \)th iteration, \( m_{i,c}^{(p)} \triangleq (m_{i,c,0}, m_{i,c,1}^{(p)}, \ldots, m_{i,c,|\Omega|-1}^{(p)}) \); and it is updated as

\[
m_{i,c}^{(p)} \triangleq G_{i,c,k}(m_i^{(0)}, m_{c,i}^{(p)}, \forall c' \in V_i)
\]

\[
= \log \frac{\Pr[b_{i,k} = 0 | m_i^{(0)}, m_{c,i}^{(p-1)}, \forall c' \in V_i \setminus \{c\}, m_{c,i,k'}^{(p-1)}, \forall k' \neq k]}{\Pr[b_{i,k} = 1 | m_i^{(0)}, m_{c,i}^{(p-1)}, \forall c' \in V_i \setminus \{c\}, m_{c,i,k'}^{(p-1)}, \forall k' \neq k]}, \quad \forall k,
\]

(4)

where \( m_i^{(0)} \) denotes the prior probabilities of the \( i \)th information symbol, input from other receiver modules (e.g., a channel decoder); \( V_i \) denotes the set of all channel nodes incident to information node \( i \).

In (4), assume that the messages \( m_i^{(0)} \) and \( m_{i,c}^{(p-1)} \), for all \( c' \) are independent random variables, then we have

\[
G_{i,c,k}(m_i^{(0)}, m_{c,i}^{(p)}, \forall c' \in V_i) = m_i^{(0)} + \sum_{c' \in V_i \setminus \{c\}} m_{c,i}^{(p)}. \quad (5)
\]

On the other hand, we have the following three different approaches, that is, \( a \)-\textit{posteriori}-probability- (APP-) based scheme, linear-MMSSE-soft-interference-cancellation-(LMMSE-SIC)-based scheme, and match-filter-soft-interference-cancellation- (MF-SIC)-based scheme, to compute (3), that is,

\[
F_{c,i,k}(y_c, m_{c,i}^{(p)}, \forall i' \in U_c)
\]

\[
= \begin{cases} 
\sum_{e' \in a_{c,k}} \exp \left(-|y_c - \sum_{e \in U_i} h_{c,e-1} x_{i'}|^2/2\sigma^2 + \sum_{k=0}^{\log_2 |\Omega|-1} b_{c,k} \cdot m_{c,i,k}^{(p-1)} / 2 \right) - m_{i,c}^{(p-1)}, & \text{for APP,} \\
\sum_{e' \in a_{c,k}} \exp \left(-|y_c - \sum_{e \in U_i} h_{c,e-1} x_{i'}|^2/2\sigma^2 + \sum_{k=0}^{\log_2 |\Omega|-1} b_{c,k} \cdot m_{c,i,k}^{(p-1)} / 2 \right) - m_{i,c}^{(p-1)}, & \text{for LMMSE-SIC, MF-SIC,} \\
\end{cases}
\]

(6)

and for LMMSE-SIC,

\[
w_{c,i} = \frac{h_{c,e-i}^*}{\sum_{e' \in U_i \setminus \{i\}} |h_{c,e-i}|^2 (1 - |\bar{x}_{c,e-i}|^2) + |h_{c,e-i}|^2 + \sigma^2}, \quad m_{c,i} = w_{c,i}^* h_{c,c-i}, \quad \nu_{c,i} = \mu_{c,i} - \mu_{c,i}^2,
\]

(7)

and for MF-SIC,

\[
w_{c,i} = \frac{h_{c,e-i}}{|h_{c,e-i}|^2}, \quad \mu_{c,i} = 1, \quad \sigma_{c,i} = \frac{\sum_{e' \in U_i \setminus \{i\}} |h_{c,e-i}|^2 (1 - |\bar{x}_{c,e-i}|^2) + \sigma^2}{|h_{c,e-i}|^2},
\]

(8)
First, consider a sparse 4-path fading channel:

4.1. SISO multipath fading channels

In this section, we analyze the factor-graph-based soft self-iterative equalizer.

**Algorithm 1:** Algorithm description of the factor-graph-based soft self-iterative equalizer.

```
Initialize: for all edges
m_{i0} = 0
for all edges
m_{i0} = F_i(y_i, m_{i0}^{(0)}, \forall i' \in \mathcal{U}_i)

Self-iterative equalize:
for p = 1 to P
/* compute messages from channel nodes to information nodes */ for all edges
m_{i0} = F_i(y_i, m_{i0}^{(p)}, \forall i' \in \mathcal{U}_i)
/* compute messages from information nodes to channel nodes */ for all edges
m_{i0} = G_i(m_{i0}^{(p)}, m_{c0}^{(p)}, \forall c' \in \mathcal{V}_i)
end

Output: /* compute information symbols' a posteriori probabilities m_i^{(p)} */ for i = 0 to T
m_i^{(p)} = \sum_{c' \in \mathcal{V}_i} m_{c0}^{(p)}
end
```

with \( \tilde{y}_c = \sum_{i \in \mathcal{U}_c \setminus \{i\}} h_{c,i} x_i - t \),
\[
\tilde{x}_i = \sum_{x_i \in \Omega} \log_{|\Omega|-1} \frac{b_{i,k} m_{x_i}^{(p-1)}}{1 + b_{i,k} m_{x_i}^{(p-1)}},
\]
where \( \delta_{i,k}^+ \) is the set defined as \( \{x_i \in \Omega \mid b_{i,k} = 0\} \), and similarly is \( \delta_{i,k}^- \); \( Q_{i,k}^+ \) is the union of \( \{x_i \in \Omega \mid \forall i' \in \mathcal{U}_c \setminus \{i\}\} \) and \( \delta_{i,k}^- \), and similarly is \( Q_{i,k}^- \). The detailed derivation of (6) is shown in the appendix.

Finally, the whole steps of the proposed equalizer are given in Algorithm 1.

4. NUMERICAL SIMULATIONS AND ANALYSIS

In this section, we analyze the factor-graph-based soft self-iterative equalizer in sparse wireless multipath channels through numerical simulations. For simplicity, we assume that channel gains remain constant in one frame and change independently from one to the other. The modulator uses the QPSK constellation with Gray mapping. Each frame contains 128 QPSK symbols per transmit antenna; proper zero prefix information symbols are inserted in each frame.

The soft equalizer is a self-iterative device; we only study the uncoded system. The performance is evaluated in terms of frame error rate (FER) versus the signal-to-noise ratio (SNR).

4.1. SISO multipath fading channels

First, consider a sparse 4-path fading channel: \( y_t = h_{0} x_t + h_3 x_{t-3} + n_t \), with \( E(|h_0|^2) = 0.7, E(|h_3|^2) = 0.2 \); thus, \( L = 4 \) and \( L' = 2 \). In Figure 3, the performance of three different approaches, (i.e., APP, LMMSE-SIC, and MF-SIC), to computing the extrinsic messages passed from channel nodes to information nodes is presented. For each scheme, total six iterations, that is, \( P = 6 \), are conducted in the self-iterative equalizer. Serving as a benchmark, the performance of the optimum maximum likelihood equalizer based on BCJR algorithm is also included in the figure. Since the factor graph of this channel is cycle free, the belief propagation algorithm theoretically is able to achieve optimum performance. Indeed, the soft iterative equalizer using APP-based message update scheme achieves the optimum performance after a few iterations. On the contrary, two low-complexity schemes, LMMSE-SIC and MF-SIC, suffer error floors at high SNRs. We remark that the prior probability input from other receiver modules (e.g., channel decoder) can lower but never eradicate such error floors; henceforth we will only consider the APP-based scheme for channel node message updating.

Now, consider a sparse 5-path fading channel: \( y_t = h_0 x_t + h_3 x_{t-3} + h_4 x_{t-4} + n_t \), where \( E(|h_0|^2) = 0.7, E(|h_3|^2) = 0.2 \), and \( E(|h_4|^2) = 0.1 \); thus, \( L = 5 \) and \( L' = 3 \). As seen in Figure 1, there exist a number of cycles with length 8 in the factor graph, where a “cycle” is defined as a close loop in the graph and its “length” is defined as the number of edges traversed by that loop. This cycle condition accounts for the marginal gap between the factor-graph-based equalization and the optimum performance, as shown in Figure 4.

4.2. MISO multipath fading channels

Equalization of MISO multipath channels falls into the group of “underdetermined” problems: at each time instance a mix-
The complexity of optimum BCJR equalizer soon becomes prohibitive for $(2 \times 3)$ MIMO channels, but it suffers considerable performance loss in $4 \times 4$ MIMO channels. Especially from the $(4 \times 4)$ MIMO case, we conclude that the direct application of the factor-graph-based equalizer may not be a good option for MIMO channels. It is seen from Figure 8 that the above observation also holds for MIMO multipath channels—as much as 2.5 dB performance loss is seen in a $(2 \times 2)$ MIMO with 3 multipaths.

**Alternative factor graph representation for MIMO multipath fading channels**

The previous simulation results and analysis has identified the difficulty in directly applying the factor-graph-based equalizer in MIMO channels. An alternative way to ameliorate this problem is to reconstruct the underlying factor graphs. Shown in Figure 9 the idea is to glue all channel nodes in the original graph $\{y_{1,t}, \ldots, y_{nR,t}\}$ that corresponds to different receiver antennas at the same time instance $t$ into...
a new channel node $y_t \triangleq [y_{1,t}, \ldots, y_{nR,t}]^T$; the channel coefficient on each edge is now an $(n_R \times 1)$ vector instead of a scalar. In doing so, the alternative factor graph still represents the same MIMO multipath systems, but the extensive short cycles due to multiple receive antennas are systematically avoided. The belief propagation algorithm can be accordingly rederived; and in single-path channels, it converges in one iteration and coincides with the optimal APP MIMO demodulator [12]. With this alternative factor graph representation, we can continue to apply the self-iterative equalizer for MIMO multipath fading channels to improve the performance. We now consider the case of $(2 \times 2)$ MIMO with 3 multipaths as an example. The FER curves are shown in Figure 10. It is seen that the resulting performance is significantly improved and approaches the performance from the optimum demodulation.

5. CONCLUSIONS

Since a factor graph is able to characterize multipath channels to per-path level, the factor-graph-based soft self-iterative equalizer with reduced computational complexity is a potential candidate for sparse multipath channel...
equalization. By numerical simulations, we have shown that the cycles in factor graphs are crucial to the convergence property of the considered soft self-iterative equalization. While being able to achieve near- optimum performance in single-input single-output (SISO) and multiple-input single-output (MISO) sparse multipath channels with mild cycle conditions, a factor-graph-based soft self-iterative equalizer may suffer noticeable performance loss in multiple-output (MIMO) multipath channels, unless proper means is taken to ameliorate the cycle conditions in factor graphs.

\section*{Appendix}

\subsection*{Derivation of (6)}

(i) For APP detection, we have

\begin{equation}
F_{c_i k}(y_c, m^{(p)}_{c_i k}, \forall k \in \mathcal{U}_c)
= \log \frac{\sum_{x_i \in \mathcal{A}_{c_i k}} P(x_i = x_i | y_c)}{\sum_{x_i \in \mathcal{A}_{c_i k}} P(x_i = x_i | y_c)} - \log \frac{P(b_{i k} = +1)}{P(b_{i k} = -1)}
\end{equation}

\begin{align}
&= \log \frac{\sum_{x_i \in \mathcal{A}_{c_i k}} P(y_c | x_i) P(x_i | y_c)}{\sum_{x_i \in \mathcal{A}_{c_i k}} P(y_c | x_i) P(x_i | y_c)} - m^{(p-1)}_{c_i k}
&= \log \frac{\sum_{x_i \in \mathcal{A}_{c_i k}} \exp \left( - | y_c - \sum_{i \in \mathcal{U}_c} h_{c_i r} x_{r} |^2 / \sigma^2 \right) \prod_{x_i \in \mathcal{A}_{c_i k}} P\left( b_{i k}^{(p-1)} \right)}{\sum_{x_i \in \mathcal{A}_{c_i k}} \exp \left( - | y_c - \sum_{i \in \mathcal{U}_c} h_{c_i r} x_{r} |^2 / \sigma^2 \right) \prod_{x_i \in \mathcal{A}_{c_i k}} P\left( b_{i k}^{(p-1)} \right)} - m^{(p-1)}_{c_i k}
&= \log \frac{\sum_{x_i \in \mathcal{A}_{c_i k}} \exp \left( - | y_c - \sum_{i \in \mathcal{U}_c} h_{c_i r} x_{r} |^2 / \sigma^2 + \sum_{k=0}^{[\Omega]-1} b_{r k} m^{(p-1)}_{c_i k} / 2 \right)}{\sum_{x_i \in \mathcal{A}_{c_i k}} \exp \left( - | y_c - \sum_{i \in \mathcal{U}_c} h_{c_i r} x_{r} |^2 / \sigma^2 + \sum_{k=0}^{[\Omega]-1} b_{r k} m^{(p-1)}_{c_i k} / 2 \right)} - m^{(p-1)}_{c_i k}.
\end{align}

(ii) For LMMSE-SIC detection, we first obtain the MMSE filtering output, given by

\begin{equation}
z_{c_i} = w_{c_i}^*(y_c - \hat{y}_c).
\end{equation}

Based on Gaussian approximation of $z_{c_i}$, the extrinsic messages can be computed by

\begin{equation}
F_{c_i k}(y_c, m^{(p)}_{c_i k}, \forall k \in \mathcal{U}_c)
= \log \frac{\sum_{x_i \in \mathcal{A}_{c_i k}} \exp \left( - | z_{c_i} - \mu_{c_i x_i} |^2 / \nu_{c_i}^2 \right) \prod_{x_i \in \mathcal{A}_{c_i k}} P\left( b_{i k}^{(p-1)} \right)}{\sum_{x_i \in \mathcal{A}_{c_i k}} \exp \left( - | z_{c_i} - \mu_{c_i x_i} |^2 / \nu_{c_i}^2 \right) \prod_{x_i \in \mathcal{A}_{c_i k}} P\left( b_{i k}^{(p-1)} \right)} - m^{(p-1)}_{c_i k}
\end{equation}

\begin{align}
&= \log \frac{\sum_{x_i \in \mathcal{A}_{c_i k}} \exp \left( - | w^*_{c_i}(y_c - \hat{y}_c) - \mu_{c_i x_i} |^2 / \nu_{c_i}^2 + \sum_{k=0}^{[\Omega]-1} b_{r k} m^{(p-1)}_{c_i k} / 2 \right)}{\sum_{x_i \in \mathcal{A}_{c_i k}} \exp \left( - | w^*_{c_i}(y_c - \hat{y}_c) - \mu_{c_i x_i} |^2 / \nu_{c_i}^2 + \sum_{k=0}^{[\Omega]-1} b_{r k} m^{(p-1)}_{c_i k} / 2 \right)} - m^{(p-1)}_{c_i k},
\end{align}

where

\begin{align}
w^*_{c_i} &= \frac{h^*_{c_i}}{\sum_{j \in \mathcal{U}_c \setminus \{i\}} | h_{c_j r_i} |^2 (1 - | x_{r_i} |^2)^2 + | h_{c_j r_i} |^2 + \sigma^2),
\mu_{c_i} &= w^*_{c_i} h_{c_i r_i},
\nu_{c_i}^2 &= \mu_{c_i} - \mu_{c_i}^2.
\end{align}

The details for obtaining $w^*_{c_i}$, $\mu_{c_i}$, and $\nu_{c_i}^2$ can be found in [7].

(iii) For MF-SIC, we simply apply the match filter to the soft interference canceled output, that is,

\begin{equation}
z_{c_i} = w^*_{c_i}(y_c - \hat{y}_c), \quad w^*_{c_i} = \frac{h^*_{c_i}}{| h_{c_i r_i} |^2}.
\end{equation}

We then approximate the MF-SIC output as Gaussian distributed, and compute extrinsic message in the same form in
Figure 10: FER performance of the soft iterative equalizer based on alternative factor graph representation in MIMO multipath fading channels ($n_T = 2$, $n_R = 2$, $L = 3$, $L' = 2$).

\[ \mu_{c,i} = 1, \]

\[ \nu_{c,i}^2 = \sum_{\ell \in U \setminus \{i\}} |h_{c,c-\ell}|^2 \left( 1 - |\hat{x}_{c-\ell}|^2 \right) + \sigma^2. \]  

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