Predicting relative energy dissipation for vertical drops equipped with a horizontal screen using soft computing techniques

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ABSTRACT

This study was designed to evaluate the ability of Artificial Intelligence (AI) methods including ANN, ANFIS, GRNN, SVM, GP, LR, and MLR to predict the relative energy dissipation ($\Delta E/E_0$) for vertical drops equipped with a horizontal screen. For this study, 108 experiments were carried out to investigate energy dissipation. In the experiments, the discharge rate, drop height, and porosity of the screens were varied. Parameters $y_c/h$, $y_d/y_c$, and $p$ were input variables, and $\Delta E/E_0$ was the output variable. The efficiencies of the models were compared using the following metrics: correlation coefficient (CC), mean absolute error (MAE), root-mean-square error (RMSE), Normalized root mean square error (NRMSE) and Nash–Sutcliffe model efficiency (NSE). Results indicate that the performance of the ANFIS_gbellmf based model with a CC value of 0.9953, RMSE value of 0.0069, MAE value of 0.0042, NRMSE value as 0.0092 and NSE value as 0.9895 was superior to other applied models. Also, a linear regression yielded CC $= 0.9933$, RMSE $= 0.0083$, and MAE $= 0.0067$. This linear model outperformed multiple linear regression models. Results from a sensitivity study suggest that $y_c/h$ is the most effective parameter for predicting $\Delta E/E_0$.

Key words: drop, relative energy dissipation, screen, soft computing

HIGHLIGHTS

- In this study, 108 experiments were conducted to investigate the relative energy dissipation ($\Delta E/E_0$) for vertical drops equipped with a horizontal screen.
- Intelligent models such as methods including ANN, ANFIS, GRNN, SVM, GP, LR, and MLR are applied to evaluate relative energy dissipation in vertical drops equipped with a horizontal screen.

INTRODUCTION

Screens are structures that are used to dissipate energy and reduce the length of the settling basin in water flows (Rajaratnam & Hurtig 2000). These structures are not a substitute for settling basins; however, they are used when settling basins are not technically or economically viable. Screens are frequently found to dissipate energy downstream of a hydraulic structure thanks to both the hydraulic jump and enhanced turbulence. To reduce kinetic energy, vertical drops are utilized. The flow under consideration is sub-critical and one of the most important design inputs is the inlet flow. However, in some cases, supercritical flow is also possible.

Extensive experimental and numerical studies have been conducted by many researchers to assess energy dissipation for these devices; (Rajaratnam & Hurtig 2000; Chamani et al. 2008; Kabiri-Samani et al. 2017; Crispino et al. 2019; Crispino et al. 2021; Daneshfaraz et al. 2021) are some examples.

Rouse (1936) pioneered research on this topic and presented an equation to calculate discharge based on the brink depth. The impact of supercritical flow upstream of a vertical drop on the hydraulic parameters was studied by Chamani & Beirami (2002). They found that the Froude number for a constant discharge was inversely proportional to the relative pool depth, downstream depth, and energy dissipation. The impact of the slope of the screens on the energy dissipation was studied by Balkiş (2004). It was found that the energy dissipation rate was unaffected by the slope.

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Esen et al. (2004) studied the effect of various physical dimensions of a drop with an upstream subcritical flow. It was observed that stairs located downstream of a vertical drop enhanced the relative downstream depth and energy dissipation. The influence of downstream slopes on the hydraulic performance of a vertical drop was assessed by Hong et al. (2010). Four different vertical drop slopes were tested and the results revealed that the slope length was increased by enhancing the slope.

Sadeghfam et al. (2015) investigated dissipation of energy in an inundated hydraulic jump with dual vertical screens. They found that the energy dissipated by the dual vertical screens is increased thanks to the formation of a hydraulic jump. Kabiri-Samani et al. (2017) and Sharif & Kabiri-Samani (2018) explored the working of hydraulic grid drop-type dissipators with regard to downstream depth and found that the relative downstream depth of the pool and energy dissipation increased with the use of grid drop-type dissipators when compared to a vertical drop.

An experimental study was employed by (Daneshfaraz et al. 2020a, 2020b, 2020c) to investigate the impact of dual horizontal screens on hydraulic behavior. The results revealed that the mechanism of flow can be transformed from supercritical to subcritical downstream of a drop through the utilization of dual horizontal screens. Using vertical screens with two porosity ratios located downstream of inclined drops was investigated by (Daneshfaraz et al. 2020a, 2020b, 2020c). Results showed that compared to a plane inclined drop, the screen also caused an increase of at least 407% and up to 903% in total relative energy dissipation. Daneshfaraz et al. (2021a, 2021b, 2021c) studied energy dissipation in gabion inclined drops. The results indicated that In all cases, downstream Froude number of the gabion inclined drop range is reduced from 4.17–8.52 to 1.28–2.64. Daneshfaraz et al. (2021a, 2021b, 2021c) investigated effect of horizontal screen diameter on hydraulic parameters of vertical drop. The finding reveal that the relative wetting length increased by increasing the relative diameter of the horizontal screen. Daneshfaraz et al. (2022) Investigated the performance of a vertical drop with a screen. Outcom of this study shows that drops equipped with screens increase the relative downstream depth.

Recently, dissipation of energy in hydraulic structures was estimated using machine learning techniques. Norouzi et al. (2019a, 2019b) employed an adaptive neuro-fuzzy inference system (ANFIS) to study the dissipation of energy among inclined drops equipped with downstream vertical screens. The results of their study revealed that the ANFIS model values of coefficient of determination ($R^2$) and root mean square error (RMSE) are 0.996 and 0.006, respectively. Thus, ANFIS has an outstanding ability to quantify the dissipation of energy related with inclined drops equipped with downstream vertical screens.

In spite of the extensive prior work, there is a paucity of research on the use of an intelligent model for estimating energy dissipation as fluid flows over a drop. Thus, the present study utilizes independent intelligent models such as Artificial Neural Networks (ANN), Adaptive Neuro-Fuzzy Inference System (ANFIS), Support Vector Machine (SVM), Generalized Regression Neural Network (GRNN), Gaussian Process regression (GP), and both multiple linear and nonlinear regression methods to evaluate energy dissipation. The ability of these methods to calculate energy dissipation will be the focus of the present study.

**METHODS**

**Experimental Set-up**

Experiments were conducted in a laboratory cascade which was 5 m in length, 0.3 m in width and with a variable depth ranging from 0.45 to 0.75 m. The test facility is located in the hydraulic laboratory at the University of Maragheh. Two pumps with a capacity of 450 liters per minute were used to generate and regulate the inlet flow. The flow was measured using a rotameter mounted on the pump with ±2% accuracy. The inlet flow was pumped into the upstream reservoir by the pump and entered the flume bypassing the baffle. Water depths were measured at five points along the channel width with a ±1 mm precision point gauge. The features of the laboratory model employed in the present study are presented in Table 1 and shown in Figure 1.

**Table 1** | Specification of the laboratory model

| Vertical drop | Screen |
|---------------|--------|
| Type          | Height (m) | Width (m) | Length (m) | Type | Porosity (%) | Thickness (m) | Length (m) |
| Glass         | 0.15-0.2-0.25 | 0.3 | 1.2 | Polyethylene | 40–50 | 0.01 | 0.7 |
108 experiments were conducted to assess dissipation of energy with varying flow rates, drop heights, and porosities of the horizontal screens.

**Calculation of energy dissipation**

The energy upstream of the vertical drop was calculated using Equation (1) from (Bakhmeteff 1932):

\[ E_u = 1.5y_c + h \]  

(1)

where \( E_u \) denotes the total energy upstream of the drop, \( h \) denotes the drop height, and \( y_c \) denotes critical depth upstream of the drop. The energy downstream of the drop is calculated using Equation (2):

\[ E_d = y_d + \frac{q^2}{2gy_d} \]  

(2)

In Equation (2), \( E_d \) denotes the total energy downstream of the vertical drop, \( y_d \) denotes the downstream depth, \( g \) is the acceleration of gravity, and \( q \) refers to the discharge per unit width. Then, for calculating the relative energy dissipation for the vertical drop structures, Equation (3) was used.

\[ \frac{\Delta E}{E_u} = \frac{E_u - E_d}{E_u} \]  

(3)

**Dimensional analysis**

Next, attention is turned to the relationship of the flow rate to the parameters on which it depends. The geometric and hydraulic parameters affecting the flow are given in Equation (4)

\[ f_1(\rho, \mu, g, Q, h, p, t, y_c, y_u, y_b, y_p, L_{mix}, \Delta E, E_u) = 0 \]  

(4)

where \( \rho \) represents the water density, \( \mu \) is the dynamic viscosity, \( Q \) signifies the discharge, \( t \) is the screen thickness, \( L_{mix} \) the mixing length, \( p \) the screen porosity ratio, \( y_u \) is the upstream drop depth, \( y_p \) the pool depth under the falling jet, \( y_b \) the drop brink depth, and \( E_u \) is the total energy upstream of the drop.

Considering \( y_u, \rho \) and \( g \) as the repeating variables, the dimensionless Equation (5) is obtained through Buckingham \( \pi \) theorem as follows:

\[ f_2 \left( Re_u, Pr_u, \frac{h}{y_u}, \frac{t}{y_u}, \frac{y_c}{y_u}, \frac{y_b}{y_u}, \frac{y_p}{y_u}, \frac{L_{mix}}{y_u}, \frac{\Delta E}{E_u}, \frac{E_u}{y_u} \right) = 0 \]  

\[ \frac{\Delta E}{y_u} = \frac{E_u - E_d}{E_u} = \frac{\Delta E}{E_u} \]  

(5)
with some simplification, Equation (6) is obtained:

\[
\hat{f}_3\left(Re_u, Fr_u, \frac{f}{H}, \frac{y_c}{H}, \frac{y_b}{H}, \frac{y_d}{H}, \frac{L_{mix}}{H}, \frac{\Delta E}{E_u}\right) = 0
\]  

(6)

where \(Re_u\) is upstream Reynolds number and \(Fr_u\) is upstream Froude number. Considering that the Reynolds number range is 10000–35000, the flow is fully turbulent and viscous effects can be neglected. The Froude number range for subcritical flow (Grant & Dawson 2001) was determined by measuring the upstream depth which was found to be in the range 0.68–0.84 for all tests. Consequently, the effect of Froude number can also be neglected (Cakir 2003; Kabiri-Samani et al. 2017).

The dependent parameters were derived as a function of the independent parameters in light of the preceding discussion and simplifications, as shown here:

\[
\frac{\Delta E}{E_u} = \hat{f}_3\left(\frac{y_c}{H}, \frac{f}{H}, \frac{y_d}{y_c}\right)
\]  

(7)

In Equation (7), \(\Delta E/E_u\) is the relative energy dissipation. The term \(y_c/H\) is the relative critical depth. The non-dimensional ratio of the relative critical depth is in the range of 0.07–0.39 and the porosity ratio of the screens used in this study are 40 and 50%.

**Artificial neural networks (ANN)**

ANN is a soft computing-based approach which can be utilized to estimate energy dissipation. The WEKA software was used to develop the ANN model which utilizes an iterative method. Factors such as momentum coefficients, learning rate, number of hidden layers, and neurons control the networks. With the ANN approach, knowledge is represented in terms of a complex weight matrix that renders it a black-box to the user. In addition, network parameters are required to be found by a trial-and-error method that is very time-consuming. Haykin (2010) has provided a detailed theoretical description of ANN. However, ANN is easy to use and it can approximate any input/output map (Salmasi et al. 2013).

Prior research, such as (Malik et al. 2020) and (Nou et al. 2019) have used the ANN method to model a monthly pan evaporation process which calculated scour depths but to our knowledge, this method has not year been applied to a problem similar to that of the current paper. Figure 2 depicts the structure of ANN with three layers including input, middle, and output layers.

**Adaptive neuro-fuzzy inference system (ANFIS)**

An adaptive network is a multilayer structure whose overall output behavior is determined by the value of a set of modifiable parameters. The adaptive network structure is comprised of a calibration of interconnected nodes where each node is considered a processing unit. The key problem in fuzzy systems design (if-then rules) are resolved through by making active use of ANN’s automatic production of rule-learning capability and parameter
optimization. The two rules of this system are also expressed by Equations (8) and (9):

If \( x \) is \( A_1 \) \& \( y \) is \( B_1 \) \& \( z \) is \( C_1 \)
Then \( f = p_1x + q_1y + k_1z + r_1 \) \hspace{1cm} (8)

If \( x \) is \( A_2 \) \& \( y \) is \( B_2 \) \& \( z \) is \( C_2 \)
Then \( f = p_2x + q_2y + k_2z + r_2 \) \hspace{1cm} (9)

One of the most regularly used fuzzy nervous systems which runs a Sugeno fuzzy system in a neural structure is ANFIS. It employs back-propagation training and minimum error squares in the training process. It is important to identify the type association function and its number in the first layer of ANFIS. Two methods of network separation and cluster separation are used in the first layer. This network includes functions such as \( \text{Trimf}, \text{Gausmf}, \text{Gbellmf}, \text{and Trapmf} \). The adaptive neural fuzzy model operates based on the change in the number of center values and the range of belonging functions in different iterations to reach the appropriate network based on the minimum error present. The ANFIS model includes the ability to estimate the rectangular side weir discharge capacity (Bilhan et al. 2013), predict side weir discharge coefficients (Shamshirband et al. 2015), and study energy decreases of the drop (Norouzi et al. 2019a, 2019b). A schematic representation of the structure of the ANFIS methodology is illustrated in Figure 3.

Support vector machine (SVM)
The SVM algorithm is one branch of kernel methods in machine learning. This type of learning system is utilized for classifying and estimating data fitting functions to minimize errors. (Vapnik 1995) developed the SVM algorithm which is based on statistical learning theory. The role of the support vector machine is to identify a function \( f(x) \) for training patterns in such a way that it has the maximum margin of training values \( y \). Thus, the SVM is a model that fits a curve of thickness \( \varepsilon \) to the data so that minimal error is observed in the fit to the experimental data. The estimation of the relationship between the dependent variable \( y \) and a set of independent variables \( x \) is necessary in a SVM regression model. The relationship between the dependent variables and \( f \) is:

\[ y = f(x) + \text{noise} \] \hspace{1cm} (10)

The function \( f \) is generated by training the SVM model on a training dataset. The training embraces a process which permanently optimizes the error function. There are two well-known models for SVM-type regression. One is known as an SVM-v model and another is the second-order SVM regression model known as SVM-\( \varepsilon \). The present study utilizes the SVM-\( \varepsilon \) model due to its widespread and successful usage in regression problems. The error function for this model is defined as follows:

\[ \frac{1}{2} W^T W + c \sum_{i=1}^{N} \eta_i^2 + c \sum_{i=1}^{N} \xi_i^0 \] \hspace{1cm} (11)
The error function should be minimized using the following constraints.

\[
W^T \Phi(x_i) + b - y \leq e + \xi_i^0
\]

\[
y_i - W^T \Phi(x_i) - b \leq e + \xi_i^0
\]

\[
\xi_i, \xi_i^0 \geq 0
\]

\[
i = 1, \ldots, N
\]

(12)

where, \(c\) denotes the capacity constant, \(W\) denotes the coefficient vector, \(W^T\) is the vector of the coefficients, \(\xi_i\) and \(\xi_i^0\) denote the deficiency coefficients, \(b\) is a constant, \(N\) denotes the training model, and \(\Phi\) is the kernel function. Amongst other kernel functions, the radial basis function kernel is the best choice. In this study, the following relation is applied:

\[
K(x_i, x) = \exp \left( -\gamma ||x_i - x||^2 \right)
\]

(13)

Diversified SVM model results have been utilized to predict vertical drop hydraulic parameters in the presence of dual horizontal screens (Daneshfaraz et al. 2020a, 2020b, 2020c). SVM models have also been used to investigate labyrinth and arched labyrinth weir discharge coefficients (Roushangar et al. 2017) and to study trapezoidal labyrinth weir discharge coefficients (Norouzi et al. 2019a, 2019b).

**Generalized regression neural network (GRNN)**

A GRNN neural network is a basic radial neural network and a general-purpose approximator for smooth functions. Provided adequate data is accessible, GRNN is capable of smoothly approximating any function. It is a three-layer neural network wherein the number of neurons in the first and the last layers equal with the dimensions of the input and output vectors. However, the number of hidden layer neurons in the GRNN model is similar to the number of observational data – unlike other networks. The normal efficacy function is utilized in each of the hidden layer neurons and the input data to this function for each neuron is the Euclidean distance between the input data and the observations of that neuron. This calculation is facilitated by Equation (14).

\[
f(X_r, b) = e^{-I^2}
\]

\[
I = ||X_r - X_b|| \times 0.836/h
\]

(14)

where, \(X_r\) signifies a vector input to the network with an unknown output, \(X_b\) signifies the input value of the observations in time and \(b\) and \(h\) of the radius parameter. Altering \(h\) will cause a change in the value of the function, and the function will find the best fit to the data. Output values of this function are in the range 0–1 so that as the Euclidean distance between the two vectors \(X_r\) and \(X_b\) approaches zero, the value of the function approaches one. As the vectors differ, the value of the function approaches zero. The GRNN neural network uses Equation (15) to calculate the output.

\[
Y_r = \frac{1}{n} \sum_{b=1}^{n} \left[ f(X_r, b) \times T_b \right]
\]

(15)

where \(T_b\) is the output value of the observations corresponding to the input vector \(b\) and \(n\) is the number of observational data.

Several prior research studies have used the GRNN method to estimate stream water temperature (Grbic et al. 2013), soil temperature (Mihoub et al. 2016a, 2016b), and GPK Weir discharge coefficients (Akbari et al. 2019).

The structure of the GRNN neural network is shown in Figure 4.

**Gaussian process model (GP)**

Gaussian Process regression models (GP) are used as a statistical tool in data-rich problems and group learning (Guermoui et al. 2013; Vapnik 2013). The basic aim of the GP technique is to prioritize the space functions (Mihoub et al. 2016a, 2016b). The GP method has been used for diverse applications including the calculation
of water temperatures (Grbic et al. 2013) and the determination of weir discharge coefficients (Akbari et al.
2019).

Linear regression (LR)
Regression analysis is a statistical method where in the relationship between one or more independent variables is used
to predict the response of a dependent variable. The expression for a multiple linear regression model is as follows:

$$y = \sum_{i=1}^{N} \beta_i X_i + \varepsilon \quad (16)$$

where $$\beta_i$$ signifies the regression coefficients, $$X_i$$ is the independent variable, $$\varepsilon$$ is eccentricity and $$N$$ is the number of independent variables. The estimation of regression coefficients is obtained through the least-squares method. LR has
been used in previous applications, such as for trapezoidal labyrinth side weir discharge coefficients (Emiroglu & Kisi
2013) and evaporation modeling (Shirgure 2012). Figure 5 displays a schematic diagram showing the algorithm.

Multiple linear regression (MLR)
Multiple non-linear regression (MLR) is applicable in cases with more than one predictor parameter. The MLR
model’s basic structure is as follows:

$$R = c_0 x_1^{c_1} x_2^{c_2} x_3^{c_3} \ldots \ldots \ldots x_n^{c_n} \quad (17)$$

where $$R$$ is the dependent variable and $$X_1, X_2, \ldots, X_n$$ are independent variables.

Performance measures
For assessing model performance, the correlation coefficient (CC), mean absolute error (MAE), root-mean-square
error (RMSE) Normalized root mean square error (NRMSE) and Nash–Sutcliffe model efficiency (NSE) are used.
The accuracy of the regression line matching the observed data is statistically measured through CC ranging from
$$-1$$ to $$+1$$. A coefficient of $$\pm 1$$ indicates that the regression line perfectly fits the observed data. The RMSE and
MAE provide a balanced evaluation of the goodness of fit of the model. These quantities are sensitive to large
relative errors that occur with low values; the perfect model would have zero value. These performance measures
are calculated by (Ghorbani et al. 2013):

$$CC = \frac{n \sum a \sum b - \left( \sum a \right) \left( \sum b \right)}{\sqrt{n(\sum a^2) - \left( \sum a \right)^2} \sqrt{n(\sum b^2) - \left( \sum b \right)^2}} \quad (18)$$

$$MAE = \frac{1}{n} | a - b | \quad (19)$$
\[ RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (a - b)^2} \]  
\[ NRMSE = \frac{RMSE}{\bar{a}} \]  
\[ NSE = 1 - \frac{\sum_{i=1}^{n} (a - b)^2}{\sum_{i=1}^{n} (a - \bar{a})^2} \]

where the symbols \( a \) and \( b \) represent the actual and predicted values, respectively. The term \( n \) indicates the number of observations and \( \bar{a} \) shows the average of the actual values.

**RESULTS**

In this study various soft computing methods were used for the prediction of \( \Delta E/E_u \) using training and testing data set for model development and validation respectively. The dataset (108 measurements) was divided into two groups. The larger group totaled 74 observations whereas the smaller group had 34 observations. The first group's 74 observations were used for the model development while the second group of 34 observations were used for validation. This training/testing process helps reduce the error and regulates against overfitting. Table 2 lists the correlation matrix of the total data set and Table 3 provides the ranges and features for both parts. The symbols \( y_c/h, y_d/y_c \), and \( p \) are independent variables and are selected as input variables whereas \( \Delta E/E_u \) is an output.

Figures 6–10 illustrate the performance of the above-discussed modeling techniques. Scattering is quantified by plotting the predicted data against the actual data and is represented with the best fit line (\( y = x \)).
Table 2 | Correlation matrix for data used in this study

| Variables | $y_c/h$ | $y_d/y_c$ | $p$ | $\Delta E/E_u$ |
|-----------|---------|-----------|-----|---------------|
| $y_c/h$   | 1       | 0.9917    | -0.0953 | -0.9928 |
| $y_d/y_c$ | 0.9917  | 1         | -0.1332 | -0.9872 |
| $p$       | -0.0953 | -0.1332   | 1    | 0.0711  |
| $\Delta E/E_u$ | -0.9928 | -0.9872 | 0.0711 | 1        |

Table 3 | Statistical measurements

| Parameters | Minimum | Maximum | Mean  | Standard Deviation | Skewness | Confidence Level (95.0%) |
|-----------|---------|---------|-------|-------------------|----------|--------------------------|
| $y_c/h$   | 0.0768  | 0.3743  | 0.2097| 0.0689            | 0.6268   | 0.0160                   |
| $y_d/y_c$ | 0.0982  | 0.4373  | 0.2413| 0.0849            | 0.6358   | 0.0197                   |
| $p$       | 0.4000  | 0.5000  | 0.4595| 0.0494            | -0.3934  | 0.0115                   |
| $\Delta E/E_u$ | 0.6321 | 0.8909 | 0.7594| 0.0609            | -0.3330  | 0.0141                   |

Training data set

| Parameters | Minimum | Maximum | Mean  | Standard Deviation | Skewness | Confidence Level (95.0%) |
|-----------|---------|---------|-------|-------------------|----------|--------------------------|
| $y_c/h$   | 0.1111  | 0.3659  | 0.2270| 0.0785            | 0.3857   | 0.0274                   |
| $y_d/y_c$ | 0.1352  | 0.4287  | 0.2629| 0.0949            | 0.3985   | 0.0331                   |
| $p$       | 0.4000  | 0.5000  | 0.4559| 0.0504            | -0.2480  | 0.0176                   |
| $\Delta E/E_u$ | 0.6372 | 0.8525 | 0.7471| 0.0680            | -0.2800  | 0.0237                   |

Testing data set

Figure 6 | Comparison of actual and predicted values of $\Delta E/E_u$ using testing data with (a) LR (b) MLR.
Figure 7 | Comparison of actual and predicted values of $\Delta E/E_u$ using testing data for (a) GP_PUK (b) GP_RBF.

Figure 8 | Comparison of actual and predicted values of $\Delta E/E_u$ using the testing dataset (a) SVM_PUK (b) SVM_RBF.
LR and MLR equations are developed using least square techniques with the help of XLSTAT software. The equations are as follow:

\[
\frac{\Delta E}{E_u} = 0.9602 - 0.6636 \left( \frac{y_c}{y_H} \right) - 0.1763 \left( \frac{y_d}{y_c} \right) - 0.0416(p)
\]  

(23)

\[
\frac{\Delta E}{E_u} = 0.5285 \left( \frac{y_c}{y_H} \right)^{-0.1111} \times \left( \frac{y_d}{y_c} \right)^{-0.1206} \times (p)^{-0.0017}
\]  

(24)

**Figure 9** | Comparison of actual and predicted values of $\Delta E/E_u$ using testing data (a) ANN (b) GRNN.

**Figure 10** | ANFIS model structure.
The agreement between LR and MLR predictions are shown in Figure 6(a) and 6(b) respectively; Predictions made using the LR equation are in better agreement with observations, with $R^2 = 0.986$. The LR equation is thus suitable for predicting $\Delta E/E_u$. Figure 6 and Table 4 indicate that LR outperforms MLR with $CC = 0.9933$, $RMSE = 0.0083$, $MAE = 0.0067$, $NRMSE = 0.0111$ and $NSE = 0.9849$.

Table 4 | Performance evaluation parameters for all models and for both training and testing datasets

| Models          | Training data set | Testing data set |
|-----------------|-------------------|------------------|
|                | CC    | RMSE | MAE   | NRMSE | NSE    | CC    | RMSE | MAE   | NRMSE | NSE    |
| LR             | 0.9939 | 0.0067 | 0.0047 | 0.0088 | 0.9878 | 0.9933 | 0.0083 | 0.0067 | 0.0111 | 0.9848 |
| MLR            | 0.9861 | 0.0101 | 0.0075 | 0.0133 | 0.9723 | 0.9850 | 0.0127 | 0.0099 | 0.0170 | 0.9640 |
| GP_PUK         | 0.9986 | 0.0032 | 0.0017 | 0.0042 | 0.9972 | 0.9949 | 0.0071 | 0.0040 | 0.0095 | 0.9888 |
| GP_RBF         | 0.9935 | 0.0077 | 0.0056 | 0.0101 | 0.9839 | 0.9930 | 0.0089 | 0.0071 | 0.0119 | 0.9824 |
| SVM_PUK        | 0.9983 | 0.0035 | 0.0013 | 0.0046 | 0.9967 | 0.9950 | 0.0072 | 0.0039 | 0.0096 | 0.9886 |
| SVM_RBF        | 0.9923 | 0.0100 | 0.0074 | 0.0131 | 0.9728 | 0.9925 | 0.0108 | 0.0083 | 0.0145 | 0.9739 |
| ANN            | 0.9977 | 0.0042 | 0.0025 | 0.0055 | 0.9952 | 0.9944 | 0.0076 | 0.0047 | 0.0102 | 0.9872 |
| GRNN           | 0.9990 | 0.0027 | 0.0016 | 0.0035 | 0.9981 | 0.9923 | 0.0090 | 0.0065 | 0.0120 | 0.9821 |
| ANFIS_trmf     | 0.9981 | 0.0038 | 0.0022 | 0.0049 | 0.9962 | 0.9946 | 0.0072 | 0.0042 | 0.0096 | 0.9885 |
| ANFIS_trapmf   | 0.9979 | 0.0039 | 0.0027 | 0.0051 | 0.9959 | 0.9958 | 0.0078 | 0.0055 | 0.0105 | 0.9864 |
| ANFIS_gbellmf  | 0.9986 | 0.0032 | 0.0021 | 0.0042 | 0.9973 | 0.9953 | 0.0069 | 0.0042 | 0.0092 | 0.9895 |
| ANFIS_gaussmf  | 0.9986 | 0.0032 | 0.0019 | 0.0042 | 0.9972 | 0.9928 | 0.0081 | 0.0046 | 0.0108 | 0.9854 |

Figure 11 | Sugeno type approach of ANFIS.
Figure 12 | Comparison of actual and predicted values of $\Delta E/E_o$ using ANFIS-based models.

Figure 13 | Actual and predicted values of $\Delta E/E_o$ for the training data-set.
The Gaussian process-based model is prepared by utilizing an iterative method. Pearson VII kernel functions and radial basis kernel functions are used. Table 4 summarizes the performance of the GP_PUK and GP_RBF models. The results indicate the superior performance of the GP_PUK model. Performance evaluation parameters for GP_PUK for predicting $\Delta E/E_u$ are: $CC = 0.9949$, $MAE = 0.0040$, $RMSE = 0.0071$, $NRMSE = 0.0095$ and $NSE = 0.9888$ for the testing stage. Figure 7(a) and 7(b) displays the results from the GP_PUK and GP_RBF models during the testing stage respectively.

Preparation of a Support Vector Machine based model is an iterative method. The model preparation involves the Pearson VII kernel function and radial basis kernel functions. Table 4 lists the performance parameters for the SVM_PUK and SVM_RBF models. The SVM_PUK model is seen to be superior to the SVM_RBF model. The SVM_Puk model for predicting $\Delta E/E_0$ results in $CC = 0.9950$, $MAE = 0.0043$, $RMSE = 0.0072$, $NRMSE = 0.0096$ and $NSE = 0.9886$ for the testing stage. Results obtained from the SVM_PUK and SVM_RBF models are plotted in Figure 8(a) and 8(b) respectively.

![Final generalized bell-shape membership functions for three inputs derived from the training process.](image-url)

**Figure 14** | Final generalized bell-shape membership functions for three inputs derived from the training process.
The preparation of a neural network-based model is also an iterative method. Table 4 also lists the performance outcomes from the ANN and GRNN models. As seen, the ANN approach is superior; the evaluation parameters for the GRNN based model for predicting $\Delta E/E_0$ are: $CC = 0.9944$, $MAE = 0.0047$, $RMSE = 0.0076$, $NRMSE = 0.0102$ and $NSE = 0.9872$. Figure 9(a) and 9(b) depicts results from the ANN and GRNN models during the testing stage, respectively.

The ANFIS based model is also an iterative method. In this study, four different types of membership functions were used: Triangular, Trapezoidal, Gaussian, and Generalized bell-shape. Figure 10 display the final structure of ANFIS model used in this study and Figure 11 indicates that the model is sugeno type. ANFIS_gbellmf based model information is listed below:

- Number of nodes: 78
- Number of linear parameters: 27
- Number of nonlinear parameters: 27
- Total number of parameters: 54
- Number of training data pairs: 74
- Number of checking data pairs: 34
- Number of fuzzy rules: 27

Table 4 lists the performance results from various ANFIS-based models and the results suggest that the generalized bell shape-based ANFIS model (ANFIS_gbellmf) outperforms the other ANFIS-based models. Performance parameters for the ANFIS_gbellmf model for predicting $\Delta E/E_0$ are: $CC = 0.9953$, $MAE = 0.0042$, $RMSE = 0.0069$, $NRMSE = 0.0092$ and $NSE = 0.9895$. ANFIS-based models during the testing stage are plotted in Figure 12. Actual and predicted values of $\Delta E/E_0$ using ANFIS_gbell MFs based model for the training data-set is shown in Figure 13. The final membership functions of the developed model are shown in Figure 14. Figure 15 gives a rule diagram for $\Delta E/E_0$ using Generalized bell-shape MFs for ANFIS. Figure 16 shows the surface diagram.
of $\Delta E/E_u$ from ANFIS for Generalized bell-shape MFs focuses on three-dimensional surface diagram of two input and output ($\Delta E/E_u$), and it can be concluded that there are nonlinear relationship between the input parameters and $\Delta E/E_u$.

**DISCUSSION**

The ANFIS_gbellmf based model has been found to perform better than the other approaches for comparing regression and soft-computing-based models. The linear regression model performs better than the MLR model for this dataset. As already shown by the results listed in Table 4, the GP model outperforms SVM-based models for predicting $\Delta E/E_u$. A Pearson VII kernel function outperforms the radial basis kernel function with GP and SVM techniques. In addition, the ANN model is better than the GRNN model. The results enlisted
Table 5 | Single factor ANOVA results

| Sr. No. | Source of Variation       | F     | P-value | F crit | Variation among groups |
|---------|---------------------------|-------|---------|--------|------------------------|
| 1       | Between Actual and LR     | 0.0284| 0.8668  | 3.9863 | Insignificant          |
| 2       | Between Actual and MLR    | 0.0031| 0.9561  | 3.9863 | Insignificant          |
| 3       | Between Actual and GP_PUK | 0.0179| 0.8940  | 3.9863 | Insignificant          |
| 4       | Between Actual and GP_RBF | 0.0134| 0.9083  | 3.9863 | Insignificant          |
| 5       | Between Actual and SVM_PUK| 0.0224| 0.8814  | 3.9863 | Insignificant          |
| 6       | Between Actual and SVM_RBF| 0.0273| 0.8693  | 3.9863 | Insignificant          |
| 7       | Between Actual and ANN    | 0.0266| 0.8709  | 3.9863 | Insignificant          |
| 8       | Between Actual and GRNN   | 0.0403| 0.8415  | 3.9863 | Insignificant          |
| 9       | Between Actual and ANFIS_trimf | 0.0110| 0.9169  | 3.9863 | Insignificant          |
| 10      | Between Actual and ANFIS_trapmf | 0.0192| 0.8902  | 3.9863 | Insignificant          |
| 11      | Between Actual and ANFIS_gbellmf | 0.0193| 0.8898  | 3.9863 | Insignificant          |
| 12      | Between Actual and ANFIS_gaussmf | 0.0018| 0.9659  | 3.9863 | Insignificant          |

Figure 17 | Box plot of applied model error distribution using the test data set.
### Table 6 | Error statistics for all applied models

| Models        | Minimum | Maximum | 1st Quartile | Median | 3rd Quartile | Mean  |
|---------------|---------|---------|--------------|--------|--------------|-------|
| LR            | 0.0203  | 0.0151  | -0.0022      | 0.0031 | 0.0090       | 0.0028|
| MLR           | 0.0296  | 0.0288  | -0.0055      | 0.0036 | 0.0071       | 0.0009|
| GP_PUK        | 0.0160  | 0.0200  | 0.0000       | 0.0010 | 0.0028       | 0.0022|
| GP_RBF        | 0.0250  | 0.0160  | -0.0028      | 0.0020 | 0.0087       | 0.0019|
| SVM_PUK       | 0.0140  | 0.0210  | 0.0000       | 0.0010 | 0.0020       | 0.0025|
| SVM_RBF       | 0.0290  | 0.0230  | -0.0045      | 0.0020 | 0.0098       | 0.0026|
| ANN           | 0.0200  | 0.0210  | 0.0007       | 0.0015 | 0.0031       | 0.0027|
| GRNN          | 0.0162  | 0.0259  | -0.0013      | 0.0023 | 0.0061       | 0.0033|
| ANFIS_trimf   | 0.0201  | 0.0190  | -0.0007      | 0.0006 | 0.0024       | 0.0017|
| ANFIS_trapmf  | 0.0176  | 0.0215  | -0.0014      | 0.0005 | 0.0061       | 0.0023|
| ANFIS_gbellmf | 0.0163  | 0.0185  | -0.0007      | 0.0011 | 0.0033       | 0.0023|
| ANFIS_gaussmf | 0.0251  | 0.0183  | -0.0008      | 0.0003 | 0.0014       | 0.0007|

### Figure 18 | Taylor Diagram (a) LR, MLR, GP & SVM (b) ANN, GRNN & ANFIS based models.
in Table 4 suggest that ANFIS_gbellmf is superior to membership-based ANFIS and other applied models. Table 4 indicates that there is no significant difference among actual and predicted values using the various models. Table 5 lists results from single-factor ANOVA. The overall error distribution is plotted using a box plot as shown in Figure 17. Consequently, over-estimation and under-estimation from the models can be corroborated through the negative and positive error values, respectively. For the applied models, the minimum, quartile, median, mean and maximum errors are listed in Table 6. The results are also displayed graphically in Figure 17. As can be seen, the maximum and minimum errors from the ANFIS_gbellmf model are: 0.0163 and 0.0185, respectively. This verifies the capability of ANFIS_gbellmf to predict $\Delta E/E_u$.

Figure 17 reflect results from the ANFIS_gbellmf based model while Figure 18 presents Taylor’s diagram for all applied models. This diagram illustrates the performance of the applied models. Values of three statistic parameters (standard deviation, correlation, and root mean square error) were calculated and comparisons between the actual and predicted results are shown for predicting $\Delta E/E_u$. Figure 18 suggests that the ANFIS_gbellmf model achieves a higher correlation with the minimum standard deviation. In addition, the Taylor’s diagram confirms that the ANFIS_gbellmf model is superior to the other models. The accuracy of the estimated data is quantified through a Taylor diagram which is a visual representation of performance assessment. Each point in this diagram represents the performance of the corresponding method; the closer the points are to the observational data point, the higher the accuracy and the lower the error.

**Sensitivity study**

The impact of each independent variable on the output is determined by a sensitivity study. A number of methods have been introduced in order to perform the sensitivity analysis. Artificial intelligence-based models and the best performing model (ANFIS_gbellmf) have been used for assessing sensitivity. The models were prepared by removing one input parameter from the input combination of the best-developed model. Table 7 lists the performance of each model in the absence of one of the inputs. The results indicate that $y_c/h$ is the most effective parameter for predicting $\Delta E/E_u$.

**CONCLUSIONS**

This study was performed to evaluate the performance of regression and soft-computing-based models for predicting $\Delta E/E_u$. ANFIS_gbellmf based model outperformed other models and has a higher correlation coefficient and a minimum of mean absolute error and root mean square error. A linear regression model is superior to MLR based models. Another observation is that the GP model outperforms SVM-based models. A Pearson VII kernel function works better than a radial basis kernel function with GP and SVM techniques. Finally, the ANN model is superior to the GRNN model. A sensitivity investigation reveals that $y_c/h$ is the most effective parameter for predicting $\Delta E/E_u$ using ANFIS_gbellmf based model. Due to the importance of relative energy dissipation for vertical drops equipped with a horizontal screen, it is recommended that numerical models and other AI methods efficiency models be used as a high-precision model to predict the energy dissipation for vertical drops equipped with a screen. It is also recommended that check the performance of horizontal grids at different heights.

**Table 7 | Sensitivity investigation based on the ANFIS_gbellmf model**

| Sr No | Input combination | Output | ANFIS_gbellmf |
|-------|-------------------|--------|---------------|
|       | $y_c/h$ | $y_d/y_c$ | $P$ | $\Delta E/E_u$ | CC | RMSE | MAE |
| 1     |         |          |      |               | 0.9953 | 0.0069 | 0.0042 |
| 2     |         |          |      |               | 0.9933 | 0.0082 | 0.0054 |
| 3     |         |          |      |               | 0.9930 | 0.0082 | 0.0056 |
| 4     |         |          |      |               | 0.9910 | 0.0098 | 0.0076 |
DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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