Singular gauge potentials and the gluon condensate at zero temperature

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We consider a new cooling procedure which separates gluon degrees of freedom from singular center vortices in SU(2) LGT in a gauge invariant way. Restricted by a cooling scale $\kappa^4/\sigma^2$ fixing the residual SO(3) gluonic action relative to the string tension, the procedure is RG invariant. In the limit $\kappa \to 0$ a pure Z(2) vortex texture is left. This minimal vortex content does not contribute to the string tension. It reproduces, however, the lowest glueball states. With an action density scaling like $a^4$ with $\beta$, it defines a finite contribution to the action density at $T = 0$ in the continuum limit. We propose to interpret this as a mass dimension 4 condensate related to the gluon condensate. Similarly, this vortex texture is revealed in the Landau gauge.

1. COSET COOLING

SU(N) gluodynamics at zero temperature is believed to confine since extended vortex degrees of freedom carrying Z(N) flux are realized in a condensed phase. In LGT one attempts to localize, configuration by configuration, vortices by center projection, mostly starting from the Maximal Center Gauge (MCG) thought to be optimal for that purpose\footnote{Institute for Theoretical Physics, University of Tübingen, Germany}. The result, however, $Z(N)$ flux squeezed into thin $P$-vortices, is irreproducible with respect to the gauge copy to which MCG fixing is applied\footnote{Research Center for Nuclear Physics, Osaka University, Osaka 567-0047, Japan}. In particular, starting from Landau gauge no vortex structure is discovered by subsequent MCG fixing which would allow to understand the area law of Wilson loops\footnote{K. Langfeld}. A gauge independent method of vortex finding is therefore highly desirable. This was the motivation to explore the capability of coset cooling\footnote{Poster presented by E.-M.I.} for this purpose, complementing other methods making use of the lowest modes of some auxiliary adjoint Higgs field (Laplacian Center Gauge\footnote{Poster presented by E.-M.I.}). Here, we will report that - instead of fixing the confining center vortices - the coset cooling method is rather localizing a minimal center vortex content living at the UV scale. It corresponds to singular fields which carry a finite action density in the continuum limit. This disagrees with the belief that, approaching the continuum limit, the lattice degrees of freedom can be reduced to gluon fields $A_\mu$ by expanding $U_{x,\mu} = \exp (i a A_{\mu}^a(x) t^a)$ around the unit element. The same observation, based on a new implementation of Landau gauge fixing at zero and non-zero temperature, is reported by K. Langfeld\footnote{K. Langfeld}.

Coset cooling is relaxation with respect to the adjoint action which is written for a given link $s_{x,\mu}^\text{gluon} = \frac{4}{3} \sum_{\bar{\nu} \in \pm 1} tr(U_{x,\mu} U_{x,\bar{\nu}} U_{x+\bar{\nu},\mu} U_{x+\mu,\bar{\nu}}).$ (1)

This action tolerates Z(2) vortex degrees of freedom on the plaquette scale. The relaxation update consists in replacing $U_{x,\mu}$ by $U_{x,\mu}^\text{cool}$ proportional to

$$\sum_{\bar{\nu} \in \pm 1} tr(U_{x,\mu} U_{x,\bar{\nu}} U_{x+\bar{\nu},\mu} U_{x+\mu,\bar{\nu}}).$$ (2)

We have used restricted coset cooling: when the local gluon action obeys $s_{x,\mu}^\text{gluon} < 8 \kappa^4 a^3$, relaxation of the given link stops. Considering Monte Carlo ensembles generated at various $\beta$ values, the result of this cooling is RG invari-
ant if and only if the cooling scale $\kappa/\sqrt{\sigma}$ is chosen independently of $\beta$. In the limit $\kappa \to 0$, coset cooling leads to a pure $Z(2)$ configuration. Otherwise, at any finite $\kappa$ (monitoring the residual gluon action density), the $Z(2)$ degrees of freedom are defined by usual center projection $U_{x,\mu} \to Z_{x,\mu} = \text{sign } \text{tr} (U_{x,\mu})$.

2. THE STATIC $Q\bar{Q}$ POTENTIAL IN COSET-COOLED LATTICE FIELDS

We consider coset-cooled lattices representing Monte Carlo ensembles for different $\beta$ as a function of the cooling scale. On one hand, for $\kappa/\sqrt{\sigma} \approx 1$ the Wilson action is already strongly concentrated on a subset of plaquettes. On the other hand, at this stage coset-cooled configurations still sustain the full $Q\bar{Q}$ force over distances $R > 0.4$ fm. The short-range Coulomb force due to gluon exchange is already wiped out while extended vortices, which are relevant for the formation of color-electric flux tubes, are still intact.

Thus the proposed cooling does not preserve the non-perturbative force at all distances. Instead, we have found a method to suppress thick confining vortices below some cooling radius. This might become useful in future studies of the confinement mechanism. In the following we will rather concentrate on the singular vortex component ($c$-vortices) disclosed by the cooling technique.

3. THE VORTEX TEXTURE AS A $d = 4$ VACUUM CONDENSATE

Consider the trace of the energy-momentum tensor $\theta_\mu^\mu$ which appears as the dimension four vacuum condensate $O_4$ in the operator product expansion. We found that $O_4 \propto a^4$ scales like $a^4$ with $\beta > 2.2$ for different values of the cooling scale $\kappa/\sqrt{\sigma}$.

Fig. 2 shows for two values of $\kappa/\sqrt{\sigma}$ the dimensionless lattice condensate $O_4(\kappa) \propto a^4 = \frac{24}{\pi^2} \left( 1 - \frac{1}{2} \text{tr} U_p \right)_{\text{cooled with scale } \kappa(3)}$ as a function of $\beta$. Already for finite $\kappa$ we find the condensate $O_4(\kappa)$ as a well defined, RG invariant function of $\kappa$ in the continuum limit. We have tried to describe this condensate as a function of $\kappa$ by a fit of the form $O_4(\kappa)/\sigma^2 = a_0 + a_1 \kappa^4$ with (fit A) and without (fit B) constant $a_0$.

Fig. 3 clearly demonstrates the presence of both $c$-vortex and gluon components. The glu-
onic contribution $\propto \kappa^4$ to the RG invariant condensate starts to dominate for $\kappa > \sqrt{\sigma}$.

The constant $a_0$ specifies the ultimate $Z(2)$ vortex content consistent with $O_4 = \lim_{\kappa \to 0} O_4(\kappa) \approx 0.10 \ldots 0.15$ GeV$^4$ which is in the ballpark of recent $SU(2)$ gluon condensate estimates. After Landau gauge fixing the density $\rho$ of defect links with $Z_{x,\mu} = -1$ has been also found to be RG invariant, $\rho \approx 0.7$ fm$^{-4}$.

4. $c$-VORTEX DOMINANCE OF THE GLUEBALL SPECTRUM

Pure gluodynamics is characterized by a wide gap between the correlation length corresponding to the lowest gauge invariant excitations (glueballs) and the confinement scale of few 10$^2$ MeV. In view of the proposed coset cooling method it is interesting to ask whether removing the coset, i.e. gluon, part distorts the glueball spectrum. This question has been answered by a comparison of the lowest glueball states in uncooled and coset-cooled gauge field configurations reported in Ref. [7]. For $\beta = 2.3, 2.4$ and 2.5 on a $8^3 \times 16$ lattice the lowest $O^+$ glueball state (with mass $m_{O^+} = 1.67(11)$ GeV) and the lowest $2^+$ glueball state ($m_{2^+} = 2.30(8)$ GeV), have been identified first on uncooled configurations. Fig. 4 demonstrates how the renormalized $O^+$ glueball correlator measured on coset-cooled configurations is fitted by the exponential slope obtained from uncooled $SU(2)$ correlator data. Similarly, the $2^+$ glueball mass is reproduced.

Figure 3. $O_4/\sigma^2$ vs. cooling scale $\kappa/\sqrt{\sigma}$

Figure 4. Glueball propagator for coset-cooled configurations compared with the fit describing the glueball for uncooled configurations

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