Path-shortening realizations of nonadiabatic holonomic gates

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Nonadiabatic holonomic quantum computation uses non-Abelian geometric phases to implement a universal set of quantum gates that are robust against control imperfections and decoherence. Until now, a number of three-level-based schemes of nonadiabatic holonomic computation have been put forward, and several of them have been experimentally realized. However, all these works are based on the same class of nonadiabatic paths, which originates from the first nonadiabatic holonomic proposal. Here, we propose a universal set of nonadiabatic holonomic gates based on an extended class of nonadiabatic paths. We find that nonadiabatic holonomic gates can be realized with paths shorter than the known ones, which provides the possibility of realizing nonadiabatic holonomic gates with less exposure to decoherence. Furthermore, inspired by the form of this new type of paths, we find a way to eliminate decoherence from nonadiabatic holonomic gates without resorting to redundancies.

I. INTRODUCTION

In the circuit model of quantum computation [1], information is processed by means of unitary transformations, i.e., quantum gates acting on qubits, and therefore the central requirement of the circuit-based quantum computation is to realize high-fidelity quantum gates. However, the practical implementation of high-fidelity quantum gates is very challenging, mainly due to control errors and decoherence. Control errors and decoherence are respectively induced by inaccurate manipulations and interactions with environments, and can lead to errors that propagate through the computation. To reduce the effects of these error sources, different kinds of robust quantum gates have been proposed, among which nonadiabatic holonomic gates play a prominent role.

Nonadiabatic holonomic gates are realized by using nonadiabatic non-Abelian geometric phases [2], i.e., nonadiabatic holonomies on the Grassmann manifold $G(N; L)$, which is the space of $L$ dimensional subspaces of an $N$ dimensional Hilbert space. These gates can be performed at high speed and depend only on the global nature of evolution paths of quantum systems, which make them robust against control errors. Due to these features, nonadiabatic holonomic gates have been considered experimentally in circuit quantum electrodynamics [3–24], nuclear magnetic resonance [6, 20], and nitrogen-vacancy (NV) centers in diamond [9, 10, 21, 22].

Although impressive progress, both theoretically and experimentally, has been made in the field of nonadiabatic holonomic quantum computation, the possibility to find nonadiabatic paths with a larger degree of flexibility is still largely unexplored. In fact, all the above mentioned works [3–24] are based on the same class of nonadiabatic paths, being characterized by an effective rotation angle of the bright state restricted to $\pi$. Thus, one interesting and challenging topic is to find new, feasible and useful nonadiabatic paths, which is of significance for applications in quantum gate design.

Here, we propose a universal set of nonadiabatic holonomic gates based on an extended class of paths that originate from the first nonadiabatic holonomic proposal. Here, we propose a universal set of nonadiabatic holonomic gates based on an extended class of nonadiabatic paths. We find that nonadiabatic holonomic gates can be realized with paths shorter than the known ones, which provides the possibility of realizing nonadiabatic holonomic gates with less exposure to decoherence. Furthermore, inspired by the form of this new type of paths, we find a way to eliminate decoherence from nonadiabatic holonomic gates without redundancies. Our proposal can be realized in various systems and is experimentally feasible. To realize it, one needs only to apply pulses with adjustable oscillation frequency, Rabi frequency, and phase, which can be realized with current experimental techniques.

II. PATH-SHORTENING SCHEME

A. Single-qubit gates

Consider a three-level system with energy eigenstates $|0\rangle$, $|1\rangle$, $|e\rangle$, and eigenvalues $\omega_0$, $\omega_1$, $\omega_e$. The states $|0\rangle$ and $|1\rangle$ are used as computational states, and $|e\rangle$ is an auxiliary state. The transition between $|j\rangle$ ($j \in \{0, 1\}$) and $|e\rangle$ is induced by the laser field $E_{\nu}(t) = \epsilon_{\nu} g_{\nu}(t) \cos \nu t$, where $\epsilon_{\nu}$ is the polarization, $g_{\nu}(t)$ is the pulse envelope function, and $\nu$ is the oscillation frequency. Thus, the system Hamiltonian in the laboratory frame is $H_{\text{lab}}(t) = H_0 + \mu \cdot [E_0(t) + E_1(t)]$, where $H_0 = -\omega_{01}|0\rangle\langle 0| - \omega_{1e}|1\rangle\langle 1|$ is the bare Hamiltonian with the energy of $|e\rangle$ set to zero and $\mu$ is the electric dipole operator. We perform a transformation to a rotating frame by using the rotation operator $V(t) = \exp[-i(\nu_0|0\rangle\langle 0| + \nu_1|1\rangle\langle 1|) t]$, which turns the system Hamiltonian $H_{\text{lab}}(t)$ into $H_{\text{rot}}(t) = \sum_{j=0,1}[\Omega_{ej}(e^{i\phi_{ej}}|j\rangle\langle j| + \text{H.c.}) - \Delta_{ej}]/2$. Here, the real-valued Rabi frequency $\Omega_{ej}$, the laser phase $\phi_{ej}$, and the detuning $\Delta_{ej}$ satisfy $\Omega_{ej} = g_{\nu}(t) |\epsilon_{\nu} (\mu \cdot e_{ej})|/2$ and $\Delta_{ej} = \omega_{ej} - \nu$. Respectively. In $H_{\text{rot}}(t)$, we have ignored rapidly oscillating terms (rotating wave approximation). By assuming $\Delta_{00} = \Delta_{e1} = \Delta$ and $\Omega_{0e} = \Omega_{e1} = \Omega$, and $\Omega_{01} = \Omega_{e0} = 0$.
and making a suitable shift of zero point energy, the Hamiltonian reads
\[
H_{\text{rot}} = \Delta |e\rangle \langle e| + \Omega (e^{i\phi}|e\rangle \langle b| + \text{H.c.}),
\] (1)
where \(\Omega = \sqrt{\Omega_0^2 + \Omega_1^2}\) and \(\phi = \phi_0\). Here, \(|b\rangle = \cos \theta(0) + \sin \theta e^{i\phi}|1\rangle\) is the bright state, with time-independent tan \(\theta = \Omega_1/\Omega_0\) and \(\phi = \phi_0 - \phi_1\). The dark state \(|d\rangle = \sin \theta(0) - \cos \theta e^{i\phi}|1\rangle\) is decoupled from the dynamics.

We use Hamiltonian \(H_{\text{rot}}\) to realize our proposal. To see this, we first briefly explain how nonadiabatic holonomies arise in unitary evolutions. Consider an \(N\) dimensional system containing an \(L\) dimensional computational subspace \(S(0) = \text{Span}(|\phi_1(0))_{i=1}^{N}\). The evolution operator driven by the Hamiltonian \(H(t)\) is a nonadiabatic holonomic gate acting on \(S(0)\) if for some time \(T\) the following two conditions are satisfied: (i) \(\sum_{T-1}^{T} |\phi_i(T)| \langle \phi_i(T)| = \sum_{T-1}^{T} |\phi_i(0)| \langle \phi_i(0)|\) and (ii) \(\langle \phi_i(t)|H(t)|\phi_j(t)| = 0, k, l, i, \ldots, L, \) where \(|\phi_i(t)| = T \exp[-i \int_0^T H(t')dt'] |\phi_i(0)|\), with \(T\) being time-ordering. The above two conditions guarantee that the evolution of the computational subspace is both cyclic (i) and geometric (ii).

To realize our proposal with Hamiltonian \(H_{\text{rot}}\), we first split the evolution into a segment-pair and analyze under which requirements the system defines a nonadiabatic holonomic gate. The segment-pair is performed on the time intervals \([0, \tau]\) and \([\tau, T]\), where \(\tau\) is the first segment ending time, coinciding with the initial time of the second segment (we assume \(\tau < T\)). The Hamiltonian of the first segment reads
\[
H_1 = \Delta_1 |e\rangle \langle e| + \Omega_1 (e^{i\phi_1}|e\rangle \langle b_1| + \text{H.c.}),
\] (2)
where \(\Delta_1, \Omega_1, \phi_1\), and \(|b_1\rangle\) are the detuning, the Rabi frequency, the laser phase, and the bright state, respectively. The dark state of \(H_2\) is \(|d_2\rangle\). \(H_2\) is defined in a frame associated with the rotation operator
\[
V_2(t) = \exp[-i (\nu_{2.0}|0\rangle \langle 0| + \nu_{2.1}|1\rangle \langle 1|) t],
\] (6)
where \(\nu_{2.0}\) and \(\nu_{2.1}\) are the laser frequencies, satisfying the relation \(\nu_{2.0} - \nu_{2.0} = \omega_{e_1} - \omega_{e_2} = \Delta_2\). Since we allow for \(\Delta_2 \neq \Delta_1\), the rotating frames of the two segments, i.e., the rotation operators \(V_1(\tau)\) and \(V_2(\tau)\), can be different. In order to compensate for this difference, the initial computational subspace of the second segment should be taken as \(V_2(\tau) V_1(\tau) S(\tau)\). By using Eqs. (3), (4), and (6), the basis states of \(V_2(\tau) V_1(\tau) S(\tau)\) read
\[
|\psi_1\rangle = e^{i(\Delta_1 - \Delta_2 t) \tau} \cos \eta_1 |\cos \theta_1 + i \sin \theta_1 \cos \eta_1\rangle |b_1\rangle,
\]
\[
-\sin \theta_1 \sin \eta_1|e\rangle,
\]
\[
|\psi_2\rangle = e^{i(\Delta_1 + \Delta_2 t) \tau} |\sin \theta_1 \sin \eta_1|e\rangle.
\] (7)
In the following, we first show how to make sure the geometric condition (ii) is satisfied for the initial subspace \(\text{Span}(|\psi_1\rangle, |\psi_2\rangle)\) of the second segment, followed by a demonstration of the cyclic condition (i) for the segment pair.

Since \(H_2\) and \(e^{iH_2t}\) commute, the geometric condition (ii) turns into the constraints \(\langle H_2^\dagger \delta \rangle = 0\), where \(\delta, \delta^\dagger \in (|\psi_1\rangle, |\psi_2\rangle)\). We discuss these four constraints one by one. First we note that, since \(|d_1\rangle \in \text{Span}(|b_1\rangle, |d_1\rangle) = \text{Span}(|b_2\rangle, |d_2\rangle)\), we can rewrite the basis states \(|\psi_1\rangle\) and \(|\psi_2\rangle\) in Eq. (8) as
\[
|\psi_1\rangle = c_1|e\rangle + c_2|b_2\rangle + c_3|d_2\rangle,
\]
\[
|\psi_2\rangle = c_4|b_2\rangle + c_5|d_2\rangle.
\] (8)
where \(c_j\) are complex numbers. By combining Eqs. (5) and (8), we find
\[
\langle \psi_1|H_2|\psi_2\rangle = c_1^* c_2 \Omega_2 e^{i\phi_2}.
\] (9)
The coefficient \(c_1\) must be nonzero in order for the evolution along the first segment to be noncyclic. Thus, the only nontrivial solution of \(\langle \psi_1|H_2|\psi_2\rangle = 0\) is \(c_4 = 0\). This implies that \(|d_1\rangle\) and \(|d_2\rangle\) are the same up to a global phase. Thus, \(\langle \psi_1|H_2|\psi_2\rangle = 0\) requires that \(H_2\) takes the form
\[
H_2 = \Delta_2 |e\rangle \langle e| + \Omega_2 (e^{i\phi_2}|e\rangle \langle b_2| + \text{H.c.}),
\] (10)
which in turn implies that the geometric constraints \(\langle \psi_2|H_2|\psi_1\rangle = 0\) and \(\langle \psi_2|H_2|\psi_2\rangle = 0\) are satisfied.

It remains to check the constraint \(\langle \psi_1|H_2|\psi_1\rangle = 0\). To this end, we define \(U_2 = e^{-iH_2(\tau - \tau)}\), being the time evolution operator corresponding to the full traversal of the second segment, and temporarily assume that the evolution satisfies condition (i), i.e., the computational subspace performs cyclic evolution on \([0, T]\). By using Eqs. (7) and (10), we first find that \(U_2|\psi_2\rangle = e^{i\phi_2}|d_1\rangle\) for some global phase \(\phi_2\). Thus, the cyclic condition (i) entails that
\[
U_2|\psi_1\rangle = e^{i\phi_1}|b_1\rangle,
\] (11)
where \(\phi_1\) is a global phase. By combining \([H_2, U_2] = 0\) and Eq. (11), one finds
\[
\langle \psi_1|U_2^\dagger H_2 U_2|\psi_1\rangle = \langle b_1|H_2|b_1\rangle = 0.
\] (12)
In other words, \( \langle \psi_1 | H_2 | \psi_1 \rangle = 0 \) is satisfied as long as the evolution generated by \( H_2 \) satisfies the cyclic condition (i).

To complete the analysis, we now demonstrate how to satisfy the cyclic condition (i). To this end, we rewrite Eq. (11) as \( U_2^{\dagger} | b_1 \rangle = e^{-i\eta_2} | \psi_1 \rangle \), and evaluate the left-hand side

\[
U_2^{\dagger} | b_1 \rangle = e^{i\theta_2 \cos \eta_2} | \cos \theta_2 - i \sin \theta_2 \cos \eta_2 \rangle | b_1 \rangle + e^{i\varphi_2} \sin \theta_2 | \sin \eta_2 \rangle | e \rangle,
\]

(13)

where \( \theta_2 = \sqrt{(\Delta_1/2)^2 + (\Delta_2/2)^2} \) and \( \tan \eta_2 = 2\Omega_2/\Delta_2 \). It is noteworthy to the parameters \( \theta_2, \eta_2 \), and \( \varphi_2 \) can be chosen independently of the corresponding parameters of the first pulse. By combining Eqs. (7) and (15) with \( U_2^{\dagger} | b_1 \rangle = e^{-i\eta_2} | \psi_1 \rangle \), one finds that as long as

\[
| \sin \theta_2 \sin \eta_2 | = | \sin \theta_1 \sin \eta_1 |,
\]

(14)

the populations satisfy \( | \langle e | U_2^{\dagger} | b_1 \rangle |^2 = | \langle \psi_1 | e \rangle |^2 \) and \( | \langle b_1 | U_2^{\dagger} | b_1 \rangle |^2 = | \langle b_1 | b_1 \rangle |^2 \). Thus, under the condition in Eq. (14), it only remains to adjust the phase \( \varphi_2 \) in order for the evolution to satisfy the cyclic condition (i). Since both \( U_2^{\dagger} | b_1 \rangle \) and \( | \psi_1 \rangle \) contain \( | e \rangle \), it follows that \( | \sin \theta_2 \sin \eta_2 | \) and \( | \sin \theta_2 \sin \eta_2 | \) are nonzero. If furthermore \( | \sin \theta_2 \sin \eta_2 | = | \sin \theta_1 \sin \eta_1 | < 1 \), it follows that \( \varphi_2 \) is a relative phase between the states \( | e \rangle \) and \( | b_1 \rangle \), and should be chosen as

\[
\varphi_2 = \varphi_1 + (\Delta_1 - \Delta_2) \tau - a - \sum_{j=1}^{2} \arg(\cos \theta_j)
\]

(15)

\[
+ i \sin \theta_j \cos \eta_j,
\]

where \( a = \pi \) if \( \sin \theta_2 \sin \eta_2 = \sin \theta_1 \sin \eta_1 \), and \( a = 0 \) if \( \sin \theta_2 \sin \eta_2 = - \sin \theta_1 \sin \eta_1 \). If \( | \sin \theta_2 \sin \eta_2 | = | \sin \theta_1 \sin \eta_1 | = 1 \), the phase \( \varphi_2 \) becomes a global phase and there is no special constraint on its value.

Until now, we have found the requirements under which the system defines a nonadiabatic holonomic gate. The holonomic gate reads

\[
U = \exp(i\beta | b_1 \rangle \langle b_1 | + | d_1 \rangle \langle d_1 |),
\]

(16)

where \( \beta = \sum_{j=1}^{2} \arg(\cos \theta_j + i \sin \theta_j \cos \eta_j) - \theta_j \cos \eta_j \) if \( | \sin \theta_j \sin \eta_j | < 1 \), and \( \beta = \varphi_1 - \varphi_2 + a \) if \( | \sin \theta_j \sin \eta_j | = 1 \). The corresponding nonadiabatic path reads

\[
S_{|b_2 \rangle | b_1 \rangle} \rightarrow S_{|b_2 \rangle | \psi_1 \rangle} \rightarrow S_{|b_2 \rangle | b_1 \rangle},
\]

(17)

where \( S_{|A \rangle | B \rangle} \) denotes the subspace spanned by states \( | A \rangle \) and \( | B \rangle \). In fact, by using the derived conditions, we can realize more flexible nonadiabatic paths. According to Eq. (12), \( H_2 \) can be used to drive the system for time \( \tau \leq t < T \), while keeping the holonomic feature. In this case, the corresponding evolution is \( S_{|b_2 \rangle | \psi_1 \rangle} \rightarrow S_{|b_2 \rangle | e \rangle} \), where \( | \psi_1 \rangle \) is a superposition state of \( | e \rangle \) and \( | b_1 \rangle \), and \( | \psi_2 \rangle = e^{i\xi} | d_1 \rangle \) with \( \xi \) being the compensation phase. Then a Hamiltonian \( H_1 \) having the form of Eq. (11) can be constructed to continue this path. By repeating, we can realize nonadiabatic paths of the form

\[
S_{|b_2 \rangle | b_1 \rangle} \rightarrow S_{|b_2 \rangle | \psi_1 \rangle} \rightarrow S_{|b_2 \rangle | e \rangle} \rightarrow \cdots
\]

(18)

where \( e^{i\gamma} \) is a geometric phase factor.

Following Eq. (13), one can construct nonadiabatic paths different from the previous ones. This can be seen from the evolution of the bright state \( | b \rangle \). Since \( | b \rangle \) evolves in the subspace \( \text{Span}\{|b \rangle, |e \rangle\} \), its evolution can be viewed by the effective Bloch sphere \( \mathcal{B} \) with \( | b \rangle \) and \( | e \rangle \) being its poles. For the previous schemes [20, 21], the bright state \( | b \rangle \) evolves along a path on \( \mathcal{B} \) that corresponds to an effective total rotation angle \( \sum_j \theta_j = \sum_j \sqrt{(\Delta_j/2)^2 + \Omega_j^2 T_j} = \pi, T_j \) being the duration time of the \( j \)-th segment. On the other hand, for the paths in Eq. (18), the effective rotation angle of \( | b \rangle \) can be smaller than \( \pi \). We illustrate this with an example. Consider a path containing two segments, where the first segment is induced by

\[
H_1 = \Delta_1 | e \rangle \langle e | + \Omega_1 (e^{i\xi_1} | e \rangle \langle b | + e^{-i\xi_1} | b \rangle \langle e |)
\]

(19)

where we assume the effective rotation angle \( \theta_1 = \sqrt{(\Delta_1/2)^2 + \Omega_1^2 T_1} = \pi/3 \), and the ratio of \( 2\Omega_1/\Delta_1 \) is 3/4. According to the derived conditions, the Hamiltonian that induces the second segment, given \( H_1 \), can be chosen to have the form

\[
H_2 = \Omega_2 (e^{i\xi_2} | e \rangle \langle b | + e^{-i\xi_2} | b \rangle \langle e |)
\]

(20)

with the phase \( \varphi_2 = \varphi_1 - 0.6\pi - \arg(5 + 3 \sqrt{3})i \) and the effective rotation angle \( \theta_2 = \Omega_2 T_2 \approx 0.24\pi \). The realized nonadiabatic holonomic gate for this path is \( \mathcal{H} = e^{i\gamma} | b \rangle \langle b | + | d \rangle \langle d | \) with \( \gamma = \pi/18 \). According to the above calculations, the total effective rotation angle \( \theta_1 + \theta_2 \) for the bright state \( | b \rangle \) is thus about 0.57\pi, which is significantly smaller than \( \pi \). More importantly, the above example shows that shorter paths can be realized, which provides the possibility of realizing nonadiabatic holonomic gates with less exposure to unwanted decoherence effects.

### B. Dynamical decoupling

Much effort has been paid focusing on realizing nonadiabatic holonomic gates with less exposure to decoherence [14, 17, 28]. This works resort to redundancies which consist of encoding logical qubits with sets of physical qubits. Here, inspired by the multi-segment form of Eq. (18), we propose a new approach to eliminate decoherence from nonadiabatic holonomic gates without using redundancies. Since the paths in Eq. (18) contain many segments, we consider interleaving these segments with a dynamical decoupling sequence [29]. Consider a three-level system experiencing decay, induced by the system-environment interaction Hamiltonian

\[
H_{SE} = \langle e | 0 \rangle (0, 0) \langle e | 0 \rangle + \langle e | 1 \rangle (1, 1) \langle e | 1 \rangle \otimes E_{c1},
\]

(21)

where \( E_{c0} \) and \( E_{c1} \) are environment operators. For \( H_{SE} \), the decoupling group can be taken as \( U(g_1, g_2, g_3) \), where \( I \) is the identity operator, \( g_1 = | 1 \rangle \langle 1 | - | e \rangle \langle e | - | 0 \rangle \langle 0 | \), \( g_2 = | e \rangle \langle e | - | 0 \rangle \langle 0 | - | 1 \rangle \langle 1 | \), and \( g_3 = | 0 \rangle \langle 0 | - | e \rangle \langle e | - | 1 \rangle \langle 1 | \). The dynamical decoupling sequence is \( I \rightarrow g_1 \rightarrow g_3 \rightarrow g_1 \rightarrow g_3 \), with \( g_1 = e^{-i\pi/2} | b \rangle \langle b | \) and \( g_3 = e^{-i\pi/2} | b \rangle \langle b | \). Consider
a four-segment path $S_{(|0\rangle,|\downarrow\rangle)} \rightarrow S_{(|0\rangle,|\uparrow\rangle)} \rightarrow S_{(|0\rangle,|\uparrow\rangle)} \rightarrow S_{(|0\rangle,|\downarrow\rangle)}$, with the segment Hamiltonians being $H_1$, $H_2$, $H_3$, and $H_4$. After the interleaf, the whole evolution reads

$$I \rightarrow H'_1 \rightarrow g_1 \rightarrow H'_2 \rightarrow g_3 \rightarrow H'_3 \rightarrow g_1 \rightarrow H'_4 \rightarrow g_3.$$  \hspace{1cm} (22)

One can see that if $H'_1 = H_1$, $H'_2 = g_1H_2g_1$, $H'_3 = g_2H_3g_2$, and $H'_4 = g_3H_4g_3$, then the evolution in Eq. \((22)\) is equivalent to $H_1 \rightarrow H_2 \rightarrow H_3 \rightarrow H_4$. One can verify that $H'_1$, $H'_2$, $H'_3$, and $H'_4$ still have the form of Eq. \((1)\). More importantly, the evolution driven by them remains holonomic. For example, at the beginning of the evolution generated by $H'_2$, the computational subspace is $g_1S_{(|0\rangle,|\downarrow\rangle)}$. Thus, $S_{(|0\rangle,|\downarrow\rangle)}H'_2g_1S_{(|0\rangle,|\downarrow\rangle)} = S_{(|0\rangle,|\downarrow\rangle)}H_2S_{(|0\rangle,|\downarrow\rangle)} = 0$ and the holonomic feature is preserved. One can similarly use dynamical decoupling to reduce dephasing of our holonomic scheme. In this case, the interaction Hamiltonian reads

$$H'_{SE} = |0\rangle\langle 0| \otimes E_0 + |1\rangle\langle 1| \otimes E_1 + |e\rangle\langle e| \otimes E_e,$$  \hspace{1cm} (23)

were $E_0$, $E_1$, and $E_e$ are environment operators. One can use the pulse $P = e^{-i\int \omega(t) dt} \otimes (|0\rangle\langle 0| + |1\rangle\langle 1| + H.c.)$ to create a common environment that protects the computational subspace. $P$ preserves the form of the Hamiltonian in Eq. \((1)\) and the holonomic feature. Thus, the dynamical decoupling idea can be used to eliminate dephasing. It is noteworthy that the used decoupling pulses for $g_1$, $g_3$, and $P$ also have the geometric feature, which makes the whole decoherence eliminating method geometric.

### C. Two-qubit gates

We next show that our paths can be used to realize two-qubit nonadiabatic holonomic gates too. We use the NV center electron spin as the target qubit and one nearby $^{13}$C nuclear spin as the control qubit. Both the electron and nuclear spin are polarized through optical pumping, which can be confirmed by optically detected magnetic resonance spectroscopy. The spins are interacting with each other through hyperfine and dipole couplings. By applying state-selective microwave pulses, one can couple the electronic spin-triplet ground states $|0\rangle$, $|1\rangle$, $|\alpha\rangle$ conditioned on the nuclear spin states $|\uparrow\rangle$, $|\downarrow\rangle$. With microwave fields with adjustable oscillation frequency, Rabi frequency, and phase, one can realize the Hamiltonian

$$H_1 = |j\rangle\langle j| \otimes \left[ \Delta_j |\alpha\rangle\langle \alpha| + \Omega_0, |\alpha\rangle\langle 0| + \Omega_1, |\alpha\rangle\langle 1| + H.c. \right]$$  \hspace{1cm} (24)

with $j = \uparrow$ or $\downarrow$, detunings $\Delta_j$, and $\Omega_0, \Omega_1, \Omega_j$ complex-valued Rabi frequencies \([10, 31, 32]\). By alternating $H_I$ and $H_1$ so as to generate a nonadiabatic multi-segment path, one can realize the two-qubit nonadiabatic holonomic gate

$$U_{nc} = \sum_{j=\uparrow, \downarrow} |j\rangle\langle j| \otimes U_j,$$  \hspace{1cm} (25)

where $U_j$ are unitary holonomic operators acting on the states $|0\rangle$ and $|1\rangle$. $U_{nc}$ may entangle the nuclear and electronic spin qubits $\text{Span}([|\uparrow\rangle, |\downarrow\rangle])$ and $\text{Span}([0], [1])$, respectively, if $U_\uparrow$ and $U_\downarrow$ are different. It is noteworthy that the above two-qubit gates can also be protected by our decoherence eliminating method.

### III. DISCUSSION

To realize our proposal, Eq. \((15)\) is a central condition, which guarantees that subsequent segments match each other. When it is satisfied, the bright state evolves cyclically and acquires a purely geometric phase factor that translates into a non-Abelian holonomy via the dependence of the bright state on the laser parameters $\theta, \phi$ \([3]\). Similarly, when realizing geometric gates in two-level systems, the basis states evolve cyclically and each acquires an Abelian geometric phase. Here, multi-segment paths, e.g., orange-slice-shaped paths, can be used (see, e.g., Ref. \([29]\)), for which conditions similar to Eq. \((15)\) exist to make sure the segments match. Considering the feasibility of Abelian geometric gates, the central condition Eq. \((15)\) is expected to be experimentally feasible.

In actual experiments, phases and detunings can typically be implemented with high accuracy, while imperfect control of duration or strength of pulses is hardest to deal with \([30]\). Generally, imperfect control errors are proportional to the effective rotation angle $\sum_j \theta_j$ unless there exists cancelation between the segment errors. Clearly, for non-split paths, such cancelation is not possible because there is only one segment. For non-split paths, the effective rotation angle is always $\pi$, while for ours, it can be less than $\pi$. This implies that for split paths, even in the cases where the segment errors do not cancel, the errors may have a less effect because the total effective rotation angle is smaller. In addition, we have also shown how to interleave the multi-segment paths with dynamical decoupling sequences. This provides the possibility to reduce decoherence without using any additional qubits. Thus, our proposal can be used to increase the fidelity of nonadiabatic holonomic gates.

In our calculations, we assume the detuning is time-independent and therefore the used pulses need to be square pulses in order to preserve the purely geometric feature of the evolution. In fact, this assumption can be relaxed and we illustrate this with the NV center and the $^{87}$Rb cold atom. For the NV center, the states $|0\rangle$, $|1\rangle$ and $|e\rangle$ are mapped into the Zeeman components $|m = -1\rangle$, $|m = +1\rangle$ and $|m = 0\rangle$, respectively. The transitions from $|m = -1\rangle$ to $|m = +1\rangle$ and $|m = 0\rangle$ are coupled by a microwave field whose frequency, amplitude and phase are adjusted by mixing with an arbitrary-waveform generator. As a result, the Hamiltonian in Eq. \((1)\) with the detuning being time-dependent can be realized \([10, 31]\). For the $^{87}$Rb cold atom, $|0\rangle$, $|1\rangle$ and $|e\rangle$ can be mapped into the Zeeman sublevels of $|F = 1, m_F = -1\rangle$, $|F = 1, m_F = +1\rangle$ and $|F = 1, m_F = 0\rangle$, respectively. Zeeman sublevels with a quantum number difference $\Delta m_F = \pm 1$ are coupled by radio frequencies or the two-photon Raman transition $(\sigma^+ - \pi)$. A second-order Zeeman effect is applied to introduce an inhomogeneous splitting between the sublevels, which allows individual control of both levels. In this case, the detuning...
can also be time-dependent \cite{32,33}. The described implementations require a high degree of pulse control, but should be within reach with current experimental technologies.

IV. CONCLUSIONS

In conclusion, we have proposed a universal set of nonadiabatic holonomic gates based on an extended class of nonadiabatic paths. Specifically, we show how to realize an extended class of nonadiabatic multi-segment paths and develop nonadiabatic holonomic gates based on them. We find that these gates can be realized with paths shorter than the known ones, which provides the possibility of realizing nonadiabatic holonomic gates with less exposure to decoherence. Furthermore, inspired by the multi-segment form of this new type of paths, we find a way to eliminate decoherence from nonadiabatic holonomic gates without redundancies. Our proposal can be realized in various systems and are feasible in experiment.

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