Statistical Analysis of Multiple Antenna Strategies for Wireless Energy Transfer

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Abstract

Wireless Energy Transfer is emerging as a potential solution for powering small energy-efficient devices. We propose strategies that use multiple antennas at a power station, which wirelessly charges a large set of single-antenna devices. Proposed strategies operate without Channel State Information (CSI), we attain the distribution and main statistics of the harvested energy under Rician fading channels with sensitivity and saturation energy harvesting (EH) impairments. A switching antenna strategy, where single antenna with full power is transmitting at the time, guarantees the lowest variance in the harvested energy, thus providing the most predictable energy source, and it is particularly suitable for powering sensor nodes with highly sensitive EH hardware operating under non-LOS (NLOS) conditions; while other WET schemes perform alike or better in terms of the average harvested energy. Under NLOS switching antennas is better, when LOS increases transmitting simultaneously with equal power in all antennas is best. Moreover, spatial correlation is not beneficial unless the power station transmits simultaneously through all antennas, raising a trade-off between average and variance of the harvested energy since both metrics increase with the spatial correlation. Moreover, the performance gap between CSI-free and CSI-based strategies decreases quickly as the number of devices increases.

I. INTRODUCTION

With the advent of the Internet of Things (IoT) era, there is an increasing interest in energy efficient technologies in order to prolong the battery life time of the devices. The recent trends
in energy harvesting (EH) techniques provide a fundamental efficient method that avoids the replacing or recharging batteries procedures, which may be costly, inconvenient or hazardous, e.g., in toxic environments, for sensors embedded in building structures or inside the human body [1]. Many types of EH schemes, according to the energy source, have been considered, based on solar, piezoelectric, wind, hydroelectric, and wireless radio frequency (RF) signals [2]. While harvesting energy from environmental sources is dependent on the presence of the corresponding energy source, RF-EH provides key benefits in terms of being wireless, readily available in the form of transmitted energy (TV/radio broadcasters, mobile base stations and handheld radios), low cost, and having small form factor implementation.

Three main transmit scenarios can be distinguished in RF-EH networks, namely Wireless Energy Transfer (WET) [3], Wireless Powered Communication Network (WPCN) [4] and Simultaneous Wireless Information and Power Transfer (SWIPT) [5]. In the first scenario a power transmitter transfers energy to EH receivers to charge their batteries, without any information exchange, while WPCN refers to those cases where the EH receiver uses the energy harvested in a first phase to transmit its information in a second phase. Finally, in the third scenario a hybrid transmitter is transferring wireless energy and information signals using the same waveform to multiple receivers. More details on each of these scenarios can be found in [6], along with a survey on energy beamforming (EB) techniques. In this work we focus on WET scenarios, that also could be seen as an element of WPCN systems [4], while readers can refer to [7] for a review and discussion on recent progress on SWIPT technologies.

A. Related Work

Many recent works have considered specifically WET and WPCN setups in different contexts and scenarios. An overview of the key networking structures and performance enhancing techniques to build an efficient WPCN is provided in [8], while authors also point out new and challenging research directions. A power beacon that constantly broadcasts wireless energy in a cellular network for RF-EH was proposed in [9]. These power beacons are deployed in conjunction with base stations to provide power coverage and signal coverage in the network, while the deployment of this hybrid network under an outage constraint on data links was designed using stochastic-geometry tools. In [10], a hybrid access point (AP) was proposed.

1This is because WPCN setups consist of WET, which is followed by a wireless information transfer phase.
where the AP broadcasts wireless power in the downlink followed by data transmission using the harvested energy in the uplink in a time-division duplex (TDD) manner. Also in TDD setups, works in [11]–[15] consider the transmission of separately short energy and information packets (stringent delay constraints) in ultra-reliable WPCN scenarios under different channel conditions, e.g., Rayleigh or Nakagami-m fading. Authors either analyze the performance of the information transmission phase [11], or optimize it by using power [12] and rate [13] control, or cooperative schemes under perfect [14] and imperfect [15] Channel State Information (CSI). Some scheduling strategies that allow a direct optimization of the energy efficiency of the network are also proposed in [16]. Additionally, an energy cooperation scheme that enables energy cooperation in battery-free wireless networks with WET is presented in [17].

Yet, WET requires shifts in the system architecture and in its resource allocation strategies for optimizing the energy supplying, thus, avoiding energy outages. In that regard, authors in [18] study the probability density function (PDF), the cumulative distribution function (CDF), and the average of the energy harvested from signals transmitted by multiple sources. Interestingly, such information allows to determine the best strategies when operating under different channel conditions. Additionally, multi-antenna EB, where the energy-bearing signals are weighted at the multiple transmit antennas before transmission, has been proposed very recently [19], [20]. The average throughput performance of EB in a WPCN, consisting in one hybrid AP with multiple antennas and a single-antenna user, is investigated in [19]. The impact of various parameters, such as the AP transmit power, the energy harvesting time, and the number of antennas on the system throughput is analyzed. In [20], authors propose an EB scheme that maximizes the weighted sum of the harvested energy and the information rate in multiple-input single-output (MISO) WPCN. They show that their proposed scheme achieves the highest performance compared to existing work. In practice, the benefits of EB in WET crucially depend on the available CSI at the transmitter. An efficient channel acquisition method for a point-to-point multiple-input multiple-output (MIMO) WET system is designed in [21] by exploiting the channel reciprocity. Authors provide useful insights on when channel training should be employed to improve the transferred energy. Meanwhile, the training design problem is studied in [22] for MISO WET systems in frequency-selective channels.
B. Contributions and Organization of the Paper

The problem of CSI acquisition in WET systems is critical and limits the practical significance of previous works. This is because WET systems are inherently energy-limited, and part of the harvested energy would need to be used for CSI acquiring purpose [23]. In fact, the required energy resources to that end cannot be neglected when there is a large number of antennas and/or if the estimation takes place at the EH side since it requires complex baseband signal processing. Even when previous problems could be addressed in some particular scenarios, there is still the problem of CSI acquisition in multi-user setups, specially in IoT use cases where the broadcast nature of wireless transmissions could be exploited for powering a massive number of devices simultaneously. In such cases, effective CSI-free strategies are of vital importance.

This paper addresses CSI-free WET with multiple transmit antennas, while assuming practical characteristics of EH hardware. The main contributions of this work can be listed as follows:

- We present and analyze several strategies for the use of multiple antennas at a dedicated power station that powers a set of RF-EH devices without any CSI. The performance analysis considers the harvested energy at the receiver, and comparisons with ideal CSI-based schemes are carried out;
- We attain the distribution and main statistics, e.g., mean and variance, of the harvested energy in correlated Rician fading channels under the operation of each of the WET schemes and ideal EH operation. These results are extended to more practical scenarios where sensitivity and saturation EH impairments come to play. The Rician fading assumption is general enough to include a class of channels, ranging from a fully random Rayleigh fading channel without line of sight (LOS) to a fully deterministic LOS channel, by varying the Rician factor $\kappa$;
- We found that switching antennas such that only one antenna with full power is transmitting at a time, guarantees the lowest variance in the harvested energy, thus providing the most predictable energy source, and it is particularly suitable for powering sensor nodes with highly sensitive EH hardwares and operating under non LOS (NLOS) conditions; while the other schemes perform better (or at least equal) in terms of the average harvested energy.
- While under NLOS it is better switching antennas, under some LOS it is better transmitting simultaneously with equal power by all antennas. Additionally, an increase in the spatial correlation is generally prejudicial, except when transmitting simultaneously with equal
power by all antennas, for which there is a trade-off between average and variance of the harvested energy since both metrics increase with the spatial correlation;

- Numerical results validate our analytical findings and demonstrate the suitability of the CSI-free over the CSI-based strategies as the number of devices increases.

Next, Section II presents the system model, while Section III introduces the WET strategies under study. Their performance under Rician fading is investigated in Section IV, while Section V presents numerical results. Finally, Section VI concludes the paper.

Notation: Boldface lowercase letters denote vectors, while boldface uppercase letters denote matrices. For instance, \( \mathbf{x} = \{x_i\} \), where \( x_i \) is the \( i \)-th element of vector \( \mathbf{x} \); while \( \mathbf{X} = \{x_{i,j}\} \), where \( x_{i,j} \) is the \( i \)-th row \( j \)-th column element of \( \mathbf{X} \). By \( \mathbf{I} \) we denote the identity matrix, and by \( \mathbf{1} \) we denote a vector of ones. The superscript \((\cdot)^T\) denotes the transpose, \( \text{det}(\cdot) \) denotes the determinant, and by \( \text{Diag}[x_1, x_2, \cdots] \) we denote the diagonal matrix with elements \( x_1, x_2, \cdots \).

The \( \ell_p \)-norm of vector \( \mathbf{x} \) is \( ||\mathbf{x}||_p = \left( \sum_i |x_i|^p \right)^{1/p} \) [24, Eq.(3.2.13)]. \( \mathbb{C} \) denotes the set of complex numbers and \( i = \sqrt{-1} \) is the imaginary unit. Meanwhile, \( x^* \) is the conjugate value of \( x \), and \( |\cdot| \) is the absolute operation, or cardinality of the set according to the case. \( \mathbb{E}[\cdot] \) and \( \text{VAR}[\cdot] \) denote expectation and variance, respectively, while \( \mathbb{P}[A] \) is the probability of event \( A \). \( \mathbf{y} \sim \mathcal{N}(\mu, \mathbf{R}) \) is a Gaussian random vector with \( \mathbb{E}[\mathbf{y}] = \mu \) and covariance \( \mathbf{R} \), \( W \sim \text{RIC}(\kappa) \) is a Rician random variable (RV) with factor \( \kappa \) [25, Ch.2], while \( V \sim \Gamma(m, a/m) \) is a gamma random variable with PDF and CDF given by

\[
f_V(v) = \frac{(m/a)^m}{\Gamma(m)} v^{m-1} e^{-mv/a}, \quad F_V(v) = 1 - \frac{\Gamma(m, mv/a)}{\Gamma(m)}, \quad v \geq 0, (1)
\]

where \( \Gamma(p) \) and \( \Gamma(p, x) \) is the complete and incomplete gamma function, respectively. Additionally, \( Z \sim \chi^2(\varphi, \psi) \) is the non-central chi-squared RV with \( \varphi \) degrees of freedom and parameter \( \psi \), thus, its PDF and CDF are given by [25, Eqs.(2-1-118) and (2-1-121)]

\[
f_Z(z) = \frac{1}{2} e^{-(z+\psi)/2} \left( \frac{z}{\psi} \right)^{\varphi/2-1} I_{\varphi/2-1}\left(\sqrt{\psi z}\right), \quad F_Z(z) = 1 - Q_{\varphi/2}\left(\sqrt{\psi}, \sqrt{z}\right), \quad z \geq 0, (2)
\]

where \( I_n \) is the \( n \)-th order modified Bessel function of the first kind [25, Eq.(2-1-120)] and \( Q \) is the Marcum Q-function [25, Eq.(2-1-122)]. According to [25, Eq.(2-1-125)] we have that

\[
\mathbb{E}[Z] = \varphi + \psi, \quad \text{VAR}[Z] = 2(\varphi + 2\psi). \quad (3)
\]

Table I summarizes the main symbols used throughout this paper.
TABLE I
Main Symbols

| Symbol | Description |
|--------|-------------|
| $T, S$ | Dedicated power station and set of sensor nodes |
| $S_j$ | $j$–th sensor node ($j$–th element of $S$) |
| $\alpha, \beta$ | Real and imaginary part of $h$, respectively |
| $w_k$ | $k$–th precoding vector for the $k$–th energy beam |
| $\vartheta_j$ | Path loss of the link $T \rightarrow S_j$ times the overall transmit power of $T$ |
| $\xi_j^1, \xi_j^2$ | Incident RF energy and harvested energy at $S_j$ |
| $\xi^0$ | Harvested energy under the ideal EH linear model |
| $\mu$ | Mean vector of real and imaginary parts of $h$ |
| $\rho$ | Spatial correlation coefficient |
| $\lambda$ | Eigenvalue of $\sigma^{-2}R$ |
| $\mathbf{B}$ | Matrix with orthogonalized eigenvectors of $\sigma^{-2}R$ |
| $\xi_{th}$ | Minimum amount of RF input power for which $S$ can operate |
| $M$ | Number of antennas at $T$ |
| $\mathbf{h}$ | Complex fading channel vector |
| $l$ | Number of energy beams transmitted by $T$ |
| $\varpi_1, \varpi_2$ | Sensitivity and saturation levels of the EH hardware |
| $g$ | Function that describes the relation between harvested energy and RF input power |
| $\eta$ | Energy conversion efficiency |
| $\kappa$ | Rician fading factor |
| $\mathbf{R}$ | Covariance matrix of elements of $\alpha$ and $\beta$ |
| $\sigma^2$ | Variance of the elements of $\alpha$ and $\beta$ |
| $\mathbf{A}$ | Diagonal matrix containing the values of $\lambda$ |
| $\varphi, \psi$ | Non central chi-squared distribution parameters |
| $\kappa_1^*, \kappa_2^*$ | $\kappa$ that maximizes the variance of the harvested energy as columns under the AA scheme and, SA or AA – CSI schemes, respectively |

II. System Model

Consider the scenario in Fig. 1 in which a dedicated power station $T$ equipped with $M$ antennas, powers wirelessly a set $S = \{S_1, S_2, \ldots, S_{|S|}\}$ of single-antenna sensor nodes located nearby. Quasi-static channels are assumed, where the fading process is considered to be constant over the transmission of a block and independent and identically distributed (i.i.d) from block to block. The fading channel coefficient between the $i$–th antenna of $T$ and the $j$–th sensor node $S_j$ is denoted as $h_{i,j} \in \mathbb{C}$, while $\mathbf{h}_j \in \mathbb{C}^{M \times 1}$ is a vector with the channel coefficients from the power station antennas to $S_j$. 

![Fig. 1. System model: dedicated power station $T$ equipped with $M$ antennas, powers wirelessly a set $S$ of single-antenna sensor nodes located nearby.](image-url)
In general, during WET $T$ may transmit with up to $l \leq M$ energy beams to broadcast energy to all sensors in $S$. Then, the received signal at $S_j$ is given by

$$y_j^{(t)} = \sqrt{\varrho_j} h_j^T \sum_{k=1}^{l} w_k x_k^{(t)} + n_j^{(t)},$$

(4)

where $t$ is the block index, $w_k \in \mathbb{C}^{M \times 1}$ denotes the precoding vector for generating the $k$-th energy beam, and $x_k^{(t)}$ is its energy-carrying signal. It is assumed that $x_k$’s are i.i.d RVs with zero mean and unit variance. Without loss of generality we set $\sum_{k=1}^{l} ||w_k||_2^2 = 1$, while $\varrho_j$ accounts for the path loss of the link $T \rightarrow S_j$ times the overall transmit power of $T$. Finally, $n_j$ is the Additive White Gaussian Noise (AWGN) at $S_j$. Then, by considering negligible the noise energy, the incident RF power at the $j$-th EH receiver is given by

$$\xi_{rf}^j = \varrho_j \sum_{k=1}^{l} |h_j^T w_k|^2.$$

(5)

Now, the harvested energy $\xi_j$, can be written as a function of $\xi_{rf}^j$ as $\xi_j = g(\xi_{rf}^j)$ where $g : \mathbb{R} \rightarrow \mathbb{R}$ is a non-decreasing function of its argument. In general $g$ is nonlinear and analytical analyses are cumbersome, but starting from the linear model the accuracy can be significantly improved by considering three main factors that limit strongly the performance of a WET receiver [12], [26]–[28]: (i) its sensitivity $\varpi_1$, which is the minimum RF input power required for energy harvesting; (ii) its saturation level $\varpi_2$, which is the RF input power for which the diode starts working in the breakdown region, and from that point onwards the output DC power keeps practically constant; and (iii) the energy efficiency $\eta \in [0, 1]$ in the interval $\varpi_1 \leq \xi_{rf}^j \leq \varpi_2$, which we assume as constant. Therefore, we can write $\xi_j$ as

$$\xi_j = g(\xi_{rf}^j) = \begin{cases} 
0, & \xi_{rf}^j < \varpi_1 \\
\eta \xi_{rf}^j, & \varpi_1 \leq \xi_{rf}^j < \varpi_2 \\
\eta \varpi_2, & \xi_{rf}^j \geq \varpi_2 \end{cases}. $$

(6)

Notice that the linear model assumed in most of the literature does not take into account the sensitivity and saturation phenomena, which is equivalent to operate with $\varpi_1 = 0$ and $\varpi_2 \rightarrow \infty$. Thus, taking these impairments into consideration in the scenarios under discussion is an important contribution from a practical perspective.

\footnote{We use the terms energy and power indistinctly, which can be interpreted as if block duration is normalized.}
III. WET STRATEGIES

First, in Subsection III-A we characterize the performance of WET for three different strategies at $T$ without any CSI, while two alternative strategies that require full CSI are presented as benchmarks in Subsection III-B.

A. WET Strategies without CSI

Since no CSI is available and $w_k$ cannot depend on the channel coefficients, $T$ does not form energy beams to reach efficiently each of the $S_j$. Therefore, for these kind of strategies it is only necessary focusing on the performance of an arbitrary user $S_j$, while also setting $l = 1$.

1) One Antenna (OA): Is the simplest strategy because only one out of $M$ antennas is used for powering the devices, thus, transmitting with full power. Then, using (5) we obtain

$$\xi_{jOA} = g(\varrho_j |h_{i,j}|^2),$$

(7)

where $i = 1$ or 2 or $\cdots$ or $M$. Notice that in this case $w_1$ is a vector of zeros with entry 1 in the $i$–th element. There is no difference whether $T$ is equipped with only one or several antennas when operating with the OA strategy.

2) All Antennas at Once (AA): The OA strategy does not exploit multiple antennas, thus, it does not take advantage of that degree of freedom. One obvious and simple alternative is transmitting with all antennas but with reduced power at each, $w_1 = (1/\sqrt{M})1_{M\times1}$, thus

$$\xi_{jAA} = g\left(\frac{\varrho_j}{M} \sum_{i=1}^{M} |h_{i,j}|^2 \right).$$

(8)

The OA and AA schemes are the extreme cases of a more general strategy where $K$ out of $M$ antennas are selected to power the sensors. As a consequence, the $K$-out-$M$’s performance is limited by that of the OA and AA strategies.

3) Switching Antennas (SA): Instead of transmitting with all antennas at once, $T$ may transmit with full power by one antenna at a time such that all antennas are used during a block. Assuming equal-time allocation for each antenna, the system is equivalent to that in which each sub-block duration is $1/M$ of the total block duration, and the total harvested energy accounts for the sum of that of the $M$ sub-blocks. That is

$$\xi_{jSA} = \frac{1}{M} \sum_{i=1}^{M} g(\varrho_j |h_{i,j}|^2).$$

(9)

Notice that any other allocation of transmit powers is not advisable since $T$ does not know how the channels are behaving.
Note that \(w_1\) in each sub-block is given as in the OA strategy, but the chosen \(i\) is different in each sub-block.

**B. Benchmark WET Strategies**

For the sake of describing some benchmark strategies, herein we consider that full CSI is available at \(T\). Sensors \(S\) could use a “small” amount of energy \(4\) to send some pilot symbols to \(T\) in order to acquire the CSI. This requires reciprocal channels, thus, \(T\) should be listening at the time of the transmission. Otherwise, \(T\) has to send the pilots and wait for a feedback from the sensor(s) informing the CSI. Whatever the case, it seems unsuitable in a setup where multiple energy constrained sensors require the powering service from \(T\), specially if we also consider the multiple access problem. Therefore, we present the following CSI-based strategies only as benchmarks for those presented in the previous subsection. We assume that \(T\) knows also the EH hardware characteristics, e.g., \(\varpi_1, \varpi_2, \eta\), of the EH sensors and the goal is to maximize the overall energy harvested by \(S\).

1) **Best Antenna** (OA – CSI): This strategy is the counterpart for the previous OA scheme when full CSI is available. In this case, the antenna that provides the greatest amount of overall harvested energy is selected out of the overall set. Therefore, \(l = 1\) and

\[
\sum_{j=1}^{\lvert S \rvert} \xi_{j\text{OA-CSI}} = \max_{i=1,\ldots,M} \sum_{j=1}^{\lvert S \rvert} g\left(\varrho_j |h_{i,j}|^2\right),
\]

\(10\) since this time \(w_1\) is a vector of zeros with entry 1 in the selected antenna index. Of course, in a multi-user system where the nodes benefit from the WET phase simultaneously, the best antenna is usually not the same for all users. Different from \(10\), another possible implementation may be that in which some users are optimized first and then the others, but in all these scenarios the complexity scales quickly with the number of users while reducing the overall performance.

2) **Best Transmit Beamforming** (AA – CSI): This strategy is the counterpart for the previous AA scheme when full CSI is available. In this case, instead of transmitting with the same power over each antenna, \(T\) precompensates through \(\{w_k\}\) for the channel and EH hardware effects before transmission such that the overall harvested energy at \(S\) is maximized. Hence,

\[
\sum_{j=1}^{\lvert S \rvert} \xi_{j\text{AA-CSI}} = \max_{\{w_k\}} \sum_{j=1}^{\lvert S \rvert} g\left(\varrho_j \sum_{k=1}^{M} |h_j^T w_k|^2\right),
\]

\(11\)

\(^4\)Coming from a short phase where \(S\) are first powered by one of the previous CSI-free strategies, or even from some residual energy after previous rounds.
where \( l \) is set to \( M \) since that is the maximum possible number of energy beams. In case the system performance maximizes with an smaller \( l \), which may be the case when \( |S| < M \), then, some of the optimum beamformers are all-zeros vectors. Notice that the OA – CSI scheme is less sensitive to CSI imperfections than the AA – CSI. This is because the former only relies on the power gain of the channel, while the latter requires the full characterization, envelope and phase, of the channel coefficients.

C. Comparison of the WET Strategies for \(|S| = 1\) and under Ideal EH Linear Model

Some useful insights come from setting \(|S| = 1\) and using the ideal EH linear model such that \( \varpi_1 = 0 \) and \( \varpi_2 \rightarrow \infty \). In this case, we denote \( \xi^0 \) as the harvested energy at the unique sensor node while we avoid using the subindex \( j \), then, (7), (8), (9), (10) and (11) can be rewritten as

\[
\xi^0_{OA} = \eta \theta |h_i|^2, \tag{12}
\]

\[
\xi^0_{AA} = \frac{\eta \theta}{M} \left| \sum_{i=1}^{M} h_i \right|^2, \tag{13}
\]

\[
\xi^0_{SA} = \frac{\eta \theta}{M} \sum_{i=1}^{M} |h_i|^2, \tag{14}
\]

\[
\xi^0_{OA-CSI} = \eta \theta \max_{i=1, \ldots, M} |h_i|^2, \tag{15}
\]

\[
\xi^0_{AA-CSI} = \eta \theta \max_{w_1} |h^T w_1|^2 = \eta \theta \sum_{i=1}^{M} |h_i|^2, \tag{16}
\]

where \((a)\) comes from setting \( w_1 = h^*/||h||_2 \) which is the optimum precoding vector for Maximum Ratio Transmission (MRT) [29] in a MISO system.

**Theorem 1.** The following relations are satisfied:

\[
\xi^0_{OA} \leq \xi^0_{OA-CSI} \leq \xi^0_{AA-CSI} = M \xi^0_{SA}, \tag{17}
\]

\[
\xi^0_{AA} \leq \xi^0_{AA-CSI} = M \xi^0_{SA}. \tag{18}
\]

**Proof.** According to (12), (15), (16) and (14) we have that

\[
\xi^0_{OA} = \eta \theta |h_i|^2 \leq \eta \theta \max_{i=1, \ldots, M} |h_i|^2 = \xi^0_{OA-CSI} \leq \eta \theta \sum_{i=1}^{M} |h_i|^2 = \xi^0_{AA-CSI} = M \xi^0_{SA}, \tag{19}
\]
where the equality in (a) is only when \( M = 1 \). Thus, (17) is satisfied. Now, for the second part of the proof we proceed from (13), (16) and (14) as follows.

\[
\frac{\xi_{AA}^0}{\eta \rho} = \frac{1}{M} \left( \sum_{i=1}^{M} |h_i|^2 \right)^{2/3} \leq \frac{1}{M} \left( \sum_{i=1}^{M} |h_i|^2 \right)^{2/3} \leq \left( \sum_{i=1}^{M} |h_i|^2 \right)^{2/3} \leq \frac{M \xi_{AA}^{0 - \text{CSI}}}{\eta \rho} = \frac{M \xi_{AA}^{0 - \text{CSI}}}{\eta \rho},
\]

(20)

where (a) comes from applying the triangular inequality and generalizing as shown in [30, Section 1.1.7], (b) follows from using each time the \( \ell_p \)-norm notation, while (c) from the inequality between the arithmetic and quadratic mean.

\[\square\]

IV. ANALYSIS UNDER RICIAN FADING

Herein we assume that channels undergo Rician fading, which is a very general assumption that allows modeling a wide variety of channels by tuning the Rician factor \( \kappa \geq 0 \), e.g., when \( \kappa = 0 \) the channel envelope is Rayleigh distributed, while when \( \kappa \rightarrow \infty \) there is a fully deterministic LOS channel. Since this work deals mainly with CSI-free WET schemes, and for such scenarios the characterization of one sensor’s performance is similar to that of the others, we focus our attention to the performance of a generic node, thus we avoid using subindex \( j \). Additionally, the performance gap between the CSI-free and CSI-based WET schemes is maximum for \(|S| = 1 \) and analyzing such scenario when CSI is available allows getting analytical expressions along with some useful insights. Previous assumptions imply that the envelope distribution of \( h_i \) is Rician distributed with factor \( \kappa \), e.g., \(|h_i| \sim \text{RIC}(\kappa)\). Specifically, the channels are Gaussian with independent real and imaginary parts, \( h = \alpha + i\beta \) with \( \alpha, \beta \sim N\left(\frac{1}{\sqrt{2}} \mu, R\right) \), where \( R \) and \( \mu \) are respectively the covariance matrix and mean vector of \( \alpha, \beta \) [31].

For performance evaluation it is enough considering \( \mu = \mu [1]_{M \times 1} \), e.g., equal mean over all the fading paths, and uniformly spatial correlated fading, such that the antenna elements are correlated between each other with coefficient \( \rho \). Thus,

\[
R = \sigma^2 \begin{bmatrix}
1 & \rho & \ldots & \rho \\
\rho & 1 & \ldots & \rho \\
\vdots & \vdots & \ddots & \vdots \\
\rho & \rho & \ldots & 1
\end{bmatrix}_{M \times M},
\]

(21)

\(^5\)Spatial correlation occurs due to insufficient spacing between antenna elements, small angle spread, existence of few dominant scatterers, and the antenna geometry. The general concept of spatial correlation is usually linked only to the specific positive spatial correlation, even when the negative correlation is also physically possible, mainly due to the use of decoupling networks and antenna geometry effects. On the other hand, notice that correlation between the coefficients \( h_i \) is highly probable in the kind of systems we are investigating here because of the short range transmissions [32]. Even though, by setting \( \rho = 0 \) we are also able of modeling completely independent fading realizations over all the antennas.
where $\sigma^2$ is the variance of each $\alpha_i, \beta_i$. In order to guarantee that $R$ is positive definite and consequently a viable covariance matrix, $\rho$ is lower bounded by $-\frac{1}{M-1}$, thus $-\frac{1}{M-1} \leq \rho \leq 1$. Additionally, factor $\kappa$ is connected to $\mu$ and $\sigma^2$ as

$$\kappa = \frac{\mu^2}{2\sigma^2},$$

and normalizing the channel power gain as $\mu^2 + 2\sigma^2 = 1$, e.g., $\mathbb{E}[|h_i|^2] = 1$, we attain

$$\sigma^2 = \frac{1}{2(1 + \kappa)}; \quad \mu^2 = \frac{\kappa}{1 + \kappa}. \quad (23)$$

A. Distribution of the Harvested Energy under Ideal EH Linear Model

Now, we proceed to characterize the distribution of the harvested energy when using each of the WET strategies analyzed in the previous section under the ideal EH linear model.

1) OA: Obviously, the spatial correlation has no impact on the performance of the OA scheme since under that scheme $T$ selects only one antenna without using any information related with the other antennas. From (12) we proceed as follows

$$\xi^0_{OA} = \eta \mathbb{E}[|h_i|^2] = \eta \theta(\alpha^2 + \beta^2) \sim \eta \theta \sigma^2 \chi^2(2, \frac{\mu^2}{\sigma^2}) \sim \eta \theta \sigma^2 \chi^2(2, 2\kappa), \quad (24)$$

where (a) comes from normalizing the variance of $\alpha$ and $\beta$ such that $\hat{\alpha}, \hat{\beta} \sim \mathcal{N}(\frac{1}{\sqrt{2} \sigma}, 1)$, (b) comes from the direct definition of a non-central chi-squared RV [25, Ch.2], and (c) follows after using (22) and (23).

2) AA: Now we focus on the performance of the AA scheme. Based on (13) we have that

$$\xi^0_{AA} = \frac{\eta \theta}{M} \sum_{i=1}^{M} \alpha_i + \sum_{i=1}^{M} \beta_i \sim \frac{\eta \theta}{M} |\hat{\alpha} + i \hat{\beta}|^2 \sim \eta \theta \sigma^2 (1 + (M - 1)\rho) |\theta + i \vartheta|^2 \sim \eta \theta \sigma^2 (1 + (M - 1)\rho) \chi^2(2, \frac{M}{1 + (M - 1)\rho} \frac{\mu^2}{\sigma^2}) \sim \eta \theta \sigma^2 \chi^2(2, 2\kappa)$$

where (a) comes from using $\hat{\alpha} = \sum_{i=1}^{M} \alpha_i$ and $\hat{\beta} = \sum_{i=1}^{M} \beta_i$ and notice that $\hat{\alpha}, \hat{\beta}$ are still Gaussian RVs [34] with mean $M \mathbb{E}[\alpha_i] = \frac{1}{\sqrt{2}} M \mu$ and variance $\sum_{i=1}^{M} \sum_{j=1}^{M} r_{i,j} = \sigma^2 (1 + (M - 1)\rho) M$ according to (21), thus, (b) follows after variance normalization such that $\theta, \vartheta \sim \mathcal{N}(\frac{\sqrt{M}}{\sqrt{2(1 + (M - 1)\rho)}} \mu, 1)$. Finally, (c) comes from using the definition of a non-central chi-squared RV [25, Ch.2], while (d) from using (22) and (23).
3) SA: Let us proceed with the analysis of the SA scheme. From (14), we write $\xi_{SA}^0$ as

$$
\xi_{SA}^0 = \frac{\eta M}{M} \sum_{i=1}^{M} (\alpha_i^2 + \beta_i^2) = \frac{\eta}{M} (\alpha^T \alpha + \beta^T \beta).
$$

(26)

Since $\alpha$ and $\beta$ are i.i.d between each other we focus on the product $\alpha^T \alpha$ and the results are also valid for $\beta^T \beta$. Let us define $v = R^{-1/2}(\alpha - \frac{1}{\sqrt{2}}\mu)$ which is distributed as $\mathcal{N}(0, I)$, then

$$
\alpha = R^{1/2}v + \frac{1}{\sqrt{2}}\mu
$$

$$
\alpha^T \alpha = (R^{1/2}v + \frac{1}{\sqrt{2}}\mu)^T (R^{1/2}v + \frac{1}{\sqrt{2}}\mu) = (v + R^{-1/2} \frac{1}{\sqrt{2}}\mu)^T R (v + R^{-1/2} \frac{1}{\sqrt{2}}\mu),
$$

(27)

where last step comes from simple algebraic transformation s. Notice that

$$
R = \sigma^2 B \Lambda B^T,
$$

(28)

which is the spectral decomposition of $R$ [35, Ch.21]. In (28), $\Lambda$ is a diagonal matrix containing the eigenvalues of $\sigma^{-2}R$ and $B$ is a matrix whose column vectors are the orthogonalized eigenvectors of $\sigma^{-2}R$. In order to find the eigenvalues, $\lambda$s, of $\sigma^{-2}R$, we require solving $\det(\sigma^{-2}R - \lambda I) = 0$ for $\lambda$, for which we proceed as follows

$$
det(\sigma^{-2}R - \lambda I)
$$

$$
= det \left( \begin{bmatrix} 1 - \lambda & \rho & \cdots & \rho \\ \rho & 1 - \lambda & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & 1 - \lambda \end{bmatrix} \right) = det \left( (1 - \lambda - \rho) \begin{bmatrix} 1 - \lambda - \rho \\ \rho \\ \vdots \\ \rho \end{bmatrix} \right)
$$

$$
\overset{(a)}{=} (1 - \lambda - \rho)^M \left( 1 + [\rho, \rho, \ldots, \rho] \begin{bmatrix} 1 - \lambda - \rho \\ \rho \\ \vdots \\ \rho \end{bmatrix} \right) \overset{(b)}{=} (1 - \lambda - \rho)^{M-1} (1 - \lambda + \rho(M-1)),
$$

(29)

where $(a)$ comes from using the Matrix determinant lemma [35], while $(b)$ follows after some algebraic manipulations. Now, two different eigenvalues are easily obtained by matching (29) with 0. These are $\lambda = 1 - \rho$ with multiplicity $M - 1$ and $\lambda = 1 + \rho(M-1)$ with multiplicity 1, thus

$$
\Lambda = \text{Diag} \left[ 1 - \rho, \cdots, 1 - \rho, 1 + (M-1)\rho \right].
$$

(30)

Meanwhile, the corresponding eigenvectors, $e$, satisfy $\sigma^{-2}Re = \lambda e$, thus,
• for $\lambda = 1 - \rho$ we have

$$\begin{bmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_M \end{bmatrix} = (1 - \rho) \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_M \end{bmatrix} \rightarrow \begin{bmatrix} e_1 + \rho \sum_{i=1}^{M-2} e_i \\ \rho e_1 + e_2 + \rho \sum_{i=1}^{M-3} e_i \\ \vdots \\ \rho \sum_{i=1}^{M-1} e_i + e_M \end{bmatrix} = \begin{bmatrix} (1 - \rho) e_1 \\ (1 - \rho) e_2 \\ \vdots \\ (1 - \rho) e_M \end{bmatrix}$$

$$\begin{bmatrix} \rho \sum_{i=1}^{M-1} e_i \\ \rho \sum_{i=1}^{M-2} e_i \\ \vdots \\ \rho \sum_{i=1}^{M} e_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \rightarrow \sum_{i=1}^{M} e_i = 0, \quad (31)$$

• and for $\lambda = 1 + \rho(M - 1)$ we have

$$\begin{bmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_M \end{bmatrix} = (1 + \rho(M - 1)) \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_M \end{bmatrix} \rightarrow \begin{bmatrix} e_1 + \rho \sum_{i=1}^{M-2} e_i \\ \rho e_1 + e_2 + \rho \sum_{i=1}^{M-3} e_i \\ \vdots \\ \rho \sum_{i=1}^{M-1} e_i + e_M \end{bmatrix} = \begin{bmatrix} (1 + \rho(M-1)) e_1 \\ (1 + \rho(M-1)) e_2 \\ \vdots \\ (1 + \rho(M-1)) e_M \end{bmatrix}$$

$$\begin{bmatrix} \rho \sum_{i=1}^{M-1} e_i \\ \rho \sum_{i=1}^{M-2} e_i \\ \vdots \\ \rho \sum_{i=1}^{M} e_i \end{bmatrix} = \begin{bmatrix} \rho e_1 e_2 \\ \rho e_1 e_3 \\ \vdots \\ \rho e_1 e_M \end{bmatrix} \rightarrow e_1 = e_2 = \cdots = e_M. \quad (32)$$

Thus, the eigenvectors associated to $\lambda = 1 - \rho$ satisfy (31), while the eigenvector associated to $\lambda = (1 + \rho(M - 1))$ satisfies (32). After orthogonalization by using the Gram-Schmidt process, and normalization, the resulting vectors still satisfy either (31) or (32) according to the case. For the latter, the resulting vector is $[1/\sqrt{M}, 1/\sqrt{M}, \ldots, 1/\sqrt{M}]$, therefore

$$B^T = \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,M} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ b_{M-1,M-1} & b_{M-1,M-2} & \cdots & b_{M-1,1} \end{bmatrix} \quad (33)$$

where $\sum_{j=1}^{M} b_{i,j} = 0$ for $j = 1, \cdots M - 1$.

Now, substituting (28) into (27) yields

$$\alpha^T \alpha = (v + (\sigma^2 \Lambda B^T) - \frac{1}{\sqrt{2} \mu}) \sigma^2 B^T (v + (\sigma^2 \Lambda B^T) - \frac{1}{\sqrt{2} \mu})$$

$$= (a) \left( B^T v + \frac{1}{\sqrt{2} \sigma} \Lambda B^T \frac{1}{\sqrt{2} \mu} \right) \sigma^2 \Lambda \left( B^T v + \frac{1}{\sqrt{2} \sigma} \Lambda B^T \frac{1}{\sqrt{2} \mu} \right)$$

$$= (b) \left( Q + \frac{1}{\sqrt{2} \sigma} \mu \right)^T \sigma^2 \Lambda \left( Q + \frac{1}{\sqrt{2} \sigma} \mu \right) \equiv \sigma^2 \sum_{i=1}^{M} \lambda_i \left( \frac{1}{\sqrt{2} \sigma} \mu \right)^2 \quad (34)$$

where $(a)$ comes after some algebraic transformations, $(b)$ follows from taking $Q = B^T v \sim \mathcal{N}(0, I)$ and $d = \Lambda^{-\frac{1}{2}} B^T \times 1_{M \times 1}$, for which using (33) and (30) yields

$$d = \left[ 0, 0, \cdots, 0, \frac{M}{1 + (M - 1) \rho} \right]^T \quad (35)$$
and finally (c) comes from setting \( \varsigma_i \sim \mathcal{N}(0, 1) \). Notice that \( \lambda_i \) is the \( i \)-th eigenvalue of \( \sigma^{-2} \mathbf{R} \).

Using (34) into (26) yields

\[
\xi_{SA}^0 = \frac{\eta \theta \sigma^2}{M} \sum_{i=1}^{M} \lambda_i \left[ \left( \varsigma_i + d_i \frac{1}{\sqrt{2} \sigma} \right)^2 + \left( \omega_i + d_i \frac{1}{\sqrt{2} \sigma} \right)^2 \right]
\]

\[
= \frac{\eta \theta \sigma^2}{M} \left[ (1 - \rho) \sum_{i=1}^{2(M-1)} \varsigma_i^2 + (1 + (M - 1)\rho) \sum_{i=1}^{2} \left( \omega_i + \sqrt{\frac{M}{2(1 + (M - 1)\rho)}} \frac{\mu}{\sigma} \right)^2 \right],
\]

\[
\xi_{SA}^0 \sim \frac{\eta \theta}{2M(1 + \kappa)} \left[ (1 - \rho) \chi^2(2(M - 1), 0) + (1 + (M - 1)\rho) \chi^2(2, \frac{2\kappa M}{2M - 1}) \right],
\]  

(36)

where (a) comes from defining \( \omega_i \sim \mathcal{N}(0, 1) \) to use when evaluating the term \( \beta^T \beta \) in (26), which has the same form given in (34), while (b) follows from using (35). In (b) we also regrouped similar terms, which allows writing (c) after using the direct definition of a non-central chi-square distribution [25, Ch.2] along with (22) and (23).

**Remark 1.** Therefore, under uniform spatial correlation, \( \xi_{SA}^0 \) is distributed as a linear combination of a chi-square RV and a non-central chi-square RV, with \( 2(M - 1) \) and \( 2 \) degrees of freedom, respectively. Unfortunately, it seems intractable finding a closed-form expression for the distribution of \( \xi_{SA}^0 \), except for

- \( \rho = 0 \), for which \( \xi_{SA}^0 \sim \frac{\eta \theta}{2M(1 + \kappa)} \left[ \chi^2(2(M - 1), 0) + \chi^2(2, 2M\kappa) \right] \sim \frac{\eta \theta}{2M(1 + \kappa)} \chi^2(2M, 2M\kappa) \), which can be easily verified by using the direct definition of a non-central chi-squared RV;

- \( \rho = -\frac{1}{M - 1} \), for which \( \xi_{SA}^0 \sim \frac{\eta \theta}{2(M - 1)(1 + \kappa)} \chi^2(2(M - 1), 0) + \frac{\eta \theta \kappa}{1 + \kappa} \). The latter term results from the fact that when \( \rho \to -\frac{1}{M - 1} \) the PDF of \( (1 + (M - 1)\rho) \chi^2(2, \frac{2\kappa M}{1 + (M - 1)\rho}) \) is 1 at \( 2M\kappa \), and 0 otherwise.

- \( \rho = 1 \), for which \( \xi_{SA}^0 \sim \frac{\eta \theta}{2(1 + \kappa)} \chi^2(2, 2\kappa) \).

**Remark 2.** For full positive correlation, e.g., \( \rho = 1 \), the performance of the SA scheme matches that of the OA. This is an expected result since even by switching antennas the energy harvested at S keeps the same.

4) **OA – CSI:** According to (15) finding the distribution of \( \xi_{OA–CSI}^0 \) is equivalent to the problem of finding the distribution of the Signal-to-Noise Ratio in a correlated Rician single-input multiple-output (SIMO) channel scenario, where the receiver with \( M \) antennas uses Selection
Combining (SC). Thus, we can directly use [37, Eq.(21)] to state

\[ F^{\xi_0}_{\text{OA-CSI}}(\eta x) = e^{-\frac{x}{\eta}} \int_0^\infty \left[ 1 - Q\left(\sqrt{\frac{2\rho t}{1-\rho}}, \sqrt{\frac{2(1+\kappa)x}{1-\rho}}\right) \right]^M e^{-t} I_0\left(2\sqrt{\frac{\kappa t}{\rho}}\right) dt, \] (37)

\[ f^{\xi_0}_{\text{OA-CSI}}(x) = \frac{d}{dx} F^{\xi_0}_{\text{OA-CSI}}(x) = \frac{M(1+\kappa)}{\eta g(1-\rho)} e^{-\frac{x}{\eta(1-\rho)}} \int_0^\infty e^{-\frac{t}{1-\rho}} I_0\left(2\sqrt{\frac{\kappa t}{\rho}}\right) \times \left[ 1 - Q\left(\sqrt{\frac{2\rho t}{1-\rho}}, \sqrt{\frac{2(1+\kappa)x}{1-\rho}}\right) \right]^{M-1} I_0\left(2\sqrt{\frac{\rho t(1+\kappa)x}{\eta(1-\rho)}}\right) dt. \] (38)

For two specific correlation setups it is even possible to get simplified expressions as follows

- For \( \rho = 0 \) the channel coefficients are i.i.d, thus,

\[ F^{\xi_0}_{\text{OA-CSI}}(\eta x) = P\left[\xi_0^{\text{OA-CSI}} < \eta x\right] = P\left[\max_{i=1,\ldots,M}\lvert h_i \rvert^2 < x\right] \overset{(a)}{=} P\left[\lvert h_i \rvert^2 < x\right]^M \equiv P\left[z < 2(1+\kappa)x\right]^M = F_Z(2(1+\kappa)x)^M, \] (39)

where in \( (a) \) we take advantage of the i.i.d property of the channel realizations when \( \rho = 0 \) and \( (b) \) comes from making \( z = 2(1+\kappa)|h_i|^2 \) where \( Z \sim \chi^2(2,2\kappa) \). Now,

\[ f^{\xi_0}_{\text{OA-CSI}}(x) = \frac{d}{dx} F^{\xi_0}_{\text{OA-CSI}}(x) = \frac{d}{dx} F_Z(2(1+\kappa)x/(\eta \rho))^M \overset{(a)}{=} \frac{2(1+\kappa)M}{\eta \rho} F_Z\left(\frac{2(1+\kappa)x}{\eta \rho}\right)^{M-1} f_Z\left(\frac{2(1+\kappa)x}{\eta \rho}\right), \] (40)

where \( (a) \) comes from using the chain rule. Notice that both, \( f_Z(x) \) and \( F_Z(x) \) are given in (2) with \( \varphi = 2 \) and \( \psi = 2\kappa \).

- For \( \rho = 1 \) the performance under the OA – CSI scheme matches that of the OA strategy since the fading behaves instantaneously equal over all the antennas.

Finally, notice that when \( \rho \) is the minimum, \( \rho = -\frac{1}{M-1} \), the performance gain of this scheme over that of the OA strategy should be the maximum.

5) AA – CSI: For the AA – CSI scheme the characterization is much easier since \( \xi^{0}_{\text{AA-CSI}} = M\xi^{0}_{\text{SA}} \) according to (20), thus, by using (36) we attain

\[ \xi^{0}_{\text{AA-CSI}} \sim \frac{\eta \Theta}{2(1+\kappa)} \left[ (1-\rho)\chi^2\left(2(M-1),0\right) + (1+(M-1)\rho)\chi^2\left(2,\frac{2\kappa M}{1+(M-1)\rho}\right) \right]. \] (41)

Notice that the analysis in Remark I can be extended to this case straightforwardly.
6) Comparisons and Remarks: Since the distribution of the harvested energy is related in all the cases with the non-central chi-squared distribution except for the OA − CSI scheme, we are able to find the mean and variance statistics according to (3). When ρ = 0 we can also provide an upper bound for the mean of ξ_{OA−CSI}^0 by using [38, The. 2.1] such that

\[ \mathbb{E}[\xi_{OA−CSI}^0] \leq \frac{1}{\eta \theta} \sum_{i=1}^{M} \mathbb{E}[|h_i|^2] + \sqrt{\frac{M-1}{M}} \sum_{i=1}^{M} \text{VAR}[|h_i|^2] + \sum_{i=1}^{M} \mathbb{E}[|h_i|^2] - \frac{1}{M} \left( \sum_{i=1}^{M} \mathbb{E}[|h_i|^2] \right)^2 \]

\[ \mathbb{E}[\xi_{OA−CSI}^0] \overset{(a)}{=} \eta \theta \left( \mathbb{E}[|h_i|^2] + \sqrt{(M-1)\text{VAR}[|h_i|^2]} \right) \overset{(b)}{=} \eta \theta \left( 1 + \sqrt{\frac{(1+2\kappa)(M-1)}{1+\kappa}} \right). \tag{42} \]

where (a) comes from using \( \sum_{i=1}^{M} \mathbb{E}[|h_i|^2] = M \mathbb{E}[|h_i|^2] \) and \( \sum_{i=1}^{M} \text{VAR}[|h_i|^2] = M \text{VAR}[|h_i|^2] \), while (b) follows after using (3). Notice that for \( M = 1 \) the equality is satisfied as expected. Regarding the variance, it is shown in [39, The. 3.1] that the best upper bound for \( \text{VAR}[\max_{i=1,...,M} |h_i|^2] \) is \( M \text{VAR}[|h_i|^2] \). However, that result is not tight for the fading distribution of our interest here, and we dispense with that result.

In Table 1 we summarize the statistics of the energy harvested under the operation of each of the WET schemes. We also include the CDF expressions, which can be easily obtained by using \( F_Z(z) \) in (2) in most of the cases. In some cases, where it was impossible a full characterization for all the values of \( \rho \), we provide the results for the case of i.i.d channel realizations where \( \rho = 0 \). Some remarks follow next.

Remark 3. For the AA scheme, both the mean and variance of the harvested energy, increase with the correlation coefficient \( \rho \). Thus, the greatest gain in the mean harvested energy occurs when channels are fully positively correlated. In fact, when channels are fully positive correlated \( S \) harvests \( M \) times more energy under the AA scheme than when using the OA strategy, but with a dispersion \( M^2 \) times greater. This can be easily corroborated also from the very definition of \( \xi_{AA}^0 \) since \( \xi_{AA}^0 = \frac{\eta \theta}{M} \sum_{i=1}^{M} h_i \overset{(a)}{=} \frac{\eta \theta}{M} M |h_i|^2 = \eta \theta M |h_i|^2 = \eta \theta M \xi_{OA}^0 \), where (a) comes from the fact that for \( \rho = 1 \) all the channel coefficients are the same, e.g., \( h_i = h_n, \forall i, n = 1, ..., M \).

For the particular value \( \rho = -\frac{1}{M-1} \), the dispersion of the harvested energy becomes 0, thus in such situations \( T \) provides a stable, non-random, energy supply to \( S \), such that \( \xi_{AA}^0 = \frac{\eta \theta M \kappa}{1+\kappa} \). Notice that \( \rho = -\frac{1}{M-1} \) guarantees the exact prediction of the harvested energy, which grows linearly with \( M \) in LOS channels. However, under Rayleigh conditions the harvested energy becomes 0, which is because of cancellation of the multiple path signals at the sensor.
Remark 4. The average energy harvested under the SA and AA − CSI schemes is constant, while the variance is a convex function of the spatial correlation coefficient, which can be easily corroborated by checking that the second derivative of $\text{VAR}[\xi_{\text{SA}}^0]$ and $\text{VAR}[\xi_{\text{AA-CSI}}^0]$ with respect to $\rho$ is non negative. The minimum variance for a given LOS parameter occurs when $\rho = -\min(\kappa, \frac{1}{M-1})$, which comes from setting the first derivative of $\text{VAR}[\xi_{\text{SA}}^0]$ or $\text{VAR}[\xi_{\text{AA-CSI}}^0]$ to 0 and solving for $\rho$, while using also the restriction $\rho \geq -\frac{1}{M-1}$. Therefore, for increasing/decreasing $\rho$ above/below − $\min(\kappa, \frac{1}{M-1})$, the variance increases.

Additionally, notice that the variance of the harvested energy under the SA scheme is

- $\frac{\eta^2 \sigma^2 (1+2\kappa)}{M (1+\kappa)^2}$ for $\rho = 0$, which is inversely proportional to the number of antennas;
- $\frac{\eta^2 \sigma^2 (1+2\kappa)}{(1+\kappa)^2}$ for $\rho = 1$, which is independent of the number of antennas;

Remark 5. Notice that $\mathbb{E}[\xi_{\text{OA}}^0] \leq \mathbb{E}[\xi_{\text{OA-CSI}}^0] \leq \eta_0 M = \mathbb{E}[\xi_{\text{AA-CSI}}^0] = M \mathbb{E}[\xi_{\text{SA}}^0]$, since for $M \geq 1$ and $\kappa \geq 0$ we have that $(1+\kappa)^2 \geq 1 + 2\kappa \rightarrow \frac{1+2\kappa}{(1+\kappa)^2} \leq 1 \rightarrow \frac{1+2\kappa}{(1+\kappa)^2} (M-1) \leq M-1 \rightarrow \frac{\sqrt{(1+2\kappa)(M-1)}}{1+\kappa} \leq \sqrt{M-1} \leq M-1 \rightarrow \mathbb{E}[\xi_{\text{OA-CSI}}^0] = 1 + \frac{\sqrt{(1+2\kappa)(M-1)}}{1+\kappa} \leq M$, which agrees with (17) of Theorem 7. We have ignored the impact of $\rho$ in the exact expression of $\mathbb{E}[\xi_{\text{OA-CSI}}^0]$ since there is no closed form, instead we used the upper bound given in (42) for the case of $\rho = 0$. And $\frac{\mathbb{E}[\xi_{\text{AA}}^0]}{\eta_0} = \frac{1+M\kappa(M-1)\rho}{1+\kappa} \leq M = \frac{\mathbb{E}[\xi_{\text{AA-CSI}}^0]}{\eta_0} = M \frac{\mathbb{E}[\xi_{\text{SA}}^0]}{\eta_0}$ holds since $M \geq 1$, and agrees with (18).

Remark 6. For $\kappa \geq 0$, the average harvested energy under the operation of the AA scheme is an increasing function of $\kappa$ since $\frac{d}{d\kappa} \mathbb{E}[\xi_{\text{AA}}^0] > 0$. For OA, SA and AA − CSI schemes, the average harvested energy does not depend on the LOS parameter. Meanwhile, the variance of
the harvested energy decreases with $\kappa$ when OA scheme is used, while it has a maximum on

$$
\kappa^*_1 = \frac{(M - 1)(1 - \rho)}{M},
$$

$$
\kappa^*_2 = \frac{(M - 1)\rho(1 - \rho)}{1 + (M - 1)\rho},
$$

when the AA scheme and, SA or AA – CSI schemes, are utilized, respectively. This is because

$$
\frac{d}{d\kappa} \text{VAR}[\xi_{\text{OA}}] < 0, \quad \frac{d}{d\kappa} \text{VAR}[\xi_{\text{AA}}] \approx \begin{cases} 0, & \kappa > \kappa^*_1 \\ > 0, & \kappa < \kappa^*_2 \end{cases},
$$

$$
\frac{d}{d\kappa} \text{VAR}[\xi_{\text{SA}}] \approx \begin{cases} 0, & \kappa > \kappa^*_2 \\ > 0, & \kappa < \kappa^*_1 \end{cases}.
$$

**Remark 7.** The average harvested energies under the CSI-based WET schemes overcome their CSI-free counterparts. But, interestingly, for i.i.d channels and $M \geq 2$, T manages to harvest more energy on average under the AA scheme than when using the OA – CSI scheme, even without considering the energy cost associated to the CSI acquisition, when

$$
M > \frac{1 + 2\kappa}{\kappa^2}. \tag{45}
$$

This is because under such condition and $\rho = 0$, $\frac{\mathbb{E}[\xi_{\text{AA}}]}{\eta} = \frac{1 + M\kappa}{1 + \kappa} > 1 + \frac{\sqrt{(1+2\kappa)(M-1)}}{1 + \kappa} \geq \frac{\mathbb{E}[\xi_{\text{OA}}]}{\eta}$ holds. Condition in (45) can also be written as $\kappa > \frac{1+\sqrt{M+1}}{M}$.

In many cases not only a high average of harvested energy is desirable but also a low variance. Let us assume for instance that the sensor requires a reliable energy supply each time that it harvests energy from $T$ transmissions, and that exceeds the value $\xi_{\text{th}}$. The value of $\xi_{\text{th}}$ may be seen as the minimum amount of energy for which $S$ can operate, which in general satisfies $\xi_{\text{th}} \geq \eta \omega_1$. In such cases a question arises: What is the more suitable WET strategy? The probability of energy outage is the performance metric to be evaluated in this case, and it is defined as $\mathbb{P}[\xi^0 < \eta \xi_{\text{th}}] = F_{\xi^0}(\eta \xi_{\text{th}})$. The more reliable scheme would be that one with the lowest probability of energy outage.

**Remark 8.** The SA strategy provides WET rounds with the lowest variance in the harvested energy, thus, it is more predictable. Additionally, it is the only strategy for which the variance of the harvested energy over multiple rounds decreases by increasing the number of antennas $M$ for $\rho < 1$. Therefore, this scheme can provide a deterministic (non random) source of energy when $M \to \infty$, e.g., $\xi_{\text{SA}}^0 = \eta \omega$.

**Remark 9.** For WET strategies without CSI and under i.i.d Rician channels, notice that
• when $\kappa \to 0$, e.g., Rayleigh fading, the energy harvested under the OA and AA schemes follows the same distribution, hence no diversity (no gain) is attained from using all antennas at once. For that scenario, the SA scheme performs best, since although with the same average harvested energy as the others, its variance is the lowest and decreases with $M$;
• when $\kappa > 0$, the average harvested energy under the AA scheme is the greatest among all WET schemes, and increases linearly with $M$. However, it is also more dispersed because of its higher variance that also increases linearly with $M$.

Figs. 2 and 3 show the PDF of the harvested energy normalized by $\eta_0$, which allows validating the analytical results by Monte Carlo simulations. Specifically, for Fig. 2 we set $\rho = 0$, $\kappa \in \{0, 3\}$ and $M \in \{2, 12\}$ and notice that the upper bound for $\mathbb{E}[\xi_0^{OA-CSI}]$ provided in (42) is also validated, and it is shown to be less tight when $M$ increases. As expected, the CSI-based schemes outperform their counterparts that do not take advantage of any information. When there is NLOS ($\kappa = 0$), it is shown that the AA and OA schemes perform the same and without any gain from increasing the number of antennas, thus, in such cases the SA scheme is the best alternative if no CSI information is available. Notice that by increasing $M$, the PDF of $\xi_{SA}^0$ gets narrower around its mean, thus, providing a more predictable energy supply. This situation holds for $\kappa > 0$, and among the CSI-free schemes the average harvested energy under the AA scheme is the greatest and increases with $M$, which is a very attractive characteristic. Meanwhile, in Fig. 3 the PDF of the harvested energy is shown when using each of the WET schemes with $M = 4$, $\kappa \in \{0, 3\}$

---

6Since analytical expressions for the distribution of the energy harvested under the SA and AA – CSI schemes were only obtained for $\rho \in \{0, 1\}$, the analytical results were identified by markers in Figs. 3b and d, while the Monte Carlo approach was used to plot the continuous lines.
and $\rho \in \{0, 0.5, 1\}$. Notice that correlation impacts negatively on the performance of both CSI-based schemes. Indeed, the greater the correlation the lesser the gains of operating with CSI, and the performance under each CSI-based scheme matches that of its CSI-free counterpart for $\rho = 1$. Comments in Remarks 2, 3 and 4 are also corroborated.

Fig. 4a shows the energy outage performance for a setup with $M = 4$ and $\kappa = 1$. Notice that for relatively small $\xi_{th}$ the schemes can be ordered according to a performance decreasing order as $\text{AA} - \text{CSI} > \text{OA} - \text{CSI} > \text{SA} > \text{AA} > \text{OA}$. Thus, the SA scheme performs the best among the CSI-free strategies, while for relatively large $\xi_{th}$ the AA scheme is superior, and it could even reach a performance greater than that achieved by the OA - CSI. The very idealistic AA - CSI scheme is the clear winner in this regard also. A more detailed analysis of the energy outage is presented in Fig. 4b, where the regions where each CSI-free WET strategy performs the best are shown for a wide range of values of $\kappa$, $\rho$ and $M \in \{2, 12\}$. The OA scheme does not appear since it is always overcome by either SA or AA strategy. Notice that SA benefits less from a greater LOS parameter and/or a greater energy threshold and/or greater spatial correlation. Finally, the number of antennas does not impact strongly on delimiting the
regions for which one scheme outperforms the other in terms of the energy outage probability.

B. Statistics of the Harvested Energy under Sensitivity and Saturation Phenomena

The statistics of the harvested energy under the ideal EH linear model, for which the harvested energy is always proportional to the incident RF power with factor $\eta$, was investigated in the previous subsection. In practice, the sensitivity and saturation phenomena limit also the EH performance. Incident signal powers below of the sensitivity level cannot be harvested, while power levels in the saturation region cannot be fully exploited. Therefore, systems with high variance in the RF incident power, e.g., all the CSI-based and the AA scheme, are more sensitive to these phenomena. In this subsection we depart from ideal results in Subsection IV-A to attain the distribution and main statistics of the harvested energy in more practical setups.

**Theorem 2.** The energy harvested under each of the schemes when $|S| = 1$ is distributed as

$$F_\xi(\xi) \cong \begin{cases} 
F_{\xi^0} (\eta \varpi_1), & 0 \leq \xi < \eta \varpi_1 \\
F_{\xi^0} (\xi), & \eta \varpi_1 \leq \xi < \eta \varpi_2 \\
1, & \xi \geq \eta \varpi_2 
\end{cases}$$

which is only an approximation when using the SA scheme. The PDF of $\xi$ can be easily derived from (46).

**Proof.** We have that $\xi^0 = \xi^0 / \eta$. Then, for the OA and AA schemes, $\xi = g(\xi^0) = g(\xi^0 / \eta)$ is satisfied according to (7) and (8). Additionally, notice that since $g$ is a non-decreasing function which saturates at some point, $\arg \max_{i=1,\ldots,M} g(\varrho |h_i|^2)$ may have several solutions, but definitely
one of them is always \( \arg \max_{i=1,...,M} \rho |h_i|^2 \). Therefore, \( \xi_{\text{OA–CSI}} = g(\xi_{\text{OA–CSI}}^0/\eta) \) is also satisfied. Similarly, for the AA–CSI scheme we have that \( \arg \max_{w_i} |h^T w_i|^2 \) is at least one of the optimum beamformers that maximizes \( g(\rho |h^T w_i|^2) \), thus, \( \xi_{\text{AA–CSI}} = g(\xi_{\text{AA–CSI}}^0/\eta) \). For the SA scheme the situation is different since in general \( \frac{1}{M} \sum_{i=1}^{M} g(\rho |h_i|^2) \neq g(\frac{\rho}{M} \sum_{i=1}^{M} |h_i|^2) \), however it could be used as an approximation that becomes tighter as \( \rho \) increases. Therefore,

\[
\xi \simeq g(\xi^0/\eta),
\]

which is satisfied with equality for all the schemes but for SA. Using (47) and (6) it is straightforward attaining (46).

In the previous subsection we attained the average and variance of the harvested energy, and consequently also for the RF incident power, under each of the WET schemes. As commented, the variance analysis allows explaining the energy outage performance in conjunction with the CDF curves. Henceforth, we focus only on the average harvested energy when considering the sensitivity and saturation phenomena. Following lemma provides an analytical results that is useful for the average harvested energy characterization given right after in Theorem 3.

**Lemma 1.** Let \( q_1(\varphi, \phi) = \mathbb{E}[g(\phi Z_1)] \) and \( q_2(\psi, \phi) = \mathbb{E}[g(\phi Z_2)] \), where \( Z_1 \sim \chi^2(\varphi, 0) \) and \( Z_2 \sim \chi^2(2, \psi) \), then

\[
q_1(\varphi, \phi) = \frac{2\eta}{\Gamma(\varphi/2)} \left( \phi \Gamma \left( 1 + \frac{\varphi}{2}, \frac{\phi}{2} \right) - \phi \Gamma \left( 1 + \frac{\varphi}{2}, \frac{\eta}{\phi} \right) + \frac{\phi}{2} \Gamma \left( \frac{\phi}{2}, \frac{\phi}{2} \right) \right),
\]

\[
q_2(\psi, \phi) \approx \frac{\eta}{\Gamma(m)} \left[ \frac{\phi(\psi+2)}{m} \right] \left( \Gamma \left( m+1, \frac{m\phi}{\phi(\psi+2)} \right) - \Gamma \left( m+1, \frac{m\phi}{\phi(\psi+2)} \right) + \omega_2 \Gamma \left( m, \frac{m\phi}{\phi(\psi+2)} \right) \right],
\]

where \( g \) is the function defined in (6) and \( m = (\psi/2 + 1)^2/(\psi + 1) \).

**Proof.** According to (2), the distribution of \( Z_1 \) can be simplified as follows

\[
f_{Z_1}(z) = \frac{1}{2} z^{\varphi/4-1/2} e^{-z/2} \lim_{\psi \to 0} \frac{I_{\varphi/2-1}(\sqrt{\psi} z)}{\psi^{\varphi/4-1/2}} \]

\[
\approx \frac{1}{2} z^{\varphi/4-1/2} e^{-z/2} \frac{1}{\Gamma(\varphi/2)} \left( z \right)^{\varphi/2-1} e^{-z/2} = \frac{1}{2\Gamma(\varphi/2)} \left( \frac{z}{\varphi/2} \right)^{\varphi/2-1} e^{-z/2},
\]

\[
F_{Z_1}(z) = 1 - Q_{\varphi/2}(0, \sqrt{z}) = 1 - \frac{\Gamma(\varphi/2, z/\varphi/2)}{\Gamma(\varphi/2)},
\]

where (a) comes from finding the limit by using l’Hôpital rule successively and (b) follows after simple algebraic transformations. For getting (51), it is required evaluating \( Q(0, \sqrt{z}) \) by using its definition and performing some limit operations or equivalently one may use result in (50).
for finding the CDF; in any case the result matches \((51)\). Now, using \((50)\) and \((51)\) along with \((6)\) to calculate \(\mathbb{E}[g(\phi Z_1)]\) we obtain

\[
\mathbb{E}[g(\phi Z_1)] = \eta\phi \int z f_{Z_1}(z)\,dz + \eta \omega_2 \int_{\omega_1/\phi}^{\infty} f_{Z_1}(z)\,dz = \frac{\eta}{\Gamma(\varphi/2)} \int \left(\frac{z}{\omega_1/\phi}\right)^{\varphi/2} e^{-z/2} \,dz + \eta \omega_2 (1-F_{Z_1}(\omega_2/\phi))
\]

\[
= \frac{\eta \phi}{\Gamma(\varphi/2)} \left(-2\Gamma(1+\varphi/2, \omega_2/\phi)\right) + \eta \omega_2 \frac{\Gamma(\varphi/2, \omega_2/(2\phi))}{\Gamma(\varphi/2)}, \quad (52)
\]

and \((48)\) follows after some simple algebraic transformations. Continuing with the second part of the proof, the PDF of \(Z_2\) can be simplified according to \(f_Z(z)\) in \((2)\) as follows

\[
f_{Z_2}(z) = \frac{1}{2} e^{-(z+\psi)/2} I_0(\sqrt{\psi} z), \quad (53)
\]

which is the power distribution under Rician fading channel with \(\kappa = \psi/2\) and average power \((\psi+2)\) \((25)\). Unfortunately, to the best of our knowledge computing \(\int z f_{Z_2}(z)\,dz\) is not analytically tractable, thus, we were not able of finding an exact closed form expression. However we can take advantage of the fact that for \(m = (\kappa+1)^2/(2\kappa+1)\), the Rician distribution approximates the Nakagami-m \((40)\), then \(Z_2 \sim \Gamma\left(\frac{(\psi+1)^2}{\psi+1}, \frac{(\psi+2)(\psi+1)}{(\psi+1)^2}\right)\) approximately. Now,

\[
\mathbb{E}[g(\phi Z_2)] = \frac{\omega_2/\phi}{\omega_1/\phi} \int z f_{Z_2}(z)\,dz + \frac{\omega_2/\phi}{\omega_1/\phi} \int_{\omega_1/\phi}^{\infty} f_{Z_2}(z)\,dz \approx \eta \phi \int \left(\frac{m z}{\psi+2}\right)^m \frac{e^{-m z/\psi}}{\Gamma(m)} \,dz + \eta \omega_2 (1-F_{Z_1}(\omega_2/\phi))
\]

\[
= -\frac{\eta \phi (\psi+2)}{m \Gamma(m)} \Gamma\left(m+1, \frac{m z}{\psi+2}\right) + \eta \omega_2 \frac{\Gamma(m, \frac{m \omega_2}{\psi+2})}{\Gamma(m)}, \quad (54)
\]

where \((a)\) comes from using \((1)\) and \((b)\) follows from solving the integral. Then, after some simple algebraic transformations we attain \((49)\).

Theorem 3. The average harvested energy under sensitivity and saturation phenomena, and for each of the WET schemes (but for OA - CSI) when \(|S| = 1\), is given by

\[
\mathbb{E}[\xi_{\text{OA}}] = q_2\left(2\kappa, \frac{\theta}{2(1+\kappa)}\right), \quad (55)
\]

\[
\mathbb{E}[\xi_{\text{AA}}] = q_2\left(\frac{2M\kappa}{1+(M-1)\rho}, \frac{1+(M-1)\rho}{2(1+\kappa)}\right), \quad (56)
\]

\[
\mathbb{E}[\xi_{\text{SA}}] \approx q_1\left(2(M-1), \frac{(1-\rho)\theta}{2M(1+\kappa)}\right) + q_2\left(\frac{2\kappa M}{1+(M-1)\rho}, \frac{1+(M-1)\rho}{2M(1+\kappa)}\right), \quad (57)
\]

\[
\mathbb{E}[\xi_{\text{AA-CSI}}] = q_1\left(2(M-1), \frac{(1-\rho)\theta}{2(1+\kappa)}\right) + q_2\left(\frac{2\kappa M}{1+(M-1)\rho}, \frac{1+(M-1)\rho}{2(1+\kappa)}\right). \quad (58)
\]
Fig. 5. Average harvested energy as a function of \( \eta \) under ideal and non-ideal EH linear model for \( M = 4 \), \( \kappa = 3 \) and \( \rho \in \{0.2, 0.8\} \). (a) AA, (b) SA, (c) OA – CSI and (d) AA – CSI.

**Proof.** According to (24) we have that \( \xi_0^{\text{OA}} / \eta \sim \frac{\rho}{2(1+\kappa)} \chi^2(2, 2\kappa) \), then, using (47) yields \( \mathbb{E}[\xi_{\text{OA}}] = \mathbb{E}[g(\xi_0^{\text{OA}} / \eta)] \), and using the results in Lemma 1 we attain (55). Similarly, using (25), (36) and (41) along with (47), we have that \( \xi_{\text{AA}} = \xi_0^{\text{AA}} / \eta \sim \frac{1+(M-1)\rho}{2(1+\kappa)} \rho \chi^2(2, \frac{2M\kappa}{1+\rho(M-1)}) \), \( \xi_{\text{SA}} \approx \xi_0^{\text{SA}} / \eta \sim \frac{\rho}{2M(1+\kappa)} \chi^2(2(1+\kappa), 0) + (1+(M-1)\rho) \chi^2(2, \frac{2M\kappa}{1+\rho(M-1)}) \), and \( \xi_{\text{AA–CSI}} = M\xi_0^{\text{SA}} / \eta \), respectively, and after using Lemma 1 we attain (56), (57) and (58). \( \square \)

Notice that a closed-form expression for the average harvested energy under the OA – CSI was not tractable under the ideal linear EH model, thus, neither here.

**V. NUMERICAL RESULTS AND DISCUSSION**

In this section we investigate the performance of the WET strategies under Rician fading conditions and assuming sensors equipped with EH hardware characteristics [41]: \( \varpi_1 = -22 \) dBm, \( \varpi_2 = -8 \) dBm and \( \eta = 35\% \). First, we consider a setup with \( |S| = 1 \) and show in

\( ^7 \) Rather than the maximum efficiency, we assume a more conservative value.
Fig. 5 the average harvested energy as a function of \( \varrho \) under ideal and non ideal EH linear model. We set \( M = 8 \) while considering scenarios LOS and NLOS, and under small and large spatial correlation. For short, we do not present the performance results of the OA scheme, but notice that they approximate the OA – CSI’s as \( \rho \) increases, therefore, the results for \( \rho = 0.8 \) in Fig. 5a are very close to those related to the OA scheme. Obviously, the average harvested energy performance is an increasing function of \( \varrho \), which is related to the average RF signal power. Notice that for the ideal EH linear model the average harvested energy increases linearly, but when operating under sensitivity and saturation phenomena that is not longer the case. The ideal EH model overestimates heavily the average harvested energy when \( \varrho \) is small or large, thus, it is only suitable for a finite range of values of \( \varrho \). This range depends on the system setup and on the utilized WET scheme. For instance, under some LOS and spatial correlation, specifically \( \rho = 0.2 \) and \( \kappa = 3 \), ideal and non-ideal EH linear models offer similar performance when \( \varrho \in [-50, -40] \) dB for SA and OA – CSI schemes, while the range is approximately shifted to the left \( 10 \log_{10}(M) \) dB when utilizing the AA and AA – CSI schemes. This is because under the ideal EH lineal model, AA and AA – CSI schemes share similar average harvested energy performance, and approximately \( M \) times greater than the performance attained under SA and OA – CSI schemes. In Subsection IV-A we showed that the average harvested energy is not a function of \( \kappa \) and \( \rho \) when operating under the ideal EH linear model with the SA and AA – CSI schemes, however, and expected from results in Theorem 3 this is not longer the case when sensitivity and saturation phenomena come to play. Additionally, notice that analytical expressions provided in Theorem 3 are corroborated also in Fig. 5. The gap between simulation and analytical results come from the fact that function \( q_2 \) in Theorem 3 is approximated according to (49), however, in all the cases analytical results capture well the impact of sensitivity and saturation impairments.

Next, we only consider the non-ideal EH linear model and we evaluate the system performance when powering multiple sensors, e.g., \( |\mathcal{S}| \geq 1 \). To that end, we consider that sensors \( \mathcal{S} \) are uniformly distributed in the disk region of radius \( R = 15 \) m [7], [8] around \( T \), thus, the distance between \( S_j \) and \( T \), denoted as \( \iota_j \), is distributed with \( f_{\iota_j}(x) = 2x/R^2 \), \( x \leq R \). We assume that \( \varrho_j = \iota_j^{-3}/100 \), which may model practical scenarios with path-loss exponent 3, transmit power of 40 dBm, and 30 dB of average signal power attenuation at a reference distance of 1 m [40]. Notice that under the CSI-free schemes, the performance metrics such as average harvested energy per user and average energy outage per user, can be easily computed by departing from results in
Section IV and performing integration over $f_{i_1}(x)$. For the CSI-based schemes, which are used only as benchmarks, analytical analyses in multi-user setups and under sensitivity and saturation phenomena are cumbersome and we resort to Monte Carlo methods and global optimization solvers when finding optimum energy beamformings. Notice that this may be impossible to implement in practice and the performance gains should decrease with the cardinality of $S$. These are the main reasons for using the CSI-free WET strategies and the scope of this work.

Fig. 6 shows the average harvested energy and outage per user as a function of the number of sensors. Since the average performances of the CSI-free schemes do not depend on $|S|$, they appear as straight lines, while the performance of the CSI-based schemes gets asymptotically worse with $|S|$. The latter is because as $|S|$ increases selecting the best antenna under the OA – CSI scheme converges to select any random antenna, e.g., OA, while for the AA – CSI scheme, the chances of creating sharp and strong beams capable of reaching all the users, decreases. Observe that the SA scheme outperforms the OA strategy in terms of average harvested energy under sensitivity and saturation phenomena, which is different from the ideal case discussed in Subsection IV-A. However, SA performs worst in terms of energy outage because of the relatively large value of $\eta \varpi_1$. As shown in Fig. 4 in order to get the advantages of the diversity order and low-variance of the SA scheme, sensors have to be able of operating at very low power levels, thus it may be suitable for sensor networks with very small values of $\varpi_1$ and requiring uninterrupted operation. Additionally, the OA – CSI scheme outperforms the OA and SA strategies in both metrics and the gap is significant when powering a small number of sensors. However, notice that as $|S|$ increases (and even for $|S| = 1$) the CSI acquisition procedures and associated energy expenditures become a problem and limit the practical benefits of the scheme. Meanwhile, the AA – CSI scheme outperforms the AA strategy specially when powering a very small number of devices, and even for $|S| = 1$ the performance gap is also not significant. According to the analyses in Subsection IV-A, the gap is even expected to decrease if any of $M, \kappa, \rho$ increases, thus, the AA scheme is recommended for powering both either a small or large set of low-power devices. Finally, it is known that analytical approximations obtained in Subsection IV-B for $|S| = 1$ hold for any value of $|S|$ when using the CSI-free schemes, and as shown in the figure, the expressions are particularly accurate for the OA and AA schemes, while not for the SA strategy for the reasons given in Theorems 2 and 3.
VI. CONCLUSION AND FUTURE WORK

In this paper, we analyzed three WET strategies that a dedicated power station equipped with multiple antennas can utilize when powering a set of single-antenna devices. These strategies operate without any CSI, which is a very practical assumption since it is very challenging to acquire CSI at the transmit side in low-power WET systems and multi-user setups. The distribution of the harvested energy when using each of the multi-antenna schemes is attained in Rician fading channels under sensitivity and saturation EH phenomena and compared to other two CSI-based benchmark strategies. We investigated the impact of spatial correlation among the antenna elements evidencing its benefits only when transmitting simultaneously with equal power by all antennas, while under the operation of the other schemes the system performance may reduce considerably when the spatial correlation increases. We found that the switching antennas strategy where only one antenna with full power is transmitting at the time, guarantees the lowest variance in the harvested energy, thus providing the most predictable energy source, and it is particularly suitable for powering sensor nodes with highly sensitive EH hardwares and operating under poor LOS conditions; while the other WET schemes perform better (or at least equal) in terms of the average harvested energy. While under NLOS it is better switching antennas, when LOS increases it is better to transmit simultaneously with equal power by all antennas. Numerical and simulation results validate our analytical findings and demonstrate the suitability of the CSI-free over the CSI-based strategies as the number of devices increases. All these are fundamental result that can be used when designing practical WET systems.
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