Manifold-driven spirals in N-body barred galaxy simulations

E. Athanassoula*

Aix Marseille Université, CNRS, LAM (Laboratoire d’Astrophysique de Marseille) UMR 7326, 13388, Marseille, France

Accepted 2012 July 19. Received 2012 July 5; in original form 2012 May 3

ABSTRACT

We discuss the properties of spiral arms in an N-body simulation of a barred galaxy and present evidence that these are manifold driven. The strongest evidence comes from following the trajectories of individual particles. Indeed, these move along the arms while spreading out a little. In the neighbourhood of the Lagrangian points they follow a variety of paths, as expected by manifold-driven trajectories. Further evidence comes from the properties of the arms themselves, such as their shape and growth pattern. The shape of the manifold arms changes considerably with time, as expected from the changes in the bar strength and pattern speed. In particular, the radial extent of the arms increases with time, thus bringing about a considerable increase of the disc size, by as much as 50 per cent in about 1 Gyr.

Key words: galaxies: evolution – galaxies: kinematics and dynamics – galaxies: spiral – galaxies: structure.

1 INTRODUCTION

In a series of papers (Romero-Gómez et al. 2006, hereafter Paper I; Romero-Gómez et al. 2007, hereafter Paper II; Athanassoula, Romero-Gómez & Masdemont 2009a, hereafter Paper III; Athanassoula et al. 2009b, hereafter Paper IV; Athanassoula et al. 2010, hereafter Paper V), we proposed a theory to explain the formation and properties of spirals and inner and outer rings in barred galaxies. According to it, the backbone of these structures are a bunch of orbits guided and confined by the invariant manifolds associated with the periodic orbits around the saddle points of the potential in the frame of reference corotating with the bar. We call our theory manifold theory, or manifold flux-tube theory.

Some of the introductory dynamics necessary to follow our work are summarized in Binney & Tremaine (2008, section 3.3.2). Manifolds and the orbits they guide are described and explained in Paper I, while a relatively lengthy summary, avoiding equations, can be found in Section 2 of Paper III. Here we analyse an N-body simulation of an evolving barred galaxy and present evidence that its spiral arms are manifold driven. In Section 2 we give a brief theoretical reminder and describe relevant manifold shapes in simple analytical potentials. In Sections 3 and 4 we present the simulation and our results and make comparisons with our theoretical predictions. Further discussion and conclusions are given in Section 5.

2 THEORETICAL REMINDERS AND EXTENSIONS

Let us model a barred galaxy potential in the simplest possible way, namely by an axisymmetric part (including the disc and the halo) and a bar, which we assume rotates with a constant angular velocity. The dynamics of this system is best studied in a frame corotating with the bar, where there are five equilibrium points at which the derivative of the potential in the rotating frame is zero. These are called Lagrangian points (L_i, i = 1, 2, ..., 5). Two of them (L_1 and L_2) are located on the direction of the bar major axis, outside the bar but near its ends. By convention, we place the bar along the x-axis, measure the angles in a counterclockwise sense and call L_1 (L_2) the Lagrangian point on the right (left) of the centre. L_1 and L_2 are saddle-point unstable, so that the families of periodic orbits around each one of them (called Lyapunov orbits; Lyapunov 1949) are, at least in their neighbourhood, also unstable. They thus cannot trap regular quasi-periodic orbits around them since any orbit with initial conditions in their immediate vicinity (in phase space) is chaotic and will have to escape their neighbourhood. Not all departure directions are, however, possible. The direction in which the orbit can escape is set by what are called the invariant manifolds.

Manifolds can be thought of as tubes that guide the motion of particles whose energy is equal to theirs. There are four manifold branches emanating from a given Lyapunov orbit (fig. 2 of Paper III), two inside corotation (inner branches) and two outside (outer branches). Along two of these branches (one inner and one outer) the mean motion is towards the region of the Lagrangian point, while along the other two it is away from it. In Papers I–V we proposed that these manifolds and the orbits they guide are the building blocks of the spirals and rings in barred galaxies.

Fig. 1 illustrates a number of useful manifold shapes and properties. In the left-hand panel, we use a potential including a spiral component. The latter exerts an additional gravitational forcing which is not symmetric with respect to the bar major axis, thus shifting the Lagrangian points away from that axis. The motion along the inner manifold branch (light green) is from the upper bar region to the left,
i.e. towards \( L_2 \), as shown by the black arrow. These manifolds then circle for a few times around \( L_2 \) and then follow the outer manifold branch (red), in the sense shown by the arrow, thereby outlining the spiral. After tracing somewhat more than 180° in azimuth, they form a loop each, as if they were bouncing off a circle with radius roughly equal to the outer distance of the zero velocity curve from the galactic centre. All displayed manifolds have the same Jacobi constant, near that of \( L_1 \) and \( L_2 \).

In the middle and right-hand panels, we use a simple bar potential with no spiral component, so that \( L_1 \) and \( L_2 \) are now on the direction of the bar major axis. The manifolds are very similar to those of the previous example, with one considerable difference, i.e. we chose now to display manifolds whose Jacobi constant is much larger than that of \( L_1 \) and \( L_2 \). Therefore, both the Lyapunov orbits and the outline of the corresponding manifolds are much larger than in the previous example (see also Paper I), so that part of the outer manifolds when reaching the vicinity of \( L_1 \) will turn inwards, becoming inner manifolds and moving leftwards along the upper part of the bar towards \( L_2 \). We plot, in blue, one of these manifolds as an illustration. Note that amongst all the red and blue manifolds, the one that turns inwards is the one which is leftmost in the last part of the trajectory before reaching \( L_1 \). This motion is characteristic of manifolds with a relatively large Jacobi constant and is explained in detail in Koon et al. (2000) and in Paper II (cf. also Paper V).

The fraction of orbits that follows such trajectories depends on the potential and on the distribution function of the orbits. In the right-hand panel we follow the evolution of the blue manifold further to later times and show that it retraces the upper outer manifold branch. Another alternative (not shown here) would be that, in the neighbourhood of \( L_2 \), instead of going upwards and following the arm, the trajectory goes downwards and follows an inner branch below the bar major axis (as e.g. in fig. 1 of Paper 5, or as the black path in the left-hand panel of fig. 3 in that paper, albeit for a different potential).

3 SIMULATION

The simulation we will discuss here has two live components, a halo and a disc, represented by a million and 200 000 particles, respectively. The former has a volume density

\[
p_0(r) = \frac{M_h}{2\pi^{3/2}} \frac{\alpha}{r^\alpha} \exp(-r^2/r_c^2) \quad r^2 + \gamma^2,
\]

where \( r \) is the radius, \( M_h \) is the halo mass, \( \gamma \) and \( r_c \) are the halo core and cut-off radii, respectively, and the constant \( \alpha \) is given by \( \alpha = (1 - \sqrt{\pi} \exp(q^2) [1 - \text{erf}(q)])^{-1} \), where \( q = \gamma/r_c \) (Hernquist 1993). We use here \( \gamma = 15 \) kpc, \( r_c = 50 \) kpc and \( M_h = 25 \times 10^{10} M_\odot \). The initial density distribution of the disc is

\[
p_\odot(R, z) = \frac{M_d}{4\pi h^2 z_0} \exp(-R/h) \sech^2 \left( \frac{z}{z_0} \right),
\]

where \( R \) is the cylindrical radius, \( M_d \) is the disc mass, \( h \) is the disc radial scale length and \( z_0 \) is the disc vertical scale thickness. We use here \( h = 3 \) kpc, \( z_0 = 0.6 \) kpc and \( M_d = 5 \times 10^{10} M_\odot \). The corresponding circular velocity curve is shown in Fig. 2. For the radial velocity dispersion of the disc particles, \( \sigma_R(R) \), we take \( \sigma_R(R) = 100\exp(-R/3h) \) km s\(^{-1}\).

The initial conditions were made using the iterative method (Rodionov, Athanassoula & Sotnikova 2009), and the simulation was run using the \texttt{GADGET2} code (Springel, Yoshida & White 2001; Springel 2005). We adopted softening lengths of 100 (200) pc for the disc (halo) and an opening angle of 0.5.

4 RESULTS

To follow the morphological evolution in this simulation, we saved the snapshots every 0.005 Gyr. For each one of them, the position angle of the bar was calculated and the snapshot rotated so as to
Figure 3. Evolution of the disc component. The time in Gyr is given in the bottom-left corner of each panel.

Figure 3. Evolution of the disc component. The time in Gyr is given in the bottom-left corner of each panel.

display the bar horizontally. In this manner, we visualize the evolution in a frame of reference corotating with the bar. An animation, using these frames and produced with the GLNEMO2 software, can be viewed in http://195.221.212.246/dynmovie/MFolds, sgs058_noorbits.avi and sgs058_polar.avi for face-on Cartesian and polar views, respectively, while selected frames are shown in Fig. 3.

For our initial conditions, and within a distance of the order of 20 kpc from the centre, the dynamics are dominated by the disc rather than the halo mass distribution. Because of this, the bar grows early on in the simulation and fast in time. It is initially very fat, but by \( t = 0.5 \) Gyr, it starts becoming longer and narrower, while two spiral arms start forming, one from each end of the bar. These spirals are trailing, and their angular extent increases with time as their tip approaches the opposite side of the bar. Until about 0.9 or 1.0 Gyr they form a grand-design two-armed structure, staying attached to the end of the bar and rotating with the same pattern speed as the bar.

Shortly after \( t = 0.9 \) Gyr, the shape of the bar undergoes strong changes – as material initially in the arms is accreted to its outer parts – while the arm–interarm density contrast drops, so that the spiral is not as clearly discernible. This is followed by a second spiral episode, qualitatively very similar to the first one and starting between 1.0 and 1.1 Gyr, i.e. a new two-armed grand-design spiral develops, again starting from one tip of the bar and extending towards the opposite bar side. This second spiral episode lasts till about \( t = 1.5 \) Gyr, and its shape and amplitude are quantitatively considerably different from those in the first episode.

The above description is in good agreement with our manifold theory results. Indeed, this theory leads naturally to two-armed trailing spiral arms (Paper IV), which start growing from the tip of the bar first outwards and then towards the opposite end of the bar (fig. 6 in Paper I). Material for these arms should come from the outer parts of the bar (Paper V). One of the extremities of each arm should be linked to \( L_1 \) or \( L_2 \), and the arms should rotate with the same pattern speed as the bar. All these developments and morphological properties are indeed seen during the evolution in our simulation, arguing strongly for a manifold origin of the spiral arms.

The main argument, however, in favour of the manifold origin of the spirals comes from the orbits of the individual particles. In density wave theory, the arms are loci of density maxima. Particles should thus traverse the arms, but stay longer in the arm than in the interarm region (Lin & Shu 1964). This is totally different from our manifold theory, where spiral arms should be a bundle of orbits guided by the manifolds, so that particles should move along the arms rather than across them.

It should thus be possible to find out which of the two theories is the main driver of the spiral structure in the simulation simply by following a number of particles. For this we use our series of snapshots in which the bar orientation is kept horizontal. At \( t = 0.8 \) Gyr, when the bar and spiral are well developed, we selected 60 particles located in the part of the spiral arm which is near the tip of the bar, roughly where we estimated by eye that the \( L_2 \) Lagrangian point would lie (cf. left-hand panel of Fig. 1). Similar results can be found by selecting particles at other times or locations, provided they are clearly in an arm at selection time. We then followed the trajectories of these particles from \( t = 0.5 \) to 1.55 Gyr and produced a sequence of 211 frames in each of which we superposed on the snapshot of all disc particles a filled white circle marking the current location of each chosen particle and a white line for its trajectory over the previous 0.175 Gyr.\(^1\)

We repeated this task for all snapshots and thus produced

\(^1\) This time range should be kept in mind when comparing the spiral shape and the early parts of orbit, because the spiral could have evolved during these 0.175 Gyr.
Manifold-driven N-body spirals

Figure 4. Nine snapshots of the disc component, on which we overlay the locations (white filled circles) and trajectories (white solid lines) of the 60 particles that we follow (see text). The time in Gyr is given in the bottom-left corner of each panel.

an animation (http://195.221.212.246:4780/dynam/movie/MFolds, sgs058_orbits.avi and sgs058_polar_orbits.avi for Cartesian and polar views, respectively). In Fig. 4 we show nine such frames which display the salient features of the particle trajectories.

Roughly from $t = 0.5$ to $0.65$ Gyr the chosen particles travel along the outer part of the bar, in the direction from $L_1$ to $L_2$, then they circle a couple of times around what should be $L_2$ (frame at $t = 0.82$) and escape its vicinity following the upper arm (times 0.90–1.18). Thus, the particle orbits stay within the arms, as expected for the manifold theory, and do not cross them as would have been expected by the density wave theory. After roughly $t = 1.1$ Gyr, the forerunners of the group of particles we follow reach the vicinity of the opposite side of the bar from which they emanated (i.e. the right-hand side, near $L_1$). From that point onwards the particles divide themselves into two groups. Some continue downwards, make a loop, as if they were bouncing off an invisible barrier, and then trace the lower spiral arm from $L_1$ to $L_2$. Others retrace the upper outline of the bar leftwards in the direction from $L_1$ to $L_2$. Once they have reached the vicinity of $L_2$, another branching occurs. Some particles go to the upper spiral arm which they follow from $L_2$ to $L_1$, while others stay in the bar (frames at $t = 1.45$ and 1.53). All the trajectories and branchings described above are very similar to those seen in Fig. 1 and in Papers I–V.

Furthermore, note that the orbits spread out as they trace the arm moving from $L_2$ to $L_1$ between $t = 0.65$ and 1.1 Gyr, leading to a gradual widening of the spiral arm. This can be already seen in Fig. 3. This widening is also expected from the manifold origin of these arms (see Fig. 1 here and Papers I and V). How much this widening is depends on the potential, so that we cannot compare Figs 1 and 4 quantitatively.

5 DISCUSSION AND CONCLUSIONS

In this Letter we followed the formation of spiral structure in an $N$-body simulation of a barred disc galaxy. We witness the formation of a short-lived and recurrent spiral structure, two episodes of which last over roughly 1 Gyr. In between these two episodes the spiral never quite vanishes, but its amplitude decreases considerably. These spirals have the same properties as the manifold spirals discussed in Papers I–V. We also followed the trajectories of
particles in the arms and found that, contrary to what would have been expected for density wave spirals, they do not cross the arms, but they move along them, starting off from the vicinity of one of the Lagrangian saddle points ($L_1$ or $L_2$) in the direction of the other one. Thus, their trajectories outline the arms. In fact, the whole of the trajectory contributes to the spiral structure, and these spirals can be called flux-tube manifold spirals.

There are several more signs, telltale of a manifold origin. For example, particles which outline the spiral have their origin in the outer part of the bar and join the spiral via the vicinity of a point whose location is compatible with that of the saddle Lagrangian points ($L_1$ or $L_2$). Furthermore, they loop in the vicinity of that point before they follow the arm. Another telltale sign of the manifold origin of the arm is that in the vicinity of each saddle Lagrangian point, the particles split into two groups, in the same way as manifolds in analytic potentials.

From all the above it becomes clear that we are witnessing manifold-driven spirals in our simulation. This – together with the good agreement between the properties of manifold-driven and observed rings and spirals discussed in Papers IV and V and by Martínez-García (2012) – argues strongly that manifolds do play a role in spiral and ring formation. Manifolds, however, are not the only possible origin of such structures. Indeed, spirals have been witnessed also in other $N$-body simulations and interpreted in terms of other theories (Sellwood & Carlberg 1984; D’Onghia, Vogelsberger & Hernquist 2012; Grand, Kawata & Cropper 2012; Sellwood 2012). We already included a note of caution on this in Paper V.

Our comparison between $N$-body and theoretical manifolds must necessarily stay qualitative. As any orbital structure work, the manifold theory relies on a few simplifications, the most important of which is that the potential is time-independent in the frame of reference corotating with the bar. On the other hand, in simulations and in real galaxies the potential evolves with time, due to redistribution of angular momentum via the resonances (e.g. Lynden-Bell & Kalnajs 1972; Weinberg 1985; Athanassoula 2002, 2003). However, if this evolution is not too fast, it should not present a problem for our manifold flux-tube theory where the arm is constituted by the whole flow of material guided by the manifolds and will only lead to a change of the manifold properties with time. Any secular change in the potential is expected to lead to a secular evolution of the manifold properties. Furthermore, our simulations show that, even when the rate of change is considerable, particle trajectories are still guided by manifolds, keep their characteristic signatures and obey the selection rules of permissible paths defined by them. Nevertheless, their shape and extent change as expected, while particles can get trapped or untrapped by these manifolds.

Such orbits as described here have also been witnessed as responses to applied analytical potentials (Danby e.g. 1965; Patsis 2006; Papers I-V). To our knowledge, however, this is the first time that the manifold theory is tested in a realistic self-consistent $N$-body simulation by following the motion of individual particles. Nevertheless, the simulation discussed here is in no way unique; manifold-driven spirals can be seen in a large number of our simulations, some of which include gas, and will be discussed elsewhere.

Comparing the disc extent between times 0.5 and 1.5 Gyr, we note a considerable increase, of the order of 50 per cent, due to the spirals. This shows that the bar and the associated manifolds can drive the overall evolution of the disc significantly. We will explore this process further in future work.

ACKNOWLEDGMENTS

We thank M. Romero-Gomez and A. Bosma for help and discussions, and J. C. Lambert for his splendid work with GLNEMO (http://projets.oamp.fr/projects/glNemo2).

REFERENCES

Athanassoula E., 2002, ApJ, 569, L83
Athanassoula E., 2003, MNRAS, 341, 1179
Athanassoula E., Romero-Gómez M., Masdemont J. J., 2009a, MNRAS, 394, 67 (Paper III)
Athanassoula E., Romero-Gómez M., Bosma A., Masdemont J. J., 2009b, MNRAS, 400, 1706 (Paper IV)
Athanassoula E., Romero-Gómez M., Bosma A., Masdemont J. J., 2010, MNRAS, 407, 1433 (Paper V)
Binney J., Tremaine S., 2008, Galactic Dynamics, 2nd edn. Princeton Univ. Press, Princeton, NJ
Danby J. M. A., 1965, AJ, 70, 501
D’Onghia E., Vogelsberger M., Hernquist L., 2012, preprint (arXiv:1203.0513)
Grand R. J. J., Kawata D., Cropper M., 2012, MNRAS, 421, 1529
Harsoula M., Kalapotharakos C., Contopoulos G., 2011, MNRAS, 411, 1111
Hernquist L., 1993, ApJS, 86, 389
Koon W., Lo M., Marsden J., Ross S., 2000, Chaos, 10, 427
Li C. C., Shu F. H.-S., 1964, ApJ, 140, 646
Lyapunov A., 1949, Ann. Math. Studies, 17
Lynden-Bell D., Kalnajs A. J., 1972, MNRAS, 157, 1
Martínez-García E., 2012, ApJ, 744, 92
Patsis P., MNRAS, 369, L56
Rodionov S. A., Athanassoula E., Sotnikova N. Y., 2009, MNRAS, 392, 904
Romero-Gómez M., Masdemont J. J., Athanassoula E., García-Gómez C., 2006, A&A, 453, 39 (Paper I)
Romero-Gómez M., Athanassoula E., Sotnikova N. Y., 2009, MNRAS, 392, 904
Rodionov S. A., Athanassoula E., Sotnikova N. Y., 2009, MNRAS, 392, 904
Sotouis P., Efthymiopoulos C., Voglis N., 2008, MNRAS, 387, 1264
Voglis N., Tsoutsis P., Efthymiopoulos C., 2008, MNRAS, 373, 280
Weinberg M. D., 1985, MNRAS, 213, 451

This paper has been typeset from a TEx file prepared by the author.

© 2012 The Author, MNRAS 426, L46–L50
Monthly Notices of the Royal Astronomical Society © 2012 RAS

2 This is not the case for the theory presented in e.g. Voglis, Tsoutsis & Effthymiopoulos (2008) or Harsoula, Kalapotharakos & Contopoulos (2011) which considers the loci of the apsidal manifold sections. It thus requires a quasi-stationary, non-evolving potential.

3 A previous attempt (Tsoutsis, Efthymiopoulos & Voglis 2008) used an unrealistic simulation, with no separate disc and halo components, just a single cylindrical-shaped component, with a vertical-to-horizontal size ratio of ~0.3 and a velocity dispersion larger than 100 km s$^{-1}$, i.e. properties very different from those of a spiral galaxy.