This work presents a complete cyclic cosmological scenario based on nonlinear magnetic field. It is constructed a model composed by five fluids namely baryonic matter, dark matter, radiation, neutrinos and a cosmological magnetic field. The first four fluids are treated in the standard way and the fifth fluid, the magnetic field, is described by a nonlinear electrodynamics. The free parameters are fitted by observational data (SNIa, CMB, extragalactic magnetic fields, etc) and by simple theoretical considerations. As result arises a cyclic cosmological model which preserves the main successes of standard big bang model and solve some other problems like the initial singularity, the present acceleration and the Big Rip.

I. INTRODUCTION

Although the inflationary scenario still maintains its status of the paradigm in Cosmology, there is an increasing interest in the alternative proposal of models displaying a bounce [1–5]. This means the possible existence of a collapsing era prior to the actual expanding phase. One of the main differences of these two proposals concerns the behavior of small perturbations and its evolution from a perfect spatially homogeneous phase to the production of inhomogeneities. There are hopes, due to a thorough analysis of these processes [6], that this question will be settled in the near future. Another important point in favor of bouncing models is that they avoid the problem of initial singularity. The present paper examines one of these scenarios which the dynamical history of the universe is partial controlled by a magnetic field. It is analyzed its main effects and its compatibility with actual observations. The main hypothesis of this model concerns the non-linearity character of magnetic field.

The effects of a nonlinear electromagnetic theory in a cosmological setting have been studied in several articles [7–14] and for different reasons. For example, in [7] the non-linearity is responsible to avoid the initial singularity, and in [8] the non-linearity is accountable for generate the recent acceleration. In these two papers the framework is the same: a cosmological magnetic field governed by a non-linear electromagnetic theory is responsible by the desirable effects.

Recently, these two effects were combined generating a cyclic cosmological toy model [15]. The purpose of the present paper is improve this model and investigate how realistic it can be. It is a well-known fact that the standard cosmological model unavoidably leads to a singular behavior of the curvature invariants in what has been termed the Big Bang. On the other hand, there are some evidences that point in the direction that the universe is undergoing an accelerated expansion [16, 17]. In principle, these two problems have no simple connection and thus does not have a unique combined solution. It should be tempted to try to unify these two questions in a single model. It will be shown that in the framework of a magnetic universe these two problems are solved at once.

In general, the cyclic cosmological models can be divided in two classes: Those which generate the cyclic behavior with non-conventional matter fields [18, 19], and those which produce the cyclicity through of extension of General Relativity [20, 22]. The model proposed here belongs to the first class where the non-conventional matter is represented by a non-linear magnetic field.

The plan of the paper is as follows. Section II presents the notion of the Magnetic Universe and its generic features. In section III it is fitted or constrained the five independent parameters presents in this models. The section IV presents the complete scenario consisting of the five eras. The paper ends in section V with some generic final comments. Moreover, it is presented in the appendix A a brief review about the average procedure and in appendix B a discussion about the model stability.

II. MAGNETIC UNIVERSE

To construct a realistic magnetic universe we start assuming that the universe is composed by the five fluids namely baryonic matter, cold dark matter, neutrinos, radiation and a cosmological magnetic field. The first four fluids will be treated in the standard way. Thus baryonic and dark matter are modeling by non-relativistic fluids (zero pressure), and neutrinos and radiation are featured as ultra-relativistic fluid (The mass of neutrinos is neglected). The fifth fluid, the magnetic field, will be described by the Lagrangian for the non-linear electrodynamics given by

\[ L_{\text{NLED}} = \alpha^2 F^2 - \frac{1}{4} \frac{\mu^2}{F} + \frac{\beta^2}{F^2}, \]

where the dimensional constants \( \alpha, \beta \) and \( \mu \) are to be determined by observation.

The nonlinear terms can be interpreted as a phenomenological approach. But a more interesting and
fundamental scenario is suppose these terms represent a classical interpretation of vacuum polarization. Indeed, Hesseinberg, Schwinger and others showed that quantum corrections due vacuum polarization changes the classical Lagrangian introducing nonlinear terms [23, 24].

Let us suppose that the five fluids are independent. Moreover, the magnetic field enter in cosmological scenario through the average procedure described in appendix A. By construction, the average of the electric part of the cosmological electromagnetic field must vanishes, i.e. $E^2 = 0$. And in this case, each term of $L_{\text{NLED}}$ becomes cosmological independent in the sense that each one behaves as a non-interacting perfect fluid (for details see [15]).

Treating baryonic matter and dark matter as a single fluid and neutrinos and radiation as another single fluid, the magnetic universe is completely featured for six independent components. In this context, the total energy density and the total pressure is given by

$$\rho_T = \rho_m + \rho_{UR} + \sum \rho_{Bi}, \quad p_T = p_m + p_{UR} + \sum p_{Bi},$$

where

$$\rho_m = \rho_{m0} \left( \frac{a_0}{a} \right)^3, \quad p_m = 0$$
$$\rho_{UR} = \rho_{UR0} \left( \frac{a_0}{a} \right)^4, \quad p_{UR} = \frac{1}{3} \rho_{UR}$$
$$\rho_{B1} = -16 \alpha^2 B_0^4 \left( \frac{a_0}{a} \right)^8, \quad p_{B1} = \frac{5}{3} \rho_{B1}$$
$$\rho_{B2} = B_0^2 \left( \frac{a_0}{a} \right)^4, \quad p_{B2} = \frac{1}{3} \rho_{B2}$$
$$\rho_{B3} = \frac{\mu^2}{4 B_0^2} \left( \frac{a}{a_0} \right)^4, \quad p_{B3} = \frac{7}{3} \rho_{B3}$$
$$\rho_{B4} = -\frac{\beta^2}{16 B_0^2} \left( \frac{a}{a_0} \right)^8, \quad p_{B4} = -\frac{11}{3} \rho_{B4}. \quad (3)$$

The real evolution of the universe depends on the values of the six constants presents in (3). However, at least in a qualitative manner, it is possible to distinguish five distinct cosmological stages just looking the form of the terms in (3). These five stages are:

- The bouncing era: this era is governed by $\rho_{B1}$ and it dominates in the earliest universe (before nucleosynthesis). In bouncing era the scale factor reaches a minimum value.
- The radiation era: this era is governed by $\rho_{UR}$ with $a(t) \sim t^{1/2}$. During this epoch occurs the primordial nucleosynthesis.
- The matter era: this era is dominated by $\rho_m$ with $a(t) \sim t^{2/3}$. It is expected that the structure formation occurs in this epoch.
- The acceleration era: in this period $\rho_{B3}$ govern the cosmic evolution. The term $\rho + 3p$ becomes negative and the universe begins to accelerate.
- The re-bouncing era: this era is dominated by $\rho_{B4}$. The term $\rho + 3p$ becomes positive once more and starts a new decelerated phase. After a while, the $a(t)$ reach a maximum and re-bounces entering in a collapsing phase.

In the section [17] it will be shown explicitly that the universe pass through of all the five stages.

III. SETTING THE SIX CONSTANTS

In order to analyze the complete scenario it is necessary to determine six constants where only five are independent. Although five independent constants are a elevated number of free parameters, this model describes all the evolution for the universe, since the bounce until the re-bounce. It keeps untouchable the main successes of the standard Big Bang model (primordial nucleosynthesis, CMB generation, etc) and solve some other problems like the initial singularity, the present acceleration and the Big Rip.

The six parameters are divided in two distinct classes: the first one, represented by $\alpha$, $B_0$ and $\rho_{UR0}$, acts on the primordial universe; and the second one, represented by $\mu$, $\beta$ and $\rho_{m0}$, affects only the recent universe. Because this difference, each class must be fixed by independent manner. Let’s start with the primordial constants.

A. Constants for the primordial universe

In this subsection it will be fixed or constrained the three primordial parameters namely $\alpha$, $B_0$ and $\rho_{UR0}$. The most simple to determine is $\rho_{UR0}$, so let's start with it.

The constant $\rho_{UR0}$ represents the nowadays energy density for the all ultra-relativistic particles. It is basically composed of radiation $\rho_\gamma$ and neutrinos $\rho_{\nu0}$. For radiation, the precisely CMB measurement [25] shows that,

$$\rho_\gamma = 4.6 \times 10^{-5} \rho_c \quad \text{with} \quad \rho_c = \frac{8 \pi G}{3 H_0^2} = 1.9 \times 10^{-9} h^{-2} \text{erg}$$

where $h \simeq 0.72$.

Theoretical considerations for a massless neutrinos [26] imply in

$$\rho_{\nu0} = 6 \times \frac{7}{16} \times \left( \frac{4}{11} \right)^{4/3} \rho_\gamma = 3.1 \times 10^{-5} \rho_c.$$
Therefore, the nowadays ultra-relativistic energy density is
\[ \rho_{UR0} = 7.7 \times 10^{-5} \rho_c. \]  

(4)

The next parameter it should be fix is \( B_0 \). As could be seen in [3], the constant \( B_0 \) acts directly only in the primordial universe through \( \rho_{B2} \). Nevertheless, its numerical value is necessary to determine the \( \alpha, \beta \) and \( \mu \) constants. Thus, \( B_0 \) affects not only the primordial universe but also, in the indirectly way, the present universe.

Physically the parameter \( B_0 \) is related with a cosmological magnetic field. Supposing the existence of a large scale magnetic field, its background mean value \( \overline{B}_0 \) must be \( \overline{B}_0 < 10^{-9} \, G \) [27]. In the cgs system, the energy density related with this magnetic field is
\[ \rho_{B0} = \frac{\overline{B}_0^2}{8\pi} = 4 \times 10^{-20} b^2 \, \text{erg}, \]

(5)

where \( b \) is just a parametrization (\( b \leq 1 \)).

On the other hand, the present energy density for the cosmological magnetic field \( \rho_{B2} \) is given by
\[ \rho_{B2} = \frac{\tilde{F}}{4} = B_0^2. \]

So, the comparison between the last equation and [5] results in
\[ B_0^2 = 4 \times 10^{-20} b^2 \, \text{erg} = 1.1 \times 10^{-11} b^2 \, \rho_c. \]

(6)

Replacing [6] and [4] in (3), a first relevant result arises: \( \rho_{B2} \) is always subdominant, i.e. the Maxwell term concerning to cosmological magnetic field is not important to the universe evolution. In fact, even the actual cosmological experiments including CMB anisotropy observation performed by WMap satellite do not have accuracy to measure a cosmological magnetic field. Perhaps, the future experiments of CMB anisotropy (e.g. Planck satellite) are able to detected it.

The last parameter necessary to fix is \( \alpha \). This parameter rules the bouncing stage, hence it is concerning at the very early universe. These features make difficult to precisely determine \( \alpha \). Therefore, instead of to fix it we just establish a range of validity for \( \alpha \). The maximum and minimum allowed values are obtained through of the relation
\[ a_{\text{nuc}} = 9.5 \times 10^{-127}, \]

(7)

For consistence with second condition the bounce should happen below the Planck scale, i.e. \( T_{\gamma\text{Bou}} \ll 10^{-10} \, GeV \). In this model, the bouncing occurs when \( \rho_{UR} + \rho_{B1} \approx 0 \) which implies in
\[ \left( \frac{a_{\text{Bou}}}{a_0} \right)^4 = \left( \frac{T_{\gamma0}}{T_{\gamma\text{Bou}}} \right)^4 = \frac{64B_0^2}{\rho_{\gamma0} \alpha^2}. \]

So,
\[ \left( \frac{T_{\gamma0}}{T_{\gamma\text{Bou}}} \right)^4 \gg 1.6 \times 10^{-127}, \]

(8)

and hence
\[ \alpha^2 \gg \frac{9.5 \times 10^{-122}}{b^2} \rho_c^{-1}. \]

At last, using (7) and (8), it is obtained the range of validity for the parameter \( \alpha \):
\[ \frac{9.5 \times 10^{-112}}{b^2} \rho_c^{-1} \ll \alpha^2 \ll \frac{9.5 \times 10^{-24}}{b^2} \rho_c^{-1}. \]

(9)

The term \( \alpha^2 F^2 \) at the Lagrangian could be interpreted as an one-loop quantum correction in the infrared regime. Indeed, it correction was obtained first by Euler-Heisenberg [23]. In this context, \( \alpha^2 \) values
\[ \alpha^2 = \left( \frac{1}{90} \right) \left( \frac{1}{137} \right)^2 \left( \frac{\hbar}{m_e c} \right)^3 \left( \frac{1}{m_e c^2} \right) \Rightarrow \]
\[ \alpha^2 \approx 4.2 \times 10^{-32} \frac{c^3}{\text{erg}} = 3.68 \times 10^{-40} \rho_c^{-1}. \]

(10)

Comparing (10) with (9) it could be seen that the both results are consistent. So, in this scenario, the bouncing stage was generated by quantum electrodynamics effects.

B. Constants for the recent universe

In this subsection, we intend to determine the three remain constants namely \( \mu, \beta \) and \( \rho_{n0} \). Differently for the three previous parameters, these constants are relevant at the same time (present universe). For this reason, they should be set simultaneously.

The procedure that was adopted is based on the fit of the model using supernova data. This procedure concerns only to recent universe, so it could be neglected.
the terms \( \rho_B1, \rho_B2 \) and \( \rho_B2 \). In this case, the luminosity distance \( d_L \) is written as

\[
d_L(z) \equiv \frac{d_L}{H_0} = (1 + z) \int_0^z \frac{dz}{H(z)}
\]

(11)

where the Hubble function \( H(z) \) is given by

\[
\frac{H(z)}{H_0} = \sqrt{\Omega_{m0} (1 + z)^3 + \Omega_{\mu0} (1 + z)^4 + \Omega_{\beta0} (1 + z)^{−8}}
\]

(12)

with the three \( \Omega \)'s defined as

\[
\Omega_{m0} \equiv \frac{\rho_{m0}}{\rho_c}, \quad \Omega_{\mu0} \equiv \frac{\mu^2}{4B_0^2\rho_c} \quad \text{and} \quad \Omega_{\beta0} \equiv -\frac{\beta^2}{16B_0^4\rho_c}.
\]

(13)

For \( z = 0 \), (12) implies in \( \Omega_{m0} + \Omega_{\mu0} + \Omega_{\beta0} = 1 \). Thus, for the present universe, the model has only two free parameters.

The set of SN1a which was used in the fit is the Union set [28]. This set collects the mainly sets of supernovas which was obtained during the last twelve years. It is constituted of 414 SN divided in 13 sub-sets where all of them were analyzed through of the same procedure. Based on this procedure, the authors excluded 107 SN, so only 307 supernovas were regarded true standard candles. It is with these 307 SN which is fitted the two parameters.

The supernova data are usually assumed as Gaussian, hence it could be used the statistical likelihood method to determine the free parameters. The standard procedure consist in to build and minimize a \( \chi^2 \) through the fit of the model’s parameter. Following the steps point out in [28], let’s start the construction of \( \chi^2 \) regarding only the statistical errors:

\[
\chi^2 = \sum_i \left[ \mu_r (m_i, s_i, c_i | M, \alpha, \beta) - \mu (z_i | \Omega_{m0}, \Omega_{\mu0}) - M \right]^2 / \sum_{jk} C_{jk} e_j e_k + \sigma^2_{tot} + \sigma^2_{int}
\]

(14)

where

\[
\mu (z_i | \Omega_{m0}, \Omega_{\mu0}) = 5 \log d_L (z_i | \Omega_{m0}, \Omega_{\mu0})
\]

and

\[
\mu_r = m_i - M + \alpha (s_i - 1) - \beta c_i
\]

(15)

where \( m_i \) is the \( B \)-band peak magnitude, \( M \) is the absolute magnitude, \( s_i \) represents the stretch in supernova lightcurve, \( c_i \) represents the color variation between \( B \) and \( V \) bands, \( \alpha \) is the stretch parameter, \( \beta \) is the color parameter and \( M \) is an additive factor which include constants like \( H_0 \). The \( C_{jk} \) matrix is the statistical covariant matrix, \( \sigma_{tot} \) is related with astrophysical dispersion and \( \sigma_{int} \) represents the intrinsical dispersion.

The term \( \sum_{jk} C_{jk} e_j e_k \) represents the uncertainties associated with \( m_i, s_i \) and \( c_i \) where \( e_j = (1, \alpha, -\beta) \). In principle \( \alpha \) and \( \beta \) are free parameters which must be fitted for each specific cosmological model. Nevertheless, the authors in [28] found out that \( \alpha \) and \( \beta \) are almost independent of cosmological parameters. So, it is assumed that \( \alpha \) and \( \beta \) are constants\(^2\), the uncertainties of \( m_i, s_i \) and \( c_i \) could be directly passed to \( \mu_r \). In this case, the \( \chi^2 \) is written as

\[
\chi^2 (\Omega_{m0}, \Omega_{\mu0}, \mathcal{M}) = \sum_{i=1}^N \left( \mu_r - \mu (z_i | \Omega_{m0}, \Omega_{\mu0}) - M \right)^2 / \sigma^2_{\mu_r} + \sigma^2_{int} + \sigma^2_{\text{tot}}
\]

(16)

where the parameter \( \mathcal{M} \) was embedded in \( \mathcal{M} \). In [28] is available the list with \( z, \mu_r, \) and \( \sigma_{\mu_r} \) for each one of the 307 SN1a.

- Gravitational lensing decreases the mode of the brightness distribution and causes increased dispersion in the Hubble diagram at high red-shift [28, 30]. The both strong and weak leaning effects engender a dispersion of \( 0.093 \times z \) in magnitude [30], and if the statistical of SN1a is large enough this uncertainty could be considered as a Gaussian error.

- All the SN1a light-curve data were corrected for extinction caused by our Galaxy. Nevertheless, these corrections contain statistical and systematic errors which act directly on the magnitude value. These statistical uncertainties will be neglected because three reasons: they only concern the SN with \( z < 0.2 \), and these ones represent approximately 20% of total set; out of Galactic plane these uncertainties value as a rule \( \sim 0.01 \); the implementation of these uncertainties is very complicated because the extinction and its errors change with the Galactic coordinates. The systematic uncertainties are treated later on.

- As point out in several papers [16, 17, 28, 31], the peculiar velocity in the host galaxy causes a error \( \sigma_v \) which varies between 100 and 500 Kms/s. In this paper, we adopt \( \sigma_v = 300 \text{ Kms/s} \). So, the uncertainty propagated to magnitude due the peculiar velocities is

\[
\sigma_{\mu_v} = \frac{5}{d_L \ln 10} \sigma_{dv} \simeq \frac{2.172 \sigma_v}{z} / c,
\]

where \( \sigma_{dv} = \sigma_v / c = 0.001 \).

\(^2\) This approximation engender an new source of error which will be regarded as a systematic error.
These three sources of uncertainty are encapsulated in $\sigma_{\text{tot}}$. Thus, the discussion above results in

$$\sigma_{\text{tot}}^2 = \sigma_{\text{int}}^2 + \sigma_{\text{err}}^2 = (0.093z)^2 + \left(\frac{2.172 \sigma_v}{z/c}\right)^2. \quad (17)$$

The last source of statistical error which was considered is the uncertainty produced by an intrinsical dispersion $\sigma_{\text{int}}$. This intrinsical dispersion is related about non corrected and/or unknown errors.

To estimate $\sigma_{\text{int}}$, it is performed a first fit with a small initial value ($\sim 0.01$ mag). Then it is determined a new $\sigma_{\text{int}}$ imposing $\chi^2 = 1$. And the last step consists in perform the same routine once more to obtain a more accurate and definitive $\sigma_{\text{int}}$. During this procedure it is not used $\sigma_{\text{tot}}$ because the intrinsical dispersion is related only with the "main" data namely $\{z, \mu, \sigma_{\mu}\}$.

1. Systematic uncertainties

The systematic uncertainties are introduced at the same way which was performed in [28]. The method consists in to introduce a gaussian distribution with a $\Delta M_s$ parameter for each present systematic uncertainty $\sigma_{\Delta M_s}$. The new parameters get in additively in the modulus of distance, i.e.

$$\mu(z|\Omega_{m0}, \Omega_{\mu0}) \rightarrow \mu(z|\Omega_{m0}, \Omega_{\mu0}) + \sum_s \Delta M_s,$$

and the previous likelihood changes to

$$\mathcal{L}_{\text{prev}}(\Omega_{m0}, \Omega_{\mu0}) \rightarrow \mathcal{L}_{\text{post}}(\Omega_{m0}, \Omega_{\mu0}, \Delta M_s)$$

where

$$\mathcal{L}_{\text{post}}(\Omega_{m0}, \Omega_{\mu0}, \Delta M_s) \equiv \mathcal{L}_{\text{prev}}(\Omega_{m0}, \Omega_{\mu0}) \prod_s e^{\frac{\Delta M_s^2}{2\sigma_{\Delta M_s}^2}}.$$

In this kind of construction, we are supposing that a specific source of systematic error leads to a "variation" in the modulus of distance. And the importance of its variation is determined through of a gaussian distribution where the standard deviation is identified with the systematic uncertainty.

At the Union set, there are two kinds of systematic errors: one which depends on the specific sub-set (e.g. due to observational effects) and the other which concerns to all set of supernovas (e.g. due to astrophysical or fundamental calibration effects). The systematic uncertainties common to all samples can be absorbed in the definition of absolute magnitude. On the other hand, it is expected a difference between the nearby ($z \sim 0.05$) and distant ($z \sim 0.5$) supernovas. So, we can cast the common systematic uncertainties to a uncertainty in the difference $\Delta M$ between absolute magnitudes of close and distant SNs. Following the steps discussed in [28], it is defined $z_{\text{div}} = 0.2$ as the point which splits the SNs in nearby and farther objects. Beside $\Delta M$, we introduce a set of 13 parameters $\Delta M_s$ which represents the systematic uncertainties for each one of the 13 sub-sets.

The complete likelihood for the model is given by

$$\mathcal{L}_{\text{post}} = e^{-\frac{\chi^2_{\text{post}}}{2}}, \quad (18)$$

where

$$\chi^2_{\text{post}} = \sum_{s=1}^{13} \sum_{i=1}^{N_s} \frac{(\mu_{\text{err}} - \mu + \Delta M_s + \Delta M_s - \mathcal{M})^2}{\sigma_{\mu_{\text{err}}}^2 + \sigma_{\text{int}}^2 + \sigma_{\Delta M_s}^2},$$

and

$$\Delta M^* = \begin{cases} \Delta M & \text{for } z > 0.2 \\ 0 & \text{for } z \leq 0.2 \end{cases}.$$ 

The Union set was split in the sum of the 13 sub-set where each sub-set has $N_s$ samples. Thus, comparing (16) with (19) it follows that $\sum_{s=1}^{13} N_s = N$.

The systematic errors arise from seven distinct sources:

- Errors due the fixation of $\alpha$ and $\beta$ parameters.
- Errors due the possible samples contamination.
- Errors due the models of supernovas light curve and $K$-corrections.
- Error due photometric zero point calibration.
- Error due a possible variation of Malmquist bias with the red-shift.
- Systematic errors due gravitational lensing effects.
- Systematic error due the normalization in the corrections for Galactic extinction maps.

Taking account all these effects, the authors of [28] reached the following uncertainties:

$$\sigma_{\Delta M} = 0.04 \quad \text{and} \quad \sigma_{\Delta M_s} = 0.033 \quad \text{for all } s.$$ 

So, the error related with each one of the 13 sub-sets are the same.

2. Results

First, we take account only the statistical uncertainties. Using the data presents in [28], it is performed a numerical calculation to determine the cosmological parameters. The procedure could be summarized in five steps:

1. Construction of $\mathcal{L}(\Omega_{m0}, \Omega_{\mu0}, \mathcal{M})$ not taking account $\sigma_{\text{tot}}$. 

2. Marginalization of $\mathcal{M}$ parameter$^3$.

3. Calculation of $\sigma_{int}$.

4. Construction of the new likelihood taking account $\sigma_{tot}$ and the $\sigma_{int}$.

5. And finally, the minimization of $\chi^2(\Omega_{m0}, \Omega_{\mu0})$.

The results are shown in figure 1.

FIG. 1: Parametric contour plot for $\Omega_{m0} \times \Omega_{\mu0}$ taking account only the statistical uncertainties. From inside to outside the contour lines correspond respectively to 68%, 95% and 99% confidence level. The dashed line represents $\Omega_{b0} = 0$.

The best fit for $\Omega_{m0}$ or $\Omega_{\mu0}$ could be established marginalizing in all the other parameters. Performing these marginalization we get in the following results:

$$\Omega_{m0} = 0.44^{+0.03}_{-0.03} \quad \text{and} \quad \Omega_{\mu0} = 1.20^{+0.35}_{-0.33} \quad (20)$$

Finally, it is determined $\Omega_{b0}$ through of the equation $\Omega_{m0} + \Omega_{\mu0} + \Omega_{b0} = 1$. Thus,

$$\Omega_{b0} = -0.64^{+0.35}_{-0.33} \quad (21)$$

The next step is to include the systematic uncertainties in the procedure. Analogously to the first case, it is performed an analytic marginalization at the “non-cosmological parameters” namely $\mathcal{M}$, $\Delta M$ and the set of $\Delta M_s$. Again, the method is straightforward but the calculation is much more cumbersome than the previous case.

Using the same steps described earlier, it is obtained the following result:

$$\Omega_{m0} = 0.448_{-0.039}^{+0.042} \quad \text{and} \quad \Omega_{\mu0} = 1.17_{-0.37}^{+0.39} \quad (22)$$

and the calculated value for $\Omega_{b0}$ is

$$\Omega_{b0} = -0.62_{-0.39}^{+0.37} \quad (23)$$

As expected, the inclusion of systematic uncertainties increases the uncertainties at the cosmological parameters.

3. Including a prior in $\Omega_{m0}$

Measurements of X-ray gas mass fraction $f_{gas}$, done in galaxy clusters allow to establish excellent values to the ratio $\Omega_{b0}/\Omega_{m0}$ [32,33]. On the other hand, primordial nucleosynthesis determine with good accuracy the actual baryon density $\Omega_b$. So, join these two facts it is possible to obtain an excellent estimation for $\Omega_{m0}$. Based on this estimation, we intend to build a gaussian prior to $\Omega_{m0}$ as follows:

$$P = \exp \left[ -\frac{(\Omega_{m0} - \bar{\Omega}_{m0})^2}{2\sigma_{m0}^2} \right],$$

where $\Omega_{m0}$ and its uncertainty are obtained through of gas mass fraction procedure.

According to [33], the $f_{gas}$ is linked with $\Omega_{b0}/\Omega_{m0}$ as shown below:

$$f_{gas}^{\Lambda CDM}(z) = \frac{KA^b(z)}{1 + s(z)} \left( \frac{\Omega_{b0}}{\Omega_{m0}} \right) \left[ \frac{d_{\Lambda CDM}^2(z)}{d_A(z)} \right]^{\frac{1}{2}},$$

$^3$ We are interested only in cosmological parameters.
where $d_A$ is the angular diameter distance given by

$$d_A(z) = \frac{1}{(1+z)} \int_0^z \frac{dz}{H(z)}. \quad (25)$$

The $f_{gas}^{\Lambda CDM}$ is the gas mass fraction for $\Lambda CDM$ model (the reference model) with the following parameters: $\Omega_{m0} = 0.3$, $\Omega_{\Lambda} = 0.7$ and $h = 0.7$. The other factors which appear in (24) are described below:

- $K$ is a constant that parameterizes uncertainties in the accuracy of the instrument calibration and X-ray modeling. Its value is $K = 1 \pm 0.1$.
- $\gamma$ is related with the non-thermic pressure support in the clusters. Its value is $1.0 < \gamma < 1.1$.
- $b(z)$ is the depletion factor, i.e. the ratio by which the baryon fraction is depleted with respect to the universal mean. It is modeled by $b(z) = b_0(1 + \alpha_0 z)$ where $b_0 = 0.83 \pm 0.04$ and $-0.1 < \alpha_0 < 0.1$.
- The function $s(z) = s_0(1 + \alpha_s z)$ describes the baryonic mass fraction in stars where $s_0 = (0.16 \pm 0.05) h_{70}^{1/2}$ and $-0.2 < \alpha_s < 0.2$.
- The $A$ factor take account the change in the subtended angle of the clusters due the difference between $\Lambda CDM$ model and the others models. According to authors in [33], this effect could be neglected for all red-shifts of interest, i.e. $A = 1$.

To estimate $\bar{\Omega}_{m0}$, it is used a set of six clusters with low red-shifts (see table I).

| Cluster classification | $z$  | $f_{gas}^{\Lambda CDM} h_{70}^{b_0}$ |
|------------------------|-----|----------------------------------|
| Abell 1795             | 0.063 | 0.1074 (0.0075)                 |
| Abell 2029             | 0.078 | 0.1117 (0.0042)                 |
| Abell 478              | 0.088 | 0.1211 (0.0053)                 |
| PKS0745-191            | 0.103 | 0.1079 (0.0124)                 |
| Abell 1413             | 0.143 | 0.1082 (0.0058)                 |
| Abell 2204             | 0.152 | 0.1213 (0.0116)                 |

TABLE I: The red-shift and $f_{gas}$ for the six galaxy clusters. The data were taken from table 3 of reference [33].

As we are considering only clusters at low red-shift, it can be neglected the dependence on $z$ of the functions $b(z)$ and $s(z)$. Thus, inverting (24) it follows:

$$\bar{\Omega}_{m0} \simeq \frac{Kb_0}{(1 + s_0) f_{gas}^{\Lambda CDM}} \left[ \frac{d^{\Lambda CDM} (z)}{d_A (z)} \right]^3 \Omega_{b0}. \quad (26)$$

Face on the other uncertainties, the variation of gamma could be neglected, i.e. $\gamma = 1$. Using primordial nucleosynthesis models and data of deuterium abundance, it is possible to derive the value for $\Omega_{b0}$ with good accuracy [34]:

$$\Omega_{b0} h^2 = 0.0214 \pm 0.002 \quad \text{with} \quad h = 0.7 \pm 0.1, \quad (27)$$

where the value of $h$ was determined in [35].

The procedure to determine $\bar{\Omega}_{m0}$ could be summarized in the following steps:

1. Fit of parameters using the set of SNIa without priors. The steps were discussed previously.
2. Calculation of $d_A(z)$ using the previous fit.
3. Determination of $\bar{\Omega}_{m0}$ and its corresponding uncertainty for each one of the six clusters.
4. Calculation of weighted mean and the mean uncertainty for $\bar{\Omega}_{m0}$.
5. Finally, it repeats the previous step but now using the prior for $\bar{\Omega}_{m0}$.

These steps was performed some times until $\bar{\Omega}_{m0}$ stabilize in

$$\bar{\Omega}_{m0} = 0.278 \pm 0.038. \quad (28)$$

The complete result including the statistical and systematic uncertainties and the prior in $\Omega_{m0}$ is shown in figure 3. The best fits for $\bar{\Omega}_{m0}$ and $\Omega_{\bar{p}0}$ are

$$\Omega_{m0} = 0.367^{+0.026}_{-0.023} \quad \text{and} \quad \Omega_{\bar{p}0} = 1.47^{+0.37}_{-0.34}, \quad (29)$$

and the calculated value for $\Omega_{\beta 0}$ is

$$\Omega_{\beta 0} = -0.84^{+0.37}_{-0.34}. \quad (30)$$

It is interesting determine explicitly the values of $\mu$ and $\beta$. Thus, replacing (29), (30) and (6) in (13) follows:

$$\mu = \sqrt{4 B_0^2 \rho_c \Omega_{\beta 0}^4} = 8.0 \times 10^{-6} \rho_c, \quad \beta = \sqrt{-16 B^2_0 \rho_c \Omega_{\beta 0}^3} = 3.98 \times 10^{-11} \rho_c^{1/2}. \quad \text{erg}$$

Since $\rho_c \sim 10^{-9} \text{erg}$, the values of $\mu$ and $\beta$ are extremely small and undetectable in the classical earth experiments. Other interesting feature of this model is that the value of $\Omega_{m0}$ is greater than the majority models presents in literature. Even with the inclusion of the prior in $\Omega_{m0}$ the difference remains. It is possible that this feature are related with the eminent deceleration provide by $\beta^2 F^2$ term. A last point important to comment is that with the inclusion of this prior, the region where $\Omega_{\beta 0} > 0$ (below to the dashed line) is excluded in $2\sigma$. It is a important feature because with positive $\Omega_{\beta 0}$ the model does not engender the re-bounce.
FIG. 3: Parametric contour plot for $\Omega_{m0} \times \Omega_{R0}$ taking account the statistical and systematic uncertainties and a prior in $\Omega_{m0}$. From inside to outside the contour lines correspond respectively to 68%, 95% and 99% confidence level. The dashed line represents $\Omega_{m0} = 0$.

**IV. THE COMPLETE SCENARIO**

Before to discuss the complete scenario, it is necessary, based on the previous section, choose the values for the six constant of this model. For the three constants associated with the primordial universe were selected $b$ for $\rho_{UR0}$, $6$ with $b = 1$ for $B_0^2$ and $\alpha = 10^{-64}\rho_c^{-1}$ suitable with $9$. For the three constants related with the recent universe were chosen the best fit for $\Omega_{m0}$, $\Omega_{R0}$ and $\Omega_{\alpha0}$ given by $29$ and $30$. This best fit is the most convenient choice to characterize the complete scenario because it incorporates more observational cosmological features than the others two. Nevertheless, using another best fit the qualitative results discussed below remain the same.

**A. The bouncing period and the primordial acceleration**

Near to bounce, the Friedmann equations could be approximated by,

$$\left(\frac{\dot{a}}{a}\right)^2 \simeq H_0^2 \left[ \Omega_{UR0} \left(\frac{a_0}{a} \right)^4 - \Omega_{\alpha0} \left(\frac{a_0}{a} \right)^8 \right]$$

and

$$\frac{\ddot{a}}{a} \simeq -H_0^2 \left[ \Omega_{UR0} \left(\frac{a_0}{a} \right)^4 - 3\Omega_{\alpha0} \left(\frac{a_0}{a} \right)^8 \right]$$

where

$$\Omega_{UR0} = \frac{\rho_{UR0}}{\rho_c} \text{ and } \Omega_{\alpha0} = \frac{16\alpha^2 B_0^4}{\rho_c}.$$

The conditions for bouncing

$$\dot{a}_{Bou} = 0 \text{ and } \ddot{a}_{Bou} > 0,$$

implies in

$$\left(\frac{a_{Bou}}{a_0} \right)^4 = \frac{\Omega_{\alpha0}}{\Omega_{UR0}},$$

which allows rewrite de Friedmann equations as

$$\left(\frac{\dot{a}}{a} \right)^2 \simeq H_0^2 \left[ \frac{\Omega_{UR0}}{\Omega_{\alpha0}} \left(\frac{a_{Bou}}{a} \right)^4 - \left(\frac{a_{Bou}}{a} \right)^8 \right]$$

(31)

and

$$\frac{\ddot{a}}{a} \simeq -H_0^2 \left[ \frac{\Omega_{UR0}}{\Omega_{\alpha0}} \left(\frac{a_{Bou}}{a} \right)^4 - 3\left(\frac{a_{Bou}}{a} \right)^8 \right].$$

(32)

The two previous equations clearly show the existence of non-relativistic matter was neglected.

The solution for (31) was studied in $15$ and has the following form:

$$a_{Bou} \simeq \left( \frac{\Omega_{\alpha0}}{\Omega_{UR0}} \right)^{1/4} a_0 \simeq 7.08 \times 10^{-21} a_0,$$

(33)

where in this calculation the contribution of non-relativistic matter was neglected.

The time scale $Q$ is defined as

$$Q \equiv \frac{H_0 \Omega_{UR0}}{\sqrt{\Omega_{\alpha0}}} = 4.1 \times 10^{20} \text{s}^{-1},$$

with $H_0 = 72 \text{ Km/s} \cdot \text{Mpc}$ - see $35$.

This solution is valid throughout the range in which the non-relativistic matter is negligible, e. g. from bounce to primeval nucleosynthesis.

The bouncing period plot is shown in figure $1$.

Just looking for this plot it is possible to realize that the primordial acceleration (inflation) engendered by the model is very restricted. In fact, the equation $32$ states that the acceleration occurs only between $a_{Bou} < a(t) < 1.32a_{Bou}$, which in usual inflationary language corresponds just to 0.3 e-folds. It rules out the standard mechanism to solve de horizon problem. Nevertheless, in this model the universe is cyclic and eternal, so in fact this problem does not exist. Much more investigate (and complicated) is the question of evolution of energy density fluctuations. This issue is directly related with model stability (see appendix B) and it will be subject of future investigations.
B. The re-bouncing period and the recent acceleration

For the present, the Friedmann equations could be approximated by,

$$\left(\frac{\dot{a}}{a}\right)^2 \simeq H_0^2 \left[ \Omega_{m0} \left(\frac{a_0}{a}\right)^3 + \Omega_{\rho0} \left(\frac{a}{a_0}\right)^4 + \Omega_{\gamma0} \left(\frac{a}{a_0}\right)^8 \right]$$

and

$$\frac{\ddot{a}}{a} \simeq -\frac{H_0^2}{2} \left[ \Omega_{m0} \left(\frac{a_0}{a}\right)^3 - 6\Omega_{\rho0} \left(\frac{a}{a_0}\right)^4 - 10\Omega_{\gamma0} \left(\frac{a}{a_0}\right)^8 \right]$$

(35)

(36)

where \( \Omega \)'s are defined in (13).

The conditions for re-bouncing are given by

$$\dot{a}_{RBou} = 0 \quad \text{and} \quad \ddot{a}_{RBou} < 0.$$

Unlike the equations of bounce, the equations above could not be expressed analytically. Even the \( a_{RBou} \) could not be expressed from \( \Omega \)'s in a simple manner. Nevertheless, numerical approaches allow us to study many features of this period. Figure 5 shows the numerical solution for (35).

From the numerical calculations involving (35) it could be determined the maximum size for the universe

$$a_{RBou} = 1.173a_0$$

(37)

and the time remaining to reach it

$$t_{RBou} - t_0 = 3.37\text{Gyr}.$$

The period of acceleration \( \Delta t_{acel} \) is obtained from (36) and results in

$$\Delta t_{acel} = 5.33\text{Gyr}.$$
FIG. 6: Plot for the scale factor $a/a_0$ in function of time $t$ in Giga years. $t = 0$ is chosen as the instant of bounce. The three times $t_1$, $t_2$ and $t_3$ split the expansion period in three eras: matter era - $10^{-1} < t < t_1$, acceleration era - $t_1 < t < t_2$ and re-bouncing era $t_2 < t < t_3$. The other two eras, radiation and bouncing eras, are too close to origin, thus it is not possible distinguish them, at least in these scales.

In terms of $a_0$ we obtain $a_{Bou} = 7.08 \times 10^{-21} a_0$ and $a_{RBou} = 1.173a_0$ where the maximum variation for the scale factor is $1.66 \times 10^{20}$. Comparing these results with those obtained previously - equations (33) and (37) - it verifies that both have the same numerical value. This confirms the validity of analysis made in sections IV A and IV B.

Another interesting analysis which can be done is the evolution of total equation of state $\omega_T$. Using the relation

$$\omega_T = \frac{p_T}{\rho_T}$$

we can study its evolution in terms of time. In most of the time during a complete cycle $|\omega_T| \leq 1$. This range include two long periods of deceleration (radiation and matter eras) and two periods of acceleration (primordial and recent). However, because of features of bouncing and re-bouncing, $\omega_T$ must eventually assume values less than $-1$ and values greater than $1$. Indeed, near to bouncing in a range $|t - t_{Bou}| < 1.1 \times 10^{-21}$s we have $\omega_T < -1$, which means a phantom equation of state. And near to re-bouncing in a range $|t - t_{RBou}| < 2.3Gyr$ we have $\omega_T > 1$, i.e. an Ekpyrotic equation of state. It is noteworthy that during all period of recent acceleration $\omega_T > -1$. This analysis confirm that this model is an explicit realization of a quintom cyclic scenario [18, 36].

V. CONCLUSION

In this paper it was constructed a cyclic cosmological model with five fluids namely baryonic and non-baryonic matter, radiation, neutrinos and a cosmic magnetic field described by a nonlinear electrodynamics. With five independent parameters it reproduces correctly the three expansion phases - radiation, matter and accelerate phases - and produces a bouncing and a re-bouncing stages. Another important point is that the main features for the model could be analyzed splitting it in two independent phases (primordial and actual) with a linking phase to connected both.

In primordial phase the relevant components are the bouncing and ultra-relativistic components characterized by the constants $\alpha$ and $\Omega_{U,R}$. The ultra-relativistic constant was fixed through CMB measurement and theoretical consideration about the neutrinos background. On the other hand, the constant $\alpha$ remains widely free. It happens because there are not much information about the pre-nucleosynthesis universe. Another way to determine $\alpha$ is to interpret the term $\alpha^2F^2$ as an one-loop quantum correction due vacuum polarization in the infrared regime [23, 24]. In this case, $\alpha^2 \approx 10^{-40}\rho_c^{-1}$ and this value is in complete agreement with the cosmological constraints (see section IV A).

Nowadays the relevant components are the non-relativistic matter, the acceleration and the re-bouncing terms. The three constants - $\Omega_m$, $\Omega_{\nu}$ and $\Omega_{\beta}$ - associated with each term was fixed using data of SNIa. Figures 1, 2 and 3 show that the re-bounce probably occurs. For the complete fit, represented by figure 3 the non-existence of re-bouncing is excluded in 95% confidence level. Other important feature for this model is the high value obtained for $\Omega_{m0}$ compared with the standard $\Lambda CDM$ model. Even with a robust prior for non-relativistic matter this feature remains. I believe that it happens due the eminent deceleration engendered by the re-bouncing term. To better clarify this point, it would be important to use another cosmological tests. This possibility will be investigated in the future.

A important question concerns about the evolution of energy density fluctuations. As point out in section IV A the model not engender an inflationary period. Nevertheless, it is not a essential problem for eternal cyclic universe since the causal connection could be established in previous cycle. Still, the subject about formation and/or dissociation of structure (linked with the evolution of energy density fluctuations) is an important topic and should be investigated in the future.

Other question concerns about the features of NLED used. As mentioned in section I the NLED was only used to describe the cosmological magnetic field while the radiation was retracted by the standard electrodynamics. So, the non-linear effects act only in the magnetic field and the description of a free radiation remains as in Maxwell theory. The main reason to adopt this approach is because quantum corrections due vacuum polarization changes the classical Lagrangian introducing nonlinear terms [23]. And the new terms only appear when we are treating with a quasi-static electromagnetic field [24]. So, the fundamental Lagrangian is the Maxwell one and the extra terms are just effects of vacuum polar-
ization. This approach is directly related with the model stability. Indeed, if we consider the $L_{NLED}$ as fundamental Lagrangian instead of Maxwell Lagrangian the model necessarily will present instabilities near to bounce (see appendix B).

For a wide point of view, we can speculate that the electromagnetic phenomenons in extremum conditions (e. g. very large and very short scale) are described by a non-linear generalization of Maxwell electrodynamics. Assuming that this generalization in a given scale could be expanded in positive and negative powers of $F \equiv F_{\mu\nu}F^{\mu\nu}$, it is necessary for convergence which $\alpha_k F^k > \alpha_{k+1} F^{k+1}$ and $\beta_k F^{-k} > \beta_{k+1} F^{-(k+1)}$ for $k > 0$. So, it can be argued that in this model the maximum and the minimum values for the scale factor limit the influence of $F^k$ terms with $|k| > 2$, i.e. terms of kind $F^3, F^4, ...$ and $F^{-3}, F^{-4}, ...$ are negligible in all cosmological time. In this context, the proposal Lagrangian is an approximation for a generic non-linear electrodynamics theory.

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**Appendix A: Review of average procedure**

The standard cosmological model, provided by the FLRW geometry, obeys the cosmological principle, which means that the 3-space is homogeneous and isotropic geometry. Thus, for compatibility with the cosmological framework, it is necessary an average procedure in the electromagnetic field [37]. As usual, we set the volumetric spatial average of a quantity $J$ by

$$\bar{J} \equiv \frac{1}{V} \int J \sqrt{-g} \, d^3x, \quad (39)$$

where $V = \int \sqrt{-g} \, d^3x$. And to generate the compatibility with the FLRW geometry, it is necessary to impose that

$$\bar{E}_i = 0, \quad \bar{B}_i = 0, \quad \bar{E}_i\bar{B}_j = 0, \quad (40)$$

$$\bar{E}_i\bar{E}_j = -\frac{1}{3} E^2 g_{ij}, \quad \bar{B}_i\bar{B}_j = -\frac{1}{3} B^2 g_{ij}. \quad (41)$$

Thus, applying the average procedure in the energy-momentum tensor produced by a $L = L(F)$, we obtain a $T_{\mu\nu}$ of a perfect fluid where

$$\rho = -L - 4 L_{\bar{F}} E^2, \quad (42)$$

$$p = L - \frac{4}{3} (2B^2 - E^2) L_{\bar{F}}, \quad (43)$$

and $L_{\bar{F}} \equiv dL/d\bar{F}$. The bar above $F$ point out that spatial average was performed.

**Appendix B: Model stability**

The purpose of this appendix is to discuss the model stability. As it is well known all flat cosmological model with bounce violates the null energy condition (NEC). On the other hand, in [35] was shown that under fairly general conditions the NEC violation necessarily implies instability (ghosts). More specifically, the authors showed that when all null vectors violate NEC (as in isotropic systems) or when superluminal excitations are not present, the NEC violation will result in instabilities. These statements extend in a straightforward manner for isotropic fluids through the Lagrangian formulation for fluid dynamics [39]. Thus, it is reasonable to ask if the proposed model is stable?

To answer this question we will restrict the analysis to a region near the bounce. In this case the relevant components are represented by the following Lagrangian:

$$L \simeq \alpha^2 F^2 - \frac{1}{4} F - \frac{1}{4} F_{UR}, \quad (43)$$

where $F$ is associated with the magnetic field and $F_{UR}$ with the ultrarelativistic constituents.

To analyze the stability of a model it is necessary to perturb this model and analyze the behavior of these perturbations. Thus, in the context of magnetic universe model, this analysis should be separated in two cases: perturbations taken before the achievement of spatial averages and perturbations taken after this achievement.

Let’s discuss the first case (perturbations taken before). The term $F^2$ arises when quantum corrections from vacuum polarization are interpreted classically [23, 24]. Indeed, Schwinger showed in [24] that for quasi-static electromagnetic fields the effects of vacuum polarization induce corrections in the classical Lagrangian and the first of these corrections is given by $\alpha^2 F^2$. It is worth noting that the complete results obtained in [23] and [24] are non-perturbative. So, the term $\alpha^2 F^2$ represents the inclusion (classical) of effects of vacuum polarization in the cosmological background. Therefore, the quantum perturbations should be done in the usual Lagrangian of electrodynamics $\frac{1}{4} F$ which does not present any instability problem. In this context, the quantization of $L \simeq \alpha^2 F^2 - \frac{1}{4} F$ or any other $L_{NLED}$ is completely meaningless.

For the second case, the perturbations must be performed in energy-momentum tensor of perfect fluid (isotropic background). So, the conclusions found in [35] can be directly applied, and therefore the violation of NEC will imply in an unstable model. Note that these perturbations should necessarily be of classical type (such as gravitational perturbations) since the spatial average has already been taken. In the magnetic universe model, this period of instability occurs only near to bounce and it quickly ends (see section IV A).

From a different point of view, we can look at [43] as a fundamental Lagrangian. Thus, it is necessary to analyze
the stability of this Lagrangian. The term \(-\frac{1}{2} F_{UR}\) will be neglected because it does not generate instabilities. So,

\[
L_B \simeq \alpha^2 F^2 - \frac{1}{4} F. \tag{44}
\]

The energy-momentum tensor of \(L_B\) is given by

\[
T_{\mu\nu} = (1 - 8\alpha^2 F) F_\mu \, a F_\nu - \left(\alpha^2 F^2 - \frac{1}{4} F\right) g_{\mu\nu},
\]

remembering that \(F_{\mu\nu}\) is determined only by magnetic field \(\vec{B}\).

To establish if there is a violation of NEC it is necessary to analyze the quantity \(T_{\mu\nu} n^\mu n^\nu\) where \(n^\mu = (\omega, \vec{n})\) is a null vector. Performing this calculation in FLRW background we obtain the following equation:

\[
T_{\mu\nu} n^\mu n^\nu = \left[1 - 8\alpha^2 F\right] \left(\vec{n} \times \vec{B}\right)^2, \tag{45}
\]

where \(F\) is function of \(B^2\). Thus, to happen NEC violation \((T_{\mu\nu} n^\mu n^\nu < 0)\) the first bracket should be negative. Another important point is that if \(T_{\mu\nu} n^\mu n^\nu < 0\) then all null vector violate NEC.

In general, the above equation is not spatially isotropic since \(B\) may depend on points in space. Indeed, it is assumed that \(\vec{B}\) has a random spatial distribution whose spatial average is determined by (41). Therefore, it is possible there are regions of space with instability (where NEC is violated for all null vectors) and other regions perfectly stable. Note that the instabilities only appear at many different points of space if, on average, the term \(8\alpha^2 F\) is of the order (or larger) of unity, i.e. \(8\alpha^2 F \gtrsim 1\), and it is exactly what happens near to bounce. So, using the bouncing condition (??) and (44) follows

\[
\rho_{Bou} \simeq \frac{1}{4} F_{Bou} - \alpha^2 F_{Bou}^2 = 0 \Rightarrow 8\alpha^2 F_{Bou} = 2,
\]

where \(\vec{F} = 2B^2\) - eq. (5).

Since, on average, the signal of (45) is determined by

\[
\left[1 - 8\alpha^2 F\right] \text{ or } \left[1 - 16\alpha^2 B^2\right],
\]

it is possible to conclude that near to bounce most regions will present unstable perturbations. However, as the universe expands the factor \(B^2 \sim a^{-2}\) decreases causing a decrease in the number of regions with unstable perturbations. Note that with the increase of \(a\) in only one order of magnitude the probability of having a region with unstable perturbations is negligible. One interesting possibility for future work is to study how the perturbations generated near to bounce evolve.

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