Muon $g - 2$ and precision electroweak physics in the MSSM

Gi-Chol Cho$^{1),2)}$, Kaoru Hagiwara$^{1)}$ and Masashi Hayakawa$^{1)}$

$^1)$ Theory Division, KEK, Tsukuba, Ibaraki 305-0801, Japan
$^2)$ Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126 Pisa, Italy

Abstract

The minimal supersymmetric extension of standard model (MSSM) is examined by analyzing its quantum effects on the precision electroweak measurements and the muon $g - 2$. We examine carefully the effects of light charginos and neutralinos that are found to improve the fit to the electroweak data. We identify two distinct regions on the $(\mu, M_2)$-plane that fit well to the electroweak data and give significant contribution to muon $g - 2$. 
The minimal supersymmetric standard model (MSSM) has been one of the extensively investigated theories beyond the standard model since it was recognized [1] that it has the potential to unify strong and electroweak gauge interactions. On the other hand, despite enormous efforts to find its signature, there has been no direct evidence for the existence of the supersymmetric particles. In this letter we study quantitatively the MSSM predictions for the muon $g - 2$ in the light of the latest precision electroweak data [2, 3], and that of the present and future $g - 2$ experiments [4].

The importance of the muon $g - 2$, conventionally denoted as $a_\mu = \frac{1}{2}(g_\mu - 2)$, has been widely discussed in the context of supersymmetric theories [5, 6, 7, 8]. The observation of the effect from the MSSM is found generally accessible with the target accuracy of the current experiment at Brookhaven National Laboratory (BNL) [4]

$$\Delta a_\mu(\text{expt}) = 4.0 \times 10^{-10}. \quad (1)$$

This accuracy is about 1/20 of the error (numerals in the parenthesis) of the current measurement [4]

$$a_\mu(\text{expt}) = 11659 \ 235 \ (73) \times 10^{-10}. \quad (2)$$

The prediction of the standard model (SM) is

$$a_\mu(\text{SM}) = 11659 \ 168.75 \ (9.56) \times 10^{-10}, \quad (3)$$

where the details of specific contributions is found in Ref. [9]. We shall come back to the theoretical uncertainties of the SM prediction not included in the above quoted error of $9.56 \times 10^{-10}$ in the summary part.

As noted above the experimental results on the precision measurements around the $Z$-pole have played an essential role in revealing the possibility of grand unification. This implies the existence of many supersymmetric particles below the TeV scale and hence we may expect to observe their quantum effects in the precision electroweak experiments as well as in the muon $g - 2$. Although no signal of supersymmetry has so far been identified, recent systematic study of the precision electroweak data found that the existence of the relatively light ($\sim 100$ GeV) charginos and neutralinos can make the MSSM fit slightly better than that of the standard model [10]. We therefore study carefully the MSSM effects on the muon $g - 2$ in the range $|\mu|, M_2 < 500$ GeV. It is found that analysis performed here could give rise to a systematic criterion to identify the preferred range of the MSSM parameters.
As we will see, the additional contribution to the muon $g - 2$ in the MSSM can be greater than the target accuracy (1) of the present experiment. Thus it is convenient to define the additional new physics contribution to the muon $g - 2$ in the unit of $10^{-10}$

$$\delta a_\mu \equiv 10^{10} \times \{a_\mu^{\text{MSSM}} - a_\mu^{\text{SM}}\}. \tag{4}$$

The present experimental data (2) can then be expressed as

$$-16 \leq \delta a_\mu \leq 149, \tag{5}$$

at the 1-$\sigma$ level. Here we add the theoretical uncertainty of 10 in the unit of $10^{-10}$ in eq. (3) linearly to the present experimental error of 73 in eq. (2).

In the MSSM $a_\mu$ receives essentially two new contributions. One comes from the chargino ($\tilde{\chi}^+_j, j = 1, 2$) and muon-sneutrino ($\tilde{\nu}_\mu$) propagation in the intermediate states, and the other from the neutralino ($\tilde{\chi}^0_j, j = 1 \sim 4$) and smuon ($\tilde{\mu}_i, i = 1, 2$) intermediate states. The chargino-sneutrino contribution can be expressed as

$$a_\mu(\tilde{\chi}^-) = \frac{1}{8\pi^2} \sum_{j=1}^{2} \left( \frac{m_\mu}{m_{\tilde{\nu}_\mu}} \sum_{i=1}^{2} \left( \frac{m_{\chi_i}}{m_{\nu_i}} G_1 \left( \frac{m_{\chi_i}^2}{m_{\nu_i}^2} \right) \left| g_{L} \tilde{\chi}^-_j \tilde{\nu}_\mu \right|^2 + \left| g_{R} \tilde{\chi}^-_j \tilde{\nu}_\mu \right|^2 \right) \right) + \frac{m_{\tilde{\chi}^-}}{m_{\tilde{\nu}_\mu}} G_3 \left( \frac{m_{\tilde{\chi}^-}^2}{m_{\tilde{\nu}_\mu}^2} \right) \text{Re} \left( \left( g_{R} \tilde{\chi}^-_j \tilde{\nu}_\mu \right)^* \right) \left( g_{L} \tilde{\chi}^-_j \tilde{\nu}_\mu \right), \tag{6a}$$

where

$$G_1(x) = \frac{1}{12(x-1)^4} [(x-1)(x^2 - 5x - 2) + 6x \ln x], \tag{6b}$$

$$G_3(x) = \frac{1}{2(x-1)^3} [(x-1)(x-3) + 2 \ln x], \tag{6c}$$

while the neutralino-smuon contribution as

$$a_\mu(\tilde{\chi}^0) = -\frac{1}{8\pi^2} \sum_{i=1}^{2} \sum_{j=1}^{4} \left( \frac{m_\mu}{m_{\tilde{\nu}_i}} \sum_{i=1}^{2} \left( \frac{m_{\chi_i}}{m_{\nu_i}} G_2 \left( \frac{m_{\chi_i}^2}{m_{\nu_i}^2} \right) \left| g_{L} \tilde{\chi}^0_\mu \tilde{\nu}_i \right|^2 + \left| g_{R} \tilde{\chi}^0_\mu \tilde{\nu}_i \right|^2 \right) \right) + \frac{m_{\tilde{\chi}^0}}{m_{\tilde{\nu}_i}} G_4 \left( \frac{m_{\tilde{\chi}^0}^2}{m_{\tilde{\nu}_i}^2} \right) \text{Re} \left( \left( g_{R} \tilde{\chi}^0_\mu \tilde{\nu}_i \right)^* \right) \left( g_{L} \tilde{\chi}^0_\mu \tilde{\nu}_i \right), \tag{7a}$$

where

$$G_2(x) = \frac{1}{12(x-2)^4} [(x-1)(2x^2 + 5x - 1) - 6x^2 \ln x], \tag{7b}$$

$$G_4(x) = \frac{1}{2(x-1)^3} [(x-1)(x+1) - 2x \ln x]. \tag{7c}$$

Here we adopt the notation of Ref. [10, 11] for the coupling constants

$$\mathcal{L} = \sum_{\alpha=L,R} g_{\alpha} F_1 F_2 S F_1 P_\alpha F_2 S, \tag{8}$$

3
where $F_1$ and $F_2$ are four-component fermion fields, $S$ denotes a scalar field, and
\[ P_L = \frac{1 - \gamma_5}{2}, \quad P_R = \frac{1 + \gamma_5}{2}. \] (9)

The charged Higgs boson contribution, which gives rise to a sizable effect in the case of $b \rightarrow s\gamma$ transition, is highly suppressed due to the small Yukawa couplings in the muon $g - 2$. The MSSM contribution to the muon $g - 2$ is most significant at large $\tan \beta$. The chargino-sneutrino loop, eq. (6), then dominates over the other in almost all region of the $(\mu, M_2)$ plane. However the neutralino-smuon loop contribution to $\delta a_\mu$ is also larger than the target accuracy (1) of the current experiment. Except for $(\mu, M_2, \tan \beta)$ which determines the chargino and neutralino masses, the former depends on the left-handed SUSY breaking slepton mass, $m_\tilde{L}$, while the latter also depends on the right-handed one, $m_\tilde{E}$, in addition. We discuss this point in more detail when we study the dependence of $\delta a_\mu$ on $m_\tilde{E}$.

In our analysis we allow the MSSM parameters to vary freely restricted only by experimental constraints from direct and indirect searches. When restricted to specific models of supersymmetry (SUSY) breaking such as the minimal supergravity scenario, or the gauge mediated supersymmetry breaking scenario, we reproduce the known results [5, 6]. First we consider the case when all the squarks are so heavy that their effects on low-energy physics disappears. In our actual numerical calculation, we set all the squark masses and the extra Higgs boson masses at 1 TeV such that they do not make worse the fit to the electroweak data [10, 12] and the $b \rightarrow s\gamma$ rate [13].

The constraints on the MSSM parameter space from the electroweak experiments have been found in Ref. [10] by using the 18 Z-pole data given by the LEP/SLC experiments [2] and the $W$-boson mass $m_W$ given by the Tevatron and LEP2 experiments [8]. The electroweak observables are affected by the supersymmetric particles through both the universal gauge boson propagator corrections and the process specific vertex/box corrections. Following the formalism introduced in Ref. [10], the universal part of the radiative corrections of the Z-pole observables can be represented by two oblique parameters $S_Z$ and $T_Z$, while the $W$-boson mass $m_W$ itself can be adopted as the third oblique parameter. The parameters $S_Z$ and $T_Z$ are expressed as the sum of the conventional $S$ and $T$ parameters [14, 15] and the $R$ parameter which measures the running effect of the Z-boson propagator correction between $q^2 = 0$ and $q^2 = m_Z^2$. The current electroweak data favors new physics that gives negative contribution both to the $S_Z$ and $T_Z$ parameters, and hence they constrain strongly the additional positive
contributions to these parameters in the MSSM. The squark contributions to the $T_Z$ parameter are generally found to be positive while they do not affect to the $S_Z$ parameter so much. The light charginos and neutralinos are found to give negative contributions to the $R$ parameter and hence they make both the $S_Z$ and $T_Z$ parameters negative, thereby improving the fit to the electroweak data. Light left-handed sleptons make $S_Z$ negative but keep $T_Z$ essentially unchanged. It has been shown that if the chargino mass is close to its lower mass bound from the LEP2 experiment, $m_{\tilde{\chi}_1^-} \sim 100$ GeV, and the squarks and sleptons are sufficiently heavy, say 1 TeV, the total $\chi^2$ in the MSSM is slightly better than that in the standard model \cite{10}:
\begin{equation}
\Delta \chi^2 \equiv \chi^2_{\text{MSSM}} - \chi^2_{\text{SM}} \sim -1.
\end{equation}
No other combinations of light supersymmetric particles are found to improve the fit to the electroweak data over the standard model.

Summing up, the precision electroweak data favors light charginos and neutralinos but disfavors light left-handed squarks and sleptons. The muon $g - 2$ is found to be sensitive to the left-handed slepton mass when there are light charginos and neutralinos. If the left-handed sleptons are relatively light, then the MSSM contribution to $g - 2$ grows but the electroweak fit worsens. Below we examine quantitatively the MSSM prediction for the muon $g - 2$ and the electroweak data, and look for the region of the MSSM parameter space which gives observable $g - 2$ effect without spoiling the good fit to the electroweak data.

Taking this circumstances in mind, we show in Fig. 1 the contours of fixed values of $\delta a_{\mu}$ defined in Eq. (3) on the $(\mu, M_Z)$-plane for a relatively large value of $\tan \beta = 50$. We set $m_L = m_E = 200$ GeV in Fig. 1(a) and $m_L = m_E = 500$ GeV in Fig. 1(b), respectively. For the gaugino masses, we adopt the “unification” condition, $M_1/M_2 = \alpha_1(M_Z)/\alpha_2(M_Z)$, for simplicity. However, the general aspect of our conclusion obtained here persists as long as the gaugino masses share a common order of magnitude. The shaded-region shows the region of the parameters already inconsistent with the mass bound for chargino \cite{10}:
\begin{equation}
m_{\tilde{\chi}_1^-} > 93 \text{ GeV}.
\end{equation}
The physical slepton masses are obtained as the eigenvalues of the slepton mass-squared matrix for each flavor ($l = e, \mu$ or $\tau$):
\begin{equation}
M^2_l = \begin{pmatrix}
M^2_{l,L} & M^2_{l,R} \\
M^2_{R,L} & M^2_{R,R}
\end{pmatrix},
\end{equation}
Figure 1: Contours of the MSSM contribution to the muon $g-2$ in the unit of $10^{-10}$ ($\delta a_\mu$) on the ($\mu, M^2$)-plane for $\tan \beta = 50$. (a) $m_\tilde{L} = m_\tilde{E} = 200$ GeV. (b) $m_\tilde{L} = m_\tilde{E} = 500$ GeV. The region enclosed by the dashed line gives $\Delta \chi^2 = \chi^2_{\text{MSSM}} - \chi^2_{\text{SM}} < 0$ for the precision electroweak data, while the region enclosed by the bold line satisfies $\Delta \chi^2 < 1$. The darkly shaded zone is excluded from the current lower bound on the chargino mass. The lightly shaded region is excluded by the currently allowed value for muon $g-2$ in eq. (5). I and II on the graphs indicate Region I in Eq. (14) and Region II in Eq. (15) respectively.

\begin{align*}
M^2_{LL} &= m^2_\tilde{L} + m^2_l - M^2_Z \cos 2\beta \left( \frac{1}{2} - \sin^2 \theta_W \right), \\
M^2_{RR} &= m^2_\tilde{E} + m^2_l - M^2_Z \cos 2\beta \sin^2 \theta_W, \\
M^2_{LR} &= (M^2_{RL})^* = m_l (A^*_l - \mu \tan \beta),
\end{align*}

where $m_l$ denotes the charged-lepton mass. In Fig. 1 we show the contours of muon $g-2$ when $A_\mu$ is set equal to zero. Effects of nonzero $A_\mu$ are discussed below.

Fig. 1(a) shows a generic feature that the MSSM contributions can be as large as $(100 \sim 300) \times 10^{-10}$ when $\tan \beta = 50$ and $m_\tilde{E} = 200$ GeV, which is much greater than the $W$- and $Z$-contributions in the SM \cite{17},

$$a_\mu(\text{weak}) = 15.1 (4) \times 10^{-10}. \quad (13)$$

This is essentially because of the large Yukawa coupling for the $\mu_R \tilde{h}^- \tilde{\nu}_\mu$ vertex at large $\tan \beta \gg 1$. Here $\mu_R$ is the right-handed muon component and $\tilde{h}^-$ is the charged component of the higgsino which couples to charged leptons and down-type quarks. The magnitude of the MSSM predictions reduces to the level of the
SM weak corrections \([13]\) at around \(\tan \beta = 3\), as shown in Fig. 2. The MSSM contribution to the muon \(g - 2\) scales roughly as \(r_L/m^2_L + r_E/m^2_E\) in the large \(\tan \beta\) case, where \(r_L\) and \(r_E\) are some constants. The dependence on \(m_L\) and \(m_E\) will be further evaluated below.

Also shown in the figures is the region where the fit to the electroweak data is better than that of the SM (the narrow region enclosed by the dashed lines). The region falls mostly into the region forbidden by the chargino mass bound (the darkly shaded regions). The region enclosed by the bold lines gives reasonably good fit to the electroweak data, where \(\Delta \chi^2\) in Eq. (10) is less than one. We find that there are essentially two regions which respect the result from the precision electroweak measurements. One region is

\[
\text{Region I : } 120 \text{ GeV} < \mu < c_1 M_2, \tag{14}
\]

where \(c_1\) is a positive number greater than one, depending on the slepton masses. The other minor region appears only when \(m_L\) is taken to be larger than about 500 GeV (see Fig. 1(b)):

\[
\text{Region II : } 100 \text{ GeV} < M_2 < 150 \text{ GeV}. \tag{15}
\]

The lightest chargino is generally higgsino-dominated in Region I while it is gaugino-dominated in Region II. Both domains correspond to the extreme regime in which the chargino mass becomes nearly equal to the experimental lower bound, around which the chargino-neutralino contributions to the oblique parameters pull both \(S_Z\) and \(T_Z\) back to negative directions. This negative contribution to \(\Delta \chi^2\) is necessary to complement the positive contribution (which worsens the fit) due to relatively small \(m_L\). Thus the MSSM with large \(\tan \beta\) gives a sizable contribution to the muon \(g - 2\) if \(|\mu|, M_2\) and \(m_L\) are all smaller than 500 GeV. Actually the present muon \(g - 2\) data \([4]\) already excludes the negative \(\mu\) region of Fig. 1.

The muon \(g - 2\) is a less powerful probe of the MSSM at smaller \(\tan \beta\). Fig. 2 shows the similar graphs as Fig. 1, but for \(\tan \beta = 3\). As in Fig. 1(a) and (b), \(m_L = m_E = 200\) GeV in Fig. 2(a), and \(m_L = m_E = 500\) GeV in Fig. 2(b). Here we are interested in whether we can find some evidence of the MSSM with small \(\tan \beta\) through the measurement of the muon \(g - 2\) in view of the other precise low energy data. Fig. 2(a) shows that the region with positive \(\mu\) and small \(M_2\) for small slepton mass \((m_L = 200\) GeV) is accessible by the present muon \(g - 2\) experiment with its target precision \([4]\), while respecting the result of the electroweak experiments.
Figure 2: Graph similar to Fig. 1 for tan $\beta = 3$. (a) $m_{\tilde{L}} = m_{\tilde{E}} = 200$ GeV. (b) $m_{\tilde{L}} = m_{\tilde{E}} = 500$ GeV. III and IV on the graphs indicate Region III in Eq. (16a) and Region IV in Eq. (16b) respectively.

Fig. 2(b) shows that new two allowed regions appear on the negative $\mu$ side of the ($\mu, M_2$)-plane for slepton masses larger than about 500 GeV. Those two regions are

Region III: $-c_2 M_2 < \mu < -80$ GeV,  
Region IV: $80$ GeV < $M_2$ < $-c_3 \mu$, $\mu < -120$ GeV,  

where $c_2$ and $c_3$ are positive numbers depending on $m_{\tilde{L}}$ and $m_{\tilde{E}}$. The MSSM correction to the muon $g - 2$ is small compared to the target precision in those regions in Fig. 2(b). However, when $m_{\tilde{E}} < m_{\tilde{L}}$, the observable enhancement might be expected in Region III and IV, as will be shown in Fig. 3(b).

Next we study the MSSM prediction to the muon $g - 2$ without assuming the universality between $m_{\tilde{L}}$ and $m_{\tilde{E}}$. In Fig. 3 we show the $m_{\tilde{L}}$-dependence of the maximally admissible $\delta a_\mu$ for (a) tan $\beta = 50$ and (b) tan $\beta = 3$ when $A_\mu = 0$. In each figure, the solid, dotted and dashed lines represent $m_{\tilde{E}} = 100$ GeV, 300 GeV and 500 GeV, respectively. We find that, for tan $\beta = 50$, the maximum of $\delta a_\mu$ is achieved when $M_2/\mu$ is nearly equal to $\pm 1$. For tan $\beta = 3$, this property also holds as long as $m_{\tilde{L}}$ is small enough (less than about 500 GeV).

It should be remarked that the precision measurements around Z-pole favor large left-handed slepton mass but is rather blind to the right-handed slepton mass. Fig. 3 tells us that the MSSM prediction of the muon $g - 2$ has sizable
Figure 3: $m_{\tilde{E}}$-dependence of the maximally possible MSSM contributions to the muon $g - 2$ in the unit of $10^{-10}$ ($\delta \alpha_\mu$) for (a) $\tan \beta = 50$ and (b) $\tan \beta = 3$ when $A_\mu = 0$. The right-handed slepton mass $m_{\tilde{E}}$ is taken to be 100 GeV (solid line), 300 GeV (dotted line) and 500 GeV (dashed line).

$m_{\tilde{E}}$-dependence when the target accuracy (1) of the current $g - 2$ experiment is taken into account. Let us recall that the chargino-sneutrino loop contribution (3) depends on $m_{\tilde{L}}$ while the neutralino-smuon loop contribution (7) depends on both $m_{\tilde{L}}$ and $m_{\tilde{E}}$. The numerical study shows that the chargino-sneutrino loop contribution decreases faster than the neutralino-smuon loop contribution if $m_{\tilde{L}}$ is increased while $m_{\tilde{E}}$ is kept fixed. Thus $\delta \alpha_\mu$ is more sensitive to relatively small $m_{\tilde{E}}$ for larger $m_{\tilde{L}}$. This is because only the neutralino-smuon loop contribution depends on $m_{\tilde{E}}$ and the lighter smuon is almost the right-handed slepton component for $m_{\tilde{E}} \ll m_{\tilde{L}}$. The neutralino-smuon loop gives nonzero contribution to $\delta \alpha_\mu$ even for $m_{\tilde{L}} \to \infty$. This explains why $\delta \alpha_\mu$ does not decouple in Fig. 3 at large $m_{\tilde{L}}$. Indeed, for $\tan \beta = 50$, $m_{\tilde{E}} = 100$ GeV and $m_{\tilde{L}} \to \infty$, the maximally possible $\delta \alpha_\mu$ is 65, which is observable in the light of the precision (1) for the muon $g - 2$ measurement. Fig. 3 shows that, on account of the constraint on $\delta \alpha_\mu$ (3), there is a possibility that the MSSM with small $\tan \beta$ for negative $\mu$ and $m_{\tilde{E}} < m_{\tilde{L}}$ can be probed by the current muon $g - 2$ experiment with the target precision (1).

So far, all our results have been shown for $A_\mu = 0$. We discuss the dependence of these results on $A_\mu$ here. We first recall that $A_\mu$ enters only in the left-right mixing of the smuon mass-squared matrix (12) for all the electroweak precision observables and the muon $g - 2$ at one-loop order. Thus $A_\mu$-dependence comes from the neutralino-smuon loop contribution. The numerical analysis shows that
\[ \delta a_\mu \text{ varies at most 5 when } A_\mu \text{ varies from } -1 \text{ to } 1 \text{ TeV for the various choices of } m_\tilde{\tau}, m_\tilde{E}, M_2 \text{ and } \mu \text{ in the allowed region at both the large and small tan } \beta. \] 

Thus our findings in Fig. 1 and Fig. 3(a) remain valid up to the target precision (3) of the current muon \( g-2 \) experiment even with nonzero \( A_\mu \). In particular the slepton mass dependence does not change quantitatively. For small \( \tan \beta \), the impact of nonzero \( A_\mu \) is comparable to the magnitude of \( \delta a_\mu \) in \( A_\mu = 0 \) case. Then, our discussion on the small \( \tan \beta \) case with \( A_\mu = 0 \) might be modified depending on the magnitude and the sign of \( A_\mu \). Note that, if \( \tan \beta \) is small, \( A_\mu \) could affect \( \delta a_\mu \) when the relative sign between \( A_\mu \) and \( \mu \) is opposite. This is because \( A_\mu \) always appears as a linear combination with \( \mu \tan \beta \) in the slepton mass-squared matrix element (12). We find that \( \delta a_\mu \) for positive \( \mu \) could be at most 15 if \( A_\mu \) is negative and large, say \( \sim -1 \) TeV.

To summarize the paper we performed the quantitative analysis on the MSSM contribution to the muon \( g-2 \) on account of the constraint from the precision electroweak data. The analysis \[10\] on the precision electroweak measurements tells us that the fit of the MSSM might be better than that of the SM when the chargino mass is close to the LEP2 bound (\( \sim 100 \) GeV) and the left-handed sfermions are heavy enough, say a few hundred GeV. Our study has been done by taking account of this point and the target precision \( 4 \times 10^{-10} \) in (2) of the current muon \( g-2 \) experiment.

At first, assuming the universality between the left- and the right-handed SUSY breaking mass parameters, \( m_\tilde{\tau} = m_\tilde{E} \), we found four regions on the \((\mu, M_2)\)-plane where the fit to the electroweak data is not worse as compared to the SM and the predicted MSSM contribution to the muon \( g-2 \) is allowed from the current experimental result. We found that the muon \( g-2 \) is significantly enhanced (of the order \( 100 \times 10^{-10} \)) in Region I (120 GeV \( < \mu < c_1 M_2 \), with \( c_1 > 1 \)) for \( \tan \beta = 50 \) and \( m_\tilde{\tau} = m_\tilde{E} = 200 \) GeV (see Fig. 1(a)). For the heavy slepton mass, \( m_\tilde{\tau} = m_\tilde{E} = 500 \) GeV, and \( \tan \beta = 50 \), Region II (100 GeV \( < M_2 < 150 \) GeV) appears in addition, where the predicted value of the muon \( g-2 \) is allowed in the current experiment but the MSSM correction is at most about \( 60 \times 10^{-10} \) in both Region I and II. (See Fig. 1(b).)

We also examined the dependence on \( m_\tilde{\tau} \) and \( m_\tilde{E} \) of those results without universality assumption. We found that \( m_\tilde{E} \)-dependence of the muon \( g-2 \) is larger than its target precision of the present experiment in the large \( \tan \beta \) case. Thus the muon \( g-2 \) complements the precision electroweak measurement which is
rather blind to $m_{\tilde{E}}$. It might be a unique possible indirect probe to give constraints on the right-handed slepton. When $m_{\tilde{E}} < m_{\tilde{L}}$, the MSSM with small tan $\beta$ is also accessible by the current $g - 2$ experiment (See Fig. 3(b)). The model falls in Region III (16a) or in Region IV (16b) on the $(\mu, M_2)$-plane for $\mu < 0$. It is also found that the MSSM contribution to the muon $g - 2$ is always affected by at most $5 \times 10^{-10}$ from the $A_\mu$-term. There is no essential change in the above results in the large tan $\beta$ case. For negative $A_\mu$ of the order 1 TeV, the MSSM with the small tan $\beta$ in Region I and II, rather than Region III and IV, can be probed by the muon $g - 2$ measurement. The detectable limit of the chargino at Tevatron Run-II is expected to be 250 GeV [18]. If the chargino is found at Tevatron, the muon $g - 2$ will enable us to select the parameters of the MSSM.

Here we remind the reader that the allowed range (5) for $\delta a_\mu$ is derived from (2) and (3). If the result in Ref. [19], which utilizes the accurate data for hadronic decay of $\tau$ (See Ref. [20] and the references therein) further with the help of quark-hadron duality in evaluation of dispersion integral, is applied instead of the result in Ref. [21], the leading order QCD correction involved in the above quantity receives $(8 \sim 9) \times 10^{-10}$ modification, resulting in

$$a_\mu^{(SM)} = 11659.160.05 \ (6.44) \times 10^{-10}.$$  \hfill (17)

This could then change the allowed range of $\delta a_\mu$ from (5) to

$$-4 < \delta a_\mu < 154.$$  \hfill (18)

In so far as we illustrate the tendency of the excluded region at large tan $\beta$ with the current experimental value (2), the difference between (5) and (18) does not matter. But such a difference will become crucial to find appropriate sets of $(\mu, M_2)$ when the experimental uncertainty is reduced to the level (1). This aspect is quite interesting from the viewpoint of searching the MSSM at large tan $\beta$. Thus the reduction of the error residing in QCD corrections to the muon $g - 2$ is a necessary task to be put forward promptly.

**Acknowledgment**

The authors thank S. Eidelman for the various useful comments on the manuscript.
References

[1] K. Hagiwara, Ann. Rev. Nucl. Part. Sci. 48 (1998) 463.

[2] The LEP Collaborations ALEPH, DELPHI, L3, OPAL, the LEP Electroweak Working Group and the SLD Heavy Flavor Group, CERN-EP/99-15.

[3] I. Riu, talk given at the XXXIVth Rencontres de Moriond, March 13-20, 1999.

[4] R. M. Carey et. al, Phys. Rev. Lett. 82 (1999) 1632.

[5] J. Ellis, J. Hagelin and D. V. Nanopoulos, Phys. Lett. B116 (1982) 283; J. A. Grifols and A. Mendez, Phys. Rev. D26 (1982) 1809; D. A. Kosower, L. M. Krauss and N. Sakai, Phys. Lett. B133 (1983) 305.

[6] U. Chattopadhyay and P. Nath, Phys. Rev. D53 (1996) 1648.

[7] J. L. Lopez, D. V. Nanopoulos and X. Wang, Phys. Rev. D49 (1994) 366; T. Moroi, Phys. Rev. D53 (1996) 6565; ibid. D56, 4424 (1997) (E); F. Borzumati, G. R. Farrar, N. Polonsky and S. Thomas, Nucl. Phys. B555 (1999) 53.

[8] M. Carena, G. F. Giudice and C. E. Wagner, Phys. Lett. B390 (1997) 234; E. Gabrielli and U. Sarid, Phys. Rev. Lett. 79 (1997) 4752; Phys. Rev. D58 (1998) 115003;

[9] M. Hayakawa and T. Kinoshita, Phys. Rev. D57 (1998) 465;

[10] G. C. Cho and K. Hagiwara, hep-ph/9912260 (1999), to appear in Nucl. Phys. B.

[11] G. C. Cho and K. Hagiwara, “The MSSM Larangian for MadGraph2”, in preparation.

[12] D. M. Pierce, J. A. Bagger, K. Matchev and R. Zhang, Nucl. Phys. B491 (1997) 3;
W. de Boer, A. Dabelstein, W. Hollik, W. Mosle and U. Schwickerath, Z. Phys. C75 (1997) 627;
P. H. Chankowski, J. Ellis and S. Pokorski, Phys. Lett. B423 (1998) 327;
J. Erler and D. M. Pierce, Nucl. Phys. B526 (1998) 53;
I. V. Gaidaenko, A. V. Novikov, V. A. Novikov, A. N. Rozanov and
M. I. Vysotsky, hep-ph/9812346.
A. Dedes, A. B. Lahanas and K. Tamvakis, Phys. Rev. D59 (1999) 015019;
G. C. Cho, K. Hagiwara, C. Kao and R. Szalapski, hep-ph/9901351.

[13] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, Nucl. Phys. B353 (1991) 591;
J. Hewett, Phys. Rev. Lett. 70 (1993) 1045;
V. Barger, M. Barger and R. J. N. Phillips, Phys. Rev. Lett. 70 (1993) 1368;
J. L. Lopez, D. V. Nanopoulos and G. T. Park, Phys. Rev. D48 (1993) 974;
N. Oshimo, Nucl. Phys. B404 (1993) 20;
F. M. Borzumati and F. Vissani, Z. Phys. C67 (1995) 513;
J. Wu, R. Arnowitt and P. Nath, Phys. Rev. D51 (1995) 1371;
J. L. Lopez, D. V. Nanopoulos, X. Wang and A. Zichichi, Phys. Rev. D51 (1995) 147.

[14] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964; Phys. Rev. D46 (1992) 381.

[15] K. Hagiwara, D. Haidt, C. S. Kim and S. Matsumoto, Z. Phys. C64 (1994) 559;
Z. Phys. C68 (1995) 352(E).

[16] S. Mihara, “Recent results from LEP-II”, talk given at KEK Theory Meeting
on Collider Physics, Jan 11-12, 1999 (Tsukuba, January 1999).

[17] A. Czarnecki, B. Krause and W. J. Marciano, Phys. Rev. Lett. 76 (1996) 3267.

[18] D. Amidei and R. Brock, “Future electroweak physics at the Fermilab Tevatron,
Report of the TeV2000 Study Group: Supersymmetric Physics”,
FERMILAB-PUB-96-046 (1996).

[19] M. Davier and A. Hocker, Phys. Lett. B435 (1998) 427.

[20] J. H. Kuhn, Nucl. Phys. Proc. Suppl. 76 (1999) 21.

[21] S. Eidelman and F. Jegerlehner, Z. Phys. C67 (1995) 585;
B. Krause, Phys. Lett. B390 (1997) 392;
R. Alemany, M. Davier and A. Hocker, Eur. Phys. J. C2 (1998) 123;