Cross Sections
for Inelastic 2-to-2 Meson-Meson Scattering

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Abstract

With quark-antiquark annihilation and creation in the first Born approximation, we study the reactions: $K\bar{K} \rightarrow K\bar{K}^*$, $K\bar{K} \rightarrow K^*\bar{K}$, $\pi K \rightarrow \pi K^*$, $\pi K \rightarrow \rho K$, $\pi \pi \rightarrow K\bar{K}^*$, $\pi \pi \rightarrow K^*\bar{K}$, $\pi \rho \rightarrow K\bar{K}$, $\pi \rho \rightarrow K^*\bar{K}$, $\rho \rho \rightarrow K\bar{K}^*$, $K\bar{K}^* \rightarrow \rho \rho$, and $K^*\bar{K} \rightarrow \rho \rho$. Unpolarized cross sections for the reactions are obtained from transition amplitudes that are composed of mesonic quark-antiquark relative-motion wave functions and the transition potential for quark-antiquark annihilation and creation. From a quark-antiquark potential that is equivalent to the transition potential, we prove that the total spin of the two final mesons may not equal the total spin of the two initial mesons. Based on flavor matrix elements, cross sections for some isospin channels of reactions can be obtained from the other isospin channels of reactions. Remarkable temperature dependence of the cross sections is found.

Keywords: Inelastic meson-meson scattering, Quark-antiquark annihilation, Quark potential model.

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I. INTRODUCTION

In hadronic matter that is created in ultrarelativistic heavy-ion collisions, various kinds of meson-meson scattering take place. The meson-meson scattering can be studied in quark degrees of freedom or meson degrees of freedom. Elastic $\pi\pi$ scattering for $I = 2$ and
elastic πK scattering for \( I = 3/2 \) have been studied in the quark interchange mechanism in the first Born approximation \[1\] and in nonperturbative schemes together with chiral perturbation theory \[2,5\]. The reactions \( \pi\pi \to \rho\rho \) for \( I = 2 \), \( KK \to K^*K^* \) for \( I = 1 \), \( \pi K \to \rho K^* \) for \( I = 3/2 \), and so on have also been studied in the quark interchange mechanism \[6,7\]. The reactions \( \pi\pi \to K\bar{K}, \rho\rho \to K\bar{K}, \pi\rho \to K\bar{K}, \) and \( \pi\rho \to K^*\bar{K} \) can be studied by quark-antiquark annihilation and creation in the first Born approximation \[8\] or through one-meson exchange in effective meson Lagrangians \[9,10\]. Furthermore, the four isospin channels, \( \pi K \to \rho K^* \) for \( I = 1/2 \), \( \pi K^* \to \rho K \) for \( I = 1/2 \), \( \pi K^* \to \rho K^* \) for \( I = 1/2 \), and \( \rho K \to \rho K^* \) for \( I = 1/2 \), are studied in the assumption that quark interchange as well as quark-antiquark annihilation and creation are dominant mechanisms \[11\]. These studies have revealed interesting features of these reactions. For example, the cross section for the inelastic meson-meson scattering governed by quark interchange increases very rapidly to a maximum value and then decreases rapidly while the center-of-mass energy of the two colliding mesons increases from threshold; however, the cross section for the inelastic meson-meson scattering governed by quark-antiquark annihilation and creation may decrease very slowly from the maximum value.

In the present work we are interested in the inelastic meson-meson scattering among \( \pi, \rho, K, \) and \( K^* \) mesons, which is assumed to be dominated by quark-antiquark annihilation and creation. The meson-meson reactions studied in Ref. \[8\] include \( \pi\pi \to \rho\rho, K\bar{K} \to K^*\bar{K}^*, K\bar{K}^* \to K^*\bar{K}^*, K^*\bar{K} \to K^*\bar{K}^*, \pi\pi \to K\bar{K}, \pi\rho \to K\bar{K}^*, \pi\rho \to K^*\bar{K}, \) and \( K\bar{K} \to \rho\rho \). But these reactions do not exhaust all the 2-to-2 meson-meson reactions among \( \pi, \rho, K, \) and \( K^* \) mesons. Hence, in the present work we study these reactions: \( KK \to K\bar{K}^*, K\bar{K} \to K^*\bar{K}, K\bar{K} \to K^*\bar{K}, K\bar{K} \to K^*\bar{K}, K\bar{K} \to K^*\bar{K}, K\bar{K} \to K^*\bar{K}, K\bar{K} \to K^*\bar{K}, K\bar{K} \to K^*\bar{K}, \rho\rho \to K^*\bar{K}, \rho\rho \to K^*\bar{K}, \rho\rho \to K^*\bar{K}, \rho\rho \to K^*\bar{K}^*, K\bar{K} \to \rho\rho, \) and \( K^*\bar{K} \to \rho\rho \). These reactions have not been studied elsewhere, and complement those reactions studied in Ref. \[8\].

This paper is organized as follows. In Sec. II we present formulas of unpolarized cross sections for 2-to-2 meson-meson reactions that are governed by annihilation of a quark-antiquark pair and creation of another quark-antiquark pair. In Sec. III we calculate
transition amplitudes with mesonic quark-antiquark wave functions and the transition potential for quark-antiquark annihilation and creation. In Sec. IV we show unpolarized cross sections for inelastic meson-meson scattering, and give relevant discussions. In Sec. V we summarize the present work.

II. CROSS-SECTION FORMULAS

It is shown by the two Feynman diagrams in Fig. 1 that the reaction $A + B \rightarrow C + D$ is caused by quark-antiquark annihilation and creation in the Born approximation. In the left diagram of Fig. 1 the quark of meson $A$ and the antiquark of meson $B$ annihilate into a gluon, this gluon creates a new quark-antiquark pair, and the new quark and the antiquark in meson $A$ combine into meson $C$ as well as the new antiquark and the quark in meson $B$ combine into meson $D$. In the right diagram of Fig. 1 the antiquark of meson $A$ and the quark of meson $B$ annihilate into a gluon, the gluon creates a new quark-antiquark pair, and the new antiquark and the quark of meson $A$ form meson $C$ as well as the new quark and the antiquark of meson $B$ form meson $D$. These are the processes that we consider in the present work.

Let $E_A (E_B, E_C, E_D)$ and $J_{Az} (J_{Bz}, J_{Cz}, J_{Dz})$ denote the energy and the magnetic projection quantum number of the angular momentum $J_A (J_B, J_C, J_D)$ of meson $A (B, C, D)$, respectively. From the four-momenta of mesons $A$ and $B$, $P_A$ and $P_B$, the Mandelstam variable $s = (P_A + P_B)^2$ is defined. The unpolarized cross section for $A+B \rightarrow C + D$ depends on $\sqrt{s}$ and temperature $T$. Let $\theta$ be the angle between $\vec{P}$ and $\vec{P}'$ which are the three-dimensional momenta of mesons $A$ and $C$ in the center-of-mass frame, respectively. The unpolarized cross section is 

$$\sigma^{\text{unpol}}(\sqrt{s}, T) = \frac{1}{(2J_A + 1)(2J_B + 1)} \frac{1}{32\pi s} \frac{|\vec{P}'(\sqrt{s})|}{|\vec{P}(\sqrt{s})|} \times \int_0^\pi d\theta \sum_{J_{Az}, J_{Bz}, J_{Cz}, J_{Dz}} |M_{aq_1q_2} + M_{aq_1q_2}|^2 \sin \theta,$$

where $M_{aq_1q_2}$ and $M_{aq_1q_2}$ are the transition amplitudes corresponding to the left diagram.
and the right diagram in Fig. 1, respectively. The transition amplitudes are given by

\[ M_{aq_1\bar{q}_2} = \sqrt{2E_A2E_B2E_C2E_D} \int \frac{d^3p_{q_1\bar{q}_1}}{(2\pi)^3} \frac{d^3p_{q_2\bar{q}_2}}{(2\pi)^3} \times \psi^+(\bar{p}_{q_3\bar{q}_4}) \psi^+(\bar{p}_{q_2\bar{q}_4}) V_{aq_1\bar{q}_2}(\vec{k}) \psi_{q_1\bar{q}_1}(\vec{p}_{q_1\bar{q}_1}) \psi_{q_2\bar{q}_2}(\vec{p}_{q_2\bar{q}_2}), \]

(2)

\[ M_{a\bar{q}_1q_2} = \sqrt{2E_A2E_B2E_C2E_D} \int \frac{d^3p_{q_1\bar{q}_1}}{(2\pi)^3} \frac{d^3p_{q_2\bar{q}_2}}{(2\pi)^3} \times \psi^+(\bar{p}_{q_3\bar{q}_4}) \psi^+(\bar{p}_{q_2\bar{q}_4}) V_{a\bar{q}_1q_2}(\vec{k}) \psi_{q_1\bar{q}_1}(\vec{p}_{q_1\bar{q}_1}) \psi_{q_2\bar{q}_2}(\vec{p}_{q_2\bar{q}_2}), \]

(3)

where \( \vec{k} \) is the three-dimensional momentum of gluon; \( V_{aq_1\bar{q}_2} \) and \( V_{a\bar{q}_1q_2} \) are the transition potentials for \( q_1 + \bar{q}_2 \rightarrow q_3 + \bar{q}_4 \) in the left diagram and \( \bar{q}_1 + q_2 \rightarrow q_3 + \bar{q}_4 \) in the right diagram of Fig. 1, respectively; \( \psi_{ab} \) and \( \vec{p}_{ab} \) are the wave function and the relative momentum of constituents \( a \) and \( b \), respectively.

III. TRANSITION AMPLITUDE

In order to calculate the transition amplitudes, we need the mesonic quark-antiquark wave functions and the transition potential for quark-antiquark annihilation and creation. The quark-antiquark annihilation and creation is shown in Fig. 2 with \( q(p_1) + \bar{q}(-p_2) \rightarrow q'(p_3) + \bar{q}'(-p_4) \), where \( p_1 \) and \( p_3 \) (\( p_2 \) and \( p_4 \)) are the four-momenta of quarks (antiquarks). The transition potential shown below is given in Ref. [8],

\[ V_{aq\bar{q}}(\vec{k}) = \frac{g_s^2}{k^2} \frac{\vec{\lambda}(34)}{2} \cdot \frac{\vec{\lambda}(21)}{2} \left( \frac{\vec{\sigma}(34) \cdot \vec{k} \vec{\sigma}(21) \cdot \vec{k}}{4m_q' m_q} - \vec{\sigma}(34) \cdot \vec{\sigma}(21) \right) \]

\[ - \frac{\vec{\sigma}(21) \cdot \vec{p}_2 \vec{\sigma}(34) \cdot \vec{\sigma}(21) \vec{\sigma}(21) \cdot \vec{p}_1}{4m_q^2} - \frac{\vec{\sigma}(34) \cdot \vec{p}_2 \vec{\sigma}(34) \cdot \vec{\sigma}(21) \vec{\sigma}(34) \cdot \vec{p}_4}{4m_q^2}, \]

(4)

where \( g_s \) is the gauge coupling constant, \( m_q \) (\( m_{q'} \)) is the mass of the initial (final) quark, \( \vec{\lambda} \) are the Gell-Mann matrices, and \( \vec{\sigma} \) are the Pauli matrices. \( \vec{\lambda}(21) \) (\( \vec{\sigma}(21) \)) mean that they have matrix elements between the color (spin) wave functions of the initial quark and the initial antiquark, and \( \vec{\lambda}(34) \) (\( \vec{\sigma}(34) \)) mean that they have matrix elements between the color (spin) wave functions of the final quark and the final antiquark.

The wave functions of mesons \( A, B, C, \) and \( D \) are individually given by

\[ \psi_A = \phi_A r e l \phi_A c o l o r \phi_A f l a v o r \chi_{S_A S_{A'}} \]

(5)
\[ \psi_B = \phi_{B\text{rel}}\phi_{B\text{color}}\phi_{B\text{flavor}}\chi_{S_B S_{Bz}}, \tag{6} \]
\[ \psi_C = \phi_{C\text{rel}}\phi_{C\text{color}}\phi_{C\text{flavor}}\chi_{S_C S_{Cz}}, \tag{7} \]
\[ \psi_D = \phi_{D\text{rel}}\phi_{D\text{color}}\phi_{D\text{flavor}}\chi_{S_D S_{Dz}}, \tag{8} \]

where \( S_i \) is the spin of meson \( i \), and \( S_{iz} \) is its magnetic projection quantum number.

According to the two diagrams in Fig. 1, \( \psi_A = \psi_{q_1\bar{q}_1}, \psi_B = \psi_{q_2\bar{q}_2}, \psi_C = \psi_{q_3\bar{q}_1} = \psi_{q_1\bar{q}_4}, \) and \( \psi_D = \psi_{q_2\bar{q}_4} = \psi_{q_3\bar{q}_2} \). The wave function of meson \( i \) is made up of the quark-antiquark relative-motion wave function \( \phi_{\text{rel}} \), the color wave function \( \phi_{\text{color}} \), the flavor wave function \( \phi_{\text{flavor}} \), and the spin wave function \( \chi_{S_i S_{iz}} \).

The transition amplitudes contain color, spin, and flavor matrix elements. The color matrix elements related to the Gell-Mann matrices are 4/9 for the two diagrams in Fig. 1. The spin matrix elements related to the Pauli matrices have been provided in Ref. [8]. Let \( M_{a q_1 \bar{q}_2 f} \) and \( M_{a q_3 \bar{q}_4 f} \) represent the flavor matrix elements that correspond to the left and right diagrams in Fig. 1, respectively. The values of the flavor matrix elements are listed in Table I where \( I \) is the total spin of the two initial or final mesons for the reactions:

\[ \begin{align*}
K\bar{K} \to K\bar{K}^*, & \quad K\bar{K} \to K^*\bar{K}, & \quad \pi K \to \pi K^*, & \quad \pi K \to \rho K, & \quad \pi\pi \to K\bar{K}^*, & \quad \pi\pi \to K^*\bar{K}, \\
\pi\pi \to K^*\bar{K}^*, & \quad \pi\rho \to K\bar{K}, & \quad \pi\rho \to K^*\bar{K}^*, & \quad \rho\rho \to K^*\bar{K}^*, & \quad K\bar{K}^* \to \rho\rho, & \quad K^*\bar{K} \to \rho\rho.
\end{align*} \]

The quark-antiquark relative-motion wave functions are given by the Schrödinger equation with a temperature-dependent potential. The experimental masses of ground-state mesons [12] are reproduced by the Schrödinger equation, while the up and down quark masses are 0.32 GeV and the strange quark mass is 0.5 GeV [13]. The temperature-dependent potential between constituents \( a \) and \( b \) is given by [13]

\[ V_{ab}(\vec{r}) = V_{si}(\vec{r}) + V_{ss}(\vec{r}), \tag{9} \]

where \( \vec{r} \) is the relative coordinate of \( a \) and \( b \). The first term \( V_{si}(\vec{r}) \) is the central spin-independent potential and depends on temperature:

\[ V_{si}(\vec{r}) = -\frac{\lambda_a}{2} \cdot \frac{\lambda_b}{2} \cdot \frac{3}{4} D \left[ 1.3 - \left( \frac{T}{T_c} \right)^4 \right] \tanh(Ar) + \frac{\lambda_a}{2} \cdot \frac{\lambda_b}{2} \cdot \frac{6\pi v(\lambda r)}{25} \exp(-Er), \tag{10} \]

where \( \lambda_a \) and \( \lambda_b \) are the quark masses, \( D \) is a dimensionless constant, \( T_c \) is the critical temperature, \( A \) is a positive constant, \( v(\lambda r) \) is the volume function, and \( \lambda r \) is a measure of the distance between the constituents.
where \( D = 0.7 \text{ GeV}, T_c = 0.175 \text{ GeV}, A = 1.5[0.75 + 0.25(T/T_c)^{10}]^{16} \text{ GeV}, E = 0.6 \text{ GeV}, \lambda = \sqrt{25/16 \pi^2 \alpha'} \) with \( \alpha' = 1.04 \text{ GeV}^{-2} \), \( \vec{\lambda}_a (\vec{\lambda}_b) \) are the Gell-Mann matrices for the color generators of constituent \( a \) (\( b \)), and the dimensionless function \( v(x) \) is given by Buchmüller and Tye in Ref. [14].

This potential \( V_{ai}(\vec{r}) \) is relevant to the temperature of hadronic matter and the distance \( r \). It shows some characteristics as follows. At very short distances \( r < 0.01 \text{ fm} \), the potential arises from one-gluon exchange plus perturbative one- and two-loop corrections. At large distances and blow the QCD phase-transition temperature \( T_c \), the color screening produced by high-temperature medium may be strong. Karsch et al. [15] have provided such a numerical quark-antiquark potential at \( r \geq 0.3 \text{ fm} \) in a temperature region from lattice QCD calculations. When the distance between the quark and the antiquark becomes large, the quark-antiquark potential at a given temperature becomes a constant value, which decreases with increasing temperature.

The second term \( V_{ss}(\vec{r}) \) in Eq. (9) is the spin-spin interaction which originates from one-gluon exchange plus perturbative one- and two-loop corrections [16], depends on constituent masses, and includes relativistic effects [17]:

\[
V_{ss}(\vec{r}) = -\frac{\vec{\lambda}_a}{2} \cdots \frac{\vec{\lambda}_b}{2} \frac{16 \pi^2}{25} \frac{d^3}{\pi^{3/2}} \exp\left(-d^2 r^2\right) \frac{\vec{s}_a \cdot \vec{s}_b}{m_a m_b} + \frac{\vec{\lambda}_a}{2} \cdots \frac{\vec{\lambda}_b}{2} \frac{4 \pi}{25} \frac{1}{r} \frac{d^2 v(\lambda r)}{dr^2} \frac{\vec{s}_a \cdot \vec{s}_b}{m_a m_b},
\]

(11)

where \( m_a \) (\( m_b \)) and \( \vec{s}_a \) (\( \vec{s}_b \)) are the mass and the spin of constituent \( a \) (\( b \)), respectively, and \( d \) is given by

\[
d^2 = d_1^2 \left[ \frac{1}{2} + \frac{1}{2} \left( \frac{4 m_a m_b}{(m_a + m_b)^2} \right)^4 \right] + d_2^2 \left( \frac{2 m_a m_b}{m_a + m_b} \right)^2,
\]

(12)

where \( d_1 = 0.15 \text{ GeV} \) and \( d_2 = 0.705 \).

### IV. NUMERICAL CROSS SECTIONS AND DISCUSSIONS

We consider the following inelastic meson-meson scattering processes that mainly take the two Feynman diagrams in Fig. 1:

\[
K\bar{K} \rightarrow K\bar{K}^*, \quad K\bar{K} \rightarrow K^*\bar{K}, \quad \pi K \rightarrow \pi K^*, \quad \pi K \rightarrow \rho K,
\]
\( \pi\pi \to K\bar{K}^*, \pi\pi \to K^*\bar{K}, \pi\pi \to K^*\bar{K}^* \),

\( \pi\rho \to K\bar{K}, \pi\rho \to K^*\bar{K}^* \),

\( \rho\rho \to K^*\bar{K}^*, K\bar{K}^* \to \rho\rho, K^*\bar{K} \to \rho\rho. \)

The total spin of \( \pi \) and \( K \) mesons does not equal the total spin of \( \pi \) and \( K^* \) mesons or of \( \rho \) and \( K \) mesons, i.e., the total spin in either \( \pi K \to \pi K^* \) or \( \pi K \to \rho K \) is not conserved. Quark interchange thus does not happen in the two reactions. \( \mathcal{M}_{a\bar{q}1\bar{q}2} \) and \( \mathcal{M}_{a\bar{q}12f} \) are proportional to the flavor matrix elements. If the transition amplitudes equal zero, the unpolarized cross section given in Eq. (1) is zero. As seen in Table 1, the flavor matrix elements for the two reactions for \( I = 3/2 \) are zero. Quark-antiquark annihilation and creation does not happen in the two reactions for \( I = 3/2 \) too. Therefore, cross sections for \( \pi K \to \pi K^* \) for \( I = 3/2 \) and \( \pi K \to \rho K \) for \( I = 3/2 \) are zero in the present work, but we still investigate \( \pi K \to \pi K^* \) for \( I = 1/2 \) and \( \pi K \to \rho K \) for \( I = 1/2 \) of which \( \mathcal{M}_{a\bar{q}12f} \) are not zero. The other reactions must involve quark-antiquark annihilation and creation, but do not involve quark interchange.

It is shown in Table 1 that only the right diagram in Fig. 1 contributes to the reactions:

\( \pi\pi \to K\bar{K}^*, \pi\pi \to K^*\bar{K}, \pi\pi \to K^*\bar{K}^* \), \( \pi\rho \to K\bar{K}, \pi\rho \to K^*\bar{K}^* \), \( K\bar{K}^* \to \rho\rho \), and \( K^*\bar{K} \to \rho\rho \). Since the flavor matrix elements for the reactions for \( I = 0 \) are \( \sqrt{6}/2 \) times the ones for \( I = 1 \), the cross sections for the reactions for \( I = 0 \) are 1.5 times the cross sections for \( I = 1 \). The cross section for \( K\bar{K} \to K^*\bar{K} \) (\( \pi\pi \to K^*\bar{K}, K^*\bar{K} \to \rho\rho \)) equals the one for \( K\bar{K} \to K\bar{K}^* \) (\( \pi\pi \to K\bar{K}^*, K\bar{K}^* \to \rho\rho \)). We thus do not plot the cross sections for \( K\bar{K} \to K^*\bar{K}, \pi\pi \to K^*\bar{K} \), and \( K^*\bar{K} \to \rho\rho \).

The gauge coupling constant is \( 2\sqrt{\frac{\alpha_s}{3}} \) for quark-antiquark annihilation and creation \[11, 14\]. According to Eq. (1), we calculate unpolarized cross sections at the six temperatures \( T/T_c = 0, 0.65, 0.75, 0.85, 0.9, \) and 0.95. In Figs. 8-12 we plot the unpolarized cross sections for the following ten channels:

\( I = 1 \) \( K\bar{K} \to K\bar{K}^* \), \( I = 0 \) \( K\bar{K} \to K\bar{K}^* \),

\( I = 1/2 \) \( \pi K \to \pi K^* \), \( I = 1/2 \) \( \pi K \to \rho K \),

\( I = 1 \) \( \pi\pi \to K\bar{K}^* \), \( I = 1 \) \( \pi\pi \to K^*\bar{K}^* \).
The last channel is endothermic at $T/T_c = 0$ and exothermic at $T/T_c = 0.65$, $0.75$, $0.85$, $0.9$, and $0.95$. The other nine channels are endothermic. The numerical cross sections for endothermic reactions are parametrized as

$$
\sigma_{\text{unpol}}(\sqrt{s}, T) = a_1 \left( \frac{\sqrt{s} - \sqrt{s_0}}{b_1} \right)^{e_1} \exp \left[ e_1 \left( 1 - \frac{\sqrt{s} - \sqrt{s_0}}{b_1} \right) \right] + a_2 \left( \frac{\sqrt{s} - \sqrt{s_0}}{b_2} \right)^{e_2} \exp \left[ e_2 \left( 1 - \frac{\sqrt{s} - \sqrt{s_0}}{b_2} \right) \right],
$$

(13)

where $\sqrt{s_0}$ is the threshold energy, and $a_1$, $b_1$, $e_1$, $a_2$, $b_2$, and $e_2$ are parameters. The numerical cross sections for exothermic reactions are parametrized as

$$
\sigma_{\text{unpol}}(\sqrt{s}, T) = \frac{\vec{P}'_2}{\vec{P}_2^2} \left\{ a_1 \left( \frac{\sqrt{s} - \sqrt{s_0}}{b_1} \right)^{e_1} \exp \left[ e_1 \left( 1 - \frac{\sqrt{s} - \sqrt{s_0}}{b_1} \right) \right] \\
+ a_2 \left( \frac{\sqrt{s} - \sqrt{s_0}}{b_2} \right)^{e_2} \exp \left[ e_2 \left( 1 - \frac{\sqrt{s} - \sqrt{s_0}}{b_2} \right) \right] \right\},
$$

(14)

The parameter values are listed in Tables 2-4. In the three tables the quantity $d_0$ is the separation between the peak’s location on the $\sqrt{s}$-axis and the threshold energy. The smaller $d_0$ is, the faster the cross section increases from zero to the peak cross section.

The quantity $\sqrt{s_z}$ is the square root of the Mandelstam variable at which the cross section is $1/100$ of the peak cross section. The quantity $\sqrt{s_z} - \sqrt{s_0} - d_0$ is the difference between $\sqrt{s_z}$ and the peak’s location on the $\sqrt{s}$-axis. The smaller $\sqrt{s_z} - \sqrt{s_0} - d_0$ is, the faster the cross section decreases from the peak cross section to zero.

A feature of the endothermic reactions in Figs. 3-11 is that the peak cross section decreases first and then increases as the temperature goes up. As the temperature increases from zero, confinement shown by the potential in Eq. (10) becomes weaker and weaker, the Schrödinger equation produces increasing meson radii, and mesonic quark-antiquark states become looser and looser. On one hand the weakening confinement with increasing temperature makes combining final quarks and antiquarks into final mesons more difficult, and thus reduces cross sections; On the other hand the increasing radii of initial mesons cause increasing cross sections as the temperature goes up. The two factors determine
the change in peak cross section with respect to the temperature. Another feature is that 
the cross section increases rapidly from zero to a maximum value when the total energy 
of the two initial mesons in the center-of-mass frame increases from the threshold energy, 
and the cross section further decreases from the maximum value or exhibits a plateau on 
the right of the peak as seen in Figs. 8 and 10.

The unpolarized cross sections for \( K \bar{K} \rightarrow K\bar{K}^* \) for \( I = 1 \) and for \( I = 0 \) are shown in 
Figs. 3 and 4, respectively. Quark-antiquark annihilation and creation takes place in the 
two isospin channels. If \( M_{a\bar{q}_1q_2} (M_{a\bar{q}_1q_2}) \) for a reaction equals zero, the left (right) diagram 
does not contribute to the reaction. It is shown from Table 1 that the two diagrams in 
Fig. 1 contribute to \( K\bar{K} \rightarrow K\bar{K}^* \) for \( I = 0 \), and only the right diagram contributes to 
\( K\bar{K} \rightarrow K\bar{K}^* \) for \( I = 1 \). The peak cross section of \( K\bar{K} \rightarrow K\bar{K}^* \) for \( I = 0 \) at a given 
temperature is more than 4 times the one for \( I = 1 \), and \( \sqrt{s_z} \) for \( I = 0 \) in Table 2 is 
roughly 2 times that for \( I = 1 \). When \( T/T_c = 0, 0.65, \) and \( 0.75 \), \( d_0 \) of \( K\bar{K} \rightarrow K\bar{K}^* \) for 
\( I = 0 \) is equal to \( d_0 \) for \( I = 1 \); when \( T/T_c = 0.85, 0.90, \) and \( 0.95 \), \( d_0 \) of \( K\bar{K} \rightarrow K\bar{K}^* \) for 
\( I = 0 \) is less than \( d_0 \) for \( I = 1 \). Therefore, the cross section for \( K\bar{K} \rightarrow K\bar{K}^* \) for \( I = 0 \) is 
larger than the one for \( K\bar{K} \rightarrow K\bar{K}^* \) for \( I = 1 \).

The unpolarized cross section for \( K\bar{K} \rightarrow \rho\rho \) for \( I = 1 \) has been shown in Fig. 15 of 
Ref. [8], and the one for \( K\bar{K}^* \rightarrow \rho\rho \) for \( I = 1 \) in Fig. 12 in the present work. At zero 
temperature the peak cross section of \( K\bar{K} \rightarrow \rho\rho \) for \( I = 1 \) is smaller than the one of 
\( K\bar{K}^* \rightarrow \rho\rho \) for \( I = 1 \); the cross section for \( K\bar{K} \rightarrow \rho\rho \) for \( I = 1 \) decreases from the peak 
cross section slower than for \( K\bar{K}^* \rightarrow \rho\rho \) for \( I = 1 \) since \( \sqrt{s_z} \) of the former is larger than 
that of the latter. When the two reactions are exothermic, the cross section for \( K\bar{K} \rightarrow \rho\rho \) 
for \( I = 1 \) decreases slower than for \( K\bar{K}^* \rightarrow \rho\rho \) for \( I = 1 \) with increasing center-of-mass 
energy of the two initial mesons from the threshold energy plus \( 10^{-4} \) GeV.

In the present work and in Ref. [8] we have studied the reactions: \( K\bar{K} \rightarrow K\bar{K}^* \), 
\( K^*\bar{K} \), and \( K^*\bar{K}^* \); \( \pi\pi \rightarrow K\bar{K} \), \( K\bar{K}^* \), \( K^*\bar{K} \), and \( K^*\bar{K}^* \); \( \pi\rho \rightarrow K\bar{K} \), \( K\bar{K}^* \), \( K^*\bar{K} \), and 
\( K^*\bar{K}^* \). As an example we compare the production of \( K\bar{K}^* \) with the production of \( K^*\bar{K}^* \) 
in the \( K + \bar{K} \) reaction. The unpolarized cross sections for \( K\bar{K} \rightarrow K^*\bar{K}^* \) for \( I = 1 \) and 
for \( I = 0 \) have been shown in Figs. 8 and 9 of Ref. [8], respectively. Quark-antiquark
annihilation and creation takes place in the two isospin channels. The peak cross section of \( K \bar{K} \rightarrow K \bar{K}^* \) for \( I = 1 \) ( \( I = 0 \) ) at a given temperature is larger than the one of \( K \bar{K} \rightarrow K^* \bar{K}^* \) for \( I = 1 \) ( \( I = 0 \) ). The Mandelstam variable \( \sqrt{s} \) corresponding to the peak cross section of \( K \bar{K} \rightarrow K \bar{K}^* \) is smaller than the one corresponding to the peak cross section of \( K \bar{K} \rightarrow K^* \bar{K}^* \) at a given temperature. Since the two reactions have the same initial mesons, the initial mesons in \( K \bar{K} \rightarrow K \bar{K}^* \) have a smaller value of \( | \vec{P} | \) corresponding to the peak cross section than in \( K \bar{K} \rightarrow K^* \bar{K}^* \). The cross section given in Eq. (1) is proportional to the inverse of \( s \ | \vec{P} \ | \). Therefore, the peak cross section of \( K \bar{K} \rightarrow K \bar{K}^* \) is larger than the one of \( K \bar{K} \rightarrow K^* \bar{K}^* \).

The total spin of the two final mesons may not equal the total spin of the two initial mesons in the reactions in the present work. This can be accounted for from a quark-antiquark potential which is equivalent to the transition potential in Eq. (4). The quark-antiquark potential is obtained from the transition potential under the Fierz transformations in Ref. [8],

\[
V_{q\bar{q}FF}(\vec{k}) = -\frac{g_F^2}{k^2} \left[ \frac{1}{3} \lambda_f^0(31) \lambda_f^0(42) + \frac{1}{2} \bar{\lambda}_f(31) \cdot \bar{\lambda}_f^T(42) \right] \times \left[ \frac{4}{9} \lambda_f^0(31) \lambda_f^0(42) - \frac{1}{12} \bar{\lambda}(31) \cdot \bar{\lambda}^T(42) \right] \left[ -\frac{3}{2} - \frac{1}{2} \bar{\sigma}(31) \cdot \sigma(42) \right] + \frac{\bar{\sigma}(42) \cdot \vec{p}_1 \sigma(42) \cdot \vec{p}_2}{8m_qm_{q'}} + \frac{\bar{\sigma}(31) \cdot \vec{p}_3 \sigma(31) \cdot \vec{p}_1}{8m_qm_{q'}} + \frac{3\bar{\sigma}(31) \cdot \vec{p}_1 \sigma(42) \cdot \vec{p}_2}{8m_q^2} - \frac{\bar{\sigma}(31) \cdot \vec{p}_1 \sigma(42) \cdot \vec{p}_4}{8m_qm_{q'}} - \frac{\bar{\sigma}(31) \cdot \vec{p}_3 \sigma(31) \cdot \vec{p}_2}{8m_qm_{q'}} + \frac{3\bar{\sigma}(31) \cdot \vec{p}_3 \sigma(42) \cdot \vec{p}_4}{8m_q^2} + \frac{\bar{\sigma}(31) \cdot \vec{p}_3 \sigma(42) \sigma(42) \cdot \vec{p}_2}{8m_qm_{q'}} + \frac{\bar{\sigma}(31) \cdot \vec{p}_3 \sigma(31) \sigma(42) \sigma(42) \cdot \vec{p}_4}{8m_q^2} - \frac{\bar{\sigma}(42) \cdot \vec{p}_4 \sigma(31) \sigma(42) \sigma(42) \cdot \vec{p}_2}{8m_qm_{q'}} - \frac{\bar{\sigma}(42) \cdot \vec{p}_4 \sigma(31) \sigma(42) \sigma(31) \cdot \vec{p}_1}{8m_qm_{q'}} \right],
\]

(15)

where \( \lambda_f^0 \) is a 3 \times 3 unit matrix in flavor space, \( \lambda_f \) are the Gell-Mann matrices that operate in flavor space, and \( \lambda^0 \) is a 3 \times 3 unit matrix in color space. The superscript \( T \) in Eqs. (15), (16), and (19)-(21) means transposition [15].

For the left diagram in Fig. 11 the quark-antiquark potential corresponding to \( q_1 + \bar{q}_2 \rightarrow \)
\( q_3 + \bar{q}_4 \) is between \( q_1 \) and \( \bar{q}_2 \),

\[
V_{a_0 q_3 q_4}(k) = -\frac{g^2}{k^2} \left[ \frac{1}{3} \lambda_0^0(31) \lambda_0^0(42) + \frac{1}{2} \bar{\lambda}(31) \cdot \bar{\lambda}(42) \right] \\
\times \left[ \frac{4}{9} \lambda_0^0(31) \lambda_0^0(42) - \frac{1}{12} \bar{\lambda}(31) \cdot \bar{\lambda}(42) \right] \left[ \frac{3}{2} - \frac{1}{2} \bar{\sigma}(31) \cdot \bar{\sigma}(42) \right] \\
+ \frac{\bar{\sigma}(42) \cdot \bar{p}_{q_4} \bar{\sigma}(42) \cdot \bar{p}_{q_4}}{8m_q m_{q_3}} + \frac{\bar{\sigma}(31) \cdot \bar{p}_{q_3} \bar{\sigma}(31) \cdot \bar{p}_{q_1}}{8m_q m_{q_3}} + \frac{3\bar{\sigma}(31) \cdot \bar{p}_{q_3} \bar{\sigma}(42) \cdot \bar{p}_{q_2}}{8m_q^2} \\
- \frac{\bar{\sigma}(31) \cdot \bar{p}_{q_1} \bar{\sigma}(42) \cdot \bar{p}_{q_4}}{8m_q m_{q_3}} - \frac{\bar{\sigma}(31) \cdot \bar{p}_{q_3} \bar{\sigma}(42) \cdot \bar{p}_{q_2}}{8m_q m_{q_3}} + \frac{3\bar{\sigma}(31) \cdot \bar{p}_{q_3} \bar{\sigma}(42) \cdot \bar{p}_{q_4}}{8m_q^2} \\
+ \frac{\bar{\sigma}(31) \bar{\sigma}(31) \cdot \bar{p}_{q_3} \bar{\sigma}(42) \cdot \bar{p}_{q_2}}{8m_q^2} \frac{\bar{\sigma}(31) \bar{\sigma}(31) \cdot \bar{p}_{q_3} \bar{\sigma}(42) \cdot \bar{p}_{q_4} \bar{\sigma}(42)}{8m_q m_{q_3}} + \frac{\bar{\sigma}(31) \bar{\sigma}(31) \cdot \bar{p}_{q_3} \bar{\sigma}(42) \cdot \bar{p}_{q_4} \bar{\sigma}(42)}{8m_q m_{q_3}} \\
- \frac{\bar{\sigma}(31) \bar{\sigma}(31) \cdot \bar{p}_{q_3} \bar{\sigma}(42) \cdot \bar{p}_{q_2}}{8m_q^2} - \frac{\bar{\sigma}(31) \bar{\sigma}(31) \cdot \bar{p}_{q_3} \bar{\sigma}(42) \cdot \bar{p}_{q_4} \bar{\sigma}(42)}{8m_q m_{q_3}} \right].
\] (16)

While we carry out the Fierz transformations, we need that quark 3 (antiquark 4) and quark 1 (antiquark 2) have the same flavor. Then, quark and antiquark masses possess \( m_{q_3} = m_{q_1} \) and \( m_{q_4} = m_{q_2} \). For convenience we use \( \bar{p}_{q_1}' = \bar{p}_{q_3} \) and \( \bar{p}_{q_2}' = \bar{p}_{q_4} \). The Hamiltonian corresponding to the left diagram in Fig. 1 is

\[
H_1 = V_{a_0 q_3 q_4}(k) + V_{q_1 q_3} + V_{q_2 q_4} \\
+ \sqrt{m_{q_1}^2 + \bar{p}_{q_1}^2} + \sqrt{m_{q_2}^2 + \bar{p}_{q_2}^2} + \sqrt{m_{q_3}^2 + \bar{p}_{q_3}^2} + \sqrt{m_{q_4}^2 + \bar{p}_{q_4}^2},
\] (17)

where \( V_{q_1 q_3} \) and \( V_{q_2 q_4} \) are the Fourier transform of \( V_{ab}(r) \) in Eq. (9). The total spin of the two mesons is

\[
\bar{S} = \bar{S}_{q_1} + \bar{S}_{q_3} + \bar{S}_{q_2} + \bar{S}_{q_4}.
\] (18)

The commutator of the \( z \) component \( S_z \) of the total spin and the Hamiltonian is

\[
[S_z, H_1] = -\frac{g^2}{k^2} \frac{1}{8m_{q_1}^2} \left( \frac{1}{3} \lambda_0^0(31) \lambda_0^0(42) + \frac{1}{2} \bar{\lambda}(31) \cdot \bar{\lambda}(42) \right) \left( \frac{4}{9} \lambda_0^0(31) \lambda_0^0(42) - \frac{1}{12} \bar{\lambda}(31) \cdot \bar{\lambda}(42) \right) \left( \frac{3}{2} - \frac{1}{2} \bar{\sigma}(31) \cdot \bar{\sigma}(42) \right) \\
\times \{ \sigma_{q_{2y}} \bar{p}_{q_{3y}} \bar{p}_{q_{4y}} - \bar{p}_{q_{3y}} \bar{p}_{q_{4y}} + p_{q_{1y}} p_{q_{2y}} + p_{q_{1y}} p_{q_{2y}} - p_{q_{1y}} \bar{p}_{q_{2y}} + \bar{p}_{q_{1y}} \bar{p}_{q_{2y}} + p_{q_{1y}} p_{q_{2y}} - p_{q_{1y}} \bar{p}_{q_{2y}} \}
\]
\[ + p'_{ix}p_{qix} + p'_{ix}P'_{qix} - p'_{ix}P_{qix} + p'_{ix}p_{qix} - p'_{ix}p_{qix} \]
\[ + \sigma_{qix}[p'_{ix}p_{qiy} - p'_{ix}p_{qiz} - p_{qix}p_{qiy} + p_{qix}p_{qiz} - p_{qix}P'_{qix} + p_{qix}p_{qix} - p_{qix}p_{qix}] \]
\[ - p'_{ix}P_{qiz} + P'_{ix}p_{qiz} + P'_{qiz}p_{qix} - P_{qiz}p_{qiz} \]
\[ + \sigma_{qix}[p'_{ix}p_{qix} - p'_{ix}p_{qiz} - p_{qix}p_{qiz} - p_{qix}p_{qix} + p_{qix}p_{qix} - p_{qix}p_{qix}] \]
\[ - p'_{ix}p_{qiz} + p'_{ix}p_{qix} - p_{qix}p_{qix} - p_{qix}p_{qix} \]
\[ + i\sigma_{qix}[3p_{qix}p_{qiz} + p_{qix}P'_{qix} - p'_{qix}p_{qix} - p_{qix}P_{qix} + p_{qix}p_{qix} + p_{qix}P_{qix} \]
\[ - p'_{qix}p_{qix} + P'_{qix}p_{qix} - P_{qix}p_{qix} + p_{qix}p_{qix} + p_{qix}P_{qix} \]
\[ + (\sigma_{qix}P_{qix} + \sigma_{qix}P_{qix})]4p_{qix}p_{qiz} - 4p_{qix}p_{qiz} - 2p_{qix}p_{qix} - 2p_{qix}p_{qix} \]
\[ - 2p'_{qix}p_{qix} + 2p'_{qix}p_{qix} + 4p_{qix}p_{qix} + 4p_{qix}p_{qix} + 2p_{qix}p_{qix} + 2p_{qix}p_{qix} \]
\[ - 2p'_{qix}p_{qix} - 2p_{qix}p_{qix} \}
\[ + i(\sigma_{qix}P_{qix} + \sigma_{qix}P_{qix})]4p_{qix}p_{qix} - 4p_{qix}p_{qix} - 2p_{qix}p_{qix} - 2p_{qix}p_{qix} \]
\[ - 2p'_{qix}p_{qix} + 2p'_{qix}p_{qix} + 4p_{qix}p_{qix} + 4p_{qix}p_{qix} + 2p_{qix}p_{qix} + 2p_{qix}p_{qix} \]
\[ - 2p'_{qix}p_{qix} - 2p_{qix}p_{qix} \}
\[ + i(\sigma_{qix}P_{qix} + \sigma_{qix}P_{qix})]3p_{qix}p_{qix} - p_{qix}p_{qix} - p_{qix}p_{qix} + 3p_{qix}P_{qix} + p_{qix}p_{qix} - p_{qix}p_{qix} \]
\[ - p'_{qix}p_{qix} + P'_{qix}p_{qix} - P_{qix}p_{qix} - p_{qix}p_{qix} + p_{qix}p_{qix} + p_{qix}P_{qix} \]
\[ + i(\sigma_{qix}P_{qix} + \sigma_{qix}P_{qix})]3p_{qix}p_{qix} - p_{qix}p_{qix} - p_{qix}p_{qix} + 3p_{qix}P_{qix} + p_{qix}p_{qix} - p_{qix}p_{qix} \]
\[ + p'_{qix}p_{qix} - p_{qix}p_{qix} - p_{qix}p_{qix} + p_{qix}p_{qix} + p_{qix}P_{qix} + p_{qix}P_{qix} \}
\[ (19) \]

where \( p_{ix}, p_{iy}, \) and \( p_{iz} \) are the three components of the momentum of incoming quark \( i (i = q_1, \bar{q}_2); p'_{ix}, p'_{iy}, \) and \( p'_{iz} \) are the three components of the momentum of outgoing quark \( i (i = q_1, \bar{q}_2); \sigma_{qix}, \sigma_{qiy}, \) and \( \sigma_{qiz} \) are the three components of \( \bar{\sigma}_{q_1} \equiv \bar{\sigma}(31); \sigma_{qix}, \sigma_{qiy}, \) and \( \sigma_{qiz} \) are the three components of \( \bar{\sigma}_{q_2} \equiv \bar{\sigma}(42); \lambda^0_{q_1f} \equiv \lambda^0(31), \lambda^0_{q_2f} \equiv \lambda^0(42), \lambda_{q_1f} \equiv \lambda_f(31), \lambda_{q_2f} \equiv \lambda_f(42), \lambda^0_{q_1} \equiv \lambda^0(31), \lambda^0_{q_2} \equiv \lambda^0(42), \bar{\lambda}_{q_1} \equiv \bar{\lambda}(31), \text{and} \bar{\lambda}_{q_2} \equiv \bar{\lambda}(42). \)
Applying the momentum conservation $\vec{p}'_{q_2} = \vec{p}_{q_1} + \vec{p}_{q_2} - \vec{p}_{q_1}'$, we get

$$[S_z, H_1] = -\frac{g_s^2}{k} \frac{1}{8m_q^2} \left( \frac{1}{3} \lambda_{q_1}^\dagger \lambda_{q_2}^0 + \frac{1}{2} \vec{\lambda}_{q_1} \cdot \vec{\lambda}_{q_2}' \right) \left( \frac{4}{9} \lambda_{q_1}^0 \lambda_{q_2}^0 - \frac{1}{12} \vec{\lambda}_{q_1} \cdot \vec{\lambda}_{q_2}' \right)$$

$$\times \left\{ \sigma_{q_2y} [p_{q_1x} p_{q_1z} + p_{q_1x}' p_{q_1z}' - p_{q_1z} p_{q_1y} - p_{q_1z}' p_{q_1y} + p_{q_2y} p_{q_1z} - p_{q_2z} p_{q_1y}] 
+ \sigma_{q_2z} [p_{q_1x} p_{q_1z} + p_{q_1x}' p_{q_1z}' - p_{q_1z} p_{q_1y} - p_{q_1z}' p_{q_1y} + p_{q_2x} p_{q_1z} - p_{q_2z} p_{q_1x}] 
+ \sigma_{q_1y} [p_{q_1x} p_{q_1z} - p_{q_1x}' p_{q_1z}' + p_{q_1z} p_{q_1y} + p_{q_1z}' p_{q_1y} - p_{q_1y} p_{q_1z} + p_{q_1y} p_{q_1z}' ] 
+ \sigma_{q_1x} [p_{q_1x} p_{q_1z} - p_{q_1x}' p_{q_1z}' + p_{q_1z} p_{q_1y} + p_{q_1z}' p_{q_1y} - p_{q_1y} p_{q_1z} + p_{q_1y} p_{q_1z}' ] 
+ i \sigma_{q_1z} \sigma_{q_2z} [-p_{q_1y} p_{q_1z} - 3p_{q_1y} p_{q_1z} - 3p_{q_1y}' p_{q_1z} + 4p_{q_1y} p_{q_1z}' - p_{q_1z} p_{q_1y} + p_{q_1z} p_{q_1y} + 2p_{q_2z} p_{q_1y} - 2p_{q_2z} p_{q_1y}] 
+ 3p_{q_1x} p_{q_1z} + 3p_{q_1x} p_{q_1z}' - 4p_{q_1x} p_{q_1z}' + p_{q_1z} p_{q_1x} - p_{q_1z} p_{q_1x} - 2p_{q_1z} p_{q_1x} 
- 2p_{q_1z} p_{q_1x} + p_{q_1z} p_{q_1x} ] + i (\sigma_{q_1y} \sigma_{q_2y} + \sigma_{q_1y} \sigma_{q_2y}) [4p_{q_1x} p_{q_1x} + 4p_{q_1x} p_{q_1x}]$$

$$- 4p_{q_1x} p_{q_1x} - 4p_{q_1y} p_{q_1y} - 4p_{q_1y} p_{q_1y} + 4p_{q_1y} p_{q_1y} + 2p_{q_1y} p_{q_1y} - 2p_{q_1x} p_{q_1x}$$

$$+ 2p_{q_2y} p_{q_2y} - 2p_{q_2z} p_{q_2x} ] + i (\sigma_{q_1y} \sigma_{q_2y} - \sigma_{q_1x} \sigma_{q_2x}) [4p_{q_1x} p_{q_1y} + 4p_{q_1x} p_{q_1y}]$$

$$- 8p_{q_1x} p_{q_1x} + 4p_{q_1y} p_{q_1x} + 4p_{q_1y} p_{q_1x} - 4p_{q_1y} p_{q_1x} - 4p_{q_2y} p_{q_2y} ] + i \sigma_{q_1z} \sigma_{q_2y} [p_{q_1z} p_{q_1z}$$

$$+ 3p_{q_1z} p_{q_1z} - 4p_{q_1z} p_{q_1z} + p_{q_1z} p_{q_1z} - p_{q_1z} p_{q_1z} - 2p_{q_2z} p_{q_1z} - 2p_{q_1z} p_{q_1z} + 4p_{q_1z} p_{q_1z} - 4p_{q_1z} p_{q_1z}$$

$$+ 4p_{q_1z} p_{q_1z} + 4p_{q_1z} p_{q_1z} - 2p_{q_1z} p_{q_1z} + 2p_{q_2z} p_{q_1z} + 2p_{q_2z} p_{q_1z} + p_{q_1y} p_{q_1z}] \right\}.$$
We may obtain the commutator 

\[
1) \text{have the same flavor, and the quark and antiquark masses satisfy } m_{q_3} = m_{q_2} \text{ and } m_{\bar{q}_4} = m_{\bar{q}_1}. \text{ The Hamiltonian corresponding to the right diagram in Fig. 1 is}
\]

\[
H_2 = V_{q_2\bar{q}_4} + V_{q_3\bar{q}_1} + V_{q_4\bar{q}_2} + \sqrt{m_{q_1}^2 + p_{\bar{q}_1}^2} + \sqrt{m_{q_2}^2 + p_{\bar{q}_2}^2} + \sqrt{m_{q_3}^2 + p_{\bar{q}_3}^2} + \sqrt{m_{q_4}^2 + p_{\bar{q}_4}^2}.
\]

We may obtain the commutator \([S_z, H_2] \text{ from } [S_z, H_1]\) in Eq. (20) by the replacement, \(q_1 \leftrightarrow q_2 \text{ and } \bar{q}_1 \leftrightarrow \bar{q}_2\). We may also get \([S_x, H_2] \text{ and } [S_y, H_2] \text{ from } [S_x, H_1] \text{ and } [S_y, H_1]\) by the replacement. \([S_x, H_2], [S_y, H_2], \text{ and } [S_z, H_2]\) may not be zero. This indicates that \(S_x, S_y, \text{ and } S_z\) may not be conserved, and the total spin may not be conserved, i.e., the total spin of the two final mesons may not equal the total spin of the two initial mesons.

V. SUMMARY

From the transition potential and the mesonic quark-antiquark relative-motion wave functions, we have calculated the unpolarized cross sections for the 2-to-2 meson-meson reactions that arise from quark-antiquark annihilation and creation in the first Born approximation. The reactions include \(K \bar{K} \rightarrow K \bar{K}^*, K \bar{K} \rightarrow K^* \bar{K}, \pi K \rightarrow \pi K^*, \pi K \rightarrow \rho K, \pi \pi \rightarrow K \bar{K}^*, \pi \pi \rightarrow K^* \bar{K}, \pi \rho \rightarrow K \bar{K}, \pi \rho \rightarrow K^* \bar{K}, \rho \rho \rightarrow K^* \bar{K}^*, K \bar{K}^* \rightarrow \rho \rho, \text{ and } K^* \bar{K} \rightarrow \rho \rho\). The Hamiltonian of the two mesons contains the quark-antiquark potential which is equivalent to the transition potential. We have derived the
commutation relations of the total spin of the two mesons and the Hamiltonian. Due to the quark-antiquark potential the commutators may not be zero, and the total spin may not be conserved in the reactions. With increasing center-of-mass energy of the two initial mesons from the threshold energy, the cross sections for the endothermic reactions increase very rapidly to the peak cross sections first, and then decrease or display plateaus for $\pi\pi \rightarrow K^*\bar{K}^*$ and $\pi\rho \rightarrow K^*\bar{K}^*$ at some temperatures. The cross sections exhibit remarkable temperature dependence. To use the cross sections conveniently, we have parametrized the numerical cross sections for the ten isospin channels of reactions. Based on the flavor matrix elements, the cross sections for the other isospin channels of reactions can be obtained from the cross sections for the ten channels.

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Figure 1: Solid (wavy) lines stand for quarks or antiquarks (gluon). The left diagram has $q_1 + \bar{q}_2 \rightarrow q_3 + \bar{q}_4$ for $A(q_1\bar{q}_1) + B(q_2\bar{q}_2) \rightarrow C(q_3\bar{q}_1) + D(q_2\bar{q}_1)$, and the right diagram has $\bar{q}_1 + q_2 \rightarrow q_3 + \bar{q}_4$ for $A(q_1\bar{q}_1) + B(q_2\bar{q}_2) \rightarrow C(q_1\bar{q}_4) + D(q_3\bar{q}_2)$. 

Figure 2: Corresponding to the process \( q(p_1) + \bar{q}(-p_2) \rightarrow q'(p_3) + \bar{q}'(-p_4) \), solid (wavy) lines stand for quarks or antiquarks (gluon). \( k \) denotes the gluon four-momentum, \( e \) its color index, and \( \tau \) its space-time index.
Figure 3: Cross sections for $K\bar{K} \rightarrow K\bar{K}^*$ for $I = 1$ at various temperatures.
Figure 4: Cross sections for $K\bar{K} \rightarrow K\bar{K}^*$ for $I = 0$ at various temperatures.
Figure 5: Cross sections for $\pi K \rightarrow \pi K^*$ for $I = 1/2$ at various temperatures.
Figure 6: Cross sections for $\pi K \rightarrow \rho K$ for $I = 1/2$ at various temperatures.
Figure 7: Cross sections for $\pi\pi \rightarrow K\bar{K}^*$ for $I = 1$ at various temperatures.
Figure 8: Cross sections for $\pi\pi \rightarrow K^*\bar{K}^*$ for $I = 1$ at various temperatures.
Figure 9: Cross sections for $\pi\rho \rightarrow K\bar{K}$ for $I = 1$ at various temperatures.
Figure 10: Cross sections for $\pi\rho \rightarrow K^*\bar{K}^*$ for $I = 1$ at various temperatures.
Figure 11: Cross sections for $\rho\rho \rightarrow K^*\bar{K}^*$ for $I = 1$ at various temperatures.
Figure 12: Cross sections for $K\bar{K}^* \rightarrow \rho\rho$ for $I = 1$ at various temperatures.
| Channel | $M_{a_1q_2f}$ | $M_{a_1q_1q_2}$ |
|---------|----------------|----------------|
| $I = 1$ $K^- K^- \rightarrow K^- K^+$ | 0 | 1 |
| $I = 0$ $K^- K^- \rightarrow K^- K^+$ | 2 | 1 |
| $I = 1$ $K^- K^- \rightarrow K^+ K$ | 0 | 1 |
| $I = 0$ $K^- K^- \rightarrow K^+ K$ | 2 | 1 |
| $I = 3/2$ $\pi K^- \rightarrow \pi K^+$ | 0 | 0 |
| $I = 1/2$ $\pi K^- \rightarrow \pi K^+$ | 0 | $\frac{3}{2}$ |
| $I = 3/2$ $\pi K^- \rightarrow \rho K$ | 0 | 0 |
| $I = 1/2$ $\pi K^- \rightarrow \rho K$ | 0 | $\frac{3}{2}$ |
| $I = 1$ $\pi\pi \rightarrow K^- K^+$ | 0 | -1 |
| $I = 0$ $\pi\pi \rightarrow K^- K^+$ | 0 | $-\frac{\sqrt{6}}{2}$ |
| $I = 1$ $\pi\pi \rightarrow K^+ K$ | 0 | -1 |
| $I = 0$ $\pi\pi \rightarrow K^+ K$ | 0 | $-\frac{\sqrt{6}}{2}$ |
| $I = 1$ $\pi\pi \rightarrow K^+ K^*$ | 0 | -1 |
| $I = 0$ $\pi\pi \rightarrow K^+ K^*$ | 0 | $-\frac{\sqrt{6}}{2}$ |
| $I = 1$ $\pi\rho \rightarrow K^- K^+$ | 0 | -1 |
| $I = 0$ $\pi\rho \rightarrow K^- K^+$ | 0 | $-\frac{\sqrt{6}}{2}$ |
| $I = 1$ $\pi\rho \rightarrow K^+ K^*$ | 0 | -1 |
| $I = 0$ $\pi\rho \rightarrow K^+ K^*$ | 0 | $-\frac{\sqrt{6}}{2}$ |
| $I = 1$ $\rho\rho \rightarrow K^- K^+$ | 0 | -1 |
| $I = 0$ $\rho\rho \rightarrow K^- K^+$ | 0 | $-\frac{\sqrt{6}}{2}$ |
| $I = 1$ $\rho\rho \rightarrow K^+ K^*$ | 0 | -1 |
| $I = 0$ $\rho\rho \rightarrow K^+ K^*$ | 0 | $-\frac{\sqrt{6}}{2}$ |
Table 2: Values of the parameters. $a_1$ and $a_2$ are in units of millibarns; $b_1$, $b_2$, $d_0$, and $\sqrt{s_z}$ are in units of GeV; $e_1$ and $e_2$ are dimensionless.

| Reactions                  | $T/T_c$ | $a_1$ | $b_1$ | $e_1$ | $a_2$ | $b_2$ | $e_2$ | $d_0$ | $\sqrt{s_z}$ |
|----------------------------|---------|-------|-------|-------|-------|-------|-------|-------|-------------|
| $I = 1 \bar{K}K \rightarrow K\bar{K}^*$ | 0       | 0.14  | 0.173 | 0.3   | 1.27  | 0.124 | 0.6   | 0.125 | 3.06        |
|                            | 0.65    | 0.1   | 0.209 | 0.2   | 0.9   | 0.142 | 0.7   | 0.15  | 2.91        |
|                            | 0.75    | 0.1   | 0.26  | 1.0   | 0.69  | 0.128 | 0.5   | 0.15  | 2.87        |
|                            | 0.85    | 0.1   | 0.03  | 0.5   | 0.4   | 0.157 | 0.51  | 0.1   | 2.82        |
|                            | 0.9     | 0.2   | 0.04  | 0.6   | 0.35  | 0.153 | 0.41  | 0.075 | 2.77        |
|                            | 0.95    | 0.3   | 0.21  | 0.5   | 0.32  | 0.035 | 0.45  | 0.05  | 2.76        |
| $I = 0 \bar{K}K \rightarrow K\bar{K}^*$ | 0       | 4.0   | 0.108 | 0.63  | 3.05  | 0.251 | 0.44  | 0.125 | 7.21        |
|                            | 0.65    | 3.3   | 0.127 | 0.67  | 1.5   | 0.267 | 0.35  | 0.15  | 6.12        |
|                            | 0.75    | 2.5   | 0.114 | 0.56  | 1.3   | 0.25  | 0.39  | 0.15  | 5.78        |
|                            | 0.85    | 1.1   | 0.059 | 0.5   | 1.4   | 0.195 | 0.45  | 0.075 | 5.58        |
|                            | 0.9     | 1.4   | 0.183 | 0.4   | 1.7   | 0.04  | 0.5   | 0.05  | 5.34        |
|                            | 0.95    | 1.6   | 0.16  | 0.33  | 2.0   | 0.03  | 0.52  | 0.025 | 5.74        |
| $I = 1/2 \pi K \rightarrow \pi K^*$ | 0       | 0.4   | 0.22  | 1.5   | 0.45  | 0.329 | 0.5   | 0.225 | 6.47        |
|                            | 0.65    | 0.3   | 0.18  | 1.0   | 0.26  | 0.275 | 0.4   | 0.25  | 5.78        |
|                            | 0.75    | 0.1   | 0.33  | 0.3   | 0.33  | 0.2   | 0.8   | 0.25  | 5.71        |
|                            | 0.85    | 0.2   | 0.17  | 0.5   | 0.04  | 0.439 | 0.4   | 0.15  | 5.81        |
|                            | 0.9     | 0.07  | 0.1   | 0.9   | 0.17  | 0.219 | 0.4   | 0.15  | 6.05        |
|                            | 0.95    | 0.1   | 0.1   | 0.5   | 0.15  | 0.308 | 0.47  | 0.15  | 6.23        |
| $I = 1/2 \pi K \rightarrow \rho K$ | 0       | 0.309 | 0.33  | 0.5   | 0.33  | 0.18  | 1.1   | 0.225 | 6.22        |
|                            | 0.65    | 0.061 | 0.84  | 0.29  | 0.29  | 0.21  | 0.8   | 0.25  | 4.81        |
|                            | 0.75    | 0.15  | 0.19  | 0.8   | 0.09  | 0.34  | 0.4   | 0.25  | 4.78        |
|                            | 0.85    | 0.055 | 0.4   | 0.5   | 0.1   | 0.14  | 0.5   | 0.15  | 4.94        |
|                            | 0.9     | 0.05  | 0.61  | 1.4   | 0.11  | 0.1   | 0.5   | 0.15  | 5.02        |
|                            | 0.95    | 0.05  | 0.71  | 1.6   | 0.12  | 0.12  | 0.5   | 0.15  | 5.37        |
Table 3: The same as Table 2, but for three other reactions.

| Reactions | $T/T_c$ | $a_1$ | $b_1$ | $e_1$ | $a_2$ | $b_2$ | $e_2$ | $d_0$ | $\sqrt{s_z}$ |
|-----------|--------|------|------|------|------|------|------|------|----------|
| $I = 1 \pi\pi \rightarrow K\bar{K}^*$ | 0.65 | 0.15 | 0.25 | 0.7 | 0.03 | 0.28 | 2.2 | 0.3 | 3.69 |
| | 0.75 | 0.06 | 0.2 | 1.0 | 0.09 | 0.24 | 0.5 | 0.2 | 3.3 |
| | 0.85 | 0.016 | 0.35 | 0.4 | 0.095 | 0.19 | 0.6 | 0.2 | 3.17 |
| | 0.9 | 0.004 | 0.41 | 0.2 | 0.06 | 0.2 | 0.6 | 0.2 | 3.0 |
| | 0.95 | 0.011 | 0.26 | 0.3 | 0.041 | 0.2 | 0.7 | 0.2 | 2.91 |
| | 0.95 | 0.01 | 0.17 | 0.3 | 0.06 | 0.22 | 0.6 | 0.25 | 2.82 |
| $I = 1 \pi\pi \rightarrow K^*\bar{K}$ | 0.65 | 0.09 | 0.22 | 1.0 | 0.3 | 0.31 | 0.7 | 0.25 | 4.76 |
| | 0.75 | 0.13 | 0.5 | 1.9 | 0.098 | 0.09 | 0.6 | 0.3 | 3.61 |
| | 0.85 | 0.024 | 0.83 | 8.0 | 0.09 | 0.225 | 0.56 | 0.3 | 3.35 |
| | 0.9 | 0.013 | 0.98 | 8.6 | 0.022 | 0.24 | 0.54 | 0.35 | 3.08 |
| | 0.95 | 0.008 | 1.13 | 7.42 | 0.012 | 0.26 | 0.5 | 0.45 | 3.03 |
| | 0.95 | 0.003 | 1.42 | 15.0 | 0.0181 | 0.41 | 0.5 | 0.35 | 3.01 |
| $I = 1 \pi\rho \rightarrow K\bar{K}$ | 0.65 | 0.55 | 0.3 | 2.6 | 0.58 | 0.174 | 0.6 | 0.3 | 2.98 |
| | 0.75 | 0.082 | 0.25 | 1.7 | 0.19 | 0.163 | 0.5 | 0.2 | 3.04 |
| | 0.85 | 0.027 | 0.22 | 1.5 | 0.11 | 0.17 | 0.5 | 0.2 | 3.09 |
| | 0.9 | 0.003 | 0.23 | 2.4 | 0.053 | 0.18 | 0.5 | 0.2 | 3.11 |
| | 0.95 | 0.005 | 0.5 | 1.3 | 0.04 | 0.16 | 0.5 | 0.2 | 3.08 |
| | 0.95 | 0.01 | 0.46 | 1.2 | 0.04 | 0.16 | 0.5 | 0.2 | 2.97 |
Table 4: The same as Table 2, but for three other reactions.

| Reactions             | $T/T_c$ | $a_1$ | $b_1$ | $e_1$ | $a_2$ | $b_2$ | $e_2$ | $d_0$ | $\sqrt{s_{x}}$ |
|-----------------------|---------|-------|-------|-------|-------|-------|-------|-------|---------------|
| $I = 1 \pi\rho \to K^*\bar{K}^*$ | 0       | 0.4   | 0.61  | 2.0   | 0.6   | 0.13  | 0.7   | 0.225 | 4.77         |
|                       | 0.65    | 0.048 | 0.78  | 3.9   | 0.073 | 0.17  | 0.6   | 0.35  | 3.96         |
|                       | 0.75    | 0.017 | 0.13  | 0.6   | 0.024 | 0.7   | 3.1   | 0.55  | 3.65         |
|                       | 0.85    | 0.0012| 0.1   | 0.63  | 0.006 | 0.65  | 1.88  | 0.75  | 3.18         |
|                       | 0.9     | 0.0019| 0.13  | 0.63  | 0.0051| 0.66  | 1.81  | 0.5   | 3.02         |
|                       | 0.95    | 0.01  | 0.544 | 1.1   | 0.006 | 0.1   | 0.5   | 0.25  | 2.9          |
| $I = 1 \rho\rho \to K^*\bar{K}^*$ | 0       | 3     | 0.1   | 0.72  | 1.49  | 0.32  | 0.5   | 0.1   | 4.23         |
|                       | 0.65    | 0.165 | 0.16  | 0.54  | 0.1   | 0.98  | 4.5   | 0.15  | 3.98         |
|                       | 0.75    | 0.016 | 1.0   | 4.0   | 0.04  | 0.26  | 0.6   | 0.25  | 3.72         |
|                       | 0.85    | 0.0104| 0.377 | 1.21  | 0.0017| 0.58  | 0.57  | 0.3   | 3.25         |
|                       | 0.9     | 0.006 | 0.29  | 2.1   | 0.01  | 0.36  | 0.78  | 0.25  | 2.9          |
|                       | 0.95    | 0.017 | 0.32  | 0.5   | 0.031 | 0.21  | 1.3   | 0.2   | 2.53         |
| $I = 1 K\bar{K}^* \to \rho\rho$   | 0       | 2.2   | 0.11  | 0.7   | 1.0   | 0.29  | 0.3   | 0.1   | 4.6          |
|                       | 0.65    | 0.18  | 0.33  | 0.23  | 0.35  | 0.05  | 0.6   | 0.05  | 4.0          |
|                       | 0.75    | 0.048 | 0.87  | 1.7   | 0.09  | 0.07  | 0.45  | 0.05  | 3.96         |
|                       | 0.85    | 0.006 | 1.18  | 4.2   | 0.012 | 0.19  | 0.51  | 0.2   | 3.56         |
|                       | 0.9     | 0.005 | 0.092 | 0.6   | 0.007 | 0.7   | 1.7   | 0.3   | 3.4          |
|                       | 0.95    | 0.002 | 1.21  | 18    | 0.01  | 0.32  | 0.5   | 0.25  | 3.28         |