On the Generation of X-Ray Photon Pairs: A Verification of the Unruh Effect?

Friedhelm Bell*
Department für Physik, Ludwig-Maximilians-Universität, D-85748 Garching, Germany

We investigate the production of entangled X-ray photons by incoherent double Compton scattering (IDCS) which seems to be an interesting alternative to coherent double Compton scattering used in earlier work. Very recently it has been proposed to use the Unruh effect as a source for photon pairs (R.Schützhold et al., Phys.Rev.Lett. 100, 091301 (2008)). We will discuss this mechanism in comparison with inverse IDCS.

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I. INTRODUCTION

The production of correlated two-photon states by parametric down conversion in nonlinear media is well known in the visible optical region. Here, down conversion is meant as the spontaneous decay of a single photon into a pair whose energies add up to the primary one. The effect is usually described as the mixing of real photons with vacuum fluctuations resulting in two real photons. The importance of entangled photons will be realised if one recognises that they are fundamental ingredients of future quantum computers or quantum cryptography. Whereas in the visible optical region rather strong sources for photon pairs are available the situation for entangled X-ray photons is very different. Motivated by a proposal of Freund and Levine\cite{Freund1986} Eisenberger and McCall\cite{Eisenberger1985} where the first who realized X-ray parametric down conversion experimentally. One of the remarkable features of both the proposal and the experiment was that the recoil of the photon pair should be overtaken by the target crystal, thus providing some kind of coherence. Whereas the Eisenberger-McCall experiment used a conventional X-ray tube as the primary photon source, the experiment was later repeated with synchrotron radiation from storage rings\cite{Adams1987}, but even then, the event rate did not exceed 0.1 photon pairs/s. Both by Eisenberger\cite{Eisenberger1985} and Adams\cite{Adams1987} the results were discussed in terms of mixing the incoming X-ray photon with vacuum fluctuations of the photon field at frequencies of the photon pair. The high power density of these fluctuations especially at X-ray energies compensates for the small optical nonlinearity of the target material, making down conversion observable. Whereas in these experiments\cite{Adams1987, Eisenberger1985} the photon pair was detected by a coincidence technique we also mention experimental work where only one of the photons was observed while the other had vanishing small energy\cite{Adams1987}.

While the explanation of entangled X-ray photon generation as described above seems to be influenced by the picture for down conversion by vacuum fluctuations a more conventional - and for our case more appropriate - understanding of the effect was already offered in the proposal of Freund and Levine: “The incoherent process of double Compton scattering first discussed by Heitler and Nordheim, in which a photon when interacting with a relativistic (and hence nonlinear) electron decays into two photons of lesser energy, is well known. We discuss here an analogous coherent phenomenon, the spontaneous parametric decay of X-rays. This process is related to double Compton scattering in the same way that ordinary Bragg diffraction relates to ordinary Compton scattering.”\cite{Freund1986}

In the following we discuss incoherent double Compton scattering (IDCS) as a source for X-ray photon pairs. It is hoped that the efficiency of this process is larger than for coherent double Compton scattering (CDCS), since stringent conditions for defining photon momentum vectors necessary to fulfill the phase matching condition in CDCS do not apply in IDCS. We shall also connect our analysis with recent publications on a very different source for correlated X-ray photons: pair production by the Unruh-Hawking effect in a non-inertial reference frame\cite{Eisenberger1985, Adams1987, Freund1986}. While this effect by itself is very important in quantum gravity it might also have relevance to our theme. Finally, we mention that IDCS plays an important role in astrophysics since it was realised that it can become the main source for soft photons in astrophysical plasmas with low baryon density\cite{Eisenberger1985}.

Last not least we indicate useful applications of entangled X-ray photons as in two photon interferometry. Instead of making a single photon probe two interfering paths, one might also employ two-photon states in interferometry. We stress that in this case an interference pattern will be observed even if both photons have different energies and arrive at different detectors (fourth-order interference experiments\cite{Eisenberger1985}), thus allowing phase shift observations at different energies $\omega_1$ and $\omega_2$. This, of course, implies mutual coherence of both photons. Since quantum efficiency of X-ray detectors is close to 100% the visibility of higher order interferences to test spin-free Bell’s inequalities of quantum theory is at maximum\cite{Eisenberger1985}. The same holds for Einstein-Podolsky-Rosen experiments where any deficiency of quantum efficiency is equivalent to influences of external degrees of freedom on pure quantum states.

*Electronic address: friedhelm.bell@baw4u.de
II. INCOHERENT DOUBLE COMPTON SCATTERING

After an order of magnitude calculation by Heitler and Nordheim\textsuperscript{13} Mandl and Skyrme\textsuperscript{14} were the first who calculated within the framework of quantum electrodynamics the differential cross section for IDCS exactly. These results have been extended and intensively discussed in the book of Jauch and Rohrlich\textsuperscript{15}. Fig. 1 shows one of the 3! equivalent third order Feynman diagrams for IDCS\textsuperscript{15}. We note that these diagrams are equivalent to the six components of the second order susceptibility calculated in nonlinear optics\textsuperscript{16}. In the following we use socalled natural units, i.e. $\hbar = m = c = 1$. As shown in Fig. 1 IDCS is a process where an incoming photon with 4-vector $k = (\omega, \mathbf{k})$ is scattered at an electron with $p = (\gamma, \mathbf{p})$ resulting in two final photons with $k_1 = (\omega_1, \mathbf{k}_1)$ and $k_2 = (\omega_2, \mathbf{k}_2)$ and the final electron with $p' = (\gamma', \mathbf{p}')$. Kinematics demand

$$p' + k_1 + k_2 - p = 0$$

(2.1)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.pdf}
\caption{Feynman diagram for double Compton scattering}
\end{ figure

We use the signature $(- + + +)$, i.e. $p^2 + 1 = 0$. The calculation holds for the lab frame and $\gamma = (1 - \beta^2)^{-1/2}$. Replacing the initial electron 4-vector by $p = (M, 0)$ and the final one by $p' = (\sqrt{M^2 + q^2}, \mathbf{g})$, where $M$ is the target mass and $\mathbf{g}$ a reciprocal lattice vector, one obtains the kinematics for CDCS.

The differential cross section is given for the emission of photons 1 and 2 into solid angles $d\Omega_1$ and $d\Omega_2$ in the direction $(\theta'_1, \varphi'_1)$ and $(\theta'_2, \varphi'_2)$. These angles refer to a polar coordinate system whose axis is the incident electron direction. The azimuths $\varphi'$ are conveniently measured from the plane of incidence formed by the vectors $\mathbf{k}$ and $\mathbf{\beta}$. We keep the notation of ref. 15. Now, let $\theta_1$ and $\theta_2$ be the angles between the directions of the outgoing photons and the incoming one and $\theta_{12}$ that between the pair, then

$$\cos \theta_1 = \cos \alpha \cos \theta'_1 + \sin \alpha \sin \theta'_1 \cos \varphi'_1$$

(2.2)

$$\cos \theta_2 = \cos \alpha \cos \theta'_2 + \sin \alpha \sin \theta'_2 \cos \varphi'_2$$

(2.3)

$$\cos \theta_{12} =$$

(2.4)

$$\cos \theta'_1 \cos \theta'_2 + \sin \theta'_1 \sin \theta'_2 \cos (\varphi'_1 - \varphi'_2)$$

holds, where $\alpha$ is the angle between $\mathbf{k}$ and $\mathbf{\beta}$. For a given energy $\omega_1$ of photon 1 the energy $\omega_2$ of photon 2 is obtained from

$$\omega_2 = \frac{\omega_1(1 - \beta \cos \alpha) - \gamma \omega_1(1 - \beta \cos \theta'_1) - \omega_1(1 - \cos \theta_1)}{\gamma(1 - \beta \cos \theta'_2) + \omega(1 - \cos \theta_2) - \omega_1(1 - \cos \theta_{12})}$$

(2.5)

Then, the triple differential cross section for IDCS reads

$$\frac{d^3 \sigma_{DC}}{d\omega_1 d\Omega'_1 d\Omega'_2} = \frac{\alpha_{QED}^2}{(4\pi)^2} \frac{r_0^2}{\gamma \omega(1 - \beta \cos \alpha)} \frac{\gamma(1 - \beta \cos \theta'_1)}{\gamma(1 - \beta \cos \theta'_2) + \omega(1 - \cos \theta_2) - \omega_1(1 - \cos \theta_{12})}$$

(2.6)

with the cross section function

$$X = 2(ab - c)[(a + b)(x + 2) - (ab - c) - 8] - 2\alpha^2 + \beta^2 - 8c + \frac{4}{AB}[(A + B)(x^2 + x) - (aA + bB)(2x + z(1 - x)) + x^3(1 - z) + 2xz] - 2p[a + c(1 - x)]$$

(2.7)

The abbreviations read

$$a = \sum_{i=1}^{3} \kappa_i; b = \sum_{i=1}^{3} \frac{1}{\kappa'_i}; c = \sum_{i=1}^{3} \frac{1}{\kappa_i \kappa'_i}$$

(2.8)

$$x = \sum_{i=1}^{3} \kappa_i; z = \sum_{i=1}^{3} \kappa_i \kappa'_i$$

$$A = \prod_{i=1}^{3} \kappa_i; B = \prod_{i=1}^{3} \kappa'_i; \rho = \sum_{i=1}^{3} \left( \frac{\kappa_i}{\kappa'_i} + \frac{\kappa'_i}{\kappa_i} \right)$$

(2.9)

and

$$\kappa'_1 = +p' \cdot k_1 =$$

(2.10)

$$\kappa_2 = -p' \cdot k_2 =$$

(2.11)

$$\kappa_3 = +p \cdot k = -\gamma(1 - \beta \cos \alpha)$$

(2.12)

$$\kappa'_1 = +p' \cdot k_1 =$$

(2.13)

$$\kappa'_2 = +p' \cdot k_2 =$$

(2.14)

$$\kappa'_3 = +p' \cdot k =$$

(2.15)

It is $r_0 = 2.82 \cdot 10^{-15}$m the classical electron radius and $\alpha_{QED} = 1/137$ the fine structure constant. The
The triple differential cross section of IDCS (eq. (2.6)) has been verified quantitatively by several authors. In the following we will apply this cross section to special cases.

**III. FIXED TARGETS**

In case of a solid state target we assume that the electrons are at rest, i.e. $\beta = 0$. This implies that the angles $(\theta_1', \varphi_1')$ and $(\theta_2', \varphi_2')$ become unimportant and the cross section will be differential with respect to $\theta_1$ and $\theta_2$. The angle between the two photons becomes

$$\cos \theta_{12} = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos (\varphi_1 - \varphi_2)$$  \hspace{1cm} (3.1)

We first calculate the cross section in case of hard X-rays, i.e. $\omega = 100$ keV. At storage rings of the 3rd generation photon fluxes of about $10^{12}$ photons/s can be achieved with a relative monochromaticity of less than $1\%$. Fig. 2 shows the triple differential cross section for $\varphi_1 = 0$, $\varphi_2 = \pi$ and $\theta_1 = \theta_2 = \theta$ as a function of $\theta$. As a typical value for the cross section we get for $\omega = \omega_2 = 42$ keV and $\theta = 2$ rad $d^3\sigma_{DC} = 8 \, nb/keV \cdot sr^2$. It is readily seen that for either $\omega_1 = 0$ or $\omega_2 = 0$ - the upper limit for $\omega_1$ - the cross section diverges reflecting the well known infrared catastrophe if QED cross sections are calculated perturbatively from S-matrix theory. It was Feynman who first showed that when radiative corrections to single Compton scattering and the infrared divergence of double Compton scattering are calculated on the same footing both divergences cancel. It is also seen that the cross section vanishes when both photons are emitted exactly in forward direction. It is easily demonstrated that

$$\omega_{1max} = \omega' = \frac{\omega}{1 + \omega' (1 - \cos \theta_1)}$$  \hspace{1cm} (3.2)

in this case the kinematics of (2.1) cannot be fulfilled. We note that the maximum energy $\omega_{1max}$ for photon 1, which corresponds to $\omega_2 = 0$, is identical with the energy $\omega'$ for single Compton scattering independently of the setting of photon detector 2. An equivalent conclusion holds for $\omega_{2max}$. Only in case that both photons are emitted in the same direction ($\theta_{12} = 0$) it is seen from (2.5) that the sum $\omega_1 + \omega_2$ of both photon energies is identical to $\omega'$ at that angle.

Setting photon detectors at the most intense cross sections, i.e. at $\theta = 2$ rad with solid angles $\Delta \Omega = 3 \cdot 10^{-2} \, sr$, assuming a 100 $\mu$m thin Al target and a detected energy bandwidth of 5% we end up with photon-pair production rates shown in Fig. 3. Between 20 and 70 keV the production rate increases from 0.1 to 1 pairs/s, which is comparable to or a factor 10 larger than that from CDCS mentioned in the Introduction. Even stronger two-photon sources might be available at forthcoming X-ray Free Electron Lasers (XFEL). The European XFEL project envisages for the SASE 1 device an average photon flux of $4 \cdot 10^{16}$ photons/s at an energy of $\omega = 12.4$ keV. Again, Fig. 4a shows the cross section for $\theta_1 = \theta_2 = \theta$ and $\varphi_1 = 0$, $\varphi_2 = \pi$ (which, in the language of nonlinear optics, is the one-mode case) and $\varphi_1 = \varphi_2 = 0$ (Fig. 4b, i.e. the two-mode case) respectively. In the latter case, both photons are emitted in the same direction. The cross sections are roughly an order of magnitude smaller than those of Fig. 2.

As in Fig. 3 we have calculated the pair production rate but now for a 10 $\mu$m thin Al target to avoid strong photo-absorption. For the cross section of Fig. 4b and $\theta = 2.1$ rad the pair production rate is plotted in Fig. 5. Between 4 and 10 keV it changes from 6 to 30 pairs/s. A remark seems to be indicated about the cross section when both photons are emitted in the same direction. The triple differential cross section describes

![FIG. 2: (Color) Isodensity plot of the triple differential cross section for double Compton scattering (fixed target) as a function of $\omega_1$ and the scattering angle $\theta$. The scale is logarithmic, i.e. the number -10 at the colour scale means $10^{-10}$ b/keV $\cdot$ sr$^2$. The plot holds for $\theta_1 = \theta_2 = \theta$, $\varphi_1 = 0$ and $\varphi_2 = \pi$. The emission of both photons is symmetric with respect to the direction of the incoming photon, whose energy is $\omega = 100$ keV.](image)

![FIG. 3: The pair production rate for the cross section of Fig. 2 as a function of photon energy.](image)
of the cross sections we argue that in this special case \( d^2\sigma_{DC}/d\omega_1d\Omega = 4\pi d^3\sigma_{DC}/d\omega_1d\Omega_1d\Omega_2 \). For the production rate of Fig. 5 this would mean an increase by a factor \( 4\pi/3 \cdot 10^{-2} = 400 \).

### IV. INVERSE DOUBLE COMPTON SCATTERING

In this chapter we consider the scattering of low energy optical laser photons at high energetic electrons with \( \gamma \gg 1 \) where two photons are generated in the exit channel. The reason is threefold: i) by this inverse IDCS extremely hard photon pairs even in the MeV range can be produced, ii) a comparison with Unruh-Hawking radiation mentioned in the Introduction can be made, and iii) a connection to undulator radiation will be drawn.

i) In case of inverse IDCS one derives from eq. (2.5) the maximum photon energy \( \omega_1^{\text{max}} \) (i.e. \( \omega_2 = 0 \)) in detector 1

\[
\omega_1^{\text{max}} = \frac{\gamma \omega_L (1 - \beta \cos \alpha)}{\gamma (1 - \beta \cos \beta_0') + \omega_L (1 - \cos \theta_1')} \tag{4.1}
\]

where \( \omega_L \) is the laser photon energy. For \( \gamma \gg 1 \) and with the approximation \( \cos \theta_1 \simeq \cos \alpha \), (4.1) can be written

\[
\omega_1^{\text{max}} = \frac{\omega_m}{1 + (\beta_1'/\beta_0')^2} \tag{4.2}
\]

where \( \omega_m = \gamma x/(1 + x) \) and \( \beta_0' = \sqrt{1 + x}/\gamma \). It holds \( x = 4\gamma \omega_L \cos^2(\alpha_0/2) \) with \( \alpha_0 = \pi - \alpha \). Eq. (4.2) is identical with the scattered photon energy of inverse single Compton scattering\(^{21}\). To calculate the pair production rate as in chapter III one has to evaluate the luminosity \( L_{e\gamma} \) for realistic conditions.

Assuming for both the electron and photon bunch perfect overlapping in space and time, the luminosity per bunch crossing and a head-on collision reads

\[
L_{e\gamma} = \frac{N_e N_{\gamma}}{2\pi (\sigma_{te}^2 + \sigma_{t\gamma}^2)} \tag{4.3}
\]

\( N_e \) and \( N_{\gamma} \) are the total electron and photon numbers within the bunches, and \( \sigma_{te}, \sigma_{t\gamma} \) the rms values of the transverse extensions of the bunches. For realistic numbers we have adopted a scenario which has been proposed for future table-top FELs\(^{22}\). Then, the number of electrons/bunch is about 1 nC or \( N_e = 10^{10} \). Assuming a laser with \( \omega_L = 2.5 \) eV, an intensity \( I_L = 10^{18} \text{W/cm}^2 \), a pulse duration \( \tau_L = 50 \text{ fs} \) and a matching of the transverse extensions of both bunches one arrives at a luminosity/electron

\[
L_{e\gamma}/N_e = \frac{I_L \tau_L}{4\pi \omega_L} = 0.06 \text{ b}^{-1} \tag{4.4}
\]

In Fig. 6 we have plotted the triple differential yield/electron \( Y = d^3\sigma_{DC}/d\omega_1d\omega_2d\Omega \) for
\[ \sigma = 300 \text{ and } \theta'_1 = \theta'_2 = \theta', \varphi'_1 = 0, \varphi'_2 = \pi. \] 
The maximum energy in forward direction amounts to \(4\gamma^2 \omega_L = 900 \text{ keV}.\) 
Employing the most intense yield \(Y\) corresponding to a cross section of \(8 \text{ mb/keV-sr}^2\) at an angle \(\gamma \theta' = 0.6\), an energy resolution of 5\% for the symmetrical case \(\omega_1 = \omega_2 = \omega_{\text{max}}/2 = 160 \text{ keV}\) and solid angles \(\Delta \Omega_1' = \Delta \Omega_2' = 10^{-6} \text{sr}\), we obtain a pair production rate

\[ Y N \Delta \Omega_1' \Delta \Omega_2' \Delta \omega_1 = 4 \cdot 10^{-5} \text{ pairs/pulse} \tag{4.5} \]

Going to the asymmetric case, i.e. \(\omega_1 \neq \omega_2\), one can easily gain an order of magnitude in yield. We also mention that for a head-on collision the two-photon pulse duration is \(\sigma_{\text{le}}/c\) with \(\sigma_{\text{le}}\) as the rms length of the electron bunch, and is thus independent of the laser pulse duration \(\tau_L\). 
In order to get a feeling about the magnitude of differential cross sections we will compare single Compton cross sections with those for double Compton scattering. Due to the finite pulse duration the laser has a rather strong energy broadening \(\Delta \omega_L = 1/\tau_L\). Thus one finds for the double differential cross section for single Compton scattering

\[ \frac{d^2 \sigma_S}{d \omega d \Omega} = \frac{d \sigma_S}{d \Omega} G(\omega) \tag{4.6} \]

where \(d \sigma_S/d \Omega\) is the cross section for inverse single Compton scattering (unpolarized incoming photons):

\[ \frac{d \sigma_S}{d \Omega} = r^2_0 \frac{\omega^2}{\kappa'} \left( \frac{\omega}{\kappa'} \right)^2 \times \left[ \frac{\kappa'}{\kappa} + \frac{\kappa}{\kappa'} + 2 \left( \frac{1}{\kappa'} - \frac{1}{\kappa} \right) + \left( \frac{1}{\kappa} - \frac{1}{\kappa'} \right)^2 \right] \tag{4.7} \]

with

\[ \omega' = \frac{\gamma \omega_L (1 - \beta \cos \alpha)}{\gamma (1 - \beta \cos \theta') + \omega_L (1 - \cos \alpha)} \tag{4.8} \]

and \(\kappa = p \cdot k = -\gamma \omega_L (1 - \beta \cos \alpha), \kappa' = p \cdot k' = -\gamma \omega' (1 - \beta \cos \theta').\) In case of a Gaussian bunch the spectral function \(G(\omega)\) reads

\[ G(\omega) = \frac{\omega_L \tau_L}{\omega' \sqrt{2\pi}} \exp \left( -\frac{1}{2} (\omega, \gamma \omega_L) \right) \tag{4.9} \]

On the other hand one obtains the double differential cross section for double Compton scattering in case where one is not interested in the second photon

\[ \frac{d^2 \sigma_D}{d \omega_1 d \Omega_1} = \int_{-1}^{+1} \int_{-1}^{+1} d \varphi_1' d \theta_1' \frac{d^2 \sigma_D}{d \omega_1 d \Omega_1 d \Omega_2} \tag{4.10} \]

For the laser and electron bunch data given above the double differential cross sections in case of head-on collision have been plotted in Fig. 7. The left side of the figure holds for single, the right one for double Compton scattering. We mention that the single Compton peak might be further broadened due to the finite emittance of the electron bunch.

ii) A very interesting method for the production of entangled X-ray photons has been proposed by Schützhold et al.\textsuperscript{8,9}. Accelerated electrons can convert virtual quantum vacuum fluctuations into real particle \(\text{pairs}\) via non-inertial scattering which can be understood as a signature of the Unruh effect\textsuperscript{23}. In a sense, Unruh radiation might be called a sister to Hawking radiation\textsuperscript{24,25}. In 1974 Hawking\textsuperscript{24} had shown that due to quantum fluctuations black holes, while embedded in a thermal bath with temperature \(T_{\text{UH}}\), may evaporate particle pairs. Unfortunately, a quantitative comparison between the two-photon yields of the Unruh-Hawking effect and inverse IDCS is difficult to make since at least for the differential yields no exact numbers are given in Fig.1 of ref.\textsuperscript{9}. But what ever the outcome of such a comparison would be one seems to await a problem: either the Unruh yield is considerably smaller than that from IDCS, then it becomes difficult to detect this effect since it belongs kinematically to the same region as IDCS. This is also demonstrated by a comparison of Fig. 7 with Fig.1 of Schützhold et al.\textsuperscript{2}. If the yield is stronger one has to explain the quantitative agreement (within a few percent) of experimental data\textsuperscript{15} with the theory for IDCS\textsuperscript{14}. Or, finally, given about the same yield for both effects, the interesting question arises wether the physical backgrounds of Unruh-Hawking radiation and IDCS are the same. If this proves right-and, in fact, such a possibility has been indicated in ref. 15 of Schützhold et al.\textsuperscript{2} the remarkable situation exists, that a signature for what later was called the Unruh effect had been verified experimentally 20 years before Unruh published his seminal paper in 1976\textsuperscript{23}. It might also be enlightening to cite a judgement by A. Ringwald: “Whether ultimately one will call this a verification of the Unruh effect or just basic quantum field theory (QED) is a matter of taste or linguistics.” Adopting this view we recognize a minor
similarity only between Fig.1 of ref.9 and our Fig.7. But we also note severe doubts of several authors, whether Unruh radiation exists at all\textsuperscript{27}.

iii) Finally, we mention that it is well known that there exists a strong relationship between an electromagnetic undulator (EMU) like a laser wave and a magnetic undulator (MU) with magnetic field strength $B_U$ used as an insertion device at synchrotron radiation places\textsuperscript{28-29}. In case of an EMU the MU parameter $K$ is replaced by the Lorentz invariant normalized laser strength $a_L$

$$K = \frac{eB_U\lambda_U}{2\pi mc} \rightarrow a_L = \frac{eE_L\lambda_L}{2\pi mc^2}$$ \hspace{1cm} (4.11)

and the MU period length $\lambda_U$ corresponds to the EMU wavelength $\lambda_L$ via\textsuperscript{28}

$$\lambda_U = \lambda_L/(\cos \alpha_0 + 1/\beta)$$ \hspace{1cm} (4.12)

i.e. for $\alpha_0 = 0$ (head-on collision) the energy $\omega_f$ of the MU fundamental in case of small $K$ is $\omega_f = 4\gamma^2\omega_U = 2\gamma^2\omega_U$ with $\omega_U = 2\pi c/\lambda_U$. For an intensity $I_L = 10^{18}$ W/cm$^2$ one obtains for linearly polarized laser light an electric field strength $E_L = (2I_L/\epsilon_0c)^{1/2} = 2.7 \cdot 10^{12}$ V/m and thus a laser parameter $a_L = 0.85$.

We note that this laser parameter induces an electron acceleration $a$ which corresponds to an Unruh-Hawking temperature\textsuperscript{23}

$$T_{UH} = \hbar a/(2\pi k_B c) = \hbar \omega_L a_L/(2\pi k_B) = 1900 \text{ K}$$ \hspace{1cm} (4.13)

and which is close to that for electrons in modern high energy lepton storage rings\textsuperscript{30} ($k_B$ =Boltzmann constant).

Within the context of our discussion it might be indicated to make a short remark about thermal bathing of electrons in a storage ring. Due to radiative spin-flip transitions within the magnetic lattice of the storage ring electrons become polarized but the polarization stays always below 100%. Bell and Leinaas\textsuperscript{30} have attributed this depolarization to the thermal Unruh effect, thus replacing the standard interpretation in terms of the well-known Sokolov-Ternov effect in QED\textsuperscript{32}. But in a recent paper E.Akhmedov and D.Singleton\textsuperscript{33} have shown, “that the laboratory observer interprets the effect as the Sokolov-Ternov effect while the non-inertial co-moving observer interprets it as the circular Unruh effect. Physically, these two effects are the same.” Thus, also in this case, the Unruh effect has been well known for a long time under a different name and has even been experimentally observed\textsuperscript{34}.

This is in accordance with statements from a very recent review article about the Unruh effect and its applications\textsuperscript{32}. "We shall emphasize that, although it is certainly fine to interpret laboratory observables from the point of view of uniformly accelerated observers, the Unruh effect itself does not need experimental confirmation any more than free quantum field theory does. It might happen, that some observables can be more easily computed and interpreted from the point of view of uniformly accelerated observers using the Unruh effect. This is a matter of convenience and not of principle". In addition, several examples are given by the authors, where first each phenomenon is discussed by plain quantum field theory adapted to inertial observers and then it is shown, how the same observables can be reproduced from the point of view of Rindler observers with the help of the Unruh effect. Specifically they write that “Schützhold et al.\textsuperscript{36} note that a Rindler photon seen to be scattered off a static charge by the Rindler observers should correspond to a pair of correlated Minkowski photons emitted from a uniformly accelerated charge as seen by the inertial observers. Although the correlated radiation of Minkowski photons could be explained by inertial observers using textbook quantum field theory, it is certainly interesting to understand these processes invoking the Unruh effect". The theory of double Compton scattering is nothing else than a description of the generation of Minkowski photon pairs in terms of textbook QED\textsuperscript{15}. Last but not least we quote a conclusion drawn by H.C. Rosu\textsuperscript{24} in his review article ‘Hawking-like Effects and Unruh-like Effects: Toward Experiments?’: “My feeling at the end of the survey is that actually the goal is not so much to try to measure a ‘Hawking’ or a ‘Unruh’ effect. Being ideal concepts/paradigms, what we have to do in order to put them to real work is to make them interfere with the many more ‘pedestrian’ viewpoints.”

The value of $a_L$ compares also well with usual MU parameters $K$. Since for $a_L > 1$ the transverse wiggling motion of the electron within the EMU starts to deviate from an harmonic oscillation with frequency $\omega_L$ due to the increasing importance of a longitudinal Lorentz force which induces an additional oscillation with frequency $2\omega_L$ (figure-of-eight motion\textsuperscript{29}), harmonics in the radia-
Fundamental is given by extended. For large MU parameters K the energy of the analogon between MU and EMU radiation can even be observed (multipole radiation). But it is well known that in strong electromagnetic fields the the electron is better described by Volkov states, since the 4-momentum of a charged particle inside an electromagnetic wave is altered due to continuous absorption and emission of photons. For a charged particle

\[ \omega_f = 2\gamma^2\omega_U / (1 + K^2 + (\gamma\theta)^2) \]  

where \( \theta \) is the emission angle. On the other hand all expressions for single and double Compton scattering discussed above have be obtained for electron plane waves in vacuum. But it is well known that in strong electromagnetic fields the electron is better described by Volkov states, since the 4-momentum of a charged particle inside an electromagnetic wave is altered due to continuous absorption and emission of photons. For a charged particle with 4-momentum \( p_\mu \) outside the field the effective 4-momentum \( q_\mu \) (quasimomentum) inside the field is  

\[ q_\mu = p_\mu - \frac{a_\mu^2}{2(k \cdot p)} k_\mu \]  

and thus \( q_\mu q^\mu = -(1 + a_\mu^2) = -m_{eff}^2 \). Due to the interaction with the laser field the electron is "dressed", i.e. it acquires an effective mass \( m_{eff} > 1 \). Inspecting the formulas of chapter II when all \( p \)'s are replaced by \( q \)'s, the most dramatic effect occurs for the kinematics of eq. (4.1). Instead of (2.5) one obtains in case of inverse IDCS (\( \gamma \gg 1 \))

\[ \omega_2 = \frac{2\gamma^2\omega_L (1 - \beta \cos \alpha) - \omega_1 (1 + a_L^2 + (\gamma\theta_f)^2) - 2\gamma\omega_1\omega_L (1 - \cos \alpha)}{1 + a_L^2 + (\gamma\theta_f)^2 + 2\gamma\omega_L (1 - \cos \alpha)} \]  

and equivalently to eq. (4.1)

\[ \omega_{max} = \frac{2\gamma^2\omega_L (1 - \beta \cos \alpha)}{1 + a_L^2 + (\gamma\theta_f)^2} \]  

which in case of head-on collisions (\( \alpha = \pi \)) becomes

\[ \omega_{max} = \frac{4\gamma^2\omega_L}{1 + a_L^2 + (\gamma\theta_f)^2} \]  

Eq. (4.18) establishes further more the equivalence of EMU-and MU-radiation, see eq. (1.13). This nonlinear QED mass-shift effect\(^{29,36}\) is not small, since for \( a_L = 0.85 \) it would predict a 40% reduction of \( \omega_{max} \). Strictly speaking, this holds for a flat-top shaped laser pulse with a unique strength \( a_L \) only. In case of a Gaussian beam one has an intensity and hence an \( a_L \)-distribution. During its encounter with the laser pulse the electron will emit radiation with a variety of mass-shifts, resulting in a so called "ponderomotive broadening"\(^{35}\) which would strongly effect the single Compton peak in Fig. 7 by increasing its width by approximately 40%. A corresponding shift in case of the 12.4 keV radiation, which we have assumed for fixed targets, turns out to be negligible small. For a lateral extension of the photon beam \( \sigma_{y} = 35 \mu m, 10^{12} \) photons/pulse and a 100 fs pulse duration\(^{29}\) we obtain a pulse intensity of \( I = 5 \cdot 10^{14} W/cm^2 \) and thus a normalized XFEL-strength \( a_L \equiv 2 \cdot 10^{-6} \ll 1 \). If it would be possible to focus the XFEL beam down to the diffraction limit \( \sigma_{y} = \lambda L \)-up to now with a method not known - one could even reach \( a_L = 0.7 \). While there are unambiguous indications of the generation of harmonics in inverse single Compton scattering\(^{35}\) a corresponding signature for the associated mass-shift discussed above has, to our knowledge, not yet been verified. Therefore, we have refused to include this effect in our analysis.

From such an analogon two conclusions can be drawn: 1) it might be useful to look for higher harmonics in inverse IDCS similar to those of inverse single Compton scattering (i.e. nonlinear Thomson scattering\(^{29,36}\)). This, in fact, would be a multi-photon process, i.e. the four vector \( k \) in (2.1) should be replaced by \( nk \) where \( n \) is the number of simultaneously absorbed laser photons\(^{35}\). 2) one may look for entangled photons in energy regions between the peaks of MU-FEL radiation. Due to the excellent brilliance of such a source one might even speculate about some transverse (spatial) coherence of the photon pairs, i.e. a correlation of the amplitudes of pair-states in transverse space of the FEL-beam\(^{38}\), thus realising a kind of “photon pair laser”\(^{12}\). In nonlinear optics such a multiple photon pair-state is called a one- or two-mode squeezed vacuum state\(^{1,40}\).

V. CONCLUSION

It has been demonstrated that IDCS is well suited as a source for entangled X-ray photons, though we admit that the increase of the yield compared to that of CDCS is not dramatic. We remark that the cross section function X of (2.3) has been obtained by averaging over the initial and summing over the final polarization vectors, i.e. the cross section holds for unpolarized radiation only. It would be therefore desirable to repeat the calculation for polarized incident light. In absence of such a calculation we argue as follows: in the rest frame of the electron the relative inelasticity for 180° backscattering is \( \Delta \omega^*/\omega^* = 2 \omega^* \). Thus, if \( 2\omega^* \ll 1 \) we expect that scattering is well described by Thomson scattering. Since
Finally, we comment on some notions. Since the total cross section of ordinary Thomson scattering is about $r_0^2$, this result can also be obtained by classical physics. Thus, Larmor radiation which, of course, includes undulator radiation and its harmonics might be called classical radiation. In contrast, IDCS (whose cross section is proportional to $a_{QED}r_0^2$), or Unruh radiation, can be termed quantum radiation. Certainly, the Unruh effect itself is an important test ground for theoreticians (L.C.B. Crispino et al. call the effect a theoretical laboratory to investigate quantum field theories in non-inertial reference frames, but it seems questionable if it should play an equivalent role for experimentalists. I am indebted to Bernhard Adams and Ralf Schützhold for valuable discussions.

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