Matched Pulse Propagation in a Three-Level System

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ABSTRACT

The Bäcklund transformation for the three-level coupled Schrödinger-Maxwell equation is presented in the matrix potential formalism. By applying the Bäcklund transformation to a constant electric field background, we obtain a general solution for matched pulses (a pair of solitary waves) which can emit or absorb a light velocity solitary pulse but otherwise propagate with their shapes invariant. In the special case, this solution describes a steady state pulse without emission or absorption, and becomes the matched pulse solution recently obtained by Hioe and Grobe. A nonlinear superposition rule is derived from the Bäcklund transformation and used for the explicit construction of two solitons as well as nonabelian breathers. Various new features of these solutions are addressed. In particular, we analyze in detail the scattering of “invertons”, a specific pair of different wavelength solitons one of which moving with the velocity of light. Unlike the usual case of soliton scattering, the broader inverton changes its sign through the scattering. Surprisingly, the light velocity inverton receives time advance through the scattering thereby moving faster than light, which however does not violate causality.
1 Introduction

The nonlinear interaction between radiation and a multi-level optical medium has received considerable interests for many years. Recently, this topic has attracted more attention in the context of lasing without inversion \cite{1, 2} and electromagnetically induced transparency (EIT) \cite{3}. EIT is a technique for rendering an otherwise optically thick medium transparent to a weak probe laser by coupling the upper level coherently to a third level by a strong laser field. The transparency for pulses propagating through an optically thick medium has been known earlier, particularly for a soliton (2π pulse) in the shape of the hyperbolic-secant type \cite{4} and through a three-level medium \cite{5}. More recently, there appeared exact analytic solutions for a pair of solitary waves, so called matched-solitary-wave pairs (MSP), propagating through a three-level medium whose invariant shapes are more general than the hyperbolic-secant type \cite{6}.

In this paper, we present a new type of analysis for the three-level coupled Schrödinger-Maxwell equation based on the matrix potential formalism. In particular, we find the Bäcklund transformation of the coupled Schrödinger-Maxwell equation in terms of the matrix potential variable and apply the Bäcklund transformation to the constant electric field background to obtain new MSP solutions which generalize the result in Ref. \cite{6}. These solutions in general describe the breakup of a MSP into another MSP with slower velocity and a soliton pulse moving with light velocity, or the reverse process of fusing a MSP and a soliton into another MSP. With a specific choice of parameters, these solutions reduce to the MSP solution in in Ref. \cite{6}, which describes steady-state propagation of MSP without breakup. The generality in the shape of a MSP solution is explained through the $SU(2) \times U(1)$ group symmetry of the three-level Λ-system with equal oscillator strengths. We show that the Bäcklund transformation also allows a nonlinear superposition rule for solitons as well as MSP solutions. An explicit formula for the nonlinear superposition is given in terms of matrix potentials and used to generate two-soliton and nonabelian breather solutions. We consider in detail the scattering of a specific type of solitons which we call “invertons”. Invertons are a pair of different wavelength solitons one of which moving with the velocity of light and the other with slower velocity. Unlike the usual case of soliton scattering, the broader inverton changes its sign during the scattering process. Surprisingly, the light velocity inverton receives time advance through the scattering thereby moving faster than light. We show that however causality is not violated.
A typical nonabelian breather describes a breathing $0\pi$-pulse $E_1(v < c)$ which afterwards transfers to the nonbreathing $E_2$ pulse moving with light velocity.

2 Matrix potential formalism of the $\Lambda$-system

Consider a $\Lambda$-configuration where level three is higher than level one and two. The system of equations governing the propagation of pulses are given by the Schrödinger equation

$$\begin{align*}
\frac{\partial c_1}{\partial t} &= i\Omega_1 c_3 \\
\frac{\partial c_2}{\partial t} &= i\Omega_2 c_3 \\
\frac{\partial c_3}{\partial t} &= i(\Omega_1^* c_1 + \Omega_2^* c_2),
\end{align*}$$

(1)

and the Maxwell equation

$$\begin{align*}
i(\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t})\Omega_1 &= l_1 c_1 c_3^* \\
i(\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t})\Omega_2 &= l_2 c_2 c_3^*.
\end{align*}$$

(2)

Here, $l_i = 2\pi N\mu_i^2 \omega_i / \hbar$; $i = 1, 2$ and $c_k$; $k = 1, 2, 3$ are slowly varying probability amplitudes for the level occupations, $\Omega_i = \mu_i E_i / 2\hbar$ are the Rabi frequencies for the transitions $i \rightarrow 3$, $E_1$ and $E_2$ are the slowly varying electromagnetic field amplitudes, $\mu_i$ is the dipole matrix element for the relevant transition and $\omega_i$ is the corresponding laser frequency, and $N$ is the density of resonant three-level atoms. For brevity, we introduce a new coordinate $z = t - x/c$, $\bar{z} = x/c$ so that $\partial \equiv \partial / \partial z = \partial / \partial t$, $\bar{\partial} \equiv \partial / \partial \bar{z} = \partial / \partial t + c \partial / \partial x$.

As in our earlier papers [7, 8], the main tool of our analysis will be using a matrix potential $g$ instead of the probability amplitudes $c_i$ in the following way; let $g$ be a $3 \times 3$ unitary matrix whose second row is the complex conjugation of probability amplitudes, i.e.,

$$g = \begin{pmatrix}
* & * & * \\
c_1^* & c_2^* & c_3^* \\
* & * & *
\end{pmatrix},$$

(3)

where the first and the third rows are to be determined later. In terms of $g$, the density matrix $\rho$ whose components are $\rho_{mk} = c_m c_k^*$ takes a simple form,

$$\rho = \frac{i}{l_1} g^{-1} \bar{T} g, \quad \bar{T} = \begin{pmatrix}
0 & 0 & 0 \\
0 & -il_1 & 0 \\
0 & 0 & 0
\end{pmatrix}.$$  (4)
The specific choice of the matrix $\bar{T}$ is not essential. One may consider an arbitrary diagonal matrix $\bar{T}$ to handle density matrices in a more general context. Note that the first and the third rows of the matrix $g$ do not affect the density matrix $\rho$. In other words, the density matrix $\rho$ is invariant under the left multiplication of $g$ by any matrix $h$,

$$g \rightarrow g' = hg,$$

which commutes with $\bar{T}$ and thus of the form

$$h = \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}.$$

At first sight, introducing the matrix $g$ with more redundant components than $c_i$’s may seem an unnecessary complication. However, this is not so. In fact, it not only manifests the symmetry group structure of the system, but it also simplifies the problem of solving differential equations. Later, we show that the Bäcklund transformation of the system, a solution generating technique, also takes a simple form in terms of $g$. The main advantage of using $g$ is that $g$ solves the Schrödinger equation identically. In order to see this, we fix the redundancy introduced by Eqs. (5) and (6). Adopting the notation for the following matrix decomposition;

$$\Sigma = \Sigma_M + \Sigma_H, \quad \Sigma_M = \begin{pmatrix} 0 & 0 & * \\ 0 & 0 & * \\ * & * & 0 \end{pmatrix}, \quad \Sigma_H = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$$

we fix the redundancy by imposing the constraint condition on $g$,

$$(g^{-1}\partial g)_H = 0, \quad (\bar{\partial}gg^{-1})_H = 0.$$

One can always solve the constraint by finding an $h$ which makes $g$ to satisfy the constraint via the transform in Eq. (5). We also parameterize the remaining components of $g^{-1}\partial g$ such that

$$g^{-1}\partial g = \begin{pmatrix} 0 & 0 & -i\Omega_1 \\ 0 & 0 & -i\Omega_2 \\ -i\Omega_1^* & -i\Omega_2^* & 0 \end{pmatrix}.$$

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$^3$The existence of such an $h$ can be proved by adopting the field theory formulation of the problem as in [8]. However, we do not need the explicit expression of $h$.

$^4$Note that $g^{-1}\partial g$ is anti-Hermitian and $g$ is unitary.
The nonzero components in Eq. (9) express the Rabi frequencies in terms of $g$. In this new parametrization, one can readily check that the Schrödinger equation (1) arises from the simple identity,

$$\partial g^\dagger = \partial g^{-1} = -g^{-1}\partial gg^{-1} = -g^{-1}\partial gg^\dagger.$$  \hspace{1cm} (10)

This situation may be compared with the ordinary electromagnetism where the static electric field $\vec{E}$ in terms of a scalar potential $\phi$, $\vec{E} = -\nabla \phi$, solves the curl-free condition $\nabla \times \vec{E} = 0$. Likewise, we solve the Schrödinger equation in terms of a matrix potential $g$ and express the electric field components $\Omega_i$ in terms of $g$ as in Eq. (9). Originally, the Schrödinger and the Maxwell equations are made of five complex, component equations in total. Three of them (the Schrödinger part) are now solved identically in terms of $g$, where $g$ is partially constrained by Eq. (8). Then, the remaining degree of $g$ can be parametrized by two unknown complex functions and the Maxwell equation changes into two component equations for these two unknown functions only. In this way, the Maxwell equation resembles the Poisson equation in electrostatics. However, we do not need explicit component expressions in this paper so we suppress them. What we need is the expression of the Maxwell equation in terms of $g$ such that

$$\bar{\partial}(g^{-1}\partial g) = Q[T, g^{-1}\bar{T}g]Q$$  \hspace{1cm} (11)

where

$$T = \begin{pmatrix} -\frac{i}{2} & 0 & 0 \\ 0 & -\frac{i}{2} & 0 \\ 0 & 0 & \frac{i}{2} \end{pmatrix}, \quad Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & l_2/l_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \hspace{1cm} (12)$$

Note that this is indeed consistent with Eq. (9). The Maxwell equation in Eq. (11) possesses a symmetry under the change: $g \rightarrow g' = gh$ where $h$ is an arbitrary constant diagonal matrix. That is, $g'$ again satisfies Eqs. (9) and (11). This symmetry becomes enhanced to a larger one when the oscillator strengths are equal ($l_1 = l_2 = s$) so that $Q$ is the identity matrix. For the $\Lambda$-system, the oscillator strength $s$ is positive which

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5 In mathematical terms, we have associated the density matrix $\rho$ with the coset $G/H = (SU(3)/SU(2) \times U(1))$ and introduced the matrix $g$ for the parametrization of $G/H$. The constraint equation on $g$ makes a specific choice for each equivalence class, and variables which parameterize equivalence classes are determined by the Maxwell equation. Since the constraint restricts the subgroup $H = SU(2) \times U(1)$-part of the variable $g$, the remaining unconstrained part of $g$ can be expressed in terms of two unknown complex functions $\varphi_1$ and $\varphi_2$. In the gauged sigma model formulation, the Maxwell equation is vector gauge invariant so that it decouples from the $H$-degree of freedom, i.e. it reduces to two complex equations only in $\varphi_1$ and $\varphi_2$. 

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we assume throughout the paper. With equal oscillator strengths, \( \tilde{h} \) can be a constant unitary matrix of the form

\[
\tilde{h} = \begin{pmatrix} h_{11} & h_{12} & 0 \\ h_{21} & h_{22} & 0 \\ 0 & 0 & h_{33} \end{pmatrix}.
\]  

(13)

In other words, \( \tilde{h} \) is a constant matrix belonging to the group \( SU(2) \times U(1) \). In terms of physical variables, this symmetry amounts to the transform,

\[
\left( \begin{array}{c} \Omega'_1 \\ \Omega'_2 \end{array} \right) = \left( \begin{array}{c} h^*_{11} & h^*_{12} \\ h^*_{21} & h^*_{22} \end{array} \right) \left( \begin{array}{c} \Omega_1 \\ \Omega_2 \end{array} \right),
\]

\[
\begin{pmatrix} c'_1 \\ c'_2 \\ c'_3 \end{pmatrix} = \begin{pmatrix} h^*_{11} & h^*_{21} & 0 \\ h^*_{12} & h^*_{22} & 0 \\ 0 & 0 & h^*_{33} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix},
\]  

(14)

where the primed variables are solutions of Eqs. (9) and (11) provided that unprimed variables are. In particular, if the unprimed solution is a single 2\(\pi\)-pulse (\( \Omega_1 = 0, \ \Omega_2 \sim \text{sech}(t - x/v)/t_\text{p} \)), the primed solution represents a simulton solution \([10]\). When the oscillator strengths are equal (\( Q = 1 \)), the theory becomes integrable and exact analytic solutions can be found. However, even for \( Q \neq 1 \), the Maxwell equation admits a Lax pair representation and infinite sequences of conserved integrals can be found \([12]\). In this paper, we assume the equal oscillator strengths so that the Maxwell equation becomes

\[
\bar{\partial}(g^{-1}\partial g) = [T, g^{-1}\bar{T}g].
\]  

(15)

3 Bäcklund transformation

The Maxwell equation in Eq. (13) is equivalent to the consistency condition: \([L_1, L_2] = 0\) of the overdetermined linear equations,

\[
L_1 \Psi = (\partial + g^{-1}\partial g + \lambda T)\Psi = 0
\]

\[
L_2 \Psi = (\bar{\partial} + \frac{1}{\lambda}g^{-1}\bar{T}g)\Psi = 0,
\]  

(16)

where \( \lambda \) is a spectral parameter. We may apply the inverse scattering method to Eq. (13) and obtain exact solutions as in \([8]\). Instead, we present in this paper an alternative, simpler method - the Bäcklund transformation (BT) - which allows a more direct construction of exact solutions. Moreover, the advantage of using the matrix potential \( g \) becomes clear when the BT is used to establish a nonlinear superposition rule for a couple of single pulses. This is similar to the linear case where the application of the usual superposition rule is easier in terms of the scalar potential rather than the electric
field itself. Let \( g_0 \) and \( \Psi_0 \) be a particular solution of Eqs. \((15)\) and \((16)\), then \( g_1 \) is also a solution of Eq. \((15)\) if it satisfies the Bäcklund transformation of type I (type I-BT):

\[
\text{Type I} - \text{BT; } g^{-1}_1 \partial g_1 - g^{-1}_0 \partial g_0 - i\eta[g^{-1}_1 g_0, T] = 0
\]

\[
i\eta \bar{\partial}(g^{-1}_1 g_0) + g^{-1}_1 \bar{T} g_1 - g^{-1}_0 \bar{T} g_0 = 0.
\]

(17)

Here, \( \eta \) is an arbitrary parameter of the transformation. Once again, the type I-BT is a set of overdetermined first order partial differential equations whose consistency requires that \( g_0 \) and \( g_1 \) should be both solutions of Eq. \((15)\). An equivalent expression of the BT is in terms of the linear function \( \Psi \) as in \((13)\). We define the Bäcklund transformation of type II (type II-BT) by:

\[
\text{Type II} - \text{BT; } \Psi_1 = \frac{\lambda}{\lambda - i\eta}(1 + \frac{i\eta}{\lambda} g^{-1}_a g_0) \Psi_0.
\]

(18)

It can be readily checked that \( g_1 \) and \( \Psi_1 \) satisfy Eqs. \((15)\) and \((16)\) provided Eq. \((17)\) holds and vice versa. The type II-BT is particularly useful for establishing a nonlinear superposition rule. Assume that \((g_a, \Psi_a)\) and \((g_b, \Psi_b)\) are two sets of solutions with BT parameters \( \eta_a \) and \( \eta_b \) respectively, solving the BT for a particular solution, \((g_0, \Psi_0)\). If we apply the BT once more to \((g_a, \Psi_a)\) with \( \eta = \eta_b \) and also to \((g_b, \Psi_b)\) with \( \eta = \eta_a \), and require that they result in the same solution (this amounts to the commutability of the diagram in Fig. 1), then we obtain from the type II-BT,

\[
\Psi = \frac{\lambda}{(\lambda - i\eta_b)}(1 + \frac{i\eta_b}{\lambda} g^{-1}_a) \frac{\lambda}{(\lambda - i\eta_a)}(1 + \frac{i\eta_a}{\lambda} g^{-1}_b) \Psi_0
\]

\[
= \frac{\lambda}{(\lambda - i\eta_a)}(1 + \frac{i\eta_a}{\lambda} g^{-1}_b) \Psi_0
\]

(19)

Equivalently, we have the nonlinear superposition of two solutions,

\[
g = (\eta_b g_a - \eta_a g_b)g_0^{-1}(\eta_b g_b^{-1} - \eta_a g_a^{-1})^{-1}.
\]

(20)

Combining this expression with the type I-BT, we obtain a useful formula for the nonlinear superposition of the Rabi frequencies,

\[
g^{-1} \partial g = \frac{1}{2}(g_a^{-1} \partial g_a + g_b^{-1} \partial g_b) + \frac{i}{2}[g^{-1}(\eta_a g_a + \eta_b g_b), T].
\]

(21)

Or,

\[
\Omega_1 = \frac{1}{2}(\Omega_1^a + \Omega_1^b) - \frac{i}{2} F_{13}
\]

\[
\Omega_2 = \frac{1}{2}(\Omega_2^a + \Omega_2^b) - \frac{i}{2} F_{23}
\]

\[
F \equiv (\eta_b g_b^{-1} - \eta_a g_a^{-1}) g_0 (\eta_b g_a - \eta_a g_b)^{-1}(\eta_a g_b + \eta_b g_a).
\]

(22)
4 Matched pulses

Now, we construct solutions by integrating the type I-BT directly. We choose the particular solution $g_0$ for a constant electric field.

$$g_0^{-1} \partial g_0 = \begin{pmatrix} 0 & 0 & -i\Omega_1 \\ 0 & 0 & -i\Omega_2 \\ -i\Omega_1^* & -i\Omega_2^* & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -i\Omega_0 \\ 0 & 0 & 0 \\ -i\Omega_0 & 0 & 0 \end{pmatrix} \equiv \Lambda$$ (23)

for a real constant $\Omega_0$, so that

$$g_0 = e^{\Lambda z} = \begin{pmatrix} \cos(\Omega_0 z) & 0 & -i\sin(\Omega_0 z) \\ 0 & 1 & 0 \\ -i\sin(\Omega_0 z) & 0 & \cos(\Omega_0 z) \end{pmatrix}.$$ (24)

If we set $f \equiv e^{-\Lambda z} g_1$, the type I-BT becomes

$$f^{-1} \partial f + f^{-1} \Lambda f - \Lambda - i\eta[f^{-1}, T] = 0$$ (25)

$$i\eta \bar{\partial} f^{-1} + f^{-1} \bar{T} f - \bar{T} = 0.$$ (26)

Since $f^{-1} \partial f$ is anti-hermitian, Eq. (25) requires that

$$[f^{-1} - f, T] = 0,$$ (27)

which we solve by taking

$$f^{-1} - f = 2i \sin \theta$$ (28)

for an arbitrary real parameter $\sin \theta$. If we rewrite $f$ in terms of another matrix $P$,

$$f \equiv e^{-\Lambda z} g_1 = \cos \theta (2P - 1) - i \sin \theta,$$ (29)
then Eqs. (28) and (29) imply that $P$ is a hermitian projection operator, i.e.

$$P^2 = P, \quad P^\dagger = P.$$  \hfill (30)

In terms of $P$, Eqs. (25) and (26) can be written by

$$\begin{align*}
(1 - P)(\partial + \Lambda - i\tilde{\eta}T)P &= 0 \\
(1 - P)(i\tilde{\eta}\bar{\partial} - \bar{T})P &= 0,
\end{align*}$$  \hfill (31)

where $\tilde{\eta} \equiv e^{i\theta}\eta$. Since $P$ is a projection operator acting on the three-dimensional space, we may express $P$ using a three-dimensional vector $\vec{s} = (s_1, s_2, s_3)$ by

$$P_{ij} = s_is_j^*/\left(\sum_{k=1}^{3} s_k s_k^*\right)$$  \hfill (32)

which transforms Eq. (31) into a linear one

$$\begin{align*}
(\partial + \Lambda - i\tilde{\eta}T)\vec{s} &= 0 \\
(i\tilde{\eta}\bar{\partial} - \bar{T})\vec{s} &= 0.
\end{align*}$$  \hfill (33)

Since $\Lambda$ and $T$ commute, this may be integrated immediately resulting that

$$s_i = \sum_{k=1}^{3} [\exp{\Delta}]_{ik}u_k, \quad \text{for} \quad \Delta = (i\tilde{\eta}T - \Lambda)z - \frac{i}{\tilde{\eta}}\bar{T}z$$  \hfill (34)

where $\vec{u} = (u_1, u_2, u_3)$ is an arbitrary complex constant vector. Explicitly,

$$\begin{align*}
\tilde{\eta} &= (\cosh \sqrt{K}z + \frac{\tilde{\eta}}{2\sqrt{K}} \sinh \sqrt{K}z)u_1 + \frac{i\Omega_0}{\sqrt{K}}(\sinh \sqrt{K}z)u_3 \\
\tilde{\eta}^2 &= \exp\left(\frac{\tilde{\eta}z}{2} - \frac{s\tilde{z}}{\tilde{\eta}}\right)u_2 \\
s_3 &= \frac{i\Omega_0}{\sqrt{K}}(\sinh \sqrt{K}z)u_1 + (\cosh \sqrt{K}z - \frac{\tilde{\eta}}{2\sqrt{K}} \sinh \sqrt{K}z)u_3,
\end{align*}$$  \hfill (35)

where $K = \tilde{\eta}^2/4 - \Omega_0^2$. Finally, Rabi frequencies are given by

$$\begin{align*}
\Omega_1 &= \Omega_0 - 2i\eta \cos \theta s_1 s_3^*/\left(\sum_{k=1}^{3} s_k s_k^*\right) \\
\Omega_2 &= -2i\eta \cos \theta s_2 s_3^*/\left(\sum_{k=1}^{3} s_k s_k^*\right)
\end{align*}$$  \hfill (36)
and probability amplitudes are obtained through Eqs. (3) and (29),
\[
c_1 = 2 \cos \theta s_1 s_2^*/(\sum_{k=1}^{3} s_k s_k^*)
\]
\[
c_2 = \cos \theta [2 s_2 s_2^*/(\sum_{k=1}^{3} s_k s_k^*) - 1] + i \sin \theta
\]
\[
c_3 = 2 \cos \theta s_3 s_2^*/(\sum_{k=1}^{3} s_k s_k^*).
\]

(37)

Note that this solution, combined with the symmetry transformation in Eq. (14), represents a rich family of single pulse solutions. For the vanishing \((\Omega_0 = 0)\), it becomes
\[
\Omega_1 = \frac{1}{N} (-2i\eta \cos \theta u_1 u_3^* \exp[ i\eta z \sin \theta] )
\]
\[
\Omega_2 = \frac{1}{N} (-2i\eta \cos \theta u_2 u_3^* \exp[ i(\eta z + \frac{s}{\eta}) \sin \theta - \frac{s}{\eta} \bar{z} \cos \theta] )
\]

(38)

and
\[
c_1 = \frac{1}{N} (2 \cos \theta u_1 u_3^* \exp[ (\eta z - \frac{s}{\eta} \bar{z}) \cos \theta - i \frac{s}{\eta} \bar{z} \sin \theta] )
\]
\[
c_2 = \frac{1}{N} ( -|u_1|^2 \exp[ \eta z \cos \theta - i\theta] + |u_2|^2 \exp[ (\eta z - 2 \frac{s}{\eta} \bar{z}) \cos \theta + i\theta] - |u_3|^2 \exp[ -\eta z \cos \theta - i\theta] )
\]
\[
c_3 = \frac{1}{N} (2 \cos \theta u_3 u_3^* \exp[ - \frac{s}{\eta} \bar{z} \cos \theta - i(\eta z + \frac{s}{\eta} \bar{z}) \sin \theta] )
\]

(39)

where
\[
N \equiv |u_1|^2 \exp[ \eta z \cos \theta] + |u_2|^2 \exp[ \eta z \cos \theta - 2 \frac{s}{\eta} \bar{z} \cos \theta] + |u_3|^2 \exp[ -\eta z \cos \theta].
\]

(40)

This solution for \(u_1 \neq 0, u_2 \neq 0, \eta \cos \theta > 0\) describes the transfer of the 2\(\pi\)-pump pulse in the limit \(x \to -\infty\):
\[
\begin{align*}
\Omega_1 & \to 0, \quad \Omega_2 \to -i\eta \cos \theta \text{sech} \Sigma_1 e^{\Sigma_2} \\
c_1 & \to 0, \quad c_2 \to \frac{e^{\Sigma_1+i\theta} - e^{-\Sigma_1-i\theta}}{e^{\Sigma_1} + e^{-\Sigma_1}}, \quad c_3 \to \cos \theta e^{-i\Sigma_2} \text{sech} \Sigma_1 \\
\Sigma_1 & = \cos \theta (\eta t - \frac{(\eta^2 + s)}{\eta} x + \phi_1), \quad \Sigma_2 = \sin \theta (\eta t - \frac{(\eta^2 - s)}{\eta} x + \phi_2)
\end{align*}
\]

(41)

to the 2\(\pi\)-Stokes pulse moving with the velocity of light in the limit \(x \to \infty\):
\[
\begin{align*}
\Omega_1 & \to -i\eta \cos \theta \text{sech}[\cos \theta \eta (t - \frac{x}{c})] \exp[ i \sin \theta \eta (t - \frac{x}{c})], \quad \Omega_2 \to 0 \\
c_1 & \to 0, \quad c_2 \to -e^{-i\theta}, \quad c_3 \to 0,
\end{align*}
\]

(42)
where the arbitrary constants $\phi_i, i = 1, 2$ determine soliton positions in time and space. In the case of $\theta = 0$, this transfer of $2\pi$-pulse has been given in [11]. For $u_1 = 0$ or $u_2 = 0$, the solution remains as the steady state $2\pi$-pulse given in Eqs. (11) or (42) respectively without causing any transfer of pulses. This steady pulse, in connection with the symmetry transform in Eq. (14), is the simulton solution [10]. The free parameter $\theta$ measures the amount of self-detuning of a pulse from the carrier frequency. The $2\pi$-Stokes pulse in Eq. (12) is the same as the usual $2\pi$-pulse ($\theta = 0$) but with the carrier frequency shifted by the amount $\delta w = \eta \sin \theta$. On the other hand, Eq. (11) shows that the $2\pi$-pump pulse receives a time independent phase factor $\exp[i(s \sin \theta/c\eta)x]$ in addition to the shift of carrier frequency. We emphasize that this detuning has nothing to do with the frequency detuning of electromagnetic fields from the resonance line. In fact, our system is on resonance and thus the parameter $\theta$ measures the self-generated detuning of each pulses. The effect of $\theta$ to a single $2\pi$-pulse is to broaden the pulse shape maintaining the $2\pi$ area of the envelope which is adjusted by the shift of the carrier frequency. Recall that due to the symmetry in Eq. (14), a more general expression for a single pulse arises as a linear mixture of $\Omega_1$ and $\Omega_2$ in Eq. (38) which possess a wide range of free parameters.

If $\Omega_0 \neq 0$, the solution describes pulses more general than the hyperbolic-secant type. For the simplicity of analysis, we assume that $\theta = 0$ and $|\eta| \geq 2|\Omega_0|$. We also restrict to the parameters; $u_1 = r_1$, $u_2 = r_2$, $u_3 = ir_3$ for real $r_i$ and rewrite Eq. (35) for the notational convenience as follows:

$$
\begin{align*}
  s_1 &= A \exp(\sqrt{K}z) + B \exp(-\sqrt{K}z) \\
  s_2 &= \sqrt{A^2 + C^2} \exp(\eta z - \frac{s\bar{z}}{\eta}) \\
  s_3 &= iC \exp(\sqrt{K}z) + iD \exp(-\sqrt{K}z)
\end{align*}
$$

(43)

where $r_2$ is chosen to be $\sqrt{A^2 + C^2}$ by an appropriate choice of the coordinate origin. The coefficients are defined by

$$
\begin{align*}
  A &= \frac{1}{2}(1 + \frac{\eta}{2\sqrt{K}})r_1 - \frac{\Omega_0}{2\sqrt{K}}r_3 \\
  B &= \frac{1}{2}(1 - \frac{\eta}{2\sqrt{K}})r_1 + \frac{\Omega_0}{2\sqrt{K}}r_3 \\
  C &= \frac{\Omega_0}{2\sqrt{K}}r_1 + \frac{1}{2}(1 - \frac{\eta}{2\sqrt{K}})r_3 \\
  D &= -\frac{\Omega_0}{2\sqrt{K}}r_1 + \frac{1}{2}(1 + \frac{\eta}{2\sqrt{K}})r_3
\end{align*}
$$
\[ K = \frac{1}{4} \eta^2 - \Omega_0^2. \]  

(44)

In the limit where \( x \to -\infty \), the solution takes an asymptotic form:

\[ \Omega_1 = -\Omega_0 \tanh \Sigma_I, \quad \Omega_2 = \frac{\eta C}{\sqrt{A^2 + C^2}} \sech \Sigma_I \]  

(45)

and

\[ c_1 = -\frac{\eta C}{\sqrt{A^2 + C^2}} \sech \Sigma_I, \quad c_2 = -\tanh \Sigma_I, \quad c_3 = \frac{iC}{\sqrt{A^2 + C^2}} \sech \Sigma_I \]

\[ \Sigma_I = \frac{2s + (\eta^2 - 2\eta \sqrt{K})}{2\eta c}(x - v_I t) \]  

(46)

where the velocity \( v_I \) is

\[ v_I = \frac{\eta^2 - 2\eta \sqrt{K}}{2s + \eta^2 - 2\eta \sqrt{K}c}, \]  

(47)

which is less than the light velocity \( c \). Comparison of electric fields in Eq. (45) with initial populations in Eq. (46) shows that two laser pulses are arranged in the so-called counterintuitive order \[9\]. In the \( x \to \infty \) limit, the asymptotic form of the solution is

\[ \Omega_1 = -\Omega_0 \tanh \Sigma_F + \Omega_1^{\Sigma}, \quad \Omega_2 = \frac{\eta C}{\sqrt{A^2 + C^2}} \sech \Sigma_F \]  

(48)

and

\[ c_1 = \frac{B}{\sqrt{B^2 + D^2}} \sech \Sigma_F, \quad c_2 = -\tanh \Sigma_F, \quad c_3 = \frac{iD}{\sqrt{B^2 + D^2}} \sech \Sigma_F \]  

(49)

where

\[ \Sigma_F = \frac{2s + (\eta^2 + 2\eta \sqrt{K})}{2\eta c}(x - v_F t) - \Delta_0 \]

\[ v_F = \frac{\eta^2 + 2\eta \sqrt{K}}{2s + \eta^2 + 2\eta \sqrt{K}c} \]

\[ \Delta_0 = \frac{1}{2} \ln \frac{B^2 + D^2}{A^2 + C^2} \]

\[ \Omega_1^{\Sigma} = \frac{\Omega_0 (r_1^2 + r_3^2) - \eta r_1 r_3}{(A^2 + C^2)e^{\Delta_0} \cosh(2\sqrt{K}(t - x/c) - \Delta_0) + AB + CD}. \]  

(50)

In the far past, this solution represents a matched-solitary-wave pair (MSP) moving with velocity \( v_I \) whose invariant shape is not the hyperbolic-secant type. Eq. [48] shows that this MSP in general breaks up into another MSP with slower velocity \( (v_F < v_I) \) and a soliton pulse moving with light velocity. Figure 2 shows explicitly this breakup behaviour with parameters, \( s = 1, \ r_1 = 0.7, \ r_3 = 1, \ \Omega_0 = 0.5 \) and \( \eta = 1.5 \).
By changing the sign of $\eta$, one could equally consider the reverse process of fusing a MSP and a soliton into another MSP. In the specific case where $K = \eta^2/4 - \Omega^2_0$ vanishes, this solution reduces to the steady state pulse which agrees with the MSP solution by Hioe and Grobe \[3\],

$$\Omega_1 = -\Omega_0 \tanh \Sigma_S, \quad \Omega_2 = \sqrt{2} \Omega_0 \text{sech} \Sigma_S$$  \hspace{1cm} (51)

and

$$c_1 = \frac{1}{\sqrt{2}} \text{sech} \Sigma_S, \quad c_2 = -\tanh \Sigma_S, \quad c_3 = \frac{i}{\sqrt{2}} \text{sech} \Sigma_S$$  \hspace{1cm} (52)

where

$$\Sigma_S = \frac{s + 2\Omega^2_0}{2\Omega_0 c}(x - v_ST), \quad v_S = 2\Omega^2_0/(s + 2\Omega^2_0).$$  \hspace{1cm} (53)

Thus, our solution generalizes the result of Hioe and Grobe and shows that MSP propagates steadily only when the velocity $v_S$ of MSP is specifically given by the parameter $\Omega_0$ as in Eq. (53). Otherwise, it breaks up as explained before. In other words, in the case of MSP where $K \neq 0$, the hyperbolic-tangent pulse $\Omega_1$ induces a partial transfer of the other soliton pulse $\Omega_2$ into the light velocity one. This is similar to the behavior of adiabaton moving with a slower velocity which also generates a light velocity solitary pulse during its formation \[11\] where the most general expression of our MSP solution arises when we incorporate the $SU(2) \times U(1)$ symmetry transform as in Eq. (14) as well as the self-detuning effect ($\theta \neq 0$). This solution contains a large number of free parameters which account for the variety of pulse shapes and initial conditions required for the atoms. The stability of this MSP solution against small perturbations is an important issue \[15\] which we plan to consider elsewhere.

5 Invertons and breathers

The nonlinear superposition rule given in Eqs. (20)-(22) allows the superposition of two MSP solutions $g_a$ and $g_b$ whenever they are obtained from $g_0$ with BT parameters $\eta_a$ and $\eta_b$ respectively. In general, $g_a$ and $g_b$ could possess different set of free parameters and the superposition rule requires only that they are obtained from the same $g_0$ through BT. If $g_a$ and $g_b$ have same set of parameters except for BT parameters, we could analytically

\[\text{We thank Referee for raising this point.}\]
continue the BT parameter in Eq. (20) to the complex case in such a way that
\[
\eta_a = \exp[i\theta_B], \quad \eta_b = \exp[-i\theta_B]
\] (54)
for a certain real parameter $\theta_B$, and that $g$ is still unitary and becomes a nonabelian breather solution. As an example, we take the one-soliton to be the MSP in the previous section with a set of parameters
\[
\Omega_0 = 0, \; u_1 = 1, \; u_2 = 1, \; u_3 = -i.
\] (55)
Then, the nonabelian breather solution has an asymptotic form for $x \to -\infty$,
\[
\Omega_1 = 0, \quad \Omega_2 = \frac{\sin 2\theta_B(\exp[(2x/c - t)\cos \theta_B]\sin(\theta_B + t\sin \theta_B) + \exp[(t - 2x/c)\cos \theta_B]\sin(\theta_B - t\sin \theta_B))}{-1 + \cos^2 \theta_B \cos(2t\sin \theta_B) - \cosh[(4x/c - 2t)\cos \theta_B] \sin^2 \theta_B}
\] (56)
and for $x \to \infty$,
\[
\Omega_1 = \frac{\sin 2\theta_B(e^{Z_C}\sin(\theta_B - Z_S) + e^{-Z_C}\sin(\theta_B + Z_S))}{-1 + \cos^2 \theta_B \cos(2Z_S) - \cosh(2Z_C) \sin^2 \theta_B}, \quad \Omega_2 = 0, \quad Z_C = (t - x/c)\cos \theta_B, \quad Z_S = (t - x/c)\sin \theta_B.
\] (57)
This describes a breathing $0\pi$-pulse $\Omega_2$ ($v < c$) which transfers to the nonbreathing $\Omega_1$ pulse moving with the velocity of light. This is a typical motion of a nonabelian breather. Figure 3 shows a breathing motion with $\theta_B = 1.2$. Different values of parameters $u_i$ in general distort the shape of $0\pi$-pulse so that the time area of the pulse is nonzero, but they lead to the essentially same type of transferring motion except for the case $u_1 = 0$ or $u_2 = 0$ where it becomes a steady state breather.

For real BT parameters $\eta_a$ and $\eta_b$, the superposed solution $g$ in general describes the scattering of two MSP solutions. Here, we concentrate only on a particular case of two soliton solutions, which we name as a pair of “invertons” due to their interesting new features. Consider one solitons $g_a$ and $g_b$ given by
\[
g_a = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos \phi_a & \sin \phi_a \\ 0 & \sin \phi_a & -\cos \phi_a \end{pmatrix}
\]
\[
\cos \phi_a = \tanh[\eta_a t - \frac{\eta_a^2 + s}{c\eta_a}x], \quad \sin \phi_a = \text{sech}[\eta_a t - \frac{\eta_a^2 + s}{c\eta_a}x]
\] (58)
and
\[
g_b = \begin{pmatrix}
\cos \phi_b & 0 & \sin \phi_b \\
0 & -1 & 0 \\
\sin \phi_b & 0 & -\cos \phi_b
\end{pmatrix}
\]
\[
\cos \phi_b = \tanh[\eta_b(t - x/c)] \\
\sin \phi_b = \sech[\eta_b(t - x/c)].
\] (59)

In terms of Rabi frequencies, these correspond to
\[
\Omega_1^{(a)} = 0, \quad \Omega_2^{(a)} = -i\eta_a \sech[\eta_a t - \frac{\eta_a^2 + s}{c\eta_a} x]
\] (60)

and
\[
\Omega_1^{(b)} = -i\eta_b \sech[\eta_b(t - x/c)], \quad \Omega_2^{(b)} = 0,
\] (61)

that is, they represent two $2\pi$-pulses with different resonance frequencies, one moving with light velocity and the other moving with slower velocity.

The nonlinear superposition rule gives rise to the superposed Rabi frequencies and probabilities such that
\[
\Omega_1 = -i\eta_b \sin \phi_b \frac{\eta_a^2 \cos \phi_a + \eta_a \eta_b (1 + \cos \phi_a) + \eta_b^2}{\eta_a^2 + \eta_b^2 + \eta_a \eta_b (1 + \cos \phi_a + \cos \phi_b - \cos \phi_a \cos \phi_b)} \\
\Omega_2 = -i\eta_a \sin \phi_a \frac{\eta_a^2 + \eta_a \eta_b (1 + \cos \phi_b) + \eta_b^2 \cos \phi_b}{\eta_a^2 + \eta_b^2 + \eta_a \eta_b (1 + \cos \phi_a + \cos \phi_b - \cos \phi_a \cos \phi_b)}
\] (62)

and
\[
c_1 = \frac{(\eta_a + \eta_b) \eta_b \sin \phi_a \sin \phi_b}{\eta_a^2 + \eta_b^2 + \eta_a \eta_b (1 + \cos \phi_a + \cos \phi_b - \cos \phi_a \cos \phi_b)} \\
c_2 = \frac{-(\eta_a^2 + \eta_b^2) \cos \phi_a - \eta_a \eta_b (1 + \cos \phi_a - \cos \phi_b + \cos \phi_a \cos \phi_b)}{\eta_a^2 + \eta_b^2 + \eta_a \eta_b (1 + \cos \phi_a + \cos \phi_b - \cos \phi_a \cos \phi_b)} \\
c_3 = \frac{-(\eta_a^2 + \eta_a \eta_b (1 + \cos \phi_b) + \eta_b^2 \cos \phi_b) \sin \phi_a}{\eta_a^2 + \eta_b^2 + \eta_a \eta_b (1 + \cos \phi_a + \cos \phi_b - \cos \phi_a \cos \phi_b)}.
\] (63)

This solution describes the scattering of two invertons where the fast moving inverton $I_F$ passes through the slow moving one $I_S$. Before the collision, their asymptotic forms are given by the following configurations;

**Inverton $I_F$** : \( \cos \phi_a = 1 \)
\[
c_1 = 0, \quad c_2 = -1, \quad c_3 = 0 \\
\Omega_1 = -i\eta_b \sech[\eta_b(t - x/c)], \quad \Omega_2 = 0
\] (64)
and

\textbf{Inverton }$I_S$ ; \quad \cos \phi_b = 1

\begin{align*}
c_1 &= 0, \quad c_2 = - \tanh[\eta_a t - \frac{\eta_a^2 + s}{c\eta_a} x], \quad c_3 = - \text{sech}[\eta_a t - \frac{\eta_a^2 + s}{c\eta_a} x] \\
\Omega_1 &= 0, \quad \Omega_2 = - i \eta_a \text{sech}[\eta_a t - \frac{\eta_a^2 + s}{c\eta_a} x].
\end{align*}

(65)

After the collision, their asymptotic forms are

\textbf{Inverton }$I_F$ ; \quad \cos \phi_a = -1

\begin{align*}
c_1 &= 0, \quad c_2 = 1, \quad c_3 = 0 \\
\Omega_1 &= i \eta_a \text{sign}(\frac{\eta_b + \eta_b}{\eta_a - \eta_b}) \text{sech}[\eta_b (t - x/c) + \delta], \quad \Omega_2 = 0
\end{align*}

(66)

and

\textbf{Inverton }$I_S$ ; \quad \cos \phi_b = -1

\begin{align*}
c_1 &= 0, \quad c_2 = - \tanh[\eta_a t - \frac{\eta_a^2 + s}{c\eta_a} x + \delta], \quad c_3 = - \text{sign}(\frac{\eta_a + \eta_b}{\eta_a - \eta_b}) \text{sech}[\eta_a t - \frac{\eta_a^2 + s}{c\eta_a} x + \delta] \\
\Omega_1 &= 0, \quad \Omega_2 = - i \eta_a \text{sign}(\frac{\eta_a + \eta_b}{\eta_a - \eta_b}) \text{sech}[\eta_a t - \frac{\eta_a^2 + s}{c\eta_a} x + \delta],
\end{align*}

(67)

where sign denotes the sign function and the phase shift parameter $\delta$ is

$$\delta = \ln|\frac{\eta_a + \eta_b}{\eta_a - \eta_b}|.$$ 

(68)

The appearance of the sign function is a novel feature of soliton scatterings in a three-level $\Lambda$ system which does not arise in the two-level atom case. A careful analysis of the above asymptotic solutions shows that the broader pulse changes its sign after the collision (thereby the name inverton) while the sharper pulse remains the same. Also, the phase shift always arises in such a way that the slow inverton receives time retardation while the fast one receives time advance. This implies that the fast inverton moving with light velocity moves faster than light through the scattering process! However, this does not violate causality. In fact, the fast inverton has infinitely stretched tails which trigger the slow inverton to transfer its energy to the fast one.

Figure 4 shows the scattering of the invertons with parameters $\eta_a = 0.5, \eta_b = 0.9$ and $s = 1$. In order to see that causality is not violated, we consider the energy conservation
law given by
\[ \frac{\partial}{\partial t} \rho = -\frac{\partial}{\partial x} j \] (69)
where
\[ \rho = |\Omega_1|^2 + |\Omega_2|^2 + |c_1|^2 + |c_2|^2 = |\Omega_1|^2 + |\Omega_2|^2 + 1 - |c_3|^2 \]
\[ j = c(|\Omega_1|^2 + |\Omega_2|^2). \] (70)

Figure 5 shows the energy transfer process between the two invertons. When the right end tail of the fast inverton \( I_F \) reaches the slow inverton \( I_S \), the energy is transferred from \( I_S \) to \( I_F \) and \( I_F \) receives phase advance. After the center of \( I_F \) reaches \( I_S \), the energy of \( I_F \) is absorbed into the \( I_S \) which results in a retarded phase shift of \( I_S \). Thus, no energy is really transferred faster than light. The scattering process is indeed a nicely balanced exchange process where the scattered invertons maintain their shapes invariant except for the change of sign. The change of polarizations of simultons in the scattering process has been known earlier [16] and the scattering of two single-frequency \( 2\pi \)-pulses with different resonance frequencies is also considered in [14]. However, the binary, sign changing behavior of invertons is a new aspect which did not receive attention in earlier works, and hopefully will find some applications in the future.

6 Discussion

In this paper, we have introduced a new formalism for the three-level coupled Schödinger-Maxwell equation based on a matrix potential variable, and through the Bäcklund transformation we obtained various new solutions which generalize previously known solutions.

We found new MSP solutions, more general than the steady state MSP found by Hioe and Grobe, which account for the generic breakup behavior of a MSP into another MSP and a light velocity soliton. Two-soliton solutions and nonabelian breathers are obtained through the nonlinear superposition rule and scattering of invertons are analyzed in detail. In this new formalism, we have demonstrated the \( SU(2) \times U(1) \) group symmetry of the three-level system with equal oscillator strengths and using that we have found a general expression of solutions which mixes pulses with different velocities. This group symmetry also accounts for the variety of steady state MSP solution by Hioe and Grobe which linearly superpose different Jacobi elliptic functions. Another new feature of our
formalism is the introduction of a self-detuning parameter $\theta$ to the solutions. For each $2\pi$-pulses, these $\theta$-values are conserved in time and survives from the scattering process. That is, it is a conserved charge which can be used to label each pulses in addition to the area of pulses. We also emphasize that our new formalism is not restricted to the three-level case but also can be extended to other multi-level cases \[8\] with the same analytic power.

Finally, we comment on matched pulse propagation through absorbing media. In this case, the time rate equation for the probability amplitude $c_3$ in Eq. (11) is replaced by $\frac{\partial c_3}{\partial t} \rightarrow \frac{\partial c_3}{\partial t} + \gamma c_3$, which implies the decaying of the excited state $|3\rangle$ at a rate $\gamma$ to states other than $|1\rangle$ and $|2\rangle$. This obviously breaks the probability conservation law; $|c_1|^2 + |c_2|^2 + |c_3|^2 = 1$ so that our matrix potential formalism does not apply to this case. Nevertheless, it is important to note that the $SU(2) \times U(1)$ group symmetry of the three-level system with equal oscillator strengths persists even in the case of an absorbing medium. This shows that recent analytic and numerical studies on matched pulse propagation through absorbing media \[17\]-\[21\] can be extended to more general initial conditions. For instance, most studies assume the standard initial condition of the population being in the ground state. The $SU(2) \times U(1)$ group symmetry as indicated in Eq. (14) maps the standard initial condition to a coherently prepared one which linearly superposes $|1\rangle$ and $|2\rangle$. This also maps two Rabi frequencies $\Omega_1$ and $\Omega_2$ via the linear transform as in Eq. (14) so that one obtains a more general description of pulse propagation. Thus, for example, when a matched pulse of equal amplitude is found, the above symmetry generates a set of new solutions with unequal amplitudes. This agrees with results in the recent numerical study \[21\], which showed the existence of stable, shape invariant matched pulses of unequal amplitudes depending on the coherent preparation of initial conditions.

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