Inflationary Baryogenesis

RAGHAVAN RANGARAJAN\(^{(a)}\) AND D.V. NANOPoulos\(^{(b),(c),(d)}\)

\(^{(a)}\) Theoretical Physics Division, Physical Research Laboratory
Navrangpura, Ahmedabad 380 009, India

\(^{(b)}\) Department of Physics, Texas A&M University
College Station, TX 77843-4242, USA

\(^{(c)}\) Astroparticle Physics Group, Houston Advanced Research Center (HARC)
Mitchell Campus, The Woodlands, TX 77381, USA

\(^{(d)}\) Chair of Theoretical Physics, Academy of Athens
Division of Natural Sciences, 28 Panepistimiou Avenue
Athens 10679, Greece

Abstract

In this letter we explore the possibility of creating the baryon asymmetry of the universe during inflation and reheating due to the decay of a field associated with the inflaton. CP violation is attained by assuming that this field is complex with a phase that varies as the inflaton evolves. We consider chaotic and natural inflation scenarios. In the former case, the complex decaying field is the inflaton itself and, in the latter case, the phase of the complex field is the inflaton. We calculate the asymmetry produced using the Bogolyubov formalism that relates annihilation and creation operators at late time to the
annihilation and creation operators at early time.
I. INTRODUCTION

Explaining the origin of the matter-antimatter asymmetry of the universe is an essential ingredient in our understanding of the history of the universe. In this article, we study the possibility of creating the baryon asymmetry of the universe by the production of particles during inflation and reheating by the decay of a complex field related to the inflaton. We consider the case where the complex decaying field is the inflaton itself as well as the case where the phase of the complex field is the inflaton, as in natural inflation. The former case is similar to chaotic inflation but with a complex inflaton. By assigning baryon number to a scalar field present during inflation and introducing a baryon number violating coupling between this field and the inflaton we find that there is a net baryon number asymmetry in the produced particles.

The notion that the inflaton plays a role in baryogenesis is not new. If the reheat temperature is above the mass of certain heavy particles, such as GUT gauge and Higgs bosons, then the latter are thermally produced and their subsequent out-of-equilibrium decays create a baryon asymmetry. The production of heavy GUT gauge and Higgs bosons or squarks by the direct decay of the inflaton when the reheat temperature and/or the inflaton mass is less than that of the heavy bosons has also been considered. Once again, the out-of-equilibrium decays or annihilations of these particles gives rise to the baryon asymmetry of the universe. In all the above scenarios CP violation enters into the couplings of the heavier bosons to lighter particles. In our scenario the baryon asymmetry is produced in the direct decay of a field associated with the inflaton. Furthermore the CP violation must manifest itself in the decay of this field. In this respect it is similar to the scenario mentioned in Ref. and discussed in more detail in Ref. in which the baryon asymmetry is created by the direct decay of the inflaton. However, unlike in Ref. , in our scenario CP violation is provided dynamically through the time dependent phase of an evolving complex inflaton or of a complex field associated with the inflaton. We explicitly calculate the asymmetry in our scenario and compare it to the baryon asymmetry of the
universe. We follow the work of Ref. [8] in which the asymmetry was calculated in the context of a universe that contracts to a minimum size, bounces back and then expands. The universe was static at both initial and late times. The B-violating coupling was $\lambda R (\phi^* \Lambda \psi + \psi^* \Lambda^* \phi)$, where $R$ is the Ricci scalar, $\phi$ carried baryon number and $\Lambda$ was a complex function of time, which provided the necessary CP violation. In this work, we have adapted the formalism of Ref. [8] to consider the asymmetry that might be created in a more realistic universe that is initially inflating and then enters a reheating phase followed by the standard evolution of the universe. Furthermore we have given a more realistic source of CP violation, namely, a time varying complex field.

Recently Ref. [9] appeared in which the authors discuss the generation of the baryon asymmetry during preheating in a scenario similar to the one discussed here. We discuss later the differences and similarities between our work and theirs.

As in Ref. [8], we consider a lagrangian consisting of two complex scalar fields $\phi$ and $\psi$. $\phi$ and $\psi$ are assumed to carry baryon number $+1$ and $0$ respectively and we assume a B-violating term

$$\lambda (\eta^2 \phi^* \psi + \eta^* \psi^* \phi), \quad (1)$$

where $\lambda$ is a dimensionless constant and $\eta$ is related to the inflaton field and is complex. The baryon number of $\phi$ and $\psi$ particles is established by their interactions with other particles in the Standard Model. The latter are not included in our lagrangian below as they do not enter into our calculations. We assume that the initial velocity of the $\eta$ field and/or the shape of its potential ensures that its phase varies as the inflaton rolls down its potential. Thus we have dynamic CP violation.

To obtain the asymmetry in our scenario we use the fact that the annihilation and

\footnote{To ensure that the baryon asymmetry created is not erased by sphaleron processes, we assume that the interaction in Eq. (1) also violates B-L. This may be achieved, for example, if $\psi$ carries no lepton number.}
creation operators for the fields \( \phi \) and \( \psi \) are not the same during the inflationary phase and at late times after reheating. However, the annihilation and creation operators at late times can be written as linear combinations of the annihilation and creation operators during the inflationary phase using the Bogolyubov coefficients. So,

\[
\tilde{a}_k^\phi = A_k a_k^\phi + B_k b_k^\phi\dagger + A' k a_k^\psi + B'_ k b_k^\psi\dagger,
\]

(2)

\[
\tilde{b}_k^\phi = C_k a_{-k}^\phi + D_k b_{-k}^\phi + C' k a_{-k}^\psi + D' k b_{-k}^\psi\dagger,
\]

(3)

where \( \tilde{a}_k^\phi \) and \( \tilde{b}_k^\phi \) are operators at late times and \( a_k^\phi \) and \( b_k^\phi \) are operators at early times in the inflationary phase. Similar expressions exist for \( \tilde{a}_k^\psi \) and \( \tilde{b}_k^\psi \). In the Heisenberg picture, if we choose the state to be the initial vacuum state, then it will remain in that state during its subsequent evolution. One can then see that the number of \( \phi \) particles and antiparticles of momentum \( k \), given by \( \langle 0 | \tilde{a}_k^\phi \tilde{a}_k^\phi | 0 \rangle \) and \( \langle 0 | \tilde{b}_k^\phi \tilde{b}_k^\phi | 0 \rangle \) respectively, are non-zero and proportional to \( |B_k|^2 + |B'_k|^2 \) and \( |C_k|^2 + |C'_k|^2 \). Furthermore, if \( |B_k|^2 + |B'_k|^2 \neq |C_k|^2 + |C'_k|^2 \) then one obtains a baryon number asymmetry. (If \( \lambda = 0 \) in Eq. 1 then \( B'_k \) and \( C'_k \) are 0 and \( |B| = |C| \), and one gets no asymmetry.)

Particle number, and correspondingly annihilation and creation operators, are well defined only in adiabatic vacuum states. However, the vacuum state in the inflationary era, i.e., our in state, must be chosen judiciously to avoid infrared divergences. This is discussed in Section III.

The framework of this article is as follows. In Section II we present the lagrangian density for the complex scalar fields \( \phi \) and \( \psi \) relevant to our calculation and obtain their equations of motion. We then write down the Fourier decomposition of \( \phi \) and \( \psi \) during the inflationary phase and during reheating. General expressions for the coefficients \( A - D' \) in Eqs. 2 and 3 have been derived in Ref. 8. We shall present these results without rederiving them and then present the general result for the baryon asymmetry of the universe. In Section III, we present the particular solutions for our scenario of a universe that undergoes exponential inflation \( (a \sim e^{Ht}) \) followed by an inflaton-oscillation dominated phase \( (a \sim t^{2/3}) \). We then calculate the total baryon asymmetry for this scenario in the context of chaotic and natural
inflation. We conclude in the last section. In the Appendix we discuss issues related to the regularisation of infrared divergences and the necessity of an infrared cutoff to satisfy the conditions of perturbation theory.

II.

Consider a lagrangian density

\[
L = \sqrt{-g}[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + g^{\mu\nu}\partial_\mu\psi^*\partial_\nu\psi - (m_\phi^2 + \xi_\phi R)\phi^*\phi - (m_\psi^2 + \xi_\psi R)\psi^*\psi]
\]

\[
+ g^{\mu\nu}\partial_\mu\eta^*\partial_\nu\eta - m_\eta^2\eta^*\eta - V(\eta) - \lambda(\eta^2\phi^*\psi + \eta^*\psi^*\phi)],
\]

where \(m_{\phi,\psi}\) are the masses of the respective fields and \(\xi_{\phi,\psi}\) are their couplings to the curvature. We have assumed that \(\eta\) is minimally coupled. \(V(\eta)\) includes all interactions of \(\eta\) other than the coupling to \(\phi\) and \(\psi\) already listed above. Below we shall consider natural inflation and chaotic inflation scenarios. In the former case the inflaton will be associated with the phase of \(\eta\). In the latter case we assume that the complex \(\eta\) field is the inflaton field. (Thus we are really considering an extension of chaotic inflation since the inflaton is now complex.) We shall assume a spatially flat Robertson-Walker metric. The equations of motion for the above fields are

\[
\ddot{\phi} + 3\left(\frac{\dot{a}}{a}\right)\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + (m_\phi^2 + \xi_\phi R)\phi + \lambda\eta^2\psi = 0,
\]

\[
\ddot{\psi} + 3\left(\frac{\dot{a}}{a}\right)\dot{\psi} - \frac{1}{a^2}\nabla^2\psi + (m_\psi^2 + \xi_\psi R)\psi + \lambda\eta^*\phi = 0.
\]

We now write

\[
\phi(x) = \sum_k e^{ik\cdot x}\frac{1}{[La(t)]^{3/2}}\phi_k(t),
\]

with \(k = \frac{2\pi}{L}(n_x, n_y, n_z)\) and a similar expression for \(\psi(x)\). The equations of motion for the Fourier coefficients \(\phi_k\) and \(\psi_k\) are (note that \(\phi_k\) and \(\psi_k\) are operators)

\[
\ddot{\phi}_k(t) + \left(\frac{k^2}{a^2} - \frac{3}{4} \frac{\dot{a}^2}{a^2} - \frac{3}{2} \frac{\ddot{a}}{a} + m_\phi^2 + \xi_\phi R\right)\phi_k(t) + \lambda\Lambda\phi_k(t) = 0,
\]

\[
\ddot{\psi}_k(t) + \left(\frac{k^2}{a^2} - \frac{3}{4} \frac{\dot{a}^2}{a^2} - \frac{3}{2} \frac{\ddot{a}}{a} + m_\psi^2 + \xi_\psi R\right)\psi_k(t) + \lambda\Lambda\psi_k(t) = 0.
\]
\[
\ddot{\psi}_k(t) + \left(\frac{k^2}{a^2} - \frac{3}{4} \frac{\dot{a}^2}{a^2} - \frac{3}{2} \frac{\ddot{a}}{a} + m_\psi^2 + \xi_\psi R\right) \dot{\psi}_k(t) + \lambda R \Lambda^* \phi_k(t) = 0, 
\]
\(\phi_k\) and \(\psi_k\) satisfy
\[
[\phi_k(t), \dot{\phi}_k(t)] = i \delta_{k,k'}, \tag{11}
\]
\[
[\psi_k(t), \dot{\psi}_k(t)] = i \delta_{k,k'}. \tag{12}
\]

To use the results derived in Ref. [8] we shall have to solve Eqs. 9 and 10 for times during inflation and during reheating. To simplify our calculations we shall assume that the fields \(\phi\) and \(\psi\) are massless and minimally coupled, i.e., \(m_{\phi,\psi} = 0, \xi_{\phi,\psi} = 0\). (Typically, a spin zero particle will obtain a mass of order \(H\) during inflation, if \(H\) is greater than its bare mass [10]. However, this does not occur if the mass is protected by a symmetry.)

To facilitate the use of perturbation theory and to be able to define an in state we assume that the B-violating interaction switches on at some time \(t_1\). Let \(t_2\) be the time when inflation ends, \(t_3\) be the time when reheating ends and \(t_f\) be the final time at which we evaluate the baryon asymmetry. We assume that B-violation vanishes after \(t_3\). The annihilation and creation operators at \(t_f\) can be expressed as linear combinations of the annihilation and creation operators at an early time \(t_i\) before \(t_1\) in the inflationary era. These relations have been derived perturbatively to order \(\lambda^2\) in Ref. [8] giving

\[
a_{f,k}^\phi = [\alpha_k^\phi(1 + i\lambda^2 H_1^\phi) - i\lambda^2 \beta_k^{\phi*} H_3^\phi] a_{i,k}^\phi + \beta_k^{\phi*} (1 - i\lambda^2 H_4^\phi) + i\lambda^2 \alpha_k^\phi H_2^\phi] b_{i,-k}^\phi \tag{13}
\]
\[
b_{f,k}^\phi = [\alpha_k^{\phi*}(1 - i\lambda^2 H_1^\phi) + i\lambda^2 \beta_k^{\phi*} H_3^\phi] b_{i,k}^\phi + \beta_k^{\phi*} (1 - i\lambda^2 H_4^\phi) - i\lambda^2 \alpha_k^{\phi*} H_2^\phi] a_{i,-k}^\phi
\]
\[
+ i\lambda[\alpha_k^{\phi*} I_4 - \beta_k^{\phi*} I_2] b_{i,k}^\phi + i\lambda[\alpha_k^{\phi*} I_3 - \beta_k^{\phi*} I_1] a_{i,k}^\phi, \tag{14}
\]
\[
a_{f,k}^{\psi*} = [\alpha_k^{\psi*}(1 + i\lambda^2 H_1^\psi) - i\lambda^2 \beta_k^{\psi*} H_3^\psi] a_{i,k}^{\psi*} + \beta_k^{\psi*} (1 - i\lambda^2 H_4^\psi) + i\lambda^2 \alpha_k^{\psi*} H_2^\psi] b_{i,-k}^{\psi*} \tag{15}
\]
\[
b_{f,k}^{\psi*} = [\alpha_k^{\psi*}(1 - i\lambda^2 H_1^\psi) + i\lambda^2 \beta_k^{\psi*} H_3^\psi] b_{i,k}^{\psi*} + \beta_k^{\psi*} (1 - i\lambda^2 H_4^\psi) - i\lambda^2 \alpha_k^{\psi*} H_2^\psi] a_{i,-k}^{\psi*}
\]
\[
+ i\lambda[\alpha_k^{\psi*} I_3 - \beta_k^{\psi*} I_1]^* b_{i,k}^{\psi*} + i\lambda[\alpha_k^{\psi*} I_2 - \beta_k^{\psi*} I_2]^* a_{i,-k}^{\psi*}. \tag{16}
\]
where

\[
I_1 = \int_{-\infty}^{\infty} dt \eta^2(t) \chi_k^{\phi^*}(t) \chi_k^{\psi}(t),
\]

\[
I_2 = \int_{-\infty}^{\infty} dt \eta^2(t) \chi_k^{\phi^*}(t) \chi_k^{\psi*}(t),
\]

\[
I_3 = \int_{-\infty}^{\infty} dt \eta^2(t) \chi_k^{\phi}(t) \chi_k^{\psi}(t),
\]

\[
I_4 = \int_{-\infty}^{\infty} dt \eta^2(t) \chi_k^{\phi}(t) \chi_k^{\psi*}(t),
\]

and

\[
H_1^\phi = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \eta^2(t) \chi_k^{\phi^*}(t)
\]

\[
\times \Delta_k^\psi(t, t') \chi_k^{\phi}(t') \eta^2(t'),
\]

\[
H_2^\phi = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \eta^2(t) \chi_k^{\phi^*}(t)
\]

\[
\times \Delta_k^\phi(t, t') \chi_k^{\phi}(t') \eta^2(t'),
\]

\[
H_3^\phi = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \eta^2(t) \chi_k^{\phi}(t)
\]

\[
\times \Delta_k^\psi(t, t') \chi_k^{\phi^*}(t') \eta^2(t'),
\]

\[
H_4^\phi = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \eta^2(t) \chi_k^{\phi}(t)
\]

\[
\times \Delta_k^\phi(t, t') \chi_k^{\phi^*}(t') \eta^2(t'),
\]

and the \(H_i^\psi\) are defined as \(H_i^\phi\) with \(\eta^2\) replaced by \(\eta'^2\) and \(\chi_k^{\phi^*}\), \(\chi_k^{\phi^*}\) and \(\Delta^\psi\) replaced by \(\chi_k^{\psi}\), \(\chi_k^{\psi^*}\) and \(\Delta^\phi\) respectively. Above, \(\chi_k^{\phi}\) and \(\chi_k^{\psi}\) are complex functions (not operators) that solve Eqs. 9 and 10, with \(\lambda\) set to 0, respectively. \(\Delta_k^\phi\) and \(\Delta_k^\psi\) are the retarded Green’s functions for Eqs. 9 and 10 respectively, i.e., they satisfy Eqs. 9 and 10 with \(\lambda\) set to 0 and a delta function \(\delta(t - t')\) on the r.h.s. of the equations. The subscripts on the coefficients \(\alpha_k\) and \(\beta_k\) and on the functions \(\chi_k\) and \(\Delta_k\) refer to \(|k|\). \(\alpha_k\) and \(\beta_k\) are complex and satisfy

\[
|\alpha_k^{\phi}|^2 - |\beta_k^{\phi}|^2 = |\alpha_k^{\psi}|^2 - |\beta_k^{\psi}|^2 = 1.
\]

We assume that the initial state at \(t_i\) is the vacuum state. Then, in the Heisenberg picture, the number of \(\phi\) particles of momentum \(k\) at \(t_f\) is given by
\[ \langle N_k^\phi(t_f) \rangle = \langle in|a_{f,k}^\phi a_{f,k}^\phi|in \rangle \]
\[ = |\beta_k^\phi|^2 + \lambda^2 \left[ |\alpha_k^\phi|^2 |I_2|^2 + |\beta_k^\phi|^2 (|I_3|^2) + 2 \text{Re}(\alpha_k^\phi \beta_k^\phi (iH_2^\phi - I_2 I_4^*) ) \right] \]  
(27)

and

\[ \langle \bar{N}_k^\phi(t_f) \rangle = \langle 0, in|b_{f,k}^\phi b_{f,k}^\phi |0, in \rangle \]
\[ = |\beta_k^\phi|^2 + \lambda^2 \left[ |\alpha_k^\phi|^2 |I_3|^2 + |\beta_k^\phi|^2 (|I_2|^2) + 2 \text{Re}(\alpha_k^\phi \beta_k^{\phi*} (iH_3^\phi + I_4 I_3^*) ) \right] \]  
(28)

where we have used

\[ 2ImH_1 = |I_1|^2 - |I_2|^2, \]  
(29)

\[ 2ImH_4 = |I_3|^2 - |I_4|^2. \]  
(30)

The baryon asymmetry for particles of momentum \( k \) at \( t_f \) is thus

\[ \Delta N_k^\phi(t_f) = \langle N_k^\phi(t_f) \rangle - \langle \bar{N}_k^\phi(t_f) \rangle = \lambda^2 (|I_2|^2 - |I_3|^2), \]  
(31)

where we have used Eq. 26 and

\[ H_2^\phi - H_3^{\phi*} = iI_1 I_3^* - iI_2 I_4^*. \]  
(32)

The reader is referred to Ref. [8] for a more detailed derivation of the above results. Note that the asymmetry does not depend on \( \alpha_k \) and \( \beta_k \) implying that the asymmetry is independent of the purely gravitational production of particles in the expanding universe. (\( \xi = 0 \) does not imply a conformally invariant universe. Therefore there is non-zero purely gravitational production of particles in our scenario but it does not contribute to the asymmetry.)

To obtain the net baryon number at \( t_f \) we sum over all momentum modes and take the continuum limit. Since we ultimately wish to express the baryon asymmetry as the baryon number density to entropy density ratio, and the baryon number does not change after \( t_3 \), we write

\[ \text{Eqs. (4.1c) and (4.9) of Ref. [8] contain typographical errors. The corrected equations are displayed here in Eqs. 33 and 32 respectively.} \]
\[ n_B(t_3) = \lim_{L \to \infty} 1/([La(t_3)]^3) \sum_k \langle \Delta N_k^\phi \rangle \]
\[ = \lim_{L \to \infty} 1/([La(t_3)]^3) (\frac{k}{2\pi})^3 \int d^3 k \langle \Delta N_k^\phi \rangle \]
\[ = 1/(2\pi^2 [a(t_3)]^3) \int_0^\infty dk k^2 \langle |I_2|^2 - |I_3|^2 \rangle / [(2\pi^2/45) g_* T_3^3], \quad (33) \]

Working in the approximation that at \( t_3 \) the inflaton completely decays and the universe instantaneously reheats to a temperature \( T_3 \), the baryon asymmetry of the universe is
\[ BAU \equiv n_B/s = \frac{\lambda^2}{(2\pi^2 [a(t_3)]^3)} \int_0^\infty dk k^2 \langle |I_2|^2 - |I_3|^2 \rangle / [(2\pi^2/45) g_* T_3^3], \quad (34) \]

where we have assumed that there is no dilution of the baryon asymmetry due to entropy production during the subsequent evolution of the universe. Note that because the effective coupling is a complex function of time the baryon asymmetry is obtained at \( O(\lambda^2) \) and not \( O(\lambda^4) \).

For standard reheating, \( t_3 \approx \Gamma^{-1} \), where \( \Gamma \) is the dominant perturbative decay rate of the inflaton. We take \( \Gamma = \frac{g^2}{8\pi} m_{inf} \), corresponding to the decay of the inflaton of mass \( m_{inf} \) to some light fermion-antifermion pair. Furthermore, \( T_3 = \left( \frac{30}{\pi^2 g_*} \right) \frac{1}{3} \rho(t_3)^{\frac{1}{3}} = 0.6 g_*^{\frac{1}{3}} (M_{Pl} \Gamma)^{\frac{1}{2}} \) \[11\], where \( \rho \) here refers to the inflaton energy density. \[12\] \( \rho(t_3) \approx \rho(t_2) [a(t_2)/a(t_3)]^3 \). We assume that the reheat temperature is not high enough for GUT B-violating interactions to be in equilibrium and wipe out the asymmetry generated in our scenario. On the other hand, we do not restrict ourselves to reheat temperatures below \( 10^8 \) GeV to avoid the gravitino problem \[13\] as we consider the possibility that the gravitino might be very light.

III.

To obtain the baryon asymmetry, we need to evaluate \( I_2 \) and \( I_3 \). This requires obtaining \( \chi^\phi_k \) and \( \chi^\psi_k \). We shall need to perform the integral for \( I_2 \) and \( I_3 \) only from \( t_1 \) to \( t_3 \) as B-

\[3\] The final temperature \( T_4 \) at the end of reheating is also a function of the interactions of the inflaton decay products which we have ignored. See Ref. \[12\] and references therein for a discussion of thermalisation of the decay products.
violation vanishes earlier than \( t_1 \) and after \( t_3 \). Solutions of Eqs. 9 and 10 for \( \lambda = 0 \) and \( a(t) = \sigma t^c \) (\( c \neq 1, -1/3 \)) and \( a(t) = \sigma e^{Ht} \) have been obtained in Ref. [14]. Using them we get

\[
\chi_{k}^{\phi,\psi} = [a(t)]^{\frac{3}{2}} \left[ c_1 \left( \frac{-1}{3a(t)^3 H} \right)^{1/2} H^{(1)}_\frac{3}{2} \left( \frac{-k}{a(t)H} \right) + c_2 \left( \frac{-1}{3a(t)^3 H} \right)^{1/2} H^{(2)}_\frac{3}{2} \left( \frac{-k}{a(t)H} \right) \right]
\]

for \( t_1 < t < t_2 \), (35)

\[
\chi_{k}^{\phi,\psi} = [a(t)]^{\frac{3}{2}} \left[ c'_1 \left( \frac{-t}{a(t)^3} \right)^{1/2} H^{(1)}_\frac{3}{2} \left( \frac{-3kt}{a(t)} \right) + c'_2 \left( \frac{-t}{a(t)^3} \right)^{1/2} H^{(2)}_\frac{3}{2} \left( \frac{-3kt}{a(t)} \right) \right]
\]

for \( t_2 < t < t_3 \). (36)

\( H_{\nu}(z) \) are Hankel functions. The commutation relations Eqs. 11 and 12 imply that

\[
\chi_{k}^{\phi,\psi} \chi_{k}^{\phi,\psi^*} - \chi_{k}^{\phi,\psi^*} \chi_{k}^{\phi,\psi} = i
\]

and therefore the constants \( c_{1,2} \) and \( c'_{1,2} \) satisfy

\[
|c_2|^2 - |c_1|^2 = 3\pi/4,
\]

(38)

\[
|c'_2|^2 - |c'_1|^2 = -3\pi/4.
\]

(39)

The constants \( c_1 \) and \( c_2 \) define an initial vacuum state in the inflationary era. If one makes a choice of the de Sitter invariant vacuum state (\( c_1 = 0 \) and \( c_2 = \sqrt{3\pi/4} \)) as the \textit{in} state then it is well known that such a state suffers from an infrared divergence. One option then is to choose the constants \( c_1 \) and \( c_2 \) appropriately so as to cancel the infrared divergences even though such states will no longer be de Sitter invariant. In Ref. [14] it is suggested that one may choose the constants \( c_1 \) and \( c_2 \) as below so as to cancel the infrared divergences:

\[
c_1 = (kt_2/a(t_2))^{-p},
\]

(40)

\[
c_2 = ((kt_2/a(t_2))^{-2p} + \frac{3\pi}{4})^{1/2},
\]

(41)

\[ ^4 \text{Note that the definition of the mode functions } \chi_k \text{ in Ref. [8] differs from that in Ref. [14] by a factor of } a^{3/2}. \]

\[ ^5 \text{Other mechanisms have also been suggested to avoid the infrared divergences. For example, see Ref. [15].} \]
with $p > 0$. (We point out in the Appendix that there is also an upper limit on $p$ that was not mentioned in Ref. [14].) As we discuss in the Appendix, the nature of the infrared divergences is slightly different in our case. Though the above choice for the constants $c_{1,2}$ with appropriately chosen values of $p$ would make the final integral over $k$ infrared finite, the integrands for the intermediate integrals over $t$ become very large for small values of $k$, irrespective of the value of $p$. This leads to a problem with perturbation theory since the latter requires that $\lambda^2 |I_{2,3}|^2$ should be less than 1. This is discussed in more detail in the Appendix. Therefore we are forced to introduce a low momentum cutoff $k_L$ to justify our use of perturbation theory. We choose $k_L$ such that $k_L/a_2 = 1/t_2$. Since the low momentum cutoff automatically regulates the integral over $k$, we then choose $c_1 = 0$ and $c_2 = \sqrt{3\pi}/4$.

Continuity conditions for $\phi(x), \dot{\phi}(x)$ and $a(t)$ at $t_2$ imply that $\chi(t)$ and $d/dt(\chi(t)/[a(t)]^{3/2})$ are continuous at $t_2$, and these boundary conditions then give us $c'_1$ and $c'_2$. We have verified that the values of $c'_1$ and $c'_2$ obtained from the continuity conditions satisfy Eq. 39.

At this stage we need to specify $\eta(t)$. If we write $\eta(t)$ as $\frac{1}{\sqrt{2}} \sigma(t) e^{i\theta(t)}$ (where $\sigma(t)$ is real), it is the time varying phase of $\eta$ that provides the CP violation necessary for creating a net baryon asymmetry.

**Chaotic Inflation**

We first consider the case of chaotic inflation in which the $\eta$ field represents a complex inflaton field. In the absence of any potential for $\theta$, the equation of motion for $\theta$ is

$$\sigma^2 \ddot{\theta} + 3H \sigma^2 \dot{\theta} + 2\dot{\sigma} \sigma \dot{\theta} = 0. \quad (42)$$

In a more realistic model $V(\eta)$ will imply a potential for $\theta$. The equation of motion for $\sigma$ is

$$\ddot{\sigma} + 3H \dot{\sigma} + m^2 \sigma - \dot{\theta}^2 \sigma = 0. \quad (43)$$

We assume that $\theta(t)$ evolves starting from an initial value of 0 at $t = t_i$ and an initial velocity $\dot{\theta}_i$. We choose $\dot{\theta}_i$ consistent with a universe dominated by the potential energy of $\sigma$.

---

6 We thank D. Lyth for pointing this out to us.
Therefore we take $\dot{\theta}_i = m/2$. During inflation, $\dot{\sigma}^2 \ll m^2 \sigma^2$ and $\sigma \sim M_{Pl}$. Hence $\dot{\sigma}/\sigma \ll H$ and we ignore the last term in the equation of motion for $\theta$. Then

$$\theta(t) = \frac{\dot{\theta}_i}{3H} \left(1 - e^{-3H(t-t_i)}\right) \quad t_i \leq t \leq t_2$$

(44)

We take $H$ to be constant during inflation and corresponding to the initial energy density of the universe with $\sigma(t_i) = 3M_{Pl}$. From above, one can see that $\dot{\theta}$ is much less than $m$ for most of the inflationary era and so we ignore the last term of Eq. 43 for this era. Invoking the slow roll approximation one may also ignore $\ddot{\sigma}$ during inflation.

During reheating the $\sigma$ and the $\theta$ fields are coupled and we can not ignore the last terms of their respective equations of motion. However, to obtain $\theta(t)$ it is simpler to first rewrite $\eta$ as $\frac{1}{\sqrt{2}}(\kappa_1 + i\kappa_2)$. Then the problem reduces to one of two uncoupled damped harmonic oscillators with solutions, $\kappa_1 = (A_1/t)\cos(mt + \alpha)$ and $\kappa_2 = (A_2/t)\cos(mt + \beta)$. Here we have assumed $H = 2/(3t)$ during reheating. $\theta(t)$ is then $\tan^{-1}(\kappa_2/\kappa_1)$. The constants $A_1$, $A_2$, $\alpha$ and $\beta$ are determined by the values of $\kappa_{1,2}$ and their time derivatives at $t_2$ which can be obtained from the values of $\theta$ and $\sigma$ and their time derivatives at $t_2$. We take $t_2 \approx 2/m$, as the inflaton starts oscillating when $3H \approx m$. $\sigma(t_2) \approx M_{Pl}/6$.

Eq. (44) implies that $\theta$ becomes nearly constant within a few e-foldings after $t_i$. If the $B$-violating interaction switches on subsequent to this then $\theta$ is approximately constant between $t_1$ and $t_2$. Furthermore, since $\dot{\theta}(t_2)$ is practically zero, there is practically no rotational motion during reheating in the absence of any potential for $\theta$. So during reheating $\theta$ takes values of $\theta_2$ and $\theta_2 + \pi$ during different phases of the oscillation of $\sigma$, where $\theta_2$ is the value at $t_2$. ($\theta$ changes discontinuously at the bottom of the potential where $\sigma$ is 0.) Since the relevant phase in $I_2$ and $I_3$ is $2\theta$ the above implies that the CP phase is practically the same for the interval $t_1$ to $t_3$ and hence one should expect very little asymmetry.

**Natural Inflation**

We now consider natural inflation, in which case $\sigma(t) = f$ where $f$ is the scale of spontaneous symmetry breaking in the natural inflation scenario. In the presence of an explicit symmetry breaking term that gives mass $m_\theta$ to the inflaton $\theta$ the equation of motion for $\theta$
\[ f^2 \ddot{\theta} + 3H f^2 \dot{\theta} + m^2 f^2 \theta = 0. \]  

(45)

We assume that \( \theta \) is constant during inflation between \( t_1 \) and \( t_2 \) and is of \( O(1) \). Our results are insensitive to \( t_1 \) for \( t_1 \) earlier than about 10 e-foldings before the end of inflation. At \( t_2 \approx 2/m_\theta \) when \( 3H \approx m \) the \( \theta \) fields starts oscillating in its potential. Between \( t_2 \) and \( t_3 \), \( \theta \) evolves as

\[ \theta(t) = \theta(t_2) \frac{t_2}{t} \cos[m_\theta(t - t_2)]. \]

(46)

Now we obtain the asymmetry numerically. Smaller the value of \( g \), longer is the period of reheating contributing to the asymmetry. But for \( g \leq 10^{-3} \), \( I_2 \) and \( I_3 \) become independent of \( g \) and then the \( g \) dependence in \( BAU \) enters through \( a(t_3) \) and the reheat temperature \( T_3 \). For \( g \leq 10^{-3} \), we get

\[ BAU = \lambda^2 g(2 \times 10^{10}). \]

(47)

As we have mentioned before, perturbation theory requires that \( \lambda |I_{2,3}|^2 \) must be less than 1. This translates into an upper bound on \( \lambda \) of \( 10^{-11} \). Then even for \( g = 10^{-3} \) we get insufficient asymmetry. Other values of \( g \) give even less asymmetry.

**IV. CONCLUSION**

In conclusion, we have discussed a mechanism for creating a baryon asymmetry during inflation and reheating. While the scenario illustrated above does not create sufficient asymmetry, it may be easily modified to accommodate a potential for \( \theta \) which can give rise to a much larger asymmetry. A possible potential for \( \theta \) for the chaotic inflation scenario is \( W(\theta) = m_\theta^2 \sigma^2(1 - \cos \theta) \), which is equivalent to tilting the inflaton potential. Unlike in the analogous axion and natural inflation models, here both \( \sigma \) and \( \theta \) would be varying with time. Hence such a potential may allow for chaotic orbits and so would have to be studied with care.
We point out here that we include both the inflationary phase and the reheating phase in our calculation. Contributions during both phases do get mixed up in the evaluation of the asymmetry because of the presence of $|I_2|^2$ and $|I_3|^2$, where the time integrals in $I_2$ and $I_3$ include both the inflationary and the reheating eras. In fact we find in the natural inflation case that though the phase is taken to be constant during the inflationary era, the net baryon asymmetry for a fixed value of $\lambda$ decreases if we do not include the inflationary era in the integrals $I_2$ and $I_3$. This indicates that one should not ignore the inflationary era when calculating the asymmetry.

While we were writing up this paper Ref. [9] appeared. In this paper the authors discuss the generation of baryon asymmetry during reheating in a scenario similar to ours. The two calculations have some differences however. Our calculation is carried out in curved spacetime while the mode functions in Ref. [9] are obtained in Minkowski space. We consider standard reheating while they consider the more complicated preheating scenario. In both calculations the source of CP violation is a time varying phase. The authors of Ref. [9] suggest that the CP violating potential for the baryonic fields may be induced by their direct coupling to the inflaton or through loop effects involving the baryonic fields and other fields and then presume a form for the phase. We provide a specific scenario in which the inflaton, or a field related to the inflaton, which is coupled to the baryonic fields, is complex and its time varying phase dynamically provides CP violation. Involving the inflaton and its phase seems to us to be a simple and a very natural approach to obtain a time dependent phase. Our calculation includes both the inflationary and the reheating eras which, as we have pointed out, seems to be appropriate for our case.

\[7\] The values of $|I_{2,3}|^2$ also decrease and so, in principle, one may obtain a slightly larger asymmetry by choosing a larger value of $\lambda$ which is still consistent with the use of perturbation theory.
ACKNOWLEDGMENTS

R. R. would like to thank Sacha Davidson, A. D. Dolgov, Salman Habib, Arul Lakshminarayan, David Lyth, Leonard Parker and U. B. Sathuvalli for useful discussions and comments. R. R. would also like to thank the University of Lancaster for their hospitality. The work of D. V. N. was partially supported by DOE grant DE-FG03-95-ER-40917.

APPENDIX

In Ref. [14] infrared divergence in $\langle \phi(x)\phi(x') \rangle$ for Robertson-Walker universes with power law expansion and exponential expansion is discussed. (In this Appendix, $\phi$ is a generic scalar field.) Now

$$\langle \phi(x)\phi(x') \rangle \sim \int d^3k e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')} \chi_{\mathbf{k}}^{FP}(\tau)\chi_{\mathbf{k}}^{FP^*}(\tau'),$$

(48)

where $\chi_{\mathbf{k}}^{FP}(\tau)$ are mode functions as defined in Ref. [14] (they are $a^{3/2}$ times the mode functions defined in this paper) and $\tau = \int^t a^{-3}(t'')dt''$. If one argues that $H_1^{(1)}(z) \sim H_1^{(2)}(z) \sim iY_1(z) \sim z^{-3/2}$ as $z \to 0$ the integrand in the above integral $\sim |c_1 - c_2|^2 k^{-3}$, which implies a divergent 2-point function unless one chooses $c_1(k)$ and $c_2(k)$ such that $|c_1 - c_2| \to 0$ fast enough as $k \to 0$. Hence the choice given in Eqs. (40) and (41) which eliminates the infrared divergences if $p > 0$. We would like to point out here that there is also an upper bound on $p$. Unless $p$ is less than 3, the term proportional to $J_1^2$ in the 2-point function, namely, $\sim |c_1 + c_2|^2 J_1^2 \sim k^{-2p+3}$ will also diverge at low values of $k$.

Unlike in Ref. [14] the divergent quantities for us are $\sim \int d^3k \chi_k^{\phi*} \chi_k^{\psi*}$ and $\sim \int d^3k \chi_k^{\phi} \chi_k^{\psi}$ and the form of the divergences is slightly different. Adapting the arguments of Ref. [14] we would choose $\frac{3}{4} < p < \frac{9}{4}$ for the constants $c_1$ and $c_2$ given in Eqs. (40) and (41) to avoid infrared divergences in the integral over momentum.

In our case we also have intermediate integrals $I_2$ and $I_3$ over time and this leads to an additional problem. The $k$ dependence in the integrand of, for example, $I_3$, is contained in

$$-(c_1 - c_2)^2 Y_1^2(z) + (c_1 + c_2)^2 J_1^2(z) + 2i(c_1^2 - c_2^2)J_1(z)Y_1(z),$$

where $z$ is the relevant argument.
of the Bessel functions depending on whether the universe is in the inflationary or the reheating era. The $k$ dependence for $I_2$ is similar. When $k$ is small the argument of the Bessel functions becomes small. For low $k$ values the first term goes as $k^{2p-3}$, the second as $k^{3-2p}$ and the third is $k$ independent. Now perturbation theory requires that $\lambda^2|I_{2,3}|^2$ be less than 1. However, since $2p-3$ and $3-2p$ cannot both be greater than 0, $I_2$ and $I_3$ will become very large at low $k$ values. Thus, without a low momentum cutoff, perturbation theory breaks down at some point for any finite value of $\lambda$. We emphasise again that this is an issue related to the validity of perturbation theory and not to the infrared divergence of the integral over momentum. The latter can be regulated by the choice of constants $c_1$ and $c_2$ mentioned above, irrespective of whether or not perturbation theory is valid.
REFERENCES

[1] D. V. Nanopoulos and S. Weinberg, Phys. Rev. D 20 (1979) 2484.

[2] A. D. Dolgov and A. D. Linde, Phys. Lett. B116 (1982) 329.

[3] S. Dimopoulos and L. Hall, Phys. Lett. B196 (1987) 135.

[4] E. W. Kolb, A. Linde and A. Riotto, Phys. Rev. Lett. 77 (1996) 4290.

[5] D. Chung, E. W. Kolb and A. Riotto, Phys. Rev. D 60 (1999) 063504.

[6] A. Albrecht, P. J. Steinhardt, M. S. Turner and F. Wilczek, Phys. Rev. Lett. 48 (1982) 1437.

[7] E. W. Kolb and M. S. Turner, “The Early Universe”, (Addison-Wesley, Redwood City, California, 1990), see Secn. 6.7.

[8] N. J. Papastamatiou and L. Parker, Phys. Rev. D 19 (1979) 2283.

[9] K. Funakubo, A. Kakuto, S. Otsuki and F. Toyoda, hep-ph/0010266, (2000).

[10] M. Dine, W. Fischler and D. Nemeschansky, Phys. Lett. B136 (1984) 169; O. Bertolami and G. G. Ross, Phys. Lett. B183 (1987) 163; E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, Phys. Rev. D 49 (1994) 6410.

[11] See Eq. 8.34 of Ref. [7].

[12] S. Davidson and S. Sarkar, hep-ph/0009078 (2000).

[13] J. Ellis, A. D. Linde, and D. V. Nanopoulos, Phys. Lett. B118 (1982) 59; M. Yu. Khlopov and A. D. Linde, Phys. Lett. B138 (1984) 265; J. Ellis, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B145 (1984) 181; J. Ellis, D. V. Nanopoulos and S. Sarkar, Nucl. Phys. B259 (1985) 175.

[14] L. H. Ford and L. Parker, Phys. Rev. D 16 (1977) 245.

[15] A. D. Dolgov, M. B. Einhorn and V. I. Zakharov, Acta Phys. Polon. B26 (1995) 65.