Highly-Efficient Focusing of Terahertz Waves with an Ultra-Thin Superoscillatory Metalens: Experimental Demonstration

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**Keywords**: metasurfaces, terahertz, superoscillations, subwavelength focusing, superoscillatory lens.

**Abstract**: The performance of an ultra-thin (*thickness < 0.04λ₀*) metasurface superoscillatory lens (metaSOL) is experimentally demonstrated in the terahertz (THz) range. The metaSOL is designed using two different hexagonal unit cells to improve the efficiency and properties of the conventional transparent-opaque zoning approach. The focusing metastructure produces, at a frequency *f_{exp} = 295 GHz*, a sharp focal spot 8.9λ_{exp} away from its output surface with a transversal resolution of 0.52λ_{exp} (≈25% below the resolution limit imposed by diffraction), a power enhancement of 18.2 dB and very low sidelobe level (−13 dB). Resolution below the diffraction limit is demonstrated in a broad
fractional operation bandwidth of 18%. The focusing capabilities of the proposed metaSOL show its potential use in a range of applications such as THz imaging, microscopy and communications.

1. Introduction

Metamaterials and their 2D version, metasurfaces, have attracted much attention owing to the possibility they offer to control phase, amplitude and polarization characteristics of electromagnetic (EM) waves. This flexibility opens new avenues to engineer complex structures able to surpass the limits imposed by natural materials.\textsuperscript{[1]} In fact, it is possible to synthesize artificial materials with precise electric and magnetic responses and thus obtain exotic EM features such as near-zero and/or negative refractive index values.\textsuperscript{[2-6]} Metamaterials and metasurfaces have been applied in multiple scenarios such as lenses, sensors, antennas, circuits and computing, among others, along with spatio-temporal modulation of their EM properties, for frequencies from acoustics to the optical range.\textsuperscript{[7-16]} These man-made materials have also found applications in the emerging THz range (0.1 to 10 THz) in a wide variety of fields such as sensing, spectroscopy and imaging, defense and security, material testing, biomedicine, to name a few.\textsuperscript{[17-23]}

Imaging devices usually suffer from the diffraction produced by EM waves, which prevents resolving subwavelength features. It is well known that the spatial resolution of conventional lenses (such as the commonly used dielectric lenses and parabolic mirrors) is in the order of $0.61\frac{\lambda_0}{NA}$,\textsuperscript{[24]} where $NA$ is the numerical aperture of the lens and $\lambda_0$ is the free space operation wavelength.

Overcoming this diffraction limit has become a hot research topic worldwide and different ways to surpass it have been proposed, such as: (i) photonic nanojets for near-field imaging using dielectric microspheres and cuboids,\textsuperscript{[25-27]} (ii) near-field optical-scanning microscopy systems, which are able to detect the high frequency components from evanescent waves using probes located within less than one wavelength from the sample surface, allowing resolutions of $\frac{\lambda_0}{20}$ for two-dimensional objects,\textsuperscript{[28,29]} (iii) high-refractive index glass microspheres combined with a conventional
microscope and placed on the sample surface, which are able to project the sample's near field nanoscopic features into the far-field, generating a magnified virtual image and demonstrating a resolution four times better than the classical diffraction limit;\cite{30} (iv) hyper-lenses, based on anisotropic metamaterials, that convert evanescent waves into propagating waves with high wave-numbers that can be processed by conventional optics.\cite{31}

Regarding the THz range, metamaterial, hybrid, plasmonic and superoscillatory lenses (SOLs) have been proposed in the past.\cite{32-36} In the case of the hybrid lens discussed in\cite{32} a focal spot size about $\lambda_0/7.5$ was demonstrated at 2 THz (only by numerical simulation). In the plasmonic lens presented in\cite{33}, it was shown (also numerically) that the plasmonic modes excited by the incident THz radiation can be concentrated into a theoretical region of around $\lambda_0/180$. Nevertheless, in both examples the focal length ($FL$) was below one wavelength. For far-field super-resolution applications, however, devices are sought to have long $FL$, large field of view ($FoV$, defined as the area of low intensity around the superoscillatory hotspot that stays below 25% the maximum power value at the focus, see supplementary material for more information),\cite{34} and good transmission characteristics, not only aiming to minimize the reflection and absorption in the device, but also to improve the focusing efficiency concentrating the electric field in the focal spot. In this context, SOLs arise as an interesting and powerful alternative, as they can focus light into a sub-wavelength hotspot located further than 10 times the wavelength in free space.\cite{37-42}

Although the focus of SOL devices can be arbitrarily small in theory, it happens that as it is made narrower, high intensity side lobes arise very close to the main spot, reducing the $FoV$ and restricting its practical application. In addition, for very sharp spots with high spatial resolution, the focusing efficiency decreases exponentially (the energy of the EM waves within the $FoV$ becomes a very low percentage of the total incident energy) and the sidelobes and sideband levels become increasingly significant (see supplementary material for a formal and graphical definition of sidelobes and sidebands).
The lens analyzed in this study is based on the design theory and methods presented in a previous article published by some of the authors of this work,\[38\] and has been conceived to overcome the challenge of balancing the trade-off between the different parameters involved. Thus, here we demonstrate experimentally a metaSOL designed and operating in the lower frequency band of the THz spectrum (~300 GHz) to: i) Enhance the transmission efficiency above 70%, improving the 50% transmission efficiency achieved with the typical transparent and opaque ring masks configuration\[43-46\] and, all the more, the 5-10% transmission efficiency of other SOL design methods such as hole arrays\[47\] and superlenses\[48\]; ii) enhance the focusing efficiency, also known as the yield parameter, defined as the ratio of the energy located in the hotspot to the full focal plane (see supplementary material for a more detailed explanation of this parameter). A yield above 50% will be imposed by design, because the energy at the focus determines the limit of how small the hotspot can be while remaining effective in imaging.\[47\] This will affect the size of the hotspot, since even for the largest-sized SOLs (with a diameter of the order of $10^3$ wavelengths) the focusing efficiency reported is only 5% at most when the hotspot is well below the diffraction limit ($\text{resolution} \approx 0.38\lambda_0/\text{NA}$);\[49\] iii) obtain a focal spot size beyond the diffraction limit with small sidelobes to get sub-wavelength focusing with a large $\text{FoV}$. The focus size is usually evaluated by means of its full width at half maximum ($\text{FWHM}$), defined as the distance at which the power distribution has been reduced to half its maximum at the focal spot whereas the sidelobe level ($\text{SLL}$) is defined as the ratio between the maximum power at the focal point and the maximum power of the first sidelobe (more details about these parameters can be found in the supplementary material); iv) obtain an ultra-thin and compact lens, with a thickness $< 0.04\lambda_0$ and diameter $< 55\lambda_0$, the metaSOL becomes 1 to 2 orders of magnitude electrically smaller than the SOLs designed for the optical spectrum\[48-50\] and up to 3 times smaller than the SOLs reported in the THz range\[37\].

As a trade-off between the focal point intensity, size and $\text{FoV}$, the engineered metaSOL achieves a transmission and focusing efficiency of $\approx 71\%$ and $\approx 73\%$ respectively, with $\text{FWHM} = 0.52\lambda_\text{exp}$, $\text{SLL}$...
= −13dB and $FoV > 5\lambda_0$. Our current work represents one of the few examples of SOLs experimentally demonstrated in the THz spectrum.

### 2. Lens design and fabrication

Although the design process was fully explained in [38], here we summarize it for the sake of clarity and completeness. Superoscillation is a phenomenon in which a signal that is globally band-limited can contain local ultra-high spatial frequencies that oscillate faster than its maximum Fourier components. This property enables its use for far-field super-resolution focusing.[51-58]

Such a superoscillatory function can be easily obtained by the superposition of wave functions. Following this reasoning and applying the Huygens-Fresnel (H-F) principle,[59] each unit cell in our design is analytically considered as a point source that radiates a cylindrical wave whose amplitude and phase is equal to its transmission coefficient ($t_n$), assuming that the excitation is done with a plane wave of unit amplitude at normal incidence. The resulting field at each point of space can be calculated by adding the fields of all sources. Mathematically, this can be written as:

$$f(r) = \sum_{n=1}^{N} \frac{|t_n|}{\sqrt{|r - r'|}} e^{i(k_0|r - r'| + \text{arg}(t_n))}$$

where $f(r)$ is the electric field at the evaluation point; $N$ is the number of source points; $r$ and $r'$ are the position vectors at the evaluation point and at the source point, respectively; $k_0$ is the wave number in free space (assuming that the lens in embedded in free space); $|t_n|$ and $\text{arg}(t_n)$ are the transmission coefficient magnitude and phase at each point, respectively.

By analogy to other superoscillatory functions[60,61,62] the H-F principle provides at the focal plane a resulting field that can show a superoscillatory behavior if the transmission coefficient magnitude and phase at each point of the lens is adequately modulated. As it is known, a superoscillatory function can be obtained by the linear superposition of wave functions, enabling the H-F model, even despite its simplicity, to develop a sub-wavelength superoscillatory focus.
The metaSOL is then analytically engineered through a modified algorithm based on the combination of a genetic method, the Binary Particle Swarm Optimization (BPSO)\[^{63}\] and the H-F principle. For this purpose, two hexagonal unit cells that cover, respectively, the even and odd zones, are designed to provide a high transmission coefficient magnitude ($|t_n| = 0.8$) while ensuring enough phase difference between them ($\mid \arg \{t_{n, \text{odd}}\} - \arg \{t_{n, \text{even}}\} \mid > 0.16\pi$ rad) at the operation frequency (fixed in the design at $f_0 = 327$ GHz, $\lambda_0 = 0.917$ mm), so the algorithm can converge and generate the focal spot at the desired $FL$ (fixed in the design at $10\lambda_0$). As demonstrated in our previous work\[^{38}\], unlike the usual lenses based on binary phase masks, where the phase difference is around $\pi$, our design enabled us to use a much smaller phase difference and to benefit from a high transmission magnitude in both cells. We performed a preliminary analytical study using the H-F principle and found that a phase difference of at least $\sim0.16\pi$ (\(\pi/6\)) rad between even and odd zones was necessary for the BPSO algorithm to converge adequately and generate the focal spot. A complete discussion can be found in\[^{38}\]

The full metaSOL and the designed unit cells are represented in Figure 1 (a,b), respectively. As in\[^{38}\], we first evaluated a cylindrical lens (see Figure 1 (c)) via the H-F approximation. Then, the final spherical lens shown in Figure 1(a) was obtained by applying rotation symmetry to the cylindrical lens solution, and thus obtaining radial zones with 18783 unit cells covering a circular area of diameter $\varnothing = 49.7$ mm ($54.2\lambda_0$) and thickness $L_z = 0.036$ mm ($0.039\lambda_0$). It must be noted that this evolution from the cylindrical to the spherical lens affected the original $FL$, from $10\lambda_0$ to $9.2\lambda_0$.

The lens was lithographically patterned on a 600 nm (0.65\(\times\)10\(^{-3}\lambda_0\)) thick aluminum layer sputtered on a polypropylene (PP) film 35 $\mu$m (0.038$\lambda_0$) thick, via a standard contact photolithography technique (CPhLT) as described in\[^{64}\]. After the CPhLT process, a defect-free sample was obtained and tightened onto a ring-shaped aluminum holder with a clear aperture diameter of 50 mm.

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Figure 1. (a) Photograph of the metaSOL mounted in a ring-shaped aluminum holder and surrounded by absorbent material. (b) Photograph taken with a confocal microscope showing the interface between the two central adjacent zones. Note the two different unit cells that compose the metasurface and their dimensions schematically depicted on top. \(L\) is the hexagonal ring slot side length; \(\Delta r\) is the hexagonal slot width; \(\alpha\) is the external radius of the hexagonal slot. (c) Diagram showing the unit cell distribution of the cylindrical lens. White and black rectangles represent a unit cell of type A and B, respectively. \(L_z\) is the metasurface thickness (i.e., the lens thickness).

3. Results

3.1. Focusing properties

The fabricated metaSOL was experimentally measured using the planar near-field characterization technique (detailed below) and then compared with analytical (H-F method) and simulation (CST Studio Suite®) results. In the experimental setup the lens was illuminated with a standard horn antenna with a moderate directivity (~20 dBi) that generated a \(y\)-polarized (see coordinate axes in Figure 1(a)) Gaussian beam with a low ellipticity ratio. The source was placed 110 mm away from the lens plane, providing non-uniform illumination, so that the difference (in decibel scale) between the electric field intensity at the central point of the lens and its edges was around 10 dB, to minimize the spillover and hence the direct transmitter-to-receiver transmission. As a receiver, an Anteral standard WR-3.4 near-field waveguide probe (model NFP-15-WR03) was used, allowing a fine characterization of the electric field.\(^{65}\) This probe has a length of 15 mm and a conical shape to...
reduce reflections. Absorbent material was used to reduce the reflections from the UG/387-U flange. Although the probe suffers from some reflections from the tip that cannot be avoided, they are sufficiently small to be neglected. Regarding the crosspolar isolation, this probe presents at least 30 dB isolation to the orthogonal polarization. Millimetre-wave extenders (VDI WR3.4-VNAX) were connected to both ports of an Agilent N5242A PNA-X Network Analyzer and the receiver probe was fixed on a motorized 2-axes translation stage. The experimental setup was placed on a planar antivibration table to minimize noise from mechanical vibrations and both the metaSOL and receiver probe were surrounded with millimetre-wave absorbing material to minimize reflections and mimic anechoic chamber conditions. Also, a wide time-domain gating was set on the Vector Network Analyzer (VNA) to reduce the effect of parasitic reflections.

Before starting the measurement process, the whole system was calibrated by placing face to face the transmitter and receiver at a distance \( z = 9 \text{ mm} \) without the metaSOL and sweeping the frequency from 260 to 350 GHz with a step \( \Delta f = 1 \text{ GHz} \) (91 frequency points). To verify that using this single measurement in the optical axis for each frequency was sufficient as a calibration, the magnitude of the electric field was also recorded in the interval from \( z = 6 \) to \( z = 14 \text{ mm} \), without the metaSOL. The ripple signal obtained by comparing the new measurements at the different \( z \) positions to the reference signal \((z = 9 \text{ mm})\) was limited to be \( \pm 0.75 \text{ dB} \) in the whole frequency range. This small variability supports the simplification assumed, necessary to speed up the measurements.

After calibration, the metaSOL was inserted in the experimental setup. The electric field (E-field) amplitude and phase was recorded by scanning \( 5 \times 5 \text{mm} \) \( xy \)-plane squares, with steps of \( \Delta x = \Delta y = 0.1 \text{ mm} \) (i.e., \( 51 \times 51 \) points) at distances from \( z = 6 \text{ mm} \) to \( z = 14 \text{ mm} \) away from the aperture plane of the lens using a step of \( \Delta z = 0.5 \text{ mm} \). Hence, 17 \( xy \)-plane frames with \( 51 \times 51 \) E-field points in each of them were obtained. At each position the frequency was swept from 260 to 350 GHz with a step \( \Delta f = 1 \text{ GHz} \). A summary of the different metaSOL parameters along with the methods used to
extract them from the experimental data is shown in the supplementary material section. In Figure 2, we represent the analytical and experimental power distribution spectral maps when the probe is moved only along the optical z-axis (x = y = 0).

**Figure 2.** (a) Analytical (H-F) and (b) measured normalized power distribution spectra along the optical z-axis. In each case, the FL values obtained (FL_{H-F} = 8.44 mm = 9.20\lambda_0, FL_{exp} = 9.00 mm = 8.85\lambda_{exp}) are represented at the working frequencies f_0 = 327 GHz and f_{exp} = 295 GHz (c) Analytical and (d) measured normalized power distribution along the frequency axis at z = FL_{H-F} and z = FL_{exp}, respectively.

From these colormaps, it is immediately noticed a redshift in the experimental response compared to the analytical results. The frequency of maximum power is f_{exp} = 295 GHz (\lambda_{exp} = 1.017 mm) with FL_{exp} = 9.05 mm = 8.9\lambda_{exp}, whereas in the analytical calculation it was f_0 = 327 GHz and FL_{H-F} = 8.44 mm = 9.2\lambda_0. This deviation between numerical and experimental results is mainly due to the Gaussian beam illumination used in the experimental characterization which differs from the plane wave excitation considered in the analytical calculations. In addition, the FL (which corresponds to the highest power region of white color in the contour plot of Figure 2 a-b) shifts away from the lens along the z axis as the frequency increases. This is typical for lenses suffering from chromatic aberration.\[66\] As we are using dispersive metamaterial unit cells, it is then expected to have such a spatial shift of the focal spot.

Concentrating now on the experimental results, it is observed that the maximum power enhancement (17.4 \pm 0.75 dB, which represents a measure of how much the metaSOL magnifies the
magnitude of the electric field at the focal spot compared to free-space propagation, see supplementary material for more information) happens in the range 275-332 GHz, with a peak of 18.15 dB at $f_{exp} = 295$ GHz, as shown in Figure 3a.

![Figure 3](image)

**Figure 3.** (a) Experimental power enhancement (in decibel scale) along the frequency range 275-332 GHz at the corresponding FL for each frequency. (b) Average experimental FWHM (red curve) and Rayleigh diffraction limit (blue curve) both normalized to $\lambda$.

The average FWHM, calculated by taking the average FWHM along radial directions around the focus, is represented in Figure 3b (red curve) and compared to the Rayleigh diffraction limit (blue curve), both normalized to the wavelength. At $f_{exp}$ the FWHM is 0.53 mm (0.52$\lambda_{exp}$) whereas the Rayleigh diffraction limit is 0.66 mm (0.65$\lambda_{exp}$). Note that each frequency has a different FL (as can be seen in Figure 2b) due to chromatic aberration and hence, the FWHM calculation is done at a different z position for each frequency. Since all the foci obtained are below the diffraction limit at the corresponding FL for each frequency, this leads to a high fractional bandwidth around 18%.

To fully characterize the focal image space of the lens at the working frequency ($f_0$ in the analytical and simulation results and $f_{exp}$ in the experimental results), additional transmittance maps were obtained. The $yz$-maps at $x = 0$ mm and $xz$-maps at $y = 0$ mm, are shown in Figure 4 (panels a-c and e-g).
Figure 4. Normalized power distribution at $f_0$ on the xz-plane (a,b), yz-plane (e,f), and xy-plane (i,j), for the analytical, (first column) and simulated (second column) lens. Normalized power distribution at $f_{exp}$ on the xz-plane (c), yz-plane (g), and xy-plane (k), for the measured results (third column). Experimentally measured normalized power distribution at $z = F_{L\text{exp}}$ along the x-axis (d) and y-axis (h). (l) Experimentally measured normalized power along the z-axis for $x = y = 0$ mm, at $f_{exp}$, where $F_{L\text{exp}}$ is marked with a dotted vertical line.

Also, xy-maps were obtained at $z = FL$ for each case ($F_{LH\text{-}F} = 9.2\lambda_0$, $F_{L\text{sim}} = 9.6\lambda_0$ and $F_{L\text{exp}} = 8.9\lambda_{exp}$) and represented in Figure 4 (panels i-k). From the results, a very clear energy concentration is observed at the corresponding focal positions ($x = y = 0$, $z = FL$) while subwavelength focusing is achieved with low sidelobes (SLL = $-13$dB) and a clear FoV.
By observing carefully the results of Figure 4, it is found that the focus shape is not totally circular. This is due to the depolarization effect, a well-known phenomenon which states that when a linearly polarized incident beam is focused by a high numerical aperture lens, the electric field in the observation plane will be commonly distorted\(^ {[67]}\). It must be also noted that a spatial widening is introduced by the receiver probe since it averages the transversal component of the E-field received on its aperture. Then, a wider focus in the experiment compared to the simulation and analytical results is expected, where an ideal point detector is assumed. Another small difference, which can be related to the different nature of the illumination sources, is found in the DoF, yielding the measured result 57\% wider than in simulations. Regarding the FoV, the experimental data reveal no relevant sidebands in the \(-2.5\lambda_0 \) to \(2.5\lambda_0\) \(xy\)-frame recorded, so at least its value must be \(FoV > 5\lambda_0\). This is in good agreement with the analytical and simulation studies where no relevant sidebands were found in a \(-100\lambda_0\) to \(100\lambda_0\) \(xy\)-frame. Note that such a large spatial range was not considered in the experimental setup as it would have increased the time needed to record each \(xy\)-frame by a factor of 40 at least. This could have introduced unavoidable uncertainties in the experiment due to the thermal drift of the devices.

The focusing properties extracted from analytical, simulation and experimental results are summarized in Table 1.

| FL \(\lambda_0\) | DoF \(\lambda_0\) | FWHM \(\lambda_0\) | FWHM \(\lambda_0\) | Ellipticity \(\lambda_0\) | Power Enh. \(\lambda_0\) | SLL \(\lambda_0\) | Yield \(\lambda_0\) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Numerical \(9.16\lambda_0\) | 1.28\(\lambda_0\) | 0.46\(\lambda_0\) | 0.46\(\lambda_0\) | 1 | 18.4 dB | -12.0 dB | 54\% |
| Simulated \(9.56\lambda_0\) | 1.51\(\lambda_0\) | 0.44\(\lambda_0\) | 0.50\(\lambda_0\) | 0.88 | 16.6 dB | -11.0 dB | 63\% |
| Measured \(8.85\lambda_{\text{exp}}\) | 2.14\(\lambda_{\text{exp}}\) | 0.54\(\lambda_{\text{exp}}\) | 0.51\(\lambda_{\text{exp}}\) | 0.94 | 18.2 dB | -13.0 dB | 73\% |

\(^{a)}FL\) is the focal length; \(^{b)}DoF\) is the depth of focus; \(^{c,d)}FWHM_{xy}\) is the full-width at half-maximum in the transversal \(x\) and \(y\) axes, respectively; \(^{e)}Ellipticity\) is the ratio between FWHM \(x\) and FWHM \(y\); \(^{f)}Power-Enh\) is the maximum power enhancement; \(^{g)}SLL\) is the sidelobe level. \(^{h)}Yield\) is the focusing efficiency. \(\lambda_0 = 0.917 \text{ mm}, \lambda_{\text{exp}} = 1.017 \text{ mm}\).

The results obtained clearly demonstrate that the metaSOL has an excellent performance. The three main objectives pursued in the design have been experimentally accomplished: (i) enhanced

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focusing efficiency ($\text{yield} \approx 73\%$); (ii) reduced side lobes that provide a clean FoV, being the power level of the sidelobes below 5% of the focus power level ($\text{SLL} = -13\,$ dB); and (iii) focal spot below the diffraction limit, ($\approx 25\%$ below the Rayleigh resolution limit).

![Figure 5](image)

**Figure 5.** (a) FWHM$_{x,y}$ (normalized to $\lambda_{\exp}$) with indication of the Rayleigh diffraction limit (normalized to $\lambda_{\exp}$) and (b) Maximum Power enhancement (dB) and SLL (dB), along the z-axis at $f_0 = 295\,$ GHz. Normalized power distribution in the xy-plane at $f_{\exp}$ for $z = 6\,$ mm (c), $z = 9\,$ mm (d), $z = 10.5\,$ mm, with indication of the sideband and the sidelobe (the last related to the sidelobe level, SLL) (e) and $z = 12.5\,$ mm (f).

Finally, the full study of the lens behavior at the operation frequency is presented in Figure 5. The normalized FWHM along both $x$ and $y$ axes at different $z$ positions is represented and compared with the Rayleigh diffraction limit in Figure 5a. The enhancement and SLL in decibel scale are depicted in Figure 5b. Moreover, four additional xy-maps at $f_{\exp}$ for different $z$ positions are shown in Figure 5c-f to complete the study of the metaSOL.
The recorded hotspots presented in Figure 5 (panels c and d) show the sharp contrast between the typical SOL focusing profile, dominated by a prominent sideband, and a focus profile very similar to the diffraction pattern of a conventional diffraction-limited lens.

On the other hand, the focus shown in Figure 5e has a $FWHM = 0.28\lambda_{\text{exp}}$, achieving a remarkable reduction of the focal spot by a factor of almost 2.5 compared to the Rayleigh diffraction limit criterion ($= 0.66\lambda_{\text{exp}}$). Of course, this super-resolution comes at a price: lower power enhancement ($= 4.8 \text{ dB}$), lower focusing efficiency ($\text{yield} \approx 0.5 \%$), higher side lobes ($\text{SLL} = +6 \text{ dB}$, sidelobes higher than the focus) and a limited $FoV$ ($FoV = 0.8\lambda_{\text{exp}}$).

Finally, in Figure 5(f) the diffraction map at $z = 12.5 \text{ mm}$ is presented and it demonstrates the ability of the metaSOL to develop small hotspots ($FWHM = 0.37\lambda_{\text{exp}}$), surpassing again the diffraction limit ($= 0.68\lambda_{\text{exp}}$) by a factor of 1.8. Contrary to the focus of Figure 5(e), this hotspot is not so narrow, but it achieves a much higher focusing efficiency, being $\text{yield} \approx 7.5 \%$, with a slightly wider $FoV$ ($\approx 1.04\lambda_{\text{exp}}$). The $\text{yield}$ value achieved must be emphasized because nowadays, among the many challenging remaining issues, the most important one is the focusing efficiency.

Moreover, even for the largest-sized SOLs reported,$^{[49]}$ the focusing efficiency is only 5% at most when the hotspot size is below $0.38\lambda_0/\text{NA}$ (in the present case, $\text{NA} = 0.94$).

For the sake of completeness and to allow direct comparison, the focusing properties extracted from the most interesting focal planes at $f_{\text{exp}}$ are summarized in Table 2.

Table 2. Summary of focal behavior at $f_{\text{exp}}$

|          | $z^a$ | FoV$^b$ | $FWHM_x^c$ | $FWHM_y^c$ | Ellipticity$^c$ | Power Enh.$^{d)}$ | SLL$^d)$ | Yield$^d)$ |
|----------|-------|---------|------------|------------|-----------------|-------------------|----------|-----------|
| Focus 1  | 9 mm  | $>5.00\lambda_{\text{exp}}$ | 0.54$\lambda_{\text{exp}}$ | 0.51$\lambda_{\text{exp}}$ | 0.94            | 18.2 dB          | -13.0 dB | $\approx 73\%$ |
| Focus 2  | 10.5 mm | 0.80$\lambda_{\text{exp}}$ | 0.32$\lambda_{\text{exp}}$ | 0.25$\lambda_{\text{exp}}$ | 0.78            | 4.8 dB           | 6.0 dB   | $0.5\%$   |
| Focus 3  | 12.5 mm | 1.04$\lambda_{\text{exp}}$ | 0.39$\lambda_{\text{exp}}$ | 0.36$\lambda_{\text{exp}}$ | 0.92            | 6.3 dB           | 0.4 dB   | $7.5\%$   |

$^a_2$ $z$ is the position where the focus is evaluated; $^b_2$ FoV is the field of view; $^c_2$ $FWHM_x$ is the full-width at half-maximum in the transversal $x$ and $y$ axes, respectively; $^d_2$ Ellipticity is the ratio between $FWHM_x$ and $FWHM_y$; $^d_2$ Power-Enh is the maximum power enhancement; $^d_2$ SLL is the sidelobe level. $^d_2$ Yield is the focusing efficiency. $\lambda_{\text{exp}} = 1.017 \text{ mm}$.  

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As observed there, a very narrow focus (Focus 2) has small power enhancement and very high SLL with a poor yield, meaning that most of the energy is outside the focus. On the contrary, sacrificing resolution (Focus 1) one gets an outstanding power enhancement with very small SLL and large yield. Focus 3 is an intermediate case between the other two.

4. Conclusions

The results obtained verify that a metaSOL in the lower THz range can be successfully designed and fabricated to overcome the classical Rayleigh diffraction limit, allowing a higher resolution than that attainable with conventional THz optics. The proposed metastructure was designed using only two hexagonal unit cells. All the numerical, simulation and experimental measurements are in good agreement demonstrating a full characterization of the focusing performance of the metaSOL. It has been shown that the fabricated metaSOL has the ability to generate a focal spot at a distance of $8.9\lambda_{\text{exp}}$ away from the metastructure achieving a sharp focus with a transversal resolution of $0.52\lambda_{\text{exp}}$ ($\approx 25\%$ below the resolution limit) a power enhancement of 18.2dB and very low sidelobes (sidelobe of $-13\text{dB}$) while its transmission and focusing efficiency are very high ($\approx 71\%$ and $\approx 73\%$, respectively). Moreover, the resolution is below the diffraction limit in a broad fractional bandwidth of 18%. Due to the advantages provided by its ultra-small thickness, lightweightness, high transmission and focusing efficiency, ease of integration and sub-diffraction focal size, the proposed metalens may find application in THz imaging systems where thin, flat, light-load, and high-performance optics with sub-diffraction focusing capabilities is demanded.

Acknowledgements

This research was funded by the Spanish Ministerio de Ciencia, Innovación y Universidades, Project RTI2018-094475-B-I00 (MCIU/AEI/FEDER, UE). V.P-P. acknowledges support from the
Newcastle University (Newcastle University Research Fellowship). S.K. acknowledges support from the Ministry of Science and Higher Education of the Russian Federation, Project No. 075-15-2020-797.

Received: ((will be filled in by the editorial staff))
Revised: ((will be filled in by the editorial staff))
Published online: ((will be filled in by the editorial staff))

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An ultra-thin metasurface-based superoscillatory lens is designed and experimentally demonstrated working within the Terahertz frequency range. The spatial resolution of the resulting focus (at distance $z = 9$ mm) is improved by 25% above the resolution limit while achieving very low side lobes, an enhancement of 18.2 dB and high yield. This lens may open new avenues for high-resolution terahertz lenses.