Scalar-tensor correlations and large-scale power suppression

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Recent measurements from the BICEP2 cosmic microwave background polarization experiment indicate the presence of primordial gravitational waves with surprisingly large amplitude. If these results are confirmed, they point to a discrepancy with temperature anisotropy power spectrum measurements and suggest that extensions to the standard cosmological model may be required to resolve the discrepancy. One intriguing extension is an anticorrelation between tensors and scalars to naturally suppress the temperature power. Here I examine this possibility and show that such a suppression is not possible in the presence of a general form of anticorrelation.

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I. INTRODUCTION

The standard Λ cold dark matter (ΛCDM) model has been remarkably successful at describing the large scale geometry, content, and thermal history of the Universe. Nevertheless, whispers of discrepancies from the standard model at the largest observable scales have been heard. In particular, there is a deficit of temperature anisotropy power at the largest angular scales in the cosmic microwave background (CMB) (with respect to the best-fit six parameter model) [1]. In addition, a roughly dipolar power asymmetry is present on multipole scales ℓ < 100 (see, e.g., [2]). These features are not of very high significance and are sensitive to a posteriori choices, so they may simply turn out to be the result of large Gaussian random field fluctuations. Nevertheless, they have attracted considerable attention as they may be hinting at extensions to ΛCDM.

The whispers of discrepancies have turned into shouts recently with the announcement of results from the BICEP2 CMB polarization experiment [3]. The BICEP team has performed a measurement of the B-mode polarization power spectrum and concluded that their results indicate the presence of a remarkably large-amplitude primordial gravitational wave power spectrum, with tensor-to-scalar ratio r = 0.16±0.06. This result must pass a number of tests before this conclusion can be widely accepted. First, it must be determined whether polarized galactic foreground emission or other systematic effects might account for the signal (see, e.g., [4]). Next, the possibility that the signal, if extragalactic, is due to something other than primordial gravitational waves, such as defects, magnetic fields, or birefringence, must be considered [5–8]. Nevertheless, the BICEP announcement potentially represents one of the most important discoveries in the history of cosmology and should be taken very seriously.

One particularly surprising aspect of the BICEP result is the apparent discrepancy with CMB temperature power spectrum measurements. The Planck satellite recently placed a 95% upper limit of r < 0.11 [9]. Such temperature power limits are based on the characteristic shape of the tensor temperature power spectrum, namely, that of a large-scale plateau which tapers off on scales smaller than ℓ ∼ 100. The absence of any visible large-scale power excess limits the possible tensor contribution to within cosmic variance. The fact that we actually observe a large-scale power deficit only exacerbates this discrepancy, raising it to approximately the 3σ level [10].

Many approaches to resolving this discrepancy, based on the assumption that the BICEP measurement is correct, are possible. Generically, they require the introduction of extra cosmological parameters which have the effect of suppressing temperature power on large scales, to compensate for a large tensor contribution. Perhaps the simplest possibility is the introduction of a negative running of the primordial scalar power spectrum tilt, as pointed out by the BICEP team themselves [3]. However, the required running is much larger than that expected in the simplest inflationary models (see, e.g., [11]). Other possibilities include the addition of an anticorrelated isocurvature component [12] or of additional neutrino species (see, e.g., [13]). Of course, an ad hoc procedure of suppressing the primordial scalar power on the largest scales is certainly a possibility. While inflationary models with such features have been discussed (see, e.g., [14–16]), they require an amplitude and cutoff scale tuned remarkably to coincide with and compensate for the tensor contribution.

One intriguing possibility is that of an anticorrelation between tensors and scalars, which might offer the possibility of naturally suppressing temperature power without the need to introduce a scale or an amplitude by hand [17]. Of course, such correlations do not occur in the simplest models of inflation, and so would necessitate the introduction of complications to the basic models (see, e.g., [18]). Nevertheless, it is worth investigating the viability of this approach. In this brief report I attempt to address the question of how well temperature power can be reduced with a general form of tensor-scalar correlation. I calculate the total temperature anisotropy power due to tensors and scalars on large angular scales.

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in the presence of such a correlation. I find that a reduction to the total measured temperature power is not possible, in agreement with a special case analyzed very recently in [19].

II. SCALAR-TENSOR CORRELATIONS

In this section, I consider the most general form that a correlation can take between tensor modes, represented by the transverse-traceless (TT) spatial metric perturbation $h_{ij}$, and scalar modes, represented here by the comoving curvature perturbation $\mathcal{R}$. Both scalar and tensor modes are described in this section by their primordial values, i.e. their values on super-Hubble scales after inflation, and hence can be treated as time independent in ordinary adiabatic models. Time dependence will be straightforwardly implemented later in the analysis.

Scalar modes are taken to satisfy the two-point correlation function

$$\langle \mathcal{R}^*(\mathbf{k})\mathcal{R}(\mathbf{k}') \rangle = \frac{2\pi^2}{k^3} P_{\mathcal{R}}(k) \delta^3(\mathbf{k} - \mathbf{k}')$$

with dimensionless power spectrum $P_{\mathcal{R}}(k)$ and comoving wave vector $\mathbf{k}$. For the tensors, if we use the standard helicity states

$$e^{\pm 2}_ij(\mathbf{k}) = \frac{1}{\sqrt{2}} \left( e^+_ij(\mathbf{k}) \pm ie^-_ij(\mathbf{k}) \right),$$

we can expand the modes according to

$$h_{ij}(\mathbf{k}) = h_\lambda(\mathbf{k}) e^{\lambda}_ij(\mathbf{k}),$$

where repeated helicity indices $\lambda = \pm 2$ are summed over. Then we can write the two-point correlator as

$$\langle h^*_\lambda(\mathbf{k}) h_\lambda(\mathbf{k}') \rangle = \frac{\pi^2}{2k^3} P_h(k) \delta^3(\mathbf{k} - \mathbf{k}') \delta_\lambda\lambda',$$

with dimensionless tensor power spectrum $P_h(k)$. The tensor-to-scalar ratio at pivot scale $k_*$ is then defined by

$$r \equiv P_h(k_*) / P_{\mathcal{R}}(k_*).$$

The TT character of the tensor perturbations implies that a tensor-scalar correlation must take the form

$$\langle h^*_ij(\mathbf{k}) \mathcal{R}(\mathbf{k}') \rangle = \frac{2\pi^2}{k^3} P_{h\mathcal{R}}(k) C_{ij}(\mathbf{k}) \delta^3(\mathbf{k} - \mathbf{k}'),$$

where the (otherwise arbitrary) spatial tensor $C_{ij}(\mathbf{k})$ satisfies

$$\mathbf{k}' C_{ij} = C'^i_j = 0,$$

and $P_{h\mathcal{R}}(k)$ is the dimensionless correlated power spectrum. Crucially, no tensor satisfying Eq. (6) can be constructed from the metric or wave vector $\mathbf{k}$, so $C_{ij}$ must correspond to a new tensor field or to a breaking of Lorentz invariance. In terms of the helicity states this correlation becomes

$$\langle h^*_\lambda(\mathbf{k}) \mathcal{R}(\mathbf{k}') \rangle = \frac{\pi^2}{k^3} P_{h\mathcal{R}}(k) D_{ij} e^{\lambda}_ij(\mathbf{k}) \delta^3(\mathbf{k} - \mathbf{k}'),$$

for arbitrary tensor $D_{ij}$. The tensor $C_{ij}$ is related to $D_{ij}$ via a projection into the TT subspace, i.e.

$$C^{kl}(\mathbf{k}) = \frac{1}{2} D^{ij} e^{\lambda}_ij(\mathbf{k}) e^{\lambda}_kl(\mathbf{k}).$$

The tensor $D_{ij}$ describes the correlation more fundamentally than $C_{ij}(\mathbf{k})$, since the former is independent of the mode direction.

Relation (5) implies that the general tensor-scalar correlation for each wave vector depends on two independent parameters, the unique components of $C_{ij}$. But the physical correlation tensor $D_{ij}$ depends effectively on five parameters, since an isotropic (trace) part of $D_{ij}$ does not contribute to correlations. Two of those five parameters can be taken to determine the orientation of the first principle axis of $D_{ij}$, and one more parameter describes the orientation of the second principle axis in the plane orthogonal to the first. Therefore, up to overall rotations, there are only two parameters in $D_{ij}$, describing, e.g., the lengths of the second and third principle axes relative to the first. Choosing the coordinate and principle axes to coincide, the most general correlation takes the form

$$D^{ij} = \alpha \hat{x}^i \hat{x}^j + \beta \hat{y}^i \hat{y}^j + \delta \hat{z}^i \hat{z}^j,$$

for constants $\alpha$ and $\beta$.

We can easily evaluate the correlations explicitly in the case $\alpha = \beta = 0$. Then we have

$$D_{ij} e^{\lambda}_ij(\mathbf{k}) = \hat{z}_i \hat{z}_j e^{\lambda}_ij(\mathbf{k}) = \delta \hat{z}^i \hat{z}^j S_{ik}(\mathbf{k}) S_{jk}(\mathbf{k}) e^{\lambda}_kl(\mathbf{k}),$$

where $S_{ij}(\mathbf{k})$ represents a standard rotation from the $\hat{z}$ to $\mathbf{k}$ directions. Writing $\mathbf{k} = (\theta_k, \phi_k)$ and using the explicit form for the rotation matrices, we find

$$D_{ij} e^{\lambda}_ij(\mathbf{k}) = \frac{1}{\sqrt{2}} \sin^2(\theta_k) = 4 \sqrt{\frac{\pi}{15}} \lambda Y_{20}(\mathbf{k}),$$

where the $\lambda Y_{lm}$ are the spin-$\lambda$ spherical harmonics. As expected, the correlation behaves like a quadrupolar spin-$\lambda$ quantity. More general correlations with nonzero $\alpha$ and $\beta$ will entail mixtures of $\lambda Y_{2m}$ with all $|m| \leq 2$, although we will not need their explicit forms.

Before concluding this section, note that positivity of total power puts a restriction on the magnitude of a tensor-scalar anticorrelation. In particular, we must have

$$\langle |\gamma h_\lambda(\mathbf{k}) + \mathcal{R}(\mathbf{k})|^2 \rangle \propto |\gamma|^2 P_h(k) + 4P_{h\mathcal{R}}(k) + 4 \text{Re} \left[ \gamma^* P_{h\mathcal{R}}(k) D_{ij} e^{\lambda}_ij(\mathbf{k}) \right] \geq 0$$

for any $\gamma$. This effectively puts a constraint on the magnitude of the correlated power, $P_{h\mathcal{R}}(k)$, for the case of anticorrelations. This correlated power was apparently treated as a free function in [17].
III. CMB TEMPERATURE ANISOTROPIES

As mentioned in the Introduction, in order to determine whether tensor-scalar anticorrelation can reduce large-angle temperature anisotropy power we must use the general correlation described above to calculate the CMB anisotropies. In this section, I will explicitly calculate the scalar Sachs-Wolfe (SW) effect anisotropies, which is a reasonable approximation to the total scalar power on large scales, where the tensor contribution is significant. Similarly, I will calculate the anisotropies due to the line of sight tensor integrated SW effect. This is a very good approximation, and only ignores effects due to noninstantaneous recombination and neutrino damping. Importantly, Ref. [17] approximated the tensor contribution as localized to the last scattering surface. In fact, the contribution is relatively broadly distributed along the line of sight, which means that the correlation is expected to be weaker than predicted in [17].

The scalar SW temperature anisotropy is simply

$$\delta T^S(\hat{n}) = -\frac{1}{5} \mathcal{R}(r_{LS} \hat{n}),$$

where $r_{LS}$ is the comoving radius to last scattering. Decomposing as usual into spherical harmonics, $\delta T(\hat{n})/T = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n})$, we find for the scalar multipole coefficients

$$a_{\ell m}^S = -\frac{1}{5} \sqrt{\frac{2}{\pi}} \int dk j_\ell(k r_{LS}) \int d\Omega(k) \mathcal{R}(k) Y_{\ell m}^*(\hat{k}),$$

for spherical Bessel function $j_\ell$. The scalar power is easily calculated to be

$$C^S_\ell = \langle a_{\ell m}^S a_{\ell m}^S \rangle = \frac{4\pi}{25} \int \frac{dk}{k} P(k) j_\ell^2(k r_{LS}),$$

which describes the familiar nearly flat SW plateau.

The line of sight temperature anisotropy due to tensors is

$$\frac{\delta T^T(\hat{n})}{T} = -\frac{1}{2} \int_E^{R} \tilde{h}_{ij} \hat{n}^i \hat{n}^j dt.$$  \hspace{1cm} (17)

Here $E$ and $R$ represent the emission point on the last scattering surface and the reception point at the origin today, respectively. Ignoring the effects of neutrino and photon anisotropic stress, the tensor fluctuations evolve according to the linearized Einstein equation

$$\ddot{h}_{ij} + 3H \dot{h}_{ij} - \frac{\nabla^2}{a^2} h_{ij} = 0,$$

with Hubble rate $H \equiv \dot{a}/a$. These perturbations evolve very slowly at early times when the modes are super-Hubble, but start to decay as the modes cross the Hubble radius. Thus the anisotropy Eq. (17) on multipole scale $\ell$ is sourced mainly by modes with $k \sim \ell/r \sim a(r)H(r)$; i.e., it is sourced along a substantial interval of the line of sight. Since we only observe to a maximum distance $r_{LS}$, we can see that the tensor temperature anisotropies are only sourced on scales larger than $\ell \sim r_{LS} a(r_{LS})H(r_{LS})^2 \sim 100$.

Expanding into helicity modes, we can factor the late-time evolution out from the primordial amplitudes by defining

$$h_\lambda(k, t) = h(k, t) h_\lambda(k),$$

with the number of arguments differentiating the functions $h_\lambda(k, t)$ and $h_\lambda(k)$ and with the time-dependent factor satisfying

$$\ddot{h}(k, t) + 3H \dot{h}(k, t) + \frac{k^2}{a^2} h(k, t) = 0.$$  \hspace{1cm} (20)

Expanding again into spherical harmonics, we find after some computation

$$a_{\ell m}^T = \frac{i^\ell}{2\sqrt{\pi}} \left( \frac{(\ell + 2)!}{(\ell - 2)!} \right)^{1/2} \int dk k^2 f_\ell(k)$$

$$\times \int d\Omega(k) h_\lambda(k) Y_{\ell m}^*(\hat{k}),$$

where

$$f_\ell(k) = \int_E^{R} \tilde{h}(k, t) j_\ell(k r_{LS}) dt.$$  \hspace{1cm} (22)

It is now a simple matter to calculate the tensor power spectrum using the orthonormality of the spin spherical harmonics and the two-point correlation function (4). The result is

$$C^T_\ell = \langle a_{\ell m}^T a_{\ell m}^T \rangle = \frac{\pi (\ell + 2)!}{4 (\ell - 2)!} \int \frac{dk}{k} P_h(k) f_\ell^2(k).$$  \hspace{1cm} (23)

This describes the expected roughly flat plateau up to $\ell \sim 100$, followed by an oscillating decay. Note the structural similarity between the scalar expressions (15) and (16) and the corresponding tensor expressions (21) and (23), with spherical harmonics in the former replaced with spin-$\lambda$ spherical harmonics in the latter.

Finally, we are in a position to calculate the (diagonal) tensor-scalar temperature anisotropy correlation. Combining Eqs. (15) and (21) gives

$$\langle a_{\ell m}^T a_{\ell m}^S \rangle = -\frac{\pi}{5\sqrt{2}} \left( \frac{(\ell + 2)!}{(\ell - 2)!} \right)^{1/2} \int \frac{dk}{k} P_h(\pi k) f_\ell(k) j_\ell(k r_{LS})$$

$$\times \int d\Omega(k) D^{ij} \epsilon_{ij}(k) Y_{\ell m}(\hat{k}) Y_{\ell m}^*(\hat{k}).$$  \hspace{1cm} (24)

Using the explicit form Eq. (11) for the special case of the correlation $D^{ij} = \delta^{ij} \tilde{\epsilon}^j$, this expression becomes

$$\langle a_{\ell m}^T a_{\ell m}^S \rangle = -\frac{4\pi^{1/2}}{5\sqrt{30}} \left( \frac{(\ell + 2)!}{(\ell - 2)!} \right)^{1/2} \int \frac{dk}{k} P_h(\pi k) f_\ell(k) j_\ell(k r_{LS})$$

$$\times \int d\Omega(k) Y_{20}(k) Y_{\ell m}(k) Y_{\ell m}^*(k).$$  \hspace{1cm} (25)
To evaluate the integrals over directions \( \hat{k} \), note that observations attempt to measure the total power at each \( \ell \) mode; i.e., they attempt to measure

\[
\frac{1}{2\ell + 1} \sum_m \langle a_{\ell m}^T a_{\ell m}^S \rangle \propto \sum_m \lambda Y_{\ell m}(k)Y_{\ell m}^*(\hat{k}).
\]  

Now, relating the spherical harmonics to the rotation matrices via

\[
Y_{\ell m}(\theta, \phi) = (-1)^m \left( \frac{2\ell + 1}{4\pi} \right)^{1/2} D_{m0}^{\ell}(\phi, \theta, 0)
\]  

(I use the sign conventions of \([20]\)), Eq. (26) becomes

\[
\sum_m \langle a_{\ell m}^T a_{\ell m}^S \rangle \propto \sum_m D_{\ell m}^{ij}(0, -\theta_k, -\phi_k)D_{m0}^{ij}(\phi_k, \theta_k, 0) = \delta_{\lambda 0} = 0.
\]  

This result used the addition theorem for rotation matrices (see, e.g., \([20]\)). Note in particular that this final result indicates that the expression (24) vanishes when summed over \( m \) regardless of the form of the coupling \( D^{ij} \). In other words, it is impossible to obtain a temperature anisotropy tensor-scalar anticorrelation to suppress the effect of tensors with the hope of reconciling the *Planck* and BICEP2 limits or measurements of the tensor-to-scalar ratio.

\[\text{IV. CONCLUSIONS}\]

Note that only the total temperature power remains unsuppressed according to the result of Eq. (28). Individual modes \( a_{\ell m} \) can be expected to be affected by the tensor-scalar correlations. Thus we generically expect the appearance of statistical anisotropy, exhibited as off-diagonal correlations \( \langle a_{\ell m}^T a_{\ell' m'}^S \rangle \). In particular, we expect quadrupolar anisotropy to be induced, i.e. couplings between \( \ell \) and \( \ell \pm 2 \), due to the spin-2 character of the tensor modes. However, there are very tight constraints on the presence of quadrupolar asymmetry in the temperature anisotropies. In particular, the *Planck* measurements are consistent with zero quadrupolar asymmetry even on scales \( \ell < 100 \) \([2]\), where the effects of tensor correlations would be important. Indeed, as mentioned in the Introduction, the most notable asymmetry is of dipolar character, which should not arise from a tensor correlation.

As a logical, if increasingly baroque, possibility, it is worth mentioning that the tensor-scalar correlation tensor \( D^{ij} \), which in this work has been assumed to be a constant, could be allowed to vary spatially. With, e.g., a linear gradient in \( D^{ij} \), we might expect that dipolar-type anisotropies could be achieved. (Note that it appears to be difficult to reconcile *Planck* with BICEP by postulating a gradient in \( r \) across our observable volume \([21]\).)

In order to exhibit analytical expressions, I used the approximations of the SW effect for scalars and the line of sight contribution for tensors. Although these are good approximations on large scales, the structure of Eq. (24) should be general in that an improved treatment of the generation of anisotropies will change the detailed form of the transfer functions (\( f_k \) for scalars and \( f_\ell \) for tensors in my approximation), but will leave the integral \( \int d\Omega \epsilon_{ij} (k) \lambda Y_{\ell m}^*(k)Y_{\ell m}^*(\hat{k}) \) unchanged. Thus the final conclusion that a suppression is not possible in the total temperature power should persist.

Finally, follow-up observations by the BICEP team, and forthcoming polarization measurements from the *Planck* satellite, will be crucial in determining whether extensions to ΛCDM are indeed needed. New measurements indicating a lower tensor-to-scalar ratio than the BICEP2 value may reconcile temperature and polarization measurements while maintaining the exciting consequences of new physics. In such a scenario the motivation for a suppression of large-scale temperature power may be removed. Only future observations will decide whether the “shouts of discrepancies” will be silenced or will lead to a new view of the Universe.

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\[\text{Note added}\]

After the appearance of the first version of this paper, a closely related paper appeared \([22]\). The results in \([22]\) are in agreement with those presented here.

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