Hub Network Design in a Cooperative Environment

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Abstract. As an important strategic decision, the design of the hub network directly affects the transportation efficiency and costs of the entire network. Sharing resources to reduce costs through cooperation is the future development of logistics and transportation networks. From the perspective of horizontal cooperation, this paper focuses on the hub location problem of two carriers sharing hub facilities in a cooperative environment. A mixed-integer programming model with minimum total network cost in a cooperative environment is constructed. Numerical examples demonstrate that the supplementary cost due to cooperation will affect the result of cooperation. When the supplementary cost is within a specific range, the cooperative network is more cost-effective than non-cooperative networks.

1. Introduction
Nowadays, there is a rapid increase in transshipment network design research in the literature. Almost all logistics transportation between countries, regions, and cities needs the support of the transshipment network. Hub structure is typical in transshipment networks that benefit from economies of scale. As an important strategic decision, the design of the hub-and-spoke network will consume a lot of time and costs, and the impact on transportation efficiency will last for a long time. Therefore, it is crucial to select the appropriate node as the hub facility of the entire network for the collection, large-scale transfer, and distribution. Nowadays, the logistics and transportation market are fiercely competitive. However, horizontal cooperation between companies can bring substantial benefits. At present, many logistics companies such as the Federal Express, China Post, and other small logistics companies are trying to explore the cooperation model.

Since the seminal paper by O'Kelly [1], many variants of hub location problems have been studied in the last few decades. After more than thirty years of development, it is still the focus of research in academia. Various criteria can be used to classify hub location problems. One is based on the allocation scheme, which defines the way of allocating non-hubs to hubs. Two primary allocation schemes are distinguished: single allocation and multiple allocation. Some scholars have investigated the hub location problems satisfying certain capacity constraints under a single allocation model [2-3] and multiple allocation model [4]. To hedge against uncertainty in the supply chain, the number of literatures that use robust optimization methods to solve hub location problems under single allocation and multiple allocation is continuously increasing [5]. Another criterion is based on hub capacity. Most studies on hub location problems usually assumed that the capacity of the hub is unlimited. The capacity of the hub will directly affect the transportation capacity and cost of the entire network. The hot issues emerging in hub location problems is the decision of the hub capacity. There is much research focusing on the capacity decision of the hub [6-8]. Furthermore, Rastani et al. [9] further considered the capacity limitation of the transfer arc.

In the design of the hub location problem, there are few studies on the cooperation model. However, multiple participating companies sharing resources through cooperation can not only improve...
transportation efficiency but also reduce service prices to enhance competitiveness. Many scholars have clarified the importance of cooperation in the supply chain and believed that horizontal cooperation is becoming a way to improve the competitiveness of logistics and transportation [10-11]. Research on cooperation has emerged in the past year or two. A horizontal cooperation method for multiple companies to jointly solve the logistics optimization problem is proposed [12-14]. Numerical experiments showed that cooperation could save up costs under certain circumstances. Depending on the degree of cooperation, Quintero-Araujo et al. [15] studied the decision-making of route and hub facility location in road transportation under three cooperation schemes of non-cooperation, semi-cooperation, and full cooperation. They analyzed how to improve the flexibility of the network to reduce the total allocation cost. However, most of the mentioned studies focus on the path or the entire process, and few horizontal cooperative studies focus on hub location.

Therefore, based on the model of capacitated hub location problem for multiple allocation. This paper studies from the perspective of horizontal cooperation and constructs a mixed-integer programming model with the target of minimizing the total cost in a cooperative environment. What needs to be determined is the location and attributes of the hub, that is, whether the hub can be shared or not. Finally, the network costs of cooperation and non-cooperation are compared. A sensitivity analysis of the essential parameters is performed.

The rest of the paper is organized as follows. In Section 2, we introduce the capacitated multiple allocation formulation in the cooperative environment used in this research. Section 3 describes the computational performance of the proposed formulations on the Australian Post (AP) dataset, and finally, Section 4 summarizes the conclusions and provides future research directions.

2. Problem Description and Model Construction

2.1. Problem Description
There are two carriers in a hub network, namely A and B. In the entire hub network, the two carriers A and B are in charge of a part of the demand points, including non-hub nodes and hub nodes. The traffic flow in the hub network is implemented by a pair of OD pairs. Each pair of OD pairs is connected by at least one hub and at most two hubs, and only the capacity limitation of the first hub needs to be considered. Each non-hub node can transmit or receive traffic flow from multiple hub nodes, which is multiple allocation mode.

2.2. Model Construction
The following notations are introduced for the model parameters and variables:

- \( N \) : set of nodes
- \( f_{ij} \) : amount of flow from origin \( i \) to destination \( j \)
- \( d_{ij} \) : distance from node \( i \) to node \( j \)
- \( \chi, \alpha, \delta \) : collection, transfer and distribution coefficients
- \( F_k \) : setup cost of establishing a hub at \( k \)
- \( Q_k \) : capacity of hub \( k \)
- \( X_{ik} \) : flow from origin \( i \) to hub \( k \)
- \( Y_{ikl} \) : flow from origin \( i \) via hub \( k \) and \( l \)
- \( Z_{ij} \) : flow from origin \( i \) to destination \( j \) via hub \( l \)
- \( H_k \) : if node \( k \) is set to be a hub, \( H_k = 1 \); otherwise, \( H_k = 0 \)
- \( I_k \) : if a hub located at \( k \) is cooperative, \( I_k = 1 \); otherwise, \( I_k = 0 \)
- \( T_k \) : if a hub located at \( k \) is non-cooperative, \( T_k = 1 \); otherwise, \( T_k = 0 \)

Among them, \( H_k = I_k + T_k \). Besides, the cooperative hubs are used by two carriers together, and the supplementary cost needs to be considered. Since forming cooperation incurs not only benefits but
also the costs of hub management and instability of partners. At this time, the costs of the cooperative hubs are not only construction costs, but also supplementary costs brought by cooperation. Therefore, we introduce a supplementary cost factor $p$ to quantify supplementary costs. The Optimized Cooperation (OC) model constructed with reference to Habibi [16] is as follows:

$$
\min z = \sum_{k \in N} (1 + p)F_k H_k + \sum_{i \in I} \sum_{k \in N} c d_{ik} X_{ik} + \sum_{k \in N} \sum_{i \in I} a d_{ik} Y_{ik} + \sum_{j \in L} \sum_{i \in I} \delta d_{ij} Z_{ij})
$$

Subject to

$$
\sum_{k \in N} X_{ik} = \sum_{j \in J} f_{ij}, \forall i \in N (2)
$$

$$
\sum_{j \in J} Z_{ij} = f_{ij}, \forall i, j \in N (3)
$$

$$
\sum_{i \in I} X_{ik} \leq Q_i H_k, \forall k \in N (4)
$$

$$
\sum_{i \in I} Y_{il} + \sum_{j \in J} Z_{ij} = \sum_{i \in I} Y_{ik} + X_{ik}, \forall i, k \in N (5)
$$

$$
X_{ik} \leq \sum_{j \in J} f_{ij} H_k, \forall i, k \in N (6)
$$

$$
Z_{ij} \leq \sum_{j \in J} f_{ij} H_i, \forall i, j \in N (7)
$$

$$
H_k = I_k + T_k, \forall k \in N (8)
$$

$$
X_{ik} \leq M(1 - T_k), \forall i \in N_A, k \in N_B (9)
$$

$$
Z_{ij} \leq M(1 - T_j), \forall i, j \in N_A, l \in N_B (10)
$$

$$
Y_{il} \leq M(1 - T_k), \forall i, l \in N_A, k \in N_B (11)
$$

$$
Y_{il} \leq M(1 - T_j), \forall i, k \in N_A, l \in N_B (12)
$$

$$
Y_{il} \leq M(1 - T_k)(1 - T_l), \forall i \in N_A, k, l \in N_B (13)
$$

$$
X_{ik}, Y_{il}, Z_{ij} \geq 0, \forall i, j, k, l \in N (14)
$$

$$
I_k, T_k, H_k \in [0, 1], \forall k \in N (15)
$$

The objective function (1) minimizes the total costs when two carriers cooperate. The first term is the setup cost of establishing hubs, and the second term is the transportation cost. Constraints (2) enforce the flow from an origin to its first hubs to be equal to the amount demanded by its destinations. Constraints (3) require the flow from an origin to its destination via all opened hubs that meet the demand of the pair. The capacity limitation of an opened hub is respected by constraints (4). Constraint (5) ensure the balance of the inflow and outflow of the hub. Constraints (6) and (7) ensure that only when the hub is established can traffic flow pass through. Constraint (8) restrict the attributes of the hub, and the established hub can either be shared or non-shared. Constraints (9-13) also restrict the attributes of the hub. When the established hub cannot be shared, no traffic will pass between the hub node and the non-hub node, and between the hubs. The constraint (14) and (15) define the domain of decision variables.
Caused by the two binary variables, constraint (13) is nonlinear. To comply with the commercial solver, constraint (13) have to be linearized. These constraints assert that both $kT$ and $lT$ have to be 0 in order to allow $Y_{ikl}$ being positive. Otherwise, it is equal to zero. Therefore, the model is linearized as follows:

The binary variable $B_{ikl}$ is introduced, and the constraint (13) can be replaced by the following equation:

$$Y_{ikl} \leq M(1-T_i -T_l + B_{ikl}) \quad \forall i \in N_A, k,l \in N_B$$  \hspace{1cm} (13.1)

$$B_{ikl} \leq (T_i + T_l)/2 \quad k,l \in N_B$$  \hspace{1cm} (13.2)

$$B_{ikl} \geq T_i + T_l - 1 \quad \forall k,l \in N_B$$  \hspace{1cm} (13.3)

$$B_{ikl} \in \{0,1\} \quad \forall k,l \in N_B$$  \hspace{1cm} (13.4)

When $T_i$ and $T_l$ are both zero, the constraint (13.2) and (13.3) limit $B_{ikl}$ to zero, and then from the constraint (13.1), $Y_{ikl}$ is valid. Therefore, constraints (13.1) - (13.4) can be used to linearize constraints (13).

3. Numerical Experiment

We use the Australian Post (AP25) dataset, a popular and well-established dataset in the context of hub location literature. The 25 nodes have been selected for testing the formulation of Optimized Cooperation (OC) model and No Cooperation (NC) model. The No Cooperation (NC) model is a capacitated hub location model under multiple allocation.

The construction cost, maximum capacity limit of each hub, traffic flow, and the transportation distance between each hub were obtained from OR-Library. The cost coefficients $\chi$, $\alpha$, and $\delta$ per unit time per unit distance are set to 3, 0.75, and 2, respectively.

In order to consider the uncertainty of the supplementary costs, this paper considers the supplementary cost factor $p$ in six cases. Select $p = \{0, 0.25, 0.5, 0.75, 1, 1.25\}$ to explore the impact of additional supplementary costs on the network. The two carriers share some hubs in the network but remain in charge of the demand in their respective networks. The 25 points are categorized into two parts randomly, which are in the charge of carrier A and carrier B respectively. And the selection result is $A = \{3, 5, 6, 7, 8, 11, 14, 16, 17, 21, 22, 25\}$, and $B = \{1, 2, 4, 9, 10, 13, 18, 19, 20, 23, 24\}$. According to randomly assigned demand points, CPLEX is used to analyze the situation of cooperation and non-cooperation, respectively. The hub location results are shown in Table 1:

|        | P=0   | P=0.25 | P=0.5  | P=0.75 | P=1   | P=1.25 |
|--------|-------|--------|--------|--------|-------|--------|
| **NC** |       |        |        |        |       |        |
|        | A = \{14\} | B = \{9 13 24\} |       |        |       |        |
| **OC** | 6 14 25 | 9 14 16 24 | 9 14 16 24 | 9 14 16 24 | 9 14 16 24 | 9 13 14 24 |
| (T*)   | (16 24) | (9 16 24) | (9 16 24) | (9 16 24) | (9 16 24) | (9 13 14 24) |

*a Hubs that cannot be shared in the selected hub.

It can be seen from Table 1 that as the supplementary cost increases, the optimal hub locations in the cooperative network gradually approaches the hub locations in the non-cooperative network. When the supplementary cost is zero, the cooperation does not increase the setup cost of the hub. All the optimal hubs can cooperate, and the two carriers can share all hubs. However, with the increase of supplementary cost, hubs that cannot be shared gradually appear in the cooperative network. In other words, the hub that cannot be shared can only be used by the carrier to which it belongs, and the remaining shared hubs can still be shared. At the same time, it can be found from the table 1 that when the supplementary cost is 1.25, the hubs decided in the cooperative network are not allowed to share at this time. The selected hubs in cooperative networks are the same as the hubs selected in the non-
cooperative network, which means the cooperative network has completely become a non-cooperative network.

Besides, with the change of supplementary cost, the total cost of the cooperative network will inevitably be changed. The changing trend of the cooperation cost of the network is shown in Figure 1. As the supplementary cost increases, the cooperation cost of the cooperative network is increasing.

![Figure 1. Cooperation cost trend of NC and OC](image)

**4. Conclusion**
Cooperation and sharing of resources is the future development direction of the hub network. Based on the capacitated multiple allocation hub location model, this paper proposes a hub location model in a cooperative environment. Considering the additional cost of establishing a shared hub due to cooperation, we introduce the concept of supplementary cost and calculate the total network cost under different supplementary costs. The numerical experiment showed that the total cost of the network increased as the supplementary cost increases, and the network will gradually change from cooperation to complete non-cooperation. We also found that the cooperative network will reduce the cost by about 14% compared with the non-cooperative network, and the additional supplementary cost during cooperation will affect the result of cooperation.

Some prospects for future works are identified. Our work considers two carriers, and there are always multiple players in reality. The cooperation of three or more players is worth further study in the future. Also, in the face of an uncertain environment, the capacity decision of the hub needs to change with the situation. Hence, cooperation becomes more difficult, which is also one of the contents that can be studied in the future.

**5. Acknowledgments**
This work was supported by the Natural Science Foundation of Shanghai (No. 18ZR1409400) and the Ministry of Education Humanities and Social Science Research Planning Fund Project (No. 18YJAZH046).

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