Some remarks on Abelian dominance

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We used a renormalisation group based smoothing to address two questions related to Abelian dominance. Smoothing enabled us to extract the Abelian heavy-quark potential from time-like Wilson loops on Polyakov gauge projected configurations. We obtained a very small string tension which is inconsistent with the string tension extracted from Polyakov loop correlators. This shows that the Polyakov gauge projected Abelian configurations do not have a consistent physical meaning. We also applied the smoothing on SU(2) configurations to test how sensitive Abelian dominance in the maximal Abelian gauge is to the short distance fluctuations. We found that on smoothed SU(2) configurations the Abelian string tension was about 30\% smaller than the SU(2) string tension which was unaffected by smoothing. This suggests that the approximate Abelian dominance found with the Wilson action is probably an accident and it has no fundamental physical relevance.

1. INTRODUCTION

It is an old idea to try to understand non-Abelian gauge theories in terms of an effective Abelian model with a smaller symmetry group. One possible way of doing this on the lattice is to isolate $U(1)^{N-1}$ link variables belonging to a maximal torus of SU($N$). This is called Abelian projection. The hope is that non-Abelian confinement might be explained as a condensation of monopoles in the resulting Abelian projected model (see e.g. \textsuperscript{2} for a recent review). If one wants to explain the non-Abelian physics in the Abelian projected system, a necessary condition is that the Abelian model has to reproduce the physical features of the non-Abelian system. This property is referred to as Abelian dominance.

The projection procedure necessarily involves some gauge fixing. In principle the physical properties of the projected system can depend on the gauge choice. Up to now the only gauge in which the Abelian projected system seems to capture the physics of the non-Abelian model is the maximal Abelian gauge \textsuperscript{3}. Here in the SU(2) case the Abelian and non-Abelian string tensions at Wilson $\beta = 2.51$ agree to within 8\% \textsuperscript{3}. In other gauges, most notably in the Polyakov gauge (where Polyakov loops are diagonalised) the situation is more controversial. Since all the Polyakov loops can be exactly diagonalised at the same time, in this case “Abelian dominance” exactly and trivially holds if the string tension is measured with Polyakov loop correlators. On the other hand due to the high level of noise on the projected configurations, it is impossible to extract the string tension from Wilson loops \textsuperscript{3}.

In this talk we discuss some related issues. The first question we address is that of the gauge choice. We use a recently proposed smoothing technique based on renormalisation group ideas \textsuperscript{5}. We can drastically reduce the short-distance fluctuations while preserving the long-distance physical properties of our configurations, most importantly the SU(2) string tension. This allows us to extract the heavy quark potential from Wilson loops on Polyakov gauge projected configurations. The resulting Abelian string tension turns out to be practically zero. This result is inconsistent with the string tension measured from
Polyakov loop correlators. It shows that the physical meaning of Polyakov gauge projected configurations is questionable.

The only gauge known to us in which approximate Abelian dominance has been found (with the Wilson action) is the maximal Abelian one. Therefore in the second part of the talk we shall concentrate only on this gauge. We study the question, how Abelian dominance depends on the details of the short-distance fluctuations in this particular gauge. Using the above mentioned smoothing on Monte Carlo generated SU(2) gauge configurations we can produce smoothed configurations with the same long-distance properties but reduced short-distance fluctuations. Comparing the Abelian string tension on the original and the smoothed configurations we can gain insight into its dependence on the short-distance details. For a more detailed account of this work the reader is referred to Ref. [6].

2. THE GAUGE CHOICE

The very idea of Abelian dominance is that the diagonal Abelian degrees of freedom can account for the physical properties of the full non-Abelian configurations. The issue of gauge fixing is definitely important here since the part of the system that we retain/discard with the Abelian projection very strongly depends on it.

Let us consider the Polyakov gauge first. On any given SU(2) configuration all the links belonging to the Polyakov loops can be diagonalised simultaneously by a suitable gauge transformation. Therefore any physical quantity derived from Polyakov loops will be trivially and exactly reproduced after Abelian projection in this gauge. In particular there is exact Abelian dominance for the string tension measured with Polyakov loop correlators [7].

A good test of whether the Polyakov gauge projected Abelian configurations capture some genuine physics would be to measure the string tension using time-like Wilson loops and compare this to the string tension obtained with Polyakov loop correlators. Unfortunately this cannot be done directly because the gauge fixing introduces so much noise that one would need a huge number of configurations to get enough statistics.

We can however use an ensemble of smoothed configurations and do all the measurements on them. We generated an ensemble of 20 12^4 configurations with the fixed point action of Ref. [5] at \( \beta = 1.5 \) which corresponds to a physical lattice spacing of 0.144 fm. After one smoothing step we measured both the full SU(2) and the Polyakov gauge projected U(1) heavy quark potential on them using time-like Wilson loops. We used the method and computer code of Heller et al. [8]. Our results are shown in Figure 1. In the SU(2) case we have a good plateau at \( T = 3 \) (this has also been confirmed on another ensemble of larger statistics) but in the U(1) case the potential decreases considerably with increasing \( T \) even at this point. One can conclude that in the \( T \to \infty \) limit the U(1) string tension is probably very close to zero.

The discrepancy is striking. We would also like to note that the static quark potential measured by Polyakov-loop correlators is exactly the same
as the full non-Abelian potential. We also note that the string tension obtained from Polyakov loop correlators and timelike Wilson loops should be the same (up to some small finite size effects). This means that two different but physically equivalent measurements of the same physical quantity give absolutely different results on the Polyakov gauge projected configurations. Our result for the Polyakov gauge strongly suggests that the physics of the Abelian projection is not only very strongly gauge dependent but in most of the arbitrarily chosen gauges the Abelian projected configurations do not even have a consistent physical meaning.

The maximal Abelian gauge (MAG) is special as it minimises the off-diagonal components of the link degrees of freedom, the ones that are discarded in the projection [2]. For this reason the MAG is a priori a better choice than the gauges that diagonalise an arbitrarily selected set of operators like the Polyakov loops.

3. ABELIAN DOMINANCE AND SHORT RANGE FLUCTUATIONS

In this section we study how Abelian dominance in the maximal Abelian gauge depends on the precise nature of short distance fluctuations.

We generated 100 $8^3 \times 12$ lattices with the fixed point action of Ref. [5] at $\beta = 1.5$ (lattice spacing $a = 0.144$ fm). At first as a check we verified that Abelian dominance holds for this ensemble. We transformed the configurations into the maximal Abelian gauge. This was done using the usual overrelaxation procedure iterated until the change in the gauge fixing action became less than $10^{-8}$ per link. After Abelian projecting these configurations the heavy quark potential was extracted from time-like Wilson loops in the same way as in the previous section. From the heavy-quark potential we obtained $\sigma_{na} = 0.123(7)$ for the non-Abelian and $\sigma_{ab} = 0.119(5)$ for the Abelian string tension in lattice units.

After this check we applied one step of smoothing to the same ensemble of SU(2) configuration and repeated the measurement of the Abelian and non-Abelian potential on the smoothed configurations. It gave $\sigma_{na} = 0.115(9)$ and $\sigma_{ab} = 0.080(10)$ for the SU(2) and the U(1) string tension respectively.

The SU(2) string tension on the smoothed configurations is essentially the same as on the unsmoothed ones, reflecting the fact that smoothing does not change the long-distance features. On the other hand, as a result of smoothing, the Abelian string tension dropped by about 30%. This shows that the Abelian string tension is very sensitive to the details of the short-distance fluctuations on the SU(2) configurations. A similar result has been found for the monopole string tension using cooling with the Wilson action [9].

It seems to us quite impossible to reconcile this fact with the expectation that the Abelian string tension is a genuine long-distance physical observable which is in some sense equivalent to the SU(2) string tension. In view of this, the approximate Abelian dominance found with Wilson action in the maximal Abelian gauge seems to be an accident rather than a fundamental physical phenomenon.

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