\( \Upsilon(nS) \) and \( \chi_b(nP) \) production at hadron colliders in nonrelativistic QCD

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\( \Upsilon(nS) \) and \( \chi_b(nP) \) (\( n=1,2,3 \)) production at the LHC is studied at the next-to-leading order in \( \alpha_s \) in nonrelativistic QCD. Feeddown contributions from higher \( \chi_b \) and \( \Upsilon \) states are all considered for \( \Upsilon(nS) \) production cross sections and polarizations. The parameters are fitted by using \( \Upsilon(nS) \) yield and polarization data, and the results are consistent with those measurements. In particular, we show that the \( \Upsilon(3S) \) polarization puzzle can be understood by a large feeddown contribution from \( \chi_b(3P) \) states. The predictions of the \( \chi_b(nP) \rightarrow \Upsilon(mS) \) \((m,n=1,2,3; n \geq m)\) feeddown fractions based on these parameters are in agreement with the newly measured data, which indicates that the prompt production of \( \Upsilon(nS) \) and \( \chi_b(nP) \) can be explained in NRQCD.

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Introduction.—Since the surprisingly large production rate of \( \psi' \) at large \( p_T \) was found by CDF in 1992\(^1\), the production of heavy quarkonium at hadron colliders has been a problem full of puzzles. While the color-octet (CO) mechanism\(^2\) at leading order (LO) in nonrelativistic QCD (NRQCD) factorization\(^3\) might explain the large production rates of \( \psi' \) and \( J/\psi \) at large \( p_T \) via gluon fusion, the predicted transverse polarizations for \( J/\psi(\psi') \) were in contradiction with the measurements that the produced \( J/\psi(\psi') \) were almost unpolarized (see Ref.\(^4\) for a comprehensive review). In recent years, significant progress has been made in the next-to-leading order (NLO) QCD calculations in NRQCD. Calculations and fits for both yield and polarization of \( J/\psi \) production are performed by three groups\(^5\), where our group presented a possible explanation for the \( J/\psi \) polarization puzzle\(^6\). By including also leading power fragmentation corrections, which improves the convergence of \( \alpha_s \) expansion at large \( p_T \), a similar explanation for the \( J/\psi \) polarization is found\(^7\).

Recently, polarizations of \( \Upsilon(1S,2S,3S) \) have been measured by CMS at the LHC\(^8\). It is interesting to study the \( \Upsilon \) production within the same framework as that for the \( J/\psi \) production and further test the interpretation for the polarization puzzle in Ref.\(^4\). Note that \( \Upsilon \) should be a more suitable system than \( J/\psi \) to apply NRQCD, since both \( \nu \) (the relative velocity of heavy quarks in heavy quarkonium) and \( \alpha_s \) are smaller for bottomonium than charmonium, and thus the double expansion in \( \alpha_s \) and \( \nu \) should converge faster for bottomonium production. Earlier studies of \( \Upsilon \) production can be found in Refs.\(^9\)\(^12\) and references therein. In Ref.\(^13\), a NLO calculation of \( \Upsilon(1S,2S,3S) \) polarizations is given, where the polarizations for \( \Upsilon(1S,2S) \) agree with the CMS measurements\(^8\), but the predicted ratio of differential cross sections of \( \chi_{b2}(1P) \) to \( \chi_{b1}(1P) \)\(^13\) is too large and inconsistent with the CMS data\(^14\). Furthermore, without considering the \( \chi_b(3P) \) feeddown, the polarization data of \( \Upsilon(3S) \) can not be explained\(^15\).

Recently, the radiative transition of \( \chi_b(3P) \) to \( \Upsilon(3S) \) was first seen by the LHCb Collaboration\(^16\). Therefore, the explanation for \( \Upsilon(1S,2S) \) and in particular \( \Upsilon(3S) \) polarizations should be reconsidered, and a proper treatment for \( \chi_b(1P,2P,3P) \) feeddown contributions is needed. One should note that the treatment of \( \chi_b(3P) \) and \( \Upsilon(3S) \) will influence the production of \( \Upsilon(1S,2S) \) through the cascaded effects. In this work, we study the prompt production of \( \Upsilon(1S,2S,3S) \) with both direct and feeddown contributions at NLO in \( \alpha_s \) in NRQCD.

The polarized cross section for a bottomonium \( H \) can be factorized as\(^4\)

\[
\sigma_{s_z,s_z} = \sum_{i,j,n} \int dx_1 dx_2 G_{i/p} G_{j/p} \langle O_n^H \rangle \sigma_{s_z,s_z}^{i,j,n} \tag{1}
\]

where \( p \) denotes either proton or anti-proton, \( G_{i/p} \) are the parton distribution functions (PDFs) of \( p \), and the indices \( i,j \) run over all the partonic species. \( \langle O_n^H \rangle \) is the long distance matrix element (LDME), with "\( n \)" denotes the color, spin and angular momentum of the intermediate \( b\bar{b} \) pair, which can be \( 3S_1^{[1,8]} \), \( 1S_0^{[8]} \) and \( 3P_0^{[8]} \) for \( \Upsilon \), and \( 3P_1^{[1]} \) and \( 3S_0^{[8]} \) for \( \chi_b \). The yield can be obtained by summing the polarized cross sections over the spin quantum number \( s_z \). The virtual corrections are calculated by using our Mathematica code\(^6\)\(^13\) and the real corrections are obtained by using the HELAC-Onia program\(^17\). We further use the CTEQ6L1 and CTEQ6M PDFs\(^18\) respectively for LO and NLO calculations. The bottom quark mass is set to be \( m_b = 4.75 \) GeV, meanwhile the renormalization, factorization, and NRQCD scales are \( \mu_r = \mu_f = \sqrt{m_T^2 + 4m_b^2} \) and \( \mu_s = m_b \).

Feeddown and \( \chi_b(nP) \).—For \( \Upsilon \) the polarization observable \( \lambda_\theta \) can be expressed as \( \lambda_\theta = \frac{d\sigma_{s_z=s_z}}{d\sigma_{s_z=-s_z}} \), where \( \sigma_{s_0} \) and \( \sigma_{s_1} \) are polarized prompt cross sections, includ-
ing both direct production and feeddown contributions from higher Υ(nS) and χ_b(nP) states. Since the transitions between Υ(nS) are dominated by the S-wave dipion modes, the feeddown of higher Υ(nS) will inherit the spin index of the mother particles. While for the χ_b(nP) feeddown, which proceeds mainly through χ_b(nP) → Υ(mS)γ, the general inheritance relations of polarizations are given in Ref. [8].

\[
\lambda_{\chi_b} = \frac{d\sigma^{\chi_b}_{00} - d\sigma^{\chi_b}_{11}}{3d\sigma^{\chi_b}_{11} + d\sigma^{\chi_b}_{00}},
\]

(2)

Similar to χ_b at NLO in α_s, the χ_{bJ} production is determined by the color-octet (CO) 3S^{[8]} and color-singlet (CS) 3P^{[1]} contributions. If CO 3S^{[8]} is dominant, which leads to transverse polarization at large p_T, the ratios of polarized cross sections become d\sigma^{\chi_b}_{00} / d\sigma^{\chi_b}_{11} = 2 : 1 and d\sigma^{\chi_b}_{12} / d\sigma^{\chi_b}_{11} = 1 / 3 : 1 / 2 : 1, and the feeddown polarization parameters in Eq. (2) are 0.20 for χ_b1 and 0.29 for χ_b2. If further including the CS 3P^{[1]} contribution, the overall polarization parameters of χ_{bJ} feeddown only change slightly. This implies that the χ_{bJ} feeddown will contribute a modest transverse polarization for Υ at large p_T.

The CS LDMEs for χ_{bJ}(nP) can be related to the derivatives of radial wave functions at the origin by

\[
\langle O^{\chi_{bJ}(nP)}(3P^{[1]}) \rangle = (2J + 1) \frac{3}{4\pi} |R_{nP}^{[1]}(0)|^2,
\]

(3)

where |R_{nP}^{[1]}(0)|^2 can be estimated in potential models. E.g. the B-T potential model [22] gives |R_{1P2P3P}^{[1]}(0)|^2 = (1.417, 1.653, 1.794) GeV^5. In fact, various potentials in Refs. [20] and [22] all indicate |R_{nP}^{[1]}(0)|^2 \approx |R_{2P}^{[2]}(0)|^2 \approx |R_{3P}^{[3]}(0)|^2. So, as a balanced approximation, we use

\[
|R_{nP}^{[1]}(0)|^2 \approx 1.653 \text{ GeV}^5, \quad n = 1, 2, 3,
\]

(4)
as input. The CO LDMEs are introduced via the ratio

\[
r_{nP} = m_b^2 \langle O^{\chi_{bJ}(nP)}(3S^{[8]}_1) \rangle / \langle O^{\chi_{bJ}(nP)}(3P^{[1]}_1) \rangle,
\]

(5)

which is independent of J since \langle O^{\chi_{bJ}(nP)}(3S^{[8]}_1) \rangle = (2J + 1) \langle O^{\chi_{bJ}(nP)}(3S^{[8]}_1) \rangle. Unlike the CS LDMEs, r_{nP} can not be estimated from potential models, but should be extracted from experimental data.

We also assume that the total decay widths of χ_{bJ}(nP), which are related to |R_{nP}^{[1]}(0)|^2, are approximately independent of n. Then, taking the partial decay widths of χ_{bJ}(nP) → Υ(mS)γ calculated in Ref. [21] and the PDG values of Br(χ_{bJ}(1P) → Υ(1S)γ) [22] as inputs, we can calculate the branching ratios Br(χ_{bJ}(2P) → Υ(2S)γ) and Br(χ_{bJ}(2P) → Υ(1S)γ), which are close to their PDG values. This implies that it may be a good approximation that the total widths of χ_{bJ}(nP) are independent of n. With this we further calculate Br(χ_{bJ}(3P) → Υ(1S, 2S, 3S)γ), which are listed in Tab. II.

| Br | n = 1 | n = 2 | n = 3 |
|----|-------|-------|-------|
| χ_{b0}(3P) → Υ(nS) | 0.24% | 0.22% | 0.50% |
| χ_{b1}(3P) → Υ(nS) | 3.31% | 3.68% | 10.44% |
| χ_{b2}(3P) → Υ(nS) | 1.92% | 1.91% | 6.11% |

TABLE I: Estimated branching ratios Br(χ_{bJ}(3P) → Υ(1S, 2S, 3S)γ) by assuming that total decay widths of χ_{bJ}(nP) are independent of n.

Prompt Υ(nS) production. Having clarified how to treat the contributions from feeddown, we now extract LDMEs of Υ(nS) and r(nP) defined in [8] by fitting both yield and polarization data of Υ(3S), Υ(2S) and Υ(1S) in turn. In the fit, we use the measured differential cross sections by ATLAS [23] and CMS [24] and the polarization data by CMS. [25] The available cross section ratio of χ_{b2}(1P) to χ_{b1}(1P) measured by CMS [25] is also included in our fit. To avoid potential non-perturbative effects in the sense that only the first two powers in the 1/p_T^2 expansion of cross section are proven to be factorized [25], we need to introduce a relatively large p_T cut for the data (for the similar case in the production of χ_b(0), see Ref. [14, 22]). In our fit, we only use data in the region p_T > 15 GeV because the χ^2/d.o.f. will increase quickly when the p_T cut becomes smaller than 15 GeV. For example, by choosing the p_T cut 7, 9, 11, 13, 15, and 17 GeV, the corresponding χ^2/d.o.f. in fitting Υ(3S) data are 6.6, 4.0, 2.3, 1.8, 1.3, and 1.0, respectively.

As a crucial point in our fit, we assume all CO LDMEs to be positive to reduce the theoretical uncertainties. We are aware that negative CO LDMEs can not be excluded in this stage, but a positive definite hypothesis is tenable as long as the data can be well described. Indeed, we see that we can fit the data well and this gives strong support to our hypothesis.

| (O(3S^{[1]})) | GeV^{-3} | λ_1 | 10^{-2}GeV^{-3} | λ_2 | 10^{-2}GeV^{-3} | λ_3 | 10^{-2}GeV^{-3} |
|------------|---------|---|-----------|---|-----------|---|-----------|
| Υ(1S) | 9.28 | -1.29 | ± 9.24 | 3.87 | ± 0.27 | 2.64 | ± 0.08 |
| Υ(2S) | 4.63 | 5.77 | ± 6.86 | 1.51 | ± 0.25 | 1.69 | ± 0.06 |
| Υ(3S) | 3.54 | 2.52 | ± 4.81 | 0.72 | ± 0.18 | 1.09 | ± 0.05 |

TABLE II: The LDMEs for Υ(1S, 2S, 3S) production. The combined LDMEs are obtained by the fit, while the CO ones are estimated by using the B - T potential model in Ref. [21].

Based on the above method, we fit CO LDMEs for Υ(1S, 2S, 3S) with χ^2/d.o.f. = 1.05, 2.38, 1.25. The dif-
FIG. 1: Differential cross sections for the experimental windows of ATLAS, CMS and CDF. From left to right: \( \Upsilon(1S), \Upsilon(2S), \Upsilon(3S) \). The contributions from direct production are denoted by dashed lines, while those from feeddown by dashed-dotted lines. The experimental data are taken from Refs. [23, 24, 27].

FIG. 2: The polarization parameter \( \lambda_3 \) in the helicity frame for the experimental widows at the LHC. From left to right: \( \Upsilon(1S), \Upsilon(2S), \Upsilon(3S) \). The contributions from direct production are denoted by dashed lines, while those from feeddown by dashed-dotted lines. The total results are denoted by the blue bands. The experimental data are taken from Ref.[9].

The differential cross section can be expressed as

\[
d\sigma = \sum_{i=1}^{3} d\hat{\sigma}_i O_i = \sum_{i=1}^{3} a_i \Lambda_i, \quad \text{with} \quad \Lambda_i = \sum_{j=1}^{3} V_{ij} O_j, \quad (6)
\]

where \( O_i \) (\( i = 1, 2, 3 \)) denote \( \mathcal{O}(s_0^{[8]}), \mathcal{O}(s_1^{[8]}) \) and \( \mathcal{O}(T_0^{[8]}) \). Matrices \( V \) for \( \Upsilon(nS) \) are obtained in the \( \chi^2 \)-fit, which are almost independent of \( n \),

\[
V \approx \begin{pmatrix}
0.96 & -0.13 & -0.24 \\
0.25 & 0.05 & 0.97 \\
0.12 & 0.99 & -0.08
\end{pmatrix}.
\]

This is because cross sections of \( \Upsilon(nS) \) depend on the same short-distance coefficients and on \( O_i \) linearly. The linearly combined LDMEs \( \Lambda_i \) are given in Tab.II, together with CS LDMEs that are estimated by using the
B-T potential model [20]. As for \( r(nP) \), the results are listed in Tab. III together with those obtained in Ref. [13] for comparison.

| \( n \) | \( r(nP) \) |
|---|---|
| 1 | 0.39 ± 0.19 |
| 2 | 0.66 ± 0.46 |
| 3 | 0.89 ± 0.59 |

This work 0.85 ± 0.11 1.58 ± 0.38

Ref. [13]

TABLE III: The values of \( r(nP) \) for \( n = 1, 2, 3 \) in this work and in Ref. [13].

In Tab. III, we see \( \Lambda_2 \) and \( \Lambda_3 \) have small uncertainties, but \( \Lambda_1 \) has huge uncertainty. This means, there are only two linear combinations of CO LDMEs can be well determined, which reflects the fact that at high \( p_T \) the cross sections have only two leading terms \( 1/p_T^2 \) and \( 1/p_T^4 \). Specifically, contributions of \( \Lambda_2 \) and \( \Lambda_3 \) correspond approximately to \( 1/p_T^2 \) and \( 1/p_T^4 \) respectively. Although \( \Lambda_1 \) is not well constrained, the obtained LDMEs in Tab. III can still give a good prediction for observables that are not sensitive to \( \Lambda_1 \). As mentioned before, we use the constraint that all \( O_i \) are positive to reduce the theoretical uncertainty. A comprehensive discussion and treatment of the large uncertainties can be found in Ref. [14]. We find the central value of \( \Lambda_2 \) decrease more quickly than that of \( \Lambda_3 \) as \( n \) increases. This explains why a higher \( \Upsilon(nS) \) tends to have a less steep \( p_T \) cross section.

The fitted differential sections and the polarization parameter \( \lambda_0 \) in the helicity frame are shown in Figs. 1 and 2, respectively, together with the relevant data [12, 23, 24]. Comparisons between our predictions of the cross sections with the CDF data [27] are also shown in the second row in Fig. 1. It is interesting to see that, both the yield and polarization data for \( \Upsilon(1S, 2S, 3S) \) can be well described, simultaneously. In particular, the good agreement with \( \Upsilon(3S) \) polarization data is achieved explicitly by the relative larger feeddown contribution from \( \chi_b(3P) \), as indicated by the relative large value of \( r(3P) \) in Tab. III. Similar cases also influence the predictions for \( \Upsilon(1S, 2S) \) polarizations and lead to better agreement with data.

The importance of \( \chi_b \) feeddown contributions in explaining the polarization data of \( \Upsilon(1S, 2S) \) has been emphasized in Ref. [13]. However, without properly treating the \( \Upsilon(3S) \) and \( \chi_b(3P) \) production, in Ref. [13] the excess \( \chi_b(1P, 2P) \) production is invoked to account for the \( \Upsilon(1S, 2S) \) polarization data, which can be seen from the relative larger values for \( r(1P) \) and \( r(2P) \) in the third row in Tab. III. Similar to the case of \( \chi_{cJ} \) production [28, 29], the ratio of differential cross sections of \( \chi_b(1P) \) and \( \chi_b(1P) \) is sensitive to the value of \( r(1P) \). In Fig. 3, we see that with the extracted value of \( r_{1P} \) in Tab. III we can well describe the measured ratio of differential cross sections of \( \chi_3^b \) to \( \chi_b(1P) \) by CMS [14], whereas the result given by Ref. [13] is inconsistent with data.

Recently, the fractions of \( \Upsilon(mS) \) (\( m = 1, 2, 3 \)) production originating from the \( \chi_b(nP) \) (\( n = 1, 2, 3; n \geq m \)) feeddown contributions, which are denoted by \( R_{\Upsilon(mS)}^{\chi_b(nP)} \), were measured by the LHCb Collaboration [13]. It is interesting to compare the data with our predictions by using the LDMEs in Tab. III and the extracted values of \( r_{nP} \) in Tab. III together with the branching ratios given in Tab. III and in the PDG [22]. The results for the LHCb window at center-of-mass energy \( \sqrt{s} = 7 \) GeV are shown in Fig. 4. For comparison, we also evaluate the fractions \( R_{\Upsilon(mS)}^{\chi_b(nP)} \) by using the parameters in Ref. [13], which are shown in Fig. 4 as the yellow bands. From Fig. 4, one can see that the \( \chi_b(1P, 2P) \) production rates predicted by Ref. [13] are too large compared with data, whereas our predictions of the production rates of both \( \chi_b(1P, 2P) \) and \( \chi_b(3P) \), which are denoted by the blue bands in Fig. 4, are roughly consistent with data. This implies that the production rates of \( \chi_b(1P, 2P, 3P) \) measured by LHCb [13] are consistent with the polarizations of prompt \( \Upsilon(1S, 2S, 3S) \) in our theoretical framework, which serves as a nontrivial test of the production mechanism of bottomonia.

Summary. We fit LDMEs for \( \Upsilon(nS) \) and \( \chi_b(nP) \) prompt production at NLO in NRQCD by using the LHC large \( p_T \) yield data [12, 23, 24] and polarization data [13]. With a systematic careful treatment of the \( \chi_b(nP) \) feeddown contribution, our results can well describe both the yield and polarization data for \( \Upsilon(nS) \) production and differential cross section ratio of \( \chi_b(1P) \) to \( \chi_b(1P) \). In particular, we find that the \( \Upsilon(3S) \) polarization puzzle can be understood by a large feeddown contribution from \( \chi_b(3P) \) states. The predictions of the feeddown fractions \( R_{\Upsilon(mS)}^{\chi_b(nP)} \) (\( m, n = 1, 2, 3; n \geq m \)) based on these LDMEs are in agreement with the experimental data recently measured by the LHCb Collaboration [13].

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