A New Benchmark For Evaluation Of Graph-Theoretic Algorithms

Andy B. Yoo† Yang Liu† Sheila Vaidya† Stephen Poole‡

†Lawrence Livermore National Laboratory, Livermore, CA 94551
‡Oak Ridge National Laboratories, Oak Ridge, TN 37831

Abstract

We propose a new graph-theoretic benchmark in this paper. The benchmark is developed to address shortcomings of an existing widely-used graph benchmark. We thoroughly studied a large number of traditional and contemporary graph algorithms reported in the literature to have clear understanding of their algorithmic and run-time characteristics. Based on this study, we designed a suite of kernels, each of which represents a specific class of graph algorithms. The kernels are designed to capture the typical run-time behavior of target algorithms accurately, while limiting computational and spatial overhead to ensure its computation finishes in reasonable time. We expect that the developed benchmark will serve as a much needed tool for evaluating different architectures and programming models to run graph algorithms.

1 Introduction

Graph algorithms have become an increasingly important and widely-used tool in a wide range of emerging disciplines such as web mining, computational biology, social network analysis, and text analysis. Typically, input to these algorithms are the graphs that consist of typed vertices and typed edges that represent the relationships between the vertices. The vertices and edges of the graphs are often associated with some attributes.

These graphs, which belong to a class of graphs called scale-free graphs (or networks) [4], have very complex structures. Furthermore, the graphs that are usually formed by fusing fragmental information obtained from many different sources like web documents and news articles tend to grow in size as more data becomes available, and thus graphs with billions of vertices and edges have become prevalent in practice. Running the graph algorithms on the large and complex graphs in an efficient and scalable way has become an increasingly important and yet challenging problem. This raises an imperative need to identify computer architectures that are best suited for solving graph theoretic problems for large complex graphs.

There are many aspects to consider in an architectural evaluation. An ideal architecture should provide high performance and good scalability for target applications. It should also be energy-efficient, cost-effective, and easy to program. In addition, it should offer reliable, large-capacity storage subsystems in order to process large data sets, for graph theoretic applications in particular. These aspects must be carefully weighed in when evaluating various machine architectures, in conjunction with the run-time behavior and resource

*This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344. A technical report of this work is also available in LLNL-TR-422519.
usage pattern of the target applications. Benchmarks have been the most commonly used architectural eval-
uation tool, since they are specifically designed to represent the key characteristics of the target algorithms 
and hence correctly reproduce their run-time behavior. A well-designed benchmark is critical to the accurate 
architectural evaluation.

Although innumerable benchmarks have been developed for a wide spectrum of applications [18, 14, 5], 
little attempts have been made in developing benchmarks for graph-theoretic applications. Many exist-
ing graph-theoretic benchmarks consist of graph data sets designed mainly for the algorithmic evaluation 
of specific graph algorithms [27, 6]. Recently, a new benchmark suite, called DARPA High Productivity 
Computer Systems (HPCS) Scalable Synthetic Compact Application (SSCA) Graph Analysis Benchmark 
(commonly known as SSCA#2 benchmark), was developed [3]. The SSCA#2 benchmark suite is comprised 
of a synthetic scale-free graph generator [10] and four kernels each of which is designed based on a small set of 
fundamental graph-related algorithms. The benchmark has found some successes as it is the only benchmark 
currently available designed specifically for architectural evaluation for graph-theoretic problems. However, 
the SSCA#2 benchmark has some notable drawbacks that make the benchmark inadequate to use in any 
rigorous performance study. First, the benchmark is not a complete representative of a wide variety of 
commonly used graph algorithms and therefore fails to provide comprehensive models for their algorithmic 
behavior. Some of the kernels in the SSCA#2 benchmark are not graph-related, but closer to to graph 
construction and min-max finding. Remaining kernels, although they certainly address fundamental and 
very important graph problems, covers only a fraction of existing graph algorithms. Second, the design flaws 
in some of the kernels prevent the benchmark from accurately modeling the real execution-time character-
istics of the targeted algorithms. For example, its kernel calculating the betweenness centrality scores [7] 
approximates the calculation by finding shortest paths between only randomly selected vertices due to high 
computation cost. This may result in inaccurate betweenness measures, and more importantly, this may 
alter the real run-time memory access characteristics of the original algorithm, since the subgraph formed 
by the shortest paths found may have significantly different structures from the original graph.

We address these issues and propose a new graph-theoretic benchmark. The benchmark is comprised of 
a graph generator and a suite of kernels. The graph generator synthesizes scale-free graphs using a very fast 
algorithm based on preferential attachment method [10, 47]. The kernels are comprehensive and designed to 
represent a wide range of important graph algorithms, including traditional graph algorithms such as search, 
combinatorial optimization, and metrics computation and graph-mining methods, like subgraph clustering 
(also known as community finding) and spectral graph algorithms, that have gained more popularity in recent 
years. The benchmark is designed to accurately model the run-time characteristics of target algorithms with 
minimal computational cost. Furthermore, its specific and detailed design allows the objective evaluation of 
the strengths and weaknesses of different machine architectures and programming models.

The paper is organized as follows. Section 2 describes some preliminaries and graph representations used 
in this paper. The proposed benchmark is described in Section 4 in greater detail. Related work is reported 
in Section 5 followed by concluding remarks in Sections 6.

2 Preliminaries

2.1 Definitions

The proposed benchmark uses undirected, weighted, colored graph $G = (V, E)$, where $V$ is the set of vertices 
and $E$ is the set of edges. Each vertex is given an integer that uniquely identifies the vertex. The vertices 
are colored in gray scale, where the darkness of the vertex is controlled by a real value (e.g., 0 for white and 
1 for black). That is, a vertex $v_i \in V$ is a pair $\langle i, c_i \rangle$, where $0 \leq i < |V|$ and $0 \leq c_i \leq 1$. Here, $i$ and $c_i$ 
represent the ID and the color of the vertex $v_i$. An edge $e = \langle u, v, * \rangle$ is said to be incident to $u$ and $v$. The 
degree of a vertex $v$ is defined as the number of (undirected) edges that are incident to $v$. An edge $e_i \in E$
is the tuple \( (u, v, w_i) \), where \( u, v \in V \) and \( w_i \) is the weight of the edge such that \( 0 \leq w_i \leq 1 \). The graph is undirected if for every \( (u, v, w) \in E \), there exists \( (v, u, w) \in E \). There are no self-loops in \( G \). That is, for every \( (u, v, w) \in E \), \( u \neq v \). Also, the \( G \) does not have multiple edges between any two vertices \( u \) and \( v \). A graph \( G' = (V', E') \) is a subgraph of \( G \), when \( V' \subseteq V \) and \( E' \subseteq E \). A subgraph \( G' = (V', E') \) is called a clique, if for any two vertices \( u, v \in V' \), \( (u, v, \ast) \in E' \).

### 2.2 Graph Representations

As stated earlier, the performance of a typical graph algorithm is mainly governed by its memory access pattern. Therefore, the graph representation has major impact on the performance of the graph algorithms. Since there can be a large number of graph representations and each of the representations has different memory behavior, it is very important to utilize the most appropriate graph representation schemes to accurately model the run-time behavior of the graph algorithms.

We adopted two of the most common graph representations, adjacency list and compact sparse row (CSR), to store graphs in our benchmark. We chose these graph representations methods for several reasons. First, because it is critical for our benchmark to be used to evaluate a variety of machine architecture with different memory capacities, we have excluded those representations, such as adjacency matrix, that usually consume a lot of memory space to store given graph. Furthermore, the graph of interest in this research, scale-free graph, is a sparse graph, and hence we mainly focused on the graph representations ideal for sparse graphs. Second, these are two of the most widely used methods for graph representation. Finally, these two methods represent graph representations at the extreme ends of spectrum of memory access patterns, where the linked list representation exhibits more random memory accesses than the CSR representation. We believe using these two widely different graph representations will provide users with ability to measure the best- and worst-case memory performance of the target architecture.

The adjacency-list graph representation is shown in Figure 1. As Figure 1 shows, the color and adjacent vertices of the vertices are stored in two arrays, Vertex Color and Adjacency. The color of a vertex \( v_i \) is stored in Vertex Color[\( i \)]. The adjacent edges and vertices of \( v_i \) are maintained as a linked list. Adjacency[\( i \)] points to the first element in the linked list. Each node in the linked list contains the ID of a vertex, \( u \), adjacent to \( v_i \) and the weight of edge that connects \( v \) and \( u \). Given a vertex ID, \( j \), the adjacent vertices and the weights of corresponding incident edges can be obtained by simply following the linked list.

This graph representation is not very efficient in terms of storage space, since an additional space is required to store a pointer for each edge. Further, a graph algorithm that processes a graph in this representation is likely to exhibit random memory access behavior and hence suffers poor performance, since...
accessing adjacent vertices needs chasing pointers to objects that are highly likely dispersed in the heap area. It should be noted, however, that it is never our intention to design the most efficient graph benchmark. Rather, our goal is to provide a good architectural evaluation tool to users.

The CSR graph representation is one of the most popular graph representation techniques, especially ideal for representing sparse graphs such as scale-free graphs. Its popularity can be attributed to the fact that it requires minimal storage space to store graphs while reducing random memory accesses. The CSR graph representation is depicted in Figure 2. In the CSR graph representation, all the edges and their associated weights are stored in two arrays, EdgeList and EdgeWeights. Therefore, the size of these arrays is equal to the total number of edges in the graph (i.e., |E|). The adjacency information is maintained through another array, Adjacency. Unlike the adjacency list representation, where each element in the Adjacency array points to list of edges that are incident to the corresponding vertex, in the CSR representation an entry in the Adjacency array maintains the starting positions in the EdgeList array for all the edges that are incident to the corresponding vertex. In Figure 2, for example, the adjacent vertices of \( v_i \) is stored in \( \text{EdgeList}[p] \) \( \ldots \text{EdgeList}[q-1] \), and the adjacent vertices of \( v_j \) is stored in \( \text{EdgeList}[q] \ldots \text{EdgeList}[l-1] \), and so on. Also, the degree of the vertex \( v_i \) is \( q - p \). The vertex colors are also stored in VertexColor as in the same was as in the adjacency list representation.

Figure 3 presents a sample graph and its corresponding CSR graph representation\(^1\). The vertex colors and edge weights are depicted in blue and red in Figure 3a, respectively. A directed graph is used in the example for the simplicity of presentation. In an undirected graph, each (undirected) edge is translated into two edges in reverse direction and represented in the same way as the directed graph case. In this example, vertices 0 and 5 do not have any outgoing edges. This is indicated by the fact that the entries in the Adjacency array for these vertices have the same value as the ones immediately following them. It should be also noted that the Adjacency array keeps one additional entry at its end basically to indicate the array boundary for the last vertex (vertex 7 in the example).

3 Graph Generator

Recent studies have revealed that many important real-world graphs belong to a special class of graphs called \textit{scale-free graphs}. Some of these scale-free graphs include World-Wide Web [8], Internet [21, 24, 37, 41], electric power grids [46], citation networks [29, 38, 42, 43], telephone call graphs [2], and e-mail network [20].

\(^1\)The adjacency list representation is not shown here, because it is relatively straightforward to represent the graph in this data structure. Interested readers should refer to [13]
It is very important to understand the behaviors of different graph algorithms on the key real-world graphs, and hence, the proposed benchmark includes a graph generator that constructs synthetic graphs that possess key characteristics of the real-world scale-free graphs [47].

The graph generator is based on well-known preferential attachment graph model [4] and offers significant advantages over the R-MAT based graph generator [10] of the SSCA#2 benchmark. Our graph generator has low computational complexity of $O(m)$, compared to $O(m \cdot \log n \cdot \log m)$ of the graph generator in SSCA#2 benchmark, where $n$ and $m$ represent the number of vertices and edges of a graph, respectively. Furthermore, our graph generator was shown to generate more realistic scale-free graphs and can be easily parallelized, compared to the SSCA#2 graph generator [47]. The preferential attachment method [19], which our graph generator is based on, follows well-known generative Barabasi-Albert (BA) model [4]. In the BA model, a scale-free graph is constructed by repeatedly creating a new vertex and then attach it to one of the existing vertices. Here, the existing vertex is selected with a probability distribution which is proportional to its current degree $^2$.

The algorithm begins with constructing a clique of a given size. Vertices in the initial clique will form a set of high-degree vertices called hubs. Selecting an existing vertex based on its current degree can incur very high computation overhead, especially for larger graphs, since entire vertex set needs to be scanned for each newly added vertex. Our graph generator maintains a list that contains the end points of all the edges in partially constructed graph (steps 5 and 15 in 1) in order to speed up this selection process. The basic idea here is to select a vertex from the list of end points. Since the number of occurrences of each vertex in the list is equal to its degree, the probability that a vertex is selected is proportional to its degree. This enables the creation of edges in a constant time and reduces the overhead considerably.

The degree of a newly added vertex follows a uniform distribution with given average vertex degree as its mean (step 10). This is to ensure that the generated graph does not have uniform structure. Each edge in the graph is associated with a weight between 0 and 1, and similarly, each vertex in the graph has a color, which is also a real value between 0 and 1 (steps 9 and 14). These values are used by some of the kernels in the benchmark, as explained later.

Parallelizing the graph generator is also very simple. In the parallel graph generator, each processing element (PE) (e.g., processes or threads) maintain its own list of end points in the same manner as in the sequential generator. The source vertices of the edges in a list are owned by a processing element that maintains the list. However, the destination vertices may belong to other processing elements. When the generation local edges is complete, a final graph synthesized by merging all the edges with the same

---

Footnote: The BA model is also called rich get richer model, since in this model those rich vertices (vertices with high degree) have higher probability to be connected to newly created vertices and hence become richer.
destination end points. Since processing elements create local edges independent from each other, this algorithm is embarrassingly parallel. Interested readers should refer [47] for more detailed description of the parallel graph generator.

4 Description of Graph Benchmark

We designed the proposed benchmark with a set of clear design goals. First, the benchmark should be comprehensive in that it should cover a set of fundamental and more commonly used graph algorithms so that it can present a collection of varying resource usage and execution patterns typical graph algorithms exhibit during their execution. Second, the benchmark should be designed in such a way that it captures and models only the essential algorithm characteristic of the target algorithms. This requires a thorough understanding of target algorithms. Third, the benchmark should be very specific about algorithms, graph representations, and all data structures used by the algorithms so that ones can make unbiased evaluation of target architecture. Finally, the benchmark should be computationally tractable so that runs can finish in a reasonable amount of time.

As the first step in designing the proposed benchmark, we surveyed the graph algorithms that are commonly used in various important applications in practice. The survey was performed to make out benchmark as comprehensive as possible by ensuring that the new benchmark covers an extensive list of key graph algorithms. Furthermore, this study should help us to great extent better understand and correctly capture the run-time behavior of the target graph algorithms. The surveyed graph algorithms are classified into 5 groups. The algorithms in each category are modeled, mainly focusing on their algorithmic and run-time characteristics, and translated into a kernel. These kernels of the proposed benchmark are described in detail in the following. The detailed description of the proposed benchmark design is given below.

4.1 Kernel 1: Graph Search

4.1.1 Graph search algorithms

Algorithms in this class pertain to traversing paths in given graph and finding the information associated with the paths and (typically) vertices on the paths. These algorithms are fundamental graph algorithms that have been applied to solving a variety of scientific and engineering problems. In this research, we surveyed many classical graph algorithms that perform a form of graph search, including breadth-first search (BFS), depth-first search (DFS), Dijkstra’s algorithm, Prim’s algorithm, Kruskal’s algorithm, and Bellman-Ford algorithm.

We will not discuss these algorithms in further detail in this paper, as the detailed description of these algorithms is widely available in the literature [13], but we should point out key characteristics of these algorithms governing their run-time behavior. Although they deal with the vertices and edges in a given graph, the primarily operate on sets obtained from the graph. The BFS algorithm, for example, accesses two sets, which maintain the vertices at the current level and all the visited vertices, when it expands the search to the next level. Similarly, other algorithms in this class also access elements in sets in one way or another. Here, the way the elements in the sets are maintained and accessed dictates the run-time memory access patterns of these algorithms. If a set is unordered (as in the case of BFS), then most likely, accesses to the elements in the sets can be done sequentially, while accesses to ordered sets involve more random memory accesses. In addition to the set operations, adjacency list, accesses to which are usually in random order, also affects their run-time memory behavior.

They certainly can be classified into more groups. The classification is done to meet our design goals, especially the computational and space constraints. These have excluded some of the graph algorithms.
It has been generally believed that graph algorithms, especially the graph search algorithms, have poor cache utilization due to their random memory accesses. Use of efficient data structures to maintain the sets (e.g., priority queues) and represent graphs (e.g., compressed sparse row (CSR) graph representation), however, may allow the search algorithms to access the memory in more sequential manner, in contrast to the popular belief. We aim at capturing this behavior correctly in this kernel.

4.1.2 Benchmark description
Algorithm 2 describes the kernel 1 in detail. The design of the kernel is essentially based on the single-source shortest paths problem [13]. Here, the length of a path is the sum of weights of the edges that comprise the path. The input to the kernel is graph, $G(V, E)$ to be searched and the source from which search begins. The kernel returns an array of size $|V|$, where $i$-th entry has the length of path from $s$ to $v_i$. There can be many variations of algorithms to find all the shortest paths from a single source, but the proposed kernel combines the behavior of the BFS algorithm and ordered set accesses.

A queue of vertices is used to maintain a set of unvisited vertices and denoted as $Q$. Since the algorithm needs to find a vertex with the smallest current path length from $s$ (line 10 in Algorithm 2), the elements in $Q$ should be maintained in an increasing order of their current path lengths. That is, $Q$ is an ordered set.

When a vertex $v$ is selected from $Q$, its path length is the minimum path length from $s$ to $v$ and will not be improved by any other paths. Hence, the vertex $v$ is removed from $Q$. Then, all of its neighboring vertices are added to $Q$, if they are not already in $Q$, followed by relaxing their path lengths. The relaxation of a vertex basically updates the current length of path to the vertex, if there exists a path of shorter length to the vertex [13]. The relaxation is performed in lines 26 to 30 in Algorithm 2.

4.2 Kernel 2: Spectral Graph Analysis

4.2.1 Spectral graph problem
The graph algorithms in this class of algorithms are related to the spectral graph theory [15, 16, 17]. Essentially, the spectral graph theory is the study of properties of a graph in relationship to the eigenvalues and eigenvectors of its corresponding adjacency matrix. In other words, the algorithms in this class treat a graph as a matrix and aim at finding the eigenvalues/eigenvectors of the matrix. Prominent exemplary algorithms in this class include page rank [36] and random walk with restart (RWR) [40]. These algorithms compute eigenvectors, each value in which is typically interpreted as the relative importance or relevance of the corresponding vertex to other vertices in the graph.

The eigenvalue/eigenvector problem has been studied quite extensively and a number of eigensolvers are available [41, 25]. Also, it is well known that the algorithms in this class generally do not scale well and their performance and scalability are greatly affected by graph representation method. We attempt to model the essence of the run-time characteristics of these algorithms in this kernel.

4.2.2 Benchmark description
The proposed kernel relies on the simple power method [26] to find an eigenvalue for given graph. The power method finds the largest (dominant) eigenvalue and its corresponding eigenvector. Though simple, the power methods works pretty well for well-conditioned sparse graphs such as the graphs targeted by the proposed benchmark. This decision is justified by our design objective to model the behavior of the algorithms in the domain of the spectral graph theory while guaranteeing the completion of the computation in a reasonable time.

The input to the kernel are graph $G(V, E)$ and threshold values that basically used to limit the number of iterations. The kernel produces an eigenvector for the largest eigenvalue as output. The eigenvector is
initialized with the same constant, but any random numbers can be used. The function Multiplication in lines 22 to 24 in the Algorithm 3 performs a matrix-vector multiplication. The convergence is determined by the condition, $\|X^{i+1} - X^i\|$. Here, $\|X\|$ denotes the $L_1$ norm of $X$. If the ratio is smaller than given threshold, $\epsilon$, then the iteration stops. Because the power method can converge very slowly, we limit the number of iterations by another input parameter, $m$, which represents the maximum number of iterations allowed.

It should be noted that implementation of this kernel must not convert given graph representation into an equivalent adjacent matrix representation. The reason being for this restriction is that although the adjacency matrix graph representation will facilitate certain operations in the kernel (i.e., lines 16 to 17 in the Algorithm 3), accessing the matrix will have memory access patterns that are very different from the accesses to other graph representations.

4.3 Kernel 3: Vertex and Edge Accesses

4.3.1 Adjacency finding

The most frequently used graph operation is without doubt, given a vertex, finding a set of its adjacent vertices. In fact, most of the graph algorithms need to access adjacency data. The graph search algorithms, for example, must access adjacent vertices to a given vertex to traverse paths. Those algorithms that explore the graph structure, such as maximal clique finding and subgraph pattern matching, also need the vertex adjacency. These algorithms typically access the data structures that store the adjacency information repeatedly to find neighboring vertices. Therefore, accesses to the adjacency information dominates the run-time memory access patterns of many graph algorithms, especially the graph algorithms that operate primarily on the adjacency data. The repeated adjacency finding operations usually result in a memory access behavior that is characterized by a combination of random and sequential memory accesses.

A graph transformation algorithm called graph hierarchicalization [1] is of particular interest because this algorithm repeatedly accesses the adjacency information, and hence, our kernel for the vertex and edge accesses is modeled based on this algorithm. The graph hierarchicalization basically transforms a graph into a hierarchy of graphs, where the graph at each level provides a view of the graph in different granularity. More specifically, the graph hierarchicalization algorithm gradually converts the given graph into a graph of smaller size by iteratively coalescing vertices and edges of the given graph. Mapping information is also created to map the vertices and edges in two adjacent graphs in the hierarchy, although this is not included in this kernel. Since the graph hierarchicalization can abstract and reduce the size of a graph, it is mainly used for graph summarization and visualization.

Another aspect we try to capture by this kernel is the behavior of the algorithms that require frequent updates to the given graph. We incorporated the updates by replacing the coalesced vertices that with a (mega) vertex that represent the coalesced vertices and reconnecting appropriate edges in this kernel.

4.3.2 Benchmark description

Input to the kernel is graph $G(V, E)$ and a constant $\gamma$, which is used to control the number of coalescing performed. Basically, the $\gamma$ is used to compute the number of vertices from which the coalescences start. As stated above, the number of coalescences is determined by $\gamma$, which in turn is used to compute the number of vertices with which the coalescences are performed (lines 3 and 7).

In each coalescence, a vertex selected from the graph in random. Then, the vertex is coalesced with its adjacent vertices. The coalesced vertices are kept in $S$. Also, the average color of the vertices in $S$ is computed for the new vertex that represents $S$. The lines from 13 to 20 rearranges the edges that are connected to those vertices in $S$ in such a way that they are connected to the new vertex for $S$. Once coalescing vertices and reconnecting edges, all the vertices in $S$ are replaced by the new vertex representing
S (lines from 21 to 24). Since this kernel alters the structure of the original graph, it is necessary to keep track of the changes made so that the original graph can be restored (line 27). It is allowed to save the entire graph disk, if bookkeeping of the changes consume too much resources in terms of time and space.

### 4.4 Kernel 4: Graph Metric

#### 4.4.1 Graph metric computation

Various metrics are measured and utilized in the area of graph mining and graph structure analysis. Some of common graph metrics include degree distribution, clustering coefficients, and modularity. Clustering coefficient of a vertex, proposed by Watts and Strogatz [45], is a strong indicator of how densely the vertex is connected with its neighboring vertices and can be used to identify the centroids of potential clusters [11]. Similarly, the modularity [35] is a metric that directly indicates the density of a subgraph and is widely used in many community finding algorithms [31, 32, 35, 33, 34, 12, 22]. The betweenness centrality [7] of a vertex, on the other hand, is a good indication of the likelihood of the vertex belonging to a cluster.

Computation of these graph metrics generally shows random memory access patterns, since it is closely related to and heavily relies on adjacency finding. We selected one of the most popular and simplest graph metrics, clustering coefficients, as a base model in designing this kernel.

#### 4.4.2 Benchmark description

This kernel takes a graph $G(V, E)$ and a constant $m$ as its input. The constant $m$ is used to control the number of vertices for which the kernel finds clustering coefficients. The computation of the clustering coefficient (in lines 6 to 16 in Algorithm is straightforward and we will omit its description. It should be noted, however, that the kernel counts each edge as a directed edge in its calculation. This is because each undirected edge is represented as two directed edges of reverse direction in the graph representations used in our benchmark.

### 4.5 Kernel 5: Global Optimization

#### 4.5.1 Global optimization problem for graphs

There are a class of graph algorithms that finds solutions by optimizing certain objective functions. Many NP-Complete graph algorithms that solve combinatorial optimization problems belong to this class of algorithms, although they are not of interest in this research mainly due to their high computational complexity. We are mainly interested in those graph algorithms which follow greedy method [13]. These algorithms compute solutions by iteratively constructing a sequence of (partial) solutions. At each iteration, a new solution is constructed in a way to improve given objective function. A well-known graph partitioning algorithm MeTis [28], for instance, partitions a given graph into a fixed number of partitions in such a way to minimize the cuts between partitions to reduce the communications on message-passing parallel computers. A classic community finding algorithm by Newman and Girvan constructs a dendrogram of vertices that results in the largest modularity value [35].

The objective functions employed by these algorithms are usually global measures that require access and process entire graph to compute. This global accesses in fact make it very difficult to parallelize this class of algorithms. The algorithms in this class are modeled in our benchmark to capture this algorithmic behavior. Our benchmark models a well-known community finding algorithm called AUTOPART [2]. This algorithm iteratively divides a given graph into a set of communities in such a way that the value of an entropy-based objective function decreases. The AUTOPART algorithm is an ideal representative algorithm for this class of algorithms to model, because it incurs relatively low computational overhead while exhibiting the run-time behavior common to the algorithms in this class.
4.5.2 Benchmark description

The input to the kernel is graph $G(V,E)$ and two constants, $\alpha$, and $m$. The $\alpha$ is mainly used to reduce the search space by filtering out a set of vertices whose color is less than $\alpha$ (line 3 in Algorithm 6). The $m$ is used to control the number of iterations to ensure the computation finishes within a reasonable amount of time.

The kernel basically partitions the vertices of $G$ into a set of clusters. The vertices are grouped together in such a way that the grouping will improve the overall density of groups. The density of a group is defined as the ratio of the number of internal edges to that of external edges. Here an internal edge refers to edge that connects two vertices in the given group of vertices. External edge, on the other hand, is an edge one of whose endpoints does not belong to the group.

The kernel maintains a set, $C$, of groups of vertices. It first selects a group $g \in C$ that has the minimum density value. Then, it tries to split $g$ into two by iteratively checking moving a vertex from $g$ to the new group will improve the overall density (lines from 12 to 24). The objective function used by kernel is defined by a function called $Objective$ in the Algorithm 6. The objective function sums up the value of Shannon’s binary entropy function [39] for each group in $C$. The kernel continues the split of $C$ until it cannot split any of the groups in $C$ (lines 18 to 20) or its iteration reaches given $m$.

5 Related Work

Although there are numerous graph algorithms reported in the literature, little work has been done in graph-centric benchmark development. The most relevant work in this area is probably the recently developed benchmark suite, called DARPA High Productivity Computer Systems (HPCS) Scalable Synthetic Compact Application (SSCA) Graph Analysis Benchmark, which is commonly known as SSCA#2 benchmark [3]. The SSCA#2 benchmark suite is comprised of a synthetic scale-free graph generator based on RMAT method [10] and four kernels. Each of the kernels is designed based on a small set of fundamental graph-related algorithms. It is the only benchmark currently available for architectural evaluation for graph-theoretic problems. However, the SSCA#2 benchmark suffers significant shortcomings. One of the biggest drawbacks is that the benchmark is not comprehensive. Also, among the four kernels in the benchmark, kernels 1 and 2 are not directly graph-related. Remaining kernels suffer design flaws that prevent the benchmark from accurately modeling the real execution-time characteristics of the targeted algorithms. In addition, its use of RMAT as a synthetic graph generator results in longer graph generation time and graphs that do not resemble common real-world scale-free graphs.

Gokhale et. al. [23] have evaluated the use of existing and emerging computing architectures, storage technologies, and programming models to solve data-intensive problems in various scientific and engineering disciplines. One of the data-intensive applications considered in their research was graph algorithms. In particular, they were interested in the graph algorithms that access very large graphs stored in external storage devices. In an effort to investigate the use of the new storage devices and identify what is needed for the efficient and scalable computation of graph applications, they developed a graph-centric benchmark that operates on out-of-core graphs. This benchmark measures the performance and scalability of the graph ingestion and search.

A library that contains a set of the most commonly used graph algorithms has been developed [30]. The library, called Boost Graph Library (BGL), enables the reuse of graph algorithms and data structures by providing the users with a generic interface that allows access to a graph’s structure while hiding the details of the implementation. Therefore, any graph algorithms or libraries that implement this interface will be able to access the BGL generic algorithms and other algorithms that use the interface. The BGL offers some of the fundamental graph algorithms that include breadth- and depth-first search algorithms, Dijkstra’s shortest path algorithm, Kruskal’s and Prim’s minimum spanning tree algorithms, and find strongly
connected components algorithms. In addition to the graph algorithms, the BGL supports common graph representations: adjacency_list and adjacency_matrix. The BGL itself can be used as a benchmark, since it supports a wide range of fundamental graph algorithms and common data structures to represent graphs. However, it lacks a means to fine control the behavior of these graph algorithms, which any real benchmark is expected to provide.

6 Conclusions

Graph has become an increasingly important and widely-used tool in a wide range of emerging disciplines such as web mining, computational biology, social network analysis, and text analysis. These graphs are usually very large and have very complex structures. Running the graph algorithms on these large and complex graphs in an efficient and scalable way has become an increasingly important and yet challenging problem, which raises an imperative need to identify computer architectures best suited for running graph algorithms. There exists, however, no good benchmark that is designed specifically for graph algorithms.

We develop and present a new graph-theoretic benchmark in this paper to address this issue. The benchmark is comprised of a very efficient graph generator and six kernels. We thoroughly studied a large number of traditional and contemporary graph algorithms reported in the literature to have clear understanding of their algorithmic and run-time characteristics. The developed kernels are comprehensive and correctly model the common behavior of a wide range of important graph algorithms. We expect that the developed benchmark will serve as a much needed tool for evaluating different architectures and programming models to run graph algorithms.

References

[1] J. Abello and J. Korn. Mgv: a system for visualizing massive multidigraphs. Transactions on Visualization and Computer Graphics, 8(1):21–38, 2002.

[2] W. Aiello, F. Chung, and L. Lu. A random graph model for massive graphs. In Proceedings of the thirty-second annual ACM symposium on Theory of computing (STOC ’00), pages 171–180, New York, NY, USA, 2000.

[3] D. Bader and K. Madduri. Design and implementation of the hpcs graph analysis benchmark on symmetric multiprocessors. In Proc. 12th Intl Conf. on High Performance Computing, 2005.

[4] A.-L. Barabasi and R. Albert. Emergence of scaling in random networks. Science, 286:509, 1999.

[5] L. Benchmarks. http://www.netlib.org/linpack/.

[6] BHOSLIB. Bhoslib: Benchmarks with hidden optimum solutions for graph problems, 2009. http://www.nlsde.buaa.edu.cn/kexu/benchmarks/graphbenchmarks.htm.

[7] U. Brandes. A faster algorithm for betweenness centrality. Journal of Mathematical Sociology, 25:163–177, 2001.

[8] A. Broder, R. Kumar, F. Maghoul, P. Raghavan, S. Rajagopalan, R. Stata, and A. Tomkins. Graph structure in the web: Experiments and models. In 9th World Wide Web Conference, 2000.

[9] D. Chakrabarti. Autopart: parameter-free graph partitioning and outlier detection. In PKDD ’04: Proceedings of the 8th European Conference on Principles and Practice of Knowledge Discovery in Databases, pages 112–124, New York, NY, USA, 2004. Springer-Verlag New York, Inc.
[10] D. Chakrabarti, Y. Zhan, and C. Faloutsos. R-mat: A recursive model for graph mining. In *In SDM*, 2004.

[11] A. Clauset. Finding local community structure in networks. *Physical Review E*, 72:026132, 2005.

[12] A. Clauset, M. E. J. Newman, and C. Moore. Finding community structure in very large networks. *Phys. Rev. E*, 70(6):066111, Dec. 2004.

[13] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein. *Introduction to Algorithms*. MIT Press, Cambridge, MA, second edition, 2001.

[14] T. P. P. Council. [http://www.tpc.org/](http://www.tpc.org/).

[15] D. Cvetković, M. Doob, and H. Sachs. *Spectra of Graphs*. 1980.

[16] D. Cvetković, M. Doob, and H. Sachs. *Recent Results in the Theory of Graph Spectra (Annals of Discrete mathematics series)*. North-Holland, 1988.

[17] D. Cvetković, P. Rowlinson, and S. Simić. *Eigenspaces of Graphs*. 1997.

[18] D. H. Bailey et al. The NAS Parallel Benchmarks. Technical Report NASA Technical Memorandum 103863, NASA Ames Research Center, 1993.

[19] S. N. Dorogovtsev and J. F. F. Mendes. *Evolution of Networks*. Oxford University Press, 2004.

[20] H. Ebel, L.-I. Mielsch, and S. Bornholdt. Scale-free topology of e-mail networks. *Physical Review E*, 66:035103, 2002.

[21] M. Faloutsos, P. Faloutsos, and C. Faloutsos. On power-law relationships of the internet topology. In *SIGCOMM*, pages 251–262, 1999.

[22] M. Girvan and M. E. J. Newman. Community structure in social and biological networks. *Proc. Natl. Acad. Sci.*, 99:7821, 2002.

[23] M. Gokhale, J. Cohen, A. Yoo, W. M. Miller, A. Jacob, C. Ulmer, and R. Pearce. Hardware technologies for high-performance data-intensive computing. *Computer*, 41(4):60–68, 2008.

[24] R. Govindan and H. Tangmunarunkit. Heuristics for internet map discovery. In *IEEE INFOCOM 2000*, pages 1371–1380, Tel Aviv, Israel, March 2000.

[25] V. Hernandez, J. E. Roman, and V. Vidal. Slepc: A scalable and flexible toolkit for the solution of eigenvalue problems. *ACM Trans. Math. Softw.*, 31(3):351–362, 2005.

[26] I. Ipsen and R. M. Wills. Analysis and computation of google’s pagerank. In *Proc. 7th IMACS International Symposium on Iterative Methods in Scientific Computing*, 2005.

[27] D. Johnson and M. Tric. Cliques, coloring and satisability. In *Second DIMACS Implementation Challenge*. American Mathematical Society, Oct. 1996.

[28] G. Karypis and V. Kumar. *MeTis: Unstructred Graph Partitioning and Sparse Matrix Ordering System, Version 2.0*, 1995.

[29] P. L. Krapivsky, S. Redner, and F. Leyvraz. Connectivity of growing random networks. *Physical Review Letters*, 85:4629, 2000.
[30] A. Lumsdaine, D. Gregor, B. Hendrickson, and J. Berry. Challenges in parallel graph processing. *Parallel Processing Letters*, 17(1):5–20, 2007.

[31] M. E. J. Newman. From the cover: The structure of scientific collaboration networks. *Proceedings of the National Academy of Sciences*, 98:404–409, 2001.

[32] M. E. J. Newman. Scientific collaboration networks ii. shortest paths, weighted networks, and centrality. *Phys. Rev. E*, 64:016132, 2001.

[33] M. E. J. Newman. Detecting community structure in networks. *European Physical Journal B*, 38:321–330, May 2004.

[34] M. E. J. Newman. Fast algorithm for detecting community structure in networks. *Phys. Rev. E*, 69(6):066133, June 2004.

[35] M. E. J. Newman and M. Girvan. Finding and evaluating community structure in networks. *Phys. Rev. E*, 69(2):026113, Feb. 2004.

[36] L. Page, S. Brin, R. Motwani, and T. Winograd. The pagerank citation ranking: Bringing order to the web, 1999.

[37] R. Pastor-Satorras, A. Vazquez, and A. Vespignani. Dynamical and correlation properties of the internet. *Physical Review Letters*, 87:258701, 2001.

[38] S. Redner. How popular is your paper? an empirical study of the citation distribution. *The European Physical Journal B*, 4:131, 1998.

[39] C. Shannon. Prediction and entropy of printed english. Technical report, The Bell System Technical Journal, 1951.

[40] H. Tong, C. Faloutsos, and J.-Y. Pan. Random walk with restart: fast solutions and applications. *Knowl. Inf. Syst.*, 14(3):327–346, 2008.

[41] Trilinos. The trilinos project. [http://trilinos.sandia.gov](http://trilinos.sandia.gov).

[42] C. Tsallis and M. P. de Albuquerque. Are citations of scientific papers a case of nonextensivity? *The European Physical Journal B*, 13:777, 2000.

[43] A. Vazquez. Statistics of citation networks, 2001. cond-mat/0105031.

[44] A. Vazquez, R. Pastor-Satorras, and A. Vespignani. Large-scale topological and dynamical properties of internet. *Physical Review E*, 65:066130, 2002.

[45] D. J. Watts and S. H. Strogatz. Collective dynamics of ——small-world— networks. *Nature*, 393(6684):440–442, June 1998.

[46] D. J. Watts and S. H. Strogatz. Collective dynamics of small-world networks. *Nature*, 393:440–442, 4 June 1998.

[47] A. Yoo and K. Henderson. Parallel massive scale-free graph generators. In *Proc. SC2006*, 2006.
Algorithm 1 Graph Generation Algorithm
1: // Create a graph G(V, E), where V and E are sets of vertices and edges of G
2: // L is list of edges
3: Create a clique C of size c
4: for all ∀ edges (u, v) ∈ C do
5: \[ L ← L ∪ \{u, v\} \]
6: end for
7: for i ← c to \(|V| − 1\), where \(|V|\) denotes the number of vertices in G do
8: \[ \text{id}[u] ← i \]
9: \[ \text{color}[u] ← \text{uniform}(0, 1) \]
10: \[ d ← \text{uniform}(1, 2 \cdot D), \text{where } D \text{ is average vertex degree of } G \]
11: for j ← 0 to \(d − 1\) do
12: \[ k ← \text{uniform}(0, \mid L\mid) \]
13: \[ v ← L_k \]
14: \[ \text{weight}[(u, v)] ← \text{uniform}(0, 1) \]
15: \[ L ← L ∪ \{u, v\} \]
16: end for
17: end for
Algorithm 2 Kernel 1: Graph Search

1: Input: $G(V, E)$ and a vertex $s$
2: Output: Array $W$, where $W[v]$ is the weight of path from $s$ to $v$
3: Create an array $W$ of size $|V|$
4: for all $\forall u \in V$ do
5: \quad $W[u] \leftarrow \infty$
6: end for
7: $W[s] \leftarrow 0$
8: $Q \leftarrow \{s\}$
9: while $Q \neq \emptyset$ do
10: \quad $u \leftarrow \text{Extract}_\text{Min}(Q, W)$
11: \quad for all $\forall v \in \text{Adjacent}(u)$ do
12: \quad \quad if $v \not\in Q$ then
13: \quad \quad \quad $Q \leftarrow Q \cup \{v\}$
14: \quad \quad end if
15: \quad \quad $w \leftarrow \text{weight of } e(u, v)$
16: \quad \quad $\text{Relax}(W, u, v, w)$
17: \quad end for
18: end while

19: function $\text{Extract}_\text{Min}(Q, W)$
20: \quad $v \leftarrow v' \in Q$ such that $W[v'] \leq W[u]$ for $\forall u \in Q$
21: \quad $Q \leftarrow Q - \{v\}$
22: \quad return $v$
23: end function

24: function $\text{Relax}(W, u, v, w)$
25: \quad if $W[v] > W[v] + \text{Weight}(u, v)$ then
26: \quad \quad $W[v] \leftarrow W[u] + \text{Weight}(u, v)$
27: \quad end if
28: end function

29: function $\text{Adjacent}(u)$
30: \quad return $\{v \mid (u, v, *) \in E\}$
31: end function
Algorithm 3 Kernel 2: Spectral Graph Analysis

1: Input: $G(V,E), \epsilon$ and $m$
2: Output: An array $X$
3: Let $n = |V|$
4: Create an array $C$ of size $n$
5: for all $i$, where $i = 0 \ldots n-1$ do
6:   $C[i] = k$, such that $k = \sum_{j=0}^{n-1} \text{Weight}(i, j)$
7: end for
8: for all $j$, where $j = 0 \ldots n-1$ do
9:   $X[0]_j \leftarrow \frac{1}{n}$
10: end for
11: $t \leftarrow 0$
12: $i \leftarrow 0$
13: $\theta \leftarrow \infty$
14: while $t < m$ and $\theta > \epsilon$ do
15:   $X[i+1] \leftarrow \text{Multiplication}(X[i])$
16:   $X[i+1] \leftarrow \frac{X[i+1]}{\|X[i+1]\|}$
17:   $\theta \leftarrow \frac{\|X[i+1] - X[i]\|}{\|X[i]\|}$
18:   $i \leftarrow i + 1$
19:   $t \leftarrow t + 1$
20: end while

21: function Multiplication($X$)
22: return $A$ such that $A_k = z$, where $z = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} w \cdot X_j$ and $(i,j,w) \in E$
23: end function
Algorithm 4 Kernel 3: Vertex and Edge Accesses

1: Input: $G(V, E)$ and $\gamma$ (reduction ratio)
2: Output: $G'(V', E')$, where $G'$ is a new coalesced graph
3: $n \leftarrow \gamma \cdot |V|$
4: $i \leftarrow 0$
5: $V' \leftarrow V$
6: $E' \leftarrow E$
7: while $i < n$ do
8:   Let $u$ be vertex randomly selected from $V'$
9:   $S \leftarrow \{u\} + \text{Adjacent}(u)$
10:  Let $u'$ be a new vertex representing $S$
11:  $c \leftarrow \sum_{v \in S} \text{Color}[v] / |S|$
12:  $\text{Color}[u'] \leftarrow c$
13:  for all $\forall v \in S$ do
14:     for all $\forall e = (v_1, v_2, w) \in E'$ such that $v = v_1$ do
15:       Update $e = (v, v_2, w)$
16:     end for
17:     for all $\forall e = (v_1, v_2, w) \in E'$ such that $v = v_2$ do
18:       Update or $e = (v_1, v, w)$
19:     end for
20:  end for
21:  for all $\forall v \in S$ do
22:     $E' \leftarrow E' - \{v\}$
23:  end for
24:  $E' \leftarrow E' \cup u'$
25:  $i \leftarrow i + 1$
26: end while
27: Restore $G$
Algorithm 5 Kernel 4: Graph Metric

1: Input: \( G(V, E) \) and \( m \)
2: Output: An array \( CC \) that contains the clustering coefficients of \( m \) vertices
3: Create array \( CC \) of size \( m \)
4: \( i \leftarrow 0 \)
5: for all \( i < m \) do
6:   Let \( u \in V \) be a vertex randomly selected from \( G \)
7:   \( S \leftarrow \text{Adjacent}(u) \)
8:   \( c \leftarrow 0 \)
9:   for all \( \forall v \in S \) do
10:      \( S' \leftarrow \text{Adjacent}(v) \)
11:      for all \( \forall v' \in S' \) do
12:         if \( v' \in S \) then
13:            \( c \leftarrow c + 1 \)
14:         end if
15:      end for
16:      \( CC[u] \leftarrow \frac{c}{|S| \cdot (|S| - 1)} \)
17:   end for
18: \( i \leftarrow i + 1 \)
19: end for
Algorithm 6 Kernel 5: Global Optimization

1: Input: $G(V, E)$, $\alpha$, and $m$
2: Output: $k$, the number of clusters that maximizes objective function
3: Let $G'(V', E')$ be a graph, where $V' = \{v \in V \mid \text{Color}(v) > \alpha\}$ and $E' = \{e = (u, v, w) \in E \mid u, v \in V\}$
4: $i \leftarrow 0$
5: $k \leftarrow 1$
6: $g_1 \leftarrow V'$
7: $C_1 = \{g_1\}$
8: while $i < m$ do
9:     $g_{\text{min}} \leftarrow g_j$ such that $g_j = \min_{g \in C_i} \text{Density}(g)$
10:    $C_i \leftarrow C_i - g_{\text{min}}$
11:    $g_{\text{new}} \leftarrow \emptyset$
12:    for all $v \in g_{\text{min}}$ do
13:        $g_{\text{new}} \leftarrow g_{\text{new}} \cup \{v\}$
14:    end for
15:    if Objective($C_i \cup g_{\text{min}}$) > Objective($C_i \cup (g_{\text{min}} - \{v\}) \cup (g_{\text{new}} \cup \{v\})$) then
16:        $g_{\text{min}} \leftarrow g_{\text{min}} - \{v\}$
17:        $g_{\text{new}} \leftarrow g_{\text{new}} \cup \{v\}$
18:    end if
19:    if $g_{\text{new}} = \emptyset$ then
20:        Stop
21:    else
22:        $C_i \leftarrow C_i \cup g_{\text{min}} \cup g_{\text{new}}$
23:        $k \leftarrow k + 1$
24:    end if
25:    $i \leftarrow i + 1$
26: end while
27: function Density($g$)
28:    $S_i \leftarrow \{e = (u, v, w) \in E' \mid u \in g \text{ and } v \in g\}$
29:    $S_e \leftarrow \{e = (u, v, w) \in E' \mid e \notin S_i \text{ and } (u \in g \text{ or } v \in g)\}$
30:    $d \leftarrow \frac{|S_i|}{|S_e|}$
31: end function
32: function Objective($C$)
33:    return $\sum_{g \in G} \text{H(Density}(g))$
34: end function
35: function H($p$)
36:    return $-p \cdot \log_2 p - (1 - p) \cdot \log_2 (1 - p)$
37: end function