I. INTRODUCTION

Cosmic ray (CR) antimatter is a potential probe of exotic high energy astrophysical phenomena and a unique diagnostic of CR propagation. Over the last decade, precise measurements of the flux of CR $e^+$ and $\bar{p}$ extending to ever higher energies were reported by the PAMELA and AMS02 experiments [1–3]. The interpretation of these measurements motivates refined theoretical consideration of astrophysical $e^+$ and $\bar{p}$, produced as secondaries in the collision of primary CRs, notably protons, with interstellar matter (ISM), notably hydrogen. Our goal in the current paper is to improve on previous calculations of the inclusive production cross section of secondaries in $pp$ collisions using recent accelerator data.

The main effect we wish to capture is the violation of radial scaling at $\sqrt{s} > 50$ GeV. As shown in Refs. [4–6], this effect leads to a factor of two increase in the astrophysical $\bar{p}$ source at $\bar{p}$ energy above a few TeV. Here we evaluate the analogous effect in the CR $e^+$ source by analysing meson production at LHC energies. Earlier $e^+$ calculations were either based on too low $\sqrt{s}$ data to see the effect [7–9] or relied on Monte-Carlo tools without direct verification in the kinematical regime relevant for astrophysics [10].

We aim to achieve $\sim 10\%$ accuracy for the astrophysical $e^+$ source at $e^+$ energy ranging from a few GeV up to multi-TeV; this accuracy goal is to be compared with the main radial scaling violation effect that is, again, about a factor of two at $E \sim 10$ TeV. As a check against earlier work, we also calculate the $\bar{p}$ source to similar accuracy.

In section II we analyse the cross sections at large $\sqrt{s}$, using results from the NA49, PHENIX, ALICE, and CMS experiments. In section III, we use these results to calculate the production rate ratio $Q_{e^+}/Q_{\bar{p}}$ for secondary $e^+$ and $\bar{p}$ produced by a spectrum of high energy protons scattering on a proton target. We show that $Q_{e^+}/Q_{\bar{p}}$ is insensitive w.r.t. uncertainties in the primary proton spectrum. At $10 < E < 100$ GeV the $e^+/\bar{p}$ flux ratio measured by AMS02 falls below the production rate ratio by about 50%, while at high energy $E > 100$ GeV the measured flux ratio coincides with the production rate ratio of the secondary source.

II. DATA ANALYSIS

Our baseline fitting formulae for inclusive hadron production in $pp$ collisions are taken from Ref. [11] (Tan&Ng), which was based on $\sqrt{s} \leq 53$ GeV data and to which we provide corrections using the following new information:

i. Tan&Ng’s formulae rely on radial scaling [12–14],

$$E \frac{d^3\sigma}{dp^3}(x_R, p_T, \sqrt{s}) \rightarrow \sqrt{s} \rightarrow \infty \frac{d^3\sigma}{dp^3}(x_R, p_T)$$

(1)

where $x_R = E^*/E_{\text{max}}$, $E^*$ is the final state hadron energy in the centre of mass (CM) frame and $E_{\text{max}}$ is the maximum attainable $E^*$. Recent accelerator data show violation of radial scaling in $pp$ collisions at $\sqrt{s} \gtrsim 50$ GeV [15–19]. The $pp \rightarrow \bar{p}$ cross section increases at high energy [20–22] as compared to [11] and other early parametrisations. We will assess the analogous effect in meson production and the resulting $e^+$ yield.

ii. In addition to the high energy end, unprecedented detailed measurements of the production cross section $\pi^+, K^+$ and $\bar{p}$ [20–22] at $\sqrt{s} = 17.2$ GeV were reported by the NA49 experiment. This value of $\sqrt{s}$ is particularly relevant for $E \sim 10 - 100$ GeV final state $\bar{p}$ and $e^+$ [23]. We incorporate this data in our formulae for hadronic cross sections.
A. $p_t$-weighted cross sections and important kinematical region

Faced with an extensive data set \[15\]-\[22\], it is instructive to bracket the final state phase space that is most relevant for secondary CR production. In the fixed-target set up of high energy CR scattering on ambient ISM, the key quantity is the conversion cross section from incoming CR proton with ISM frame energy $E_p$ to outgoing secondary particle with ISM frame energy $E$,

$$
\frac{d\sigma(E_p,E)}{dE} = 2\pi \int_0^\pi d\theta p_t \left( E \frac{d\sigma}{dp_t^3} \right) (x_R, p_t, \sqrt{s}),
$$

(2)

where $\theta$ denotes the angle between the incoming proton and outgoing secondary in the ISM frame. The Lorentz-invariant differential cross section $E \frac{d\sigma}{dp_t^3}$ decreases sharply with increasing $p_t$, with the $p_t$-weighted cross section $p_t \left( E \frac{d\sigma}{dp_t^3} \right)$ peaking around average $(p_t) \sim 0.2 - 0.4$ GeV. For $E \gg (p_t)$, $m$, where $m$ is the mass of the final state hadron of interest, we can simplify the integral as

$$
\int_0^\pi d\theta p_t \left( E \frac{d\sigma}{dp_t^3} \right) (x_R, p_t, \sqrt{s}) 
\simeq \frac{1}{p_t} \int_0^\infty dp_t \left( E \frac{d\sigma}{dp_t^3} \right) (x_R|_{p_t=0}, p_t, \sqrt{s}),
$$

(3)

where $x_R|_{p_t=0}$ is computed at $p_t = 0$ and only depends on $E$ and $E_p$. This exercise shows that, in the high energy regime, the $p_t$-weighted mean cross section with fixed $x_R$ is the most important quantity for secondary CR production, allowing one to average over the detailed $p_t$ dependence reported by the experiments.

Next, we consider the relevant range of $x_R$. Consider as a representative example the cross section parametrization \[5\] :

$$
E \frac{d^3\sigma}{dp_t^3} (x_R, p_t) = f_0 e^{-\frac{m}{p_t}} (1 - x_R)^n.
$$

(4)

Typical parameters are $(p_t) \simeq 0.2 - 0.4$ GeV and $n \simeq 5 - 7$. In the limit where $m^2/E^2$, $p_t^2/E^2 \ll 2m_p/E_p$, the astrophysical source term $Q(E)$ can be written as

$$
Q(E) \propto \int_E^\infty dE_p J_p(E_p) \frac{d\sigma(E_p,E)}{dE} 
\simeq 2\pi f_0 (p_t)^2 J_p(E) \int_0^{1} dx_R x_R^{-2} (1 - x_R)^n,
$$

(5)

where $J_p$ denotes the CR proton flux and we assumed $J_p \propto E_p^{-\gamma}$. With $n \simeq 5 - 7$ and $\gamma \simeq 2.7$, the $x_R$ integrand selects the range $\sim 0.1 - 0.4$.

To summarise, we are most interested in the cross section for secondary product energy in the range $E \geq 10$ GeV. In this range, the relevant information is contained in the $p_t$-weighted mean invariant cross section at fixed $x_R$, where furthermore the relevant range of $x_R$ is $\sim 0.1 - 0.4$.

B. Hadron production cross section

In this section we discuss the hadronic cross section in the light of recent collider experiments. We take the cross section fits by Tan&Ng as baseline, and derive corrections to this formula.

A comment is in order regarding the intermediate hyperon contribution to $\bar{p}$. In $pp$ collisions, $\bar{p}$ are generated promptly or by the decay of (relatively) long-lived hyperons, notably $\Lambda$ and $\Sigma^\pm$. The Tan&Ng $\bar{p}$ fit includes the hyperon contributions. On the other hand, recent experiments such as NA49 report the prompt antiproton cross section in which the contribution of intermediate hyperon states is removed. Thus, when comparing experimental $\bar{p}$ cross section data and fits we need to specify whether the hyperon contribution is subtracted or not.

For the purpose of astrophysical calculations, of course, our eventual concern is the total $\bar{p}$ cross section including the hyperon contributions. In this section, however, we find it convenient to concentrate first on the prompt $\bar{p}$ production cross section, deferring an analysis of the hyperon contribution to App. \[3\].

1. NA49 experiment

The NA49 experiment reported measurements in a wide kinematic regime. Fig. \[1\] shows measurements of the $p_t$-weighted cross section, presented as ratio between NA49 data and the Tan&Ng’s formulae in given $x_F$ bins\(^1\). We use data from \[20\], \[21\] and \[22\] for $\pi^+$, $K^\pm$ and $\bar{p}$ respectively. For each point, statistical and systematic errors are both at the level of 10%.

As we discussed, the most relevant kinematic region to determine CR flux is $x_F = 0.1 - 0.4$. In this region, Fig. \[1\] shows that apart from an overall factor the fitting functions of Tan&Ng are consistent with the NA49 results for all final states with the possible exception of $K^{-}$ (the latter being quantitatively irrelevant for the secondary $e^+$ calculation). Motivated by this result, we introduce a scaling factor $\xi_H(\sqrt{s})$ for each hadron $H = \pi^+, K^\pm, \bar{p}$, and parametrize the cross section as

$$
E \frac{d^3\sigma_H}{dp^3} = E \frac{d^3\sigma_H}{dp^3} \bigg|_{\text{Tan&Ng}} \times \xi_H(\sqrt{s}).
$$

(6)

We take $\xi_{\pi^+} = \xi_{K^\pm} = 0.9$ and $\xi_{\bar{p}} = 0.8$ at $\sqrt{s} = 17.2$ GeV.

Note that the prompt $\bar{p}$ cross section from NA49 is off by $\sim 20\%$ from the inclusive Tan&Ng fit: this is not a discrepancy, but is mainly due to the hyperon contribution.

\(^1\) NA49 data are provided in terms of the Feynman parameter $x_F = 2p_t^L/\sqrt{s}$ (where $p_t^L$ is the hadron longitudinal momentum in the CM frame) instead of $x_R$, so we consider the $p_t$-weighted cross section at fixed $x_F$. 
present in the Tan&Ng fit while being subtracted from NA49 data. Accounting for this correction we find, in fact, that the inclusive Tan&Ng fit is in good agreement with that deduced from NA49 data.

2. High energy experiments

Next, we analyse the high energy data to determine the behaviour of $\xi_H$ at large $\sqrt{s}$. The scaling factors $\xi_H$ are calibrated to reproduce the $p_t$-weighted cross section of Eq. (9) evaluated on the high energy experimental data. Fig. 2 shows the $\sqrt{s}$ dependence of ratios of $p_t$-weighted cross sections for $\pi^+$, $K^\pm$ and $\bar{p}$ between high energy data and the Tan&Ng [11] formulae. Solid lines indicate the correction functions $\xi_H$. Black, orange, green, blue and red points correspond to NA49, PHENIX, ALICE and CMS data respectively, with estimated systematic uncertainties. The yellow points represent data sets used in Tan&Ng fitting paper [11].

are consistent with these data to within $\sim \pm 30\%$, comparable to the internal variation between the results of individual analyses in this data set, and we assign this uncertainty to the orange points.

We find that the correction functions

$$\xi_{\pi^+}(\sqrt{s}) = \begin{cases} 0.9 & (\sqrt{s} < 50 \text{ GeV}) \\ 0.9 + 0.18\log(\sqrt{s}/50 \text{ GeV})^2 & (\sqrt{s} \geq 50 \text{ GeV}) \end{cases}$$

(7)

$$\xi_{\pi^+}(\sqrt{s}) = \begin{cases} 0.8 & (\sqrt{s} < 50 \text{ GeV}) \\ 0.8 + 0.11\log(\sqrt{s}/50 \text{ GeV})^2 & (\sqrt{s} \geq 50 \text{ GeV}) \end{cases}$$

(8)

$$\xi_{K^\pm}(\sqrt{s}) = \xi_{\pi^+}(\sqrt{s}),$$

(9)

reproduce the experimentally determined $p_t$-weighted cross sections in the range $\sqrt{s} \leq 7 \text{ TeV}$.

Several comments are in order. First, the PHENIX $\bar{p}$ data [19] in Fig. 2 exhibit larger uncertainty compared to most of the other measurements, and the central values are indeed correspondingly off by $\sim 50\%$, 30\% for $\sqrt{s}=62.4$ and 200 GeV from the fit. To estimate the $p_t$-weighted $\bar{p}$ cross section from [19] we start with the data without feed-down correction, as the feed-down corrected cross section is found to be lower by a factor of a few in low $p_t$ bins, which appears broadly inconsistent with the remaining data set. To estimate the feed-down corrected result, we subtract 30\% off the inclusive result, as suggested by our analysis in App. B. The $\bar{p}$ systematic uncertainties quoted in [19] are sizeable, notably in the lower $p_t$ region, due to the feed-down correction and take maximally $\sim 50\%$, 30\% for $\sqrt{s}=62.4$ and 200 GeV. In Fig. 2 we assign these conservative uncertainty estimates of 50\% and 30\% to these data. In addition to the feed-down uncertainty, the $p_t$ range covered by the $\bar{p}$
cross section data in [19] is limited, starting from $p_t = 0.6$ GeV. This means that the $p_t$-weighted cross section estimate derived from these data is based on a kinematically sub-dominant region for astrophysical purposes.

Second, we comment on the $K_S$ contribution to the $\pi$ cross section. In the analysis of Fig. 2 we assume that the $\pi$ cross sections reported by the experiments are prompt and do not include $\pi$ from $K_S$ decay. The NA49 and CMS experiments explicitly state that $\pi$ from $K_S$ decay are discriminated in their analyses. On the other hand, the treatment in the PHENIX and ALICE experiment is unclear. This makes 5% ambiguity of the points from PHENIX and ALICE experiments in Fig. 2. In practice, this ambiguity is not quantitatively important for the determination of fitting formula.

Finally, we comment on the $x_R$ dependence in the high $\sqrt{s}$ regime. The high energy experimental data from [15, 17, 19] is only specified at mid-rapidity ($x_R \simeq 0$). This means that our fit could fail to reproduce the $x_R$ dependence in the high $\sqrt{s}$ regime. Fixing this caveat would require cross section data at non-zero $x_R$ (forward region) in the high $\sqrt{s}$ regime.

C. Comparison to previous work

In Fig. 3 we show the secondary source terms for $\bar{p}$ and $\pi^+$, assuming $pp$ production from a power-law primary proton flux $J_p \propto E_p^{-3}$, comparing our results to the fitting formulae of [5] and Tan&Ng. For $\bar{p}$ production, we now include the contributions from both hyperon decay and decay in flight of $\bar{n}$, using the procedure defined in App. B. The Black line shows the $\bar{p}$ source term ratio between that obtained using the fit of Ref. [5] (denoted ‘Winkler’) and ours. The blue dotted (red dashed) line shows the $\bar{p}$ ($\pi^+$) source term ratio between Tan&Ng [11] and ours. The deviation from radial scaling, assumed in Tan&Ng, is clear at high energy.

III. THE $e^+/\bar{p}$ FLUX RATIO

Ref. [24] pointed out that the production rate ratio $Q_{e^+}/Q_{\bar{p}}$ provides a model-independent upper bound to the flux ratio of high-energy secondary CR $e^+$ and $\bar{p}$:

$$\frac{J_{e^+}(R)}{J_{\bar{p}}(R)} < \frac{Q_{e^+}(R)}{Q_{\bar{p}}(R)}, \quad (10)$$

where the source terms for secondary $\bar{p}$ and $e^+$ produced in $pp$ collisions are:

$$Q_{\bar{p}}(E_{\bar{p}}) = 2 \int_{E_{\bar{p}}}^{\infty} dE_{p} 4\pi J_p(E_{p}) \frac{d\sigma_{pp\rightarrow \bar{p}}}{dE_{\bar{p}}}(E_{\bar{p}}; E_{p}), \quad (11)$$

$$Q_{e^+}(E_{e^+}) = \int_{E_{e^+}}^{\infty} dE_{p} 4\pi J_p(E_{p}) \frac{d\sigma_{pp\rightarrow e^+}}{dE_{e^+}}(E_{e^+}; E_{p}). \quad (12)$$

This upper bound only depends on the inclusive production cross sections and the shape of proton cosmic ray flux $J_p$.

We are now in position to extend the calculation of $Q_{e^+/\bar{p}}(R)/Q_{\bar{p}}(R)$ to high energy, and compare with the latest CR data. In Fig. 4 we show the upper bound predicted for different assumptions on the primary proton flux in the spallation region. The $e^+/\bar{p}$ flux ratio measured by AMS-02 is consistent with the upper bound and saturates it at high energy (for proton flux coinciding with the locally measured proton flux).

Recent calculations of the high-energy secondary CR $\bar{p}$ flux [24, 25], using up to date $\bar{p}$ production cross section consistent with our results here and calibrated to agree with AMS-02 B/C data, are consistent with the CR $\bar{p}$ flux measured by AMS-02. These results are reproduced in App. C. The significance, in connection with Fig. 4, is that the observed flux of CR $e^+$ at $R > 100$ GV coincides with the expected flux of secondary $e^+$, that would be expected if radiative energy loss became unimportant in

Note that (i) the factor 2 in $Q_{\bar{p}}$ comes from decay in flight of $\bar{n}$, and (ii) the normalization in our definition for $Q_{e^+/\bar{p}}$ here is somewhat different than in, e.g., Refs. [24, 25]. This is for ease of presentation and is of no consequence for the source ratio.
the propagation at these energies. Achieving such low level of energy loss would require that the characteristic secondary CR propagation time drops below a few Myr at $R > 100$ GV.

A comparison of the source ratio $Q_{e^+}/Q_{\bar{p}}$ to the observed $e^+/\bar{p}$ flux ratio was also presented in Ref. [20], which found results for $Q_{e^+}/Q_{\bar{p}}$ smaller than our value by $\sim 30\%$ in the energy range $10 - 1000$ GeV. This led Ref. [20] to argue that $e^+$ energy losses may be negligible at all energies (rather than only at $E \gtrsim 100$ GeV), as suggested by our Fig. 4. We have not been able to reproduce the origin of this discrepancy.

IV. CONCLUSIONS

We presented an analysis of inclusive $\bar{p}$, $\pi$, and $K$ production in pp collisions. Our main goal was to implement recent experimental data for meson production, in particular the effect of radial scaling violation manifest at LHC energies and recent detailed kinematical data from the NA49 experiment at intermediate energy, in semi-analytic fits used for the calculation of the astrophysical secondary production of $e^+$. We provide fitting formulae that, combined with earlier results from Tan&Ng [31], allow to compute the astrophysical production of $e^+$ and $\bar{p}$ up to the multi-TeV range with an estimated uncertainty of $\sim 20\%$.

The $e^+/\bar{p}$ flux ratio reported by AMS-02 is found to coincide with the secondary source production rate ratio $Q_{e^+}/Q_{\bar{p}}$ at high-energy $E > 100$ GeV. This coincidence may be considered as a hint for a secondary origin for CR $e^+$ and $\bar{p}$, as it would be a fine-tuned accident in models that advocate new primary sources for either antimatter CR species.

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Appendix A: Neutral kaon contributions

In this section we calculate the final state $e^+$ contribution coming from the decay of $K^0_L$ mesons. This contribution has been neglected in the literature, although the corresponding cross section is comparable to that for charged kaons which was previously taken into account. $K^0_L$ mesons are long-lived ($\tau_{K^0_L} \simeq 15$ m) in the collider set-up, so that $\pi^+$ from $K^0_L$ decay are not included in the fitting formula of the inclusive $\pi^+$ cross section. In addition, the $K^0_L$ semi-leptonic decay contributes directly to $e^+$ and $\mu^+ (\rightarrow e^+)$ production.

We consider the following decay channels [29]:

\begin{align}
\text{Br}(K^0_L \rightarrow \pi^+ e^+ \nu_e) &= 40.55 \%, \\
\text{Br}(K^0_L \rightarrow \pi^+ \mu^+ \nu_\mu) &= 27.04 \%, \\
\text{Br}(K^0_L \rightarrow \pi^+ \pi^- \pi^0) &= 12.54 \%.
\end{align}

We approximate and simplify the kinematics of $K^0_L$ three-body decays, assigning each of the decay products an energy of $m_K/3$ in the $K^0_L$ rest frame and ignoring muon polarisation. We approximate the $K^0_L$ production cross section to match that of $K^+$. The $e^+$ spectrum from boosted $\mu^+$ is given in Ref. [30] and the $e^+$ spectrum from boosted $\pi^+$ is given in Ref. [31].

The kaon contribution to astrophysical secondary $e^+$ production is highlighted in Fig. 5. The $K^0_L$ contribution amounts roughly to $5\%$ of the total $e^+$ source.

Appendix B: Antiproton cross section including anti-hyperon contributions

In this section we analyse the hyperon contribution to the inclusive $\bar{p}$ production cross section. We denote the Lorentz-invariant differential cross section as $f$:

\begin{equation}
f_{\#} = E^4 \sigma \frac{d^3P_{\#}}{dp^3}.
\end{equation}

The astrophysically relevant inclusive $f^\text{tot}_{\bar{p}}$, which includes effects from $\bar{n}$ and hyperon decays, can be decomposed in the following way:

\begin{align}
f^\text{tot}_{\bar{p}} &= f_{\bar{p}} + f_{\bar{n}}, \\
f_{\bar{p}} &= f_{\bar{p}}^0 + f_{\bar{p}}^\Lambda + f_{\bar{p}}^\Sigma, \\
f_{\bar{n}} &= f_{\bar{n}}^0 + f_{\bar{n}}^\Lambda + f_{\bar{n}}^\Sigma.
\end{align}
where \( f_0 \) indicates the prompt contribution and \( f_A, f_\Sigma \) denote contribution from the hyperon decay. Neglecting isospin violation, we assume \( f_0^p = f_0^n \). To set a rough scale for the effect we’re after here, the analysis in Sec. II shows \( f_p^p \approx 0.8 f_{\text{Tan}^*\text{Ng}}^\Lambda \) at low \( \sqrt{s} \), where \( f_{\text{Tan}^*\text{Ng}}^\Lambda \) includes the hyperon decay contribution. (See Eq. (8) and Fig. 2.)

1. Anti-hyperon production cross section at the NA49 experiment

NA49 [20] results indicate that the kinematical distribution of anti-hyperons produced in pp collisions is somewhat different from that of anti-nucleons\(^3\). We introduce \( x_R \)-dependent functions \( g_B(x_R) \) with \( B = \Lambda, \Sigma^\pm \), and parametrize the hyperon contributions as

\[
\begin{align*}
    f_0^\Lambda &= f_{\text{Tan}^*\text{Ng}}^\Lambda g_\Lambda(x_R) \text{Br}(\Lambda \rightarrow pX), \\
    f_0^\Sigma &= f_{\text{Tan}^*\text{Ng}}^\Sigma g_\Sigma(x_R) \text{Br}(\Sigma \rightarrow nX), \\
    f_p^\Lambda &= f_{\text{Tan}^*\text{Ng}}^\Lambda g_\Lambda^0(x_R) \text{Br}(\Sigma^+ \rightarrow pX), \\
    f_p^\Sigma &= f_{\text{Tan}^*\text{Ng}}^\Sigma g_\Sigma^0(x_R) \text{Br}(\Sigma^+ \rightarrow nX) + f_{\text{Tan}^*\text{Ng}}^\Sigma g_\Sigma^+(x_R) \text{Br}(\Sigma^- \rightarrow nX). 
\end{align*}
\]

The branching fractions for hyperon decays are \( \text{Br}(\Lambda \rightarrow pX) \approx 0.64 \), \( \text{Br}(\Lambda \rightarrow nX) \approx 0.36 \), \( \text{Br}(\Sigma^+ \rightarrow pX) \approx 0.52 \), \( \text{Br}(\Sigma^+ \rightarrow nX) \approx 0.48 \) and \( \text{Br}(\Sigma^- \rightarrow nX) \approx 1 \) [22]. Summing up, we obtain

\[
f_0^p \approx f_{\text{Tan}^*\text{Ng}}^\Lambda [1.6 + g_\Lambda + g_\Sigma^- + g_\Sigma^+].
\]

We neglect momentum difference between parent and daughter particle since their mass difference is \( \lesssim 20\% \).

Let us determine \( g_B(x_R) \). NA49 analysis [20] (see Fig. 22 there) offers the differential multiplicity \( dn/dx_F \) for \( \Lambda, \bar{\Lambda}, \Sigma^+, \Sigma^- \), defined as

\[
\frac{dn}{dx_F}(x_F) = \frac{\pi}{2 \sigma_{\text{inel}}} \sqrt{s} \int dp_t f_\ast \frac{E}{E}. \tag{B10}
\]

Uncertainties of \( dn/dx_F \) are not presented, but a typical error estimate of \( \sim 20\% \) can be inferred from the analysis in [5].

Although the definition of \( x_R(= E^*/E_{\text{max}}^*) \) and \( x_F(= 2p_t^2/\sqrt{s}) \) are different, their difference is of the order of \( p_t^2/s \) or \( m_p^2/s \). Thus, \( g_B(x_R) \) can be determined from the observation of \( dn/dx_F \). As discussed in section II, 0.1 \( \lesssim x_R \lesssim 0.4 \) is the important kinematical region to determine secondary cosmic ray production. In this region, the \( p_t \) dependence on \( E \) becomes weak and \( dn/dx_F \) is determined by \( p_t \) weighted averaged cross-section. In this respect, we find that \( dn/dx_F \) is a directly relevant quantity for secondary cosmic ray production. Then, it is reasonable to estimate

\[
g_B(x_R) = \left( \frac{dn_B}{dx_F} \right) \left( \frac{dn_p}{dx_F} \right)_{T\text{an}^*\text{Ng}}, \tag{B11}
\]

with \( B = \Lambda, \bar{\Lambda}, \Sigma^+, \Sigma^- \) or \( \bar{p} \).

Following Ref. [22] we assume the relation

\[
\frac{dn_{\Sigma^-}}{dx_F} \approx 0.8 \frac{dn_{\Lambda}}{dx_F} \frac{dn_{\Sigma^+}}{dx_F}. \tag{B12}
\]

Then, we expect

\[
g_{\Sigma^-} \approx 0.8 \frac{dn_{\Lambda}}{dx_F} \frac{dn_{\Sigma^+}}{dx_F} g_{\Sigma^+}. \tag{B13}
\]

We assume a similar relation for \( \Sigma^+ \):

\[
g_{\Sigma^+} \approx 0.8 \frac{dn_{\Lambda}}{dx_F} \frac{dn_{\Sigma^+}}{dx_F} g_{\Sigma^-}. \tag{B14}
\]

To obtain \( g_B \) (with \( B = \Lambda, \Sigma^\pm \)), we fit the \( x_F \) dependence shown in the NA49 analysis by the following form:

\[
g_B = a(1 - x_R)^n. \tag{B15}
\]

We found \((a, n) = (0.13, -3), (0.038, -3), (0.028, -2)\) well fit \( \Lambda, \Sigma^-, \Sigma^+ \) respectively.

Fig. 6 shows the \( x_F \) dependent \( \bar{B} \equiv dn_{\bar{B}}/dx_F \). Solid and dashed lines correspond to NA49 values and our fitting function, respectively.

2. Multiplicity of anti-hyperons at large \( \sqrt{s} \)

For relatively small \( \sqrt{s} < 50 \text{ GeV} \), we expect that Eq. (B9) holds with weak \( \sqrt{s} \) dependence. This is because, empirically, radial scaling applies at small \( \sqrt{s} \).
However, when we consider large $\sqrt{s} > 50$ GeV, we have to consider the violation of radial scaling.

Ref. [5] showed that the ratio between the multiplicity of anti-hyperons and $\bar{p}$ is not constant as function of $\sqrt{s}$. Following [5], we introduce $\sqrt{s}$ dependence as an overall factor to the hyperon contributions,

$$f_{\bar{p}}^{tot} \simeq f_{\bar{p}}^{Tan} \times \left[ 1.6 \times \xi(\sqrt{s}) + (g_{\Lambda} + g_{\Sigma^-} + g_{\Sigma^+}) \times \kappa(\sqrt{s}) \right].$$

(B16)

Here $\kappa(\sqrt{s})$ satisfies $\lim_{s \to 0} \kappa(\sqrt{s}) = 1$, and deviates from unity at large $\sqrt{s}$.

We define the ratio between the multiplicity of $\bar{\Lambda}$ and $\bar{p}$ at midrapidity:

$$\frac{\bar{\Lambda}}{\bar{p}} = \frac{dn_{\bar{\Lambda}}}{dn_{\bar{p}}} \bigg|_{x_F=0}.$$  \hspace{0.5cm} (B17)

For simplicity, we assume that $\bar{\Lambda}$, $\bar{\Sigma}^\pm$ have the same scaling law for their multiplicity. By using this assumption, we take $\kappa(\sqrt{s})$ as

$$\kappa(\sqrt{s}) = \frac{\bar{\Lambda}/p}{(\bar{\Lambda}/p)(0)}.$$  \hspace{0.5cm} (B18)

Finally, we analyse the ratio $\bar{\Lambda}/\bar{p}$ using data from STAR [33, 34], ALICE [15, 35], and CMS [18, 36] which provided multiplicity ratios at mid-rapidity. NA49 also provided differential multiplicity at the mid-rapidity; we assume an uncertainty of 20% from the uncertainty in the feed-down correction. This gives us $\bar{\Lambda}/\bar{p} = 0.24 \pm 0.05$ at $\sqrt{s} = 17.2$ GeV form NA49 experiment.

Fig. 7 shows our result. For comparison, we also show $\bar{\Lambda}/\bar{p}$ as found in [5]. Our results can be fitted by the following formula:

$$\frac{\bar{\Lambda}}{\bar{p}} = 0.24 + \frac{0.37}{1 + ((146 \text{ GeV})^{2/s})^{0.9}}.$$  \hspace{0.5cm} (B19)

### Appendix C: Secondary $\bar{p}$

Fig. 8 shows the secondary $\bar{p}$ cosmic ray flux predicted by our cross section formula, calculated under the assumption that the mean target column density traversed by CR protons; He; nuclei such as B, C, and O; and $\bar{p}$ is the same as function of magnetic rigidity [24]. The column density used in the calculation is extracted from $B/C$ data using fragmentation cross sections as specified in [23].

The simple estimate in Fig. 8 is consistent the AMS-02 $\bar{p}$ data [8]. The calculation is sensitive to a number of systematic uncertainties. The blue region shows the uncertainty of the solar modulation parameter $\phi = (0.2 - 0.8)$ GV. The grey region shows the result of varying the spectral index of proton CR above 300 GV. We vary $\gamma_p$ in the range of 2.6–2.8 where $J_p \propto E_p^{-\gamma_p}$; this should represent the possibility that the CR proton spectrum in the regions dominating secondary $\bar{p}$ production may not be identical to the locally measured spectrum. The solid green lines show the result of varying the $C \rightarrow B$ fragmentation cross section by $\pm 20\%$. Finally, the dashed dark lines represent $\bar{p}$ production cross section uncertainty of $\pm 20\%$.

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**FIG. 6:** $x_F$ dependent $B \equiv dn_B/dx_F$. The solid lines and dashed ones correspond to NA49 values and our fitting function, respectively.

**FIG. 7:** $\bar{\Lambda}/\bar{p}$ ratio in proton-proton collision at mid-rapidity.

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For simplicity, we assume that $\bar{\Lambda}$, $\bar{\Sigma}^\pm$ have the same scaling law for their multiplicity. By using this assumption, we take $\kappa(\sqrt{s})$ as

$$\kappa(\sqrt{s}) = \frac{\bar{\Lambda}/p}{(\bar{\Lambda}/p)(0)}.$$  \hspace{0.5cm} (B18)

Finally, we analyse the ratio $\bar{\Lambda}/\bar{p}$ using data from STAR [33, 34], ALICE [15, 35], and CMS [18, 36] which provided multiplicity ratios at mid-rapidity. NA49 also provided differential multiplicity at the mid-rapidity; we assume an uncertainty of 20% from the uncertainty in the feed-down correction. This gives us $\bar{\Lambda}/\bar{p} = 0.24 \pm 0.05$ at $\sqrt{s} = 17.2$ GeV form NA49 experiment.

Fig. 7 shows our result. For comparison, we also show $\bar{\Lambda}/\bar{p}$ as found in [5]. Our results can be fitted by the following formula:

$$\frac{\bar{\Lambda}}{\bar{p}} = 0.24 + \frac{0.37}{1 + ((146 \text{ GeV})^{2/s})^{0.9}}.$$  \hspace{0.5cm} (B19)

### Appendix C: Secondary $\bar{p}$

Fig. 8 shows the secondary $\bar{p}$ cosmic ray flux predicted by our cross section formula, calculated under the assumption that the mean target column density traversed by CR protons; He; nuclei such as B, C, and O; and $\bar{p}$ is the same as function of magnetic rigidity [24]. The column density used in the calculation is extracted from $B/C$ data using fragmentation cross sections as specified in [23].

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**FIG. 6:** $x_F$ dependent $B \equiv dn_B/dx_F$. The solid lines and dashed ones correspond to NA49 values and our fitting function, respectively.

**FIG. 7:** $\bar{\Lambda}/\bar{p}$ ratio in proton-proton collision at mid-rapidity.
FIG. 8: Cosmic ray $\bar{p}$ flux for several cross section formulae. In all cases, we use the mean traversed target column density extracted from B/C data using nuclear fragmentation cross sections as specified in [23]. Solid black line shows the prediction using our fit. Dotted and dashed black lines show the result when using the fit from Tan&Ng [11] and Winkler [4], respectively. Other bands and lines show various sources of systematic uncertainty; see text for details.

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