Gravitational Higgs Mechanism: The Role of Determinantal Invariants

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Abstract

The Higgs mechanism for gravity, as proposed by ’t Hooft in arXiv:0708.3184[hep-th], can be augmented by including determinantal invariants. We analyze the effects of determinantal invariants in such a set up. We find that the part of the potential that depends on the determinantal invariants, if obtains a specific exponential form in terms of its argument, may not affect the graviton mass calculated.
1 Introduction

Spontaneous local symmetry breaking acquires mass to the vector bosons via Higgs mechanism. This argument is quite similar to give a mass to the graviton via gravitational Higgs mechanism upon spontaneous diffeomorphisms breaking. Lately there have been demanding interests on exploring the gravitational Higgs mechanism. Moffat proposed the spontaneous symmetry breaking in the local Lorentz invariance due to vacuum expectation values with different studies [1]. ’t Hooft [2] studied the Brout-Englert Higgs mechanism using four scalar field to fix the gauge in the four-dimensional spacetime, by playing the role of preferred flat coordinates. We see Kakushadze [3] derived an equation for Pauli-Fierz mass term in terms of potential by using Einstein-Hilbert action to the linear order, with spontaneously broken diffeomorphisms by scalars. Very recently Demir and Pak [4] refound an equation for the mass term by using a more general action to quadratic order, with diffeomorphisms breaking via vacuum expectation values.

There are also several other studies considering gravitational Higgs mechanism with different context [5], [6], [7], [8], [9], [10], [11].

The Pauli-Fierz theory [12] linearized about Minkowski background is ghost (negative energy) free. For this theory the mass term in the lagrangian must have the form

\[-\frac{m^2}{4}(h_{\mu\nu} h^{\mu\nu} - \xi h^2)\] (1)

the trace component \( h \equiv h_{\mu}^{\mu} \) decouple for \( \xi = 1 \). To have a physically meaningful massive graviton, the non-unitary degrees of freedom must be eliminated.

The aim of this paper is to consider the Einstein-Hilbert action with two potentials, introduced in the next section, in the linearized background upon diffeomorphisms spontaneously broken by four scalars as in ’t Hooft setup without non-unitary states for massive graviton. Mainly the effect of determinantal potential on the Pauli-Fierz mass term gives physically meaningful graviton mass for a specific solution of that potential. Notation that used in this paper can be defined as follows; the induced metric field \( g^\phi_{\mu\nu} \), and its determinant \( det g^\phi_{\mu\nu} \) are introduced by the effect of the scalar coordinates \( \phi^a(x) = mx^a (a = 0, 1, 2, 3) \).

\[ g^\phi_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b \] (2)
\[ \sqrt{-detg^\phi_{\mu\nu}} = \epsilon^{\alpha\beta\gamma\rho}\partial_\alpha\phi^0\partial_\beta\phi^1\partial_\gamma\phi^2\partial_\rho\phi^3 \] (3)

here \( \epsilon^{\alpha\beta\gamma\rho} \) is Levi-Civita alternating symbol, and completely anti-symmetric tensor density of weight +1. In the weak gravitational field the metric tensor \( g_{\mu\nu} \), can be expand up to the linear order of \( h_{\mu\nu} \)

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \] (4)

its inverse

\[ g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} \] (5)

and its determinant

\[ g = det(g_{\mu\nu}) = -1 - h \] (6)
The metric determinant $g$ is a scalar density of weight +2. The flat induced metric, its inverse, and its determinant with the same argument can be defined respectively as follows

$$g^\phi_{\mu\nu} = m^2 \eta_{\mu\nu} \quad (7)$$

$$g^{(\phi)\mu\nu} = \frac{1}{m^2} g^\mu\nu \quad (8)$$

$$g^\phi = \det(g^\phi_{\mu\nu}) = -m^8 \quad (9)$$

by considering scalar fields as

$$\phi^a = m x^a$$

The outline of the paper is as follows. The action, which includes the Einstein-Hilbert term in addition to kinetic potential [3] and the potential of determinantal invariant is introduced in section 2. From this action constraint on the potentials in the massless phase is given in section 3, and the linearized equation of motion for massive graviton is derived in section 4. Finally it is completed with a conclusion in section 5.

2 Action for graviton

From the set up of [3] and [4] one can consider the following action:

$$S = M_p^{D-2} \int d^D x \sqrt{-g} [R - V(y) + F(f)] \quad (10)$$

where $R = g_{\mu\nu} R^{\mu\nu}$ is the curvature scalar, and

$$y = g^\phi_{\mu\nu} g^{\mu\nu}, f = \det g^\phi_{\mu\nu} / \det g_{\mu\nu} \quad (11)$$

are kinetic and determinantal invariants, respectively. 't Hooft [2] in the case of the four-dimension used the four scalar fields to fix the gauge

$$\phi^a = m x^a \quad (12)$$

In this paper all discussions will be in the four dimensional spacetime, $D = 4$. Thus, in the case of four dimension, the variation of equation (10) w.r.t the metric $g_{\mu\nu}$ gives

$$\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) =$$

$$V'(y) g^\phi_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \left[F(f) - \frac{2m^8}{g} F'(f) - V(y)\right] \quad (13)$$

where the primes denotes the derivatives of potentials with respects to their invariants.
3 Massless case

In the massless phase of graviton we have
\[ \phi^a = 0, \quad g^\phi_{\mu \nu} = 0, \]
and \( g_{\mu \nu} = \eta_{\mu \nu} \). In this case the curvature scalar \( R \) vanish with the constrain on the potentials as follows
\[ F(0) - V(0) = 0 \]
(15)
The vacuum solution of potentials will be discussed in the next section.

4 Massive case

In the Minkowski background \( (g_{\mu \nu} = \eta_{\mu \nu}) \) by contracting the equation (13), we have
\[ y V'(y) + 2 \left[ F(f) + 2m^8 F'(f) - V(y) \right] = 0 \]
(16)
From this equation of motion \( y \) can be identified as the function of potentials \( F(f), V(y) \) and their first derivatives as follows
\[ y = \frac{2 \left[ V(y) - F(f) - 2m^8 F'(f) \right]}{V'(y)} \]
(17)
Denoting such a solution as \( y_* \), so, for this solution we have \( y_* = 4m^2 \), and also \( f_* = m^8 \). Then we have
\[ y_* = \frac{2 \left[ V(y_*) - F(f_*) - 2m^8 F'(f_*) \right]}{V'(y_*)} \]
(18)
If we set
\[ F(f) + 2m^8 F'(f) = 0 \]
(19)
the equation (17) becomes same as the Kakushadze’s (24) in [3]. It is very clear to see that, the solution is exponential for \( F(f) \):
\[ F(f) = Ae^{-(1/2m^8)f} \]
(20)
Where \( A \) is a constant with respect to \( f \). Einstein’s equation of motion satisfies the general covariance principle since it has tensorial form. So the physical quantities should be coordinate independent. Under the coordinate transformation
\[ x^\mu \rightarrow x^\mu - \varepsilon^\mu \]
where \( \varepsilon^\mu \) is infinitesimal. The scalar fluctuation can be gauged away by using the diffeomorphisms
\[ \delta \phi^a = \nabla_\mu \phi^a \varepsilon^\mu \]
(22)
\[ \delta h_{\mu \nu} = \nabla_\mu \varepsilon_\nu + \nabla_\nu \varepsilon_\mu \]
(23)
after breaking the diffeomorphisms spontaneously by
\[ \delta \phi^a = 0. \] (24)

Considering the linearized Einstein equation (field equations for \( h_{\mu\nu} \)) propagating in the Minkowski background, we have
\[ y_{\mu\nu} \equiv m^2 \eta_{\mu\nu}, \det g^a_{\mu\nu} \equiv -m^8 \] (25)
\[ y = m^2 \eta_{\mu\nu} g^{\mu\nu} = 4m^2 - m^2 h + \ldots \] (26)
\[ f = -m^8 / g = m^8 - m^8 h + \ldots \] (27)

Hence the linearized form of equation (13) reads:
\[ (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) = \]
\[ \frac{1}{4} \eta_{\mu\nu} [V(y_*) - F(f_*) - 8m^8 F'(f_*) - 4m^4 V''(y_*) - 4m^16 F''(f_*)] h \]
\[ + \frac{1}{2} [F(f_*) + 2m^8 F'(f_*) - V(y_*)] h_{\mu\nu} \] (28)

For decouple of scalar ghost state, the first term of the right hand side of equation (28) must produce
\[ V(y_*) - 4m^4 V''(y_*) - F(f_*) - 8m^8 F'(f_*) - 4m^16 F''(f_*) = 0 \] (29)

We can write equation (28) in the general form
\[ (\partial_\alpha \partial_\nu h_\alpha^\mu + \partial_\alpha \partial_\mu h_\alpha^\nu - \partial_\mu \partial_\nu h - \partial_\alpha \partial_\mu h_{\mu\nu} - \eta_{\mu\nu} \partial_\beta \partial_\alpha h^{\beta\alpha} + \eta_{\mu\nu} \partial_\gamma \partial_\delta h^{\gamma\delta}) \]
\[ = m_g^2 [\xi \eta_{\mu\nu} h - h_{\mu\nu}] \] (30)

by introducing
\[ \xi = \frac{1}{2} \left( \frac{2m^4 [V''(y_*) + m^{12} F''(f_*)] + 3m^8 F'(f_*)}{V(y_*) - F(f_*) - 2m^8 F'(f_*)} \right) \] (31)
\[ m_g^2 = V(y_*) - F(f_*) - 2m^8 F'(f_*) \] (32)

where \( m_g \) is the mass of graviton. It is clear to see that, in the limit of \( m_g^2 \rightarrow 0 \), the solution of homogenous form of differential equation (32) is equal to the (20). After eliminating scalar ghost and vector ghost states, the equation of motion (30) becomes
\[ h = 0 \] (33)
\[ \partial^\mu h_{\mu\nu} = 0 \] (34)
\[ \partial_\alpha \partial_\mu h_{\mu\nu} - m_g^2 h_{\mu\nu} = 0 \] (35)
The equation of motion (35) describes the massive graviton (spin-2) particle without ghost state in the linearized approximation. In this situation the equations of motion are in the form of [3] and [4], but the equation (32) for Pauli-Fierz combination of mass term is shifted as follows

\[ - \delta m_{\gamma}^2 = F(f_*) + 2m^8 F'(f_*) \] (36)

This shift is proportional to \( F(f) \), its first derivative, and the mass of the scalar. It is interesting to see, if this shift in mass is ignored, then the solution for \( F(f) \) is exponential as mentioned in equation (20). Namely, the effects of the determinantal invariants can be avoided if the corresponding potential term has the form in the same equation. Further insight into the potential one can see that the linearized form must be

\[ F(f) = Ae^{1/2(h-1)} \] (37)

Also to determine the nature of potentials; for example, the non-trivial of vacuum solution, with the exponential form of \( F(f) \), relates two potentials to the cosmological constant, \( F(0) = V(0) \equiv \Lambda \), if \( A = \Lambda \). Hence, one can determine the vacuum phase of the potentials.

5 Conclusion

The results indicates that with the proposed action for massive graviton is not different from the results of [3] and [4], but the Pauli-Fierz mass term, by adding the potential of determinantal invariant \( F(f) \) to the Einstein-Hilbert action with potential of kinetic invariant \( V(y) \), becomes as in equation (32). Subsequently, the potential \( F(f) \), affecting the Pauli-Fierz mass term, is exponential when \( \delta m_{\gamma}^2 = 0 \). Thus, if one has the potential of determinantal invariant as in equation (20), the graviton mass does not effected by the exponential form of that potential. In the vacuum phase the solution for determinantal potential is constant, and equal to the cosmological constant, \( \Lambda \).

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