INTRODUCTION TO THE SPECIAL VOLUME
“MATHEMATICS OF NONLINEAR ACOUSTICS: NEW APPROACHES IN ANALYSIS AND MODELING”

Over the last 12–15 years, there has been a resurgence of interest in the study of nonlinear acoustic phenomena. Using the tools of both classical mathematical analysis and computational physics, researchers have obtained a wide range of new results, some of which might be described as remarkable. As with almost all trends in science, the reasons for this revival are varied: they range from practical applications (e.g., the need to improve our understanding of high-intensity ultrasound); to the development of numerical schemes which are better at capturing the physics of nonlinear compressible flow; to new acoustic models which lend themselves to study by analytical methods.

This volume presents a collection of papers in which mathematical tools and methods are applied to problems formulated around nonlinear acoustic models. As such, the majority of contributions deals with modeling; in particular, the investigation of existing, as well as newly proposed, models using analytic and/or numerical tools.

In the paper by Christov, the equations of motion for lossless compressible fluids described by a quadratic equation of state are reformulated under so-called Green–Naghdi theory. The author achieves an exact reduction in terms of the scalar acoustic potential and the (scalar) thermal displacement. Shock formation via acceleration wave blow up is studied numerically, thereby illustrating prior analytical results on finite-time amplitude blow-up.

Gaididei, Rasmussen, Christiansen and Sørensen in their contribution investigate oscillating shock waves in a tube using a higher-order weakly nonlinear acoustic model. Based on an augmented travelling wave ansatz for the fluid’s velocity potential, they find analytical approximations to the numerically observed single shock waves in an infinitely long tube. Using perturbation theory, approximate solutions for the off-resonant case of the driven system are also determined.

Propagation of acoustic and thermal waves in classical perfect gases under a coupled, weakly-nonlinear system (first derived by Blackstock) is considered in the paper by Jordan. Blackstock’s system is compared analytically and numerically with a simpler, weakly-nonlinear model whose constituent equations are not coupled. In particular, the author determines traveling wave solutions and analyzes the structure of the resulting solution profiles, including those admitted by the temperature and specific entropy. Results for the special case of the Navier–Stokes equations corresponding to Becker’s assumption are also provided and details on the extension of Becker’s work by others are noted.

Morro studies flow and wave propagation in mixtures. He establishes the complete set of balance equations for chemically-reacting fluid mixtures and shows that the diffusion flux relative to the barycentric reference satisfies a first-order, nonlinear differential equation, which implies that the diffusion flux is given by a balance equation, not by a constitutive assumption at the outset. This balance equation
for the diffusion flux makes the system of equations for the mixture hyperbolic, provided heat conduction and viscosity are disregarded.

The contribution by Norton and Purrington investigates heat deposition due to finite amplitude propagation through a dispersive medium, e.g., human tissue. To this end, the Westervelt equation is coupled to the Pennes bioheat equation. Additionally, the authors replace the typical thermoviscous loss mechanism in the Westervelt equation with a causal Time Domain Propagation Factor (TDPF), which incorporates the effects of both attenuation and dispersion. It is then shown that non-linear effects can be important and that the proper treatment of dispersion results in significant differences vis-à-vis modeling the medium as an ordinary thermoviscous fluid.

Wave propagation in a poroacoustic medium is the topic of the paper by Rossmanith and Puri. They begin with a weakly-nonlinear model based on the Brinkman equation and reduce it to the damped Burgers equation via the unidirectional approximation. These authors derive an approximate traveling wave solution to this equation and then study the effects upon it of varying the Darcy coefficient, Reynolds number, and coefficient of nonlinearity. A similar approach is then employed to study Cole’s problem involving the damped Burgers equation, for which an approximation is obtained using energy-based arguments.

Finally, Kaltenbacher and Peichl consider an optimization problem arising in lithotripsy, namely the shape design for the arrangement of a transducer array to generate and focus high intensity ultrasound. The resulting shape optimization problem is considered in the context of a classical model of nonlinear acoustics, the Westervelt equation. The paper concentrates on a rigorous derivation and justification of the shape derivative, based on the variational framework from Ito et al. (2008), as well as additional regularity results.

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