Scheduling prophylactic maintenance of engineering systems of residential buildings

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Abstract. The simplest model for the technical operation of residential buildings, which considers two main functions for servicing the housing stock - scheduled prophylactic inspection and repair of technical objects, as well as the elimination of sudden malfunctions of technical equipment, in particular emergency ones is proposed. It is considered that the service team can start a scheduled preventive repair and inspection only when all requests for sudden malfunctions are satisfied. The main parameters of the model are: the average time between occurrences of equipment malfunctions, the average elimination time of such malfunctions, and the average time for prophylactic inspection and repair of one technical object. Based on the methods of the queuing theory, the system’s characteristics that determine the quality of its work, as well as the boundaries of the change of parameters at which the system copes with the work from the standpoint of a particular criterion are defined.

1. Introduction

Maintenance of residential buildings is a set of measures that ensure the highest reliability of all elements and systems of a building (see, for example, [1] – [12]). The main element of the technical operation of residential buildings is a system of scheduled prophylactic inspections and repairs. However even with its rational organization, there is always a positive probability of failure of building elements, which depends not only on the aging factors of the structure. Failure can be caused by accidental circumstances, for example, an unacceptable pressure increase in heating systems, cold and hot water supply systems, etc.

Thus, two main functions for the maintenance of the housing stock can be distinguished:
- inspection of the condition of residential buildings, prophylactic maintenance and repair work,
- elimination of emergency situations and satisfaction of tenants' applications for the elimination of various malfunctions. We will call these works and the corresponding calls emergencies.

The goal of the managing company is, on the one hand, to prevent the formation of a too long queue of emergency calls, and on the other, to complete all planned prophylactic maintenance work. This article is devoted, firstly, to clarifying the conditions for the company's resources under which this goal is reachable, and secondly, to building a kind of an optimal behavior.

Our analysis relies on queuing theory, which is part of the probability theory that emerged from applications and widely used in solving applied problems in various fields. Many studies both in the technology of a construction industry and in the organization of managerial activity are based on a probabilistic approach, in particular, on the results of queuing theory (see, for example, [1], [13] -
[18]). The model proposed in this article is not classical and refers to the so-called systems with vacancies, the study of which began relatively recently (see, for example, [19] - [24]). We assume that scheduled inspection and repair work only starts when there are no emergency calls. If such work has begun, then with respect to emergency calls, the device (work team) becomes unavailable until the end of this cycle of planned work, which means a vacancy.

2. Materials and methods

2.1. The problem and methods

Suppose that the management company (MK) of housing and communal services (HCS) has one or more teams of specialists to ensure the functioning of technical equipment (heat supply, water supply, ventilation, etc.) of residential buildings. These teams have two main tasks - eliminating sudden equipment failures and conducting prophylactic inspections and repairs to ensure the necessary level of reliability of the relevant technical systems. The solution of these problems starts with the collection and processing of statistical data, which allow to obtain estimates of the parameters that determine the functioning of the system. These parameters are: \( \lambda^{-1} \) - the average time between successive moments of the occurrence of sudden equipment malfunctions, \( \nu^{-1} \) - the average repair time for such malfunctions, and \( \mu^{-1} \) - the average time of prophylactic inspection and repair. We assume that the team can start such an inspection only when there are no requests for emergency equipment repairs. The task of the MK is to make a plan for prophylactic inspections in which, on the one hand, during a certain time \( T \), the necessary number of objects \( N \) would be checked and restored, and on the other hand, the average number of requests for repair of sudden breakdowns would not exceed the specified level \( \delta \).

As is shown below, this problem may not be feasible for some values of \( \lambda \), \( \mu \), and \( \nu \). In such a situation, MK has to make organizational decisions, for example, to increase the number of specialists. Our analysis is based on queuing theory methods. For simplicity, this article assumes that there is only one work team, i.e. one device in the queuing system. The transition to the multichannel case has no fundamental obstacles, but is quite complicated technically. This will be done in the following papers.

2.2. Description of the mathematical model

Consider the simplest situation, when the MK has one team of specialists. We make the following assumptions.

- Intervals between emergency repair requests are independent exponentially distributed random variables \( \{a_n\}_{n=1}^{\infty} \), i.e. \( P(a_n \leq x) = 1 - e^{-\lambda x} \). This means that the flow of requirements incoming to the service system is Poisson (see, for example, [21]) and mathematical expectation \( E a_n = \lambda^{-1} \).
- Emergency repair times for breakdowns \( \{\xi_n\}_{n=1}^{\infty} \) are independent exponentially distributed random variables, i.e. \( P(\xi_n \leq x) = e^{-\nu x}, E \xi_n = \nu^{-1} \).
- There is one service team that can be busy either with repairing sudden breakdowns, or with prophylactic inspection or equipment repair. This means that there is one device at the service system.
- The team may begin prophylactic inspection and repair only if there are no request for emergency service. We will call such requests requirements (or customers) of the first type.

At moments when the team is exempted from the requirements of the first type, so-called delay periods begin, also having an exponential distribution with parameter \( \alpha \). If during this period no requirements of the first type were received, the team proceeds to a prophylactic inspection and repair of the next facility, which lasts a random time \( \eta \), which is called a device vacancy in the queuing theory. During the vacancy, emergency requirements may come, according to the Poisson flow with the intensity \( \lambda_1 \leq \lambda \). Note that the \( \lambda_1 < \lambda \) if there are impatient customers or an additional group of specialists that serves extra-urgent (emergency) calls when the team is busy with scheduled repairs.

Suppose that the MK wants to organize the maintenance of buildings so that the average number of calls \( \bar{q} > 0 \) for urgent repairs in the system does not exceed a certain \( \delta > 0 \) and the average number
\( \bar{n}(T) \) of facilities preventive inspection and repair of which is completed in time \( T \) is not less than \( N \), i.e.

\[ \bar{n}(T) > N(T). \tag{1} \]

To solve this problem, we need to express \( q \) in terms of the previously introduced parameters and the distribution function \( F(x) = P(\eta \leq x) \).

### 3. Results

We introduce a random process \( q(t) \), which is the number of emergency calls at time \( t \) served by a team or waiting service. From queuing theory (see, for example, [23], [24]) it follows that there are limits \( \lim_{t \to \infty} P(q(t) = j) = P_j, \ j = 0, 1, \ldots \) and \( P_j > 0, \sum_{j=0}^{\infty} P_j = 1 \) if and only if \( \rho = \frac{\lambda}{\nu} < 1 \). Otherwise \( q(t) \xrightarrow{t \to \infty} \infty \). It is assumed further that the stability condition \( \rho < 1 \) is satisfied. We define the generating function \( P(Z) = \sum_{j=0}^{\infty} Z^j P_j \) for the limit distribution. Then the average number of calls in the system \( \bar{q} = P'(1) \).

Using queuing theory methods ([21], [22]), we can obtain an expression for a function \( P(z) \):

\[ P(z) = p \frac{d(z)\mu}{\lambda} + (1 - p) \frac{1 - \rho}{1 - \rho z} \frac{1 - \rho d(z)}{1 + \rho d(z)} \tag{2} \]

Here

\[ d(z) = \frac{1 - f(\lambda_1(1-z))}{1-z}, \]

\[ f(\zeta) = \int_0^\infty e^{-\zeta x} dF(x), \quad \mu^{-1} = E\eta = \int_0^\infty xdF(x), \]

\[ \rho_1 = \frac{\lambda_1}{\nu}, \quad p = \frac{1 - \rho}{\eta + 1 - \rho \rho_1}. \]

Differentiating \( P(z) \) with respect to \( z \) and assuming \( z = 1 \), from (2) we obtain the average value \( \bar{q} \) of the number of emergency calls in the system

\[ \bar{q} = \frac{p \lambda_1 \mu \eta^2}{2} + (1 - p) \frac{z^{1+\lambda_1(1-\rho)\eta^2} + 2z\rho}{z(1-\rho)(\eta + \lambda_1 \mu)} \tag{4} \]

\[ E\eta^2 = \int_0^\infty x^2 dF(x). \]

Our next goal is to obtain a formula for the average number of completed scheduled repairs (or vacancies) for a sufficiently large time \( t \).

Let \( \Theta_n \) – the moment when \( n \)-th vacancy starts and \( \tau_n = \Theta_{n+1} - \Theta_n, (n = 0, 1, 2, \ldots, \Theta_0 = 0) \). Then \( \{\tau_n\}_{n=1}^{\infty} \) - sequence of independent identically distributed random variables. Note that exactly one vacancy occurs during the interval \( (\Theta_n, \Theta_{n+1}) \), that is a scheduled inspection and repair of one object. Based on the results of the queuing theory ([23], [24]), the average value \( E\tau_n = \bar{\tau} \) can be calculated. It is defined by the expression

\[ \tau = E\tau_n = (1 - \rho)^{-1} \left( \alpha^{-1} + \mu^{-1} (1 - \rho + \rho_1) \right). \tag{5} \]

We can also find a variance \( D\tau_n \), but its formula is too complicated and is not given here. Denote \( n(t) \) – the number of completed vacancies (or scheduled repairs) during time \( t \). From the elementary renewal theorem ([23]) we obtain that the mathematical expectation \( En(t) \) has the representation

\[ \bar{n}(t) = En(t) = \frac{t}{\bar{T}} (1 + o(1)), \quad t \to \infty \tag{6} \]
Sometimes a more accurate result of [25] is useful

\[
\lim_{t \to \infty} P \left( \frac{n(t) - \tau}{\sqrt{T} \sqrt{\tau}} < x \right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy.,
\]

i.e., the distribution of the normalized random variable \( n(t) \) converges to the normal law as \( t \to \infty \).

4. Discussions

In this section, we want to understand what conditions must be imposed on the parameters of the system in order to ensure the fulfillment of inequalities (1), i.e., carry out the required number of scheduled repairs, avoiding a large number of pending emergency calls. We start with the first inequality in (1). The value of \( \bar{q} \) is defined by formula (4), which is too complicated for the analysis. Therefore, we restrict ourselves to sufficient conditions for the inequality \( \bar{q} < \delta \). Since \( q \) monotonically increases by \( \alpha \), this can be done by going to the limit at to (4) as \( \alpha \to \infty \) and taking this limit as \( q \).

Then we go to the following condition

\[
\frac{\lambda_1 \mu \eta^2}{2} + \frac{\rho_1}{(1-\rho)(1-\rho+\rho_1)} < \delta.
\]

For the second inequality in (1) we set \( N = \gamma T \). Due to (5) and (6) in these notations, it takes the form

\[
\alpha^{-1} + \mu^{-1}(1 - \rho + \rho_1) < (1 - \rho)\gamma^{-1}.
\]

If the system parameters \( \lambda, \lambda_1, \alpha, \mu, \gamma, \eta \) satisfy the inequalities (8) and (9), then we can assume that the team satisfactorily copes with the tasks. If at least one of the conditions (8), (9) is not satisfied, managing actions should be taken. For example, to increase the processing speed of the urgent calls, which will reduce \( \rho_0 \), or to increase the number of emergency calls routed to the emergency team during vacancy, which will reduce \( \lambda \) and \( \rho_0 \). If (9) is not satisfied, the period of proceeding of scheduled repairs \( T \) can be increased which will lead to increase of \( \gamma^{-1} \).

To illustrate, we consider the simplest case when the vacancy time has an exponential distribution with the parameter \( \mu \) and \( \lambda_1 = \lambda \). Then from (9) we find

\[
\rho < 1 - \gamma \left( \frac{1}{\alpha} + \frac{1}{\mu} \right) = \beta_1
\]

In view of this inequality, a sufficient condition for fulfillment of (8)

\[
\rho < \delta \left( \frac{\gamma}{\mu} + \frac{1}{\gamma \left( \frac{1}{\alpha} + \frac{1}{\mu} \right)} \right)^{-1} = \beta_2
\]

So, (8) and (9) are satisfied if \( \rho < \min(\beta_1, \beta_2) \).

5. Conclusions

Like any scientific model, the queuing system proposed in the article is a reflection of the process of interest to us and is used for control and prediction purposes. The main function of the scientific model is not to describe phenomena, but to explain it. The model should help to figure out how some aspects of the phenomenon affect other aspects or the phenomenon as a whole. The model only partially reflects reality. A model can be considered good if, despite its incompleteness, it is able to predict the effect of changes of the system on the overall efficiency of the entire system. If conclusions, obtained using the model, do not satisfy the researcher, the model can be complicated by the introduction of
additional connections and parameters. However, we note, that this complication does not always increase the adequacy of the model, since it is necessary to evaluate statistically a large number of new parameters, which can lead to significant errors. We considered the simplest model, assuming that there is only one working team. This restriction can be removed in two ways. If there are m teams, we get an m-channel queuing system, the analysis of which is very complicated and closed expressions for the characteristics can only be obtained with \( m = 2 \). Moreover, the formulas are very cumbersome.

Another approach is to divide the entire set of serviced objects into m subsets, so the \( j \)-th team maintains objects from the \( j \)-th set \( j = 1, 2, ..., t \). Then we have \( m \) single-channel systems and we can use the results obtained in this article.

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