Constraining the Detailed Balance Condition in Hořava Gravity with Cosmic Accelerating Expansion

Chien-I Chiang\textsuperscript{1,2,}\textsuperscript{♯}, Je-An Gu\textsuperscript{2†}, Pisin Chen\textsuperscript{1,2,3,4‡}

\textsuperscript{1}Department of Physics, National Taiwan University, Taipei 10617, Taiwan, R.O.C.,
\textsuperscript{2}Leung Center for Cosmology and Particle Astrophysics, National Taiwan University, Taipei 10617, Taiwan, R.O.C.,
\textsuperscript{3}Graduate Institute of Astrophysics, National Taiwan University, Taipei 10617, Taiwan, R.O.C.,
\textsuperscript{4}Kavli Institute for Particle Astrophysics and Cosmology, SLAC National Accelerator Laboratory, Menlo Park, CA 94025, U.S.A.

\textsuperscript{♯}b95203016@ntu.edu.tw
\textsuperscript{†}jagu@ntu.edu.tw
\textsuperscript{‡}pisinchen@phys.ntu.edu.tw

Abstract: In 2009 Hořava proposed a power-counting renormalizable quantum gravity theory. Afterwards a term in the action that softly violates the detailed balance condition has been considered with the attempt of obtaining a more realistic theory in its IR-limit. This term is proportional to $\omega R^{(3)}$, where $\omega$ is a constant parameter and $R^{(3)}$ is the spatial Ricci scalar. In this paper we derive constraints on this IR-modified Hořava theory using the late-time cosmic accelerating expansion observations. We obtain a lower bound of $|\omega|$ that is nontrivial and depends on $\Lambda_W$, the cosmological constant of the three dimensional spatial action in the Hořava gravity. We find that to preserve the detailed balance condition, one needs to fine-tune $\Lambda_W$ such that $-2.29 \times 10^{-4} < (c^2 \Lambda_W)/(H_0^2 \Omega_{DE}) - 2 < 0$, where $H_0$ and $\Omega_{DE}$ are the Hubble parameter and dark energy density fraction in the present epoch, respectively. On the other hand, if we do not insist on the detailed balance condition, then the valid region for $\Lambda_W$ is much relaxed to $-0.39 < (c^2 \Lambda_W)/(H_0^2 \Omega_{DE}) - 2 < 0.12$. We find that although the detailed balance condition cannot be ruled out, it is strongly disfavored.
1. Introduction

In 2009 Hořava proposed a power-counting renormalizable quantum gravity theory, which is non-relativistic in the high-energy, or UV, limit and recovers the Lorentz symmetry in the low-energy, or IR, limit [1, 2]. Various aspects of this theory have been widely pursued [3–30]. In addition to the theory itself, its various implications in cosmology have also drawn wide attention [31–34]. Specifically, there has been a large amount of effort in studying the cosmological perturbations [35–43], black hole properties [44–54], gravitational waves [55–59], dark energy phenomenology [60–63], observational constraints on the parameters of the theory [64, 65], and many others.

With regard to the background space-time expansion, Hořava gravity in principle modifies the GR Friedmann equation with additional terms stemming from its non-conventional gravity, thereby contributing to the dark sector. Accordingly, the current and future cosmic observations may provide significant constraints on Hořava gravity, especially when connecting it with the cosmic accelerating expansion.

In this context it was found that the effective Friedmann equation derived from Hořava gravity with the detailed balance condition would include a cosmological constant (CC) term and a radiation-like $a^{-4}$ term [31, 32], where $a$ is the scale factor in the Friedmann-Robertson-Walker (FRW) metric. This radiation-like term originates from the fourth-order spatial derivative terms in the Hořava action. On the other hand, if the detailed balance condition is violated, then there would be other dark terms induced.

Although the gravity action of the Hořava theory with the detailed balance condition recovers that of GR in the IR limit, its solution may not be so. For example, in [66] it was shown that the black hole solution does not recover the usual AdS-Schwarzschild black
hole solution in GR. In order to attain a more desirable IR behavior without abandoning the simplicity provided by the detailed balance condition in the UV limit, several authors introduced a soft violation of the detailed balance condition [1, 67, 68], namely, a term proportional to the spatial three-curvature $R^{(3)}$. In particular, an IR-modified Hořava theory that accommodates flat Minkowski vacuum was studied in [68]. The exact solutions of spherical symmetry with and without matter were obtained in [69].

A question then naturally arises: to what extent can the Hořava gravity violate the detailed balance condition? One possible means to address this question would be to derive constraints on the IR-modification terms and those obeying the detailed balance condition from cosmological observations. For this purpose we consider a cosmological model studied in [60] based on the IR-modified Hořava gravity, with the FRW metric describing the background space-time and with the energy content that includes radiation and dust matter.

In [60] Park showed that the Friedmann equation of this cosmological model contains additional $a^{-4}$, $a^{-2}$, and CC terms beyond that in GR. Park identified these terms as the effective dark energy (DE) that is responsible for the cosmic acceleration. The observations about the expansion history can in principle constrain the behavior of the (effective) dark energy and thereby constrain the Hořava gravity. To constrain this Hořava Effective Dark Energy (HEDE) by observations, an efficient approach is via a phenomenological parametrization of the relevant physical quantities that have been well studied. Once the relation between the model parameters and the phenomenological parameters is established, the constraints on the model can be obtained from those on the phenomenological parameters that have been derived from observations. Park considered the widely used Chevallier-Polarski-Linder (CPL) parameterization of the equation of state of dark energy [70, 71],

$$w_{\text{DE}}(a) \equiv p_{\text{DE}}/\rho_{\text{DE}} = w_0 + w_a (1 - a),$$  \hspace{1cm} (1.1)

where the constraints on the phenomenological parameters $w_0$ and $w_a$ from the updated observations have been well studied (see, for example, [72–76]).

Park [60] explored the feasibility of HEDE by considering three best-fit values of $(w_0, w_a)$ obtained in [72, 73] where a non-flat universe was considered. It was suggested by Park that the existence of some HEDE models that satisfy these three best-fit values indicates the validity of HEDE. We note, however, that in principle the dark energy density of HEDE is determined once the values of $(w_0, w_a)$ are given. As will be shown in the present paper, these three best-fit models predict the dark energy densities that are much smaller than that required by observations, and have thus already been ruled out.

In the present paper we pursue a more comprehensive test of the IR-modified Hořava theory and its resultant HEDE based on the current observations. We particularly emphasize that a complete test of HEDE based on the cosmic expansion must take into consideration not only the evolution of the dark energy density $\rho_{\text{DE}}(a)$, which involves both the present value of $\Omega_{\text{DE}}$ and the equation of state $w_{\text{DE}}(a)$, but also the present value of the fractional density $\Omega_k$ of the spatial curvature.
Specifically, the present density fraction of dark energy $\Omega_{DE}$ should be around 0.74 and that of the spatial curvature $\Omega_k < 0.01$ [75]. In addition, since in the dark sector the radiation-like term would be dominant in the early universe, its energy density must be smaller than the true radiation energy density $\Omega_r$, otherwise HEDE would predict a later epoch of the matter-radiation equality and that in turn would be in conflict with the cosmic microwave background (CMB) and the big-bang nucleosynthesis (BBN) observational results. As will be shown in the present paper, the observational constraint on the dark energy equation of state $w_{de}$, together with the above three observational requirements, very tightly constrains HEDE. These observational constraints on HEDE favor the violation of detailed balance of Hořava gravity. We will present a lower bound on the extent of the violation.

This paper is organized as follows. In Sec. 2 we give a brief review of HEDE, a cosmological model based on the IR-modified Hořava theory. In Sec. 3 we discuss the general strategy that we utilize for the model test, which involves an approximate relation between the model parameters and the phenomenological parameters. In Sec. 4 we investigate the observational constraints on HEDE that are presented in the phenomenological parameter space. In Sec. 5 we show how the constraints on the phenomenological parameters are transcribed into that on the model parameters, and investigate its impact on the IR-modified Hořava gravity. In Sec. 6 we show the evolution patterns of the effective dark energy in the HEDE models that are consistent with observations. We conclude in Sec. 7.

2. Setup of the Model

To be self-contained, in this section we give a brief review of the IR-modified Hořava-gravity cosmological model that was investigated by Park [60]. The action of the IR-modified Hořava gravity reads

$$S_g = \int dt d^3x \sqrt{g}N \left[ \frac{2}{\kappa^2} (K_{ij}K^{ij} - \lambda K^2) - \frac{\kappa^2}{4\nu^4} C_{ij}C^{ij} + \frac{\kappa^2}{2\nu^2} \epsilon^{ijk} R^{(3)}_i \nabla_j R^{(3)}_k \right] + \frac{\kappa^2\mu^2}{8} R^{(3)}_{ij} R^{(3)}_{ij} + \frac{\kappa^2\mu^2}{8(3\lambda - 1)} \left( \frac{4\lambda - 1}{4}(R^{(3)})^2 - \Lambda_W R^{(3)} + 3\Lambda_W^2 \right) + \frac{\kappa^2\mu^2\omega}{8(3\lambda - 1)} R^{(3)} \right],$$

(2.1)

where the extrinsic curvature

$$K_{ij} = \frac{1}{2N} \left( \dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right)$$

(2.2)

(the dot denotes the time derivative), the Cotton tensor

$$C^{ij} = \epsilon^{ik\ell} \nabla_k \left( R^{(3)}_{\ell j} - \frac{1}{4} R^{(3)} \delta_{ij} \right),$$

(2.3)

and $\kappa, \lambda, \nu, \mu, \Lambda_W, \omega$ are constant parameters. Note that on the right-hand side of Eq.(2.1) the last term proportional to $\omega R^{(3)}$ induces the soft violation of the detailed balance condition. For a homogeneous and isotropic universe we consider a FRW metric of the form

$$ds^2 = -c^2 dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2/R_0^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right],$$

(4.1)
where \( k = +1, 0, -1 \) corresponds to a closed, a flat, and an open universe, respectively, and \( R_0 \) is the radius of spatial curvature of the universe in the present epoch. Assuming that the matter contribution is in the form of an ideal fluid with energy density \( \rho \) and pressure \( p \), Park obtained \([45]\)

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{\kappa^2}{6(3\lambda - 1)} \left[ \rho \pm \frac{3\kappa^2\mu^2}{8(3\lambda - 1)} \left( \frac{-k^2}{R_0^2a^4} + \frac{2k(\Lambda_W - \omega)}{R_0^2a^2} - \Lambda_W^2 \right) \right], \tag{2.5}
\]

\[
\frac{\dot{a}}{a} = \frac{\kappa^2}{6(3\lambda - 1)} \left[ -\frac{1}{2}(\rho + 3p) \pm \frac{3\kappa^2\mu^2}{8(3\lambda - 1)} \left( \frac{k^2}{R_0^2a^4} - \Lambda_W^2 \right) \right], \tag{2.6}
\]

where the analytic continuation \( \mu^2 \rightarrow -\mu^2 \) for \( \Lambda_W \) has been employed \([45, 66, 77]\). The upper (lower) sign corresponds to the case where \( \Lambda_W < 0 \) (\( \Lambda_W > 0 \)).

Comparing them with the Einstein equations derived from GR with the FRW metric,

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3c^2} (\rho_m + \rho_{\text{DE}}) - \frac{c^2k}{R_0^2a^2}, \tag{2.7}
\]

\[
\frac{\dot{a}}{a} = -\frac{4\pi G}{3c^2} [(\rho_m + \rho_{\text{DE}}) + 3(p_m + p_{\text{DE}})], \tag{2.8}
\]

one can connect the Hořava parameters \( \kappa, \lambda, \mu, \Lambda_W, \omega \) with the speed of light \( c \), the Newton’s constant \( G \), and the effective dark energy density \( \rho_{\text{DE}} \) and pressure \( p_{\text{DE}} \), although the connection is not unique. Park defined the fundamental constants \( c \) and \( G \) as

\[
c^2 = \frac{\kappa^4\mu^2|\Lambda_W|}{8(3\lambda - 1)^2}, \quad G = \frac{\kappa^2c^2}{16\pi(3\lambda - 1)}. \tag{2.9}
\]

To ensure the positivity of the dark energy density as required by observations, we consider the case where \( \Lambda_W > 0 \) and identified the dark energy density and pressure as

\[
\rho_{\text{DE}} = \frac{3c^4}{16\pi G\Lambda_W} \left( \frac{H_0^2\Omega_k^2}{c^2a^4} - \frac{2H_0^2\omega\Omega_k}{c^2a^2} + \Lambda_W^2 \right), \tag{2.10}
\]

\[
p_{\text{DE}} = \frac{3c^4}{16\pi G\Lambda_W} \left( \frac{H_0^4\Omega_k^2}{3c^2a^4} + \frac{2H_0^2\omega\Omega_k}{3c^2a^2} - \Lambda_W^2 \right), \tag{2.11}
\]

where \( \Omega_k = -(c^2k)/(R_0^2H_0^2) \). The equation of state parameter is then

\[
w_{\text{DE}} \equiv \frac{p_{\text{DE}}}{\rho_{\text{DE}}} = \frac{H_0^2\Omega_k^2}{3H_0^4\Omega_k^2} - \frac{2c^2H_0^2\omega\Omega_k}{6c^2H_0^2\omega\Omega_k} - \frac{3c^4\Lambda_W^2}{3c^2a^4}. \tag{2.12}
\]

From Eq. (2.10) we obtain

\[
\frac{\rho_{\text{DE}}}{\rho_c} = \left( \frac{H_0^2\Omega_k^2}{2c^2\Lambda_W} \right) \frac{1}{a^4} - \left( \frac{\Omega_k\omega}{\Lambda_W} \right) \frac{1}{a^2} + \frac{c^2}{2H_0^2} \Lambda_W \equiv \Omega_1 a^{-4} + \Omega_2 a^{-2} + \Omega_3, \tag{2.13}
\]

\[
\approx \Omega_1 a^{-4} + \Omega_2 a^{-2} + \Omega_3, \tag{2.14}
\]

where the critical energy density \( \rho_c = (3H_0^2c^2)/(8\pi G) \) and

\[
\Omega_1 = \frac{H_0^2\Omega_k^2}{2c^2\Lambda_W} \geq 0, \quad \Omega_2 = -\frac{\Omega_k\omega}{\Lambda_W}, \quad \Omega_3 = \frac{c^2\Lambda_W}{2H_0^2} > 0. \tag{2.15}
\]
As shown by the above formulae, this effective dark energy consists of three components that are radiation-like, curvature-like and CC-like, respectively. It involves three model parameters: \( \{ \Omega_k, \omega, \Lambda_W \} \). The first parameter stems from the FRW metric ansatz, and the last two from the Hořava gravity action. Note that for a flat universe this effective dark energy behaves as a CC and accordingly this model is the same as the CC dark energy model in GR, i.e. flat \( \Lambda \)CDM, which is consistent with all the current observational results. In the present paper we will consider a nonzero \( \Omega_k \) for possible deviation from \( \Lambda \)CDM.

### 3. Model Parameters and Phenomenological Parameters

To compare the Hořava Effective Dark Energy (HEDE) model with observational results, one effective approach is to employ a phenomenological parametrization of the relevant quantities, whose observational constraints have been well studied, as a mediator to facilitate the comparison. The observational constraints on the model can be obtained from those on the phenomenologically parametrized quantities by invoking an approximate relation between the model parameters and the phenomenological parameters. In HEDE the model parameters include \( \{ \Omega_k, \omega, \Lambda_W \} \), which determine the dark energy behavior. That is, the dark energy density \( \rho_{DE}(a) \), as well as \( w_{DE}(a) \), is a function of \( \{ \Omega_k, \omega, \Lambda_W \} \). Phenomenologically, the evolution of dark energy is determined by the present density fraction \( \Omega_{DE} \) and its equation of state parameter \( w_{DE} \). Here we invoke the widely used CPL parameterization \([70, 71]\) of \( w_{DE} \) in Eq. (1.1): \( w_{DE} = w_0 + w_a(1 - a) \). In addition, since the spatial curvature \( \Omega_k \) is involved in the model parameters, it should also be included in the phenomenological parameter space when connecting to the model space. In summary, the phenomenological parameters are \( \{ \Omega_k, \Omega_{DE}, w_0, w_a \} \). Accordingly, we have a three-dimensional model parameter space and a four-dimensional phenomenological parameter space. A mapping between them is required for constraining the model via the observational constraints on the phenomenological parameters.

The relation between \( \Omega_{DE} \) and \( \{ \Omega_k, \omega, \Lambda_W \} \) can be obtained from Eq. (2.13) by setting \( a = 1 \):

\[
\Omega_{DE}(\Omega_k, \omega, \Lambda_W) = \frac{H_0^2 \Omega_k^2}{2c^2 \Lambda_W} - \frac{\Omega_k \omega}{\Lambda_W} + \frac{c^2}{2H_0^2} \Lambda_W. \tag{3.1}
\]

Following Park [60], we connect the model parameters \( \{ \omega, \Lambda_W \} \) with the phenomenological parameters by firstly expanding \( w_{DE} \) in (2.12) around \( a = 1 \) as

\[
w_{DE} = w_{DE}|_{a=1} - w'_{DE}|_{a=1} (1 - a) + \cdots, \tag{3.2}
\]

where the prime denotes the derivative with respect to \( a \), and then identify \( w_{DE}|_{a=1} \) and \( -w'_{DE}|_{a=1} \) as \( w_0 \) and \( w_a \) in the CPL parameterization. As a result, the approximate relation between \( \{ \omega, \Lambda_W \} \) and \( \{ w_0, w_a \} \) reads

\[
w_0(\Omega_k, \omega, \Lambda_W) = \frac{H_0^4 \Omega_k^2 + 2c^2 H_0^2 \omega \Omega_k - 3c^4 \Lambda_W^2}{3H_0^4 \Omega_k^2 - 6c^2 H_0^2 \omega \Omega_k + 3c^4 \Lambda_W^2}, \tag{3.3}
\]

\[
w_a(\Omega_k, \omega, \Lambda_W) = -\frac{8c^2 H_0^2 \Omega_k (H_0^2 \omega \Omega_k^2 - 2c^2 H_0^2 \omega \Omega_k \Lambda_W + c^4 \omega \Lambda_W^2)}{3 (H_0^4 \Omega_k^2 - 2c^2 H_0^2 \omega \Omega_k + c^4 \Lambda_W^2)^2}. \tag{3.4}
\]
or equivalently,
\[ \omega = -\frac{(1 - 2w_0 - 3w_0^2 - w_a)H_0^2\Omega_k}{(1 + 4w_0 + 3w_0^2 + w_a)c^2}, \tag{3.5} \]
\[ \Lambda_W^2 = \frac{(-1 + 9w_0^2 + 3w_a)H_0^2\Omega_k^2}{3(1 + 4w_0 + 3w_0^2 + w_a)c^4}. \tag{3.6} \]

The required mapping between \( \{\Omega_k, \omega, \Lambda_W\} \) and \( \{\Omega_k, \Omega_{DE}, w_0, w_a\} \) is given by Eqs. (3.1), (3.3) and (3.4). It maps the 3D model space to a 3D hypersurface in the 4D phenomenological parameter space.

Regarding the observational constraints, as stated in Sec. 1, we assume \( \Omega_{DE} \approx 0.74 \), \( \Omega_k < 0.01 \) [75], \( \Omega_1 < \Omega_r \), and the constraint on \( w_{DE} \) obtained in [74]. For simplicity, we fix \( \Omega_{DE} = 0.74 \). This reduces the dimension of the phenomenological parameter space from four to three: \( \{\Omega_k, w_0, w_a\} \), and that of the model space from three to two, i.e. a 2-dimensional surface in the 3D space \( \{\Omega_k, \omega, \Lambda_W\} \). This 2D surface corresponds to the relation \( \Omega_k = \Omega_k(\omega, \Lambda_W) \) obtained from Eq. (2.13) with \( \rho_{DE}(a = 1)/\rho_c = 0.74 \). In this case the relations in Eqs. (3.1), (3.3) and (3.4) become \( \Omega_{DE} = \Omega_{DE}(\omega, \Lambda_W) \), \( w_0 = w_0(\omega, \Lambda_W) \) and \( w_a = w_a(\omega, \Lambda_W) \). They map the reduced 2D model space \( \{\Omega_k(\omega, \Lambda_W), \omega, \Lambda_W\} \) to a 2D surface \( \{\Omega_k(\omega, \Lambda_W), w_0(\omega, \Lambda_W), w_a(\omega, \Lambda_W)\} \) in the reduced 3D phenomenological parameter space \( \{\Omega_k, w_0, w_a\} \). Then, the observational constraint on the model is represented by the intersection of this 2D surface and the well-studied, observationally allowed regions of the 3D phenomenological parameter space. We note that the spatial curvature is well constrained. The upper bound of \( |\Omega_k| \) is around 0.01 and could be even smaller [75]. Thus, the allowed region in the 3D phenomenological parameter space is very thin in the \( \Omega_k \) direction. Accordingly, as a good approximation after imposing the constraint \( \Omega_k < 0.01 \), we will simply consider the constraints on the \( \{w_0, w_a\} \) plane when taking the above-mentioned intersection that presents the valid region of the model.

4. Constraints on the Phenomenological Parameter Space

As commented in [60], for the sake of self-consistency one should require \( \Lambda_W^2 \geq 0 \). From Eq. (3.6), this in turn requires that

\[ \{w_a > -1 - 4w_0 - 3w_0^2, w_a \geq (1 - 9w_0^2)/3\} \]

or

\[ \{w_a < -1 - 4w_0 - 3w_0^2, w_a \leq (1 - 9w_0^2)/3\}. \tag{4.1} \]

To simplify the following calculations, we define

\[ A = 1 + 4w_0 + 3w_0^2 + w_a, \tag{4.2} \]
\[ B = -1 + 9w_0^2 + 3w_a. \tag{4.3} \]

The self-consistency condition in Eq. (4.1) then reads

\[ \{A > 0, B \geq 0\} \quad \text{or} \quad \{A < 0, B \leq 0\}. \tag{4.4} \]
The valid region on the \( \{w_0, w_a\} \) plane for this self-consistency condition is presented in Figure 1 by the shaded area.

Figure 1: The valid (shaded) region for the self-consistency condition \( \Lambda_W^2 > 0 \).

Rewriting Eqs. (3.5) and (3.6) in terms of \( A \) and \( B \), we have

\[
\omega = \frac{3A + B - 8}{6A} \left( \frac{H_0^2 \Omega_k}{c^2} \right), \quad \Lambda_W = \sqrt{\frac{B}{3A}} \left( \frac{H_0^2}{c^2} \right) |\Omega_k|, \quad (4.5)
\]

Substituting Eq. (4.5) into Eq. (3.1), we obtain

\[
|\Omega_k| = \text{sgn}(A) \frac{\sqrt{3}}{4} \sqrt{AB} \Omega_{DE}, \quad (4.6)
\]

where \( \text{sgn}(A) \) denotes the sign of \( A \). For a positive dark energy density, this relation requires \( A \geq 0 \), which, as combined with Eq. (4.4), leads to

\[
\{ A > 0, B \geq 0 \}, \quad (4.7)
\]

thereby excluding the bottom middle shaded area in Figure 1.

The current observations suggest \( \Omega_{DE} \approx 0.74 \) and \( |\Omega_k| < 0.01 \) [75]. With Eqs. (4.6) and (4.7), these two requirements give a stringent constraint on the parameters \( A \) and \( B \) (i.e. \( w_0 \) and \( w_a \)):

\[
0 < AB < \left( \frac{4 \times 0.01}{\sqrt{3} \times 0.74} \right)^2 \approx 9.74 \times 10^{-4}. \quad (4.8)
\]

This constraint largely shrinks the allowed region in the parameter space, which is presented by the shaded region in Figure 2.
As indicated in Eq. (2.14), the effective dark energy sector of the (IR-modified) HEDE model consists of a radiation-like ($\Omega_1$), a curvature-like ($\Omega_2$), and a constant ($\Omega_3$) term. In the early universe when $a$ is small, the radiation-like term dominates the effective dark energy sector. Such a term, if too large, would cause the epoch of the matter-radiation equality happened at a time that is later than that suggested by the observational results about CMB and BBN. Specifically, this term may contribute to the effective relativistic degrees of freedom in the early universe. In the CMB analysis the radiation is usually subdivided into two categories: (i) photons and (ii) effective neutrinos (including neutrinos and other effective relativistic particles). Accordingly, 

$$\Omega_r = \Omega_\gamma + \Omega_\nu = \Omega_\gamma (1 + 0.2271 N_{\text{eff}}), \quad (4.9)$$

where $\Omega_r$ is the present radiation energy density fraction, and $N_{\text{eff}}$ is the number of effective neutrino species. The WMAP results suggest that $\Omega_r \approx 8.47 \times 10^{-5}$ and $N_{\text{eff}} \approx 4$ [75]. Accordingly the effective neutrinos have a comparable contribution to the radiation energy density. Regarding the radiation-like term from Horava gravity as a source of the effective neutrinos, we obtain an upper bound of $\Omega_1$ given by $0.2271 N_{\text{eff}} \Omega_\gamma$, which is of the same order as $\Omega_r$. This leads us to impose the constraint,

$$\Omega_1 < \Omega_r \approx 8.47 \times 10^{-5}. \quad (4.10)$$

Substituting Eqs. (4.5) and (4.6) into the definition of $\Omega_1$ in Eq. (2.15), we obtain $\Omega_1 = 3|A|\Omega_{DE}/8$, from which and Eq. (4.7) the above constraint requires

$$0 < A < \frac{8\Omega_r}{3\Omega_{DE}} \approx 3.05 \times 10^{-4}. \quad (4.11)$$

This tightly constrained region is presented in Figure 3 by the black area which is so narrow that it looks like a black curve. In this figure we also show the constraint given in Eq. (4.8),
which is presented by the gray area. For $w_0 < -1/3$, it largely overlaps with the black narrow region.

In addition, in Figure 3 we show the $1\sigma$ (long-dashed contour) and the $2\sigma$ (dot-dashed contour) observational constraints of $w_0$ and $w_a$ obtained in [74] from the combined data set that includes the SN-Ia data from the Constitution Set, the CMB measurement from the five-year WMAP, and the BAO measurement from SDSS and 2dFGRS. The intersection of all the above-mentioned allowed regions gives the valid IR-modified Hořava Effective Dark Energy model, which is the black narrow region enclosed by the long-dashed ($1\sigma$) or the dot-dashed ($2\sigma$) contour.

Figure 3: Various constraints on the $w_0$-$w_a$ plane for the IR-modified Hořava Effective Dark Energy (HEDE) model. The constraints are given in Eqs. (4.8) and (4.11), and in [74]. The long-dashed and the dot-dashed contour respectively present the $1\sigma$ and the $2\sigma$ constraint on $\{w_0, w_a\}$, which is obtained in [74] from the current SN Ia, CMB and BAO data. The narrow gray region presents the constraint in Eq. (4.8) for the requirement $|\Omega_k| < 0.01$. The narrow black region that nearly overlaps with the narrow gray region for $w_0 < -1/3$ presents the constraint in Eq. (4.11) for the requirement $\Omega_l < \Omega_r$. The intersection of these three kinds of allowed regions gives the valid HEDE model. The three black dots denote the best-fit models Park considered in [60], none of which is in the valid region.
5. Constraining the IR-modified Hořava Gravity

To constrain the IR-modified Hořava gravity, here we transfer the observational constraints on the phenomenological parameters to the \textit{HEDE} model via the mapping between the phenomenological parameter space and the model space. This mapping is given in Eqs. (3.5) and (3.6), i.e., in Eq. (4.5), where $|\Omega_k|$ is a function of $A$ and $B$, as given in Eq. (4.6), after we fix $\Omega_{DE} = 0.74$. With Eqs. (4.6) and (4.7) substituted into Eq. (4.5), this mapping can be rewritten as

\begin{align}
\tilde{\omega} &= s_k \frac{\sqrt{3}}{24} (3A + B - 8) \sqrt{B/A}, \\
\tilde{\Lambda}_W &= \frac{1}{4} B,
\end{align}

where the two dimensionless parameters $\tilde{\omega}$ and $\tilde{\Lambda}_W$ are defined as

\begin{align}
\tilde{\omega} &= \frac{c^2 \omega}{H_0^2 \Omega_{DE}}, \quad \tilde{\Lambda}_W = \frac{c^2 \Lambda_W}{H_0^2 \Omega_{DE}},
\end{align}

and $s_k$ denotes the sign of $\Omega_k$.

In the valid region, i.e. the black narrow region enclosed by the $2\sigma$ contour in Figure 3, we have the constraints $0 < A < 3.05 \times 10^{-4}$, as required in Eq. (4.10), and $6.44 < B < 8.48$. From the above mapping and the constraints on $A$ and $B$, we obtain

\begin{align}
\tilde{\omega} (\epsilon, \tilde{\Lambda}_W) &= \frac{s_k}{2} \left[ (\tilde{\Lambda}_W - 2) + \epsilon \right] \sqrt{\frac{\tilde{\Lambda}_W}{\epsilon}},
\end{align}

where

\begin{align}
0 < \epsilon &\equiv \frac{3}{4} A < 2.29 \times 10^{-4}, \\
-0.39 < \tilde{\Lambda}_W - 2 < 0.12.
\end{align}

For more details, the constraint on $\{\tilde{\omega}, \tilde{\Lambda}_W\}$ can be read as follows.

\begin{align}
-\infty < s_k \tilde{\omega} < s_k \tilde{\omega}(\epsilon_{max}) < 0 \quad &\text{as} \quad \tilde{\Lambda}_W \leq 2 - \epsilon_{max}, \\
-\infty < s_k \tilde{\omega} < s_k \tilde{\omega}(\epsilon_{max}) > 0 \quad &\text{as} \quad 2 - \epsilon_{max} < \tilde{\Lambda}_W < 2, \\
0 < s_k \tilde{\omega} < \sqrt{\epsilon_{max}/2} \quad &\text{as} \quad \tilde{\Lambda}_W = 2, \\
\sqrt{\tilde{\Lambda}_W (\tilde{\Lambda}_W - 2)} < s_k \tilde{\omega} < \infty \quad &\text{as} \quad 2 < \tilde{\Lambda}_W \leq 2 + \epsilon_{max}, \\
0 < s_k \tilde{\omega}(\epsilon_{max}) < s_k \tilde{\omega} < \infty \quad &\text{as} \quad \tilde{\Lambda}_W > 2 + \epsilon_{max},
\end{align}

where $\epsilon_{max} = 2.29 \times 10^{-4}$. This constraint is presented in Figure 4, where the dark region and the light region correspond to $s_k = +$ and $s_k = -$ , i.e. $\Omega_k > 0$ and $\Omega_k < 0$, respectively.
As shown in Figure 4, $\tilde{\Lambda}_W$ is restricted to values between 1.61 and 2.12, and $|\tilde{\omega}|$ has a nontrivially lower limit but no upper limit. We note that for $\tilde{\omega} = 0$ the allowed region is almost a point, specifically,

$$-2.29 \times 10^{-4} < \tilde{\Lambda}_W - 2 < 0. \tag{5.8}$$

That is, in the case where detailed balance is preserved ($\omega = 0$) we need to fine-tune the value of $\Lambda$ for the HEDF model to be consistent with observational results. Thus, the cosmological test strongly disfavors, although does not rule, the Hořava action that preserves detailed balance. On the contrary, for large $|\tilde{\omega}|$ the full range of $\tilde{\Lambda}_W$, (1.61, 2.12), is allowed. Moreover, there is no upper limit to $|\tilde{\omega}|$. Accordingly, the observational results suggest the breaking of the detailed balance condition.

Note that the curvature-like effective energy term in Eq. (2.14), as originated from the soft violation of detailed balance, remains finite when $\tilde{\omega}$ goes to infinity. This can be seen in the following.

$$\Omega_2 = -\frac{\Omega_k \omega}{\Lambda_W} = \frac{\Omega_{DE}}{8} (8 - 3A - B), \tag{5.9}$$

where $\Omega_{DE} \approx 0.74$, $0 < A < 3.05 \times 10^{-4}$ and $6.44 < B < 8.48$ for the valid region. We emphasize that even though at the action level the magnitude of the soft violation can be arbitrarily large with no upper limit, the corresponding effective energy may still be tightly constrained.
6. Possible Behavior of the Effective Dark Energy

Recall Eqs. (2.13) and (2.14), the HED E consists of the radiation-like ($\Omega_1 a^{-4}$), the curvature-like ($\Omega_2 a^{-2}$) and the constant-like ($\Omega_3$) sectors:

$$\frac{\rho_{\text{DE}}}{\rho_c} = \left(\frac{H_0^2 \Omega_k^2}{2c^2 \Lambda_W}\right) \frac{1}{a^4} + \left(\frac{\Omega_k \omega}{\Lambda_W}\right) \frac{1}{a^2} + \frac{c^2}{2H_0^2} \Lambda_W$$

$$\equiv \Omega_1 a^{-4} + \Omega_2 a^{-2} + \Omega_3. \quad (6.1)$$

Substituting Eqs. (3.5), (3.6) and (4.6) into Eq. (2.15), we obtain

$$\Omega_1 = \frac{3\Omega_{\text{DE}}}{8} (1 + 4w_0 + 3w_a^2), \quad (6.3)$$

$$\Omega_2 = -\frac{3\Omega_{\text{DE}}}{4} (-1 + 2w_0 + 3w_0^2 + w_a), \quad (6.4)$$

$$\Omega_3 = \frac{\Omega_{\text{DE}}}{8} (-1 + 9w_0^2 + 3w_a). \quad (6.5)$$

Note that $\Omega_1 + \Omega_2 + \Omega_3 = \Omega_{\text{DE}}$ as required. The requirement $\Omega_{\text{DE}} = 0.74$ and the constraint on $\{w_0, w_a\}$ give an allowed region in the $\{\Omega_1, \Omega_2, \Omega_3\}$ space: a plane $\Omega_1 + \Omega_2 + \Omega_3 = 0.74$ bounded by the box $\{0 < \Omega_1 < 8 \times 10^{-6}, -0.05 < \Omega_2 < 0.15, 0.59 < \Omega_3 < 0.78\}$ corresponding to the $2\sigma$ contour on $\{w_0, w_a\}$.

To show the possible evolution patterns of the HED E, we consider three sample cases, DE.1, DE.2 and DE.3, corresponding to three points in the narrow valid region in Figure 3. The values of $\{w_0, w_a; \Omega_1, \Omega_2, \Omega_3\}$ in these three cases are as follows.

|       | $w_0$     | $w_a$     | $\Omega_1$     | $\Omega_2$     | $\Omega_3$     |
|-------|-----------|-----------|----------------|----------------|----------------|
| **DE.1** | $-1.00$   | $1.14 \times 10^{-4}$ | $3.18 \times 10^{-5}$ | $-6.35 \times 10^{-5}$ | $0.740$ |
| **DE.2** | $-0.95$   | $9.27 \times 10^{-2}$  | $4.24 \times 10^{-5}$ | $5.54 \times 10^{-2}$ | $0.685$ |
| **DE.3** | $-0.90$   | $1.70 \times 10^{-1}$  | $3.18 \times 10^{-6}$ | $1.11 \times 10^{-1}$ | $0.629$ |

The evolution of the energy density in these three cases is shown in Figure 5. These three cases have the same density fraction at present, $\Omega_{\text{DE}} = 0.74$, and share similar evolution patterns in the late times up to $\ln(1 + z) \approx 0.25$. DE.1 resembles $\Lambda$CDM at low redshifts. Its energy density remains nearly constant for $\ln(1 + z) < 2.0$. This is because $\Omega_1$ and $\Omega_2$ are both extremely small in this case. In the cases of DE.2 and DE.3 the energy densities increase with $z$ rapidly for $\ln(1 + z) > 0.25$ due to the $\Omega_2$ term. Generally speaking, it is the difference in $\Omega_2$ that makes the evolution patterns distinct from each other in the interval $0.25 < \ln(1 + z) < 3.5$. In the early universe, the radiation-like term in HED E would dominate. Hence for $\ln(1 + z) > 5$ the slopes of $\ln(\rho/\rho_c)$ versus $\ln(1 + z)$ in different cases are roughly the same. The value of $\Omega_1$ determines the value of $\ln(\rho/\rho_c)$ at high redshifts. The cases with larger $\Omega_1$ have larger energy densities in the early times. Nevertheless, in the early times the energy density of HED E should be smaller than that of radiation, as required in Eq. (4.10).

In Figure 6 we show the redshift dependence of the ratio $\rho_i/\rho_t$, where $\rho_i$ stands for the energy density of DE.1, DE.2, DE.3, matter or radiation, and $\rho_t$ for the total energy density.
Figure 5: The energy density evolution patterns in three sample cases. The solid, the dot-dashed and the double-dot-dashed line correspond to $\text{DE}.1$, $\text{DE}.2$ and $\text{DE}.3$, respectively.

Figure 6: The redshift dependence of the contribution of different energy contents to the total energy density. The solid, dot-dashed, double-dot-dashed, long-dashed, and short-dashed lines correspond to $\text{DE}.1$, $\text{DE}.2$, $\text{DE}.3$, matter and radiation, respectively.

density. The evolution of the three cases nearly coincide with each other after the time when $\ln(1 + z) \approx 0.5$ (i.e. $z \approx 0.65$), around which the crossing between the HED and the matter energy density at low redshift happens. This is because after the crossing the constant-like ($\Omega_3$) term dominates the dark sector, while these three cases have similar $\Omega_3$. Before this crossing, the three cases behave differently. For $\ln(1+z) < 4$ the contribution of the HED to the total energy decreases with $z$, and the decreasing is more rapid in $\text{DE}.1$ than in $\text{DE}.2$ and $\text{DE}.3$. The main reason is that $\Omega_2$ in $\text{DE}.1$ is negative. In general, $\rho_{\text{DE}}/\rho_\Lambda$ decreases with $z$ more rapidly for smaller $\Omega_2$. For $\ln(1+z) > 4$ (i.e., for $z > 50$), the
The contribution of the *HEDE* to the total energy increases with $z$. This is a reflection of the dominance of the radiation-like ($\Omega_1$) term over the other two terms in the dark sector. The slope of this increment and the contribution of the *HEDE* at high redshifts are determined by the value of $\Omega_1$. In the case with larger $\Omega_1$, $\rho_{DE}$ increases with $z$ more rapidly and $\rho_{DE}/\rho_t$ is larger for $\ln(1+z) > 4$. However, the contribution of the *HEDE* would be smaller than that of radiation in the early times, as required in Eq. (4.10).

In Figure 7 we show the redshift dependence of the contribution of the three terms, $\{\Omega_1(1+z)^4, \Omega_2(1+z)^2, \Omega_3\}$, in the *HEDE* energy density for the three cases in Eq. (6.6). For all the three cases, the constant-like ($\Omega_3$) term dominates the dark sector in the late times, while the radiation-like ($\Omega_1$) term dominates in the early times. Whether there exists a period of the dominance of the curvature-like ($\Omega_2$) term depends on the magnitude of $\Omega_2$. For *DE.1*, the contribution of the term $\Omega_2(1+z)^2$ is much smaller than the other two terms. Hence in Figure 7a the line corresponding to this term cannot be seen. For the *DE.2* and *DE.3* cases, the curvature-like term dominates the dark sector during a “middle age”. The precise period of such “middle age” varies from case to case. In general,
for larger $\Omega_2$, the dominance of the curvature-like term starts earlier and ends later, with larger contribution from this term to the \textit{HEDE}.

7. Summary and Discussion

In this paper we test the IR-modified Hořava gravity from the cosmological point of view, in particular, from the viewpoint of cosmic expansion. We conclude that the Hořava gravity with soft violation of the detailed balance condition is consistent with the current observational results on the expansion history of the universe. Specifically, this gravity theory can generate the late-time cosmic acceleration with the behavior that is well consistent with observations. We note that the Hořava gravity with the detailed balance condition, though not ruled out, requires fine-tuning $\Lambda_W$ such that $-2.29 \times 10^{-4} < (c^2 \Lambda_W)/(H_0^2 \Omega_{DE}) - 2 < 0$ in order to fit the observational data. This result, together with previous studies [66, 78], suggests that the breaking of the detailed balance condition, at least softly, is necessary to render Hořava gravity a more realistic IR-limit.

We obtained the observational constraints on two model parameters, $\Lambda_W$ and $\omega$, i.e., the cosmological constant of the three dimensional Einstein-Hilbert action and the coefficient of the soft violation term. The parameter $\Lambda_W$ is well-constrained and it should be of the order of the inverse square of the Hubble length, $H_0^2/c^2$. More precisely, we found that $\Lambda_W$, in units of $(H_0^2/c^2)$, is bounded within a small range, $(1.61 \Omega_{DE}, 2.12 \Omega_{DE})$, i.e., $(1.19, 1.57)$ after imposing $\Omega_{DE} = 0.74$. On the other hand, we obtained a lower bound, but without an upper bound, to $|\omega|$ with regard to the extent of the soft violation of the detailed balance condition. The lower bound depends on $\Lambda_W$, and in most cases it is also around the order of the inverse square of the Hubble length, $H_0^2/c^2$.

With our more comprehensive investigation into the cosmic-expansion test of Hořava gravity, we found the Hořava effective dark energy (\textit{HEDE}) much more restrictive than that deduced in [60]. Specifically, the allowed parameter space $\{w_0, w_a\}$ is now much smaller. It is a narrow strip beside the parabola $1 + 4w_0 + 3w_0^2 + w_a = 0$ around $(w_0, w_a) = (-1, 0)$. The energy density of the \textit{HEDE}, with different values of the model parameters, can give rise to a cosmological constant as well as non-constant behaviors. For the latter cases with non-constant energy density, the main difference between the models therein is the evolution of the dark energy density in the “middle age”. This is because during the “middle age” the curvature-like ($\Omega_2$) term dominates, while the range of allowed $\Omega_2$ is not small, namely from $-0.04$ to $+1.03$. As a result the dark energy behavior in the non-constant cases almost coincide with each other at high redshifts. This is because in the early times the dark energy is dominated by the radiation-like ($\Omega_1$) term that is highly restricted, namely $\Omega_1 \lesssim 8.47 \times 10^{-5}$.

In our analysis we compare \textit{HEDE} with observational results by invoking the CPL parametrization of the equation of state of dark energy: $w_{DE} = w_0 + w_a(1-a)$, as a mediator. We then transfer the constraint on the phenomenological parameters $\{\Omega_k, \Omega_{DE}, w_0, w_a\}$ to that on the model parameters $\{\Omega_k, \omega, \Lambda_W\}$ via an approximate relation between $\{w_0, w_a\}$ and $\{\Omega_k, \omega, \Lambda_W\}$. Naively, since the dimension of the phenomenological parameter space is larger than that of the model space, the 4D parameter space seems flexible enough to
accommodate the mapping from the 3D model space. Nevertheless, to be prudent, the validity of this approximation should be checked. In particular, it is important to verify that the two energy densities, \( \rho_{\text{HEDE}}(z; \Omega_k, \omega, \Lambda_W) \) and \( \rho_{\text{CPL}}(z; \Omega_{\text{DE}}, w_0, w_a) \), are consistent with each other, where \( \rho_{\text{HEDE}} \) is given in Eqs. (2.10) or (2.13) and

\[
\rho_{\text{CPL}} = \rho_0 e^{3w_a(a-1)a^{-3(1+w_0+w_a)}}. \tag{7.1}
\]

The approximation is valid when the difference between these two energy densities is significantly smaller than the observational accuracy in the relevant redshift range. This consistency check requires further investigation. One way to avoid the possible incompatibility between the model space and the phenomenological parameter space is to use the model to fit data directly, e.g., invoking the \( \chi^2 \) fitting to obtain the observational constraints on the model parameters \( \{\Omega_k, \Lambda_W, \omega\} \). This is under our investigation and will be reported in our follow-up paper.

Hořava gravity, an interesting alternative gravity theory that breaks the Lorentz symmetry, should ideally be constrained by observations and experiments ranging from microscopic, solar, astronomical, to cosmological scales. From the cosmological point of view, a modified gravity theory changes not only the cosmic expansion history but also the structure formation. In the present paper we have shown how the IR-modified Hořava gravity can be tightly constrained by the observations about the cosmic expansion. In addition to the expansion history, we expect the observations about the cosmic structures, such as galaxy surveys and weak lensing observations, would also provide important constraints on Hořava gravity. This is worthy of further investigations.

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