Precise Branching Ratio of $^{24m}$Al Beta Decay

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Abstract. The branching ratio of the isomeric $\gamma$ decay of $^{24m}$Al has been measured to be 69.6(7)% which is much smaller than the previously accepted value of 82.5(30)%. As a result, the branching ratio to the $^{24}$Mg ground state increases up to 24.1(7)% assuming that the other $\beta$-decay branching ratios are the previously accepted values. The half-life of $^{24m}$Al was also precisely determined to be 130.9(13) ms. The $B$(GT) value from the ground state of $^{24}$Mg to the $^{24m}$Al of 0.0577(16) deduced from the $\beta$ decay is now in good agreement with that deduced from charge-exchange reactions.

1. Introduction

A $\beta$ decay measurement provides important information on the structure of mother and daughter nuclei such as $ft$ values, energy levels, and spin-parity. The $\beta$-decay $ft$ values enables us to deduce the Gamow-Teller(GT) transition strength $B$(GT) when the transition has $\Delta L = 0$ and $\Delta S = 1$. The $B$(GT) can be deduced not only from $\beta$-decay $ft$ values but also from charge-exchange reactions [1]. The $B$(GT) values obtained in these two methods are usually in good agreement. However, the $B$(GT) value from $^{24}$Mg($0^+$, g.s.) to $^{24m}$Al($1^+$, 426 keV) of 0.024(8) deduced from the $\beta$-decay $ft$ value [2] is about half of the value of 0.050(1) from ($p, n$) reaction [3] and 0.054(1) from ($^3$He, t) reaction [4] for the same transition. In Ref. [2], they adopted previous value of 2.3(6) as the ratio between the branching ratio to the ground state and to the first exited state [5], since $\beta$ branching to the $^{24}$Mg ground state could not be measured. In order to clarify this disagreement, we studied the decay of $^{24m}$Al using a $\beta$- and $\gamma$-ray spectrometer at a modern fragment-separator facility. Figure 1 shows the decay scheme for the $^{24m}$Al with the branching ratios and log $ft$ values give in Ref [2, 6]. We measured the absolute branching ratio of the isomeric $\gamma$ decay with the total $\beta$ branch obtained by counting the number of $\beta$ rays.

2. Experiment

The experiment was carried out at HIMAC synchrotron and fragment-separator facility [7]. A $^{24m}$Al beam was produced through the charge-exchange reaction of a 100A MeV $^{24}$Mg primary
beam impinging on a 4.5mm-thick polyethylene (CH$_2$) target. The charge-exchange reaction on a proton is expected to provide a larger isomeric ratio and a larger production cross section than that on a Be target due to the pure inverse ($p, n$) reaction. The beam duration was about 100 ms. We got the beam every 6 or 10 seconds. The $^{24m}$Al beam was separated by the fragment separator with two dipole magnets and a 1.2mm-thick wedge-shape Al degrader. The intensity and purity of the $^{24m}$Al beam was about 1000 particles per pulse and 75%, with contaminants of 23% $^{23}$Mg and 2% $^{22}$Na.

The experimental setup of the $\beta$- and $\gamma$-ray spectrometer is shown in Fig. 2. The secondary beam was defined by a Pb collimator with a hole of 20 mm $\phi$. A 0.5mm-thick plastic scintillator (PS) placed after the Pb collimator enables us to count the number of incident heavy ions. The beam was implanted into an active stopper made of a 5mm-thick plastic scintillator with an area of 35 mm $\times$ 37 mm for $\beta$ rays. The scintillator signals were read out by two photomultiplier tubes. Because a coincidence of two signals reduced noises caused by dark current, we could set the detection threshold low. A HPGe detector with an efficiency of 50% relative to the standard NaI crystal and an energy resolution of 2.2 keV (FWHM) at 1333 keV of $^{60}$Co was installed behind the active stopper. A 8mm-thick Cu absorber was placed between the active stopper and the Ge detector to prevent $\beta$ rays from entering the Ge detector.

We measured the number of 426-keV $\gamma$ rays and the $\beta$ rays to determine the branching ratio of the isomeric $\gamma$ transition. Figure 3 shows the $\gamma$-ray energy spectra of the Ge detector. The upper line shows the singles spectrum and the lower line shows the spectrum in coincidence with $\beta$ rays. As shown in Fig. 4, the detection efficiency for the 426-keV $\gamma$ ray was determined to be 0.0175(4) using the known $\beta$-delayed $\gamma$ rays of $^{22}$Mg and $^{23}$Mg. These beams with the same range in the active stopper as the $^{24}$Al beam were used for the calibration. To determine the energy dependence, radioactive sources such as $^{22}$Na, $^{60}$Co, $^{133}$Ba, $^{137}$Cs, and $^{152}$Eu were also used. The result of GEANT simulation shown by the solid line is in good agreement with these data. A high $\beta$-ray detection efficiency for the active stopper of 99(1)% was obtained by comparing between the number of single $\gamma$ rays and the number of $\gamma$ rays gated by the signal of $\beta$ rays in the stopper. One reason for so high efficiency is that the $\beta$-ray energy deposit $\Delta E$ is large enough by implanting the $^{24m}$Al beam in the center of the 5mm-thick active stopper.

Figure 1. Decay scheme with branching ratios ($BR$) and log $ft$ values given in Ref. [2, 6]. Our results are shown by bold letters.
and another reason is that the detection threshold of the active stopper was set low enough. The $\beta$-ray energy spectrum for the active stopper is shown in Fig. 5. Similar efficiencies were also obtained using the $^{22}\text{Mg}$ and the $^{23}\text{Mg}$ beams. The GEANT simulation estimates that the 7(1)$\%$ of the 426-keV $\gamma$ rays emit additional photons in the active stopper by the Compton scattering process, and they pretend as if they are from the $\beta$-decay. This effect was taken into account in the determination of the $\beta$-decay counts.

**Figure 3.** The $\gamma$-ray energy spectra. Upper line shows the singles spectrum and lower line shows the spectrum in coincidence with $\beta$ rays.

**Figure 4.** The $\gamma$-ray detection efficiency determined by using standard $\gamma$ sources and the $^{22}\text{Mg}$ and $^{23}\text{Mg}$ radioactive ion beam.

**Figure 5.** The $\beta$-ray energy spectrum from active stopper made of 5mm-thick plastic scintillator. Solid circles show the experimental result. Solid and Dashed lines show the GEANT simulated energy deposit of $\beta$ ray and 426-keV $\gamma$ ray by Compton scattering, respectively.

**Figure 6.** The $\beta$- and $\gamma$-ray time spectra. Solid squares are time spectrum of $\beta$ ray. Solid circles are time spectrum of 426-keV $\gamma$ ray.
3. Analysis and Discussion

The decay time spectra for the $\beta$ rays of the active stopper and the 426-keV $\gamma$ ray of the Ge detector are shown in Fig. 6. The decay-time analysis of the $\beta$ decay including the isomeric $\gamma$ transition is different from that of a simple $\beta$ decay. Let us consider a cocktail beam with the number of $^{24m}$Al $N_m$ and the number of $^{24g}$Al $N_g$ is implanted in the stopper at $t = 0$. The count rate of the isomeric $\gamma$ ray $N^\gamma(t)$ and that of all the $\beta$ rays $N^\beta(t)$ are given by

$$N^\gamma(t) = \epsilon^\gamma_m R \lambda_m N_m e^{-\lambda_m t}$$

$$N^\beta(t) = \epsilon^\beta_m (1 - R) \lambda_m N_m e^{-\lambda_m t} + \epsilon^\beta_g R \lambda_g N_g \frac{\lambda_m}{\lambda_m - \lambda_g} \left( e^{-\lambda_g t} - e^{-\lambda_m t} \right) + \epsilon^\beta_g \lambda_g N_g e^{-\lambda_g t},$$

where $R$, $\lambda_m$, $\lambda_g$, $\epsilon^\gamma_m$, $\epsilon^\beta_m$, and $\epsilon^\beta_g$ are the branching ratio of the isomeric $\gamma$ transition, the decay constant of $^{24m}$Al ($\ln(2)/131$ ms), that of $^{24g}$Al ($\ln(2)/2.053$ s), the detection efficiency of the $\gamma$ ray, that of the $\beta$ ray of $^{24m}$Al, and that of the $\beta$ ray of $^{24g}$Al, respectively. In Eq. (2), the first term is for the $\beta$ decay of the $^{24m}$Al, the second term is for the $\beta$ decay of the $^{24g}$Al derived from the isomeric $\gamma$ decay of $^{24m}$Al, and the third term is for the $\beta$ decay of the $^{24g}$Al derived from the beam. In order to fit Eq. (1) and Eq. (2) to the experimental data, it is useful to replace them by

$$N^\gamma(t) = A^\gamma_m e^{-\lambda_m t}$$

$$N^\beta(t) = A^\beta_m e^{-\lambda_m t} + A^\beta_g e^{-\lambda_g t}$$

where $A^\gamma_m$, $A^\beta_m$, and $A^\beta_g$ are defined as

$$A^\gamma_m = \epsilon^\gamma_m R \lambda_m N_m$$

$$A^\beta_m = \epsilon^\beta_m (1 - R) \lambda_m N_m - \epsilon^\beta_g \left( \frac{\lambda_g}{\lambda_m - \lambda_g} \right) R \lambda_m N_m$$

$$A^\beta_g = \epsilon^\beta_g \lambda_g N_g + \epsilon^\beta_g \left( \frac{\lambda_g}{\lambda_m - \lambda_g} \right) R \lambda_m N_m.$$  

Utilizing the relation of Eq. (5) and Eq. (6), the branching ratio $R$ is given by

$$R = \frac{(A^\gamma_m/\epsilon^\gamma_m)}{(1 + \alpha)(A^\beta_m/\epsilon^\beta_m) + (A^\beta_g/\epsilon^\beta_g)}.$$  

where the correction factor $\alpha$ is described as

$$\alpha = \frac{\epsilon^\beta_g}{\epsilon^\beta_m} \left( \frac{\lambda_g}{\lambda_m - \lambda_g} \right).$$  

If $\lambda_g$ was much smaller than $\lambda_m$, the correction factor $\alpha$ could be zero. However, the fact that $\alpha$ is 0.068 in this case is not negligibly small compared with our desired precision of $R$.

By fitting Eq. (3) to the $\gamma$-decay time spectrum and by fitting Eq. (4) to the $\beta$-ray spectrum, we obtained $A^\gamma_m$, $A^\beta_m$, and $A^\beta_g$. Here, the background of the other gamma rays which are derived from natural backgrounds and the $\beta$ decay of $^{23}$Mg were taken into account properly. The dead-time effect of the data taking system was corrected properly by using the artificial 100-Hz trigger clock. Pile-up effect was negligibly small. Substituting obtained values into Eq. (8), the branching ratio $R$ has been determined to be 69.6(7)% as shown in Table 1. The uncertainty mainly comes from the uncertainty of $\gamma$-ray detection efficiency and that of the estimation of
Compton scattering probability in the active stopper. The internal conversion coefficient for this transition is negligibly small. The present value is a factor four more precise than previous values of 82.5(30)% [2] and 78(3)% [8]. Our value differs from them over the error bars. The precise half-life $T_{1/2}$ of $^{24m}$Al is also determined to be 130.9(13) ms by analyzing the $\gamma$-decay time spectrum. The present value is similar to the previous values [6] within the error bars, and is twice more precise than the weighted average of them. The isomeric ratio of the beam was found to be 84.4(18)% by comparing between $A_\gamma$ and $A_\beta$. The branching ratio to the 1369-keV, $2^+$ state in $^{24}$Mg was also obtained to be 3.5(5)%, which indicates that the ratio of the $\gamma$-ray intensity of 1369 keV to that of 426 keV is in good agreement with the previous value given in Ref. [2]. The other intensities of $\gamma$ rays with high energies could not be determined due to their small detection efficiencies and small branching ratios.

| Table 1. Branching ratio of $^{24m}$Al decay in units of %. |
|-----------------------------------|-----------------|-----------------|-----------------|
| $^{24}$Al($1^+$, g.s.) | $^{24}$Mg($0^+$, g.s.) | $^{24}$Mg($2^+$, 1369 keV) |
| Present | 69.6(7) | 24.1(7)* | 3.5(5)* |
| J. Honkenen et al.[2] | 82.5(30)** | 10(3)** | 4.4(5)** |
| T. Shibata et al.[8] | 78(3) | – | – |
| A. J. Armini et al.[5] | 93(2) | 4.4(12) | 1.9(5) |

*These values were used relative $\gamma$-ray intensities given in Ref. [2]. **These values were used relative branching ratio of $^{24}$Mg($0^+$, g.s.) to $^{24}$Mg($2^+$, 1369 keV) given in Ref. [5].

The branching ratio from the $^{24m}$Al($1^+$, 426 keV) to the $^{24}$Mg($0^+$, g.s.) was determined to be 24.1(7)%, by assuming that the other $\beta$-ray intensities feeding to the exited states of $^{24}$Mg is 8.9(3)% of the isomeric $\gamma$ decay (see Ref. [2]). The obtained branching ratio is 2.4 times larger than the previous one of 10.1(28)%. The log $ft$ value of 5.297(13) is now smaller than the previous value of 5.68(15). The $B(\text{GT})$ value can be derived by the relationship

$$B(\text{GT}) = (2J_f + 1) \frac{K}{(g_A/g_V)^2 ft}$$  \hfill (10)

where $J_f$ is the spin of final state, a constant $K = 6143.6(17)$ second [9], and a coupling constant ratio $g_A/g_V = -1.270(3)$ of the axial vector current to the vector current of weak interaction [10]. The $B(\text{GT})$ value of 0.0577(16) have been calculated from our $\beta$-decay study as shown in Table 2. The difference between the $B(\text{GT})$ value deduced from the $\beta$ decay and that deduced from the charge-changing reactions is now comparable to the differences for other transitions in the light mass region.

| Table 2. $B(\text{GT})$ values from $^{24}$Mg ($0^+$, g.s.) to $^{24m}$Al ($1^+$, 426 keV). |
|-----------------------------------|-----------------|-----------------|-----------------|-----------------|
| $\beta$ decay (present) | $\beta$ decay (previous) | ($p, n$) reaction [3] | ($^3$He, $t$) reaction [4] |
| $B(\text{GT})$ | 0.0577(16) | 0.024(8) | 0.050(1) | 0.054(1) |
The observation of M3 transition in light nuclei is very rare. Especially, a pair of mirror nuclei $^{24m}$Al and $^{24m}$Na is unique in that they both decay with the isomeric M3 transition. Therefore, the data of the mirror nuclei are valuable for theoretical test of the magnetic octupole matrix elements. The M3 transition strength $B(M3)$ for $^{24m}$Na [6] has been already determined precisely. The $B(M3)$ value for $^{24m}$Al changes over 10% as shown in Table 3. The experimental ratio of $B(M3)$ for $^{24m}$Na to $B(M3)$ for $^{24m}$Al becomes consistent with the theoretical ratio by a shell-model calculation [11], although the theoretical prediction of the absolute values still overestimates the experimental values by 50%.

Table 3. Comparison of $B(M3)$ values in $^{24m}$Al and $^{24m}$Na (in units of $\mu^2 N \text{ fm}^4$).

|        | $^{24m}$Al | $^{24m}$Na | $^{24m}$Na/$^{24m}$Al |
|--------|-----------|------------|------------------------|
| Experiment (compilation) [6] | 269(13)   | 1038(5)    | 3.9(2)                 |
| Experiment (present) | 231(3)    | 4.50(7)    |                        |
| Shell-model calculation [11] | 344       | 1538       | 4.47                   |

4. Summary
The $\beta$-$\gamma$ spectroscopy of $^{24m}$Al has been carried out by using a secondary $^{24m}$Al beam with high purity and high isomeric ratio at HIMAC synchrotron and fragment-separator facility. The branching ratio of $^{24m}$Al isomeric $\gamma$ decay was precisely determined to be 69.6(7)% as a result, the branching ratio of the $\beta$ decay to the $^{24}$Mg ground state was obtained to be 24.1(7)%, where relative $\gamma$-ray intensities of previous values were assumed. By using this branching ratio, the $B(\text{GT})$ value from the $^{24}$Mg($0^+$, g.s.) to the $^{24m}$Al($0^+$, 426 keV) of 0.0577(16) was deduced, which is in good agreement with the $B(\text{GT})$ values deduced from ($p,n$) and ($^3$He, t) charge-exchange reactions. The half-life $T_{1/2}$ of $^{24m}$Al was determined to be 130.9(13) ms precisely. The newly obtained $B(M3)$ value of the $^{24m}$Al isomeric $\gamma$ decay made the ratio of the experimental $B(M3)$ values between $^{24m}$Na and $^{24m}$Al isomeric $\gamma$ transition consistent with the theoretical prediction by a shell-model calculation.

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