Simplified Model for Analyzing Shortwave Solar Effects on Indoor Thermal Comfort

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Abstract. Shortwave solar irradiance through building windows may have significant impacts on indoor thermal comfort, especially in near-window zones. Such effects change with intensity and spectral variations of the solar irradiance incident on building windows, which is related to the day of the year, time of day, orientation and dimension of the window, and atmospheric conditions. To assess the effects on thermal comfort, we derived a variable - mean radiant temperature delta based on a proposed spectrally-resolved method to represent the quantity of shortwave solar irradiance incident on occupants and be incorporated into PMV (predicted mean votes)-based thermal comfort models. By characterizing the variations of the calculated PMV values under different solar conditions, the influencing factors to indoor thermal comfort by shortwave solar irradiance were obtained and analyzed. Last, upon a series of parametric settings and numerical analysis, simplified statistical regression models were also established to directly predict spectrally-resolved mean radiant temperature delta and PMV values. This could be convenient and extensively to estimate the solar effects on indoor thermal comfort within the near-window zones.

1. Introduction

Measuring and understanding human thermal comfort under a certain condition is crucial for configuring the heating and cooling system, which controls the indoor thermal environment and also consumes a large fraction of building energy. The predicted mean vote (PMV) model is typically used to predict human thermal comfort by six major parameters, including air temperature and mean radiant temperature (MRT) [1]. MRT measures the effect of longwave radiation on indoor thermal comfort. However, the solar effects in the near-window zones have not been addressed well in the current PMV model and thermal comfort estimation methods. The shortwave solar effect comes from the transmitted solar irradiance through building windows and is then absorbed by the human skin. To incorporate the shortwave effect in the PMV model, a transfer quantity from the shortwave to the longwave effect (MRT delta) has to be obtained and added to MRT [2]. As the solar irradiance, the transmittance of windows, and the skin absorptance are all spectral-dependent parameters, simple parameters representing the full spectrum of radiometric quantities in the conventional estimation method may not be accurate, especially when the wavelength-dominant glazing systems are involved [3]. Therefore, a spectrally-resolved method has been developed and demonstrated to be necessary for calculating the MRT delta [4, 5].

The computation of the MRT delta can be divided into six nodes, as shown in Figure 1. The first one is the intensity and spectrum of the direct, diffuse, and reflected solar irradiance, which is related to...
the solar position and atmospheric conditions. The second is the penetration of the incident solar irradiance through the window horizontally and vertically [6], which is confined by the dimension and orientation of the window, the position of the occupant, and the sun. The third is the spectral transmittance of the window system. The fourth is the effective body surface exposed to solar radiation, which is related to the posture of the occupant. The fifth is the achievable proportion of the effective body surface, which is 0.5 for diffuse and reflected irradiance. Also, direct irradiance is related to the relation of human orientations and solar positions. The sixth node is the spectral absorptance of human skins. Because each node is related to various factors and also has strong interdependencies, the synergic effect in terms of spectral variation of the solar irradiance, the window optical properties, and the skin make the calculation quite complicated. Thus, simplifying calculation and predicting MRT delta with an easier method is necessary, especially when a rapid estimation is needed for the near-window zones. As there are too many influencing factors, this paper only presented the analysis of the roles of the day of the year (DOY), the time of day (TOD), the orientation of windows, and the atmospheric conditions. Specifically, atmospheric conditions in this work mainly referred to aerosol optical depth (AOD, ranges from zero to one) and precipitation levels, which are also the important atmospheric factors determining the variation of the solar spectra. With the variation of DOY and TOD, solar position (e.g. Zenith angle) varies. Thus, the penetration of the solar irradiance into indoor varies for different orientations of windows. The penetration of the solar irradiance is evaluated in the vertical direction by the fraction of the body exposed to the sun ($f_{bes}$) and horizontal direction by the indicator of direct solar penetration from the horizontal direction ($f_{toggle}$). For penetration evaluation from the horizontal direction, we only concern if direct solar irradiance could enter indoor, and thus $f_{toggle}$ is a binary variable which equals one if direct solar irradiance could penetrate evaluated from the horizontal direction. $f_{toggle}$ is obtained by comparing accessible angle constrained by window dimension and occupant position and the solar horizontal angle relative to the front of a person (SHARP) [5].

![Image](image.png)

**Figure 1.** Six nodes of solar transfer.

For exploring the influencing factors, the MRT delta and PMV values under various simulated conditions with different combinations were calculated upon the spectrally-resolved method. To simulate the solar spectra incident on the vertical building window, SPCTRL2 software provided by the National Renewable Energy Laboratory (NREL) was adopted [7]. SPCTRL2 provides direct and diffuse irradiance from 305 nm to 4000 nm and supports user inputs of DOY, TOD, atmospheric conditions, and orientation of the targeted surface (i.e., windows in this work). In this paper, different values of DOY, TOD, AOD, precipitation, and orientation of the window were used to simulate solar
spectra via SPCTRL. Subsequently, combined with the hypothesized window optical properties and skin’s spectral absorptance features, we calculated the MRT delta. Last, the PMV values were calculated based on the calculated MRT delta values and different indoor operative temperatures. Statistical analysis was also performed based on the datasets obtained from the above solar simulation and numerical analysis, and a simplified model for predicting the PMV values driven by the shortwave solar effects in near-window zones was obtained.

2. Methods

MRT delta values were simulated under different design conditions: four DOYs × average 6.5 TODs × four orientations × two precipitations × two AODs, thus forming 416 combination conditions. We chose the DOYs of 70, 160, 228, and 365. During each DOY, we also selected one TOD every two hours during sunshine hours. Specifically, during the DOYs 70, 228, and 365, the TODs of 8, 10, 12, 14, 16, 18 were selected, while in the DOY 160, the TODs of 6, 8, 10, 12, 14, 16, 18, and 20 were chosen. In each TOD during each DOY, four window orientations, facing north, east, south, and west, were considered. In addition, the lowest and highest values of the precipitation and AOD in 36 days (which were selected by choosing three days in each month in a year) were also explored. In detail, the selected values for AOD are 0.0269 and 0.9759, and the selected values for the precipitation are 0.4664 and 3.7188. Furthermore, with the variations of each condition, solar zenith angle, solar azimuth angle, air mass, \( f_{hes} \), SHARP, and \( f_{togg} \) were recorded. These parameters are not only dependent variables changed with variations of the design conditions but also crucial intermediate variables in the calculation of the MRT delta values.

Furthermore, some other influencing variables were pre-assumed or kept constant in the current numerical analysis. In particular, the location for the analysis was State College, Pennsylvania (40.8° N, 77.9° W). The turbidity was set to be 1.14, indicating the constant sky condition. The albedo of the outdoor ground surface was assumed at 0.2. Also, the occupant was hypothesized to face the window and sat in the middle of a window with a distance of 0.5 m. Meanwhile, the window size was 2.8 m by 2.8 m with no shading system and sill height of 0.5 m, and the indoor floor reflectance was set at 0.5. Upon these hypothesized settings, several characteristics related to the user and environment could be obtained. First, under the assumption that the occupant-faced window and was located in the middle of the window, the accessible angle constrained by window dimension and occupant position was 70°, and thus \( f_{togg} \) was equal to zero when SHARP is larger than 70°, otherwise \( f_{togg} \) would be one [5]. Second, as the occupant was seated, the fraction of the human body surface exposed to the surrounding environment was 0.696 [8]. Third, the fraction of the sky vault exposed to the occupant was 0.61, which was mainly determined by the window dimension. Lastly, in this simulation, a single-pane window was used with an average solar transmittance of 0.549, and white skin was also assumed with an average skin absorptance of 0.570. This human skin absorptance was calculated upon the reflectance of human skin.

Regarding the PMV calculation, we took operative temperature, MRT delta, humidity, airflow speed, metabolic rate, and clothing insulation into account. Among them, four were set as constant across the calculation: 50% relative humidity, 0.1 m/s air speed, 1 met metabolic rate, and 0.6 clo clothing insulation. As for operative temperature, which is the average of air temperature and MRT in this work [9], 18°C to 27°C were used corresponding to each calculated MRT delta.

3. Results and discussion

3.1. Variations of MRT delta

The MRT delta values in each window orientation at different TODs across DOYs are shown in Figure 2. Firstly, the variations are different across window orientations. The MRT delta for the north-facing windows is relatively smaller compared with the other three orientations due to the generally low solar irradiance. Similar trends could be found in the other three orientations. The more direct solar irradiance is, the higher the MRT delta appears. In the MRT delta calculations, such relationships are reflected in
a parameter $f_{toggle}$ that is derived from the window orientation and solar azimuth angle. Its variations are depicted in Figure 2 as well. The transparent points in these curves are conditions with $f_{toggle}$ of zero. We could clearly find out from this figure that the conditions with small MRT delta values are typically those with zero $f_{toggle}$ values. Thus, $f_{toggle}$ would be used as an important predictor to determine the MRT delta.

Second, during a day, the MRT delta values increase at first and then decrease, and the variations change with DOY. This variation is related to the zenith angle because the zenith angle decreases at first and then increases during a day and the variation is related to DOY, which is similar to the trend of MRT delta values with different changing directions. For intermediate variables, $f_{bes}$ is negatively proportional to the zenith angle (correlation is -0.96), and air mass is calculated from the zenith angle. As the relationship between zenith angle and air mass is not linear, variable transformation or polynomial terms of zenith angle could be reasonable components in the prediction.

For the precipitation and AOD, as the inconvenience of achieving the exact values of the two variables, these two variables would be transformed to binary variables in the prediction. Thus, we only need to judge if the variables are extreme.

![Figure 2. MRT delta of each window orientation at different TODs across DOYs. The value shown for each point is the solar zenith angle.](image)

3.2. Model for MRT delta prediction

Based on the analysis above and our previous analysis [5], significant variables include solar zenith angle, $f_{toggle}$ and AOD. A model was fitted by multiple linear regression for predicting the MRT delta, as shown in equation (1). The R-squared value for the model is 0.9620, and the mean squared error (MSE) is 1.6463.

$$MRT\ deltaa = -2.2506 + 0.2190\text{Zenith} - 0.0024\text{Zenith}^2 + 33.1966f_{\text{toggle}}$$
$$+ 1.5873I(AOD) - 0.2847\text{Zenith} \times f_{\text{toggle}} - 9.2838f_{\text{toggle}} \times I(AOD)$$

As $f_{toggle}$ is determined by SHARP, the calculation of SHARP based on solar azimuth angle and orientation of the window is shown in Table 1. The zenith angle is accessible online with latitude, longitude, DOY, and TOD as the input. As mentioned above, AOD is the only significant predictor related to atmospheric variations for MRT delta under atmospheric variations. $I(AOD)$ is equal to one.
when AOD has bigger values while zero otherwise. Typically, AOD has bigger values due to the hazy atmosphere caused by dust, smoke, pollution, and so on.

### Table 1. SHARP calculation.

| Orientation | Azimuth angle (°) | SHARP (°) |
|-------------|------------------|-----------|
| North       | 0-180            | =Azimuth angle |
|             | 180-360          | =360- Azimuth angle |
|             | 0-90             | =90- Azimuth angle |
| East        | 90-270           | = Azimuth angle-90 |
|             | 270-360          | =450- Azimuth angle |
| South       | 0-180            | =180- Azimuth angle |
|             | 180-360          | =Azimuth angle-180 |
|             | 0-90             | =90+ Azimuth angle |
| West        | 90-270           | =270- Azimuth angle |
|             | 270-360          | =Azimuth angle-270 |

### 3.3. Model for PMV prediction

PMV values are related to the MRT values and operative temperatures in our simulation. Thus, a model predicting PMV values based on MRT values and operative temperatures was also fitted. Among 416 calculated MRT delta values from 416 conditions, 41 values were chosen to fit the model. The calculated PMV values at operative temperatures from 18°C to 27°C by 41 conditions (41 MRT delta values) are shown in Figure 3. At each condition, PMV is increasing with the operative temperature, and the PMV and operative temperature follow a linear relationship with an R-squared value of 1 within each condition. However, the slopes and intercepts of the fitted lines were not the same. The differences of the intercepts are related to the conditions, which have different MRT delta values. Meanwhile, the differences in the slopes indicate that the operative temperatures and conditions have interactions. Thus, to fit a common formula predicting the PMV values, MRT delta and the interaction of operative temperature and MRT delta are needed in addition to the operative temperature. The fitted formula is shown in equation (2) with an R-squared value of 0.9997 and an MSE of 0.0505. The fitted lines under each condition by the common formula are also plotted in Figure 3 (black lines) for comparisons.

\[
PMV = -9.9180 + 0.1254MRT \text{ delta} + 0.3791T_o + 0.0026MRT \text{ delta} \times T_o
\]  

(2)

**Figure 3.** PMV values under operative temperatures from 18 °C to 27 °C, and fitted lines for PMV values.
4. Conclusions
Analyzing shortwave solar effects on indoor thermal comforts involves multiple influencing factors and thus has a complex process, especially using the spectrally-resolved method. To simplify the calculation, a linear regression model with an R-squared value of 0.9620 was proposed to predict the MRT delta with the changes of DOY, TOD, window orientation, and atmosphere. The predictors in this model are zenith angle, $f_{toggie}$, and binary AOD. Among them, $f_{toggie}$ is determined by SHARP that could be simply calculated according to the orientation of the window and the solar azimuth angle as provided in Table 1. Furthermore, a simplified model with an R-squared value of 0.9997 for predicting the PMV values upon the MRT delta and operative temperature change was also established. By these two simplified regression models, the shortwave solar effects on indoor thermal comforts in the near-window zone could be easily estimated when the DOY, TOD, window orientation, and atmospheric conditions are given. The simplified calculation would be helpful for the situations that rapid estimations are needed. As multiple factors influencing incident solar irradiance other than DOY, TOD, window orientation, and atmosphere were assumed with constant values and constrained with specific settings, the models fitted in this paper are only applied to limited cases. More factors would be explored in models in our future work.

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