Non-Determinism and the Lawlessness of ML Code

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ABSTRACT
Legal literature on machine learning (ML) tends to focus on harms, and as a result tends to reason about individual model outcomes and summary error rates. This focus on model-level outcomes and errors has masked important aspects of ML that are rooted in its inherent non-determinism. We show that the effects of non-determinism, and consequently its implications for the law, instead become clearer from the perspective of reasoning about ML outputs as probability distributions over possible outcomes. This distributional viewpoint accounts for non-determinism by emphasizing the possible outcomes of ML. Importantly, this type of reasoning is not exclusive with current legal reasoning; it complements (and in fact can strengthen) analyses concerning individual, concrete outcomes for specific automated decisions. By clarifying the important role of non-determinism, we demonstrate that ML code falls outside of the cyberlaw frame of treating “code as law,” as this frame assumes that code is deterministic. We conclude with a brief discussion of what work ML can do to constrain the potentially harm-inducing effects of non-determinism, and we clarify where the law must do work to bridge the gap between its current individual-outcome focus and the distributional approach that we recommend.

1 INTRODUCTION
Legal decision logic bears some resemblance with the logic of mathematical functions in that both map inputs to outputs. When adjudicating a particular case, a magistrate assembles the available evidence, which they supply as parameters to legal rules, whose application ultimately results in a decision. Just as with mathematical functions, there can be variations in input parameters, which correspond to variations in outcomes. Kolber [24] takes this functional analogy a step further, classifying the correspondences between legal inputs and outputs into “smooth” and “bumpy” types. A smooth relationship is one for which gradual changes in inputs map to gradual changes in outputs. Bumpy relationships, in contrast, exhibit discontinuities: Slight variations in inputs can map to large variations in outputs.1 Machine learning (ML) — a discipline within the mathematical tradition — unsurprisingly seems to follow a similar logic. Classification problems, for example, resemble Kolber [24]’s concept of bumpiness; varied, continuous inputs become discretized outputs. Determining loan-worthiness, for example, is bumpy because a classification model maps personal data to a binary outcome in the set {grant_loan, reject_loan}.

This comparison between the work of law and that of ML, in which both are reasoned about as functions, is deceptively attractive. At first glance, it seems to mirror the decades-long literature in cyberlaw that has considered the law and “if/then” code rules2 to be complementary modalities that regulate and mediate human experience [2, 10, 22, 29]. It is thus perhaps intuitive and appealing to consider stretching this analogy further: To treat the mathematical-functional-similarity of the law and ML as a rationale for christening ML as the latest type of code-imbued regulator. To stretch this even further, if ML can be fashioned to design new “microdirectives” or usher in a new era of “personalized law,” as some legal scholars contend [9, 20], then perhaps ML could breathe new life into the succinct cyberlaw refrain that “code is law” [31, 37]. That is, rather than using this widely-quoted shorthand to stand in for the more-precise (but still abbreviated) “code is constitutive of law” [2, p. 675], ML code could literally be used to generate law.

And yet, while it might be appealing to take these steps to connect the nascent field of ML law with its older cyberlaw sibling, upon deeper examination, the comparison between ML and the law via functions does not hold up. For one, as much legal scholarship acknowledges, the mechanism by which ML translates from inputs to outputs fundamentally differs from the analogous mechanisms in the law [3, 11, 12, 23, 27, 32, 33]. The law has a variety of mechanisms — rules, standards, factors tests, etc. — each accompanied with justifications for (and amendments concerning) their use, as well as a long record in jurisprudence of their application to specific cases. In contrast, ML may behave like a function, but we often do not understand how that function works. In ML systems, we can have full access to both the inputs and subsequent outputs, while having no clear understanding of how the mapping from one to the other occurred. In other words, unlike the law, ML functions defy explanation and reasonable justification, which in turn raises fundamental questions about the legitimacy of using ML as a decision-making tool and muddies the ability to determine accountability when these tools cause harms [15]. In short, ML’s problem with “explainability” shows how the analogy essentially and inescapably falls short; both the law and ML may behave like functions, but functions that are fundamentally different in kind.

This analogy falls short in another fundamental way—one that is significant enough for us to pause attempting to close the loop between cyberlaw “code is law scholarship” and legal scholarship concerning ML, but has thus-far remained under-explored. Code that follows “if/then” logic — the type of code addressed in cyberlaw literature [2, 10, 22, 29] — is deterministic: It specifies behaviors that to execute (the “then”) when certain, specified conditions (the “if”) are met. Importantly, the code of ML models does not execute “if/then” rules. Instead, ML is statistical in nature; it exhibits

1For example, it is reasonable to contend that tort law should be smooth, with the amount of harm caused exhibiting a direct and continuous relationship with the degree of compensation owed. However, in practice, tort law is often bumpy, exhibiting discrete outputs: Defendants are either liable to provide full compensation (regardless of the particular degree of contributing to harm), or they are not liable at all [24, p. 673].

2Either as a type of architecture [29, 31] or a modality on its own [22].
stochasticity and non-determinism. We explore the meaning of these terms in detail later in this paper. For now, it suffices to provide an intuition: Deterministic code ensures that computing with the same inputs yields the same outputs; stochasticity and non-determinism, in contrast, can cause two similar (even identical) training procedures to produce vastly different performance results in practice [21, 35]. In remainder of this paper, we explain how stochasticity and non-determinism are integral to the behavior of ML systems, and thus, while currently under-explored in legal scholarship, understanding this behavior is essential for reasoning about questions concerning the regulability, legitimacy, and accountability of ML decision-making tools.

Our first contribution is to show that the emphasis on individual errors and error rates in existing legal scholarship is concealing other important issues in ML that are rooted in non-determinism. While focusing on individual outcomes and error rates for specific models is important — and intuitive, given that it parallels case-based analysis in the law — it nonetheless provides a limited view of behavior of ML. We show that the effects of non-determinism, and consequently its implications for the law, instead become clearer from the perspective of reasoning about ML outputs as probability distributions over possible outcomes. The key difference is that this distributional viewpoint accounts for non-determinism by providing a window into the possible outcomes of ML. Importantly, this type of reasoning is not exclusive with current legal reasoning; it complements (and in fact can strengthen) analyses concerning individual, concrete outcomes for specific automated decisions (Section 2). By clarifying the important role and potential effects of non-determinism, we then demonstrate that ML code falls outside the cyberlaw frame, which assumes deterministic code (Section 3). Lastly, we conclude with a brief discussion of what work ML can do to constrain the potentially harm-inducing effects of non-determinism, and we clarify where the law must do work to bridge the gap between its current case-based analysis of ML systems and the distributional analysis that we recommend (Section 4).

2 FROM THINKING ABOUT ERROR RATES TO REASONING ABOUT DISTRIBUTIONS OVER OUTCOMES

Legal literature concerning the empirical performance of ML tools tends to focus on issues of accuracy [28] [6, pp. 1249-50] [8, pp. 9,12] [10, p. 1253]. This work typically evaluates ML in terms of individual decision outcomes in relation to the harms these outcomes cause, and uses summary error rates to draw conclusions about a particular model’s accuracy. Solely focusing on the accuracy of specific inference outcomes and summary rates can conceal other important issues implicated by non-determinism, which are also important factors to consider in legal analyses of ML technology.

In this section, we show how a different viewpoint for analyzing ML can reveal additional concerns salient to legal scholars studying the implications of deploying ML tools. In particular, we explain how moving away from analyses of individual outcomes to thinking distributions over possible outcomes can expand legal scholars’ understanding of the behavior of ML tools. In particular, reasoning about distributions over possible outcomes highlights the important role of non-determinism in ML, and in turn the potential harms non-determinism can cause. In fact, as we will show via simple synthetic examples, the presence of non-determinism in code that involves stochasticity calls into question essential assumptions about the fundamental nature of accuracy in ML.

2.1 Example 1: Distributions over Individual Outcomes

We first consider an ML system that aims to determine individuals’ creditworthiness by predicting their credit scores. The developers write a snippet of code to achieve this task — a procedure for training models to predict individuals’ credit scores. The execution of this code to actually train a model exhibits stochasticity: Running this one piece of code multiple times will result in multiple trained models that vary in comparison to one another. For example, if we were to take many such models and supply them with the same individual as input, the corresponding outputs would yield a distribution over possible credit score outcomes for that individual. We illustrate this in Figure 1 for two individuals. In other words, since this process yields a distribution over possible credit scores for each individual — and not just a single credit score — determining an individual’s credit score is not a deterministic function of the code written by the engineer to train ML models. Rather, credit score for an individual is a function of the procedure that this code can execute; it is a function of executing model training, which exhibits stochasticity and thus a distribution of possible outcomes.

Figure 1: Synthetic probability distributions for two individuals’ possible predicted credit scores.
The viewpoint of distributions over possible outcomes shown in Figure 1 illustrates a problem. For the two individuals depicted in the graph, their credit score distributions overlap (shown in purple). This means that it is possible that there is some subset of models, produced by the stochastic training process, for which we cannot distinguish between these two individuals in terms of their credit scores. And yet, in looking at each of their distributions overall, there are all clearly cases where they do not overlap and are thus clearly distinguishable. That is, from its distributional perspective, this figure shows that it is possible to produce models that suggest contradictory results: Some models are able to distinguish these individuals via different credit scores, while it is possible that some models are not able to discern a difference. Instead of either always being able to clearly distinguish these two individuals via different credit scores, or being able to always treat them as having the same credit score, both contradictory possibilities are suggested by this distributional viewpoint.

This ambiguity complicates what accuracy means for a model produced by this training process, because it is not clear what a “correct” model should do with respect to how it views these two individuals. Is it “correct” to model them as distinguishable, or “correct” to model them as indistinguishable, in terms of their credit scores? It is impossible to say with 100% certainty, since there is no notion of ground truth credit score. Put differently, this figure indicates that there is a meta-problem of not being able to draw a firm line between correctness and incorrectness for models trained by this process. This issue of being unable to draw a clear boundary between correctness and incorrectness illustrates how model output decisions can exhibit non-determinism: For the different inputs, depending on the model, the outputs for those inputs may be distinguishable or may be indistinguishable.

After we have selected a particular model to use for predicting credit scores, we can think about deterministic outputs. That is, by picking a particular model that encodes a specific function, we have locked in a deterministic score for each individual. We can move from reasoning about distributions over possible outcomes of credit scores for individuals, to thinking about deterministic, concrete outcomes (which are conditional on the model we have chosen). Legal literature on ML tends to engage with models at this stage — where there is one chosen, specific model. Given one specific model, with deterministic outcomes for each individual, it becomes possible to perform analyses concerning the inaccuracy of individual outcomes, associated harms, and metrics like error rates to capture summary information about a model’s overall performance across a sample of inputs. But at the distributional level conveyed in Figure 1, concepts like accuracy remain a slippery concept. In reasoning about possible model outcomes, rather than specific model outcomes, this level makes the potential areas of uncertainty in trained models more transparent [4]; it clarifies how the possibility of different outcomes can have the effect of muddling the distinction between correctness and incorrectness.

2.2 Example 2: Distributions over Models
Reasoning over distributions of outcomes does not just apply to thinking about how outcomes for fixed individual inputs may vary based on choice of model. This view can also help reason about non-determinism concerning models trained from same stochastic training process. Figure 2 shows distributions over model outcomes for two models trained using the exact same procedure. Because this training procedure is stochastic, non-determinism plays a role in causing variation in each trained model’s distribution over possible outcomes. Without non-determinism, the distributions for both models’ outcomes in Figure 2 would completely overlap.

Ideally, even with non-determinism, repeated runs of similar or identical training processes would result in outcome distributions that are reasonably similar to each other (as, one could argue, is the case in Figure 2, since the curves roughly overlap). If the models’ distributions do not vary too much, then at least we can be confident that picking any of them as the specific model to deploy is a reasonable choice, as each model indicates performance roughly representative of all the models that were trained. In other words, we do not have to contend with the issue of drawing a line between which models are correct and which are incorrect, because all of the models are effectively the same. Regardless, the fact that models are not completely identical requires us to shift our thinking about ML. This distributional view clarifies that we should be thinking of one run of an ML training process as learning a possible distribution over possible outcomes, rather than the singularly correct distribution over possible outcomes.

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3This is in contrast to applications for which we can reasonably say that there is a ground truth, such as a computer vision system that distinguishes between cats and dogs, an example input is either a cat or a dog, not both.

5We could also visualize this problem as distributions over outcomes, in which we treat each outcome on the x-axis as the trained model. From the resulting single distribution over models, we could get a sense of how models trained from same process vary.

6Of course, they are not exactly the same, varying on individual inputs in either an arbitrary fashion or, perhaps, varying in a manner correlated with an attribute, e.g. varying more for a particular gender or racial group. Neither of these regimes—arbitrariness and discrimination—are tolerated by the law.

Figure 2: Synthetic distributions of model outcomes for two models trained for the same task, using the same algorithm and data. Non-determinism in the training process yields different distributions of model outcomes.
non-determinism [5, 14, 16, 21, 36, 38]. In such cases, it is will not necessarily be clear if there is a representative model in the group—if there is a model that is more "correct" than the others. Once again, due to non-determinism, drawing a firm boundary between correctness and incorrectness is ill-defined. As with the example in Figure 1, this example similarly raises questions of how to legitimately pick a model that we can be confident will yield robust and reliable performance. This question is not just of theoretical relevance. In practice, non-determinism can cause resulting model outcome distributions to vary so much that, for a particular input, models can yield wildly inconsistent results.

To make this more concrete, we describe an example in the ML literature that demonstrates the effects of such non-determinism. In recent work, Forde et al. [21] and Qian et al. [35] investigated how the impact of non-determinism on training models using similar training procedures could impact model fairness. Qian et al. [35] published an extensive empirical study, in which they repeatedly trained models with identical training procedures. They then compared the resulting model outcome distributions to see how much they varied. In particular, they computed common algorithmic fairness metrics to probe how fairness measurements varied for different models produced by identical training procedures. They found that fairness measurements could vary by 12.6%, in fact in some cases was significant enough for one trained model to pass US legal compliance rules, while another could violate required fairness thresholds [35, p. 2].

In other words, Qian et al. [35] illustrates clearly how non-determinism can have a significant impact on fairness in the distribution of possible modeling outcomes. This result indicates that picking any one specific model to deploy — which then could be examined in terms of individual errors and error rates, fairness-related or otherwise — is a non-trivial task. Non-determinism necessarily has an unpredictable role in the specific outcomes of training models. When this unpredictability leads to wide variability in metrics like fairness, this then raises fundamental questions not just about the fairness of particular models, but about the fairness of the process by which those models were trained. Put simply, if it is possible for models trained in the same way to exhibit vastly different fairness levels, how can we be sure (especially when training just a few models under limited computational resource budgets) that the one model we have selected to deploy in practice is representative of what is (at least close to) maximally possible in terms of fairness?

Questions like these are not clear from looking at individual outcomes or error rates for single models alone. Instead, it is looking at distributions over outcomes that raises questions about the legitimacy model-producing processes, through indicating how the resulting models from those processes can vary in important ways. This distributional view provides information that can help us interrogate whether the process for training ML models for a specific task is robust enough to justify the use of any such model produced from that process. By robust we mean that, even in the presence of non-determinism, the resulting variation in the behavior of possible ML models — whether variation in model outcome distributions, or variation in outcomes across models for particular individual inputs — is not arbitrary or the product of happenstance.

The example of Qian et al. [35] arguably does not meet this definition of robustness, given the large variance in fairness metrics across the distribution of models they produced. This becomes especially clear when one considers how such variance in fairness could impact due process [28] [6, pp. 1249-50] [8, pp. 9, 12] [10, p. 1253]— if a particular chosen model by chance demonstrates poor performance with respect to fairness, in turn leading to a greater number of unfair individual outcomes in practice.

3 NON-DETERMINISTIC CODE IS LAWFUL

In moving from looking at individual errors and model error rates to reasoning about distributions of outcomes, we have seen how the non-determinism inherent in ML can raise key questions concerning the legitimacy of using ML-driven processes in decision-making. We have seen, too, how non-determinism can directly effect harms at the individual level, in cases in which a training process is not sufficiently robust to guarantee that its resulting models behave similarly for key metric, such as fairness. In short, our discussion thus-far has indicated that non-determinism can have significant, detrimental effects on the behavior of ML code.

And yet, the stochasticity that drives this non-determinism is the very thing that makes it possible for ML to model complex phenomena. This seems to present an inherent contradiction. That is, significant variance across fairness metrics for models produced via identical training procedures almost sounds like a software bug; and yet, the non-determinism that enables results like this are a not a bug, but a feature of ML.9 As we briefly discussed in Section 1, the non-determinism due to ML’s statistical nature functions more like a feature than a bug because it is necessary for solving problems that are too complex to specify exhaustively using "if/then" deterministic rules [34].

Moreover, as we saw in Section 2, non-determinism can make it very difficult to reason about the difference between correctness and incorrectness in ML program behaviors, thus making accuracy a fuzzy concept that is difficult to pin down.10 And yet, in the existing legal literature on ML, the issue of inaccuracy and accuracy, particularly at the individual model level, has been a dominant theme [3, 6, 8, 11, 18, 19, 28]. For the law to adequately contend with non-determinism, we have argued that the legal literature must shift to also consider the viewpoint of distributions over outcomes, as this viewpoint indicates how non-determinism fundamentally problematizes our understanding of accuracy.

Based on this prior discussion, we now argue that this will also require shift in the dominant thread of cyberlaw thinking that echoes the refrain that "code is law"11. In brief, "code as law" stands in for the idea that code does the work of law; code, like the law, is a modality for regulating and mediating human behavior [22, 29]. As Grimmelmann [22] summarizes in more detail, "code is law" captures the idea that "software itself can be effectively regulated by major social institutions, such as businesses or governments. ... If

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9For work on the elusive boundary between bugs and inherent features in ML, please refer to Cooper et al. [15]. More generally, delineating what constitutes a bug for randomized programs is a philosophical question, which has long remained unresolved in the Programming Languages research community [25, 26].

10It is also worth noting that the approximate computing concept of the trade-off between accuracy and efficiency [13], and more generally using a temporal lens to analyze outcomes [39], further complicates our understanding of accuracy in ML.

11This phrase, which originated from work in Reidenberg [37], has been further developed and revised [22, 29, 30], and then ultimately codified in Lessig [31]. It has since been partially adapted to account for the new kinds of experiences that ML (particularly robotics) will mediate [1, 7].
other institutions can regulate software, and software can regulate individual behavior, then software provides these institutions an effective way to shape the conduct of individuals’ [22, p. 1721].

In the extensive literature that has followed from Lessig [31]’s codification of the concept, various scholars have built on and problematized different aspects of “code is law” [2, 7, 22], such that it has ultimately remained a resonant and powerful frame for thinking about technology. However, the work that contends with this concept tends to (often implicitly) assume a deterministic view of code. It considers code to be a set of automated “if/then” rules that ensure consistent decisions as it works to enable and constrain human behavior [22, pp. 1728–1732] [2, p. 676] [10, p. 1253]. In this view, code can concretely specify rule-like (rather than standard-like) relationships between inputs and outputs that are “free from ambiguity” [22, p. 1723]. Put simply, this conception of code maps nicely to “if/then” rules that resemble those in the law. Yet, as we have seen throughout this paper, the assumption of deterministic code does not hold for ML. Due to its statistical nature, ML model code does not operate by deterministic “if/then” rules. Instead, due to non-determinism, it is as if both the “if” and the “then” are fuzzy — they are not specifiable in concrete terms. It is therefore natural to ask: What does non-deterministic code do to an idea of “code as law” that is predicated on determinism?

We attempt an answer in a (sort-of) proof by contradiction. We begin by assuming that “code as law” still holds for the non-deterministic code of ML. From there, then, we would need to consider what it would mean for the law to similarly exhibit non-determinism. And this is where “code is law” immediately starts to break down. In the ideal case, the law should have deterministic outcomes based on its inputs. It can exhibit variation in the relationships between inputs and outputs, but it should not be the case that there is randomness or arbitrariness in those relationships [24, pp. 665-666]. In practice, non-determinism can of course occur in the law: Judicial discretion means that, given similar inputs, outputs can vary across judges (or even within the same judge). But unlike in ML, the legal system embodies answerability. There are actors in the system who must step forward and answer for their decisions; they must provide explanations and are subject to cross-examination [6, 40]. Answerability in part functions to remove randomness and arbitrariness from the law. In the long run, the system undergoes an ongoing process of legitimation. In other words, the law has mechanisms for recourse, which effectively can serve to root out non-determinism. In other words, unlike ML, law treats non-determinism like a bug, not a feature.

This indicates a fundamental incompatibility for understanding ML code as law. The law can do work to avoid non-determinism, while ML inherently relies on it. Because of ML’s inherent non-determinism, and its resultant unpredictability, code, unlike law, evades regulation. To borrow a phrase from Jack Balkin, such “code is lawless” [1, p. 52], since the unpredictability that results from non-determinism presents key problems for thinking of code as being constitutive of law.

4 CONCLUSION: CLOSING THE GAP BETWEEN THE DISTRIBUTIONAL AND THE INDIVIDUAL

Non-deterministic code may itself be lawless, but this does not mean we should entirely avoid its use [16] and that we can do nothing to better regulate its deployment in practice. On the ML side, we can strive to develop tools that obtain some measure of consistency — e.g., similar model outcome distributions across training runs — even in the presence of non-determinism. The current push for more robust ML is in fact working to develop algorithms that leverage non-determinism to learn complex decision surfaces, but also provably have bounded effects on, for example, variance in model-training outcomes. In short, ML can do work to tighten distributions, to provide theoretical limits on error (that then have to be met in practice), and to characterize rigorous trade-offs between computational resource usage for training models and how robust resulting models can be. These are all rich areas of research in ML, all of which become better-appreciated when understanding ML from a distributional perspective.

While ML can work to tighten distributions and improve robustness, non-determinism will always remain feature, not a bug. Legal scholarship thus needs to attend to the role of distributions over outcomes, in order to fully appreciate how stochasticity contributes to uncertainty and non-determinism in the behavior of ML systems. As we have seen through brief examples concerning unfairness and due process, uncertainty and non-determinism, not just individual outcomes, can themselves implicate harms. Since the law will necessarily focus on harms, its work will be to close the gap between these two essential ways of viewing ML — to ensure that the law is able to reason about distributional aspects, in such a way that these aspects serve to clarify how they relate to individual outcomes. The law must find ways to bring the distributional and the individual together, such that it can successfully bring ML to account for the harms it causes.

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