Ratio in Ratio Type Exponential Strategy for the Estimation of Population Mean

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Abstract
This paper is an attempt to develop an estimator for finite population mean. Motivated by Kiregyera (1984), a ratio in ratio type exponential strategy is developed for estimation of population mean in double sampling for stratification. To compare with relevant considered estimators, expressions for bias and mean squared error of the developed estimator have been derived. The developed estimator has been compared with usual unbiased estimator, Ige and Tripathi (1987), ratio estimator and ratio type exponential estimator given by Tailor et al (2014) theoretically as well as empirically.

Keywords: Bias, ratio in ratio type strategy, double sampling for stratification (DSS), finite population mean.

1 Introduction
Estimation is very common in almost all fields including agriculture, economics, population studies, consumer market etc. In the field of agriculture, government or any research organization or any one may be interested to know total or average production of a crop per district. This problem in
statistics, especially in sampling theory, is addressed as estimation of population mean. Thus in the field of agriculture total or average production of any crop can be estimated by different estimators in different suitable sampling techniques.

Ratio type estimators assume that population mean of auxiliary variate is known. But in various practical applications it is unknown. In this type of situations, double sampling procedure is used in which first a large sample is drawn to estimate population mean of auxiliary variate then a sample of required size is selected either from a large sample or directly from the population. Recently, Singh et al. (2012), Sharma and Tailor (2014) and Mehta and Tailor (2020) contributed significantly in double sampling.

Application of stratified random sampling is possible only if both sampling frame as well as and strata weights are known. Non-availability of strata weights compel researchers to think about use of double sampling for stratification. For example a shopkeeper of school uniforms want to estimate the average sizes of uniforms to sale in the locality. Suppose, he wants to apply stratified random sampling, he must have an idea about distribution of population according to gender i.e. the makeup of the gender in the locality. When such information is lacking, it restricts the application of stratified random sampling. This restriction about application of stratified random sampling shifts researchers on use of double sampling for stratification.

In DSS, a larger sample is drawn and stratified to estimate strata weights. Then, using simple random sampling without replacement (SRSWOR), a sample from each stratum is taken, and information on both the study and auxiliary variables are recorded for each stratum.

DSS technique was developed by Neyman (1938). Classical ratio and product estimators for population were studied in DSS technique initially by Ige and Tripathi (1987). Ratio & Product type exponential estimators in DSS technique were developed by Tailor et al. (2014). Tailor and Lone (2014) worked out a ratio-cum-product estimator for population mean.

Singh and Nigam (2020a, 2020b) suggested Ratio-Ratio type exponential estimator and Product-Product type exponential estimator for population mean.

Chand (1975) envisaged an idea of chaining of ratio estimators by using the ratio estimator of population mean of auxiliary variate i.e. of $\bar{X}$ based on known population mean of another auxiliary variate $z$ i.e. $\bar{Z}$. Kiregyera (1984) studied a ratio in regression estimator for population mean. Lone et al. (2020) studied an alternative to ratio and product type estimators of finite population mean in double sampling for stratification. Tailor and Janbandhu (2020)
extended the work of Chand (1975) using Bahl and Tuteja (1991) estimator and developed a chain- ratio type exponential estimator for population mean in double sampling procedure.

Work cited above, motivates authors to consider the problem of estimation of finite population mean in DSS technique with development of a ratio in ratio type exponential estimator of population mean.

1.1 Technical Procedure of DSS
Let $P$ be a population of size $N$ such that $P = [P_1, P_2, P_3, \ldots, P_N]$. From this population $P$

- a larger sample $s_1$ of size $n'$ using SRSWOR is drawn,
- $s_1$ is classified into strata of units $n'_h$ in $h$th stratum and
- after stratification, a sample of size $n_h$ units is selected from each stratum that constitutes the $n$ sized DSS sample.

Here such

$$n = \sum_{h=1}^{L} n_h, \quad n' = \sum_{h=1}^{L} n'_h, \quad n_h = v_h n'_h \ (0 < v_h < 1), \quad h = 1, 2, \ldots, L.$$ 

1.2 Reviewing a Few Established Estimators
Let $y$ as a study variable and $x$ and $z$ as auxiliary variables with

$$\bar{X} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} x_{hi}, \quad \bar{Y} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} y_{hi}, \quad \bar{Z} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} z_{hi}$$

are population mean of $y$, $x$ and $z$ respectively.

Here objective is to develop an estimator for population mean of the study variable.

In DSS technique, usual unbiased estimator of $\bar{Y}$ is defined by

$$\bar{y}_{ds} = \sum_{h=1}^{L} w_h \bar{y}_h.$$ 

(1)

The classical ratio estimator of $\bar{Y}$, given by Cochran (1940) were studied in DSS technique by Ige and Tripathi (1987) as

$$\bar{Y}_{R}^* = \bar{y}_{ds} \bar{x}_d^*,$$ 

(2)
where \( \bar{x}_{ds} = \sum_{h=1}^{L} w_h \bar{x}_h \) and \( \bar{y}_{ds} = \sum_{h=1}^{L} w_h \bar{y}_h \) are unbiased estimators of \( \bar{X} \) and \( \bar{Y} \) respectively based on second phase sample.

Ratio-type exponential estimator of \( \bar{Y} \), using exponential function, were envisaged by Bahl-Tuteja (1991) in simple random sampling as

\[
\hat{Y}_{Re} = \bar{y} \exp \left( \frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right). \tag{3}
\]

Bahl-Tuteja (1991) estimator \( \hat{Y}_{Re} \) was studied by Tailor et al. (2014) in DSS technique as

\[
\hat{Y}_{ds \text{Re}} = \bar{y}_{ds} \exp \left( \frac{\bar{x}' - \bar{x}_{ds}}{\bar{x}' + \bar{x}_{ds}} \right). \tag{4}
\]

where \( \bar{x}' = \sum_{h=1}^{n_h} w_h \bar{x}'_h \) is an unbiased estimator of \( \bar{X} \) based on first phase sample.

### 2 Developed Estimator

Motivated by Kiregyera (1984), a ratio in ratio-type exponential estimator for finite population mean using known population mean of the second auxiliary variable \( z \) in DSS technique is developed as

\[
\hat{Y}_{CRR}^{ds} = \bar{y}_{ds} \exp \left( \frac{\bar{x}' \left( \frac{\bar{Z}}{\bar{Z}} \right) - \bar{x}_{ds}}{\bar{x}' \left( \frac{\bar{Z}}{\bar{Z}} \right) + \bar{x}_{ds}} \right). \tag{5}
\]

where \( \bar{z}' = \sum_{h=1}^{n_h} w_h \bar{z}'_h \) is an unbiased estimator of \( \bar{Z} \) based on first phase sample.

The bias and \( MSE \) of the developed estimator \( \hat{Y}_{CRR}^{ds} \) can be easily obtained by assuming

\[
\bar{y}_{ds} = \bar{Y} (1 + e_o), \quad \bar{x}_{ds} = \bar{X} (1 + e_1), \quad \bar{x}' = \bar{X} (1 + e'_1) \quad \text{and} \quad \bar{z}' = \bar{Z} (1 + e'_2)
\]

such that \( E(e_o) = E(e_1) = E(e'_1) = E(e'_2) = 0 \) and

\[
E(e_o^2) = \frac{1}{Y^2} \left[ S_y^2 \left( \frac{1 - f}{n'} \right) + \frac{1}{n'} \sum_{h=1}^{L} W_h S_{yh}^2 \left( \frac{1}{Vh} - 1 \right) \right],
\]

\[
E(e_1^2) = \frac{1}{X^2} S_x^2 \left( \frac{1 - f}{n'} \right),
\]
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\[E(e_1^2) = \frac{1}{X^2} \left[ S_x^2 \left( \frac{1-f}{n'} \right) + \frac{1}{n'} \sum_{h=1}^{L} W_h S_{xh} \left( \frac{1}{v_h} - 1 \right) \right],\]

\[E(e_2^2) = \frac{1}{Z^2} S_z^2 \left( \frac{1-f}{n'} \right),\]

\[E(e_0e_1) = \frac{1}{YX} \left[ \frac{1-f}{n'} S_{yx} + \frac{1}{n'} \sum_{h=1}^{L} W_h S_{yxh} \left( \frac{1}{v_h} - 1 \right) \right],\]

\[E(e_1e'_1) = \frac{1}{X^2} S_x^2 \left( \frac{1-f}{n'} \right),\]

\[E(e_0e'_1) = \frac{1}{YX} \left( \frac{1-f}{n'} \right) S_{yx}, \quad E(e_0e'_2) = \frac{1}{YZ} \left( \frac{1-f}{n'} \right) S_{yz},\]

\[E(e_1e'_2) = \frac{1}{XZ} \left( \frac{1-f}{n'} \right) S_{xz}, \quad E(e'_1e'_2) = \frac{1}{XZ} \left( \frac{1-f}{n'} \right) S_{xz}\]

where

\[f = \frac{n'}{N}, \quad S_x^2 = \frac{1}{N-1} \sum_{h=1}^{L} \sum_{i=1}^{N_h} (x_{hi} - \bar{X}_h)^2,\]

\[S_y^2 = \frac{1}{N-1} \sum_{h=1}^{L} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)^2,\]

\[S_z^2 = \frac{1}{N-1} \sum_{h=1}^{L} \sum_{i=1}^{N_h} (z_{hi} - \bar{Z}_h)^2,\]

\[S_{xh}^2 = \frac{1}{N_h-1} \sum_{i=1}^{N_h} (x_{hi} - \bar{X}_h)^2,\]

\[S_{yh}^2 = \frac{1}{N_h-1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)^2,\]

\[S_{zh}^2 = \frac{1}{N_h-1} \sum_{i=1}^{N_h} (z_{hi} - \bar{Z}_h)^2,\]
\[ S_{yx} = \frac{1}{N-1} \sum_{h=1}^{L} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)(x_{hi} - \bar{X}_h), \]

\[ S_{yz} = \frac{1}{N-1} \sum_{h=1}^{L} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)(z_{hi} - \bar{Z}_h), \]

\[ S_{xz} = \frac{1}{N-1} \sum_{h=1}^{L} \sum_{i=1}^{N_h} (x_{hi} - \bar{X}_h)(z_{hi} - \bar{Z}_h). \]

Substituting the above values in (5), the developed estimator \(\hat{\bar{Y}}_{ds}^{CRR}\) can be expressed as

\[ \hat{\bar{Y}}_{ds}^{CRR} - \bar{Y} = \bar{Y} \left[ \frac{1}{2}(2e_0 + e'_1 - e'_2 - e_1) + \frac{1}{8}(3e_1^2 - e_1^2 + 3e_2^2) \right. \]

\[ \left. - 4e_0e_1 + 4e_0e'_1 - 4e_0e'_2 - 2e_1e'_1 + 2e_1e'_2 - 2e'_1e'_2 \right] \]

(6)

Finally, approximately up to the first degree, bias of \(\hat{\bar{Y}}_{ds}^{CRR}\) is obtained as

\[ B(\hat{\bar{Y}}_{ds}^{CRR}) = \left[ \frac{1}{8n'f} \sum W_h \left( \frac{1}{v_h} - 1 \right) \frac{1}{X} (3R_1S_{xh}^2 - 4S_{yxh}) \right. \]

\[ + \left( \frac{1-f}{8n'} \right) \frac{1}{Z} (3R_2S_{zh}^2 - 4S_{yz}) \]

(7)

Square and expectation of (6) provides \(MSE\) of the developed estimator \(\hat{\bar{Y}}_{ds}^{CRR}\) as

\[ E[\hat{\bar{Y}}_{ds}^{CRR} - \bar{Y}]^2 = \bar{Y}^2 E \left[ \frac{2e_0 + e'_1 - e'_2 - e_1}{2} \right]^2 \]

\[ MSE(\hat{\bar{Y}}_{ds}^{CRR}) = \frac{1}{4} \bar{Y}^2 E[4e_0^2 + e'_1^2 + e'_2^2 + e_1^2 + 4e_0e'_1 - 4e_0e'_2 \]

\[ - 4e_0e_1 - 2e'_1e'_2 - 2e_1e'_1 + 2e_1e'_2] \]

\[ MSE(\hat{\bar{Y}}_{ds}^{CRR}) = \frac{\bar{Y}^2}{4} \left\{ \frac{4}{\bar{Y}^2} \left( \frac{1-f}{n'} \right) \right. \]

\[ + \left. \frac{1}{n'} \sum W_h \left( \frac{1}{v_h} - 1 \right) S_{yh}^2 \right\} \]
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\[
+ \frac{1}{X^2} S_x^2 \left( \frac{1 - f}{n'} \right) + \frac{1}{Z^2} S_z^2 \left( \frac{1 - f}{n'} \right) \\
+ \frac{1}{X^2} \left\{ S_x^2 \left( \frac{1 - f}{n'} \right) + \frac{1}{n'} \sum W_h \left( \frac{1}{v_h} - 1 \right) S_{xh}^2 \right\} \\
+ \frac{4}{YX} S_{yx} \left( \frac{1 - f}{n'} \right) - \frac{4}{YZ} \left( \frac{1 - f}{n'} \right) S_{yz} \\
- \frac{2}{XZ} \left( \frac{1 - f}{n'} \right) S_{xz} - \frac{2}{X^2} \left( \frac{1 - f}{n'} \right) S_x^2 \\
+ \frac{2}{XZ} \left( \frac{1 - f}{n'} \right) S_{xz} \\
\text{MSE}(\hat{Y}_{CRR}^{ds}) = \left[ S_y^2 \left( \frac{1 - f}{n'} \right) + \frac{1}{n'} \sum W_h \left( \frac{1}{v_h} - 1 \right) \\
\times \left( S_{yh}^2 + \frac{R_1 S_{zh}^2}{4} - R_1 S_{ysh} \right) \\
+ \left( \frac{1 - f}{4n'} \right) \left( R_2 S_z^2 - 4R_2 S_{yz} \right) \right].
\]

Finally, approximately up to the first degree, \( \text{MSE} \) of \( \hat{Y}_{CRR}^{ds} \) is obtained as

\[
\text{MSE}(\hat{Y}_{CRR}^{ds}) = \left[ S_y^2 \left( \frac{1 - f}{n'} \right) + \frac{1}{4n'} \sum W_h \left( \frac{1}{v_h} - 1 \right) \\
\times (4S_{yh}^2 + R_1 S_{zh}^2 - 4R_1 S_{ysh}) \\
+ \left( \frac{1 - f}{4n'} \right) \left( R_2 S_z^2 - 4R_2 S_{yz} \right) \right].
\]

(8)

3 Comparisons of Estimators

In this section, the developed estimator is being compared with relevant considered estimators from their efficiency point of view.
In DSS, the variance of unbiased estimator \( \tilde{y}_{ds} \), MSE of Ige & Tripathi (1987) and Tailor et al. (2014) estimator are respectively given by

\[
V(\tilde{y}_{ds}) = S_y^2 \left( \frac{1 - f}{n'} \right) + \frac{1}{n'} \sum_{h=1}^{L} W_h S_{yh}^2 \left( \frac{1}{\nu_h} - 1 \right), \tag{9}
\]

\[
\text{MSE}(\hat{Y}_{R}) = S_y^2 \left( \frac{1 - f}{n'} \right) + \frac{1}{n'} \sum_{h=1}^{L} W_h \left( \frac{1}{\nu_h} - 1 \right)
\times (S_{yh}^2 + R^2 S_{xh}^2 - 2R_1 S_{yxh}), \tag{10}
\]

\[
\text{MSE}(\hat{Y}_{Re}) = S_y^2 \left( \frac{1 - f}{n'} \right) + \frac{1}{n'} \sum_{h=1}^{L} W_h \left( \frac{1}{\nu_h} - 1 \right)
\times \left[ S_{yh}^2 + \frac{R_1^2}{4} S_{xh}^2 \left( 1 - \frac{\beta_{yxh}}{R_1} \right) \right]. \tag{11}
\]

Comparison of (8), (9), (10) and (11) exhibits that the developed chain ratio type exponential estimator \( \hat{Y}_{CRR} \) would perform better in terms of efficiency then

(i) \( \tilde{y}_{ds} \) if

\[
(1 - f)(R^2 S_z^2 - 4R_2 S_{yz}) < \sum W_h \left( \frac{1}{\nu_h} - 1 \right) (4R_1 S_{yxh} - R_1^2 S_{xh}^2) \tag{12}
\]

(ii) Ige-Tripathi (1987) estimator \( \hat{Y}_{R} \) if

\[
(1 - f)(R^2 S_z^2 - 4R_2 S_{yz}) < \sum W_h \left( \frac{1}{\nu_h} - 1 \right) (3R_1^2 S_{xh}^2 - 4R_1 S_{yxh}) \tag{13}
\]

(iii) Tailor et al. (2014) estimator \( \hat{Y}_{Re} \) if

\[
(1 - f)(R^2 S_z^2 - 4R_2 S_{yz}) < \sum W_h \left( \frac{1}{\nu_h} - 1 \right) (3R_1 S_{yxh}). \tag{14}
\]

4 Empirical Illustrations

In this section, two natural population data sets have been considered to test the performance of the developed estimator as compared to considered estimators with the help of numerical illustration. The description of considered natural population data sets is given below:
4.1 Population I – [Source: Singh and Choudhary (1971), p. 177]

$Y$: Productivity (MT/Hectare),
$X$: Production in ‘000 Tons and
$Z$: Area in ‘000 Hectare,

| Parameter | Stratum I | Stratum II |
|-----------|-----------|------------|
| $N_h$     | 10        | 10         |
| $n_h$     | 4         | 4          |
| $n_h$     | 6         | 6          |
| $\bar{Y}_h$ | 264.00   | 214.70     |
| $\bar{X}_h$ | 939.00  | 1121.50    |
| $\bar{Z}_h$ | 263.20   | 202.90     |
| $S_{yh}$  | 149.53    | 192.02     |
| $S_{zh}$  | 389.67    | 1165.20    |
| $S_{xh}$  | 162.85    | 178.54     |
| $S_{yzh}$ | 53277.00  | 68650.00   |
| $S_{yxh}$ | 23798.00  | 33841.00   |
| $S_{xzh}$ | 58729.00  | 60376.00   |
| $S_{y}^2$ | 31814.87  |            |
| $S_{x}^2$ | 31692.05  |            |
| $S_{yz}$  | 29562.58  |            |

4.2 Population II [Source: Murthy (1967), p. 228]

$Y$: Outcome,
$X$: Fixed capital and
$Z$: No. of workers,

| Parameter | Stratum I | Stratum II |
|-----------|-----------|------------|
| $N_h$     | 5         | 5          |
| $n_h$     | 2         | 2          |
| $n_h$     | 4         | 4          |
| $\bar{Y}_h$ | 1925.80  | 3115.60    |
| $\bar{X}_h$ | 214.40   | 333.80     |
| $\bar{Z}_h$ | 51.80    | 60.60      |
| $S_{yh}$  | 615.92    | 340.38     |
| $S_{zh}$  | 74.87     | 66.35      |
| $S_{xh}$  | 0.75      | 4.84       |
| $S_{yzh}$ | 39360.68  | 22356.50   |
| $S_{yxh}$ | 411.16    | 1536.24    |
| $S_{xzh}$ | 38.08     | 287.92     |
| $S_{y}^2$ | 668351.00 |            |
| $S_{x}^2$ | 34.84     |            |
| $S_{yz}$  | 1668.23   |            |
Table 1  

| Estimator | Population 1 | Population 2 |
|-----------|--------------|--------------|
| \( \bar{y}_{ds} \) | 100.00 | 100.00 |
| \( \hat{Y}_{ds}^{R} \) | 81.60 | 160.95 |
| \( \hat{Y}_{ds}^{Re} \) | 89.23 | 91.62 |
| \( \hat{Y}_{ds}^{CRR} \) | 164.45 | 198.76 |

Table 2  

| Comparisons | Population 1 | Population 2 |
|-------------|--------------|--------------|
| 1. \( MSE(\hat{Y}_{ds}^{CRR}) < MSE(\bar{y}_{ds}) \) | \(-35207.31 < 659428057.5 \) | \(-45841.35 < 28495362481 \) |
| 2. \( MSE(\hat{Y}_{ds}^{CRR}) < MSE(\hat{Y}_{ds}^{R}) \) | \(-35207.31 < 32779.90 \) | \(-45841.35 < 134415.76 \) |
| 3. \( MSE(\hat{Y}_{ds}^{CRR}) < MSE(\hat{Y}_{ds}^{Re}) \) | \(-35207.31 < 21244.76 \) | \(-45841.35 < 851363.89 \) |

5 Conclusions  

Present paper suggests a ratio in ratio type exponential estimator for population mean by replacing the sample mean \( x' \) by its ratio estimator using known population mean of another variable that works as auxiliary variable for the first auxiliary variable. As the usual ratio estimator provides better efficiency as compared to simple mean estimator, here instead of sample mean based on first phase sample a ratio estimator using known population mean of the another auxiliary variate \( Z \) i.e \( \bar{Z} \) has been used. Empirical illustrations given in Section 4 provides the evidence in favour of the above mentioned concept used in the development of \( \hat{Y}_{ds}^{CRR} \). Table 1 exhibits that estimator \( \hat{Y}_{ds}^{CRR} \) has the maximum percent relative efficiency as compared to all other considered estimators in both population data sets given by Singh and Choudhary (1971) and Murthy (1967). Table 2 provides the empirical values of the conditions under which the developed estimator has less \( MSE \). Table 2 also shows that all the conditions obtained in Section 3 are satisfied that reflects in Table 1 in terms of highest percent relative efficiency of the developed estimator \( \hat{Y}_{ds}^{CRR} \). Hence, \( \hat{Y}_{ds}^{CRR} \) is advised for practical use in the field for the estimation of population mean in comparison to usual unbiased estimator, ratio estimator given by Ige and Tripathi (1987) and ratio type exponential estimator suggested by Tailor et al. (2014) in case DSS technique if the conditions obtained in Section 3 are met.
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