A Global Modeling Approach for Load Forecasting in Distribution Networks

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Abstract—Efficient load forecasting is needed to ensure better observability in the distribution networks, whereas such forecasting is made possible by an increasing number of smart meter installations. Because distribution networks include a large amount of different loads at various aggregation levels, such as individual consumers, transformer stations and feeders loads, it is impractical to develop individual (or so-called local) forecasting models for each load separately. Furthermore, such local models ignore the strong dependencies between different loads that might be present due to their spatial proximity and the characteristics of the distribution network. To address these issues, this paper proposes a global modeling approach based on deep learning for efficient forecasting of a large number of loads in distribution networks. In this way, the computational burden of training a large amount of local forecasting models can be largely reduced, and the cross-series information shared among different loads can be utilized. Additionally, an unsupervised localization mechanism and optimal ensemble construction strategy are also proposed to localize/personalize the forecasting model to different groups of loads and to improve the forecasting accuracy further. Comprehensive experiments are conducted on real-world smart meter data to demonstrate the superiority of the proposed approach compared to competing methods.

Index Terms—Load forecasting, smart meter, global model, distribution networks, deep learning.

I. INTRODUCTION

Load forecasting at the system level plays a crucial role in the energy industry as accurate forecasts are of vital importance for the planning and operation of both power systems as well as business entities [1]. Due to the integration of new smart-grid technologies, such as demand-response and distributed energy resources, load forecasting is also becoming increasingly important at various levels of the distribution networks as most of these technologies require accurate load forecasts for efficient energy management [2]. To support these developments, an increasing number of smart meters are being deployed at the individual consumer level and higher in the network, e.g., in medium voltage/low voltage (MV/LV) transformer stations or LV feeders. As a result, scalable forecasting techniques applicable to a large number of measuring points are in high demand.

Traditional forecasting approaches that work well at the system level and include both deterministic and probabilistic load forecasting methods, such as [3] and [4], are typically not well suited for load forecasting in distribution networks, where large amounts of electricity demand time series at multiple network levels and multiple sites need to be modeled efficiently. System-level demand typically consists of the aggregated demand of thousands or millions of consumers. By the law of large numbers, such demand is much less volatile than the LV demand, and hence, it is easier to predict [5]. This assertion was, for example, validated in [6] through the introduction of an empirical scaling law that describes load forecasting accuracy at varying levels of aggregation. It was shown that for various forecasting methods, larger numbers of aggregated consumers consistently resulted in better forecasting performance. Furthermore, the comprehensive review of load forecasting methods in LV networks from [7] suggested that the biggest difference between system-level and distribution network forecasting is due to: (i) the load volatility at lower voltage levels, where the data uncertainty significantly impacts the forecasting performance [6]; and (ii) the large number of forecasts needed in the distribution networks. These differences clearly warrant a different approach to load forecasting when attempting to cater to the characteristics of the distribution networks.

Traditional load forecasting techniques rely on (so-called) local modeling, where one prediction model is developed for each given time series. These models work well when forecasts need to be made for a small number of time series but are less suitable for forecasting larger groups of time series, which is the case in distribution network load forecasting. Additionally, local models commonly suffer from the small sample size problem and tend to overfit, especially when using heavily parameterized (e.g., deep-learning based) prediction models. To mitigate these drawbacks, manual feature engineering is adopted with local models in an effort to find suitable data representations that can efficiently encode the most salient cues of the time-series data [8]. However, manual feature engineering often limits the accuracy of load forecasting since finding simple yet informative data representations is not always feasible. Additionally, feature engineering also adversely affects scalability, as human supervision is typically needed to capture the main characteristics of the given time series. Future smart grids that need to forecast the load at many sites, hence, cannot afford to rely on such human supervision.

To address the shortcomings of local models, so-called global modeling approaches started to appear recently in the literature [8]. Unlike local modeling solutions, their global counterparts consider all time series as the same regression task and fit a single model to all time series in the given input set. Thus, global models are trained on a large number of time series simultaneously so that the cross-series information can be exploited when learning the forecasting model. In the load forecasting literature, leveraging the global modeling approach is still very sparse. Shi et al. [9], for example, proposed a pooling-based deep RNN model for household forecasting, where consumers were split into different groups randomly, and the electrical load of each group was forecasted separately. Voß et al. [10] compared two forecasting strategies, i.e., one...
model for all consumers and one model for each consumer, for individual consumer load forecasting and reported that using a single model for all consumers yielded superior performance.

Global models have also been studied beyond load forecasting. Researchers at Uber [11] trained a single neural network on Uber data and applied their model to unrelated time-series data from the M3 dataset (approx. 1500 time series from various domains) and reported highly competitive performance without re-training. These results not only point to the impressive generalization capabilities of global modeling techniques but also have considerable implications for future research in this area. As global models generalize well across time series with diverse characteristics, a single generic model could (ideally) be used for various time series forecasting tasks. Specifically, in the energy industry a single model could, for example, be utilized for forecasting the load at different sites, even if historical data for a certain site is not available. Global modeling approaches have shown great potential in the last few years by winning the prestigious M4 and M5 forecasting competitions [12], [13]. Various deep learning-based time series forecasting models, such as N-BEATS [14], DeepAR [15], ES-LSTM [16], and DeepTCN [17], have been proposed recently and they all utilize global modeling when learning the forecasting models.

While global modeling techniques have been successfully applied to various problem domains, very limited emphasis was given so far to electricity demand forecasting. Moreover, comprehensive investigations into (optimal) solutions for efficiently forecasting the loads of individual consumers as well as different consumer aggregates, such as transformer station (TS) or feeders, are still largely missing from the literature. In this paper, we want to fill this gap by proposing an efficient and effective way to forecast large groups of electricity demand time series in distribution networks using global modeling and deep learning. As a result, we make the following main contributions in this work:

1) We propose a global modeling approach based on deep learning for efficient forecasting of a large number of loads in distribution networks at various levels of aggregation;
2) We improve the performance of the global model further by introducing an unsupervised localization mechanism and optimal ensemble in the forecasting framework;
3) We conduct comprehensive case studies on real-world data to verify the superiority of the global model, localization, and ensemble process of the proposed method.

The rest of this paper is organized as follows: Section II introduces the global forecasting problem; Section III provides the framework and details of the proposed method; Section IV conducts comprehensive case studies on a real-world dataset; Section V presents the final conclusions and provides directions for future work.

II. PROBLEM STATEMENT

Load forecasting in the distribution network consists of forecasting large sets of electricity demand time series at multiple network levels and multiple sites. Given the characteristics and requirements of future smart grids, such forecasting should be done for thousands of measuring points, rendering existing (computationally expensive) forecasting models, which commonly model each given time series through a specific regression problem, impractical. This fact provides a very strong motivation for research into load forecasting models that: (i) scale better with the number of time series considered, (ii) generalize well over time series with diverse statistical properties, and (iii) produce reliable load forecasts for large groups of time series data. In this section, we formally discuss load forecasting models and introduce a global framework for time series modeling that addresses most of the shortcomings discussed above.

A. Problem Formulation

Let \( \mathcal{Y} = \{y_{i,1:T_i}^{N}\}_{i=1}^{N} \) represent a set of \( N \) time series, where \( y_{i,1:T_i} = [y_{i,1}, y_{i,2}, \ldots, y_{i,T_i}]^T \in \mathbb{R}^{T_i} \) stands for an univariate time series, \( y_{i,t} \in \mathbb{R} \) is the value of \( i \)-th time series at timestamp \( t \), and \( T_i \) denotes the length \( i \)-th of the series. Furthermore, let \( H \in \mathbb{N}^+ \) and \( K \in \mathbb{N}^+ \) denote the forecasting horizon and the number of lags considered, respectively. In it’s simplest form, the goal of short-term load forecasting (STLF) is to predict the vector of future values \( \hat{y}_{i,T_i+1:H} = [\hat{y}_{i,T_i+1}, \hat{y}_{i,T_i+2}, \ldots, \hat{y}_{i,T_i+H}]^T \in \mathbb{R}^{H} \) given past observations, or formally:

\[
\hat{y}_{i,T_i+1:H} \rightarrow \hat{y}_{i,T_i+1:H},
\]

where \( \hat{y}_{i,T_i+1:H} \) stands for a point forecast of the \( i \)-th time series. In the considered problem domain, the time series data typically consists of active power measurements measured at various levels of aggregation in the distribution network, i.e., individual consumers, various aggregates representing transformer stations and feeders.

B. Local and Global Modeling

From a modeling perspective, time series forecasting methods can, in general, be partitioned into methods that utilize either local or global modeling. An overwhelming majority of univariate time series forecasting methods developed in the last few decades use local modeling [8], where each time series is considered independently from all others. With this approach, each of the time series in a given set is assumed to come from a different data generating process and is, therefore, modeled individually, as a separate regression problem. As a result, a distinct forecasting model is estimated for each series. Conversely, global modeling considers all time series within the same regression task and fits a single univariate forecasting function to all of the time series in the set. This approach makes a strong assumption that all time series in the given set come from the same data generating process. Consequently, global (time series) modeling techniques are able to overcome some of the main limitations of local modeling approaches, i.e., (i) they prevent over-fitting because of larger sample size is used during training, (ii) they scale gracefully w.r.t. to the number of time series modeled, and (iii) they exploit cross-series information by sharing parameters across time series resulting in better performing models with highly competitive generalization capabilities [8].

Let \( X \) denote a feature matrix, where \( X \in \mathbb{R}^{m \times p} \), and let \( Y \) denote the corresponding target matrix, where \( Y \in \mathbb{R}^{m \times H} \). Here, \( m \) stand for the number of samples of a given time series and \( p \) for the number of features (e.g., lag values, categorical features etc.) in \( X \). The samples in \( X \) and \( Y \) for a given
time series $y$ are commonly created using a rolling window approach. Additionally, assume that such feature matrices and corresponding targets are available for a set of $N$ distinct time series, i.e., $\{X_i\}_{i=1}^N$ and $\{Y_i\}_{i=1}^N$. In a local forecasting approach, $N$ models are trained in total and one function $f_i(\cdot)$ with model parameters $\theta_i$ is estimated for each time series in the set, as follows:

$$f_i(X_i; \theta_i) = Y_i.$$  \hfill (2)

Conversely, in a global forecasting approach, samples of all time series are first stacked together, so that:

$$X = \begin{bmatrix} X_1 \\ X_2 \\ ... \\ X_N \end{bmatrix}, \quad Y = \begin{bmatrix} Y_1 \\ Y_2 \\ ... \\ Y_N \end{bmatrix},$$  \hfill (3)

and one global forecasting function $f(\cdot)$ with model parameters $\theta$ is then estimated for all time series in the set:

$$f(X; \theta) = Y.$$  \hfill (4)

Such global models represent powerful tools for time-series forecasting that can be further improved through various localization strategies [8]. Global models, to the best of our knowledge, have not yet been widely considered for the problem of load forecasting despite their immense potential for this task. Our main contribution in this work, therefore, lies in the introduction of the global modeling framework for load forecasting and a new STLF approach designed around this framework. As we show in the experimental section through rigorous experimental evaluations, the proposed approach leads to state-of-the-art performance while exhibiting several desirable characteristics not available with existing local models.

\section*{III. METHODOLOGY}

This section presents the novel framework and details for load forecasting using global time series modeling.

\subsection*{A. Proposed Framework}

The framework for forecasting large groups of electricity demand time series is shown in Fig. 1 and consists of four distinct steps. In the first step, a pool of time series is used to fit a (single) global model to all available time series data.

This process models the global data characteristics and is strongly related to existing transfer learning strategies where knowledge obtained when solving one problem is utilized in a different but related problem domain. In the second step, a clustering procedure is applied to identify data samples (clusters) that share specific data characteristics not necessarily captured by the learned global model. These data clusters serve as additional sources of information that supplement the information already considered by the global model. The identified clusters are then utilized in the third step for fitting multiple localized models on subsets of time series in the input pool. This step is needed to infuse the initial global model with information on cluster-specific data characteristics and further improve its generalization capabilities. Additionally, it also avoids potential locally optimal solutions that could arise if the forecasting models were trained on the data clusters directly. In the fourth step, the localized forecasts for each time series are combined into the final prediction by building an optimal forecasting ensemble. Details on the outlined steps are given in the following sections.

\subsection*{B. Deep Learning-based Global Modeling}

Given a set of time-series data, the proposed framework first learns a global forecasting model $f(\cdot)$ with model parameters $\theta$, as detailed in Eq. (4). Because the entire input set of time series data, i.e., $\{X_i\}_{i=1}^N$ and $\{Y_i\}_{i=1}^N$, can be utilized for training, global models can typically be more heavily parameterized than their local counterparts. As a result, such models are more difficult to overfit while offering superior performance for a wider range of input data due to the larger model capacity. While arbitrary forecasting models could be used as the basis for the proposed framework, we select the recent N-BEATS model [14] for this task due to its excellent performance for various time-series prediction problems. However, note that the proposed framework is model agnostic and can be implemented with arbitrary backbones.

\textbf{Model Description.} Following existing literature [18], [7], we use load lags and categorical features as the input to the forecasting model. The use of load lags allows us to exploit historical data of each time series for the forecasting task, whereas categorical features encoding the month, the day of the week, and the hour in a day, are employed to model seasonality.

Load lags are created using a rolling window approach (with $K$ lags) for each time series separately, and then stacked.
together into the combined lag feature matrix $X_{lags}$ for all time series in the input set. Similarly, categorical features for the month, the day of the week, and the hour of the day are created by first extracting relevant information from each time series at timestamp $t$, and then stacking the values together for all time series in the set. Finally, one-hot encoding is used to generate the categorical feature matrix $X_{exog}$. Thus, the global model in the proposed framework aims to implement the following mapping:

$$f : X \mapsto Y,$$

where $X = [X_{lags}, X_{exog}]$, and $Y$ is a target matrix with a horizon of $H$. To learn the model we use a standard Mean Absolute Error (MAE) learning objective with L1 regularization to control the model complexity, i.e.:

$$L = \|Y - f(X, \theta)\|_1 + \lambda\|\theta\|_1,$$

where $\lambda$ is a regularization factor and $\hat{Y} = f(X, \theta)$ are the model predictions.

**Deep Learning Backbone.** The N-BEATS architecture is used as the backbone for the global model $f(\cdot)$. It consists of a series of base architectural blocks linked together through residual connections, as shown in Fig. 2.

Each of the $R$ blocks in the model is comprised of $L$ fully-connected (FC) layers with ReLU activations and two linear layers that generate two distinct outputs, i.e., 1) the (partial) forecast $\hat{Y}^{(r)}$, and 2) an estimate of the block’s original (lag) input $\hat{X}_{lags}$, also called the backcast. The first block in the model takes the lag values $X_{lags}$ as well as the categorical features $X_{cat}$ as input, whereas the remaining blocks only accept the (recursive) residuals of the lags, i.e., $X_{lags}^{(r)} = X_{lags}^{(r-1)} - X_{lags}^{(r-1)}$, as the basis for their predictions. Here, $r$ denotes the block index, where $r \in \{1, 2, \ldots, R\}$ and no superscript is used for the input of the model. To generate the final prediction for a given input feature matrix $X$, the N-BEATS model aggregates the forecasting outputs of all $R$ blocks of the model, i.e. [14]:

$$\hat{Y} = \sum_{r=1}^{R} \hat{Y}^{(r)}.$$  

In our extended design, the prediction of the first N-BEATS block $\hat{Y}^{(1)}$ makes full use of the historical time series information as well as the categorical features extracted from the given set of time series. The remaining blocks, on the other hand, refine the initial prediction with higher-order predictions based on residual lags values only. It is also important to note that the extended model inherits all of the characteristics of the original architecture, such as transparent gradient flow during training (due to the double residual design) and interpretability through the aggregation of partial forecasts $\hat{Y}^{(r)}$.

**Implementation Details.** To learn the model parameters we minimize the objective from Eq. (6) using the Adam optimizer [19]. The L1 regularization factor $\lambda$ is set to 0.0001. When training the initial global model, the learning rate is initially set to $lr = 0.001$ and reduced 3 times by a factor of 10 every time the validation loss plateaus. When training the localized models, we start with a lower learning rate of $lr = 0.0001$ and reduce it by a factor of 10 every 20 epochs. Training is stopped after the validation loss stops decreasing. We consider 30 minute forecasts which result in a dimensionality of the categorical features of 67 (12 for each month, 7 for each day of the week, and 48 for each hour in a day). We also do not share weights across the blocks of the N-BEATS model as advocated by its authors, as this was found to result in better forecasting performance. The model itself is implemented with a total of 3 blocks and 3 FC hidden layers in each block and each FC layer having 512 units.

**C. Consumer Clustering**

The global forecasting model described in the previous section is readily applicable to all time series from the input set $\{Y_i\}_{i=1}^{N}$. However, given its global nature, the forecasting performance may still be sub-optimal for time-series data with specific characteristics. To accommodate such time series and further improve forecasting performance, the proposed framework uses a localization mechanism that rely on time series clustering. The main idea behind this mechanism is to identify time-series data with common (but distinct) characteristics and then to adapt/localize the learned global model with respect to the identified clusters.

To accommodate the clustering procedure, we first reparametrize the time series data in $\{Y_i\}_{i=1}^{N}$ and compute descriptive features encoding high-level time series characteristics for each time series in the set. Following the work from [20], we compute the mean, the variance, first order of auto-correlation, trend, linearity, and a number of additional features for each time series and use the extracted feature as the basis for clustering. We utilize a hierarchical clustering procedure based on $K$-means and cluster splitting (akin to the Linde-Buzo-Gray (LBG) algorithm [21]), which results in the following cluster hierarchy:

$$\{Y_i\}_{i=1}^{N} = \{\pi_0^{(l)} \cup \pi_1^{(l)}\}; l = 1,$$

$$\{Y_i\}_{i=1}^{N} = \{\pi_0^{(l)} \cup \pi_1^{(l)} \cup \pi_2^{(l)}\}; l = 2,$$

$$\vdots$$

$$\{Y_i\}_{i=1}^{N} = \{\pi_0^{(l)} \cup \pi_1^{(l)} \ldots \cup \pi_{C-1}^{(l)}\}; l = C - 1,$$

The reader is referred to [20] for details on the complete feature set.
Algorithm 1: Model localization for cluster hierarchy

Input: Cluster hierarchy: \{\pi_{i}^{(l)}\}, for \(l = 1, 2, \ldots, C - 1\)
Output: Localized models \{f_{\pi_{i}}^{(l)}\}

1. for \(l = 1\) to \(C - 1\) do
   2. for \(i = 0\) to \(l\) do
      3. Get subsets \(X_{\pi_{i}}^{(l)}\) and \(Y_{\pi_{i}}^{(l)}\) from \(\{Y_{k}\}_{k=1}^{N}\)
      4. Load global model \(f(\cdot)\) with parameters \(\theta\)
      5. Optimize \(f(\cdot)\) through Eq. (6) using \(X_{\pi_{i}}^{(l)}\) and \(Y_{\pi_{i}}^{(l)}\)
      6. Store resulting model \(f_{\pi_{i}}^{(l)}\) with parameters \(\theta_{\pi_{i}}^{(l)}\)

where \(\pi_{i}^{(l)}\) represents the \(i\)-th subset of the time series data at the \(l\)-th level of the cluster hierarchy and \(\bigcap_{i=1}^{l} \pi_{i}^{(l)} = \emptyset\). Note that a hierarchical clustering procedure is selected for time series partitioning because it offers a convenient and, most importantly, reproducible way of generating time series subsets as opposed to standard approaches that exhibit a certain level of uncertainty due to the initialization procedure.

D. Global Model Localization

The data clusters from Eq. (8) are utilized to localize the global forecasting model by adapting it based on the identified subsets of the overall time series data. This type of model localization strategy exhibits several desirable characteristics: (i) it is generally applicable and fully unsupervised, i.e., it does not rely on any prior knowledge or data labels, (ii) it still allows to control the forecasting complexity by sharing parameters across groups of times series, and (iii) it enables improved forecasting performance by fine-tuning the initial global model to the specifics of the clustered times series. It is important to note at this point that localized global models are formally still global models as they are trained on a set of times series and not a single time series at the time.

Let \(\pi_{i}^{(l)}\) represent the \(i\)-th subset of the complete time series data identified by the clustering procedure described above at the \(l\)-th level of the hierarchy. Furthermore, let the corresponding feature matrix be denoted as \(X_{\pi_{i}}^{(l)}\) and the target matrix as \(Y_{\pi_{i}}^{(l)}\). The localization strategy used in our framework can then be defined as a model adaptation procedure that estimates a new set of model parameters for the time series in \(\pi_{i}^{(l)}\), so that:

\[
f_{\pi_{i}}^{(l)}(X_{\pi_{i}}^{(l)}, \theta_{\pi_{i}}^{(l)}) = Y_{\pi_{i}}^{(l)}.
\]

The resulting model \(f_{\pi_{i}}^{(l)}\) is then a localized version of the global model \(f\) and is expected to provide better forecasting performance on the subset \(\pi_{i}^{(l)}\). It needs to be noted that the localized model is learned by initializing the backbone architecture (N-BEATS in this work) with the global model parameters \(\theta\) and then fine-tuning the model on the data from \(\pi_{i}^{(l)}\). While the localized models could theoretically also be learned from scratch, such a strategy could easily face similar issues as existing local models and is prone to overfitting, poor generalization and local maxima.

The presented localization mechanism is applied to the complete cluster hierarchy from Eq. (8) and generates a hierarchy of localized models, as described in Algorithm 1.

E. Building Ensemble of Forecasts

Let \(x = [x_{lag}, x_{out}]\) represent a feature vector corresponding to a specific time series of the \(i\)-th cluster \(\pi_{i}^{(l)}\) from the \(l\)-th level in the cluster hierarchy. The forecast for the feature vector at the \(l\)-th level \(\hat{y}^{(l)}\) is then computed through the following gate superposition, i.e.:

\[
\hat{y}^{(l)} = \sum_{i=0}^{l} \delta_{i}(x)f_{\pi_{i}}^{(l)}(x, \theta_{\pi_{i}}^{(l)}),
\]

where the gate \(\delta_{i}(\cdot)\) in the above equation is a Dirac function of the following form:

\[
\delta_{i}(x) = \begin{cases} 
1; & \text{if } x \in \pi_{i}^{(l)} \\
0; & \text{otherwise}
\end{cases}
\]

To combine the forecasts from the global model and the \(C - 1\) forecasts from the cluster hierarchy (illustrated in Fig. 3), we utilize a simple bottom-up selection procedure. Following established literature [22], an ensemble of models is created by first generating all \(C\) forecasts for all considered time series and evaluating their performance on a (hold out) validation set. Next, a ranked list is generated by sorting the models according to their performance. Finally, a sequential model selection procedure is employed to find the optimal ensemble. This sequential procedure first combines the first and second best-performing models by averaging their forecasts. If the performance on the validation set increases, the third-performing model is added, and the evaluation procedure is repeated. Models are then sequentially added until the performance stops increasing or the entire set of models is exhausted. To ensure optimal performance across the entire time series set, model ensembles are built for each time series in the set separately. The entire procedure is summarized within Algorithm 2.

IV. EXPERIMENTS AND RESULTS

In this section, we report experimental results on a real-world dataset that demonstrate the performance and characteristics of the proposed forecasting framework. Specifically, we show experiments and corresponding results that: (i) compare the global modeling framework to its local counterpart, (ii) analyze the proposed localization strategy in comparison with baseline alternatives, (iii) ablate various components of the framework to demonstrate their contribution, and (iv) benchmark the proposed solution against competing state-of-the-art forecasting techniques from the literature.
A. Dataset Description

We use the dataset published by Commission for Energy Regulation (CER) in Ireland [23] for the experiments. The dataset contains load profiles of over 6000 residential consumers and small & medium enterprises for approximately one and a half years (from July 1, 2009 to December 31, 2010) with a half-hourly resolution. As we are also interested in the forecasting performance on consumer aggregates at the higher network levels, e.g., transformer stations or feeder loads, three additional aggregates, representing small, medium and large transformer stations are created. To generate realistic load aggregates from the initial pool of individual load time series, a total of 1000 time series corresponding to the following groups are created for the experiments, i.e.:

- Individual consumers (single): 250 time series of individual consumers;
- Small transformer stations (sTS): 250 time series each having 50 consumers in an aggregate;
- Medium transformer stations (mTS): 250 time series each having 100 consumers in an aggregate;
- Large transformer stations (lTS) 250 time series each having 200 consumers in an aggregate.

Fig. 4 shows an example of a weekly profile for 4 randomly chosen time series, each taken from one consumer group. It can be seen that the variability in the daily profiles decreases with the level of aggregation. The load of the individual consumer (grey line) is highly volatile, whereas the load of the large transformer station, where there are 200 consumers in an aggregate, is much less volatile and is, therefore, expected to be easier to forecast.

We perform STLFW with the horizon of \( H = 48 \) (24 hours) and create dataset samples using a rolling window approach. We use one year of data for training, the following 12 weeks for validating, and the final 12 weeks for testing. Hyper-parameters are tuned using the validation set and a time-based dataset split is adopted for the experiments, where the training, validation, and testing sets are consecutive in time.

B. Performance Metrics

Forecasts are evaluated using the Mean Absolute Scaled Error (MASE)², which is a standard measure for comparing forecast accuracy across multiple time series [24]. MASE scores greater than one indicate that the forecasts are worse, on average than (seasonal) naive models.

Let \( y_{i,T+h} \) be the corresponding model prediction. MASE is calculated by dividing the Mean Absolute Error (MAE) of a model forecast on the test set with the in-sample MAE of a naive seasonal model calculated over both the training and test data. In our case, the naive seasonal model takes values from a previous week to predict the load in the current week at the same time instance (observations measured one week or \( S \) periods in the past). The formal definition of MASE is given in Eq. (14), i.e.:

\[
\text{MAE}_{\text{test}} = \frac{1}{H} \sum_{h=1}^{H} |y_{T+h} - \hat{y}_{T+h}| \quad (12)
\]

\[
\text{MAE}_{\text{naive}} = \frac{1}{H} \sum_{h=1}^{H} |y_{T+h} - y_{T+h-S}| \quad (13)
\]

\[
\text{MASE} = \frac{\text{MAE}_{\text{test}}}{\text{MAE}_{\text{naive}}} \quad (14)
\]

Note that we omit the time series index \( i \) in the above equation for brevity. In the experiments, MASE scores are computed for every time series in the given set and the average across all time series is then reported as the final performance indicator for the evaluated forecasting models.

²Additional results using the Normalized Mean Absolute Error (NMAE) and Mean Absolute Percentage Error (MAPE) are provided in the Appendix.
C. Global vs. Local Modeling

We first compare the forecasting performance of the proposed global and the established local modeling frameworks to demonstrate the benefits of modeling time series globally and illustrate the main characteristics of the global forecasting models. In the local setting, one (N-BEATS) model is trained for each time series, which results in 1000 models trained in total. In the global setting, on the other hand, a single (global) model is trained for all 1000 time series. We note that only the first step of the overall framework is considered at this stage (without any model localization or ensembles). To balance model complexity against the amount of available training data and to ensure reasonable generalization, we design the local models with 32 units per layer in each stack of the N-BEATS backbone and use 512 units per layer for the global model. This configuration was determined through hyper-parameter optimization performed during the model development. As a result of the presented setup, each local model has 78,848 parameters, resulting in a total of 78,848,000 parameters that have to be estimated for the local modeling framework, as there are 1000 time series in the input set. The global model, on the other hand, has a total of 2,684,544 parameters that are shared between all 1000 time series. Thus, the global model needs approximately $1000 \times 30 \times$ fewer parameters than the local models in total to forecast the entire set of time series.

Overall Performance. The performances of the global and local models across the different types of consumer aggregates are provided in Table I. The global modeling framework ensures 2.83% better forecasts on average over all consumer aggregates than the local models in terms of MASE. It can be seen that for all except the individual consumers, global modeling clearly outperforms its local counterpart and yields consistent performance gains. For the time series corresponding to the individual consumers, global modeling performs slightly worse with a performance drop of 0.28%.

Fig. 5 shows the improvement of the global over the local models in percentage for each time series and aggregate type separately using the Empirical Cumulative Relative Frequency (ECDF). Here, each dot represents the improvement for one time series, where the improvement in percentage is calculated as $(\text{MASE}_{\text{local}} - \text{MASE}_{\text{global}}) \times 100\%$. The global approach performs better than the local one on 35%, 95%, 98% and 98% of the time series for the single, small, medium, and large TSs, respectively. The improvement is significantly better for the small, medium, and large TSs, whereas in the case of single consumers, the local approach performs better. We ascribe this result to the fact that the global model needs to account for a large group of time series and, therefore, exhibits superior performance on time series that are less volatile. Nonetheless, as we show later, the global models can further improve the reported performance by using various localization strategies.

Different Horizons. Next, we investigate the behavior of the global and local modeling frameworks with respect to the length of the forecasting horizon $H$. The results in Fig. 6 show that for the first forecasted time step $t_0$ (for one half-hour ahead), the global approach outperforms the local approach in terms of MASE by 3.2%, 7.4%, 8.7%, and 7.7% for the single, small, medium and large TSs, respectively. The improvement decreases with the increase in the forecasting horizon. With the largest horizon, the improvement of the global approach over the local one saturates between 3 and 4% for the small, medium, and large TSs, whereas for the single consumers, the MASE difference stays between 0 and $-0.5\%$.

Dataset Size. Because the global forecasting models are learned on data of all time series in the given input set, an extremely large amount of data is in general available for training. When building local models, a standard approach in the load forecasting literature is to use at least one year of data for training and a fixed window of data for testing. With global modeling, the amount of training data increases linearly with the number of time series in the input set. In our case, this means that the global model has $1000 \times 358$ more samples available for training than each of the local models.

As a result, it is expected that such global models can be learned well even if the training data is subsampled.

To validate this assumption, we analyze the performance of the global models with smaller training sample sizes per time series. We keep the test data fixed to 3 months but sample the training data from the entire (one year) history of data during the model learning stage. Specifically, we use a sampling procedure that reduces the amount of training data by a factor of $12 \times$. The results in Table II show that even when using $12 \times$ less data points, the performance decreases by only 0.35%, whereas the training time decreases by approximately 4 times, i.e., training the model takes 2 days instead of 8 on the utilized hardware. Additionally, we observe that even when trained on the subsampled data, the global approach still outperforms the local approach that uses the whole history of data. This represents a highly desirable characteristic that can help to mitigate some of the data quality issues seen with the current smart meter infrastructure, as the model does not need the whole history of data of each smart meter if the training is done within a global modeling framework. Based on this insight, we use the presented subsampling in all following experiments as it significantly decreases the training time, while having a minimal impact on the overall forecasting performance.

D. Model Localization and Ablations

In the next series of experiments, we consider the entire proposed framework, including the model localization and final ensembles. We present a series of experiments that: (i)
TABLE II: Impact of training dataset size on global modeling

| Training data     | Single | sTS  | mTS  | lTS  | All   |
|-------------------|--------|------|------|------|-------|
| Whole dataset     | 0.8011 | 0.7546 | 0.7115 | 0.6784 | 0.7364 |
| Subsampled by 12x | 0.8048 | 0.7574 | 0.7146 | 0.6829 | 0.7399 |

Figure 6: Improvement of global modeling over local modeling with different forecasting horizons.

Figure 7: Performance improvement caused by the proposed model localization procedure.

The results of the experiments with all model variants are presented in Table III. To further demonstrate the value of the proposed localization procedure, we additionally include results for the Singleton case, where each time series is considered a separate cluster and the global model is localized only with the training data of the time series, on which the forecasts are performed. This configuration results in $N$ localized models, where $N$ is again the number of time series in the input set. For comparison purposes, we also report Baseline results obtained with the global model trained without localization on the subsampled training data (see Table II).

We can see that fine-tuning the global model for each time series separately (Singleton) significantly degrades the performance (MASE increases by 4.46%) and is even worse on time-series characteristics as proposed in Subsection III-C (denoted as TF) but also features based on mean daily profiles where each time series is represented with a vector of size $3 \times 48$, consisting of a mean daily profile of a working day, Saturdays and Sundays, that are stacked together (denoted as DF) [25].

Cluster Algorithm. We evaluate two clustering algorithms: K-Means and DBSCAN [26]. Preliminary experiments show that DBSCAN cannot identify any useful clusters when representing time series with mean daily profiles; therefore, we apply DBSCAN only on features based on time series characteristics. As a result, three clustering approaches are considered for the comparative analysis: (i) K-Means using features based on mean daily profiles, (ii) K-Means using time series characteristics (as proposed for our framework), and (iii) DBSCAN using time series characteristics.

Forecast Generation. When generating forecasts from the localized model, we investigate three competing strategies, where we: (i) identify the optimal number of clusters for all time series in the input set (ALL hereafter) and make predictions in accordance with Eq. (10) using the same fixed number of clusters for all given time series, (ii) identify the optimal number of clusters for each time series in the input set separately and make predictions based on Eq. (10) using a different input set partitioning for each time series (BEST hereafter), and (iii) create an ensemble from the cluster hierarchy, as proposed for our framework (denoted as ENS).

The most significant improvement is for individual consumers identified clusters. We, therefore, evaluate different options for each time series supplied to the clustering algorithm, alternatives and benchmark them against the proposed forecasting approach, we consider various other parts of the framework, and (iii) compare some of our design choices to selected alternatives.

Model Localization. First, we explore the effect of the proposed model localization strategy on the overall forecasting performance. To this end, we start with the global model evaluated in the previous subsection and localize it based on subsets of the time series data identified through a clustering procedure. Finally, we build an ensemble of forecasts using the procedure described in subsection III-E. Fig. 7 shows the improvement observed due to the localization procedure for each aggregate type using ECDF plots. It can be seen that for almost all the series, localization improves forecasting performance. The most significant improvement is for individual consumers (grey dots), where the infusion of additional information through the localization procedure contributes towards much more convincing forecasting results. To get better insight into the impact of the proposed localization strategy, we show in Fig. 8 the improvements caused by localization as a function of the length of the forecasting horizon $H$. It can be seen that localization significantly improves forecasting performance on all horizons and for all aggregate types.

Design Alternatives and Ablations. To demonstrate the efficacy (and contribution) of the design choices made with the proposed forecasting approach, we consider various other alternatives and benchmark them against the proposed solution. In general, the performance of the localization strategies relying on data clustering depends on (i) the representation of each time series supplied to the clustering algorithm, (ii) the characteristics of the selected clustering algorithm, and (iii) the way the final forecasts are generated/combined from the identified clusters. We, therefore, evaluate different options for each of these factors, i.e.:

- Representation. We use feature-based clustering for all experiments, where a set of features is first extracted from the time series and a clustering procedure is applied on top of the extracted features. We explore features based
than the performance of the local models in Table I. Additionally, we observe that in all three cases, localization using groups of similar time series performs better than using localization for each time series separately (Singleton) which can easily lead to over-fitting. We also see that in general (i) model localization improves performance over the baseline global model, (ii) features based on time-series characteristics produce better results than features based on daily profiles, and (iii) the ensemble approach consistently yields the lowest overall MASE scores among all considered forecast-generation strategies. Considering all tested model variants, it can be concluded that the proposed localization approach (K-Means+TF+ENS – shaded gray) and the corresponding design choices all contribute to the increased forecasting performance and lead to the best overall results. The bottom line in Table III shows the improvement of the proposed approach over the baseline. As can be seen, the biggest improvements are observed when forecasting the load of individual consumers, with a performance gain of 3.5%.

Understanding the Clustering. The localization strategy used in the proposed approach relies on time series clustering. To better understand the clustering procedure, we conduct a detailed analysis using the closely related approach from the previous experiment that does not leverage ensembling to generate the final forecasts. Specifically, we use K-Means+TF+ALL as the additional strategy for the analysis, which uses the same clustering hierarchy as the proposed approach but generates the final forecasts using only the (single) cluster level from the hierarchy that performed best across all time series. The results in the top part of Fig. 9 show the value of MASE as a function of the number of clusters for this approach, whereas the results in the bottom part show the improvement over the baseline model, i.e., the global without localization. Here, we evaluate MASE on all time series and for each cluster number separately. It can be seen that the localization procedure utilizing between 2 and 8 clusters actually worsens performance. Conversely, performance starts increasing when the number of clusters is at least 9. The best results are observed with 15 clusters, where the average MASE score is worse by 1.6% compared to the proposed approach. These results suggest that it is critical to identify a suitable data granularity for the clustering procedure as performance gains can only be expected with a sufficient number of clusters utilized for the localization step.

E. Comparison with the State-of-the-Art

Finally, we compare the proposed global forecasting approach, i.e., with and without model localization, with the following competing state-of-the-art models:

- **N-BEATS** proposed in [27], but without the exogenous features added in this paper. Instead of ensembling multi-

TABLE III: Performance comparison of different global model localization strategies in term of MASE.

| Clustering Algorithm | Representation | Forecast Generation | Single | sTS | mTS | ITS | All |
|----------------------|----------------|---------------------|--------|-----|-----|-----|-----|
| Baseline             | n/a            | n/a                 | 0.8048 | 0.7574 | 0.7146 | 0.6829 | 0.7399 |
| Singleton            | n/a            | n/a                 | 0.7828 | 0.7994 | 0.7844 | 0.7713 | 0.7845 |
| K-Means              | DF             | ALL                 | 0.7939 | 0.7595 | 0.7211 | 0.6888 | 0.7408 |
| K-Means              | DF             | BEST                | 0.7844 | 0.7518 | 0.7121 | 0.6802 | 0.7321 |
| K-Means              | DF             | ENS                 | 0.7823 | 0.7460 | 0.7058 | 0.6712 | 0.7263 |
| DBSCAN               | TF             | ALL                 | 0.8048 | 0.7574 | 0.7146 | 0.6829 | 0.7399 |
| DBSCAN               | TF             | BEST                | 0.7795 | 0.7575 | 0.7146 | 0.6829 | 0.7336 |
| DBSCAN               | TF             | ENS                 | 0.7757 | 0.7550 | 0.7154 | 0.6844 | 0.7326 |
| K-Means              | TF             | ALL                 | 0.7795 | 0.7574 | 0.7231 | 0.6912 | 0.7378 |
| K-Means              | TF             | BEST                | 0.7753 | 0.7540 | 0.7160 | 0.6845 | 0.7325 |
| K-Means              | TF             | ENS                 | 0.7698 | 0.7444 | 0.7047 | 0.6694 | 0.7221 |

Improvement [%] | 3.50 | 1.30 | 0.99 | 1.35 | 1.78 |

Fig. 8: Performance improvement due to model localization as a function of the forecasting horizon.

Fig. 9: Forecasting performance as a function of the number of clusters used for the K-Means+TF+ALL localization strategy.
TABLE IV: Comparison with competing state-of-the-art forecasting models (MASE).

| Model          | Single | sTS   | mTS   | lTS   | All   |
|----------------|--------|-------|-------|-------|-------|
| N-BEATS [27]   | 0.8306 | 0.7775| 0.7253| 0.6920| 0.7564|
| DeepAR [15]    | 0.7725 | 0.7891| 0.7241| 0.6832| 0.7422|
| Ours w/o localization | 0.8048 | 0.7574| 0.7146| 0.6829| 0.7389|
| Ours w/ localization | 0.7698 | 0.7444| 0.7047| 0.6694| 0.7221|

V. CONCLUSIONS

The load forecasting literature is currently dominated by forecasting techniques that rely on local modeling, where each given time series is modeled independently from all others. While such techniques have shown good performance for STLF tasks, the expected scale of future smart grids will soon render them impractical. To address this shortcoming, we presented in this paper a novel framework for STLF based on deep learning that, different from existing solutions, relies on global modeling to capture the characteristics of a large group of time series simultaneously. Based on this framework, a novel approach to load forecasting was introduced that not only utilizes global time series modeling but also adopts a powerful model localization strategy. The proposed approach was evaluated in comprehensive experiments and in comparison to state-of-the-art techniques from the literature. The experimental results showed that the proposed approach outperformed all competing models by a considerable margin across different aggregate types.

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APPENDIX

In the main part of the paper, we reported all results only in terms of MASE scores to keep the presentation uncluttered. In this Appendix, we now also report the main results in terms of additional performance indicators regularly used in the STLF literature, i.e., the Normalized Mean Average Error (NMAE) & the Mean Absolute Percentage Error (MAPE) [28]. Since both performance scores correspond to error measures, lower scores again indicate better forecasting performance. The results reported in this section look at: (i) the comparison between local and global modeling for STLF, (ii) the impact of model localization on performance, and (iii) additional analyses related to the clustering procedure that forms the basis for the model localization strategies used in the proposed framework.

A. Local vs. Global Modeling

The first results in Tables V and VI compare the performance of the proposed global modeling approach to its local counterpart in terms of MAPE and NMAE, respectively, using the same experimental setup as in Section IV-C. Thus, the global model is built based on the first step of the proposed approach only. Note that for the MAPE results, we don’t report forecasting performance for the individual consumers, as this error cannot be computed for a single time series. For the NMAE error, performance is reported for all aggregate types.

TABLE V: Performance evaluation of the local and global modeling frameworks to STLF using MAPE. All\textsuperscript{1} indicates the mean MAPE across all aggregate types but without the individual consumers.

| Modeling framework | Single | sTS | mTS | ITS | All\textsuperscript{1} |
|--------------------|--------|-----|-----|-----|----------------|
| Local modeling     | n/a    | 10.2987 | 8.9845 | 7.5793 | 8.9542 |
| Global modeling    | n/a    | 9.7199 | 8.2061 | 6.8396 | 8.2552 |

TABLE VI: Performance evaluation of the local and global modeling frameworks to STLF using NMAE. The global modeling framework yields an overall performance improvement and also ensures performance gains across all aggregate types except the individual consumers.

| Modeling framework | Single | sTS | mTS | ITS | All |
|--------------------|--------|-----|-----|-----|-----|
| Local modeling     | 0.5688 | 0.1069 | 0.0910 | 0.0746 | 0.2103 |
| Global modeling    | 0.5708 | 0.1025 | 0.0857 | 0.0701 | 0.2088 |

Overall, the results show similar behavior as with the MASE scores. Global modeling in general improves performance for all aggregate types with both MAPE and NMAE scores. The only exception here are the results for the individual consumers when forecasting performance is measured in terms of NMAE. Due to the variability of this type of time series, the local modeling has a slight edge over the global modeling. However, as we show in the next section, using a localization procedure on top of the global model results in superior performance on this type of time series as well.

B. Impact of Localization

To further demonstrate the importance of localization in the global modeling framework, Tables VII and VIII summarize the forecasting performance of the global modeling framework with and without localization in terms of MAPE and NMAE scores, respectively. The reported results correspond to the proposed ensemble-based localization procedure.

As can be seen, the localization improves performance across all aggregate types, with the biggest performance gains being observed for the time series that correspond to the individual consumers (Single). These observations are consistent with the observations made in the main part of the paper, where MASE was used as a performance indicator instead of MAPE and NMAE. The results show that the global model, after localization (as proposed in this paper), convincingly outperforms the local modeling framework regardless of the aggregate type.

C. Additional Clustering Analyses

In the main part of the paper, we presented an analysis of the clustering procedure needed for the model localization step and showed that a sufficient clustering granularity is needed to ensure that the localization mechanism actually contributes to better forecasting performance. Here, we report additional analyses of the clustering-based localization strategies to further justify the design choices made with our global forecasting approach. We again report all results by considering MASE as the main performance indicator and use the same experimental protocol as in Section IV.

First, we analyze how well a particular number of clusters performs across the time series in the input set. For this experiment, we first determine the optimal number of clusters for each time series separately using the $K$-Means+TF+BEST strategy and then observe for what fraction of time series a certain number of clusters results in the best overall performance. The results of this analysis are reported in Fig. 10 - separately for each aggregate type. It can be seen that for the individual consumers, for example, the model localization procedure with 19 clusters is the optimal choice for 14% of the time series in this group. In the case of sTSs, mTSs and ITSs, we can see that a single cluster (i.e., the global model without localization) provides the best forecasting performance in most cases (for 24.8%, 56.4%, 75.6% of all cases for the sTSs, mTSs and ITSs, respectively). These results suggest that
model localization with a larger number of time-series subsets is more important for more volatile time series, but also that an adaptive procedure is needed to ensure good performance across time series with diverse characteristics, such as those caused by different aggregate types. Such a procedure is, for example, provided by the proposed ensemble approach.

Last but not least, we perform a detailed analysis of the proposed ensemble approach, i.e., $K$-Means+TF+ENS. Our approach averages the partial forecasts from Eq. (10) and\(^3\), therefore, combines the predictions from multiple localized models that capture the characteristics of the input time series data at different levels of granularity. The results in Fig. 11 again show the fraction of time series, for which the combination of a certain number of models results in optimal performance. It can be seen that for 66% of time series corresponding to individual consumers the combination of models from two levels in the cluster hierarchy in the ensemble yields the best forecasts. For $s$TSs and $m$TSs the combination of models from three levels is optimal for 54% and 50% of the time series, respectively. In the case of $l$TSs the combination of 4 models is optimal for 42.4% of the time series. These results suggest that combining models using an adaptive strategy that is able to account for specific data characteristic is important for the final forecasting performance and is, therefore, utilized in the proposed approach.

\(^3\)Recall that each partial forecast is generated from the localized models from one level in the cluster hierarchy.