Synthetic magnetic fields for cold erbium atoms

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The implementation of the fractional quantum Hall effect in ultracold atomic quantum gases remains, despite substantial advances in the field, a major challenge. Since atoms are electrically neutral, a key ingredient is the generation of sufficiently strong artificial gauge fields. Here we theoretically investigate the synthetization of such fields for bosonic erbium atoms by phase imprinting with two counter-propagating optical Raman beams. Given the non-vanishing orbital angular momentum of the rare earth atomic erbium species in the electronic ground state and the availability of narrow-line transitions, heating from photon scattering is expected to be lower than in the frequently investigated atomic alkali species. We give a parameter regime for which strong synthetic magnetic fields with good spatial homogeneity are predicted. We also estimate the size of the Laughlin gap for typical experimental parameters of a two-dimensional atomic erbium microcloud.

Our analysis shows that cold rare earth atomic ensembles are highly attractive candidate systems for experimental explorations of the fractional quantum Hall regime.

I. INTRODUCTION

To this date the quantum Hall effect is an active frontier of research as it is the hallmark of systems with topological order. For two-dimensional electron gases both the integer (IQHE) and the fractional quantum Hall effect (FQHE) have been observed in strong magnetic fields [1, 2]. The key characteristics of a gapped quantum Hall systems is its non-trivial topology, characterized by a non-vanishing Chern-number. Associated with this are a number of fascinating features such as topologically protected edge states. Atoms are electrically neutral, but for these systems magnetic fields have been emulated using trap rotation [3, 4], lattice shaking [5], and phase imprinting via the photon recoil [6–8] methods.

There are two physical regimes for bosonic atoms in a synthetic magnetic field. If the applied artificial field is small the ground state remains a Bose-Einstein condensate (BEC), with a macroscopic wavefunction. The artificial (FQHE) have been observed in strong magnetic fields this ground state having anyonic character [12]. Particularly interesting since some of the excitations above flux quanta, \( qB/h \), it has been pointed out that the ground state is a Laughlin state [10, 11], which is particularly interesting since some of the excitations above this ground state having anyonic character [12].

The current experimental challenge lies in generating synthetic gauge fields for atoms that are strong enough to reach the fractional quantum Hall regime. To surpass technical limitations of direct atomic cloud rotation schemes arising from e.g. the metastability of the atomic cloud one can apply phase imprinting methods via Raman manipulation [13–16] for the generation of gauge fields. Phase imprinting methods at the present stage have used alkali atoms with an \( S \)-electronic ground state \((L = 0)\), for which Raman transitions are only possible between ground state sublevels due to the additional fine- and hyperfine structure, see Fig. 1(a). This impedes the use of laser detunings above the fine structure splitting of the upper electronic state, and leads to a limitation of the possible coherence time. In principle, atoms with a \( P \)-electronic ground state, see Fig. 1(b) for a corresponding level scheme, would be attractive candidates, however corresponding atomic species, as the oxygen atom, are technically difficult to laser cool due to technically inconvenient UV electronic transition wavelengths and a large number of required repumping lasers.

Cui et al., specifically considering the case of dysprosium atoms and a \( \sigma^+ - \pi \) optical polarization configuration of Raman beams inducing a \( \Delta m = 1 \) ground state coupling scheme, proposed the use of lanthanide atoms for the generation of gauge fields [25]. Corresponding atomic species for the cases of dysprosium, erbium, and thulium fulfill the requirement of an orbital angular momentum \( L > 0 \) in the electronic ground state. Moreover, for these atomic systems laser cooling and the production of degenerate atomic quantum gases is feasible, see Refs. [17–21] for experimental work realizing both atomic Bose-Einstein condensates or degenerate Fermi gases using corresponding isotopes, or a cold atomic gas for the thulium case. When tuning into the vicinity of suitable electronic transitions, one expects that Raman transitions with far-off-resonant optical beams and cor-
respondingly low heating rates become possible. While the ratio between the upper state linewidth $\Gamma$ and a detuning of order of the upper state fine structure splitting for the rubidium alkali atom case is $\sim 10^{-3}$, a linewidth to detuning ratio $\Gamma/\Delta \simeq 10^{-7}$ seems feasible in rare earth atomic systems, see also a corresponding estimation in earlier work \cite{22}. Similarly, in these systems one expects that state dependent dipole trapping with long coherence times can be realized. Another, in many cases favorable property of many lanthanides is their large magnetic moment, giving rise to a large dipole-dipole interaction \cite{23, 24}.

In the present work, we present a scheme for the generation of artificial magnetic fields for erbium atoms using phase imprinting from two counter-propagating Raman beams with opposite circular polarization, inducing a $\Delta m = 2$ Raman coupling between ground state Zeeman sublevels, in the presence of a transverse gradient of a real magnetic field. The latter is needed to break time-reversal symmetry, a necessary ingredient of the quantum Hall effect. For suitable values of the two-photon Rabi frequency, we obtain a spatially very uniform synthetic magnetic field. We consider some examples for possible submerged shell transitions of the erbium atom to implement the Raman coupling. Finally, we derive estimates for the Laughlin gap for a two-dimensional atomic erbium quantum gas subject to the strong synthetic magnetic field.

In the following, chapter II discusses the generation of light-induced magnetic fields in a three-level system, and chapter III specifies this for some atomic erbium narrow-line transitions. Subsequently, chapter IV gives an estimation of the size of the Laughlin gap in the system, and chapter V closes with conclusions.

\section*{II. SYNTHETIC MAGNETIC FIELDS FOR THREE-LEVEL ATOMS}

For the sake of simplicity, we start by considering the generation of a synthetic magnetic field for a three-level atom with two stable ground state levels $|g_{+1}\rangle$ and $|g_{-1}\rangle$ and one spontaneously decaying excited state level $|e_{0}\rangle$, as shown in Fig. 1(b). The index used in the state notation denotes the corresponding Zeeman quantum number. The discussed implementation follows the work of Spielman \cite{16} developed for alkali atoms, however we here consider the atom to be driven by two far detuned counter-propagating laser beams in a $\sigma^+ - \sigma^-$ polarization configuration, which results in Raman coupling between ground state sublevels $|g_{+1}\rangle$ and $|g_{-1}\rangle$ with $\Delta m = 2$. The general idea is to construct a Hamiltonian offering an atomic dispersion that mimics that of a charged particle in the presence of a position-dependent vector potential $A^*$, so that a synthetic magnetic field $B^* = \nabla \times A^*$ emerges. This can be achieved with a transversal gradient of the (real) magnetic field, which leads to a two-photon detuning $\delta = \omega_+ - \omega_- - \omega_0$, where $\omega_+$ and $\omega_-$ denote the laser frequencies with corresponding polarizations and $\omega_0$ is the energetic difference between $|g_{+1}\rangle$ and $|g_{-1}\rangle$ that is position-dependent. Assume both the magnetic field $B = B_x \vec{e}_x$ and the counter-propagating laser beams oriented along the $x$-axis and a magnetic field gradient along the $y$-axis, see also Fig. 1(c). We now have $B_x(y) = B_{0,x} + y \partial B_x/\partial y$, which realizes a position-dependent Raman detuning $\delta(y) = 2 g \mu_B y \partial B_x/\partial y$, where $g$ is the atomic Landé $g$-factor and $\mu_B$ the Bohr magneton.

For the counter-propagating laser beam configuration $\sim 2\hbar k_L$ momentum per Raman transition is transferred to the atoms, with $k_L = k_L \vec{e}_z$ and $k_L = 2\pi/\lambda$. In the following, $|g_{+1}, \vec{p}\rangle$ denotes an atom in the internal state $g_{+1}$ and momentum $\vec{p}$. The resulting effective Hamiltonian for an atomic system confined to the $x$-$y$ plane can be written in the basis of the coupled levels $|g_{-1}, \hbar(\vec{k} + \vec{k}_L)\rangle$ and $|g_{+1}, \hbar(\vec{k} - \vec{k}_L)\rangle$ as

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{(a) Reduced level scheme of alkali atoms, with an $S$ electronic ground state ($L = 0$). (b) Reduced level scheme for a transition from a ground state with $L = 1$ to an electronically excited state with $L' = 0$. This system gives an example for an electronic transition starting from a higher orbital angular momentum ground state, for which even with radiation far detuned from the electronically excited state Raman transitions between different ground state spin projections become possible (the ground state $|g_0\rangle$ is shown in gray, because it is not relevant for the atom-light coupling here). For the shown case of $L = 1$, the Raman transitions can be driven with a $\sigma^+ - \sigma^-$ optical polarization configuration, inducing Raman transitions with $\Delta m = 2$. (c) Schematic for synthetization of an artificial magnetic field for atoms using optical driving with two counter-propagating Raman beams and a transverse gradient of the (real) magnetic field.
}
\end{figure}
The vector potential

The Hamiltonian for the lower dressed state energy level can be approximated as

\[
\hat{H}_{\text{eff}} \approx E_0 + \frac{\hbar^2 (k_x - k_{\text{min},y}(y))^2}{2m^*} + \frac{\hbar^2 k_y^2}{2m}, \quad (2a)
\]

where \(k_{\text{min},y}(y)\) denotes the wavevector at which the described minimum of the dispersion curve occurs, and \(m^*\) denotes an effective mass for the motion along the \(x\)-direction. In the second equation (Eq. 2b) we have used the replacement \(k_{\text{min},y}(y) = q^* A_x^*(y)/\hbar\), where \(A_x^*\) is the synthetic vector potential discussed above and \(q^*\) is a synthetic charge. Given the transverse detuning variation from the gradient of the (real) magnetic field, we readily expect a non-vanishing value of the synthesized magnetic field along the \(z\)-axis: \(B_x^* = -\partial A_x^*(y)/\partial y = -\hbar/q^* \partial k_{\text{min},y}(y)/\partial y\), with \(\delta' = \partial \delta/\partial y = 2q_E B_x^*/\hbar\delta y\) as the detuning gradient. We arrive at \(B_x^* = -\hbar \delta'/q^* \partial k_{\text{min},y}(y)/\partial \delta\). Near \(y = 0\), for which \(\delta \approx 0\), the synthetic vector potential \(A_y^*\) varies linearly with the two-photon detuning \(\delta\), and correspondingly the transverse position \(y\) (Fig. 2(c)). The generated synthetic magnetic field has a maximum at \(y = \delta(y) = 0\). Fig. 2(d) shows the spatial variation along the \(y\)-axis, where we have assumed \(q^* = e\) and \(\delta'/2\pi = 2.66 \text{kHz}/\mu\text{m}\), as obtained e.g. with \(g = 1\) and a gradient of the real magnetic field of 9.5 \(\text{G/cm}\). The assumed experimental parameters for the magnetic field gradient, and also the obtained magnitude and spatial variation of the synthetic magnetic field are comparable to the case of the rubidium experiment of [16]. The different transferred momentum of the Raman transitions with counter-propagating laser beams introduces modifications of order below a factor of 2.

III. SYNTHETIC MAGNETIC FIELDS FOR THE ATOMIC ERBIUM CASE

We now discuss the possibility to generate synthetic magnetic fields for a specific rare earth system, atomic errium, for the transitions \(4f^{12}6s^2(\text{Er}) \rightarrow 4f^{11}(4f_{15/2}^0)5d_{5/2}6s^2(15/2,5/2)^2\), with \(J' = 5, 6\) and 7 at transition wavelengths of \(\lambda = 877 \text{ nm}\), \(\lambda = 847 \text{ nm}\), and \(\lambda = 841 \text{ nm}\) respectively. These transitions excite an electron within the incompletely filled “submerged” \(f\)-shell of the atom and all have a relatively small natural linewidth, e.g. \(\Gamma/2\pi = 8.0 \text{ kHz}\) for the \(J = 6 \rightarrow J' = 7\) transition near \(\lambda = 841 \text{ nm}\) wavelength [26]. Given the comparatively large energy difference to neighbouring levels in terms of the linewidths, the systems are very attractive for Raman manipulation with far detuned optical beams. Ultimately, we expect the heating rate to be limited e.g. by the off-resonant contribution from the strong \((\Gamma_{\text{blue}}/2\pi \approx 28 \text{ MHz})\) blue cooling transition near...
401 nm, which is off-resonant by an amount of order of the optical frequency. This sets a limit on the usable detuning from the upper state to the narrow-line transition of order $\Delta / \Gamma \approx 10^7$, and within this limit we assume in the following that off-resonant contributions from other excited states can be neglected.

Despite of the small scattering rate for radiation correspondingly tuned in the vicinity of such an inner-shell transition, scalar, vector, and tensor polarizabilities become comparable [27]. We assume a nuclear spin of $I = 0$, as is the case for all stable bosonic erbium isotopes (e.g. $^{168}$Er), so that $F = J$. As the $^{3}H_{6}$ ground state of atomic erbium possesses a total angular momentum of $J = 6$ (with $L = 5$, $S = 1$), $13 m_F$-sublevels exist. Our assumed Raman coupling scheme uses a $\sigma^+$ – $\sigma^-$ configuration, coupling only sublevels with $\Delta m_F = \pm 2$, so that 7 ground state sublevels ($|g_{\alpha}\rangle$), with $m_F = \alpha$ and $\alpha = -6, -4, \ldots, 6$, are coupled by the Raman beams, see Fig. 3(a) for the coupling scheme in the case of the $J = 6 \rightarrow J' = 7$ transition.

The laser electric field is $\vec{E} = E_0, x_+ \cos(\omega t - k_+ x) + E_0, x_- \cos(\omega t + k_- x)$, where $E_0, x_\pm$ denotes the field amplitudes of the $\sigma^+$, $\sigma^-$ polarized optical beams and $x_\pm$ are the corresponding unit polarization vectors.

The relative strength of the coupling between a certain ground state sublevel $|g_{\alpha}\rangle$ component and an excited state component $|e_{\alpha}\rangle$ with $\alpha = n \pm 1$ is characterized by the corresponding Clebsch-Gordan coefficient $c_{\alpha\alpha}$. For a list of Clebsch-Gordan coefficients for the three erbium transitions mentioned above, see the appendix.

The laser coupling between levels can be written in the form $\Omega_{\pm} c_{\alpha\alpha \pm 1} = \langle e_{\alpha \pm 1} | \vec{d} | g_{\alpha} \rangle E_0, \pm / \hbar$, where $\vec{d}$ denotes the dipole operator, and $\Omega_+, \Omega_-$ the Rabi frequencies for the $\sigma^+$, $\sigma^-$ polarized waves respectively for a transition with a Clebsch-Gordan coefficient of unity. For a large detuning $\Delta$ from the excited levels, the upper states can be adiabatically eliminated, and we arrive at an effective interaction Hamiltonian

$$\hat{H}'_{\text{eff}} = \frac{\hbar^2}{2m} \sum_{\alpha = -6}^{6} \frac{6}{\alpha / 2 \in \mathbb{Z}} \left[ \omega_{AC, \alpha} - \frac{\alpha^2}{2} \right] |g_{\alpha}\rangle \langle g_{\alpha}|$$

$$+ \frac{4}{2} \hbar \Omega_{R, \alpha \alpha + 2} |g_{\alpha}\rangle \langle g_{\alpha+2}| e^{-i 2 \hbar k L x},$$

where $\Omega_{R, \alpha \alpha \pm 2} = c_{\alpha \alpha \pm 1} c_{\alpha \alpha \pm 1} \Omega_{\pm \Omega_{\pm}}/(2 \Delta)$ denote effective two-photon Rabi frequencies between ground state sublevels and $\omega_{AC, \alpha} = (c_{\alpha 000}^2 + c_{\alpha 2}^2 + c_{\alpha -2}^2)/(2 \Delta)$ is the AC Stark shift of the ground state sublevels. As we choose a large detuning $\Delta$, any excited-state shifts $\delta_{\alpha}$ can be neglected. In the basis of eigenstates $|g_{\alpha}, \vec{p} + \alpha \hbar \vec{k}_L\rangle$ with $\alpha = -6, -4, \ldots, 6$, where $\vec{p} = \hbar \vec{k}$, Eq. 3 can be written more explicitly using the matrix form

$$\hat{H}'_{\text{eff}} = \begin{pmatrix}
H_{-6,-6} & \hat{\Omega}_{-6,-4} & 0 & 0 & 0 & 0 & 0 \\
\hat{\Omega}_{-4,-6} & H_{-4,-4} & \hat{\Omega}_{-4,-2} & 0 & 0 & 0 & 0 \\
0 & \hat{\Omega}_{-2,-4} & H_{-2,-2} & \hat{\Omega}_{-2,0} & 0 & 0 & 0 \\
0 & 0 & \hat{\Omega}_{0,-2} & H_{0,0} & \hat{\Omega}_{0,2} & 0 & 0 \\
0 & 0 & 0 & \hat{\Omega}_{2,0} & H_{2,2} & \hat{\Omega}_{2,4} & 0 \\
0 & 0 & 0 & 0 & \hat{\Omega}_{4,2} & H_{4,4} & \hat{\Omega}_{4,6} \\
0 & 0 & 0 & 0 & 0 & \hat{\Omega}_{6,4} & H_{6,6}
\end{pmatrix},$$

where $H_{\alpha, \alpha} = \hbar (\omega_{AC, \alpha} - \alpha \delta / 2) + \hbar^2 ((k_+ + \alpha k_L)^2 + k_0^2) / 2m$ and $\hat{\Omega}_{\alpha, \alpha \pm 2} = \hbar \Omega_{R, \alpha \alpha \pm 2} / 2$. To find the eigenenergies of the multi-level system, we numerically solve the eigen-system (4). Fig. 3(b) shows the energy dispersion curves for $\delta = 0$ of the uncoupled system, and Fig. 3(c) and 3(d) for different values of $\Omega_0 = 8 E_L / \hbar$ and $96 E_L / \hbar$ respectively, where $\Omega_R = \Omega_+ \Omega_-/(2 \Delta)$ denotes the effective two-photon Rabi frequency for Clebsch-Gordan coefficients of unity.. We are interested in the dispersion of the lowest energetic eigenstate. While for the lower value of the two-photon Rabi coupling (Fig. 3(c)) the curve has multiple minima, the plot shown in Fig. 3(d) with $\Omega_R = 96 E_L / \hbar$ depicts a smooth, near parabolic dispersion of the low energy dressed state. More generally, for the $J = 6 \rightarrow J' = 7$ transition we find that for Rabi frequencies $\hbar \Omega_R \gtrsim (m_{F, \text{max}} \hbar k^2)/(2m) = m_{F, \text{max}} E_L$, with $m_{F, \text{max}} = 6$, corresponding to the recoil energy associated with the momentum difference between atoms in an outermost and a central Zeeman sublevel, the dispersion can be approximated as $E(\delta) = E_0 + \hbar (k_+ - k_{\text{max}}(\delta))^2/(2m^*)$ for not too large values of the detuning $\delta$. With the identification $A_0^2(\delta) = \hbar k_{\text{max}}(\delta)/q^*$ and noting that in the presence of the transverse gradient of the real magnetic field $\delta = \delta(y)$, we find that one can describe the atomic dynamics also in the multi-level case by an effective Hamiltonian of the form of Eq. 2b. In addition, a scalar potential emerges. The blue line in Fig. 4(a)
shows the dependence of the generated synthetic vector potential versus $\delta$, which varies smoothly between $-6\hbar k_L/\omega^2$ and $6\hbar k_L/\omega^2$. Fig. 4(b) shows the corresponding synthetic magnetic field for a detuning gradient of $\delta'/2\pi = 21$ kHz/$\mu$m, as obtained with a gradient of the real magnetic field of 70.3 G/cm for the erbium case with $g = 1.166$. Importantly, the synthetic magnetic field is spatially very uniform over a relatively large distance ($\sim 10 \mu$m, see below for further discussion), with additional peaks at the edge. For smaller values of the two-photon Rabi frequency the synthetic magnetic field loses spatial homogeneity, and for values below the multiphoton recoil even becomes spiky (see inset of Fig. 4(b)), as understood from the multiple minima of the dispersion curve in this parameter regime (see e.g. Fig. 3(c)).

The above value for the gradient of the real magnetic field ($\partial B_z/\partial y = 70.3$ G/cm) was chosen to with the noted parameters ($\Omega_R = 96 E_L/h$, $J = 6 \rightarrow J' = 7$ transition) reach a ratio of $\hbar \omega_c/E_L = 1$ (with $E_L/(2\pi h) \simeq 1.68$ kHz in the center, where $\omega_c = eB_z/m^2$ denotes the value of the cyclotron frequency, which following earlier work ([22]) is a desirable parameter regime for the observation of fractional quantum Hall physics in such systems. For the magnetic length we find $l_{\text{mag}} = \sqrt{\hbar/m_c} \approx 0.19 \mu$m, yielding an area of order $A \sim 2\pi l_{\text{mag}}^2$ per flux quantum, or an atomic area density $n \approx 1/(4\pi l_{\text{mag}}^2) \approx 2 \mu$m$^{-2}$ at half filling. The area of spatial homogeneity shown in Fig. 4(b) of $\sim 10 \mu$m diameter in a circular 2D geometry should thus be sufficient to load up to $\sim 200$ atoms into a Laughlin state. The red dashed line in Fig. 4(b) for comparison gives the spatial variation of the synthetic magnetic field for the case of an idealized three-level system (Fig. 1(b)), with parameters as to also obtain $\hbar \omega_c = E_L$ at $y = 0$.

A ratio $\Delta/I = 10^7$ is achieved for the $J = 6 \rightarrow J' = 7$ atomic erbium transition for a detuning $\Delta/(2\pi) \approx 80$ GHz. Both the required Raman beam intensity of $\sim 2.3$ W/mm$^2$ and the described value of the magnetic field gradient are experimentally well achievable. For the quoted parameters we have $\Omega_R|_{0, \pm 2} = c_0|_{\pm 1} \pm \delta \pm 1 \Omega_R = \sqrt{2/13} \cdot 3\sqrt{5}/96 \cdot 96 E_L/h \approx 26 E_L/h$, which is roughly about a factor two above the value investigated for rubidium in [16]. On the other hand, the Clebsch-Gordan coefficients for the $\sigma^+ - \sigma^-$ polarization configuration considered here are more favorable than for the $\sigma^+ - \pi$ case investigated in the rubidium works, so one may expect the ratio of Rabi coupling and spontaneous scattering at comparable detuning for the erbium and rubidium cases to be roughly comparable. As noted above, the lanthanide case is expected to allow for larger values of $\Delta/I$, which should reduce the influence of spontaneous scattering. Given that for a smooth variation of the low energy dispersion curve with a single minimum $\Omega_R$ should be above $\sim m_{\text{F,max}} E_L/h$, from the point of a low spontaneous scattering rare earth atoms with not too high values of $m_{\text{F,max}}$ seem advantageous, although this limits the magnitude of the achievable synthetic magnetic flux.

We have also investigated the use of the $J = 6 \rightarrow J' = 5$ and $J' = 6$ components of the $4f^{12}6s^2(5\text{H}_6) \rightarrow 4f^{11}4d_{5/2}5d_{5/2}6s^2(15/2, 5/2, 5/2)$, erbium transition to implement synthetic magnetic fields. The top panel of Fig. 5 gives dispersion curves for the $J = 6 \rightarrow J' = 5, 6, 7$ transitions ((a) - (c)) for $E_L = h \omega_c$ and $\Omega_R = 96 E_L/h$, and the middle and lower panels the detuning dependence of the derived synthetic vector potential and the spatial variation of the synthetic magnetic field respectively.

| $J' = 5$ | $J' = 7$ |
|----------|----------|
| $\Omega_R/E_L$ | $\delta'/2\pi \partial B_z/\partial y$ | $I$ | $\delta'/2\pi \partial B_z/\partial y$ | $I$ |
| 32 | 9.18 | 28.14 | 0.72 | 10.43 | 31.94 | 0.78 |
| 64 | 15.78 | 48.36 | 1.43 | 14.99 | 45.92 | 1.55 |
| 96 | 24.11 | 73.88 | 2.15 | 20.96 | 70.34 | 2.33 |
While for the $J = 6 \rightarrow J' = 5$ component we, similarly as in the $J = 6 \rightarrow J' = 7$ case, for $\Omega_R = 96E_L/\hbar$ expect to reach a spatially quite uniform synthetic magnetic field and obtain $\hbar\omega_c = E_L$ in the center with comparable parameters for the transverse magnetic field gradient, for the $J = 6 \rightarrow J' = 6$ case the synthetic field essentially reduces to a single spike in the center. This is understood from the for this component less favorable variation of Clebsch-Gordan coefficients with the Zeeman quantum number, with relatively small couplings near the center of the Zeeman diagram ($|m_F| \approx 0$). Thus, the lowest energetic dispersion curve has two, rather than a single, minimum. Tab. I gives a comparison of the required gradients of the real magnetic field to reach a value of the cyclotron frequency $\hbar\omega_c/E_L = 1$ at $y = 0$ for different values of the two-photon Rabi frequency $\Omega_R$ for both the $J = 6 \rightarrow J' = 5$ and $J = 6 \rightarrow J' = 7$ transitions. As described above, for the lower values of $\Omega_R$, while requiring smaller Raman beam intensities and gradients of the real magnetic field, the spatial homogeneity of the synthetic gauge field reduces.

**IV. LAUGHLIN-GAP**

In order to observe the FQHE all atoms subject to the synthetic gauge field have to be in the lowest Landau level (LLL). The following calculations assume a LLL with a filling factor of $\nu = 1/2$ and use theory results from [11]. In the presence of interactions, the true ground state then becomes a highly correlated Laughlin state. To allow for a selective loading by adiabatic mapping from e.g. an initial Bose-Einstein condensate, the energetic gap to the next excited state, the so-called Laughlin gap $\Delta E_{LG}$, should be sufficiently large.

Given experimental limits on the experimentally realizable magnitude of the gauge field, the use of small atom numbers may seem desirable, with disk-shaped microtraps – with the confinement along the axis of the synthetic magnetic field (i.e. the $z$-axis) being sufficiently strong to restrict the atomic dynamics to the two transverse directions (i.e. in the $x$-$y$ plane).

In the case of $N = 4$ atoms per microtrap the Laughlin gap was estimated to $\Delta E_{LG} \approx 0.16\hbar g_{\text{int}}$, where $g_{\text{int}} = \sqrt{2\pi/m}\ell_s$ is the 2D interaction coefficient, $a_s$ is the s-wave scattering length and $\ell_s = \sqrt{\hbar/m\omega_c}$ is the confinement length in $z$-direction, with $\omega_c$ being the corresponding trapping frequency. The disk-shaped configuration can e.g. be realized by the dipole potential induced by a far-detuned one-dimensional standing wave, whose wave-length is denoted as $\omega_{\text{trap}}$ in the following. In this configuration we have $\omega_c/(2\pi) = \sqrt{2U_0/m}/\omega_{\text{trap}}$, where $U_0$ denotes the trap depth. For $\omega_{\text{trap}} = 1.064 \mu$m and a typical trap depth $U_0 = 50E_{L,\text{trap}}$ with $E_{L,\text{trap}} = h^2/(2m\ell_{\text{trap}}^2)$ we arrive at $\ell_s \approx 64 \mu$m and $\omega_c/(2\pi) = 14.8 \text{ kHz}$. For $\hbar\omega_c = E_L$, as we expect to be achievable with the parameters described in section III, we arrive at a Laughlin gap of $\Delta E_{LG} \approx h \cdot 720 \text{ Hz}$ for the case of a Raman beams wavelength tuned to near the $J = 6 \rightarrow J' = 7$ transition and a s-wave scattering length of $a_s = 200a_0$ [28], where $a_0$ is Bohr’s radius. For larger atom number the predicted size of the Laughlin gap slightly reduces, and in the asymptotic case ($N \gg 1$) reaches $\Delta E_{LG} \approx 0.14g_{\text{int}}$, corresponding to $h \cdot 450 \text{ Hz}$ for the above parameters. For the corresponding gap sizes adiabatic loading from a Bose-Einstein condensate seems realistic. Use of large values of the atom number per trap can be experimentally feasible. Here one benefits from the incompressibility of the Laughlin phase, pushing quasi-holes to the outer trap regions. This is a useable configuration when applying spatially resolved detection techniques only monitoring the central trap region.

**V. CONCLUSIONS**

We have investigated the laser-induced synthetization of gauge fields in the atomic erbium lanthanide system. A configuration with two counter-propagating oppositely circular polarized Raman beams was shown to be an attractive approach for both on $J = 6 \rightarrow J' = 5$ and $J' = 7$ narrow-line atomic erbium transitions. In the presence of a transverse gradient of the real magnetic field, strong synthetic magnetic fields with good spatial homogeneity.
are predicted to be possible, with estimated photon scattering rates roughly two orders of magnitude lower than in implementations with alkali atomic systems. We have moreover estimated the size of the expected Laughlin gap for typical experimental parameters, showing that rare earth atomic systems are attractive candidates for experimental investigations of fractional quantum Hall physics.

On the theory side, an important topic for future work remains to derive the form of the ground state in the presence of both the synthetic magnetic field and dipolar interactions [29].

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Appendix

In Tab. II the Clebsch-Gordan coefficients for the here relevant ground state sublevels of the three transitions with $J = 6 \rightarrow J^\prime = 5, 6, 7$ respectively are listed. Here $m_F$ denotes the ground state sublevel from which a transition to $m_F^\prime = m_F + 1$ (with $\Delta m_F = +1$) or $m_F - 1$ (with $\Delta m_F = -1$) originates.

| $m_F$ | $-6$ | $-4$ | $-2$ | $0$ | $2$ | $4$ | $6$ |
|-------|------|------|------|----|----|----|----|
| $J = 6 \rightarrow J^\prime = 5$ | | | | | | | |
| $\Delta m_F = +1$ | $\sqrt{\frac{15}{37}}$ | $\sqrt{\frac{15}{37}}$ | $\sqrt{\frac{15}{37}}$ | $\frac{1}{\sqrt{37}}$ | $\frac{1}{\sqrt{37}}$ | | |
| $\Delta m_F = -1$ | $\frac{1}{\sqrt{37}}$ | $\frac{1}{\sqrt{37}}$ | $\frac{1}{\sqrt{37}}$ | $\sqrt{\frac{15}{37}}$ | $\sqrt{\frac{15}{37}}$ | | |
| $J = 6 \rightarrow J^\prime = 6$ | | | | | | | |
| $\Delta m_F = +1$ | $\frac{1}{\sqrt{13}}$ | $-\sqrt{\frac{15}{13}}$ | $\frac{1}{\sqrt{13}}$ | $\frac{1}{\sqrt{13}}$ | $-\sqrt{\frac{15}{13}}$ | | |
| $\Delta m_F = -1$ | $\frac{1}{\sqrt{13}}$ | $\sqrt{\frac{15}{13}}$ | $\frac{1}{\sqrt{13}}$ | $\frac{1}{\sqrt{13}}$ | $\sqrt{\frac{15}{13}}$ | | |
| $J = 6 \rightarrow J^\prime = 7$ | | | | | | | |
| $\Delta m_F = +1$ | $\frac{1}{\sqrt{13}}$ | $\sqrt{\frac{15}{13}}$ | $\frac{1}{\sqrt{13}}$ | $\frac{1}{\sqrt{13}}$ | $\sqrt{\frac{15}{13}}$ | $\frac{1}{\sqrt{13}}$ | |
| $\Delta m_F = -1$ | $\frac{1}{\sqrt{13}}$ | $\sqrt{\frac{15}{13}}$ | $\frac{1}{\sqrt{13}}$ | $\frac{1}{\sqrt{13}}$ | $\sqrt{\frac{15}{13}}$ | $\frac{1}{\sqrt{13}}$ | |

TABLE II. Relevant Clebsch-Gordan coefficients for the three transitions $J = 6 \rightarrow J^\prime = 5, 6, 7$.

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