TuSeRACT: Turn-Sample-Based Real-Time Traffic Signal Control

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Abstract

Real-time traffic signal control systems can effectively reduce urban traffic congestion but can also become significant contributors to congestion if poorly timed. Real-time traffic signal control is typically challenging owing to constantly changing traffic demand patterns, very limited planning time and various sources of uncertainty in the real world (due to vehicle detection or unobserved vehicle turn movements, for instance). SURTRAC (Scalable URban TRAffic Control) is a recently developed traffic signal control approach which computes delay-minimising and coordinated (across neighboring traffic lights) schedules of oncoming vehicle clusters in real time. To ensure real-time responsiveness in the presence of turn-induced uncertainty, SURTRAC computes schedules which minimize the delay for the expected turn movements as opposed to minimizing the expected delay under turn-induced uncertainty. Furthermore, expected outgoing traffic clusters are communicated to downstream intersections. These approximations ensure real-time tractability, but degrade solution quality in the presence of turn-induced uncertainty. To address this limitation, we introduce TuSeRACT (Turn Sample based Real-time trAffic signal ConTrol), a distributed sample-based scheduling approach to traffic signal control. Unlike SURTRAC, TuSeRACT computes schedules that minimize expected delay over sampled turn movements of observed traffic, and communicates samples of traffic outflows to neighbouring intersections. We formulate this sample-based scheduling problem as a constraint program, and empirically evaluate our approach on synthetic traffic networks. We demonstrate that our approach results in substantially lower average vehicle waiting times as compared to SURTRAC when turn-induced uncertainty is present.

Keywords: intelligent transportation systems, real-time traffic signal control, stochastic online scheduling, agent communication under uncertainty, sample average approximation

1 Introduction

Traffic signals contribute significantly to urban traffic congestion due to three reasons: (i) poor allocation of green light time along different directions; (ii) lack of coordination between traffic lights; (iii) inability to respond to real-time traffic patterns. Improvements to traffic signal mechanisms are believed to provide the biggest reductions in traffic congestion and increase in effective capacity of existing road networks.
Providing improvements to traffic signal mechanisms in real-time is challenging because of constantly changing demand patterns, trade-off between local and global delay, uncertainty associated with vehicle turns and the scale of incoming traffic.

Recent research on SURTRAC (Scalable Urban Traffic Control) [Xie et al., 2012b,a] has introduced mechanisms for real-time adaptive traffic signal control. SURTRAC is a real-time, distributed, schedule driven approach, where each intersection independently schedules clusters of incoming vehicles along different directions (referred to as phases) and communicates projected expected outgoing traffic clusters to traffic signals at neighboring intersections. Due to its scalability and effectiveness, SURTRAC was deployed to control traffic signals in real time and was shown to provide significant improvements over existing mechanisms.

Despite its scalability and effectiveness, SURTRAC has two fundamental limitations: (a) To ensure real-time response given the vehicular movement uncertainties (due to changing demand patterns, turn movements), instead of minimizing expected delay, SURTRAC minimizes delay for the expected scenario of vehicular movement uncertainty. This approximate optimization objective (of minimizing delay for expected scenario) is particularly limiting when traffic on a road segment moves out of the intersection along different road segments in same or different phases; (b) In order to ensure scalable coordination among traffic signals, expected traffic projection (and not actual traffic projection) is communicated to traffic signals at neighboring intersections.

To address these key limitations, we introduce TuSeRACT (Turn Sample driven Real-time traffic signal Control), a traffic sample driven distributed scheduling approach to traffic signal control. Unlike SURTRAC, TuSeRACT:

- optimizes expected delay over a set of turn samples; and
- communicates samples of actual traffic projection to traffic signals at neighbouring intersections.

This is achieved through a combination of two key contributions. First, we provide a Constraint Programming (CP) formulation that is based on Sample Average Approximation (SAA) [Kleywegt et al., 2002] and employs turn samples for controlling each individual traffic signal. Second, we provide a novel communication mechanism between traffic signals at neighboring intersections that is based on samples of outgoing traffic.

On multiple benchmark problems considered in the literature, we demonstrate that TuSeRACT is able to significantly outperform the leading approach for real-time traffic signal control, SURTRAC. We were able to reduce the expected delay consistently by up to 60% (and on average around 35%) across multiple configurations of traffic lights (considered in literature) with multiple traffic conditions (low, medium, high) and on multiple instantiations of traffic movements.

2 Background

2.1 Real-Time Traffic Signal Control

Real-time traffic signal control systems aim to give right-of-way to competing streams of oncoming traffic in a manner that minimizes vehicular waiting time while ensuring safety and fairness. We now describe the real-time traffic signal control problem.

We consider traffic signals at intersections with a set of multi-lane incoming entry roads and outgoing exit roads, as shown in Figure 1. Flow of traffic from an entry road to an exit road defines
Figure 1: An intersection with multiple entry and exit approaches. Arrows indicate the direction of traffic flow.

Figure 2: Phase design for a 4-phase signalized intersection. Each arrow represents a turn movement, i.e. traffic flow from an entry road to an exit road at the intersection. Turn movements which can safely occur simultaneously are grouped into a phase.

A turn movement. Turn probability $T_{e,f}$ is the probability that a vehicle will turn from an entry road $e$ to an exit road $f$ of an intersection. For each entry road $e$ of the intersection, $\sum_f T_{e,f} = 1$. In practice, turn probabilities are estimated as moving averages using recently observed turn movements.

At each intersection, non-conflicting turn movements are grouped into a set of phases $I$, as shown in Figure 2. Each phase $i \in [1, |I|]$ gives right-of-way to a set of traffic flows which can safely proceed through the intersection concurrently. The intersection phase design thus maps each permissible turn movement to a unique phase.

Traffic signals are usually required to cyclically give right-of-way to a fixed sequence of predefined phases $I$, i.e. the next phase to phase $i$ is given by:

$$next(i) = (i + 1) \mod |I|$$

Each phase $i$ has a variable duration (or green time) $g \in [G_i^{min}, G_i^{max}]$, where $G_i^{min}$ is the minimum green time and $G_i^{max}$ is the maximum allowed green time. To ensure safety, a fixed inter green or yellow time $Y_i$ is applied after each phase $i$, during which no phase has right-of-way. The combination of $G_i^{min}, G_i^{max}$ and $Y_i$ is jointly referred to as the traffic signal timing specification.
A traffic signal timing plan specifies the duration of each phase \( i \in I \) for multiple cycles, and adheres to the traffic signal timing specification. Real-time traffic signal control systems continually formulate, implement and re-evaluate traffic signal timing plans at specific decision points. At each decision time point \( t \), a traffic signal timing plan is formulated for the time \([t, t + H_{opt}]\), where \( H_{opt} \) is the optimization horizon. The computed traffic signal timing plan aims to minimize waiting time for vehicles approaching the intersection. Each traffic signal timing plan is based on the traffic signal timing specification, the traffic arrival distribution for a limited observation horizon \( H_{obs} \) and the initial conditions at \( t \). The initial conditions at \( t \) include \( i_t \), the phase which has right-of-way at the decision point and \( g_t \), the time for which \( i_t \) has been green. The initial conditions determine the feasibility of extension of the current traffic signal timing plan at \( t \).

The computed traffic signal timing plan is then implemented up to the next decision point \( t' \), where \( t' \leq t + H_{opt} \). It is then re-evaluated and re-extended in a short execution interval before \( t' \). The extent of commitment to formulated plans varies across approaches. We consider systems which simply make an extension decision for the current phase \( i_t \) at the decision point \( t \). At each decision point, the system implements one of the following decisions based on the computed traffic signal timing plan:

- The current phase \( i_t \) is terminated and the next phase \( next(i_t) \) is given right-of-way for a minimum green time \( G_{min}^{next(i_t)} \) after an intergreen time \( Y_{i_t} \). The next decision point \( t' = t + Y_{i_t} + G_{min}^{next(i_t)} \).
- The current phase \( i_t \) is extended for a duration \( ext > 0 \) and the next decision point \( t' = t + ext \).

### 2.2 Challenges

Lower commitment to formulated traffic signal timing plans result in increased responsiveness to changing traffic patterns, but creates a computational challenge by limiting planning time. The computational challenge is compounded by large observation horizons and complex phase designs. Additionally, real-time traffic signals must cope with the challenges posed by uncertainty in the real world, which arise due to error in vehicle detection and unknown vehicle routes, as turn movements for observed vehicles are not known.

In this paper, we focus on the challenge posed by uncertainty due to vehicle turn movements. Uncertainty due to vehicle turn movements can manifest as:

- **Uncertainty in planning:** In cases where vehicles on an entry edge \( e \) can exit the intersection in multiple, competing phases (as shown in Figure 3c), turn probabilities must be accounted for while estimating traffic demand along competing phases of the intersection. Planning for green time allotted to competing phases is contingent on this demand estimate.

- **Uncertainty in communication:** Uncertainty in vehicle turn movements also poses a challenge when planning is distributed, and projected traffic outflows must be communicated to neighbouring intersections, which use these outflows to adjust their own signal timing plans. As shown in Figure 3b, vehicles on an entry edge \( e \) could exit the intersection in the same phase, but on to different exit edges. While there is no uncertainty in planning in this case, turn probabilities need to be accounted for while estimating the traffic outflows on exit edges.
2.3 Schedule-Driven Traffic Signal Control

Scalable Urban Traffic Control (SURTRAC) is a recently developed real-time, distributed traffic signal control system which uses a schedule-driven approach to traffic signal control. [Xie et al., 2012b] formulate the traffic signal control problem as a single-machine scheduling problem, where each intersection is treated as a machine, and clusters of oncoming vehicles along competing routes are treated as jobs to be scheduled. Intersections in the traffic network independently and asynchronously compute delay-minimizing schedules for the vehicle clusters in their respective observation horizons. To ensure responsiveness to fluctuating traffic demands, these plans are re-evaluated after a brief period of commitment. Each intersection communicates projected traffic outflows to its immediate neighbours.

We now describe various components of SURTRAC. These components form the base for our sampling-based, schedule-driven approach to traffic signal control.

2.3.1 Aggregate Traffic Representation

SURTRAC represents oncoming traffic as clusters of vehicles. This representation captures the non-uniformity of traffic flows (as compared to macroscopic flow-based representations) while still remaining computationally efficient (as compared to microscopic vehicle-based representations). Aggregating vehicles into clusters also reduces the impact of sensing errors.

A cluster of vehicles $c$ is characterized by the tuple $\langle n_c, arr_c, dep_c \rangle$, where $n_c$ is the number of vehicles in the cluster, $arr_c$ is the estimated arrival time of first vehicle in the cluster at the stop line of the intersection, and $dep_c$ is the estimated departure time of the last vehicle in the cluster. $dur_c = dep_c - arr_c$ is the time the cluster requires to cross the stop line. Arrival and departure times of vehicles on an entry road are calculated by ignoring inter-vehicular effects. The arrival time $arr_c$ of a vehicle is calculated as $d/v_f$ where $d$ is the distance of the vehicle from the stop line of the intersection and $v_f$ is the free-flow-speed, the desired driver speed in the absence of congestion. All departure and arrival times are rounded to a time resolution $\Delta$.

A cluster sequence $C$ is a sequence of clusters sorted in increasing order of arrival times at the intersection, $C = (c_1, ..., c_K)$, $arr_k \geq arr_{k-1}$.

Sensed vehicular data along each entry road $e$ of the intersection is clustered into a cluster sequence $C_e$ as follows: The queued vehicles along $e$ are grouped into queue clusters. Arriving
vehicles are clustered by dividing the observation horizon $H_{\text{obs}}$ into buckets of duration $\text{samp}$, the sampling interval. Vehicles arriving in interval $h$ are clustered together. $C_e$ is formed by combining the queued cluster and arriving clusters into an ordered cluster sequence. The resulting cluster sequence is further aggregated by combining arriving clusters if the time gap between them is within a specified threshold $\text{thc} \geq 0$.

Cluster sequences along all entry roads of the intersection are collectively termed as the road-flow, $C_{RF} = (C_1, \ldots, C_E)$ where $C_e$ is the observed cluster sequence along entry road $e$.

Since multiple entry roads can have right-of-way in a single phase, and since the intersection ultimately computes phase-timing plans, the cluster sequences which can be concurrently scheduled through the intersection are combined to give per-phase cluster sequences. The per-phase cluster sequences obtained by merging sequences from the per-road roadflow are collectively termed as the inflow, $C_{IF} = (C_{i1}, \ldots, C_{i|I|})$ where $C_i$ is the cluster sequence along phase $i$. The resulting inflow is used to compute a schedule of clusters through the intersection.

In the case where vehicles on an entry road leave the intersection in multiple phases (as shown in Figure 3c), turn movements must be accounted for while converting roadflow to inflow. SURTRAC uses turn proportions to assign the expected number of vehicles to each phase, resulting in the expected inflow $E[C_{IF}]$. SURTRAC then computes a delay-minimizing schedule for the expected inflow.

### 2.3.2 Scheduling Strategy

SURTRAC formulates the traffic signal control problem as a cluster scheduling problem. A delay-minimizing schedule of clusters along all phases is computed using the inflow, and this implies the desired phase schedule. Non-divisibility of clusters is assumed, which reduces the problem to a cluster sequencing problem. The computed schedule is subject to precedence constraints on cluster sequences along each phase.

All permissible cluster sequences form the scheduling search space. A forward dynamic programming algorithm is used to search for the optimal schedule in this search space. To improve search efficiency (while sacrificing optimality), SURTRAC can group partial schedules and uses an dominance criterion to eliminate partial schedules. As a result, it can compute near-optimal schedules for observed traffic at the order of milliseconds.

### 2.3.3 Commitment

SURTRAC extends or terminates the phase at decision point based on the delay-minimizing phase switch implied by the computed cluster schedule. In case of termination, a cluster schedule is recomputed after a minimum green time of the next phase has elapsed. In case of extension, the phase is extended to let one cluster proceed through the intersection, after which the schedule is recomputed. Since SURTRAC does not account for the maximum green time constraints during scheduling, a check before implementation ensures that the extension decision does not violate these constraints.

### 2.3.4 Communication

Once a schedule is computed, each intersection computes the projected outgoing cluster sequences or outflow $C_{OF}^f$ for each of each exit road $f$. These outflows are then communicated to the downstream neighbours along each of these exit edges. Neighbouring intersections then append these
projected outflows to their locally observed roadflows, essential expanding their observation horizon.

Since the maximum green time constraint is not accounted for during scheduling, all schedules are corrected for violations to ensure that only feasible outflows are computed to neighbours.

When observed clusters on an entry road can exit the intersection using multiple roads, SURTRAC uses turn proportions to assign the expected number of vehicles to the projected outflow along each exit road \( f \). Expected outflow \( E[C_{OF}^f] \) communicated to the neighbour along exit road \( f \), for all exit roads of the intersection.

### 2.3.5 Uncertainty

As discussed, in cases where uncertainty due to turn movements arises during planning SURTRAC uses turn probabilities to compute the expected observed cluster inflow.

SURTRAC computes a delay-minimizing cluster schedule for the expected inflow \( E[C_{IF}] \), effectively minimizing the delay \( D \) for the expected inflow over all permissible schedules. More formally, when planning under uncertainty due to unknown vehicle turn movements, SURTRAC solves the following optimization problem:

\[
\min_{x \in X} D(E[C_{IF}], x) \quad (1)
\]

where \( X \) is the set of permissible schedules of clusters in the observed inflow (the scheduling search space) or equivalently, the set of permissible signal timing plans.

While this approximation allows efficient computation of schedules even under turn-induced uncertainty, it is not equivalent to the stochastic optimization problem arising due to turn-induced uncertainty. In other words, it does not schedule inflow clusters in a manner which minimizes the expected delay over the scheduled random traffic inflow. This stochastic optimization can be formalized as:

\[
\min_{x \in X} E_T[D(C_{IF}, x)] \quad (2)
\]

where \( T \) is the turn probability distribution, and the inflow \( C_{IF} \) is a random variable due to its dependence on \( T \).

To address this, we propose and investigate the potential of a sampling-based approach to the aforementioned cluster scheduling problem under turn-induced uncertainty.

We also provide a sampling-based communication protocol to address communication under uncertainty, in which case SURTRAC communicates expected projected outflows to neighbours.

Figure 4: An example of turn proportions in a 4-phase intersection
Figure 5: SURTRAC: Planning and communication under turn movement uncertainty. Here, we illustrate planning and communication as handled at a single intersection. We assume that the intersection has a 4-phase design (as shown in Figure 2), and that turn proportions from each entry road are as shown in Figure 4. For simplicity, we illustrate SURTRAC’s approach using three vehicles on a single entry road of the intersection.

(a) Observe arriving and queued vehicles on each entry road
(b) Cluster observed vehicles on each entry road by proximity (per-road clusters)

(c) From each cluster, assign the expected number of vehicles to each permissible exit road according to turn proportions
(d) Derive per-phase clusters from per-road clusters using the phase design

(e) Compute a delay-minimizing schedule for the expected per-phase clusters (according to constraints on phase duration and phase order)

(f) Communicate expected vehicle counts and departure times to immediate downstream neighbours along all exit roads based on the computed cluster schedule and the cluster composition
3 Turn-Sample-Based Real-Time Traffic Signal Control

We now propose an alternative real-time, distributed, schedule-driven, sample-based approach to traffic signal control. Our approach is based on the framework defined by SURTRAC in that it is a distributed approach that treats traffic signal control as a cluster scheduling problem. However, it aims to solve the stochastic optimization problem arising due to unobserved vehicle routes.

Our approach to the stochastic optimization problem defined in Equation 2 is based on sample average approximation (SAA) [Kleywegt et al., 2002]. SAA allows us to approximate the expectation of a quantity involving probabilistic components by averaging it over several realizations of the probabilistic component. It thus allows us to transform the stochastic optimization problem into a deterministic optimization problem.

Here, we approximate the expected delay \( D \) caused by a signal timing plan \( x \) by averaging the delay it causes over \( N \) samples of the inflow \( C^{IF} \). This transforms the original stochastic optimization problem into the following deterministic problem:

\[
\min_{x \in X} \frac{1}{N} \sum_{n=1}^{N} D(C^{IF}_n, x) \quad (3)
\]

where each \( C^{IF}_n \) is a realization of the inflow generated using turn probability distribution \( T \).

Our approach tackles uncertainty associated with unknown vehicle turn movements by solving this resulting sampling-based deterministic optimization problem. At each decision step, each intersection independently samples turn movements for observed vehicles, and computes a signal timing plan which minimizes the average delay across these samples.

We now describe the various components of our sampling-based approach.

3.1 Traffic Representation and Sampling

Like SURTRAC, our approach ultimately represents oncoming traffic as clusters of vehicles, and aggregates clusters into per-phase cluster sequences (inflows), which are then used to compute a delay-minimizing schedule for the observed clusters. However, our approach handles uncertainty associated with turn movements by computing a signal timing plan which minimizes the average delay over multiple samples of the inflow. The samples are generated while aggregating sensed vehicular data into inflows, and this intermediate process differentiates our traffic representation from SURTRAC’s, despite both approaches using the same framework. We now formally describe our sampling process.

Like SURTRAC, we use sensed vehicular data from each entry road of the intersection. Sensor data for each entry road specifies the positions of all observed vehicles on the road in terms of distances from the stop line of the intersection.

While SURTRAC clusters sensor data on each entry road into roadflows, which are then transformed into per-phase inflows, we directly create per-phase inflows from sensor data by sampling an outgoing exit edge for each vehicle. For each observed vehicle on entry edge \( e \), we independently sample an exit edge \( f \) according to the given turn probabilities \( T_{e,f} \). As mentioned, turn probability \( T_{e,f} \) is the probability that a vehicle on entry road \( e \) will exit the intersection using exit road \( f \) and \( \sum_f T_{e,f} = 1 \forall e \). Since the intersection phase design maps each permissible turn to a single phase, sampling an exit road (or a turn) for a vehicle is equivalent to sampling the phase in which each vehicle will leave the intersection. A complete sample drawn by each intersection
at each decision point is the random vector of sampled exit roads for all observed vehicles at an intersection. \( N \) such samples are drawn.

Post sampling, each observed vehicle can be characterised by the tuple \((e, f, i, e, f, d)\), where \( e \) is the entry edge on which the vehicle is observed, \( f \) is the exit edge sampling using \( T, i, e, f \) is the phase in which the turn movement from \( e \) to \( f \) is given right-of-way (exit phase) and \( d \) is the sensed distance between the vehicle and the stop line of the intersection. Vehicle positions \( d \) are then converted to vehicle arrival times \( arr = d / v_f \), where \( v_f \) is the free-flow-speed.

Vehicles are segregated into phases based on their sampled exit phases, and their arrival times are then used to aggregate them into clusters in the same manner as SURTRAC. As vehicles are aggregated into phases, the composition of each cluster is recorded. We record the tuple \((arr, f)\) for each vehicle in the cluster, where \( arr \) is the estimated arrival time of the vehicle at the stop line and \( f \) is the sampled exit edge. We define cluster composition \( \gamma \) as the list of \((arr, f)\) tuples for all vehicles in the cluster, \( \gamma = (\text{arr}_1, f_1), (\text{arr}_2, f_2), \ldots) \).

Each cluster \( c \) is then characterized by the tuple \((|c|, arr_c, dep_c, \gamma_c)\).

This sampling and clustering process is repeated \( N \) times at each decision point. As a result, we get a sample set \( S \) of \( N \) sampled inflows, \( S = (C_{1f}^1, \ldots, C_{Nf}^N) \). Our objective is to compute a signal timing plan which minimizes the average delay across \( S \). Each sampled inflow \( C_{nf}^i \) specifies a cluster sequence along each phase \( i, C_{nf}^i = (C_{n,1}, \ldots, C_{n,|I|}) \).

### 3.2 Scheduling Strategy

As described, our approach to the cluster scheduling problem under uncertainty is based on sample average approximation, where we aim to compute a signal timing plan which minimizes the average delay across sampled traffic inflows. We formulate this deterministic, sample-based cluster scheduling problem (formalized in Equation 3) as a constraint program, and solve it using the IBM ILOG CP Optimizer.

The chief differences between our approach and SURTRAC’s scheduling model are:

- Our approach is based on sampling and the objective is to minimize the average delay across the sampled traffic inflows, while SURTRAC minimizes the delay for the expected traffic inflow.
- We formulate the problem as a preemptive scheduling problem, while SURTRAC treats the problem as a non-preemptive scheduling problem. That is, we do not assume the non-divisibility clusters.
- We formulate and solve the scheduling problem as a constraint program, while SURTRAC solves the scheduling program by searching an abstract scheduling search space using a forward dynamic programming algorithm.

We now describe our constraint programming formulation of the sampling-based, preemptive cluster scheduling problem.

#### 3.2.1 Input

- Sampled traffic inflows: Our approach requires samples of inflows (per-phase cluster sequences), which are generated using observed vehicle data and turn probabilities, as described previously. We use sample set \( S \) of \( N \) sampled inflows, \( S = (C_{1f}^1, \ldots, C_{Nf}^N) \). Each
interval variables
We model our decision variables using the notion of interval variables, a CP Optimizer feature which allows us to intuitively model scheduling problems in terms of intervals of time. An interval variable \( i \) is an interval of time \([\text{start}(i), \text{end}(i)]\), where \( \text{start}(i) \) and \( \text{end}(i) \) are integral. \( \text{length}(i) \) is defined as \( \text{end}(i) \) \( - \) \( \text{start}(i) \). An interval variable can be optional, i.e., it may or may not be present in the solution.

Our scheduling strategy must ultimately compute a common signal timing plan across all inflow samples, and scheduled departure times (or outgoing clusters) for each inflow sample. We define the following decision variables for these components:

- Phase intervals: We define the interval variable \( P_{i,j} \forall i, j \) which represents phase \( i \) in cycle \( j \).
- Outgoing cluster intervals: Since we assume that clusters are divisible, we assume that fragments of a cluster may be scheduled in any cycle. We define the interval variable \( C_{n,i,k,j} \forall n, i, k, j \) which represents the fragment of cluster \( c_{n,i,k} \), scheduled in cycle \( j \) (if any). \( \text{start}(C_{n,i,k,j}) \) represents the time at which the cluster fragment starts leaving the intersection by crossing the stop line. Since the complete cluster may exit the intersection in less than \( |J| \) cycles, these intervals are optional. The absence of \( C_{n,i,k,j} \) implies that no fragment of \( c_{n,i,k} \) is scheduled in cycle \( j \).

3.2.2 Decision Variables
We model our decision variables using the notion of interval variables, a CP Optimizer feature which allows us to intuitively model scheduling problems in terms of intervals of time. An interval variable \( i \) is an interval of time \([\text{start}(i), \text{end}(i)]\), where \( \text{start}(i) \) and \( \text{end}(i) \) are integral. \( \text{length}(i) \) is defined as \( \text{end}(i) \) \( - \) \( \text{start}(i) \). An interval variable can be optional, i.e., it may or may not be present in the solution.

Our scheduling strategy must ultimately compute a common signal timing plan across all inflow samples, and scheduled departure times (or outgoing clusters) for each inflow sample. We define the following decision variables for these components:

- Phase design: We assume that each intersection has a pre-defined set of phases \( I = (1 \ldots |I|) \).
- Signal timing specification: We assume that the order of phases in \( I \) is fixed. For each phase \( i \), its duration \( g_i \in [G_i^{\text{min}}, G_i^{\text{max}}] \) and an intergreen time of \( Y_i \) must be applied after it.
- Optimization horizon: We require the observed clusters to be cleared in a set of cycles \( J = (1 \ldots |J|) \), where each cycle specifies the duration for all phases in \( I \) subject to phase order and phase duration constraints. \( J \) is used to limit the finish time of the computed signal timing plans, and includes the cycle under way at decision point. If \( |J^*| \) is the optimal number of cycles required to clear the observed traffic, any estimate \( |J| \geq |J^*| \) can be used to limit the finish times of the computed schedules.
- Initial conditions: To determine the feasibility of extension of the current signal timing plan, we require the following at each decision time point \( t \):
  - \( t \): Current time at the decision point. Treating the decision point as the origin, we set \( t = 0 \).
  - \( i_t \): Current phase, i.e., phase which has right-of-way at decision point.
  - \( g_t \): Current phase duration, i.e., the time for which the current phase has been green.

\[ C_{n,i,k,j} = (C_{n,i,1}, \ldots, C_{n,i,|I|}), \text{ where } C_{n,i} \text{ is the cluster sequence along phase } i \text{ in sample } n. \text{ Each cluster sequence } C_{n,i} = (c_{n,i,1}, \ldots, c_{n,i,k}), \text{ a tuple of clusters along phase } i, \text{ ordered by cluster arrival times. Cluster } c_{n,i,k} \text{ is cluster } k_i \text{ along phase } i \text{ in sample } n, \text{ and is characterised by } (|c_{n,i,k}>, arr_{n,i,k}, dur_{n,i,k}, \gamma_{n,i,k}). \] \( C_{n,i} \) is ordered by cluster arrival times, \( arr_{n,i,k_i} \geq arr_{n,i,(k-1)_i} \).
3.2.3 Constraints

The computed signal timing plan and outgoing cluster departures are constrained by the signal timing specification, the initial conditions at the decision point and also the observed traffic inflow. We now formulate these constraints in our model.

- Phase duration: The duration of each phase is constrained by a minimum green and a maximum green time predefined by the signal timing specification.

\[
\text{length}(P_{i,j}) \in [G_{i}^{\text{min}}, G_{i}^{\text{max}}] \quad \forall i, j
\]  

(4)

- Phase order and intergreen time: In each cycle, the phases \( I \) must be given right-of-way in a fixed order per the signal timing specification, and a fixed intergreen time must be applied between consecutive phases.

\[
\text{start}(P_{i,j}) = \text{end}(P_{i-1,j}) + Y_{i-1} \quad \forall i \neq 1, j
\]  

(5)

- Cycle order: The cycles \( J \) occur in a fixed order, and a fixed intergreen time is applied between consecutive cycles.

\[
\text{start}(P_{i,j}) = \text{end}(P_{|I|,j-1}) + Y_{|I|} \quad \forall j \neq 1
\]  

(6)

- Initial conditions: The current phase \( i \) at decision time \( t \) determines the feasibility of extension of the current signal timing plan. We formulate this by constraining the start and end times of \( i \) in the current cycle \( j = 1 \).

\[
\text{start}(P_{i,1}) = t - g_t \\
\text{end}(P_{i,1}) \geq t
\]  

(7) (8)

- Cluster fragments: We assume that clusters are divisible, and that fragments of a single cluster can be scheduled in different cycles.

The length of each fragment of a cluster is constrained by the length of the cluster:

\[
\text{length}(C_{n,i,k,i}) \in [1, \text{dur}_{n,i,k}] \quad \forall n, i, k, j
\]  

(9)

We ensure that each cluster completely leaves the intersection in at most \( J \) cycles:

\[
\sum_{j} \text{length}(C_{n,i,k,i}) = \text{dur}_{n,i,k} \quad \forall n, i, k
\]  

(10)

- Cluster departure: Outgoing clusters can be scheduled to exit the intersection only after they arrive at the stop line:

\[
\text{start}(C_{n,i,k,i}) \geq \text{arr}_{n,i,k} \quad \forall n, i, k, j
\]  

(11)

Fragment \( C_{n,i,k,i,j} \) if scheduled, must be scheduled in the appropriate phase \( P_{i,j} \):

\[
\text{start}(C_{n,i,k,i,j}) \geq \text{start}(P_{i,j}) \quad \forall n, i, k, j
\]  

(12)

\[
\text{end}(C_{n,i,k,i,j}) \leq \text{end}(P_{i,j}) \quad \forall n, i, k, j
\]  

(13)
• Cluster precedence among clusters in the same phase: Any fragment of cluster \( c_{n,i,k_i} \) can be scheduled only all fragments of \( c_{n,i,(k-1)_i} \) have been scheduled. In other words, \( c_{n,i,k_i} \) can be scheduled only after \( c_{n,i,(k-1)_i} \) has completely exited the intersection.

\[
presenceOf(C_{n,i,k_i}) \rightarrow not(presenceOf(C_{n,i,(k-1)_i}))
\]

\( \forall n, i, k_i \neq 0, j, j' = j + 1, j + 2, \ldots, |J| \)  
(\( j' \) is any succeeding cycle)

\[
\text{end}(C_{n,i,(k-1)_i,j}) \leq \text{start}(C_{n,i,k_i,j})
\]

\( \forall n, i, k_i \neq 0, j \)

3.2.4 Objective

Our objective is to compute a single signal timing plan which minimizes the cumulative waiting time across the vehicles in all the inflow samples:

\[
\min \sum_{n,i,k_i,j} \left( \text{start}(C_{n,i,k_i,j}) - \text{arr}_{n,i,k_i} \right) \ast \left( |c_{n,i,k_i,j}| \ast \frac{\text{size}(C_{n,i,k_i,j})}{\text{dur}_{n,i,k_i}} \right)
\]

\( (\text{start}(C_{n,i,k_i,j}) - \text{arr}_{n,i,k_i}) \) represents the delay incurred by cluster fragment \( C_{n,i,k_i,j} \), and \( (\frac{|c_{n,i,k_i,j}| \ast \text{size}(C_{n,i,k_i,j})}{\text{dur}_{n,i,k_i}}) \) represents the number of vehicles in that fragment.

3.3 Commitment

Like SURTRAC, our approach extends or terminates the current phase \( i_t \) at the decision point \( t \). The extension decision is based on the planned phase schedule, specifically the planned end point \( \text{end}(P_{i_t,1}) \) of the current phase. If \( \text{end}(P_{i_t,1}) = t \), the current phase is terminated and the next decision point \( t' = t + Y_{i_t} + C_{\text{min}}^{\text{next}(i_t)} \). If \( \text{end}(P_{i_t,1}) > t \), the current phase is extended up to \( \text{end}(P_{i_t,1}) \) and the next decision point \( t' = \text{end}(P_{i_t,1}) \).

Our commitment to the computed schedule differs from SURTRAC’s in that:

• Our extension decision need not be checked for violations of the maximum green time constraint, since we account for this constraint while planning.

• In case the current phase is to be extended, we allot additional green time as planned by the schedule (and may allow multiple additional clusters to pass through in the current phase before replanning). On the other hand, SURTRAC extends the current phase to let one cluster proceed through the intersection, after which the schedule is recomputed.

3.4 Sample-Based Communication

Like SURTRAC, our approach to traffic signal control is that of distributed, local planning with communication with neighbours. Intersections independently compute signal timing plans for
traffic in their respective observation horizons and communicate projected outflows to neighbouring intersections. This allows neighbouring intersections to expand their observation horizon. Limiting communication to immediate neighbours makes both approaches scalable to large road networks.

The key communication-related difference between both approaches lies in the form of traffic outflows communicated to downstream neighbours, especially in cases where turn-induced uncertainty exists. Based on the traffic inflow and the computed phase schedule, SURTRAC computes expected outflow cluster sequences along each exit road of the intersection. These are then communicated to downstream neighbours along the respective exit roads. At the neighbouring intersections, the outflows are appended to the observed roadflows to increase the observation horizon.

Our approach is to communicate multiple samples of vehicle departure times to neighbouring intersections along the sampled exit roads as opposed to communicating a single expected outflow cluster sequence along each exit road. Vehicle departure times for each of the $N$ samples are computed using the cluster delays computed during planning (derived from cluster departure times), and the cluster composition which specifies the arrival time of each vehicle in the cluster. The computed departure times for the vehicles is then simply communicated along the exit roads sampled for the vehicles.

At each neighbouring intersection, this is equivalent to receiving $N'$ samples ($N' \leq N$) of vehicle arrival times along each entry edge. As described, each intersection uses the locally observed sensor data to sample exit edges for observed vehicles, and the sampling process is repeated $N$ times. Before generating sample $n$, $n \in [1, N']$, sample set $n$ of the received non-local vehicle arrival times is appended to the locally observed temporal arrival distribution in order the increase the observation horizon. Exit edges are then sampled for all vehicles in this extended observation horizon.

## 4 Performance Evaluation

We empirically evaluate our approach on the following synthetic road networks: (1) an isolated intersection (2) a 1x5 grid network (3) a 5x5 grid network. We use networks from [Xie et al., 2012b,a] after appropriate modifications to allow turning. All simulations are run on Simulation of Urban MObility [Behrisch et al., 2011], an open source traffic simulation package.

We implement and evaluate two versions of our approach: Sample-based cluster scheduling without communication (U-TuSeRACT) and sample-based scheduling with communication with neighbouring intersections (C-TuSeRACT). We use the IBM ILOG CP Optimizer to solve the sample-based scheduling problem, and plan only on distinct inflows to improve efficiency. Every optimization problem is solved using a single thread of CP Optimizer with a solver compute time limit of 5 seconds.

We implement two versions of SURTRAC to serve our baselines: SURTRAC without communication among neighbouring intersections (U-SURTRAC) [Xie et al., 2012b] and SURTRAC with communication between neighbours (C-SURTRAC) [Xie et al., 2012a].

We use mean vehicle waiting time as an indicator of solution quality, and assess our approach using the change in mean waiting time relative to SURTRAC.

We vary road lengths (observation horizon) and phase designs across networks in order to evaluate our approach in a variety of scenarios. For each network, we define a traffic demand
profile which specifies... (run simulations on three traffic demand levels. Given pre-defined turn proportions which remain static throughout the simulation, we generate a fixed set of vehicle routes for each demand level. For each demand level, traffic is generated for 15 minutes and the problem horizon extended till all vehicles have cleared the network (as in [Guilliard et al., 2016]).

Each of these set of routes is used to generate 20 test instances (where routes are fixed, but vehicle arrival times vary), which we then evaluate our approach on.

We evaluate our approach on each instance with 5 independent and identically distributed sample sets, which we generate offline using the fixed routes and the preset turn proportions. For each sample set, we run TuSeRACT with \{1, 5, 10, 20, 30\} samples. This amounts to 25 runs of each version of TuSeRACT per test case, and 500 runs per demand level.

We generate samples offline to study the effect of adding additional samples to an existing sample set. In practice, vehicle turns would be sampled online, at each decision step online.

We assume simple vehicle models - vehicles travel at the constant speed of 10 m/s and queued vehicles are discharged at a saturation flow rate of $N_{lane}/2.5$ vehicles / second after a startup lost time of 3.5s, where $N_{lane}$ is the number of lanes along the incoming road. We assume all oncoming vehicles are passenger vehicles with length = 5m. We cluster them at a sampling interval $samp = 1s$, and use a threshold of 3s to further aggregate clusters by proximity.

For each phase across the simulations, we set $G_{min}$ to 5 seconds, $G_{max}$ to 55 seconds and $Y$ to 5 seconds.

The time resolution used by our implementation of SURTRAC is $\Delta_S = 0.5s$ while that used by our approach $\Delta_T = 1s$. We use an optimization horizon of 3 cycles (including the current cycle) to limit schedules computed by TuSeRACT.

For both approaches, we assume that:

- Vehicles can be detected exactly along the full observation horizon (usually the full incoming road segment in our cases). In other words, we ignore sensor detection error.

- Turn proportions remain static throughout a simulation and are known exactly by each intersection. In practice, these turn proportions are estimated as moving averages based on recently observed turn movements.

- Planning and communication are instantaneous. For both approaches, planning and communication time are counted outside the simulation and have no impact on the simulation. We make this simplifying assumption to focus solely on the solution quality of the two approaches. We discuss the trade-off between solution quality and real-time tractability in the context of both the approaches, but do not aim to address it in this paper. Here, we aim to investigate whether sampling-based traffic signal control can reduce waiting times when planning under turn-induced uncertainty.

4.1 Isolated Intersection

We evaluate our approach on a two-lane, two-way single intersection (6a). We assume that all incoming roads are 300m long (equivalent to a 30s observation horizon), and that vehicles can turn left, right or travel straight through from each incoming road. We assume that 60% of the vehicles do not turn, and 20% turn right and left respectively. We assume this is a four phase intersection as shown in Figure 2. We use this network to evaluate our core approach while planning under uncertainty.
We generate routes according to the demand profile in Figure 6b, where we show the distribution of the total demand over the incoming approaches over the traffic generation period. We use this profile to generate test cases for three demands: 900 vehicles/hour, 1350 vehicles/hour and 1800 vehicles/hour.

Figure 6: Experimental Setup for an Isolated Intersection

(a) Arrows indicate the direction of vehicle movement, and permissible turns are shown alongside the incoming approaches.

(b) Demand flow profile

To evaluate performance, we report the percentage change in mean waiting time relative to U-SURTRAC, averaged over all test cases and sample sets (Figure 7). Our experiments show that U-TuSeRACT results in significantly lower mean vehicle waiting times with respect to U-SURTRAC for most settings. We observe that U-TuSeRACT is provide significant reductions in average vehicular waiting time over U-SURTRAC: 40% on average. The number of samples required to produce this substantial improvement is small. Although a single sample is not sufficient across all demand levels, using 5 samples consistently provides a 35-40% reduction in delay across all demand levels. Increasing the sample count beyond 5 does not affect the performance gain in the low demand scenario. However, the performance gain drops as sample count is increased beyond 5 in the case of higher demand levels because increasing the sample count also poses a tougher computational challenge to U-TuSeRACT. To verify that this degradation is due to insufficient compute time, we run simulations on all test cases (and one sample set) with a solver time limit of 30 seconds. Results show that performance gain indeed improves with increased compute time (Figure 8).
4.2 Arterial Network

We next evaluate our approach on an 5 intersection arterial network, as shown in Figure 9a. All road lengths in this case are 250m, equivalent to a 25s observation horizon. As shown, we permit turning on only one intersection, which services the given turn movements in three phases. No turning occurs at other intersections, which are all two-phase intersections. For communication, we set the horizon extension to 20s. We generate traffic for demand levels 900 vehicles/hour, 1200 vehicles/hour and 1500 vehicles/hour according to the demand profile shown in Figure 9b.

Our results (Figure 10a, Figure 10b) show that our approach provides significant reductions in delay over SURTRAC, however, C-TuSeRACT does not seem to outperform U-TuSeRACT. One
reason for this is that the demand in these cases causes queue spillover which propagates to neighbouring intersections. SURTRAC proposes a spillover prevention strategy in [Xie et al., 2012a], but we do not explore it here.

Figure 9: Experimental Setup for an Arterial Network

(a) Permissible turn movements shown alongside respective incoming roads.

(b) Demand Profile
4.3 5x5 Grid Network

Finally, we evaluate our performance on a synthetic 5x5 network, shown in Figure 11a. All road lengths are 75m, except for one set of roads, which is 25m long, and one set of boundary edges which are 150m long. Figure 11a indicates the road lengths of the network. We generate demand for levels 4000 vehicles/hour, 5000 vehicles/hour and 6000 vehicles/hour for this network. Figures 12c, 12b and 12a show the results of our experiments. As for other networks, TuSeRACT provides a significant reduction in delay over SURTRAC, although performance does drop as the demand and the number of samples is increased.
Figure 11: Experimental Setup for a 5x5 Grid Network

(a) Intersections represented by filled circles are 4-phase intersections where through, left and right turns are permissible from each incoming road. The other intersections are 2-phase intersections. Through traffic movements are permissible at all intersections, and additional turn movements (if any) are shown alongside the respective roads.

(b) Demand Profile
5 Conclusion

In this paper, we propose a sampling-based approach to traffic signal control in the presence of vehicle turn-induced uncertainty. We show experimentally that our approach provides significant reductions in delay over a competing approach which makes an approximation to solve the turn-induced stochastic optimization problem in order to remain real-time tractable. While we are yet to investigate the real-time tractability of our approach, initial experiments show that sampling is a promising approach to tackle uncertainty in this domain, and that significant performance improvement can be achieved over an existing approach with very few samples.
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