Dimension-Six Proton Decays
in the Modified Missing Doublet $SU(5)$ Model

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Dimension-five operators for nucleon decays are suppressed in the modified missing doublet (MMD) model in the supersymmetric $SU(5)$ grand unification. We show that nonrenormalizable interactions decrease the unification scale in the MMD model, causing a significant increase in the nucleon decay rate of dimension-six operators. We find that the theoretical lower bound on the proton lifetime $\tau(p \to e^+\pi^0)$ is within the observable range at Super-Kamiokande.

The doublet-triplet splitting is one of the most serious problems in the supersymmetric (SUSY) grand unified theory (GUT). Various interesting resolutions to this difficult problem have been proposed. Among these, the modified missing doublet (MMD) $SU(5)$ model is the most attractive, since it explains very naturally the small value of the $SU(3)_C$ gauge coupling constant $\alpha_3(m_Z) = 0.118 \pm 0.004$ by GUT-scale threshold corrections. Furthermore, the dangerous dimension-five operators for nucleon decays are strongly suppressed owing to an additional chiral symmetry in the MMD model.

If the nucleon decay due to the dimension-five operators is outside the reach of Super-Kamiokande, the dimension-six nucleon decays such as $p \to e^+\pi^0$ are clearly important to test the GUT. In this paper, we point out that introduction of nonrenormalizable operators may decrease the GUT scale by a factor of about 3 for a reasonable parameter region in the MMD $SU(5)$ model. As a consequence, nucleon decays due to GUT gauge-boson exchanges become $\sim 100$ times faster than the previous estimate. The present analysis, therefore, suggests that the theoretical lower bound for proton decay of the dominant channel $p \to e^+\pi^0$ is within the observable range at the Super-Kamiokande detector.

The original missing doublet (MD) model in the SUSY $SU(5)$ GUT consists of the following chiral supermultiplets:

$$\psi_i(10), \phi_i(5^*),$$
$$H(5), \bar{H}(5^*), \theta(50), \bar{\theta}(50^*), \Sigma(75),$$

where $i(=1-3)$ represents the family index. This original MD model is incomplete, since the $SU(5)$ gauge coupling constant $\alpha_5(\equiv g_5^2/4\pi)$ blows up below the gravitational scale $M_G \simeq 2.4 \times 10^{18}$ GeV, because of the large representations for $\theta$, $\bar{\theta}$ and $\Sigma$. Unfortunately, the MMD $SU(5)$ model cannot be excluded even if nucleon decays are not observed by the Super-Kamiokande experiments.
One may avoid this unwanted situation by taking a mass for $\theta$ and $\bar{\theta}$ at the gravitational scale. However, in this case the colored Higgs mass $M_{Hc}$ becomes smaller than $\sim 10^{15}$ GeV. This leads to too rapid nucleon decays through dimension-five operators.\textsuperscript{10)

The MMD $SU(5)$ model\textsuperscript{2}) solves the above problem by imposing an additional chiral (Peccei-Quinn) symmetry. To incorporate the Peccei-Quinn symmetry in the original MD model, we introduce a new set of chiral supermultiplets, 

\[ H'(5), \ H'(5^*), \ \theta'(50), \ \bar{\theta}'(50^*). \]

We then assume the $U(1)_{PQ}$ charges for each multiplet as

\[
\begin{align*}
\psi_1(10) &\rightarrow e^{i\alpha/2}\psi_1(10), & \phi_1(5^*) &\rightarrow e^{i\beta/2}\phi_1(5^*), \\
H(5) &\rightarrow e^{-i\alpha}H(5), & \bar{H}(5^*) &\rightarrow e^{-i\alpha+\beta/2}\bar{H}(5^*), \\
H'(5) &\rightarrow e^{i\alpha+\beta/2}H'(5), & \bar{H}'(5^*) &\rightarrow e^{i\alpha}\bar{H}'(5^*), \\
\theta(50) &\rightarrow e^{i\alpha}\theta(50), & \bar{\theta}(50^*) &\rightarrow e^{i\alpha+\beta}\bar{\theta}(50^*), \\
\theta'(50) &\rightarrow e^{-i\alpha+\beta}\theta'(50), & \bar{\theta}'(50^*) &\rightarrow e^{-i\alpha}\bar{\theta}'(50^*), \\
\Sigma(75) &\rightarrow \Sigma(75),
\end{align*}
\]

with $3\alpha + \beta \neq 0$.

We take the masses of $\theta, \bar{\theta}$ and $\theta', \bar{\theta}'$ at the gravitational scale $M_G$ to maintain the perturbative description of the SUSY GUT. Then, we have only two pairs of Higgs multiplets, $H, \bar{H}$ and $H', \bar{H}$, and one Higgs $\Sigma$ in addition to three families of quark and lepton multiplets below the scale $M_G$.

The 75-dimensional Higgs $\Sigma$ has the following vacuum expectation value causing the desired $SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$ breaking:

\[
\begin{align*}
\langle \Sigma \rangle^{(\alpha\beta)}_{(\gamma\delta)} &= \frac{1}{2} \left\{ \delta^\alpha_\gamma \delta^\beta_\delta - \delta^\alpha_\delta \delta^\beta_\gamma \right\} V_\Sigma, \\
\langle \Sigma \rangle^{(ab)}_{(cd)} &= \frac{3}{2} \left\{ \delta^a_c \delta^b_d - \delta^a_d \delta^b_c \right\} V_\Sigma, \\
\langle \Sigma \rangle^{(aa)}_{(bb)} &= -\frac{1}{2} \left\{ \delta^a_b \delta^a_b \right\} V_\Sigma,
\end{align*}
\]

where $\alpha, \beta, \cdots$ are the $SU(3)_C$ indices and $a, b, \cdots$ the $SU(2)_L$ indices. Integration of the heavy fields, $\theta, \bar{\theta}'$ and $\theta', \bar{\theta}'$, generates Peccei-Quinn invariant masses of the colored Higgs multiplets as

\[ M_{H_c}H_c^\alpha H_c^{\alpha} + M_{H_c}H_c^{\alpha}\bar{H}_c^{\alpha}. \]

with

\[ M_{H_c} \simeq 48G_H G_{H'} V_{\Sigma}^2, \quad M_{H_c} \simeq 48G_{H} G_{H'} V_{\Sigma}^2 \]

Here, the coupling constants $G_H, \ G_{H'}, \ G_{H}'$ and $G_{H}$ are expected to be $O(1)$ (see Ref. 2) for their definitions). Note that dimension-five operators for nucleon decays are completely suppressed as long as the Peccei-Quinn symmetry is unbroken.
The two pairs of Higgs doublets, $H_f, \bar{H}_f$ and $H'_f, \bar{H}'_f$, on the other hand, remain massless. They acquire masses through the Peccei-Quinn symmetry breaking. As shown in Ref. 2) we choose $U(1)_{PQ}$ charges for the Peccei-Quinn symmetry breaking fields so that only one pair of Higgs doublets, $H'_f$ and $\bar{H}'_f$, has a mass $M_{H'_f}$ of the order of the Peccei-Quinn scale $\sim (10^{10} - 10^{12})$ GeV. Thus, the model is nothing but the SUSY standard model below the Peccei-Quinn scale. The Peccei-Quinn symmetry breaking generates dimension-five operators which are, however, proportional to $M_{H'_f}/(M_{H_c}M_{\bar{H}_c})$.

We are now ready to discuss the renormalization group (RG) equations for the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge coupling constants $\alpha_3, \alpha_2$ and $\alpha_1$ in the standard model. It has been pointed out in Ref. 11) that the GUT scale and the colored Higgs mass are determined independently by using the gauge coupling constants at the electroweak scale. For the case of the MMD model we have $^2)$ at the one-loop level

$$
(3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1})(m_Z) = \frac{1}{2\pi} \left\{ \frac{12}{5} \ln \frac{M_{H'_c}^{\text{eff}}}{m_Z} - 2 \ln \frac{m_{\text{SUSY}}}{m_Z} - \frac{12}{5} \ln(1.7 \times 10^4) \right\},
$$

(5)

and

$$
(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(m_Z) = \frac{1}{2\pi} \left\{ 12 \ln \frac{M_V^2 M_\Sigma}{m_Z^3} + 8 \ln \frac{m_{\text{SUSY}}}{m_Z} + 36 \ln(1.4) \right\}.
$$

(6)

Here, the effective mass for the colored Higgs satisfies $M_{H'_c}^{\text{eff}} = M_{H_c}M_{\bar{H}_c}/M_{H'_f}$. Note that the last values in Eqs. (5) and (6) come from the mass splitting among the components in $\Sigma(75)$. $M_V$ is the mass of the GUT gauge bosons ($M_V = 2\sqrt{6}g_5 V_\Sigma$) $^*$) and $M_\Sigma$ is the mass of the heaviest component of $\Sigma(75)$ ($M_\Sigma = (10/3)\lambda_{75} V_\Sigma$). The definition of the Yukawa coupling constant $\lambda_{75}$ is given in Ref. 2).

To perform a quantitative analysis we use two-loop RG equations between the electroweak and the GUT scale. For GUT scale threshold corrections we use the one-loop result. We use a mass spectrum derived from the minimum supergravity instead of the common mass $m_{\text{SUSY}}$ for superparticles to calculate one-loop threshold corrections at the SUSY-breaking scale. Using the experimental data $\alpha_{em}^{-1}(m_Z) = 127.90 \pm 0.09, \sin^2 \theta_W(m_Z) = 0.2314 \pm 0.0004$ $^{12}$) and $\alpha_3(m_Z) = 0.118 \pm 0.004, ^7$) we obtain the GUT scale as

$$
9.5 \times 10^{15} \text{ GeV} \leq (M_V^2 M_\Sigma)^{1/3} \leq 2.3 \times 10^{16} \text{ GeV}.
$$

(7)

The dimension-six operators for nucleon decays depend on $M_V$, and hence Eq. (7) is not sufficient to calculate the decay rate. Fortunately, the lower bound on $M_V$ is given by the following consideration. The upper bound $M_\Sigma$ is determined by requiring that the Yukawa coupling $\lambda_{75}$ never blows up below the gravitational

*$^1$ We find a mistake in the form of $M_V$ in Ref. 2).
scale. By solving the one-loop RG equation for $\lambda_{75}$, we find\(^*\)
\[
M_{\Sigma} \leq 1.3M_V,  \tag{8}
\]
which, together with Eq. (7), leads to
\[
M_V \geq 8.7 \times 10^{15} \text{ GeV}.  \tag{9}
\]
This gives the lower bound for the proton lifetime as
\[
\tau(p \rightarrow e^+\pi^0) = 2.9 \times 10^{35} \times \left( \frac{M_V}{10^{16} \text{ GeV}} \right)^4 \left( \frac{0.0058 \text{ GeV}^3}{\alpha} \right)^2 \text{ years} \geq 1.6 \times 10^{35} \text{ years}.  \tag{10}
\]
Here, $\alpha$ is the hadron matrix element,\(^**\) and we have used the lattice value $\alpha = 0.0058 \text{ GeV}^3$.\(^{13}\)

One may consider that threshold corrections from some additional particles at the GUT scale could decrease the GUT scale. However, if one introduces additional multiplets at the GUT scale, the $SU(5)$ gauge coupling constant tends to blow up below the gravitational scale. Simple choices may be pairs of $5 + 5^*$ and/or pairs of $10 + 10^*$. We easily see from Eq. (6) that pairs of $5 + 5^*$ do not change the GUT scale $(M_V^2 M_{\Sigma})^{1/3}$. As for $10 + 10^*$, we have found that no significant change of the GUT scale is obtained by their threshold corrections for a natural range of mass parameters.

We are now led to consider nonrenormalizable interactions suppressed by $1/M_G$. The most dominant operator contributing to the gauge coupling constants $\alpha_3$, $\alpha_2$ and $\alpha_1$ is
\[
\int d^2 \theta \frac{\delta}{M_G} W_B^A \cdot W_D^C \Sigma^{(BD)}_{(AC)} + \text{h.c.},  \tag{11}
\]
where $A, B, \cdots$ are $SU(5)$ indices. Here, $W$ is the $SU(5)$ fieldstrength superfields whose kinetic term is given by
\[
\int d^2 \theta \frac{1}{8g_5^2} W_B^A \cdot W_A^B + \text{h.c.}  \tag{12}
\]
The above nonrenormalizable term yields corrections to $\alpha_i$ ($i = 1 - 3$) at the GUT scale as
\[
\alpha_3^{-1} = \alpha_5^{-1} - 4 \left( 4\pi \delta \frac{V_\Sigma}{M_G} \right),  \\
\alpha_2^{-1} = \alpha_5^{-1} - 12 \left( 4\pi \delta \frac{V_\Sigma}{M_G} \right),  \\
\alpha_1^{-1} = \alpha_5^{-1} + 20 \left( 4\pi \delta \frac{V_\Sigma}{M_G} \right).  \tag{13}
\]
\(^*\) When $\lambda_{75}$ is much larger than $g_5$ at the gravitational scale, it rapidly converges to the quasi-infrared fixed point at the GUT scale, and on the fixed point $M_{\Sigma}/M_V = 1.28$.

\(^**\) See Ref. 10) for the definition of $\alpha$. 

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\(10\) for the definition of $\alpha$. 

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Fig. 1. The theoretical lower bounds on the proton lifetime $\tau(p \to e^+\pi^0)$ as a function of the parameter $(\delta V_\Sigma/M_G) \times 10^2$. Here, we have assumed a lattice value of the hadron matrix element $\alpha = 0.0058$ GeV$^3$. The shaded area is excluded by the recent Super-Kamiokande experiments ($\tau(p \to e^+\pi^0) > 7.9 \times 10^{32}$ years).

Then, Eqs. (5) and (6) are modified as

$$
(3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1})(m_Z) = \frac{1}{2\pi} \left\{ \frac{12}{5} \ln \frac{M_{H_c}^{\text{eff}}}{m_Z} - 2 \ln \frac{m_{\text{SUSY}}}{m_Z} - 48 (8\pi^2 \frac{V_\Sigma}{M_G}) - \frac{12}{5} \ln(1.7 \times 10^4) \right\},
$$

(14)

$$
(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(m_Z) = \frac{1}{2\pi} \left\{ 12 \ln \frac{M_{\Sigma}^2 M_{\Sigma}}{m_Z^3} + 8 \ln \frac{m_{\text{SUSY}}}{m_Z} + 144 (8\pi^2 \frac{V_\Sigma}{M_G}) + 36 \ln(1.4) \right\}.
$$

(15)

We see that the GUT scale $(M_{\Sigma}^2 M_{\Sigma})^{1/3}$ decreases easily by a factor $\sim 3$ for $\delta V_\Sigma/M_G = 0.0035$. As a consequence, the effective mass for the colored Higgs multiplets $M_{H_c}^{\text{eff}}$ increases by almost 240 times compared with the values for $\delta = 0$. This is rather desirable to suppress the dimension-five operators for nucleon decays since they are proportional to $1/M_{H_c}^{\text{eff}}$. Note that the presence of the nonrenormalizable operator

* Introduction of a similar nonrenormalizable operator to Eq. (11) suppressed by $1/M_G$ does not change the GUT scale in the minimal SUSY $SU(5)$ GUT.
in Eq. (11) lowers further the value of $\alpha_3(m_Z)$ for $\delta > 0$. This over-reduction of $\alpha_3(m_Z)$ is, however, compensated by increasing $M_{\text{He}}^{\text{eff}}$.

In conclusion, we have shown in this paper that introduction of the nonrenormalizable interaction may decrease the GUT scale by a factor $\sim 3$ in the MMD $SU(5)$ model. In Fig. 1, we show the theoretical lower bounds on the proton lifetime $\tau(p \rightarrow e^+\pi^0)$. We see that they lie within the observable range at Super-Kamiokande for a reasonable parameter region $\delta \sim O(1)$. A recent report from Super-Kamiokande shows that one event of the mode $e^+\pi^0$ still survives the cut criterion for backgrounds. If this is indeed a real signal for $p \rightarrow e^+\pi^0$ decay, we may be led to a serious consideration of physics at the gravitational scale.

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