Lambda Oscillations and the Conservation Laws
A. Widom and Y.N. Srivastava
Physics Department, Northeastern University, Boston MA, USA

ABSTRACT: Lowe, Bassalleck, Burkhardt, Rusek, Stephenson, and Goldman assert (under the assumption that secondary decay vertices exhibit amplitude interference at fixed space-time points) that for $\pi^- + p^+ \rightarrow \Lambda + K^0$ Lambda oscillations disappear. Under the same assumption, we find that conservation of energy and conservation of momentum also disappear. Quantum oscillations occur for quite ordinary particles for reasons which are discussed.

1: Introduction

In two particle “in” and two particle “out” scattering, for which

$$\pi^- + p^+ \rightarrow \Lambda + K^0,$$  \hspace{1cm} (1a)

is a special case, there is a well known theorem that the spatial wave functions (for fixed total energy and fixed total momentum) factor into a product of wave functions; one for the “center of mass coordinate” $R$, and one for the “relative coordinate” $r$,

$$\Psi_{total}(r_1, r_2) = \phi(R)\psi(r).$$  \hspace{1cm} (1b)

Discrete quantum numbers are left implicit. While the wave function $\phi(R)$ is in principle a “wave packet”, it is still conventional to use a plane wave in a very big box and write $\phi(R) = \exp\{i(P_{total} \cdot R)/\hbar\}$. It is also usual to work in the center of mass frame in which case $P_{total} = 0$. This convention will be followed below.

Hence, in the center of mass frame $P_{total} = 0$ (and for the moment neglecting life time effects), the conservation laws of energy and momentum dictate for Eqs.(1) that the “in” wave function is given by

$$\Psi_{in}(r_p, r_\pi) = \psi_{in}(r_p - r_\pi).$$  \hspace{1cm} (2a)

and that the “out” wave function is given by

$$\Psi_{out}(r_K, r_\Lambda) = \psi_{out}(r_K - r_\Lambda).$$  \hspace{1cm} (2b)

With Eq.(2b) in mind, we point out the following

Theorem: If a wave function $\psi_{out}(r_K - r_\Lambda)$ oscillates in the coordinate $r_K$, then the same wave function oscillates in the coordinate $r_\Lambda$

We trust that the proof is obvious. If the $K^0$ oscillates in space, then the $\Lambda$ oscillates in space. And this will be reflected in the positions of the secondary vertices of the joint distribution of both particles via $|\Psi_{out}(r_K, r_\Lambda)|^2$.

With life-time effects included, we discussed\[1\] $\Lambda$ oscillations in previous work. More recently, Lowe, Bassalleck, Burkhardt, Rusek, Stephenson, and Goldman\[2\] find that the Lambda oscillations disappear, but “normal” Kaon oscillations are left in tact. To get to...
the root cause of the differences in opinion, it is then simply a matter of locating the point at which momentum and energy are no longer conserved in the argument of Lowe et. al., and this we shall do in what follows.

2: Momentum Conservation

Shown schematically below is the outgoing state of Eqs.(1)

\[
\begin{align*}
\Lambda & \quad \leftarrow \quad (\text{out}) \quad \rightarrow \quad K^0.
\end{align*}
\]

Note that in the center of mass frame

\[
P_{\text{total}} = p_\Lambda + p_K = 0,
\]

so that

\[
\exp\left\{i\left(p_\Lambda \cdot r_\Lambda + p_K \cdot r_K\right)/\hbar\right\} = \exp\left\{i\left(p_K \cdot (r_K - r_\Lambda)/\hbar\right)\right\}.
\]

In decomposing the process in Eq.(3) into various possibilities, and then superimposing amplitudes for these possibilities, we enforce strict conservation of momentum, which means in the center of mass system that Eqs.(4) must be enforced. The “long” and the “short” of it are shown below

\[
\left(\begin{array}{ccc}
\Lambda & -p_L & p_L \\
\text{L} & \text{(out)} & \text{L} \\
\Lambda & -p_S & p_S \\
\text{S} & \text{(out)} & \text{S}
\end{array}\right)\]

and the Λ momentum is always equal and opposite to the K momentum be it a long \(K_L\) with mass \(M_L\) and momentum \(p_L\) or a short \(K_S\) with a mass \(M_S\) and momentum \(p_S\). In any case, from momentum conservation laws in Eqs.(4), superpositions of all the processes yield the form of the wave function \(\psi_{\text{out}}(r_K - r_\Lambda)\). It is simply impossible to construct a wave function of that form which oscillates in the coordinate \(r_K\) but does not oscillate in the coordinate \(r_\Lambda\).

3: Proper Times

The violation of the four momentum conservation law is slipped into the argument of Lowe et. al. by considerations of “proper time”. Here is the story of proper time:

Once upon a time there were two twins who led different life styles. One twin (along with laboratory observers) hardly moved at all, while the other twin moved around pretty fast. Much to the shock of the laboratory observers, the fast twin seemed younger than the slow twin. The fast twin’s proper time was never the same as the laboratory observers’ proper time, and it was not even the same as the proper time that professor Einstein taught the laboratory observers to calculate

\[
c^2\tau^2 = c^2t^2 - |r|^2.
\]

Excuse us for telling the wrong (classical) story. Einstein already explained to all of us that the two twins read out two different proper times so there is no paradox!
Please let us start again. Once upon a time one electron was fired at two slits, so the electron split into two virtual twins who later met and combined again at the same counter behind the slits. One virtual twin went through slit 1 and arrived at the counter in her proper time $\tau_1$. The other virtual twin went through slit 2 and arrived at the same counter with her proper time $\tau_2$. Nevertheless, it was really only one electron but with two proper times and Einstein never liked this story. Bohr told Einstein that there was no paradox! The laboratory observers where shocked to learn that the virtual twin electrons had the same energy, traveled different distances to the same counter, but arrived at this counter at the same laboratory time. Bohr told the laboratory observers to stop thinking in classical terms! The two paths, and two proper times, and the diffraction oscillation from a beam of such electrons in the counters behind the slits, were all simply a consequence of the wave function and the interference phase $\theta_{12} = (mc^2/\hbar)(\tau_2 - \tau_1)$ where $m$ is the electron mass. If one bounces electrons off a crystal, then reflected outgoing waves oscillate. Low energy electron diffraction (LEED) is measured every day. Ordinary particles can oscillate!

Now in high energy physics, one measures a vertex position $r$ and a momentum $p$ and deduces for a particle of mass $M$ what is often called proper time,

$$\tau_{\text{experimental}} = (M|\mathbf{r}|/|\mathbf{p}|).$$

(7)

The important point is that Eq.(7) would be true if the particle were classical. But what if the particle is quantum mechanical? It would be hard to maintain from the viewpoint of the uncertainty principle that both $p$ and $r$ are exactly known or that both $E$ and $t$ are exactly known. Furthermore, the laboratory observers do not even attempt measure the time $t$ in Eq.(6). They measure a “proper time” as defined in Eq.(7).

Consider the $K^0$ particle in Eqs.(1). This meson has “two proper times” just like the electron going through two slits; i.e.

$$\tau_L = (M_L|\mathbf{r}_K|/|\mathbf{p}_L|), \quad \tau_S = (M_S|\mathbf{r}_K|/|\mathbf{p}_S|),$$

(8)

and the laboratory observers might like to know that neither of these proper times need be the classical proper time in Eq.(6) which is not measured anyway. Nobody even tries to measure $t$ in Eq.(6). If you conserve total energy for both the “long” and the “short” alternatives,

$$\sqrt{c^2|\mathbf{p}_L|^2 + M_L^2c^4} + \sqrt{c^2|\mathbf{p}_L|^2 + M_L^2c^4} = \sqrt{c^2|\mathbf{p}_S|^2 + M_S^2c^4} + \sqrt{c^2|\mathbf{p}_S|^2 + M_S^2c^4},$$

(9)

then it is evident that you can have neither $M_L$ the same as $M_S$ nor $|\mathbf{p}_L|$ the same as $|\mathbf{p}_S|$. For the interference phase of the $K^0$,

$$\theta_K = c^2(M_L\tau_L - M_S\tau_S)/\hbar,$$

(10)

let us define

$$\bar{\tau}_K = (1/2)(\tau_L + \tau_S), \quad \bar{M} = (1/2)(M_L + M_S),$$

(11a)

$$\Delta M_K = (M_L - M_S), \quad \Delta \tau_K = (\tau_L - \tau_S).$$

(11b)
Then the two terms in Eq.(10) yield

$$\hbar \theta_K = c^2 (\tilde{r}_K \Delta M_K + \tilde{M} \Delta \tau_K),$$

which depend on total energy. Lowe et. al. do not find that $\theta_K$ depends on total energy for the very good reason that they do not conserve total energy and they do not maintain the two proper times $\tau_L$ and $\tau_S$ for the Kaon. One cannot conserve total energy and find only one proper time for the oscillating $K^0$. Elimination of one of the two proper times also eliminates conservation of energy.

The $\Lambda$ in Eqs.(1) oscillates, but not because the mass is split. Like an oscillating diffracted electron wave bouncing off a crystal in LEED experiments, the interference phase for the $\Lambda$ has only the proper time differences

$$\hbar \theta_{\Lambda} = c^2 M_{\Lambda} \Delta \tau_{\Lambda} = c^2 M_{\Lambda}^2 |r_{\Lambda}| \{(1/|p_S|) - (1/|p_L|)\}.$$  

Conservation of energy and momentum demands that $|p_L|$ be different than $|p_S|$ so the Lambda has a non-zero interference phase $\theta_{\Lambda}$. This too depends on total energy if one conserves total energy.

4: Conclusion

It is perhaps unfortunate that quantum “oscillations” in high energy physics seems to refer to some esoteric particles rather than plain quantum interference or diffraction which can occur for any quantum particles. One might say, “the Kaon is very strange; see how it oscillates?”. But the electron is not very strange, and it oscillates too! The reason that quantum interference occurs is that quantum particles do not move on well defined space-time paths. For the problem at hand conservation of energy and conservation of momentum dictate an internal wave function such that the joint probability $|\Psi_{out}(r_K, r_{\Lambda})|^2$ oscillates in both coordinates.

Acknowledgements

A. Widom is grateful to professor J. Lowe for many communications.

References

[1] Y.N. Srivastava, A. Widom and E. Sassaroli, Phys. Lett. B334, 436 (1995).
[2] J. Lowe, B. Bassalleck, H. Burkhardt, A. Rusek, G.J. Stephenson Jr., and T. Goldman, “No Lambda Oscillations” hep-ph/9605234.