Landau Diamagnetism in Noncommutative Space and the Nonextensive Thermodynamics of Tsallis

Ömer F. Dayi\textsuperscript{a,b} * and Ahmed Jellal\textsuperscript{a} †

\textsuperscript{a} Feza Gürsey Institute, P.O. Box 6, 81220, Çengelköy, Istanbul, Turkey.
\textsuperscript{b} Physics Department, Faculty of Science and Letters, Istanbul Technical University, 80626 Maslak–Istanbul, Turkey.

Abstract

We consider the behavior of electrons in an external uniform magnetic field $\vec{B}$ where the space coordinates perpendicular to $\vec{B}$ are taken as noncommuting. This results in a generalization of standard thermodynamics. Calculating the susceptibility, we find that the usual Landau diamagnetism is modified. We also compute the susceptibility according to the nonextensive statistics of Tsallis for $(1-q) \ll 1$, in terms of the factorization approach. Two methods agree under certain conditions.

*E-mail: dayi@itu.edu.tr and dayi@gursey.gov.tr.
†E-mail: jellal@gursey.gov.tr.
1 Introduction

The natural appearance of noncommutativity in string theories [1] and the observation that ordinary and noncommutative gauge theories are equivalent [2] have increasingly led to attempts to study physical problems in noncommutative spaces. Apparently, formulation of quantum mechanics in noncommutative spaces [3], [4] may provide some hints for understanding the physical consequences of noncommuting space coordinates.

Although noncommuting coordinates are operators even at the classical level, one can treat them as commuting by replacing operator products with $\ast$–products. This approach was developed in order to formulate quantum mechanics without appealing to operators ([5] and the references therein). However, it is more appropriate to perform calculations with noncommutative coordinates without introducing new machinery. This approach allows us to generalize classical as well as quantum mechanics without altering their main physical interpretations and to recover the usual results when noncommutativity is switched off.

There exist some physical systems which do not obey the rules of the standard thermodynamics (some of them are mentioned in [6]). One may hope to find a remedy for these anomalies by generalizing Boltzmann–Gibbs statistics by adopting noncommutative coordinates. At this point, we cannot offer a general answer as to whether this is always possible. Nevertheless, we can obtain insights by studying a specific example.

Calculation of susceptibility for electrons moving in an external uniform magnetic field leads to Landau diamagnetism [7], which is a consequence of quantization. We wish to study this system, which may provide some evidence on whether the generalization of the usual thermodynamics due to noncommutativity of space coordinates is useful when Boltzmann–Gibbs statistics is not applicable. One can choose a gauge such that the motion in the direction of the magnetic field remains intact. It is then convenient to calculate the susceptibility in a setting where the coordinates which are perpendicular to the external magnetic field are taken as noncommuting, but the one in the direction of the external magnetic field is kept commuting.

On the other hand Tsallis [8], [6] proposed a nonextensive statistics, which is a candidate for curing some of the problems that appear in standard thermodynamics. It is therefore interesting to investigate whether the statistics resulting from noncommutativity of the coordinates is
somehow connected to the nonextensive statistics of Tsallis.

After a brief review of the quantization of an electron moving in an external uniform magnetic field and the related coherent states, we calculate the partition function, the magnetization and the susceptibility, taking the coordinates perpendicular to the magnetic field as noncommuting.

Although Landau diamagnetism according to Tsallis nonextensive statistics was studied in [9], the method adopted therein is not suitable for our purposes. We therefore, present another method to calculate the susceptibility according to the nonextensive statistics of Tsallis by retaining the leading terms in \((q - 1) \ll 1\). These two methods yield different results.

We then show that in the high temperature limit the susceptibility obtained in the noncommuting space and the one based on Tsallis nonextensive statistics agree if their parameters satisfy a certain relation.

## 2 Preliminaries

An electron moving in the external uniform magnetic field \(\vec{B} = \vec{\nabla} \times \vec{A}\), neglecting spin, is described by the Hamiltonian

\[
H = \frac{1}{2m}(\vec{p} + \frac{e}{c}\vec{A})^2. \tag{1}
\]

We let the magnetic field \(\vec{B}\) be in the \(z\)-direction by choosing the symmetric gauge:

\[
\vec{A} = (-\frac{B}{2}y, \frac{B}{2}x, 0). \tag{2}
\]

After imposing the usual canonical commutation relations between the coordinates \(\vec{r}\) and the momenta \(\vec{p}\), the operators

\[
a = \frac{1}{\sqrt{2m\hbar\omega}} \left( p_x - \frac{m\omega}{2}y \right) - i \left( p_y + \frac{m\omega}{2}x \right), \tag{3}
\]

and its hermitian conjugate \(a^\dagger\) can be shown to satisfy

\[
[a, a^\dagger] = 1, \tag{4}
\]

where \(\omega = \frac{eB}{me}\) is the cyclotron frequency.
In the $z$–direction the motion is free, however, the appropriate Hamiltonian for the motion in the $(x, y)$–plane becomes

$$H \perp = \hbar \omega (a \dagger a + \frac{1}{2}).$$

(5)

Eigenstates of (5) can simultaneously be taken to be eigenstates of angular momentum in the $z$–direction. Although, one can construct coherent states reflecting this fact [10], for our purposes it is sufficient to consider the normalized coherent states $|\alpha >$ defined as

$$|\alpha > \equiv \exp \left( - \frac{|\alpha|^2}{4l^2} - i \sqrt{\frac{m \omega}{2 \hbar}} a \dagger \right) |0 >,$$

(6)

where $l = (\frac{\hbar}{m \omega})^{\frac{1}{2}}$ is the magnetic length and $\alpha$ is a complex parameter. As usual the ground state $|0 >$ is defined to satisfy $a |0 > = 0$, and $|\alpha >$ are eigenstates of the annihilation operator:

$$a |\alpha > = \left( \frac{\hbar}{2m \omega} \right)^{\frac{1}{2}} \frac{\alpha}{il^2} |\alpha >,$$

(7)

These states were used in [10] to calculate the susceptibility for a cylindrical body which results in the usual Landau diamagnetism.

3 Landau diamagnetism in noncommutative space

Because of choosing the symmetric gauge (2), the motion along the $z$–direction is free and the motion in the plane perpendicular to the magnetic field $\vec{B}$ is described by (3). Let us keep the $z$ coordinate commuting, but take the coordinates perpendicular to the magnetic field $\vec{B}$ as noncommuting

$$[\hat{x}, \hat{y}] = i\theta,$$

(8)

where the constant $\theta$ is real. Noncommutativity can be imposed by treating the coordinates as commuting, but introducing the star product

$$\ast \equiv \exp \frac{i \theta}{2} \left( \hat{x} \hat{y} - \hat{y} \hat{x} \right).$$

(9)

Now, we deal with the commutative coordinates $x$ and $y$ but replace the products with the star product (9). For example, instead of the commutator (3) one introduces the Moyal bracket

$$x \ast y - y \ast x = i \theta.$$

(10)
We quantize this system in terms of the standard canonical quantization by establishing the usual canonical commutation relations \([r_i, p_j] = i\hbar\delta_{ij}\). However, eigenvalue equations should be modified. According to this receipt, in the symmetric gauge (2), the transverse part of the Hamiltonian (1) acting on an arbitrary function \(\Psi(\vec{r}, t)\) yields

\[ H_{\perp} \ast \Psi(\vec{r}, t) = \frac{1}{2m} \left[ \left( p_x - \frac{eB}{2c} y \right)^2 + \left( p_y + \frac{eB}{2c} x \right)^2 \right] \ast \Psi(\vec{r}, t) \equiv H_{nc} \Psi(\vec{r}, t), \]  

(11)

where, in terms of \(\kappa = \frac{eB\theta}{4c}\), we defined

\[ H_{nc} = \frac{1}{2m} \left[ \left( (1 + \kappa) p_x - \frac{eB}{2c} y \right)^2 + \left( (1 + \kappa) p_y + \frac{eB}{2c} x \right)^2 \right]. \]  

(12)

We obtain the appropriate creation and annihilation operators for this system, \(\tilde{a}^\dagger\) and \(\tilde{a}\), from (3) by replacing \(\omega\) with the modified cyclotron frequency \(\tilde{\omega}\):

\[ \tilde{\omega} \equiv (1 + \kappa)\omega. \]  

(13)

They satisfy the usual commutation relation

\[ [\tilde{a}, \tilde{a}^\dagger] = 1. \]

The modified transverse Hamiltonian \(H_{nc}\) can be written as

\[ H_{nc} = \hbar\tilde{\omega}(\tilde{a}^\dagger\tilde{a} + \frac{1}{2}). \]  

(14)

Reminiscent of noncommutativity is only in the frequency \(\tilde{\omega}\). In fact we construct the coherent states \(|\tilde{\alpha}\rangle\) of the noncommutative case from \(|\alpha\rangle\) by the replacement \(a \rightarrow \tilde{a}, a^\dagger \rightarrow \tilde{a}^\dagger, \omega \rightarrow \tilde{\omega}\) in (3). Obviously, they satisfy

\[ \tilde{a}|\tilde{\alpha}\rangle = \left( \frac{\hbar}{2m\tilde{\omega}} \right)^{\frac{1}{4}} \frac{\alpha}{\tilde{l}^{\frac{1}{2}}} |\tilde{\alpha}\rangle, \]  

(15)

where \(\tilde{l} = (\frac{\hbar}{m\tilde{\omega}})^{\frac{1}{4}}\).

The creation and annihilation operators \(\tilde{a}^\dagger\) and \(\tilde{a}\) are the usual ones appearing in quantum mechanics. Thus, the total Hamiltonian

\[ H_t = \frac{p_z^2}{2m} + H_{nc}, \]

can be used to define the partition function in noncommutative coordinates in the standard way as

\[ Z_{nc} = \text{Tr} e^{-\beta H_t}, \]  

(16)
where $\beta = 1/kT$.

Now, by following the arguments of [10], the partition function $Z_{nc}$ for a cylindrical body of volume $V$ can be written as

$$Z_{nc} = \frac{V}{\lambda^3} \frac{\beta \hbar \omega}{2} e^{-\frac{\beta \hbar \omega}{2}} \int_0^\infty d^2 \alpha < \tilde{\alpha} | e^{-\beta \hbar \omega \tilde{a} \tilde{a}} | \tilde{\alpha} >,$$

where $\lambda = \left( \frac{2 \pi \hbar^2 \beta}{m} \right)^{\frac{1}{2}}$ is the thermal wavelength. By performing the calculation it leads to

$$Z_{nc} = \frac{V}{\lambda^3} \frac{\beta \hbar \omega}{2} \frac{(1 + \kappa)}{\sinh \left( \frac{\beta \hbar \omega}{2} (1 + \kappa) \right)},$$

where we emphasized the $\kappa$ dependence. We should adopt the standard definitions for the free energy and the magnetization:

$$F_{nc} = -\frac{n}{\beta} \ln Z_{nc}, \quad M_{nc} = -\frac{\partial F_{nc}}{\partial B},$$

where $n$ is the total number of particles. Thus, we compute the magnetization as

$$M_{nc} = \frac{n \hbar e}{mc} (1 + 2 \kappa) \left[ \frac{1}{\beta \hbar \omega (1 + \kappa)} - \frac{1}{2} \coth \left( \frac{\beta \hbar \omega}{2} (1 + \kappa) \right) \right].$$

The susceptibility defined by

$$\chi_{nc} = \frac{1}{n} \frac{\partial M_{nc}}{\partial B},$$

can be calculated to yield

$$\chi_{nc} = \frac{\hbar e}{mc} 2 \kappa \left[ \frac{1}{\beta \hbar \omega (1 + \kappa)} - \frac{1}{2} \coth \left( \frac{\beta \hbar \omega}{2} (1 + \kappa) \right) \right] - \left( \frac{\hbar e}{mc} \right)^2 \beta (1 + 2 \kappa)^2 \left[ \frac{1}{\beta \hbar \omega (1 + \kappa)} \right]^2 + \frac{1}{4} \left[ 1 - \coth^2 \left( \frac{\beta \hbar \omega}{2} (1 + \kappa) \right) \right].$$

In the high temperature limit, $\beta \ll 1$, we get

$$\chi_{nc} = \chi_L \left[ 1 + 6 \kappa + 6 \kappa^2 \right],$$

where the usual Landau diamagnetism is

$$\chi_L = -\frac{1}{3} \left( \frac{\hbar e}{2mc} \right)^2 \beta.$$

Noncommutativity arising in string theories is due to a background field strength [11]. This leads to the fact that the related noncommutativity parameter is positive. However, when we
introduce the noncommutativity in an ad hoc manner in terms of the commutator (8), there is no restriction on the sign of \( \theta \). In spite of that, for the consistency of our formalism \( \tilde{\omega} \) should be positive, which yields the condition \( \kappa = \frac{eB}{4c} > -1 \). For these values of \( \kappa \) (or \( \theta \)) the system is diamagnetic, except for the values \(-0.8 < \kappa < -0.2\), where \( \chi_{nc} \) becomes positive. This is an interesting fact which deserves a more detailed study.

4 Landau diamagnetism according to the nonextensive statistics of Tsallis

To deal with the systems which do not obey the rules of the usual Boltzmann-Gibbs statistics Tsallis proposed a nonextensive generalization of thermodynamics [8],[6]. In this formalism internal energy can be constrained in three different ways which result in different definitions of partition function [11],[12]. We take the definition of the partition function to be

\[
Z_q = \left[ 1 - (1 - q)\beta \sum_n E_n \right]^{1 \over 1 - q},
\]

where the parameter \( q \) is a real number and \( E_n \) are energy eigenvalues. This definition possesses some undesirable properties and the most satisfactory definition which we denote as \( \tilde{Z}_q(\beta) \) is given in terms of normalized \( q \)-expectation values [12]. Nevertheless, there exists a map [12] \( \beta \to \beta' \) such that \( \tilde{Z}_q(\beta) = Z_q(\beta') \). Moreover, \( \beta' \) is an increasing function of \( \beta \) [13], so that the high temperature limit can equivalently be taken as \( \beta' \ll 1 \).

The free energy \( F_q \) is also modified:

\[
F_q = -\frac{1}{\beta} \frac{Z_q^{1-q}}{1-q} - \frac{1}{1-q},
\]

but the magnetization \( M_q \) and the susceptibility \( \chi_q \) are still given by

\[
M_q = -\frac{\partial F_q}{\partial B}, \quad \chi_q = \frac{1}{n} \frac{\partial M_q}{\partial B}.
\]

We wish to investigate whether the generalized statistics obtained due to noncommutativity of space and the nonextensive statistics of Tsallis possess some common features. Obviously, we can have insights by studying the specific example of the previous section, namely electrons
in the uniform magnetic field $\vec{B}$ according to the nonextensive statistics of Tsallis and then comparing their results.

Calculation of generalized partition functions exactly is a hard task [14] and it is not always possible to achieve. We use the approximation which is known as the factorization approach [15]. It is valid for high and low temperatures without any restriction on $q$ and for small number of states for $(1 - q) \ll 1$ [16]. However, its validity can be enlarged to other values of $q$ as far as some other conditions are satisfied [17], [18].

Our aim is to calculate the generalized partition function $Z_q$ for a cylindrical body of length $L$ and radius $R$, which is oriented along the $z$-direction, using the Hamiltonian (1). The transverse part of the Hamiltonian, by choosing the symmetric gauge (2), is given as in (5). By using the factorization approach the generalized partition function $Z_q$ can be written as

$$ Z_q \simeq \frac{L}{\hbar} \left( \frac{m}{2\pi\beta} \right)^{\frac{1}{2}} \left[ 1 - \frac{\beta \hbar \omega}{2} (1 - q) \right]^{\frac{1}{1-q}} Z_\perp, \quad (27) $$

where $Z_\perp$ is taken as

$$ Z_\perp = \frac{R^2}{4\pi l^4} \int d^2 \alpha < \alpha | [1 - (1 - q) \beta \hbar \omega a^\dagger a]^{\frac{1}{1-q}} | \alpha >. \quad (28) $$

The coherent states $|\alpha >$ are the eigenstates of $a$ satisfying (7). Thus, if we write the operators appearing in (28) in normal ordered form the computation will become manageable. Fortunately, it is known that any function $f$ of $a^\dagger a$ can be written in the normal ordered form as [19]

$$ f(a^\dagger a) = \sum_{r=0}^{\infty} \sum_{s=0}^{r} \frac{(-1)^s f(r-s)}{(r-s)! s!} a^\dagger^r a^s. \quad (29) $$

Actually, for an exponential function it yields

$$ e^{\xi a^\dagger a} = \sum_{r=0}^{\infty} \frac{(e^\xi - 1)^r}{r!} a^\dagger^r a^r, \quad (30) $$

where $\xi$ is a constant. By making use of (29) and (2) we can write (28) as

$$ Z_\perp = \frac{R^2}{4\pi l^4} \int d^2 \alpha \sum_{r=0}^{\infty} C_r \left( \frac{|\alpha|^2}{2l^2} \right)^r, \quad (31) $$

where the coefficients $C_r$ are given by

$$ C_r = \sum_{s=0}^{r} \frac{(-1)^s}{(r-s)! s!} \left[ 1 - (1 - q) \beta \hbar \omega (r-s) \right]^{\frac{1}{1-q}}. \quad (32) $$
These coefficients can be expanded in \((1 - q)\) as

\[
C_r = \sum_{s=0}^{r} \frac{(-1)^s}{(r-s)!s!} \sum_{n=0}^{\infty} \frac{(-1)^n}{(1 - q)^n n!} \left( \sum_{k=1}^{\infty} \frac{(1 - q)^k}{k} (\beta \hbar \omega)^k (r - s)^k \right).
\]  \quad (33)

The terms to second order in \((1 - q)\) can be evaluated explicitly

\[
C_r = \sum_{s=0}^{r} \frac{(-1)^s}{(r-s)!s!} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left\{ (\beta \hbar \omega)^n + (1 - q) \frac{n}{2} (r - s)^{n+1} (\beta \hbar \omega)^n + \right.

+(1 - q)^2 \left\{ \frac{n(n - 1)}{8} + \frac{n}{3} \right\} (r - s)^{n+2} (\beta \hbar \omega)^n + \ldots \right\}.
\]

We retain the terms to first order in \((1 - q)\) and utilize (30) to obtain the compact form

\[
C_r \simeq \frac{(e^{-\beta \hbar \omega} - 1)^r}{r!} - \frac{1}{2} \frac{\partial^2}{\partial \beta^2} \left[ \frac{(e^{-\beta \hbar \omega} - 1)^r}{r!} \right].
\]  \quad (34)

Now, we write \(Z_\perp\) as

\[
Z_\perp = R^2 4\pi l^4 \left( 1 - \frac{(1 - q)}{2} \beta^2 \frac{\partial^2}{\partial \beta^2} \right) \int d^2 \alpha \exp \left[ - \frac{1}{2 l^2} (1 - e^{-\beta \hbar \omega}) \right],
\]  \quad (35)

which can be evaluated to yield

\[
Z_\perp = \frac{R^2}{2l^2} \left( 1 - \frac{(1 - q)}{2} \beta^2 \frac{\partial^2}{\partial \beta^2} \right) \left( \frac{1}{1 - e^{-\beta \hbar \omega}} \right).
\]  \quad (36)

By keeping the dominant terms in \((1 - q)\), the first factor appearing in (27) can be written as

\[
\left[ 1 - \frac{\beta \hbar \omega}{2} (1 - q) \right]^{\frac{1}{r!}} \simeq e^{-\frac{\beta \hbar \omega}{2}} - \frac{(1 - q)}{2} \left( \frac{\beta \hbar \omega}{2} \right)^2 e^{-\frac{\beta \hbar \omega}{2}}.
\]  \quad (37)

Substitution of the approximated values (36) and (37) into (27) yields

\[
Z_q \simeq \frac{V}{\lambda^3} \frac{\beta \hbar \omega}{2} \left\{ \frac{1}{\sinh(\beta \hbar \omega / 2)} - \frac{(1 - q)}{2} \left[ \frac{(\beta \hbar \omega / 2)^2}{\sinh(\beta \hbar \omega / 2)} + e^{-\frac{\beta \hbar \omega}{2}} \beta^2 \frac{\partial^2}{\partial \beta^2} (1 - e^{-\beta \hbar \omega})^{-1} \right] \right\}.
\]  \quad (38)

We can write it in the compact form

\[
Z_q \simeq Z_L \left[ 1 - \frac{(1 - q)}{2} \Sigma(\beta, \omega) \right],
\]  \quad (39)

where \(Z_L\) is the partition function of free electrons in the magnetic field \(\vec{B}\) according to the standard thermodynamics

\[
Z_L = \frac{V}{\lambda^3} \frac{\beta \hbar \omega}{2} \frac{1}{\sinh(\beta \hbar \omega / 2)},
\]  \quad (40)
and the first order modification is
\[ \Sigma(\beta, \omega) = (\beta h \omega)^2 \left[ \frac{1}{4} + \frac{1}{2} e^{-\frac{\beta h \omega}{2}} \frac{\cosh(\beta h \omega/2)}{\sinh^2(\beta h \omega/2)} \right]. \tag{41} \]

Because of dealing with the values of \( q \) such that \((1 - q) \ll 1\), we retain the terms to first order in \((1 - q)\) for the free energy \( F_q \)
\[ F_q \simeq -\frac{1}{\beta} \left[ \ln Z_L - \frac{(1 - q)}{2} \Sigma(\beta, \omega) - \frac{(1 - q)}{2} (\ln Z_L)^2 \right], \tag{42} \]
after inserting (39) into (25). It is now straightforward to calculate the magnetization \( M_q \) and the susceptibility \( \chi_q \) of the nonextensive case by making use of the definitions (26), which lead to
\[ M_q \simeq M_L - \frac{(1 - q)}{2} \frac{1}{\beta} \frac{\partial \Sigma}{\partial B} + (1 - q) M_L \ln Z_L, \tag{43} \]
\[ \chi_q \simeq \chi_L - \frac{(1 - q)}{2} \left[ \frac{1}{\beta} \frac{\partial^2 \Sigma}{\partial B^2} + \frac{2\beta}{n^2} M_L^2 + 2 \chi_L \ln Z_L \right], \tag{44} \]
where \( M_L = \frac{\partial \ln Z_L}{\partial B} \) and \( \chi_L = \frac{1}{n} \frac{\partial M_L}{\partial B} \) are the magnetization and the susceptibility according to the standard extensive formalism which can be approximated in the high temperature limit, \( \beta \ll 1 \), as
\[ M_L \simeq -\frac{1}{6} \frac{neh^2 \omega}{mc} \beta, \tag{45} \]
\[ \chi_L \simeq -\frac{1}{3} \left( \frac{he}{2mc} \right)^2 \beta. \tag{46} \]

Moreover, the leading terms of \( \Sigma(\beta, \omega) \) for high temperatures are
\[ \Sigma(\beta, \omega) \simeq 2 - \beta h \omega + \frac{7}{3} \left( \frac{\beta h \omega}{2} \right)^2. \tag{47} \]

For \( \beta \ll 1 \), the third term of (14) behaves like \( \beta^3 \) so that it should be ignored with respect to the second term behaving like \( \beta \). We then conclude that according to the nonextensive formalism of Tsallis the standard Landau diamagnetism is modified as
\[ \chi_q \simeq \chi_L \left[ 1 + (1 - q)(7 + \ln Z_L) \right], \tag{48} \]
for \( \beta \ll 1 \) as well as retaining only the terms to first order in \((1 - q)\).
5 Conclusions

The modified susceptibility in noncommuting coordinates (23) for $\beta \ll 1$ and the one obtained according to the nonextensive formalism of Tsallis (48) for $\beta \ll 1$ and $(1 - q) \ll 1$, in terms of the factorization approach, coincide if their parameters are related as

$$\kappa + \kappa^2 = (1 - q) \left( \frac{7 + \ln Z_L}{6} \right).$$  (49)

We recall that $\kappa$ is given in terms of the noncommutativity parameter $\theta$ as $\kappa = \frac{eB\theta}{4c}$. Thus, the generalization of the standard thermodynamics obtained due to noncommutativity of coordinates covers, in certain limits, the nonextensive formalism of Tsallis for Landau diamagnetism. We would like to emphasize that this result is obtained in terms of some approximations and it is far from being an exact result.

Obviously, it may happen that some deformations of quantum mechanical systems other than letting the coordinates be noncommuting may lead to similar relations. Nevertheless, this does not alter the fact that our results indicate that the generalized statistics due to noncommutative coordinates is a candidate to deal with the systems which do not obey the rules of the standard thermodynamics. However, it is not guaranteed that the results obtained in this specific example can be extrapolated to other physical systems. To have a better understanding of the roles of noncommuting coordinates in generalization of the standard thermodynamics one should study more physical systems in noncommuting coordinates.

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