THE PROSPECTS
FOR EXTENDED PHASE SPACE APPROACH
TO QUANTIZATION OF GRAVITY

T. P. Shestakova
Department of Theoretical and Computational Physics, Rostov State University,
Sorge St. 5, Rostov-on-Don 344090, Russia
E-mail: shestakova@phys.rsu.ru

Abstract

A brief review of main features of the new approach named “quantum geometrodynamics in extended phase space” is given and its possible prospects are discussed. Gauge degrees of freedom are treated as a subsystem of the Universe which affects the evolution of the physical subsystem. Three points can be singled out when the gauge subsystem shows itself as a real constituent of the Universe: a chosen gauge condition determines the form of equation for the physical part of wave function, the form of density matrix and the measure in physical subspace. An example is considered when a physically relevant choice of gauge condition leads to almost diagonal density matrix. The analogy between a transition to another reference frame (another basis in physical subspace) and a transition to accelerating reference frame in Rindler space is suggested.

1. Introduction

My talk is devoted to a new approach which was proposed by G. M. Vereshkov, V. A. Savchenko and me [1, 2, 3, 4] and named “quantum geometrodynamics in extended phase space”. The results of our work have already been reported at several conferences (see, for example, [6, 7]), in particular, at “COSMION-99” [5]. So the goal of this my talk is to give a review of main features of the new approach and to discuss its possible prospects.

A search for a new approach was inspired by the well-known problems of the Wheeler – De-Witt quantum geometrodynamics such as the problem of time and the problem of reparametrization noninvariance. It is a generally known fact that, while at the classical level different forms
of gravitational constraints corresponding to various choice (parametrizations) of gauge variables are equivalent, at the quantum level they lead to non-equivalent conditions on a wave function. One can find in the literature different solutions to the Wheeler – DeWitt equation which correspond to different parametrizations of a gauge variable and are not reducible to each other \[8, 9, 10, 11, 12, 13, 14\]. A choice of the gauge variable (as a rule, it is the lapse function \(N\)) is inevitable and, as was shown in our papers \[4, 5\], implies some choice of a reference frame. Indeed, the choice of gauge variables together with imposing any additional conditions on these variables define equations for \(g_{\mu\nu}\) components of the metric which fix a reference frame according to Landau and Lifshitz. One could consider the condition of independence of \(N\) on physical degrees of freedom \[15\] as such an additional condition on the gauge variable, but even in this case it would imply the choice of a certain reference frame. It becomes especially important in case of simple cosmological models when we deal with the only gauge degree of freedom \(N\). It has been emphasized by some authors that the (3+1)-slicing in the Arnowitt – Deser – Misner formalism is equivalent to some kind of gauge fixing and there is the inconsistency between the (3+1)-slicing of a global manifold and the requirement for a wave function to be invariant under space diffeomorphisms and time displacement \[16, 17, 18\]. All the above leads to the conclusion that we failed to construct quantum geometrodynamics which would be gauge invariant in a strict mathematical sense.

It may seem that these mathematical problems are typical for canonical approach developed by Dirac for constrained systems and the situation would be different if one turned to a more powerful path integral quantization of gauge theories of Batalin, Fradkin and Vilkovisky (BFV). As a matter of fact, in the path integral approach one faces the same problems in another appearance. The BFV effective action is constructed in such a way that dynamics in extended phase space (EPS) turns out to be equivalent the Dirac generalized Hamiltonian dynamics. The main role is given to gravitational constraints. The Wheeler – DeWitt equation can be obtained in this approach as a consequence of the requirement of the BRST invariance of a state vector, \(\Omega |\Psi\rangle = 0\) (for simple cosmological models \(\Omega\) is a linear combination of the constraints). On the other hand, the Wheeler – DeWitt equation can be derived from the BFV path integral under the assumption of its BRST invariance which is ensured by asymptotic boundary conditions on ghosts and Lagrange multipliers (see \[10\]). So, the Wheeler – DeWitt equation, even if derived from the path integral by formal procedure, inherits all the problems we have met in canonical quantization: the inevitability of choice of parametrization, etc. Moreover, one should impose asymptotic boundary conditions which are justified only in asymptotically flat spacetime, while already in case of a close universe or a universe with another non-trivial topology we have no grounds to impose asymptotic boundary conditions.
The next point which I would like to mention is the difference between the group of transformations generated by gravitational constrains and that of gauge transformations of the Einstein theory. In fact, already at the classical level we deal with two different theories of gravity which are invariant under different groups of transformations in Lagrangian and Hamiltonian formulations. In the path integral quantization these transformations define the structure of ghost sectors which also appear to be different. The two formulations could enter into agreement only in a gauge-invariant sector which can be singled out by asymptotic boundary conditions; the later ones must supposedly pick out trivial solutions for ghosts and Lagrange multipliers to ensure gauge-invariant dynamics. However, if one consider the Universe as a system which, in general, does not possess asymptotic states, one have to pose the question: what formalism should one prefer? What are consequences of the fact that we consider the path integral without asymptotic boundary conditions? What will be a role of gauge degrees of freedom, which were traditionally considered as redundant, in this new approach?

2. Hamiltonian dynamics in EPS: our approach

It is more convenient to illustrate our approach for a simple minisuperspace model with a gauged action

\[ S = \int dt \left\{ \frac{1}{2} v(\mu, Q) \gamma_{ab} \dot{Q}^a \dot{Q}^b - \frac{1}{v(\mu, Q)} U(Q) + \pi_0 \left( \dot{\mu} - f_{,a} \dot{Q}^a \right) - iw(\mu, Q) \dot{\theta} \dot{\bar{\theta}} \right\}. \]  

Here \( Q = \{Q^a\} \) stands for physical variables such as a scale factor or gravitational-wave degrees of freedom and material fields, and we use an arbitrary parametrization of a gauge variable \( \mu \) determined by the function \( v(\mu, Q) \). In the case of isotropic universe or the Bianchi IX model \( \mu \) is bound to the scale factor \( a \) and the lapse function \( N \) by the relation

\[ \frac{a^3}{N} = v(\mu, Q). \]  

\[ w(\mu, Q) = \frac{v(\mu, Q)}{v_{,\mu}}; \quad v_{,\mu} \overset{\text{def}}{=} \frac{\partial v}{\partial \mu}. \]  

\( \theta, \bar{\theta} \) are the Faddeev–Popov ghosts after replacement \( \bar{\theta} \rightarrow -i\bar{\theta} \).

In the class of gauges not depending on time

\[ \mu = f(Q) + k; \quad k = \text{const}, \]  

which can be presented in a differential form,

\[ \dot{\mu} = f_{,a} \dot{Q}^a, \quad f_{,a} \overset{\text{def}}{=} \frac{\partial f}{\partial Q^a}. \]
the Hamiltonian can be obtained in a usual way, according to the rule $H = P \dot{Q} - L$, where $(P, Q)$ are the canonical pairs of extended phase space, by introducing momenta conjugate to all degrees of freedom including the gauge and ghost ones,

$$H = P_a \dot{Q}^a + \pi \dot{\mu} + \bar{\rho} \dot{\theta} + \dot{\bar{\rho}} - L = \frac{1}{2} G^{\alpha \beta} P_\alpha P_\beta + \frac{1}{v(\mu, Q)} U(Q) - \frac{i}{w(\mu, Q)} \bar{\rho} \rho,$$

(2.6)

where $\alpha = (0, a)$, $Q^0 = \mu$,

$$G^{\alpha \beta} = \frac{1}{v(\mu, Q)} \left( \begin{array}{c} f_{,a} \ f_{,a}^a \\ f_{,a}^a \\ \gamma_{ab} \end{array} \right).$$

(2.7)

Varying the effective action (2.1) with respect to $Q^a, \mu, \pi$ and $\theta, \bar{\theta}$ one gets, correspondingly, motion equations for physical variables, the constraint, the gauge condition and equations for ghosts. The extended set of Lagrangian equations is complete in the sense that it enables one to formulate the Cauchy problem. The explicit substitution of trivial solutions for ghosts and the Lagrangian multiplier $\pi$ to this set of equations turns one back to the gauge-invariant classical Einstein equations.

It is not difficult to check that the system of Hamiltonian equations in EPS

$$\dot{P} = -\frac{\partial H}{\partial Q}; \quad \dot{Q} = \frac{\partial H}{\partial P}$$

(2.8)

is completely equivalent to the extended set of Lagrangian equations, the constraint and the gauge condition acquiring the status of Hamiltonian equations. The idea of extended phase space is exploited in the sense that gauge and ghost degrees of freedom are treated on an equal basis with other variables. This gave rise to the name “quantum geometrodynamics in extended phase space”.

Let us emphasize that the Hamiltonian dynamics is constructed here in a different way than in the BFV approach. For example, the Hamiltonian constraint is modified and looks like follows:

$$\dot{\pi} = \frac{1}{v^2(\mu, Q)} \left[ \frac{1}{2} \left( P_a P^a + 2 \pi f_{,a}^a P^a + \pi^2 f_{,a} f_{,a}^a \right) + U(Q) \right] - \frac{i}{w^2(\mu, Q)} w_{,\mu} \bar{\rho} \rho.$$  

(2.9)

The gauge-dependent terms can be eliminated making use of trivial solutions for $\pi$ and ghosts. Together with the restriction on the class of admissible parametrization, $v(\mu, Q) = \frac{u(Q)}{\mu}$, it reduces the constraint to the form

$$\mathcal{T} = 2u(Q) \frac{1}{u(Q)} P_a P^a + \frac{1}{u(Q)} U(Q) = 0.$$  

(2.10)

Thus, the Hamiltonian constraint (2.10) can be restored by means of asymptotic boundary conditions. We come to the conclusion that the Dirac primary and secondary constraints
\( \pi = 0, \quad T = 0 \) correspond to a particular situation when it is possible to single out the trivial solutions for \( \pi \) and ghosts by asymptotic boundary conditions. In this sense, both the Dirac and the BFV quantization schemes are applicable to systems with asymptotic states only.

Since the Hamiltonian dynamics in EPS is completely equivalent to Lagrangian dynamics, the group of transformations in EPS corresponds to the group of gauge transformations in the Lagrangian formalism. One can construct the BRST generator,

\[
\Omega = w(Q, \mu) \pi \dot{\theta} - H \theta = -i \pi \rho - H \theta. \tag{2.11}
\]

It is easy to check that (2.11) generates transformations in EPS which are identical to the BRST transformations in the Lagrangian formalism. On the other hand, the generator (2.11) does not coincide with the one constructed according to prescriptions by BFV which for the present model has an especially simple form:

\[
\Omega_{\text{BFV}} = \eta^\alpha \mathcal{G}_\alpha = T \theta - i \pi \rho, \tag{2.12}
\]

where \( \mathcal{G}_\alpha = (\pi, \ T) \) is the full set of constraints. As was mentioned above, the Wheeler–DeWitt equation \( T |\Psi\rangle = 0 \) immediately follows from the requirement of BRST invariance \( \Omega_{\text{BFV}} |\Psi\rangle = 0 \) due to arbitrariness of BFV ghosts \( \{\eta^\alpha\} \).

Because of the difference in groups of transformations the BFV charge (2.12) turns out to be irrelevant in this consideration. At the same time, the “new” BRST generator (2.11) cannot be presented as a combination of constraints and does not lead to the Wheeler–DeWitt equation.

This makes us to look in a new light at the status of BRST invariance. We have a theory in EPS which, after imposing a gauge condition, is still invariant under global BRST transformation. However, this simple example shows that after quantization of the theory the requirement of BRST invariance is not, in general, a remedy to restore the broken gauge invariance.

### 3. The general solution to the Schrödinger equation and the role of gauge degrees of freedom

Let us turn now to the quantization procedure. Independently of our notion of gauge invariance or noninvariance of the theory, the wave function should obey some Schrödinger equation. We derive the Schrödinger equation from the path integral with the effective action (2.1) and without asymptotic boundary conditions by a standard method originated by Feynman [19, 20].

So, the Schrödinger equation is a direct mathematical consequence of the fact that we consider the Universe as a system without asymptotic states. For the present model it reads

\[
i \frac{\partial \Psi(\mu, Q, \theta, \bar{\theta}; t)}{\partial t} = H \Psi(\mu, Q, \theta, \bar{\theta}; t), \tag{3.1}
\]
where
\[ H = -\frac{i}{\omega} \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} - \frac{1}{2M} \frac{\partial}{\partial Q} M G^{\alpha \beta} \frac{\partial}{\partial Q^\beta} + \frac{1}{v} (U - V); \] (3.2)
the operator \( H \) corresponds to the Hamiltonian in EPS \((2.6)\). \( M \) is the measure in the path integral,
\[ M(\mu, Q) = v^k (\mu, Q) w^{-1}(\mu, Q); \] (3.3)
\( K \) is a number of physical degrees of freedom; the wave function is defined on extended configurational space with the coordinates \( \mu, Q^a, \theta, \bar{\theta} \).

\( V \) is a quantum correction to the potential \( U \), that depends on the chosen parametrization \((2.2)\) and gauge \((2.4)\) (its explicit form has been given, e. g. in \([6, 7]\)).

The general solution to the Schrödinger equation has the following structure:
\[ \Psi(\mu, Q, \theta, \bar{\theta}; t) = \int \Psi_k(Q, t) \delta(\mu - f(Q) - k)(\bar{\theta} + i\theta) dk. \] (3.4)

It is a superposition of eigenstates of a gauge operator,
\[ (\mu - f(Q)) |k\rangle = k |k\rangle; \quad |k\rangle = \delta(\mu - f(Q) - k). \] (3.5)

It can be interpreted in the spirit of Everett’s “relative state” formulation. In fact, each element of the superposition \((3.4)\) describe a state in which the only gauge degree of freedom \( \mu \) is definite, so that time scale is determined by processes in the physical subsystem through functions \( v(\mu, Q), f(Q) \) (see \((2.2), (2.3)\)), while \( k \) being determined by initial clock setting. Indeed, according to \((2.4)\), the parameter \( k \) gives an initial condition for the variable \( \mu \). The function \( \Psi_k(Q, t) \) describes a state of the physical subsystem for a reference frame fixed by the condition \((2.4)\). It is a solution to the equation
\[ i \frac{\partial \Psi_k(Q; t)}{\partial t} = H_{(\text{phys})} \Psi_k(Q; t), \] (3.6)
\[ H_{(\text{phys})} = \left[ -\frac{1}{2M} \frac{\partial}{\partial Q^a} M \gamma^{ab} \frac{\partial}{\partial Q^b} + \frac{1}{v} (U - V) \right] \right|_{\mu=f(Q)+k}. \] (3.7)

The dependence of \( \Psi_k(Q, t) \) on \( k \) is not fixed by the equation \((3.6)\) in the sense that \( \Psi_k(Q, t) \) can be multiplied by an arbitrary function of \( k \). On the other side, one cannot choose the function \( \Psi_k(Q, t) \) to be not depending on \( k \), since in this case one would obtain a non-normalizable, non-physical state. The normalization condition for the wave function \((3.4)\) reads
\[ \int \Psi^*(\mu, Q, \theta, \bar{\theta}; t) \Psi(\mu, Q, \theta, \bar{\theta}; t) M(\mu, Q) d\mu d\theta d\bar{\theta} \prod_a dQ^a = \]
\[ \int \Psi_k^*(Q, t) \Psi_k(Q, t) \delta(\mu - f(Q) - k) \delta(\mu - f(Q) - k') M(\mu, Q) dk dk' \prod_a dQ^a = \]
\[ = \int \Psi_k^*(Q, t) \Psi_k(Q, t) M(f(Q) + k, Q) dk \prod_a dQ^a = 1. \tag{3.8} \]

It is normalizable under the condition that \( \Psi_k(Q, t) \) is a sufficiently narrow packet over \( k \). Even from the classical point of view a gauge condition cannot be fixed absolutely precisely, so that the wave function \((3.4)\) would depend on the gauge conditions through \( \delta \)-function. We should rather consider a narrow enough packet over \( k \) to fit a certain classical \( \bar{k} \) value:

\[
\Psi(\mu, Q, \theta, \bar{\theta}; t) = \frac{1}{\sqrt{2i\alpha\sqrt{\pi}}} \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2\alpha^2} (k - \bar{k})^2 \right] \Psi_k(Q, t) \delta(\mu - f(Q) - k) (\bar{\theta} + i\theta) dk = \frac{1}{\sqrt{2i\alpha\sqrt{\pi}}} \exp \left[ -\frac{1}{2\alpha^2} (\mu - f(Q) - \bar{k})^2 \right] \Psi_k(Q, t) (\bar{\theta} + i\theta). \tag{3.9} \]

Our main aim is to give a description of a physical Universe, so our next step will be the construction of a density matrix

\[
\rho(Q, Q', t) = \int \Psi^*(\mu, Q, \theta, \bar{\theta}; t) \Psi(\mu, Q', \theta, \bar{\theta}; t) M(\mu, Q) d\mu d\theta d\bar{\theta}. \tag{3.10} \]

For the wave function \((3.9)\) the expression for density matrix reads

\[
\rho(Q', t) = \exp \left( -\frac{1}{4\alpha^2} [f(Q) - f(Q')]^2 \right) \Psi_k^*(Q, t) \Psi_k(Q', t) \times \times M \left( \frac{1}{2} [f(Q) + f(Q')] + \bar{k}, Q \right). \tag{3.11} \]

The normalization condition for the density matrix is

\[
\int \rho(Q, Q, t) \prod_a dQ^a = \int \Psi_k^*(Q, t) \Psi_k(Q, t) M(f(Q) + \bar{k}, Q) \prod_a dQ^a = 1, \tag{3.12} \]

it corresponds to the condition \((3.8)\).

By introducing a certain gauge condition we determine a gauge subsystem of the Universe which affects properties of physical Universe. From a mathematical point of view we can single out three points where the gauge subsystem shows itself as a real constituent of the Universe:

- A chosen gauge condition determines the form of the equation \((3.6)\) for the physical part of the wave function \( \Psi_k(Q, t) \), in particular, an effective quantum potential.

- The density matrix \((3.11)\) describing physical subsystem of the Universe also depends on the chosen gauge condition through the factor \( \exp \left( -\frac{1}{4\alpha^2} [f(Q) - f(Q')]^2 \right) \). It plays an important role when analyzing the question under what conditions the Universe can behave in a classical manner.

- The measure \( M \left( f(Q) + \bar{k}, Q \right) \) in physical subspace also depends on the gauge condition, so that any changes of the gauge condition result in changes of the measure. In other words, if we determined the gauge subsystem in some different way, it would reflect on the structure of physical subspace.
4. Problems and prospects

In this concluding part of my talk I shall consider in more detail these three points.

• One can draw an analogy between solutions to the equation (3.6) for the physical part of the wave function corresponding to different reference frames and solutions to the Wheeler – DeWitt equations in different parametrization mentioned in the Introduction.

In general, one can seek a solution to Eq. (3.6) in the form of superposition of stationary state eigenfunctions:

\[ \Psi_k(Q, t) = \sum_n c_n \Psi_{kn}(Q) \exp(-iE_nt); \]  

(4.1)

\[ H_{(phys)} \Psi_{kn}(Q) = E_n \Psi_{kn}(Q). \]  

(4.2)

The eigenvalue \( E \) corresponds to a new integral of motion that emerges as a result of fixing a gauge condition and characterizes the gauge subsystem. On the other hand, in some papers [12, 13, 14] it has been suggested to reduce the Wheeler – DeWitt equation to a stationary Schrödinger equation of the form \( H\Psi = E\Psi \) (see also [16, 17, 18], where the canonical formalism is modified in such a way that a new Hamiltonian constraint becomes parabolic resulting in a Schrödinger equation and corresponding eigenvalue problem for a wave function). The origin of the Hamiltonian eigenvalue is different in different approaches, but it would be interesting to compare a physical sense of these solutions. In any case, there still remains the problem of choice of parametrization, or, strictly speaking, a preferable reference frame.

• About 15 years ago a number of papers were published where the notion of decoherence in quantum cosmology was widely discussed. Decoherence is believed to be one of necessary requirements for the system (here we mean the Universe as a whole) could be regarded as classical [21]. It implies a transition from a pure state to some mixed state described by diagonal density matrix which could be considered as a result of interaction with some environment [22, 23]. The application of this idea to quantum cosmology implies splitting the Universe into two subsystems, one of which is a system under investigation and the other plays the role of environment. It has been proposed to consider some modes of scalar, gravitational and other fields as an environment. However, there is no natural way to split the Universe into two subsystems.

Our investigation [7] demonstrated that the gauge subsystem can be supposed to be a candidate for the role of an environment. Gauge degrees of freedom are not observable
directly, but only by its influence on the physical subsystem. The expression (3.11) shows that under some gauge condition the density matrix becomes about diagonal. A good example is given by the condition for the lapse function $N$:

$$N = a + \frac{1}{a^3}.$$  \hspace{1cm} (4.3)

(here $a$ is a scalar factor). This gauge condition is rather interesting in some respects. For large $a$ we have $N = a$ (conformal time gauge), while in the limit of small $a$ the condition (4.3) can be rewritten as $Na^3 = 1$. The latter corresponds to the constraint on metric components $\sqrt{-g} = \text{const}$. This constraint is known to lead to the appearance of $\Lambda$-term in the Einstein equations \[21, 5\]. So the condition (4.3) describes in the limit of small $a$ an exponentially expanded early universe with $\Lambda$-term while in the limit of large $a$ we have a Friedmann universe in the conformal time gauge $N = a$. For the gauge (4.3) $f(a) = a + \frac{1}{a^3}$, and in the limit of large $a$ the density matrix would have a Gaussian peak, $\exp\left[-\frac{1}{4\alpha^2}(a - a')^2\right]$, i. e. the Universe would demonstrate a classical behaviour in a good approximation.

In our approach the Universe may behave quasi-classically only in a spacetime region where one can introduce a certain gauge condition. If spacetime manifold consists of regions covered by different coordinate charts, so that one should introduce different reference frames in these regions, the Universe cannot behave in a classical manner nearby borders of these regions. It is in accordance with the fact that the quasiclassical approximation is also not valid nearby borders of these regions. We can consider it as another indication that the description of reference frames should not be ignore when one deals with nontrivial topology of spacetime.

- As was said above, any changes of a gauge condition result in changes of the measure $M\left(f(Q) + k, \bar{Q}\right)$ and, in general, in the structure of physical subspace. Varying of the gauge condition means that from the basis (3.5) one goes to another basis (4.4),

$$\langle \mu - f(Q) - \delta f(Q), k \rangle = k |k\rangle; \quad |k\rangle = \delta (\mu - f(Q) - \delta f(Q) - k).$$  \hspace{1cm} (4.4)

The Hamiltonian in physical subspace (3.7) whose form also depends on chosen parametrization and gauge then could acquire additional terms, so that with respect to the original subspace with the basis (3.5) the Hamiltonian may, in general, acquire an anti-Hermitian part. It would means that the transition to another basis (4.4) would have an irreversible character from the point of view of the observer in the reference frame corresponding to the basis (3.5). On the other side, the observer in the reference frame corresponding to
the basis \[1.3\] would not experience any irreversible changes and, from his point of view, the evolution of the Universe would be unitary. There is a certain analogy between the transition from one to another basis in physical subspace and a transition to accelerating reference frame in Rindler space. In the latter case one can formally associate some entropy with the area of an accelerating horizon \[25, 26\]. However, for the observer in a non-accelerating reference frame the entropy would be zero and time evolution would be described by an unitary operator. In the both cases we can see that physical situations are observer dependent. But, in the case of nontrivial topology which was mentioned above, we have to pass on to another reference frame and another basis in physical subspace. It rather resembles the situation with casual horizons when the very existence of black hole entropy poses the question if evolution is still unitary.

Further investigation of these questions requires development of the formalism beyond simple models with finite degrees of freedom. The generalization of the formalism for time dependent gauges, when the structure of physical subspace is continuously changing, also deserves careful attention. It is another prospect of the presented approach.

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