Scale-free gravitational collapse as the origin of \( \rho \sim r^{-2} \) density profile – a possible role of turbulence in regulating gravitational collapse

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ABSTRACT

Astrophysical systems, such as clumps that form star clusters share a density profile that is close to \( \rho \sim r^{-2} \). We prove analytically this density profile is the result of the scale-free nature of the gravitational collapse. Therefore, it should emerge in many different situations as long as gravity is dominating the evolution for a period that is comparable or longer than the free-fall time, and this does not necessarily imply an isothermal model, as many have previously believed. To describe the collapse process, we construct a model called the turbulence-regulated gravitational collapse model, where turbulence is sustained by accretion and dissipates in roughly a crossing time. We demonstrate that a \( \rho \sim r^{-2} \) profile emerges due to the scale-free nature of the system. In this particular case, the rate of gravitational collapse is regulated by the rate at which turbulence dissipates the kinetic energy such that the infall speed can be 20-50\% of the free-fall speed (which also depends on the interpretation of the crossing time based on simulations of driven turbulence). These predictions are consistent with existing observations, which suggests that these clumps are in the stage of turbulence-regulated gravitational collapse. Our analysis provides a unified description of gravitational collapse in different environments.

Key words: hydrodynamics – turbulence – gravitation – methods:analytical – galaxies: star formation

1 INTRODUCTION

In many astrophysical systems, the underlying density structures are close to spherical symmetric, which allows one to define radial profiles where the radially-averaged mean gas density \( \rho \) is a function of the radius \( r \). In systems like star cluster-forming molecular clumps, the radial density profile is very close to \( \rho \sim r^{-2} \) (e.g. Mueller et al. 2002; Evans 2003; Wyrowski et al. 2012; Palau et al. 2014; Wyrowski et al. 2016; Csengeri et al. 2017; Zhang & Li 2017), and this has become even more clear after one has constructed density profiles of these clumps with a higher spatial dynamical range using observations obtained with different instruments/interferometer configurations (e.g. Csengeri et al. 2017; Zhang & Li 2017).

The theoretical interpretation of this profile remains unclear. Shu (1977) studied the collapse of singular isothermal gas spheres, and found that \( \rho \sim r^{-2} \) emerges as a result of hydrostatic balance, and during the collapse a somewhat shallower density profile is established. Presumably, because of historical reasons, this had lead to the misconception that a \( \rho \sim r^{-2} \) density profile must indicate some pressure support. However, one should always bear in mind that expected radial profile depends on many factors, including the equation of state as well as the initial and boundary conditions. Even after considering all these factors, the solutions might not be physical (see e.g. Whitworth & Summers 1985, for a thorough treatment). Another common problem with the studies during 1960-1980s was that the authors typically assumed an isothermal equation of state, and in systems such as star-forming regions, gas motion is highly supersonic such that the thermal pressure is not dominant. This makes it difficult compare observations with their model predictions.

Interestingly, the \( \rho \sim r^{-2} \) profile seems to emerge in a variety of situations where we do not expect the isothermal condition to hold. For example, in molecular clumps that form star clusters, the ram-pressure of the turbulent motion is much larger than sound velocity (which is related to the thermal motions of the gas). However, observations still indicate that a \( \rho \sim r^{-2} \) profile emerges. There are also mod-
els that can produce the $\rho \sim r^{-2}$ profile with some free-fall (Larson 1969; Naranjo-Romero et al. 2015). One should explain why we still observe $\rho \sim r^{-2}$ in these very different conditions – what is the cause of the $\rho \sim r^{-2}$ profile?

In this paper, assuming a spherically-symmetric geometry, we present a simple scaling analysis, and propose that $\rho \sim r^{-2}$ profile is a natural consequence of scale-free gravitational collapse, where, after some time, the local infall time becomes linked to the dynamical time of the gas. We back up our analytical argument with an analytical model that describes the interplay between turbulence and gravity, and prove that a $\rho \sim r^{-2}$ should emerge if the kinetic energy injected into the system due to gravitational collapse and dissipates through turbulence. The model quantitatively explains observations of these pc-scale molecular clumps. In short, the emergence of a $\rho \sim r^{-2}$ profile does not necessarily imply an isothermal model, but should be regarded as a generic feature of scale-free gravitational collapse.

2 ORIGIN OF THE $\rho \sim r^{-2}$ PROFILE

We use the method of scaling analysis (Barenblatt 1996) to derive the expected radial profile during the gravitational collapse. We consider a spherical gas clump. In the clump, collapse had already occurred such that within the range between $r_{in}$ and $r_{out}$, the system reaches a quasi-stationary state, where the density structure can be described as $\rho(r)$ where $r$ is the radius, $r_{in}$ is the inner radius and $r_{out}$ is the outer radius. How one determines $r_{in}$ and $r_{out}$ will be explained later. We are not studying how the structures are formed, but are interested in what the structure would be had the clump been given enough time to relax.

We further impose the constraint that the mass inflow rate at $r_{out}$ is identical to the mass inflow rate at the inner boundary $r_{in}$. In reality, $r_{in}$ corresponds to the radius within which gravity of the central objects dominates, and $r_{out}$ is the radius beyond which the gas is no longer gravitationally bound (see also Li (2017) where they presented a Jeans-like formula to derive $r_{out}$). We further assume that $r_{in} \ll r_{out}$ (e.g. $r_{in}$ is sufficiently small so that we can neglect the effect of $r_{in}$ on the density slope). Due to the lack of a characteristic scale in addition to $r_{out}$, we expect the density distribution to take the form of a power-law where

$$\rho = \rho_0 (r/r_0)^{-k_\rho} \sim r^{-k_\rho} ,$$

where $\rho_0$ and $\rho_0$ are normalisation factors. The mass enclosed in a region of radius $r$ is

$$m_{\text{enclosed}} \approx \int_{r_{in}}^{r} 4\pi \rho dr \sim r^{3-k_\rho} .$$

At radius $r$, the radial acceleration is

$$a_r \approx Gm_{\text{enclosed}}/r^2 \sim r^{-1-k_\rho} ,$$

the virial velocity (velocity at which the systems is virialised) is

$$v_{\text{ff}} \approx \sqrt{Gm_{\text{enclosed}}/r} \sim r^{1-k_\rho/2} .$$

and the free-fall time is

$$t_{\text{ff}} \approx \sqrt{m/G\rho} \sim r/v_{\text{ff}} \sim r^{k_\rho/2} .$$

When gravity is dictating the evolution of the system, the infall velocity at radius $r$ must be proportional to the free-fall velocity $v_{\text{ff}} \sim r^{1-k_\rho/2}$. Our clump has a mass inflow rate of

$$m \approx 4\pi^2 \rho v_{\text{infall}} \sim r^2 \rho v_{\text{ff}} \sim r^{3-3k_\rho/2} .$$

To drive the radial density profile of such a self-similar infalling system, we demand that the mass inflow rate $m$ to be a constant ($m \sim r^0 \approx \text{Constant}$) at different radii. Using Eq. 6, we have

$$3 - 3k_\rho/2 = 0 \rightarrow k_\rho = 2, \text{thus, } \rho \sim r^{-2} ,$$

which explains the $r^{-2}$ profile. Finally, we remind the reader that at a given radius $r$, $v_{\text{infall}} \sim v_{\text{ff}}$ automatically implies that the infall time $t_{\text{infall}} = r/v_{\text{infall}}$ is positively related to the free-fall time $t_{\text{ff}} = r/v_{\text{ff}}$.

We briefly explain what determines $r_{in}$: From Eq. 3, assuming $\rho \sim r^{-2}$, we expect a radial gravitational acceleration that scales with $r^{-3}$. A necessary condition for our analysis to be applicable is that the acceleration estimated by Eq. 3 should be accurate there. In reality (e.g. the case of star cluster-forming clumps), at very small radii, gravity from the central proto star-cluster dominates, and we expect our results to be accurate beyond a given radius where the deviation of acceleration from what one expects from Eq. 3 is not significant (as has been discussed in Murray & Chang (2015)).

To sum up, we have demonstrated that a spherical system that undergoes self-similar infall tend to relax to $\rho \sim r^{-2}$ during a self-similar infall, if (1) the system is gravitationally bound (2) the system has sufficiently relaxed and has reached a quasi-stationary state, (3) the amount of mass is roughly conserved during the infall (e.g. the gas consumption due to star formation during the infall is negligible), and (4) the mass of the central object does not cause the acceleration to deviate from the one predicted by Eq. 3 by much.

These conditions are satisfied at least approximately in many situations, and can explain the $r^{-2}$ profile found in previous papers (e.g. Larson 1969; Penston 1969; Naranjo-Romero et al. 2015). What is universal among these models is that after some relaxation processes, the infall time of the gas become linked to the free-fall time of the gas, and this leads to the $\rho \sim r^{-2}$ profile. However, it remains to be determined what influences the infall speed.

In the next section, we construct a simple analytical model that involves the interplay between turbulence dissipation and gravitational collapse, and demonstrate that the ratio between the infall speed and free-fall speed is related to the number of stars formed, and from this we can derive the dependence of the form $f = f(k_\rho)$.

2 Here, it is not straightforward to derive the value of $f$ directly. It is easier to consider some simple cases: For free-fall collapse $f$ can approach and even exceed unity and otherwise $f \leq 1$. In Sec. 3, we will demonstrate how turbulence can regulate the collapse to $f < 1$, where value of $f$ determined by the energy dissipation efficiency of the turbulence.

3 Where the analytical result of Penston (1969) is $\rho \sim r^{-12/7}$, and 12/7 is almost –2.
ratio between the turbulence dissipation rate and gravitationally collapse rate.

3 TURBULENCE REGULATED GRAVITATIONAL COLLAPSE OF MOLECULAR CLUMPS

We establish a concrete physical model to describe how the interplay between turbulence and gravity gives rise to a \( \rho \sim r^{-2} \) profile, and determine the rate of gravitational collapse. The model shares a common view with many of the previous ones, e.g. Murray & Chang (2015); Lee & Hennebelle (2016); Li (2017).

We consider a clump with a density profile \( \rho(r) \) and an infall velocity profile \( v_{\text{infall}}(r) \). In essence, the structure of molecular interstellar medium is complicated, and the close-to-spherical geometry which we adapt is an approximation, which we believe is a reasonable one: Observationally, these structures are indeed close to be spherical. One good example is the well-resolved nearby star-cluster forming region NGC1333. These observations thus provide an empirical justification to our assumption. Theoretically, combined with some flow asymmetry, gravity is believed to be able to drive collapse away from sphericity, producing sheets, and then filaments (Lin et al. 1965; Zel'dovich 1970). This mechanism should explain the existence of filamentary structures in ISM (Gómez & Vázquez-Semadeni 2014). However, it has been later shown that the effect can be weakened by the presence of a steep density gradient (Goodman & Binney 1983; Lai & Goldreich 2000). Since our clumps are centrally-condensed, we expect the density gradient stabilisation to be effective, thus the deviation from sphericity due to gravity should not be severe for our case. Besides, a turbulent pressure can also help to reduce the radial density asymmetry. These justify our spherical assumption.

We also assume that the profiles are are independent of time \( t \). This corresponds to the situation where there is a continuous mass inflow starting from \( r_{\text{out}} \) to \( r_{\text{in}} \). When mass gets accumulated at the centre, we expect \( \rho \) to increase also with the time. In these cases, our analysis is still valid as long as the time at which \( \rho \) increases is much longer than the dynamical time of the system.

The evolution of the clumps is dominated by a combination of turbulence and gravity. Gravity is responsible for driving the turbulent motion (Field et al. 2008; Klessen & Hennebelle 2010; Robertson & Goldreich 2012; Murray & Chang 2015), and the turbulence motion decays within a few crossing times (Stone et al. 1998; Mac Low et al. 1998). The role of turbulence is to dissipate the energy generated by gravitational at different scales.

We assume that at radius \( r \), the turbulent velocity dispersion \( \sigma_v \) is determined by the virial velocity \( \sigma_v \approx r \). This corresponds to the case where the clumps are supported by the virialised quasi-random motions which are generally assumed to be turbulence, and accretion is the major source of the turbulence (Field et al. 2008; Elmegreen & Burkert 2010; Klessen & Hennebelle 2010).

Previously, Murray & Chang (2015) carried out a detailed analysis of gravitational collapse in a turbulent medium using a 1D model. In their model, turbulence is driven by adiabatic heating (Robertson & Goldreich 2012). Here, we assume that at different radii, gas in the clump is virialized. Our view is supported by the fact that observationally, these clumps are found to be virialized over multiple scales\(^4\), as a theoretical treatment, it is appropriate to invoke a virialised turbulence model and remove the dependence of the results on the initial turbulence seed assumed in the previous model. Due to the presence of the infall, gas in the clumps can never establish a complete virial equilibrium: if the infall time is comparable to the dynamical time, infall motion can not be neglected; if the infall time is longer than the dynamical time, the infall motion is slow and can be neglected as a first-order approximation, where the gas can stay close to the virial equilibrium. In observations, the infall speed is a fraction of the free-fall speed (e.g. Evans 2003; Wyrowski et al. 2012, 2016), thus the assumption of virial equilibrium is still valid given the presence of the infall.

In a real molecular clump, star formation has some effects on the gas evolution. Generally speaking, star formation has two effects: First, it consumes mass, and second, it provides feedback. Because it is believed that the star formation efficiency is a strong function of gas density (Padoan et al. 2014), it is only able to consume gas efficiently at the very inner part of the clumps and acts as a mass sink, and this has already been properly considered in our model. Star formation also produces feedback. In general, the feedback effects include wind-driven molecular outflows, stellar winds and supernova explosions. The feedback from wind-driven molecular outflows is mostly contributed from low-mass stars. These outflows might drive turbulence (Li & Nakamura 2006; Matzner 2007). However, the effects are considered to be relatively weak (Matzner & Jumper 2015). In essence, the efficiency of the outflow feedback depends on the outflow geometry. According to some more recent models, the outflowing gas is contained in a thin layer (Li et al. 2013), the coupling between the outflow and the environment is inefficient and the impact of these outflows on the large-scale clump dynamics can be neglected. Wind from massive stars and supernova explosions can be very powerful and will disrupt the clumps at some stage. Here, we are only interested in the clump structures before the disruption occurs. Thus, in spite of its simplicity, our model should be able to capture the general behaviour of the gas flow in the observed molecular clumps.

3.1 Parameterization

We parameterize the density structure as

\[
\rho(r) = \rho_0 \left( \frac{r}{r_0} \right)^{-k_v},
\]

(8)

The infall velocities at different radii are parameterised as

\[
v_{\text{infall}}(r) = v_0 \left( \frac{r}{r_0} \right)^{-k_v}.
\]

(9)

\(^4\) See e.g. Li et al. (2015b) where they pointed out that the value of the virial parameter is related to how one selects regions. If the regions are properly defined, all the regions in the Perseus and Ophiuchus that they studied are gravitationally bound. This is consistent with previous results, e.g. Goodman et al. (2009).
where $v_0$ is a normalisation parameter whose value will be determined later.

We work with Eulerian coordinates. At $(r, r + \Delta r)$, the shell have a mass of

$$m_{\text{shell}} = 4\pi r^2 \rho(r) \Delta r = 4\pi \rho_0 r_0^2 \left( \frac{r}{r_0} \right)^{3-k_r} \Delta r ,$$

and the mass enclosed in such a shell is

$$m_{\text{enclosed}} = \frac{4\pi \rho_0 r_0^3}{3} \left( \frac{r}{r_0} \right)^{3-k_r} .$$

We assume that gas in each shell is approximately virialized,

$$\sigma_{\text{v,3D}} = \sqrt{\frac{GM_{\text{enclosed}}}{r}} ,$$

where $\sigma_{\text{v,3D}}$ is the three-dimensional velocity dispersion $^5$. The crossing time can be estimated as $^6$

$$t_{\text{cross}} \approx \frac{2 \times \beta \times r}{\sigma_{\text{v,3D}}} \approx \frac{2 \times \beta \left( \frac{r^3}{GM_{\text{enclosed}}(r)} \right)^{1/2}}{\sigma_{\text{v,3D}}} ,$$

where the factor of 2 comes from the fact that the diameter is two times the radius. $\beta$ is introduced as the ratio between the driving scale of the turbulence and the clump diameter. If the driving scale of the turbulence is comparable to the clump diameter $\beta = 1$; if the driving scale of the turbulence is much smaller, $\beta < 1$ (details concerning the driving of turbulence has been described in Stone et al. 1998). In this paper, we assume that the driving scale of the turbulence is comparable to the clump diameter, where $\beta = 1$. Note that the value of $\beta$ is also dependent on the interpretation of the crossing time in the simulations. Our referee pointed out that Fourier driving involves sine waves that contains both positive and negative velocities, and thus the actual crossing time for a mode of wavelength equal to the box size may be the time necessary for crossing only half of the box length. We have chosen $\beta = 1$, where the driving scale is comparable to the cloud size. Our assumption is consistent with the discussion in Stone et al. (1998). The kinetic energy of such a shell is

$$E_{\text{kin}} \approx \frac{1}{2} m_{\text{shell}} \sigma_v^2 = \frac{1}{2} \frac{G \pi \rho_0 r_0^2}{3 - k_p} \left( \frac{r}{r_0} \right)^{4-k_r} \Delta r .$$

We note that although the shell thickness $\Delta r$ is introduced here as a parameter, we are interested in the radially-averaged means, and our holds exactly as $\Delta \rightarrow 0$.

### 3.2 Mass and energy conservation in a clump

We derive the clump structure using mass and energy conservation. Because of mass conservation, the mass inflow rate $\dot{m} = 4\pi r^2 \rho_{\text{infall}}$ should stay constant at different radii, thus

$$\dot{m} = 4\pi r^2 \rho_{\text{infall}} = 4\pi r^2 \rho_0 \left( \frac{r}{r_0} \right)^{3-k_r} \approx \frac{r^2}{k_r - k_v} = \text{Constant} ,$$

which gives

$$2 - k_p - k_v = 0 .$$

The rate of gravitational energy release from infall is

$$\dot{E}_{\text{gravity}} = \frac{G m_{\text{enclosed}} m_{\text{shell}} \dot{m}_{\text{infall}}}{r^2} = \frac{(4\pi \rho_0)^3 G^2 \rho_0^3}{3 - k_p} \left( \frac{r}{r_0} \right)^{3-k_r} \Delta r .$$

The turbulent energy of such a shell is expected to decay within $n$ (which is believed to be around 0.8, Stone et al. 1998; Mac Low et al. 1998; Lemaster & Stone 2009) crossing times, which gives an energy dissipation rate

$$\dot{E}_{\text{kin}} \approx \frac{E_{\text{kin}}}{n \times t_{\text{cross}}} = \frac{G \pi r_0^2 \rho_0^2}{4 \pi} \left( \frac{r}{r_0} \right)^{4-k_r} \Delta r .$$

We require that energy conservation holds exactly for each shell, and this gives

$$E_{\text{kin}}(r) = E_{\text{gravity}}(r) ,$$

which implies (using Eq. 17 and Eq. 18)

$$k_p = 2 \quad (20)$$

$$k_v = 0 \quad (21)$$

$$v_0 = \left( \frac{4\pi G \rho_0^2}{r_{\text{infall}}} \right)^{1/2} \times \frac{1}{4n\beta} .$$

Observationally, $\rho_{\text{clump}}$ and $E_{\text{clump}}$ can be measured. We can use these values to determine the normalisations of Eq. 8.

$$\rho_0 = \frac{1}{4\pi r_0^2} \left( \frac{E_{\text{clump}}}{r_{\text{clump}}} \right) ,$$

and

$$E_{\text{infall}} = \left( \frac{G M_{\text{clump}}}{r_{\text{infall}}} \right)^{1/2} \times \frac{1}{4n\beta} .$$

If one defines the free-fall velocity of such a clump as $v_f = \sqrt{G M_{\text{clump}}/r_{\text{clump}}}$, the predicted infall velocity is always a fraction $(1/4n\beta)$ of the free-fall velocity, where $\beta$ is determined by turbulence driving. We assume that the driving scale is comparable to the clump diameter, such that $\beta = 1$ (see discussions followed Eq. 13). Although the general conclusion that an inefficient energy dissipation will lead to an a slow gravitational collapse is true, the above derivation has a couple of uncertainties. One noticeable one is the turbulence energy dissipation rate estimate. Here we are applying the energy dissipation rate estimated in simulations such as Stone et al. (1998); Mac Low et al. (1998); Mac Low (1999) to our model. Using the simulation results leads to two issues: first, the physical condition in the numerical simulation has to be identical to the physical condition of the
clumps, and second, the crossing time defined in the simulations and in observations must be comparable. The simulations have parameters that are chosen to resemble that of the molecular ISM, and we expect the results from the simulation to be accurate in this sense. However, in the clumps, the presence of a radial density gradient might physically affect the energy dissipation process and change the dissipation rate. How strong this effect is remains undetermined. Another uncertainty comes from the way one defines the crossing time: The crossing time formula involves a scale and the velocity dispersion at that scale. In both the simulations and our model, the velocity dispersion is the three-dimensional velocity dispersion. Therefore the velocity dispersion estimate is not a major source of uncertainty. However, it is not straightforward to find a unique way to compare $l$: in our formalism we have assumed that at radius $r$, the relevant scale is simply the diameter $2r$, and in the simulation of Stone et al. (1998); Mac Low et al. (1998). $l$ is the driving scale. In other words, one of the assumption in our estimate of turbulence energy dissipation rate is we have assumed that the driving scale of the turbulence is comparable with the clump radius. Although this is plausible, it is still dependent on the nature of turbulence in the clumps, which is not fully clear.

3.3 Universality of the $\rho \sim r^{-2}$ profile

The turbulence-regulated gravitational collapse model provides us an excellent example to understand the relevance of the analytical arguments presented in Sec. 2 to real systems. First, in our analytical study, we only made use of the scale-free nature of gravitational collapse, and here we take the interplay between turbulence and gravity into account. Since the explicit inclusion of energy dissipation rate estimates due to equilibrium cascade in the model does not introduce additional scales, we recover our $\rho \sim r^{-2}$ profile expected in the previous section. Second, by writing down detailed equations describing the interplay between gravitational contraction and turbulence dissipation in a centrally-condensed clump, we are able to analytically derive the relation between turbulence dissipation efficiency and infall: the infall velocity derived in Eq. 22 is slower than the free-fall velocity by a factor of $4 \times n$. In our clumps, the infall velocity is slower than the free-fall velocity because the turbulence dissipation time $t_{\text{dissipation}}$ is longer than what it required to be if the system undergoes free-fall. To put it in a different from:

$$\frac{v_{\text{infall}}}{v_{\text{ff}}} \leq 1 \frac{t_{\text{dissipation}}}{4 t_{\text{crossing}}},$$

(23)

where the rate of gravitational collapse is linked to the efficiency of turbulence dissipation. The collapse is slow in turbulence-supported clumps due to the inefficiency of energy dissipation.

7 In the text of Stone et al. (1998); Mac Low et al. (1998); Mac Low (1999), the author did not explicitly specify whether they have used 1D or 3D velocity dispersion. However, the kinetic energy densities are estimated using the formula $1/2 \rho v_{\text{ff}}^2$ instead of $3/2 \rho v_{\text{ff}}^2$, suggesting that the authors are using the 3D velocity dispersion.

4 OBSERVATIONAL CORRESPONDENCE

4.1 $\rho \sim r^{-2}$ profile in star-cluster forming clumps

The $r^{-2}$ profile has been widely seen in observations of star cluster-hosting clumps (e.g. Mueller et al. 2002; Evans 2003; Wyrowski et al. 2012; Palau et al. 2014; Wyrowski et al. 2016; Csengeri et al. 2017; Zhang & Li 2017). Observations also found that the infall speed is a fraction of the free-fall speed (Evans 2003; Wyrowski et al. 2016, 2012). How well can these be explained by existing theories?

Quite a few models can explain a $\rho \sim r^{-2}$-like density profile. There are two categories of models. The first category includes those without infall. The most famous one is the Shu (1977) solution where the author attributed the profile to the fact that the system must achieve hydrostatic equilibrium before the collapse starts. The other set of models involves infall. These include models that involve free-fall collapse (Larson 1969; Penston 1969; Vázquez-Semadeni et al. 2000; Ballesteros-Paredes et al. 2011; Girichidis et al. 2014; Naranjo-Romero et al. 2015; Donkov & Stefanov 2017), where the $r^{-2}$ profile is found to be the analytical solution (such as the case of (Larson 1969; Naranjo-Romero et al. 2015; Donkov & Stefanov 2017)), as well as models that involve gravity and turbulence, including the turbulence-regulated gravitational collapse model presented above.

Our general analysis has suggested that the $\rho \sim r^{-2}$ profile is a generic signature of scale-free gravitational collapse, and this is independent on whether the collapse is free-fall or not. After accepting the analysis, it is no longer a surprise that a universal profile has emerged in these different settings. This has two implications for the interpretation of observations: first, the $\rho \sim r^{-2}$ profile do not necessarily imply some pressure support, as Shu (1977) might had believed. It emerges in models that involve free-fall collapse and turbulence-regulated gravitational collapse as well. Second, even if a clump is found to be collapsing, based on the $r^{-2}$ profile, it is difficult distinguish between a pure free-fall and a turbulence-regulated collapse.

To further distinguish these infall models, we should look at the infall velocity. According to Sec. 3, if turbulence is regulating the collapse, we expect to observe an infall velocity that is much smaller than the free-fall velocity. This is very different from the free-fall models where the infall velocity should be close to the free-fall velocity. In observations, the velocities are found to be around $\sim 5 - 30\%$ of the free-fall velocity (e.g. Evans 2003; Wyrowski et al. 2012, 2016), suggesting that some mechanism is regulating the collapse.

According to Sec. 3, if turbulence dissipates in $n$ crossing times. Assuming that the driving scale of turbulence is comparable to the clump diameter, $\beta \approx 1$ (Eq. 13), the infall velocity should be roughly $1/4n$ times the free-fall velocity. The turbulence in the clumps is believed to dissipate within around $n = 0.7$ crossing time (for a Mach 7 hydrodynamic turbulence) to $n = 1.2$ crossing time (for a Mach 1.2 hydrodynamic turbulence, according to Lemaster & Stone (2009)). This would imply that the infall speed should be roughly $1/4n \approx 20\%$ to $31\%$ of the free-fall speed. This is comparable to the infall speed seen in observations (which is $\sim 5 - 30\%$ of the free-fall velocity) 8. Thus the observations and our

8 Note that although including a magnetic field can also change
turbulence-regulated gravitational collapse model is largely consistent. However, one needs to bear in mind that there are sources whose infall velocity is too small to be explained by turbulence regulation. These sources might have a different nature, and this issue remains to be understood. In general, gravitational collapse regulated by turbulence dissipation can quantitatively explain the slowness of gravitational infall.

4.2 Connection between $\rho \sim r^{-2}$ profile and the surface density PDF

A convenient way to quantify a structure is to derive the density PDF (Probability distribution function). Since the density profile in our model is a power-law, it is natural to expect the clumps to have power-law density PDFs (as has been discussed in Kritsuk et al. (2011))\(^9\). It is indeed the case: the $\rho \sim r^{-2}$ profile implies a surface density PDF of $P(\Sigma) \sim \Sigma^{-2}$ where $\Sigma$ is surface density (Girichidis et al. 2014; Li & Burkert 2016)\(^10\), and interestingly, this is consistent with fact that in observations (e.g. Kainulainen et al. 2009; Lombardi et al. 2015, Li & Burkert 2016).

5 CONCLUSIONS

The $\rho \sim r^{-2}$ profile has been seen in many self-gravitating systems, and the interpretation of this profile has been ambiguous. In many cases, these profiles are interpreted using an isothermal solution or free-fall collapse. In this paper, based on a simple scaling analysis, we argue that this profile can be created in a wide range of situations where, after some initial relaxation processes, the time for gravitational collapse is linked to the local free-fall time. When this condition has been reached, the collapse will occur in a self-similar fashion, which leads to the $\rho \sim r^{-2}$ profile. This explains why the $r^{-2}$ profile has been found in simulations of different settings.

We further develop an analytical model called turbulence-regulated gravitational collapse model that incorporates the interplay between turbulence dissipation and driving of turbulence by gravitational collapse. Using constraints such as mass and energy conservation, we find that the stationary solution should exhibit the $\rho \sim r^{-2}$ profile. The infall speed should be a fraction of the free-fall speed, and this is mainly caused by the fact that the rate of gravitational collapse is regulated by the rate at which turbulence dissipates the kinetic energy ($v_{\text{infall}}/v_{\text{ff}} \approx 1/4 \times t_{\text{dissipation}}/t_{\text{crossing}}$), where, according to Lemaster & Stone (2009), $t_{\text{dissipation}} \approx 0.7-1.1 \times t_{\text{crossing}}$. Turbulence dissipation is able to regulate the gravitational collapse such that the infall speed is 20-30% of the free-fall speed, although this is still dependent on the driving of turbulence, as can be seen in Eq. 13 and Eqs. 22. This turbulence-regulated gravitational collapse model roughly explains the range of infall speed observed for parsec-scale molecular clumps in the Milky Way.

Some final notes from the author: 1. Dr. Y-Q Lou pointed out that apart from the anabatic heating, there are numerous instabilities through which turbulence can be generated (Lou & Lian 2012). 2. Similar to our Sec. 2, Burkert (2017) is another example where insights can be obtained by exploiting the continuity equation. 3. A discussion with Prof. S. Donkov suggests that similar to our case, the $\rho \sim r^{-2}$ profile can be achieved with a combination of (Donkov & Stefanov 2017).

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