An Area-based Intensity Measure for Incremental Dynamic Analysis

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Abstract

Incremental Dynamic Analysis (IDA) is a parametric analysis method based on nonlinear dynamic analysis. It involves performing nonlinear dynamic analysis of the structural model to a suite of ground motion records, each scaled to multiple levels of intensity (as measured by Intensity Measure, IM), and recording the responses (measured by Engineering Demand Parameter, EDP). An IDA curve combines the intensity measure of a site-specific ground motion with structural responses from nonlinear dynamic analysis of the given structure. It is thus an unavoidable fact that the IDA curves display large record-to-record variability. This observed dispersion relies closely on the IM used. In other words, different IMs can lead to various results from the IDA method. The aim of this paper is to find a proper IM with an advantage in efficiency for IDA analysis. An area-based IM, \( A(T_1) \), which considers the period elongation due to nonlinear deformation was proposed. A 6-story reinforced concrete frame was modeled to compare the efficiency of PSDA to assess the mean annual frequencies (MAF) by incremental dynamic analysis (IDA). It is the goal given a selected intensity measure (IM) can be acquired of exceeding a certain level of EDP, \( \lambda_{\text{edp}}(edp) \), which can be obtained by the total probability theorem as follows (Shome et al., 1998):

\[
\lambda_{\text{edp}}(edp) = \int_{\text{edp}} \int_{\text{IM}, \text{M}, \text{R}} P(\text{EDP} > \text{edp} | \text{IM}, \text{M}, \text{R}) f(\text{IM} | \text{M}, \text{R}) f(\text{M}, \text{R}) d\text{d} \text{m}(\text{im}) \tag{1}
\]

where \( \int_{\text{edp}} \int_{\text{IM}, \text{M}, \text{R}} P(\text{EDP} > \text{edp} | \text{IM}, \text{M}, \text{R}) f(\text{IM} | \text{M}, \text{R}) d\text{d} \text{m}(\text{im}) \) means the probability of EDP exceeding the level, \( \text{edp} \), given IM, magnitude (M) and distance (R); \( f(\text{IM} | \text{M}, \text{R}) \) is the conditional probability density function of the IM given M and R; and \( f(\text{M}, \text{R}) \) is the joint probability density function of M and R.

It is obvious from Eq. 1 that efforts should be made select the suitable EDP and IM to obtain \( \lambda_{\text{edp}}(edp) \). It is well known that the maximum inter-story drift of structures correlates well with structural damage and it is the most popular EDP to investigate the seismic vulnerability of buildings (Lin et al., 2011). The maximum inter-story drift is thus chosen as an EDP in this paper.

Another parameter, a desirable IM, should contain some characteristic properties. One of the most important properties of IMs is efficiency. An efficient IM can decrease the discreteness of the IDA results which means reducing the number of ground motions. Consequently, fewer nonlinear time-history analyses are needed to estimate the median EDP versus IM, saving the computing time. Sufficiency is another property of IM. A sufficient IM is conditionally statistically independent of ground motion characteristics like magnitude M and distance R. Thus, Eq. 1 can be simplified as Eq. 2:

\[
\lambda_{\text{edp}}(edp) = \int_{\text{edp}} P(\text{EDP} > \text{edp} | \text{IM}) d\lambda_{\text{im}}(\text{im}) \tag{2}
\]

Scaling robustness also plays a significant role in IDA. If an IM is robust, the results will not be biased in the estimation of the seismic demand as the IM scales.

Keywords: incremental dynamic analysis; intensity measure; area-based; efficiency; spectral acceleration value
up or down.

Because different IMs can lead to various results, many forms of IMs have been investigated in existing studies. Peak ground acceleration (PGA) has a long history and it is the most widely used IM by many researchers. Bazzurro et al. (1998) proposed $S_a(T)$, the elastic spectral acceleration at period $T_1$ (the fundamental period or the 1st period of a structure, hereinafter the same), as an IM. Vamvatsikos and Cornell (2002) summarized and compared both IMs of PGA and $S_a(T)$. It was proved that $S_a(T)$ is more efficient than PGA. However, $S_a(T)$ fails to perform well when the higher modes effect of structures cannot be ignored or the period elongation needs to be considered due to inelastic deformations.

As $S_a(T_1)$ cannot provide information about spectrum at other regions besides $T_1$, improved IMs based on spectral shape were studied. Cordova et al. (2000) introduced an IM $S^*$ that includes a second spectral acceleration value at the period $T_1$ to consider the inelastic behavior of structures. $T_1$ is recommended to be twice that of the fundamental period $T_0$. Similarly, Lin et al. (2010) proposed $S_{n1}$, choosing $CT_1$ for the second spectral acceleration value and the parameter $C$ is 1.5 from the ASCE/SEI 7-05 Standard (ASCE 2006). They both focused on a single value to represent period elongation. However, it is impossible for a structure to reach a fixed period after yielding under any earthquake load case. As a result, such IMs cannot precisely present the structural responses at all period ranges. Therefore, an IM, which depends on multi-values of spectral acceleration beyond fundamental period $I_{np}$, was proposed by Bojórquez and Iervolino (2010) which were proved to be more efficient than $S_a(T)$. $S_{n,\text{avg}}$, a part of $I_{np}$, is composed of 10 spectral acceleration values from $S_a(T_1)$ to $S_a(2T_1)$. It is a trend that a desirable IM will contain more spectral acceleration values reflecting the information on ground motions and structures. Recently, Adeli et al. (2012) suggested the area under acceleration spectrum at the range of a period from $1.2T_n$ to $1.5T_1$ as an IM (hereafter referred to as $A_0$), where $T_n$ is the period of the mode in which the cumulative modal mass participation exceeds 95%. $A_0$ can be treated as a kind of acceleration spectrum intensity and essentially, it is the sum of a large amount of spectral acceleration values with small intervals.

On the other hand, Lin et al. (2010) introduced $S_{n2}$ in the same paper for the structure of which the fundamental period is beyond 1.5s to take into account the higher modes effect by multiplying the spectral acceleration values in power function. And $S_{n2}$ considers the first two periods in one direction. Asgarian et al. (2012) proposed a similar IM but includes the first three periods. Besides, Zhou et al. (2013) introduced IMs called $S_{12}$ and $S_{123}$ to consider the first two and three periods respectively. Differently, the indexes of $S_{12}$ and $S_{123}$ are the weight of modal mass participation of the first two/three periods to reduce the value at higher modes because of its lower participation. In addition, Lu et al. (2013) proposed $\bar{S}_a$ that deals with the first $n$ periods in one direction equally without any index. And the relationship between the number $n$ and the fundamental period of structures $T_1$ was also deduced in order to make $\bar{S}_a$ more widely available.

All the IMs above that combine structures and ground motions by the calculation of spectral acceleration values can be regarded as spectral shape-based IMs. In this paper, an improved area-based IM called $A(T_1)$ was proposed for the structure dominated by the first mode effect. To simplify the calculation of area-based IM $A_0$, the area at the interval $[T_1,cT_1]$ under acceleration spectrum was used (see Fig.1.). There are two ways to define the value of parameter $c$: 1) 1.5 refers to the ASCE/SEI 7-05 Standard; 2) 2.0 refers to the experience from shaking table tests that a structure was expected to collapse when the fundamental period extended to 2$T_1$. Both cases were discussed in this paper compared to the existing IMs mentioned above.

All the IMs here can be divided into four groups (see Table 1.): Group 1 is composed of IMs in common use: PGA and $S_a(T_1)$. Group 2 considers the higher modes effect containing $S_{n2}$ and $S_{123}$; Group 3 considers the nonlinear property of structures including $S^*$, $S_{n1}$ and $S_{n,\text{avg}}$; and Group 4 includes the original area-based IM $A_0$ and the improved IM in this paper, $A(T_1)$.

2. Description of the Building

A 6-story reinforced concrete (RC) frame is modeled in this study. The plan layout is shown in Fig.2. The height of the first story is 4.2 m and the others are 3.6m. The dimensions for the frame columns and beams are 550 mm × 550 mm and 300 mm × 550 mm, respectively. The concrete strength grade of the members is C35 (compressive strength $f_{ck}$ is 23.4 MPa). The strength grade of longitudinal and stirrup bars is HRB335 (tensile strength $f_{ak}$ is 455 MPa and the modulus is $2.0 \times 10^5$ MPa) and

![Acceleration response spectrum](image-url)

**Fig.1.** Physical Diagram of $A(T_1)$

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Table 1. Intensity Measures for Discussion

| IM     | Description/expression                                                                 |
|--------|----------------------------------------------------------------------------------------|
| Group 1|                                                                                       |
| PGA   | peak ground acceleration                                                              |
| $S_a(T_i)$ | elastic spectral acceleration at the fundamental period $T_i$ (hereinafter the same) of a structure |
| Group 2|                                                                                       |
| $S_{12}$ | $S_{12}=S_a(T_i)^\alpha \cdot S_a(T_i)^\beta$, $\alpha=0.5$, $T_i$ is the fundamental period of a structure in one direction |
| $S_{31}$ | $S_{31}=S_a(T_i)^\alpha \cdot S_a(T_i)^\beta$, $\alpha=0.5$, $T_i$ is the fundamental period of a structure in one direction |
| $S_{avg}$ | $S_{avg}=(\Pi S_a(T_i))^{\frac{1}{n}}$, $n=10$, $T_i=2T_i$, $T_i$ is the equal interpolation point between $T_i$ and $2T_i$ |
| Group 3|                                                                                       |
| $A_{i}$ | area under acceleration spectrum at the interval $[1.2T_i, 1.5T_i]$, $T_i$ is the period of the mode in which the cumulative modal mass participation exceeds 95% |
| $A(T_i)$ | area under acceleration spectrum at the interval $[T_i, 1.5T_i]$ |
| $A(T_2)$ | area under acceleration spectrum at the interval $[T_2, 2T_2]$ |

HPB300 (tensile strength $f_{th}$ is 420 MPa and the modulus is $2.0 \times 10^5$ MPa), respectively. The frame is designed in accordance with Chinese codes by elastic design software PKPM. And the matching analytical model was built by software Perform-3D. According to the modal analysis results from Perform-3D, the fundamental period of the frame is 0.892s and modal mass participation is 84%. The second period in the same direction is 0.282s with a modal mass participation of 10%. Cumulative modal mass participation for the first two periods reaches nearly 95%, thus $T_2$ can be treated as $T_m$ in $A_i$.

For steel material, a bilinear stress-strain model was adopted without considering cyclic degradation. For concrete material, two types of models were used: one for unconfined concrete at the edge of a cross section based on the Chinese Code for Design of Concrete Structures (GB 50010-2010, 2010); another for confined concrete in the core area based on the stress-strain model proposed by Mander et al. (1988).

Because the stress-strain relationship of the concrete material in Perform-3D consists of straight lines instead of a curve, turning points should be found to fit the curve. In this paper, a model modified by Moehle et al. (2011) was used to simulate the state of concrete in compression (see Fig.4.).

3. Modeling of Elements

The analytical model was established by software Perform-3D with inelastic elements. A rigid diaphragm action is assumed for the frame. In this paper, beams and columns were modeled by a plastic zone model (Computers and Structures, Inc., 2006a; Computers and Structures, Inc., 2006b) as shown in Fig.3. This model has a fiber segment at each end, with a length of 0.5 times that of the member depth, and an elastic segment for the rest of the element. The fiber segment consists of two kinds of material, steel and concrete, the dimension and position of which are based on the reinforcement information calculated from the software PKPM.

4. Selection of Ground Motions

Recommended Seismic Design Criteria for New Steel Moment-Frame Buildings (FEMA 350, 2000) suggests that 10 to 20 ground motions are required for IDA. Thus in this study, 15 sets of ground motions...
were selected from Pacific Earthquake Engineering Research (PEER) for the frame based on the Chinese Code for Seismic Design of Buildings (GB 50011-2010, 2010) which requires the spectral acceleration at main periods of structures to conform to the standard response spectrum. Fig. 5. shows the 5% damped acceleration spectrum of the 15 selected ground motions and the detail information is described in Table 2.

![Acceleration Spectrum of Ground Motions](image)

### Table 2. Information of Ground Motions

| No. | Event                      | Station                          | Year |
|-----|----------------------------|----------------------------------|------|
| GM-1| Borrego Mtn                | San Onofre-SO Cal Edson          | 1968 |
| GM-2| Imperial Valley-06         | El Centro Array #12              | 1979 |
| GM-3| Chalfant Valley-04         | Zack Brothers Ranch              | 1986 |
| GM-4| Whittier Narrows-01        | Covina-S Grand Ave               | 1987 |
| GM-5| Whittier Narrows-02        | LA-Baldwin Hills                 | 1987 |
| GM-6| Loma Prieta                | Coyote Lake Dam (Downst)         | 1989 |
| GM-7| Loma Prieta                | Fremont-Emerson Court            | 1989 |
| GM-8| Northridge-01              | Jensen Filter Plant Generator    | 1994 |
| GM-9| Northridge-01              | LA-Pico & Sentous                | 1994 |
| GM-10| Northridge-01              | LA-Wonderland Ave                | 1994 |
| GM-11| Northridge-01              | Lakewood-Del Amo Blvd            | 1994 |
| GM-12| Chi-Chi, Taiwan-02         | TCU119                           | 1999 |
| GM-13| Chi-Chi, Taiwan-05         | CHY088                           | 1999 |
| GM-14| Chi-Chi, Taiwan-05         | HWA024                           | 1999 |
| GM-15| Chi-Chi, Taiwan-05         | TCU102                           | 1999 |

Fig. 5. Acceleration Spectrum of Ground Motions

### 5. Analysis for Intensity Measures

After ground motions were selected, the PGA of those ground motions were scaled with the increment of 100 gal each step from 70 gal to the value when the maximum inter-story drift, $\theta_{\text{max}}$, reaches 0.1 or the tangent slope equals 20% of the elastic slope which means the structure reaches the collapse capacity point according to the IDA method. Nonlinear time-history analysis of the frame was performed. The IDA result of the studied frame in terms of PGA and $\theta_{\text{max}}$ is presented in Fig. 6.

Generally, an efficient IM can reduce the dispersion of results and enhance the computing efficiency which is more practical and expedient for engineering projects. Therefore the efficiency is given full attention for comparison in this paper.

![IDA Curves](image)

Fig. 6. IDA Curves

Cornell et al. (2002) indicated that the relationship between IM and EDP can be expressed as Eq. 3:

$$\overline{EDP} = a IM^b$$  \hspace{1cm} (3)

where $\overline{EDP}$ is the mean value of engineering demand, $a$ and $b$ are regression coefficients. And this expression can be transformed to perform a linear regression of logarithms of $IM$ and $EDP$:

$$\ln(\overline{EDP}) = \ln a + b \ln IM$$  \hspace{1cm} (4)

The efficiency can be estimated by the standard deviation, $\beta_{E_{DP,IM}}$ through regression analysis:

$$\beta_{E_{DP,IM}} = \sqrt{\frac{\sum [(\ln(EDP) - \ln(aIM^b))^2]}{n-2}}$$  \hspace{1cm} (5)

where the discrete data $EDP_i$ is obtained from the $i^{th}$ nonlinear time history. Because the number of the data is fixed, $n$ is constant. Thus, the estimation of efficiency can be simplified into $\Sigma [(\ln(EDP) - \ln(aIM^b))^2]$, i.e., the residual sum of squares. The lower the residual sum of squares is, the more efficient the analytical results are.

The results can be seen in Fig. 7. and Table 3. The expression of Eq. 4 is adapted to a concise equation, $y = mx + n$. 

![Graph](image)
In Table 3., it is PGA, rather than $S_a(T_1)$ which has an advantage in efficiency by referring to previous research (Vamvatsikos and Cornell, 2002). Ground motion selection can be the factor that the spectral acceleration value at the point of the fundamental period of each ground motion is controlled by standard
response spectrum, which means the ratio of $S_a(T_1)$ and PGA is controlled. As PGA is scaled up, $S_a(T_1)$ goes up correspondingly. However, the ratio cannot be fixed definitely and there will be a deviation among ground motions for the same PGA. Moreover, the scaling is based on PGA. As a result it might lead to an approximate or even better result for PGA. Baker and Cornell (2005) also suggest that the parameter $\varepsilon$, an indicator of the 'shape' of the response spectrum, should be considered when selecting ground motions. Thus PGA can replace the position of $S_a(T_1)$ due to ground motion selection.

From results in Group 2, the residual sums of squares of IMs are large, showing little influence compared with traditional IMs from Group 1, for considering the higher modes effect. Those IMs are even less efficient than PGA. The main reason probably lies in the domination of the first mode effect of the frame as the corresponding modal mass participation is beyond 80%, which implies that the effect on higher modes can be ignored.

In Table 3., another important conclusion that can be obtained is that IMs from Group 3 and Group 4 have smaller residual sums of squares for taking period elongation into account. From $S_a$ and $S_{a1}$ to $A(T_1)$, with the number of spectral acceleration values of IMs at the range of elongation zone increasing from a single one to multiple values, the IM is more efficient. And the residual sum of squares of $A(T_1)$ are 21.47 for $A(T_1)$ ($c=1.5$) and 20.98 for $A(T_1)$ ($c=2$) which are smaller than other IMs (see Fig.8.). By comparison, the range of period from $T_1$ to $2T_1$ is preferred for the area-based IM. Furthermore, from the results in Group 4, the efficiency of original area-based IM $A_{in}$ is far behind $A(T_1)$, in spite of the advantage compared to PGA and $S_a(T_1)$. This indicates that the area in the range of the higher modes zone of IM cannot provide extra efficiency. On the contrary, it even reduces the efficiency in terms of the IM that only considers the period elongation. Hence, the improved or simplified area-based IM is recommended in this study, especially for structures dominated by the first mode effect.

6. Conclusions
In this paper, an improved area-based IM was proposed to find a proper spectral IM to improve the efficiency of analysis in IDA. A 6-story reinforced concrete frame was modeled and 15 sets of ground motions were selected for IDA. The residual sum of squares is the index to measure the efficiency of eight existing IMs and the proposed one. Because of the domination of the first mode effect of the frame, IMs considering higher modes effect fail to perform well while IMs with the spectral acceleration values in the range of the elongation zone are more efficient than the widely used IMs, PGA and $S_a(T_1)$. With the number of the spectral acceleration values of IMs increasing, $A(T_1)$ is proved to be the most efficient. And it is a trend that the spectral shape will be the main ground motion feature expressing the engineering demand of structures.

One should note that the limitation still exists in this study that the frame model is by no means representative of all types of structures and it is the first mode domination. Further work is needed to generalize the solution and other factors such as higher modes should also be taken into account in area-based IMs.

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