Quantum control gates with weak cross-Kerr nonlinearity

Qing Lin

College of Information Science and Engineering,
Huaqiao University (Xiamen), Xiamen 361021, P.R.China.

Jian Li
Department of Physics, Southeast University, Nanjing 211189, P. R. China

Abstract

In this paper, with the weak cross-Kerr nonlinearity, we first present a special experimental scheme called controlled-path gate with which the realization of all possible bipartite positive-operator-value measurements of two-photon polarization states may be nearly deterministic. Following the same technique, the realization of quantum control gates, including the controlled-NOT gate, Fredkin gate, Toffoli gate, arbitrary controlled-U gate, and even arbitrary multi-controlled-U gate, are proposed. The corresponding probabilities are 1/2, 1/8, 2/23, etc. respectively. Only the coherent states are required but not any ancilla photons, and no coincidence measurement are required which results in these gates are scalable. The structures of these gates are very simple, and then we think they are feasible with the current experimental technology in optics.

PACS numbers: 03.67.Lx, 42.50.Ex

*Electronic address: qlin@mail.ustc.edu.cn
I. INTRODUCTION

In the quantum computation, quantum control gates play a very important role. It was proven that two-qubit unitary gates and single-qubit gates are sufficient for universal quantum computation [1]. In linear optics, many schemes are provided for the realization of two-qubit unitary gates, for example, controlled-NOT (CNOT) gates [2] or controlled-phase gates [3]. However, some of these gates work on the coincidence basis which results in these gates are not scalable, i.e., these gates can not be used to realize multi-qubit gates and then the universal computation. Moreover, all these gates are probabilistic which result in the probability of the realization of universal computation may be tiny, for the reason that many two-qubit unitary gates required. For example, quantum Fredkin gate can be constructed by five CNOT gates and some single-qubit gates [4], and the probability of CNOT gate is only 1/4 in linear optics [2], then the probability of Fredkin gate is \(4^{-5} = 9.8 \times 10^{-4}\). To avoid the inefficient, more efficient even deterministic gates must be looked for. Fortunately, with the weak cross-Kerr nonlinearity, a parity projector [5] and a deterministic CNOT gate [6] had been proposed, and then the universal computation can be realized deterministic in principle. However, the universal computation and even a multi-qubit gate may be need too many CNOT gates, then the structure may be too complex to be realized in optics. Alternatively, it is interesting to look for some multi-qubit gates with simple structure even though the probability is not unit. In this paper, we will present the quantum control gates with very simple structure, and we think these gates may be more feasible with the current experimental technology.

This paper is organized as follows. In sec.II, we first propose a scheme of a gate we call it controlled-path (C-path) gate with the weak cross-Kerr nonlinearities, and then use this gate to realize all possible bipartite positive-operator-value measurements (POVMs) of two-photon polarization states. In addition, this technique is developed to realize the CNOT gate, Fredkin gate, Toffoli gate, controlled-U (CU) gate and even multi-controlled-U (MCU) gate. Sec.III is for conclusion remarks.
II. QUANTUM CONTROL GATE

Before we outline our schemes of quantum control gates, we briefly review the useful weak cross-Kerr nonlinearity which has been used in Refs. [5, 6, 7, 8]. Suppose a non-linear weak cross-Kerr interaction between a signal state (photonic qubit) $|\psi\rangle = c_1 |0\rangle + c_2 |1\rangle + c_2 |2\rangle$ and a coherent state $|\alpha\rangle$. After the evolution, the output state is,

$$|\psi\rangle |\alpha\rangle \rightarrow c_1 |0\rangle |\alpha\rangle + c_2 |1\rangle \left|\alpha e^{i\theta}\right\rangle + c_2 |2\rangle \left|\alpha e^{2i\theta}\right\rangle,$$

where $\theta$ is induced by the nonlinearity. Through a general homodyne-heterodyne measurement of the phase of the coherent state, the signal state $|\psi\rangle$ will be projected into a definite number state or superposition of number states. Because the measurement can be performed with high fidelity, the projection is nearly deterministic. This technique has first been used to realize a parity projector [5], and then a CNOT gate [6]. It provides a new route to new quantum computation [7]. The requirement for this technique is $\alpha \theta > 1$ [7], where $\alpha$ is the amplitude of the coherent state. Even with the weak nonlinearity ($\theta$ is small), this requirement can be satisfied with large amplitude of the coherent state, then this requirement may be feasible with current experimental technology. Our schemes of quantum control gates also work with the weak cross-Kerr nonlinearity.

A. C-path gate

Firstly, we discuss the C-path gate. Here, we use the polarization of photons as qubit and define the horizontally (vertically) linear polarization $|H\rangle$($|V\rangle$) as the qubit $|0\rangle$($|1\rangle$). Consider a two-qubit initially prepared in the state $|\Psi\rangle = \alpha |H\rangle_1 |H\rangle_2 + \beta |H\rangle_1 |V\rangle_2 + \gamma |V\rangle_1 |H\rangle_2 + \delta |V\rangle_1 |V\rangle_2$, where $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. In a C-path gate, the paths of the first photon are controlled by the second photon. The experimental setup is shown in Fig.1. The control photon is transmitted through a balanced Mach-Zehnder (M-Z) interferometer formed by two polarizing beam splitters (PBS$_1$, PBS$_2$) which let the photon $|H\rangle$ be passed and the photon $|V\rangle$ be reflected, while the target photon is injected into a 50:50 beam splitter (BS). The two photons combined with a coherent state $|\alpha\rangle$ interact with the cross-Kerr nonlinearities, such that a phase shift will be induced in the coherent state. Suppose the control photon induces a controlled phase shift $-\theta$, while the target photon induces a
controlled phase shift $\theta$, then the input state $|\Psi\rangle |\alpha\rangle$ will evolve to the follows:

$$
\frac{1}{\sqrt{2}} \left( \alpha |H\rangle_{1}^{S_{1}} |H\rangle_{2} + \beta |H\rangle_{1}^{S_{2}} |V\rangle_{2} + \gamma |V\rangle_{1}^{S_{1}} |H\rangle_{2} + \delta |V\rangle_{1}^{S_{2}} |V\rangle_{2} \right) |\alpha\rangle \\
+ \frac{1}{\sqrt{2}} \left( \alpha |H\rangle_{1}^{S_{2}} + \gamma |V\rangle_{1}^{S_{2}} \right) |H\rangle_{2} |\alpha e^{-i\theta}\rangle + \frac{1}{\sqrt{2}} \left( \beta |H\rangle_{1}^{S_{1}} + \delta |V\rangle_{1}^{S_{1}} \right) |V\rangle_{2} |\alpha e^{i\theta}\rangle,
$$

(2)

where the superscripts $S_{1}, S_{2}$ denote the paths of the first photon. Through a general homodyne-heterodyne measurement (X homodyne measurement), the two-photon state will be projected into the following state,

$$
\alpha |H\rangle_{1}^{S_{1}} |H\rangle_{2} + \beta |H\rangle_{1}^{S_{2}} |V\rangle_{2} + \gamma |V\rangle_{1}^{S_{1}} |H\rangle_{2} + \delta |V\rangle_{1}^{S_{2}} |V\rangle_{2}.
$$

(3)

Here we only retain the case that no phase shift induced in the coherent state, and the success probability is $P_{\text{succ}}^{C_{P}} = 1/2$. If a switch (S) which will exchange the two photons and a phase shift conditionally controlled by the homodyne detection through a classical feedforward are applied, this C-path gate is nearly deterministic, i.e., $P_{\text{succ, max}}^{C_{P}} = 1$. By the same way, one can implement a multi-controlled-path gate in which multiple qubits control the paths of the other qubits.

This C-path gate is very useful in the quantum computation for the reason that many quantum control gates (for example, CNOT gate, Fredkin gate, etc.) can be realized by some operations performed in the different paths of the target photons. These schemes of quantum control gates will be discussed in the following. Now we discuss the first use of this control-path gate. If we place a half wave plate (HWP, set at 22.5°-Hadamard gate) in the path of the control photon which is shown in the dashed line of the Fig.1, the following state can be achieved,

$$
\frac{1}{\sqrt{2}} \left( \alpha |H\rangle_{1}^{S_{1}} + \beta |H\rangle_{1}^{S_{2}} + \gamma |V\rangle_{1}^{S_{1}} + \delta |V\rangle_{1}^{S_{2}} \right) |H\rangle_{2} + \frac{1}{\sqrt{2}} \left( \alpha |H\rangle_{1}^{S_{2}} - \beta |H\rangle_{1}^{S_{1}} + \gamma |V\rangle_{1}^{S_{2}} - \delta |V\rangle_{1}^{S_{1}} \right) |V\rangle_{2}.
$$

(4)

If the detection of the control photon infers its polarization is $|H\rangle$, the initial state $|\Psi\rangle$ has been transferred onto the following state of a single photon in the Hilbert space of its polarization and path states,

$$
|\Phi\rangle = \alpha |HS_{1}\rangle + \beta |HS_{2}\rangle + \gamma |VS_{1}\rangle + \delta |VS_{2}\rangle.
$$

(5)
The success probability is $P_{\text{succ}}^{CT} = 1/2$. If a classical feedforward phase shift $\pi$ is induced to the path $S_2$ when the detection infers the polarization of the control photon is $|V\rangle$, the success probability will increase to 1.

The transformation $|\Psi\rangle \rightarrow |\Phi\rangle$ is crucial for the realization of all possible bipartite POVMs of two-photon polarization states in Ref. [9]. In their scheme, a special three-photon entangled state created by a quantum Fredkin gate and a teleportation process of five photons are required for this transformation. It is evident that our scheme is better than their scheme in the amount of resource, the complexity of the operations, and the great advantage of our scheme is the success probability is nearly unity which makes the realization of all possible bipartite POVMs of two-photon polarization states nearly deterministic.

B. CNOT gate

Secondly, we discuss the CNOT gate. Suppose two photons initially prepared in the state $|\Psi\rangle$, and the CNOT gate can be described by the following transformation,

$$|\Psi\rangle \rightarrow \alpha |H\rangle_1 |H\rangle_2 + \beta |H\rangle_1 |V\rangle_2 + \gamma |V\rangle_1 |V\rangle_2 + \delta |V\rangle_1 |H\rangle_2.$$ (6)

The experimental setup is shown in Fig.2, here the first photon is the control photon which is transmitted through a balanced M-Z interferometer formed by two PBSs (PBS$_1$, PBS$_2$), while the target photon is also transmitted through a balanced M-Z interferometer formed by two BSs (BS$_1$, BS$_2$) whose transmissivity (reflectivity) are $T_1, T_2$ ($R_1, R_2$) respectively. A single-photon operation $\sigma_x$ is performed in one arm. With the cross-Kerr nonlinearities and a X homodyne measurement associated with the classical feedforward, the following states can be achieved in the output,

$$\sqrt{T_1 R_2} (\alpha |H\rangle_1 |H\rangle_2 + \beta |H\rangle_1 |V\rangle_2) + \sqrt{R_1 T_2} (\gamma |V\rangle_1 |V\rangle_2 + \delta |V\rangle_1 |H\rangle_2),$$ (7)

or

$$\sqrt{R_1 T_2} (\alpha |H\rangle_1 |H\rangle_2 + \beta |H\rangle_1 |V\rangle_2) + \sqrt{T_1 R_2} (\gamma |V\rangle_1 |V\rangle_2 + \delta |V\rangle_1 |H\rangle_2).$$ (8)

Compared with the Eq. (6), it is immediately to find that the CNOT operation is completed when the condition $\sqrt{T_1 R_2} = \sqrt{R_1 T_2}$ is satisfied, and the success probability $P_{\text{succ}}^{\text{CNOT}} = 2T_1 R_2$. It is easy to find that the maximum success probability is $P_{\text{succ,max}}^{\text{CNOT}} = 1/2$ when
\[ T_1 = R_2 = 1/2. \] Compared with the scheme proposed by Nemoto et al. [6], our scheme is probabilistic but no ancilla photons are required.

### C. Fredkin gate

Thirdly, we discuss the Fredkin gate which is also called controlled-swap gate. Consider a single photon (control photon) in the state \( |\psi\rangle = \alpha |H\rangle + \beta |V\rangle \) (\( |\alpha|^2 + |\beta|^2 = 1 \)), and two photons (target photons) in the state \( |\phi\rangle = p_1 |\Psi^+\rangle + p_2 |\Psi^-\rangle + p_3 |\Phi^+\rangle + p_4 |\Phi^-\rangle \) (\( \sum_i |p_i|^2 = 1 \)), where \( \{|\Psi^\pm\rangle, |\Phi^\pm\rangle\} \) are the Bell states. A Fredkin gate can be described by the following transformation,

\[
|\psi\rangle |\phi\rangle \rightarrow \alpha |H\rangle \left( p_1 |\Psi^+\rangle + p_2 |\Psi^-\rangle + p_3 |\Phi^+\rangle + p_4 |\Phi^-\rangle \right) \\
+ \beta |V\rangle \left( p_1 |\Psi^+\rangle - p_2 |\Psi^-\rangle + p_3 |\Phi^+\rangle + p_4 |\Phi^-\rangle \right),
\]

that is, if the control photon is in the state \(|H\rangle\), the target two photons are unchanged; while the control photon is in the state \(|V\rangle\), a swap operation is implemented to the target two photons. For the reason that, only the singlet state \(|\Psi^-\rangle\) is antisymmetric while the other three Bell states are symmetric, the swap operation only results in a phase shift \( \pi \) to the state \(|\Psi^-\rangle\) while the other states unchanged. Our scheme of the Fredkin gate is shown in Fig.3. The control photon is transmitted through a balanced M-Z interferometer formed by two PBSs (PBS1, PBS2), while the two target photons are transmitted through a balanced M-Z interferometer formed by two BSs (BS1, BS2 or BS3, BS4) whose transmissivity (reflectivity) are \( T_1, T_2 \) or \( T_3, T_4 \) (\( R_1, R_2 \) or \( R_3, R_4 \)) respectively. In addition, a balanced M-Z interferometer (in the dashed line of Fig.3) formed by two BSs (BS5, BS6) associated with a phase shift \( \pi \) in one arm is required. The Hong-Ou-Mandel interference in this M-Z interferometer yields the following transformation [14]:

\[
|\Psi^-\rangle \rightarrow -|\Psi^-\rangle; \quad |\Psi^+\rangle \left( |\Phi^\pm\rangle \right) \rightarrow |\Psi^+\rangle \left( |\Phi^\pm\rangle \right).
\]

Compared with the above two schemes, we change the phase shift induced by the control photon to be \(-2\theta\), while the phase shift is \( \theta \) for the two target photons. If the cross-Kerr nonlinearities are used and we retain the case that no phase shift induced in the coherent state, we will achieve the following state in the output:
\[
\sqrt{T_1R_2R_3T_4} \alpha |H\rangle (p_1 |\Psi^+\rangle + p_2 |\Psi^-\rangle + p_3 |\Phi^+\rangle + p_4 |\Phi^-\rangle) \\
+ \sqrt{R_1T_2T_3R_4} \beta |V\rangle (p_1 |\Psi^+\rangle - p_2 |\Psi^-\rangle + p_3 |\Phi^+\rangle + p_4 |\Phi^-\rangle).
\]

(11)

Compared with the Eq.(9), the Fredkin gate is realized when the condition \(\sqrt{T_1R_2R_3T_4} = \sqrt{R_1T_2T_3R_4}\) is satisfied, then the success probability is \(P_{\text{succ}}^{\text{Fredkin}} = T_1R_2R_3T_4\). Hence the maximum success probability is \(P_{\text{succ}}^{\text{Fredkin}} = 1/16\) when \(T_1 = R_2 = R_3 = T_4 = 1/2\). Moreover, if a M-Z interferometer which is same to the M-Z interferometer in the dashed line associated with a phase shift \(\pi\) conditionally controlled by the homodyne detection (the phase of the coherent state is \(\pm 2\theta\)) through a classical feedforward, is implemented in the outputs of BS_2 and BS_4, the probability may be \(P_{\text{succ, max}}^{\text{Fredkin}} = 1/8\).

Now we compare our scheme of Fredkin gate with the previous schemes. In 1989, Milburn used the cross-Kerr nonlinearities to realize the Fredkin gate [10], however, its cross-Kerr nonlinearities operate in single photon level, so it requires huge nonlinearities which is a great challenge for the current experimental technology. In linear optics, two types of Fredkin gate, heralded gate and post-selected gate, had been proposed [11, 12, 13]. Exclusive of the requirement of ancilla photons and small probability, the shortcomings of these gates are obvious. The heralded Fredkin gates require single-photon detectors which is also a great challenge for the current technology, and the post-selected Fredkin gates work on the coincidence basis which results in these gates are not scalable. Compared with these schemes, only the coherent states are required in our scheme, and the structure is so simple that we think it is feasible with the current technology.

D. Toffoli gate, CU gate and MCU gate

A little change that a CNOT gate or arbitrary two-qubit unitary gate replaces the setups in the dashed line of the Fig.3, associated with appropriate transmissivities of the four beam splitters, is enough for the realization of the Toffoli gate or the CU gate. In the following, we calculate the probability of Toffoli gate and the CU gate. For the Toffoli gate, two coherent states are required because a CNOT gate is included in this scheme. Consider a single photon (control photon) in the state \(|\psi\rangle = \alpha |H\rangle + \beta |V\rangle (|\alpha|^2 + |\beta|^2 = 1)\), and two photons (target photons) in the state \(|\phi\rangle = q_1 |HH\rangle + q_2 |HV\rangle + q_3 |VH\rangle + q_4 |VV\rangle (\sum_i |q_i|^2 = 1)\). Suppose
that the transmissivities (reflectivities) of the four BSs are \( T_1, T_2, T_3, T_4 \) \( (R_1, R_2, R_3, R_4) \) respectively, now the modified scheme of the Fredkin gate will evolve the initial state \( |\psi\rangle |\phi\rangle \) to the follows (here we also retain the case that no phase shift induced in the coherent state),

\[
\sqrt{T_1 R_2 R_3 T_4} |H\rangle (q_1 |HH\rangle + q_2 |HV\rangle + q_3 |VH\rangle + q_4 |VV\rangle) \\
+ \frac{1}{\sqrt{2}} \sqrt{R_1 T_2 T_3 R_4} |V\rangle (q_1 |HH\rangle + q_2 |HV\rangle + q_3 |VV\rangle + q_4 |VH\rangle),
\]

where the coefficient \( 1/\sqrt{2} \) is induced by the CNOT gate. The Toffoli gate is completed when the condition \( \sqrt{T_1 R_2 R_3 T_4} = \frac{1}{\sqrt{2}} \sqrt{R_1 T_2 T_3 R_4} \) is satisfied. The success probability is \( P_{\text{Toffoli}}^{\text{succ}} = T_1 R_2 R_3 T_4 \). Choose \( T_1 = R_2 = R_3 = T_4 = \frac{1}{\sqrt{2}+1} \), the success probability may be \( P_{\text{Toffoli}}^{\text{succ}} = \frac{1}{23} \). Similarly, a CNOT gate conditional controlled by the homodyne detection (the phase of the coherent state is \( \pm 2\theta \)) through a classical feedforward is implemented in the outputs of BS\(_2\) and BS\(_4\), the probability may be \( P_{\text{Toffoli}}^{\text{succ,max}} = \frac{2}{23} \).

In linear optics, two types of Toffoli gate, heralded gate and post-selected gate, had been proposed \[13, 15\]. Similarly, exclusive of the requirement of ancilla photons and small probability, the uses of single-photon detectors and the coincidence measurement limit their use in the universal computation. These shortcomings are not exist in our scheme, and the simple structure makes it much feasible with current technology.

The realization of CU gate is similar, and the success probability is determined by the probability of the arbitrary unitary gate (suppose as \( 1/p \)) which can be realized by some CNOT gates and single-qubit gates, and the transmissivities of the four beam splitters. The condition for the CU gate is \( \sqrt{T_1 R_2 R_3 T_4} = \frac{1}{\sqrt{p}} \sqrt{R_1 T_2 T_3 R_4} \). Also choose \( T_1 = R_2 = R_3 = T_4 = \frac{1}{\sqrt{p}+1} \), the success probability of CU gate may be \( P_{\text{CU}}^{\text{succ}} = \left( \frac{1}{\sqrt{p}+1} \right)^4 \), and it may be \( P_{\text{CU}}^{\text{succ,max}} = 2 \left( \frac{1}{\sqrt{p}+1} \right)^4 \) with some additional setups similar to the Toffoli gate. In addition, it is straightforward to develop this technique to the realization of MCU gate which is shown in Fig.4. The realization is described in the following. The control photons are all transmitted through a balanced M-Z interferometer formed by two PBSs respectively, while the target photons are all transmitted through a balanced M-Z interferometer formed by two BSs respectively. Next, similar to the setups in the dashed line of Fig.3, in one arm of all the M-Z interferometers form by the BSs, we implement a multi-qubit unitary gate which can be realized by the quantum control gates described above. Assisted by some coherent states and the weak cross-Kerr nonlinearity, the MCU gate can be realized associated with
the appropriate transmissivities of the BSs. Compared with the realization of MCU gate with many CNOT gates and single-qubit gates, our scheme can reduce the complexity of the realization greatly.

III. CONCLUSION

In this paper, with the weak cross-Kerr nonlinearity, we first present a special experimental scheme called C-path gate with which the realization of all possible bipartite POVMs of two-photon polarization states can be simpler and nearly deterministic. Following the same technique, the schemes of the realization of quantum control gates have been proposed, including the CNOT gate (1/2), Fredkin gate (1/8), Toffoli gate (2/23), CU gate and even MCU gate. All these gates are scalable with the certain probabilities which are larger than those gates in linear optics. Less resource are required and the structures of these gates are so simple that we think they are feasible with current technology and may be useful for the realization of universal computation in optics.

Acknowledgments

The author would like to thank Dr Pieter Kok and Bill Munro for their helpful discussions. The author Qing Lin was funded by the HuaQiao University Foundation, China (Grant No. 07BS406).

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge) (2000).

[2] T. B. Pittman, M. J. Fitch, B. C Jacobs and J. D. Franson, Phys. Rev. A, 68, 032316 (2003); J. L. O’Brien, G. J. Pryde, A. G. White, T. C. Ralph and D. Branning, Nature (London), 426, 264 (2003); Z. Zhao, A. N. Zhang, Y. A. Chen, H. Zhang, J. F. Du, T. Yang and J. W. Pan, Phys. Rev. Lett. 94, 030501 (2005); R. Okamoto, H. F. Hofmann, S. Takeuchi and K. Sasaki, Phys. Rev. Lett. 95, 210506 (2005); X. H. Bao, T. Y. Chen, Q. Zhang, J. Yang, H. Zhang, T. Yang and J. W. Pan, Phys. Rev. Lett. 98, 170502 (2007).
[3] N. K. Langford, T. J. Weinhold, R. Prevedel, K. J. Resch, A. Gilchrist, J. L. O'Brien, G. J. Pryde and A. G. White, Phys. Rev. Lett. 95, 210504 (2005); N. Kiesel, C. Schmid, U. Weber, R. Ursin and H. Weinfurter, Phys. Rev. Lett. 95, 210505 (2005).

[4] J. A. Smolin and D. P. DiVincenzo, Phys. Rev. A, 53, 2855 (1996).

[5] S. D. Barrett, P. Kok, K. Nemoto, R. G. Beausoleil, W. J. Munro and T. P. Spiller, Phys. Rev. A, 71, 060302(R) (2005).

[6] K. Nemoto and W. J. Munro, Phys. Rev. Lett. 93, 250502 (2004).

[7] T. P. Spiller, K. Nemoto, S. L. Braunstein, W. J. Munro, P. van Loock and G. J. Milburn, New J. Phys. 8, 30 (2006).

[8] P. Kok, Phys. Rev. A 77, 013808 (2008).

[9] S. E. Ahnert and M. C. Payne, Phys. Rev. A 73, 022333 (2006).

[10] G. J. Milburn, Phys. Rev. Lett. 62, 2124 (1989).

[11] Y. X. Gong, G. C. Guo and T. C. Ralph, Phys. Rev. A, 78, 012305 (2008).

[12] J. Fiurášek, Phys. Rev. A, 78, 032317 (2008).

[13] J. Fiurášek, Phys. Rev. A, 73, 062313 (2006).

[14] C. K. Hong, Z. Y. Ou and L. Mandel, Phys. Rev. Lett. 59, 2044 (1987).

[15] T. C. Ralph, K. J. Resch and A. Gilchrist, Phys. Rev. A 75, 022313 (2007).
FIG. 1: Controlled-path gate with weak cross-Kerr nonlinearity. Assisted by the switch (S) and the phase shift conditional controlled by the homodyne detection through a classical feedforward, this gate is nearly deterministic. If the setups in the dashed line are used, this gate can be used to realize all possible bipartite positive-operator-value measurements of two-photon polarization states nearly determinately.
FIG. 2: CNOT gate with the weak cross-Kerr nonlinearity. Assisted by a classical feedforward, this gate can be implemented with the probability $1/2$. 
FIG. 3: Fredkin gate with the weak cross-Kerr nonlinearities. The setups in the dashed line will complete the transformation $|\Psi^-\rangle \rightarrow -|\Psi^-\rangle$; $|\Psi^+\rangle$ ($|\Phi^\pm\rangle$) $\rightarrow$ $|\Psi^+\rangle$ ($|\Phi^\pm\rangle$). Associated with the nonlinearities and appropriate transmissivities of four beam splitters, the Fredkin gate is realized with the probability 1/16. If some additional setups are used, the probability will increase to 1/8. For details, see text.

FIG. 4: Multi-control-U gate.