GAMMA-RAY STUDIES OF BLAZARS: SYNCHRO-COMPTON ANALYSIS OF FLAT SPECTRUM RADIO QUASARS

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ABSTRACT

We extend a method for modeling synchrotron and synchrotron self-Compton radiations in blazar jets to include external Compton (EC) processes. The basic model assumption is that the blazar radio through soft X-ray flux is nonthermal synchrotron radiation emitted by isotropically distributed electrons in the randomly directed magnetic field of outflowing relativistic blazar jet plasma. Thus, the electron distribution is given by the synchrotron spectrum, depending only on the Doppler factor $\delta_D$ and the mean magnetic field $B$, given that the comoving emission region size scale $R_b \lesssim c\delta_D t_v/(1+z)$, where $t_v$ is the variability time and $z$ is the source redshift. Generalizing the approach of Georgopoulou, Kirk, & Mastichiadis to arbitrary anisotropic target radiation fields, we use the electron spectrum implied by the synchrotron component to derive accurate Compton-scattered $\gamma$-ray spectra throughout the Thomson and Klein–Nishina regimes for EC scattering processes. We derive and calculate accurate $\gamma$-ray spectra produced by relativistic electrons that Compton-scatter (1) a point source of radiation located radially behind the jet, (2) photons from a thermal Shakura–Sunyaev accretion disk, and (3) target photons from the central source scattered by a spherically symmetric shell of broad-line region gas. The calculations of broadband spectral energy distributions from the radio through $\gamma$-ray regimes are presented, which include self-consistent $\gamma\gamma$ absorption on the same radiation fields that provide target photons for Compton scattering. The application of this baseline flat spectrum radio/$\gamma$-ray quasar model is considered in view of data from $\gamma$-ray telescopes and contemporaneous multiwavelength campaigns.

Key words: galaxies: active – galaxies: jets – gamma rays: theory – methods: analytical – radiation mechanisms: non-thermal

Online-only material: color figures

1. INTRODUCTION

A class of radio-loud blazar active galactic nuclei (AGNs) that emit luminous fluxes of $\gtrsim 100$ MeV–GeV $\gamma$ rays was discovered with the EGRET on the Compton Gamma Ray Observatory (Hartman et al. 1992, 1999; Fichtel et al. 1994). This result clarified the nature of 3C 273, which was first identified as a $\gamma$-ray emitting AGN in COS-B satellite data (Hermsen et al. 1977). The $\gamma$ rays from blazars are certainly nonthermal in origin and associated with the radio jets formed by the supermassive black holes that power these sources. The largest subclass of EGRET AGNs is moderate redshift ($z \approx 1$) flat-spectrum radio quasars (FSRQs) with blazar properties, including apparent superluminal motion, rapidly variable optical emission, high polarization, and intense broadened optical emission lines. Another subclass of the EGRET AGNs consists of $\gamma$-ray emitting BL Lacertae (BL Lac) objects, which are generally at lower redshifts ($z \approx 0.1$–0.3) and, by definition, have weak or absent optical emission lines in their spectra. The X-ray selected BL Lac (XBL) subset were discovered to have weak or absent optical emission lines in their spectra. Another subclass of the EGRET AGNs consists of $\gamma$-ray emitting flat-spectrum radio quasars (FSRQs) with blazar properties, the common superluminal nature of the first identified $\gamma$-ray blazars, namely 3C 273, 3C 279, and PKS 0528+134, led Dermer et al. (1992) to propose a Compton-scattering origin for the $\gamma$ rays. In this model, jet electrons Compton-scatter accretion-disk photons that intercept the jet plasma. The nonthermal jet electrons can also scatter internal synchrotron photons to produce a synchrotron self-Compton (SSC) component (Bloom & Marscher 1996). Given the broadened emission lines in the spectra of FSRQs, accretion-disk radiation scattered by the surrounding gas of the broad-line region (BLR) will provide a further source of target photons to be scattered to $\gamma$-ray energies (Sikora et al. 1994), as will radiation from a surrounding dusty torus (Kataoka et al. 1999; Blażejowski et al. 2000). The accretion-disk and scattered radiation will attenuate jet $\gamma$-rays through $\gamma\gamma$ pair-production attenuation (Becker & Kafatos 1995; Blandford & Levinson 1995). Expressions for the $\gamma$-ray spectral energy distributions (SEDs) of blazars produced by Compton scattering processes have been derived and calculated for many specific models of the black hole/blazar jet environment. In the case of external accretion-disk photons as the target photon source, where the accretion disk is described by an optically thick, geometrically thin thermal Shakura & Sunyaev (1973) accretion disk, Compton-scattered $\gamma$-ray spectra were calculated in the Thomson regime by Dermer & Schlickeiser (1993, 2002). Calculations of the Thomson-scattered spectra for a quasi-isotropic target radiation field formed by BLR gas or hot dust were made by Sikora et al. (1994), Dermer et al. (1997), and Blażejowski et al. (2000). Detailed numerical calculations, including both accretion-disk and scattered radiation fields, have been made by, e.g., Kusunose & Takahara (2005), Böttcher & Bloom (2000), and Böttcher & Reimer (2004).

Compton scattering in the Klein–Nishina regime is not so simple to treat compared to analyses restricted to the Thomson regime, but is unavoidable for blazar analysis in the era of

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the Fermi Gamma-Ray Space Telescope (formerly known as GLAST) and the ground-based γ Cherenkov telescopes. For surrounding isotropic radiation fields in the stationary frame of the blazar AGN, Georganopoulos et al. (2001) suggested to transform the comoving electron distribution to the stationary frame and then scatter the target photons to γ-ray energies, using the formula first derived by Jones (Jones 1968; Blumenthal & Gould 1970). This approach is generalized in this paper to surrounding anisotropic radiation fields.

The usual spectral modeling approach proceeds by injecting power-law electrons and evolving these particles while they produce the output synchrotron and Compton-scattered radiation (e.g., Dermer & Schlickeiser 1993; Böttcher et al. 1997). For example, Moderski et al. (2005) calculate electron energy evolution and spectral formation throughout the Thomson and Klein–Nishina regimes for different ratios of synchrotron and isotropic radiation field energy densities. They show that reduced Compton losses in the Klein–Nishina regime compared to synchrotron losses can lead to spectral hardening of the synchrotron component in the optical/X-ray regime (noted earlier by Dermer & Atoyan 2002). A difficulty in this approach is that the electron energy-loss rate depends on the photon spectrum of the comoving radiation field, not just the total radiation energy density, and this field evolves with time. The modeler is faced with the prospect of simultaneously fitting the synchrotron and Compton components. The acceleration scenario may well be oversimplified, and non–power-law particle injection distributions could be more realistic than power-law injection spectra, e.g., due to nonlinear effects in Fermi acceleration. Moreover, a separation between the acceleration and radiation zones may not be justified.

Here, we extend a method of blazar analysis recently proposed for TeV blazars (Finke et al. 2008) that avoids these difficulties. For a standard γ-ray blazar model, where isotropically distributed electrons spiral in a randomly oriented magnetic field with mean magnetic field strength B in the fluid frame, the measured synchrotron flux directly reveals the electron spectrum responsible for the synchrotron radiation. The only uncertainties are the mean magnetic field B, the comoving size scale R′ c of the emitting region, and the Doppler factor δD = [Γ(1 − β cos θ)]−1 (Γ = 1/√1 − β2 is the bulk Lorentz factor of the outflow). With this electron spectrum, we then Compton-scatter target photons of the surrounding radiation fields using the head-on approximation to the total Compton cross section (Dermer & Schlickeiser 1993), valid when the electron Lorentz factor γ ≫ 1. This generalizes the approach of Georganopoulos et al. (2001) to surrounding anisotropic radiation fields. The temporally evolving electron spectrum in blazars can be derived in this approach from simultaneous multil wavelength blazar data. Values of B, δD, and jet power can then be deduced. The related treatment for XBLs applied to PKS 2155–304, including more details about the derivation of the electron spectrum from the synchrotron component, the derivation and calculations of the SSC component, and internal γ-ray opacity by the synchrotron photons, is given by Finke et al. (2008).

Analysis of blazar SEDs using this approach is presented in Section 2, where formulae to calculate Compton-scattered internal and external radiation and a δ-function approximation for γγ opacity from the internal radiation field are given. Derivations of the Compton-scattered spectrum for specific examples of external radiation fields consisting of a monochromatic point source of radiation radially behind the jet, a Shakura–Sunyaev disk model, and a model BLR radiation field are derived in Section 3. Discussion of the results is found in Section 4.

2. ONE-ZONE SYNCHROTRON/SYNCHROTRON SELF-COMPTON MODEL WITH γγ OPACITY

We consider a one-zone model for blazar flares. Multiple zones could still be allowed, but the product of the duty cycle and number of zones would have to be small enough that interference of emissions from the different regions would still permit rapid variability. In this case, the emission would still predominantly arise from a single zone. Distinct zones could also emit the bulk of their radiation in different wavebands. In this case, the copatriality assumption often made in blazar modeling would not apply. In this regard, correlated variability data are essential to test the underlying assumptions made when a one-zone model is employed. Slowly varying radio/IR synchrotron and hard X-ray and low-energy γ-ray Compton emissions could involve extended emission regions.

A radiative event from the source emission region that varies on a comoving timescale t′ c ≥ R′ c/ c is related to the observed variability timescale through the relation t c = δDtc/(1 + z), where z is the redshift; thus the comoving blob radius is R′ c ≲ cδDtc/(1 + z). The inequality allows us to neglect light-travel time effects from different parts of the emitting volume and avoid integrations over source volume. Within this zone, the nonthermal electrons with isotropic pitch angle distribution are described by the total comoving electron number spectrum N′ c(γ′) in terms of the comoving electron Lorentz factor γ′. The magnetic field is assumed to be randomly oriented in the comoving fluid frame. The relativistic electrons that gyrate in this field radiate nonthermal synchrotron radiation, observed as the low-energy component in blazar SEDs.

Note that throughout this paper, two versions of the Heaviside function are used: H(x) = 0 for x < 0 and H(x) = 1 for x ≥ 0; as well as H(x; x1, x2) = 1 for x1 ≤ x ≤ x2 and H(x; x1, x2) = 0 everywhere else.

2.1. Synchrotron and Self-Compton Components

The νFν synchrotron radiation spectrum can be approximated by the expression

\[ f_{e}^{\text{syn}} \approx \frac{e}{6\pi d_{L}^{2}} c\sigma_{T}U_{B}^{1/3}N_{*}^{\delta D}(\gamma_{s}), \]

(1)

where

\[ \gamma_{s} = \left( \frac{\epsilon(1 + z)}{\delta D} e_{B}^{\epsilon} \right)^{1/3}. \]

(2)

dL = dL(z) is the luminosity distance, c is the speed of light, \( \sigma_{T} \) is the Thomson cross section, z is the source redshift, and the comoving magnetic-field energy density of the randomly oriented comoving field with comoving mean intensity \( B_{c} \) is \( U_{B} \equiv B^{2}/8\pi \). We use \( \epsilon \) and \( e' \) to refer to the dimensionless photon energy in the observer and comoving frame, respectively.

Here and throughout this paper, unprimed quantities refer to the observer’s frame and primed quantities refer to the frame comoving with the AGN’s jet, with the exception being \( B_{c} \), the comoving magnetic field. Inverting this expression gives the comoving electron distribution

\[ N_{c}(\gamma_{s}) = V_{e}^{\delta D} f_{e}^{\text{syn}} \approx \frac{6\pi d_{L}^{2} N_{*}^{\delta D}}{c\sigma_{T}U_{B}^{1/3}}. \]

(3)
where

\[ \epsilon_{\text{syn}} = \frac{\delta p \epsilon_B \gamma'^2}{1 + z}, \]  

(4)

\( \epsilon_B = \frac{B}{B_{\text{cr}}} \) is the ratio of \( B \) and the critical magnetic field \( B_{\text{cr}} = m_e c^2/eh \approx 4.41 \times 10^{13} \) G (Dermer & Schlickeiser 2002), and \( V'_{\text{p}} = \frac{4 \pi R_{\text{g}}^3}{3} \) is the comoving volume of the blob. Note that \( U_B = \epsilon_B^2 U_{\text{rel}} = \epsilon_B^2 B_{\text{cr}}^2/8\pi \). Equation (3) gives a good representation to the source electron distribution when the \( \nu F_{\nu} \) spectral index \( a < 4/3 \) (i.e., for spectra softer than \( a = 4/3 \), adopting the convention \( f_\nu \propto \epsilon^a \)) and away from the high-energy cutoff of the spectrum (see Finke et al. 2008, for comparison).

The SSC \( \nu F_{\nu} \) flux is given by

\[ f_{\nu}^{\text{SSC}} = \frac{\delta \epsilon_{\text{D}}}{\delta L} \epsilon' L_{\text{SSC}}^{\epsilon'}(\epsilon', \Omega_\epsilon). \]  

(5)

The formula of Jones (1968; see also Blumenthal & Gould 1970) gives the SSC \( \nu F_{\nu} \) flux,

\[ f_{\nu}^{\text{SSC}} = \frac{3}{4} c \sigma_T \epsilon_{\nu} \frac{\delta \epsilon_{\text{D}}}{\delta L} \int_{0}^{\infty} d\epsilon' \frac{u(\epsilon')}{\epsilon'^2} \times \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} d\gamma' \frac{N'(\gamma')}{\gamma'^2} F_C(q', \Gamma_\epsilon), \]  

(6)

where

\[ F_C(q', \Gamma_\epsilon) = \left[ 2q' \ln q' + (1 + 2q')(1 - q') \right] + 1\left( \frac{\Gamma_\epsilon q'}{2} \right)^2 \left( (1 + \Gamma_\epsilon q') - (1 - q') \right) H \left( q'; \frac{1}{4\gamma'^2}, 1 \right), \]  

(7)

\[ q' = \frac{\epsilon'/\gamma'}{\Gamma_\epsilon(1 - \epsilon'/\gamma')}, \quad \text{and} \quad \Gamma_\epsilon = 4\epsilon'/\gamma'. \]  

(8)

The synchrotron photons provide a target radiation field with spectral energy density

\[ u(\epsilon') = \epsilon' m_e c^2 n_{\text{sync}}(\epsilon') = \frac{3d_\epsilon^2 f_{\nu}^{\text{syn}}}{c^2 R_{\text{g}}^2 \delta \epsilon_{\text{D}} \epsilon'^2}, \]  

(9)

using Equation (2). The scattered photon energy in the comoving frame is related to the observed photon energy \( h\nu = m_e c^2 \epsilon \) by the relation

\[ \epsilon_{\nu} = \left( 1 + \frac{z}{\delta} \right) \epsilon = \frac{\epsilon_{\epsilon}}{\delta}. \]  

(10)

From the limits on the integration over \( \gamma' \) implied by the limits on \( q' \) we find

\[ \gamma'_{\text{min}} = \frac{1}{2} \epsilon_{\epsilon} \left( 1 + \sqrt{1 + \frac{1}{\epsilon'_{\epsilon}} - 1} \right), \]  

(11)

and

\[ \gamma'_{\text{max}} = \frac{\epsilon'_{\epsilon} \epsilon_{\epsilon}'}{\epsilon'_{\epsilon} - \epsilon_{\epsilon}} H(\epsilon'_{\epsilon} - \epsilon_{\epsilon}) + \gamma' \epsilon_{\epsilon} H(\epsilon_{\epsilon}' - \epsilon'_{\epsilon}). \]  

(12)

(see Finke et al. 2008, for a detailed derivation of synchrotron/SSC models and application to XBLs). Here, the maximum lepton Lorentz factor injected into the radiating fluid is \( \gamma'_{\epsilon} \). The \( \nu F_{\nu} \) SSC spectrum is therefore given by

\[ f_{\nu}^{\text{SSC}} = \left( \frac{3}{2} \right)^{3} \frac{d_\epsilon^2 \epsilon'^2}{R_{\text{g}}^2 c \delta \epsilon_{\text{D}} \epsilon U_B} \int_{0}^{\infty} d\epsilon' \frac{f_{\nu}^{\text{syn}}}{\epsilon'^{3/2}} \times \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} d\gamma' \frac{F_C(q', \Gamma_\epsilon)}{\gamma'^{5/2}} f_{\nu}^{\text{syn}}, \]  

(13)

where \( \tilde{\epsilon} \equiv (1 + z) \epsilon \) and \( \tilde{\epsilon} \equiv \frac{\delta \epsilon_{\text{D}} \epsilon_{\text{D}} \gamma'^2}{1 + z} \). The maximum \( \nu F_{\nu} \) SSC flux at photon energy \( \epsilon_{\text{pk}}^{\text{SSC}} \) can be approximated in the Thomson limit by the expression

\[ f_{\nu}^{\text{SSC}} \approx \frac{24\pi d_\epsilon^2 (1 + z)^2}{(\delta \epsilon_{\text{D}} B_{\text{tv}})^2 c^3} \left( f_{\nu}^{\text{syn}} \right)^2, \]  

(14)

where the peak frequencies are related by

\[ \epsilon_{\text{pk}}^{\text{SSC}} = \sqrt{\frac{\epsilon_{\epsilon}^{\text{SSC}} \epsilon_{\epsilon} \delta \epsilon_{\text{D}}}{1 + z}}. \]  

(15)

(Tavecchio et al. 1998; Finke et al. 2008). Here, \( f_{\nu}^{\text{syn}} \) is the \( \nu F_{\nu} \) peak of the synchrotron component, which reaches its maximum at \( \epsilon = \epsilon_{\epsilon}^{\text{pk}} \).

Second-order SSC takes place when the SSC photons are again Compton-scattered by electrons in the same blob, and may account for superquadratic variability of the \( \gamma \)-ray flux with respect to the synchrotron flux (Perlman et al. 2008). In principle, these photons can again be Compton-scattered to arbitrarily higher orders, though higher order scatterings are negligible due to Klein–Nishina effects. Calculating second-order SSC can be done accurately by replacing \( f_{\nu}^{\text{syn}} \) with \( f_{\nu}^{\text{SSC}} \) in Equation (13), so that

\[ f_{\nu}^{\text{SSC,2}} = \left( \frac{3}{2} \right)^{3} \frac{d_\epsilon^2 \epsilon'^2}{R_{\text{g}}^2 c \delta \epsilon_{\text{D}} \epsilon U_B} \int_{0}^{\infty} d\epsilon' \frac{f_{\nu}^{\text{SSC}}}{\epsilon'^{3/2}} \times \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} d\gamma' \frac{F_C(q', \Gamma_\epsilon)}{\gamma'^{5/2}} f_{\nu}^{\text{SSC}}, \]  

(16)

2.2. \( \gamma \gamma \) Opacity

Gamma-ray photons are subject to \( \gamma \gamma \) attenuation by synchrotron photons produced in the radiating plasma, by ambient photons in the environment of the black hole (starred frame), and by photons of the intergalactic radiation field. The \( \gamma \gamma \) pair-production cross section

\[ \sigma_{\gamma \gamma}(s) = \frac{1}{2} \pi r_{\gamma}^2 \left( 1 - \beta_{\text{cm}}^2 \right) \left[ (3 - \beta_{\text{cm}}^2) \ln \left( \frac{1 + \beta_{\text{cm}}}{1 - \beta_{\text{cm}}} \right) - 2\beta_{\text{cm}} (2 - \beta_{\text{cm}}^2) \right]. \]  

(17)

(Jauch & Rohrlich 1976; Nikishov 1961; Gould & Schréder 1967; Brown et al. 1973), where \( \gamma_{\text{cm}} \) is the center-of-momentum frame Lorentz factor of the produced electron and positron,

\[ \beta_{\text{cm}} = \left( 1 - \gamma_{\text{cm}}^{-2} \right)^{1/2} = \sqrt{1 - s^{-1}}, \]  

(18)

\[ s = \gamma_{\text{cm}}^2 = \frac{\epsilon_{\epsilon} \epsilon_{\epsilon} (1 + z)}{1 - \cos \psi}, \]
and \( r_e = e^2/m_e c^2 \cong 2.8179 \times 10^{-13} \text{ cm} \) is the classical electron radius. The interaction angle \( \psi \), given by the relation

\[
\cos \psi = \mu \mu_s + \sqrt{1 - \mu_s^2} \sqrt{1 - \mu^2} \cos(\phi - \phi_s), \tag{19}
\]
is the angle between the directions of the photon detected with the energy \( \epsilon_1 \) and the target photon with the energy \( \epsilon_s \).

The absorption probability per unit path length is

\[
\frac{d\tau\gamma\gamma(\epsilon_1)}{dx} = \int d\Omega_s \left(1 - \cos \psi\right) \int_0^\infty d\epsilon_s \ n_{ph}(\epsilon_s, \Omega_s) \sigma_{\gamma\gamma}(s). \tag{20}
\]

For absorption by synchrotron photons within the radiating volume, \( \epsilon_s \to \epsilon' \) and \( \epsilon_1 \to \epsilon_1' = (1 + z)\epsilon_1/\delta_0, \) and the target synchrotron radiation field is given by Equation (9). In this case, the optically thin \( \gamma\gamma \) optical depth integrated over path-length. For absorption by ambient photons in the vicinity of the AGN, \( \epsilon_s \) is the photon energy in the AGN rest frame. For cosmic \( \gamma\gamma \) absorption, the target photons are given by the spectrum of the intergalactic background light, which evolves with redshift. In the latter two cases, the intrinsic spectrum is modified by the factor \( \exp(-\tau\gamma\gamma) \).

3. COMPTON-SCATTERED EXTERNAL RADIATION FIELDS

In the one-zone model, the flux spectrum of Compton-scattered external radiation fields is given by the Compton spectral luminosity \( \epsilon_s L_C(\epsilon_s, \Omega_s) \) according to the relation

\[
f^C_e = \frac{\epsilon_s L_C(\epsilon_s, \Omega_s)}{d_I^2}, \tag{22}
\]

where \( \epsilon_s \equiv (1 + z)\epsilon \), from Equation (10), and \( \Omega_s = \Omega \). The latter equality means that the photon direction is not deflected in transit to the observer. The Compton spectral luminosity is given by

\[
\epsilon_s L_C(\epsilon_s, \Omega_s) = m_e c^3 \epsilon_s^2 \int d\Omega_s \int_0^\infty d\epsilon_s \ n_{ph}(\epsilon_s, \Omega_s) \int d\Omega_e \times \int_1^\infty dy N_e(\gamma, \Omega_s) \times (1 - \cos \psi) \frac{d\sigma_{\epsilon\gamma}(\bar{\epsilon})}{d\epsilon_s} \times \delta(\Omega_e - \Omega_s), \tag{23}
\]

having already introduced the approximation that the scattered photon travels in the same direction as the relativistic scattering electron, i.e., \( \Omega_s = \Omega_e \). Because of this approximation, the cosine of the angle \( \psi \) is given by Equation (19). The invariant collision energy

\[
\bar{\epsilon} \equiv \gamma \epsilon_s (1 - \sqrt{1 - 1/\gamma^2 \cos \psi}) \cong \gamma \epsilon_s (1 - \cos \psi) \tag{24}
\]
because \( \gamma \gg 1 \). The relation \( n_{ph}(\epsilon_s, \Omega_s) = n(\epsilon_s, \Omega_s)/(m_e c^2 \epsilon_s) \) gives the specific spectral number density of target photons with energy \( \epsilon_s \), the starred quantities referring to the frame stationary with respect to the black hole.

The Compton cross section in the head-on approximation is given by

\[
\frac{d\sigma_{\epsilon\gamma}}{d\epsilon_s} \cong \frac{\pi r_e^2}{\gamma \epsilon} \Xi \left(\epsilon_s, \frac{\epsilon}{\gamma \epsilon} \frac{2\gamma \epsilon}{1 + 2 \epsilon} \right) \tag{25}
\]

(Dermer & Schlickeiser 1993; Dermer & Böttcher 2006), where

\[
\Xi \equiv y + y^{-1} - \frac{2 \epsilon_s}{\gamma \epsilon y} + \left(\frac{\epsilon_s}{\gamma \epsilon y}\right)^2, \tag{26}
\]

\[ y \equiv 1 - \frac{\epsilon_s}{\gamma \epsilon} \tag{27}\]

and \( \bar{\epsilon} \) is given by Equation (24). The Compton spectral luminosity in the head-on approximation becomes

\[
\epsilon_s L_C(\epsilon_s, \Omega_s) = c \pi r_e^2 \epsilon_s^2 \int d\Omega_s \int_0^{\epsilon_s/2} d\epsilon_s \ u_s(\epsilon_s, \Omega_s) \frac{d\sigma_{\epsilon\gamma}(\bar{\epsilon})}{d\epsilon_s} \times \int_0^\infty d\gamma \ y^{-2} N_e(\gamma, \Omega_s) \Xi. \tag{28}
\]

The lower limit on the electron Lorentz factor \( \gamma_{\text{low}} \) and the upper limit \( \epsilon_{s, \text{hi}} \) implied by the kinematic limits on \( y \) are

\[
\gamma_{\text{low}} = \frac{x + 1 + 2}{\epsilon_s \epsilon_s (1 - \cos \psi)}, \tag{29}
\]

and

\[
\epsilon_{s, \text{hi}} = \frac{2 \epsilon_s}{1 - \cos \psi}. \tag{30}
\]

Equation (28) is the starting point to calculate accurate Compton-scattered spectra involving relativistic electrons and external photon fields with arbitrary anisotropies. In contrast to the comoving electron spectrum used in the SSC calculation, the calculation of Compton-scattered radiation uses the electron spectrum \( N_e(\gamma, \Omega) \) and the target photon spectrum defined in the stationary frame (Georganopoulos et al. 2001). The invariant phase volume \( d\nu = dV/\bar{d}\bar{p} \) for relativistic particles is given by

\[
\frac{dN}{d\nu} = \frac{dN}{dV d\bar{p}} = \frac{1}{(m_e c)^3} \frac{1}{\gamma^2} \frac{dN}{dy d\Omega d\nu} = \text{inv}, \tag{31}
\]

implying that

\[
N_e(\gamma', \Omega) = \frac{\gamma'^2}{\gamma^2} \frac{dV}{d\nu'} N_e(\gamma', \Omega') = \delta_D N_e(\gamma', \Omega'), \tag{32}
\]

noting that \( dV/d\nu' = dt'/dt = \delta_D \), and \( \gamma = \delta_D \gamma' \) when \( \gamma' \gg 1 \), required for the head-on approximation. For an isotropic comoving distribution of electrons, \( N_e(\gamma, \Omega) = \delta_D N_e(\gamma'/4\pi) \). Hence

\[
\epsilon_s L_C(\epsilon_s, \Omega_s) = \frac{c r_e^2}{4} \epsilon_s^2 \delta_D \int_0^{2\pi} d\phi_s \int_1^1 d\mu_s \int_{\epsilon_{s,\text{hi}}}^{\infty} d\epsilon_s \ u_s(\epsilon_s, \Omega_s) \frac{d\sigma_{\epsilon\gamma}(\bar{\epsilon})}{d\epsilon_s} \times \int_0^\infty d\gamma \ y^{-2} N_e(\gamma, \Omega_s) \Xi, \tag{33}
\]

or

\[
f^C_e = \epsilon_s L_C(\epsilon_s, \Omega_s) = \frac{c r_e^2}{4\pi d_I^2} \epsilon_s^2 \delta_D \int_0^{2\pi} d\phi_s \int_1^1 d\mu_s \int_0^{\epsilon_{s,\text{hi}}} d\epsilon_s \ u_s(\epsilon_s, \Omega_s) \epsilon_s \int_0^\infty d\gamma \ N_e(\gamma/\delta_D) \gamma^2 \Xi. \tag{34}
\]
In terms of the measured synchrotron $\nu F_\nu$ spectrum, Equation (3), the source Compton spectrum for EC scattering in a standard one-zone model for blazars is, in general, given by the fourfold integral

$$f_\epsilon^{EC} = \left( \frac{3}{4} \right)^2 \frac{\epsilon_r^2 \delta_D^2}{U_B} \int_0^{2\pi} d\phi_j \int_0^1 d\mu_s \int_0^{\epsilon_s,\epsilon_s} d\epsilon_* \frac{u_\epsilon(\epsilon_*, \Omega_\epsilon)}{\epsilon_*^2}$$

$$\times \int_{\gamma_{low}}^{\infty} d\gamma \frac{f_{\gamma, syn}}{\gamma^5} \Xi,$$  \hspace{1cm} (35)

with

$$\epsilon_s \equiv \frac{\epsilon_B \gamma^2}{(1 + z) \delta_D},$$  \hspace{1cm} (36)

using Equation (3). The number of integrations can obviously be reduced by choosing symmetrical target photon geometries.

### 3.1. Point Source Radially Behind Jet

First, we consider the flux when nonthermal electrons in a relativistic jet Compton scatter photons from a point source of radiation, isotropically emitted and located radially behind the outflowing plasma jet. For a monochromatic point source with luminosity $L_0$ and energy $\epsilon_0$, the spectral luminosity can be expressed as

$$L_\epsilon(\epsilon_s) = L_0 0(1 - \epsilon_0).$$  \hspace{1cm} (37)

The SED of the target photon source at distance $r$ from the point source is therefore given by

$$u(\epsilon_s, \Omega_\epsilon) = \frac{L_0}{4\pi r^2 c} \frac{\delta(\epsilon_s - \epsilon_0)}{2\pi}.$$  \hspace{1cm} (38)

Substituting Equation (38) into Equation (33) and solving gives

$$\epsilon_s L_C(\epsilon_s, \Omega_\epsilon) = \frac{r^2 \epsilon_s^2 L_0 \delta_D^3}{16\pi \epsilon_0^3} \int_{\gamma_{low}}^{\infty} d\gamma \frac{N_\gamma(\gamma / \delta_D)}{\gamma^2} \Xi.$$  \hspace{1cm} (39)

Using Equation (22), Equation (39) becomes

$$f_\epsilon^{C, pt} = \frac{r^2 \epsilon_s^2 L_0 \delta_D^3}{16\pi \epsilon_0^3} \int_{\gamma_{low}}^{\infty} d\gamma \frac{N_\gamma(\gamma / \delta_D)}{\gamma^2} \Xi,$$  \hspace{1cm} (40)

or with Equation (3),

$$f_\epsilon^{C, pt} = \frac{3^2}{8\pi} \frac{L_0 \epsilon_r^2 \delta_D^2}{c^2 U_B \epsilon_0^3} \int_{\gamma_{low}}^{\infty} d\gamma \frac{\epsilon_s^3}{\gamma^5} \Xi,$$  \hspace{1cm} (41)

where $\epsilon_s$ is defined in Equation (36), $\Xi$ is defined by Equation (26) with $\epsilon_s$ replaced by $\epsilon_s \equiv \gamma_0 (1 - \mu_s)$, and

$$\gamma_{low} = \frac{\epsilon_s}{2} \left[ 1 + \sqrt{1 + \frac{2}{\epsilon_0 \epsilon_s (1 - \mu_s)}} \right].$$  \hspace{1cm} (42)

Equations (40) and (41) give the Compton-scattered spectrum from a point source of radiation located radially behind the jet, generalizing the Thomson-regime result (Dermer et al. 1992) to include scattering in the Klein–Nishina regime. A scattered disk component should be found in all blazar models, with its importance strongly dependent on the distance $r$ of the jet from the accretion disk. The point-source approximation gives the least upscattered flux in the Thomson limit, and an extended disk having the same power as a point source will give a more intense flux. At sufficiently large jet heights $r \gg \Gamma^4 R_g$, defining the far field, where $R_g \equiv GM/c^2 \cong 1.5 \times 10^{13} M_\odot$ cm is the gravitational radius, the Shakura–Sunyaev disk can be described as a point source radially behind the jet. Photons from large disk radii are important in the near field $r \ll \Gamma^4 R_g$ (Dermer & Schlickeiser 2002).

#### 3.1.1. Reduction to the Thomson Regime

We now derive the Thomson limit for the $\nu F_\nu$ spectrum, Equation (40). Because we consider relativistic electrons $\gamma \gg 1$, we are restricted to the condition $\gamma_{low} \gg 1$, which occurs according to Equation (42) when either $\epsilon_s \gg 1$ or $\epsilon_s / \epsilon_0 (1 - \mu_s) \gg 1$. The Thomson condition can be expressed as $\epsilon_s \ll \gamma$, which is guaranteed when $\epsilon_s \ll \gamma_{low}$, in which case $\epsilon_0 \epsilon_s (1 - \mu_s) \ll 1$. Another statement of the Thomson condition is that $\gamma_0 (1 - \mu_s) \ll 1$ which, with $\epsilon_s \ll \gamma$, again implies that $\epsilon_0 \epsilon_s (1 - \mu_s) \ll 1$. Thus,

$$\gamma_{low} \rightarrow \frac{\epsilon_s}{\sqrt{2 \epsilon_0 (1 - \mu_s)}}.$$  \hspace{1cm} (43)

For the scattering kernel, Equation (26), $\epsilon_s \ll \gamma$ and $y \rightarrow 1$ in the Thomson regime, so

$$\Xi \rightarrow \Xi_T \equiv 2 - 2 \left( \frac{\epsilon_s}{\gamma \epsilon} \right) + \left( \frac{\epsilon_s}{\gamma \epsilon} \right)^2.$$  \hspace{1cm} (44)

Away from the endpoints of the spectrum, $\epsilon_s \ll \gamma_0 \epsilon$ and $\Xi \rightarrow 2$. Hence

$$f_\epsilon^{pt, T} = \frac{3}{4} \frac{\sigma_T L_0}{(4\pi r_d L_t)^2} \left( \frac{\epsilon_s}{\epsilon_0} \right)^2 \delta_D^3 \int_{\gamma_0}^{\infty} d\gamma \frac{N_\gamma(\gamma')}{\gamma'^2},$$  \hspace{1cm} (45)

defining

$$\delta_D \gamma' \equiv \frac{\epsilon_s}{\sqrt{2 \epsilon_0 (1 - \mu_s)}}.$$  \hspace{1cm} (46)

For the comoving electron distribution, Equation (3), in the power-law form

$$N_\gamma(\gamma') = K' \gamma'^{-p} H(\gamma' \setminus \gamma_1', \gamma_2'),$$  \hspace{1cm} (47)

Equation (45) becomes

$$f_\epsilon^{pt, T} = \frac{3}{4} \frac{\sigma_T L_0 K'}{(p + 1) (4\pi r_d L_t)^2} \left( \frac{\epsilon_s}{\epsilon_0} \right)^2 \delta_D^{p+3}$$

$$\times \left[ \max(\gamma', \gamma_1')\gamma'^{-(p+1)} - \gamma_2'^{-(p+1)} \right] \rightarrow \frac{3}{4} \frac{\sigma_T L_0 K'}{(p + 1) (4\pi r_d L_t)^2}$$

$$\times \left( \frac{\epsilon_s}{\epsilon_0} \right)^{(3-p)/2} \delta_D^{p+3} [2(1 - \mu_s)]^{(p+1)/2},$$  \hspace{1cm} (48)

where the final expression applies in the regime $\gamma_1' \ll \gamma' \ll \gamma_2'$. This can be written as

$$f_\epsilon^{pt, T} \approx \left( \frac{3}{p + 1} \right) \delta_D^0 (1 - \mu_s)^2 \left( \frac{\sigma_T}{4\pi r_d^2 L_t} \right) \left( \frac{L_0}{4\pi r_d^2} \right) \gamma'^3 N_\gamma(\gamma'),$$  \hspace{1cm} (49)

which can be compared with the Thomson-regime expression

$$f_\epsilon^{pt, T} \equiv \left( \frac{1}{2} \right) \delta_D^0 (1 - \mu_s)^2 \left( \frac{\sigma_T}{4\pi r_d^2 L_t} \right) \left( \frac{L_0}{4\pi r_d^2} \right) \gamma'^3 N_\gamma(\gamma'),$$  \hspace{1cm} (50)

(Dermer et al. 1992; Dermer & Schlickeiser 1993, 2002), where

$$\gamma' = \frac{1}{\delta_D} \sqrt{\frac{\epsilon_0 (1 + z)}{\epsilon_0 (1 - \mu_s)}} = \sqrt{2} \gamma.$$  \hspace{1cm} (51)
3.1.2. Accurate Thomson Regime Spectrum

Equation (49) for the Thomson-scattered spectrum was derived assuming $\Xi_T = 2$, away from the endpoints of the spectrum. Using the full expression for $\Xi_T$ and a power-law electron distribution gives an accurate expression for the spectrum of a localized jet of isotropically entrained electrons Thomson scattering a point source, monochromatic radiation field that enters the jet from behind. The result is

$$f_{e,pl,T} = \frac{\pi r_e^2 L_0}{(4 \pi r_d L)^2} \left( \frac{\epsilon_s}{\epsilon_0} \right)^2 \delta_D \int_{\beta_{in}}^\infty dy \left( \frac{2}{\gamma^2 - 2s/\gamma^2} \right) N'_s(\gamma'),$$

where

$$s \equiv \frac{\epsilon_s}{\epsilon_0 (1 - \mu_s)} = 2 \delta_D^2 \gamma^2.$$

(52)

For the power-law electron distribution $N'_s(\gamma')$, Equation (47), the accurate analytic Thomson-regime $\nu F_\nu$, flux from an isotropic monochromatic point source radiation field located behind the jet is

$$f_{e,pl,T} = \frac{3}{4} \sigma_T L_0 K' \left( \frac{\epsilon_s}{\epsilon_0} \right)^2 \delta_D \left[ \frac{1}{1 + p} \left( \gamma_1^{-(1+p)} - \gamma_2^{-(1+p)} \right) - \frac{s}{3(3+p)} \left( \gamma_1^{-(3+p)} - \gamma_2^{-(3+p)} \right) + \frac{s^2}{2(5+p)} \left( \gamma_1^{-(5+p)} - \gamma_2^{-(5+p)} \right) \right],$$

where

$$\gamma_1 = \max(\delta_D \gamma', \delta_D \bar{\gamma})$$

and $\gamma_2 = \delta_D \gamma'$. In the asymptotic limit $\gamma_1' \ll \bar{\gamma} \ll \gamma_2'$, Equation (54) approaches

$$f_{e,pl,T} \rightarrow A(p) \left( \frac{\sigma_T}{4 \pi d_L^2} \right) \left( \frac{L_0}{4 \pi r^2} \right) \delta_D (1 - \mu_s)^2 \bar{\gamma}^3 N'_s(\bar{\gamma}),$$

where

$$A(p) \equiv 3 \left( \frac{1}{1 + p} - \frac{2}{3 + p} + \frac{2}{5 + p} \right).$$

(55)

The values of $A(p) = 1.0, 0.657$, and 0.5 for $p = 1, 2$, and 3, respectively. For the Thomson approximation away from the endpoints of the spectrum, given by Equation (49), the corresponding coefficient is $3/(p+1)$.

3.1.3. Solution with Compton Cross Section

From Equations (39) and (22),

$$f_{e,pl,C} = \frac{\pi r_e^2 L_0}{(4 \pi r_d L)^2} \left( \frac{\epsilon_s}{\epsilon_0} \right)^2 \delta_D \int_{\beta_{in}}^\infty dy \frac{N'_s(\gamma/\delta_D)}{\gamma^2} \left( y + y^{-1} - \frac{2 \epsilon_s}{\gamma \epsilon_y} \right)^2.$$

(57)

Introducing $u = \epsilon_s/\gamma$ and $v = \epsilon_s \epsilon_0 (1 - \mu_s)$ and changing variables to $u$ gives, for the power-law electron distribution, Equation (47),

$$f_{e,pl,C} = \frac{\pi r_e^2 L_0 K'}{(4 \pi r_d L)^2} \left( \frac{\epsilon_s}{\epsilon_0} \right)^2 \delta_D^{3+p} \epsilon_s^{-1+(p)} I_C,$$

where

$$I_C = \int_{u_1}^{u_2} du \left[ u^p - u^{p+1} + \frac{u^p}{1 - u} - \frac{2u^{p+2}}{v(1 - u)} + \frac{u^{p+4}}{v^2(1 - u)^2} \right].$$

(59)

and $u_1 = \epsilon_s \delta_D/\gamma_2'$, $u_2 = \epsilon_s \min \left( \frac{1}{\gamma_{in}}, \frac{\delta_D}{\gamma_1'} \right)$. The series solution of Equation (59) is given by

$$I_C = \left\{ \frac{u^{p+1}}{p+1} - \frac{u^{p+2}}{p+2} + \sum_{i=0}^\infty \frac{u^{i+p+1}}{v(i+p+3)} \right\} \left\{ \frac{u^2}{v^2} - \frac{1}{i+p+5} \right\}. $$

(60)

Equation (59) can be solved analytically for integral $p$.

3.2. Shakura–Sunyaev Accretion-Disk Field

For the emission spectrum of an accretion disk surrounding the supermassive black hole, we consider the cool, optically thick blackbody solution of Shakura & Sunyaev (1973). The disk emission is approximated by a surface radiating at the blackbody temperature associated with the local energy dissipation rate per unit surface area, which is derived from considerations of viscous dissipation of the gravitational potential energy of the accreting material. The accretion luminosity is defined in terms of the Eddington ratio

$$\ell_{Edd} = \frac{\eta \dot{m} c^2}{L_{Edd}}.$$

(61)

where $\eta \sim 0.1$ is the efficiency to transform accreted matter to escaping radiant energy. The Eddington luminosity $L_{Edd} = 1.26 \times 10^{36} M_8$ ergs s$^{-1}$, where the mass of the central supermassive black hole is $M = 10^8 M_\odot$, and the black hole is accreting mass at the rate $\dot{m}$ (gm s$^{-1}$).

For steady flows where the energy is derived from the viscous dissipation of the gravitational potential energy of the accreting matter, the radiant surface-energy flux

$$\frac{d\mathcal{E}}{dAdt} = \frac{3GM \dot{m}}{8\pi R^3} \phi(R).$$

(Shakura & Sunyaev 1973), where

$$\phi(R) = \left[ 1 - \beta_i (R_i/R)^{1/2} \right],$$

(63)

$\beta_i \equiv 1$, and $R_i = 6GM/c^2$ for the Schwarzschild metric. Integrating Equation (62) over a two-sided disk gives $\eta = 1/12$. Assuming that the disk is an optically thick blackbody, the effective temperature of the disk can be determined by equating Equation (62) with the surface energy flux $\sigma_{SB} T^4(R)$. A monochromatic approximation for the mean photon energy $m c^2 (\epsilon(R)) \equiv k_B T(R)$ at radius $R$ of the accretion disk with mean temperature $T(R)$ is given by

$$m c^2 (\epsilon(R)) \equiv 2.70 k_B T(R) \equiv 2.70 k_B \left[ \frac{3GM \dot{m} \phi(R)}{8\pi R^3 \sigma_{SB}} \right]^{1/4} \equiv 137 \left( \frac{\ell_{Edd}}{M_8 \dot{m}} \right)^{1/4} \bar{R}^{-3/4} \text{ eV},$$

(64)

so

$$\langle \epsilon(\bar{R}) \rangle \equiv 2.7 \times 10^{-4} \xi \bar{R}^{-3/4},$$

(65)

where

$$\xi \equiv \left( \frac{\ell_{Edd}}{M_8 \dot{m}} \right)^{1/4}.$$

(66)
Here, $\theta = \arccos \mu_*$ is the angle between the directions of the jet and the photon that intercepts the jet, and
\[
\tilde{R} = \frac{R}{R_g} = \tilde{r} \sqrt{\mu_*^2 - 1}, \tag{67}
\]
where $\tilde{r} = r/R_g$. Lengths marked with a tilde are normalized to $R_g$. The final expression in Equation (64) is valid when $\tilde{R} \gg \tilde{R}_{\text{min}}$, though we assume it is reasonably accurate to $\tilde{R} \approx \tilde{R}_{\text{min}}$.

The intensity of the Shakura–Sunyaev accretion-disk model along the jet axis is given by
\[
I_\epsilon^{\text{SS}}(\Omega; \tilde{R}) \equiv \frac{3GMm}{16\pi^2 R^3} \varphi(\tilde{R}) \delta \left( \epsilon - \frac{2.7k_B}{m_c c^2} T(\tilde{R}) \right). \tag{68}
\]
(Shapero & Teukolsky 1983, Chapter 14). Thus,
\[
I_\epsilon^{\text{SS}}(\Omega; \tilde{R}) = \frac{3}{16\pi^2} \frac{\ell_{\text{Edd}} L_{\text{Edd}}}{\eta R^2} \varphi(\tilde{R}) \delta(\epsilon - \langle \epsilon(\tilde{R}) \rangle). \tag{69}
\]
(Dermer & Schlickeiser 2002). Substituting Equation (69) into Equation (33), using the relation $I_\epsilon(\Omega) = c \epsilon(\Omega)$, gives
\[
f_\epsilon^{\text{SS}} = \frac{3^2}{2^9 \pi^3} \frac{\sigma_T c^3}{2 R_g} \eta \delta_D \int_0^{2\pi} d\phi_\epsilon
\times \int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \frac{\varphi(\tilde{R})}{(\mu_*^2 - 1)^{3/2} (\epsilon(\tilde{R}))^2} \int_0^{\gamma_{\text{min}}} d\gamma \gamma^{-2} N'_\epsilon(\gamma/\delta_D)
\times \left[ y + y^{-1} - \frac{2\epsilon_s}{\gamma - \gamma y} + \left( \frac{\epsilon_s}{\gamma - \gamma y} \right)^2 \right]. \tag{70}
\]
The integration over angle in Equation (70) is limited by the inner radius of the accretion disk, so that $\mu_{\text{max}} = \left[ 1 + (6/\tilde{r})^2 \right]^{-1/2}$ for a Schwarzschild black hole,
\[
\tilde{\epsilon} = \tilde{\epsilon}(\gamma, \epsilon_\gamma, \psi) = \gamma \langle \epsilon(\tilde{R}) \rangle (1 - \cos \psi), \tag{71}
\]
and
\[
\gamma_{\text{low}} = \frac{\epsilon_s}{2} \left( 1 + \sqrt{1 + \frac{2}{(\epsilon(\tilde{R}))(1 - \cos \psi)} \right). \tag{72}
\]
The other limit on the angular integration arises because of the restriction given by Equation (30), so that
\[
\langle \epsilon(\tilde{R}) \rangle < \frac{2\epsilon_s}{1 - \cos \psi}, \tag{73}
\]
which restricts the integral to a maximum value of $\tilde{R}$ and therefore $\mu_* \lesssim \mu_{\text{min}}$. In the calculation of $\cos \psi$, Equation (19), we take $\phi_\epsilon = 0$ without loss of generality because of the assumed azimuthal symmetry of the accretion-disk emission.

The result for the accretion-disk radiation field scattered by isotropic, relativistic jet electrons is a threefold integral, reduced from a fourfold integral by approximating the disk blackbody spectrum by its mean thermal energy at different radii. When expressed in terms of the measured synchrotron $\nu F_\nu$ spectrum using Equation (3), the result for the accretion-disk radiation field is
\[
f_\epsilon^{\text{SS}} = \frac{3^3}{2^8 \pi^3} \frac{\epsilon_s^2}{c R_g^2 U_B} \frac{\ell_{\text{Edd}} L_{\text{Edd}}}{\eta R^2} \delta_D \int_0^{2\pi} d\phi_\epsilon
\times \int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \frac{\varphi(\tilde{R})}{(\mu_*^2 - 1)^{3/2} (\epsilon(\tilde{R}))^2} \int_0^{\gamma_{\text{min}}} d\gamma \gamma^{-5} f_s^{\gamma \gamma}(\gamma + y^{-1} - \frac{2\epsilon_s}{\gamma - \gamma y} + \left( \frac{\epsilon_s}{\gamma - \gamma y} \right)^2), \tag{74}
\]
recalling the definitions of $\gamma'_\epsilon$ and $\epsilon$ from Equations (2) and (36), respectively. This can be reduced to a twofold integral by approximating a typical scattering as occurring at $\phi_\epsilon = \pi/2$, so that $\tilde{\epsilon} = \epsilon(1 - \mu_* \epsilon_s)$. Because it is feasible to perform the threefold integral numerically, we show results of the more accurate calculations.

### 3.2.1. Regimes in Compton-Scattered Accretion-Disk Spectra

The limiting behaviors of the Compton-scattered spectra can be understood based on simple $\delta$-approximations in the Thomson regime. EC scattering of the Shakura–Sunyaev disk with radiant luminosity $L_\gamma = \ell_{\text{Edd}} L_{\text{Edd}}$ in the near field, i.e., $\tilde{r} \ll \Gamma^4$, can be approximated as
\[
f_\epsilon^{\text{ECF}} \approx \frac{\delta_D^2 L_d}{2B^2 R_g^2 c^2} f_s^{\gamma \gamma}, \tag{75}
\]
where
\[
\epsilon^{\text{syn}} = \epsilon_s(\tilde{r}) = \frac{2\epsilon_s}{\delta_D(\sqrt{3})} \equiv 3 \times 10^{-10} \frac{\epsilon S(B)}{\delta_D} \left( \frac{M_8 \eta}{L_{\text{Edd}}} \right)^{1/4}. \tag{76}
\]
In the far field ($\tilde{r} \gg \Gamma^4$),
\[
f_\epsilon^{\text{EFF}} \approx \frac{3}{8} \frac{L_d}{\gamma_c B^2 \delta_D} f_s^{\gamma \gamma}, \tag{77}
\]
where
\[
\tilde{\epsilon}^{\text{syn}} = 3.2 \times 10^{14} \frac{d^4}{d^8} \epsilon^{\text{syn}} M_8 \left( \frac{\eta}{L_{\text{Edd}}} \right)^{1/4}. \tag{78}
\]
(Dermer & Schlickeiser 2002).

### 3.2.2. $\gamma \gamma$ Opacity from Accretion-Disk Photons

Photons from the accretion disk will interact with high energy $\gamma$-rays to produce electron–positron pairs, modifying the very high energy (VHE; multi-GeV–TeV) $\gamma$-ray spectrum by a factor of $e^{-\tau_{\gamma \gamma}}$. The absorption opacity $\tau_{\gamma \gamma}$ can be calculated by inserting the photon density, $n_{\gamma\gamma}$, for an accretion disk into Equation (20) and integrating $x$ from $r$ to $r_0$. For a Shakura–Sunyaev accretion disk, the photon density is given by
\[
n_{\gamma\gamma}^{\text{SS}}(\epsilon_s, \Omega; \tilde{R}) = \frac{\ell_{\text{Edd}}^{\text{SS}}(\Omega; \tilde{R}}{\epsilon_s m_c c^3}, \tag{79}
\]
where $I_\epsilon^{\text{SS}}(\Omega; \tilde{R})$ is given by Equation (69). For an azimuthally symmetric accretion disk, the optical depth to $\gamma \gamma$ pair-production attenuation for a photon with the observed energy $\epsilon_1$ traveling outward along the jet axis starting at the height $\tilde{r}$ is given by
\[
\tau_{\gamma \gamma}(\epsilon_1, \tilde{r}) \equiv 3 \times 10^6 \frac{\ell_{\text{Edd}}^{1/4}}{\eta^{3/4}} M_8 \frac{1}{\tilde{r}^{3/4}} \int_{\tilde{r}}^{\infty} d\tilde{x} \frac{\varphi(\tilde{R})}{(1 + R^2/x^2)^{3/2}} \left[ \frac{\sigma_{\gamma \gamma}(\tilde{x})}{\pi r^2} \right] (1 - \mu_*), \tag{80}
\]
where $\tilde{s} \equiv \langle \epsilon(\tilde{R}) \rangle \epsilon_1 (1 + z) (1 - \mu_\ast)/2$ and $\mu_\ast = 1/\sqrt{1 + \tilde{R}^2/\xi^2}$.

For the Shakura–Sunyaev disk extending to the innermost stable orbit of a Schwarzschild black hole, one sees from Equation (80) and the definition of $\langle \epsilon(\tilde{R}) \rangle$, Equation (65), and $\xi$, Equation (66), that

$$
\frac{t_{SS}^{\epsilon_1} (\epsilon_1, \tilde{r})}{M_8} \propto \text{function of } \xi \text{ and } \tilde{r}.
$$

(81)

A first-order correction to $\gamma\gamma$ opacity for a photon traveling at a small angle $\theta_\gamma = \arccos \mu_\ast \ll 1$ along the jet axis is obtained by replacing $\mu_\ast$ with $\mu_\ast \mu_\ast$, which implicitly assumes that typical interactions take place at azimuth $\phi_\gamma = \pi/2$. A detailed calculation of the $\gamma\gamma$ opacity from accretion-disc photons is given by Becker & Kafatos (1995).

### 3.2.3. Numerical Results

The electron distribution is assumed to be well represented by the Band et al. (1993)-type function

$$
N_e(\gamma') = K_e' H(\gamma'; \gamma_{\text{min}}', \gamma_{\text{max}}') \left[ \gamma'^{p_1 - p_0} \exp(\gamma'/\gamma_0) \times H(\nu/p_1 - p_0; \gamma_0 - \gamma') \left[ 1 + (\nu/p_2 - p_0)\gamma_0 \right]^{p_2 - p_0} \gamma'^{p_2 - p_1} \exp(\nu/p_1 - p_2) \right].
$$

(82)

This distribution is essentially two smoothly joined power laws with the power-law number indices $p_1$ and $p_2$, and the low- and high-energy cutoffs, $\gamma_{\text{min}}'$ and $\gamma_{\text{max}}'$, respectively, in the electron spectrum. For illustrative purposes, we take $p_1 = 2$ and $p_2 = 4$, or a break by 2 units in the electron spectrum. This can be compared to a break by 1 unit expected for synchrotron and Thomson losses. Our approach is, however, to use the flaring synchrotron spectrum to imply the underlying flaring electron distribution without regard to specific acceleration and radiation processes. When the nonthermal electron distribution is obtained from analysis of blazar data, then the underlying jet physics that gives rise to the inferred electron spectrum can be examined.

The total jet power in the stationary frame of the host galaxy is given by

$$
P_j = 2\pi R_j^2 \beta \Gamma^2 c u_{\text{tot}} = P_{j,\text{par}} + P_{j,B}
$$

(83)

(Celotti & Fabian 1993; Celotti et al. 2007; Finke et al. 2008), where $u_{\text{tot}}'$ is the total energy density in the jet, $P_{j,\text{par}}$ is the jet power from particles, and $P_{j,B}$ is the jet power from the magnetic field. Here, the factor of 2 takes into account that the jet is two-sided. For synchrotron-only emission, it is expected that the jet will be in equipartition and $P_{j,\text{par}} \approx P_{j,B}$, which will minimize the jet power. In order to explain the Compton-scattered component, however, the energy density in electrons can be different than the magnetic field energy density.

We performed a parameter study by varying model parameters with respect to a baseline model, with baseline parameters given in Table 1. We consider a $10^8 M_\odot$ supermassive black hole jet source located at redshift $z = 1$. The jet is radiating at $10^3 R_\odot$ from the black hole and has the Doppler factor $\delta D = 25$ and the bulk Lorentz factor $\Gamma = 25$, so that the angle of the jet direction to the line of sight is $\theta_0 \approx 1/\Gamma$. The mean magnetic field is 1 G, and the variability time $t_{\text{var}} = 10^4$ s, corresponding to $\approx 10 \times$ the light crossing time for the Schwarzschild radius of a $10^8 M_\odot$ black hole. The jet opening angle $\theta_j \approx R_j/\tilde{r} \lesssim c \theta_D t_{\text{var}}/(1 + z) \lesssim 0.25 \approx 15^\circ$ for standard parameters. While varying the parameters, the synchrotron spectrum was kept relatively constant by varying $K'_e$ and $\gamma_0$ following the relations given in Section 3.2.1 (except when changing angle). This was done with a $\chi^2$ fitting technique to the baseline synchrotron spectrum (see Finke et al. 2008). The transition between the near field and far field takes place at $\tilde{r} \approx \Gamma^4$, so that the baseline height of the jet is in the near field.

The Compton-scattered accretion-disc spectra are calculated from Equation (70). Figure 1 shows the effects of changing the magnetic field. With increasing $B$, fewer electrons are required to make the same synchrotron flux, so that both the first and second-order SSC flux, and the flux of the Compton-scattered accretion-disc component decrease with increasing $B$. In all of these models, the second-order SSC emission is overwhelmed by the EC component and is not visible. The overall levels of all Compton-scattered components are $\propto B^{-2}$.

With increasing disk power $\epsilon_{\text{EDD}}$, the accretion-disc radiation and also the Compton-scattered accretion-disc component become progressively larger, as shown in Figure 2. Note that the temperature of the disk photons at a given radius increases...
SSC flux decreases, but the larger value of internal photon energy densities decrease. Consequently, the synchrotron flux at the same flux level, so the electron and becomes larger and fewer electrons are required to make the For larger Doppler factors and fixed variability times, the radius
\[ r = 1.6 \times 10^4 R_g \]
follows from Equations (14) and (75), for the following reasons.

EC flux decreases enough. Figure 4 shows how variations in viewing angle. Contrary to Figures 1–5, we let the synchrotron flux vary. In this calculation, \( \Gamma = 25 \), and the observation angle and therefore \( \delta_D \) changes. It is interesting to note that the SSC flux varies least with changes in viewing angle, while the Comptonized disk component changes most rapidly. The ratios of the various components are explained by noting that \( \delta_D \) and \( \delta_D \), respectively. The different observing angles \( \theta = 1/\Gamma, 2/\Gamma, 3/\Gamma, 4/\Gamma \) correspond to Doppler factors \( \delta_D = 25, 10, 5, \) and 2.94, respectively. Separate spectral components are shown for the \( \theta = 4/\Gamma \) case.

The beaming factor for synchrotron radiation is \( \propto \epsilon_{\text{Edd}}^{1/4} \). Figure 3 displays the dependence of the blazar SED on height \( r \) of the jet. As the blob’s distance from the disk increases, the level of the Comptonized disk radiation decreases. In both the cases of changing the \( \epsilon_{\text{Edd}} \) and \( r \) the SSC components are unaffected; eventually, the Compton-scattered disk flux falls below the SSC fluxes. The second-order SSC is visible when the EC flux decreases enough. Figure 4 shows how variations in \( \gamma_{\text{min}} \) affect the low energy part of the synchrotron, SSC, and EC components.

In Figure 5, the Doppler factor is varied from the baseline value of 25, with the same approximate synchrotron flux. As \( \delta_D \) increases, the SSC component decreases, while the Compton-scattered accretion-disk component increases. This behavior follows from Equations (14) and (75), for the following reasons. For larger Doppler factors and fixed variability times, the radius becomes larger and fewer electrons are required to make the synchrotron flux at the same flux level, so the electron and internal photon energy densities decrease. Consequently, the SSC flux decreases, but the larger value of \( \delta_D \) means that the external target photon field becomes more intense. Thus, the Compton-scattered accretion disk radiation increases with increasing \( \delta_D \). Also, as the Doppler factor increases, the SSC component is shifted to lower energies while the Compton-disk component is shifted to higher energies, in accordance with Equations (15) and (76), respectively.

Figure 6 illustrates how the blazar SED is affected by changes in viewing angle. Contrary to Figures 1–5, we let the synchrotron flux vary. In this calculation, \( \Gamma = 25 \), and the observation angle and therefore \( \delta_D \) changes. It is interesting to note that the SSC flux varies least with changes in viewing angle, while the Comptonized disk component changes most rapidly. The ratios of the various components are explained by noting that \( \delta_D \) and \( \delta_D \), respectively. The beaming factor for synchrotron radiation is \( \propto \epsilon_{\text{Edd}}^{1/4} \) in the flat portion of the \( \nu F_\nu \), with the convention that the flux density \( F_\nu \propto \nu^{-\alpha} \). The relative magnitudes of the SSC components are explained from Equation (14), noting that \( f_{\nu,\text{syn}}^\text{SSC} \propto \delta_D^2 \), so that \( f_{\nu,\text{syn}}^\text{SSC} \propto \delta_D^2 \). The peak flux of the scattered disk component in the near-field regime, \( f_{\nu,\text{syn}}^\text{ECNF} \propto \delta_D^6 \), from Equation (75).
A larger viewing angle or smaller Doppler factor implies a smaller size scale of the emitting region for a given variability time $t_v$. The characteristic Thomson-scattering depth is $n'_e c \tau_R \propto R_b^{-2}$ for a constant comoving electron spectrum $N'_e(\gamma')$ (described in our calculations by Equation (82)) in a region of size $R_b$. Consequently, the suppression of the SSC component due to Doppler boosting is offset by the increased scattering depth for larger observing angles.

Figure 7 shows a calculation of the accretion-disk opacity $\tau_{\gamma\gamma}(\epsilon, \tilde{r})$ for photons traveling along the jet axis, using Equation (80) and the parameters given in Table 1. Here, we assume that the Shakura–Sunyaev disk reaches to the innermost stable orbit ($\tilde{R}_i = 6$) of a Schwarzschild black hole. At larger jet heights, the accretion-disk opacity declines and only increasingly higher energy photons are subject to attenuation due to the threshold condition. The effect of increasing $\xi$ is to increase the mean accretion-disk photon energy at a given radius, from Equation (66), and therefore lower the energy at which $\gamma$ rays can be attenuated. At $\tilde{r} = 10^3$, only $\gtrsim 10$ TeV photons are attenuated by the disk radiation field (with $\xi_{\text{edd}} = 1$). The $\tau_{\gamma\gamma}$ corrections have not been included in the SED calculations, but are only important at $v \gtrsim 10^{22}$ Hz.

In all of our models the jet is particle-dominated ($P_{\gamma, \text{par}} \gg P_{\gamma, B}$) and the magnetic field is below its equipartition value. The total jet power, also divided into magnetic field and particle powers, is shown in Table 2. This is typical of results from modeling the observed X-ray and $\gamma$-ray emission of TeV blazars with the SSC component (Finke et al. 2008).

### 3.3. External Isotropic Radiation Field

This is the case treated by Georganopoulos et al. (2001), where jet electrons scatter a surrounding external isotropic radiation field $u_\nu(\epsilon, \Omega) = u_\nu(\epsilon)/4\pi$. By integrating Equation (28) over the angle variables, recognizing that $d\mu d\phi = 2\pi d\cos\psi$ for this geometry, one obtains

$$f_{\nu, \text{iso}} = \frac{3}{4\pi} \frac{c^2 \sigma T e^2}{d^2} \int_0^\infty d\epsilon u_\nu(\epsilon) \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} d\gamma N'_e(\gamma/\delta_D) F_c(q, \Gamma_e),$$

where $F_c(q, \Gamma_e)$ is given by Jones’s formula (Jones 1968), Equation (7). Because the scattering is taking place in the stationary frame,

$$q = \epsilon_{\gamma}/\Gamma_e (1 - \epsilon_{\gamma}/\Gamma_e),$$

with $\Gamma_e = 4 \epsilon_{\gamma}^{1/2}$ and, as before, $\epsilon_{\gamma} = (1 + z)\epsilon$. The limits $\gamma_{\text{min}}$ and $\gamma_{\text{max}}$ are given by

$$\gamma_{\text{min}} = \frac{1}{2} \epsilon_{\gamma} \left( 1 + \sqrt{1 + \frac{1}{\epsilon_{\gamma}^2}} \right)$$

and

$$\gamma_{\text{max}} = \frac{\epsilon_{\gamma} \epsilon_s}{\epsilon_{\gamma} - \epsilon_s} H(\epsilon_{\gamma} - \epsilon_s) + \gamma_2 H(\epsilon_{\gamma} - \epsilon_s).$$

For the case of the cosmic microwave background radiation (CMBR),

$$u_\nu(\epsilon_s) = u_{\nu, \text{CMBR}}(\epsilon_s, \Omega_e) = \frac{2m_e c^3}{\lambda_C^3} \frac{\epsilon_s^3}{\exp(\epsilon_s/\Theta) - 1},$$

where $\Theta = kT/m_e c^2$ is the dimensionless temperature of the blackbody radiation field, and $T = 2.72(1 + z)$ K. Substituting Equation (88) into Equations (22) and (33) gives

$$f_{\nu, \text{CMBR}} = \frac{3m_e c^3 T^4 \sigma T^4 \delta_D^3}{2H^3 \lambda_C^3} \int_0^{\gamma_{\text{min}}} d\gamma \gamma^{-2} N'_e(\gamma/\delta_D) F_c(q, \Gamma_e).$$

---

**Table 1**

| Parameter | Symbol | Value |
|-----------|--------|-------|
| Redshift  | $z$    | 1     |
| Bulk Lorentz factor | $\Gamma$ | 25 |
| Doppler factor | $\delta$ | 25 |
| Magnetic field | $B$ | 1 G |
| Variability timescale | $t_v$ | $10^3$ s |
| Low-energy electron spectral index | $p_1$ | 2 |
| High-energy electron spectral index | $p_2$ | 4 |
| Minimum electron Lorentz factor | $\gamma_{\text{min}}$ | $10^7$ |
| Maximum electron Lorentz factor | $\gamma_{\text{max}}$ | $10^7$ |
| Accretion efficiency | $\eta$ | 10^{-1} |
| Ratio of $B$ to equipartition field | $B/B_{\text{eq}}$ | 0.3 |
| Jet power in magnetic field | $P_{j, B}$ | $7 \times 10^{34}$ erg s^{-1} |
| Jet power in particles | $P_{j, \text{par}}$ | $10^{46}$ erg s^{-1} |

**Table 2**

| $B$ [G] | $\delta_D$ | $\gamma_{\text{min}}$ | $L_B$ [$10^{34}$ erg s^{-1}] | $L_{\text{par}}$ [$10^{35}$ erg s^{-1}] | $L_{\text{tot}}$ [$10^{35}$ erg s^{-1}] |
|---------|------------|----------------------|-------------------------------|----------------------------------|----------------------------------|
| 1       | 25 100    | 6.6                  | 11                            | 11                               |
| 1       | 15 100    | 0.85                 | 36                            | 36                               |
| 1       | 20 100    | 2.7                  | 19                            | 19                               |
| 1       | 30 100    | 14                   | 7.5                           | 7.6                              |
| 1       | 35 100    | 25                   | 5.3                           | 5.5                              |
| 0.5     | 25 100    | 1.6                  | 27                            | 27                               |
| 2       | 25 100    | 26                   | 4.8                           | 5.0                              |
| 4       | 25 100    | 110                  | 2.0                           | 3.1                              |
| 1       | 25 30     | 6.6                  | 18                            | 18                               |
| 1       | 25 300    | 6.6                  | 5.7                           | 5.7                              |
| 1       | 25 600    | 6.6                  | 3.0                           | 3.0                              |

Note.

First model is baseline model.
Equation (89) gives an accurate spectrum of radiation made when jet electrons with number spectrum $N_e'(\gamma')$ Compton-scatter blackbody photons (see Tavecchio et al. 2000; Dermer & Atoyan 2002; Böttcher et al. 2008).

3.4. Scattered BLR Radiation Field

The BLR is thought to consist of dense clouds with a specified covering factor that can be determined from AGN studies. These clouds intercept central-source radiation to produce the broad emission lines in broad-line AGN (Kaspi & Netzer 1999). The diffuse gas will also Thomson scatter the central source radiation. The scattered radiation provides an important source of target photons that jet electrons scatter to $\gamma$-ray energies (Sikora et al. 1994, 1997). This radiation field also attenuates $\gamma$ rays produced within the BLR (Blandford & Levinson 1995; Levinson & Blandford 1995; Böttcher & Dermer 1995; Donea & Protheroe 2003; Liu & Bai 2006; Reimer 2007; Sitarek & Bednarek 2008).

Here, we calculate the angular distribution of the Thomson-scattered radiation from a shell of gas with density

$$n_e(R) = n_0 \left( \frac{R}{R_0} \right)^\xi H(R; R_1, R_0)$$

(90)

extending from the inner radius $R_1$ to the outer radius $R_0$ (see Figure 8; the calculation of fluorescence atomic-line radiation differs by considering dense clouds with a volume filling factor). The shell is assumed to be spherically symmetric with a power-law radial density distribution and radial Thomson depth $\tau_T = \sigma_T \int_{R_1}^{R_0} dR n_e(R)$. The Thomson-scattered spectral photon density can be estimated by noting that a fraction $\approx r_n(r)\sigma_T$ of the central source radiation is scattered with ambient photon density $n_{ph}(\epsilon_s; r) = N_{ph}(\epsilon_s)/4\pi r^2c$ is scattered, giving a target scattered radiation field

$$n_{ph}(\epsilon_s; r) \approx n_e(r)\sigma_T \frac{N_{ph}(\epsilon_s)}{4\pi r^2c}$$

(91)

when $R_1 \lesssim r \lesssim R_0$. Here, $N_{ph}(\epsilon_s) = L(\epsilon_s)/(mc^2\epsilon_s)$ is the central source photon-production rate, assumed to radiate isotropically, and $L(\epsilon_s)$ is its spectral luminosity.

A more accurate calculation of the Thomson-scattered photon density is obtained by integrating the expression

$$n_{ph}(\epsilon_s; R) = \int dV \frac{n(\epsilon_s; \vec{R})}{4\pi x^2c}$$

(92)

over volume, where $dV = R^2d\phi d\mu dR$ and $x^2 = R^2 + r^2 - 2rR\cos\theta$ (Figure 8). Assuming that the photons are isotropically Thomson-scattered by an electron without change in energy, so that $n(\epsilon_s; R) = N_{ph}(\epsilon_s)\sigma_T n_e(R)/(4\pi R^2c)$, then

$$n_{ph}(\epsilon_s; r) = \frac{L(\epsilon_s)\sigma_T}{8\pi m_e c^3 \epsilon_s} \int_0^\infty d\mu \int_0^\infty dR n_e(R) \frac{n_e(R)}{x^2}$$

$$= \frac{L(\epsilon_s)\sigma_T}{8\pi m_e c^3 \epsilon_s} \int_0^\infty dR n_e(R) \ln \frac{R+r}{R-r}$$

(93)

from a spherically symmetric electron density distribution (see Gould 1979; Böttcher & Dermer 1995, for a time-dependent treatment). In the case of an isotropic, uniform surrounding medium, $n_e(R) = n_o$, $\xi = 0$, $R_1 = 0$, and $R_0 \rightarrow \infty$ in Equation (90), and Equation (93) gives $n_{ph}(\epsilon_s; r) = 3.324N_{ph}(\epsilon_s)\sigma_T n_0/(8\pi cr)$, noting that the integral $\int_0^\infty \ln |(1+u)/(1-u)|/u du \approx 3.324$. Thus, the approximation given by Equation (91) is accurate to within a factor of $\approx 2$ for this case.

The angle-dependent scattered photon distribution $n_{ph}(\epsilon_s, \mu_s; r)$ can be derived by imposing a $\delta$-function constraint on the angle $\theta_s = \arccos \mu_s$, in Equation (92) (see also Donea & Protheroe 2003), so that

$$n_{ph}(\epsilon_s, \mu_s; r) = \frac{\sigma_T N_{ph}(\epsilon_s)}{8\pi cr} \times \int_{-\mu_s}^{\mu_s} d\mu \int_0^\infty dg n_s(g, r)\delta[\mu_s - \bar{\mu}_s(\mu, g)]$$

$$= \int_{-\mu_s}^{\mu_s} d\mu \frac{N_{ph}(\epsilon_s)}{8\pi cr}$$

(94)

after changing variables to $g = R/r$. From Figure 8, $\theta_{max} = \pi - \theta_s$, so that $\mu_{min} = \cos \theta_{max} = \pi - \theta_s$ and $-\mu_s \leq \mu \leq 1$. The law of sines gives $R^2(1-\mu^2) = x^2(1-\mu_s^2)$, with the result

$$\bar{\mu}_s(\mu, g) = \frac{\pm(1-g\mu)}{\sqrt{1+g^2-2g\mu}}$$

(95)

Transforming the $\delta$-function in $\mu_s$ to a $\delta$-function in $g$ gives, after solving,

$$n_{ph}(\epsilon_s, \mu_s; r) = \frac{\sigma_T N_{ph}(\epsilon_s)}{8\pi cr} N(\mu_s, r),$$

(96)

where

$$N(\mu_s, r) = N(\mu_s, n(r)) \equiv \int_{-\mu_s}^{\mu_s} d\mu n_s(\bar{g}, r) \sqrt{1+\bar{g}^2-2\bar{g}\mu/(1-\mu^2)}$$

(97)

(units of $N$ are $1/L^3$), and

$$\bar{g} = \frac{\bar{g}(\mu, \mu_s)}{-\mu(1-\mu_s^2) + \mu_s\sqrt{(1-\mu^2)(1-\mu_s^2)}}$$

(98)

Figure 9 shows the angle dependence of the scattered radiation field in the stationary frame for this idealized geometry.
for the parameters given in the figure caption. As can be seen, the scattered radiation field is nearly isotropic when \( r < R_\odot \), and starts to display increasing asymmetry peaked in the outward direction at increasingly greater heights. When \( r > R_\odot \), all scattered radiation is outwardly directed. Most of the scattered material is near the inner edge when \( \zeta = -2 \), so that the intensity of the scattered radiation field is largest at \( r \lesssim R_\odot \). By contrast, the intensity of the scattered radiation field is not so enhanced toward the inner regions when \( \zeta = 0 \).

The Compton-scattered radiation spectrum is given, in general, by Equation (34). Substituting Equation (96) for the angular distribution of the target photon source, for a monochromatic photon source

\[
N_{\text{ph}}(\epsilon_s) = \frac{L_0(\epsilon_s - \epsilon_\odot)}{m_e c^2 \epsilon_s},
\]

(99)

the \( \nu F_\nu \) flux

\[
j_{\nu}^{\text{EC,scat}}(r) = \frac{(\pi r_s^2)^2 L_0 \delta^3}{12 \pi^2 d_t^2 r} \left( \frac{\epsilon_s}{\epsilon_s^*} \right)^2 \int_{\max(1-1.12\epsilon_s/\epsilon_s^*/)}^{1} d\mu_s N(\mu_s, r)
\]

\[\times \int_{\gamma_{\text{low}}}^{\infty} d\gamma N'(\gamma/\delta_D) \gamma^2 \Theta(\gamma/\delta_D - 1) d\gamma \]

(100)

In this expression,

\[
\gamma_{\text{low}} \equiv \frac{\epsilon_s}{2} \left[ 1 + \sqrt{1 + \frac{2}{\epsilon_s \epsilon_s(1 - \mu_s)}} \right]
\]

(101)

(compare Equation (29)). Substituting Equation (96) into Equation (20) for a monochromatic photon source, Equation (99), gives

\[
\tau_{\gamma\gamma}(\epsilon_1, r) = \frac{\sigma_T L_0}{8 \pi m_e c^3 \epsilon_\odot} \int_{r}^{\infty} \frac{dx}{x}
\]

for the opacity of a photon with the measured energy \( \epsilon_1 \) emitted outward along the jet axis at height \( r \). The \( \gamma\gamma \) opacity vanishes when \( \epsilon_1 \approx 1/\epsilon_s(1+z) \) due to the \( \gamma\gamma \) pair-production threshold.

### 3.4.1. Thomson-Scattered Isotropic Monochromatic Radiation Field

Substituting \( n_s(\epsilon_s, \Omega_s) = n_0 \delta(\epsilon_s - \epsilon_\odot)/4\pi \) for an isotropic monochromatic radiation field in Equation (35) gives, with \( \Xi \to 2 \) for the Thomson regime away from the endpoints of the spectrum,

\[
f_{\nu}^{\text{T,iso}} = \frac{c^2 \pi r_s^2 \delta_D^3 \mu_0}{4\pi d_t^2} \left( \frac{\epsilon_s}{\epsilon_s^*} \right)^2 \int_{\max(1-1.12\epsilon_s/\epsilon_s^*/)}^{1} d\mu_s
\]

\[\times \int_{\gamma_{\text{low}}}^{\infty} d\gamma N'(\gamma/\delta_D) \gamma^2 \Theta(\gamma/\delta_D - 1) d\gamma \]

(103)

We consider Compton upscattering, i.e., \( \epsilon_s > \epsilon_\odot \) for the power-law electron spectrum given by Equation (47). The Thomson approximation is only valid far from the endpoints. The asymptotes for the Thomson-scattered spectrum of a surrounding isotropic, monochromatic radiation field therefore become

\[
f_{\nu}^{\text{T,iso}} \approx \frac{2 c^2 \pi r_s^2 \mu_0}{4 \pi d_t^2 (p+1)} K' \left( \frac{\epsilon_s}{\epsilon_s^*} \right)^{2} \delta_D^2 \gamma^{-1-p} \gamma_D^{-1} d\gamma
\]

for \( \epsilon_s \lesssim \epsilon_\odot \ll 4 \epsilon_s(\delta_D^2 \gamma_D')^2 \),

(104)

and

\[
f_{\nu}^{\text{T,iso}} \approx \frac{2 p+3}{(p+1)(p+3)} \frac{c^2 \pi r_s^2 \delta_D^3 \mu_0}{4 \pi d_t^2} K' \left( \frac{\epsilon_s}{\epsilon_s^*} \right)^{3-p/2} \delta_D^2 \gamma^{-1} \gamma_D^{-1} d\gamma
\]

for \( 4 \epsilon_s(\delta_D^2 \gamma_D')^2 \lesssim \epsilon_\odot \ll 4 \epsilon_s(\delta_D^2 \gamma_D')^2 \).

(105)

Note the different beaming factors in the two asymptotes. Equation (105) agrees with the Thomson expression derived by Dermer et al. (1997), Equation (22), to within factors of order unity.

### 3.4.2. Numerical Calculations

We now present calculations of the SED of FSRQ blazars, including the EC-scattering component formed by jet electrons that scatter target photons which themselves were previously scattered by BLR material. This EC BLR component is not found in conventional synchrotron/SSC models of blazars, and so distinguishes standard model blazar BL Lac and FSRQs. The inclusion of the \( \gamma\gamma \) opacity through the scattered radiation makes the calculation self-consistent.

For the simplified shell geometry depicted in Figure 8, we calculate the EC-scattering component using Equation (100), and calculate the \( \gamma\gamma \) opacity using Equation (102). To simplify the calculations, the spectrum of the radiation scattered by the BLR is assumed to be monochromatic with energy \( \approx 50 \text{ eV} \), corresponding to the mean energy from the accretion-disk radiation for the standard model. Results of such a calculation are shown in Figure 10, using parameters for a standard FSRQ blazar model given in Table 1. The constant-density BLR is confined between 100 and 200 \( R_\odot \), and the Thomson depth through the BLR is \( \tau_T = 0.1 \). In Figure 10(a), the emission region of the jet is far outside the BLR, at 1000 \( R_\odot \). Therefore
most of the BLR photons encounter the jet from behind, and the rate of tail-on scatterings is suppressed by the rate factor. Consequently, the flux of the EC BLR component is much less that the flux of the EC disk component. Figure 10(b) presents a similar calculation, except that the BLR now extends to \(10^3 R_g\). The outer radius, \(R_o\), of the BLR is varied, keeping its Thomson depth, \(\tau_T = 0.01\), constant.

As expected, the EC BLR component is significantly enhanced compared to Figure 10(a).

There is very little \(\gamma\gamma\) absorption by the accretion-disk radiation or the BLR radiation when the jet is found outside the BLR. On the other hand, when the jet is within the BLR, there can be significant \(\gamma\gamma\) opacity. This is shown in Figure 11, where we use the standard parameters for the jet and a BLR with \(\tau_T = 0.01\), except that the outer radius, \(R_o\), of the BLR is varied. The effects of \(\gamma\gamma\) attenuation by the scattered radiation field are shown. When \(R_o \ll r\), the jet height, then the threshold is suppressed except for the highest energy \(\gamma\) rays. When the blob lies within the BLR, \(r \lesssim R_o\), the opacity from the scattered BLR radiation is significant and the \(\gamma\gamma\) opacity large for \(\gamma\) rays with \(\epsilon \sim 2/\epsilon_s\), which for our monochromatic radiation field

with \(\epsilon_s \approx 10^{-4}\), is at \(v \approx 10^{24}\) Hz (\(\approx 4\) GeV). When \(R_o \gg r\), the \(\gamma\gamma\) opacity, proportional to \(r n_s(\epsilon_s; r)\), declines \(\propto R_o^{-1}\) for constant Thomson depth, as can be seen from Equation (91). Note that the EC \(\gamma\) rays are not very sensitive to the changes in the BLR parameters, since most of the emission is formed by the EC disk component.

The effect of changing the radial gradient of BLR scattering material is shown in Figure 12. For a steeper density gradient, \(\zeta = -2\), and a constant Thomson depth, the material is concentrated near the inner edge of the BLR at \(R_i\), so the changes in the \(\gamma\gamma\) opacity are most dramatic when \(R_o \approx r\) due to geometric effects. When \(R_i \ll r \ll R_o\), the radiation field is essentially unchanged for different values of \(R_o\), and so also is the \(\gamma\gamma\) opacity.

4. DISCUSSION AND SUMMARY

We have presented accurate expressions for modeling synchrotron and Compton-scattered radiation from the jets of AGNs that include target radiation fields from the accretion disk and BLR. This extends our technique for modeling synchrotron and
SSC emission (Finke et al. 2008) to include EC scattering in a relativistic jet of thermal radiation from the accretion disk, and accretion-disk radiation Thomson-scattered by electrons in the BLR. These formulae use the full Compton cross section and are accurate throughout the Thomson and Klein–Nishina regimes at any angle with respect to the jet axis, so can also be used to model $\gamma$-ray emission from radio galaxies, e.g., M87 (Aharonian et al. 2006). We also derive expressions for the $\gamma\gamma$ opacity through the same scattered radiation field that serves as a target photon source for the jet electrons. The expressions, Equations (80) and (102), for opacity from the accretion-disk and scattered radiation field assume, however, that the photon travels along the jet axis, though it is straightforward to derive the more general case.

In the results presented here (Figures 1–6 and 10–12) we have chosen parameters for demonstration purposes that give an exaggerated EC component, particularly a high $\ell_{\text{eff}}$. Lowering the disk luminosity lowers the radiation considerably, as seen in Figure 2. The disk radiation field in FSRQ blazars can be seen when the nonthermal blazar radiation is in a low state, as in the cases of 3C 279 (Pian et al. 1999), 3C 454.3 (Raiteri et al. 2007), and most clearly, 3C 273 (e.g., von Montigny et al. 1997). These observations can be used to assign the accretion-disk luminosity when modeling a specific blazar, though the disk brightness could also vary during the flaring epoch.

The models presented here do, however, have limitations. They do not yet include enhancements from secondary cascade radiation initiated by $e^\pm$ pairs formed by $\gamma$ rays interacting with lower energy radiation from the disk and the BLR. For the parameters considered here, this would not make a significant difference in the calculated SEDs because the energy flux of the attenuated radiation is a small fraction of the escaping flux. But even in this case, the re-injected pairs from the attenuated radiation will be isotropized if the re-injection occurs outside the relativistic flow, and will then make only a small contribution to the Doppler-boosted radiation.

Spectral features result from $\gamma\gamma$ absorption by BLR radiation, as seen in Figure 12. By assuming hard primary $\gamma$-ray emission components, Aharonian et al. (2008) argue that hard intrinsic spectra from blazars such as 1ES 1101-232 (Aharonian et al. 2007) could be formed through $\gamma\gamma$ attenuation. If the primary TeV radiation originates from an underlying jetted electron distribution, then a consistent model requires that a $\gamma\gamma$-ray spectrum formed by Compton-scattering processes arises from the same radiation field responsible for $\gamma\gamma$ absorption. As our calculations show, soft Compton-scattered TeV spectra are formed due to Klein–Nishina effects in scattering, whether from the accretion disk or from photons scattered by the BLR. Thus, either an extremely bright GeV component would be expected in the scenario of Aharonian et al. (2008; compare to our Figures 11 and 12), which would easily be detected with Fermi, or a hard primary $\gamma$-ray spectrum must originate from other processes. Moreover, if this explanation was correct, then blazars with stronger broad emission lines should have harder VHE $\gamma$-ray spectra than weak-lined BL Lacs.

Cascade radiation induced by ultrarelativistic hadrons could inject high-energy leptons to form a hard radiation component, though a sufficiently dense target field for efficient photohadronic losses will itself severely attenuate the TeV radiation (Atoyan & Dermer 2003). Depending on the underlying acceleration model, a distinctive hadronic signature could appear at $\gamma$-ray energies only, though one would expect that both electrons and protons would be accelerated synchronously.

Our calculations also do not as yet include absorption by the diffuse extragalactic background light (EBL), which would be important for EGRET $\gamma$-ray FSRQs, which have a broad redshift distribution with a mean value $\langle z \rangle \sim 1$, larger than the mean value $\langle z \rangle \sim 0.3$ for BL Lacs (Mukherjee et al. 1997). In the cases where the Compton-scattered spectra decline steeply due to Klein–Nishina effects (e.g., Figure 1), the issue of EBL absorption is secondary. In the one FSRQ, 3C 279 at $z = 0.536$, detected from 80 to $\sim 300$ GeV energies with MAGIC (MAGIC Collaboration 2008), EBL effects cannot be neglected. For the models studied here, the soft calculated $\gamma$-ray spectra cannot in any case account for the measured, let alone intrinsic, spectrum of 3C 279. Based on the simultaneous optical—X-ray—VHE $\gamma$-ray SED of 3C279 during the MAGIC detection, Böttcher et al. (2009) show that one-zone leptonic models have severe problems explaining the flare, and require either extremely high Doppler factors or magnetic fields well below equipartition. Correlated X-ray, Fermi, and TeV campaigns will offer opportunities to apply the results developed in this paper.

In conclusion, we have derived expressions to model FSRQ blazars that self-consistently include $\gamma\gamma$ attenuation on the same target photons that are Compton-scattered by the relativistic electrons. By assuming that the lower energy, radio–UV emission is nonthermal synchrotron radiation, then the underlying electron distribution can be determined given the Doppler factor, magnetic field, and size scale of the emission region. The equations derived here can be used to calculate the $\gamma$-ray emission spectrum from SSC and EC-scattering processes from this electron distribution. Separately, one can determine whether the inferred electron distribution can be derived from specific acceleration scenarios. Complementary to the technique of injecting electron spectra and cooling, this method can be applied to multiwavelength data sets to analyze high-energy processes in the jets of AGN.

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ERRATUM: “GAMMA RAY STUDIES OF BLAZARS: SYNCHRO-COMPTON ANALYSIS OF FLAT SPECTRUM RADIO QUASARS” (2009, ApJ, 692, 32)

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The photo-absorption due to γ-rays interacting with broad line region (BLR) photons was calculated incorrectly in Figures 11 and 12 of the published version of this article. Corrected figures can be found in Figures 1 and 2. We are grateful to Dahai Yan (Yunnan University) for bringing this error to our attention. Absorption effects from the BLR will clearly have less of an observable

![Figure 1](image1.png)

**Figure 1.** Corrected version of Figure 11.

![Figure 2](image2.png)

**Figure 2.** Corrected version of Figure 12.

(A color version of this figure is available in the online journal.)
effect than we concluded, particularly for the $\xi = -2$ case. However, for the $\xi = 0$ case, a spectral break is still clearly seen. All our other conclusions remain unchanged.

We also point out a few errors which do not affect our results, as the correct expressions were used in our calculations. Equation (88) of the published article should read

$$\frac{u_*(\epsilon_*)}{4\pi} = u_{bb}(\epsilon_*, \Omega_*) = \frac{2m_e c^2}{\lambda C_0^3} \frac{\epsilon_*^3}{\exp(\epsilon_*/\Theta) - 1}$$

i.e., there was an extra factor of $c$ in the numerator.

The equation in the text just above Equation (93) should read

$$\dot{n}(\epsilon_*; \tilde{R}) = \dot{N}_{\text{ph}}(\epsilon_*) \sigma_{\text{eff}}(R)/(4\pi R^2),$$

i.e., there was an extra factor of $c$ in the denominator.

Equation (103) should read

$$f_{T, \text{iso}} = \frac{c \pi r_0^2 \delta D \epsilon_*}{4\pi d_L^2} \left( \frac{\epsilon_*}{\epsilon_r} \right)^2 \int_{\max(-1,1-2\epsilon_*/\epsilon_s)}^1 d\mu_*
\times \int_{\sqrt{\epsilon_*/\epsilon_s}^{1-\mu_*}}^{\infty} d\gamma \gamma^2 N_e'(\gamma/\delta_D)$$

i.e., $\delta_D$ should be raised to the power 3 rather than $3 + p$.

In Equation (104), the conditional should read

$$\epsilon_* \lesssim \epsilon_r \ll 4\epsilon_* (\delta_D^2 \gamma_1)^2,$$

and in Equation (105) the conditional should read

$$4\epsilon_* (\delta_D^2 \gamma_1)^2 \lesssim \epsilon_* \ll 4\epsilon_* (\delta_D^2 \gamma_2)^2.$$