Signatures of Fractional Quantum Hall States in Topological Insulators

Dong-Xia Qu1, Y. S. Hor2, and R. J. Cava3

1 Lawrence Livermore National Laboratory, Livermore, CA 94550, USA
2 Department of Physics, Missouri University of Science and Technology, Rolla, MO 65409, USA
3 Department of Chemistry, Princeton University, New Jersey 08544, USA.

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The fractional quantum Hall (FQH) state is a topological state of matter resulting from the many-body effect of interacting electrons and is of vast interest in fundamental physics [1, 2]. The experimental observation of topological surface states (SSs) in three-dimensional bulk solids has allowed the study of a correlated chiral Dirac fermion system, which can host a single Dirac valley without spin degeneracy [3, 4]. Recent theoretical studies suggest that the fractional quantum Hall effect (FQHE) might be observable in topological insulators [11, 12]. However, due to the dominant bulk conduction it is difficult to probe the strong correlation effect in topological insulators from resistivity measurements [8, 9]. Here we report the discovery of FQH states in Bi2Te3 from thermopower measurements. The surface thermopower is ten times greater than that of bulk, which makes possible the observation of fractional-filled Landau levels in SSs. Thermopower hence provides a powerful tool to investigate correlated Dirac fermions in topological insulators. Our observations demonstrate that Dirac topological SSs exhibit strongly correlated phases in a high magnetic field, and would enable studies of a variety of exotic fractional quantum Hall physics and other correlated phenomena in this newly discovered chiral Dirac system.

Topological insulators (TIs) are a new class of quantum states of matter with topologically protected conducting SSs, arising from the topology of the bulk electronic band structure [13–17]. There are two distinguishing features of topological surface states. One is the existence of an odd number of Dirac cones on each surface, and the other is the helical spin arrangement [5, 6, 10]. Theoretically, the relativistic nature of Dirac fermions is believed to significantly modify the electron-electron interactions, with the possibility to produce more robust ground states at the n = 1 Landau level (LL) in TIs than in conventional two-dimensional electron systems [20–23]. The unique spin texture and the coexistence of non-insulating bulk states also raise the intriguing question of whether TIs may host exotic FQH states owing to the non-trivial Berry’s phase [10], huge Zeeman energy [24], and the screening effect from bulk carriers [11]. The potential realization of more stable non-Abelian FQH states in TIs is of practical interest for topological quantum computing [12, 25].

There have been magnetoresistance measurements at n = 0 and 1 LLs in (Bi1–xSbx)2Se3 [3] and at n = 4 and 5 LLs in Bi2Se3Te [18]. However, the sub-integer oscillations at n = 1 LL in (Bi1–xSbx)2Se3 can only be resolved in the second-derivative trace and their best linear fit intersects the filling-factor axis at 0 instead of 1/2, inconsistent with a Dirac spectrum. Such a property makes the exact origin of these oscillations unclear [3]. On the other hand, the FQHE in TIs is theoretically precluded in the |n| > 1 LLs [11], suggesting the features in Bi2Se3Te unlikely to be ascribable to the FQHE. So far, transport studies of FQHE in TIs have been limited to samples with mobility below 3,000 cm²/Vs and the SS conduction is susceptible to conducting bulk states.

To explore the existence of FQH states in TIs, we present thermoelectric measurements on the Bi2Te3 crystals. The surface mobility of these crystals is up to 14,000 cm²/Vs [3], comparable to the Hall mobility (30,000 cm²/Vs) of high quality graphene where the FQHE has been recently discovered [26]. We first examined the dependence of thermopower $S_{xx}$ on temperature $T$ in both metallic and nonmetallic samples (Fig. 1, insets). Though $S_{xx}$ shows a low-$T$ peak in all these samples, the peak of the nonmetallic samples Q1 and Q2 is significantly stronger than that of the metallic sample M1. These observed peaks indicate the occurrence of phonon-drag effect that is expected to appear at ~ 29 K in high purity Bi2Te3 crystals [27]. It has been demonstrated that the phonon-drag thermopower from a two-dimensional (2D) conducting layer on a three-dimensional (3D) crystal can display giant quantum oscillations due to the phonon intra- and inter-LL scattering in the presence of a strong magnetic field. In such a 3D system, surface electrons are dragged by non-equilibrium 3D phonons of the whole specimen, while in a purely 2D system such as graphene, electrons of a wavevector $k$ can only interact with 2D phonons of a wavevector $q < 2k$. In addition, the bulk thermopower is considerably suppressed due to the existence of two types of bulk carriers with opposite signs [8]. Therefore, we expect that the magneto-thermopower of SSs is orders of magnitude larger than that of bulk in the high field limit. The thermopower measurement thus provides a powerful tool to elucidate the nature of the topological SSs that is difficult to be probed by the conductance measurement.

*All correspondence should be addressed to D. Qu (qu2@llnl.gov)
As shown below, the extracted bottom surface Q2. Similar results were observed in sample Q1, with the top and bottom surface index fields one piece of sample (Fig. 2c). Hence, we can pinpoint \( \nu_{\mu}/H < \) crystal into bulk samples with a thickness faces. Similar effect has been seen in strained HgTe 3D degeneracy is lifted between top (+) and bottom (−) surfaces. This splitting indicates that the than the oscillating structure in the low field limit.

Figure 1, a and b, show the thermopower response \( -S_{xx} \) versus the inverse magnetic field 1\( /H \) in samples Q1 and Q2, respectively. Large LL oscillations begin to emerge at \( H > 8 \) T and their amplitude becomes smaller as \( T \) decreases from 7 to 3.5 K. A prominent feature of these oscillations is that at \( 1/H < 0.06 \) T\(^{-1}\) sharp dips (black arrows) appear, with an apsidal spacing smaller than the oscillating structure in the low field limit.

It is illuminating to compare the oscillations in \( -S_{xx} \) and the Shubnikov-de Hass (SdH) effect in the conductance tensor \( G_{xx} \), which was confirmed to arise from the 2D SSs in the previous study. As shown in Fig. 2, a and b, the extrema in \( -S_{xx} \) coincide with the extrema in \( \Delta G_{xx} \). This occurs because both \( -S_{xx} \) (\( S_{xx} < 0 \) for electron-like carriers) and \( \sigma_{xx} \) peak when the Fermi level \( (E_F) \) aligns with each LL, whereas vanish when \( E_F \) lies between LLs. Furthermore, we observe pronounced LL splitting near 1\( /H = 0.061, 0.102, \) and 0.142 T\(^{-1}\) (gray dashed lines in Fig. 2b). This splitting indicates that the degeneracy is lifted between top (+) and bottom (−) surfaces. Similar effect has been seen in strained HgTe 3D TIs. Here, a weak Te composition gradient in Bi\(_2\)Te\(_3\) breaks the inversion symmetry and generates displaced Dirac points (see the inset of Fig. 2a). By cleaving the crystal into bulk samples with a thickness \( t = 20 \sim 100 \) \( \mu_m \), we obtain slightly different surface carrier densities, which then leads to two sets of Landau filling factor \( \nu \) in one piece of sample (Fig. 2c). Hence, we can pinpoint the top and bottom surface index fields \( B_{\nu^+} \) and \( B_{\nu^-} \) from the periodic spacing of strong (black dashed lines) and weak (grey dashed lines) minima in \( \Delta G_{xx} \) for sample Q2. Similar results were observed in sample Q1, with its bottom surface cleaved adjacent to the top surface of sample Q2. As shown below, the extracted bottom surface carrier concentration in Q1 is almost the same as the top surface carrier concentration in Q2. The Fermi energy levels relative to the Dirac point in samples Q1 and Q2 are sketched in Fig. 2d.

In terms of the carrier concentration \( n_e \) on one surface, \( B_{\nu} \) is related to \( \nu \) by

\[
B_{\nu} = \frac{n_e \phi_0}{(\nu - \gamma)}
\]

where \( \phi_0 = h/e \) is the magnetic flux quanta, \( h \) Planck’s constant, \( e \) the charge of electron, and \( \gamma \) the filling factor shift. A shift with \( \gamma = 0 \) corresponds to a conventional spectrum, whereas a deviation from the zero-shift with \( \gamma = 1/2 \) implies a Dirac spectrum. The 1/2 arises from the \( n = 0 \) LL at the Dirac point. In the following, we label the filling factors as \( \nu^s \), where \( s = \pm \) indexes the top and bottom SSs. With the \( B_{\nu}^{-1} \) identified in both \( -S_{xx} \) and \( \Delta G_{xx} \), we plot them against integers (triangles and circles in Fig. 3, a and b). The slopes of the linear-fit to the data yield the carrier concentration \( n_e = 9.27 \times 10^{11} \) and 7.37 \( \times 10^{11} \) cm\(^{-2}\), with the Fermi wavevector \( k_F = 0.034 (0.031) \) and 0.030 (0.027) Å\(^{-1}\), for the top (bottom) SSs in samples Q1 and Q2, respectively. The linear-fit intercepts the \( \nu \) axis at \( \gamma = 0.45 \pm 0.02 \) in Q1 and 0.67 \pm 0.02 in Q2, consistent with a Dirac dispersion. Hence, we are again convinced that the 2D Dirac states give rise to the LL indexing shown in Fig. 2. Furthermore, the weak-field Hall anomaly provides an independent measurement of the average surface wavevector. The value of \( k_F \) derived from surface Hall conductance is in reasonable agreement with the quantum oscillation analysis.

We then illustrate the fine structure of \( -S_{xx} \) in the range of \( 1 < \nu < 2 \) for samples Q1 and Q2 (Fig. 3, c and d). We observed narrow, reproducible minima at the fields \( B_{\nu^s} \) (dashed lines) with \( \nu^s = 5/3 \). In addition to the minima at \( \nu^s = 5/3 \), a valley-like structure at \( \nu^s = 9/5 \) is discernible in Q1, and the dip at \( \nu^s = 9/5 \) becomes prominent in Q2 as the temperature decreases. As seen in Fig. 3, a and b, the fractional Landau fillings lie on a straight line with the integer ones. To examine the oscillating profiles of the most pronounced sub-integer structures, we plot the \( -S_{xx} \) versus the filling factor calculated as \( \nu = n_e \phi_0 B^{-1} + \gamma \) (Fig. 4, a and b). The \( -S_{xx}(\nu) \) traces obtained at various \( n_e \) are almost overlapped, and their minima are all located around \( \nu = 1.67 \pm 0.02 \) (traces are displaced for clarity). Both features strongly suggest that the observed high-field structures are associated with the FQH states at \( n = 1 \) LL. Based on the theory of thermopower in the QHE regime, a minimum in \( \sigma_{xx} \) should accompany a minimum in thermopower. As shown in Fig. 2b (black arrows), two sub-integer dips are clearly resolved in \( \Delta \sigma_{xx} \) with \( \nu^s \) close to \( \pm 5/3 \). Noting that for conventional 2D electron gas in the FQH regime, \( \rho_{xy} \) should be quantized with \( (\nu e^2/h)^{-1} \) at filling factor \( \nu \). Unfortunately, the present of the bulk conduction channel does not allow us to measure exact Hall magnitude. Even in most resis-

FIG. 1: Magneto-thermopower in high magnetic fields. (a) The thermopower response \( -S_{xx} \) versus \( 1/H \) in sample Q1 at temperatures \( T = 3.5, 5, \) and 7 K. (b) \( -S_{xx} \) versus \( 1/H \) in sample Q2. The insets show the \( T \) dependence of the thermopower profiles in samples Q1 and M1 in (a) and in sample Q2 in (b). The arrows mark the sub-integer dips resolved in \( 1/H > 0.06 \) T\(^{-1}\). The dashed lines indicate the oscillating minima in the low field regime.
negative TIs, Hall quantization is obscured by dominate bulk contribution and no quantized Hall plateaus have so far been observed.

In spite of the limited data set, we may roughly estimate the lower bound of the gap energy of the 5/3 state \((\Delta_{5/3})\) from the T dependence of the thermopower and resistivity minima, both of which scale as \(e^{-\Delta_{5/3}/2T}\). As shown in Fig. 2b and Fig. 3, c and d, the 5/3\(^+\) state persists until 5 K, indicating \(\Delta_{5/3} > 5\) K. This value is more than an order of magnitude larger than the corresponding gap \((\Delta_{3/2})\) in the GaAs system with a much higher mobility \([20]\), but comparable to the gap in graphene with a similar mobility \([21]\). This is not surprising because the \(n = 1\) LL in the topological SSs is a mixture of the \(n = 0\) and \(1\) LLs in non-relativistic systems. It makes the FQH states in the \(n = 1\) LL in the Dirac system more robust than those in the GaAs system \([11, 20–23]\). Compared with graphene, the surface states in Bi\(_2\)Te\(_3\) has only one Dirac valley with no spin degeneracy, analogous to a completely four-fold degeneracy lifted graphene system where the FQH states do not mix between spin- and valley-bands. Moreover, the helical spin texture of topological SSs and the presence of conductive bulk states may lead to enhanced effective Coulomb interactions, rendering the FQH states even more robust in TIs \([11]\).

We next extract the surface thermopower \(S_{xx}^s\) from the observed thermopower response. The measured thermopower tensor \(S_{ij}\) can be expressed as the sum,

\[
S_{ij} = \sum_{k=x,y} \rho_{ik}(\alpha_{kj}^b + \alpha_{kj}^s)/l
\]

where \(\rho_{ij}\) is the total resistivity tensor, \(\alpha_{ij}^l = \sum_k \sigma_{ik}^l S_{kj}^l\) with \(l = b\) or \(s\), the bulk or surface thermoelectric conductivity tensor, and \(\sigma_{ij}^l\) the bulk or surface conductivity tensor. Since \(\rho_{xx} \gg \rho_{yz}, \sigma_{xx}^b \gg \sigma_{yz}^b, \sigma_{xx}^s \gg \sigma_{yz}^s\) for nonmetallic Bi\(_2\)Te\(_3\) in the high-field regime, \(S_{xx}^s\) can be approximated as

\[
S_{xx}^s = S_{xx}^b + 1/l \rho_{xx} G_{xx}^s S_{xx}^s
\]

where \(S_{xx}^b\) is the bulk thermopower and \(G_{xx}^s\) the surface conductance. The bulk thermopower only gives rise to a featureless background. The \(\rho_{xx} G_{xx}^s/l\) term can be obtained from the resistivity measurements. We find that the maximum magnitude of \(\rho_{xx} G_{xx}^s/l \sim 0.027\) and 0.01 in Q1 and Q2, respectively. From Eq. (3), we can extract the \(-S_{xx}^s\) versus \(H\) in Q1 and Q2 (Fig. 4, c and...
The fractional structures resolved at the integer-filling factors are marked by dotted lines. The fractional structures resolved at $\nu < 2$ are indicated by arrows.

The peak magnitude of $-S_{xx}$ is in the range of 0.5 to 2.0 mV K$^{-1}$, which is more than an order of magnitude higher than that of the bulk $\sim 30 \mu$V K$^{-1}$ at 5 K. Unlike conventional 2D systems where the thermopower magnitude roughly displays a linear field dependence, the surface thermopower at higher order LLs such as $n = 4$ is comparable or even greater than that of lower LLs ($n = 3$). This giant oscillating magnitude and the specific field profile of the surface thermopower can be understood within the scenario of the 2D Dirac electron and 3D phonon interaction.

Because of the relativistic dispersion of topological surface states, the wave function $\Psi_n$ of a Dirac electron in the $n$th LL is the superposition of the $n$th and $(n - 1)$th LL wave functions of a non-relativistic electron. The mixture nature of the wave function significantly modifies the electron-phonon matrix element in the $n \geq 1$ LLs, leading to a thermopower profile different from an ordinary 2D system.

Using a general model given in ref. 29 and the wave function for topological SSs, we numerically simulate the thermopower induced by the integer Landau quantization in Q1 and Q2, with the phonon mean free path treated as a fitting parameter. (Fig. 4, c and d, green curves). We include the average LL broadening width $\Gamma = 3.5$ and 7 meV for Q1 and Q2, respectively. The simple electron-phonon interaction model (Eq. S5) does not capture the fractional features, as FQHE is not included in the model. However, it reproduces the index field position and the oscillation magnitude from the integer Landau quantization. This suggests that the observed giant integer Landau oscillations can be explained by 2D Dirac fermion and 3D phonon interaction. A more comprehensive FQHE framework is needed to model the fractional-filling states in thermopower response.

By performing thermopower measurements, we have resolved fractional Landau quantization of SSs at $\nu = 5/3$ and 9/5. The observed gap energy at the 5/3 state is ten times larger than that of the non-relativistic electron systems. The demonstration of the FQH states in the topological surface bands opens the door to future studies of fractional quantum Hall effect physics in the topological insulator, which is expected to display strong correlation effects between chiral Dirac fermions.

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