Hadronic Spectra and
Kaluza-Klein Picture of the World

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Abstract
A manifestation of Kaluza-Klein picture in hadronic spectra is discussed. We argue that the experimentally observed structures in hadronic spectra confirm the Kaluza-Klein picture of the world.

“... the simpler the presentation of a particular law of Nature, the more general it is ...”

Max Planck, Nobel Lecture, June 2, 1920

1 Introduction

Dear Colleagues.

It seems that here is just the place where we could remember one of the greatest physicists of the last XX Century, I mean German physicist Max Planck. My experience in science allows me to definitely share Max Planck opinion in the above written fragment of his Nobel Lecture. Following this opinion I’d like to present here new very simple and at the same time very general physical law concerning the structure of hadronic spectra.

Although the modern strong interaction theory formulated in terms of known QCD Lagrangian is commonly accepted, this theory do not allow us to make an appreciable breakthrough in the problem of calculating the masses of compound systems so far mainly because that problem is a significantly non-perturbative one. In other words, this means that our theoretical understanding of low-energy QCD spectroscopy is far from desired. Even the best currently performed lattice computations in QCD cannot help us to understand the exact nature of the real hadron spectrum.

All of you know that the strong interactions are characterized by multi-particle production. The dynamics of the multi-particle systems with a necessity contains the so called many-body forces. Many-body forces are fundamental forces which take place in the multi-particle systems where the number of particles is greater than two, and they are responsible for the dynamics of the production processes. For example, the three-body forces are responsible for the dynamics of one-particle inclusive reactions; see Ref. [1] and

1Extended version of the talk presented at Xth International Conference on Hadron Spectroscopy HADRON ’03, August 31 – September 6, 2003, Aschaffenburg, Germany.
A description of the many-body forces requires the use of multidimensional spaces, and as a consequence we cannot construct the strong interactions theory in a self-consistent way without the use of multidimensional spaces. Therefore, it seems it would be natural to formulate the strong interactions theory in a multidimensional space from the very beginning.

The idea to use the multidimensional spaces in fundamental physics is not new: famous works of Kaluza and Klein were the first ones where this idea has been elaborated. The original idea of Kaluza and Klein is based on the hypothesis that the input space-time is a \((4 + d)\)-dimensional space \(M_{(4+d)}\) which can be represented as a tensor product of the visible four-dimensional world \(M_4\) with a compact internal \(d\)-dimensional space \(K_d\)

\[
M_{(4+d)} = M_4 \times K_d.
\]

The compact internal space \(K_d\) is space-like one i.e. it has only spatial dimensions which may be considered as extra spatial dimensions of \(M_4\). An especially simple example of \(M_{(4+d)}\) is a space with the factorizable metric. In accordance with the tensor product structure of the space \(M_{(4+d)}\) the metric may be chosen in a factorizable form. This means that if \(z^M = \{x^\mu, y^m\}, (M = 0, 1, \ldots, 3 + d, \mu = 0, 1, 2, 3, m = 1, 2, \ldots, d)\), are local coordinates on \(M_{(4+d)}\) then the factorizable metric looks like

\[
ds^2 = G_{MN}(z) dz^M dz^N = g_{\mu\nu}(x) dx^\mu dx^\nu + \gamma_{mn}(x, y) dy^m dy^n,
\]

where \(g_{\mu\nu}(x)\) is the metric on \(M_4\).

In the year 1921, Kaluza proposed a unification of the theory of gravity and the Maxwell theory of electromagnetism in four dimensions starting from the theory of gravity in five dimensions. He assumed that the five-dimensional space \(M_5\) had to be a product of a four-dimensional space-time \(M_4\) and a circle \(S_1\): \(M_5 = M_4 \times S_1\). It was shown that the zero mode sector of the Kaluza model is equivalent to the four-dimensional theory which describes the Gilbert-Einstein gravity with a four-dimensional general coordinate transformations and the Maxwell theory of electromagnetism with a gauge transformations.

Recently some models with extra dimensions have been proposed to attack the electroweak quantum instability of the Standard Model known as hierarchy problem between the electroweak and gravity scales. However, it is obviously that the basic idea of the Kaluza-Klein scenario may be applied to any model in Quantum Field Theory. As illustrative example, let us consider the simplest case of \((4+d)\)-dimensional model of scalar field with the action

\[
S = \int d^{4+d}z \sqrt{-G} \left[ \frac{1}{2} (\partial_M \Phi)^2 - \frac{m^2}{2} \Phi^2 + \frac{G_{(4+d)}}{4!} \Phi^4 \right],
\]

where \(G = \det |G_{MN}|\), \(G_{MN}\) is the metric on \(M_{(4+d)} = M_4 \times K_d\), \(M_4\) is pseudo-Euclidean Minkowski space-time, \(K_d\) is a compact internal \(d\)-dimensional space with the characteristic size \(R\). Let \(\Delta_{K_d}\) be the Laplace operator on the internal space \(K_d\), and \(Y_n(y)\) are ortho-normalized eigenfunctions of the Laplace operator

\[
\Delta_{K_d} Y_n(y) = -\frac{\lambda_n}{R^2} Y_n(y),
\]

and \(n\) is a (multi)index labelling the eigenvalue \(\lambda_n\) of the eigenfunction \(Y_n(y)\). A \(d\)-dimensional torus \(T^d\) with equal radii \(R\) is an especially simple example of the compact internal
space of extra dimensions $K_d$. The eigenfunctions and eigenvalues in this special case look like

$$Y_n(y) = \frac{1}{\sqrt{V_d}} \exp \left( i \sum_{m=1}^{d} n_n y^m / R \right),$$

$$\lambda_n = |n|^2, \quad |n|^2 = n_1^2 + n_2^2 + \ldots n_d^2, \quad n = (n_1, n_2, \ldots, n_d), \quad -\infty \leq n_m \leq \infty,$$

where $n_m$ are integer numbers, $V_d = (2\pi R)^d$ is the volume of the torus.

To reduce the multidimensional theory to the effective four-dimensional one we write a harmonic expansion for the multidimensional field $\Phi(z)$

$$\Phi(z) = \Phi(x, y) = \sum_n \phi^{(n)}(x) Y_n(y).$$

The coefficients $\phi^{(n)}(x)$ of the harmonic expansion (5) are called Kaluza-Klein (KK) excitations or KK modes, and they usually include the zero-mode $\phi^{(0)}(x)$, corresponding to $n = 0$ and the eigenvalue $\lambda_0 = 0$. Substitution of the KK mode expansion into action (2) and integration over the internal space $K_d$ gives

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} (\partial_\mu \phi^{(0)})^2 - \frac{m^2}{2} (\phi^{(0)})^2 + \frac{g}{4!} (\phi^{(0)})^4 + 

+ \sum_{n \neq 0} \left[ \frac{1}{2} (\partial_\mu \phi^{(n)}) (\partial^\mu \phi^{(n)}) - \frac{m_n^2}{2} \phi^{(n)} \phi^{(n)*} \right] + \frac{g}{4!} (\phi^{(0)})^2 \sum_{n \neq 0} \phi^{(n)} \phi^{(n)*} \right\} + \ldots.$$

For the masses of the KK modes one obtains

$$m_n^2 = m^2 + \frac{\lambda_n}{R^2},$$

and the coupling constant $g$ of the four-dimensional theory is related to the coupling constant $G_{(4+d)}$ of the initial multidimensional theory by the equation

$$g = \frac{G_{(4+d)}}{V_d},$$

where $V_d$ is the volume of the compact internal space of extra dimensions $K_d$. The fundamental coupling constant $G_{(4+d)}$ has dimension [mass]$^{-d}$. So, the four-dimensional coupling constant $g$ is dimensionless one as it should be. Eqs. (7,8) represent the basic relations of Kaluza-Klein scenario. Similar relations take place for other types of multidimensional quantum field theoretical models. From four-dimensional point of view we can interpret each KK mode as a particle with the mass $m_n$ given by Eq. (7). We see that in accordance with Kaluza-Klein scenario any multidimensional field contains an infinite set of KK modes, i.e. an infinite set of four-dimensional particles with increasing masses, which is called the Kaluza-Klein tower. Therefore, an experimental observation of series KK excitations with a characteristic spectrum of the form (4) would be an evidence of the existence of extra dimensions. So far the KK partners of the particles of the Standard Model have not been observed. In the Kaluza-Klein scenario this fact can be explained by a microscopic small size $R$ of extra dimensions ($R < 10^{-17} cm$); in that case the KK excitations may be produced only at super-high energies of the scale $E \sim 1/R > 1 TeV$. 
Below this scale only homogeneous zero modes with \( n = 0 \) are accessible ones for an observation in recent high energy experiments. That is why, there is a hope to search the KK excitations at the future LHC and other colliders.

We have calculated early \[2\]

\[
\frac{1}{R} = 41.481 \text{MeV}, \tag{9}
\]

or

\[
R = 24.1 \text{GeV}^{-1} = 4.75 \times 10^{-13} \text{cm}. \tag{10}
\]

If we relate the strong interaction scale with the pion mass

\[
G_{(4+d)} \sim \frac{10}{[m_\pi]^d}, \tag{11}
\]

then

\[
g \sim \frac{10}{(2\pi m_\pi R)^d}, \tag{12}
\]

end

\[g(d = 1) \sim 0.5.\]

On the other hand

\[g_{\text{eff}} = g_{\pi NN} \exp(-m_\pi R) \sim 0.5, \quad (g^2_{\pi NN}/4\pi = 14.6). \tag{13}\]

So, \( R \) has a clear physical meaning: the size \(10\) just corresponds to the scale of distances where the strong Yukawa forces in strength come down to the electromagnetic ones. Moreover,

\[M \sim R^{-1} (M_{\text{Pl}}/R^{-1})^{2/(d+2)} |_{d=6} \sim 5 \text{ TeV}. \tag{14}\]

Mass scale \(14\) is just the scale accepted in the Standard Model, and this is an interesting observation as well. Actually mass scale \(14\) means that the gravity effects may be detected at the future LHC collider.

### 2 Peculiarities of Kaluza-Klein excitations

From the formula for the masses of the KK modes

\[m_n = \sqrt{m^2 + \frac{n^2}{R^2}} \]

we obtain

\[m_n = m + \delta m_n, \quad \delta m_n = \frac{n^2}{2mR^2}, \quad n << mR, \tag{15}\]

and this just corresponds to the spectrum of potential box with the size which is equal to the size of internal compact extra space. In other case we have

\[m_n = n\omega + \delta m_n, \quad \delta m_n = \frac{m\alpha^2}{2n}, \quad \omega \equiv \frac{1}{R}, \quad \alpha^2 \equiv mR, \quad \alpha^2 << n, \tag{16}\]
and here we come to the (quasi)oscillator (quasi, because \( n \) instead of \( n + 1/2 \) for one-dimensional case) and (quasi)Coulomb (quasi, because \( 1/n \) instead of \( 1/n^2 \) and \( \alpha^2 = mR \) instead of \( \alpha^2 = (1/137)^2 \) spectra. Clearly, we can neglect the (quasi)Coulomb contribution in the region \( n \gg \alpha^2 \equiv mR \).

It is a very remarkable fact that KK modes of relativistic origin, being made with a quantization of finite moving in the space of extra dimensions, interpolate the non-relativistic spectrum of a potential box and the oscillator spectrum.

The spectrum of two(a and b)-particle compound system is defined in fundamental (input) theory by the formula

\[
M_{ab}^n = m_a + m_b + \delta m_{ab}^n(m_a, m_b, G_{4+d}).
\]

(17)

The goal of the fundamental theory is to calculate \( \delta m_{ab}^n(m_a, m_b, G_{4+d}) \). We have not the solution of that problem in strong interaction theory because this is a significantly non-perturbative problem. However, in the framework of Kaluza-Klein approach we can rewrite the above formula in an equivalent form

\[
M_{ab}^n = m_{a,n} + m_{b,n} + \delta m_{ab,n}(m_{a,n}, m_{b,n}, g),
\]

(18)

where \( m_{a,n}, m_{b,n} \) are KK modes of particles \( a \) and \( b \), and we can calculate using four-dimensional perturbation theory for the quantity \( \delta m_{ab,n}(m_{a,n}, m_{b,n}, g) \). Moreover, because \( \delta m_{ab,n}(m_{a,n}, m_{b,n}, g) < m_{a(b),n} \), we can put with a high accuracy

\[
M_{ab}^n \approx m_{a,n} + m_{b,n},
\]

(19)

and this fact allows one to formulate a global solution of the spectral problem in hadronic spectroscopy.

### 3 On global solution of the spectral problem

According to Kaluza and Klein we suggest that the input (fundamental) space-time \( \mathcal{M}_{(4+d)} \) is represented as

\[
\mathcal{M}_{(4+d)} = M_4 \times \mathcal{K}_d.
\]

Let \( \lambda_n \) are characteristic numbers of the Laplace operator on \( \mathcal{K}_d \) with a characteristic size \( R_\mathcal{K} \)

\[
\Delta_{\mathcal{K}_d} Y_n(y) = -\frac{\lambda_n}{R_\mathcal{K}^2} Y_n(y).
\]

Let \( \lambda_\mathcal{K} \) be the set of all characteristic numbers of the Laplace operator

\[
\lambda_\mathcal{K} \equiv \{ \lambda_n : n \in \mathbb{Z}^d \equiv \underbrace{\mathbb{Z} \times \mathbb{Z} \times \cdots \times \mathbb{Z}}_d \}.
\]

(20)

There is one-to-one correspondence

\[
\mathcal{K} \iff (R_\mathcal{K}, \lambda_\mathcal{K}).
\]

Let us consider a compound hadronic system \( h \) which may decay into some channel

\[
h \rightarrow a + b + \cdots + c.
\]

(21)
We introduce the spectral mass function of the given channel by the formula

\[
M_{ab...c}(R_K, \lambda_n a, \lambda_n b, \cdots, \lambda_n c) = \sqrt{m_a^2 + \frac{\lambda_n a}{R_K^2}} + \sqrt{m_b^2 + \frac{\lambda_n b}{R_K^2}} + \cdots + \sqrt{m_c^2 + \frac{\lambda_n c}{R_K^2}}.
\] (22)

Now we build the Kaluza-Klein tower:

\[
t_{ab...c}(K) \equiv t_{ab...c}(R_K, \lambda_K) \overset{\text{def}}{=} \left\{ M_{ab...c}(R_K, \lambda_n a, \lambda_n b, \cdots, \lambda_n c) : \lambda_n i \in \lambda_K \right\},
\] (23)

\[(i = a, b, \ldots, c).\]

After that we build the Kaluza-Klein town as a union of the Kaluza-Klein towers corresponding to all possible decay channels of the hadronic system \( h \)

\[
T_h(K) \equiv T_h(R_K, \lambda_K) \overset{\text{def}}{=} \bigcup_{\{ab...c\}} t_{ab...c}(R_K, \lambda_K).
\] (24)

We state:

\[
\boxed{M_h \in T_h(K)}.
\] (25)

Let \( \mathcal{H} \) be the set of all possible physical hadronic states. We build the hadronic Kaluza-Klein country \( C_\mathcal{H}(K) \) by the formula

\[
C_\mathcal{H}(K) \overset{\text{def}}{=} \bigcup_{h \in \mathcal{H}} T_h(K).
\] (26)

The whole spectrum of all possible physical hadronic states we denote \( M_\mathcal{H} \)

\[
M_\mathcal{H} \overset{\text{def}}{=} \left\{ M_h : h \in \mathcal{H} \right\}.
\] (27)

We state:

\[
\boxed{M_\mathcal{H} \in C_\mathcal{H}(K)}.
\] (28)

The formulae (25) and (28) provide the global solution of the spectral problem in hadronic spectroscopy.

Here we have to make some clarifying remarks. First of all, in the construction of the global solution among all possible decay channels of the hadronic system \( h \) there have to be taken into account only those channels which contain the fundamental particles and their different multi-particle compound systems in the final states, as it should be. An appearance of non-zero KK modes of the fundamental particles and their compound systems in the final states of the decay channels is forbidden by the construction. For example, the decay channel

\[
h \rightarrow a^* + b + \cdots + c,
\] (29)

where \( a^* \) is a non-zero KK mode of the fundamental particle \( a \), cannot be used in the construction. The decay channel

\[
h \rightarrow A + b + \cdots + c,
\] (30)

where \( A \) is some multi-particle compound system which may decay into some channel with the fundamental particles \( a_i(i = 1, 2, \ldots, k) \) in the final state

\[
A \rightarrow a_1 + a_2 + \cdots + a_k,
\] (31)
is admissible one by the construction. But the decay channel

\[ h \leftrightarrow A^* + b + \cdots + c, \]  

(32)

where \( A^* \) denotes some non-zero KK mode of \( A \), is forbidden. In other words, the underlying physical principle in the construction of the global solution was the principle of non-observability of non-zero KK modes of the fundamental particles and their compound systems. According to that principle non-zero KK modes of the fundamental particles may manifest themselves only virtually during an interaction, for example, when they are staying in a compound system. Non-zero KK modes of the fundamental particles living in a compound system define the main properties of a compound system such as the mass and the life time of the system. As mentioned above, an interaction of KK modes is weak, therefore we can calculate with a high accuracy the mass of a compound system as a simple sum of the masses of KK modes. Moreover, weakly interacting KK modes result very narrow widths of the compound states, and this phenomenon is observed at the recent experiments.

The dynamics of the compound systems decays is physically transparent: Non-zero KK modes of the constituents make a transition to zero KK modes, and we observe zero KK modes as decay products. In the framework of such decay dynamics we can estimate the widths of the compound states

\[ \Gamma_n \sim \frac{\alpha_{\text{eff}}}{2} \cdot \frac{n}{R} \cdot O(1) \sim 0.4 \cdot n \text{ MeV}, \]  

(33)

where \( n \) is the number of KK excitation. The broad peaks in the hadronic spectra we interpret as an envelope of the narrow peaks predicted by Kaluza-Klein scenario.

We shown in the previous section that non-zero KK modes look like the states of a particle in confine potentials. Such particle might be considered as a quasi-particle which cannot be observed without the destroying a confine potential. A quasi-particle becomes a real particle by a transition of a non-zero KK mode to a zero KK mode which is equivalent to the destroying a confine potential, and we observe a zero KK mode i.e. a real fundamental particle as a decay product. This consideration justifies the underlying physical principle in the construction of the global solution. In fact, we present here quite a new look on the Kaluza-Klein picture as a whole.

4 Comparison with the experimental data

In papers [2, 3, 4, 6, 8, 12] we have verified the global solution on the set of experimental data with two-nucleon system, two-pion system, three-pion system, strange mesons, charmed and charmed-strange mesons and found out that the solution accurately described the experimentally observed hadronic spectra. Here we extract from the previous papers the main Tables with an addition of some novel ones.

The Table 1 extracted from Ref. [3] contains theoretically calculated Kaluza-Klein tower of KK-excitations for two-nucleon system, taking into account that Kaluza-Klein scenario predicts \( M_{n}^{pp} = M_{n}^{p\bar{p}} \), and the experimentally observed mass spectra of proton-proton and proton-antiproton systems above elastic threshold. There are the references in [3] where the experimental data have been extracted from. As it is seen from Table 1, the nucleon-nucleon dynamics at low energies provides a quite remarkable confirmation
of Kaluza-Klein picture. Moreover, Kaluza-Klein scenario predicts a special sort of (super)symmetry between fermionic (dibaryon) and bosonic states, which is very nontrivial fact, and Table 1 contains an experimental confirmation of this fact as well. We very hope that the empty cells in Table 1 will be filled in the future experimental studies.

The Kaluza-Klein tower of KK-excitations for two-pion system, extracted from Ref. [4], is shown in Table 2 where the comparison with experimentally observed mass spectrum of two-pion system is also presented; see details in [4]. Here we have only one empty cell \( M_{1}^{\pi\pi} \approx (1112 - 1114) \), and this is a subject for a further careful analysis of two-pion system. Besides, we would like to add the reference [5], which contains a review of known previous results concerning the ABC-particle observed for the first time in 1961 at Berkeley in the reaction \( pd \rightarrow \alpha X^0 \). A more precise experimental study in Ref. [5] compared to the experiments performed before allowed to establish four states in two-pion system with the masses \( M \approx 310 \text{ MeV}, M \approx 350 \text{ MeV}, M \approx 430 \text{ MeV}, M \approx 550 \text{ MeV} \). The Table 2 shows that there is a quite remarkable correspondence of the calculated KK excitations for two-pion system with the experimentally observed mass spectrum of two-pion system, which we consider as an additional strong evidence of Kaluza-Klein picture of the world. It should especially be pointed out that the \( f_2(0^+2^+) \)-mesons \( (M_{f_2} = 1272 \pm 8 \text{ MeV}) \) and \( f_2(2175 \pm 20 \text{ MeV}) \) investigated by IHEP group under direction of Yu.D. Prokoshkin accurately agree with the calculated values and excellently incorporated in the scheme of systematics provided by Kaluza-Klein picture.

The Kaluza-Klein tower built for three-pion system in Ref. [6] is shown in Table 3 where the comparison with experimentally observed mass spectrum of three-pion system is also presented. Certainly, here we have a much more poor experimental data set compared to the case of two-pion system. Nevertheless, again we see from Table 3 that there is a quite remarkable correspondence of the calculated KK excitations for three-pion system with the experimental data where such data exist, and this fact can also be considered as an additional evidence of Kaluza-Klein picture of the world. It is pleased for us to emphasize that the experimental measurement of the \( a_2 \) meson mass made by Protvino VES Collaboration [5] with the best world precision

\[
M(a_2) = 1311.3 \pm 1.6(\text{stat}) \pm 3.0(\text{syst}) \text{ MeV}
\]

is in excellent agreement with the theoretically calculated value

\[
M_{10}^{\pi^+\pi^-\pi^0} = 1311.55 \text{ MeV}.
\]

The same is true for the \( \omega_3 \) meson where the theoretically calculated mass of KK excitation in \( 3\pi^0 \) system \( M_{13}^{3\pi^0} = 1667.68 \text{ MeV} \) is in a very good agreement with PDG AVERAGE value \( M(\omega_3) = 1667 \pm 4 \text{ MeV} \). Moreover, it is very interesting to point out that theoretical calculation of KK excitations in \( \rho\pi \) system by the formula

\[
M_{n}^{\rho\pi} = \sqrt{m_\rho^2 + \frac{n^2}{R^2}} + \sqrt{m_\pi^2 + \frac{n^2}{R^2}}, \quad (n = 1, 2, 3, \ldots),
\]

where we use \( m_\rho = 769.3 \text{ MeV} \) for the \( \rho \) meson mass from PDG, gives \( M_{10}^{\rho\pi} = 1310.28 \text{ MeV} \) and \( M_{10}^{\rho\pi} = 1311.67 \text{ MeV} \) which accurately agree with the experimental measurement of the \( a_2 \) meson mass provided by VES Collaboration. This means that \( a_2 \) meson may
manifest itself as a configuration of $\rho \pi$ system in the main, and this is a very nontrivial fact. For example, that statement is not true for the $\omega_3$ meson. The Kaluza-Klein tower built for $\rho \pi$-system is shown in Table 4. VES Collaboration gives $M(\pi_1(1^{-+})) = 1610 \pm 20 \text{ MeV}$, ($\Gamma = 290 \pm 30 \text{ MeV}$, see [7]), which is excellently incorporated in Table 4.

Table 5 corresponds to the Kaluza-Klein tower of KK excitations for two-kaon system built in Ref. [8]. This Table apart of old experimental data contains new states recently observed in $K_S^0 K_S^0$-system and presented at this Conference in the talks [9, 10]. It is very nice to emphasize that all new states here are in excellent agreement with the values predicted in the scheme of the systematics provided by Kaluza-Klein approach.

Table 6 extracted from Ref. [8] shows the Kaluza-Klein tower of KK excitations for $K \pi$-system, and experimentally observed states are also presented here. Again we see from Table 6 that there is a quite remarkable correspondence of the calculated KK excitations for $K \pi$-system with the experimentally observed states of strange mesons.

Kaluza-Klein towers of KK excitations for the $K \pi\pi$-system and $K \rho$-system are shown in Table 7 and Table 8 extracted from [8] where the comparison with experimentally observed mass spectrum has been presented too. As in previous history we see from Tables 7–8 a quite remarkable correspondence of the calculated KK excitations for $K \pi\pi$-system and $K \rho$-system with the masses of the states where such states are experimentally observed. Many empty cells in Tables 7–8 indicate a wide field in experimental study of the $K 2\pi$-system.

We can see new recently observed states $D_{sJ}(2317)$ decaying to $D_s \pi^0$ and $D_{sJ}(2457)$ decaying to $D_s^* \pi^0$ (discussed in comprehensive review talk presented at this Conference by Belle Collaboration [11]) in Tables 9–10 extracted from Ref. [12], where the Kaluza-Klein towers of KK excitations for the $D_s \pi$-system and $D_s^* \pi$-system are presented. We would also like to point out that $D_{sJ}(2317)$ state may occupy $M_{22}^{K^*K}$-Storey in the Kaluza-Klein tower of KK excitations for the $K^*K$-system; see Table 11. The most impressive fact from the view point of our developed theoretical conception is the first observation of a very narrow charmonium state with a mass of $3871.8^{+0.7}_{-0.8} (\text{stat}) \pm 0.4 (\text{syst}) \text{ MeV}$ which decays into $\pi^+\pi^-J/\psi$ [11, 13]. It has been stressed that the $\pi^+\pi^-$ invariant mass for the $M(3872)$ signal region concentrate near the $\rho$ mass [13]. Such state really exists, and it lives just on the second storey in the Kaluza-Klein tower of KK excitations for the $\rho J/\psi$-system; see Table 12. As it is seen from Table 12 there is a wonderful agreement of experimentally measured mass with theoretically calculated one.

5 Mein Ruf to search new states

As it was mentioned above, we have performed an analysis of experimental data on mass spectrum of the states containing strange mesons and compared them with the calculated values provided by Kaluza-Klein scenario [8]. By this way we have found out quite an interesting correspondence shown below

7-storey:

\[(?)\sigma(650) \in M_7^{\pi\pi}(640 - 644) \quad \longleftrightarrow \quad K^*(892) \in M_7^{K\pi}(893 - 898),\]

15-storey:
17-storey:
\( f_2(1275) \in M_{15}^{\pi\pi}(1273 - 1275) \leftrightarrow K_2^*(1430) \in M_{15}^{K\pi}(1431 - 1434) \),
\( f_0,2(1430) \in M_{17}^{\pi\pi}(1435 - 1438) \leftrightarrow K_2^*(1580) \in M_{17}^{K\pi}(1579 - 1582) \),
\( f_0(1522) \in M_{18}^{\pi\pi}(1518 - 1520) \leftrightarrow K^*(1680) \in M_{18}^{K\pi}(1654 - 1657) \),
\( f_0,? (1580) \in M_{19}^{\pi\pi}(1599 - 1601) \leftrightarrow K_3^*(1780) \in M_{19}^{K\pi}(1730 - 1733) \),
\( f_0,? (1580) \in M_{19}^{\pi\pi}(1599 - 1601) \leftrightarrow K_3^*(1780) \in M_{19}^{K\pi}(1730 - 1733) \),
22-storey:
\( \eta_{\pi,2}(1840) \in M_{22}^{\pi\pi}(1845 - 1846) \leftrightarrow K_0^*(1950) \in M_{18}^{K\pi}(1960 - 1963) \),
\( f_4(1935) \in M_{23}^{\pi\pi}(1927 - 1928) \leftrightarrow K_4^*(2045) \in M_{23}^{K\pi}(2038 - 2040) \),
\( \rho_5(2250) \in M_{27}^{\pi\pi}(2256 - 2257) \leftrightarrow K_5^*(2380) \in M_{27}^{K\pi}(2352 - 2354) \).

From this correspondence it follows that \( K\pi\)-system looks like a system built from two-pion system by replacement of some one pion with a kaon. In fact, all experimentally observed hadronic states in \( K\pi\)-system have the corresponding partners in two-pion system. However, some hadronic states in two-pion system do not have the corresponding strange partners in \( K\pi\)-system experimentally observed so far. That is why the further study of \( K\pi\)-system is quite a promising subject of the investigations.

Concerning a three-pion system we have found out that
\( a_2(1311) \in M_{10}^{3\pi}(1309 - 1313), \quad a_2(1311) \in M_{10}^{\rho\pi}(1310 - 1312) \).

Moreover, we predict the strange partner of \( a_2\)-meson which we would like to call as \( a_2^s\)-meson
\( a_2^s(1520) \in M_{10}^{K2\pi}(1517 - 1523), \quad a_2^s(1520) \in M_{10}^{K\rho}(1519 - 1522) \).

Apart of isospin \( a_2^s(1520)\)-meson may have the same quantum numbers as \( a_2(1311)\)-meson. We call up to search \( a_2^s(1520)\)-meson and other strange partners of the three-pion states experimentally observed till now [11]. In this respect it seems the factory with an intensive kaon beams would be a very good device to realize such program. However, we would like to especially emphasize that recently observed states in \( K_S^0 K_S^0 \) system reported at this Conference by ZEUS Collaboration [10] seem indicate on the possibility to observe at HERA the \( a_2^s(1520)\)-meson in \( K_S^0 \pi^+ \pi^- \) system where the invariant mass of \( \pi^+ \pi^- \) system concentrated near the \( \rho\)-peak.
6 One comment

In the consideration made above we have used the simplest form of torus for the internal compact extra space and considered only diagonal elements in Kaluza-Klein towers. In fact, we have established the non-trivial physical principle according to which KK modes of decay products preferably paired up in compound system when they lived on one and the same storey in Kaluza-Klein tower. However, there are an exceptional cases. For example, $\rho$ and $\omega$ mesons appear as the non-diagonal elements of the Kaluza-Klein towers:

$$m_{\rho} \in M_{\pi,n,m}^{\pi^+\pi^-} = \sqrt{m_{\pi^+}^2 + \frac{n^2}{R^2}} + \sqrt{m_{\pi^-}^2 + \frac{m_{\rho}^2}{R^2}},$$  \hspace{1cm} (35)

$$M_{n,m}^{\pi^+\pi^-} (n_{\pi^+} = 12, m_{\pi^-} = 4) = 766.97\text{MeV}, \quad M_{n,m}^{\pi^+\pi^-} (n_{\pi^+} = 13, m_{\pi^-} = 4) = 773.85\text{MeV},$$

$$M_{n,m}^{\pi^0\pi^0} (n = 13, m = 4) = 769.78\text{MeV}, \quad M_{n,m}^{\pi^+\pi^0} (n_{\pi^+} = 13, m_{\pi^0} = 4) = 770.92\text{MeV},$$

and

$$m_{\omega} \in M_{n,m,k}^{\pi^+\pi^-\pi^0} = \sqrt{m_{\pi^+}^2 + \frac{n^2}{R^2}} + \sqrt{m_{\pi^-}^2 + \frac{m_{\rho}^2}{R^2}} + \sqrt{m_{\pi^0}^2 + \frac{k^2}{R^2}},$$  \hspace{1cm} (36)

$$M_{n,m,k}^{\pi^+\pi^-\pi^0} (n_{\pi^+} = 5, m_{\pi^-} = 6, k_{\pi^0} = 5) = 782.80\text{MeV}.$$  

In general, as it follows from the observed hadron spectrum, the non-diagonal elements of the Kaluza-Klein towers are physically suppressed. Actually, the architecture of the hadronic Kaluza-Klein towns is unambiguously defined by an internal compact extra space with its geometry and shapes, and we have to learn much more about the geometry and shapes of a compact internal extra space. However, one very important point in Kaluza-Klein picture is established now in a reliable way: The size of the internal compact extra space define the global characteristics of the hadronic spectra while the masses of the constituents are the fundamental parameters of the compound systems which the elements of the global structures being. A knowledge of the true internal compact extra space is a knowledge of the Everything that is the God. Our consideration made above shown that we found out a good approximation to the true internal extra space. In our opinion, the global goal of the Natural Philosophy and the fundamental particle\&nuclear physics as its part, in the future, will be in that to perceive the true internal extra space.

7 Conclusion

We have shown that one simple formula with one fundamental constant described more than 120 experimentally observed hadronic states. This is the most impressive fact in our developed theoretical conception\cite{15}. We did not ascribed the quantum numbers to the predicted states because we have made only model independent predictions concerning the masses of the states which have related only with the existence of a compact internal extra space.

The performed analysis allows us to conclude with a confidence: The experimentally observed structures in hadronic spectra reveal the existence of the extra dimensions and confirm the Kaluza-Klein picture of the world.

I began my talk with the saying of Max Planck, and I would like to finish the talk with the saying of Max Planck as well.
For it fell to this (atom) theory to discover, in the quantum action, the long-sought key to the entrance gate into the wonderland of spectroscopy, which since the discovery of spectral analysis had obstinately defied all efforts to breach it.

Max Planck, Nobel Lecture, June 2, 1920

We could paraphrase Max Planck and say that discovery of the fundamental scale of internal extra space with its geometry and shapes provides the long-awaited key to the entrance gate into the wonderland of hadronic spectroscopy, which since the discovery of strong forces had obstinately defied all efforts to open it.

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Table 1: Kaluza-Klein tower of KK excitations of $pp(p\bar{p})$ system and experimental data.

| n  | $M_{pp}^n$ MeV | $M_{exp}^{pp}$ MeV | $M_{exp}^{p\bar{p}}$ MeV | n  | $M_{pp}^n$ MeV | $M_{exp}^{pp}$ MeV | $M_{exp}^{p\bar{p}}$ MeV |
|----|----------------|--------------------|--------------------------|----|----------------|--------------------|--------------------------|
| 1  | 1878.38        | 1877.5 ± 0.5       | 1873 ± 2.5               | 15 | 2251.68        | 2240 ± 5            | 2250 ± 15                |
| 2  | 1883.87        | 1886 ± 1           | 1870 ± 10                | 16 | 2298.57        | 2282 ± 4            | 2300 ± 20                |
| 3  | 1892.98        | 1898 ± 1           | 1897 ± 1                 | 17 | 2347.45        | 2350                | 2340 ± 40                |
| 4  | 1905.66        | 1904 ± 2           | 1910 ± 30                | 18 | 2398.21        | 2380 ± 10           |                         |
| 5  | 1921.84        | 1916 ± 2           | ~ 1920                   | 19 | 2450.73        | 2450 ± 10           |                         |
|    |                | 1926 ± 2           |                          | 20 | 2504.90        | ~ 2500              |                         |
| 6  | 1941.44        | 1937 ± 2           | 1939 ± 2                 | 21 | 2560.61        |                    |                         |
|    |                | 1942 ± 2           | 1940 ± 1                 | 22 | 2617.76        | ~ 2620              |                         |
|    |                | ~1945              | 1942 ± 5                 | 23 | 2676.27        |                    |                         |
| 7  | 1964.35        | 1965 ± 2           | 1968                     | 24 | 2736.04        | 2735                | 2710 ± 20                |
|    |                | 1969 ± 2           | 1960 ± 15                | 25 | 2796.99        |                    |                         |
| 8  | 1990.46        | 1980 ± 2           | 1990_{-30}^{+15}         | 26 | 2859.05        | 2850 ± 5            |                         |
|    |                | 1999 ± 2           |                        | 27 | 2922.15        |                    |                         |
| 9  | 2019.63        | 2017 ± 3           | 2020 ± 3                 | 28 | 2986.22        |                    |                         |
| 10 | 2051.75        | 2046 ± 3           | 2040 ± 40                | 29 | 3051.20        |                    |                         |
|    |                | ~2050              | 2060 ± 20                | 30 | 3117.04        |                    |                         |
| 11 | 2086.68        | 2087 ± 3           | 2080 ± 10                | 31 | 3183.67        |                    |                         |
|    |                |                    | 2090 ± 20                | 32 | 3251.06        |                    |                         |
| 12 | 2124.27        | ~2122              | 2105 ± 15                | 33 | 3319.15        |                    |                         |
|    |                | 2121 ± 3           | 2110 ± 10                | 34 | 3387.90        | 3370 ± 10           |                         |
|    |                | 2129 ± 5           | 2140 ± 30                | 35 | 3457.28        |                    |                         |
| 13 | 2164.39        | ~2150              | 2165 ± 45                | 36 | 3527.25        |                    |                         |
|    |                | 2172 ± 5           | 2180 ± 10                | 37 | 3597.77        | 3600 ± 20           |                         |
| 14 | 2206.91        | 2192 ± 3           | 2207 ± 13                | 38 | 3668.81        |                    |                         |
Table 2: Kaluza-Klein tower of KK excitations for two-pion system and experimental data.

| n | $M_{n}^{\pi^0\pi^0}$ MeV | $M_{n}^{\pi^0\pi^\pm}$ MeV | $M_{n}^{\pi^\pm\pi^\pm}$ MeV | $M_{exp}^{\pi\pi}$ MeV |
|---|---|---|---|---|
| 1 | 282.41 | 286.80 | 291.21 | $\sim$ 300 |
| 2 | 316.87 | 320.80 | 324.73 | 322 $\pm$ 8 |
| 3 | 367.18 | 370.58 | 373.98 | 370 $\pm$ i356 |
| 4 | 427.78 | 430.71 | 433.64 | 430 $\pm$ i325 |
| 5 | 494.92 | 497.45 | 499.99 | 506 $\pm$ 10 |
| 6 | 566.26 | 568.48 | 570.70 | 585 $\pm$ 20 |
| 7 | 640.41 | 642.38 | 644.34 | 650 $\pm$ i370 |
| 8 | 716.50 | 718.26 | 720.01 | 732 $\pm$ i123 |
| 9 | 793.96 | 795.55 | 797.13 | 780 $\pm$ 30 |
| 10 | 872.44 | 873.88 | 875.33 | 870 $\pm$ i370 |
| 11 | 951.68 | 953.00 | 954.32 | 955 $\pm$ 10 |
| 12 | 1031.50 | 1032.72 | 1033.94 | 1015 $\pm$ 15 |
| 13 | 1111.78 | 1112.92 | 1114.05 | |
| 14 | 1192.43 | 1193.49 | 1194.55 | 1165 $\pm$ 50 |
| 15 | 1273.38 | 1274.37 | 1275.36 | 1275.4 $\pm$ 1.2 |
| 16 | 1354.57 | 1355.50 | 1356.43 | 1359 $\pm$ 40 |
| 17 | 1435.96 | 1436.84 | 1437.72 | 1434 $\pm$ 18 |
| 18 | 1517.53 | 1518.36 | 1519.19 | 1522 $\pm$ 25 |
| 19 | 1599.24 | 1600.02 | 1600.81 | 1593 $\pm$ 8 $\pm$ 29 $^{47}$ |
| 20 | 1681.07 | 1681.82 | 1682.57 | 1678 $\pm$ 12 |
| 21 | 1763.00 | 1763.72 | 1764.43 | 1768 $\pm$ 21 |
| 22 | 1845.03 | 1845.71 | 1846.40 | 1854 $\pm$ 20 |
| 23 | 1927.14 | 1927.79 | 1928.45 | 1921 $\pm$ 8 |
| 24 | 2009.32 | 2009.94 | 2010.57 | 2010 $\pm$ 60 |
| 25 | 2091.56 | 2092.16 | 2092.76 | 2086 $\pm$ 15 |
| 26 | 2173.85 | 2174.43 | 2175.01 | 2175 $\pm$ 20 |
| 27 | 2256.19 | 2256.75 | 2257.31 | $\sim$ 2250 |
| 28 | 2338.58 | 2339.12 | 2339.66 | $\sim$ 2330 |
| 29 | 2421.01 | 2421.53 | 2422.05 | 2420 $\pm$ 30 |
| 30 | 2503.47 | 2503.97 | 2504.48 | 2510 $\pm$ 30 |
Table 3: Kaluza-Klein tower of KK excitations for three-pion system and experimental data.

| n  | $M_{n}^{3\pi^0}$ MeV | $M_{n}^{\pi^0\pi^0}$ MeV | $M_{n}^{\pi^0\pi^0\pi^0}$ MeV | $M_{n}^{3\pi\pi}$ MeV | $M_{\exp}^{3\pi}$ MeV |
|----|----------------|----------------|----------------|----------------|----------------|
| 1  | 423.62         | 428.02         | 432.42         | 436.81         | $\eta(0^{-+})[547]$ |
| 2  | 475.30         | 479.23         | 483.17         | 487.10         |                 |
| 3  | 550.77         | 554.17         | 557.57         | 560.98         | 1194 ± 14       |
| 4  | 641.68         | 644.60         | 647.53         | 650.46         | 1311.3 ± 1.6    |
| 5  | 742.38         | 744.91         | 747.44         | 749.98         | 1419 ± 31       |
| 6  | 849.40         | 851.61         | 853.83         | 856.05         |                 |
| 7  | 960.62         | 962.58         | 964.55         | 966.51         | $\eta'(0^{-+})[958]$ |
| 8  | 1074.75        | 1076.51        | 1078.26        | 1080.02        |                 |
| 9  | 1190.95        | 1192.53        | 1194.12        | 1195.70        | 1194 ± 14       |
| 10 | 1308.66        | 1310.10        | 1311.55        | 1312.99        | 1311.3 ± 1.6    |
| 11 | 1427.51        | 1428.84        | 1430.16        | 1431.49        | 1419 ± 31       |
| 12 | 1547.25        | 1548.47        | 1549.69        | 1550.91        |                 |
| 13 | 1667.68        | 1668.81        | 1669.94        | 1671.08        | 1667 ± 4        |
| 14 | 1788.65        | 1789.71        | 1790.76        | 1791.82        | 1801 ± 13       |
| 15 | 1910.07        | 1911.06        | 1912.05        | 1913.04        |                 |
| 16 | 2031.86        | 2032.79        | 2033.72        | 2034.65        | 2030 ± 50       |
| 17 | 2153.95        | 2154.83        | 2155.70        | 2156.58        | 2090 ± 30       |
| 18 | 2276.29        | 2277.12        | 2277.95        | 2278.78        |                 |
| 19 | 2398.85        | 2399.64        | 2400.43        | 2401.22        |                 |
| 20 | 2521.69        | 2522.35        | 2523.10        | 2523.85        |                 |
| 21 | 2644.50        | 2645.22        | 2645.93        | 2646.65        |                 |
| 22 | 2767.54        | 2768.23        | 2768.91        | 2769.59        |                 |
| 23 | 2890.71        | 2891.36        | 2892.02        | 2892.67        |                 |
| 24 | 3013.97        | 3014.60        | 3015.23        | 3015.86        |                 |
| 25 | 3137.33        | 3137.94        | 3138.54        | 3139.14        |                 |
| 26 | 3260.78        | 3261.36        | 3261.94        | 3262.52        |                 |
| 27 | 3384.29        | 3384.85        | 3385.41        | 3385.97        |                 |
| 28 | 3507.87        | 3508.41        | 3508.95        | 3509.49        |                 |
| 29 | 3631.51        | 3632.03        | 3632.55        | 3633.08        |                 |
| 30 | 3755.21        | 3755.71        | 3756.21        | 3756.72        |                 |
Table 4: Kaluza-Klein tower of KK excitations for $\rho\pi$ system and experimental data.

| n  | $M_{\rho\pi^0}^{n}\text{MeV}$ | $M_{\rho\pi^\pm}^{n}\text{MeV}$ | $M_{\rho\pi}^{\exp}\text{MeV}$ |
|-----|---------------------------------|---------------------------------|---------------------------------|
| 1   | 911.62                          | 916.02                          |                                 |
| 2   | 932.19                          | 936.13                          |                                 |
| 3   | 962.89                          | 966.29                          |                                 |
| 4   | 1000.88                         | 1003.81                         |                                 |
| 5   | 1044.23                         | 1046.76                         |                                 |
| 6   | 1091.69                         | 1093.91                         |                                 |
| 7   | 1142.48                         | 1144.45                         |                                 |
| 8   | 1196.07                         | 1197.83                         |                                 |
| 9   | 1252.08                         | 1253.67                         |                                 |
| 10  | 1310.23                         | 1311.67                         |                                 |
| 11  | 1370.28                         | 1371.60                         |                                 |
| 12  | 1432.05                         | 1433.27                         |                                 |
| 13  | 1495.37                         | 1496.50                         |                                 |
| 14  | 1560.10                         | 1561.16                         |                                 |
| 15  | 1626.12                         | 1627.11                         |                                 |
| 16  | 1693.32                         | 1694.25                         |                                 |
| 17  | 1761.58                         | 1762.46                         |                                 |
| 18  | 1830.83                         | 1831.66                         |                                 |
| 19  | 1900.98                         | 1901.77                         |                                 |
| 20  | 1971.95                         | 1972.70                         |                                 |
| 21  | 2043.67                         | 2044.39                         |                                 |
| 22  | 2116.10                         | 2116.78                         |                                 |
| 23  | 2189.16                         | 2189.81                         |                                 |
| 24  | 2262.81                         | 2263.44                         |                                 |
| 25  | 2337.00                         | 2337.60                         |                                 |
| 26  | 2411.69                         | 2412.27                         |                                 |
| 27  | 2486.85                         | 2487.41                         |                                 |
| 28  | 2562.43                         | 2562.97                         |                                 |
| 29  | 2638.41                         | 2638.93                         |                                 |
| 30  | 2714.76                         | 2715.27                         |                                 |
Table 5: Kaluza-Klein tower of KK excitations for two-kaon system and experimental data.

| n  | $M_{n}^{2K}$ MeV | $M_{n}^{K_{0}K_{\pm}}$ MeV | $M_{n}^{2K_{\pm}}$ MeV | $M_{exp}^{2K}$ MeV |
|----|-----------------|-----------------|-----------------|-----------------|
| 1  | 998.80          | 994.81          | 990.83          |                  |
| 2  | 1009.08         | 1005.14         | 1001.20         |                  |
| 3  | 1025.99         | 1022.11         | 1018.24         | 1019.417±0.014   |
| 4  | 1049.21         | 1045.42         | 1041.63         |                  |
| 5  | 1078.32         | 1074.64         | 1070.95         | $X(1070)^{(\ast)}$ |
| 6  | 1112.87         | 1109.30         | 1105.73         |                  |
| 7  | 1152.37         | 1148.93         | 1145.48         |                  |
| 8  | 1196.33         | 1193.01         | 1189.69         |                  |
| 9  | 1244.27         | 1241.08         | 1237.89         |                  |
| 10 | 1295.76         | 1292.69         | 1289.63         | $\sim 1300^{(\ast\ast)}$ |
| 11 | 1350.38         | 1347.44         | 1344.50         |                  |
| 12 | 1407.77         | 1404.96         | 1402.14         |                  |
| 13 | 1467.62         | 1464.91         | 1462.21         |                  |
| 14 | 1529.62         | 1527.02         | 1524.43         | $1537^{+9}_{-8}^{(\ast\ast)}$ |
| 15 | 1593.53         | 1591.04         | 1588.55         |                  |
| 16 | 1659.13         | 1656.74         | 1654.34         | 1655 ± 17        |
| 17 | 1726.22         | 1723.92         | 1721.62         | $1726 \pm 7^{(\ast\ast)}$ |
| 18 | 1794.64         | 1792.43         | 1790.22         |                  |
| 19 | 1864.24         | 1862.11         | 1859.99         | $1864.1 \pm 1.0$ |
| 20 | 1934.89         | 1932.85         | 1930.80         |                  |
| 21 | 2006.49         | 2004.52         | 2002.54         | $X(2000)^{(\ast)}$ |
| 22 | 2078.93         | 2077.03         | 2075.12         |                  |
| 23 | 2152.14         | 2150.30         | 2148.45         |                  |
| 24 | 2226.02         | 2224.24         | 2222.46         |                  |
| 25 | 2300.53         | 2298.81         | 2297.08         |                  |
| 26 | 2375.60         | 2373.93         | 2372.26         |                  |
| 27 | 2451.17         | 2449.56         | 2447.94         |                  |
| 28 | 2527.21         | 2525.64         | 2524.08         |                  |
| 29 | 2603.67         | 2602.15         | 2600.63         |                  |
| 30 | 2680.52         | 2679.04         | 2677.57         |                  |

The states labelled by (*) in the $K_{S}K_{S}$ system have been reported at this Conference in the talk of E. Fadeeva; see [9]. The states labelled by (**) in the same $K_{S}K_{S}$ system have been reported at this Conference in the talk of M. Barbi (ZEUS Collaboration at HERA); see [10].
Table 6: Kaluza-Klein tower of KK excitations for $K\pi$ system and experimental data.

| n  | $M_{n}^{K^{0}\pi^{0}}$ MeV | $M_{n}^{K^{0}\pi^{\pm}}$ MeV | $M_{n}^{K^{\pm}\pi^{0}}$ MeV | $M_{n}^{K^{\pm}\pi^{\pm}}$ MeV | $M_{exp}^{K\pi}$ MeV |
|----|-----------------------------|-----------------------------|-----------------------------|-----------------------------|------------------------|
| 1  | 640.60                      | 645.00                      | 636.62                      | 641.02                      |                        |
| 2  | 662.97                      | 666.91                      | 659.03                      | 662.97                      |                        |
| 3  | 696.58                      | 699.99                      | 692.71                      | 696.11                      |                        |
| 4  | 738.50                      | 741.42                      | 734.71                      | 737.63                      |                        |
| 5  | 786.62                      | 789.16                      | 782.93                      | 785.47                      |                        |
| 6  | 839.57                      | 841.79                      | 836.00                      | 838.22                      |                        |
| 7  | 896.39                      | 898.36                      | 892.95                      | 894.91                      | $K^{*}(892)$           |
| 8  | 956.42                      | 958.17                      | 953.10                      | 954.85                      |                        |
| 9  | 1019.12                     | 1020.70                     | 1015.93                     | 1017.51                     |                        |
| 10 | 1084.10                     | 1085.54                     | 1081.03                     | 1082.48                     |                        |
| 11 | 1151.03                     | 1152.35                     | 1148.09                     | 1149.41                     |                        |
| 12 | 1219.64                     | 1220.86                     | 1216.82                     | 1218.04                     |                        |
| 13 | 1289.70                     | 1290.83                     | 1287.00                     | 1288.13                     |                        |
| 14 | 1361.03                     | 1362.08                     | 1358.43                     | 1359.48                     |                        |
| 15 | 1433.45                     | 1434.44                     | 1430.97                     | 1431.95                     | $K^{*}_{0,2}(1430)$    |
| 16 | 1506.85                     | 1507.78                     | 1504.46                     | 1505.39                     |                        |
| 17 | 1581.09                     | 1581.97                     | 1578.79                     | 1579.67                     | $K_{2}(1580)$          |
| 18 | 1656.08                     | 1656.91                     | 1653.87                     | 1654.70                     | $K^{*}(1680)$          |
| 19 | 1731.74                     | 1732.53                     | 1729.61                     | 1730.40                     | $K_{3}^{*}(1780)$     |
| 20 | 1807.98                     | 1808.73                     | 1805.93                     | 1806.68                     |                        |
| 21 | 1884.75                     | 1885.46                     | 1882.77                     | 1883.49                     |                        |
| 22 | 1961.98                     | 1962.67                     | 1960.08                     | 1960.76                     | $K^{*}_{0}(1950)$      |
| 23 | 2039.64                     | 2040.29                     | 2037.80                     | 2038.45                     | $K^{*}_{4}(2045)$      |
| 24 | 2117.67                     | 2118.30                     | 2115.89                     | 2116.52                     |                        |
| 25 | 2196.04                     | 2196.65                     | 2194.32                     | 2194.92                     |                        |
| 26 | 2274.72                     | 2275.30                     | 2273.06                     | 2273.64                     |                        |
| 27 | 2353.68                     | 2354.24                     | 2352.07                     | 2352.63                     | $K^{*}_{5}(2380)$      |
| 28 | 2432.90                     | 2433.44                     | 2431.33                     | 2431.87                     |                        |
| 29 | 2512.34                     | 2512.86                     | 2510.82                     | 2511.34                     |                        |
| 30 | 2592.00                     | 2592.50                     | 2590.52                     | 2591.02                     |                        |
Table 7: Kaluza-Klein tower of KK excitations for $K\pi\pi$ system and experimental data.

| n  | $M_n^{K02\pi0}$ MeV | $M_n^{K02\pi^\pm}$ MeV | $M_n^{K^\pm2\pi^0}$ MeV | $M_n^{K^\pm2\pi^\pm}$ MeV | $M_{exp}^{K2\pi}$ MeV |
|----|----------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| 1  | 781.81               | 790.61                  | 777.83                  | 786.62                  |                         |
| 2  | 821.41               | 829.27                  | 817.47                  | 825.33                  |                         |
| 3  | 880.17               | 886.98                  | 876.30                  | 883.10                  |                         |
| 4  | 952.39               | 958.24                  | 948.60                  | 954.45                  |                         |
| 5  | 1034.08              | 1039.15                 | 1030.39                 | 1035.46                 |                         |
| 6  | 1122.70              | 1127.14                 | 1119.13                 | 1123.57                 |                         |
| 7  | 1216.60              | 1220.53                 | 1213.15                 | 1217.08                 |                         |
| 8  | 1314.67              | 1318.18                 | 1311.35                 | 1314.86                 |                         |
| 9  | 1416.10              | 1419.27                 | 1412.91                 | 1416.08                 |                         |
| 10 | 1520.32              | 1523.21                 | 1517.25                 | 1520.14                 |                         |
| 11 | 1626.87              | 1629.51                 | 1623.93                 | 1626.57                 | 1629 ± 7                |
| 12 | 1735.39              | 1737.83                 | 1732.57                 | 1735.01                 | 1730 ± 20               |
| 13 | 1845.59              | 1847.86                 | 1842.89                 | 1845.15                 | ~ 1840                 |
| 14 | 1957.24              | 1959.36                 | 1954.65                 | 1956.76                 |                         |
| 15 | 2070.14              | 2072.12                 | 2067.66                 | 2069.63                 |                         |
| 16 | 2184.13              | 2186.00                 | 2181.74                 | 2183.60                 |                         |
| 17 | 2299.07              | 2300.83                 | 2296.78                 | 2298.53                 |                         |
| 18 | 2414.85              | 2416.51                 | 2412.64                 | 2414.30                 |                         |
| 19 | 2531.36              | 2532.93                 | 2529.23                 | 2530.81                 |                         |
| 20 | 2648.51              | 2650.01                 | 2646.46                 | 2647.96                 |                         |
| 21 | 2766.25              | 2767.68                 | 2764.27                 | 2765.70                 |                         |
| 22 | 2884.50              | 2885.86                 | 2882.59                 | 2883.96                 |                         |
| 23 | 3003.21              | 3004.51                 | 3001.36                 | 3002.67                 |                         |
| 24 | 3122.33              | 3123.58                 | 3120.55                 | 3121.80                 |                         |
| 25 | 3241.82              | 3243.03                 | 3240.10                 | 3241.30                 |                         |
| 26 | 3361.65              | 3362.81                 | 3359.98                 | 3361.14                 |                         |
| 27 | 3481.78              | 3482.90                 | 3480.16                 | 3481.28                 |                         |
| 28 | 3602.19              | 3603.27                 | 3600.62                 | 3601.70                 |                         |
| 29 | 3722.85              | 3723.89                 | 3721.32                 | 3722.37                 |                         |
| 30 | 3843.73              | 3844.74                 | 3842.25                 | 3843.26                 |                         |
Table 8: Kaluza-Klein tower of KK excitations for $K\rho$ system and experimental data.

| n  | $M_n^{K^0\rho}$ MeV | $M_n^{K^+\rho}$ MeV | $M_{exp}^{K\rho}$ MeV |
|----|---------------------|---------------------|-----------------------|
| 1  | 1269.82             | 1265.83             |                       |
| 2  | 1278.30             | 1274.36             | 1273 ± 7              |
| 3  | 1292.29             | 1288.42             |                       |
| 4  | 1311.59             | 1307.81             |                       |
| 5  | 1335.93             | 1332.24             |                       |
| 6  | 1365.00             | 1361.43             |                       |
| 7  | 1398.46             | 1395.01             | 1402 ± 7              |
| 8  | 1435.99             | 1432.67             | 1414 ± 15             |
| 9  | 1477.24             | 1474.05             | ~ 1460                |
| 10 | 1521.89             | 1518.82             |                       |
| 11 | 1569.63             | 1566.69             |                       |
| 12 | 1620.19             | 1617.37             |                       |
| 13 | 1673.29             | 1670.58             |                       |
| 14 | 1728.70             | 1726.10             | 1717 ± 27             |
| 15 | 1786.20             | 1783.71             | 1776 ± 7              |
| 16 | 1845.59             | 1843.20             |                       |
| 17 | 1906.71             | 1904.41             |                       |
| 18 | 1969.39             | 1967.18             | 1973 ± 8 ± 25         |
| 19 | 2033.48             | 2031.35             |                       |
| 20 | 2098.86             | 2096.81             |                       |
| 21 | 2165.42             | 2163.44             |                       |
| 22 | 2233.05             | 2231.14             |                       |
| 23 | 2301.66             | 2299.82             |                       |
| 24 | 2371.16             | 2369.38             |                       |
| 25 | 2441.49             | 2439.76             |                       |
| 26 | 2512.57             | 2510.90             |                       |
| 27 | 2584.34             | 2582.72             |                       |
| 28 | 2656.75             | 2655.18             |                       |
| 29 | 2729.75             | 2728.22             |                       |
| 30 | 2803.29             | 2801.81             |                       |
Table 9: Kaluza-Klein tower of KK excitations for $D_s^\pm\pi$ system and experimental data.

| n | $M_n^{D_s^\pm\pi^0}$ MeV | $M_n^{D_s^\pm\pi^\pm}$ MeV | $M_{\text{exp}}^{D_s^\pm\pi}$ MeV |
|---|--------------------------|-----------------------------|----------------------------------|
| 1 | 2110.64                  | 2115.04                     |                                 |
| 2 | 2129.18                  | 2133.11                     |                                 |
| 3 | 2156.52                  | 2159.92                     |                                 |
| 4 | 2189.87                  | 2192.80                     |                                 |
| 5 | 2227.35                  | 2229.89                     |                                 |
| 6 | 2267.80                  | 2270.02                     |                                 |
| 7 | 2310.50                  | 2312.47                     |                                 |
| 8 | 2355.02                  | 2356.77                     |                                 |
| 9 | 2401.06                  | 2402.65                     |                                 |
| 10| 2448.44                  | 2449.88                     |                                 |
| 11| 2497.02                  | 2498.34                     |                                 |
| 12| 2546.70                  | 2547.92                     |                                 |
| 13| 2597.40                  | 2598.53                     |                                 |
| 14| 2649.07                  | 2650.13                     |                                 |
| 15| 2701.66                  | 2702.65                     |                                 |
| 16| 2755.14                  | 2756.07                     |                                 |
| 17| 2809.45                  | 2810.33                     |                                 |
| 18| 2864.58                  | 2865.41                     |                                 |
| 19| 2920.50                  | 2921.29                     |                                 |
| 20| 2977.17                  | 2977.92                     |                                 |
| 21| 3034.59                  | 3035.30                     |                                 |
| 22| 3092.72                  | 3093.40                     |                                 |
| 23| 3151.54                  | 3152.19                     |                                 |
| 24| 3211.03                  | 3211.66                     |                                 |
| 25| 3271.18                  | 3271.78                     |                                 |
| 26| 3331.95                  | 3332.53                     |                                 |
| 27| 3393.34                  | 3393.90                     |                                 |
| 28| 3455.33                  | 3455.87                     |                                 |
| 29| 3517.90                  | 3518.42                     |                                 |
| 30| 3581.02                  | 3581.53                     |                                 |

$D_{sJ}(2317)$

$D_s^*(2112)$
Table 10: Kaluza-Klein tower of KK excitations for $D_s^{\pm}\pi$ system and $D_{sJ}(2457)$-meson.

| n | $M_n^{D_s^{\pm}\pi^0}$ MeV | $M_n^{D_s^{\pm}\pi^\pm}$ MeV | $M_{exp}^{D_s^{\pm}\pi}$ MeV |
|---|---|---|---|
| 1 | 2254.01 | 2258.41 | |
| 2 | 2272.46 | 2276.39 | |
| 3 | 2299.65 | 2303.05 | |
| 4 | 2332.80 | 2335.73 | |
| 5 | 2370.02 | 2372.55 | |
| 6 | 2410.14 | 2412.36 | |
| 7 | 2452.47 | 2454.43 | $D_{sJ}^+(2457)$ |
| 8 | 2496.56 | 2498.31 | |
| 9 | 2542.12 | 2543.70 | |
| 10 | 2588.96 | 2590.41 | |
| 11 | 2636.96 | 2638.28 | |
| 12 | 2686.01 | 2687.23 | |
| 13 | 2736.04 | 2737.17 | |
| 14 | 2786.99 | 2788.05 | |
| 15 | 2838.82 | 2839.81 | |
| 16 | 2891.50 | 2892.43 | |
| 17 | 2944.98 | 2945.86 | |
| 18 | 2999.24 | 3000.07 | |
| 19 | 3054.26 | 3055.05 | |
| 20 | 3110.01 | 3110.76 | |
| 21 | 3166.46 | 3167.18 | |
| 22 | 3223.61 | 3224.30 | |
| 23 | 3281.43 | 3282.08 | |
| 24 | 3339.90 | 3340.53 | |
| 25 | 3399.00 | 3399.61 | |
| 26 | 3458.72 | 3459.30 | |
| 27 | 3519.04 | 3519.60 | |
| 28 | 3579.95 | 3580.49 | |
| 29 | 3641.42 | 3641.94 | |
| 30 | 3703.44 | 3703.94 | |
Table 11: Kaluza-Klein tower of KK excitations for $K^*K$ system
and $D_{sJ}(2317)$-meson.

| n  | $M_n^{K^*K^0}$ MeV | $M_n^{K^*K^+}$ MeV | $M_{exp}^{K^*K}$ MeV |
|----|--------------------|--------------------|---------------------|
| 1  | 1392.02            | 1392.48            | $h_1(?1^{+-})$      |
| 2  | 1400.05            | 1400.53            |                     |
| 3  | 1413.30            | 1413.82            |                     |
| 4  | 1431.57            | 1432.15            | $f_1(0^{+1^{++}})$  |
| 5  | 1454.63            | 1455.27            |                     |
| 6  | 1482.18            | 1482.89            | $\eta(0^{+0^{--}})$ |
| 7  | 1513.94            | 1514.71            | $f_1(0^{+1^{++}})$  |
| 8  | 1549.58            | 1550.42            |                     |
| 9  | 1588.80            | 1589.70            |                     |
| 10 | 1631.30            | 1632.27            | $\eta_2(0^{+2^{--}})$ |
| 11 | 1676.82            | 1677.83            |                     |
| 12 | 1725.08            | 1726.14            |                     |
| 13 | 1775.85            | 1776.95            |                     |
| 14 | 1828.91            | 1830.04            |                     |
| 15 | 1884.06            | 1885.22            |                     |
| 16 | 1941.12            | 1942.29            |                     |
| 17 | 1999.92            | 2001.11            |                     |
| 18 | 2060.32            | 2061.51            |                     |
| 19 | 2122.17            | 2123.38            |                     |
| 20 | 2185.37            | 2186.57            |                     |
| 21 | 2249.79            | 2251.00            |                     |
| 22 | 2315.35            | 2316.55            | $D_{sJ}(2317)$      |
| 23 | 2381.94            | 2383.14            |                     |
| 24 | 2449.49            | 2450.68            |                     |
| 25 | 2517.92            | 2519.10            |                     |
| 26 | 2587.17            | 2588.34            |                     |
| 27 | 2657.17            | 2658.33            |                     |
| 28 | 2727.87            | 2729.01            |                     |
| 29 | 2799.22            | 2800.35            |                     |
| 30 | 2871.17            | 2872.28            |                     |
Table 12 Kaluza-Klein tower of KK excitations for $\rho J/\psi$ system and $D_{sJ}(3872)$-meson.

| $n$ | $M_{n}^{\rho J/\psi}$ MeV | $M_{exp}^{\rho J/\psi}$ MeV |
|-----|--------------------------|-----------------------------|
| 1   | 3867.57                  |                             |
| 2   | 3871.74                  | 3871.8±0.7±0.4             |
| 3   | 3878.67                  |                             |
| 4   | 3888.30                  |                             |
| 5   | 3900.58                  |                             |
| 6   | 3915.41                  |                             |
| 7   | 3932.73                  |                             |
| 8   | 3952.42                  |                             |
| 9   | 3974.39                  |                             |
| 10  | 3998.54                  |                             |
| 11  | 4024.75                  |                             |
| 12  | 4052.92                  |                             |
| 13  | 4082.95                  |                             |
| 14  | 4114.74                  |                             |
| 15  | 4148.19                  |                             |
| 16  | 4183.22                  |                             |
| 17  | 4219.74                  |                             |
| 18  | 4257.68                  |                             |
| 19  | 4296.95                  |                             |
| 20  | 4337.48                  |                             |
| 21  | 4379.23                  |                             |
| 22  | 4422.11                  |                             |
| 23  | 4466.09                  |                             |
| 24  | 4511.11                  |                             |
| 25  | 4557.11                  |                             |
| 26  | 4604.07                  |                             |
| 27  | 4651.93                  |                             |
| 28  | 4700.65                  |                             |
| 29  | 4750.21                  |                             |
| 30  | 4800.57                  |                             |