Pedagogical notes on Black Holes, de Sitter space and bifurcated horizons

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Abstract

I discuss black hole evaporation in two different coordinate systems and argue that the results of the two are compatible once one takes the holographic principle into account. de Sitter space is then discussed along similar lines. Finally I make some remarks about smooth initial conditions in GR, which evolve to space-times with bifurcate horizons, and emphasize the care one must take in identifying spaces of solutions of General Relativity which belong to the same quantum theory of gravity. No really new material is presented, but the point of view I take on all 3 subjects is not widely appreciated.
1 Black hole evaporation

The string theoretic arguments for unitarity in black hole evaporation have been accepted by many researchers and dismissed by others. Even those who accept them tend to feel unease about the absence of a more local picture of how Hawking’s paradox is resolved. Some time ago, W. Fischler and I [1] proposed such a resolution, but it has not garnered much attention. It seems to me that by simply adding a little bit of information, namely the holographic principle, about the non-perturbative formulation of quantum gravity, one comes to a satisfactory resolution of the paradox without all of the heavy machinery of string theory.

The demonstration proceeds by tackling the singular region of the space-time geometry head on, and showing that the holographic principle [1] provides a novel interpretation of it, which ends up being completely equivalent to the Schwarzschild observer’s point of view.

This equivalence will also motivate some remarks about the nature of the states in the two pictures of the geometry, which are even more important for appreciating the nature of the quantum theory of de Sitter space.

In Schwarzschild coordinates, the eternal black hole metric is

\[ ds^2 = -dt^2 f(r) + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \]

where \( f(r) = (1 - \frac{R_S}{r}) \), and \( d\Omega^2 \) is the metric on the unit two-sphere. Near the horizon, where \( r = R_S + x \), this becomes

\[ ds^2 = -dt^2 \frac{x}{R_S} + xR_S\left(\frac{dx}{x}\right)^2 + R_S^2 d\Omega^2. \]

This is the same as

\[ ds^2 = R_S^2 [e^y(-dt^2 + dy^2) + d\Omega^2], \]

where we have made all coordinates dimensionless and committed the unforgivable sin of using the same letter for dimensionless and dimension-full time. The near horizon limit is \( y \to -\infty \).

\[ ^1 \] Throughout this note I will be referring to the Strong form of the Holographic Principle advocated by Fischler and myself. This is the assumption that the density matrix referred to in the covariant entropy bound is the maximally uncertain density matrix on the Hilbert space associated with the region bounded by the holographic screen.
As a consequence, states with arbitrary energy according to a local geodesic observer, have arbitrarily small energy as measured by the supported observer at infinity. This is the famous redshift of signals coming from an observer falling into a black hole. The supported observer never sees anything fall through the horizon, but signals from things near the horizon become harder and harder to detect. Equally important is that the range of the $y$ coordinate near the horizon is infinite, and that the metric there is conformal to flat space. When we use the boundary conditions at the horizon which correspond to the Hartle-Hawking\cite{2} vacuum state on the extended Kruskal manifold, quantum fields in this geometry have an infinite number of arbitrarily low energy states, all concentrated near the horizon.

This contradicts the Bekenstein-Hawking formula, which assigns a finite entropy to the black hole. The holographic principle states, in this context, simply that the BH formula is correct, and that quantum field theory over-counts the near horizon states by an infinite amount.

Now let us introduce another coordinate system. Consider the $t = 0$ slice of the Schwarzschild coordinates and time-like geodesics that penetrate this slice orthogonal to it. Follow along each of these geodesics, measuring time according to the local proper time. Label each point on the new time slices by the same coordinates that label the intersection of the same geodesic with the $t = \tau = 0$ slice. It is important, and true for the Schwarzschild metric, that the time-like geodesics never cross. This gives us a metric of the form

$$ds^2 = -d\tau^2 + R_S^2 \left( \frac{R^2 + 1}{R^2} \right)^{1/2} \left( \frac{dr}{dR} \right)^2 dR^2 + r^2(R, \tau) d\Omega^2.$$ 

The Schwarzschild radial coordinate $r$ is defined as a function of $\tau$ and $R$ by

$$\frac{\tau}{R_S} = \pm (R^2 + 1) \left[ \frac{r}{R_S} - \frac{r^2}{(R_S^2(R^2 + 1))} \right]^{1/2}$$

$$+(R^2 + 1)^{3/2} \cos^{-1}(\left[ \frac{r}{R_S(R^2 + 1)} \right]^{1/2}).$$

These are called Novikov coordinates\cite{3}, and they cover the entire Kruskal manifold just as well as Kruskal coordinates. They also match to the Schwarzschild coordinates at infinity. The metric, as well as the Hamiltonian of quantum fields in this coordinate system, is time dependent. Inside the horizon the time dependence becomes singular at a finite proper time of order $R_S$. This is the surface of Schwarzschild coordinate $r = 0$. We expect more and more field theoretic degrees of freedom to be excited as we approach the singularity.

It is important to note that any finite spatial coordinate $R$ maps to a point inside the horizon at large enough $\tau$. The Novikov coordinates map out a congruence of time-like geodesics, each of which starts with zero velocity at $t = \tau = 0$, and some value of $R$ and the angles on the two sphere. Any such geodesic is attracted by the black hole and falls through the horizon and encounters the singularity. For a given $R$, the time it takes to hit the singularity is $\sim R$. However, it is only for the last $\sim R_S$ of that time, that this coordinate point is inside the horizon. Thus, at a given time $\tau$ there is a value $R(\tau)$, such that all $R > R(\tau)$ are outside the horizon. Below we will discuss time scales of order $\tau \sim R_S$ and use the phrase inside the horizon to mean “those values of $R$ which are inside the horizon at this time”. Quantum field theory can be split into degrees of freedom localized in the interior
and exterior regions, and in this coordinate system there is no funny buildup of states on the horizon. We will now argue that the QFT description is not valid, in these coordinates, in the interior.

It is a consequence of the singularity that no causal diamond inside the horizon has a holographic screen area larger than the area of the black hole horizon. The strong holographic principle then implies that the system inside the horizon has a finite number of states, contrary to the prediction of quantum field theory.

On the time scale $R_S$ the geometrical picture of the spatial geometry of the interior of the black hole, in Novikov coordinates, breaks down. The black hole singularity $r = 0$ is a space-like surface that occurs at fixed Novikov time of order $R_S$. To understand the spatial geometry, suppress one coordinate on the two sphere, and think of the standard picture of the geometry of a static star as a bowl shaped depression, with the matter in the star at the bottom of the bowl. In spherical gravitational collapse, the bowl gets deeper and its cross section gets narrower, reaching infinite depth and zero width in a time of order $R_S$.

Consider a collection of “sputniks”, thrown into the black hole to map out its interior geometry. We can view these as a physical construction of Novikov coordinates in the interior. If two of these satellites are thrown into the black hole with a small time difference, they are out of causal contact. The space between them stretches faster than the speed of light. If, as in the synchronous coordinates, they are thrown in at precisely the same time, they will instead crash into each other in a time no bigger than $R_S$, if they have any width at all. More mathematically, it is well known that small classical anisotropic fluctuations grow rapidly as the singularity is approached, and that there is rapid quantum particle creation for quantum fields placed in this background. The obvious conclusion is that, in Novikov coordinates, the whole quantum field theory in a fixed space-time background approximation is breaking down rapidly as the singularity is approached. For $R_S \sim 10^{-9}\text{sec} \sim 10\text{cm}$, the black hole evaporation time is of order the age of the observed universe. Thus, in Novikov coordinates, the singularity is encountered long before the external observer can see enough Hawking radiation to make any conclusion about the internal state of the black hole.

The time dependent Hamiltonian in Novikov coordinates inside the horizon, is not integrable. It commutes with the space-like Schwarzschild Killing vector $\partial_t$, and with the generators of $SO(3)$, but has no other conservation laws. This fact, and the holographic principle, are all we need to understand the behavior of the system inside the horizon on times scales of order $R_S$. There is no meaning to the interior time evolution for larger times. In this regime, it is a good approximation to treat the interior of the black hole as an isolated system. Hawking radiation is transferring degrees of freedom to the external world, but for large $R_S$, this process is slow and its effect on the interior dynamics can be neglected.

Consider the subspace of the interior Hilbert space with fixed values of the conserved quantities. The fact that the system is not integrable is equivalent to the statement that the Lie algebra generated by the time dependent Hamiltonians $H(\tau)$ is the algebra of all Hermitian matrices on this subspace. We will also assume that this remains true near the singularity at $\tau = \tau_s = kR_S$. An example of a way in which this could fail is a behavior like

$$H(\tau) \sim H_s(\tau - \tau_s)^{-p}.$$

In this case the behavior near the singularity would become integrable and the eigenstates
of $H_s$ would be preserved by the evolution, up to a rapidly oscillating phase. There is no indication of this kind of behavior in the field theory Hamiltonians in the black hole background. Indeed, in field theory the Hilbert space can be viewed in the the Fock space basis of in-falling particles. As the singularity is approached we can excite more and more particles of higher and higher energy, and the state vector wanders off into the highest entropy part of the Fock space. Of course, in field theory, the Hilbert space is infinite dimensional, so the evolution runs off to infinity and the system is truly singular.

By contrast, if we accept the bound on the number of states implied by the Strong Holographic Principle, the space of normalizable states is a compact manifold, $CP^N$. The one parameter non-integrable groupoid of unitary transformations $Te^{-i \int^t ds H(s)}$ describes a chaotic trajectory on this manifold. The singularity in the flow implies that each point in $CP^N$ is visited an infinite number of times as the singularity is approached. The time averaged state of the system rapidly approaches the maximally uncertain density matrix on the finite dimensional space of black hole interior states. Although the QFT in curved space-time description of the dynamics has completely broken down, very simple general principles allow us to conclude what the time averaged behavior of the system is, with no need to find a precise description of the interior Hamiltonian.

Indeed, from a more ambitious point of view, we may say that there is no meaning to the effective Hamiltonian for this localized region, besides the general behavior we have described. The Principle of General Covariance tells us that we should be able to describe the interior of the black hole with many different coordinate systems. A small sample of those would be systems of synchronous coordinates which differ from Novikov coordinates only by the choice of the portion of the initial space-like slice inside the horizon. The Hamiltonians $H(\tau)$ in this new set of coordinates will not commute with those in Novikov coordinates, but will have the same general properties. The detailed time dependence of the state will differ, but the time averaged behavior will be identical. Thus, one could view the conclusion that the time averaged density matrix of the black hole interior rapidly approaches the maximally uncertain density matrix on a Hilbert space of dimension determined by the strong holographic principle, as the only generally covariant fact about the black hole interior.

In any of these Novikovoid coordinate systems one can describe, in the QFT approximation, the Hilbert space of the region outside the black hole horizon. For large $R_S$ the Fock space basis of multi-particle states is the best basis in which to think about the time evolution. The Arnowitt-Deser-Misner Hamiltonian, which is defined as a surface term at infinity, acts on this exterior Hilbert space. The holographic principle tells us that we should think of this particle system as interacting with another finite dimensional quantum system. On time scales of order $R_S$, the discussion of the previous paragraphs tell us that this system approaches a time averaged state which is the maximally uncertain finite dimensional density matrix. All of these states have the same ADM energy, so it makes sense to say that the black hole interior is in its micro-canonical ensemble at energy equal to the black hole mass. Since its entropy, $\pi(R_SM_P)^2$, is large, the interaction of field theory with this system should be well described by thermodynamics.

The rest is history. We have discovered that, given very generic properties of the holo-

If the number of states is large, and the Hamiltonian $H_s$ sufficiently generic, then this integrability is illusory, and the conclusion we will draw is still valid after time averaging.
graphic description of the interior, the physics predicted for the exterior on a time scale greater than $R_S$ and less than $R_S^2 M_P^2$ is identical to that in Schwarzschild coordinates. The fearsome transition between pure and mixed states has been accomplished by the traditional pragmatic method of time averaging the state of an ergodic system. On time scales of order the black hole evaporation time, one would need a more sophisticated model which allows the exchange of degrees of freedom between the interior and exterior systems. The only models I know which come anywhere near achieving this goal are those of [6].

There are a number of important lessons we have learned along the way. Perhaps the most important is that we should not expect to find a detailed microscopic quantum description of the interior of a black hole. In general relativity, local physics is gauge dependent. When a local region is well described in terms of quantum particles traveling through a classical space-time, the notion of a gauge independent local description is approximately meaningful. Within the black hole interior, there are no semi-classical observables to catch hold of. No basis of the Hilbert space is preferred and the only invariant conclusion we can come to is that the density matrix of the system is maximally uncertain.

Next we emphasize the fact that the black hole singularity has been “resolved” by quantum mechanics, without removing it. Indeed the singular behavior of the time dependent finite dimensional Hamiltonian is crucial to the conclusion that the density matrix rapidly approaches the micro-canonical ensemble as the singularity is approached. This is quite different from the resolution of time-like singularities by the addition of IR quantum degrees of freedom, which has been the hallmark of the string theory treatment of singularities.

Another important feature of our discussion has been the coincidence between the description of the gauge invariant physics in the Schwarzschild and Novikov coordinate systems. The misleading infinity in the state count is encountered in both systems, and is cut off by fiat, using the holographic principle. The description of the physics of black hole interior states (which the Schwarzschild observer calls near horizon states) is different because the set of observables used to probe them in these different coordinate systems are different, and do not commute. However, in both coordinate systems we conclude that the state of these degrees of freedom is well approximated by a high entropy density matrix. More detailed quantum information about these states is really available only to the external observer. Ultimately what this means is that there is an S-matrix in asymptotically flat space, which completely describes the formation and evaporation of black holes. It is unlikely that anyone will ever find even an approximate description of that S-matrix, which goes beyond the inclusive information first calculated by Hawking. For the truly interior description, we have argued that the detailed microscopic description of the dynamics appears in the semi-classical approximation to depend strongly on the choice of interior time slicing. Since there is no operational way to actually construct these coordinate systems, those descriptions seem somewhat meaningless, except that one finds the same time averaged density matrix near the singularity, for all of them. The Schwarzschild description of interior states as near horizon states, leads immediately to the conclusion that black hole evaporation is a unitary process. The holographic principle allows us to come to an identical conclusion in coordinate systems that are smooth at the horizon.

In the early 1990s there was a renewed burst of activity related to the paradoxes of Hawking radiation. I believe that it was about this time that the idea of a “nice slice” coordinate system was first realized to be crucial to Hawking’s argument that black hole
evaporation violates unitarity. The nice slice coordinates are similar to Novikov coordinates, in that they are smooth at the horizon, but the interior time coordinate is related to the exterior one on the same time slice by a large Lorentz boost. When time of order the evaporation time $R_S^3 M^2_\text{P}$ has passed in the exterior, the interior time is still significantly less than $R_S$. This enables one to claim that it is reasonable to use QFT to describe the whole slice. One then claims that information that an advocate of unitarity would insist is encoded in the outgoing radiation, is in fact encoded in field degrees of freedom in the interior portion of the slice, which commute with all the exterior radiation fields.

There have been significant disagreements in the literature, even among authors of the same paper [4], about whether the sub-Planckian time intervals between slices in the interior, invalidates the use of quantum field theory in these coordinates. I will not add more verbiage to this controversy, but simply claim that our discussion shows that one can use less contrived coordinate systems, all smooth at the horizon, to come to conclusions opposite to those indicated by doing field theory on nice slices. Like many other overly contrived confections, nice slices are profoundly misleading. Given our discussion, one can no longer state that the Schwarzschild coordinate view of black hole evaporation is misleading because it uses coordinates that are singular at the horizon.

I want to end this section by emphasizing again that our discussion is only approximate and does not extend to a description of the full process of Hawking evaporation of a black hole. In order to do that one would have to find a method in which one could allow a finite quantum system interacting with a QFT to “sublime” and transmit almost all of its degrees of freedom into outgoing radiation. It’s clear that if this can be done at all, the proper description of the finite system would be a cutoff version of the near horizon states in Schwarzschild coordinates. The problem that bedevils any such attempt is that in order to describe the system properly, one has to let the background geometry change in order to conserve ADM energy. I suspect that the only complete description of the evaporation process is through the S-matrix of a model of quantum gravity in asymptotically flat space. From the practical point of view we should admit that computing that S-matrix should be at least as difficult as a complete S-matrix description of the byproducts of a thermonuclear explosion starting from a description of the initial state of some colliding nuclei.

A more local description of black hole physics might be achievable with the formalism of holographic space-time [7]. There the fundamental object is the quantum version of a causal diamond with finite area holographic screen. This is just a finite dimensional Hilbert space, which for large dimension encodes the area of the screen through the Bekenstein-Hawking relation. The intersection between two diamonds is described by a common tensor factor in their Hilbert spaces. A holographic space-time is a collection of Hilbert spaces, with overlap prescriptions, and a consistent unitary dynamics. One can encode local concepts approximately in this formalism. For example, a time-like trajectory is encoded as a nested sequence of Hilbert spaces, each a tensor factor of its successor.

For a black hole, consider a causal diamond $I$, which has its past tips prior to the black hole formation. $I$ lies mostly inside the horizon, but its outer boundary sticks out of the horizon over a range of solid angles, centered at some solid angle $\vec{\Omega}$. Now consider a causal diamond $E$, which lies mostly outside the horizon, but has part of its boundary dipping down into the horizon and incorporating the portion of the boundary of $I$ outside the horizon. Now consider a collection of such pairs of diamonds tiling the sphere. That is, the intersection of
any pair of diamonds $E_i$ and $E_j$ is zero, and any point on the horizon is contained in some $E_i$. We think of the collection of $E_i$ as the quantum system corresponding to the “stretched horizon”.

I believe that it is possible to prove that the areas of the holographic screens of the maximal diamonds contained in the overlaps between $I_i$ and $E_i$ is $\geq$ the area of the horizon, because all 2-surfaces inside the horizon are trapped. If one relies on the geometrical/QFT picture of the black hole interior, one would argue that there is important information in $I_i$ that is causally disconnected from the overlap. Indeed, one can think of $E_i$ as the causal diamond of an observer that is supported near the horizon for a long time, and eventually falls into the black hole. The future tip of $E_i$ must be on the singularity, but there are timelike trajectories in $E_i$ that can stay outside the horizon for a very long time. Such an observer is causally disconnected from observers that fell into the hole much earlier. However, our discussion has shown that for times of order $R_S$ in Novikov coordinates, one should not apply the QFT picture to the interior of the black hole. Instead it is a finite dimensional quantum system, which revisits every one of its states an infinite number of times as $\tau$ approaches the singularity. The degrees of freedom in $I_i$ may be only a tensor factor in that interior Hilbert space, but the collection of all $I_i$ encompasses all of it.

In the holographic space-time approach, there is room in the collection of $E_i$ to encode all of the information in the collection of $I_i$. We have argued that the dimension of the tensor product of these Hilbert spaces is large enough to teleport the state of the interior into a state in $\otimes_i E_i$. Furthermore, the rules of the holographic formalism say that $E_i$ contains a tensor factor identical to $I_i$, and whose state is a unitary transform of the state in $I_i$ (part of the definition of the quantum space-time is the specification of this unitary transformation). We have had to give up a quasi-local description of $I_i$ in order to achieve this teleportation, but that is all. There are larger causal diamonds, describing the trajectories of observers that remain outside the black hole forever, which intersect with the $E_i$ and can transmit all of the information about the black hole interior, out to infinity. This process is perfectly compatible with an approximate local field theory description.

Thus, following the prescriptions of holographic space-time, and prescribing that the Hilbert spaces describing the black hole interior have a singular, ergodic, time dependent Hamiltonian, of the type described above we get a picture of black hole evaporation with only two lacunae. The first is that there is not yet a derivation of the dynamics of QFT in curved space-time as an approximation to a consistent holographic space-time. We have identified the correct kinematic variables to describe particle physics but do not yet have a consistent holographic dynamics, which is close to that prescribed by QFT.

The holographic formalism may provide a way to get around the second lacuna, the peculiar phenomenon of sublimation of degrees of freedom, which we referred to above. In our heuristic discussion of QFT in Rindler coordinates, we encountered different variables of the system, whose time dependence became singular at different Novikov times. In the holographic description, the singular Hamiltonian acts only on the Hilbert space of internal states. The external Hilbert spaces $E_i$ use a different time variable to describe the dynamics.

It is always worth keeping in mind that the black hole doesn’t really last forever. One can easily choose $E$ so that it contains timelike trajectories which stay outside the horizon for times of order the evaporation time.
of the black hole state, but agree with the conclusion that its time averaged density matrix is micro-canonical.

The inner boundary of the causal diamond described by $E_i$, but its outer boundary can reach null infinity. Thus, there is no barrier, in principle, to construct a dynamics in which a subset of the degrees of freedom describing $E_i$ are in equilibrium for a long time, but then turn into radiation. In the holographic description, different causal diamonds are independent quantum systems, with their dynamics constrained to agree, up to conjugation by a unitary, on overlaps. Causal diamonds of some in-falling observers have overlaps with those of some supported observers, which consist of a region close to the horizon. The consistency conditions are satisfied if the two descriptions both conclude that the finite dimensional system describing the overlap, has a time averaged density matrix that is maximally uncertain.

2 de Sitter space

I will keep this section brief, since I have discussed this material in many recent articles. We will examine two interesting coordinate systems for dS space. The first, called static coordinates, is the analog of the Schwarzschild coordinate system for a black hole. It has the form

$$ds^2 = -dt^2 f(r) + \frac{dr^2}{f(r)} + r^2 d\Omega^2,$$

with

$$f(r) = (1 - \frac{r^2}{R^2}).$$

Near the horizon $f(r)$ has a linear zero, and we can transform to coordinates like $(t, y)$ for the black hole. The Killing vector $\partial_t$, which has norm 1 at $r = 0$ becomes null at the horizon, so we again see an infinite pileup near the horizon of very low energy states of QFT on this background. As before, the holographic principle suggests that this infinity is cut off, and that the total entropy of the static patch is $\pi(RM_P)^2$.

In global coordinates, the metric has the form

$$ds^2 = -d\tau^2 + R^2 \cosh^2(\tau/R) d\Omega^3,$$

where $d\Omega^3$ is the metric on the unit 3-sphere. Here the infinity shows up as the infinite spatial volume of the 3-spheres as $|t| \to \infty$. At late times, it looks like we get an infinite number of copies of the spatial region covered by the static coordinates. Note however that at $\tau = 0$ there are only two copies, and as we will see in the next section, the second copy is just a trick for computing thermal Green’s functions in the static region. Since quantum field theory is unitary, there are no more quantum states in the large $\tau$ region than there are at $\tau = 0$. In field theory, this is because there are an infinite number of states in any volume, no matter how small. The expansion of the universe can convert field theory states of any large momentum, into low momentum states.

As in asymptotically flat or AdS spaces, we can obtain useful information about the quantum theory by investigating perturbations that do not disturb the asymptotic behavior. Since most ways of foliating this geometry give compact spatial sections, the asymptotic regions to be considered are past and future infinity.
To get an idea of the constraints on such perturbations, consider the exercise of setting small masses $m$ on each point of the sphere, i.e. making the “co-moving observers” of global coordinates into physical particles. If we do this at global time $T$, and space the masses by the particle’s Compton wavelength (since in a quantum theory, no particle can be localized more precisely than that), then at $t = 0$ the particle number density is

$$m^3 \cosh^3(T/R),$$

and the 00 component of the stress tensor is exponentially large if $T \gg R$. In other words, long before $t = 0$, the back reaction on the geometry of the test masses becomes important. In order to avoid this, we must make $m \sim \cosh^{-1}(T/R)$ at time $T$. This strongly suggests that, if we want to preserve dS asymptotics in the future, we must not try to fill the apparently huge volumes of space available in the past with matter. Rigorous results along these lines have been obtained in [9] [10]. The conclusion of those studies is that if one inserts too much matter in the infinite past, then a singularity forms before $t = 0$. If the singularity can be confined within a marginally trapped surface of radius $< 3^{-1/2}R$, this can be viewed as a black hole excitation of dS space, but if not, the whole space-time experiences a Big Crunch and we are no longer within the class of asymptotically dS space-times.

Thus considerations strictly confined to global coordinates give us similar constraints on the total entropy of asymptotically dS space-time, as those we find in the static patch. As in the black hole example, the infinite number of horizon volumes in global coordinates is mirrored by the infinite number of near horizon states in static coordinates. Both infinities are cut off by the holographic principle.

We can get a little more insight into this by thinking about how many of the states in the static patch can be thought of as localized particles in the bulk. The field theory entropy is of order

$$S \sim M_c^3 R^3,$$

where $M_c$ is an ultraviolet cutoff. The energy of a typical state in this ensemble is

$$E \sim M_c^4 R^3,$$

and it has a Schwarzschild radius

$$R_S \sim \frac{M_c^4 R^3}{M_P^2},$$

so if we insist that the system does not contain black holes whose size scales like $R$ (since most black hole states are not well modeled by field theory) then we find the field theoretic entropy is bounded by

$$S < (RM_P)^{3/2}.$$

This indicates that the total dS entropy is large enough to accommodate $(RM_P)^{1/2}$ copies of the maximal set of field theory degrees of freedom in a single horizon volume. This is, plausibly, the correct cutoff on the global coordinate picture of an infinite number of horizon volumes in unconstrained QFT.

To summarize, simple arguments, in either global or static coordinates, suggest that dS space is a quantum system with a finite dimensional Hilbert space, in accordance with the Strong Holographic Principle.
Both the eternal black hole and de Sitter space have bifurcate horizons. The space-time geometry consists of two causally disconnected regions separated by a pair of horizons. There is a space-like surface on which the two regions touch at a single point, so that, in QFT the Hilbert space can be thought of as a tensor product\(^4\) of Hilbert spaces of states localized in the two separate regions.

W. Israel \([11]\), following the seminal work of Hartle and Hawking, first pointed out the proper interpretation of this peculiar feature of these geometries in the quantum theory. Both the eternal black hole and dS space are thermal equilibrium states. The QFT physics in such a state is contained in the thermal correlators

\[
\text{Tr} \ e^{-\beta H} T \phi(x_1, t_1) \ldots \phi(x_n, t_n).
\]

For equal times, the static thermal correlators are easily computed in terms of a Euclidean path integral. In principle, more general correlators can be obtained by analytic continuation, but analytic continuation of an approximate formula often misses crucial aspects of the real time physics, and in particular hydrodynamic behavior, in which the results of small perturbations acting over long time periods can be crucial. Instead, the real time physics is usually represented by the Keldysh-Mahantappa-Schwinger closed time path formalism. This is a Lorentzian signature path integral over a complex contour. It computes in-in expectation values, unlike the standard Lorentzian path integral, which produces in-out transition amplitudes. By making insertions on different portions of the complex contour, one can compute time ordered expectation values, or anti-time ordered ones, or retarded commutators, or Wightman functions etc.

A particular form of the closed time path integral, called Thermo-field theory, computes thermal expectation values as pure state expectation values in an entangled state of two copies of the original system. The idea is to consider the state

\[
|\Psi_\beta\rangle = \sum_n e^{-\frac{\beta E_n}{2}}|n\rangle_1 \otimes |n\rangle_2,
\]

for a system whose Hamiltonian is

\[
H = H_1 - H_2.
\]

This is a zero energy state, and we can consider ordinary expectation values in this state of operators \(e.g.\) that operate only in system 1. It’s easy to see that these are just the thermal expectation values exhibited above.

Israel \([11]\) pointed out that the Hawking-Hartle state of QFT on the Kruskal extension of a black hole was precisely such a Thermo-field state, and that this was an explanation for the bizarre fact that time runs in opposite directions in the two branches of the manifold.

\(^4\)Actually, there are technical subtleties in this statement having to do with UV divergences. We will ignore those in the sequel, primarily because a truly quantized theory of gravitation cuts them off.
Maldacena [12] has recently revived interest in this interpretation in connection with AdS black holes and [13] has extended Israel’s result to the Bunch-Davies vacuum of dS space.

All of this is straightforward and well known. There is another class of solutions with bifurcate horizons that has been investigated in recent literature. These are the Guth-Farhi solutions [14] and similar solutions in AdS space [15]. There are two characteristics of these solutions that are different from either eternal black holes or dS space. They evolve from smooth initial data on a space-like slice. The future asymptotic region looks like a black hole embedded in either asymptotically flat or AdS space. The singular region of the black hole has a second horizon, the exterior region of which is a dS space with a black hole embedded in it.

Since they evolve from smooth initial data, one might have interpreted solutions like these as demonstrations that one can “create a universe in the laboratory”. There is an implicit violation of unitarity in this phrase. It is clear that the observer in the asymptotically flat or AdS region can only see an impure density matrix in the asymptotic future, because its state is entangled with the state in the dS region. Furthermore, the entropy of the dS universe is not bounded by the entropy of the black hole in the asymptotically flat or AdS region.

Fortunately, we do not have to assume that the smooth initial conditions, which lead to this bifurcated future infinity, define a quantum state in our theory. Guth and Farhi proved that all such solutions have a singularity at finite proper time in their past, and their paper is entitled “An obstacle to creating a universe in the laboratory”. In asymptotically flat space-time the observables of a quantum theory of gravity are all encoded in a scattering matrix. No scattering data on past infinity, can ever produce the Guth-Farhi initial data on a space-like slice. Thus, despite its smoothness and despite the fact that it falls off at infinity, this solution has nothing to do with the scattering matrix of quantum gravity in asymptotically flat space.

Our experience with eternal black holes and de Sitter space suggests an alternate explanation of the meaning of this state in the quantum theory. Consider a density matrix $\rho$ for a quantum system whose Hilbert space is $\mathcal{H}_1$. Given any other quantum system whose Hilbert space $\mathcal{H}_2$ has a dimension larger than or equal to that of $\mathcal{H}_1$, we can construct $\rho$ by tracing over $\mathcal{H}_2$ in some entangled state

$$|\Psi\rangle = \sum_{m,n} c_{mn}|\psi_m\rangle_1|\psi_n\rangle_2.$$  

$$\rho = \sum_{m,k,n} c_{mn}^* c_{kn} |\psi_k\rangle_1\langle\psi_m|_2.$$  

Neither $\mathcal{H}_2$ nor the choice of entangled state is uniquely specified by $\rho$. If $\rho$ has less than maximal entropy in $\mathcal{H}_1$, then we can even make do with a lower dimensional Hilbert space $\mathcal{H}_2$. In quantum information theory, constructions like this are used by adherents of the *Church of the Larger Hilbert Space* (CLHS), and clever choices of the entangled state can simplify arguments and calculations. What is clear is that no such construction implies in quantum mechanics that the particular Hilbert space $\mathcal{H}_2$ or choice of the entangled state, has anything to do with the quantum theory in $\mathcal{H}_1$. Even those kneeling in the pews of the CLHS would laugh you out of the room if you suggested as much. An impure density matrix
is a statement of ignorance. That ignorance can be caused by the correlation of the degrees of freedom we measure with any number of other systems. There is no way to tell which one.

So I would like to suggest that all solutions of Einstein’s equations with two causally disconnected future infinities, even if they evolve from smooth initial data, be viewed as possibly artificial ways of constructing a density matrix for one system by entangling it with another. Such solutions do not imply that one of the quantum systems can be realized as a state in the other. One simply cannot create a universe in the laboratory.

Indeed, as I have argued elsewhere [16], if I consider a potential with an asymptotically flat solution and a dS solution, and attempt to throw energy in from infinity in order to form a bubble in which the field lies near the dS minimum over a sphere of radius $R$, then before $R$ becomes as large as the dS radius, the entire region is engulfed in a black hole. This procedure cannot realize a Guth-Farhi solution, because of the theorem of [14].

Thus, if we restrict attention to solutions whose initial conditions extrapolate smoothly to past infinity in Minkowski space, with small localized perturbations corresponding to incoming wave packets, we will never find a solution with a bifurcate horizon. The Cosmic Censorship conjecture is properly stated as the claim that the future asymptotics of all such solutions corresponds to outgoing wave packets, plus a finite number of finite area black holes. In the quantum theory the black holes will decay and the S-matrix for wave packets will be unitary.

A similar result may be conjectured for asymptotically AdS space. Here we must be careful to impose the correct boundary conditions at spatial infinity in order to be sure we are discussing solutions that correspond to states in a given theory, rather than perturbations of the original theory by a local operator on the boundary [17]. The appropriate boundary conditions for scattering in AdS space have been discussed by [18].

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