Entangled two atoms through different couplings and the thermal noise

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The entanglement of two atoms is studied when the two atoms are coupled to a single-mode thermal field with different couplings. The different couplings of two atoms are in favor of entanglement preparation: it not only makes the case of absence entanglement with same coupling appear entanglement, but also enhances the entanglement with the increasing of the relative difference of two couplings. We also show that the diversity of coupling can improved the critical temperature. If the optical cavity is leaky during the time evolution, the dissipative thermal environment is benefit to produce the entanglement.

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I. INTRODUCTION

Environment noise can result in decoherence of quantum system. In order to overcome the decoherence, some authors proposed a number of approaches such as a quantum error correction [1] and a decoherence-free subspace [2]. Another direct method is to employ the environment noise to produce entanglement [3-12]. In solid state system, the thermal entanglement in Heisenberg model is an example of using the thermal environment [3-6]. It has been found that the anisotropy in XY Heisenberg chain can be used to increase the critical temperature [5]. In cavity QED, the cavity loss or the thermal field can also induce entanglement between two atoms [7-11]. In Ref. [7], using quantum jump approach, the authors described a scenario where entanglement between two atomic systems can be induced via continuous observation of the cavity loss. Working in different initial state (different from Ref. [7]) and employing a master equation, in Ref. [11], the author proposed a scheme to prepare two atomic entanglement. Two two-level atoms can be entangled through interacting with a single-mode thermal field [9] [10]. But in the studies on cavity QED [8], for simplicity of the calculation, the couplings $g_i (i = 1, 2)$ between the two atoms and the field are assumed equal. Although, in Ref. [7] [11], their calculation also applied to $g_1 \neq g_2$, they just had discussed the equal couplings in their plots and had not discussed the effect of two atoms with different couplings. In fact, the coupling rate $g_i$ depends on the atom position $r_i$; due to the randomness of the atom position $r_i$, it is very difficult to control the same couplings between different atom [13]. The truth that the anisotropy in two-qubit Heisenberg model can increase the critical temperature motivate us to consider whether different couplings $g_i$ of the two atoms result in some novel properties in cavity QED.

In this paper, we study the system that the two atoms with different coupling constants are coupled to the optical cavity which is initially in thermal field state. When the atoms are initially in $|ee\rangle$ and the cavity is initially in a single-mode thermal field, with the same coupling constant they are not entangled [9] but if with different coupling constant they can be entangled. We analyse the dependence of the relative difference of two couplings on the entanglement. We find that there is a possibility to obtain more entanglement through increasing the relative difference of two couplings. We show that the diversity of coupling can improved the critical temperature at which entanglement disappears. We also study the case of a cavity with a steady leak during the time evolution, it is found that the dissipative cavity field is benefit to produce the entanglement.

II. THE EFFECT OF THE DIVERSITY COUPLING OF THE TWO-ATOM

In order to make clear the function of different couplings, we first consider a quantum system composed of two two-level atoms interacting with a single-mode thermal field, which can be produced by a leaky cavity in thermal equilibrium at temperature $T$. After preparation the initial cavity field, the cavity stop to leak. The cavity mode is assumed to be resonant with the atomic transition frequency. Under the rotating wave approximation, the Hamiltonian in the interacting picture is

$$H_I = g_1(a\sigma_1^+ + a^+\sigma_1^-) + g_2(a\sigma_2^+ + a^+\sigma_2^-)$$

(1)

where $a$ ($a^+$) denotes annihilation (creation) operator of the cavity field; the atomic transition operators are $\sigma_i^- = |g_i\rangle_i\langle e_i|$ and $\sigma_i^+ = |e_i\rangle_i\langle g_i|$: $g_1$ and $g_2$ are the two coupling constants for the atom $i (i = 1, 2)$. For expression the diversity of the two couplings, we let
\[ g = \frac{g_1 + g_2}{2}, \quad \gamma = \frac{g_1 - g_2}{g_1 + g_2} \]  

where \( g \) denote average coupling and \( \gamma \) express the relative difference of the two atomic couplings. Because of the identity of the two atoms, the range of \( \gamma \) is between 0 and 1. The Hamiltonian can be rewritten as

\[ H_t = g(1 + \gamma)(a\sigma_1^+ + a^+\sigma_1^-) + g(1 - \gamma)(a\sigma_2^+ + a^+\sigma_2^-) \]  

The single-mode thermal field with its mean photon number \( \bar{n} \) is a mixture of Fock states. It take the form

\[ \rho_c = \sum_n \frac{\bar{n}^n}{(1 + \bar{n})^{n+1}} |n\rangle\langle n|, \]

and \( \bar{n} = \left(e^{\frac{\hbar\omega}{k_B T}} - 1\right)^{-1} \) relates to the environment temperature. Entanglement of two atoms can be measured by concurrence \( C \) which is written as \([14][15]\)

\[ C = \max(0, 2 \max\{\lambda_i\} - \sum_{i=1}^{4} \lambda_i), \]

where \( \lambda_i \) is the square roots of the eigenvalues of the matrix \( R = \rho S \rho^* S \), \( \rho \) is the density matrix, \( S = \sigma_1^y \otimes \sigma_2^y \) and * stands for complex conjugate. The concurrence is available no matter what \( \rho \) is pure or mixed. We will numerical calculate the entanglement between the two atoms.

In Ref. [9][10], for simplicity they assumed the atoms 1 and 2 couple to a single-mode thermal field with the same coupling constant. They found that if the two atoms initially are both in excited states, the two atoms could not be entangled neither in one-photon process nor in two-photon process. In our simulation, if \( \gamma = 0 \), we also can not find entanglement. But when we chose \( \gamma = 0.4 \), the result is reversed which is shown in Fig. 1. It is obvious that under the condition of different couplings the two atoms (initial in \(|ee\rangle\) ) can be entangled. This is contrary to the case of the same coupling constants [9][10]. When the two atoms initially are both in excited state, it had been shown that the atom and the field are always entangled despite of the presence of the other atom [9]. It is the atom-field entanglement that result in the decoherence between the two atoms. We can understand it from the same couplings: because their couplings are the same, their rabi oscillation also have the same steps, so the two atoms have no correlation, i.e., there is no entanglement between them. If the couplings are not the same, when they resonate with the cavity, they are no longer with the same step. Through interacting with cavity, the two atoms are entangled. As we state above, the different couplings are more closely to experiments. So, the existing entanglement with different couplings maybe more easy achieved. Fig. 1 also show that the vacuum state ( \( \bar{n} = 0 \) ) induces the maximum entanglement. With the increase of the mean photon number, the entanglement between two atoms decreases. At a certain value of \( \bar{n} \), the entanglement disappears; we call that \( \bar{n} \) as critical average photon number and the temperature (corresponding to the \( \bar{n} \)) is called as critical temperature.

Increasing \( \gamma \) to 0.8, we plot the entanglement of two-atom in Fig.1b. Comparing Fig.1a with Fig.1b, one can find that the maximum entanglement of Fig. 1b is larger than that of Fig. 1a, that is, one can obtain much more strong entanglement with \( \gamma = 0.8 \). On the other hand, in Fig. 1a the critical average photon number is below 4 and in Fig.1b when \( \bar{n} = 5 \), the maximum entanglement is about 0.15, that is, the critical average photon number is larger than 5. So, the critical temperature of Fig. 1b is larger than that of Fig.1a. Whether can we conclude from Fig.1 that the larger the relative difference of two couplings, the larger the critical average photon number, the larger the critical temperature? The answer is no. Because the rabi oscillation of the atoms is relevant to the couplings, the entanglement is not a monotonic function of \( \gamma \). It will be shown in Fig. 2 and we will analyse it laterly. But at least in some ranges we can say that through increasing \( \gamma \), the critical temperature can be improved. The conclusion, in a certain extent, is similar to that the anisotropy in Heisenberg XY chain can increase critical temperature of the thermal entanglement [5]. The difference is that there, the critical temperature of the thermal entanglement is monotonously increased with the increasing of the anisotropy parameter, here due to the rabi oscillation, the critical temperature is not a monotonous function of the two couplings. But with the increasing of \( \gamma \), there are the possibility and the tendency to increasing the critical temperature (we will explain it next).

Because every atom periodically entangles and disentangles with the cavity field, and the periodicity relates to the coupling, so the entanglement between two atoms also exhibit periodicity and its periods are also related to the two couplings. We can observe this periodicity in Fig.1. For make clear the dependence of the entanglement on \( \gamma \), we plot the entanglement as a function of \( \gamma \) and \( t \) in Fig. 2 for the atomic initial state \(|ee\rangle\), where the average photon number \( \bar{n} = 1 \). We can see that for \( \gamma = 0 \) no entanglement exist which is coincident with the Ref. [9]. But with the increasing of \( \gamma \), in some time intervals one can see the large entanglement. Of cause, due to existing many peaks.
(existing up the hill and down the hill), there are either the extent in which entanglement is increased with the increasing of \( \gamma \) or the extent in which the entanglement is decreased with the increasing of \( \gamma \). Although the relation between the entanglement and \( \gamma \) is not monotonous, the main trend of entanglement is increased with the increasing of \( \gamma \). In other word, the maxima values of entanglement is increased with the increasing of \( \gamma \). Thus, if we can keep the relative large \( \gamma \), even the two atoms interact with thermal field, we can also obtain relative large entanglement. On the other hand, since the average photon number is a measure of the cavity classicality, the larger it is, the smaller the entanglement, i.e., with the increase of the mean photon number, the entanglement gradually decreases to zero. If in the extent in which entanglement is increased with the increasing of \( \gamma \), we can reckon the critical average photon number is increased with the increasing of \( \gamma \), thus the critical temperature is increased. If the span of \( \gamma \) is relative large such as from 0.4 to 0.8, the maxima entanglement is increased, the critical temperature, of cause, is increased.

The two atoms with initial state \(|ee\rangle\), which can not be entangled if they have the same coupling constants \([9][10]\), can be entangled when they are different in coupling constant. The initial state \(|eg\rangle\) is the best case which the entanglement could be the relative best but is far smaller than 1 \([9][10]\). If the coupling constants are different, what will happen? We directly numerical simulate the entanglement as a function of \( \gamma \) and \( t \) in Fig. 3. In Fig. 2 and Fig. 3, we both can see that when \( \gamma = 1 \) the entanglement do not exist. For \( \gamma = 1 \), only the one atom interact with the cavity and the other atom has no coupling with the field, so, there is no entanglement between them. When the two-atom is initially in the state \(|eg\rangle\) with \( \gamma = 0 \) is exact the case which had been discussed in Ref. \([9]\). From Fig. 3, we can observe entanglement when \( \gamma = 0 \), and it is different from the case of initial state \(|ee\rangle\) with \( \gamma = 0 \). But the entanglement in Fig. 3 when \( \gamma = 0 \) is not the largest. With the increasing of \( \gamma \), the maximum values of entanglement are also increased. There are still existing many “hills”, that is, there are either the extent in which entanglement is increased with the increasing of \( \gamma \) or the extent in which the entanglement is decreased with the increasing of \( \gamma \). Therefore, we can also obtain strong entanglement and improve the critical temperature through increasing \( \gamma \) in some extent. Thus, although Fig. 3 is obviously different from Fig. 2, the effects of \( \gamma \) on increasing critical temperature and improved entanglement are exactly the same in these two kinds of initial states.

### III. THE INFLUENCE OF THERMAL NOISE ON THE ATOM-ATOM ENTANGLEMENT

When the single-mode cavity is in thermal equilibrium with its environment due to the leakage, the cavity is in thermal field \([16]\). In section 2 and in Ref. \([9][10]\), the thermal field is just as initial cavity state; in the process of evolution, the cavity stop leaking. Since the cavity is leaky, why we assumed it stop to leaking? Now, we let it continue to leak in the later time evolution.

The master equation governing the time evolution of the global system is given by \((\hbar = 1)\)

\[
\dot{\rho} = i[\rho, H] + \mathcal{L}(\rho),
\]

where the Hamiltonian still have the form of Eq.(3). The Liouvillean is given by

\[
\mathcal{L}(\rho) = \kappa(\bar{n} + 1)(2a\rho a^+ - a^+ a \rho - \rho a^+ a) + \kappa \bar{n}(2a^+ \rho a - aa^+ \rho - \rho aa^+)
\]

We chose \(\kappa = 0.4, \gamma = 0.4\) and simulate the case of the two atoms are initially in \(|ee\rangle\) and the cavity is initially in the thermal state. Figure 4 show the entanglement as a function of average photon number and time \(t\). We notice that the critical average photon number is larger than 5, and in Fig. 1 \(\bar{n}\) is smaller than 5. To a thermal field state, the dissipative environment can increase the critical average photon number. Furthermore, with the time evolution, the entanglement is no longer periodic appearing zero but gradually reaches their asymptotic values, and the asymptotic values decrease with the increasing of \(\bar{n}\). When the cavity field initially is in thermal state, the dissipation of the cavity virtually can be considered as a driver of the cavity, so, with the evolution the entanglement is gradually increased. This is contrary to the general case in which the entanglement is destroyed by environment due to the decoherence. After all the system is a open one, so the entanglement is no longer periodic appearing zero but gradually reaches their asymptotic values. So, after long time evolution, we can obtain a strong and steady entanglement. Therefore, as long as the evolution time is enough, it is not necessary to precise control the interaction time. In experiment, precise control of the interacting time is very difficult to achieve \([17]\). We summarize that the dissipative environment not only improved the critical temperature but also provide us a steady entanglement.

### IV. CONCLUSION

We discuss the entanglement induced by a single-mode heat environment when the two atoms are coupled to the optical cavity with different coupling constants. When the atoms are initially in \(|ee\rangle\), with the same coupling constant
they could not be entangled [9] but if with different coupling constant they can be entangled. Even to the initial state $|eg\rangle$ in which the two atoms can be entangled with the same coupling, the different couplings are avail to produce atom-atom entanglement. Through the analysis about the dependence of entanglement on the relative difference of two couplings $\gamma$, we find that by increasing $\gamma$ we can obtain strong entanglement. We also show that the diversity of coupling can improved the critical temperature, which is very similar to that the anisotropy in Heisenberg XY chain can increase the critical temperature of thermal entanglement [5]. We study the leak of the cavity on the entanglement during the time evolution. It is found that the keeping on leaking is benefit to produce the entanglement. It not only can improves the critical temperature but also provid us a relative steady and strong entangled state.

Figure captions:
Fig.1. Entanglement as a function of average photon number $\bar{n}$ and $t$ when the pair of atoms is initially prepared in the state $|ee\rangle$ and the field is initially in the thermal state, where (a) : $\gamma = 0.4$, (b):$\gamma = 0.8$; for all plots $g = 1$.

Fig. 2. The dependence of entanglement on the relative difference of the two atomic couplings $\gamma$ and time $t$ for the initial atomic state $|ee\rangle$ when $g = 1$, $\bar{n} = 1$.

Fig. 3. The same as Fig. 2 but for the initial atomic state $|eg\rangle$.

Fig. 4. The effect of dissipative environment on the atom-atom entanglement when the atoms are initially in $|eg\rangle$ for $\kappa = 0.4$, $g = 1$.

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