Strong decays of double-charmed pseudoscalar and scalar $c\bar{c}u\bar{d}$ tetraquarks

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The strong decays of the pseudoscalar and scalar double-charmed tetraquarks $T_{cc\bar{c}d}^+$ and $\bar{T}_{cc\bar{c}d}^+$ are investigated in the framework of the QCD sum rule method. The mass and coupling of these exotic four-quark mesons are calculated using the QCD two-point sum rule approach by taking into account vacuum condensates of the quark, gluon, and mixed local operators up to dimension ten. Our results for masses $m_{T}= (4130 \pm 170)$ MeV and $m_{\bar{T}} = (3845 \pm 175)$ MeV demonstrate that these tetraquarks are strong-interaction unstable resonances and decay to conventional mesons through the channels $T_{cc\bar{c}d}^+ \rightarrow D^+ D^*(2007)^0$, $D^0 D^*(2010)^+$ and $\bar{T}_{cc\bar{c}d}^+ \rightarrow D^+ D^0$. Key quantities necessary to compute partial width of these decay modes, i.e. the strong couplings of two $D$ mesons and a corresponding tetraquark $q_i$, $i = 1, 2$ and $G$ are extracted from the QCD three-point sum rules. The full width $\Gamma_T = (129.9 \pm 23.5)$ MeV demonstrates that the tetraquark $T_{cc\bar{c}d}^+$ is a broad resonance, whereas the scalar exotic meson with $\Gamma_{\bar{T}} = (12.4 \pm 3.1)$ MeV can be classified as relatively narrow state.

I. INTRODUCTION

Double-charmed tetraquarks as exotic mesons are already in the agenda of high energy physics. Their properties were studied in a more general context of double-heavy mesons built of a heavy diquark $QQ$ and heavy or light antidiquarks [1, 4, 11, 12, 15, 17]. A main question addressed in these basic papers was whether such four quarks can form bound states or exist as unstable resonances. It was demonstrated that exotic mesons $QQ\bar{q}\bar{q}$ might be stable provided that the mass ratio of constituent quarks $m_Q/m_q$ is large enough. In this sense, tetraquarks with a diquark $bb$ are more promising candidates to be stable exotic mesons than ones containing a $bc$ or $cc$ pair. In fact, the isoscalar $J^P = 1^+$ tetraquark $T_{bb\bar{u}\bar{d}}$ is expected to lie below the two $B$-meson threshold and is strong-interaction stable state [4]. The situation with $T_{bc\bar{q}q'}$ and $T_{cc\bar{q}q}$ is not quite clear: they may exist either as bound or resonant states.

In the following years the chiral quark model, dynamical and relativistic quark models, other theoretical schemes of high energy physics were used to calculate spectroscopic parameters of the doubly charmed tetraquarks $\bar{T}_{cc\bar{c}d}^+$ and $T_{cc\bar{c}d}^+$. Their production in ion, proton-proton and electron-positron collisions, in $B_c$ and $\Xi_{bc}$ decays was investigated as well [3, 13, 14]. In the framework of the QCD sum rule method the axial-vector tetraquarks $QQ\bar{q}\bar{q}$ were explored in Ref. [14]. In accordance with obtained results the mass of $T_{bb\bar{u}\bar{d}}$ is below the open bottom threshold and, hence, it cannot decay directly to conventional mesons. Within the same method tetraquarks with quantum numbers $J^P = 0^-, 0^+, 1^-$ and $1^+$, and quark contents $QQ\bar{q}\bar{q}$ were studied in Ref. [13].

Recent intensive investigations of double-heavy tetraquarks were inspired by the discovery of doubly charmed baryon $\Xi_{ccu} = ccu$ [16]. The mass of this particle was utilized as an input information in a phenomenological model to evaluate masses of the tetraquarks $T_{bb\bar{u}\bar{d}}^{-}$ and $T_{cc\bar{c}d}^+$. It was confirmed once more that the axial-vector isoscalar state $T_{bb\bar{u}\bar{d}}^{-}$ is stable against strong and electromagnetic interactions, whereas the tetraquark $T_{cc\bar{c}d}^+$ can decay to $D^0 D^+$ mesons. A conclusion on a stable nature of $T_{bb\bar{u}\bar{d}}^{-}$ was drawn also in Refs. [18, 19].

The spectroscopic parameters and widths of the doubly charmed pseudoscalar tetraquarks $T_{cc\bar{c}d}^{++}$ and $T_{cc\bar{c}d}^{+-}$, which bears two units of the electric charge, were calculated in Ref. [20]. Obtained results showed that these exotic mesons are rather broad resonances. Various aspects of double-charmed tetraquarks were also analyzed in the publications [21, 22, 25, 26, 27, 28, 29, 30, 31, 32].

In the present work we investigate the pseudoscalar and scalar tetraquarks $T_{cc\bar{c}d}^+$ and $\bar{T}_{cc\bar{c}d}^+$. First, we calculate their spectroscopic parameters using the QCD two-point sum rule method, and in this process take into account nonperturbative contributions up to dimension ten. Our studies demonstrate that these exotic mesons are unstable resonances, which decay strongly to conventional mesons. The kinematically allowed decay modes $T_{cc\bar{c}d}^+ \rightarrow D^+ D^*(2007)^0$, $T_{cc\bar{c}d}^+ \rightarrow D^0 D^*(2010)^+$, and $\bar{T}_{cc\bar{c}d}^+ \rightarrow D^0 D^+$ are analyzed and their partial widths are found. To this end, we consider the strong couplings of two $D$ mesons to tetraquarks, which are key quantities of the analysis, and extract their values from the three-point QCD sum rules. Obtained predictions are used to estimate the full width of the tetraquarks $T_{cc\bar{c}d}^+$ and $\bar{T}_{cc\bar{c}d}^+$. This work has the following structure: In Sec. III we calculate the mass and coupling of the tetraquarks $T_{cc\bar{c}d}^+$ and $\bar{T}_{cc\bar{c}d}^+$. Here we provide details of calculations for the
II. MASS AND COUPLING OF THE PSEUDOSCALAR AND SCALAR TETRAQUARKS $T_{cc,\Sigma}^+$ AND $\bar{T}_{cc,\Sigma}^+$

As it it has been noted above, the mass and coupling of the tetraquarks $T_{cc,\Sigma}^+$ and $\bar{T}_{cc,\Sigma}^+$ (in what follows denoted by $T$ and $\bar{T}$, respectively) can be evaluated by means of the QCD two-point sum rule method. The essential component of this approach is the interpolating current, which should be composed of relevant diquark fields and has the quantum numbers of the original particle. There are different currents that meet these requirements [15]. For the pseudoscalar tetraquark $T$ with two identical $c\bar{c}$-quarks we choose a structure made of the heavy pseudoscalar and light scalar diquarks

$$J(x) = \epsilon^T_{\alpha}(x) Cc_{\alpha}(x) \pi_{\alpha}(x) \gamma_5 C\bar{c}_{\beta}(x).$$

The interpolating current for the scalar tetraquark $\bar{T}$ can be constructed from the heavy and light axial-vector diquark fields [21]

$$\bar{J}(x) = \epsilon^T_{\alpha}(x) C\gamma_\mu c_{\alpha}(x) [\pi_{\mu}(x) \gamma_\mu C\bar{c}_{\beta}(x)],$$

where $\epsilon = \epsilon^{abc\gamma de}$. In expressions above $a, b, c, d$ and $e$ are color indices and $C$ is the charge-conjugation operator.

The QCD two-point sum rules to evaluate the spectroscopic parameters of the tetraquark $T$ should be derived from the correlation function

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0 | T\{J(x)\bar{J}(0)\} | 0 \rangle.$$  

After replacement $J(x) \rightarrow \bar{J}(x)$ the similar correlator can be written down for the second particle $\bar{T}$. Below we give details of calculations for the mass $m_T$ and coupling $f_T$, and provide only final results for the state $T$.

To extract the desired sum rules from the correlation function $\Pi(p)$, one has, first of all, to express it in terms of the tetraquarks’ physical parameters and, by this way determine their phenomenological side $\Pi^{\text{phys}}(p)$. The function $\Pi^{\text{phys}}(p)$ can be derived by inserting into the correlation function $\Pi(p)$ a full set of relevant states, carrying out integration over $x$ in Eq. (3), and isolating the contribution of the ground-state particle $T$. As a result of these manipulations we get

$$\Pi^{\text{phys}}(p) = \frac{\langle 0 | J| T(p) \rangle \langle T(p)| J^\dagger | 0 \rangle}{m_T^2 - p^2} + \ldots,$$

which contains the contribution of the ground-state particle written down explicitly, as well as effects due to higher resonances and continuum states: the latter in Eq. (4) is denoted by dots.

The correlation function $\Pi^{\text{phys}}(p)$ can be recast into a more simple form if one introduces the matrix element of the pseudoscalar tetraquark

$$\langle 0 | J| T(p) \rangle = \frac{f_T m_T^2}{2m_c^2}.$$  

Then, we find

$$\Pi^{\text{phys}}(p) = \frac{1}{4m_c^2} \frac{f_T^2 m_T^4}{m_T^2 - p^2} + \ldots$$

In general, to continue calculations one should choose in $\Pi^{\text{phys}}(p)$ a Lorentz structure and fix the corresponding invariant amplitude. Because in the case under discussion $\Pi^{\text{phys}}(p)$ has the trivial structure $\sim I$, the amplitude $\Pi^{\text{phys}}(p^2)$ equals to the function from Eq. (4).

The QCD side of the sum rules $\Pi^{\text{OPE}}(p)$ can be found by computing the correlation function in terms of the quark propagators. To this end, we insert the interpolating current $J(x)$ to the expression (3), and after contracting the relevant quark fields find

$$\Pi^{\text{OPE}}(p) = i \int d^4x e^{ipx} \text{Tr} \left[ \gamma_5 S_{\mu}^b(x) \gamma_5 S_{\mu}^a(-x) \right] \text{Tr} \left[ S_{c}^{b/}(x) S_{c}^{a/}(x) + S_{c}^{d/}(x) S_{c}^{b/}(x) \right].$$

Here $S_{c}(x)$ and $S_{u(d)}(x)$ are the heavy $c$- and light $u(d)$-quark propagators, explicit expressions of which can be found, for example, in Ref. [20]. In Eq. (4) we also introduce the shorthand notation

$$\bar{S}(x) = CST(x)C.$$

By equating the amplitudes $\Pi^{\text{phys}}(p^2)$ and $\Pi^{\text{OPE}}(p^2)$, applying the Borel transformation to both sides of this expression, and performing the continuum subtraction we get an equality, which can be used to derive sum rules for the mass $m_T$ and coupling $f_T$. The Borel transformation suppresses contribution of higher resonances and continuum states, and generates a dependence of the sum rules on a new parameter $M^2$. The continuum
subtraction allows one by invoking the assumption on the quark-hadron duality to replace an unknown physical spectral density $\rho^{\text{phys}}(s)$ by $\rho^{\text{OPE}}(s)$, which is calculable as an imaginary part of $\Pi^{\text{OPE}}(p)$. A price paid for this simplification is appearance in the sum rules the continuum threshold parameter $s_0$ that separates from one another the ground-state and continuum contributions to $\Pi^{\text{OPE}}(p^2)$.

To derive the final sum rules we use this equality, as well as one obtained from the first expression by applying the operator $d/d(-1/M^2)$. As a result, we get

$$m_T^2 = \frac{\int_{4m_c^2}^{s_0} ds \rho^{\text{OPE}}(s)e^{-s/M^2}}{\int_{4m_c^2}^{s_0} \rho^{\text{OPE}}(s)e^{-s/M^2}},$$

and

$$f_T^2 = \frac{4m_c^2}{M^2} \int_{4m_c^2}^{s_0} \rho^{\text{OPE}}(s)e^{(m_T^2-s)/M^2}.$$ 

As we have noted above, Eqs. (9) and (10) depend on the auxiliary parameters $M^2$ and $s_0$. Their values are related to a problem under analysis and should be fixed to satisfy constraints, which we explain below. But, the sum rules contain also various vacuum condensates that are universal for all of problems:

$$(\bar{q}q) = -(0.24 \pm 0.01)^3 \text{ GeV}^3,$$

$$m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2,$$

$$\langle \sigma Gq \rangle = m_0^2(\bar{q}q),$$

$$\frac{\alpha_s G^2}{\pi} = (0.012 \pm 0.004) \text{ GeV}^4,$$

$$g_s^3 G^3 = (0.57 \pm 0.29) \text{ GeV}^6.$$ 

In numerical computations we use this information on vacuum condensates and the c-quark mass $m_c = 1.275^{+0.025}_{-0.035} \text{ GeV}$. Our studies prove that the working regions for the parameters

$$M^2 \in [4, 6] \text{ GeV}^2, \quad s_0 \in [20, 22] \text{ GeV}^2,$$

meet all restrictions imposed on $M^2$ and $s_0$.

The regions (12) are extracted from analysis mainly of a pole contribution to correlator and convergence of the sum rules. The pole contribution

$$\text{PC} = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)},$$

where $\Pi(M^2, s_0)$ is the Borel-transformed and subtracted invariant amplitude $\Pi^{\text{OPE}}(p^2)$, is one of important quantities to extract limits on the Borel parameter $(M^2_{\text{min}}, M^2_{\text{max}})$. In accordance with our computations at $M^2_{\text{min}} = 4 \text{ GeV}^2$ it amounts to 0.7, whereas at $M^2_{\text{max}} = 6 \text{ GeV}^2$ is 0.37, which can be considered as a nice result for four-quark mesons. But, at the same time, a lower limit of the Borel parameter depends on the convergence of the operator product expansion (OPE). Restrictions imposed on $M^2$ by convergence of OPE can be analyzed by means of the ratio

$$R(M^2_{\text{min}}) = \frac{\Pi^{\text{DimN}}(M^2_{\text{min}}, s_0)}{\Pi(M^2_{\text{min}}, s_0)}.$$ 

Here $\Pi^{\text{DimN}}(M^2, s_0)$ is a contribution to the correlation function arising from the last term (or from few last terms) in OPE. Numerical analysis proves that for $\text{DimN} = \text{Dim}(8 + 9 + 10)$ this ratio is $R(4 \text{ GeV}^2) = 0.2$, which guarantees the convergence of the sum rules. It is worth noting that the lower boundary of the Borel window is determined from joint analysis of PC and $R(M^2_{\text{min}})$, i.e. the maximum accessible pole contribution is limited by the convergence of OPE. Additionally, at minimum of the Borel parameter the perturbative term amounts to 68\% of the total result exceeding the nonperturbative contributions.

In general, quantities extracted from the sum rules should not depend on the auxiliary parameters $M^2$ and $s_0$. In real calculations, nevertheless we observe a residual dependence of $m_T$ and $f_T$ on them. Hence, the choice of $M^2$ and $s_0$ should minimize these non-physical effects as well. The working windows for the parameters $M^2$ and $s_0$ satisfy also these conditions. In Figs. (11) and (12) we plot the mass $m_T$ and coupling $f_T$ as functions of $M^2$ and $s_0$, which allows one to see uncertainties generated by the sum rule computations. It is seen that both $m_T$ and $f_T$ depend on $M^2$ and $s_0$, which is a main source of the theoretical uncertainties inherent to the sum rule computations. For the mass $m_T$ these uncertainties are small $\pm 4\%$, because the ratio in Eq. (13) cancels some of these effects. But even for the coupling $f_T$ the ambiguities do not exceed $\pm 20\%$ part of the central value.

Our calculations lead to the following results:

$$m_T = (4130 \pm 170) \text{ MeV},$$

$$f_T = (0.26 \pm 0.05) \times 10^{-2} \text{ GeV}^4.$$ 

The prediction for $m_T$ confirms that $T$ can be interpreted as a member the pseudoscalar multiplet of the double-charmed tetraquarks. In fact, parameters of other members of this multiplet $T^{++}_{cc\pi\pi}$ and $T^{++}_{cc\pi\pi}$ were calculated in Ref. [20]. The mass splitting between these two states 125 MeV is connected with replacement $\pi \rightarrow \gamma$ in their quark contents. The similar replacement $\pi \rightarrow \gamma$ in $T^{++}_{cc\pi\pi}$ generates the tetraquark $T$. Comparing now the mass 4265 MeV of $T^{++}_{cc\pi\pi}$ with $m_T = 4130 \text{ MeV}$ we
find 135 MeV mass difference between these two particles. In other words, the state $T$ occupies an appropriate place in the pseudoscalar multiplet of the double-charmed tetraquarks, which we consider as important consistency check of the present result.

Let us also note that $m_T$ is considerably lower than $(4430 \pm 130)$ MeV predicted in Ref. [15] for the pseudoscalar tetraquark with the same quark content and structure. This discrepancy presumably stems from quark propagators, in which some of higher-dimensional nonperturbative terms were neglected, and also from a choice of the working regions for the parameters $M^2$ and $s_0$.

The mass and coupling of the state $\bar{T}$ can be calculated by the similar manner. The difference here is connected with the matrix element of the scalar particle

$$\langle 0 | \bar{J} | T(p) \rangle = f_{\bar{T}} m_{\bar{T}},$$

which leads to the replacement $4m_s^2/m_{\bar{T}}^2 \rightarrow 1/m_{\bar{T}}^2$ in the sum rule for the coupling $f_{\bar{T}}$ [10]. The QCD side of new sum rules is given by the expression

$$\Pi^{\text{OPE}}(\mu) = i \int d^4 x e^{ipx} \bar{c} \gamma^\nu (\gamma^\nu \bar{c}') \mu \text{Tr} \left[ \gamma_{
u \delta} S_{\nu d}(x) \gamma_{\mu \delta} S_{\mu c}(x) \right] - \text{Tr} \left[ \gamma_{\nu \delta} S_{\nu c}(x) \gamma_{\mu \delta} S_{\mu c'}(x) \right].$$

The new function $\Pi^{\text{OPE}}(\mu)$ modifies also the spectral density $\rho^{\text{OPE}}(s)$. The remaining steps have been explained above, therefore we provide final information about the range of the parameters used in computations

$$M^2 \in [3, 4] \text{ GeV}^2, \ s_0 \in [19, 21] \text{ GeV}^2,$$

and obtained predictions

$$m_{T} = (3845 \pm 175) \text{ MeV},$$

$$f_{\bar{T}} = (1.16 \pm 0.26) \times 10^{-2} \text{ GeV}^4.$$
It is necessary to note that at $M^2_{\text{max}} = 4 \text{ GeV}^2$ the pole contribution exceeds 0.16 which is acceptable when considering the four-quark mesons, whereas at minimum $M^2_{\text{min}} = 3 \text{ GeV}^2$ it reaches 0.7. The convergence of the operator product expansion at $M^2_{\text{min}} = 3 \text{ GeV}^2$ is also guaranteed, because $R(3 \text{ GeV}^2) = 0.03$. Our result for $m_T$ is very close to the prediction (3870 ± 90) MeV obtained in Ref. [21].

III. STRONG DECAYS OF THE TETRAQUARKS $T^{+}_{ce,\Sigma}$ AND $\bar{T}^{+}_{ce,\Sigma}$

Masses of the tetraquarks $T$ and $\bar{T}$ are large enough to make their strong decays to ordinary mesons kinematically allowed processes. The mass of $T$ is $(58 \pm 29) \text{ MeV}$ below (we refer only to central value of $m_T$) the $S$-wave $D^+ D^*_0(2400)^0$ limit, but is $255 \text{ MeV}$ above the open-charm $D^+ D^*(2007)^0$ and $D^0 D^*(2010)^+$ thresholds and, hence $T$ can decay in $P$-wave to these conventional mesons. The exotic state $\bar{T}$ decays in $S$-wave to a pair of $D^* D^0$ mesons, because its mass $m_{\bar{T}}$ exceeds 110 MeV the corresponding border. The $P$-wave decays of $T$ require a master particle to be considerably heavier than $3845 \text{ MeV}$ which is not the case.

Below we consider in a detailed form the decay $T \rightarrow D^+ D^*(2007)^0$, and present final results for remaining modes. Our goal here is to calculate the strong coupling corresponding to the vertex $T D^+ D^*(2007)^0$. To derive the QCD three-point sum rule for the coupling and extract its numerical value one begins from analysis of the correlation function

$$\Pi^\mu(p, p') = i^2 \int d^4x d^4y e^{i(p'y-px)} \langle 0 | T \{ J^{D^*}_\mu(y) \times J^D(y) \} | 0 \rangle. \quad (20)$$

Here $J(x)$, $J^D(x)$ and $J^{D^*}(x)$ are the interpolating currents for the tetraquark and mesons $D^+$ and $D^*(2007)^0$, respectively. The $J(x)$ in given by Eq. [11], whereas for the remaining two currents we use

$$J^{D^*}(x) = \bar{u}(x)i\gamma_\mu c(x), \quad J^D(x) = \bar{d}(x)i\gamma_5 c(x). \quad (21)$$

The four-momenta of the tetraquark $T$ and meson $D^*(2007)^0$ are $p$ and $p'$, then the momentum of the meson $D^+$ is $q = p - p'$. We follow the standard prescriptions of the sum rule method and calculate the correlation function $\Pi^\mu(p, p')$ using both physical parameters of the particles involved into a process and quark-gluon degrees of freedom. Separating the ground-state contribution to the correlation function (20) from contributions of higher resonances and continuum states, for the physical side of the sum rule $\Pi^\mu_{\text{phys}}(p, p')$ we get

$$\Pi^\mu_{\text{phys}}(p, p') = \langle 0 | J^{D^*_\mu} | D^{*0}(p') \rangle \langle 0 | J^D | D^+(q) \rangle \frac{(p'^2 - m_{D^*}^2)(q'^2 - m_D^2)}{(p^2 - m_T^2)} \times \frac{\langle D^+(q) D^{*0}(p') | T(p) \rangle | T(p) | J^D | 0 \rangle}{(p^2 - m_T^2)} + \ldots \quad (22)$$

The function $\Pi^\mu_{\text{phys}}(p, p')$ can be further simplified by expressing matrix elements in terms of the mesons' physical parameters. To this end, we introduce the matrix elements

$$\langle 0 | J^{D^*_\mu} | D^{*0} \rangle = m_D f_D \epsilon_\mu, \quad \langle 0 | J^D | D^+ \rangle = m_{D^*} f_{D^*} \epsilon_\mu. \quad (23)$$

where $m_D, m_{D^*}$, and $f_D, f_{D^*}$ are the masses and decay constants of the mesons $D^+$ and $D^*(2007)^0$, respectively. In Eq. (23) $\epsilon_\mu$ is the polarization vector of the meson $D^*(2007)^0$. We model $\langle D(q) D^{*0}(p') | T(p) \rangle$ in the form

$$\langle D^+(q) D^{*0}(p') | T(p) \rangle = g_1(q^2) \epsilon_\mu, \quad (24)$$

and denote by $g_1(q^2)$ the strong coupling of the vertex $T(p) D(q) D^{*0}(p')$. Then it is not difficult to see that

$$\Pi^\mu_{\text{phys}}(p, p') = g_1(q^2) \frac{m_D^2 f_D m_{D^*} f_{D^*}}{2m_T^2 (p'^2 - m_{D^*}^2)(q'^2 - m_D^2)} \times \frac{1}{(p^2 - m_T^2)} \left( \frac{m_T^2 - m_{D^*}^2}{2m_{D^*}^2} \right) \left( \frac{q^2 - p'^2}{2m_D^2} \right) + \ldots \quad (25)$$

The correlation function $\Pi^\mu_{\text{phys}}(p, p')$ has two Lorentz structures $\sim p'_\mu$ and $\sim q_\mu$. We choose to work with the invariant amplitude $\Pi^\mu_{\text{phys}}(p^2, p'^2, q^2)$ corresponding to $\sim p'_\mu$. The double Borel transformation of this amplitude over variables $p^2$ and $p'^2$ forms the phenomenological side of the sum rule. To find the QCD side of the three-point sum rule we compute $\Pi^\mu(p, p')$ in terms of the quark propagators and get

$$\Pi^\mu_{\text{phys}}(p, p') = i^2 \int d^4x d^4y e^{i(p'y-px)} \left\{ \text{Tr} \left[ \gamma_\mu S^{i\alpha}(y-x) \times \tilde{S}^{i\alpha}(x) \right] \gamma_5 \right\} \times \frac{1}{(p^2 - m_T^2)} \left( \frac{m_T^2 - m_{D^*}^2}{2m_{D^*}^2} \right) \left( \frac{q^2 - p'^2}{2m_D^2} \right) + \ldots \quad (26)$$

The correlation function $\Pi^\mu_{\text{phys}}(p, p')$ is calculated with dimension-five accuracy, and has the same Lorentz structures as $\Pi^\mu_{\text{phys}}(p, p')$. The double Borel transformation $\Pi^\mu_{\text{phys}}(p^2, p'^2, q^2)$ is the invariant amplitude that corresponds to the term $\sim p'_\mu$, constitutes the second part of the sum rule. By equating $\Pi^\mu_{\text{phys}}(p^2, p'^2, q^2)$ and Borel transformation of $\Pi^\mu_{\text{phys}}(p^2, p'^2, q^2)$, and performing continuum subtraction we find the sum rule for the coupling $g_1(q^2)$.

The Borel transformed and subtracted amplitude $\Pi^\mu_{\text{phys}}(p^2, p'^2, q^2)$ can be expressed in terms of the spectral
density $\tilde{\rho}(s, s', q^2)$ which is proportional to the imaginary part of $\Pi^{OPE}(p, p')$

$$\Pi(M^2, s_0, q^2) = \int_{m_i^2}^{s_0} ds \int_{m_i^2}^{s_0'} ds' \tilde{\rho}(s, s', q^2) \times e^{-s/M_2^2} e^{-s'/M_2^2},$$

(27)

where $M^2 = (M_1^2, M_2^2)$ and $s_0 = (s_0, s_0')$ are the Borel and continuum threshold parameters, respectively. Then, the sum rule for $g_1(q^2)$ is determined by the expression

$$g_1(q^2) = \frac{4m_2^2m_1D_m}{\int_{M_1^2}^{M_2^2} \int_{M_1^2}^{M_2^2} m_2^2 - m_1^2 - q^2} \times e^{m_2^2/M_2^2} e^{m_1D_m/M_2^2} \Pi(M^2, s_0, q^2).$$

(28)

The coupling $g_1(q^2)$ is a function of $q^2$ and, at the same time, depends on the Borel and continuum threshold parameters. For simplicity, we do not show its dependence on these parameters, introduce new variable $Q^2 = -q^2$ and denote the obtained function as $g_1(Q^2)$.

The sum rule (28) contains masses and decay constants of the final mesons: these parameters are collected in Table I. For the masses of $D$ mesons we use information from Ref. 27. A choice for the decay constants of the pseudoscalar and vector $D$ mesons is a more complicated task. They were calculated using various models and methods 28, 32. Predictions obtained in these papers sometimes differ from each other considerably. Therefore, for the decay constant of the pseudoscalar $D$ mesons we use the available experimental result, whereas for the vector mesons—the QCD sum rule prediction from Ref. 31.

To carry out numerical analysis of $g_1(Q^2)$, apart from the spectroscopic parameters of $D$ mesons, one needs also to fix $M^2$ and $s_0$. The restrictions imposed on these auxiliary parameters are standard for sum rule computations and have been discussed above. The windows for $M_1^2$ and $s_0$ correspond to the $T$ channels, and coincide with the working regions $M_1^2 \in [4, 6]$ GeV$^2$ and $s_0 \in [20, 22]$ GeV$^2$ determined in the mass calculations. The next pair of parameters $(M_2^2, s_0')$ is chosen to be within the limits

$$M_2^2 \in [3, 5] \text{ GeV}^2, \quad s_0' \in [6, 8] \text{ GeV}^2.$$  

(29)

The strong coupling $g_1(Q^2)$ extracted from the sum rule demonstrates a residual dependence on $M^2$ and $s_0$, which is a main source of theoretical errors. The working intervals for the auxiliary parameters are chosen in such a way that to minimize these uncertainties. For an example, in Fig. 3 we plot the coupling $g_1(Q^2)$ as a function of the Borel parameters $M_1^2$ and $M_2^2$. It is seen that when changing $M^2$ the coupling $g_1(Q^2)$ is a subject to some variations, which nevertheless remain within allowed limits.

The width of the decay under analysis should be computed using the strong coupling at the $D^*$ meson’s mass-shell $q^2 = m_{D^*}^2$, which is not accessible to the sum rule calculations. We evade this difficulty by employing a fit function $F_1(Q^2)$ that for the momenta $Q^2 > 0$ coincides with QCD sum rule’s predictions, but can be extrapolated to the region of $Q^2 < 0$ to find $g_1(-m_D^2)$. In the present work to construct the fit function $F_1(Q^2)$ we use the following analytic form

$$F_1(Q^2) = F_0 \exp \left[ c_1 \frac{Q^2}{m_T^2} + c_2 \left( \frac{Q^2}{m_T^2} \right)^2 \right],$$

(30)

where $F_0$, $c_1$ and $c_2$ are fitting parameters. Numerical analysis allows us to fix $F_0 = 1.74, c_1 = 0.83$ and $c_2 = -0.38$. In Fig. 4 we depict the sum rule predictions for $g_1(Q^2)$ and also provide $F_1(Q^2)$: a nice agreement between them is evident.

This function at the mass-shell $Q^2 = -m_D^2$ gives

$$g_1 \equiv F_1(-m_D^2) = 4.21 \pm 0.65.$$  

(31)

The width of decay $T \to D^+ D^*(2070)^0$ is determined by the simple formula

$$\Gamma[T \to D^+ D^*(2070)^0] = \frac{g_1^2 \lambda^3 (m_T, m_{1D'}, m_D)}{8\pi m_D^2},$$

(32)

where

$$\lambda(a, b, c) = \frac{1}{2a} \sqrt{a^2 + b^2 + c^2 - 2(a^2 b^2 + a^2 c^2 + b^2 c^2)}.$$ 

(33)

Using the strong coupling from Eq. 31, it is not difficult to evaluate width of the decay $T \to D^+ D^*(2070)^0$

$$\Gamma[T \to D^+ D^*(2070)^0] = (64.3 \pm 16.5) \text{ MeV}.$$  

(34)

The second process $T \to D^0 D^*(2010)^+$ can be considered by the same manner. Corrections have to be made in the physical side and matrix elements are trivial. The QCD side of the sum rule in the approximation $m_a = m_d = 0$, which we use in this paper, coincides with $\Pi^{OPE}_a(p, p')$. The Borel and threshold parameters $M^2$ and $s_0$ are chosen as in the first channel. The differences are connected with the spectroscopic parameters of produced mesons $D^0$ and $D^*(2010)^+$. These factors modify numerical predictions for $g_2(Q^2)$, which is the strong coupling of the vertex $TD^0 D^*(2010)^+$, and change the fit function $F_2(Q^2)$. For parameters of $F_2(Q^2)$ we get $F_0^2 = 5.11, c_1^2 = 0.83$ and $c_2^2 = -0.38$. The result for the partial width of the decay $T \to D^0 D^*(2010)^+$ reads

$$\Gamma[T \to D^0 D^*(2010)^+] = (65.6 \pm 16.8) \text{ MeV}.$$  

(35)

The decay of the scalar four-quark meson $\bar{T} \to D^+ D^0$ is the last process to be considered in this Section. To extract the sum rule for the strong coupling $G(q^2)$ corresponding to the vertex $\bar{T} D^+ D^0$ of the scalar tetraquark $\bar{T}$ and pseudoscalar $D^+$, $D^0$ mesons, we start from the correlation function,

$$\Pi(p, p') = i^2 \int d^4 x d^4 y e^{i(p' y - p x)} \langle 0 | T \{ J^D(y) \times J^{D^0}(0), \bar{J}(x) \} | 0 \rangle,$$

(36)
where $\tilde{J}(x)$ and $J^D(x)$ are the interpolating currents of the particles $\tilde{T}$ and $D^+$ defined by Eqs. [2] and [21], respectively. As the interpolating current of the pseudoscalar meson $D^0$ we use

$$J^{D^0}(x) = \overline{\psi}(x)i\gamma_5\epsilon^l(x).$$

(37)

Then it is not difficult to get the physical side of the sum rule

$$\overline{\Pi}^{\text{Phys}}(p, p') = \frac{m_{D^0} f_{D^0}}{m_c} \int \frac{d^4q}{(q^2 - m_D^2)} \langle D^0(p)|\overline{\psi}(p)\gamma_\mu J^{D^0}(q)\psi(p')\rangle G(q^2) \frac{1}{(q^2 - m_D^2)} + \ldots$$

(38)

Introducing the new matrix elements

$$\langle 0|J^{D^0}\rangle D^0(q) = m_{D^0} f_{D^0},$$

$$\langle D^0(q)|D^0\rangle D^0(p') = G(q^2)(p \cdot p'),$$

(39)

where $m_{D^0}$ and $f_{D^0}$ are $D^0$ meson’s mass and decay constant, one can rewrite $\overline{\Pi}^{\text{Phys}}(p, p')$ in terms of the physical parameters

$$\overline{\Pi}^{\text{Phys}}(p, p') = G(q^2) \frac{m_{D^0} f_{D^0} f_{\tilde{T}}^m}{2m_c^2 m_D^2} \frac{m_{D^0}^2 f_D}{(q^2 - m_{D^0}^2)} \left( m_{D^0}^2 + m_D^2 - q^2 \right) + \ldots$$

(40)

We find the QCD side in the form:

$$\overline{\Pi}^{\text{OPE}}(p, p') = i^2 \int d^4x d^4y e^{i(p' y - px)} \epsilon \epsilon \{ \text{Tr} \left[ \gamma_5 S_{\overline{c} c}^\mu(y - x) \right]$$

$$\times \gamma_\mu S_{\overline{d} d}^\mu(x) \gamma_\nu S_{\overline{c} c}^\nu(x - y) \} - \text{Tr} \left[ \gamma_5 S_{\overline{c} c}^\mu(y - x) \right]$$

$$\times \gamma_\mu S_{\overline{d} d}^\mu(x) \gamma_\nu S_{\overline{c} c}^\nu(x - y) \}.$$ (41)

The following manipulations with $\overline{\Pi}^{\text{Phys}}(p, p')$ and $\overline{\Pi}^{\text{OPE}}(p, p')$ are standard operations which lead to the sum rule

$$G(q^2) = \frac{2m_c^2 m_D^2 f_D f_{\tilde{T}}}{m_{D^0}^2 m_D^2} \frac{q^2 - m_{D^0}^2}{m_D^2 + m_D^2 - q^2}$$

$$\times e^{m_{D^0}^2/M^2} e^{m_D^2/M^2} \overline{\Pi}(M^2, s_0, q^2).$$

(42)

In numerical calculations the auxiliary parameters for $\overline{T}$ and $D^+$ channels are chosen as in Eqs. [18] and [29], respectively. The parameters of the fit function $F_3(Q^2)$ are equal to $F_0^3 = 0.31 \text{ MeV}^{-1}$, $c_1^3 = -1.15$ and $c_2^3 = 0.92$, which at the mass-shell $Q^2 = -m_{D^0}^2$ leads to the strong coupling

$$G(-m_{D^0}^2) = (0.43 \pm 0.07) \text{ GeV}^{-1}.$$ (43)

The width of this decay is determined by the expression

$$\Gamma(\overline{T} \to D^+ D^0) = \frac{G^2 m_D^2}{8\pi} \lambda \left( 1 + \frac{\lambda^2}{m_D^2} \right),$$

(44)

where $\lambda = (m_{\overline{T}}, m_D, m_{D^0})$. Numerical computations predict:

$$\Gamma(\overline{T} \to D^+ D^0) = (12.4 \pm 3.1) \text{ MeV}.$$ (45)

TABLE I: Parameters of $D$ mesons produced in the decays of the tetraquarks $T$ and $\overline{T}$.

| Parameters | Values (in MeV units) |
|------------|-----------------------|
| $m_{D^0}$  | 1864.83 ± 0.05        |
| $m_D$      | 1869.65 ± 0.05        |
| $m_{1D^*}$ (D$^*(2007)^0$) | 2006.85 ± 0.05 |
| $m_{2D^*}$ (D$^*(2010)^+$) | 2010.26 ± 0.05 |
| $f_D$      | 203.7 ± 1.1           |
| $f_{D^*}$  | 263 ± 21              |

FIG. 3: The strong coupling $g_1(Q^2)$ as a function of the Borel parameters $M^2 = (M_1^2, M_2^2)$ at the fixed $(s_0, s_0^*) = (21.7 \text{ GeV}^2, Q^2 = 5 \text{ GeV}^2)$.

FIG. 4: The sum rule predictions and fit function for the strong coupling $g_1(Q^2)$. The star shows the point $Q^2 = -m_{D^0}^2$.

The partial width of these decays are main result of the present section.
In this work we have explored features of the doubly charmed pseudoscalar and scalar tetraquarks $T$ and $\bar{T}$. We have calculated their masses and couplings, as well as found partial widths of their strong decays. Our result for $m_T$ has allowed us to interpret the resonance $T$ as a member of the pseudoscalar multiplet of double-charmed tetraquarks. Saturating its full width by decays $T \rightarrow D^+D^*(2007)^0$ and $T \rightarrow D^0D^*(2010)^+$ it is possible to find

$$\Gamma_T = (129.9 \pm 23.5) \text{ MeV}. \quad (46)$$

Other members of this multiplet are tetraquarks $T_{cc\bar{s}s}^{++}$ and $T_{cc\bar{d}d}^{++}$, which were explored in Ref. [20]. These tetraquarks together with $T$ form correct pattern of the pseudoscalar multiplet. Indeed, their masses differ from each other by $\sim 125$ MeV connected with $s$-quark(s) in their contents or with its absence. The exotic mesons with full widths $\Gamma[T_{cc\bar{s}s}^{++}] = (302 \pm 113) \text{ MeV}$ and $\Gamma[T_{cc\bar{d}d}^{++}] = (171 \pm 52) \text{ MeV}$ are broad resonances. The width $\Gamma_T$ differs from $\Gamma[T_{cc\bar{s}s}^{++}]$ considerably, but is comparable with $\Gamma[T_{cc\bar{d}d}^{++}]$. Therefore, we classify the pseudoscalar tetraquark $T$ as a broad resonance.

The scalar double-charmed tetraquark $\bar{T}$ with full width $\Gamma_\bar{T} = (12.4 \pm 3.1) \text{ MeV}$ is relatively narrow state. This resonance is a member of a doubly charmed scalar tetraquarks' multiplet. Parameters of other members, their strong decays and, in particular, full widths can provide valuable information on properties of this multiplet.

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