Topology change in quantum gravity

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A particular approach to topology change in quantum gravity is reviewed, showing that several aspects of Stephen’s work are intertwined with it in an essential way. Speculations are made on possible implications for the causal set approach to quantum gravity.

\footnote{1 To appear in the proceedings of Stephen Hawking’s 60\textsuperscript{th} birthday conference, Cambridge University, Jan. 2002}
1 Introduction

The challenge facing anyone giving a talk in celebration of Stephen Hawking’s contributions to physics is to convey something of the epic breadth of his work in a very short time. The subject of topology change in quantum gravity provides an opportunity to do just that. Topology change is not only a subject to which Stephen has directly made many seminal contributions but also one which, looked at from a particular point of view, weaves together several major themes of his work over the years. This is the point of view I will describe.

The framework for topology change I will set out exists in what might be called a “top down” approach to quantum gravity. By this I mean that we take what we know – general relativity, continuum spacetime, quantum field theory and so forth – and try to put them together as best we can, preserving as much structure as possible. Most workers in the field believe that this approach will not, ultimately, be adequate for quantum gravity and that we will need a new “bottom up” theory for which we postulate new fundamental structures and principles and in which continuum spacetime is an emergent phenomenon. Of course it may turn out that the rules we now choose to apply to our top down calculations will be shown to be wrong when we have the underlying theory of quantum gravity, there is that risk, but as a strategy it seems unavoidable. How could we hope to deduce the new principles completely unguided? We proceed in the hope that our top down calculations are giving us clues in our search for the correct bottom up theory. And contrariwise, our beliefs about the eventual form of the bottom up theory will inform the choices we must make in those top down calculations. There’s a rich and complicated process of cross-influence involved in the whole endeavour.

In this spirit, we can hope that topology change will be a particularly fruitful area to study, since it is generally believed that it does not occur in classical general relativity and so would be a genuinely quantum phenomenon – one in which the underlying theory must leave its indelible mark in the realm of the continuum.

I have not attempted to give a comprehensive set of references but have tended to cite papers where more details can be found on some of the main arguments.
2 A top down framework for topology change

What we will mean by a topology change is a spacetime based on an n-dimensional manifold, $M$, with an initial spacelike $n - 1$ dimensional hypersurface, $\Sigma_0$, and a final spacelike hypersurface, $\Sigma_1$, not diffeomorphic to $\Sigma_0$. For simplicity in what follows we take $\Sigma_0$ and $\Sigma_1$ to be closed and $M$ compact so that the boundary of $M$ is the disjoint union of $\Sigma_0$ and $\Sigma_1$. This restriction means effectively that we’re studying topology changes that are localized. For example, a topology change from $\mathbb{R}^3$ to $\mathbb{R}^3$ with a handle – an $S^2 \times S^1$ – attached can be reduced to the compact case because infinity in both cases is topologically the same. On the other hand, a topology change from $\mathbb{R}^3 \times C$ to $\mathbb{R}^3 \times C'$ where $C$ and $C'$ are two non-diffeomorphic Calabi-Yau’s, say, will not be able to be accommodated in the current scheme because it involves a change in the topology of infinity.

Following Stephen [1, 2], I prefer the Sum Over Histories (SOH) approach to quantum gravity which can be summarized in the following formula for the transition amplitude between the Riemannian metric $h_0$ on $(n - 1)$-manifold $\Sigma_0$ and the Riemannian metric $h_1$ on $(n - 1)$-manifold $\Sigma_1$:

$$\langle h_1 \Sigma_1 | h_0 \Sigma_0 \rangle = \sum_M \int_g \omega(g) .$$

(1)

The sum is over all $n$-manifolds, $M$, called cobordisms – whose boundary is the disjoint union of $\Sigma_0$ and $\Sigma_1$ and the functional integral is over all metrics on $M$ which restrict to $h_0$ on $\Sigma_0$ and $h_1$ on $\Sigma_1$. Each metric contributes a weight, $\omega(g)$, to the amplitude and we’ll hedge our bets for now on the type of metrics in the integral and hence the precise form of the weight.

It’s clear from this that the SOH framework lends itself to the study of topology change as it readily accommodates the inclusion of topology changing manifolds in the sum. Despite the fact that we may not be able to turn (1) into a mathematically well-defined object within the top down approach, if even the basic form of this transition amplitude is correct then we can already draw some conclusions. We can say that a topology change from $\Sigma_0$ to $\Sigma_1$ can only occur if there is at least one manifold which interpolates between them, in other words if they are cobordant. This does not place any restriction on topology change in 3+1 spacetime dimensions since all closed three-manifolds are cobordant, but it does in all higher dimensions: not all closed four-manifolds are cobordant, for example. We can also say that even if cobordisms exist, there must also exist appropriate metrics on at least one
cobordism and so we come to the question of what the metrics should be. There are many possibilities and just three are listed here.

1. Euclidean (i.e. positive definite signature) metrics. This choice is of course closely associated with Stephen and the whole programme of Euclidean quantum gravity [3]. It is to this tradition and to Stephen’s influence that I attribute my enduring belief that topology change does occur in quantum gravity. Indeed, Euclidean (equivalently, Riemannian) metrics exist on any cobordism and it would seem perverse to exclude different topologies from the SOH.

2. Lorentzian (i.e. (−, +, +, . . . +) signature) metrics. With this choice we are forced, by a theorem of Geroch [4], to contemplate closed timelike curves (CTC’s or time machines). Geroch proved that if a Lorentzian metric exists on a topology changing cobordism then it must contain CTC’s or be time non-orientable. Stephen has been at the forefront of the study of these causal pathologies, formulating his famous Chronology Protection Conjecture [5] (see also Matt Visser’s contribution to this volume). Stephen and Gary Gibbons also proved that requiring an $SL(2, \mathbb{C})$ spin structure for fundamental fermi fields on a Lorentzian cobordism produces a further restriction on allowed topology changing transitions [6, 7].

3. Causal metrics. By this I mean metrics which give rise to a well-defined “partial order” on the set of spacetime events. A partial order is a binary relation, $\prec$, on a set $P$, with the properties:

- (i) transitivity: $(\forall x, y, z \in P)(x \prec y \prec z \Rightarrow x \prec z)$
- (ii) irreflexivity: $(\forall x \in P)(x \not\prec x)$.

A Lorentzian metric provides a partial order via the identification $x \prec y \Leftrightarrow x \in J^-(y)$, where the latter condition means that there’s a future directed curve from $x$ to $y$ whose tangent vector is nowhere spacelike (a “causal curve”), so long as the metric contains no closed causal curves. The information contained in the order $\prec$ is called the “causal structure” of the spacetime. By Geroch’s theorem, we know there are no Lorentzian metrics on a topology changing cobordism that give rise to a well-defined causal structure. But, there are metrics on any cobordism which are Lorentzian almost everywhere which do [8].
These metrics avoid Geroch’s theorem by being degenerate at a finite number of points but the causal structure at the degenerate points is nevertheless meaningful. Describing these metrics will be the job of the next section.

We will plump for choice 3, causal metrics, in what follows. There are many reasons to do so but the one I would highlight is that it is the choice which fits in with a particular proposal for the underlying theory, namely causal set theory, in which it is the causal structure of spacetime, over all its other properties, that is primary and will persist at the fundamental level. We will return to causal sets later. This choice, of causal spacetimes in the SOH, cannot be deduced logically from any top down considerations. It is a choice informed by a vision of what kind of theory quantum gravity will be when we have it.

## 3 Morse metrics and elementary topology changes

Morse theory gives us a way of breaking a cobordism into a sequence of elementary topology changes [9, 10, 8]. On any cobordism $M$ there exists a Morse function, $f : M \to [0,1]$, with $f|_{\Sigma_0} = 0, f|_{\Sigma_1} = 1$ such that $f$ possesses a set of critical points \( \{p_k\} \) where $\partial_a f|_{p_k} = 0$ and the Hessian, $\partial_a \partial_b f|_{p_k}$, is invertible. These critical points, or Morse points, of $f$ are isolated and, because $M$ is compact, there are finitely many of them. The index, $\lambda_k$, of each Morse point, $p_k$ is the number of negative eigenvalues of the Hessian at $p_k$. It is the number of maxima in the generalized saddle point at $p_k$ if $f$ is interpreted as a height function. For spacetime dimension $n$, there are $n+1$ possible values for the indices, $(0,1,\ldots,n)$. A cobordism with a single Morse point is called an elementary cobordism.

Three elementary cobordisms for $n = 2$ are shown in figure 1. They are the $\lambda = 2$ yarmulke in which a circle is destroyed, the $\lambda = 1$ trousers in which two circles join to form a single circle and the $\lambda = 0$ time reverse of the yarmulke in which a circle is created from nothing. (The upside-down trousers is in fact also a $\lambda = 1$ elementary cobordism: locally the Morse point looks the same as the regular trousers with one maximum and one minimum.) For higher spacetime dimensions, $n$, the generalizations of these are easy to visualize: the index $n$ yarmulke (or its time reverse of index 0) is half an $n$-sphere, the index 1 trousers (or its time reverse of index $n-1$) is an
$n$-sphere with three balls deleted creating three $S^{n-1}$ boundaries. For $n > 3$ qualitatively different types of Morse point exist with at least two maxima and two minima, \( i.e. \lambda \neq 0, 1, n-1, n \). These are impossible to draw but we will encounter some examples in the next section.

Using a Morse function, \( f \), on \( M \) we can construct “Morse metrics” which are Lorentzian everywhere except at the Morse points where they are zero. The precise form is not important here, but roughly the Morse function is used as a time function as you’d expect. These Morse metrics are our candidates for inclusion in the SOH for quantum gravity.

Now, there is important counterevidence to the claim that topology change occurs in quantum gravity. This is work which shows that the expectation value of the energy-momentum tensor of a massless scalar field propagating on a 1 + 1 Morse trousers is singular along the future light cone of the Morse point \([11, 12]\). In addition, one can look instead at the in-out matrix element of the energy momentum tensor and one finds a singularity along both the future and past light cones of the Morse point (calculation described in \([8]\)). This last result in particular, if it can be extended to all Morse metrics on the 1 + 1 trousers, can be taken to suggest that in the full SOH expression for the transition amplitude, integrating out over the scalar field first will leave an expression for the effective action for \( g \) that is infinitely sensitive to fluctuations in \( g \). Thus, destructive interference between nearby metrics will suppress the contribution of any metric on the trousers. To be sure, this is a heuristic argument that would need to be strengthened but suppose it is valid. Would this mean, as DeWitt has argued, that all topology change is suppressed? The answer is not necessarily, especially if the following two conjectures hold.

The first conjecture is based on the idea that it is a certain property, called “causal discontinuity”, of the causal structure of the 1 + 1 trousers that is the origin of the bad behaviour of quantum fields on it. So the conjecture

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\lambda=2 \\
\lambda=1 \\
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Figure 1: Three elementary cobordisms for $n = 2$ and $\lambda = 2, 1, 0$: the yarmulke, trousers and time-reverse of the yarmulke.
(Sorkin) is that quantum fields will be singular on causally discontinuous spacetimes but well-behaved on causally continuous spacetimes. The second conjecture (Borde and Sorkin) is that only Morse metrics containing index 1 or \( n - 1 \) points (trousers type) are causally discontinuous.

So what is causal discontinuity? When I was first learning about these things I was excited to discover that Stephen himself invented the concept in work with R.K. Sachs [13]. That paper is a piece of hard mathematical physics but there is a physically intuitive way of understanding the concept. Roughly, a spacetime is causally discontinuous if the causal past, or future, of a point changes discontinuously as the point is moved continuously in spacetime. We can see from figure 2 that it is very plausible that the 1 + 1 trousers is causally discontinuous: an observer down in one of the legs will have a causal past that is contained only in that leg, but as the observer moves up into the waist region, as they pass the future light cone of the Morse point, their causal past will suddenly get bigger and include a whole new region contained in the other leg. Interestingly, Stephen and R.K. Sachs conclude their paper by saying, “[T]here is some reason, but no fully convincing argument, for regarding causal continuity as a basic macrophysical property.” Which view ties in nicely with the first of the two conjectures.

Figure 2: The 1 + 1 trousers with part of the future light cone of the Morse point, \( p \). The other part goes up the back. The past light cone of the Morse point also has two parts, one down each leg.
4 Good and bad topology change

If we assume the two conjectures of the previous section hold, and that the argument about the consequent suppression of causally discontinuous metrics in the SOH is valid, the implication is that cobordisms which admit only Morse metrics containing index 1 and/or $n-1$ points will be suppressed in the sum over manifolds. We can use this to draw conclusions about many interesting topology changing processes in quantum gravity. They divide into “good” ones which can occur and “bad” ones which do not. The results in this section and the next are taken from a series of papers on topology change [14, 15, 16, 17].

Good processes include the pair production of black holes of different sorts in a variety of scenarios worked on by many people including Stephen (see the contribution by Simon Ross in this volume). For example, in 4 spacetime dimensions, the manifold of the instanton that is used to calculate the non-extremal black hole pair production rate [18] admits a Morse function with a single index 2 Morse point. The pair production of Kaluza-Klein monopoles [19] is also good, as is the nucleation of spherical bubbles of Kaluza-Klein ($n-5$)-branes in magnetic fluxbrane backgrounds [20]. The decay of the Kaluza-Klein vacuum [21] is good, which is slightly disappointing: one might have hoped that it would be stabilized by these considerations. We know, however, that the cobordism for KK vacuum decay is the same as that for pair production of KK monopoles [22] and so if the latter is a good process so must the former be.

The Big Bang, or creation of an $(n-1)$-sphere from nothing via the yarmulke, is good. Notice that in this way of treating topology change as a sequence of elementary changes, the universe, if created from nothing, must start off as a sphere. No other topology is cobordant to the empty set via an elementary cobordism. The conifold transition in string theory [23] where a three-cycle shrinks down to a point and blows up again as a two-cycle is good. Indeed, the shrinking and blowing up process traces out the seven-dimensional cobordism (each stage of the process is a level surface of a corresponding Morse function) and the fact that it is a three-cycle that degenerates and a two-cycle that blows up tells us that the index of the cobordism is three.

Bad topology changes include spacetime wormholes where an $S^3$ baby universe is born by branching off a parent universe, the epitome of a trousers cobordism. Stephen founded the study of baby universes and spacetime
wormholes \[24\] within the Euclidean quantum gravity framework where our present considerations do not apply. However, if one takes the view that Euclidean solutions – instantons – are to be thought of as devices for calculating transition amplitudes which are nevertheless defined as sums over real, causal, spacetimes, then the badness of the trousers would be counterevidence for the relevance of spacetime wormholes.

Another bad topology change is the pair production or annihilation of topological geons, particles made from non-trivial spatial topology. This deals a serious blow to the hope that the processes of pair production and annihilation of geons could restore to geons the spin-statistics correlation that they lack if their number is fixed \[25\].

In 1+1 and 2+1 spacetime dimensions, all topology changes except for the yarmulkes and their time-reverses are bad ones. This raises the question, is this not in conflict with string theory and the finiteness of topology changing amplitudes in 2+1 quantum gravity \[26\]? For the former, Lenny Susskind has argued that the infinite burst of energy when a loop of string splits into two can be absorbed as a renormalisation of the string coupling constant \[27\]. Martin Roček takes an alternative view, that in first quantized string theory, choosing to integrate over causal metrics on the world sheet would be analogous to choosing paths in the SOH for relativistic quantum mechanics that move only forward in time which would be inconsistent \[28\]. In the latter case, it seems that in the first order, frame-connection formalism suitable for 2+1 gravity, topology change can occur even as a classical process since the relationship between the frame and connection is an equation of motion and can hold even at points where the frame is degenerate \[29\]. In this case, it is no surprise that quantum amplitudes for topology changing processes in 2+1 gravity are non-zero. In fact, it becomes something of a puzzle why we do not see topology change on macroscopic scales all the time if it is an allowed classical process. It would seem that the first order formalism and the metric formalism are genuinely different theories of gravity and distinguishing between them might be an observational issue.

5 Progress on the Borde-Sorkin conjecture

Having looked at some of the consequences of the conjectures, we can ask how plausible they are. There is fragmentary evidence for the conjecture that causal discontinuity leads to badly behaved quantum fields but causally
continuous topology changes allow regular quantum field behaviour [8, 30]. A key investigation that needs to be done is of quantum field theory on a four dimensional spacetime with an index two point, which is conjectured to be regular.

On the other hand we are well on the way to proving the Borde-Sorkin conjecture that Morse spacetimes are causally continuous if and only if they contain no index 1 or n − 1 points. I will sketch the basic ideas involved in the progress made to date.

If we think about the causal structure around the Morse point, p, of the 1 + 1 trousers, it seems intuitive that the causal past of p should contain two separate parts, one down each “leg” of the trousers. And the causal future of the Morse point also divides into two lobes, one up the front and one up the back of the trousers. It’s also intuitive that the causal discontinuities of the 1 + 1 trousers should be related to the disconnectedness of the causal past and future of p in the neighbourhood of p. Flattening out the crotch region, we should obtain a causal structure that looks like that shown in figure 3. There is a special metric for the 1 + 1 trousers in which the causal structure

![Figure 3: The causal structure in the neighbourhood of the Morse point, p, of the 1 + 1 trousers. The past (future) of p consists of the two regions P_1 and P_2 (F_1 and F_2). The “elsewhere” of p divides into four regions S_1, . . . S_4.](image)

can be proved to be exactly as shown: the past (future) of the Morse point p consists of the two regions P_1 and P_2 (F_1 and F_2).
There are two types of causal discontinuity here. The first type is when an observer starting in $P_1$, say, crosses the past light cone of $p$ into $S_1$, say. As it does so the causal future of the observer, which at first contains regions in both $F_1$ and $F_2$, jumps so that it no longer contains any points in $F_2$. The second type is when an observer in $S_1$, say, crosses the future light cone of $p$ into $F_1$. As this happens, the causal past of the observer which contained no points in $P_2$ suddenly grows to contain a whole new region in $P_2$.

The special metric in which this behaviour can be demonstrated exactly generalizes to higher dimensions and all Morse indices. For dimension $n$ and index $\lambda \neq 0, n$ (no yarmulkes for now), the causal past and future of $p$ are obtained from figure 3 by rotating it around the $x$-axis by $SO(n - \lambda)$ and around the $y$-axis by $SO(\lambda)$. We see that when $\lambda = 1$ the past of $p$ remains in two disconnected pieces and when $\lambda = n - 1$ the future of $p$ remains in two pieces. But when $\lambda \neq 1, n - 1$ then both the future and past of $p$ become connected sets.

These suggestive pictures can be turned into a proof that, for these special metrics, index 1 and $n - 1$ Morse points produce causal discontinuities and the other indices do not. We can further show that, not just these special metrics, but any index 1 and $n - 1$ Morse metric is causally discontinuous. It is also true that any Morse metric on the yarmulke is causally continuous. It remains to be proved that any Morse metric on a $\lambda \neq 0, 1, n - 1, n$ elementary cobordism is causally continuous.

6 Looking to the future

I will end by speculating on possible implications that these results might have for a particular proposal for quantum gravity, namely causal set theory, the approach championed by Rafael Sorkin [31, 32, 33]. The basic hypothesis is that the underlying fundamental substructure of spacetime is a discrete object called a causal set, which is, roughly, a Planck density random sampling of the causal structure of spacetime. Mathematically, a causal set, $C$, is a partial order (thus it satisfies the conditions of transitivity and irreflexivity given previously) which is also “locally finite” meaning that the set $\{z \in C : x \prec z \prec y\}$ has finite cardinality for all pairs of points $x$ and $y$ in $C$.

How could a causal set hope to be the basic stuff of spacetime? How could such a truly discrete entity, with not a real number in sight, underlie
a continuum spacetime with its topology, differential structure and metric? The answer lies in work by Stephen and others that shows that for Lorentzian manifolds satisfying a certain causality condition (marginally stronger than the absence of closed causal curves) the causal structure determines the metric up to an overall conformal factor. Since this result is not as well known as it should be, I’ll give a few details here. Stephen proved that if \((M, g)\) and \((M', g')\) are Lorentzian spacetimes and \(f : M \to M'\) is a homeomorphism where \(f\) and \(f^{-1}\) preserve future directed continuous null geodesics then \(f\) is a smooth conformal isometry \([34]\). Malament used this result to show that if \((M, g)\) and \((M', g')\) are past and future distinguishing spacetimes (this condition means that distinct points have non-equal pasts and futures) and \(F : M \to M'\) is a bijection such that \(x \in I^+(y)\) if and only if \(F(x) \in I^+(F(y))\) \(\forall x, y \in M\), then \(F\) is a smooth conformal isometry \([35]\).

Finally one can show that a bijection that preserves the causal structure, \(J^+\) (i.e., \(F : M \to M'\) such that \(x \in J^+(y)\) if and only if \(F(x) \in J^+(F(y))\) \(\forall x, y \in M\)) also preserves the chronological structure \(I^+\) when the spacetimes are distinguishing \([36]\). Notice that in the final form of the result, the bijection is not required even to be continuous or have any properties apart from being causal structure preserving.

Given this powerful result, it seems plausible, even reasonable to suppose that a Planck scale “discretization” of the causal structure should encode all information about a spacetime at length scales above the Planck length. The missing conformal factor, or volume information, is fixed by making the correspondence that the volume of a region counts the number of causal set elements contained in the region.

The causal set hypothesis is both conservative and radical. It is conservative in that it takes the belief that many workers in quantum gravity hold that spacetime is discrete at the fundamental level and the theorem that causal structure is nearly all the metric and puts them together in the most obvious way: the underlying structure is a discretization of the causal structure. The hypothesis then ties up, in a most satisfying way, the question of how the remaining spacetime information is provided, because the correspondence of Volume \(\sim\) Number can only be made due to the discreteness of the causal set. The details of the hypothesis include the prototype of a solution to the knotty problem of how to discretize spacetime whilst maintaining local Lorentz invariance, which solution – roughly that the discretization is random – is conjectured by Sorkin to be essentially unique. Causal set theory is nevertheless radical because it proposes that, fundamentally, there is only a
local finitude with a partial order. Dimension, manifold, topology, differentiable structure, spacetime metric, spacetime causal structure, perhaps also matter, would all be unified in terms of order. To whet the appetite even further, let me mention that causal set theory was used to make a surprising prediction which has subsequently been verified by observation: namely a prediction of the current order of magnitude of a non-zero cosmological constant [32, 33, 37].

Could the picture of topology change I have sketched tell us anything about causal set calculations or vice versa? To see how it could, I will discuss two examples.

One is the process of black hole evaporation which Stephen discovered, the understanding of which is going to play a key role in the development of any successful theory of quantum gravity. As far as the continuum theory goes, the nearest we can come to a spacetime description of black hole formation and evaporation is the conjectured Carter-Penrose diagram shown in figure 4. What could be the description in quantum gravity of this crucial process? One motivation for Stephen’s study of baby universes was that a spacetime wormhole, or trousers, was the most obvious candidate for a spacetime description in the Euclidean approach, with the matter falling into and the radiation emitted from the black hole residing in the baby universe. The approach we have taken here, however, suggests that trousers cannot occur and so we must look for a different description.

Figure 4: The Carter-Penrose conformal diagram of the formation and evaporation of a black hole.
According to the causal set hypothesis, the underlying reality is a causal set, we know that much. Where we have a continuum description, the causal set must be manifold-like and where the continuum description breaks down, close to the singularity, the causal set will have no continuum approximation and only the causal set itself will be a good description of what is happening there. We don’t yet have a quantum dynamics for causal sets but we do have a family of physically motivated “warm up” dynamical models to play with. These are classical, stochastic models in which a causal set grows, element by element, in a probabilistic way that respects (the discrete versions of) general covariance and classical causality, the “Rideout-Sorkin” (RS) models [38]. Joe Henson has proved that in these RS models every element in a causal set almost surely has an element to its future, with the corollary that every point has an infinite future set [39]. This result might be called, “Causal Immortality”.

If this prediction persists in the quantum theory of causal sets we will then know what to expect in a causal set description of black hole formation and evaporation. The causal set will have a continuum approximation corresponding to almost all of the Carter-Penrose conformal diagram but close to the singularity the causal set will cease to be manifold-like. It won’t, however, end there: the causal set will extend on into the future. It’s possible that the part of the causal set that is “born” in the singularity will continue to be non-manifold-like forever but, alternatively, it may become manifold-like again. (Note, these words “continue”, “forever” and “become” are merely suggestive, they are not supposed to imply a physically meaningful background “time”.) In the latter case it would then be a “baby universe”, disconnected from the region in which the black hole formed and evaporated, though not a baby universe created by a continuum wormhole.

It is tempting to speculate yet further that this might realise something like Smolin’s idea of cosmological natural selection [40], with new universes with different coupling constants being created, not inside black holes but when black holes evaporate. The causal structure of the black hole singularity in the formation-evaporation spacetime would be the following. If the singularity represents a single causal set point s and if \( F(s) \) is the future set of s, then any point \( x \in F(s) \) would have the property that a point related to x must be related to s. This says that nothing can influence the future of s except through s and means that s is a so-called “past post” as far as \( F(s) \) is concerned (a “post” is a point in a causal set to which all points in the set are related). Now, posts are responsible for a cosmological
renormalisation of the coupling constants which characterize the dynamics of causal sets, at least in the classical RS models \cite{41, 42} and it seems that a similar renormalisation would take place when there’s a past post.

The second example is an investigation into how causal discontinuity might be ruled out by causal set dynamics. First we need a characterization of causal discontinuity in terms of the causal order alone. This might be done by looking at the various equivalent conditions for causal discontinuity in the continuum that Stephen and R.K. Sachs have provided \cite{13}. Alternatively, thinking about causal sets which arise by randomly sampling causally discontinuous spacetimes, it seems that one possible characteristic that they share is the existence of an element $x$, which has a (past or future) relation, which if severed would make a big change in the size of the past or future set of $x$. One would have to make this more precise but intuitively it seems to make sense. Then one would hope to be able to predict that causal sets with such a property do not occur. This would have to be done in the, as yet unknown, quantum theory of causal sets but to see broadly how it might work we could ask whether we are able make such a prediction in the RS models. In these models, one can prove things like Henson’s result that every point has a future relation. Graham Brightwell has also shown that an infinite antichain (an antichain is a totally unordered subset) almost surely doesn’t occur \cite{39}. It seems possible that one could prove something like “the probability that a point $x$ will have a future set which has a large subset which will be cut off from $x$ by the severing of a single relation is zero.”

So, for the future, there is much to be done on the specific conjectures mentioned, in order to put this whole approach to topology change on a firmer footing. And for the broader picture, I think that things look very interesting and promising for the causal set approach to quantum gravity. Although Stephen has not worked directly on this area, his work on global causal analysis is one of the crucial underpinnings of the programme. His work is central to the proof that the causal structure of a spacetime encodes most of the information about that spacetime. He has also been one of the main proponents of the Sum Over Histories approach to quantum gravity which will be at the heart of developments on a quantum dynamics for causal sets. This is because a causal set has an essentially “spacetime” character: it’s hard to see how one could make any headway with an attempt at the space+time split required for a canonical quantization for example. In a sense, causal sets provide an explanation for Stephen’s Chronology Protection Conjecture because one can “predict” that there will be no CTC’s (this is
slightly more than a case of simply assuming what we want to prove – the
irreflexivity condition on the causal set can be dropped but a spacetime with
CTC’s will nevertheless never be an approximation to a transitive digraph
even when it has closed loops [43]). Of course, Stephen’s work on black hole
thermodynamics will have a major role to play in any proposed theory of
quantum gravity but it is particularly important in causal set theory since
one of the main motivations for believing in an underlying discrete structure
at all is the finiteness of the black hole entropy [37]. Altogether, it wouldn’t
be going too far to say that the influence of Stephen’s work can be seen in
the very foundations of the causal set approach. I hope there will be some
successes of causal set theory to report for Stephen’s 70th birthday.

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