FURTHER STUDY ON U(1) GAUGE INVARIENCE RESTORATION

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abstract

To further investigate the applicability of the projection scheme for eliminating the unphysical divergence $s/m_e^2$ due to U(1) gauge invariance violation, we study the process $e^- + W^+ ightarrow e^- + \bar{t} + b$ which possesses advantages of simplicity and clearness. Our study indicates that the projection scheme can indeed eliminate the unphysical divergence $s/m_e^2$ caused by the U(1) gauge invariance violation and the scheme can apply to very high energy region.

It is well known that when an unstable particle exists at the s-channel as an intermediate boson or fermion, one must introduce its width, otherwise the integration over final states would blow up as long as the energy $\sqrt{s}$ is high enough. The commonly adopted form is the Breit-Wigner form which modifies the denominator of the propagator $(q^2 - M^2)$ into $(q^2 - M^2 + i\Gamma)$ where $q, M, \Gamma$ are the momentum, mass and width of the unstable particle respectively. In the simplest way, the width $\Gamma$ can take its measured value. However, if there is a t-channel $\gamma$–propagator in the Feynman diagram at the same time, the introduction of $\Gamma$ violates the U(1) gauge invariance.

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This violation is due to an ill-treatment i.e. an unreasonable truncation of the perturbative expansion.

This violation would cause divergent terms such as $\log s/m^2_e$ and $s/m^2_e$. This divergence only occurs near the $e^−−$forward scattering, i.e. small $\theta$ region, where $\theta$ is the angle between momenta of incoming and outgoing electrons. The small measure at vicinity of $\theta \to 0$ makes the logarithmic divergence ($\log s/m^2_e$) benign which can only bring up negligible changes to the calculations, on contraries, the linear term $s/m^2_e$ is intolerable and causes the theoretical evaluation of the corresponding cross section unreasonably large by many orders.

Therefore, to obtain physical results, one should restore or at least partly restore the U(1) gauge invariance, namely eliminate the disastrous divergent power term $s/m^2_e$. Many suggestions to remedy this problem were discussed in literatures [1, 2] for the typical process $e^+e^- \to e^- + \bar{\nu} + u + \bar{d}$ where an s-channel W-boson propagator exists and its Breit-Wigner form violates the U(1) gauge invariance. Later Paravassiliou and Pilaftsis [3] carried out a systematic study and restored the U(1) gauge invariance based on the pinch technique. In their work, the width effects are compensated in principle.

For practical applications, we need some simple schemes. Argyres et al. introduced lepton and quark loops and the absorptive part of the triangles compensates the effect of $\Gamma_W M_W$ which violates the gauge invariance [4].

Instead, we suggested a "projection scheme" which can project out the troublesome U(1) gauge violation terms which lead to the unphysical divergence $s/m^2_e$ [5]. This scheme is to choose a special gauge, then in the theoretical calculations the $s/m^2_e$ terms disappear automatically, in other words, the U(1) gauge invariance is partly restored. Even though the logarithmic divergence $\log s/m^2_e$ remains, it is benign, so that one can tolerate such terms. Moreover, the contribution from the logarithmic divergence to the cross section is negligible (up to less than 1%) and physical results can be obtained in this scheme (see later part of this work for more
For further study the violation and restoration of the U(1) gauge invariance and probe the applicability of the projection scheme, we investigate the process $e^- + W^+ \to e^- + t + \bar{b}$. The concerned Feynman diagrams for this process at the tree level are presented in Fig.1. The process has some obvious advantages. First, it is a three-body final state case, so the final-state integration is much simpler than $e^+e^- \to e^- + \bar{\nu} + u + \bar{d}$. Thus it is easier to study the U(1) gauge invariance and restoration and that is the aim of this work. Secondly, in this case the concerned s-channel W-propagator is connected to $t\bar{b}$ which are real particles in the final states, so that the intermediate W-boson can never be on its shell and the final state integration is convergent anyway, thus an involvement of $i\Gamma_W M_W$ in the W-propagator is not needed at all for obtaining accurate results. In other words, we can be convinced that the results obtained without introducing the Breit-Wigner modification of the W-propagator are the physical ones (to order of $O(\alpha_s^0)$). Our purpose of this work is to study the U(1) gauge invariance, so that we can take the results obtained without the $i\Gamma_W M_W$ as the a standard and then compare the results with $i\Gamma_W M_W$ existing in the propagator to it for testifying the applicability and accuracy of our scheme.

In the following, we will give detailed discussions on this problem of U(1) gauge invariance violation and restoration, mainly show how to obtain reasonable physical results in our scheme.

(1) The U(1) gauge invariance violation in $e^- + W^+ \to e^- + t\bar{b}$.

In Fig.1.a, if the propagator of the s-channel W-boson is written in the Breit-Wigner form as

$$\frac{1}{p^2_+ - M_W^2 + i\Gamma_W M_W},$$

we can check the U(1) gauge invariance, i.e. the Ward-identity.
We write the reaction amplitude as

\[ M = \bar{u}_e(p_1)\gamma^\mu \frac{-i}{q^2} u(p_2)(T_a + T_b + T_c)_\mu + \Delta M, \]

where \((T_a + T_b + T_c)_\mu\) correspond to the effective vertex of the first three Feynman diagrams in Fig.1, and the fourth one \(\Delta M\) is irrelevant to the U(1) problem. Then, the Ward identity is

\[ A \equiv q^\mu \cdot (T_a + T_b + T_c)_\mu = 0, \tag{1} \]

where \(q^\mu\) is the 4-momentum of the virtual t-channel photon.

A straightforward derivation shows

\[
A = q^\mu \cdot (T_a + T_b + T_c)_\mu \\
= \left[ (p_2^2 - p_W^2) \frac{1}{p_+^2 - M_W^2 + i\gamma'} - 1 \right] \bar{u}_e(p_2) \gamma^\alpha (1 - \gamma^5) v_b(p_3), \tag{2}
\]

where we take \(\gamma_W = \Gamma_W M_W\) following the notation of [4] and other symbols are marked in Fig.1 explicitly.

\(A\) is not zero unless

\[ \gamma_W = 0, \quad \text{or} \quad p_W^2 = M_W^2 - i\gamma'_W \quad \text{and} \quad \gamma'_W = \gamma_W. \]

Because the incoming W-boson is a real on shell particle, usually

\[ p_W^2 = M_W^2 \]

is required. The extra \(i\gamma'_W\) represents a deviation from its central value. In next section, we demonstrate that in a stringent way to restore the U(1) gauge invariance this extra term is necessary as long as the s-channel W-propagator is modified to the Breit-Wigner form, by contrast, in Sec.4, we show that with the projection scheme one does not need to worry about it after all.

(2) Restoration of U(1) gauge invariance.
One of the practical ways to restore the gauge invariance was given by Argyres et al. They considered the contribution from the radiative correction to the $WW\gamma$ vertex. They proved that the absorptive part of a triangle consisting of quarks and leptons compensates the effects of $i\gamma_W$, which violates the U(1) gauge invariance, then with this extra contribution, the Ward identity holds again.

In their derivation for $e^+e^- \rightarrow e^-\bar{\nu}ud$, only one W-boson is at s-channel (Fig.1.a), so that according to the Cutkosky cutting rule, only one cut exists and it corresponds to the cut-1 in Fig.2. In our case $e^- + W^+ \rightarrow e^- + t + \bar{b}$, both cut-1 and cut-2 can exit non-zero. Observation manifests that the cut-1 gives an absorptive part of the triangle which compensates the imaginary part of the W-boson propagator, i.e. $i\gamma_W$, whereas the cut-2 is also non-zero and corresponds to $i\gamma'_W$. Argyres et al. proved that the cut 1 fully compensates of the imaginary part of the s-channel W-boson $i\gamma_W$, so a non-zero contribution of cut-2 needs to be compensated by other sources, otherwise, the Ward identity would be violated again. Thus as claimed above, the on-shell condition of the incoming W-boson would be changed to $p_W^2 = M_W^2 + i\gamma'_W$. In general, $\gamma_W$ does not need to be equal to $\gamma'_W$. From eq.(3) one can notice that if we write $p_W^2 = M_W^2 + i\gamma'_W$ and let $\gamma_W = \gamma'_W$, the fermion triangle is not needed and the Ward identity holds automatically. However, as $i\gamma_W \neq i\gamma'_W$, eq.(3) tells us that the Ward identity is violated at tree level and we can restore it by taking into account of higher order correction, i.e. the absorptive part of the fermionic triangle. Obviously, the cut-1 of Fig.2 compensates $i\gamma_W$ whereas, the cut-2 would compensate $i\gamma'_W$.

Moreover, since in process $e^+e^- \rightarrow e^- + \bar{\nu} + u + \bar{d}$, the outgoing quark and antiquarks are $u$ and $\bar{d}$, whose masses are small and can be neglected, due to the CVC and PCAC theorems, $p_\alpha u(p_3)\gamma_\alpha(1 - \gamma_5)v_b(p_4) \approx 0$ where $p_+ = p_3 + p_4$. Then the corresponding Lorentz structure for $\Gamma_{\alpha\beta\mu}$ does not contain terms proportional to $p_+^\mu$. But in our case, for the vertex $Wt\bar{b}$, this simple relation does not exist and the Lorentz structure becomes a bit more complicated.

The detailed procedure about the derivation of the cut-1 absorptive part of the fermionic triangle
was given in \[4\], so here we only present the results corresponding to the $Wt\bar{b}$ vertex, as

$$(\Delta \Gamma^{\alpha\beta\mu})^{(1)} = C_0^{(1)} p_+^\beta p_-^\mu + C_1^{(1)} q_+^\alpha p_-^\mu + C_2^{(1)} g_+^{\beta\alpha} p_-^\mu + C_3^{(1)} g_+^{\mu\beta} q_-^\alpha + C_4^{(1)} g_+^{\mu\alpha} p_-^\beta,$$

(3)

where the superscript (1) denotes the quantities for the cut-1.

Comparing with the corresponding expressions for $Z^{\alpha\beta\mu}$ given in \[4\], one can notice that there are two more terms in (3). Moreover, the expressions of $C_0^{(1)} \sim C_5^{(1)}$ are much more complicated than that of \[3\], in fact, the derivations are similar, even though very tedious. The explicit expressions of $C_0^{(1)} \sim C_5^{(2)}$ are presented in the appendix.

The cut-1 and cut-2 are geometrically symmetric to the virtual photon line in the triangle diagram, so that while deriving the absorptive part of the loop corresponding to cut-2, one obtains $C_0^{(2)} \sim C_5^{(2)}$ by an exchange of $p_+ \leftrightarrow p_-$. However, since $p_-$ is associated with the incoming W-boson, we have

$$\epsilon_\beta p_-^\beta = 0,$$

the terms related to $p_-^\beta$ disappear and the Lorentz structure for cut-2 is exactly the same as that of \[3\] where $Wud$ vertex was under consideration.

The total absorptive contribution of the fermionic triangle is

$$\Delta \Gamma^{(tot)}_{\alpha\beta\mu} = \Delta \Gamma^{(1)}_{\alpha\beta\mu} + \Delta \Gamma^{(2)}_{\alpha\beta\mu},$$

(4)

$\Delta \Gamma^{(1)}_{\alpha\beta\mu}$ and $\Delta \Gamma^{(2)}_{\alpha\beta\mu}$ would be in opposite sign, because of a fermionic exchange. If they were equal in magnitude, they would cancel each other. This observation is consistent with eq.(3). Obviously, in fact, they are not equal in magnitude, and they would cancel $i\gamma_W$ and $i\gamma'_W$ respectively as discussed above.

Argyres et al. proved in a convincing way that the existence of the absorptive part of the fermionic triangle fully compensates the effect of $i\gamma_W$ and thus restores the U(1) gauge invariance. Therefore, if one adds $\Delta \Gamma_{\alpha\beta\mu}$ to the tree expression, the U(1) gauge invariance is guaranteed
and their results indicated that the troublesome term $s/m_e^2$ does not exist in evaluating the cross section.

However, as noted that the expressions of $\Delta \Gamma^{(tot)}_{\alpha\beta\mu}$ is very complicated and its practical applications are restricted. We introduced a simple scheme as the "projection scheme" which can eliminate the linear divergent $s/m_e^2$ automatically.

In the following section, we will discuss its application to $e^- + W^+ \rightarrow e^- t\bar{b}$ process.

(3) The application of the projection scheme.

The projection scheme is described in all details in [5]. For completeness, here we give a brief introduction to the scheme.

In fact, the U(1) gauge invariance i.e. the Ward identity guarantees an exact cancellation among unphysical large numbers and only physical quantities which may be many orders smaller than the large numbers remain. Therefore, even small violation of the gauge invariance fails the delicate cancellation and leads to disastrous divergent terms such as $s/m_e^2$. It is well understood that this gauge invariance violation is due to an ill-truncation of the perturbative expansion, so is not physical. If we first confirm the gauge invariance, we can replace

$$l_\mu \equiv \bar{u}_e(p_2)\gamma_\mu u_e(p_1), \quad (5)$$

by

$$l'_\mu = l_\mu - cq_\mu, \quad (6)$$

Since $q_\mu T^\mu = 0$ where $T_\mu$ is the effective vertex of Fig.1, the replacement is trivial if there is no gauge invariance violation. However, when $i\gamma_\nu W$ exists this treatment plays an crucial role for eliminating the linear divergence. We choose the coefficient $c$ according to

$$\frac{\delta}{\delta c} max(|l'_0|^2, |l'_1|^2, |l'_2|^2, |l'_3|^2) = 0, \quad (7)$$
and obtain
\[ c = \frac{q_0 l_0 + q_1 l_1 + q_2 l_2 + q_3 l_3}{q_0^2 + q_1^2 + q_2^2 + q_3^2} \]  
(8)

where \( k_i \) are in the Euclidean space \([5, 6]\). Because
\[ q_\mu l^\mu = 0, \]
in the Minkovsky space, we have
\[ c = \frac{2k_0q_0}{||k||^2}, \]
where \( ||k||^2 \equiv k_0^2 + k_1^2 + k_2^2 + k_3^2 \) is a positive definite quantity. This scheme, in principle, is to select a special gauge and it is aimed to eliminate the components of \( l_\mu \) which are parallel to \( q_\mu \) in the Euclidean space.

By the projection scheme, the problem of large number cancellation is avoided, the troublesome divergent \( s/m_e^2 \) does not appear at all. This scheme is embedded in the program FDC which is designed for calculating such cross sections \([7]\). With the program FDC, we calculate the differential and total cross sections of \( e^- + W^+ \rightarrow e^- + t + \bar{b} \) for various energies. The results are tabulated bellow where for a comparison, we list the numbers corresponding to the results of the standard (i.e. without \( i\gamma_W \)), without using the projection scheme and with the scheme respectively.

| \( \sqrt{s} \) (GeV) | 200 | 500 | 800 | 1200 | 1600 | 2000 |
|-----------------|-----|-----|-----|------|------|------|
| "standard"      | \(6.44 \times 10^{-2}\) | \(6.92 \times 10^{-1}\) | 1.04 | 1.30  | 1.46  | 1.57  |
| without PJ      | \(1.18 \times 10^2\)   | \(8.32 \times 10^5\)   | 7.18 \( \times 10^6\) | 2.98 \( \times 10^7\) | 1.30 \( \times 10^8\) | 3.21 \( \times 10^8\) |
| with PJ         | \(6.44 \times 10^{-2}\) | \(6.92 \times 10^{-1}\) | 1.04 | 1.30  | 1.46  | 1.57  |

The notation in the table 1, "standard" means that results obtained without introducing \( i\gamma_W \) in the W-boson propagator and also without employing the projection scheme; "without PJ" means the results obtained with \( i\gamma_W \) but without using the projection scheme and "with PJ"
denotes the results with $i\gamma_W$ and the projection scheme. The unit of the cross section is in GeV$^{-2}$. Our results are also graphed in Fig.3 and Fig.4.

In Fig.3, one can see that the differential cross section $d\sigma/d\cos \theta$ tends to infinity as $\theta \to 0$. That indicates a collinear divergent property of the process. It is also noted that with the projection scheme, even though $d\sigma/d\cos \theta$ is still singular at $\theta = 0$, the degree of singularity is decreased compared to that without projection scheme. One can expect that this singularity of differential cross section would not blow up the total cross section. The curves in Fig.4 confirms this point.

As discussed above, $i\gamma_W$ does not need to be added in this process, so the corresponding solution without $i\gamma_W$ can serve as the standard, i.e. the real physical solution. In Fig.4, it corresponds to curve-1. When the projection scheme is not employed and the Breit-Wigner form of the W-propagator is taken, we observe that the results shown in curve-2 are $7\sim8$ orders larger than the standard. It indicates that such results are not physical. The curve-3 corresponds to the results where the Breit-Wigner form is taken and the projection scheme is employed. We notice that the curve-3 coincides with the standard (curve-1) perfectly, it can also be seen from the data in table 1. Namely we almost cannot distinguish between them.

(4) Discussion.

The U(1) gauge invariance violation brings up difficulties for correctly evaluating the cross sections. Without carefully handling this problem, all obtained results are not reliable at all. In principle, one should restore the U(1) gauge invariance by involving the loop effects. The complete restoration which includes higher order contributions is difficult, even though possible. Thus a practical method which is applicable to theoretically calculating the cross sections of all processes where a t-channel virtual photon exists and collinear divergence would cause disastrous consequence, is favored.
The projection scheme is not aimed to restore the U(1) gauge invariance, but to eliminate the troublesome $s/m^2$, while $\log(s/m^2)$ remains and contributes negligibly to the total cross section. Our numerical results indicate that even for $\sqrt{s} = 2000$ GeV, the consistency of the results with the ”standard” is perfectly satisfactory. Therefore, this scheme is applicable for evaluating cross sections at very high energies.

Similar schemes were proposed [8], compared to theirs, our scheme is easier in practical applications, but in principle, all schemes are parallel.

Moreover, the same scheme is embedded in the FDC program and applied to evaluating the cross section of $e^+e^- \rightarrow e^+e^-$. Since this process serves as a standard reaction to testify the working condition of a collider and software, accurate evaluation of its cross section is crucially important. Application of the projection scheme is proved to give satisfactory results.

Our conclusion is that the projection scheme is very useful for evaluating the cross sections where U(1) gauge invariance violation might be brought up by the Breit-Wigner form of propagators of unstable intermediate bosons or fermions. The accuracy is satisfactory for a very wide range of energy. This scheme can be widely applied in the numerical analysis of the future experiments.

References

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Appendix

\((\Delta \Gamma^{\alpha\beta\mu})^{(1)} = C_0 p_+^\alpha p_+^\beta p_+^\mu + C_1 q^\alpha q^\beta q^\mu + C_3 g^\mu\nu p_+^\alpha + C_4 g^\mu\nu q^\alpha + C_5 g^{\mu\nu} p_+^\beta, \quad (9)\)

where

\[
\begin{align*}
C_0 & = C_{00} + C_{01} f_0, \\
C_1 & = C_{10} + C_{11} f_0, \\
C_2 & = C_{20} + C_{21} f_0, \\
C_3 & = C_{30} + C_{31} f_0, \\
C_4 & = C_{40} + C_{41} f_0, \\
C_5 & = C_{50} + C_{51} f_0. \\
\end{align*}
\]

(10)

The coefficients \(C_{00}, C_{11}\) and the function \(F_0\) are given below explicitly.

\[
C_{00} = \frac{1}{\lambda^2} \left( \frac{-40}{3} p_+ \cdot q - \frac{52}{3} p_+^2 + 20 q^2 \right) + \frac{1}{\lambda^2} \left( \frac{152}{3} p_+ \cdot q q_+^2 q^2 - \frac{26}{3} p_+^4 q^2 - 4 p_+ \cdot q q_+^4 - 18 p_+^2 q^4 \right)
\]
\[
C_{01} = -4 + \frac{1}{\lambda} \left( 4p_+ \cdot q p^4_+ q^4 + 8p_+ \cdot q q^2 - 10p^2_+ q^2 - 6q^4 \right) \\
+ \frac{1}{\lambda^2} \left( 3p_+ \cdot q p^4_+ q^2 - 9p_+ \cdot q p^2_+ q^4 - 9p^4_+ q^4 + 2p_+ \cdot q q^6 - 7p^4_+ q^6 \right) \\
+ \frac{1}{\lambda^3} \left( 5p_+ \cdot q p^6_+ q^4 + 30p_+ \cdot q p^4_+ q^6 - 20p^6_+ q^6 \right) \\
+ 5p_+ \cdot q p^2_+ q^8 - 20p^4_+ q^8); \\
\]

(11)

\[
C_{10} = \frac{1}{\lambda} \left( -\frac{100}{3} p^2_+ \right) \\
+ \frac{1}{\lambda^2} \left( -\frac{4}{3} p_+ \cdot q p^4_+ q^2 + 20p_+ \cdot q p^2_+ q^2 - \frac{20}{3} p^4_+ q^2 + 8p^2_+ q^4 \right) \\
+ \frac{1}{\lambda^3} \left( -10p_+ \cdot q p^6_+ q^2 - 30p_+ \cdot q p^4_+ q^4 + 30p^6_+ q^4 + 10p^4_+ q^6 \right); \\
\]

(13)

\[
C_{11} = \frac{1}{\lambda} \left( 8p_+ \cdot q q^2_+ - 6p^4_+ + 8p^2_+ q^2 \right) \\
+ \frac{1}{\lambda^2} \left( 12p_+ \cdot q p^4_+ q^2 - 10p^6_+ q^2 - 18p^4_+ q^4 - 4p^2_+ q^6 \right) \\
+ \frac{1}{\lambda^3} \left( 20p_+ \cdot q p^6_+ q^4 - 5p^8_+ q^4 + 20p_+ \cdot q p^4_+ q^6 - 30p^6_+ q^6 - 5p^4_+ q^8 \right); \\
\]

(14)

\[
C_{20} = \frac{1}{\lambda} \left( -\frac{8}{3} p_+ \cdot q p^2_+ + 6p^2_+ q^2 \right) \\
+ \frac{1}{\lambda^2} \left( -2p_+ \cdot q + 2p^2_+ \right) + \frac{1}{\lambda} \left( -2p_+ \cdot q p^2_+ q^2 \right) \\
+ \frac{1}{\lambda^2} \left( 3p_+ \cdot q p^4_+ q^4 - p^6_+ q^4 + p_+ \cdot q p^2_+ q^6 - 3p^4_+ q^6 \right); \\
\]

(16)

\[
C_{30} = 4 + \frac{1}{\lambda} \left( \frac{16}{3} p_+ \cdot q p^2_+ + 4p_+ \cdot q q^2 - 6p^2_+ q^2 \right) \\
+ \frac{1}{\lambda^2} \left( -2p_+ \cdot q p^4_+ q^2 - 2p_+ \cdot q p^2_+ q^4 + 4p^4_+ q^4 \right); \\
\]

(17)

\[
C_{31} = -6p_+ \cdot q - 2p^2_+ + 8q^2 \\
+ \frac{1}{\lambda} \left( -2p_+ \cdot q p^2_+ q^2 - p^4_+ q^2 - 2p_+ \cdot q q^4 + 3p^2_+ q^4 \right) \\
+ \frac{1}{\lambda^2} \left( 3p_+ \cdot q p^4_+ q^4 - p^6_+ q^4 + p_+ \cdot q p^2_+ q^6 - 3p^4_+ q^6 \right); \\
\]

(18)

\[
C_{40} = \frac{1}{\lambda} \left( 4p_+ \cdot q p^2_+ + \frac{2}{3} p^4_+ - 4p^2_+ q^2 \right) \\
+ \frac{1}{\lambda^2} \left( -4p_+ \cdot q p^4_+ q^2 + 2p^6_+ q^2 + 2p^4_+ q^4 \right); \\
\]

(19)

\[
C_{41} = 2p^2_+ + \frac{1}{\lambda} \left( 2p_+ \cdot q p^2_+ - 4p_+ \cdot q p^2_+ q^2 - 2p^4_+ q^2 + 2p^2_+ q^4 \right) \\
+ \frac{1}{\lambda^2} \left( p_+ \cdot q p^6_+ q^2 + 3p_+ \cdot q p^4_+ q^4 - 3p^6_+ q^4 - p^4_+ q^6 \right); \\
\]

(20)
\[ C_{50} = \frac{1}{\lambda} \left( \frac{40}{3} p_+ \cdot q p_+^2 + \frac{2}{3} p_+^4 - 10 p_+^2 q^2 \right) + \frac{1}{\lambda^2} \left( -6 p_+ \cdot q p_+^4 q^2 + 2 p_+^6 q^2 - 2 p_+ \cdot q p_+^2 q^4 + 6 p_+^4 q^4 \right); \] 
\[ (21) \]

\[ C_{51} = 2 p_+ \cdot q - 4 p_+^2 \\
+ \frac{1}{\lambda} \left( 2 p_+ \cdot q p_+^4 + 2 p_+ \cdot q p_+^2 q^2 - 6 p_+^4 q^2 - 2 p_+^2 q^4 \right) \\
+ \frac{1}{\lambda^2} \left( p_+ \cdot q p_+^6 q^2 + 6 p_+ \cdot q p_+^4 q^4 - 4 p_+^6 q^4 + p_+ \cdot q p_+^2 q^6 - 4 p_+^4 q^6 \right). \] 
\[ (22) \]

We also present
\[ f_0 = \frac{1}{\sqrt{\lambda}} \log \frac{p_- \cdot q + \sqrt{\lambda}}{p_- \cdot q - \sqrt{\lambda}}, \]
\[ (23) \]

and
\[ \lambda = (p_+ \cdot q)^2 - p_+^2 q^2. \]

Figure Caption

Fig1. The Feynman diagrams at tree level for $e^- + W^+ \rightarrow e^- + t + \bar{b}$.

Fig.2. The fermionic triangle. The cut-1 and cut-2 are drawn and they correspond to different absorptive parts of the loop.

Fig.3. The differential cross section $d\sigma/d\cos\theta$ vs. $\theta$ with $\sqrt{s} = 1000$ GeV, the curve-1 stands for the standard, the curve-2 for the results without PJ and the curve-2 for that with PJ.

Fig.4. The total cross sections $\sigma$ vs. $\sqrt{s}$. The curve-1 stands for the standard, the curve-2 for the results without PJ and the curve-3 for that with PJ.