Can a strong magnetic background modify the nature of the chiral transition in QCD?

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Abstract

The presence of a strong magnetic background can modify the nature and the dynamics of the chiral phase transition at finite temperature: for high enough magnetic fields, comparable to the ones expected to be created in noncentral high-energy heavy ion collisions at RHIC and the LHC, the original crossover is turned into a first-order transition. We illustrate this effect within the linear sigma model with quarks to one loop in the \( \overline{\text{MS}} \) scheme for \( N_f = 2 \).

Key words: Phase diagram of QCD, Chiral transition, Magnetic field
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Strong magnetic fields are known to produce remarkable physical effects in macroscopic compact astrophysical objects, such as illustrated by magnetars [1]. More recently, it has been proposed that very high magnetic fields could be created in noncentral high-energy heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC), at Brookhaven, and the Large Hadron Collider (LHC), at CERN, affecting observables that could provide a signature of the presence of CP-odd domains in the presumably formed quark-gluon plasma (QGP) phase [2] via a mechanism of separation of charge. Estimates point to magnetic fields \( B \sim 10^{19} \) Gauss (\( eB \sim 6m_{\pi}^{2} \)), a very intense magnetic background from the point of view of quantum chromodynamics (QCD).

Besides its role in the separation of charge in CP-odd domains, one can ask a simpler and no less interesting question about the possible effects of high magnetic fields on hot quark matter [3]: can a strong magnetic background modify the nature of the chiral transition in QCD? We show below that the answer is yes, and that, for high enough magnetic fields, comparable to the ones expected to be created in noncentral high-energy heavy ion collisions at RHIC and the LHC, the original crossover, as predicted by lattice simulations, is turned into a first-order transition [3].
Modifications in the vacuum of quantum electrodynamics and QCD have also been investigated within different frameworks, mainly using effective models, especially the NJL model, and chiral perturbation theory, but also resorting to the quark model and certain limits of QCD. Most treatments have been concerned with vacuum modifications by the magnetic field, though medium effects were considered in a few cases. More recently, effects on the dynamical quark mass and on the quark-hadron transition were also considered.

To investigate the effects of a strong magnetic background on the nature and dynamics of the chiral phase transition at finite temperature, and vanishing chemical potential, we adopt as an effective theory the linear sigma model coupled to quarks (LSM) with two flavors, \( N_f = 2 \), defined by the lagrangian

\[
\mathcal{L} = \overline{\psi}_f \left[ i \gamma^\mu \partial_\mu - g (\sigma + i \gamma_5 \vec{\tau} \cdot \vec{\pi}) \right] \psi_f + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi}) - V(\sigma, \vec{\pi}) ,
\]

where \( V(\sigma, \vec{\pi}) = \frac{1}{4} (\sigma^2 + \vec{\pi}^2 - v^2)^2 - \hbar \sigma \) is the self-interaction potential for the mesons, exhibiting both spontaneous and explicit breaking of chiral symmetry. The \( N_f = 2 \) massive fermion fields \( \psi_f \) represent the up and down constituent-quark fields \( \psi = (u, d) \). The scalar field \( \sigma \) plays the role of an approximate order parameter for the chiral transition, being an exact order parameter for massless quarks and pions. The latter are represented by the pseudoscalar field \( \vec{\pi} = (\pi^0, \pi^+, \pi^-) \), and it is common to group together these meson fields into an \( O(4) \) chiral field \( \phi = (\sigma, \vec{\pi}) \). In what follows, we implement a simple mean-field treatment with the customary simplifying assumptions (see, e.g., Ref. [20]). Quarks constitute a thermalized fluid that provides a background in which the long wavelength modes of the chiral condensate evolve. Hence, at \( T = 0 \), the model reproduces results from the usual LSM without quarks or from chiral perturbation theory for the broken phase vacuum. In this phase, quark degrees of freedom are absent (excited only for \( T > 0 \)). The \( \sigma \) field is heavy, \( M_\sigma \sim 600 \text{ MeV} \), and treated classically. On the other hand, pions are light, and fluctuations in \( \pi^+ \) and \( \pi^- \) couple to the magnetic field, \( B \), as will be discussed in the next section, whereas fluctuations in \( \pi^0 \) give a \( B \)-independent contribution that we ignore, for simplicity. For \( T > 0 \), quarks are relevant (fast) degrees of freedom and chiral symmetry is approximately restored in the plasma for high enough \( T \). In this case, we incorporate quark thermal fluctuations in the effective potential for \( \sigma \), i.e. we integrate over quarks to one loop. Pions become rapidly heavy only after \( T_c \) and their fluctuations can, in principle, matter since they couple to \( B \). The parameters of the lagrangian are chosen such that the effective model reproduces correctly the phenomenology of QCD at low energies and in the vacuum, in the absence of a magnetic field. Standard integration over the fermionic degrees of freedom to one loop, using a classical approximation for the chiral field, gives the effective potential in the \( \sigma \) direction \( V_{\text{eff,}\sigma} = V(\phi) + V_q(\phi) \), where \( V_q \) represents the thermal contribution from the quarks that acquire an effective mass \( M(\sigma) = g|\sigma| \). The net effect of the term \( V_q \) is correcting the potential for the chiral field, approximately restoring chiral symmetry for a critical temperature \( T_c \sim 150 \text{ MeV} \).

Assuming that the system is now in the presence of a strong magnetic field background that is constant and homogeneous, one can compute the modified effective potential following the procedure outlined in Ref. [3]. In what follows, we simply provide some of the main results. For definiteness, let us take the direction of the magnetic field as the \( z \)-direction, \( \vec{B} = B \hat{z} \). The effective potential can be generalized to this case by a
simple redefinition of the dispersion relations of the fields in the presence of $\vec{B}$, using the minimal coupling shift in the gradient and the field equations of motion. For this purpose, it is convenient to choose the gauge such that $A^\mu = (A^0, \vec{A}) = (0, -By, 0, 0)$. Decomposing the fields into their Fourier modes, one arrives at eigenvalue equations which have the same form as the Schrödinger equation for a harmonic oscillator potential, whose eigenmodes correspond to the well-known Landau levels. The latter provide the new dispersion relations

$$p_{on}^2 = p^2_z + m^2 + (2n + 1)|q|B, \quad p_{on}^2 = p^2_z + m^2 + (2n + 1 - \sigma)|q|B,$$

for scalars and fermions, respectively, $n$ being an integer, $q$ the electric charge, and $\sigma$ the sign of the spin. Integrals over four momenta and thermal sum-integrals are modified accordingly, yielding sums over the Landau levels.

In our effective model, the vacuum piece of the potential will be modified by the magnetic field through the coupling of the field to charged pions. To one loop, and in the limit of high $B$, $eB >> m^2$, one obtains (ignoring contributions independent of the condensates)

$$V^{\nu}_V + V^{\pi}_V = -\frac{2m^2 eB}{32\pi^2} \log 2. \quad (3)$$

Thermal corrections are provided by pions and quarks. However, the pion thermal contribution as well as part of the quark thermal contribution are exponentially suppressed for high magnetic fields, as has been shown in Ref. [3]. The only part of the quark thermal piece that contributes is

$$V^T_q = -N_c \frac{eBT^2}{2\pi^2} \left[ \int_{-\infty}^{+\infty} dx \ln \left( 1 + e^{-\sqrt{x^2 + M_q^2/T^2}} \right) \right], \quad (4)$$

where $N_c = 3$ is the number of colors. Therefore, the effective potential is corrected by the contributions in (3) and (4). The presence of the magnetic field enhances the value of the chiral condensate and the depth of the broken phase minimum of the modified effective potential, a result that is in line with those found within different approaches (see, for instance, Refs. [10,12,14]).

At RHIC, estimates by Kharzeev, McLerran and Warringa [2] give $eB \sim 5.3m^2_\pi$. (For the LHC, there is a small increase). So, we adopt $eB \sim 6m^2_\pi$ as a reasonable estimate. Fig. 1 displays the effective potential for $eB \sim 10m^2_\pi$ at different values of the temperature to illustrate the phenomenon of chiral symmetry restoration via a first-order transition. For lower values of the field, the barrier is smaller. In Fig. 2, we show a zoom of the effective potential for $eB \sim 6m^2_\pi$ at a temperature slightly below the critical one. This figure highlights the presence of a first-order barrier in the effective potential. For a magnetic field of the magnitude that could possibly be found in non-central high-energy heavy ion collisions, one moves from a crossover scenario to that of a weak first-order chiral transition, with a critical temperature $\sim 30\%$ higher [3].

Although Lattice QCD seems to indicate a crossover instead of a first-order chiral transition at vanishing chemical potential, a strong magnetic background might modify this picture dramatically. In particular, the results presented here might be relevant for
the physics of the primordial QCD transition, where the description of phase conversion is generally via bubble nucleation, assuming a first-order transition [21]. For heavy ion collisions at RHIC and LHC, although the barrier in the effective potential seems to be quite small, it can probably trap most of the system down to the spinodal explosion, altering the dynamics of phase conversion and its relevant time scales [22]. This opens a new direction in the study of the phase diagram for strong interactions, with a rich phenomenology awaiting to be investigated [3]. This work was partially supported by CAPES, CNPq, FAPERJ and FUJB/UFRJ.

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