We propose a new class of spontaneous baryogenesis models that does not produce baryon isocurvature perturbations. The baryon chemical potential in these models is independent of the field value of the baryon-generating scalar, hence the scalar field fluctuations are blocked from propagating into the baryon isocurvature. We demonstrate this mechanism in simple examples where spontaneous baryogenesis is driven by a non-canonical scalar field. The suppression of the baryon isocurvature allows spontaneous baryogenesis to be compatible even with high-scale inflation.
1 Introduction

The three basic ingredients for creating the asymmetry between baryons and antibaryons from an initially symmetric state was laid out by Sakharov [1]. However, the third condition of a deviation from thermal equilibrium can actually be traded for a breaking of the CPT symmetry. This is the idea of spontaneous baryogenesis [2], which typically invokes a scalar field derivatively coupled to the baryon current in the form \((\partial_\mu \phi) j_\mu^B\). With such an interaction, the time derivative of a coherent scalar \(\dot{\phi}\) spontaneously breaks CPT and shifts the spectrum of baryons relative to that of antibaryons by an amount \(\mu \propto \dot{\phi}\). As a consequence, baryogenesis is allowed even in thermal equilibrium if baryon number nonconserving processes occur rapidly. This mechanism has been implemented in various particle physics models, e.g., [3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

The scalar condensate \(\phi\) which drives spontaneous baryogenesis can leave further imprints in the subsequent cosmology [13]. One such example is the baryon isocurvature perturbation [14], since super-horizon field fluctuations of \(\phi\) sourced during inflation give rise to spatial fluctuations of the baryon-to-photon ratio. Hence the current observational bounds on isocurvature perturbations provide strong constraints on spontaneous baryogenesis scenarios. Here, the amplitude of the field fluctuation is set by the scale of inflation, therefore the bound on the baryon isocurvature can be translated into an upper bound on the inflation scale. In particular, for the minimal spontaneous baryogenesis scenario where the scalar \(\phi\) possesses a quadratic potential, the bound on baryon isocurvature from measurements of the cosmic microwave background (CMB) constrains the inflationary Hubble rate as \(H_{\text{inf}} \lesssim 10^{12} \text{GeV} [13]\). This implies that if inflationary gravitational waves are detected in the near future, thus confirming an inflation scale higher than \(10^{12} \text{GeV}\), then the minimal scenario of spontaneous baryogenesis would be ruled out. As there are a variety of ongoing and upcoming experiments in search of primordial gravitational waves, it is of great interest to investigate whether high-scale inflation rules out spontaneous baryogenesis in general.
The production of the baryon isocurvature in spontaneous baryogenesis is due to the fact that the relative shift between the baryons and antibaryons is set by the scalar velocity, $\mu \propto \dot{\phi}$, which takes slightly different values among different patches of the universe in the presence of the scalar field fluctuations. However, there are situations where the fluctuations in the field value do not necessarily lead to fluctuations in the field velocity. It was pointed out in [13] that for cases where the scalar possesses non-quadratic potentials with inflection points, the fluctuations in the scalar velocity is suppressed if the initial position of the scalar field happens to lie close to an inflection point. We should also mention that there have been other proposals which may evade the isocurvature constraint; these include compensating the baryon isocurvature with cold dark matter isocurvature if the scalar serves as dark matter [13], stabilizing the scalar in a false vacuum during inflation [10], driving spontaneous baryogenesis by domain walls [12], or by the derivative of the Ricci scalar instead of a scalar field [15].

In this paper we propose a new class of spontaneous baryogenesis models that is free from baryon isocurvature perturbations. The basic idea is to block the field fluctuations of the baryon-generating scalar $\phi$ from propagating into the baryon isocurvature by invoking a combination of a derivative coupling $f(\phi)(\partial_\mu \phi) j_\mu^B$ and a scalar potential $V(\phi)$ that renders the product $f(\phi)V'(\phi)$ constant. Then, since the scalar slowly rolling along the potential possesses a velocity of $\dot{\phi} \propto -V'(\phi)$, the induced shift in the baryon/antibaryon spectra,

$$\mu \propto f(\phi) \dot{\phi} \propto f(\phi)V'(\phi) = \text{const.,}$$

is independent of the scalar field value. Therefore the resulting baryon number becomes spatially homogeneous even though the scalar itself possesses field fluctuations.

We obtain a set of reasonably simple models where the situation of (1.1) is realized, by considering a scalar field with a non-canonical kinetic term. We discuss various special properties of such models, and in particular we will find that a successful baryogenesis is allowed even with high-scale inflation. Here, let us remark that the inflection point model of [13] also satisfies the condition (1.1) at special points along the scalar potential. However in the new class of models presented in this paper, the suppression happens generically without the need of tuning the initial position of the scalar. Furthermore, the striking feature that the baryon asymmetry is independent of the scalar field value provides predictive power to the model.

The paper is organized as follows. We first briefly explain how the condition (1.1) can be satisfied by a non-canonical scalar in Section 2. Then we study in detail two example models in Sections 3 and 4. We conclude in Section 5.

## 2 Models with Non-Canonical Scalars

We begin by briefly describing a class of models where a non-canonical scalar field drives spontaneous baryogenesis without producing baryon isocurvature. The Lagrangian of the model consists of a scalar with a non-canonical kinetic term, a mass term, and a coupling to the divergence of a baryon current of the form

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \sqrt{1 + \left( \frac{\phi}{\lambda} \right)^{2n}} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 - \left( \frac{\phi}{f} \right)^n \nabla^\mu j^\mu_B \right],$$

(2.1)
where \( n \) is a positive integer, while \( \lambda, f, \) and \( m \) are mass scales. For small field values \( |\phi| \ll \lambda \), the kinetic term is almost canonical. On the other hand in the large field limit of \( \phi \gg \lambda \), the kinetic term becomes approximately proportional to \( \phi^n (\partial \phi)^2 \) and thus it can be made canonical by redefining the field as

\[
\sigma \propto \phi^{\frac{n+2}{2}}. \tag{2.2}
\]

In terms of this canonically normalized field, the originally mass term serves as a power-law potential of

\[
V = \frac{1}{2} m^2 \phi^2 \propto \sigma^{\frac{4}{n+2}}. \tag{2.3}
\]

Hence the time derivative of the scalar that slowly rolls along this potential depends on the field value as

\[
\dot{\sigma} \propto - \frac{\partial V}{\partial \sigma} \propto \sigma^{-\frac{n-2}{n+2}}. \tag{2.4}
\]

The coupling to the baryon current, after integration by parts, can also be rewritten in terms of the canonical field as

\[
-\sqrt{-g} \left( \frac{\phi}{f} \right)^n \nabla_\mu j^\mu_B \Rightarrow \sqrt{-g} \left\{ \partial_\mu \left( \frac{\phi}{f} \right)^n \right\} j^\mu_B \propto \sigma^{\frac{n+2}{n+2}} (\partial_\mu \sigma) j^\mu_B. \tag{2.5}
\]

Here one sees that the condition (1.1) is satisfied for the canonical field, therefore the shift in the baryon/antibaryon spectra is independent of the field value,

\[
\mu \propto \sigma^{\frac{n+2}{n+2}} \dot{\sigma} \propto \sigma^0. \tag{2.6}
\]

Thus it is clear that the field fluctuations of \( \sigma \) do not source fluctuations in the baryon number.

This mechanism of suppressing the baryon isocurvature is simplest to understand in the case of \( n = 2 \): Here the potential in the large field limit is linear, \( V \propto \sigma \), whose constant tilt renders the scalar velocity \( \dot{\sigma} \) homogeneous. Since the derivative coupling also takes a linear form \( \partial_\mu \sigma j^\mu_B \), the spectrum shift is simply \( \mu \propto \dot{\sigma} \), which is guaranteed to be homogeneous. The suppression of the baryon isocurvature in linear potentials was also pointed out in [13], however, as is clear from the above discussion, the suppression happens for an arbitrary positive integer \( n \).

In the following sections, we study in detail the model (2.1) and its variant, focusing on cases of \( n = 1 \) and 2. In addition to the suppression of the baryon isocurvature, the scalar undergoes nonstandard evolution after baryogenesis, and as a consequence the model behaves quite differently from the usual spontaneous baryogenesis scenarios.

### 3 \( n = 1 \) : Spontaneous Baryogenesis with Fractional Power Terms

In this section we analyze spontaneous baryogenesis with the case of \( n = 1 \) in (2.1), where the Lagrangian of a real scalar \( \phi \) reads

\[
S = \int d^4 x \sqrt{-g} \left[ -\frac{1}{2} \nabla_\mu \phi^\mu \nabla_\nu \phi^\nu - \frac{1}{2} m^2 \phi^2 - \frac{\phi}{f} \sum_i c_i \nabla_\mu j^\mu_i \right]. \tag{3.1}
\]

Here \( \lambda, m, f \) are mass scales, \( c_i \) is a dimensionless coefficient, and \( j^\mu_i \) represents the current of a particle/antiparticle pair \( i \) which has baryon number \((-)B_i\) for the (anti)particle. The time
component $j_i^0 = n_i - \bar{n}_i$ represents the difference in the number density between the particle and antiparticle, and the sum $\sum_i$ runs over all particle species coupled to $\phi$.

Let us briefly comment on the possible origin of the above action. A nice example where the non-canonical kinetic term can arise is through the Nambu–Goto action of a brane, as in the D-brane monodromy inflation models of [16, 17]. See also [18] which invoked similar kinetic terms in the context of inflation. Regarding the coupling to the divergence of the current, such a term can arise, for instance, from anomalous couplings to the SU(2) gauge fields. (However in such cases the coupling term would only be effective when sphalerons are in equilibrium [12, 19].) The mass scale $f$ in the coupling can be related to the scale of new physics; for example, in the models of [3, 6, 7], the scalar $\phi$ is a pseudo-Nambu–Goldstone boson of the broken baryon number, with $f$ being the associated symmetry breaking scale. Although in this paper we do not specify the origin of the scalar and its coupling to the baryon current, we will restrict our attention to field ranges of $\phi$ that do not exceed $f$. Furthermore, we assume $f$ to be larger than the Hubble expansion rate and the cosmic temperature during the relevant epochs. Some more discussions on the validity of the effective field theory will be provided in Section 3.7.

As for the scale $\lambda$ in the kinetic term, we suppose it to follow a hierarchy of $\lambda \ll f$. In the small field limit of $|\phi| \ll \lambda$, the field $\phi$ is almost canonical and the system reduces to the usual case studied in most spontaneous baryogenesis models. On the other hand, in order to study the large field regime of $\lambda \ll |\phi| \lesssim f$, let us focus on positive field values $\phi > 0$ and introduce a new field,

$$\sigma = \frac{2}{3} \frac{\phi^{3/2}}{\lambda^{1/2}},$$

with which the action is rewritten as

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \sqrt{1 + \left( \frac{2 \lambda}{3 \sigma} \right)^{4/3} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma} - \frac{1}{2} \left( \frac{3}{2} \right)^{4/3} m^2 \lambda^{2/3} \sigma^{4/3} + \frac{2 \lambda}{3 \sigma} \left( \frac{1}{3} \partial_\mu \sigma \right)^{1/3} f \sum_i c_i j_i^\mu \right].$$

Here, for the coupling term to the current, we have performed an integration by parts and dropped the total derivative. In the large field regime of $\phi \gg \lambda$, or $\sigma \gg \lambda$, the almost canonical field is $\sigma$ and it has a potential with a fractional power,

$$V(\sigma) = \frac{1}{2} \left( \frac{3}{2} \right)^{4/3} m^2 \lambda^{2/3} \sigma^{4/3},$$

as well as a fractional power-law coupling to the current.

### 3.1 Qualitative Picture

Now let us analyze spontaneous baryogenesis with the above model. We illustrate the cosmological history and the scalar field dynamics in Figure 1: Some time after cosmic inflation, the universe undergoes reheating. Then supposing some baryon number nonconserving processes to be in equilibrium, the baryon asymmetry is produced as the scalar field rolls along its potential. The baryon
number eventually freezes in when the baryon number nonconserving processes fall out of equilibrium. After baryogenesis, as the Hubble friction becomes weaker, the scalar starts to oscillate about the minimum of its potential.

We suppose the scalar field to be initially located in the large field regime:

$$\lambda \ll \phi_\ast \ll f,$$

where we have used $\phi_\ast$ to denote the scalar field value during inflation when the CMB pivot scale $k_\ast$ leaves the horizon. Then, as we will explicitly show, the oscillation amplitude $\bar{\phi}$ at the beginning of the oscillation is larger than $\lambda$ and hence the scalar initially undergoes anharmonic oscillations along the fractional power-law potential (3.4). After $\bar{\phi}$ becomes smaller than $\lambda$, the oscillation becomes harmonic.

### 3.2 Scalar Field Dynamics

The universe is considered to initially undergo inflation, then to be effectively matter-dominated (MD), and after reheating to be radiation-dominated (RD). Spontaneous baryogenesis is supposed to happen during the RD epoch. We describe the cosmological background in terms of a flat FRW universe,

$$ds^2 = -dt^2 + a(t)^2 dx^2,$$

whose Hubble expansion rate follows

$$\frac{\dot{H}}{H^2} = -\frac{3(1 + w)}{2},$$

with $w = -1, 0, 1/3$ during the inflation, MD, and RD epochs, respectively. An overdot denotes a derivative in terms of the cosmological time $t$.

The scalar field is considered to have a negligible effect on the expansion of the very early universe. Therefore we require the initial field value of the canonical field to be sub-Planckian,

$$\sigma_\ast < M_p, \quad \text{i.e.,} \quad \phi_\ast < \left(\frac{3}{2}\right)^{2/3} M_p^{2/3} \lambda^{1/3},$$

Figure 1: Schematic of the scalar field dynamics (not to scale).
otherwise the scalar could dominate the universe. The equation of motion of a homogeneous scalar field, i.e. \( \sigma = \sigma(t) \), reads

\[
\gamma(\sigma)(\dot{\sigma} + 3H\dot{\sigma}) - \frac{1}{2\gamma(\sigma)} \left( \frac{2}{3} \right)^{7/3} \frac{4/3\sigma^2}{\sigma^{7/3}} + \frac{\left( \frac{2}{3} \lambda \right)^{1/3}}{f} \sum_i c_i \nabla_{\mu} j_{i}^{\mu} = 0,
\]

(3.9)

where \( \gamma(\sigma) \) is defined as

\[
\gamma(\sigma) = \sqrt{1 + \left( \frac{2}{3} \lambda \sigma \right)^{4/3}}.
\]

(3.10)

The scalar field dynamics prior to the onset of the oscillations is approximately given by

\[
\frac{3(3+w)}{2} H \dot{\sigma} \simeq -V'(\sigma).
\]

(3.11)

One can check that this expression provides a good approximation to the equation of motion (3.9) when

\[
\left( \frac{\lambda}{\sigma} \right)^{4/3}, \frac{V''(\sigma)}{H^2}, \frac{\left| \sum_i c_i \nabla_{\mu} j_{i}^{\mu} \right|}{m^2 \lambda^{1/3} f \sigma^{2/3}} \ll 1.
\]

(3.12)

These conditions can be understood as the requirements of, respectively, large field value \( \phi^2 \gg \lambda^2 \), small effective mass compared to the Hubble rate, and negligible backreaction from the baryons during baryogenesis. We also note that the approximation (3.11) is actually an attractor while the conditions (3.12) are satisfied; see e.g. the analyses in Appendix A of [20].

Here we have discussed a homogeneous scalar, however we remark that since the scalar’s effective mass is initially much lighter than the Hubble rate (cf. (3.12)), the scalar field actually obtains spatial fluctuations on super-horizon scales during inflation. When discussing cosmological perturbations in the following sections, we will take into account the scalar field fluctuations by noting that the initial field value during inflation is slightly different among different patches of the universe by \( \delta \sigma \sim H_{\text{inf}}/2\pi \). We also note that throughout this paper we will focus on cases where the field fluctuations can be treated as small perturbations, hence we suppose

\[
\sigma_* > H_{\text{inf}}, \quad \text{i.e.,} \quad \phi_* > \left( \frac{3}{2} \right)^{2/3} H_{\text{inf}}^{2/3} \lambda^{1/3},
\]

(3.13)

for the scalar field value during inflation.

### 3.3 Baryon Asymmetry

The coherent background of \( \dot{\phi} \) spontaneously breaks the CPT symmetry, and thus sources a relative shift in the energy spectra of the particles and antiparticles through the coupling term \((\partial_{\mu} \phi/f) c_i j_i^\mu\). When the (anti)particles \( i \) are in thermal equilibrium, the energy shift can be interpreted as particles obtaining an effective chemical potential of

\[
\mu_i = -c_i \frac{\dot{\phi}}{f} = -c_i \left( \frac{2}{3} \lambda \right)^{1/3} \frac{\dot{\sigma}}{f} = \frac{c_i m^2 \lambda}{5 f H}.
\]

(3.14)

and \(-\mu_i\) for antiparticles. In the far right hand side, we used the slow-varying approximation for the scalar velocity (3.11) in a RD universe \((w = 1/3)\). Here one clearly sees that the chemical potential
is independent of the scalar field value $\sigma$, as was discussed around (2.6). The chemical potential gives rise to a baryon asymmetry, when there are baryon violating processes occurring rapidly. Supposing all the particle species $i$ to be relativistic fermions and ignoring their masses, the difference in the number densities of the particles and antiparticles is

$$j_0^i = n_i - \bar{n}_i = \frac{g_i}{6} \mu_i T^2 \left\{ 1 + \mathcal{O} \left( \frac{\mu_i}{T} \right)^2 \right\}, \quad (3.15)$$

where $g_i$ represents the internal degrees of freedom of the (anti)particle $i$, and we have assumed the chemical potential to be much smaller than the cosmic temperature, i.e. $\mu_i^2 \ll T^2$. Hence the ratio between the baryon number density $n_B = \sum_i B_i(n_i - \bar{n}_i)$ and the entropy density

$$s = \frac{2\pi^2}{45} g_{*s} T^3 \quad (3.16)$$

is obtained as

$$\frac{n_B}{s} = \frac{15}{4\pi^2} \sum_i B_i g_i \mu_i \quad (3.17)$$

This ratio freezes in after the baryon violating interactions fall out of equilibrium, given that there are no further baryon or entropy production afterwards. Hence the ratio at the decoupling of the baryon violating interactions $(n_B/s)_{\text{dec}}$ should coincide with the present value $(n_B/s)_0 \approx 8.6 \times 10^{-11}$ measured by Planck [21]. Hereafter we denote the decoupling temperature by $T_{\text{dec}}$, and also use the subscript “dec” for quantities measured at decoupling. On the other hand for quantities in the present universe, we use the subscript “0”. (Here one also sees from (3.17) that the assumption of $\mu_{i,\text{dec}}^2 \ll T_{\text{dec}}^2$ is justified in the case of $(n_B/s)_{\text{dec}} \approx 8.6 \times 10^{-11}$, unless $\sum_i B_i g_i / g_{*s}$ takes an extremely small value.)

Using (3.14) and the relation between the Hubble rate and the temperature in a RD universe:

$$3M_p^2 H^2 = \rho_i = \frac{\pi^2}{30} g_s T^4 \quad (3.18)$$

the baryon-to-entropy ratio (3.17) at decoupling is obtained as

$$\left| \frac{n_B}{s} \right|_{\text{dec}} = \frac{9}{\pi^3} \left( \frac{5}{8} \right)^{1/2} \sum_i B_i g_i \frac{M_p m^2 \lambda}{g_{s,\text{dec}} g_{*s,\text{dec}} T_{\text{dec}}^3 f} \quad (3.19)$$

This result should be contrasted to the baryon asymmetry created in usual spontaneous baryogenesis scenarios where $n_B/s$ is proportional to some powers of the scalar field value $\sigma$, and thus carries isocurvature perturbations of $\delta n_B/n_B \sim \delta \sigma/\sigma \sim H_{\text{inf}}/(2\pi\sigma)$. In our case, as long as the scalar follows the slow-varying solution (3.11) during baryogenesis, the $\sigma$ dependence drops out of the baryon number. As a consequence, the inhomogeneities in $\sigma$ are blocked from propagating into those of the baryons, and thus the baryon isocurvature perturbation is strongly suppressed.

However, we should also remark that the error in the slow-varying approximation (3.11) can source subleading contributions to the baryon asymmetry, which can produce a tiny but non-vanishing baryon isocurvature. We will estimate this effect numerically later when we discuss the parameter space of the model. Let us further note that, if the scalar ever dominates the universe after creating the baryon asymmetry, then this would also lead to baryon isocurvature perturbations. This issue will be discussed in Section 3.5.
Before closing this subsection, let us rewrite the requirement of negligible backreaction from the baryons, i.e. the third of the slow-varying conditions (3.12), using (3.14), (3.15), and (3.18). Ignoring the spatial components of \( j^\mu_i \) and the time derivative of \( g_\ast \), the condition is rewritten as

\[
\frac{\sum_i c_i^2 g_i}{10} \left( \frac{\lambda}{\sigma} \right)^{2/3} \left( \frac{T}{f} \right)^2 \ll 1. \tag{3.20}
\]

Hence we see that, as we have been assuming \( f > T \), the backreaction from the baryons is guaranteed to be negligible in the large field regime \( \sigma \gg \lambda \) (unless \( \sum_i c_i^2 g_i \) takes a large value).

### 3.4 The Fate of the Scalar

#### Onset of Oscillations

Up until the end of baryogenesis, the scalar velocity \( \dot{\sigma} \) is nonzero but the field value itself is effectively frozen. This is clearly seen from the slow-varying approximation (3.11) giving

\[
\left| \frac{\dot{\sigma}}{H\sigma} \right| = \frac{2}{3 + w} \frac{V''(\sigma)}{H^2}, \tag{3.21}
\]

which is much smaller than unity while \( V'' \ll H^2 \). However after the decoupling, as the Hubble friction becomes weaker, the scalar eventually starts to oscillate about its potential minimum. Note here that if the oscillation starts prior to decoupling, the baryon asymmetry would be extremely suppressed as the effective chemical potential becomes tiny when averaged over the oscillations. (See [6, 7] and Appendix of [13] for detailed discussions on this.) Thus we require the scalar to start the oscillation after decoupling, and discuss the fate of the oscillating scalar condensate in this subsection.

We begin by defining the ‘onset’ of the scalar oscillation as when the field excursion during one Hubble time becomes comparable to the distance to the potential minimum, i.e.,

\[
\left| \frac{\dot{\sigma}}{H\sigma} \right| \bigg|_{\text{osc}} = 1, \tag{3.22}
\]

and denote quantities measured at this moment by the subscript “osc”. Then by supposing the slow-varying approximation (3.11) to be valid until the onset of the oscillation, we obtain

\[
H_{\text{osc}}^2 = \frac{1}{5} \left( \frac{3}{2} \right)^{1/3} \left( \frac{\lambda}{\sigma_{\text{osc}}} \right)^{2/3} m^2, \tag{3.23}
\]

where we used \( w = 1/3 \) as we are interested in cases where the scalar starts its oscillation during the RD era.

The field value at the onset of the oscillation \( \sigma_{\text{osc}} \) can further be expressed in terms of field values at earlier times. For this purpose, let us rewrite the slow-varying solution (3.11) as

\[
d\sigma/V'(\sigma) = -2dt/(3(3+w)H),
\]

and integrate both sides through the inflation, MD, and RD epochs using (3.7); integrating from when the pivot scale \( k_\star \) exits the horizon during inflation until the onset of the oscillation, one obtains

\[
\left( \frac{3\sigma_\star}{2\lambda} \right)^{2/3} - \left( \frac{3\sigma_{\text{osc}}}{2\lambda} \right)^{2/3} = \frac{N_\star}{3} \frac{m^2}{H_{\text{inf}}^2} + \frac{2}{27} \left( \frac{m^2}{H_{\text{reh}}^2} - \frac{m^2}{H_{\text{inf}}^2} \right) + \frac{1}{20} \left( \frac{m^2}{H_{\text{osc}}^2} - \frac{m^2}{H_{\text{reh}}^2} \right). \tag{3.24}
\]
Here, \( N_* \) denotes the number of \( e \)-folds between the horizon exit of \( k_* \) and the end of inflation, and \( H_{\text{reh}} \) denotes the Hubble rate at reheating. Hence, supposing a hierarchy between the Hubble rates as
\[
H_{\text{osc}}^2 \ll H_{\text{reh}}^2, \quad \frac{3}{20N_*} H_{\text{inf}}^2,
\] and using the expression (3.23) for \( H_{\text{osc}} \), one finds
\[
\sigma_{\text{osc}} = \left( \frac{6}{7} \right)^{3/2} \sigma_*, \quad \text{i.e.,} \quad \phi_{\text{osc}} = \frac{6}{7} \phi_*.
\] (3.26)
This explicitly shows that, given that the scalar is located in the large field regime during inflation, i.e. \( \phi_* \gg \lambda \), then the oscillation also starts with an amplitude \( \phi_{\text{osc}} \gg \lambda \) so that the oscillation is initially anharmonic.\(^1\)

One can also rewrite the requirement that the scalar should start its oscillation after decoupling by using (3.23) and (3.26) as
\[
\left( \frac{H_{\text{osc}}}{H_{\text{dec}}} \right)^2 = \frac{63}{2\pi^2} \frac{1}{g_{\text{dec}}} \frac{M_p^2 m^2 \lambda}{T_{\text{dec}}^4 \phi_*} < 1,
\] (3.28)
where we expressed the Hubble rate at decoupling in terms of the temperature.

**Scalar Abundance**

Let us now compute the energy density of the oscillating scalar field. During the anharmonic oscillations along the fractional power-law potential\(^2\) \( V \propto \sigma^{4/3} \), the scalar density redshifts as\(^3\)
\[
\rho_\phi \propto a^{-12/5},
\] (3.29)
and thus the oscillation amplitude damps as
\[
\bar{\sigma} \propto a^{-9/5}, \quad \text{i.e.,} \quad \bar{\phi} \propto a^{-6/5},
\] (3.30)
When the amplitude becomes as small as \( \bar{\phi} \lesssim \lambda \), the scalar undergoes harmonic oscillations along the quadratic potential, hence the scalings become
\[
\rho_\phi \propto a^{-3}, \quad \bar{\phi} \propto a^{-3/2}.
\] (3.31)
\(^1\)The field excursion during the \( N_* \) \( e \)-foldings in the inflation epoch, \( \Delta \phi_* \), can be computed by integrating \( d\sigma/V'(\sigma) = -dt/(3H_{\text{inf}}) \) as
\[
\frac{\Delta \phi_*}{\phi_*} = \frac{N_* m^2 \lambda}{3 H_{\text{inf}}^2 \phi_*} = \frac{20N_*}{21} \left( \frac{H_{\text{osc}}}{H_{\text{inf}}} \right)^2,
\] (3.27)
where we also used (3.23) and (3.26) upon obtaining the far right hand side. Hence one sees that the condition of \( H_{\text{osc}}^2 \ll 3H_{\text{inf}}^2/(20N_*) \) in (3.25) implies that the scalar field is effectively frozen during inflation. If this condition is violated, then the scalar may start oscillating before the end of inflation. We also note that this condition is similar to the second of the slow-varying condition (3.12) during inflation, but with an additional factor of \( N_* \).
\(^2\)We have defined \( \sigma \) to describe the \( \phi > 0 \) regime, but one can similarly introduce an almost canonical field for the large field regime in the negative side \( \phi < 0 \).
\(^3\)A canonical scalar field that coherently oscillates (mostly) along a power-law potential \( V \propto \varphi^s \) can be described as a perfect fluid with an equation of state parameter \( w = (s-2)/(s+2) \) when averaged over the oscillation. Hence its energy density redshifts as \( \rho_\varphi \propto a^{-6s/(s+2)} \) in an expanding universe.
Let us denote the time when the amplitude becomes $\bar{\phi} = \lambda$ by $t_q$, and quantities measured at this time by the subscript “q”. We assume the time $t_q$ to be during the RD epoch. While the scalar undergoes anharmonic oscillations, the relation between the entropy density and the scalar amplitude is obtained from (3.30) as $s \propto a^{-3} \propto \bar{\phi}^{5/2}$, yielding

$$\frac{s_q}{s_{osc}} = \left( \frac{\lambda}{\bar{\phi}_{osc}} \right)^{5/2}.$$  

(3.32)

This, combined with the harmonic redshifting (3.31) and $\rho_{\phi q} \simeq m^2 \lambda^2/2$ gives the scalar density during the harmonic oscillations,

$$\rho_{\phi} = \rho_{\phi q} s_{q} \simeq \frac{1}{2} m^2 \lambda^2 \frac{s_{osc}}{s_q} \frac{s_{osc}}{s_q} = \frac{1}{2} \frac{s}{s_{osc}} \frac{m^2 \bar{\phi}_{osc}^{5/2}}{\lambda^{1/2}} \quad \text{for} \quad t \geq t_q.$$  

(3.33)

In a RD universe, the entropy density can be expressed in terms of the Hubble rate using (3.16) and (3.18). Further using (3.23) and (3.26), the energy density of the oscillating scalar while the universe is RD can also be expressed as

$$\rho_{\phi} \simeq \frac{1}{2} \left( \frac{10}{3} \right)^{3/4} \left( \frac{6}{7} \right)^{13/4} \frac{g_{s*D}}{g_{s*osc}} \frac{g_{*osc}}{g_*} \left( \frac{g_{s*osc}}{g_*} \right)^{3/4} \frac{H^{3/2} m^{1/2} \phi_{13/4}^*}{\lambda^{5/4}}$$  

(3.34)

for $t_q \leq t < t_{eq}$, where $t_{eq}$ denotes the time of matter-radiation equality.

**Decay of Scalar**

After the harmonic oscillation begins, the scalar eventually decays away through the derivative coupling $(\partial_{\mu} \phi) j^{\mu}/f$. The decay modes depend on the particles constituting the current $j^{\mu}$, however the derivative coupling typically includes anomalous couplings to $W \tilde{W}$ and $Z \tilde{Z}$. Here we do not specify the decay channel, and instead parameterize the decay rate of the scalar by

$$\Gamma_{\phi} = \frac{\beta}{64 \pi^3} \frac{m^3}{f^2},$$

(3.35)

with a dimensionless constant $\beta$ which is typically smaller than unity.

The scalar decays when the Hubble expansion rate becomes comparable to the decay rate, i.e., around when $H = \Gamma_{\phi}$. If this happens during $t_q \leq t < t_{eq}$, then the ratio between the scalar energy density and the total density of the universe right before the decay is estimated from (3.34) as

$$r \equiv \left. \frac{\rho_{\phi}}{3 M_p^2 H^2} \right|_{H = \Gamma_{\phi}} = \frac{2^6 \cdot 3^{3/2} \cdot 5^{3/4} \pi^{3/2}}{7^{13/4} \beta^{3/2}} \frac{g_{s*D}}{g_{s*osc}} \frac{g_{s*osc}}{g_*} \left( \frac{g_{s*osc}}{g_*} \right)^{3/4} \frac{f \phi_{13/4}^*}{M_p^2 m^{5/4}},$$

(3.36)

where $g_{(s)*D}$ corresponds to the effective relativistic degrees of freedom right before the decay. Upon obtaining this expression we have considered a RD universe, hence the oscillating scalar (which behaves as pressureless dust) is assumed to be subdominant, i.e. $r < 1$. On the other hand if $r$ as

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4The anharmonic oscillations along the potential $V \propto \sigma^{4/3}$, which is flatter than a quadratic, may lead to the formation of localized configurations of the oscillating scalar field [22, 23, 24]. Here we assumed that such oscillons do not form, but if they do, the final scalar density could be modified from (3.34).
expressed in (3.36) exceeds unity, then it would signify that the scalar dominates the universe before decaying away. However in such cases, the entropy production by the decay of the dominant scalar would dilute the already produced baryon asymmetry.\(^5\)

Here one may wonder whether the scalar could serve as dark matter if it survives until the present; however one can show that a stable scalar would overclose the universe. Neglecting for the moment the decay, the scalar abundance today is computed by substituting the present day entropy density \(s_0\) into (3.33), which can be expressed as
\[
\Omega \phi h^2 = \frac{\rho_\phi h^2}{3M_p^2 H_0^2} \approx 300 \times \left( \sum_i B_i c_i g_i \right)^{-3} \left( \frac{g_{s*} \phi^*}{106.75} \right)^{-5/4} \left( \frac{g_{s*} \phi^*}{106.75} \right)^{3/4} \left( \frac{g_{s*} \phi^*}{106.75} \right)^{-1}
\]
\[
\times \left( \frac{(n_B/s)_{0}}{8.6 \times 10^{-11}} \right)^3 \left( \frac{f}{\lambda} \right) \left( \frac{\phi^*}{\lambda} \right)^{1/2} \left( \frac{H_{\text{dec}}}{H_{\text{osc}}} \right)^{11/2} \left( \frac{f}{T_{\text{dec}}} \right)^2.
\]

Here we have used (3.19) as the baryon-to-entropy ratio today, and also (3.18), (3.28). The dimensionless Hubble constant \(h\) is defined as \(H_0 = 100h\ \text{km sec}^{-1}\ \text{Mpc}^{-1}\). Each of the last four parentheses in the second line is larger than unity, as we have been assuming the initial field value to lie within the range of \(\lambda \ll \phi^* \lesssim f\), the onset of oscillations to be after decoupling, i.e. \(H_{\text{osc}} < H_{\text{dec}}\), and the decoupling temperature to satisfy \(T_{\text{dec}} < f\) (cf. discussions below (3.1)). Hence as long as \(\sum B_i c_i g_i \sim 1\) and \(g_{s*} \sim 100\) at the relevant times, then the (hypothetical) relic abundance of a scalar that creates the observed baryon asymmetry would be as large as \(\Omega \phi h^2 \gtrsim 300\). This indicates that, were it not for the decay, the scalar would dominate the universe well before the standard matter-radiation equality.

3.5 Constraint on Curvaton-like Behaviors

It should also be noted that if the scalar dominates or comes close to dominating the universe before decaying, it would create curvature perturbations à la curvatons [25, 26, 27, 28]. In such cases, although our model blocks baryon isocurvature during baryogenesis, the creation of the curvature perturbations in the end gives rise to baryon isocurvature. Here we estimate this effect and place an upper bound on the density ratio \(r\) upon decay (3.36) as well as the inflation scale.

From (3.34) we see that the scalar density at decay depends on the scalar field value during inflation as \(\rho_\phi|_{H=\Gamma_\phi} \propto \phi^*^{13/4} \propto \sigma_*^{13/6}\). Thus the curvature perturbation produced by the scalar is estimated, up to linear order in the field fluctuation, as
\[
\zeta_\phi \sim \min(r, 1) \left( \frac{\delta \rho_\phi}{\rho_\phi} \right)_{H=\Gamma_\phi} = \min(r, 1) \frac{13}{6} \frac{\delta \sigma_*}{\sigma_*}.
\]

During inflation the field \(\sigma\) is nearly canonical with an effective mass much lighter than the Hubble rate, therefore its fluctuation power spectrum is \(P_{\delta \sigma_*} = (H_{\text{inf}}/2\pi)^2\). The curvature perturbation

\(\footnote{A possibility that we do not pursue in this work is that a baryon asymmetry much larger than in the present universe is originally produced, but gets diluted by the scalar domination. Although the parameter space for such scenarios is expected to be quite narrow, as the scalar can impact the cosmological expansion history by once dominating the universe, it would be interesting to investigate such cases with an eye towards the observational signals the scalar may leave.}

\(\footnote{Strictly speaking, \(g_{s*}\) can also depend on \(\phi_\star\), but we ignore this effect here.}

11
created after the production of the baryon asymmetry sources a baryon isocurvature of \( S_{B\gamma} = 3\zeta_\phi \) (see e.g. Eq. (5.6) of [13]), hence the resulting baryon isocurvature power on the pivot scale is

\[
P_{B\gamma}(k_\star) \sim \left\{ \min(r, 1) \frac{13}{2} \frac{H_{\text{inf}}}{2\pi\sigma_*} \right\}^2 \lesssim 3 \times 10^{-9}.
\] (3.39)

In the far right hand side, we have shown the current upper limit on a scale-invariant and uncorrelated baryon isocurvature perturbation from the Planck CMB results [29]. Thus we obtain an order-of-magnitude constraint on the curvaton-like behavior of the scalar field as

\[
\min(r, 1) \frac{H_{\text{inf}}}{\sigma_*} \lesssim 10^{-4}.
\] (3.40)

One can also require that the curvature perturbation (3.38) produced by the scalar should not exceed the observed amplitude \( P_{\zeta}(k_\star) \sim 2 \times 10^{-9} \), however this gives a bound weaker than (3.40).

### 3.6 Parameter Space

We now look into the parameter space of our model that allows a successful baryogenesis. Let us recall the requirements.

First of all, we have considered a cosmological history where inflation is followed by reheating, and then by the decoupling of the baryon violating interactions, i.e. \( H_{\text{inf}} > H_{\text{reh}} > H_{\text{dec}} \). The baryon isocurvature can be suppressed if the scalar is initially located in the large field regime of \( \lambda \ll \phi_* \lesssim f \). Here, the scale of \( f \) is assumed to be higher than the inflationary Hubble rate as well as the reheating temperature, i.e. \( f > H_{\text{inf}}, T_{\text{reh}} \). This in particular implies \( f > T_{\text{dec}} \). Moreover, the initial value of the canonical field is required to be larger than the field fluctuations obtained during inflation, i.e. \( \sigma_* > H_{\text{inf}} \) (cf. (3.13)), but sub-Planckian, i.e. \( \sigma_* < M_p \) (cf. (3.8)), to avoid the scalar itself from driving inflation or dominating the universe. Then the baryon asymmetry is generated as (3.19), given that the scalar initially obeys the slow-varying conditions (3.12), and also that the baryon violating interactions decouple before the scalar starts its oscillations, i.e. \( H_{\text{dec}} > H_{\text{osc}} \) (cf. (3.28)). Upon computing \( \phi_{\text{osc}} \) we have also assumed a hierarchy between the Hubble rates as \( H_{\text{osc}}^2 \ll H_{\text{reh}}^2 \) and \( H_{\text{osc}}^2 \ll 3H_{\text{inf}}^2/(20N_*) \) (cf. (3.25)); the latter condition implies that the scalar field value is frozen during inflation.

Here, it is easy to check that the three slow-varying conditions of (3.12) follow from the other requirements: The first condition is nothing but the large field requirement. The second condition is guaranteed to hold at least until around decoupling from \( H_{\text{dec}} > H_{\text{osc}} \). As for the third condition which is required during baryogenesis, we have already seen in (3.20) that it is automatically satisfied for \( \lambda \ll \phi_* \) and \( f > T \).

In order to avoid the baryon asymmetry from being diluted after the scalar starts to oscillate, the scalar is required to decay away before dominating the universe, i.e. \( r < 1 \), where the density ratio \( r \) at decay is given in (3.36). Here the scalar is considered to decay during its harmonic oscillations, thus we have assumed its decay rate to satisfy \( \Gamma_\phi < H_q \), where \( H_q \) can be obtained from (3.32). The condition \( r < 1 \) guarantees the scalar to decay before the matter-radiation equality, but in order for the scalar not to spoil Big Bang Nucleosynthesis (BBN), we further require the scalar to either decay before BBN, or have negligible density during the period. The ratio \( r \), combined with the
inflation scale, is further bounded as (3.40) from the constraint on the curvaton-like behavior of the scalar.

In Figure 2 we show the viable parameter space in the \( m - f \) plane. Here the dimensionless parameters are chosen as \( \beta = 1 \), and the relativistic degrees of freedom is considered to be \( g_{*s} = 106.75 \) from the time of decoupling until the decay of the scalar. We have also fixed the decoupling temperature \( T_{\text{dec}} \) from the requirement that the final baryon-to-entropy ratio (3.19) matches the present value of \( (n_B/s)_0 \approx 8.6 \times 10^{-11} \). Furthermore, the values of \( \lambda \) and \( \phi_* \) are fixed in terms of \( f \) so that they satisfy \( \lambda \ll \phi_* \lesssim f \); we show example cases where \( \lambda = 10^{-4} f \), \( \phi_* = f \) (left panel) and \( \lambda = 10^{-4} f \), \( \phi_* = 10^{-2} f \) (right panel). The solid lines in the figures represent the conditions that do not involve the inflation scale \( H_{\text{inf}} \), which are the requirements of decoupling before the onset of the scalar oscillations \( H_{\text{osc}} > H_{\text{inf}} \) (green line), the scalar decay before dominating the universe \( r < 1 \) (purple line), and \( f > T_{\text{dec}} \) (pink line). The triangular regions surrounded by these solid lines satisfy all three requirements. We also note that in the entire triangular regions, the field value is sub-Planckian \( \sigma_* < M_p \), and the decay rate of the scalar satisfies \( H_{\text{BBN}} < \Gamma_\phi < H_q \) so that the scalar decays after starting its harmonic oscillations, but before BBN.

The allowed windows further shrink when taking into account the conditions that involve the inflation scale \( H_{\text{inf}} \). The colored regions show the parameter space satisfying all conditions, under fixed values of \( H_{\text{inf}} \). Here we have also numerically computed the baryon isocurvature perturbations that arise due to deviations from the slow-varying solution, which was discussed below (3.19). We numerically solved the scalar’s equation of motion starting from initial conditions of \( \sigma_* \) that vary by \( H_{\text{inf}}/2\pi \), and estimated the baryon isocurvature by computing the difference arising in the chemical potential \( \mu \propto \dot{\phi} \) at the time of decoupling.\(^7\) In corners of the parameter space where some conditions are only marginally satisfied (such as cases where the slow-varying conditions (3.12) start to break down at decoupling), the baryon isocurvature can become non-negligible, as we see below.

In the left panel for \( \lambda = 10^{-1} f \) and \( \phi_* = f \), we show the viable parameter space under \( H_{\text{inf}} = 10^9 \text{ GeV} \) (red shaded region), \( 10^{10} \text{ GeV} \) (yellow shaded region), and \( 10^{11} \text{ GeV} \) (blue shaded region). Note that the regions overlap with each other; in particular the left part of the blue region is on top of the yellow region, whose left part is on top of the red region. Thus the red region actually occupies most of the triangular region surrounded by the solid lines. The right-side boundaries of each colored regions are determined by the condition of \( H_{\text{inf}} > H_{\text{osc}} \), while the upper left boundaries are from the constraint (3.40) on the curvaton-like behavior. In the lower left corner of the triangular region, the ratio \( H_{\text{inf}}/\sigma_* \) is not too small, and moreover the condition \( H_{\text{dec}} > H_{\text{osc}} \) is only marginally satisfied.\(^8\) This implies that the scalar at the time of decoupling is starting to deviate from the slow-varying solution, and hence a non-negligible fraction of the large field fluctuations can leak into the baryon isocurvature. Comparison of the numerically computed baryon isocurvature with the \textit{Planck} bound trims off the lower left corners of the windows for \( H_{\text{inf}} = 10^{10} \text{ GeV} \) and \( 10^{11} \text{ GeV} \), as shown in the

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\(^7\)We remark that the value of the reheating temperature, as long as it satisfies the conditions mentioned above, has little effect on the amplitude of the baryon isocurvature. We also note that we have neglected the backreaction from the baryons in the numerical computations.

\(^8\)If \( H_{\text{osc}} \) is close to \( H_{\text{dec}} \), the rapid rolling of the scalar towards the end of baryogenesis may also lead to a large time-variation of the chemical potential \( \mu \). It was discussed in [30] that in such cases the computation of the baryon production can be modified.
$H_{\inf} = 10^9$ GeV
$H_{\inf} = 10^{10}$ GeV
$H_{\inf} = 10^{11}$ GeV

Figure 2: Parameter space for the $n = 1$ spontaneous baryogenesis model in the $m - f$ plane. The viable parameter space is shown by the colored regions, where the different colors correspond to different inflation scales. The solid lines represent the requirements for the model: decoupling before the onset of scalar oscillations $H_{\text{dec}} > H_{\text{osc}}$ (green), decay of the scalar before dominating the universe $r < 1$ (purple), and $f > T_{\text{dec}}$ (pink). The black dashed lines indicate where the model may become sensitive to the UV theory. The other conditions are explained in the text.

In the right panel for $\lambda = 10^{-4} f$ and $\phi_* = 10^{-2} f$, we show the viable parameter space for $H_{\inf} = 10^{11}$ GeV (red shaded region), $10^{12}$ GeV (yellow shaded region), and $10^{13}$ GeV (blue shaded region). The right-side boundaries of the windows for $H_{\inf} = 10^{11}$, $10^{12}$ GeV are from the condition $H_{\text{osc}}^2 < 3H_{\inf}^2/(20N_*)$, while the right-side boundary for $H_{\inf} = 10^{13}$ GeV is from $H_{\inf} > H_{\text{dec}}$ (which of these conditions is stronger depends on the inflation scale). The upper left boundaries for all three cases are from the constraint (3.40) on the curvaton-like behavior, and the lower left boundaries are from the constraint on the baryon isocurvature arising from deviations from the slow-varying trajectory. The parameter window here exists for inflation scales up to $H_{\inf} \sim 10^{14}$ GeV, which implies that spontaneous baryogenesis can be compatible even with high inflation scales close to the current observational upper limit [29].

Note also that each point in the colored regions possesses a range for the reheating scale that satisfies $H_{\inf} > H_{\text{reh}}$ and $f > T_{\text{reh}} > T_{\text{dec}}$. However, we should also remark that the allowed range of $T_{\text{reh}}$ becomes small in regions close to the boundaries where the conditions such as $H_{\inf} > H_{\text{dec}}$ or $f > T_{\text{dec}}$ are only marginally satisfied. In such corners of the parameter space, one will have to require an instantaneous reheating and/or the baryon violating processes to decouple soon after reheating.

We further remark that the model may become sensitive to the UV completion of the theory.
in some parts of the parameter space. Such regions are indicated by the black dashed lines in the figures, which we will discuss in the next section.

### 3.7 Power Counting Estimate of Cutoff

Although the main focus of the present paper is on the phenomenology of spontaneous baryogenesis with a $\phi$-independent chemical potential, let us briefly comment on the validity of the theory (3.1) by estimating its cutoff scale, above which the effective field theory breaks down and new physics should intervene. Here we estimate a lower bound on the cutoff through power counting analyses as was discussed in, e.g., [31, 32, 33]. The cutoff depends on the background scalar value, hence let us focus on the large field regime $\sigma \gg \lambda$ during spontaneous baryogenesis, and expand the almost canonical $\sigma$ around its background value as $\sigma = \sigma_{\text{bg}} + \hat{\sigma}$. Then the scalar potential (3.4) yields operators involving $p$ powers of $\hat{\sigma}$ as

$$m^2 \lambda^2 \frac{\hat{\sigma}^p}{\sigma_{\text{bg}}^{p-\frac{4}{3}}}.$$

(3.41)

where we ignored numerical coefficients. Hence from operators with dimension $p$ larger than four, the energy scales above which perturbation theory breaks down can be read off as

$$\Lambda_1(p) \sim \left( \frac{\sigma_{\text{bg}}^{p-\frac{4}{3}}}{m^2 \lambda^2} \right)^{\frac{1}{p-4}}.$$

(3.42)

Likewise, the derivative coupling to the baryon current in (3.3), after integration by parts, yields operators with $p \geq 5$ of the form

$$\frac{\lambda^4}{f} \frac{\hat{\sigma}^{p-4}}{\sigma_{\text{bg}}^{p-\frac{14}{3}}} \sum_i c_i \nabla_{\mu} j_{i\mu},$$

(3.43)

with which perturbative analyses are expected to break down above

$$\Lambda_2(p) \sim \left( \frac{f \sigma_{\text{bg}}^{p-\frac{14}{3}}}{\lambda^4} \right)^{\frac{1}{p-4}}.$$

(3.44)

These estimates imply the cutoff of the theory as $\Lambda = \min_{p \geq 5}(\Lambda_1(p), \Lambda_2(p))$. Here, note that both $\Lambda_1(p)$ and $\Lambda_2(p)$ asymptote to $\sigma_{\text{bg}}$ in the large $p$ limit.

Representing the typical energy scale during baryogenesis by the decoupling temperature, let us now compare it to $\Lambda$. In Figure 2, by taking $\sigma_{\text{bg}} = \sigma_*$, we have indicated where $T_{\text{dec}} = \Lambda$ by the black dashed lines; $T_{\text{dec}} < \Lambda$ is satisfied on the left sides of the lines. For the chosen sets of parameters, and further in the regions of $m < f$ where the allowed windows exist, $\Lambda_{1,2}(p)$ are either independent of $p$, or become smaller for larger $p$. Hence $\Lambda = \Lambda_{1,2}(p \to \infty) \sim \sigma_{\text{bg}}$. In Figure 2(a), one sees that $T_{\text{dec}}$ is safely below $\Lambda$ in the entire allowed window. On the other hand in Figure 2(b), $T_{\text{dec}}$ exceeds $\Lambda$ in some part of the window; there the model may be sensitive to the UV completion of the theory. One can further estimate the cutoff in the small field regime $|\phi| \ll \lambda$ during the scalar oscillations and require it to be larger than the oscillation energy $\rho_{\phi}^{1/4} \sim m^{1/2}\phi^{1/2}$, but this turns out to yield weaker constraints on the allowed windows compared to those during baryogenesis.
However, we also remark that the constraint of $T_{\text{dec}} < \Lambda$ should be considered as a rather conservative bound, as the typical energy scales of the scattering processes described by the operators (3.41) and (3.43) may actually be lower than $T_{\text{dec}}$. Furthermore, provided the theory is endowed with some extra symmetries, the naive power counting can fail and the cutoff can be much higher than the $\Lambda$ estimated above. We leave a more detailed analysis of these issues, including explicit computations of the scattering amplitudes, for future work.

4 n = 2: Spontaneous Baryogenesis with Linear Terms

In this section we study spontaneous baryogenesis driven by a scalar with a $Z_2$ symmetric action of

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \left\{ 1 + \left( \phi \lambda^2 \right) \right\} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 - \frac{1}{2} \left( \phi \frac{f}{f^*} \right)^2 \sum_i c_i \nabla_\mu j_i^\mu \right], \quad (4.1)$$

which is phenomenologically similar to the case of $n = 2$ in (2.1). The field that becomes canonical in the large field regime of $\phi \gg \lambda$ is

$$\sigma = \frac{1}{2} \phi^2 \lambda, \quad (4.2)$$

with which the action is rewritten as, after integrating by parts the coupling term,

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \left( 1 + \frac{\lambda}{2\sigma} \right) g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - m^2 \lambda \sigma + \frac{\lambda \partial_\mu \sigma}{f^2} \sum_i c_i j_i^\mu \right]. \quad (4.3)$$

Hence this model in the large field regime reduces to a theory with a linear potential as well as a linear coupling to the baryon current.

The basic picture is the same as for the $n = 1$ case studied in the previous section, hence here we just list the relevant results that can be obtained from similar computations. The final baryon-to-entropy ratio is now

$$\frac{n_B}{s}_{\text{dec}} = \frac{9}{2 \pi^3} \left( \frac{5}{8} \right)^{1/2} \frac{\sum_i B_i c_i g_i}{g_{s\text{dec}} g_{s\text{dec}}} \frac{M_p m^2 \lambda^2}{T_{\text{dec}}^3 f^2}, \quad (4.4)$$

and one can also check that the ratio between the Hubble rates at the onset of the oscillations and decoupling, which should be smaller than unity, is

$$\left( \frac{H_{\text{osc}}}{H_{\text{dec}}} \right)^2 = \frac{45}{\pi^2} \frac{1}{g_{s\text{dec}}} \frac{M_p^2 m^2 \lambda^2}{T_{\text{dec}}^3 f^2} < 1. \quad (4.5)$$

During the anharmonic oscillation along the linear potential, the scalar density and oscillation amplitude redshift as

$$\rho_\phi \propto a^{-2}, \quad \sigma \propto a^{-2}, \quad \phi \propto a^{-1}, \quad (4.6)$$

and the scalar density after the harmonic oscillation begins is obtained as

$$\rho_\phi \approx \frac{2^{11/4}}{5^{3/2}} \frac{g_{s*}}{g_{s\text{osc}}} \left( \frac{g_{s\text{osc}}}{g_*} \right)^{3/4} \frac{H_{\text{osc}}^3/2 m^{1/2} \phi_{*}^{3/2}}{\rho_{*}^{5/2}} \frac{\lambda^{5/2}}{\phi_{*}^{9/2}} \quad \text{for} \quad t_q \leq t < t_{eq}, \quad (4.7)$$

16
Figure 3: Parameter space for the $n = 2$ spontaneous baryogenesis model in the $m - f$ plane. The viable parameter space is shown by the colored regions, where the different colors correspond to different inflation scales. The solid lines represent the requirements for the model: decoupling before the onset of scalar oscillations $H_{dec} > H_{osc}$ (green), decay of the scalar before dominating the universe $r < 1$ (purple), and $f > T_{dec}$ (pink). The black dashed line indicates where the model may become sensitive to the UV theory. The other conditions are explained in the text.

As in the case of $n = 1$, were it not for the decay, the oscillating scalar would overclose the universe well before the standard matter-radiation equality. However, since the Lagrangian for $\phi$ is now $Z_2$ symmetric, the scalar condensate could be stable and thus be disastrous for cosmology.\textsuperscript{9}

Hence let us suppose that the $Z_2$ symmetry is slightly broken so that the minima of the quadratic potential and the derivative coupling term are misaligned by $\Delta \phi \sim \lambda$. Then, expanding the coupling term around the potential minimum gives rise to $\sim (\lambda \phi / f^2) \sum_i c_i \nabla_\mu j_i^\mu$, providing a decay channel for $\phi$. Expressing the decay rate by

$$\Gamma_\phi = \frac{\beta}{64\pi^3} \frac{m^3 \lambda^2}{f^4}, \quad (4.8)$$

with a dimensionless parameter $\beta$ which is typically smaller than unity, one can compute the density ratio right before the decay of the scalar as

$$r \equiv \frac{\rho_\phi}{3 M_p^2 H^2} \bigg|_{H = \Gamma_\phi} = \frac{2^{23/4} \pi^{3/2}}{3 \cdot 5^{3/2}} \frac{g_{s*D}}{g_{s*osc}} \left( \frac{g_{s*osc}}{g_{s*D}} \right)^{3/4} \frac{f^2 \phi_\star^{9/2}}{M_p^2 m^{7/2}}, \quad (4.9)$$

given that the scalar decays during $t_q \leq t < t_{eq}$.

\textsuperscript{9}Although, depending on the parameters, the scalar $\phi$ may dissipate its energy through the $\phi^2 \nabla_\mu j^\mu$ interaction. For detailed discussions on $Z_2$ symmetric scalars, see e.g. [34] and references therein.
The baryon-to-entropy ratio (4.4) and the decay rate (4.8) in the $n = 2$ model are suppressed by powers of $(\lambda/f)$ compared to the case of $n = 1$, cf. (3.19) and (3.35). As a consequence, the parameter space is narrower for $n = 2$. The $m - f$ parameter space for the $n = 2$ model is shown in Figure 3. The dimensionless parameters are chosen as $\sum_i B_i c_i g_i = 1$, $\beta = 1$, $g(\sigma)* = 106.75$, and the decoupling temperature $T_{\text{dec}}$ is fixed by normalizing the baryon-to-entropy ratio (4.4) to the present value $(n_B/s)_0 \approx 8.6 \times 10^{-11}$. Furthermore, $\lambda$ and $\phi*$ are fixed as $\lambda = 10^{-1} f$, $\phi* = f$. The conditions that do not involve $H_{\text{inf}}$ are represented by the solid lines: $H_{\text{dec}} > H_{\text{osc}}$ (green line), $r < 1$ (purple line), and $f > T_{\text{dec}}$ (pink line). The triangular region surrounded by these solid lines satisfy the three requirements, as well as $\sigma* < M_P$ and $H_{\text{BBN}} < \Gamma_T < H_0$. After further taking into account the conditions on $H_{\text{inf}}$, then the viable parameter space satisfying all conditions are shown as the colored regions. In each colored region, the inflation scale is fixed to $H_{\text{inf}} = 10^9 \text{GeV}$ (red shaded region), $10^{10} \text{GeV}$ (yellow shaded region), $10^{11} \text{GeV}$ (blue shaded region). Here, the constraints that set further boundaries inside the triangular region are: $H_{\text{inf}} > H_{\text{dec}}$ (right-side boundary for $H_{\text{inf}} = 10^9 \text{GeV}$), the constraint on the curvaton-like behavior (upper left boundaries), and the constraint on the baryon isocurvature arising from deviations from the slow-varying trajectory (lower boundaries for $H_{\text{inf}} = 10^{10}, 10^{11} \text{GeV}$). Comparing with Figure 2(a) of the $n = 1$ case, one sees that the parameter window is now narrower, which can also be seen from comparing the powers of $\lambda$ in the conditions (4.5), (4.9) with the corresponding (3.28), (3.36). (Note also the dependence of $T_{\text{dec}}$ on $\lambda$ after fixing the baryon-to-entropy ratio (4.4), (3.19).) When the ratio $\lambda/f$ is smaller than $10^{-1}$, the window for $n = 2$ becomes much narrower than for $n = 1$.

We have also estimated the cutoff of the theory by power counting. For the parameters in the figure, both the derivative coupling and the $Z_2$-breaking terms give a cutoff in the large field regime during baryogenesis as $\Lambda \sim \sigma*$. The condition $T_{\text{dec}} < \Lambda$ is satisfied on the left side of the black dashed line, hence one sees that $T_{\text{dec}}$ is below the cutoff in the entire allowed window.

5 Conclusions

In this work, we proposed a new class of spontaneous baryogenesis models that does not produce baryon isocurvature perturbations during the generation of the baryon asymmetry. The basic idea is that when the baryon-generating scalar possesses a potential and a coupling satisfying the condition (1.1), the induced baryon chemical potential is independent of the scalar field and thus becomes spatially homogeneous.\textsuperscript{10} We demonstrated this mechanism in models that involve non-canonical scalar fields with actions of the form (2.1) which, after canonical normalization, reduces to a theory satisfying (1.1). The cosmological history in these non-canonical models is similar to that of vanilla spontaneous baryogenesis, except for that the scalar field undergoes anharmonic oscillations along a non-quadratic potential and hence its density redshifts in a specific way. We analyzed in detail the baryon generation and the cosmology for the models of $n = 1$ and 2, and investigated the parameter space that gives rise to successful baryogenesis. We found that our models allow scalar masses and couplings, as well as decoupling and inflation scales, in regions that can be quite different from

\textsuperscript{10}One may wonder whether a similar technique could also be used to suppress isocurvature perturbations of scalar field dark matter, such as axions. In order to achieve this, the field fluctuation should be blocked from producing fluctuations in the dark matter density $V(\sigma)$, which seems difficult unless there is overtaking such that fields starting higher up in the potential reaches the potential minimum faster.
those in vanilla scenarios (see e.g. [13] for comparison). In particular, the suppression of the baryon isocurvature allows spontaneous baryogenesis with inflation scales up to $H_{\text{inf}} \sim 10^{14} \text{GeV}$. This implies that, even if inflationary gravitational waves are detected in the near future, spontaneous baryogenesis would not be ruled out.

Clearly an important direction for further study is to realize the setup for isocurvature suppression within fundamental physics constructions. The Nambu–Goto action of branes could serve as a potential candidate for realizing the non-canonical kinetic term, as in the D-brane monodromy inflation models of [16, 17]. We should also note that the runnings of the couplings may affect the condition (1.1). In this paper we have discussed the validity of the effective field theory using power counting arguments, however it is also important to analyze in more detail the stability of the theory against radiative corrections. We leave these investigations for future work.

Although we have focused on non-canonical scalar fields, the mechanism of suppressing the baryon isocurvature is much more general. It would also be interesting to explore other ways to realize the scalar-independent chemical potential of (1.1). We stress that in these models without baryon isocurvature, the resulting baryon asymmetry is independent of the scalar field value. This gives predictive power to the model, especially in cases where the baryon-generating scalar arises as a Nambu–Goldstone boson of a spontaneously broken symmetry and thus has random initial conditions.

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