Random field disorder and charge order driven quantum oscillations in cuprates

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In the pseudogap regime of the cuprates, $\mathbb{Z}_2$ symmetry can be broken by charge order. Quenched disorder due to potential scattering, can, as a point of principle, be treated, as a random field Ising model. The resulting local, glassy dynamics is shown to lead to an electronic non-Fermi liquid, and, if it is strong enough, can preclude quantum oscillations. This is because the electron spectral function will have no quasiparticle peak and the effective mass diverges at the Fermi surface. The result holds in both two and three dimensions, even though the three dimensional random field Ising model has a phase transition. In contrast to charge density, $d$-density wave order, reflecting staggered currents, is more impervious to potential disorder because potential scattering does not directly couple to currents. However, in Landau theory, there is a fourth order coupling of the square of the $d$-density wave order parameter and the square of the charge order. This induced charge order will similarly be affected by random field effects, which, however, being a higher order effect may be weak enough to corrupt the Fermi liquid behavior.

I. INTRODUCTION

It is generally agreed that the phenomena of high field quantum oscillations, in a variety of cuprate superconductors reflects a Fermi surface reconstruction. If this is so, it points to a Fermi liquid ground state, at least in the sense of adiabatic continuity. As to the precise nature of this reconstruction, there appears to be no general agreement or even understanding. Recent experiments in which an incommensurate charge density wave (ICDW) order has been observed have shown that the order is short-ranged. Strict ICDW does not have a sharply defined Fermi surface and, as a consequence, there are no quantum oscillations that are truly periodic function of $1/B$, where $B$ is the magnetic field.

Here we shall show that even commensurate charge density wave (CDW), chosen for simplicity to be of period-2, in the presence of disorder, may not be able to explain Fermi surface reconstruction and consequently quantum oscillations. The reason, in short, is that the ubiquitous potential disorder can strongly couple to CDW order, leading to a non-Fermi liquid electron spectral function with no quasiparticles. If this is the case, the principal order could not be CDW. The $d$-density wave may be a more likely candidate because potential disorder cannot directly couple to currents. We focus here on CDW in $2D$, but also extend the results for $3D$; the conclusions are the same.

The period-2 CDW breaks $\mathbb{Z}_2$ symmetry. We can therefore adopt a simple model, the random field Ising model (RFIM) to describe CDW coupled to disorder. The disorder in a RFIM drives fluctuations on many length scales, and consequently many time scales, producing a glassy dynamics. The overall structure of our reasoning has the flavor of “proof by contradiction”, but of course it is not rigorous. The overall structure of our reasoning has the flavor of “proof by contradiction”, but of course it is not rigorous. The plan of the paper is as follows: in Section II we derive the susceptibility of the RFIM and in Section III the electron Green function. Section IV describes the three dimensional generalization and finally in Section V we present our con-

![FIG. 1. Current pattern for $Q = (\pi/2, \pi/2)$, reproduced from Ref. 15.](image-url)
II. THE RFIM AND THE IMAGINARY PART OF THE SUSCEPTIBILITY

Independent of its origins, the RFIM Hamiltonian is relevant to any $\mathbb{Z}_2$ symmetry broken state with quenched disorder:

$$H_0 = -J \sum_{<ij>} s_i^z s_j^z - \sum_i h_i^z s_i^z$$  \hspace{1cm} (1)

where $J$ is the coupling between the Ising spins, and $\{ h_i^z \}$ is a collection of uncorrelated, uniformly distributed (rectangular distribution) random variables with zero mean and variance $\sigma^2$. The notation $<ij>$ denotes nearest neighbors. The assumption that the random fields $h_i^z$ are "uniformly distributed" may have subtle significance. An extreme choice of distribution for the random fields, such as a bimodal distribution, can even change the order of a phase transition. That said, the current understanding of the theory of the RFIM suggests that the most significant effects of the random field are controlled by the variance of the distribution; the only free parameter in our Hamiltonian is therefore $\zeta = \frac{\sigma}{J}$.

Analogous to the thermally driven fluctuation of the system at finite temperature, the ground state of the RFIM has disorder driven fluctuations at zero temperature. These fluctuations have a distribution of length scales, $p(L)$, which arise from domain walls in the ground state of the RFIM (see FIG. 1). FIG. 3 and FIG. 4. The distribution $p(L)$ controls the quantum mechanical tunneling in many-dimensional WKB theory:

$$\text{Im} \chi(\omega) \sim \int dL p(L) \delta(\omega - \omega_0 e^{-e^{L^\alpha}})$$  \hspace{1cm} (2)

This phenomenological argument captures the essential glassy characteristics produced by the distribution of length scales $p(L)$. In principle, the attempt frequency $\omega_0$, $\alpha$, and $c$ are microscopic parameters which could be left undetermined in this phenomenological approach. However, for a compact volume $\alpha$ is the dimension, $D$. Following prior work, we numerically calculate the ground state of the purely classical $H_0$ by converting the lattice into a flow network and solving the minimum-cut problem, for which there are efficient algorithms. By careful choice of the parameters of the flow-network, each cut is made to correspond to a spin configuration, and the minimal cut therefore corresponds to the ground state configuration.

For various finite system sizes, the probability that a domain wall exists in the ground state, $P_{\text{dw}}$, is determined by averaging over many disorder realizations. To help understand the meaning of this quantity, we remark that, in the one-dimensional case, a domain wall is just a spin flip. In the disorder-free case, creating a single spin flip costs energy $\sim 2J$, an energy that, by Jordan-Wigner transformation, can be thought of as a fermion gap. The spin flip can move throughout the system at no energy cost. In higher dimensions, the analogy is less precise, but the presence of a domain wall can be thought of as the collapse of the gap in the

![FIG. 2. Scaling of $P_{\text{dw}}$ in the one-dimensional case. The collapse works as expected for $x = (J/\sigma)^2/L = \zeta^2/L$; the logarithmic correction factor $(1 + A \log L)$ is found to have little effect: $A = 0 \pm 0.3$. The best fit parameters to Equation (4) are $x_0 = 0.0596 \pm 0.0007$, $\lambda = -0.0103 \pm 0.0005$ and $\theta = 0.034 \pm 0.002$. All points correspond to averages performed over 2048 disorder realizations.

Ising system. Most importantly, the size of the domains is controlled by locations of these domain walls. In particular, $P_{\text{dw}}$ is the cumulative distribution function of the ordered domains or "clusters", and

$$p(L) = \frac{dP_{\text{dw}}}{dL}.$$  \hspace{1cm} (3)

We elaborate here on the calculation of the frequency-dependent imaginary part of the susceptibility $\chi(\omega)$. For weak enough interactions, the clusters of the ground state will continue to behave as a single coherent spin, flipping entirely or not at all. We assume that the clusters are sufficiently sparsely distributed such that the behavior of each cluster is independent of other clusters. A disorder average eliminates any wave vector dependence, and makes the susceptibility local. The $\omega \to 0$ behavior will therefore be dominated by the clusters, whose distribution $p(L)$ we have already found. We relate the quantum mechanical tunneling at a frequency $\omega$ to a cluster of a particular size, determined by the exponent $\alpha$, in a many-dimensional WKB limit.

Previous numerical work reproduced in Fig. 3 found that, in 2D, $P_{\text{dw}}$ lies on a universal curve,

$$P_{\text{dw}} \approx f(x) = \frac{1}{(1 + \exp[\frac{\omega_0 x}{\lambda}])^\theta}$$  \hspace{1cm} (4)

where $x = \log L - (\zeta/\zeta_0)^2$ and $\zeta_0$ is a numerically fit. Although such a scaling is unexpected, it is not unprecedented: in 1D, the Imry-Ma argument finds the ordered domains scale like $L \sim (J/\sigma)^2$. The surprising feature is that, when we define the $x$ axis to be $\log L - (\zeta/\zeta_0)^2$, all points collapse onto
a single universal curve. We find good agreement with a previous study at the special value $P_{dw} = 1/2$. The scaling of domain sizes is seen by setting $x$ to some constant. From the fit given by Equation (4), and taking the $\omega \to 0$ limit, we find

$$\text{Im} \chi(\omega) = \frac{\omega_0}{\omega} \left( \log \frac{\omega}{\omega_0} \right)^\psi$$

(5)

where the exponent $\psi = 1 + 1/(\lambda\alpha) > 1$. Furthermore, in 3D, by allowing the exponent 2 in the definition of $x$ to vary, a very similar scaling form with $x = \log L - (\zeta/\zeta_0)^k$ leads to an excellent collapse. A comparison of these three cases is made in Fig. 2, Fig. 3, and Fig. 4, where up to 2048 disorder realizations were employed. For the larger system sizes in three dimensions, fewer realizations were used: for $L = 40$ and $L = 80$, 1024 realizations were used; for $L = 160$, 512 disorder realizations were used.

In these calculations, scaling parameters (i.e., $\zeta_0$ and $k$) are varied to produce the best collapse. The coefficient $\zeta_0$ sets some scale for the strength of the disorder, and the exponent $k$ controls how sensitive the sizes of the domains are to disorder. In three dimensions, it is clear that, with $k \sim 5$, the domains depend very strongly upon the strength of the disorder, presumably a residual signature of the ordering transition. The collapse itself is fit to the aforementioned asymmetric sigmoid, Eq. (4), producing the best fit parameters $\theta$, $x_0$, and $\lambda$. The $x_0$ and $\lambda$ parameters control the center and width of the sigmoid, respectively, while $\theta$ controls the sigmoid’s asymmetry. Physically, $x_0$ determines the onset of the occurrence of domain walls, and $\lambda$ roughly how quickly the regime is dominated by the existence of at least one domain wall (all, of course, in terms of the scaling parameter $x$). Once $P_{dw}$ is fit to this form, the distribution of clusters $p(L)$ can be found. One of the principal results of the present paper is that this numerical result also holds in three dimensions, addressed in detail in Section IV. The exponential scaling of domain sizes appears to be a generic feature leading to glassy dynamics.

III. THE ELECTRON SELF ENERGY

The itinerant electrons’ self-energy is assumed to be determined by the interaction with the bath of fluctuations of the RFIM. In particular, we find the one-loop correction of FIG. 5 relating the imaginary part of the frequency dependent susceptibility $\text{Im} \chi(\omega)$ to the fermion self-energy $\Sigma(\omega)$. In this section, the self energy $\Sigma$ of the electrons is calculated to leading order in perturbation theory, assuming some coupling $\gamma$ of the RFIM fluctuations to the electrons. We use the form of $\chi$ in Equation (5) and find the one-loop correction to the self energy, as schematically illustrated in FIG. 5, in a reduced graph expansion. Note that it is not necessary to use the matrix formalism corresponding to the charge order, because, as we shall show, there are no quasiparticles, hence no possible Fermi surface reconstruction. It therefore suffices to examine the coupling of an electron to the fluctuating bath.
The Fermi and Bose functions $f(q, \omega - \omega') \Im \chi(q, \omega')$ of the RFIM. In terms of the energy of quasiparticles $\omega$,

$$\Im \Sigma(\omega) = -\gamma^2 \int \frac{d\omega'}{\pi} \sum_q \Im G(k - q, \omega - \omega') \Im \chi(q, \omega') \times [b(\omega') + f(\omega - \omega')]$$

(6)

The Fermi and Bose functions $f(\omega)$ and $b(\omega)$ restrict the $\omega'$ integration to $[0, \omega]$ in the zero-temperature limit we are considering, making the integral vanish for $\omega < 0$. Because the susceptibility is local, and therefore independent of $q$, the problem simplifies enormously. The sum over momenta leads to a factor of the density of states at the Fermi surface, $\nu$:

$$\Im \Sigma(\omega) = -\gamma^2 \nu \int_0^\omega \Im \chi(\omega') d\omega'$$

$$= -\gamma^2 \nu \chi(0) \omega \int_0^\omega \frac{d\omega'}{\omega'} \left(\log \frac{\omega}{\omega'}\right)^\psi$$

$$= -\frac{\omega}{\psi - 1} \left(\log \frac{\omega}{\omega}\right)^{\psi - 1}$$

(7)

where $\Sigma_0 = \gamma^2 \nu \chi(0) \omega$ and $\omega > 0$. Next, we obtain the real part of the self-energy correction via the Kramers-Kronig relation:

$$\Re \Sigma(\omega) = \frac{2}{\pi} \int_0^\infty \frac{\omega'}{\omega^2 - \omega'^2} \Im \Sigma(\omega') d\omega'$$

$$= -\frac{2\Sigma_0}{(\psi - 1)} \frac{1}{\omega^2} \left(\log \frac{\omega}{\omega}\right)^{\psi - 1}$$

(8)

where we have introduced a cutoff $\Lambda$. If the logarithm is slowly varying, we can approximate it as a constant with $\omega' = \omega$; however, for this approximation to be valid $\psi$ cannot be too large compared to unity. Then,

$$\Re \Sigma(\omega) \approx \frac{2\Im \Sigma(\omega)}{\pi} \ln \frac{\omega}{\omega} = -\frac{2\Sigma_0}{\pi(\psi - 1)} \left(\ln \frac{\omega}{\omega}\right)^{\psi - 1}$$

(9)

where we have taken the largest possible value of the cutoff, $\Lambda \rightarrow \omega_0$, and discarded the subdominant terms in the $\omega \rightarrow 0$ limit. The physical values of $\psi$ in two and three dimensions are 2.78 and 1.94, respectively. Thus, in the limit $\omega \rightarrow 0$, the real and the imaginary part of the self energy vanishes.

FIG. 5. Leading order (one-loop) self energy graph. The fermion couples to the bath of RFIM fluctuations.

However, they have unusual non-Fermi liquid forms apparent in the spectral function as a function of frequency, given by

$$A(k, \omega) = \frac{1}{\pi} \Im G_R(k, \omega) = \frac{1}{\pi} \Im \frac{1}{\omega - \xi_k - \Sigma(\omega)}$$

$$= \frac{\Im \Sigma(\omega)}{\pi (\omega - \xi_k - \Re \Sigma(\omega))^2 + (\Im \Sigma(\omega))^2}$$

(10)

where $\xi_k = \hbar v_F (k - k_F)$. The zero-frequency behavior illustrates the failure of the conventional Fermi-liquid approach: put $\Omega = \frac{1}{\imath \omega_0 / \omega}$ to find

$$A(k, \omega) = C \left(\frac{\psi - 1}{\Sigma_0 \pi} \left[\Omega^{\psi - 1} + \left(\frac{\psi - 1}{\Sigma_0} (\omega - \xi) \Omega^{(1 - \psi)/\nu} + \frac{2\pi}{\pi} \Omega^{(\psi - 3)/\nu}\right)^2\right]^{-1}\right.$$

(11)

Putting $\omega = k_F$ and taking the $\omega \rightarrow 0$ limit,

$$A(k_F, \omega) \rightarrow \frac{\pi}{4} \frac{\psi - 1}{\Sigma_0} \omega^{3 - \psi}$$

(12)

we find that $A \rightarrow 0$, for the physical values of $\psi < 3$, leading to an extremely slow falloff going like a fractional power of a logarithm, as seen in FIG. 6. Furthermore, and despite the slow falloff, the quasiparticle weight always vanishes as $\omega \rightarrow 0$:

$$Z^{-1} = 1 - \Re \frac{\partial \Sigma}{\partial \omega} = 1 + \frac{2\Sigma_0}{\pi \omega} \omega - \frac{2}{\pi} \left(\ln \frac{\omega}{\omega_0}\right)^{\psi - 1}$$

(13)

Indeed, any such quasiparticle, would have divergent effective mass:

$$\frac{m^*}{m} = \frac{v_F}{v_F'} = 1 + \frac{1}{v_F} \Re \frac{\partial \Sigma}{\partial k} \omega = 1 + \Re \frac{\partial \Sigma}{\partial \omega} = Z^{-1}$$

(14)
IV. THREE DIMENSIONAL EXTENSION

In three dimensions (and higher) an order-to-disorder phase transition is expected at zero temperature and finite \( \zeta = \frac{J}{\sigma} \). We find the disorder phase transition at \( \zeta_c = 0.446 \pm 0.001 \), with critical exponents \( \beta = 0.02 \pm 0.01 \) and \( \nu = 1.5 \pm 0.2 \), in good agreement with previous results\(^2\) (see Fig. 7). Our primary purpose in performing this numerical calculation is to find \( P_{\text{dw}} \), a fit of which is shown in Fig. 3.

As in the two dimensional case, the distribution of domain sizes scales exponentially with the coupling strength, though in \( (J/\sigma)^k \) the exponent becomes \( \sim 5 \), indicating a very strong dependence on the system size. Also of note is that the scale parameter \( \xi_0 \) is not far from the phase transition \( \zeta_c \). It is possible these values are not coincidental, but we are aware of no analytical explanation. Although the plot includes data from both sides of the order-disorder phase transition, it is important to understand that the rise in \( P_{\text{dw}} \) is not a symptom of this phase transition. We shall attempt to explain this effect.

The Imry-Ma argument shows that the behavior of large domains dictates the formation of ordered and disordered phases in the RFIM\(^5\). For finite systems smaller than these domains, the ordering can therefore be left undestroyed. Even for strong disorder, small enough systems can order. Conversely, for infinite systems which permit ordered phases (i.e., \( d > 2 \)), even weak disorder creates (exceedingly rare) disordered regions of otherwise ordered phases. Although these disordered domains do not destroy the overall, long-range order, these domains permit low energy excited modes.

It is important to understand the difference between the two and three dimensional distribution of domain walls, shown in Fig. 2 and Fig. 3, respectively. In the two dimensional case (or the disordered three dimensional case), there is no overall order in the system; some regions spontaneously order, and \( P_{\text{dw}} \) characterizes the sizes of those regions. In the three dimensional case on the ordered side of the phase transition, there are regions that spontaneously disorder by electing a spin direction counter the overall trend. Despite their differences in origin, these regions still respond to a coupling with the itinerant electrons in the same way. Using the value of \( \psi \) derived from the three-dimensional fits, all calculations, including \( \chi(\omega) \) in Equation (6), the spectral function \( A \) in Equation (12), and the effective mass in Equation (14) are otherwise unchanged. In three dimensions, a non-Fermi liquid forms.

Comparison to Mean Field Theory

Previous work\(^{13,22}\) reproduced here in Fig. 3 found that in two dimensions the correlation length

\[
\xi_{2D} (J/\sigma) \sim \exp \left[ A \left( \frac{J}{\sigma} \right)^2 \right]
\]

with \( A \approx 1.8 \). There are many physically realistic situations in which a stack of two-dimensional layers only interact weakly with each other\(^{23} \). We can model this with the Hamiltonian

\[
H_{\text{stacked}} = -J_{||} \sum_{\langle ij \rangle_{xy}} s_i^z s_j^z - J_z \sum_{\langle ij \rangle_z} s_i^z s_j^z - \sum_i h_i^z s_i^z
\]

where \( J_{||} \) is the in-plane coupling, \( J_z \) the interplane coupling, \( \langle ij \rangle_z \) denotes nearest neighbors in the \( z \) direction, and \( \langle ij \rangle_{xy} \) the neighbors in the \( xy \) plane. The random fields \( h_i^z \) are as before.

Numerically, we fix a particular value of \( J_z \), and then vary \( J_{||} \) to identify the phase boundary in the \( J_z-J_{||} \) plane, see...
If, however, we consider the 3D system as a stacked coupled 2D planes, then from mean field theory we find the simple result:

\[ J_z \gtrsim J_{||}/\xi_{2D}^2 \]  

(17)

which disagrees catastrophically with the numerical results in the weak \( J_{||} \) limit. Due to the exponential scaling of the domain sizes for both \( J_z/\sigma \) and \( J_{||}/\sigma \), it is not numerically feasible to explore regions much beyond those shown in the figure. Unfortunately, any numerical attack on the limiting behavior of the phase transition, therefore, will be restricted.

The qualitative features are readily understood. When \( J_{||} \to 0 \) the system decouples as 1D RFIM, which cannot order. On the other hand if \( J_z \to 0 \) the 2D RFIM cannot also order but has an exponentially large crossover scale. It is in the latter regime that the fully 3D and stacked 2D results overlap.

V. CONCLUSION

The effect of random field disorder is significant even in the apparently well ordered materials of high temperature superconductors, but its effect is quite different for the two orders: CDW and DDW. DDW modulates bond currents: a Hartree-Fock calculation of DDW is given by Laughlin. Because the \( \mathbb{Z}_2 \) symmetry is broken for period-2 CDW, it is susceptible to random field disorder and, as we have found here, destroying the Fermi surface in both two and three dimensions. The form of the non-Fermi liquid behavior is quite unusual. Truly incommensurate order in the presence of disorder is far too complex and was not addressed in the present work.

When CDW order is exposed to \( \mathbb{Z}_2 \) symmetry breaking disorder (and treated as an RFIM), glassy dynamics develop, as demonstrated by the susceptibility \( \text{Im} \chi(\omega) \) of Eq. [5]. Physically, the glassy dynamics develop from the wide range of scales on which domain walls exist in the ground state of RFIM, characterized by a numerically fit parameter \( \lambda \). No Fermi-surface reconstruction can in principle occur, precluding quantum oscillations driven by charge order. Important caveats are that the parameter \( \Sigma_0 \) is not too small and CDW correlations are not long-ranged.

In contrast, DDW couples more weakly to disorder, and the the quantum oscillations will be more robust even though the disorder breaks translational symmetry in terms of staggered bond currents. It is common belief that DDW has not received confirmation from neutron (see, however, Mook et al.) or NMR measurements (see, however, Ref. [30]). However, the observed weak CDW involves such a small motion of the atoms, it is hard to believe that it could be the cause of a large magnitude pseudogap. In any case, the short range nature of the CDW combined with RFIM disorder cannot explain quantum oscillations. As a third option, if we neglect disorder and assume very long-ranged CDW, (perhaps infinitely long-ranged), Fermi surface reconstruction and quantum oscillation has been showed to be possible. The current experiments, however, do not appear to support these assumptions.

It would be remiss of us not to remark on the directions in which further research must be carried out. We have not discussed yet the incommensurate DDW along with parasitically generated charge order, nor have we discussed the self-consistent analysis of the electron Green function or even a self-consistent derivation of the imaginary part of the susceptibility of the RFIM. We shall return to these topics in the near future, but the main conclusions should remain unchanged.

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In two dimensions, the Imry-Ma argument gives no information. Nonetheless, the domain wall roughening which controls the destruction of the ordered phase in 2D, still relies on large domains.\textsuperscript{50}