Are valence quarks rotating?

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Abstract
We suggest to compare the deep inelastic scattering structure functions measured in the unpolarized charged-lepton scattering off a transversely polarized proton and off a longitudinally polarized proton at larger Bjorken variable $x$, one may get a direct evidence concerning the valence quark orbital angular momentum in the proton. This possible rotating effect is estimated by using a proton-ensemble model.

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The European Muon Collaboration (EMC) result [1] indicates that only a small fraction of the nucleon spin is due to quark spins. There have been many attempts to explain this (second) EMC effect from the fundamental theory. The key to understand the spin structure of the nucleon is the orbital angular momentum (OAM) of quarks and gluons. However, progress in this direction has been hindered by a number of difficulties in measuring.

In this letter we suggest a direct observation of the valence quark OAM: comparing the deep inelastic scattering (DIS) structure functions measured in the unpolarized charged-lepton scattering off a transversely polarized proton and off a longitudinally polarized proton at larger Bjorken variable \( x \), one may get a direct evidence concerning the orbital motion of the valence quarks. Our idea is straightforward: Fermi motion momentum \(( \sim 200 MeV)\) of a nucleon inside a nucleus is small comparing with a larger longitudinal momentum of this nucleon at the infinite momentum frame in DIS, however, such motion distorts the quark momentum distributions in the nucleon and it has been clearly observed by so called the (first) EMC effect [2]. The reason is that the quark distributions at larger \( x \) are small values, any perturbations on them can be sensitively appeared in the ratio of the structure functions with and without perturbation. Therefore, we also can expect that the orbital motion of a valence quark in a transversely polarized proton will add to the longitudinal motion of this quark and smears its distribution. On the other hand, there is no such smearing effect in a longitudinally polarized proton since the rotating plane is perpendicular to the \( z \)-direction. Thus, the ratio of the inclusive structure functions in the above two different polarized protons will appear a like-EMC effect at larger \( x \). We believe that such evidence has existed in the experimental data base and we have not used it.

For estimating the smearing effect of the orbital motion in the valence quark distribu-
tions, we construct a proton-ensemble to analogy Fermi motion in the nucleus. Considering a DIS event (Fig.1a), where a virtual photon probes a valence quark in the polarized proton. Because the parton OAM $\vec{L}$ in a proton is always restricted by a general condition $L_z + s = 1/2$ and it is irrelevant to the independent motions of the partons, therefore, we assume that the collective rotation of the partons can be separated from other single particle (intrinsic) motion at the adiabatic approximation. We denote the velocities of these two motions at center of mass (c.m) system of the proton are $\vec{u}_R$ and $\vec{v}_{in}$, respectively. Now we image that this targeted proton $P$ is replaced by an equivalent non-rotating proton $P^*$, which moves with velocity $\vec{v}_R$ but keeps all other non-rotating structures of the proton $P$ (Fig.1b). The probe can not distinguish two scattering events in Fig. 1a and 1b, however, we realize the separation of the orbital rotating motion from other intrinsic motions of the struck quark and avoid the unknown parameter—the mass of the current quarks. After A-times measurements, we have a proton-ensemble $\{P^*_A\}$, which is consisted by A-protons $P^*$ (Fig. 1b). The normalized distribution of the equivalent proton $P^*$ inside the proton-ensemble $\{P^*_A\}$ at its c.m system is $\rho_\xi(\vec{k}_R)$, where $\xi$ indicates the polarized state of the original proton and $\vec{k}_R = m_N \vec{v}_R$. We project this distribution to the plus-component in the light-front version, one can get the probability of an equivalent proton $P^*$ caring the fractional momentum $y = A\vec{k}_R^+/P_{\{P^*_A\}}^+$

$$f_{R,\xi}(y) = \int d^3\vec{k}_R \rho_\xi(\vec{k}_R) \delta(y - A\vec{k}_R^+/P_{\{P^*_A\}}^+).$$  \hfill (1)

We denote the quark momentum distribution in the equivalent proton $P^*$ as $F_q(z, Q^2)$ and that in the proton-ensemble as $F_{q,\xi}(x, Q^2)$, they have relation

$$F_{q,\xi}(x, Q^2) = \int dy f_{R,\xi}(y) F_q(x/y, Q^2),$$  \hfill (2)

where
\[ x = \frac{A k_{\text{in}}^+}{P_{(p^*)}^+}; \quad z = \frac{x}{y} = \frac{k_q^+}{k_R}. \] (3)

For the longitudinally polarized proton with respect to its motion, situation becomes simple. Since

\[ \rho_L(\vec{k}_R) = \rho_L(k_{R,x}, k_{R,y}) \delta(k_{R,z}), \] (4)

we have

\[ f_{R,L}(y) = \delta(1 - y). \] (5)

On the other hand, for the transversely-polarized proton,

\[ \rho_T(\vec{k}_R) = \rho_T(k_{R,y}, k_{R,z}) \delta(k_{R,x}). \] (6)

We take \( A \) as a large number. Since the rotating momentum is small comparing with a larger longitudinal momentum in the infinite momentum frame, its distribution from 0 to \( k_{R,\text{max}} \) is narrow. For simplicity we assume that the distribution \( \rho_T(k_{R,y}, k_{R,z}) \) presents roughly a uniform distribution from \( \sqrt{k_{R,y}^2 + k_{R,z}^2} = 0 \) to \( k_{R,\text{max}} \), i.e.,

\[ \rho_T(k_{R,y}, k_{R,z}) = \frac{1}{\pi k_{R,\text{max}}^2} \theta(k_{R,\text{max}} - \sqrt{k_{R,y}^2 + k_{R,z}^2}). \] (7)

Thus, we have

\[ f_{R,T}(y) = \begin{cases} \frac{2}{\pi} \left( \frac{m_N}{k_{R,\text{max}}} \right)^2 \sqrt{\left( \frac{k_{R,\text{max}}}{m_N} \right)^2 - (y - 1)^2} & \text{if } 1 - k_{R,\text{max}}/m_N \leq y \leq 1 + k_{R,\text{max}}/m_N \\ 0 & \text{otherwise} \end{cases} \] (8)

We use the ratio
to describe the rotating effect of the valence quarks, where $F_{2,T}(x, Q^2)$ (or $F_{2,L}(x, Q^2)$) is the structure function of the unpolarized lepton scattering off a transversely polarized proton (or off a longitudinally polarized proton). A convenient method is to measure

$$R_I^x > 0.4 = \frac{F_{2,T}(x, Q^2)}{F_{2,L}(x, Q^2)},$$

(9)

where $F_2(x, Q^2)$ is the structure function of the unpolarized proton, although the expected effect in $R_{II}$ is about $2/3$ of $R_I$.

For estimating the rotating effect, we use a simple parametrization of the valence quark distribution $xV(x, Q^2) = 3B^{-1}(0.5, 4)x^{0.5}(1-x)^3$, where B is the Beta function. In Fig. 2 we present the results using $k_{R, max} = 20, 50, 100$ and $200 MeV$, which corresponding to the maximum rotating velocities $v_{R, max} = 0.02, 0.05, 0.11$, and 0.21 in the natural unit $\hbar = c = 1$. Following a consideration in classical mechanics, we regard the target proton as a rigid body rotating with angular velocity $\omega = v_{R, max}/R_N$. Thus we can estimate the contributions of the parton OAM to the proton spin in the above mentioned examples, they are $L/(1/2\hbar) = I\omega/(1/2\hbar) = 8\%, 20\%, 40\%$ and $80\%$, respectively. Although these predictions are naive and we do not expect the precise OAM-information from them, however, any results deviating (even non-deviating) from unity in Eq. (9) are instructive for understating "spin crisis".

In conclusion, through the relations between two kinds of EMC effects (i.e., the contributions of Fermi motion to the first EMC effect and the spin structure of the proton in the second EMC effect), we suggest to compare the deep inelastic scattering structure functions measured in the unpolarized charged-lepton scattering off a transversely polarized proton and off a longitudinally polarized proton at larger Bjorken variable $x$, one may
get a direct evidence concerning the valence quark orbital motion. This possible rotating effect is estimated by using a proton-ensemble model.
References

[1] EMC Collaboration, J. Ashman et al., Phys. Lett. B202, 603 (1988); Nucl. Phys. B328, 1 (1989).

[2] EMC Collaboration, J. J. Aubert et al., Phys. Lett. B123, 275 (1983); R.G. Arnold et al., Phys. Rev. Lett. 52, 727 (1984).
Fig. 1 (a) A DIS event of the unpolarized charged-lepton scattering off a polarized proton $P$, where the struck quark has rotating velocity $\vec{v}_R$ and other intrinsic velocity $\vec{v}_{in}$; (b) An equivalent DIS event corresponding to (a), where target proton $P^*$ moves with $\vec{v}_{in}$ without rotation; $A$-times measurements construct a proton-ensemble $\{P^*\}_A$. 
Fig. 2 The ratio $R_I$ in Eq. (9), which presents the smearing effect of different orbital angular momenta of the valence quarks with parameter (from right to left) $k_{R,\text{max}} = 20, 50, 100$ and $200 \text{MeV}$. 