Extending Heisenberg’s measurement–disturbance relation to the twin-slit case.

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Heisenberg’s position-measurement–momentum-disturbance relation is derivable from the uncertainty relation $\sigma(q)\sigma(p) \geq \hbar/2$ only for the case when the particle is initially in a momentum eigenstate. Here I derive a new measurement–disturbance relation which applies when the particle is prepared in a twin-slit superposition and the measurement can determine at which slit the particle is present. The relation is $d \times \Delta p \geq 2\hbar/\pi$, where $d$ is the slit separation and $\Delta p = D_M(P_f, P_i)$ is the Monge distance between the initial $P_i(p)$ and final $P_f(p)$ momentum distributions.

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1. Introduction

There is a fundamental ambiguity in Heisenberg’s uncertainty relation which dates back to its birth in the famous 1927 paper\(^{(1)}\). Here Heisenberg introduced the relation in the context of a position measurement by a $\gamma$-ray microscope, as (in my notation)

$$\epsilon_q \times \delta p \sim \hbar, \quad (1)$$

where $\epsilon_q$ is “the precision with which the value $q$ is known (say the mean error of $q$)” and $\delta p$ is “the discontinuous change of $p$ in the Compton effect.” By “mean error” Heisenberg evidently meant root-mean-square error, or its equivalent, and I will follow this use. The relation (1) we may call the Heisenberg measurement–disturbance relation. As Heisenberg says,

The instant the position is determined ... the electron undergoes a discontinuous change in momentum. This change is the greater the ... more exact the determination of the position.

The roles of $p$ and $q$ in this description are clearly not symmetric. But in the same work Heisenberg talks about the uncertainty relation as referring to “simultaneous determination of two canonically conjugate quantities”, which is a different statement. Not long after Heisenberg, Weyl\(^{(3)}\) put this latter statement on a rigorous footing as

$$\sigma(q)\sigma(p) \geq \hbar/2. \quad (2)$$

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Here $\sigma(q), \sigma(p)$ are the simultaneous values of the standard deviations of $q, p$. Following modern use, I will call this an uncertainty relation.

What link is there between the uncertainty relation (2) and Heisenberg’s measurement–disturbance relation (1)? It seems fairest to let Heisenberg speak for himself. In his most complete description of his position, contained in his 1930 book (4), he first gives a derivation of the relation (2). This derivation, using only “the mathematical scheme of quantum theory and its physical interpretation” says nothing about momentum transfer or position measurement. It is only in the next section, “Illustrations of the Uncertainty Relations”, that he introduces these ideas, and does so very carefully:

The uncertainty principle refers to the degree of indeterminacy in the possible present knowledge of the simultaneous values of various quantities with which quantum theory deals; it does not restrict, for example, the exactness of a position measurement alone or a velocity measurement alone. Thus suppose that the velocity of a free electron is known, while the position is completely unknown. Then the principle states that any subsequent observation of the position will alter the momentum by an unknown and indeterminable amount such that after carrying out the experiment our knowledge of the electronic motion is restricted by the uncertainty relation.

As Heisenberg appeared to be well aware, this is the only statement about momentum disturbance and position measurement error which one can make as a logical consequence of the uncertainty relation (2). The standard deviations $\sigma(p)$ and $\sigma(q)$ refer to the state of the particle after the measurement. It is only because the particle state prior to the measurement was a momentum eigenstate (having zero dispersion in momentum) that one can equate $\sigma(p)$ with $\delta p$, the mean momentum disturbance. Likewise it is only because the particle prior to the measurement had a completely undefined position that one can equate $\sigma(q)$ with the mean error $\epsilon q$ of the position measurement. To see this latter point, consider the case where the particle is not in a momentum eigenstate, but instead localized at two narrow slits of width $a$, separated by a distance $d$. Then a position measurement with an error of order $d$ will resolve the two slits, and the particle will become localized at one of them. The resulting final standard deviation in the position $\sigma(q) \sim a$ is not related to the error of the measurement $\sim d$.

In the derivation of Eq. (2) in Ref. (4), Heisenberg does not explicitly use the commutation relations

$$[q, p] = i\hbar,$$

but rather the Fourier-transform relation between the position and momentum representations. It should especially be noted that there is no hint in Heisenberg’s work that the more general Robertson uncertainty relation (5)

$$\sigma(A)\sigma(B) \geq \frac{1}{2} |\langle [A,B] \rangle|$$

leads to a more general measurement–disturbance relation. Instead, there are good reasons for maintaining that it does not. If, unlike $p$ and $q$, the quantities $A$ and $B$ are not canonically conjugate, then preparing the system in an eigenstate of $A$ need not ensure that all values of $B$ are equally likely prior to the measurement. After the measurement, the standard deviation $\sigma(A)$ can still be identified with the disturbance in the quantity $A$ caused by the measurement. However
the quantity $\sigma(B)$ is not necessarily the accuracy of the measurement of $B$, because there may have been some information regarding the value of $B$ prior to the measurement (the argument follows the same lines as given above for the twin-slit case). The lesson is that the Heisenberg measurement–disturbance relation is of quite particular content.

In his actual illustrations in the 1930 book (including the famous $\gamma$-ray microscope from the 1927 paper), Heisenberg seeks to give an intuitive interpretation of the momentum transfer (in terms of the Compton recoil and such like). But he never claims to rigorously prove any momentum–disturbance relation other than

$$\epsilon_q \times \delta p \geq \hbar/2$$

which applies when the particle is initially in a momentum eigenstate (or at least a state with negligible momentum dispersion).

The problem is that one often wishes to consider position measurements, and hence momentum transfers, in situations in which the particle is not in a momentum eigenstate. A particular example of interest which I have already mentioned is that of the twin slits. This was one of the subjects of the Bohr–Einstein debates\(^8\) and, more recently, has been surrounded by the controversy over whether there is a momentum transfer (of order commensurate with the uncertainty relation) concomitant with determining which slit a particle passes through. Scully, Englert and Walther\(^6\) prove that if the particle is already localized at one of the slits then there need be no such momentum transfer. On this basis they say that it is particle-wave complementarity, rather than the uncertainty principle, which explains the loss of the interference pattern when one measures which slit the particle went through. Storey, Tan, Collett and Walls\(^7\) on the contrary have claimed that there is always a transverse (that is, in the direction of the line connecting the slits) momentum disturbance at least equal to $\hbar/d$, where $d$ is the slit separation. In this they uphold the opinion of Bohr\(^8\) that one can regard complementarity as being enforced by the uncertainty principle. Further exchanges are found in Refs.\(^9\),\(^10\).

It was pointed out by myself and Harrison\(^11\) that the basis of the disagreement lay in a difference over the definition of momentum transfer. As noted above, the momentum transfer is in general defined unambiguously only if the particle is initially in a momentum eigenstate. The calculations of Scully \textit{et al.} concern the local momentum transfer. This is the momentum transfer which can be seen in the shift or broadening of the momentum distribution of a particle already localized at one of the slits. By contrast, the momentum transfer distribution considered by Storey \textit{et al.} can perhaps be best characterized as the potential momentum transfer. It would be an actual momentum transfer if the particle were initially in a momentum eigenstate.

In Ref.\(^12\) myself and Harrison, together with Collett, Tan, Walls and Killip, showed that by using the Wigner function formalism one can identify different types of momentum transfer, which we called local and nonlocal. The local momentum transfer corresponds to the concept of momentum disturbance used by Scully, Englert and Walther\(^6\). We showed, in agreement with the claims of Scully \textit{et al.}, that this may indeed be zero and that the momentum transfer in the theorem of Storey \textit{et al.} was not relevant to this calculation. On the other hand, we showed that a particular measure of the nonlocal momentum transfer is always greater than $\pi \hbar/2d$, and this is derived in the same manner as the theorem of Storey \textit{et al.}\(^7\). It is this nonlocal momentum transfer which caused (in the Wigner function formalism) the loss the loss of the interference fringes.
In this work I want to revisit this question afresh. Rather than concentrating on the issue of interference and the loss of it, I propose to look at the question in the following context. As noted above, Heisenberg was the first to derive a rigorous measurement–disturbance relation (5), in the case when a particle is initially in a momentum eigenstate. Now consider a different situation. The particle, rather than having an equal probability amplitude of being at all points \( q \), now has an equal probability of being at just two points (or at least two identical small regions) separated by a distance \( d \). A measurement which can distinguish between these two regions (hereafter known as slits) must have a discrimination length scale \( \sim d \). Therefore we expect that if any extension of Heisenberg’s measurement–disturbance relation is possible, the particle’s momentum should be disturbed by an amount \( \sim \hbar/d \).

The aim of this paper is to show that it is indeed possible to derive a relation of this sort. I would not call the relation I derive a Heisenberg relation, because Heisenberg’s measurement–disturbance relation is based on the so-called Heisenberg uncertainty relation (5). The extension of this relation to the twin-slit case need not be based on that particular theorem, in particular because the measure of the momentum transfer cannot be based on the standard deviation. Nevertheless, it will be based in the formalism of quantum mechanics, including the conjugate relation of position and momentum, just as much as Heisenberg’s measurement–disturbance relation was. The momentum transfer I calculate in this paper is different from any of those in Refs.(6),(7),(12). Moreover, the measure of momentum transfer I propose uses only the momentum distributions of the particle before and after the measurement. As such it could not be criticized as being merely a potential momentum transfer. Despite the importance of nonlocal momentum transfer in the recent work of Ref.(13), the issue of locality or nonlocality is irrelevant to this work and will not be discussed.

2. Describing the Measurement

The starting point for the calculation is the initial wavefunction for the twin-slit case, which can be written as

\[
\psi_i(q) = 2^{-1/2} [\phi_a(q) + \phi_a(q - d)].
\]  

Here \( \phi_a(q) \) is a wavefunction parameterized by a positive real number \( a \) such that its width scales as \( a \) and

\[
\lim_{a \to 0} |\phi_a(q)|^2 = \delta(q).
\]

For example,

\[
\phi_a(q) = (2\pi a^2)^{-1/4} \exp(-q^2/4a^2)
\]

would do, and I will use this form for some specific calculations. For this example the state (6) is normalized only in the limit \( a \to 0 \), but that is all that we need.

In the momentum representation (indicated by a tilde), the initial state is, up to an irrelevant phase factor,

\[
\tilde{\psi}_i(p) = \tilde{\phi}_a(p) \sqrt{2} \cos \frac{pd}{2\hbar},
\]
where
\[ \tilde{\phi}_a(p) = \frac{1}{\sqrt{2\pi\hbar}} \int dq \, e^{-ipq/\hbar} \phi_a(q). \] (10)

The momentum probability distribution is therefore
\[ P_i(p) = \left(1 + \cos \frac{pd}{\hbar} \right) \mathcal{E}(p), \] (11)
where
\[ \mathcal{E}(p) = |\tilde{\phi}_a(p)|^2. \] (12)

It is the oscillations of period $2\pi\hbar/d$ under the envelope $\mathcal{E}(p)$ which are evidence of the coherent superposition of the particle being at the two slits at $q = 0, d$.

The effect of a position measurement on the particle’s wavefunction is to change it into
\[ \psi_f(q) = N^{-1/2}_\xi O_\xi(q) \psi_i(q), \] (13)
where $O_\xi(q)$ is a function relating to a particular measurement result $\xi$ and
\[ N_\xi = \int dq |O_\xi(q)\psi_i(q)|^2 \] (14)
is the probability for obtaining that result. Obviously the sum over all probabilities $N_\xi$ must equal unity. We are interested in the case where the particular result $\xi$ successfully distinguishes between the two slits. Then $O_\xi(q)$ must be zero in the region of one of the slits. Without loss of generality we may take it to be zero for $q \approx d$. Thus the final wavefunction is
\[ \psi_f(q) = N^{-1/2}_\xi O_\xi(q) 2^{-1/2} \phi_a(q). \] (15)

Creating a MacLaurin expansion of $\ln O_\xi(q)$ yields
\[ \psi_f(q) \propto \exp(\alpha q + \beta q^2 + \ldots) \phi_a(q), \] (16)
where $\alpha, \beta, \ldots$ are complex numbers.

Taking the particular form of $\phi_a(q)$ in Eq. (8), and ignoring the unwritten higher order terms in Eq. (16) we get
\[ \tilde{\psi}_f(p) \propto \exp \left[ (\alpha - ip/\hbar)^2 / (a^2 + 4\beta) \right]. \] (17)

Thus in the limit $a \to 0$ we find the final momentum distribution to be
\[ P_f(p) = \lim_{a \to 0} |\tilde{\psi}_f(p)|^2 = \frac{a}{\hbar} \sqrt{\frac{2}{\pi}} \exp \left[ -a^2(p/\hbar - k)^2 \right], \] (18)
which has been normalized. Here $k = \text{Im}(\alpha)$. The real part of $\alpha$, and the whole of $\beta$ (and also the higher order terms) become irrelevant when the limit $a \to 0$ is taken. In fact, the Gaussian form of the original wavefunction $\phi_a(q)$ is not required for this result and we can take the final momentum probability distribution to be more generally
\[ P_f(p) = \mathcal{E}(p - \hbar k), \] (19)
where $\mathcal{E}(p)$ is as defined in Eq. (12).
3. Quantifying the Momentum Transfer

From the preceding section we see that the momentum probability distributions before and after the position measurement which distinguishes between the two slits are respectively

\[ P_i(p) = \left(1 + \cos \frac{pd}{\hbar}\right) \mathcal{E}(p), \]
\[ P_f(p) = \mathcal{E}(p - \hbar k), \]

where \( \mathcal{E}(p) \) is an envelope which is arbitrarily smooth and broad. Obviously the two momentum distributions are different and so one would be justified in saying that there must have been some momentum transfer. One approach to quantifying this transfer would be to compare the moments of the two distributions. However, for the case \( k = 0 \), the mean and standard deviation of \( P_i(p) \) and \( P_f(p) \) are identical\(^{(12)} \). This case corresponds to the scheme proposed by Scully et al\(^{(6)} \), in which the local momentum transfer (which is the only momentum transfer with which they are concerned) is zero. While comparing the moments of \( P_i(p) \) and \( P_f(p) \) may identify the local momentum transfer (or lack of it)\(^{(12)} \), it is evidently not a good measure of the overall change in the momentum distribution.

The question we thus face is, what is a better way to quantify momentum transfer? One could seek a measure based on the measurement function \( O_\xi(q) \), as done by Storey et al\(^{(7)} \). The problem with this approach is that it is independent of the initial state. This would seem to imply that there is necessarily a momentum transfer even if the particle is already localized at one slit. While not logically impossible\(^{(11)} \), this is hard to accept on physical grounds given that Scully et al\(^{(6)} \),\(^{(9)} \),\(^{(10)} \) have shown that the momentum distributions may be unchanged by the measurement in that case. This difficulty can be overcome by turning to the Wigner function formalism\(^{(12)} \), as mentioned above. Nevertheless the simpler solution would seem to be to abandon \( O_\xi(q) \) altogether and seek a (non-moment-based) measure which involves only the initial and final momentum distributions.

Our question is then very closely related to one considered by the 18th century French mathematician, Monge\(^{(14)} \). The problem involves the transportation of a mass of soil from a given configuration (e.g. a heap) to another configuration (e.g. a dike) by haulage. As an idealization, we can assume that the mass of soil is divided into an arbitrarily large number of identical small loads each of which is hauled separately. A particular strategy for shifting the soil will therefore be characterized by an average distance over which the loads are hauled (vertical displacement being assumed negligible). Monge’s problem is to minimize this average distance. The haulage strategy which achieves this is known as the Monge plan and the resulting distance is known as the Monge distance. The Monge distance so defined is a good metric (in the mathematical sense) over the space of all possible soil distributions. The distance resulting from a non-optimal strategy is not a good metric, since inefficient workers could move loads of soil backwards and forwards over a large distance without changing the overall distribution of soil at all.

In modern probability theory, the Monge distance is used to define a metric over the space of probability distributions\(^{(15)} \). For simplicity, consider only distributions in one real variable. Imagine dividing up the area under the two curves into infinitesimal elements of equal area. Then the Monge distance is the minimum mean distance over which elements of the first probability distribution can be shifted so as to transform it into the second probability distribution. Clearly the Monge distance has the same dimension as the variable whose distribution we are considering.
There are generalizations of Monge’s distance (such as the Fréchet distance, which is the minimum root-mean-square distance), but there is no particular reason to prefer them for this problem.

For the case at hand, the Monge distance between the initial $P_i(p)$ and final $P_f(p)$ momentum distributions has the dimensions of momentum, and can in fact be identified with the average of the absolute value of the momentum transfer by the measurement. In this one-dimensional case, the Monge plan is to transport the infinitesimal elements along the line without changing their order. Translating these words into mathematics, the Monge distance $D_M$ is given by

$$D_M(P_f, P_i) = \int_0^1 d\lambda |F_i^{-1}(\lambda) - F_f^{-1}(\lambda)|. \quad (22)$$

Here $F^{-1}(\lambda)$ is defined by

$$F^{-1}(F(p)) \equiv p, \quad (23)$$

where $F(p)$ is the fiducial distribution

$$F_{i/f}(p) = \int_{-\infty}^{p} dp' P_{i/f}(p'). \quad (24)$$

By a change of variable Eq. (22) becomes

$$D_M(P_f, P_i) = \int_{-\infty}^{\infty} dp' |F_i(p') - F_f(p')|. \quad (25)$$

In our case we have

$$F_i(p) = \mathcal{F}(p) + \frac{\hbar}{d}\mathcal{E}(p) \sin \frac{pd}{\hbar} - \frac{\hbar}{d} \int_{-\infty}^{p} \sin \frac{pd'}{\hbar} d\mathcal{E}(p') \quad (26)$$

$$F_f(p) = \mathcal{F}(p - \hbar k) \quad (27)$$

where $\mathcal{F}(p)$ is the fiducial distribution of $\mathcal{E}(p)$. Now since $\mathcal{E}(p)$ has a width of order $\hbar/a \gg \hbar/d$, the size of the three terms in Eq. (26) are of order $1, a/d, (a/d)^2$ respectively. As I will show, the first-order term yields a finite contribution to $D_M$, so the second-order term can be neglected.

Substituting the expressions for the fiducial distribution into Eq. (25) yields

$$\int_{-\infty}^{\infty} dp' \left| \int_{p'-\hbar k}^{p'} \mathcal{E}(p'') dp'' + \frac{\hbar}{d}\mathcal{E}(p') \sin \frac{pd'}{h} \right|. \quad (28)$$

Using the fact that $ka, a/d \ll 1$, in the limit $a \to 0$ we can replace this expression by

$$D_M(P_f, P_i) = \int_{-\infty}^{\infty} dp' \mathcal{E}(p') \left| \hbar k + \frac{\hbar}{d} \sin \frac{pd}{h} \right| = \left| \hbar k + \frac{\hbar}{d} \sin \frac{pd}{h} \right|, \quad (29)$$

where the average is over one period of the sinusoidal function.

This last expression can be evaluated analytically for any $k$. However, in order to derive a relation between the slit separation $d$ and the momentum transfer $D_M$ we are interested only in the minimum over all $k$. It is not difficult to verify that the minimum occurs for $k = 0$, and has the value

$$D_M^\text{min}(P_f, P_i) = \frac{d}{\pi \hbar} \int_0^{\pi \hbar/d} dp' \frac{\hbar}{d} \sin \frac{pd}{h} = \frac{2\hbar}{\pi d}. \quad (30)$$
Since this is the smallest possible value for $D_M$ we can thus write a new momentum-disturbance relation

$$D_M(P_f, P_i) \geq \frac{2\hbar}{\pi d},$$

where the equality can clearly be attained.

4. Conclusion

As explained in the introduction, Heisenberg’s measurement–disturbance relation (5) between the accuracy of a position measurement $\epsilon_q$ and the momentum disturbance $\delta p$ applies only when the particle can be treated as being initially in a momentum eigenstate. In any other situation it can be used only heuristically, not rigorously. In this paper I have considered one of these other situations, a particular one which is of continuing interest, the twin slit. Here the particle is prepared in an equal-amplitude superposition of being at both the upper and lower slit, separated by a distance $d$. The measurement in this case simply distinguishes between these two possibilities. In this paper, I have argued that a good measure for the momentum disturbance $\Delta p$ is the Monge distance $D_M(P_f, P_i)$ between the momentum distributions before and after the measurement. Using this measure I have derived a new (twin-slit) measurement–disturbance relation:

$$d \times \Delta p \geq \frac{2\hbar}{\pi}.$$  

It should be emphasized that $d$ and $\Delta p$ are not standard deviations for $q$ and $p$ for any state of the particle. Unlike the Heisenberg measurement–disturbance relation (5), the relation (32) is not derived from Eq. (2). Thus the fact that the right-hand-side of Eq. (32) is different from that of Eq. (5) by a numerical factor of $4/\pi$ is not surprising and is of no particular significance. However, I think it is justifiable to call Eq. (32) an extension of the Heisenberg relation because the origin of the relation is exactly the same, namely the conjugate relation between the position and momentum of a particle. This conjugate relation is expressed in the Fourier transform which takes one from the position representation to the momentum representation, and which gives rise to the oscillations of period $2\pi \hbar/d$ in the initial momentum distribution of Eq. (11) which are erased by the measurement. The smaller the separation between the two slits, the more accurate the position measurement must be to resolve them, and the larger the momentum transfer.

With regard to the use of the Monge distance as a measure of the momentum transfer, it might be questioned whether this has any “physical” (rather than mathematical) justification for this measure. It turns out that the answer is yes, if one is prepared to accept the Bohmian interpretation\(^{16}\) of quantum mechanics as physical. This idea is explored extensively in Ref.\(^{17}\), where I show that the individual trajectories taken by particles under Bohmian mechanics can be traced both with and without a measurement for the twin-slit case. In the far-field, the velocities of these particles can be compared in these two cases, and the momentum change caused by the position measurement computed. If one defines $\Delta p$ to be the absolute value of the momentum change, averaged over all of the possible initial starting points of the particle, then one finds that this measure obeys exactly the inequality (32).
Having mentioned the Bohmian interpretation, it is worth re-emphasizing that the results of this paper are completely independent of one’s interpretation of quantum mechanics. The momentum disturbance $\Delta p$ is just the integral of the absolute value of the difference between the fiducial momentum distributions with and without the measurement. This integral is no harder to calculate from experimental data than the standard deviation of the momentum distribution which appears in the original Heisenberg measurement–disturbance relation (5).

Finally, one might wonder whether the result in this paper points the way towards a more general measurement–disturbance relation which would hold not only for the momentum eigenstate and twin-slit cases but for all initial conditions. Unfortunately I think the answer is no. The problem is not one of the definition of momentum transfer (the one used in this work would seem to be generally applicable), but of the measurement error. As a trivial example, if the initial state is sufficiently well localized in position then a position measurement of finite error may have no effect on the state whatsoever, so that there will be no momentum disturbance $\Delta p$. What distinguishes the momentum eigenstate and twin-slit cases is that in these cases a position measurement does have a clear effect on the particle. In the first case the measurement error $\epsilon_q$ can be chosen arbitrarily; in the second case it is the slit separation $d$ which is the relevant length scale (providing the two slits are distinguished by the measurement). It may be possible to work out measurement–disturbance relations for other particular examples, but it seems doubtful that they would supply any more insight than can be obtained from considering the two obvious cases.

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