Fragility analysis for vehicle derailment on railway bridges under earthquakes

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Abstract With the rapid development of high-speed railways around the globe, the safety of vehicles running on bridges during earthquakes has been paid more attention to. In the design of railway bridges, in addition to ensuring the safety of the bridge structure in earthquake, the vehicle safety should also be ensured. Previous studies have focused on the detailed analysis of vehicle derailment on bridges, proposing complex numerical algorithms for wheel–rail contact analysis as well as for parametric analysis, but they are inconvenient for designers. Intensity measure (IM) used in performance-based earthquake engineering is introduced in this study. A method to evaluate the vehicle safety on bridges under earthquakes is proposed with respect to the optimal IM. Then, the vehicle derailment case of the Kumamoto earthquake in Japan verifies the decoupling method of vehicle–bridge interaction model. In the assessment of vehicle derailments, eight IMs are systematically compared: the IMs of bridge deck motion are generally better than those of ground motion; the variation coefficient of spectral intensity of the bridge deck is the smallest at different frequencies. Finally, the derailment fragility cloud map is presented to evaluate the vehicle safety on bridges during earthquakes.

Keywords Vehicle–bridge interaction model · Decoupling method · Earthquake · Vehicle derailment · Intensity measure

1 Introduction

As high-speed railways develop rapidly around the globe, they will inevitably cross seismic zones. Meanwhile, the upgrading of railway design standards requires an extensive use of bridges or viaducts to ensure the smoothness of the tracks. Especially for high-speed railway lines above 200 km/h, bridge sections is more than those for traditional railway lines. For example, the length of bridges on the Beijing–Shanghai high-speed railway is more than 80% of the total line. Vehicles are more likely to run on bridges in earthquakes [1].

The safety of vehicle on bridges in earthquakes has been extensively investigated. Yang and Wu [2] evaluated train stability resting and moving on bridges subjected to seismic loads. Xia et al. [3] investigated the effect of vehicle speed and seismic wave propagation speed on the dynamic response of the vehicle–bridge interaction (VBI) system. Gong et al. [4] investigated the safety of vehicles traversing a three-tower cable-stayed bridge under an earthquake. Tanabe et al. [5] developed the DIASTARS software to investigate the dynamic response of VBI model under earthquakes. Xiao et al. [6] used the El-Centro seismic record to study the derailment mechanism of high-speed trains under earthquakes. Montenegro et al. [7] evaluated the safety of vehicles running on bridges under earthquakes. They found that even moderate-intensity earthquakes can threaten the running safety of vehicles. Ju [8] investigated the derailment of high-speed vehicles moving on multi-span simply-supported bridges.

The studies above focused on the detailed time history analysis of vehicle derailment on the bridge, which requires complex wheel–rail contact numerical algorithms and extensive parameter analysis, inconvenient for engineering design. Moreover, the results of deterministic
time history analysis are dependent on the selected ground motion samples. However, ground motions are highly uncertain, and the safety of vehicles running on bridges is related to the selected ground motion samples. Therefore, there is an urgent need for a practical method to evaluate the vehicle safety.

In the seismic analysis of structural engineering, the uncertainty of ground motion is expressed by the intensity measure (IM). The ground motion IM contains a lot of information related to the characteristics of ground motion. Existing studies [9–11] have shown that an appropriate IM ensures a more accurate evaluation of seismic performance and a sufficient reduction in the variability of structural response prediction.

The selection of an appropriate IM has been addressed by a number of researchers [12–15]. Vamvatsikos and Cornell [16] showed that, for medium- and short-period structures, pseudo-spectra acceleration at the natural period of structures can effectively reduce the discreteness of the calculated structural responses. Kazantzis and Vamvatsikos [17] evaluated eight IMs using the incremental dynamic analysis method for high-rise reinforced concrete frames and low-rise steel frames, respectively. They concluded that the geometric mean of the spectral acceleration values within a certain period range is an appropriate IM.

The selection of an optimal IM is investigated for school building [18], RC moment frames [19, 20], concrete dams [12], highway bridges [21, 22] and monolithic free-standing columns [23], whereas it has yet not to be explored for vehicles running on bridges. The objective of this study is to develop a practical method to evaluate the vehicle running safety on bridges under earthquakes. Firstly, the seismic responses of vehicles on different frequency bridges are identified to illustrate the differences of the dominated vibration modes of vehicles. Secondly, a comprehensive comparison of the IMs for bridge deck motions and ground motions is performed to select the optimal IM. It is found that the spectral intensity of the bridge deck motion performs best among the candidate IMs. Finally, the derailment fragility cloud map is presented to evaluate the vehicle safety, and the method is verified using two other ground motions.

2 Vehicle–bridge interaction model

The VBI model contains vehicle–track submodel and bridge submodel, as shown in Fig. 1. The running safety of vehicles on the bridge during earthquakes involves the dynamic analysis of a vehicle, a track, and a bridge. The vehicle–track model [24] and bridge model are introduced briefly in this section.
similar to the moving section simplification in Refs. [25, 26]. This method uses the steady deflection of the rail induced by a concentrated force $P$ as the shape function of the rail (see Fig. 2). This shape function is associated with each wheel and is assumed to move forward with the train. In Fig. 2, $E_r$ is the elasticity modulus of rail material, $I_{rv}$ is the bending moment of inertia about the lateral axis of the rail section, and $k_{rv}$ is the vertical stiffness of the supporting pad under the rail per unit length. The oscillator has two DOFs: lateral and vertical. The modal mass $M_{rv}$, modal damping $C_{rv}$ and modal stiffness $K_{rv}$ in the vertical direction are calculated by the Rayleigh–Ritz method [25], and the modal mass $M_{rh}$, modal damping $C_{rh}$ and modal stiffness $K_{rh}$ in the lateral direction can be derived in a parallel way. Then, the mass $M_r$, damping $C_r$ and stiffness $K_r$ of this oscillator are obtained:

$$M_r = \text{diag}\{M_{rv}, M_{rh}\},$$
$$C_r = \text{diag}\{C_{rv}, C_{rh}\},$$
$$K_r = \text{diag}\{K_{rv}, K_{rh}\}.$$  

The dynamic equations of the rail structures are

$$M_r \ddot{Y}_r + C_r \dot{Y}_r + K_r Y_r = F_{rw} + F_{rb},$$
$$F_{rb} = K_r Y_d + C_r \dot{Y}_d,$$  (3)

where $M_r$, $C_r$ and $K_r$ are the mass, damping and stiffness matrices of the track structures; $\dot{Y}_r$, $\ddot{Y}_r$ are $Y_r$, the acceleration, velocity and displacement vectors, respectively; $F_{rw}$ is the force vector acting on the track from the vehicle; $F_{rb}$ is the force acting on the track from the bridge deck; $Y_d$ and $\dot{Y}_d$ are deck displacement and velocity beneath the moving rail.

Given intensive interactions between the wheel and rail during earthquakes, a fully nonlinear wheel–rail contact model is established. This model should capture the wheel–rail separation and impact of the uplifted wheel against the rail. The trace curve method [27] was used to find the contact point between wheel and rail. The wheel–rail normal force was calculated using the nonlinear Hertz theory [28]. The friction in the tangent direction was calculated using Shen–Hedrick formula [29]. A detailed description of wheel–rail contact model was shown in Appendix. The new explicit integration method [30] was used to expedite the solution of the vehicle–track system. To ensure the stability of the numerical algorithm, the time step was taken as $0.1$ ms for the vehicle–track system.

### 2.2 Bridge model

At present, most bridge structures on railways are simply supported. In this work, a series span of simply supported bridges are adopted. In the vehicle–bridge interaction model, the bridge is modeled by the finite element method (FEM), as shown in Fig. 3a. The girders and piers of the bridge structures are modeled as the 3-D beam element consisting of two nodes. Each node has six DOFs: three translations and three rotations. The beam span is 32 m with a box section. The bridge deck is modeled into a longitudinal girder that simulates all the bridge mechanical properties. The bridge is composed of 60 spans with a total length of 1920 m. For simplicity, the pier heights are kept constant for the whole bridge structure. The sectional properties of the girder and pier are listed in Table 1. The concrete modulus of elasticity for girder and pier is 35.5 and 33.0 GPa,
respectively. For the sectional properties listed in Table 1, the lateral mode frequency of the bridge is 2.9 Hz.

The dynamic equations for the bridge structure are

\[ M_b \ddot{u}(t) + C_b \dot{u}(t) + K_b u(t) = F_{bv}(t) + F_{bseis}(t), \]  

where \( M_b, C_b, \) and \( K_b \) are the mass, damping and stiffness matrices of the bridge structure, respectively; \( F_{bv} \) is the force of the rail acting on the bridge, and \( F_{bseis} \) is the seismic force acting on the bridge.

The locations of rail are also shown in Fig. 3b, CHN60 rail profiles with a rail gauge of 1.435 m, and the rails have a cant of 1/40. The bridge and the rails are connected by linear spring and linear damping. Therefore, when the vibration displacement and velocity of the bridge deck beneath the moving rail are determined, the bridge–rail interaction force is calculated. The deck displacement beneath the moving rail can be calculated from bridge deck response by

\[ Y_{d,j}^L = Y_{d,j}^R = Y_b(x_j) - H_r \cdot \theta_b, \]  
\[ Z_{d,j}^L = Z_b(x_j) + L_r^L \cdot \theta_b(x_j), \]  
\[ Z_{d,j}^R = Z_b(x_j) - L_r^R \cdot \theta_b(x_j), \]

where \( Y_{d,j}^L \) and \( Y_{d,j}^R \) are the lateral displacement of left rail and right rail, respectively; \( Z_{d,j}^L \) and \( Z_{d,j}^R \) are the vertical displacement of left rail and right rail, respectively; \( Y_b(x_j), Z_b(x_j), \) and \( \theta_b(x_j) \) are lateral, vertical, and torsional displacement at the centroid of the bridge deck section; \( x_j \) is the longitudinal location of the \( i \)-th wheelset on the bridge deck which varies with time as the vehicle moves along the bridge; \( H_r \) is the vertical coordinate position of rail bottom to the centroid of bridge deck; \( L_r^L \) and \( L_r^R \) are the lateral coordinate positions of rail bottom to the bridge deck centroid.

### 2.3 Displacement-based derailment criterion

Currently, there are two main categories of derailment criteria: wheel–rail force-based criterion and wheel–rail displacement-based criterion. The former under earthquake conditions is conservative [31]. Moreover, real derailment does not necessarily occur when the force-based criterion is met. Under high-intensity earthquake actions, frequent separations and collisions between the wheel and the rail could occur, making the force-based derailment criteria unavailable.

In this work, the displacement-based derailment criterion is used to evaluate the vehicle safety on bridges during earthquakes [32]. As indicated in Fig. 4, when the vertical wheel lift reaches a height of 28 mm, the wheel is no longer constrained by the rail, and the vehicle may derail at any time. This critical state is called displacement derailment based on wheel lift [32].

### 3 The decoupled method of VBI system under earthquakes

#### 3.1 Steps of the decoupled method

Due to the complexity of the vehicle–bridge interaction model, no commercial software is available to date for dynamic analysis. To simplify the evaluation of the vehicle safety running on bridges under earthquakes, a decoupling method of the vehicle–bridge interaction has been suggested in Japanese codes [33, 34].

In the codes, the bridge is simplified into a single-degree-of-freedom (SDOF) system with the only lateral response, as shown in Fig. 5 [35]. The bridge responses caused by earthquakes were firstly obtained without vehicles on the bridge. Then, the absolute displacement responses of bridge deck were used as the input excitations to the vehicle–track system to evaluate the vehicle running safety. This decoupling method is suitable for engineering applications.
The equivalent stiffness $k_{eq}$ of the SDOF system is calculated by the push-over method [35, 36]. The equivalent mass $m_{eq}$ of the SDOF system is obtained by

$$m_{eq} = m_u + 0.3m_p,$$

(6)

where $m_u$ represents the mass of the superstructure supported by the abutment, which is generally taken as the sum mass of the girder and the constant load; $m_p$ the mass of the pier. The equivalent period of the SDOF system is calculated by

$$T_{eq} = 2\pi \sqrt{\frac{m_{eq}}{k_{eq}}}. $$

(7)

### 3.2 Feasibility of lateral excitation model in the decoupling method

There are two underlying assumptions in Japanese codes [33, 34]: (1) the vehicle has a marginal effect on the absolute seismic response of bridge under earthquakes; (2) the SDOF system can represent the dynamic behavior of the overall structure well. To illustrate the feasibility of the decoupling method, the track displacement excitations of the lateral excitation (LE) model are compared with those of the vehicle–bridge interaction (VBI) model, as Fig. 6 shows. Note that the track displacement excitations are the absolute deck displacements beneath the moving rail in the vehicle–bridge interaction model.

The bridge model of a lateral frequency 2.9 Hz in Sect. 2.2 is adopted. The Northridge earthquake in 1994 recorded at Beverly Hills-14145 Mulhol [37] serves as the
input. Figure 6a shows the comparison of track displacement for the same car (1st car); Fig. 6b shows the comparison of track displacement for different cars (1st car and 8th car).

In VBI model, the track displacement excitations of the front wheelset are almost identical to that of the back wheelset for the same car. For the front wheelset of the 1st car and the back wheelset of the 8th car, the track excitations are almost identical. The differences of track excitation to vehicle between LE model and VBI model are acceptable.

### 3.3 Verification of decoupling method for VBI model

To verify the decoupling method, a practical track excitation under the lateral component of the Kumamoto earthquake was input to confirm whether the derailment accident on bridges could be reproduced. Reference [38] presented the track displacement excitation to vehicles during the earthquake (see Fig. 7). It should be noted that the track displacement excitations are the deck displacement response beneath the moving rail in the VBI model.

As mentioned above, in VBI model, the track displacement excitations of the front wheelset are almost identical to that of the back wheelset. Thus, the front wheelset and back wheelset excited by the same displacement would have little effect on the calculation results. The track displacement excitation is a lateral excitation in the orthogonal direction to the track. In the decoupling method, the displacement excitation is an external excitation for the vehicle–track system. Then, the dynamic analysis of vehicle–track system can be obtained by solving Eqs. (1–3).

Figure 8 shows the comparison of time history of the vertical wheel–rail forces during the earthquake. Figure 8a and b shows the simulation results of vertical wheel–rail forces for the left and right wheels [38], respectively. Figure 8c and d shows the vertical wheel–rail forces for the left and right wheels by numerical simulation, respectively.

Figure 9 shows the comparison of time history of the vertical wheel lift. Figure 9a and b depicts the simulation results of vertical wheel lift for the left and right wheels in Ref. [38], respectively. Figure 9c and d shows the vertical wheel lift for the left and right wheels by numerical simulation, respectively.

As seen from the comparisons above, the derailment accident on bridges induced by Kumamoto earthquake can be reproduced by the numerical simulations using the decoupling method. The waveforms of wheel–rail forces and wheel lift obtained from the numerical simulations are similar to those in Ref. [38], indicating the accuracy of the decoupled method of VBI model.

### 4 Frequency and mode of railway vehicles

A stopper with an initial gap (0.04 m) in the lateral direction between the trucks and the car-body is included in the vehicle model. Thus, the vehicle is a nonlinear model. The frequency response curve is related to the excitation amplitude. A larger input acceleration amplitude triggers the lateral stopper, which is much stiffer than the secondary springs. The natural frequency of the vehicle becomes larger. The stopper is apparently activated in the event of vehicle derailment during earthquakes. To obtain accurate vehicle frequency during earthquakes more, the input acceleration should be large enough so that the lateral stopper is activated. The amplitude of the input acceleration excitation to vehicle is 3 m/s² in this section.

In Fig. 10a, the horizontal axis represents the frequency of the excitation, which varies from 0.1 to 3 Hz. A wide range of frequencies is adopted to include the vehicle natural frequencies ranging between 0.7 and 2.1 Hz [39, 40]. When the excitation frequency is very small (0.1 Hz), the external excitation changes very slowly. The lateral displacement of the vehicle system can be assumed as the static deformation under this input excitation. At different excitation frequencies, the normalized displacement is the ratio of the displacement amplitude of car-body under each excitation frequency to the static deformation.

As indicated in Fig. 10a, when the excitation frequency is very small (0.1 Hz), the lateral displacement amplitude of the car-body is 0.11 m, and the normalized displacement of the car-body is 1. With the increase of excitation frequency, the amplitude gradually increases. The excitation frequency corresponding to the maximum lateral displacement is 0.66 Hz. That is, the lateral fundamental frequency of the vehicle system is 0.66 Hz. Figure 10b shows the vehicle mode (rocking motion) corresponding to this frequency. When the excitation frequency exceeds 0.9 Hz, the displacement amplitude of the vehicle is less than the static deformation (the normalized displacement of the car-body is less than 1), and the excitation above this frequency may have a suppressive effect on the vehicle response.
**Fig. 8** Comparison of time history of the vertical wheel–rail forces: (a) left wheel in Ref. [38]; (b) right wheel in Ref. [38]; (c) left wheel by simulation; (d) right wheel by simulation.

**Fig. 9** Comparison of time history of the wheel lift: (a) left wheel in Ref. [38]; (b) right wheel in Ref. [38]; (c) left wheel by simulation; (d) right wheel by simulation.
5 Seismic responses of vehicle on bridges at different frequencies

The seismic responses of vehicles on bridges of different frequencies differ greatly. Therefore, we combine the vehicle frequency and vibration mode (Sect. 4) to analyze the vehicle responses. The external excitations are derived from the absolute seismic response of the bridge deck. The lateral frequency of the bridge is 0.7, 2 and 4 Hz, respectively. The Northridge ground motion in 1994 [37], recorded at Beverly Hills-14145 Mulhol is used as the input. The PGA is set as 2 m/s², which is design acceleration amplitude in seismic intensity of VIII zone. The acceleration time history and displacement response spectrum of the ground motion are shown in Fig. 11.

The absolute displacement responses of bridges at different frequencies are shown in Fig. 12. In Fig. 12, the first three curves are the absolute seismic responses of the bridge at the frequencies of 0.7, 2 and 4 Hz, respectively; the last one shows the time history of ground motion displacement.

As indicated in Fig. 12, when the lateral frequency of the bridge is low, the absolute displacement response of bridge deck is similar to that of a sine wave with a frequency being the fundamental lateral frequency of the bridge. When the lateral frequency of the bridge is high (above 4 Hz), the absolute seismic response of bridge deck is close to the input ground motion.

The vertical and lateral wheel–rail forces and displacements at different frequencies are shown in Fig. 13. The last curve in each subfigure means that the vehicle runs on the ground, and are directly excited by the earthquake. In this...
case, the track excitations (deck displacements beneath the moving rail) are replaced by the displacement excitation of the earthquake.

As Fig. 13 shows, when the bridge frequency is low (around 0.7 Hz), the peaks of the vertical force and the lateral force occur at the same moment. The close peaks of the vertical force and the lateral force imply that a rocking motion takes place. The peak lateral force is relatively small compared to that of the vertical force, because the vehicle response on the bridge of a low frequency (0.7 Hz) is dominated by the rocking motion (vehicle rocking frequency is 0.66 Hz), with no violent collisions occurring in the wheel–rail lateral direction.

As the bridge frequency increases, the vehicle seismic responses on the bridge change significantly. For example, at 2 Hz, the peaks of the vertical and the lateral force do not occur simultaneously, different from the

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**Fig. 12** The absolute displacement responses of bridge at different frequencies

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**Fig. 13** Vehicle responses on bridge at different frequencies: **a** wheel–rail force; **b** wheel–rail vertical displacement; **c** wheel–rail lateral displacement
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1. Trend observed at low frequency (0.7 Hz). The vertical force peaks delay to the peaks of the lateral force. For the wheel–rail force, the maximum reduction amplitude of vertical force is similar to that of 0.7 Hz. However, the amplitude of the lateral force is nearly two times larger than that of 0.7 Hz. For the wheel–rail relative displacement, the amplitude of vertical displacement changes significantly due to the separation between the wheel and rail for the bridge frequency of 2 Hz, while a small change in the amplitude of the lateral displacement is found. This shows that there is a violent lateral collision between the wheel and rail, i.e., the vehicle response is no longer dominated by a rocking motion.

When the bridge frequency is higher than 4 Hz, the vehicle seismic responses (including wheel–rail forces and displacement) on the bridge are similar to the last curves in Fig. 13, because the absolute displacement response of the bridge is close to the input earthquake itself, and the response of the vehicle mainly depends on the frequency of the earthquakes. In short, the responses of vehicles on bridges at different frequencies vary considerably under earthquakes.

6. Intensity measures

6.1 Ground motion records

Given the randomness of ground motions, the far-field record set of FEMA P695 [37] is used, which contains 22 natural seismic records. The ground motion records used here are from various earthquake events around the world, with their magnitudes, names and recording stations listed in Table 2. The ground motion records were selected based on the following criteria: (1) the magnitude is greater than 6.5; (2) no more than two records are taken from the same earthquake event; (3) ground motion records are from soft rock and stiff soil sites (predominantly site class C and D conditions).

The acceleration response spectra of the 22 seismic records are given in Fig. 14. The horizontal axis represents the natural period $T_n$, and the vertical axis represents the acceleration response spectra $A/a_{g0}$ normalized with respect to peak ground acceleration $a_{g0}$. As indicated in Fig. 14, the acceleration response spectra of the seismic records show a reasonable dispersion due to the uncertainty of earthquakes.

6.2 Candidate intensity measures

Ground motions are very complex. Structure damages due to ground motions are related to the amplitude, frequency content and time duration. An appropriate intensity measure (IM) should be able to comprehensively reflect the three factors.

In this work, two categories of IMs are used, corresponding to time history and response spectra of ground motions. The IMs of time history ignore the structural characteristics of the bridge or vehicles, called as non-structure-specific IMs. The second category of IMs depends on the structural characteristics, called as structure-specific IMs. The representative IMs chosen from the two categories are listed in Table 3.

### Table 2 Ground motion records

| No. | Earthquake event | Recording station          | Magnitude |
|-----|------------------|-----------------------------|-----------|
| 1   | Northridge       | Beverly Hills-14145 Mulhol  | 6.7       |
| 2   | Northridge       | Canyon Country-W Lost Cany  | 6.7       |
| 3   | Duzce, Turkey    | Bolu                        | 7.1       |
| 4   | Hector Mine      | Hector                      | 7.1       |
| 5   | Imperial Valley  | Delta                       | 6.5       |
| 6   | Imperial Valley  | El Centro Array #11         | 6.5       |
| 7   | Kobe, Japan      | Nishi-Akashi                | 6.9       |
| 8   | Kobe, Japan      | Shin-Osaka                  | 6.9       |
| 9   | Kocaeli, Turkey  | Duzce                       | 7.5       |
| 10  | Kocaeli, Turkey  | Arcelik                      | 7.5       |
| 11  | Landers          | Yermo Fire Station          | 7.3       |
| 12  | Landers          | Coolwater                   | 7.3       |
| 13  | Loma Prieta      | Capitola                    | 6.9       |
| 14  | Loma Prieta      | Gilroy Array #3             | 6.9       |
| 15  | Manjil, Iran     | Abbar                       | 7.4       |
| 16  | Superstition Hills | El Centro Imp. Co. Cent | 6.5       |
| 17  | Superstition Hills | Poe Road (temp)            | 6.5       |
| 18  | Cape Mendocino   | Rio Dell Overpass-FF        | 7.0       |
| 19  | Chi-Chi, Taiwan  | CHY101                      | 7.6       |
| 20  | Chi-Chi, Taiwan  | TCU045                      | 7.6       |
| 21  | San Fernando     | LA Hollywood Stor FF        | 6.6       |
| 22  | Friuli, Italy    | Tolmezzo                    | 6.5       |
In Table 3, \(a(t)\), \(v(t)\) and \(d(t)\) are the acceleration, velocity and displacement of the ground motion, respectively; \(S_a\), \(S_v\) and \(S_d\) are the pseudo-acceleration, pseudo-velocity and displacement spectra, respectively. During the calculation of the response spectra of the ground motion, the damping ratio \(\xi = 5\%\).

Different from the condition that the vehicles run on the ground, vehicles running on bridges are excited directly by the absolute seismic response of the bridge deck. The excitation of the bridge deck may be more relevant to the vehicle safety on the bridge than the ground motion excitation. Thus, the IMs of the two excitations (ground motion and bridge deck motion) are presented in Table 4 for evaluation. In Table 4, the subscript \(g\) denotes the IMs of ground motion, and the IMs without a subscript \(g\) represent the IMs of the bridge deck motion.

### 7 Selection of an appropriate IM

#### 7.1 Selection criteria for IMs

In this paper, the incremental dynamic analysis (IDA) method is used to determine the critical IMs for vehicle derailment. Firstly, the PGAs of all ground motion records in Table 2 are scaled to \(1 \text{ m/s}^2\). Then, the IDA method is used to amplify the PGA of the ground motion step by step with an increment \(0.1 \text{ m/s}^2\). For each amplified PGA, the amplitude of wheel lift can be obtained using the decoupling method in Sect. 3.1. Figure 15 shows the wheel lift of vehicles against the ground motion PGA. The blue line represents ground motion No. 5, and the red line ground motion No. 6 in Table 2. The lateral frequency of the bridge is \(2 \text{ Hz}\), and the vehicle running speed is \(300 \text{ km/h}\).

As indicated in Fig. 15, the wheel lift increases with the PGA. When the vehicle wheel lift reaches \(28 \text{ mm}\), the magnified PGA is called critical PGA. Utilizing the critical PGA, the remaining seven IMs in Table 3 are derived. The absolute seismic response of the bridge deck is calculated by inputting the ground motion with the critical PGA into the bridge system. Then, the eight IMs of bridge deck motion in Table 4 are obtained. For bridges at different frequencies, the critical IM under 22 ground motions, denoted as \(IM_{ci} (i = 1, 2, \ldots, 22)\), are derived in this way.

The choice of an appropriate IM is very crucial for the accuracy of the probability-based seismic assessment of a structure. An appropriate IM is measured by its efficiency.
Fig. 16 CV of the IMs for the critical vehicle derailment: a ground motion; b bridge deck motion

Fig. 17 The influence of track irregularity on the SI limit values: a ground motion No. 1; b ground motion No. 2

Fig. 18 The influence of vehicle speed on the SI limit values: a ground motion No. 1; b ground motion No. 2
The efficiency of an IM is typically assessed from the dispersion of the engineering demand parameter (EDP) at a given IM level. For vehicle systems, the occurrence of derailment is far more important than wheel lift values. The consequences of vehicle behavior are not significantly influenced by the amplitude of the wheel lift, provided it is smaller than the 28 mm. Thus, IM efficiency can be directly measured from the coefficient of variation (CV) of the IM \( IM_i \) (\( i = 1, 2, \ldots, 22 \)): the smaller the CV, the more efficient the IM. The CV is defined as the ratio of the standard deviation to the mean value:

\[
CV = \frac{\hat{\beta}}{\hat{\theta}},
\]

where \( \hat{\beta} \) denotes the standard deviation, \( \hat{\theta} \) the mean value, and \( n \) the number of ground motions.

### 7.2 Performance of candidate IMs

The performance of the candidate IMs for evaluating vehicle derailment on bridges is investigated. A wide range of bridge frequencies with 0.5–5 Hz is chosen to cover most simply supported bridges in high-speed railway engineering. At different frequencies, eight candidate IMs for ground motion and bridge deck motion are systematically compared to evaluate the vehicle running safety on bridges during earthquakes.

Figure 16 shows the CV of the IMs for the critical state of vehicle derailment on bridges. The CV of the critical IMs for bridge deck motion is smaller than that for the ground motion. This is because the vehicle system is excited directly by the seismic response of the bridge deck. Compared to ground motion excitations, bridge deck excitation is more strongly correlated to vehicle derailment. For the IMs of bridge deck motion, spectral-type IMs perform better than peak-type IMs. Both SI and ASI have good performance for bridges at different frequencies, with a maximum coefficient of variation of 0.225 for ASI and 0.19 for SI. Thus, the IM of bridge deck motion SI is suggested in this study to evaluate the vehicle running safety on bridges during earthquakes.

### 8 Simplified assessment method for vehicle safety

#### 8.1 Influence of track irregularity on safety limits of SI

The track irregularity is obtained from the Germany power spectral density with low disturbance. As indicated in Fig. 17, track irregularity has a negligible effect on safety limit \( S_{IL} \) when the displacement-based derailment criterion is used.

#### 8.2 Influence of vehicle speed on safety limits of SI

Figure 18 shows the safety limit \( S_{IL} \) for bridges at different frequencies when the vehicle runs at 150 and 300 km/h. As indicated in Fig. 18, the vehicle speed has a negligible effect on \( S_{IL} \). This is due to the fact that vehicle derailments on the bridge during earthquakes are controlled mainly by
the lateral dynamic behavior of the vehicle, and it is less influenced by the longitudinal motion (speed). However, the consequences of a vehicle derailment can be very serious when the vehicle speed is high.

8.3 Probabilistic assessment of vehicle safety on bridges

To simplify the evaluation of vehicle safety running on bridges during earthquakes, the seismic fragility analysis based on the incremental dynamic analysis method is introduced.

Figure 19 a shows the safety limits $SI_{Li}$ ($i = 1, 2, \ldots, 22$) of the vehicle derailment on the bridges under 22 ground motions in Table 2. It is assumed that the critical $SI_{Li}$ ($i = 1, 2, \ldots, 22$) follow a lognormal distribution. The mean and standard deviation of the logarithmic values of $SI_{Li}$ are described by $\ln \alpha$ and $\beta$ [41]:

$$\ln \alpha = \frac{1}{n} \sum_{i=1}^{n} \ln SI_{Li},$$  \hspace{1cm} (11)

$$\beta = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\ln(SI_{Li}/\alpha))^2}. \hspace{1cm} (12)$$

![Fig. 20 Seismic information: a acceleration time history; b acceleration response spectra](image)

![Fig. 21 Validation of the derailment fragility cloud map: a for Kern County ground motion; b for San Fernando ground motion](image)

![Fig. 22 Fitting equation of the mean value of the $SI_L$](image)
Then, the probability of vehicle derailment on the bridges for a given $SI$ is

$$p_t = \phi \left( \frac{\ln SI - \ln \alpha}{\beta} \right), \quad (13)$$

where $p_t$ is the probability of vehicle derailment on the bridges; $\phi(\cdot)$ is the standard normal cumulative distribution function.

For a bridge at different frequencies, the fragility curve of vehicle derailment is obtained with Eq. (13). Figure 19b shows the derailment fragility cloud map, which indicates the derailment probability of vehicles running on bridges with different frequencies.

In the seismic design of railway bridges, for a given bridge structure, the lateral bridge frequency can be calculated by Eq. (7). Then, the SI of bridge deck under earthquakes can be easily obtained under earthquakes. Checking the SI in Fig. 19b can easily determine the probability of vehicle derailment. This method only requires the calculation of SI for the bridge deck motions under earthquakes, and avoid the time-consuming numerical simulation.

To verify the proposed method, another two seismic motions were used, and their acceleration time histories and acceleration response spectra are shown in Fig. 20.

For the two ground motions, the numerical simulation results of the vehicle–track system are shown in Fig. 21, where the derailment probability is taken from Fig. 19b. As the figure shows, the real vehicle derailments mostly correspond to the cases of the derailment probability larger than 50%. This demonstrates the accuracy of the vehicle derailment fragility cloud map.

For convenience, using the mean value of $SI_L$ could achieve a rough assessment result of vehicle safety. In Fig. 22, the mean value of $SI_L$ is fitted by a simple polyline equation Eq. (14). When the SI of bridge deck motion is less than the $SI_{L, \text{mean}}$, the vehicle running safety on bridges is ensured. Otherwise, the vehicle may be derailed.

\[
\begin{align*}
SI_{L, \text{mean}} &= -8f + 10 & 0.5 \leq f \leq 0.75 \\
SI_{L, \text{mean}} &= 4 & 0.75 < f \leq 2 \\
SI_{L, \text{mean}} &= 0.8f + 2.4 & 2 < f \leq 3.25 \\
SI_{L, \text{mean}} &= 5.3 & 3.25 < f \leq 5 
\end{align*}
\] \quad (14)

\section{9 Conclusions}

The IM used in performance-based earthquake engineering (PBEE) is introduced in this study, and the goal is to select an optimal IM for vehicle safety running on bridges. A practical method to evaluate the vehicle running safety on bridges under earthquakes is proposed with respect to the optimal IM. The main conclusions are as follows:

(a) The response of vehicles on bridges at different frequencies varies considerably under seismic actions. When the bridge frequency is low (around 0.7 Hz), the response of the vehicles is dominated by rocking, with no violent collisions in the lateral direction between the wheels and rails.

(b) Compared with the ground motion IMs, the IMs of the bridge deck motion have a higher correlation with vehicle derailment. The SI of the bridge deck motion has the smallest CV among the candidate IMs. Thus, the SI is suggested to evaluate the vehicle running safety on bridges during earthquakes.

(c) When using the displacement-based derailment criterion under strong earthquakes, track irregularity and vehicle speed have a marginal effect on the safety limits $SI_L$.

(d) The vehicle running safety on the bridges can be assessed during earthquakes using the derailment fragility cloud map. This method is quite convenient and accurate and avoids the time-consuming numerical calculation.

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\section*{Appendix}

\subsection*{Geometric model of wheel–rail contact}

According to the trace curve method, the contact point of wheel and rail is considered to lie on a spatial trace line. Figure 23 shows the wheel–rail spatial contact geometry. The wheel–rail contact point $O_R$ can be calculated by

\[
\begin{align*}
x &= x_B + l_x R_w \tan \delta_R \\
y &= y_B - \frac{R_w}{1 - l_y} (l_x^2 \tan \delta_R + l_y m) \\
z &= z_B - \frac{R_w}{1 - l_z} (l_x l_y \tan \delta_R - l_z m)
\end{align*}
\] \quad (15)
where $R_w$ and $\delta_R$ are the wheel rolling radius and the contact angle at the wheel tread, respectively; $m = \sqrt{1 - l_y(1 + \tan^2 \delta_R)}$; $l_x$, $l_y$ and $l_z$ are the $X$, $Y$ and $Z$ direction cosines,

$$
\begin{align*}
    l_x &= -\cos \phi_w \sin \psi_w \\
    l_y &= \cos \phi_w \cos \psi_w \\
    l_z &= \sin \phi_w
\end{align*}
$$

(16)

$x_B$, $y_B$ and $z_B$ are the coordinates of the rolling circle center $B$,

$$
\begin{align*}
    x_B &= d_w l_x \\
    y_B &= d_w l_y + Y_w \\
    z_B &= d_w l_z
\end{align*}
$$

(17)

$\phi_w$, $\psi_w$ and $Y_w$ are the roll angle, yaw angle and lateral displacement of wheelset; $d_w$ is the lateral distance of the rolling circle from wheelset center.

When $\phi_w$, $\psi_w$ and $Y_w$ are certain, a series of the wheel rolling circles can be determined by changing $d_w$. Then, the wheel–rail contact trace curve can be determined.

### Wheel–rail normal force model

The nonlinear Hertzian elastic contact theory was used to calculate the wheel–rail normal contact forces:

$$
P_N(t) = \left[ \frac{1}{G} \delta N(t) \right]^2.
$$

(18)

where $\delta N(t)$ is the elastic compressing amount at the wheel–rail contact point; $G$ is the wheel–rail contact coefficient.

For the wheel with cone tread,

$$
G = 4.57 \times R^{-0.149} \times 10^{-8} \text{ (m/N}^{2/3})
$$

(19)

For the wheel with worn tread,

$$
G = 3.86 \times R^{-0.115} \times 10^{-8} \text{ (m/N}^{2/3})
$$

(20)

where $R$ is the radius of the wheel. As can be seen, to obtain the wheel–rail normal force, the normal compressing amount at the wheel–rail contact point for each moment $t$ should be calculated.

### Wheel–rail creep force model

Based on the Kalker’s linear creep theory, the wheel–rail longitudinal creep force $F_x$, the lateral creep force $F_y$, and the spin creep torque $M_z$ are described by

$$
\begin{align*}
    F_x &= f_{11} \xi_x \\
    F_y &= f_{22} \xi_y - f_{23} \xi \phi \\
    M_z &= f_{23} \xi_y - f_{33} \xi \phi
\end{align*}
$$

(21)

where $\xi_x$, $\xi_y$ and $\xi \phi$ are the longitudinal, lateral and spin creepages, respectively; $f_{ij}$ represents the creep coefficients,

$$
\begin{align*}
    f_{11} &= G(ab)C_{11} \\
    f_{22} &= G(ab)C_{22} \\
    f_{23} &= G(ab)^{3/2}C_{23} \\
    f_{33} &= G(ab)^2C_{33}
\end{align*}
$$

(22)

$G$ is the combined shear modulus of the wheel and rail materials; $a$ and $b$ are the long and short semi-axil lengths of the wheel–rail contact patches, respectively; $C_{ij}$ are the Kalker’s creep coefficients, which depend on the ratios of the semi-axil lengths of the wheel–rail contact patches.

Note that the Kalker’s linear creep theory is only available for small creepages situation. For large creepages situations, the linear relationship between creep forces and creepages will no longer be applicable. In this case, a nonlinear modification is adopted, which is available for the calculation of wheel–rail creep forces under the condition of large creepages.

The resultant force of the longitudinal creep force and the lateral creep force is

$$
F = \sqrt{F_x^2 + F_y^2}.
$$

(23)

Define the nonlinear modified coefficient for the creep forces as

$$
\xi = \frac{F'}{F},
$$

(24)

where
\[ F' = \begin{cases} \mu N \left[ \frac{F}{\mu N} - \frac{1}{3} \left( \frac{F}{\mu N} \right)^2 + \frac{1}{27} \left( \frac{F}{\mu N} \right)^3 \right] & \text{if } F \leq 3\mu N; \\ \mu N & \text{if } F > 3\mu N. \end{cases} \] (25)

\( \mu \) is the friction coefficient between the wheel and the rail.

The modified longitudinal creep force, lateral creep force and spin creep torque are

\[ F'_x = \epsilon F_x, \quad F'_y = \epsilon F_y, \quad M'_c = \epsilon M_c. \] (26)

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