Acceleration at Relativistic Shocks
in Gamma-Ray Bursts

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Abstract

Most recent extragalactic models of gamma-ray bursts consider the expansion of a relativistic blast wave, emanating from a solar-mass type progenitor, into the surrounding interstellar medium as the site for their activity. The popular perception is that the optical afterglows result from the external shock interface, while the prompt transient gamma-ray signal arises from multiple shocks internal to the expansion. This paper illustrates a number of acceleration properties of relativistic and ultrarelativistic shocks that pertain to GRB models, by way of a standard Monte Carlo simulation. Computations of the spectral shape, the range of spectral indices, and the energy gain per shock crossing are presented, as functions of the shock speed and the type of particle scattering.

1 Introduction

The gamma-ray burst (GRB) field has burgeoned in the last few years, and particularly after the discovery (e.g. van Paradijs, et al. 1997; Costa, et al. 1997; Frail, et al. 1997) of optical, radio and X-ray transient counterparts, and the subsequent identification of a cosmological redshift via atomic absorption features in the 8th May 1997 afterglow (Metzger et al. 1997). Theoretical interpretations abound, mostly focusing on some variation of a blast wave expansion impacting on the interstellar medium (ISM) surrounding the burst (e.g. Meszaros & Rees 1993). Supersonic blast wave impact upon the surrounding ISM guarantees relativistic shock formation, basically a relativistic version of supernova remnants; the dissipation of the ram pressure kinetic energy via diffusive particle acceleration in the shock is the commonly-invoked means of converting the bulk motion into a viable supply of energy for radiative purposes. Principal quantities generally appearing in radiation emission models of GRBs include the Lorentz factor $\Gamma_1$ of the shock, and the spectral index of the electron (and perhaps ion) population. Furthermore, modelling of ultra-high energy cosmic ray production by GRBs (e.g. Waxman 1995; Vietri 1995) requires knowledge of the mean energy gain a particle experiences in traversing the shock. To date, GRB models of both the transient gamma-ray event and fireball afterglows have invoked only the crudest notions of Fermi acceleration. The aim of this presentation is to probe these shock acceleration properties, which are pertinent to GRB blast wave models, using results from a Monte Carlo simulation of diffusive acceleration.

The Monte Carlo technique we employ here has been described in detail in numerous expositions (Ellison, Jones & Eichler 1981; Jones and Ellison 1991; Baring, Ellison & Jones 1993; Ellison, Baring, & Jones 1996). The simulation technique is a kinematic model. Particles are injected upstream and allowed to convect into the shock, colliding with postulated scattering centers (presumably magnetic irregularities) along the way. As they diffuse between the upstream and downstream regions, they continually gain energy. An important property of the model is that it treats thermal particles like accelerated ones, making no distinction between them. Hence, as the accelerated particles start off as thermal ones, this technique automatically injects particles from the thermal population into the acceleration process. One valuable consequence of this unified treatment is that modification of the shock hydrodynamics by the accelerated population can easily be incorporated. Such non-linear hydrodynamics are omitted in the present test-particle application, though they will probably be an important aspect of the GRB acceleration problem given their relevance to the modelling of supernova remnant emission (e.g. Baring et al. 1999).
The test-particle results presented here use a guiding-center version of the Monte-Carlo technique, older than our latest codes which compute particle gyro-orbits exactly rather than just track the center of gyration. The guiding-center approach, which is detailed in Baring, Ellison & Jones (1993), is often expedient, and is entirely appropriate to plane-parallel shock applications (where the field lies along the shock normal, i.e. $\Theta_{B1} = 0$), which form the focus here. This method is precisely that implemented in Ellison, Jones & Reynolds’ (1990, hereafter EJR90) treatment of parallel relativistic shocks, thereby providing a principal motivation for adhering to a similar approach. The updated code replicates results obtained in EJR90. Following EJR90, both large angle scattering (LAS) and pitch angle diffusion (PAD) will be implemented here. For LAS, the mean-free path $\lambda$ in the fluid frame is constrained to be proportional to a particle’s gyroradius $r_g$, in accord many previous expositions (e.g. EJR90, Baring, Ellison & Jones 1993, hereafter BEJ93; Ellison, Baring, & Jones 1996), while for PAD we set $\lambda$ to be independent of $r_g$, following EJR90.

2 Results

The primary purpose of this paper is to extend the work of EJR90 to ultrarelativistic shocks, and to explore spectral properties of results from our simulation in the context of gamma-ray bursts. Representative ion distributions obtained in the rest frame of the shock are depicted in Figure 1 for $\Gamma_1 = 5$. The notations used are $\Gamma_1$ ($\Gamma_2$) for the Lorentz factor of the upstream (downstream) flow speed $u_1$ ($u_2$) in the shock rest frame, and $r$ for the velocity compression ratio in this frame. The value $r = 3$ is chosen to mimic expectations from an ultrarelativistic $\gamma = 4/3$ gas, though other values are possible due to possible mildly-relativistic nature of the downstream gas. The upstream gas temperature and magnetic field are kept sufficiently low to maintain the strength of high Mach number shocks and reduce the number of parameters to which results are sensitive.

The left panel of Figure 1 illustrates how a smooth power-law spanning many decades can be obtained at energies not too far in excess of thermal ones. The spectrum is somewhat steeper, though not markedly so, than the canonical $E^{-2}$ particle distribution for strong non-relativistic shocks. For slower shock speeds, the PAD-generated power-laws are steeper still (e.g. Kirk & Schneider 1987; EJR90), while for faster shocks appropriate to GRB scenarios, the spectral index $\sigma$ saturates around the value of 2.2 when $\Gamma_1 \gg 1$, as found by Bednarz & Ostrowski (1998). Index determinations are listed in Table 1, and while here we replicate Bednarz & Ostrowski’s (1998) asymptotic behaviour, we also find that $\sigma$ monotonically decreases with $\Gamma_1$, in contradiction to their findings in the range around $\Gamma_1 \sim 3$. Since the simulation reproduces the analytic results of Kirk & Schneider (1987) at $u_1 = 0.9c$ of $\sigma$ considerably higher than 2.2, our findings of monotonicity appear plausible.

| $\Gamma_1$ | $u_1/c$ | $u_2/c$ | PAD | LAS |
|-----------|--------|---------|-----|-----|
| 2.29      | 0.9    | 2.34    | 1.81|
| 3         | 0.9428 | 2.23    | 1.59|
| 5         | 0.9798 | 2.22    | 1.49|
| 9         | 0.9938 | 2.20    | 1.41|
| 27        | 0.9993 | 2.19    | -   |
| 81        | 0.9999 | 2.18    | -   |

Table 1: Asymptotic spectral indices $\sigma$ for plane-parallel ($\Theta_{B1} = 0^\circ$) relativistic shocks of various Lorentz factors $\Gamma_1$ and compression ratio $r = 3$ for the cases of pitch angle diffusion (PAD), and large angle scattering (LAS). Simulational uncertainties in predicting $\sigma$ are typically of the order of 1–2% for PAD, and twice that for LAS. Spectra for the $\Gamma_1 = 5$ case are exhibited in Figure 1.
Figure 1: Omni-directional fluxes for protons (of mass $m_p$) accelerated in relativistic shocks of Lorentz factor $\Gamma_1$ (i.e. upstream flow speed $c[1-1/\Gamma_1^2]^{1/2}$ and velocity compression ratio $r = 3$ in the shock rest frame. These high Mach number, test-particle shocks are plane-parallel ($\Theta_{B1} = 0^\circ$). The fluxes represent differential (in energy) distributions multiplied by the particle speed (i.e. $\approx c$), as measured in the shock frame somewhat downstream of the shock discontinuities. Cases corresponding to two types of scattering are illustrated, namely pitch angle diffusion (PAD; Left Panel), and large angle scattering (LAS; Right Panel). In each case, in addition to the total downstream spectrum, the maximum energies corresponding to 1, 3, 5, and 7 shock crossings are labelled, indicating the generally monotonic increase in energy that is a signature of the Fermi mechanism. Observe that for PAD, this increase falls far short of an amplification by a factor of $\Gamma_1^2$ at higher energies. The asymptotic power-law behaviours, which are achieved at energies $\gg \Gamma_1 m_p c^2$, are indicated.

The right panel of the Figure depicts a spectrum obtained for LAS with similar parameters. Two striking features emerge: (i) that the asymptotic power-law is generated only at much higher energies than in the PAD case, and (ii) when it is obtained, it is much flatter than for PAD. Both are essentially due to the prompt removal of particles by single large angle scatterings from the narrow Lorentz cone of directions for which the ions can remain upstream of the shock. PAD and LAS generate different angular distributions ahead of a relativistic shock, and the fact that these result in dissimilar spectral indices is widely understood (e.g. see EJR90 and references therein). The spectral structures above the thermal peak for LAS correspond to contributions from successive shock crossings in a manner somewhat like the structure seen at mildly suprathermal energies in non-relativistic shocks (e.g. BEJ93). These structures start out flat due to a kinematic spread induced by LAS, and then slowly steepen and merge into the power-law continuum. The trend for LAS is again that for faster shocks, $\sigma$ declines (see Table 1). However, any putative saturation could not be demonstrated numerically, due to difficulties in generating good statistics at energies well in excess of $10^{20}$ eV.

The maximum energy of particles in the downstream region for 1, 3, 5 and 7 shock-crossings are also depicted as vertical dashed lines in Figure 1. For LAS, it is clear that this scales as $\Gamma_1^2$, and energy amplification per shock crossing that is widely (and erroneously) quoted in GRB model literature. This represents the maximum amplification, and the mean amplification is much less, declining with increasing energy. This contention follows immediately from the tendency of the spectral plateaux
to steepen with energy, given that the probability of convection downstream of the shock drops with increasing energy. For PAD, even the maximum amplification falls far short of $\Gamma_1^2$, and saturates to a factor of order unity at high energies. This can be seen as follows. For a downstream particle of speed $\beta c$ (in the shock frame) that crosses upstream, its velocity angle relative to the shock normal must satisfy $\beta \cos \theta > \beta_2$ ($\sim 1/3$). This yields a range of possible upstream fluid-frame Lorentz factors $\gamma_F$ given by $\gamma_F(1 + \beta_1 \beta_2) < \gamma < \gamma_F(1 + \beta_1 \beta_1)$, beamed in a narrow cone around $\cos \theta_F = 1$. Pitch angle diffusion gradually widens this distribution till $1 - \cos \theta_F' \sim 1/(2\Gamma_1^2)$, at which point the ions convect downstream again with a shock frame Lorentz factor $\gamma' = \gamma_F \Gamma_1(1 - \beta_F \beta_1 \cos \theta_F')$. It is then trivial to determine that $\gamma'/\gamma \sim 2$ ($\ll \Gamma_1^2$), a result noted by Gallant & Achterberg (1999).

The implications of these results for gamma-ray burst modelers are the following. First, liberal use of $\Gamma_1^2$ amplification factors in shock crossings when estimating maximum energies obtainable in relativistic shocks is inappropriate. This impacts contentions (Waxman 1995; Vietri 1995) that GRBs can generate ultra-high energy cosmic rays, as do reductions in acceleration times seen when $\Gamma_1 \gg 1$ (e.g. see EJR90, and references therein). The disparate nature of the spectral indices between PAD and LAS cases is also of concern to the GRB community. While PAD yields a narrow range of indices more-or-less commensurate with inferences from both prompt gamma-ray emission and delayed X-ray/optical afterglows, the structured, flat LAS spectra may be at odds with GRB data. Furthermore, given the broad dynamic range, huge differences would arise in flux predictions for different wavebands. While the results presented here are for protons, one expects similar spectral properties for (and hence the concerns for emission from) electrons, since they too are relativistic when $\Gamma_1 \gg 1$ and hence readily resonate with Alfvén and whistler modes. Bednarz & Ostrowski (1996) argue in favor of PAD operating in shocks with $\Gamma_1 \gg 1$. We believe the situation not to be transparent. The definition of PAD in the context of relativistic shocks is effectively that angular deflections are substantially within the Lorentz loss-cone of half-angle $1/\Gamma_1$. As soon as deflections exceed this small value, spectra from our simulations quickly flatten to reproduce the LAS ones exhibited here. Hence the critical issue as to whether PAD or LAS operates in GRBs is contingent upon the typical magnitude of particle deflections in field turbulence associated with relativistic shocks. This nontrivial question will be the subject of future investigation.

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