Target normal spin asymmetry and charge asymmetry for $e\mu$
elastic scattering and the crossed processes

E. A. Kuraev, V. V. Bytev, Yu. M. Bystritskiy

JINR-BLTP, 141980 Dubna, Moscow region, Russian Federation

E. Tomasi-Gustafsson

DAPNIA/SPhN, CEA/Saclay, 91191 Gif-sur-Yvette Cedex, France

(Dated: November 27, 2018)

Abstract

Two kinds of asymmetry arise from the interference of the Born amplitude and the box-type amplitude corresponding to two virtual photons exchange, namely charge-odd and one spin asymmetries. In case of unpolarized particles the charge-odd correlation is calculated. It can be measured in combination of electron muon and positron muon scattering experiments. The forward-backward asymmetry is the corresponding quantity which can be measured for the crossed processes. In the case of polarized muon the one-spin asymmetry for annihilation and scattering channels has been calculated. The additional structure function arising from the interference is shown to suffer from infrared divergencies. The background due to electroweak interaction is discussed.

PACS numbers:
I. INTRODUCTION

The motivation of this work is to give an accurate description of the process $e\bar{\mu} \rightarrow e\bar{\mu}(\gamma) e\bar{e} \rightarrow \mu\bar{\mu}(\gamma)$ in frame of QED, in order to provide a basis for the comparison with experimental data. High precision experiments on the processes $e\bar{e} \rightarrow \tau\bar{\tau}$ and $e\bar{e} \rightarrow p\bar{p}$ are planned in future $c-\tau$ facilities. Moreover, the possibility of colliding $e\mu$ beam facilities has been discussed in framework of programs on verification of Standard Model (SM) prediction.

The obtained results can also be applied, as a realistic model, to electron (positron) scattering on a point-like hadron (proton), based on arguments given in this paper.

It is known that in Born approximation the differential cross-section of elastic proton-electron scattering

$$e(p_1) + p(p) \rightarrow e(p'_1) + p(p')$$

(1)
can be expressed in terms of two proton form factors, $F_{1,2}(q^2)$, which are functions of a single argument, the momentum transfer squared, $q^2 = t$.

Taking into account two (and more) photon exchanges, (TPE), leads to a generalization of the Born picture, namely the amplitude of $ep$ scattering depends on two Mandelstam variables, the total energy $s$ and $t$. The virtual photon Compton scattering amplitude is a rather complex object, which can be expressed in terms of 12 chiral amplitudes. Nevertheless, taking into account parity conservation and omitting the terms of order $m_e/m_\mu$, (which are responsible for chirality violation) reduce the number of relevant amplitudes to three $^1$:

$$M^{(2)} = \frac{i\alpha^2}{t} \bar{u}(p'_1)\gamma_\mu u(p_1) \times \bar{u}(p') \left[ F_1(s,t)\gamma_\mu \frac{F_2(s,t)}{2M} \gamma_\mu \hat{q} + \frac{1}{t} F_3(s,t)(\hat{p}_1 + \hat{p}'_1)(p + p')_\mu \right] u(p),$$

with $q = p_1 - p'_1$, $s = 2p_1p$.

The explicit calculation, given in this paper, permits one to extract the individual contributions $F_1$, $F_2$, and $F_3$, in frame of QED. The infrared divergency is cancelled when the relevant soft photon emission is correctly taken into account.

Charge-odd and backward-forward asymmetries appear naturally from the interference of one and two photon exchange amplitudes in frame of QED and SM due to $Z_0$-boson exchange in Born approximation. But at the energy range reachable at $c-\tau$ factories, the relevant contribution of SM type is $^2$:

$$\frac{d\sigma^{odd}_{Z}}{d\sigma_{QED}} \approx \frac{s}{M_Z^2} a_\alpha a_\alpha \approx 5 \cdot 10^{-5}, \quad 3 < \sqrt{s} < 5 \text{ GeV}. \quad (2)$$
which is quite small compared to QED effects. The accuracy of results given below is determined by

\[ \mathcal{O} \left( \frac{m_e^2}{m_\mu^2}, \frac{m_e^2}{m_\tau^2}, \frac{m_e^2}{m_p^2} \right) \sim 0.1\% \quad (3) \]

and the contribution of higher orders of QED \( \alpha/\pi \approx 0.5\% \). Moreover we assume that all the velocities of the final heavy particles are finite in the annihilation as well as in the scattering channels. This is the reason why Coulomb factors are neglected.

Our paper is organized as follows. In sections II the annihilation channel and the scattering channel \( e^- \rightarrow \mu^- \) are considered in sections III and XIV respectively. In section XV we take into account the soft photon emission and construct charge-odd and forward-backward asymmetries. In section XVI we analyze the crossing relation between two channels. Explicit form for additional structure \( G_3 \) for annihilation channel is given in section XVII. In section XVII the one-spin asymmetries are investigated. The results are summarized in the Conclusions.

II. PROCESS \( e^+ + e^- \rightarrow \mu^+ + \mu^- (\gamma) \)

At first, we consider the process of creation of \( \mu^+ \mu^- \) pairs in electron-positron annihilation:

\[ e^+(p_+) + e^-(p_-) \rightarrow \mu^+(q_+) + \mu^-(q_-). \quad (4) \]

The cross-section in the Born approximation, can be written as:

\[ \frac{d\sigma_B}{d\Omega_{\mu^-}} = \frac{\alpha^2}{4s} \beta(2 - \beta^2 + \beta^2 c^2), \quad (5) \]

with \( s = (p_+ + p_-)^2 = 4E^2, \beta^2 = 1 - \frac{4m_e^2}{s}, E \) is the electron beam energy in center of mass reference frame (implied for this process below), \( m, m_e \) are the masses of muon and electron, \( c = \cos \theta \), and \( \theta \) is the angle of \( \mu^- \)-meson emission to the electron beam direction.

The interference of the Born amplitude

\[ M_B = \frac{i4\pi\alpha}{s} \bar{v}(p_+)\gamma_\mu u(p_-)\bar{u}(q_-)\gamma_\mu v(q_+), \]

with the box-type amplitude \( M_B \), results in parity violating contributions to the differential cross section, i. e. the ones, changing the sign at \( \theta \rightarrow \pi - \theta \). As a consequence of charge-odd correlations we can construct:

\[ A(\theta, \Delta E) = \frac{d\sigma(\theta) - d\sigma(\pi - \theta)}{d\sigma_B(\theta)}. \quad (6) \]
FIG. 1: Feynman diagrams for two-photon exchange in $e\bar{e} \rightarrow \mu\bar{\mu}$ process: box diagram (a) and crossed box diagram (b).

Here we take into account as well the emission of an additional soft real photon with energy not exceeding some small value $\Delta E$, so that $A(\theta, \Delta E)$ is free from the infrared singularities.

Part of the results presented here were previously derived by one of us (E. A. K.) in Ref. [3], and partially published in [4].

There are two box-type Feynman amplitudes (Fig. 1). We calculate only one of them, the uncrossed diagram (Fig. 1a) with matrix element

$$M_a = i\alpha^2 \int \frac{d^4k}{i\pi^2} \frac{\bar{u}(q_-)Tv(q_+) \times \bar{v}(p_+)Zu(p_-)}{(\Delta)(Q)(P_+)(P_-)},$$

$$\Delta = (k - \Delta)^2 - m_e^2, \quad Q = (k - Q)^2 - m^2, \quad P_\pm = (k \mp P)^2 - \lambda^2,$$

with $\lambda$-"photon" mass and

$$T = \gamma_\alpha(\hat{k} - \hat{Q} + m)\gamma_\beta, \quad Z = \gamma_\beta(\hat{k} - \hat{\Delta})\gamma_\alpha,$$

$$\Delta = \frac{1}{2}(p_+ - p_-), \quad Q = \frac{1}{2}(q_+ - q_-), \quad P = \frac{1}{2}(p_+ + p_-).$$

We will assume

$$m^2 = \frac{s}{4}(1 - \beta^2) \sim s \sim -t \sim -u.$$

The explicit form of kinematical variables used below is:

$$\Delta^2 = -P^2 = -\frac{s}{4}, \quad Q^2 = -\frac{1}{4}s\beta^2, \quad \sigma = \Delta Q = \frac{1}{4}(u - t),$$

$$u = (p_+ - q_-)^2 = -\frac{s}{4}(1 + \beta^2 + 2\beta c), \quad t = (p_+ - q_-)^2 = -\frac{s}{4}(1 + \beta^2 - 2\beta c).$$

The contribution to the cross section of the amplitude arising from the crossed Feynman
The scalar, vector and tensor loop momentum integrals are defined as:

\[ J; J_\mu; J_{\mu \nu} = \int \frac{d^4k}{i \pi^2} \frac{1}{(\Delta)(Q)(P_+)(P_-)} \]

Using symmetry properties, the vector and tensor integrals can be written as:

\[ J_\mu = J_\Delta \cdot \Delta_\mu + J_Q \cdot Q_\mu, \]

\[ J_{\mu \nu} = K_0 g_{\mu \nu} + K_P P_\mu P_\nu + K_Q Q^\mu Q_\nu + K_{\Delta \mu} \Delta_\nu + K_x (Q_\mu \Delta_\nu + Q_\nu \Delta_\mu). \]

The quantity \( R(s, t) \) can be expressed as a function of polynomials \( P_i \) as:

\[ R = P_1 J + P_2 J_\Delta + P_3 J_Q + P_4 K_0 + P_5 K_\Delta + P_6 K_Q + P_7 K_P + P_8 K_x, \]

where the explicit form of polynomials is given in Appendix A. Using the explicit expression for the coefficients \( J_\Delta, ..., K_x \) (See Appendix B) we obtain

\[ R(s, t) = 4(\sigma - \Delta^2)(2\sigma - m^2)F + 16(\sigma - \Delta^2)(\sigma^2 + (\Delta^2)^2 - m^2\Delta^2)J \\
+ 4[(\Delta^2)^2 - 3\Delta^2\sigma + 2\sigma^2 - m^2\sigma]F_Q + 4[2(\Delta^2)^2 - 2\Delta^2\sigma + 2\sigma^2 - m^2\Delta^2]F_\Delta \\
+ 4[(\Delta^2)^2 + \Delta^2\sigma + m^2\Delta^2]G_Q + 4[-(\Delta^2)^2 + \sigma^2 - 2m^2\Delta^2]H_Q, \]

with the quantities \( F \div H_Q \) given in Appendix B. Finally the charge-odd part of differential cross section has the form

\[ \left( \frac{d\sigma_{\text{virt}}^{ee}(s, t)}{d\Omega_\mu} \right)_{\text{odd}} = -\frac{\alpha^3 \beta}{2\pi s} D^{\text{ann}}, \]

\[ D^{\text{ann}} = \frac{1}{s} [R(s, t) - R(s, u)] = (2 - \beta^2 + \beta^2 \epsilon^2) \ln \left( \frac{1 + \beta c}{1 - \beta c} \right) \ln \frac{s}{\lambda^2} + D^{\text{ann}} \]
\[
\mathcal{D}_{\nu}^{ann} = \left(1 - 2\beta^2 + \beta^2 c^2\right) \left[\frac{1}{1 + \beta^2 + 2\beta c} \left(\frac{\ln \frac{1 + \beta c}{2} + \ln \frac{s}{m^2}}{2}\right)\right] \\
- \frac{1}{1 + \beta^2 - 2\beta c} \left(\frac{\ln \frac{1 - \beta c}{2}}{2} + \ln \frac{s}{m^2}\right) \\
+ \beta c \left[\phi(\beta) \left(\frac{1}{2\beta^2} - 1 - \frac{\beta^2}{2}\right) - \frac{1}{\beta^2} \ln \frac{s}{m^2} - \frac{\pi^2}{6} + \frac{1}{2} \ln^2 \frac{s}{m^2}\right] \\
- \frac{1}{2} \ln^2 \frac{1 - \beta c}{2} - \frac{1}{2} \ln^2 \frac{1 + \beta c}{2} + \text{Li}_2 \left(\frac{1 + \beta^2 + 2\beta c}{2(1 + \beta c)}\right) + \text{Li}_2 \left(\frac{1 + \beta^2 - 2\beta c}{2(1 - \beta c)}\right) \\
- \frac{m^2}{s} \left[\ln \frac{1 - \beta c}{2} - \ln \frac{1 + \beta c}{2} + 2\text{Li}_2 \left(\frac{1 + \beta^2 + 2\beta c}{2(1 + \beta c)}\right) - 2\text{Li}_2 \left(\frac{1 + \beta^2 - 2\beta c}{2(1 - \beta c)}\right)\right],
\]
where \(\phi(\beta) = sF_Q\), \(F_Q\) is given in Appendix B and
\[
\text{Li}_2(z) = -\int_0^z \frac{dx}{x} \ln(1 - x)
\]
is the Spence function. The quantity \(\mathcal{D}_{\nu}^{ann} - \mathcal{D}_{\nu}^{ann}\) suffers from infrared divergences, which will be compensated taking into account the soft photons contribution (see below).

**III. SCATTERING CHANNEL**

Let us consider now the elastic electron muon scattering
\[
e(p_1) + \mu(p) \rightarrow e(p'_1) + \mu(p')
\]
which is the crossed process of \([2]\). The Born cross section is the same for the scattering of electrons and positrons on the same target. Taking the experimental data from the scattering of electron and positron on the same target (muon or proton), one can measure the difference of the corresponding cross-sections which is sensitive to the interference of the one and two photon exchange amplitudes. For the case of proton target, in the Laboratory (Lab) frame, the differential cross section as a function of the energy of the initial electron, \(E\) and of the electron scattering angle, \(\theta_e\), was derived in Ref. \([5]\):
\[
\frac{d\sigma^{ep}}{d\Omega} = \frac{\alpha^2 \cos^2 \frac{\theta_e}{2}}{4E^2 \sin^4 \frac{\theta_e}{2}} \rho \left[\frac{F_E^2 + \tau F_M^2}{1 + \tau} + 2\tau F_M^2 \tan^2 \frac{\theta_e}{2}\right],
\]
where
\[
\rho = 1 + \frac{2E}{m} \sin^2 \frac{\theta_e}{2}, \quad \tau = \frac{-t}{4m^2} = \frac{E^2}{m^2 \rho} \sin^2 \frac{\theta_e}{2},
\]
and it is known as the Rosenbluth formula. The Sachs electric and magnetic proton form factors, \(F_E\) and \(F_M\) are related to the Pauli and Dirac form factors by \(F_E = F_1 - \tau F_2\),
For the scattering on muon, one replaces $F_1 = 1$, $F_2 = 0$ and Eq. (21) becomes
\[
\frac{d\sigma_{\mu}^{\mu}}{d\Omega} = \frac{\alpha^2 (s^2 + u^2 + 2tm^2)}{2m^2 \rho^2 t^2}, \quad s = 2p_1 p = 2mE, \quad t = -2p_1 p'_1, \quad u = -2p p'_1 = -\frac{s}{\rho}. \quad (22)
\]
The charge-odd contribution to the cross section of e\mu−elastic scattering is:
\[
\left(\frac{d\sigma_{\mu}^{\mu}}{d\Omega}\right)_{\text{odd}} = -\frac{\alpha^3}{2\pi m^2 \rho^2} Re \left(\mathcal{D}^{sc}\right),
\]
\[
\mathcal{D}^{sc} = \frac{1}{t} [\mathcal{D}(s, t) - \mathcal{D}(u, t)] = \frac{2}{t^2} [s^2 + u^2 + 2tm^2] \ln \frac{-u}{s} \ln \frac{-t}{\lambda^2} + \mathcal{D}_{\text{virt}}^{sc} \quad (23)
\]
with
\[
\mathcal{D}_{\text{virt}}^{sc} = \frac{s - u}{t} \left[ \frac{1}{2} \ln \frac{-t}{m^2} - \frac{\tau}{1 + \tau} \ln \frac{-t}{m^2} + m^2 \tilde{F}_Q \left(6\tau + 2 - \frac{2\tau^2}{1 + \tau}\right) \right] + \frac{s}{t} \left[ -\ln \frac{s}{-t} + \pi^2 + 2\text{Li}_2 \left(1 + \frac{m^2}{s}\right) \right] - \frac{u}{t} \left[ -\ln \frac{u}{t} + 2\text{Li}_2 \left(1 + \frac{m^2}{u}\right) \right]
\]
\[
+ \frac{(1 - 2\tau)}{(-4\tau)} \left[ 2 \ln \frac{s}{-u} \ln \frac{-t}{m^2} + \ln \frac{-u}{m^2} - \ln \frac{s}{m^2} + \pi^2 \right.
\]
\[
+ 2\text{Li}_2 \left(1 + \frac{m^2}{s}\right) - 2\text{Li}_2 \left(1 + \frac{m^2}{u}\right) \left] + \left(2m^2 - \frac{su}{t}\right) \left[ \ln \frac{s - u}{m^2 + s - m^2 + u} \right] \right. \quad (24)
\]
with the help of the following relation:
\[
m^2 \tilde{F}_Q = -\frac{1}{4\sqrt{\tau}(1 + \tau)} \left[ \pi^2 + \ln(4\tau) \ln x + \text{Li}_2 \left(-2\sqrt{\tau x}\right) - \text{Li}_2 \left(\frac{2\sqrt{\tau}}{\sqrt{x}}\right) \right], \quad (25)
\]
where
\[
x = \frac{\sqrt{1 + \tau} + \sqrt{\tau}}{\sqrt{1 + \tau} - \sqrt{\tau}}.
\]

### IV. SOFT PHOTON EMISSION

In this section the emission of soft real photons in the Lab reference frame for e\mu- scattering is calculated. Following Ref. [6], the odd part of cross section
\[
\frac{d\sigma^{\text{soft}}}{d\sigma_0} = -\frac{\alpha}{4\pi^2} \cdot 2 \int \frac{d^3k}{\omega} \left( \frac{p'_1}{p'_1 k - p_1} - \frac{p_1}{p_1 k} \right) \left( \frac{p'}{p' k - p} - \frac{p}{p k} \right) s_0, \omega < \Delta \epsilon \quad (26)
\]
must be calculated in the special reference frame $S_0$, where the sum of the three-momenta of proton and of the recoil proton is zero $\vec{z} = \vec{k} + \vec{p}' = 0$. Really, in this frame, the on-mass shell condition of the scattered muon $\delta[(z - k)^2 - m^2], z = p_1 + p - p'_1$ does not depend on
the direction of the emitted photon. The photon energy can be determined as the difference of the energy of the scattered electron and the corresponding value for the elastic case: the maximum value of the photon energy $\Delta \varepsilon$ in the $S_0$ frame is related with the energy of the scattered electron, detected in the Lab frame $\Delta E$ as (see [6], Appendix C),

$$\Delta \varepsilon = \rho \Delta E.$$  

(27)

The calculation of the soft photon integral with $\omega < \Delta \varepsilon$ can be performed using t’Hooft and M. Veltman approach (see [7], Section 7). We find

$$\frac{d\sigma_{\text{soft}}}{d\Omega} = -\frac{\alpha^3}{\pi} \frac{(s^2 + u^2 + 2tm^2)}{2m^2 \rho^2 t^2} \left\{ 2 \ln \rho \ln \left[ \frac{(2\rho \Delta E)^2}{\lambda^2 x} \right] + D_{\text{soft}}^{\text{sc}} \right\},$$

$$D_{\text{soft}}^{\text{sc}} = -2 \text{Li}_2 \left( 1 - \frac{1}{\rho x} \right) + 2 \text{Li}_2 \left( 1 - \frac{\rho}{x} \right), \quad \rho = \frac{s}{-u},$$

(28)

which is in agreement with Ref. [6].

The sum $(d\sigma_{\text{virt}} + d\sigma_{\text{soft}})_{\text{odd}}$ has the form

$$\left( \frac{d\sigma_{\text{virt}}}{d\Omega_e} + \frac{d\sigma_{\text{soft}}}{d\Omega_e} \right)_{\text{odd}} = -\frac{\alpha^3}{2 \pi m^2 \rho^2} \frac{(s^2 + u^2 + 2tm^2)}{t^2} \left[ -2 \ln \rho \ln \left( \frac{(2\rho \Delta E)^2}{-tx} \right) + \Xi \right],$$

$$\Xi = \text{Re} \left[ -\frac{t^2 D_{\text{virt}}^{\text{sc}}}{s^2 + u^2 + 2tm^2} - D_{\text{soft}}^{\text{sc}} \right].$$

(29)

and it is independent from the photon mass $\lambda$.

The function $\Xi$ is shown in Fig. 2 as a function of $\cos \theta_e$ for given $E/m$.

FIG. 2: $\Xi(s, \cos \theta)$ for $E = 5m$ (dashed line) and $E = 10m$, $m$ is muon mass.
The ratio between the difference and the sum (corresponding to the Born cross section) of the cross sections for $e^\pm\mu$ scattering is:

$$\frac{\frac{d\sigma}{d\Omega}(e^-\mu_\mu) - \frac{d\sigma}{d\Omega}(e^+\mu_\mu)}{\frac{d\sigma}{d\Omega}(e^+\mu_\mu) + \frac{d\sigma}{d\Omega}(e^-\mu_\mu)} = \frac{\alpha}{\pi} \left[ \Xi - 2\ln \rho \ln \left( \frac{2\rho \Delta E}{t^x} \right) \right].$$

(30)

The odd contributions to the differential cross section for the process $e^+ + e^- \rightarrow \mu^+ + \mu^-$, due to soft photon emission, has the form:

$$(d\sigma_{soft}^{e^+\mu^-\mu^+\gamma})_{odd} = d\sigma_0 \left( -\frac{\alpha}{4\pi^2} \right) 2 \int \frac{d^3k}{\omega} \left( \frac{p_+}{p_- k} + \frac{p_+}{p_+ k} \right) \left( \frac{q_+}{q_+ k} - \frac{q_-}{q_- k} \right)_{S_0, \omega < \Delta \varepsilon}.$$  

(31)

Again, the integration must be performed in the special frame $S_0$, where $\bar{p}_+ + \bar{p}_- - \bar{q}_+ = \bar{q}_- + \bar{k} = 0$. In this frame we have

$$(q_- + k)^2 - m^2 = 2(E_+ + \omega)\omega \approx 2m\omega = (p_+ + p_- - q_+)^2 - m^2 = 4E(E - \varepsilon_+),$$

$$E - \varepsilon_+ = \frac{m}{2E} \Delta \varepsilon.$$  

(32)

In the elastic case $E - \varepsilon^e_+ = 0$ and the photon energy in the Lab system is

$$\Delta E = \varepsilon^e_+ - \varepsilon_+ = \frac{m}{2E} \Delta \varepsilon.$$  

(33)

The t’Hooft-Veltman procedure for soft photon emission contribution leads to:

$$\frac{d\sigma_{soft}^{ann}}{d\Omega} = \frac{d\sigma_0}{d\Omega} \frac{2\alpha}{\pi} \ln \left( \frac{4E\Delta E}{m\lambda} \right)^2 \ln \frac{1 + \beta c}{1 - \beta c} + D_s^{ann}$$

(34)

with

$$D_s^{ann} = \frac{1}{2} \text{Li}_2 \left( \frac{-2\beta(1 + c)}{(1 - \beta)(1 - \beta c)} \right) + \frac{1}{2} \text{Li}_2 \left( \frac{2\beta(1 - c)}{(1 + \beta)(1 - \beta c)} \right) - \frac{1}{2} \text{Li}_2 \left( \frac{-2\beta(1 - c)}{(1 - \beta)(1 + \beta c)} \right) - \frac{1}{2} \text{Li}_2 \left( \frac{2\beta(1 + c)}{(1 + \beta)(1 + \beta c)} \right).$$  

(35)

The total contribution (virtual and soft) is free from infrared singularities and has the form

$$\frac{d\sigma_{ann}}{d\Omega} = \frac{\alpha^3 \beta}{2\pi s} (2 - \beta^2 + \beta^2 c^2) \Upsilon, \quad \Upsilon = 2 \ln \frac{1 + \beta c}{1 - \beta c} \ln \left( \frac{2\Delta E}{m} \right) + \Phi(s, \cos \theta),$$

$$\Phi(s, \cos \theta) = D_s^{ann} - \frac{D^{ann}}{2 - \beta^2 + \beta^2 c^2}. $$

(36)

The quantity $\Phi(s, \cos \theta)$ is presented in Fig. 3.

The relevant asymmetry can be constructed from (6)

$$A = \frac{4\alpha}{\pi} \Upsilon.$$  

(37)
FIG. 3: $\Phi(s, \cos \theta)$, for $s = 10m^2$ (dashed line) and $s = 20m^2$, $m$ is muon mass.

V. CROSSING SYMMETRY

In this section we formally consider the relations between the kinematical variables in the scattering and in the annihilation channel, $e^+ + e^- \rightarrow p + \bar{p}$. The reduced form of the differential elastic $ep$ scattering cross section, commonly used, is defined as $\sigma_{\text{red}} = \tau F_M^2 + \varepsilon F_E^2$ and it is related to the differential cross section by:

$$\frac{d\sigma}{d\Omega} = \sigma_M \sigma_{\text{red}}, \quad \sigma_M = \frac{\alpha^2 \cos^2 \frac{\theta_e}{2}}{4E^2 \sin^4 \frac{\theta_e}{2}} \rho \varepsilon (1 + \tau), \quad \varepsilon = \frac{1}{1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2}}.$$  \hspace{1cm} (38)

where $\varepsilon$ is the polarization of the virtual photon, and varies from $\varepsilon = 0$, for $\theta_e = \pi$ to $\varepsilon = 1$, for $\theta_e = 0$.

The crossing relation between the scattering channel $e + p \rightarrow e + p$ and the annihilation channel $e^+ + e^- \rightarrow p + \bar{p}$ consists in replacing the variables of the scattering channel $s = 2p_1p = 2Em$ and $Q^2 = -t$ according to

$$m^2 + s \rightarrow t = -2E^2(1 - \beta c), \quad Q^2 \rightarrow -s = -4E^2, \quad \cos \theta = \cos \vec{p}_- \cdot \vec{q}_-.$$  \hspace{1cm} (39)

where $\theta$ is the angle of the antiproton with respect to the incident electron, in the center of mass system (CMS).

The following relation holds, for the annihilation channel:

$$\cos^2 \theta = \frac{(t - u)^2}{s(s - 4M^2)}, \quad s + t + u = 2m^2.$$  \hspace{1cm} (40)

On the other hand, in the scattering channel, one has:

$$\frac{1 + \varepsilon}{1 - \varepsilon} = \cot^2 \frac{\theta_e}{2} + 1 = \frac{(s - u)^2}{Q^2(Q^2 + 4M^2)}, \quad Q^2 = s + u.$$  \hspace{1cm} (41)
Therefore one proves the validity of the crossing relation:

\[
\cos \theta = \sqrt{\frac{1 + \varepsilon}{1 - \varepsilon}} \equiv y, \tag{42}
\]

based on the analytical continuation from the annihilation channel to the scattering one. This relation was derived in Refs. [8, 9]. Using this relation and the property of the $2\gamma$ contribution to the annihilation cross-section

\[
\left( \frac{d\sigma}{d\Omega} (\theta) \right)_{2\gamma} = - \left( \frac{d\sigma}{d\Omega} (\pi - \theta) \right)_{2\gamma}
\]

i.e.

\[
\left( \frac{d\sigma}{d\Omega} (\theta) \right)_{2\gamma} = \cos \theta f(\cos^2 \theta, s)
\]

the authors of Ref. [9] built the ansatz for $2\gamma$ contribution to $ep$-elastic scattering

\[
\frac{d\Delta\sigma}{d\Omega_e} (e^- p \rightarrow e^- p) = y f(y^2, Q^2); \quad f(y^2, Q^2) = c_0(Q^2) + y^2 c_1(Q^2) + y^4 c_2(Q^2) + \ldots \tag{43}
\]

This property follows from the change of the sign of the contribution for virtual and real photon emission when the $(s \leftrightarrow u)$ transformation is applied (see Eqs. and relation (41)).

This form of the contribution of the $1\gamma \otimes 2\gamma$ interference to the differential cross section derives explicitly from C-invariance and crossing symmetry of electromagnetic interactions and excludes any linear function of $\varepsilon$ for a possible parameterization of such contribution.

Let us note that not only the elastic channel must be taken into account: the interference of the amplitudes corresponding to the emission of a photon by electron and by proton must be considered, too.

Evidently, the relations derived above are valid for the considered processes with electrons and heavy lepton setting respectively $F_E = F_M = 1$.

VI. DERIVATION OF THE ADDITIONAL STRUCTURE: ANNIHILATION CHANNEL

Let us start from the following form of the matrix element for the process $e^+(p_+) + e^-(p_-) \rightarrow \mu^+(q_+) + \mu^-(q_-)$ in presence of $2\gamma$ exchange:

\[
M_2 = \frac{i\alpha^2}{s} \bar{v}(p_+) \gamma_{\mu} u(p_-) \times \bar{u}(q_-) \left( G_1 \gamma_{\mu} - \frac{G_2}{m} \gamma_{\mu} \hat{P} + 4 \frac{1}{s} G_3 \hat{Q}_{\mu} \right) v(q_+), \tag{44}
\]

where the amplitudes $G_i$ are complex functions of the two kinematical variables $s$, and $t$.

To calculate the structure $G_3$ from the $2\gamma$ amplitude (see Eq. (44)), both Feynman diagrams (Figs. 1a and 1b) must be taken into account. Similarly to Section II, only one of
them can be calculated explicitly (the uncrossed one), whereas the other can be obtained from this one by appropriate replacements.

To extract the structure $G_3$ we multiply Eq. (44) subsequently by
\[
\bar{u}(p_-)\gamma_\lambda v(p_+) \times \bar{v}(q_+)\gamma_\lambda u(q_-),
\]
\[
\bar{u}(p_-)\hat{Q}v(p_+) \times \bar{v}(q_+)u(q_-),
\]
and perform the summation on fermions spin states.

Solving the algebraical set of equations we find
\[
G_1^a = \frac{1}{\beta^4 \sin^4 \theta} \left\{ (8B^a + A^a \beta^2 \sin^2 \theta)(1 - \beta^2 \cos^2 \theta) - 4C^a \beta \cos \theta \left[ 2 - \beta^2(1 + \cos^2 \theta) \right] \right\},
\]
\[
G_2^a = \frac{1}{\beta^4 \sin^4 \theta} \left\{ \beta(1 - \beta^2)(A^a \beta \sin^2 \theta - 8C^a \cos \theta) + 4B^a \left[ 2 - \beta^2(1 + \cos^2 \theta) \right] \right\},
\]
\[
G_3^a = \frac{1}{\beta^4 \sin^4 \theta} \left[ -A^a \beta^2 \sin^2 \theta \cos \theta - 8B^a \cos \theta + 4\beta C^a(1 + \cos^2 \theta) \right],
\]
with
\[
A^a = \int \frac{d^4k}{i\pi^2} (\Delta)(Q)(P_+)(P_-) \frac{1}{s} Tr(\hat{p}_+Z\hat{p}_-\gamma_\lambda) \times \frac{1}{4} Tr [(q_- + m)T(\Delta)(P_+)(P_-) \frac{1}{s} Tr(\hat{p}_+Z\hat{p}_-\gamma_\lambda),
\]
\[
B^a = \int \frac{d^4k}{i\pi^2} (\Delta)(Q)(P_+)(P_-) \frac{m}{s} Tr(\hat{p}_+Z\hat{p}_-\hat{Q}) \times \frac{1}{4} Tr [(\hat{q}_- + m)T(\Delta)(P_+)(P_-) \frac{1}{s} Tr(\hat{p}_+Z\hat{p}_-\hat{Q}),
\]
\[
C^a = \int \frac{d^4k}{i\pi^2} (\Delta)(Q)(P_+)(P_-) \frac{1}{s} Tr(\hat{p}_+Z\hat{p}_-\hat{Q}) \times \frac{1}{4} Tr [(\hat{q}_- + m)T(\Delta)(P_+)(P_-) \frac{1}{s} Tr(\hat{p}_+Z\hat{p}_-\hat{Q}).
\]

The explicit value for $G_3^a$ is:
\[
G_3^a = \frac{2s}{\beta^3 (1 - c^2)^2} \left\{ \frac{1}{2} G_Q(1 - c^2)\beta^3(1 - \beta c)
+ \frac{1}{2} H_Q\beta^2(1 - c^2) \left[ c(-3 + 5\beta^2) - \beta - \beta c^2 \right]
+ F_{\Delta} c \left[ 1 - 4\beta^2 + 2\beta^4 + c^2 \beta^2(3 - 4\beta^2) - 2\beta c(1 - 2\beta^2) \right]
+ F_Q \beta \left[ -c^2 + \beta c \left( -\frac{1}{2} - 4\beta^2 c^2 + \frac{5}{2} c^2 \right) + \beta^2 \left( -\frac{1}{2} + 2\beta^2 c^2 + \frac{3}{2} c^2 \right) \right]
- 2Js\beta^2 c(1 - c^2)(1 - \beta^2)(1 - \beta c)
+ F c \left[ 1 + \beta^2 c^2 - 2\beta^4 - 4\beta^4 c^2 + \beta c(-3 + 4\beta^2 + 2\beta^4 + \beta^2 c^2) \right] \right\}.
\]

The contributions from the crossed Feynman diagram can be obtained from Eqs. (48) by:
\[
(A^b, B^b, C^b)_{crossed} = -[A^a, B^a, C^a(\cos \theta \rightarrow -\cos \theta)]_{uncrossed}.
\]

As one can see, in the quantities $G_1$, $G_2$, and $G_3$ infrared divergencies are present.
VII. ONE SPIN ASYMMETRY

Let us consider now the process of electron interaction with a heavy lepton (point-like proton). For clearness the expressions are written for the proton case. For the interaction of electrons with heavy leptons, $\mu$ or $\tau$, one should take $G_E = G_M = 1$ and use the relevant mass replacement.

The target spin asymmetry for heavy fermions production process $e^+(p_+) + e^-(p_-) \to p(q_+) + \bar{p}(q_-)$ (in CMS frame) is defined as

$$\frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = (\vec{a}\vec{n})R_n,$$

where $\vec{a}$ is the proton polarization vector, $\vec{n} = (\vec{q} \times \vec{p}) / |\vec{q} \times \vec{p}|$ is the unit vector normal to the scattering plane, $d\sigma^\uparrow$ is the cross section of processes with proton polarization vector $\vec{a}$, $d\sigma^\downarrow$ is the cross section of processes with proton polarization vector $-\vec{a}$. Thus the denominator of the left-hand side in Eq. (50) is the unpolarized cross section of process $e^+e^- \to p + \bar{p}$ which is well-known \[10\] (for the case of proton-antiproton creation):

$$\frac{d\sigma_{e^+e^- \to p\bar{p}}}{d\Omega} = \frac{\alpha^2\beta}{4s} \left[ (1 + \cos^2 \theta) |G_M|^2 + (1 - \beta^2) |G_E|^2 \sin^2 \theta \right],$$

with $\beta = \sqrt{1 - \frac{4M^2}{s}}$ is the velocity of proton in c.m. frame, $s$ is the total energy square and $\theta$ is the angle between vectors $\vec{q}$ and $\vec{p}$.

The difference of cross sections in (50) is originated by the $s$-channel discontinuity of interference of the Born-amplitude with TPE amplitude

$$d\sigma^\uparrow - d\sigma^\downarrow \sim Re \sum \left( A_{\text{elastic}}^+ \cdot A_{\text{TPE}} + A_{\text{elastic}}^+ \cdot A_{\text{TPE}}^+ \right).$$

Using the density matrix of final proton $u(p)\bar{u}(p) = (\hat{p} + M)(1 - \gamma_5 \hat{a})$ one gets

$$Re \sum \left( A_{\text{elastic}}^+ \cdot A_{\text{TPE}} + A_{\text{elastic}}^+ \cdot A_{\text{TPE}}^+ \right) = 32 \frac{(4\pi\alpha)^3 (2\pi i)^2}{s\pi^2} Re(Y),$$

$$Y = \int \frac{dk}{i\pi^2} \frac{1}{(\Delta)(Q)(+)(-)} \times \frac{1}{4} Tr \left[ \hat{p} \gamma^\alpha \hat{p}' \gamma^\mu \left( \hat{k} - \hat{\Delta} \right) \gamma^\nu \right] \times$$

$$\times \frac{1}{4} Tr \left[ (\hat{p} - M)(-\gamma_5 \hat{a}) \gamma_\alpha (\hat{p}' + M) \gamma_\nu \left( \hat{k} - \hat{Q} + M \right) \gamma_\mu \right].$$

Performing the loop-momenta integration the right-hand side of Eq. (53) can be expressed in terms of basic integrals (see Appendix B)

$$Re(Y) = 4M(a, \Delta, Q, P) Im \left( F_Q - G_Q + H_Q \right),$$

(54)
FIG. 4: Asymmetry $R_n$ for the case of structureless proton for energies $s = 5$ GeV$^2$ (dashed line) and $s = 15$ GeV$^2$ (solid line).

where $(a, \Delta, Q, P) \equiv \varepsilon^{\mu\nu\rho\sigma} a_\mu \Delta_\nu Q_\rho P_\sigma = (\sqrt{s}/2)^3 (\vec{a} \vec{n}) \beta \sin \theta$. Using the expressions listed in Appendix B we have:

$$\text{Im} (F_Q - G_Q + H_Q) = \frac{\pi}{s} \psi(\beta) = \frac{\pi}{s\beta^2} \left( \frac{1 - \beta^2}{\beta} \ln \frac{1 + \beta}{1 - \beta} - 2 \right). \quad (55)$$

Thus, after standard algebra, the following expression for spin asymmetry can be obtained for the processes $e^+ + e^- \rightarrow \vec{p} + \vec{p}$:

$$R_n = 2\alpha \frac{M}{\sqrt{s}} \frac{\beta \psi(\beta) \sin \theta}{2 - \beta^2 \sin^2 \theta}. \quad (56)$$

and it is shown in Fig. 4 as a function of $\theta$ at several values of $s$ for the case of structureless proton.

Such considerations apply to the scattering channel when the initial protons is polarized. Similarly to (55), one finds

$$\text{Im}_s \left( \bar{F}_Q - \bar{G}_Q + \bar{H}_Q \right) = -\frac{\pi}{s + M^2}. \quad (57)$$

(note that the $s$-channel imaginary part vanishes for the crossed photon diagram amplitude).

The contribution of the polarization vector appears in the same combination

$$(a, \Delta, Q, P) = \frac{1}{2} (a, p, p_1, q) = \frac{M E^2}{2\rho} \sin \theta (\vec{a} \vec{n}). \quad (58)$$
The single spin asymmetry for the process $e^- + \vec{p} \rightarrow e^- + p$ (the initial proton is polarized) has the form:

$$\frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = (\tilde{a}\tilde{n})T_n$$

(59)

with

$$T_n = \frac{\alpha}{2M^2} \frac{s^2}{s + M^2} \frac{1 + \tau}{\rho\sigma_{\text{red}}} \frac{\epsilon}{\sin \theta \tan^2 \frac{\theta}{2}}.$$  

(60)

This quantity is shown in Fig. 5 for the case of structureless proton as a function of $\theta$, for two values of $s$. The asymmetry decreases when the c.m.s. energy growth, so on experiment it is useful to measure the asymmetry near the threshold of proton (or heavy lepton) production.

VIII. CONCLUSIONS

We calculated QED radiative corrections to the differential cross-section of the processes $e^+ + e^- \rightarrow \mu^+ + \mu^- (\gamma)$, $e^\pm + \mu \rightarrow e^\pm + \mu(\gamma)$, arising from the interference between the Born and the box-type amplitudes. The relevant part of soft photon emission contribution, which eliminates the infrared singularities, was also considered.
According to the previous discussion, the box Feynman diagram, calculated in QED frame, can also be applied to lepton-hadron interaction. In the present work, we considered structureless Born approximation for annihilation and scattering channels. A realistic case with Born and box diagram depending on form-factors will be considered further (in preparation).

Angular asymmetry, charge asymmetry as well as target spin asymmetry were calculated. These quantities are free from infrared and electron mass singularities. Numerical applications show that these observables are large enough to be measured (see Figs. 2-5).

The charge-asymmetry properties of radiative corrections in the annihilation channel induce non-trivial terms in the cross section due to crossing symmetry relations (see (43)). The parametrization (44) for the contribution to the matrix element arising from box-type diagrams in terms of three additional functions \( G_i(s, t), i = 1, 2, 3 \) suffers from infrared divergencies, which can be eliminated by taking into account soft photon emission expressed in terms of structures \( G_1, G_2, G_3 \). This procedure results in replacing \( \ln(m/\lambda) \) with \( \ln \Delta \).

The results obtained here, for the processes \( e^\pm p \rightarrow e^\pm p(\gamma) \) are particularly interesting in view of the experiments planned at Novosibirsk [11] and at JLab [12] as well as \( e^+ e^- \rightarrow NN(\gamma) \), which can be investigated at Frascati [13] and Beijing [14].

IX. APPENDIX A: TRACE CALCULATION.

The explicit expressions for the polynomials \( P_i \) are:

\[
P_1 = 8\{- (\Delta^2)^3 - \Delta^2 \sigma^2 + 2\sigma^3 + [(\Delta^2)^2 - 2\Delta^2 \sigma]m^2\},
\]

\[
P_2 = 16[(\Delta^2)^2 \sigma - \sigma^3 + \Delta^2 \sigma m^2],
\]

\[
P_3 = 8\{2(\Delta^2)^2 \sigma - 2\sigma^3 + m^2[(\Delta^2)^2 + 2\Delta^2 \sigma - \sigma^2] + \Delta^2 m^4\},
\]

\[
P_4 = 8[5(\Delta^2)^2 - 6\Delta^2 \sigma + 5\sigma^2 - 5\Delta^2 m^2],
\]

\[
P_5 = 8[(\Delta^2)^3 - 2(\Delta^2)^2 \sigma + \Delta^2(\sigma^2 - m^2 \Delta^2)],
\]

\[
P_6 = 8\{ (\Delta^2)^3 - 2(\Delta^2)^2 \sigma + \Delta^2 \sigma^2 + m^2[-(\Delta^2)^2 - 2\Delta^2 \sigma + 2\sigma^2] - 2\Delta^2 m^4\},
\]

\[
P_7 = 8[-(\Delta^2)^3 + 2(\Delta^2)^2 \sigma - \Delta^2 \sigma^2 + (\Delta^2)^2 m^2],
\]

\[
P_8 = 8\{ -(\Delta^2)^3 + (\Delta^2)^2 \sigma - 3\Delta^2 \sigma^2 + 3\sigma^3 - m^2[(\Delta^2)^2 + 3\Delta^2 \sigma]\}. \tag{61}
\]
X. APPENDIX B: USEFUL INTEGRALS.

In the calculation of $e p$ scattering we use the following set of scalar integrals with three and four denominators [3].

\[
F_\Delta = \frac{-i}{\pi^2} \int \frac{d^4k}{(\Delta)(P_+)(P_-)} = \frac{1}{s} \left[ \frac{n^2}{6} + \frac{1}{2} \ln^2 \frac{s}{m_c^2} \right],
\]

\[
F_Q = \frac{-i}{\pi^2} \int \frac{d^4k}{(Q)(P_+)(P_-)} = \frac{1}{s\beta} \left[ \frac{1}{2} \ln^2 \frac{1 - \beta}{2} - \frac{1}{2} \ln^2 \frac{1 + \beta}{2} + \text{Li}_2 \left( \frac{1 + \beta}{2} \right) - \text{Li}_2 \left( \frac{1 - \beta}{2} \right) \right],
\]

\[
H = \frac{-i}{\pi^2} \int \frac{d^4k}{(\Delta)(Q)(P_+)} = G = \frac{-i}{\pi^2} \int \frac{d^4k}{(\Delta)(Q)(P_-)} = -\frac{1}{2(m^2 - t)} \left[ \ln^2 \frac{m^2 - t}{m^2} + \left( 2 \ln \frac{m^2 - t}{m^2} + \ln \frac{m^2}{m_e^2} \right) \ln \frac{s}{m^2} - \ln^2 \frac{m^2 - t}{m^2} \right]
\]

\[
H_P = H + \frac{1}{m^2 - t} \left( -\ln \frac{m^2}{m_c^2} - \frac{m^2 - t}{t} \ln \frac{m^2 - t}{m^2} \right),
\]

\[
H_Q = H + \frac{1}{m^2 - t} \left( \ln^2 \frac{m^2}{m_e^2} + 2 \ln \frac{m^2 - t}{m^2} \right),
\]

\[
F = \frac{1}{2} s J - G = -\frac{1}{2(m^2 - t)} \left[ \left( 2 \ln \frac{m^2 - t}{m^2} + \ln \frac{m^2}{m_e^2} \right) \ln \frac{s}{m^2} - \ln^2 \frac{m^2 - t}{m^2} \right] + \ln \frac{m^2 - t}{m^2} \ln \frac{m^2}{m_e^2} + 2 \text{Li}_2 \left( -\frac{t}{m^2 - t} \right),
\]

\[
J = \frac{-i}{\pi^2} \int \frac{d^4k}{(\Delta)(Q)(P_+)(P_-)} = -\frac{1}{s(m^2 - t)} \left[ \left( 2 \ln \frac{m^2 - t}{m^2} + \ln \frac{m^2}{m_e^2} \right) \ln \frac{s}{\lambda^2} \right].
\]

The terms proportional to $m^2_s/s$, $m^2_s/m^2_\mu$ were neglected. Notations follow Ref. [8].

The vector integrals with three denominators are:

\[
\frac{1}{i\pi} \int \frac{k^\mu d^4k}{(\Delta)(Q)(P_+)} = H_{P\mu} + H_{\Delta\mu} + H_Q\mu, \quad H_Q = \frac{1}{t} \ln \frac{m^2 - t}{m^2},
\]

\[
H_{\Delta} = \frac{1}{m^2 - t} \left( -\ln \frac{m^2}{m_c^2} - \frac{m^2 - t}{t} \ln \frac{m^2 - t}{m^2} \right),
\]

\[
H_P = H + \frac{1}{m^2 - t} \left( \ln \frac{m^2}{m_e^2} + 2 \ln \frac{m^2 - t}{m^2} \right),
\]

\[
\frac{1}{i\pi} \int \frac{k^\mu d^4k}{(\Delta)(P_+)(P_-)} = G_{\Delta\mu}, \quad G_{\Delta} = \frac{1}{s} \left( -2 \ln \frac{s}{m_c^2} + \frac{1}{2} \ln^2 \frac{s}{m_e^2} + \frac{\pi^2}{6} \right),
\]

\[
\frac{1}{i\pi} \int \frac{k^\mu d^4k}{(Q)(P_+)(P_-)} = G_Q\mu, \quad G_Q = \frac{1}{s - 4m^2} \left( -2 \ln \frac{s}{m_c^2} + s F_Q \right).
\]

Four denominator vector and tensor integrals were defined in [14]. The relevant coefficients are:

\[
J_{\Delta} = \frac{1}{2d} \left[ (F + F_{\Delta}) \sigma - Q^2 (F + F_Q) \right],
\]

\[
J_Q = \frac{1}{2d} \left[ (F + F_Q) \sigma - \Delta^2 (F + F_{\Delta}) \right], \quad F = \frac{1}{2} s J - G, \quad d = \Delta^2 Q^2 - \sigma^2.
\]
\[ K_0 = -\frac{1}{2\sigma} \left[ \sigma(F - G + H_P + H_\Delta + H_Q) + H_\Delta (\sigma - \Delta^2) - H_Q (\sigma - Q^2) \\
+ 2P^2 J_\Delta (\Delta^2 - 2\sigma) + \Delta^2 G_\Delta - Q^2 G_Q - 2P^2 Q^2 J_Q \right]. \]

\[ K_\Delta = -\frac{1}{2\sigma d} \left[ Q^2 \sigma(G - F - H_P - 3H_\Delta + 6P^2 J_\Delta) \\
+ (\Delta^2 Q^2 + \sigma^2)(H_\Delta - 2P^2 J_\Delta - G_\Delta) - Q^4 (H_Q - 2P^2 J_Q - G_Q) \right]. \]

\[ K_P = \frac{1}{2P^2 \sigma} \left[ 2\sigma(H_\Delta - 2P^2 J_\Delta + H_P + \frac{1}{2} F - \frac{1}{2} G) + Q^2 (H_Q - 2P^2 J_Q - G_Q) \\
- \Delta^2(H_\Delta - 2P^2 J_\Delta - G_\Delta) \right], \]

\[ K_Q = -\frac{1}{2\sigma d} \left[ -\Delta^2 \sigma A_P + 2\Delta^4 A_\Delta + (\sigma^2 - 2\Delta^2 Q^2) A_Q \right], \]

\[ K_x = -\frac{1}{2d} \left( \sigma A_P + Q^2 A_Q - 2\Delta^2 A_\Delta \right), \]

where we used

\[ A_\Delta = H_\Delta + 2\Delta^2 J_\Delta - G_\Delta, \quad A_Q = H_Q + 2\Delta^2 J_Q - G_Q, \quad A_P = F - G + H_P + 3H_\Delta + 6\Delta^2 J_\Delta. \]  

\[ (65) \]

[1] M. I. Golberger, Y. Nambu and R. Oehme, Ann. Phys. (N.Y.) 2, 226 (1957)

S. D. Drell and J. D. Sullivan, Phys. Lett. 19, 516 (1965).

[2] P. Van Nieuwenhuizen, Nucl. Phys. B 28, 429 (1971),

I.B. Khriplovich, Sov. J. Nucl. Phys. 17, 298 (1973)

[3] E. A. Kuraev and G. Meledin, Preprint 76-91 (1976), Novosibirsk.

[4] E. A. Kuraev and G. Meledin, Nucl. Phys. B 122, 485 (1977).

[5] M. N. Rosenbluth, Phys. Rev. 79, 615 (1950).

[6] L. C. Maximon and L. A. Tjon, Phys. Rev. C 62, 054320 (2000).

[7] G. t’Hooft and M. Veltman, Nucl. Phys. B 153, 365 (1979).

[8] M. P. Rekalo, E. Tomasi-Gustafsson and D. Prout, Phys. Rev. C 60, 042202 (1999).

[9] M. P. Rekalo and E. Tomasi-Gustafsson, Eur. Phys. J. A 22, 331 (2004).

[10] B. L. Ioffe, L. N. Lipatov, V. A. Khoze, ‘Deep inelastic processes’ (in russian), Moscow, Energoatom Izdat, 1983. A. Z. Dubnickova, S. Dubnicka and M. P. Rekalo, Nuovo Cim. A 109, 241 (1996).

[11] D. Nikolenko, private communication; J. Arrington et al., nucl-ex/0408020
[12] Jefferson Laboratory proposal PR-04-019, (R. Suleiman, L. Penchev, R. Gilman, and C. Perdrisat spokespersons) 'Measurement of the Two-Photon Exchange Contribution in ep Elastic Scattering Using Recoil Polarization'.

[13] M. Mirazita et al., Letter of Intent, Frascati, 2005

[14] W. G. Li [BES Collaboration], Prepared for 32nd International Conference on High-Energy Physics (ICHEP 04), Beijing, China, 16-22 Aug 2004.