D’yakonov-Perel’ spin relaxation under electron-electron collisions in QWs

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The D’yakonov-Perel’ mechanism of spin relaxation is connected with the spin splitting of the electron dispersion curve in crystals lacking a center of symmetry. In a two-dimensional noncentrosymmetric system, e.g. quantum well or heterojunction, the spin splitting is a linear function of $k$, at least for small values of $k$. We demonstrate that the spin relaxation time $\tau_s$ due to the spin splitting is controlled not only by momentum relaxation processes as widely accepted but also by electron-electron collisions which make no effect on the electron mobility. In order to calculate the time $\tau_s$ taking into account the electron-electron scattering we have solved the two-dimensional kinetic equation for the electron spin density matrix. We show how the theory can be extended to allow for degenerate distribution of the spin-polarized two-dimensional electron gas.

I. INTRODUCTION

In recent years the electron spin relaxation processes have received much attention in connection with various spintronics applications. For free electrons in quantum well structures the following four mechanisms of spin decoherence are most important (see Refs. [1,2] and references therein): the Elliot-Yafet, D’yakonov-Perel’, Bir-Aronov-Pikus mechanisms and electron spin-flip scattering by paramagnetic centers. This paper is devoted to the D’yakonov-Perel’ spin relaxation (DPSR) in which case the spin relaxation time, $\tau_s$, is related to the spin splitting and given by Ref. [3]

$$\tau_s^{-1} \propto (\Omega^2_k) \tau .$$

Here $\Omega_k$ is the effective Larmor frequency describing the splitting of the electron spin dispersion branches, the angle brackets mean averaging over the electron energy distribution and $\tau$ is a microscopic electron relaxation time. In a two-dimensional (2D) system lacking a center of a symmetry, e.g. quantum well or heterojunction, the frequency $\Omega_k$ is linear in $k$. In this case the time $\tau_s$ was usually identified with the momentum relaxation time that determines the electron mobility Refs. [4-7]. In Ref. [8] we have shown that the inverse time $\tau_s^{-1}$ is determined not only by the momentum scattering rate but also by an independent contribution from electron-electron collisions which make no effect on the electron mobility. Really, electron-electron collisions change the direction of $k$ and $\Omega_k$ and, therefore, they control the D’yakonov-Perel’ spin relaxation exactly in the same way as any other scattering processes do.

The paper is organized as follows. Section 2 contains the discussion of mechanisms of the spin splitting of the electron subbands in quantum wells and electron spin decoherence due to spin splitting. Sec. 3 describes the D’yakonov-Perel’ spin relaxation mechanism in terms of the kinetic theory, in Sec. 4 we analyze the electron-electron collision integral for the spin-polarized electron gas, and Sec. 5 describes the solution of the kinetic equation with the account of electron-electron collisions.

II. SPIN SPLITTING OF ELECTRON SUBBANDS

In the parabolic approximation the effective electron Hamiltonian in the zinc-blende-based (001)-grown quantum well (QW) can be written as

$$\mathcal{H} = \frac{\hbar^2 k^2}{2m} + \beta_1 (\sigma_y k_y - \sigma_z k_z) + \beta_2 (\sigma_z k_y - \sigma_y k_z) .$$

(2)

where $x||[100], y||[010], \beta_{1,2}$ are constants, $m$ is the in-plane effective mass, $\sigma_l$ are the Pauli matrices and $k_n$ are the components of the 2D electron wavevector, $k$. The terms with higher powers in $k$ are not considered here. In the symmetrical QWs (the $D_{2d}$ point group) the linear-$k$ spin-dependent term proportional to $\beta_1$ is allowed only, it is called the bulk-inversion asymmetry (BIA) term. In asymmetrical QW structures characterized by the $C_{2v}$ point-group symmetry, there exists another spin-dependent contribution proportional to $\beta_2$ and called the structure-inversion asymmetry (SIA) term or the Rashba term Refs. [10,11] (see also Ref. [12] and references therein). The structure asymmetry can be related with non-equivalent normal and inverted interfaces, external or built-in electric fields, compositionally stepped QWs etc.

It is convenient to introduce the Cartesian coordinates $x' || [1\bar{1}0], y' || [110], z || [001]$ which allow to write a sum of the BIA and SIA terms in the form

$$\mathcal{H}_{cl}(k) = (\beta_- \sigma_x k_{y'} + \beta_+ \sigma_y k_{x'}) / 2 ,$$

(3)
where $\beta_{\pm} = 2(\beta_2 \pm \beta_1)$. The effective Larmor frequency $\Omega_k$ is defined by $H_{c1} = (\hbar/2)\Omega_k \cdot \sigma$, its components are

$$\Omega_{k, x'} = \beta_- k_{y'}/\hbar, \quad \Omega_{k, y'} = \beta_+ k_{x'}/\hbar, \quad \Omega_{k, z} = 0.$$  

(4)

The spin splitting at $k = (k_{x'}, k_{y'})$ is $h\Omega_k = \sqrt{\beta_+^2 k_{x'}^2 + \beta_-^2 k_{y'}^2}$. If only one of the linear-$k$ terms, BIA or SIA, is nonzero then $\beta_+^2 = \beta_+^2 \equiv \beta^2$ and the splitting $h\Omega_k = \beta k$ is angular independent.

Consider an electron gas occupying the lowest conduction subband $1$ and assume that, at the moment $t = 0$, the electrons are spin-polarized in the same direction along, say, the growth axis $z$. Owing to the spin-splitting of the electron subband the electron spin in the state with the wavevector $k$ precesses around the axis $\Omega_k$ which lies in the interface plane, see Eq. (4). In case of the large splitting, $|\Omega_k| \gg 1/\tau$, the spin of electron in the state $k$ will oscillate with time as $s_z(t) = s_z(0) \cos \Omega_k t$. In the case of $\Omega_k$ being isotropic in the QW plane, the spin polarized electrons which occupy the circle of the fixed radius in the $k$ space show the similar oscillatory behavior for their $s_z$ component. However, if $\beta_+^2 \neq \beta_+^2$ and/or the electrons occupy states with different values of $|k|$, the scatter in $\Omega_k$ results in a fast non-exponential decay of $s_z$.

D’yakonov and Perel’ Ref. 3 were the first to show that the processes of electron-momentum scattering result in the slowing off the spin decoherence caused by the spin splitting. In the collision-dominated (“motional-narrowing”) limit $|\Omega_k| \ll 1/\tau$, this results in an exponential decay of the average spin: $\bar{s}_z(t) = \bar{s}_z(0) \exp(-t/\tau_s)$, where $\tau_s$ is given by Eq. (4). The dimensionless coefficient in Eq. (4) can be obtained from the solution of kinetic equation Refs. 4, 7. This approach is valid as long as the energy relaxation time $\tau_s$ is short as compared to $\tau_z$.

### III. DPSR IN TERMS OF THE KINETIC THEORY

In the frame of kinetic theory, the electron distribution in the wavevector and spin spaces is described by a $2 \times 2$ spin-density matrix

$$\hat{\rho}_k = f_k + s_k \cdot \sigma.$$  

(5)

Here $f_k = \text{Tr}(\hat{\rho}_k/2)$ is the average occupation of the two spin states with the wavevector $k$, or distribution function of electrons in the $k$-space, and the average spin in the $k$ state is $s_k = \text{Tr}[\hat{\rho}_k(\sigma/2)]$. If we neglect the spin splitting then, for arbitrary degeneracy of an electron gas with non-equilibrium spin-state occupation but equilibrium energy distribution within each spin branch, the electron spin-density matrix can be presented as

$$\hat{\rho}^0_k = \left\{ \exp\left[ \frac{E_k - \mu - \mu_+ (\sigma o_s)}{k_B T} \right] + 1 \right\}^{-1},$$  

(6)

where $E_k = \hbar^2 k^2 / 2m$, $k_B$ is the Boltzmann constant, $T$ is the temperature, $o_s$ is the unit vector in the spin polarization direction, $\mu_\pm = \mu \pm \mu$ are the effective Fermi energies for electrons with the spin component $1/2$ or $-1/2$ along $o_s$ so that the energy distribution functions of electrons with the spin $\pm 1/2$ are given respectively by

$$f_{k, \pm} = \left[ \exp\left( \frac{E_k - \mu_\pm}{k_B T} \right) + 1 \right]^{-1}.$$  

Note that Eq. (6) can be rewritten in the equivalent form Ref. 3

$$\hat{\rho}^0_k = f^0_k + s^0_k \cdot \sigma = \frac{1}{2} \left[ f_{k, +} + f_{k, -} + (f_{k, +} - f_{k, -}) (\sigma o_s) \right].$$

The densities $n_\pm$ of 2D electrons with a particular spin can be related with the effective Fermi energies by

$$n_\pm = \frac{m}{2\pi \hbar^2} k_B T \ln \left( 1 + e^{\mu_\pm/k_B T} \right).$$  

(7)

If the spin splitting is non-zero but small compared to $\hbar/\tau$, the distribution function $\text{Tr}[\hat{\rho}_k/2] = f_k^0$ does not change, whereas the spin vector obtains a correction $\delta s_k = s_k - s_k^0$ proportional to the spin splitting. Therefore, the spin-density matrix may be presented as

$$\hat{\rho}_k = \hat{\rho}^0_k + \delta s_k \cdot \sigma.$$  

(8)
The quantum kinetic equation for the spin-density matrix taking into account the electron-electron collisions has the form

$$\frac{\partial \hat{\rho}_k}{\partial t} + \frac{i}{\hbar} [\mathcal{H}_{el}(k), \hat{\rho}_k] + \hat{Q}_k\{\hat{\rho}\} = 0,$$

(9)

where \([P, R] = PR - RP\), \(\mathcal{H}_{el}(k)\) is the linear-\(k\) Hamiltonian, and the third term in the left-hand side is the collision integral or the scattering rate, in this equation it is a 2×2 matrix. It follows from Eq. (9) that the pseudovector kinetic equation for \(\mathbf{s}_k\) can be written as

$$\frac{d\mathbf{s}_k}{dt} + \Omega_k \times \mathbf{s}_k + Q_k\{\mathbf{s}\} = 0,$$

(10)

where \(Q_k\{\mathbf{s}, f\} = (1/2)\text{Tr}(\sigma Q_k\{\rho\})\). In the following we ignore spin flips under scattering. Then, say, for the elastic scattering one has

$$Q_k\{F\} = \sum_{k'} W_{k'k}(F_{k'} - F_k),$$

where \(W_{k'k}\) is the probability rate for the electron transition from the state \(k\) to \(k'\). The collision integral for electron-electron scattering is considered in the next section. Here it suffices to note that, for the distribution \(\hat{\rho}_k^0\), the collision integral vanishes identically. This integral also vanishes after the summation over \(k\) which allows, in particular, to derive from Eq. (10) the following equation of balance for the total average spin \(\mathbf{S}_0 = o_s(n_+ - n_-)\):

$$\frac{d\mathbf{S}_0}{dt} + \sum_k \Omega_k \times \delta \mathbf{s}_k = 0.$$

(11)

The angular dependence of the non-equilibrium correction \(\delta \mathbf{s}_k\) is the linear combination of \(\cos \Phi_k = k_{x'}/k\) and \(\sin \Phi_k = k_{y'}/k\), where \(\Phi_k\) is the angle between \(k\) and the axis \(x'\). Retaining in the kinetic equation (10) terms proportional to the first angular harmonics we obtain the equation for \(\delta \mathbf{s}_k\) with the inhomogeneous term linear in \(\mathbf{S}_0\). Then one can substitute the solution in the second term of Eq. (11). The final result is that the tensor of inverse spin relaxation times, \(1/\tau_{x'y'}\), is diagonal in the coordinate system \(x', y', z\) and given by Refs. 4-13

$$\frac{1}{\tau_{x',x'}} = \frac{1}{2} \left( \frac{\beta_+}{\hbar} \right)^2 \langle k^2 \tau_p \rangle, \quad \frac{1}{\tau_{y',y'}} = \frac{1}{2} \left( \frac{\beta_-}{\hbar} \right)^2 \langle k^2 \tau_p \rangle, \quad \frac{1}{\tau_{z,z}} = \frac{1}{\tau_{x',x'}} + \frac{1}{\tau_{y',y'}},$$

(12)

where \(\tau_p\) is the momentum relaxation time. If among the two contributions, BIA and SIA, to the spin splitting one is dominant and \(|\beta_+| = |\beta_-|\), the spin relaxation times are interconnected by Ref. 4

$$\tau_{x',x'} = \tau_{y',y'} = 2 \tau_{z,z}.$$

Interplay between the BIA and SIA contributions can lead to a giant spin relaxation anisotropy Ref. In particular, if these contributions coincide, \(\beta_1 = \beta_2\), so that \(\beta_- = 0\) one has \(\tau_{x',x'} = \tau_{y,y'} = \tau_{z,z} = \infty\). In the case \(\beta_1 = -\beta_2\) the coefficient \(\beta_- = 0\), the time \(\tau_{x',x'}^0\) is infinite and \(\tau_{y,y'}^0\) coincides with \(\tau_{z,z}^0\).

**IV. ELECTRON-ELECTRON COLLISIONS IN QWS**

Here we will write the electron-electron collision term \(\hat{Q}_k\{\hat{\rho}\}\) in the general case of arbitrary spin-density matrix \(\hat{\rho}_k\) (in particular, arbitrary degeneracy and arbitrary distribution of spin in the \(k\)-space). For this purpose we remind that the matrix element of the Coulomb scattering \(\mathbf{k}, \mathbf{s}_k + \mathbf{k'}, \mathbf{s}_{k'} \rightarrow \mathbf{p}, \mathbf{s}_p + \mathbf{p'}, \mathbf{s}_{p'}\) is given by

$$M(\mathbf{p}, \mathbf{s}_p; \mathbf{p'}, \mathbf{s}_{p'} | \mathbf{k}, \mathbf{s}_k; \mathbf{k'}, \mathbf{s}_{k'} ) = V_{k-p} \delta_{s_p,s_k} \delta_{s_{p'},s_{k'}} - V_{k-p'} \delta_{s_p,s_{k'}} \delta_{s_{p'},s_k},$$

(13)

where \(s_k, s_{k'}, \ldots = \pm 1/2\), \(V_q\) is a Fourier transform of the 2D Coulomb potential of the electron-electron interaction

$$V_q = \frac{2\pi e^2}{\omega_0 \Sigma},$$

(14)

e is the elementary charge, \(\omega_0\) is the dielectric constant, and \(\Sigma\) is the sample area in the interface plane; in the following we set \(\Sigma = 1\). Equation (13) takes into account both the direct and exchange Coulomb interaction.
In order to present $\hat{Q}_k\{\hat{\rho}\}$ in a compact form we introduce the $2 \times 2$ unit matrix $I^{(1)}$ and Pauli matrices $\sigma^{(1)}_\alpha$ ($\alpha = x, y, z$) for the spin coordinates $s_p, s_k$ and a similar set of four matrices, $I^{(2)}$ and $\sigma^{(2)}_\alpha$, for the spin coordinates $s_{p'}, s_{k'}$. One can check that Eq. (13) allows the following matrix representation

$$\hat{M} = A I^{(1)} I^{(2)} + B \sigma^{(1)} \cdot \sigma^{(2)},$$

(15)

$$A = V_{k-p} - \frac{1}{2} V_{k-p'}, \quad B = -\frac{1}{2} V_{k-p'}.$$  

(16)

Now the collision term for the electron spin-density matrix can be presented as

$$\hat{Q}_k\{\rho\} = \frac{\pi}{2 \hbar} \sum_{k' pp'} \delta_{k+k', p+p'} \delta(E_k + E_{k'} - E_p - E_{p'}) \text{Tr}_2 G(p, p'; k, k'),$$

(17)

$$G(p, p'; k, k') =$$

\[\begin{array}{l}
\hat{M}(I^{(1)} - \hat{\rho}^{(1)}_p)(I^{(2)} - \hat{\rho}^{(2)}_p) \hat{M}(I^{(1)} - \hat{\rho}^{(1)}_{p'}) \hat{M}(I^{(2)} - \hat{\rho}^{(2)}_{p'}) \hat{M} - \\
\hat{M} \hat{\rho}^{(1)}_p \hat{\rho}^{(2)}_p \hat{M}(I^{(1)} - \hat{\rho}^{(1)}_{p'}) (I^{(2)} - \hat{\rho}^{(2)}_{p'}) - (I^{(1)} - \hat{\rho}^{(1)}_{p'}) (I^{(2)} - \hat{\rho}^{(2)}_{p'}) \hat{M} \hat{\rho}^{(1)}_p \hat{\rho}^{(2)}_p \hat{M}.
\end{array}\]

(18)

Here the spin-density matrices $\hat{\rho}^{(1)}(k) = I^{(1)} f_k + \sigma^{(1)} s_k, \hat{\rho}^{(2)}(k') = I^{(2)} f_{k'} + \sigma^{(2)} s_{k'}$ etc., the symbol $\text{Tr}_2$ means the trace over the spin variable 2. After the trace is found the index 1 in $\text{Tr}_2 G(p, p'; k, k')$ can be omitted. In order to derive Eqs. (17), (15) we used the standard diagram technique.

Instead of equation (9) for the spin-density matrix one can use a scalar equation for the distribution function $f_k$ in the form

$$\frac{df_k}{dt} + Q_k\{f, s\} = 0,$$

(19)

and a equation for the spin-distribution vectorfunction as

$$\frac{ds_k}{dt} + \Omega_k \times s_k + Q_k\{s, f\} = 0,$$

(20)

where

$$Q_k\{f, s\} = \frac{1}{2} \text{Tr}_1 [\hat{Q}_k\{\hat{\rho}\}],$$

(21)

$$= \frac{\pi}{4 \hbar} \sum_{k' pp'} \delta_{k+k', p+p'} \delta(E_k + E_{k'} - E_p - E_{p'}) \text{Tr}_1 \text{Tr}_2 [G(p, p'; k, k')] \cdot$$

$$Q_k\{s, f\} = \frac{1}{2} \text{Tr}_1 [\sigma^{(1)} \hat{Q}_k\{\hat{\rho}\}],$$

(22)

$$= \frac{\pi}{4 \hbar} \sum_{k' pp'} \delta_{k+k', p+p'} \delta(E_k + E_{k'} - E_p - E_{p'}) \text{Tr}_1 \text{Tr}_2 [\sigma^{(1)} G(p, p'; k, k')] .$$


For the analysis of the general equations (14), (15), (21), (22) we consider below few particular cases.

*Spin-unpolarized electrons.* In this case $s_k \equiv 0$ and the spin-density matrix reduces to a product of the unit $2 \times 2$ matrix and the distribution function $f_k$. Taking into account that

$$\hat{M}^2 = (A^2 + 3B^2) I^{(1)} I^{(2)} + 2B(A - B) \sigma^{(1)} \cdot \sigma^{(2)},$$

we come to the conventional collision term

$$Q_k\{f\} = \frac{2\pi}{\hbar} \sum_{k' pp'} \delta_{k+k', p+p'} \delta(E_k + E_{k'} - E_p - E_{p'}) (A^2 + 3B^2)$$

(23)
\[ \times [f_k f_{k'} (1 - f_p) (1 - f_{p'}) - f_p f_{p'} (1 - f_k) (1 - f_{k'})] . \]

Note that according to Eq. \( \text{10} \) one has
\[ A^2 + 3B^2 = V^2_{k-p} + V^2_{k-p'} - V_{k-p} V_{k-p'} \]
which is one-forth of the function \( R \) introduced in Eq. (2.4b) in Ref. \( \text{10} \) and the above collision term agrees with the equation (2.4a) in the cited paper. It is worth to note that in the sum \( \sum \) \( V^2_{k-p} + V^2_{k-p'} \) can be replaced by \( 2V^2_{k-p} \).

Electrons polarized along the same axis. By using the coordinate system with \( z \) parallel to the electron spin polarization one has \( s_{h,x} = s_{h,y} = 0 \) and the spin-density matrix is a diagonal matrix with the diagonal components \( f_{k,s} (s = \pm 1/2) \). It follows then that the products \( \hat{\rho}^{(1)}(k) \hat{\rho}^{(2)}(k') \) and \( [I^{(1)} - \hat{\rho}^{(1)}(p)][I^{(2)} - \hat{\rho}^{(2)}(p')] \) are diagonal as well. We can take into account the spin conservation
\[ M(p, s_3; p', s_4 | k, s_1; k', s_2) \propto \delta_{s_3 + s_4, s_1 + s_2} \]
and use the identity
\[ M(p, s_3; p', s_4 | k, s_1; k', s_2) M(p, s_3; p', s_4 | k, -s_1; k', s_2) = 0 . \]
This allows, in agreement with Ref. \( \text{12} \), to rewrite the collision term for \( f_{k,s} \) as
\[ \frac{2\pi}{\hbar} \sum_{k', p'} \sum_{s' s_1 s_2} \delta_{k+k', p+p'} \delta(E_k + E_{k'} - E_p - E_{p'}) W(p, s_1; p', s_2 | k, s; k', s') \]
\[ \times [f_{k,s} f_{k',s'} (1 - f_{p,s_1})(1 - f_{p',s_2}) - f_{p,s_1} f_{p',s_2} (1 - f_{k,s}) (1 - f_{k',s'})] . \]
Here
\[ W(p, s; p', s | k, s; k', s) = (A + B)^2 = (V_{k-p} - V_{k-p'})^2 , \]
\[ W(p, s; p', -s | k, s; k', -s) = (A - B)^2 = V^2_{k-p} , \]
\[ W(p, -s; p', s | k, s; k', -s) = (B)^2 = V^2_{k-p'} , \]
and other values of \( W \) with \( s_1 + s_2 \neq s + s' \) are zero.

Low electron polarization. If the average electron spin \( s_k \) is small as compared with the occupation probability \( f_k \) then, in the equation for \( f_k \), one can ignore the spin polarization at all and use Eq. \( \text{23} \), while in the equation for \( s_k \), one can retain in the collision term only the contribution linear in \( s_k \). The linearized collision term is given by
\[ Q_k \{ s, f \} = \frac{2\pi}{\hbar} \sum_{k', p, p'} \delta_{k+k', p+p'} \delta(E_k + E_{k'} - E_p - E_{p'}) \]
\[ \times [(V^2_{k-p} + V^2_{k-p'} - V_{k-p} V_{k-p'}) s_k F(k'; p, p') - V_{k-p} V_{k-p'} s_{k'} F(k; p, p')] \]
\[ - V_{k-p} (V_{k-p} - V_{k-p'}) s_p F(p'; k, k') - V_{k-p'} (V_{k-p'} - V_{k-p}) s_{p'} F(p; k, k')] , \]
where
\[ F(k_1, k_2, k_3) = f_{k_1} (1 - f_{k_2}) (1 - f_{k_3}) + (1 - f_{k_1}) f_{k_2} f_{k_3} = f_{k_1} (1 - f_{k_2} - f_{k_3}) + f_{k_2} f_{k_3} . \]
Equation \( \text{24} \) can be transformed to
\[ Q_k \{ s, f \} = \frac{2\pi}{\hbar} \sum_{k', p, p'} \delta_{k+k', p+p'} \delta(E_k + E_{k'} - E_p - E_{p'}) \]
\[ \times [(2V^2_{k-p} [s_k F(k'; p, p') - s_{p} F(p'; k, k')]) - \]
\[ - V_{k-p} V_{k-p'} [s_k F(k'; p, p') + s_{k'} F(k'; p, p') - 2s_{p} F(p'; k, k')] . \]
Here the term proportional to $2V_{k-p}^2$ is due to the direct Coulomb interaction whereas the term proportional to $V_{k-p}V_{k-p'}$ comes from the exchange interaction.

**Non-degenerate 2D electron gas.** In this case the function $F(k_1; k_2, k_3)$ reduces to $f_{k_1}$ and the collision terms are as follows

$$Q_k \{ f, s \} = \frac{2\pi}{\hbar} \sum_{k' p' p} \delta_{k+k', p+p'} \delta(E_k + E_{k'} - E_p - E_{p'})$$

$$\times [(2V_{k-p}^2 - V_{k-p}V_{k-p'})(f_k f_{k'} - f_p f_{p'}) - V_{k-p}V_{k-p'}(s_k \cdot s_{k'} - s_p \cdot s_{p'})] ,$$

$$Q_k \{ s, f \} = \frac{2\pi}{\hbar} \sum_{k' p' p} \delta_{k+k', p+p'} \delta(E_k + E_{k'} - E_p - E_{p'})$$

$$\times [2V_{k-p}^2 (s_k f_{k'} - s_p f_{p'}) - V_{k-p}V_{k-p'} (s_k f_{k'} + s_{k'} f_k - 2s_p f_{p'})] .$$

Neglecting the exchange interaction given by the term proportional to the $V_{k-p}V_{k-p'}$ the scattering rate $Q_k \{ f, s \}$ is independent on the electron spin distribution. Both $Q_k \{ f, s \}$ and $Q_k \{ s, f \}$ take a simple form

$$Q_k \{ f, s \} = \sum_{k' p' p} W_{p' p, k k'} (f_k f_{k'} - f_p f_{p'}) , Q_k \{ s, f \} = \sum_{k' p' p} W_{p' p, k k'} (s_k f_{k'} - s_p f_{p'}) ,$$

where $W_{p' p, k k'}$ is the probability rate for the scattering of a pair of electrons from the $k, k'$ states to the $p, p'$ states

$$W_{p' p, k k'} = \frac{2\pi}{\hbar} \delta_{k+k', p+p'} \delta(E_k + E_{k'} - E_p - E_{p'}) 2V_{k-p}^2 ,$$

an additional factor of 2 takes into account the double degeneracy of the electronic states.

**V. ELECTRON-ELECTRON SCATTERING TIME CONTROLLING THE DPSR**

In what follows we consider only a non-degenerate 2D electron gas in which case the zero-approximation spin-density matrix $\bar{\rho}^0_k$ can be written as

$$\bar{\rho}^0_k = f_k^0 (1 + 2s \cdot \sigma) ,$$

where $f_k^0$ is the Boltzmann distribution function, and $s$ is the average spin per electron, $S/(n_+ + n_-)$. The nonequilibrium correction $\delta s_k$ satisfies the equation

$$\Omega_k \times (2f_k^0 \bar{s}_0) + Q_k \{ \delta s, f^0 \} = 0 .$$

It should be noted that in the 2D case the collision term does not allow the quasi-elastic and relaxation time approximations and Eq. (30) must be solved directly.

The $\alpha$-component of the vector product $\Omega_k \times \bar{s}_0$ can be represented as $(\Omega_k \times \bar{s}_0)_\alpha = \Lambda_{\alpha\beta\gamma} k_\beta \bar{s}_0 \gamma$, where the third-rank tensor $\Lambda$ in general case of both BIA and SIA linear-$k$ terms has four nonzero components

$$\Lambda_{xxxx} = -\Lambda_{xxxx} = \beta_+ / \hbar , \quad \Lambda_{yyyy} = -\Lambda_{yyyy} = \beta_- / \hbar .$$

The function $(1/k_\beta)Q_k \{ k_\beta F_k, f^0 \}$ is independent of the azimuthal angle $\Phi_k$ (here $F_k$ is an arbitrary function of $k = |k|$) as the operator $Q_k \{ \delta s_\alpha, f^0 \}$ conserves the angular distribution in the $k$ space. In such case the solution may be written as follows

$$\delta s_\alpha(k) = -\Lambda_{\alpha\beta\gamma} k_\beta \bar{s}_0 \gamma , k_T \tau_{ee}^* e^{i\mu/k_B T} v(K) .$$

Here we introduced the dimensionless wavevector $K = k/k_T$, $k_T = (2m_{eB}/\hbar^2)^{1/2}$,

$$\tau_{ee}^* = \frac{\hbar k_B T \omega^2}{e^4 N} ,$$

$$\mu = \frac{2\pi k_B T}{\hbar^2} ,$$

$$\tau_{ee} = \frac{\hbar k_B T \omega^2}{e^4 N} ,$$

$$\tau_{ee} = \frac{\hbar k_B T \omega^2}{e^4 N} .$$
\[ N = n_+ + n_- \text{ and } v(K) \text{ satisfies the equation} \]
\[
K e^{-K^2} = \int d^2 K' \int d^2 P \tilde{W}_{PP',KK'} \left( v(K)e^{-K'^2} - \cos \Theta v(P)e^{-P^2} \right), \tag{33}
\]
where \( \Theta \) is the angle between \( K \) and \( P \), \( P' = K + K' - P \),
\[
\tilde{W}_{PP',KK'} = \frac{1}{|K - P|^2} \delta(K^2 + K'^2 - P^2 - P'^2). \]

Inserting Eq. (31) into Eq. (11) one obtains after summation over \( k \) the principal values of the tensor of reciprocal spin relaxation times
\[
\frac{1}{\tau_{xx'}} = \left( \frac{\beta_x k_T}{\hbar} \right)^2 \tau, \quad \frac{1}{\tau_{yy'}} = \left( \frac{\beta_y k_T}{\hbar} \right)^2 \tau, \quad \frac{1}{\tau_{zz}} = \frac{1}{\tau_{xx'}} + \frac{1}{\tau_{yy'}}. \tag{34}
\]
The time \( \tau \) which controls the spin relaxation is given by
\[
\tau = \tau_{ee}^* I, \quad I = \frac{1}{2} \int_0^\infty v(K)K^2dK. \tag{35}
\]
The parameter \( \tau_{ee}^* \) is present also in the \( ee \)-scattering time which determines the rate of energy exchange between 2D electrons Ref. 12.

The function \( v(K) \) was expanded in series using a basis set \( l_n(\epsilon) = \sqrt{\frac{2}{\pi}} \exp(-\epsilon)L_n(2\epsilon) \), where \( L_n(\epsilon) \) are the Laguerre polynomials and \( \epsilon = K^2 \). The expansion was substituted into the right-hand side of Eq. (33), the integration has been performed by Monte-Carlo method. The problem was reduced to a set of linear inhomogeneous equations for the expansion coefficients of \( v(K) \). The resulting value of \( I \) in Eq. (33) was found to be \( \approx 0.027 \). Allowance for the exchange interaction leads to a slight increase of this value to \( \approx 0.028 \).

**Comparison with ionized impurities scattering.** If the ionized impurities of the same concentration \( N \) lie inside the 2D layer then the corresponding transport time is given by \( \tau_{tr} = (2/\pi^2)\tau_{ee}^* \) (see Ref. 16). The spin relaxation time, \( \tau_{ee}^* \), controlled by scattering by the ionized impurities is given by Eq. (34) where \( \tau \) is changed by \( \tau_{tr}/2 \), thus the ratio of the spin relaxation time, \( \tau_{ee}^* \), governed by electron-electron collisions and the time \( \tau_{ee}^* \) is \( \approx 3.6 \), i.e. the elastic scattering by impurities is less efficient. If the doped layer is separated from the quantum well by a spacer, the influence of the Coulomb potential of ionized impurities on \( \tau_{ee}^* \) is reduced and eventually can be neglected.

The above result was obtained for the 2D Coulomb potential \( U^{2D}(\rho) = e^2/\epsilon \rho \), where \( \rho \) is the distance between the electrons in the interface plane. It is this potential that leads to the Fourier transform given by Eq. (14). In order to analyze the role of the quasi-2D character of the electron wave function confined in a QW of the finite thickness \( a \) one can replace \( U^{2D}(\rho) \) by the effective potential obtained by averaging the three-dimensional Coulomb potential as follows
\[
U(\rho) = \frac{e^2}{\epsilon \rho} \int \frac{\varphi_{e1}^2(z)\varphi_{e1}^2(z')}{\sqrt{\rho^2 + (z - z')^2}} dz dz',
\]
where \( \varphi_{e1}(z) \) is the electron envelope function at the lowest conduction subband \( e1 \). The straightforward calculation shows Ref. 15 that for the conduction band offset \( V_c \gg h^2/ma^2 \) and \( k_Fa < 1 \) a value of the time \( \tau = \tau_{ee}^* I \) increases with widening the QW as the electron-electron interaction becomes weaker, but an order of magnitude of \( I \) and \( \tau \) remains the same as in the exact 2D case.

**Conclusion. Future Work**

We have shown that electron-electron collisions control the D’yakonov-Perel’ spin relaxation in the same way as any scattering processes do. The electron-electron collision integral has been derived for an arbitrary degeneracy and spin distribution of the 2D electron gas. The calculations has been performed for non-degenerate 2D and quasi-2D electrons confined in a quantum well. Calculations of the spin relaxation time controlled by electron-electron collisions in bulk semiconductors are in progress. An important next step is the extension of the calculations from non-degenerate to degenerate spin-polarized electron gas.

In agreement with our theory, the latest optical spin-dynamic measurements in an \( n \)-doped GaAs/AlGaAs QW of high mobility give an experimental evidence of the electron-electron scattering to randomize the electron spin precession Ref. 14.
Note in conclusion that the time $\tau = \tau^*_e I$ can be related physically with the momentum relaxation time of an electron by equilibrium holes of the density $N$ if the electron and hole effective masses are assumed to coincide. In the electron collisions with holes the directed electron momentum is transferred to the hole gas and decays within the time $\sim \tau$. The work was supported by RFBR, and by the programs of the Russian Ministry of Sci. and the Presidium of the Russian Academy of Sci.

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