Free Surface Behavior of a Liquid Metal under the Imposition of a High Frequency Magnetic Field

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(Received on April 4, 2005; accepted on April 22, 2005)

Free surface behavior of a liquid metal under the imposition of a high frequency magnetic field has been examined. The high frequency magnetic field was generated by imposing a high frequency current on a coil surrounding the vessel filled with a liquid gallium. The magnetic field penetrated into a liquid gallium only from an upper surface due to the shielding effect of the copper vessel. A shallow standing wave on a free surface of a gallium was generated by a mechanical oscillator and its behavior was measured by a laser level sensor. The amplitude of the standing wave decreases with increase in the intensity and the frequency of the magnetic field. A damping behavior of the standing wave just after stopping the mechanical oscillation was also examined. The larger the intensity and the frequency of the magnetic field are, the more quickly the wave decays.

KEY WORDS: magnetic field; perturbation; free surface; surface wave; liquid metal.
that the temperature variation was kept within a few Kelvins in all the runs. A standing wave was generated in the center section by use of a mechanical oscillator. Its half amplitude without a magnetic field, $A_0$, was about 3 mm. The wave motion was measured by using a laser level sensor in the vicinity of the copper thin plates where the loop of the standing wave was located. A power spectrum of the wave motion was obtained by use of a fast Fourier transform analyzer.

The intensities and frequencies of the magnetic field used in this experiment and the electromagnetic skin layers, defined by equation (1) are listed in Table 1.

$$\delta = \frac{1}{\mu \sigma f_m}$$  \hspace{1cm} (1)

In this equation, $\mu$ is a magnetic permeability, $\sigma$ is an electrical conductivity and $f_m$ is a frequency of a high frequency magnetic field. Two frequencies, 40 kHz and 450 kHz were adopted in this experiment. The amplitude of the standing wave excited by the mechanical oscillator under the no magnetic field condition was the same order with the electromagnetic skin layer of a liquid gallium in the case of 40 kHz, while it was much larger than the electromagnetic skin layer in the case of 450 kHz. The electromagnetic skin layer of a solid copper was much less than the thickness of the copper vessel (2 mm) in both the frequencies. Thus, the magnetic field can not penetrate into the liquid gallium from both the side walls and the bottom wall of the copper vessel, and it can penetrate from the upper free surface into the liquid gallium.

3. Influence of High Frequency Magnetic Field

3.1. Magnetic Field Distribution

The axial component of the magnetic flux density was measured along the axial direction of the coil at the level just above the liquid gallium surface. No difference between magnetic field profiles in 40 kHz and 450 kHz was observed. The sharp peaks of the magnetic field were observed at the positions of the thin copper plates set for dividing the vessel as shown in Fig. 2. Between the thin copper plates, the intensities of the magnetic field measured at the places corresponding to the coil tube appear about 15 percent larger than that at the places corresponding to the coil tube gap.

3.2. Perturbation on Metal Surface

Perturbation on the liquid gallium surface was induced by imposing the high frequency magnetic field under the condition without the mechanical oscillation. They were measured using the laser level sensor. The perturbation in the case without the magnetic field and with the magnetic field of 0.015 T, 450 kHz are shown in Fig. 3.
induced by the imposition of the magnetic field could be caused by the non-uniformity of the magnetic field distribution and/or the Joule loss. Averaged amplitude was evaluated under the different intensities of the magnetic field from the measured data. The result is shown in Fig. 4. Increasing the magnetic flux density increases the amplitude in both the frequencies, 40 kHz and 450 kHz. The largest amplitude of the perturbation observed in the experiment was less than 1mm.

3.3. Amplitude of Standing Wave

In all of the experiments, the amplitude of the standing wave under no magnetic field were set to be 3 mm. Then, the influence of the perturbations due to the non-uniformly distributed magnetic field and/or the Joule loss mentioned in the previous section can be ignored since the amplitude of the standing wave was much larger than that of the perturbations. A mechanical oscillator excited a standing wave in the center section of the vessel with the wave length of 200 mm, which was corresponding to a double length of the center section. The intrinsic frequency of the standing wave under no magnetic field was 2.5 Hz as shown in Fig. 5.

Based on the small amplitude wave theory, the frequency, has been formulated as a function of a wave length, as following:

\[ f = \frac{g}{2\pi\lambda} \left( \frac{2\pi\rho}{\lambda^2} \right) \tanh \left( \frac{2\pi d}{\lambda} \right) \]  

where \( d \) is a depth of liquid, \( g \) is acceleration of gravity, \( \gamma \) is surface tension and \( \rho \) is density, respectively. The first term in the right hand side bracket in this equation indicates the gravity term while the second term indicates the surface tension term. The critical wave length in which the gravity term equals with the surface tension term and corresponding frequency are calculated as 0.022 m and 8.46 Hz, respectively in this experimental condition. On the other hand, the ratio of the metal depth to the wave length, 0.15 is much less than unity. Then, the standing wave excited in this experiment is a gravity dominant shallow wave.

The surface wave motion under the different intensity of the magnetic field was measured. Typical experimental results of the 450 kHz magnetic field are shown in Fig. 6. The amplitude decreases by imposing the magnetic field. The relations between the intensity of the magnetic field and the amplitude of the standing wave are summarized in Fig. 7. The amplitude of the standing wave decreases with increase in the intensity of the magnetic field in the both frequencies and its tendency seen in 450 kHz is more intensive than that in 40 kHz. The suppression of the amplitude is not observed until the intensity of the magnetic field exceeds 0.015 T in the case of 40 kHz.

3.4. Decaying Behavior of Wave Motion

By stopping the mechanical oscillation, the standing wave motion decays due to not only the viscous loss in the liquid gallium and the friction between the vessel wall and the liquid gallium but also electromagnetic phenomena. The displacement from the equilibrium surface of a liquid metal, \( \eta \), which decays exponentially with oscillation is shown in Fig. 8, and can be expressed by

\[ \eta = A \exp(-Dt) \cos 2\pi ft \]  

where \( D \) is a decay constant and \( f \) is the frequency of the oscillation.
where $A$ is an amplitude of the standing wave before stopping the mechanical oscillation and $D$ is a damping constant, respectively. The damping constant $D$, in this experiment was evaluated from the slope of the envelope in the semilogarithmic plot of the wave amplitude versus the elapsed time after stopping the oscillator as shown in Fig. 9. The damping constant under no magnetic field, $D_0$, was estimated at $0.35 \text{ s}^{-1}$ as the averaged value in this experimental system. To evaluate the effect of the magnetic field on the wave damping, the difference between the damping constants with and without magnetic field is newly defined as an effective damping constant, in which the damping effects due to viscosity and friction are eliminated. The effective damping constant, $D-D_0$, increases with increase in magnetic flux density as shown in Fig. 10. The relationship between the non-dimensional damping constant, $D/D_0$ and the imposed magnetic field is shown in Fig. 11 with the approximate equations obtained by minimum mean square method. The damping constant is experimentally related with the magnetic field as following

$$D = D_0 \exp(cB), \quad c = 23 \text{ for } 450 \text{ kHz} \quad \text{and} \quad c = 21 \text{ for } 40 \text{ kHz}$$

The coefficient, $c$ in the approximate equation for 450 kHz magnetic field is slightly larger than that for 40 kHz magnetic field. That is, the damping effect of the magnetic field with 450 kHz is more intensive than that with 40 kHz. The results given in Figs. 7, 10 and 11 imply that a high frequency magnetic field has the function to damp the shallow wave motion.

4. Conclusion

It has been confirmed that a high frequency magnetic field has the function to suppress a shallow wave motion. The knowledge obtained in this experimental work is the followings.

(1) The high frequency magnetic field induced by a coil current introduces the small perturbations on a liquid metal surface due to a non-uniformity of a magnetic field distribution.

(2) The amplitude of the standing wave decreases with increase in the intensity and the frequency of a magnetic field.

(3) The damping constant increases with increase in the intensity and the frequency of a magnetic field.

Nomenclature

- $A$: Amplitude of a wave (m)
- $A_0$: Amplitude of a wave without magnetic field (m)
- $B$: Magnetic flux density (T)
- $c$: Coefficient in equation (4) (T$^{-1}$)
- $D$: Damping constant (s$^{-1}$)
- $D_0$: Damping constant under no magnetic field (s$^{-1}$)
- $d$: Depth of liquid metal (m)
- $f$: Frequency of standing wave (Hz)
- $f_m$: Frequency of magnetic field (Hz)
- $g$: Acceleration of gravity (m/s$^2$)
- $t$: Time (s)
- $w$: Thickness of Cu vessel (m)
δ: Electromagnetic skin layer (m)
η: Displacement from the equilibrium surface (m)
λ: Wave length (m)
μ: Magnetic permeability (H/m)
σ: Electrical conductivity (S/m)
γ: Surface tension (N/m)
ρ: Density (kg/m³)

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