NUMERICAL SIMULATION WITH LIGHT
WILSON-QUARKS

I. MONTVAY
Deutsches Elektronen-Synchrotron DESY
Notkestr. 85, D-22603 Hamburg, Germany
E-mail: istvan.montvay@desy.de

The computational cost of numerical simulations of QCD with light dynamical
Wilson-quarks is estimated by determining the autocorrelation of various quantities. In test runs the expected qualitative behaviour of the pion mass and coupling
at small quark masses is observed.

1. Introduction
In Nature there exist three light quarks (u, d and s) which determine hadron
physics at low energies. Numerical simulations on the lattice have to deal
with them – which is not easy because the known simulation algorithms
slow down substantially if light fermions are involved.

At present most dynamical ("unquenched") simulations are performed
with relatively heavy quarks, in case of Wilson-type lattice fermions typi-
cally at masses above half of the strange quark mass, and then chiral
perturbation theory (ChPT)\(^1\) is used for extrapolating the results to the
small u- and d-quark masses. This extrapolation is better under control if
the dynamical quarks are as light as possible.

In this talk I report on some recent work of the qq+q Collaboration
concerning numerical simulations with light Wilson-quarks\(^2,3,4\). We used
the two-step multi-boson (TSMB) algorithm\(^5\) which turned out to be rela-
tively efficient for light fermions in previous investigations of supersym-
metric Yang-Mills theory. (For a review with references see ref.\(^6\)).

2. Estimates of computational costs
In numerical Monte Carlo simulations the goal is to produce a sequence
of statistically independent configurations which can be used for obtaining
estimates of expectation values of different quantities. A measure of inde-
dependence is provided by the values of the integrated autocorrelation lengths in the configuration sequence, usually denoted by $\tau_{\text{int}}^Q$. This depends on the quantity $Q$ of interest and gives the distance of statistically independent configurations.

![Figure 1](image-url)  
Figure 1. Power fit of the average plaquette autocorrelation given in units of $10^6 \cdot \text{MVM}$ as a function of the dimensionless quark mass parameter $M_r$. The best fit of the form $c M_r^z$ is at $c = 7.92(68)$, $z = -2.02(10)$.  

The $\text{qq+q}$ Collaboration has recently performed a series of test runs on $8^3 \cdot 16, 12^3 \cdot 24$ and $16^4$ lattices with $N_f = 2$ and $N_f = 2 + 1$ quark flavours. The quark masses were in the range $\frac{1}{6} m_s < m_q < 2 m_s$ and the autocorrelations of several quantities as average plaquette, smallest eigenvalue of the fermion matrix, pion mass and coupling etc. have been determined. The error analysis and integrated autocorrelations in the runs have been obtained using the linearization method of the ALPHA collaboration\[7\].

The computational cost of obtaining a new, independent gauge configuration in an updating sequence with dynamical quarks can be parametrized,
for instance, as

$$C = F \left( r_0 m_\pi \right)^{-z_\pi} \left( \frac{L}{a} \right)^{z_L} \left( \frac{r_0}{a} \right)^{z_a}.$$  (1)

Here $r_0$ is the Sommer scale parameter, $m_\pi$ the pion mass, $L$ the lattice extension and $a$ the lattice spacing. The powers $z_{\pi,L,a}$ and the overall constant $F$ are empirically determined. The unit of “cost” can be, for instance, the number of necessary fermion-matrix-vector-multiplications (MVMs) or the number of floating point operations to be performed. For an example on the quark mass dependence of the cost see figure 1 which is taken from ref.2. This shows that the quark mass dependence in case of the average plaquette is characterized by a power $z_\pi \simeq 4$. For other quantities, as the smallest eigenvalue of the fermion matrix and the pion mass, a smaller power $z_\pi \simeq 3$ is observed.

Other tests on the lattice volume and lattice spacing dependence showed a surprisingly mild increase in both directions if compared to the data2 on $8^3 \cdot 16$ lattice at $a \simeq 0.27 \text{ fm}$. Some results on the volume dependence of the average plaquette autocorrelation $\tau^\text{ plaq}_{\text{int}}$ are given in the first four lines of table 1. The runs with label (e16) and (E16) belong to almost the same

| label | lattice | $\beta$ | $\kappa$ | $\tau^\text{ plaq}_{\text{int}}$ [flop] |
|-------|---------|---------|---------|---------------------------------|
| (e)   | $8^3 \cdot 16$ | 4.76    | 0.190   | $4.59(37) \cdot 10^{13}$ |
| (e16) | $16^3$   | 4.76    | 0.190   | $7.5(1.3) \cdot 10^{14}$ |
| (h)   | $8^3 \cdot 16$ | 4.68    | 0.195   | $1.7(6) \cdot 10^{14}$ |
| (h16) | $16^3$   | 4.68    | 0.195   | $1.10(17) \cdot 10^{15}$ |
| (E16) | $16^3$   | 5.10    | 0.177   | $2.1(4) \cdot 10^{14}$ |

quark mass ($M_r \simeq 1.4$) but have by a factor of about 1.5 different lattice spacing. A typical expectation for the power governing the lattice spacing dependence is $z_a = 2$ which would imply by a factor of 2.25 larger value for (E16) than for (e16). Compared to run (e) $z_L = 4$ and $z_a = 2$ would imply for (E16) an increase by a factor 36 instead of the actual factor $\simeq 4$. The observed relative gain is partly due to some improvements of the simulation algorithm (see ref.3) and is, of course, very welcome in future simulations.
Test of $\chi$PT logarithms on $8^3 \times 16$

![Figure 2. Fits of the pseudoscalar meson mass-squared with the one-loop ChPT formula.](image)

3. Chiral logarithms?

The behaviour of physical quantities, as for instance the pseudoscalar meson ("pion") mass $m_\pi$ or pseudoscalar decay constant $f_\pi$ as a function of the quark mass are characterized by the appearance of chiral logarithms. These chiral logs, which are due to virtual pseudoscalar meson loops, have a non-analytic behaviour near zero quark mass of a generic form $m_q \log m_q$. They imply relatively fast changes of certain quantities near zero quark mass which are not seen in present data\textsuperscript{8,9,10,11,12}.

Although we have rather coarse lattices ($a \simeq 0.27$ fm) and, in addition, up to now we are working with unrenormalized quantities – without the $Z$-factors of multiplicative renormalization – it is interesting to see that the effects of chiral logs are qualitatively displayed by our data. Fits with the ChPT-formulas (see, for instance, ref.\textsuperscript{13,14}) are shown in figures 2 and 3. These are taken from ref.\textsuperscript{4,15} where the fit parameters are also quoted. To see the expected qualitative behaviour with chiral logarithms in numerical simulations at small quark masses is quite satisfactory but for a quantitative determination of the ChPT parameters one has to go to smaller lattice spacings.
Test of $\chi$PT logarithms on $8^3 \times 16$

Figure 3. Fits of the pseudoscalar meson decay constant with the one-loop ChPT formula.

Bibliography

1. J. Gasser and H. Leutwyler, *Annals Phys.* **158**, 142 (1984).
2. $q\bar{q}+q$ Collaboration, F. Farchioni, C. Gebert, I. Montvay and L. Scorzato, *Eur. Phys. J.* **C26**, 237 (2002); hep-lat/0206008.
3. $q\bar{q}+q$ Collaboration, F. Farchioni, C. Gebert, I. Montvay and L. Scorzato, hep-lat/0209038.
4. $q\bar{q}+q$ Collaboration, F. Farchioni, C. Gebert, I. Montvay and L. Scorzato, hep-lat/0209038.
5. I. Montvay, *Nucl. Phys.* **B466**, 259 (1996); hep-lat/9510042.
6. I. Montvay, *Int. J. Mod. Phys.* **A17**, 2377 (2002); hep-lat/0112007.
7. ALPHA Collaboration, R. Frezzotti, M. Hasenbusch, U. Wolff, J. Heitger and K. Jansen, *Comput. Phys. Commun.* **136**, 1 (2001); hep-lat/0009027.
8. CP-PACS Collaboration, A. Ali-Khan et al., *Phys. Rev.* **D65**, 054505 (2002); hep-lat/0105015.
9. UKQCD Collaboration, C.R. Allton et al., *Phys. Rev.* **D65**, 054502 (2002); hep-lat/0107021.
10. JLQCD Collaboration, S. Aoki et al., *Nucl. Phys. Proc. Suppl.* **106**, 224 (2002); hep-lat/0110179.
11. JLQCD Collaboration, S. Hashimoto et al., hep-lat/0209091.
12. CP-PACS Collaboration, Y. Namekawa et al., hep-lat/0209073.
13. H. Leutwyler, *Nucl. Phys. Proc. Suppl.* **94**, 108 (2001); hep-ph/0011049.
14. S. Dürr, hep-lat/0208051.
15. C. Gebert, PhD Thesis, Hamburg University, 2002.