Probing Planck scale physics with quantum Love

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Future gravitational wave detectors have been projected to be able to probe the nature of compact objects in great detail. In this work, we study the potential observability of the Planck length physics with the tidal deformability of the compact objects in an inspiraling binary. We find that despite the error in the Planck scale distance resolution being exponentially sensitive to errors in the Love number, it is possible to probe them with extreme mass ratio inspirals. We also consider the consequences of the Love number not being sensitive to the Planck scale logarithmically. We discuss how the quantum effects can affect the gravitational wave observables in that scenario too.

Introduction— The discovery of gravitational waves (GWs) [1, 2] paved the way towards probing fundamental physics. These observations provided a fillip to tests of General Relativity (GR) in the strong-field regime [3, 4]; e.g., stringent bounds on the mass of the graviton and violations of Lorentz invariance have been placed [5–7]. As a result, GWs have become very important in the context of fundamental physics. Various possible distinction between black holes (BHs) and other exotic compact objects (ECOs) based on tidal deformability [8–12], tidal heating [10, 13–16], multipole moments [13, 17–21], echoes in postmerger [22–30] and electromagnetic observations [28, 31–34] has been proposed in the literature. One of the very intriguing questions in fundamental physics is how gravity behaves in the quantum regime. Since GWs bring information from the very close vicinity of BHs, it is expected that GWs may shed some light on this mystery [35–41]. The idea behind such expectations follows from the fact that the Planck scale physics may affect the tidal Love numbers of the compact objects [11, 12]. As compact objects coalesce, the information of the tidal Love number gets imprinted on the emitted GWs. Hence, with a sufficiently sensitive detector, it is possible to extract the information of the very small values of tidal Love number which determines the nature of the compact object, i.e., a BH with zero Love number or of a horizonless object with non-zero Love number.

As a result, GWs can be considered to act as a gravitational microscope of near horizon physics [42]. Unfortunately, in [43] it was argued that the statistical error can stand as a challenge to the precession measurement. It was also argued that in the Planckian regime quantum noise is likely to populate the observation, making it impossible to measure below a certain length scale. We here build on these works. We will demonstrate that the extreme mass-ratio inspirals (EMRIs) have the potential to have sufficient accuracy to probe in that scale. As a result, statistical limitations described in previous works can be avoided in certain cases and limitations from the quantum aspects can be used to gain information regarding the nature of the bodies.

Tidal deformability— Consider a binary with mass of the $i$th component to be $m_i$ in the inspiral phase. We can model these systems using the post-Newtonian (PN) theory, which is a weak-field/slow-velocity expansion of the field equations. The emitted GWs from such systems can be modeled in the frequency domain as,

$$h(f) = A(f)e^{i(\psi_{PP}(f) + \psi_{TH}(f) + \psi_{TD}(f))},$$

(1)

where $f$ is the GW frequency, $A(f)$ is the amplitude in the frequency domain. $\psi_{PP}(f)$ is the contribution to the GW Fourier phase while treating the objects as spinning test particles, $\psi_{TH}(f)$ is the contribution due to tidal heating, and $\psi_{TD}(f)$ is the contribution due to their tidal deformability. In several works it has been argued that $\psi_{TH}(f)$ and $\psi_{TD}(f)$ can be used to probe the nature of the compact objects. As a result it can be used as a distinguisher between black holes (BHs) and exotic compact objects (ECOs). In this work we will focus only on $\psi_{TD}(f)$.

To leading PN order, one can show that this contribution is

$$\psi_{TD}(f) = -\frac{117}{8} \frac{(1 + q)^2}{q} \frac{\tilde{\Lambda}}{m^5} v^5,$$

(2)

where $v = (\pi mf)^{1/3}$ is the velocity, with $m = m_1 + m_2$ the total mass, and

$$26\tilde{\Lambda} = (1 + 12/q)\lambda_1 + (1 + 12q)\lambda_2,$$

(3)

where, $\lambda_i = \frac{2}{3}k_i m_i^2$ with $k_i$ the ($\ell = 2$, electric-type) tidal Love numbers and $q = m_1/m_2$ is the mass ratio.

$\delta - k$ relation— Tidal Love numbers are the response of a body to an external tidal field. It explicitly depends on the details of the internal structure of the compact object. It has been argued that for the BHs of GR, the Love number vanishes [43–45]. Other compact objects unlike BHs, have a non-zero Love number. Depending on the

1 It can be non-zero for non-axisymmetric perturbation of rotating BHs [46]. Check Ref. [47] for results in detailed numerical simulations.
properties of the object it can vary widely. According to their equation of state, matter anisotropy, and fluid nature, neutron stars can have Love numbers of \( \mathcal{O}(10^2) \) [48–57] and similarly for the boson stars [8, 9]. Love numbers of highly compact ECOs scales as \( \sim 1/\log(\ell) \), where \( \delta \equiv r_s - r_H = r_H(1 + \epsilon) \), where \( r_s \) is the actual surface position of the ECO, and \( r_H \) is the surface position of the horizon if it were a BH [8].

Hence the Love number of these ECOs vanishes logarithmically with compactness. By motivated this finding, it was argued in Ref. [10] that this logarithmic behavior can be used to possibly probe the Planck scale physics near the horizon (surface) of a BH (ECO). This logarithmic behaviour translates to the \( \delta - k \) relation as follows (caveats is discussed later) [10],

\[
\delta = r_s - r_H = r_H e^{-1/k}
\]  

(4)

Deviation of Planckian order \( (\delta \sim \mathcal{O}(10^{-35}) \) ) meters) corresponds to \( k \sim 10^{-2} \) for masses of the BH ranging in the range \( (10^{7} - 10^{9}) M_\odot \) [10, 42]. Therefore by measuring small \( k \), Planck scale physics can be probed.

Measuring quantum noise– In such case it would seem that the only limitation that would disallow us from achieving such a feat is the sensitivity of the detectors. Therefore, in principle, with sufficiently sensitive detectors, that can measure \( k \) with better accuracy than \( \sim 10^{-2} \), we will be able to probe Planckian physics. However, in [42] it has been argued that it is unlikely to be the case as quantum noise of \( \delta \) will populate at that level. As a result, the error in \( \delta \) will get modified as [42],

\[
\frac{\sigma^\text{Tot}_\delta}{\delta} = \sqrt{\left(\frac{\sigma^\text{Stat}_\delta}{\delta}\right)^2 + \frac{a^2 \ell^2_{pl}}{\delta^2}}
\]

\[
= \sqrt{\left(\frac{\sigma^\text{Stat}_\delta}{r_H}\right)^2 + \frac{1}{k^2} \left(\frac{\sigma^\text{Stat}_\delta}{k}\right)^2} + \frac{a^2 \ell^2_{pl}}{\delta^2}
\]  

(5)

where, \( \delta \) and \( r_H \) is the estimated value of \( \delta \) and \( r_H \) from the observation. Stat is the shorthand for statistical error. The error induced by quantum noise is \( a \ell_{pl} \). Where \( \ell_{pl} \) is the Planck length and \( a \) is a number \( \sim \mathcal{O}(1) \). We will discuss it in detail later.

If we assume that the error would indeed behave in this manner, we can definitely use it to estimate \( a \). This will help us in measuring the quantum noise. This will be possible to do since other parameters can be measured independently. From the observation we will have \( \sigma^\text{Stat}_\delta \), \( \sigma^\text{Stat}_\delta \), \( M \), \( \tilde{\chi} \). This can be used to estimate \( \sigma^\text{Stat}_\delta \), \( r_H \). From the observation the inferred value of Love number \( k \) will also be available. Therefore, if we can have an estimation of \( \sigma^\text{Stat}_k \) then we can estimate the \( a^2 \).

This is possible by injecting synthetic signal in detectors with \( k \) and other observed parameters. Running a Bayesian estimation on that we can have an estimation of the statistical error, which is an artifact of the observation. With sufficiently sensitive detector \( \sigma^\text{Stat}_k \) can be reduced to very small values. By estimating those values from simulations we can estimate the systematic error, which is arising from the quantum nature. Having an estimation of \( a^2 \) can lead us to understand the quantum states near horizon. For this purpose, in next section we will investigate if it is possible to reduce the statistical error sufficiently in the future detectors.

Observability– The Einstein Telescope (ET) [58] and cosmic Explorer [59] are third generation detectors with high sensitivity. We did a Fisher matrix calculation to estimate the error \( (\Delta k) \) in \( k \), in these detectors, using gwbench [60]. We found the percentage error in \( k(=0.005) \) to be very high to be well measured.

On the other hand, extreme mass ratio inspirals (EMRIs) are one of the promising sources of GW which will be observed with the future space-based Laser Interferometer Space Antenna (LISA) [61]. The emitted GW from these systems can stay in the detector band from months to year. As a result, despite being small, with LISA we will be able to measure the tidal Love numbers of supermassive BHs in EMRI, quite precisely. This has the potential to open up the possibility to probe the nature of these bodies “up to the Planck scale”.

To estimate the effect of the Love number of these supermassive bodies in EMRI, we calculate dephasing as a function of \( k \). We ignore the contribution of the secondary body. The primary body’s mass is considered to be \( m_1 = M \) and the dimensionless spin is \( \chi \). A useful estimator to describe the effects of \( k \) in the phase is the total number of GW cycles \( (\equiv N) \) that accumulates within a given frequency band of the detectors. In terms of the frequency-domain phase \( \psi_{TD}(f) \) this can be defined as,

\[
N \equiv \frac{1}{2\pi} \int_{4 \text{ mHz}}^{\text{ISCO}(M, \chi)} df \frac{d^2 \psi_{TD}(f)}{df^2}
\]

(6)

Using the expression of phase in Eq. (6) we calculate dephasing. In Fig. 1 and Fig. 2 we show the magnitude of dephasing \( (\delta \phi) \) in radian, as a function of \( k \). As expected, the dephasing increases linearly with \( k \). The results are consistent with the expectations discussed in Ref. [62]. The black dashed horizontal line represents \( \delta \phi = 1 \) radian. Dephasing \( \delta \phi > 1 \) represents a strong effect. The result implies that the EMRIs can be the potential sources that will be sensitive to the Planck scale physics.

To measure Planck scale physics it is required that \( \sigma^\text{Stat}_\delta < \delta \), where \( \sigma^\text{Stat}_\delta \) and \( \delta \) is the statistical error and the inferred value of \( \delta \). If we ignore the quantum effects (which will be discussed later) then the leading order error will arise due to the statistical error in \( k \). Hence,

\[
\frac{\sigma^\text{Stat}_\delta}{\delta} \sim \frac{\sigma^\text{Stat}_k}{k^2}.
\]

(7)

\footnote{For an invariant definition of \( \delta \) check [42].}
Assuming Eq. (4) to be valid, for $\tilde{k} \sim .005 (.01)$ to probe sub-Planckian effects it is required that $\delta^{\text{Stat}}_{\epsilon} < 2.5 \times 10^{-3} (10^{-4})$. From Fig. 1 and Fig. 2 it seems achievable with EMRIs.

Validity of $\delta - k$ relation—In this section, we will argue that Eq. (4) is unlikely to hold in the context of GW observation. It is not justified to assume that $k \rightarrow 1/|\log(\epsilon)|$ scaling will be valid in the Planck scale. This result has been derived assuming classical gravity. To probe Planck scale physics, it is necessary for $\epsilon$ to be of Planckian order. This means that the compact objects for which the surface is at $\epsilon \neq 0$ should be almost as compact as a BH. The conventional matter should collapse if it is distributed in such close proximity. The origin of Planckian $\epsilon$ must be therefore exotic matter or quantum effects. Since the modification is happening at the Planck level it is most likely that the origin would be quantum effects.

Hence, these systems are not “classical” systems to begin with. As a result, the quantum behavior should become important. Consequently, it will become necessary to take into account of the quantum properties of the states of the system to find the sub-leading contribution to the leading order classical results. These sub-leading “quantum-corrections” most likely will be the interaction between the quantum observables at the Planck scale and the classical fields (discussed later). In such a case, the $\delta - k$ relationship is likely to get modified by $k \sim 1/|\log(\epsilon)| + k_q(\epsilon)$, where the first term arises solely from the classical physics and the second term arises due to the quantum corrections. Therefore even though the first term starts to go to zero for very small $\epsilon$, the second term survives and captures the details of the quantum nature. For BH as $k = 0$ classically, quantum effects can introduce nonzero $k_q$, resulting in $k = k_q$.

Even though this behavior will be present for the values of $\epsilon$ that represents the Planck scale, it does not imply apriori that this contribution will be present only in the Planck scale. It brings the question, that from which value of $\epsilon = \epsilon_q$ this behavior becomes important. If the compact objects are not sufficiently compact i.e. $\epsilon_{ECO} \gg \epsilon_q$, then these corrections could be unimportant in the context of probing Planck scale physics and also $k_q \rightarrow 0$. If $\epsilon_q \sim \epsilon_{P}$, where $\epsilon_P$ represents Planckian $\epsilon$, then this definitely becomes important to take this into account. Only the investigations from the quantum gravity side can answer these questions.

Quantum Love changes classical Love—Due to the presence of an external tidal field a nonzero quadrupole moment $Q$ (multipole moment) gets induced on the bodies. In linear regime it is proportional to the external tidal field $E$, where the proportionality constant is the tidal Love number ($\lambda$). Throughout our calculations, we will suppress the indices and any non-scalar tensor will be represented by boldface. Therefore the tidal deformability can be defined as,

$$Q = -\lambda E$$

(8)

where, $\lambda = \frac{2}{3} km^5$, with $k$ and $m$ being the tidal Love number and the mass of the body (note $m$ is not the total mass of a binary as was assumed before).

To find the contribution of the quantum effects we will consider quantum operators for all physical observables. We will assume none of the operators have zero eigenvalue, hence they are invertible. We will separate the classical contribution and quantum fluctuation as,
\[ \lambda \to \lambda + \lambda_c \hat{I} \]
\[ \hat{Q} \to \hat{Q} + \hat{Q}_c \hat{I} \]
\[ \hat{\epsilon} \to \hat{\epsilon} + \hat{\epsilon}_c \hat{I} \]

(9)

where \( \lambda_c, \hat{Q}_c, \hat{\epsilon}_c \) are the classical contribution to the observables, and \( \hat{I} \) (\( \hat{I} \)) is the tensor (scalar) identity operator. We will also assume that Eq. (8) is valid in this regime, but in the sense of quantum operators\(^4\). Hence, it can be expressed as,

\[ \hat{Q} + \hat{Q}_c \hat{I} = -\lambda_c \hat{\epsilon}_c \hat{I} - \hat{\epsilon} \lambda_c - \hat{\epsilon} \hat{\epsilon}_c - \hat{\epsilon} \hat{\epsilon}_c. \]

(10)

Using this relation it is possible to identify the expressions of the classical contributions as well as the quantum contributions as,

\[ \lambda_c = -\frac{Q_c}{\hat{\epsilon}_c}, \quad \hat{\lambda} = -\left( \frac{\hat{\epsilon} \lambda_c + \hat{Q}}{\hat{\epsilon}_c + \hat{\epsilon}} \right). \]

(11)

Note, \( \hat{\epsilon} \) represents quantum corrections to the classical value of the external tidal field. Hence, this quantum correction represents quantum correction of the external body’s mass and the separation. Ignoring \( \hat{\epsilon} \), simplified expressions can be found as,

\[ \lambda_c = -\frac{Q_c}{\hat{\epsilon}_c}, \quad \hat{\lambda} = -\frac{\hat{Q}}{\hat{\epsilon}_c}. \]

(12)

This result is equivalent to the expressions used in Ref. \([11, 12]\). We will assume that the state of the system is \( |\Psi\rangle \) and we will suppress the \( \Psi \) while writing the expectation value with respect to \( |\Psi\rangle \). As a result, deformability gets modified as,

\[ \lambda = \lambda_c + \langle \hat{\lambda} \rangle \equiv \lambda_c + \lambda_q, \]

where \( \langle \rangle \) represents expectation value, and,

\[ \lambda_q = -\left( \frac{\hat{\epsilon} \lambda_c + \hat{Q}}{\hat{\epsilon}_c + \hat{\epsilon}} \right) \approx -\frac{\langle \hat{Q} \rangle}{\hat{\epsilon}_c}. \]

(14)

In the second term we have ignored the quantum fluctuation of the external tidal field. Using this expression the systematic error in \( \lambda \) arising from the quantum nature can be expressed as,

\[ \sigma_{\lambda}^{\text{sys}} = \sqrt{\left( \frac{\hat{\epsilon} \lambda_c + \hat{Q}}{\hat{\epsilon}_c + \hat{\epsilon}} + \lambda_q \right)^2} \]

(15)

We have established that the statistical error in \( k \) for EMRIs will be lower. Therefore observability of quantum noise solely will depend on the value of the standard deviation of the fluctuation of \( \hat{k} \). To find corresponding result in \( k \), we separate out each observables in to its classical and quantum parts as follows,

\[ k \to \hat{k} + \hat{I} k_c \]
\[ m \to \hat{m} + \hat{I} m_c \]

Using \( \lambda = \frac{2}{3} km^5 \) we find,

\[ \lambda_c = \frac{2}{3} m^5 k_c, \]
\[ \hat{k} = \left( \frac{3 \lambda}{2m^2} - \frac{5k_c \hat{m}}{m_c} - \frac{15\lambda \hat{m}}{2m^6} \right) + \mathcal{O}(\hat{m}^2) \]
\[ \lambda_q = \frac{2}{3} m^5 \left( k_q + 5k_c \frac{\langle \hat{m} \rangle}{m_c} + 5 \frac{\langle \hat{k} \rangle}{m_c} \right), \]

where, \( k_q \equiv \langle \hat{k} \rangle \).

If we separate out the mean value from \( \hat{k} \) as \( \hat{k} = \hat{x} + \hat{I} k_q \) then the error takes the simplified following form,

\[ \frac{\sigma_{k}^{\text{sys}}}{k} = \frac{\sqrt{\langle x^2 \rangle}}{k}. \]

(18)

Note, a knowledge of the quantum state of the body will not only allow to estimate \( k_q \) but also \( \sigma_{k}^{\text{sys}} \). Therefore if the systems do have quantum corrections, to measure its effect we have two observables to measure, namely the \( k_q \) and \( \sigma_{k}^{\text{sys}} \). Since with EMRI statistical error will be low, it can help us measure the systematic error.

Note, having an effective relation \( k_c \sim 1/|\ln \epsilon| \), where \( \epsilon \) is some effective length scale, does not necessarily imply that this relation would hold in terms of the quantum operator. If it does hold then it would imply that the mass and the induced quadrupole moment are related to the surface position as,

\[ \hat{k} = -\frac{\hat{Q}}{\hat{\epsilon}_c} \frac{3}{2m^5} \left( 1 - 5 \frac{\hat{m}}{m_c} \right) + 5k_c \frac{\hat{m}}{m_c} + \mathcal{O}(\hat{m}^2) = -\frac{1}{\ln(\frac{\hat{\lambda}}{\hat{\epsilon}_c})}, \]

where we have ignored the quantum fluctuation of the external field.

If this relationship does not hold then even though there will be systematic error arising from quantum effects, it will not be related to the fluctuation of the Planckian length. Value of the \( \epsilon \) will represent some effective length scale from which quantum effects become important and resulting in non-zero \( k_q \) and \( \sigma_{k}^{\text{sys}} \). As we have already demonstrated the capacity of EMRIs to measure very small values of \( k \), it can be said that EMRIs

\( \text{\textsuperscript{4}} \) It is likely that there will be some modification due to quantum effects. But for the current work we will ignore such contributions.
can really act as the GW microscopes (probably Planck scope is more suitable term!).

Note, there is a degeneracy in the definition of $k$ [8, 43, 48]. Therefore depending on the definition of $k$, $\lambda \propto k_{\text{CFMPR}} R^5$ or $\lambda \propto k_{\text{BBR}} R^5$ [43, 48]. In our work we considered the definition in Ref. [8], as connection with Planck scale physics is evident in this definition. However, most of the discussions in this work does not depend on one of the definitions. Therefore, while defining $k$ this issue needs to be resolved. If the definition in Ref. [43, 48] is considered then $\bar{n}$ will be replaced by $\bar{R}$ in the equations.

Discussion–We have explored the resolving power of the gravitational micro(Planck)scope which can be used to probe near horizon physics at radial position $\delta$ with tidal Love number $k$. The presence of environment effects could impact the GW signal [63–66] and exclusion of them may lead to erroneous measurements of Love numbers. These should be taken into account to properly assess the potential of LISA. It is also required to study in detail from the theoretical standpoint the possible origin of these systems and stability [67].

In this paper, we have explored the possibility of using the ECO relation between $k$ and $\delta$ to probe Planck scale physics with EMRI. Our result suggests that it is possible for EMRIs to bring information in this regard. We have also argued that it is most likely that this relation will get modified due to quantum effects. Measuring such deviation can help us probe the quantum nature. We have also constructed a general formalism to take into account of the quantum effects. From the constructed formalism, it is evident that even if Eq. (4) is not valid, there will be quantum signatures on the observables, at least in principle. We discussed how it should be estimated and showed that in EMRIs they can have observable contribution. To achieve our conclusions we have assumed the binary to be in an equatorial circular orbit, which is unlikely to be true for EMRIs. This should be investigated in the future.

Therefore it is high time to explore these avenues from the quantum gravity side. Finding possible effects of quantum gravity, as well as detailed numerical studies of coalescence of compact objects that has quantum gravity contribution near its surface. This as a result will lead to proper quantification of quantum gravity effects on the GW observables.

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