Fermi-liquid approach for description of initial stage of fragmentation at heavy nuclei collisions

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Abstract

A mechanism is proposed for initial stage of instability development that can induce the fragmentation of nuclear matter, arising as a result of collisions of non-relativistic heavy nuclei. Collision of heavy nuclei is simulated as a collision of two unbounded Fermi-liquid “drops”.

The instability origination in such a system is related to propagation of increasing oscillations in the nuclear matter. These oscillations can exist in a resting Fermi-liquid: modified Landau zero sound, modified spin and isospin waves, combination of these more simple waves. These instabilities are analogous to the beam instability in ordinary electron plasma. Behavior features of the obtained oscillation increase increments are provided. They can be used as indication for experimental confirmation of the proposed mechanism of fragmentation at nuclear collisions. Directions along which nuclear matter “jets” can be expected are specified.

1 Introduction

The fragmentation phenomenon, i. e. the simultaneous decay of the excited nucleus into lighter nuclei (fragments) and separate particles, has been known for a rather long time. However the scientific interest to this phenomenon have been rapidly growing lately due to the experiments on collisions of heavy fast nuclei, which were carried out in different scientific centers of the world (CERN, Switzerland; Dubna, Russia; Berkley, Oak Ridge, USA; Hamburg, Germany; Orsay, Caen, France). This interest is caused by the prospect to confirm our notions about inner structure of nuclei and the character of inter-nucleon interactions and also by the possibility to obtain new fundamental knowledge about the origin of intranuclear and intranucleon forces.

Nuclear matter formed by the collision of heavy nuclei is an unstable object decaying within a very short time. The decay of arisen nuclear matter is accompanied by the forming

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of new lighter nuclei and nuclear fragments (so-called nuclear fragmentation process). Nowadays while describing the evolution of nuclear matter, formed after heavy nuclei collision, one usually considers two scenarios (see for example [1, 2]). According to the first one the state of statistical equilibrium can be reached in the nuclear matter (the thermalization of the nuclear matter) permitting the thermodynamic description of the fragmentation process by means of the phase transition theory methods. According to the second scenario the resulting nuclear matter is essentially unstable matter, in which the processes allow only the dynamic description.

In the present paper we assume the latter scenario. Each colliding nucleus is a system of nucleons and it can be considered as a generalized Fermi-liquid, whose particles have spin and isospin (distinguishing protons and neutrons) degrees of freedom. Let us note that the Fermi-liquid approach of Landau and Silin [3, 4] is widely applied in theoretical nuclear physics in order to describe the properties of heavy nuclei [5, 6]. In solving the specific problems of theoretical nuclear physics it is often assumed that heavy nuclei (with \( A > 100 \)) can be considered as the infinite drops of nucleonic Fermi liquid in cases when surface effects are unimportant. Then such assumption enables us to simplify complicated mathematical treatment without violation of the final results. Note, that theoretical approach for finite Fermi liquids was elaborated by A.B.Migdal (see [5] and references therein).

In the present paper we also suppose the nuclear matter, formed due to heavy nuclei collisions, to be the Fermi-liquid consisting of particles with spin and isospin degrees of freedom. The inter-particle interaction in this kind of nuclear liquid is described by such parameters as Landau amplitudes, by analogy with the normal Fermi-liquid theory [3, 4]. So in order to consider the fragmentation formed after collision we come to the less complicated problem of interaction of two drops of nucleon infinite Fermi-liquid moving relatively to each other. Note that description of collisions of heavy ions as two drops of infinite quark Fermi-liquids leading to formation of quark-gluon plasma was analyzed in paper [7].

We have demonstrated that interactions of two nucleon Fermi-liquids in the system can originate a number of instabilities associated with increased oscillations of modes existing in the ordinary Fermi-liquid. Such waves include a modified Landau zero sound, modified spin waves, waves related to excitation of isospin degrees of freedom, as well as more complex waves related to a combination of the mentioned simple waves [6]. The present instabilities are analogous to the known ”beam” instability in the ordinary electron plasma, occurring when charged particle beams pass through the plasma [8, 9]. Since the mentioned instabilities can evolve exactly after the mutual penetration of colliding Fermi-liquid drops and formation of united Fermi-liquid, we consider the formed system as an object which is far from the statistical equilibrium and requiring the dynamical description. The dynamics of such system in this work is described by the kinetic equation for the one-particle distribution function for nucleons in collisionless approximation, which is widely used in the theory of ordinary Fermi-liquid [3, 4]. Such approach for phenomena we consider is valid in the case when the characteristic time of instability evolution is less in comparison with the characteristic time of relaxation in our system. In many cases such approach allows the analytic description of the initial stage of instability development in formed nuclear matter without numerical calculations. Note that the use of mentioned kinetic equation restricts our consid-
eration to non-relativistic nucleon energies. The modification of the kinetic equation for the description of evolution of the nuclear matter with relativistic nucleons is given in [10, 11]. In these papers the development of filamentary instabilities in the nuclear matter at the stage of the mutual penetration of the two colliding nuclei was considered on the basis of the kinetic equation for relativistic nucleons.

Use of this approach allows us to describe the initial stage of instability development in nuclear matter, formed after the nuclear collision and to obtain the dispersion equations for zero sound oscillations in such system (analogue of zero sound oscillations in ordinary Fermi-liquid). The solutions of equations for small Landau amplitudes are investigated in details in case of collision of nuclei in normal state as well as in case of collisions of heavy excited nuclei. The expressions for the increments of zero-sound oscillations in nuclear matter are found. We have shown that those increments have characteristic logarithmic dependence on the excitation energy of nucleus when the nuclear matter is formed due to the collision of heavy excited nuclei. This characteristic dependence can be used for experimental proof or disproof of the mechanism of initiation of instabilities in colliding nuclei system.

We suppose the confirmation of predictions given in present paper should be expected for example in experiments on nuclear collisions Gd+U or Xe+Sn (INDRA, [2, 12]) with incoming nucleus energy above 145 MeV per nucleon.

2 Basic kinetic equations of two colliding Fermi-liquids

A solution of the formulated problem on kinetics of two nucleon Fermi-liquids collisions is based on the assumption that the Fermi-liquid energy is a functional $E(f)$ of one-particle distribution function $f_i(x, p, t)$ of quasiparticles (nucleons; here index “$i$” is used to denote a quasiparticle spin and isospin, $x$ is a coordinate and $p$ is a quasiparticle momentum, $t$ is a time). We want to emphasize that in the Fermi-liquid theory the introduced energy functional plays the role analogous to Hamiltonian in the microscopic theory.

Evolution of one-particle distribution function $f_i(x, p, t)$ in the collisionless approximation ($\omega \tau_r \gg 1$, $\omega$ is a frequency, $\tau_r$ is a relaxation time) is governed by the kinetic equation

$$\frac{\partial f_i}{\partial t} + \frac{\partial \varepsilon_i}{\partial p} \frac{\partial f_i}{\partial x} - \frac{\partial \varepsilon_i}{\partial x} \frac{\partial f_i}{\partial p} = 0, \quad (1)$$

where quasiparticle energy $\varepsilon_i(x, p, f)$, determining kinetics of the nucleon system, repre-
sents a variational derivative of the energy functional over the one-particle distribution function
\[ \varepsilon_i(x, p, f) = \frac{\delta E(f)}{\delta f_i(x, p)}. \]

A specific form of quasiparticle energy \( \varepsilon_i(x, p, f) \) as a functional of the distribution function in the Fermi-liquid theory is not known. Therefore, let us introduce functions of quasiparticle interaction \( F_{ij} \) (generalized Landau amplitudes), which represent a linear reaction of the quasiparticle energy on small deviation \( \delta f_i(x, p, t) \) of distribution function \( f_i(x, p, t) \):
\[ \delta \varepsilon_i(x, p, t) = \sum_j \int d\tau' \int dx' F_{ij}(x - x', p, p') \times \]
\[ \times \delta f_j(x', p', t), \quad (2) \]

where \( d\tau = d^3p / (2\pi \hbar)^3 \). Functions \( F_{ij} \), representing second order variational derivatives of the energy functional with respect to the one-particle distribution function, are main characteristics of the theory that can be experimentally determined (in this connection see, for example \[13, 14\]). In view of short-range interaction forces between nucleons, from here on we will consider that the functions \( F_{ij} \) have the following form:
\[ F_{ij}(x - x', p, p') = F_{ij}(p, p') \delta(x - x'), \quad (3) \]

where quantities \( F_{ij}(p, p') \) (we will call them also Landau amplitudes) do not depend on coordinates. Note that assumption \[3\] is valid only if we neglect the Coulomb forces induced by the proton charge. Linearizing equation \[1\] near a stationary quasi-equilibrium state (see below), described by distribution function \( f_{0i} \), we can go over to Fourier components of deviations \( \delta f_i = f_i - f_0 \) of the one-particle distribution functions from their equilibrium values
\[ \tilde{f}_i(p, \omega, k) = \int d^3x \int dt \delta f_i(x, p, t) \exp(i\omega t - ikx) \]
\[ = \int d^3x \int dt \tilde{f}_i(x, p, t) \exp(i\omega t - ikx) \]
\[ \tilde{f}_i(p, \omega, k) = \int d^3x \int dt \tilde{f}_i(x, p, t) \exp(i\omega t - ikx). \quad (4) \]

As a result, using \[2\] we get the following kinetic equation for \( \delta \tilde{f}_i(p, \omega, k) \):
\[ \left( \omega - \frac{k}{\hbar} \frac{\partial \varepsilon_{0i}}{\partial p} \right) \tilde{f}_i(p, \omega, k) + \]
\[ + k \frac{\partial f_{0i}}{\partial p} \int d\tau' \sum_j F_{ij}(p, p') \tilde{f}_j(p', \omega, k) = 0, \quad (5) \]

where amplitudes \( F_{ij}(p, p') \) in accordance with \[2, 3\] are given by the formula
\[ F_{ij}(p, p') = \frac{\delta^2 E(f)}{\delta f_i(p) \delta f_j(p')} \bigg|_{f_i = f_{0i}} \]

and \( \varepsilon_{0i} \) is the energy of noninteracting nucleons,
\[ \varepsilon_0 = \frac{p^2}{2m} \quad (m \text{ is the nucleon mass}). \]

If we assume that interaction between nucleons is invariant with respect to the spin and isospin transformations, then a structure of Landau amplitudes \( F \) for the nucleon liquid will be of the following form:
\[ F = I^{(s)} I^{(i)} F^{(0)} + I^{(i)} (\sigma \sigma) F^{(s)} + \]
\[ + I^{(s)} (\tau \tau) F^{(i)} + (\sigma \sigma) (\tau \tau) F^{(si)} \quad (6) \]

where \( F^{(0)}, F^{(s)}, F^{(i)}, F^{(si)} \) are generalized Landau amplitudes, \( I^{(s)}, \sigma \) are unit matrix and Pauli matrices in the spin space, \( I^{(i)}, \tau \) are a
unit matrix and Pauli matrices in the isospin space. Accordingly in equations (3) there will be one of the Landau amplitudes from expression (6), depending on which oscillations are considered (oscillations of density, spin density, isospin density or spin-isospin density). Thereby, hereinafter we will deal with only one linearized kinetic equation

\[ \left( \omega - k \frac{\partial \varepsilon_0}{\partial \mathbf{p}} \right) \tilde{f}(\mathbf{p}, \omega, \mathbf{k}) + k \frac{\partial f_0}{\partial \mathbf{p}} \int d\tau' F\left( \mathbf{p}, \mathbf{p}' \right) \delta f(\mathbf{p}, \omega, \mathbf{k}) = 0, \]

(7)

where we will bear in mind \( F(\mathbf{p}, \mathbf{p}') \) as one of the amplitudes from (3).

Now let us obtain the expression for a quasi-equilibrium distribution function from kinetic equation (7). The equilibrium distribution functions of the resting drops of Fermi-liquid will be denoted by \( f_0(1) (\mathbf{p}) \) and \( f_0(2) (\mathbf{p}) \). Then in the case when the first drop is resting and the second one (impacting) is moving with velocity \( \mathbf{u} \), equilibrium distribution functions will be \( f_0(1) (\mathbf{p}) \) and \( f_0(2) (\mathbf{p} - m\mathbf{u}) \), respectively. This implies that the quasi-equilibrium distribution function of two colliding unbounded Fermi-liquids consisting of identical particles has the form

\[ f_0(\mathbf{p}) = \alpha_1 f_0(1) (\mathbf{p}) + \alpha_2 f_0(2) (\mathbf{p} - m\mathbf{u}). \]

(8)

Obviously that this function obeys the kinetic equation (7).

We want to explain a meaning of coefficients \( \alpha_1 \) and \( \alpha_2 \) appearing in this expression. With this aim we will introduce auxiliary distribution functions \( g_0(\mathbf{p}), g_0(1) (\mathbf{p}), g_0(2) (\mathbf{p}) \) normalized to unity (and because of that permitting a probability interpretation) and related to each other analogously to relation (5):

\[ g_0(\mathbf{p}) = q_1 g_0(1) (\mathbf{p}) + q_2 g_0(2) (\mathbf{p} - m\mathbf{u}). \]

(9)

From the normalization condition of these functions to unity it follows that values \( q_1 \) and \( q_2 \) satisfy the equality

\[ q_1 + q_2 = 1. \]

(10)

From here we see that coefficients \( q_1 \) and \( q_2 \) must be interpreted as probabilities that an arbitrary selected particle belongs to either a system with distribution function \( g_0(1) (\mathbf{p}) \), or to a system with the distribution function \( g_0(2) (\mathbf{p}) \). However, in this case probabilities \( q_1 \) and \( q_2 \) must be specified by relative frequencies of the events of finding objects with distribution functions \( g_0(1) (\mathbf{p}) \) and \( g_0(2) (\mathbf{p}) \). If we are dealing with two colliding beams of nuclei, then for probabilities \( q_1 \) and \( q_2 \), we can obtain the following expressions (see (10))

\[ q_1 = N_1 / (N_1 + N_2), \quad q_2 = N_2 / (N_1 + N_2), \]

(11)

where \( N_1 \) and \( N_2 \) are densities of nuclei in the first and second beams respectively.

As it was previously noted the above given argumentation implies the normalization of the distribution functions to unity. In the case under consideration this requirement is not fulfilled for functions \( f_0(\mathbf{p}), f_0(1) (\mathbf{p}), f_0(2) (\mathbf{p}) \) because

\[ \int d\tau f_0(\mathbf{p}) = n, \quad \int d\tau f_0(1) (\mathbf{p}) = n_1, \quad \int d\tau f_0(2) (\mathbf{p}) = n_2, \]

(12)

where \( n \) is density of number of particles in the system consisting of two colliding Fermi-liquids; \( n_1 \) and \( n_2 \) are densities of number of particles...
in each of these Fermi-liquids. But assuming in (9) that
\[ g_0(p) = \frac{f_0(p)}{n}, \quad g_0^{(1)}(p) = \frac{f_0^{(1)}(p)}{n_1}, \]
\[ g_0^{(2)}(p) = \frac{f_0^{(2)}(p)}{n_2}, \]
and introducing denotations
\[ \alpha_1 = nq_1/n_1, \quad \alpha_2 = nq_2/n_2, \] (13)
we can arrive at the expression (8).

3 Zero-sound dispersion equation in a simple model

In order to solve kinetic equation (7) we will use a simple model, where the Landau amplitudes do not depend on momenta \( p, p' \). We also assume that the colliding Fermi-liquids have zero temperature, so that the equilibrium distribution functions of separate drops are given by the formulas
\[ f_0^{(1)}(p) \equiv \theta (\varepsilon_{1F} - \varepsilon), \quad f_0^{(2)}(p) \equiv \theta (\varepsilon_{2F} - \varepsilon), \]
\[ \varepsilon = \frac{p^2}{2m} = \varepsilon_0(p), \] (14)
where \( \theta(\varepsilon) \) is theta function, \( \varepsilon_{1F}, \varepsilon_{2F} \) are Fermi energies of the colliding drops. It is evident from (14) that in this case a difference between nucleus types can be found only in their different Fermi energies (note that for heavy nuclei \( \varepsilon_{1F} \approx \varepsilon_{2F} \)).

Within used assumptions linearized kinetic equation (7) can be written as:
\[ \tilde{\delta f}(p, \omega, k)(\omega - kv) + \]
\[ + k \frac{\partial f_0(p)}{\partial p} F \int \! d\tau \tilde{\delta f}(p, \omega, k) = 0, \] (15)
where \( v = \frac{\partial \varepsilon}{\partial p} = \frac{p}{m} \), and in accordance with (8), (14)
\[ \frac{\partial f_0}{\partial p} = -\alpha_1 v \delta (\varepsilon_{1F} - \varepsilon) - \alpha_2 (v - u) \times \]
\[ \times \delta \left( \varepsilon_{2F} - \frac{m(v - u)^2}{2} \right). \] (16)

The most general solution of equation (15) looks as
\[ \tilde{\delta f}(p, k, \omega) = \]
\[ = -F \{\omega - kv + io\}^{-1} k \frac{\partial f_0(p)}{\partial p} \delta \varphi(\omega, k) + \]
\[ + \delta A(p, k) \delta (\omega - vk), \] (17)
where we denoted
\[ \delta \varphi(\omega, k) \equiv \int \! d\tau \tilde{\delta f}(p, k, \omega), \] (18)
and \( \delta A(p, k) \) are arbitrary functions satisfying the condition (see (11))
\[ \delta A^*(p, k) = \delta A(p, -k). \] (19)

Formula (17) permits to find a value of \( \delta \varphi(\omega, k) \) in terms of the functions \( \delta A(p, k) \):
\[ \delta \varphi(\omega, k) = \varepsilon^{-1}(\omega, k) \tilde{\delta A}(\omega, k), \] (20)
where (see (19))
\[ \tilde{\delta A}(\omega, k) = \int \! d\tau \delta A(p, k) \delta (\omega - kv), \] (21)
\[
\delta A^* (\omega, \mathbf{k}) = \delta A (-\omega, -\mathbf{k}).
\]

Substituting further (20) into (17), we will have the following relation for \( \tilde{\delta} f (\mathbf{p}, \mathbf{k}, \omega) \):

\[
\tilde{\delta} f (\mathbf{p}, \mathbf{k}, \omega) = \delta A (\mathbf{p}, \mathbf{k}) \delta (\omega - \mathbf{k} \mathbf{v}) - F \{ \omega - \mathbf{k} \mathbf{v} + io \}^{-1} \times \tilde{\varepsilon}^{-1} (\omega, \mathbf{k}) \delta A (\omega, \mathbf{k}) \frac{\partial f_0 (\mathbf{p})}{\partial \mathbf{p}},
\]

where \( \tilde{\varepsilon} (\omega, \mathbf{k}) \) in (20), (22) is defined by the formula

\[
\tilde{\varepsilon} (\omega, \mathbf{k}) = \tilde{\varepsilon}^* (-\omega, -\mathbf{k}) = \tilde{\varepsilon}_1 (\omega, \mathbf{k}) + i \tilde{\varepsilon}_2 (\omega, \mathbf{k}) = 1 + F k \int d \tau \frac{\partial f_0 (\mathbf{p})}{\partial \mathbf{p}} \{ \omega - \mathbf{k} \mathbf{v} + io \}^{-1} \delta A (\omega, \mathbf{k}) \frac{\partial f_0 (\mathbf{p})}{\partial \mathbf{p}}.
\]

(23)

For the case of the charged Fermi-liquid \( \tilde{\varepsilon} \) represents a complex dielectric permittivity (see, for example, [15]). As is known the presence of an imaginary additional component in \( \tilde{\varepsilon} (\omega, \mathbf{k}) \) indicates damping or increase of wave amplitudes. The wave dispersion law \( \omega (\mathbf{k}) = \omega_0 (\mathbf{k}) + i \gamma (\mathbf{k}) \) should be found from the equation

\[
\varepsilon (\omega_0 (\mathbf{k}) + i \gamma (\mathbf{k}) , \mathbf{k}) = 0,
\]

with decrement (increment) \( \gamma (\mathbf{k}) \) determined by the imaginary part of quantity \( \tilde{\varepsilon} (\omega, \mathbf{k}) \). This is a reason why weakly damping or weakly increasing oscillations in the system

\[
|\omega_0 (\mathbf{k})| >> |\gamma (\mathbf{k})|
\]

can exist only on condition that

Using the formula

\[
(z + io)^{-1} = P \frac{1}{z} - i \pi \delta (z)
\]

(24)

(where \( P \) is a symbol of the principal value), and taking into account (16), we will write functions \( \tilde{\varepsilon}_1 (\omega, \mathbf{k}), \tilde{\varepsilon}_2 (\omega, \mathbf{k}) \) (see (23)) in the following form:

\[
\tilde{\varepsilon}_1 (\omega, \mathbf{k}) = 1 + \alpha_1 F \left\{ 1 - \frac{\omega}{2k_1} \ln \left| \frac{\omega + k v_1}{\omega - k v_1} \right| \right\} + \frac{\alpha_2 v_2 F}{v_1} \left\{ 1 - \frac{\omega - k u \cos \alpha}{2k_2} \ln \left| \frac{\omega - k u \cos \alpha + k v_2}{\omega - k u \cos \alpha - k v_2} \right| \right\},
\]

\[
\tilde{\varepsilon}_2 (\omega, \mathbf{k}) = \frac{\pi}{2} F \left\{ \alpha_1 \frac{\omega}{k v_1} \theta \left( 1 - \left| \frac{\omega}{k v_1} \right| \right) + \alpha_2 \frac{v_2}{v_1} \frac{\omega - k u \cos \alpha}{k v_2} \theta \left( 1 - \left| \frac{\omega - k u \cos \alpha}{k v_2} \right| \right) \right\}.
\]

(25)

(27)

Here \( F \) is a dimensionless Landau amplitude

\[
F = \frac{F m v_1^2}{2 \pi^2 \hbar^3},
\]

(28)

velocities \( v_1 F, v_2 F \) are given by relations

\[
\varepsilon_1 F = \frac{m v_1^2}{2}, \quad \varepsilon_2 F = \frac{m v_2^2}{2}
\]

(29)

and angle \( \alpha \) is an angle between directions of wave vector \( \mathbf{k} \) and velocity of impacting drop.
In accordance with (24) - (27) a dispersion equation has the form
\[ \tilde{\varepsilon}_1(\omega_0(k), k) = 0, \]
or
\[ 1 + \alpha_1 \mathcal{F} \left\{ 1 - \frac{s}{2} \ln \frac{s+1}{s-1} \right\} + \frac{1}{\eta} \alpha_2 \mathcal{F} \times \]
\[ \times \left\{ 1 - \frac{1}{2} (\eta s - s_0) \ln \frac{\eta s - s_0 + 1}{\eta s - s_0 - 1} \right\} = 0, \]
(30)
and the decrement (increment) is given by
\[ \gamma(k) = \left\{ \frac{\partial \tilde{\varepsilon}_1(\omega, k)}{\partial \omega} \right\}_{\omega=\omega_0}^{-1} \tilde{\varepsilon}_2(\omega, k), \]
(31)
\[ \left\{ \frac{\partial \tilde{\varepsilon}_1(\omega, k)}{\partial \omega} \right\}_{\omega=\omega_0} = \frac{s}{\omega_0} \left\{ - \frac{\alpha_1 \mathcal{F}}{2} \ln \frac{s+1}{s-1} + \frac{\alpha_1 \mathcal{F} s}{s^2 - 1} + \frac{\alpha_2 \mathcal{F}}{2} \ln \frac{\eta s - s_0 + 1}{\eta s - s_0 - 1} + \frac{\alpha_2 \mathcal{F}}{\eta (\eta s - s_0)^2 - 1} \right\}. \]
(33)

4 Solution of the dispersion equation for zero-sound oscillations

A dispersion law of the zero-sound oscillations \( \omega = \omega_0(k) \) is determined by the solution of equation (30). Note first of all that this equation (in the same way as the relation for \( \gamma(k) \), see (31), (33)) is invariant with respect to a simultaneous substitution \( s \to -s, \ s_0 \to -s_0, \) (see (32)). This condition shows that propagation of the zero-sound wave is possible in opposite directions with the same increment or decrement. Consequently, let us consider \( s > 0 \) for the sake of definiteness. Since equation (30) is quite complicated in its general form, we will make certain assumptions in order to obtain a solution.

It is known that in the ordinary Fermi-liquid propagation of undamped zero-sound oscillations at zero temperature is possible only under the condition \( \mathcal{F} > 0, \ s > 1 \) (in this connection see [16, 17]). Otherwise oscillations will quickly damp. Let us make an assumption that
\[ \mathcal{F} > 0, \ s > 1 \]
(34)
and demonstrate that in this case oscillations can increase or damp due to existence of the impacting drop. Naturally, one can easily see from (31) that possibility of propagation of weakly increased or weakly damped oscillations in the case, determined by inequalities (33), is related to the fulfillment of the condition
\[ |\eta s - s_0| < 1. \]
(35)
If inequalities (34), (35) are valid, quantity \( \tilde{\varepsilon}_2 (\omega_0, k) \equiv \tilde{\varepsilon}_2 (s) \) has the form:

\[
\tilde{\varepsilon}_2 (s) = \alpha_2 \frac{\pi}{2\eta} \left( \eta s - s_0 \right) F.
\] (36)

Let us study the solution of dispersion equation (30) with small Landau amplitudes \( F, F \ll 1 \) and at the conditions (34), (35). Since in this case quantity \( s \) is close to unity, \( s \approx 1 + \delta s \), equation (30) can be written as

\[
1 + \alpha_1 F + \frac{\alpha_2}{\eta} F \left( \eta - s_0 \right) \times \\
\ln \left\{ \frac{1 + (\eta - s_0)}{1 - (\eta - s_0)} \right\} = \frac{\alpha_1}{2} F \ln \frac{2}{\delta s},
\]

whereof

\[
\delta s = 2 \left\{ 1 + (\eta - s_0) \right\} \frac{\alpha_2}{\alpha_1^2 (\eta - s_0)} \times \\
\exp \left\{ -2 \left( 1 + \frac{\alpha_2}{\alpha_1 \eta} \right) \right\} \exp \left\{ -\frac{2}{\alpha_1 F} \right\},
\] (37)

where

\[
s = 1 + \delta s.
\]

Taking into account (33), for conditions (34), (35) we have

\[
\left\{ \frac{\partial \tilde{\varepsilon}_1}{\partial \omega} \right\}_{\omega = \omega_0} \approx \frac{1}{\omega_0} \frac{\alpha_1 F}{2\delta s}, \quad \omega_0 = kv_1 F.
\]

Further using formulas (31), (36), (37), we have arrived at the following relation for \( \gamma (k) \):

\[
\frac{\gamma (k)}{\omega_0} \approx 2\pi \frac{\alpha_2}{\eta \alpha_1} (\eta - s_0) \times \\
\times \left\{ 1 + \frac{(\eta - s_0)}{1 - (\eta - s_0)} \right\} \times \\
\times \exp \left\{ -\frac{2}{\alpha_1 F} \left( 1 + \alpha_1 F + \frac{\alpha_2 F}{\eta} \right) \right\},
\] (38)

where \( |\eta - s_0| < 1, \quad \omega_0 = kv_1 F, \quad F \ll 1. \)

According to (4), (24) the condition \( \gamma (k) > 0 \), or \( 0 < (\eta - s_0) < 1 \), corresponds to the oscillation damping, whereas the condition \( \gamma (k) < 0 \), or \( -1 < (\eta - s_0) < 0 \) corresponds to the oscillation increase, meaning instability development. The latter inequality can be written (in view of (32)) as \( \eta < s_0 < \eta + 1 \), or taking into account \( \eta = \sqrt{\varepsilon_1 F / \varepsilon_2 F} \)

\[
v_1 F < u \cos \alpha < v_1 F + v_2 F,
\] (39)

where \( \alpha \) is an angle between vectors \( u \) and \( k \).

It is evident from (38) that when approaching to the right boundary of interval (39), that is if

\[
s_0 \rightarrow \eta + 1, \quad s = 1 + \delta s
\] (40)

the increment \( \gamma (k) \) is unrestrictedly increasing. However, it is evident that for the case (40) we need to solve more correctly dispersion equation (30), which in this approximation and keeping in mind \( s \rightarrow 1 \) has the form:

\[
1 + \alpha_1 F + \frac{\alpha_2 F}{\eta} = -\frac{1}{2} \alpha_1 F \ln \frac{\delta s}{2} - \frac{\alpha_2 F}{2\eta} \ln \frac{\eta \delta s}{2}.
\]

The solution of this equation is given by the relation:
\[ \delta s = 2\eta \frac{\alpha_2}{\eta \alpha_1 + \alpha_2} \times \exp \left\{ -2\eta \frac{\alpha_2}{(\eta \alpha_1 + \alpha_2)\mathcal{F}} \left[ 1 + \alpha_1 \mathcal{F} + \frac{\alpha \mathcal{F}}{\eta} \right] \right\}. \] (41)

Remembering that according to (33) under conditions (31), (36), (40) the relation for \( \frac{\partial \tilde{\varepsilon}_1}{\partial \omega} \) is

\[ \frac{\partial \tilde{\varepsilon}_1}{\partial \omega} \approx \frac{1}{\omega_0} \frac{\eta \alpha_1 + \alpha_2 \mathcal{F}}{2\delta s}, \]

and considering (31), (36), (4), we can obtain the following formula for increment \( \gamma(k) \):

\[ \frac{\gamma(k)}{\omega_0} \approx -2\pi \frac{\alpha_2}{\eta \alpha_1 + \alpha_2} \times \exp \left\{ -2\eta \frac{\alpha_2}{(\eta \alpha_1 + \alpha_2)\mathcal{F}} \left[ 1 + \alpha_1 \mathcal{F} + \frac{\alpha \mathcal{F}}{\eta} \right] \right\}, \] (42)

\( \mathcal{F} \ll 1, \ \omega_0 \approx kv_{1F}, \ s \approx 1, \ s_0 \lesssim \eta + 1, \) where quantities \( s_0, \ \eta, \ s \) are defined as before by relations (32). Taking into account that \( (\eta \alpha_1 / \eta \alpha_1 + \alpha_2) \ll 1 \), comparing relations (38) and (42) one can easily conclude that when \( \mathcal{F} \ll 1 \), the maximum value of the increment is reached for the upper limit of the interval (39), i.e., when

\[ \sqrt{E \cos \alpha} \lesssim \sqrt{\varepsilon_{1F}} + \sqrt{\varepsilon_{2F}}, \] (43)

where \( E \) is the kinetic energy per one nucleon in the impacting drop.

5 Temperature impact

So far we have discussed the problem of propagation of weakly increasing or damping zero-sound oscillations, assuming that the colliding Fermi-liquid drops have zero temperature (see (14)). However, when two equilibrium Fermi-liquid drops collide and the temperature of each one is non-zero, a scenario of weak instability development can have significant peculiarities.

In order to solve this problem we need to modify relations (30), (31), determining frequency and increments (decrements) of zero-sound oscillations in conformity with the case of the colliding equilibrium Fermi-liquid drops with the non-zero temperature. In this case the equilibrium distribution functions of the drops are as follows (see (8) - (13)):

\[ f_0^{(1)}(p) = \left\{ \exp \left[ \left( \frac{p^2}{2m} - \varepsilon_{1F} \right) / T_1 \right] + 1 \right\}^{-1}, \]

\[ f_0^{(2)}(p - mu) = \left\{ \exp \left[ \left( \frac{(p - mu)^2}{2m} - \varepsilon_{2F} \right) / T_2 \right] + 1 \right\}^{-1}, \] (44)

where \( T_1, T_2 \) are temperatures of the resting and impacting drops, respectively. The given below relations should hold true for the drops

\[ (T_1/\varepsilon_{1F}) \ll 1, \ (T_2/\varepsilon_{2F}) \ll 1. \] (45)

The relations determine possibility to apply the Fermi-liquid description to systems of strong interacting fermions (in the present case - nucleons). Taking into account formulas (44) and inequalities (45), one can make low temperature expansion in expression (23), that can result in the following equation for determination of frequency \( \omega_0 = skv_{1F} \) of the zero-sound oscillations in the main approximation over parameters \( (T_1/\varepsilon_{1F}), (T_2/\varepsilon_{2F}) \) (see [18]):

\[ \varepsilon_1(s) \equiv \tilde{\varepsilon}_{10}(s) + \tilde{\varepsilon}_{1T}(s) = 0, \]
\[ \tilde{\varepsilon}_{10}(s) = 1 + \alpha_1 \mathcal{F} \left\{ 1 - \frac{s}{2} \ln \left| \frac{s+1}{s-1} \right| \right\} + \alpha_2 \frac{1}{\eta} \mathcal{F} \left\{ 1 - \frac{1}{2} (\eta s - s_0) \ln \left| \frac{\eta s - s_0 + 1}{\eta s - s_0 - 1} \right| \right\} = 0, \]

\[ \tilde{\varepsilon}_{1T}(s) = \]

\[ = \frac{-\alpha_1 \mathcal{F}}{2} P \int_0^\infty \frac{zd\zeta}{e^\zeta + 1} \left( 2 + (s^2 - 1) \right) \frac{(\varepsilon_1/T_1)^2 - z^2}{\varepsilon_1 - z} - \frac{\alpha_2 \mathcal{F}}{2\eta} P \int_0^\infty \frac{zd\zeta}{e^\zeta + 1} \left( 2 + [(\eta s - s_0)^2 - 1] \right) \frac{(\varepsilon_2/T_2)^2 - z^2}{\varepsilon_2 - z}, \]

where \( P \) means as before the symbol of the principal value. In the main approximation the expression for quantity \( \tilde{\varepsilon}_2(s) \), defining in accordance with (27), (31) a weak increase or damping of the zero-sound oscillations, can be put as:

\[ \tilde{\varepsilon}_2(s) = \frac{\pi}{2} \mathcal{F} \left\{ \alpha_1 s f_0 \left( \frac{\varepsilon_1/F}{T_1} (s^2 - 1) \right) + \frac{\alpha_2 (\eta s - s_0)}{\eta} f_0 \left( \frac{\varepsilon_2/F}{T_2} [ (\eta s - s_0)^2 - 1] \right) \right\}, \]

where

\[ f_0(\varepsilon/T) = \left\{ e^{\varepsilon/T} + 1 \right\}^{-1}. \]

Under these conditions dispersion equation (46), meeting these conditions, in the main approximation over \( \delta s \) will get the form:

\[ \mathcal{F} \ll 1, \quad s \gtrsim 1, \quad s_0 \lesssim \eta + 1, \quad (\varepsilon_2/F/T_2) \gg 1, \]

\[ \delta s (\varepsilon_2/F/T_2) \ll 1, \quad \delta s = s - 1 \ll 1. \]

In order to obtain this equation we used the asymptotic estimate of the integral, when \( |t| \ll 1 \)

\[ P \int_0^\infty \frac{zd\zeta}{(e^\zeta + 1) (t^2 - z^2)} \rightarrow I_0 + \frac{1}{2} \ln |t| - I_1 t^2, \]
The integrals having exactly the same form describe the temperature impact on dispersion equation (46). The solution of equation (50) is:

\[
I_0 = \frac{1}{4} - \int_1^\infty \frac{dz}{z(e^z + 1)} + \frac{1}{2} \int_0^1 \frac{dz}{z} \left\{ \tanh \frac{z}{2} - \frac{z}{2} \right\} \approx 0.07, \\
I_1 = \int_1^\infty \frac{dz}{z^3(e^z + 1)} - \int_0^1 \frac{dz}{z^3} \left\{ \tanh \frac{z}{2} - \frac{z}{2} \right\} \approx 0.11.
\]

The solution of equation (50) is:

\[
\delta s = 2 \left( \frac{\varepsilon_{2F}}{T_1} \right) \eta \alpha_1 \times \exp \left\{ -\frac{2Q_1}{\alpha_1 F} \right\}, \quad (52)
\]

\[
Q_1 \equiv 1 + \alpha_1 F + \frac{\alpha_2 F}{\eta} (1 - I_0).
\]

In accordance with (49) this solution must satisfy relation \( \delta s (\varepsilon_{2F}/T_2) \ll 1 \), whence, remembering that \( \varepsilon_{2F}/T_2 \gg 1 \), it is easy to determine a "temperature" range, when solution (52) exists for \( \delta s \):

\[
1 \ll \varepsilon_{2F}/T_2 \ll \exp \left\{ \frac{2\eta Q_1}{(\eta \alpha_1 + \alpha_2) F} \right\}. \quad (53)
\]

Noting further that under (50) the formula holds

\[
\left\{ \frac{\partial \tilde{\xi}}{\partial \omega} \right\}_{\omega = \omega_0} \approx \frac{1}{\omega_0} \frac{\alpha_1 F}{2\delta s}, \quad \omega_0 = k v_1 F
\]

and using the fact that when \( T_1 \to 0 \) and \( s \gg 1 \)

\[
\tilde{\varepsilon}_2 = \frac{-\pi \alpha_2}{4 \eta} F, \quad (54)
\]

we can arrive at the following expression for increment \( \gamma_k (s) \), considering (51),

\[
\frac{\gamma_k}{\omega_0} = -\pi \frac{\alpha_2}{\eta \alpha_1} \left( \frac{4 \varepsilon_{2F}}{T_1} \right) \eta \alpha_1 \times \\
\times \exp \left\{ -\frac{2}{\alpha_1 F} \left[ 1 + \alpha_1 F + \frac{\alpha_2 F}{\eta} (1 - I_0) \right] \right\}, \quad (55)
\]

which is true at the conditions (49), (53). In order to obtain formula (54), we have used expression (47), and also have taken into account that (see (48))

\[
f_0 \left( \frac{\varepsilon_{1F}}{T_1} (s^2 - 1) \right) \to \theta (1 - s^2) = 0, \quad s \gtrsim 1,
\]

\[
\approx f_0 \left( \frac{2\varepsilon_{2F}}{T_2} \eta \delta s \right) \approx f_0 (0) = \frac{1}{2},
\]

\[
s_0 \lesssim \eta + 1.
\]

Now let us consider a case when the impacting Fermi-liquid drop has zero temperature, \( T_2 = 0 \), and the resting drop has temperature \( T_1 \). We assume that conditions (compare with (49)) are valid:

\[
\mathcal{F} \ll 1, \quad s \gtrsim 1, \quad s_0 \lesssim \eta + 1, \quad (\varepsilon_{1F}/T_1) \gg 1,
\]

\[
\delta s (\varepsilon_{1F}/T_1) \ll 1, \quad \delta s = s - 1 \ll 1. \quad (56)
\]
In this case the dispersion equation in the main approximation over \( \delta s \) considering asymptotic estimate (51) has the form:

\[
\tilde{\varepsilon}_1 (s) \approx 1 + \alpha_1 F (1 - I_0) + \frac{\alpha_2 F}{\eta} - \frac{\alpha_1 F}{2} \ln \frac{4 \varepsilon_{1T}}{T_1} + \frac{\alpha_2 F}{2\eta} \ln \frac{\eta \delta s}{2} = 0.
\] (57)

A solution of this equation is given by

\[
\delta s = \frac{2}{\eta} \left( \frac{\varepsilon_{1F}}{T_1} \right) \frac{\eta \alpha_1}{\alpha_2} \times \exp \left\{ -\frac{2\eta Q_2}{\alpha_2 F} \right\},
\] (58)

\[
Q_2 \equiv 1 + \alpha_1 F (1 - I_0) + \frac{\alpha_2 F}{\eta},
\]

and here, we should remember that in accordance with (56) the relation \( \delta s (\varepsilon_{1F}/T_1) \ll 1 \) must be valid, and a "temperature" condition of existence of such solution is determined by the inequality (compare with (53)):

\[
1 \ll \frac{\varepsilon_{1F}}{T_1} \ll \exp \left\{ \frac{2\eta Q_2}{(\eta \alpha_1 + \alpha_2) F} \right\}.
\] (59)

Noting further that for \( s \gtrsim 1, s_0 \lesssim \eta + 1 \) and \( T_2 = 0 \) the following formulas are valid

\[
f_0 \left( \frac{\varepsilon_{1F}}{T_1} (s^2 - 1) \right) \approx f_0 \left( \frac{2\varepsilon_{1F}}{T_1} \delta s \right) \approx f_0 (0) = \frac{1}{2},
\]

\[
f_0 \left( \frac{\varepsilon_{2F}}{T_2} [(\eta s - s_0)^2 - 1] \right) \xrightarrow{T_2 \to 0} \theta (1 - (\eta s - s_0)^2)
\]

\[
f_0 \left( \frac{\varepsilon_{2F}}{T_2} [(\eta s - s_0)^2 - 1] \right) \xrightarrow{T_2 \to 0} 1,
\]

the expression for \( \tilde{\varepsilon}_2 \), determining in accordance with (31) the damping or increase of the zero-sound oscillations, can be put in a form

\[
\tilde{\varepsilon}_2 = \frac{\pi}{4\eta} F \{ \eta \alpha_1 - 2\alpha_2 \}.
\] (60)

Bearing in mind that quantity \( \{ \partial \tilde{\varepsilon}_1 / \partial \omega \}_{\omega = \omega_0} \) in compliance with (57) is given by the formula

\[
\left\{ \frac{\partial \tilde{\varepsilon}_1}{\partial \omega} \right\}_{\omega = \omega_0} \approx \frac{1}{\omega_0} \frac{\alpha_2 F}{2\eta} \frac{\delta s}{\eta \alpha_2} \times
\]

\[
\times \exp \left\{ -\frac{2}{\alpha_2 F} \left[ 1 + \alpha_1 F (1 - I_0) + \frac{\alpha_2 F}{\eta} \right] \right\},
\] (61)

\[
\omega_0 = kv_{1F}.
\]

As is easy to see, the formula (62) in contrast to (55), which is correct when \( T_1 = 0, T_2 \neq 0 \), determines either a coefficient of damping (when \( \eta \alpha_1 > 2\alpha_2 \)), or increase (when \( \eta \alpha_1 < 2\alpha_2 \)) of the zero-sound oscillations. Such non-invariance of (55), (62) relative to interchanging of the impacting and resting drops with a corresponding change of their characteristics \( \varepsilon_{1F} \leftrightarrow \varepsilon_{2F}, T_1 \leftrightarrow T_2 \) (in other words the absence of the Galilean invariance in the system of the collided drops) is not in any sense paradoxical. This is because within the developed Fermi-liquid description model the interaction of quasiparticles is not invariant with respect to the Galilean transformations (see (2), (3)).

Therefore, the relation \( \eta \alpha_1 < 2\alpha_2 \) that can be written as

\[
\alpha_1 \sqrt{\varepsilon_{1F}} < 2\alpha_2 \sqrt{\varepsilon_{2F}},
\] (63)
along with (56), (59) determines existence conditions of weakly increasing oscillations with increment \( \gamma_k \) in the system

\[
\frac{\gamma_k}{\omega_0} \approx \pi \frac{\eta \alpha_1 - 2 \alpha_2}{\eta \alpha_2} \left( 4 \frac{\varepsilon_{1F}}{T_1} \right)^{\alpha_2} \times \exp \left\{ - \frac{2 \eta}{\alpha_2 F} \left[ 1 + \alpha_1 F (1 - I_0) + \frac{\alpha_2 F}{\eta} \right] \right\}
\]  

(64)

and coefficient \( I_1 \) is determined by (51).

The solution of equation (66), since \((A/B) \ll 1\), will be unambiguous and positive when \( A \geq 1 \):

\[
\delta s = \frac{A}{B} + \sqrt{\frac{A^2}{4B^2} + \frac{A - 1}{B}}, \quad A \geq 1. \quad (68)
\]

Focusing on the explicit form of coefficients \( A \) and \( B \), it is easy to prove that the conditions

\[
\delta s (\varepsilon_{1F}/T_1) \ll 1, \quad \delta s (\varepsilon_{2F}/T_2) \ll 1
\]

are automatically fulfilled.

Proceeding from (31), (33) and noting that in accordance with (47), (65), (66) the following formulas hold

\[
\varepsilon_2 \approx \frac{\pi}{4\eta} F \{ \eta \alpha_1 - \alpha_2 \},
\]

\[
\left\{ \frac{\partial \tilde{\varepsilon}_1}{\partial \omega} \right\}_{\omega = \omega_0} \approx \frac{1}{\omega_0} \sqrt{A^2 + 4 B (A - 1)},
\]

we will attain the following expression for coefficient \( \gamma_k \):

\[
\frac{\gamma_k}{\omega_0} \approx \frac{\pi}{4\eta} F \frac{\eta \alpha_1 - \alpha_2}{\sqrt{A^2 + 4 B (A - 1)}}, \quad \omega_0 = kv_{1F}. \quad (69)
\]

It is clear that when \( \eta \alpha_1 < \alpha_2 \), or

\[
\alpha_1 \sqrt{\varepsilon_{1F}} < \alpha_2 \sqrt{\varepsilon_{2F}}, \quad (70)
\]

quantity \( \gamma_k \) represents an instability increment of the zero-sound oscillations.

Expression (69) takes a simple form

\[
\frac{\gamma_k}{\omega_0} \approx \frac{\pi}{4\eta} F \frac{\eta \alpha_1 - \alpha_2}{\alpha_1 \ln (\varepsilon_{1F}/T_1) + \alpha_2 \ln (\varepsilon_{2F}/T_2)} \quad (71)
\]
or

\[ \frac{\gamma_k}{\omega_0} \approx \frac{\pi}{4\eta} F (\eta \alpha_1 - \alpha_2), \quad \omega_0 = kv_1 F \]

when \( A \approx 1 \), that is (see (67))

\[ \alpha_1 \ln (\varepsilon_1 F/T_1) + \alpha_2 \ln (\varepsilon_2 F/T_2) \approx \frac{2}{F}. \]  

(72)

\[ \frac{\gamma_k}{\omega_0} \approx \frac{\pi (\eta \alpha_1 - \alpha_2)}{32 \eta I_1} \left\{ \frac{F}{2} \left[ \alpha_1 \ln (\varepsilon_1 F/T_1) + \alpha_2 \ln (\varepsilon_2 F/T_2) \right] - 1 \right\}^{-1/2} \left\{ \alpha_1 \left( \frac{\varepsilon_1 F}{T_1} \right)^2 + \eta \alpha_2 \left( \frac{\varepsilon_2 F}{T_2} \right)^2 \right\}^{-1}. \]  

(73)

Expressions for increments (71), (73) look even simpler, if we consider collision of identical nuclei having the same temperatures (see below formulas (75), (76)). Comparison of expressions (38), (42) with (55), (62), (71), (73), with taking into account relations (63), (70) allows to come to a conclusion that for the colliding Fermi-liquid drops with nonzero temperatures the instability increments are higher compared to the ones for the colliding drops with zero temperatures. In particular, for the collision of the Fermi-liquid drops having comparable temperatures \( T_1 \sim T_2 \) \((T_1/\varepsilon_{1F}) \ll 1, (T_2/\varepsilon_{2F}) \ll 1)\), when condition (72) is fulfilled, the increase increment of the zero-sound oscillations has a power like character of dependence over the Landau amplitude \( F \ll 1 \), whereas for other considered here cases (except the one leading to (73)), the degree of the increment smallness is given by exponential multipliers of the type \( \exp(-\lambda/F) \), \( F \ll 1 \), where \( \lambda \) is a certain constant. If we recall that in our consideration collision of two fast (but not relativistic) nuclei is simulated by collision of the two Fermi-liquid drops, then the provided comment means that the instabilities, related to increase of the zero-sound oscillations in the system of the two collided excited nuclei (i.e. nuclei with nonzero temperatures), must develop much more intensively compared to the instabilities in the system of the collided unexcited nuclei.

6 Discussion of results

Up to now we have studied the case of small Landau amplitudes \( F \ll 1 \). As it was proved in this case the dispersion equations of the zero-sound oscillations allow for an analytical solution in the perturbation theory over small \( F \). Analytical solutions of dispersion equations can be obtained also for the case of large Landau amplitudes, \( F \gg 1 \). However, if we consider these solutions from the viewpoint of the problem about instability development in the system of collided non-relativistic nuclei, it would be easy to prove that a solution of the dispersion equations in the perturbation theory over a small parameter \( 1/F \) will lead us beyond the non-relativistic approximation, which was one of the main assumptions in the present work.

Naturally, it is easy to prove that the solutions of dispersion equation (30) are within the range of large \( s, s \gg 1 \) (note at once that when
s \gg 1$, it is possible to neglect temperature dependent components in equation (46). But in accordance with (31), (33), (47) for existence of instabilities the following condition must be satisfied $s_0 > \eta s$, or

$$\sqrt{\varepsilon/\varepsilon_F \cos \alpha} > s \gg 1, \quad (74)$$

where $\varepsilon$ is kinetic energy per one nucleon in the impacting drop. In inequality (74) we took into account that for heavy nuclei $\varepsilon_1 F \approx \varepsilon_2 F = \varepsilon_F$, meaning, $\eta \approx 1$. Remembering that for the heavy nuclei $\varepsilon_F \approx 36$ MeV, and the non-relativistic approximation requires the nucleon kinetic energy to be small compared to the rest energy $mc^2 \sim 1$ GeV, $(\varepsilon/mc^2 \ll 1)$, it is easy to see that relation (74) can not be satisfied under the non-relativistic approximation. Therefore, in this work we do not consider the case of large Landau amplitudes.

In this article we have restricted ourselves by consideration of positive Landau amplitudes. When Landau amplitudes are negative, the dispersion equation of the zero-sound oscillations can be solved only by numerical methods. This is because in this case a real and imaginary parts of the frequency are comparable with each other and the analytical methods, based on the perturbation theory, become inapplicable. The numerical solution of the dispersion equations for the negative Landau amplitudes is not included into the present work, which has as its object to demonstrate a principal possibility of instability development in the system of the colliding heavy nuclei, induced by propagation and increase of the zero-sound oscillations.

Formulas (71), (73) could be used as a proof of possible development of just such instabilities in the system of the heavy colliding nuclei. When two identical nuclei collide ($\varepsilon_1 F = \varepsilon_2 F = \varepsilon_F, \eta = 1, T_1 \approx T_2 = T$), formula (71) takes the form:

$$\gamma_k \approx \frac{\pi}{\omega_0} \frac{\alpha_1 - \alpha_2}{2 \alpha_1 + \alpha_2} \frac{1}{\ln (\varepsilon_F/T)}, \quad \omega_0 = k v_F. \quad (75)$$

In this case the expression (73) is significantly simplified too:

$$\frac{\gamma_k}{\omega_0} \approx \frac{\pi}{32 I_1} \frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2} (\varepsilon_F/T)^{-2} \times \times \{(1/2) \mathcal{F}(\alpha_1 + \alpha_2) \ln (\varepsilon_F/T) - 1\}^{-1/2}, \quad (76)$$

As it is we known temperature $T$ can be related to the excitation energy of nucleus $U$ by an approximate formula (in this connection see, for example, [19, 20, 21, 22])

$$T \approx \sqrt{aU}, \quad (77)$$

where $a$ is a certain constant, which should be determined from experimental data (see in particular [19, 20, 21]). For example, in accordance with [19] for a nucleus with mass number $A = 115, a \approx 1/8$ MeV; $A = 181, a \approx 1/10$ MeV. Note that within the framework of the Fermi-gas model of nucleus quantity $a$ can be found from formula [22]

$$a = \frac{4 \varepsilon_F}{A \pi^2}. \quad (77)$$

Following this formula for $\varepsilon_F \approx 36$ MeV and $A = 115, A = 181$, we have $a \approx 0, 124$ MeV and $a \approx 0, 0795$ MeV, respectively.

If we consider that the instabilities, associated with the increase of amplitudes of the zero-sound oscillations, are responsible for fragmentation, then specific time of fragmentation $\tau_f$ must be of the order of the value reciprocal to increment $\gamma_k$

$$\tau_f \sim 1/\gamma_k. \quad (78)$$
Thus, if we could experimentally measure the fragmentation time of nuclei $\tau_f$, then the specific dependence of the time from excitation energy $U$ would be governed by the formula

$$\tau_f \sim C \ln \left( \frac{\varepsilon_F}{\sqrt{aU}} \right), \quad C = \frac{2}{\pi} \frac{(\alpha_1 + \alpha_2)}{(\alpha_1 - \alpha_2)}$$

(79)

when relation (72) holds, and by the formula

$$\tau_f \sim C \left( \frac{\varepsilon_F}{\sqrt{aU}} \right)^2 \left\{ \mathcal{F} (\alpha_1 + \alpha_2) \ln \left( \frac{\varepsilon_F}{\sqrt{aU}} \right) - 2 \right\},$$

(80)

when condition $A \gg 1$, leading to (73) is valid.

According to (75)-(78) this would show that the proposed mechanism of the instability development in the system of the colliding heavy nuclei is working. Besides, if such instability development mechanism is realized, it can result in nucleus fragmentation and consequently, according to (72), (79), (80) we could estimate both values of frequencies of oscillations of developed instabilities and values of the Landau amplitudes for the nuclear matter formed as a result of the collision. It is should be noted that the Landau amplitudes are parameters of the Fermi-liquid approach of the present work and, generally speaking, they don’t necessarily coincide with the Landau amplitudes for certain heavy nuclei, values of which are contained in some published papers (see, for example [13, 14]).

However, if we make an assumption that values of the Landau amplitudes in the nuclear matter, formed as a result of the collision of two fast heavy nuclei, are close to the corresponding values of amplitudes of certain nuclei, then we can estimate values of increments for specific cases and provide suggestions about range of use for the results of the present work. Work [14] gives grounds to make a conclusion that the value of Landau amplitude $F_0^{(0)}$, determining oscillations of the nucleus density (see (6)), is within the range of $-0.25$ and $0.5$. These values of the Landau amplitudes from the positive part of this range, when $\varepsilon_1 \approx \varepsilon_2 = 36 \, \text{MeV}$, are fully within the frame of the assumptions made in present work.

The values of amplitudes $F_0^{(s)}$, $F_0^{(i)}$ (see (6)), determining oscillations of the spin and isospin densities in the nucleus (see [13, 14]) are also in compliance with the assumptions of our work:

$$F_0^{(s)} \approx 0, 27 \div 0, 35, \quad F_0^{(i)} \approx 0, 59 \div 0, 72.$$  

As for the negative values of amplitude $F_0$ and values of amplitude $F_0^{(si)}$ (see [13, 14])

$$F_0^{(si)} \approx 1, 26 \div 1, 36,$$

determining zero-sound oscillations of the spin-isospin density, these cases as we have already said require numerical calculations. This is the reason why they are not included into this work.

Finally, let us study the question about peculiarities in behavior of a non-equilibrium distribution function of two colliding nucleon drops in the momentum space. This will allow to determine a direction of nucleus fragment release as a result of instability development in such a system with respect to a direction of velocity of the moving drop.

It follows from (17), that deviation of distribution function $\delta f (p, k, \omega)$ from the equilibrium distribution function is determined by the equation

\[ \]
\[
\delta f(p, k, \omega) = -F \left\{ \frac{\alpha_1}{\omega - kv + io} k \left( \frac{\partial f_0^{(1)}(p)}{\partial p} \right) + \frac{\alpha_2}{\omega - kv + io} k \left( \frac{\partial f_0^{(2)}(p - mu)}{\partial p} \right) \right\} \times \int d\tau' \delta f(p', k, \omega) + \delta A(p, k) \delta(\omega - vk),
\]

(81)

where derivatives \( \frac{\partial f_0^{(1)}(p)}{\partial p} \), \( \frac{\partial f_0^{(2)}(p - mu)}{\partial p} \) are given by (16). According to (4), (81) the non-equilibrium distribution function can have maximum, given the following conditions are met

\[
(\omega - kv)|_{v = v_{1F}} = 0,
\]

\[
(\omega - ku - k(v - u))|_{v = v_{2F}} = 0,
\]

\[
v = \frac{p}{m}.
\]

Since \( \omega \approx \omega_0 = skv_{1F} \), these conditions can be written as

\[
s - \cos \theta = 0, \quad \eta s - s_0 - \cos \beta = 0, \quad (82)
\]

where \( \theta \) is an angle between directions of vectors \( k \) and \( v \), \( \beta \) is an angle between directions of vector \( k \) and vector \( v - u \). The first of these conditions is not fulfilled because \( s > 1 \). As we have previously mentioned a damping or increase of the zero-sound oscillations can take place only when \( |\eta s - s_0| < 1 \). This means that the second condition in (82) can be fulfilled. The maximum value of increment \( \gamma_k \) is reached when \( \eta s - s_0 \sim -1 \) (see (11)). Under condition \( \varepsilon_{1F} \approx \varepsilon_{2F} = \varepsilon_F (\eta = 1) \) and \( s \geq 1 \) it corresponds to the condition

\[
s_0 = \frac{u}{v_F} \cos \alpha \approx 2 \quad (83)
\]

(\( \alpha \) is as before an angle between directions of vectors \( u \) and \( k \)) and, hence

\[
u \geq 2v_F. \quad (84)
\]

Relation \( \eta s - s_0 \sim -1 \) is accordance with (82) means that vectors \( k \) and \( v - u \) are anti-parallel

\[
v - u = - |v - u| \frac{k}{k},
\]

or, bearing in mind that \( |v - u| = v_F \),

\[
v - u = -v_F \frac{k}{k}.
\]

Multiplying scalarwise this equation by vector \( v - u \), with taking into account (83), we can obtain the following relation for cosine of angle \( \theta_0 \) between directions of vectors \( v \) and \( u \)

\[
\cos \theta_0 = \frac{u^2 - 2v_F^2}{vu},
\]

Hence, remembering that the module of vector \( v \) is within the region of values \( u - v_F \) and \( u + v_F \), and also that by (84)

\[
u^2 - 2v_F^2 < 1, \quad \frac{u^2 - 2v_F^2}{u(u + v_F)} \geq 1,
\]

we have

\[
\frac{u^2 - 2v_F^2}{u(u + v_F)} \leq \cos \theta_0 \leq 1.
\]

Taking into consideration the inequality (84) the latter equation can be written in the following form

\[
\frac{1}{3} \leq \cos \theta_0 \leq 1. \quad (85)
\]
Thus, according to the results of this paper in the case when the inequality \(84\) is true the jets of nuclear matter at the collision of heavy nuclei should be expected along the directions defined by the condition \(85\). In the case when the inequality \(84\) is breaking, the angular distribution of the outgoing matter should be close to an isotropic one.

Let us come back to the problem collisionless approximation for the description of dynamics of nuclear matter formed by the collision of heavy nuclei. The main condition of its applicability \(1\) from the point of view of the results of this paper is the inequality \(\omega_0 \tau_r \gg 1\), where \(\omega_0 = skv_F\) and \(\tau_r\) is a relaxation time. It is necessary also that the characteristic times of the instabilities’ development \(\tau_f \sim 1/\gamma\) (where \(\gamma\) are the increments found in this paper) should be smaller or have the same magnitude as the relaxation time \(\frac{1}{\gamma} \lesssim \tau_r\). The latter equation can be written down like \(\frac{\gamma}{\omega_0 \omega_0 \tau_r} \lesssim 1\). Thus, the equations

\[\omega_0 \tau_r >> 1, \quad \frac{\gamma}{\omega_0 \omega_0 \tau_r} \lesssim 1.\]

\[\omega_0 >> \gamma \gtrsim 1/\tau_r, \quad t << 1/\gamma \lesssim \tau_r.\]

are the applicability conditions of the collisionless approximation for the describing the initial stage of development of the instabilities in the nuclear matter, formed by the heavy nuclei collisions. Let us remark that the kinetic equation without taking into account collisions between particles had been used by some authors for the description of the dynamics of the matter formed by the heavy nuclei collisions (see in this case \([1, 2, 23]\), and references therein). As a rule this equation was written in the mean field approximation. For example the spinodal fission in the expanding nucleon Fermi-liquid being was investigated in \([23]\) on the basis of such kinetic equation. However, as it is known, the average field approach needs considerable numeric calculations. The usage of the Fermi-liquid approach with phenomenological parameters of interaction (Landau amplitudes) for the describing the dynamics of the nuclear matter allows in many cases to use analytic treatment without the help of numerical methods.

As it was repeatedly noted in this paper the application of Fermi-liquid approach is justified while describing the properties of heavy nuclei. For this reason the confirmations of the mechanism of origin of instabilities in the nuclear matter suggested in this paper, one can expect in experiments at the collisions of heavy fast (but non-relativistic) nuclei. These might be the reactions Gd+U or Xe+Sn (INDRA, \([2, 12]\)) at the energies of the incoming nucleus more than 145 MeV per nucleon.

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