Analysis of Chinese Zither Modes Based on Finite Element Analysis

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Abstract. This paper makes a modal analysis of Chinese zither by using the finite element analysis method. By combining traditional Chinese national musical instruments with computer simulation technology, it studies the modal structure of Chinese zither according to the method of "data model - establishing FEM model - assigning material parameters - establishing constraints - solving the case". Through modal analysis, it not only verifies the validity of computer simulation method, but also provides data support and theoretical support for the improvement of Chinese zither structure and the structural research of traditional Chinese plucked instruments.

1. Introduction
Chinese zither is the representative of the cultural heritage of the China. As a traditional Chinese plucked instrument, it has more than two thousand years history. The external form of zither is simple, but structure is extremely complex[1]. Although most of the modern methods of making zither can be made by machine, they still cannot be separated from the manual method. Therefore, the selection of timber, shape and the production planning of musical instruments are based on the experience of the zither maker. They are well-made and well-used materials, so the production process of each zither factory also demonstrates different style.

2. The structural characteristics of the zither
Chinese zither has mellow timbre, long lasting charm and wide gamut, and has become the leader of traditional plucked instruments. These characteristics of tone colour and timbre are closely related to their structural characteristics[2]. Chinese zither consists of panel board, baseboard (or rear panel), sound hole, bulkhead (zither inside), curb plate, front and rear YueShan, perforation, string, zither code (Yanzhu), zither box, string nail and other parts. As shown in the Figure 1:
Figure 1. zither structure

There are one or more sound holes of different shapes on the baseboards, usually three. The shape and position of the sound hole play a certain role in the change of resonance, volume and tone quality of the zither. The hollow part between the panel boards and the baseboards has a partition, which plays a supporting role in the structure of the zither and makes its structure stable. In order to decorate, the four sides of zither are often made of precious wood, such as rosewood, red sandalwood and broad-leaved yellow sandalwood[3].

3. Finite element model description of zither structure

3.1. Instruction of finite element analysis of Chinese zither

With the rapid development of finite element technology, modal simulation analysis method has been widely concerned and applied in engineering structures such as aircraft, ships and automobiles. This method of analysis combined with structural vibration has great advantages in solving structural vibration and analyzing the relationship between structure and sound. The structure of zither is complex, so it is difficult to get its vibration characteristics through theoretical analysis. It is usually analyzed by finite element method in engineering application[4]. In the process of analysis, the structure of the zither is simplified properly. It uses the finite element analysis software to establish the finite element mesh model of the three-dimensional structure of the zither, reasonably chooses the wood material parameters, and calculates the structural response of the zither at various frequencies. The pre-processing software uses HyperMesh 14.0, the solver uses Optistruct, and the post-processing software uses HyperView 14.0.

3.2. Modal analysis finite element method

3.2.1 Structure discretization. This step is exactly the same as static analysis. Because of their different analysis contents, they may have different requirements for grid form. For example, in static analysis, it is required to refine the mesh in the stress concentration area, but in dynamic analysis, because the natural frequency and the main mode of vibration are mainly related to the mass and stiffness distribution of the structure, it requires the whole structure to adopt the uniform mesh form as far as possible.

3.2.2 Element analysis. The task of element analysis is still to establish element characteristic matrix and form element characteristic equation. In dynamic analysis, the element characteristic matrix includes not only stiffness matrix, but also mass matrix and damping matrix. The virtual displacement principle is used to establish the element characteristic matrix. Under the action of dynamic load, the virtual displacement of element nodes is assumed for any instantaneous moment $\{\delta \dot{q}\}^e$. Then the
corresponding virtual displacement \( \{ \delta \dot{d} \} \) and virtual strain \( \{ \delta \varepsilon \} \) are generated in the element. At this time, the virtual strain energy generated in the element is:

\[
\delta U = \iiint_V \{ \delta \varepsilon \}^T \{ \sigma \} dV
\]  
(1)

\( \{ \sigma \} \) is the stress of the node, \( dV \) is the element of volume.

In this case, the element is not only subjected to dynamic loads, but also to inertial forces \(- \rho \{ \ddot{d} \} dV\) and \(- \nu \{ \dot{d} \} dV\), \( \rho \) is the material density and \( \nu \) is the linear damping coefficient. Therefore, the virtual work done by external forces is as follows:

\[
\delta W = \iiint_V \{ \delta d \}^T \{ P_r \} dV + \iiint_V \{ \delta d \}^T \{ P_s \} dA + \iiint_V \{ \delta d \}^T \{ P_c \} - \iiint_V \rho \{ \dot{d} \}^T \{ \ddot{d} \} dV - \iint \nu \{ \dot{d} \}^T \{ \dot{d} \} dV
\]  
(2)

\( \{ P_r \} \) is the dynamic force acting on the element.

\( \{ P_s \} \) is the dynamic surface force acting on the element.

\( \{ P_c \} \) is the dynamic concentrated force acting on the element.

\( V \) element volume;

\( A \) element area;

\( \{ d \} = [N]\{ q \}^e \), \( \{ \varepsilon \} = [B]\{ q \}^e \)

\([N]\) shape function matrix, which is only a function of coordinates, has nothing to do with time.

\([B]\) strain matrix, in which each non-zero element is a constant determined by the node coordinates:

\[
\{ \dot{d} \} = [N]\{ \dot{q} \}^e \\
\{ \ddot{d} \} = [N]\{ \ddot{q} \}^e \\
\{ \delta \dot{d} \} = [N][\delta \dot{q}]^e \\
\{ \delta \varepsilon \} = [B][\delta \dot{q}]^e
\]  
(3)

According to the principle of virtual displacement, there is the following formula

\[
\delta U = \delta W
\]  
(4)

By substituting and sorting (1), (2) and (3) into (4), the equation of motion of the element can be obtained.

\[
[m]^e\{ \ddot{q} \}^e + [C]^e\{ \dot{q} \}^e + [k]^e\{ q \}^e = \{ R(t) \}^e
\]  
(5)

\[
[k]^e = \iiint_V [B]^T[D][B]dV
\]  
(6)

\[
[k]^s = \iiint_V [B]^T[D][B]dV
\]  
(7)

\[
[c]^e = \iiint_V [N]^T\nu[N]dV
\]  
(8)

\([D]\) elastic matrix is determined by elastic modulus and Poisson's ratio, which is independent of coordinates. \([k]^e, [m]^e, [c]^e\) are the stiffness matrix, mass matrix and damping matrix, which are the characteristic matrices that determine the dynamic performance of the element.

\[
\{ R(t) \}^e = \iiint_V [N]^T\{ P_r \} dV + \iint [N]^T\{ P_s \} dA + [N]^T\{ P_c \}
\]  
(9)
(9) is called the element node load array. It is the result of the displacement of the physical force, surface force and concentrated force acting on the element to the element node.

3.3. Global matrix integration
The task of global matrix integration is to assemble the characteristic matrices of each element into the characteristic matrices of the whole structure so as to obtain the global equilibrium equation.

\[
[M]\ddot{q} + [C]\dot{q} + [K]q = \{R(t)\}
\]

In the formula, \(q\) is an n-order array composed of displacement components of all nodes.
\(n\) the total degree of freedom of the structure.
\(\{R(t)\}\) the node load array;
\[
\{R(t)\} = \sum_{i=1}^{n}\{R_i(t)\}
\]

\(i\) the number of nodes:
\([K]\) the stiffness matrix of the structure.
\([M]\) the mass matrix of the structure.
\([C]\) the damping matrix of the structure.

The stiffness matrix, mass matrix and damping matrix of the structure are all composed of the corresponding matrix of the element.

3.4. The modal extraction method of Radioss
In general, we choose the first four methods: Subspace method, Block Lanczos method, Power Dynamics method and Reduced method. Unsymmetric method, Damped method and QRDamped method are only used in special cases. Modal extraction methods mainly depend on the size of the model (relative to the computing power of the computer) and specific application occasions. Comparing these four methods synthetically, the finite element model of the zither contains more nodes and units, and combined with the model and computer hardware configuration, this paper chooses the block Lanczos method for modal analysis[5].

3.5. Chinese zither mode
Chinese zither is modeled by 2nd-order tetrahedron element, the basic element size is 3mm. The string is simulated by spring element. The zither has a finite element model, the number of model elements is 612767, and the number of nodes is 162264, as shown in Figure 2.

![Figure 2. Chinese zither finite element model](image)

Paulownia density is 2.52X10^{-10}T/mm³, elastic modulus is 5000MPa and poisson's ratio is 0.49, as shown in Table 1. The total mass of Chinese zither is 7.579Kg.
Table 1. Material parameters of Chinese zither finite element model

| Material  | Elastic modulus (MPa) | Poisson's ratio | Density (T/mm³) |
|-----------|-----------------------|-----------------|-----------------|
| Paulownia | 5000                  | 0.49            | 2.52X10-10      |

4. Chinese zither modal result

Modal analysis is a common method used by many scholars at home and abroad to study musical instruments. By calculating the free mode of the zither, the modal results within the first 400 Hz of the model are obtained. The first 12 order modal results are shown in Figure 3-5.

Figure 3. 1 ~ 4 mode

Figure 4. 5~8 mode
5. Conclusion

Using the method of computer virtual simulation, this paper uses Hypermesh software to analyze the modal of the finite element model of the zither and to explore the vibration characteristics of the musical instrument scientifically, which is the first step in the simulation study of the national musical instrument. Vibration modal analysis of zither is only one part of the problem of frequency domain characteristics of vibration. The simulation analysis of noise of the zither needs to be further explored.

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