Gaseous bubble dynamics in a pulsational viscous flow

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Abstract. The paper is devoted to the investigation of gaseous bubble dynamics in a pulsational viscous flow. Behaviour of the bubble suspended in a viscous liquid of different density and located near the rigid wall of oscillating container is studied. The effect of the bubble and host liquid characteristics and vibration parameters on the interaction force is examined.

1. Introduction
If a solid particle is suspended in a liquid which fills a vibrating container then a non-zero average vibrational force acting on the particle from a pulsating liquid arises if densities of particle and host liquid are different. This force could lead to a paradoxical behavior of a solid particle in a pulsational flow under non-zero gravity: the particle more dense than the liquid could rise and less dense particle could sink [1-5]. For the translational vibrations, the average vibrational force acting on the particle from the pulsating fluid is subjected to the fast decrease with the growth of distance between the particle and wall [1-5]. In the case of deformable inclusion translational vibrations of container could also influence the average shape of inclusion, flattening it in the direction of vibrations [6]. The goal of the present paper is to study the effect of viscosity on the dynamics of deformable inclusion in a pulsating liquid.

2. Governing equations. Numerical method
Let us consider the dynamics of a deformable inclusion suspended in a viscous liquid of different density in a vibrating container. Investigation is made numerically by level-set method proposed in [7-9]. This method is based on the representation of a two-layer system as one medium with the parameters depending on a marker-function. In this case, the interface diffuses into transition layer with sharply changing parameters. The capillary force is approximated by volumetric force which is introduced in the following way:

\[ F_c = -\alpha K(\phi)\delta(\phi)\nabla\phi, \quad K(\phi) = \text{div} \hat{n}, \quad \hat{n} = \nabla\phi / |\nabla\phi| \] (1)

where \( \hat{n} \) is the unit vector normal to the interface, \( \phi \) is the marker-function, \( K(\phi) \) is the local curvature of the interface, \( \delta(\phi) \) is the Dirac delta-function, \( \alpha \) is the surface tension coefficient. Taking into account the force \( F_c \) we may perform numerical simulation of the behaviour of two-phase system in the framework of single-phase approach.
In the present paper the behaviour of a two-phase system consisting of gaseous bubble and host liquid in a closed cavity subjected to the translational vibrations with frequency $\omega$ and amplitude $b$ is studied applying the approach described above. Both the gaseous and liquid phase are assumed to be incompressible. The governing equations describing the behaviour of a fluid with non-uniform density in the reference frame of oscillating container are written down in the dimensionless form as:

$$\frac{\partial (\rho v^i)}{\partial t} + (\nabla v^i)\rho v^i = -\nabla p + \frac{1}{Re} \text{Div} \, \bar{\sigma} - a \rho \cos t \, j \, \frac{1}{We} \, \bar{F}, \quad (2)$$

$$\frac{\partial \phi}{\partial t} + \sqrt{\dot{\phi}} = 0, \quad \alpha_j \left( \frac{\partial v_j}{\partial x} + \frac{\partial v_i}{\partial x_j} \right), \quad \rho = \bar{\rho} + (1 - \bar{\rho}) \eta H(\phi), \quad \eta = \bar{\eta} (1 - \bar{\eta}) H(\phi),$$

where $j$ is the unit vector of the vibration axis, $H(\phi)$ is the Heaviside function.

The equations contain the following dimensionless parameters: density and viscosity ratios $\bar{\rho} = \rho / \rho_l$, $\bar{\eta} = \eta / \eta_l$, $R = R_v / l$, Reynolds number $Re = \omega l^2 / \nu$, Weber number $We = \rho \omega l^3 / \alpha$, dimensionless vibration amplitude $a = b / l$ ($l$ is linear dimension of the cavity).

On the rigid walls of container the no-slip conditions are imposed.

We assume that in the absence of vibrations the inclusion has the shape of long circular cylinder, the container is a long cylinder of square cross-section and the axes of the container and inclusion are parallel to each other. This allows to consider the problem in two-dimensional formulation.

To solve momentum equation projection method allowing to exclude the pressure is used. When solving transport equation, the problem of numerical diffusion arises which does not allow to perform the calculations for long enough period of time. This difficulty is avoided applying reinitialization procedure. Most of the calculations was performed using the mesh 100x100, which was selected from the results of test calculations made with different number of nodes.

3. Numerical results

First series of calculations was made for the physical parameters which correspond to the two-phase system of liquid deuterium and its saturated vapour at temperature $T = T_c = 500\text{mK}$ close to the critical point $T_c = 33K$. The compressibility and phase change effects were neglected. In this case the density of liquid phase is $\rho_l = 44.93 \text{kg/m}^3$, the density of gaseous phase is $\rho_g = 18.46 \text{kg/m}^3$, the viscosities of both media are the same $\nu = 1.07 \times 10^{-7} \text{m}^2/\text{s}$, the surface tension coefficient is $\alpha = 2.89 \times 10^{-4} \text{N/m}$. The linear dimension of container is $l = 1.5\text{mm}$ and the bubble radius is $R = 1/10$. In most of the calculations the frequency of vibrations was taken to be $38.5 \text{Hz}$, and their amplitude 0.375 mm. The values of the dimensionless parameters which correspond to the parameter values indicated above are: $\bar{\rho} = \bar{\eta} = 0.42, Re = 5080, We = 303, a = 0.25$. Initially the bubble is at rest and its mass center is located at the distance equal to two bubble radii from one of the horizontal walls of container (fig.1). The direction of vibrations is parallel to that wall and initial acceleration of the bubble is determined by the vibrational acceleration.

Numerical results for the parameters and initial conditions indicated above are presented in fig. 2. As one can see from this figure, under action of vibrations the bubble performs forced translational oscillations with the frequency equal to the vibration frequency. Additionally, accelerated average motion of the bubble to the nearest wall of container is observed. This motion is related to the appearance of average vibrational attraction force. The formula for average vibrational attraction force in the case of non-deformable inclusion was obtained analytically in [10-11] neglecting the viscosities of media. Appearance of the vibrational attraction of inclusion to the nearest wall is attributed to the Bernoulli effect. Since the parameter values used in the first series of our calculations well correspond to the validity range of the high frequency approach used in [10-11] it was expected to observe the behaviour similar to the one described in [10-11]. The calculations confirmed this hypothesis. The
results of our calculations made at different vibration amplitudes and frequencies and the inclusion radii well correspond to [10-11].

![Figure 1](image1.png)

**Figure 1.** Problem configuration

![Figure 2](image2.png)

**Figure 2.** Temporal evolution of the dimensionless vertical coordinate of the bubble centroid for $\bar{\rho} = 0.42$, $\bar{\eta} = 0.42$, $We = 303$, $Re = 5080$, $a = 0.25$ (vertical coordinate of wall equals to 1)

The goal of the second series of the calculations was to investigate the effect of viscosity on the interaction between the bubble and wall. These calculations were made for the same parameter values as in the first series except for the viscosity which is varied in the range $1.07 \times 10^{-7} \text{ m}^2/\text{s}$ - $1.07 \times 10^{-5} \text{ m}^2/\text{s}$. The results are presented in figure 3. As one can see, at low viscosities (curve 1 and curve 2) the bubble is subjected to the strong attraction to the nearest wall. For the viscosity ten times greater than the lowest value (curve 3), the attraction force is substantially weaker and only small displacement of the bubble to the rigid wall is observed. With further growth of the viscosity, attraction is replaced by the repulsion of the bubble from the wall (curves 4 and 5).

![Figure 3](image3.png)

**Figure 3.** Temporal evolution of the dimensionless vertical coordinate of the bubble centroid for $\bar{\rho} = 0.42$, $\bar{\eta} = 0.42$, $We = 303$, $Re = 5080$, $a = 0.25$: 1 - $Re = 5080$, 2 - $Re = 1016$, 3 - $Re = 508$, 4 - $Re = 254$, 5 - $Re = 50.8$

![Figure 4](image4.png)

**Figure 4.** Dimensionless repulsion force versus the dimensionless distance from the wall for $\bar{\rho} = 0.42$, $\bar{\eta} = 0.42$, $We = 303$, $a = 0.25$, $Re = 101.6$
To define the dependence of the vibrational repulsion force on the distance from the rigid wall we used the balance method. For that we studied the dynamics of the bubble under the coupled action the vibrations and gravity. For our configuration the buoyancy force is directed to the upper wall. As a result, at any initial conditions a quasi-equilibrium state is attained at which the bubble performs small stationary oscillations around a position where the vibrational repulsion force is balanced by the buoyancy force. The calculations show that the repulsion force grows with the decrease of distance from the wall but remains finite even for the very small distances (fig.4). The magnitude of the repulsion force grows with the increase of the viscosity.

4. Conclusions

The paper focuses on the viscosity effect on the interaction of gaseous bubble suspended in liquid of different density with oscillating rigid wall. Numerical simulation shows that under vibrations the bubble could be either attracted to the rigid wall (at low viscosities) or repelled from the wall (at high viscosities). Appearance of the vibrational attraction of inclusion to the nearest wall is related to the Bernoulli effect: increase of the pulsational flow velocity between inclusion and the wall leads to the lowering of the pressure in this area which finally results in the attraction of inclusion to the wall. Repulsion of inclusion from the wall at high viscosities might be explained by the pulsational flow deceleration and consequently the growth of the pulsational pressure between inclusion and the wall.

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