MASS AND MAGNETIC DISTRIBUTIONS IN SELF-GRAVITATING SUPER-ALFVÉNIC TURBULENCE WITH ADAPTIVE MESH REFINEMENT

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ABSTRACT

In this work, we present the mass and magnetic distributions found in a recent adaptive mesh refinement magnetohydrodynamic simulation of supersonic, super-Alfvénic, self-gravitating turbulence. Power-law tails are found in both mass density and magnetic field probability density functions, with $P(\rho) \propto \rho^{-1.6}$ and $P(B) \propto B^{-2.7}$. A power-law relationship is also found between magnetic field strength and density, with $B \propto \rho^{0.5}$, throughout the collapsing gas. The mass distribution of gravitationally bound cores is shown to be in excellent agreement with recent observation of prestellar cores. The mass-to-flux distribution of cores is also found to be in excellent agreement with recent Zeeman splitting measurements. We also compare the relationship between velocity dispersion and density to the same cores, and find an increasing relationship between the two, with $\sigma \propto \rho^{0.25}$, also in agreement with the observations. We then estimate the potential effects of ambipolar diffusion in our cores and find that due to the weakness of the magnetic field in our simulation, the inclusion of ambipolar diffusion in our simulation will not cause significant alterations of the flow dynamics.

Key words: ISM: clouds – ISM: kinematics and dynamics – ISM: magnetic fields – magnetohydrodynamics (MHD)

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1. INTRODUCTION

Understanding the details of star formation is one of the great open problems in astrophysics today. One of the central questions is the role of magnetic fields. Two opposite paradigms have been explored during the study of this problem. The earliest paradigm argues for strong fields that dominate the process (Shu et al. 1987; Mouschovias 1987a, 1987b) and yield their dominance by way of ambipolar diffusion (AD); the other extreme argues for relatively weak fields that at most alter the thickness of shocks (Padoan & Nordlund 1999; Padoan et al. 2007), with the majority of the details being determined by the turbulence.

Central to the development of the second paradigm has been the development of more advanced computing hardware and software. Only a handful of simulations have been performed that incorporate all three necessary aspects: turbulence, gravity, and magnetic fields; notable are Gammie et al. (2003), Li et al. (2004), Heitsch et al. (2001), Vázquez-Semadeni et al. (2005), Padoan & Nordlund (2011), and Price & Bate (2008, 2010). The enormous range of scales at work in this problem have motivated the development of high dynamic range techniques, namely, adaptive mesh refinement (AMR); Fromang et al. 2006; Collins et al. 2010) and the addition of magnetohydrodynamics (MHD) to smoothed particle hydrodynamics (SPH; Price & Monaghan 2004; Price 2010). These techniques are beginning to yield interesting results on the nature of MHD in star formation (Price & Bate 2008; Dib et al. 2010).

In this work, we focus on the distribution of mass and magnetic fields in a super-Alfvénic simulation using high-resolution AMR simulations. Some of the data from the simulation presented here have already been used to examine projection effects in estimating the mass–size and linewidth–size relations in Shetty et al. (2010). Section 2 will introduce the numerical algorithm and discuss the initial conditions and parameters of the simulation. In Sections 3 and 4, we present the volume and mass-weighted density probability density function (PDF) for both density and magnetic field. In Section 5, we discuss the relation between mass density and magnetic field strength. In Section 6, we discuss the relationship between line-of-sight magnetic field strength and column density (typically used as a proxy for the mass-to-flux ratio $M/\Phi$, where $M$ is the mass, and $\Phi$ is the magnetic flux) and compare to recent observations. In Section 7, we present the mass distribution of cores and compare to observations of prestellar cores. In Section 8, we examine the potential effects of AD. We show that, due to a manner in which both magnetic field strength and velocity dispersion increase with density, it is not likely to contribute significantly to the collapse dynamics. In Section 9, we will conclude.

2. NUMERICAL MODEL

2.1. Simulation Software

The simulation was performed with the MHD extension of Enzo by Collins et al. (2010). It employs the MHD Godunov solver of Li et al. (2008) to solve the MHD equations; to constrain $\nabla \cdot B$, it employs the constrained transport (CT) scheme of Gardiner & Stone (2005); the AMR is done with the divergence-free interpolation of Balsara (2001). An isothermal equation of state (EOS) and ideal MHD are assumed throughout.

Refinement is performed such that the Jeans length

$$\lambda_J = \frac{\pi c_s}{G\rho},$$

where $c_s$ is the sound speed and $G$ is the gravitational constant, is resolved by four grid zones everywhere (Truelove et al. 1997). Thus, whenever the density in a zone is above the...
“Truelove density”

\[ \rho_T = \frac{\pi^2 c_s^2}{16 G \Delta x^2}, \]

that zone is flagged for refinement. The maximum number of levels is four. With a base grid of \(128^3\), this gives a maximum effective resolution of 2048\(^3\).

2.2. Simulation Parameters

We began with a uniform density and magnetic field with no self-gravity, and stirred with a Gaussian random field with no compressional modes, with power distributed in a top-hat between \(1 \leq k \leq 2\) and mean Mach number 8.9. Driving is done in the same manner as that in Mac Low (1999), such that the energy injection rate is constant. After a number of dynamical times, gravity was activated. This time is \(t = 0\). Driving continued for the duration of the collapse.

The initial uniform fields have a ratio of gas-to-magnetic pressure of \(\beta = 8 \pi c_s^2 (\rho)/(B^2) = 22.2\), but the mean squared field strength has been amplified by the driving so that when gravity is turned on at \(t = 0\), \(\beta = 0.2\). This yields an Alfvén Mach number of 2.8 at \(t = 0\). Units have been chosen so that the virial parameter \(\alpha = 5 \sigma^2 R/(3GM) \approx 1\), where \(\sigma\) is the three-dimensional velocity dispersion, \(M\) is the total mass in the box, and \(R\) is half the width of the box. The virial parameter is used as a measure of the relative balance of kinetic and gravitational energy. The typical parameter used to measure the relative effects of magnetic and gravitational energy is the mass-to-flux ratio in units of the critical value for support. If we use the critical value for a sheet, as in Nakano & Nakamura (1978), \((M/\Phi)_\text{c} = (4 \pi^2 G)^{-1/2}\), the uniform (pre-turbulence) cube gives us

\[ \lambda = (M/\Phi)/(M/\Phi)_\text{c} = 18.7. \]

Mass to flux is a reasonable quantity to parameterize magnetic support for simple geometries, such as spheres and disks. However, for situations like the one presented here, in both the turbulent box at \(t = 0\) and the pre-turbulent cores discussed later, the spatial structure of the objects in question is not easily defined. We find it more instructive to examine the ratio of energies. For the sake of comparison to \(M/\Phi\), we define

\[ \lambda_E = \sqrt{E_G/E_B}, \]

where

\[ E_G = G \int \frac{\rho(r)\rho(r')}{|r - r'|} d^3r d^3r', \]

\[ E_B = \frac{B^2}{8 \pi} d^3r. \]

For the initial conditions at \(t = 0\), we have

\[ \lambda_E = 6.5. \]

For reference, a sphere with critical mass \(M_c = \Phi/(2\pi \sqrt{G})\), \(\lambda_E = 0.2\).

In a similar vein, if we define \(\alpha_E = 2E_K/E_G\) as the ratio of measured gravitational energy to total kinetic energy \(E_K = \frac{1}{2} \int \rho v^2 d^3r\), we find for our data at \(t = 0\)

\[ \alpha_E = 0.64. \]

2.3. Physical Scaling

Although these simulations are scale-free, we use a box size of 10 pc and density of 300 cm\(^{-3}\) for the analysis presented here. This gives a resolution on the finest zones of 1000 AU and a mean mass density \(\rho_0 = 1.2 \times 10^{-21} \text{ g}\). The sound speed is set to be 0.2 km\(\text{s}^{-1}\), giving a temperature of \(\approx 10 \text{ K}\). Mean and RMS magnetic fields are 0.6 and 2.7 \(\text{ \mu G}\), respectively, with the mean along the \(z\) direction. These units give a total mass of \(1.2 \times 10^4 M_\odot\).

Rescaling can be found with the following relations:

\[ t_{\text{ff}} = \sqrt{\frac{3\pi}{32G\rho}} \]

\[ L \propto T^{1/2} \rho^{-1/2} \]

\[ M \propto T^{3/2} \rho^{-1/2} \]

\[ \sigma \propto T^{1/2} \]

\[ B \propto T^{1/2} \rho^{1/2}. \]

2.4. Core Definition

We select cores with the algorithm described by Smith et al. (2009) which selects topologically connected density isosurfaces at a spacing of \(\delta \rho = \rho_i/\rho_{i-1}\) that have no substructure, that is, a contour at \(\rho_{i-1}\) that contains zero or one contour above \(\rho_i\). This definition is similar to that used by Padoan et al. (2007) and Schmidt et al. (2010), both of whom reported no significant effects due to variations in \(\delta \rho\), provided \(\delta \rho < 1.16\). In our simulation, \(\delta \rho = 1.12\). A core is determined to be bound if its gravitational energy is at least twice as large as the sum of kinetic, thermal, and magnetic energies. All analysis has been performed with the AMR analysis package yt (Turk et al. 2011).

All analysis in this section takes place at a single snapshot, \(t = 0.75 t_{\text{ff}}\). At this time, there are 148 cores that match the definition of bound. Figure 1 shows a projection of density at this time step in the left panel and a close-up of a portion of the column density map in the right panel with core boundaries drawn. All cores drawn are associated with topologically isolated, gravitationally bound objects, but some have their features washed out by projection effects.

3. DENSITY PDF

One of the most prominent consequences of supersonic turbulence is the log-normal distribution of densities (Vazquez-Semadeni 1994; Padoan et al. 1997a, 1997b; Scalo et al. 1998; Passot & Vázquez-Semadeni 1998; Nordlund & Padoan 1999; Klessen 2000; Padoan & Nordlund 2002, 2011; Federrath et al. 2008b). This has been used to predict several properties of star formation, including the initial mass function (IMF) of stars (Padoan & Nordlund 2002; Padoan et al. 2007), brown dwarf frequency (Padoan & Nordlund 2004), and the star formation rate (Krumholz & McKee 2005). Here, we will discuss the PDF one expects to see from isothermal turbulence, and what has been seen in our simulations with the inclusion of self-gravity.

The central limit theorem states that the sum of a sufficiently large number of uncorrelated events will form a Normal, or Gaussian, distribution. A corollary of this is: a sufficiently large number of random \textit{multiplicative} events will form a log-normal distribution. This distribution has been experimentally verified.
in a large number of different simulations, both pure hydro (Vázquez-Semadeni 1994; Padoan et al. 1997b; Kritsuk et al. 2007, 2009b) and super-Alfvénic MHD (Li et al. 2004; Lemaster & Stone 2008; Kritsuk et al. 2009a).

The log-normal distribution is given by

\[ P(x) \propto \frac{1}{\sqrt{2\pi \sigma^2}} \exp\left[\frac{(\ln x - \mu)^2}{2\sigma^2}\right] d\ln x, \tag{12} \]

where \( x = \rho/\rho_0 \) is the over density and \( \mu = -\sigma^2/2 \) is the mean of \( \ln x \). For pure hydrodynamical turbulence,

\[ \sigma = \sqrt{\ln(1 + b^2M^2)}, \tag{13} \]

where \( b \) has been determined numerically to lie between 0.3 and 0.4 (Padoan et al. 1997b; Federrath et al. 2008b; Kritsuk et al. 2007; Beetz et al. 2008; Kritsuk et al. 2010; Federrath et al. 2010). The value \( b \) has been shown to vary by as much as a factor of three, depending on the ratio of solenoidal to compressive forcing (Federrath et al. 2008b, 2010).

For driven MHD turbulence, Lemaster & Stone (2008) find that

\[ \sigma_{\text{LM08}} = \sqrt{[-0.72 \ln(1 + 0.5M^2) + 0.2]}, \tag{14} \]

and is insensitive to the magnetic field strength.

Figure 2 shows density PDF \( P(\rho) \) for two snapshots; at \( t = 0 \) (left) and \( t = 0.75t_{\text{ff}} \) (right). In both plots, the solid line is the measured PDF and the dashed line is the fit to a log normal. Table 1 shows the fit parameters. The addition of self-gravity causes the PDF to widen and the mean to decrease. We find that \( b = 0.3 \) for our initial conditions, similar to Kritsuk et al. (2007), but \( b = 0.5 \) for the collapsed snapshot; we also find that the dispersion, \( \sigma_{\text{LM08}} \), found by Lemaster & Stone (2008) is in better agreement with the collapsed state of the simulation.

When turbulence simulations are performed in the presence of self-gravity, several authors (Klessen 2000; Slyz et al. 2005; Vázquez-Semadeni et al. 2008; Federrath et al. 2008a; Kritsuk et al. 2011) find that the log-normal PDF underestimates the high-density tail of the measured PDF. Slyz et al. (2005) fit the high-density tail to a power law with an index of \(-1.5\). Klessen (2000) does not fit a power law, but the resolution of those simulations is much lower than what we present here. Vázquez-Semadeni et al. (2008) mention the existence of a power law, but say nothing further. Observationally, Kainulainen et al. (2009) found power-law wings in column density distributions of active star-forming regions, and a similar high-density power law has been seen in Aquila (P. André 2010, private communication). Our work is the first reported case in an MHD simulation.

Kritsuk et al. (2011) find an extended two-part power law, with the index \(-1.7 \) at intermediate densities, and \(-1 \) at high densities. They provide the first explanation of this power law, associating it with a self-similar collapse. Self-similar collapse solutions have been examined by a number of authors (Larson 1969; Penston 1969; Shu 1977; Whitworth & Summers 1985), and we will examine three potential solutions here: isothermal collapse, pressure-free collapse, and the expansion wave generated from inside-out collapse. Each solution is characterized by power-law density profile, \( \rho \propto r^{-n} \), with \( n_{\text{IP}} = -2 \) for isothermal collapse, \( n_{\text{EW}} = -1.5 \), and the isothermal expansion wave that develops after the formation of the singularity, and \( n_{\text{PP}} = -12/7 \approx -1.7 \) for pressure-free collapse. These in turn yield power indices for the density PDF, \( P(\rho) \propto \rho^n \), of \( n_{\text{IP}} = -1.5 \), \( n_{\text{EW}} = -2 \), and \( n_{\text{PP}} = -1.75 \).

Figure 3 shows both volume-weighted PDF \( P(\rho) \) and mass-weighted PDF \( M(\rho) \), with the power-law fits of \(-1.64 \) and \(-0.64 \), respectively, in the range of \( \rho = 10–1000 \). The dashed line in Figure 3 is the same curve as the solid line in Figure 2, but here with the mid- to high-density power law emphasized. This power law breaks down above a density of 1000, likely due to resolution effects, as the maximum resolvable density in the simulation, according to the aforementioned resolution criterion (Truelove et al. 1997), is \( \rho/\rho_0 = 1623 \). The exponent we find is quite close to the \(-1.7 \) found by Kritsuk et al. (2011). Federrath et al. (2008a) also measure the density PDF from a driven, self-gravitating turbulence simulation (without magnetic fields) with a set of Lagrangian tracer particles. The tracer particles in their work also follow a power-law tail, with an index of \(-0.6 \pm 0.1 \) (C. Federrath 2010, private communication) consistent with our mass-weighted \( M(\rho) \). Our result lies between the exponents expected for isothermal collapse and pressure-free collapse.

This is consistent with the dominance of gravitational energy over thermal and magnetic energies, though not their complete absence. It is a bit more shallow than the expansion wave solution.
Figure 2. Density PDFs of the initial conditions at $t = 0$ (left) and after $t = 0.75 t_{\text{ff}}$ (right). Both are fit to log-normal distributions (dashed lines).

Figure 3. Volume-weighted (left) and mass-weighted (right) density PDFs at $t = 0.75 t_{\text{ff}}$, with power-law fits.

Table 1

| $t/t_{\text{ff}}$ | $M$  | $\mu$ | $\sigma$ | $\chi^2$ | $b$  | $\sigma_{\text{lin}}$ |
|-------------------|------|-------|----------|----------|------|----------------------|
| 0                 | 8.15 | -0.80 | 1.35     | $4.6 \times 10^{-6}$ | 0.27 | 1.67                 |
| 0.75              | 8.57 | -1.86 | 1.74     | $1.6 \times 10^{-6}$ | 0.52 | 1.65                 |

4. MAGNETIC PDF

Figure 4 shows the volume-weighted PDF of the magnetic field at $t = 0$ in a semi-log plot. In agreement with Padoan & Nordlund (1999), we find a roughly exponential tail at high field strength. This intermittent distribution in the magnetic field is caused by the large field amplifications by the strong, three-dimensional compressions made possible by the large kinetic energy, relative to the magnetic energy, of the super-Alfvénic turbulence.

The left panel of Figure 5 shows the PDF for the magnetic field strength, $P(B)$ for $t = 0$ (solid line) and $t = 0.75 t_{\text{ff}}$ (dashed line), here in a log–log plot. The most significant aspect of this figure is the strong power-law tail after the collapse has evolved. Gravitational collapse amplifies the peak magnetic field tail after the collapse has evolved. Gravitational collapse amplifies the peak magnetic field strength by two orders of magnitude (in fact a factor of 320), and creates a prominent power-law tail. This power-law tail is fit by $P(B) \propto B^{-x}$, with $x = -2.74$. As seen in Section 5, as well as other simulations (Li et al. 2004) and observations (Bertoldi & McKee 1992; Crutcher 1999), $P \propto B^2$. Using this and the expected density PDF as discussed in Section 3, we find that $x_{\text{LP}} = -3$ for isothermal collapse, $x_{\text{EW}} = -4$ for the expansion wave, and $x_{\text{PF}} = -3.5$ for the pressure-free solution. This clearly favors self-similar isothermal collapse as the basic physical paradigm for the high-density gas in our simulation. Details will be discussed in the following section, where we measure the relationship between $B$ and $\rho$ in our simulation.

Figure 4. Semi-log plot of $P(B)$ at $t = 0$. A roughly exponential tail can be seen at high field strength, consistent with earlier super-Alfvénic turbulence simulations.

The right panel of Figure 5 shows the mass-weighted PDF, $M(B)$ for the same snapshots as the left panel. If we fit a power law to the same field strength range that was used for $P(B)$, we find $M(B) \propto B^{-0.4}$. However, the power law in $M(B)$ is not nearly as well defined as for $P(B)$. This is due to two related effects: the power-law relation between $\rho$ and $B$ is less well defined at field strengths above $B = 100 \mu G$ (see Figure 7) and the power law in density breaks down above $\rho/\rho_0 > 1000$, likely due to resolution effects (see Figure 3).

5. FIELD STRENGTH VERSUS DENSITY

Figure 6 shows a contour plot of magnetic field strength versus density, colored by a fraction of mass in each $(B, \rho)$
Figure 5. Magnetic field PDF $P(|B|)$ (left) and mass-weighted PDF $M(|B|)$ (right) for $t = 0$ (solid line) and $t = 0.75t_{ff}$ (dashed line).

Figure 6. Magnetic field strength vs. density for all zones in the simulation for $t = 0$ (left) and $t = 0.75t_{ff}$ (right). Color field shows total mass fraction in each $(B, \rho)$ bin.

bin. The left panel shows $t = 0$, before the action of gravity. As in Padoan & Nordlund (1999), the upper envelope is matched by a power law roughly of the form $B \propto \rho^{0.4}$, and the scatter is quite large in both $B$ and $\rho$. The large scatter in field strength is due to the fact that only the component of the field perpendicular to a shock is amplified, but due to the weak nature of the magnetic field, the relative orientations of $\vec{B}$ and the shock are not correlated. Padoan & Nordlund (1999) demonstrated that in models with larger magnetic field strength (sub-Alfvénic turbulence), the flow is constrained to move primarily along the field lines, significantly decreasing the field strength amplification power of the turbulence, and decreasing the scatter in the magnetic field strength. The right panel in Figure 6 shows the $B-\rho$ relation at $t = 0.75t_{ff}$, after gravity has taken effect. Several features are noticeable. Both $B$ and $\rho$ are amplified by several orders of magnitude by the gravity, and the initial scatter in the distribution remains imprinted on the self-gravitating distribution. The relation gets more shallow at $\rho/\rho_0 > 10^3$, which is the same density at which the power-law slope seen in $P(\rho)$ changes. As before, this is likely due to resolution effects. Another feature of Figure 6 is that the upper envelope slope increases sharply at $\rho/\rho_0 > 5$. Above this density, gravitational collapse has overtaken the dynamics, as evidenced by the onset of the power law in the density PDF. As these motions are three-dimensional contractions, rather than the one-dimensional compression due to shocks, the amplification is stronger than what is possible from the turbulence alone.

Figure 7 shows a mass-weighted average $B$ as a function of density bin, which has been fit to a power law between densities of 10 and 1000. We find

$$B \propto \rho^{0.48}. \quad (15)$$

This behavior has been predicted or observed by several other authors: Fiedler & Mouschovias (1993) predicted $B \propto \rho^\kappa$, with $\kappa = 0.44-0.5$, though this was done in a quasi-static collapse model with AD; both Bertoldi & McKee (1992) and Crutcher (1999) found a similar result in dense molecular cores and interpreted it as constancy of the Alfvén speed; Li et al. (2004) found this behavior for central density and magnetic field strengths, and Scott & Black (1980) found this both for the central density in certain portions of spherical collapse. Figure 6 demonstrates that this behavior is endemic to the entire collapse process, not just the high-density collapsed objects, and happens from turbulent initial conditions as well as isolated spheres.

6. MASS-TO-FUX RATIO

Figure 8 shows line-of-sight magnetic field strength, $B_{los}$, versus column density, $N$, for three populations of cores: CN Zeeman splitting measurements of Falgarone et al. (2008), OH Zeeman splitting measurements of Troland & Crutcher (2008), and gravitationally bound cores in our simulation. Color shows $\lambda_E = \sqrt{E_G/E_B}$, our adopted proxy for the mass-to-flux ratio. The left plot shows $t = 0$, the right plot shows $t = 0.75t_{ff}$. In the simulated points, the line of sight is taken along each of the three coordinate axes, so that there are three points in the plot for each simulated core; $B_{los}$ is the density-weighted average of each field quantity, and $N = M/A$, the total computed mass of the core divided by the area projected along that axis. The
observational points are currently the best data available to relate magnetic field strength to mass, and some of the only magnetic field measurements for high-density protostellar cores. The ratio between these quantities is often used as a proxy for the mass-to-flux ratio $\lambda$:

$$\lambda = \frac{c_g N(H_2) \sqrt{G}}{B_{\text{los}} c_\phi},$$

where $c_g = (1/2, 1/3)$ is a geometrical correction for (spherical, sheet) projection effects, and $c_\phi = (0.12, 1/2\pi)$ is a correction found numerically for equilibrium configurations of a (sphere, sheet), respectively (Bourke et al. 2001). Here, we directly compare $B_{\text{los}}$ and $N(H_2)$. This allows us to eliminate the need for either correction factor and compare the results of our model directly to the observations.

We find that our cores and the observed cores have almost the same distribution in the $B_{\text{los}}$–$N$ space. This shows that the early evolution of prestellar cores is well reproduced with isothermal super-Alfvénic turbulence and self-gravity. Simulations without self-gravity by Lunttila et al. (2008) successfully reproduce the lower density OH measurements, but lack the density range to reproduce the CN observations. This is also seen in the left panel of Figure 8, which shows the initial time $t = 0$, at which only the effects of turbulence are felt by the gas. The inclusion of self-gravity and the large range of density scales allowed by AMR reproduces the higher density CN points.

The color of the simulated points corresponds to $\lambda_E = \sqrt{\rho G \Sigma}$, non-spherical analog of $\lambda = (M/\Phi)/(M/\Phi)_c$. Li et al. (2004) found that $\lambda > 10$ for all cores in question. We find that $\lambda_E > 1$ for all objects, which, using the spherical case as a guide, is analogous to $\lambda > 5$, so our results are in reasonable agreement with theirs.

The simulated points at $t = 0.75t_{\text{ff}}$ in Figure 8 are best fit by a power law

$$B_{\text{los}} = N^{0.57}.$$

Collapse that preserves mass to flux would have an exponent of one, by Equation (16). Since ideal MHD preserves $M/\Phi$ along a flux tube, this indicates that flow along the field lines must be responsible for some of the dynamics as discussed by Padoan & Nordlund (1999). The relationship in Equation (17) is also expected from Figure 6, demonstrating that the local properties of the core are dictated by the global flow properties.

7. CORE MASS FUNCTION

One of the open questions in star formation is the origin of the stellar IMF. Salpeter (1955) first measured this and fit it to a power law:

$$dN = 0.03 \left( \frac{M}{M_\odot} \right)^{\alpha} dM,$$

$$\alpha = -2.35.$$  

This fit was done between 1 and 10 $M_\odot$. The exact value of the exponent in the 1–10 $M_\odot$ range is still under investigation (Scalo 2005), though recent measurements give the range of $\alpha$ to be between $-2.3$ and $-2.8$.

It has been proposed that the IMF and core mass function (CMF) are directly related to one another, either directly (Motte et al. 1998) or with some fraction of each core lost in the final collapse and accretion phase (Enoch et al. 2008). This implies that the IMF is determined by the global or large-scale processes of star formation, in our model the combined effects of turbulence and gravity, as in the model of Padoan & Nordlund (2002). Alternative models have the IMF set by local physics, once protostars have formed within the prestellar condensations. These models include the competitive accretion model of Bonnell et al. (2001), wherein the population of neighboring protostars influences the final mass of any given star, and the models of Shu et al. (1987) or Myers (2010), where prestellar outflows halt or slow the inflow of gas onto the protostar.

Figure 9 shows the mass distribution for all bound cores. The fit to the high end of the distribution is $n(M) \propto M^{-2.1\pm0.6}$. The fit was performed and error was determined by fitting a power law between the peak and the highest bin for a succession of bin counts between 5 and 25. We find good agreement between our slope and the IMF slopes mentioned above, and the slope of $-2.3 \pm 0.6$ measured for the CMF by Enoch et al. (2008).

Further agreement with the CMF is seen in Figure 10, which shows the cumulative mass function $N(>M) = \int_M^{\infty} n(M) dM$ for our bound cores (gray line), all cores in our data (dashed line) and the prestellar cores from Peruse, Ophichus, and Serpens presented in Enoch et al. (2009; black line). Here, we have scaled both populations to the Bonnor–Ebert mass:

$$M_{\text{BE}} = \frac{1.18 c_s^4}{G^3 \rho_0^2}.$$
The observed points used a Bonnor–Ebert mass of 1.5 $M_\odot$, which corresponds to a background density of $\approx 9000$ cm$^{-3}$ at 10 K. This is somewhat higher than the mean density in these clouds, but not unreasonable for the ambient density immediately surrounding the cores, which have mean densities of $\approx 10^5$ cm$^{-3}$. The simulated cores used the mean density in the box, which in the scaling used throughout this paper gives $M_{BE} = 10 M_\odot$, though strictly speaking this is a free value. The observational data are all prestellar cores from the 1.1 mm Bolocam survey of Perseus, Serpens, and Ophiuchus presented in Enoch et al. (2006), Young et al. (2006), and Enoch et al. (2007), respectively, that do not have an associated infrared source in the Spitzer c2d catalog (Enoch et al. 2008).

The majority of these objects can reasonably be assumed to be self-gravitating, based on comparisons of the Perseus cores to kinematic information from the molecular line survey of the same region by Rosolowsky et al. (2008). As our simulation only attempts to model the prestellar core phase of star formation, and not the formation of the actual star itself, this sample of objects is the best observational counterpart for comparison. The bound cores in our simulation show outstanding agreement with the observed cores.

The dashed line in Figure 10 shows the insensitivity of our mass distribution to clump selection. The distributions show excellent agreement between the three populations, demonstrating that slight modifications to clump definition or the definition of “gravitationally bound” do not alter the basic result.

In order to make contact with the models of Padoan & Nordlund (2002) and Padoan et al. (2007), Figure 11 shows the observed cores (black line) along with bound clumps from our initial conditions. These models posit that the mass distribution is due to gravitationally bound density enhancements caused by the turbulence alone, and they use the Bonnor–Ebert mass as a proxy for gravitational binding. The agreement in Figure 11 is not as striking as that of the previous figure, as the slope is somewhat higher (that is, more cores at lower masses) and the peak mass is not as high. However, the general behavior is quite similar. This indicates that the effects of self-gravity, namely, fragmentation and accretion, provide higher order modifications to the initial seed fluctuations provided by the turbulent fluctuations.

### 8. IMPLICATIONS FOR DIFFUSION

Historically, diffusion of material past magnetic fields, specifically through the mechanism of AD, has been an enormously important aspect of star formation theory (Mouschovias 1987a; Shu et al. 1987). In the star formation model dominated by magnetic support, the conservation of $M/\Phi$ implies that if a magnetic field is strong enough to support a core initially, it is for all time. This requires a mechanism for removing magnetic support, and the most relevant source of removal is AD. In the model we are examining in this work, the magnetic field is not strong enough to support the cloud as a whole against gravity, nor is it strong enough to significantly alter the dynamics of the turbulence. We additionally do not include the effects of AD. In this section, we estimate to what degree AD would effect the dynamics of the solution, as this is a point in parameter space not often studied with AD simulations.

AD is the process by which the neutral particles, which are not directly tied to the magnetic field, diffuse past the ions, which are tied to the magnetic field. The velocity of this drift, $v_{AD}$, is determined by the balance of the ion–neutral drag force and the Lorentz force

$$v_{AD} = \frac{1}{4\pi} (\nabla \times B) \times B \approx \frac{B_{rms}^2}{4\pi \ell},$$

where $\gamma_{AD}$ is the coupling coefficient between ions and neutrals, $\rho_i$ is the ion mass density, and $\rho_n$ is the neutral mass density.

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Figure 9. Mass distribution of bound cores. The fit line is $N(M) \propto M^{-2.1\pm0.6}$, consistent with both IMF and CMF measurements. (A color version of this figure is available in the online journal.)

Figure 10. Cumulative mass distributions for our bound cores (black) and the prestellar cores from Enoch et al. (2008; gray), relative to the Bonnor–Ebert mass. Here, we have normalized the observed cores to $M_{BE} = 1.5 M_\odot$ for comparison.

Figure 11. Mass distribution of starless cores from Enoch et al. (2008) along with bound clumps from our initial conditions.
We define the AD Reynolds number

$$R_{AD} = \frac{v_f}{v_{AD}} \approx \gamma_{AD} \chi_i \rho^2 \sigma_{1D} \ell B_{rms}^{-2}$$  \tag{21}$$

as in McKee & Khersonsky (1995), see also the discussion in McKee et al. (2010), where $v_f$ is a typical flow velocity in a core, for which we use the one-dimensional velocity dispersion, $\chi_i$ is the ionization fraction, and we have assumed $\rho_n \approx \rho$ so that $\rho_i \rho_n = \chi_i \rho^2$. $R_{AD}$ measures the relative importance of AD in a particular object. Cores with large $R_{AD}$, that is, where ion-neutral drift speed is low relative to the dynamical speed, will experience a low amount of AD. Equation (21) demonstrates that $R_{AD}$ for a given object is determined by essentially two processes: self-gravitating MHD determines $\rho, \sigma_{1D}, \ell$, and $B_{rms}$; interstellar chemistry and the local neighborhood of radiation sources determine $\gamma_{AD}$ and $\chi_i$. The former is modeled in our simulation; the latter must be approximated. Further study must be done to fully incorporate AD, MHD, radiation, ionization chemistry, and AMR. However, we can put upper and lower bounds on the effects of AD by using the dynamics captured here and reasonable assumptions about the ionization state of the gas.

We follow McKee et al. (2010) for the treatment of $\gamma_{AD}$ and $\chi_i$. We assume that the dominant ion carrier is HCO$^+$, which gives the coupling coefficient as

$$\gamma_{AD} = \frac{1.9 \times 10^{-9}}{m_i + m_i} = 3.7 \times 10^{13} \gamma^* \text{cm}^3 \text{s}^{-1} \text{g}^{-1}$$  \tag{22}$$

(Draine et al. 1983), where $\gamma^*$ allows us to easily treat other charge carriers. McKee et al. (2010) find $\chi_i$ to be

$$\chi_i = 2.25 \times 10^{-6} \chi_i^{n_{H,3}^{-1/2}}$$  \tag{23}$$

where $n_{H,3} = n_{H} / (10^3 \text{ cm}^3)$ is the density of hydrogen nuclei and

$$\chi_i^* = \frac{m_i}{29 \text{ amu}} \left( \frac{C_i}{630 \text{ cm}^{-3/2} \text{ s}^{1/2}} \right) \left( \frac{\xi_{CR}}{3 \times 10^{-17} \text{ s}^{-1}} \right)$$  \tag{24}$$

is a correction for differences in model details. Here, $m_i$ is the mass of the primary ionization agent (HCO$^+$ in our case), $C_i$ corrects for discrepancies in the recombination rate due to, for example, polycyclic aromatic hydrocarbons or charged grains, and $\xi_{CR}$ is the cosmic-ray ionization rate. See McKee et al. (2010) and references therein for a discussion on factors contributing to $\chi_i^*$.

The ionization fraction is observed to vary greatly in nature and seems to show no significant correlation with density; rather, it seems to have more to do with local environment, namely, optical depth in FUV (Padoan et al. 2004), proximity to ionizing cosmic-ray sources (Dalgarno 2006), and possibly local turbulence (Xie et al. 1995). Equation (23) assumes that cosmic rays dominate the ionization, which implies a relatively high optical depth to FUV, with $A_V > 4$ (McKee 1989).

In order to separate dynamical effects, that we have fully modeled, from chemistry effects that are somewhat ambiguous, we define $\hat{R}_{AD}$ such that

$$R_{AD} = \chi_i^* \gamma^* n_{H,3}^{-1/2} \hat{R}_{AD}.$$  \tag{25}$$

$\hat{R}_{AD}$ is a fiducial estimate of $R_{AD}$, with detailed effects being put into the first three terms. Figure 12 shows the $\hat{R}_{AD}$ (filled circles) and $R_{AD}$ (open circles) versus mean density for all bound clumps. The left plot shows the initial conditions, while the right plot shows our evolved snapshot. The filled circles ($\hat{R}_{AD}$) serve two purposes: first, they represent contribution to $R_{AD}$ from purely dynamical effects, which we have modeled completely; second, they provide an upper limit for $R_{AD}$, one in which the ionization fraction is constant. This would be the case, for instance, if turbulent mixing in the cores were extremely efficient. There are three salient features in these plots. First, the initial conditions have quite large values for $\hat{R}_{AD}$, 70 < $\hat{R}_{AD}$ < 500, which would limit the applicability of AD in the early formation of the cores. Second, $\hat{R}_{AD}$ for $t = 0.75 \text{ ff}$ reaches very large values: a few clumps have $\hat{R}_{AD} < 100$, but the higher density clumps reach values as large as $3 \times 10^5$. Third, $\hat{R}_{AD} \propto n$, which implies that as collapse progresses, the importance of AD decreases quickly.

The open circles in Figure 12 represent the case where the optical depth is high enough ($A_V > 4$; McKee et al. 2010) that cosmic rays are the sole source of ionization. In this case, $\chi_i \propto n^{-1/2}$, which gives $R_{AD} = n^{-1/2} \hat{R}_{AD}$. This reduces $R_{AD}$ for all clumps, but the major features still hold: most clumps have $R_{AD} > 1$; the high-density cores can still reach $R_{AD} > 10^3$, and $R_{AD}$ still increases with density, albeit more slowly, as $R_{AD} \propto n^{1/2}$. This represents an extreme case of shielding; reduced opacity due to the clumpiness of the medium (Padoan & Nordlund 2004) will increase the ionizing radiation, and turbulent mixing will mix the ionized surface layers further into the cores (Xie et al. 1995). Both effects will increase $\chi_i$, in turn increasing $R_{AD}$. Detailed study of this behavior requires the inclusion of both AD and radiative transfer, the latter of which can be extremely computationally demanding.

The reason for this increase in $\hat{R}_{AD}$ with density can be seen by examining the terms in Equation (21). The velocity and size terms do not depend strongly on density, with $\sigma_{1D} \propto \rho^{0.25}$ and $\ell \propto \rho^{-0.32}$. The positive correlation between velocity dispersion and density can be seen in Figure 13, and is shown for our gravitationally bound cores, as well as the OH and CN Zeeman cores. This positive correlation can be understood as being due to a combination of the converging acceleration from the core’s self-gravity, as well as dynamical interactions between cores. The negative relationship between size and density can be understood as the cores’ masses clustering around a characteristic mass, here a few $M_{BE}$. This leaves $R_{AD} \propto \chi_i \rho^2 B_{rms}^{-2}$, which by Equation (15) reduces simply to

![Figure 12. AD Reynolds number in two forms: $\hat{R}_{AD}$ (filled circles) assumes constant ionization fraction, while $R_{AD}$ (open circles) assumes ionization falls off with $n^{-1/2}$, as it would for cosmic-ray-dominated ionization. The left panel is $t = 0$, while the right panel is $t = 0.75 \text{ ff}$.](image)
We find in Sections 3 and 4 that power-law tails develop for both high density and high magnetic field in volume- and mass-weighted PDFs, \( P(\rho) \propto \rho^{-1.64} \) and \( P(B) \propto B^{-2.74} \), respectively. The volume-weighted density PDF is consistent with the prediction of self-similar isothermal collapse, which predicts \( P(\rho) \propto \rho^{-1.5} \) and \( P(B) \propto \rho^{-3} \), though pressureless collapse is not ruled out (Kritsuk et al. 2011).

The relationship between the magnetic field and the density also shows a power-law behavior \( B \propto \rho^{0.48} \) throughout the gas, consistent with the findings of Li et al. (2004), who found a similar behavior in the peak density/field relation in cores. This then allows us to explain the magnetic PDF, by combining this result with the density PDF.

Gravitationally bound cores found in our simulation were compared against several observational surveys. Comparisons with the most recent Zeeman splitting measurements of Troland & Crutcher (2008) and Falgarone et al. (2008) show that the mass-to-flux ratio in our simulations agrees in value and behavior with those found observationally. The relationship between field strength and column density is fit to a power law, \( B \propto N^{0.52} \), demonstrating that significant mass to flux, thus magnetic support, is lost due to motion along the field lines. We also demonstrated an increase in velocity dispersion with increasing density, in agreement with the same observed cores. In our cores, we find \( \sigma \propto \rho^{0.25} \). Thus, we find good agreement with the three key energetic ingredients that determine the dynamics of collapse.

Comparing our CMF to that of prestellar cores in Enoch et al. (2008), we again find excellent agreement. A slope of \( 2.1 \pm 0.6 \) agrees with their fit value of \( 2.3 \pm 0.6 \), and cumulative mass distributions line up almost identically. The relatively good match between the observed CMF and the observed IMF indicates that the IMF is determined well before the onset of nuclear burning, at a relatively low (compared to the protostar) density. A multiplicative offset of \( \geq 1/4 \) is seen between the cores of Enoch et al. (2008) and the observed IMF, indicating that as much as \( 3/4 \) of the mass is lost in the final collapse phase. However, this is only an upper limit to the lost fraction, as the peak of the observed CMF is heavily influenced by its completeness limit. Bound cores in our simulation agree with the observed CMF extremely well, indicating that super-Alfvénic turbulence and gravity are primarily responsible for the structure of the mass distribution of the CMF, and ultimately the IMF.

Finally, we estimate the effects of ambipolar diffusion in the prestellar cores. We find that as cores collapse, the Lorentz force, which increases the rate of diffusion, grows more slowly than the combined effects of ion–neutral coupling and velocity dispersion, which both decrease the rate of diffusion. Thus, as collapse proceeds, the ideal MHD approximation becomes more realistic and will likely continue to do so until isothermal EOS breaks down and a hydrostatic core is formed and the ensuing disk amplifies the magnetic field strength. This result is for a single value of Mach number \( M \), virial parameter \( \kappa_\ast \), and Alfvén Mach number \( M_A \). Future study will be required to determine if this is a universal behavior of a coincidental point in parameter space and improve the estimation of the ionization fraction as a function of density.

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