STUDY OF THE REACTION $np \to np\pi^{+}\pi^{-}$ AT INTERMEDIATE ENERGIES

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Abstract

The reaction $np \to np\pi^{+}\pi^{-}$ was studied at the various momenta of incident neutrons. It was shown that the characteristics of the reaction at the momenta above 3 GeV/c could be described by the model of reggeized $\pi$ exchange (OPER). At the momenta below 3 GeV/c, it was necessary to use additionally the mechanism of one baryon exchange (OBE).

1 Introduction: study of inelastic np interactions at accelerator facility of LHEP JINR

The data about inelastic np interactions were obtained due to irradiation of 1m hydrogen bubble chamber (4$\pi$ geometry) by quasimonochromatic neutron beam ($\delta P < 2.5\%$) at the following incident momenta:

$P_0=1.25, 1.43, 1.73, 2.23, 3.10, 3.83, 4.10$ and 5.20 GeV/c

The unique of fullness and precision data are obtained [1]. It permits to carry out the detailed study of inelastic $np$ interactions in a wide region of energies.

2 The reaction $np \to np\pi^{+}\pi^{-}$ at $P_0 > 3$ GeV/c

This reaction is characterized by:

- plentiful production of the $\Delta$-resonance (see Fig.2),
- large peripherality of the secondary nucleons.

To study the mechanism of the reaction it was chosen the model of reggeized $\pi$ exchange (OPER), developed in ITEP [2].
The advantages of OPER model are:
- small number of free parameters (3 in our case),
- wide region of the described energies (2÷200 GeV),
- calculated values are automatically normalized to the reaction cross-section.

The following main diagrams correspond to the reaction $np \rightarrow np\pi^+\pi^-$ within the framework of OPER model:

Matrix element for the diagrams a, b and c from Fig. 3 is written in the following form:

$$M_1 = T_{\pi N \rightarrow \pi N}F_2T_{\pi N \rightarrow \pi N}/(t - m^2_\pi),$$

where $T_{\pi N \rightarrow \pi N}$ - amplitude of elastic $\pi N \rightarrow \pi N$ scattering off mass shell,
$F_2$ - form-factor, going away off mass shell of $T_{\pi N \rightarrow \pi N}$ amplitudes,
$1/(t - m^2_\pi)$ - $\pi$-meson propagator.

The data of elastic $\pi N \rightarrow \pi N$ were taken from PWA [3].

The analysis shows, that interference between diagrams 3a, 3b and 3c is negligible [4].

The study has shown that it is not necessary to take into account the contribution of the "hanged" diagrams (Fig.) into the reaction cross-sections at $P_0 < 10$ GeV/:

It was shown in [5] that the use of some specific cuts permits to select the kinematic region of the reaction $np \rightarrow np\pi^+\pi^-$ in which the contribution of the diagrams 3a, 3b and 3c consists up to 95 % at $P_0 > 3$ GeV/c.
Fig. 2: The distributions of \( M_{p\pi^+} \) and \( M_{n\pi^-} \) from the reaction \( np \rightarrow np\pi^+\pi^- \) at \( P_0 = 3 \text{ GeV/c} \)

Fig. 5 shows some distributions for the reaction \( np \rightarrow np\pi^+\pi^- \) for this region at \( P_0 = 5.20 \text{ GeV/c} \) (solid curves - results of calculations using OPER model).

But the diagrams shown in Fig.3 are insufficient to describe totally the characteristics of the reaction \( np \rightarrow np\pi^+\pi^- \). It is necessary to take into account the diagrams of the following type:

the matrix element for which is written in the following form:

\[
M_3 = G\bar{u}(q_N)\gamma_5 u(Q_N)F_1(T_{\pi N \rightarrow \pi\pi N}/(t - m^2_{\pi})),
\]

where \( T_{\pi N \rightarrow \pi\pi N} \) - off mass shell amplitudes of inelastic \( \pi N \rightarrow \pi\pi N \) - scattering that are known much worse than elastic \( T_{\pi N \rightarrow \pi N} \) amplitudes. Therefore it is necessary to do a parametrization of the inelastic \( \pi N \rightarrow \pi\pi N \)-scattering (see Appendix).

It permits to get a good description of the experimental characteristics of the reaction \( np \rightarrow np\pi^+\pi^- \) at \( P_0 = 5.20 \text{ GeV/c} \) (Fig. 7) taking into account OPER diagrams shown in Fig. 3 and Fig. 6 :
Figure 3: OPER diagrams 2×2 for the reaction \( np \rightarrow np\pi^+\pi^- \)

Figure 4: "Hanged" OPER diagram for the reaction \( np \rightarrow np\pi^+\pi^- \)

3 The reaction \( \bar{p}p \rightarrow \bar{p}p\pi^+\pi^- \) at \( P_0 = 7.23 \) GeV/c

Using OPER model we try to describe the experimental distributions from the reaction \( \bar{p}p \rightarrow \bar{p}p\pi^+\pi^- \) at \( P_0 = 7.23 \) GeV/c [5].

It is observed a good agreement between experimental data and theory in Fig.8.
Figure 5: Distributions for the reaction $np \rightarrow np\pi^+\pi^-$ at $P_0=5.20$ GeV/c obtained due to specific cuts.

4 The reaction $np \rightarrow np\pi^+\pi^-$ at $P_0 < 3$ GeV/c

The study of effective mass spectra of $np$ - combinations at $P_0=1.73$ and $2.23$ GeV/c (Fig.9) shows the clear peak close the threshold ($M_{np} = m_n + m_p$) that can not be described within the framework of OPER-model using the diagrams from Fig.3 and Fig.6.

The model of Regge poles with baryon exchange and nonlinear trajectories, suggested in [6] was used to describe these features. The following diagrams of one baryon exchange (OBE) were taken into account within the framework of this model:

The vertex function of elastic $np \rightarrow np$ scattering was calculated using the data from [7].

The vertex functions of $\Delta N \rightarrow np$, $NN \rightarrow \Delta N$ and $\Delta N \rightarrow \Delta N$ scattering were calculated corresponding to [8]. In result one can get the good description of the experimental distribution from the reaction $np \rightarrow np\pi^+\pi^-$ at $P_0 = 1.73$ and $2.23$ GeV/c (Fig.9 and Fig.11).
The other reactions of np interactions are scheduled to study by means of OPER model:

\begin{align*}
np \rightarrow pp\pi^- & \quad \text{vertex functions } 1 \times 2 \\
np \rightarrow pp\pi^-\pi^0 & \quad \text{vertex functions } 2 \times 2 \text{ and } 1 \times 3 \\
np \rightarrow pp\pi^+\pi^-\pi^- & \quad \text{vertex functions } 2 \times 3 \\
np \rightarrow pp\pi^+\pi^-\pi^-\pi^0 & \quad \text{vertex functions } 3 \times 3 \\
np \rightarrow np\pi^+\pi^-\pi^+\pi^- & \quad \text{vertex functions } 3 \times 3 \\
\end{align*}

Similar reactions of \( pp, \bar{p}p \) and \( \pi N \) interactions also can be described by OPER model. The following reactions were simulated for HADES experiment:

\begin{align*}
pp \rightarrow pp\pi^+\pi^- & \quad \text{at } T_{\text{kin}}=3.5 \text{ GeV} \\
np \rightarrow np\pi^+\pi^- & \quad \text{at } T_{\text{kin}}=1.25 \text{ GeV} \\
np \rightarrow npe^+e^- & \quad \text{at } T_{\text{kin}}=1.25 \text{ GeV with vertex function of } \gamma N \rightarrow Ne + e^- .
\end{align*}

Since the \( \pi N \rightarrow \pi N \) and \( \pi N \rightarrow \pi\pi N \) vertex functions are taken in helicity representation it seems to be perspective to use OPER model for description of the reaction with polarized particles.
6 Conclusion

Reaction $np \rightarrow np^{+}\pi^{-}$ is characterized by the plentiful production of the $\Delta$ resonance and the large peripherality of the secondary particles. The experimental data are successfully described by the further development of OPER model.

However at $P_0 < 3$ GeV/c it is necessary to take into account another mechanism of the reaction (such as OBE).

OPER model permits to describe another $N(\bar{N}) - N$ reactions with the production of some $\pi$-mesons. The further development of OPER-model can be very promising to describe the production of $e^+e^-$-pairs in hadronic interactions.

OPER model can be used as an effective tool to simulate various reactions of hadronic interactions.
Figure 8: Distributions for the reaction $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$ at $P_0 = 7.23$ GeV/c

Figure 9: The distributions of $M_{np}$ for the reaction $np \rightarrow np\pi^+\pi^-$ at $P_0 = 2.23$ GeV/c (left) and 1.73 GeV/c (right).

7 Appendix: Parametrization of $\pi N \rightarrow \pi\pi N$ reactions

Within the framework of Generalized Isobar Model (GIM) $[9]$ $\pi N \rightarrow \pi\pi N$ reactions are described as quasi-two body ones ($a + b \rightarrow c + d$):

- $\pi N \rightarrow N^*(\Delta^*) \rightarrow \Delta\pi$,
- $\pi N \rightarrow N^*(\Delta^*) \rightarrow N\rho$,
- $\pi N \rightarrow N^*(\Delta^*) \rightarrow N\epsilon$,
- $\pi N \rightarrow N^*(\Delta^*) \rightarrow N_{1440}\pi$,

with the consequent decays:

- $\Delta \rightarrow N\pi$. 

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Figure 10: OBE diagrams for the reaction $np \rightarrow np\pi^+\pi^-$

\[
\begin{align*}
\rho &\rightarrow \pi\pi, \\
\rho &\rightarrow \pi\pi, \\
N^*_1(1440) &\rightarrow N\pi \\
\end{align*}
\]

The spin and isospin relations were taken account.

The parameters of the following resonances (**** and ***) were taken from Review of Particle Properties:

\[
\begin{align*}
N^*_1(1440) &P_{11} & D^*(1600) &P_{33} \\
N^*_1(1520) &D_{13} & D^*(1620) &S_{31} \\
N^*_1(1675) &D_{15} & D^*(1700) &D_{33} \\
N^*_1(1680) &F_{15} & D^*(1900) &S_{31} \\
N^*_1(1720) &P_{13} & D^*(1905) &F_{35} \\
N^*_1(2000) &F_{15} & D^*(1910) &P_{31} \\
N^*_1(2080) &D_{13} & D^*(1920) &P_{33} \\
N^*_1(2190) &G_{17} & D^*(1940) &D_{33} & D^*(1950) &F_{37} \\
\end{align*}
\]

The spin and isospin relations were taken account.
Figure 11: Distributions for the reaction $np \to np\pi^+\pi^-$ at $P_0=1.73$ GeV/c

For quasi two-body reactions like $a + b \to c + d$ one can write

$$d\sigma = \frac{1}{(2S_a + 1)(2S_b + 1)} \left( \frac{2\pi}{p} \right)^2 \sum_{\lambda_i} | \langle \lambda_d \lambda_c | T | \lambda_b \lambda_a \rangle |^2 \times dPS,$$

$$\langle \lambda_d \lambda_c | T | \lambda_b \lambda_a \rangle = \frac{1}{4\pi} \sum_j (2j + 1) \langle \lambda_d \lambda_c | T_j | \lambda_b \lambda_a \rangle e^{i(\lambda - \mu)\phi} d^{j}_{\lambda \mu}(\theta).$$

where $\lambda = \lambda_a - \lambda_b$, $\mu = \lambda_c - \lambda_d$ - helicity variables,

$d^{j}_{\lambda \mu}(\theta)$ - rotation matrixe,

$dPS$ - phase space element.

The polarization components of the particles $c$ and $d$ from the reaction $a + b \to c + d$ is suitable to express through the elements of the spin density matrix (for example, for particle $d$):

$$\rho^{d}_{m\nu m'} = \frac{1}{N} \sum_{\lambda_c \lambda_b \lambda_a} < m\lambda_c | T | \lambda_b \lambda_a >^* < m\lambda_c | T | \lambda_b \lambda_a >$$

where normalization factor for $Sp\rho=1$:

$$N = \sum_{m\lambda_c \lambda_b \lambda_a} < m\lambda_c | T | \lambda_b \lambda_a >^2.$$
Example:

\[
\pi + N \rightarrow N^{*}_{1680} \rightarrow \Delta + \pi \rightarrow (N + \pi) + \pi
\]

\[
< \lambda_{\Delta}|T|\lambda_{N}> = C^{3,0,\frac{1}{2},-\lambda_{\Delta}}_{\frac{5}{2},-\lambda_{N}} C^{1,0,\frac{1}{2},-\lambda_{\Delta}}_{\frac{5}{2},-\lambda_{N}} d^{\frac{5}{2},-\lambda_{N},-\lambda_{\Delta}}(\theta) \times R_{J},
\]

where \( R_{J} \) is taken in Breit-Wigner form.

Then it is easy to get the angular distribution of \( \Delta \) (in CMS):

\[
\frac{d\sigma(s,t)}{d\Omega} \sim (1 + 2\cos^{2}\theta_{\Delta}) |R_{J}|^{2} = (1 + 2\cos^{2}\theta_{\Delta}) \times BW(\sqrt{s}, M_{R}, \Gamma_{R})
\]

If particle \( d \) is unstable: \( d \rightarrow \alpha + \beta (d \rightarrow \Delta + \pi) \) then in the rest system of the particle \( d \):

\[
W_{\Delta} = \frac{3}{4\pi} \left\{ \rho_{33} \sin^{2}\theta + \frac{1}{3} \rho_{11} (1 + 3\cos^{2}\theta) - \frac{2}{\sqrt{3}} \Re \rho_{3-1} \sin^{2}\theta \cos \phi - \frac{2}{\sqrt{3}} \Re \rho_{31} \sin \theta \cos \phi \right\}
\]

- is the normalized angular distribution of the decay products.

To compare with experimental data the following cross-sections were calculated using GIM (Fig.12):

One can see a satisfactory description of cross-sections, except \( \pi^{+}p \rightarrow np^{+}\pi^{+} \). May be it is necessary to take into account S-wave of \( \pi^{+}\pi^{+} \) scattering with \( I=2 \) in GIM.

Some distributions of the reaction \( \pi^{-}p \rightarrow n\pi^{+}\pi^{-} \) were calculated at various energies to study a quality of the application of GIM (Fig.13):

It is observed a good agreement between experimental data and theory.

References

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Figure 12: Cross-sections of the $\pi n \rightarrow \pi \pi N$ reactions.

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Figure 13: Some distributions from the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$ at $T_{kin}=1.0$ GeV [10]