The structure of the Hoyle state via a measurement of the ‘Hoyle Rotational Band’ in $^{12}$C

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Abstract. We have measured the $^{12}$C($\gamma$,3$\alpha$) reaction with an Optical Time Projection Chamber (O-TPC) detector operating with the CO$_2$(80%) + N$_2$(20%) gas mixture and gamma-ray beams from the HI$\gamma$S facility of the TUNL at Duke. We measured complete angular distributions (between 9.1 - 10.7 MeV) from which we determine the cross section yield curve and E1-E2 relative phases leading to an unambiguous identification of the second 2$^+$ state in $^{12}$C at 10.03(11) MeV. The observed spectrum of $^{12}$C below 12 MeV including the 2$^+_2$ observed in this work resembles the rotation-vibration spectrum predicted for a triangular shape oblate spinning top in which the Hoyle state is the first vibrational breathing mode of the triangular three alpha particle system. The predicted rotation-vibration spectrum of a triangular shape oblate spinning top (with a D$_{3h}$ symmetry) allows us to compare the moment of inertia of the predicted Hoyle rotational band to the ground state rotational band and in this way extract the model radius parameter of the Hoyle state of 3.22(8) fm ($\sqrt{2}$ time the r.m.s. radius of the ground state) which is close to the radius extracted from $^{12}$C(x,x') data.

1. Introduction

The second 0$^+$ state at 7.654 MeV in $^{12}$C that was first predicted (in 1953) by Hoyle [1] plays a central role in the study of clustering in nuclei. Ever since Brink suggested that the Hoyle state is a very extended object with the structure of three alpha-particles arranged in a linear chain [2] many theoretical models were developed to describe the Hoyle state and the structure of $^{12}$C. One of the issues of great current interest is whether the Hoyle state is an extended object with an rms radius considerably larger than the ground state of $^{12}$C. Indeed clustering is thought to occur in nuclear matter with low density (e.g. one third of the central density of nuclei). As such for an extended Hoyle state clustering can occur throughout the nucleus and not as typically found on the surface of nuclei. In this paper we deduce the rms radius of the Hoyle state from the moment of inertia of the rotational band built on top of the Hoyle state and we conclude that the rms radius of the Hoyle state is considerably (35%) larger than the ground state.

A number of models have been proposed to describe the structure of $^{12}$C including a group theoretical U(7) model with a D$_{3h}$ symmetry [3], a microscopic Fermionic Molecular Dynamic (FMD) model [4] together with ‘BEC-like’ cluster model [5], ab-initio No Core Shell Model (NCSM) [6], ab-initio No Core Simplectic Model (NCSpM) [7], and ab-initio Lattice Effective Field Theory (L-EFT) [8, 9]. It is shown by the ab-initio NCSpM calculations that the Hoyle
state can only be described by up to 20 $\hbar \omega$ shell model space [7], suggesting that it is an extended alpha-clustering configuration.

Current models differ on the shape of the Hoyle state. In the U(7) model and the FMD model the Hoyle state is predicted to be an oblate triangular three alpha-particle configuration. In the frame of the L-EFT theory [9] the Hoyle state is primarily of the bent-arm chain (or obtuse triangular) shape. Both ab-initio NCSpM calculations [7] and L-EFT calculations [9] predict the Hoyle state to be of deformed prolate shape. In addition while the FMD model predicts an rms radius of the Hoyle state (3.48 fm) that is $\sqrt{2}$ larger than the ground state (2.39 fm) [4], calculations in the frame of the ab-initio L-EFT theory [9] predicts an rms radius (2.4 fm) equal (within the predicted error bar) to the rms radius of the ground state of $^{12}$C and the ab-initio NCSpM calculations [7] predict an rms radius (2.93 fm) that is only 22% larger than the ground state of $^{12}$C.

The three alpha-particle structure of $^{12}$C naturally leads to models that utilize triangular geometry [3, 4, 8, 9]. Such triangular systems are ubiquitous in physics including the $X_3$ molecular system [10], and the three quark system [11, 12] and their spectrum resemble the one predicted by the oblate spinning top with a $D_{3h}$ symmetry [10, 11]. The rotation-vibration spectrum predicted for the triangular three alpha-particle oblate spinning top appears to be a very useful phenomenological framework to discuss the (rotation-vibration) spectrum of the three alpha-particle system of $^{12}$C [3]. It is phenomenological schematic model that serves as a useful guide in which to discuss the essential degrees of freedom of the three alpha-particle system. In nuclei unlike molecules the energy scale of the vibrational and rotational states is similar leading to large mixing among rotation-vibration states [3].

For example in the U(7) model the Hoyle state is predicted to be the first vibrational breathing mode of the (oblate) deformed state on top of which a rotational band is predicted which is very similar to the ground state rotational band with the same triangular geometry. In contrast for example the FMD model [4] and the ‘BEC-like’ cluster models [5] do not predict the $2^+_1$ to be a member of a rotational band. The EFT calculations [9] on the other hand predict a Hoyle rotational band [9]. The $1^-$ state at 10.84 MeV is predicted by the U(7) model to be the bending mode of the three alpha-particle system with a rotational band that includes the $1^-$ and almost degenerate $2^+$ and $2^-$ states.

2. Measurement of the $^{12}$C$(\gamma, 3\alpha)$ Reaction
The current measurement of the $^{12}$C$(\gamma, 3\alpha)$ reaction was performed at the HI$\gamma$S facility that produces an intense, nearly monoenergetic gamma-ray beam by Compton backscattering photons from a free-electron laser [13]. Beam energy spreads of 300 - 350 keV were measured at beam energies between 9.1 and 10.7 MeV, with on-target intensities of $\approx 2 \times 10^8 \, \gamma/\text{sec}$. The beam intensity was measured by detecting neutrons from the $d(\gamma, n)p$ reaction using a D$_2$O target, cross-calibrated against a large NaI(Tl) detector. The energy profile of the beam was measured using a large HPGe detector, and the spectra were unfolded using a Monte Carlo technique [15, 16]. The alignment of the detector with respect to the beam was achieved using a gamma camera and lead absorbers placed in the front and back of the detector [17] as discussed in [14].

An optical-readout time projection chamber (O-TPC) operating at 100 torr with the gas mixture of CO$_2$(80%) + N$_2$(20%) described in [14], was used. The O-TPC consists of a time projection chamber with an opto-electronic chain to record the image of each event. The O-TPC was used as target material as well as to detect the outgoing alpha-particles from the $^{12}$C$(\gamma, 3\alpha)$ reaction.

The events recorded in the O-TPC include protons from the $^{14}$N$(\gamma, p)$ reaction, alpha particles from the $^{16,18}$O$(\gamma, \alpha)$ and the $^{12}$C$(\gamma, 3\alpha)$ reactions and cosmic rays. Nearly all (98%) of the $^{12}$C dissociation events proceed via the $^{12}$C$(\gamma, \alpha_0)^8$Be and the subsequent (immediate) decay of the
ground state of $^8$Be by two co-linear alpha-particles. As discussed in [14] all events were easily distinguished from the $^{12}$C($\gamma$, 3$\alpha$) events except for the $^{16}$O($\gamma$, $\alpha$) events. The recorded energy and track of the $^{12}$C and $^{16}$O dissociation events are very similar and the events are separated using the line shape analysis described in [14]. Each measured time projection signal was fit using the calculated line shapes of the $^{12}$C($\gamma$, $\alpha_0$)$^8$Be and $^{16}$O($\gamma$, $\alpha$)$^{12}$C events as discussed in [14]. The goodness-of-fit parameters, $\chi^2_C$ and $\chi^2_O$ of the predicted line shapes of $^{12}$C($\gamma$, $\alpha_0$)$^8$Be and $^{16}$O($\gamma$, $\alpha$)$^{12}$C respectively, were used to classify the event. Discrimination between $^{12}$C($\gamma$, $\alpha_0$)$^8$Be and $^{16}$O($\gamma$, $\alpha$)$^{12}$C events is carried out using the fitted line shape of the time projection signals.

Complete angular distribution of $^{12}$C($\gamma$, $\alpha_0$)$^8$Be events were measured at seven energies between 9.1 and 10.7 MeV. The events recorded in the O-TPC (in three dimension) are transformed to the ($\theta$, $\phi$) coordinate system as discussed in [14]. The angular distributions were fit in terms of $E_1$ and $E_2$ amplitudes and their relative phase $\phi_{12}$ [18]. Since angular information was available for each $^{12}$C($\gamma$, 3$\alpha$) event individually, unbinned maximum likelihood fits were used to avoid losing information through binning. This also reduced the number of fitting parameters, as the fitting function is normalized to unity. An angular distribution at a gamma-ray beam energy of 9.6 MeV is shown in Fig. 1. The solid curve is the fitted angular distribution calculated for $a(E2) = 96.75\%$ and $\phi_{12} = 80.3^\circ$. For almost all beam energies the angular distributions were dominated by the $E2$ component.

The total cross section yield curve was deduced from the angular distribution measured at seven different gamma-ray beam energies between 9.1 and 10.7 MeV, as shown in Fig. 2 with error bars that include both statistical and systematic uncertainties. The systematic uncertainties associated with each measured cross section are dominated by a 5% uncertainty in the gamma-ray beam intensity. In Fig. 2a we show the separated E1 and E2 cross section components measured at these energies together with the calculated Breit-Wigner resonances with energy-dependent level shifts and widths [19], convoluted with the measured gamma-ray
beam energy distribution. Coulomb wave functions were calculated using the continued-fraction expansion technique [20] with \( R_0 = 1.4 \, fm \). Each Breit-Wigner term includes three free parameters: the partial widths \( \Gamma_{\alpha}, \Gamma_{\gamma} \) and the resonance energy.

The E2 cross section data allow us to unambiguously identify a \( 2^+ \) resonance at 10.03(11) MeV with a total width of 800(130) keV and gamma-decay width of 60(10) meV; \( B(E2 : 2^+_2 \rightarrow 0^+) = 0.73(13) \, e^2 fm^4 \) or 0.45 W.u. The measured \( B(E2 : 2^+_2 \rightarrow 0^+) \) is close to prediction of the FMD model (0.46 \( e^2 fm^4 \)) [21], and smaller than predicted in the L-EFT calculations [2(1) \( e^2 fm^4 \)] [9]. Note that the slight difference between the maximum of the calculated cross section (at 9.8 MeV) and the resonance energy (at 10.03 MeV) is due to the energy-dependent widths used in the fit. The highest energy data point at 10.7 MeV is not consistent with this
single resonance hence in order to estimate the error in the measured resonance energy we also analyzed our data including another $2^+$ that was previously suggested at 11.1 MeV leading to total error of the resonance energy of 110 keV.

3. Conclusion
The measured $2^+_2$ state at 10.03(11) MeV reported in this work lies 2.38(11) MeV above the $0^+_1$ Hoyle state which is approximately half the excitation energy of the $2^+_1$ state of $^{12}$C. Since the U(7) model predicts the ground state rotational band and the Hoyle bands to arise from the same geometrical shape we conclude that in this model the radius parameter of the Hoyle state is approximately √2 larger than the r.m.s. radius of the ground state. This conclusion is in fact consistent with (but slightly larger than) the r.m.s. radius of the Hoyle state [2.89(4) fm] measured using $^{12}$C(x,x') data [23]. Indeed all thus far extracted radius parameter of the Hoyle state including using the moment of inertia as discussed here, or from $^{12}$C(e,e') data [4], or from $^{12}$C(x,x') data [23], are all model dependent. Our inability to measure elastic scattering off (the very short lived) Hoyle state will continue to inhibit a measurement of the r.m.s. radius of the Hoyle state.

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