Denoising Method for Underwater Acoustic Signals Based on Sparse Decomposition

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Abstract. In order to improve the problem of relatively low signal-to-noise in the extraction of underwater acoustic signals under the background of strong interference, a sparse decomposition-based underwater acoustic signal denoising method is proposed. The main work is as follows: First, the signal is decomposed into a complete dictionary that can reflect the characteristics of the signal environment through the singular value decomposition method. Then, a cyclic shift is used to construct a signal matrix and an initial dictionary. A newly generated super-complete dictionary is obtained through training and updating. The atoms in the dictionary matrix are correlated and orthogonalized by an adaptive orthogonal matching pursuit method. Finally, the linear combination of the atoms that can best reflect the characteristic information of the underwater acoustic signal is used to reconstruct the underwater acoustic signal to achieve the purpose of denoising. The noise filtering underwater noise signal is simulated and compared with the traditional filtering method. The simulation results show that this method improves the signal-to-noise ratio of the signal after sparse decomposition and reconstruction of the original underwater acoustic signal, and has a certain denoising ability under strong noise and various types of reverberation interference.

1. Introduction

Underwater acoustic signal processing has always been an important research area for underwater moving target detection, target positioning, target recognition, and underwater acoustic communication. In the fields of signal noise reduction and signal fault detection, many scholars have used k iterative singular value decomposition (K-SVD) algorithm \cite{1} to sparsely decompose the signal, and transform the signal through a sparse representation model to a complete dictionary and sparse coefficient matrix phase. Way of multiplication \cite{2}. The method in this paper uses adaptive orthogonal matching tracking (OMP) to decompose and reconstruct the sparsely represented signal, and improves the signal SNR.

2. Dictionary learning and orthogonal matching pursuit algorithm

The dictionary learning method is generally the K-SVD method \cite{3}, proposed by Michal Aharon. The singular value decomposition is performed on the signal error components, and the update condition selection dictionary atom with the smallest error component is selected through multiple iterations to select the optimal solution of the objective function \cite{4}.

Literature \cite{5} studied the sparse signal OMP algorithm measured linearly with a small amount of noise, and the conclusion showed that the method can effectively recover the signal characteristics within the maximum range when the non-zero amount of the signal is at the minimum amplitude.
Reference [6] promoted the OMP algorithm for sparse signal reconstruction efficiency, and selected the correct index during each iteration. The number of iterations was controlled to a certain range, and the signal restoration performance was improved. Reference [7] used the L1 minimization method and the matching pursuit method to recover the signal algorithm, which can guarantee the speed of the OMP algorithm and the accuracy of the reconstructed signal.

3. Denoising processing algorithm in this paper

3.1. Underwater acoustic signal model

The mathematical model of the chirped underwater acoustic signal can be established as follows[8]:

\[ f_{LFM}(t) = A \exp \left( 2\pi j \left( \frac{B}{T} \times \frac{1}{2} t^2 \right) \right) \quad t \in \left( -\frac{T}{2}, \frac{T}{2} \right) \]  

(1)

In the formula, \( A \) is the amplitude of the underwater acoustic signal, and \( t \) is the time variable of the sampling point of the underwater acoustic signal, in seconds s. The sampling interval lasts from \(-T/2\) to \(T/2\) for a total of \( N = 1024/2 \) sampling points. \( T \) is the time width of the signal pulse, and \( B \) is the bandwidth of the underwater acoustic signal.

3.2. Signal sparse decomposition and parameter estimation principles

The principle of signal sparse decomposition is to combine the best atoms that represent the original characteristics of the signal to solve the following objective function optimization problems:

\[
\min_{D,X} \{ \| Y - DX \|_F^2 \} \quad \text{s.t.} \quad \forall k, \|x_k\|_0 \leq T_0
\]  

(2)

In formula (2), \( Y \) represents a sample set of signals, \( D \) is an over-complete dictionary of signals, and \( X \) is the optimal coefficient matrix, corresponding to \( X \) when formula (2) takes the minimum value. \( T_0 \) is the maximum number of non-zero elements in the signal sparse representation.

The first step is to assume: \( D \in R^{m \times k}, x \in R^n, y \in R^n, Y = \{ y_i \}_{i=1}^N, X = \{ X_i \}_{i=1}^N, D \) represents an over-complete dictionary, \( y \) is the corresponding training chirp underwater acoustic signal, \( x \) is the coefficient vector of the training chirp underwater acoustic signal obtained by the OMP algorithm, \( Y \) is the set of \( N \) training chirp underwater acoustic signals, \( X \) The corresponding is the set of \( Y \) solution vectors.

In the second step, the sample set decomposition can be described by formula (3):

\[
\| Y - DX \|_F^2 = \left\| Y - \sum_{j=1}^{K} d_j^T x_j \right\|_F^2 = \left\| E_k - d_k x_k \right\|_F^2
\]  

(3)

Formula (3) Sparse matrix \( X \) and dictionary \( D \) are decomposed into the sum of \( k \) matrices of rank 1, \( d_k \) is the vector of the K-th column of the updated dictionary \( D \), and \( x_k \) is the K-th row of the sparse matrix \( x \) multiplied by the K-th column \( d_k \) Vector, \( E_k \) is the decomposition error matrix of the sample set. The following parameter definitions are introduced to achieve signal singular value decomposition (SVD). \( w_k = \{ i \mid 1 \leq i \leq K, x_k^T (i) \neq 0 \} \), \( x_k^T = x_k^T \Omega_k, Y_R^k = Y \Omega_k, E_R^k = E_k \Omega_k, w_k \) is a set of atomic indices represented by signal sparseness, \( \Omega_k \) is a matrix with a size of \( N \times l(w_k) \), \( l(w_k) \) is the length of the set \( w_k \), the element at \( \Omega_k \) is 1 at position \( (w_k(i),i) \), and the elements at other positions of matrix are 0, \( x_R^k \), \( Y_R^k \), and \( E_R^k \) As element \( x_k \), signal set \( Y \), and decomposition error \( E_k \), the corresponding zero element is removed. Equation (3) is written as:

\[
\left\| E_k \Omega_k - d_k x_k \Omega_k \right\|_F^2 = \left\| E_k^R - d_k x_R^k \right\|_F^2
\]  

(4)
Then perform SVD decomposition on $E_k$ to get: $E_k = U\Delta V^T$. The first column of $U$ is exchanged for the previous $d_k$ and the corresponding atom $D$ in $d_k (k = 1, 2, \ldots, k)$ is updated by column to obtain a new dictionary.

3.3. Propose constructing signal matrix and initial dictionary
Because the K-SVD algorithm requires an over-complete dictionary, the condition is that the number of columns of the signal matrix must be greater than the number of rows of the signal matrix. The signal is usually converted to the Hankel matrix [9], in order to improve the calculation efficiency, and the signal matrix will be within the range of the nodes in a certain time series. Therefore, a method of cyclically moving a matrix and representing signal characteristics is proposed. This method cyclically moves the entire vector to generate many new signal vectors, which are similar in structure to the original signal vector.

Assuming that the total number of cyclic movements is $m$ times, the resulting matrix forms an underwater acoustic signal matrix, which is defined as:

$$X = [X_0, X_1, \ldots, X_m] \in \mathbb{R}^{T \times (m+1)}$$

(5)

$T$ is the number of rows in the matrix, $(m+1)$ is the number of columns in the matrix. When the value of $m$ is smaller, the number of elements contained in the signal matrix is smaller. When an appropriate value of $m$ is selected, the calculation amount of the algorithm is effectively reduced.

The atoms obtained by the time-domain averaging method cannot form the entire over-complete dictionary, but only randomly select some atoms from the signal matrix to generate an initial state dictionary containing the first $m+1$ atoms.

3.4. Adaptive OMP algorithm for processing underwater acoustic signals
The adaptive OMP algorithm is divided into a function traversal phase and a temporary solution update phase[10]. During the traversal phase of the $k$-th iteration, the condition for searching for the best atom is to find the atom with the largest inner product.

$$\left| r^{k-1}, d_{jk} \right| = \sup_{d \in d_k} \left| r^{k-1}, d_i \right|$$

(6)

In formula (6), $r^{k-1}$ is the residual component after the $k$-th iteration, and $d_i$ represents the $i$-th atom in the dictionary. $d_{jk}$ represents the $k$-th atom extracted by the function, and the atom index in the $d$ dictionary is stored in the objective function sample set $Y^k$. In the update phase of the temporary solution, the objective function $\|Y - D_X X\|^2_F$ is minimized under the supporting sample set $Y^k$, and $D_{Y^k}$ is defined as a matrix $D_k$ composed of columns belonging to the supporting sample set.

This translates the problem into a minimization of the objective function $\|Y - D_{Y^k} X_{Y^k}\|^2_F$, where $X_{Y^k}$ is a nonzero element. The objective function’s second derivative is set to zero, and the objective function minimization solution is expressed as:

$$D_{Y^k}^t \left( D_{Y^k} X_{Y^k} - Y \right) = -D_{Y^k}^t r^k = 0$$

(7)

When $r^k = Y - D_{Y^k} X^k = Y - D_{Y^k} X_{Y^k}$, this operation can make the residual $r^k$ orthogonal to the previously extracted atoms, which can reduce the reuse of atoms in subsequent iterations. $d_{sk}$ is represented as the $i$-th atom in the dictionary $D_{sk}$, and $X_{Y^k}$ is represented as the coefficient corresponding to the $i$-th atom $d_{sk}$. The final signal $Y$ can be written as: $Y = \sum_{i=1}^{k} X_{Y^i} d_{Y^i} + r^k$. 

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It is easier to observe that \( r^k \) is orthogonal to \( D_{y^i} \), which is \( r^k d_{y^i}^T = 0, k = 1, 2, \cdots, k. \) \( D_{y^i} \) corresponds to a super-complete dictionary, which is also orthogonal when the signal atomic frequencies in the dictionary are different, that is \( d_{y^i} d_{y^j}^T = 0, i \neq j, i, j = 1, 2, \cdots, k. \) \( d_{y^i} \) is a unit vector, which is \( \|d_{y^i}\|^2 = 1. \) From the perspective of energy distribution, the signal can be described as:

\[
\|Y\|_2^2 = \left\| \sum_{i=1}^{k} X_{y^i} d_{y^i} + r^k \right\|_2^2 = \sum_{i=1}^{k} X_{y^i}^2 \|d_{y^i}\|_2^2 + \|r^k\|_2^2 = \sum_{i=1}^{k} X_{y^i}^2 + \|r^k\|_2^2
\]

(8)

The premise of the termination of the traditional OMP algorithm is that the number of iterations reaches a certain target value or the residual is less than a certain value. Equation (8) shows that the largest coefficient in the atomic representation plays the largest role in the harmonic representation and component modulation process. Therefore, the termination condition of the adaptive OMP algorithm is proposed. It is assumed that after the \( k \)-th iteration of the coefficient matrix, the maximum element is \( x_i \) and the minimum element is \( x_k \). After testing, when it is \( x_i > \alpha x_k (n \geq 3), \) the iteration process is terminated. At this time, a valid signal can be extracted from the noisy underwater acoustic signal.

3.5. LFM underwater acoustic signal feature extraction process

Input: The length of the measured signal is \( N = 1024/2 \), a well-constructed super-complete dictionary \( D \), the number of times the signal matrix is shifted \( m \), the initial dictionary randomly selects the number of atoms \( n \).

initialization: \( k = 0 \)

Step 1 Build a super-complete Gabor dictionary.

Step 2 Rotate the vector to construct a signal matrix. \( x_0 = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix} \)

Step 3 The signal matrix after \( m \) cyclic shifts is expressed as: \( X = [X_0 \ X_1 \ \cdots \ X_m] \in \mathbb{R}^{r \times (m+1)} \)

Step 4 Column average is performed on \( m+1 \) matrices to generate an initial state dictionary of \( m+1 \) atoms.

Step 5 The adaptive OMP algorithm searches for the best matching atom.

Step 6 When the residual \( r^k \) is less than a certain threshold, the loop ends when the coefficient matrix passes through \( k \)-th iterations and the maximum element \( x_i \) is greater than a certain multiple of the minimum element \( x_k \). If the conditions are not met, repeat steps 2 ~ 6.

Step 7 An optimized dictionary \( D_o \) and coefficient matrix \( A_x \) are obtained. \( X = D_o \times A_x \)

Step 8 Shift restore the denoised signal. \( x = \begin{bmatrix} x_1' & x_2' & \cdots & x_m' \end{bmatrix} \)

4. Simulation experiment and result analysis

The index parameters for evaluating the filtering effect of underwater acoustic signals are signal-to-noise ratio SNR, mean square error MSE, and waveform similarity NCC[11].

4.1. Analysis of Noise Reduction Performance of Chirp Signal

The ideal linear FM underwater acoustic signal [12] bandwidth is 80KHz, the pulse time width is \( 10 \times 10-3S \), the underwater acoustic signal sampling frequency factor is 160 KHz, the carrier frequency is...
80 KHz, the signal FM slope is 8 KHz/S, and the number of sampling points of the signal pulse It is N = 1024/2.

After adding a Gaussian white noise with a variance of 0.35 units to the original underwater acoustic signal, the waveform of the underwater acoustic signal has been basically distorted as shown in Fig. 1 (a). The waveform of the underwater signal filtered by the method in this method is shown in Fig. 2 (a). It can be shown from the waveform diagram that the method can accurately restore the original signal waveform under the signal-to-noise ratio of -3.75dB.

### 4.2. Evaluation index of denoising effect of chirp signal
In order to reflect the comparison result between the denoised underwater acoustic signal and the ideal LFM underwater acoustic signal, the SNR of the proposed method is compared with the empirical mode decomposition denoising and wavelet denoising methods. Figure 4 shows the changes of MSE and NCC using the method in this paper.

The higher the signal-to-noise ratio of the signal, the smaller the mean square error as much as possible, and the closer the waveform similarity coefficient is to 1, indicating that the better the filtering effect. Fig.4 is a graph of the change law of the evaluation indexes MSE and NCC when a Gaussian white noise ranging from 0 to 1 is gradually added to the chirp signal.

### 5. Concluding remarks
This paper proposes a method for denoising underwater acoustic signals based on sparse decomposition[13], which transforms the problem of denoising the chirp signal into a corresponding objective function optimization problem. An adaptive orthogonal matching pursuit (OMP) algorithm is
proposed. When the maximum and minimum elements of the coefficient matrix meet a certain threshold range after the k-th iteration, the iteration process is terminated to obtain the coefficient matrix corresponding to the optimized dictionary\cite{14}. Then the noisy chirped underwater acoustic signal is restored\cite{15}.

The shortcoming of the algorithm in this paper is that when the signal-to-noise ratio is lower than a certain value, the effect of signal restoration becomes worse. This is because the matching conditions may not be optimal. Therefore, the optimal selection of dictionary and the refinement of constraint blocks are the focus of the next study.

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