Concept selection and Kinematic modeling of hand rehabilitation robotic device

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Abstract. Rehabilitation using an exoskeleton is one of the widely accepted methods for the recovery of lost motor abilities. Most of the present research concentrates on either soft or rigid exoskeletons. In this paper, the design of a five-finger exoskeleton is discussed along with the advantage of soft and rigid bodied exoskeletons. Concept selection for the designs is done using the standard procedure of screening and scoring. Forward and inverse kinematic modeling of thumb and index finger is performed with Denavit-Hartenberg (DH) method.

1. Introduction

Every year, more than 15 million people are affected by stroke, which is the second leading cause of motor dysfunction after dementia. Several cerebrovascular diseases and neuromuscular disorders, such as stroke, epilepsy, dementia, paralysis, impaired motor control with symptoms ranging from spontaneous distracting gestures to inhibited movements. There are almost 9 million stroke survivors who typically face motor dysfunction. Movement disorders primarily decrease the quality of life of the patient, because the ability of a person to move is important to achieve essential daily living activities (ADL). Fortunately, there are many choices for restoring upper-hand mobility, including orthotics and physical rehabilitation. In the case of neurologically related disorders, the effectiveness of physical rehabilitation depends primarily on the onset, duration, strength, and task orientation of the training [1]. It also relies on the patient's health status and commitment to regain motor functions [2]. Intense repetitions of motor movements represent a major load on therapists during physical therapy. There seems to be a need for a robotic system that physically supports it.

The exoskeleton concept emerged in the late 19th century, but in the 1960’s, the prototype named Hardiman appeared [3]. Initially, Hardiman was designed for military purposes to increase the wearer's strength and performance. Exoskeleton research has improved dramatically since the 1990s, due to work proposed by the Berkeley Robotics Human Engineering Laboratory and other research groups [4]. Exoskeletons usually have both physical and cognitive human-robot interface (HRI) elements for physical assistive devices. It is therefore important to understand the biomechanics and neuroscience behind neuromuscular diseases and recovery. To design soft-rigid robotic based systems, understanding anatomy is also important. Robotic exoskeletons are structures used to recover the user's physical abilities [5].

There are two types of exoskeleton based on mechanical nature, i.e., rigid bodied exoskeleton and soft-bodied exoskeleton. Rigid body exoskeletons were robust and assisted the body stiffly [7]. In contrast with the soft-bodied exoskeleton, due to simpler system dynamics, the rigid bodied exoskeleton tolerates a simpler control system design [8,9]. This allows more complicated cases of deception performed with relative ease. For high force/torque transmission requirements, rigid bodied exoskeletons are useful. Rigid exoskeletons were hard, requiring high torque actuators to be combined

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with them and having a larger source of power [10,11]. Compared to rigid body exoskeletons, soft-bodied exoskeletons are smaller, less costly, and have less inertia. The limitation of the soft-bodied exoskeleton is that power transmission is complex and difficult. There is no rigid external frame for these systems, hence they depend on the skeleton framework of the consumer to provide the rigid frame. This makes reduced power requirements and allows for longer running times compared to rigid bodied exoskeletons with the same power source [12]. To overcome the limitation of completely soft-bodied systems, a possible middle ground having both rigid and soft parts is essential for an effective exoskeleton design as proposed by park et al. [13]. In the design of HANDEXOS [14], they had described the mechanical design of the index finger module along with dynamic modeling and some preliminary experimentation. Some misalignment related problem is also explained in the design. Hong kai et al. created a soft robotic glove and tested the efficiency of actuators as well as a glove for healthy participants in terms of kinetic and kinematic assistance. Besides, surface electromyography and radiofrequency recognition techniques were used to identify the user's intent to trigger or deactivate the glove [15]. A poly-articular tendon drive mechanism to mimic the mechanical compliance of human fingers was proposed by Hasegawa et al. They also proposed a new dual-sensing system mechanism and a new bioelectric potential-based switching control algorithm so that it could be easily used by the wearer [16]. Cempini et al. suggested a self-alignment design mechanism to absorb misplacement of human/robot joint axes and developed a novel thumb opposition mechanism. The kinematic design and actuation were addressed in the Hx exoskeleton [17] with theoretical and experimental validation of its efficiency. The PI control, which is based on the pressure value of the balloon sensor [18], is used to perform the power-assist motion. An embedded controller-dependent robotic hand module is defined by Tong et al. for each finger, it has 5 finger assemblies capable of driving 2 degrees of freedom (DOFs) at the same time. It is actuated by a linear actuator; at the metacarpophalangeal (MCP) joint, the finger assembly achieves a 55º range of motion (ROM) and 65º ROM at the proximal interphalangeal (PIP) joint. For opening-closing of hand, they have used electromyography (EMG) signals which helps in detecting the intention of the user [19]. The actuation systems in an exoskeleton are not inexpensive, particularly if the exoskeleton allows the actuator to be mounted at the user's joints, in which the actuator at one joint must endure the load of the actuator at the next joint, and so on. Researchers have solved this flaw by using cable transmission systems, which allow the exoskeleton to mount its actuator on a fixed foundation, reducing the weight of the device and lowering its price [20].

This paper introduces the design and development of a novel hand rehabilitation robotic system after considering the advantages and limitations of soft and rigid bodied exoskeletons by integrating both to achieve a positive result. This paper is structured as follows. Section 2 introduces conceptual designs for a hand rehabilitation robot. Section 3 explains concept selection and section 4 presents the design of the selected hand rehabilitation robotic device. Section 5 explains kinematic modeling, section 6 explains inverse kinematic modeling, and section 7 is the conclusion.

2. Conceptual designs for a hand rehabilitation robot

The conceptual design is a rough approximation of technology and its philosophies of operation [21]. It gives clarity in the design and helps to identify which technology will be suitable for its effective working. Some designs have been developed based on a literature survey by giving priority to the research gaps explained in the introduction. Figure 1 shows the developed concepts using solid works.

Concept 1 is a webbed design in which the supports at each joint are firm and lightweight, whereas concept 2 is highlighted with a very thin design which will be compactable for almost all possible thickness of different biomechanical dimensions of fingers. Concept 3 is having less surface contact with fingers and the joints are pinned. In concept 4 the design is based on a double-slotted mechanism which will help in adjusting the lengths between each finger joint. Concept 5 is a modified version of concept 3 in which some part modification and elimination of extra materials are done. Whereas concept 6 is a slotted mechanism that helps to adjust the link lengths of each joint, and it is having fewer material requirements compared to others.

Concepts 1, 2, and 3 are having 21 joints and 23 DOF whereas concepts 4, 5, and 6 consist of 17 joints and 21 DOF. As shown in figure 1, concept 01 is bulky while concept 02 is most compact compared to others.
To achieve smooth contact and fixing, a silicon fabric strip is planned to place between the exoskeleton and the human fingers which is also used to tie it. Here soft fabrics are not shown however they will be integrated into the later stage of fabrication. This will help to achieve the advantages of a soft-bodied exoskeleton. All the concepts comprise of revolute joints with their link arrangements different for each concept. It is planned to drive the different concepts using the belt and pulley mechanism.

![Conceptual designs](image)

**Figure 1.** Conceptual designs (concept 1, 2, 3, 4,5, and 6 are represented as a, b, c, d, e, and f respectively).

3. **Concept selection**

Concept selection is done in two steps i.e., concept screening and concept scoring. Both concept screening and concept scoring stages follow a six-step process. The first step deals with the preparation of the selection criteria. Then rating and ranking of the concepts followed by combining and improving the concepts. After that selection and reflection on the results and processes is carried out. From the developed concepts, the most suitable concept needs to be selected. The matrices are used in the idea selection process as visual guides for finding consensus among the team members who construct the concepts. Matrices correspond to the needs and other decision criteria, based on the requirements the best concept is selected through a certain standard procedure. Criteria selected for the concept scoring and screening are ease of handling, weight, aesthetics, ergonomics, ease of manufacture, Degrees of Freedom (DOF) and ease of actuation.

3.1. **Concept screening**

Screening is a simple, rough assessment aimed at generating a few reasonable options. Its evaluation is done using a screening matrix as a guide. In concept screening, the concepts inferior to standard reference is given as “-” sign, and concept superior are given as “+” sign, and concept equal to the reference is given as “0”. Then based on the net score ranking is given, the concept with the highest net score is the best design and the lowest net score is the worst design. While analyzing the net score it is visible that concept 2 is having the highest score followed by concept 6,1,3,5,4.
Table 1. Concept screening.

| Selection criteria          | Concept 01 | Concept 02 | Concept 03 | Concept 04 | Concept 05 | Concept 06 |
|-----------------------------|------------|------------|------------|------------|------------|------------|
| Ease of handling           | -          | +          | -          | -          | -          | 0          |
| Aesthetics                  | +          | +          | -          | -          | 0          | 0          |
| Ease of Manufacture         | +          | +          | 0          | -          | -          | 0          |
| Weight                      | -          | +          | 0          | -          | -          | 0          |
| Ergonomics                  | -          | 0          | -          | 0          | 0          | 0          |
| Degrees of freedom          | 0          | 0          | +          | 0          | 0          | 0          |
| Ease of actuation           | -          | -          | -          | 0          | 0          | 0          |
| Sum +’s                     | 2          | 4          | 1          | 0          | 0          | 0          |
| Sum 0’s                     | 1          | 2          | 2          | 2          | 4          | 8          |
| Sum −’s                     | 4          | 1          | 4          | 5          | 3          | 0          |
| Net score                   | -2         | 3          | -3         | -5         | -3         | 0          |
| Rank                        | III        | I          | IV         | VI         | V          | II         |
| Continue?                   | Yes        | Yes        | Yes        | No         | Yes        | Yes        |

( Modify )

Figure 2. Modified Concept 2.

Concept 2 is modified with optimized thickness. Small pulleys are incorporated at each joint such that the actuation of the system using tendon drive will be easier and more practical. For the proper seating, the design is modified and some hallow portions are given to reduce the weight as shown in figure 2.

3.2. Concept scoring

Scoring is also similar to screening, whereas it carefully analyses few concepts and selects the best one out of them. After concept screening concept 04 is rejected due to the lowest net score and the remaining concepts were undergone concept scoring. Weightage for each criterion is given based on the literature survey. The scores for each concept is obtained by multiplying the weightage and rating. The equation for weighted score is given as $S_j = \sum_{i=1}^{n} r_{ij} w_i$ where, $r_{ij}$ is the raw rating for concept j for the ith criterion, $w_i$ is the weighting for ith criterion, n is the number of criteria, $S_j$ is the total score for concept j. More weightage is given for the criteria such as ease of handling, weight, and ease of manufacture.
And rest of the criteria as given equal weightage. It is found that concept 2 having the highest net score. Hence it is screened as the best concept and selected for development.

**Table 2. Concept scoring.**

| Selection criteria | Weightage | Concept 01 Rating | Concept 02 Rating | Concept 03 Rating | Concept 05 Rating | Concept 06 Rating |
|--------------------|-----------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Ease of handling   | 20%       | 3.0               | 3.0               | 3.0               | 3.0               | 3.0               |
| Aesthetics         | 10%       | 4.0               | 4.0               | 3.5               | 3.5               | 4.0               |
| Ease of Manufacture| 20%       | 4.0               | 4.5               | 4.5               | 4.5               | 4.0               |
| Weight             | 20%       | 3.0               | 0.6               | 5.0               | 0.9               | 3.0               |
| Ergonomics         | 10%       | 3.5               | 0.35              | 3.5               | 0.35              | 3.5               |
| Degrees of freedom | 10%       | 4.0               | 4.5               | 0.45              | 3.0               | 0.3               |
| Ease of actuation  | 10%       | 3.0               | 0.3               | 3.0               | 0.3               | 3.5               |
|                   |           |                   |                   |                   |                   |                   |
| Net Score          | 3.45      | 4.1               | 3.3               | 3.2               | 3.8               |
| Rank               | NO        | Selected          | NO                | NO                | NO                |
| Continue?          | NO        | Selected          | NO                | NO                | NO                |

4. **Design of selected hand rehabilitation device**

This study mainly focuses on the upper limb rehabilitation of the five fingers in the right hand except for the wrist joint. The human hand consists of thumb, index, middle, ring, and little finger, in which the structure of four fingers except thumb is similar. The fingers are made up of digits, each digit is having two segments the phalangeal segment and the metacarpophalangeal segment. Thumb has one interphalangeal (IP) joint and a metacarpophalangeal (MCP) joint which has a flexion/extension motion. The selected concept 2 design is having 3 joints and 4 DOF for an index, middle, ring, and little finger. Whereas the thumb is having 3 joints and 4 DOF. Thumb differs from the other fingers due to the presence of an intermetacarpal joint. The human lower hand up to the wrist is mainly having 16 joints and 23 DOF.

In the selected model (Figure 3), the dimension of each link is done by considering the standard biomechanics data. The design of the thumb and index fingers has 4 DOF and 4 joints. Soft silicone strips are used to fix the rigid links over the fingers. The cable-driven mechanism is used to actuate the robot. Pulleys are provided at each joint to wound the cables over it, and cables are actuated by using servo motors hence they can get desired motions.

![Figure 3. Selected hand rehabilitation robotic device.](image-url)
4. Kinematic modeling

The use of the robot's kinematic equations to evaluate the end effector's position and orientation depending on the defined values of the joint parameters is known as forward kinematics. Forward kinematics is used to calculate the relationship between the pose of the end effector with respect to joint parameters. Here forward and inverse kinematics of thumb and index finger is explained in the following section.

4.1. Forward kinematic modeling of the thumb

The DH frame representation of thumb which is having the global frame \{0\} at the wrist joint is shown in figure 4. All the fingers are having local frames \{1\} and \{2\} at the metacarpal joint. The frame at the proximal joint is \{3\} and at the distal joint is \{4\}. Frame \{1\} and \{2\} at the metacarpal joint has the same origin as shown in figure 4. The inter metacarpal (IMCP) joint is represented as jt1 and jt2, the proximal and distal joints are represented as jt3 and jt4, respectively. Joint rotations are represented as \(\theta\) whereas \(\theta_{t2}, \theta_{t3}, \theta_{t4}\) and \(\theta_{t5}\) are joint rotation at jt1, jt2, jt3, and jt4 respectively.

![Figure 4. DH frame representation of Thumb.](image)

| Link | \(\theta\) | \(d\) | \(a\) (mm) | \(\alpha\) |
|------|----------|------|---------|------|
| 1    | \(\theta_{t1}=34.2^\circ\) (fixed) | 0    | 54.37   | 0\(^\circ\) | \(0_{T_{t1}}\) |
| 2    | \(\theta_{t2}\) | 0    | 0       | 90\(^\circ\) | \(1_{T_{t2}}\) |
| 3    | \(\theta_{t3}\) | 0    | 48.68   | 90\(^\circ\) | \(2_{T_{t3}}\) |
| 4    | \(\theta_{t4}\) | 0    | 15.88   | 0\(^\circ\) | \(3_{T_{t4}}\) |
| 5    | \(\theta_{t5}\) | 0    | 25.54   | 0\(^\circ\) | \(4_{T_{t5}}\) |

Based on biometric data [22] the distance between each joint is taken and it is as follows, the distance between the global and local frame is \(L1\) (54.37mm) and the distance between frame \{1\} and \{2\} is \(L3\) (48.68mm). Distance between frame \{3\} and \{4\} is \(L4\) (15.88mm) and the length of a distal digit is \(L5=25.54\)mm. Angle about the z-axis, from x0 to x1 at global frame is fixed and is \(\theta_{\text{thumb}}=34.25^\circ\)
which is taken as the initial position and it is measured from the Solidworks platform. DH parameter tabulated in table 3. Where “θ” is the angle about the previous z-axis, from the previous x to next x. “d” is offset along the previous z to common normal and where “a” is the length of common normal.

The final transformation matrix of thumb is represented as $0_{T_{t5}}$. It is calculated as the product of $0_{T_{t1}}$, $1_{T_{t2}}$, $2_{T_{t3}}$, $3_{T_{t4}}$ and $4_{T_{t5}}$ as shown in equation (1). $0_{T_{t1}}$ represents the transformation matrix of the first link and correspondingly $1_{T_{t2}}$, $2_{T_{t3}}$, $3_{T_{t4}}$ and $4_{T_{t5}}$ represents the transformation matrix of the remaining links.

$$0_{T_{t5}} = 0_{T_{t1}} \times 1_{T_{t2}} \times 2_{T_{t3}} \times 3_{T_{t4}} \times 4_{T_{t5}}$$  (1)

The general format of the final transformation matrix ($0_{T_{t5}}$) of thumb is represented in equation 2.

$$0_{T_{t5}} = \begin{bmatrix} a_{11}, & a_{12}, & a_{13}, & a_{14} \\ a_{21}, & a_{22}, & a_{23}, & a_{24} \\ a_{31}, & a_{32}, & a_{33}, & a_{34} \\ a_{41}, & a_{42}, & a_{43}, & a_{44} \end{bmatrix}$$  (2)

Where, $a_{11}$, $a_{12}$, $a_{13}$, $a_{14}$ are the elements of the final transformation matrix of thumb and their values given in appendix 1.

4.2. Forward kinematic modeling of the Index finger

DH model of the index finger is very much similar to that of the thumb. It is having one MCP joint, an IP joint, and a distal joint. MCP joint represented as ji0 and ji1, proximal and distal joints is represented as ji2 and ji3 respectively. $\theta_{i2}$, $\theta_{i3}$, $\theta_{i4}$ and $\theta_{i5}$ are joint rotation at ji0, ji1, ji2, and ji3 respectively. Figure 5 shows the DH frame representation of the index finger.

From the biometric data [22] the distance between each joint is taken and it is as follows, the distance between the global and local frame is L01 (106.47mm) and the distance between frame {1} and {2} is L03 (29mm). Distance between frame {3} and {4} is L04 (24mm) and the length of a distal digit is L05 (30mm). Angle about the z-axis, from x0 to x1 at global frame is fixed and is $\theta_{index1} = 13.98^\circ$ which is calculated from the Solidworks model. DH parameters are tabulated in table 4.

![Figure 6. DH frame representation of Index finger.](image-url)
The general end-effector pose is given by equation 5.

\[ AE = \begin{bmatrix} nx & ox & ax & px \\ ny & oy & ay & py \\ nz & oz & az & pz \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (5)

The final forward transformation matrix is given by equation 5. That is equated with equation 6 to find the joint angles. For the effective mathematical solving inverse of the transformation matrix of the first link \((O_T)_1^{-1}\) is multiplied with equation 1. The general equations for finding inverse kinematics are given as follows

\[
\begin{align*}
0_{T5} * (O_{T1})^{-1} & = 1_{T2} * 2_{T3} * 3_{T4} * 4_{T5} \\
0_{T5} * (O_{T1})^{-1} * (O_{T2})^{-1} & = 2_{T3} * 3_{T4} * 4_{T5} \\
0_{T5} * (O_{T1})^{-1} * (O_{T2})^{-1} * (O_{T3})^{-1} & = 3_{T4} * 4_{T5} \\
0_{T5} * (O_{T1})^{-1} * (O_{T2})^{-1} * (O_{T3})^{-1} * (O_{T4})^{-1} & = 4_{T5}
\end{align*}
\] (6)

After equating RHS and LHS elements of the set of equations (6), the inverse kinematics of thumb is calculated and given in appendix 3. Since all four fingers having similar design structures the inverse formulae were found to be the same. Whereas \(\theta_1\) values are different i.e., \(\theta_{index1} = 34.25\), \(\theta_{middle1} = 0\), \(\theta_{ring1} = -12.68\)º, and \(\theta_{little1} = -25.68\)º.

| Link | \(\Theta\) | \(d\) | \(a\) (mm) | \(\alpha\) |
|------|-----------|-------|------------|----------|
| 1    | \(\theta_{index1} = 13.98\)º (fixed) | 0     | 106.47     | 0º       | \(0_{T1}\) |
| 2    | \(\theta_{i2}\) | 0     | 0          | 90º      | \(1_{T2}\) |
| 3    | \(\theta_{i4}\) | 0     | 29         | 0º       | \(2_{T3}\) |
| 4    | \(\theta_{i5}\) | 0     | 24         | 0º       | \(3_{T4}\) |
| 5    | \(\theta_{i6}\) | 0     | 30         | 0º       | \(4_{T5}\) |

Since index, middle, ring, and little finger have similar design structures the DH parameter is found to be the same. The first joints of each finger are fixed at the local frame and their angle \(\theta_1\) is measured from the Solidworks model. Angles for each finger are \(\theta_{index1} = 13.98\)º, \(\theta_{middle1} = 0\)º, \(\theta_{ring1} = -12.68\)º, \(\theta_{little1} = -25.68\)º.

The final transformation matrix for the index finger \(0_{T15}\) is calculated from equation (4)

\[
0_{T15} = 0_{T1} * 1_{T2} * 2_{T3} * 3_{T4} * 4_{T5} \] (3)

\[
0_{T15} = \begin{bmatrix} b11, b12, b13, b14 \\ b21, b22, b23, b24 \\ b31, b32, b33, b34 \\ b41, b42, b43, b44 \end{bmatrix} \] (4)

Where, \(b11, b12, \ldots, b44\) are the elements of the final transformation matrix of an index finger, and their values are given in appendix 2.

5. Inverse Kinematic modeling

The inverse kinematics is done for finding the joint angles when the pose of the end-effector is known.

\[ AE = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \]
\[ \theta_{ring1} = -12.68, \quad \theta_{little1} = -25.68. \] Inverse formulae for the remaining four fingers are calculated and given in appendix 4.

6. Conclusion
A design for a hand exoskeleton robotic device for rehabilitation has been presented. The selection of the concept is done through concept screening and concept scoring for six conceptual designs. Forward and inverse kinematic analysis of the selected concept is performed by using Denavit-Hartenberg (DH) modeling. The use of lightweight materials like aluminum alloy and silicon strips will help the design framework to take the advantage of both soft and rigid structures. Actuation of the joints can be made possible with the help of tendon-driven cables which is pending future work. Several challenges are there in developing a fully equipped exoskeleton, the major challenge is to detect the intention of the user and act accordingly. Implementation of electromyography (EMG) or electroencephalogram (EEG) signal will help to detect the user intention up to a certain level.

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\[ a11 = \cos(\theta_1 + \theta_2) \] 
\[ a12 = \sin(\theta_1 + \theta_2) \] 
\[ a21 = \cos(\theta_3 + \theta_4 + \theta_5) \] 
\[ a22 = \sin(\theta_3 + \theta_4 + \theta_5) \]

\[ a31 = \cos(\theta_1 + \theta_2) \] 
\[ a32 = \sin(\theta_1 + \theta_2) \] 
\[ a41 = 1 \] 
\[ a42 = 0 \] 
\[ b23 = \cos(\theta_1 + \theta_2) \] 
\[ b24 = \sin(\theta_1 + \theta_2) \] 
\[ b33 = 0 \] 
\[ b34 = 0 \] 
\[ b43 = 0 \] 
\[ b44 = 1 \]
\[ \theta_{t3} = \tan^{-1}\left( \frac{ax}{\cos(\theta_{t1}+\theta_{t2})az} \right); \]  
\[ \theta_{t34} = \theta_{t3} + \theta_{t4} = \tan^{-1}\left( \frac{[(−ax\cos(\theta_{t1}+\theta_{t2}))−(ay\sin(\theta_{t1}+\theta_{t2}))]}{az} \right); \]  
\[ \theta_{t45} = \theta_{t4} + \theta_{t5} = \tan^{-1}\left( \frac{(ny\cos(\theta_{t1}−\theta_{t2}))−(nx\sin(\theta_{t1}−\theta_{t2}))}{(ay\cos(\theta_{t1}−\theta_{t2}))−(ox\sin(\theta_{t1}−\theta_{t2}))} \right); \]  
\[ \theta_{t4} = \theta_{t34} - \theta_{t3} = \tan^{-1}\left( \frac{[(−ax\cos(\theta_{t1}+\theta_{t2}))−(ay\sin(\theta_{t1}+\theta_{t2}))]}{az} \right), \quad \tan^{-1}\left( \frac{ax}{\cos(\theta_{t1}+\theta_{t2})az} \right); \]  
\[ \theta_{t5} = \theta_{t45} - \theta_{t4} = \tan^{-1}\left( \frac{(ny\cos(\theta_{t1}−\theta_{t2}))−(nx\sin(\theta_{t1}−\theta_{t2}))}{(ay\cos(\theta_{t1}−\theta_{t2}))−(ox\sin(\theta_{t1}−\theta_{t2}))} \right), \quad \tan^{-1}\left( \frac{ax}{\cos(\theta_{t1}+\theta_{t2})az} \right); \]  

**Appendix 4**

\[ \theta_2 = \tan^{-1}\left( \frac{-ax}{ay} \right); \]  
\[ \theta_3 = \tan^{-1}\left( \frac{nz}{oz} \right); \]  
\[ \theta_{34} = \theta_{t3} + \theta_{t4} = \tan^{-1}\left( \frac{[(−ay\sin(\theta_{t1}+\theta_{t2}))−(ax\cos(\theta_{t1}+\theta_{t2}))]}{az} \right); \]  
\[ \theta_{345} = \theta_{t3} + \theta_{t4} + \theta_{t5} = \tan^{-1}\left( \frac{[(−ax\cos(\theta_{t1}))−(nx\sin(\theta_{t1}))]}{(ny\cos(\theta_{t1}))−(ox\sin(\theta_{t1}))} \right); \]  
\[ \theta_{4} = \theta_{34} - \theta_{3} = \tan^{-1}\left( \frac{[(−ax\cos(\theta_{t1}))−(nx\sin(\theta_{t1}))]}{(ny\cos(\theta_{t1}))−(ox\sin(\theta_{t1}))} \right), \quad \tan^{-1}\left( \frac{nz}{oz} \right); \]  
\[ \theta_{5} = \theta_{345} - \theta_{34} = \tan^{-1}\left( \frac{[(−ay\sin(\theta_{t1}+\theta_{t2}))−(ax\cos(\theta_{t1}+\theta_{t2}))]}{(nz\cos(\theta_{t1}))−(ox\sin(\theta_{t1}))} \right), \quad \tan^{-1}\left( \frac{ax}{\cos(\theta_{t1}+\theta_{t2})az} \right); \]