AdS/dS CFT Correspondence

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Abstract

We propose and study a holographic dual of the type IIB superstring theory of $\text{AdS}_5 \times S^5$ in terms of the $\mathcal{N} = 4$ superconformal Yang-Mills theory on $\text{dS}_4$. We use the bulk to boundary formalism to evaluate the boundary correlation functions and verify that it agrees with the expected result in $\text{dS}$ conformal field theory. The gauge theory is expected to be UV finite and enjoy exact $SL(2, \mathbb{Z})$ strong-weak duality. As the string theory Green-Schwarz sigma model carries an infinite number of classically conserved charges, it also suggest that the superconformal Yang-Mills theory is integrable and deserves further studies.
1 Introduction

de Sitter space plays a central role in modern theoretical physics. It is not only relevant for the description of the late time cosmology. It is also widely believed that the universe has underwent a period of inflationary expansion described by a quasi-de Sitter metric. As such it is of utmost importance to understand the quantum dynamics of de Sitter space. Despite intensive research, e.g. [1, 2, 3] for some of the approaches, the problem of quantum gravity in de Sitter space remains open. In fact even the much less ambitious problem of a quantum field theory dynamics in de Sitter space is already quite a nontrivial problem. Although the problem of the definition of Hamiltonian and particles for a quantum field theory (QFT) in a generic curved spacetime [4] can be dealt with in perturbation theory for QFT in de Sitter space, it is not necessary so in the strongly coupled regime, in which case we also expect new phenomena may arise. Besides, the understanding of the long time secular effects in de Sitter space is another important problem that called for a treatment beyond the usual perturbation scheme [6, 7, 8]. It has been speculated that [9, 10] the infrared (IR) quantum effects in dS space could provide a screening effect on the cosmological constant and offers a solution to the cosmological constant problem. One of the motivations of this work is the desire for a better understanding of the dynamics of quantum field theories in de Sitter spacetime beyond perturbation theory.

A powerful tool in this regard is the AdS/CFT correspondence [11]. The holographic nature of quantum gravity was originally suggested by the discovery of the thermodynamic nature of the black hole mechanics, most notably the Bekenstein-Hawking formula for black hole entropy [12, 13]. The AdS/CFT correspondence provides the first explicit example of how holography can be realized in string theory [11, 14, 15]. Over the years, it has also been used as an important tool to learn about the physical properties of various quantum field theories in the strongly coupled nonperturbative regime. See, for example, the reviews [16, 17, 18]. In some cases, when sufficient amount of supersymmetry or integrability is present, even exact results are possible [19, 20]. In the case of de Sitter space, gauge/gravity dual has been studied in [21]-[44]. In particular, evidence of dynamical phase transition for confining gauge theory on de Sitter space was found in [35]; entanglement of entropy for strongly coupled field theories on de Sitter space with a gravity dual was computed in [37] and their results suggested that the FRW cosmologies is contained in the field theory description. One notes that for the kind of questions addressed in these previous works, the gauge/gravity correspondence was only needed to be considered in the generic sense without having to spell out in details the involved string theory and the boundary field theory. Nevertheless it is certainly interesting to have a concrete duality so that one can ask precise questions of other kinds. Another motivation of this work is to construct such a more precise gauge/gravity correspondence in de Sitter spacetime.
The construction of global supersymmetric field theory in four dimensional de Sitter spacetime is however impossible [45, 46]. First of all, there does not exist Majorana Killing spinor on de Sitter spacetime, which are necessary for the construction of supersymmetry. Moreover, the usual de Sitter superalgebra has no unitary representation. These no-go theorems are likely to be the reason why a holographic duality involving a supersymmetric field theory on de Sitter spacetime has not been constructed. However it has been realized quite recently that the no-go theorem can be bypassed if global superconformal symmetry is considered instead of global supersymmetry. In particular the $\mathcal{N} = 4$ superconformal non-abelian Yang-Mills theory on dS$_4$ has been constructed [47]. The existence of this theory is also anticipated from the works of [48, 49]. This superconformal Yang-Mills theory is expected to enjoy exact $SU(2,2|4)$ supersymmetry [47]. One might also expect that this superconformal theory may share some of the remarkable properties such as integrability and S-duality like its cousin, the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on Minkowski spacetime, which would then allow exact results to be obtained. Hence it’s gauge/gravity correspondence would be a perfect laboratory to see how some of the nontrivial results of the AdS/CFT correspondence would extend in the presence of a time dependent background spacetime. The desire to study the properties of the $\mathcal{N} = 4$ superconformal Yang-Mills theory on de Sitter spacetime is another motivation of this work.

As it turns out, de Sitter spacetime can be obtained as the boundary of the AdS spacetime. In the Poincare patch of the AdS space, a boundary of Minkowski spacetime with a conformal structure was created [14]. However one may also choose a dS-sliced coordinates (8) of the AdS$_5$ spacetime and obtain dS$_4$ as the boundary manifold [50]. Based on this observation and the results of [47], we propose in this paper the AdS/dS CFT correspondence: Type IIB string theory on AdS$_5 \times S^5$ with boundary condition imposed on the boundary dS$_4$ is dual to the $\mathcal{N} = 4$ superconformal $SU(N)$ Yang-Mills theory on dS$_4$.

The plan of the paper is as follows. In section 2 we review the global definition of dS and AdS spacetime, and the coordination of AdS which give rises to Minkowski or de Sitter spacetime as boundaries. We also describe the $\mathcal{N} = 4$ superconformal Yang-Mills theory and some general properties of conformal field theory on de Sitter spacetime. In section 3, we introduce the bulk to boundary propagator formulation of the AdS/CFT correspondence [14] [15], generalized for a general bulk metric. In sections 4 and 5, we apply this formulation to compute the boundary correlators for conformal operators of scalar and spin 1/2 types. We show that the results obtained agree with the de Sitter conformal field theory. The paper is ended with further discussion in section 6.
2 AdS/dS CFT Correspondence

Our proposal is that Type IIB string theory on \( \text{AdS}_5 \times S^5 \) with boundary condition specified on the boundary \( \text{dS}_4 \) is dual to the \( \mathcal{N} = 4 \) superconformal \( SU(N) \) Yang-Mills theory on \( \text{dS}_4 \). Below let us spell out some of the basic elements of the duality.

2.1 dS Embedding in AdS

The \((d+1)\)-dimensional Anti-de Sitter space \( \text{AdS}_{d+1} \) is a maximally symmetric space with negative cosmological constant. It can be most easily defined by an embedding

\[
-X_0^2 + X_1^2 + \ldots + X_d^2 - X_{d+1}^2 = -L^2, \tag{1}
\]

in the \((d+2)\)-dimensional flat space \( \mathbb{R}^{d+2} \) with the metric \( \eta_{MN} = \text{diag}(-1, 1_d, -1) \). Here \( L \) is the radius of the AdS space and the cosmological constant is given by

\[
\Lambda_0 = \frac{d(d-1)}{2L^2}. \tag{2}
\]

\( \text{AdS}_{d+1} \) is invariant under the group \( SO(2, d) \) as both the embedding metric and the embedding equations are invariant under this transformation. Similarly the de Sitter space \( \text{dS} \) is a maximally symmetric space with positive cosmological constant. For \( d \)-dimensional dS space, it is given by the hyperboloid

\[
-Y_0^2 + Y_1^2 + \ldots + Y_d^2 = L^2, \tag{3}
\]

in the flat space \( \mathbb{R}^{d+1} \) with the metric \( \eta_{MN} = \text{diag}(-1, 1_d) \). The cosmological constant is

\[
\Lambda_1 = \frac{(d-1)(d-2)}{2L^2}, \tag{4}
\]

and \( \text{dS}_d \) has the symmetry group \( SO(1, d) \).

In the standard application of AdS/CFT correspondence, one uses the Poincare coordinates

\[
X_0 = \frac{r}{2} \left[ 1 + \frac{x^2 - t^2 + L^2}{r^2} \right], \quad X_i = \frac{L x_i}{r}, \quad i = 1, \ldots, d - 1,
\]

\[
X_d = \frac{r}{2} \left[ 1 + \frac{x^2 - t^2 - L^2}{r^2} \right], \quad X_{d+1} = \frac{L t}{r}, \tag{5}
\]
in which case the AdS metric takes the form

\[ ds^2 = \frac{L^2}{r^2} (dr^2 + dt^2 + dx_i^2), \quad r \geq 0. \]  

(6)

It is clear that each constant \( r \)-slice describes a copy of Minkowski space. The reason of the constraint \( r \geq 0 \) is because of the singularity at \( r = 0 \) and so the metric (6) can cover only half of the AdS space. Hence the Poincare patch describes a patch of the AdS space with boundary consisting of a copy of the Minkowski space \( M_d \) at \( r = 0 \), together with a single point \( P \) at \( r = \infty \). This fact has been of crucial importance in the prescription of [14, 15] for the realization of the holography of gravity in AdS space.

It is interesting to note that the \((d+1)\)-dimensional Anti-de Sitter space \( \text{AdS}_{d+1} \) also admit a coordinate patch with \( d \)-dimensional de Sitter space \( \text{dS}_d \) slicing. This fact has been used in the construction of braneworld with cosmological constant [50]. The embedding of \( \text{dS}_d \) can be realized with the following change of coordinates of the \( \text{AdS}_{d+1} \):

\[ X_{d+1} = L \cosh \frac{z}{L}, \quad X_\mu = Y_\mu \sinh \frac{z}{L}, \quad \mu = 0, 1, \ldots, d, \]  

(7)

with \( Y_\mu \) satisfying (3) and describes de Sitter space \( \text{dS}_d \). In this coordinate patch, the AdS metric takes the form

\[ ds^2 = dz^2 + \sinh^2 \left( \frac{z}{L} \right) ds_{\text{dS}}^2, \quad z \geq 0. \]  

(8)

The metric (8) describes a portion of the AdS space with boundary consisting of a copy of the de Sitter space \( \text{dS}_d \) at \( z = \infty \), together with a single point at \( z = 0 \). An explicit solution of the constraint (3) is given by

\[ Y_0 = \frac{\sinh Ht}{H} - \frac{1}{2} H x_i^2 e^{-Ht}, \quad Y_i = x_i e^{-Ht}, \quad Y_d = \frac{\cosh Ht}{H} - \frac{1}{2} H x_i^2 e^{-Ht}, \]  

(9)

with \( H = 1/L \) and \( i = 1, \ldots, d-1 \). This gives the \( \text{dS}_d \) metric in terms of the planar coordinates \((t, x^i)\):

\[ ds_{\text{dS}}^2 = -dt^2 + e^{-2Ht} dx_i^2. \]  

(10)

The normalized distance \( P \) between two points \( X \) and \( X' \) in the ambient space

\[ P(X, X') := \frac{\eta_{MN} X^M X'^N}{L^2}, \]  

(11)

is a convenient quantity that can be used to express the geodesic distance \( \mathcal{D} \) between any two points in the AdS or dS. For time like separated points in AdS, \( P \) is related to the
geodesic distance $\mathcal{D}$ between two points $X, X'$ by $P = -\cos(\mathcal{D}/L)$. For points in the same causal diamond in dS, the relation is $P = \cos(\mathcal{D}/L)$. For AdS$_{d+1}$, $P$ is given by

$$P_{\text{AdS}}(X, X') = -\frac{1}{\xi},$$

where

$$\xi^{-1} = \frac{r^2 + r'^2 + (x_i - x'_i)^2 - (t - t')^2}{2rr'}$$

in the Poincare coordinates (5); and

$$\xi^{-1} = \cosh Hz \cosh Hz' - \sinh Hz \sinh Hz' \times P_{\text{dS}}(x^\mu, x'^\mu)$$

in the dS planar coordinates (7), (9). Here

$$P_{\text{dS}} := \cosh H(t - t') - \frac{e^{-H(t+t')}}{2} H^2(x_i - x'_i)^2$$

is the normalized distance in dS$_d$. Note that $P_{\text{dS}} = 1$ for coincident points, therefore it is more convenient to consider the following quantity

$$\sigma^2(x, x') := e^{-H(t+t')}(x_i - x'_i)^2 - \frac{\cosh H(t - t') - 1}{H^2/2}$$

as a measurement of distance between any two points $(t, x_i), (t', x'_i)$ in dS$_d$. In general it is

$$\frac{P_{\text{dS}} - 1}{H^2} = -\frac{1}{2}\sigma^2.$$

The quantity $\sigma^2$ has the property that it coincides with the proper distance in Minkowski spacetime in the flat space limit $H \to 0$,

$$\sigma^2 \to |x - x'|^2 := -(t - t')^2 + (x - x')^2.$$  

In terms of the conformal time $x^0 = H^{-1} \exp(HT)$, the dS metric (10) can be written as

$$ds^2 = \frac{1}{H^2 x_0^2} (-dx_0^2 + dx_i^2)$$

and it is

$$\sigma^2 = \frac{(x_\mu - x'_\mu)^2}{H^2 x_0 x'_0},$$

where indices are raised and lowered by the Minkowski metric $\eta_{\mu\nu}$. 

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2.2 \( \mathcal{N} = 4 \) superconformal Yang-Mills theory

Let us review the \( \mathcal{N} = 4 \) superconformal Yang-Mills theory on dS\(_4\) constructed in [47]. There the metric is taken to be of the form (19). Crucial to the construction of [47] is the existence of conformal Killing spinor on dS\(_4\). Unlike the Killing spinor equation, the conformal Killing spinor defined by the equation

\[
\left( D_\mu - \frac{1}{4} \gamma_\mu \mathcal{D} \right) \epsilon = 0
\]

is compatible with the Majorana condition on spinor. This can be solved and one obtain the conformal Killing spinors on dS\(_4\)

\[
\epsilon(x) = \frac{1}{\sqrt{Hx^0}} (\eta_0 + x^\mu \gamma_\mu \eta_1),
\]

where \( \eta_0, \eta_1 \) are arbitrary Majorana spinors. This gives \( \mathcal{N} = 1 \) superconformal symmetry in dS\(_4\) and corresponds to a basis of 8 real supercharges.

The \( \mathcal{N} = 4 \) maximal superconformal Yang-Mills theory on dS\(_4\) contains the gauge potentials \( A^a_\mu \), four Majorana gauginos \( \lambda^a_\alpha \) and six real scalars \( X^a_i \), where the indices \( a \) is in the adjoint of the gauge group \( SU(N) \). The Lagrangian is \( \mathcal{L} = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 \) where

\[
\mathcal{L}_2 = -\left[ \frac{1}{4} F^{\mu\nu}_{a} F^{a\mu\nu} + \bar{\lambda}^{a\alpha} \gamma^\mu D_\mu P_L \lambda^a_\alpha + \frac{1}{2} D_\mu X^a_i D^\mu X^a_i + H^2 X^a_i X^a_i \right],
\]

\[
\mathcal{L}_3 = -\frac{1}{2} f^{abc} X^a_i \left[ C_i^{\alpha\beta} \bar{\lambda}^b_\alpha P_L \lambda^c_\beta + C_{i\alpha\beta} \bar{\lambda}^{ba} P_R \lambda^{c\beta} \right],
\]

\[
\mathcal{L}_4 = -\frac{1}{4} f^{abc} f^a_{b'c'} X^b_i X^c_j X^{b'}_i X^{c'}_j.
\]

Here \( P_L, P_R \) are chiral projectors, \( C_i \) are the six ‘t Hooft instanton matrices:

\[
C_1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 0 & -\sigma_3 \\ \sigma_3 & 0 \end{pmatrix}, \quad C_3 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix},
\]

\[
C_4 = -i \begin{pmatrix} 0 & i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix}, \quad C_5 = -i \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad C_6 = -i \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}
\]

and \( \sigma_i \) are the Pauli matrices. Note that \( C_1, C_2, C_3 \) are real, \( C_4, C_5, C_6 \) are imaginary.

The action admits an \( SU(4) \) R-symmetry and the superconformal symmetry:

\[
\delta A^a_\mu = -\epsilon^a \gamma_\mu P_L \lambda^a_\alpha - \bar{\epsilon}_a \gamma_\mu P_R \lambda^{a\alpha},
\]

\[
\delta X^a_i = -\epsilon^a P_L C^{\alpha\beta}_i \lambda_\beta - \bar{\epsilon}_a P_R C_{i\alpha\beta} \lambda^{a\beta},
\]

\[
\delta \lambda^a_\alpha = \frac{1}{2} \gamma^{\mu\nu} F^a_{\mu\nu} \epsilon_\alpha - \gamma^\mu D_\mu X^a_i (P_L C^{\alpha\beta}_i \epsilon_\beta + P_R C_{i\alpha\beta} \epsilon^\beta) - \frac{1}{2} X^a_i (P_R C^{\alpha\beta}_i \mathcal{D} \epsilon_\beta + P_L C_{i\alpha\beta} \mathcal{D} \epsilon^\beta)
\]

\[
-\frac{1}{2} f^{abc} X^b_i X^c_j [(C_i C_j)^{\alpha\beta} P_R \epsilon^\beta + (C_i C_j)_{\alpha}^{\beta} P_L \epsilon_\beta],
\]
where \( P_L \epsilon_\alpha, P_R \epsilon^\alpha \) are an \( SU(4) \) quartet of Majorana conformal Killing spinors. Here \((C_i C_j)^\alpha_\beta := C_i^{\alpha\gamma} C_j^{\gamma\beta}\) and \((C_i C_j)_\alpha^\beta := C_i^{\alpha\gamma} C_j^{\sigma\beta}\). Due to its large amount of superconformal symmetry, the theory is expected to be UV finite. Since there is no massless minimal coupled field, it is also expected that there is no IR divergence. Hence the theory is expected to enjoy exact \( SU(2,2|4) \) supersymmetry. Adding a \( \theta \)-term and restore the Yang-Mills coupling \( g \), one expect the theory also enjoy exact \( SL(2,\mathbb{Z}) \) strong-weak duality, just as the type IIB superstring theory does.

The moduli space of the theory can be easily worked out. With fermions and the gauge fields set to zero. The equation of motion for the scalar fields read

\[
- D_\mu D^\mu X_i + 2 H^2 X_i - [X_j, [X_i, X_j]] = 0. \tag{31}
\]

One class of solution (static) is given by product of fuzzy spaces described by \( SU(2) \). A more interesting solution is

\[
X_i = e^{Ht} Z_i, \tag{32}
\]

where \( Z_i \) are diagonal. This describes an expanding \( R^6 \) and is expected from the form \((10)\) of the de Sitter metric used.

### 2.3 Conformal field theory in dS space

The \( SO(1, d) \) isometries of de Sitter space \( dS_d \) is generated by the generators:

\[
L_{AB} = Y^A \frac{\partial}{\partial Y^B} - Y^B \frac{\partial}{\partial Y^A}, \quad A, B = 0, 1, \cdots, d, \tag{33}
\]

which acts linearly on the de Sitter hyperboloid. In terms of the dS space conformal coordinates \((19)\), the de Sitter isometries are generated by the spatial rotations \( J_{ij} \), dilation \( D \), spatial translations \( P_i \) and special conformal transformation \( K_i \):

\[
J_{ij} = -iL_{ij} = -i(x_i \partial_j - x_j \partial_i), \quad \text{where } i = 1, \cdots, d - 1, \tag{34}
D = -iL_{0d} = -ix^\mu \partial_\mu, \tag{35}
P_i = -i(L_{id} + L_{0i}) = -iH^{-1} \partial_i, \tag{36}
K_i = i(L_{id} - L_{0i}) = -2iH x_i x^\mu \partial_\mu - iH x^2 \partial_i. \tag{37}
\]

The corresponding finite transformations are:

\[
x'_i = \Lambda_i^j x_j, \quad \Lambda \in SO(d - 1) \ \text{rotations}, \tag{38}
\]

\[
x'^\mu = \lambda x^\mu, \tag{39}
\]

\[
x'_i = x_i + a_i, \tag{40}
\]

\[
x'^\mu = \frac{x^\mu + b^\mu x^2}{1 + 2b_\mu x^\mu + b^2 x^2}, \quad b^\mu = (0, b^i). \tag{41}
\]
As the de Sitter metric is related to the flat space Minkowski metric by a Weyl factor, the conformal symmetry of dS is the same \(SO(2,d)\) as that of the Minkowski spacetime, and is obtained by adding to the dS isometries the generators: the Lorentz boosts \(J_0\), time translation \(P^0\) and the special conformal transformation \(K^0\). The corresponding finite transformations are:

\[
\begin{align*}
    x'^\mu &= \Lambda_\mu^\nu x^\nu, \quad \text{nonvanishing } \Lambda_0^\mu, \Lambda_0^0 = \text{Lorentz boost}, \\
    x'^0 &= x^0 + a^0, \quad \text{(42)} \\
    x'^\mu &= \frac{x^\mu + b^\mu x^2}{1 + 2b_\mu x^\mu + b^2 x^2}, \quad b^\mu = (b^0, 0) \quad \text{(44)}
\end{align*}
\]

and the metric transforms as

\[
ds^2 \rightarrow ds'^2 = \Lambda^2(x)ds^2,
\]

where, respectively,

\[
\Lambda(x)^2 = \left(\frac{x_0}{\Lambda_0^\mu x^\mu}\right)^2, \quad \left(\frac{x_0}{x_0 + a_0}\right)^2, \quad 1. \quad \text{(46)}
\]

As usual, the special conformal transformations can be constructed out of translations and the inversion

\[
x^\mu \rightarrow x'^\mu = \frac{x^\mu}{x^2} := \left(\frac{1}{x}\right)^\mu. \quad \text{(47)}
\]

Inversion is an isometry and induces an \(SO(1,d-1)\) rotation on vector

\[
\frac{\partial}{\partial x'^\mu} = x^2 M_\mu^\nu(x) \frac{\partial}{\partial x^\nu}, \quad \text{(48)}
\]

where

\[
M_\mu^\nu(x) := \delta_\mu^\nu - 2 \frac{x_\mu x^\nu}{x^2} \quad \text{(49)}
\]

and satisfies \(M_\alpha^\alpha M_\beta^\beta \eta_{\alpha\beta} = \eta_{\mu\nu}\). For a spinor in fundamental representation, inversion induces the transformation

\[
\psi'(x) = S_\alpha^\beta(x) \psi_\beta \left(\frac{1}{x}\right), \quad \text{(50)}
\]

where the matrix \(S_\alpha^\beta(x)\) satisfies

\[
S^\dagger(x) \Gamma^\mu S(x) = M_\nu^\mu \Gamma^\nu \quad \text{(51)}
\]

and is given by

\[
S(x) = \frac{\Gamma_\mu x^\mu}{|x|}. \quad \text{(52)}
\]
In a conformal field theory, scalar (with respect to the rotation group) operator $O$ of conformal dimension $\Delta$ satisfies under conformal transformation $x \to x'$ as

$$O'(x') = \frac{1}{|\Lambda(x)|^\Delta} O(x). \quad (53)$$

For two such operators, dS invariance implies that their 2 point function must be function of the geodesic distance $\sigma(x, y)^2$. Furthermore, invariance under the conformal transformations implies that

$$\langle O_1(x) O_2(y) \rangle = \begin{cases} \frac{C_{12}}{\sigma(x, y)^{2\Delta}} & \text{if } \Delta_1 = \Delta_2 = \Delta, \\ 0 & \text{if } \Delta_1 \neq \Delta_2. \end{cases} \quad (54)$$

As for operator $O^\alpha(x)$ of dimension $\Delta$ in spin 1/2 representation, it satisfies under translation in $x^0$:

$$O'^\alpha(x') = \frac{1}{|\Lambda(x)|^\Delta} S^\alpha_{\beta}(x) O^\beta(x\mu). \quad (55)$$

Together with its property under inversion

$$O'^{\alpha}(x') = S^{\alpha}_{\beta}(x) O^{\beta}(x\mu), \quad (56)$$

one can easily show that the two point function for any two such operators is

$$\langle O_1^{\alpha}(x) O_2^{\beta}(y) \rangle = \frac{D_{\alpha \beta}(x, y)}{\sigma(x, y)^{2\Delta}} \delta_{\Delta_1, \Delta_2}, \quad (57)$$

where $\Delta = \Delta_1$. Here $D$ satisfies the relation

$$D(x, y) = S^\mu(x) D \left(\frac{1}{x}, \frac{1}{y} \right) S(y) \quad (58)$$

and is given by

$$D_{\alpha \beta}(x, y) = \frac{(x - y)_\mu \Gamma^\mu}{|x - y|}. \quad (59)$$

Constraints on 3 and higher point functions of dS CFT can be similarly worked out.

### 3 Bulk Propagators and Boundary Correlators: General Setup

One of the very basic tools for the study of gauge/gravity correspondence is the bulk to boundary formalism [14, 15]. In this section we develop this formalism for a general bulk
metric. Most of what we do in section 3.1 is a simple generalization of known results in the literature. The general result obtained in section 3.2 is new.

Consider a \((d + 1)\)-dimensional manifold \(\mathcal{M}\) with boundary and the metric

\[
ds^2 = g_{MN} dy^M dy^N.
\]  

Without loss of generality, we assume that the metric has an expansion of the form near the boundary,

\[
ds^2 = dz^2 + \gamma_{\mu\nu}(z, x) dx^\mu dx^\nu, \quad \gamma_{\mu\nu}(z, x) = p^2(z) h_{\mu\nu}(x),
\]  

for some function \(p(z)\) and \(h_{\mu\nu}(\mu, \nu = 0 \cdots, d - 1)\) is the boundary metric. The boundary is supposed to be at some location \(z = a\) in this coordinate system.

### 3.1 Scalar case

First let us consider the case of a real scalar field with the action

\[
I(\phi) = -\frac{1}{2} \int d^{d+1}y \sqrt{|g|} (g^{MN} \partial_M \phi \partial_N \phi + m^2 \phi^2),
\]  

where \(g\) is the absolute value of the determinant of the matrix \(g_{MN}\). Performing an integration by parts leads to

\[
I(\phi) = -\frac{1}{2} \int d^d x \sqrt{|g|} n^M \partial_M \phi + \frac{1}{2} \int d^{d+1}y \sqrt{|g|} \phi (\Box - m^2) \phi,
\]  

where \(n^M := \partial x^M / \partial z\) is the normal vector to the boundary surface. For our metric \(\text{[61]}\), the only nonvanishing component is \(n^z = 1\). The first term of \(\text{[63]}\) is a boundary term and the second term gives the equations of motion

\[
(\Box - m^2) \phi = 0, \quad \Box := \frac{1}{\sqrt{|g|}} \partial_M (\sqrt{|g|} g^{MN} \partial_N ),
\]  

where \(\Box\) is the \((d + 1)\)-dimensional d’Alembertian operator of \(\mathcal{M}\).

When evaluated on shell, the bulk term in \(\text{[63]}\) vanishes and only the boundary term contributes. It is clear that the boundary contribution depends on the boundary behavior of the solution of the equation of motion. Without loss of generality, consider a solution of \(\phi\) with the following leading asymptotic behavior near the boundary,

\[
\phi \sim f(z) \phi_0(x), \quad z \sim a,
\]  

\(10\)
for some function $f$ of $z$. It is then convenient to introduce the bulk to boundary propagator $K$ defined by the following differential equation

$$
\Box K(z, x, x') = 0 , \quad (66)
$$

and has the boundary behavior

$$
\lim_{z \to a} K(z, x, x') \sim f(z) \frac{\delta^{(d)}(x - x')}{\sqrt{h}} , \quad (67)
$$

where $h$ is the absolute value of the determinant of the boundary metric $h_{\mu\nu}$ and $\sim$ denotes the leading order contributing term in the sense of distribution. For convenience, we have chosen to include a volume factor $\sqrt{h}$ in our definition of $K$, making it scalar. The introduction of the bulk to boundary propagator allows us to write $\varphi(z, x)$ as

$$
\varphi(z, x) = \int d^d x' \sqrt{h(x')} K(z, x, x') \varphi_0(x') , \quad (68)
$$
in agreement with the definition (67). If we substitute (68) into (63), we get

$$
I(\phi) = -\frac{1}{2} \int d^d x \sqrt{h(x)} d^d x' \sqrt{h(x')} \varphi_0(x) G(x, x') \varphi_0(x') , \quad (69)
$$

with $G(x, x')$ defined as

$$
G(x, x') := \lim_{z \to a} \left( f(z) p^{d}(z) \partial_z K(z, x, x') \right) . \quad (70)
$$

According to the prescription of AdS/CFT correspondence [14, 15], the 2-point function of the dual field theory is given by $G(x, x')$. The equation (70) expresses it in terms of the normal derivative of the bulk-to-boundary propagator $K$ at the boundary.

The key goal now is to obtain the bulk-to-boundary propagator. In the original works [14, 15], this is obtained by solving the differential equation (66) directly. There is however a more effective way. Let us introduce the Green function for the bulk

$$
(-\Box + m^2) G(z, x; z', x') = \frac{1}{\sqrt{g}} \delta^{(d)}(x - x') \delta(z - z') . \quad (71)
$$

Using the Green’s identity, one can easily obtain the solution of the scalar Klein-Gordon equation (64) in terms of the Green function as

$$
\varphi(z, x) = \int d^d x' \varphi_0(x') \sqrt{\gamma(z', x')} \left( G \partial_{z'} f(z') - f(z') \partial_{z'} G \right) \bigg|_{z' \to a} . \quad (72)
$$
Comparing with (68), the bulk-to-boundary propagator $K$ can be written in terms of the Green function as
\[
K(z, x, x') = \lim_{z' \to a} p^d(z') \left( G(z, x; z', x') \partial_z f(z') - f(z') \partial_z G(z, x; z', x') \right)
\] (73)
and subsequently the two point function $G(x, x')$ can be obtained in terms of $K$ using (70). This formula displays clearly how the bulk physics, as encoded in the bulk Green function, is translated to the physics on the holographic field theory through the boundary data: the asymptotic behavior $f$ of the field and of the metric volume factor $p^d(z)$ near the boundary. The relation of the propagator with the Green function and its derivative, turns out to be always scaled in such a way that the limit at the boundary is finite. Higher point functions can be obtained by the Witten diagrams [14].

### 3.2 Spin 1/2 case

The action for a massive free spin 1/2 fermion on our $(d + 1)$-dimensional space $\mathcal{M}$ reads
\[
I_0 = \int d^{d+1}x \sqrt{g} \bar{\psi} (\slashed{D} - \mu) \psi,
\] (74)
where $g_{MN}$ is the metric on $\mathcal{M}$. Without loss of generality, we assume $\mu > 0$. The Dirac equation of motion reads
\[
(\slashed{D} - \mu) \psi = 0, \quad \bar{\psi} (-\slashed{D} - \mu) = 0.
\] (75)

The action (74) vanishes on-shell. For the application of AdS/CFT, one needs to supplement it with the boundary action [51]
\[
\frac{1}{2} \int_{\partial \mathcal{M}} d^d x \sqrt{\gamma} \bar{\psi} \psi,
\] (76)
where $\partial \mathcal{M}$ is the boundary of $\mathcal{M}$ and the metric takes the form (61) near the boundary. In practical calculation, as various quantities in $I_b$ are divergent near the boundary, one needs to consider it as the limit
\[
I_b := \lim_{z \to a} \frac{1}{2} \int_{\partial \mathcal{M}} d^d x \sqrt{\gamma} \bar{\psi} \psi.
\] (77)

The necessity of the boundary term was justified in [53] which demonstrated that only then the variational principle for the fermionic action is well defined: it ensures that by decomposing the spinor $\psi$ in terms of the eigenvalues of say, the Gamma matrix $\Gamma^z$, the on-shell action is not a function of both components, since the regularity of the solution
on $\mathcal{M}$ restrict that only half the components of the spinor $\psi$ can be prescribed on the boundary.

Now let us consider a solution $\psi$ with the leading asymptotic behavior near the boundary

$$\psi \sim f(z)\psi_0(x), \quad z \sim a,$$

where $f(z)$ is a function and $\psi_0$ is a spinor living on the boundary. It is easy to see that for positive $\mu$, the non-normalizable mode is obtain from $\psi_0$ of negative chirality:

$$\Gamma^z\psi_0 = -\psi_0.$$  \hfill (79)

The fermionic bulk to boundary propagator $S(z, x, x')$ from a point $(z, x^\mu)$ in the interior to a point $x'^\mu$ on the boundary is defined by the differential equation

$$(\not D - \mu)S = 0 \quad \text{ (80)}$$

and the boundary behavior

$$\lim_{z \to a} S(z, x, x') \sim f(z) \frac{\delta^{(d)}(x - x')}{\sqrt{h}}.$$ \hfill (81)

It allows us to write the on-shell configuration of $\psi$ as

$$\psi(z, x) = \int d^d x' \sqrt{h(x')} S(z, x, x') \psi_0(x'),$$ \hfill (82)

in agreement with the definition (81). Substituting (82) into $I_b$, we obtain

$$I_b = \int d^d x' \sqrt{h(x')} d^d x'' \sqrt{h(x'')} \bar{\psi}_0(x') \mathcal{G}(x', x'') \psi_0(x''),$$ \hfill (83)

where

$$\mathcal{G}(x', x'') := \lim_{z \to a} \frac{1}{2} \int d^d x \sqrt{\gamma(z, x)} S^\dagger(z, x, x') S(z, x, x'').$$ \hfill (84)

As $S$ behaves like a delta function near the boundary, the integral picks up its contribution from the two regions: $x \sim x'$ and $x \sim x''$ and we obtain

$$\mathcal{G}(x', x'') = \lim_{z \to a} \frac{1}{2} f(z) p^d(z) (S^\dagger(z, x'', x') + S(z, x', x'')).$$ \hfill (85)

This formula is analogous to (70) and gives the fermionic two point function in terms of the fermionic bulk to boundary propagator $S$. To find $S$, one can try to solve for it directly from the defining equations (80) and (81). This has been carried out in [51, 52, 53] for the original AdS/CFT correspondence with Minkowski CFT living on the boundary. However this is not necessary as we only need $S$ near the boundary in (85). In the next section,
we will show that $S$ in \[S(85)\] can be obtained in terms of the scalar bulk to boundary propagator $K$:

$$S = \mathcal{D} K.$$  \hspace{1cm} (86)

As noted in [50], apart from having Minkowski spacetime $M_4$ and de Sitter spacetime $dS_4$ as boundaries, it is also possible to consider another coordinate patch of the AdS$_5$ and have AdS$_4$ as boundary. In this case boundary dual theory is a superconformal Yang-Mills theory living on AdS$_4$. The general results obtained here can be applied to all these cases.

4 \textbf{Scalar 2-Point Function in dS Dual Field Theory}

In the above we have shown how the bulk to boundary propagator and the two point correlators in the boundary field theory can be derived from a knowledge of the bulk Green function. In this section, we consider different boundaries of the AdS space and will use these formulas to show how the same AdS bulk physics could manifest itself differently on the boundary field theories.

In the standard AdS/CFT correspondence which employ the Poincaré coordinate patch \[\xi\] of the AdS space, one can put the metric in the form \[\gamma\] by writing $r = \exp(-z/L)$. It is

$$\sqrt{-\gamma} = \frac{1}{r^d} = e^{dz/L}. \hspace{1cm} (87)$$

Near the boundary $r' \to 0$ or $z' \to \infty$, from \[\xi\] we have

$$\xi \approx 2rr' \to 0. \hspace{1cm} (88)$$

To compute the bulk to boundary propagator $K$ from \[\xi\], we need to know the Green function near the boundary. For AdS$_{d+1}$ in the Poincare coordinates, we have, see for example \[\gamma\],

$$L^{d-2}G(X, X') = \frac{C_\Delta}{2^\nu \Gamma(\Delta)} \left(\frac{\xi}{2}\right)^\Delta \left(\frac{\Delta + 1}{2}, \frac{\Delta + 1}{2}, \nu + 1; \xi^2\right), \hspace{1cm} (89)$$

where

$$C_\Delta := \frac{\Gamma(\Delta)}{\pi^{\frac{d}{2}} \Gamma\left(\Delta - \frac{d}{2}\right)} \hspace{1cm} (90)$$

and $\Delta$ can be either $\Delta_+$ or $\Delta_-$

$$\Delta_\pm := \frac{d}{2} \pm \nu, \hspace{1cm} \nu := \sqrt{\frac{d^2}{4} + m^2L^2}. \hspace{1cm} (91)$$
Using the properties
\[
\lim_{x \to 0} F(a, b, c; x) = 1, \quad \frac{d}{dx} F(a, b, c; x) = \frac{ab}{c} F(a + 1, b + 1, c + 1; x),
\]
(92)
of the hypergeometric functions, we get
\[
\partial_{x'} (\xi^\Delta F) = -\frac{\Delta}{L} \xi^\Delta + O(\xi^{\Delta+1})
\]
near the boundary. Now it is the non-normalizable mode with the asymptotic behavior
\[
f = r^\Delta f_0
\]
that defines a field at the boundary. It is easy to see that in order to get the desired boundary behavior (67) for \( K \), we need to adopt the root \( \Delta = \Delta_+ \) in the Green function (89) and we obtain the bulk to boundary propagator (up to a constant)
\[
K = \left( \frac{r}{r^2 + |x - x'|^2} \right)^\Delta.
\]
(93)
It follows immediately from (70) that
\[
G(x, x') = \frac{1}{|x - x'|^{2\Delta}},
\]
(94)
which is the expected form of the two point function for operator of dimension \( \Delta \) in CFT living on Minkowski spacetime.

Next we consider the dS slicing (8) of AdS. To compute the 2-point function in the corresponding boundary conformal field theory, we use the fact that the Green function is a scalar and therefore invariant under coordinate transformations. The AdS\(_{d+1}\) metric \( g \) we consider is
\[
ds^2 = dz^2 + \sinh^2(Hz) \, ds_{\text{dS}}^2,
\]
(95)
and has a dS boundary with metric \( h \), where \( \sqrt{g} = \sinh^d(Hz)\sqrt{h} \). Near the boundary \( z' \to \infty \), (14) gives
\[
\xi = \frac{e^{-H(z+z')}}{H^2 \rho^2 / 8} \to 0,
\]
(96)
where
\[
\rho^2 = \sigma^2 + e^{-2Hz} \cdot \frac{1 - H^2 \sigma^2 / 4}{H^2 / 4}
\]
(97)
and \( \sigma^2 \) is given by (20). Working similarly as above, we need the asymptotics of the non-normalizable mode near the boundary \( z \to \infty \) in the dS slicing. In our coordinates, the Klein-Gordon operator \( \Box \) is given by
\[
\Box = \partial_z^2 + \frac{dH}{\tanh Hz} \partial_z + \frac{1}{\sinh^2 Hz} \Box_{\text{dS}}, \quad \Box_{\text{dS}} = \frac{1}{\sqrt{-h}} \partial_\mu (\sqrt{-h} h^{\mu \nu} \partial_\nu).
\]
(98)
Consider a solution of the form \( \varphi(z, x) = f(z) \varphi_0(x) \), near the boundary we obtain the second order ordinary differential equation

\[
f'' + dH f' - m^2 f = 0 ,
\]

with the non-normalizable solution

\[
f = e^{-\Delta - H z}, \quad \Delta_\pm := \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + \frac{m^2}{H^2}}.
\]

The bulk to boundary propagator is given by (up to a constant),

\[
K = \left( \frac{e^{-Hz}}{\rho^2} \right)^\Delta, \quad \text{with } \Delta = \Delta_+
\]

and we obtain from (70) the two point function

\[
G(x, x') = \frac{1}{\sigma(x, x')^{2\Delta}}.
\]

This is the expected form of the two point function for operators of dimension \( \Delta \) of a conformal field theory in dS spacetime.

## 5 Fermion 2-Point Function in dS Dual Field Theory

### 5.1 Flat space conformal field theory

Let us first consider the canonical case \( \mathbb{G} \) of AdS in Poincare coordinates, with Minkowski spacetime located at the boundary at \( r = 0 \). The vielbein reads

\[
e^A_M = \frac{1}{r} \delta^A_M ,
\]

with the curved indices \( M = (r, \mu) \) and flat indices \( A = (r, a) \) having range \( a, \mu = 0, \ldots, d - 1 \). For convenience, we will assume \( L = 1 \) in this subsection. \( L \) can be restored easily by dimensional analysis. The Dirac operator is given by

\[
\slashed{D} = e^A_M \Gamma^A (\partial_M + \frac{1}{2} \omega_M^{BC} \Sigma_{BC}) = r \Gamma^r \partial_r + r \Gamma^\mu \partial_\mu - \frac{d}{2} \Gamma^r ,
\]

where the matrices \( \Gamma^A = (\Gamma^r, \Gamma^\mu) \) are the flat space gamma matrices:

\[
\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}, \quad \eta_{AB} = \text{diag}(1, \eta_d).
\]
The Dirac equation (75) in AdS\(_{d+1}\) reads
\[
\left( \Gamma^r \left( r \partial_r - \frac{d}{2} \right) + r \Gamma^\mu \partial_\mu - \mu \right) \psi = 0 .
\] (106)

The asymptotic behavior of the on-shell mode is governed by the behavior of the Dirac operator near the boundary, we obtain
\[
\psi \sim r^{\tilde{\Delta}} \psi_0(x), \quad r \to 0,
\] (107)
where
\[
\tilde{\Delta}_\pm := \frac{d}{2} \pm \mu \Gamma^r.
\] (108)

In order for this mode to be non-normalizable, it is necessary to take the boundary spinor \(\psi_0\) to be of negative chirality
\[
\Gamma^r \psi_0 = -\psi_0
\] (109)
and \(\mu > d/2\) such that \(\tilde{\Delta}_- < 0\).

To construct \(S\), it is useful to note that the Dirac operator \(\slashed{D}\) satisfies the following relation
\[
\slashed{D}^2 = \left( \Box + \frac{d^2}{4} \right) \mathbf{1} - r \Gamma^\mu \Gamma^r \partial_\mu ,
\] (110)
where
\[
\Box = r^2 \partial_r^2 + r(1 - d) \partial_r + r^2 \partial_\mu^2
\] (111)
is the d’Alembertian on AdS\(_d\) space in the metric (6). (110) can be written in the more convenient form
\[
\slashed{D}^2 + (\slashed{D} - \mu) \Gamma^r = \left( \Box + \frac{d^2}{4} \right) + \left( r \partial_r - \frac{d}{2} - \mu \Gamma^r \right) .
\] (112)

We propose to construct the fermion bulk to boundary propagator \(S\) as follow:
\[
S = (\slashed{D} + \mu + \Gamma^r)K + \delta ,
\] (113)
where
\[
K = \left( \frac{r}{\rho^2} \right)^{\Delta_+} \text{ with } \rho^2 := r^2 + (x - x')^2 ,
\] (114)
is the bulk to boundary propagator for an auxiliary scalar field of mass \(m\);
\[
\delta := -(\slashed{D} - \mu)^{-1} \left( r \partial_r - \tilde{\Delta}_- \right)K .
\] (115)
We remark that our notation for $\tilde{\Delta}_{\pm}$ takes into account that our boundary spinor $\psi_0$ lives in the $\Gamma^r = -1$ sector. It is easy to see that in order for $S$ to satisfy the equation of motion (80), $m$ has to be given by
\[ m^2 = \mu^2 - \frac{d^2}{4}. \] (116)
This relation is also precisely what is needed to guarantee that $S$ satisfies the desired boundary condition. In fact, since $K$ has the boundary behavior
\[ \lim_{r \to 0} K \sim r^{\Delta - \delta(d)}(x - x'). \] (117)
As a result, $\delta$ vanishes at the boundary since $\Delta_{\pm} = \tilde{\Delta}_{\pm}$ (due to (116) and $\Gamma^r = -1$ when acting on $\psi_0$). Now, it is easy to check that
\[ \partial K = \mu K - 2\tilde{\Delta}_- \frac{r}{\rho} U K, \] (118)
where
\[ U := \frac{r \Gamma^r + (x - x')\mu \Gamma^\mu}{\rho}. \] (119)
Note that $U^2 = 1$. As one approaches the boundary, $K$ imposes $x = x'$ implying
\[ \frac{r}{\rho} \to 1, \quad U \to \Gamma^r, \] (120)
and so $S$ satisfies the same (up to proportional constant) boundary condition as $K$:
\[ \lim_{r \to 0} S \sim r^{\tilde{\Delta} - \delta(d)}(x - x'). \] (121)

In principle one can use (113) to compute the full expression of $S$. However for our purpose of computing the two point function function using (85), this is not necessary as we only need to know $S$ near the boundary. Note that the $\mu$ and $\Gamma^r$ term of (113) do not contribute to the boundary action (83) since
\[ \bar{\psi}_0 \psi_0 = 0, \quad \bar{\psi}_0 \Gamma^r \psi_0 = 0, \] (122)
where we have used the fact that $\psi_0$ and $\bar{\psi}_0$ has opposite chirality. Also, since $\delta = 0$ at the boundary. As a result, the form of $S$ to be used in (85) is given by
\[ S = \partial K. \] (123)
Since only the second term in (118) contributes, so we obtain the fermionic two point function in the conformal field theory living on the boundary flat space
\[ G(x, x') = \frac{(x - x')\mu \Gamma^\mu}{|x - x'|^2 \tilde{\Delta}_+} \frac{1}{|x - x'|^2 \Delta_+}. \] (124)
The result (124) was first obtained by [51] using the AdS/CFT correspondence.
5.2 dS conformal field theory

Next let us consider the AdS_{d+1} space with a dS boundary with the metric
\[
ds^2 = dz^2 + \sinh^2 Hz (-dt^2 + e^{-2Ht}dx_i^2).
\]

The vielbein \(e^M_A\) is
\[
e^M_z = \delta^M_z, \quad e^M_t = \frac{1}{\sinh Hz} \delta^M_t, \quad e^M_i = \frac{e^{Ht}}{\sinh Hz} \delta^M_i.
\]

The non-zero spin connection elements \(\omega^{AB}_M\) are
\[
\omega^{t\,z}_M = H \cosh Hz \delta_{tM}, \quad \omega^{i\,z}_M = H \cosh Hz e^{-Ht} \delta_{iM}, \quad \omega^{i\,t}_M = He^{-Ht} \delta_{iM}
\]
and the Dirac operator reads
\[
\mathcal{D} = \Gamma_z \partial_z + \frac{1}{\sinh Hz} \left( \Gamma^t \partial_t + e^{Ht} \Gamma^i \partial_i \right) - \frac{H(d-1)}{2 \sinh Hz} \Gamma^t + \frac{dH}{2 \tanh Hz} \Gamma^z.
\]

We note in passing that
\[
\mathcal{D} = \Gamma_z \partial_z + \frac{1}{\sinh Hz} \mathcal{D}_{\text{dS}} + \frac{dH}{2 \tanh Hz} \Gamma^z.
\]

The asymptotic behavior of the solution to the Dirac equation can be easily worked out. It is
\[
\psi \sim e^{-\tilde{\Delta}z} \psi_0(x), \quad z \to \infty,
\]
where here
\[
\tilde{\Delta} \pm := \frac{d}{2} \pm \mu \Gamma^z.
\]

In order for this mode to be non-normalizable, we need
\[
\Gamma^z \psi_0 = \psi_0
\]
and \(\mu > d/2\).

The general solution of the Dirac equation can be written in the form (82) with the boundary spinor \(\psi_0\) satisfying \(\Gamma^z \psi_0 = \psi_0\). As before we note the useful relation:
\[
\mathcal{D}^2 - (\mathcal{D} - \mu) H \coth Hz \Gamma^z = \left( \Box + \frac{d^2 H^2}{4 \tanh^2 Hz} - \frac{(d-3)(d+1)H^2}{4 \sinh^2 Hz} \right)
- H \coth Hz \left( \partial_z - \mu \Gamma^z + \frac{dH}{2 \tanh Hz} \right) + \frac{He^{Ht}}{\sinh^2 Hz} \Gamma^i \Gamma^i \partial_i \Gamma^z.
\]
where
\[ \Box = \partial_z^2 - \frac{1}{\sinh^2 Hz} \partial_t^2 + \frac{e^{2Ht}}{\sinh^2 Hz} \partial_i^2 + \frac{dH}{\tanh Hz} \partial_z + \frac{(d-1)H}{\sinh^2 Hz} \partial_t \] (134)
is the d’Alembertian on AdS space with the metric (125). This allows us to construct the fermion bulk to boundary propagator \( S \) as
\[ S = (\mathcal{D} + \mu - H \coth Hz \Gamma^z)K + \delta , \] (135)
where
\[ K = \left( \frac{e^{-Hz}}{\rho^2} \right)^{\Delta_+} \] (136)
is the bulk to boundary propagator for an auxiliary scalar field of mass \( m \);
\[ \delta := -(\mathcal{D} - \mu)^{-1}\left( q(z) - H \coth Hz \left( \partial_z + H \tilde{\Delta}_- \right) + \frac{He^{Ht}}{\sinh^2 Hz} \Gamma^i \tilde{\Gamma}^j \partial_j \right)K , \] (137)
and
\[ q(z) := \frac{dH^2 \tanh Hz - 1}{2 \tanh^2 Hz} - \frac{(2d+3)H^2}{4} \frac{1}{\sinh^2 Hz} . \] (138)
It is easy to see that one needs again (116) in order for \( S \) to satisfy its defining differential equation. One also easily see that \( \delta \) vanishes at the boundary and \( S \) satisfies the desired boundary condition.

To calculate the boundary two point function using (85), we only need to know \( S \) near the boundary, which is \( S = (\mathcal{D} + \mu - H \coth Hz \Gamma^z)K \). As terms that are proportional to the unit matrix or \( \Gamma^z \) do not contribute to (83), we can drop them in \( S \) and we obtain again the relevant expression (86) for \( S \). In terms of conformal coordinates, it is
\[ S = \frac{1}{2} e^{-Hz} x_0 \Gamma^\mu \partial_\mu K - \frac{H(d-1)}{2} \Gamma^0 e^{-Hz} K \] (139)
for large \( z \).

Now the first term of (139) is equal to \(-\Delta_+ b K D\), where \( b \) is the function \( b := e^{-Hz}/\rho \) and \( D \) is the matrix \( D := x^0 x^\mu \partial_\mu \sigma^2/(2\sigma) \). Near the boundary, \( K \) imposes \( \sigma = 0 \) and so \( b = 2/H \) and
\[ D = \frac{(x - x')_\mu \Gamma^\mu}{|x - x'|} . \] (140)
It is then clear that the second term in (139) is sub-leading compared to the first term. As a result, the boundary two-point function is given by (up to a constant)
\[ G(x, x') = \frac{D}{\sigma^2 \Delta_+} . \] (141)
This agrees with the result (57) for a conformal field theory in dS spacetime.
6 Discussion

In this paper, we have proposed an AdS/CFT correspondence of the Type IIB string theory on $\text{AdS}_5 \times S^5$ in terms of a superconformal field theory on the dS$_4$ boundary. As a first step, we have provided evidence of it by showing that the boundary correlators of the dS conformal field theory can be reproduced from the AdS bulk dynamics. It would be interesting to compute the higher point functions and see if there is any nonrenormalization theorem for chiral operators as in [54]. It would also be interesting to look at minimal surfaces with different boundary conditions and compute the interacting potential between quarks anti-quarks and study properties of the entanglement entropy in a quantum field theory in a time dependent background.

According to the holographic principle [55,56] (see also [57] for a review), the full description of quantum gravity in a region requires only a quantum field theory living at the boundary. Gauge/gravity correspondence is a nice illustration of the holographic principle. In the original works of [14,15], a boundary flat $R^4$ was created when the Poincare patch of the AdS spacetime was considered. In this paper we considered a different coordinate patch with a different boundary, and showed that the same bulk dynamics (e.g. the same IIB supergravity equations in the classical supergravity limit), with different boundary conditions would results in different holograms. This is of course in consistent with the statement of the holographic principle. Nevertheless this is an aspect of the holographic principle that has not been emphasized much so far in the literature. In the full non-nonperturbative formulation of string theory on $\text{AdS}_5 \times S^5$, all the different boundaries and boundary conditions should be contained in the moduli space of the theory itself. This implies that the different dual quantum field theories may also be considered to be contained in some bigger theory of quantum field theories.

On the $\text{AdS}_5 \times S^5$ side, the existence of the Lax pair and an infinite set of classically conserved nonlocal charges are properties of the Green-Schwarz string sigma model [19]. Note that these properties were established using the global AdS space without referencing to the Poincare coordinate system. As such, the construction [19] of charges also apply in our case. One may speculate that the $\mathcal{N} = 4$ superconformal Yang-Mills on dS$_4$ may also be integrable in some of its sectors. This is an interesting aspect that deserves further studies.
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