Abstract

We present some arguments in favour of that the Kaluza-Klein picture of the world has been confirmed in the experiments at very low energies where the nucleon-nucleon dynamics has been studied. Early predicted $L$-particle is related to the new scale of strong nucleon-nucleon forces in the spirit of Kaluza-Klein approach. It is shown that KK excitations remarkably describe the experimentally observed mass spectrum of diproton system.

1 Introduction: Kaluza-Klein Picture

The original idea of Kaluza and Klein is based on the hypothesis that the input space-time is a $(4+d)$-dimensional space $\mathcal{M}_{(4+d)}$ which can be represented as a tensor product of the visible four-dimensional world $M_4$ with a compact internal $d$-dimensional space $K_d$

$$\mathcal{M}_{(4+d)} = M_4 \times K_d.$$ (1)

The compact internal space $K_d$ is space-like one i.e. it has only spatial dimensions which may be considered as extra spatial dimensions of $M_4$. In according with the tensor product structure of the space $\mathcal{M}_{(4+d)}$ the metric may be chosen in a factorizable form. This means that if $z^M = \{x^\mu, y^m\}$, $(M = 0, 1, \ldots, 3 + d, \mu = 0, 1, 2, 3, m = 1, 2, \ldots, d)$, are local coordinates on $\mathcal{M}_{(4+d)}$ then the factorizable metric looks like

$$ds^2 = G_{MN}(z)dz^Mdz^N = g_{\mu\nu}(x)dx^\mu dx^\nu + \gamma_{mn}(x, y)dy^m dy^n,$$

where $g_{\mu\nu}(x)$ is the metric on $M_4$.

In the year 1921, Kaluza proposed a unification of the Einstein gravity and the Maxwell theory of electromagnetism in four dimensions starting from Einstein gravity in five dimensions. He assumed that the five-dimensional space $\mathcal{M}_5$ had to be a product of a four-dimensional space-time $M_4$ and a circle $S_1$: $\mathcal{M}_5 = M_4 \times S_1$. After that the metric $G_{MN}(z)$ in $\mathcal{M}_5$ can be decomposed into 10 components describing Einstein gravity tensor $G_{\mu\nu} \rightarrow g_{\mu\nu}$, four components $G_{\mu 5} \rightarrow A_\mu$ forming an electromagnetic gauge field, and one component $G_{55} \rightarrow \phi$ representing a scalar field. It was shown that the zero mode sector of the Kaluza model (the model is actually a five-dimensional Einstein gravity) is equivalent to the four-dimensional theory which describes the Einstein gravity with a four-dimensional general coordinate transformations and the Maxwell theory of electromagnetism with a gauge transformations. In his original work, Kaluza assumed the zero mode of scalar field had to be positive constant in order to insure a positive energy.
Recently some models with extra dimensions have been proposed to attack the electroweak quantum instability of the Standard Model known as hierarchy problem between the electroweak and gravity scales. To illustrate the main idea how the hierarchy problem may be solved in a theory with extra dimensions let us consider the Einstein (4 + \(d\))-dimensional gravity with the action

\[
S_{(4+d)} = \frac{1}{16\pi G_{(4+d)}} \int d^{4+d}z \sqrt{-G} R(\mathcal{M}_{(4+d)}),
\]

where \(G = \det |G_{MN}|\), and the Ricci scalar curvature \(R(\mathcal{M}_{(4+d)})\) is defined by the metric \(G_{MN}\). By the mode expanding and integrating over \(K_d\) one obtains the four-dimensional action

\[
S_4 = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R(M_4) + \text{non-zero modes},
\]

where \(g = \det |g_{\mu\nu}|\), \(g_{\mu\nu}\) is Einstein gravity tensor on \(M_4\), and the Ricci scalar curvature \(R(M_4)\) is defined by the metric \(g_{\mu\nu}\), \(G_N\) is four-dimensional gravitational Newton constant related with the fundamental constant \(G_{(4+d)}\) by the following equation

\[
G_N = \frac{1}{V_d} G_{(4+d)}, \tag{2}
\]

where \(V_d\) is the volume of the compact internal space \(K_d\). We can rewrite Eq. (2) in terms of the four-dimensional Planck mass \(M_{Pl} = G_N^{-1/2}\) and a fundamental mass scale of the \((4 + d)\)-dimensional gravity \(M^{d+2} = G_{(4+d)}^{-1}\)

\[
M_{Pl}^2 = V_d M^{d+2}. \tag{3}
\]

The latter formula is often cited as the reduction formula. If \(R\) is a characteristic size of \(K_d\) then \(V_d \sim R^d\), i.e. we suppose \(V_d = C_d R^d\), where \(C_d\) is some constant. Eq. (3) gives

\[
M_{Pl} = C_d^{1/2} M(MR)^{d/2}. \tag{4}
\]

Now, it’s clear that, if the size \(R\) of the compact internal space \(K_d\) is large compared to the fundamental length \(M^{-1}\), the Planck mass is much larger than the fundamental gravity scale. Going further on, if we suppose that the fundamental gravity scale is of the same order as the electroweak scale, \(M \sim 1\, \text{TeV}\), then a huge gap between \(M_{Pl}\) and \(M_{EW}\) is resulted from the large size of the internal extra space \(K_d\). So, the hierarchy problem replaces by the hierarchy \(R/M^{-1} \sim (M_{Pl}/M)^{2/d}\) and becomes the problem to explain why the size \(R\) of extra space \(K_d\) is large. From Eq. (4), assuming that \(M \sim 1\, \text{TeV}\), it follows

\[
R \sim M^{-1} \left(\frac{M_{Pl}}{M}\right)^{2/d} \sim 10^{32/d} \cdot 10^{-17}\, \text{cm}. \tag{5}
\]

or

\[
R^{-1} \sim M \left(\frac{M}{M_{Pl}}\right)^{2/d} \sim 10^{3-32/d}\, \text{GeV}.
\]

An exceptional case \(d = 2\) gives \(R \sim 1\, \text{mm},\) \((R^{-1} \sim 10^{-4}\, \text{eV})\) and this is certainly a quite interesting observation. In the case \(d = 1\) we have \(R \sim 10^{15}\, \text{cm}\), i.e. this case is excluded by unacceptable large value of \(R\). For \(d > 2\) one obtains

\[
d = 3, \quad R \sim 4.6 \times 10^{-7}\, \text{cm}, \quad R^{-1} \sim 20\, \text{eV}
\]
\(d = 6, \quad R \sim 2.2 \times 10^{-12} \text{cm}, \quad R^{-1} \sim 4.6 \text{MeV}\)

It is obviously that the basic idea of the Kaluza-Klein scenario may be applied to any model in Quantum Field Theory. As example, let us consider the simplest case of \((4+d)\)-dimensional model of scalar field with the action

\[
S = \int d^{4+d}z \sqrt{-G} \left[ \frac{1}{2} \left( \partial_{\mu} \Phi \right)^2 - \frac{m^2}{2} \Phi^2 + \frac{G_{(4+d)}}{4!} \Phi^4 \right],
\]

where \(G = \det |G_{MN}|, G_{MN}\) is the metric on \(M_{(4+d)} = M_4 \times K_d, M_4\) is pseudo-Euclidean Minkowski space-time, \(K_d\) is a compact internal \(d\)-dimensional space with the characteristic size \(R\). Let \(\Delta_{K_d}\) be the Laplace operator on the internal space \(K_d\), and \(Y_n(y)\) are ortho-normalized eigenfunctions of the Laplace operator

\[
\Delta_{K_d} Y_n(y) = -\frac{\lambda_n}{R^2} Y_n(y),
\]

and \(n\) is a (multi)index labeling the eigenvalue \(\lambda_n\) of the eigenfunction \(Y_n(y)\). \(d\)-dimensional torus \(T_d\) with equal radii \(R\) is an especially simple example of the compact internal space of extra dimensions \(K_d\). The eigenfunctions and eigenvalues in this special case look like

\[
Y_n(y) = \frac{1}{\sqrt{V_d}} \exp \left( i \sum_{m=1}^d n_m y^m / R \right),
\]

where \(n_m\) are integer numbers, \(V_d = (2\pi R)^d\) is the volume of the torus.

To reduce the multidimensional theory to the effective four-dimensional one we write a harmonic expansion for the multidimensional field \(\Phi(z)\)

\[
\Phi(z) = \Phi(x, y) = \sum_n \phi^{(n)}(x) Y_n(y).
\]

The coefficients \(\phi^{(n)}(x)\) of the harmonic expansion are called Kaluza-Klein (KK) excitations or KK modes, and they usually include the zero-mode \(\phi^{(0)}(x)\), corresponding to \(n = 0\) and the eigenvalue \(\lambda_0 = 0\). Substitution of the KK mode expansion into action and integration over the internal space \(K_d\) gives

\[
S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \left( \partial_{\mu} \phi^{(0)} \right)^2 - \frac{m^2}{2} (\phi^{(0)})^2 + \frac{g}{4!} (\phi^{(0)})^4 + \right. \\
\left. + \sum_{n \neq 0} \left[ \frac{1}{2} \left( \partial_{\mu} \phi^{(n)} \right) \left( \partial^{\mu} \phi^{(n)} \right)^* - \frac{m_n^2}{2} (\phi^{(n)}\phi^{(n)*}) + \frac{g}{4!} (\phi^{(0)})^2 \sum_{n \neq 0} \phi^{(n)}\phi^{(n)*} \right] \right\} + \ldots.
\]

For the masses of the KK modes one obtains

\[
m_n^2 = m^2 + \frac{\lambda_n}{R^2},
\]

and the coupling constant \(g\) of the four-dimensional theory is related to the coupling constant \(G_{(4+d)}\) of the initial multidimensional theory by the equation

\[
g = \frac{G_{(4+d)}}{V_d},
\]
where \( V_d \) is the volume of the compact internal space of extra dimensions \( K_d \). The fundamental coupling constant \( G_{(4+d)} \) has dimension [mass]\(^{-d} \). So, the four-dimensional coupling constant \( g \) is dimensionless one as it should be. Eqs. (11,12) represent the basic relations of Kaluza-Klein scenario. Similar relations take place for other types of multidimensional quantum field theoretical models. From four-dimensional point of view we can interpret each KK mode as a particle with the mass \( m_n \) given by Eq. (11). We see that in according with Kaluza-Klein scenario any multidimensional field contains an infinite set of KK modes, i.e. an infinite set of four-dimensional particles with increasing masses, which is called the Kaluza-Klein tower. Therefore, an experimental observation of series KK excitations with a characteristic spectrum of the form \( \sigma_{11} \) would be an evidence of the existence of extra dimensions. So far the KK partners of the particles of the Standard Model have not been observed. In the Kaluza-Klein scenario this fact can be explained by a microscopic small size \( R \) of extra dimensions \( (R < 10^{-17} \text{cm}) \); in that case the KK excitations may be produced only at super-high energies of the scale \( E \sim 1/R > 1 \text{TeV} \). Below this scale only homogeneous zero modes with \( n = 0 \) are accessible ones for an observation in recent high energy experiments. That is why, there is a hope to search the KK excitations at the future LHC and other colliders.

However, as we have mentioned above, the solution of hierarchy problem motivated vice versa to introduce the internal extra space with a large size \( (R \sim 1 \text{mm}) \). To resolve this obvious contradiction recently it has been proposed a remarkable idea of “brane world picture” according to which all matter fields (except gravity) are localized on a three-dimensional submanifold – brane – embedded in fundamental multidimensional space. In the brane world scenario extra dimensions may have large and even very large size; they may also have experimentally observable effects. Even though the models with the brane world scenario may rather seem as exotic ones, nevertheless, they provide a base for a nontrivial phenomenological issues related to the fundamental problems in particle physics and cosmology. We refer with a pleasure the interested reader to the excellent review articles [1, 2] and many references therein.

In this note we would like to argue in favour of that the extra dimensions have been observed for a long time in the experiments at very low energies where the nucleon-nucleon dynamics has been studied. In this respect, we would like to show that the structure of proton-proton total cross section at very low energies has a clear signature of the existence of the extra dimensions.

## 2 Global structure of proton-proton total cross section

Recently a simple theoretical formula describing the global structure of proton-proton total cross-section in the whole range of energies available up today has been derived. The fit to the experimental data with the formula was made, and it was shown that there is a very good correspondence of the theoretical formula to the existing experimental data obtained at the accelerators [3, 4].

Moreover it turned out there is a very good correspondence of the theory to all existing cosmic ray experimental data as well. The predicted values for \( \sigma_{pp}^{\text{tot}} \) obtained from theoretical description of all existing accelerators data are completely compatible with the values obtained from cosmic ray experiments [5]. The global structure of proton-proton total cross section is shown in Fig. 1 extracted from paper [5].
The theoretical formula describing the global structure of proton-proton total cross section has the following form

\[ \sigma_{pp}^{tot}(s) = \sigma_{as}^{tot}(s) \left[ 1 + \left( \frac{c_1}{\sqrt{s - 4m_N^2}} R_0^3(s) - \frac{c_2}{\sqrt{s - s_{thr}} R_0^3(s)} \right) (1 + d(s)) \right]_{s>s_{thr}} + \text{Res}(s) \],

\[ R_0^2(s) = \left[ 0.40874044 \sigma_{as}^{tot}(s)(mb) - B(s) \right] (GeV^{-2}), \]
\[ \sigma_{as}^{tot}(s) = 42.0479 + 1.7548 \ln^2(\sqrt{s}/20.74), \]
\[ B(s) = 11.92 + 0.3036 \ln^2(\sqrt{s}/20.74), \]
\[ c_1 = (192.85 \pm 1.68)GeV^{-2}, \quad c_2 = (186.02 \pm 1.67)GeV^{-2}, \]
\[ s_{thr} = (3.5283 \pm 0.0052)GeV^2, \]
\[ d(s) = \sum_{k=1}^{8} \frac{d_k}{s^{k/2}}, \quad \text{Res}(s) = \sum_{i=1}^{N} \frac{C_R^i s_R^i \Gamma_R^i}{\sqrt{s - 4m_N^2} [(s - s_R^i)^2 + s_R^i \Gamma_R^i/2]} \].

For the numerical values of the parameters \(d_i (i = 1, \ldots, 8)\) see original paper [4]. The mathematical structure of the formula is very simple and physically transparent: the total cross section is represented in a factorized form; one factor describes high energy asymptotics of total cross section and it has the universal energy dependence predicted by the general theorems in local Quantum Field Theory (Froissart theorem); the other factor is responsible for the behaviour of total cross section at low energies and it has a complicated resonance structure. The nontrivial feature of the formula is the presence of the new “threshold” \(s_{thr} = 3.5283 \, GeV^2\) which is near the elastic one.

Some information concerning the diproton resonances is collected in Table 1. The positions of resonances and their widths, listed in Table 1, were fixed in our fit, and only relative contributions of the resonances \(C_R^i\) have been considered as free fit parameters. Fitted parameters \(C_R^i\) obtained by the fit are listed in Table 1 too.
Table 1. Diproton resonances.

| $m_R$(MeV) | $\Gamma_R$(MeV) | Refs. | $C_R$(GeV$^2$) |
|------------|-----------------|-------|----------------|
| 1937 ± 2   | 7 ± 2           | 6     | 0.058 ± 0.018  |
| 1947(5) ± 2.5 | 8 ± 3.9       | 7     | 0.093 ± 0.028  |
| 1955 ± 2   | 9 ± 4           | 6     | 0.158 ± 0.024  |
| 1965 ± 2   | 6 ± 2           | 6     | 0.138 ± 0.009  |
| 1980 ± 2   | 9 ± 2           | 6     | 0.310 ± 0.051  |
| 1999 ± 2   | 9 ± 4           | 6     | 0.188 ± 0.070  |
| 2008 ± 3   | 4 ± 2           | 6     | 0.176 ± 0.050  |
| 2027 ± ?   | 10 − 12         | 8     | 0.121 ± 0.018  |
| 2087 ± 3   | 12 ± 7          | 6     | −0.069 ± 0.010 |
| 2106 ± 2   | 11 ± 5          | 6     | −0.232 ± 0.025 |
| 2127(9) ± 5 | 4 ± 2           | 6     | −0.222 ± 0.056 |
| 2180(72) ± 5 | 7 ± 3         | 6     | 0.131 ± 0.015  |
| 2217± ?   | 8 − 10          | 8     | 0.112 ± 0.031  |
| 2238 ± 3   | 22 ± 8          | 6     | 0.221 ± 0.078  |
| 2282 ± 4   | 24 ± 9          | 6     | 0.098 ± 0.024  |

Our fitting curve concerning low-energy region is shown in Fig. 2. We also plotted in Fig. 3 the resonance structure of proton-proton total cross section at low energies without the experimental points but with dashed line corresponding the “background” where all resonances are switched off. As it is seen from this Figure there is a clear signature for the diproton resonances. We may conclude that the diproton resonances are confirmed by the data set for proton-proton total cross section at low energies from statistical point of view by the good fit [8].

From the global structure of proton-proton total cross-section it follows that the new “threshold”, which is near the elastic one, looks like a manifestation of a new unknown particle:

$$\sqrt{s_{thr}} = 2m_p + m_{\mathcal{L}}, \quad m_{\mathcal{L}} = 1.833 \text{ MeV}. \quad (13)$$

This particle was called as $\mathcal{L}$-particle from the word lightest. It should be emphasized that we predicted the position of the new “threshold” with a high accuracy. Of course, the natural questions have been arisen. What is the physical nature and dynamical origin of $\mathcal{L}$-particle? Could $\mathcal{L}$-particle be related to the experimentally observed diproton resonances spectrum? In the next section we’ll give the answers to these questions.

### 3 $\mathcal{L}$-particle in Kaluza-Klein world

Here we would like to apply the main issues of Kaluza-Klein approach to our concrete case. Let us assume that $\mathcal{L}$-particle is related to the first KK excitation in the diproton system. Using formula (11) for the masses of KK modes, we can calculate the scale (size) $R$ of the compact internal extra space. So, starting from the formula

$$\sqrt{s_{thr}} = 2m_p + m_{\mathcal{L}} = 2\sqrt{m_p^2 + \frac{1}{R^2}}, \quad (14)$$
Figure 2: The proton-proton total cross-section versus $\sqrt{s}$ at low energies. Solid line corresponds to our theory predictions.

Figure 3: The resonance structure for the proton-proton total cross-section versus $\sqrt{s}$ at low energies. Solid line is our theory predictions. Dashed line corresponds to the “background” where all resonances are switched off.

one obtains

$$\frac{1}{R} = \sqrt{m_{L}(m_{p} + \frac{1}{4}m_{L})} = 41.481\,MeV,$$

(15)

where $m_{p} = 938.272\,MeV$ for the proton mass and Eq. (13) for the mass of $L$-particle have been used. From Eq. (15) it follows

$$R = 24.1\,GeV^{-1} = 4.75\times10^{-13}\,cm.$$

(16)

It should be emphasized a remarkable fact: the size (16) just corresponds to the scale of distances where the strong Yukawa forces in strength come down to the electromagnetic
forces
\[ g_{\text{eff}} = g_{\pi NN} \exp(-m_\pi R) \sim 0.5, \quad (g_{\pi NN}^2/4\pi = 14.6). \]

On the other hand, for the fundamental mass scale calculated by formulae (3) or (4) with account of size (16) in the case \( d = 6 \) we find
\[ M \sim R^{-1} \left( \frac{M_{\text{Pl}}}{R^{-1}} \right)^{2/(d+2)} \bigg|_{d=6} \sim 5 \text{ TeV}. \] (17)

Mass scale (17) is just the scale accepted in the Standard Model, and this is an interesting observation as well.

Going further on, let us build the Kaluza-Klein tower of KK excitations by the formula
\[ M_n = 2 \sqrt{m_p^2 + \frac{n^2}{R^2}} , \quad (n = 1, 2, 3, \ldots) \] (18)

and compare it with the observed irregularities in the spectrum of mass of the diproton system.\(^1\) The result of the comparison is shown in Table 2. As it is seen from the Table 2, there is a quite remarkable correspondence of the Kaluza-Klein picture with the experiment.

Now, let us suppose that effective bosons \( B_n \) with the masses \( m_n = n/R \) related to KK-excitations of a proton may have an effective Yukawa-type interaction with the fermions
\[ L_{\text{eff}} = g_{\text{eff}} \bar{\psi}_f O \psi_f B_n, \] (19)

where \( f \) denotes some fermion, for example lepton or quark. If \( m_n > 2m_f \) then effective bosons may decay into fermion-antifermion pair. For the partial width of such decay in the lowest order over coupling constant we have
\[ \Gamma_n = \frac{\alpha_{\text{eff}} m_n}{2} F_O(x_n^2), \] (20)

where \( \alpha_{\text{eff}} = g_{\text{eff}}^2/4\pi \), \( x_n^2 = m_f^2/m_n^2 \), \( F_O(x^2) = (1 - 4x^2)^{3/2} \) for \( O = 1 \) and \( F_O(x^2) = (1 - 4x^2)^{1/2} \) for \( O = \gamma_5 \). In that case one obtains an estimation
\[ \Gamma_n \sim n \cdot 0.4 \text{ MeV}. \] (21)

It’s clear from the physics under consideration that a life time of the diproton resonances will be defined by the decays of effective bosons \( B_n \). This is a remarkable fact that crude estimation (21) is in a good agreement with an experiment and gives an explanation of (super)narrowness of dibaryons peaks. Moreover, estimation (21) shows that the larger the dibaryon mass is, the larger is the width of the dibaryon.

Certainly, we have considered here the simplest case of Kaluza-Klein picture: The built KK-tower corresponds to either one-dimensional compact extra space or d-dimensional equal radii torus with the constraint
\[ n = \sqrt{n_1^2 + n_2^2 + \ldots n_d^2} = 1, 2, 3, \ldots, \] (22)

where \( n_i (i = 1, \ldots, d) \) are integer numbers.

\(^1\)Similar formula has been discussed in the literature [17] but with a different physical interpretation.
Table 2. Kaluza-Klein tower of KK excitations of diproton system.

| n  | $M_n$ (MeV) | $M_{pp}^{exp}$ (MeV) | Refs. |
|----|-------------|----------------------|-------|
| 1  | 1878.38     | 1877.5 ± 0.5         | [9]   |
| 2  | 1883.87     | 1886 ± 1             | [6]   |
| 3  | 1892.98     | 1898 ± 1             | [6]   |
| 4  | 1905.66     | 1904 ± 2             | [10]  |
| 5  | 1921.84     | 1916 ± 2             | [6]   |
|    |             | 1926 ± 2             | [10]  |
| 6  | 1941.44     | 1937 ± 2             | [6]   |
|    |             | 1942 ± 2             | [10]  |
|    |             | ∼1945                | [7]   |
| 7  | 1964.35     | 1965 ± 2             | [6]   |
|    |             | 1969 ± 2             | [11]  |
| 8  | 1990.46     | 1980 ± 2             | [6]   |
|    |             | 1999 ± 2             | [6]   |
| 9  | 2019.63     | 2017 ± 3             | [6]   |
| 10 | 2051.75     | 2035 ± 8             | [16]  |
|    |             | 2046 ± 3             | [6]   |
|    |             | ∼2050                | [12]  |
| 11 | 2086.68     | 2087 ± 3             | [6]   |
| 12 | 2124.27     | ∼2122                | [12]  |
|    |             | 2121 ± 3             | [13]  |
|    |             | 2129 ± 5             | [6]   |
| 13 | 2164.39     | 2140 ± 9             | [16]  |
|    |             | ∼2150                | [12]  |
|    |             | 2172 ± 5             | [6]   |
| 14 | 2206.91     | 2192 ± 3             | [13]  |
|    |             | 2217                 | [8]   |
|    |             | 2220                 | [15]  |
| 15 | 2251.67     | 2238 ± 3             | [6]   |
|    |             | 2240 ± 5             | [13]  |
| 16 | 2298.57     | 2282 ± 4             | [6,14]|
| 17 | 2347.45     | 2350                 | [15]  |

The constraint (22) corresponds to the special (Diophantus!) selection of the states. It’s clear that in general case of generic extra compact manifold we would have a significantly more wealthy spectrum of KK-excitations. We could imagine that there exist such extra compact manifold with a suitable geometry where KK-excitations of a few input fundamental entities (proton, electron, photon, etc.) would provide the experimentally observed spectrum of all particles, their resonances and nuclei states. As we hope, it would be possible to find in this way the global solution of the Spectral Problem. Anyhow, we believe that such perfect extra compact manifold with a beautiful geometry and its good-looking shapes exist.
4 Conclusion

In this short article we have presented some arguments in favour of Kaluza and Klein ideas genius which are waiting their time in high-energy experiments at future colliders. In fact, we have shown that Kaluza-Klein picture of the world has been confirmed in the experiments at very low energies where the nucleon-nucleon dynamics has been studied. Geniuly simple formula (18) provided by Kaluza-Klein approach so accurately describes the mass spectrum of diproton system, it’s very nice; certainly, it is not an accidental coincidence.

Here we have concerned the simplest model where the protons were considered as a scalar particles. It is well known that account of fermionic degrees of freedom may result the nontrivial problems related to both the index and the kernel of Dirac operator on a generic compact manifold. However, since the kernel of Dirac operator is equal to the kernel of its square, we can say with confidence that account of fermionic degrees of freedom for a proton will not change our main conclusion. This conclusion is that the existence of the extra dimensions was experimentally proved for a long time, but we did not understand it. Now, seems we understand it.

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