Spontaneous frequency shift and phase retardation of coupled Josephson oscillations in cuprate superconductors

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Abstract

We examine coupling interactions used to synchronize macroscopic Josephson oscillations induced in intrinsic Josephson junction (IJJ) mesa stacks made of Bi2212 single crystals. Synchronized emissions of terahertz electromagnetic (EM) waves are detected under common voltage and current bias operations of two connected mesas with close individual emission frequencies, while uncoupled and bimodal emissions are frequently observed in two mesas with different individual emission frequencies. Detailed observations of the polarization of the EM wave emitted when two mesas are biased in parallel or series allow us to reveal the coupling matrix components, which include ratios of synchronized IJJs in the mesas and phase retardation between the macroscopic Josephson oscillations. This finding stimulates systematic survey on polarization of EM wave emitted from synchronized multiple mesa devices in order to realize powerful emissions from superconductors.

I. INTRODUCTION

The interaction between multiple oscillators is one of the long-term concerns for human being, beginning with Huygens, who considered the synchronous vibrations of pendula hanging from beams in a ship during the Age of Discovery. In our modern world, physically intuitive examples of synchronization among multiple oscillators include: multiple metronomes on a clipboard, synchronous flickering of certain types of fireflies, and electric generators connected to a power network [1, 2]. In the quantum mechanics, the phenomenon of superconductivity is also a synchronous phenomenon of electrons as oscillators, in which a large number of Cooper pairs maintain their coherence; in Josephson junctions, the phase of the order parameter is manifested in the form of current [3, 4].

Various phenomena attributed to couplings between Josephson junctions have been observed in cuprate superconductor single crystals, in which a strong modulation of order parameter along the c-axis attributed to the layered crystal structure exists [5]. In such systems, known as intrinsic Josephson junctions (IJJ), the excitation of a Josephson plasma wave along the ab-plane [6, 7] induces macroscopic current oscillations on the crystal surface...
owing to the interplay with the alternating current (AC) Josephson effect and their synchronous oscillations accompanied by the inductive and capacitive couplings among stacked IJJs [8–11], which combine to cause electromagnetic (EM) radiation into space [12–14]. Intensive research has been carried out since 2007 on Josephson plasma emission (JPE) using mesa-structured devices formed from Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (Bi2212) single crystals [15–24], and further research on the possibility of a novel and practical terahertz (THz) source have been reported [25–37].

One promising method to increase the radiation intensity is to synchronize multiple mesas to emit radiation. As an initial attempt, when two mesas were connected in series, individual and coupled emissions observed at different voltages applied to the series circuit were reported, with the coupled emission intensity being higher than the sum of the individual intensities [38]. Next, for a device with up to three mesas connected in parallel, an intensity of 0.61 mW was achieved, which is roughly proportional to the square of the number of mesas to be synchronized, and it was concluded that the emission of the electric wave is a synchronous coherent radiation [39].

Very recently, the authors proposed a method for analyzing the synchronous phenomena by estimating the phase difference between the Josephson plasma waves excited in a mesa structure device from precise observations of the polarization of the emitted EM wave [40, 41]. This technique allows us to determine the contribution of each mesa to the synchronized emission under biasing multiple mesas, where temperature and current distributions have been less investigated [42]. It is crucial to compare the emission features for parallel and series connections of a pair of mesas. However, there have been no reports of investigations on the differences between them in identical devices. In the case of a radiating mesa in which a thousand stacked IJJs oscillate coherently as a macroscopic Josephson junction, the common voltage bias (CVB) produced by the parallel connection should induce synchronous oscillations, whereas the common current bias (CCB) produced by the series connection corresponds to the interplay between two microscopic IJJs [43, 44].

In this paper, we discuss the difference in inter-mesa coupling when two of three mesas are operated by CVB and CCB modes, based on the polarization observation of EM waves emitted from mesas in individual and coupled emission states. A CVB mostly generates coupled emissions with unimodal frequency spectra, whereas a CCB tends to generate uncoupled emissions with bimodal frequency spectra. The coupled emission state is a superposition
of the individual emissions and perturbations corresponding to the inter-mesa coupling. The coefficients of the linear combination with the bases of the individual oscillation states compose the mesa interaction matrices, i.e., the states of the harmonically oscillating plasmons inside the superconducting substrate that are responsible for the interaction between the nonlinear Josephson plasma oscillations in the mesas. A systematic survey of the interaction matrix will lead to the development of powerful THz sources by controlling the mesa-to-mesa synchronization and will stimulate emergence of superconducting devices that enable THz quantum communication.

II. EXPERIMENT

In this study, we discuss EM wave emissions from three mesas (B, C, and E) formed on a Bi2212 single crystal grown by the floating zone method using photolithography and argon ion milling [23, 28]. The device is shown in Fig. 1(a). The basic properties, such as the mesa shape, $T_c$, and current–voltage characteristics of the three mesas, are summarized in the Supplementary Materials. Parallel and series connections can simultaneously energize a pair of mesa structures connected by a Bi2212 substrate single crystal. For a CVB, the wires connected to the two mesas were short-circuited at the outside of the cryostat, and a voltage was applied between the electrode and the substrate crystal, so that the same voltage was applied to the connected mesas, as shown in Fig. 1(a). On the other hand for a CCB, a bias voltages was applied between the two mesas, so it was not ensured that the same voltage was applied to each mesa. The EM wave emitted was detected by a silicon bolometer cooled by liquid helium [28], and the intensity was evaluated by the bolometer output voltage and the oscillation frequency measured using an FTIR spectrometer with a split mirror [19, 45, 46]. For the polarization evaluation, the Stokes polarization parameters (SPPs) were estimated from the transmitted intensity of an optical system combining a polarizer and a rotating achromatic quarter wave plate (QWP) as described in Ref. [40, 47].
III. RESULTS AND DISCUSSION

A. Current–voltage characteristics and emission intensity

Figure 1(b) shows the current–voltage–emission ($I - V - E$) characteristics of mesa B and E under CVB. This situation is represented by B $\parallel$ E hereinafter. In comparison to data for individual mesas B and E (Fig. 1(c) and Fig. S1 in the Supplementary Materials), the current–voltage characteristics (IVC) for B $\parallel$ E is elongated twice with respect to the current, and the emission intensity indicates more complicated multi-modal behavior as a function of the total bias current (Fig. 1(b) right panel) [48]. In this case, two intensity maxima were observed: a broad maximum at $I = 48$ mA, and a rather sharp maximum at $I = 26$ mA. The former current is understood as the sum of the bias current for the maximum emission intensity of mesas B and E. The latter current is due to multivalent $I - V - E$ characteristics with respect to the voltage [19, 23]. One mesa emits intensively at a current of 22 mA, while the other little emits at a few mA under biasing at the same voltage. In the subsequent discussion, spectra and polarization data for high-bias emissions are employed for comparison between simultaneous and individual emissions. It is important to note that the bias current for which the maximum emission for B $\parallel$ E is attained (47.3 mA) is slightly larger than the sum of those for B and E (21.9 and 21.7 mA, respectively). Furthermore, the voltage applied to B $\parallel$ E for the maximum emission is 0.90 V, which is slightly lower than those for B and E (at 0.98 and 0.96 V, respectively).

A similar plot for the CCB of B and E (B$-E$) is shown in Fig. 1(c). The low-voltage part of the IVC for B$-E$ largely overlap the IVC for B. It was also found that the bolometer responses for B and B$-E$ between 15 and 35 mA with increasing current are superposable. This clearly indicates that only B contributes to the emission power, despite the CCB of B$-E$. In the same current range, with decreasing current, on the other hand, significant increase in bolometer response of B$-E$ in comparison to B was observed. The maximum emission was attained at $I = 18.4$ mA, and local intensity maxima were found at $I = 22.6$ and 26.4 mA (indicated by solid and open triangles, respectively), which were slightly higher than those of the individuals B and E. The net mesa voltage for B$-E$ at the maximum emission power was $V = 1.89$ V, which was slightly lower than the sum of the bias voltages for the individual emissions.
B. Spectral characteristics

The spectra of B, C, E, B∥E, and B−E are shown in Figs. 2(a) and (b). Unimodal spectra were observed not only for individual operations but also for both CVB and CCB, suggesting that multiple mesas are coupled and radiating synchronously. The observed unimodal spectra were as follows: individual (B: 0.43–0.57, C: 0.50–0.55, E: 0.50–0.55 THz), B and E (CVB: 0.47–0.53, CCB: 0.46–0.5 THz), and C and E (CVB: 0.47–0.53, CCB: 0.48–0.61 THz). The frequencies of the emission spectra as functions of the averaged voltage applied to the mesas $V$ are shown in Fig. 2(c). The linear fittings to the plots provide the number, $N$, of IJJs contributing to emissions. For the BE pair, in which $N$s for individual emissions of B and E are very close as ∼ 870, the frequency regions for parallel and series connections largely overlap, and their $N$ is approximately 10% smaller than those for B and E individuals. For simultaneous bias of B and C (CVB: 0.45–0.58, CCB: 0.43–0.55 THz), bimodal spectra were observed, as shown in Fig. 2(b). At this time, the mesas are presumed to be radiating independently. In particular, in the combination of B and C, a unimodal spectrum is observed only at 0.45 THz for CVB, while bimodal spectra were found in the other CVBs and all of the CCBs. The $N$ for individual C is distributed between 780 and 810, which is more than 10% smaller than that for B and E individuals.

The result that CVB operations tend to be coupled can be understood as follows. For a CVB, an equivalent voltage was applied to the two mesas. Although it is not promised to apply an equal voltage to the serially connected IJJs composing a mesa structure, the inductive coupling among the stacked IJJs expanding the whole mesa-stack results in an equal voltage being applied because of the Josephson oscillation of an identical frequency. Consequently, an equal voltage is applied to all contributing IJJs, and then Josephson plasma waves of a single frequency are excited. The BE and CE pairs, for which a unimodal spectrum is always observed, are presumed to have a relatively strong coupling between the mesas.

It is intriguing that the coupling in the BE and CE pairs with larger separation is relatively stronger than that in the combination of BCs resulting in synchronous oscillations. The reasons for this are discussed first. The most important factor is the commensurability of the inter-mesa separation $d$ with respect to the mesa width $w$, which is a dominant factor in the emission frequency of a single mesa. When we consider that the coupling is mediated by Josephson plasmons, zero-voltage Josephson harmonic oscillations inside the
superconducting substrate underneath the mesas, which has been observed as microwave absorption [7], the interference between the plasmons and the nonlinear Josephson oscillations excited in the mesas determines the coupling. Here, using $w = 67\mu m$ (approximate average of widths of the mesas listed in Table SI of the Supplementary Materials), $d/w$ = values of 1.3 (BC), 6.1 (BE), and 3.8 (CE) are obtained. For combinations with $d/w$ closer to an integer, the effective coupling attributed to the strength of the frequency entrainment of the nonlinear Josephson plasma oscillation is more pronounced. The second most important factor is the similarity in the device geometry, including electrodes: mesas B and E are roughly mirror-symmetric, and a large overlap in the emission frequency regions of individually biased cases are observed. In other words, for the cases that the device shapes are closer to being equivalent and the emission frequencies are closer, even weak perturbations may cause frequency entrainments. In addition, the CE pair is symmetric for the mesas and their vicinities (electrodes, not shown in the figure, are far from mirror-symmetric). In summary, the BE pair provides coupled phase dynamics between two oscillators with close eigenfrequencies while the CE pair provides them between two oscillators with different eigenfrequencies of more than 10%.

C. Polarization properties

Next, we discuss polarization. Using the polarization analyzer introduced in Ref. [40], we obtain the transmitted intensity as a function of the QWP angle, as shown in Fig. 3(a). Mesas C, E, C–E, and C||E were found to be biased at an approximate voltage 0.88 V, for example. Based on these results, the Stokes polarization parameters SPPs $(S_0, S_1, S_2, S_3) = S_0(1, S_1, S_2, S_3)$ can be estimated, and the polarization ellipse which is the locus of the electrical field vector, can be obtained. The estimated polarization ellipses for C, E, C–E, and C||E are shown in Fig. 3(b). The measurements were performed for B, C, and E and their combinations under individual and simultaneous bias cases. For all of the individual cases and most of the simultaneous bias cases, polarization ellipses with the major axis along the short side of the mesa (x-axis) were obtained. However, for the combination of B and C, where bimodal spectra were obtained, trajectories with relatively small axial ratios were obtained. These are attributable to mixed polarizations caused by uncoupled emissions.
D. Linear combination method to describe a coupled emission

Let us consider the coupled emission of the BE pair as an example. If we write electric field emitted from the individually biased mesas B and E as $|B⟩ = E_B \exp[i(\omega_B t + \delta_B)]$ and $|E⟩ = E_E \exp[i(\omega_E t + \delta_E)]$, respectively, where $E_B$, $\omega_B$, and $\delta_B$ are the electric field vector, angular frequency, and the phase of an EM wave emitted from mesa B, respectively. The simultaneous bias in the parallel connection can be written as follows:

$$|B \parallel E⟩ = \alpha |B⟩ + \beta |E⟩,$$

where $\alpha$ and $\beta$ are complex. This is based on the idea that the observed coupled emission $|B \parallel E⟩$ is a superposition of the EM waves emitted from mesas B and E under CVB, referred as $|B'⟩$ and $|E'⟩$, which are results of perturbation arising from the coupling interaction $V_{B\parallel E}$ to $|B⟩$ and $|E⟩$, respectively. Thus,

$$\begin{pmatrix} |B'⟩ \\ |E'⟩ \end{pmatrix} = V_{B\parallel E} \begin{pmatrix} |B⟩ \\ |E⟩ \end{pmatrix} = \begin{pmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \end{pmatrix} \begin{pmatrix} |B⟩ \\ |E⟩ \end{pmatrix},$$

where $\alpha = \alpha_1 + \alpha_2$ and $\beta = \beta_1 + \beta_2$. Writing $|B \parallel E⟩ = E_{B\parallel E} \exp[i(\omega_{B\parallel E} t + \delta_{B\parallel E})]$, we obtain $\alpha(\omega_B) = |\alpha| \exp[i(\omega_B' t + \delta_B')]$ and $\beta(\omega_E) = |\beta| \exp[i(\omega_E' t + \delta_E')]$ with $\omega_{B\parallel E} = \omega_B + \omega_E = \omega_B' + \omega_E'$ and $\delta_{B\parallel E} = \delta_B + \delta_E' = \delta_B + \delta_E$. Here, $\omega_B'$ and $\delta_E'$ are the frequencies and phase changes of mesa B associated with synchronization, which indicates the magnitude of the inter-mesa perturbation. It is considered that $|\alpha|$ and $|\beta|$ correspond to the ratios of the number of IJJs contributing to for the coupled emission ($N_B'$ and $N_E'$) with respect to those for the individual emissions ($N_B$ and $N_E$); i.e. $|\alpha| = N_B'/N_B$ and etc. $\delta_γ = \arg(\beta/\alpha) = \delta_E' - \delta_B'$ is the phase difference of the coupling perturbation for mesa B and E, which may related to the separation between the two mesas and the wavelength of the simultaneous oscillations of the two mesas. In the following, the validity of determining $\alpha$ and $\beta$ is discussed with either (i) fixing bases as specific polarizations of individual emissions in Sec. III E or (ii) taking bases as continuous valuables as a function of the basis frequency in Sec. III F.

E. Fixed bias analysis results and discussion

Let us assume that the bases are taken at the maximum $S_0$ of the individual emissions. This assumption is reasonable given the approximation that the polarization of the emission
does not change with the bias voltage. More precisely, the polarization may change with respect to the emission frequency because the polarization is determined by the matching between frequency and mesa shape. For all coupled emissions, $\delta_\gamma$ and $|\alpha|, |\beta|$, with the bases listed in Table SI are plotted together with measured SPPs in Fig. 4. Here, $|\alpha|$, and $|\beta|$ must not be much more than unity because it is assumed that all IJJ of the stack participating in the emission, with a maximum $S_0$ of individual emissions. The distributions of $\delta_\gamma$ for all combinations show two groups: e.g., for the $B \parallel E$ case, $\delta_\gamma$ below or above 90 degrees at approximately 450 GHz. Between the groups, $\delta_\gamma$ shows a significant change, despite small differences in SPPs as shown in Fig. 4. This behavior indicates that the continuous evolution of polarization within a group should be discussed within a group, that is the assumption of discussion such as number of contributing IJJs $N$ is different between the groups. With respect to the frequency evolution of expected phase difference due to inter-mesa propagation $\delta_d = 2\pi ndf_s/c_0$ drawn in the middle panels of Fig. 4 (a-d), no systematic change in $\delta_\gamma$ was found. Here, $n = 4.1$ is a refractive index of Bi2212 and $c_0$ is the light velocity in vacuum. This is interpreted to mean that separation between mesas is not an essential component for synchronization.

The analysis discussed above relies on three specific measured polarizations: a coupled emission with respect to two individual emissions. Considering the known facts concerning IJJ THz emission, i.e., that emission intensity and polarization are sensitive to the temperature distribution of the device [26], it is natural to presume that the temperature distribution of the simultaneous emission is different from the individual emissions. This discrepancy results in significant deviation of $|\alpha|$ and $|\beta|$ from unity. We employ bases in the following paragraphs, as continuous functions defined by polynomial fittings to experimental results.

**F. Continuous valuables as a function of basis frequency**

We considered polarization as a continuous function of the basis frequency to investigate the basis selection. We fit quadratic functions for the mesa voltage dependence of the SPP for the individually biased cases, and then regarded the fitted functions as the basis for a function of frequency. Figure S3 (c-d) shows the measured SPPs of B, C, and E and fitted parabolic curves as functions of the individual emission frequency $f_i$. The curves lie within the error bars of the measured data. Considering a measured coupled emission with
a specific frequency represented by Eq. (1) with continuous functions of \( f_s \), \( \alpha(f_i) \) and \( \beta(f_i) \) can be derived with a restriction on \( f_i \) for each basis.

\[
\begin{pmatrix}
\alpha \\
\beta
\end{pmatrix}
= (|B\rangle, |E\rangle)^{-1} |B \parallel E\rangle.
\]

(3)

Here, let us assume that the same \( \omega_i \) is taken for bases, that is \( \omega_B = \omega_E \) for the BE pair. The details of the calculation are described in the Supplementary Materials. To compare CVB and CCB operations, obtained coefficients at \( f_s = 514 \text{ GHz} \) of \( B \parallel E \) and \( f_s = 511 \text{ GHz} \) of \( B-E \) are shown in Figs. 5 (a) and (b), respectively. Qualitative differences in the \( f_i \) evolutions of the coefficients are found between CVB and CCB despite close \( f_s \)’s.

First, \( f_s = f_i \) cases (no change in frequency between the individual and coupled emissions) is discussed. The vertical bars in Fig. 5 represent the coupled emission frequencies; thus the crossing points to \( |\alpha|, |\beta|, \) and \( \delta \gamma \) provide solutions. In (a), both \( |\alpha| \) and \( |\beta| \) are close to 1 but \( \delta \gamma \) close to \(-\pi \) arises the small \( S_0 \). In (b), the large \( |\alpha| \) and \( |\beta| \) imply that the present assumption, e.g. the basis selection, is inappropriate. The selected solutions to have \( |\alpha| \) and \( |\beta| \) close to 1 are presented in Table II(b). Here, we notice that \( \delta \gamma \) depends on the combination (BE or CE) and connection (CVB or CCB). The results show that coupled emissions are attained by adjusting the phase difference between the two bases. The decreases in \( |\alpha| \) and \( |\beta| \) from unity may be attributed to the partial synchronization of IJJs inside a mesa with another mesa. If this coupling is mediated by the substrate, the lower part of the mesa tended to participate to the synchronization.

Next, let us discuss the cases of \( f_s \neq f_i \). Here, we assume that the number of synchronizing IJJs inside the mesa does not change with the coupled emission; therefore, the basis frequency \( \omega \) is determined by the minimum \( S = (|\alpha| - 1)^2 + (|\beta| - 1)^2 \) (sum of squared deviations from 1) then \( \alpha \) and \( \beta \) are obtained. For \( |B \parallel E\rangle \), the minimum of \( S \) is found at 519 GHz. The frequency change attributed to the coupling perturbation \( f_s - f_i \) is estimated to be \(-5 \text{ GHz} \), which is approximately 1 % of \( f_i \). Concerning the other coupled emissions, where \( f_s \) is within the range of basis frequency \( f_i \), the estimated difference between \( f_i \) and \( f_s \) is less than 10 GHz.

Compared with methods that select a basis from measured polarizations with different frequencies discussed first, more reasonable results for \( |\alpha| \) and \( |\beta| \) are obtained by taking the basis as a continuous function. We presume that this is due to the assumption that identical frequencies for bases are needed to compose a linear combination representing the coupled
emission. In previous studies, coupled emissions have been discussed by comparing singly biased individual emissions with different frequencies, and the frequency change accompanied by inter-mesa coupling has remained unknown. The present work clearly demonstrates that frequency entrainment occurs and that each mesa provides an EM wave identical to the singly biased case but at the entrained frequency when multiple mesas oscillate synchronously. Thus, the coupled emission is observed as a superposition of the entrained individual emissions such as \( |B'\rangle = \alpha_1 |B\rangle + \beta_1 |E\rangle \) and \( |E'\rangle = \alpha_2 |B\rangle + \beta_2 |E\rangle \). So evaluating SPPs of \( |B'\rangle \) and other entrained emissions allows us to determine elements of inter-mesa coupling perturbation matrix, e.g. \( V_{B||E} \). Here, we propose to obtain polarization mapping with respect to the azimuth angle from the device to estimate SPPs of each perturbed bases, for example. Further systematic measurements of the polarization mappings with respect to external perturbations such as local temperature increase may reveal quantum-mechanically entangled properties of THz EM waves emitted from the pair of mesas coupled by the superconducting Josephson plasmon inside the substrate.

The systematic difference between CVB and CCB observed in the present study presents quantitative differences of media to promote the synchronous radiation; i.e., the Josephson plasmon excited the base crystal. Confining to the case of \( f_s = f_i \), the coupling interaction for CVB conveys \( \delta_\gamma \), whereas that for CCB may adjust \( |\alpha| \) and \( |\beta| \) to have the identical \( f_i \) in addition to \( \delta_\gamma \). To attain highest emission power by approaching to in-phase synchronization, \( \delta_\gamma \) holds an essential relevance for CVB, and the number of synchronously oscillating IJJ s within a mesa is another relevance for CCB. This is consistent with the \( I - V - E \) data shown in Figs. 1(c) and S2(b). Here the highest detection power was observed at current less than 30 mA, at which a large part of IJJ s composing mesa E may be zero resistance. Further investigations on CVB and CCB systems with more mesas in combination with artificially-fabricated Josephson junction arrays \([49]\) leads to multi-pixel two-dimensional JPE mesa array devices and a global coupling system of Josephson oscillators \([50]\) which represents synchronous phenomena within a mesa consisting of thousands IJJs.

IV. SUMMARY

Terahertz electromagnetic wave emission from two simultaneously biased mesas of Bi2212 IJJ stacks on a superconducting substrate was investigated by intensity, frequency, and polar-
ization measurements as functions of bias voltage, combination of mesas, and parallel/series connections. Coupled and uncoupled emissions tended to be observed in parallel and series connections of the mesas, respectively. The frequencies of the uncoupled emissions largely overlapped the individual emissions, whereas the frequencies of the coupled emissions were systematically varied. Electromagnetic waves from pairs of coupled mesas can be represented by the superpositions of two individual emissions at entrained frequencies and phases, irrespective of parallel and series connections, rather than by linear combinations of observed individual emissions with different frequencies. Further systematic investigations involving changes in inter-mesa coupling and mesa geometries are required to fully assess the features of the perturbation matrix to control the inter-mesa synchronous phenomena and attain a high-power terahertz source from superconducting IJJ stacks.

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TABLE I. Measured properties of coupled emissions for B∥E, B−E, C∥E, and C−E (a) and determined values of $\alpha$ and $\beta$ with continuous bases of $f_s = f_i$ (b), and $f_s \neq f_i$ (c). Here, $f_s$ and $f_i$ denote simultaneous and individual emission frequencies.

|                     | B ∥ E | B − E | C∥E | C − E |
|---------------------|-------|-------|-----|-------|
| $P_{\text{max}}$ (mV) | 0.737 | 0.799 | 0.502 | 0.668 |
| $I_{\text{max}}$ (mA)   | 47.3  | 18.3  | 26.0 | 25.4  |
| $V_{\text{max}}$ (V)    | 0.903 | 1.89  | 1.01 | 1.89  |
| $R_{\text{Electrode}}$ (Ω) | 13.55 | 18.29 | 12.03 | 19.58 |
| Averaged $N$           | 798   | 815   | 869  | 850   |
| $f_s = f_i$ (GHz)      | 514   | 522   | 526  | 557   |
| $|\alpha|$             | 1.25  | 0.402 | 1.71 | 0.136 |
| $|\beta|$              | 1.16  | 1.22  | 1.47 | 0.737 |
| $\delta, \gamma$ (deg.) | 165   | 143   | 139  | 144   |
| $f_s$ (GHz)            | 522   | 507   | 526  | 495   |
| $f_i$ (GHz)            | 519   | 519   | 538  | 523   |
| $|\alpha|$             | 1.10  | 0.472 | 1.54 | 0.69  |
| $|\beta|$              | 0.93  | 1.33  | 1.42 | 0.74  |
| $\delta, \gamma$ (deg.) | 162   | 151   | 143  | 33    |
FIG. 1. (a) A microscope picture of the device (left) and a schematic drawing of the device and their connections for $B \parallel E$ (right, orange) and $B - E$ (yellow). Separations between mesas ($d$) are 87.5 (BC), 412 (BE), and 256 (CE) $\mu$m. (b,c) Current-voltage characteristics and bolometer response of $B \parallel E$ (b, color-coded), $B$ (c, black), and $B - E$ (c, violet). Solid and open triangles in the right panel of (c) point “local maxima” and “broad peak”, respectively.
FIG. 2. FTIR spectra of coupled simultaneous emissions (a) and uncoupled simultaneous emissions (b) together with individual emissions at vicinity mesa voltages. In (c), peak frequencies of emission spectra are plotted as functions of averaged mesa voltage: net voltage divided by 2 for series connections. Thick straight lines mean the ac Josephson relation with corresponding number of IJJ's, N. Thin gray lines are drawn for Ns with a step of 20.
FIG. 3. Polar plot of transmission intensity (a) and polarization ellipses (b) of C, E, and their combinations. Curves in (a) are least-square fitting curves to derive SPPs. Ellipses in (b) represent locus of electric vector of emitted EM waves at the corresponding mesa voltages. Color correspondences are same as (a). Rectangle contours are scaled outlines of the mesas.
FIG. 4. Frequency evolution of Stokes polarization parameters, \( S_0, \tilde{S}_1, \tilde{S}_2, \tilde{S}_3, DOP \) (top of three), \( \text{arg}(\beta/\alpha) \) (middle), \(|\alpha|, |\beta|\) (bottom) for B \( \parallel E \) (a), B–E (b), C\( \parallel E \) (c), and C–E (d). Frequencies are estimated from bias voltages via the ac Josephson relation with values listed in Table I.
FIG. 5. Frequency evolution of $\text{arg}(\beta/\alpha)$ (green, right axis), $|\alpha|$, $|\beta|$ (black and red, left axis) with continuous bases for $B\parallel E$ (a) and $B-E$ (b). Vertical purple bars represents frequencies of coupled emissions.