Inflaton field fluctuations from gauge-invariant metric fluctuations during inflation

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Abstract

The evolution of the inflaton field fluctuations from gauge-invariant metric fluctuations is discussed. In particular, the case of a symmetric $\phi_c$-exponential potential is analyzed.

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I. INTRODUCTION

An attractive proposal concerning the first moments of the observable universe is that of chaotic inflation [1]. At some initial epoch, presumably the Planck scale, the scalar field existing in nature are roughly homogeneous and dominate the energy density. Their initial values are random, subject to the constraint that energy density is at the Planck scale. Among them is the inflaton field $\varphi$, which is distinguished from the noninflaton fields by the fact that the potential is relatively flat in its direction. This field would be responsible for the inflationary expansion of the universe. Inflationary model [2] solves several difficulties which arise from the standard cosmological model, such as the horizon, flatness, and monopole problems. Furthermore, it provides a mechanism for the creation of primordial density fluctuations needed to explain the structure formation in the universe. Stochastic inflation [3–8] is a very interesting approach to inflation that has played an important role in inflationary cosmology in the last two decades. This approach gives the possibility of making a description of the matter field fluctuations in the infrared (IR) sector by means of the coarse-grained matter field [9], that describes the inflationary universe on cosmological (super Hubble) scales. Since these perturbations are classical on super Hubble scales, in this sector one can make a standard stochastic treatment for the coarse-grained inflaton field. The IR sector is very important because the spatial inhomogeneous would explain the present

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day observed matter structure in the universe at cosmological scales. Matter fluctuations are responsible for metric fluctuations in the universe around the background Friedmann-Robertson-Walker (FRW) metric. The theory of linearized gravitational perturbations in an expanding universe is a very important subject of study in modern cosmology. It is used to describe the growth of structure in the universe, to calculate the predicted microwave background fluctuations, and in many other considerations. The growth of perturbations in an expanding universe is a consequence of gravitational instability. A small overdensity will exert an extra gravitational attractive force on the surrounding matter. Consequently, the perturbation will increase and will in turn produce a larger attractive force. In an expanding universe the increase in force is partially counteracted by the expansion. This, in general, results in power-law growth rather than exponential growth of the perturbations. Mathematically, the problem of describing the growth of small perturbations in the context of general relativity reduces to solving the Einstein equations linearized on an expanding background [10].

In particular, the inflaton field fluctuations are responsible for metric fluctuations around the background FRW metric. When metric fluctuations do not depend on the gauge, the perturbed globally flat isotropic and homogeneous universe is described by [10]

\[ ds^2 = (1 + 2\psi) \, dt^2 - a^2(t)(1 - 2\Phi) \, dx^2, \]

where \( a \) is the scale factor of the universe and \((\psi, \Phi)\) are the gauge-invariant (GI) perturbations of the metric. In the particular case where the tensor \( T_{\alpha\beta} \) is diagonal, one obtains: \( \Phi = \psi \) [10]. The field \( \Phi \) is called relativistic potential and describes the scalar GI metric fluctuations. The coordinate system (1) is more convenient for the investigation of density perturbations than the usual synchronous system.

We consider a semiclassical expansion for the inflaton field \( \phi(\vec{x}, t) = \phi_c(t) + \phi(\vec{x}, t) \) [7], with expectation values \( \langle 0|\phi|0 \rangle = \phi_c(t) \) and \( \langle 0|\phi|0 \rangle = 0 \). Here, \( |0\rangle \) is the vacuum state. Due to \( \langle 0|\Phi|0 \rangle = 0 \), the expectation value of the metric (1) gives the background metric that describes a flat FRW spacetime: \( \langle ds^2 \rangle = dt^2 - a^2 dx^2 \).

The Einstein equations can be linearized in terms of \( \phi \) and \( \Phi \) and the resulting equations for matter and metric fluctuations are

\[ \frac{1}{a^2} \nabla^2 \Phi - 3H \dot{\Phi} - \left( \dot{H} + 3H^2 \right) \Phi = \frac{4\pi}{M_p^2} \left( \phi_c \dot{\phi} + V' \phi \right), \]

\[ \frac{1}{a} \frac{d}{dt} (a\Phi) = \frac{4\pi}{M_p^2} \left( \dot{\phi_c} \phi \right), \]

\[ \ddot{\phi} + 3H \dot{\phi} - \frac{1}{a^2} \nabla^2 \phi + V''(\phi_c)\phi + 2V'(\phi_c)\Phi - 4\dot{\phi_c} \dot{\Phi} = 0, \]

where \( a \) is the scale factor of the universe and prime and overdots denote respectively the derivatives with respect to \( \phi_c \) and time. Furthermore, and \( \dot{H} = \dot{a}/a \) give us the Hubble parameter.

In this paper we are aimed to study the evolution of the inflaton field fluctuations \( \phi \) from GI metric fluctuations. To make it, firstly we must to solve the equations (2) and (4) for \( \Phi \), to be able the solution for \( \phi \) in eq. (3).
II. INFALTON FLUCTUATIONS FROM GI METRIC FLUCTUATIONS

The dynamics of $\phi_c$ on the background FRW metric is given by the equations

$$\ddot{\phi}_c + 3H\dot{\phi}_c + V'(\phi_c) = 0, \quad \dot{\phi}_c = -\frac{M_p^2}{4\pi}H'.$$  \hfill (5)

Furthermore, the scalar potential can be written in terms of the Hubble parameter

$$V(\phi_c) = \frac{3M_p^2}{8\pi} \left[ H^2 - \frac{M_p^2}{12\pi} (H')^2 \right].$$  \hfill (6)

If we replace eq. (3) in (2), we obtain the Klein-Gordon like equation for the GI metric fluctuations $\Phi$

$$\ddot{\Phi} + \left( H - \frac{2\ddot{\phi}_c}{\phi_c} \right) \dot{\Phi} - \frac{1}{a^2} \nabla^2 \Phi + 2 \left( \dot{H} - \frac{\ddot{\phi}_c}{\phi_c} \right) \Phi = 0.$$  \hfill (7)

The equation (7) can be simplified by introducing the redefined field $Q = e^{1/2 \int [H - 2\ddot{\phi}_c/\phi_c]} dt \Phi$.

If we make it, we obtain

$$\ddot{Q} - \frac{1}{a^2} \nabla^2 Q = \left[ \frac{1}{4} \left( H - 2\ddot{\phi}_c/\phi_c \right)^2 + \frac{1}{2} \frac{\dot{H}}{a^2} \frac{d}{dt} \left( \frac{\ddot{\phi}_c}{\phi_c} \right) - \frac{1}{2} \left( \dot{H} - \frac{\ddot{\phi}_c}{\phi_c} \right) \right] Q = 0.$$  \hfill (8)

This field can be expanded in terms of the modes $Q_k = e^{\lambda_k \cdot \vec{x}} \xi_k(t)$

$$Q(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[ \alpha_k Q_k(\vec{x}, t) + \alpha_k^\dagger Q_k^*(\vec{x}, t) \right],$$  \hfill (9)

where $\alpha_k$ and $\alpha_k^\dagger$ are the annihilation and creation operators that comply with the commutation relations

$$[\alpha_k, \alpha_{k'}^\dagger] = \delta^{(3)}(k - k'),$$  \hfill (10)

$$[\alpha_k, \alpha_{k'}] = [\alpha_k^\dagger, \alpha_{k'}^\dagger] = 0.$$  \hfill (11)

The equation for the modes $Q_k$ is

$$\ddot{Q}_k + \omega_k^2(t) \ Q_k = 0,$$  \hfill (12)

where $\omega_k^2 = a^{-2} (k^2 - k_0^2)$ is the squared time dependent frequency for each $k$-mode and $k_0$ separates the infrared ($k \ll k_0(t)$) and ultraviolet ($k \gg k_0(t)$) sectors

$$\frac{k_0^2}{a^2} = \frac{1}{4} \left( H - 2\ddot{\phi}_c/\phi_c \right)^2 + \frac{1}{2} \left[ \dot{H} - \frac{\ddot{\phi}_c}{\phi_c} \right] - \frac{1}{2} \left( \ddot{H} - \frac{\dddot{\phi}_c}{\phi_c} \right)$$  \hfill (13)

Furthermore, the field $Q$ obeys the following commutation law: $[Q(\vec{x}, t)\dot{Q}(\vec{x}', t)] = i\delta^{(3)}(\vec{x} - \vec{x}')$, so that the modes $\xi_k$ are renormalized by the expression
\[ \dot{\xi}_k^* \xi_k - \ddot{\xi}_k \xi_k^* = i. \quad (14) \]

The inflaton field oscillates around the minimum of the potential at the end of inflation. Due to this fact the solutions of the eq. (12) when \( \dot{\phi}_c = 0 \) and \( \ddot{\phi}_c = 0 \) are very important. When \( \dot{\phi}_c = 0 \) we obtain that \( Q_k = 0 \), but the solutions for \( \Phi_k \) are given by

\[ \Phi_k = a^{-1} \Phi_k^0, \quad (15) \]

where \( \Phi_k^0 \) is the initial amplitude for \( \Phi_k \), for each wavenumber \( k \). This means that the amplitude of each mode \( \Phi_k \) decreases with the expansion of the universe. On the other hand, when \( \ddot{\phi}_c = 0 \), the field is at the minimum of the potential. At this moment the equation (12) takes the form

\[ \ddot{Q}_k + \left[ \frac{k^2}{a^2} - \left( \frac{H^2}{4} - \frac{3}{2} \dot{H} \right) \right] Q_k = 0, \quad (16) \]

where \( \Phi_k = a^{-1/2} Q_k \).

Now we can write the equation (3) in terms of the field \( Q \). Once we know the modes \( Q_k \), the modes \( \phi_k \) for the gauge-invariant inflaton fluctuations will be determined by

\[ \phi_k = \frac{M_p^2}{4\pi \alpha \phi_c} e^{-\frac{1}{2} \int \left[ H - \frac{2}{\dot{\phi}_c} \right] dt} \left\{ Q_k \left[ \frac{H}{2} + \frac{2}{\phi_c} \ddot{\phi}_c \right] + \dot{Q}_k \right\}. \quad (17) \]

Hence, if we assume that slow-roll conditions [11] are fulfilled (it should be before the reheating period), the fluctuations for energy density will be

\[ \frac{\delta \rho}{\rho} \simeq \frac{V'}{V} \left\langle \phi^2 \right\rangle_{GI}, \quad (18) \]

where the squared fluctuations of \( \phi \) are

\[ \left\langle \phi^2 \right\rangle_{GI} = \frac{1}{2\pi^2} \int dk k^2 \phi_k(t) \phi_k^*(t). \quad (19) \]

Here, the modes \( \phi_k \) are given by the eq. (17).

**III. POWER-LAW INFLATION FOR A SYMMETRIC \( \phi_C \)-POTENTIAL**

To illustrate the formalism we can examine a scalar potential given by \( V(\phi_c) = V_0 \ e^{2\alpha |\phi_c|} \), where \( \alpha^2 = \frac{4}{M_p^2} \) gives the relationship between \( \alpha \) and the power of the expansion \( p \) for a scale factor that increases as \( a \sim t^p \). The Hubble parameter is given by \( H(t) = p/t \), or, in term of \( \phi_c \)

\[ H_c = \frac{\pi}{M_p} \left( \frac{32V_0}{12\pi - \alpha^2 M_p^2} \right)^{1/2} e^{\alpha |\phi_c|}, \quad (20) \]

where \( V_0 = \frac{3M_p^4}{8\pi} H_c^2 \left[ \frac{12\pi - M_p^2 \alpha^2}{12\pi} \right] \) and \( H_e = p/t_e \) is the value of the Hubble parameter at the end of inflation. Furthermore, the evolution for \( |\phi_c(t)| \) is
\[ |\phi_c(t)| = |\phi_0| - \frac{1}{\alpha} \ln \left( \frac{t}{t_0} \right), \quad (21) \]

where \( t \geq t_0 \). Since \( \dot{\phi}_c = -\text{sgn}(\phi_c) \frac{1}{\alpha} \) and \( \ddot{\phi}_c = \text{sgn}(\phi_c) \frac{1}{\alpha^2} \) [we assume \( \text{sgn}(\phi_c) = \pm 1 \) for \( \phi_c \) positive and negative, respectively], the evolution for \( \Phi \) will be described by the equation

\[ \ddot{\Phi} + \frac{(p+2)}{t} \dot{\Phi} - \frac{1}{a^2} \nabla^2 \Phi = 0. \quad (22) \]

Now we can make the transformation \( Q = \Phi e^{\int (p+2)t^{-1} dt} \) so that the differential equation for \( Q \) yields

\[ \ddot{Q} - \frac{1}{a^2} \nabla^2 Q - \left[ \frac{p}{2} \left( \frac{p}{2} + 1 \right) t^{-2} \right] Q = 0. \quad (23) \]

The general solution for the modes \( Q_k(t) \) is

\[ Q_k(t) = C_1 \sqrt[\nu]{\frac{t}{t_0}} H^{(1)}_{\nu}(x(t)) + C_2 \sqrt[\nu]{\frac{t}{t_0}} H^{(2)}_{\nu}(x(t)), \quad (24) \]

where \((C_1, C_2)\) are constants, \((H^{(1)}_{\nu}[x], H^{(2)}_{\nu}[x])\) are the Hankel functions of (first, second) kind with \( x(t) = \frac{t_0^{\nu}}{a_0(p-1)} \) and \( \nu = \frac{p+1}{2(p-1)} \). Using the renormalization condition \((14)\), we obtain the Bunch-Davis vacuum [12] solution \((C_1 = 0, C_2 = \sqrt{\frac{\pi}{2(p-1)}})\)

\[ Q_k(t) = \sqrt{\frac{\pi}{2}} \sqrt[\nu]{\frac{t}{t_0(p-1)}} H^{(2)}_{\nu}(x(t)), \quad (25) \]

In the UV sector the function \( H^{(2)}_{\nu}[x] \) adopts the asymptotic expression (i.e., for \( x \gg 1 \))

\[ H^{(2)}_{\nu}[x] \approx \sqrt{\frac{2}{\pi x}} \left[ \cos \left( x - \nu \pi/2 - \pi/4 \right) - i \sin \left( x - \nu \pi/2 - \pi/4 \right) \right] , \quad (26) \]

whilst on the IR sector (i.e., for \( x \ll 1 \)) it tends asymptotically to

\[ H^{(2)}_{\nu}[x] \approx \frac{1}{\Gamma(\nu + 1)} \left( \frac{x}{2} \right)^\nu - \frac{i}{\pi} \Gamma(\nu) \left( \frac{x}{2} \right)^{-\nu}. \quad (27) \]

The \( \Phi \)-squared field fluctuations on the IR sector are \((\langle \Phi^2 \rangle)_{IR} = \frac{1}{2} \int_{t_0}^{e_k(t)} dd k^2 |\Phi_k|^2 \), and becomes

\[ \langle \langle \Phi^2 \rangle \rangle_{IR} \approx \frac{1}{4} \left\{ \begin{array}{l} \frac{t_0^{p}}{2a_0(p-1)} 2^\nu \left[ a_0 \sqrt[\nu]{\frac{\pi}{2}} \right] \frac{2(p-1)}{\pi t_0 \Gamma^2(p-1)(3p-1)} \\ \frac{t_0^{p}}{2a_0(p-1)} 2^\nu \left[ a_0 \sqrt[\nu]{\frac{\pi}{2}} \right] \frac{2(p-1)}{\pi t_0 \Gamma^2(p-1)(3p-1)} \\ + \frac{\Gamma^2(\nu) \left[ \frac{t_0^{p}}{2a_0(p-1)} \right]^{-2\nu} \left[ a_0 \sqrt[\nu]{\frac{\pi}{2}} \right] \frac{2(p-2)}{\pi^3 t_0 (p-3)}} {\pi^3 t_0 (p-3)} \end{array} \right\} \quad (28) \]
where $\epsilon = k^{(IR)}_{\text{max}}/k_p \ll 1$ is a dimensionless constant, $k^{(IR)}_{\text{max}} = k_0(t_*)$ at the moment $t_*$ when the horizon entry and $k_p$ is the Planckian wavenumber (i.e., the scale we choose as a cut-off of all the spectrum). The power spectrum on the IR sector is $P_{\Phi}|_{IR} \sim k^{3-2\nu}$. Note that $(\langle \Phi^2 \rangle)_{IR}$ increases for $p > 2$, so that to the IR squared $\Phi$-fluctuations remain almost constant on cosmological scales we need $p \approx 2$. We find that a power close to $p = 2$ give us a scale invariant power spectrum (i.e., with $\nu \simeq 3/2$ for $(\langle \Phi^2 \rangle)_{IR}$. Furthermore, density fluctuations for matter energy density are given by $\delta \rho/\rho = -2\Phi$, so that $\langle \delta \rho^2 \rangle^{1/2}/\langle \rho \rangle \sim \langle \Phi^2 \rangle^{1/2}$.

On the other hand, in the UV sector these fluctuations are given by
\[
(\langle \Phi^2 \rangle)_{UV} \simeq \frac{a_0}{4t_0^{p+1} \pi^2} \left\{ \frac{k_p^2}{t^2} - \frac{a_0^2}{t^{2p}} \left[ \frac{p}{2} \left( \frac{p}{2} + 1 \right) \right] \right\} t^{3-2\nu}.
\] (29)

The power spectrum in this sector go as $P_{\Phi}|_{UV} \sim k^4$. We observe from eq. (29) that $(\langle \Phi^2 \rangle)_{UV}$ increases during inflation for $p > 3$. From the results (28) and (29) we obtain that $1 < p \leq 2$, because a power-law $p > 2$ could give a very inhomogeneous universe on cosmological scales. Since $(\langle \Phi^2 \rangle)_{UV} \geq 0$, we obtain the condition
\[
k_p^2 - \frac{a_0^2}{t^{2(p+1)}} \left[ \frac{p}{2} \left( \frac{p}{2} + 1 \right) \right] \geq 0.
\] (30)

If $a_0 = H_0^{-1}$ ($H_0$ is the initial value of the Hubble parameter), inflation should ends at $t = t_e$, where
\[
t_e \simeq \left[ \frac{k_p H_0}{\sqrt{\frac{p}{2} \left( \frac{p}{2} + 1 \right)}} \right]^{\frac{1}{p-1}}.
\] (31)

For example, for $k_p H_0 = 10^{11} M_p$ and $p = 2$, we obtain $t_e \simeq 5.8 \times 10^{10} M_p^{-1}$.

Furthermore, from eq. (17) we obtain the solutions for the modes $\phi_k(t)$
\[
\phi_k(t) = \text{sgn}(\phi_c) M_p \sqrt{\frac{1}{8t_0 p (p-1)}} \left[ t^{-(p+1)/2} H_{\nu+1}^{(2)}[x(t)] - k \frac{t_0^p}{a_0} t^{-(3p-1)/2} H_{\nu+1}^{(2)}[x(t)] \right],
\] (32)

where $H_{\nu+1}^{(2)}[x(t)]$ takes the asymptotic expressions (26) and (27) for $x \gg 1$ and $x \ll 1$, respectively. The $k$-modes for the inflaton field fluctuations on the IR and UV sectors are given respectively by
\[
(\phi_k(t))_{IR} \simeq \text{sgn}(\phi_c) M_p \left\{ \frac{1}{\Gamma(\nu+1) \left( \frac{t_0^p}{2a_0 (p-1)} \right)^\nu} \right\}^{1/2} \left[ t^{-1} e^{-i \left[ \frac{t_0^p}{a_0(p-1)} - \nu \pi^{1/(p+1)} \right]} k^{-1/2} - \left( \frac{t_0^p}{a_0} \right) t^{-p} e^{-i \left[ \frac{t_0^p}{a_0(p-1)} t^{p-1} \right]} k^{1/2} \right],
\] (33)

\[
(\phi_k(t))_{UV} \simeq \text{sgn}(\phi_c) M_p \left[ \frac{a_0}{2 p t_0^{p+1} \pi} \right] t^{-1} e^{-i \left[ \frac{t_0^p}{a_0(p-1)} - \nu \pi^{1/(p+1)} \right]} k^{-1/2} - \left( \frac{t_0^p}{a_0} \right) t^{-p} e^{-i \left[ \frac{t_0^p}{a_0(p-1)} t^{p-1} \right]} k^{1/2},
\] (34)
so that the squared $\phi$-fluctuations on both sectors are

\[
\langle \phi^2 \rangle_{IR} \approx A \; t^{2(p-2)},
\]

\[
\langle \phi^2 \rangle_{UV} \approx B_1 \; t^{-2} + B_2 \; t^{-2p} + B_3 \; t^{-(p+1)} - B_4 \; t^{2(p-2)},
\]

where the constants $A$, $B_1$, $B_2$, $B_3$ and $B_4$ are

\[
A = \frac{M_p^2}{8\pi^4 t_0 (p-1)^2 p A_2^2 A_1^2 a_0^2} \left\{ \pi^2 A_3^{2^{p-1}} A_1^2 a_0^2 \left( \frac{t_0}{a_0 (p-1)} \right)^{\frac{p^2-2}{p-1}} \right\} \left[ 16 \pi^p p - 2 \frac{\pi^p}{p-1} \right] 
\]

\[
+ a_0^2 A_3^{2^{p-2}} A_2^2 A_1 \left( \frac{p+1}{2(p-1)} \right) \left[ \frac{a_0}{t_0^p} (p-1) \right]^{\frac{p^2-2}{p-1}} \left[ 2 \frac{\pi^p}{p-1} p - 16 \pi^p \right] 
\]

\[
+ A_3^{2^{p-2}} A_2 A_1^2 a_0^2 \Gamma \left( \frac{p+1}{2(p-1)} \right) \left[ \frac{a_0^2}{t_0^p} (p^2 - p+1) \right]^{\frac{p^2-2}{p-1}} \left[ 416 \pi^p - 2 \frac{\pi^p}{p-1} p \right] 
\]

\[
+ a_0^2 A_1^2 A_3^2 a_3^{3^{p-2}} \left[ \frac{a_0}{t_0^p} (p-1) \right]^{\frac{p^2-2}{p-1}} \left[ 2 \frac{\pi^p}{p-1} p - 4 \pi^p \right] 
\]

\[
+ \pi^2 \left( \frac{t_0^p}{a_0 (p-1)} \right)^{\frac{3p-1}{p-1}} t_0^{2p} A_3^{3^{p-3}} A_2^2 \left[ 42 \frac{(7-1)p}{p-1} p - 32 \frac{(7-1)p}{p-1} \right] 
\]

\[
+ \pi^2 a_0 \frac{t_0^p}{a_0 (p-1)} \left[ \frac{t_0}{a_0 (p-1)} \right]^{\frac{2p}{p-1}} f_{3^p}^{3^{p-2}} A_2 A_1 [2 - 3p] \right\}, \quad (37)
\]

\[
B_1 = \frac{M_p^2}{16\pi^3 t_0^p} k_p^2, \quad (38)
\]

\[
B_2 = \frac{M_p^2 t_0^p k_p^4}{32\pi^3 a_0^3}, \quad (39)
\]

\[
B_3 = \frac{M_p^2}{12\pi^3 t_0 a_0 p} \left( k_p \frac{t_0^p}{a_0} \right), \quad (40)
\]

\[
B_4 = \frac{a_0^2}{t_0^p} \left[ \frac{1}{2} \left( \frac{p^2}{4} + \frac{p}{2} \right) + \frac{1}{4} \left( \frac{p^2}{4} + \frac{p}{2} \right)^2 + \frac{2}{3} \left( \frac{p^2}{4} + \frac{p}{2} \right)^{3/2} \right], \quad (41)
\]

where $A_1 = \Gamma \left( \frac{5p-3}{2(p+1)} \right)$, $A_2 = \Gamma \left( \frac{3p-1}{2(p-1)} \right)$, $A_3 = \frac{a_0^2}{t_0^p} p(p+1)$ and $k_p$ is the wave number at the Planckian scale. Note that for a scale invariant $(\langle \Phi^2 \rangle_{IR} - power spectrum with p = 2$ (i.e., for $\nu = 3/2$), the squared inflaton fluctuations on the infrared sector $(\langle \phi^2 \rangle_{IR}$ and the late times squared fluctuations on the ultraviolet sector $(\langle \phi^2 \rangle_{UV}$ are constant.

**IV. FINAL COMMENTS**

In this paper we have studied the evolution of the inflaton field fluctuations in a symmetric $\phi_c$-exponential potential, from GI metric fluctuations previously renormalized by eq. (14). Metric fluctuations are here considered in the framework of the linear perturbative corrections. The scalar metric perturbations are spin-zero projections of the graviton, which
only exists in nonvacuum cosmologies. The issue of gauge invariance becomes critical when we attempt to analyze how the scalar metric perturbations produced in the early universe influence a background globally flat isotropic and homogeneous universe. This allows us to formulate the problem of the amplitude for the scalar metric perturbations on the evolution of the background FRW universe in a coordinate-independent manner at every moment in time. Note that we have not considered back-reaction effects which are related to a second-order metric tensor fluctuations. In the power-law expanding universe here studied the GI metric fluctuations are well described by the field Φ and predicts a scale invariant power spectrum on the IR sector for \( p = 2 \) [13]. The interesting of the result here obtained is that for \( p = 2 \) the inflaton field fluctuations result to be scale invariant on the IR sector, but also on the UV sector if we consider a cut-off \( k_p \) on the Planckian scale. Hence, the problem of the UV divergence for the inflaton field fluctuations would be avoided, but also the problem of the temporal increasing of such that fluctuations on small scales, because \( \langle \phi^2 \rangle_{UV} \) becomes squeezed at the end of inflation.

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