$R^2$ corrections to holographic Schwinger effect

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We study $R^2$ corrections to the holographic Schwinger effect in an AdS black hole background and a confining D3-brane background, respectively. The potential analysis for these backgrounds is presented. The critical values for the electric field are obtained. It is shown that for both backgrounds increasing the Gauss-Bonnet parameter the Schwinger effect is enhanced. Moreover, the results provide an estimate of how the Schwinger effect changes with the shear viscosity to entropy density ratio, $\eta/s$, at strong coupling.

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I. INTRODUCTION

It is well known that in quantum electrodynamics (QED) the virtual particles can turn into real ones when an external strong electric field is applied. This non-perturbative phenomenon is known as the Schwinger effect. The production rate $\Gamma$ for a weak-coupling and weak-field condition has been studied in [1] long time ago. Later, it was generalized to the case of arbitrary-coupling and weak-field regime in [2], that is

$$\Gamma \sim \exp\left(-\frac{\pi m^2}{eE} + \frac{e^2}{4}\right),$$

where $E$ is the external electric field, $m$ is the electron mass, $e$ is the elementary electric charge. Actually, the Schwinger effect is not unique to QED but usual for QFTs coupled to an U(1) gauge field. AdS/CFT, namely the duality between a string theory in the AdS space and a conformal field in the physical space-time, can realize a system that coupled with an U(1) gauge field [3–5]. Therefore, it is of great interest to study the Schwinger effect in the context of AdS/CFT. The pair production of the $W$ bosons, at large $N_c$ and large 't Hooft coupling $\lambda \equiv g^2 Y M N_c$, was obtained by Semenoff and Zarembo in their seminal work [6]

$$\Gamma \sim \exp\left(-\frac{\sqrt{\lambda}}{2}\left(\sqrt{\frac{E}{E_c}} - \sqrt{\frac{E}{E_c}}\right)^2\right),$$

with

$$E_c = \frac{2\pi m^2}{\sqrt{\lambda}},$$

where $E_c$ is the critical electric field. Interestingly, in this case $E_c$ completely agrees with the DBI result. After [6], there are many attempts to address the Schwinger effect in this direction. For example, the pair production for the general backgrounds has been investigated in [7]. The Schwinger effect in confining backgrounds is discussed in [8]. The potential analysis for Schwinger effect is addressed in [9]. The pair production in constant electric and magnetic fields has been analyzed in [10]. Investigations are also extended to some AdS/QCD models [11, 12]. Other related discussions can be found, for example, in [13–22]. For a review on this topic, see [23].

In general, string theory contains higher derivatives corrections due to the presence of stringy effect. Although very little is known about the forms of higher derivative corrections, given the vastness of the string landscape one expects that generic corrections can occur [24]. Motivated by this, some quantities have been investigated in conformal field theories dual to gravity with higher derivative corrections. Such as $\eta/s$ [25, 26], heavy quark potential [28], imaginary part of potential [29], drag force [30], and jet quenching parameter [31].

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As the calculation of the holographic Schwinger effect is very related to string theory, it is natural to consider various stringy corrections, such as $R^2$ corrections. In this paper, we would like to analyze how $R^2$ corrections affect the Schwinger effect. In addition, it was argued [32] that $\eta/s \geq 1/4\pi$ can be violated in theories with $R^2$ corrections, therefore, the connection between the shear viscosity and the Schwinger effect in these $R^2$ theories may be an interesting fact that comes for free in holography. These are the main motivations of the present work.

The paper is organized as follows. In the next section, we briefly review the backgrounds with $R^2$ corrections. In section 3, we perform the potential analysis for the AdS black hole background with $R^2$ corrections and evaluate the critical electric field from the DBI action. In section 4, we study the Schwinger effect in a confining D3-brane background with $R^2$ corrections as well. The last part is devoted to conclusion and discussion.

II. SETUP

Let us briefly review the backgrounds with curvature-squared corrections given in [33]. Restricting the gravity sector in the $AdS_5$ space, the leading order higher derivative corrections can be written as

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g}[R + \frac{12}{L^2} + L^2(\alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})],$$

where $G_5$ is the five dimensional Newton constant, $R_{\mu\nu\rho\sigma}$ represents the Riemann tensor, $R_{\mu\nu}$ denotes the Ricci tensor, $R$ stands for the Ricci scalar, $L$ refers to the radius of $AdS_5$ at leading order in $\alpha_i$ where one has assumed that $\alpha_i \sim \frac{\alpha'}{L^2} \ll 1$. Other terms with factors of $R$ or additional derivatives can be suppressed by higher powers of $\frac{\alpha'}{L^2}$ [25]. However, at leading order only $\alpha_3$ is unambiguous while $\alpha_1$ and $\alpha_2$ can be arbitrarily varied by a metric redefinition [25, 27]. To avoid this problem, one works with the Gauss-Bonnet gravity in which $\alpha_i$ can be fixed in terms of a single parameter, $\lambda_{GB}$. The action of Gauss-Bonnet gravity in four dimensions is given by

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g}[R + \frac{12}{L^2} + \frac{\lambda_{GB}}{2} L^2(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})],$$

where $\lambda_{GB}$ is constrained in the range

$$-\frac{7}{36} < \lambda_{GB} \leq \frac{9}{100},$$

where the upper range is determined to avoid causality violation in the boundary [26] and the lower bound comes from requiring the boundary energy density to be positive-definite [35].

The black brane solution of the Gauss-Bonnet gravity is [36]

$$ds^2 = -m^2 \frac{r^2}{L^2} f(r) dt^2 + \frac{r^2}{L^2} d\vec{x}^2 + \frac{L^2}{r^2} \frac{dr^2}{f(r)},$$

with

$$f(r) = \frac{1}{2\lambda_{GB}} [1 - \sqrt{1 - 4\lambda_{GB}(1 - \frac{r_h^4}{r^4})}],$$

and

$$m^2 = \frac{1}{2}(1 + \sqrt{1 - 4\lambda_{GB}}),$$

where $r$ is the radial coordinate describing the 5th dimension. $r = r_h$ is the event horizon and $r = \infty$ is the boundary. The plasma temperature is

$$T = \frac{m r_h}{\pi L^2}.$$  

Moreover, $\lambda_{GB}$ can be related to $\eta/s$ by [25, 27]

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - 4\lambda_{GB}),$$

one can see that $\eta/s > \frac{1}{4\pi}$ is violated for $\lambda_{GB} > 0$. Also, increasing $\lambda_{GB}$ leads to decreasing $\eta/s$. 

III. ADS BLACK HOLE BACKGROUND

We now follow the calculations of [9] to study the Schwinger effect for the background metric of (7). The Nambu-Goto action is

$$S = T_F \int d\tau d\sigma \mathcal{L} = T_F \int d\tau d\sigma \sqrt{g}, \quad T_F = \frac{1}{2\pi \alpha'},$$

(12)

where $T_F$ is the fundamental string tension. $g$ denotes the determinant of the induced metric on the string world sheet with

$$g_{\alpha\beta} = g_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta},$$

(13)

where $g_{\mu\nu}$ is the metric, $X^\mu$ represents the target space coordinates.

Using the static gauge

$$x^0 = \tau, \quad x^1 = \sigma,$$

(14)

and assuming that the coordinate $r$ depends only on $\sigma$

$$r = r(\sigma),$$

(15)

one obtains the induced metric $g_{\alpha\beta}$ as

$$g_{00} = \frac{m^2 f(r)}{L^2}, \quad g_{01} = g_{10} = 0, \quad g_{11} = \frac{r^2}{L^2} + \frac{L^2 \dot{r}^2}{r^2 f(r)},$$

(16)

with $\dot{r} = \frac{dr}{d\sigma}$. Then the lagrangian density is found to be

$$\mathcal{L} = \sqrt{\frac{m^2 f(r)r^4}{L^4} + m^2 \dot{r}^2}.$$  

(17)

Now that $\mathcal{L}$ does not depend on $\sigma$ explicitly, so the corresponding Hamiltonian is a constant, that is

$$\mathcal{L} - \frac{\partial \mathcal{L}}{\partial \dot{r}} \dot{r} = \text{constant}. $$

(18)

Considering the boundary condition at $\sigma = 0$, 

$$\dot{r} = 0, \quad r = r_c, \quad (r_h < r_c < r_0),$$

(19)

where we have assumed that the probe D3-brane is put at an intermediate position ($r = r_0$) between the horizon and the boundary. It was shown that this manipulation can yield a finite mass [6].

To proceed, one finds

$$\frac{f(r)r^4}{\sqrt{f(r)r^4 + L^4 \dot{r}^2}} = \sqrt{f(r_c)}r_c^4,$$

(20)

with

$$f(r_c) = \frac{1}{2\lambda_{GB}} \left[ 1 - \sqrt{1 - 4\lambda_{GB}(1 - \frac{r_h^4}{r_c^4})} \right].$$

(21)

Then a differential equation is derived

$$\dot{r} = \frac{dr}{d\sigma} = \sqrt{\frac{r^4 f(r)[r^4 f(r) - r_c^4 f(r_c)]}{L^4 r_c^4 f(r_c)}}.$$  

(22)

By integrating the separate length of the test particle pair is obtained

$$x = \frac{2L^2}{ar_0} \int_1^{1/a} dy \sqrt{\frac{f_1(y_c)}{y^4 f_1(y)[y^4 f_1(y) - f_1(y_c)]}},$$

(23)
with
\[ f_1(y) = \frac{1}{2\lambda_{GB}} \left[ 1 - \sqrt{1 - 4\lambda_{GB}(1 - \frac{b^4}{a^4y})} \right], \quad f_1(y_c) = \frac{1}{2\lambda_{GB}} \left[ 1 - \sqrt{1 - 4\lambda_{GB}(1 - \frac{b^4}{a^2})} \right], \]  
(24)

where we have introduced the following dimensionless quantities
\[ y = \frac{r}{r_c}, \quad a = \frac{r_c}{r_0}, \quad b = \frac{r_h}{r_0}. \]  
(25)

On the other hand, inserting (22) into (17) one gets the sum of potential energy (PE) and static energy (SE)
\[ V_{PE+SE} = 2T_F m r_0 a \int_1^{1/a} dy \sqrt{\frac{y^4 f_1(y)}{y^4 f_1(y) - f_1(y_c)}}, \]  
(26)

Next, we calculate the critical electric field. The DBI action is given by
\[ S_{DBI} = -T_{D3} \int d^4x \sqrt{-\text{det}(G_{\mu\nu} + F_{\mu\nu})}, \quad T_{D3} = \frac{1}{g_s (2\pi)^3 \alpha'}, \]  
(27)

where \( T_{D3} \) is the D3-brane tension.

By virtue of (7), the induced metric \( G_{\mu\nu} \) reads
\[ G_{00} = -\frac{m^2 r^2}{L^2} f(r), \quad G_{11} = \frac{r^2}{L^2}, \quad G_{22} = \frac{r^2}{L^2}, \quad G_{33} = \frac{r^2}{L^2}. \]  
(28)

According to [?], \( F_{\mu\nu} = 2\pi\alpha' F_{\mu\nu} \), one finds
\[ G_{\mu\nu} + F_{\mu\nu} = \begin{pmatrix} -\frac{m^2 r^2}{L^2} f(r) & 2\pi\alpha' E & 0 & 0 \\ -2\pi\alpha' E & \frac{r^2}{L^2} & 0 & 0 \\ 0 & 0 & \frac{r^2}{L^2} & 0 \\ 0 & 0 & 0 & \frac{r^2}{L^2} \end{pmatrix}, \]  
(29)

which yields
\[ \text{det}(G_{\mu\nu} + F_{\mu\nu}) = -\left( \frac{r^4}{L^4} \right)^2 [m^2 f(r) - \frac{(2\pi\alpha')^2 E^2 L^4}{r^4}], \]  
(30)

where we have assumed that the electric field \( E \) is turned on along the \( x^1 \)-direction \[8\].

Inserting (30) into (27) and setting the D3-brane at \( r = r_0 \), one gets
\[ S_{DBI} = -T_{D3} \frac{r_0^4}{L^4} \int d^4x \sqrt{m^2 f(r_0) - \frac{(2\pi\alpha')^2 E^2 L^4}{r_0^4}}, \]  
(31)

with
\[ f(r_0) = \frac{1}{2\lambda_{GB}} \left[ 1 - \sqrt{1 - 4\lambda_{GB}(1 - \frac{b^4}{a^2})} \right]. \]  
(32)

It is required that the square root in (31) is non-negative
\[ m^2 f(r_0) - \frac{(2\pi\alpha')^2 E^2 L^4}{r_0^4} \geq 0. \]  
(33)

which leads to
\[ E \leq T_F \frac{r_0^2}{L^2} m \sqrt{f(r_0)}. \]  
(34)

Finally, we end up with the critical field \( E_c \) in the AdS black hole background with \( R^2 \) corrections
\[ E_c = T_F \frac{r_0^2}{L^2} m \sqrt{\frac{1}{2\lambda_{GB}} \left[ 1 - \sqrt{1 - 4\lambda_{GB}(1 - \frac{b^4}{a^2})} \right]}, \]  
(35)
one can see that $E_c$ depends on the temperature as well as the Gauss-Bonnet parameter.

To move on, we study the total potential. As a matter of convenience, we define a parameter

$$\alpha \equiv \frac{E}{E_c}. \tag{36}$$

Then from $\eqref{eq:Pe}$ and $\eqref{eq:Se}$ one finds the total potential $V_{\text{tot}}$ as

$$V_{\text{tot}} = V_{P_{\text{E+SE}}} - E x$$

$$= 2T_F m r_0 a \int_1^{1/a} dy \left[ \frac{y^4 f_1(y)}{y^4 f_1(y) - f_1(y_c)} \right]$$

$$- \frac{2T_F m r_0 a \alpha}{a} \int_1^{1/a} \frac{1}{2\lambda_{GB}} \left[ 1 - \sqrt{1 - 4\lambda_{GB}(1 - b^4)} \right] dy \left[ \frac{f_1(y_c)}{y^4 f_1(y) - f_1(y_c)} \right]. \tag{37}$$

To compare with the Einstein case in $\eqref{eq:Einstein}$, we set $b = 0.5$ and $T_F r_0 = L^2 / r_0 = 1$. In Fig.1, we plot the total potential $V_{\text{tot}}$ as a function of the inter-distance $x$ with two different values of $\lambda_{GB}$, the left panel is for $\lambda_{GB} = -0.1$ and the right one is for $\lambda_{GB} = 0.05$. In all of the plots from top to bottom $\alpha = 0.8, 0.9, 1.0, 1.1$, respectively. From the figures, we can see that there is a critical electric field at $\alpha = 1$ ($E = E_c$), in agreement with $\eqref{eq:Einstein}$.

To see the effect of $R^2$ corrections on the potential barrier, we plot $V_{\text{tot}}$ against $x$ at $\alpha = 0.8$ with different values of $\lambda_{GB}$ in the left panel of Fig.2. We can see that increasing $\lambda_{GB}$ leads to decreasing the height and the width of the barrier. As we know, the higher the barrier, the harder the produced pair escapes to infinity. Therefore, we conclude that by increasing $\lambda_{GB}$ the Schwinger effect is enhanced.
Moreover, to show the effect of \( R^2 \) corrections on \( E_c \), we plot \( V_{tot} \) against \( x \) at \( \alpha = 1 \) with different \( \lambda_{GB} \) in the right panel of Fig.2. It is found that the barrier vanishes for each plot implying the vacuum becomes unstable. In fact, that the barrier of each plot disappears at \( \alpha = 1 \) can be strictly proved, i.e, we can calculate the derivative of \( V_{tot} \) at \( x = 0 \) as,

\[
\left. \frac{dV_{tot}}{dx} \right|_{x=0} = (1 - \alpha) m T_F \sqrt{f(r_0)}. \tag{38}
\]

**IV. CONFining D3-brane background**

In this section we analyze the Schwinger effect in a confining D3-brane background with \( R^2 \) corrections. The metric is given by [36]

\[
ds^2 = -\frac{r^2}{L^2} dt^2 + \frac{r^2}{L^2} (dx^1)^2 + \frac{r^2}{L^2} (dx^2)^2 + \frac{m^2}{L^2} f(r) \frac{r^2}{L^2} (dx^3)^2 + \frac{L^2}{r^2} \frac{dr^2}{f(r)}, \tag{39}
\]

with

\[
f(r) = \frac{1}{2\lambda_{GB}} [1 - \sqrt{1 - 4\lambda_{GB}(1 - \frac{r^4_h}{r^4})}], \tag{40}
\]

and

\[
m^2 = \frac{1}{2}(1 + \sqrt{1 - 4\lambda_{GB}}), \tag{41}
\]

where \( r_h \) is the inverse compactification radius in the \( x^3 \)-direction.

Similar to the previous section, we call again the inter-distance and the sum of potential energy and static energy as \( x \) and \( V_{PE+SE} \), respectively. One finds

\[
x = \frac{2L^2}{r_0 a} \int_1^{1/a} \frac{dy}{y^2 \sqrt{(y^4 - 1)f_1(y)}}, \tag{42}
\]

and

\[
V_{PE+SE} = 2T_F r_0 a \int_1^{1/a} \frac{y^2 dy}{\sqrt{(y^4 - 1)f_1(y)}}, \tag{43}
\]

where \( f_1(y) \) is defined in [24].

The next step is to evaluate the critical electric field. Using [39], we have the induced metric as

\[
G_{\mu\nu} = \begin{pmatrix}
-\frac{r^2}{L^2} & 2\pi \alpha' E & 0 & 0 \\
-2\pi \alpha' E & \frac{r^2}{L^2} & 0 & 0 \\
0 & 0 & \frac{r^2}{L^2} & 0 \\
0 & 0 & 0 & m^2 f(r) \frac{r^2}{L^2}
\end{pmatrix}, \tag{44}
\]

Then we get

\[
det(G_{\mu\nu} + F_{\mu\nu}) = -m^2 f(r) \frac{r^4}{L^4} \left[ \frac{r^4}{L^4} - (2\pi \alpha')^2 E^2 \right], \tag{45}
\]

where the electric field \( E \) is turned on along the \( x^1 \)-direction as well.

Substituting [40] into [41] and setting the D3-brane at \( r = r_0 \), one has

\[
S_{DBI} = -m T_{D3} \frac{r_0^4}{L^4} \int d^4 x \sqrt{f(r_0)[1 - \frac{(2\pi \alpha')^2 L^4}{r_0^4} E^2]]. \tag{46}
\]
FIG. 3: $V_{tot}$ against $x$. Left: $\lambda_{GB} = -0.1$, from top to bottom $\alpha = 0.1, 0.25, 0.6, 1.0, 1.3$; Right: $\alpha = 0.6$, from top to bottom $\lambda_{GB} = -0.1, 0.01, 0.05$, respectively.

Obviously,

$$f(r_0) > 0.$$  \hspace{1cm} (48)

To avoid (47) being ill-defined, one needs only

$$1 - \frac{(2\pi \alpha')^2 L^4}{r_0^4} E^2 \geq 0,$$  \hspace{1cm} (49)

results in

$$E \leq T_F \frac{r_0^2}{L^2}.$$  \hspace{1cm} (50)

Therefore, the critical field $E_c$ in the confining D3-brane background with $R^2$ corrections is obtained

$$E_c = T_F \frac{r_0^2}{L^2}.$$  \hspace{1cm} (51)

one can see that in this case $E_c$ is not affected by the $R^2$ corrections. This is because the electric field is turned on $x^1$-direction while the $x^3$-direction, related to $\lambda_{GB}$, is compactified [8].

Likewise, the total potential is

$$V_{tot} = V_{PE+SE} - Ex$$

$$= 2T_Fr_0a \int_1^{1/a} \frac{y^2 dy}{\sqrt{(y^4 - 1)f_1(y)}} - 2T_F\alpha r_0 \int_1^{1/a} \frac{dy}{y^2 \sqrt{(y^4 - 1)f_1(y)}}.$$  \hspace{1cm} (52)

where $\alpha$ is defined in (36).

According to the analysis of [8], there are two critical values for the electric field in the confining D3-brane background, $E_c = \frac{T_Fr_0^2}{L^2}$ and $E_s = \frac{T_Fr_0^2}{L^2}$. When $E > E_c$, the potential barrier vanishes and no tunneling occurs implying the vacuum becomes unstable. When $E_s < E < E_c$, the potential barrier is present and the Schwinger effect can be described as a tunneling process. When $E < E_s$, the potential tends to diverge at infinitely and no Schwinger effect occurs.

Let us discuss results. In the left panel of Fig.3, we plot $V_{tot}$ versus $x$ with $\lambda_{GB} = -0.1$ by setting $b = 0.5$ and $2L^2/r_0 = 2T_Fr_0 = 1$, as follows from [8]. Other cases with different $\lambda_{GB}$ have similar picture. From the figures, one can see that there indeed exist two critical values for the electric field: one is at $\alpha = 1 (E = E_c)$, the other is at $\alpha = 0.25 (E = E_s = 0.25E_c)$.

Likewise, to study $R^2$ corrections to the potential barrier, we plot $V_{tot}$ versus $x$ at $\alpha = 0.6$ with different $\lambda_{GB}$ in the right panel of Fig.3. One can see that as $\lambda_{GB}$ increases both the height and width of the potential barrier decrease. Thus, one concludes that increasing $\lambda_{GB}$ enhances the Schwinger effect, consistently with the findings of [13].
FIG. 4: $V_{tot}$ against $x$. Left: $\alpha = 0.25$; Right: $\alpha = 1$. In all of the plots from top to bottom $\lambda_{GB} = -0.1, 0.01, 0.05,$ respectively.

Also, to show the effect of $R^2$ corrections on the two critical electric fields, we plot $V_{tot}$ versus $x$ at $\alpha = 0.25$ and $\alpha = 1$ with different $\lambda_{GB}$ in Fig.4. From the left panel, one can see that by varying $\lambda_{GB}$ the potential always becomes flat at $\alpha = 0.25$, which means the critical field $E_s$ is not modified by $R^2$ corrections. This is consistent with the value of $E_s$ defined as $E_s = \frac{T r^2 r^2}{2}$. From the right panel, one finds that the barrier vanishes for each plot at $\alpha = 1$, in agreement with the DBI result.

V. CONCLUSION AND DISCUSSION

In this paper, we have investigated $R^2$ corrections to the holographic Schwinger effect in an AdS black hole background and a confining D3-brane background, respectively. The critical values for the electric field were obtained. It is shown that for both backgrounds increasing the Gauss-Bonnet parameter the Schwinger effect is enhanced. Moreover, the critical electric field $E_c$ is dependent on $\lambda_{GB}$ in the AdS black hole background but not affected by it in the confining D3-brane background.

In addition, the results may provide an estimate of how the Schwinger effect changes with $\eta/s$ at strong coupling. From (11) one knows that increasing $\lambda_{GB}$ leads to decreasing the $\eta/s$ thus making the fluid becomes more "perfect". On the other hand, increasing $\lambda_{GB}$ leads to increasing the Schwinger effect. Therefore, one concludes that at strong coupling as $\eta/s$ decreases the Schwinger effect is enhanced.

Finally, we should admit that we cannot predict a result for $\mathcal{N} = 4$ SYM theory because the first higher derivative correction is related to $R^4$ terms but not $R^2$. We leave this for further study.

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