Shear viscosity of $\beta$-stable nuclear matter

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Abstract

Viscosity plays a critical role in determining the stability of rotating neutron stars. We report the results of a calculation of the shear viscosity of $\beta$-stable matter, carried out using an effective interaction based on a state-of-the-art nucleon-nucleon potential and the formalism of correlated basis functions. Within our approach the equation of state, determining the proton fraction, and the nucleon-nucleon scattering probability are consistently obtained from the same dynamical model. The results show that, while the neutron contribution to the viscosity is always dominant, above nuclear saturation density the electron contribution becomes appreciable.

Key words:
nuclear matter, effective interaction, neutron stars
21.65.-f, 24.10.Cn, 26.60.-c

1. Introduction

The quantitative description of transport properties of nuclear matter is relevant to the understanding of a variety of neutron star properties. Thermal conductivity is one of the driving factors of the cooling process, while electrical conductivity is relevant to the ohmic dissipation of magnetic fields in the star interior. In rotating stars, a crucial role is also played by viscosity, that determines the possible onset of the gravitational-wave driven instabilities first predicted by Chandrasekhar in the 1970s [1, 2].

Gravitational radiation (GR) is emitted when a non-radial oscillation mode of the star is excited by an internal or external perturbation. In non rotating neutron stars the emission of GR is a dissipative process, leading to the damping of the oscillation, while in rotating stars the effect can be quite different. At the end of the 1970s, Friedman and Schutz [3] proved...
that, due to the mechanism discussed in Refs.\cite{1,2}, all perfect fluid rotating stars are in fact unstable. More recently, Andersson\cite{4} and Friedman and Morsink\cite{5} demonstrated that all the so-called r modes, i.e., the oscillations of rotating stars whose restoring force is the Coriolis force, are driven unstable by GR in all perfect fluid stars. On the other hand, if matter in the star interior does not behave as a perfect fluid, dissipative processes, such as viscosity, can damp the modes responsible of the instability, or even suppress them completely. Hence, knowledge of the viscosity of neutron star matter is required to determine whether a mode is stable or unstable.

The main limitation of most analyses of the damping of neutron-star oscillations\cite{6} lies in the lack of consistency between the dynamical models used to obtain the equation of state (EOS), describing the equilibrium properties of the star, and those employed to describe transport properties. In their seminal paper, Cutler and Lindblom\cite{6} combined a variety of EOS, resulting from different theoretical approaches, with the pioneering estimates of the shear viscosity coefficient of neutron star matter obtained in the 1970s by Flowers and Itoh, who used the Landau-Abrikosov-Khalatnikov\cite{7,8} formalism and the neutron-neutron collision probability estimated from the measured scattering phase shifts\cite{9,10}.

Nuclear many body theory provides a consistent framework to obtain the in-medium nucleon-nucleon (NN) cross section and the transport coefficients of nuclear matter from realistic NN potentials, using either the $G$-matrix\cite{11} or the CBF\cite{12} formalism. In both approaches one can define a well-behaved effective interaction, suitable for use in standard perturbation theory in the Fermi gas basis and allowing for a unified treatment of equilibrium and non-equilibrium properties\cite{11,12,13,14,15}.

The CBF effective interaction has been employed to carry out a calculation of the shear viscosity of pure neutron matter\cite{12}. However, a more realistic model of neutron star matter must allow for the presence of protons and electrons. As matter density increases, the electron chemical potential may also exceed the muon rest mass, making the appearance of muons energetically favorable.

In this Letter, we discuss the generalization of the approach of Ref.\cite{12} to the case of $\beta$-stable matter consisting of neutrons, protons and electrons.
2. Formalism

The application of the Abrikosov-Khalatnikov [8] formalism to the calculation of the shear viscosity of matter consisting of neutrons, protons and electrons in $\beta$-equilibrium was first developed by Flowers and Itoh [9, 10]. Within their approach, the NN scattering rate due to strong interactions is modeled using the measured free space cross section, thus neglecting all modifications caused by the presence of the nuclear medium.

In Refs. [9, 10], the calculation of the transport coefficients of $\beta$-stable matter is carried through a straightforward generalization of the case of pure neutron matter. In a multicomponent system the Boltzmann-Landau equation takes the form

$$ \frac{\partial n_\alpha}{\partial t} + \nabla \cdot \left( n_\alpha \frac{\partial \epsilon_{p\alpha}}{\partial \mathbf{p}} - n_\alpha \frac{\partial \epsilon_{p\alpha}}{\partial \mathbf{p}} \right) = \sum_\beta I_{\alpha\beta}, \quad (1) $$

where $n_\alpha = n_{p\alpha}(\mathbf{r}, t)$ denotes the distribution of quasiparticles of type $\alpha$ ($\alpha = n, p, e$), carrying momentum $\mathbf{p}$ and energy $\epsilon_{p\alpha}$. The form of the collision term in the right hand side of the above equation clearly shows that, in principle, all binary collisions, involving both like and unlike quasiparticles, must be taken into account.

The shear viscosity, defined as the coefficient of the momentum flux tensor appearing in the left hand side of Eq. (1), can be written as [10]

$$ \eta = \eta_n + \eta_p + \eta_e, \quad (2) $$

the contribution associated with quasiparticles of type $\alpha$ being given by [9, 10, 16, 17]

$$ \eta_\alpha = \frac{1}{5} \rho_\alpha m_\alpha^* v_{F\alpha}^2 \tau_\alpha \frac{2}{\pi^2 (1 - \ell_{\alpha\alpha})} C(\ell_{\alpha\alpha}). \quad (3) $$

In the above equation, $\rho_\alpha$, $m_\alpha^*$ and $v_{F\alpha}$ denote the density, effective mass and Fermi velocity, respectively, while the quasiparticle lifetime $\tau_\alpha$ is given by

$$ \tau_\alpha = \frac{4\pi^4}{m_\alpha^* T^2 \sum_\beta m_\beta^* \langle W_{\alpha\beta} \rangle}, \quad (4) $$

where $T$ is the temperature, and

$$ \ell_{\alpha\alpha} = \frac{\sum_\beta \langle W_{\alpha\beta} L_{\alpha\beta}^\alpha \rangle}{\sum_\beta \langle W_{\alpha\beta} L_{\alpha\beta} \rangle}. \quad (5) $$
In Eq. (5), $W_{\alpha\beta}$ denotes the probability of collisions between quasiparticles of type $\alpha$ and $\beta$. In the low temperature limit, underlying the Landau-Abrikosov-Khalatnikov approach, scattering processes can only involve quasiparticles carrying momenta close to the Fermi momentum. As a consequence, at fixed baryon density, $\rho = \rho_p + \rho_n$, and proton fraction $x = \rho_p/\rho$, $W_{\alpha\beta}$ only depends on two angular variables, $\theta$ and $\phi$, and the averages in Eqs.(4) and (5) are defined as

$$\langle F \rangle = \int \frac{d\Omega}{4\pi} F(\theta, \phi). \quad (6)$$

The quantities $L_{\alpha\beta}^\alpha$ and $L_{\alpha\beta}^\beta$ are also functions of the angles $\theta$ and $\phi$. Their explicit expressions, as well as that of the factor $C(\ell_{\alpha\alpha})$, are given in Refs. [9, 10].

3. Results

For any given baryon density, the calculation of the shear viscosity requires the knowledge of the proton fraction $x$, determined by the conditions of $\beta$-equilibrium and charge neutrality

$$\mu_n - \mu_p = \mu_e \quad , \quad \rho_p = \rho_e \quad , \quad (7)$$

where $\mu_{\alpha}$ denotes the chemical potential of quasiparticles of type $\alpha$.

In this work, the proton and neutron chemical potentials have been computed within the Hartree-Fock approximation

$$\mu_{\alpha} = \epsilon_{\alpha}(p_{F\alpha}) \quad , \quad (8)$$

where the Fermi momentum is given by $p_{F\alpha} = (3\pi^2\rho_{\alpha})^{1/3}$, using the single particle spectrum obtained from the CBF effective interaction of Ref. [12] in the Hartree-Fock approximation

$$\epsilon_{\alpha}(p) = x_{\alpha} \left\{ \frac{p^2}{2m} + \rho \sum_{\beta} x_{\beta} \int d^3x \left[ \langle v_{\text{eff}} \rangle_D - \langle v_{\text{eff}} \rangle_E \ell(p_{F\alpha}x) \cos p \cdot x \right] \right\} \quad . \quad (9)$$

In the above equation, $m$ denotes the nucleon mass, $\ell(x) = 3(\sin x - x \cos x)/x^3$ and the spin averaged direct and exchange matrix elements of the effective interaction are given by

$$\langle v_{\text{eff}} \rangle_D = \frac{1}{2} \sum_{\sigma_\alpha \sigma_\beta} \langle \alpha \beta|v_{\text{eff}}|\alpha \beta \rangle \quad , \quad \langle v_{\text{eff}} \rangle_E = \frac{1}{2} \sum_{\sigma_\alpha \sigma_\beta} \langle \alpha \beta|v_{\text{eff}}|\beta \alpha \rangle \quad . \quad (10)$$
Figure 1: Solid line: single particle spectrum evaluated using Eq. (9) and the effective interaction of Ref. [12]. The diamonds show the results of Ref. [21], obtained using the FHNC approach and a realistic nuclear Hamiltonian.

The effective interaction \( v_{\text{eff}} \) is based on a truncated version of the NN potential referred to as Argonne \( v_{18} \) [18], providing an excellent fit of deuteron properties and the full Nijmegen phase shift database. It also includes the effects of interactions involving three- and many-nucleon forces, described according to the approach originally proposed in Ref. [19]. The EOS of symmetric nuclear matter and pure neutron matter obtained using the dynamical model of Ref. [12] turn out to be in fairly good agreement with the results of the state-of-the-art calculations of Ref. [20], carried out using the full Argonne \( v_{18} \) potential.

In Fig. 1, the single particle spectrum of symmetric nuclear matter at equilibrium density obtained from Eq. (9) is compared to the results of Ref. [21], carried out within the Fermi Hyper-Netted Chain (FHNC) approach using a realistic nuclear Hamiltonian.

Figure 2 shows the baryon density dependence of the proton fraction resulting from the numerical solution of Eqs. (7), carried out assuming that electrons can be described using the single particle spectrum of the relativistic Fermi gas. The effective masses \( m_{\alpha}^* \) appearing in Eqs. (3) and (4) can be readily obtained from the single particle energies through

\[
\frac{1}{m_{\alpha}^*} = \frac{1}{p} \frac{d\epsilon(p)}{dp}.
\]
The scattering probabilities employed in our calculations take into account both strong and electromagnetic interaction. Nuclear interactions contributing to $W_{nn}=W_{pp}$ and $W_{np}=W_{pn}$ have been described within the dynamical model of Ref. [12], while the calculation of $W_{ep}=W_{pe}$, $W_{en}=W_{ne}$ and the electromagnetic part of $W_{pp}$ have been carried following Refs. [9, 10].

Figure 3 shows the temperature-independent quantities $\eta_{T} T^{2}$, as well as $\eta T^{2}$, plotted as a function of baryon density. It appears that the proton viscosity $\eta_{p}$ is always very small, due to their low density and mobility. On the other hand, in spite of the fact that $\rho_{p}=\rho_{e}$, the electron viscosity $\eta_{e}$ turns out to be much higher, as electrons are ultra-relativistic. As expected, the dominant contribution is $\eta_{n}$, as the neutron fraction is larger than 90% over the whole range of baryon density. For $\rho \lesssim \rho_{0}$, $\rho_{0}=0.16$ fm $^{-3}$ being the equilibrium density of symmetric nuclear matter, the total viscosity can be identified with the neutron contribution. The electron contribution becomes barely visible only at larger density.

The main difference between the case of a multi-component fluid and that of pure neutron matter, discussed in Ref. [12], lies in the larger number of channels available in collision processes, which implies a shorter quasiparticle lifetime, leading in turn to a lower viscosity. This feature is illustrated by the diamonds of Fig. 3 showing the energy dependence of the viscosity of...
Figure 3: Baryon density dependence of the quantity $\eta T^2$ for protons (dotted line), electrons (dot-dashed line) and neutrons (dashed line) in charge neutral $\beta$-stable matter. The solid line corresponds to the total $\eta T^2$ (see Eq. (2)). For comparison, the diamonds show $\eta T^2$ of pure neutron matter, corresponding to $x_p = 0$.

pure neutron matter, corresponding to $x_p = 0$.

To gauge the dependence of our results on the composition of matter, we have repeated the calculations using a larger proton fraction, obtained by multiplying the results shown in Fig. 2 by a factor two. Note that changing the proton fraction amounts to using a different model of nuclear dynamics. Larger proton fractions correspond to a larger contribution of the symmetry term to nuclear matter energy.

As shown in Fig. 4, in this case the electron viscosity exceeds neutron viscosity at values of baryon density $\rho \sim 0.2 \text{ fm}^{-3}$, just above nuclear matter saturation density, corresponding to an electron fraction of $\sim 8\%$.

4. Conclusions

The results of our work show that inclusion of medium effects on NN scattering leads to a sizable suppression of the collision probability, producing in turn an enhancement of the shear viscosity of nuclear matter.

The role played by screening of the bare NN interaction, mainly due to short range correlations, was already pointed out in Ref. [12] for the case of pure neutron matter. Our study of charge-neutral $\beta$-stable matter consisting of neutrons, protons and electrons, carried out using the same
effective interaction, suggests that, while the proton contribution to viscosity can be safely neglected, the electron contribution is appreciable. In the case of dynamical models predicting a larger proton fraction, typically $x_p \gtrsim 8\%$ at $\rho \sim \rho_0$, it may in fact become dominant at densities exceeding nuclear matter saturation density. Overall, due to the availability of a larger number of reaction channels, the viscosity of $\beta$-stable matter turns out to be lower than that of pure neutron matter.

All the above considerations are based on the tenet that neutrons, protons and electrons behave as normal Fermi liquid. In the region of density and temperature in which neutrons become superfluid, their contribution to the viscosity vanishes and the electron contribution takes over.

In their study of the effect of viscosity on neutron star oscillations, the authors of Ref. [6] took into account the onset of superfluidity writing the viscosity in the form

$$\eta_s = [1 - \Theta(\rho, T)]\eta + \Theta(\rho, T)\eta_e,$$

where

$$\Theta(\rho, T) = \begin{cases} 0 & T > T_c \\ 1 & T < T_c \end{cases},$$

$T_c = T_c(\rho)$ being the critical temperature, taken from Ref.[22]. It has to be pointed out, however, that a fully consistent analysis of the stability of rotating neutron stars requires that the EOS, determining the proton fraction,
the viscosity coefficient and the critical temperature be all obtained from the same dynamical model. The effective interaction approach appears to be ideally suited to pursue this project.

Numerical calculations of the \( ^1S_0 \) superfluid gap carried out using the effective interaction of Ref.\([12]\) yield a critical temperature \( T_c \sim 2 \times 10^{10} \) K at density \( \rho \sim 0.04 \) fm\(^{-3}\), typical of the neutron star inner crust \([23]\), in fairly good agreement with the results of Ref.\([22]\). The extension of this study to the case of pairing in \(^3P_2\) states is currently being carried out.

As a final remark, it is worth mentioning that a more realistic model of neutron star matter should include muons, whose appearance is likely to be energetically favored at densities above nuclear saturation density. However, compared to electrons, muons have larger mass, and therefore lower mobility. As a consequence, taking into account their contribution to the viscosity is not expected to significantly affect the conclusions of our work.

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