Absorption in a Particle Oscillations

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Abstract

In the framework of field approach with the finite time interval the particle oscillations in a medium are considered. The absorption in a final states do not lead to dramatic suppression of $ab$ transitions. Also we touch on the problem of infrared singularities. The approach under study is infrared-free. It is shown that "correction" published recently [A.Gal, Phys. Rev. C61 (2000) 028201] is clearly wrong.

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1 Introduction

The theory of ab oscillations [1] are based on single-particle model. The interaction of particles $a$ and $b$ with the matter is described by potentials $U_{a,b}$. $ImU_b$ is responsible for loss of $b$-particle intensity. The wave functions $\Psi_{a,b}$ are given by equations of motion. The index of refraction, the forward scattering amplitude $f(0)$ and potential are related to each other, so later on the standard approach is referred to as potential model.

In some instances there is a need to consider the ab conversion in the matter followed by reaction

\[ (a - \text{medium}) \rightarrow (b - \text{medium}) \rightarrow b + c \rightarrow f. \]

(1)

Here $c$ is the particle of medium (one should sum over all $c$-particles); $b + c \rightarrow f$ represents the reaction. The whole process (ab transition, b-medium interaction) takes place in the same layer of matter. $b$-particle absorption is essential and $ImU_b$ cannot be ignored. An example is the $n\bar{n}$ transitions [2] in the medium followed by annihilation

\[ (n - \text{medium}) \rightarrow (\bar{n} - \text{medium}) \rightarrow f, \]

(2)

where $f$ are the annihilation products which should be detected (see Fig.1a). Standard calculation (see, for example [3,4] and Section 2) predicts a dramatic suppression of this process due to $\bar{n}$-medium interaction. Below it is shown that process (2) is qualitatively equivalent to free-space process given in Fig.2:

\[ n \rightarrow \bar{n} \rightarrow \bar{p}e^+\nu. \]

(3)

Therefore, process (3) should be drastically suppressed by the decay in a final state, what is incorrect. We cite a body of other arguments (see Sec.6) which point to the fact that potential approach is reasonable as starting point only.

In the framework of field approach with finite time interval (FTA) we perform the direct calculation of the processes shown in Fig.1. The connection between FTA and $S$-matrix theory is studied. The approach reproduces all the results in neutrino oscillations in which $Imf_{\bar{b}}(0)$ is ignored as well as standard calculation of the process (2). However, we disagree with standard calculation, because, in our opinion, the absorption is described improperly in this case.

2 Potential model

In the standard approach the $n\bar{n}$ transitions in the medium are described by Schrödinger equations

\[
(i\partial_t - H_0)\Psi_n(x) = \epsilon\Psi_n(x),
\]

\[
(i\partial_t - H_0 - V)\Psi_\bar{n}(x) = \epsilon\Psi_\bar{n}(x),
\]

\[
H_0 = -\nabla^2/2m + U_n,
\]

\[
V = U_\bar{n} - U_n = ReU_\bar{n} - i\Gamma/2 - U_n, \tag{4}
\]

$\epsilon = (m_2 - m_1)/2 = 1/\tau_{n\bar{n}}$. Here $m$ and $U_n = const$ are the neutron mass and potential, respectively; $U_\bar{n}$ and $\Gamma \sim 100$ MeV are the optical potential and annihilation width of $\bar{n}$; $\tau_{n\bar{n}}$ is a free-space $n\bar{n}$ oscillation time; $m_{1,2}$ are the masses of the stationary states $n_{1,2}$. $\epsilon$ is a small parameter. The initial conditions are $\Psi_n(0, x) = n(0, x)$, $\Psi_\bar{n}(0) = 0$, where $n(x) = V^{-1/2}\exp(-i\epsilon_n t + iP_n x)$, $\epsilon_n = P_n^2/2m + U_n$. 

2
For analogy with field approach we introduce the evolution operator $U(t) = I + iT(t)$. For $V = \text{const.}$ in the lowest order in $\epsilon$ we obtain matrix elements $U_{ii}(t) = \langle n(0) | \Psi_n(t) >$ and $T_{ni}(t) = \langle n(0) | \Psi_n(t) >$:

$$
T_{ni}(t) = (\epsilon/V)(\exp(-iVt) - 1),
$$
$$
T_{ii}(t) = i(\epsilon/V)^2[1 - iVt - \exp(-iVt)].
$$

The probability to find antineutron $W_{\bar{n}}(t)$ is

$$
W_{\bar{n}}(t) = |T_{\bar{n}i}(t)|^2 = (\epsilon/|V|)^2[1 - 2\cos(ReVt)e^{-\Gamma t/2} + e^{-\Gamma t}].
$$

The unitarity condition gives

$$
1 = |U_{ii}(t)|^2 + W_{\bar{n}}(t) + W_{\text{ann}}(t),
$$

where $W_{\text{ann}}(t)$ is the probability to find the annihilation products. In the potential model the probability of $n\bar{n}$ conversion $W_{\text{pot}}$ is defined by equation

$$
W_{\text{pot}}(t) = 1 - |U_{ii}(t)|^2 = 2\text{Im}T_{ii}(t).
$$

For brevity, we put below

$$
V = -i\Gamma/2, \quad \Gamma t \gg 1.
$$

Then

$$
W_{\text{pot}}(t) \approx 4\epsilon^2 t/\Gamma.
$$

Noting that free-space $n\bar{n}$ transition probability is

$$
W_f = \epsilon^2 t^2
$$

for suppression factor $R_{\text{pot}}$ we have $R_{\text{pot}} = W_{\text{pot}}/W_f = 4/\Gamma t \ll 1$.

We would like to stress that Eq.(7) is automatically fulfilled only for correct model with Hermitian operators. Also recall that instead of realistic operator of $\bar{n}$-medium interaction $\hat{U}_{\bar{n}}$ the parametrization

$$
\hat{U}_{\bar{n}} = \text{Re}U_{\bar{n}} - i\Gamma/2 = \text{const}
$$

is used. On the reasons given below we abandon the approximation (12). In this case the usual procedure connected with diagonalization of mass matrix cannot be realized. However, the result do not depend on the basis. All existing calculations have been done in $n - \bar{n}$ representation.

### 3 Field approach

Let us consider the process (2) in the framework of field approach. The interaction Hamiltonian involves two terms:

$$
H_{n\bar{n}}(t) = \epsilon \int d^3x(\bar{\Psi}_n \Psi_n + \text{H.c.}),
$$

$$
H(t) = \text{(all } n - \text{medium interactions) } - U_n,
$$

$H_f = H_{n\bar{n}} + H$. Here $\Psi_n$ and $\bar{\Psi}_{\bar{n}}$ are the fields of $n$ and $\bar{n}$, respectively; $m_{\bar{n}} = m_n = m$. The background nuclear matter field $U_n$ is included in unperturbed Hamiltonian $H_0$; quadratic term
produce all the results in neutrino oscillations. (Commonly, in neutrino oscillations

Substituting this expression in Eq.(14) one obtains Eq.(6). This means that FTA should reproduce all the results in neutrino oscillations. (Commonly, in neutrino oscillations

H_{n\bar{n}} included in H_I. The sole physical distinction with model (4) lies in description of $\bar{n}$-medium interaction $H$. If it is putted that $H(t) = \int d^3x(-i\Gamma/2)\bar{n}\Psi \Psi \bar{n}$ (what is at least unjustified), then the potential model results are reproduced (see below). Field approach allows one to calculate directly the off-diagonal terms.

Let us $U_n = ReU_{\bar{n}} = 0$, $\Gamma = \Gamma_\beta$, $H = H_\beta$, where $\Gamma_\beta$ and $H_\beta$ are the width and Hamiltonian of free-space decay $\bar{n} \rightarrow \bar{p}e^+\nu$. In this case Eqs.(4) and (13) describe the free-space process (3) in the framework of single-particle model and field approach, respectively. (As with process (2), the neutron decay is excluded. In this connection the free-space process (3) is imaginary one, which is inessential.) Then according to Eq.(10) the process (3) is drastically suppressed: $W_{pot}(t) \approx 4e^2t/\Gamma_\beta$. So in the field approach this process (see Fig.2) should be also suppressed by the decay in a final state, what seems unrealistic.

For Fig.1a (and, therefore, for Fig.2) it was shown [5] that the process probability is $W(t) \approx W_f(t)$. Below this result is derived from consideration of more general process shown in Fig.1b. However, first of all we verify FTA, namely, reproduce the potential model results (5).

4 Calculation

First of all we consider the $n\bar{n}$ transitions with $\bar{n}$ in the final states ($\bar{n}$ are detected). The similar problem takes place in neutrino oscillations. Due to the zero momentum transfer in the vertex corresponding to $H_{n\bar{n}}(t)$ the process amplitude is singular

$$M_s = \epsilon \frac{1}{\epsilon_n - p_n^2/2m - U_n} M \sim 1/0,$$

where $M$ is the amplitude of $\bar{n}$-medium interaction. (The amplitude of potential model is considered in Sec.5.) For solving the problem the approach with finite time interval was proposed [4]. In the lowest order in $\epsilon$ we have

$$<\bar{n}0|U(t) - I|0n> = iT_{\bar{n}n}(t) = (-i) <\bar{n}_p0| \int_0^t dt_\beta H_{n\bar{n}}(t_\beta) + T^{\bar{n}}(t - 0) \int_0^{t_k} dt_\beta H_{n\bar{n}}(t_\beta) |0n_p>,$$

$$T^{\bar{n}}(t - t_\beta) = \sum_{k=1}^\infty (-i)^k \int_{t_\beta}^t dt_1... \int_{t_\beta}^{t_{k-1}} dt_k H(t_1)...H(t_k),$$

where $|0n_p>$ and $|0\bar{n}_p>$ are the states of the medium containing the neutron and antineutron with 4-momenta $p = (\epsilon_n, p_n)$. Taking into account that $H_{n\bar{n}} |0n_p> = \epsilon |0\bar{n}_p>$, we change the order of integration [4] and obtain

$$T_{\bar{n}n}(t) = -\epsilon t - \epsilon \int_0^t dt_\beta iT_{\bar{n}n}(t - t_\beta),$$

$$iT_{\bar{n}n}(\tau) = <\bar{n}_p0| T^{\bar{n}}(\tau) |0\bar{n}_p>,$$

where $\tau = t - t_\beta$. The $\bar{n}$-medium interaction is separated out in block $T^{\bar{n}}_{ii}(\tau)$.

For verification of FTA we calculate $T^{\bar{n}}_{ii}(\tau)$ in the framework of potential model: $V = const$, $H(t) = V(t) = \exp(iH_0t)V \exp(-iH_0t) = V$. We have

$$iT^{\bar{n}}_{ii}(\tau) = U^{\bar{n}}_{ii}(\tau) - 1 = \exp(-iV\tau) - 1.$$

Substituting this expression in Eq.(14) one obtains Eq.(6). This means that FTA should reproduce all the results in neutrino oscillations.
is ignored.) One further important test of FTA (calculation of $T_{ii}(t)$) have been given in [5]. Therefore, the FTA was verified by the example of exactly solvable potential model. It is involved in $\langle \bar{n}_p 0 | T^n(\tau) | 0\bar{n}_p \rangle$ as a special case.

Let us consider the process (2) wherein annihilation products are detected, namely, the $n\bar{n}$ transitions in the nuclear matter (Fig.1a). Here, $\Gamma \sim 100$ MeV and $\bar{n}$ inevitably annihilates. We consider the more general problem. We calculate Fig.1b on the interval $(t/2, -t/2)$. As a result it will be shown that: (a) When $q \to 0$ ($q$ is 4-momenta of particle escaped in the $n\bar{n}$ transition vertex), the result converts to one corresponding to Fig.1a, which is interesting for us. (b) When $q \neq 0$ and $t \to \infty$, the result coincides with S-matrix one. Such scheme allows to verify and study the FTA. (c) The functional structure of $W_{pol}(t)$ is wrong.

Let us consider the imaginary process

$$n \to \bar{n} + \Phi.$$  

For decay to be permissible in vacuum put $m_{\bar{n}} = m - 2m_\Phi$. The corresponding process in nuclear matter is shown in Fig.1b. This is a nearest analogy to the process under study. Instead of Eqs.(13) we have $H_f = H'_{n\bar{n}} + H$,

$$H'_{n\bar{n}}(t) = e^i \int d^3x (\bar{\Psi}_n \Phi^* \Psi_n + H.c.),$$  

(17)

where $H'_{n\bar{n}}$ is the Hamiltonian corresponding to decay (16). (For Fig.1a $H'_{n\bar{n}} \to H_{n\bar{n}}$.) In a manner like the calculation of $T_{ii}(t)$ we have

$$T_b(t) = - < \Phi_q f | T^{\bar{n}}(t) \int_{-t/2}^{t/2} dt_\beta H'_{n\bar{n}}(t_\beta) e^{-\alpha t_\beta} | 0n_p >,$$

(18)

$$T^{\bar{n}}(t) = \sum_{k=1}^{\infty} (-i)^k \int_{-t/2}^{t/2} dt_1 ... \int_{-t/2}^{t_k-1} dt_k H(t_1)...H(t_k) = T \exp(-i \int_{-t/2}^{t/2} dt_1 H(t_1)) - 1.$$  

(19)

Here $< f |$ represents the annihilation products with $(n)$ mesons. Multipliere $\exp(-\alpha | t_\beta |)$, $\alpha > 0$ is introduced for realization of adiabatic hypothesis. One obtains

$$T_b(t) = i e^i \frac{1}{\Delta q - i\alpha} < f | T^{\bar{n}}(t) | 0\bar{n}_{p-q} > NF,$$

(20)

$$F = e^{-\alpha | t_k |} - e^{-\alpha t/2} e^{-i\Delta q(t_k+t/2)},$$  

(21)

$\Delta q = q_0 - 2m_\Phi + (p_n - q)^2/2m_{\bar{n}} - p^2/2m$. Here $| 0\bar{n}_{p-q} >$ is the state of the medium containing the $\bar{n}$ with 4-momenta $p-q$. The normalizing factors of the wave functions of $\bar{n}$ and annihilation mesons are included in $< f | T^{\bar{n}}(t) | 0\bar{n}_{p-q} >$ and the other those in the multiplier $N$.

Nonsingular diagram. - Since $\Delta q \neq 0$ the limit $t \to \infty$ can be considered. Then $F = \exp(-\alpha | t_k |)$ and $T_b$ is the usual S-matrix element. FTA reproduces the S-matrix result, as we set out to prove.

It is easy to estimate the widths corresponding to Fig.1b and free-space decay (16):

$$\Gamma_b \approx e^2 \Gamma/(2\pi^2),$$

$$\Gamma_{free} \approx e^2 m_\Phi/(2\pi),$$

where we have put $m_\Phi/m \ll 1$. The $t$-dependence is determined by exponential decay law

$$W_{b,free} = 1 - e^{-\Gamma_b,free t} \sim \Gamma_b,free t.$$
These formulas will be needed below.

*Singular diagram.* - Let us return to $T_b(t)$ and consider the formal limit $q \to 0$. For $n\bar{n}$ transition Hamiltonian the adiabatic hypothesis should not be used: $\alpha = 0$ (see below). Now factor $F/\Delta q$ is

$$\frac{[1 - (1 - i\Delta q(t/2 + t_k))]\Delta q}{\Delta q} = i \int_{-t/2}^{t_k} dt_\beta.$$  

Changing the integration order [4] and going to interval $(t, 0)$ one obtains

$$T_b(t) = -\epsilon N \int_0^t dt_\beta <f| T_\bar{n}(t - t_\beta) |0\bar{n}_p>. \quad (22)$$

At a point $q = 0$, $H_{n\bar{n}}'(t) = H_{n\bar{n}}(t)$ and $N = 1$. We have

$$T_b(t) \to T_a(t) = -\epsilon \int_0^t d\tau i T_{\bar{n}}^i(\tau),$$

$$iT_{\bar{n}}^i(\tau) = <f| T_\bar{n}(\tau) |0\bar{n}_p>. \quad (23)$$

$T_{\bar{n}}^i(\tau)$ is an exact annihilation amplitude. $T_a(t)$ coincides with the second term of Eq.(14) with the replacement $<i| = <\bar{n}_p0| \to <f|$. This can be considered as a test for $T_a(t)$. (In the Ref.[5] only Fig.1a was calculated. Also it was shown that there is a double counting in the potential model.)

5 **Infrared singularities and S-matrix problem formulation**

The amplitude of potential model obtained by means of $S$-matrix approach is not singular. From microscopic theory standpoint the reason is as follows. Due to zero momentum transfer in the $\epsilon$-vertex this amplitude contains singular propagator of $\bar{n}$. However, it also contains block $T_{\bar{n}}^i$ which is a sum of zero angle rescattering diagrams of $\bar{n}$. As a result the self-energy part $\Sigma = V$ appears (see Eq.(22) of Ref.[5]). We are interesting in off-diagonal matrix elements which do not contain above mentioned sum ($T_{\bar{n}}^i$ instead of $T_{\bar{n}}^i$) and hence diverges, because one singular propagator after $\epsilon$-vertex appears in any case. (Note, that formal sum of series over $\epsilon$ gives meaningless self-energy part $\Sigma \sim 1/0$.)

FTA is infrared-free. It naturally connected with conditions of experiment. Really, measurement of any process corresponds to some interval $\tau$. So it is necessary to calculate $U_{fi}(\tau)$. The replacement $U(\tau) \to S(\infty)$ is justified when the main contribution gives some region $\Delta < \tau$, so that

$$U_{fi}(\tau > \Delta) = U_{fi}(\infty) = U_{fi} = S_{fi} = \text{const.} \quad (24)$$

The expressions of this type are the basis for all $S$-matrix calculations. In principle, the three cases are possible.

1. There is bound to be asymptotic regime, however it is not achieved automatically. Then the adiabatic hypothesis is used and usual scheme realizing in the field theory or non-stationary theory of scattering takes place. Fig.1b corresponds to this case.

2. There is no asymptotic regime. Example is provided by oscillation Hamiltonian $H_{n\bar{n}}(t)$. $S$-matrix approach is inapplicable. We have usual non-stationary problem. Because of this, for Fig.1a the result has been obtained in the framework of FTA. In the vertex corresponding to $H_{n\bar{n}}(t)$ the adiabatic hypothesis was not used.
3. For the interaction Hamiltonian $H_f(t)$ the asymptotic is reached automatically without resort to adiabatic hypothesis. An example of this type is $T_{ii}^\alpha(\tau): T_{ii}^\alpha(\tau \gg 1/\Gamma) = i$.

For non-diagonal matrix elements the asymptotic should be reached as well:

$$T_{fi}^\alpha(\tau > 1/\Gamma) = T_{fi}^\alpha = \text{const.}$$ (25)

Really, annihilation of $\bar{n}$ in nuclear matter can be considered as a system decay with the characteristic time $\sim 10^{-23}$ s. On the other hand the observation time $t = T_0 \sim 1$ yr [6] is far more then that in the experiments on particle decays. So in our case expression (24) is more obvious then for any free-space reaction or decay. Recall that all calculations in the particle physics based on condition (24). (For process (3) it has a form $T_{fi}^\beta = \text{const}$, where $T^\beta$ is matrix of $\beta$-decay.) Besides, for $T_{ii}^\alpha$ it was verified by direct calculation.

Since $t \gg 1/\Gamma$, by means of (25) one obtains

$$T_a(t) \approx -ict T_{fi}^\alpha.$$ (26)

$T_{fi}^\alpha$ can be calculated in the framework of $S$-matrix theory. However, in our case we know that the $\bar{n}$-nucleus decay probability is $W^\alpha = \sum_{f \neq i} |T_{fi}^\alpha|^2 = 1$. Finally

$$W_{ann}(t) \approx \sum_{f \neq i} | -ict T_{fi}^\alpha |^2 = \epsilon^2 t^2 W^\alpha = \epsilon^2 t^2 = W_f.$$ (27)

Due to the annihilation channel $n\bar{n}$ conversion is practically unaffected by the medium. So $\tau_{\bar{n}n} \sim T_{\bar{n}n}$, where $T_{\bar{n}n}$ is the oscillation time of neutron bound in a nucleus. The t-dependence of $W_{ann}$ is determined by that of more slow subprocess of the $n\bar{n}$ conversion $W_f \sim t^2$. Formally, quadratic dependence follows from expression for $T_a(t)$.

All the results have been obtained by means of formal expansions. They are valid at any finite $t$. Consequently, the singularity of $S$-matrix amplitude $M_s$ is a direct result of incorrectness of the formulation of the $S$-matrix problem. If $t \to \infty$ Eq.(26) diverges just as $M_s$ does. So infrared singularities point to the fact that there is no asymptotic regime. Conversely, the analysis of condition (24) suggests the method for handling. The lack of asymptotic leads to FTA which is infrared-free. (There is no asymptotic regime for free-space $K^0\bar{K}^0$-oscillations as well. In our opinion, it makes sense to look at the calculation of $\Delta m = m_L - m_S$ (GIM mechanism) from this standpoint.)

6 Discussion

From Eq.(10) it is seen that:

1. $W_{pot} \sim 1/\Gamma$, whereas Fig.1 gives the inverse dependence $| T_{a,b} |^2 \sim | T_{fi}^\alpha |^2$ (see Eqs.(20), (26)). Such structure is typical and for $f \neq i$ uniquely determined.

2. The $t$-dependence of the process probability in the medium and vacuum is different: $W_{pot} \sim t$, $W_f \sim t^2$. It is beyond reason to such fundamental change. In our calculation the $t$-dependence is the same: when $q = 0$ (Fig.1a), $W_{ann} \sim t^2$ and $W_f \sim t^2$; when $q \neq 0$ (Fig.1b), $W_b \sim t$ and $W_{free} \sim t$.

3. In the medium the $n\bar{n}$ transition suppressed by a factor $R_{pot} = W_{pot}/W_f = 4/\Gamma t$. If $t = T_0 = 1.3$ yr [6] ($T_0$ is observation time in proton-decay type experiment) and $\Gamma = 100$ MeV then $R_{pot} \sim 10^{-31}$ which seems absolutely unreal. The distribution (27) gives $R_a \sim 1$. For related problems typical suppression factor is $R \sim 1$. The process (16) suppressed by a factor
$R_b = \Gamma_b/\Gamma_{\text{free}} \approx 1/\pi$, where the value $m_b = \Gamma \sim 100$ MeV was used. The realistic example is a pion production $pn \rightarrow pp\pi^-$ in vacuum and on neutron bound in nucleus. When $\epsilon_x$ is in the region of resonance, the pion absorption due to interaction in the final state is very strong. This effects on the number of pions emitted from the nucleus, but not on the fact of pion formation inside nucleus, which is interesting for us.

(4) By means of diagram technique it is easy to get the $S$-matrix amplitude of potential model $M_{\text{pot}}$. From the expression for $M_{\text{pot}}$ (see Eq.(22) of Ref.[5]) it is seen that there is a double counting with respect to $H$.

Points (1)-(4) suggest that the functional structure of $W_{\text{pot}}$ is wrong. When annihilation decay probability of $\bar{n}$-nucleus $W^n(\tau) = \sum_{f \neq i} |T^n_{fi}(\tau)|^2 \rightarrow 1$, the error increases up to 100 percent resulting in change of functional structure: $W_{\text{ann}} \sim t^2 \rightarrow W_{\text{pot}} \sim t$. As $W^n(\tau) \rightarrow 0$, that is $Im U_{\bar{n}} \rightarrow 0$, the error also tends to zero and Eqs.(4) give an exact result.

Solving Eqs.(4) by method of Green functions one can get

$$T_{ni}(t) = i\epsilon \int_0^t dt_1 \int_0^{t_1} d\tau U^n_{ii}(\tau),$$

$$T_{ni}(t) = -\epsilon \int_0^t d\tau U^n_{ii}(\tau),$$

(28)

where $U^n_{ii}(\tau)$ is defined by Eq.(15). It is seen that the main contribution gives the region $\tau < 2/\Gamma$. However, in this region it is meaningless to speak about potential $U_{\bar{n}}$.

(The self-energy part $\Sigma = V$ is due to of zero angle rescattering of $\bar{n}$. Such scheme is artificial, because $\sigma_{\text{ann}}^{nn} > \sigma_{elN}^{nn}$ and in the first act of $\bar{n}$-medium interaction annihilation takes place. Also recall that for sufficiently early times the exponential decay law is violated [7].)

One might hope that correct result can be provided by appropriate parametrization. However, $U_{\bar{n}}$ has been fitted to radically different problem. Really, the coupled Eqs.(4) give rise to the following equation:

$$(\partial_t^2 + i\partial_t (V + 2H_0) - H_0^2 - H_0 V + \epsilon^2)\Psi_n = 0.$$  

(29)

$\Psi_n$ is suffice to get $W_{\text{pot}}$. Meantime, $U_{\bar{n}}$ is fitted to $\bar{p}$-atom and low energy scattering data. These problems are described by stationary equations of Schrodinger type

$$(\nabla^2/2m + i\Gamma/2)\Psi_{\bar{n}} = -E\Psi_{\bar{n}}.$$  

(30)

The distinctions between Eqs.(29) and (30) are obvious. Eq.(29) is even not Schrodinger type. It describes $n$ rather then $\bar{n}$. For Eq.(29) the parameter $U_{\bar{n}}$ is uncertain; the physical sense of $Im U_{\bar{n}}$ is not clear. The $\Gamma$-dependence of the results is inverse: $W_{\text{pot}} \sim 1/\Gamma$, whereas Eq.(30) gives $W \sim 1 - \exp(-\Gamma t)$. The structure $\int_0^t d\tau U^n_{ii}(\tau)$ (see Eqs.(28)) appears only in the oscillation problem. It is necessary to know $U^n_{ii}(\tau)$ at a scale $\tau \sim 1/\Gamma$. So it is beyond reason to use $U_{\bar{n}}$ in the Eqs.(29),(4), because it was fitted to absolutely different problem.

Let us explain this point. Fitting potential $U_{\bar{n}}$ we fit the matrix elements associated with Eq.(30). However, these matrix elements differ radically from those involved in Eqs.(7),(8). For example, in the Born approximation the zero angle $\bar{p}$-nucleus scattering amplitude is $f(0) \sim \int d^3x U_{\bar{n}}(x)$. (Compare with Eqs.(28).) Thus from a pure phenomenological standpoint $T_{ni}(t)$ is uncertain as well. It can be calculated only by means of correct model with Hermitian operators.

For $\bar{n}$ in nuclear matter the $t$-dependent equation is

$$(i\partial_t - m + i\Gamma/2)\Psi_{\bar{n}} = 0,$$  

(31)

The realistic example is

and in the first act of $\bar{n}$-medium interaction annihilation takes place. Also recall that for sufficiently early times the exponential decay law is violated [7].)
\[ \Psi_n(0) = 1. \] From the condition of probability conservation we have \( W_{ann} = 1 - |\Psi_n|^2 = 1 - \exp(-\Gamma t). \) \( \Gamma \) is extracted from experiment. When \( \Gamma \) increases, \( W_{ann} \) increases as well; so it will also be for oscillation problem. The potential model gives inverse tendency (10). In the problem corresponding to Eq.(31) the value of \( \Gamma \) affects the branching ratio of channels only.

Taking into account above given analogy with process (3) we conclude: when \( ImU_n \neq 0, W_{pot} \) is wrong. The scheme based on Eqs.(12),(7) is improper. The parametrization \( \hat{U}_n \sim -i\Gamma/2 \) is very useful for Eqs.(30),(31) i.e., for the problems with prepared \( \bar{n} \). However, it is incompatible with a realistic \( \bar{n} \)-nuclear dynamic characteristic of \( n\bar{n} \) conversion.

Finally, we point to an important difference between calculations. Since the asymptotic of \( U^\bar{n}_{ii}(\tau) \) is \( U^\bar{n}_{ii}(\tau \gg 1/\Gamma) \to 0, \) the values of \( T_{ii} \) and \( W_{pot} \) are defined by behaviour of \( U^\bar{n}_{ii}(\tau) \) at a scale \( \tau \sim (10^{-24} - 10^{-23}) \text{ s}, \) which is very "undesirably". For off-diagonal matrix elements the picture is inverse: \( T^\bar{n}_{f\neq i}(\tau \gg 1/\Gamma) \to \text{const} \neq 0. \) (Otherwise, the corresponding process probability is \( W^\bar{n}_{f\neq i}(\tau) = |T^\bar{n}_{f\neq i}(\tau)|^2 = 0. \) As a result the main contribution to Eq.(23) gives the region \( 1/\Gamma < \tau < t \) (rather then \( \tau < 2/\Gamma \)), resulting in Eqs.(26),(27). Considering off-diagonal matrix elements we obviate the principal difficulties mentioned above.

In fact the problem is very original. From a view-point of optical potential it is that we deal with the system of time-dependent coupled equations. In consequence of this the results are expressed through the t-dependent matrix elements, which should be calculated beyond the potential model. Hence we come to the field approach. However, the oscillation Hamiltonian corresponds to 2-tail (because of this the single-particle model is used) which leads to infrared singularities. To avoid them the problem is considered on the finite time interval. So we come to the time-dependent description again.

7 Comment on Gal’s paper

In the paper [8] our result was improperly "corrected". The probability to find the annihilation products is \( W_{ann}(t) \approx e^{2t^2W} \) (see Eq.(27)), where \( W^n \) is the \( \bar{n} \)-nucleus decay probability \( W^n = 1 - \exp(-\Gamma t) \approx 1; e^{2t^2} \) is the free-space \( n\bar{n} \) transition probability. "Correction" is that instead of \( W^n \) the probability to find \( \bar{n} \) (Eq.(6)) is substituted. As a result \( W_{ann}(t) \approx e^{2t^2W/\Gamma} \). (See Eqs.(31)-(33) of Ref.[8] II) We read: The probability to find annihilation products=(the \( n\bar{n} \) transition probability)×(the probability to find \( \bar{n} \)-nucleus). Obviously, this "correction" is only trivial error. The rest of the "results" [8] are known since 1980. For example, Eq.(27) of Ref.[8] (the final result) coincides with Eq.(2) of Ref.[5] (the beginning of this paper). In the Ref.[8] the Eqs. (4) are solved. This solution is known since Eiler’s time. The essence of the problem is in the description of the dynamic of the process, i.e., in approximation (12).

In the abstract [8] we read:"...within a simple model which respects unitarity". Unitarity within a model with "anti-Hermitian" operator \( U_n = -i\Gamma/2 \)?

Also Gal writes: ”Dover, Gal, Richard [9] pinpointed errors in Ref.[4].” In fact, the situation is inverse. The simple calculation given in [9] confirms the FTA and was already done in our first paper [4] (zero angle rescattering diagrams of \( \bar{n} \)) as well as in Ref.[5] (see Pgs. R1884, R1885). Since the authors [8,9] “overlooked” all of this we cite here only one paragraph from Ref.[5]: ”The authors [9] substitute \( H = -i\Gamma/2 \) in Eq.(4) of Ref.[5] and obtain \( W_{pot} \). On the basis of this and only this they refute the result of Ref.[4]. In other words they refute our result because it differs from the potential model one.”

Unusual logic. The procedure mentioned above is a verification of FTA by the example of exactly solveble (but incorrect) potential model. We abandon the approximation (12), i.e., the
potential description of $\bar{n}$-medium interaction in principal, what was clearly pointed in [4,5].

8 On an enhancement of oscillations

In the region $\Gamma t \gg 1$

$$W_{\text{ann}}(t)/W_{\text{pot}}(t) \sim -\text{Im} V t \sim \Gamma t \gg 1$$

what means an enhancement of oscillations as compared with standard result. Certainly, this is a record value. For other problems an enhancement factor will provoke a lesser skepticism. In fact the suppression factor $R_{\text{pot}} \sim 10^{-31}$ is surprising.

When $\Gamma t < 1$, annihilation products as well as $\bar{n}$ in the final state can be detected. In this case condition (24) is not fulfilled. Nevertheless, the qualitative picture remains as before: in the standard approach $\text{Im} U_{\bar{n}}$ leads to suppression of oscillations, in our one this is not the case.

The presence of open channels of $\bar{n}$-medium interaction (which are described by $\text{Im} U_{\bar{n}}$ in the potential model) do not lead to suppression of oscillations.

For large part of actual problems the both approaches give an identical result. For two-step processes of the type (1), when $b$-particle absorption is essential, the principal disagreement takes place. We do not want to make the categorical conclusions. Nevertheless, two points are obvious: (1) The potential approach should be considered only as starting point. (2) The only assumption used in our calculation is expression (24). However, this condition is basic to the all $S$-matrix calculations.

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**Figure caption**

Fig.1. (a) $n\bar{n}$ transition in a medium followed by annihilation; (b) The same as in Fig.(a), but with escaping of particle in the $n\bar{n}$ transition vertex.

Fig.2. Free-space process $n \to \bar{n} \to \bar{p}e^+\nu$. 


