Constraining sterile neutrinos with a low energy beta-beam

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We study the possibility to use a low energy beta-beam facility to search for sterile neutrinos by measuring the disappearance of electron anti-neutrinos. This channel is particularly sensitive since it allows to use inverse beta decay as detection reaction; thus it is free from hadronic uncertainties, provided the neutrino energy is below the pion production threshold. This corresponds to a choice of the Lorentz $\gamma \simeq 30$ for the $^4$He parent ion. Moreover, a disappearance measurement allows the constraint of sterile neutrino properties independently of any CP violating effects. A moderate detector size of a few 100 tons and ion production rates of $\sim 2 \cdot 10^{13} \text{s}^{-1}$ are sufficient to constrain mixing angles as small as $\sin^2 2\theta = 10^{-2}$ at 99% confidence level.

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INTRODUCTION

A large number of experiments have now convincingly demonstrated that active neutrinos, i.e. left handed neutrinos which interact via W and Z exchange, can change their flavor. More recently, KamLAND [1] is providing direct evidence for neutrino oscillations. The oscillation of three active neutrinos describes the global neutrino data very well, see e.g. [2]. The fact that neutrinos oscillate implies that at least two of the mass eigenstates have a non-zero mass. Most models to accommodate massive neutrinos introduce right handed neutrinos, i.e. states which do not couple to W and Z bosons. These right handed neutrinos can either directly provide a Dirac mass term or they mediate the seesaw mechanism [3]. In the latter case, the right handed neutrino tends to be very heavy with $m_R \sim 10^{12} - 10^{15}$ GeV. However, in the most general scenario there is a 6x6 mass matrix whose entries are essentially unknown. To obtain the physical neutrino masses at scales below the electroweak phase transition this mass matrix has to be diagonalized and its eigenvalues are the neutrino masses relevant for low energy observations. Since the entries of the mass matrix are not known it can gives rise to any spectrum of eigenvalues and thus neutrino masses. The need to describe the oscillation of three active neutrinos implies that at least three eigenvalues have to be of $O(1 \text{eV})$ or less. However, there can be $0 - 3$ additional small eigenvalues corresponding to light neutrino states, which due to the Z decay width bound would have to be sterile. Thus, we conclude that sterile neutrinos are theoretically well motivated by the observation of neutrino mass. Furthermore, we see that there can be one or more sterile neutrinos which are light enough to play a role in neutrino oscillations. Note, that all these considerations are entirely independent of any experimental claims to have seen sterile neutrinos, like, for example, the one by LSND [4].

In this work we explore how well sterile neutrinos can be constrained by a dedicated oscillation experiment based on a low energy beta-beam facility. For other physics which can be studied using a low energy beta-beam, e.g. see [6]. The proposed experiment is a disappearance experiment and will be sensitive to $\Delta m^2 = 0.5 - 50 \text{eV}^2$ and can probe mixing angles as small as $\sin^2 2\theta = 10^{-3} - 10^{-2}$. The oscillation probability for our purposes is a two flavor $\nu_e \to \nu_s$ oscillation and the survival probability of $\nu_e$ is given by

$$P_{ee} = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}.$$  (1)

The most sensitive experiments in this mass range looking for the disappearance of $\nu_e$ have been the reactor neutrino experiments: Bugey [7] and Chooz [8]. The Bugey bound is valid in the range $\Delta m^2 = 0.1 - 1 \text{eV}^2$ and it can constrain $\sin^2 2\theta$ below $5 \cdot 10^{-2}$. The bound from Chooz is $\sin^2 2\theta \simeq 10^{-1}$ for $\Delta m^2 > 0.01 \text{eV}^2$.

EXPERIMENTAL SETUP

The concept we propose here is based on using a pure $\bar{\nu}_e$ beam from the beta decay of completely ionized radioactive ions circulating inside a storage ring. The Lorentz boost $\gamma$ of these ions will be chosen such that the resulting $\nu_e$ have an energy below the pion production threshold. In this case, the by far most likely reaction is inverse beta decay on free protons. In this experiment we have an accurate theoretical understanding of the neutrino flux, spectrum and cross section. The detector will be placed so close to the neutrino source that oscillation will happen within the detector itself. Thus the different parts of the detector will effectively act as near and far detector. This allows the cancellation of most systematical errors in a similar fashion as in modern reactor neutrino experiments.

1 We know from the invisible decay width of the Z boson [2], that there are 3 active neutrinos with $m < m_Z/2$.  

\[\text{\small INTRODUCTION, EXPERIMENTAL SETUP, etc.} \]
In the storage ring and the beta-beam complex, a total of five years.

The idea of using beta decay of exotic nuclei to produce well-defined neutrino beams was introduced. The basic observation is, that if a beta decaying nucleus is moving with a Lorentz $\gamma \gg 1$, the isotropic neutrino emission will be collimated into a beam. We assume that $3 \cdot 10^{12}$ ions are injected per second into the storage ring and the beta-beam complex operates for $1.6 \cdot 10^7$ s per calendar year and our experiment runs for a total of five years.

We consider a magnetic field $B = 5 \, T$ inside the storage ring. Taking $\gamma = 30$ this yields magnetic radius $r = 56 \, m$ for the $\text{He}^{++}$ ion. The useful fraction, $f$, of decays then is given by the ratio of the length of the straight section, $S$, to the overall circumference of the ring, $f = \frac{S}{2 \pi r}$. With $S = 10 \, m$, we obtain $f = 2.7\%$.

**Event rate calculation**

Figure 1 shows a schematic of the setup we consider. In this scheme we will have a cylindrical detector whose symmetry axis is aligned with the straight section of the storage ring. The free parameters are (see figure 1): the length of the straight section, $S$, the distance between the front end of the storage ring and the front end of the cylindrical detector ($Y$), the radius ($R$) and the length ($D$) of the detector. In the following, we will derive a general expression for the neutrino event rate as a function of these free parameters.

Neglecting the small Coulomb corrections to the beta-spectrum\(^2\), the lab frame neutrino beta-beam flux per unit length of the straight section in units of $\text{s}^{-1}\text{MeV}^{-1}\text{sr}^{-1}\text{m}^{-1}$ emitted at an angle $\theta$ with the beam axis is described by

$$\phi^{\text{NC}}(E, \theta) = \frac{1}{4 \pi m_e^2 f} \frac{1}{\gamma(1 - \beta \cos \theta)} (E_0 - E)^2 \sqrt{(E_0 - E)^2 - m_e^2},$$

where $m_e$ is the electron mass, $E_0$ is the electron total end-point energy and $E^*$ is the rest frame energy of the emitted neutrino\(^4\). $f$ is the phase space factor associated with the beta decay of the nucleus. $\gamma$ is the Lorentz boost such that $E^* = \gamma E (1 - \beta \cos \theta)$, $E$ being the neutrino energy in the lab frame. $g \equiv N_0 / S$ is the number of useful decays per unit time per unit length of the straight section.

To calculate the resulting number of events in a cylindrical detector of radius $R$ and length $D$ aligned with the beam axis it is necessary to integrate over the length $S$ of the straight section of the storage ring and the volume of the detector. Here we assume that the beam is perfectly collinear and has no transverse extension\(^1\). The un-oscillated event rate in a detector placed at a distance $Y$ from the storage ring is given by

$$dN/dt = n \phi \frac{\gamma}{2 (1 - \beta \cos \theta)} \int_0^S dS \int_0^D dD \int_0^{2\pi} d\theta 2\pi \sin \theta \int_{E_{\min}}^{E_\max} dE \phi^{\text{NC}}(E, \theta) \sigma(E),$$

where $\tan \theta' (x, \ell) = \frac{R}{\ell x + \ell}$ and $E' = \frac{E_0 m_0}{m_e \gamma (1 - \beta \cos \theta)}$.

Note, that the baseline, relevant for oscillations, is $L = Y + x + \ell$. Here, $n$ represents the number of target nuclei per unit detector volume, $\varepsilon$ is the detector efficiency which is taken to be unity in our calculation. $E_{\min}$ denotes the energy threshold for our detection method. We work with a threshold of 25 MeV which

\(^2\) We checked that these corrections are negligible for our purposes.

\(^4\) Note, that the beam size is of order $10^{-2} \, m$, whereas all other length scales are of order $\sim 10 \, m$. 

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**FIG. 1:** Schematic of the detector and storage ring.

**FIG. 2:** The un-oscillated event rate as a function of neutrino energy. The event rate has been calculated including all geometrical effects and with a luminosity of $3 \cdot 10^{12} \, \text{ions} \, \text{s}^{-1}$. The different lines show the result for different values of the length of the straight section, $S$, as indicated by the labels next to each line.
oscillated event rates for the considered energies. Figure 2 shows the resulting un-
tion \[10\], which is the predominant reaction channel at \(\sigma\) of neutrinos. The red line assumes that the neutrinos are all generated at in one point in the middle of the straight section. The blue (dashed) line includes all geometrical effects and the detector resolu-
eV.

FIG. 3: This figure shows the ratio of oscillated to un-
oscillated events as a function of the reconstructed \(L/E\). In upper panel, \(\Delta m^2 = 10\) eV\(^2\) and in lower panel \(\Delta m^2 = 40\) eV\(^2\). The value of mixing term \(\sin^2 2\theta = 0.1\). The red (solid) line includes all geometrical effects and the detector resolution, whereas the blue (dashed) line assumes a point source of neutrinos.

ensures that our events are well above the backgrounds. \(\sigma(E)\) stands for the inverse beta decay reaction cross section \[10\], which is the predominant reaction channel at the considered energies. Figure 2 shows the resulting un-
oscillated event rates for \(S = 10\) m, \(Y = 50\) m, \(R = 3.6\) m and \(D = 28.7\) m. This corresponds to a detector mass of 1 kton.

Optimization of the geometry

The goal is to obtain an experimental configuration which has optimal sensitivity to the disappearance of \(\bar{\nu}_e\) corresponding to a mass squared difference \(\Delta m^2 = 1 - 10\) eV\(^2\). For this optimization we fixed the detector mass at 1 kton, thus \(D\) is entirely determined by \(R\) or vice versa. We tested the values \(Y = \{30, 50, 70, 90\}\) m. We studied variations in \(\gamma\) from 20 − 35. This range was cho-

We found that within those options the configuration with \(\gamma = 30\), \(S = 10\) m, \(L = 50\) m and \(D = 28.7\) m is optimal.

Using these numbers, figure 3 shows the resulting ra-
tio of oscillated to un-oscillated event rates for two dif-
ferent vales of \(\Delta m^2\) as a function of the reconstructed \(L/E\). The blue line assumes that the neutrinos are all generated at in one point in the middle of the straight section. The red line fully accounts for all the geometry effects. Clearly, for \(\Delta m^2 = 10\) eV\(^2\) (upper panel), sev-
oral oscillation periods can be resolved. A comparison between the amplitudes of those periods allows to can-

Detector

The detector we envisage is essentially similar to the MiniBooNE detector \[11\], however with a cylindrical shape. The important background will all be beam based, since beam-off backgrounds will be well measured and cosmic ray events can be tagged with high efficiency as was done in similar near surface experiments \[11\]. The beam energy has been selected to be below threshold for pion production, therefore there are few channels available for neutrino interactions. Only charged cur-
rent quasi-elastic scattering on carbon and electron elas-
tic scattering can mimic the signal of the inverse beta decay primary positron. These event types will experi-
ence the same disappearance rates due to oscillations, but the neutrino energy reconstructed under the inverse beta decay hypothesis would be systematically less than the true neutrino energy. At these energies the cross sections for these background interactions are smaller by at least an order of magnitude, so any effect they might have on the measured oscillation parameters, in particular \(\sin^2 2\theta\) would be at most 10\%, and they can be accounted for and corrected.

To further reduce the impact of these beam based back-
grounds, the detector should be optimized for the detec-
tion of inverse beta decay. Typically this is done by tag-
ging the primary positron with the free neutron capture in delayed coincidence with a mean lifetime of \(\sim 200\) \(\mu s\) in undoped organic scintillator. However, observing the 2.2 MeV gamma ray from neutron capture on hydrogen can be a significant challenge in a detector tuned to see events in the 50 to 200 MeV range. Additionally, the long delay time will put several beam bunches between the primary and secondary events and will therefore increase the probability of false tags. Adding gadolinium to the scintillator would help somewhat by increasing the tag energy to 8 MeV and reducing the capture time to \(\sim 30\) \(\mu s\). Nevertheless, even if the neutron tag was highly
efficient, at least some of the quasi-elastic events on carbon will also have a correlated neutron capture tag. Another possibility is to design the detector to be sensitive to the positron direction. We expect the elastic scattering events to be peaked in the very forward direction, while the quasi-elastic carbon events will have a much broader angular distribution. The angular distribution of the hydrogen inverse beta decay events will fall somewhere in between. Sensitivity to the angular distribution can be achieved by reducing the scintillation light to a point where Čerenkov light can be distinguished.

To achieve an energy resolution of 10% or better over the 50 to 200 MeV range should be possible with a photocathode coverage of 10% if we assume approximately equal parts Čerenkov and scintillation light [11]. Position and timing resolutions, needed to correlate events with beam bunches, should be achievable at the half meter and 10 ns level. With a bunch spacing of 100 ns or greater, this resolution would provide sufficient space between bunches to demonstrate the rejection of non-beam backgrounds. At this level of detector resolution the $L/E$ uncertainty is fully dominated by the unknown production backgrounds. This improves sensitivity to $\Delta m^2 > 10 \text{ eV}^2$. Thus length of the straight section can effectively be used to tune the experiment to the desired range of $\Delta m^2$ and it can be envisaged to have running periods with different straight section lengths within the same setup. The lower panel shows variations of the systematic error. Again our default setup with systematic error of 0.01 is shown as red, solid line. At very small value of systematic error of 0.001, the sensitivity (dash-dotted blue line) becomes essentially independent of $\Delta m^2$, once the first oscillation maximum can be observed. Note, that this very small value of systematic error of 0.001 probably is not attainable in a real experiment. For more realistic values of systematic error in the range 0.005 – 0.05, the sensitivity limit does not change for $\Delta m^2 \leq 10 \text{ eV}^2$, which is due to the cancellation of the normalization error between different oscillation maxima. This is also illustrated in figure [3].

Figure [1] shows the obtainable sensitivity in the $\sin^2 2\theta - \Delta m^2$ plane at 99% CL. Our default configuration is shown as red solid line in both panels. The setup proposed here improves on the existing limit for $\Delta m^2 \geq 0.2 \text{ eV}^2$. In the range $1 \text{ eV}^2 < \Delta m^2 < 10 \text{ eV}^2$ the improvement is one order of magnitude or better. The upper panel shows how the sensitivity changes with varying the length of the straight section. A longer straight section (dash-dotted line) implies a large fraction of useful decays and thus better statistics. At the same time the $L/E$ resolution is reduced. As a result the sensitivity extends to smaller mixing angles (higher statistics) but at smaller $\Delta m^2$ (resolution). The analogous arguments holds also for a shorter straight section (dashed line), which yields smaller statistics but better resolution. This improves sensitivity to $\Delta m^2 > 10 \text{ eV}^2$. Thus the length of the straight section can effectively be used to tune the experiment to the desired range of $\Delta m^2$ and it can be envisaged to have running periods with different straight section lengths within the same setup. The lower panel shows variations of the systematic error. Again our default setup with systematic error of 0.01 is shown as red, solid line. At very small value of systematic error of 0.001, the sensitivity (dash-dotted blue line) becomes essentially independent of $\Delta m^2$, once the first oscillation maximum can be observed. Note, that this very small value of systematic error of 0.001 probably is not attainable in a real experiment. For more realistic values of systematic error in the range 0.005 – 0.05, the sensitivity

**RESULTS**

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**FIG. 4:** Exclusion plots of sensitivity to active-sterile oscillations in a near detector low energy beta-beam set-up. These are 99% CL limits (1 dof). In upper panel, we vary the straight section of the storage ring while in the lower panel, we show the performance taking various values of expected systematic error of the considered setup. The pink (dash-dot-dotted line) line shows the current, combined limit on $1 - P_{\bar{e}e}$ from Bugey [7] and Chooz [8].
FIG. 5: Exclusion plots of sensitivity to active-sterile oscillations in near detector low energy beta-beam set-up. These are 99% CL limits (1 dof). In upper panel, we choose different values of detector mass in the range 0.25 to 2 kton. Lower panel depicts the performance of the set-up for different values of total injected ions. The pink (dash-dot-dotted) line shows the current, combined limit on $1 - P_{ee}$ from Bugey [7] and Chooz [8].

**CONCLUSIONS**

We have studied a near detector setup at a low-$\gamma$ beta-beam facility for its ability to constrain the disappearance of electron anti-neutrinos for mass squared differences $\Delta m^2 = 1 - 10 \text{ eV}^2$. The key point is, that for a suitably chosen geometry several oscillation maxima occur within the same detector and thus a disappearance measurement at the sub-percent level becomes possible without requiring a stringent control on systematic errors. We focused on using a beam from the decay of $^6\text{He}$, which produces electron anti-neutrinos. This allows to use inverse beta decay as detection reaction and we can exploit the well defined relationship between the positron and neutrino energy. Thus, we have a very clean sample of electron anti-neutrino events. We carefully optimized the geometry and beam energy and found that $\gamma = 30$ yields the best sensitivity while still having the bulk of neutrinos below the pion production threshold. In order to have sufficient resolution in $L/E$ we had to reduce the length of the straight section down to 10 m, which makes this setup unique. Note, that this allows our experiment to run parasitically in a low energy beta-beam facility since we use only around 3% of all ions. For a conservative beam luminosity of $3 \cdot 10^{12}$ ions s$^{-1}$ and detector mass of 1 kton we obtain a sensitivity to $\sin^2 2\theta \simeq 10^{-2}$ (99% CL) for $\Delta m^2 = 1 - 10 \text{ eV}^2$.

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