Galactic Rotation Described with Bulge+Disk Gravitational Models

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Abstract

Observations reveal that mature spiral galaxies consist of stars, gases and plasma approximately distributed in a thin disk of circular shape, usually with a central bulge. The rotation velocities quickly increase from the galactic center and then achieve a constant velocity from the core to the periphery. The basic dynamic behavior of a mature spiral galaxy, such as the Milky Way, is well described by simple models balancing Newtonian gravitational forces against the centrifugal forces associated with a rotating thin axisymmetric disk. In this research, we investigate the effects of adding central bulges to thin disk gravitational models.

Even with the addition of substantial central bulges, all the critical essential features of our thin disk gravitational models are preserved. (1) Balancing Newtonian gravitational and centrifugal forces at every point within the disk yields computed radial mass distributions that describe the measured rotation velocity profiles of mature spiral galaxies successfully. (2) There is no need for gravity deviations or “massive peripheral spherical halos of mysterious Dark Matter”. (3) The calculated total galactic masses are in good agreement with star count data. (4) The addition of central bulges increases the calculated total galactic masses, possibly more consistent with the presence of galactic gases, dust, grains, lumps, planets and plasma in addition to stars. (5) Compared with the light distribution, our mass distributions within the disk are larger out toward the galactic periphery which is cooler with lower opacity/emissivity (and thus darker). This is apparent from edge-on views of galaxies which display a dark disk-line against a much brighter galactic halo.

1 Introduction

1.1 Observational Knowledge of Radial Galactic Rotation Profiles

Telescopic images of mature spiral galaxies reveal most of the stars, gas and plasma reside in an approximately circular disk that is very thin compared with its radius, often with the presence of a central bulge. The data on galactic rotational velocity profiles (Refs.[1]-[6]) of mature spiral galaxies are characterized by a rapid increase from the galactic center, reaching a nearly constant velocity from the outer core to the outer periphery. These basic measured features may be idealized as

\[ V(r) = 1 - e^{-r/R_c}, \]

where \( V(r) \) denotes the dimensionless rotational velocity measured in units of maximum asymptotic rotational velocity \( V_0 \) and \( r \) the radial coordinate from the galactic center. The parameter \( R_c \) is a description from the data of the various “core” radii of different galaxies. Typical galactic
rotational profiles described by (1) are displayed in Fig 1. As indicated by the measurement data, the rotation velocity typically rises linearly from the galactic center (as if the local mass was in rigid body rotation), and then reach an approximately constant (flat) velocity out to the galactic periphery.

The observed galactic rotation curves (Eq(1) and Fig.1) can not (Ref.[7]) be explained by simply applying the so-called orbital velocity law, derived for a spherically symmetric gravitational field applicable to the Keplerian rotation of our solar-planet system (where most mass is located at the center), but not to galaxies with substantial mass distributed in a disk-like shape. In fact, the galactic mass distribution calculated by the orbital velocity law applied to these constant (flat) galactic rotation curves yields an increasing mass density with radius, contrary to the measured galactic luminosity curves which decrease exponentially with radius.

1.2 Thin-Disk Gravitational Models with Bulge Added

For a thin rotating galactic disk, we impose a balance between the Newtonian gravitational forces and centrifugal forces at each and every point. Because the gravitational field of a thin disk is not spherically symmetric, the orbital velocity law is not applicable. As illustrated by Feng & Gallo [8] [9], an axisymmetric thin disk gravitational model successfully describes the basic rotational dynamics of mature spiral galaxies with a mass density decreasing from the center to periphery. And the calculated total galactic masses are in good agreement with star count data.

For simplicity, the observed central bulge in mature spiral galaxies was not considered in previous idealized thin disk gravitational models by Feng & Gallo [8] [9]. In this paper, consistent with observations, we add the gravitational effects of central spherical bulges to thin disk models. We do not address the mechanism(s) maintaining the spherical shape against gravitational and centrifugal forces in this publication [10]. As observed, the central spherical bulge is implicitly assumed to rotate at the same radial speeds as the disk (in cylindrical coordinates), but this feature is not explicitly addressed because it does not affect the computational results. Only the gravitational effects of this assumed central bulge and the disk are computed in solving for the rotating disk mass distributions.

In detail, it is assumed the bulge has a spherically symmetric mass density decreasing with radial distance (in spherical coordinates) via a Gaussian function \( e^{-\beta r^2} \) where \( \beta \) is a positive adjustable parameter. This Gaussian function is convenient since changing one parameter \( \beta \) allows us to vary the size of the spherical bulge relative to the disk to examine the effects of the bulge size on galactic disk rotation. Our final generic results are not sensitive to the details of this Gaussian assumption. Our model of the entire galaxy consists of a variable superposition of two components: an axisymmetric thin disk and a spherically symmetric central bulge. Both the size and mass of the bulge relative to the disk are independently varied to examine the effects of the bulge on galactic disk rotation.

2 Governing Equations

Similar to the treatment of Feng & Gallo [8] [9], the equation of force balance in an axisymmetric thin disk including a spherically symmetric central bulge is written as

\[
\int_0^1 \left[ \int_0^{2\pi} \frac{\hat{r} \cos \phi - r}{(\hat{r}^2 + r^2 - 2 \hat{r} r \cos \phi)^{3/2}} \right] \rho(\hat{r}) h \hat{r} d\hat{r} - \frac{M_b}{r} \int_0^r \frac{e^{-\beta \hat{r}^2} \hat{r}^2 d\hat{r}}{\hat{r}^2} + A \frac{V(r)^2}{r} = 0,
\]

where all the variables are made dimensionless by measuring lengths (e.g., the radial coordinate \( r \), the radial coordinate as the variable of integration \( \hat{r} \), and the thickness of disk \( h \)) in units of...
the outermost galactic radius \( R_g \), mass of bulge \( (M_b) \) in units of the total galactic mass \( (M_g) \), disk mass density \( (\rho) \) in units of \( M_g/R^3_g \), and rotational velocity \([V(r)]\) in units of the maximum galactic rotational velocity \( V_0 \). The disk thickness \( h \) is assumed to be constant and small in comparison with the galactic radius \( R_g \). Actually the physically meaningful quantity here is the combined variable \( (\rho h) \) that represents the effective surface mass density on the thin disk. As long as the disk thickness \( h \) is much smaller than \( R_g \), its mathematical effect is inconsequential to the value of \( (\rho h) \). The gravitational forces of the finite series of concentric rings is described by the first term (double integral) while the centrifugal forces are described by the third term. The second term represents the effects of the spherically symmetric central buldge.

In (2), we call the dimensionless parameter \( A \) “galactic rotation parameter”, given by

\[
A \equiv \frac{V_0^2 R_g}{M_g G},
\]

where \( G \) denotes the gravitational constant, \( R_g \) is the outermost galactic radius, and \( V_0 \) is the maximum asymptotic rotational velocity.

As described by Feng & Gallo [8,9], both \( \rho(r) \) and \( A \) can be determined from a given \( V(r) \) and a given (but varied) value of \( M_b \) by solving an equation system including (2) and a conservation constraint for constant total mass of the galaxy \( M_g \), e.g.,

\[
2\pi \int_0^1 \rho(\hat{r}) h \hat{r} d\hat{r} = 1 - M_b.
\]

Assuming a bell-shape mass density distribution for the spherically symmetric bulge described by a Gaussian function, the bulge mass density is given by

\[
\rho_b(r) = \frac{M_b}{4\pi \int_0^1 e^{-\beta \hat{r}^2} d\hat{r}} e^{-\beta r^2},
\]

where \( \beta \) is a positive adjustable parameter. The bulge mass density is assumed to end at the galaxy rim \( (r = 1) \), so that

\[
4\pi \int_0^1 \rho_b(\hat{r}) \hat{r}^2 d\hat{r} = M_b.
\]

Figure 2 illustrates several bulge mass density distributions \( \rho_b(r) \) at various values of \( \beta \) and \( M_b \). Note the bulge mass density \( \rho_b(r) \) is spherically symmetric, whereas the disk mass density \( \rho(r) \) is only axisymmetric and \( r \) denotes the radial disk coordinate in the cylindrical coordinate system used in the present computations.

3 Computational Techniques

To facilitate numerical computation, we discretize the governing equations (2) and (4) by dividing the one-dimensional problem domain \([0,1]\) into a finite number of line segments called (linear) elements. As described by Feng & Gallo [8], each element covers a subdomain confined by two end nodes, e.g., element \( i \) corresponds to the subdomain \([r_i, r_{i+1}]\) where \( r_i \) and \( r_{i+1} \) are the nodal values of \( r \) at nodes \( i \) and \( i+1 \), respectively. With each of the \( N - 1 \) elements mapped onto a unit line segment \([0,1]\) in the \( \xi \)-domain (i.e., the computational domain), \( N \) independent residual equations can be obtained from the collocation procedure, i.e.,

\[
\sum_{n=1}^{N-1} \int_0^1 \left[ \frac{E(m_i)}{\hat{r}(\xi) - r_i} - \frac{K(m_i)}{\hat{r}(\xi) + r_i} \right] \rho(\xi) h\hat{r}(\xi) \frac{d\hat{r}}{d\xi} d\xi + \frac{1}{2} \left[ AV(r_i)^2 - \frac{M_b}{r} \int_0^1 e^{-\beta \hat{r}^2} \hat{r}^2 d\hat{r} \right] = 0,
\]

(6)
where $K(m)$ and $E(m)$ denotes the complete elliptic integrals of the first kind and second kind, with
\[ m_i(\xi) \equiv \frac{4\dot{r}(\xi)r_i}{[\dot{r}(\xi) + r_i]^2}. \]  

(7)

The $N$ residual equations (6) can be used to compute either the $N$ nodal values of $V(r_i)$ from given distribution of $\rho(r_i)$ or the distribution of $\rho(r_i)$ from a given set of $V(r_i)$, with given values of $A$ and $h$. Without loss of generality, the value of $h$ is assumed to be 0.01 as comparable with that observed for the Milky Way galaxy. If the constraint equation (4) is also used with a discretized form
\[ 2\pi \sum_{n=1}^{N-1} \int_0^1 \rho(\xi) h\dot{r}(\xi) \frac{d\dot{r}}{d\xi} d\xi - 1 + M_b = 0, \]  

(8)

the value of $A$ can also be determined as part of the numerical solution.

These generally applicable equations are conveniently used for computing variables, even when analytical formulas are available for some special cases. Hence, a unified treatment for all cases is established for convenient comparison and analysis. Moreover, as discussed by Feng & Gallo [8], imposing a boundary condition at the galactic center $r = 0$ for continuity of derivative of $\rho$, i.e., in discretized form
\[ \rho(r_1) = \rho(r_2), \]  

(9)

is desirable for obtaining high-quality numerical solutions.

With the adjustable parameters such as $R_c$, $\beta$, and $M_b$ specified and mathematical singularities properly treated, linear equations (6) and (8) for $N + 1$ unknowns can be solved with a standard matrix solver, e.g., by Gauss elimination [11].

4 Computational Results for Bulge+Disk Models

Here the effects of bulge parameters on galactic disk mass distributions compatible with measured rotational velocities are explored. Attention is focused on the Milky Way galaxy which has a rotation velocity profile (1) closely represented in Figure 1 with $R_c = 0.015$ (cf. Feng & Gallo [8, 9]).

4.1 Disk Mass Density Distributions Calculated with Constant $\beta = 100$ but Various Bulge Masses $M_b$

Figure 3 shows the disk mass density distributions computed with constant $\beta = 100$ but various bulge masses ($M_b = 0, 0.1, 0.2$, and 0.25). All these curves are for the Milky Way galaxy with a rotation curve with $R_c = 0.015$ according to (1). Note that $M_b = 0$ represents a thin disk without a bulge. With increasing bulge mass $M_b$, a localized decrease of disk mass density appears where the bulge mass has significant density around the galactic center. A local minimum develops around $r = 0.1$ for $M_b \geq 0.2$. When the value of $M_b$ is further increased, the nodal values of disk mass density $\rho$ around local minimum may become negative which is physically unacceptable. Thus, our computational results demonstrate an upper limit for the bulge mass $M_b$ corresponding to a given bulge size as characterized by the value of $\beta$. This is consistent with reality. In all bulge + disk cases, the measured galactic rotation profiles are accurately reproduced.

Another noteworthy feature in Figure 3 is that the bulge influence on the disk mass distribution diminishes beyond $r = 0.2$, where the bulge reaches its effective edge (cf. Figure 2).
4.2 Disk Mass Density Distributions Computed with Various Combinations of Bulge Parameters $M_b$ and $\beta$

Figure 4 shows the computed disk mass density distributions for various combinations of bulge masses $M_b$ and $\beta$. For reference, the unlabeled curve is for $M_b = 0$ which represents a thin disk without a bulge. In all cases, the appropriate calculated galactic rotation parameters $A$ are indicated. All these curves are for the Milky Way galaxy with $R_c = 0.015$. In all bulge + disk cases, the measured galactic rotation profiles are accurately reproduced.

5 Total Galactic Mass

The measured rotational velocity profiles $V(r)$ includes knowledge of maximum rotational velocity $V_0$ and galactic radius $R_g$. With the computed value of the galactic rotation parameter $A$, the total galactic mass $M_g$ can be calculated as

$$M_g = \frac{V_0^2 R_g}{AG}.$$  \hfill (10)

To check viability we investigate the idealized rotational velocity profile $V(r)$ of our own Milky Way galaxy shown in Figure 1 with core radius $R_c = 0.015$. From galactic rotation measurements, the parameters appropriate for the Milky Way galaxy are $R_c = 0.015$, $V_0 = 2.5 \times 10^3 (m/s)$, and $R_g = 10^5$(light-years) $= 9.46 \times 10^{20}$($m$). In Table 1, the total galactic mass $M_g$ of the Milky Way galaxy is calculated for a wide range of bulge masses $M_b$ and bulge sizes $\beta$, and the corresponding computed values of galactic rotation parameter $A$. The total mass of the Milky Way galaxy is then determined from (10) to be in the range

$$M_g = 2.8 - 3.2 \times 10^{11}$(solar-mass)$.$

These values are in very good agreement with Milky Way star counts of 100 billion. In Table 1, note that a large increase in bulge mass $M_b$ only yields a small increase (10 percent) in the total galactic mass $M_g$ over the disk-only case (no bulge) ($M_b = 0$). However, these larger galactic masses may possibly be more compatible with reality since the galaxies also contain gases, dust, grains, lumps, planets and plasma, all in addition to stars.

However, we emphasize the essential physics of galactic rotation is gravitationally controlled by the ordinary baryonic matter within thin galactic disks. The central bulge has minor effects because the comparatively small amount of matter in the outer regions of the disk are gravitationally more effective in controlling the rotational dynamics of the galactic periphery.

6 Ordinary Baryonic Matter versus Dark Matter

To theoretically describe the measured rotational velocity curves of spiral galaxies, there are three very different approaches and conclusions.

(1) Ordinary Baryonic Matter. We assume Newtonian gravity/dynamics and computationally solve for mass distributions that successfully duplicate the measured rotational velocities. These mass distributions decrease roughly exponentially from the galactic center in the central core, but then decrease more slowly (inversely with radius) towards the periphery. This decrease is slower than the measured light distribution. Thus there is ordinary baryonic matter within the galactic disk distributed towards the cooler periphery with lower emissivity/opacity and therefore darker. Our view is consistent with edge-views of galaxies which exhibit a dark disk line against a much
Table 1: For the Milky Way galaxy, the total galactic mass \( M_g \) (in units of solar-mass) is calculated from data and various bulge parameters. From the measured galactic rotation curve, \( R_c = 0.015 \), \( R_g = 9.46 \times 10^{20} \) (m), and \( V_0 = 2.5 \times 10^5 \) (m/s). A range of values of bulge parameters \((M_b, \beta)\) are examined along with the corresponding computed values of the galactic rotation parameter \( A \). The computed total galactic mass \( M_g \) increases only 10 percent over the disk-only case (no bulge) \((M_b = 0)\) in spite of large increases of the examined bulge mass \( M_b \).

| \( M_b \) | \( \beta \) | \( A \) | \( M_g \) (solar-mass) |
|---|---|---|---|
| 0 | 1.57 | 2.84 \times 10^{11} |
| 0.1 | 500 | 1.57 | 2.84 \times 10^{11} |
| 0.1 | 100 | 1.56 | 2.86 \times 10^{11} |
| 0.15 | 200 | 1.56 | 2.86 \times 10^{11} |
| 0.2 | 200 | 1.56 | 2.86 \times 10^{11} |
| 0.25 | 100 | 1.54 | 2.89 \times 10^{11} |
| 0.3 | 50 | 1.52 | 2.93 \times 10^{11} |
| 0.4 | 20 | 1.47 | 3.03 \times 10^{11} |
| 0.5 | 10 | 1.39 | 3.21 \times 10^{11} |

brighter galactic halo. There are no mysteries in this rational scenario based on verified physics (Refs. [12]-[31]).

(2) Dark Matter. By contrast, others inaccurately assume the galactic mass distributions follow the measured light distributions (approximately exponential), and then the measured rotational velocity curves are not duplicated. But this assumption of a simple direct relationship between light intensity and mass is very inaccurate because it is not based on sound physical principles. This so-called Mass/Light ratio is inaccurate since both the temperature and opacity/emissivity are important but ignored variables. These deficiencies are clear from edge-on views of spiral galaxies where a dark galactic line is obvious against a bright galactic background, revealing the substantial radial temperature gradient across the galaxy. There is no simple direct relationship between mass and light, and such an assumption is grossly over-simplified (Refs. [35]-[60]).

With this inaccurate assumption, the discrepancy between measured and calculated velocity profiles are particularly severe beyond the galactic core. To alleviate this discrepancy, speculations are invoked re “massive peripheral spherical halos of mysterious Dark Matter” But no significant matter has been detected in this untenable unstable gravitational halo distribution. This speculated Dark Matter is “mysterious” since it does not interact with electromagnetic fields (light) nor ordinary matter except through gravity. This Dark Matter must have other abnormal (non-baryonic) properties to maintain its peripheral spherical shape against the galactic rotation and gravitational attraction of ordinary matter. Many unverified “mysteries” are invoked as solutions to real physical phenomena (Refs. [35]-[60]).

(3) Modified Gravity. Possible deviations from Newtonian gravity/dynamics have been proposed, but there is no independent experimental evidence of such deviations. Our use of Newtonian gravity/dynamics with sound computational techniques has proven successful to explain the observed flat rotation curves (Refs. [61]).

Conclusion. We conclude our approach utilizing Newtonian gravity/dynamics and computationally solving for the ordinary baryonic mass distributions within the galactic disk simulates reality and agrees with data.
7 Nature of Ordinary Baryonic Matter

Our approach yields higher total galactic masses in agreement with star counts. Concurrently, our mass density distributions also yield more mass distributed out towards the cooler galactic periphery. We wish to conjecture about the nature of some of this ordinary baryonic matter which appears dark. In addition to stars, this material is some combination of dust, grains, lumps, planets, plasma and gases.

Consider hydrogen gas. Since the temperature is higher in the galactic core and cooler towards the galactic periphery, we expect more ionized hydrogen (plasma) in the hotter core, then more atomic hydrogen away from the core, and finally more molecular hydrogen out towards the cooler periphery. This molecular hydrogen is often ignored, although measurements have revealed its presence (Ref.[62]). However, quantitative estimates of its density vary widely. We note that molecular hydrogen is a naturally dark material since it has very low emission and absorption coefficients due to its high molecular structural symmetry. Combined with its presence towards the cooler periphery, we qualitatively believe this molecular hydrogen (Refs.[63]-[64]) is one component of the ordinary baryonic dark matter in the disk periphery we have found from analysis of galactic rotation profiles.

8 Limitations and Strengths of Disk and Bulge+Disk Models

Our Disk and Bulge+Disk gravitational models do not address many important features such as spiral structure, plasma effects, galactic formation, galactic evolution, galactic jets, black holes, relativistic effects, galactic clusters, etc..

It is well known (Ref.[7]) that the internal gravitational behavior of a thin disk is much different than a sphere. This distinctly different behavior of disks enables our models to describe the rotational dynamics of mature spiral galaxies, and their total galactic masses, even with the addition of substantial central spherical bulges. The reason is that matter in the outer disk periphery is gravitationally closer and stronger in controlling the rotation in those outer regions, even compared with the substantial central spherical bulges which are more distant and gravitationally weaker.

Our gravitational models have finite radial extent. Beyond the galactic radius, we assume the density has dropped to the inter-galactic level, which is approximately spherically symmetric and thus no longer affects the galactic dynamics. We mention this because some others (Refs.[7]-[35]), have taken the relevant integrals to infinity, which we think is inappropriate.

In our approaches, we balance the gravitational forces against the centrifugal forces at each and every point within the disk. Thus, our solutions for the mass distributions and total galactic mass satisfy the rotational velocity measurements and ensure stability within the same context as similar calculations for our Solar System and Earth satellites. Some previous authors obtain solutions that are not gravitationally stable because they obtain incorrect mass distributions and incorrect galactic masses and do not satisfy the measured rotational profiles. Thus, their solutions are unstable, whereas our solutions are stable within the Newtonian context.

Plasma effects are certainly active in the formation and evolution of galaxies from the original hot plasma (Refs.[65]-[67]). However, for mature spiral galaxies, the free plasma density has dropped to levels sufficiently low that plasma does not affect the predominantly gravitational galactic dynamics. This is evidenced in our own Solar System in which gravitational dynamics dominate even with the observed effects of solar wind, coronal mass ejections, auroras, comet tails, etc. The plasma in our Sun is stabilized by gravitational forces, even though plasma effects are very active within the Sun itself. Since our Solar System is approximately 1/3 distance from the center of our Milky Way galaxy, we have our Solar System evidence for the dominance of gravitational forces
within our own Milky Way galaxy, at least at this radial distance and beyond to the periphery. We expect plasma phenomena to be more active in the hotter central galactic core.

Summarizing, both our Disk and Bulge+Disk models are sufficient to describe the rotational dynamics of mature spiral galaxies and their total galactic masses. Disk models with an additional central Bulge yield higher total galactic masses.

9 Conclusions

Even with the addition of substantial central galactic bulges, all the critical essential features of our thin disk gravitational models are preserved. (1) Balancing Newtonian gravitational and centrifugal forces at every point within the disk yields computed radial mass distributions that describe the measured rotation velocity profiles of mature spiral galaxies successfully. (2) There is no need for gravity deviations or “massive peripheral spherical halos of mysterious Dark Matter”.

(3) The calculated total galactic masses are in good agreement with star count data. (4) The addition of central bulges increases the calculated total galactic masses, possibly more consistent with the presence of galactic gases, dust, grains, lumps, planets and plasma in addition to stars. (5) Compared with the light distribution, our mass distributions within the disk are larger out toward the galactic periphery which is cooler with lower opactiy/emissivity (and thus darker). This is apparent from edge-on views of galaxies which display a dark disk-line against a much brighter galactic halo.

Most previous research assumes a galactic density decreasing exponentially with radius out to the galactic periphery, analogous to the measured light distribution. But this assumption (by others) is inaccurate since both the temperature and opacity/emissivity are important but ignored variables. There is no simple relationship between mass and light. These prior models do NOT describe the measured velocity profiles, and speculations are invoked re halos of mysterious Dark Matter or gravitational deviations to compensate. The Dark Matter must have “mysterious” (non-baryonic) properties because there is no evidence of its existence and it is not responding to gravitational, centrifugal and electromagnetic forces in any known manner. By contrast, our results indicate no massive peripheral spherical halos of mysterious Dark Matter and no deviations from simple gravity. Our total galactic mass determinations are also in reasonable agreement with data.

The controversy is summarized as follows.

We believe there is ordinary baryonic matter within the galactic disc distributed more towards the galactic periphery which is cooler with lower opacity/emissivity (and therefore darker).

Others believe there are massive peripheral spherical halos of mysterious Dark Matter surrounding the galaxies.

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Figure 1: Typical Galactic Rotational Velocity Profiles $V(r)$ idealized from measurements for $R_c = 0.015, 0.03, 0.05, \text{ and } 0.1$. 
Figure 2: Bulge mass density distributions $\rho_b(r)$ from (5) are displayed for reasonable bulge parameters we have examined ($\beta = 10, 100, 200, \text{and } 500$ with $M_b = 0.5, 0.25, 0.15, \text{and } 0.1$).
Figure 3: Disk mass density $\rho(r)$ computed with $\beta = 100$ and various bulge masses. $M_b = 0, 0.1, 0.2,$ and $0.25$. Note the case of $M_b = 0$ represents a thin disk without a bulge. All these curves are for the Milky Way galaxy with $R_c = 0.015$. In all cases, the appropriate calculated galactic rotation parameters $A$ are indicated. In all these bulge + disk cases, the measured galactic rotation profiles are accurately reproduced.
Figure 4: Disk mass density $\rho(r)$ computed for various combinations of bulge parameters. For $M_b = 0.1$, $\beta = 500$, and $A = 1.57$. For $M_b = 0.15$, $\beta = 200$, and $A = 1.56$. For $M_b = 0.25$, $\beta = 100$, and $A = 1.54$. For $M_b = 0.5$, $\beta = 10$, and $A = 1.39$. For comparison, the curve without label is for $M_b = 0$ which represents a disk without a bulge. In all cases, the appropriate calculated galactic rotation parameters $A$ are indicated. The constant value of $R_c = 0.015$ is utilized which represents the Milky Way galaxy. In all bulge + disk cases, the measured galactic rotation profiles are accurately reproduced.