Abstract

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QCD DESCRIPTION OF PARTICLE SPECTRA UP TO LEP-1.5 ENERGIES AND THE RUNNING OF $\alpha_s$

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Recent results on the energy spectra in QCD jets are reported. Within the Modified Leading Logarithmic Approximation (MLLA) and the Local Parton Hadron Duality (LPHD) model one finds a very good description of the $e^+e^-$ data from the lowest up to the LEP 1.5 energies. A model with fixed $\alpha_s$ can be excluded, already in certain finite energy intervals. The fits also extrapolate smoothly into the region of small particle energies, in particular, the data follow a scaling prediction for the low energy limit derived from the colour coherence of the soft gluon emission.

1 Introduction

One of the important predictions of perturbative QCD on the intrinsic structure of jets concerns the energy spectra of particles. The particles of low energy $E$ are not multiplied with increasing jet energy $E_{\text{jet}}$ because of colour coherence in the cascading process and this yields the bell shaped spectrum in the variable $\xi = \log \frac{E_{\text{jet}}}{E}$, the so-called "lump-backed plateau".

Predictions on the spectrum have been carried out in the MLLA which takes into account the leading double logarithmic (DLA) results and all next-to-leading corrections of order $\sqrt{\alpha_s}$. Terms of higher order are included as well, although not completely, which allow to fulfill the initial condition for the parton cascade at threshold. These predictions, at the parton level, involve only two parameters: the QCD scale $\Lambda$ which determines the running of the coupling $\alpha_s$ and the transverse momentum cut-off $Q_0$ of the gluon emission. Remarkably, the observed hadron spectra are rather well described by the spectrum of partons if a low value $Q_0 \sim 250$ MeV of the order of hadronic masses is used and this observation has led to the LPHD hypothesis. As a justification of this approach is not yet available at a fundamental level it seems important to determine its range of applicability and its limitations. Also one would like to know to what extent the predictions are sensitive to QCD as the underlying theory. In this contribution the sensitivity of the energy spectra to the running $\alpha_s$ and the colour coherence - two properties specific to QCD as a field theory - are addressed. More details can be found in refs. 2, 4, 7.

2 Moment Analysis of Particle Spectra

To analyse the effect of the running $\alpha_s$ it is convenient to work with the moments of the $\xi$-spectrum of particles which evolve independently of each other with energy. They are defined by

$$< \xi^q > = \int d\xi \xi^q D(\xi, Y, \lambda) / N $$

where we use the logarithmic variables $Y = \log(E_{\text{jet}}/Q_0)$ and $\lambda = \log(Q_0/\Lambda)$; the normalization is by the multiplicity $N$. We consider finally the cumulant moments $K_q$ which are obtained from the $< \xi^q >$ moments by

$$K_q = < \xi > = \xi, \quad K_2 = \sigma^2 = < (\xi - \xi)^2 >, \quad K_3 = < (\xi - \xi)^3 >, \quad K_4 = < (\xi - \xi)^4 > - 3\sigma^4, \ldots $$

also one introduces the reduced cumulants $\kappa_q = K_q/\sigma^q$, in particular the skewness $\kappa_3$ and the kurtosis $k = \kappa_4$. The cumulant moments $K_q(Y, \lambda)$ behave at high energies like

$$K_q(Y, \lambda) = K_q(Y_0, \lambda) +$$

$$+ \int_{Y_0}^Y dy \left( - \frac{\partial}{\partial \omega} \right)^q \gamma_\omega[\alpha_s(y, \lambda)] \bigg|_{\omega=0}$$

where $Y_0$ is the initial energy. Here $\gamma_\omega[\alpha_s(y, \lambda)]$ denotes the anomalous dimension of the Laplace transform of the $\xi$-spectrum $dn/d\xi \equiv D(\xi, Y, \lambda)$. Eq. (3) shows directly the sensitivity of the cumulant moments to the running of $\alpha_s$. In particular, for fixed coupling $K_q \sim Y$ at high energies. In our applications, we take into account the initial condition at threshold for the jet to consist of only one parton which implies $K_q = 0$ at $Y_0 = 0$.

The MLLA prediction for the $\xi^q$ moments can be written for arbitrary $Q_0$ and $\Lambda$ as:

$$< \xi^q > = \frac{1}{N} \sum_{k=0}^{q} \binom{q}{k} (N_1 L_k^{(q)} + N_2 R_k^{(q)})$$

(2)
where \( N_1, N_2, L_k^{(q)} \) and \( R_k^{(q)} \) are known functions of \( a = 11N_c/3 + 2n_f/3N_c^2, b = (11N_c - 2n_f)/3 \), and \( \lambda \) where \( n_f, N_c \) denote the numbers of flavours and colours. Comparing these formulae to the experimental data on moments, the two parameters coincide, i.e., \( Q_0 = \Lambda \), or \( \lambda = 0 \). In this case the formulae simplify and the moments can be expressed in terms of the parameter \( B = a/b \) and the variable \( z = \sqrt{16N_cY/b} \) as:

\[
\frac{<\xi^q>}{Y_q} = P_0^{(q)}(B + 1, B + 2, z) + \frac{2B + 2(z)}{zB + 1(z)} P_1^{(q)}(B + 1, B + 2, z)
\]

where \( P_0^{(q)} \) and \( P_1^{(q)} \) are polynomials of order \( 2(q - 1) \) in \( z \).

These expressions extrapolate smoothly to threshold, where they are determined by the initial condition for a single parton. Similarly one can derive the complete MLLA results for fixed \( \alpha_s \), which simplify for high energies to

\[
\bar{\xi}_{fix} = \left[ 1 + \frac{\eta}{\bar{\gamma}_0} \right] \frac{Y}{2}, \quad \sigma_{fix}^2 = \frac{\bar{\gamma}_0^2}{4\pi^2} Y
\]

\[
s_{fix} = -\frac{3\eta}{\gamma_0 \sqrt{\bar{\gamma}_0 Y}}, \quad k_{fix} = \frac{3(4\eta^2 - \bar{\gamma}_0^2)}{\gamma_0^2} \frac{1}{\bar{\gamma}_0^2 Y}
\]

where \( \eta = \alpha_s^2/8N_C, \bar{\gamma}_0 = \sqrt{\bar{\gamma}_0^2 + \eta^2} \) and \( \bar{\gamma}_0 = \sqrt{2N_C\alpha_s}/\pi \).

### 3 Relating parton and hadron spectra

Strictly speaking the QCD results are only reliable for \( E \gg Q_0 \). In order to determine the moments one has to integrate the spectra over the full range and extrapolate to the soft limit. The experimental particle densities are usually given as function of momenta. The same kinematic limits of parton and hadron spectra is obtained if the same effective mass \( Q_0 \) is assigned to the charged particles which also limits the energy of the partons \( (E_h = E_p \geq Q_0) \).

Furthermore, a common behaviour of the parton and hadron spectra near the boundary can be obtained if the spectra are related by

\[
E_h \frac{dn(\xi_E)}{dp_h} = K_h E_p \frac{dn(\xi_E)}{dp_p} \equiv K_h D(\xi_E, Y, \lambda)
\]

4 Comparison with experimental data

The moments \( <\xi^q> \) are determined from the spectra \( Edn/dp \) vs. \( \xi_E \) after appropriate transformation of the measured \( x_p = 2p/\sqrt{s} \) spectra of charged particles using \( E = \sqrt{p^2 + Q_0^2} \) and therefore depend on \( Q_0 \). First a fit of the two parameters \( Q_0 \) and \( \lambda \) is obtained by comparing the moments for a selected \( Q_0 \) with the theoretical predictions from Eq. (4) for different \( \lambda \). The best agreement with the data was obtained for \( Q_0 \approx \lambda \approx (270 \pm 20) \text{ MeV} \). It was not possible to obtain a satisfying description for the fixed \( \alpha_s \) case. Choosing \( \gamma_0 = 0.64 \) and the same \( Q_0 = 270 \text{ MeV} \) a good description of the multiplicity \( \bar{N} \) and the slope of \( \bar{\xi} \) vs. \( Y \) could be obtained but not for the other quantities. In Fig. 1 we show the evolution of the first cumulant moments \( K_q \) with energy for running \( \alpha_s \) and fixed \( \alpha_s \). Note that these predictions depend only on the two parameters \( Q_0 \) and \( \lambda \) which actually coincide in the fit. The absolute normalization of the moments is given at threshold \( Y_0 = 0 \) by \( K_q = 0 \).

As this is an application of perturbative QCD to an extreme limit one could make the weaker (more conventional) assumption and choose a higher starting energy, say \( Y_0 = 2 \left( \sqrt{s} \approx 4 \text{ GeV} \right) \). In this case for each moment an extra free parameter \( K_q(Y_0) \) had to be introduced which, however, would not improve the fit for running \( \alpha_s \) essentially; a backward evolution from \( Y_0 \) would again reproduce approximately the initial condition at threshold.

Therefore, the initial condition yields the highly constrained fit with only two parameters (the normalization parameter \( K_h \) only enters the multiplicity \( \bar{N} \), not shown here).

In case of fixed \( \alpha_s \) one may ask the same ques-
Figure 1: The first four cumulant moments $K^q$ of the charged particles’ energy spectra $Edn/d\eta$ vs. $\xi_E$, shown as a function of $Y$ for $Q_0 = 270$ MeV. Predictions of the Limiting Spectrum of MLLA with running $\alpha_s$ (solid lines), of MLLA with fixed $\alpha_s (= 0.214)$ (dashed lines) and of MLLA with fixed $\alpha_s$ normalized by hand to the experimental point at $\sqrt{s} = 44$ GeV (dotted lines) are also shown; in all cases $n_f = 3$ (from Ref. 4).

Figure 2: Charged particle inclusive momentum distribution at LEP-1.5 from ALEPH (diamonds), DELPHI (squares) and OPAL Collaborations (triangles) in comparison with theoretical predictions of the Limiting Spectrum with $Q_0 = 270$ MeV (solid line). Dashed lines show the predictions of the Limiting Spectrum after correction for kinematical effects (from Ref. 5).

The dependence of the number of flavours $n_f$ has been studied as well. As the jet evolution is dominated at all energies by the gluon emission at the lower transverse momenta, the choice $n_f = 3$ is a good approximation up to LEP energies.

The recent LEP-1.5 data are also well accounted for by the same theoretical scheme as is shown in Fig. 2. Here the experimental $\xi$-spectrum is shown in comparison with the limiting spectrum (setting $\xi_p = \xi_E$) which terminates at $Y \sim 5.3$ and after rescaling $\xi_E \rightarrow \xi_p$ with relation (3) which takes into account the boundary effects. Also the moments of the distribution and the position of the maximum are well reproduced by the theoretical predictions at this energy.
5 Soft limit of energy spectrum

Having observed the good fit of the QCD-LPHD prediction not only towards high but also towards low \textit{cms} energies \( \sqrt{s} \) we now turn to the behaviour of the fits at low particle energy \( E \). As the emission of very soft gluons from all other partons in the jet is coherent the production rate of such particles is nearly energy independent. Indeed, the analytic results in the DLA and MLLA show the scaling behaviour of the spectrum in the soft limit and the production rate is essentially determined by the colour charge of the initial partons. Remarkably, the observed hadrons follow the trend of this prediction as can be seen in Fig. 3 which shows the invariant particle density as a function of particle energy \( E = \sqrt{p^2 + Q_0^2} \) at \( Q_0 = 270 \text{ MeV} \) at various \textit{cms} energies in comparison with the MLLA predictions (from Ref. 6).

6 Conclusions

The particle energy spectra are well described by the QCD parton shower in the MLLA assuming a close similarity between partons and hadrons according to the LPHD approach. This similarity appears to work not only for the highest but also down to rather low \textit{cms} energies as well as to low particle energies. This agreement is by no means trivial. For example, replacing the running \( \alpha_s \) by the fixed \( \alpha_s \) yields predictions quite incompatible with experiment, also if the threshold for the onset of a fixed \( \alpha_s \) regime is increased. The scaling prediction derived from colour coherence for the soft particles works well for hadrons.

This may suggest to view the final stage of hadron production through resonances, as explicitly incorporated into Monte Carlo hadronization models, as being dual to the partonic evolution with running coupling down to the low scale of a few hundred MeV.

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