Comparing Quantum Gravity Models: String Theory, Loop Quantum Gravity, and Entanglement gravity versus $SU(\infty)$-QGR

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Abstract

In a previous work [1] we proposed a new model for Quantum Gravity (QGR) and cosmology, dubbed $SU(\infty)$-QGR. One of the axioms of this model is that Hilbert spaces of the Universe and its subsystems represent $SU(\infty)$ symmetry group. In this framework, the classical spacetime is interpreted as being the parameter space characterizing states of the $SU(\infty)$ representing Hilbert spaces. Using quantum uncertainty relations, it is shown that the parameter space - the spacetime - has a 3+1 dimensional Lorentzian geometry. Hereafter a review of $SU(\infty)$-QGR, including the demonstration that its classical limit is Einstein gravity, we compare it with several QGR proposals, including: string and M-theories, loop quantum gravity and related models, and QGR proposals inspired by holographic principle and quantum entanglement. The purpose is to find their common and analogous features, even if they apparently seem to have different roles and interpretations. The hope is that such exercise gives a better understanding of gravity as a universal quantum force and clarifies the physical nature of the spacetime. We identify several common features among the studied models: importance of 2D structures; algebraic decomposition to tensor products; special role of $SU(2)$ group in their formulation; necessity of a quantum time as a relational observable. We discuss how these features can be considered as analogous in different models. We also show that they arise in $SU(\infty)$-QGR without fine-tuning, additional assumptions, or restrictions.

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1 Introduction and results

Several fundamental questions about gravity and spacetime are not still answered by general relativity or by various attempts to find a consistent quantum description for gravitational interaction. The most daunting of these issues are the dimension of spacetime, which is usually considered to be the observed (3+1) without any explanation for its origin. Moreover, general relativity and Einstein gravity do not specify what is the nature of spacetime, except that it is curved in presence of matter and energy. Most QGR models treat spacetime as a physical entity, which despite being coupled to matter, has an independent existence. Indeed, often quantization of gravitational interaction, which is necessary in a Universe with quantum matter [2], is interpreted as inevitability of a quantized spacetime. There are, however, multiple evidence against this conclusion:

- It is demonstrated [3] that Einstein equation can be obtained from the second law of thermodynamics and holographic principle, that is the proportionality of entropy inside a null (light-like) surface to its area rather than volume [4, 5, 6, 7]. Holographic behaviour has been also observed in many-body systems with negligible gravity [8, 9]. These observations confirm the conclusion of [3] that Einstein equation should be considered as equation of state. This interpretation and universality of gravitational interaction imply that what is perceived as space and its geometrical properties, such as distance and curvature, represent the state of its matter content. Thus, it seems that spacetime and matter are inseparable aspects of the same physical reality/entity.

- Even without holographic principle the fact that energy-momentum tensor of matter - the source of gravitational interaction - depends on the spacetime metric means that spacetime and matter are more intertwined than, for instance, bosonic gauge fields and their matter source in Yang-Mills models.

- In Quantum Field Theory (QFT) spacetime or its dual energy-momentum mode space (but not both at the same time) are used as indices to keep track of the continuum of matter and radiation. Giving the fact that in a quantum realm the classical vacuum - the apparently empty space between particles - can be described as a sea of virtual - off-shell - quantum states [10], means that we could completely neglect the physical space - the perceived 3-dimensional space. This were possible if we could identify, tag, and order all real and virtual particles, for instance by using the strength of their mutual quantum entanglement [11, 12, 13, 14, 22] or interaction strength [18]. In this view, the classical Einstein equation could be interpreted as an equation of state, which dynamically modifies parameter (index) space according to variation of interactions and entanglement between particles, and with respect to a relational quantum clock [15].

- It is useful to remind that in most QGR models the dimension of spacetime is considered as a parameter and little attempt is made to explain why it has the observed value.
In the last decade or so progress in quantum information theory has motivated construction of QGR models which are not based on the quantization of a classical theory. They are sometimes called Quantum First models in the literature \cite{17}. In addition, progress in quantum information has highlighted the crucial role of the division of the Universe to parts - subsystems - and a proper mathematical definition for what can be considered as a distinguishable quantum (sub)system. This concept has special importance for gravity, because as much as we know from general relativity, it is a universal force, coupling everything to the rest of the Universe. Indeed, we will see later in this work that some QGR models struggle to find a naturally factorized - tensor product - Hilbert space in which each factor can be considered as presenting the Hilbert space of a subsystem. In QFTs without gravity subsystems are particles or their collections. A priori the same concept can be applied to QGR. However, in the strong coupling limit of QGR spacetime/gravity and matter may be indistinguishable. Therefore, it is necessary to have physically and mathematically well defined description of may be called a distinguishable subsystem of the Universe. We also remind that tensor product of subsystems is not only important for gravitational interaction, but also for the meaningful definition of locality, quantum clock, quantum information flow and relative entropy, renormalization flow, and holographic principle. None of these concepts would make sense without mathematical and physical notion of distinguishable subsystems.

In \cite{1, 16} we proposed a model for a quantum Universe which can be classified in quantum first category. Here we call it $SU(\infty)$-QGR. A brief review of this model is given in Sec. 2. It is a fundamentally quantum model, in the sense that its axioms come from quantum physics and its formulation is not a quantized version of a classical model. It does not include in its foundation, neither explicitly nor implicitly, a background spacetime or ingredients from Einstein general relativity, such as entropy-area relation. It is shown that both spacetime and Einstein equation emerge from quantum properties. The physical space is identified with the space of indices parameterizing Hilbert and Fock spaces of the Universe and their subsystems. Einstein equation presents the projection of relational evolution of subsystems on the parameter space - the only observable when experiments do not have sufficient sensitivity to observe quantum field of gravitational interaction.

In other Quantum First proposals usually a background spacetime is implicitly present in their axioms. Examples of such models are those described in \cite{13, 18}. There is also implicit assumption of a physical space in models based on holographic principle - a hypothesis inspired from semi-classical general relativity \cite{19, 20} such as \cite{14, 22}. Indeed, it is obvious that holography without a geometrical space is meaningless. By contrast, in $SU(\infty)$-QGR the classical physical space and time genuinely emerge from quantum structure of the model and the assumption that any physical entity must be inside the Universe.

Quantum First QGRs and other modern approaches to QGR at first sight seem very different from each others. However, history of science is full of cases where seemingly different theories and interpretations were finally turned up to present the same physical concept viewed in different perspectives. The best example is Schrödinger’s wave mechanics and Heisenberg’s matrix mechanics approaches to quantum mechanics, which were later proved to be equivalent. For this reason, any new theory should look for what it has in common with other relevant models, and what new concepts or interpretations it is proposing. Such verification is particularly necessary for new QGR proposals, because QGR has been under intensive investigation for close to a century. Moreover, giving the fact that at present none of the proposals is fully satisfactory or has observational support, a better understanding of common aspects of different candidates may provide a direction and path to further developments, and eventually to the true model, unless all the proposals are completely irrelevant.

In this work we compare $SU(\infty)$-QGR with some of popular approaches to QGR, namely: symplectic models, including Loop Quantum Gravity (LQG) and related models; string theory and its closely related matrix models (M-theory) and Anti-de Sitter-Conformal Field Theory (AdS/CFT) duality - more generally gauge-gravity duality; and models based on the holographic principle and quantum entanglement. Although our purpose is to find similarities and analogous features of among these
models, this investigation also clarify their principle differences, which may be equally useful for further theoretical development, and eventually discriminating or constraining these models in experiments and observations.

We do not consider more traditional approaches such as canonical quantization [48, 49, 25] (see e.g. [50, 27] for a review) and ADM 3+1 method [28]. After decades of research, it is now clear that they do not lead to a consistent and renormalizable theory. Other models omitted here are QGR models based on non-commutative spacetimes, models based on the quantum history interpretation of quantum mechanics, and the causal sets. These models are based on postulates that fundamentally deviate from those of $SU(\infty)$-QGR and their comparison with the latter is meaningless.

We begin by presenting a summary of the results of this work in the subsection 1.1. Its purpose is to provide a quick overlook of comparison results. Therefore, if there are unclear points, the reader should refer to Sec. 3 for more explanation.

In Sec. 2 and its subsections we briefly review $SU(\infty)$-QGR and show that the common features of QGR models arise naturally and without fine-tuning or addition of new assumptions to the initial axioms. Details of the comparison between models are discussed in Sec. 3. For each model we first briefly remind its main features. Then, we compare them with those of $SU(\infty)$-QGR. It is obvious that detailed and technical description of models and their variants, about which in some cases thousands of papers and numerous text books are written, is out of the scope of the present work. The aim of short reminds here is to introduce features and notations used for the comparison with $SU(\infty)$-QGR. Sec. 3.1 reviews several background independent QGR models, including Ponzano-Regge model and LQG. Quantum First models are reviewed and compared with $SU(\infty)$-QGR in Sec. 3.2. We compare string and M-theories, and gauge-gravity duality conjecture with $SU(\infty)$-QGR in Sec. 3.3. A short outline is given in Sec. 4.

1.1 Summary of comparison results

Form comparison of $SU(\infty)$-QGR proposal with some of other approaches to QGR we recognize a series of similar aspects, symmetries, and structures, which despite their different roles and interpretations in different models can be considered as analogous and common. Here we should emphasize that what we call similarity or analogy should not be interpreted as one-to-one correspondence. For instance, decomposition of $SU(\infty)$ to $SU(2)$ factors in $SU(\infty)$-QGR is not the same operation as discretizing space to tetrahedra weighed by spins on their edges. Nonetheless, they have analogous mathematical descriptions - in this case a spin network. If the QGR models reviewed in this work contain at least some of the features and properties of the true theory, they should be, most probably, reflected in these common or analogous characteristics.

The common features that we found in the models investigated in Sec. 3 can be summarized as the followings:

Presence of 2-dimensional spaces or structures in the construction of models:

In some QGR models 2D spaces are used to construct a quantized space. They are either 2D boundary of a symplectic geometry consisting of connected tetrahedra with nonzero curvature at vertices, or 2D worldsheets/membranes, embedded in a multi-dimensional space. These structures are treated as fundamental objects of the models - similar to particles in QFTs without gravity - and despite significant differences in their interpretation in different models, they have crucial role in generation of what is perceived as spacetime and gravity. In these models 2D structures are usually postulated and considered as physical entities. In this respect $SU(\infty)$-QGR is an exception, because diffeo-surfaces emerge from axioms and symmetries, and are considered as
properties rather than being physical objects\textsuperscript{2}.

Extended nature of 2D structures has a crucial role in making QGR models renormalizable and in preventing singularities. In $SU(\varpi)$-QGR this property is reflected in the fact that by definition a diffeo-surface cannot be shrunk to a point, otherwise $SU(\varpi)$ symmetry would be represented trivially.

\textbf{Decomposition to an algebraic tensor product:}

Universe is a composite system and by definition, the Hilbert space of composite quantum systems is decomposed to tensor product of the Hilbert spaces of their subsystems \textsuperscript{29}. Therefore, it is normal that an algebraic tensor product structure emerges, in one way or another, in the construction of QGR models. However, the most crucial tensor product structures in background independent models such as LQG and related models are spin-networks associated to the symplectic geometry and quantization of space. Specifically, their Hilbert space consists of all embedding of spin-weighted graphs - spin networks - generating the symplectic geometry states\textsuperscript{3}. For this reason tensor products in spin networks cannot be interpreted as division to separable subsystems. This aspect is also shared by Quantum First models based on the entanglement.

In string and matrix theories, tensor products emerge in separation of compactified and non-compactified fields or as special configuration of string condensate in the form of D-branes (Moyal-Weyl solution). This can be interpreted as regarding spacetime and particles/matter fields as separate subsystems. Of course one may consider ensemble of strings, or more generally membranes or their presentation as large matrices as subsystems. However, they structures live in a higher dimensional space. In perturbative formulation this space has to be flat and it is not clear how strings/membranes interactions can generate it. In non-perturbative M-theory and matrix formulation, strings/membranes are frozen in a brane condensate in order to explain the observed (3+1)D spacetime. Their quantum fluctuations are treated similar to fields in QFTs, with \textit{particles} as fundamental subsystems, and the $D = 10$ dimensional background is static and unobservable.

\textbf{$SU(2)$ group and spin network:}

$SU(2)$ symmetry and/or its representations have a special role in most QGR models. In particular, they intervene in the construction of quantized geometry, because $SU(2) \cong SO(3)$ is the coordinate symmetry of the physical space.. Exceptions are string theory and $SU(\varpi)$-QGR . Although $SU(2)$ group and its representations are extensively used in the formulation of $SU(\varpi)$-QGR , it remains a purely mathematical utility without prior connection to the structure of classical spacetime.

\textbf{A hidden or explicit $SU(\varpi)$ symmetry:}

In models based on the symplectic construction of space the number of cells - usually tetrahedra - has to be considered to go to infinity to obtain a continuum at large distance scales - low energies. As these cells are indistinguishable from each others, the Hilbert space and dynamics of these models is invariant under $SU(\varpi)$ group defined on $\mathbb{R}$, rather than $\mathbb{C}$ considered in $SU(\varpi)$-QGR.

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\textsuperscript{2}Notice that the issue of what makes an abstract entity a \textit{physical} object is rather philosophical. In practice, in mathematical formulations of physical phenomena all entities are abstract but related to what can be measured. Thus, in this sense they can be considered as \textit{physical}.

\textsuperscript{3}The physical space can be considered as the state which is observed in the measurements.
String-gauge duality (M-theory) conjecture \cite{30, 31, 32} identifies Yang-Mills models with large number of colors $N_c$ with string states. For $N_c \to \infty$ the symmetry of the Yang-Mills theory is $SU(\infty)$. Indeed, in matrix model implementation of this conjecture, the fundamental objects are $N \times N$, $N \to \infty$ matrices, and $SO(D)$ symmetry of fundamental $D = 10$ dimensional space according to string theory can be interpreted as a special case of internal symmetries $G$ in $SU(\infty)$-QGR. However, despite these similarities, interpretation of matrices in M-theory and $SU(\infty)$-QGR are very different. Matrix models do not explore $SU(\infty)$ symmetry and only use large matrices as representation of strings worldsheets or membranes in a special state of the fundamental background spacetime. By contrast, in $SU(\infty)$-QGR no special configuration is necessary to explain the observed (3+1)D spacetime. The tensor product $SU(\infty) \otimes G$ provides mathematical requirements for division to subsystems \cite{29} and guarantees the existence of a perturbative expansion for both gravitational and matter sectors, without any constraint on the internal symmetry $G$. On the other hand, as $SU(\infty) \times G \cong SU(\infty)$, the model has also a non-perturbative limit.

**Emergence of time and evolution as relative and relational phenomena:**

A relational clock and its associated time parameter are necessary in the most QGR approaches except in string theory, in which time, space, and matter are treated in a same manner and are included in the foundation of the model.

These common properties demonstrate that despite their apparent differences QGR candidates are more similar than probably expected.

All the common features discussed here arise naturally and straightforwardly in $SU(\infty)$-QGR. Nonetheless, this model is a new proposal and much more must be done and understood about it before it can be considered as a genuine contender of a consistent and testable quantum gravity model. In particular, its predictions for the puzzle of black hole information loss and its predictions for future experiments seeking the detection of decoherence by quantum gravitational interactions should be investigated.

## 2 A brief review of $SU(\infty)$-QGR

In this section we briefly summarize axioms, structure, and constituents of $SU(\infty)$-QGR. Only mathematical formulations necessary for the comparison with other QGR models are presented here.

### 2.1 Axioms and algebra

The $SU(\infty)$-QGR is based on 3 well motivated assumptions:

1. Quantum mechanics is valid at all scales and applies to every entity, including the Universe as a whole;

2. Any quantum system is described by its symmetries and its Hilbert space represents them;

3. The Universe\(^4\) has infinite number of independent degrees of freedom, that is mutually commuting observables.

\(^4\)Here by Universe we mean everything causally or through its quantum correlation observable. Independent quantum observables correspond to mutually commuting hermitian operators applied to the Hilbert space. Their subspace is homomorphic to the Cartan subspace of the symmetry group of the quantum system, here the whole Universe \cite{42}.
These axioms might seem trivial and generic. Here we briefly argue that they are not:

Axiom 1 is not trivial because some QGR models extend or restrict quantum mechanics and/or QFT\(^5\) in order to accommodate QGR, see Sec. 3.2.1 for a brief review of these models.

Axiom 2 is added to the above list because in postulates of quantum mechanics, as defined by Dirac [33] and von Neumann [34], the Hilbert space is an abstract Banach space and no relation to symmetries is explicitly mentioned. Axioms of quantum mechanics with symmetry as a foundational concept are described in [42]. Of course, in practice the Hilbert space is chosen such that it represent symmetries. However, this is due to the fact that the choice of Hilbert space for a quantum system is motivated by the configuration space of its classical limit and its symmetries. If we want to construct a fundamentally quantum model without referring to a corresponding classical system, we must specify how the Hilbert space is defined.

Axiom 3 defines the symmetry of the system - the Universe, which as explained above is the basis for all other properties of the quantum system. Of course QFTs by definition have infinite number of observables/degrees of freedom - one or more at each point of the spacetime. However, in SU(\(\infty\))-QGR there is no spacetime and the model is constructed as an abstract quantum system defined exclusively by its symmetry and its representation by the Hilbert space.

On the hindsight, simplicity of these axioms is their advantage and in the following subsections we briefly review what can be concluded from this apparently generic assumptions. It should be reminded that we do not have any observed evidence of quantum gravity. Thus, sophisticated and designed axioms of some QGR models look rather imaginative, and one wonders why nature should have selected them among many other possibilities.

### 2.2 Representation of SU(\(\infty\)) group and Hilbert space

The last assumption means that the Hilbert space of the Universe \(H_U\) is infinite dimensional and represents SU(\(\infty\)) symmetry group, that is:

\[
\mathcal{B}[H_u] \cong SU(\infty)
\]

where the sign \(\cong\) means homomorphism and \(\mathcal{B}[H_u]\) is the space of bounded linear operators acting on \(H_U\). Generators \(\hat{L}_{lm}, \ l \geq 0, \ |m| \leq l\) of \(\mathcal{B}[H_u]\) satisfy the Lie algebra:

\[
[\hat{L}_{lm}, \hat{L}_{l'm'}] = i f^{nmn'}_{lm,l'm'} \hat{L}_{n'n'}
\]

where structure coefficients \(f^{nmn'}_{lm,l'm'}\) can be determined using properties of spherical harmonic functions, see e.g. [35] for more details. The reason for this property is that SU(\(\infty\)) can be decomposed to tensor products of SU(2):

\[
\hat{L}_{lm} = \mathcal{R} \sum_{i_\alpha=1, \ldots, N} a^{(m)}_{i_1, \ldots, i_l} \sigma_{i_1} \cdots \sigma_{i_l} \quad (l, m) \ | \ l = 0, \ldots, \infty; \ -l \leq m \leq +l
\]

where \(\sigma_{i_\alpha}\)'s are \(N \rightarrow \infty\) representations of Pauli matrices [35] and \(\mathcal{R}\) is a normalization constant. Coefficients \(a^{(m)}\) are determined from expansion of spherical harmonic functions with respect to spherical description of Cartesian coordinates [35].

The model is quantized using dual of its Hilbert space \(H^*_U\) and its space of bounded linear operators \(\mathcal{B}[H^*_U]\):

\[
[\hat{L}_a, \hat{J}_b] = -i \delta_{ab} \hbar, \quad \hat{J}_a \in \mathcal{B}[H^*_U]
\]

\(^5\)We remind that QFT is not a model by its own. It is a specialized formulation of quantum mechanics suitable for description of many-body systems in spacetimes, specially in a Lorentz invariant manner.
\(\hbar\) is the Planck constant.

It is known that \(SU(x)\) is homomorphic to area preserving diffeomorphism of compact 2D surfaces \([35, 36, 37, 38, 39]\). From now on we use the shorthand name \textit{diffeo-surface} for the surfaces which their area preserving diffeomorphism is homomorphic to \(SU(x)\) of interest. Diffeo-surfaces with different genus correspond to non-equivalent (non-isometric) representations of \(SU(x)\) \([38, 39]\). These surfaces, and thereby \(\mathcal{B}[\mathcal{H}_U] \cong SU(x)\) are parameterized by two angular parameters \((\theta, \phi)\). On the other hand, \(SU(x)\) algebra is homomorphic to Poisson bracket of spherical harmonic functions, which for \(\hbar = 1\) and dimensionless operators can be written as:

\[
\hat{L}_{lm} = i \left( \frac{\partial Y_{lm}}{\partial \cos \theta} \frac{\partial}{\partial \phi} - \frac{\partial Y_{lm}}{\partial \cos \theta} \frac{\partial}{\partial \phi} \right) = i \sqrt{|g^{[2]}|} \epsilon^{\mu \nu} (\partial_\mu Y_{lm}) \partial_\nu, \quad \mu, \nu \in \{\theta, \phi\} \tag{5}
\]

\[
\hat{L}_{lm} Y_{lm'} = -i \{Y_{lm}, Y_{lm'}\} = -i f^{lm}_{lm', m''} Y_{lm''} \tag{6}
\]

\[
\{f, g\} \equiv \frac{\partial f}{\partial \cos \theta} \frac{\partial g}{\partial \phi} - \frac{\partial f}{\partial \phi} \frac{\partial g}{\partial \cos \theta}, \quad \forall \ f, g \tag{7}
\]

In this representation of \(\mathcal{B}[\mathcal{H}_U]\) vectors of the Hilbert space \(\mathcal{H}_U\) are complex functions of \((\theta, \phi)\). If \(\hat{L}_{lm}\) (or equivalently \(\hat{J}_{lm}\)) operators are normalized by a constant factor proportional to \(\frac{\hbar}{M_P}\), where \(M_P\) is a mass scale - presumably Planck mass - the r.h.s. commutation relation (2) becomes zero for \(\hbar \to 0\) or \(M_P \to \infty\) and the algebra of observables becomes Abelian, as in the classical mechanics. Thus, only when \(\hbar \neq 0\) and \(M_P < \infty\) the model presents a quantum system. This property establishes an inherent relationship between gravity and quantumness, as suggested in \([40]\).

### 2.2.1 \(SU(2)\) in \(SU(x)\)-QGR

The symmetry group \(SU(2)\) has a special place in many QGR models, including in \(SU(x)\)-QGR where it is used for Cartan decomposition of \(SU(x)\) and description of its representations \([35, 36, 37, 38, 39]\). In particular, it allows to expand members of \(SU(x)\) as a linear function of spherical harmonic functions, analogous to an infinite spin chain. Consequently, generators of \(SU(x)\) are described by spin quantum numbers \((l, m)\). This representation is more suitable for practical applications than abstract complex functions of two angular parameters \((\theta, \phi)\). Nonetheless, one can easily transform one representation to the other, see e.g. appendices in \([1]\).

We should emphasize that despite the importance of \(SU(2)\) for \(SU(x)\)-QGR , it is not anything more than a mathematical tool. In fact, using the relation:

\[
SU(N) \cong SU(N - K) \otimes SU(K) \tag{8}
\]

the group \(SU(x)\) can be decomposed to tensor products of any \(SU(N),\ N < \infty\) by repeated application of (8). Decomposition (3) corresponds to the case of \(K = 2\). It is the smallest non-Abelian special unitary group which can be used in the Cartan decomposition of \(SU(x)\).

### 2.3 Subsystems of the Universe

In \([1]\) it is shown that the quantum Universe as defined in the previous section is static and trivial. This is not a surprise, because there is no time parameter or a subsystem which plays the role of a quantum clock. On the other hand, according to a corollary in a description of quantum mechanics in which symmetry is considered to be foundational \([42, 43]\), this quantum system must inevitably be decomposable to subsystem. To this end, the Hilbert space must be factorized such that subsystems satisfy conditions defined in \([29]\). They include, among other things, factorization of the system’s symmetry group and its representations. Using properties of \(SU(x)\) \([16]\), in particular its multiplication \([39]\):

\[
(SU(x))^n \cong SU(x) \quad \forall \ n > 0 \tag{9}
\]
in [1, 16] it is demonstrate that Hilbert spaces of subsystems have the general form of:

\[ B[\mathcal{H}_s] \cong SU(\infty) \times G \]  

(10)

where \( \mathcal{H}_s \) indicates the Hilbert space of a subsystem and \( G \) is a finite rank symmetry group. The presence of internal symmetries in the Standard model of particle physics is the main motivation for existence of \( G \). Other motivations are discussed in [1]. Like any other quantum system, observables of a subsystem is defined by (10) are hermitian members of \( B[\mathcal{H}_s] \).

The Hilbert space of all subsystems is the tensor product of representations of the symmetry of subsystems (10). Using (9), the Hilbert space of the ensemble of subsystems is:

\[ B[\bigotimes_s \mathcal{H}_s] \cong \left( SU(\infty) \times G \right)^{N_{\infty}} \cong SU(\infty) \times G^{N_{\infty}} \]  

(11)

As \( G^{N_{\infty}} \cong SU(\infty) \), (11) is consistent with (1). Moreover, (11) shows that \( SU(\infty) \) factor of the Hilbert space can be considered as common to all subsystems. Thus, it has a role analogous to that of classical spacetime for all entities in the Universe.

2.3.1 Parameter space of subsystems

In addition to the emergence of an internal symmetry, the division of this quantum Universe induces a size or more precisely an area scale. Indeed, although the preserved area of one diffeo-surface is irrelevant for its diffeomorphism as representation of \( SU(\infty) \) group, it becomes important when parameter spaces of multiple systems with this symmetry, including the Universe as a whole, are compared. This is analogous to comparing finite intervals on a line with each others. An infinite line alone is scale invariant. But lengths of finite intervals can be compared with each others. This operation induces a length scale for the finite intervals and thereby for the whole line. Therefore, after division to subsystems the parameter space of \( SU(\infty) \) part of the Hilbert spaces of subsystems will depend on a third dimensionful parameter that we call \( r \). It is measured with respect to a reference subsystem. Diffeo-surfaces of subsystems can be considered to be embedded in this 3D space. Notice that quantum state of a subsystem does not necessarily have a fixed \( r \), and can be a superposition of pointer states with fixed \( r \).

Finally, to make the above setup dynamical, a relational dynamics and evolution à la Page & Woottter [15] or similar methods, see e.g. [44] for a review, can be introduced by selecting one of the subsystems as a quantum clock. Variation of states of other subsystems are compared with the variation of state of the clock and is parameterized by a time parameter \( t \). We interpret this 4D parameter space, which is homomorphic to \( \mathbb{R}^{4 \mid 1} \) as the classical spacetime shared by all subsystems of the Universe. Of course the Hilbert space of every subsystem also has a factor representing its internal symmetry \( G \), as shown in eq. (10). As \( SU(\infty) \) and \( G \) are considered to be orthogonal, their parameter spaces and actions on the states are separable. Specifically, \( G \) transformations are performed locally to states \( |t, r, \theta, \phi\rangle \), similar to a Yang-Mills gauge field defined on the classical spacetime. Due to this analogy, we identify the parameter space of the \( SU(\infty) \) symmetry with the classical spacetime.

2.4 Relation to classical geometry and Einstein equation

Using Mandalestam-Tam uncertainty relation [45], a quantity proportional to quantum fidelity of two close states \( \rho \) and \( \rho_1 = \rho + d\rho \) of subsystems (except reference and clock) can be defined [1]:

\[ ds^2 \equiv Q(\tilde{H}, \rho) dt^2 = \text{tr}(\sqrt{d\rho} \sqrt{d\rho}^\dagger), \quad Q(\rho, \tilde{H}) \equiv \frac{1}{2} |\text{tr}(\sqrt{\rho} \tilde{H} \sqrt{\rho}^\dagger)| \]  

(12)

where \( \tilde{H} \) is a Hamiltonian operator that generates the evolution of subsystems for the selected quantum clock associated to the time parameter \( t \). Notice that here we have assumed that internal symmetry
states of $\rho$ and $\rho_1$ are the same. We also remind that integrating out reference and clock subsystems
makes state of other subsystem mixed and they should be treated as open quantum systems [46].

The infinitesimal quantity $ds$ is a scalar of both the Hilbert space of subsystems and its parameter space. Due
to the similarity of $ds$ to affine separation in Riemann geometry in the rest frame of subsystems, we can identify
the two quantities up to an irrelevant normalization constant. Then, in an arbitrary reference frame of the parameter space $ds$ can be expanded as:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \quad (13)$$

where $x^\mu$ is a point in the parameter space and $g_{\mu\nu}$ is the local metric. Using Mandalestam-Tam
inequality, in [1] it is proved that the signature of metric $g_{\mu\nu}$ of the parameter space must be negative.
Notice that the presence of a trace operator in the r.h.s. of (12) means that its l.h.s. is independent
of the reference frame of the parameter space. This can be proved by expanding operators $\rho$ and $d\rho$
in an arbitrary basis $|t, r, \theta, \phi\rangle$ of the Hilbert space and calculating the trace in (12). Tracing amounts
to integration over parameters $(t, r, \theta, \phi)$. Thus, $ds$ is independent of spacetime parametrization
and coordinates $x^\mu$ in (13) should be considered as representative or average parameters of the quantum
state $\rho$. In general relativity integration over the affine displacement $ds$ generates the world line of
the system in the spacetime. Quantum systems do not follow a path in the classical phase space. Nonetheless,
the world line generated by integration of $ds$ defined in (12) or (13) for the quantum
subsystem can be interpreted as projection of the averaged path of the state in the Hilbert space into
its parameter space - the spacetime.

These properties of $SU(\infty)$-QGR show that we have to find a pure quantum definition for extreme
objects such as black holes, because their general relativity definition through their metric is highly
degenerate and does not define their quantum state. Moreover, spacetime singularities of classical
black holes may be irrelevant when they are considered as a many-body quantum state. Equations
(12-13) are obtained from uncertainty principle [1]. Thus, similar to quantum mechanics, singularity of
metric (13) can be interpreted as indistinguishability of states, or in other words infinite uncertainty.
In any case, this important topic needs more investigation, once a proper quantum definition of a
black hole is found.

Notice that here $t$ and $r$ are considered as classical values. More generally measurements preformed
on the clock and on the subsystems relative to a reference to determine $r$ do not need to be projective.
Such cases are intensively studied in the literature for general quantum systems [44, 46] and we leave
their application to $SU(\infty)$-QGR to future works.

2.4.1 Lorentz invariance of the parameter space

A remark about Lorentz invariance of the parameter space is in order. In (13) this property is
manifest. But, given the different origins of time, distance, and angular coordinates in $SU(\infty)$-QGR
and their interpretation as classical spacetime, the question arises whether their $\mathbb{R}^{(3+1)}$ space is Lorentz
invariant. The answer to this question is positive for following reasons:

- Choices of a quantum clock and a reference subsystem for comparison between diffeo-surfaces are
  arbitrary. Change of these choices amount to changing corresponding parameters.

- Division to subsystems is not rigid and may change with change of clock and reference subsystem
  such that they respect necessary conditions defined in [29]. Thus, changing $t$ and $r$ in general
  lead to modification of $SU(\infty)$ parameters $(\theta, \phi)$ and each of the new parameters $(t', r', \theta', \phi')$
  would be a function of old parameters $(t, r, \theta, \phi)$.

- By definition the ensemble of subsystems must generate the static 2D Universe irrespective of how
  subsystems are defined and parameterized. This condition imposes Lorentz and diffeomorphism
  invariance on the parameter space - the spacetime.
2.5 Evolution

In this section we briefly review dynamics of the Universe as a whole and after its division to subsystems. We begin by constructing a symmetry-invariant functional for the whole Universe and then its modification when subsystems are taken into account.

2.5.1 The whole Universe

According to symmetry description of quantum mechanics foundation [42], the Universe as a whole is static and in a sort of equilibrium state. Specifically, using properties of $SU(N)$ groups, a $SU(\infty)$-invariant functional consisting of elements of $\mathcal{H}_U$ and $B[\mathcal{H}_U]$ has the following form:\(^\text{8}\)

$$\mathcal{L}_U = \int d^2\Omega \left[ \frac{1}{2} \sum_{a, b} L^a(\theta, \phi) L_b(\theta, \phi) \text{tr}(\hat{L}_a \hat{L}_b) + \frac{1}{2} \sum_a \left( L_a(\theta, \phi) \text{tr}(\hat{L}_a \rho) + C.C. \right) \right],$$

$$d^2\Omega = \sqrt{|g^{(2)}|} d(\cos \theta) d\phi$$  (14)

where $a = (l, m)$ or $(\theta', \phi')$, as explained in [1]. C-number amplitude $L_a$ determines the contribution of $SU(\infty)$ generator $\hat{L}_a$. We remind that the integration over angular coordinates of diffeo-surface is part of tracing operation, because generators (5) of the $SU(\infty)$ symmetry are defined at each point of the diffeo-surface. By definition the whole Universe is in a pure state, because there is noting nothing outside, which could have been possibly traced out. Therefore, its density matrix can be written as $\rho = |\Psi\rangle \langle \Psi|$. In [1] it is explicitly shown that, as expected, applying variational principle with respect to amplitudes $L_a$ and components of the density matrix $\rho$ leads to a vacuum state as the equilibrium solution.

The action $\mathcal{L}_U$ is a formal description. In particular, it does not clarify how amplitudes $L_a$’s and density operator $\rho$ change under application of $SU(\infty)$ group and reparametrization of diffeo-surface to preserve $\mathcal{L}_U$. This subject is described in details in [16], and as it is very important for the interpretation of the model as QGR and establishment of its relation with classical gravity, here we review the findings in some extent.

We first remind that the surface element $d^2\Omega$ in (14) is invariant under reparametrization of angular coordinates. Thus, each term in the integrand must be reparametrization invariant. Moreover, for $SU(N)$ groups $\text{tr}(\hat{L}_a \hat{L}_b) = C_a \delta_{ab}$, where $C_a$ is a constant. Therefore, phases of amplitudes $L_a$ in the first term of (14) are irrelevant. In addition, in the second term $\rho$ is hermitian and without loss of generality generators $\hat{L}_a$ can be chosen to be hermitian too. Thus, phases of $L_a$’s are irrelevant and $L_a$’s can be considered to be real fields defined on the 2D diffeo-surface.

Amplitudes $L_a$ must be invariant under translation $\theta \rightarrow \theta + \theta_0$, $\phi \rightarrow \phi + \phi_0$ for arbitrary constant shift of the coordinates origin by $\theta_0$ and $\phi_0$, and rigid rotation of the frame. This means that $L_a$ must have a differential form with respect to coordinates $(\theta, \phi)$. Considering, in addition, the invariance under non-commutative $SU(\infty)$ symmetry, we find that the first term in (14) should have the form of a 2D Yang-Mills Lagrangian for $SU(\infty)$. Thus, $\mathcal{L}_U$ can be written as [1]:

$$\mathcal{L}_U = \int d^2\Omega \left[ \frac{1}{2} \text{tr}(F_{\mu\nu} F^{\mu\nu}) + \frac{1}{2} \text{tr}(\Omega \rho) \right], \quad \mu, \nu \in \cos \theta, \phi$$  (15)

$$F_{\mu\nu} \equiv F^a_{\mu\nu} \hat{L}_a = [D_\mu, D_\nu], \quad D_\mu = (\hat{\partial}_\mu - \Gamma_\mu) \mathbb{1} - \sum_a A^a_\mu \hat{L}_a, \quad \Gamma_\mu \equiv F^a_{\mu\nu} F^a_{\nu\mu} = L^*_{\alpha} L_\alpha, \quad \forall a.$$  (16)

\(^{8}\)More generally trace of multiplications of any number of generators of $SU(N)$ is invariant under $SU(N)$ transformation. However, their values are not independent and can be determined from structure coefficients. Using only the lowest order nonzero trace makes (14) equivalent to classical Lagrangian in QFT, despite the fact that the model does not come from a classical one.
where \( D_\mu \) is 2D covariant and gauge preserving derivative with an appropriate 2D connection \( \Gamma_\mu \) (in \( F^{\mu\nu} \) the connection will be canceled). Exact expression of the differential operator \( \mathcal{B} \) depends on the representation of 2D Euclidian group by the state |\( \Psi \rangle \). For a scalar-type state \( \mathcal{B} = \overline{D}_\mu D^\mu \) and for a spinor-type state \( \mathcal{B} = i\alpha^0 \sigma^i e_i^\mu \overline{D}_\mu \), where \( \alpha^i \) \( i = \{1,2\} \) are two of the \( N \rightarrow \infty \)-dimensional representation of Pauli matrices and \( e_i^\mu \) is the third Pauli matrix; \( \sigma^i \)'s are zweibeins (analogous to vierbein in 2D). We remind that in 2D spaces the 2-form \( F^a_{\mu\nu} \) has only one independent nonzero component. Therefore, the number of degrees of freedom in the two sides of (17) is the same.

In Yang-Mills models the field strength \( F_{\mu\nu} \) is a gauge invariant measurable. Moreover, in (15) variation of \( L^a \)'s and thereby \( F_{\mu\nu} \) can be compensated by a diffeomorphism transformation of the compact 2D surface i.e the variation of \( g^{\mu\nu} \). On the other hand, up to a global rescaling of the area of the diffeo-surface this transformation can be considered as application of \( SU(\infty) \) under which \( F_{\mu\nu} \) is invariant. As we discussed before, the area of diffeo-surface of the whole Universe is not measurable. In this sense the first term in (15) is topological and can be identified, up to an irrelevant normalization constant, with the Euler class of the compact 2D diffeo-surface:

\[
\int d^2 \Omega \, \text{tr}(F^{\mu\nu} F_{\mu\nu}) \equiv \int d^2 \Omega \, R^{(2)}
\]

where \( R^{(2)} \) is the 2D Ricci scalar of the parameter space - the diffeo-surface. This result is not surprising because a single indivisible quantum system is trivial [42]. In a geometrical view, if local details of the Universe are not distinguishable, only its global - topological - properties may characterize its states. In the present model the relevant global property is the topology of the diffeo-surface, corresponding to non-homomorphic representations of the \( SU(\infty) \) group [38, 39].

Equation (18) establishes the relation between \( SU(\infty) \)-QGR and classical gravity. Specifically, it shows that if quantum operators \( F^a_{\mu\nu} L^a \) cannot be distinguished or observed, their overall effects is observed as variation of geometry of the parameter space. We can also interpret the r.h.s. of (18) as the projection of dynamics of the quantum Universe onto its parameter space.

### 2.5.2 Evolution of subsystems

When the Universe is divided to subsystems and a reference subsystem and a quantum clock are selected, it is still possible to construct a \( SU(\infty) \) invariant action functional similar to (15). It will depend on two additional parameters \( r \) and \( t \). They reflect the fact that \( SU(\infty) \) symmetry is now respected not only by the whole Universe, but also by its subsystems, which have acquired a new relative observable \( r \) with respect to a selected reference subsystem, and their relative evolution is measured by a clock parameter \( t \) with respect to a selected quantum clock. In addition, a full action must include terms invariant under the internal symmetry group of subsystems \( G \). The formal description of this functional is [1]⁷:

\[
\mathcal{L}_{U_r} = \frac{1}{4\pi L_P^2} \int d^4 x \sqrt{-g} \left[ \frac{1}{4} \sum_{l,m,l',m'} \text{tr}(L^a_{lm}(x)L_{l'm'}(x)L_{lm}^\dagger L_{l'm'}^\dagger) + \sum_{l,m,a} \text{tr}(L_{lm}(x)T_a(x)L_{lm} \otimes \hat{T}_a) + \sum_{l,m} L_{lm} \text{tr} (\hat{L}_{lm} \otimes \mathbb{1}_G \rho_s(x)) + \sum_{a,b} \text{tr} (T_a^s(x)T_b(x)\hat{T}_a \hat{T}_b) + \frac{1}{2} \sum_a T_a \text{tr}(\mathbb{1}_{SU(\infty)} \otimes \hat{T}_a \rho_s(x)) \right].
\]

(19)

where \( T^a \)'s are generators of the finite rank internal symmetry of subsystems \( G \), and \( L_P \equiv \sqrt{\hbar G_N/c^2} \) is the Planck length. Notice that we have used Cartesian frame for coordinates of the 4D parameter space - the spacetime - and explicitly shown dimensionful coupling constant of \( SU(\infty) \) symmetry.

---

⁷Normalization of (19) is different from the expression in [1]. In the present normalization amplitudes \( L_{lm} \) and \( T_a \) are dimensionless.
Following the same line of arguments given for (14) about the invariance of parameter space under coordinate transformations, invariance under $SU(\infty)$ transformations, and demonstration that the resulting action has the form of a Yang-Mills model with $SU(\infty)$ symmetry, we conclude that (19) has the form of a Yang-Mills model for $SU(\infty) \times G$ symmetry in the $\mathbb{R}^{(3+1)}$ curved parameter space - spacetime - of the subsystems:

$$\mathcal{L}_{U_s} = \int d^4 x \left[ \frac{1}{4} \text{tr}(F^{\mu\nu} F_{\mu\nu}) + \frac{1}{4} \text{tr}(G^{\mu\nu} G_{\mu\nu}) + \frac{M}{2} \text{tr}(\mathcal{B} \rho_s) \right], \quad \mu, \nu \in 0, 1, 2, 3 \quad (20)$$

$$F_{\mu\nu} = F_{\mu\nu}^L \equiv [D_\mu, D_\nu], \quad D_\mu = \partial_\mu - \Gamma_\mu - \sum_{lm} A_{\mu}^{lm} \hat{L}_{lm}, \quad F_{\mu\nu}^L F_{\nu\mu}^L = L_{lm}^L L_{lm}^L. \quad (21)$$

$$G_{\mu\nu} = G_{\mu\nu}^T \equiv [D'_\mu, D'_\nu], \quad D'_\mu = \partial_\mu - \Gamma_\mu - \sum_a B_{\mu}^a \hat{T}^a, \quad G_{\mu\nu}^a G_{\nu\mu}^a = T_a^s T^a. \quad (22)$$

The dimensionful constant $M \propto M_\text{P}$, and similar to $\mathcal{B}$ its value depends on the representation of Lorentz group of the parameter space - spacetime - realized by subsystems states $\rho_s$. The expression for $\mathcal{B}$ would be similar to the examples given in Sec. 2.5.1 with an additional interaction term for $G$ symmetry. Equations (12 - 13) show how the metric of the parameter space is related to quantum states of the subsystem obtained from action (22).

### 2.5.3 Classical limit

When experiments are not sensitive to quantum field strength $F^{\mu\nu}$ of the $SU(\infty)$ symmetry, only its effect on the geometry of the $(3+1)$D parameter space - the spacetime - described by (12 - 13) would be observable. Using (18), in Appendix (A) we show that in classical limit the pure $SU(\infty)$ term in (20) can be approximated by the 4D Ricci scalar $R^{(4)}$, which its integration over the 4D parameter space is no longer topological:

$$\int d^4 x \text{tr}(F^{\mu\nu} F_{\mu\nu}) \overset{\text{classical limit}}{\longrightarrow} \propto \int d^4 x R^{(4)} \quad (23)$$

This last step finalizes our demonstration that in $SU(\infty)$-QGR Einstein equation is a property of the parameter space characterizing the underneath quantum states of the Universe and its subsystems - matter. It confirms that Einstein equation should be considered as an equation of state [3] and its quantization, as well as the quantization of spacetime are meaningless. Moreover, as the quantum gravity interaction has the form of a Yang-Mills model, its effect at high energies should resemble to additional gauge interactions on a curved spacetime. Therefore, it is also meaningless to talk about quantum corrections to the Einstein equation. In any case, it is well established that any change in Einstein equation can be considered as a change in geometry part or matter part, corresponding to Jordan or Einstein frame, respectively.

We should remark that in the above formulation it is assumed that multiple copies of the quantum clock are available for estimating the average value of an observable used to define the time parameter $t$. In other words the clock is tomographically complete. This is not a necessity, and time and/or relative distance may be quantified by non-projective measurements. We leave the investigation of such general case to future works. We do not discuss either the origin of dark energy/cosmological constant in this framework here, because it may depend on the quantum aspects of clock and reference and the fact that after their selection the Universe must be considered as an open quantum system.

A note is in order about the finding that in $SU(\infty)$-QGR quantum gravity is a Yang-Mills model. This means that its mediator quantum field is a vector - in the parameter space - rather than the observed spin-2 graviton field of the classical Einstein gravity. Nonetheless, the relation (23) shows that there is no contradiction between the two observations. This is analogous to the predictions of the early models for strong interaction before the discovery of QCD model. Due to the strong coupling at low energies and confinement of constituent partons, the observations seemed to show a nonlocal and geometrically
extended interaction analogous to a string. We now know that this phenomenological interpretation is wrong, and the confusion is caused by non-perturbative nature of the QCD interaction at energy scales lower than $\Lambda_{QCD}$. In the same manner, the deformation of spacetime, which in general relativity is interpreted as gravity, is generated by relative variation of quantum states of all constituents of the Universe, and the local metric and curvature of the parameter space - spacetime - present their average effect.

2.6 Summary of $SU(\infty)$-QGR model and its properties

We conclude this section by summarizing the $SU(\infty)$-QGR model and what is found about QGR in this framework so far:

- Assuming that Hilbert spaces of the Universe and its subsystems represent $SU(\infty)$ symmetry, we showed that the Hilbert space of the Universe as a whole can be parameterized by 2 continuous parameter. When the Universe is divided to subsystems presenting a finite rank symmetry group $G$, and a quantum reference subsystem and a quantum clock are chosen, 2 additional parameters arise: a relative distant and a relative time à la Page & Wootter or equivalent proposals.

- We interpreted the above 4D parameter space as the classical spacetime and demonstrated that its signature must be negative, i.e. it has a Lorentzian metric. Moreover, as it is a parameter space, its quantization is meaningless.

- The coordinate independent affine parameter of the spacetime is related to the variation of the quantum state of the subsystems.

- We defined symmetry invariant functionals over the Hilbert space of the Universe as a whole, and over those of its subsystems. They plays the role of an action for the evolution of the Universe and its subsystems, respectively.

- The action has the form of Yang-Mills gauge theories on the parameter space for both $SU(\infty)$ and subsystem specific (internal) finite rank $G$ symmetry. Thus, at quantum level like other forces mediator boson of gravity is spin-1.

- We showed that the action functional for the whole Universe is static. Moreover, its purely $SU(\infty)$ Yang-Mills part is topological and proportional to the Euler constant, i.e. integral over the 2D Ricci scalar curvature. The constant of the proportionality is not an observable.

- When the Universe is divided to subsystems, in the classical limit when the quantum Yang-Mills vector field of the $SU(\infty)$ symmetry cannot be detected, the purely $SU(\infty)$ Yang-Mills part of the action functional will be proportional to the 4D Ricci scalar curvature. Therefore, the classical limit of $SU(\infty)$-QGR is the Einstein gravity and the observed spin-2 graviton is a classical effective field.

- This important prediction should be testable with future quantum experiments, for instance those seeking decoherence or entanglement initiated by quantum gravity.

3 Comparison with other quantum gravity proposals

In this section we compare $SU(\infty)$-QGR with LQG and related models, string theories and related models, AdS/CFT conjecture, and several Quantum First models. This list is far from covering all the QGR proposals, so do the citations. In particular, non-commutative spacetime models, causal sets, and quantum gravity models based on the quantum histories are not compared with $SU(\infty)$-QGR.
because of their fundamental differences. Nonetheless, some of them are briefly mentioned because of their connection with models reviewed here.

For each model we first remind its main assumptions and results, only for the purpose of fixing notations necessary for the comparison with $SU(x)$-QGR. We should emphasize that for the sake of briefness various new ideas and methods added to the original construction of these models are not explored here. Giving the fact that some of these proposals have been intensively under investigation for decades, their detailed review and comparison with $SU(x)$-QGR need a much extended work than this article. Moreover, our goal here is finding common features of the models rather than assessing their performance. For these reasons only the most foundational aspects and results of the models are considered and compared with those of $SU(x)$-QGR. We should also remind that $SU(x)$-QGR is a recent and under development proposal and its properties are not fully investigated. For this reason, its comparison with other QGR models is limited to what is known about it. Notably, its application to various QGR related phenomena is left to future works.

As we discussed in the Introduction, due to the close relation between gravity and geometry of space-time in the classical general relativity and Einstein gravity, finding a quantum model for gravitational interaction has been usually considered to be equivalent to quantization of spacetime as a physical entity. A notable difference between $SU(x)$-QGR and other QGR models is the absence of a quantized background or quantized spacetime. This unique feature becomes fundamental when one tries to compare this model with other QGR proposals. Indeed, a direct comparison cannot be made. Thus, the purpose of this work is to investigate whether there are comparable or analogous features in these models. For instance, $SU(2)$ group is present in the construction of many QGR models, including $SU(x)$-QGR. Our aim is to clarify the origin of these sort of similarities, and investigate whether they are superficial and unrelated, or reflect deep relation among models, despite their apparent differences.

### 3.1 Background independent models

Following the failure of coordinate dependent canonical quantization of Einstein-Hilbert equation [47, 48, 49] (see e.g. [50] for a review) and ADM (3+1)D description of Einstein equation and its quantization [28], in 1961 Tullio Regge proposed a discrete but coordinate independent description of Einstein gravity [51]. This model is the basis of most background independent QGR models. For this reason we briefly review it here.

#### 3.1.1 Regge discrete geometry

According to this model a curved two or higher dimensional space can be approximately considered as flat everywhere except on the triangulated 2D surfaces - 2-simplexes. In particular, 3D or (2+1)D curved spaces can be approximated by sticking together tetrahedra with Euclidean or Lorentzian geometry in their bulk. The deficit angle of a vertex in the bulk of space is $\varepsilon = 2\pi - \sum_{f} \theta_f$ where $\theta_f$ is the angle of triangle (face) $f$ attached to vertex $v$, see e.g. [52] for a review of Regge calculus. For vertices sitting on the boundary of the symplectic surface the deficit angle is $\varepsilon = \pi - \sum_{f} \theta_f$. The discretized gravity Regge action is:

$$S_{\text{Regge}} = \sum_{e} l_{e} \varepsilon_{e}$$

where index $e$ run over all edges, $l_{e}$ is length of the edge $e$, and $\varepsilon_{e}$ is the deficit angle of the vertex opposite to it. In Regge action tetrahedra edges can take any positive real value.

#### 3.1.2 Ponzano-Regge 3D QGR

In 1968 Ponzano and Regge proposed a 3D discretized quantum geometry model [53] based on the Regge action $S_{\text{Regge}}$. They showed that if in (24) $l_{e}$’s are chosen to be quantized spin, that is $l_{e} = \frac{\hbar}{2\pi} \theta$, then

$$S_{\text{Ponzano-Regge}} = \sum_{e} l_{e} \varepsilon_{e}$$

where $\varepsilon_{e}$ is the deficit angle of the vertex opposite to $e$. This action is invariant under a gauge transformation of the form $l_{e} \rightarrow l_{e} e^{i\alpha}$, where $\alpha$ is a real number.

### 3.2 Background dependent models

For the purpose of this section we restrict ourselves to the so-called pure spinor approach, considering the classical $10D$ supergravity (11) theory as the starting point. Although this work is restricted to pure spinor approach, our analysis is rather general and can be extended to other QGR models.

#### 3.2.1 Pure spinor approach

In the pure spinor approach [54], the gravitino field is defined in terms of a spinor field $\psi_{\alpha}$, where $\alpha$ is the spinor index. The gravitino field $\psi_{\alpha}$ is a solution of the supergravity equations of motion and is related to the graviton field $h_{\mu\nu}$ by the following relation:

$$h_{\mu\nu} = 2\partial_{\mu} \partial_{\nu} \psi_{\alpha}$$

where $\partial_{\mu}$ are the spatial partial derivatives. The gravitino field $\psi_{\alpha}$ is a spinor field, and its components are given by:

$$\psi_{\alpha} = \gamma_{\alpha} \phi$$

where $\phi$ is the scalar field and $\gamma_{\alpha}$ are the Gamma matrices. The scalar field $\phi$ is a solution of the supergravity equations of motion and is related to the metric field $g_{\mu\nu}$ by the following relation:

$$g_{\mu\nu} = 2\partial_{\mu} \partial_{\nu} \phi$$

where $\partial_{\mu}$ are the spatial partial derivatives. The scalar field $\phi$ is a scalar field, and its components are given by:

$$\phi = \phi_{\mu}$$

where $\phi_{\mu}$ are the scalar field components. The scalar field $\phi$ is a solution of the supergravity equations of motion and is related to the metric field $g_{\mu\nu}$ by the following relation:

$$g_{\mu\nu} = 2\partial_{\mu} \partial_{\nu} \phi$$

where $\partial_{\mu}$ are the spatial partial derivatives. The scalar field $\phi$ is a scalar field, and its components are given by:

$$\phi = \phi_{\mu}$$

where $\phi_{\mu}$ are the scalar field components. The scalar field $\phi$ is a solution of the supergravity equations of motion and is related to the metric field $g_{\mu\nu}$ by the following relation:

$$g_{\mu\nu} = 2\partial_{\mu} \partial_{\nu} \phi$$

where $\partial_{\mu}$ are the spatial partial derivatives. The scalar field $\phi$ is a scalar field, and its components are given by:

$$\phi = \phi_{\mu}$$

where $\phi_{\mu}$ are the scalar field components. The scalar field $\phi$ is a solution of the su...
\( j_e, j_e \in \{0, 1/2, 1, 3/2, \cdots \} \) and \( j_e \)'s of each face satisfy triangle rule:

\[
|j_1 - j_2| \leq j_3 \leq j_1 + j_2
\]

their 6j symbol will be nonzero and approximately equal to the cosine of Regge action.

Partition function of the Ponzano-Regge QGR is constructed from multiplication of the positive exponent of the cosine of Regge action for all tetrahedra, weighted, and summed over all configurations of spins:

\[
Z_{PR} = \lim_{N \to \infty} \sum_{j_e \leq N} A^N_j(N) \prod_{e \in S_1} (-1)^{2j_e} (2j_e + 1) \prod_{e \in S_3} (-1)^{-j_e} \sum_{e=1, \cdots, 6} j_e \begin{pmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{pmatrix}
\]

Ponzano-Regge discrete quantum gravity was the first evidence of a close relation between gravity in 3D space or (2+1)D spacetime and representations of \( SU(2) \) group. This relation was later confirmed by the introduction of Ashtekar variables [54] in the framework of (3+1)D ADM formulation for quantization of gravity. In fact, as we explain in the following sections, the concept of triangulation and associating spins to edges of triangles comes up in one way or another in other QGR models, as well.

### 3.1.3 Ashtekar variable and Loop Quantum Gravity

Loop Quantum Gravity (LQG) [56, 57] can be considered as continuum limit of symplectic QGR models [55]. It uses ADM (3+1)-D formalism with background-independent Ashtekar variables [54]. They consist of a spin connection 2-form \( \omega^a_i(x) \), defined on the product of a 3D Euclidean manifold and a \( SU(2) \) group manifold - more precisely a \( SU(2) \) bundle on a 3D Euclidean manifold - and triads \( E^a_i \) such that \( E^a_i E^b_i = \delta^a_j \delta^b_k \) where \( i = 1, 2, 3 \) is coordinate index of the Euclidean space and \( a = 1, 2, 3 \) indicates generators of \( SU(2) \) symmetry group. They replace coordinates and metric as dynamical variables. In the quantized model their dual variables are respectively \( E^a_i \) and gauge field \( A^a_i = \omega^a_i + \gamma K^a_i \), where \( K^a_i = K_{ij} E^i j / \sqrt{|n|} \), \( K_{ij} \) is extrinsic curvature tensor of the 3D space, \( h \) is determinant of the metric of physical 3D space, and \( \gamma \) is the Immirzi constant [58].

### 3.1.4 \( SU(2) \) symmetry, degeneracies and observables in LQG

Although metric, and thereby coordinates, are apparently present in the definition of Ashtekar variables, their choice do not affect geometry of space and its quantization. The reason is that space curvature is described by \( SO(3) \cong SU(2) \) transformation of a rigid frame, rather than deformation of the metric. Specifically, the rigid frame rotates when it is transported across the curved space manifold. On the other hand, the freedom of choice of the rigid frame at each point of the 3D manifold means that its \( SO(3) \cong SU(2) \) symmetry is a gauge symmetry. Thus, \( A^a_i \) and \( E^a_i \) include more degrees of freedom than \( g_{\mu \nu} \) in the (3+1)D classical gravity. This is evident from counting the number of components of these fields.

To eliminate degeneracies observables of LQG and spin network (or foam) [59, 60, 61] - its discretized version - are quantized topological quantities generated by Wilson loops [62]. This is why the model is called Loop QGR, and one of its most remarkable prediction is the quantization of area [62]. This feature establishes the relation between LQG formulation using continuous Ashtekar variables, spin network as its approximation, and symplectic geometry of Ponzano-Regge: Quantized surfaces have non-trivial \( SU(2) \) holonomy and triangulated 3D space à la Regge becomes a manageable approximation, including essential properties of a quantized curved space with meaningful continuum limit.
3.1.5 Analogies between foundations of LQG and related models with $SU(\infty)$-QGR

In $SU(\infty)$-QGR conserved areas of diffeo-surfaces and their comparison induce an area (length) scale in the model, without being quantized. Moreover, $E^i_l$ fields are analogous to amplitudes $L_{l,m}$ in $SU(\infty)$-QGR. In fact, in [16] we show that in order to be invariant under coordinate transformations of the parameter space, these amplitudes must be differential operators in the parameter space. Indices $(l, m)$ are analogous to the internal $SU(2)$ symmetry of triads. However, in contrast to Ashtekar variables, their values are obtained from ensemble of representations of $SU(2)$ factors in the decomposition of $SU(\infty)$ in equation (3). This property is similar to Ponzano-Regge and spin network where edges of tetrahedra are weighed by spins. However, in $SU(\infty)$-QGR both $l$ and $m$ quantum numbers of $SU(2)$ representations are involved in the action of the model and they are not constrained. The reason is that in contrast to LQG and Ponzano-Regge models, in $SU(\infty)$-QGR the Hilbert space does not represent a real space geometry.

3.1.6 Hilbert spaces of LQG and related models

6j symbols consist of summation over weighted multiplication of 4 Wigner 3j symbols. In turn 3j symbols are proportional to Clebsch-Gordan coefficients $6j$ symbols consist of summation over weighted multiplication of 4 Wigner 3j symbols. In turn 3j symbols are proportional to Clebsch-Gordan coefficients $6j$. This property is similar to Ponzano-Regge and spin network where edges of tetrahedra are weighed by spins. However, in $SU(\infty)$-QGR both $l$ and $m$ quantum numbers of $SU(2)$ representations are involved in the action of the model and they are not constrained. The reason is that in contrast to LQG and Ponzano-Regge models, in $SU(\infty)$-QGR the Hilbert space does not represent a real space geometry.

Considering the expansion (3) of $SU(\infty)$ group, it is clear that the Ponzano-Regge partition function $Z_{PR}$ includes special configurations of a quantum system which its Hilbert space represents $SU(\infty)$ symmetry, namely states that can be arranged as tetrahedra in a 3D space. This observation can be extended to other models based on a symplectic representation of space such as LQG, spin network, and Group Field Theories (GFT). Indeed, [63] describes explicit construction of the Hilbert space of a single tetrahedron in LQG/spin network by associating $SU(2)$ operators to edges of the tetrahedron. State of a unit cell of space - sometimes called atom of space - is generated by application of these operators to a vacuum state, such that the projection (amplitude) of the total spin of the tetrahedron is equal to its associated 6j symbol. This procedure can be extended to ensemble of $N \rightarrow \infty$ tetrahedra content of space, which can be also considered as spin-weighted graphs [64]. Thus, we conclude that state generator operators, and thereby Hilbert spaces of Discrete QGR (DQGR) models such as Ponzano-Regge and LQG models, which we collectively call $\mathcal{H}_{DQGR}$, are subspaces of the Hilbert space of a quantum system with $SU(\infty)$ symmetry, such as $SU(\infty)$-QGR.

3.1.7 Kinematical and physical Hilbert spaces and reality conditions

It is useful to remind that 3j, 6j, and fundamental representation of $SU(2)$ are in general defined on the field of complex numbers. By contrast, a partition function or path integral over geometries of the physical space or spacetime, which should approaches to Einstein gravity in the limit of $\hbar \rightarrow 0$ must be real valued [64, 65]. Moreover, due to the degeneracies discussed in Sec. 3.1.4, the Hilbert spaces $\mathcal{H}_{DQGR}$ of LQG and related models are not physical, but Kinematical [64]. The Hilbert space of physical states $\mathcal{H}_{phys}$ containing quantized background independent geometries is a subspace of $\mathcal{H}_{DQGR}$, that is $\mathcal{H}_{DQGR} \supset \mathcal{H}_{phys}$. However, it is in general difficult to construct $\mathcal{H}_{phys}$ explicitly [64]. In addition, demonstration of diffeomorphism and Lorentz invariance of physical states is not straightforward and one might expect violation of Lorentz invariance in QGR models with discretized space [66]. Indeed, diffeomorphism invariance of DQGR is explicitly shown only for special cases [67, 68].

Even in DQGR/LQG models that preserve Lorentz invariance dispersion relation of gravitational waves [69] and electromagnetic radiation [70] may deviate from general relativity. However, both of these deviations are stringently constrained [71, 72, 73]. Moreover, Immirzi parameter may affect...
interaction of fermions [74], and thereby induces a *fifth force* type effect on matter. This effect is also constrained by various tests of gravity [75].

Complexities analogous to nonphysical states in the formulation of LQG and related models do not arise in $SU(\pi)$-QGR. The parameters defining the Hilbert space, namely $(t,r,\theta,\phi)$ are real, and by construction their redefinition - in other words diffeomorphism of the parameter space - corresponds to change of the Hilbert space’s basis by application of a unitary transformation - a member of $SU(\pi)$ symmetry group of the subdivided quantum Universe. We also notice that although $SU(\pi)$-QGR, Ponzano-Regge model, LQG, spin network, and their extension to GFT share $SU(2)$ symmetry in their construction, in practice all of them, except $SU(\pi)$-QGR, use only the Casimir operator of $SU(2)$. The reason is that eigen states $m$ of azimuthal projection of spin vector induce a preferred direction or in other words a frame, which these models want to avoid.

### 3.1.8 Time and matter in LQG

Similar to $SU(\pi)$-QGR, in LQG and related models time must be considered as a relational observable. One way of making the model dynamic is to consider time as the classical affine parameter of histories [76, 77] or path integrals in the quantized physical space [78]. Although in such setups Lorentz and diffeomorphism invariance is not trivial, it may be achievable [78, 67, 68].

Describing time by histories needs a *historian* - a reference subsystem with respect to which histories are defined. But, construction of background independent QGR models do not clarify how to satisfy necessary conditions for division of a quantum system [29]. In fact kinematical Hilbert space $H_{DQGR}$ seems to be inseparable [64]. Specifically, division of the Hilbert space to orthogonal blocks, which could be considered as subsystems, needs an additional symmetry, because in these models $SU(2)$ is inherently related to gravity. We might consider tetrahedra as the most fundamental *atomic subsystem* [79]. However, to discriminate one tetrahedron as reference, there must be selection criterion, thus another symmetry - observable. This issue is directly related to the fact that LQG and related models do not consider matter fields - a symmetry orthogonal to space - in their foundations. Although, a time parameter and matter fields can be easily added to the Einstein gravity Lagrangian described as a function of Ashtekar variables and their duals, see e.g., [83], the foundational issue of time definition in LQG and related models is not fully solved. Attempts to solve this problem, for instance through quantization of phase space [80, 81, 82], indeed include matter and/or symmetries orthogonal to diffeomorphism symmetry.

### 3.1.9 Non-perturbative characteristic of LQG and related models

The origin of subsystem definition issue in background independent models is their non-perturbative approach to QGR. Division to subsystems needs a criteria for breaking the Hilbert space or its parameter space to distinguishable sectors. Such operation implies the possibility of a perturbative description of the system at some scale. However, in LQG and related models, in absence of matter there is no natural covariant rule for a quantum gravitational perturbative expansion. This observation clarifies why there is no inherent way to include matter in these models. In fact, division to subsystems; emergence of a quantum clock, inclusion of matter in the foundation of the model, and existence of both perturbative and non-perturbative regimes are related. In $SU(\pi)$-QGR they are naturally implemented in the construction of the model through special form of its symmetries.

### 3.1.10 Outline of comparison between background independent models and $SU(\pi)$-QGR

In conclusion, although $SU(2)$ symmetry plays an essential role in the construction of background independent models and $SU(\pi)$-QGR, its role and *raisons d’être* in these models are very different. Notably, in LQG, GFT, and other symplectic models it is strictly related to the assumption of a
physical 3D quantum space. Nonetheless, spin network realization of LQG can be considered as a subspace of the Hilbert space of $SU(\omega)$-QGR, in which with additional relations - entanglement - between representations of $SU(2)$ components are considered. Both background independent models and $SU(\omega)$-QGR rely on the definition of a relative time or histories, which need division of the Universe to subsystem. In $SU(\omega)$-QGR this concept is built in the construction of the model and provides the necessary ingredients for definition of a quantum subsystem as clock and inclusion of matter fields.

3.2 Quantum approaches to QGR

Inherently quantum approaches - called Quantum First by some authors [18] - are relatively recent arrivals into the jungle of QGR proposals and $SU(\omega)$-QGR can be classified in this group. For this reason it is crucial to investigate its similarities and differences with other models in this category.

A shared characteristic of Quantum First models is the absence of a classical spacetime as a foundational concept in their axioms - or at least this is the claim. Consequently, it has to emerge down the road from more primary properties and structures of an abstract quantum system. It is useful to remind that the concept of an emergent spacetime is not limited to these models. The possibility that spacetime may not be a fundamental entity is also considered by other QGR candidates as well, see e.g. [84, 85, 86]. Specifically, it is suggested that a quantum Lorentz invariant spacetime orthogonal to internal gauge symmetries may emerge in QGR models based on the extension of the Poincaré group and gauge symmetries [107, 108, 109]. The idea of spacetime emergence is also explored by models in which, in one way or another, thermodynamics and quantum gravity are unified [90, 91, 92]. These models seem to have little common aspects with $SU(\omega)$-QGR and we do not discuss them further here.

In absence of any hint about the quantum nature of gravity, for instance its Hilbert space, and its relationship with classical gravity and other interactions, Quantum First models usually use priors inspired from semi-classical gravity, in particular from properties of semi-classical physics of black holes. Based on these priors two categories of Quantum First models other than $SU(\omega)$-QGR can be distinguished:

- Models that consider locality and causality as indispensable for QGR: Some of these models need modification of standard quantum mechanics;
- Models inspired by black hole entropy and its relationship with holographic principle and AdS/CFT duality.

3.2.1 Modified quantum mechanics and locality

Locality is considered to be crucial for describing black holes, their thermodynamics [19, 4] and its puzzles [20, 21]. More generally, causality and observed finite speed of information propagation in both classical general relativity and QFT implies some degree of locality in any interaction, including QGR. For these reasons, locality and its close relationship with the definition of subsystems as localized entities in the Universe have been the motivation of authors of [93, 94, 18] for proposing a generalized quantum mechanics. Specifically, history description of quantum mechanics [95, 96] is generalized in [93] to define coarse-grained histories as a bundle of fine-grained histories (path integrals). They replace the Hilbert space of quantum mechanics, which in a QGR framework corresponds to a spacelike surface during an infinitesimal time interval, defined with respect to a reference clock. In turn, sets of histories present subspaces of the bundle. Presumably, in this model not only the state of a system, but also its whole Hilbert space changes with time.
Inspired by generalized quantum mechanics, \cite{94} proposes an alternative way to implement locality in what is called \textit{universal} quantum mechanics. In analogy with the bundle space of \cite{93} it extends the space of physical states to provide additional labeling, such as \textit{in} and \textit{out} states in curved spacetimes \cite{97}. In addition, labels can be interpreted as time or labels of states in a multiverse, as needed. Physical states can be considered as \textit{local} in this extended state space.

These models and other QGR proposals based on the quantum histories, see e.g. \cite{98,99} and references therein, have little common features with SU(\(\infty\))-QGR, which is strictly based on the highly tested standard quantum mechanics. The reason for having reviewed them here is their role in the development of further models with some similarities with SU(\(\infty\))-QGR, which we will review in the following subsections.

### 3.2.2 QGR from locality and causality

Localization of quantum mechanics in \cite{93} does not specify an explicit implementation procedure. Nonetheless, motivated by this model \cite{18,100,17,101} propose a road-map for realization of this concept in what they call Local Quantum Field Theories (LQFT). In these QFT models observables convey quantum information only locally. Here we call the corresponding QGR proposal LQFT-QGR.

In quantum systems with infinite degrees of freedom, such as in QFTs, spacetime sector of the Hilbert space cannot be factorized to disconnected (untangled) subspaces without violating causality. Such quantum systems are said to have Type III operator algebra in the classification of \cite{102,103}. For this reason, in LQFT-QGR the division to subsystems is performed algebraically. Specifically, it is assumed that for any region of spacetime \(U\) there is an extension \(U_e\). Observables \(\hat{A}\) and \(\hat{A}\) are defined such that they have nonzero support respectively on \(U\) and \(\bar{U}_e\), where \(\bar{U}_e\) is the complementary space of \(U_e\). Under these conditions \(\hat{A}\) and \(\bar{\hat{A}}\) are assumed to be disentangled in a specific vacuum:

\[
\langle U_e | \hat{A} \hat{A} | U_e \rangle = \langle 0 | \hat{A} | 0 \rangle \langle 0 | \bar{\hat{A}} | 0 \rangle
\]

(27)

The vacua \(|U_e\rangle\) and \(|0\rangle\) are related by a Bogoliubov transformation. This definition is considered to provide a sort of localization without factorization of the Hilbert space. However, it is evident that this algebraic structure is not in general diffeomorphism invariant and observable operators \{\(\hat{A}\)\} and \{\(\hat{A}\)\} must satisfy specific conditions to retain their invariance and physical meaning \cite{101}. Seeking such operators, \cite{100,101} find that in analogy with gauge invariant Wilson loops in Yang-Mills theories, diffeomorphism invariant operators \(\Phi_{\Gamma} \in \{\hat{A}\}\) are nonlocal structures, which depend only on the spacetime connection \cite{100}. Specifically:

\[
\Phi_{\Gamma}(x) = \phi(x^\mu + V_\Gamma^\mu) \quad \text{ (28)}
\]

where \(\Gamma\) is a path running from point \(x\) of the spacetime to infinity, and \(V_\Gamma^\mu\) is the integral of an expression depending on the metric of spacetime along the path \(\Gamma\). An explicit expression for \(V_\Gamma^\mu\) is obtained for the weak coupling limit of semi-classical gravity in \cite{101}.

### 3.2.3 Comparison of LQFT-QGR with SU(\(\infty\))-QGR

In LQFT-QGR two essential concepts for QGR, namely division of the Universe to subsystems and carriers of quantum information are considered to be the same. In this respect, the model is similar to SU(\(\infty\))-QGR, that is carriers of information are matter/radiation fields and their internal symmetries, which is orthogonal to diffeomorphism of spacetime and a necessary criteria for division of the Universe to subsystems. However, the two models are conceptually very difference. In LQFT-QGR subsystems are somehow localized in spacetime. By contrast, in SU(\(\infty\))-QGR spacetime is not the quantum Universe and no locality condition is imposed on subsystems/particles. In fact, in SU(\(\infty\))-QGR locality and causality are not postulated. As we discussed in Sec. 2.4, they arise from quantum
uncertainties. Moreover, interpretation of coordinates in (13) as average or expectation values, shows that in agreement with quantum mechanics observations, locality in general is an approximation.

LQFT-QGR and $SU(x)$-QGR share the absence of a classical dynamics in their foundation. Moreover, both models are a type of QFT on a curved spacetime, which plays the role of a parameter space. Their difference is in the definition of observable fields: LQFT-QGR constrains field operators to realize special algebraic structures and a sort of locality, whereas in $SU(x)$-QGR both gravity and matter sectors are quantum fields similar to QFTs without gravity. In addition, in $SU(x)$-QGR spacetime genuinely emerges, whereas in LQFT-QGR it is implicitly postulated and present from the beginning. Although in contrast to many other QGR proposals spacetime per se is not quantized, the model offers no explanation for its origin, its dimension, or properties of its metric, or its relationship with other quantum fields.

**Type III algebra in LQFT-QGR and $SU(x)$-QGR** Operators indexed or parameterized by $\mathbb{R}^n$ cannot be divided to subsets associated to limited regions of the indices, if the whole algebra has to be invariant under diffeomorphism [102, 103]. It is why a symmetry orthogonal to diffeomorphism is necessary for tagging and fulfilling conditions for definition of quantum subsystem [29].

As QFTs, both LQFT-QGR and $SU(x)$-QGR are Type III quantum systems. In $SU(x)$-QGR the inseparability of continuous operators is reflected in the common $SU(x)$ symmetry of all subsystems, including the Universe as a whole, and the need for a factorized finite rank internal symmetry. By contrast LQFT-QGR considers strict locality as a foundational concept and tries to use nontrivial topological structures as a replacement for tagging and identifying subsystems. However, at present there is no evidence for the possibility of such algebraic structures in QFTs, except for the solutions obtained in the weak coupling regime of semi-classical gravity [101]. Moreover, although topological structures are observed in condensed matter, they are extremely fragile. By contrast, symmetry breaking or emergence, as requested in $SU(x)$-QGR, is widespread in nature. We also notice that topological structures proposed by LQFT-QGR are different from those used in LQG as observables. In LQG Wilson loops do exist because of axioms and construction of the model. By contrast, the existence of such operators LQFT-QGR are in large extend a conjecture. The model explored in [101] for such structures is semi-classical and includes perturbative Einstein equation, which is non-renormalizable and cannot be considered as a genuine QGR.

3.2.4 QGR and emergent spacetime from entropy and holography

Another set of conjectures used for getting insight into QGR without considering an underlying classical dynamics is the holographic principle [4, 5, 6] and gauge-gravity duality conjecture [104, 106], specially in the form of AdS/CFT duality, see Sec. 3.3.3 for more details. Notice that this conjecture should not be confused with models that try to quantize gravity by extending gauge group of the Standard Model, such that it includes Lorentz and Poincaré symmetries [107, 108, 109].

Motivation for the holography conjecture [4, 5, 6] is the proportionality of semi-classical black hole entropy to area of its horizon, rather than to its volume [19, 20]. According to holography conjecture there is an upper limit on the amount of quantum information contained inside the bulk of a region of spacetime [4]. It is proportional to the area of its boundary and is maximal for black holes [19, 20]. This conjecture is not limited to gravitational systems and similar behaviour is observed in other many-body quantum systems, if a suitable null (light-like) boundary surface can be defined [7]. In particular, entanglement entropy of some low dimensional many-body quantum systems at critical point, that is when the system is scale invariant and behaves conformally, is calculable analytically, and the results show that they follow holographic principle [110, 8, 9].

AdS/CFT duality conjecture [30, 111] posits that quantum properties of the boundary of a spacetime region in the limit that it can be approximated by a conformal QFT can be related to geometry and
QGR of the bulk if its background geometry is AdS.

Inspired by these conjectures, [13] considers two quantum systems with a quantum CFT living on their common boundary. An analogy is established between the reduction of entanglement entropy and exchanged quantum information between the two systems when their boundary is shrunk, and the reduction of gravitational interaction with increasing distance. To understand this analogy, imagine squeezing a rubber bar in the middle. More it is squeezed, more material is pushed to the two ends and smaller becomes the surface connecting them until the bridge breaks and the two parts separate. Of course, this analogy is very far from being a QGR model. Nonetheless, it has motivated construction of QGR models using entanglement entropy as the origin of what is classically perceived as geometrical distance.

### 3.2.5 Entanglement-Based Models (EBM) of quantum gravity

A more systematic approach to construction of a spacetime from entropy-area law is proposed in [14, 22], where spacetime metric and geometry emerge from tensor decomposition of the Hilbert space of the Universe to entangled subspaces. This model is based on several axioms, see [22] for the complete list. They include:

1. A preferred tensor decomposition of the Hilbert space $\mathcal{H}$ [of the Universe], where each factor $\mathcal{H}_i$ presents Hilbert space of a point or a small space around a point of space:

   $$\mathcal{H} = \bigotimes_i \mathcal{H}_i$$  \hspace{1cm} (29)

2. There is what is called Redundancy Constrained (RC) states for each subset of the Hilbert space $B \subset \mathcal{H}$, considered to be a subspace of physical space. Its entropy is assumed to be:

   $$S(B) = \frac{1}{2} \sum_{i \in B, j \not\in B} I(i : j)$$  \hspace{1cm} (30)

   $$I(i : j) = S(i) + S(j) - S(i \cup j)$$  \hspace{1cm} (31)

   where $I(i : j)$ is the mutual information of subsystems $i$ and $j$. This construction replaces area-law axiom considered in [13, 14].

3. It is assumed that the system is in an entanglement equilibrium state, when subsystems are in RC states. Under small perturbations the entropy of $B$ is assumed to be conserved. This means that the total entropy is conserved. Moreover, when states deviate from RC, their entropy can be decomposed to entropy of a fiducial RC state and a subleading component, interpreted as an effective field theory. The two components cancel each other to preserve the total entropy.

It is clear that axiom 1 is constructed such that the Hilbert space $\mathcal{H}$ presents physical space. Thus, we conclude that similar to LQFT-QGR in this model the space does not really emerge, but its existence is postulated. Moreover, we notice that the definition of subsystems is loose and does not explicitly respect necessary conditions [29]. It is why this axiom explicitly states that factorization is static and somehow is preferred. But it is not specified what is the criteria for its selection.

Axiom 3 replaces action and variation principle that in classical mechanics and QFT models lead to dynamics and field equations, respectively. In addition, according to this axiom RC states can be considered as a background around which a perturbation is performed. Indeed, the model does not consider highly non-RC states and applies only to weak gravity cases [22].

The structure described by above axioms can be considered as an information graph, which its vertices are factors of the Hilbert space and its edges are weighted by mutual information $I(i : j)$ of subsystems.
corresponding to factors of the Hilbert space. This graph is analogous to discrete geometry in Ponzano-Regge, spin network, and LQG.

To complete the geometrical interpretation, the area of information graph or its subgraphs must be related to entanglement information. In [13, 14] this connection is established by assuming holographic principle. However, when RC structure is assumed [22], according to one of the axioms of the model (axiom 3 in [22]), the area associated to a subspace $B$ of the space is:

$$A(B, \bar{B}) = \frac{G_N}{2} I(B : \bar{B})$$

(32)

where $G_N$ is the Newton constant (for $\hbar = 1$ and $c = 1$) and $\bar{B}$ is the complementary of $B$. Although the area $A$ associated to a subspace of the Hilbert space is not the boundary of a bulk space, the inspiration from holographic principle is evident. This axiom and Radon transform is used to describe area as a function of the entropy of factors $\hat{H}_i\forall i$ of the Hilbert space and define a background metric. Perturbation of this metric are interpreted as the perturbation of quantum state of the physical space. Additionally, variation of the entanglement graph geometry is used as a clock to which a Hamiltonian and an operator analogous to energy-momentum can be associated. The latter can be considered as an effective field theory generating subleading entropy of states, which are perturbatively deviated from RC states. Finally, by comparing this formulation with general relativity and using Radon transform, [22] argues that Einstein equation can be concluded.

3.2.6 Comparison of EBM with $SU(\infty)$-QGR

We find that EBM is more similar to $SU(\infty)$-QGR - in spirit rather than construction - than other models. Here we briefly highlight their common features.

Factorization of the Hilbert space and division to subsystems The importance of division of the Hilbert space to factors presenting subsystems is crucial in both models. However, as remarked earlier, in EBM the division is considered to be rigid and preferred. This is in strict opposition to the approach of $SU(\infty)$-QGR. The reason behind the special factorization is again the absence of a concrete criteria to discriminate between factors - subsystems.

We notice that the issue of how to divide the Universe and its Hilbert space to quantum subsystems generally arises in quantum approach to QGR due to foundational requirements [42], and in some other QGR models for various reasons. Model makers use different schemes to deal with this crucial matter. For instance, they introduce topological structures - as in LQG and LQFT-QGR; or simply consider a fixed decomposition without addressing its origin, as in EBM. $SU(\infty)$-QGR assumes an orthogonal finite rank symmetry - presumably from symmetry breaking or emerging - to fulfill general conditions for division of a quantum system to subsystems according to the criteria defined by [29]. Although the nature and origin of this symmetry is not specified in the construction of $SU(\infty)$-QGR, properties of $SU(\infty)$ symmetry, notably equations (8, 9) facilitate the interpretation of the Universe as a many-body quantum system, in which based on our knowledge from condensed matter, a symmetry of the form (10) can arise relatively easily. More importantly, in $SU(\infty)$-QGR the finite rank symmetry is associated to matter. In this way, matter and space become intertwined and inseparable. This is not the case in EBM, LQFT-QGR or LQG and related models.

Geometry and classical gravity Another common aspect between EBM and $SU(\infty)$-QGR is the explicit dependence of the space geometry on the quantum state - through entanglement entropy in EBM and through fidelity in $SU(\infty)$-QGR. However, emergence, construction, and physical meaning of the space in the two models are very different. In EBM of [14, 22] factors of the Hilbert space are considered to present points or regions of the physical space and the information graph is interpreted
as a symplectic geometry, which in the continuum limit can be considered as a quantized space. Therefore, although the existence of a physical space is not explicitly mentioned in the axioms, it is implicitly behind the factorization of the Hilbert space. By contrast, in $SU(\infty)$-QGR space genuinely emerges as parameter space of $SU(\infty)$ representations.

A consequence of these differences is that $SU(\infty)$-QGR has an explicit explanation for the dimension of spacetime, where as in EBM dimension of the space(time) is not fixed. In fact, the information graph can be embedded in any space with dimension $d \geq 2$. Notice that the relation between area of a subgraph (subsystem) and its entanglement entropy with its complementary in (32) does not restrict the graph to be planar - not even locally. A priori every vertex - that is every factorized subsystem of the Hilbert space - can be entangled to all other subsystems. In [14] it is assumed that the number of entangled subsystems to a vertex - corresponding to the number of edges attached to it - is limited. Nonetheless, their number can be large and the graph rules do not constrain their mutual angle. Thus, in contrast to Ponzano-Regge and LQG, in which spins associated to edges of the symplectic space must satisfy triangle constraint at each vertex, the information graph in EBM can be embedded in a multi-dimensional space. For these reasons, $d$ is considered as a stochastic parameter determined from averaging over geometries of many information graphs [14]. On the other hand, spacetime dimension is a fundamental quantity which affects many observables in particle physics and cosmology at all energy scales. So far no evidence of an extra/infra or stochastic dimension is detected.

In $SU(\infty)$-QGR the relationship between affine parameter, metric, and quantum fidelity in equation (12) naturally relates ensemble of parameters (not just distance or area) to quantum states of the subsystems. In both EBM and $SU(\infty)$-QGR Einstein equation remains classical and is obtained from relationship between quantities with underlying quantum origin.

### Analogy between distance and entanglement

In both models an area quantity emerges and it has a crucial role for their interpretation as QGR. In $SU(\infty)$-QGR it emerges from comparison of the preserved areas of diffeo-surfaces of subsystems with an arbitrary reference subsystem. In EBM it is postulated in (32), where a dimensionful area/distance parameter is mandatory. Although, the way a scale emerges in these models is very different, in both cases it is related to the division of Universe to subsystems. Indeed, in EBM entanglement and its associated entropy are meaningful only when multiple quantum systems are present. In $SU(\infty)$-QGR division to subsystems is necessary to make the conserved area of diffeo-surfaces relevant and measurable.

In addition to difference in the manner that a dimensionful scale arises in these models, there is another important difference. In $SU(\infty)$-QGR the area is related to geometry of the compact parameter space of representations of $SU(\infty)$ symmetry of subsystems. Thus, it is a well defined and unique measurable for each subsystem relative to a reference. By contrast, quantification of entanglement and relative quantum information is not unique and various definitions, e.g. von Neumann or Rényi entropy can be used, and each of them has its own merit and applications. EBM models of [13, 14, 22] do not specify which one of these entropies should be used or what is rationale for preferring one to others, or whether different definitions should be interpreted as different choices of coordinates.

### 3.3 String theory, M-theory, and AdS/CFT duality in 3 and higher dimensions

String theory and related models are without doubt the most intensively studied QGR proposals. Although some of Quantum First models are inspired by (super)string theories and AdS/CFT duality conjecture, string theories are not properly speaking Quantum First. Their perturbative formulation is a quantized 2D sigma model, originally proposed for describing strong nuclear interaction [113]. Non-perturbative formulation of string models, also called M-theory, and its realization as matrix model has the form of a (super)Yang-Mills QFT.

In the recent decades new approaches to string theories are extensively studied in the literature and
various concepts and structures are added to their initial construction. Their list includes: D-branes
states [114]; String condensation and its relation with p- and D-brane solutions [114, 115, 116, 117],
important for the conjectured non-perturbative formulation of string models also called the M-theory;
And AdS/CFT duality [30, 112], which is the simplest case of gauge-gravity conjecture [118, 31, 32]
and closely related to M-theory and matrix models. Nonetheless, the basic structure of (super)string
theories and their properties continue to be considered as foundational and established knowledge
for development of these more advanced theories. In particular, M-theory uses the 10D or (9+1)D
Euclidean or Minkowski spacetimes, respectively. This is the fundamental dimension of spacetime
in perturbative superstring theories. Similarly, the first evidence of AdS/CFT correspondence was
discovered for D3 brane models in a 10D compactified \( AdS_5 \times S^5 \) background spacetime [120]. Thus,
due to the importance of perturbative formulation of string models, in this section we first briefly
remind their findings and how they compare with \( SU(p|q) \)-QGR. Then, we review and compare M-
theory and its matrix realization, and AdS/CFT conjecture.

3.3.1 Perturbative string theories and their comparison with \( SU(\infty) \)-QGR

As extended literature and textbooks on string theory related subjects such as [113, 119] are available,
we do not review these models in details and only remind their most important properties used for
comparison with \( SU(\infty) \)-QGR wherever they are necessary.

2D surfaces in string theory and \( SU(\infty) \)-QGR

Overlooking all the complexities of string and
superstring theories, they can be summarized as 2D quantum gravity of a conformal quantum sigma
model. Here quantum gravity means summation over all possible geometries of their 2D worldsheet -
more generally a membrane. In this view of string theories, their most evident common feature with
\( SU(\infty) \)-QGR is the crucial role of 2D surfaces and their diffeomorphism in their construction. As
we will see in more details in Sec. 3.3.2, even in non-perturbative approaches such as M-theory and
p- and D-brane models, the 2D surfaces do not lose their crucial role and are implicitly present and
represented by large matrices.

On the other hand, the role, properties, and interpretation of 2D surfaces in these theories and
\( SU(\infty) \)-QGR are profoundly different. In string theories 2D worldsheets of strings or more generally
membranes are quantized and summation over their geometries is interpreted as path integral of 2D
quantum gravity. By contrast, diffeo-surfaces in \( SU(\infty) \)-QGR are not an independent physical entities,
neither they are quantized. They are associated to quantum states of the Universe and its content
- subsystems in the same way that in QFT a charge or spin is associated to a particle but it is not
the particle. Deformations of diffeo-surfaces do not correspond to different (quantum)-gravitational
states, but rather represent members of the symmetry group of shared by all quantum subsystems,
including the whole Universe.

String sigma model

As a sigma model both bosonic and fermionic quantum fields live on the
2D worldsheet of strings [113, 119]. In superstring models they are interpreted as coordinates of a
\( n \)-dimensional quantum spacetime and their supersymmetric counterparts, respectively. One can
equally interpret the worldsheet of a string as a 2D extended membrane embedded or emerged in an
\( n \)-dimensional spacetime. Additionally, string theories are in general 2D Conformal Field Theories
(CFT). This means that they are invariant under rescaling of both worldsheet 2D coordinates and
local rescaling of the fields. This double conformality is a necessary condition for eliminating central
charge and anomalies, which arise when these models are quantized [119]. Cancellation of these
unwanted elements limits the spacetime (target space) dimension to \( n = 26 \) for bosonic strings or to
\( n = 10 \) in superstring models. More generally, the sigma model can be any CFT with Kac-Moody
algebra having the same number of degrees of freedom as bosonic and supersymmetric models. As
mentioned earlier, the value of fundamental - rather than observed - spacetime dimension obtained
from sigma model formulation of superstrings is taken for granted in further developments of these models. Interestingly, $n = 1$ model is also a consistent quantum model [113]. The single (super)field in such model cannot be interpreted as a background spacetime, but they are studied as a decoupled sector in matrix formulation of string theory [121].

In the framework of $SU(\infty)$-QGR, the string setup - without quantization of 2D worldsheet/membrane - can be considered as a special state for a quantum system with $SU(\infty)$ symmetry. The sigma model of strings - without constraints arising from conformal symmetry and quantization - can be interpreted as special states for subsystems with an internal symmetry $G$. Quantization of such a state in the framework (super)string theory restricts the internal symmetry $G$ to groups allowed by the cancellation of anomalies. For instance, $G$ may be identified with: 10D sigma model of superstring models and/or its $SO(32)$ or $E_8$ internal symmetry; symmetries of the low energy corresponding $N = 4$ supergravity in 11D; or symmetries of compactified coordinates or quantum fluctuations of D-brane solutions in M-theory. The origin of these similarities can be traced back to the Virasoro algebra of string fluctuations, which is a subalgebra of Surface-preserving Diffeomorphism (SDiff) of a torus $SDiff(T^2)$ and the fact that the latter is a representation of $SU(\infty)$ group [122, 123, 37].

One of the main advantages of string theory to canonical QGR is its renormalizability and absence of UV singularity, owed to the extended nature of strings. Although details are not yet worked out for $SU(\infty)$-QGR, from its Yang-Mills action we expect that it be renormalizable. Moreover, UV singularity should not arise, because the distance between subsystems is related to the relative area of their diffeo-surfaces, which by definition cannot shrink to a point - equivalent to zero distance, other $SU(\infty)$ would be represented trivially. This feature should play the role of a build-in ultra-violet cut-off without introducing any fixed scale.

Curved spacetime and gravity in string theory

The issue of a curved target (field) spacetime in string theory does not have an analogy in $SU(\infty)$-QGR. However, solutions proposed to overcome this problem and their role in non-perturbative formulation of string theory in the framework of M-theory can be compared with $SU(\infty)$-QGR.

In sigma model formulation of string theories quantization is consistent only when the geometry of the field (target) space - interpreted as fundamental spacetime - is flat. This means that only metric perturbations around this Minkowski background are quantum mechanically meaningful. Although in the framework of perturbative string theories a string gas has been considered - specially for the purpose of describing cosmological perturbations [124] - the inherently intertwined nature of spacetime and strings may make it impossible to consider them as separately evolving entities. There are, nonetheless, exceptions. AdS/CFT duality conjecture, discussed in more details in Sec. 3.3.3, is proved for AdS$_3$ space, and is considered as evidence for consistent formulation of string theory, at least in some curved background spaces.

Another way to overcome the issue of curved target space is considering special configurations/solutions for the dynamics of strings in the target space. These solutions usually include localization of perturbative string modes. For instance, extremity of open string can be restricted to move on a $p < D$ dimensional subspace of the target space, called a p-brane or more general solutions in the form of D-branes [114]. The induced geometry on p/D-branes can be curved. In this framework, the observed $(3 + 1)$ dimensional spacetime can be a brane in $D$-dimensional fundamental target space. In the same manner, D-branes can be formed from condensation of closed strings, but they may be unstable [115, 116, 117]. D0 branes are another class of interesting configuration of Yang-Mills gauge fields in the target space. They correspond to coordinate independent configurations, which may change with time [104] or be static [105]. These models are studied in the framework of gauge-gravity duality conjecture [104] and M-theory. These models have more common aspects with $SU(\infty)$-QGR than perturbative string models and we review them in more details in the next subsection.
3.3.2 M-theory and matrix theories

M-theory and matrix models are developed as candidates for non-perturbative formulation of string theory, see e.g. [125, 126] for review. Using various concepts, including large $N$ expansion of perturbative QFTs [127] and holography principle, it is conjectured that non-perturbative type II string theories can be described as $U(N)$ supersymmetric Yang-Mills theories and present quantum states of type IIA strings in D0 background [104, 128].

BFSS [104] matrix model - called also D1+0 brane - is a 10D super Yang Mills model, obtained from compactification of one dimension of 11D super gravity effective field theory of string theory at low energy limit. It is reduced to 1+0 dimension by assuming that all the fields in the model are independent of 9 spatial coordinates. The dependence on the last coordinate is removed in what is called D0 energy limit. It is reduced to 1+0 dimension by assuming that all the fields in the model are independent of 9 spatial coordinates.

The action of IKKT model is defined as:

$$ S_{IKKT}[X] = \frac{1}{g^2} \text{Tr} \left( \frac{i}{4} [X^a, X^b] [X^c, X^d] \eta_{ac} \eta_{bd} - \frac{i}{2} \bar{\psi}_\alpha (C \sigma^{ab} [A_a, \psi_\beta]) \right) $$

where $X^a$, $a = 0, \ldots, 9$ and $\bar{\psi}_\alpha$, $\alpha = 1, \ldots, 16$ are hermitian $N \times N$ matrices representing $SO(D = 10)$ (Euclidean) or $SO(D - 1, 1)$ (Minkowski); $\eta_{ab}$ is the metric of a flat Minkowski or Euclidean 10D space; $\sigma^a$'s are $16 \times 16$ Pauli matrices for $D=10$ space; and $C$ is charge conjugate operator of the same dimension. The action $S_{IKKT}[X]$ is similar to that of type IIB superstring in Schilg gauge [130].

It is assumed that $N \rightarrow \infty$ such that $Ng^2 = g^2 < \infty$. Therefore, $X^a$ and $\psi_\alpha$ are respectively bosonic and fermionic $N \rightarrow \infty$ dimensional representations of non-Abelian $SO(D)$ (or $SO(D-1,1)$). Although the original motivation for this model has been the string theory, it can be considered without referring to strings and are also studied in $D \neq 10$ [129, 126]. Using variation principle, one can obtain field equations for $X^a$ and $\psi_\alpha$. In particular, considering only bosonic Yang-Mills sector, the dynamic equation for $X^a$ is:

$$ [X^a, [X^b, X^c]] \eta_{ac} = 0 $$

(34)

Giving the finite rank of Yang-Mills symmetry group of the model and large dimension of its representation by $X^a$, equation (34) has many solutions. Aside trivial commuting matrices, solutions having the form:

$$ [Y^\mu, Y^\nu] = i \theta^{\mu\nu} \mathbb{1}, \quad \mu, \nu = 0, \ldots, 2n - 1, \quad 2n \leq D $$

(35)

where $\theta^{\mu\nu}$ is an anti-hermitian constant matrix $\theta^{\mu\nu} = -\theta^{\nu\mu}$, provide reduction of space dimension and a quantized noncommutative geometry [131] for these branes. Specifically, in case of Moyal-Weyl solution, the background 10D space is static with $Y^\mu = \bar{Y}^\mu$, $\mu = 0, \ldots, 2n - 1$ and $Y^i = 0, i = 2n, \ldots, D - 1$. To define quantum fluctuations $Y^a$ matrices are decomposed as:

$$ Y^a = \bar{Y}^a + (A^\mu, 0) + (0, \phi^i) $$

(36)

Although $A^\mu$ can be considered as a $U(1)$ gauge group, it actually belongs to gravity sector and cannot be identified as $U(1)$ symmetry of the Standard Model. Matter fields in matrix models can arise like in string theory by compactifying $D - 2n$ fields, see e.g. [125] for a review, or in case of Moyal-Weyl type solutions, by considering $k$ coinciding branes, see e.g. [114, 132, 126] for a review. Assuming quantum superposition of fluctuations of $k$ branes as defined in (36), they are locally invariant under $SU(k)$ symmetry. Thus, in this approach $A^\mu$ and $\phi^i$ can be expanded as:

$$ A^\mu = -\theta^{\mu\nu} A^\nu_b (\bar{Y}) T^b, \quad \phi^i = \phi^i_b (\bar{Y}) T^b $$

(37)

where $T^b$'s are generators of adjoint representation of $SU(k)$. As indicated earlier, due to the fundamental non-commutation nature of $Y^a$ this construction cannot accommodate a $U(1)$ symmetry. In the same manner fermion fields can be constructed, but they would be in adjoint representation of
$SU(k)$, because in 10D they are supersymmetric partners of coordinates $X^a$. For $n=2$ this setup lead to a quantum noncommutative $\mathbb{R}^4_\theta \subset \mathbb{R}^D$ space identified as the physical spacetime.

In contrast to Randall-Sundrum type brane models, in matrix theories quantum fluctuations of geometry and matter do not propagate to the 10D bulk. Therefore, matrix theories, and more generally models based on the condensation of string modes to branes, are not dependent on the 10D background geometry. This solves the problem of string formulation in curved spaces discussed in Sec. 3.3.1. But, in matrix models only fluctuations of background (target) 10D space are observables. Therefore, it is not clear what is the role of unobservable static (in some models) of the 10D fundamental background / target space. Moreover, D-branes may decay [115, 116, 117] and stability of overall setup is not certain. In any case, these field/string solutions are special configurations, many of them are plausible [133], and it is not clear why nature should prefer the one corresponding our Universe.

Finally, the low energy effective action of matrix models is a modified version of Einstein equation [126, 134, 135], which is stringently constrained, specially with gravitational waves [136].

**Comparison of matrix models with $SU(\infty)$-QGR** There are many similarities between matrix models and $SU(\infty)$-QGR, but also significant differences. In both models the fundamental objects are $N \rightarrow \infty$ matrices. Matrix models are pure (super)Yang-Mills, and $X^a$ and $\psi^i$ in (33) are $N \times N$ matrices in adjoint representation of an internal finite rank symmetry. By contrast, $SU(\infty)$-QGR is constructed from a Hilbert space and includes both square and column matrices as primary entities, in adjoint and fundamental representations of both $SU(\infty)$ and internal symmetries.

In matrix theory the large dimension of matrices is inspired by large color and loop number limit of QFT [127], conjectured to present strong coupling regime. In $SU(\infty)$-QGR the motivation is rather cosmological and based on the observed large number of degrees of freedom in the Universe. These apparently different motivations converge to each other because for a perturbative estimation of observables, for instance $S$-matrix, up to a given degree of precision, one has to take into account more loops, virtual particles, and their degrees of freedom for stronger couplings. The assumption of $SU(\infty)$-QGR that every subsystem of the universe represents infinite degrees of freedom is an explicit realization of the above concept. $SU(\infty)$-QGR uses the above axiom as a foundation for constructing other aspects of the model. Moreover, based on the observed spontaneous breaking and emergence of symmetries in many-body systems, see e.g. [137, 138], it defines subsystems according to the well established criteria in quantum information theory [29]. The formulation of the model, specially in what concerns the universal quantum gravitational interaction, is completely independent of internal symmetries of subsystems (particles or fields), which are not constrained by the model. In fact, considering that when all constituents interact with each others the symmetry is $G \rightarrow SU(\infty)$, one expects that many other smaller rank symmetries should have nonzero probability to arise in intermediate states, where the number of effectively coupled or entangled subsystems is finite.

By contrast, in matrix models the large dimension of matrices remains as a background concept and does not directly intervene in the construction of symmetries and dynamics. The latters are in a large extent inspired or concluded from superstring theory. Indeed, large matrices in these models present a membrane or worldsheet of a string [35, 122, 123]. Despite the fact that matrix models can be considered as stand-alone and what they consider as fundamental spacetime can have other dimension than 10 of superstrings, see e.g. [129], BFSS and IKKT models and their variants are mostly constructed and studied in 10D Euclidean or Minkowski space. In any case, even without referring to string theory, the presence of extra-dimensions in matrix models is inevitable for introduction of matter and other interactions. They are considered to be the quantum fluctuations of a D0 condensate [126] (and references therein) or compactified dimensions [125] (and references therein). However, similar to many QGR proposals and in contrast to $SU(\infty)$-QGR, matrix models do not provide any explanation for the observed dimension of spacetime.
In summary, both M-theory (matrix models) and SU(\(\infty\))-QGR emphasize on the importance of SU(\(\infty\)) in a quantum description of gravity. But, they diverge in many details. In particular, in matrix models SU(\(\infty\)) symmetry is not explored. The manner internal symmetries arise in these models and constrained are very different. Finally, the large dimension of fundamental spacetime - the target space - in M-theory does not have an analogue in SU(\(\infty\))-QGR.

3.3.3 Anti-de Sitter - Conformal Field Theory (AdS-CFT) duality

According to AdS-CFT duality conjecture [30], and more generally gauge-gravity duality [31, 32] in M-theory, there is a one to one correspondence between quantum states of a suitable quantum CFT living on the boundary of a region of the spacetime and supergravity (string theory) in its AdS bulk.

This conjecture is closely related to the holographic principle, but there is not yet a general proof for it, except in (2+1)D spaces [111]. Specifically, consider a conformal field theory on the (1+1)D space \(\mathbb{R} \times S^1\) boundary of an AdS\(_3\) spacetime. Define two complementary subsystems \(A\) and \(B\) divided along \(\mathbb{R}\) axis of the bulk (see figure 1 of [111]). The Hilbert space of the quantum CFT is factorized to \(\hat{H}_A \otimes \hat{H}_B\) and entanglement entropy between \(A\) and \(B\) is defined as \(S(A) = -\text{tr}(\rho_A \ln \rho_A)\), where \(\rho_A\) is the density matrix of \(A\) when the state of \(B\) is traced out.

It is proved [111] that the static entanglement entropy, that is at \(t = \text{constant}\), between the two subsystems is proportional to the length of the geodesic (null) curve passing inside the AdS\(_3\) and joining the 2-point cross-section on the \(t = \text{constant}\) \(S^1\) boundary. More generally, for an AdS\(_{d+2}\) spacetime the entanglement entropy is conjectured to be:

\[
S(A) = \frac{\text{Area of } \gamma_A}{4G_N^{d+2}}
\]

where \(\gamma_A\) is the \(d\)-dimensional minimal (geodesic) boundary surface and \(G_N\) is the Newton constant. Additionally, it is shown that \(S(A) \to 0\) only when the size of the system goes to infinity \([8, 9]\). This case corresponds to when the two subsystems are infinitely separate from each other.

We notice that the definition of subsystems in \([30, 111]\) is geometric. This is an important point, because as we discussed in Sec. 3.2.3, QFTs have Type III algebra and Lorentz invariant quantum subsystems cannot be defined by division of their support spacetime. Thus, \(A\) and \(B\) are not properly speaking subsystems and diffeomorphism invariant. It is not clear whether and how this issue affects the AdS/CFT duality conjecture, specially in higher dimensional spaces for which a proof is not available.

For \(d = 2\) the AdS \(\cong \mathbb{R} \times \mathbb{R} \times S^d\) geometry is homomorphic to the simplest geometry of parameter space in SU(\(\infty\))-QGR after division of the Universe to subsystems. For this case, relation with a CFT on the boundary in the framework of SU(\(\infty\))-QGR can be understood as the following: For the whole Universe or an approximately isolated subsystem the size of the diffeo-surface of its SU(\(\infty\)) symmetry is approximately irrelevant for its observables. This property can be interpreted as an approximate conformal symmetry, that is scaling invariance of the parameter space of the system and its pull-back into the Hilbert space. Considering an external quantum clock, at a given time the parameter space of such an isolated subsystem is approximately 2D and its quantum dynamics is approximately a 2D CFT. Its operators generate a Virasoro algebra, which is a subalgebra of SDiff(T(2)) \(\cong SU(\infty)\) \([122, 123, 37]\). Invariance by scaling means that any arbitrary diffeomorphism can be bring back to a surface preserving one.

\(^8\)Notice that geometrical division to \(A\) and \(B\) and factorization of their Hilbert space is in general valid only in a given frame because as we discussed in Sec. 3.2.2 QFT’s are Type III.
4 Outline

Comparison of several popular QGR models with \(SU(\infty)\)-QGR proposal in this work finds a number of common or analogous features between them. We discussed the origin of these properties and showed that they arise in \(SU(\infty)\)-QGR either from its axioms or can be concluded from them. Giving simple axioms and systematic and natural emergence of common features in \(SU(\infty)\)-QGR, this model may help to clarify some of puzzling properties of other QGR models.

\(SU(\infty)\)-QGR is not yet applied to physical phenomena in which QGR may be involved, such as:

- Hawking radiation;
- Spacetime singularities;
- Information loss paradox of black holes;
- Topology of parameter space of the model identified as the classical spacetime and whether it can be changed;
- Particle physics at Planck scale.

Regarding the last item in this list, as mentioned in Sec. 2.5.2, we expect more unbroken gauge symmetries at high energies, which can be also interpreted as the internal symmetry \(G\). Therefore, a crucial task is to determine how internal symmetries vary with energy scale \(\Lambda\). Particle physics experiments show that \(G|_{\Lambda \sim 1\text{TeV}} = SU(3) \times SU(2) \times U(1)\), i.e. the Standard Model symmetry. But, signature of interactions at higher energies may be smeared by the physics at lower energies [139]. Nonetheless, if many-body high energy states behave similar to low energy many-body systems, the analogy may help to find best criteria for detecting signatures of phase transitions due to symmetry transition at high energies in (astro-)particle physics experiments or cosmological observations. Other topics in the above list would be the subject of future investigations.

A Classical limit of \(SU(\infty)\) Yang-Mills of subsystems

We first review some of properties of curvatures of (pseudo)Riemannian manifolds. For any Riemannian or pseudo-Riemannian manifold \(\mathcal{M}, g\) of dimension \(d \geq 2\) equipped with a Levi-Civita connection \(\nabla\) the Riemann curvature (1,3) tensor at point \(p \in \mathcal{M}\) is defined as:

\[
R_p(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]}Z
\]  

(39)

Vector fields \(X, Y, Z \in T\mathcal{M}_p\) and \(T\mathcal{M}_p\) is the tangent space at \(p\). When \(X, Y, Z\) are chosen to be \(\hat{e}_i \equiv \partial/\partial x^i\) basis of the tangent space for coordinates \(x^i, i = 0, \ldots, d - 1\), one recover the usual coordinate dependent definition of the Riemann curvature tensor (we drop \(p\) because it corresponds to the point with coordinates \(x^i\):

\[
R_i(\hat{e}_j, \hat{e}_k)\hat{e}_l = R^l_{kij},
\]  

(40)

\[
\varepsilon_{ijkl} R_{ijkl} \equiv R(\hat{e}_i, \hat{e}_j, \hat{e}_k, \hat{e}_l) \equiv \left( R(\hat{e}_i, \hat{e}_j)\hat{e}_k, \hat{e}_l \right) = g_{mi} R^m_{kij}
\]  

(41)

Using the notation defined in (41) for (0,4) Riemann curvature tensor sectional curvature \(K(\Pi) = K(X,Y)\) with respect to a 2D plane \(\Pi \subset T\mathcal{M}_p\) containing two vectors \(X, Y \in T\mathcal{M}_p\) at \(p \in \mathcal{M}\) is defined as:

\[
K(\Pi) \equiv K(X,Y) = \frac{R_p(X,Y,X,Y)}{\langle X \rangle \langle X,Y \rangle - \langle X,Y \rangle^2}
\]  

(42)
Notice that $K(\Pi)$ is independent of the choice of $X$ and $Y$ and depends only of the plane passing through them. It can be shown that $\langle R_p(X,Y)Z,W\rangle$ can be expanded with respect to sectional curvatures [140]. Using relations between different curvature tensors of a Riemannian manifold, Ricci scalar at point $p \in M$ is defined as [141]:

$$R(p) = \sum_{i \neq j} R_p(e_i, e_j, e_i, e_j) = \sum_{i \neq j} K_p(e_i, e_j)$$  \hspace{1cm} (43)

where $e_i, i = 0, \ldots, d-1$ is an orthonormal basis of $T_pM$. From (43) we conclude that there is only one sectional curvature at each point of a 2D surface and it is equal to its Ricci scalar curvature.

In order to extend the relation (18) between $SU(\infty)$ Yang-Mills action and Ricci scalar curvature of its diffeo-surface of an isolated quantum system with $SU(\infty)$ symmetry to large number of such systems when the Universe is divided to subsystems, we have to integrate over their contribution. In Sec. 2.3.1 we showed that the parameter space of subsystems is $(3+1)$D dimensional. Applying definitions of curvature tensors to this parameter space, each sectional curvature in (43) can be interpreted as Ricci curvature of the diffeo-surface of a subsystem and the summation in the r.h.s. of (43) and integration over the volume of the parameter space amounts to taking into account all subsystems of the Universe. Therefore, the r.h.s. of (23) corresponds to the r.h.s. of (18) when the Universe is divided to subsystems. In the same way, the l.h.s. of (23) corresponds to the l.h.s. of (18) when the contribution of subsystems are calculated separately. However, (23) is valid only in the classical limit, because as discussed in Sec. 2.4, after the selection of reference and clock the ensemble of remaining subsystems should be considered as an open quantum system. Therefore, (23) is an approximation, valid only in the classical limit where the reference and the clock can be considered as classical.

References

[1] Ziaeepour, H. Making a Quantum Universe: Symmetry and Gravity MDPI Universe J., (2021) Special Issue 80 Years of Professor Wigner’s Seminal Work “On Unitary Representations of the Inhomogeneous Lorentz Group” [arXiv:2009.03428].

[2] Eppley, K.; Hanna, E. The Necessity of Quantizing the Gravitational Field. Found. Phys. 7, (1977) 51.

[3] Jacobson, T. Thermodynamics of Spacetime: The Einstein Equation of State. Phys. Rev. Lett. 75, (1995) 1260, [arXiv:gr-qc/9504004].

[4] Bekenstein, J.D. Statistical black-hole thermodynamics. Phys. Rev. D 12, (1975) 3077.

[5] t’Hooft, G. Dimensional Reduction in Quantum Gravity. (1993), [arXiv:gr-qc/9310026].

[6] Susskin, L. The World as a Hologram. JMP 36, (1995) 6377, [arXiv:hep-th/9409089].

[7] Bousso, R. The holographic principle. Rep. Mod. Phys. 74, (2002) 825, [arXiv:hep-th/020310].

[8] Holzhey, C.; Larsen, F.; Wilczek, F. Geometric and Renormalized Entropy in Conformal Field Theory. Nucl. Phys. B 424, (1994) 443, [arXiv:hep-th/9403108].

[9] Calabrese, P.; Cardy, J. Entanglement Entropy and Quantum Field Theory. J. Stat. Mech. 0406, (2004) P06002, [arXiv:hep-th/0405152].

[10] Ziaeepour, H. Issues with vacuum energy as the origin of dark energy. Mod. Phys. Lett. A 27, (2012) 1250154, [arXiv:1205.3304].

[11] Rovelli, C. Quantum mechanics without time: A model. Phys. Rev. D 42, (1990) 2638.

[12] Markopoulou, F. Space does not exist, so time can. (2009), [arXiv:0909.1861].
[13] Van Raamsdonk, M. Building up spacetime with quantum entanglement. *Gen. Rel. Grav* **42**, (2010) 2323, *Gen. Rel. Grav* **42**, (2010) 2323; *Int. J. Mod. Phys. D* **19**, (2010) 2429, [arXiv:1005.3035].

[14] Cao, C.; Carroll, S.M.; Michalakis, S. Space from Hilbert Space: Recovering Geometry from Bulk Entanglement. *Phys. Rev. D* **95**, (2017) 024031, [arXiv:1606.08444].

[15] Page, D.N.; Wootters, W.K. Evolution without evolution: Dynamics described by stationary observables. *Phys. Rev. D* **27**, (1983) 2885.

[16] In preparation (2021).

[17] Giddings, S.B. Quantum-first gravity. *Found. Phys.* **49**, (2019) 177, [arXiv:1803.04973].

[18] Giddings, S.B. Hilbert space structure in quantum gravity: an algebraic perspective. *J. High Energy Phys.* **2015**, (2015) 1, [arXiv:1503.08207]

[19] Bekenstein, J.D. Black Holes and Entropy. *Phys. Rev. D* **7**, (1973) 2333.

[20] Hawking, S. Break of predictability in gravitational collapse. *Phys. Rev. D* **14**, (1976) 246.

[21] Zurek, W.H. Entropy Evaporated by a Black Hole. *Phys. Rev. Lett.* **49**, (1982) 1683.

[22] Cao, C.; Carroll, S.M. Bulk Entanglement Gravity without a Boundary: Towards Finding Einstein’s Equation in Hilbert Space. *Phys. Rev. D* **97**, (2018) 086003, [arXiv:1712.02803].

[23] Dewitt, B. Quantum Theory of Gravity. I. The Canonical Theory. *Phys. Rev.* **160**, (1967) 1113.

[24] Hartle, J.B.; Hawking, S.W. Wave function of the Universe. *Phys. Rev. D* **28**, (1983) 2960.

[25] Wheeler, J.A. On the nature of quantum geometrodynamics. *Annals Phys.* **2**, (1957) 604.

[26] Rocci, A. On first attempts to reconcile quantum principles with gravity. *J. Phys. Conf. Ser.* **470**, (2013) 012004, [arXiv:1309.7336]..

[27] Kiefer, C. Quantum geometrodynamics: Whence, whither? *Gen. Rel. Grav* **41**, (2009) 877, [arXiv:0812.0295].

[28] Arnowitt, R.; Deser, S.; Misner, C. Dynamical Structure and Definition of Energy in General Relativity. *Phys. Rev.* **116**, (1959) 1322, [arXiv:gr-qc/0405109]

[29] Zanardi, P.; Lidar, D.; Lloyd, S. Quantum tensor product structures are observable-induced. *Phys. Rev. Lett.* **92**, (2004) 060402, [arXiv:quant-ph/0308043].

[30] Maldacena, J.M. The Large N Limit of Superconformal Field Theories and Supergravity. *Adv. Theor. Math. Phys* **2**, (1998) 231, [arXiv:hep-th/9711200].

[31] Witten, E. Anti De Sitter Space And Holography. *Adv. Theor. Math. Phys* **2**, (1998) 253, [arXiv:hep-th/9802150].

[32] Aharony, O.; Gubser, S.S.; Maldacena, J.; Ooguri, H.; Oz, Y. Large N Field Theories, String Theory and Gravity. *Phys. Rep.* **323**, (2000) 183, [arXiv:hep-th/9905111].

[33] Dirac, P.A.M.: The Principles of Quantum Mechanics, Oxford University Press (1958).

[34] Von Neumann, J.: Mathematical Foundation of Quantum Theory, Princeton University Press, (1955).

[35] Hoppe, J. Quantum Theory of a Massless Relativistic Surface and a Two-dimensional Bound State Problem. Ph.D. Thesis, MIT, Cambridge, MA, USA, (1982).
[36] Floratos, E.G.; Iliopoulos, J.; Tiktopoulos, G. A note on $SU(\infty)$ classical Yang-Mills theories. *Phys. Lett. B* **217**, (1989) 285.

[37] Hoppe, J. Diffeomorphism Groups, Quantization, and $SU(\infty)$. *Int. J. Mod. Phys. A* **4**, (1989) 5235.

[38] Hoppe, J.; Schaller, P. Infinitely Many Versions of $SU(\infty)$. *Phys. Lett. B* **237**, (1990) 407.

[39] Zuenger, Y. Why Matrix theory works for oddly shaped membranes. *Phys. Rev. D* **64**, (2001) 086003, [arXiv:hep-th/0106030].

[40] Ziaeepour, H. And what if gravity is intrinsically quantic ? *J. Phys. Conf. Ser.* **174**, (2009) 012027, [arXiv:0901.4634].

[41] Su, Z-Y. A Scheme of Cartan Decomposition for su(N). (2006), [arXiv:quant-ph/0603190].

[42] Ziaeepour, H. Foundational role of symmetry in Quantum Mechanics and Quantum Gravity. in “Quantum Mechanics: Theory, Analysis, and Applications”, Nova Science Publishers Inc., New York (2019), [arXiv:1305.4349].

[43] Ziaeepour, H. Symmetry as a foundational concept in Quantum Mechanics. *J. Phys. Conf. Ser.* **626**, (2015) 012074, [arXiv:1502.05339].

[44] Hoehn, P.A.; Smith, A.R.H.; Lock, M.P.E. The Trinity of Relational Quantum Dynamics. (2019), [arXiv:1912.00033].

[45] Mandelstam, L.; Tamm, I. The Uncertainty Relation Between Energy and Time in Non-relativistic Quantum Mechanics. *J. Phys. (USSR)* **9**, (1945) 249.

[46] Hoehen, Ph; Lock, M.P.E.; Ali Ahmad, S.; Smith, A.R.H.; Galley, T.D. Quantum Relativity of Subsystems. (2021), [arXiv:2103.01232].

[47] Rosenfeld, L. Zur Quantelung der Wellenfelder. *Annal der Physik* **397**, (1930) 113.

[48] Dewitt, B. Quantum Theory of Gravity. I. The Canonical Theory. *Phys. Rev.* **160**, (1967) 1113.

[49] Hartle, J.B.; Hawking, S.W. Wave function of the Universe. *Phys. Rev. D* **28**, (1983) 2960.

[50] Rocci, A. On first attempts to reconcile quantum principles with gravity. *J. Phys. Conf. Ser.* **470**, (2013) 012004, [arXiv:1309.7336]..

[51] Regge, T. General Relativity without Coordinates. *Nuovo Cimento* **19**, (1961) 558.

[52] Gambini, R.; Pullin, J. Consistent discretization and canonical classical and quantum Regge calculus. *Int. J. Mod. Phys. D* **15**, (2006) 1699, [arXiv:gr-qc/0511096].

[53] Ponzano,, G.Regge, T. Semiclassical limit of Racah coefficients. p1-58; in Spectroscopic and group theoretical methods in physics, ed. F. Bloch, North-Holland Publ. Co. Amsterdam, (1968).

[54] Ashtekar, A. New Variables for Classical and Quantum Gravity. *Phys. Rev. Lett.* **57**, (1986) 2244.

[55] Immirzi, G. Quantum Gravity and Regge Calculus. *Nucl. Phys. B* Proc.Suppl **57**, (1997) 65, [arXiv:gr-qc/9701052].

[56] Rovelli, C. *Quantum Gravity*; Cambridge University Press: Cambridge, UK, (2004).

[57] Ashtekar, A.; Lewandowski, J. Background Independent Quantum Gravity: A Status Report. *Class. Quant. Grav.* **21**, (2004) R53, [arXiv:gr-qc/0404018].
[58] Immirzi, G. Real and complex connections for canonical gravity. *Class. Quant. Grav.* **14**, (1997) L177, [arXiv:gr-qc/9612030].

[59] Barrett, J.W.; Crane, L. Relativistic spin networks and quantum gravity. *J. Math. Phys.* **39**, (1998) 3296, [arXiv:gr-qc/9709028].

[60] Barrett, J.W.; Crane, L. A Lorentzian Signature Model for Quantum General Relativity. *Class. Quant. Grav.* **17**, (2000) 3101, [arXiv:gr-qc/9904025].

[61] Livine, E.R. Projected Spin Networks for Lorentz connection: Linking Spin Foams and Loop Gravity. *Class. Quant. Grav.* **19**, (2002) 5525, [arXiv:gr-qc/0207084].

[62] Ashtekar, A.; Rovelli, C.; Smolin, L. Weaving a Classical Metric with Quantum Threads. *Phys. Rev. Lett.* **69**, (1992) 237.

[63] Rovelli, C.; Colosi, D.; Doplicher, L.; Fairbairn, W.; Modesto, L.; Noui, Karim Background independence in a nutshell. *Class. Quant. Grav.* **22**, (2005) 2971, [arXiv:gr-qc/0408079].

[64] Smolin, L. An invitation to loop quantum gravity. in Quantum Theory and Symmetries. Ed. P.C. Argyres, et al., World Scientific (2004), [arXiv:hep-th/0408048].

[65] Maran, S.K. Reality Conditions for Spin Foams. (2005), [arXiv:gr-qc/0511014].

[66] Collins, J.; Perez, A.; Sudarsky, D.; Urrutia, L.; Vucetich, H. Lorentz invariance and quantum gravity: an additional fine-tuning problem? *Phys. Rev. Lett.* **93**, (2004) 191301, [arXiv:gr-qc/0403053].

[67] Gambini, R.; Pullin, J. Emergent diffeomorphism invariance in a discrete loop quantum gravity model. *Class. Quant. Grav.* **26**, (2009) 035002, [arXiv:0807.2808].

[68] Ashtekar, A. Some surprising implications of background independence in canonical quantum gravity. *Gen. Rel. Grav.* **41**, (2009) 1927, [arXiv:0904.0184].

[69] Bojowald, M.; Mortuza Hossain, G. Loop quantum gravity corrections to gravitational wave dispersion. *Phys. Rev. D* **77**, (2008) 023508, [arXiv:0709.2365].

[70] Girelli, F. Hinterleitner, F. Major, S.A. Loop Quantum Gravity Phenomenology: Linking Loops to Observational Physics. *SIGMA* **8**, (2012) 098, [arXiv:1210.1485].

[71] Abdo, A.A.; Ackermann, M.; Ajello, M.; Asano, K.; Atwood, W.B.; Axelsson, M.; Baldini, L.; Ballet, J.; Barbiellini, G.; Baring, M.G.; et al. A limit on the variation of the speed of light arising from quantum gravity effects. *Nature* **462**, (2009) 331, [arXiv:0908.1832].

[72] The LIGO Scientific Collaboration Tests of General Relativity with GW170817. *Phys. Rev. Lett.* **123**, (2019) 011102, [arXiv:1811.00364].

[73] The LIGO Scientific Collaboration Tests of General Relativity with the Binary Black Hole Signals from the LIGO-Virgo Catalog GWTC-1. *Phys. Rev. D* **100**, (2019) 104036, [arXiv:1903.04467].

[74] Perez, A.; Rovelli, C. Physical effects of the Immirzi parameter. *Phys. Rev. D* **73**, (2006) 044013, [arXiv:gr-qc/0505081].

[75] Bergé, J.; Pernot-Borràs, M.; Uzan, J.-P.; Brax, P.; Chlun, R.; Métris, G.; Rodrigues, M.; Touboul, P. MICROSCOPE’s constraint on a short-range fifth force. (2021), [arXiv:2102.00022].

[76] Gaul, M.; Rovelli, C. Loop Quantum Gravity and the Meaning of Diffeomorphism Invariance. *Lect. Notes Phys.* **541**, (2000) 277, [arXiv:gr-qc/9910079].
[77] Halliwell, J.J.; Walden, P. Invariant Class Operators in the Decoherent Histories Analysis of Timeless Quantum Theories. *Phys. Rev. D* **73**, (2006) 024011, [arXiv:/0509013].

[78] Reisenberger, M.; Rovelli, C. Spacetime states and covariant quantum theory. *Phys. Rev. D* **65**, (2002) 125016, [arXiv:gr-qc/0111016].

[79] Terno, D.R. Quantum information in loop quantum gravity. *J. Phys. Conf. Ser.* **33**, (2006) 469, [arXiv:gr-qc/0512072].

[80] Giesel, K.; Thiemann, T. Algebraic Quantum Gravity (AQG) IV. Reduced Phase Space Quantisation of Loop Quantum Gravity. *Class. Quant. Grav.* **27**, (2010) 175009, [arXiv:0711.0119].

[81] Husain, V.; Pawlowski, T. Time and a physical Hamiltonian for quantum gravity. *Phys. Rev. Lett.* **108**, (2012) 141301, [arXiv:1108.1145].

[82] Giesel, K.; Vetter, A. Reduced Loop Quantization with four Klein-Gordon Scalar Fields as Reference Matter. (2016), [arXiv:1610.07422].

[83] Gielen, S.; Oriti, D. Cosmological perturbations from full quantum gravity. *Phys. Rev. D* **98**, (2018) 106019, [arXiv:1709.01095].

[84] Wilczek, F. Riemann-Einstein Structure from Volume and Gauge Symmetry. *Phys. Rev. Lett.* **80**, (1998) 4851, [arXiv:hep-th/9801184].

[85] Seiberg, N. Emergent Spacetime. in *The Quantum Structure of Space and Time*. Ed: J. Harvey, p. 163 World Scientific (2007), [arXiv:hep-th/0601234].

[86] Westman, H.; Sonego, S. Coordinates, observables and symmetry in relativity. *Annals Phys.* **324**, (2009) 1585 [arXiv:0711.2651].

[87] Wilczek, F. Riemann-Einstein Structure from Volume and Gauge Symmetry. *Phys. Rev. Lett.* **80**, (1998) 4851, [arXiv:hep-th/9801184].

[88] Torres-Gomez, A.; Krasnov, K. Gravity-Yang-Mills-Higgs unification by enlarging the gauge group. *Phys. Rev. D* **81**, (2010) 085003, [arXiv:0911.3793].

[89] Barrett, J.W.; Kerr, S. Gauge gravity and discrete quantum models. (2013), [arXiv:1309.1660].

[90] Padmanabhan, T. Gravity and the Thermodynamics of Horizons. *Phys. Rep.* **406**, (2005) 49, [arXiv:gr-qc/0311036].

[91] Padmanabhan, T. Gravity as an emergent phenomenon: A conceptual description. *AIP Conf. Proc.* **939**, (2007) 114, [arXiv:0706.1654].

[92] Verlinde, E.P. On the Origin of Gravity and the Laws of Newton. *J. High Energy Phys.* **1104**, (2001) 029, [1001.0785].

[93] Hartle, J.B. Generalizing Quantum Mechanics for Quantum Spacetime. in *The Quantum Structure of Space and Time*: ed. by D. Gross, M. Henneaux, A. Sevrin, World Scientific, Singapore, (2007), [arXiv:gr-qc/0602013].

[94] Giddings, S.B. Universal quantum mechanics. *Phys. Rev. D* **78**, (2008) 084004, [arXiv:0711.0757].

[95] Griffiths, R.B. Consistent Histories and the Interpretation of Quantum Mechanics. *J. Stat. Phys.* **36**, (1984) 219.

[96] Isham, C.J. Quantum Logic and the Histories Approach to Quantum Theory. *J. Math. Phys.* **35**, (1994) 2157, [arXiv:gr-qc/9308006].
[97] Birrell, N.D.; Davies, P.C.W. Quantum Fields in Curved Space. Cambridge University Press: Cambridge, UK, (1982).

[98] Henson, J. Quantum Histories and Quantum Gravity. J.Phys.Conf.Ser 174, (2009) 012020, [arXiv:0901.4009].

[99] Hartle, J.B. Quantum Multiverses. arXiv (2018), [arXiv:1801.08631].

[100] Donnelly, W.; Giddings, S.B. How is quantum information localized in gravity? Phys. Rev. D 96, (2017) 086013, [arXiv:1706.03104].

[101] Donnelly, W.; Giddings, S.B. Gravitational splitting at first order: Quantum information localization in gravity. Phys. Rev. D 98, (2018) 086006, [arXiv:1805.11095].

[102] von Neumann, J. Mathematische Grundlagen der Quantun mechanik, Springer, Berlin (1932).

[103] Yngvason, J. The Role of Type III Factors in Quantum Field Theory. Rept.Math.Phys. 55, (2005) 135, [math-ph/0411058].

[104] Banks, T.; Fischler, W.; Shenker, S.H.; Susskind, L. M Theory As A Matrix Model: A Conjecture. Phys. Rev. D 55, (1997) 5112, [arXiv:hep-th/9610043].

[105] Ishibashi, N.; Kawai, H.; Kitazawa, Y.; Tsuchiya, A. A Large-N Reduced Model as Superstring. Nucl. Phys. B 498, (1997) 467, [arXiv:hep-th/9612115].

[106] Aharony, O.; Gubser, S.S.; Maldacena, J.; Ooguri, H.; Oz, Y. Large N Field Theories, String Theory and Gravity. Phys. Rep. 323, (2000) 183, [hep-th/9905111].

[107] Wilczek, F. Riemann-Einstein Structure from Volume and Gauge Symmetry. Phys. Rev. Lett. 80, (1998) 4851, [arXiv:hep-th/9801184].

[108] Torres-Gomez, A.; Krasnov, K. Gravity-Yang-Mills-Higgs unification by enlarging the gauge group. Phys. Rev. D 81, (2010) 085003, [arXiv:0911.3793].

[109] Barrett, J.W.; Kerr, S. Gauge gravity and discrete quantum models. arXiv (2013), [arXiv:1309.1660].

[110] ’t Hooft, G. Dimensional Reduction in Quantum Gravity. (1993), [arXiv:gr-qc/9310026].

[111] Ryu, S.; Takayanagi, T. Holographic Derivation of Entanglement Entropy from AdS/CFT. Phys. Rev. Lett. 96, (2006) 181602, [arXiv:hep-th/0603001].

[112] Maldacena, J. The gauge/gravity duality, in Black Holes in Higher Dimensions Ed. G. Horowitz, Cambridge University Press, (2012) [arXiv:1106.6073].

[113] Green, M.B.; Schwarz, J.H.; Witten, E. Superstring Theory I & II; Cambridge University Press: Cambridge, UK, (1987).

[114] Polchinski, J. TASI lecture on D-branes. [arXiv:hep-th/9611050].

[115] Adams, A.; Polchinski, J.; Silverstein, E. Don’t Panic! Closed String Tachyons in ALE Spacetimes. J. High Energy Phys. 0110, (2001) 029, [arXiv:hep-th/0108075].

[116] Karczmarek, J.L.; Strominger, A. Closed String Tachyon Condensation at c=1. J. High Energy Phys. 0405, (2004) 062, [arXiv:hep-th/0403169].

[117] Green, D. Nothing for Branes. J. High Energy Phys. 0704, (2007) 025, [arXiv:hep-th/0611003].

[118] Gubser, S.S.; Klebanov, I.R.; Polyakov, A.M. Gauge Theory Correlators from Non-Critical String Theory. Phys. Lett. B 428, (1998) 105, [arXiv:hep-th/9802109].
[119] Polchinski, J.  *String Theory I & II*; Cambridge University Press: Cambridge, UK, 2005.

[120] Maldacena, J.M.; Strominger, A. Semiclassical decay of near extremal fivebranes. *J. High Energy Phys.* **9712**, (1997) 008, [arXiv:hep-th/9710014].

[121] Bousso, R.; Mints, A.L. Holography and entropy bounds in the plane wave matrix model. *Phys. Rev. D* **73**, (2006) 126005, [arXiv:hep-th/0512201].

[122] Floratos, E.F.; Iliopoulos, J. A Note on the Classical Symmetries of the Closed Bosonic Membranes. *Phys. Lett. B* **201**, (1988) 237.

[123] Antoniadis, I.; Ditsas, P.; Floratos, E.F.; Iliopoulos, J. New Realizations of the Virasoro Algebra as Membrane Symmetries. *Nucl. Phys. B* **300**, (1988) 549.

[124] Nayeri, A.; Brandenberger, R.H.; Vafa, C. Producing a Scale-Invariant Spectrum of Perturbations in a Hagedorn Phase of String Cosmology. *Phys. Rev. Lett.* **97**, (2006) 021302, [arXiv:hep-th/0511140].

[125] Konechny, A. Schwarz, A. Introduction to M(atrix) theory and noncommutative geometry. *Phys. Rep.* **360**, (2002) 353, [arXiv:hep-th/0012145].

[126] Steinacker, H. Emergent Geometry and Gravity from Matrix Models: an Introduction. *Class. Quant. Grav.* **27**, (2010) 133001, [arXiv:1003.4134].

[127] ’t Hooft, G. A Planar Diagram Theory for Strong Interactions. *Nucl. Phys. B* **72**, (1974) 461.

[128] Dijkgraaf, R.; Verlinde, E.; Verlinde, H. Matrix String Theory. *Nucl. Phys. B* **500**, (1997) 43, [arXiv:hep-th/9703030].

[129] Kawahara, N.; Nishimura, J.; Takeuchi, S. High temperature expansion in supersymmetric matrix quantum mechanics. *J. High Energy Phys.* **0712**, (2007) 103, [arXiv:0710.2188].

[130] Schild, A. Classical null strings. *Phys. Rev. D* **16**, (1977) 1722.

[131] Connes, A.; Douglas, M.R.; Schwarz, A. Noncommutative Geometry and Matrix Theory: Compactification on Tori. *J. High Energy Phys.* **02**, (1998) 003, [arXiv:hep-th/9711162].

[132] Steinacker, H. Covariant Field Equations, Gauge Fields and Conservation Laws from Yang-Mills Matrix Models. *J. High Energy Phys.* **02**, (2009) 044, [arXiv:0812.3761].

[133] Kumar, J. A Review of Distributions on the String Landscape. *Int. J. Mod. Phys. A* **21**, (2006) 3441, [arXiv:hep-th/0601053].

[134] Brahma, S.; Brandenberger, R.; Laliberte, S. Emergent Cosmology from Matrix Theory. [arXiv:2107.11512].

[135] Steinacker, H. Gravity as a Quantum Effect on Quantum Space-Time. [arXiv:2110.09336].

[136] The LIGO-Virgo Collaboration Tests of General Relativity with Binary Black Holes from the second LIGO-Virgo Gravitational-Wave Transient Catalog. *Phys. Rev. D* **103**, (2021) 122002, [arXiv:2010.14529].

[137] Anderson, P.W. Absence of Diffusion in Certain Random Lattices. *Phys. Rev.* **109**, (1958) 1492.

[138] Koma, T.; Tasak, H. Symmetry Breaking and Finite-Size Effects in Quantum Many-Body Systems. *J. Stat. Phys.* **76**, (1994) 745.

[139] Ziaeepour, H. QCD Color Glass Condensate Model in Warped Brane Models. *Grav. Cosmol. Suppl.* **11**, (2005) 189, [hep-ph/0412314].
[140] Kühnel, W. Differential Geometry. Third Edition AMS, Rhode Island, (2010).

[141] Gallier, J Differential Geometry and Lie Groups, Vol. I. Springer, (2020)