We show that for each M-theory background, having subspaces with metrics of given type, there exist M2-brane configurations, which in appropriate limit lead to two-spin magnon-like energy-charge relations, established for strings on $AdS_5 \times S^5$, its $\beta$-deformation, and for membrane in $AdS_4 \times S^7$.

**Keywords:** M-theory, M/field theory correspondence, spin chains.

1 Introduction

One of the predictions of the AdS/CFT correspondence is that the string theory on $AdS_5 \times S^5$ should be dual to $\mathcal{N} = 4$ Super Yang Mills (SYM) theory in four dimensions [1], [2], [3]. The spectrum of the string states and of the operators in SYM should be the same. The recent checks of this conjecture beyond the supergravity approximation are connected to the idea to search for string solutions, which in semiclassical limit (large conserved charges) are related to the anomalous dimensions of certain gauge invariant operators in the planar SYM [4], [5]. On the field theory side it was established that the corresponding dilatation operator is connected to the Hamiltonian of integrable Heisenberg spin chain [6].

In a recent paper [7], Hofman and Maldasena explored a specific semiclassical limit for strings on $R \times S^2$ subspace of $AdS_5 \times S^5$ and related it to the spin chain magnon states. This limit leads to significant simplifications, and thus allows for further improvement of our knowledge about the string/gauge spectrum duality. More specifically, the ”giant magnon” solution obtained in [7] is a string with energy $E$ and spin $J$, which in the limit $E, J \rightarrow \infty$, $(E - J)$-finite, obey the energy-charge relation

$$E - J = \frac{\sqrt{\lambda}}{\pi} \cos \theta_0,$$

\[\text{1e-mail: bozhilov@inrne.bas.bg}\]
where \( \lambda \) is the 't Hooft coupling, proportional to the square of the string tension \( T \), and the geometric angle \( \theta_0 \) is identified with the magnon momentum \( p \) on the spin chain side through the equality

\[
\cos \theta_0 = | \sin(p/2) | .
\]

In [8], N. Dorey proposed dispersion relation describing magnon \textit{bound states}

\[
E - J = \sqrt{Q^2 + \frac{\lambda}{\pi^2} \sin^2(p/2)},
\]

(1.1)

where \( Q \) is the number of the constituent magnons, which should correspond on the string theory side to the two-spin energy-charge relation

\[
E - J_2 = \sqrt{J_1^2 + \frac{\lambda}{\pi^2} \sin^2(p/2)}.
\]

(1.2)

The folded string solution used in [8] as confirmation of the above proposal, in the limit

\[
E, J_2 \to \infty, \quad E - J_2, J_1 - \text{finite},
\]

gives

\[
E - J_2 = \sqrt{J_1^2 + \frac{4\lambda}{\pi^2}} = 2\sqrt{\left(\frac{J_1}{2}\right)^2 + \frac{\lambda}{\pi^2}}.
\]

(1.3)

As far as the folded string configuration is symmetric, this state was interpreted as consisting of two excitations with momenta \( p = \pm \pi \), each carrying half of the total angular momentum (spin) \( J_1 \). The conclusion drawn was that then (1.3) agrees with (1.1). In a subsequent paper [11], N. Dorey et al. was able to find string solution, which gives exactly the relation (1.2) after the identification \( p = 2 \tan^{-1}(1/k) \), where \( k \) is a free parameter. The same result has been obtained in [12]-[18], by identifying different parameters in the string solutions with \( p \), or by purely group theoretic means [19]. Evidently, the general structure is [13]

\[
E - J_2 = \sqrt{J_1^2 + k^2 \lambda},
\]

(1.4)

where \( k \) is a constant depending on the particular solution.

The above results have been obtained for strings moving on the type IIB \( AdS_5 \times S^5 \) background. However, it turns out that relation of the type (1.4) also holds for strings on the \( \beta \)-deformed \( AdS_5 \times S^5 \) [20]. The difference with (1.2) is in the shift

\[
\frac{p}{2} \to \frac{p}{2} - \pi \beta,
\]

where \( \beta \) is the deformation parameter [16], [17].

\(^2\)Obtained in [9], [10].
The influence of the NS-NS field on the two-spin giant magnon has been also examined [21]. The resulting changes in (1.4) are: new constant $k^2$ and

$$J_1^2 \rightarrow \text{const} J_1^2.$$  

For further investigations of the giant magnon properties see [22]-[28] and references therein.

In this letter, we will show that there exist string configurations, which satisfy magnon-like dispersion relations of the type

$$E - AJ_2 = \sqrt{BJ_1^2 + CT^2}, \quad T^2 \sim \lambda,$$

depending on three parameters $A$, $B$ and $C$. Moreover, our main result is that the equality (1.5) also holds for specific M2-brane configurations in M-theory. Such solution has been already found for membrane moving on a subspace of $AdS_4 \times S^7$ [29].

2 Two-spin magnon-like relations from M-theory

We will work with the following gauge fixed membrane action and constraints [30], which coincide with the gauge fixed Polyakov type action and constraints after the identification (see for instance [31]) 2

$$2\lambda^0 T_2 = L = \text{const}:$$

$$S_M = \int d^3\xi L_M = \frac{1}{4\lambda^0} \int d^3\xi \left[ G_{00} - \left( 2\lambda^0 T_2 \right)^2 \det G_{ij} \right],$$

$$G_{00} + \left( 2\lambda^0 T_2 \right)^2 \det G_{ij} = 0,$$

$$G_{0i} = 0.$$  

In (2.1)-(2.3), the metric induced on the membrane worldvolume $G_{mn}$ is given by

$$G_{mn} = g_{MN} \partial_m X^M \partial_n X^N,$$

where $g_{MN}$ is the target space metric. The equations of motion for $X^M$, following from (2.1), are as follows ($G \equiv \det G_{ij}$)

$$g_{LN} \left[ \partial_0^2 X^N - \left( 2\lambda^0 T_2 \right)^2 \partial_i \left( G G^{ij} \partial_j X^N \right) \right]$$

$$+ \Gamma_{L,MN} \left[ \partial_0 X^M \partial_0 X^N - \left( 2\lambda^0 T_2 \right)^2 G G^{ij} \partial_i X^M \partial_j X^N \right] = 0,$$

where

$$\Gamma_{L,MN} = g_{LK} \Gamma_{MN}^K = \frac{1}{2} \left( \partial_M g_{NL} + \partial_N g_{ML} - \partial_L g_{MN} \right)$$

are the components of the symmetric connection corresponding to the metric $g_{MN}$.  

3
If we split the target space coordinates as $x^M = (x^\mu, x^a)$, where $x^\mu$ are those on which the background does not depend, the conserved charges are given by the expression

$$Q_\mu = \frac{1}{2\lambda^2} \int d\xi^1 d\xi^2 g_{\mu N} \partial_0 X^N.$$  \hspace{1cm} (2.6)

Now, let us turn to our particular tasks. Consider backgrounds of the type

$$ds^2 = c^2 \left[-dt^2 + c_1^2 d\theta^2 + c_2^2 \cos^2 \theta d\varphi_1^2 + c_3^2 \sin^2 \theta d\varphi_2^2 + c_4^2 f(\theta) d\varphi_3^2\right],$$  \hspace{1cm} (2.7)

where $c, c_1, c_2, c_3, c_4$ are arbitrary constants, and $f(\theta)$ takes two values: $f(\theta) = 1$ and $f(\theta) = \sin^2 \theta$. We embed the membrane into (2.7) in the following way

$$X^0(\xi^m) \equiv t(\xi^m) = \Lambda_0^0 \xi^0, \quad X^1(\xi^m) = \theta(\xi^2),$$

$$X^2(\xi^m) \equiv \varphi_1(\xi^m) = \Lambda_0^2 \xi^0,$$

$$X^3(\xi^m) \equiv \varphi_2(\xi^m) = \Lambda_0^3 \xi^0,$$

$$X^4(\xi^m) = \varphi_3(\xi^m) = \Lambda_1^4 \xi^1,$$

$$\mu = 0, 2, 3, 4, \quad a = 1, \quad \Lambda_0^a, \ldots, \Lambda_4^a = \text{constants}.$$  \hspace{1cm} (2.8)

This ansatz corresponds to M2-brane extended in the $\theta$-direction, moving with constant energy $E$ along the $t$-coordinate, rotating in the planes defined by the angles $\varphi_1, \varphi_2$, with constant angular momenta $J_1, J_2$, and wrapped along $\varphi_3$. The computations show that for this embedding, the constraints (2.3) and the equations of motion for the membrane coordinates $X^\mu(\xi^m)$ are satisfied identically. Moreover, it turns out that the remaining constraint (2.2) is first integral of the equation of motion for $X^a = X^1 = \theta$. That is why, it remains to solve the differential equation (2.2) only.

We begin with the case $f(\theta) = 1$, when (2.2) reduces to

$$K\theta^2 + V(\theta) = 0,$$

$$K = -(2\Lambda^0 T_2 c^2 c_1 c_4 \Lambda_1^4)^2,$$

$$V(\theta) = c^2 \left\{ (\Lambda_0^0)^2 - (\Lambda_0^2 c_2)^2 - (\Lambda_0^3 c_3)^2 - (\Lambda_0^4 c_2)^2 \right\} \sin^2 \theta.$$  \hspace{1cm} (2.9)

From (2.9) one obtains the turning point ($\theta' = 0$) for the effective one dimensional motion

$$M^2 = \frac{(\Lambda_0^0)^2 - (\Lambda_0^2 c_2)^2}{(\Lambda_0^3 c_3)^2 - (\Lambda_0^4 c_2)^2}.$$  \hspace{1cm} (2.10)

The solution of (2.9) is

$$\xi^2(\theta) = \frac{2\Lambda^0 T_2 c c_1 c_4 \Lambda_1^4 \sin \theta}{M \left[ (\Lambda_0^3 c_3)^2 - (\Lambda_0^4 c_2)^2 \right]^{1/2}} F_1(1/2, 1/2, 1/2; 3/2; \sin^2 \theta, \frac{\sin^2 \theta}{M^2}),$$  \hspace{1cm} (2.11)

where $F_1(a, b_1, b_2; c; z_1, z_2)$ is one of the hypergeometric functions of two variables. On this solution, the conserved charges (2.6) take the form ($Q_0 \equiv -E$, $Q_2 \equiv J_1$, $Q_3 \equiv J_2$, $Q_4 = 0$)

$$\frac{E}{\Lambda_0^0} = \frac{2\pi^2 T_2 c^2 c_1 c_4 \Lambda_1^4}{\left[ (\Lambda_0^3 c_3)^2 - (\Lambda_0^4 c_2)^2 \right]^{1/2}} F_1(1/2, 1/2; 1; M^2),$$  \hspace{1cm} (2.12)
\[
\frac{J_1}{\Lambda_0^2} = \frac{2\pi^2 T_2 c^3 c_1 c_2^2 c_4 A_1^4}{[(\Lambda_0^3 c_3)^2 - (\Lambda_0^2 c_2)^2]^{1/2}} 2 F_1(-1/2, 1/2; 1; M^2), \tag{2.13}
\]
\[
\frac{J_2}{\Lambda_0^2} = \frac{2\pi^2 T_2 c^3 c_1 c_2^2 c_4 A_1^4}{[(\Lambda_0^3 c_3)^2 - (\Lambda_0^2 c_2)^2]^{1/2}} \left[ 2 F_1(1/2, 1/2; 1; M^2) - 2 F_1(-1/2, 1/2; 1; M^2) \right], \tag{2.14}
\]

where \(2 F_1(a, b; c; z)\) is the Gauss’ hypergeometric function.

Our next aim is to consider the limit, in which \(M\) tends to its maximum value: \(M \to 1_-\). In this case, by using (2.10) and (2.12)-(2.14), one arrives at the energy-charge relation

\[
E - \frac{J_2}{c_3} = \sqrt{\left( \frac{J_1}{c_2} \right)^2 + (4\pi T_2 c^3 c_1 c_4 A_1^4)^2}, \tag{2.15}
\]

for

\[
E, J_2/c_3 \to \infty, \quad E - J_2/c_3, J_1/c_2 - \text{finite}. \tag{2.16}
\]

Now, we are going to consider the case \(f(\theta) = \sin^2 \theta\) (see (2.7)), when (2.2) takes the form

\[
\tilde{K} \theta^2 + V(\theta) = 0, \tag{2.17}
\]

\[
\tilde{K} = -(2\lambda^0 T_2 c^2 c_1 c_4 A_1^4)^2 \sin^2 \theta = K \sin^2 \theta,
\]

where \(V(\theta)\) and correspondingly \(M^2\) are the same as in (2.9) and (2.10). The solution of (2.17) is given by the equality

\[
\xi^2(\theta) = \frac{\lambda^0 T_2 c c_4 A_1^4 \sin^2 \theta}{M [(\Lambda_0^3 c_3)^2 - (\Lambda_0^2 c_2)^2]^{1/2}} F_1(1, 1/2, 1/2; 2; \sin^2 \theta, \frac{\sin^2 \theta}{M^2}), \tag{2.18}
\]

and is obviously different from the previously obtained one. The computations show that on (2.18) the conserved charges (2.6) are as follows

\[
\frac{E}{\Lambda_0} = \frac{2\pi T_2 c^3 c_1 c_4 A_1^4}{[(\Lambda_0^3 c_3)^2 - (\Lambda_0^2 c_2)^2]^{1/2}} \ln \left( \frac{1 + M}{1 - M} \right), \tag{2.19}
\]

\[
\frac{J_1}{\Lambda_0^2} = \frac{2\pi T_2 c^3 c_1 c_2^2 c_4 A_1^4}{[(\Lambda_0^3 c_3)^2 - (\Lambda_0^2 c_2)^2]^{1/2}} \left[ \frac{1 - M^2}{2} \ln \left( \frac{1 + M}{1 - M} \right) + M \right], \tag{2.20}
\]

\[
\frac{J_2}{\Lambda_0^2} = \frac{2\pi T_2 c^3 c_1 c_2^2 c_4 A_1^4}{[(\Lambda_0^3 c_3)^2 - (\Lambda_0^2 c_2)^2]^{1/2}} \left[ \frac{1 + M^2}{2} \ln \left( \frac{1 + M}{1 - M} \right) - M \right]. \tag{2.21}
\]

Taking \(M \to 1_-\), one sees that it corresponds again to the limit (2.16), and the two-spin energy-charge relation is

\[
E - \frac{J_2}{c_3} = \sqrt{\left( \frac{J_1}{c_2} \right)^2 + (2\pi T_2 c^3 c_1 c_4 A_1^4)^2}, \tag{2.22}
\]

which differs from (2.15) only by a factor of 4 in the second term on the right hand side.
It is instructive to compare the above results with the string case by using the same approach. To this end, for correspondence with the membrane formulae, we will use the Polyakov action and constraints in diagonal worldsheet gauge

\[ S_S = \int d^2 \xi \mathcal{L}_S = \int d^2 \xi \frac{1}{4 \lambda_0} \left[ G_{00} - \left( 2 \lambda^0 T \right)^2 G_{11} \right], \]

\[ G_{00} + \left( 2 \lambda^0 T \right)^2 G_{11} = 0, \]

\[ G_{01} = 0, \]

where

\[ G_{mn} = g_{MN} \partial_m X^M \partial_n X^N, \quad \partial_m = \partial / \partial \xi^m, \quad m = (0, 1), \quad M = (0, 1, \ldots, 9). \]

The usually used conformal gauge corresponds to \( 2 \lambda^0 T = 1. \)

An appropriate string theory background is

\[ ds^2 = c^2 \left[ -dt^2 + c_1^2 d\theta^2 + c_2^2 \cos^2 \theta d\varphi_1^2 + c_3^2 \sin^2 \theta d\varphi_2^2 \right]. \]  

(2.23)

We consider string embedding in (2.23) of the type

\[ X^0(\xi^m) \equiv t(\xi^m) = \Lambda_0^0 \xi^0, \quad X^1(\xi^m) = \theta(\xi^1), \]

\[ X^2(\xi^m) \equiv \varphi_1(\xi^m) = \Lambda_0^2 \xi^0, \]

\[ X^3(\xi^m) \equiv \varphi_2(\xi^m) = \Lambda_0^3 \xi^0, \quad \Lambda_0^0, \Lambda_0^2, \Lambda_0^3 = \text{constants}. \]  

(2.24)

This ansatz corresponds to string extended in the \( \theta \)-direction, moving with constant energy \( E \), and rotating in the planes given by the angles \( \varphi_1, \varphi_2 \), with constant angular momenta \( J_1, J_2 \). Our calculations show that in the limit (2.16), the string configuration (2.24) is characterized by the following magnon-like relation

\[ E = \frac{J_2}{c_3} = \sqrt{\left( \frac{J_1}{c_2} \right)^2 + (4T c_2 c_1)^2}. \]  

(2.25)

Obviously, the two-spin energy-charge relations (2.15), (2.22) for membranes and (2.25) for strings are of the same type.

3 Discussion

We have shown here that for each M-theory background, having subspaces with metrics of the type (2.7), there exist M2-brane configurations given by (2.8), which in the limit (2.16) lead to the two-spin, magnon-like, energy-charge relations (2.15) and (2.22).

Examples for target space metrics of the type (2.7) are several subspaces of \( R \times S^7 \), contained in the \( \text{AdS}_4 \times S^7 \) solution of M-theory. As we already noticed in the introduction, a membrane configuration has been found in [29], corresponding to membrane moving on one of the possible \( \text{AdS}_4 \times S^7 \) subspaces, with the desired properties. Namely, the background metric is given by

\[ ds^2 = (2l_p R)^2 \left\{ -dt^2 + 4 \left[ d\psi^2 + \cos^2 \psi d\varphi_1^2 + \sin^2 \psi \left( \cos^2 \theta_0 d\varphi_2^2 + \sin^2 \theta_0 d\varphi_3^2 \right) \right] \right\}. \]  

6
where the angle $\theta$ is fixed to an arbitrary value $\theta_0$, and the background 3-form field on $AdS_4$ vanishes. The obtained two-spin, magnon-like, energy-charge relation is

$$E - \frac{J_2}{2 \cos \theta_0} = \sqrt{\left(\frac{J_1}{2}\right)^2 + \left[2\sqrt{\pi} T_2 (l_p R)^3 \Lambda_1^4 \sin \theta_0 \right]^2},$$

and it corresponds to $c = 2l_p R$, $c_1 = 2$, $c_2 = 2$, $c_3 = 2 \cos \theta_0$, $c_4 = 2 \sin \theta_0$ in (2.22).

Moreover, it is not difficult to see that there exist 4 different subspaces of $R \times S^7$ of the type (2.7), when one of the isometry coordinates $\phi_1$, $\phi_2$, $\phi_3$ or $\phi_4$ equals zero, for which membrane embedding of the type (2.8) ensures the existence of 12 solutions with semiclassical behavior described by (2.15) or (2.22), corresponding to different values of the parameters $c$, $c_1$, ..., $c_4$.

Let us show that this is indeed the case. To this end, we parameterize the metric on $R \times S^7$ subspace of $AdS_4 \times S^7$ as follows

$$ds^2 = (2l_p R)^2 \left\{-dt^2 + 4 \left\{d\psi_1^2 + \cos^2 \psi_1 d\phi_1^2 + \sin^2 \psi_1 \left[d\psi_2^2 + \cos^2 \psi_2 d\phi_2^2 + \sin^2 \psi_2 \left(d\theta^2 + \cos^2 \theta d\phi_3^2 + \sin^2 \theta d\phi_4^2\right)\right]\right\}\right\}.$$

If we fix $\phi_4 = 0$, we will have two subcases for which the metric will be of the type (2.7): $(\psi_1, \theta)$ fixed to $(\psi_1^0, \theta_0)$,

$$ds_1^2 = (2l_p R)^2 \left\{-dt^2 + 4 \left\{\cos^2 \psi_1^0 d\phi_1^2 + \sin^2 \psi_1^0 \left[d\psi_2^0 + \cos^2 \psi_2^0 d\phi_2^2 + \sin^2 \psi_2^0 \cos^2 \theta_0 d\phi_3^2\right]\right\}\right\},$$

and $(\psi_2, \theta)$ fixed to $(\psi_2^0, \theta_0)$,

$$ds_2^2 = (2l_p R)^2 \left\{-dt^2 + 4 \left\{d\psi_1^2 + \cos^2 \psi_1 d\phi_1^2 + \sin^2 \psi_1 \left[d\psi_2^0 + \cos^2 \psi_2^0 d\phi_2^2 + \sin^2 \psi_2^0 \cos^2 \theta_0 d\phi_3^2\right]\right\}\right\}.$$

The appropriate membrane embedding of the type (2.8) for the background given by $ds_1^2$ is

$$X^0(\xi^m) = t(\xi^m) = \Lambda_0^0 \xi^0,$$

$$X^1(\xi^m) = \phi_1(\xi^m) = \Lambda_1^0 \xi^1,$$

$$X^2(\xi^m) = \psi_2(\xi^2),$$

$$X^3(\xi^m) = \phi_2(\xi^m) = \Lambda_3^0 \xi^0,$$

$$X^4(\xi^m) = \phi_3(\xi^m) = \Lambda_4^0 \xi^0.$$

It corresponds to $J_{\phi_1} = 0$, $(J_{\phi_2}, J_{\phi_3}) \neq 0$. In the limit $M \to 1_-$, $J_{\phi_2}$ is finite, whereas $J_{\phi_3} \to \infty$. The energy-charge relation $E(J_{\phi_1}, J_{\phi_2})$ is particular case of the one in (2.15), because $ds_1^2$ conform to $f = 1$ in (2.7). It reads

$$E - \frac{J_{\phi_3}}{2 \sin \psi_1^0 \cos \theta_0} = \sqrt{\left(\frac{J_{\phi_2}}{2 \sin \psi_1^0}\right)^2 + [2\sqrt{\pi} T_2 (l_p R)^3 \Lambda_1 \sin \psi_1^0 \cos \psi_1^0]^2}.$$
For the background described by $ds^2$, there are two possible embeddings of the type (2.8). They are

\begin{align*}
X^0(\xi^m) &= t(\xi^m) = \Lambda_0^0 \xi^0, \\
X^1(\xi^m) &= \psi_1(\xi^2), \\
X^2(\xi^m) &= \varphi_1(\xi^m) = \Lambda_0^2 \xi^0, \\
X^3(\xi^m) &= \varphi_2(\xi^m) = \Lambda_0^3 \xi^0, \\
X^4(\xi^m) &= \varphi_3(\xi^m) = \Lambda_0^4 \xi^0,
\end{align*}

and

\begin{align*}
X^0(\xi^m) &= t(\xi^m) = \Lambda_0^0 \xi^0, \\
X^1(\xi^m) &= \psi_1(\xi^2), \\
X^2(\xi^m) &= \varphi_1(\xi^m) = \Lambda_0^2 \xi^0, \\
X^3(\xi^m) &= \varphi_2(\xi^m) = \Lambda_0^3 \xi^0, \\
X^4(\xi^m) &= \varphi_3(\xi^m) = \Lambda_0^4 \xi^0.
\end{align*}

For the first case, $(J_{\varphi_1}, J_{\varphi_2}) \neq 0$, $J_{\varphi_1} = 0$. In the limit $M \to 1-\cdot J_{\varphi_1}$ is finite, while $J_{\varphi_2} \to \infty$. For the second case, $(J_{\varphi_1}, J_{\varphi_3}) \neq 0$, whereas $J_{\varphi_2} = 0$. In the above mentioned limit, $J_{\varphi_1}$ is finite, $J_{\varphi_3} \to \infty$. The energy-charge relations $E(J_{\varphi_1}, J_{\varphi_2})$ and $E(J_{\varphi_1}, J_{\varphi_3})$ are particular cases of the relation (2.22), because $ds^2$ correspond to $f = \sin \theta$ in (2.7). The explicit expressions for $E(J_{\varphi_1}, J_{\varphi_2})$ and $E(J_{\varphi_1}, J_{\varphi_3})$ are given by

\begin{align*}
E &= \frac{J_{\varphi_2}}{2 \cos \psi_2^0} = \sqrt{\left(\frac{J_{\varphi_2}}{2}\right)^2 + \left[2^6 \pi T_2 (l_p R)^3 \Lambda_1^4 \sin \psi_2^0 \cos \theta_0\right]^2},
\end{align*}

and

\begin{align*}
E &= \frac{J_{\varphi_3}}{2 \sin \psi_3^0 \cos \theta_0} = \sqrt{\left(\frac{J_{\varphi_3}}{2}\right)^2 + \left[2^6 \pi T_2 (l_p R)^3 \Lambda_1^4 \cos \psi_3^0\right]^2}.
\end{align*}

Thus, we showed that for $\varphi_4 = 0$ there exist three membrane configurations with the searched properties. By performing the same analysis for the subspaces defined by $\varphi_1 = 0$, $\varphi_2 = 0$ or $\varphi_3 = 0$, one can find another nine membrane solutions with the same type of semiclassical behavior.

More examples for target space metrics of the type (2.7), for which there exist the membrane configurations (2.8) giving rise to two-spin magnon-like energy-charge relations, can be found for instance in different subspaces of the $AdS_7 \times S^4$ solution of M-theory and not only there.

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