Influence of temperature correlations on phase dynamics and kinetics of ultrathin lubricant film

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The melting of ultrathin lubricant film is studied by friction between atomically flat surfaces. The fluctuations of lubricant temperature are taken into account defined by Ornstein-Uhlenbeck process. The phase diagrams and portraits are calculated for the cases of second-order and first-order transitions — the melting of amorphous and crystalline lubricants. It is shown that in the first case the stick-slip friction domain appears, dividing the regions of dry and sliding friction. In the second case the three stick-slip friction domains arise characterized by transitions between dry, metastable and stable sliding friction. The increase of correlation time of the lubricant temperature fluctuations leads to increasing of the frictional surfaces temperature needed for realization of sliding friction. The singular point, meeting the mode of dry friction, has complex character of stability. The stationary states, corresponding to the stable and metastable sliding friction, are presented by the focus-type singular points.

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I. INTRODUCTION

The problems of the sliding friction continue to attract a considerable attention due to vast application in engineering of flat solid surfaces divided by thin film of lubricant. It is found experimentally that during the process of friction the liquid film becomes progressively thinner, at first its physical properties change gradually (quantitatively), and then the changes acquire the sharp (qualitative) character. The qualitative changes consist of non-Newtonian mechanism of flow and replacement of the ordinary melting by glass transition, however, the lubricant film continues to behave itself like liquid. In tribology such behaviour is called the "mixed lubrication", which represents the intermediate mode of friction characterized by transition from volume properties of lubricating material to the boundary ones.

The boundary mode of friction, described in the proposed work, is realized in the case of ultrathin lubricant films with thickness less than four diameters of molecules at smooth surfaces or asperities, high loads, and low shear rates. It is characterized by the following changes of static (equilibrium) and dynamic properties of lubricant — simple unstructured Newtonian liquid:

- non-fluidlike (non-Newtonian) properties: transition between liquid and solid phases, appearance of new liquid-crystalline states, epitaxially induced long-range ordering;
- tribological properties: absence of flow until yield point or critical shear stress reached, solidlike behaviour of lubricant characterized by defect diffusion and dislocation motion, shear melting, boundary lubrication.

Experiments with mica, silica, metal oxide, and surfactant monolayer surfaces, between, which organic liquids and aqueous solutions were placed, have shown that there are transformations between the different types of dynamic phases during sliding. They manifest themselves in appearance of intermittent (stick-slip) friction, which is characterized by periodic transitions between two or more dynamic states during the stationary sliding and is the major reason for destruction and wear of frictional elements. Thus, molecularly thin lubricant films undergo more than one type of transition that results to existence of different types of stick-slip motion.

In particular, the experimental investigation of rheological properties of lubricant film were allowed to explain the nature of above described anomalous features of boundary friction. Besides, with this aim the theoretical presentation of lubricating material was used as the viscoelastic matter characterized by heat conductivity. So, the observed phenomenology of ultrathin lubricant film is successfully described in on the basis of viscoelastic matter approximation and Ginzburg-Landau equation, where order parameter defines the shear melting and freezing. The phase diagram is calculated defining the domains of sliding, stick-slip, and dry friction in the plane temperature — film thickness.

Along the line , within the framework of Lorenz model for description of viscoelastic matter the transition of ultrathin lubricant film from solidlike into liquidlike state is shown to occur due to both thermodynamic and shear melting. The cooperative description of these processes is carried out as a result of self-organization of the shear stress and stain, and the lubricant temperature. The additive noises of above quantities are introduced for building the phase diagrams, where fluctuations intensities and frictional surfaces temperature define the domains of sliding, stick-slip, and dry friction. In reference the conditions are found at which the stick-slip friction regime corresponds to the intermittency mode in-
herent in self-organized criticality phenomenon.

However, in spite of the fact that, as a rule, temperature determines the lubricant state, the question remains opened on influence of its correlated fluctuations on the process of friction. Here, it is shown that internal fluctuations of temperature, which are described by the Ornstein-Uhlenbeck process, result in complication of dynamic phase diagram. The shear stress distribution function, the phase diagrams and portraits are calculated for the cases of second-order and first-order transitions — the melting of amorphous and crystalline lubricant (Sects. III and IIB), respectively. The different stick-slip friction modes are predicted to occur. The influence of correlation time value of the studied fluctuations is described on the phase diagram. The investigation of kinetic modes of boundary friction is carried out using the phase plane method. The self-similar phase dynamics and kinetics of lubricant film are investigated.

II. CONTINUOUS TRANSITION

In the previous work on the basis of rheological description of the viscoelastic medium the system of kinetic equations has been obtained, which define the mutually coordinated evolution of the shear components of the stress $\sigma$ and the strain $\varepsilon$, and the temperature $T$ in ultrathin lubricant film during friction between atomically flat mica surfaces. Let us write these equations using the measure units

$$
\sigma_s = \left( \frac{\rho c_v \eta_0 T_c}{\tau_T} \right)^{1/2},
$$

$$
\varepsilon_s = \frac{\sigma_s}{G_0} \left( \frac{\tau_\varepsilon}{\tau_T} \right)^{1/2} \left( \frac{\rho c_v \tau_\varepsilon \varepsilon}{\eta_0} \right)^{1/2},
$$

for variables $\sigma$, $\varepsilon$, $T$, respectively, where $\rho$ is the mass density, $c_v$ is the specific heat capacity, $T_c$ is the critical temperature, $\eta_0 \equiv \eta(T = 2T_c)$ is the typical value of shear viscosity $\eta$, $T_\varepsilon = \rho T^2 c_v / \kappa$ is the time of heat conductivity, $I$ is the scale of heat conductivity, $\kappa$ is the heat conductivity constant, $\tau_\varepsilon$ is the relaxation time of matter strain, $G_0 \equiv \eta_0 / \tau_\varepsilon$:

$$
\tau_\sigma \dot{\sigma} = -\sigma + g \varepsilon,
$$

$$
\tau_\varepsilon \dot{\varepsilon} = -\varepsilon + (T - 1) \sigma,
$$

$$
\tau_T \dot{T} = (T_c - T) - \sigma + \sigma^2 + \lambda(t).
$$

Here the stress relaxation time $\tau_\sigma$, the temperature $T_c$ of atomically flat mica friction surfaces, and the constant $g = G / G_0$ are introduced, where $G$ is the lubricant shear modulus. Replacement of $\varepsilon / \tau_\sigma$ by $\partial \varepsilon / \partial t$ reduces Eq. (2) to the Maxwell-type equation for viscoelastic matter approximation that is widely used in the theory of boundary friction. The relaxation behaviour of viscoelastic lubricant during the process of friction is described also by Kelvin-Voigt equation (3) and (11). It takes into account the dependence of the shear viscosity on the dimensionless temperature $\eta = \eta_0 / (T - 1)$ [12]. Equations (2) and (3) represent jointly the new rheological model. It is worth noting that rheological properties of lubricant film are investigated experimentally for construction of phase diagram [3]. Equation (4) represents the heat conductivity expression, which describes the heat transfer from the friction surfaces to the layer of lubricant, the effect of the dissipative heating of a viscous liquid flowing under the action of the stress, and the reversible mechanic- and calorific effect in linear approximation. Equations (2) – (4) coincide with the synergetic Lorenz system formally [13, 14], where the shear stress acts as the order parameter, the conjugate field is reduced to the shear strain, and the temperature is the control parameter. As is known this system can be used for description of the thermodynamic phase and the kinetic transitions.

The purpose of this work is to study the phase dynamics and kinetics of boundary friction at introduction into Eq. (4) the stochastic source $\lambda(t)$ representing the Ornstein-Uhlenbeck process:

$$
\langle \lambda(t) \rangle = 0, \quad \langle \lambda(t) \lambda(t') \rangle = \frac{I}{\tau_\lambda} \exp\left( -\frac{|t - t'|}{\tau_\lambda} \right),
$$

where $I$ is the temperature fluctuations intensity, $\tau_\lambda$ is the time of their correlation.

Following [15], let us define the physical sense of value $I$. The time correlation of dimensionless temperature of lubricant is determined by the average value of product

$$
\varphi(\tau) = \langle (\Delta T(t) \Delta T(t + \tau) \rangle = \langle (\Delta T)^2 \rangle \exp(-\zeta |\tau|).
$$

Here $1 / \zeta$ fixes the relaxation time for setting of equilibrium. Supposing that in formulas (6) and (7) the moments of time are connected by equality $t + \tau = t'$, we get:

$$
\varphi(t' - t) = \langle \Delta T(t) \Delta T(t') \rangle = \langle (\Delta T)^2 \rangle \exp(-\zeta |t' - t|).
$$

Substituting of average square of temperature fluctuations $\langle (\Delta T)^2 \rangle = T^2 / c_v$ in expression [5] gives

$$
\langle \Delta T(t) \Delta T(t') \rangle = \frac{T^2}{c_v} \exp(-\zeta |t' - t|).
$$

Comparing this formula with the second equality [6], we obtain:

$$
\lambda(t) = \Delta T(t), \quad \tau_\lambda = \frac{1}{\zeta}, \quad I = \frac{T^2}{c_v \zeta}.
$$

Consequently, the value of noise intensity $I$ is defined by the temperature and the heat capacity $c_v$ of lubricant.
At the first glance it can appear, that in the phase diagrams $T_c(I)$ presented further, the every value $I$ meets the unique $T_c$, and instead of regions, there the curve has to exist in pointed out coordinates. The motion along this curve describes the evolution of the system. However, it does not so, since parameter $T_c$ represents the thermostat temperature whose arbitrary change does not determine the temperature $T$ uniquely. It is necessary only to suppose that as a result of system’s self-organization the defined value $T_c$ can meet the manifold of values $T$ and, in accordance with (10), the intensities $I$ in the different moments of time. Besides, the variation of $c_o$ leads to the change of $I$ in the course of time. Thus, it implies the existence of phase diagram. It is possible also to change the intensity $I$ due to the arbitrary choice of parameter $\zeta$ characterizing the concrete system. The last means that the systems exist, where the noise does not influence substantially on their time behaviour, and the ones, where fluctuations influence critically.

However, here we do not restrict ourselves by such approach and understand temperature fluctuations in more wide sense. It is related to that, as a rule, the thermal influence is caused by different external stochastic sources. Besides, it is possible to interpret the noise with the help of fluctuations $T$, as well as of $T_c$. This presentation of noise is often used for its modeling in real systems [10].

In reference [8] a melting of ultrathin lubricant film by friction between atomically flat mica surfaces has been represented as a result of action of spontaneously appearing shear stress leading to the plastic flow. This is caused by the heating of friction surfaces above the critical value $T_{c_0} = 1 + g^{-1}$. Thus, according to such approach the studied solidlike-liquidlike transition of lubricant film occurs due to both thermodynamic and shear melting. The initial reason for this self-organization process is the positive feedback of $T$ and $\sigma$ on $\varepsilon$ [see Eq. (3)] conditioned by the temperature dependence of the shear viscosity leading to its divergence. On the other hand, the negative feedback of $\sigma$ and $\varepsilon$ on $T$ in Eq. (4) plays an important role since it ensures the system stability.

According to this approach the lubricant represents a strongly viscous liquid that can behave itself similar to the solid — has a high effective viscosity and still exhibits a yield stress [2, 11]. Its solidlike state corresponds to the shear stress $\sigma = 0$ because Eq. (2), describing the elastic properties at steady state $\partial\sigma/\partial t = 0$, falls out of consideration. Equation (5), containing the viscous stress, reduces to the Debye law describing the rapid relaxation of the shear strain during the microscopic time $\tau_\varepsilon \approx a/c \sim 10^{-12}$ s, where $a \sim 1$ nm is the lattice constant or the intermolecular distance and $c \sim 10^3$ m/s is the sound velocity. At the same time the heat conductivity equation (6) takes on the form of simplest expression for temperature relaxation that does not contain the terms representing the dissipative heating and the mechanic-and-caloric effect of a viscous liquid.

Equation (6) describes the flow of lubricant with velocity $V = \eta \partial\varepsilon/\partial t$ due to action of appearing viscous shear stress. Moreover, in accordance with Ref. [7] in the absence of shear deformations the temperature mean-square displacement is defined by equality $\langle u^2 \rangle = T/G\alpha$. The average shear displacement is found from the relationship $\langle u^2 \rangle = \sigma^2 a^2/\mathcal{G}^2$. The total mean-square displacement represents the sum of these expressions provided that the thermal fluctuations and the stress are independent. Above implies that the transition of lubricant from solidlike to fluidlike state is induced both by heating and under influence of stress generated by solid surfaces at friction. This agrees with examination of solid state instability within the framework of shear and dynamic disorder-driven melting representation in absence of thermal fluctuations. It is assumed that the film becomes more liquidlike and the friction force decreases with the temperature growth due to decreasing activation energy barrier to molecular hops. Besides, the friction force decreases with increasing velocity at the contact $V = \eta \varepsilon/\partial t$ because the latter leads to the growth of the shear stress $\sigma$ according to the Maxwell stress-strain $\varepsilon$ relationship: $\partial\sigma/\partial t = -\sigma/\tau_\sigma + G\partial\varepsilon/\partial t$.

This work is devoted to study of the stochastic source $\lambda(t)$ influence on the evolution of the stress $\sigma(t)$. In accordance with experimental data for the organic lubricant [3, 9] the stress relaxation time at normal pressure is equal to $\tau_\sigma \sim 10^{-10}$ s. Since ultrathin lubricant film consists of less than four molecular layers, the temperature relaxes to the value $T_c$ during the time satisfying the inequality $\tau_T \ll \tau_\sigma$. Therefore, we will suppose that conditions are fulfilled

$$\tau_\sigma \approx \tau_\varepsilon \gg \tau_T, \quad (11)$$

at which lubricant temperature $T$ follows the change of the shear components of stress $\sigma$ and strain $\varepsilon$. Then, it is possible to select a small parameter and to put $\tau_T T \simeq 0$ in Eq. (4). As a result, we obtain the expression for temperature:

$$T = T_c - \varepsilon - \sigma^2 + \lambda(t). \quad (12)$$

Let us give for the system (2), (3), and (12) the more simple form, reducing it to the single equation for the shear stress $\sigma(t)$. For this purpose it is necessary to express $\varepsilon$ and $T$ via $\sigma$. Differentiating with respect to time the equation for strain $\varepsilon$ that is obtained from (2), we get equation for $\dot{\varepsilon}$. Substitution of these expressions for $\varepsilon$, $\dot{\varepsilon}$ and the equality (12) in (3) gives evolution equation in canonical form of equation for nonlinear stochastic oscillator of the van der Pole generator type:

$$m\ddot{\sigma} + \gamma(\sigma)\dot{\sigma} = f(\sigma) + \phi(\sigma)\lambda(t), \quad (13)$$

where the coefficient of friction $\gamma$, the force $f$, the amplitude of noise $\phi$, and the parameter $m$ are defined by expressions

$$\gamma(\sigma) \equiv \frac{1}{g} \left[ \tau_\varepsilon + \tau_\sigma (1 + \sigma^2) \right], \quad f(\sigma) \equiv \sigma \left( T_c - 1 - g^{-1} \right) - \sigma^3 \left( g^{-1} - 1 \right), \quad \phi(\sigma) \equiv \sigma, \quad m \equiv \frac{\tau_\sigma \tau_\varepsilon}{g}. \quad (14)$$
Let us find the distribution function of the stress $\sigma$. With this aim, we will use the method of effective potential \[17, 18\]. As a result, the Fokker-Planck equation is obtained
\[
\frac{\partial P}{\partial t} = -\frac{\partial}{\partial \sigma} \left( D^{(1)} P \right) + \frac{\partial}{\partial \sigma} \left( D^{(2)} \frac{\partial P}{\partial \sigma} \right). \tag{15}
\]
It is expressed in terms of coefficients
\[
D^{(1)} = \frac{1}{\gamma} \left\{ f - \phi^2 \frac{\partial \gamma^{-1}}{\partial \sigma} - \phi \frac{\partial}{\partial \sigma} \left( \frac{2I}{\gamma} + I\tau_\lambda \right) \right\}, \tag{16}
\]
\[
D^{(2)} = \frac{\phi^2}{\gamma} \left( \frac{I}{\gamma} + 2I\tau_\lambda \right). \tag{17}
\]
In the stationary case the solution of Eq. \[15\] leads to the distribution
\[
P(\sigma) = Z^{-1} \exp \{-E(\sigma)\}, \tag{18}
\]
which is fixed by effective potential
\[
E(\sigma) = -\int_0^\sigma \frac{D^{(1)}(x)}{D^{(2)}(x)} dx, \tag{19}
\]
and normalization constant
\[
Z = \int_0^\infty d\sigma \exp \left( \int_0^\sigma \frac{D^{(1)}(x)}{D^{(2)}(x)} dx \right). \tag{20}
\]
The stationary shear stress is found from the extremum condition of distribution \[18\]
\[
\frac{D^{(1)}(\sigma)}{D^{(2)}(\sigma)} = 0. \tag{21}
\]
According to Fig. 1 the distribution \[18\] has pronounced maxima whose positions are determined by the set of parameters $\tau_\sigma$, $\tau_\varepsilon$, $\tau_\lambda$, $g$, $I$, and $T_e$. At the small values of friction surfaces temperature $T_e$ a single maximum is realized at the point $\sigma = 0$ meeting the dry friction mode (curve 1). With $T_e$ growth the two maxima appear at points $\sigma = 0$ and $\sigma \neq 0$ (curve 2), the first of them corresponds to the dry friction, the second one — to the sliding. Here the stick-slip friction mode, characterized by transitions between the indicated stationary regimes, is realized. With the further growth of $T_e$ the zero maximum of $P(\sigma)$ disappears and the maximum at $\sigma \neq 0$ remains only (curve 3), i.e., lubricant becomes liquidlike.

Supposing in \[21\] $\sigma = 0$, we find the critical value of friction surfaces temperature
\[
T_{e0} = \frac{1 + g}{g} + \left( \tau_\lambda + 2g \tau_\varepsilon + \tau_\sigma \right) I \tag{22}
\]
providing transition to the sliding mode. It is seen that $T_{e0}$ grows at the increase of noise intensity $I$ and correlation time $\tau_\lambda$. The values of relaxation times of the shear stress and strain influence by reverse manner. The domains of dry (DF), sliding (SF), and stick-slip (SS) modes of friction are realized in the phase diagram presented in Fig. 2. The growth of correlation time $\tau_\lambda$ results in the increase of value $T_e$, corresponding to the tricritical point $T$ at defined intensity $I$. Thus, the region of dry friction broadens, while sliding and stick-slip friction becomes more hardly realized.

Apparently, the increase of the sheared surfaces temperature $T_e$ transforms lubricant to the sliding friction mode. It can be understood considering Eq. \[18\] that describes the damping oscillations. Here the surfaces temperature is included only in expression for driving force $f$, which increases with growth of $T_e$. As is known, the liquid state can correspond to the oscillation mode with large amplitude, but the solid state can not. With the increasing value of effective force in \[14\] the amplitude of oscillations increases, and longer oscillation process is realized to the moment of setting of the certain friction mode in the system.

For studying the dynamics of change of friction modes it is enough to represent the distribution $P(\sigma)$ by position of its maximum $\dot{\sigma}$. This is achieved by the use of the path integrals formalism \[18\], within the framework of which the extreme values $\dot{\sigma} = \dot{\sigma}(t)$ of the initial distribution function \[18\] evolve in accordance with the effective distribution
\[
\Pi(\dot{\sigma}, \dot{\sigma}, t) \propto \exp \left( -\int \Lambda(\dot{\sigma}, \dot{\sigma}, t) dt \right). \tag{23}
\]
Here the Onsager-Machlup function $\Lambda$, acting as the Lagrangian of Euclidean field theory, is the subject for determination.

Equation \[18\] can be transformed to the form:
\[
\frac{\partial P}{\partial t} = -\frac{\partial}{\partial \sigma} \left[ \left( D^{(1)} + \frac{dD^{(2)}}{d\sigma} \right) P \right] + \frac{\partial^2}{\partial \sigma^2} \left( D^{(2)} P \right). \tag{24}
\]
For finding of the $\Lambda(\dot{\sigma}, \dot{\sigma}, t)$ dependence we write down the differential Langevin equation
\[
d\dot{\sigma} = \left( D^{(1)} + \frac{dD^{(2)}}{d\sigma} \right) dt + \sqrt{2D^{(2)}} dW(t) \tag{25}
\]
corresponding to the Fokker-Planck equation \[24\]. Here the stochastic differential $dW(t)$ represents the Winner process with the properties of white noise:
\[
\langle dW(t) \rangle = 0, \quad \langle (dW(t))^2 \rangle = dt. \tag{26}
\]
The feature of stochastic equations is that differential $dW(t)$ can not be obtained by the simple division of Eq. \[25\] by $\sqrt{2D^{(2)}}$. With this aim, it is necessary to pass from the random process $\dot{\sigma}(t)$ to the white noise $x(t)$ related with the initial Jacobian $dx/d\dot{\sigma} = (2D^{(2)})^{-1/2}$. Then, substitution of Eq. \[25\] into the Ito stochastic differential
\[
dx = \frac{dx}{d\dot{\sigma}} d\dot{\sigma} + \frac{1}{2} \frac{d^2x}{d\dot{\sigma}^2} (d\dot{\sigma})^2, \tag{27}
\]
taking into account \[24\], leads to the expression
\[
\frac{dx}{d\sigma} = \frac{d}{d\sigma} \left( D^{(1)} + \frac{dD^{(2)}}{d\sigma} \right) + \frac{d^2x}{d\sigma^2} D^{(2)} \right) dt
\]
+ \frac{d}{d\sigma} \sqrt{2D^{(2)}} dW(t). \tag{28}
\]

Here the terms are neglected whose order exceeds with transition to the variable \(\dot{\tilde{\sigma}}\) and comparing with (23), we arrive at Lagrangian equation

\[
\text{Eq. (30) describes the system behaviour in accordance with potential energy}
\]
with respect to \(\tilde{\sigma}\). Consequently, for further consideration it is necessary to replace the expression \(U\) by \(E(\sigma)\) \[23\]. In this case, Eq. \[30\] describes the system behaviour with distribution \[18\] obtained above.

The system kinetics is defined by Euler–Lagrange equation

\[
\frac{\partial \Lambda}{\partial \sigma} - \frac{\partial}{\partial t} \frac{\partial \Lambda}{\partial \dot{\sigma}} = \frac{\partial R}{\partial \sigma}.
\tag{32}
\]

Within the white noise presentation the dissipative function has the simplest form \(R = \dot{\sigma}^2 / 2\) and it is transformed to

\[
R = \frac{\dot{\sigma}^2}{4D^{(2)}}
\tag{33}
\]

with transition to the variable \(\dot{\tilde{\sigma}} = (2D^{(2)})^{1/2} \dot{\sigma}\). Substituting in \[22\] the equalities \[20\], \[19\], \[23\], we arrive at differential equation

\[
\ddot{\tilde{\sigma}} + \dot{\tilde{\sigma}}^2 (D^{(2)})^{1/2} + \dot{\tilde{\sigma}} - 2\frac{D^{(1)}}{D^{(2)}} D^{(2)} = 0. \tag{34}
\]

At the steady state supposing in \[24\] \(\dot{\tilde{\sigma}} = 0\), the extremum condition \[21\] of distribution \[18\] is obtained. Note that the distribution maximum corresponds to the minimum of effective potential, and the distribution minimum — to its maximum.

Consider kinetics of the system using the phase plane method for analysis of Eq. \[24\]. The phase portraits are presented in Fig. 3 corresponding to the curves of Fig. 1.

The region of dry friction (DF) (Fig. 3a) is characterized by the presence of the one singular point \(D\), which corresponds to the maximum of probability \(P(\sigma)\) at \(\sigma = 0\). This point is non-standard and it requires interpretations. It is located at the origin of coordinates, and phase trajectories are curved around it, so that the system never comes to the value \(\sigma = 0\), i.e., this point is not stationary. Consider the system behaviour at the arbitrary initial condition. According to the phase trajectories the system evolves to the zero value of stress. Thus, if at initial conditions the growth rate of stress is positive it decreases to the zero at first (during this time the stress increases). Then, the stress value decreases asymptotically to the zero with the increase of its decrease rate. Presumably, the described situation meets to divergence of the probability \(P(\sigma)\) at point \(\sigma = 0\). This is related with infinite growth of decrease rate of stress variation at going of the system to \(\sigma = 0\). Let us assume that the system has reached the point, at which decreasing rate becomes critical. In such case the increase of stress value is expected, which is accompanied by the change of rate sign and transition of the system in the positive phase plane domain. Further, again the decreasing of stress occurs, and the described situation repeats oneself. Alternatively, the sign of stress variation rate does not change, and only its decreasing takes place. This moves the system on a neighboring phase trajectory, along which it approaches to the zero stress more rapidly. As a result, in the course of time the oscillation mode of dry friction is set in the vicinity of point \(\sigma = 0\) at the arbitrary initial conditions. Thus, the oscillations amplitude is small, and lubricant has solidlike structure.

The phase portrait of the system, characterizing the region of stick-slip friction (SS), is shown in Fig. 3b. The three singular points appear here: \(D\), the saddle \(N\), and the stable focus \(F\). As well as above, the point \(D\) is realized at the origin of coordinates and it corresponds to the dry friction mode. The saddle \(N\) meets the minimum of \(P(\sigma)\) and it is unstable stationary point. It is worth noting that at the initial value of the shear stress on the right-hand side from point \(N\) and \(\sigma = 0\), the sliding mode of friction is set in the system during time. If the initial value of stress appears on the left-hand side from saddle \(N\), the dry friction is set in similar case. Thus, the point \(N\) separates two maximums of distribution function \(P(\sigma)\). The focus \(F\) corresponds to the non-zero maximum of stress distribution function, i.e., it describes the liquidlike state of lubricant. The damping oscillations, corresponding to this point, mean that lubricant becomes more liquidlike and more solidlike periodically. However, the stable sliding friction is set always. Presumably, these oscillations are conditioned by
III. INFLUENCE OF DEFORMATIONAL DEFECT OF MODULUS

Actually, the shear modulus introduced (in terms of the relaxation time $\tau_\sigma$) in Eq. (22), depends on the stress value. This leads to the transition of the plastic deformation mode to the plastic one. It takes place at characteristic value of the stress $\sigma_p$, which does not exceed the value $\sigma_\sigma$ (in other case the plastic mode is not manifested). For consideration of deformational defect of the modulus we will use $\tau_p(\sigma)$ dependence proposed in \[5\], instead of $\tau_\sigma$. As a result, Eq. (22) takes the form:

$$\tau_p' = -\sigma \left( 1 + \frac{\theta - 1}{1 + \sigma} \right) + g_\omega \varepsilon, \quad (35)$$

where the relaxation time for the plastic mode $\tau_p = \eta_\sigma / \Theta$ is introduced ($\eta_\sigma = \tau_\sigma G$ is the effective viscosity, $\Theta$ is the hardening factor), $\theta = \Theta / G < 1$ is the parameter describing the ratio of tilts of the plastic and the Hookean sections of the deformation curve, $g_\omega = G^2 / \Theta G_0$ and $\alpha = \sigma_p / \sigma_\sigma$, are the constants. Then, within the framework of approximation \[11\] the system \[25\], \[3\], and \[4\], as well as above, is reduced to equation (cf. \[13\]):

$m \ddot{\sigma} + \gamma(\sigma) \dot{\sigma} = f(\sigma) + \phi(\sigma) \lambda(t), \quad (36)$

where the coefficient of friction $\gamma$, the force $f$, the amplitude of noise $\phi$, and the parameter $m$ are defined by expressions

$$
\gamma(\sigma) \equiv \frac{1}{g} \left[ \tau_\varepsilon \left( 1 + \frac{\theta - 1}{1 + \sigma} \right) + \tau_p \left( 1 + \sigma^2 \right) \right], \\
f(\sigma) \equiv \sigma \left[ T_e - 1 - \frac{1}{g} \left( \frac{\theta - 1 + \sigma}{1 + \sigma} \right) \right] - \sigma^2 \left[ \frac{1}{g} \left( \frac{\theta - 1 + \sigma}{1 + \sigma} \right) \right] - 1, \quad (37)
$$

According to the effective potential method \[17\], \[18\] we obtain the Fokker–Planck equation \[10\] with coefficients $D^{(1)}$ and $D^{(2)}$:

$$
D^{(1)} = \frac{1}{\gamma} \left\{ \sigma \left[ T_e - 1 - \frac{1}{g} \left( \frac{\theta - 1 + \sigma}{1 + \sigma} \right) \right] \right\}, \\
D^{(2)} = \frac{1}{\gamma} \left( \gamma - \frac{\tau_p}{\theta - 1 + \sigma^2} + \sigma \right), \quad (38)
$$

Using Eqs. (21), (38), and (39), the equality is found with the same meaning as \[22\], which gives the boundary of existence of distribution \[11\] maximum at zero value of stress corresponding to the solidlike state of lubricant

$$
T_{\varepsilon e} = \frac{\theta - 1 + \sigma}{\sigma} + \left( \frac{\tau_\lambda}{\theta - 1 + \sigma} + \tau_p \right) I. \quad (40)
$$

The $P(\sigma)$ dependencies are shown in Fig. 4 for the different modes of friction. Curves 1–5 correspond to the domains of dry (DF), stick-slip (SS), stick-slip and sliding (SS+SF), metastable and stable sliding (MSF+SF), and sliding friction (SF), respectively. The phase diagram and portraits are presented in Figs. 5 and 6 corresponding to the curves of Fig. 4.

The most complex form of $P(\sigma)$ function is inherent in SS+SF region (curve 3 in Fig. 4). Here the solidlike, metastable and stable liquidlike lubricant states coexist meeting the maximum of $P(\sigma)$. It means the realization possibility of intermittent (stick-slip) friction, at which the periodic transitions occur between the dynamic modes corresponding to these states. In the MSF+SF domain of stick-slip motion the metastable and stable sliding can periodically change each other (curve 4 in Fig. 4). It is characteristic that transition from SS+SF to MSF+SF region is accompanied by disappearance of dry friction in the system. The domain of dry friction (DF) broadens, and the region of sliding friction (SF) decreases with growth of correlation time $\tau_\lambda$ of noise.

The phase portrait of dry friction region (DF) is similar to that is inherent in continuous transformation (Fig. 3a). It implies that DF domain is equivalent at the taking into account of the modulus defect and without it.

The phase portrait describing the region of stick-slip friction (SS) is similar to the characteristic one at continuous transformation (Fig. 3b). The basic their difference is that here the trajectories around focus are considerably more elongated along both coordinates axes. It means the greater stability of sliding friction.

The most complex region (SS+SF) is represented by the phase portrait shown in Fig. 6a. The five singular points are realized here: $D$, $N$, $N'$, the stable focuses $F$, $F'$. As well as above, saddles correspond to the minimum of $P(\sigma)$ dependence. Point $D$ meets the solidlike state of lubricant. Stable focus $F$ determines the first non-zero maximum of probability. It is apparent that the oscillations are weakly pronounced around this
point. In this mode lubricant represents the very viscous liquid, because in such type of fluid at presence of noise the oscillations are damped strongly. Actually, the point $F$ corresponds to the small values of stress, and with its decreasing the lubricant becomes more viscous, and it is transformed into the solidlike state at $σ = 0$. Thus, using phase portraits it is possible to give explanation to that the liquidlike state of lubricant, but not the solidlike one, corresponds to the large values of shear stress. Focus $F'$ meets the second non-zero maximum of $P(σ)$ function, and there are oscillations with large amplitude around it. This implies the fluidlike state of lubricant and, accordingly, sliding. The last point is in large distance along abscissa axis from all others ones. This mode of friction is most probable only, since points $D$ and $F$, corresponding to the dry and metastable sliding friction, have large stability and probability of realization also. From here the conclusion follows that lubricant can undergo periodic transitions (stick-slip) between the modes corresponding to the points $D$, $F$, and $F'$. Since these modes are stable and separated by the pronounced minimums of distribution function $P(σ)$ (by saddles), the transitions between them is necessary to expect after large intervals of time.

The phase portrait of MSF+SF domain is represented in Fig. 6b. There are three singular points — the stable focuses $F$, $F'$, and the saddle $N$. The latter is similar to the described above saddles and it meets the minimum of probability dependence on the stress. Point $F$ corresponds to the first maximum of distribution, which describes metastable sliding mode (MSF), and $F'$ — the second maximum, which defines stable sliding (SF). There are only insignificant oscillations around the focus $F$, however, lubricant in this mode is less viscous liquid than in the vicinity of the point $F'$ in Fig. 6a. At the origin of coordinates the singular point is absent, and the dry friction is not realized. Focus $F'$ is similar to described in Fig. 6a, however, it’s “attraction” domain is more stretched along both axes that means the larger fluidity of lubricant and stability of this mode. Therefore in comparison with the previous case, here the arising of sliding friction is more probable.

SF region is represented by the phase portrait, which is similar to the described for the continuous transformation (Fig. 3c). Here the one stable focus $F$ is realized, characterized by oscillations in it’s vicinity, representing stable sliding friction. The basic difference is that in this case oscillations take place with large amplitude that implies strong fluidity of lubricant and pronounced stability of such mode. However, as well as in all above considered situations, in the course of time in lubricant the stationary shear stress is set corresponding to the maximum of the initial distribution $P(σ)$.

In the basic equations (43) the shear stress $σ$ stands in the first power. However, in general case it’s exponent $a$ may be fractional ($0 < a < 1$), but not intenger:

\[
\begin{align*}
τ_σ \dot{σ} &= −a' + gε, \\
τ_ε \dot{ε} &= −ε + (T − 1)σ', \\
τ_T \dot{T} &= (T_c − T) − a'ε + σ^{2a} + λ(t).
\end{align*}
\]

Taking into account the additive noises of the shear stress and strain, and the temperature of lubricant film it has been shown [10] that such system describes the self-similar mode for which the characteristic scale of shear stress is absent [20]. This regime is determined by the homogeneous distribution function

\[
P(y) = y^{-2a}P(σ), \quad y = σσ_s.
\]

In particular, the value $2a = 1.5$ corresponds to the self-organized criticality mode, at which, unlike the phase transition, the process of self-organization does not require the external influence ($T_c = 0$) and occurs spontaneously [11, 21].

The self-similar behaviour of lubricant film is studied taking into account fluctuations of its temperature defined by Ornstein-Uhlenbeck process. It is shown that the fluctuations of lubricant temperature result in disappearance of sliding friction region at presence of dry and stick-slip friction domains in both cases of second-order and first-order transitions. In the second case the stick-slip motion arises characterized by three stationary values of shear stresses at which dry, metastable and stable sliding friction are realized. The increase of correlation time of lubricant temperature fluctuations leads to increasing of frictional surfaces temperature needed for realization of stick-slip friction.

The study of Eqs. (41) – (43) shows that the phase portraits for similar domains reproduce above considered qualitatively. However, there are substantial differences. The fractional Lorenz system at $a ≠ 1$ and $I ≠ 0$ results in presence of the singular point $D$ in phase portraits, which corresponds to the solidlike state of lubricant. Besides, the variation of $a$ leads to the complication of $P(σ)$ dependence, and as a result, to more complex form of phase portraits. Within the determined friction mode at decreasing of $a$ the increase of abscissas of the stable focuses is observed. Consequently, the weakening of fractional feedbacks in the Lorenz-type models results in the increase of lubricant fluidity and the reducing of friction. However, in the systems described by fractional exponent $a$ the dry friction is realized always. Thus, it is impossible to assert that such systems more preferable to friction decrease than linear systems.

IV. CONCLUSION

The above consideration shows that increase of temperature of frictional surfaces $T_c$, at presence of colored noise of lubricant temperature, can be accompanied by self-organization of elastic and thermal fields leading to
the mode of sliding friction. Indeed, the correlation degree of lubricant temperature change plays the substantial role. If the correlation time $\tau_\lambda$ increases, the growth of friction surfaces temperature is necessary for transition from dry to sliding friction mode at the fixed intensity $I$ of temperature fluctuations. In the case of continuous transformation at small intensity $I$ this transition takes place avoiding the region of intermittent friction, i.e., has the form of second-order transition — the melting of amorphous lubricant. In the reverse case of large $I$ the first-order transition is realized corresponding to the melting of crystalline lubricant.

At setting of the sliding friction mode in the system the damping oscillations arise and the shear stress relaxes to the stationary value fixed by the probability distribution $P(\sigma)$. The amplitude of these oscillations increases with growth of stationary values of shear stress. It means that large shear stress $\sigma$ corresponds to the liquidlike structure of lubricant. The solidlike state of lubricant is described by the singular point $D$ at the origin of coordinates that has complex character of stability and corresponds to the divergence of probability $P(\sigma)$. The oscillations near this point are absent.

For description of first-order transition the shear modulus defect is taken into account. It is shown that the change of temperature fluctuations intensity $I$ and frictional surfaces temperature $T_e$ can transform the system from the dry friction mode to the sliding. The latter arises at two values of shear stress. Accordingly, the three singular points, that define the stationary values of stress, appear in phase portraits — non-standard point $D$ at zero stress and two stable focuses at non-zero ones. The intermittent (stick-slip) mode of friction can be realized as a result of transitions between solidlike, metastable and stable liquidlike lubricant states, which are described by zero and non-zero singular points.

Taking into consideration the nonlinear relaxation of the shear stress and fractional feedbacks in the Lorenz system it has been shown that the region of sliding friction is realized in the phase diagram only in absence of temperature fluctuations. In this case the singular point $D$ is always present in phase portraits, which corresponds to the solidlike state of lubricant and dry friction. Besides, the change of fractional exponent can complicate the phase diagram, increasing the number of friction domains, in accordance with experimental data [3].

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Figure captions

to the paper by A. V. Khomenko
"Influence of temperature correlations on phase
dynamics and kinetics of ultrathin lubricant film"

FIG. 1. The distribution function of the shear stress for
the second-order transition at $g = 0.2$, $\tau_{\sigma} = \tau_{\varepsilon} = 0.1$, $\tau_{\lambda} = 0.2$, and $I = 5$. The curves 1, 2, 3 correspond to the temperatures $T_e = 5, 16, 20$, respectively.

FIG. 2. The phase diagram corresponding to the parameters of Fig. 1 with the domains of dry (DF), sliding (SF), and stick-slip (SS) friction ($T$ is the tricritical point).

FIG. 3. The phase portraits corresponding to the parameters of Fig. 1: (a) DF mode corresponds to the curve 1 in Fig. 1; (b) SS — curve 2 in Fig. 1; (c) SF — curve 3 in Fig. 1.

FIG. 4. The distribution function of shear stress for the first-order transition at $\tau_p = \tau_{\varepsilon} = \tau_{\lambda} = 0.1$, $\theta^{-1} = 7$, $\alpha = 0.3$, $g_{\theta} = 0.4$, $I = 4.5$. The curves 1 – 5 correspond to the temperatures $T_e = 16, 21, 23.25, 24, 26$, respectively.

FIG. 5. The phase diagram corresponding to the parameters of Fig. 4 with the domains of dry (DF), sliding (SF), and stick-slip (SS, MSF+SF, SS+SF) friction modes.

FIG. 6. The phase portraits corresponding to the parameters of Fig. 4: (a) SS+SF mode corresponds to the curve 3 in Fig. 4; (b) MSF+SF — curve 4 in Fig. 4.
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5
