J. Slonczewski invented spin-transfer effect in layered systems in 1996. Among his first predictions was the regime of “windmill motion” of a perfectly symmetric spin valve where the magnetizations of the layers rotate in a fixed plane keeping the angle between them constant. Since “windmill” was predicted to happen in the case of zero magnetic anisotropy, while in most experimental setups the anisotropy is significant, the phenomenon was not a subject of much research. However, the behavior of the magnetically isotropic device is related to the interesting question of current induced ferromagnetism and is worth more attention. Here we study the windmill regime in the presence of dissipation, exchange interaction, and layer asymmetry. It is shown that the windmill rotation is almost always destroyed by those effects, except for a single special value of electric current, determined by the parameters of the device.

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between the moments). The exchange torque acting on $m_2$ is $-\mathbf{T}_{ex}$ since we are dealing with an internal interaction between two moments.

The spin-transfer torques $\tau_{1,2}$ are given by

$$\tau_1 = u_1 \cdot (m_1 \times [n_2 \times m_1])$$

$$\tau_2 = -u_2 \cdot (n_2 \times [m_1 \times n_2])$$

with torque strengths

$$u_i = \frac{\hbar I}{2e} g_i [(n_1 \cdot m_2)].$$

Here $I$ is the electric current flowing form magnet 2 to magnet 1, $e$ is the (negative) electron charge, and $g_i [(n_1 \cdot m_2)]$ are material and device specific spin-polarization factors. For negligible spin-relaxation in the non-magnetic spacer between the magnets one has $g_1 = g_2$ (see Ref. [2]). Note that both $u_{1,2}$ are positive when electrons flow from magnet 2 to magnet 1. The minus sign in front of the right hand side of Eq. (4) reflects the symmetry of spin-transfer torque.

First, we rewrite Eqs. (11) and (2) so that time derivatives are on the left hand side only. Defining $T_1 = \mathbf{T}_{ex} + \tau_1$, $T_2 = -\mathbf{T}_{ex} + \tau_2$, we get for $i = 1, 2$

$$(1 + \alpha_i^2)\dot{m}_i = \gamma \left( T_i + \frac{\alpha_i}{m_i} [m_i \times T_i] \right).$$

It is convenient to introduce vectors $\nu = [m_2 \times m_1]$, $l_1 = [m_1 \times [m_2 \times m_1]$, $l_2 = [m_2 \times [m_1 \times m_2]$. Then

$$m_1 = A_1 \nu + B_1 l_1,$$

$$m_2 = -A_2 \nu + B_2 l_2,$$

with

$$A_1 = \frac{\gamma}{1 + \alpha_1^2} \left( J - \frac{\alpha_1 u_1}{m_1 m_2} \right),$$

$$A_2 = \frac{\gamma}{1 + \alpha_2^2} \left( J + \frac{\alpha_2 u_2}{m_1 m_2} \right),$$

$$B_1 = \frac{\gamma}{(1 + \alpha_1^2) m_1} \left( \frac{\alpha_1 J}{m_1 m_2} - \frac{u_1}{m_1 m_2} \right),$$

$$B_2 = \frac{\gamma}{(1 + \alpha_2^2) m_2} \left( \frac{\alpha_2 J}{m_1 m_2} - \frac{u_2}{m_1 m_2} \right).$$

Consider now the total magnetic moment of the system $M = m_1 + m_2$ and calculate the derivative $dM^2/dt$. Using Eq. (6) and the properties $(m_1 \cdot l_1) = 0$, $(m_1 + m_2) \cdot l_1 = \nu^2$, we find

$$\frac{dM^2}{dt} = 2C [m_1 \times m_2]^2$$

with constant coefficient $C$ that depends on material parameters and spin-transfer strengths

$$C = B_1 + B_2 = \gamma \left( \frac{\alpha_1}{(1 + \alpha_1^2) m_1} + \frac{\alpha_2}{(1 + \alpha_2^2) m_2} \right) J + \frac{\gamma}{m_1 m_2} \left( \frac{u_1}{(1 + \alpha_1^2) m_1} - \frac{u_2}{(1 + \alpha_2^2) m_2} \right).$$

Since $[m_1 \times m_2]^2$ is always positive, except in parallel or antiparallel configurations, we can conclude that after a transient period the magnetic configuration will reach either the state of maximal $M$ (i.e., parallel state) for $C > 0$, or the state of minimal $M$ (i.e, antiparallel state) for $C < 0$. Since in both collinear states $\mathbf{T}_{ex} = 0$ and $\tau_{1,2} = 0$, the system will come to rest and no “windmill” motion will happen. For small spin transfer torques $u_{1,2}$ the final state will be determined by the sign of $J$ and, as expected, the device will end up in a configuration corresponding to the minimum of exchange energy.

The marginal case $C = 0$ is the only situation when the “windmill” is possible. According to Eq. (9), the value of $C$ linearly depends on electric current $I$ through $u_{1,2}$. The only exception is the singular case when device parameters satisfy $g_1/[(1 + \alpha_1^2) m_1] = g_2/[(1 + \alpha_2^2) m_2]$, and $C$ is current-independent. Thus in general one can achieve the windmill regime by tuning the current exactly to the “marginal” value $I_w$, such that $C(I_w) = 0$. Note that this value corresponds to a spin transfer strength of $u_w \sim \alpha J m_1 m_2$, and since $\alpha \ll 1$ the required spin torque is much smaller than the exchange torque. The situation is similar to the switching regime, where spin transfer effect works against the magnetic anisotropy. In both cases critical values of spin torque are proportional to the small Gilbert damping coefficient.

The original discussion of the windmill regime in Ref. [2] assumed $J = 0$, $\alpha_{1,2} = 0$, and $m_1 = m_2$. It was found that magnetic moments rotate in the plane spanned by vectors $m_1$ and $m_2$ at the initial moment, and the angle $\theta$ between them remains constant. How will the windmill motion look in the general situation? At $C = 0$ the total magnetic moment is conserved, $M^2 = m_1^2 + m_2^2 + 2(m_1 \cdot m_2) = \text{const}$, thus $\theta$ is constant in general case as well. Since magnitudes of $m_1$ and $m_2$ are also fixed, constant $\theta$ implies that both vectors will rotate with the same angular velocity

$$\dot{m}_i = [\omega \times m_i]$$

(cf. the theorem on the motion of a rigid body with a fixed point). To find $\omega$ we expand it in the basis of vectors $(\nu, m_1, m_2)$ as $\omega = a \nu + b_1 m_1 + b_2 m_2$ with unknown coefficients $a$ and $b_{1,2}$. Substituting this form of $\omega$ into Eqs. (11), using expressions (7) for $\dot{m}_i$, the fact that for the marginal value of current one has $B_1 = -B_2 = B_w$, and properties $[\nu \times m_1] = -l_1$, $[\nu \times m_2] = l_2$, we find $a = -B_w$, $b_1 = A_2$, and $b_2 = A_1$

$$\omega = B_w [m_1 \times m_2] + A_2 m_1 + A_1 m_2.$$

Since $\dot{\omega} = [\omega \times \omega] = 0$, $\omega$ is an invariant of motion, determined by the initial conditions.

Since $\alpha_{1,2} \ll 1$ and $u_{we} \sim \alpha J$ at the marginal point, we can make approximations in expressions (8) and use $A_1 \approx A_2 \approx \gamma J$, $B_1 \approx \gamma (\alpha_1 J m_1 m_2 + u_1)/m_1^2 m_2$, $B_2 \approx \gamma (\alpha_2 J m_1 m_2 - u_2)/m_2^2 m_1$. Equation $C = B_1 + B_2 = 0$
then gives the following values at the marginal point

\[ I_w \approx \frac{2e}{h} \left( \frac{m_1 \alpha_2 + m_2 \alpha_1}{m_1 g_2 - m_2 g_1} \right) J m_1 m_2, \quad (13) \]

\[ B_w \approx \frac{\gamma \alpha_1 g_2 + \alpha_2 g_1}{m_1 g_2 - m_2 g_1} J. \]

Note that approximation \( u_w \sim \alpha J \) is violated when parameters are close to the degenerate situation \( m_1 g_2 - m_2 g_1 = 0 \). This is the situation when \( C \) is independent of the current and the windmill regime cannot be achieved. Far away from the degenerate situation one has

\[ \omega \approx \gamma J \left( \frac{\alpha_1 g_2 + \alpha_2 g_1}{m_1 g_2 - m_2 g_1} \right) [m_1 \times m_2] + (m_1 + m_2). \]

The first term in parentheses is smaller than the second one by a factor of \( \alpha \ll 1 \). Thus vectors \( m_1 \) precess approximately around the total magnetic moment \( M = m_1 + m_2 \). In the presence of damping and spin-transfer \( M \) is not conserved and performs a small angle rotation around the constant vector \( \omega \). In this respect the motion is very different from Slonczewski's situation at \( J = 0 \), where \( M \) was performing \( 360^\circ \) rotations around \( [m_1 \times m_2] \). The \( J = 0 \) rotations, however, require the fulfillment of a condition \( g_1/[(1+\alpha_1^2)m_1] = g_2/[(1+\alpha_2^2)m_2] \), which is, in particular, satisfied in a completely symmetric valve considered in Ref. 2.

Finally, we return to Eq. (9) and investigate the \( C \neq 0 \) case. It is convenient to rewrite (11) in terms of \( x = \cos \theta \)

\[ \dot{x} = C m_1 m_2 (1 - x^2). \]

The solution reads

\[ x(t) = \cos \theta(t) = \tanh \left( \frac{t + t_0}{\text{sgn}(C) T_*} \right), \]

with

\[ T_* = \frac{1}{|C(t)| m_1 m_2}; \quad (15) \]

and parameter \( t_0 \) determined by the initial angle, \( \cos \theta_0 = \tanh(t_0/T_*) \). We conclude that as \( t \to \infty \) the system approaches a collinear configuration with a current dependent characteristic time \( T_* \). The latter diverges in the vicinity of the marginal current \( I_w \).

In conclusion, we studied the motion of a two layer spin-transfer device with zero magnetic anisotropy. We show that in the presence of damping, layer asymmetry, and exchange interaction between the layers the windmill rotation decays with characteristic time constant \( T_* \). The decay time depends on the current pumped through the device and diverges at a "marginal" current \( I_w \). For \( I \neq I_w \) the system reaches either a parallel or an antiparallel state after a transient period. Exactly at the marginal point \( I = I_w \) the system performs a perpetual generalized windmill motion.

Interestingly, precession motion analogous to the windmill regime was also found in multilayers and bilayers with magnetic anisotropy. In those systems it exists not at a singular point, but in the whole range of current values. Thus, rather unexpectedly, anisotropy can be advantageous for the windmill regime.

Finally, coming to the discussion of the current induced ferromagnetism, we see that in a two magnet device current can induce both ferromagnetic and antiferromagnetic order. However, the situation with only two magnets can be special, and it is necessary to consider devices with three and more magnets to predict what happens in the system of many isotropic paramagnetic impurities under the influence of spin-transfer torques.

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