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\( \Xi \)-nucleus potential for \( \Xi^- \) quasifree production in the \( ^9\text{Be}(K^-, K^+) \) reaction

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We study phenomenologically a \( \Xi^- \) production spectrum of the \( ^9\text{Be}(K^-, K^+) \) reaction at 1.8 GeV/c within the distorted-wave impulse approximation using the optimal Fermi-averaged \( K^- p \to K^+ \Xi^- \) amplitude. We attempt to clarify properties of a \( \Xi^- \)-nucleus potential for \( \Xi^-\Lambda\text{Li} \), comparing the calculated spectrum with the data of the BNL-E906 experiment. The results show a weak attraction in the \( \Xi^- \)-nucleus potential for \( \Xi^-\Lambda\text{Li} \), which can sufficiently explain the data in the \( \Xi^- \) quasifree region. The strength of \( V_{\Xi^-} = -17 \pm 6 \) MeV is favored within the Woods-Saxon potential, accompanied by the reasonable absorption of \( W_{\Xi^-} = -5 \) MeV for \( \Xi^- p \to \Xi^- n \) and \( \Xi^- p \to \Lambda\Lambda \) transitions in nuclear medium. It is difficult to determine the value of \( W_{\Xi^-} \) from the data due to the insufficient resolution of 14.7 MeV FWHM. The energy dependence of the Fermi-averaged \( K^- p \to K^+ \Xi^- \) amplitude is also confirmed by this analysis, and its importance in the nuclear \( (K^-, K^+) \) reaction is emphasized.

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I. INTRODUCTION

Recently, Nakazawa et al. [1] reported the first evidence of a bound state of the \( \Xi^-\text{He} \) system that was identified by the “KISO” event in the KEK-E373 experiment. This result supports that the \( \Xi^- \)-nucleus potential has a weak attraction of \( V_{\Xi^-} \simeq -14 \) MeV in the Woods-Saxon (WS) potential, as suggested by previous analyses [2–4]. However, there still remains an uncertainty about the nature of the \( S = -2 \) dynamics caused by \( \Xi^-N \) interaction and \( \Xi^-N-\Lambda\Lambda \) coupling in nuclei due to the limit to the available data. More experimental information is needed for the understanding of \( \Xi^- \) hypernuclei. Recently, Nagae et al. [5] have performed an accurate observation of the \( \Xi^- \) production spectrum in double-charge exchange reactions (\( K^- , K^+ \)) on \( ^{12}\text{C} \) targets at 1.8 GeV/c in the J-PARC E05 experiment, and their analysis is now ongoing. The double-charge exchange reactions such as \( (K^- , K^+) \) on nuclear targets produce neutron-rich \( \Xi^- \) hypernuclei, e.g., the neutron excess of \( (N - Z)/(N + Z) = 0.25 \) for a \( \Xi^-\text{Li} \) system, which is populated on \( ^9\text{Be} \). The behavior of \( \Xi^- \) in the neutron-excess environment is strongly connected with the nature of neutron stars [6] in which the baryon fraction is found to depend on properties of hypernuclear potentials [7].

Kohno [8] examined theoretically \( \Xi^- \) production spectra for the quasifree (QF) interaction region in the \( (K^- , K^+) \) reactions on \( ^9\text{Be} \) and \( ^{12}\text{C} \) targets in the semiclassical distorted-wave method, using the \( \Xi^- \)-nucleus potential derived from the next-to-leading order (NLO) in chiral effective field theory. However, it has shown that the calculated \( \Xi^- \) QF spectrum on \( ^9\text{Be} \) seems to be insufficient to reproduce the experimental data, so that quantitative information on the \( \Xi^- \)-nucleus potential for \( \Xi^-\Lambda\text{Li} \) (\( \Xi^-\text{He} \)) may be unreliable.

In this paper, we investigate phenomenologically the \( \Xi^- \) QF spectrum produced via the \( ^9\text{Be}(K^-, K^+) \) reaction at 1.8 GeV/c in order to extract valuable information on the \( \Xi^- \)-nucleus (optical) potential for the \( \Xi^-\Lambda\text{Li} \) system from the data of the BNL-E906 experiment [8,9]. We attempt to clarify properties of the \( \Xi^- \)-nucleus potential for \( \Xi^-\Lambda\text{Li} \) and to understand a mechanism of the \( \Xi^- \) QF spectrum in comparison with the data [9]. Thus we demonstrate the calculated \( \Xi^- \) QF spectrum in the \( ^9\text{Be}(K^-, K^+) \) reaction within the distorted-wave impulse approximation (DWIA), taking into account the energy dependence of the Fermi-averaged \( K^- p \to K^+ \Xi^- \) amplitude in the optimal Fermi-averaging procedure [10,11].

II. CALCULATIONS

A. Distorted-wave impulse approximation

Let us consider production of \( \Xi^- \) hypernuclei in the nuclear \( (K^- , K^+) \) reaction. According to the Green’s function method [12] in the DWIA, an inclusive \( K^+ \) double-differential laboratory cross section of the \( \Xi^- \) production on a nuclear target with a spin \( J_A \) (its \( z \) component \( M_A \)) [13–15] is given by

\[
\frac{d^2\sigma}{d\Omega dE} = \frac{1}{[J_A]} \sum_{M_A} S(E),
\]

where \([J_A] = 2J_A + 1\). The strength function \( S(E) \) is written as

\[
S(E) = -\frac{1}{\pi} \text{Im} \sum_{a\alpha'} \int dr dr' F_{\Xi}^{a\alpha'}(r) G_{\Xi}^{a\alpha'}(E; r, r') F_{\Xi}^{a\alpha'}(r'),
\]
where \( G_{\Sigma}^{\omega} \) is a complete Green’s function for a \( \Sigma \) hypernuclear system, \( F_{\Sigma}^{\omega} \) is a \( \Sigma \) production amplitude defined by

\[
F_{\Sigma}^{\omega} = \beta^2 \bar{f}_{K^{-p}\rightarrow K^{+}Z^{-}} \chi_{pK^{-}}^{-}(\alpha \mid \tilde{\psi}_{p} \mid \Psi_{\Lambda}),
\]

and \( \alpha (\alpha') \) denotes the complete set of eigenstates for the system. The kinematical factor \( \beta \) denotes the transition from a two-body \( K^{-}p \) laboratory system to a \( K^{-} \)-nucleus laboratory system. \( \bar{f}_{K^{-p}\rightarrow K^{+}Z^{-}} \) is a Fermi’s factor in nuclear medium [11,14,15]. To reduce ambiguities in the distorted results of the s.p. energies of the nucleons and the root-mean-square potential from the data.

Recently, the authors [10] have found the strong energy dependence for the structure of \( K^{-} \)-nucleus final states are obtained by solving the light nuclear system [18], leading to an effective momentum transfer \( q \approx 390–600 \text{ MeV}/c \) in the nuclear \( (K^{-}, K^{+}) \) reaction for \( K^{+} \) forward-direction angles of \( \theta_{ab} = 1.5^\circ–8.5^\circ \) at \( \sqrt{s} = 1.8 \text{ GeV}/c \), we simplify the computational procedure for \( \chi_{pK^{-}}^{-} \) and \( \chi_{pK^{-}}^{+} \), using the eikonal approximation [15]. To reduce ambiguities in the distorted waves, we adopt the same parameters used in calculations for the \( \Lambda \) and \( \Sigma^{-} \) QF spectra in nuclear (\( \pi^{\pm}, K^{\mp} \)) and \( (K^{-}, \pi^{\mp}) \) reactions [11,16,17]. Here we used the total cross sections of \( \sigma_{K^{-}} = 28.9 \text{ mb} \) for the \( K^{-} \)N scattering and \( \sigma_{K^{+}} = 19.4 \text{ mb} \) for the \( K^{+} \)N scattering, and \( \alpha_{K^{-}} = \alpha_{K^{+}} = 0 \), as the distortion parameters. We also take into account the recoil effects, which are very important to estimate the hypernuclear production cross section for a light nuclear system [18], leading to an effective momentum transfer having \( q_{\text{eff}} \approx (1 - 1/A)q \approx 0.80q \) for \( A = 9 \).

Recently, the authors [10] have found the strong energy dependence for the \( K^{-}p \rightarrow K^{+}Z^{-} \) reaction in the nuclear medium, together with the angular dependence for \( \theta_{ab} \). Therefore, we emphasize that such behavior of \( \bar{f}_{K^{-p}\rightarrow K^{+}Z^{-}} \) plays a significant role in explaining the shape of the spectrum in the nuclear \( (K^{-}, K^{+}) \) reaction [10] as well as those in the nuclear \((\pi^{\pm}, K^{\mp})\) reactions [11,16,17]. Because \( \bar{f}_{K^{-p}\rightarrow K^{+}Z^{-}} \), modifies the spectral shape including the \( \Sigma^{-} \) QF region widely, one must extract carefully information concerning the \( \Sigma \)-nuclear potential from the data.

### III. \( \Sigma \)-Nucleus Potential

The \( \Sigma \)-nucleus final states are obtained by solving the Schrödinger equation

\[
\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + U_{\Sigma}(r) + U_{\text{Coul}}(r) \right] \Psi_{\Sigma} = E \Psi_{\Sigma},
\]

where \( \mu \) is the \( \Sigma \)-nucleus reduced mass, \( U_{\Sigma} \) is the \( \Sigma \)-nucleus potential, and \( U_{\text{Coul}} \) is the Coulomb potential. The \( \Sigma \)-nucleus potential for \( \Sigma^{-} \rightarrow ^{\Lambda} \) is given by

\[
U_{\Sigma}(r) = V_{\Sigma}(r) + iW_{\Sigma}(E, r) = \left( V_{0}^{\Sigma} + iW_{\Sigma}(g(E)) \right) f(r),
\]

with the assumption of the WS form

\[
\begin{align*}
 f(r) &= \left[ 1 + \exp \left( \frac{(r - R + a)}{\alpha} \right) \right]^{-1},
\end{align*}
\]

where \( R = r_{01/2}^{\alpha/4} \) and \( a \) denote a radius and a diffuseness of the potential, respectively. \( V_{0}^{\Sigma} \) is a strength parameter for the real part of the potential; \( W_{\Sigma}^{\alpha} \) is a strength parameter for the imaginary part of the potential, which denotes the \( \Sigma^{-} \) absorption processes including the \( \Sigma^{-} \rightarrow \Sigma^{0}n \) and \( \Sigma^{-} \rightarrow \Lambda \) reactions. \( g(E) \) is an energy-dependent function that increases linearly from 0.0 at \( E = E_{\text{in}}(\Lambda) \) to 1.0 at \( E = 20 \text{ MeV} \) with respect to the \( \Sigma^{-} \) threshold, as often used in nuclear optical models [24], where \( E_{\text{in}}(\Lambda) = -23.3 \text{ MeV} \) corresponds to the \( \Lambda \) emitted threshold.

The ground state of \( ^{\Lambda} \rightarrow ^{\Lambda} \) is a bound state at the neutron binding energy of \( B_{n} = 2.03 \text{ MeV} \) with respect to the
The parameters of the resultant WS form in Eq. (6) are \( r_0 = 0.783 \) fm, \( a = 0.722 \) fm, and \( R = r_0A_{\text{core}}^{1/3} = 1.57 \) fm, which reproduce the radial shape of the form factor very well; the rms radius of the potential denotes

\[
\langle r^2 \rangle_{V_z}^{1/2} = \left[ \frac{1}{2} \int r^2 V_z(r)dr \right]^{1/2} = 2.81 \text{ fm.} \tag{7}
\]

On the other hand, the spreading imaginary parts of \( W_0^\Sigma \) may represent complicated continuum states of \( \Lambda_\Lambda \Lambda \), \( 9^g \Lambda \), \( ^9\Lambda \), and \( ^9\Lambda \). Considering the states of \( ^8\Lambda \) and \( ^{10}\Lambda \), we choose carefully the MHO size parameters of \( b_r = 1.42 \pm 0.06 \) fm and \( b_p = 1.95 \pm 0.06 \) fm with center-of-mass and nucleon-size corrections, adjusting the matter rms radius of \( \langle r^2 \rangle_{m}^{1/2} = 2.39 \pm 0.06 \) fm [25]. Following the procedure in Ref. [17], we use the WS form with the parameters of \( r_0 \) and \( a \) adjusted to give a best least-squares fit to the radial shape of the form factor obtained by folding a Gaussian range of \( \Delta r = 1.2 \) fm into the matter MHO density distribution [17]. The parameters of the resultant WS form in Eq. (6) were obtained by

\[
\langle r^2 \rangle_{m}^{1/2} = 2.39 \text{ fm.} \tag{7}
\]

We attempt to determine the strength parameters of \( V_0^\Sigma \) and \( W_0^\Sigma \) in Eq. (5) phenomenologically in comparison with the data of the \( ^7\Lambda (K^-, K^+) \) reaction. Figure 1 shows the real and imaginary parts of the \( \Sigma \) nuclear potential for \( \Sigma^{-}8\text{Li} \), choosing the reasonable strengths of \( V_0^\Sigma = -17 \) MeV and \( W_0^\Sigma = -5 \) MeV, as we discuss in Sec. IV.

**IV. RESULTS**

\[ \chi^2 \] fitting

Tamagawa et al. (BNL-E906 Collaboration) reported the experimental data of the \( \Sigma^{-}8\text{Li} \) QF spectra for the \( K^+ \) forward-direction angles of \( \theta_{\text{lab}} = 1.5^\circ -8.5^\circ \) in the \( ^7\Lambda (K^{-}, K^{+}) \) reactions at the incident \( K^- \) momentum of \( p_K^- = 1.8 \) GeV/c [9]. The average cross section \( \sigma_{\text{lab}}(1^\circ -8.5^\circ) \) in the laboratory frame was obtained by

\[
\sigma_{\text{lab}}(1^\circ -8.5^\circ) \equiv \int_{\theta_{\text{lab}} = 1.5^\circ}^{8.5^\circ} \frac{d^2 \sigma}{dp_K^-d\Omega} d\Omega \int_{\theta_{\text{lab}} = 1.5^\circ}^{8.5^\circ} d\Omega, \tag{8}
\]

with the detector resolution of 14.7 MeV full width at half maximum (FWHM) [9]. The strength parameters of \( V_0^\Sigma \) and \( W_0^\Sigma \) in Eq. (5) should be adjusted appropriately to reproduce the data of \( \sigma_{\text{lab}}(1^\circ -8.5^\circ) \).

We consider the \( \Sigma^{-} \) QF spectrum for \( \Sigma^{-}8\text{Li} \) hypernuclear states with \( J^P = 1/2^- \) and \( T_B = 3/2 \), using the Green’s function method [12], in order to be compared with the data of the \( ^7\Lambda (K^{-}, K^{+}) \) reaction at the BNL-E906 experiment [9]. Calculating the spectra for \( \theta_{\text{lab}} = 1.5^\circ -8.5^\circ \), we estimate the average cross section for the corresponding \( \sigma_{\text{lab}}(1.5^\circ -8.5^\circ) \) in Eq. (8). To make a fit to the spectral shape of the data, we introduce a renormalization factor of \( f_s \) into the absolute value of the calculated spectrum because the eikonal distortion and the amplitude of \( \int f_{K^-\Sigma^-} d\Omega \) would have some ambiguities [10,15]. The detector resolution of 14.7 MeV FWHM is also taken into account. We obtain the values of \( \chi^2 \) for fits to the data points of \( N = 17 \) in \( p_K^- = 1.07 -1.39 \) GeV/c, varying the strengths of \( (V_0^\Sigma, W_0^\Sigma) \) and \( f_s \); we assume the value of 0.018 \( \mu b/(\text{sr MeV} c^{-1}) \) as a constant background. Thus we estimate the average cross section in Eq. (8), calculating the spectra for \( \theta_{\text{lab}} = 1.5^\circ -8.5^\circ \) in the parameter region of \( V_0^\Sigma = (-36)--(18) \) MeV by a 6-MeV energy step and \( W_0^\Sigma = (-10)--0 \) MeV by a 2-MeV energy step. The 1-MeV energy step is taken in the estimation near the \( \chi^2_{\min} \) point.

Figure 2 displays the contour plots of \( \chi^2 \)-value distributions for \( \sigma_{\text{lab}}(1.5^\circ -8.5^\circ) \). The minimum value of \( \chi^2 \) is found to be \( \chi^2_{\min} = 15.2 \) at \( V_0^\Sigma = -17 \) MeV, \( W_0^\Sigma = -5 \) MeV, and \( f_s = 0.940 \), leading to beltlike regions of \( \Delta \chi^2 = 2.30, 4.61, \) and 9.21, which correspond to 68%, 90%, and 99% confidence.
levels for two parameters, respectively, where \( \Delta \chi^2 = \chi^2 - \chi_{\text{min}}^2 \). We find that the value of \( \chi^2 \) is almost insensitive to \( W_0^{\Xi} \). This fact implies that the parameter of \( W_0^{\Xi} \) cannot be determined from the BNL-E906 data due to the insufficient resolution of 14.7 MeV FWHM. Nevertheless, we recognize that the calculated spectrum for \( V_0^{\Xi} \approx -17 \) MeV seems to be in good agreement with the data when \( W_0^{\Xi} \approx -5 \) MeV; it gives the minimum value of \( \chi^2/N = 15.2/17 = 0.89 \) and the standard deviation of \( \sigma \approx 6 \) MeV. In Table I, we list the reduced \( \chi^2 \) values of \( \chi^2/N \) in calculations when \( V_0^{\Xi} = -30 \), \(-24 \), \(-18 \), \(-12 \), \(-6 \), 0, and \(+12 \) MeV, and \( W_0^{\Xi} = -10 \), \(-5 \), and 0 MeV, comparing the calculated spectra with the data. Note that the absolute values of the calculated cross section can explain the magnitude of the data, as seen by \( f_i \approx 0.9 \)–1.0.

Figure 3 shows the absolute values of the calculated spectrum for \( \delta_{1.5-8.5}^{17} \) in the best-fit calculation and compares them with the data of the BNL-E906 experiment at \( p_{K^+} = 1.07-1.39 \) GeV/c. We recognize that an attraction in the \( \Xi^- - \bar{\Xi} \) potential is needed to reproduce the data. The contribution of \( p \)-hole configurations is larger than that of \( s \)-hole configurations in the \( \Xi^- - \bar{\Xi} \) QF region of \( p_{K^+} = 1.2-1.4 \) GeV/c, whereas the former is similar to the latter in the region of \( p_{K^+} < 1.2 \) GeV/c where the recoil momentum grows into \( q > 540 \) MeV/c. Consequently, we confirm that the \( \Xi \) potential for \( \Xi^- - \bar{\Xi} \) has a weak attraction in the real part of the WS potential with \( r_0 = 0.738 \) fm and \( a = 0.722 \) fm:

\[
V_0^{\Xi} = -17 \pm 6 \text{ MeV} \quad \text{for} \quad W_0^{\Xi} = -5 \text{ MeV}.
\]

This potential provides the ability to explain the \( ^9 \text{Be}(K^-, K^+) \) data at the BNL-E906 experiment. Several authors [2,3] have

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\( V_0^{\Xi} \) & \( W_0^{\Xi} \) & \( N = 17 \) data points & \( \chi^2/N \) & \( f_i \) \\
(MeV) & (MeV) & & & \\
\hline
+12 & 0 & 69.8/17 & 0.988 \\
0 & 0 & 37.6/17 & 0.964 \\
-6 & 0 & 26.4/17 & 0.951 \\
-12 & 0 & 18.9/17 & 0.939 \\
-18 & 0 & 15.6/17 & 0.927 \\
-24 & 0 & 16.8/17 & 0.914 \\
-30 & 0 & 22.8/17 & 0.902 \\
+12 & -5 & 58.3/17 & 0.999 \\
0 & -5 & 30.7/17 & 0.975 \\
-6 & -5 & 21.8/17 & 0.962 \\
-12 & -5 & 16.5/17 & 0.950 \\
-17 & -5 & 15.2/17 & 0.940 \\
-18 & -5 & 15.3/17 & 0.938 \\
-24 & -5 & 18.4/17 & 0.925 \\
-30 & -5 & 26.1/17 & 0.913 \\
+12 & -10 & 49.0/17 & 1.010 \\
0 & -10 & 25.7/17 & 0.985 \\
-6 & -10 & 18.9/17 & 0.973 \\
-12 & -10 & 15.6/17 & 0.961 \\
-18 & -10 & 16.3/17 & 0.948 \\
-24 & -10 & 21.1/17 & 0.936 \\
-30 & -10 & 30.4/17 & 0.923 \\
\hline
\end{tabular}
\caption{The \( \chi^2 \)-fitting for various strength parameters, \( V_0^{\Xi} \) and \( W_0^{\Xi} \), in the WS potential with \( r_0 = 0.738 \) fm and \( a = 0.722 \) fm for \( \Xi^- - \bar{\Xi} \). The value of \( \chi^2/N \) and the renormalization factor \( f_i \) are obtained by comparing the calculated spectrum with the data of the \( \chi^2 \) values of the average cross sections of \( \delta_{1.5-8.5}^{17} \) for \( p_{K^+} = 1.07-1.39 \) GeV/c. The data were taken from Ref. [9].}
\end{table}
TABLE II. Binding energies $B_{\Xi^-}$ and widths $\Gamma_{\Xi^-}$ of the $\Xi^-$-nucleus ($n\ell$) bound states for $\Xi^-\cdot 3\text{Li}$ ($\Xi^-\cdot \text{He}$). The strengths of $V_0^{\Xi^-} = -17$ MeV and $W_0^{\Xi^-} = -2.5 (-1.5)$ MeV are used in the WS potential for the $\Xi^-$ bound region. These values are estimated in combination with the $\Xi$-nucleus potential $U_{\Xi} = V_{\Xi} + iW_{\Xi}$ and the Coulomb potential $U_{\text{Coul}}$.

| $(n\ell)$ | $V_{\Xi} + U_{\text{Coul}} + iW_{\Xi}$ | $V_{\Xi} + U_{\text{Coul}}$ | $V_{\Xi}$ | $U_{\text{Coul}}$ |
|-----------|---------------------------------|----------------|----------------|----------------|
|           | $-B_{\Xi^-}$ (MeV) | $\Gamma_{\Xi^-}$ (MeV) | $\text{rms}$ | $-B_{\Xi^-}$ (MeV) | $\text{rms}$ | $-B_{\Xi^-}$ (MeV) | $\text{rms}$ | $-B_{\Xi^-}$ (MeV) | $\text{rms}$ |
| $W_0^{\Xi^-} = -2.5$ MeV |                      |                    |                 |                  | |
| $1S$      | $-1.851$               | $1.118$            | $3.95$          | $-1.897$         | $3.96$       | $-0.475$         | $5.56$       | $-0.255$         | $14.6$    |
| $2S$      | $-0.122$               | $1.3 \times 10^{-2}$ | $29.3$         | $-0.122$         | $29.5$       | —                | —           | $-0.066$         | $53.4$    |
| $2P$      | $-0.068$               | $3.1 \times 10^{-4}$ | $43.4$          | $-0.068$         | $43.4$       | —                | —           | $-0.067$         | $44.0$    |
| $W_0^{\Xi^-} = -1.5$ MeV |                      |                    |                 |                  | |
| $1S$      | $-1.880$               | $0.669$            | $3.95$          | $-1.897$         | $3.96$       | $-0.475$         | $5.56$       | $-0.255$         | $14.6$    |
| $2S$      | $-0.122$               | $8.0 \times 10^{-3}$ | $29.4$         | $-0.122$         | $29.5$       | —                | —           | $-0.066$         | $53.4$    |
| $2P$      | $-0.068$               | $1.9 \times 10^{-4}$ | $43.4$          | $-0.068$         | $43.4$       | —                | —           | $-0.067$         | $44.0$    |

attempted to determine the values of $V_0^{\Xi^-}$ for fits to the shape and magnitude of the $\Xi^-$ QF spectra from the data of the $^3\text{C}(K^-, K^+)\text{He}$ reaction [3]. They have suggested that the $\Xi$-nucleus potential has a weak attraction of $V_0^{\Xi^-} \simeq -14$ MeV in the WS potential. It is shown that the results of Eq. (9) in our analysis are considerably consistent with the results of the previous studies [2,3].

### B. $\Xi^-$-nucleus bound states

In Table II, we show the numerical results of binding energies and widths of the $\Xi^-$-nucleus ($n\ell$) bound states for $\Xi^-\cdot 3\text{Li}$, where $(n\ell)$ denotes the principal and angular momentum quantum numbers for the relative motion between $\Xi^-$ and $^3\text{Li}$. By solving the Schrödinger equation of Eq. (4) with the WS potential $U_{\Xi}$ and the finite Coulomb potential $U_{\text{Coul}}$, we obtain a complex eigenvalue as a Gamow state,

$$E_{\text{nt}} = -B_{\Xi^-} - i\frac{\Gamma_{\Xi^-}}{2},$$

where $B_{\Xi^-}$ and $\Gamma_{\Xi^-}$ denote a binding energy and a width of the bound state, respectively. When we use $V_0^{\Xi^-} = -17$ MeV in the WS potential, we confirm that there exists a very shallow $\Xi^-$ $(1S)$ bound state due to the weak attraction in the $\Xi$-nucleus potential even if the Coulomb potential is switched off; the binding energy accounts for $B_{\Xi^-}(1S) = 0.475$ MeV and the rms radius of $(r^2)^{1/2} = 5.56$ fm. When the Coulomb potential is switched on, the binding energy is significantly shifted downward in comparison with the corresponding Coulomb eigenstate, as seen in Table II. Thus this state is often regarded as a “Coulomb-assisted” $\Xi^-$-nucleus bound state; $B_{\Xi^-}(1S) = 1.897$ MeV and $(r^2)^{1/2} = 3.96$ fm.

A $\Xi^-$ hyperon bound state in nuclei must be absorbed by strong interaction via the $\Xi^- p \rightarrow \Lambda \Lambda$ conversion process. To estimate the width of the $\Xi^-$ bound state, we assume the value of $W_0^{\Xi^-} = -2.5$ MeV, which corresponds to the strength of $W_2(E)$ at the $\Xi^-$ threshold ($E = 0.0$ MeV). Thus we obtain the width of $\Gamma_{\Xi^-}(1S) = 1.118$ MeV, together with $B_{\Xi^-}(1S) = 1.851$ MeV. When we use $W_0^{\Xi^-} = -1.5$ MeV arising from the $\Xi^- p \rightarrow \Lambda \Lambda$ conversion in the $\Xi N$ NLO potential [27,28], we obtain $\Gamma_{\Xi^-}(1S) = 0.669$ MeV and $B_{\Xi^-}(1S) = 1.880$ MeV (see also Sec. V B).

### V. DISCUSSION

#### A. Effects of the real part of the $\Xi$-nucleus potential

To see effects of the attraction in the $\Xi$-nucleus potential for $\Xi^-\cdot 3\text{Li}$, we discuss the shapes and magnitudes of the calculated spectra. Figure 4 shows the absolute values of the calculated spectra for $\sigma_{1.5-8.5}$ in the $\Xi^-$ QF region, using various strengths of $V_0^{\Xi^-}$. We find that the shape and the magnitude of the calculated spectrum are considerably sensitive to the value of $V_0^{\Xi^-}$. This confirms that the value of $\chi^2/N$ is significantly changed by $V_0^{\Xi^-}$. The peak position of the QF spectrum is scarcely shifted downward for $p_{K^+}$, as $V_0^{\Xi^-}$ changes from $-24$ to $+12$ MeV, whereas the magnitude of this peak is slightly reduced by $7.4\%$. There is a difference between the spectra of $V_0^{\Xi^-} = (-24)$–(-12) MeV in the momentum

![Figure 4](https://example.com/figure4.png)

FIG. 4. Shapes and magnitudes of the calculated spectra for $\sigma_{1.5-8.5}$ in the $^9\text{Be}(K^-, K^+)\text{He}$ reaction at $p_{K^+} = 1.80$ GeV/c, depending on the strengths of $V_0^{\Xi^-} = -24$, $-12$, $0$, and $+12$ MeV in the WS potential with $W_0^{\Xi^-} = -5$ MeV. The spectra are folded with a detector resolution of 14.7 MeV FWHM.
region of $p_{K^+} > 1.2$ GeV/c, corresponding to the region of lower energies $E < 140$ MeV. On the other hand, the shapes and magnitudes of the spectra with $V_0^\Sigma = (-24) + (12)$ MeV become similar to each other in the region of higher energies $E > 140$ MeV ($p_{K^+} < 1.2$ GeV/c).

**B. Validity of the imaginary part of the $\Sigma$-nucleus potential**

In Sec. IV A, we have found that the shapes and magnitudes of the calculated spectra are not so sensitive to the value of $W_0^\Sigma$ when we change $W_0^\Sigma = (-10) - 0$ MeV in the imaginary part of the $\Sigma$-nucleus potential. This is because a mask of $W_0^\Sigma$ is inevitable due to the insufficient resolution of 14.7 MeV FWHM. Thus we recognize that it is difficult to determine the value of $W_0^\Sigma$.

According to the procedure of Gal, Toker, and Alexander [29], we examine theoretically an appropriate parameter for $W_0^\Sigma$ from a viewpoint of the first-order optical $(\rho \sigma)$ potential,

$$U^{(1)}(r) = t_{Z - p} \rho_\sigma(r) + t_{Z - n} \rho_n(r),$$

in terms of the effective two-body $\Sigma N$ elastic $t_{ZN}$ scattering matrices in the laboratory frame, where $\rho_{\sigma,n}(r)$ are the proton and neutron densities of the core nucleus. By the optical theorem $4\pi \text{Im} f_{ZN}(0) = k_\Sigma^2 \sigma_{\text{tot}}$ and considering collisions of zero energy $\Sigma$ with bound nucleons, we obtain the imaginary part $W_{Z}^{(1)}$ of the optical potential involving the $\Sigma^- p \rightarrow \Sigma^0 n$ and $\Sigma^- p \rightarrow \Lambda \Lambda$ conversions, which is given by

$$W_{Z}^{(1)}(r) = -(v_{Z - p}\sigma(\Sigma^- p \rightarrow \Sigma^0 n, \Lambda \Lambda)\rho_\sigma(r)/2,$$

where $v$ is the relative velocity of a $\Sigma^- p$ pair, and $(\cdots)$ indicates nuclear medium corrections to the free space value of $\sigma$ arising from Fermi averaging, binding effects, and the Pauli principle, etc. The cross section is well approximated up to 300 MeV/c in the laboratory system by the form

$$v_{Z - p}\sigma = (v_{Z - p}\sigma)_{0}/(1 + \alpha v),$$

with the two representative parametrizations of $(v_{Z - p}\sigma)_{0} = 25$ mb and $\alpha = 18$ for the $\Sigma^- p \rightarrow \Sigma^0 n$ and $\Sigma^- p \rightarrow \Lambda \Lambda$ reactions, fitting to $v_{Z - p}\sigma$, which are given by the $\Sigma N$ NLO potential [27,28]. Taking into account the closure assumption and nuclear medium corrections [29], we obtain $(v_{Z - p}\sigma) = 7.02$ mb within the Fermi gas model. Using the relation between $(\sigma\rho)$ and Im$b$, where $b$ is the effective parameter of a complex scattering length for $\Sigma^- p$, we roughly estimate

$$\text{Im} b = \mu(\sigma\rho)/8\pi = 0.078 \text{ fm},$$

of which the value corresponds to $W_0^\Sigma = -6.2$ MeV in the WS potential. We find that this value is similar to $W_0^\Sigma = -5$ rm MeV for the minimum value of $\chi_{\text{min}}$, as shown in Fig. 2. If we replace the momentum distributions of the Fermi gas model by those of the s.p. shell model for the finite nuclei, the results may not change. Therefore, we believe that the $\Sigma$-nucleus potential with $V_0^\Sigma = -17$ MeV and $W_0^\Sigma = -5$ MeV is appropriate to the study of the $\Sigma^- QF$ spectrum in the $^9\text{Be}(K^+, K^+)$ reaction at $p_{K^+} = 1.8$ GeV/c.

Considering the same manner for only the $\Sigma^- p \rightarrow \Lambda \Lambda$ conversion [27,28], we also obtain $(v_{Z - p}\sigma)_{0} = 4.5$ mb and $\alpha = 20$. Thus we estimate $\text{Im} b = 0.018$ fm, which corresponds to $W_0^\Sigma = -1.5$ MeV. Such a small absorption of

![FIG. 5. Comparison of the calculated spectra for $\sigma_{\Sigma^- p K^+}$ with the data of the $^9\text{Be}(K^+, K^+)$ reaction at $p_{K^+} = 1.80$ GeV/c [9], using the WS potential with $V_0^\Sigma = -17$ MeV and $W_0^\Sigma = -5$ MeV. Solid and dashed curves denote the spectra obtained by the optimal and standard Fermi-averaged $K^- p \rightarrow K^+ \Sigma^-$ amplitudes for $\Sigma^- p \rightarrow K^+ \Sigma^-$, respectively. A dot-dashed curve denotes the spectrum obtained by $\beta(d\sigma/d\Omega)^{\text{opt}} = \text{constant}$. The spectra are folded with a detector resolution of 14.7 MeV FWHM.

$W_0^\Sigma \simeq -1$ MeV may be acceptable because the $\Sigma^- p \rightarrow \Lambda \Lambda$ coupling is recently predicted to be rather small [8,30].

**C. Verification of the optimal Fermi-averaged $K^- p \rightarrow K^+ \Sigma^-$ amplitude**

In a previous paper [10], we emphasized the importance of the energy dependence of the $K^- p \rightarrow K^+ \Sigma^-$ amplitude of $\bar{K}_{K^- p K^+}^{\Sigma^-}$ arising from the optimal Fermi-averaging procedure [11] in the nuclear $(K^-, K^+)$ reaction. We discuss the calculated $\Sigma^- QF$ spectra involving the energy dependence of $\bar{K}_{K^- p K^+}^{\Sigma^-}$ in comparison with the data of the $^9\text{Be}(K^-, K^+)$ reaction in the BNL-EN906 experiment. To see the importance of the energy dependence of $\bar{K}_{K^- p K^+}^{\Sigma^-}$, we also estimate the spectrum in the DWIA using the “standard” Fermi-averaged cross section $(d\sigma/d\Omega)^{\text{opt}}$ for the $K^- p \rightarrow K^+ \Sigma^-$ reaction, which may be given by

$$\left(d\sigma/d\Omega\right)^{\text{av}}_{\text{lab}} = \int dp_N \rho(p_N) \left(d\sigma/d\Omega\right)^{\text{free}}_{\text{lab}},$$

where $\rho(p_N)$ is a proton momentum distribution in the target nucleus, and $(d\sigma/d\Omega)^{\text{opt}}_{\text{lab}}$ is the differential cross section for the $K^- p \rightarrow K^+ \Sigma^-$ reaction in free space. This spectrum is proportional to $\beta(d\sigma/d\Omega)^{\text{opt}}_{\text{lab}}$, indicating the energy dependence of $\beta$, whereas the value of $(d\sigma/d\Omega)^{\text{opt}}_{\text{lab}}$ at each $\theta_{\text{lab}}$ becomes constant in Eq. (15). In Table III, we show the calculated values of $(d\sigma/d\Omega)^{\text{opt}}_{\text{lab}}$ and $(d\sigma/d\Omega)^{\text{free}}_{\text{lab}}$. Figure 5 displays the calculated $\Sigma^- QF$ spectra obtained by the optimal and standard Fermi-averaged $K^- p \rightarrow K^+ \Sigma^-$ amplitudes in the $^9\text{Be}(K^-, K^+)$ reaction at $p_{K^+} = 1.8$ GeV/c, together with the spectrum obtained by $\beta(d\sigma/d\Omega)^{\text{opt}}_{\text{lab}} = \text{constant}$, omitting the energy dependence of $\beta$. We find that the energy dependence of $\bar{K}_{K^- p K^+}^{\Sigma^-} \rightarrow \Sigma^- p \Lambda \Lambda$ acts on the shape and magnitude of the QF spectrum remarkably, and it makes its width narrower.
If we use a constant value for \( \bar{f}_{\Xi^- p \rightarrow K^+ \Xi^-} \) in our calculations, the shape and the magnitude of the calculated \( \Xi^- \) QF spectrum cannot explain the data qualitatively. We show clearly that the optimal Fermi averaging for the \( K^- p \rightarrow K^+ \Xi^- \) reaction provides a good description of the energy dependence of the \( \Xi^- \) QF spectrum in the nuclear (\( K^+, K^- \)) reaction\[10\]. Therefore, we recognize that the optimal Fermi-averaged amplitude for \( \bar{f}_{\Xi^- p \rightarrow K^+ \Xi^-} \) is essential to explain the shape and the magnitude of the spectrum including the \( \Xi^- \) QF region with a wide energy range. Thus it is required that we extract information concerning the \( \Xi^- \)-nucleus potential carefully from the data of the experimental spectrum.

## VI. SUMMARY AND CONCLUSION

We have studied phenomenologically the \( \Xi^- \) production spectrum of the \( ^9\text{Be}(K^-, K^+) \) reaction at 1.8 GeV/c within the DWIA using the optimal Fermi-averaged \( K^- p \rightarrow K^+ \Xi^- \) amplitude. We have attempted to clarify properties of the \( \Xi^- \)-nucleus potential for \( \Xi^- \)-\(^9\text{Li} \), comparing the calculated spectrum with the data of the BNL-E906 experiment. We have performed the \( \chi^2 \) fitting to the \( N = 17 \) data points for \( \theta_{1.5\text{–}8.5} \), varying the strength parameters of \( V_0^\Xi \) and \( W_0^\Xi \) in the WS potential.

In conclusion, we show the weak attraction in the \( \Xi^- \)-nucleus potential for \( \Xi^- \)-\(^9\text{Li} \), which provides the ability to explain the data for the \( \Xi^- \) QF region in the \( ^9\text{Be}(K^-, K^+) \) reaction at 1.8 GeV/c, consistent with analyses for previous experiments\[1,3\]. The attraction of \( V_0^\Xi = -17 \pm 6 \text{ MeV} \) is favored within the WS potential, accompanied by the reasonable absorption of \( W_0^\Xi = -5 \text{ MeV} \) for the \( \Xi^- p \rightarrow \Xi^- n \) and \( p \rightarrow \Lambda \Lambda \) transitions in nuclear medium, although it is difficult to determine the value of \( W_0^\Xi \) from the data due to the insufficient resolution of 14.7 MeV FWHM. The importance of the energy dependence of the Fermi-averaged \( K^- p \rightarrow K^+ \Xi^- \) amplitude is confirmed by this analysis. The detailed analysis is also required for the J-PARC E05 experiment of the \( ^{12}\text{C}(K^-, K^+) \) reaction at 1.8 GeV/c\[5\]. This investigation is a subject for future research.

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### Table III. Comparison of the standard Fermi-averaged differential cross sections \((d\sigma/d\Omega)^{\text{std}}\) for the \( K^- p \rightarrow K^+ \Xi^- \) reaction at \( p_{K^-} = 1.8 \text{ GeV/c} \) with the differential cross sections \((d\sigma/d\Omega)^{\text{free}}\) for the \( K^- p \rightarrow K^+ \Xi^- \) reaction in free space\[10\]. The values are in units of mb/sr.

| \(\theta_{\text{lab}}\) | 1° | 2° | 3° | 4° | 5° | 6° | 7° | 8° | 9° |
|---------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \((d\sigma/d\Omega)^{\text{std}}\) | 55.4 | 54.7 | 53.5 | 51.9 | 50.0 | 47.8 | 45.5 | 43.1 | 40.7 |
| \((d\sigma/d\Omega)^{\text{free}}\) | 67.1 | 65.9 | 64.1 | 61.7 | 58.8 | 55.6 | 52.3 | 49.0 | 45.8 |

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