1 Introduction

The resummation of large logarithms associated with wide angle soft gluon emissions has been investigated for the last 20 years. For certain observables the contributions from non-global logarithms have to be taken into account. One of the simplest of these non-global observables is the ‘gaps-between-jets’ cross-section. This is the cross-section for producing a pair of high transverse momentum jets ($Q$) with a restriction on the transverse momentum of any additional jets radiated in between the two jets, i.e. $k_T < Q_0$ for emissions in the gap. This observable has been studied and has been measured at HERA and the Tevatron.

In the original calculations of the gaps-between-jets cross section, all those terms $\sim \alpha_s^n \ln^n(Q/Q_0)$ that can be obtained by dressing the primary $2 \to 2$ scattering in all possible ways with soft virtual gluons were summed. The restriction to soft gluons implies the use of the eikonal approximation. Let us focus on quark-quark scattering from now on. The corresponding resummed cross-section can be written

$$\sigma = M^\dagger S_V M \quad \text{with} \quad M = \exp \left( -\frac{2\alpha_s}{\pi} \frac{Q}{Q_0} \int \frac{dk_T}{k_T} \Gamma \right) M_0. \quad (1)$$

Here, $M$ is the all-orders $qq \to qq$ amplitude (a 2-component vector in colour space), $M_0$ is the hard scattering amplitude and $S_V$ represents the cut. The anomalous dimension matrix $\Gamma$. 

We identify a source of super-leading logarithms in the gaps-between-jets observable at hadron colliders. These new contributions are expected to generally appear in non-global observables in QCD and are connected with the presence of Coulomb phase terms.
incorporates the effect of dressing a $qq \to qq$ amplitude with a virtual gluon in all possible ways. It receives contributions from two distinct regions of the loop-integral: the first corresponds to an on-shell gluon (to which one can assign a rapidity) and is identical, but with opposite sign, to the contribution from a real gluon. The second contribution, sometimes referred to as the ‘Coulomb gluon’ contribution’ is purely imaginary ($i\pi$ terms) and stems from the region where the emitting parton is on-shell. Eq. (1) therefore corresponds to the independent emission of soft gluons, i.e. the iterative dressing of the $2 \to 2$ process with a softer gluon: due to perfect real/virtual cancellation outside the gap (the first line of fig. 1 shows two contributions) one only has to consider virtual gluons in the gap and the Coulomb terms.

Figure 1: Illustrating the cancellation (and miscancellation) of soft gluon corrections.

However, there is another source of leading logarithms. Let us consider the two diagrams in the second line of fig. 1. A real gluon (which is outside the gap by the definition of our observable) emits a softer real or virtual gluon. The real-virtual cancellation is guaranteed only for the softest gluon. Since real gluons above $Q_0$ are forbidden in the gap, the two diagrams do not completely cancel; the left diagram with the virtual gluon being in the gap and its $k_T$ being larger than $Q_0$ survives. The non-global nature of our observable has prevented the soft gluon cancellation which is necessary in order that eq. (1) should be the complete result.

It is therefore necessary to include the emission of any number of soft gluons outside the gap region (real and virtual) dressed with any number of virtual gluons within the gap region. Clearly it is a formidable challenge to sum all leading logarithms, mainly because of the complicated colour structure. Progress has been made, working in the large N approximation. Here, we keep the exact colour structure but instead we only compute the cross-section for one gluon outside the gap region. This can be viewed as the first term in an expansion in the number of out-of-gap-gluons.

## 2 Super-leading logarithms

In order to extract the leading logarithms we consider soft gluons strongly ordered in transverse momentum. The cross-section for one gluon outside and any number of gluons inside the gap is split into two parts corresponding to a virtual or real out-of-gap gluon:

$$\sigma_1 = -\alpha \int_{Q^2}^{Q_0} \frac{dk_T}{k_T} \int_{out} \frac{dy \, d\phi}{2\pi} \left( A_V + A_R \right), \quad \bar{\alpha} \equiv \frac{2\alpha_s}{\pi}$$

(2)
\[ A_R = M_0^\dagger \exp \left( -\bar{\alpha} \int_{k_T}^{Q} \frac{dk_T'}{k_T'} \Gamma^{\dagger} \right) \mathbf{D}_\mu^\dagger \exp \left( -\bar{\alpha} \int_{Q_0}^{k_T} \frac{dk_T'}{k_T'} \Lambda^{\dagger} \right) S_R \]

\[ \exp \left( -\bar{\alpha} \int_{Q_0}^{k_T} \frac{dk_T'}{k_T'} \Lambda \right) \mathbf{D}_\mu \exp \left( -\bar{\alpha} \int_{k_T}^{Q} \frac{dk_T'}{k_T'} \Gamma \right) M_0, \]  

\[ A_V = M_0^\dagger \exp \left( -\bar{\alpha} \int_{Q_0}^{k_T} \frac{dk_T'}{k_T'} \Gamma^{\dagger} \right) S_V \exp \left( -\bar{\alpha} \int_{k_T}^{Q} \frac{dk_T'}{k_T'} \Gamma \right) \gamma \exp \left( -\bar{\alpha} \int_{k_T}^{Q} \frac{dk_T'}{k_T'} \Gamma \right) M_0 + \text{c.c..} \]  

$\mathbf{D}_\mu$ and $\gamma$ are the matrices that represent the emission of a real and a virtual gluon ($k_T, y, \phi$) outside the gap, respectively. The major new ingredient is the matrix $\Lambda$ which incorporates the dressing of the $qq \rightarrow qgg$ process with a virtual gluon. The emission of the out-of-gap gluon is sandwiched between two exponentials: this accounts for all possible positions of the out-of-gap gluon within a chain of any number of $k_T$-ordered gluons within the gap.

The phase space of the out-of-gap gluon in eq.2 includes the configurations where it is collinear to either of the external quarks. One might suppose that the corresponding divergences cancel among $A_R$ and $A_V$. This is true in case of the final state collinear limit. However, in the limit of the out-of-gap gluon becoming collinear to one of the initial state quarks, i.e. $|y| \rightarrow \infty, k_T > Q_0$, there is no cancellation:

\[ [A_V + A_R]|_{|y|\rightarrow\infty} \neq 0. \]  

In particular, $(A_V + A_R)$ becomes independent of $y$ in that limit. This has severe consequences. As the out-of-gap region stretches to infinity in rapidity, the integral eq.2 is divergent as it stands. This divergence however indicates that one needs to go beyond the soft approximation when considering the out-of-gap gluon. As the energy of the out-of-gap gluon is $E_g = 1/2 k_T(e^y + e^{-y})$ and $k_T > Q_0$ the limit $|y| \rightarrow \infty$ implies $E_g \rightarrow \infty$. Insisting on $E_g$ being smaller than the centre-of-mass energy $E_{CMS} = 2Q \cosh \Delta/2$ ($\Delta$ is the rapidity interval between the final state quarks) then translates into $y < \ln(Q/k_T)$. The integration in eq.2 therefore gives $\alpha_s \ln^2(Q/Q_0)$ whereas each gluon within the gap only contributes $\alpha_s \ln(Q/Q_0)$. We have obtained a hitherto unknown super-leading logarithmic contribution, i.e. a term $\sim \alpha_s^2 \ln^{n+1}(Q/Q_0)$.

In a more rigorous treatment the out-of-gap gluon is considered in the collinear (but not soft) approximation. The ‘plus prescription’ for the collinear divergence appears accompanied by an additional term that generates the super-leading logarithm. The additional logarithm can therefore be seen as extra collinear logarithm due to the failure of the ‘plus prescription’ for gluons with $k_T > Q_0$; collinear logarithms in the gaps-between-jets cross-section can be summed into the pdf’s only up to scale $Q_0$.

The super-leading logarithms are formally more important than any leading-logarithmic result. Their numerical impact, though, still has to be investigated (for first results see 8). There is a couple of remarks to be made.

- The miscancellation eq.5 and hence the super-leading logarithm is intimately connected with the Coulomb phase terms. If one artificially switches off the $i\pi$ terms in the evolution matrices, then there is full cancellation in eq.5. Moreover, the super-leading logarithm makes its appearance at the lowest possible order in $\alpha_s$, i.e. at $O(\alpha_s^4)$ relative to the Born cross-section. More explicitly, the $O(\alpha_s)$ and $O(\alpha_s^2)$ corrections to the Born cross-section simply never involve more than one $i\pi$ term and hence any $i\pi$ terms must cancel since the cross-section is real. The first candidate order at which two factors of $i\pi$ can appear is
therefore $O(\alpha_s^3)$. However, the addition of the gluon with the lowest $k_T$ can never generate a net factor of $i\pi$ since any such factors must cancel between the two diagrams where the lowest $k_T$ gluon lies either side of the cut. The first $i\pi$ terms and the first super-leading logarithm appear in case of four soft gluons:

$$\sigma_1 \sim \sigma_0 \left( \frac{2\alpha_s}{\pi} \right)^4 \ln^5 \left( \frac{Q}{Q_0} \right) \pi^2 Y. \quad (6)$$

Here, $Y$ is the size of the rapidity gap and $\sigma_0$ is the Born cross-section.

- At $O(\alpha_s^4)$ the super-leading logarithms arise from one out-of-gap gluon at maximum. The proof of this is based on the fact that two or more out-of-gap gluons imply two or less gluons that can provide $i\pi$ terms. We have shown above that this is not enough for the $i\pi$ factors to appear in the cross-section and we know that only if $i\pi$ terms survive there is a non-cancellation in the initial state collinear limit.

- At higher orders in $\alpha_s$ more gluons can be outside the gap. At each order there is a maximum number of out-of-gap gluons that we expect to provide the maximum power of the logarithm stemming from the region where they all become collinear to either of the initial state partons. At each higher order we then have an additional $\alpha_s \ln^2(Q/Q_0)$ in such a configuration. What appears as super-leading logarithms as compared to the single logs, $\alpha_s^n \ln(Q/Q_0)^n$ would thus constitute a series of double logarithms, $\alpha_s^n (\ln^2(Q/Q_0))^n$. To resum these double logarithms a deeper understanding of the colour evolution of multiparton systems seems necessary.

- The new super-leading contributions are not restricted to the gaps-between-jets observable. In event shape variables, for instance, the requirement of the variable to be smaller than some value translates into a restriction of the phase space available for real gluons, i.e. a gap. We expect the super-leading logarithms to arise generally in non-global observables.

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