Cooper Pairing Revisited

V.C. Aguilera-Navarro,\textsuperscript{a} M. Fortes,\textsuperscript{b} M. de Llano\textsuperscript{c}
F.J. Sevilla\textsuperscript{b}

\textsuperscript{a}Depto. de Física, UNICENTRO, 85015, Guarapuava, PR, Brazil
\textsuperscript{b}Instituto de Física, UNAM, 01000 México, DF, Mexico
\textsuperscript{c}Instituto de Investigaciones en Materiales, UNAM, 04510 México, DF, Mexico

1 Introduction

Doubtlessly the most central notion in superconductivity, for both low and high transition temperatures $T_c$, is that of Cooper pairs (CPs) \cite{1} that form among the underlying electron (or hole) charge carriers of the many-electron system. And yet, it is perhaps the least understood concept. Shortly after the publication of the BCS theory \cite{2} of superconductivity, charged pairs observed in magnetic flux quantization experiments with 3D conventional \cite{3}\cite{4}, and much later with quasi-2D cuprate \cite{5} superconductors, suggested CPs as an indispensable ingredient, regardless of the inability of BCS theory \textit{per se} to describe high-$T_c$ superconductivity, as now seems an almost universal consensus. Cooper pairing is no less central, albeit with a different interfermion interaction, to \textit{neutral-atom superfluidity} as in liquid $^3$He \cite{6}, and presumably also to ultracold \textit{trapped alkali Fermi gases} such as $^6$Li \cite{7} and $^{40}$K \cite{8} where pairing is expected to occur as well. More recently, studies of quantum degenerate Fermi gases consisting of neutral $^{40}$K atoms and their so-called Feshbach \textquotedblleft resonance superfluidity" have appeared \cite{9}-\cite{13}, but assuming quadratically-dispersive CPs and ignoring hole pairs—two fundamental shortcomings as we now illustrate.

In Sec. 2 we recall how the Bethe-Salpeter (BS) many-body equation (in the ladder approximation), treating both 2p and 2h pairs on an equal footing, implies that the ordinary CP problem [based on an ideal Fermi gas (IFG)
ground state (the usual “Fermi sea”) does not possess stable energy solutions; in Sec. 3 we sketch how CPs based not on the IFG-sea but on the BCS ground state survive, along with the usual trivial sound mode, as nontrivial “generalized” or “moving” CPs, linear in total or center-of-mass-momentum (CMM) in leading order, that are positive energy resonances with an imaginary energy term implying finite-lifetime effects. The nontrivial “moving CP” solution, though often confused [14] with it, is physically distinct from the trivial sound mode solution sometimes called the Anderson-Bogoliubov-Higgs (ABH) [15], ([16] p. 44), [17][18] collective excitation mode. The ABH mode is also linear in CMM in leading order, and reduces to the IFG ordinary sound mode in zero coupling. All this occurs in both the 3D study outlined in Ref. [19] as well as in 2D [20]. Sec. 4 offers conclusions.

Provided CPs are bosons, as they indeed [21] turn out to be, our results will in general be crucial for Bose-Einstein condensation (BEC) scenarios employing BF models of superconductivity, not only in exactly 2D as with the Berezinskii-Kosterlitz-Thouless [22][23] transition, but also down to (1 + \( \epsilon \))D which characterize the quasi-1D organo-metallic (Bechgaard salt) superconductors [24]-[26]. This is contrary to well-entrenched perceptions (see, e.g., Ref. [27]) that BEC is impossible in 2D.

2 Ordinary Cooper pairing

The original “ordinary” CP problem [1] (with the BCS model two-electron interaction) is defined in an \( N \)-electron system for two electrons above the Fermi energy \( E_F \) but within a thin shell of energy \( \hbar \omega_D \). Only there do they suffer a constant attraction \(-V < 0\), giving rise to the familiar result for the negative energy of the pair, relative to the energy \( 2E_F \) of the interactionless pair,

\[
\mathcal{E}_0 = -\frac{2\hbar \omega_D}{e^{2/\lambda} - 1} \quad \xrightarrow{\lambda \to 0} \quad -2\hbar \omega_D e^{-2/\lambda}.
\]

Here \( \lambda \equiv g(E_F)V \) is a dimensionless coupling constant with \( g(E_F) \) the density of fermionic states (for each spin) evaluated at \( E_F \). The equality in (1) is exact in 2D for all coupling—as well as in 1D or 3D provided only that \( \hbar \omega_D \ll E_F \) so that \( g(\epsilon) \simeq g(E_F) \), a constant that can be taken outside an energy integral.
However, the original CP problem neglects the effect of two-hole (2h) CPs treated *simultaneously* on an equal footing with two-electron, or two-particle (2p), CPs—as Green’s functions [28] can naturally guarantee. On the other hand, the BCS condensate consists of equal numbers of 2p and 2h *Cooper* correlations. This was already evident, though scarcely emphasized, from the perfect symmetry about $\mu$, the electron chemical potential, of the well-known Bogoliubov [29] $v^2(\epsilon)$ and $u^2(\epsilon)$ coefficients [see just below (10) later on], where $\epsilon$ is the electron energy. Indeed, our prime motivation comes from the fact established recently [30] that the BCS condensate is a BEC condensate for equal numbers of 2p and 2h pairs, in the limit of weak coupling. Additional empirical motivation comes from the unique but unexplained role played by *hole* charge carriers in the normal state of superconductors in general [31]. Even further motivation stems from the ability of the “complete (in that both 2h- and 2p-CPs are allowed in varying proportions) BF model” of Refs. [30],[32]-[33] to “unify” both BCS and BEC theories as special cases, and to predict substantially higher $T_c$’s than BCS theory without abandoning electron-phonon dynamics. Compelling evidence for a significant presence of this dynamics in high-$T_c$ cuprate superconductors from angle-resolved photoemission spectroscopy data has recently been reported [34].

In dealing with the many-electron system we assume the BCS model interaction in the form with double Fourier transform

$$\nu(|k_1 - k'|) = -(k_F^2/k_1 k_1')V \quad \text{if} \quad k_F - k_D < k_1, \quad k_1' < k_F + k_D,$$

(2)

and $= 0$ otherwise. As before $V > 0$, $\hbar k_F \equiv mv_F$ the Fermi momentum, $m$ the effective electron mass, $v_F$ the Fermi velocity, and $k_D \equiv \omega_D/v_F$ with $\omega_D$ the Debye frequency. The usual physical constraint $\hbar \omega_D \ll E_F \equiv \hbar^2 k_F^2/2m$ then implies that $k_D/k_F \equiv \hbar \omega_D/2E_F \ll 1$.

The bound-state BS wavefunction equation [19] in the ladder approximation with both particles and holes for the original IFG-based CP problem using an interaction such as (2) is

$$\Psi(k,E) = -\left(\frac{i}{\hbar}\right)^2 G_0(K/2 + k, \xi_K/2 + E) G_0(K/2 - k, \xi_K/2 - E) \times$$

$$\times \frac{1}{2\pi i} \int_{-\infty}^{+\infty} dE' \frac{1}{L^d} \sum_{k'} \nu(|k - k'|) \Psi(k', E')$$

(3)
where $L^d$ is the “volume” of the $d$-dimensional system; $K \equiv \mathbf{k}_1 + \mathbf{k}_2$ is the CMM and $k_1 \equiv k_1 - k_2$ the relative wavevectors of the 2e bound state whose wavefunction is $\Psi(k, E)$; $E_K \equiv E_1 + E_2$ is the energy of this bound state while $E \equiv E_1 - E_2$, and $G_0(K/2 + \mathbf{k}, E/2 + E)$ is the bare one-fermion Green’s function given by

$$G_0(k_1, E_1) = \frac{\hbar}{i} \left\{ \frac{\theta(k_1 - k_F)}{-E_1 + \epsilon_{k_1} - E_F - i\varepsilon} + \frac{\theta(k_F - k_1)}{-E_1 + \epsilon_{k_1} - E_F + i\varepsilon} \right\} \tag{4}$$

where $\epsilon_{k_1} \equiv \hbar^2 k_1^2/2m$ and $\theta(x) = 1$ for $x > 0$ and $= 0$ for $x < 0$, so that the first term refers to electrons and the second to holes. Figure 1 shows all Feynman diagrams for the 2p, 2h and ph wavefunction $\psi_+, \psi_-$ and $\psi_0$, respectively, that emerge in the general (BCS-ground-state-based) problem to be discussed later. For the present IFG-based case, diagrams in shaded rectangles do not contribute. Since the energy dependence of $\Psi(k, E)$ in (3) is only through the Green’s functions, the ensuing energy integrals can be evaluated directly in the complex $E'$-plane and yield, for interaction (2),

$$(2\xi_k - \mathcal{E}_0)\psi_k = V \sum_{k'}^\prime \psi_{k'} - V \sum_{k''}^\prime \psi_{k'} \tag{5}$$

where $\psi_k$ is the resulting wavefunction after the energy integration. Here $\xi_k \equiv \hbar^2 k^2/2m - E_F$ while $\mathcal{E}_0$ is the (unknown) eigenvalue energy. The single prime over the first (2p-CP) summation term denotes the restriction $0 < \xi_{k'} < \hbar\omega_D$ (i.e., above the IFG “sea”) while the double prime in the last (2h-CP) term means $-\hbar\omega_D < \xi_{k'} < 0$ (i.e., below the IFG sea). Without this latter term we have Cooper’s Schrödinger-like equation [1] for 2p-CPs whose implicit wavefunction solution is clearly $\psi_k = (2\xi_k - \mathcal{E}_0)^{-1}V \sum_{k'}^\prime \psi_{k'}$. Since the summation term is constant, performing that summation on both sides allows canceling the $\psi_k$-dependent terms, leaving the eigenvalue equation

$$\sum_k^\prime (2\xi_k - \mathcal{E}_0)^{-1} = 1/V.$$ 

This is one equation in one unknown $\mathcal{E}_0$; transforming the sum to an integral over energies immediately gives (1). This corresponds to the usual negative-energy, infinite-lifetime stationary-state bound pair. For $K \geq 0$ the CP eigenvalue equation is just

$$\sum_k^\prime (2\xi_k - \mathcal{E}_K + \hbar^2 K^2/4m)^{-1} = 1/V.$$ 

Since a CP state of energy $\mathcal{E}_K$ is characterized only by a definite $K$ but not definite $\mathbf{k}$, in contrast to a “BCS pair” defined [Ref. [2], Eqs. (2.11) to (2.13)] with fixed $\mathbf{K}$ and $\mathbf{k}$ (or equivalently definite $\mathbf{k}_1$ and $\mathbf{k}_2$). This
renders CPs legitimate bosons [21]. Without the first summation term in (5) the same result in $E_0$ (1) for 2p-CPs follows for 2h-CPs (apart from a sign change).

In contrast with Cooper’s equation neglecting hole-pairs, the complete CP equation (5) cannot be derived from an ordinary (non-BS) Schrödinger-like equation in spite of its simple appearance. A more general technique such as the BS equation that includes both particles (in this case electrons) and holes is needed. To solve it for the unknown energy $E_0$, let the rhs of (5) be defined as $A - B$, with $A$ relating to the 2p-pair term and $B$ to the 2h-pair term. Then the unknown $\psi_k$ becomes

$$\psi_k = (A - B)/(2\xi_k - E_0) \quad \text{or equivalently} \quad \psi(\xi) = (A - B)/(2\xi - E_0)$$

whence

$$A \equiv \lambda \int_{-h\omega_D}^{h\omega_D} d\xi \psi(\xi) = \frac{1}{2}(A - B)\lambda \int_{-E_0}^{2h\omega_D - E_0} dz/z \equiv (A - B)x,$$

$$B \equiv \lambda \int_{-h\omega_D}^{0} d\xi \psi(\xi) = \frac{1}{2}(A - B)\lambda \int_{-2E_0}^{-E_0} dz/z \equiv (A - B)y.$$

The integrals are readily evaluated giving $x \equiv \frac{1}{2}\lambda \ln(1 - 2h\omega_D/E_0)$ and $y \equiv -\frac{1}{2}\lambda \ln(1 + 2h\omega_D/E_0)$. As $A$ and $B$ still contain the unknown $\psi(\xi)$ let us eliminate them. Note that equations (7) are equivalent to two equations in two unknowns $A$ and $B$, or

$$(1 - x)A + xB = 0 \quad \text{and} \quad -yA + (1 + y)B = 0.$$ 

These readily lead to the single equation $1 - x + y = 0$, which on inserting the definitions for $x$ and $y$ becomes

$$1 = \frac{1}{2}\lambda \ln[1 - (2h\omega_D/E_0)^2] \quad \text{which gives} \quad E_0 = \pm i2h\omega_D/\sqrt{e^{2/\lambda} - 1}.$$

As the CP energy is pure-imaginary, there is an obvious instability of the CP problem when both particle- and hole-pairs are included. This transcendant result dates back to the late 50’s and early 60’s and was reported in Refs. [16] p. 44 and [35] Sec. 33, where, however, the well-known pure 2p and 2h special
cases just stressed was not discussed. Clearly then, the original CP picture is meaningless if particle- and hole-pairs are treated on an equal footing, as consistency demands. Curiously, this result has been largely ignored in the entire literature since then.

A dramatic analogue of the nontriviality of such consistency is found in the very high temperature treatment of relativistic BEC [36], where pair production becomes possible and creates antibosons in addition to more bosons. Here, BEC must take into account $N$ antibosons of charge, say, $-q$ along with the $N$ bosons of charge $q$. In units such that $\hbar \equiv c \equiv k_B \equiv 1$ the boson energy is $\varepsilon_K = (K^2 + m_B^2)^{1/2}$. Charge conservation requires that not only $N \equiv N_0(T) + \sum_{K \neq 0} \exp \beta(\varepsilon_K - \mu_B) - 1$ be constant but rather $N-\tilde{N}$, where $\tilde{N}$ is the same as $N$ but with $+\mu_B$ instead of $-\mu_B$. If $\rho \equiv q(N - \tilde{N})/L^3 \equiv qn$ is the net conserved charge density, it is shown in Ref. [36] that $T_c = (3|n|/m_B)^{1/2}$ and that the condensate fraction $n_0/n = [1 - (T/T_c)^3]$. This is qualitatively different from the better-known results assuming only $N$ constant, which are the mass-independent $T_c = [\pi^2 n/\zeta(3)]^{1/3}$ and $n_0/n = [1 - (T/T_c)^3]$. This example exhibits the strikingly dramatic effect of including or not antiparticles (analogous to holes in the nonrelativistic case).

3 Generalized Cooper pairing

However, a BS treatment not about the IFG sea but about the BCS ground state (which we refer to as “generalized” Cooper pairing) vindicates the CP concept, and adds something new. This is equivalent to starting not from the IFG unperturbed Hamiltonian but from the BCS one. Thus, (4) is replaced by

$$G_0(k_1, E_1) = \frac{\hbar}{i} \left\{ \frac{v_{k_1}^2}{-E_1 + E_{k_1} - i\varepsilon} + \frac{u_{k_1}^2}{-E_1 + E_{k_1} + i\varepsilon} \right\}$$

(9)

where $E_k \equiv \sqrt{\xi_k^2 + \Delta^2}$ with $\Delta$ the fermionic gap, $v_k^2 \equiv \frac{1}{2}(1 - \xi_k/E_k)$ and $u_k^2 \equiv 1 - v_k^2$ are the Bogoliubov functions [29]. As $\Delta \to 0$ these three quantities become $|\xi_k|$, $\theta(k_1 - k_F)$ and $\theta(k_F - k_1)$, respectively, making (9) become (4) as expected. Substituting $G_0(k_1, E_1)$ by $G_0(k_1, E_1)$ corresponds to rewriting the total Hamiltonian so that the pure-kinetic-energy unperturbed Hamiltonian
is replaced by the BCS one. The remaining terms are then assumed suitable to a perturbation treatment. Experimental support for this can be found precisely in Refs. [3]-[5], and its physical justification lies in recovering both the expected ABH sound mode (which contains the BCS $T = 0$ gap equation) and the finite-lifetime effects of moving CPs. In either 3D [19] or 2D [20] the BCS-based BS equation yields two distinct solutions: a) the usual trivial ABH sound solution and b) a highly nontrivial “moving CP” solution. In either case the BS formalism consists of a set of three coupled equations, one for each (2p, 2h and ph) channel wavefunction for any spin-independent interaction such as (2). However, the ph channel decouples, leaving only two coupled wavefunction equations for the ABH solution in 2D which we consider first. We focus here on 2D because of its interest [37] for quasi-2D cuprate superconductors.

The equations involved are too lengthy even in 2D and will be derived in detail elsewhere, but for the trivial or ABH sound solution they boil down to the single expression

$$\frac{1}{2\pi} \lambda \hbar v_F \int_{k_{F-k_D}}^{k_{F+k_D}} dk \int_0^{2\pi} d\varphi \left\{ u_{K/2+k} u_{K/2-k} + v_{K/2+k} v_{K/2-k} \right\} \times$$

$$\times \left[ \frac{v_{K/2+k} u_{K/2-k}}{E_K + E_{K/2+k} + E_{K/2-k}} + \frac{u_{K/2+k} u_{K/2-k}}{-E_K + E_{K/2+k} + E_{K/2-k}} \right] = 1$$

(10)

where $\varphi$ is the angle between $K$ and $k$. Here $k_D$ is defined as below (2) and as before $\lambda \equiv V g(E_F)$ with $g(E_F) \equiv m/2\pi\hbar^2$ the constant 2D electronic DOS and $V$ is defined in (2). The ABH collective excitation mode energy $E_K$ must then be extracted from this equation. For $K = 0$ it is just $E_0 = 0$ (Ref. [16] p. 39) and (10) rewritten as an integral over $\xi \equiv \hbar^2 k^2/2m - E_F$ reduces to the familiar BCS $T = 0$ gap equation $\int^\infty_0 d\xi/\sqrt{\xi^2 + \Delta^2} = 1/\lambda$ for interaction (2) which gives $\Delta = \hbar \omega_D / \sinh(1/\lambda)$. Returning to the ABH energy $E_K$ equation (10) and Taylor-expanding $E_K$ about $K = 0$, and then taking $\Delta$ small, leaves

$$E_K = \frac{\hbar v_F}{\sqrt{2}} K + O(K^2) + o(\lambda)$$

(11)

where $o(\lambda)$ refers to interfermion interaction terms that vanish as $\lambda \to 0$. Note that the leading term is just the ordinary sound mode in an IFG whose
sound speed \( c = v_F / \sqrt{d} \) in \( d \) dimensions. This result also follows (trivially) on solving for \( c \) in the familiar thermodynamic relation \( dP/dn = mc^2 \) involving the zero-temperature IFG pressure \( P = n^2[d(E/N)/dn] = 2nE_F / (d + 2) = 2Cd n^{2/d+1} / (d+2) \) where the constant \( C_d \) will drop out. Here the IFG ground-state energy \( E = dNE_F / (d + 2) \) was used along with \( E_F \equiv \hbar^2k_F^2 / 2m = C_d n^{2/d} \) while \( n \equiv N/L^d = k_F^d / d^{2d-2} \pi^d / 2 \Gamma(d/2) \) is the fermion-number density. The derivative \( dP/dn \) finally gives \( c = \hbar k_F / m \sqrt{d} \equiv v_F / \sqrt{d} \) which in 2D is just the coefficient of the first term of (11).

The nontrivial or moving CP solution of the BCS-ground-state-based BS treatment, which is entirely new, leads to the pair energy \( \mathcal{E}_K \) which in 2D is contained in the equation

\[
\frac{1}{2\pi} \hbar v_F \int_{k_F-k_D}^{k_F+k_D} dk \int_0^{2\pi} d\varphi u_{K/2+k} v_{K/2-k} \times \\
\times \left\{ u_{K/2-k} v_{K/2+k} - u_{K/2+k} v_{K/2-k} \right\} \frac{E_{K/2+k} + E_{K/2-k}}{-\mathcal{E}_K^2 + (E_{K/2+k} + E_{K/2-k})^2} = 1,
\]

(12)

In addition to the pp and hh wavefunctions (depicted diagrammatically in Ref. [19] Fig. 2), diagrams associated with the ph channel give zero contribution at \( T = 0 \). A third equation for the ph wavefunction describes the ph bound state but turns out to depend only on the pp and hh wavefunctions. Taylor-expanding \( \mathcal{E}_K \) in (12) in powers of \( K \) around \( K = 0 \), and introducing a possible damping factor by adding an imaginary term \( -i\Gamma_K \) in the denominator, yields to order \( K^2 \) for small \( \lambda \)

\[
\pm \mathcal{E}_K \simeq 2\Delta + \frac{\lambda}{2\pi} \hbar v_F K + \frac{1}{9} \frac{\hbar v_F}{k_D} e^{1/\lambda} K^2 - i\hbar v_F K \left[ \frac{\lambda}{\pi} + \frac{1}{12k_D} e^{1/\lambda} K \right] + O(K^3)
\]

(13)

where the upper and lower sign refers to 2p- and 2h-CPs, respectively. A linear dispersion in leading order again appears, but now associated with the bosonic moving CP. Figure 2a graphs the exact moving CP real energy (full curves) extracted from (12), along with its leading linear-dispersion term (short-dashed) and this plus the next (quadratic) term (long-dashed) from (13). The interaction parameter values used for (2) were \( \hbar \omega_D / E_F = 0.05 \) (a typical value for cuprates) and the two values \( \lambda = \frac{1}{4} \) (lower set of curves) and \( \frac{1}{2} \) (upper set), so that \( \mathcal{E}_0 / E_F \equiv 2\Delta / E_F = 2\hbar \omega_D / E_F \sinh(1/\lambda) \). This has values \( \simeq 0.004 \) (for \( \lambda = \frac{1}{4} \)) and 0.028 (for \( \lambda = \frac{1}{2} \)), (marked as dots in the
Remarkably enough, the linear approximation (short-dashed lines in figure) is better over a wider range of $K/k_F$ values for weaker coupling in spite of a larger and larger partial contribution from the quadratic term in (13). This peculiarity also emerged from the ordinary CP treatment with the BCS model interaction [38] in both 2D and 3D, and with a (regularized) delta interaction in 2D [39] and in 3D [40], and might suggest the expansion in powers of $K$ to be an asymptotic series that should be truncated after the linear term. For reference we also plot the linear term $\hbar v_F K / \sqrt{2}$ of the sound solution (11) (thick long-dashed line starting at origin). The positive-energy 2p-CP resonance of width $\Gamma_K$ has a lifetime from (13) of $\tau_K \equiv \hbar / 2 \Gamma_K = \hbar / 2 \left[ (\lambda/\pi) h v_F K + (h v_F / 12 k_D) e^{1/\lambda} K^2 \right]$ diverging only at $K = 0$, and falling to zero as $K$ increases; see Fig. 2b. Thus, “faster” moving CPs are shorter-lived and eventually break up, while “non-moving” ones are (infinite-lifetime) stationary states. The real linear term $(\lambda / 2\pi) h v_F K$ in (13) contrasts sharply with the coupling-independent leading-term $(2 / \pi) h v_F K$ that follows [38] from the original CP problem neglecting holes, which if graphed in Fig. 2a would almost coincide with the ABH term $h v_F K / \sqrt{2}$ and have a slope about 90% smaller. Fig. 2c depicts analogies of a 3D potential problem for the original CP energy (top), as well as for $K > 0$ (middle) and $K = 0$ (bottom) BCS-based BS CPs.

As in Cooper’s [1] original equation, our BCS-based BS moving CPs are characterized by a definite $K$ and not also by definite $k$ as the pairs discussed by BCS [2]. Hence, the objection does not apply that CPs are not bosons because BCS pairs with definite $K$ and $k$ (or equivalently definite $k_1$ and $k_2$) have creation/annihilation operators that do not obey the usual Bose commutation relations [Ref. [2], Eqs. (2.11) to (2.13)]. In fact, (12) shows that a given “generalized” CP state labeled by either $K$ or $E_K$ can accommodate (in the thermodynamic limit) an indefinitely many possible BCS pairs with different $k$’s; see Ref. [21]. A recent electronic analog [41] of the Hanbury Brown-Twiss photon-effect experiment suggests electron pairs to be definitely bosons.

4 Conclusions

Hole pairs treated on a par with electron pairs play a vital role in determining the precise nature of nontrivial CPs even at zero temperature—only when
based not on the usual IFG “sea” but on the BCS ground state. Their treatment with a Bethe-Salpeter equation gives purely-imaginary-energy CPs when based on the IFG, and when based on the BCS ground state gives positive-energy, resonant-state CPs with a finite lifetime for nonzero CMM. This is instead of the more familiar negative-energy stationary states of the original IFG-based CP problem that neglects holes, as sketched just below (5). The BS “moving-CP” dispersion relation (13), on the other hand, is gapped by twice the BCS energy gap, followed by a linear leading term in the CMM expansion about $K = 0$. This linearity is distinct from the better-known but trivial one (11) associated with the sound or ABH collective excitation mode whose energy vanishes at $K = 0$.

Thus, instead of the quadratic $\hbar^2 K^2 / 2(2m)$ assumed for CPs in Refs. [9]-[13],[30],[32],[42]-[44], among many others, BF models based on the correct CP linearity for the boson component can give BEC for all $d > 1$, including exactly 2D, and thus in principle address not only quasi-2D cuprate but also quasi-1D organo-metallic superconductors.

Acknowledgments MdeLl and MF thank UNAM-DGAPA-PAPIIT (Mexico), grant # IN106401, and CONACyT (Mexico), grant # 27828 E, for partial support. MdeLl is grateful for travel support through a grant to Southern Illinois University at Carbondale from the U.S. Army Research Office.

References

[1] L.N. Cooper, Phys. Rev. 104, 1189 (1956).

[2] J. Bardeen, L.N. Cooper and J.R. Schrieffer, Phys. Rev. 108, 1175 (1957).

[3] B.S. Deaver, Jr. and W.M. Fairbank, Phys. Rev. Lett. 7, 43 (1961).

[4] R. Doll and M. Nährauer, Phys. Rev. Lett. 7, 51 (1961).

[5] C.E. Gough, et. al., Nature 326, 855 (1987).

[6] D. Vollkardt and P. Wölfle, *The Superfluid Phases of Helium 3*, (Taylor & Francis, London, 1990).
[7] K.E. Strecker, et. al., Phys. Rev. Lett. 91, 080406 (2003).
[8] M. Holland, et. al., Phys. Rev. A 61, 053610 (2000), and refs. therein.
[9] M.J. Holland, et. al., Phys. Rev. Lett. 87, 1204061 (2001).
[10] E. Timmermans, et. al., Phys. Lett. A 285, 228 (2001).
[11] M.L. Chiofalo, et. al., Phys. Rev. Lett. 88, 090402 (2002).
[12] Y. Ohashi and A. Griffin, Phys. Rev. Lett. 89, 130402 (2002).
[13] L. Pitaevskii and S. Stringari, Science 298, 2144 (2002).
[14] M. Randeria, asserted at CMT20 Workshop (Poone, India, 1996).
[15] P.W. Anderson, Phys. Rev. 112, 1900 (1958).
[16] N.N. Bogoliubov, V.V. Tolmachev and D.V. Shirkov, Fortschr. Phys. 6, 605 (1958); and A New Method in the Theory of Superconductivity (Consultants Bureau, NY, 1959).
[17] P.W. Higgs, Phys. Lett. 12, 132 (1964).
[18] L. Belkhir, M. Randeria, Phys. Rev. B 49, 6829 (1994).
[19] M. Fortes, et. al., Physica C 364-365, 95 (2001).
[20] V.C. Aguilera-Navarro, et. al., to be published. cond-mat/0306726.
[21] M. de Llano, F.J. Sevilla, S. Tapia and J.R. Clem, to be published. cond-mat/0307258.
[22] V.L. Berezinskii, Sov. Phys. JETP 34, 610 (1972).
[23] J.M. Kosterlitz and D.J. Thouless, J. Phys. C6, 1181 (1973).
[24] D. Jérôme, Science 252, 1509 (1991).
[25] J.M. Williams, et. al., Science 252, 1501 (1991).
[26] H. Hori, Int. J. Mod Phys. B 8, 1 (1994).
[27] A.A. Abrikosov, asserted at CMT26 Workshop (Luso, Portugal, 2002).

[28] A.L. Fetter and J.D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[29] N.N. Bogoliubov, N. Cim. 7, 794 (1958).

[30] V.V. Tolmachev, Phys. Lett. A 266, 400 (2000).

[31] J. Hirsch, Physica C 341-348, 213 (2000) and also www.iitap.iastate.edu/htcu/forum.html#Q3.

[32] M. de Llano and V.V. Tolmachev, Physica A 317, 546 (2003).

[33] J. Batle, et. al., Cond. Matter Theories 18 (in press) (2003); BCS and BEC Finally Unified: A brief review. cond-mat/0211456.

[34] A. Lanzara, et. al., Nature 412, 510 (2001).

[35] A.A. Abrikosov, L.P. Gorkov, and I.E. Dzyaloshinskii, *Methods of Quantum Field in Statistical Physics* (Dover, NY, 1975) § 33.

[36] H.E. Haber and H.A. Weldon, Phys. Rev. Lett. 46, 1497 (1981).

[37] B.D. Brandow, Phys. Repts. 296, 1 (1998).

[38] M. Casas, et. al., Physica C 295, 93 (1998).

[39] S.K. Adhikari, et. al., Phys. Rev. B 62, 8671 (2000).

[40] S.K. Adhikari, et. al., Physica C 351, 341 (2001).

[41] P. Samuelsson and M. Büttiker, Phys. Rev. Lett. 89, 046601 (2002).

[42] J.M. Blatt, *Theory of Superconductivity* (Academic, New York, 1964).

[43] R. Friedberg and T.D. Lee, Phys. Rev. B 40, 6745 (1989).

[44] R. Friedberg, T.D. Lee, and H.-C. Ren, Phys. Lett. A 152, 417 and 423 (1991).
Figure captions.

1. Wavefunction Feynman diagrams for 2p ($\psi_+$), 2h ($\psi_-$) and ph ($\psi_0$) bound states arising from the BS equations. Shaded rectangles designate diagrams that do not contribute in the IFG-based case.

2. a) Exact “moving” 2p-CP (real) energy $E_K$ (in units of $E_F$) in 2D from (12) (full curves), compared with its linear leading term (short-dashed lines) and its linear plus quadratic expansion (long-dashed curves) both from (13), $vs.$ CMM wavenumber $K$ (in units of $k_F$), for interaction (2) parameters $\lambda = \frac{1}{4}$ (lower set of curves) and $\frac{1}{2}$ (upper set of curves), and $\hbar \omega_D/E_F = 0.05$. For reference, leading linear term (11) of trivial ABH sound mode is also plotted (lower thick dashed line). b) 2p-CP lifetime as defined in text. c) Analogy of BCS-based BS 2p-CPs with various states in a 3D potential problem, as discussed in text.
Figure 1. Wavefunction Feynman diagrams for $2p (\psi_+), 2h (\psi_-)$ and ph ($\psi_0$) bound states arising from the BS equations. Shaded rectangles designate diagrams that do not contribute in the IFG-based case.
