Design of Smooth Switching LPV Controller Based on Coprime Factorization and $H_\infty$ Performance Realization

WEILIN WU$^1$, WEI XIE$^{1,2}$, TOSHIO EISAKA$^3$, AND LIEJUN LI$^4$

$^1$College of Electronic Information, Guangxi University for Nationalities, Nanning 530006, China
$^2$Guangdong Provincial Molecular Material Polymer Laboratory, Guangzhou 510641, China
$^3$Division of Information and Communication Engineering, Kitami Institute of Technology, Kitami 090-8507, Japan
$^4$School of Mechanical and Automotive Engineering, South China University of Technology, Guangzhou 510641, China

Corresponding author: Wei Xie (weixie@scut.edu.cn)

This work was supported in part by the Science and Technology Program of Guangzhou under Grant 201904010441, in part by the Open Fund of the Ministry of Education Key Laboratory of High-Efficiency Near-Net-Shaping Technology and Equipment for Metallic Materials (Class B), in part by the National Engineering Research Center for Metallic Materials, South China University of Technology, under Grant 20200012, and in part by the National Natural Science Foundation of China under Grant 62006052.

ABSTRACT This paper discusses the problem of designing smooth switching control based on the coprime factorization method. Aiming at the instantaneous chattering phenomenon generated by the linear parameter varying (LPV) controller during the switching moment, the moving region of the gain-scheduling variables is divided into a specified number of local subregions with overlapped region. Riccati inequality is used to solve the central controller for $H_\infty$ performance. The Youla free parameters are designed using the coprime factorization for each parameter sub-region by considering the system’s global and local performance requirements. Youla free parameter switching is used to improve the smoothness of switching and suppress transient response disturbance. Finally, the effectiveness of the method is verified by a simulation example.

INDEX TERMS Smooth switch control, linear parameter varying, $H_\infty$ performance realization, coprime factorization.

I. INTRODUCTION

Since Shamma [1] proposed the linear parameter varying (LPV) system, it has solved the shortcomings of traditional variable gain control technology. The LPV system can better describe the nonlinear and time-varying characteristics of complex physical systems. In recent decades, it has quickly become an interesting research topic in control theory [2]–[4]. Typically, a single controller is designed to control an LPV plant. When the time-varying parameters change in an extensive range, the single LPV controller usually causes conservative control performance. Lu and Wu [5] proposed a design method of switching the LPV controller to solve this problem. The method of switching LPV controllers was quickly applied to some work, such as ultra-high-speed aircraft [6], [7], Aero-Engine [8]–[10], satellite attitude control [11]. Postma and Nagamune [12] used a switching LPV controller to regulate the air-fuel ratio. By dividing the parameter range into smaller sub-regions, a separate LPV controller was designed for each sub-region and switched according to the operating point in [12]. The LPV controller was found by solving the convex optimization problem of linear matrix inequality (LMI). Besides, Zhao and Nagamune [13] proposed an output feedback switching LPV controller design method. The switching controller can ensure the stability and gain performance of the closed-loop system when the measurement scheduling parameters are inaccurate.

To overcome the transient response of the switching controller at the switching moment, Chen et al. [14] proposed a smooth switching LPV controller, which attracted the interest of researchers, and the research results have been applied to various engineering fields [7], [15]. A smooth switching gain scheduling controller was designed to solve the problem of large-scale offshore wind turbines operating within the full wind speed range in [16]. Also, Chen [17] considered the LPV control of the delayed switching state feedback. It linearly interpolated the controller variables on the switching
surface to achieve smooth switching during the switching process and switching on the overlap region. However, this method cannot quantitatively evaluate the switching smoothness and obtain the relative stability in overlapped subregions. Besides following the idea of linear interpolation of the control matrix on the switching surface, Hanifzadegan and Nagamune [18] introduced the smoothness index and imposed constraints on the controller’s derivative matrix to make up for the defects in [17].

Coprime factorization approach for control and design of systems can be traced back to the literature [19]. Both Youla parametrization and dual Youla parametrization are based on the doubly coprime factorization [20]. Quadrat [21], [22] proposed Q parameters through the double coprime factorization of the transfer matrix. The controller designed by Youla parametrization can improve the system’s robust stability and reject the adverse effects caused by existing disturbances. In [23], a Youla parameterized adaptive controller was proposed for the mechanical systems to deal with the unknown vibration caused both by the deterministic disturbance and the random disturbance. Bianchi and Sánchez-Peña [24] proposed a switching LPV controller design method based on the idea of Youla parametrization. The controller design is decomposed into two steps, one focused on ensuring global stability and the other on achieving the desired performance in each subset. But it does not consider the smooth switching strategy. It is very complicated to calculate the Youla parameters.

This work presents a new smooth switching technology to design the switching of Youla parameters. Compared with the literature [25], the novelty of this method is that we introduce a parameter overlap division method to improve the smoothness of switching. Also, we use the coprime factorization technique to obtain the Youla parameters of each sub-area from the controller. Then, the Youla parameter of the overlapping area is obtained by linear interpolation of the Youla parameter of the two adjacent sub-regions. The contribution of this work is to use the Youla parameter switch Q instead of the switch controller to achieve smooth switching. The flexibility and freedom of the switching controller are increased, and the transient response disturbance during the switching is suppressed.

This paper is organized as follows. Section II presents the definition and the problem statement of controller design. Section III details the process of the smooth switching LPV controller which designed by Youla parametrization. Conclusions and future research directions are discussed in Section IV.

II. DEFINITION AND PROBLEM STATEMENT

We consider the following LPV plant

\[
\begin{align*}
\dot{x}(t) &= A(\theta)x(t) + B_1(\theta)w(t) + B_2(\theta)u(t), \\
z(t) &= C_1(\theta)x(t) + D_{11}(\theta)w(t) + D_{12}(\theta)u(t), \\
y(t) &= C_2(\theta)x(t) + D_{21}(\theta)w(t) + D_{22}(\theta)u(t),
\end{align*}
\]

where \( \theta := [\theta_1, \ldots, \theta_J]^T \) is the vector of scheduling parameters, \( x \in \mathbb{R}^n \) is the state vector, \( w \in \mathbb{R}^{n_w} \) is the vector of exogenous inputs, \( u \in \mathbb{R}^{n_u} \) is the control input vector, \( z \in \mathbb{R}^{n_z} \) is the vector of output signals related to the performance of the control system, \( y \in \mathbb{R}^{n_y} \) is the measured output vector. The system matrices are the continuous and bounded function of the parameter \( \theta \) and can be measured in real time.

Assumption 1: \( (A(\theta), B_2(\theta), C_2(\theta)) \) triple is parameter dependent, stabilizable and detectable for all \( \theta \).

Assumption 2: The matrix functions \( [B_1^i(\theta) \ D_{12}^i(\theta)] \) and \( [C_2^i(\theta) \ D_{21}^i(\theta)] \) have full row ranks for all \( \theta \).

Assumption 3: \( D_{22}(\theta) = 0 \).

Suppose that \( \dot{\theta} \) and its rate of change \( \dot{\theta} \) are bounded

\[
\Theta = \{ \theta \in \mathbb{R} : \underline{\theta} \leq \theta \leq \bar{\theta} \},
\]

\[
V = \{ \dot{\theta} \in \mathbb{R} : \nu \leq \dot{\theta} \leq \bar{\nu} \},
\]

and for the switching LPV controller design, we can divide the interval \( \Theta \) into \( N \) sub-region, including overlapped regions and non-overlapped regions, as shown in Figure 1, where \( \Theta_i \) is defined by the time-varying parameters of subregions.

\[
\Theta_i = \{ \theta \in \mathbb{R} : \underline{\theta}_i \leq \theta \leq \bar{\theta}_i \}, \quad i \in N.
\]

FIGURE 1. The interval of time-varying parameters of local overlap characteristics.

The overlapped regions are composed of two adjacent sub-regions, which can be expressed as

\[
\Theta_{i,i+1} = \{ \theta \in \mathbb{R} : \underline{\theta}_{i+1} \leq \theta \leq \bar{\theta}_i \}, \quad i \in N.
\]

The non-overlapped region \( \Theta_{i,i} \) is the ith subregion except for the overlapped subregion, which can be expressed by the formula (4)

\[
\Theta_{i,i} = \begin{cases} 
\{ \theta \in \mathbb{R} : \underline{\theta}_i \leq \theta \leq \bar{\theta}_i \}, & i = 1, \\
\{ \theta \in \mathbb{R} : \underline{\theta}_{i-1} \leq \theta \leq \bar{\theta}_{i+1} \}, & i = 2, 3, \ldots, N - 1, \\
\{ \theta \in \mathbb{R} : \underline{\theta}_{i-1} \leq \theta \leq \bar{\theta}_i \}, & i = N. 
\end{cases}
\]

In this paper, we define \( \sigma(t) : [0, \infty) \rightarrow N = \{1, 2, \ldots, J\} \) as the switching signal of the system, which is determined by the time-varying parameters \( \theta \). Therefore, switching between controllers only occurs on the boundary of the time-varying parameter subregion.

Then, some important lemmas are provided as follows:

Lemma 1: Given an LPV plant \( P_{22}(\theta) \) with state space realization (5)

\[
P_{22}(\theta) = \begin{bmatrix} A(\theta) & B_2(\theta) \\ C_2(\theta) & D_{22}(\theta) \end{bmatrix},
\]

where
let a coprime factorization of the system $P_{22}(\theta)$ from (5) and a stabilizing controller $K(\theta)$ be given by

$$P_{22} = N(\theta)M^{-1}(\theta) = \tilde{M}^{-1}(\theta)\tilde{N}(\theta),$$

$$K(\theta) = U_0(\theta)V_0(\theta)^{-1} = \tilde{V}_0(\theta)^{-1}\tilde{U}_0(\theta),$$

where the matrices $N(\theta), M(\theta), \tilde{N}(\theta), \tilde{M}(\theta), U(\theta), V(\theta), \tilde{U}(\theta), \tilde{V}(\theta)$ satisfy the double Bezout equation (8).

$$\begin{bmatrix}
M(\theta) & U_0(\theta) \\
N(\theta) & V_0(\theta)
\end{bmatrix}
\begin{bmatrix}
\tilde{V}_0(\theta) & -\tilde{U}_0(\theta) \\
-\tilde{N}(\theta) & \tilde{M}(\theta)
\end{bmatrix}
= \begin{bmatrix}
\tilde{V}_0(\theta) & -\tilde{U}_0(\theta) \\
-\tilde{N}(\theta) & \tilde{M}(\theta)
\end{bmatrix}
\begin{bmatrix}
M(\theta) & U_0(\theta) \\
N(\theta) & V_0(\theta)
\end{bmatrix} = \begin{bmatrix} I & 0 \\
0 & I \end{bmatrix}. \tag{8}
$$

Then, the set of all regular controllers that are stable in the system can be parameterized as

$$K(Q(\theta)) = F_1(J_k(\theta), Q(\theta)).$$

**Proof:** Assume that the controller $K(\theta)$ is an observer-based feedback controller given by

$$K(\theta) = \begin{bmatrix} \tilde{A}(\theta) - L(\theta) & 0 \\ F(\theta) & I \end{bmatrix}, \tag{9}
$$

where

$$\tilde{A}(\theta) = A(\theta) + B_2(\theta)F(\theta) + L(\theta)C_2(\theta) + L(\theta)D_{22}(\theta)F(\theta).$$

One possible way to construct the eight stable coprime matrices in (6) is then

$$\begin{bmatrix} M(\theta) & U_0(\theta) \\
N(\theta) & V_0(\theta)
\end{bmatrix} = \begin{bmatrix} A_F(\theta) & B_2(\theta) - L(\theta) \\
F(\theta) & I
\end{bmatrix},
$$

$$A_F(\theta) = A(\theta) + B_2(\theta)F(\theta),
$$

$$C_F(\theta) = C_2(\theta) + D_{22}(\theta)F(\theta),
$$

$$\begin{bmatrix} \tilde{V}_0(\theta) & -\tilde{U}_0(\theta) \\
-\tilde{N}(\theta) & \tilde{M}(\theta)
\end{bmatrix} = \begin{bmatrix} A_L(\theta) - B_1(\theta) & L(\theta) \\
F(\theta) & I
\end{bmatrix},
$$

$$A_L(\theta) = A(\theta) + L(\theta)C_2(\theta),
$$

$$B_L(\theta) = B_2(\theta) + L(\theta)D_{22}(\theta).$$

Based on the above coprime factorization of the system $P_{22}(\theta)$ and the controller $K(\theta)$. The parameterized formulas of all stabilizing controllers are obtained so that the system’s stability can be guaranteed by a stable parameter $Q(\theta)$ [26]

$$K(Q(\theta)) = U(Q(\theta))V(Q(\theta))^{-1}, \tag{10}
$$

where

$$U(Q(\theta)) = U_0(\theta) + M(\theta)Q(\theta),
$$

$$V(Q(\theta)) = V_0(\theta) + N(\theta)Q(\theta),
$$

$$Q(\theta) \in RH_\infty,$$

or by using the left decomposition form

$$K(Q(\theta)) = \tilde{V}(Q(\theta))^{-1}\tilde{U}(Q(\theta)), \tag{11}
$$

where

$$\tilde{U}(Q(\theta)) = \tilde{U}_0(\theta) + Q(\theta)\tilde{M}(\theta),
$$

$$\tilde{V}(Q(\theta)) = \tilde{V}_0(\theta) + Q(\theta)\tilde{N}(\theta),
$$

$$Q(\theta) \in RH_\infty.$$

Using the Bezout equation (8), the controller (10) or (11) can be realized by the lower linear fractional transformation in the parameter $Q(\theta)$,

$$K(Q(\theta)) = F_1(J_k(\theta), Q(\theta)),
$$

where $J_k(\theta)$ is given as

$$J_k(\theta) = \begin{bmatrix}
U_0(\theta)V_0(\theta)^{-1} & \tilde{V}_0(\theta)^{-1} \\
V_0(\theta)^{-1} & -V_0(\theta)^{-1}N(\theta)
\end{bmatrix}. \tag{12}
$$

**Remark 1:** The formula (12) is the standard state-space $H_\infty$ solution suggested by Doyle et al. [27]. This shows that the standard state-space $H_\infty$ solution with a free parameter can also be obtained using constrained coprime factorization. If the controller can be obtained by Riccati methods, we can parameterize all stabilizing $H_\infty$ controllers with free parameter $Q$ using the proposed doubly coprime factorization.

**Lemma 2** [28]: For the system (1), a set of stabilizing controllers $K_i(\theta), i = 1, 2, \cdots, p$, for the system can be implemented as

$$K_0(Q_i(\theta)) = F_1(J_k(\theta), Q_i(\theta)) \tag{13}
$$

where $J_{k0}$ is formed as similar to (12) and the parameter $Q_i(\theta)$ is given by

$$Q_i(\theta) = \tilde{U}_i(\theta)V_0(\theta) - \tilde{V}_i(\theta)U_0(\theta), \quad i = 1, \cdots, p. \tag{14}
$$

or

$$Q_i(\theta) = \tilde{V}_i(\theta)(K_i(\theta) - K_0(\theta))V_0(\theta), \quad i = 1, \cdots, p.$$

**III. MAIN RESULTS**

A. CENTER CONTROLLER DESIGN

The following Riccati inequality is used to solve the central controller with $H_\infty$ control performance:

$$\psi(X) := A^T(\theta)X(\theta) + X(\theta)A(\theta) + C_1^T(\theta)C_1(\theta) + X(\theta)(\gamma^2B_1(\theta)B_1^T(\theta) - B_2(\theta)B_2^T(\theta))X(\theta)
$$

$$+ \sum_{i=1}^{p} \delta_i \frac{\partial X(\theta)}{\partial \delta_i} < 0, \tag{15}
$$

$$\psi(Y) := A_{imp}(\theta)Y(\theta) + Y(\theta)A_{imp}^T(\theta) + B_1(\theta)B_1^T(\theta) + Y(\theta)(\gamma^2X(\theta)B_2(\theta)B_2^T(\theta)X(\theta) - C_2^T(\theta)C(\theta))$$

$$\times Y(\theta) + \sum_{i=1}^{p} \delta_i \frac{\partial Y(\theta)}{\partial \delta_i} < 0, \tag{16}
$$

where $A_{imp}(\theta) = A(\theta) + \gamma^{-2}B_1(\theta)B_1^T(\theta)X(\theta), X(\theta), Y(\theta)$ are the solutions of Riccati’s inequality, $\gamma$ is some prescribed performance level.
By solving the inequations (15) and (16), a central controller with $H_\infty$ control performance can be obtained

$$K_\infty(\theta) = \begin{bmatrix} \hat{A}(\theta) + \hat{B}_2(\theta)F(\theta) + L(\theta)C_2(\theta) & -L(\theta) \\ F(\theta) & 0 \end{bmatrix}$$

(17)

where

$$\hat{A}(\theta) = A_{imp}(\theta) + \gamma^{-2}Y(\theta)X(\theta)B_2(\theta)B_2^T(\theta)X(\theta),$$

$$\hat{B}_2(\theta) = B_2(\theta) + \gamma^{-2}Y(\theta)X(\theta)B_2(\theta),$$

$$L(\theta) = -Y(\theta)C_2(\theta), \quad F(\theta) = -B_2^T(\theta)X(\theta).$$

Remark 2: In this paper, we need to use Schur complementary lemma to transform Riccati inequality into LMI to obtain the set of all stabilization controllers can be represented by the Youla free parameter $Q_i(\theta)$

$$K(Q_i(\theta)) = F_i(J_k(\theta), Q_i(\theta)),$$

so that the Youla free parameter of smooth switching controller can be designed by Eq.(18)

$$Q(\theta) = \begin{cases} Q_{i,i}(\theta) & \theta \in \Theta_{i,i}, \quad i \in N_i, \\ Q_{i,i+1}(\theta) & \theta \in \Theta_{i,i+1}, \quad i \in N_{i-1} \end{cases}$$

(18)

where the state space expression of $Q_i(\theta)$ is

$$Q_i = \begin{pmatrix} (A(\theta) + L_i(\theta)C_2(\theta)) & B_2(\theta) + L_i(\theta)D_{22}(\theta) & -L_i(\theta) \\ F_i(\theta) & 0 & \end{pmatrix}$$

$$\times \begin{pmatrix} A(\theta) + B_2(\theta)F_0(\theta) & -L_0(\theta) \\ C_2(\theta) + D_{22}(\theta)F_0(\theta) & 0 \\ F_0(\theta) & I \end{pmatrix}$$

Proof: Since the central controller $K_\infty(\theta)$ is solved by Riccati inequalities (15) and (16), then it is a stable controller that satisfies $H_\infty$ control performance. According to the Lemma 1, it can be parameterized with coprime factorization.

The variation range of time-varying parameters is divided into several sub-intervals, and the LPV controller is designed for each parameter sub-region. Since the set of all stabilizing controllers can be represented by a Youla parameter $Q$, the controller can be represented by $K_i(\theta) = F_i(J_k(\theta), Q_i(\theta))$, where $F_i$ is the lower fraction transformation.

The region is divided into the overlapped regions and non-overlapped regions. As shown in Figure 1, non-overlapped sub-interval Youla parameter is $Q_{i,i}(\theta)$, overlapped sub-interval Youla parameter is $Q_{i,i+1}(\theta)$.

According to the formula (14), $Q_i(\theta)$ is defined as

$$Q_i(\theta) = \tilde{U}_i(\theta)\tilde{V}_i(\theta) - \tilde{U}_i(\theta)U_0(\theta)$$

$$= \tilde{V}_i(\theta)(K_i(\theta) - K_0(\theta))V_0(\theta).$$

To guarantee the Bezout characteristics similar to the formula (8), define the coprime factor as shown in the formula (19), by constructing $Q_i(\theta) \in RH_\infty$, and each $Q_i(\theta)$ is stable, the Youla parameter of the non-overlapped region can be constructed as

$$Q_i(\theta) = -\begin{bmatrix} \tilde{V}_i(\theta) & -\tilde{U}_i(\theta) \\ \end{bmatrix} \begin{bmatrix} U_0(\theta) \\ V_0(\theta) \end{bmatrix},$$

(19)

we can get

$$Q_i(\theta) = \begin{bmatrix} \prod_{i=1}^{N_i} A(\theta) + B_2(\theta)F_0(\theta) & -L_0(\theta) \\ F_i(\theta) & F_0(\theta) \end{bmatrix}.$$
where,
\[ \prod_{i=1}^{11} = A(\theta) + L_i(\theta)C_2(\theta), \]
\[ \prod_{i=12} = B_2(\theta)F_0(\theta) - L_i(\theta)C_2(\theta), \]

\( F_0(\theta) \) is the feedback gain of the central solution, and \( A(\theta) + B_2(\theta)F_0(\theta) \) is stable. \( L_0(\theta) \) is the observer gain with stable central solution, \( A(\theta) + L_0(\theta)C_2(\theta) \) is stable. \( F_i(\theta) \) is the feedback gain corresponding to each subregion of the parameter interval, \( L_i(\theta) \) is the observer gain corresponding to each subregion.

The Youla parameter \( Q_{i,i+1}(\theta) \) for overlapped subintervals is obtained by linear interpolation of the parameters \( Q_i(\theta) \) and \( Q_{i+1}(\theta) \) of two adjacent subintervals. We construct the Youla parameter of the overlapped region through a convex combination, then the Youla parameter of the overlapped region is

\[ Q_{i,i+1}(\theta) = \alpha_1 Q_{i+1}(\theta) + \alpha_2 Q_i(\theta), \]

where \( \alpha_1 + \alpha_2 = 1 \).

It is converted into a state-space expression that can be described as

\[
\begin{bmatrix}
\dot{x}_{Q_{i+1}} \\
\dot{x}_{Q_i}
\end{bmatrix} = 
\begin{bmatrix}
A_{Q_{i+1}}(\theta) & 0 \\
0 & A_{Q_i}(\theta)
\end{bmatrix}
\begin{bmatrix}
x_{Q_{i+1}} \\
x_{Q_i}
\end{bmatrix} + 
\begin{bmatrix}
B_{Q_{i+1}}(\theta) \\
B_{Q_i}(\theta)
\end{bmatrix} \zeta
\]

\( u = \left[ \alpha_1 C_{Q_{i+1}}(\theta) \alpha_2 C_{Q_i}(\theta) \right] \begin{bmatrix} x_{Q_{i+1}} \\ x_{Q_i} \end{bmatrix} + \left[ \sum_{l=1}^{2} \alpha_i D_{Q_l}(\theta) \right] y \)  

Then, the state space of the Youla parameter of the smooth switch controller in the overlapped region can be realized as follows

\[ Q_{i,i+1}(\theta) = \begin{bmatrix} A_{Q_{i+1}}(\theta) & 0 \\ 0 & A_{Q_i}(\theta) \end{bmatrix} \begin{bmatrix} B_{Q_{i+1}}(\theta) \\ B_{Q_i}(\theta) \end{bmatrix} \begin{bmatrix} 2 \sum_{i=1}^{2} \alpha_i D_{Q_i}(\theta) \end{bmatrix} \begin{bmatrix} \alpha_1 C_{Q_{i+1}}(\theta) \\ \alpha_2 C_{Q_i}(\theta) \end{bmatrix} \]

Therefore, the Youla free parameter of smooth switching can be expressed as

\[ Q(\theta) = \begin{cases} Q_{i,i}(\theta) & \theta \in \Theta_{i,i}, \ i \in N_i, \\ Q_{i,i+1}(\theta) & \theta \in \Theta_{i,i+1}, \ i \in N_{i-1}, \end{cases} \]

where

\[ Q_{i,i}(\theta) = Q_i(\theta) = \begin{bmatrix} \tilde{V}_i(\theta) \\ -\tilde{U}_i(\theta) \end{bmatrix} \begin{bmatrix} U_0(\theta) \\ V_0(\theta) \end{bmatrix}, \]

\[ Q_{i,i+1}(\theta) = \alpha_1 Q_{i+1}(\theta) + \alpha_2 Q_i(\theta), \]

\[ \alpha_1 = \frac{\theta - \tilde{\theta}_{i-1}}{\tilde{\theta}_{i+1} - \tilde{\theta}_{i-1}}, \quad \alpha_2 = 1 - \alpha_1. \]

Algorithm Given an LPV Plant Satisfying Assumptions (1-3), the Following Steps Lead to a Parameterized Controller

1. Solving central controller with Riccati inequalities (15) and (16).
2. According to Figure 1, the variation range of parameters is divided into subregions with overlaps.
3. Getting the parameter \( Q \) of Youla by the coprime factorization method.
4. According to the change of parameters, the switch of Youla parameter \( Q \) is performed.

The framework for parameterization of switching LPV controller can be summarized in the following.

Remark 3: The advantage of this design method is that we design the Youla parameters of each sub-regions from the controller based on the coprime factorization technology, and introduce the parameter overlap division method to improve the smoothness of the switching. It can increase the flexibility and freedom of the switching controller and effectively suppress the disturbance during the switching. However, this method has the disadvantage that the controller’s parameterization will increase the complexity of the controller and increase the amount of calculation.

IV. EXAMPLE

In this section, two examples are given to verify the effectiveness of the method in this article.

Example 1: We apply the technique of smooth switching LPV controllers to the system model of Firebee aircraft. The Firebee aircraft is a morphing aircraft, which is a complex parameter variable system that depends on the shape and structure. It can be modeled as an LPV system, and its scheduling parameter is the rate of change of the wing sweep angle. The parameters of this example can be found in [29].

\[
\begin{bmatrix}
\dot{x} \\
\dot{z}
\end{bmatrix} = 
\begin{bmatrix}
A(\theta) & 0_{2 \times 2} & B_2(\theta) \\
C_1(\theta) & I_2 & 0_{3 \times 2} & D_{12}(\theta) & I_2 \\
0_{1 \times 2} & I_2 & 0_{2 \times 1} & 0_{2 \times 1} & 1
\end{bmatrix}
\begin{bmatrix}
x \\
z \\
y \\
w \\
u
\end{bmatrix}
\]

where

\[ A(\theta) = \begin{bmatrix}
0.2255\theta - 1.3967 \\
0.0876\theta^2 - 0.4889\theta - 0.3775 \\
0.4489\theta - 0.8229
\end{bmatrix}, \quad B_2(\theta) = \begin{bmatrix}
0.0034\theta - 0.001638 \\
0.00053\theta^2 - 0.04415\theta - 0.143984
\end{bmatrix}, \quad C_1(\theta) = \begin{bmatrix}
I_2 \\
0_{1 \times 2} & D_{12}(\theta) = \begin{bmatrix}
0_{2 \times 1} & 1
\end{bmatrix}
\]

The state variable \( x = [\Delta \alpha, \Delta \beta]^T \) represents the angle of attack and the pitch rate. The control input \( u = \Delta \delta \) denotes the elevator deflection.

We get \( \Theta \in [0, 3] \), the variation rate \( V = [-0.2, 0.2] \), when we solve Riccati’s inequality, the desired performance is chosen as \( \gamma_0 = 1.5 \). We divide the parameter into several overlapped regions.

\[ \Theta_1 = [0, 1.2], \quad \Theta_2 = [0.8, 2.7], \quad \Theta_3 = [2.4, 3.0]. \]
The robust LPV control system structure of the Firebee aircraft is shown in Figure 3.

![FIGURE 3. Robust LPV control system structure.](image)

where $W_e$ is the tracking error weight, $W_\omega$ is the measurement noise weight, $W_u$ is the control weight, each weighting function is chosen as

$$W_e = \frac{s/100 + 1}{s + 3}I_3, \quad W_\omega = \text{diag}(0.01, 0.05),$$

$$W_u = \frac{s + 0.1/100}{0.1(s + 0.001)}.$$

It is assumed that the variation rate of the wing sweep angle is

$$\theta(t) = 1.5 + 1.5 \sin(1/15)t - 0.5\pi - (5/15), \quad 0 \leq t \leq 60,$$

as shown in Fig.4. When the smooth switching controller is used, there are four times of switching in 21.6s, 25.6s, 38s and 42.6s respectively, also marked with a small circles in Fig.4.

![FIGURE 4. Parameter trajectory used in the simulations.](image)

Then, we perform simulations on the nonlinear model of morphing aircraft using the switching Youla parameter $Q$ and switching controllers $K$, respectively. The simulation results are shown in Fig.5 and Fig.6.

Fig.5 shows the controller output response. The blue solid line represents switching Youla parameter $Q$. The red dotted line represents switching controller $K$ without Youla parameterization optimization. We can see that the control input with the interference of switching Youla parameter $Q$ is smaller than switching controller $K$ when switching occurs.

![FIGURE 5. The responses of controller output compare switching Q with switching K.](image)

Fig.6 shows the system output response. It can be observed that using the switching controllers $K$. The system output has sudden undesirable transient behavior at the switching point. In contrast, using the switching Youla parameter $Q$, the anti-interference ability of the system output response from the switching $Q$ is better than the switching $K$.

**Example 2:** To further demonstrate the effectiveness of the algorithm proposed above, another example is given.

\[
\dot{x}(t) = \begin{bmatrix}
-1 & 0 & 1 \\
0 & -0.5 & \theta \\
0 & \theta & -0.5
\end{bmatrix} x(t) + \begin{bmatrix}
1 \\
0.5 \\
0.2
\end{bmatrix} w(t) + \begin{bmatrix}
0 \\
1 \\
1
\end{bmatrix} u(t)
\]

\[
z(t) = \begin{bmatrix}
0 & 0.5 & 1
\end{bmatrix} x(t)
\]

\[
y(t) = \begin{bmatrix}
1 & 0 & 10
\end{bmatrix} x(t) + 0.2w(t)
\]

(26)

where $\theta$ is a real parameter satisfying $-2 \leq \theta \leq 2$ and can be measured online, $w(t)$ is external input signal. The parameter region is divided into two overlapped subsets: $[-2, 0.7], [-0.2, 2]$, the overlapping region is $[-0.2, 0.7]$. The time-varying parameter trajectory is shown in Fig. 7.
Switching occurs when the parameter trajectories cross the switching surface at 14.5s, 16.75s, 43.25s and 45.5s. The reference input select the step signal to enter at 5 seconds.

The simulation results are shown in Fig.8 and Fig.9. The dotted line represents the method of non-smooth switching in literature [24], and the solid line represents the method in this paper. It is obvious that the anti-disturbance capability of smooth switching is better than non-smooth switching.

V. CONCLUSION

This paper has investigated the problem with designing a smooth switching LPV controller based on Youla parameterization. The design of the switched controller mainly involves two steps. Firstly, the Riccati inequality is used to solve the central controller that satisfies the $H_{\infty}$ control performance. The range of parameters is divided into several overlapping subregions, and an LPV controller is designed for each subregion. Secondly, each sub-area controller is parameterized by using the coprime factorization method to obtain the corresponding Youla parameters. The switch of Youla parameter $Q$ is used to replace the switch controller to achieve smooth switching. This method can suppress the disturbances during switching, which satisfies the higher degree of freedom and flexibility to achieve better control performance. To verify the effectiveness of the proposed method, a morphing aircraft model is used as a simulation example. By applying the proposed smooth switching controller to the morphing aircraft, the flight system exhibits excellent stability and robustness and can switch smoothly. The method proposed in this paper can be extended to nonlinear systems, adaptive control systems, and other fields. Meanwhile, these results can be applied to the lathe vibration system, magnetic fluid deformable mirror system, and intelligent transportation system (ITS).

REFERENCES

[1] J. S. Shamma, “Analysis and design of gain scheduled control systems,” Ph.D. dissertation, Massachusetts Inst. Technol., Cambridge, MA, USA, 1988.
[2] Q. Wu, Z. Liu, F. Liu, and X. Chen, “LPV-based self-adaption integral sliding mode controller with $L_2$ gain performance for a morphing aircraft,” IEEE Access, vol. 7, pp. 81515–81531, 2019.
[3] W. Zhang, M. Xia, and J. Zhu, “LPV modeling and identification of unsteady aerodynamics for fast maneuvering aircrafts,” IEEE Access, vol. 7, pp. 92436–92443, 2019.
[4] D. Yang, G. Zong, and S. K. Nguang, “$H_{\infty}$ bumpless transfer reliable control of Markovian switching LPV systems subject to actuator failures,” Inf. Sci., vol. 512, pp. 431–445, Feb. 2020.
[5] B. Lu and F. Wu, “Switching LPV control designs using multiple parameter-dependent Lyapunov functions,” Automatica, vol. 40, no. 11, pp. 1973–1980, 2004.
[6] D. Xiao, M. Liu, Y. Liu, and Y. Lu, “Switching control of a hypersonic vehicle based on guardian maps,” Acta Astronautica, vol. 122, pp. 294–306, May/Jun. 2016.
[7] T. He, G. G. Zhu, S. S.-M. Swei, and W. Su, “Smooth-switching LPV control for vibration suppression of a flexible airplane wing,” Aerosp. Sci. Technol., vol. 84, pp. 895–903, Jan. 2019.
[8] D. Yang, G. Zong, and H. Karimi, “$H_{\infty}$ refined anti-disturbance control of switched LPV systems with application to aero-engine,” IEEE Trans. Ind. Electron., vol. 67, no. 4, pp. 3180–3190, Apr. 2020.
[9] D. Yang and J. Zhao, “$H_{\infty}$ output tracking control for a class of switched LPV systems and its application to an aero-engine model,” Int. J. Robust Noninear Control, vol. 27, no. 12, pp. 2102–2120, Aug. 2017.
[10] T.-J. Liu, X. Du, X.-M. Sun, H. Richter, and F. Zhu, “Robust tracking control of aero-engine rotor speed based on switched LPV model,” Aerosp. Sci. Technol., vol. 91, pp. 382–390, Aug. 2019.
[11] H. Xu, Z. Wei, L. Yuquan, and Z. Yang, “Fault tolerant control with switched LPV method based on hysteresis strategy and an application to a microsatellite model,” in *Proc. 34th Chin. Control Conf. (CCC)*, Hangzhou, China, Jul. 2015, pp. 6153–6158.

[12] M. Postma and R. Nagamune, “Air-fuel ratio control of spark ignition engines using a switching LPV controller,” *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 5, pp. 1175–1187, Sep. 2012.

[13] P. Zhao and R. Nagamune, “Switching LPV controller design under uncertain scheduling parameters,” *Automatica*, vol. 76, pp. 243–250, Feb. 2017.

[14] P.-C. Chen, S.-L. Wu, and H.-S. Chuang, “The smooth switching control for TORA system via LMIs,” in *Proc. IEEE Int. Conf. Control Autom.*, Xiamen, China, Jun. 2010, pp. 1338–1343.

[15] H. Cheng, C. Dong, Q. Wang, and W. Jiang, “Smooth switching linear parameter-varying fault detection filter design for morphing aircraft with asynchronous switching,” *Trans. Inst. Meas. Control*, vol. 40, no. 8, pp. 2622–2638, May 2018.

[16] P.-C. Chen, C.-H. Chiang, C.-H. Hsu, and K.-H. Chen, “Smooth switching gain-scheduled control for large scale offshore wind turbine under full wind-speed conditions,” in *Proc. Int. Conf. Syst. Sci. Eng.*, Puli, Taiwan, Jul. 2016, pp. 1–4.

[17] P.-C. Chen, “The design of smooth switching control with application to V/STOL aircraft dynamics under input and output constraints,” *Asian J. Control*, vol. 14, no. 2, pp. 439–453, 2012.

[18] M. Hanifzadegan and R. Nagamune, “Smooth switching LPV controller design for LPV systems,” *Automatica*, vol. 50, no. 5, pp. 1481–1488, May 2014.

[19] H. H. Rosenbrock, *State-Space and Multivariable Theory*. London, U.K.: Nelson, 1970.

[20] I. Mahtout, F. Navas, V. Milanés, and F. Nashashibi, “Advances in Youla–Kučera parametrization: A review,” *Annu. Rev. Control*, vol. 49, pp. 81–94, Jan. 2020.

[21] A. Quadrat, “On a generalization of the Youla–Kučera parametrization. Part I: The fractional ideal approach to SISO systems,” *Syst. Control Lett.*, vol. 50, no. 2, pp. 135–148, Oct. 2003.

[22] A. Quadrat, “On a generalization of the Youla–Kučera parametrization. Part II: The lattice approach to MIMO systems,” *Math. Control, Signals, Syst.*, vol. 18, no. 3, pp. 199–235, Aug. 2006.

[23] F. Qian, Z. Wu, M. Zhang, T. Wang, Y. Wang, and T. Yue, “Youla parameterized adaptive vibration control against deterministic and band-limited random signals,” *Mech. Syst. Signal Process.*, vol. 134, pp. 106359–106370, Dec. 2019.

[24] F. D. Bianchi and R. S. Sánchez-Peña, “A novel design approach for switched LPV controllers,” *Int. J. Control*, vol. 83, no. 8, pp. 1710–1717, Aug. 2010.

[25] W. Wu, W. Xie, and L. Li, “Switching linear parameter-varying controller design with $H_\infty$ performance based on Youla parameterization,” *IEEE Access*, vol. 8, pp. 184765–184773, 2020.

[26] H. Niemann and J. Stoustrup, “Gain scheduling using the Youla parameterization,” in *Proc. 38th IEEE Conf. Decis. Control*, Phoenix, AZ, USA, Dec. 1999, pp. 2306–2311.

[27] J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis, “State-space solutions to standard $H_2$ and $H_\infty$ control problems,” *IEEE Trans. Autom. Control*, vol. 34, no. 8, pp. 831–847, Aug. 1989.

[28] H. Niemann, J. Stoustrup, and R. B. Abrahamsen, “Switching between multivariable controllers,” *Optim. Control Appl. Methods*, vol. 25, no. 2, pp. 51–66, Mar. 2004.

[29] W. Jiang, C. Dong, and Q. Wang, “A systematic method of smooth switching LPV controllers design for a morphing aircraft,” *Chin. J. Aeronaut.*, vol. 28, no. 6, pp. 1640–1649, Dec. 2015.