Social Sourcing: Incorporating Social Networks Into Crowdsourcing Contest Design

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Abstract—In a crowdsourcing contest, a principal holding a task posts it to a crowd. People in the crowd then compete with each other to win the rewards. Although in real life, a crowd is usually networked and people influence each other via social ties, existing crowdsourcing contest theories do not aim to answer how interpersonal relationships influence people’s incentives and behaviors and thereby affect the crowdsourcing performance. In this work, we novelly take people’s social ties as a key factor in the modeling and designing of agents’ incentives in crowdsourcing contests. We establish two contest mechanisms by which the principal can impel the agents to invite their neighbors to contribute to the task. The first mechanism has a symmetric Bayesian Nash equilibrium, and it is very simple for agents to play and easy for the principal to predict the contest performance. The second mechanism has an asymmetric Bayesian Nash equilibrium, and agents’ behaviors in equilibrium show a vast diversity which is strongly related to their social relations. The Bayesian Nash equilibrium analysis of these new mechanisms reveals that, besides agents’ intrinsic abilities, the social relations among them also play a central role in decision-making. Moreover, we design an effective algorithm to automatically compute the Bayesian Nash equilibrium of the invitation crowdsourcing contest and further adapt it to a large graph dataset. Both theoretical and empirical results show that the new invitation crowdsourcing contests can substantially enlarge the number of participants, whereby the principal can obtain significantly better solutions without a large advertisement expenditure.

Index Terms—Crowdsourcing, social network, Bayesian game, mechanism design, contest theory, social recruiting.

I. INTRODUCTION

A. Background

Crowdsourcing is the action of an organisation outsourcing its task to an undefined and generally large crowd of people using an open call for participation [1]. Contest has proven to be a very successful approach in many platforms for crowdsourcing. A contest is a social and economic interaction where a group of agents exert costly and irretrievable effort in order to win prizes and the principal needs to make decision of judiciously awarding prizes among these agents based on their performances. It can be modeled as a game in which agents strategize on how to contribute effort and compete against each other. The applications of contest design have varied in human and multi-agent systems, ranging from political campaigns, sports, advertising, school admission, labour contract, R&D competition to crowdsourcing. The development of contest design theory has been specially advanced over recent years in artificial intelligence, fueled by the need for various internet online services applications.

In a crowdsourcing contest, a principal posts a task on a platform and announces a monetary reward that he is willing to pay for a winning solution. Participants submit solutions to the platform, and the principal chooses the best solution and awards the prize. For example, in the world’s largest competitive software development portal TopCoder.com, software development phases are realised through contests in which the crowd of developers compete with each other for the best innovation. Popular projects on GitHub can also provide a bounty to elicit better code from users. The Netflix Prize contest awarded one million dollars to an outstanding team that created the most accurate algorithm for predicting user ratings. Social recruitment on LinkedIn or Recruiter.com can also be seen as a crowdsourcing contest where the recruiters try to acquire staff with the highest ability from a crowd.

B. Challenges

The crowdsourcing contest is essentially about how to elicit efforts from a crowd of people and how to allocate rewards among them. In real life, a crowd is usually networked, and people influence other people via social ties. However, existing crowdsourcing contest applications and theories do not formally consider how interpersonal relationships influence people’s incentives and decisions, and thereby affect the overall performance of the crowdsourcing. Without considering the social relations, incentive design in crowdsourcing contests is inadequate. This is because an individual can exert her influence on a crowdsourcing not only by contributing her own solution, but also via her relations with other people. For instance, it is commonly seen that people share information with their friends about job opportunities, investment opportunities, challenges, contests or money-making tasks. However, the state-of-the-art crowdsourcing contests do not aim to answer why people share information with others. They do not treat people’s social influences as a key factor in behavioral reasoning and incentive design. By the current contest models, the principal can only convene some people via her own effort. Talents with high ability may not know the task and will not contribute, which results in a limited quality of the crowdsourcing.

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C. Our Contribution

We propose a novel crowdsourcing contest model considering that people may invite their friends to contribute to a task. Our model is essentially a Bayesian game on graph where each node’s behavior has two dimensions: whether to contribute her ability and which neighbors to invite. This is a non-typical high-dimensional contest model and is different from traditional contests where the principal can only call together agents via her effort.

Based on this Bayesian game model, we establish two invitation mechanisms for crowdsourcing contests by which a principal can impel agents to invite their friends to contribute. Each mechanism has three steps: (i) agents register in the contest, and they can invite friends to register; (ii) agents contribute their solutions; (iii) the principal chooses the winners and allocates the rewards. The two mechanisms have different rules and different information exposure. The first one publishes the dynamically changing number of registered agents. It has a symmetric Bayesian Nash equilibrium (BNE) where every agent’s strategy is simple to play: when the number of registered agents surpasses a specific value, invite all neighbors but not contribute; otherwise, do not invite anyone but contribute. The second mechanism allows agents to observe the social graph. This empowers them to make deliberative decisions taking into consideration of social relations. This mechanism admits asymmetric BNE, where every agent has her own threshold to decide whether to contribute, and all agents prefer to invite all neighbors. We also devise a recursive algorithm for the computation of the complex asymmetric BNE.

We also conduct extensive crowdsourcing contest experiments on a Facebook network dataset. Both theoretical and experimental results show that, with the invitation model and the mechanisms, the crowdsourcing contest principal can convene as many people as possible but without a large amount of advertisement expenditure. Thus the hidden valuable people in the social network get the chance to participate even though they are not known directly by the principal. The principal obtains significantly better solutions without a large extra payment on propagation.

D. Related Works

The works on contest design are traced back to [2] and [3], which studied rent-seeking problems. Another pioneering work [4] showed that rank-order tournaments could serve as an efficient scheme to elicit effort from strategic agents. Optimal contest design, which aims at eliciting better submissions, dates back to [5] and has recently been investigated in a wide variety of settings in both economics and artificial intelligence.

For instance, some works focused on the effects of contest success functions (e.g. [6], [7]), some studied the prize structures in contests (e.g. [8], [9]), and some tried to find the optimal contest under different output-skill-effort functions and maximizing goals (e.g. [10], [11]). There is also a valuable topic about multi-stage contests [12]. With the concept of crowdsourcing burgeoning with the development of internet, many works embed contest research into the crowdsourcing scenario, such as prize structures [13], [14], [15], [16], multiple contests on crowdsourcing platforms [17], and crowdsourcing effectiveness under specific schemes [18], [19]. A comprehensive survey of crowdsourcing contests is in [20] and [21].

A recent branch of works on contest theory proposed the simple contest mechanisms [11], [22], [23]. The simple contest captures a common scenario that an agent’s submission quality is intrinsic and cannot be adjusted. Thus agents in a simple contest only need to decide whether to participate or not. For example, in a job interview, all the candidates would do their best and the performances only depend on their abilities. The previous works on simple contest all deal with a fixed set of agents. There are several models of simple contest having been discussed, including models of single or multiple winners [11], certain or uncertain performance evaluation [24], parallel or sequential competing order [25] and so on. In this work, we also follow the basic ideas of simple contest design but design a game and mechanisms in a more realistic setting: the domain of agents is expanded from a fixed set to all agents in the social network, where the principal can only reach a portion of the whole crowd.

There has also been works about query propagation in social networks. In this problem, each node is offered with a reward by its parent, and offers a smaller reward to its children for the propagation of a query. The first model formulating the query propagation on a random network was given by [26]. Later works such as [27], [28], [29] extended this work to more general conditions. The famous 2009 DARPA Network Challenge witnessed the outstanding achievements of this series of works [30]. Some up-to-date results (e.g. [31], [32]) extended the scenario from query to contest, where the propagation of crowdsourcing through social ties plays an important role. Regarding the worker recruitment crowdsourcing, a recent work [33] divided the task duration into multiple cycles and dynamically updated the propagating and completing rewards. Besides the query incentive network problems, researchers also proposed auction-based models for crowdsourcing [34]. There have been some very recent works on auction mechanism design in social networks [35], [36], [37], [38]. As far as we know, all these works concentrate on introducing social networks into typical auction problems, which are quite different from contest design. Meanwhile, the above existing works for crowdsourcing did not provide a general game-theoretical analysis for the incentive mechanism design. They also didn’t identify the Bayesian Nash equilibrium of crowdsourcing considering network structure’s impact [39].

II. INVITATION CONTEST MODEL

This section defines the invitation contest as a Bayesian game and characterises the Bayesian Nash equilibrium. Then, combining with graph theory, we represent several structural notions which will be used throughout the paper.

A. Bayesian Game for Invitation Contest

Consider a crowd of agents $N = \{1, \ldots, n\}$. The social network underlying these agents is a graph $G = (N, E)$ where
the edge set $E$ is the social relationships among agents in $N$. Denote agent $i$’s neighbor set by $E_i$. Two nodes can reach each other directly if there is an edge linking them, but the structure of the whole graph is unobservable by any agent. There is a principal $p \in N$, who has a task and wants to obtain high-quality solutions. Principal $p$ holds a contest to find high-ability agents and to acquire solutions. Without other agents’ help, $p$ can only call together a small portion of agents in $N$, which she can directly influence (either by advertisement or notification).

Any agent can participate in the contest by contributing her own effort, or by inviting her neighbors, or both, or neither. An agent contributing her effort will incur a cost $c > 0$ while making an invitation is costless. An agent could be rewarded for providing a good solution to the task or for inviting a high-ability agent. An agent’s ability $q_i$ is a value from a continuous interval $[l, u]$ where $l \geq 0$ is the lower bound of agents’ abilities and $u$ is the upper bound. The higher the value of an agent’s ability is, the more proficient the agent is. Assume the abilities of all agents are i.i.d. with probability density function (pdf) $f(q)$ and cumulative distribution function (cdf) $F(q)$.

Each agent’s ability is her private information and can not be observed by other agents. That is, the game is with incomplete information. Therefore, we model the contest in social networks as a Bayesian game [40], which is the conventional model for games with incomplete information.

Definition 1: A Bayesian game is a tuple $(N, q, f, s, \pi)$ where $N$ is the set of agents; $q = (q_1, \ldots, q_n)$ is the profile of agents’ abilities; $f: q \to \mathbb{R}$ is the joint distribution of agents’ abilities; $s = (s_1, \ldots, s_n)$ is the profile of agents’ strategies, where $s_i : q_i \to (a_i, e_i)$ is a mapping from ability $q_i$ to contribution action $a_i$ and invitation action $e_i$; $\pi = (\pi_1, \ldots, \pi_n)$ is the profile of agents’ utility functions, where $i$’ utility function $\pi_i : q \times s \to \mathbb{R}$.

Especially, agent $i$’s strategy $s_i : q_i \to (a_i, e_i)$ shows the two-dimensional decision: one is the contribution action $a_i \in \{q_i, 0\}$, meaning that if an agent contributes, she contributes with her full ability. Such a contribution setting has a wide range of contest applications [11], [41]. The other dimension is the invitation action $e_i \in \mathcal{P}(E_i)$, i.e., which friends (neighbors in $G$) to invite. $\mathcal{P}(E_i)$ is the powerset of $i$’s neighbor set $E_i$. Note that an agent being invited does not necessarily mean she must contribute. Also, an agent inviting some friends does not necessarily mean she does not contribute.

B. Equilibrium Concept

In classic games with complete information, it is assumed that an agent perfectly knows which kind of opponents she is playing with. The Bayesian game we construct for a contest in social networks is more realistic because the uncertainty of agents’ types (ability in contests) is captured. In Bayesian games, an agent’s ability cannot be observed by any other agent. Still, all agents’ abilities are drawn from the prior joint probability distribution $f = p(q_1, \ldots, q_n)$, which is common knowledge.

Using the Bayes rule, conditioning on knowing her own ability and the prior $f = p(q_1, \ldots, q_n)$, an agent can compute the conditional distribution $p(q_{-i} | q_i)$, which quantifies the joint probability of her opponents’ ability profile $q_{-i} = (q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_n)$. With the conditional distribution known, agent $i$ can compute the expected payoff for each of her strategy $s_i(q_i) = (a_i, e_i)$. Denote the strategy profile of all agents except $i$ by $s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$, we can have the solution concept Bayesian Nash equilibrium.

Definition 2: The strategy profile $s^* = (s^*_1, \ldots, s^*_n)$ for the contest in social networks is a Bayesian Nash equilibrium (BNE) if for every $i \in N$, her two-dimensional strategy is

$$
s^*_i(q_i) = \arg \max_{s^*_i} \sum_{q_{-i}} p(q_{-i} | q_i) \pi_i(s^*_i(q_i), s^*_{-i}(q_{-i})).
$$

Strategy profile $s^*$ is a BNE if no agent $i$ can increase her expected utility by unilaterally deviating from this profile. Recall that $p(q_{-i} | q_i)$ can be computed by $i$ via Bayes rule.

Remark 1: In Bayesian games, each agent’s objective is to maximize the expected utility, i.e., the sum of many different $\pi_i$ weighted by $p(q_{-i} | q_i)$. Agents do not intend to maximize any specific payoff $\pi_i$. Under some profile $q_{-i}$, the value of $\pi_i$ could be low, and even negative. However, this does not necessarily mean the expected utility is low.

Finding the BNE for the Bayesian game according to Eq.(1) is like solving a linear combination of many games with complete information. However, it is worth noting that, the difficulty of this work lies in the two-dimensional strategy of each agent, where one dimension is her personal attribute $a_i$, and the other is her social attribute $e_i$. These two dimensions of actions are entangled when agents make decisions and are also affected by other agents’ contributions and invitations. The invitation contest is essentially a complex Bayesian game on graph, which has not been formalized and tackled before. These make our problem much more challenging than solving a typical Bayesian game.

C. Invitation Graph and Ranking Tree

With agents’ invitations, the task information propagates in the social network $G$. All the invited agents and all inviting relations form a subgraph $\mathcal{H} = (U \cup \{p\}, \mathcal{E}') \subseteq G$, where the invited agent set $U \subseteq N$ and the inviting relation set $\mathcal{E}' \subseteq \mathcal{E}$. We call $\mathcal{H}$ the invitation graph.

Now we identify some other terminologies, which will be crucial for the contest mechanism design and equilibrium analysis in the following sections. The relation beat depicts the ability ranking of agents, while the relation lead captures the structural ranking of the nodes in $\mathcal{H}$.

Definition 3 (Beat): Agent $i$ beats agent $j$, if $i$ and $j$ both contribute and the contribution qualities $q_i > q_j$, or if $i$ contributes but $j$ does not.

With agents’ invitations taken into consideration, besides the ability, an agent’s position in the network also plays a crucial role. We use the following notion to depict the relative position of any two agents in the invitation graph.

Definition 4 (Lead): Agent $i$ leads agent $j$ (formally, $i \succ j$), if $j$ cannot know the task when $i$ doesn’t invite any neighbor.
That is, \( i \) is a cut node in \( \mathcal{H} \) and if she does not propagate the task information, then all the routes from the principal \( p \) to \( j \) are broken, resulting in that \( j \) cannot know the task and will not contribute. Denote by \( D_i \) the set of all agents led by \( i \), and by \( C_i \) the set of all agents leading \( i \). Note that both \( D_i \) and \( C_i \) exclude \( i \) herself. When \( i \) leads \( j \), we have \( i \in C_j, j \in D_i \) and \( D_j \subset D_i \).

The leading/led relations among all agents in any given invitation graph \( \mathcal{H} \) can be described by a ranking tree \( \mathcal{T} \). In a ranking tree, the root is the principal \( p \), and other nodes are invited agents. The nodes in \( C_i \) forms the branch from \( p \) to \( i \). Every node in this branch is \( i \)’s leader. All the nodes in the sub-tree rooted at \( i \) form the set \( D_i \). Note that by definition, in \( \mathcal{T} \), the branch \( C_i \) is a set of cut nodes, but it is not necessary that these cut nodes are adjacent in \( \mathcal{H} \).

Given the social network of a crowd, there could be different invitation graphs depending on different strategy profiles of all agents. But with a specific invitation graph, the ranking tree is unique. We show an example of the above notions in Fig. 1 and Example 1.

**Example 1:** (1) Let Fig. 1(a) be the social network where agents \( a \) and \( d \) invite no one but other agents invite all their neighbors. (2) Then the invitation graph is in Fig. 1(b). From this invitation graph, we have \( \mathcal{U} = \{a, b, c, d, e, f\} \), \( \mathcal{C}_b = \emptyset \), \( \mathcal{D}_b = \{c, d, e, f\} \), \( \mathcal{C}_c = \{b\} \) and \( \mathcal{D}_c = \{f\} \). (3) The ranking tree generated from this invitation graph is shown in Fig. 1(c).

Specifically, \( b \) does not directly link to \( e \) in \( \mathcal{H} \) while \( b \) leads \( e \) in \( \mathcal{T} \). This is because \( \mathcal{T} \) is not simply a sub-graph extracted from \( \mathcal{H} \); rather it is re-constructed based on the leading relation in Definition 4. After removing \( b \), there will be no route from \( p \) to \( e \), thus \( b \) leads \( e \). Comparatively, albeit \( c \) directly links to \( e \) in \( \mathcal{H} \), only removing \( c \) will not break the information flow from \( p \) to \( e \). Thus \( c \) does not lead \( e \) in \( \mathcal{T} \).

### III. SIMPLE INVITATION MECHANISM

In this section, we design the first invitation contest mechanism in which the number of participants is real-time public information, and the award phase has a naive and straightforward rule. We prove that this mechanism admits a simple Bayesian Nash equilibrium: each agent either contributes her full ability or invites all her friends, but not both, nor somewhere in between. The equilibrium strategy of this mechanism is very simple for agents to implement. Therefore, we call it simple invitation mechanism (SIM).

**A. Contest Mechanism Characterization**

Simple invitation mechanism (SIM) is designed to have the following phases.

1. **Registration phase:** Agents register in the contest, and they can invite friends to register. The principal publishes the total number of registered agents in real time.
2. **Competition phase:** Registered agents finally decide whether to contribute their abilities or not.
3. **Award phase:** The agent contributing the highest ability wins prize \( \mathcal{M} \), and each leading node of the winner is awarded a tiny invitation bounty \( \epsilon \).

In the registration phase, the principal publishes the registered agent number in real time. This is the information exposed to all the agents, which will be considered when they strategize on their contribution actions and invitation actions. During the registration and competition phases, agents play a Bayesian game, and the utilities are realized in the award phase. Note that in the registration phase, agents can also think about whether to contribute, but their final decisions are made in the competition phase. We show the allocation rule of the award phase in Algorithm 1.

### Algorithm 1 Allocation Rule in Award Phase of SIM

**Input:** invited agents \( \mathcal{U} \), invitation graph \( \mathcal{H} \), agents’ contribution qualities \( q_i, \forall i \in \mathcal{U} \)

**Output:** winner \( w \), \( w \)’s leading agent set \( \mathcal{C}_w \)

1. initialize \( w = \text{nil}, \mathcal{C}_w = \emptyset \)
2. if there is at least one contribution then
   1. compute \( w \leftarrow \arg \max_{i \in \mathcal{U}} q_i \), tie-break if needed;
   2. for \( i \in \mathcal{U} \) do
      1. if \( i \neq w \) and \( i \) leads \( w \) then
         1. \( \mathcal{C}_w \leftarrow \mathcal{C}_w + \{i\} \);
   3. return \( w, \mathcal{C}_w \).

Now we analyze agents’ incentives and behaviors under SIM, which will further help us to characterize the BNE of SIM. We will show that every agent has a common threshold \( r \) in a given invitation graph. If an agent \( i \)’s ability \( q_i \) is above this threshold, then she prefers to contribute; otherwise, she prefers not to contribute. We will further prove that, in the BNE of SIM, every agent \( i \)’s strategy (i.e., \( s_i : q_i \mapsto (a_i, e_i) \)) has a very simple form: if the number of registered agents is less than her critical value, her best decision is contributing \( (a_i = q_i) \) but inviting no neighbor \( (e_i = \emptyset) \); otherwise, her best decision is inviting all neighbors \( (e_i = \mathcal{E}_i) \) but not contributing \( (a_i = \emptyset) \).

**B. Ability Threshold for Contributing**

Consider an agent \( i \) in a given invitation graph with ability value \( q_i \). Given any strategy of another invited agent \( j \), the probability that \( i \) beats \( j \) when contributing is \( P(a_j < q_i) \). This value is non-decreasing and continuous in \( q_i \). Besides, denote the probability that \( i \) does not win but is a leading node of winner \( w \) when contributing by \( P(i \in \mathcal{C}_w) \).

If agent \( i \) contributes, her expected utility \( \pi_i \) is the sum of the expected prize of being the winner and the expected...
invitation bounty of leading the winner minus the cost for contributing, that is
\[ \pi_i(a_i = q_i) = M \cdot \prod_{j \in U_{-i}} P(a_j < q_i) + \epsilon \cdot P(i \in C_w) - c, \]
where \( U_{-i} \) is the set of all invited agents other than \( i \).

If agent \( i \) does not contribute, her expected utility can only be from the expected invitation bounty of leading the winner, that is
\[ \pi_i(a_i = 0) = \epsilon \cdot P'(i \in C_w), \]
where \( P'(i \in C_w) \) is the probability \( i \) is a leading node of the winner when \( i \) does not contribute. \( P'(i \in C_w) \) is not identical to \( P(i \in C_w) \).

Agent \( i \) will contribute iff \( \pi_i(a_i = q_i) > \pi_i(a_i = 0) \). Moreover, since we assume the invitation bounty \( \epsilon \) is very small comparing with \( M \), then approximately, \( i \) will contribute when
\[ \prod_{j \in U_{-i}} P(a_j < q_i) > \frac{c}{M}. \tag{2} \]
Since \( P(a_j < q_i) \) is non-decreasing and continuous in \( q_i \), the left side of Eq.(2) is also non-decreasing and continuous in \( q_i \). Therefore, there must be a threshold \( r_i \in [l, u] \) and \( i \) will contribute iff \( q_i > r_i \). Such a strategy using this threshold is a best response for every agent against any given strategy profile of her opponents.

Proposition 1 (Ability Threshold): In the simple invitation mechanism, every agent contributes iff her ability \( q_i > r_i \), where \( r_i \) is her ability threshold. If all agents share an identical \( r_i = r \), this is a common threshold.

In what follows, we will show that under SIM, there exists a common threshold \( r \) that, in equilibrium, every agent with ability \( q_i > r \) will contribute, and every agent with ability \( q_i \leq r \) does not contribute. We will derive this common threshold \( r \) in an explicit form.

C. Beating and Winning Probabilities

The key to prove the existence of a common threshold in the BNE is a typical game-theoretical reasoning scheme that, for an arbitrary agent \( i \), given that all other agents’ thresholds are \( r \), then \( i \) also has this threshold \( r \). Note that we do not assume \( r_i = r \) as a premise, and we even do not assume \( i \) has any threshold. But eventually, we will prove that, \( i \) indeed has a threshold and it equals \( r \). We first calculate the probability that agent \( i \) beats the other agents.

Consider any other agent \( j \) who has threshold \( r \). According to Definition 3, for any \( q_i \in [l, u] \), there are two cases where \( i \) beats \( j \): one is they both contribute but \( i \) is better, i.e., \( r < q_j < q_i \); the other is \( i \) contributes but \( j \) doesn’t, i.e., \( q_j \leq r \). Note that we do not assume \( i \) has threshold \( r \), so it is not necessary that \( r < q_j \). To sum up, if \( i \) beats \( j \), it means \( q_j \leq \max\{q_i, r\} \). Then the cumulative probability that \( i \) beats \( j \) is
\[ P(q_j \leq \max\{q_i, r\}) = F_{q_j}(\max\{q_i, r\}) \tag{3} \]
Under i.i.d. assumption, multiplying this cumulative probability over all \( j \), we directly have the following lemma and corollary.

Lemma 1: The probability that agent \( i \) wins under SIM is \( \prod_{j \in U_{-i}} F_{q_j}(\max\{q_i, r\}) \), where \( q_i \) is her ability, \( r \) is the ability threshold, and \( U_{-i} \) is the set of all invited agents other than \( i \).

Corollary 1: Under the i.i.d. assumption of agents’ abilities, the probability for \( i \) to win is \( F(\max\{q_i, r\})^{\mid U_{-i} \mid - 1} \), and the probability for \( i \) not to win is \( 1 - F(\max\{q_i, r\})^{\mid U_{-i} \mid - 1} \).

Note that, value of this winning probability still depends on the number of \( j \) in \( U \) (i.e. the size of \( U \)), and agents’ invitations may change this value. But on a specific invitation graph obtained in the registration phase of SIM, we do know the size of \( U \). Next we will derive the formula of \( r \).

D. Common Threshold in Symmetric Equilibrium

Now we analyze agent \( i \)'s contribution behavior under SIM. There are two cases of \( i \)'s contribution. The first case is \( i \) contribute with her ability \( q_i \). In this case, agent \( i \)'s utility is composed of two parts: one is the expected benefit from winning prize \( M \) and the other is that from leading the winner. According to Corollary 1, the former happens with a probability
\[ p^w_i(q_i, e_i) = F(\max\{q_i, r\})^{\mid U \mid - 1}, \]
and the later happens with a probability
\[ p^l_i(q_i, e_i) = \left( 1 - F(\max\{q_i, r\})^{\mid U \mid - 1} \right) \cdot \frac{|D_i|}{|U_{-i}|}. \]
Recall that \( D_i \) is the set of all agents led by \( i \) and the fraction \( |D_i|/|U_{-i}| \) denotes the probability that a random agent is led by \( i \). Thus the product is the probability that \( i \) leads the winner.

To sum up, the expected utility for \( i \) to contribute is
\[ \pi_i(q_i, e_i) = M \cdot p^w_i(q_i, e_i) - c + \epsilon \cdot p^l_i(q_i, e_i). \tag{4} \]
The other case is \( i \) doesn't contribute. She may still benefit from leading the winner. The probability that there exists a winner is \( 1 - F(r)^{\mid U \mid - 1} \). This is just the probability that there exists at least one agent contributing. Then the probability that the winner is led by \( i \) is
\[ p^l_i(0, e_i) = \left( 1 - F(r)^{\mid U \mid - 1} \right) \cdot \frac{|D_i|}{|U_{-i}|}. \]
Thus the expected utility for \( i \) not to contribute is:
\[ \pi_i(0, e_i) = \epsilon \cdot p^l_i(0, e_i). \tag{5} \]

To ensure that agent \( i \) has incentive to contribute, the difference between Eq.(4) and Eq.(5) should be non-negative, that is \( M \cdot p^w_i(q_i, e_i) - c + \epsilon \cdot p^l_i(q_i, e_i) - p^l_i(0, e_i) \geq 0 \). Since comparing with the prize \( M \), the tiny bounty is very small, then approximately, the binding constraint for \( i \) to contribute is \( p^w_i(q_i, e_i) = \frac{c}{M} \), which results in the common threshold as
\[ r = F^{-1} \left( \frac{|U \mid - 1}{| U_{-i} |} \cdot \frac{c}{M} \right). \tag{6} \]
The threshold in Eq.(6) is derived by the binding condition. For an arbitrary agent \( i \), the difference between her contributing utility in Eq.(4) and non-contributing utility in Eq.(5) is positive if and only if her ability \( q_i > r \).

With this threshold existing, for any agent, her best strategy is: to contribute if her ability is above this threshold, and not to contribute otherwise.

**Proposition 2 (Common Threshold):** under SIM, there is a symmetric Bayesian Nash equilibrium where every agent contributes iff her ability is above the common ability threshold Eq.(6).

**E. Invitation in Symmetric Equilibrium**

Proposition 2 shows that the ability threshold \( r \) governs agents’ contributing decisions in the competition phase. Now the remaining question is: for each agent \( i \), how to decide whom to invite in the registration phase? Recall that under SIM, the total number of registered agents is public knowledge, thus next we will show how every agent’s strategy \( s_i \) is depending on this registration number.

If \( q_i > r \), on the one hand, according to Proposition 2, agent \( i \) will contribute. The utility for contributing in Eq.(4) is decreasing in the size of \( U \). This means when \( q_i > r \), agent \( i \) is better not to invite anyone. On the other hand, by Eq.(6), \( q_i > r \) is equivalent to \( q_i > F^{-1}(\frac{|U|-1}{\sqrt{V^2}}) \). Inversing this function we have \( |U| < \log_F(q_i) \frac{\sqrt{e}}{\sqrt{2}} + 1 \). This means when the total number of invited agents is less than a critical number \( t(q_i) = \log_F(q_i) \frac{\sqrt{e}}{\sqrt{2}} + 1 \), agent \( i \) is better not to invite anyone. Note that, although \( t(q_i) \) is increasing in \( q_i \), for any fixed agent \( i \) with ability \( q_i \), the critical number \( t(q_i) \) is a constant.

When \( q_i < r \), agent \( i \) will not contribute. Now the only way she can obtain some benefit is being a leader of the winner. In this case, \( i \) needs to invite as many neighbors as possible, because inviting more neighbors can only increase the chance of being a leader of the winner. This is intuitive. Consider that if agent \( i \) does not contribute, according to Eq.(5), \( i \)’s expected utility is

\[
\pi_i(0, e_i) = \epsilon \cdot p_i(0, e_i) = \epsilon \cdot \left( 1 - F(r) \right) \frac{|D_i|}{|U|}. 
\]

Replacing \( r \) in this formula by Eq.(6), we have \( \pi_i(0, e_i) = \epsilon \cdot (1 - \frac{1}{\sqrt{e}}) \cdot \frac{|D_i|}{|U|} \), where \( D_i \subseteq U \), and \( |U| - |D_i| \) is a value independent of \( i \)’s invitation behavior. This indicates that \( \pi_i(0, e_i) \) is monotonically increasing in \( |D_i| \). Moreover, we know that \( |D_i| \) is non-decreasing in \( |e_i| \). Therefore, when \( i \) does not contribute, she will invite as many neighbors as possible. These discussions finally give us Theorem 1.

**Theorem 1: The Bayesian Nash equilibrium under SIM is:**

for an arbitrary agent \( i \) with ability \( q_i \), if the number of invited agents is less than a critical number \( t(q_i) = \log_F(q_i) \frac{\sqrt{e}}{\sqrt{2}} + 1 \), then \( i \) contributes but invites no one; otherwise \( i \) invites all her neighbors but does not contribute.

**Remark 2: What Theorem 1 indicates is:** under SIM, the only thing mattering is the dynamically increasing number of registered agents. (i) During the registration phase, any agent only needs to monitor the increasing number of registered agents. Before it surpasses her critical number, she should invite no one. Once it surpasses her critical number, she should invite every neighbor. (ii) Then in the competition phase, she contributes only if she did not invited anyone during the registration phase.

In summary, SIM is very simple for agents to play and it will result in an interesting phenomenon: the growth of the registered population scares off some low-ability agents, who then try their best to make the population grow. Thus, SIM can provide a nice incentive whereby low-ability agents try their best to invite, while high-ability agents all contribute their abilities. Eventually, the high-ability agents in a very large crowd stand out, whereby a high-quality solution is obtained.

IV. COLLECTIVE INVITATION MECHANISM

SIM is simple for agents to play, its equilibrium is easy for the principal to predict, and it can also elicit high quality solutions. However, according to Theorem 1 and Remark 2, high-ability agents under SIM may still prefer not to invite anyone, which may limit the highest quality of the solutions. In this section, we design another crowdsourcing contest where all agents in the crowd prefer to invite their neighbors. We call it collective invitation mechanism (CIM).

A. Contest Mechanism Characterization

We first characterize the workflow of CIM. Like SIM, it also has three phases, but the information exposure and award rule by the mechanism are different.

1) **Registration phase**: Agents register in the contest and they can invite their friends to register. The principal collects the invitation record of each registered agent.

2) **Competition phase**: At the beginning of this phase, the principal constructs and publishes the whole invitation graph \( H \) to all registered agents. Then each agent decides whether to contribute her ability or not.

3) **Award phase**: Each agent who beats all the agents not led by herself is awarded a prize \( M \). Agents who do not contribute have no reward.

There are two differences of CIM comparing with SIM. One is information exposure. CIM does not publish any information in the registration phase, but publishes \( H \) when the competition phase starts. \( H \) contains not only the number of registered agents but also the structural information among them. The other difference is that CIM may select out multiple high-ability agents and award each a prize \( M \). The allocation rule of the award phase is in Algorithm 2.

Note that \( H \) is constructed by the principal using the invitation record of every agent. The principal can collect each agent’s invitation record in various ways. For instance, she can ask an agent to report whom she invited and who invited her. If two agents’ reports are consistent, then there is an edge between them in \( H \). The principal can also collect the invitation record via certain technologies such as blockchain or digital signature.

For ease of notation, denote \( P_i = U - D_i - \{i\} \). According to line-5 in Algorithm 2, only agents in \( P_i \) can be competitors of \( i \), but the set \( P_i \) is not affected by \( i \)’s own invitation. If \( i \) beats all \( j \) in \( P_i \), she will be selected out as a winner.
Throughout this section, \( P_i \) will be used as a key factor for analyzing agents’ behaviors. What is the intuition of designing such an award rule? Recall that by definition, \( D_i \) is the set of agents who can join the contest only when \( i \) invites her neighbors. If a mechanism allows a competition between \( i \) and agents from \( D_i \), then \( i \) will not invite anyone. Therefore, ensuring that any agent \( i \) does not compete with agents in \( D_i \) can provide her with incentives to invite.

According to the algorithm, there are two basic properties of CIM. Firstly, if there is a unique best contributor, then she wins prize \( M \). Secondly, the winners other than the best contributor must be her leaders. This is because for any winner who is not the best contributor, the best contributor cannot be in \( P_i \), otherwise \( i \) cannot be a winner.

We will analyze agents’ behaviors under CIM. Different from SIM whose equilibrium is all agents share a common threshold, we will prove that, under CIM, there exists an asymmetric Bayesian Nash equilibrium where each agent’s best response to the opponents is a threshold strategy and different agents have different thresholds. We will give an explicit form of agents’ thresholds.

### B. Strategy and Utility

First we will show that under CIM, the equilibrium strategies are also threshold strategies such that every agent contributes only if her ability surpasses her threshold.

With a fixed invitation graph, consider an agent \( i \) with ability \( q_i \). Given a strategy of another agent \( j \), the probability that \( i \) beats \( j \) when \( i \) contributes is \( P(a_j < q_i) \). This value is non-decreasing and continuous in \( q_i \). Under the i.i.d. assumption, the expected utility for \( i \) to contribute is

\[
\pi_i(q_i) = M \cdot \prod_{j \in P_i} P(a_j < q_i) - c, \tag{7}
\]

which is also non-decreasing and continuous in \( q_i \). Using this utility as well as the continuity and monotonicity of \( P \), we can obtain the following proposition.

**Proposition 3:** Under CIM, the Bayesian Nash equilibrium is a threshold equilibrium, in which each agent contributes only if her ability surpasses her own threshold.

**Proof:** First consider the case that \( i \) contributes with the upper-bound ability \( q_i = u \). No matter what \( j \) does, it is always that \( P(a_j < u) = 1 \). Thus Eq.(7) degenerates to

\[
\pi_i(u) = M - c > 0,
\]

while \( i \)'s utility when not contributing is 0. It means an agent with a close-to-upper-bound ability always contributes.

Next consider a case that agent \( i \) contributes with the lower-bound ability \( q_i = l \). With this lowest ability, \( i \) can beat \( j \) only when \( j \) does not contribute, which happens with the probability \( P(a_j = 0) < 1 \). Now Eq.(7) becomes

\[
\pi_i(l) = M \cdot \prod_{j \in P_i} P(a_j < l) - c.
\]

The sign of this utility is not determined. (1) If \( \pi_i(l) \leq 0 \), since \( \pi_i(q_i) \) is non-decreasing and continuous in \( q_i \) and we already know \( \pi_i(u) > 0 \), then there must be a value \( r_i \in [l, u) \) such that \( \pi_i(r_i) = 0 \). And \( i \) contributes only when her ability \( q_i > r_i \). (2) When \( \pi_i(l) > 0 \), since \( q_i \geq l \) and \( \pi_i(q_i) \) is non-decreasing and continuous in \( q_i \), \( i \) will always contribute. We call such agent \( i \) unconditional contributor under the second condition. The unconditional contributor is a special kind of agent under CIM. Appendix B.1 discusses more about them. For the unconditional contributors, we can simply set their thresholds as the lower-bound, i.e., \( r_i = l \). This does not change any results.

This proposition is intuitive. If an agent \( i \) does not contribute, her utility is 0. If she contributes, her utility \( \pi_i(q_i) \) is a non-decreasing and continuous function of her ability \( q_i \). Specifically, if she has the highest ability \( q_i = u \), then \( \pi_i \) is positive. The non-decreasing monotonicity of \( \pi_i \) ensures that there is a threshold value \( r_i \geq l \) such that \( \pi_i(q_i) \geq 0 \) only if \( q_i \geq r_i \). Proposition 3 says that we only need to focus on the threshold strategies. Note that the thresholds for different agents are not necessarily identical under CIM.

With this proposition and Definition 3 we can further know that, the probability for \( i \) to beat \( j \) by contributing is \( P(q_j \leq \max\{q_i, r_j\}) = F_{q_i}(\max\{q_i, r_j\}) \). Therefore under the i.i.d. assumption, Eq.(7) can be reformulated into

\[
\pi_i(q_i) = M \cdot \prod_{j \in P_i} F_{q_j}(\max\{q_i, r_j\}) - c. \tag{8}
\]

This formula is an agent’s expected utility regarding others’ thresholds. In the next subsection, it will be used as a key component for analysing agents’ invitation incentives.

### C. Patterns of Asymmetric Threshold

Proposition 3 is of great importance. It restricts the strategy space to the threshold form. Now based on structural analysis of leading relations among agents, we describe several basic patterns about the asymmetric thresholds of all agents.

The following lemma leverages the structural position of any agent, and it shows that threshold \( r_i \) of any agent \( i \) has an upper-bound.

**Lemma 2:** For any agent \( i \) under CIM, her ability threshold \( r_i \leq F^{-1} (\frac{1}{|P|} \sqrt{|P|}) \), where \( P_i = U - D_i - \{i\} \).

**Proof:** Prove by contradiction. Let \( s_i = F^{-1} (\frac{1}{|P|} \sqrt{|P|}) \). Suppose that there exists an agent \( i \) whose \( r_i > s_i \). When
Thus \( i \) will contribute with \( q_i < r_i \). This contradicts that \( r_i \) is the threshold.

Here the only variable is \(|P_i|\), which is the number of \( i \)'s competitors. These agents are not affected by \( i \)'s invitation. This means the upper-bound of an agent's threshold is independent from her invitation decision. Moreover, if two agents have leading relation, then the following lemma can be used to compare their thresholds.

**Lemma 3:** In the Bayesian Nash equilibrium of CIM, if agent \( i \) leads agent \( j \), then \( j \) has higher threshold than \( i \).

**Proof:** Suppose two agents \( i \) and \( j \) where \( i \) leads \( j \), then \( j \in D_i, D_j \subset D_i \) by definition, and thus \( P_i \subset P_j \). First consider \( j \). Since \( r_j \) is the threshold of \( j \), if the \( j \)'s ability \( q_j = r_j \), then her utility of contributing is

\[
\pi_j(q_j = r_j) = M \cdot \prod_{i \in P_j} F_{q_i} \left( \max \{ r_j, q_j \} \right) - c \geq 0.
\]

The product in this equality can be factorized as

\[
\prod_{k \in P_j} F_{q_k} \left( \max \{ r_k, r_j \} \right) \cdot \prod_{k \in P_j, k \not\in P_i} F_{q_k} \left( \max \{ r_k, r_j \} \right).
\]

(9)

Now consider \( i \). If \( i \)'s ability \( q_i = r_j \), then her utility of contributing is

\[
\pi_i(q_i = r_j) = M \cdot \prod_{k \in P_i} F_{q_k} \left( \max \{ r_k, r_j \} \right) - c,
\]

where the product is exactly the same as the first term in Eq.(9). Since the second term in Eq.(9) is a probability less than 1, we have \( \pi_i(q_i = r_j) > \pi_j(q_j = r_j) \geq 0 \). The utility of \( i \) can only be 0 if she does not contribute. This means if \( q_i = r_j, i \) will contribute. Therefore, \( r_i < r_j \).

This lemma shows that, we can rank different agents’ thresholds via sorting them according to the leading relations. This further reflects the fact that the structural information can help us to calculate the equilibrium behaviors. Specifically, agents not leading anyone must be leaf nodes in the ranking tree. The following definition formally defines the leaf agents.

**Definition 5 (Leaf Agent):** An agent \( i \) is an leaf agent if she leads no one (i.e. \( D_i = \emptyset \)).

Note that a leaf agent in the ranking tree does not necessarily mean she has no friends. It only means this agent is not a cut node in the invitation graph. For an agent of the leaf type, unilaterally altering her invitation does not change the group of invited agents. Proposition 4 proves that all leaf agents \( z \) have an identical threshold, and the proof details can be found in Appendix B.2.

**Proposition 4:** In the Bayesian Nash equilibrium under CIM, every agent \( z \) who is a leaf node in the ranking tree of the invitation graph has an identical threshold \( r_z = F^{-1} \left( \frac{|U|}{\sqrt{M}} \right) \), which is the highest among all agents’ thresholds.

We know that for all leaf agents, the value \(|P_i| = |U| - 1\), which is the largest among all agents. Combining with Lemma 2, we can know a leaf agent’s threshold is larger than any non-leaf agent’s threshold. Moreover, the above proposition holds for any \( H \), including an extreme case where every agent is of leaf type (i.e., there is no cut node in \( H \)) and all agents share a common threshold.

**Remark 3:** In the Bayesian Nash equilibrium under CIM, (i) An agent contributes only if her ability surpasses her threshold. (ii) As long as some agents are not of leaf type, there is no common threshold. (iii) Leaf agents have the highest threshold.

D. Invitation in Asymmetric Equilibrium

The three points in Remark 3 are straightforward from the previous results. Point (i) is from Proposition 3, point (ii) is from Lemma 3, and point (iii) is from Proposition 4. Especially, from point (iii), it is apparent that under CIM, no agent wants to be of leaf type, which incentivizes each agent to invite her neighbors so as to make herself leading some agents. The following Lemma formally proves this.

**Lemma 4:** Under CIM, for any agent, she can decrease her threshold by inviting her neighbors.

**Proof:** Consider an arbitrary agent \( i \). Given the other agents’ invitations, when \( i \) invites no one, she becomes leaf type. Suppose in this case the whole agent set is \( U \) and \( i \)'s threshold is \( r_i \). When \( i \) invites some (or all) of her neighbors, suppose the whole agent set is \( U' \) and her threshold is \( r_i' \). It is straightforward that \( U \subseteq U' \).

When \( i \) invites no one, \( i \) is a leaf agent. By Proposition 4, we have \( r_i = F^{-1} \left( \frac{|U|}{\sqrt{M}} \right) \) and \( F(r_i') \left| \frac{|U|}{\sqrt{M}} \right| = \frac{1}{\sqrt{M}} \). Note that \( r_i \) is also the upper-bound threshold shown in Lemma 2.

When \( i \) invites her neighbors, if \( U = U' \), \( i \) is still of leaf type and \( r_i' = r_i \). Otherwise, \( U \subset U' \), but \( i \)'s competitor set is still \( P_i \) and some agents in \( P_i \) would still be of leaf type. According to Proposition 4, a leaf agent \( z \in P_i \) has the threshold \( r_z' = F^{-1} \left( \frac{|U'|}{\sqrt{M}} \right) \). Since \( |U| < |U'| \), it is easy to know \( r_z' > r_i \). Based on this, if \( i \)'s ability \( q_i = r_i \), then her utility for contributing and inviting is

\[
\pi_i(U', q_i = r_i) = M \cdot F_{q_i} \left( \max \{ r_i', r_i \} \right) \cdot \prod_{k \in P_i, k \not\in P_i} F_{q_k} \left( \max \{ r_k, r_i \} \right) - c \geq M \cdot F_{q_i} \left( r_i' \right) \cdot \prod_{k \in P_i, k \not\in P_i} F_{q_k} \left( r_i \right) - c
\]

\[
= M \cdot \prod_{k \in P_i} F_{q_k} \left( r_i \right) - c = 0.
\]

In the above formula, line-2 classifies competitors into two groups, i.e., leaf agent \( z \) and other agents. Line-3 to line-4 is by the previous result that \( r_i' > r_i \). The equality part in line-4 is by the facts that \(|P_i| = |U| - 1\) and \( r_i = F^{-1} \left( \frac{|U|}{\sqrt{M}} \right) \) in Proposition 4.

Now we know \( \pi_i(U', q_i = r_i) > 0 \). Since \( \pi_i \) is monotonically increasing in \( q_i \), there must be a smaller value \( r_i' < r_i \).
that makes $\pi_i(U', q_i = r'_i) = 0$. This $r'_i$ is the threshold when $i$ invites her neighbors. Therefore, an agent can decrease her threshold by inviting her neighbors.

This lemma points out the fact that inviting neighbors reduces the requirement on an agent’s ability. The intuition is: when an agent invites her neighbors, there is a chance for her to lead some agents and avoid being a leaf agent who is proved to have the upper-bound threshold. The following theorem goes one step further and formally shows that in equilibrium, all agents will invite someone.

**Theorem 2 (Collective Invitation):** Under CIM, every agent invites some or all of her neighbors in Bayesian Nash equilibrium.

**Proof:** By Lemma 4, agent $i$’s threshold when she invites is less equal than that when she doesn’t invite, i.e., $r'_i \leq r_i$. Given $q_i$, there are three cases of the order of $q_i, r_i$ and $r'_i$.

1) $q_i < r'_i \leq r_i$. In this case, $i$ doesn’t contribute either she makes invitation or not. Then $\pi_i(U', q_i) = \pi_i(U, q_i) = 0$, where $U'$ is the agent set when $i$ invites, and $U$ is that when $i$ does not invite.

2) $r'_i < q_i \leq r_i$. In this case, if she doesn’t make any invitation in the registration phase, $i$ won’t contribute in the competition phase, and then her utility is 0; if she invites in the registration phase, $i$ will contribute in the competition phase and then obtain utility greater than 0. Therefore, $\pi_i(U', q_i) > \pi_i(U, q_i)$.

3) $r'_i \leq r_i < q_i$. In this case, $i$ will contribute in the competition phase no matter whether she has invited or not. $r_i$ is $i$’s threshold when she is a leaf agent, thus by the threshold upper-bound in Lemma 2, for any $k \in P_i$, $r_k \leq r_i$. Thus, $r_k < q_i$, and

$$\pi_i(U', q_i) = \mathcal{M} \cdot \prod_{k \in P_i} F_{q_k}(\max(r'_k, q_i)) - c \geq \mathcal{M} \cdot \prod_{k \in P_i} F_{q_k}(q_i) - c = \mathcal{M} \cdot \prod_{k \in P_i} F_{q_k}(\max(r_k, q_i)) - c = \pi_i(U, q_i).$$

Considering all these the cases, inviting friends is better off for every agent under CIM.

From Theorem 2, we know that CIM provides a good incentive such that in the Bayesian Nash equilibrium, every agent would invite some, if not all, of her neighbors.

**E. Contribution in Asymmetric Equilibrium**

Now we derive an expression of an arbitrary agent’s threshold. For an arbitrary $i$, if her ability just meets her threshold, i.e., $q_i = r_i$, her utility is 0. Then Eq.(8) can be rewritten as

$$\pi_i(r_i) = \mathcal{M} \cdot \prod_{j \in J_i} F_{q_j}(r_j) \cdot \prod_{j \in Q_i} F_{q_j}(r_j) - c = 0, \quad (10)$$

where $J_i$ is the set of $i$’s competitors whose thresholds are larger than $r_i$, and $Q_i$ is the set of $i$’s competitors whose thresholds are not larger than $r_i$. That is, $P_i = J_i \cup Q_i$. Under the i.i.d. assumption, $F$ is identical for different $j$. Then we can rewrite Eq.(10) as

$$F(r_i)_{Q \cup J} = \frac{c}{\mathcal{M}} \prod_{j \in J_i} F(r_j).$$

This formula together with Proposition 4 immediately leads to the following theorem, which is our major result.

**Theorem 3 (Private Threshold):** In the Bayesian Nash equilibrium of CIM, the ability threshold of any leaf agent $z$ is

$$r_z = F^{-1}\left(\frac{c}{\sqrt{\mathcal{M}}} \prod_{j \in J_z} F(r_j)\right), \quad (11)$$

while that of any non-leaf agent $i$ is

$$r_i = F^{-1}\left(\frac{c}{\sqrt{\mathcal{M}}} \prod_{j \in J_i} F(r_j)\right). \quad (12)$$

The special case where $|Q_i| = 0$ is handled in Appendix B.1. Recall that $J_i$ is the set of competitors whose thresholds are greater than $i$’s. Theorem 3 tells us that, to decide $r_i$, we need to know $r_j$ of each $j \in J_i$, as well as the size of $Q_i$. For this purpose, we can devise an algorithm to traverse all nodes and recursively compute each $r_i$ by leveraging Theorem 3.

So far, we have shown how agents make invitation decisions in the registration phase and also given the formula of their ability thresholds. But still, we have not seen the detailed invitation behaviors and contributions from agents in equilibrium.

In the next section, we will propose a computational method to characterize the agents’ equilibrium behaviors under CIM.

**V. Equilibrium Computation**

Now we propose a computational method for automatically searching the asymmetric equilibrium of CIM. We first show that it is possible to rank agents’ ability thresholds even without knowing the values of these thresholds. The key idea is to find another computable numerical value which is monotone in $r_i$. Since this mediator value has a one-to-one correspondence relation with the agent’s threshold, we can rank the thresholds by ranking agents’ mediator values.

We then design an algorithm to compute the agents’ thresholds in equilibrium by using this ranking and the ability threshold’s formula in Eq.(12). These thresholds finally determines the equilibrium and the performance of CIM.

**A. Monotonicity of Thresholds**

In the competition phase of CIM, for agent $i$, if all the agents led by her (i.e. $D_i$) do not have enough ability to contribute, we say $i$ is a leader of mediocre (lom). Denote the probability for $i$ to be a lom by $p_{lom}^i$, which is calculated as

$$p_{lom}^i = P(\forall k \in D_i, q_k < r_k) = \prod_{k \in D_i} F_{q_k}(r_k). \quad (13)$$

Recall that $F_{q_k}(r_k)$ is the probability that $k$’s ability is below her threshold $r_k$, thus the product is the probability that no agent in $D_i$ can contribute.

The intuition behind the probability $p_{lom}^i$ is: if agent $i$ has a small $p_{lom}^i$, it is more likely that $i$ invites some contributors who have enough ability. An agent with small $p_{lom}^i$ has a
more important social position, for the subtree she leads is more competitive. Therefore, the invitation from an agent with small $p_{i}^\text{lom}$ is more likely to scare off her competitors (i.e. the thresholds of agents in $P_i$ are high due to $i$’s invitation). When $i$’s competitors are scared off, $i$’s threshold becomes smaller. The following two lemmas give a formal proof for that agents with small $p_{i}^\text{lom}$ has low thresholds.

**Lemma 5:** For any two agents $i$ and $j$ who don’t have leading relation (i.e. $i \not= j$ and $j \not= i$), if $p_{i}^\text{lom} < p_{j}^\text{lom}$, then $r_i < r_j$.

**Lemma 6:** If $p_{i}^\text{lom} = p_{j}^\text{lom}$, then $r_i = r_j$.

These two lemmas are simple but profound, they are crucial for our major result. Recall that our final objective is to compute agents’ thresholds in the asymmetric BNE, where each agent has her own threshold. $p_{i}^\text{lom}$ is a breakthrough point. This is because, given a ranking tree, different agents’ $p_{i}^\text{lom}$ can be computed in a bottom-up recursive way. More importantly, these two lemmas reveal that, to sort agents’ thresholds in equilibrium, we only need to rank their $p_{i}^\text{lom}$, which is more operational. This makes it possible to design a recursive algorithm for equilibrium computation. The proofs of these lemmas use similar techniques, but they need some calculation and are space consuming. We left the proofs into Appendix C1 and C2.

From Lemma 3, Lemma 5 and Lemma 6, it’s easy to derive that, in any equilibrium under CIM, $\forall i, j \in U(i \not= j)$,

\[
\begin{align*}
 r_i = r_j \iff p_{i}^\text{lom} = p_{j}^\text{lom}, \\
 r_i < r_j \iff p_{i}^\text{lom} < p_{j}^\text{lom}.
\end{align*}
\]  

(14)

Thus we can directly have the following Proposition 5 which serves as a corner stone for designing an algorithm to compute the BNE of CIM.

**Proposition 5 (Monotonicity of Threshold):** In the Bayesian Nash equilibrium of CIM, for any two agents $i$ and $j$, $r_i < r_j$ iff $p_{i}^\text{lom} < p_{j}^\text{lom}$.

The significance of Proposition 5 is, it shows that if we want to rank all agents’ thresholds, we just need to rank the values of all $p_{i}^\text{lom}$, which is more easily accessible than $r_i$.

**B. Recursive Algorithm**

Based on Theorem 3 and the fact that the leaf agents have the highest threshold, we design Algorithm 3 to compute the Bayesian Nash equilibrium of CIM.

The algorithm has two key temp sets $A$ and $T$, which are updated iteratively. Set $A$ stores all agents whose thresholds are already known and set $T$ contains agents whose thresholds will be computed in the current iteration. In each iteration, the current $T$ is computed with information from the previous $A$. Then the obtained current $T$ is added to $A$.

The algorithm has four components. In initialization, the ranking tree $T$ is generated. Set $A$ is initialized with all leaf agents (i.e. leaf nodes in $T$), whose common threshold is computed by Eq.(11). In each iteration, using the thresholds of agents in $A$, the second component checks whose $p_{i}^\text{lom}$ values are ready to compute (line 10), and computes them by Eq.(13) (line 11). After that, in the third component, the obtained $p_{i}^\text{lom}$ values are sorted (line 13), and agents with the largest $p_{i}^\text{lom}$ are selected out and moved to the temp set $T$ (line 14). Finally, still using thresholds of agents in $A$ and by Eq.(12), the fourth component computes the thresholds of all nodes in $T$, and then attaches $T$ to $A$. The following workflow explains this algorithm in detail.

1) **Initialization** (lines 1 - 7): With the set of all invited agents $U$ and the invitation graph $\mathcal{H}$, it is easy to compute the leaf agent set $Z$, and $D_1$ and $P_1$ for every $i$. It is also shown that the threshold of leaf agents $Z$ can be directly computed by Eq.(11), which is done in the loop.

2) **Computing** $p_{i}^\text{lom}$ (lines 9 - 12): If the thresholds $r_k$ of $i$’s descendants $k \in D_i$ are all known, then using Eq. (13), we can calculate $p_{i}^\text{lom}$ for agent $i$. Lines 9 - 10 traverse $U$ and find every $i$ whose threshold is not yet obtained (i.e., $r_i = \text{nil}$) but can be computed (i.e., $\forall k \in D_i, r_k \neq \text{nil}$). Line 11 computes $p_{i}^\text{lom}$ of these agents.

3) **Sorting** $p_{i}^\text{lom}$ (lines 13 - 14): With Proposition 5, we can sort $r_i$ via sorting $p_{i}^\text{lom}$. Given all the agents whose $r_i$ is not known but $p_{i}^\text{lom}$ have been obtained, this component ranks them and selects out those with the highest $p_{i}^\text{lom}$.

4) **Computing** $r_i$ (lines 15 - 18): With the largest $p_{i}^\text{lom}$ agents, this component computes $r_i$ for these agents following Eq.(12). In each iteration of the while loop, there is a new set of the largest $p_{i}^\text{lom}$ agents $T$. Set $A$ is updated in each iteration, and gradually expands until all agents’ thresholds have been obtained.

### Algorithm 3 Computing Threshold in BNE of CIM

**Input:** invited agents $U$, invitation graph $\mathcal{H}$

**Output:** thresholds $r = (r_1, \ldots, r_i, \ldots, r_U)$

1. generate the ranking tree $T$;
2. compute $D_i$ and $P_i$ for all $i \in U$;
3. initialize flag $\phi_i = 0$ and $r_i = \text{nil}$ for all $i \in U$;
4. find leaf node set $Z$;
5. for each leaf node $i \in Z$ do
   6. assign $r_i$ with the value in Eq.(11);
7. initialize threshold-known node set $A = Z$;
8. while $A \neq U$ do
   9. for each $i$ whose $\phi_i = 0$ and $r_i = \text{nil}$ do
      10. if $\forall k \in D_i, r_k \neq \text{nil}$ then
         11. compute $i$’s $p_{i}^\text{lom}$ value by Eq.(13);
         12. set $i$’s flag to $\phi_i = 1$;
   13. rank all $i$ whose $\phi_i = 1$ by descending $p_{i}^\text{lom}$;
   14. move all top $i$ into temp set $T$;
   15. for all $i \in T$ do
      16. use $J_i = A \cap P_i$ to compute $r_i$ by Eq.(12);
      17. reset $\phi_i = 0$;
   18. update $A = A \cup T$; then reset $T = \emptyset$;
9. return $r$.

We would like to emphasize to the readers that, in each iteration of the while loop, we only select out the highest $p_{i}^\text{lom}$ agents to form $T$ and compute their thresholds $r_i$. Recall that $r_i$ is monotone in $p_{i}^\text{lom}$ (subsection V-A). Therefore,
the thresholds computed in any previous iterations must be higher than that computed in the current iteration. Technically, in each iteration of the while loop, every \( r_i \) is computed using the previously obtained opponent set \( J_i \), in which all agents must have higher thresholds than \( i \). Such a computation is an implementation our major theoretical result Theorem 3. It is also the spirit for the recursion in Algorithm 3. Given the above analysis about the while loop and the inner loops, it is not hard to know the time complexity of Algorithm 3 is \( \mathcal{O}(n^3) \) where \( n \) represents the crowd size. The detailed analysis is in Appendix C3.

With Proposition 5, Theorem 3 and the characterization of Algorithm 3, now we summarize the result for the computation of BNE under CIM.

**Theorem 4:** Under CIM, it is a Bayesian Nash equilibrium that every agent invites some or all of her neighbors and their ability thresholds are computed by Algorithm 3.

### C. Efficient Computation on Large Graphs

In this subsection, we first analyze to what extent agents are willing to invite friends and whether it is possible that "all agents invite all neighbors" forms the Bayesian Nash equilibrium. We then adapt Algorithm 3 to large graphs.

Recall in Eq.(8), the expected utility for \( i \) to contribute with ability \( q_i \) depends on the value of equilibrium thresholds of \( i \)'s competitors \( j \in P_i \). By definition of Bayesian Nash equilibrium, we need to verify that inviting all is the best response when all other agents invite all. This means we need to examine whether there exists a profitable unilateral deviation from “fully inviting” to “partially inviting”. One needs to traverse every agent’s whole invitation action space (i.e. the power set of the neighbor set), which is exponential to the number of neighbors. Thus for large graphs, it is crucial to simplify this procedure.

According to subsection II-C, if different invitation graphs have identical ranking tree structure, then these different graphs have identical input for Algorithm 3. It is significantly important, the type-checking work can be done efficiently with a bottom-up recursion algorithm.

Generally, for two agents \( i \) and \( j \) of the same type, if there is that \( r_{d_i} = r_{d_j} \) for every pair of \( (d_i, d_j) \in \mathcal{R}_{i,j} \), then by Proposition 5, we know \( p_i^\text{loc} = p_j^\text{loc} \). Moreover, according to the definition of \( p_i^\text{loc} \) in Eq.(13), we have

\[
\prod_{h \in D_{d_i}} F_{q_h}(r_h) = \prod_{k \in D_{d_j}} F_{q_k}(r_k).
\]

Since \( r_{d_i} = r_{d_j} \) and \( \mathcal{R}_{i,j} \) is bijective, we also have:

\[
\prod_{d_i \in D_i} F_{q_{d_i}}(r_{d_i}) = \prod_{d_j \in D_j} \left( F_{q_{d_j}}(r_{d_j}) \cdot \prod_{h \in D_{d_j}} F_{q_h}(r_h) \right)
= \prod_{d_j \in D_j} \left( F_{q_{d_j}}(r_{d_j}) \cdot \prod_{k \in D_{d_j}} F_{q_k}(r_k) \right)
= \prod_{d_j \in D_j} F_{q_{d_j}}(r_{d_j}).
\]

This yields \( p_i^\text{loc} = p_j^\text{loc} \), which finally gives us \( r_i = r_j \) by Lemma 6. With the same process we can inductively determine the type of each agent in the ranking tree, whereby the following theorem is obtained.

**Theorem 5:** In the Bayesian Nash equilibrium of CIM, agents of the same type have an identical threshold. The threshold of every type is computed via a bottom-up recursion on the ranking tree.

Theorem 5 shows the property of contribution behaviors of the agent types. With Algorithm 3 and Theorem 5, we have the following corollary, which reveals the property of invitation behaviors of the agent types.

**Corollary 2:** Given all agents fully invite all their neighbors, if agent \( i \) has no incentive to deviate from "fully inviting", then every agent of \( i \)'s type has no incentive to deviate from "fully inviting".

![Fig. 2. An example ranking tree](image-url)
Fig. 3. Equilibrium performances of invitation contests mechanisms SIM and CIM over 1000 different tasks. We compare them with traditional contest mechanism with no invitation (MN), as well as a naive random invitation contest (RI) where each agents has a probability of 10% to invite all her neighbors. Both SIM and CIM provide strong incentive for agents to invite, while MN and RI do not provide incentive for invitation.

We illustrate this corollary with a detailed example in Appendix D1. With Theorem 5 and Corollary 2, we can further simplify Algorithm 3 by only calculating threshold of one agent of each type, rather than all agent of that type. This significantly reduces the computation complexity, making it possible to compute BNE for large graphs.

VI. EXPERIMENTS
We implement our contest mechanisms on a well-known Facebook network dataset with 4039 nodes and 88234 edges [42]. We randomly chose three nodes as principals. Their degrees are 5, 25, and 153, respectively, representing that they have different levels of social resources. On this network, we implement four different contest mechanisms, including SIM, CIM, a traditional contest mechanism with no invitations (abbreviated as MN), as well as a naive random invitation contest in which the probability for any agent to invite her neighbors is 0.1 (abbreviated as RI).

For each of the three principals and each of the four contest mechanisms, we simulate 1000 different tasks. That is, there are totally $3 \times 4 \times 1000 = 12000$ contests simulated in our experiments. For each single task, nodes’ abilities $q_i$ are sampled from the exponential distribution $E(\lambda = 1)$. This sampling well captures that people’s abilities for different tasks may show a large diversity in crowdsourcing while people with high abilities are very rare. With the simulation data, we evaluate the performances of SIM and CIM in the following key criteria: the overall performance in Fig. 3, the crowd coverage and the budget feasibility in Table I, as well as the dynamics of the highest solution quality in Fig. 4.

A. Overall Performance and Comparison
In Fig. 3, each column is for a fixed principal with a certain degree; each row is for a contest mechanism, and from the top row to the bottom row, we have MN, RI, SIM, and CIM, respectively. Each subfigure is the statistical result of 1000 different tasks. The $x$-axis is the bins for the highest quality in each task. The $y$-axis is the proportion of the bins in 1000 tasks. Each colored chart depicts the distribution of the best solution within 1000 tasks.

Note that the random invitation contest RI is an analog to the real-world invitation among people, which works as a naive baseline for verifying SIM and CIM. Note that, RI’s invitation probability 0.1 is high enough compared with people’s invitation probability in a real contest.

1) The best solutions under SIM and CIM are much better than those under MN and RI. It can be seen by comparing each row in a column. MN and RI can never surpass SIM or CIM, even for the principal with a very large degree. This is because SIM and CIM provide strong incentives for inviting and contributing.

2) Both SIM and CIM are robust against the principals’ social resource diversity. This can be seen by comparing the columns in the third and fourth rows. The best solution distribution does not change too much for different principals. This robustness makes SIM and CIM not only work well for companies or big organizations, but also have advantages for principals who have limited social resources. This is especially suitable for websites like Github or StackExchange where most principals are normal users with small degrees.

3) In all the subfigures, the black bars above $x = 0$ depict the chances that no agent contributes under this mechanism. We can see that MN, RI, and SIM all suffer from these failure chances. In sharp contrast, CIM completely solves this problem.
B. Crowd Coverage and Budget Feasibility

Recall that according to the theoretical result about SIM (see Theorem 1 and Remark 2), in BNE, low-ability agents try their best to invite while high-ability agents contribute but do not invite. Thus it is possible that not all agents are invited into the contest under SIM. To inspect to what extent agents in a crowd can be invited, we simulate 1000 tasks using SIM on the Facebook dataset. It is observed that in almost all 1000 contests, all agents in the crowd are invited. Among these contests, the ratios that all agents are invited range from 98.8% to 99.4%, depending on the principal’s degree. The detailed statistics are in the middle box of Table I. For example, the item 4038 : 98.8% means that in 98.8% of the 1000 tasks, all the 4038 agents are invited.

Regarding the crowd coverage under CIM, it is even more satisfactory, in all the experiments under CIM, we find that every agent always fully invites all her neighbors. More importantly, from all 1000 contests under CIM, we find that when all the other agents invite all their neighbors, “inviting all her neighbors” is i’s best response. This indicates that, every agent invites all her neighbors should be a Bayesian Nash equilibrium of CIM. This is a strong evidence for that CIM performs even better than what Theorem 3 asserts. It provides a very strong incentive for invitation.

Regarding the budget, since SIM only selects out one winner, it is always budget feasible. For CIM, in 98.6% of all the 1000 tasks, only one winner is selected. This indicates that, although CIM may select out multiple winners and award each of them, such multi-winner outcomes are very rare. Specifically, even when there is more than one winner, in all the experiments of CIM, the principal only needs to award at most two winners. These results are in the right box of Table I. For example, the item 1 : 98.3% means among all 1000 contests under CIM, 988 contests select out only 1 winner. Moreover, the item 2 : 1.2% means all the remaining 12 contests select out only 2 winners. This means CIM not only guarantees high-quality solutions; it is also nearly budget-feasible since in expectation, the principal’s expenditure is very close to her budget for one prize.

C. Dynamics of Highest Quality

In this part, we will reveal how the population size n of the crowd N affects the highest quality of the solutions elicited by these mechanisms. By the notion highest quality, we mean the quality of the best solution among all the contributed solutions from agents. Given a number n of agents with i.i.d. abilities, we will first find the upper bound of the performance of any crowdsourcing contest. Then, we will numerically study how the highest qualities in our mechanisms evolve as n increases and compare them with this upper bound.

Consider n agents with an arbitrary network structure. The highest ability among the n agents is the largest order statistic $q_{(n)}$ whose cdf is

$$
\hat{F}_{q_{(n)}}(x) = \prod_{i \in [n]} P(q_i \leq x) = \prod_{i \in [n]} F_{q_i}(x) = F(x)^n.
$$

Thus the PDF of $q_{(n)}$ is

$$
\hat{f}(x) = \frac{d\hat{F}(x)}{dx} = n \cdot F(x)^{n-1} \cdot f(x).
$$

And the expected value of $q_{(n)}$ is

$$
E(q_{(n)}) = \int_0^\infty x \cdot \hat{f}(x)dx = \int_0^\infty n \cdot x \cdot F(x)^{n-1} \cdot f(x)dx.
$$

This is the expectation of the highest ability in a crowd of n agents. It is also the upperbound of the feasible highest quality in any crowdsourcing contest. Traditional crowdsourcing contests cannot reach this upperbound, simply because the number of agents in them cannot reach the value $n$.

To verify the worst-case performance of SIM and CIM, we choose a node with very small degree (d = 5) as the principal. If there is no invitations (i.e., by traditional contests), the task posted by the principal can only acquire d = 5 agents. If we try our best to invite while high-ability agents contribute (see Theorem 1 and Remark 2), in BNE, low-ability agents try their best to invite while high-ability agents contribute but do not invite. Thus it is possible that not all agents are invited into the contest under SIM. To inspect to what extent agents in a crowd can be invited, we simulate 1000 tasks using SIM on the Facebook dataset. It is observed that in almost all 1000 contests, all agents in the crowd are invited. Among these contests, the ratios that all agents are invited range from 98.8% to 99.4%, depending on the principal’s degree. The detailed statistics are in the middle box of Table I. For example, the item 4038 : 98.8% means that in 98.8% of the 1000 tasks, all the 4038 agents are invited.

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C. Dynamics of Highest Quality

In this part, we will reveal how the population size n of the crowd N affects the highest quality of the solutions elicited by these mechanisms. By the notion highest quality, we mean the quality of the best solution among all the contributed solutions from agents. Given a number n of agents with i.i.d. abilities,
the participant population expands gradually. At the very beginning of the contest, only the principal’s 5 neighbors participate and the highest quality is low. Then it increases rapidly before \( n = 1000 \). After that, it grows steadily with the increasing population size.

Both invitation mechanisms show excellent performances. The expectation of the highest quality they provide is close to the upper bound of any mechanism’s feasible quality. Moreover, CIM performs even better and is more stable than SIM. We do not show the performances of the existing crowdsourcing contest models since they do not have incentive design, and their population sizes can not be guaranteed in BNE. In most cases, their participant populations are limited to a fixed small group.

VII. CONCLUSION

Existing crowdsourcing contest models only consider the agents who directly link to the principal, while a large amount of agents in the social network who do not know the task could not participate. We extend contest design theory to social network environments and introduce new contest mechanisms whereby agents are impelled to invite their friends. Our new invitation model and contest mechanisms capture the commonly seen scenarios where an agent can exert influence on the crowdsourced task not only by her own ability but also via her social influence. In contrast to traditional crowdsourcing contest methods, our new mechanisms can discover high-ability agents who hide deeply in the social network, which significantly improves the crowdsourcing quality. Our work is the first attempt to identify how social network structure affects agents’ incentives in the equilibrium of a crowdsourced task. Future research could be the extensions of the invitation contests under complex settings, such as sequential contests, contests with uncertain performance, and so on. Another future direction is to investigate the change in the principal’s profit when more agents are invited into the contest. Moreover, applying invitation contest mechanisms to various scenarios, such as privacy-preserving crowdsourcing [43] and agent learning crowdsourcing [44], is also good topics.

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