Effect of $NN$ correlations on predictions of nuclear transparencies for protons, knocked-out in high $Q^2$ $(e, e'p)$ reactions

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Abstract

We study the transparency $T$ of nuclei for nucleons knocked-out in high-energy semi-inclusive $(e, e'p)$ reactions, using an improved theoretical input, discussed by Nikolaev et al. We establish that neglect of $NN$-correlations between the knocked-out and core nucleons reduces nuclear transparencies by $\approx 15\%$ for light, to $\approx 10\%$ for heavy nuclei. About the same is predicted for transparencies, integrated over the transverse or longitudinal momentum of the outgoing proton. Hadron dynamics predicts a roughly constant $T$ beyond $Q^2 \approx 2$ GeV$^2$, whereas for all targets the largest measured data point $Q^2 = 6.7$ GeV$^2$ appears to lie above that plateau. Large error bars on those data-points preclude a conclusion regarding the onset of colour transparency.

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I. INTRODUCTION.

In the past few years there has been a keen interest in high-energy semi-inclusive (SI) $A(e,e'p)X$ reactions, from which one wishes to extract the transparency $T$ of nuclei for a knocked-out nucleon. Special interest centers on the possibility that from some threshold on, and initially growing with $Q^2$, the medium is far more transparent than hadronic dynamics predicts. QCD in fact predicts such behaviour. Depending on the circumstances, it allocates to the state of a knocked-out proton during the time it passes a nucleus, a small-sized non-hadronic component. For it the medium has an anomalously large (colour) transparency (CT) [1].

The recently published SLAC NE18 experiment with electron 4-momentum transfers $1 \leq Q^2(\text{GeV}^2) \leq 6.7$ on various targets [2,3], has been considered to be a possible testing ground for the onset of CT. An obvious prerequisite for its detection is a demonstration that standard hadronic dynamics is not capable to account for the data. This is only possible if both available data and predictions are accurate.

The above explains maybe the veritable plethora of published predictions which not infrequently refer to different dynamic input elements or kinematic details [4–14]. Moreover, while data have been taken and analyzed, there had been no definitive information available on experimental details such as cuts on missing mass and momenta, or detector acceptances. This rendered uncertain, which calculated transparency should ultimately be compared with data and invited tests of the sensitivity of predicted transparencies for extreme 'theoretical' cuts [12].

The present note is an outgrow of work by Nikolaev and co-workers, especially Refs. [6,7] and also of [12]. In all, one considers SI reactions where the projectile excites high-lying core states. Application of closure over those states leads to the SI coincidence cross section, integrated over the electron energy loss. The corresponding transparency will for brevity be called the energy integrated one, $T^E$.

In general one considers situations where cuts have been applied, i.e. when the underlying
cross sections are integrated over the entire proton momentum, or over some component of it. The corresponding transparencies will be called ‘momentum integrated transparencies’, to be denoted by $T^{E,P}$. Those have been studied by Nikolaev et al in $T^{E,P}$ who report small effects if $NN$-correlations are retained.

In addition to $T^E$, the transparencies $T^{E,P_z}, T^{E,P_\perp}$ which are integrated over components of the proton momentum have been studied in $T^{E,P}$. Partly on the basis of results in Ref. $T^{E,P}$ for the different $T^{E,P}$, $NN$-correlations have from the onset been disregarded in $T^{E,P}$. A comparison of data with predictions for finite momentum cuts can be found in $T^{E,P}$.

The evaluation of the various transparencies discussed in $T^{E,P}$ is the cleanest thus far. However, the judgment that $NN$-correlations may be disregarded seems to be at variance with other estimates on $T^{E,P}$, $T^{E,P}$ and on $T^{E,P}$ $T^{E,P}$. The same is reported in $T^{E,P}$ for low-energy loss SI reactions, not permitting closure. Additional material may be found in $T^{E,P}$.

There is thus still lacking an actual calculation of the effect of $NN$-correlations in the otherwise satisfactory description of Nikolaev et al in $T^{E,P}$. In the present note we attempt a numerical assessment of their relevance. In the end we compare predictions with the NA18 data and discuss the possibility that the latter show the onset of colour transparency.

II. HADRON DYNAMICS FOR UNRESTRICTED AND PARTIALLY
   RESTRICTED TRANSPARENCIES.

Without re-derivations we shall in the following borrow results from $T^{E,P}$, (from here on denoted by I) and cite equations by their number in I. We start from the assumption that the $eN$-cross section can be factored out from the SI cross section and focus on the remaining SI response or structure function $S^{SI}$. The latter contains inelastic form factors, connecting the target ground state to excited of a highly excited core and a high-energy proton, scattered in the field of the core (I.1).

A considerable simplification results, if the excitation of high core states permits closure

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over those to be performed. The result for the SI response per nucleon is (cf. I.3)

\[ S_{SI}(q, \omega, p) = \delta(\omega + \langle \Delta \rangle - e_p) \int \int d\mathbf{r}_1 d\mathbf{r}_1' e^{i\mathbf{p}^T(\mathbf{r}_1 - \mathbf{r}_1')} \rho_1(\mathbf{r}_1, \mathbf{r}_1') \tilde{R}(q, \mathbf{r}_1, \mathbf{r}_1') \]  

(1a)

\[ \tilde{R}(q, \mathbf{r}_1, \mathbf{r}_1') = \left( \prod_j \int d\mathbf{r}_j \right) \frac{\rho_A(\mathbf{r}_1, \mathbf{r}_1'; \mathbf{r}_j)}{\rho_1(\mathbf{r}_1, \mathbf{r}_1')} \left[ 1 + \gamma(q, \mathbf{r}_1 - \mathbf{r}_1; \mathbf{r}_1' - \mathbf{r}_1) \right] \]  

(1b)

\[ = \left( \prod_j \int d\mathbf{r}_j \right) \frac{\rho_A(\mathbf{r}_1, \mathbf{r}_1'; \mathbf{r}_j)}{\rho_1(\mathbf{r}_1, \mathbf{r}_1')} \left[ 1 + \sum_{l \geq 2} \gamma(q, \mathbf{r}_1 - \mathbf{r}_1; \mathbf{r}_1' - \mathbf{r}_1) + \ldots \right] \]  

(1c)

The above SI response depends parametrically on the momentum-energy loss \((q, \omega)\) of the incident electron, the momentum and total energy \(p, e_p\) of the knocked-out proton and on some average excitation energy \(\langle \Delta \rangle\). The momentum of the struck proton (minus the missing momentum) is \(p' = p - q\). The following dynamical input elements are required for an evaluation of (1):

1) \(n\)-particle density matrices \(\rho_n(\mathbf{r}_1, \mathbf{r}_1'; \mathbf{r}_j)\), diagonal in all coordinates except in the one of the struck particle, which has arbitrarily been chosen to be 1.

2) The operator \(\gamma\) which describes the FSI factor \(\tilde{R}\) in (1b), and which in turn is governed by the scattering of the knocked-out proton with the core. Eq. (1c) is its multiple-scattering expansion of (1b) in terms of the off-shell scattering of '1' from individual core nucleons.

In view of the large momentum transfers imparted onto the knocked-out nucleon, its scattering from the core is conveniently described by Glauber theory (cf. [17]). By way of illustration we give the result for \(\gamma\) in Eq. (1) for scattering on an isolated core-nucleon at the origin \((\mathbf{r} = \mathbf{b}, z)\)

\[ \gamma(q, \mathbf{r}, \mathbf{r}') = \left( 1 + \Gamma_{q}^{off}(\mathbf{b}, z) \right) \left( 1 + \Gamma_{q}^{off}(\mathbf{b}', z') \right) - 1 \]  

(2a)

\[ = \Gamma_{q}^{off}(\mathbf{b}, z) + \left[ \Gamma_{q}^{off}(\mathbf{b}', z') \right]^* + \Gamma_{q}^{off}(\mathbf{b}, z) \Gamma_{q}^{off}(\mathbf{b}', z')^* \]  

(2b)

The generally off-shell scattering is in the impact representation described by a corresponding off-shell profile \(\Gamma^{off}\). For diffractive \(NN\) interactions the off-shell profile may approximately be related to their on-shell analog \(\Gamma = \Gamma^{on}\)

\[ \Gamma_{q}^{off}(\mathbf{b}, z) \approx \theta(-z) \Gamma_{q}(\mathbf{b}) \]  

(3a)

\[ \Gamma = e^{i\chi_{q}(\mathbf{b})} - 1 \approx -\frac{\sigma_{q}^{tot}}{2}(1 - i\tau_{q})A(q, \mathbf{b}), \]  

(3b)
\[ A(q, b) \approx \frac{(Q_0^0)^2}{4\pi} \exp\left[-(bQ_0^0/2)^2\right] \]  

(3c)

where we used standard NN scattering parameters: the \(NN\) total cross section \(\sigma_{\text{tot}}\), the ratio \(\tau\) of real and imaginary parts of the forward \(NN\) elastic scattering amplitude and \(Q_0^{-1}\), the range of \(\Gamma_q(b)\); zero-range corresponds to \(A(q, b) \to \delta^2(b)\). In the impact parameter representation 'partial' inelastic contributions \(\sigma_{q,\text{inel}}(b)\) to elastic \(NN\) scattering are as follows related to the imaginary part of the eikonal phases \(\chi\) in (3b):

\[ \sigma_{q,\text{inel}}(b) \equiv 1 - e^{-2i\chi(b)} \approx A(q, b)\sigma_{q,\text{inel}}^0 \]

\[ \sigma_{q,\text{inel}}^0 = \sigma_{q,\text{tot}} - \sigma_{q,\text{el}}^0 \]  

(4)

Since we shall limit ourselves to FSI caused by binary collisions (i.e. contributions in (1c) up to first order in \(\gamma\)) we need only discuss the two lowest densities. As in I, we parameterize as follows the non-diagonal single-particle density matrix

\[ \rho_1(r_1, r_1') \approx \rho(S) \int dS' \rho_1(s, S') \approx \rho(r_1)\Sigma(s) \]

\[ \Sigma(s) = \int \frac{d^3p}{(2\pi)^3} n(p)e^{-ipS}, \]  

(5)

where \(S = (r_1 + r_1')/2; s = r_1 - r_1'\). \(\rho(r) = \rho_1(r, r)\) and \(n(p)\) are the single nucleon density and momentum distributions.

More problematic is the half-diagonal \(\rho_2\). Only recently have calculations been made for nuclear matter and some light nuclei \[19,20\]. For our purposes suffices the following interpolating approximation

\[ \rho_2(r_1, r_1'; r_2) = \rho_1(r_1, r_1')\rho_2(r_2)\zeta_2(r_1, r_1'; r_2) \]

\[ \zeta_2(r_1, r_1'; r_2) \approx \sqrt{g(r_1 - r_2)g(r_1' - r_2)}, \]  

(6)

with \(g(r)\) the pair distribution function (see \[22\] for some remarks on the use of (6) for fermions). Using (4) and (3), Eqs. (1) become

\[ S^{SI}(q, \omega, p) = \delta(\omega + \langle \Delta \rangle - ep) \int dse^{ipS}\Sigma(|s|) \int dr_1 \rho(r_1)\tilde{R}(q, r_1, s_z) \]

\[ = \delta(\omega + \langle \Delta \rangle - ep) \int dse^{ipS}\Sigma(|s|)\tilde{G}(q, s) \]  

(7a)

\[ \tilde{G}(q, s) = \int dr_1 \rho(r_1)\tilde{R}(q, r_1, s) \]  

(7b)
The above will be confronted with the standard Plane Wave Impulse Approximation (PWIA), where by definition the knocked-out proton exits without being affected by the medium. The corresponding response is obtained from (7) putting \( \tilde{R}, \tilde{G} \rightarrow 1 \)

\[
S^{SI,PWIA}(q, \omega, p) = \delta(\omega + \langle \Delta \rangle - e_{p}) \int ds e^{ip' \cdot s} \Sigma(|s|) \int dr_{1} \rho(r_{1}) = \delta(\omega + \langle \Delta \rangle - e_{p}) n(p') \quad (8)
\]

We define \( T^{SI} \) as the ratio of the experimental yield and its theoretical PWIA approximation or, equivalently, as the ratio of the corresponding SI responses. In transparencies, cuts are applied in the full cross section and in the corresponding PWIA approximation. Cuts in cross sections should be applied both from (7), (8) one then obtains for a coincidence experiment

\[
T(q, \omega, p) \equiv \frac{S^{SI}(q, \omega, p)}{S^{SI,PWIA}(q, \omega, p)} = \frac{\int ds e^{ip' \cdot s} \Sigma(|s|) \tilde{G}(q, s)}{\int ds e^{ip \cdot s} \Sigma(|s|)} \quad (9a)
\]

\[
= [n(p')]^{-1} \int ds e^{ip' \cdot s} \Sigma(|s|) \tilde{G}(q, s)
\]

\[
= [n(p')]^{-1} \int \frac{dp''}{(2\pi)^3} n(p'' - p') G(q, p''), \quad (9b)
\]

with \( G \) the Fourier transform of \( \tilde{G} \). The absence of \( \omega \) in (9) is an artifact of the applied closure. It causes \( T \rightarrow T^{E} \), with \( T^{E} \) the transparency, when SI cross sections, integrated over the electron energy loss \( \omega \) are used.

Eq. (9) is formally like (I.14). The difference lies in the approximation used in I, used for its evaluation. Eq. (9) shows, that the operator \( \gamma \) in (1) for the coincidence response, is composed of two off-shell scattering phases \( e^{i\chi^{off}(q, b, z)} = \Gamma^{off}(q, b, z) + 1 \) at different \( (b, z) \). In I it has been assumed that it is a reasonable approximation to assume those scatterings to proceed for identical impact parameters \( b \), but different \( z \). Nikolaev et al have since pointed out that this restriction may be relaxed \( \tilde{(4)} \) and they have evaluated (9) in a mean-field approximation for nuclei, disregarding spatial correlations. However, one may proceed as in I where those have been retained, keeping as in \( \tilde{(4)} \) different trajectories in \( \gamma \), Eq. (1).

Limiting ourselves to binary collisions, one first defines the following Final State Interaction (FSI) component phases
\[ \tilde{\Omega}(q, r_1, r'_1) = -(A - 1) \frac{\sigma_{tot}^2}{2} (1 - i\tau) \int_{-\infty}^{0} dz \int d^2 b A(b) \rho(r_1 - r) \zeta_2(r; r' - s) \]
\[ \rightarrow -(A - 1) \frac{\sigma_{tot}^2}{2} (1 - i\tau) \int_{-\infty}^{0} dz \rho(b_1, z - z_1) \zeta_2(0, z; s_\perp, z - s_z) \]  
(10a)

\[ \tilde{\Omega}'(q, r_1, r'_1) = (\Omega(q, r'_1, r_1)^* \]
\[ = -(A - 1) \frac{\sigma_{tot}^2}{2} (1 + i\tau) \int_{-\infty}^{0} dz \rho(b_1 - s_\perp, z_1 - z - s_z) \zeta_2(0, z; s_\perp, z + s_z) \]  
(10b)

\[ \tilde{M}(q, r_1, s) = (A - 1) \sigma_{tot el} e^{-s_\perp^2 Q_0^2 / 8} \left[ \int_{-\infty}^{0} dz J - \theta(-s_z) \int_{-|s_z|}^{0} dz J \right] \]
\[ J = \rho \left( b_1 - s_\perp / 2, z_1 - z \right) \zeta_2 \left( s_\perp / 2, z; -s_\perp / 2, z - s_z \right) \]  
(10c)

where the second equation (10a) is the zero range limit of the first one. Those define the FSI factors in the first cumulant approximation [23]

\[ \tilde{R} = \exp[\tilde{\Omega} + \tilde{\Omega}' + \tilde{M}] \]
\[ \tilde{G} = \int dr_1 \tilde{R}(q, r_1, r'_1) \]  
(11)

Substitution of (11) into (9) provides the coincidence SI transparency \( T^E \) as function of missing proton momenta.

The neglect of NN-correlations amounts to putting \( \zeta_2 \) in (10), thus

\[ \tilde{\Omega}_0(q, r_1, r'_1) = -(A - 1) \frac{\sigma_{tot}^2}{2} (1 - i\tau) \int_{-\infty}^{0} dz \int d^2 b A(b) \rho(r_1 - r) \]
\[ \rightarrow -(A - 1) \frac{\sigma_{tot}^2}{2} (1 - i\tau) \int_{z_1}^{\infty} d\tilde{z} \rho(b_1, \tilde{z}) \]  
(12a)

\[ \tilde{\Omega}'_0(q, r_1, r'_1) = (\Omega_0(q, r'_1, r_1)^* \]
\[ = -(A - 1) \frac{\sigma_{tot}^2}{2} (1 + i\tau) \int_{z_1}^{\infty} d\tilde{z} \rho(b_1 - s_\perp, \tilde{z} - s_z) \]  
(12b)

\[ \tilde{M}_0(q, r_1, s) = (A - 1) \sigma_{tot el} e^{-s_\perp^2 Q_0^2 / 8} \left[ \int_{-\infty}^{0} dz J - \theta(-s_z) \int_{-|s_z|}^{0} dz J \right] \]
\[ J = \rho \left( b_1 - s_\perp / 2, z_1 - z \right) \]  
(12c)

In I we have tested several relaxations on the proton momentum in the above \( T \), moving from semi-inclusive to ever more inclusive reactions. Also here we consider the same when yields for fixed \( q \) are integrated over energy loss as well as the same for unobserved proton momentum, transverse to \( \hat{q} \), i.e. (cf. I.15)
\[ T^{E, P \perp}(q, p_z) \equiv \frac{\int d^2 p_\perp S^{SI}(q, p) S^{P \perp}(q, p)}{\int d^2 p_\perp S^{SI, P \perp}(q, p)} = \frac{\int d s \varepsilon^{ip_z s_z} \Sigma(s_z) \tilde{G}(q, s_z)}{\int d s \varepsilon^{ip_z s_z} \Sigma(s_z)} \]  

(13)

The same for a maximal \( p_z \) cut reads

\[ T^{E, P}(q, p) \equiv \frac{\int dp_\perp S^{SI}(q, p) S^{P \perp}(q, p)}{\int dp_\perp S^{SI, P \perp}(q, p)} = \frac{\int d s \varepsilon^{iP_z s_z} \Sigma(|s_z|) \tilde{G}(q, |s_z|)}{\int d s \varepsilon^{iP_z s_z} \Sigma(|s_z|)} \]  

(14)

III. RESULTS AND DISCUSSION.

Using the parameters of Table I in I, we have computed transparencies under varying experimental and theoretical conditions. The discussion here will be limited to \( T^E \), Eqs. (9), and \( T^{E, P \perp} \), Eq. (13), and both use energy-integrated cross sections. Their difference lies in the transverse proton momentum \( p_\perp \) which in the former case is observed and in the latter is not.

Figs. 1a-c show the above mentioned transparencies for C, Fe and Au for a selected value \( p_z = q \) and as function of the four \( Q^2 \) values of the NE18 experiment. In each case we show results with \( NN \)-correlations included and with those removed, i.e. using (11) in respectively (10) and (12). In addition we added the result worked out in I and [9] for the above mentioned approximation where the proton-trajectories in \( \gamma \), Eq. (1), having equal impact parameters \( b_1 \). The following emerges:

1) For the smallest measured \( Q^2 = 1 \) GeV\(^2 \) the equal impact parameter Ansatz considerably overestimates the proper \( T^E \). For all other data points the overestimate is less than 10\%, which is not small on the scale of effects one wishes to investigate.

2) As has also been observed in I, taking a maximal cut in \( p_\perp \) leads only to minor changes in \( T \).

3) The inclusion of \( NN \)-correlations in \( T^E, T^{E, P \perp} \) based on binary collision FSI, enhances both chosen transparencies in a hardly \( Q^2 \)-independent fashion by \( \approx 15\% \) for C to \( \approx 10\% \) for Au.

The latter outcome is well-established and numerically significant, yet no clear-cut conclusions follow from a comparison of data. Predictions when \( NN \)-correlations are included
or in their absence are both within error bars, and from the current data one cannot prove
the need for the above correlations.

On the theoretical side there is uncertainty about the influence of higher order contribu-
tions. For instance Nikolaev et al \[6\] estimated the effect of ternary collisions between
the knocked-out proton and a pair of correlated core nucleons on $T^{E,P}$ and claim that those
about halve correlation effects in binary collisions FSI alone. We have already remarked
in I that not all ternary collision terms in (1), quadratic in $\gamma$ have been retained in \[3\].
It is desirable to include corrections due to FSI from higher order collision, but a reliable
calculation may well be difficult.

Regarding the data, with the exception of C, the lowest $Q^2$ data points agree with
predictions and reflect the relatively low $NN$ cross sections, which enter (9) and (13). Their
increase for increasing energy to fairly constant values leads to a predicted plateau in $T$. The
reported increase of $T$ for large $Q^2$ falls therefore outside hadronic predictions. A similar
observation with much greater accuracy may mark the onset of Colour Transparency, but
the large error bars preclude such a conclusion at the moment.

We conclude by comparing the above formalism for SI processes and the one used in \[22\]
for totally inclusive $A(e, e'/X$ ones. Both emphasize the Final State Interaction of a struck
proton with the core and their descriptions have therefore common features. It is therefore
gratifying to see a similar measure of agreement between data and predictions for both type
of reactions.
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Figure Captions.

Fig. 1a. Transparencies of C for a proton, knocked out in $C(e,e'p)X$ for NE18 kinematics and $p_z = q$, as function of $Q^2$. Drawn and dotted lines correspond to $\mathcal{T}^E$, Eq. (9), with the effect of $NN$-correlations included, respectively neglected. Long dashes and dash-dotted curves are pararle results for $\mathcal{T}^{E,P\perp}$, Eq. (13). Short dashes are results from I for (9) including correlations, assuming $b'_1 = b_1$ in (1). Data are from O’Neill et al [3] and differ slightly from previously published data by Makins et al [2].

Fig. 1b. Same as Fig. 1a for Fe. Data are from O’Neill et al [3].

Fig. 1c. Same as Fig. 1a for Au.
