Optimization of Mechanical Impedance Matchers for Parametric Transducers in Gravitational Wave Spherical Detectors

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Abstract. A resonant-mass spherical gravitational wave detector is been built at the Physics Institute of Sao Paulo University which will be part of a detection network with a similar one that is under construction in The Netherlands. The goal of such detector is to reach a sensitivity of $10^{-21}$ in $\hbar$ at frequencies close to 3200 Hz within a bandwidth of 200 Hz. Its expected sensitivity will be close to the quantum limit and for such sensitivity to be reached several parameters must be optimized. One of them is the calibration of the mechanical impedance matchers, which are mechanical oscillators assembled on the sphere with the purpose of coupling the oscillations of the sphere surface to the electromechanical transducers that are used as motion sensors. This work has the goal to optimize such mechanical coupling, consequently the detector sensitivity, using a microwave parametric transducer for which an important variable is the electrical quality factor. The study involves the simulation of the oscillation of the mechanical parts of the detector with its all 17 frequency modes while checking whether these modes behave as predicted by the theory. We found that the simulation works adequately and that most of the frequency modes investigated are in good agreement with the theory about vibration quadrupole modes in spheres. Other modes did not fit so well in the theory and we comment on this in the text.
1. Introduction

SCHENBERG is a spherical resonant-mass gravitational wave detector [1, 2, 3] being built at the Department of Materials and Mechanics of the University of Sao Paulo, Brazil. According to the theory of general relativity the sphere’s quadrupole modes are the ones expected to be excited by a gravitational wave (GW) signal and theoretically they are exactly degenerate.

The sphere, with 65 cm in diameter and weighting 1.15 ton, is made of a copper-aluminum alloy [4] with 94% Cu and 6% Al. Figure 1 shows a cut view of this detector with some of its characteristics. Theoretical calculations yield a value of 3170.4 Hz for the frequency of the degenerate quadrupole modes of a bare sphere [5].

- Mass 1,15 ton
- Diameter 65 cm
- Alloy 94%Cu-6%Al
- Transducers: μwave parametrics

Figure 1: Schematics of the SCHENBERG detector.

The detector will have six transducers [6, 7] arranged on the sphere’s surface in a half-dodecahedron distribution; these sensors will be located as if in the center of the six connected pentagons in a dodecahedron surface, following the studies by Merkowitz and Johnson [8] confirmed by Magalhaes and collaborators [9]. The transducers and the suspension hole are located in the sphere surface as indicated in figure 2. SCHENBERG’s sphere modes as published in [10] were measured after holes were drilled to accommodate the suspension and future transducers. The values found were (in Hz) 3172.5, 3183.0, 3213.6, 3222.9 and 3240.0.

Figure 2: Position of the transducers and the suspension hole.
When the modes of the system sphere without hole plus six transducers are determined using a theoretic model their frequencies are grouped into three quintuplets (frequencies, in Hz, of 2842.3, 3140.8 and 3527.7) and two singlets (frequencies, in Hz, of 2936.9 and 3424.1) [5]. The quintuplets are located in the sides and at the center of the bandwidth, and the singlets are located between the quintuplets.

By analyzing the signal from such sensors the amplitudes and the direction of the incoming gravitation wave can be obtained [11, 12]. A similar detector is been built in The Netherlands, called MiniGrail [13], with a frequency close to SCHENBERG’s.

The Brazilian group has decided to use as motion sensors microwave parametric transducers, like the one used in the Australian GW detector NIOBÉ [14]. In this kind of transducer a superconducting cavity is pumped with monochromatic resonant microwaves and when the size of the cavity changes due to the vibration (one of the cavity wall is connected to the sphere by the mechanical impedance matcher) two side bands are created in the microwave signal that leaves the cavity. The amplitude of the side band is proportional to the amplitude of the sphere vibration.

Between the sphere surface and the transducer an impedance matcher is placed, composed by a series of mechanical oscillators with the same frequency of the detector and with decreasing effective masses, the last of these being the microwave cavity wall itself. The function of such device is to act as a mechanical amplifier (the amplitude is amplified by the square root of the mass ratio) as well as a mechanical filter so that only signals within the relevant bandwidth are sensed by the transducer.

It is expected that a multimode mechanical impedance matcher for the transducer will have several advantages over the single mode transducer [15, 16, 17, 18, 19], either parametric, inductive or capacitive. For this reason such a matcher is planned to be used in SCHENBERG.

In order to propose improvements based on the practical tests that shall start soon, we decided to perform simulations that would allow us to estimate how changes in the parameters of the mechanical impedance matcher could influence its match to the antenna. The results of our studies are presented in the following sections.

2. The Two-Mode Mechanical Impedance Matcher

We chose a design for the impedance matcher which was simple to model with a finite element program and whose parameters could easily be changed: a mechanical matcher with a disk supported by three rings that act as springs, its second mode being a membrane located at the disk's center of mass (see figure 3). Although this is not the same shape of the actual transducer used in the experiment its matching to the sphere is physically similar, modeled by springs and masses. The design used in this simulation is expected to be used in the next detector improvement phase.

The material chosen for the impedance matcher is pure niobium with the following properties: Young’s modulus: \( E = 1.05 \times 10^{11} \) Pa; specific mass: \( \rho = 8580.00 \) kg/m\(^3\); Poisson’s coefficient: \( \nu = 0.38 \).

![Figure 3: Impedance matcher used in the finite elements simulation. The figure on the left shows the cut perspective and on the right is the full perspective. The membrane is located in the lower disks of the figures.](image-url)

After we defined the design of the transducer we then had to determine the dimensions of the diaphragm for the first simulation. The mass of the disk was chosen to be equal to the the
desired one (initially 30g) and its initial diameter was chosen as 20 mm. To find the initial thickness of the membrane we used the formula for membrane frequency found in the book by Blevins [20]. As this formula does not take into account the correct boundary conditions we used it to find only the first frequency which would then be refined in the finite element modeling. The intention behind these determinations was to tune the impedance matcher as close as possible to the frequency of the sphere quadrupole modes in a way that it would work as a resonant transducer.

3. The Simulations

3.1. The Bare Sphere

Using the finite elements program Solid Works Simulation 2010 in all simulations, we first simulated the sphere alone based on SCHENBERG’s spherical antenna as described in the first paragraph of Section 1 with no holes drilled in it. The parameters we used for the Cu-Al alloy were: Young’s modulus: \( E = 1.303 \times 10^{11} \) Pa; specific mass: \( \rho = 8077.5 \) kg/m\(^3\); Poisson’s coefficient: \( \nu = 0.364 \).

We found that the frequencies for the five quadrupole modes were (in Hz) 3189.7, 3190.0, 3190.8, 3191.2 and 3192.1. These numbers are within the band 3190.8 \( \pm \) 1.0 Hz. The very small standard deviation (about 0.03%) indicates that the modes are degenerate as theory predicts. The small differences from the average value of 3190.8 Hz are interpreted as due to numerical limitations in the finite elements modeling. This average value differs from the theoretical prediction mentioned in Section 1 (3170.4 Hz) by 0.6%, possibly due to small differences in the values of physical parameters.

The simulations show clearly that the antenna changes its shape from a sphere into an ellipsoid during its vibration at all frequencies as is also predicted from theory.

As mentioned in Section 1, measurements of SCHENBERG’s sphere modes in the presence of holes yielded an average of 3206.4 Hz with a splitting of nearly 1%. It is reasonable to speculate that the lower (less than 0.5%) average frequency obtained in the above simulation is mainly due to the fact that no holes were drawn in it. Nevertheless, the simulated frequencies and the measured ones are quite close to each other, around 3198.6 \( \pm \) 0.1 Hz.

3.2. Sphere without Suspension Hole plus Six Matchers

Six impedance matchers were include in the model positioned on the sphere surface as shown in figure 4. They were located on the sphere surface as in figure 2 and were fastened to the sphere surface by the solid disk. They were aggregated into the same structure of the sphere in the finite element program.

![Figure 4: A two-mode impedance matcher positioned on the sphere's surface.](image)

Then the simulation was run with six impedance matchers distributed on the full sphere, and the frequencies obtained in the simulation are shown in table 1.
Table 1: Frequencies (in Hz) found on the simulation of the system full sphere plus six two-modes impedance matchers. The frequencies labeled phase correspond to the situation in which the disk and the membrane move in phase and the ones labeled counter phase imply that they move in phase opposition.

| Mode Frequencies (Hz) | Phase Characteristics |
|-----------------------|-----------------------|
| 3098.208              | in phase              |
| 3098.574              | in phase              |
| 3099.689              | in phase              |
| 3099.953              | in phase              |
| 3100.313              | in phase              |
| 3101.513              | in phase              |
| 3156.595              | in phase              |
| 3156.725              | in phase              |
| 3256.697              | counter phase         |
| 3157.249              | counter phase         |
| 3157.343              | counter phase         |
| 3206.034              | counter phase         |
| 3208.252              | counter phase         |
| 3209.792              | counter phase         |
| 3211.047              | counter phase         |
| 3211.460              | counter phase         |
| 3212.595              | counter phase         |

The frequencies in table 1 follow the theoretical pattern presented in Section 1: three quintuplets (frequencies, in Hz, around 3099, 3156 and 3211) and two singlets (frequencies near 3101 Hz and 3206 Hz). Based on previous arguments, differences between theory and simulation are believed to be due mainly to the values of the physical parameters used and minimally due to numerical limitations.

3.3. Sphere with Suspension Hole plus Six Impedance Matchers

A suspension hole was include in the sphere drawing for a new simulation and the results are shown in table 2. The membrane thickness used was 63 μm.

The frequencies in table 2 follow a pattern similar to the case in the preceding section: three quintuplets (frequencies, in Hz, around 3082, 3138 and 3187) and two singlets (frequencies near 3085 Hz and 3184 Hz).
Table 2: Frequencies found in the simulation for the system sphere with suspension hole plus six two-modes impedance matchers. The frequencies labeled *phase* correspond to the situation in which the disk and the membrane move in phase and the ones labeled *counter phase* imply that they move in phase opposition.

| Mode Frequencies (Hz) | Phase Characteristics |
|-----------------------|-----------------------|
| 3080.784              | in phase              |
| 3081.498              | in phase              |
| 3082.119              | in phase              |
| 3082.460              | in phase              |
| 3083.612              | in phase              |
| 3085.366              | in phase              |
| 3119.850              | in phase              |
| 3120.150              | in phase              |
| 3038.042              | counter phase         |
| 3138.157              | counter phase         |
| 3153.345              | counter phase         |
| 3148.637              | counter phase         |
| 3186.637              | counter phase         |
| 3187.147              | counter phase         |
| 3187.276              | counter phase         |
| 3187.722              | counter phase         |
| 3189.185              | counter phase         |

4. Conclusions
The simulation yielded patterns predicted theoretically and results close to measured values, which is a good evidence that the simulation is working properly.

Two frequency singlets and three quintuplets were identified even when we included a hole in the simulated sphere (Section 3.3). They were shifted from the values obtained in the no-hole case (Section 3.2), again in good agreement with the theory about vibration quadrupole modes in spheres since the small holes are expected to slightly change the body's mode frequencies. We found that the singlets are very close to the side quintuplets while theoretically their positions was closer to the half distance between the two nearest quintuplets. The central quintuplet is not degenerate because its average frequency is 3133.9 Hz with a standard deviation of 0.5%; this significant deviation suggests physical causes for the non-degeneracy, probably because of the presence of the suspension hole. The side quintuplets can be considered degenerate.

More work is necessary to understand why the singlets are dislocated from the position they have in the no-hole case (more symmetrical with respect to the quintuplets), and why the side quintuplets are degenerate.

The impedance matcher design presented here may be machined in a single piece and this will allow the system to have a high mechanical quality factor. Also, it is easy to change masses and frequencies of this impedance matcher by small changes in masses, membrane thickness and the rings used as springs making the tuning process easier.

In a future work we plan to include in the simulations the holes with which the transducers are attached to the sphere.
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