Directed closure measures for networks with reciprocity

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The study of triangles in graphs is a standard tool in network analysis. Real-world networks are often directed and have a significant fraction of reciprocal edges. Direction leads to many complexities, and it can be difficult to “measure” this structure meaningfully. We propose a collection of directed closure values that are analogues of the classic transitivity measure (the fraction of wedges that participate in triangles). Our study of these values reveals a wealth of information of the nature of direction. For instance, we immediately see the importance of reciprocal edges in forming triangles and can measure the power of transitivity. Surprisingly, the chance that a wedge is closed depends heavily on its directed structure. We also observe striking similarities between the triadic closure patterns of different web and social networks.

I. INTRODUCTION

The study of triangles is by now a classic tool in the analysis of large-scale networks. The focus on triangles has its roots in a variety of fields: in social sciences as a manifestation of various theories, in physics as local measures of clustering, in biology as motifs. Yet most contemporary data mining and massive graph analysis methods first convert real-world interaction data (think of this as a graph with attributes) into an undirected graph, and then work on this graph. It is a major challenge to account for the attributes on edges.

Most social networks, web networks, and product networks are all truly directed networks. Moreover, directed networks often have a significant percentage of reciprocal edges. Newman et al. [1] shows that the fraction of such edges in commonly studied graphs is quite high, and subsequent studies underlined the importance of such edges in virus/news spreading and understanding the network formation [2–4].

The set of triangles (and wedges) involving directed and reciprocal edges is rich and holds information about the underlying dynamics [5–9]. But it is challenging to make sense of this information and also compare different graphs (from varied sources) along these metrics.

We focus on a directed graph (digraph) \( G = (V, E) \). In a standard digraph, all edges are just ordered pairs of vertices of the form \( (i, j) \). We will think of the graph as having two different types of edges: directed and reciprocal. A reciprocal edge is technically a pair \( \{(i, j), (j, i)\} \), which we merge into a single reciprocal edge. We do not think of a reciprocal edge as containing two directed edges, but consider it to be a special edge on its own. Observe that reciprocal edges are essentially undirected. The total number of edges is the sum of the number of directed edges and reciprocal edges. In our figures, reciprocal edges are depicted as double-headed arrows. We define reciprocity of a graph, \( r \), as the ratio of the number of reciprocal edges to the total number of edges. Note that our definition is slightly different than that of [1].

A wedge is a pair of edges that share an endpoint, and a triangle is a set of three (unparallel) edges that are incident on a set of three vertices. We have 6 different types of wedges and 7 different types of triangles. We give more details about these structures in §II. We give the list of directed wedges and triangles with reciprocity in Figure 1. The earliest construction of this list is by Holland and Leinhardt [5].

\[ r = \frac{\text{number of reciprocal edges}}{\text{total number of edges}} \]

\[ (1) \]

\[ (i): \text{out} \quad \quad (ii): \text{in-out} \quad \quad (iii): \text{in} \quad \quad (iv): \text{reciprocal-in} \quad \quad (v): \text{reciprocal-out} \quad \quad (vi): \text{double-reciprocal} \]

\[ (a) \text{Directed wedges} \]

\[ (a): \text{simple acyclic} \quad \quad (b): \text{simple cycle} \]

\[ (c): \text{out-reciprocal} \quad (d): \text{one-reciprocal cycle} \]

\[ (e): \text{in-reciprocal} \quad (f): \text{two-reciprocal cycle} \]

\[ (g): \text{three-reciprocal cycle} \]

\[ (b) \text{Directed triangles} \]

FIG. 1: Directed structures
A. Main results of this paper

We generalize the classic notion of transitivity (pg. 243 of [10]), which is also called the global clustering coefficient, to digraphs. As described formally in §IIA, this yields a set of 15 directed closure values that provide a triadic pattern of a digraph. We perform experiments on a set of publicly available datasets. We present the directed closure information in a succinct form that allows comparison of different graphs. This leads to a series of observations.

- **Heterogeneity of closure:** We find the closure fractions of wedges vary greatly depending on the wedge type. In-wedges ((iii) in Figure 1a) usually dominate the graph but are rarely closed. In many cases, all other wedge types close frequently.
- **Reciprocity induces closure:** For every graph we analyze, the presence of a reciprocal edge in a wedge greatly increases the chance of closure. In other words, wedges with reciprocal edges participate in triangles more frequently than (uniform) random wedges.
- **The power of transitivity:** Simple and one-reciprocal cycles ((b) and (d) in Figure 1b) are very infrequent. These triangles have relations \( u \rightarrow v, v \rightarrow w \) (where either of these might be reciprocated), but \( u \) does not point to \( v \). The fact that they are so rare suggest the power of transitivity in the underlying dynamics. This appears to validate the importance of transitivity, as posited by Holland and Leinhardt [5] in the social science community. (Recent results of Leskovec et al. [11] in signed networks make comparable observations). These observations also underscore the importance of reciprocity, since it distinguishes triangles without transitivity from those that have it.
- **The non-randomness of direction:** We consider a simple random model of direction in an underlying undirected graph and compute directed closure values for this model. The predictions from this are significantly different from the actual data, showing that our findings indicate a deep directed structure in real-world networks. These observations show the importance of direction and reciprocity, which is not emphasized enough in analyses of social networks. Designing meaningful measures related to directed triangles and interpretable presentations is an important step in understanding digraphs. This work is a step in that direction. The wealth of information that is obtained by looking at directed closure values shows the importance of these measures.

B. Previous work

The earliest study of directed triads with reciprocity, to our knowledge, is in the social sciences, by Holland and Leinhardt [5]. They explicitly list the 16 different possible triads (including the 3 patterns with at most one edge) and count them in various social networks of the time. They also try to measure the effects of reciprocity in network formation. This is called the triad census. Skvoretz [12] and Skvoretz et al. [13] use these numbers of predict various biases in network formation. In a more recent study, Faust [8] computes a triad census on many graphs to compare their structure. Most of this work has been restricted to small data sets (at most hundreds of nodes). Finding such triads has been referred to as motif finding in the bioinformatics community [6]. Directed triangle counts have been used to define enhanced modularity measures [14]. Simpler versions of triad census counts have also been used to analyze gaming data [9]. Szell et al. also perform triadic analysis on gaming data [15, 16].

Much work on triangles has been done in the physics community. A classic local measure of triangle density is the clustering coefficient, introduced by Watts and Strogatz [17]. Fagiolo [7] proposes a local clustering coefficient measure for directed networks, though it ignores reciprocity. Ahnert and Fink [18] construct “clustering coefficients signatures” from these measures and classify directed networks. Recent work of Winkler and Reichardt [19] discusses the occurrence of the 16 different induced subgraphs on 3 vertices. They show the interdependence of these frequencies, and give \( Z \)-scores (with respect to an ERGM graph distribution) for real graphs. This is closely related to our study, though they do not focus on closure ratios specifically. We demonstrate how these ratios unearth common trends for diverse real networks, that raw counts do not reveal.

Leskovec et al. [11] study signed networks and validate (and extend) the theory of balance [20, 21]. They study the behavior of signed triangles to show that theory of balance does not suffice to explain networks. They also look at direction, but their datasets do not involve much reciprocity. It would be interesting to combine their work with our measures of directed closures.

II. THE DIRECTED CLOSURES

We begin with some notation and introduction to the directed structures in Figure 1a and Figure 1b. We use small Roman numerals to index the types of wedges, and small Latin letters for triangles. Furthermore, \( \psi \) is used to denote a variable wedge type, and \( \tau \) for a variable triangle type. We also give some names for further reference. (Holland and Leinhardt [5] have a naming scheme for directed triads that involve a triple of numbers with a letter. We deviate from this notation because it is easier to remember names than 3 digit codes.)

We stress that these types form a partition of all wedges and triangles. Since reciprocal edges are special, we do not think of (say) the reciprocal-out wedge containing an out wedge.

For each vertex \( v \), we have three associated degrees: the indegree, outdegree, and reciprocal degree. These are denoted by \( d_v^-, d_v^+, \) and \( d_v^\psi \). The total degree \( d_v = d_v^- + d_v^+ + d_v^\psi \). We mention some of the salient features...
of these directed structures.

- **Basic vs reciprocal structures:** The structures without reciprocal edges form the first rows in both Figure 1a and Figure 1b. There are only 3 types of wedges and 2 types of triangles, underscoring the importance of reciprocity.

- **Cyclic relations:** Triangle types (b), (d), (f), (g) all contain a cycle, and there is a progression of 0, 1, 2, and 3 reciprocal edges.

- **The table of \( \chi(\psi, \tau) \) values:** Different triangle types naturally contain different types of wedges. This information is summarized by the function \( \chi(\psi, \tau) \), which we define as the number of type \( \psi \) wedges in type \( \tau \) triangles. The list of nonzero values of \( \chi(\psi, \tau) \) is provided in Table I. Each row contains the wedge information of that triangle type. There are 15 nonzero entries in this table. 

| Triangle types \( \tau \) | Wedge types \( \psi \) | i | ii | iii | iv | v | vi |
|---------------------------|------------------|---|----|-----|----|---|----|
| a                         | m               | 1 | 1 | 1   |     |    |    |
| b                         | n               | 3 |    |     |    |    |    |
| c                         | o               | 1 | 1 | 1   |     |    |    |
| d                         | p               | 1 | 2 |     |    |    |    |
| e                         | q               | 1 | 1 | 1   |     |    |    |
| f                         | r               | 1 | 1 |     |    |    |    |
| g                         | s               | 3 |    |     |    |    |    |

**TABLE I:** Number of occurrences of each wedge type per triangle type: \( \chi(\psi, \tau) \).  

- **Wedge counts:** For vertex \( v \), let \( W_v, \psi \) be the set of \( \psi \)-wedges centered at \( v \). It is routine to compute \( |W_v, \psi| \) given the degrees of \( v \). This is summarized in Table II.

| \( \psi \) | \( \frac{d_v^\psi}{2} \) | \( d_v^\psi d_v^\psi \) | \( \frac{d_v^\psi}{2} \) | \( d_v^\psi d_v^\psi \) | \( d_v^\psi d_v^\psi \) | \( \frac{d_v^\psi}{2} \) |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( W_v, \psi \) |                     |                 |                 |                 |                 |                 |

**TABLE II:** Number of wedges per vertex for each wedge type.

**A. \( (\psi, \tau) \)-closure**

The transitivity (or global clustering coefficient) is defined as \( 3|T|/|W| \) (\( T \) is the set of triangles and \( W \) is the set of wedges). Semantically, this is the fraction of wedges that participate in triangles.

In the undirected setting, a wedge is called closed if it participates in a triangle and open otherwise. We say that a \( \psi \)-wedge is \( \tau \)-closed if the open wedge participates in a type \( \tau \) triangle. The \( (\psi, \tau) \)-closure, \( \kappa_{\psi, \tau} \), is the fraction of \( \psi \)-wedges that are \( \tau \)-closed. Formally, let \( W_\psi \) be the set of \( \psi \)-wedges and \( T_\tau \) be the set of \( \tau \)-triangles.

\[
\kappa_{\psi, \tau} = \frac{\chi(\psi, \tau)|T_\tau|}{|W_\psi|}
\]

The number of \( \psi \)-wedges in \( \tau \)-triangles is \( \chi(\psi, \tau)|T_\tau| \). Note that if a type \( \tau \) triangle contains no type \( \psi \) wedge, then this quantity is just zero because of \( \chi(\psi, \tau) \). As mentioned earlier, there are 15 non-trivial \( (\psi, \tau) \)-closures.

**B. Representations**

We create a directed closure chart that combines all the \( \kappa_{\psi, \tau} \) values. We give an example for the web-Google [22] graph in Figure 2. The bars on the \( x \)-axis are indexed by the different wedge types, and the \( y \)-axis is \( \kappa_{\psi, \tau} \). We make a stacked bar chart with the different closure values, where the triangle types are shown in 7 different colors. For example, the blue part of the first bar is the fraction of out-wedges closing into trans triangles (\( \kappa_{1,0} \)). Some of the salient features follow.

1. **Single closure value:** Consider some wedge type and triangle type (say out-wedge and simple acyclic-triangle). The value \( \kappa_{1,0} \) is shown by the height of the blue part of the first bar. The height of the blue part in the second bar shows the fraction of in-out-wedges that are closed into a simple acyclic-triangle.

2. **Total closure of wedge type:** The total height of the bar is total fraction of closed wedges of that type. For example, we see that in-wedges close infrequently.

3. **Percentage of wedge type:** Underneath the wedge pictures is the percentage of that wedge type.

4. **Percentage of triangle type:** Underneath the legend for triangles is the percentage of that triangle type.

5. **Undirected transitivity:** The value of \( \kappa \) is marked by a thick dashed line.

**III. OBSERVATIONS ON CLOSURE CHARTS**

We analyze the directed closure properties of various real graphs, whose properties are presented in Table III. In this table, \( |V| \), \( |E| \), \( |W| \), and \( |T| \) correspond to the number of vertices, edges, wedges, and triangles, respectively. The reciprocity, \( r \), is the fraction of total edges that are reciprocal edges. The undirected transitivity \( (3|T|/|W|) \) is given by \( \kappa \).

**A. Similarities of directed closures**

Figure 2, Figure 3, and Figure 4 have the closure charts for three different web graphs: web-Google, web-Stanford, and web-BerkStan [22]. These graphs have vertices for webpages and directed edges for web links. Figure 5, Figure 6, and Figure 7 have the charts for three
TABLE III: Properties of the graphs

| Graph Name          | $|V|$   | $|E|$   | $|W|$   | $|T|$   | $r$   | $\kappa$ |
|---------------------|-------|-------|--------|--------|-------|--------|
| amazon0505          | 410K  | 3357K | 73M    | 3951K  | 0.55  | 0.162  |
| soc-Slashdot0902    | 82K   | 870K  | 75M    | 603K   | 0.84  | 0.024  |
| web-Stanford        | 282K  | 2312K | 3944M  | 11330K | 0.28  | 0.009  |
| web-BerkStan        | 685K  | 7601K | 27983M | 64691K | 0.25  | 0.007  |
| wiki-Talk           | 2394K | 5021K | 12594M | 9204K  | 0.14  | 0.002  |
| web-Google          | 876K  | 5105K | 727M   | 13392K | 0.31  | 0.055  |
| soc-Epinions1       | 76K   | 509K  | 74M    | 1624K  | 0.41  | 0.066  |
| web-NotreDame       | 326K  | 1470K | 305M   | 8910K  | 0.52  | 0.088  |
| youtube-links       | 1158K | 4945K | 1474M  | 3057K  | 0.79  | 0.066  |
| flickr-links        | 1861K | 22614K| 14675M | 548659K| 0.62  | 0.112  |
| soc-livejournal     | 5284K | 76938K| 7519M  | 310877K| 0.73  | 0.124  |

Social networks [22]. The vertices of soc-Epinions correspond to the members of Epinions, a consumer review site. A directed edge between users shows a trust relationship originating from one user (these are signed by trust/distrust, which we ignore). The vertices of soc-Slashdot [22] are users and edges represent tagging as friend or foe. The vertices of soc-livejournal [23, 24] are Slashdot users with edges denoting friendship (which is one-way).

Observe the uncanny similarity of the closure charts web graphs, despite them being from different sources (and different sizes). The color patterns are remarkably similar, showing similar distributions of different closures. The social networks show more variation, but the overall structure of the charts is not far from the web graphs. In general, we note that in-wedges rarely close and reciprocal wedges close much more frequently.

B. Heterogeneity of closure

The heterogeneity of wedge closure is quite clear from all the closure charts. Focus on the web graphs. Other than in-wedges, all other wedge types close frequently. The undirected transitivity is always below 0.05, but specific wedge types close more than 50% of the time (shown by the total height of the bar). In-wedges form a dominant majority of all wedges (more than 98%) but close infrequently. Indeed, the low value of transitivity is explained by the high percentage yet low closure of in-wedges.

The picture is not as dramatic in the social networks, but there is some variation in closures over the wedge types. Quite consistently, in-wedges do not close and double-reciprocal-wedges close more frequently.

C. Effect of reciprocity on closure rates

How does reciprocity change the chance of closure? Observe that in, in-out, and out-wedges contain no reciprocal edges, reciprocal-in and reciprocal-out-wedges have exactly 1 reciprocal edge, and double-reciprocal has 2 reciprocal edges. As the charts clearly indicate, having reciprocal edges increases the chance of closure a wedge. We do a comprehensive calculation on a variety of graphs in Figure 8.

Consider a graph and choose $k$ from $\{0, 1, 2\}$. Fix the set of wedges with $k$ reciprocal edges, and look at the fraction of those that close (into any triangle). This gives the data presented in Figure 8. Observe how there is consistently a monotonic (and often dramatic) increase in closure fractions as reciprocity increases. The average of chance of closure for a wedge without reciprocal edges is only 3%. But this number goes to 23% if one of the edges is reciprocal and further increases to 38% when both edges are reciprocal. This finding is consistent with the earlier reports about reciprocal edges, indicating stronger ties between two vertices [1–4]. It also underscores how important it is to consider direction in networks.

D. The power of transitivity

Throughout the closure charts, one notices the infrequency of simple cycle and one-reciprocal cycle triangles. These are colored light blue and yellow, and one can see how little of those colors are present (or one can directly look at their percentages). Let us focus on triangles that contain a cycle showing a “cyclic” relationship. These are exactly simple cycle, one-reciprocal cycle, two-reciprocal cycle, and three-reciprocal cycle triangles. (These are given in light blue, yellow, brown, and pink, respectively.) Now consider transitive relations that are not reciprocated. For example, $A$ connects to $B$ who connects to $C$, but $A$ does not connect to $C$. When a triangle contains a cycle, a reciprocated transitive relationship creates a reciprocal edge.

Since simple cycle triangles have no reciprocal edges, there are three transitive relations that are not reciprocated. Analogously, for one-reciprocal cycle-triangle, there are two such unreciprocated relations. And for two-reciprocal cycle and three-reciprocal cycle triangles, these numbers are one and zero.

So we ask, when a triangle contains a cycle, does it...
FIG. 2: Directed closure for web-Google

FIG. 3: Directed closure for web-Stanford

FIG. 4: Directed closure for web-BerkStan

FIG. 5: Directed closure for soc-Epinions1

FIG. 6: Directed closure for livejournal

FIG. 7: Directed closure for soc-Slashdot0902
FIG. 8: How reciprocity increases closure rates: the x-axis goes over various graphs. The colored bars correspond to wedges with 0, 1, or 2 reciprocal edges. The y-axis gives the fraction of those wedges that close (into any triangle).

FIG. 9: Power of transitivity: For each graph in our collection, we plot the percentages of (different) triangle types containing a cycle. Each bar corresponds to a single graph, and the stacked bar charts give the percentages of the 4 different triangle types. Note the dominance of pink and brown (three-reciprocal cycle and two-reciprocal cycle triangles).

contain unreciprocated transitive relations? One would think that a cycle indicates a strong tie between three vertices, and so reciprocation is expected. This is exactly what we see in Figure 9, quite strongly over practically all graphs. Almost all triangles with a cycle are either two-reciprocal cycle or three-reciprocal cycle triangles. We almost never see any simple cycle triangles, shown by the lack of light blue in Figure 9. Again, this is more evidence that reciprocal edges play an important role in graph structure. The results demonstrate that the power of transitivity of real world networks. One can observe that social relationships carried forward two steps (as a transitive relation) almost always lead to reciprocation.

IV. NULL MODELS FOR \((\psi, \tau)\)-CLOSURE

In the previous section, we made several observations about the \((\psi, \tau)\)-closure rates in real graphs. How significant are these results? Can they be explained merely by the reciprocity of a graph? We discuss a null hypothesis, somewhat similar to that in [19]. This is based on assigning the type of each edge only based on the reciprocity of the graph. We start by making the graph undirected and add direction/reciprocity randomly as follows. If \((u, v)\) is an undirected edge, we make it reciprocal with probability \(r\); we direct it from \(u\) to \(v\) with probability \((1-r)/2\), we direct from \(v\) to \(u\) with probability \((1-r)/2\).

Based on this model, the probabilities of an undirected wedge and/or triangle being of a certain type are presented in Table IV and Table V. We note the distinction from [19]. We start with the original (undirected) graph and add direction/reciprocity, as opposed to constructing the undirected graph in an ER fashion.

Table V reveals that observations of the previous section cannot be explained by randomness or reciprocity. For instance, if we compare the expected fractions of the last two triangles two-reciprocal cycle and three-reciprocal cycle, we see that two-reciprocal cycle should be more frequent when the reciprocity, \(r < 0.75\). Even though this condition holds in most of the graphs in our data set, we observe the contrary behavior in real data sets, and three-reciprocal cycle generally is more frequent than two-reciprocal cycle. Another observation is about simple cycle triangles. According to our null model, sim-
Directed Closure

FIG. 10: Closure chart of web-Google for random directions: We consider the undirected version of web-Google graph and added one-way and reciprocal edges according the random null model. Observe that the total closure for each wedge is identical, and how different this is from Figure 2.

ple acyclic and simple cycle triangles have the same dependence of reciprocity, and simple acyclic triangles are expected to be only 3 times more frequent than simple cycle triangles. However, simple acyclic triangles are much more frequent in practice. In other words, the null model can explain the sparsity of simple cycle triangles, but not their near absence.

Figure 10 illustrates how the directed closure chart would look when direction is random. We take the undirected version of web-Google graph and add one-way and reciprocal edges according the null model. If we compare this figure to Figure 2, we see a totally different distribution, pointing to the significance of our results. Here, we are only presenting the results for web-Google for brevity, but we observed the same trend in all other graphs.

Finally, in Figure 11 we look at two triangle types, out-reciprocal and one-reciprocal cycle, whose dependences on reciprocity are the same, but one is over-represented, while the other is under represented, compared to the expectation of the null hypothesis. Type out-reciprocal triangles are overrepresented in all graphs except web-NotreDame and youtube-links, while one-reciprocal cycle triangles are underrepresented in all graphs.

All these results show that the direction in triangles reveals a special structure, which cannot be explained by randomness or reciprocity.

V. CONCLUSIONS

We perform a detailed study of directed triangles in massive networks, by defining the set of directed closure measures. These quantities reveal a surprising amount of information about digraphs. We observe heterogeneity in closure rates of different wedges, strong effect of reciprocity in closure rates, the power of transitivity in the structure of triangles. Our results also show that these observations cannot be explained merely by randomness or reciprocity. We hope that this paper leads the way in deeper studies into digraphs.

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FIG. 11: Deviations from the null model: For various graphs, we plot the fraction of triangles of a given type together with what is predicted by the random null model. This is done for the out-reciprocal and one-reciprocal cycle triangles. Observe the large differences, showing that directed triangle distributions are far from random.

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