Entanglement amplification between superposed detectors in flat and curved spacetimes

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We consider an entanglement harvesting protocol between two Unruh-deWitt detectors in quantum superpositions of static trajectories in the static de Sitter and thermal Minkowski spacetimes. We demonstrate for the first time that the spatial superposition of each detector’s path allows entanglement to be harvested from the quantum field in regimes where it would be otherwise impossible for detectors on classical trajectories. Surprisingly, for detectors on sufficiently delocalised trajectories in a thermal bath, the amount of harvested entanglement grows with the temperature of the field, violating a no-go theorem derived by Simidzija et. al. (Phys. Rev. D 98, 085007). Furthermore, we discover that mutual information harvesting is inhibited by the presence of quantum interference between the superposed detector trajectories.

I. INTRODUCTION

Entanglement describes the non-classical correlations that can exist within quantum systems, and is a fundamental property of the vacuum state in relativistic quantum field theory (QFT). In this regard, it underpins several important phenomena including the Unruh effect [1] and Hawking radiation [2], while from an information-theoretic perspective, it can be utilised as a physical resource which enables non-classical protocols such as quantum teleportation [3, 4].

Extracting entanglement from the vacuum state of quantum fields is a well-studied problem in the literature. Following the pioneering work of Valentini [5] and Reznik et. al. [6, 7] emerged the field of entanglement harvesting, which studies how initially uncorrelated quantum systems, interacting locally with the ambient background field, can become entangled. Entanglement harvesting has been shown to unveil novel information about the structure of entanglement in quantum fields and its dependence upon spacetime dimensionality [8], curvature [9–12], topology [13], and the presence of horizons [14]. It has also been studied in experimentally feasible settings, see for example [15].

In the studies mentioned, it is common to model the detectors using idealised, two-level systems (such as an atom) whose coupling to the ambient scalar field is a simple approximation to the light-matter interaction. This model, known as the Unruh-deWitt detector [16], has been recently applied to the problem of entanglement harvesting in the presence of indefinite causal order [17]. More specifically, the authors of [17] consider detectors whose interaction with the field occurs in a quantum-controlled superposition of temporal orders. Intriguingly, it was shown that the presence of temporal superposition enhances the ability of stationary detectors to extract entanglement from the Minkowski vacuum state. In particular, the no-go theorem of [18], which states that UdW detectors with Dirac-delta switching functions and arbitrary spatial profiles and detector-field couplings cannot harvest entanglement, was shown to be violated using the temporal superposition.

In this paper, we apply the quantum-controlled UdW detector model to entanglement harvesting in two well-known spacetimes; the static patch of de Sitter spacetime, and thermal Minkowski spacetime (i.e. flat Minkowski spacetime with a finite-temperature quantum field). We show that the presence of superposition – not only the temporal superposition of the detector’s interaction with the field, but also for detectors travelling in spatial superpositions of spacetime trajectories – enhances the ability of such detectors to harvest entanglement, compared to analogous scenarios with detectors on single trajectories. The presence of the superposition introduces quantum interference between the different field regions that each detector interacts with, suppressing the local excitations that they individually perceive. This typically allows the detectors to extract entanglement in regimes where it would be otherwise impossible for detectors travelling on classical trajectories. In fact, we demonstrate a violation of the no-go theorem derived by Simidzija et. al. in [19], which asserts that the entanglement harvested by detectors interacting with a thermal field state will decrease monotonically with the temperature of the field. Since the theorem was derived under the tacit assumption of classical detector trajectories, our result shows that standard intuition (namely that higher temperatures detrimentally affect quantum entanglement) fails when extending the motion of detectors to include superpositions. We also study the harvesting of mutual information, which measures the total amount of correlations between the detectors including classical correlations and non-entanglement quantum correlations. Interestingly, we discover that mutual information harvesting is inhibited by the presence of the aforementioned quantum interference effects.

Our paper is outlined as follows: in Sec. II, we review

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where we assume

$$|\Psi\rangle = D_{1|1}(\tau)\Psi(x(\tau))$$

which is a weak coupling constant, \(\eta(\tau)\) is a switching function, \(\sigma(\tau) = |e\rangle\langle e|e^{i\Omega \tau} + H.c\) is the interaction picture Pauli operator, acting on the detector Hilbert space. To model

the detector as travelling in a superposition of trajectories, we follow recent works [20–23] by introducing a control degree

of freedom, \(|\chi\rangle\), to the initial state of the detector, so that the

initial detector-field state is described by

$$|\Psi\rangle_{CFD} = |\chi\rangle |\psi\rangle |g\rangle$$

where

$$\rho_D = \left(1 - \mathcal{P}_D \right) 0 \mathcal{P}_D + \mathcal{O}(\lambda^4)$$

Upon tracing out the field degrees of freedom, it can be straightforwardly shown that the density matrix of the detector

system is given by

$$\mathcal{P}_{ij,D} = \int \mathcal{D}\tau \mathcal{D}\tau' |\psi\rangle |\phi(x(\tau))\phi(x(\tau'))\rangle |\psi\rangle$$

which are two-point field correlation functions evaluated between the trajectories \(x_1(\tau)\) and \(x_j(\tau')\). Equation (9) contains

local terms evaluated along the individual trajectories of the

superposition, as well as cross-correlation interference terms, evaluated between different trajectories. Notably, these

interference terms would be inaccessible to detectors travelling on classical, localised worldlines. If, instead of measuring

the control in a superposition basis, one traces out the control

state, the final detector state is a probabilistic mixture of the

individual contributions along each trajectory in the superpo-

sition, without the interference terms between them:

$$\mathcal{P}_{D} = \frac{1}{N} \sum_{i=1}^{N} \mathcal{P}_{i, D}.$$ 

As we show in Sec. IV, the interference terms suppress the local

excitations experienced by the individual detectors, which

enhances their capacity to extract entanglement from the vac-

uum state.

### III. Entanglement Harvesting with Quantum-Controlled Detectors

We now consider a composite system of two point-like

UdW detectors, coupled to the real, massless scalar field along

a quantum-controlled superposition of trajectories. The inter-

This yields the final detector-field state,

$$|\Psi\rangle_{FD} = \frac{1}{N} \sum_{i=1}^{N} \hat{U}_i |0\rangle |g\rangle.$$ 

where the second-order terms in \(\lambda\) contain time-ordered inte-

grals, denoted by \(\mathcal{T}\). Acting \(\hat{U}\) on the initial state and measur-

ing the control in the superposition state \(|\chi\rangle\) yields the final
detector-field state,

$$|\Psi\rangle_{CFD} = |\chi\rangle |\psi\rangle |g\rangle$$

where

$$\hat{H}_{int} = \lambda \eta(\tau) \sigma(\tau) \hat{\Phi}(x(\tau))$$

$$|\chi\rangle = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |i\rangle$$

$$\hat{H}_{int} = \sum_{i=1}^{N} \hat{H}_i \otimes |i\rangle \langle i|$$

$$\hat{H}_i = \lambda \eta_\tau \sigma_\tau \hat{\Phi}(x_\tau)$$

$$\hat{U} = \sum_{i=1}^{N} \hat{U}_i \otimes |i\rangle \langle i|$$

to second-order in perturbation theory, where

$$\hat{U}_i = 1 - i \int d\tau \hat{H}_i - \int_T d\tau' d\tau' \hat{H}_i(\tau) \hat{H}_i(\tau') + \mathcal{O}(\lambda^3)$$

contains field operators evaluated along the \(i\)th trajectory of

the superposition. The evolution of the detector-field-control

system can be obtained by expanding the time-evolution oper-

ator, \(\hat{U}\),

The detector-field coupling is described via the interaction Hamiltonian,

$$\hat{H}_{int} = \lambda \eta(\tau) \sigma(\tau) \hat{\Phi}(x(\tau))$$

where \(\lambda\) is a weak coupling constant, \(\eta(\tau)\) is a switching function,

$$\rho_D = \left(1 - \mathcal{P}_D \right) 0 \mathcal{P}_D + \mathcal{O}(\lambda^4)$$

to leading order in \(\lambda\), where

$$\mathcal{P}_D = \frac{\lambda^2}{N^2} \sum_{i,j=1}^{N} \mathcal{P}_{ij,D} = \frac{\lambda^2}{N^2} \left\{ \sum_{i,j=1}^{N} \mathcal{P}_{ij,D} + \sum_{i\neq j} \mathcal{P}_{ij,D} \right\}$$

local terms evaluated along the individual trajectories of the

superposition, as well as cross-correlation interference terms, evaluated between different trajectories. Notably, these

interference terms would be inaccessible to detectors travelling on
classical, localised worldlines. If, instead of measuring the
control in a superposition basis, one traces out the control
state, the final detector state is a probabilistic mixture of the
individual contributions along each trajectory in the superpo-
sition, without the interference terms between them:

$$\mathcal{P}_D = \frac{1}{N} \sum_{i=1}^{N} \mathcal{P}_{i, D}.$$ 

As we show in Sec. IV, the interference terms suppress the local

excitations experienced by the individual detectors, which

enhances their capacity to extract entanglement from the vac-

uum state.

### II. Unruh-deWitt Detectors in Superpositions of Trajectories

The standard UdW detector model considers a two-level system, whose internal states \(|g\rangle, |e\rangle\) with energy gap \(\Omega\), couple to the real, massless scalar field \(\Phi(x(\tau))\) initially in the state \(|\psi\rangle\), along the worldline \(x(\tau)\). The detector-field coupling is described via the interaction Hamiltonian,

$$|\Psi\rangle_{CFD} = |\chi\rangle |\psi\rangle |g\rangle$$

where

$$|\chi\rangle = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |i\rangle$$

where \(i\) are orthogonal states, and the interaction Hamiltonian takes on the modified form,

$$|\Psi\rangle_{CFD} = |\chi\rangle |\psi\rangle |g\rangle$$

where

\(\mathcal{P}_D = \frac{1}{N} \sum_{i=1}^{N} \mathcal{P}_{i, D}.

As we show in Sec. IV, the interference terms suppress the local

excitations experienced by the individual detectors, which

enhances their capacity to extract entanglement from the vac-

uum state.
action Hamiltonian is now given by
\[
\hat{H}_{\text{int}} = \sum_{D=A,B} \sum_{i=1}^{N} \hat{H}_{D_i} \otimes |i\rangle \langle i|
\]  
(13)
where as in Eq. (4),
\[
\hat{H}_{D_i} = \lambda \eta_{D_i}(\tau)\sigma_D(\tau)\Phi(x_{D_i}(\tau))
\]  
(14)
describes the part of the interaction between the \(D\)th detector along the \(i\)th trajectory of the superposition. To second-order in perturbation theory, the time-evolution operator is
\[
\hat{U} = \sum_{i=1}^{N} \hat{U}_i \otimes |i\rangle \langle i|
\]  
(15)
where
\[
\hat{U}_i = 1 + \hat{U}^{(1)}_i + \hat{U}^{(2)}_i + \mathcal{O}(\lambda^3)
\]  
(16)
and \(\hat{U}^{(k)}_i\) is the \(k\)th order term of the Dyson series expansion,
\[
\hat{U}^{(1)}_i = -i \int d\tau \left( \hat{H}_{A_i}(\tau) + \hat{H}_{B_i}(\tau) \right)
\]  
(17)
\[
\hat{U}^{(2)}_i = - \int_{\tau_1}^{\tau_2} d\tau d\tau' \left( \hat{H}_{A_i}(\tau) + \hat{H}_{B_i}(\tau) \right) \left( \hat{H}_{A_i}(\tau') + \hat{H}_{B_i}(\tau') \right).
\]  
(18)

The initial state of the system is simply
\[
|\Psi\rangle_{\text{CFD}} = |\chi\rangle |\psi\rangle |g\rangle_A |g\rangle_B.
\]  
(19)

Again, let us assume that the control system is measured in its initial state, \(|\chi\rangle\). The conditional state of the system after measuring the control in the state \(|\chi\rangle\) is
\[
\langle \chi | \hat{U} | \Psi \rangle_{\text{CFD}} = \frac{1}{N} \sum_{i=1}^{N} \left\{ |\psi\rangle |g\rangle_A |g\rangle_B + \langle \chi | \hat{U}^{(1)} | \Psi \rangle_{\text{CFD}} + \langle \chi | \hat{U}^{(2)} | \Psi \rangle_{\text{CFD}} \right\}
\]  
(20)
where
\[
\langle \chi | \hat{U}^{(1)} | \Psi \rangle_{\text{CFD}} = -i\lambda \int d\tau e^{i\Omega} \left( \eta_{A_i}(\tau)\Phi(x_{A_i}(\tau)) |e\rangle_A |g\rangle_B + \eta_{B_i}(\tau)\Phi(x_{B_i}(\tau)) |g\rangle_A |e\rangle_B \right) |\psi\rangle
\]  
(21)
\[
\langle \chi | \hat{U}^{(2)} | \Psi \rangle_{\text{CFD}} = -\lambda^2 \int_{\tau_1}^{\tau_2} d\tau d\tau' e^{-i\Omega(\tau-\tau')} \left( \eta_{A_i}(\tau)\eta_{B_i}(\tau') \Phi(x_{A_i}(\tau)) \Phi(x_{B_i}(\tau')) + (A \leftrightarrow B) \right) |\psi\rangle |g\rangle_A |g\rangle_B
\]  
(22)
yielding
\[
\hat{\rho}_D = \left( \begin{array}{cccc}
1 - \mathcal{P}_A - \mathcal{P}_B & 0 & 0 & \mathcal{M} \\
0 & \mathcal{P}_B & \mathcal{L} & 0 \\
0 & \mathcal{L}^* & \mathcal{P}_A & 0 \\
\mathcal{M}^* & 0 & 0 & 0
\end{array} \right)
\]  
(23)
upon tracing out the field degrees of freedom. Note that the \(|g\rangle \langle e|_A \otimes |g\rangle \langle e|_B\) element of the density matrix, \(\mathcal{M}\), is obtained from the purely second-order term in the Dyson series expansion (18) and only contains correlations between the \(i\)th trajectories of the two detectors. The \(|e\rangle \langle g|_A \otimes |g\rangle \langle e|_B\) and \(|g\rangle \langle e|_A \otimes |e\rangle \langle g|_B\) terms in \(\hat{\rho}_D\) (\(\mathcal{L}\) and its conjugate) are obtained via a product of the first-order terms in (17) and contain cross-correlations between the \(i\)th and \(j\)th trajectories of both detectors. Specifically
\[
\mathcal{P}_D = \frac{\lambda^2}{N^2} \sum_{i,j=1}^{N} \int \int d\tau d\tau' \left( \eta_{D_i}(\tau)\eta_{D_j}(\tau') e^{-i\Omega(\tau-\tau')} \mathcal{W}(x_{D_i}(\tau), x_{D_j}(\tau')) \right)
\]  
(24)
\[
\mathcal{L} = \frac{\lambda^2}{N^2} \sum_{i,j=1}^{N} \int \int d\tau d\tau' \left( \eta_{B_i}(\tau)\eta_{A_j}(\tau') e^{-i\Omega(\tau-\tau')} \mathcal{W}(x_{A_i}(\tau), x_{B_j}(\tau')) \right)
\]  
(25)
\[
\mathcal{M} = -\frac{\lambda^2}{N} \sum_{i=1}^{N} \int \int d\tau d\tau' e^{-i\Omega(\tau+\tau')} \left( \eta_{A_i}(\tau)\eta_{B_i}(\tau') \mathcal{W}(x_{A_i}(\tau), x_{B_i}(\tau')) + \eta_{B_i}(\tau)\eta_{A_i}(\tau') \mathcal{W}(x_{B_i}(\tau), x_{A_i}(\tau')) \right).
\]  
(26)

As before, the terms \(\mathcal{P}_D\) are local transition probabilities for the \(D\)th detector. The terminology of ‘local’ in this context must be understood as pertaining to the transition probability of an individual detector, recalling that these likewise contain
terms evaluated ‘locally’ (in a spatiotemporal sense) along each trajectory of the superposition, as well as ‘non-locally’ between the trajectories (i.e. the interference terms). The so-called entangling term, $M$, is a function of the trajectories of both detectors. In particular, it is a sum of the non-local field correlations between the $i$th trajectories of detector $A$ and $B$. Finally, it should be noted that in our model for the control, the entangling term is identical to that in which the detectors traverse classical worldlines. This is true when the separation of the $i$th superposed trajectories between detectors $A$ and $B$ are equal to the comparable separation of detectors $A$ and $B$ when they traverse classical paths.

To quantify the entanglement of the final bipartite detector state, we utilise the concurrence, which is defined as [24]

$$C(\rho_B) = 2\max\left\{0, |M| - \sqrt{P_A P_B} \right\}.$$  (27)

From Eq. (27), we see that the amount of entanglement extracted from the field is a competition between the entangling term, $M$, and the geometric mean of the local transition probabilities of the two detectors, $\sqrt{P_A P_B}$. For entanglement to be extracted from the field, the non-local correlations probed by the detectors must exceed the local field fluctuations perceived along their respective (superposed) trajectories.

Another quantity of interest is the mutual information harvested by the detectors, which quantifies the total amount of correlations – both quantum and classical – between them. For bipartite quantum systems, the mutual information is given by the relative entropy between the state of the system and the tensor product of the reduced states of the subsystems,

$$I(\rho_{AB}) = S(\rho_{AB}||\rho_A \otimes \rho_B).$$  (28)

From the reduced density matrix obtained in Eq. (23), the mutual information between two UdW detectors to leading order in $\lambda$, is [19]

$$I(\rho_{AB}) = \mathcal{L}_+ \log \mathcal{L}_+ + \mathcal{L}_- \log \mathcal{L}_- - \mathcal{P}_A \log \mathcal{P}_A - \mathcal{P}_B \log \mathcal{P}_B.$$  (29)

where

$$\mathcal{L}_\pm = \frac{1}{2} \left\{ \mathcal{P}_A + \mathcal{P}_B \pm \sqrt{(\mathcal{P}_A - \mathcal{P}_B)^2 + 4|\mathcal{L}|^2} \right\}.$$  (30)

For detectors which have vanishing concurrence but non-zero mutual information, we can infer that the correlations are either classical, or come from non-entanglement quantum correlations known as quantum discord. Note in particular that the mutual information contains the off-diagonal $\mathcal{L}$ terms of the detector density matrix. Interestingly, $\mathcal{L}$ contains Wightman functions evaluated along the $j$th trajectory of detector $A$, and the $i$th trajectory of detector $B$, which include correlations that are not directly probed by the detectors, upon inclusion of the quantum-controlled superposition. These terms can be understood as the leading order contributions to the classical correlations between the detectors [25].

### IV. Entanglement Harvesting in de Sitter Spacetime

#### A. Field-theoretic and geometric details

dS de Sitter spacetime is a maximally symmetric spacetime of constant positive curvature that is a solution to the Einstein field equations with cosmological constant $\Lambda$. We shall write the inverse de Sitter length $l = \sqrt{\Lambda/3}$ [26–29], as it allows us to refer to $l$ interchangeably with the spacetime curvature, since it is related to the Ricci scalar via $R = 12l^2$.

The de Sitter manifold can be described by the 5-dimensional hyperboloid,

$$-Z_0^2 + Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 = 1/l^2.$$  (31)

The hyperboloid is embedded within a flat 5-dimensional geometry,

$$ds^2 = -dZ_0^2 + dZ_1^2 + dZ_2^2 + dZ_3^2 + dZ_4^2.$$  (32)

with coordinates $(Z_0, Z_1, Z_2, Z_3, Z_4)$. A particularly convenient coordinate system useful for field-theoretic calculations are the static coordinates $(T, r, \theta, \phi)$, given by

$$Z_0 = \sqrt{1/l^2 - r^2} \sinh(lT)$$  (33)

$$Z_1 = \sqrt{1/l^2 - r^2} \cosh(lT)$$  (34)

$$Z_2 = r \cos \theta$$  (35)

$$Z_3 = r \sin \theta \cos \phi$$  (36)

$$Z_4 = r \sin \theta \sin \phi$$  (37)

with the corresponding metric given by

$$ds^2 = -(1 - l^2 r^2) dT^2 + \frac{1}{1 - l^2 r^2} dr^2 + r^2 d\Omega_5^2.$$  (38)

where $d\Omega_5^2 = d\theta^2 + \sin^2 \theta d\phi^2$. The coordinates Eq. (33)–(37) only cover part of the entire de Sitter spacetime, a region known as the static patch (for a useful visualisation, see [27]). A test particle in this spacetime experiences an acceleration with magnitude,

$$\alpha = \frac{lr}{\sqrt{l^2 - r^2}}.$$  (39)

Evidently, both the curvature of the spacetime and the radial position of the test particle influence its acceleration. Furthermore, as $l \to 0$ or $r \to 0$, the acceleration vanishes.

We consider a massless, conformally coupled real scalar field since the Wightman function takes on the simple form [30, 31],

$$\mathcal{W}_{ds} = -\frac{1}{4\pi^2} \frac{1}{(Z_0 - Z'_0)^2 - \Delta Z_i^2 - i\varepsilon}$$  (40)
function: W

where $\Delta Z^2_i = (Z_i - Z_i')^2 + (Z_2 - Z_2')^2 + (Z_3 - Z_3')^2 + (Z_4 - Z_4')^2$ and $\varepsilon$ is an infinitesimal regularisation constant. We consider detectors superposed along the trajectories

$$
(r, \theta_1, \phi), (r, \theta_2, \phi)
$$

for detector A

$$
(r', \theta_1', \phi'), (r', \theta_2', \phi')
$$

for detector B respectively (i.e. a two-trajectory superposition, for simplicity), such that the $i$th set of trajectories are separated by the Euclidean (entangling) distance [32]

$$
L_{mi} = 2r \sin \left( \frac{\theta_{mi}}{2} \right)
$$

where $\theta_{mi} = \theta_i - \theta_i'$. In our analysis, we take $\theta_{m} := \theta_{m1} = \theta_{m2}$ for simplicity. This also means that the entangling term of the bipartite detector density matrix is identical to the classical trajectory case. We make use of three kinds of Wightman function: $W_{ds}$, a self-correlation term evaluated locally along each trajectory in the superposition, for each detector, $W_{ds-s}$, evaluated non-locally between the superposed trajectories of a single detector, which acts as an interference term between the trajectories, and $W_{ds-m}$, evaluated between the $i$th trajectories of the respective detectors, $A$ and $B$, and carries information about the non-local field correlations probed by the two detectors.

Explicitly, the Wightman functions are given by

$$
W_{ds}(s) = -\frac{\beta^2}{16\pi^2} \frac{1}{\sinh^2(\beta s/2 - i\varepsilon)}
$$

$$
W_{ds-s}(s) = -\frac{\beta^2}{16\pi^2} \frac{1}{\sinh^2(\beta s/2 - i\varepsilon) - (\beta L_s/2)^2}
$$

$$
W_{ds-m}(s) = -\frac{\beta^2}{16\pi^2} \frac{1}{\sinh^2(\beta s/2 - i\varepsilon) - (\beta L_m/2)^2}
$$

where $\beta = (1 - r^2)^{-1/2}$. Now, let us assume for illustration that a single detector interacts with the field mediated by a Gaussian switching function in the infinite interaction-time limit (that is, the detector-field interaction is effectively constant). It can be straightforwardly shown that the transition probability of a single detector in this limit is proportional to the Planck spectrum [33],

$$
P_D = \frac{\lambda^2 \Omega}{2\pi} \frac{1}{e^{\pi \Omega / \beta} - 1}
$$

meaning that the field is populated by thermal particles at the Gibbons-Hawking temperature

$$
T_{ds} = \frac{\beta}{2\pi}
$$

In Eq. (45), we have also defined the superposition distance, $L_s$, as

$$
L_s = 2r \sin \left( \frac{\theta_s}{2} \right)
$$

which is the Euclidean distance separating the $i$th and $j$th trajectories of the superposition, where $\theta_s = \theta_i - \theta_j$ is the angular separation between them.

The simple form of the Wightman functions allows for the direct numerical evaluation of the transition probability and the entangling term in the reduced density matrix. In the following analysis, we consider entanglement harvesting between detectors in spatial superpositions, as well scenarios where the detector-field interaction occurs in a superposition of temporal order. We compare our results with scenarios without the superposition, demonstrating in general, that superposition enhances the amount of harvested entanglement.

B. Spatial superpositions

We consider first detectors in a superposition of two trajectories, where the initial state of the control is given by

$$
|\chi\rangle = \frac{1}{\sqrt{2}} (|1\rangle + |2\rangle).
$$

Likewise, we assume that the final measurement of the control in the state $|\chi\rangle$. We take the switching function to be identical along each trajectory, namely a Gaussian centred at $\tau = 0$,

$$
\eta_D(\tau) = \exp \left\{ -\frac{\tau^2}{2\sigma^2} \right\}
$$

where $\sigma$ is a characteristic timescale for the interaction. The excitation probability for both detectors is given by

$$
P_D = \frac{\lambda^2 \sqrt{\pi \sigma^2}}{2} \left\{ \int_{-\infty}^{\infty} ds \frac{e^{-s^2/4\sigma^2}}{\sinh^2(\beta s/2 - i\varepsilon)} - \left( \beta L_s/2 \right)^2 \right\}
$$

where the contribution from ‘local’ excitations (i.e. along the individual trajectories) and ‘non-local’ interference terms is evident. Meanwhile, the entangling term takes the form

$$
M = -2\lambda^2 \sqrt{\pi \sigma^2} e^{-\sigma^2 \Omega^2} \int_0^\infty ds \frac{e^{-s^2/4\sigma^2}}{\sinh^2(\beta s/2 - i\varepsilon) - (\beta L_m/2)^2}.
$$

In Fig. 1, we have plotted the concurrence between the detectors as a function of the superposition distance, $L_s/\sigma$. For the detectors in superposition, the concurrence is a monotonically increasing function of the superposition distance, asymptoting to a fixed value at large distances. Since the entangling term is identical for both vanishing and non-vanishing superposition distances, we can trace this behaviour to the inter-
FIG. 1. Concurrence, $C_{AB}/\lambda^2$ as a function of the superposition distance, $L_s$, for (a) $l\sigma = 0.1$, (b) $l\sigma = 0.175$ (c) $l\sigma = 0.195$ and (d) $l\sigma = 0.198$. The dashed lines represent the concurrence harvested by detectors on classical trajectories corresponding to cases (a), (b); for (c) and (d), it is impossible for such detectors to harvest entanglement. The other parameters we have used are $r/\sigma = 5$, $\Omega\sigma = 0.05$, $L_m/\sigma = 1.36$.

As before, the superposition of each detector's path inhibits the local excitations it perceives, and larger separations produce a more significant suppression of these terms. Remarkably, the presence of the interference terms actually causes the transition probability to decay as the detectors approach the horizon. That is to say that as the temperature of the field increases, the excitation probability of the detectors decreases, the excitation probability of the detectors decreases rapidly, a generic feature of non-local field correlations. This behaviour has been observed in other contexts, and has been referred to as the anti-Unruh effect [34, 35] (for flat spacetime) and the anti-Hawking effect [36] (in the vicinity of a black hole). This behaviour is explored in greater detail in a companion paper (in preparation). Here, we find that such an effect enables entanglement harvesting in regimes that are not be possible for detectors on classical paths.

In Fig. 2, we have plotted the concurrence as a function of the detector energy gap, $\Omega\sigma$. In general, the concurrence rapidly decays for negative energy gaps (i.e. detectors initialised in their excited state). As before, the presence of the superposition amplifies the concurrence, which again, can be understood in terms of a suppression effect in the local noise term, $\sqrt{P_A P_B}$, for larger superposition distances (see inset). At large energy gap, this term becomes independent of the superposition distance.

In Fig. 3, we have plotted the concurrence as a function of the radial coordinate, $r/\sigma$ for a fixed spacetime curvature, $l\sigma$. As before, the superposition of each detector's path inhibits the local excitations it perceives, and larger separations produce a more significant suppression of these terms. Remarkably, the presence of the interference terms actually causes the transition probability to decay as the detectors approach the horizon. That is to say that as the temperature of the field increases, the excitation probability of the detectors decreases, in the regimes shown. Such a behaviour has been observed in other contexts, and has been referred to as the anti-Unruh effect [34, 35] (for flat spacetime) and the anti-Hawking effect [36] (in the vicinity of a black hole). This behaviour is explored in greater detail in a companion paper (in preparation). Here, we find that such an effect enables entanglement harvesting in regimes that are not be possible for detectors on classical paths.

FIG. 2. Concurrence, $C_{AB}/\lambda^2$ as a function of the energy gap, $\Omega\sigma$, comparing the entanglement harvested by detectors on classical trajectories, with those in superposition. The plot displays the concurrence for detectors with $\theta = 0, \pi/8, \pi/4, \pi/2$ (from yellow to dark blue). The inset compares the entangling term, $M$ (dashed line), with the local noise term, $\sqrt{P_A P_B}$. The other parameters used are $a\sigma = 0.15, r/\sigma = 5$, and $\theta_m = \pi/12$.

FIG. 3. Concurrence, $C_{AB}/\lambda^2$ as a function of the radial coordinate, $r/\sigma$ where we have compared detectors on classical trajectories with detectors in superposition, with (blue) $\theta = \pi/8$, (brown) $\theta = \pi/4$, and (orange) $\theta = \pi/2$. The dashed line in the main figure is the concurrence for detectors on classical trajectories. The other parameters used are $l\sigma = 0.2$, and $\Omega\sigma = 0.02$. The inset compares the entangling term, $M$ (black line), with the local noise term, $\sqrt{P_A P_B}$, where the colours correspond to those shown in the main figure.

C. Temporal superpositions

Thus far, we have considered detectors travelling on spatially delocalised trajectories. In particular, each detector was initialised in a superposition of physical paths, separated by a fixed Euclidean distance. We now turn to detectors traveling along single trajectories whose interaction with the field occurs in a temporal superposition.
In the case of temporal superposition, the temporal switching of each detector occurs in a quantum-controlled superposition. As before, the control state is initialised and then measured in the superposition state

$$|\chi_+\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle).$$

(54)

In the following, we refer to such detectors as interacting with ‘non-classical’ spacetime regions (i.e. having switching functions subjected to quantum indeterminacy). There are two scenarios of particular interest. The first is what was labelled in [17] as a past-future superposition, in which the detectors are jointly switched on along a common spacelike slice, but in a quantum-controlled superposition of times (i.e. the centre-time of the Gaussian). The switching functions of the two detectors take the form,

$$\eta_{A_1}(\tau) = \eta_{B_1}(\tau) = \exp\left\{ -\frac{(\tau + \tau_0)^2}{2\sigma^2} \right\},$$

(55)

$$\eta_{A_2}(\tau) = \eta_{B_2}(\tau) = \exp\left\{ -\frac{(\tau - \tau_0)^2}{2\sigma^2} \right\}. \tag{56}$$

Here $-\tau_0$ (which we refer to as the superposition time-delay, defined with respect to the detector frame) is the time-delay of the switching function with respect to the origin. The second scenario, labelled cause-effect superposition [17], is one in which the causal order of the interactions of the two detectors with the field is indefinite. This is achieved by a superposition of the causal arrangements of the two detectors; that is, detector $A$ switched on in the causal past of detector $B$, in superposition with detector $B$ switched on in the causal past of detector $A$. The switching functions take the form,

$$\eta_{A_1}(\tau) = \eta_{B_2}(\tau) = \exp\left\{ -\frac{(\tau + \tau_0)^2}{2\sigma^2} \right\},$$

(57)

$$\eta_{A_2}(\tau) = \eta_{B_1}(\tau) = \exp\left\{ -\frac{(\tau - \tau_0)^2}{2\sigma^2} \right\}. \tag{58}$$

Both the past-future and cause-effect superpositions represent scenarios possessing indefinite causal order. In [17], it was shown that detectors interacting with the Minkowski vacuum in a quantum-controlled superposition of temporal orders (i.e. centre switching times) can harvest entanglement under generic conditions, violating the no-go theorem of [18] for detectors activated only once. The setup in [17] also represents a toy model for the quantum switch [37], a scenario where operations on a quantum system occur in a superposition of temporal orders, which was first proposed within the framework of the process matrix formalism. The following analysis represents a next step in understanding the effect of indefinite causal order upon the ability for detectors to harvest entanglement, with the eventual aim to apply it to gravitationally-induced indefinite causal order [38].

1. Past-future superposition

We first consider entanglement harvesting with detectors in a past-future superposition. The local transition probability and entangling terms are respectively given by

$$P_D = \frac{\lambda^2 \sqrt{\pi \sigma^2}}{2} \left\{ \int_{-\infty}^{\infty} ds \frac{e^{-i\Omega s} e^{-s^2/4\sigma^2}}{\sinh^2(\beta s/2 - i\epsilon)} + e^{-\tau_0^2/\sigma^2} \int_{-\infty}^{\infty} ds \frac{e^{-i\Omega s} e^{-s^2/4\sigma^2} \cosh(s\tau_0/\sigma^2)}{\sinh^2(\beta s/2 - i\epsilon)} \right\}, \tag{59}$$

and

$$M = -2\sqrt{\pi \sigma^2} \cos\left(\frac{\tau_0 \Omega}{2}\right) e^{-\sigma^2 \Omega^2} \times \int_0^{\infty} ds \frac{e^{-s^2/4\sigma^2}}{\sinh^2(\beta s/2 - i\epsilon)} - (\beta L_m/2)^2. \tag{60}$$

Notably, the entangling term is an oscillatory function of the centre interaction time, $\tau_0$, and the energy gap, $\Omega$. As $\tau_0$ is varied, the entangling term will experience periodic resonances, produced by interference between the field regions that the detector interacts with in temporal superposition. Unlike cause-effect superposition, the entangling term does not decay with $\tau_0$. This is because the $i$th interaction region of detector $A$ and $B$ remain at a fixed distance in spacetime, independent of $\tau_0$. Finally, for detectors with classical switching functions, the concurrence is independent of $\tau_0$, since the Wightman functions for the cases considered are time-translation invariant.

FIG. 4. The concurrence $C_{AB}/\lambda^2$ harvested by detectors in a past-future temporal superposition, as a function of the superposition time-delay $\tau_0/\sigma$ for $l/\sigma = 0.2$, $r/\sigma = 2.5$, $\Omega \sigma = 1$, $L_m = 3.54$. Detectors on classical trajectories in this regime cannot harvest entanglement. The inset compares the entangling term $M$ (orange curve), with the local noise term, $\sqrt{P_A P_B}$ as a function of $\tau_0/\sigma$.

In Fig. 4, we have plotted the concurrence harvested by the detectors as a function of the superposition time-delay, $\tau_0/\sigma$. In the regime considered, entanglement harvesting is possi-
able for detectors in superposition, but impossible for detectors with classical switching functions. We notice that the entangling term oscillates periodically as a function of $\tau_0/\sigma$, as was already inferred from the form of Eq. (60). The transition probability is also oscillatory and decays to an equilibrium value as $\tau_0/\sigma$ grows large. This equilibrium value is half that of a single detector; only the ‘local’ contributions to $P_D$ are non-vanishing at large separations. For $\tau_0/\sigma = 0$, there is no superposition of interaction times, and the transition probabilities equal that of a single detector. Correspondingly, we find that entanglement harvesting is generally inhibited for $|\tau_0/\sigma| \sim 0$. Interestingly, for $|\tau_0/\sigma| \ll 1$ (on either side of $|\tau_0/\sigma| \sim 0$), the local field excitations of each detector are significantly inhibited, which allows for an enhancement of the concurrence, indicated by the sharp peaks near the origin in Fig. 4. As $|\tau_0/\sigma|$ grows, the temporal distance between the interaction regions likewise grows, and the interference terms in the transition probability decay to zero. Hence, the equilibrium value for the transition probability is simply half of that for a single detector interacting with the field in a localised spacetime region.

In Fig. 5, we have plotted the concurrence as a function of the energy gap, $\Omega\sigma$, and the superposition time-delay, $\tau_0/\sigma$. In particular, the bottom graph displays the regions of the parameter space for which larger amounts of entanglement are harvested by the quantum-controlled detectors, in comparison to detectors on classical trajectories. Some features of note include the amplification of entanglement harvesting at small negative energy gaps, as well as the regions near $|\tau_0/\sigma| \sim 0$ for which the resonant trough in the transition probability amplifies the concurrence between the detectors.

2. Cause-effect superposition

For the cause-effect superposition, the transition probability of each detector takes the same form as the past-future superposition, Eq. (59). The entangling terms of each detector takes the same form as the past-future superposition, and the transition probability decays with classical switching functions. Furthermore, the cause-effect superposition enables in general, a greater amount of entanglement harvesting at periodic values of $\tau_0/\sigma$, which amplifies the concurrence obtained by the past-future detector superposition of the top figure, compared with an analogous setup without the superposition. The coloured regions are those in which superposed detectors can harvest entanglement where classical detectors cannot, and vice versa for the white regions.

![FIG. 5. (top) Plot of the concurrence, $C_{AB}/\lambda^2$, as a function of the past-future superposition time-delay $\tau_0/\sigma$ and the energy gap, $\Omega\sigma$, with $\sigma = 0.2$, $r/\sigma = 2.5$, $L_m/\sigma = 0.65$. (bottom) Plot of the difference in the concurrence obtained by the past-future detector superposition of the top figure, compared with an analogous setup without the superposition.](image)

In Fig. 6, we have plotted the concurrence as a function of $\tau_0/\sigma$, comparing both the cause-effect arrangement and the scenario where the detectors $A$ and $B$ interact with the field in classical spacetime regions, with a time-delay.

Unlike the past-future superposition, which amplified entanglement harvesting at periodic values of $\tau_0/\sigma$, the cause-effect superposition enables in general, a greater amount of entanglement to be harvested, compared with detectors with classical switching functions. Furthermore, the cause-effect superposition also allows entanglement to be harvested in regimes where it would not be possible with classical detectors.

By construction, the interference terms equal the local contributions when $\tau_0/\sigma = 0$, and hence $\sqrt{P_A P_B}$ is maximised.
FIG. 6. Concurrence, $C_{AB}$, harvested by detectors with classical switching times (orange) compared to a cause-effect superposition of interaction times (blue), as a function of $\tau_0/\sigma$. The dashed lines correspond to $L_m = 3.53$, while the solid lines correspond to $L_m = 2.5$. The insets show the entangling term (blue) and local noise (orange) for (a) $L_m = 2.5$ and (b) $L_m = 3.53$. The other parameters we have used are $l\sigma = 0.2$, $r/\sigma = 2.5$, $\Omega\sigma = 1$. For small superposition distances, $|\tau_0/\sigma| \ll 1$, the transition probability of each detector experiences an anti-resonance (i.e. dipping below the $|\tau_0/\sigma| \gg 1$ equilibrium value), amplifying the concurrence between the detectors. For larger time-delays, the entangling term decays rapidly and the detectors can no longer harvest entanglement.

In Fig. 7, we compare the concurrence harvested by the detectors for (a) detectors in a cause-effect superposition and (b) detectors interacting with the field in classically defined spacetime regions. In general, the detectors in superposition can harvest more entanglement than the detectors with classically defined interaction regions; the presence of the superposition also yields non-vanishing concurrence at larger values of $\tau_0/\sigma$ than would be possible for the analogous classical-switching setup. Hence, the interference between the different spacetime regions probed by the delocalised detectors allows entanglement harvesting to occur at larger time-delays than would be possible in the corresponding classical case, as evidenced by the oscillations in Fig. 7(a).

V. HARVESTING MUTUAL INFORMATION

A. Spatial superpositions

So far, we have studied the harvesting of entanglement by detectors in superposition. We now turn to the harvesting of mutual information (29). For spatial superpositions, the local transition probability remains the same, but now we also require the mutual information term $\mathcal{L}$. These are given respectively by

$$
\mathcal{P}_D = \frac{\lambda^2 \sqrt{\pi \sigma^2}}{2} \left\{ \int_{-\infty}^{\infty} ds \, e^{-s^2/4\sigma^2} e^{-i\Omega s} \frac{1}{\sinh^2(\beta s/2 - i\epsilon)} + \int_{-\infty}^{\infty} ds \, e^{-s^2/4\sigma^2} e^{-i\Omega s} \frac{1}{\sinh^2(\beta s/2 - i\epsilon) - (\beta \mathcal{L}_s/2)^2} \right\}
$$

(63)
and

\[
\mathcal{L} = \frac{\lambda^2 \sqrt{\pi} \sigma^2}{2} \left\{ \int_{-\infty}^{\infty} dx \frac{e^{-x^2/\sigma^2} e^{-i 2x}}{\sinh^2(\beta x/2 - i 2x) - (\beta L_s/2)^2} + \int_{-\infty}^{\infty} dx \frac{e^{-x^2/\sigma^2} e^{-i 2x}}{\sinh^2(\beta x/2 - i 2x) - (\beta L_s/2)^2} \right\}
\]

where we have defined the ‘mutual information distance’ as

\[
L_i = 2r \sin \left( \frac{\theta_i}{2} \right)
\]

where \(\theta_i\) is the difference in the angular coordinate between the \(i\)th trajectory of detector \(A\) and the \(j\)th trajectory of detector \(B\) \((i \neq j)\).

In Fig. 8, we have plotted the mutual information as a function of the superposition distance for different values of the de Sitter curvature, \(l \sigma\). As with entanglement harvesting, the amount of mutual information between the detectors decreases with increasing \(l \sigma\). That is, as the curvature of the spacetime increases, the detectors effectively approach the cosmological horizon (since \(r \sigma\) is fixed) and the background temperature of the field increases. Interestingly, the mutual information (for detectors in superposition) behaves oppositely to the concurrence; rather than growing with increased superposition distance, \(I(\rho_{AB})\) decreases. Apart from this, the decay of the mutual information as the detectors approach the horizon accords with similar results found recently in [39], which showed that a black hole horizon likewise extinguishes correlations encoded in \(I(\rho_{AB})\).

In Fig. 9, we have plotted the mutual information as a function of the energy gap, \(\Omega \sigma\). Compared with the concurrence, which vanished for large negative energy gaps, the mutual information oscillates in a decaying manner as \(\Omega \sigma\) becomes more negative. The mutual information harvested by detectors on classical trajectories is typically larger than that obtained for detectors in superposition (and always so, for positive energy gaps); however for certain values of \(\Omega \sigma\), the superposed detectors harvest slightly more mutual information than the classical-trajectory counterparts.

Finally, in Fig. 10, we have plotted the mutual information as a function of \(r/\sigma\), in a universe with fixed curvature. In a similar manner to the concurrence, we discover that the mutual information between the detectors decays as the detectors approach the cosmological horizon. Furthermore, detectors in superposition generally harvest less mutual information, complementing the result shown in Fig. 8. As already noted, we find that detectors on classical trajectories can in general, harvest larger amounts of mutual information compared with detectors in superpositions of trajectories. Moreover, in contrast to the previous section on entanglement harvesting, we find that larger superposition distances actually
inhibit the harvesting of mutual information from the de Sitter vacuum. To explain this behaviour, we conjecture a comparable but opposite effect to that discovered in [19]. There, it was found that mutual information harvesting from finite-temperature field states is amplified at larger temperatures (i.e. as the local noise experienced by the individual detectors increases). In our case, the quantum interference effects introduced by the superposition suppress local noise, thus leading to an inhibition of the mutual information harvested by the detectors.

B. Temporal superpositions

In contrast to entanglement harvesting, the mutual-correlation terms, $L$, are identical for both the past-future and cause-effect superpositions. They are given by

$$
L = \frac{\lambda^2}{2} \sqrt{\pi \sigma^2} \left\{ \int_{-\infty}^{\infty} ds \frac{e^{-s^2/4\sigma^2} e^{-\beta \lambda s}}{\sinh^2(\beta s/2 - i \epsilon) - (\beta L_m/2)^2} + \int_{-\infty}^{\infty} ds \frac{e^{-s^2/4\sigma^2} e^{-\beta \lambda s} \cosh(s \sigma_0/\sigma^2)}{\sinh^2(\beta s/2 - i \epsilon) - (\beta L/2)^2} \right\}
$$

where the local transition probabilities are identical to the expressions previously stated. In Fig. 11, we have plotted the mutual information (orange) as a function of the superposition time-delay, $\tau_0/\sigma$. The concurrence obtained for the past-future superposition (scaled by 1/4), is displayed in blue. We find that the mutual information oscillates with $\tau_0/\sigma$, eventually decaying to an equilibrium value for $|\tau_0/\sigma| \gg 1$. Interestingly, the mutual information peaks in regions where the entanglement vanishes, for example at $\tau_0/\sigma = 0$ and $|\tau_0/\sigma| \simeq 2.5$. Contrarily, we also discover that the troughs in the mutual information occur at the peaks of the concurrence, for example at $|\tau_0/\sigma| \simeq 1.5$. This oscillatory behaviour indicates that the interference effects experienced by the detectors are highly sensitive to the precise field regions probed by the interaction. In particular in the regime $|\tau_0/\sigma| \lesssim 5$, the correlations between the detector approximately swap between mutual information and entanglement.

In Fig. 12, we have plotted the mutual information as a function of the energy gap, $\Omega \sigma$, and the time-delay, $\tau_0/\sigma$. The presence of the temporal superposition enhances mutual information harvesting in the $|\tau_0/\sigma| \ll 1$ and negative energy gap regions, compared to detectors interacting with the field in classical spacetime regions. Similarly in Fig. 13, we have plotted the mutual information as a function of the entangling
distance, $L_m/\sigma$, and the time-delay. Only at large separations does the presence of the superposition enhance mutual information harvesting, over and against detectors on classical spacetime trajectories.

We have thus far considered scenarios where the control system is measured in the same state as it was initially prepared in. More generally, we perform the measurement in some arbitrary superposition basis, given by

$$|\chi\rangle = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} e^{-\phi_i} |i\rangle$$ (67)

where $\phi_i \in \mathbb{R}$, so that the final detector-field state is given by

$$|\Psi\rangle_{FD} = \frac{1}{N} \sum_{i=1}^{N} e^{i\phi_i} |\hat{U}_i|0\rangle |\gamma\rangle_B$$ (68)

and $\hat{U}_i$ is defined in Eq. (6).

For detectors in a superposition of two trajectories, the local transition probability and entangling term are given respectively by

$$P_D = \frac{\lambda^2 \sqrt{\pi \sigma^2}}{2} \left\{ \int \frac{ds}{\sinh^2(\beta s/2 - i\epsilon)} \right.$$

$$\left. + \cos(\phi_1 - \phi_2) \int \frac{ds}{\sinh^2(\beta s/2 - i\epsilon - (\beta L_m/2)^2)} \right\}$$ (69)

and

$$M = \lambda^2 \sqrt{\pi \sigma^2} \int_0^{\infty} ds \frac{e^{-s^2/4\sigma^2} (1 + \cos(\phi_1 - \phi_2))}{\sinh^2(\beta s/2 - i\epsilon) - (\beta L_m/2)^2}.$$ (70)

For brevity, we consider only the entanglement harvested by the detectors in this scenario. We notice immediately that if the final state of the control is orthogonal to the initial state, namely

$$|\chi\rangle = \frac{1}{\sqrt{2}} (|1\rangle - |2\rangle)$$ (71)

then $M = 0$ (i.e $\phi_1 - \phi_2 = \pi$), implying that it is impossible for the detectors to harvest entanglement.

In Fig. 14, we have plotted the concurrence between the two detectors as a function of the relative phase $\phi_1 - \phi_2$, and the radial coordinate, $r/\sigma$. We find that the concurrence is maximised when the control is measured in its initial state; that is, $\phi_1 - \phi_2 = 2\pi n$ where $n \in \mathbb{Z}$. As noted above, for a relative phase of $\pi$ (and values near this) between the control states, the concurrence vanishes completely.

VI. FINITE-TEMPERATURE ENTANGLEMENT HARVESTING IN MINKOWSKI SPACETIME

We now consider entanglement harvesting with finite-temperature fields in flat Minkowski spacetime. It is well-known that the single detectors on classical trajectories respond identically to Gibbons-Hawking radiation in the de Sitter conformal vacuum and thermal radiation in flat Minkowski spacetime. It was shown by Ver Steeg and Menicucci [9] that by introducing a second detector, these spacetimes can in principle be distinguished through the amount of harvested entan-


A. Spatial superpositions

The expressions for the local excitation probabilities, $P_D$, and the entangling term, $\mathcal{M}$, take the same form as those used for the de Sitter case, apart from the relevant Wightman functions. In Fig. 15, we have plotted the concurrence between the detectors as a function of the superposition distance, $L_s$, for different values of $\kappa \sigma$. One immediately notices the similarity between Fig. 15 and Fig. 1 whereby the concurrence increases in an asymptotic fashion as the superposition distance increases. Again, we obtain this result because at large superposition distances, the interference terms in the detector transition probabilities, $P_{1,D}$, vanish (i.e., the correlations between the superposed interaction regions of each detector decay with the superposition distance). Note especially that the contributions to $P_D$ from the individual trajectories of the superposition are independent of $L_s$, unlike the interference terms. This further emphasises the phenomenon of enhanced entanglement harvesting in the presence of superposition, and we conjecture that it is also true for more generic scenarios.

In Fig. 16, we have plotted the concurrence harvested by the detectors as a function of the energy gap, $\Omega \sigma$. We find that entanglement harvesting is generally inhibited for negative energy gaps, corresponding to detectors initialised in their excited state. This closely resembles the behaviour of the concurrence for detectors in de Sitter spacetime. Although the entangling term is symmetric with respect to $\Omega$, the transition probability grows unbounded as $\Omega \sigma$ becomes more negative (see the inset of Fig. 16). Again, we notice that the concurrence between the detectors is amplified for detectors in spatial superpositions, and most notably so for small energy gaps, $|\Omega \sigma| \ll 1$.

In Fig. 17, we have plotted the concurrence as a function of $\kappa \sigma$, comparing detectors travelling on classical trajectories with those in quantum-controlled spatial superpositions. For sufficiently small superposition distances, the concurrence decays with increasing temperature, as the noise induced by

In the following, we consider two UdW detectors, each prepared in a superposition of static trajectories separated by the superposition distance $L_s$, with the $i$th trajectories each separated by the entangling distance, $L_m$. The Wightman functions evaluated along the individual trajectories are identical to the de Sitter case, which means that single detectors on classical trajectories respond identically to the field in both these spacetimes (with the identification $\beta \leftrightarrow \kappa$). However the non-local Wightman functions take a slightly different form, and are given by [43]

\begin{align}
\mathcal{W}_{\text{th},i}(s) &= \frac{\kappa \left( \coth \frac{s}{2} (L_s - s') + \coth \frac{s}{2} (L_s + s') \right)}{16\pi^2 L_s} \\
\mathcal{W}_{\text{th},m}(s) &= \frac{\kappa \left( \coth \frac{s}{2} (L_m - s') + \coth \frac{s}{2} (L_m + s') \right)}{16\pi^2 L_m}
\end{align}

where we have defined $s' = s - i\epsilon$.

\[\text{FIG. 14.} \quad \text{Concurrence, } C_{AB}/\lambda^2, \text{ between the detectors, as a function of the relative phase } \varphi_1 - \varphi_2 \text{ and the radial coordinate, } r/\sigma. \text{ We have used } l/\sigma = 0.2, \Omega/\sigma = 0.02, \theta_m = \pi/6 \text{ and } \theta_i = \pi/2. \text{ The white line shows the contour of zero concurrence.}\]

\[\text{FIG. 15.} \quad \text{Concurrence, } C_{AB}/\lambda^2, \text{ harvested by the detectors, as a function of } L_s/\sigma \text{ for } L_m/\sigma = 1.25, 1.5, 1.75, 2, 2.25 \text{ (from top to bottom). The dashed lines represent the concurrence between detectors on classical trajectories. We have used the parameters } l/\sigma = 0.2, \Omega/\sigma = 0.2.\]

\[\text{FIG. 16.} \quad \text{Concurrence, } C_{AB}/\lambda^2, \text{ for different values of } \kappa \sigma. \text{ One immediately notices the similarity between Fig. 15 and Fig. 1 whereby the concurrence increases in an asymptotic fashion as the superposition distance increases. Again, we obtain this result because at large superposition distances, the interference terms in the detector transition probabilities, } P_{1,D}, \text{ vanish (i.e., the correlations between the superposed interaction regions of each detector decay with the superposition distance). Note especially that the contributions to } P_D \text{ from the individual trajectories of the superposition are independent of } L_s, \text{ unlike the interference terms. This further emphasises the phenomenon of enhanced entanglement harvesting in the presence of superposition, and we conjecture that it is also true for more generic scenarios.}\]
FIG. 16. Concurrence, $C_{AB}$, as a function of the energy gap, $\Omega \sigma$, for $L_s/\sigma = 0, 1, 2, 5$ (top to bottom). The inset shows the local noise terms against the entangling term, $M$ (dashed line). The other parameters we have used are $L_m/\sigma = 1$ and $\kappa \sigma = 0.2$.

FIG. 17. Concurrence, $C_{AB}$ as a function of $\kappa \sigma$ where we have plotted $L_s/\sigma = 0, 1, 2, 5$ top to bottom (orange to blue lines). The inset compares the local noise term against the entangling term (dashed line) for $L_s/\sigma = 0, 2, 5$ top to bottom (blue and orange plots, respectively). The other parameters we have used are $L_m/\sigma = 1, \Omega \sigma = 0.1$.

The presence of thermal particles dominates over the correlations between the two detectors. Nevertheless, there is still an advantage of using detectors in superposition, due to the relative suppression of the local noise terms. Quite remarkably, we observe that for sufficiently large superposition distances ($L_s/\sigma \gg 1$), and temperatures, ($\kappa \sigma \gg 1$), the concurrence reaches a global minima and grows with increasing temperature.

To understand this, note that although $M$ grows with increasing temperature, and for detectors on classical trajectories, it is eventually overtaken by the local noise term, $\sqrt{P_A P_B}$ for small superposition distances. However for sufficiently large superposition distances, the interference terms die out, and the transition probability approaches half its value obtained for a classical trajectory. This has the effect of suppressing the noise terms below the entangling term, even for large values of $\kappa \sigma$.

These terms grow linearly for large $\kappa \sigma$, which leads us to the counter-intuitive conclusion, that (to second-order perturbation theory) for detectors in sufficiently distant spatial superpositions, the amount of entanglement harvested from the field grows with increasing temperature. This result demonstrates (to our knowledge) the first violation of the no-go theorem derived by Simidzija in [19], where it was shown that for a thermal field state, (a) the amount of entanglement harvested by identical UdW detectors decreases with the temperature, and (b) that there always exists a threshold temperature above which identical UdW detectors cannot harvest any entanglement. This theorem was derived in the regime of perturbation theory for detectors with arbitrary switching functions and spatial profiles, and applies regardless of the dimension of the spacetime or the mass of the field [19]. Our results reveal that the theorem relies on an additional tacit assumption that
trajectories of the detectors are classical.

The density plots of Fig. 18 further illustrate the difference between detectors on superposed trajectories and those on classical paths. Clearly, the detectors in superposition can become entangled in a significantly broader region of the parameter space, and for sufficiently small energy gaps, $|\Omega\sigma| \ll 1$, the concurrence grows with the temperature of the field. Note especially that the detector transition probability $P_D$ (including the constituent 'local' and 'interference' contributions) behaves as we should expect with increasing temperature; that is, it increases monotonically with $\kappa\sigma$. In other words higher temperatures amplify excitations in the superposed detector, which would typically inhibit entanglement harvesting in a classical detector. However alongside the amplification of excitations with increasing superposition distance, $\mathcal{L}_s$. For sufficiently large $\mathcal{L}_s$, this suppression is such that $\sqrt{P_A P_B}$ is always smaller than $\mathcal{M}$, term even for higher and increasing temperatures.

Due to the unbounded growth in these quantities, perturbation theory will eventually break down. It would be interesting to see how higher-order terms in $\lambda$ modify the above results.

VII. MUTUAL INFORMATION HARVESTING IN THERMAL MINKOWSKI SPACETIME

We finally study the mutual information harvested by detectors in thermal Minkowski spacetime. The expressions for the $\mathcal{L}$ term are identical to those derived for the de Sitter scenario, with a replacement of the relevant Wightman functions.

Fig. 19 displays the mutual information harvested by the detectors at fixed entangling distance, as a function of $\Omega\sigma$. The mutual information behaves in a similar manner to that already observed for the detectors in de Sitter spacetime, decaying as for large negative energy gaps in an oscillatory manner.

In Fig. 20, we have plotted the mutual information as a function of $\kappa\sigma$. We find that the mutual information grows with the temperature of the field, which corroborates the result of [19], which also found a similar result. As expected from our previous results, the presence of the superposition suppresses the total amount of mutual information between the detectors, relative to the case where the detectors travel on classical trajectories. In the regime we have considered, both the entanglement and the mutual information grow unbounded with the temperature of the field.

![Fig. 19. Mutual information, $I(\rho_{AB})/\tilde{\lambda}^2$, harvested by the detectors as a function of the energy gap, $\Omega\sigma$, $\mathcal{L}_s/\sigma = 0.2, 2.5$ (corresponding to orange, blue respectively). The inset shows a larger range of negative energy gaps, for which the superposition enhances mutual information harvesting in certain regimes. The other parameters used are $\mathcal{L}_m/\sigma = 1$, $\kappa\sigma = 5$.](image)

In Fig. 21, we have plotted the mutual information harvested by the detectors as a function of the entangling distance, $\mathcal{L}_m/\sigma$, for different values of the superposition distance $\mathcal{L}_s/\sigma$. Notably, while the presence of the superposition generally suppresses the amount of mutual information between the detectors, $I(\rho_{AB})/\tilde{\lambda}^2$ is not merely a monotonic function of $\mathcal{L}_s/\sigma$, as shown in the inset in Fig. 21. For small values of $\mathcal{L}_s/\sigma$, $I(\rho_{AB})/\tilde{\lambda}^2$ decays in a similar fashion to that found for the de Sitter mutual information. However for intermediate values, we see a small increase in $I(\rho_{AB})/\tilde{\lambda}^2$, which explains why at small values of $\mathcal{L}_m/\sigma$, the mutual information harvested by detectors with $\mathcal{L}_s/\sigma = 2.5$ is larger than that harvested by the detectors with $\mathcal{L}_s/\sigma = 1.25$.

Finally, in Fig. 22, we have plotted the mutual information between the detectors as a function of the entangling distance and energy gap. We notice that for large negative energy gaps, the mutual information decays with $\mathcal{L}_m/\sigma$ but then revives to form a small peak, before decaying again to zero. This is a different regime to that shown in Fig. 21, which examined $I(\rho_{AB})/\tilde{\lambda}^2$ for detectors with positive energy gap. Notably, for large negative energy gaps and entangling distances, the mutual information is significantly enhanced.

VIII. CONCLUSION

It was already shown in [20–22], that the superposition of an UdW detector’s path enables it to probe the global fea-
FIG. 21. Mutual information, $I(\rho_{AB})/\tilde{\lambda}^2$, harvested by the detectors, with $L_s/\sigma = 0, 1.25, 2.5, 5$ (from black to orange). The other parameters we have used are $\Omega\sigma = 0.1, \kappa\sigma = 0.2$. The inset displays the mutual information as a function of the superposition distance, $L_s/\sigma$ for $L_m/\sigma = 3.75$ (and all other parameters equal).

FIG. 22. Mutual information, $I(\rho_{AB})/\tilde{\lambda}^2$, harvested by the detectors in a spatial superposition of trajectories as a function of $\Omega\sigma$ and $L_m/\sigma$. In this plot, we have used the parameter settings $l\sigma = 0.2$ and $L_s/\sigma = 2.5$.

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