A PRELIMINARY STUDY ON THE ESTABLISHMENT OF OCEAN TIDE MODELS OVER THE SOUTH CHINA SEA FROM T/P ALTIMETRY

BAO Jingyang
CHAO Dingbo
LI Jia~heng

KEY WORDS satellite altimetry; the South China Sea; tide model

ABSTRACT On the basis of the characteristic of the perfect spatial distribution of the T/P altimeter data, a spatial harmonic tidal analysis is performed, which transfers tidal harmonic constants $H$ and $g$ of each constituent into a pair of parameters: the cosine part $U$ and sine part $V$. And each part is expanded into a polynomial. The polynomial coefficients are estimated with altimeter data upon the least squares criteria. Thus the models of principal tidal waves in the South China Sea are established. 72 cycles of T/P data from cycle II through 82 are included in the calculation. The models are evaluated with different approaches and data set. The conclusions are that the tide models can provide partial tide amplitudes with 3 cm accuracy, and that phase lags deviation of those tides with amplitude larger than 10 cm are within ±10°.

1 Introduction

Since 1960’s, especially during the last two decades, many tidalists have studied on the tidal wave systems of the South China Sea. Ye Anle, et al. (1983), Shen Yujiang, et al. (1985), Fang Guohong, et al. (1994), and Cao Deming, et al. (1997) simulated the tidal field in this area based on the numerical hydro-dynamical tidal equation. Large discrepancies exist between these models due to the differences in resolutions, open boundary conditions and model simplifying.

Through measuring the distance between the sea level and the satellite, the technique of altimetry provides an effective method of studying sea level morphology and its variation in the ocean, in other words, it works as the vast of tide gauges over the ocean. Since the 1980’s, a lot of researches on ocean tide extracting are reported. On the basis of Geosat observations, the first utility altimetry tidal model was established in 1990. Comparing with other altimeter, TOPEX/POSEIDON (T/P for short) has a more accurate orbit and lower surveying errors. 9.9156 days of exact repeat cycle design guarantees the aliasing periods of leading ocean tides shorter than 180 days, which are beneficial to separating constituents. With the parallel development of the tidal hydrodynamic numerical calculation and data assimilation theories, since 1994 more than 20 new generation global ocean models have been developed. Although these global scale models can provide 2-3 cm accuracy level of tidal parameters (harmonic constant or orthogonal weights) in the oceans. On continent shelves and in margin seas, the differences between the models and ground truth taken from tide gauges are too large to be accepted. Few researches with altimetry techniques in the study area have been reported. The only exception is the ones given by Mazzega (1994).
Geodetists concern with tide models for the following reasons. On one hand, satellite geodetic techniques (especially the altimetry) provide a powerful approach for tide observation. On the other hand, the tidal information is very important for the correction of altimeter data (the tidal correction reaches more than 80% of all the corrections) and beneficial to satellite orbit calculation, gravity field model establishment and sea level variations study.

In this paper, a three degree polynomial function is taken as base function, and all altimeter data distribute in the study area are adopted for estimating the coefficients based on the least square criterion. Then the model is evaluated with some approaches.

2 Data, method and model

2.1 Data for calculation

From 11 through 82 cycles of T/P observation data are used in the tide inversion. The study area covers 5°-20° N and 109°20'-120° E, with the exception of the areas separated by islands for the consideration of the dynamic characteristics.

![Fig. 1 Distribution of T/P ground tracks](image)

The preprocessing of the altimeter data include various corrections recommended by GDR users’ handbook except for the ocean tide correction. About 20 cm bias between TOPEX and POSEIDON (Deng Xiaoli, et al., 1996) are compensated for POSEIDON observations.

14 tide gauges listed in the Admiral Tidal Tables are selected for evaluating our tide models, all those tide gauges are selected with the following principle. The tidal gauges should be near (shorter than 200 km) the altimetry ground track or in the study area. And the tidal parameters should be derived from real observations (not inferred). We analyzed 9 sets of observation series collected from the South China Sea basin with harmonic method, which provides a good reference to be compared for our models.

2.2 Approach to data processing

Conventionally hourly tidal observation series taken from a tide gauge is analyzed with a harmonic or response method. In harmonic approaches, aliasing problem is seldom considered because the sampling frequencies are higher than those of all the principal tides. When the tidal frequencies satisfy the Rayleigh criterion, for one year of hourly tidal observation series, 8 main tides (Q1, O1, P1, K1, N2, M2, S2, k2) or more can be separated successfully.

Altimetry techniques provide a vast time series of sea level height covering the oceans and seas. Each points along the ground track acts as a tide gauge. On all these "gauges", the data sampling frequency is 1 per revolution (excepting on cross-over points), and these sampling fashion makes diurnal and semidiurnal tides aliasing into long period signals. Thus the separation of tides depends on the aliasing periods. For example, to separate constituents M2 and S2, about 3 years of T/P observations will be required, and 9 years of observation is essential for P1 and k2 to be separated.

Fortunately, the available of neighboring parallel ground track and the intersection of the track make things easy, for the reason that the observations with other sampling fashions provide more information about the phase change. In our research, a polynomial base function is adopted for describing the state of tidal field in the study domain. Thus the tidal harmonic constants are replaced by the model coefficients, and each observation acts as a sampling of the model. The establishment of the model is transferred to parameter estimation problem based on the least squares approach.
ciple is as those empirical global harmonic tide models established with T/P data, which are based on bins of observations or crossover data. The approach used in this study is more effective because the trend of the tidal field is considered.

The height of a partial tide in a given point can be described with a single frequency vibration:

$$\xi(\varphi, \lambda, t) = H(\varphi, \lambda) \cos[\sigma t + \chi(\lambda) + \nu - g(\varphi, \lambda)]$$

where $H$ represents amplitude and $g$ phase lag referring to Beijing time system. Both of them consist of a pair of harmonic constants of a constituent. $\sigma$ is phase velocity of the tide, $t$ is time referring to a selected origin, $\chi$ is astronomy phase of the corresponding expanding term of tidal potential in Greenwich system, and $f$ and $u$ are nodal correction factors.

For convenience, the harmonic tidal constants are transferred to the cosine part and sine part:

$$U = H \cos g$$
$$V = H \sin g$$

According to tidal expanding theory and tidal calculating experiences, the regular tidal variation can be expressed as the sum of 8 principal partial tides heights relative to the mean sea level. These tides are $Q_1, O_1, P_1, K_1, N_2, M_2, S_2$ and $k_2$. Considering the non-tidal sea level change factors and observation errors, the observed instantaneous sea level height is written as follows:

$$h(t) = \bar{h} + \sum_{i=1}^{8} \left( f_i \cos(\sigma_i t + \chi_i + u_i) U_i + f_i \sin(\sigma_i t + \chi_i + u_i) V_i \right) + \epsilon = PA$$

where $P = (1, f_1 \cos(\sigma_1 t + \chi_1 + u_1), f_1 \sin(\sigma_1 t + \chi_1 + u_1), \ldots, f_8 \sin(\sigma_8 t + \chi_8 + u_8))$ = $\left[ P_1, P_2, P_3, \ldots, P_{17} \right]^T$

$\alpha = (\bar{h}, U_1, V_1, \ldots, V_8)^T = (a^1, a^2, a^3, \ldots, a^{17})^T$

$h$ is the mean sea level height above geoid or a priori mean sea level; $\epsilon$ is the combined effect of non-tidal perturbations and observation errors.

For establishment of our tidal model in regional area with satellite altimeter data, each term of $\alpha$ is described with a function of geographical location.

$$a^i = f(\varphi, \lambda)$$

Here, a third order polynomial is adopted as base function to be approximated.

$$a^i = a_0^i + a_1^i \Delta \lambda + a_2^i \Delta \varphi + a_3^i \Delta \lambda^2 + a_4^i \Delta \lambda \Delta \varphi + a_5^i \Delta \varphi^2 + a_6^i \Delta \lambda^3 + a_7^i \Delta \lambda^2 \Delta \varphi + a_8^i \Delta \lambda \Delta \varphi^2 + a_9^i \Delta \varphi^3 = Q \beta^i$$

where $Q = (1, \Delta \lambda, \Delta \varphi, \ldots, \Delta \varphi^3)$

$$\beta^i = (a_0^i, a_1^i, a_2^i, \ldots, a_9^i)^T$$

$\Delta \varphi = \varphi - \varphi_0, \Delta \lambda = \lambda - \lambda_0$

$(\varphi_0, \lambda_0)$ is the node which, in our study, is located on 115° E and 13° N.

In this way, the observation function of sea level height for altimetry is in the form of:

$$\xi(\varphi, \lambda, t) = A(\varphi, \lambda, t) \chi$$

where $A(\varphi, \lambda, t) = [P_1(\lambda, t)Q(\varphi, \lambda), P_2(\lambda, t)Q(\varphi, \lambda), \ldots, P_{17}(\lambda, t)Q(\varphi, \lambda)]$

for a specific observation, the corresponding vector of the design matrix is represented with

$$A_j = [P_{1j}Q, P_{2j}Q, \ldots, P_{17j}Q]$$

Involving all the observations, we have

$$Q = BX + \Delta$$

where $B = \left[ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \right]$ $n$ denotes the number of the observations.

In the sense of the least square, the solution is estimated as

$$X = (B^TB)^{-1}B^TL$$

2.3 Model assessment approaches

Some items are adopted for model assessment, which are maximum amplitude and phase lag biases relative to the reference data, standard deviation of the amplitude and phase lag, and combined constituent prediction error (rms), and the sum of the rms written as rss of $m$ leading constituents. These items are described with the following formulae.

1) Standard deviations of amplitude and phase lag relative to reference data:

$$m_H = \left[ \frac{1}{N} \sum_{i=1}^{N} (H_i - H^0_i)^2 \right]^{1/2}$$

$$m_\varphi = \left[ \frac{1}{N} \sum_{i=1}^{N} (\varphi_i - \varphi^0_i)^2 \right]^{1/2}$$

2) Combined constituent predicting error (rms):

$$rms = \left[ \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2} \left[ (U_i - U^0_i)^2 + (V_i - V^0_i)^2 \right] \right]^{1/2}$$
3) The sum of \( \text{rms} \) of many tides (\( \text{rss} \))
\[
\text{rss} = \left( \sum_{j=1}^{N} \text{rms}_j^2 \right)^{1/2}
\]
(10)

where \( N \) is the number of point on which the comparison is performed, and the superscript 0 denotes the reference data.

With the above items, the comparisons are performed in three approaches, the comparison between results derived from various period of observation, the comparison between \( \text{T/P} \) results and in-situ truth of tide gauges, and the comparison between our results with global ones.

3 Results and their analyses

3.1 Comparison between results from different time span of \( \text{T/P} \) observations

The time span of \( \text{T/P} \) observations from cycle 11 through 82 is divided into two sub sets: the set of cycle 11 to 46 and the set of cycle 47 to 82. The results from the two sub sets are compared with each other and each of them is compared with the combined result from cycle 11 through 82. All the comparisons are performed on 129 \( 1^\circ \times 1^\circ \) grids. The statistics results are listed in Table 1.

The above results infers that the results derived with each yearly series are close enough to each other, which reflects the stability of the model coefficients and the harmonic constants. Moreover, the results from each yearly series coincide with ones from combined 2 years series better, so the combined results can be thought as the average of 2 yearly results, which verify the efficiency of this model to relieve the effects of aliasing. From above analysis we can conclude that the yearly \( \text{T/P} \) results are on the 2 cm accuracy level.

3.2 Comparison with coastal tidal gauge truth

The value obtained from the models established with 2 years of \( \text{T/P} \) observations are compared with the data in the admiralty tidal tables on 14 tidal gauge locations. The statistics is listed in Table 2.

The maximum differences of 4 main tides derived from the model with the in-situ truth listed in the tidal table are less than 10 cm for amplitudes, and the standard deviations of amplitudes and phase lags are within 5 cm and 30° respectively. The combined predicting error of each partial tide is within 5 cm, the sum of \( \text{rms} \) of 4 main tides is 8.363 cm. Considering the data quality of the tidal table, the results of this work are perfect even on the coast near this study area.

3.3 Comparison with in-situ truth on the island tidal gauges

For more meticulous evaluation of our models, 9 island tidal gauges are selected for the comparison, the statistic results are listed in Table 3.
In the inverse area, the model is more close to those in-situ truth, the standard deviations of all the 8 tidal amplitudes are less than 2 cm, and the maximum amplitude differences are within 3 cm. For all of the 4 partial tides with amplitude larger than 10cm, the phase lag deviations are within 10°.

3.4 Comparison with other global and regional tide models

Compared with existing tidal charts, Schwiderski model has too large differences from the results of this study and ground truth, and Mazzega model gives larger estimates for the leading tides in the South China Sea. This model is close to that of Ye Anle's and that of Fang Guohong's.

Taking the SR95.1 model as an example of the new generation global ocean tide models, the comparison is performed on 602 0.5 × 0.5° grids, the results are listed in Table 4.

From Table 4, we can conclude that the differences between our models and SR95.1 ones are obvious and they manifest oneself in systemic pattern.

4 Tidal charts

Co-amplitude charts of constituent $O_1, K_1, M_2$, $S_2$ based on the models developed in this paper are shown in Fig 2.
5 Conclusion

The polynomial is effective to model the tide fields of the center part of the South China Sea and to solve the aliasing problems of the T/P sampling.

Through various comparisons, the models are verified. In the center part of the study area, the accuracy of amplitudes of principal partial tides reaches 3cm, and the standard deviation of phase lags are within 10°. In coastal area an acceptable results are also obtained.

The models developed in this paper can be used to correct tidal signals in altimetry data set for the researches of gravity field, various scales of ocean circulation and sea surface topography.

References

1 Ye A L, Robinson I S. Tidal dynamics in the South China Sea. Geophys J Roy. Astro. Soc., 1983, 72(3): 691-707
2 Callahan P S. TOPEX/POSEIDON project GDR user’s handbook. JPL Rep. D-8944, Jet Propul, Lab. , Pasadena, Calif., 1992
3 Cao D M, Fang G H, Huang Q Z, et al. Tidal regime in the Nanhaa sea area and its adjacent southwest waters. Beijing: Oceanologa et Limnologaw Sinica , 1997, 28(2): 198-208(in Chinese)
4 Fang G H, Zheng W Z, Chen Z Y, et al. The analysis and prediction of tides and tidal currents. Beijing: Ocean Press, 1986(in Chinese)
5 Fang G H, Cao D M, Huang Q Z, et al. The numerical simulation of tides and tidal currents in the South China Sea. Acta Oceadetica, 1994, 116(2): 1-12(in Chinese)
6 Bao J Y, Chao D B, Li J C, et al. The harmonic analysis of tides on the TOPEX/POSEIDON cross-overs in the South China Sea. Acta Geodeltica, 2000, 22(1): 17-23(in Chinese)
7 Mazzega P, Berge M. Ocean tides in the Asian semi-enclosed seas from TOPEX/POSEIDON altimeter measurements. J. Geophys. Res., 1994, 99: 24 867-24 881
8 Deng X L, Chao D B, Chen J Y, et al. A preliminary process of TOPEX and POSEIDON data in the China Sea and neighbour. Acta Geodetica et Cartographica Sinica, 1996, 25(3): 226-232(in Chinese)
9 Shen Y J, Hu D M, Mei L M, et al. A numerical calculation of tides in the South China Sea. Transactions of Oceanology and Limnology, 1985, 1: 1-12(in Chinese)