The next big step in our understanding of particle physics will be the uncovering of the electro-weak symmetry breaking (EWSB) mechanism. The present and upcoming collider experiments (Fermilab and LHC) will be able to test the Standard Model (SM) Higgs mechanism. New physics is widely expected at around the TeV scale if the Higgs mass is not to receive large radiative corrections and require severe fine-tuning. A stringent constraint on the SM mechanism of EWSB is the tight structure of flavor changing (FC) interactions: tree-level FC neutral currents are forbidden and charged currents are controlled by the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix

\[
V = \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \frac{\lambda}{2} & A\lambda^2(\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^2(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}.
\]

Within the SM, the CKM matrix is the only source of FC interactions and of CP violation. There is no reason, in general, to expect that new physics (needed to stabilize the Higgs mass) at the TeV scale will be in the basis wherein the quark mass matrix is diagonal. This reasoning gives rise to another fundamental problem in particle physics, namely the flavor puzzle i.e. unless the scale of new physics is larger than \(10^3\) TeV it causes large FCNC effects especially for the \(K - \bar{K}\) system. Thus flavor physics provides constraints on models of new physics up to scales that are much much larger than what is accessible to direct searches at colliders such as the Tevatron or the LHC. Flavor physics is therefore expected to continue to provide crucial information for the interpretation of any physics that LHC may find.

In this decade significant progress has been made in our understanding of flavor physics, thanks in large part to the spectacular performance of the two asymmetric B-factories. For the first time, we learned that the CKM paradigm of CP violation is able to simultaneously account for the observed CP-violation in the K and B-systems up to an accuracy of \(O(20\%)\). Impressive as this is, it is still important to understand that this leaves a lot of room for new physics. Indeed, as more data from B-factories became available and also as the accuracy in some key theoretical calculations was attained, several 2-3 \(\sigma\) hints of new physics have emerged. While this clearly does not represent an unambiguous signal for new physics, it does mean that efforts need to continue both on the experiment and on the theory front to seek greater clarity with regard to these anomalies.

In this context use of semileptonic decays in all traditional analysis of the Unitarity Triangle (UT) to date is a concern. The inclusive \(b \rightarrow u\) transitions are not governed by any symmetry and as a result are a special challenge for continuum methods. Exclusive decays are in principle amenable to the lattice and steady, but unfortunately rather slow, progress is being made. The fact that for both \(b \rightarrow c\) and for \(b \rightarrow u\) inclusive and exclusive methods disagree by \(\approx 2\sigma\) casts a shadow of doubt on the results of the UT analysis. This is especially aggravated by the fact that use of the input from \(\epsilon_K\), representing the indirect CP violation from the \(K_L \rightarrow \pi\pi\), into the UT is exceedingly sensitive to \(V_{ub}\), scaling as the fourth power. These observations motivate us to seek alternative approaches, which we will provide herein, though we want to emphasize that we are not suggesting that efforts to improve our understanding of semi-leptonic decays be discontinued but rather that alternate methods of analysis for the UT could be very valuable and should also be developed.

Recent improvements especially in the lattice calculation of \(B_K\) \[2, 3, 4, 5\], led to the appearance of a ~ 2\(\sigma\) tension that can be interpreted as new physics in \(B_d\) and/or in K mixing \[6, 7, 8, 9, 10\]. The interplay of \(\epsilon_K\) with \(\Delta M_{B_s}/\Delta M_{B_d}\) and \(S_{\psi K}\) (time–dependent CP asymmetry in \(B \rightarrow J/\psi K_s\)) is at the heart of the tension. The inclusion of \(|V_{ub}|\) from semileptonic \(b \rightarrow u\ell\nu\) decays, tends to favor a scenario with new physics in kaon mixing \[11\]. An important difficulty with these analyses is the long standing discrepancy between the extraction of \(|V_{ub}|\) and \(|V_{td}|\) from exclusive and inclusive semileptonic decays alluded to above. From the inspection of Table \[\] one sees that inclusive and exclusive determinations dif-

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Unitarity Triangle Without Semileptonic Decays

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Use of semi-leptonic decays has become standard in constraining the Unitarity Triangle. Bearing in mind that precise calculations of these are very challenging, we propose an entirely new approach. In particular the \(|V_{cb}| + \epsilon_K\) constraint, which depends extremely sensitively on \(|V_{cb}|\) in the traditional method, is replaced by the interplay between \(\epsilon_K\), \(\text{BR}(B \rightarrow \tau\nu)\) and \(\Delta M_{B_s}\). It is found that even in this method tensions with the Standard Model persist at the \(\sim 1.8\sigma\) level. Furthermore, improvements on the \(B \rightarrow \tau\nu\) branching ratio and on the lattice determination of \(f_{B_s}\) increase the effectiveness of this method significantly.

Semileptonic Decays

\[V = \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \frac{\lambda}{2} & A\lambda^2(\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^2(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}.
\]
fer at the 2σ level. While Ref. [2] demonstrated that $|V_{ub}|$ can be dropped from the fit without affecting the observed tension, it is usually believed that $|V_{cb}| \approx \Lambda^2$ from semileptonic decays is essential in order to use $\varepsilon_K$ (because of its $\Lambda^4$ dependence).

Bearing all this in mind, in this letter we propose a new approach to the UT analysis, wherein no use of semileptonic decays is made. In particular, we show that the new approach to the UT analysis, wherein no use of semileptonic decays is essential in order to use $\varepsilon_K$ (that the impact of the approach we champion in this letter remains largely unaffected even if the lattice errors could be employed can be found in Refs. [18, 19]. We follow the approach of Refs. [7, 10] and utilize the averages calculated in Ref. [11] with some exceptions: we include inclusive $|V_{ub}|$, albeit with an additional 10% model uncertainties [20]; we take a simple (not weighted) average of the determinations of $\xi$ from Fermilab/MIILC [21] and HPQCD [22]. Also we take the central value of $f_B, \sqrt{B}_s$ from Ref. [11] but adopt the uncertainty quoted in Ref. [22]. We adopt this conservative stance to show that the impact of the approach we champion in this letter remains largely unaffected even if the lattice errors are not as small as currently claimed in the literature.

We summarize the inputs we use in Table I. Below we first present explicitly only those formulas that are relevant to the traditional analysis which uses semi-leptonic decays:

\begin{equation}
\Delta M_{B_s} = \chi_f f_B^2 B_{B_s} A^2 \lambda^4
\end{equation}
\begin{equation}
\Delta M_{B_d} = \chi_d f_B^2 B_{B_d} A^2 \lambda^6 (\eta^2 + (-1 + \rho)^2)
\end{equation}
\begin{equation}
\Delta M_{B_s} = \frac{m_{B_s} \xi^2 \lambda^2}{m_{B_s} \eta^2 + (-1 + \rho)^2}
\end{equation}
\begin{equation}
|\varepsilon_K| = 2 \chi f_B B_{B_s} \lambda^6 (A^2 \lambda^4 (\rho - 1) \eta_2 S_0(x_t) + A^2 (\eta_2 S_0(x_t) - \eta_1 S_0(x_t)))
\end{equation}
\begin{equation}
BR(B \to \tau \nu) = \chi_f f_B^2 |V_{ub}|^2 \approx \chi_f f_B^2 A^2 \lambda^6 (\rho^2 + \eta^2)
\end{equation}
where we expanded in $\lambda$ and defined

$$
\chi_q = G_F^2 m_W^2 m_B \eta B S_0(x_t)/(6\pi^2),
$$

$$
\chi_\epsilon = (G_F^2 m_W^2 f_K^2 m_K)/(12\sqrt{2}\pi^2 \Delta m_K^{\exp}),
$$

$$
\chi_\tau = G_F^2 m_W^2 m_{B/+(8\pi\Gamma_{B/})}^2/(1-m_c^2/m_b^2)^2.
$$

The 68% C.L. allowed regions in the $(\rho, \eta)$ plane are shown in Fig. 1 where we show explicitly that the $\epsilon_K$, $B \to \tau \nu$ (pink) and $|V_{ub}|$ (yellow) constraints require $|V_{cb}|$ in order to be drawn independently. In particular we obtain:

$$
|V_{ub}|_{\text{fit}} = (3.61 \pm 0.13) \times 10^{-3},
$$

$$
\text{BR}(B \to \tau \nu)_{\text{fit}} = (0.87 \pm 0.11) \times 10^{-4},
$$

$$
|\sin 2\beta|_{\text{fit}} = 0.766 \pm 0.036,
$$

where each quantity is extracted by removing the corresponding direct determination form the fit (for $V_{ub}$ this means removing information from semileptonic $b \to u\ell \nu$ decays [17]). Note that $|V_{ub}|_{\text{fit}}$ is close to the determination from exclusive decays and that BR$(B \to \tau \nu)_{\text{fit}}$ is smaller than the corresponding world average $(1.43 \pm 0.37) \times 10^{-4}$ [13]. The result in Eq. (10) is driven by the updated value for the $B$ decay constant. The relatively low p–value [23] ($p = 15\%$) of the overall fit has been interpreted in terms of new physics in either $K$ or in $B_d$ mixing [6] [7] [8] [9] [10] [11].

Adopting the model independent parametrizations

$$
|\epsilon_{K}^{NP}| = C_\epsilon |\epsilon_{K}^{SM}|,
$$

$$
M_{12}^{NP} = e^{i\theta_\delta} M_{12}^{SM},
$$

$$
\text{BR}(B \to \tau \nu)^{NP} = r_H \text{BR}(B \to \tau \nu)^{SM},
$$

where $M_{12}^{NP}$ is the matrix element of the effective Hamiltonian between $B_d$ and $\bar{B}_d$ meson states and in the SM we have $(C_\epsilon, r_H) = 1$ and $\theta_\delta = 0$. We obtain

$$
C_\epsilon = 1.28 \pm 0.14 \Rightarrow (2.0\sigma, p = 58\%)
$$

$$
\theta_\delta = -(4.0 \pm 1.8)^\circ \Rightarrow (2.2\sigma, p = 61\%)
$$

$$
r_H = 1.7 \pm 0.5 \Rightarrow (4.0\sigma, p = 29\%).
$$

The above results are mutually exclusive in the sense that they are obtained by allowing either new physics in $K$, or in $B_d$ mixing or in $B_u \to \tau \nu$ and point to a $\sim 2\sigma$ hint for new physics. The quoted p–values have to be compared with SM result ($p_{SM} = 15\%$). The lower p–value for new physics in $B \to \tau \nu$ indicates that the tension in the fit is only partially lifted by new contributions to $B \to \tau \nu$.

Removing semileptonic decays Recall that inclusive and exclusive $b \to (c, u)\ell \nu$ decays are tree–level Standard Model processes and, therefore most likely, are quite insensitive to the presence of new physics. The $|V_{ub}|$ constraint translates into a determination of the parameter $A$ of the CKM matrix. Knowledge of the latter is critical in order to extract information from $|V_{ub}| \propto A$, $\text{BR}(B \to \tau \nu) \propto A^2$ and $\epsilon_K \propto A^4$ (see Eqs. (4,5)). The $(\rho, \eta)$ regions allowed by each of these three observables is obtained with the inclusion of $|V_{ub}|$. Without any information on $A$ these bands would cover the whole $(\rho, \eta)$ plane. The main role of the determination of $|V_{ub}|$ is to limit the amount of new physics contributions to the phase of $B_d$ mixing; in fact, an upper limit on $|V_{ub}|$ implies an upper limit on $S_{\psi K} = \sin 2(\beta + \theta_\delta)$ [24] [25]. Presently inclusive and exclusive determinations of $|V_{ub}|$ and $|V_{ub}|$ differ at the $2\sigma$ level (see Table III). The exclusion of the $|V_{ub}|$ constraint is not critical any longer to the presence of the $2\sigma$ effects in Eqs. (15) [16] as emphasized recently in [7]. In particular the prediction that we obtain for the $B_u$ mixing phase in the no–$V_{ub}$ scenario reads $|\sin 2\beta|_{\text{fit}} = 0.840 \pm 0.056$ deviating by $2.8\sigma$ from its direct determination. On the other hand, $|V_{cb}|$ appears to be central: employing only its exclusive (inclusive) determination, the $2.0\sigma$ significance of the extraction of $C_\epsilon$ shifts to $2.5\sigma (1.6\sigma)$; similarly the $2.2\sigma$ effect in $B_d$ mixing shifts to $2.9\sigma (1.4\sigma)$.

We now come to elaborating on the new approach that we are advocating here in which no use of semi-leptonic decays will be made. Note that the critical issue is the determination of $A$ from $|V_{ub}|$. We find that the interplay of $\epsilon_K$, $\text{BR}(B \to \tau \nu)$ and $\Delta M_{B_s}$ results in a fairly strong constraint on the $(\rho, \eta)$ plane even without using semileptonic decays at all. A simple way to understand
as is the case for the large tan β=1500 region. In the MSSM, chargino loops contributions to the charged Higgs can only reduce the BR(τν) level only the large tan β is, in turn, excluded by $B\rightarrow Dτν$ data both in the 2HDM and in the MSSM (we follow the numerical analysis of Ref. [27]). At 95% C.L. the solution $X_H=0$ opens up, corresponding to large $M_{H^\pm}$. In the 2HDM the $B\rightarrow X_sγ$ constraint implies $m_{H^\pm}>295$ GeV [28]. In the MSSM, chargino loops contributions to the $b\rightarrow sγ$ amplitude can compensate charged Higgs effects: the bound on the charged Higgs depends strongly on the chosen point in the supersymmetric parameter space [29].

Let us now discuss the dominant sources of uncertainties in this analysis. In the following table we list the most relevant inputs, their errors and their impact on $ε_K$ (as it follows from Eqs. [4], [18] and [19]):

| $X$ | $\hat B_K$ | $|V_{ub}|$ | $f_{B_d}\hat B_{1/2}^d$ | BR($B\rightarrow τν$) | $f_B$ |
|-----|------------|--------|-----------------|-------------------|------|
| $δX$ | 4% | 2.5% | 6.9% | 26% | 5% |
| $δε_K$ | 4% | 10% | 27.6% | 52% | 20% |

First of all, note that the impact of $\hat B_K$ on the error is
subdominant. The use of the semileptonic $b \to c$ constraint results in a $\sim 10\%$ determination of $\varepsilon_K$, roughly half of the uncertainty obtained by employing only $\Delta M_{B_s}$ (i.e.: $f_{B_s, \hat{B}_s^{1/2}}$). A calculation of $f_{B, \hat{B}_s^{1/2}}$ at the 2.5% level would reduce the overall uncertainty on $\varepsilon_K$ to 10%; a calculation at the 1% level would impact $\varepsilon_K$ at the same level as $\hat{B}_K$. At first sight, the impact of $B \to \tau \nu$ seems irrelevant. Fortunately the non–trivial dependence of $\text{BR}(B \to \tau \nu)$ on $\rho$ and $\eta$ implies a certain degree of orthogonality between the constraints [18] and [19], as can be seen explicitly in the upper panel of Fig. 2. A numerical estimate of the impact of this constraint can be obtained by removing it from the fit and recalculating the overall p–value: we obtain $p = 43\%$, meaning that no hint of new physics is observed. The experimental uncertainty on the $B \to \tau \nu$ branching ratio is therefore an important ingredient of this analysis. Once the latter reaches the 10% level, improvements on $f_B$ will be relevant as well. We summarize this discussion in Fig. 4 and in the following table:

| $\delta_\tau$ | $\delta_s$ | $p_{SM}$ | $\theta_d \pm \delta\theta_d$ | $\theta_d^\ast \delta\theta_d$ |
|--------------|-------------|----------|--------------------------------|-------------------------------|
| 26% *6.8%    | 32%         | -8(4.6)  | 87% 1.8$\sigma$               |
| 26% 2.5%     | 3.3%        | -9(6.3)  | 85% 2.7$\sigma$               |
| 26% 1%       | 0.1%        | -10(0.3)| 84% 3.4$\sigma$               |
| 10% *6.8%    | 0%          | -8(2.8)  | 87% 3.1$\sigma$               |
| 5% *6.8%     | 0.08%       | -8(2.3)  | 87% 3.8$\sigma$               |
| 10% 2.5%     | 0.1%        | -9(2.5)  | 84% 3.7$\sigma$               |
| 10% 1%       | 0.004%      | -9(2.6)  | 83% 4.3$\sigma$               |
| 5% 2.5%      | 0.004%      | -9(1.2)  | 84% 4.4$\sigma$               |
| 3% 1%        | 0.00009%    | -9(1.9)  | 82% 5.0$\sigma$               |

where $\delta_\tau = \delta\text{BR}(B \to \tau \nu)$, $\delta_s = \delta(f_{B, \hat{B}_s^{1/2}})$ and * denotes the current uncertainties. The values $\delta_\tau = (10, 3)\%$ correspond to a super–B factory [31, 32, 33] while the reduction of $\delta_s$ is a purely theoretical (i.e. lattice) issue. Note also that lattice calculations of matrix elements relevant for decay constants or for $(K^0, B_d, B_s)$ oscillations and therefore for $\delta_s$, do not require momentum injection unlike the calculation of the semileptonic form factors and to that extent are simpler. However, we stress again that we are not suggesting abandoning the traditional approach with use of semileptonic decays, but rather in addition making concerted efforts towards improved lattice determination of $f_{B, \hat{B}_s^{1/2}}$ and also of the $\text{BR}(B \to \tau \nu)$. These should provide valuable redundancy in our quest for new physics through flavor studies even in the LHC era. Finally we would like to stress that the main focus of the present letter is to propose a new clean strategy to implement simultaneously $K$ and $B_d$ mixing constraints on the $(\rho, \eta)$–plane and that our projections on the reach of this method depend solely on improving $\delta_\tau$ and $\delta_s$ and are quite insensitive to the rest of the inputs summarized in Table 4 in particular, the assumed errors in lattice computations.

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