Solution Concepts in Hierarchical Games with Applications to Autonomous Driving

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Abstract

With autonomous vehicles (AV) set to integrate further into regular human traffic, there is an increasing consensus of treating AV motion planning as a multi-agent problem. However, the traditional game theoretic assumption of complete rationality is too strong for the purpose of human driving, and there is a need for understanding human driving as a bounded rational activity through a behavioral game theoretic lens. To that end, we adapt three metamodels of bounded rational behavior; two based on Quantal level-k and one based on Nash equilibrium with quantal errors. We formalize the different solution concepts that can be applied in the context of hierarchical games, a framework used in multi-agent motion planning, for the purpose of creating game theoretic models of driving behavior. Furthermore, based on a contributed dataset of human driving at a busy urban intersection with a total of ~4k agents and ~44k decision points, we evaluate the behavior models on the basis of model fit to naturalistic data, as well as their predictive capacity. Our results suggest that among the behavior models evaluated, modeling driving behavior as pure strategy NE with quantal errors at the level of maneuvers with bounds sampling of actions at the level of trajectories provides the best fit to naturalistic driving behavior.

INTRODUCTION

Motion planners are a critical component of autonomous vehicle (AV) architecture, and the decisions made by the algorithms impact the safety of road users, such as pedestrians, cyclists, and other human-driven vehicles. Traditional approaches to motion planning have typically treated the problem as a single-agent problem; in this perspective, a vehicle interacts with the environment (in simulation or on-field setting), possibly with the help of recorded human-driven trajectories, and plans its actions by optimizing over its objectives while taking into account the dynamic obstacles in the vicinity (Schwarting, Alonso-Mora, and Rus 2018; Ilievski et al. 2019). However, in reality human driving is a complex system with a symbiotic relation among agents, where actions of a vehicle influence the future actions of other road users and vice versa. More recently, there has been a focus towards treating motion planning of AVs as a multi-agent problem with game-theoretic solutions to AV decision making (Fisac et al. 2019; Sadigh et al. 2016; Camara et al. 2018; Li et al. 2018). Such approaches can account for heterogeneous objectives in a group of vehicles in a traffic scene and identify equilibrium solutions that guide the actions of the AV. Given that the movement dynamics of a vehicle is in continuous domain, it is intuitive to model the dynamics as a differential game, an approach adopted by multiple models in the literature (Fridovich-Keil et al. 2019; Sadigh et al. 2016; Wang et al. 2019). However, the applicability of such games as a general purpose planner is limited by the trade off between the computational burden and expressivity; cases where efficient solutions exist in a multi-agent setting restrict the behavior of the agents to only linear dynamics (Fridovich-Keil et al. 2019). As an alternative, (Fisac et al. 2019) models the planning problem as a hierarchical game where the game is decomposed into two levels; a long-horizon strategic game that can model richer agent behavior and a short-horizon tactical game with simplified information structure. Although hierarchical games are well suited and show promising results for planning in AV, for the models to be applicable in real world situations, we need to understand how well the stationary concepts in the game match naturalistic human driving behavior. It is well known that in many realistic settings, the theoretical fixed point of Nash equilibrium is a poor predictor of human behavior (Goeree and Holt 2001); therefore, it is necessary to investigate if the same is true for human driving behavior too. In absence of that information, we do not know what to optimize for.

Behavioral game theory provides a framework to analyse decision making in a naturalistic setting and models of behavior that often have higher predictive power than Nash equilibria (Camerer 2011). A key element in behavioral game theory is bounded rationality, where the conventional game-theoretic notion of agents as fully rational is relaxed to allow for sub-optimal behavior. Such behavior may arise from limitations in cognitive reasoning, or error-prone actions (Samuelson 1995). Driving is a cognitively demanding job that requires situational awareness and sophisticated visuomotor co-ordination, added on to individual habits, biases, and preferences; and it is not hard to imagine that driving at its core is a bounded rational activity. Consequently, it becomes essential for AV game theoretic planners to be able to characterize the bounded rational behavior in human driv-
ing: for example, if humans are prone to making error in judgement when the signal is about to turn red from amber at a busy intersection, then the AV planner should take that into account since the safety of the AV decision is conditioned on the error made by the human driver. Therefore, developing a game-theoretic planner for an AV is a multi-step process, broadly involving a) selection of the right behavior model and equilibrium concepts for other road agents, b) estimation of the parameters of the model, and c) generation of a safe maneuver and trajectory after accounting for the model and its parameters. In this paper, we primarily focus on the first two aspects. Wright and Leyton-Brown developed a general framework of analysing and estimating parameters of popular behavioral game theory models based on observations of game play focussing on two models of behavior, i.e., Quantal Level-k (QLk) and Poisson-Cognitive Hierarchy (P-CH) (Wright and Leyton-Brown 2012). Although QLk and P-CH do not capture all types of bounded rationality that one can think of in the case of human driving, such as the ones that arise from sampling the actions of other agents, the framework developed in (Wright and Leyton-Brown 2012) nevertheless can be applied to a wider set of behavior models including the ones we develop in this paper.

In this paper we make the following contributions: (i) We formalize the concept of a hierarchical game that connects research in traffic psychology with motion planning for autonomous driving. (ii) We extend three models of behavior from behavioral game theory, two based on Quantal level-k and one based on Nash equilibrium with quantal errors, and demonstrate the possible solution concepts that can be applied to hierarchical games. (iii) In order to better understand strategic and non-strategic decision making in human driving, we compare 25 game theoretic models based on a cross sectional study of human drivers at a busy urban intersection with a total of 3913 agents and 43765 decision points. We make this dataset, which is one of the largest multi-agent behavior dataset of human driving, publicly available as a contribution.

**Hierarchical games**

Prior to recent focus in autonomous driving, there has been considerable body of research on modeling driving behavior within the field of traffic psychology with a long history of treating driving behavior as a hierarchical model (Keskinen et al. 2004, Van der Molen and Bötticher 1988, Lewis-Evans 2012, Michon 1985). A primary motivation of a hierarchical decomposition is that drivers have different motivations and risk judgements in each level of the hierarchy, and the functional decomposition into a hierarchical system allows for modelling the risk and safety considerations separately at each level. Motion planners in autonomous vehicles also follow a similar hierarchical pattern of decomposition; a high level route planner plan is given to a behavior planner, which sets up the tactical maneuvers for a lower level trajectory planner, which in turn generates the trajectory profile for the vehicle controller after respecting its nonholonomic constraints. In addition to the motivation mentioned earlier, treating the problem of planning as a hierarchical system is also driven by computational efficiency as previously shown in (Fisac et al. 2019).

In a multi-agent setting, this means that the planning problem has to be extended to the notion of a hierarchical game, which we formalize further below. A hierarchical game is formulated by

- A set of $N$ agents indexed by $i ∈ \{1, 2, 3, \ldots, N\}$.
- A set of $K$ levels indexed by $\kappa ∈ \{1, 2, 3, \ldots, K\}$.
- A set of actions $A_{i, \kappa}$ available to each agent $i$ at level $\kappa$.
- A strategy $s_i$ for agent $i$ is a $K$-tuple $s_i = (a_{i,1}, a_{i,2}, \ldots, a_{i,K})$ where $a_{i,\kappa} ∈ A_{i,\kappa}$ and the strategy space of $s_i$ is $\prod_{\kappa \in K} A_{i,\kappa}$.
- A set of states $X_i$ of agent $i$ in level 1, and an initial mapping function $f_{i,1} : X_i → \mathcal{P}(A_{i,1})$ that maps the initial state of the agent to the available actions in level 1, where $\mathcal{P}(\cdot)$ is the power set.
- Set-valued functions $f_{i,j} : \prod_{j=1}^{\kappa-1} A_{i,j} → \mathcal{P}(A_{i,\kappa})$ for each agent $i$ that maps a partial strategy $(a_{i,1}, a_{i,2}, \ldots, a_{i,\kappa-1})$ to $\mathcal{P}(A_{i,\kappa})$ and gives the set of available actions to $i$ in level $\kappa > 1$ for the partial strategy till level $\kappa - 1$.
- A set of $N$ payoff (utility) functions $U = \{u_i(s_i, s_{-i})\}$, where $-i$ refers to all agents other than $i$.

The hierarchical game imposes a total ordering in actions $A_i = \{A_{i,1}, A_{i,2}, \ldots, A_{i,K}\}$ of a given agent, and along with $f_{i,\kappa}$ induces a game tree, as shown in Fig. 1. The frequency at which a hierarchical game is instantiated $(\Delta t_p)$ and the time horizon of each strategy $(\Delta t_h)$ are exogenous to the model. Each node is labeled $n_{i,\kappa,j}$, where $i$ and $\kappa$ are the agent and level indices, and $j$ is the node identifier within level $\kappa$. This general formulation of a hierarchical game does not prescribe a fixed information structure, and allows the designer to set an information structure that is appropriate to the environment and situation they want to model. For example, (Fisac et al. 2019) model a lane change scenario where an AV merges into a lane occupied by a human driven vehicle as a Stackelberg game, with the AV being the leader and the human driven vehicle responding to the action of the AV. In situations where assignment of a leader and a follower is unclear or that assumption is too strong, the agents
might not have perfect information on the state of the play. Fig. 1 illustrates a 2-agent 2-level scenario as an example where an AV (indexed as 1) executes a free right turn on red at a signalized intersection (in a situation similar to id:14 in Fig. 2), while a human driven vehicle (id:26 and re-indexed as 2 in Fig. 1) approaches cross path from left to right. The AV can either decide to turn (T) or wait (W) for the cross path vehicle to pass, i.e., $f_{1,1}(X_1) = A_{1,1} = \{T, W\}$. The human driven vehicle (id:26) can either slow down (D) or choose not to slow down (U), $f_{2,1}(X_2) = A_{2,1} = \{D, U\}$. Since either agent does not have perfect information about what the other agent is about to do next, agent 2 does not know whether they are in node $n_{2,1,1}$ or $n_{2,1,2}$ (connected by the information set $I_1(1)$). This imperfection of information is also reflected at the trajectory level (level 2 actions), where each agent can only distinguish between the nodes in level 2 that follow from their own chosen actions in level 1, but not from the ones that follow from the other agent’s level 1 decision ($T_1(2), J_1(5)$). It becomes apparent from this structure that the game has no proper subgame, and the game reduces to a simultaneous move game. It is well understood that a way to solve such games is by reduction to normal form. However, as we shall see, the hierarchical game has additional constraints that allow solving the game in Fig. 1 also through backward induction. To designate the nodes where utilities accumulate at each level in the backward induction process, we label a set of nodes in each level $\kappa$ as level roots $L(\kappa) = \{n_{i,\kappa,j} | \text{parent}(n_{i,\kappa,j}) \notin N_\kappa\}$ where $N_\kappa$ is the set of nodes in level $\kappa$. In other words, the set of level roots contain nodes in each level $\kappa$ whose parent is not in level $\kappa$. Therefore, $L(1) = \{n_{1,1,1}\}$ and $L(2) = \{n_{1,2,1}, n_{1,2,2}, n_{1,2,3}, n_{1,2,4}\}$. Algorithm 1 shows the standard backward induction process adapted to the hierarchical game. The algorithm starts at the bottom most level $\kappa$ and recursively moves up the tree by solving the level games $G_\kappa$ at every level. At each level, a simultaneous move level game $G_\kappa$ is instantiated from each node in $L(\kappa)$. These level games are constructed by first extracting $\sigma_i(n)$, which gives the partial pure strategy for agent $i$ that lies on the branch from the root node of the game tree $L(1)$ to node $n \in L(\kappa)$. $f_{i,\kappa}$ gives the available actions for each agent $i$ in the current level $\kappa$, and these actions form the domain of available strategies in the level game $G_\kappa$. The utilities depend on the level of the game; for level game $G_{\kappa=K}$ the utilities are same as the game utility $U$, whereas

### Algorithm 1: Backward induction for a hierarchical game

| Result: $S^*_\kappa, V^*_\kappa$ |
|---|
| for $\kappa := K; \kappa = 1$; $\kappa := \kappa - 1$ do |
| for $n \in L(\kappa)$ do |
| $S^*_{\kappa,n}, V^*_{\kappa,n} \leftarrow$ solve $G_\kappa \{\prod_{i=1}^N f_{i,n}(\sigma_i(n))$ |
| $\kappa = K?U; V^*_{\kappa+1,L(\kappa+1)}$ |
| end |
| end |

for level games $G_{\kappa<K}$ are solved based on the game values $V^*_{\kappa+1,L(\kappa+1)}$ from the game $G_{\kappa+1}$ solved in the previous iteration. Note that the pseudocode shows only the case where a single solution and game value $(S^*_{\kappa,n}, V^*_{\kappa,n})$ is propagated up the hierarchy. In the case of multiple solutions for the level games, the strategies and values have to be tracked and repeated for each solution. The solutions and game value $S^*_{\kappa,n}, V^*_{\kappa,n}$ depend on the solution concept used for the individual level game, and this is discussed in detail later under Solution concepts.

One can see that the backward induction process is very similar, if not same as solving for subgame perfect equilibria in multi-stage games with stages being replaced by levels in the hierarchy (Tadelis 2013). However, we cannot call it that since the level games are not subgames in the game tree. The reason why the backward induction works though is because the mapping functions $f_{i,n}$ eliminate strategies for every agent $i$ that are not direct successors of the partial strategies $\sigma_i(n) \cdot \sigma_{-i}(n)$, essentially breaking any information set within a level $\kappa$ that spans across two separate level roots in $L(\kappa)$. More intuitively, this mimics the elimination of hypothetical strategies where in level 1 a vehicle may think about slowing down, but in level 2 chooses a trajectory that speeds up; and the fact that this cannot happen is part of the common knowledge among the agents in the game.

### Game structure

In this section, we describe the details of the game structure used in our study, including the number of agents, actions/strategies, and utilities.

**Relevant agents and available actions.** Since we are interested in investigating decision making in the most critical tasks at a signalized intersection (such as unprotected left turns and right turns on red), at each time step $\Delta t_p = 1s$, we setup a hierarchical game with an action plan horizon of
time constraints to make a decision (which is in the order of milliseconds), the situation is ripe for bounded rationality to be in play. Osborne and Rubinstein takes a view of bounded rationality that emerges from agents’ employing a mental process to sample other agents’ actions and respond based on the imagined outcome of those samples (Osborne and Rubinstein 1998). In our case, this is akin to a vehicle sampling a set of trajectories of other agents and responding in accordance to the sampled trajectories. Naturally, one may imagine that some sampling procedures make more sense than others. We now briefly mention the sampling procedures used in our experiments, and the intuitive reasoning behind each.

At each time step when the game tree is instantiated, agents are currently reasoning over. To construct the trajectory sample, we select lattice endpoints along the lane centerline and use a piecewise constant acceleration model to generate the final trajectory. In the subsequent sections, we refer to this sampling scheme that produces a single trajectory as $S(1)$ sampling. With a little more cognitive bandwidth, along with the $S(1)$ trajectory sample, they can also sample trajectories that form the extreme ends of the bounded level-2 action space of other agents.

These trajectories are bounded spatially by the lane boundaries and temporally by the upper and lower bounds on the velocity limits of the level 1 action they correspond to. We refer to this scheme as bounds sampling $S(1 + B)$. This set of trajectories indicate what other agents might do in normative (i.e. following the rules as captured by the piecewise constant acceleration model) as well as in the extreme case but still within the physical limitations of the vehicle. The final sampling scheme lies in between the two schemes. Similar to $S(1 + B)$, this scheme includes the $S(1)$ trajectory; however, the rest of the trajectories are sampled from a multivariate Gaussian distribution with $\mu = [x_{S(1)}, y_{S(1)}, v_{S(1)}]$, and an unit diagonal covariance matrix, where $(x_{S(1)}, y_{S(1)}, v_{S(1)})$ is the lattice endpoint corresponding to the $S(1)$ trajectory. We refer to this scheme as $S(1 + G)$ and the samples include the normative behavior that comes from $S(1)$ along with variations in the path and velocity of the vehicle but not to the extremes that were captured in the $S(1 + B)$ scheme.

Utilities. To determine the utility structure, we draw from motivational aspects of driver behavior modelling in traffic psychology literature (Summala 1988). In general, driving motivations can be broadly classified into inhibitory and excitatory. Whereas excitatory motivations drive a driver to make progress towards reaching the destination, inhibitory motivations are the balancing factors that account for mitigating crashes and mental stress. In our case, the degree of progress a driver can make based on a selected trajectory $a_{i, 2}$ is the excitatory utility $u_{v, exc}(a_{i, 2})$ as determined by the trajectory length $\|a_{i, 2}\|$, $u_{v, exc}(a_{i, 2}) = \min(\|a_{i, 2}\|, 1)$, where $d_o$ is a constant and can be interpreted as the distance to goal or crossing the intersection. Inhibitory utility is based on the minimum distance gap of the trajectory to other vehicles $u_{v, inh}$ as well as respecting pedestrian’s right of way $u_{p, inh}$. The final form of the utility function is

$$u_i(a_{i, 2}, a_{i, 2}) = W \cdot [u_{v, inh}(a_{i, 2}, a_{i, 2}) \ u_{p, inh}(a_{i, 2}) \ u_{v, exc}(a_{i, 2})]$$

$$u_{v, inh}(a_{i, 2}, a_{i, 2}) = \int \text{erf} \left[ \frac{|d(a_{i, 2}, a_{i, 2}) - \theta|}{\sigma \sqrt{2}} \right] N(\theta; d_o, a_{i, 2}, a_{i, 2}, \sigma)d\theta$$

Sigmoidal functions are a popular family of functions that map a safety surrogate metric, e.g., distance gap $d(a_{i, 2}, a_{i, 2})$, into an utility interval (Fishburn 1970). For $u_{v, inh}$, we first fix a minimum safe distance gap $d_{s, a_{i, 2}, a_{i, 2}}$ based on the task (left turn, right turn, etc.) of the agents in the game. The value of the safe distance gap determines the location $\theta$ of the sigmoidal function (erf). However, since the conception of what is considered safe may vary in a population of drivers, we let $\theta$ to be a random variable that is normally distributed with $\mu = d_{s, a_{i, 2}, a_{i, 2}}$ and constant variance $\sigma$ determining the scale of the sigmoidal function. The choice of erf as the sigmoidal function is a mathematical convenience since the Gaussian integral of the erf in $u_{v, inh}(a_{i, 2}, a_{i, 2})$ evaluates to another sigmoidal erf$\left(\frac{d(a_{i, 2}, a_{i, 2}) - d_{s, a_{i, 2}, a_{i, 2}}}{2\sigma}\right)$ (cf. supplementary ma-
Solution concepts

A key element that influences solution concepts in games is the manner in which each agent reasons over the strategies of other agents. In non-strategic behavior models, agents do not explicitly model other agents in the game and respond solely on the basis of their own utility structure \((\text{Wright and Leyton-Brown } 2020)\). Strategic agents, on the other hand, perform some reasoning over the strategies of other agents and respond accordingly.

The first category of behavior models we consider is the Quantal level-k (QLk) model \((\text{Wright and Leyton-Brown } 2012)\). QLk models the population of agents as a mix of strategic and non-strategic agents, with strategic agents having an iterated cognitive hierarchy of reasoning. Strategic agents in QLk use Quantal Best Response (QBR) function, often expressed as a logit response \(\pi_{i}^{\text{QBR}}(a_{i}, s_{-i}, \lambda) = \frac{\exp(\lambda u_{i}(a_{i}, s_{-i}))}{\sum_{a_{i}} \exp(\lambda u_{i}(a_{i}, s_{-i}))}\), where \(s_{-i}\) represent the pure or mixed strategies of other agents and \(\lambda\) is the precision parameter that can account for errors in agent response with respect to utility differences.\(^{1}\) When \(\lambda \to 0\), the mixed response is a uniform random distribution, whereas \(\lambda \to \infty\) makes the response equivalent to best response. Level-0 agents are non-strategic (NS) agents who choose their actions uniformly at random, whereas Level-1 agents are strategic (S) agents who believe that the population consists solely of Level-0 agents, and their response is a QBR response to Level-0 agents’ actions. In the original QLk model, level-0 agents follow an uniform distribution mixed strategy; however, in our case we use an expanded definition of level-0 agents presented in \((\text{Wright and Leyton-Brown } 2014)\), where instead of an uniform distribution, the level-0 agents’ strategies follow more intuitive yet non-strategic response, such as best response (BR) or maximin (MM) response. We believe that the expanded definition of the level-0 agents suit our situation much better, since it is unrealistic to expect a driver to choose actions purely at random from their available actions. Even with this expanded definition, these are still non-strategic since agent responses depend purely on their own utilities and do not rely on a strategic reasoning over other agents’ utilities \((\text{Wright and Leyton-Brown } 2020)\).

Table 1: Distribution of strategic (S) and non-strategic (NS) behavior in level games \(G_{1}\) and \(G_{2}\) in three metamodels QL0, QL1, and PNE-QE.

| NS | NS | S+NS | NS | S | NS |
|----|----|------|----|---|----|
| QL0 | QL1 | PNE-QE |

Table 1: Distribution of strategic (S) and non-strategic (NS) behavior in level games \(G_{1}\) and \(G_{2}\) in three metamodels QL0, QL1, and PNE-QE.

into levels \(s_{i} = (a_{i,1}, a_{i,2})\), the manner in which an agent reasons over strategies in one level might not be the same as the reasoning process in another level. Therefore, instead of a single solution concept in the game of Fig. 3, the level games \(G_{2}\) can have a different solution concept than the one in game \(G_{1}\). In our models, we let agents have a cognitively less demanding non-strategic response in \(G_{2}\), and a more deliberative strategic response in \(G_{1}\). This choice is similar to one taken in \((\text{Fiasc et al. }2019)\), and reflects the natural process where it is easier for drivers to reason strategically over the strategy space of discrete maneuvers than over the space of infinitely many trajectories.

We consider two metamodels of behavior under QLk: QL0 and QL1. We refer to them as metamodels, since they can be further refined based on the choice of response function and sampling schemes to create concrete models. In QL0 models, we restrict the population to be solely level-0 responders in both \(G_{1}\) and \(G_{2}\). In QL1, the population consist of a mix of level-0 and level-1 responders in \(G_{1}\) and level-0 responders in \(G_{2}\). (Table 1)

For models of non-strategic behavior, we use two response functions; best response (BR) and best worst-case or maximin response (MM). The model for BR is:

\[
\begin{align*}
a^{*}_{i,\kappa} &= \arg \max_{a_{i,\kappa}, a_{-i,\kappa}} u_{i}(a_{i,\kappa}, a_{-i,\kappa}) \\
\pi_{i}(a_{i,\kappa}) &= \frac{\exp(\lambda_{i} \cdot u_{i}(a_{i,\kappa}, a_{-i,\kappa}))}{\sum_{a_{i,\kappa}} \exp(\lambda_{i} \cdot u_{i}(a_{i,\kappa}, a_{-i,\kappa}))}
\end{align*}
\]

\[(2)\]

where \(a^{*}_{i}\) is the pure strategy utility maximizing action for \(i\). The model for non-strategic MM response is:

\[
\begin{align*}
a^{*}_{i,\kappa} &= \arg \max_{a_{i,\kappa}, a_{-i,\kappa}} u_{i}(a_{i,\kappa}, a_{-i,\kappa}) \\
\pi_{i}(a_{i,\kappa}) &= \frac{\exp(\lambda_{i} \cdot u_{i}(a_{i,\kappa}, a_{-i,\kappa}))}{\sum_{a_{i,\kappa}} \exp(\lambda_{i} \cdot u_{i}(a_{i,\kappa}, a_{-i,\kappa}))}
\end{align*}
\]

\[(3)\]

Equations 2 and 3 are relaxations that translate the pure strategy action to a noisy response \(\pi_{i}(a_{i,\kappa})\) based on the precision parameter \(\lambda_{i}\) and sensitivity to \(i\)’s utility difference with respect to opponent actions that maximizes \(i\)’s utility for BR and minimizes for MM.

In QL1 metamodel, the population consists of a mix of level-0 and level-1 agents. Level-0 agents in this population follow non-strategic behavior as formulated earlier and level-

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1 In this formulation, the symbols \(s_{i}\) and \(a_{i}\) are strategies and actions of a game in a general sense, and not related to the symbols used specifically in the formulation of hierarchical games earlier.
the observed strategy

\(X\) agents best responds quantally to level-0 agents' behavior. With the expanded definition of level-0 agents as non-strategic bounded rational agents, there is a design choice to be made on what level-1 agents believe about level-0 agents. They can either consider level-0 agents bounded rational having mixed response of Equations 2 and 4 or level-1 agents can consider level-0 agents to be pure strategy rational responders based on Equations 1 and 2. We choose the later to align with the original QLk model, where agents modeling other agents as bounded rational agents are observed only at a higher cognitive level (level-2 and above).

In QLk models, mixed population is modeled as uniform population of bimodal mixture behavior. Therefore, if the proportion of level-0 and level-1 agents is \(\alpha\) and \(1 - \alpha\) respectively, then the QL1 model response in \(G_1\) is the mixed strategy response

\[
\pi_{QL1}(a_{i,1}) = \alpha \cdot \pi_{QL0}(a_{i,1}) + (1 - \alpha) \cdot \pi_{QBR}(a_{i,1}, a_{-i,1}, \lambda_i)
\]

(5)

where \(\pi_{QL0}(a_{i,1})\) is the left hand side of the equation 2 or 4 and \(a_{-i,1}\) is the solution to equations 1 or 3 for each of the other agents.

The final metamodel we consider is a generalization of pure strategy Nash equilibrium with noisy response. In this metamodel, agents follow a non-strategic model in \(G_2\), and a strategic model in \(G_1\) as described below.

\[
a_{i,1} = \arg\max_{a_{i,1}} u_i(a_{i,1}, a_{-i,1})
\]

(6)

\[
\pi_i(a_{i,1}) = \frac{\exp[-\lambda_i \cdot \min_{a_{-i,1}} (\lambda_i, a_{-i,1}) (u_i^* - u_i(a_{i,1}, a_{-i,1}))]}{\sum_{a_{i,1}} \exp[-\lambda_i \cdot \min_{a_{-i,1}} (\lambda_i, a_{-i,1}) (u_i^* - u_i(a_{i,1}, a_{-i,1}))]}
\]

(7)

where \(u_i^* = u_i(a_{i,1}^*, a_{-i,1}^*)\). In the above model, agents respond according to pure strategy Nash equilibrium \(a_{i,1}^*\), but in error may choose actions \(a_{i,1} \notin a_{i,1}^*\) based on the sensitivity to the difference in the utility of the action and an equilibrium action. We refer to this model as pure strategy Nash equilibrium with quantal errors (PNE-QE). The formulation is similar to Quantal Response Equilibrium (QRE), yet with key differences. In QRE, strategic reasoning occurs in a space of mixed responses and the precision parameter is part of common knowledge in the game. In our model, reasoning over opponent strategies is in pure strategy action space and the precision parameter is endogenous to each agent; therefore, when an agent reasons about the strategies of other agents, their parameters do not play a role. (Crawford, Costa-Gomes, and Iriberri 2013).

Based on the choice of the metamodel, the response function, and the sampling scheme, we get 25 different behavior models (B), cf. Fig. 3 which we evaluate in the next section.

**Estimation of game parameters.** Our dataset contains instances of \(D\) (~23k) hierarchical games, instantiated at a frequency of 1s with the state variables \(X_i\) along with the observed strategy \(s_i = (a_{i,1}^*, a_{i,2}^*)\) for every agent in each game. For a game \(g \in \{(G_k, b_j) | k \in \{1, 2\}, b \in B, j = \{1, ..., D\}\}\) identified by the level game, the behavior model \(b\), and the game index \(j\), we note the errors in actions with respect to the pure strategy responses in the games as \(\Delta U_g = \{e_{i,g} | e_{i,g} = \min_{a_{i,g}} [u_i(a_{i,g}^*, a_{-i,g}^*) – u_i(a_{i,g}^*, a_{-i,g})]\},\) where \(a_{i,g}^*\) are the solutions to Equations 1 or 3 for non-strategic models and 5 for PNE-QE model. We verified the existence of pure strategy NE for all \(G_1\) games in \(D\). \(\Delta U_g\) follows an exponential distribution based on the game’s precision parameter for non-strategic and PNE-QE models, and a mixed exponential distribution for QL1 in \(G_1\). We model the precision parameters \(\lambda_{i,g}\) in individual game \(g\) to be a function of the agents’ state vector \(X_{i,g}\) in the game. Therefore, to estimate the value of \(\lambda_{i,g}\) we fit a generalized linear model \(glm(\epsilon_{i,g} \sim \beta X_{i,g})\) for QL1 with Gamma \((k=1)\) family and inverse link, which models \(\epsilon_{i,g}\) as an exponentially distributed random variable with \(E[\epsilon_{i,g}] = \frac{1}{\lambda_{i,g}}\) and \(Var[\epsilon_{i,g}] = \frac{1}{\lambda_{i,g}^2}\), \(\beta\) is the model co-efficient, solved through maximum likelihood estimate based on the data in \(\Delta U_g\). The prediction of the \(glm\) model gives the mean and standard error of \(\lambda_{i,g}^{-1}\) based on the state observation \(X_{i,g}\). For the mixed exponential distribution in QL1 model, once we estimate the individual precision parameters of 5 we use iterative gradient ascent to solve for \(\alpha\) by maximizing the likelihood function \(\Sigma \nu_{a_{i,1}} \ln(\pi_{QL1}(a_{i,1}^*))\).

**Experiments.**

In our experiment we study naturalistic human driving behavior and based on the structure of the hierarchical game, we evaluate which behavior model captures human driving better, both in terms of model fit and predictive accuracy. Our dataset contains traffic observations collected from a drone camera at a busy urban intersection during mid-day traffic. The dataset contains a total of 3649 vehicles and 264 pedestrians, including their centimetre-accurate trajectory estimates. We analyse the decision making in right turning and left turning vehicles, which results in a total of 23119 hierarchical games. In our experiments, we study behaviors after setting \(W = [0.25 0.5 0.25]\), thereby giving more importance to pedestrian inhibitory actions and set the value of \(d_g = 100\) m. In particular we answer the following research questions:

**RQ1** Which solution concept provides the best explanation for the observed naturalistic data?

**RQ2** How do state factors influence the precision parameters in the games?

**RQ3** How does the choice of the response function in the lower level game \(G_2\) affect the higher level solutions in \(G_1\)?

**RQ1.** We address this question in three ways; with respect to the (i) parameter values in the model, (ii) predictive accuracy in unseen data, and (iii) model fit. Fig. 4 shows the mean and standard error of \(\lambda_{i,g}\) estimates in level games \(G_1\) for the set of behavior models. Models are indexed by their metamodel followed by the choices of response functions in \(G_1\); \(G_2\) followed by the sampling scheme used in \(G_2\). For
PNE-QE models, the response function in $G_2$ is omitted for $S(1)$ sampling since the hierarchical game only consists of $G_1$ games; and in those cases each agent has a single choice under each level-2 root. We perform our analysis of RQs 1 and 2 based on $G_1$, and discuss the impact of the choice of $G_2$ solution concepts as a part of RQ3. Higher $\lambda$ values indicate agents following a strategy that is closer to pure strategy responses of the models. For QL1 models, Fig. 3 shows the estimates of the precision parameter of level-1 responders. The mean and standard deviation of the proportion of level-1 responders in Q1 models are $\alpha = 0.519 \pm 0.02$. Overall, PNE-QE:BR$_S(1+B)$, PNE-QE model with best response in $G_2$ with bounds sampling) show highest value of the precision parameter ($\lambda = 190.8 \pm 0.57$). Next, we evaluate model fit using Akaike information criterion (AIC) values, which are noted in the figure in brackets. PNE-QE models with bounds sampling of trajectories have lowest AIC values (-185.87 and -187.34 for PNE-QE:BR$_S(1+B)$, PNE-QE:MM$_S(1+B)$), indicating the best fit among the models based on the criteria. Alternatively, model selection can also be guided by their predictive power in unseen situations. For evaluation based on this criterion, we use random subsampling with 75:25 training and testing split and 30 runs. The model parameters are estimated based on the observations in the training set, and the predictive accuracy is measured on the basis of the log likelihood of the observed actions in the testing set. Fig. 3 shows the log likelihood of the observed $G_1$ actions in the testing set as predicted by each model, along with the standard deviation. In general, we observe that QL1 and PNE-QE models have slightly better predictive accuracy than QL0 models. However, the difference between models are not as pronounced as in the case of other metrics, such as AIC and precision parameter. Overall, the results indicate that based on the three evaluation criteria combined (precision parameter, AIC, and predictive performance), pure strategy Nash equilibria, especially with bounds sampling of trajectories, is still a good model of decision making at the level of maneuvers, but with a noisy response; and this noise can be modelled with a quantal error model that is sensitive to the utility difference to a sample NE.

**RQ2.** In this research question we study the impact of the state factors on the precision parameter. We rank each state factor based on their relative impact on the precision parameter based on the fitted glm (the list of state factors are described in supplementary material S3). The factors that are found to have the most impact are **Segment** (the area of the intersection the vehicle is in) and the state of the traffic light. When vehicles are on a right turn execution segment, they show an average $\lambda$ increase of 165.62. Compared to the left turn task, right turns are more self-paced (in North America), where a driver has more time available to perceive their surroundings and execute the turn when it is safe to do so, which may be one reason for the higher precision. Similarly, when the traffic light is amber or red, there is an increase in $\lambda$ of 136 compared to when the light is green. Other than right turning vehicles, most movement in amber or red light are vehicles slowing down to stop at red light or left turning vehicles finishing executing their turn. In these situations, given that the severity of making a wrong decision, we observe higher rationality in drivers. These results show that there is much more variation within individual models depending on situational circumstances (as indicated by the impact of state factors) compared to the variation in mean $\lambda$ values across different models.

**RQ3.** Within the three behavior metamodels, we investigate the impact of the choice of the response function in $G_2$ on the precision parameter in $G_1$. For QL0:BR models, the choice of response function in $G_2$ is statistically non-significant (Dunn’s test, $p$-value=0.19). However, for all other models, the choice of $G_2$ response function is significant, and choosing a best response function at the level of trajectories results in a higher value of $\lambda$ in level games $G_1$ compared to maximin response for all the models.

**Conclusion**

We formalize the concept of a hierarchical game and develop the various solution concepts that can be applied to a hierarchical game by adapting popular behavioral game theoretic metamodels (QLk and PNE-QE). We evaluated the behavior models based on a large contributed dataset of human driving at a busy urban intersection. Our results show that among the behavior models evaluated, modeling driving behavior as pure strategy NE with quantal errors at the level of maneuvers along with bounds sampling of trajectories provides the best fit to naturalistic driving behavior. Additionally, right turning vehicles demonstrate a higher precision in their behavior with respect to the behavior models, than left-turning ones and there is a high impact of situational circumstances on the models. We identify two main directions for future work. The first one is to analyze the sensitivity of the solutions of the behavior models to different weights $W$ of the
excitatory and inhibitory motivations and to addition of socio-
cal utility norms (Schwarting et al. 2019). The second one is
to perform a more extensive analysis of the effect of situ-
tional context on the behavior models to determine which
models work better under what circumstances.

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Ethics statement
Research in self-driving cars or autonomous vehicles has
broad impact on transportation and society in general. Mem-
ers of the public have a stake in the development of AVs
since the algorithms and the processes that go into the de-
velopment of AVs impact the safety of everyone as road users.
The main goal of our paper is to understand human driving
behavior in a multi-agent setting in order to make it easier
to evaluate how decisions made by AV motion planning al-
gorithms impact other road users. Although the approaches
developed in the paper are well suited to be used for the pur-
pose of verification and testing of AV motion planners, there
are ethical impacts that should be taken into consideration
while applying the models.
First, driving behaviors that fall under errors or off-
equilibrium behaviors are considered off-equilibrium only
with respect to a specific utility structure. Our ability to
predict the utility of motivations at an individual level is
severely limited, and this limitation needs to be acknowl-
 edged and taken into account. For example, the quantita-
tive value an individual driver, for whom their car is a vital
commodity for their source of livelihood, assigns to driving
safely, (as modelled through surrogate safety metrics) may be very different from another individual who owns multiple cars and uses their vehicle only for casual commute. In addition, there are several factors, such as, socioeconomic status, disability, access to insurance, etc., that play a role in shaping the driving behavior of an individual.

Secondly, as shown in the paper, off-equilibrium behaviors that we observe in the behavior models can be modelled as an exponential distribution; i.e., the probability of behaviors that lie away from the equilibrium reduces the further the behavior is from on-equilibrium behavior. Since we can estimate the parameters of this distribution, it may be tempting to evaluate models solely through quantitative risk metrics that are derived from this distribution. However, along with such an analysis, there is also a need to be more transparent and investigate the situational context in which the low probability events occur. Due to the same factors mentioned earlier, choosing a certain behavior profile for an AV may adversely impact a segment of road-users, such as older people or people with disability, disproportionately while keeping the overall risk at a population level within a prescribed threshold. Therefore, the use of behavior models in practical AV development needs to be accompanied with not only the information about objective risk metrics but also how the chosen behavior profile impacts vulnerable sections of road users.