Bound States Can Stabilize Electroweak Strings

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ABSTRACT:

We show that the electroweak $Z$–string can be stabilized by the presence of bound
states of a complex scalar field. We argue that fermions coupled to the scalar field of the
string can also make the string stable and discuss the physical case where the string is
coupled to quarks and leptons. This stabilization mechanism is expected to work for other
embedded defects and also for unstable solutions such as the sphaleron.
It is now known that vortex solutions\(^1\) may be embedded in almost any field theoretic model that exhibits spontaneous symmetry breaking\(^2\). In particular, two distinct vortex solutions are known to be embedded in the standard electroweak model\(^2,3,4,5\). These are called the \(\tau\)–string and the \(Z\)–string in the literature. The \(\tau\)–string is conjectured to be unstable for all values of the parameters while the \(Z\)–string has been shown to be stable only for \(\sin^2 \theta_W \approx 1\). The \(Z\)–string in the standard electroweak model with \(\sin^2 \theta_W = 0.23\) is unstable\(^6\) and remains so even at high temperatures such as would be present in the early universe\(^7\).

If the string solutions are indeed unstable under all circumstances, their relevance to physical processes would probably be negligible. However, in this letter we shall show that the strings can be stabilized by the presence of other scalar or fermionic fields in the theory. The idea behind this result is quite simple to understand, especially if one is aware of the reason that permits the existence of non-topological solitons\(^8\). Suppose that we have a theory in which the Higgs mechanism is responsible for generating the mass of a certain scalar. Then, after the symmetry breaking, the Higgs field gives the scalar a mass but the back-reaction of the scalar field on the Higgs field is to try and prevent the Higgs field from acquiring its vacuum-expectation-value (vev). In other words, the scalar would rather live in a region where the Higgs field vanishes since the mass of the scalar field is zero wherever the Higgs field is zero. But the center of the string is precisely a region where the Higgs field vanishes. Therefore the scalar likes to accumulate on the string and tends to maintain the string configuration with its region of vanishing Higgs field - that is, the scalar adds to the stability of the configuration. Yet another way of stating this idea is that \textit{the string is a “bag” in which the scalar prefers to sit and, hence, hold together.}

In what follows, we shall only consider the case of a scalar field interacting with the electroweak \(Z\)–string. To start with, we shall describe the effect of scalar bound states on semilocal strings\(^9\) where it is fairly clear that the stability improves due to the bound state.
This in itself shows that the electroweak $Z$–string will become more stable when it has scalar bound states since, after all, the $Z$–string is nothing but the semilocal string when $\sin^2\theta_W = 1$. However, we go further and explicitly examine the case of the $Z$–string with a scalar bound state. Our results suggest that it may be possible to get stable $Z$–strings even when $\sin^2\theta_W = 0.23$.

This does not immediately imply that stable $Z$–strings occur in the standard electroweak model since there is no extra scalar field in this model. However, the standard model does contain leptons and quarks which will also have bound states on the string. We expect the arguments of the previous paragraphs to apply in this case too since, once again, it is favorable for the fermion to sit in the string “bag” and to prevent the bag from decaying. Whether the lepton and quark bound states are sufficient to stabilize the $Z$-string is another story that needs detailed investigation. We hope to undertake this task in the near future.

The Lagrangian that yields semilocal string solutions with an additional complex scalar field is:

$$L_{sl} = (D^\mu \phi)^\dagger (D_\mu \phi) + (\partial^\mu \chi)^\ast (\partial_\mu \chi) - \frac{1}{4} F_{Z\mu\nu}^Z F^{Z\mu\nu} - V(\phi, \chi)$$ (1)

where,

$$V(\phi, \chi) = \lambda_1 \left( \phi^\dagger \phi - \frac{\eta^2}{2} \right)^2 + \lambda_2 |\chi|^4 + 2\lambda_3 (\phi^\dagger \phi \pm m^2) \chi^* \chi.$$ (2)

The field $\phi$ is a global $SU(2)$ doublet carrying a gauged $U(1)$ charge, while $\chi$ is a single complex field. The covariant derivative is defined by,

$$D_\mu = \partial_\mu + \frac{i}{2} \alpha Z_\mu.$$ (3)

There are two approaches to finding solutions that describe a string with a non-trivial $\chi$ configuration. The first is that, for the negative sign in (2) and for some values of the parameters, the string configuration together with $\chi = 0$ is unstable, and the stable
ground state solution is one that has a non-trivial \( \chi \) condensate on the string\(^{10} \). It may be speculated that the presence of a condensate\(^{11} \) might improve the stability of the string. Indeed we have checked that there is an improvement in string stability due to a condensate but the improvement is only marginal and is certainly not enough to stabilize the string when \( \sin^2 \theta_W = 0.23 \). The second approach is to consider the string in the presence of \( \chi \) particles - that is, the string with \( \chi \) bound states. The \( \chi \) particles carry a conserved \( U(1) \) global charge which is derived from the conserved current

\[
j^\mu = \frac{i}{2}(\chi^* \partial^\mu \chi - \chi \partial^\mu \chi^*) .
\]

Hence, we consider a string in the presence of a certain amount of global \( U(1) \) charge. This, together with cylindrical symmetry, leads to the following ansatz for \( \chi \):

\[
\chi = e^{i\omega t} \psi(r) ,
\]

where \( r \) is the cylindrical radial coordinate. The charge per unit length along the \( z \)-direction in this configuration is:

\[
q = 2\pi \omega \int dr \ r \psi^2 .
\]

We will look at solutions of the equations of motion following from (1) that consist of a semilocal string and a fixed amount of \( U(1) \) charge. In accordance with the usual ansatz for the semilocal string\(^9 \), we take,

\[
\phi = \begin{pmatrix} 0 \\ f(r)e^{i\theta} \end{pmatrix} , \quad Z_\mu = -\frac{v(r)}{r} \hat{e}_\theta .
\]

It is now convenient to rescale the fields and coordinates to make them dimensionless:

\[
P = \sqrt{2} \frac{\eta}{f} , \quad V = \frac{\alpha}{2} v , \quad w = 4\sqrt{\lambda_3} \frac{\psi}{\alpha \eta} , \quad R = \frac{\alpha \eta}{2\sqrt{2}} r .
\]

Then the equations of motion are:

\[
P'' + \frac{P'}{R} - (1 - V)^2 \frac{P}{R^2} + \beta(1 - P^2)P - w^2 P = 0 ,
\]
\[ V'' - \frac{V'}{R} + 2(1 - V)P^2 = 0 , \quad (10) \]

\[ w'' + \frac{w'}{R} - \lambda w^3 - \gamma (P^2 - \delta^2)w = 0 \quad (11) \]

where primes denote derivatives with respect to \( R \). The parameters entering these equations are defined by:

\[ \beta = \frac{8\lambda_1}{\alpha^2} = \frac{m_H^2}{m_Z^2} , \quad \lambda = \frac{\lambda_2}{\lambda_3} , \quad \gamma = \frac{8\lambda_3}{\alpha^2} = \frac{m_\chi^2}{m_Z^2} , \quad \delta^2 = \frac{2}{\eta^2} \left( m^2 + \frac{\omega^2}{2\lambda_3} \right) . \quad (12) \]

Here \( m_H, m_Z \) and \( m_\chi \) denote the masses of the \( \phi, Z \) and \( \chi \) particles respectively. In addition to the equations (9), (10) and (11), we also have the constraint that the rescaled (dimensionless) charge is some fixed non-zero constant. Therefore,

\[ \bar{q} \equiv \frac{\omega}{\eta \sqrt{\lambda_3}} \int dR \ R \ [w(R)]^2 = \text{constant} . \quad (13) \]

The boundary conditions on \( P, V \) and \( w \) are: \( P(0) = 0, P(\infty) = 1, V(0) = 0, V(\infty) = 1, w'(0) = 0 \) and \( w(\infty) = 0 \).

So far we have been looking at the unperturbed string plus bound state solution. Now we turn to the stability analysis.

The stability analysis of the semilocal string\(^\text{12}\) can be reduced to an analysis of the perturbation in the upper component of \( \phi \) alone. Even in the presence of a bound state, this remains true since it is the upper component of \( \phi \) which provides a channel for the string to unwind on the vacuum manifold. If the upper component of \( \phi \) was forced to remain zero, the semilocal string would be identical to the Nielsen-Olesen string which we know to be topologically stable even in the presence of other fields. Hence, it is sufficient to examine perturbations in \( \phi_1 \) - the rescaled upper component of \( \phi \). Furthermore, it is sufficient to consider \( \phi_1 \) to be real and a function of the radial coordinate alone\(^\text{6,13}\).

The energy variation due to the perturbation \( \phi_1 \) is:

\[ \delta E = \eta^2 \pi \int dRR \left[ \phi_1'^2 + M^2(R)\phi_1^2 \right] , \quad (14) \]
where,

\[ M^2(R) = \frac{V^2}{R^2} + \beta (P^2 - 1) + w^2. \] (15)

It is immediately obvious that the presence of a bound state improves the stability of the semilocal string since the contribution to \( M^2 \) coming from \( w \) is always positive. In the absence of a bound state, we know that the semilocal string is stable only for \( 0 \leq \beta \leq 1 \). Hence, a bound state on the semilocal string will stabilize the string for values of \( \beta \) larger than 1. A quantitative statement about the stability of the “bound semilocal string”, however, requires a numerical analysis since the bound state will also back-react on the unperturbed string configuration. Here, since we are primarily interested in the electroweak string, we simply remark that our numerical analysis confirms that the semilocal string can be stabilized for \( \beta > 1 \) if we include a suitable bound state on the string.

The electroweak string will have the same field configuration as the semilocal string, with all the gauge fields except the Z set to zero. To analyze its stability, we use the results of Ref. 6, where it is shown that the stability issue reduces to asking if there are any negative eigenvalues (\( \Omega \)) to the Schrödinger equation:

\[
-\frac{1}{R} \frac{d}{dR} \left( R \frac{d\zeta}{Q \frac{dR}{P}} \right) + U(R)\zeta = \Omega \zeta
\]

with

\[ U(R) = \frac{1}{Q} \frac{P^2}{P^2} + \frac{2S}{R^2P^2} + \frac{1}{R} \frac{d}{dR} \left( \frac{R \frac{P'}{Q \frac{P}}}{P} \right) \] (17)

\[ Q = (1 - 2cos^2\theta_W V)^2 + 2cos^2\theta_W R^2 P^2 \] (18)

\[ S = \frac{P^2}{2} - cos^2\theta_W \frac{V'^2}{Q} + \frac{R}{2} \frac{d}{dR} \left[ \frac{V' (1 - 2cos^2\theta_W V)}{Q} \right]. \] (19)

The boundary conditions on \( \zeta \) are: \( \zeta(0) = 1, \zeta'(0) = 0 \) and \( \zeta(\infty) = 0 \).

We have first found the unperturbed configuration using (9), (10) and (11) subject to the constraint (13) by using numerical relaxation techniques. Then we have solved (16) by
a numerical shooting method which allows us to check if $\Omega$ is positive or negative. In all our numerical work we have taken $\beta = 0.40$ ($m_H = 58$ GeV) - this satisfies the experimental constraint $\beta > 0.38$ obtained by LEP. In Fig. 1 we show the minimum value of $\sin^2 \theta_W$ that is required for a string with a certain amount of charge to be stable in the case when $\lambda = 0 = m$ and $\gamma = 1$. (The plot is not very sensitive to the value of $\gamma$.) This plot shows that the stability of the electroweak $Z$–string greatly improves in the presence of bound states and stability at lower values of $\sin^2 \theta_W$ may be achieved by putting enough charge on the string. Although our numerical analysis was not able to find a stable solution for the physical case $\sin^2 \theta_W = 0.23$, we feel that a more extensive exploration of parameter space might result in a stable solution in this case also. On the other hand, this issue is not very relevant since the truly physical model does not contain an extra scalar field like $\chi$.

The reader may wonder if the introduction of the field $\chi$ could have introduced some new instability in the configuration. As was shown in Ref. 25 in the case of cylindrical non-topological solitons, there is a possible instability in the distribution of charge along the string. We have checked that this instability is also present in the electroweak string with $\chi$ bound states at least in the case when $m^2 = 0$. A little thought, however, reveals that the instability is peculiar to our toy model and would be absent in the physically realized model. This is because $\chi$ and $\phi$, being spin 0 fields, lead to an attractive force between the $\chi$ charges, so that the unperturbed linear distribution of charge on the string is unstable to clumping up into spherical distributions. In the physical model, however, there are gauge fields (spin 1) present in the theory which would lead to repulsive forces between the bound charges and would prevent the clumping instability$^{15}$.

The most pertinent question at this juncture is if the standard electroweak model also admits stable bound electroweak strings. In this case one needs to look at fermionic bound states with the standard number of quarks and leptons and with the experimentally
determined parameters. The physical argument - in which we view the string as a bag - applies to fermions also and so fermionic bound states will also improve the stability of electroweak strings. One difference with the bosonic case is that fermions obey the Pauli exclusion principle and so every additional fermion that we put in the string bag must occupy a different quantum state. This makes it somewhat less energy efficient to pack fermions onto the string\textsuperscript{16}.

There are some additional (technical) difficulties in investigating the bound $Z$–string in a realistic setting. The first such difficulty is that the quarks and leptons carry electromagnetic charge and the electromagnetic field of the bound state must also be taken into account in the stability analysis. A second difficulty is that the stability analysis must necessarily include the neutrinos since these couple to the perturbations of the Higgs field. Both these difficulties promise to make the realistic stability analysis an Herculean task.

The stabilizing effect of bound states would be felt by other embedded defects as well\textsuperscript{17}. In particular, one may ask if bound states can stabilize the electroweak $\tau$–string. It would also be of some interest to study the effects of bound states on embedded monopole configurations\textsuperscript{2}. Given the rather general stability analyses of monopoles\textsuperscript{18,19}, it would be worthwhile to see how bound states can fail to stabilize the embedded monopole. Another unstable configuration that might be stabilized by the presence of bound states is the sphaleron\textsuperscript{5}.

Finally we would like to make one more comment that is relevant for cosmology and the observational prospects for electroweak defects. A loop of electroweak string has two distinct instabilities: the first is the field-theoretic instability that we have discussed above and the second is a dynamical instability against collapse. In this letter we have argued that the presence of bound states on the string can protect the string against the field-theoretic instability. These bound states also add to the energy density of the string without correspondingly adding to the pressure. This implies that the dynamics of bound strings
should be similar to that of wiggly strings\textsuperscript{20} or to current carrying superconducting strings - depending on the nature of the charge on the string. It is also known from previous work that currents\textsuperscript{10} on the string can protect the loop against dynamical collapse\textsuperscript{21,22,23,24}. These facts together suggest the possibility that there are stable, static ring configurations (also, vortons\textsuperscript{22}) in the standard electroweak model. If the sphaleron is also stabilized by bound states, it would imply additional particle-like solutions in the model.

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11. Note the distinction between “condensate” and “bound state”. A condensate is the ground state configuration of $\chi$ in the background of the string while a bound state requires the presence of $\chi$ particles.

17. The simplest example of an embedded defect is a domain wall in a global $U(1)$ model.

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16. In both the bosonic and fermionic cases, the addition of more charge is expected to lead to diminishing returns in improved stability. This is because the string “swells” as we increase the charge and this costs energy.
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