Migration reversal of soft particles in vertical flows

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Abstract – Non-neutrally buoyant soft particles in vertical microflows are investigated. We find for light soft particles in downward Poiseuille flow cross-streamline migration (CSM) to off-center streamlines and for heavy particles CSM to the center. In both cases a reversal of the vertical flow direction and the related shear gradient causes a reversal of the migration direction. This gravitational driven CSM of soft particles occurs also in linear shear flows: heavy (light) particles migrate antiparallel (parallel) to the shear gradient. The surprising, flow-induced migration (reversal) is characterized by simulations and analytical approximations, confirming our plausible explanation of the effect. This might be applied for separating particles.

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Introduction. – Microfluidics is a rapidly evolving cross-disciplinary field, ranging from basic physics to a large variety of applications in life science and technology [1–3]. The blooming subfield of the dynamics of neutrally buoyant soft particles in suspension and their cross-streamline migration (CSM) in rectilinear shear flows plays a central role for blood flow, (blood) cell and DNA sorting or polymer processing among others [2–5]. In contrast, little is known about the dynamics of non-neutrally buoyant soft particles in rectilinear flows. We demonstrate in this work a novel migration reversal for such particles.

Segre and Silberberg reported in 1961 on CSM of neutrally buoyant rigid particles in finite Reynolds-number flows through pipes [6]. When particles and channels approach the micrometer scale, fluid inertia does not matter and particles follow the Stokesian dynamics. In this limit CSM occurs not for rigid but for soft particles for example in rectilinear flows [7,8], whereby the flows fore-aft symmetry is broken, requiring intra-particle hydrodynamic interaction [7]. Such symmetry breaking occurs also near boundaries via wall-induced lift forces [3,8,9] or by space-dependent shear rates, so that dumbbells [7], droplets [10,11], vesicles and capsules [12–14] exhibit CSM even in unbounded flow. Such parity-breaking mechanisms may be also accompanied by a viscosity contrast [15] or chirality [16]. Recently, CSM was found for asymmetric soft particles in time-dependent linear shear flow [17] and that soft particles are actuated even in a homogeneous but time-dependent flow by taking particle inertia into account [18].

Heavy rigid particles in a finite Reynolds number flow downward or upward in a gravitational field migrate away from or to the tube center, respectively [19]. Effects of axial forces along the tube axis on particles and their CSM in finite Reynolds number flows were studied in refs. [20–22] and effects of axial (electrical) forces on (charged) polymers in Poiseuille flows in refs. [23,24]. In the wide field of sedimentation, characteristic deformations of sedimenting heavy vesicles have been explored only recently [25], but very little is known about the dynamics non-neutrally buoyant soft particles in vertical Stokes flows.

Here we show that during sedimentation in a channel heavy soft microparticles migrate to its center, while rigid particles do not [26]. Moreover, heavy (light) soft particles migrate antiparallel (parallel) to the shear gradient in vertical rectilinear Stokes flows, as shown in fig. 1. This dependence of the CSM-direction on the shear gradient direction is obtained by approximate analytical calculations and by numerical simulations for soft capsules and ring polymers. We also provide a plausible qualitative explanation of CSM of non-neutral particles: It is based on the interplay between the shear-induced orientation of the elliptically shaped soft capsule (ring) together with its anisotropic friction coefficients.

Model and approach. – To investigate the cross-stream migration of non-neutrally buoyant soft particles
Poiseuille flow along the \( y \) when with the Neo-Hookean potential \( V \) under the impact of a vertical gravitational force \( \vec{F}_g \). The simulations reveal that the capsule displays (a) a migration to the wall and (b) a migration to the wall when \( \vec{u}_0 \parallel \vec{F}_g \) or (b) a migration to the wall when \( \vec{u}_0 \perp \vec{F}_g \). Parameters: see footnote 1.

under the impact of a vertical gravitational force \( \vec{F}_g = F_0 \vec{e}_y \) in rectilinear flows, we use the capsule and ring as particle representatives, exposed to a vertical 3D, plane Poiseuille flow along the \( y \)-axis

\[
\vec{u}_0(x) = \vec{u}_0 \left[ 1 - \left( x/d \right)^2 \right] \vec{e}_y
\]

with its two confining boundaries at \( x_d = \pm d \) and \( u_0 \geq 0 \) the maximum flow velocity at the channel center.

The capsule and ring are described by a bead-spring model consisting of \( N \) beads. The surface of the capsule is triangulated with the beads at the vertices (see Supplementary material.pdf given as Supplementary Material (SM)). Their Stokesian dynamics [27] is described by the coupled equations (1 \( \leq i \leq N \)),

\[
\vec{r}_i = \vec{u}_0(\vec{r}_i) + \sum_1^N \vec{H}_{ij} \cdot \vec{F}_j,
\]

whereby the inertia of the beads is neglected due to the small Stokes number (see SM). The particle center is given by \( r_c = \sum_{i=1}^N r_i/N \).

The force on bead \( j \) is given by \( \vec{F}_j = -\nabla_j V(\vec{r}) + \vec{F}_g \) with \( V(\vec{r}) \) referring to the total potential, and \( \vec{H}_{ij} \) denoting the mobility matrix, as specified in the following.

For a capsule, the total potential is \( V(\vec{r}) = V_{NH} + V_0 + V_0 \) with the Neo-Hookean potential \( V_{NH} \), suited to describe rubber-like materials with a constant surface shear-elastic modulus \( G \) [28,29], and a bending potential \( V_b \)

\[
V_b = -\kappa_c/2 \sum_{i,j} \left( 1 - \cos \beta_{i,j} \right)
\]

with bending elasticity \( \kappa_c \). The angle \( \beta_{i,j} \) is formed by the two normal surface vectors of nearest-neighbor triangles with beads at the triangle corners [30]. The potential \( V_i = -k_i/2(V(t) - V_0)^2 \) keeps the capsule’s instantaneous volume \( V(t) \) close to the reference volume \( V_0 = \pi R^2/4 \) of a spherical capsule of radius \( R \) with volume stiffness \( k_i \).

The total potential of the ring of radius \( R \) reads \( V(\vec{r}) = V_0 + V_b \), using a harmonic potential \( V_b = k \sum_{i=1}^N (R_i - b)^2 \) with spring constant \( k \), equilibrium bond length \( b \), and \( R_i = |\vec{r}_i| \) the magnitude of the bond vector \( \vec{R}_i = r_i - r_{i+1} \) of the next-nearest beads \( i \) and \( i + 1 \). The bending potential \( V_0 \) with a bending stiffness \( \kappa_e \) is given by

\[
V_b = -k_e/2 \sum_{i=1}^N \ln (1 + \cos \beta_i) ,
\]

with the angle \( \beta_i \) defined via \( \cos \beta_i = \vec{e}_{R_{i-1}} \cdot \vec{e}_{R_i} \) by the bond unit vectors \( \vec{e}_{R_i} = \vec{R}_i/|\vec{R}_i| \).

The Blake tensor \( H_{ij} \), describing the hydrodynamic interaction (HI) between beads \( i \) and \( j \) in the presence of a single plane boundary within the \( yz \)-plane [31], reads

\[
H_{ij}(\vec{r}_i, \vec{r}_j) = \frac{\pi \eta R \delta_{ij}}{16 h_{ij}} \left[ \frac{1}{3} \delta_{\alpha\beta} + \frac{1}{6} \eta \bar{\delta}_{ij} \right] \left( \frac{1}{R_{ij}^3} \right),
\]

The first term in eq. (5) describes the bulk HI via the Oseen tensor [27] with \( e_{R_{ij}} = R_{ij}/|R_{ij}| \) and \( R_{ij} = r_i - r_j \); \( \eta \) and \( \alpha \) refer to the viscosity, respectively, bead radius, and \( \alpha, \beta \in \{ x, y, z \} \).

The term \( 3H_{ij}(\vec{r}_i, \vec{r}_j) \) is the source singlet due to the HI of the mirror bead \( j \), given by eq. (6) for \( i \neq j \) with \( \vec{R}_{ij} \) replaced by \( \vec{R}_{ij} = \vec{R}_i - \vec{R}_j \), \( e_{R_{ij}} = R_{ij}/|R_{ij}| \).

The last two terms in eq. (5) refer to the Stokes doublet (D)

\[
D_{ij} \left( \frac{\vec{r}_i - \vec{r}_j}{d_{ij}} \right) = \frac{e^{\beta_{ij}}}{4 \pi \eta R_{ij}^3} \left[ \frac{1}{3} \delta_{\alpha\beta} - \frac{2}{3} \eta e_{R_{ij}} e_{R_{ij}} \right],
\]

and source doublet (SD)

\[
SD_{ij} \left( \frac{\vec{r}_i - \vec{r}_j}{d_{ij}} \right) = \frac{1}{4 \pi \eta R_{ij}^3} \left[ h_{ij} (1 - 2 \delta_{\beta\delta}) \right] \times \left[ \delta_{\alpha\beta} e_{R_{ij}} - \delta_{\alpha\delta} e_{R_{ij}} + \delta_{\beta\delta} e_{R_{ij}} - 3 e_{R_{ij}} e_{R_{ij}} \right].
\]

Screening effects of a second wall are included by superposition of the HI of the single walls, and provide reasonable results for a channel width to particle size ratio larger than 5 [32]. In simulations where wall-HI is turned off, the mobility \( H_{ij} \) in eq. (5) reduces to the Oseen tensor \( 3H_{ij} \).
We use throughout this work (dimensionless) parameters given in footnote 1. \( \tau_c = \frac{\eta R}{G} \) and \( \tau_R = \frac{\eta R^3}{\kappa R} \) are the typical relaxation time of a capsule and a ring, respectively. In both cases the capillary number \( \text{Ca} = \frac{\gamma}{\kappa} \) (with \( \tau = \tau_c \) or \( \tau_R \)) is a measure for the particle deformation, with the shear gradient \( \gamma = \partial_x u_0 e_x \). Further information on the modeling is provided in the SM.

**Explanation of \( \mathbf { F }_g \)-induced CSM.** – We provide at first a qualitative picture of the \( \mathbf { F }_g \)-induced migration, with the generic results of the simulation already shown in fig. 1, and derive subsequently the \( \mathbf { F }_g \)-induced CSM velocity, obtained for small capsule deformations.

A. **Qualitative analysis of CSM.** Figure 2 shows how a soft heavy capsule and ring (\( \mathbf { F }_g < 0 \)) are deformed by the local shear gradient \( \gamma \) of plane Poiseuille flows \( u_0(x) \), whereby the snapshots are from Stokesian dynamics simulations: (a) \( u_0 \downarrow \mathbf { F }_g \) and (b) \( u_0 \uparrow \mathbf { F }_g \). For moderate \( \gamma \), their shape is nearly ellipsoidal or elliptic, respectively, with their major axis being inclined at an angle \( \theta \) with \( \mathbf { F }_g \) and \( \mathbf { F }_g \); the sign of \( \theta \) depends on the sign of the local \( \gamma \) [29], as shown in fig. 2(a) and (b). The drag coefficients \( \zeta_{\gamma \perp} \) of the distorted capsule and ring are different along and perpendicular to their major axis with \( \zeta_{\parallel} < \zeta_{\perp} \). Therefore a vertical force \( \mathbf { F }_g \) acting on the obliquely oriented bodies causes a slanted migration velocity \( \mathbf { v } \) with an angle \( \alpha \) to \( \mathbf { F }_g \) and with its transverse component \( v_m \). The sign of \( v_m \) and, hence, the CSM direction depends on the sign of the local shear rate \( \gamma(x_c) \), which is controlled by the direction of \( \mathbf { F }_g \).

We note that the \( \mathbf { F }_g \)-driven CSM of soft particles occurs for any kind of vertical flow with a finite local \( \gamma \), including linear shear flow. This is to be contrasted to the well-known CSM of neutrally buoyant soft particles (\( \mathbf { F }_g = 0 \)), where a spatial-dependent shear gradient is required to drive the bulk migration [12,13]. When soft particles migrate towards the wall, the CSM gets balanced by lift forces due to the wall-HI [9], so that the migration stops at a certain wall distance, as shown below.

B. **Analytical approximation of CSM.** We now derive an analytical expression of the CSM velocity \( v_m \) for the case where a total force \( \mathbf { F }_g = \mathbf { F }_g e_y \) acts on a shear-distorted capsule. Motivated by our simulations, including the snapshots in fig. 2, we approximate the spatially varying \( \gamma(x) \) by a constant \( \gamma(x_c) \) across the capsule, i.e., the Poiseuille flow is locally replaced by a linear shear flow. The assumption of a linear shear flow entails that the capsule shape is an ellipsoid with a major axis.

Under these prerequisites, the respective anisotropic drag coefficients can be obtained by calculating first the three different axes of a Neo-Hookean capsule from [28,29]

\[
\begin{align*}
   r^2 &= x^2 + y^2 + z^2 = R^2 + \frac{25}{3} \text{Ca} \gamma T \cdot \mathbf{r} + O(\text{Ca}^2), \\
   J &= \frac{1}{2} \left[ (\nabla \otimes \mathbf{u}_0) + (\nabla \otimes \mathbf{u}_0)^T \right] 
\end{align*}
\]

with the lengths \( d_1 \) of the major and \( d_2 \) of the minor axis. They deviate from the radius \( R \) of a spherical capsule,

\[
d_{1,3} = \left[ 1 + \frac{25}{6} \text{Ca} \right]^{-1/2} R, \quad d_2 = R, \quad d_{1,3}^{(a)} \quad d_2 = R, \quad 10^4 \%
\]

with the \( d_{1,3} \)-axis inclined to \( \mathbf{u}_0 \) at an angle \( \theta \approx \pi/4 \) [29,33].

To obtain an analytical expression for the drag coefficients, we further assume rotational symmetry along the capsule’s major \( a \)-axis [33] with \( a = d_1 \) and approximate the minor axis by \( b = (d_2 + d_3)/2 \). Using Perrin’s formulas [33], the drag coefficients associated with the \( a \)-major
and $b$-minor axis are given by

$$\zeta_\parallel = 16\pi \nu b \left[ \frac{(2\beta^2 - 1) \ln \left( \frac{\beta+\sqrt{\beta^2 - 1}}{\beta-\sqrt{\beta^2 - 1}} \right)}{B^2} - \frac{2\beta}{B} \right]^{-1}, \quad (11)$$

$$\zeta_\perp = 16\pi \nu b \left[ \frac{(2\beta^2 - 3) \ln(\beta+\sqrt{B})}{B^2} + \frac{\beta}{B} \right]^{-1}. \quad (12)$$

with $B = \beta^2 - 1$ and $\beta = a/b$. Decomposing $\vec{F}_g$ into its components $\vec{F}_{g,\parallel}$ and $\vec{F}_{g,\perp}$, parallel and perpendicular to the major $a$-axis, the CSM velocity along the $x$-axis is

$$v_m = \mathbf{v} \cdot \mathbf{e}_x \quad \text{with} \quad \mathbf{v} = \frac{\vec{F}_{g,\perp}}{\zeta_\perp} + \frac{\vec{F}_{g,\parallel}}{\zeta_\parallel}. \quad (13)$$

A Ca-expansion of $\zeta_{||,\perp}$ provides the leading order of the Ca-dependence of $v_m$, and is given by

$$v_m = \frac{5}{96\pi \eta Re} F_g Ca + O(Ca^2) \quad (14)$$

with $v_m \sim \gamma$ being constant in linear shear flow. Equation (14) gives for the spatial-dependent Ca($x$) of a plane Poiseuille flow, Ca($x_c$) = $-\frac{2\mu x_c}{\mu G d^2}$, the CSM velocity

$$v_m \approx -\frac{5}{48\pi G d^2} \bar{F}_g u_0 x_c, \quad (15)$$

showing how the migration direction can be controlled by the relative orientation of $u_0$ and $F_g = \bar{F}_g \mathbf{e}_y$, depending on the location of the capsule center $x_c$ within the channel.

The $\bar{F}_g$-induced migration velocity $v_y = \mathbf{v} \cdot \mathbf{e}_y$ along the streamlines is determined by the Stokes drag and a deformation dependent correction term $\propto Ca$. For Poiseuille flow it is given by

$$v_y \approx \frac{\bar{F}_g}{6\pi \eta R} \frac{5}{144} \pi d^2 G \frac{u_0 x_c}{\bar{F}_g u_0 x_c}. \quad (16)$$

When $u_0 \uparrow \bar{F}_g$ and $\bar{F}_g$ sufficiently large, the capsule’s $v_y$-velocity can become antiparallel to the flow direction with the onset obtained from $v_y \geq u_0(x_c,y_c)$, i.e.,

$$\bar{F}_g \geq \frac{144\pi \eta R d^2 G u_0}{25R u_0 x_c} \left[ 1 - \frac{x_c^2}{d^2} \right]. \quad (17)$$

Simulations of CSM in unbounded flow. – With our basic understanding of the $\bar{F}_g$-driven CSM from the previous section, we extend our study of the CSM of non-neutrally buoyant soft particles by Stokesian dynamics simulations of heavy soft capsules and rings ($F_g < 0$), at first in unbounded Poiseuille flow.

Figure 3 shows for up ($u_0 \uparrow$) and down flow ($u_0 \downarrow$) the CSM velocity $v_m$ for a capsule (left panel) and a ring (right panel), extracted from the slope of the transverse trajectory $x_c(t)$, as a function of $x_c \sim Ca$. Here and in the following sections $v_m$ is a superposition of the conventional CSM and the $\bar{F}_g$-induced one. The capsule’s $\bar{F}_g$-values are varied in the range $0.01:0.03$ and correspond to $10\%–30\%$ higher density than the surrounding liquid$^2$. Note, that $v_m$ is shown in fig. 3 only in the positive range $0 < x_c < d$, because $v_m$ changes only the sign for $-d < x_c < 0$.

For comparison the conventional CSM velocity for neutrally buoyant particles is also shown in fig. 3(a)–(d) (solid lines), which is directed to the flow center for up and down flow $[12–14]$. With a parallel force $u_0 \parallel \bar{F}_g$ as in fig. 3(a) and (c) a capsule and a ring exhibit increased center-migration ($v_m < 0$). This is no longer the case when $u_0 \uparrow \bar{F}_g$, as in fig. 3(b) and (d), where the capsule and ring migrate away from the flow center, $v_m > 0$, if $F_g$ is sufficiently large. This is the case in the range $F_g > 0.001$ for the parameters in footnote $^3$ and a capsule at least 1% heavier than the liquid; otherwise the conventional center migration takes over. Importantly, the $\bar{F}_g$-induced CSM

$^2$A number of biological cells are about 5%–15% heavier than water [34] and salt-loaded capsules are even heavier.
velocity \( v_m \) can be by a factor 10 larger than for neutrally buoyant particles (solid lines), as shown in fig. 3.

As explained above, the \( F_g \)-driven CSM relies on the interplay between \( F_g \) and the anisotropic friction of soft particles, once they are deformed and obliquely oriented by a local shear gradient. This means firstly that we get a similar dependence of \( v_m \) on Ca for soft capsules and rings, which shows the generic feature of the \( F_g \)-driven migration. Secondly it means the \( F_g \)-driven migration works also in a vertical linear shear flow indicated in fig. 2; fig. 3 may also be used to read off the expected CSM velocities \( v_m \) for known \( \text{Ca} \)-values of the linear shear rate.

For a capsule or a ring lighter than the fluid one has \( F_g > 0 \), i.e., the migration is reversed for a given \( u_0 \), as predicted by eq. (15). The important point to bear in mind is that the direction of the CSM is solely controlled by the relative orientation of \( u_0 \) and \( F_g \), resulting in a migration towards the center when they are parallel or to the wall in the anti-parallel case.

**Wall effects on CSM.** The previous exploration of CSM of heavy particles in unbounded vertical-plane Poiseuille flow allowed us to identify the key mechanism of the \( F_g \)-induced CSM in bulk flow. In a bounded Poiseuille flow lift forces induced by the hydrodynamic wall-interaction (wall-HI) are expected [9] and their effects on CSM are analyzed now.

**CSM of a soft sedimenting capsule.** We first look at a heavy capsule, freely sedimenting in a vertical-plane channel when \( u_0 = 0 \). In the absence of inertia, it is well-known that rigid particles sedimenting between the two walls do not display any CSM [26]. This changes once the particle has a certain softness (measured for a Neo-Hookean capsule by the parameter \( G \)) with trajectories \( y_c-x_c \), shown in fig. 4 for various values of \( G \); the capsule always migrates away from the wall towards the center. It migrates the faster the softer the capsule is (small \( G \)). For example, a capsule of diameter 100 \( \mu \text{m} \) and 10\% more dense than the liquid migrates from \( x_c = 0.36 \text{mm} \) to \( x_c = 0.23 \text{mm} \) in fig. 4 (red curve) away from the wall, while covering a vertical distance of \( \sim 2.7 \text{mm} \). Hence, the sedimentation or elevation of heavy and light soft particles leads to particle focusing in the channel center.

It may surprise that the capsule migrates at all, considering that \( u_0 = 0 \). Recall, that the \( F_g \)-induced CSM requires 1) a particle deformation (leading to an anisotropic friction), accompanied by its inclination and 2) a finite \( F_g \) driving the oblique particle relative to the (resting) flow. Indeed, all these requirements are met: the deformation and inclination of the sedimenting particle is mediated by the wall-HI, which is stronger for those parts of the capsule in proximity to the wall; as a result, the parts further away from the wall experience less wall friction. The different drag forces across the capsule causes its deformation towards an ellipsoid with its major axis being oblique to the wall, while \( F_g \) drives the CSM of the anisotropic body.

**CSM in bounded Poiseuille flow.** The wall-induced repulsive lift forces cause a sustained center-migration. We
expect that this affects the interplay of $F_g$ and $u_0$, and, therefore, the $F_g$-induced CSM of soft particles, as shown in fig. 5 for the capsule (left panel) and ring (right panel). When $u_0 \downarrow F_g$, cf. fig. 5(a) and (c), capsules and rings continue to migrate to the center ($v_m < 0$), but now at a higher CSM velocity as a result of the repulsive wall-HI and distinct in proximity of the wall.

In turn, for $u_0 \uparrow F_g$ in fig. 5(b) and (d) the migration to the wall of both particles gets slowed down by wall-HI. Moreover, the repulsive lift forces may become strong enough that the CSM halts ($v_m = 0$) at a certain equilibrium distance to the wall, at $x_{eq} < d$, that decreases with decreasing $F_g$. Figure 6(a) shows the capsule trajectory in an unbounded and bounded Poiseuille flow, in the latter case the capsule approaches its stationary position $x_{eq} = 0.85$ $d$. However, both trajectories are rather close as long as the capsule is sufficiently far away from the wall until wall-HI becomes strong enough. Interestingly, the halt position $x_{eq}$ of the CSM can be controlled by the flow strength $u_0$, as demonstrated in fig. 6(b) for different values of $F_g$ with $x_{eq}$ being located further away from the wall the larger $u_0$. For sufficiently small $F_g$, the ordinary CSM mechanism due to a spatial shear gradient may become dominant, in which case the capsule migrates towards the center $x_{eq} = 0$ when $u_0 > 0.7$.

Comparison of different approaches. – For the $F_g$-driven CSM of soft particles in (un)bounded Poiseuille flow we relied up to now on assumptions: For the analytics leading to eq. (15), we used a body force. For our Stokesian dynamics the wall-HI is included via a modified mobility matrix in eqs. (5)–(8) [31] and the external forces $F_g$ act onto the beads on the capsule’s surface.

We now check the validity of these approximations. Therefore we compare in fig. 7 the $v_m$-Ca characteristics of a soft heavy capsule, obtained by both approaches, with results from the Lattice Boltzmann method (LBM) for a sufficiently small Reynolds number Re = 0.8. The LBM solves the Navier-Stokes equation and therefore natively captures wall-HI [35]. We model $F_g$ within LBM either as a surface (dashed line) or a force on fluid inside the capsule (solid line). This leads to a quantitatively slightly different capsule dynamics, but gives qualitatively a similar migration behavior, as shown in fig. 7.

All approaches in fig. 7 show a good quantitative agreement for small capillary numbers up to Ca ≲ 0.1 with $v_m \propto$ Ca consistent with eq. (15). Beyond linear response the analytical $v_m$ deviates from the simulations as expected. The $v_m$ obtained via Stokesian dynamics and LBM display the same qualitative behavior over the entire range of Ca and, importantly, reproduce the stop of the CSM ($v_m = 0$) due to wall-HI at a similar wall distance $x_{eq} \approx 0.85$ $d$.

Conclusions. – We identified and explained a novel cross-streamline migration (CSM) for soft microparticles that is driven by the vertical gravitational force $F_g$ acting on particles heavier ($F_g \uparrow$) or lighter ($F_g \downarrow$) than the fluid in vertical Poiseuille (or linear shear) flows at low Reynolds number. It complements the CSM driven by the spatially varying shear gradient across a soft particle in Poiseuille flow [12–14]. We analyzed this CSM for soft capsules and ring polymers.

The shear gradient in Poiseuille or linear shear flow leads to elliptically shaped soft particles and to their oblique orientation to $F_g$ and the streamlines. $F_g$ causes via the anisotropic drag of the particle its CSM velocity $v_m$ and the sign of $v_m$ depends on the shear-gradient direction. The $F_g$-driven CSM is expected to apply for non-neutrally buoyant soft microparticles in general. Heavy particles in
downward Poiseuille flow ($F_g \downarrow \downarrow u_0$) migrate to the flow center and sufficiently light particles ($F_g \uparrow \uparrow u_0$) away from it up a stationary point apart from the wall. Reversing the downward Poiseuille flow into an upward one reverses the migration directions too. This allows a flow-controlled particle positioning in the range $0 < x_c < d$. For small capsule deformations $\delta_m$ is proportional to the product of $F_g$ and the local capillary number $Ca$, $\delta_m \propto F_g Ca$. This trend is confirmed by numerical results for finite Ca.

Since the $F_g$-driven contribution to CSM is considerably larger than the conventional CSM [12–14], it opens efficient strategies for manipulation and sorting methods of non-neutrally buoyant soft particles in fluid flows, complementing those for neutrally buoyant particles of different dynamical properties [36]. For instance, many biological cells have about a 5%–15% higher mass density than water [34]. Healthy or malignant cells often have the same mass density but a different stiffness (different Ca) [37]. Therefore, they can be separated by the $F_g$-driven CSM. Furthermore, soft particles can be focused at different Ca- (stiffness-) dependent positions $x_{eq}$ between the flow center and the channel wall. This CSM mechanism is expected to work also at finite values of the Reynolds number (see also ref. [22]).

Having a tool of positioning soft particles according to their weight or stiffness along the entire channel cross-section via the vertical flow enables studies e.g. on the dynamics of non-neutrally buoyant soft particles under conditions of high or low shear rates via micro-focused synchrotron XSAS or optical microscopy [38].

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