Fusion dynamics of heavy-ion reactions using density dependent relativistic nucleon-nucleon potential

M. Bhuyan1,2,*, Shilpa Rana3,†, Raj Kumar3,‡ and S. K. Patra4§
1Department of Physics, Faculty of Science, University of Malaya, Kuala Lumpur 50603, Malaysia
2Institute of Research and Development, Duy Tan University, Da Nang 550000, Vietnam
3School of Physics and Materials Science, Thapar Institute of Engineering and Technology, Patiala 147004, India and
4Institute of Physics, Sachivalaya Marg, Sainik School, Bhubaneswar 751005, India
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In-medium effects are introduced in the microscopic description of the effective nucleon-nucleon (NN) interaction potential entitled DDR3Y in terms of the density-dependent nucleon-meson couplings within the Relativistic-Hartree-Bogoliubov (RHB) approach. The nuclear densities of the interacting target and projectile nuclei and NN potentials are obtained for non-linear NL3∗ and TM1 parameter sets within the relativistic mean-field approach and density-dependent DDME1 and DDME2 parameter sets within Relativistic-Hartree-Bogoliubov (RHB) formalism. The DDR3Y NN potential and the densities are used to obtain the nuclear potential by adopting the double folding approach. This nuclear potential is further used to probe the fusion dynamics within the ℓ−summed Wong model a few even-even systems leading to the formation of light, heavy and superheavy nuclei. The calculations are also performed for density-dependent DDM3Y and the density-independent M3Y and R3Y NN interaction potentials for the sake of comparison. We observed that the DDR3Y NN potential gives a better overlap with the experimental data as compared to non-relativistic M3Y and DDM3Y NN potentials. From the comparison of R3Y and DDR3Y interactions, it is manifested that the inclusion of in-medium effects in the NN interaction raises the fusion barrier and consequently decreases the fusion and/or capture cross-section. Moreover, the nuclear densities, as well as relativistic R3Y NN potential obtained for the more recent NL3∗ RMF parameter set, are observed to give a comparatively better fit to the experimental data.

I. INTRODUCTION

The interaction barrier generated by two colliding nuclei is of the essence in understanding the complex nuclear reaction dynamics. The formulae for the long-range repulsive Coulomb and centrifugal potentials formed between two interacting heavy ions are straightforward, whereas short-range attractive nuclear potential evaluation is ambiguous. Despite numerous theoretical efforts, the understanding of nuclear interaction in the total interaction potential is still fuzzy [1–5]. The double-folding model [6] is one of the widely used techniques that turned out to give a satisfactory description of the real part of the nucleus-nucleus as well as α−nucleus optical potentials. In the double folding model, the nuclear optical potential is obtained by integrating the densities of the interacting nuclei over an effective nucleon-nucleon (NN) interaction potential. The widely adopted choices of the effective NN interaction potential are M3Y (Michigan 3 Yukawa) interactions which were developed to fit the G-matrix elements of Reid [7] and Paris [8] NN potentials on an oscillator basis. Later, the density dependence was included in the original M3Y interactions to account for the higher-order exchange effects and the Pauli blocking exchange effects. Moreover, the density-independent M3Y interactions were also observed to fail in saturating the cold nuclear matter (NM) [7, 8]. Consequently, numerous density-dependent M3Y NN interactions were developed with density-dependent parameters fitted to reproduce the saturation properties of NM.

Parallel to the M3Y potential, recently in Refs. [9–11], the relativistic NN potential was derived within the well-established relativistic mean-field (RMF) formalism and entitled R3Y NN potential. Furthermore, this relativistic R3Y NN potential was employed to study the cluster radioactivity [9–11] and fusion dynamics [12–14] of various even-even, even-odd, and odd-odd reactions leading to the synthesis of heavy and superheavy nuclei. From these Refs. [9–14], one can conclude that the results obtained from R3Y NN interactions provide relatively better overlap to the experimental data. It is worth mentioning that the above-discussed R3Y NN interaction is obtained for the density-independent linear and nonlinear relativistic parametrization. Here our main aim is to introduce the medium effects in the R3Y NN-potential for density-dependent relativistic parametrization within Relativistic-Hartree-Bogoliubov Approach, which will be analogous to the density-dependent M3Y (DDM3Y) potential.

Unlike the non-relativistic M3Y NN interactions, the microscopic R3Y NN potential is given in terms of meson masses and coupling constants. Furthermore, the reliable method to introduce medium dependence in the relativistic R3Y NN interaction potential is to include the density-dependent coupling constants, which is different from the DDM3Y NN potential. For example, the medium effect was introduced in M3Y po-
tential through a weighted function of density [7, 8], however for R3Y NN potential, the medium effect will be included through nucleon-nucleon coupling constants within the Relativistic-Hartree-Bogoliubov (RHB) approach [15, 16]. The density-dependent DME1 and DDME2 parameter sets are used in the present study to obtain the density-dependent R3Y NN potential entitled DDR3Y potential. Furthermore, the DDR3Y potential, along with the densities from the RHB approach, is used to estimate the fusion dynamics of various heavy-ion reactions. Here, we have chosen six reaction systems namely, $^{16}\text{O}+^{24}\text{Mg}$, $^{38}\text{Si}+^{92}\text{Zr}$, $^{48}\text{Ca}+^{36}\text{Zr}$, $^{48}\text{Ca}+^{154}\text{Sm}$, $^{16}\text{O}+^{208}\text{Pb}$ and $^{48}\text{Ca}+^{238}\text{U}$, forming light, heavy and super-heavy nuclei to examine the application of DDR3Y NN potential in description of fusion mechanism. The results of newly introduced DDR3Y NN potential obtained for DDME1 [15] and DDME2 [16] parameter sets within RHB approach are also compared with the density-independent R3Y NN potential obtained for nonlinear NL3* [17] and TM1 [18] RMF parameter sets. Moreover, the traditional Reid M3Y and the density-dependent M3Y (DDM3Y) [6, 19, 20] are also considered for the comparison. The fusion and/or capture cross-section is obtained within the $\ell$-summed Wong model [21, 22] and results are also compared with the available experimental data [23–28].

The paper is organized as follows: The details of the theoretical formalism adopted in the present analysis are discussed in section III. In the present analysis, we have considered the density-dependent R3Y NN potential comparable to the M3Y has also been recently derived from relativistic mean-field (RMF) formalism. The present analysis aims to introduce the in-medium effects in the relativistic R3Y NN potential. The density-dependent R3Y (DDR3Y) NN potential will be obtained from the relativistic-Hartree-Bogoliubov (RHB) model. This M3Y, DDM3Y, R3Y, and DDR3Y NN potentials are discussed in detail in the upcoming subsections.

A. M3Y and DDM3Y Nucleon-Nucleon Interaction Potentials

The explicit medium dependence in the original M3Y interaction was introduced [6, 19, 20, 29] via multiplying it by a density dependent weighted factor $F(\rho)$,

$$V_{\text{eff}}^{M3Y}(\rho, r) = F(\rho)V_{\text{eff}}^{M3Y}(\rho).$$

Here, $V_{\text{eff}}^{M3Y}(r)$ is the radial dependent M3Y NN interaction. In the present analysis, we have considered the well-adopted Reid M3Y [7] interaction which is written as the sum of three Yukawa terms as,

$$V_{\text{eff}}^{M3Y}(r) = 7999e^{-4r}/4r - 2140e^{-2.5r}/2.5r + J_{00}\delta(r).$$

Here, $J_{00}(E)\delta(r)$ is the long range one-pion exchange potential (OPEP). The different versions of the density dependent $F(\rho)$ factor are developed in literature with parameters fitted to reproduce the saturation properties of the nuclear matter. Here, we have considered the BDM3Y-type which is written as,

$$F(\rho) = C[1 - \alpha\rho^2]$$

As mentioned above, the parameters $C$, $\alpha$ and $\beta$ are adjusted to match the nuclear matter saturation properties. Here, we have adopted BDM3Y1 ($C=1.2253$, $\alpha = 1.5124$ $fm^3$ and $\beta = 1.0$) version [29] of the density dependent Reid M3Y NN interaction as it yields nuclear incompressibility value $K = 232$ MeV which is comparable to the values given by NL3* ($K = 258.8$ MeV) [17], TM1 ($K = 281$ MeV) [18], DDM3Y1 ($K = 244.5$ MeV) [15] and ($K = 250.89$ MeV) [16] parameter sets considered here. The density $\rho$ entering in Eq. (5) is taken as sum of projectile and target densities at the midpoint of the nucleon-nucleon separation distance. This procedure is known as the frozen density approximation (FDA) [6, 19, 20, 29–31] and is well adopted in the folding model.

II. THEORETICAL FORMALISM

A large number of nucleons are involved in the nuclear fusion of two interacting heavy ions. The interaction potential formed between these colliding heavy ions plays a vital role in elucidating the complex fusion mechanism. The total interaction potential can be written as the sum of three terms,

$$V_{\text{int}}(R) = V_{C}(R) + V_{t}(R) + V_{n}(R).$$

Here, R is the separation distance between the interacting nuclei. The terms $V_{C}(R) = ZeZ_{t}e^{2}/R$ and $V_{t}(R) = \hbar^{2}\ell(\ell + 1)/2\mu R^{2}$ are the repulsive Coulomb and centrifugal potentials, respectively. $Z_{p}$ and $Z_{t}$ symbolize the charge numbers of projectile and target nuclei, and $\mu$ is the reduced mass. The last term in Eq. (1) denotes the short-range and attractive nuclear potential, which is calculated within the double-folding approach [6] as,

$$V_{n}(\vec{R}) = \int \rho_{p}(\vec{r}_{p})\rho_{t}(\vec{r}_{t})V_{\text{eff}}\rho, r = |\vec{r}_{p} - \vec{r}_{t} + \vec{R}|$$

$$d^{3}\vec{r}_{p}d^{3}\vec{r}_{t}.$$  

Here, bold symbols $\rho_{p}$ and $\rho_{t}$ denote the total nuclear density (i.e. sum of proton and neutron densities) distributions of the interacting projectile and target nuclei, respectively. $V_{\text{eff}}^{M3Y}(\rho, r)$ is the effective nucleon-nucleon (NN) interaction potential. As discussed above, the density-independent M3Y (Michigan 3 Yukawa) NN interactions are well-adopted, and density-dependent M3Y (DDM3Y) NN interactions are also developed to include the in-medium effects. The relativistic R3Y NN potential comparable to the M3Y has also been recently derived from relativistic mean-field (RMF) formalism.
B. R3Y and DDR3Y Nucleon-Nucleon Interaction Potentials

The RMF models have emerged to be very reliable in the description of structural properties of the finite nuclei, not only in the β-stable regions but also in the regions of extreme isospin asymmetry lying close to the drip lines. A phenomenological Lagrangian density [12, 13, 17, 18, 32–39] describing the nucleon-mesons many body system can be written as,

\[
\mathcal{L} = \bar{\psi} \left( i \gamma^\mu \partial_\mu - M \right) \psi + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 \\
- \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 - \frac{1}{4} \bar{\psi} \gamma^\mu \psi - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} \\
+ \frac{1}{2} m^2_w \omega^\mu \omega_\mu + \frac{1}{4} \xi_3 (\omega^\mu \omega_\mu)^2 - \frac{1}{4} \bar{\psi} \gamma^\mu \psi \omega_\mu \\
- \frac{1}{4} \tilde{B}^{\mu\nu} \tilde{B}_{\mu\nu} + \frac{1}{2} m^2_{\rho^\mu} \rho_\mu - \frac{1}{4} \bar{\psi} \gamma^\mu \tau \psi - \bar{\rho}^\mu \\
- \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \epsilon \bar{\psi} \gamma^\mu \left( \frac{1 - \tau_3}{2} \right) \psi A_\mu. 
\]  

(6)

Here, \(m_\sigma, m_w, m_\rho\) are the masses of the corresponding \(\sigma, \omega, \) and \(\rho\) mesons, which mediate the interaction between the nucleons of mass \(M\) denoted by the Dirac spinor \(\psi, g_\sigma, g_\omega, \) and \(g_\rho\) are the nucleon-meson coupling constants and \(g_2, g_3, \) and \(\xi_3\) denote the non-linear meson self interaction constants. The quantities \(\tau\) and \(\tau_3\) in Eq. (6) denote the isospin and its third component, respectively. \(\Omega^{\mu\nu}, \tilde{B}^{\mu\nu}\) and \(F^{\mu\nu}\) symbolize the field tensors for \(\omega, \rho\) and photons, respectively and are written as,

\[
F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu 
\]

(7)

\[
\Omega^{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu 
\]

(8)

and

\[
\tilde{B}^{\mu\nu} = \partial_\mu \tilde{\rho}_\nu - \partial_\nu \tilde{\rho}_\mu. 
\]

(9)

Here, \(A_\mu\) is the electromagnetic field. The equations of motions for the Dirac nucleons and mesons can be derived using the Euler-Lagrange equations in the mean-field approximation and are written as,

\[
\left( -i \alpha \nabla + \beta (M + g_\sigma \sigma) + g_\omega \omega + g_\rho \tau_3 \rho_3 \right) \psi = \epsilon \psi, \\
\left( -\nabla^2 + m_\sigma^2 \right) \sigma(r) = -g_\sigma \rho_\sigma(r) - g_2 \sigma^2(r) - g_3 \sigma^3(r), \\
\left( -\nabla^2 + m_\omega^2 \right) \omega(r) = g_\omega \rho(r) - \xi_3 \omega^3(r), \\
\left( -\nabla^2 + m_\rho^2 \right) \rho(r) = g_\rho \rho_3(r). 
\]

(10)

The relativistic effective nucleon-nucleon interaction entities as R3Y potential [9–11] is derived from the mean-field equations for the mesons under the limit of one meson exchange. The so evaluated microscopic R3Y NN potential is written in terms of masses and coupling constants of different meson fields. The relativistic R3Y NN potential is competent to the medium-independent M3Y NN interaction potential and has been applied successfully in exploring the various nuclear phenomenon. More details can be found in Refs. [9–14, 49]. In these recent studies [9–13, 49], the density independent M3Y as well as R3Y NN potentials are used and these studies manifest that the R3Y NN potential obtained for non-linear RMF parameter sets give comparatively better overlap with the available experimental data. As discussed above, the explicit density dependence was introduced in the M3Y NN potential to account for the in-medium effects, which also results in the better description of infinite nuclear matter properties. In this direction, it is crucial and interesting to introduce the medium dependence in the relativistic R3Y NN potential. Here within relativistic mean-field formalism, it is not necessary to multiply a weighted density function with the NN potential to make it medium dependant as shown for M3Y potential in Eq. (3). To include the medium effect on relativistic NN potential (R3Y), one need to consider the density-dependant parametrization within Relativistic-Hartree-Bogoliubov (RHB) Approach, where the nucleon-meson couplings are medium dependence. It is worth mentioning that this is the first time, where we have chosen the well-established Relativistic-Hartree-Bogoliubov (RHB) approach to establish the density dependent microscopic R3Y NN potential. In RHB approach, the couplings of \(\sigma, \omega\) and \(\rho\) mesons to the nucleon fields (i.e. \(g_\sigma, g_\omega\) and \(g_\rho\)) are defined as [15, 16, 50–52],

\[
g_\nu(\rho) = g_\nu(\rho_{\text{sat}}) f_\nu(x) |_{x=\sigma, \omega}, 
\]

(11)

where

\[
f_\nu(x) = a_1 \left(1 + b_1 (x + d_1)^2 \right) \left(1 + c_1 (x + d_1)^2 \right) 
\]

(12)

and

\[
g_\nu(\rho) = g_\nu(\rho_{\text{sat}}) \exp[\epsilon_\nu(x - 1)]. 
\]

(13)

Here, \(x = \rho/\rho_{\text{sat}}, \) with \(\rho_{\text{sat}}\) is the baryon density of symmetric nuclear matter at saturation. The five constraints \(f_1(1) = 1, f_\nu(0) = 0, \) and \(f_\nu(1) = f_\nu'(1)\) reduce the number of independent parameters in Eq. (12) from eight to three. All the independent parameters (the mass of \(\sigma\) meson and coupling parameters) are obtained to fit the ground state properties of finite nuclei as well as the properties of symmetric and asymmetric nuclear matter. In present analysis, we have adopted the well-known DDME1 [15] and DDME2 parameter sets [16] to study the fusion mechanism of various reactions. The density dependent R3Y NN potential \(V_{\text{eff}}^{R3Y}(r, \rho)\) in terms of density dependent meson-nucleon coupling constants defined above can be written as,

\[
V_{\text{eff}}^{R3Y}(r, \rho) = \frac{[g_\omega(\rho)]^2 e^{-m_\omega r}}{4\pi} \frac{[g_\omega(\rho)]^2 e^{-m_\omega r}}{4\pi} + \frac{[g_\rho(\rho)]^2 e^{-m_\rho r}}{4\pi} - \frac{[g_\rho(\rho)]^2 e^{-m_\rho r}}{4\pi} + J_{\text{iso}} \delta(r). 
\]

(14)

The expression for DDR3Y in Eq. (14) is identical form as that of R3Y NN potential used in previous studies of Refs. [9–14, 49]. Here in DDR3Y, the nucleon-meson...
coupling constants are density dependent, which is constant in case of R3Y NN potential. The relativistic DDR3Y NN potential obtained for DDME1 (black) and DDME2 (orange) within Relativistic-Hartree-Bogoliubov approach. The results for density-independent R3Y NN potential is calculated for non-linear NL3$^*$ (blue) and TM1 (wine) parameter sets within relativistic mean-field formalism and non-relativistic M3Y NN potential for shake of comparison. All the calculated results for NN potentials, which is essential to reproduce the saturation properties of infinite nuclear matter. In relativistic-Hartree-Bogoliubov approach, the medium dependent nucleon-meson vertices are introduced instead of non-linear self-interaction terms and more details can be found in Refs. [15–18]. As DDME1 and DDME2 parameter sets do not contain any self-interacting non-linear terms in meson field, we have also given the R3Y potential for NL3$^*$ parameter set without the non-linear self-interaction terms (dashed line) for the sake of comparison. It can be observed from Fig. 1 that the R3Y NN potential obtained for NL3$^*$ parameter set without the non-linear terms shows the deepest pocket, followed by DDME2 and DDME1 parameter sets at saturation density. This indicates that the inclusion of non-linear meson interaction terms gives a repulsive core to the NN potentials which is essential to reproduce the saturation properties of infinite nuclear matter. In relativistic-Hartree-Bogoliubov approach, the medium dependent nucleon-meson vertices are introduced instead of non-linear self-interaction terms and more details can be found in Refs. [15–18]. Following this, the actual influence of density-dependent R3Y NN potential (see Eq. 14) within the RHB approach will be encompassed through nuclear potential within double folding model.

To evaluate the nuclear potential between two interacting nuclei, the density ($\rho$) entering in Eqs. (11-14) is obtained within the relaxed density approximation (RDA) [53, 54] at the midpoint of the inter-nucleon separation. In RDA, the nuclear fusion at around and below barrier are assumed to be a slow process that allows the relaxation of proton and neutron densities [53, 54]. There is another parallel method that exists to introduce the density for a composite system, so-called the frozen density approximation (FDA) [6, 19, 20]. The FDA and RDA are widely adopted in the double folding approach for M3Y potential as shown in Eq. (5) and Skyrme Energy Density approach, respectively. Here we have used the densities of the target and projectiles from relativistic energy density functional for various parametrizations. Here in Fig. 2, we have shown a comparison of DDME2 density ($\rho$) at the midpoint of inter-nucleon separation for an illustrative case of $^{48}$Ca+$^{238}$U system calculated within FDA (red) and RDA (black) for a representative case. It can be observed from the Fig. 2 that $\rho$ obtained within FDA for lower values of $r/2$ is much higher than the nuclear matter saturation density ($\rho_{sat} = 0.152 fm^{-3}$). This is because, in FDA, the density at a fixpoint of space is given by the sum of nucleon densities of each nucleus [6, 19, 20, 29–31], which exceeds the $\rho_{sat} = 0.152$ at smaller the inter-nucleon distances. This problem can be resolved by adopting the RDA in which the density at any space do not surpass the equilibrium density of nuclear matter [53, 54].

To assess the validity of FDA and RDA in evaluating the DDM3Y and the newly introduced DDR3Y NN potentials, we have estimated the nuclear potential obtained without non-linear coupling terms. See text for details.
within the double folding approach. Fig. 3 shows the nuclear potential calculated using DDM3Y (dashed lines) and DDR3Y (solid lines) within the FDA (red) and RDA (black) for the illustrative case of $^{48}\text{Ca}+^{238}\text{U}$ system. Here, it can be noticed that the difference between nuclear potential obtained for DDM3Y NN potential obtained within FDA and RDA is almost negligible. However, there is a remarkable difference between the nuclear potentials obtained within RDA and FDA for DDR3Y NN potential. The FDA is observed to give a comparably more attractive core to NN potential as compared to the RDA. This is because, in the case of DDM3Y, the nuclear-meson couplings of $\sigma$, $\omega$, and $\rho$-mesons are density-dependent (see Fig. 1 in [15]). Thus the FDA approximation which gives $\rho >> \rho_{\text{sat}}$ at smaller $r/2$ becomes inappropriate for obtaining the nuclear potential using DDR3Y NN potential. However, both RDA, as well as FDA, give similar results in the case of DDM3Y. Following all these observations, we have adopted the RDA [53, 54] for DDR3Y whereas the traditional FDA [6, 19, 20] is used for DDM3Y. The above discussed M3Y, DDM3Y, R3Y and DDR3Y NN potentials along with the nuclear density distributions are obtained within the RMF model for non-linear NL3* and TM1 parameter sets and RHB approach for DDME1 and DDME2 parameter sets. The densities and corresponding NN potential are used to obtain the nuclear potentials (only one representative case in Figs. (2) and (3) for DD-ME2 parameter set) within double folding procedure. The nuclear potentials thus obtained are used to study the fusion mechanism of various systems within $\ell$-summed Wong model described in the upcoming subsection.

C. Fusion/Capture Cross-Section

C. Y. Wong [22] gave a simplified expression for the fusion/capture cross-section which excludes the actual angular momentum dependence of the fusion barrier characteristics. This Wong formula was later refined in [21] to account for the actual modifications entering the fusion barrier due to its angular momentum dependence. The extended $\ell$-summed Wong model [21], which describes the cross-section in terms of $\ell$-partial wave is written as,

$$\sigma(E_{\text{c.m.}}) = \pi \frac{\ell_{\text{max}}}{\hbar^2} \sum_{\ell=0}^{\ell_{\text{max}}} (2\ell + 1) P_{\ell}(E_{\text{c.m.}}). \quad (15)$$

Here, and $k = \sqrt{2\mu E_{\text{c.m.}}/\hbar^2}$ and $E_{\text{c.m.}}$ denotes the energy of target-projectile system in the center of mass frame. In present study, $\ell_{\text{max}}$ values are extracted from the sharp cut-off model [55] for above barrier energies and extrapolated for below barrier energies. The quantity $P_{\ell}$ symbolize the transmission coefficient. Using the Hill-Wheeler [56] approximation of symmetric parabolic barrier, $P_{\ell}$ can be written in terms of barrier height ($V_B^\ell$) and curvature $\hbar\omega_{\ell}$ as,

$$P_{\ell} = \left[1 + \exp\left(\frac{2\pi(V_B^\ell - E_{\text{c.m.}})}{\hbar\omega_{\ell}}\right)\right]^{-1}. \quad (16)$$

The barrier characteristics such as the barrier height ($V_B^\ell$) and barrier position ($R_B^\ell$) can be evaluated easily once the total interaction potential (see Eq. (1)) is defined, i.e.,

$$\left.\frac{dV_T}{dR}\right|_{R=R_B^\ell} = 0. \quad (17)$$

$$\left.\frac{d^2V_T}{dR^2}\right|_{R=R_B^\ell} \leq 0. \quad (18)$$

Further, the barrier curvature ($\hbar\omega_{\ell}$) can also be evaluated at $R = R_B^\ell$ corresponding to the barrier height $V_B^\ell$ as,

$$\hbar\omega_{\ell} = \hbar \left|\frac{d^2V_T}{dR^2}\right|_{R=R_B^\ell}/\mu\frac{1}{2}. \quad (19)$$

These barrier characteristics of the total interaction potential are obtained using M3Y, DDM3Y, R3Y, and DDR3Y NN interaction potential described in detail above. Finally, the fusion/capture cross-section is determined using the extended and more precise version of the Wong formula (Eq. (15)). A detailed description of the results obtained is provided in the upcoming section.

III. RESULTS AND DISCUSSION

This section aims to assess the application of medium-dependent relativistic DDR3Y NN potential (see Eq.
**TABLE I.** The positions $R_B$ (in fm) and heights $V_B$ (in MeV) of the fusion barriers obtained using R3Y, DDR3Y, M3Y, and DDM3Y NN potentials folded with nuclear densities obtained within RHB and RMF approaches for all the reactions under study.

| Reaction | $^{16}$O+$^{44}$Mg | $^{28}$Si+$^{92}$Zr | $^{48}$Ca+$^{96}$Zr | $^{48}$Ca+$^{154}$Sm | $^{16}$O+$^{208}$Pb | $^{48}$Ca+$^{238}$U |
|----------|------------------|------------------|------------------|------------------|------------------|------------------|
| Nuclear Potential | $R_B$ | $V_B$ | $R_B$ | $V_B$ | $R_B$ | $V_B$ | $R_B$ | $V_B$ | $R_B$ | $V_B$ | $R_B$ | $V_B$ |
| R3Y-NL3$^*$ | 8.9 | 14.46 | 10.8 | 70.02 | 11.7 | 92.87 | 12.5 | 134.83 | 12.2 | 73.17 | 13.5 | 185.50 |
| R3Y-TM1 | 8.7 | 14.75 | 10.7 | 71.00 | 11.5 | 94.19 | 12.3 | 136.63 | 12.0 | 74.11 | 13.3 | 187.74 |
| DDR3Y-DDME1 | 8.7 | 14.81 | 10.5 | 71.61 | 11.4 | 95.10 | 12.2 | 137.86 | 11.9 | 74.80 | 13.2 | 189.78 |
| DDR3Y-DDME2 | 8.6 | 14.87 | 10.5 | 71.79 | 11.4 | 95.21 | 12.2 | 138.00 | 11.9 | 74.90 | 13.2 | 189.90 |
| DDM3Y-NL3$^*$ | 8.3 | 15.40 | 10.0 | 74.67 | 10.9 | 98.49 | 11.8 | 142.24 | 11.5 | 76.89 | 12.7 | 196.44 |
| DDM3Y-TM1 | 8.2 | 15.58 | 10.0 | 75.22 | 10.9 | 99.17 | 11.7 | 143.17 | 11.4 | 77.36 | 12.7 | 194.46 |
| DDM3Y-DDME1 | 8.4 | 15.21 | 10.1 | 74.01 | 11.0 | 97.88 | 11.8 | 141.44 | 11.6 | 76.43 | 12.8 | 194.66 |
| DDM3Y-DDME2 | 8.3 | 15.30 | 10.1 | 74.33 | 11.0 | 98.17 | 11.8 | 141.83 | 11.5 | 76.65 | 12.8 | 194.89 |
| M3Y-NL3$^*$ | 8.4 | 15.22 | 10.3 | 73.29 | 11.0 | 97.50 | 11.9 | 142.08 | 11.6 | 76.31 | 12.8 | 194.18 |
| M3Y-TM1 | 8.3 | 15.38 | 10.2 | 73.74 | 11.0 | 98.11 | 11.9 | 142.44 | 11.6 | 76.74 | 12.7 | 195.11 |
| M3Y-DDME1 | 8.5 | 15.05 | 10.3 | 72.73 | 11.1 | 96.98 | 11.8 | 140.55 | 11.7 | 75.88 | 12.9 | 193.36 |
| M3Y-DDME2 | 8.4 | 15.13 | 10.3 | 73.00 | 11.1 | 97.23 | 11.9 | 140.89 | 11.6 | 76.09 | 12.8 | 193.76 |

**FIG. 4.** (Color online) The total interaction potential $V_T$ (MeV) at $ℓ = 0$ fm as a function of radial separation $R$ for $^{48}$Ca+$^{238}$U system calculated using the M3Y (dash double dotted lines), DDM3Y (solid lines), R3Y (dashed lines) and DDR3Y (dotted lines) NN potentials. The different colours are for parameter sets as labeled in the figure. See text for details.
DDME2 nuclear potentials. Further investigation of Fig. 3 shows that the relativistic DDR3Y NN potential gives more attractive nuclear interaction potential as compared to DDM3Y NN potential. For a more comprehensive study, the repulsive Coulomb potential is added to all the 12 sets of nuclear potentials mentioned above.

Fig. 4 shows the barrier regions of the s-wave ($\ell = 0\hbar$) total interaction potentials obtained for $^{48}\text{Ca}+^{238}\text{U}$ reaction using all the 12 combinations of nuclear densities and effective NN potentials. The positions $R_B$ (in fm) and heights $V_B$ (in MeV) of fusion barrier for all the six reactions under study are listed in Table I. Here, DDR3Y-DDME1 and DDR3Y-DDME2 signify that both the medium-dependent NN potentials as well as the nuclear densities are obtained within the RHB approach for DDME1 and DDME2 parameter sets, respectively. The BDM3Y1 version of the density-dependent Reid NN interactions (denoted as DDM3Y) is considered here since it gives the nuclear incompressibility (K) value comparable to that given by RMF parameter sets considered in the present study. It can be observed from Fig. 4 as well Table I that the R3Y-NL3* nuclear potential gives the lowest barrier whereas the DDM3Y-TM1 provides the highest fusion barrier for all the considered heavy-ion reactions. From the comparison of barrier heights obtained for R3Y and newly introduced DDR3Y as well as those obtained for M3Y and DDM3Y NN potentials, it is observed that the inclusion of density dependence in the effective NN potential increases the fusion barrier. However, the difference between the barrier characteristics obtained for DDR3Y and R3Y is more prominent than that for DDM3Y and M3Y NN potentials. This is because, in DDR3Y, the medium-dependence is introduced microscopically in terms of the density-dependent meson-nucleon couplings. Further, comparing the results given by different nuclear density distributions folded with the same M3Y NN potentials, it is noted that densities obtained for DDME1 and TM1 parameters sets give the lowest and highest barrier, respectively. Also, the total potentials obtained within DDME1 and DDME2 almost overlap in the barrier region, with DDME2 giving a slightly higher barrier. The characteristics of the total interaction potentials obtained for different NN potentials and nuclear densities are further used to calculate the fusion probability.

The $\ell$–summed Wong model equipped with relativistic-Hartree-Bogoliubov and relativistic mean-field approaches is used to evaluate the fusion/capture cross-section for all the six reactions under study. The $\ell$–values are obtained using sharp cut-off model [55] at the above barrier center of mass energies and are extrapolated for the below-barrier region. The fusion/capture cross-section $\sigma$ (nb) as function of center of mass-energy ($E_{c.m.}$) is shown in Fig. 5 calculated using 12 different nuclear potential listed in Table I. It can be noted from Fig. 5 that among R3Y, DDR3Y, M3Y, and DDM3Y, the highest cross-section is obtained for the R3Y NN potential, whereas the DDM3Y yields the lowest cross-section for all the systems. From the comparison of results obtained for NL3*, TM1, DDME1, and DDM3Y densities folded with the same NN potential (M3Y and DDM3Y), it is observed that the DDME1 and TM1 densities give the highest and lowest cross-section, respectively.

On comparing the cross-section obtained for newly developed medium-dependent DDR3Y with that obtained for R3Y NN potential, it is observed that fusion/capture cross-section decreases on introducing the density-dependence in relativistic NN potential.

The experimental data are taken from Refs. [23–28] for comparison as shown in Fig. 5. The medium-independent R3Y NN potential, as well as nuclear densities obtained for the non-linear NL3* RMF parameter set, are observed to give a comparatively better fit to the experimental data than the other sets of nuclear potential. As discussed above, the inclusion of in-medium effects in the DDR3Y NN potential within the RHB approach decreases the cross-section which results in the under-estimation of experimental data. Both the non-relativistic M3Y and DDM3Y NN potentials are also observed to underestimate the fusion/capture cross-section for all the systems under study. For $^{28}\text{Si}+^{92}\text{Zr}$ (Fig. 5(b)), $^{48}\text{Ca}+^{96}\text{Zr}$ (Fig. 5(c)) and $^{48}\text{Ca}+^{238}\text{U}$ (Fig. 5(f)) systems, the R3Y-NL3* is also observed to slightly underestimate the cross-section at below barrier energies. However, these results may be improved by the inclusion of structure effects such as nuclear deformations and orientations, which are not taken into account in the present study for the sake of simplicity. Moreover, the difference between cross-sections obtained using different nuclear potentials ceases progressively as we move towards the higher center of mass energies. This is because, at above barrier energy-regions, the effects of nuclear structure get diminished considerably, and only the angular momentum effects persist [12, 13]. Also, the difference between cross-sections obtained for different RMF and RHB parameter sets as the mass of compound nucleus formed in the reaction increases. This indicates that a relevant choice of nuclear potential becomes more and more important as we move towards the exotic regions of the nuclear chart.

IV. SUMMARY AND CONCLUSIONS

The density dependence is introduced in the description of relativistic R3Y effective NN potential. The medium-dependent DDR3Y NN potential is obtained within the framework of the relativistic-Hartree-Bogoliubov (RHB) approach first time for the well-known DDME1 and DDME2 parameter sets. This newly developed DDR3Y NN potential is further used to obtain the nuclear potential within the double folding approach and also the fusion/capture cross-section within the $\ell$–summed Wong model. The medium-independent relativistic R3Y NN potential obtained within relativistic mean-field (RMF) formalism for NL3* and TM1 pa-
FIG. 5. (Color online) The cross-section $\sigma$ (mb) for all target projectile combinations considered in the present study by calculated using the M3Y (dash double dotted lines), DDM3Y (solid lines), R3Y (dashed lines) and DDR3Y (dotted lines) NN potentials. The different colours are for parameter sets as labeled in the figure. See text for details.

The parameter sets, as well as the non-relativistic M3Y and DDM3Y NN potentials, are also considered for the comparison. The comparison of fusion barrier characteristics and Fusion/capture cross-section obtained within different forms of NN potential (R3Y, DDR3Y, M3Y and DDM3Y) is carried out for six reactions namely $^{16}$O+$^{24}$Mg, $^{28}$Si+$^{92}$Zr, $^{48}$Ca+$^{96}$Zr, $^{48}$Ca+$^{154}$Sm, $^{16}$O+$^{208}$Pb and $^{48}$Ca+$^{238}$U leading to the formation of light, heavy and superheavy compound nuclei.

From the comparison of barrier characteristics and cross-section obtained for R3Y and newly introduced DDR3Y as well as those obtained for M3Y and DDM3Y NN potentials, it is observed that the inclusion of density dependence in the effective NN potential increases the fusion barrier, which consequently decreases the fusion/capture cross-section. However, the difference between results obtained for R3Y and DDR3Y is considerably more prominent than those obtained for M3Y and DDM3Y NN potentials. This is because the medium-dependence is introduced in DDR3Y via density-dependent meson-nucleon couplings, unlike DDM3Y, where the density-dependent is introduced through a weighted function. Further, from the comparison of cross-sections obtained using different nuclear potentials with the experimental data, it is noticed that DDR3Y NN potential underestimates the fusion/capture cross-section for all the considered reactions. However, the match between the experimental and theoretical cross-section is better for DDR3Y NN potentials than M3Y and DDM3Y NN potentials. The medium-independent R3Y NN potential and nuclear densities obtained for the NL3* RMF parameter set are observed to provide a reasonable fit to the experimental data for all the systems under study. Moreover, the difference between different RMF and RHB parameter sets increases with the increase in the mass number of the compound nuclei. Thus a more systematic study involving more reactions forming heavy and superheavy nuclei can be carried out for comprehensive analysis of the effects of different nuclear densities and NN potential on the fusion mechanism.
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