Measurement of the top quark mass in the dilepton channel using \( m_{t2} \) at CDF

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We present measurements of the top quark mass using $m_{T^2}$, a variable related to the transverse mass in events with two missing particles. We use the template method applied to $t\bar{t}$ dilepton events produced in $p\bar{p}$ collisions at Fermilab’s Tevatron Collider and collected by the CDF detector. From a data sample corresponding to an integrated luminosity of 3.4 fb$^{-1}$, we select 236 $t\bar{t}$ candidate events. Using the $m_{T^2}$ distribution, we measure the top quark mass to be $M_{\text{top}} = 168.0^{+4.0}_{-4.0}$ (stat.) $\pm 2.9$ (syst.) GeV/$c^2$. By combining $m_{T^2}$ with the reconstructed top quark mass distributions based on a neutrino weighting method, we measure $M_{\text{top}} = 169.3^{+2.7}_{-3.2}$ (stat.) $\pm 3.2$ (syst.) GeV/$c^2$. This is the first application of the $m_{T^2}$ variable in a mass measurement at a hadron collider.

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I. INTRODUCTION

Models in numerous, well-motivated theoretical frameworks make predictions for new phenomena at hadron colliders such as the Tevatron and the Large Hadron Collider (LHC) \[1, 2\]. Within each framework, one can construct a number of qualitatively different models consistent with data. Thus, when discoveries are made at a hadron collider, we face the inverse problem of how one maps back to the underlying theory responsible for the new phenomena \[1, 3\]. A potentially powerful observable to discriminate among models and to extract the mass of new particles, when the new phenomenon produces a pair of new particles with large missing energy signatures, is the $m_{T2}$ variable \[4, 5\]. The $m_{T2}$ variable is based on transverse mass in events with two missing particles.

The top quark is the heaviest known elementary particle with a mass approximately 40 times larger than the mass of its isospin partner, the bottom quark ($b$). The large top quark mass ($M_{\text{top}}$) produces significant contributions to electroweak radiative corrections. Therefore, top quark mass measurements are important tests of the standard model and provide constraints on the Higgs boson mass. In the dilepton channel, $t\bar{t}$ pair production is followed by the decay of each top quark to a $W$ boson and a $b$ quark where both $W$ bosons then decay to charged leptons ($e$ or $\mu$) and neutrinos. Events in this channel thus contain two leptons, two $b$ quark jets, and two undetected neutrinos. The measurement of $M_{\text{top}}$ using complementary techniques tests and improves our understanding of this important parameter in the standard model \[6\].

In this letter, we present the first measurement of the mass of the top quark using the $m_{T2}$ distribution with $t\bar{t}$ events in the dilepton channel \[7\]. We use this channel because it has decay products similar to possible new phenomena where undetected particles are created. We compare this method with two others that were previously used: the reconstructed top quark mass using the neutrino weighting algorithm ($m_{\text{NWA}}^{T}$) \[8, 9\] and the scalar sum of transverse energies of jets, leptons, and missing transverse energy ($E_{T}$) \[10\] in the event ($H_{T}$) \[11\]. We also measure the top quark mass using pairs of observables ($(m_{T2}, m_{\text{NWA}}^{T})$ and $(m_{T}, H_{T})$) simultaneously.

II. THE $m_{T2}$ VARIABLE

Many models contain heavy, strongly-interacting particles with the same conserved charge or parity that result in weakly-interacting, stable particles in the final state. A hadron collider would pair-produce these colored particles, which then decay into standard model particles along with a pair of undetectable weakly interacting particles, so that the generic experimental signature is large missing transverse momentum accompanied by multiple energetic jets and leptons \[10\]. In this final state, we can define $m_{T2}$ as

$$m_{T2}(m_{\text{invis}}) = \min_{p_{T}^{(1)}, p_{T}^{(2)}} \left[ \max[m_{T}(m_{\text{invis}}; p_{T}^{(1)}), m_{T}(m_{\text{invis}}; p_{T}^{(2)})] \right],$$

where $m_{T}$, the transverse mass of each parent particle, is defined as

$$m_{T}(m_{\text{invis}}; p_{T}^{\text{invis}}) = \sqrt{m_{\text{invis}}^{2} + 2(E_{T}^{\text{invis}}p_{T}^{\text{invis}} - p_{T}^{\text{vis}}) \cdot p_{T}^{\text{invis}}}.$$  

Here $\text{invis}$ and $\text{vis}$ represent the individual undetected (invisible) and detected (visible) particles, respectively, $p_{T}^{(1)}$ and $p_{T}^{(2)}$ are transverse momenta of two invisible particles and $m_{\text{invis}}$ is the mass of the invisible particle. The minimization is performed with the constraint $p_{T}^{(1)} + p_{T}^{(2)} = p_{T}^{\text{missing}}$, where the magnitude of $p_{T}^{\text{missing}}$ is constrained to the missing transverse momentum.

The quantity $m_{T2}$ represents a lower bound on the mass of the parent particle. Using the $m_{T2}$ distribution, we can extract the mass of this parent particle \[12\] in a similar way to the precise measurement of the $W$ boson mass \[13\] where an event contains one charged lepton ($e$ or $\mu$) and a neutrino, with the latter not being detected.

III. EXPERIMENT AND DATA

We use a sample of $t\bar{t}$ candidates in the dilepton channel, corresponding to 3.4 fb$^{-1}$ of proton-antiproton collisions at $\sqrt{s} = 1.96$ TeV, collected using the CDF II detector \[14\]. This is a general-purpose detector designed to study $p\bar{p}$ collisions at the Fermilab Tevatron. A charged-particle tracking system, consisting of a silicon microstrip tracker and a drift chamber, is immersed in a 1.4 T magnetic field. Electromagnetic and hadronic calorimeters surround the tracking system and measure particle energies. Drift chambers and scintillators, located outside...
TABLE I: Expected and observed numbers of signal and background events assuming \( t\bar{t} \) production cross section \( \sigma_{t\bar{t}} = 6.7 \pm 0.8 \) pb and \( M_{\text{top}} = 175 \text{ GeV} / c^2 \). Uncertainties quoted capture the uncertainties on the theoretical cross section, the statistics of data in the \( Z \) mass window, the jet energy scale, the luminosity, the fake rates, and the statistics of the MC samples.

| Source                      | non-tagged | tagged |
|-----------------------------|------------|--------|
| Diboson                     | 15.2 ± 2.3 | 0.6 ± 0.1 |
| Drell-Yan                   | 31.1 ± 3.5 | 1.7 ± 0.2 |
| QCD multijets               | 31.2 ± 8.7 | 4.5 ± 1.3 |
| Total Background            | 77.5 ± 9.8 | 6.8 ± 1.3 |
| \( t\bar{t} \) with \( \sigma_{tt} = 6.7 \) pb | 68.7 ± 6.8 | 88.4 ± 8.2 |
| Total (Predicted)           | 146.2 ± 11.9 | 95.2 ± 8.3 |
| Observed (3.4 fb\(^{-1}\)) | 149        | 86     |

the calorimeters, detect muon candidates.

We select events consistent with the \( t\bar{t} \) dilepton decay topology.
We require two oppositely charged lepton candidates with \( p_T > 20 \) GeV / c with one isolated \( \ell \) lepton candidate in the central region \((|\eta| < 1)\) of the detector, and another isolated or non-isolated lepton candidate in the central region, or isolated electron candidate in the forward region \((1.0 < |\eta| < 2.0)\). We also require \( E_T \) exceeding 25 GeV, and at least two jets with \( E_T > 15 \) GeV and \( |\eta| < 2.5 \). To further reject backgrounds, we request \( H_T > 200 \) GeV. We also require the variables of interest to be consistent with the top quark hypothesis by demanding \( 20 \) GeV / \(c^2 < m_{T2} < 300 \) GeV / \(c^2 \) and \( 100 \) GeV / \(c^2 < m_{T2}^{\text{SWA}} < 350 \) GeV / \(c^2 \). The criteria select 236 \( t\bar{t} \) candidate events.

The primary sources of background production are Drell-Yan, diboson, and QCD multijet events. We estimate the rate of the Drell-Yan events with a calculation based on simulated events using the ALPGEN [16] v2.10 Monte Carlo (MC) generator and the rate of diboson events with a PYTHIA [17] v6.216 calculation. For the Drell-Yan \( Z+\)jets processes, we normalize the MC sample by matching the number of \( Z \) events predicted and observed in the \( Z \) mass region between 76 GeV / \(c^2 \) and 106 GeV / \(c^2 \). We use data to estimate the rate of background events from QCD multijet production where an event has one real lepton and one of the jets misidentified as another lepton (fake). In measuring the top quark mass, we divide the \( t\bar{t} \) candidate sample into events with and without secondary vertex \( b \) tags [18], which have very different purity. We only attempt to \( b \) tags the two highest \( E_T \) jets. Table I summarizes the composition of background events and the expected numbers of \( t\bar{t} \) and background events. We estimate the \( t\bar{t} \) signal event rates using PYTHIA v6.216 with CTEQ5L [19] parton distribution functions at leading order with a full detector simulation [20].

To calculate \( m_{T2} \) of a \( t\bar{t} \) dilepton event [21], we first identify all possible configurations corresponding to different assignments of jets to \( b \) quarks and combinations of quarks and leptons. The two most energetic jets in an event are considered to have originated from the \( b \) quarks. For each configuration, we calculate the transverse mass of each top quark \(( t \rightarrow b\nu \ell \) using Eq. [2]

\[
m_T = \sqrt{m^{2}_{bl} + m^{2}_{T\nu} + 2 (E_{T\nu}^{bl} - p_T^{bl} \cdot p_T^{\nu})},
\]

where \( m_{bl} \) and \( p_T^{bl} \) denote the invariant mass and transverse momentum of the bottom-quark jets and charged lepton (bl) system, \( m_T \), \( E_{T\nu}^{bl} \) and \( E_T^{\nu} \) are the mass and transverse momentum of the neutrino, and \( E_T^{bl} \) and \( E_T^{\nu} \) are the transverse energies of the bl system and neutrino:

\[
E_T^{bl} = \sqrt{|p_T^{bl}|^2 + m_{bl}^2} \quad \text{and} \quad E_T^{\nu} = \sqrt{|p_T^{\nu}|^2 + m_{\nu}^2}.
\]

We then calculate \( m_{T2} \) using Eq. [1] with the assumption \( m_{\nu} = 0 \), and for all possible parton assignments. We select the smallest value for each event. Figure 1 shows simulated \( m_{T2} \) distributions for various top quark masses for the combined non-\( b \)-tagged and \( b \)-tagged sample, which demonstrates that \( m_{T2} \) is sensitive to \( M_{\text{top}} \), and thus can be used to measure it.

\[ \text{FIG. 1: The } m_{T2} \text{ distributions from } t\bar{t} \text{ dilepton Monte Carlo events that pass the selection criteria for three input values of the top quark mass. Each distribution is normalized to have unit area.} \]

IV. MASS FIT

We estimate the probability density functions (PDFs) of signals and background using the kernel density estimation (KDE) [21, 22] that constructs the PDF without any assumption of a functional form. For the mass measurement with two observables, we use the two-dimensional KDE that accounts for the correlation between the two observables. First, at discrete values of \( M_{\text{top}} \) from 130 GeV / \(c^2 \) to 220 GeV / \(c^2 \) with increments of 0.5 GeV / \(c^2 \) in the region immediately above and below 175 GeV / \(c^2 \) to 5 GeV / \(c^2 \) near the extreme mass values, we estimate the PDFs for the observables from 76 \( t\bar{t} \) MC.
samples. Each sample consists of 0.6M to 4.8M generated events, with 1M events corresponding to a luminosity of 150 fb$^{-1}$, assuming a $t\bar{t}$ cross section of 6.7 pb$^{23}$. We smooth and interpolate the MC distributions to find PDFs for arbitrary values of $M_{\text{top}}$ using the local polynomial smoothing method$^{24}$. We fit the distributions of the observables in the data to the signal and background PDFs in an unbinned extended maximum likelihood fit$^{25}$ where we minimize the negative logarithm of the likelihood using MINUIT$^{26}$. The likelihood is built for the $b$-tagged and non-$b$-tagged categories separately and then combined by multiplying the two categories. We find the statistical uncertainty on $M_{\text{top}}$ by searching for the points where the negative logarithm of the likelihood minimized with respect to all other parameters deviates by 0.5 units from the minimum. Ref.$^{22}$ provides detailed information about this technique.

We test the mass fit procedures using 3,000 pseudo-experiments for each of 14 different top quark masses ranging from 159 GeV/$c^2$ to 185 GeV/$c^2$ with almost 2 GeV/$c^2$ step size. In each experiment, we select the numbers of background events from a Poisson distribution with a mean equal to the expected numbers of background events in the sample and the numbers of signal events from a Poisson distribution with a mean equal to the expected numbers of signal events assuming a $t\bar{t}$ pair production cross section of 6.7 pb. The distributions of the average mass residual (deviation from the input top mass) and the width of the pull (the ratio of the residual to the uncertainty reported by MINUIT) for simulated experiments show that the measured top quark mass is on average 0.26±0.10 GeV/$c^2$ lower than the true top quark mass and has no dependence on $M_{\text{top}}$ in the $m_{T2}$ measurements. We correct the measurement for this bias. No such bias is observed with the combined ($m_{T2}$, $m_{T2}^{\text{NWA}}$) measurement. In all cases, the fit on average correctly estimates the statistical uncertainties, based on the pull width distribution being consistent with unity. For $M_{\text{top}}$ = 175 GeV/$c^2$, we expect the statistical uncertainties on $M_{\text{top}}$ to be 4.0 GeV/$c^2$ with $m_{T2}$, 3.4 GeV/$c^2$ with $m_{T2}^{\text{NWA}}$, 5.4 GeV/$c^2$ with $H_T$, 2.9 GeV/$c^2$ with ($m_{T2}$, $m_{T2}^{\text{NWA}}$) combined, and 3.2 GeV/$c^2$ with ($m_{T2}^{\text{NWA}}$, $H_T$) combined.

V. SYSTEMATIC UNCERTAINTIES

We examine a variety of systematic effects that could affect the measurement by comparing MC simulated experiments in which we vary relevant parameters within their systematic uncertainties. The dominant source of systematic uncertainty is the light quark jet energy scale (JES)$^{27}$. We vary JES parameters within their uncertainties in both signal and background MC generated events and interpret the shifts as uncertainties. The $b$-jet energy scale systematic uncertainty arising from our modeling of $b$ fragmentation, $b$ hadron branching fractions, and calorimeter response captures the additional uncertainty not taken into account in the light quark jet energy scale. The uncertainty arising from the choice of MC generator is estimated by comparing MC simulated experiments generated with PYTHIA and HERWIG$^{28}$. We estimate the systematic uncertainty due to modeling of initial-state gluon radiation (ISR) and final-state gluon radiation (FSR) by extrapolating uncertainties in the $p_T$ of Drell-Yan events to the $t\bar{t}$ mass region$^{29}$. We estimate the systematic uncertainty due to parton distribution functions by varying the independent eigenvectors of the CTEQ6M$^{30}$ parton distribution functions, varying $\Delta QCD$, and comparing CTEQ5L$^{19}$ with MRST$^{21}$ parton distribution functions. In estimating the systematic uncertainty associated with uncertainties in the top quark production mechanism, we vary the fraction of top quarks produced by gluon-gluon annihilation from 6% to 20%, corresponding to the one standard deviation upper bound on the gluon fusion fraction$^{32}$. We estimate systematic uncertainties due to the lepton energy and momentum scales by propagating shifts in electron energy and muon momentum scales within their uncertainties. Background shape systematic uncertainties account for the variation of the background composition. In addition, we change the shape of the Drell-Yan background sample according to the difference in the missing energy distribution observed in data and simulation, and the shape of the QCD multijet model. We estimate the multiple hadron interaction systematic uncertainties to account for the fact that the average number of interactions in our MC samples are not equal to the number observed in the data. We extract the mass dependence on the number of interactions in MC pseudo-experiments by dividing our MC samples into subsamples with different number of interactions. We then multiply the slope of the result by the difference in the number of interactions between MC events and data and treat that as a systematic uncertainty.

It has been suggested that color reconnection (CR) effects could cause a bias in the top quark mass measurement and interpretations at the level of 0.5 GeV/$c^2$ [33]. We estimate uncertainties arising from CR effects using the PYTHIA 6.4 MC generator, which includes CR effects and other new features in modeling the underlying event, initial and final state radiation, and parton showering. We generate two MC samples, one using tune A [34], which is very similar to the tune for CDF nominal MC generations, the other using $A_{CR}$ [32], which includes CR into the tune A. We take the difference in the extracted mass between these two MC samples as a systematic uncertainty. We measure the difference to be 0.6 GeV for ($m_{T2}$, $m_{T2}^{\text{NWA}}$) combined, and 0.7 GeV for $m_{T2}$ alone. As a cross check, we generate two other MC samples, one using tune S$_0$ [35] and the other using NOCR [33], which include all of the new features with and without CR. We find a similar mass difference between the two samples.

Table 11 summarizes the sources and estimates of systematic uncertainties. The total systematic uncertainties, adding them in quadrature, are 2.9 GeV/$c^2$.
with \( m_{T2} \), 3.8 GeV/c\(^2\) with \( m_{tNWA} \), 5.7 GeV/c\(^2\) with \( H_T \), 3.2 GeV/c\(^2\) with \((m_{T2}, m_{tNWA})\) combined, and 3.8 GeV/c\(^2\) with \((m_{tNWA}, H_T)\) combined. The \( m_{T2} \) method has a jet energy scale uncertainty significantly smaller than \( m_{tNWA} \), resulting in the smallest total systematic uncertainty. Including both statistical and systematic uncertainties, we conclude that \( m_{T2} \) is one of the best observables for the \( M_{top} \) measurement, comparable to the measurement using \( m_{tNWA} \). Using both \( m_{T2} \) and \( m_{tNWA} \), we expect to achieve a 10% improvement in overall uncertainty over using \( m_{T2} \) alone.

### VI. RESULTS

We apply a likelihood fit to the data using observables discussed in this article. Figure 2 shows the one-dimensional log-likelihoods for \( m_{T2} \) and \((m_{T2}, m_{tNWA})\) combined. Figure 3 shows the distributions of the observables used for the \( M_{top} \) measurements overlaid with density estimates using \( tt \) signal events with \( M_{top} = 169 \) GeV/c\(^2\) and the full background model. The fit results are summarized in Table III. The extracted masses are consistent with each other and the statistical uncertainties are consistent with predictions from MC pseudo-experiments.

In conclusion, we present the top quark mass measurements in the dilepton channel using \( m_{T2} \). In 3.4 fb\(^{-1}\) of CDF data, we measure \( M_{top} \) using \( m_{T2} \) to be

\[
M_{top} = 168.0 \pm 2.9 \text{ (syst.) GeV/c}^2
\]

and using both \( m_{tNWA} \) and \( m_{T2} \) to be

\[
M_{top} = 169.3 \pm 3.2 \text{ (syst.) GeV/c}^2
\]

This is consistent with the most precise published result in this channel from the CDF [35] and D0 [36] collaborations. We expect further improvements in \( M_{top} \), with these variables as CDF accumulates about a factor of 3 more data during Tevatron Run II. The measurements in this article are the first application of the \( m_{T2} \) variable to data, and demonstrate that \( m_{T2} \) is a powerful observable for the mass measurement of the top quark in the dilepton channel. The methods described in this article will be applicable to other measurements at the Tevatron and soon at CERN’s Large Hadron Collider for discriminating new physics models and measuring the mass of heavy particles that decay into weakly interacting particles such as dark matter candidates.

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FIG. 2: Negative log-likelihood distributions.

FIG. 3: Distributions of the three variables used to estimate the top quark mass, showing the $b$-tagged and non-$b$-tagged samples separately. The data are overlaid with the predictions from the KDE probability distributions using the top quark mass $M_{\text{top}} = 169$ GeV/$c^2$ and full background model.

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TABLE III: Summary of top quark mass measurements with different observables. In the right-hand $M_{\text{top}}$ column, we combine in quadrature the statistical and systematic uncertainty in order to compare the precision of the different methods.

| Observables | $M_{\text{top}}$ (GeV/c$^2$) | $M_{\text{top}}$ (GeV/c$^2$) |
|-------------|-------------------------------|-------------------------------|
| $m_{T2}$    | $168.0^{+4.8}_{-4.0}$ (stat.) $\pm 2.9$ (syst.) | $168.0^{+5.6}_{-5.0}$ |
| $m_{NWA}$   | $169.4^{+3.3}_{-3.2}$ (stat.) $\pm 3.8$ (syst.) | $169.4^{+5.0}_{-5.0}$ |
| $H_T$       | $168.8^{+5.1}_{-6.6}$ (stat.) $\pm 5.7$ (syst.) | $168.8^{+7.6}_{-8.7}$ |
| $m_{NWA}$ and $m_{T2}$ | $169.3^{+2.7}_{-2.9}$ (stat.) $\pm 3.2$ (syst.) | $169.3^{+4.2}_{-4.2}$ |
| $m_{NWA}$ and $H_T$     | $169.6^{+2.8}_{-2.9}$ (stat.) $\pm 3.8$ (syst.) | $169.6^{+4.7}_{-4.8}$ |

the transverse plane of the detector, $E_T (p_T^{\text{missing}})$.

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