A simple extension of SM that can explain the \((g - 2)_{\mu}\) anomaly, small neutrino mass and dark-matter.

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Abstract

In this work we propose a simple extension of standard-model (SM) by adding nine new particles to it. One heavy lepton \((f)\) singlet under the \(SU(3)_c \times SU(2)_L\) carrying Lepton Number \((N_e, N_\mu, N_\tau) = (0, 1, 0)\) and charged under the \(U(1)_Y\) with \(Y = -2\) and transforming under a discrete symmetry as \(f \to -f\). One scalar \((\phi_2)\), singlet under all the SM gauge groups and transforming under the discrete symmetry as \(\phi_2 \to -\phi_2\) which does not develops a non zero vacuum-expectation-value (VEV). One more scalar \((\phi_3)\), singlet under all the SM gauge groups and invariant under the discrete symmetry which develops a non zero VEV \((v_3)\) and gives masses to \(f\), \(\phi_2\) and neutrinos. Three right-handed neutrinos \((\nu_R)\) and three left-handed Majorana neutrinos \((s_L)\). With these new additional particles added to SM we have been able to give explanations to the long standing \((g - 2)_{\mu}\) anomaly as well as the smallness of neutrino masses by the inverse see-saw mechanism. And also in this model we have a very suitable scalar dark-matter (DM) candidate in \(\phi_2\) with allowed mass as high as 53 GeV, although due to large Yukawa coupling, its contribution to the DM relic density turn out to be too small and so it can account only a small fraction of the DM relic density of the universe.

1 Introduction.

Dirac equations for a charged spin half muon predicts a magnetic moment, \(\vec{M} = g_\mu \frac{e}{2m_\mu} \vec{S}\), with the gyromagnetic ratio \(g_\mu = 2\). But quantum loop effects leads to small calculable correction to the \(g_\mu\), which make its true value deviates from 2, parameterized by the anomalous magnetic moment of muon

\[
a_\mu = \frac{g_\mu - 2}{2}.
\]
The $a_\mu$ can be accurately measured and in a given model such as Standard Model (SM), it can be precisely predicted. Hence $a_\mu$ is a very good laboratory to test SM predictions at its quantum loop level. Any deviation from SM prediction would signal presence of New Physics (NP), with current sensitivity reaching up to mass scale of $\mathcal{O}(\text{TeV})$ [1][2]. The latest PDG world average of $a_\mu$ is $\[3\]

\[a_\mu^{\text{exp}} = 11659209.1(5.4)(3.3) \times 10^{-10}.\] (2)

There are presently two upcoming experiments at Fermi-Lab and JPARC, where their goal is to improve the $a_\mu$ measurement by a factor of four or higher.

The SM prediction of $a_\mu$ is usually given as

\[a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{Had}},\] (3)

where the dominant QED part has been calculated in SM to 5 loop and is $\[4\]

\[a_\mu^{\text{QED}} = 116584718.95(0.08) \times 10^{-11},\] (4)

the electroweak (EW) part has been calculated in SM to 3 loop and is $\[3\]

\[a_\mu^{\text{EW}} = 153.6(1.0) \times 10^{-11},\] (5)

and the Hadronic (quark and gluon) part contains the main theoretical uncertainties and is $\[4\]

\[a_\mu^{\text{Had}}(\text{LO}) = 7015(42)(19)(3) \times 10^{-11},\] (6)

and

\[a_\mu^{\text{Had}}(\text{NLO}) = 7(26) \times 10^{-11}.\] (7)

Therefore combining the QED, EW and Hadronic parts together we have the SM prediction of $a_\mu$ $\[3\]

\[a_\mu^{\text{SM}} = 116591803(1)(42)(26) \times 10^{-11},\] (8)

where the errors are due to the EW, lowest-order hadronic, and higher-order hadronic contributions, respectively. The difference between the experimental value and the SM value is

\[\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 288(63)(49) \times 10^{-11},\] (9)

and when all the errors are added in quadrature, the deviation of the experimental value from the SM prediction amount to $3.6 \sigma$ $\[3\]$.
There has been many NP models proposed to explain the anomaly in $a_\mu$, one of them being the contribution to it from supersymmetry particles \[1\], another NP model proposed is the “dark photon” scenarios given in \[5\]-\[7\]. It has also been shown in \[8\]-\[9\] that the lepton specific (Type-X) two-Higgs-doublet model (2HDM) can give significant enhancement of $a_\mu$. More recently in \[10\], it has been shown that in a muon specific 2HDM, the anomaly can be reduced within 1σ provided $\tan\beta$ is very large. For more comprehensive coverage see \[13\] and the recent review by the PDG group \[3\] and the references there in.

2 Model.

2.1 Lagrangian.

To SM we add three new particles, a $SU(3)_c \times SU(2)_L$ singlet heavy lepton (f) transforming under a discrete symmetry as $f \rightarrow -f$ and charged under the SM $U(1)_Y$ with $Y = -2$ carrying Lepton-number $(N_e, N_\mu, N_\tau) = (0, 1, 0)$. One $SU(3)_c \times SU(2)_L \times U(1)_Y$ singlet scalar ($\phi_2$) also transforming under the discrete symmetry as $\phi_2 \rightarrow -\phi_2$, whose Vacuum-Expectation-Value (VEV) is zero. And one more $SU(3)_c \times SU(2)_L \times U(1)_Y$ singlet scalar ($\phi_3$) transforming under the discrete symmetry as $\phi_3 \rightarrow +\phi_3$, which develops a non zero VEV $v_3$, and gives masses to f and $\phi_2$ and neutrinos. All other SM particles are invariant under the discrete symmetry transformation. Then the Yukawa sector which is invariant under all the above symmetry transformations is given as

$$L_{Yukawa} = Y_2 \bar{\mu}_R f L \phi_2 + Y_3 \bar{f}_R f L \phi_3 + h.c. \quad (10)$$

Now constrains from heavy charged lepton searches \[14\], which is mainly focus on $f^\pm \rightarrow \nu W^\pm$ or $f^\pm \rightarrow l^\pm Z$ decay types, has ruled out $m_f \leq 100.8$ GeV at 95% CL. So from Eqs.\((10)\), it is clear that if $\phi_2$ develops VEV, then f will mix with $\mu$ to diagonalize the mixed mass generated from the first term in the Eqs.\((10)\) and therefore the bounds from heavy charged lepton searches applies. But if $\phi_2$ does not develops VEV, then heavier lepton f will not mix with the $\mu$ and the bounds from the search for heavy charged leptons does not apply as it can not decay into the final states that those experiments looked for.

Then the general Lagrangain invariant under all the symmetry transformations for the new particles can be written as

$$L_{NP} = L_{Kinetic} - V(H, \phi_2, \phi_3) + L_{Yukawa} \quad (11)$$

where

$$L_{Kinetic} = \bar{f} \gamma^\mu (iD_\mu) f + (i\partial_\mu \phi_2)^\dagger (i\partial^\mu \phi_2) + (i\partial_\mu \phi_3)^\dagger (i\partial^\mu \phi_3) \quad (12)$$
As mentioned previously, if LHC is sensitive to Higgs decays such as $h \rightarrow \text{missing mass}$, then from Eqs. (12) we can express the interaction of Higgs decay into missing masses i.e

$$V(H, \phi_2, \phi_3) = m^2 H^\dagger H + m_2^2 \phi_2^\dagger \phi_2 + m_3^2 \phi_3^\dagger \phi_3 + \lambda_2/2(H^\dagger H)^2 + \lambda_3/2(\phi_3^\dagger \phi_3)^2 + m_{13}(H^\dagger H)\phi_3 + m_{23}(\phi_2^\dagger \phi_2)\phi_3 + \mu_3\phi_3^\dagger \phi_3. \tag{13}$$

Now since we require that the VEV of $\phi_2$ to be zero, that can be guaranteed if $m_2 = 0$ and $m_{23}$, $\lambda_{12}$, $\lambda_{23}$ and $\lambda_2$ are all real and of same sign. If LHC is sensitive to the Higgs decays such as $h \rightarrow$ missing masses, then the couplings $\lambda_{12}$ and $\lambda_{13}$ can introduce terms such as $\lambda_{12}(v_0 + h)^2\phi_2^\dagger \phi_2$ and $\lambda_{13}(v_0 + h)^2(v_3 + h_3)^2$ which, besides contributing to the masses of $\phi_2$, $\phi_3$ and Higgs itself, can induce Higgs decay into missing masses i.e $h \rightarrow$ missing masses $(\phi_2^\dagger \phi_2)$ and $h \rightarrow$ missing masses $(h_3h_3)$ provided $m_h > 2m_{\phi_3}$, where $v_0$ and $v_3$ are the VEV of the SM Higgs and $\phi_3$ respectively. One main constrain on the mass of the $h_3$ comes from the onshell Z decay $Z \rightarrow \gamma h_3$ via the triangle loop, which can affect the Z width, since no deviation has been reported in the Z decay width, we can avoid the decay if we require $m_{h_3} > m_Z$.

With the $U(1)_Y$ gauge boson $B_\mu$ expressed in terms of $Z_\mu$ and $A_\mu$ as

$$B_\mu = - \sin \theta_W Z_\mu + \cos \theta_W A_\mu, \tag{14}$$

and from Eqs. (12) we can express the interaction of $Z_\mu$ and $A_\mu$ with $\bar{f} \gamma^\mu f$ current as

$$\bar{f} \gamma^\mu i(-i \frac{g'}{2} B_\mu) f = -e \bar{f} \gamma^\mu (\tan \theta_W Z_\mu + A_\mu) f, \tag{15}$$

where $g' = \frac{e}{\cos \theta_W}$. From the above equation we can see that this heavier muon can be produced in colliders as $e^+ e^- \rightarrow f^+ f^-$ at $e^+ e^-$ colliders and as $pp \rightarrow Z^* / \gamma^* \rightarrow f^+ f^-$ at LHC.

2.2 Scalar quartic couplings.

Imposing the condition $m_2 = 0$ and $m_{23}$, $\lambda_{12}$, $\lambda_{23}$ and $\lambda_2$ are all real and of same sign, to make sure the $\phi_2$ does not develop a non zero VEV, we can write the scalar potential from Eqs. (13) as

$$V(H, \phi_2, \phi_3) = m^2 H^\dagger H + m_2^2 \phi_2^\dagger \phi_2 + m_3^2 \phi_3^\dagger \phi_3 + \lambda_2/2(H^\dagger H)^2 + \lambda_3/2(\phi_3^\dagger \phi_3)^2 + m_{13}(H^\dagger H)\phi_3 + m_{23}(\phi_2^\dagger \phi_2)\phi_3 + \mu_3\phi_3^\dagger \phi_3. \tag{16}$$

As mentioned previously, if LHC is sensitive to Higgs decays such as $h \rightarrow$ missing mass $(\phi_2^\dagger \phi_2)$ and $h \rightarrow$ missing mass $(h_3h_3)$, then the couplings $\lambda_{12}$ and $\lambda_{13}$ can be probed at LHC provided
$m_h > 2m_{\phi_3}$. A special case that is interesting is where the couplings $\lambda_{12}$ and $\lambda_{13}$ are very small and terms containing them can be neglected, then the SM Higgs decouples from the all the new particles. In this limit, the mass generation of $\phi_3$ is similar to SM Higgs as can be deduced from the form of the potential above with $\lambda_{12} \approx 0$ and $\lambda_{13} \approx 0$, which is similar to SM Higgs potential except the term $\lambda_{23}(\phi_2^\dagger \phi_2)(\phi_3^\dagger \phi_3)$. This term give mass to the $\phi_2$ where we have $\phi_3 = v_3 + h_3$ after it develops a non zero VEV $v_3$, and also it generate a $\lambda_{23}2v_3(h_3)(\phi_2^\dagger \phi_3)$ term which allows the $h_3$ to decay into $\phi_2 \bar{\phi}_2$ as $h_3 \rightarrow \phi_2 \bar{\phi}_2$. And the form of the scalar potential of $\phi_3$ has the structure of the inflaton field.

2.3 Contribution to $(g-2)_\mu$ in the model.

In [12], the contribution from a scalar ($\phi$) and a charged lepton $f$ to the muon anomalous magnetic moment has been calculated in an arbitrary gauge for an interaction Lagrangian given as

$$L = \bar{\mu}(C_S + C_P \gamma^5)f \phi,$$

and they give in Eqs.(11) of [12]

$$[a_\mu] = \frac{-g_fm_y^2}{8\pi^2} \int_0^1 dx Q_\phi(x),$$

where

$$Q_\phi(x) = \frac{[C_S^2(x^2 - x^3 + \frac{m_f^2}{m_\mu^2}x^2) + C_P^2(m_f \rightarrow -m_f)]}{m_\mu^2x^2 + (m_f^2 - m_\mu^2)x + m_{\phi_2}^2(1 - x)}.$$ (19)

For the case relevant to the model in this work the above formula reduces to

$$[a_\mu]_{NP} = \frac{m_y^2Y_f^2}{16\pi^2} \int_0^1 dx \frac{x^2 - x^3}{m_\mu^2x^2 + (m_f^2 - m_\mu^2)x + m_{\phi_2}^2(1 - x)},$$

(20)

where we have $C_S = -C_P = \frac{Y_f}{2}$ and $q_f = -1$. Now $m_{\phi_2} \geq m_f$ is not allowed because then we will have a stable long live charged particle ($f^\pm$) in our model, which will contradict the non-observations of any signatures from such a heavy long lived charged particles in the past experiments. So the most reasonable condition that do not contradict any observational fact is to assume $m_f >> (m_\mu, m_{\phi_2})$, which allow $\phi_2$ to be a possible dark matter candidate. With this condition on the masses, the NP contribution to the muon anomalous magnetic moment is given as

$$\delta a_\mu = [a_\mu]_{NP} \approx \frac{m_y^2Y_f^2}{16\pi^2} \frac{1}{2} - \frac{1}{3} = \frac{m_y^2Y_f^2}{6 \times 16\pi^2 m_f^2}.$$ (21)

Then from the 1$\sigma$ of the central value of $\delta a_\mu$ from the Eqs.[12] we get

$$\frac{Y_f}{m_f} = 0.0133 \text{ GeV}^{-1},$$ (22)
and from the perturbativity condition, i.e \( Y^2 \leq 1 \), within 1\( \sigma \) of the central value of \( \delta a_\mu \), the maximum allowed value of \( m_f \) is 75.19 GeV. Non observations of tracts of any long lived heavy charged particles in the past experiments indicates that \( m_f \) should be close to this upper limit, as then its Yukawa coupling would be large, and it could have decayed immediately after its production and no observable tracts are left. However in this case if \( m_Z > 2m_{\phi_2} \) then the Z decay \( Z \to \phi_2\phi_2 \), via the triangle loop, can occur and expected to be large and so this case is already ruled out by the total Z decay width. For the case where \( m_f \sim m_{\phi_2} \gg m_\mu \), within 1\( \sigma \) of the central value of \( \delta a_\mu \), we get \( \frac{Y_2}{m_f} = 0.0188 \text{GeV}^{-1} \) with maximum allowed value of \( m_f \) (also \( m_{\phi_2} \)), from the perturbativity i.e \( Y_2 = 1 \), is 53.19 GeV, so \( Z \to \phi_2\phi_2 \) is forbidden by kinematics with the condition \( m_f > m_{\phi_2} + m_\mu \) satisfied.

### 2.4 Small neutrino masses and dark-matter.

If we add three right handed SM singlets \( \nu_R \) to the three left handed neutrinos in SM, then neutrinos can have Dirac mass term given as

\[
\mathcal{L}_M^D = \bar{\nu}_R M_d \nu_L + h.c
\]  

(23)

It has been borne out by many experimental observations and theoretical estimates that neutrinos have very small masses of \( \mathcal{O}(10^{-10}) \) Gev. The particle content of the model given in this work allows us to write more general Yukawa couplings for the neutrinos given as

\[
Y_{sij} \bar{\nu}_R s_L^j \phi_3 + Y_{mij} \bar{s}_Li s_Lj \phi_3 + h.c
\]  

(24)

where indices \( i, j \) refers to the lepton generation and \( s_Li \) are neutral leptons which are singlet under the SM gauge groups. When \( \phi_3 \) developes a non zero VEV, these Yukawa interaction terms can give masses to the respective fermions and can provide a simple explanation for the smallness of neutrino mass through the inverse-seesaw mechanism. In inverse-seesaw, on top of Dirac neutrino mass \( m_{\nu_1} \), to generate small neutrino mass, we introduce a new left handed SM singlet neutral fermion \( s_L \) having a Dirac mass \( M_s \) with \( \nu_R \) and a small lepton number breaking Majorana mass \( \eta \) (inverse seesaw mechanism) [15], with the final mass term given as

\[
\mathcal{L}_{\text{inverse-seesaw}} = \bar{\nu}_R M_d \nu_L + \bar{\nu}_R M_s s_L + s_L^T \eta s_L + h.c
\]  

(25)

where \( M_\nu = v_0 Y_{\text{Dirac}} \), \( M_s = v_3 Y_s \) and \( \eta = v_3 Y_\eta \) are \( 3 \times 3 \) mass matrices with \( v_0 \) SM Higgs VEV and \( v_3 \) is the VEV of \( \phi_3 \). In the simplest case we can take the mass matrices such that \( U_R^T M_d U_L = \text{diag}(m_{d1}, m_{d2}, m_{d3}) \), \( U_R^T M_s O_s = \text{diag}(m_{s1}, m_{s2}, m_{s3}) \) and \( O_\eta^T \eta O_s = \text{diag}(\eta_1, \eta_2, \eta_3) \) where \( U_R \) and \( U_L \) are the unusual unitary matrices that diagonalize the Dirac mass matrix while \( O_s \) is an orthogonal...
matrix that diagonalize the $\eta$ mass matrix with $M_s$ such that it is diagonalized by a unitary matrix $U_R^1$ from left and an orthogonal matrix $O_F$ from the right and the indices 1, 2 and 3 refers to the lepton generation. Then Eqs. (25) reduces to

$$L_{inverse-seesaw} = \sum_{i=1}^{3} (\bar{\nu}_{Ri} \bar{s}_{Li}^2) \begin{pmatrix} m_{d_i} & m_{s_i} \\ 0 & \eta_i \end{pmatrix} \begin{pmatrix} \nu_{Li} \\ s_{Li} \end{pmatrix} + h.c.,$$

(26)

and each of these mass matrices can be diagonalized as

$$\begin{pmatrix} m_{h_i} & 0 \\ 0 & m_{l_i} \end{pmatrix} = \begin{pmatrix} \cos(\lambda_i) & \sin(\lambda_i) \\ -\sin(\lambda_i) & \cos(\lambda_i) \end{pmatrix} \begin{pmatrix} m_{d_i} & m_{s_i} \\ 0 & \eta_i \end{pmatrix} \begin{pmatrix} \cos(\lambda_i) & -\sin(\lambda_i) \\ \sin(\lambda_i) & \cos(\lambda_i) \end{pmatrix}.$$

(27)

Then the smallness of the light neutrino mass for each generation comes from smallness of $\eta_i$ and is given as [10]

$$m_{l_i} \approx \eta_i \frac{m_{d_i}^2 + m_{s_i}^2}{m_{d_i}^2 + m_{s_i}^2} \leq O(10^{-10}) \text{ GeV and } m_{h_i} \approx \frac{m_{d_i}^2 + m_{s_i}^2}{m_{d_i}}.$$

(28)

where $\eta_i$ can be taken at the scale of the breaking of $U(1)_{Lepton}$ (Lepton number) with $\sin(\lambda_i) = \frac{m_{s_i}}{\sqrt{m_{d_i}^2 + m_{s_i}^2}}$ and $\cos(\lambda_i) = \frac{m_{d_i}}{\sqrt{m_{d_i}^2 + m_{s_i}^2}}$ and $m_{l_i}$ and $m_{h_i}$ refers to the masses of light and heavy neutrinos respectively. In the limit $m_{s_i} >> m_{d_i}$, we have $\nu_{l_i} = -\sin(\lambda_i)(\nu_{Li} + \nu_{Ri}) + \cos(\lambda_i)(s_{Li}^2 + s_{Li}) \approx -O(1)(\nu_{Li} + \nu_{Ri}) + O(\frac{m_{s_i}}{m_{d_i}})(s_{Li}^2 + s_{Li}) \approx -\nu_{l_i}$ and similarly $\nu_{h_i} \approx O(1)(s_{Li} + s_{Li}) + O(\frac{m_{s_i}}{m_{d_i}})$ where $\nu_{l_i}$ is the light neutrino and $\nu_{h_i}$ is the heavy neutrino for each generation denoted by the index $i$. In this limit the light neutrinos mainly consist of $\nu_{Li} + \nu_{Ri}$ and the heavy neutrinos consist mainly of $s_{Li}^2 + s_{Ri}$. Then we have $\nu_{Li} \approx -P_L \nu_{l_i} + O(\frac{m_{d_i}}{m_{s_i}})P_L \nu_{h_i}$ and so corrections to neutrino oscillation due to heavy neutrino ($\nu_{h_i}$) is at the order of $O(\frac{m_{s_i}}{m_{d_i}})$ where $P_L = \frac{1}{2}(1 - 5^\gamma)$. Due to interactions in Eqs. (24) and Eqs. (10), $h_3$ can decay into light neutrinos as $h_3 \rightarrow \bar{\nu}\nu$ and photons as $h_3 \rightarrow \gamma\gamma$ via the triangle loop respectively, and so only $\phi_2$ will be a stable neutral scalar that can contribute to the Dark-Matter (DM) of the universe. In the case where $m_{s} \sim m_{\phi_2} >> m_{\mu}$, within 1$\sigma$ of the central value of $\delta a_{\mu}$, the mass of the scalar DM ($m_{\phi_2}$) is allowed to be as high as about 53 GeV. But it turns out that for large Yukawa couplings, such as in the case of our model, the DM annihilation crosssection is too large that $\phi_2$ could only contribute a very small fractions of the DM relic density of the universe [10].

3 Production and signature in future searches.

Since the $f\bar{f}$ current interact with electromagnetic and neutral weak gauge bosons $\gamma$ and $Z$ respectively as given in Eqs. (15), we can produce $f^+f^-$ pair in the colliders as

$$e^+e^- \rightarrow \gamma^*/Z^* \rightarrow f^+f^-$$

(29)
at $e^+e^-$ colliders and as
\[ pp \rightarrow \gamma^*/Z^* \rightarrow f^+f^- \] (30)

at LHC. But due to non-observation of tracts of long lived charged particles in the collider experiments in the past, it is reasonable to assume that the heavy lepton $f$ are very short lived particles, i.e $Y_2 \approx 1$, and so according to $\frac{Y_2}{m_f} = 0.0188 \text{ GeV}^{-1}$, its mass should be close to maximum allowed value of about 53.19 GeV. Then the heavy lepton decay very quickly after its production into $f^\pm \rightarrow \mu^\pm \phi_2$ and it would have left no tracts, but since $\phi_2$ is stable, light and very long lived neutral particle, possible candidate for DM, it would have been mistaken with the direct production of muon pair as $e^+e^-/pp \rightarrow \gamma^*/Z^* \rightarrow \mu^+\mu^-$ especially at LHC where only part of the energy carried by protons estimated from Parton-distribution-functions (PDF) is transfered to the final $\mu^+\mu^-$ state. However at $e^+e^-$ colliders, since the total energy carried by the $e^+e^-$ pair are transfered to $f^+f^-$ pair in full, we can look for key signal at $e^+e^-$ colliders as
\[ e^+e^- \rightarrow \gamma^*/Z^* \rightarrow f^+f^- \rightarrow \mu^+\mu^- + \text{missing mass} \ (\phi_2\phi_2). \] (31)

In the $e^+e^-$ colliders, the final state $\mu^+\mu^- + \text{missing mass} \ (\phi_2\phi_2)$ of NP can be differentiated from the SM final state $\mu^+\mu^-$ pretty easily by measuring the missing mass in the final $\mu^+\mu^-$ state relative to the total energy of the initial $e^+e^-$ pair, and so $e^+e^-$ colliders running at 110 GeV or higher center of mass energies are very suitable to detect the presence of $f^+f^-$ particles. Another key signature at LHC would be the triangle loop final state
\[ pp \rightarrow \gamma^*/Z^* \rightarrow \gamma + \phi_3 \rightarrow \gamma + (\phi_3 \rightarrow \bar{\nu}\nu \text{ or } \tilde{\phi}_2\phi_2) \rightarrow \gamma + \text{missing energy}. \] (32)

In case of fermion $f$ affecting the $Z$ decay $Z \rightarrow \bar{\mu}\mu$ via the triangle loop [17], we have for $m_f = 53.19$ GeV and $m_{\phi_2} = 53$ GeV, $Br(Z \rightarrow \bar{\mu}\mu)_{NP} = 2.19 \times 10^{-6}$ compare to the PDG average [19] of $Br(Z \rightarrow \bar{\mu}\mu)_{Exp} = (3.366 \pm 0.007)\%$, so NP contribution is an order of magnitude smaller than the experimental error.

Similar heavy leptons carrying electron lepton number ($f_e$) and tau lepton numbers ($f_\tau$) can exist but for the $f_e$ if its coupling with $\phi_2$ is close to the perturbative limit then its mass should be very large to accommodate the fact that SM accounts very well for the $a_e$. For the $f_\tau$ the parameters are less constrained because $a_\tau$ is not measured accurately yet.

4 Conclusions.

In this work we proposed a simple extension of SM by adding nine additional new particles to it, a heavy lepton $f$ only charged under the SM gauge group $U(1)_Y$ with $Y = -2$ with $f \rightarrow -f$ under a
discrete symmetry transformation carrying Lepton Number \((N_e, N_\mu, N_\tau) = (0, 1, 0)\). One scalar \(\phi_2\), singlet under all the SM gauge groups and \(\phi_2 \rightarrow -\phi_2\) under the discrete symmetry with zero VEV. One more scalar \(\phi_3\), singlet under all the SM gauge groups and invariant under the discrete symmetry which develops a non zero VEV \((v_3)\) and participates in the mass generations of \(\phi_2\), f, neutrinos and SM Higgs as well as itself. Three right-handed neutrinos \((\nu_R)\) and three left-handed Majorana neutrinos \((s_L)\). With these nine additional particles added to SM we have been able to give explanations to the long standing \((g-2)_\mu\) anomaly as well as the smallness of neutrino masses by the inverse see-saw mechanism. And also in this model we have a very suitable scalar dark-matter (DM) candidate in \(\phi_2\) with allowed mass as high as about 53 GeV, although due to large Yukawa coupling, its contribution to the DM relic density turn out to be too small and so it can account only a small fraction of the DM relic density of the universe. And we have also analyzed some possible means of production and signature that can show up in LHC and present and future \(e^+e^-\) colliders. We proposed the main production and signature can be found in the processes \(e^+e^-/pp \rightarrow f^+f^- \rightarrow \mu^+\mu^- + \text{missing energy (} \overline{\phi_2}\phi_2\) and \(pp \rightarrow \gamma^*/Z^* \rightarrow \gamma + \text{missing energy (} \phi_3 \rightarrow \bar{\nu}\nu \text{ or } \overline{\phi_2}\phi_2\) via the triangle loop of fermion \(f\). So discovery potential lies in the capability of a particular collider to measure the missing energy in the final state. In that sense we think \(e^+e^-\) colliders are much better suited for discovery of the signatures of these new particles than LHC. In \(e^+e^-\) colliders, initial total energy in the CM of the \(e^+e^-\) pair is transferred to the final \(\mu^+\mu^-\) pair in full, we can detect the missing energy easily where as at LHC, only partons in the protons participate whose energy are only known in terms of PDF and therefore its sensitivity towards missing energy will be reduced.

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