Study on Resistance Loss of Wire Rope over Pulley

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Abstract. There are many factors affecting the resistance loss of wire rope over pulley, such as tension of wire rope, diameter of pulley, envelop angle and so on. In this paper, based on the mechanical model of wire rope over pulley, the bending effect of wire rope is considered. And the non-linear equations of exit tension with entry tension, pulley radius, envelop angle and bending stiffness are constructed. At last, the influence of various factors on resistance coefficient is analysed.

1. Introduction
There is no other product that can replace the wire rope as most popular carrying part[1-3]. Generally, it is accepted that the transmission efficiency of wire rope is very high. The average efficiency is as high as 96% with sliding bearings and 98% with rolling bearings. But as we know, even in the rolling bearing pulley system, the 2% loss through a pulley can be neglected, but if through the pulley system composed of six rolling bearings the loss of efficiency is about 11.5%, which is enough to affect the operation of mechanical safety. However, it is very common that wire rope passes through several pulleys repeatedly in the engineering. Therefore, it is necessary to carry out in-depth research on the loss efficiency of wire rope.

In addition, with the increase of the working load, the section area of the wire rope and the diameter of the pulley are increasing. As a result, the bending stiffness of steel wire rope has more and more influence on the transmission loss over pulley. At the same time, the existing research of wire rope transfer efficiency focuses on the pulley in the hoisting mechanism, namely the envelop angle of wire rope and pulley is 180°, and the research on different envelop angle is very rare [4]. In this paper, the relationship between bending stiffness and transfer efficiency of wire rope is studied under different envelop angle.

2. Resistance of flexible wire rope over pulley
Suppose the radius of pulley is R, the envelop angle of wire rope over pulley is α, and the forward direction of the wire rope is shown in figure 1. The entry tension is P and the exit tension is T.

The definition of wire rope transmission resistance loss is: the force that prevents the wire rope moving along the pulley, and whose direction is the same with axis of wire rope (ΔF = T-P). The force determines the working loss of the wire rope.
Figure 1. The schematic diagram of wire rope over pulley.

For the flexible wire rope, it is fully attached to the pulley when they are in contact. And the torque is in balance state under the force of tension $T$, $P$ and axis friction, as shown in figure 1. The equation of moment balance is:

$$T \cdot R = P \cdot R + \mu (T + P) \sin(\alpha/2) r_d$$  \hspace{1cm} (1)

Where the $r_d$ is the radius of pulley bearing, and $\mu$ is the bearing friction coefficient. Then the exit tension $T$ is:

$$T = P \left( \frac{R + \mu r_d}{R - \mu r_d} \sin \frac{\alpha}{2} \right) = P \left[ 1 + 2 \mu \frac{r_d}{R} \sin \frac{\alpha}{2} \right]$$  \hspace{1cm} (2)

The exit tension is only related to entry tension, friction coefficient, radius of pulley and envelop angle.

It is simple that use equation (2) to analyse the resistance, but many important factors are not considered, such as the bending stiffness of wire rope. So there are errors in the engineering application.

3. Bending effect of wire rope

3.1. Calculation of bending stiffness

The bending stiffness $EI$ of wire rope is related to the geometrical size, structure, rope core and other factors of the wire rope, which is difficult to calculate with the structural analysis [5]. At the same time, the bending stiffness of wire rope changes with the tension $T$. When the tension increases, the tightness between the wires or the wire layers increases, so the bending stiffness increases.

After analysing experimental data, Malinowski obtained the formula of bending stiffness of different types of wire rope [6]:

$$EI = (\alpha_1 \sigma_p + \alpha_2) d_i^4$$  \hspace{1cm} (3)

In the equation (3), $\alpha_1$ and $\alpha_2$ is the structural test coefficient of the wire rope; $P$ is the average tensile stress of the rope; $d_i$ is the diameter of wire rope (because of the different structure of wire rope, here $d_i$ is taken by the equivalent diameter of sum area of wires).

$$d_i = \frac{d_0}{\sqrt{k}}, \quad \sigma_p = \frac{T}{A} = \frac{T}{\pi d_0^2/4}$$  \hspace{1cm} (4)

Where $d_0$ is the nominal diameter of wire rope and $k$ is the filling coefficient.

From this, the relationship between the bending stiffness $EI$ and tension $T$ of different specifications of wire rope is obtained:
3.2. The bending deformation of the wire rope over pulley

Considering the influence of the wire rope bending stiffness, the actual envelop angle $\beta$ is less than that $\alpha$ of the flexible condition. The difference of envelop angle at the exit is $\gamma$, and at the entry is $\zeta$, as shown in figure 3. Because of the different tension at the entry and exit of the pulley, the envelop angle is not about the y-axis symmetric, that is, the $\gamma$ and $\zeta$ are different.

For example, when $d_0 = 18\text{mm}, 22\text{mm}, 26\text{mm}$, tension (kN) and bending stiffness $EI$ (N•mm$^2$) are shown in figure 2.

$$EI(T) = \left(\alpha_1 \frac{T}{\pi d_0^2/4} + \alpha_2 \frac{d_0^4}{k^2}\right) = \alpha_1 \frac{4d_0^2}{\pi k^2} T + \alpha_2 \frac{d_0^4}{k^2}$$  \hspace{1cm} (5)$$

Figure 2. The relation of tension $T$ with $EI$.

Figure 3. Influence of bending rigidity on wire rope.

Figure 4. Bending of the wire rope at the exit.

Figure 5. Wire rope tension in local coordinate system.

The local coordinate system $o'x'y'$ is established at where the wire rope is detached from the pulley. The angle of tension $T$ and $x'$ is $\gamma$. Do a line that is parallel to the tension $T$ through the origin $o'$ of the local coordinate system. The distance between the distal end of wire rope and the straight line is $e_T$, as shown in figure 4. The $e_T$ can characterize the effect of bending effect on the contact between wire rope and pulley, which is denoted as offset.

For the convenience of expression, the local coordinate variables are still denoted by $x$ and $y$.
In local coordinate system, the bending deformation of deflection \( w \) is formed because of the tension \( T \) of wire rope, and the bending angle is \( \frac{T}{EI} \). So, the tension \( T \) can be resolved into a component perpendicular to the wire rope and the tangential component of the wire rope (figure 5):

\[
T_n = T \sin(-\gamma - w') \tag{6}
\]

\[
T_t = T \cos(-\gamma - w') \tag{7}
\]

The relationship between the wire rope bending moment \( M \) and the vertical distribution force \( T_n \) is:

\[
\frac{d^2 M}{dx^2} = T_n = T \sin(-\gamma - w') \tag{8}
\]

In the case of small deformation, there is \( M = EIw'' \), \( \sin(-\gamma - w') \approx -\gamma - w' \). Therefore, the equation (8) can be represented as fourth-order linear nonhomogeneous differential equation:

\[
w^{(4)} + \frac{T}{EI} w^{(3)} + \frac{T}{EI} T' \gamma = 0 \tag{9}
\]

The characteristic equation of this equation is:

\[
\lambda^4 + \frac{T}{EI} \lambda = 0.
\]

The root of the characteristic equation is:

\[
\lambda_0 = 0, \quad \lambda_1 = -\frac{1}{3} \frac{T}{EI} e^{\frac{x}{3}}, \quad \lambda_2 = \frac{1}{3} \frac{T}{EI} e^{-\frac{x}{3}}, \quad \lambda_3 = \frac{1}{3} \frac{T}{EI}.
\]

Therefore, the general solution of the equation is:

\[
W = c_0 + c_1 e^{-\sqrt{\frac{T}{EI}} x} + c_2 e^{\sqrt{\frac{T}{EI}} x} \cos\left(\sqrt{\frac{3}{2} \frac{T}{EI} x}\right) + c_3 e^{\sqrt{\frac{T}{EI}} x} \sin\left(\sqrt{\frac{3}{2} \frac{T}{EI} x}\right).
\]

At the same time, the equation also has a special solution \( \bar{w} = -\gamma x \). So the solution of the nonhomogeneous differential equation can be expressed as the combination of the particular solution and homogeneous equation general solution:

\[
w(x) = W + \bar{w} = c_0 + c_1 e^{-\sqrt{\frac{T}{EI}} x} + c_2 e^{\sqrt{\frac{T}{EI}} x} \cos\left(\sqrt{\frac{3}{2} \frac{T}{EI} x}\right) + c_3 e^{\sqrt{\frac{T}{EI}} x} \sin\left(\sqrt{\frac{3}{2} \frac{T}{EI} x}\right) + c_4 \gamma x.
\]

According to the boundary conditions: \( w(0) = 0, \ w'(0) = 0, \ w'(+\infty) = -\tan \gamma \).

It can be concluded that: \( c_0 = \tan \gamma \sqrt{\frac{EI}{T}}, \ c_1 = -\tan \gamma \sqrt{\frac{EI}{T}}, \ c_2 = 0, \ c_3 = 0, \ c_4 = -\frac{\tan \gamma}{\gamma} \).

Therefore, the deflection curve of wire rope in local coordinate system is:

\[
w(x) = \tan \gamma \sqrt{\frac{EI}{T}} - \tan \gamma x - \tan \gamma \sqrt{\frac{EI}{T}} e^{-\sqrt{\frac{T}{EI}} x} \tag{10}
\]

It can be seen from equation (10) that the deflection \( w \) is closely related to the end tension \( T \) and bending stiffness \( EI \), and the wire rope is rapidly approaching the line after it is detached from the pulley.

The offset \( e_T \) is the distance between the deflection as \( x \) going to infinity and the tangent direction of the tension \( T \) through the zero point:

\[
e_T = \sin \gamma \sqrt{\frac{EI}{T}} \tag{11}
\]

Therefore, \( e_T \) decreases with the increase of tension \( T \), and increases with the increase of bending stiffness \( EI \), which is consistent with physical phenomena.
3.3. The envelop angle of wire rope over pulley
In the part that wire rope and pulley are completely fitted, bending deformation is produced on the wire rope by contact pressure $N$ from pulley, and the curvature radius is $R$. So the wire rope bending moment at the position detached from pulley can be represented as:

$$ M_{\text{int}} = \frac{EI}{R} \quad (12) $$

As shown in figure 6, the external bending moment of the wire rope at the position detached from pulley is the product of the tension $T$ and the offset $e_T$:

$$ M_{\text{out}} = T \cdot e_T = T \sin \gamma \sqrt{\frac{EI}{T}} \quad (13) $$

![Diagram](image)

Figure 6. The torque of the wire rope at the exit of the pulley.

According to the equal of internal and external bending moment $M_{\text{int}}=M_{\text{out}}$, it can be obtained:

$$ e_T = \frac{EI}{TR}, \quad R \sin \gamma = \left(\frac{EI}{T}\right)^2 \quad (14) $$

Therefore, the actual envelop angle of wire rope is:

$$ \beta = \alpha - \sin^{-1}\left[\frac{1}{R} \left(\frac{EI}{T}\right)^2\right] - \sin^{-1}\left[\frac{1}{P} \left(\frac{EI}{T}\right)^2\right] \quad (15) $$

It can be seen, when the tension $T$, $P$, or pulley radius $R$ tend to infinity, or bending stiffness $EI = 0$ (fully flexible wire rope), there is $\beta = \alpha$, that is the wire rope will not escape the pulley in advance but fit perfectly with the pulley. With the increase of bending stiffness $EI$, the fitting degree of the wire rope with pulley is getting smaller and smaller.

3.4. The influence of bending effect on resistance
In the equilibrium state, the bending deformation of the wire rope at entry and exit of the pulley is the same when considering the bending rigidity, as shown in figure 3. Therefore, the torque balance equation of the pulley center is as follows:

$$ T \cdot (l_T + e_T) = P \cdot (l_P + e_P) + \mu (T + P) \sin(a/2) \cdot r_d \quad (16) $$

Among them, $l_T$ is the distance from the entry as the starting point, parallel to the tension $T$ direction to the center of the center; $l_P$ is the distance from the exit as the starting point, parallel to the tension $P$ direction to the center of the center. So there are:

$$ l_T = R \cos(\gamma) \quad (17) $$

$$ l_P = R \cos(\xi) \quad (18) $$

Substituting equations (17) and (18) into equation (16), combining equations (14) and (15), the nonlinear equation is obtained:
\[ T \sqrt{R^2 - \left( \frac{EI}{T} \right)^2} - P \sqrt{R^2 - \left( \frac{EI}{P} \right)^2} - \mu(T + P)\sin\left(\frac{\alpha}{2}\right)r_d = 0 \]  \hspace{1cm} (19)

As shown in equation (19), when the entry tension \( P \) is known, the exit tension \( T \) can be calculated by this equation.

4. The resistance of wire rope considering bending rigidity

According to above analysis, the resistance of wire rope over pulley is related with entry and exit tension, friction coefficient, pulley radius, envelop angle and bending stiffness. Below, the change of resistance is analysed under different influencing factors.

First of all, define the drag coefficient (resistance loss over pulley):

\[ \delta = \frac{T - P}{P} \]  \hspace{1cm} (20)

4.1. The variety of drag coefficient with entry tension.

When the diameter of wire rope \( d_0 = 22 \) mm, the pulley radius \( R = 200 \) mm, the envelop angle \( \alpha = 120^\circ \), the change of resistance coefficient is listed with entry tension \( P = 10\)kN ~ 100kN.

The bearing friction coefficient of pulley \( \mu \) is greatly influenced by bearing type, bearing load, rotating speed, lubrication method, etc [7]. Here, \( \mu = 0.02 \), and the pulley bearing radius \( r_d = 40\)mm.

It can be seen from figure 7 that the drag coefficient of the fully flexible wire rope is not changed with the entry tension \( P \) when the envelop angle and wheel diameter are known. The resistance coefficient of the bending stiffness \( EI \) is increased with the increase of the entry tension, and gradually becomes stable. But its value is close to the full flexibility, and the maximum error is not more than 0.4%. It can be seen that the drag coefficient of wire rope is less affected by the entry tension.

![Figure 7. The relationship between the drag coefficient \( \delta \) and the tension \( P \).](image)

In the following analysis, the variation of the resistance over pulley under the influence of the main parameters such as the envelop angle, wheel diameter and rope diameter is shown.

4.2. The variety with envelop angle

When the diameter of wire rope \( d_0 = 22 \) mm, the pulley radius \( R = 200 \) mm, and the entry tension \( P = 100\)kN, the change of resistance coefficient is listed with envelop angle \( \alpha = 20^\circ ~ 180^\circ \).
As shown in figure 8, when the entry tension $P=100\text{kN}$, the drag coefficient increases with the increase of the envelop angle, but the growth tends to be slow. When the $\alpha = 180^\circ$, the resistance coefficient of maximum is 0.8%.

4.3. The variety with pulley radius
When the diameter of wire rope $d_0 = 22\text{ mm}$, the envelop angle $\alpha = 120^\circ$, and the entry tension $P = 100\text{kN}$, the change of resistance coefficient is listed with pulley radius $R = 100\text{ mm} \sim 400\text{ mm}$.

As shown in figure 9, the drag coefficient decreases rapidly with the increase of pulley radius. When the pulley radius $R=100\text{mm}$, the drag coefficient reaches the maximum, close to 1.4%; When the pulley radius increased to $R=400\text{mm}$, the drag coefficient decreased to less than 0.4%. Thus, increasing the diameter of pulley can reduce the drag coefficient.

5. Conclusions
In engineering application, the situation of wire rope over pulley is very extensive. In this paper, based on the mechanical mode of wire rope over pulley, which considers the influences of bending rigidity, the nonlinear equations of the exit tension with entry tension and bending stiffness, friction coefficient, pulley radius and envelop angle are established. It is found by single factor comparison that the drag coefficient of wire rope is proportional to the entry tension and the envelop angle, which is inversely proportional to the radius of the pulley and the diameter of the wire rope. In addition, the entry tension and the diameter of the wire rope have less influence on the drag coefficient, and the envelop angle and pulley radius have a great influence. In the actual project, it can reduce the resistance loss of the
wire rope over pulley by increasing the pulley radius or reducing the envelop angle of the wire rope with pulley.

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