Duality picture of Superconductor-insulator transitions on Superconducting nanowire

Kazumasa Makise¹, Hirotaka Terai¹, Yukihiro Tominari², Shukichi Tanaka¹ & Bunju Shinozaki²

In this study, we investigated the electrical transport properties of niobium titanium nitride (NbTiN) nanowires with four-terminal geometries to clarify the superconducting phase slip phenomena and superconducting-insulator transitions (SIT) for one-dimensional superconductors. We fabricated various nanowires with different widths and lengths from epitaxial NbTiN films using the electron beam lithography method. The temperature dependence of resistance $R(T)$ below the superconducting transition temperature $T_c$ was analyzed using thermal activation phase slip (TAPS) and quantum phase slip (QPS) theories. Although the accuracy of experimental data at low temperatures can deviate when using the TAPS model, the QPS model thoroughly represents the $R(T)$ characteristic with resistive tail at low temperatures. From the analyses of data on $T_\alpha$, we found that NbTiN nanowires exhibit SIT because of the change in the ratio of kinetic inductance energy and QPS amplitude energy with respect to the flux-charge duality theory.

The state-of-the-art superconducting quantum computer consists of superconductor-insulator-superconductor tunnel junctions called Josephson junctions because it is a commonly used for superconducting digital circuits and quantum qubit for quantum computing devices. Recently, one-dimensional (1D) superconducting nanowires (SNWs) are being considered to develop superconducting computing devices. In order to realize novel devices using nanowires, it is necessary to clarify the superconducting transport characteristics that depend on the disorder, wire length $L$, width $w$, and many other parameters. In such low-dimensional superconductors, quantum effects slightly influence the superconducting characteristics. Especially, in our nanowires, there is a significant role in the properties of SNWs. The effect of TAPS to resistance $R(\Omega)$ sharply decays as a result of the temperature drop below $T_c$. On the other hand, the QPS represents the residual resistance required to suppress the superconductivity at $T = 0$ K. Numerous studies have been conducted on the various materials that can be used for SNWs. However, there still are fundamental problems in determining the effect of TAPS and QPS on the temperature dependency of resistance $R(T)$ because SNW specimens with the same material can present different $R(T)$ characteristics. Further, some investigations show no evidence of QPS behavior at very low temperatures. Another attractive subject of investigation for SNWs is clarifying some critical values corresponding to SIT, namely, the normal state critical resistance $R^N$ or $R^N/L$, or other characteristics. Several research groups have reported the characteristics of $R(T)$ below $T_c$ and SIT for MoGe SNWs. For $R^N/L$, separating 1D specimens into superconducting and insulator phases, the $R^N(L)$, takes the almost same values in the region $30 < (R^N/L) < 100$. On the other hand, $R(T)$ characteristics show different behaviors owing to the TAPS or QPS mechanisms. Although Marković et al. and Tinkham et al. observed broad transition with a resistive tail due to the QPS at low temperatures, others fit the TAPS model to data without the resistive tail. As for $R^N$, it is expected to take the superconducting quantum resistance $R_Q = \hbar/4e^2 \approx 6.45 \text{ K}\Omega$ for Cooper pairs.
suggestion is conformed for short wires by phenomenological model and experimentally confirmed for specimens with a length of \( L < 200 \text{ nm} \). However, the data for longer specimens are inconsistent in that they exhibit \( R^\parallel > R^\perp \) and show superconductivity. Thus, there is no consensus between experiments and theories on the exact role of \( \text{SIT} \) of 1D-SNWs.

Recently, Mojoo et al. proposed an idea that the concept of flux-charge duality can relate the QPS with Josephson tunneling if the roles of phase and charge are interchanged. They discussed the crossover value between superconducting and insulating states at low temperatures as a function of ratio \( \alpha = E_J/E_k \), where \( E_J \) and \( E_k \) are the QPS energy and inductive energy, respectively. Increasing \( E_J \) leads to a transition from inductively superconducting regime where \( E_J \approx E_k \) to a capacitive insulating regime where \( E_J \gg E_k \). Further, they succeeded in showing the phase boundary between the superconducting and insulating state of data for MoGe SNWs by assuming \( \alpha = 0.3 \).

To observe the quantum phase slip in SNWs, specimens are required to be homogeneous and satisfy the condition \( 4\pi w \leq \xi \), where \( \xi \) is superconducting coherence length. Further, poor links due to inhomogeneities in the superconducting wires can cause residual resistance at low temperatures as pointed by Altomare et al. In the present report on homogenous nanowires, the \( R(T) \) characteristics of NbTiN SNWs in a broad range of the \( R^\parallel/L \) were investigated from the viewpoint of QPS mechanisms. We analyzed the data from the superconducting and insulating phase diagrams based on the flux-charge duality model using the relation \( (R^\parallel/R_0)/(L/\xi) \) versus \( L/\xi \) with a suitable parameter \( \alpha \) and other parameters in theories.

Experimental Procedure

Superconducting NbTiN films were firstly prepared by deposition at ambient temperatures on (100)-MgO substrates by DC reactive magnetron sputtering. The background pressure of the chamber was maintained below \( 2.0 \times 10^{-5} \text{ Pa} \). The relative amounts of argon and nitrogen were controlled by mass flow controller during sputtering. The total pressure was maintained at 2 mm torr and the substrate was not heated intentionally during deposition. Details of preparation procedures and films quality of NbTiN thin films are previously reported. 

The NbTiN SNWs were fabricated from 2D films with \( d = 5 \text{ nm} \) by a conventional electron beam lithography method and a reactive ion etching method with CF4 plasma. The ranges of \( L \) and \( w \) of nanowires are \( 250 \leq L \leq 1000 \text{ nm} \) and \( 10 \leq w \leq 30 \text{ nm} \), respectively. To eliminate the influence of the contact resistance, measurements of transport properties were performed by four-probe method. The normal state resistance, \( R^N \), is defined as the sample resistance at 20K. The \( T_c^N \) and \( H_{c2}^N \) were defined as the point at which the \( R^N \) reached half its value.

Results and Discussion

Figure 1(a) shows the scanning electron microscopy image of typical NbTiN SNWs. Figure 1(b) presents the characteristics of \( R(T) \) for various SNWs with different values of \( w \) and \( L \). Superconducting SNWs that have \( dR/dT > 0 \) under low temperatures and low \( R(0) \) characteristics, experience the initial drop of \( R(0) \) almost at the same temperature owing to the superconducting transition. An increase in \( R^N \) causes \( T_c \) and the residual resistance to monotonically decrease and increase, respectively. Prior to the detailed discussions on the SIT of 1D specimens from a viewpoint of quantum phase transition, we will present some transport properties of the present SNWs from the characteristics of low dimensional superconductors.

Figure 2(a) shows a typical \( R(T) \) of the NbTiN SNWs with \( L = 500 \text{ nm} \) and \( w = 20 \text{ nm} \) at various external magnetic fields. With an increase in \( H \), \( T_c \) monotonically decreases without field-tuned SIT even at \( H = 9 \text{ T} \). On the other hand, as shown in the inset, the 2D NbTiN with almost the same thickness \((\approx 5 \text{ nm})\) shows that the field-tuned SIT occurs around \( 5–6 \text{ T} \) where many vortexes in the film appear to transition into super-fluid states of vortexes in the dirty boson scenario. The present result for NbTiN SNWs suggests that this nucleation of vortexes in the dirty boson scenario causes \( R \) to vanish at some points in the SNWs owing to the fluctuation, and it recovers the phase slip by \( 2\pi \). There are two mechanisms for the phase slip, TAPS and QPS. According to the TAPS model, \( dV/dL \) and \( R_{\text{TAPS}}(T) \) are expressed by \( c_2 \).
$$dV/dI = (\hbar \Omega/4e^2k_BT) \exp \left[ -\Delta F(T)/k_BT \right] \cos h (I/I_0),$$

(2)

and

$$R_{\text{TAPS}} = (dV/dI)_{I=0} = (\hbar \Omega/4e^2k_BT) \exp \left[ -\Delta F(T)/k_BT \right]$$

(3)

where \( \Omega = (L/\xi)(\Delta F/k_BT)^{1/2}/(1/\tau_{\text{GL}}) \), \( \Delta F(T) = 0.83k_BTc(R_Q/R_N)(L/\xi_0)(1 - T/T_c)^{3/2} \) is the energy barrier, \( I_{0,\text{TAPS}} = (4ek_BT)/h \) and \( \tau_{\text{GL}} = [\pi \hbar/8k_BTc(T_c - T)] \) is the relaxation time of G-L equation. On the other hand, \( R_{\text{QPS}}(T) \) is expressed as

$$R_{\text{QPS}}(T) = \beta R_Q S_{\text{GZ}} \frac{L}{\xi(T)} e^{-S_{\text{GZ}}}$$

(4)

where \( S_{\text{GZ}} = \eta R_Q \frac{L}{R_N \xi(T)} \) is the normalized unit measured by \( R_Q \) and \( \xi(T) \), and \( \beta \) and \( \eta \) are fitting parameters held constant on the order of unity\(^{30,31}\). For \( \xi(T) \), the expression \( \xi(T) = 0.907\xi_0(0) \left( 1 + (1 - 0.25r) \xi_0(0)/r \right)^{1/2} (1 - r^2)^{-1/2} \) is adopted\(^3\). The \( dV/dI \) is also given by

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**Figure 1.** (a) Scanning electron microscopy image of a NbTiN SNW with 10 nm-width. (b) \( R(T) \) for in series of NbTiN SNWs within ranges of 250–1000 nm length \( L \) and 10–30 nm width \( w \).
where \( I_{0,QPS} \) is expected to have a different temperature dependence from \( I_{0,TAPS} = (4e\kappa_0/h)T \). Figure 3 shows the superconducting transport properties for the NbTiN SNW with \( d = 5 \text{ nm}, w = 10 \text{ nm}, L = 500 \text{ nm}, T_c = 10.0 \text{ K}, \) and \( R_N = 5.0 \text{ k}\Omega \). Figure 3(a) shows \( R(T) \) from the measurements of current-bias (○), and the \( dV/dI \) at \( I \approx 0 \) shown in Fig. 3(b). The calculation made using the TAPS model shown by the dashed line (---) cannot explain \( R(T) \) characteristic except for temperatures close to \( T_c \) and unfortunately, the theory strongly deviates from the data below 8 K. This discrepancy suggests that transport properties of the present NbTiN SNWs are incompatible with TAPS theory at low temperatures. The solid line is calculated using Eq. (4) with parameters of \( \xi(0) = 8 \text{ nm}, \beta = 0.0013, \) and \( \eta = 0.024 \) in order to fit theory with \( R(T) \) data under a broad temperature range.

The calculation using the QPS model agrees accurately with the resistive tail in the range of 5 magnitude orders. Figure 3(b) represents the \( I-V \) characteristics at temperatures of 2 K, 5 K, 6 K, 7 K, 8 K, and 9 K. The \( I-V \) curves have shown nonlinear characteristic in the superconducting region below the 9 K. This \( I \)-dependence of \( dV/dI \) at each temperature agrees well with the term of \( \cosh(I/I_{0,QPS}) \) in the Eq. (5) as shown by the solid lines. From the fitting procedure, we obtained the temperature dependences of \( R \) and \( I_{0,QPS} \). The inset in Fig. 3(b) shows the \( I_{0,QPS}(T) \), which is almost independent of temperature as shown by the dotted line. This discrepancy with the TAPS model shown by the solid line is consistent with the experimental result that \( R(T) \) cannot be explained by the TAPS model.

Before investigating the \( R^2/L \) ratio dependence on \( T_c \) in order to clarify the SIT of NbTiN SNWs, we will analyze the data using the theory based on the dynamically enhanced Coulomb repulsion competing the attractive interactions between electrons. The theoretical expression for \( T_c \) is given by a simple formula as a function of sheet resistance \( R_s \) (resistance for unit area) with the parameter \( \gamma = 1/\ln(k_BT_0T_{c0}7/1.14h) \), where \( r \) is the electron elastic scattering time. Figure 4 represents the \( R_s \) dependence of \( T_c \) for both nanowire specimens and 2D specimens. \( T_{c0} \) values are expected to be independent of \( w \) and \( L \) as shown in Fig. 5, because the \( T_c \) on the vertical axis is normalized by \( T_{c0} = 11.0 \text{ K} \) of pure 2D films. Although \( R_{so} \) of 2D specimens was controlled by changing the
thickness, $R_{\text{sq}}$ of SNW specimens with different $w$ was controlled by changing the length $L$ and by keeping the thickness constant at $\approx 5$ nm for all SNWs. The dotted line is calculated by using the theory for impure 2D system in order to fit the data of $dV/dI$ in (b). Dotted and solid lines are calculated using Eq. (2) and Eq. (4), respectively. (b) Bias current dependence of differential resistance under various temperatures below $T_c$. Solid lines are calculated using Eq. (3). The inset shows the temperature dependency of $I_0$, The dashed line shows $I_0(T) = (4\pi k T/h) T$ predicted by the TAPS theory. Solid line $I_0$ = constant is a reference point.

**Figure 3.** (a) $R(T)$ for the NbTiN SNW with $d = 5$ nm, $w = 10$ nm and $L = 500$ nm. Open circles are resistances from the low bias current measurement and squares show resistances obtained by applying the theory to the data of $dV/dI$ in (b). Dotted and solid lines are calculated using Eq. (2) and Eq. (4), respectively. (b) Bias current dependence of differential resistance under various temperatures below $T_c$. Solid lines are calculated using Eq. (3). The inset shows the temperature dependency of $I_0$. The dashed line shows $I_0(T) = (4\pi k T/h) T$ predicted by the TAPS theory. Solid line $I_0$ = constant is a reference point.
Figure 5 shows $T_c$ as functions of $R(20\,\text{K} > T_c) \equiv R_N/L$ (upper axis) and $1/A$ (lower axis), where the cross section area $A$ is derived from the relation $R_N/L = \rho/A$ using the resistivity value of $\rho = 0.9\,\mu\Omega\text{m}$ determined before the fabrication of 2D films. The $T_c$ was defined as the temperature at which the resistance takes one half of $R(20\,\text{K})$. For specimens with different size of $w$ and $L$, data of $T_c$ concurs well onto a single curve in a broad $R_N/L$ (or $1/A$) range. This fact expresses that the quantity $R_N/L$ ratio is a sufficient parameter to describe the impurity

**Figure 4.** $R_{sq}$ dependence of $T_c$ for NbTiN film and nanowire specimens. Dotted and solid lines are calculated from theories based on the dynamically enhanced Coulomb repulsion in dirty systems for the film and nanowire, respectively.

**Figure 5.** $R_N/L$ (upper horizontal axis) and $1/A$ (down horizontal axis) dependencies of $T_c$ for NbTiN SNWs. Mark (●) represents the $T_c$ for 2D specimens. The broken line represents $T_c = T_{c0}\exp(-8.75/A)$ determined from data. The dotted line is the reference point. Arrows, ↑ and ↓ indicate the SIT points.
dependence of $T_c$ for the 1D system and for 2D system in the case $R^N$ is replaced with $R_{\text{eq}}$. The $T_c$ slowly decreases below the value $R^S/L \approx 50\Omega/\text{nm}$ when $R/L$ increases, and rapidly decreases in the range above $50-60\,\Omega/\text{nm}$. For a relation between $T_c$ and $R^S/L$ in Fig. 5, Marković et al. proposed a simple criterion for the crossover value of $(R^S/L)_{\text{c.o}}$ separating an insulating state from superconducting state. If the wire resistance at $T=0$ due to the quantum phase slip is comparable to $R^S$, the resistance drop does not appear to sustain the value of $R^N$ even at very low temperatures. Using Eq.(4) at $T=0$, they obtained the normalized resistance due to quantum phase slips as

$$R_{\text{SL}} = 2\pi \exp\left[-0.33/R_{\text{NL}}\right],$$

where the resistances $R_{\text{eq}}$ and conductance $R_{\text{eq}}$ are measured in units of quantum resistance $R_Q$ and length in units of coherence length $\xi$, namely, $R_{\text{eq}} = (R/Q)(\xi/L)$ and $R_{\text{eq}} = (R^S/R_Q)(\xi/L)$. From Eq. (6), by equating $R_{\text{eq}}$ to the $R^S$, we obtain the value $(R^S/L)_{\text{c.o}} = R_Q/13.3\approx 4850\Omega/\xi(0)$. As for the present series of which $\xi(0) \approx 9\,\text{nm}$, we obtain $(R^S/L)_{\text{c.o}} \approx 54\Omega/\text{nm}$. This value is almost the same one denoted by the arrow ↑ separating the superconductor and insulator phases in Fig. 5.

From $1/A$ dependence of $T_c$ in Fig. 5, we can quantitatively discuss on the critical diameter $D_c$ of NbTiN SNW at SIT. The dotted line in Fig. 4 represents the relation $T_c = T_{c0} \exp\left(-u/A\right)$, where $T_{c0} = 11.0\,\text{K}$ and $u = 8.75\,\text{nm}^2$ are the transition temperature of 2D film and the parameter determined from the fitting procedure, respectively. $1/A$ dependence of $T_c$ has also been reported in Mo$_{78}$Ge$_{22}$ and Mo$_{50}$Ge$_{50}$ SNWs that is having greater $\alpha$ than that of the present NbTiN. If this relation for NbTiN SNW is valid for board range $1/A$, the $D_c \propto \exp\left(-u/A\right)$ value is expected to be smaller than $\approx 2\,\text{nm}$. On the other hand, the $T_c(1/A)$ characteristic drastically decreases around $1/A \approx 0.06\,\text{nm}^{-1}$, giving $D_c \approx (4A/\pi)^{1/2} \approx 4.6\,\text{nm}$ denoted by the arrow ↓, which is approximately half of the 2D NbTiN coherence length $\xi(0) \approx 9\,\text{nm}$ determined from the relation $\xi^2(0) = \frac{\hbar c}{2eT_c}dH_{c2}(T)\bigg|_{T_c=T_c}$. This estimation suggests that the restricted geometry of SNW allows smaller critical diameter than $\xi(0)$ for 2D specimens.

Now, we will discuss of size dependence of $T_c$ and SI phase diagram for the present SNWs. To clarify the 1D SIT mechanisms of NbTiN SNWs, we illustrated the SI phase diagram in Fig. 6 according to the Chakravarty–Schmid–Bulgadaev theory based on the interaction of QPS and dissipative environment. A relation between $L$ and $L/R^N$ has been reported for MoGe SNWs. The author claimed that the SIT boundary is given by a condition $R^S = R_Q = 6.45\,\text{k}\Omega$. However, the present NbTiN SNWs specimens with $L > 500\,\text{nm}$ do not satisfy this condition as shown by the dashed line, that is, specimens with $L > 500\,\text{nm}$ show superconductivity through $R^S$ is larger than $R_Q$. Although the $R(T)$ characteristic of NbTiN SNWs can be explained by the QPS theory as discussed in Figs 4 and 5, the phase diagram shown in Fig. 6 suggests that the SI boundary depends on the length of the nanowire.

QPS and the Josephson effects in SNWs are related to each other by a concept of duality transformation. According to this concept, the SI is determined by the ratio between the strength of QPS amplitude energy $E_a$ and SNW inductive energy $E_L$. Both energies are given by $E_a = eV_{\phi}/\pi = a(L/\xi)k B_T(R_Q/R_c)\exp\left(-bR_Q/R_c\right)$ and $E_L = s_0/2L = 17.4k B_T(R_Q/R^S)$, where the $R_c = R^S\xi/L$ is the resistance of the SNW over an appropriate length, $L_0 = 0.18\hbar R^S/\Phi_0$ is the kinetic inductance of the wire, $\Phi_0 = h/2e$ is the flux quantum, and $a$ and $b$ are constants of order one. According to Moomij et al., it is expected that SI occurs at condition $E_a/E_L = (a\lambda^2/17.4)$.

**Figure 6. Phase diagram of $R^S/L$ versus $L$ for NbTiN SNWs.** Squares (■) and triangles (▲) represent insulating and superconducting wires, respectively. The solid line indicates the boundary of SIT estimated from experimental results of $R$-$T$ characteristics. The dotted line is the boundary of SIT expected from the Chakravarty–Schmid–Bulgadaev theory. (see text.).
Figure 7. Phase diagram of $L/\xi(=\lambda)$ versus $r_\xi = (R^s/R_0)/(L/\xi)$ for NbTiN SNWs. (■) and (▲) are the same marks in Fig. 6. Lines denoted by (---), (—) and (——) are calculated from Eq. (7) with input parameters $(b, c) = (0.14, 0.05)$, $(b, c) = (0.23, 0.2)$ and $(b, c) = (0.28, 0.5)$, respectively.

exp$(-br_\xi)=\alpha_\lambda$, where $\lambda = L/\xi$ and $r_\xi = R_0/R_\xi(=R^s\xi/R_\xi L)$ are the normalized length and resistance, respectively. From the equation for $E_s/E_{i\lambda}$, we obtain the $\lambda$ dependence of $r_\xi$ as

$$r_\xi(\lambda) = b/\ln(a\lambda^2/17.4\alpha_\lambda) \equiv b/\ln(c\lambda^2/17.4).$$

(7)

Figure 7 shows the $r_\xi(\lambda)$ for all same NbTiN SNWs shown in Fig. 6, where $\xi = 8$ nm is used. To show a boundary separating the superconducting phase from the insulator phases, we calculate $r_\xi(\lambda)$ from Eq. (7) with input parameters $b$ and $c$. Although data are not so large to make clear the boundary, we attempt to find reasonable values for $b$ and $c$ assuming that the theoretical line must go through a reliable point of $r_\xi(\lambda) \approx 0.6$ at $\lambda = L/\xi \approx 62$ for analysis. The red-broken, red-solid, and black-solid lines are typically calculated from Eq. (7) to divide the data into superconducting and insulator phases with the use of parameters $(b, c) = (0.14, 0.05)$, $(b, c) = (0.23, 0.2)$ and $(b, c) = (0.28, 0.5)$, respectively. When we take into account the theoretical suggestions that the strength $p$ of three parameters $a$, $b$ and $\alpha_\lambda$ is given as 0.1 $< p < 1$. Further, the first combination of $(b, c) = (0.14, 0.05)$ corresponds to $(a_\lambda, a, b) = (0.025, 0.5, 0.14)$ due to the definition $c = a_\lambda a$, in Eq. (4). This critical ratio $a_\lambda = 0.025$ is very small compared with the theoretical suggestion. On the other hand, other combinations shown by solid and dotted lines give reasonable values for $a$ and $b$, namely, $0.1 < a < 0.25$ and $0.23 < b < 0.28$ for $\alpha_\lambda = 0.5$. These values are comparable to those analyzed by Mooij et al. for Mo/Ge SNWs data.

Conclusion

We investigated the transport properties of superconducting NbTiN SNWs in a wide range of $R_0^s/L$ using four-probe method to eliminate the contact resistance. The $R(T)$ characteristic with resistive tail below $T_c$ for SNWs with high values of $R_0^s/L$ can be well explained by the QPS theory. With the increasing $R_0^s/L$, the behavior of the $R(T)$ characteristic changes from superconducting to insulating. $R(T)$ exhibits superconducting-insulator transition near $(R_0^s/L)_c \approx 60\,\Omega$/nm, which agrees well with the prediction based on the QPS model by Marković et al. As for the $S-I$ phase boundary for the NbTiN SNWs, the phase diagram $L/R_0^s$ vs. $L$ is inconsistent with Chakravarty–Schmid–Bulgadaev theory, which has succeeded in describing the SIT for short MoGe SNWs. On the other hand, the analysis based on the model for the SNW which is being dual element up to Josephson junction, suggests that the separation of the superconducting and insulator phases may be controlled by the ratio of QPS amplitude energy $E_0$ and inductive energy of SNW $E_{i\lambda}$ $E_s/E_{i\lambda}$. For the present NbTiN series, we observed that SIT may occur at $0.2 < E_s/E_{i\lambda} < 0.5$.

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**Author Contributions**
K.M. performed most of the experiments and analyzed the data. H.T. contributed to the growth of nanowires. Y.T. and S.T. contributed to the experiments. All of the text of the paper was written jointly K.M. and B.S.

**Additional Information**

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