Tunable valley splitting and bipolar operation in graphene quantum dots

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Quantum states in graphene are four-fold degenerate: two fold in spins, and two fold in valleys. Both degrees of freedom can be utilized for qubit preparations. In our bilayer graphene quantum dots, we demonstrate that the valley g-factor $g_v$, defined analogously as the spin g-factor $g_s$ for valley splitting in perpendicular magnetic field, is tunable by over a factor of 4 from 20 to 90. We find that larger $g_v$ results from larger electronic dot sizes, determined from the charging energy. This control is achieved by adjusting voltages on merely two gates, which also allows for tuning of the dot-lead tunnel coupling. On our versatile device, bipolar operation, charging our quantum dot with charge carriers of the same or the opposite polarity as the leads, can be performed. Dots of both polarity are tunable to the first charge carrier by action of the plunger gate, such that the transition from an electron to a hole dot can be observed. By adding more gates, this system can easily be extended to host double dots.

I. INTRODUCTION

A local minimum or maximum in the electronic band structure of a solid is called a valley. These valleys are relevant for the electronic properties of a number of materials, such as Si [1], AlAs [2], graphene and other 2D materials [3]. Energies of valley states can be tuned by strain, a feature utilized by the semiconductor industry to increase carrier mobility [4]. For the purpose of constructing qubits, the valley degree of freedom can be treated similarly to orbital and spin degrees of freedom [5]. So far, few experimental reports exist in this direction [6].

Theories for spin qubits [7] foresee local tunability of the electronic g-factor $g_s$, relevant for the Zeeman splitting of energy levels in a magnetic field. Such tunability enables different qubits to be selectively addressed, with the magnetic field as their common control. This has been achieved in several experiments [8][11] with changes in $g_s$ of a few percent. For valley qubits, one would like to tune the valley g-factor $g_v$, which characterizes the valley splitting that is linear in low magnetic fields. Such tunability is however thus far limited [12].

Graphene quantum dots (QDs) have been proposed as spin qubits with long spin coherence times [13] due to weak spin-orbit interaction and hyperfine coupling. As an alternative, the two valleys of graphene offer a pseudo-spin degree of freedom, making graphene QDs also amenable for realizing valley qubits. Experiments on graphene quantum point contacts have already demonstrated that $g_v$, arising from orbital magnetic moments related to the Berry curvature, can be tuned by gate voltages by a factor of three [14].

Here we demonstrate that, $g_v$ in a few-carrier bilayer graphene (BLG) QD can be in situ tuned via purely electrostatic means in a controlled and systematic fashion by a factor of 4.5 from $g_v = 20$ to 90. We find experimentally that the tunability of $g_v$ is related to the electronic size of our QD, which scales with its charging energy. Theory relates the value of $g_v$ and its tunability to the same Berry curvature effects found relevant in quantum point contacts [15][16]. We show that the results of detailed modeling are in qualitative and semi-quantitative agreement with the experiment. Tuning of the charging energy and hence of $g_v$ is achieved with a carefully designed and yet straightforward three-gate geometry, which provides us also with control over the tunnel coupling between the QD and its leads, as well as over the polarity of charge carriers in the dot. Such control allows for bipolar operation of the QD, enabling us to observe the transition from an electron dot to a hole dot through the first carrier states. This device geometry provides a straightforward extension to double quantum dots.

II. TUNABLE VALLEY SPLITTING

The essence of our main result is highlighted in Fig. 1(a), which is accessible without a detailed description of our BLG QDs. The single particle level spectra for the QDs show energy splitting in a perpendicular magnetic field for the first four hole states lowest in energy. Indicated by the gray lines, this splitting is linear at low fields, and corresponds to the energy splitting between neighboring levels belonging to the $K^-$ and $K^+$ valleys. The valley g-factor $g_v$ is extracted from $\Delta E_{K^-,K^+} = g_v \mu_B B \perp$. The splitting in (a,ii) is evidently much larger than that in (a,i) (quantitatively, the $g_v$ values differ by a factor of 2.5), where (i) and (ii) are taken for QDs formed with the same gates but at two differ-
FIG. 1. (a) Single particle energy level spectrum of a QD at (i) $V_{BG} = 4.8\, V$, $V_{SG} = -3.44\, V$, $V_{L,R} = 0.0\, V$ and (ii) $V_{BG} = 3.0\, V$, $V_{SG} = -2.45\, V$, $V_{L,R} = 4.0\, V$ in perpendicular magnetic field $B_{L}$. Extracted from conductance maps shown accordingly as insets. Extraction of energy levels is performed similarly as in Ref. [17, 18] (see Appendix A 3 for more details), such that resonances are vertically shifted to touch their nearest neighbors at points with the smallest separations. Levels extracted from the 1st, 2nd, 3rd and 4th hole-state Coulomb resonance peaks are plotted in red, blue, green and yellow, respectively. Gray lines indicate the linear valley splitting. (b) 3D illustration of the van der Waals hetero-structure device with layers labeled and gates shown: split gates in yellow, and finger gates L, M and R in green. Edge contacts (orange) are fabricated at two ends of the channel. (c) Schematics of a hole dot formed in a n-type channel, with no action from gates L and R. (i) Top-view of the channel where split gates are outlined in solid and finger gates in dashed lines. Red, blue and white indicate n-type, p-type, and gapped regions, respectively. (ii) Corresponding conduction (blue) and valence (red) band edge structure along the channel. The extent of the probability distribution of the QD hole state $|\Psi(x)|^2$ is sketched by a dashed line.

ent gate voltage configurations. We attribute this salient difference in $g_c$ to small (i) and large (ii) electronic dot sizes. We now start with a detailed description of how these results are obtained, and later demonstrate an even larger tuning range for $g_c$ by a factor of 4.5.

A schematic illustration of the device is shown in Fig. 1(b) (see Appendix A 1 for details on sample fabrication). With the mutual action of a graphite back-gate voltage $V_{BG}$, and a top-gate voltage, we can alter both the Fermi energy $E_F$, and the size of the BLG band-gap $\Delta_{gap}$, below the respective top gates. Top gates here can be either the split gates (yellow, separated by 100 nm) or the finger gates (green, 20 nm in width and separated from each other by 75 nm center to center). The band-gap $\Delta_{gap}$ scales with the strength of the displacement field perpendicular to the BLG sheet [19, 21]. Here we operate at positive $V_{BG}$, inducing n-type charge carriers in the bulk BLG regions.

The situation is depicted in Fig. 1(c,i). For a fixed positive $V_{BG}$, negative split-gate voltages $V_{SG}$ open a band-gap underneath the split gates and tune $E_F$ into this gap, confining $I$ to be along the n-type conducting channel as indicated by the arrow. We then operate our finger gates at fixed $V_{SG}$. Negative voltages $V_M$ on gate M locally generate a p-type island in the channel. Thus a p-type QD is formed with naturally arising p-n junctions as tunnel barriers to the n-type leads, with gate M as its plunger gate. In this regime, we observe signatures of Coulomb blockade in the quantum dot, with sharp conductance resonances as a function of $V_M$ and fully suppressed current in-between. The corresponding band edge structure along the channel axis is depicted in (c,ii). The tunnel barriers are tuned by voltages $V_{L,R}$ applied to the barrier gates L and R (dark green), thus allowing for control of the electronic size of this quantum dot, and also its tunnel coupling to the leads.

We find experimentally that the dot-lead tunnel coupling can be tuned by $V_{L,R}$, where positive $V_{L,R}$ leads to a more strongly coupled dot, and negative $V_{L,R}$ to a more decoupled one. The peak conductance $G_{max}$ can be tuned by three orders of magnitude ($10^{-3} e^2/h - e^2/h$), and tunneling rates $\Gamma$ by a factor of 30 (10 GHz – 300 GHz). Details about measurements and extraction of these numbers are presented in Appendix B 1.

Changes in $V_{L,R}$ also affect the quantum dot size. As an indicator of its size, the dot’s charging energy $E_{ch}$ is extracted from Coulomb diamond measurements of the first hole. The inset of Fig. 2(b) shows $E_{ch}$ linearly decreasing by a factor of four (representing an increase in the QD size) for two different back gate voltages as $V_{L,R}$ is varied from negative to positive over the available tuning range. Reliable extraction of smaller $E_{ch}$ is limited by the smearing of the resonance peaks and instability of the dots due to their stronger coupling to the leads.
Dots formed at lower $V_{BG}$ and thus at a weaker displacement field have systematically smaller $E_{ch}$ and thus larger sizes, due to the smaller BLG gap $\Delta_{gap}$.

To obtain a rough estimate for the quantum dot size, the charging energy $E_{ch}$ is converted into radius, assuming the QD to be a circular disk-like capacitor of radius $r$, embedded deep in a mixture of insulating hBN and amorphous Al2O3, with self-capacitance $C = 8\epsilon_{0r}$. A decrease in $E_{ch}$ from 11.4 meV to 4.1 meV therefore corresponds to an increase in dot size from a radius of 24 nm to 67 nm. In reality, our QDs vary in aspect ratios: As $V_{BG}$ and $V_{SG}$ are kept the same, the extent of the QD across the channel remains roughly constant while $V_{L,R}$ expands and contracts the dot only along the channel.

Our high tunability is enabled by the narrow finger gates with small separations in-between. The former ensures formation of small enough structures, whereas the latter allows for sufficient control over the bands not only directly underneath but also in-between the finger gates by stray fields.

The band-edge structure schematics shown in Fig. 2(a) illustrate the above experimental observations of our QD under more negative (i) and more positive (ii) $V_{L,R}$. Similar to Fig. 1(c,ii), $V_{BG}$ and $V_{SG}$ are kept constant, and a negative $V_{M}$ has lifted the bands up in energy to form a $p$-type dot, with $p$-$n$ junctions as tunnel barriers. When more negative barrier voltages $V_{L,R}$ are then applied, the bands are raised in energy underneath gates L and R; $\Delta_{gap}$ is also increased due to the larger displacement field. As illustrated in (i), these changes effectively induce shallower, and more importantly, wider $p$-$n$ junctions along the channel, such that the tunnel barriers become more opaque. On the other hand, when $V_{L,R}$ is more positive in (ii), bands underneath L and R are lowered in energy and $\Delta_{gap}$ is decreased. The $p$-$n$ junctions become steeper and narrower, forming more transparent tunnel barriers. As a result, the dot-lead tunnel coupling increases in case (ii) [decreases in case (i)] at more positive (more negative) $V_{L,R}$, giving rise to broader (narrower) Coulomb resonance peaks with higher (lower) peak conductance.

Tunable dot size with $V_{L,R}$ then follows straightforwardly. With more opaque (transparent) barriers from more negative (positive) $V_{L,R}$, charges are more (less) confined in the $p$-type dot. Sketched with dashes in Fig. 2(a), the extent of the probability distribution $|\Psi(x)|^2$ of the QD therefore spreads less (more) into the leads, forming an effectively smaller (larger) dot. According to experimental data, this effect dominates over the a priori anticipation that the cross-capacitance between gates L and R with gate M will decrease (increase) the dot size with more positive (negative) $V_{L,R}$.

With the understanding of the tuning of our quantum dot size, we are now in a position to explain how we obtain our tunable valley g-factors. At fixed $V_{BG}$ and $V_{SG}$, we measure Coulomb resonances for the first four holes as a function of the plunger gate voltage $V_{SG}$ in perpendicular magnetic field $B_{\perp}$, for QDs formed at a set of different $V_{L,R}$. Examples of such measurements are shown as in-
sets of Fig. 1(a), with carrier occupancy labeled. We convert the $V_M$-axis into energy, using lever arms determined from Coulomb diamond measurements at corresponding gate voltage configurations. We then extract the single particle level spectrum, by subtracting from the conductance resonance positions (i.e. the energy levels) a magnetic field independent charging energy (see Appendix A3 for details). Examples of the resulting spectra are shown in Fig. 1(a) for dots with charging energies $E_{ch} = 9.2$ meV for (i) and $E_{ch} = 4.8$ meV for (ii). Indicated by the gray lines, at sufficiently low magnetic fields, the lowest two energy levels, being states of the $K^-$ and $K^+$ valleys, split linearly in energy $\Delta E_{K^-,K^+} = g_v \mu_B B$. From $\Delta E_{K^-,K^+}$ for the first and the second level, before the crossing with the third level occurs, we determine the valley g-factor $g_v$.

The extracted values of $g_v$ are plotted against the corresponding charging energies $E_{ch}$ in Fig. 2(b). Evidently, dots with lower charging energy (i.e. larger electronic size) have a systematically larger $g_v$. The tuning range is as large as a factor of 4.5 varying from $g_v = 90$ at $E_{ch} = 4.1$ meV ($r = 67$ nm) to $g_v = 20$ at $E_{ch} = 11.4$ meV ($r = 24$ nm). These results encompass values obtained in previous measurements of $g_v$ in BLG QDs [17] [18] [23].

We present calculations of the dot’s electronic structure in support of our experimental observations. We describe electrostatic confinement by a smooth potential [10] where we elongate the radius in both directions (the $x$- or the $y$- axis) to model rotationally symmetric (elliptic) QDs. See Appendix C for details of the model. The valley splitting $\Delta E_{K^-,K^+} = g_v \mu_B B$ is expressed in terms of the valley g-factor $g_v$, which arises from the orbital magnetic moment $M$ in BLG k-space, originating from the material’s non-trivial Berry curvature. In BLG the band edges at the three-fold symmetrical mini-valleys possess the largest $M$ [top-left inset to Fig. 2(c)]. For sufficiently large dots, increasing the dot’s radius along one or both axes in real space squeezes the wave-functions into the mini-valleys in momentum space, which then dominates the amount of $M$ being picked up. Hence, as shown in Fig. 2(c), $g_v$ increases with increasing dot sizes as the contributions from the mini-valleys become dominant. For non-circular dots (closer to our experimental operations as discussed before), $g_v$ depends on the elongation direction. Our experimental observation corresponds well with predictions in the large-dot regime (see Appendix C for more data as a function of the dot size and a discussion of the small-dot regime).

III. BIPOLAR OPERATION

The exquisite device control achieved with gates L, M and R not only enables us to tune the valley splitting, but also allows us to explore QDs with carriers of both polarities at will. In the previous section we considered a $p$-type dot in an $n$-type channel, with $p$-$n$ junctions as its tunnel barriers. Now we consider the situation where the dot is defined by gapped regions underneath the barrier gates L and R, with $E_F$ tuned into the gap. In this configuration, bipolar operation of the quantum dot is possible and the transition from a quantum dot occupied with the first electron to one occupied with the first hole can be observed by tuning gate $M$ only.

To this end, we operate the quantum dot at $V_{BG} = 3.7$ V and $V_{SG} = -2.92$ V, forming again a $n$-type channel between the split gates, and insulating regions beneath them. Negative $V_{L,R}$ depletes the electrons locally and tunes $E_F$ into the band gap below gates L and R. A conductive region below gate $M$ is thereby isolated from
the leads, which becomes the QD with charge carriers trapped inside. Tuning $V_M$ to sufficiently negative values enables us to change the type of charge carriers in the dot from electrons to holes, thereby facilitating bipolar operation of the dot. Due to the close separation of the finger gates, finite cross-talk exists between the plunger gate $M$ and the barrier gates $L$ and $R$. In order to mitigate the cross-talk and to keep the tunnel coupling approximately constant, we linearly correct $V_{L,R}$ when $V_M$ is swept, instead of operating at constant $V_{L,R}$. Details are shown in Appendix B 2.

The transition between the electron and the hole dot can be seen from the finite bias measurement shown in Fig. 3(a), taken at the dashed line on Fig. 6. Carrier occupancy is labeled in the corresponding diamonds. The transition of polarity from a $n$-type to a $p$-type quantum dot is identified as the largest diamond-shaped region (labeled 0) extending from $V_M = -5.2$ V to $-5.6$ V. Well-defined Coulomb diamonds with clearly visible excited states are shown for both electrons and holes for the first three carriers. We find similar lever arms and charging energies for carriers on both sides of the gap, indicating a similar geometry for the dots of the two polarities. The fine resolution of a rich group of excited states is a strong indication of the cleanliness and the high quality of our QD. From the size of the diamond labeled with carrier occupancy 0, we extract $30 \text{meV}$ as the energy separation between the first electron and the first hole state, providing a lower bound estimate of the band gap in the dot.

A schematic representation of the dots with both polarities is sketched corresponding below the measurements in Fig. 3(b). In the previous section, adjusting $V_{L,R}$ over a large range tuned the width of the $p$-$n$ junction. Now $V_{L,R}$ changes the size of the band gap and the position of $E_F$ in the gap, and thus tunes the height of the tunnel barriers. The range of $V_{L,R}$ suitable for the formation of such a dot is smaller compared to the range in effect in the previous section, as we are limited by the requirement that regions underneath gates $L$, $R$ need to remain in the gap. The dot-lead coupling strength for such a dot is tunable by $V_{L,R}$, which has been demonstrated in previous work [24], though there the first carrier state was not observed due to the large separation between barrier and plunger gates. In our case, a negative $V_M$ depletes the QD into the few-electron regime [sketched in (b,ii)], and eventually depletes it after removal of the last electron. More negative $V_M$ then adds single holes one-by-one into the now $p$-type dot [sketched in (b,i)]. During the preparation of this manuscript, we became aware of similar measurements on this bipolar operation, with a different device geometry [23].

Within the same device, but using five finger gates instead of three, we are also able to form bipolar double dots. Repeating the tuning procedure discussed above, and choosing appropriate barrier voltages $V_B$, both dots are tunable in carrier occupation to the last electron/hole. Moreover, just like for the single dot case, control over the dot-lead and also the dot-dot tunnel coupling is possible. Figure 4(a) presents the charge stability diagram of this bipolar double dot taken at barrier gate voltages $V_{B,L} = -5.5$ V, $V_{B,M} = -5.7$ V and $V_{B,R} = -5.5$ V, as a function of the two plunger gate voltages $V_{P,L}$ for the left and $V_{P,R}$ for the right dot. We perform this measurement at a finite source-drain bias voltage of $1 \text{mV}$ to enhance the visibility of the triple points.

Four different combinations of double dot polarity are observed, with finite-bias triangles visible in all four quadrants, separated by the gap of low conductance. Labeled in the center of the conductance map in Fig. 4(a), with corresponding schematics shown below in (b), we form (i) electron-hole (ii) electron-electron (iii) hole-hole and (iv) hole electron double dots, respectively, with $n$-type leads.
IV. CONCLUSION

We have presented a BLG device, allowing for in situ formation of bipolar quantum dots with tunable coupling strengths, electronic dot sizes and valley g-factors. The valley g-factor can be tuned by a factor of 4.5 from $g_v = 20$ to 90 with increasing electronic dot sizes. This dependence is brought about by the increasing influence of the high magnetic moments of the orbital mini-valleys, in qualitative agreement with theoretical calculations. The system can be straight-forwardly extended to host double dots, and potentially other multi-dot systems, by simply adding more gates. Our versatile BLG QD with widely tunable valley g-factor paves the way for promising valley qubits in graphene.

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Appendix A. MATERIALS AND METHODS

1. Sample fabrication

The device is fabricated as described in [17, 25], and schematically depicted in Fig. 1(b). Built with the dry-transfer technique [20], the van der Waals heterostructure stack lies on a silicon chip with 280 nm surface SiO2. The stack consists of a bottom graphite back gate [dark gray in Fig. 1(b)], and on top of it a BLG flake (black) encapsulated in 38 nm thick bottom and 20 nm thick top hBN flakes (blue). Ohmic edge contacts (red) with Cr and Au of 10 and 60 nm thickness, respectively, are evaporated after etching through the top hBN flake with reactive ion etching. A pair of 5 nm thick Cr, 20 nm thick Au split gates (yellow), separated by 100 nm are deposited on top, defining a 1 μm long channel. Separated by a layer of 30 nm thick amorphous Al2O3 (light gray) grown by atomic layer deposition, finger gates (green) labeled L, M and R of 20 nm in width, and 5 nm Cr and 20 nm Au in thickness, lay across the channel defined by the split gates. Nearest neighbor finger gates are separated by 75 nm from center to center.

2. Measurement condition

We perform AC (standard lock-in techniques) and DC measurements, in He$^3$ / He$^4$ dilution refrigerators, at electronic temperatures of around 100 mK.

3. Extracting single particle level spectra

Similar to [17, 18], we extract single particle level spectra from Coulomb resonance peak positions by following their evolution in perpendicular magnetic field $B_\perp$ [insets in Fig. 1(a)]. Lorentzian functions are fitted to each peak, obtaining peak positions in plunger gate voltage $V_M$. These gate voltage values are then converted into energies, using lever arms $\alpha$’s extracted from Coulomb diamond measurements taken at corresponding gate voltage configurations ($V_M$, $V_L$,$R$, $V_BG$, and $V_SG$). The first and the second level are converted with lever arms $\alpha_1$ extracted from diamonds of the first carrier state, and $\alpha_2$ extracted from the second carrier state, respectively. The third and the fourth level are converted using an average of $\alpha_1$ and $\alpha_2$. In practice, $\alpha_1$ and $\alpha_2$ differ from each other within the error range.

The energy difference between the nearest neighboring levels is the corresponding addition energy, which is the energy required to add a carrier to the QD. This energy consists of not only the single particle energy difference, but also the charging energy stemming from the Coulomb interaction, which we assume to be independent of magnetic field. When two single particle levels are degenerate, the addition energy comprises of only the charging energy. We therefore assume the minimal difference between nearest neighboring levels to be the energy of the first level at zero magnetic field. Examples of the resulting single particle energy spectra are presented in Fig. 1(a). This way of extraction sees a lifting of spin degeneracy at zero magnetic field.

Appendix B. SUPPORTING DATA

1. Tunable tunnel coupling

As well as electronic QD size discussed in the main text, dot-lead tunnel coupling can also be tuned by barrier gate voltages $V_L$,$R$. At $V_BG = 4.8$ V, $V_SG = −3.44$ V, Coulomb resonance peaks of the first hole at different
FIG. 5. (a) Coulomb resonances of the first hole at $V_{BG} = 4.8\,\text{V}$ and $V_{SG} = -3.44\,\text{V}$, plotted in energies converted into units of temperature, for varying barrier gate voltages $V_{L,R}$, scaled by their peak conductance. Solid lines present results of Lorentzian fits. (b) Peak conductances $G_{max}$ (green) and tunneling rates $\Gamma$ (blue) of the first hole as a function of $V_{L,R}$, for dots formed at $V_{BG} = 4.8\,\text{V}$ and $V_{SG} = -3.44\,\text{V}$ (circles), and $V_{BG} = 3.0\,\text{V}$ and $V_{SG} = -2.45\,\text{V}$ (squares).

$V_{L,R}$ are shown in Fig. 5(a). Lorentzian functions are fitted for each peak. Conductance of each trace is scaled with peak conductance $G_{max}$ for clarity. Plunger gate voltage $V_M$ is converted into energy using lever arms $\alpha_1$ extracted for the first hole from Coulomb diamonds measured at corresponding gate voltage configurations. It is then scaled with $k_B$. At more positive $V_{L,R}$ the dot is more strongly coupled to the leads due to the more transparent tunnel barriers as discussed in the main text, giving rise to wider peaks, and vice versa.

From the Lorentzian fits, peak conductance $G_{max}$ and full width half maxima (FWHM) (as a measure of tunneling rates) are extracted. The latter is converted into energies (and then into frequencies) using lever arms $\alpha_1$. The result is plotted in Fig. 5(b): with more positive $V_{L,R}$, peak conductance $G_{max}$ increases from $\sim 10^{-3}\,e^2/h$ to $\sim e^2/h$ by three orders of magnitudes, and tunneling rates $\Gamma$ from $\sim 10\,\text{GHz}$ to $\sim 100\,\text{GHz}$, indicating a more strongly coupled dot to the leads.

2. Single dot charge stability diagram

Figure 6 shows the conductance map, where barrier gate voltages $V_{L,R}$ are swept together with the plunger gate voltage $V_M$, at $V_{BG} = 3.7\,\text{V}$ and $V_{SG} = -2.92\,\text{V}$. At less negative $V_{L,R} > -4\,\text{V}$, the system is open with $n$-type channel. Barrier gates L and R lower the electron density, but are not enough to deplete regions underneath them into the gap. When $V_{L,R}$ become more negative, corresponding to the upper half of the conductance map, the BLG gap $\Delta_{gap}$ is opened and $E_F$ is tuned into this gap, isolating charges from the leads. Applying more negative $V_M$ from right to left on the map, electrons are removed consecutively from the $n$-type dot until depletion, and consecutive Coulomb resonances can be seen, with the first electron state indicated by the red arrow. This situation is depicted in Fig. 5(b,ii). Starting from depletion, more negative $V_M$ consecutively adds holes into the now $p$-type dot, depicted in Fig. 5(b,i). The red arrow point to the first hole state. When barriers are biased more negatively, in the lower half of the map, they too drive the BLG beneath them into $p$-type, forming a series of $p$-$n$-$p$ triple dots when $V_M$ is less negative at the bottom right of the map; and forming a large $p$-type dot together with gate M when $V_M$ is more negative, at the bottom left of the map. The two insets depict the situations at corresponding locations of the map.

Instead of being completely perpendicular to the $V_M$-axis or to the $V_{L,R}$-axis, the Coulomb resonance lines come at an angle, as can be seen from the map, demonstrating the finite cross-talk between the barrier gates.
L and R and the plunger gate M. This cross-talk stems from the close separation (75nm) between gates L, M, and R. To mitigate the effect of such cross-talk and to keep the barrier strength roughly constant, we perform bipolar operation [shown in Fig. 3(a)] at the dashed line on the map.

Appendix C. THEORETICAL MODELS

We model the BLG QD, formed with the help of electrostatic split gates [16, 24, 25, 27], by a smooth confinement potential, \( U(r) \), and a gap profile, \( \Delta(r) \), which enter in the single-electron four-band Hamiltonian [28, 29],

\[
H_k = \begin{pmatrix}
U \pm \frac{1}{2} \Delta & \pm v_3 \pi & 0 & \pm v_\gamma \pi \\
\pm v_3 \pi & U \pm \frac{1}{2} \Delta & \pm v_\gamma \pi & 0 \\
0 & \pm v_\gamma \pi & U \pm \frac{1}{2} \Delta & \gamma_1 \\
\pm v_\gamma \pi & 0 & \gamma_1 & U \mp \frac{1}{2} \Delta
\end{pmatrix},
\]

where

\[
U(x, y) = \frac{U_0}{\cosh \sqrt{\frac{(x^2+y^2)}{L^2}}},
\]

\[
\Delta(x, y) = \Delta_0 - \frac{0.3 \Delta_0}{\cosh \sqrt{\frac{(x^2+y^2)}{L^2}}},
\]

Eq. (C1) numerically in a suitable basis of localized states. We choose the eigenstates of the two-dimensional harmonic oscillator (products of wave functions \( \psi_n(x) = N_n e^{-\frac{1}{2}(\alpha x)^2} \mathcal{H}_n(\alpha x) \), where \( N_n = \sqrt{\frac{\alpha}{\sqrt{n!}} \sqrt{\pi^2 n!}} \) is the normalization constant and \( \alpha \) is a scaling factor of unit length\(^{-1}\); we adapt \( \alpha \) to the potential \( U(x) \) mimicked by a parabolic potential at the bottom of \( U \)). The basis states are then given by

\[
\psi_{\eta \mu, 1} = \begin{pmatrix}
\psi_\eta(x) \psi_\mu(y) \\
0 \\
0 \\
0
\end{pmatrix}, \quad \psi_{\eta \mu, 2} = \begin{pmatrix}
0 \\
\psi_\eta(x) \psi_\mu(y) \\
0 \\
0
\end{pmatrix},
\]

\[
\psi_{\eta \mu, 3} = \begin{pmatrix}
0 \\
0 \\
\psi_\eta(x) \psi_\mu(y) \\
0
\end{pmatrix}, \quad \psi_{\eta \mu, 4} = \begin{pmatrix}
0 \\
0 \\
0 \\
\psi_\eta(x) \psi_\mu(y)
\end{pmatrix}.
\]

For every set of system parameters we construct the matrix corresponding to Hamiltonian \( H_k \) in the basis given in Eq. (C3) and obtain the energy spectrum by diagonalization. The spectrum is converged when the energy levels change no more upon including a higher number of basis states.

For the lowest single-particle dot state, \( \Psi \), we compute its valley g-factor, \( g_v \), by estimating how much orbital angular momentum is picked up in momentum space [14, 16]

\[
g_v = \frac{2}{\mu_B} \int dK |M_z(K)| |\Psi(K)|^2,
\]

where \( \mu_B \) is the Bohr magneton. The factor 2 in Eq. (C4) stems from the fact that the total valley splitting \( \Delta E_{K^+, K^-} \) is twice that of a single valley as \( M \) is of equal magnitude but opposite sign in the two valleys.

For a circularly symmetric dot \( (a \equiv b \) in Eq. (C1), we show the dependence of \( g_v \) on the dot size in Fig. 7, alongside with the momentum space distribution of the lowest dot state, \( \Psi \). In this figure, we chose the parameters \( U_0 = -2 \text{meV} \) and \( \Delta_0 = 60 \text{meV} \) (which amounts to a gap of \( \Delta = 42 \text{meV} \) at the dot’s center) in Eq. (C1) as well as \( a = b = 1 \), while varying \( L \). For small dots, increasing the dot size reduces the support of the wave function in momentum space. Consequently, the states pick up less orbital momentum in Eq. (C4) and the valley g-factor reduces. For larger dots, the bottom of the band is more shallow and the three mini-valleys of the BLG dispersion influence the dot state. In this regime, the larger the dot, the more the state is squeezed into the mini-valleys, where the orbital magnetic moment is maximal. Hence the valley g-factor increases with dot size. The latter regime is favored by large dot sizes, shallow confinement potentials and large gaps in the BLG dispersion [16].

To model elliptically elongated dots in the \( x \) or \( y \) direction, we vary the parameters \( a \) and \( b \) in Eq. (C1) respectively. In an elliptical QD, the ground state’s wave functions are squeezed mainly into one (elongation along the \( x \)-axis) or two (elongation along the \( y \)-axis) of the three mini-valleys, leading to a further increase of the valley g-factor.
FIG. 7. The dot wave functions in the $K^+$ valley and valley g-factor as a function of the dot size for a circularly symmetric dot with a gap of $\Delta = 42$ meV at the center. The insets show the corresponding BLG dispersion, $E$, of the first conduction band (top) and orbital magnetic moment, $M$ (bottom) in valley $K^+$.

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