Fuzzy controller for the yaw and velocity control of the Guanay II AUV

J. González*, S. Gomáriz*, C. Batlle**, C. Galarza*

*SARTI Research Group, Electronics Department, Universitat Politècnica de Catalunya. Rbla. de l’Exposició, 61-69. Neapolis Building, 08800, Vilanova i la Geltrú, Spain. (e-mail: julian.gonzalez.agudelo@upc.edu).
**Institute of Industrial and Control Engineering, and Department of Applied Mathematics IV. Universitat Politècnica de Catalunya.

Abstract: This work presents in detail the fuzzy control design for yaw and velocity control of an autonomous underwater vehicle. This control has been developed from the mathematical description of the hydrodynamic model of the vehicle, which is studied and discussed from different situations of forward velocity. The model is linearized and several linear controls are designed for their actuation at certain situations, in a way that the fuzzy control allows to handle those controls globally.

Keywords: Autonomous vehicles; PID control; Piecewise linear analysis; Fuzzy control; Mathematical model.

1. INTRODUCTION

The Guanay II is an autonomous underwater vehicle (AUV) used for the observation of environmental variables. It is developed by the SARTI Group of the Technical University of Catalonia UPC (see Gomáriz et al. 2014). In order to achieve autonomous navigation it is necessary to know in detail the hydrodynamic model of the vehicle, and at the same time to have a good controller to actuate the different thrusters. The movement of the Guanay II is due to 3 thrusters, one at the center to give the main propulsion, and another two located at the fins in order to give a torque to the turns.

Regarding controllers to control the heading and velocity there are several technics in the literature. One type is the Gain Scheduled controller proposed by Silvestre and Pascoal (2007). It is an interpolator of linear controllers which are designed for different velocities. Regarding fuzzy controllers, several papers use it for obstacle avoidance, like in Dong et al. (2005) and Liu et al. (2012), who design memberships in function of the distance to the obstacle and the forward velocity. On the other hand, there are other works focused on using the advantages of Gain Scheduling controllers but applying fuzzy to manage them, like in Zhang et al. (2012) and Jun et al. (2012), who develop a fuzzy controller for AUVs in a way that it manages several linear controllers to be actuated at specific conditions of velocity. However, they don’t calculate the parameters of the controller using an analytic procedure but using a linguistic interpretation. In Reddy et al. (2010) we can see a comparative of the fuzzy controller respect gain scheduling. They show that the fuzzy controller has a similar behaviour and performance with respect to the gain scheduling controller.

Thus, due to the good performance of the fuzzy controller serving as interpolator of linear controllers, we propose in this work the use of a fuzzy controller of type TSK in order to manage different linear controllers designed for specific conditions of forward velocity. In the same way, we present an analytic development to calculate different parameters. The fuzzy controller allows us to establish activation zones (understood as the effective contribution of one rule of the controller) which can be controlled through fuzzy sets (Driankov et al. 1996; Takagi and Sugeno 1985).

This work is structured as follows. Section 2 shows a brief description of the hydrodynamic model of the Guanay II. Section 3 presents the linearization at different velocities. Section 4 shows the linear controller for the yaw. Section 5 deals with the linear controllers to control the forward velocity. Section 6 shows the fuzzy controller used as an interpolator of linear controllers. Section 7 shows and discusses the obtained results. Finally, section 8 states the conclusions.

2. VEHICLE DYNAMICS

The Guanay II is a vehicle that moves over the surface and makes vertical immersions in order to get water profiles. Thus, we will only consider three degrees of freedom for its modeling, that is, the movement over the surface: surge, sway and yaw. In this order, the position vector and velocity vector become:

\[
\eta = [n \ e \ \psi]^T \quad \nu = [u \ v \ r]^T
\]  

(1)

where \( n \) is the forward displacement, \( e \) is the lateral displacement, \( \psi \) is the yaw of the vehicle, \( u \) is the forward velocity, \( v \) is the lateral velocity and \( r \) is the yaw velocity.

Following the work of Fossen (2002), the general model for navigation can be expressed as

\[
(M_{RB} + M_A)\nu + (C_{RB} + C_A)\nu + (D_1 + D_n)\nu = \tau
\]  

(2)

where \( M_{RB} \) is the rigid body inertia matrix, \( M_A \) is the added mass matrix, \( C_{RB} \) is the rigid body Coriolis and centripetal matrix, \( C_A \) is the hydrodynamic matrix of Coriolis and centripetal force, \( D_1 \) is the linear damping matrix, \( D_n \) is the nonlinear damping matrix, and \( \tau \) is the force and torque of the thrusters.
These matrices are described by equations (3-8). They depend on the velocities \((u, v, r)\) and a series of hydrodynamic coefficients (see table 1).

\[
M_{RB} = \begin{bmatrix}
m & 0 & 0 \\
0 & m & 0 \\
0 & 0 & I_z \end{bmatrix}
\]

\[
M_A = -\begin{bmatrix}
x_u & 0 & 0 \\
0 & y_o & y_r \\
0 & 0 & N_o & N_r \end{bmatrix}
\]

\[
C_{RB} = \begin{bmatrix}
0 & 0 & -mu \\
0 & 0 & mu \\
mv - mua & 0
\end{bmatrix}
\]

\[
C_A = \begin{bmatrix}
x_u & 0 & 0 \\
0 & y_o u + y_r r \\
(Y_o u + Y_r r) & X_o u & 0
\end{bmatrix}
\]

\[
D_I = -\begin{bmatrix}
x_u & 0 & 0 \\
0 & y_o & y_r \\
0 & N_o & N_r
\end{bmatrix}
\]

\[
D_n = -\begin{bmatrix}
X_{[u][u][u]} & 0 & 0 \\
0 & Y_{[v][v][v]} & Y_{[r][r][r]} \\
0 & N_{[r][r][r]} & N_{[r][r][r]}
\end{bmatrix}
\]

| Parameter | Value | Units |
|-----------|-------|-------|
| \(m\)     | 168.2000 | kg |
| \(I_z\)   | 52.7000  | kg\cdot m^2 |
| \(X_o\)   | -452.6809 | kg |
| \(Y_o\)   | -415.9546 | kg |
| \(Y_r\)   | -518.5884 | kg\cdot m/rad |
| \(N_o\)   | 18.3217  | kg\cdot m |
| \(N_r\)   | -231.9244 | kg\cdot m^2/rad |
| \(X_{[u]}\)| -0.2992  | kg/s |
| \(Y_{[v]}\)| -45.3418 | kg/s |
| \(Y_{[r]}\)| 105.3055 | kg\cdot m/rad |
| \(N_{[r]}\)| -3.3604 | kg\cdot m/s |
| \(N_{[r]}\)| -56.9292 | kg\cdot m^2/rad |
| \(X_{[u]}\) | -152.6010 | kg/m |
| \(Y_{[v]}\) | -399.4271 | kg/m |
| \(Y_{[r]}\) | -899.5759 | kg\cdot m^2/rad |
| \(N_{[r]}\) | 21.1978  | kg |
| \(N_{[r]}\) | -284.1011 | kg\cdot m^2/rad |

The propulsion and torque are calculated as

\[
\tau = \begin{bmatrix}
\text{Prop} \\
\text{Torque}
\end{bmatrix} = \begin{bmatrix}
X_{main} + X_{lft} + X_{rgt} \\
\alpha_{fin} (X_{lft} - X_{rgt})
\end{bmatrix}
\]

where \(X_{main}\) is the force of the main thruster, \(X_{lft}\) is the force of the right thruster, \(X_{rgt}\) is the force of the left thruster, and \(\alpha_{fin}\) the distance of the lateral thrusters to the central axis, which is 0.5m.

3. LINEARIZATION

Linearization is a good tool to simplify the model and at the same time to design linear controllers. In the case of vehicles the model is linearized around the velocity. For this, we assume that the Guanay II navigates with \(v\) and \(r\) small values that can be neglected. Thus, the working point is

\[
u_0 = [u_0 \ 0 \ 0 ]^T, \quad (10)
\]

and the variables around this point are

\[
u = \nu_0 + \Delta \nu = [u_0 + \Delta u, \Delta v, \Delta r]^T. \quad (11)
\]

Thus, the dynamic model, described in the last section, can be expressed by three equations. Replacing (2) in (3-8), we obtain that

\[
\begin{bmatrix}
-2[u_0]X_{[u][u]} - X_u & 0 & 0 \\
0 & -Y_v & (m - \Delta u)u_0 - Y_r \\
0 & 0 & (m - X_u)u_0 - N_r \\
0 & 0 & -N_o
\end{bmatrix} \Delta \nu + 
\begin{bmatrix}
m - X_u & 0 & 0 \\
0 & m - Y_v & 0 \\
0 & 0 & -N_o & -N_r
\end{bmatrix} \Delta V 
\]

\[
\Delta \nu = \Delta \nu^P + \Delta \nu^T, \quad (12)
\]

The terms \(\Delta \nu, \Delta r, \Delta u^2, \Delta r^2\) were neglected because they products of multiplications of small numbers near zero. Using this equation we can calculate different transfer function applying the Laplace transform. The two principal transfer functions of interest are the variation of the yaw respect to the torque applied, and the forward velocity respect to the propulsion applied. In this order we obtain that

\[
G_{\psi}(s) = \frac{s}{\text{Torque}(s)} = \frac{m (m - Y_v) s - Y_r}{A s^3 + B s^2 + C s}, \quad (13)
\]

\[
G_u(s) = \frac{v(s)}{\text{Prop}(s)} = \frac{1}{(m - X_u) s - 2[u_0] X_{[u][u]} - X_u}, \quad (14)
\]

where

\[
A = (m - Y_v)(I_z - N_r) - Y_r N_r \\
B = (m - X_u)(N_r - Y_r) u_0 - (m - Y_r) N_r + \\
- (I_z - N_r) Y_r - Y_r N_r - N_r Y_r \\
C = Y_r (Y_r u_0 + N_r) + \\
+ (Y_r - X_u) u_0 + N_r ((m - X_u) u_0 - Y_r)
\]

The subindex \(u_0\) in the notation \(G_{\psi}(s)\) and \(G_u(s)\) represents the velocity where the model is linearized. Following, we opted for two velocities: 0.3 and 2m/s, which have been selected as an abstraction of “low” and “high” velocity respectively. Using the coefficients of the Guanay II we obtain

\[
G_{\psi}(s)_{0.3} = \frac{0.003323 s^2 + 0.000258}{s^3 + 0.8107 s^2 + 0.0583 s}, \quad (16)
\]

\[
G_{\psi}(s)_{2.0} = \frac{0.003323 s + 0.000258}{s^3 + 4.0350 s^2 + 0.7354 s}, \quad (17)
\]

\[
G_u(s)_{0.3} = \frac{0.001611}{s + 0.148}, \quad (18)
\]

\[
G_u(s)_{2.0} = \frac{0.001611}{s + 0.9836}. \quad (19)
\]

4. LINEAR CONTROLLER FOR THE YAW

In this section we propose different linear controls, since a non-linear system can be seen as a piecewise linear model. For this, it is necessary to introduce the following definition.

Definition. It is said that a set of linear controls is \textit{zonally differentiated} if it can be shown that each control is more optimal than the others in a specific zone. That is, one control
has good performance in the conditions for which it was designed, but not the others (which have good performance in other zones). For marine vehicles, these zones represent the different forward velocities \( u \) for which the model is linearized.

In this section we develop a set of zonally differentiated controls in order to control the yaw angle of the Guanay II AUV for different forward velocities.

The yaw control consists in a definition of a reference \( \psi_{ref} \) which is compared with the actual angle \( \psi \) in order to establish an error. The principal idea is to bring this error to zero using a controller \( C(s) \) that actuates the lateral thrusters. The general block diagram can be seen in figure 1.

![Block diagram to control the yaw](image)

Fig. 1. Block diagram to control the yaw (\( \psi \)).

Denoting by \( H_\psi(s) \) the transfer function of the yaw respect to the reference, it can be calculated as:

\[
H_\psi(s) = \frac{\psi(s)}{\psi_{ref}(s)} = \frac{C(s) G_\psi(s)}{1 + C(s) G_\psi(s)}
\]  

(20)

### 4.1 P controller

A proportional controller consists of scaling the error \( e_\psi \) with a constant \( k \) to actuate the thrusters:

\[
C_P(s) = k
\]  

(21)

This type of controller is good to control the yaw since for this type of plant zero error is guaranteed in steady state in a presence of a step as input. A simple calculation of the final value theorem is enough to show it. The closed loop poles can be calculated through \( H_\psi(s) \) equating its denominator to zero:

\[
1 + C_P(s) G_\psi(s) = 0
\]

(23)

For the controller design, the breakaway point is the best position for the poles because the dominant pole is as far away from the imaginary axis, which gives a fast response. Also, as they are real, the response does not present overshoot or oscillations. This point is achieved when two poles are the same.

### 4.2 PD controller

A proportional-derivative controller (PD) consists of a gain for the yaw error \( e_\psi \) and a zero which allows fast response:

\[
C_{PD}(s) = k_d s + k_p
\]  

(22)

where the subscript PD denotes "proportional-derivative" and \( u_0 \) is the velocity taken in the linearization of \( G_\psi(s) \) to design the controller.

This type of control has the great advantage that it can move the two dominant poles to more negative values, which represents better performance.

The root locus has two types of diagrams for stable systems:

1. Locate the zero between the two poles. In this case, the dominant pole will go to the zero, and the other to infinity. The advantage is that no matter the gain, the resulting poles are real. The disadvantage is that the resulting position of the dominant pole, and consequently the speed of response, is limited by the position of the zero.

2. Locate the zero at the left of the two poles. In this case, the movement of the poles has three sections: First an approach of poles until a breakaway point; second two complex branches enclosing the zero in an oval; and third one real pole moving to the zero and the other to infinity. The advantage with this design is that the two poles can be at the left of the zero, and consequently has a fast response. The disadvantage is that the fast response requires high gains, and also the possibility of having conjugated poles that create large oscillations.

The closed loop poles can be calculated through \( H_\psi(s) \) equating its denominator to zero:

\[
1 + C_P(s) G_\psi(s) = 0
\]

\[
A s^3 + (B + (m - Y_e) k_d)a^2 + (C + (m - Y_e) k_p - Y_e k_d)s - Y_e k_p = 0
\]  

(24)

For the controller design, it is better to locate the zero at the left of the two poles in order to obtain a fast response. And the gain should be a constant which leads the poles to the breakaway point at the left of the zero. Similarly to the P controller design, this point is achieved when two poles are the same.

### 4.3 Zonally differentiated controllers

We have found that the PD controller is the best option. However, the proportional controller is useful for high velocities because the Guanay II has low power to turn and the controller gives a big amount of gain. In this order, we opted to select a PD controller for low velocities, that is 0.3m/s; and a P controller for high velocities, that is 2m/s. Using (22) and (24), we obtain that

\[
C_{PD_{0.3}}(s) = 408.8266(s + 0.8039),
\]

\[
C_P(s) = 1043.8723.
\]  

(25)

(26)

Figure 2 shows the root locus for each case: When the vehicle travels at 0.3m/s the first controller, \( C_{PD_{0.3}} \), moves the two poles to -1 and the second, \( C_{P_{2.0}} \), to -0.4 ± 1.8i; in this case it is clear the benefits of the first controller. On the other hand, when the vehicle travels at 2m/s the first controller moves the dominant pole to -0.3 while the second controller moves the two poles to -2; in this case, the benefits of the second controller are clear.
Regarding the response in time, figure 3 shows the step response using these controllers. When the vehicle travels at 0.3m/s, \( C_{PD}(0.3) \) yields a better performance than the second one, which has an underdamped and slow response. For the second case when the vehicle travels at 2m/s. The controller \( C_{P}(2.0) \) yields a faster response than the first one.

In conclusion, we can say that this set of controllers is zonally differentiated, which was what we wanted to design.

5. LINEAR CONTROLLER FOR THE FORWARD VELOCITY

The velocity control consists in a definition of a reference \( u_{ref} \), which is compared with the actual velocity \( u \) in order to establish an error. The principal idea is to bring this error to zero using a controller \( T(s) \) that actuates the thrusters. The general block diagram can be seen in figure 4.

![Fig. 4. Block diagram to control the forward velocity](image)

Denoting by \( H_u(s) \) the transfer function of the velocity respect to the reference, it can be calculated as

\[
H_u(s) = \frac{u(s)}{u_{ref}(s)} = \frac{T(s) G_u(s)}{1 + T(s) G_u(s)}.
\]  

(27)

It is easy to see that this plant needs at least a proportional integral controller (PI) because it needs an integrator to guarantee zero error in steady state for a step response. In this way, we will set this controller as follows:

\[
T_{PI}(s) = \frac{k_p s + k_i}{u_0 s}
\]  

(28)

where the subscript PI denotes "proportional-integral" and \( u_0 \) is the velocity taken in the linearization of \( G_u(s) \) to design the controller.

Notice that the use of a PI controller implies a root locus with two poles and one zero, which is similar to the disposition obtained in the yaw control using a PD controller. In this order, the same design process can be used for this controller. Figure 5 shows the root locus of two controllers calculated for two transfer functions: at 0.3m/s and 2m/s. The main idea is to be zonally differentiated, in order to have the best pole disposition using the correct controller at the right conditions of velocity.

For the first transfer function, \( G_{u.0.3}(s) \), the poles are moved to -0.35 using the controller \( T_{PI}(0.3) \), while they are moved to -0.34+0.68i using the controller \( T_{PI}(2.0) \). The real poles represent a better option compared with the complex conjugated poles. On the other hand, in the second transfer function, \( G_{u.2.0}(s) \), the dominant pole is moved to -0.08 using the controller \( T_{PI}(0.3) \), which is very near to the imaginary axis, while the poles are moved to -1.4 using the controller \( T_{PI}(2.0) \), which guarantees fast response. Notice that this design moves the poles at the right breakaway point and not at the left; this is because the left breakaway point needs a large gain which compromises the zonal differentiation (a large gain implies better performance at the first transfer function \( G_{u.0.3}(s) \)).

![Fig. 5. Root locus for \( G_u(s) \) at 0.3m/s and 2m/s when are used the PI controllers.](image)
6. TSK FUZZY CONTROLLER

Alternatively to classical control engineering, it seems convenient to simulate the behaviour of a person who is able to control the given process. We call this development of a model of a human ‘control expert’ knowledge-based analysis. To make such an analysis the expert may be questioned directly. The expert then specifies his knowledge in form of linguistic rules. Instead of directly interviewing the expert it is also possible to observe his behaviour and extract from the observation protocol the necessary information. The results of this procedure can be used to provide appropriate (linguistic) rules that control the process (see Kruse et al. 1994, and Driankov et al. 1996).

In the case of the Guanay II this control can be used to control the velocity and the yaw. The above results have shown the importance of using different controllers depending of the forward velocity \( u \). This control expert can be used to change between the different controllers, in other words, fuzzy control is presented as an interpolator controller. A block diagram of this concept is presented in figure 7.

There are two fuzzy blocks. The first one has the mission of modifying the parameters of \( T(s) \) in order to control the velocity. The input taken in this case is \( u_{ref} \). Notice that we could use \( u \) as input, but the reference \( u_{ref} \) performs as well as the step response with the simulations made using the lineal controllers, while \( u \) changes over the time causing different responses.

The second block must modify the parameters of \( C(s) \) in order to control the yaw. In this case the input is the current velocity \( u \) which is the basis of the designed controllers.

6.1 Fuzzification

The velocities \( u \) and \( u_{ref} \) need to be transformed to a linguistic terms in order to be controlled in a fuzzy way. The Guanay II needs two connotations: 0.3m/s and 2m/s, considered as low and high velocity respectively. Figure 8 shows a graphical representation of the memberships.

6.2 Decision logic

The expert specifies his knowledge in form of linguistic rules. The advantage of this conception is the ease to define the rules and outputs. For instance, in the case of the Guanay II we know that some controllers have to be used at low velocities (see equations 25 and 29), and other controllers at high velocities (see equations 26 and 30). In this order, the linguistic rules to control the yaw are:

\[
\begin{align*}
R_1: & \text{ if } u \text{ is low then } C(s) = C_{PD}(0.3) \\
R_2: & \text{ if } u \text{ is high then } C(s) = C_{PD}(2.0)
\end{align*}
\]

and the linguistic rules to control the velocity are:

\[
\begin{align*}
R_1: & \text{ if } u_{ref} \text{ is low then } T(s) = T_{PD}(0.3) \\
R_2: & \text{ if } u_{ref} \text{ is high then } T(s) = T_{PD}(2.0)
\end{align*}
\]

6.3 Defuzzification

For the defuzzification we use a Takagi-Sugeno-Kang (TSK) fuzzy controller, which uses analytic functions instead of linguistic terms (see Kruse et al. 1994). Its importance stems from its ability to be adapted to the development of linear controllers, like the controllers P, PD and PI of the last sections. To use it, we calculate a different weight for each velocity and we compute the following equations.

\[
\begin{align*}
\alpha_m = \min\{\mu_{k1,m}(x_1), \ldots, \mu_{k,n,m}(x_n)\} \\
\gamma = \frac{1}{k} \sum_{m=1}^{k} \alpha_m \cdot f_m(x_1, \ldots, x_n)
\end{align*}
\]

where \( \mu_{k,m} \) is the membership of \( R_m \) for the different inputs, and \( f_m \) are the functions used (in this case the values of the linear controllers).

7. RESULTS

Several simulations with the nonlinear model were made in order to test the performance of the fuzzy controller. Figure 9 shows the step response for the yaw using the different controllers and traveling at different velocities.
When the vehicle is traveling at 0.3m/s the fuzzy controller equals the response of the controller \( C_{P(0.3)} \) which has a good settling time (3.8s) and no overshoot. However, the controller \( C_{P(2.0)} \) yields an overshoot of 26.1%. There is a similar characteristic when the vehicle is traveling at 2m/s; the fuzzy controller is similar to the response of the controller \( C_{P(2.0)} \) which has a good settling time (4.2s) and no overshoot. However, the controller \( C_{P(0.3)} \) has an undesired settling time, 19.4s. Finally, when the vehicle is traveling at 1m/s the fuzzy controller yields an intermediate response between the linear controllers.

![Fig. 9. Comparative of the step response for the yaw using the fuzzy controller and linear controllers.](image)

Fig. 9. Comparative of the step response for the yaw using the fuzzy controller and linear controllers.

Figure 10 shows the step response for the forward velocity using the different controllers and different references. When the reference is 0.3m/s the fuzzy controller is similar to the response of the controller \( T_{P(0.3)} \) which has a settling time of 10.6s and overshoot of 8.9%. However, the controller \( T_{P(2.0)} \) yields a high overshoot, 31.7%. There is a similar characteristic when the reference is 2m/s; the fuzzy controller performs as well as the response of the controller \( T_{P(2.0)} \) which has a settling time of 2.1s and overshoot of 3.4%. However, the controller \( T_{P(0.3)} \) has an undesired settling time, 19.8s. Finally, when the reference is 1m/s the fuzzy controller yields an intermediate response between the linear controllers.

![Fig. 10. Comparative of the step response for the forward velocity using the fuzzy controller and linear controllers.](image)

Fig. 10. Comparative of the step response for the forward velocity using the fuzzy controller and linear controllers.

8. CONCLUSIONS AND FUTURE WORK

In conclusion, these results show the advantage of using the TSK fuzzy controller, because it adapts the controller to the correct zone depending of the forward velocity, while the linear controllers only have good responses in a specific zone. As future work, studies about physical limitations, like the maximum power to thrust and to turn, will be included. Furthermore, a rigorous development about the global stability will be included.

ACKNOWLEDGMENT

This work has been funded by the Spanish Ministry of Education and Science and the European Union (FEDER), projects nº CGL2013-42557-R and DPI2011-25649.

REFERENCES

Dong, T., Liao, X., Zhang, R., Sun, Z., Song, Y. (2005). Path tracking and obstacles avoidance of uavs – fuzzy logic approach. Fuzzy Systems, 2005. The 14th IEEE International Conference on. pp. 43-48.

Driankov, D., Hellendoorn, H., Reinfrank, M. (1996). An introduction to Fuzzy Control. Springer.

Fossen, T. I. (2002). Marine Control Systems Guidance, Navigation, and Control of Ships, Rigs and Underwater Vehicles. Marine Cybernetics. Trondheim, Norway.

Gomáriz, S., Masmitjà, I., González, J., Masmitjà, G., Prat. J. (2014). GUANAY-II: an autonomous underwater vehicle for vertical/horizontal sampling. Journal of Marine Science and Technology. Springer Japan.

Kruse R., Gebhardt, J., Klawonn, F. (1994). Foundations of Fuzzy Systems. John Wiley & Sons, Inc. New York, USA.

Jun, S. W., Kim, D. W., Lee, H. J. (2012). Design of t-s fuzzy-model-based diving control of autonomous underwater vehicles: Line of sight guidance approach. Control, Automation and Systems (ICCAS), 2012 12th International Conference on. pp. 2071-2073.

Liu, S., Wei, Y., Gao, Y. (2012). 3d path planning for auv using fuzzy logic. Computer Science and Information Processing (CSIP), 2012 International Conference on. pp. 599-603.

Reddy, B., Srinivas, Y., VenkataRamesh, E. (2010). Analytical structures of gain-scheduled fuzzy pi controllers. Industrial Electronics, Control Robotics (IECR), 2010 International conference on. pp. 122-128.

Silvestre, . Pascoal, A. (2007). Depth control for the infante auv using gain-scheduled reduced order output feedback. Science Direct, 883-895.

Takagi, T., Sugeno, M. (1985). Fuzzy identification of systems and its applications to modelling and control. IEEE Transactions on SMC. 15 (1), 116-132.

Zhang, W., Wang, H., Bian, X., Yan, Z., Xia, G. (2012). The application of self-tuning fuzzy pid control method to recovering auv. Oceans, 2012. pp. 1-5.