Mechanism Design for Base Station Association and Resource Allocation in Downlink OFDMA Network

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Abstract

We consider a resource management problem in a multi-cell downlink OFDMA network whereby the goal is to find the optimal combination of (i) assignment of users to base stations and (ii) resource allocation strategies at each base station. Efficient resource management protocols must rely on users truthfully reporting privately held information such as downlink channel states. However, individual users can manipulate the resulting resource allocation (by misreporting their private information) if by doing so can improve their payoff. Therefore, it is of interest to design efficient resource management protocols that are strategy-proof, i.e. it is in the users’ best interests to truthfully report their private information. Unfortunately, we show that the implementation of any protocol that is efficient and strategy-proof is NP-hard. Thus, we propose a computationally tractable strategy-proof mechanism that is approximately efficient, i.e. the solution obtained yields at least $\frac{1}{2}$ of the optimal throughput. Simulations are provided to illustrate the effectiveness of the proposed mechanism.

Index Terms

Heterogenous Network, Mechanism Design, Resource Allocation, Base Station Association, Approximation Bounds, Computational Complexity, Nash Equilibrium, Price of Anarchy

I. INTRODUCTION

We consider a downlink OFDMA network with multiple base stations (BSs) serving a group of users. The BSs operate on non-overlapping spectrum bands in frequency division duplex (FDD) mode. The objective is to find the best per-BS resource allocation strategy and the user-BS assignment to achieve spectral efficiency and load balancing across the networks. This problem is well motivated by many practical networks such as the multi-technology heterogenous networks (HetNet) [1], [2], the IEEE 802.22 Wireless Regional Area Network (WRAN) [3] or a Wi-Fi network with multiple access points [4]. For example, in the HetNet, multiple wireless access technologies such as Wi-Fi, LTE or WiMAX are available for the same region. These networks operate on different spectrum bands and all utilize OFDMA for downlink transmission. Integrating these different radio access technologies and making them available to the user devices can significantly increase the overall spectral efficiency as well as achieve load balancing across different networks. The mobile users can choose from one of the technologies/networks for association, and they can switch between different technologies/networks to avoid congestion (i.e., “vertical handoff” operation, see [1]). The user-network assignment and the per-network resource allocation need to be performed jointly to achieve optimal network-wide resource allocation.

There are three major challenges for optimal resource allocation in such networks.

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1) When operating in FDD mode, the network requires the users to measure and report the downlink channel states for efficient resource allocation. An untruthful user may incorrectly report this information for its own benefit. The possibilities of various forms of untruthfulness in user behaviors in wireless networks have been recently noted (see e.g., [5]–[8]). Reference [6] has discovered that certain commercial 802.11 devices are specially manipulated to trick the access points for better rates. Such manipulation can take the form of a non-uniform backoff procedure [6] or falsely reported traffic priorities [7]. As suggested in [5], in FDD cellular networks it is also possible to manipulate the devices’ channel feedback procedure, as the compliance testing is usually limited to a few standardized scenarios. It is projected that the users of future networks may have stronger ability (and incentives) for manipulating their devices. See [5] for detailed literature review in this direction. The presence of the untruthful users can significantly reduce the overall system performance and limit network access to truthful users.

2) Even assuming the users truthfully report their channel states, finding the global optimal resource allocation is still computationally intractable (this result will be shown in Section II).

3) There is no central entity to compute and enforce a desired user-network assignment and network-wide resource allocation [4]. Consequently, a good resource allocation scheme must possess the following features: i) it should provide efficient utilization of the spectrum; ii) it must be strategy-proof, i.e., it is in the users’ best interests to truthfully reveal their private information; iii) it is distributedly implementable, in the sense that both the BSs and the users can take part in the scheme with only local information and local computation.

A. Literature Review

The problem of finding an optimal resource allocation for OFDMA network when the user-BS assignment is fixed and there is complete information on channel states has been widely studied, e.g., [9]–[11]. Various algorithms are developed to solve the BS’s utility maximization problems by allocating the transmission power and the channels in optimal ways. The joint problem of BS assignment and resource allocation in OFDMA network has been analyzed under complete information and the ability to enforce decisions from a centralized standpoint, for example, [12], [13]. However, in many practical networks there are no entities capable of performing the centralized decision making. Another strand of the literature deals precisely with this case by using non-cooperative game theory [14]–[17]. Users selfishly compute their power control and cell site selection strategies to maximize their own utilities. With proper design of the utility functions, equilibrium solutions can be obtained in distributed fashion. However, complete information on the channel states and/or the utility functions is assumed. The overall efficiencies of the identified equilibrium solutions are not characterized.

There are many recent works that design mechanisms for resource allocation problems in networks with strategic users and/or incomplete information [5], [18], [19]. It is commonly assumed that resources are divisible and that there is a closed-form expression that describes the interdependency of users’ decisions. In contrast, in our problem the resource allocation decisions are mixed in nature, i.e., decisions such as user-BS assignment and channel assignment are discrete (not divisible), while the BSs’ power allocation decision is continuous (divisible). In addition, the interdependency in users’ decisions is only implicitly characterized as the solution to the optimal resource management problem at each BS. As a result, the problem considered in this paper does not adequately fit into any of the frameworks considered in the above cited papers. We mention that the recent work [5] considered an incomplete information setting similar to ours, in which the FDD network lacks the true channel states due to the false report by the users. The objective though is to design optimal user scheduling algorithms, which is different from the objective of the present paper.
Lower bounds of the efficiency of the Nash Equilibrium (NE), or the price of anarchy, have been analyzed for network resource allocation games. Reference [20] considered a routing game in which the inefficiency is due to the selfishness of the users. Reference [21] analyzed a network utility maximization problem in which the strategic behavior of the users leads to efficiency loss. For both of the above cases the optimal system level problems can be solved globally, whereas in our case the overall problem is already difficult to solve. In [22], Vetta discussed the lower bounds of the NEs for a family of non-cooperative games assuming a special structure of the users’ utility functions. Applications of this latter result in communication and sensor networks include [23] and [24]. However, these works use highly stylized utility functions so that the result in [22] can be directly used.

B. Contributions

The main contributions of our paper are summarized as follows.

• We show that solving the joint BS assignment and resource allocation problem, even with truthful users, is NP-hard. In most existing complexity results for resource management in wireless communications (e.g., [25], [26]), the hardness of the problem is mainly due to the possibility of strong interference among the users. In contrast, in our problem the hardness lies in its mixed (discrete and continuous) formulation.

• The complexity of solving the joint problem optimally prevents the implementation of any mechanism that is both efficient and strategy-proof. To ensure tractability, we design a novel mechanism that implements an approximately optimal strategy. In the proposed mechanism each user dynamically selects a BS to maximize its own utility, and the BSs implement the celebrated Vickrey-Clark-Groves mechanism (VCG) [27]–[29] to allocate resource under the current user-BS assignment profile.

• We show that our mechanism achieves at least $\frac{1}{2}$ of the optimal throughput. This result relies on a key observation that the per-BS optimal throughput admits certain submodular property. Importantly, the obtained efficiency bounds hold true with or without the presence of the untruthful users. To the best of our knowledge, there has been no reported work that characterizes the efficiency of the equilibrium solutions for the considered problem.

The rest of the paper is organized as follows. Section II formulates the problem and provides its complexity status. Section III and IV describe the mechanism for the resource management problem as well as its distributed implementation. Section V gives some extensions of the algorithm. Section VI provides simulation results. Section VII concludes the paper.

Notations: We use bold faced characters to denote vectors. We use $x[i]$ to denote the $i$th element of vector $x$. We use $x_{-i}$ to denote the vector $[x[1], \ldots, x[i-1], x[i+1], \ldots, x[N]]$. We use $[y, x_{-i}]$ to denote a vector $x$ with its $i$th element replaced by $y$. We use $\lor$ to denote componentwise maximization: $x \lor y \triangleq \{z[i] = \max\{x[i], y[i]\}, \forall i\}$. $N \setminus i$ defines a subset of $N$: $N \setminus i \triangleq \{j : j \in N, j \neq i\}$. The main notations used in this paper are listed in Table I.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a service area with a set $N \triangleq \{1, 2, \ldots, N\}$ of users served by a set $W \triangleq \{1, 2, \ldots, W\}$ of BSs (or networks). Each BS $w$ operates on a set of channels $K_w$, with the bandwidth of each channel equally set to be $\Delta f_w$. Let $K \triangleq \cup_{w \in W} K_w$ denote the set of all channels. Suppose any two channels do not overlap, i.e., $K_w \cap K_v = \emptyset$, $\forall w \neq v$. Such assumption is justified for example in the multi-technology HetNet or in the IEEE 802.22 cognitive radio Wireless Regional Area Network (WRAN) [3]. In the latter network, a particular geographical region may be served by multiple service providers (SPs), or by multiple Access Points (APs) installed by a single SP. When operating in the
“normal mode”, the APs/SPs that serve the same region indeed operate on non-overlapping portions of the available spectrum, by using proper spectrum etiquette protocols (see Section 6.22 in [3]).

Let \( \{ |h_i^k|^2 \}_{k \in \mathcal{K}_w} \) denote the channel gains of the set of channels from BS \( w \) to user \( i \); Let \( \{ n_i^k \}_{k \in \mathcal{K}} \) denote the set of measured noise powers at user \( i \) on different channels. Both the channel gains and the noise powers are considered as private information to the users, as in the FDD mode they are measured at the mobile devices and then fed back to the BSs.

Define a length \( N \) vector \( \mathbf{a} \) as the association profile in the network, with its \( i \)th element \( a[i] = w \) indicating that user \( i \) is associated to BS \( w \). Define \( \mathbf{a}_{\cdot -i} \triangleq [a[1], \cdots, a[i-1], a[i+1], \cdots a[N]] \) as an association profile in which user \( i \) drops out of the network. For each BS \( w \), denote the set of associated users as \( \mathcal{N}_w(\mathbf{a}) \triangleq \{ i : a[i] = w \} \), which is a function of \( \mathbf{a} \).

In a downlink OFDMA network, a BS \( w \in \mathcal{W} \) can transmit to a single user \( i \in \mathcal{N}_w(\mathbf{a}) \) on a given channel \( k \in \mathcal{K}_w \). Let \( \beta_w \triangleq \{ \beta^k_w \}_{k \in \mathcal{K}_w} \) be a feasible channel assignment scheme for BS \( w \), i.e., \( \beta^k_w = i \in \mathcal{N}_w(\mathbf{a}) \) means channel \( k \) is assigned to user \( i \). Let \( \mathbf{p}_w \triangleq \{ p^k_w \}_{k \in \mathcal{K}_w} \) be a feasible power allocation scheme for BS \( w \): \( \mathbf{p}_w \geq 0 \), \( \sum_{k \in \mathcal{K}_w} p^k_w \leq \bar{p}_w \), where \( \bar{p}_w \) is the power budget for BS \( w \). Let \( \beta \triangleq \{ \beta_w \}_{w \in \mathcal{W}} \) and \( \mathbf{p} \triangleq \{ \mathbf{p}_w \}_{w \in \mathcal{W}} \).

Let us define \( r_i(\beta, \mathbf{p}, \mathbf{a}) \) as the transmission rate that user \( i \) can obtain under the resource allocation scheme \( (\beta, \mathbf{p}, \mathbf{a}) \). With continuous rate adaptation, this rate can be expressed as:

\[
 r_i(\beta, \mathbf{p}, \mathbf{a}) = \frac{\sum_{k \in \mathcal{K}_{\mathbf{a}[i]}} |h_i^k|^2 p^k_w}{\tau n_i} \left( 1 + \frac{|h_i^k|^2 p^k_w}{\tau n_i} \mathbf{1}\{ \beta^k_w = i \} \right)
\]

(1)

where \( \mathbf{1}\{ \cdot \} \) is the indicator function; \( \tau \) is the capacity gap which is determined by the target Bit Error Rate (BER) as: \( \tau = \frac{-\ln(5\text{BER})}{1.5} \) (see [30]).

The objective of the resource allocation is to find the tuple \( (\beta, \mathbf{p}, \mathbf{a}) \) that achieves efficient spectrum utilization within each BS/network while balancing the loads across different BSs/networks. Mathematically, we formulate the overall resource allocation problem as follows

\[
 \max_{\mathbf{a}, \beta, \mathbf{p}} \sum_{w \in \mathcal{W}} \alpha_w \sum_{i \in \mathcal{N}_w(\mathbf{a})} r_i(\beta, \mathbf{p}, \mathbf{a}) \\
 \text{s.t. } \mathbf{a}[i] \in \mathcal{W}, \quad \forall \ i \in \mathcal{N}, \quad \beta^k_w \in \mathcal{N}_w(\mathbf{a}), \quad \forall \ k \in \mathcal{K}_w, \quad \forall \ w \in \mathcal{W}, \quad \mathbf{p}_w \geq 0, \quad \sum_{k \in \mathcal{K}_w} p^k_w \leq \bar{p}_w, \quad \forall \ w \in \mathcal{W}.
\]

TABLE I

| Notation | Description |
|----------|-------------|
| \( \mathcal{N} \) | The set of all users |
| \( \mathcal{K} \) | The set of all channels |
| \( |h_i^k|^2 \) | The channel gain of user \( i \) on channel \( k \) |
| \( p^k_w \) | The transmit power of BS \( w \) on channel \( k \) |
| \( \beta^k_w \) | The user assignment of BS \( w \) on channel \( k \) |
| \( \mathbf{a} \) | The association profile |
| \( r_i(\mathbf{a}) \) | The rate assigned to user \( i \) |
| \( R(\mathbf{a}) \) | The optimal system throughput under \( \mathbf{a} \) |
| \( \mathcal{U}_i(\mathbf{a}) \) | The utility of user \( i \) |
| \( T_i(\mathbf{a}) \) | The tax imposed on user \( i \) |

The load balancing property of this formulation is manifested by: 1) introducing the association as a decision variable for the users; 2) including the weighting factors \( \{ \alpha_w \geq 0 \}_{w=1}^W \) in the objective. The first factor enables the users to effectively avoid congestion by switching to light-loaded BSs in
a timely fashion, while the second factor allows the network operator to further shift the traffic to the BSs with larger weights.

A. The BSs’ Resource Management: Optimal Channel Assignment and Power Allocation

We first describe each BS’s optimal resource management strategy. Let us assume that each BS $w$ has perfect knowledge of downlink normalized channel states of its associated users $c_w \triangleq \{h_{kw}^k\}_{i \in N_w(a), k \in C_w}$ (this assumption will be relaxed later).

First consider a simple case where an equal power allocation strategy is used, that is: $\tilde{p}_{kw}^k = \frac{\tilde{p}_w}{|K_w|}, \forall k \in C_w$. Each BS $w$ then optimizes its throughput by picking a suitable user to serve on each channel. Mathematically, it solves the following channel assignment (CA) problem

$$\max_{\beta_{kw}} \alpha_w \sum_{i \in N_w(a)} r_i(\beta, p, a) \quad \text{(CA)}$$

s.t. $\beta_{kw} \in N_w(a), \forall k \in C_w$.

The optimal solution to this problem is to assign each channel to the best user [9]:

$$(\beta_{kw})^* = i^*, \text{ where } i^* \in \arg \max_{i \in N_w(a)} \frac{|h_{ki}|^2}{\tau n_k^i}. \quad (2)$$

On the other hand, if the BSs can optimize both its channel assignment and power allocation, then a BS $w$ solves the following channel assignment and power allocation (CAPA) problem:

$$\max_{\beta_{kw}, p_{kw}} \alpha_w \sum_{i \in N_w(a)} r_i(\beta, p, a) \quad \text{(CAPA)}$$

s.t. $\sum_{k \in C_w} p_{kw}^k \leq \tilde{p}_w, p_{kw}^k \geq 0, \beta_{kw} \in N_w(a), \forall k \in C_w$.

The optimal solution to this problem can be written in closed form [11]:

$$(\beta_{kw})^* = i^*, \text{ where } i^* \in \arg \max_{i \in N_w(a)} \frac{|h_{ki}|^2}{\tau n_k^i},$$

$$\left( p_{kw}^k \right)^* = \left[ \lambda - \frac{r_n^w}{\tau n_k^i} \right]^+, \lambda \left( \sum_{k \in C_w} \left( p_{kw}^k \right)^* - \tilde{p}_w \right) = 0 \quad (3)$$

where $\lambda \geq 0$ is the dual variable associated with the power budget constraint. We note that the power constraint is binding at the optimal solution: $\sum_{k \in C_w} (p_{kw}^k)^* = \tilde{p}_w$.

With some abuse of notations, we use $r_i(a)$ to denote the optimal rate for user $i$ obtained by using either the CA or CAPA strategy (the actual strategy used will be indicated using a superscript CA or CAPA when necessary). When $a$ is fixed, we denote the weighted optimal throughput of BS $w$ by: $R_w(a) \triangleq \alpha_w \sum_{i \in N_w(a)} r_i(a)$.

B. The Overall Resource Management Problem: Complexity Status

In this subsection, we investigate the complexity status of the throughput optimization problem (SYS). A tuple $(\beta^*, p^*, a^*)$ is an optimal solution of the problem (SYS) only if each BS uses the CAPA strategy. Although finding the CAPA solution is easy when the user-BS association is fixed, the problem turns out to be intractable when the association becomes an optimization variable. The proof of this theorem is provided in the Appendix.

**Theorem 1:** Finding the optimal solution to the problem (SYS) is strongly NP-hard.

When each BS is restricted to allocate resource by channel assignment only, then the CA strategy is optimal for the per-BS problem. In this case, the problem (SYS) is still intractable.
Corollary 1: When the CA strategy is used for resource allocation within each BS, finding the best BS association that maximizes the system throughput is strongly NP-hard.

The above results establish the complexity status for the problem (SYS) in a general network configuration. In a special case where each BS operates on a single channel, the problem is equivalent to a maximum weighted matching problem, which is efficiently solvable.

Corollary 2: When $|K_w| = 1$, $\forall w \in W$, problem (SYS) is polynomial time solvable.

III. MECHANISM DESIGN FOR JOINT BS ASSOCIATION AND RESOURCE ALLOCATION

The previous section analyzes the per-BS and the overall resource allocation problem assuming complete information at the network side. However, in an FDD network, there is an intrinsic asymmetry in the available information at the BSs and at the users, as the downlink channel and the noise powers are measured by the users. Strategic (selfish) users can exploit such asymmetry of information for their own benefit by tampering with the devices if necessary [5]. We now provide a simple illustration of the potential inefficiency caused by the manipulation of channel state information.

Example 1: Consider the network consisting of 1 BS and 2 users with 3 channels. Let the noise power $n_{ik} = 1$ for all $i, k$, and let $\tau = 1$. Assume that the BS has a total power of 3. The channel gains are listed in Table II.

| Channel Gains for Example 1. |
|-----------------------------|
| $|h_1^1|^2$ | $|h_1^2|^2$ | $|h_1^3|^2$ |
| user 1 ($i = 1$) | 2 | 2 | 1 |
| user 2 ($i = 2$) | 0.5 | 0.5 | 2 |

When all the users report truthfully, and when the CA strategy is used, user 1 will be scheduled on channel 1 and 2, while user 2 will be scheduled on channel 3. A throughput of $3 \log(1 + 2) \approx 3.29$ nats/s can be obtained. When user 1 remains truthful but user 2 becomes selfish, and it falsely reports its channels as $(3, 3, 2)$ (instead of $(0.5, 0.5, 2)$), the BS will assign all the channels to user 2. After the channel assignment, the actual rate that user 2 obtains still depends on its true channels. Thus a throughput of $2 \log(1 + 0.5) + \log(1 + 2) \approx 1.91$ nats/s will be obtained, which is only about 58% of the optimal system throughput. In contrast, user 2’s untruthful behavior leads to its own rate increase of over 70%, at the expense of starving user 1.

A. The VCG mechanism

The optimization of system performance when strategic users have private information can be formulated as a mechanism design problem. Assuming users have quasilinear utility, a system of incentives (interference taxes) may be put in place in order to align individual users’ preferences with the goal of optimizing system performance. The goal therefore is to find the interference taxes that support the implementation of efficient resource allocation in dominant strategies, i.e. for each user, the truthful revelation of channel state information is optimal regardless of the information reported by all other users. The search for mechanisms is typically restricted to the class of direct mechanisms in which users report their private information to a third-party, which in turn allocates resources and implements a system of incentives via taxes.

1 Such rate can be achieved via the use of rateless codes. We refer the readers to [5] Section II for detailed explanation of achieving such rate when BSs do not have perfect knowledge of the actual channel.
The celebrated VCG mechanism achieves this goal by having users report their privately held information on channel states to a central controller (CU), who computes the globally optimal solution of (SYS) given the reported information. The CU then assigns the users to the BSs and a given rate according to the solution of the optimal solution of (SYS). Each user, when attempting to manipulate the allocation of resources by misreporting channel state information, is penalized for the deterioration of system performance for all other users. This is the basis for the VCG mechanism being strategy-proof, i.e. for each individual user, truthful revelation of channel state information is optimal regardless of the information reported by all other users. It should be also emphasized here that any other direct and strategy-proof mechanism implementing the solution to (SYS) is an instance of the VCG mechanism with interference taxes modified by a constant (see [31 Corollary 5.1]). Unfortunately, in the previous section we showed that finding the the global optimal solution of (SYS) is an NP-hard problem. Thus, a computationally tractable direct mechanism cannot be both strategy-proof and efficient.

Our strategy for designing a computationally tractable mechanism is to relax the requirement of optimality so that an approximately optimal solution of (SYS) can be implemented in dominant strategies. In the mechanism proposed below, tractability is achieved by: (i) decentralizing the resource allocation decisions to each BS and implementing the VCG mechanism in a per BS basis (section III-B below); (ii) allowing the users to dynamically adjust their choices of association (section III-C below).

B. Implementing VCG at each BS with Fixed Association

We formally describe the implementation of the VCG mechanism for given user-BS association profile $a$. Recall that optimal per-BS strategies were described in Section II-A.

Define the normalized channels as $h_{i,w} \triangleq \{ k^i_{v} \}_{k \in K_w}^i$; $h_{i-w} \triangleq \{ h_{j,w} \}_{j \in N_a \setminus i}^i$. Let $h_w \triangleq [h_{i,w}, h_{i-w}]$. Define user $i$’s reported normalized channels as $h_{i,w}$. Define $h_{i-w}$ and $h_w$ similarly. When we take untruthfulness into consideration, a user $i$’s rate depends on the following two terms: 1) the reported normalized channel, denoted as $h_w$, by which the BS makes the resource allocation decision; 2) the actual normalized channel $h_{i,w}$, by which user $i$ experiences the actual rate. We signify such dependencies by using $r_i(a; h_{i,w}, h_w)$ to denote user $i$’s rate. If the information reported by the users is $\hat{h}_w$, a tax $T_i(a; \hat{h}_w)$ will be levied upon user $i$, and its net utility is

$$U_i(a; h_{i,w}, \hat{h}_{-i}) = \alpha_w r_i(a; h_{i,w}, h_w) - T_i(a; \hat{h}_w).$$

The tax assessed on user $i$ is computed based on the reported channels. It is given as the total rate improvement that the set of remaining users $N_a \setminus i$ can obtain if user $i$ leaves BS $w$:

$$T_i(a; \hat{h}_w) \triangleq \sum_{\substack{j \in N_a \setminus i \atop \text{optimal throughput when } i \text{ drops out}}} \alpha_w r_j(a; \hat{h}_{j,w}, \hat{h}_{-i,w}) - \sum_{\substack{j \in N_a \setminus i \atop \text{optimal throughput of other users when } i \text{ is present}}} \alpha_w r_j(a; h_{j,w}, h_w).$$

The tax expressed in (5) ensures that each user has an incentives to act truthfully. To see why this is note that when all the users truthfully report their channels, BS $w$ can maximize its throughput. For example, the optimal throughput is better than that achieved when user $i$ reports untruthfully:

$$r_i(a; h_{i,w}, [\hat{h}_{i,w}, h_{i-w}]) + \sum_{j \in N_a \setminus i} r_j(a; h_{j,w}, h_{i-w}) \leq \sum_{j \in N_a \setminus i} r_j(a; h_{j,w}, h_w)$$

(6)

From this property we can derive the following key inequality

$$r_i(a; h_{i,w}, \hat{h}_w) + \sum_{j \in N_a \setminus i} r_j(a; \hat{h}_{j,w}, \hat{h}_w) \leq \sum_{j \in N_a \setminus i} r_j(a; h_{j,w}, \hat{h}_w) + \sum_{j \in N_a \setminus i} r_j(a; h_{j,w}, [\hat{h}_{i,w}, h_{i-w}]).$$

(7)
The sum of the two terms in the right hand side of (7) represents the sum rate achieved by all the users in BS \( w \) assuming the actual channels are \( \{h_{i,w}, \hat{h}_{i,w}\} \), and the users also truthfully report \( \{h_{i,w}, \hat{h}_{i,w}\} \). Consequently, this inequality follows from the fact that the BS’s resource management strategy maximizes the sum rate when the channels are truthfully reported.

Now suppose user \( i \) reports untruthfully (i.e., \( h_{i,w} \neq \hat{h}_{i,w} \)), then we have

\[
U_i(a; h_{i,w}, \hat{h}_{i,w}) \\
\overset{(a)}{=} r_i(a; h_{i,w}, \hat{h}_{i,w}) - \left( \sum_{j \in \mathcal{N}_w(a \setminus i)} r_j(a_{-i}; \hat{h}_{j,w}, \hat{h}_{i,w}) - \sum_{j \in \mathcal{N}_w(a) \setminus i} r_j(a; \hat{h}_{j,w}, \hat{h}_{i,w}) \right) \\
\overset{(b)}{\leq} r_i(a; h_{i,w}, [h_{i,w}, \hat{h}_{i,w}]) + \sum_{j \in \mathcal{N}_w(a \setminus i)} r_j(a; \hat{h}_{j,w}, [h_{i,w}, \hat{h}_{i,w}]) - \sum_{j \in \mathcal{N}_w(a \setminus i)} r_j(a_{-i}; \hat{h}_{j,w}, \hat{h}_{i,w}) \\
\overset{(c)}{=} U_i(a; h_{i,w}, [h_{i,w}, \hat{h}_{i,w}])
\]

where \((a)\) and \((c)\) are from the definitions of the utility in (4); \((b)\) is from (7). We have then established the desired result.

In the reminder of this paper, we will assume that the VCG mechanism is implemented at each BS. Thus, we will simply write \( r_i(a) \) instead of \( r_i(a; h_{i,w}, \hat{h}_{i,w}) \). The user \( i \)'s tax term [5] and utility term [4] can be simplified as (assuming \( a[i] = w \))

\[
T_i(a) \overset{\Delta}{=} \sum_{j \in \mathcal{N}_w(a \setminus i)} \alpha_w r_j(a_{-i}) - \sum_{j \in \mathcal{N}_w(a) \setminus i} \alpha_w r_j(a) \quad \quad (8)
\]

\[
U_i(a) \overset{\Delta}{=} \alpha_w r_i(a) - T_i(a) = \sum_{j \in \mathcal{N}_w(a)} \alpha_w r_j(a) - \sum_{j \in \mathcal{N}_w(a \setminus i)} \alpha_w r_j(a_{-i}). \quad \quad (9)
\]

In summary, by using the VCG mechanism within each BS, all the users will act truthfully, which in turn allows the BSs to optimally implement their resource allocation strategies. It is important to note here, that even in the ideal scenario where all the users behave truthfully, the tax and utility function defined in (8) and (9) are still extremely useful. As will be seen in the subsequent sections, they lead to simple and efficient network-wide resource allocation.

C. The User-BS Association Game

Suppose users are allowed to autonomously select which BS to connect to. Assuming each BS implements a VCG mechanism, we are left with a user-BS association game. We will occasionally use the superscripts CA or CAPA to specify the strategies used by the BSs. Let us define a non-cooperative BS association game as: \( \mathcal{G} \overset{\Delta}{=} \{N, \{\chi_i\}_{i \in N}, \{U_i(\cdot)\}_{i \in N}\} \), where \( \chi_i = \mathcal{W} \) is the strategy space of user \( i \); \( U_i(\cdot) \) is the utility of user \( i \) as defined in (9).

Interestingly, unlike most conventional games, in game \( \mathcal{G} \), the interdependencies of the users’ strategies are only implicitly given. For example, suppose \( a[i] = w, a[j] = q \). In order to assess the impact of user \( i \)'s change of association from BS \( w \) to BS \( q \) on user \( j \)'s utility, BS \( q \)'s resource allocation problem (either CA or CAPA) needs to be solved. There is no closed-form expression governing the users’ interdependencies. This unique property of the game makes our subsequent analysis, particularly the efficiency of the NE of game \( \mathcal{G} \), very involved.

We first characterize the utility function \( U_i(a) \) and the tax function \( T_i(a) \).

**Proposition 1:** When all the BSs use either the CA or the CAPA strategy, \( T_i(a) \geq 0, \forall i \in N \). Moreover, the users’ utility functions are bounded: \( 0 \leq U_i(a) \leq \alpha_{a[i]} r_i(a) \).

**Proof:** For notational simplicity, let \( w = a[i] \) be the association of user \( i \). Suppose that the BSs use the CA strategy. Observe that as each channel vacated by user \( i \)'s departure will be re-assigned to some user \( j \in \mathcal{N}_w(a) \setminus i \), and the assignment of all the other channels remains the same. Consequently, the
Theorem 2 asserts that the efficiency of the NEs. Central to the derivation of such lowerbound is certain submodular property of the optimal system weighted throughput. In the following, we further provide a lower bound for the CA problem (cf. (2)) implies:

Rearranging terms, and plugging in the definition of tax in (8), we have:

\[
\alpha_{i}^{\text{CA}} \geq \sum_{j \in N_{w}(a_{-i})} \alpha_{w} r_{j}^{\text{CA}} (a_{-i}) \leq \sum_{j \in N_{w}(a)} \alpha_{w} r_{j}^{\text{CA}} (a) + \alpha_{w} r_{i}^{\text{CA}} (a).
\]

D. Characterization of Nash Equilibria

In this subsection we present a series of results characterizing the pure NEs for game \( \mathcal{G} \).

Example 2 illustrates that it is the interference tax imposed by the BSs that ensures the existence of the pure NE for game \( \mathcal{G} \). In fact, such tax also guarantees the efficiency of the outcome of the game. Theorem 2 asserts that the maximum weighted throughput achievable by all the NEs is the same as the optimal system weighted throughput. In the following, we further provide a lower bound for the efficiency of the NEs. Central to the derivation of such lower bound is certain submodular property of the

| TABLE III |
| --- |
| **CHANNEL GAINS FOR EXAMPLE 2: CA CASE.** |
| CA case | \( |k_{i}^{2}| \) | \( |h_{i}^{2}| \) | \( |h_{i}^{2}| \) | \( |h_{i}^{2}| \) |
| i=1 | 2 | 0.1 | 2.2 | 0.1 |
| i=2 | 0.5 | 2.5 | 0.1 | 2.6 |
| i=3 | 0.1 | 2.4 | 2.3 | 0.2 |

Example 2 illustrates that it is the interference tax imposed by the BSs that ensures the existence of the pure NE for game \( \mathcal{G} \). In fact, such tax also guarantees the efficiency of the outcome of the game. Theorem 2 asserts that the maximum weighted throughput achievable by all the NEs is the same as the optimal system weighted throughput. In the following, we further provide a lower bound for the efficiency of the NEs. Central to the derivation of such lower bound is certain submodular property of the
per-BS throughput function $R_w(\cdot)$. Note that $R_w(a)$ depends on the association profile $a$ only through the set of associated users $N_w(a)$. We can then rewrite $R_w(a)$ as $R_w(N_w(a))$, which is expressed as a function of the set of associated users. Then we say that $R_w(\cdot)$ is submodular if the following is true

$$R_w(G \cup \{i\}) - R_w(G) \leq R_w(M \cup \{i\}) - R_w(M), \quad \forall \, i \in N, \quad \forall \, M \subseteq G \subseteq N.$$  

(13)

The submodularity implies that there is a marginal decrease of throughput when the total number of associated users increases. In [22], Tse and Hanly have shown that for a fixed power allocation policy without the total power constraint, the capacity of a fading multiple access channel is a submodular function. However, in our case showing the submodularity of the throughput $R_w(\cdot)$ is much more involved, as our resource allocation is the solution to the underlying optimization problems, hence it is dynamic with respect to the set of associated users.

Once the submodularity property is shown, we can utilize a result from Vetta [22] to obtain the desired lower bound. In particular, reference [22] introduces the notion of valid-utility games, for which lower bounds for the efficiency of the NE is $\frac{1}{2}$. We will show that our BS selection game $G$ belongs to the family of valid-utility games.

**Theorem 3:** The weighted system throughput achieved in any NE of the game $G$ must be at least half of that achieved under the optimal user-BS assignment.

**Proof:** It is easy to check that $R_w(\cdot)$ has a monotonicity property: $R_w(M) \leq R_w(G)$, $\forall \, M \subseteq G$. We then claim that $R_w(\cdot)$ satisfies (13). We only give proof for the (more difficult) CAPA case, the CA case is a straightforward extension. For simplicity of notations, we let BS $w$ operate on all channels $K$, set $n_j^k = 1$ for all $j, k$, and let $a_{w} = 1$.

Fix two sets $M, G$ with $M \subseteq G$, fix an arbitrary user $i$ with arbitrary channel gains. Define three vectors $g, m, h \in \mathbb{R}_+^K$, with their elements given as

$$g[k] = \max_{j \in K} |h_j^k|^2, \quad m[k] = \max_{j \in M} |h_j^k|^2, \quad h[k] = |h_i^k|^2.$$  

(14)

Note $g$ and $m$ represent the best channel gain on each channel for the set of users $G$ and $M$, respectively. From the fact that $M \subseteq G$, we have that $m \leq g$. Note that the throughput obtained by BS $w$ using the CAPA strategy is dependent on the set of associated users only through the best channel vector. As a result, we can also express $R_w(G)$ as $R_w(g)$, and $R_w(G \cup \{i\})$ as $R_w(g \lor h)$. In this notation, the submodularity property (13) is equivalent to

$$R_w(g \lor h) - R_w(g) \leq R(m \lor h) - R_w(m), \quad \forall \, h \geq 0, \quad g \geq m \geq 0.$$  

(15)

In the same token, the monotonicity of $R_w(\cdot)$ can be expressed as

$$R_w(g) \geq R_w(m), \forall \, g \geq m \geq 0.$$  

(16)
We then present a sufficient condition for (15) which is easier to verify. Let $e_k$ be a $K \times 1$ unit vector with its $k^{th}$ element being 1. Write $h = \sum_{k=1}^{K} e_k h[k]$. Then we have

$$R_w(g \lor h) - R_w(g) = \left[ R_w(g \lor \sum_{k=1}^{K} e_k h[k]) - R_w(g \lor \sum_{k=1}^{K-1} e_k h[k]) \right] \cdots + \left[ R_w(g \lor e_1 h[1]) - R_w(g) \right]$$

$$R_w(m \lor h) - R_w(m) = \left[ R_w(m \lor \sum_{k=1}^{K} e_k h[k]) - R_w(m \lor \sum_{k=1}^{K-1} e_k h[k]) \right] \cdots + \left[ R_w(m \lor e_1 h[1]) - R_w(m) \right].$$

In order for (15) to be true, it is sufficient that for all $k \in K$, the following is true

$$R_w(g \lor e_k h[k]) - R_w(g) \leq R_w(m \lor e_k h[k]) - R_w(m), \quad \forall \ h \geq 0, \ g \geq m \geq 0. \quad (17)$$

Condition (17) allows us to verify the submodular condition on a channel by channel basis. Partition the set $K$ into two sets: $Q = \{k | m[k] = g[k]\}$, $\overline{Q} = \{k | m[k] < g[k]\}$. We can show that (17) is true for all $k \in Q$ and $k \in \overline{Q}$. The proof for this result is given in the Appendix.

To this point we have shown that $R_w(\cdot)$ is submodular and monotone. From Proposition 11 we have that $\sum_{w} \sum_{i \in N_c(a)} U_i(a) \leq \sum_{w} R_w(a)$. Additionally, the definition of $U_i(\cdot)$ ensures that it is equal to the difference of the system throughput with and without user $i$ (cf. (9)). As a result, game $G$ is a valid utility game, and we can apply [22, Theorem 3] to deduce that any NE of the game achieves at least a half of the optimal weighted throughput. This completes the proof.

We emphasize that all the results derived in Section III-C and III-D hold true regardless of the presence of the untruthful users, as long as the BSs implement the taxation for each user as specified in (8) and (9). This is because the association game $G$ is built upon the assumption that the BSs use the VCG mechanism, and that the users are always truthful.

IV. A DYNAMIC MECHANISM

In this section we introduce a mechanism that allows the users and the BSs to jointly compute a NE of the game $G$, which is a high quality solution for the joint BS selection and resource allocation problem. All the results in this section are applicable to both games $G^{CA}$ and $G^{CAPA}$. Suppose each user maintains a length $M$ memory that operates in a first in first out fashion. Each user’s memory is used to store its best associations in the last $M$ iterations.

We first briefly describe the main steps of the proposed mechanism. It alternates between a BS optimization step and a user optimization step. When it is the BSs’ turn to act, based on the current set of associated users, each of the BSs optimally allocates the resources in its own cell using the VCG mechanism (cf. Section II-A and II-B). From the system perspective, in this step, the tuple $(\beta, p)$ is updated while holding the association fixed. When it is the users’ turn to act, each of them first computes its current best BS (in terms of achieved individual utility) according to the current association profile. It then pushes the best BS into its memory, and randomly samples one BS from its memory for actual association. As we will see later, in this step, $a$ is updated while fixing $(\beta, p)$. The sampling step is the key to establish the convergence of the proposed mechanism. The proposed mechanism is detailed in Table VI, where the superscript $(t)$ denotes the iteration number.

One benefit of the proposed mechanism is that it allows the users to update at the same time without explicit coordination of their update sequences. A possible alternative is a sequential implementation that allows a single user to update in each iteration. However such scheme is undesirable as it requires significant coordination efforts among all the users/BSs.

An important feature of the mechanism is that each of its steps can be implemented in a distributed fashion. The following two assumptions on the network are needed for such purpose:
TABLE VI
THE PROPOSED MECHANISM

| S1) Initialization: Let t=0, let the users choose their nearest BSs. |
| S2) BS Optimization: Based on current \( \mathbf{a}^{(t)} \), each BS implements a VCG mechanism. |
| S3) User Optimization: For each user \( i \in \mathcal{N} \):
  | S3-1) Compute the Best BS: Compute \( BR_i(\mathbf{a}^{(t)}) \); if \( BR_i(\mathbf{a}^{(t)}) \neq \emptyset \), randomly select \( w_i^{*(t)} \in BR_i(\mathbf{a}^{(t)}) \); otherwise, set \( w_i^{*(t)} = \mathbf{a}^{(t)}[i] \)
  | S3-2) Update Memory: Shift \( w_i^{*(t)} \) into the front of the memory; if \( t \geq M \), shift \( w_i^{*(t-M)} \) out from the end of the memory
  | S3-3) Determine the Next BS Association: Uniformly sample the user \( i \)'s memory; obtain a BS index as \( \mathbf{a}^{(t+1)}[i] \)
| S4) Continue: If \( \mathbf{a}^{(t+1)} = \mathbf{a}^{(t+1-m)} \) for \( m = 1, \ldots, M \), stop. Otherwise, let \( t=t+1 \), go to S2). |

1) Local channel information is known by each BS. That is, each BS \( w \) has the knowledge of \( \{h_{k_i}^{[2]}\}_{k \in \mathcal{K}_w, i \in \mathcal{N}_w} \), but not the channels related to other BSs. Note that in an FDD system, this information is obtained via user feedback, the truthfulness of which is ensured by implementing the per-BS VCG mechanism;

2) Each BS has a feedback channel to all the potential users.

Under the above assumptions, the mechanism can be implemented distributedly. In the BSs’ optimization step, the BSs compute the taxes and perform their per-cell resource allocation (cf. Section II-A and II-B). They are not required to have the knowledge of the operational conditions or channel states related to other BSs. In the users’ optimization step, to compute the set \( BR_i(\mathbf{a}^{(t)}) \), each user \( i \) needs to know \( U_i([w, \mathbf{a}^{(t)}_{-i}]), \forall w \) (cf. (12)). Each of these quantities can be expressed as

\[
U_i([w, \mathbf{a}^{(t)}_{-i}]) = \alpha_w r_i([w, \mathbf{a}^{(t)}_{-i}]) - T_i([w, \mathbf{a}^{(t)}_{-i}]).
\]  

Both terms in (18) can be computed by BS \( w \) and fed back to user \( i \). To compute the first term in (18), BS \( w \) solves its resource allocation problem with the set of users \( \mathcal{N}_w([w, \mathbf{a}^{(t)}_{-i}]) \). To obtain the second term in (18), BS \( w \) solves its per-cell problem with and without user \( i \) (cf. (9)). Again only local channel states are needed. It is important to note that to carry out the proposed mechanism, the users do not need to know the “summary” information (the tax and the rate estimates) from the BSs.

In practice, the users may only switch to a new BS if it offers significantly higher utility, because each of such switch induces costs such as message passing. Let us use \( c_i \) to denote such cost for user \( i \). When switching costs are included into the decision process, in each iteration of the mechanism, \( w^* \in BR_i(\mathbf{a}^{(t)}) \) implies \( U_i([w^*, \mathbf{a}^{(t)}_{-i}]) > U_i(\mathbf{a}^{(t)}) + c_i \). This modification could reduce the number of iterations needed for convergence (since the users are now less willing to change association), but could also reduce the system throughput achieved by the identified NE.

The convergence property of the proposed mechanism is provided in the following theorem. The details of the proof is relegated to the Appendix.

**Theorem 4:** When choosing \( M \geq N \), the BS association mechanism produces a sequence \( \{\mathbf{a}^{(t)}\}_{t=1}^{\infty} \) that converges to a NE of game \( \mathcal{G} \) with probability 1 (w.p.1).

V. DISCUSSIONS AND EXTENSIONS

To this point we have assumed that the BSs are interested in maximizing the per-BS throughput. Such assumption allows the BSs to have closed-form solution to their optimization problems, and it leads to important properties such as submodularity of the throughput functions. Our work can be extended to cases where the BSs allocate resources using general utility functions as well.
First we mention that all the previous properties of the mechanism can be straightforwardly generalized to the case where each BS \( w \) aims to maximize a weighted throughput of the form \( \sum_{i \in N_w} \gamma_i r_i \). The set of weights \( \{ \gamma_i \geq 0 \}_{i=1}^{N} \) can be adjusted adaptively by the BSs over time to ensure fairness among the users’ time-averaged transmission rates (see e.g., [33]).

Consider an alternative case in which BS \( w \) is interested in finding the best channel assignment to achieve the proportional fairness (PF). The per-BS problem is then given by [10]

\[
\max_{\beta_w} \sum_{i \in N_w(a)} \alpha_w \log (r_i(\beta, p, a)) \quad \text{(CA-PF)}
\]

s.t. \( \beta_w \in N_w(a), \forall k \in K_w \).

This problem generally does not admit a closed-form solution, and the BS needs to perform numerical search to obtain the optimal solutions (see [10] for a set of efficient search algorithms). Let us use \( r_i^\text{PF}(a) \) to denote the resulting transmission rate for user \( i \). Following (9), each user \( i \) in cell \( w \) has the following utility \( U_i^\text{PF}(a) \equiv \alpha_w \log(r_i^\text{PF}(a)) - T_i^\text{PF}(a) \), where \( T_i^\text{PF}(a) \equiv \alpha_w \sum_{j \in N_w(a) \setminus i} \log(r_j^\text{PF}(a-i)) - \alpha_w \sum_{j \in N_w(a)} \log(r_j^\text{PF}(a)) \). We can now construct a PF association game \( G^\text{PF} \) with each user’s utility function given as \( U_i^\text{PF}(a) \). Similarly as in Theorem 2, we can show that the optimal association profile \( \mathbf{a}^* = \arg \max_a \sum_{i \in W} \alpha_i \log(r_i^\text{PF}(a)) \) must be a NE of this game. Our proposed mechanism can be applied for finding the NE of this game.

In general, most of the properties of the mechanism (except for the efficiency lower bounds of the NE) can be extended to networks with the following properties: 1) The BSs operate independently (using orthogonal time-frequency resources); 2) The utility function chosen by each BS is separable among the associated users; 3) The optimal solutions of the per-BS optimization problem can be found. However, it is not clear at this point whether the monotonicity and the submodularity conditions can be carried over to the case of general utility functions. Showing such properties under general utility functions is left as a future work.

VI. Simulations

In this section, we present simulation results to demonstrate the performance of the proposed algorithm. Both indoor and outdoor network scenarios are considered.

A. An Indoor Network Scenario

We have the following settings for this part of the simulation. Let us denote a \( 50m \times 50m \) indoor area as \( A \); denote the \( 25m \times 25m \) central area of \( A \) as \( C \); define the border of \( A \) as \( B \). Define the parameter \( 0 \leq D \leq 1 \) as the distribution factor of the users/BSs: 1) \( D \times 100\% \) of the users and BSs are randomly placed in \( A \); 2) the rest of the users are randomly placed in \( C \) and the rest of the BSs are randomly placed on \( B \). When \( D \) is small, the subset of BSs that are located at the center of the area become hotspots and are likely to be congested. See Fig. 11 for an illustration. Let \( d_{i,w} \) denote the distance between user \( i \) and BS \( w \). The channels between user \( i \) and BS \( w \), \( \{ h_{k,i,w} \}_{k \in K_w} \), are generated independently from the complex Gaussian distribution \( \mathcal{CN}(0, \sigma_i^2) \), with \( \sigma_i^2 = L_{i,w}/PL_{i,w} \). The random variable \( L_{i,w} \) models the shadowing effect, i.e., \( 10 \log 10(L_{i,w}) \sim \mathcal{N}(0,64) \) is a real Gaussian random variable. The variable \( PL_{i,w} \) is the pathloss between BS \( w \) and user \( i \). To model the pathloss in the indoor scenario, the office environment model [33] is used. The key simulation parameters are given in Table VII. The performance of the proposed algorithm will be compared with the algorithm that first assigns the users to their nearest BSs, and then optimally perform the per-BS resource allocation. Note that this algorithm separates the process of association and per-cell resource allocation, hence in most cases
TABLE VII
SIMULATION PARAMETERS FOR THE INDOOR NETWORK.

| Parameters          | Values                       | Parameters          | Values                       |
|---------------------|------------------------------|---------------------|------------------------------|
| $\bar{p}_w$         | 23 dBm                       | Total Bandwidth     | 80 MHz                       |
| Pass Loss (dB)      | $PL_{i,w} = PL(1) + 26 \log_{10}(\frac{d_{i,w}}{1}) + 14.1$ | $\alpha_w$         | 1                            |
| BER                 | $10^{-6}$                    | Length of Memory    | 10                           |
| Operating Frequency | 1.9 GHz                      | Noise Power         | $-100$ dBm/Hz                |

This gives degraded system performance. Throughout this subsection, the CAPA strategy will be adopted for per-BS resource allocation.

The first set of experiments evaluate the convergence performance of the proposed mechanism. Fig. 2 plots 3 realizations of the evolution of system throughput. This figure demonstrates the ability of the algorithm to “track” the equilibrium solutions. The algorithm takes a few iterations to converge to new equilibria when the following events occur at iteration 100: 1) 10 (randomly placed) new/old users enter/leave the system; 2) all of the users’ channel gains are re-generated (with the locations of the users and BSs unchanged).

In Fig. 3, we evaluate the averaged convergence time for the algorithm. We highlight its “tracking” ability by adding a number of new users and by randomly re-generating all the users’ channel gains after an equilibrium has been reached. The algorithm is able to track the equilibrium much faster than performing a complete restart. In Fig. 4, we plot the averaged convergence time of the algorithm with $N = 30$, $K = 512$. We observe that the convergence time is decreasing with $D$. This phenomenon is intuitive because when $D$ is close to 1, the event of congestion is less likely to happen as on average the communication load is evenly distributed among the BSs. On the contrary, when $D$ becomes small, those BSs located in the interior of the area are likely to be congested. Large portions of the users will then seek for alternative choices, which results in longer convergence time. Additionally, when taking the switching costs into consideration, the algorithm converges significantly faster.

The second set of experiments intend to evaluate the throughput performance of the proposed algorithm. We first investigate a relatively small network with 10 users, 64 channels and $1 - 4$ BSs, and compare the performance of the proposed algorithms to the global optimal solution of the problem (SYS) (obtained by an exhaustive search). The results are shown in Fig. 5. We see that the proposed algorithm, abbreviated as Distributed BS Association (DBSA), performs well with little throughput loss. In contrast, the nearest BS algorithm performs poorly.

We then evaluate the performance of the algorithm in larger networks with 30 users, up to 8 BSs and 512 channels. Fig. 6 shows the comparison of the averaged performance of the proposed algorithm and the nearest BS algorithm. Due to the prohibitive computation time required, we are unable to obtain the optimal system throughput in this case. We instead compute a (strict) upper bound of the maximum throughput assuming that the users can connect to multiple BSs simultaneously. We refer to this as the multiple-connectivity network. We also observe that when we take the switching costs into consideration ($c_i = 1$ Mbps for all $i$), there is a slight decrease in system throughput.

In Fig. 7, we show the distribution of the per-BS rates achieved by the proposed algorithm and the nearest BS algorithm. From the figure we see that the proposed algorithm is able to distribute the throughput to different BSs fairly, while the nearest BSs algorithm may result in severe unbalance of the BSs’ loads (some BSs may experience heavy traffic while the rest of the BSs may become idle).
B. An Outdoor Multicell Cellular Network Scenario

In this section we demonstrate the performance of the proposed algorithm in a multicell OFDMA cellular network. Standard cellular network parameters are used for the simulation, see Table VIII. Again frequency selective channels with a Rayleigh fading component and 8 dB log-normal fading component are simulated. Users are assumed to be distributed uniformly in the entire network. Throughout this subsection, the system level PF objective is optimized, thus the CA-PF strategy discussed in Section V is used for the per-BS resource allocation. The solution to the problem (CA-PF) is computed using Algorithm 1 in [10]. The main purpose of this experiment is to evaluate the performance of the proposed algorithm when some of the key assumptions guaranteeing the theoretical properties of the algorithm no longer hold true. Note that in order for the proposed algorithm to work in this network setting, inter-cell interference should be treated as noise. That is, user $i$’s noise power on channel $k$, $n^k_i$, should include both the environmental noise power and the inter-cell interference power.

| Parameters                     | Values                          |
|--------------------------------|---------------------------------|
| Cell layout                    | Hexagonal, 7 cells, 3 sectors per cell |
| BS-BS distance                 | 2.8 km                          |
| Frequency Reuse                | 1                               |
| $p_{w}$                        | 49 dBm                          |
| Pass Loss Model (dB)           | $PL_{i,w} = 128.1 + 36.7 \log_{10}(d_{i,w})$ |
| BER                            | $10^{-6}$                       |
| Total Bandwidth                | 10 MHz                          |
| Noise Power                    | $-169$ dBm/Hz                   |
| Multipath Time Delay Profile   | ITU-R M.1225 PedA               |
| Number of channel (FFT size)   | 64                              |
| Length of Individual Memory    | $M = 10$                        |

We first show the convergence of the algorithm. In the considered cellular network, different BSs transmit using the same spectrum bands. Consequently our theoretical analysis of the convergence is no longer valid. However, convergence is still observed empirically. See Table IX for the comparison of the convergence speed with and without the switching costs $\{c_i\}$.

We then demonstrate the throughput performance of the algorithm. We compare the proposed algorithm with the nearest BS algorithm and the “Greedy-0” algorithm proposed in [35], which is a centralized algorithm that finds a good user-BS association by successively perturbing the user-BS association locally. In Table X and Fig. 9 we see that the proposed algorithm compares favorably with the other algorithms both in terms of system throughput and fairness levels. Each entry in the table is obtained via an average of 200 randomly generated networks.

|                  | DBSA | DBSA $c_i = 0.1$ Mbps | DBSA $c_i = 0.5$ Mbps |
|------------------|------|-----------------------|-----------------------|
| $N = 10$         | 55   | 33                    | 21                    |
| $N = 30$         | 65   | 38                    | 25                    |
| $N = 50$         | 70   | 40                    | 23                    |

$^2$Most of the network parameters are taken from [33]. In the present work only single antenna systems are simulated.
TABLE X
COMPARISON OF THE SYSTEM THROUGHPUT OF DIFFERENT ALGORITHMS

|               | DBSA | DBSA (c_i = 0.1 Mbps) | DBSA (c_i = 0.5 Mbps) | Greedy-0 | Nearest |
|---------------|------|-----------------------|-----------------------|----------|---------|
| N=20          | 97.86 Mbps | 93.08 Mbps | 90.13 Mbps | 82.23 Mbps | 63.31 Mbps |
| N=40          | 117.9 Mbps | 115.1 Mbps | 109.3 Mbps | 105.7 Mbps | 89.00 Mbps |
| N=60          | 135.9 Mbps | 129.9 Mbps | 125.1 Mbps | 119.5 Mbps | 104.8 Mbps |

Additionally, we evaluate the performance of the algorithm when only noisy channel information is available. In particular, we consider the situation in which the normalized channel magnitude estimated by the users is subject to a zero mean estimation error \[ h_i^k \] \[ n_i^k \]. Let \( |\tilde{h}_i^k|^2 \triangleq \frac{|h_i^k|^2}{n_i^k} + \epsilon_i^k \) denote the estimation error. Suppose the estimated normalized channels are used by the BSs for resource allocation. To evaluate the performance loss due to the channel inaccuracy, we introduce a term called Channel Error Ratio (CER) to quantify the strength of the channel error: \( \text{CER}_i^k \triangleq 10 \log_{10} \left( \frac{|h_i^k|^2}{n_i^k \sigma_i^k} \right) \). In Fig. 10, we plot the system throughput performance with different values of the CER. We observe that the overall throughput degrades slightly with such inaccurate channel information.

**VII. CONCLUSION**

In this work, we studied a resource management problem in a multi-cell network with strategic/selfish users. We propose a novel mechanism that implements a strategy-proof and approximately optimal scheme in dominant strategies. Utilizing a key submodularity property of the per-BS throughput function, we characterized the efficiency of the proposed mechanism. As a future work, we will study the case in which there is limited (low rate) feedback from the users to the BSs. In this case feedback strategy needs to be designed in conjunction with the BSs’ and the users’ strategies. A new approximation ratio needs to be derived for this more practical scenario. We also plan to generalize the submodularity property to other utility functions and to the case of MIMO networks. Another interesting extension of the current work is to include the users that are hostile instead of non-cooperative. Strategies that are different from pricing are needed in this case to counter the untruthfulness induced by the hostility.

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**IX. APPENDIX**

A. Proof of Theorem 7

Without loss of generality, we consider the case where \( \alpha_w = 1, n_i^k = 1, \forall w, k, i \). This claim is proved based on a polynomial time transformation from 3-SAT problem, which is a known NP-complete problem. The 3-SAT problem is described as follows. Given \( Q \) disjunctive clauses \( C_1 \cdots, C_Q \) defined on \( M \) Boolean variables \( X_1, \cdots, X_M \), i.e., \( C_q = T_{q1} \vee T_{q2} \vee T_{q3} \) with \( T_{q} \in \{X_1, \cdots, X_M, \bar{X}_1, \cdots, \bar{X}_M\} \), the question is to check whether there exists a truth assignment for the Boolean variables such that all clauses are satisfied. Define \( \pi(C_q) \) as the set of terms contained in clause \( C_q \), and define \( \mathcal{I}(T) \) as the index of a term \( T \)’s corresponding variable, i.e., if \( C_m = \bar{X}_1 \lor \bar{X}_2 \lor X_4 \), then \( \pi(C_m) = \{\bar{X}_1, \bar{X}_2, X_4\} \), and \( \mathcal{I}(\bar{X}_1) = 1 \).

Given any instance of 3-SAT problem with \( Q \) clauses and \( M \) variables, we construct an instance of the multi-BS network with \( (2Q + 1)M \) users and \( 2M + Q \) BSs. For each variable \( X_m \), we do the
following: 1) associate with it two variable BSs $X_m, \bar{X}_m$; 2) associate with it $(2Q + 1)$ variable users \{\(x^1_m, \ldots, x^Q_m\), \(\bar{x}^1_m, \ldots, \bar{x}^Q_m\)\} and \(y_m\); 3) set the individual maximum power of the BSs $X_m, \bar{X}_m$ to be $Q$, respectively; 4) let each of the BSs $X_m, \bar{X}_m$ operate on $Q$ channels. For each clause $C_q$ we do the following: 1) associate with it a single BS $C_q$; 2) set the maximum power of BS $C_q$ to be 1; 3) let BS $C_q$ operate on a single channel. Denote the $k^{th}$ channel in the set $K_w$ as $\bar{K}_w(k)$. The channel gains between various users and BSs are given as follows.

$$h_{y_m}^k = \begin{cases} 3, & w = X_m \text{ or } w = \bar{X}_m, \forall k \in K_w \\ 0, & \text{otherwise}, \end{cases} \quad h_{\bar{y}_m}^k = \begin{cases} 2, & w = X_m, k = K_w(q) \\ 1, & w = C_q, X_m \in \pi(C_q), \forall k \in K_w \\ 0, & \text{otherwise}, \end{cases}$$

\begin{align}
|h_{y_m}^k|^2 &= \begin{cases} 3, & w = X_m \text{ or } w = \bar{X}_m, \forall k \in K_w \\ 0, & \text{otherwise}, \end{cases} \\
|h_{\bar{y}_m}^k|^2 &= \begin{cases} 2, & w = X_m, k = K_w(q) \\ 1, & w = C_q, \bar{X}_m \in \pi(C_q), \forall k \in K_w \\ 0, & \text{otherwise}. \end{cases}
\end{align}

(19)

For example, for a clause $C_q = X_1 \lor \bar{X}_2 \lor X_3$, the constructed network is shown in Fig. 11

In the following, for notational simplicity, for a term $T \in \pi(C_q)$, we will use the short hand notation $T$ to denote its corresponding variable BSs (instead of $X_{I(T)}$ or $\bar{X}_{I(T)}$), use the notation $y_{I(T)}$ and \{\(I_{I(T)}^1, \ldots, I_{I(T)}^Q\)\} for its corresponding variable users. Take an arbitrary clause $C_q$ and a term $T \in \pi(C_q)$. Two important observations can be made at this point. Both of them are the direct consequence of our network construction, and we omit their proofs due to space limitation.

**Observation 1:** If at the optimal solution, BS $T$ selects its corresponding variable user $y_{I(T)}$ for transmission, then the subset of variable users \{\(I_{I(T)}^1, \ldots, I_{I(T)}^Q\)\} will not be selected by BS $T$ at the optimal solution. A direct consequence of this observation is that the maximum throughput that the BS pair $T, \bar{T}$ can achieve is $2Q + Q \log 3$.

**Observation 2:** Assuming $T \in \pi(C_q)$, then the maximum throughput achievable by the BSs \{\(T, \bar{T}, C_q\)\} is $2Q + Q \log 3 + 1$. This throughput is achieved if and only if BS $T$ selects user $y_{I(T)}$, BS $\bar{T}$ selects users \{\(I_{I(T)}^1, \ldots, I_{I(T)}^Q\)\} and BS $C_q$ selects the user $I_{I(T)}^q$.

Our main claim is that the given 3-SAT instance is satisfiable if and only if the constructed network achieves a throughput of at least $Q + 2MQ + MQ \log 3$.

Suppose the 3-SAT problem is satisfiable. Then for each clause $C_q$ there is a term $T^*_q \in \pi(C_q)$ such that $T^*_q = 1$. Let the variable BS $T^*_q$ select user $y_{I(T^*_q)}$, let the variable BS $\bar{T}^*_q$ select all users \{\(I_{I(T^*_q)}^1, \ldots, I_{I(T^*_q)}^Q\)\}, and let the clause BS $q$ select the variable user $I_{I(T^*_q)}^q$. By Observation 2 the total throughput of variable BSs $T^*_q, \bar{T}^*_q$ and the clause BS $C_q$ is $2Q + Q \log 3 + 1$. As a result, the overall system throughput is $2MQ + MQ \log 3 + Q$.

Conversely, suppose a throughput of $2MQ + MQ \log 3 + Q$ is achieved. From Observation 1 we see that the maximum throughput for a variable BS pair $X_m, \bar{X}_m$ is $2Q + Q \log 3$. As we have $M$ variable BS pairs and $Q$ clause BSs, then each clause BS must achieve throughput 1 in order for the system to achieve the throughput $M(2Q + Q \log 3) + Q$. Observation 2 implies that this is only possible if for each clause BS $C_q$, there is at least one clause BS, say $T^*_q \in \pi(C_q)$ that transmits to the corresponding variable user $y_{I(T^*_q)}$. Set \{\(T^*_q\}_{q=1}^Q\} all to 1, then each clause $C_q$ will be satisfied. Consequently, the resulting 3-SAT problem is satisfied.

**B. Proof of Theorem 2**

We prove this theorem by contradiction. Suppose $a^* \in \arg \max_a R(a)$, but $a^*$ is not a pure NE. Then there must exist a user $i$ such that $BR_i(a^*) \neq \emptyset$. Choose $\bar{w} \in BR_i(a^*)$, and define a new association profile $\bar{a} = [\bar{w}, a^*_{-i}]$. Let $w^* = a^*[i]$. We show that user $i$'s unilateral change of association has the
same effect on its own utility as well as on the system throughput

\[ U_i(\bar{a}) - U_i(a^*) = \sum_{j \in N_{w'}(\bar{a})} \alpha_{w'} r_j(\bar{a}) - \sum_{j \in N_{w'}(\tilde{a}_{-i})} \alpha_{w'} r_j(\tilde{a}_{-i}) - \left( \sum_{j \in N_{w'}(a^*)} \alpha_{w'} r_j(a^*) - \sum_{j \in N_{w'}(\tilde{a}_{-i})} \alpha_{w'} r_j(\tilde{a}_{-i}) \right) \]

\[ \overset{(b)}{=} \left( \sum_{j \in N_{w'}(\bar{a})} \alpha_{w'} r_j(\bar{a}) + \sum_{j \in N_{w'}(\tilde{a}_{-i})} \alpha_{w'} r_j(\tilde{a}_{-i}) \right) - \left( \sum_{j \in N_{w'}(a^*)} \alpha_{w'} r_j(a^*) + \sum_{j \in N_{w'}(\tilde{a}_{-i})} \alpha_{w'} r_j(\tilde{a}_{-i}) \right) \]

\[ \overset{(c)}{=} \sum_{w \in W} \alpha_w (R_w(\bar{a}) - R_w(a^*)) = R(\bar{a}) - R(a^*) \]

(20)

where (a) is from the definition of the utility function (9); (b) is due to the fact that \( a^*_{-i} = \tilde{a}_{-i} \); (c) is due to \( N_{w'}(\tilde{a}_{-i}) = N_{w'}(\bar{a}), N_{w'}(\bar{a}_{-i}) = N_{w'}(a^*) \), and \( R_w(\bar{a}) = R_w(a^*), \forall w \neq \bar{a}, w^* \). From the assumption, user \( i \) prefers to switch to BS \( \bar{a} \), then \( U_i(\bar{a}) > U_i(a^*) \). This combined with (20) yields \( R(\bar{a}) > R(a^*) \), which is a contradiction to the optimality of \( a^* \). Then we have \( a^* \in \arg \max_a R(a) \) is a NE for game \( G \).

\[ \square \]

C. Proof of Theorem 3

We show that (17) is true for all \( k \in Q \) and \( k \in \overline{Q} \).

**Step 1** We first argue that for all \( k \in Q \), (17) is true. When \( h[k] \leq m[k] = g[k] \), (17) is trivially true as both sides of it evaluate to zero. We then focus on the case \( h[k] > m[k] = g[k] \).

Assuming \( h[k] > m[k] = g[k] \), we have that \( g + e_k h[k] = m + e_k h[k] = g + e_k \) for some constant \( c \geq 0 \). Thus, for all \( k \in Q \) and \( h[k] > m[k] = g[k] \), to show the inequality (17), it suffices to show the following *decreasing difference* property

\[ R_w(g + \delta \times e_k) - R_w(g) \leq R_w(m + \delta \times e_k) - R_w(m), \forall \delta \geq 0, \text{ and } g \geq m \geq 0. \]

(21)

From (39), we know that whenever the function \( R_w(x) \) is differentiable with respect to \( x[k] \), the decreasing difference property of (21) is equivalent to the following property

\[ \lim_{\delta \to 0} \frac{R_w(g + \delta \times e_k) - R_w(g)}{\delta} \leq R_w(m + \delta \times e_k) - R_w(m), \forall \delta \geq 0, \text{ and } g \geq m \geq 0. \]

(22)

In what follows, we prove that for any \( k \) with \( h[k] > m[k] = g[k] \), the limit in (22) exists and is non-positive. To this end, a closer look at the function \( R_w(\cdot) \) is necessary. Let \( p^k_w \) denote the power allocation for channel \( k \) when the best channel gain vector is \( g \), and let \( \lambda_g \) denote the corresponding dual variable. From the CAPA strategy, we have that \( \sum_{k=1}^{K} p^k_w = \sum_{k \in K_g} \lambda_g - \frac{1}{|g[k]|} = \bar{p}_w \), which implies that

\[ \lambda_g = \frac{1}{|K_g|} \left( \bar{p}_w + \sum_{k \in K_g} \frac{1}{g[k]} \right). \]

(23)

Using this expression as well as the expression for \( p^k_w \), we have that

\[ R_w(g) = |K_g| \log \left( \frac{\bar{p}_w + \sum_{k \in K_g} \frac{1}{g[k]}}{|K_g|} \right) + \sum_{k \in K_g} \log(g[k]) = |K_g| \log(\lambda_g) + \sum_{k \in K_g} \log(g[k]). \]

(24)

We argue that when \( m \leq g \), we must have \( \lambda_g \leq \lambda_m \). Otherwise, if \( \lambda_g > \lambda_m \), due to the fact that \( \frac{1}{|g[k]|} \leq \frac{1}{m[k]} \), we must have \( p^k_w > p^k_m, \forall k \in K_m \), which implies \( \sum_{k \in K_m} p^k_w > \sum_{k \in K_m} p^k_m = \bar{p}_w \), a violation.
of the total power constraint.

Take any channel \( k^* \) with \( h[k^*] > m[k^*] = g[k^*] \), and define the best channel gains after the increase on channel \( k^* \) as \( m^* = m + e_k \times \delta \) and \( g^* = g + e_k \times \delta \), respectively. Let \( K_m \) and \( K_g \) denote the set of active channels. Comparing \( K_m \) and \( K_g \), we have the following four cases: m1) there exists an \( \epsilon > 0 \) such that for all \( 0 \leq \delta \leq \epsilon, K_m = K_m^\ast \); m2) for all \( \delta > 0, K_m \supset K_m^\ast \); m3) for all \( \delta > 0, K_m \subset K_m^\ast \); m4) for all \( \delta > 0, K_m \neq K_m^\ast \). Similarly, we have four cases g1)–g4) comparing the sets \( K_g \) and \( K_g^\ast \). In the following we give the expression for \( \lim_{\delta \to 0} R_w(m + \delta x_k) - R_w(m) \) for each of the cases m1)–m4).

We first consider case m1). In the neighborhood of \( 0 < \delta \leq \epsilon, R_w(m^\ast) \) can be expressed as

\[
R_w(m^\ast) = |K_m| \log \left( \frac{\tilde{p}_w + \sum_{k \in K_m} \frac{1}{m[k]}}{|K_m|} \right) + \sum_{k \in K_m} \log(m^\ast[k]), \quad \forall 0 \leq \delta \leq \epsilon.
\]

Consequently, in the neighborhood of \( 0 < \delta \leq \epsilon \), we have

\[
\lim_{\delta \to 0^+} \frac{R_w(m^\ast) - R_w(m)}{\delta} = -\frac{|K_m|((m[k^\ast])^2)}{\tilde{p}_w + \sum_{k \in K_m} \frac{1}{m[k]}} + \frac{1}{m[k^\ast]} = -\frac{1}{\lambda_m(m[k^\ast])^2} + \frac{1}{m[k^\ast]}.
\]

We then consider case m2). This case is shown by 4 steps.

Step (m2-1): We first show \( k^\ast \in K_m \). If on the contrary, \( \lambda_m - \frac{1}{m[k^\ast]} < 0 \), then due to the continuity of \( \tilde{p}_m^k \) with respect to \( m \), there must exist an \( \epsilon > 0 \) such that for all \( 0 < \delta < \epsilon, \tilde{p}_m^k < 0 \), which is equivalent to \( \lambda_m - \frac{1}{m[k^\ast]} < 0 \). This implies \( K_m = K_m^\ast \) for \( 0 < \delta < \epsilon \), which contradicts the assumption that \( K_m \supset K_m^\ast \), for all \( \delta > 0 \).

Step (m2-2) We then argue that \( k^\ast \in K_m^\ast \). Assume the contrary, then \( \lambda_m - \frac{1}{m[k^\ast]} < 0 \). From the previous step, we see that \( \lambda_m - \frac{1}{m[k^\ast]} \geq 0 \). Then it must be the case that \( \lambda_m > \lambda_m^\ast \). Due to the fact that for all other channels \( k \neq k^\ast, m[k] = m^\ast[k] \), then we have \( \tilde{p}_w = \sum_{k=1}^K \tilde{p}_m^k > \sum_{k=1}^K \tilde{p}_m^k = \tilde{p}_w \), a contradiction.

Step (m2-3) We have argued that \( k^\ast \) must remain in the active set. Then for \( \epsilon \) small enough, there must exist a single channel \( \hat{k} \neq k^\ast \) such that \( \hat{k} \in K_m \) but \( k \notin K_m^\ast \), for all \( 0 \leq \delta \leq \epsilon \). The dual variables \( \lambda_m \) and \( \lambda_m^\ast \) can be expressed as

\[
\lambda_m = \frac{1}{|K_m|} \left( \tilde{p}_w + \sum_{k \in K_m} \frac{1}{m[k]} \right), \quad \lambda_m^\ast = \frac{1}{|K_m| - 1} \left( \tilde{p}_w + \sum_{k \in K_m \setminus \{\hat{k}, k^\ast\}} \frac{1}{m[k]} + \frac{1}{m[k^\ast]} + \frac{1}{\delta} \right).
\]

The difference between the above two dual variables is given by

\[
0 \leq \lambda_m - \lambda_m^\ast = \frac{-\lambda_m + \frac{1}{m[k]} + \frac{1}{m[k^\ast]} - \frac{1}{m[k^\ast]+\delta}}{|K_m| - 1}
\]

where \((a)\) is from the fact that \( m^\ast \geq m \), and use the same argument in the paragraph following \[24\]. Note that \( \hat{k} \in K_m \), then \( \lambda_m - \frac{1}{m[k]} \geq 0 \). Combine this with \[27\], we have that \( \frac{1}{m[k^\ast]} - \frac{1}{m[k^\ast]+\delta} \geq \lambda_m - \frac{1}{m[k]} \geq 0 \) for arbitrary small \( \delta > 0 \). Then it must be true that \( \lambda_m - \frac{1}{m[k]} = 0 \).

Step (m2-4) Using the result obtained in Step (m2-3) and the rate expression \[24\], we can express

\[
\text{If for all } \delta > 0, \text{ multiple channels leave } K_m, \text{ then they must have the same magnitude--a probability } 0 \text{ event. Our argument can also be applied to this degenerate case, but for the sake of notational simplicity, we only present the single } \hat{k} \text{ case.}
\]
the difference of the rate $R_w(m^*)$ and $R_w(m)$ as

$$R_w(m^*) - R_w(m) = (|K_m| - 1) \log \left( \frac{(\tilde{p}_w + \sum_{k \in K_m \setminus \{k^*\}} \frac{1}{m[k^*]} + \frac{1}{m[k^*] + \delta})|K_m|m[k^*]}{(|K_m| - 1)|K_m|} \right) + \log \left( \frac{m[k^*] + \delta}{m[k^*]} \right)$$

$$= (|K_m| - 1) \log \left( \frac{|K_m| + \frac{m[k^*]}{m[k^*] + \delta} - \frac{m[k^*]}{m[k^*]} - 1}{|K_m| - 1} \right) + \log \left( \frac{m[k^*] + \delta}{m[k^*]} \right)$$

where in $(a)$, $(b)$ we have used the fact that $\lambda_m = \frac{1}{m[k^*]}$. Using L’Hôpital’s rule, we obtain

$$\lim_{\delta \to 0^+} \frac{R_w(m^*) - R_w(m)}{\delta} = \lim_{\delta \to 0^+} \frac{-\frac{m[k^*]}{(m[k^*] + \delta)^2}}{1 + \frac{1}{|K_m| - 1} - \frac{m[k^*]}{m[k^*]}} + \frac{1}{(m[k^*] + \delta)}$$

$$= -\frac{m[k^*]}{(m[k^*])^2 + \frac{1}{m[k^*]}} = -\frac{1}{\lambda_m \lambda_g (m[k^*])^2} + \frac{1}{m[k^*]}.$$  

For the cases m3–m4), the derivation is similar to the cases of m1–m2). The key observation is still that the channel $k^*$ must satisfy $k^* \in K_m$ and $k^* \in K_m^*$, and that the channel $k$ that leaves or joins the set $K_m^*$ must satisfy $\lambda_m = \frac{1}{m[k^*]}$. For these cases, (29) again holds true.

Fix $\delta < 0$, and redo the above analysis by switching the role of $m$ and $m^*$ for all four possible cases, we can obtain $\lim_{\delta \to 0^+} \frac{R_w(m^*) - R_w(m)}{\delta} = \frac{1}{\lambda_m (m[k^*])^2} + \frac{1}{m[k^*]}$. Consequently, we have that for all $k^*$ that satisfies $h[k^*] > m[k^*] = g[k^*]$, the following is true

$$\lim_{\delta \to 0^+} \frac{R_w(m \lor e_{k^*} \times \delta) - R_w(m)}{\delta} = -\frac{1}{\lambda_g (g[k^*])^2} + \frac{1}{g[k^*]},$$

For case g1-g4), the exact same argument leads to the same result. In summary, we obtain

$$\lim_{\delta \to 0^+} \frac{R_w(g^*) - R_w(g) - R_w(m^*) - R_w(m)}{\delta} = \frac{1}{\lambda_g (g[k^*])^2} + \frac{1}{\lambda_m (m[k^*])^2} - \frac{1}{m[k^*]}.$$  

Recall that $k^* \in Q$, which means that $m[k^*] = g[k^*]$. Using the fact that $g \geq m$, and $\lambda_g \leq \lambda_m$, we conclude that (22) is true for all $k$ with $h[k] > m[k] = g[k]$.  

**Step 2** We then argue that for any channel $k \in \tilde{Q}$, (17) must be true. For any $g \geq m \geq 0$, pick $k \in \tilde{Q}$, we have the following three cases: 1) $h[k] \leq m[k]$; 2) $m[k] < h[k] < g[k]$; 3) $h[k] \geq g[k]$. Verifying case 1)–case 2) is straightforward. For case 3) we have

$$R_w(m \lor e_k h[k]) - R_w(m) = R_w(m \lor e_k h[k]) - R_w(m \lor e_k g[k]) + R_w(m \lor e_k g[k]) - R_w(m) \geq R_w(m \lor e_k h[k]) - R_w(m \lor e_k g[k])$$

where the inequality is due to the monotonicity property. It is sufficient to show

$$R_w(g \lor e_k h[k]) - R_w(g) \leq R_w(m \lor e_k h[k]) - R_w(m \lor e_k g[k]), \ \forall \ h \geq 0, \ g \geq m \geq 0.$$  

Let $\tilde{m} = m \lor e_k g[k]$ and $h = g + \delta_k e_k$, for some $\delta_k > 0$. Clearly $\tilde{m}[k] = g[k]$. Then to show (33), it is sufficient to show that for all $k$ such that $\tilde{m}[k] = g[k]$, we have

$$R_w(g + \delta_k e_k) - R_w(g) \leq R_w(\tilde{m} + \delta_k e_k) - R_w(\tilde{m}), \ \forall \ \delta \geq 0, \ g \geq \tilde{m} \geq 0,$$

which reduces to the case in Step 1) (cf. condition (21)). We also have that (17) is true. Combining with our argument in Step 1), we conclude that (17) is true for all $k \in K$.  

\[\]
D. Proof of Theorem 2

Let \( c^{(t)} \) denote the better-reply association at time \( t \): \( c^{(t)}[i] = w^*_i(t) \). Define two sets \( C \) and \( A \): 
\[ c \in C \Rightarrow c \text{ appears infinitely often (i.o.) in } \{c^{(t)}\}_{t=1}^{\infty}, \text{ and } a \in A \Rightarrow a \text{ i.o. in } \{a^{(t)}\}_{t=1}^{\infty}. \]

The first claim is that there exist \( a^* \in A \) that is a pure NE for game \( G \). Observe that the sets \( |A| > 0 \) and \( |C| > 0 \) due to the finiteness of the possible association profiles. Suppose \( |A| = 1 \), then the single element in \( A \), say \( a^* \), must be a NE. Suppose \( |A| > 1 \), and choose \( c^* \in C \). Pick a time \( t \) such that \( c^{(t)} = c^* \). Note that \( c^*[i] \) is in the front of the memory for each user \( i \), then with probability at least \( (1/M)^N \), \( a^{(t+1)} = c^* \). This implies \( c^* \in A \). If \( c^* \) is a NE, then our claim is proved. If \( c^* \) is not a NE, we will show that with positive probability, we can construct a finite sequence that leads to a NE. To this end, consider the following steps of operation.

**Step 1**: With probability at least \( (1/M)^N \), \( a^{(t+1)} = c^* \). Because \( c^* \) is not a NE, then there exists an \( i \in N \) such that \( c^{(t+1)}[i] \neq c^{(t)}[i] \). Similarly as in the proof of Theorem 3, we can show that 
\[ R(a^{t+1}) < R \left( \{c^{(t+1)}[i], a^{(t+1)}_i\} \right). \]

With probability at least \( (1/M)^N \), every user \( j \neq i \) samples \( c^{(t)}[j] \), which is now at the second slot in the memory, while user \( i \) samples \( c^{(t+1)}[i] \). This event leads to \( a^{(t+2)} = \{c^{(t+1)}[i], a^{(t+1)}_i\} \), and we have \( R(a^{(t+2)}) > R(a^{(t+1)}) \). Put index \( i \) into a set \( U : U = \{i\} \). Note in this stage, we have: \( a^{(t+2)}[i] = c^{(t+1)}[i] \). Continue this process, until we reach a time \( t + n \leq t + N \) such that only users in the set \( U \) are willing to switch, i.e., \( \forall j \in E, c_j^{(T)} = a_j^{(T)} \). Note that the requirement \( M \geq N \) ensures that for all \( i \), the set of best responses \( \{c^{(m)}[i]\}_{m=t}^{(t+n)} \) is still in user \( i \)'s memory. Let \( T = t + n \). Let \( E = N \setminus U \).

**Step 2**: Observe that for all \( i \in U \), there must exist a constant \( k_i \) such that \( 0 < k_i < n \leq N \) and that its current association \( a^{(T)}[i] \) is sampled from its \( k_i \)th memory, i.e., \( c^{(T-k_i)}[i] = a^{(T)}[i] \). Pick \( q \in U \) that has the largest \( k_i \) and is willing to switch at time \( T : q = \arg \max_{i \in U, c^{(T)}[i] \neq a^{(T)}[i]} {k_i} \). We can now shift \( c^{(T-N)} \) out of the memory and still be able to construct \( a^{(T+1)} = \{c^{(T)}[q], a^{(T)}_q\} \) with positive probability, because all the elements in \( a^{(T)}_q \) must have been appeared once in \( \{c^{(t)}\}_{t=T-N+1}^{T} \). Move \( q \) out of \( U \) and into \( E \), let \( T = T + 1 \) and continue Step 2) until only users in the set \( E \) are willing to switch. Change the role of \( U \) and \( E \), and continue Step 2).

Repeating Step 2), we construct a sequence \( \{R(a^{(t+1)})\} \) that is strictly increasing. Due to the finiteness of the choice of \( a \), there must exist a finite time instance \( T^* \) after which it is not possible to find an association that differs from \( a^{(T^*+T^*)} \) with a single element and still have strict better system throughput. Consequently, \( a^* = a^{(T^*+T^*)} \) is an equilibrium profile. Thus, with positive probability, a NE profile \( a^* \) appears after \( a^{(t+1)} \) in finite steps. Because \( a^{(t+1)} = c^* \), with \( c^* \) happens i.o., we must also have \( a^* \) i.o., that is, \( a^* \in A \). The claim is proved.

The next claim is that the algorithm converges to \( a^* \) with probability 1. Let \( \{t_k\}_{k=1}^{\infty} \) denote the subsequence of \( \{t\} \) in which \( a^* \) happens. Define the event: \( C_k \triangleq \bigcap_{t=1}^{M} \{a^{(t+l)} = a^*\} \), that is, starting from a time \( t_k \), \( a^* \) appears \( M + 1 \) times consecutively. When \( C_k \) happens, we have: 1) at time \( t_k + M + 1 \), \( c_{(t_k+M+1)} = a^* \) because \( BR_i(a^*) = a^*[i] \forall i \); 2) \( a^{(t_k+M+l)} = a^* \) for all \( l \geq 1 \) because after time \( (t_k + M + l) \), each user \( i \)'s memory will solely consist of \( a^*[i] \). Note that if \( a^{(t_k)} = a^* \), appears, with probability at least \( (1/M)^N \), \( a^{(t_k+1)} = a^* \). This implies \( \Pr(C_k) \geq (1/M)^{N \times M} \). Let \( C_k^c \) denote the complement set of \( C_k \). We have:

\[
\Pr \left( \bigcap_{t=1}^{T-1} C_k^c \right) = \lim_{T \rightarrow \infty} \Pr \left( \bigcap_{t=1}^{T} C_k^c \right) = \lim_{T \rightarrow \infty} \prod_{k=1}^{T-1} (1 - \Pr(C_k)) \leq \lim_{T \rightarrow \infty} \left( 1 - \left( \frac{1}{M} \right)^{N \times M} \right)^{T-1} = 0. \tag{35}
\]

This says \( \Pr(a^{(t)} \text{ converges to some } a^* \in A^* \text{ eventually}) = 1. \)
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Fig. 1. Illustration of the simulation setting with $N = 10$, $W = 6$, $D = 0.3$.

Fig. 2. Three realizations of throughput. $K = 512$, $N = 20$, $W = 8$.

Fig. 3. Averaged convergence time v.s. number of users. Each point in this figure is averaged over 200 random networks. $W = 8$, $K = 512$, $D = 0.4$.

Fig. 4. Averaged convergence time v.s. number of BSs with and w/o costs. Each point in this figure is averaged over 200 random networks. $c_i = 1$ Mbit/sec, $N = 30$, $K = 512$, $D = \{0.2, 0.5, 0.8\}$.
Fig. 5. Averaged system throughput v.s. number of BSs by different algorithms and the maximum achievable throughput. Each point in this figure is averaged over 100 random networks. $N = 10$, $K = 64$, $D = \{0.4, 0.8\}$.

Fig. 6. Averaged system throughput v.s. number of BSs by different algorithms. $N = 30$, $K = 512$, $D = \{0.2, 0.5, 0.8\}$. Each point in this figure is averaged over 200 random networks.

Fig. 7. Empirical CDF of the per-BS rate. Each curve in this figure composed of the rate of the BSs over 100 random networks. $W = 8$, $N = 30$, $K = 512$, $D = \{0.2, 0.8\}$.

Fig. 8. Averaged throughput ratio (CAPA over nearest neighbor) v.s. distribution factor $D$. $N = 30$, $K = 512$, $W = \{2, 4, 8, 10, 12\}$. Each point in this figure is averaged over 200 random networks.

Fig. 9. Comparison of the empirical CDF of the users’ rates in the multicell cellular networks. $N = 40$. Each curve in this figure is the CDF of the users’ rates in 100 generations of the network.
Fig. 10. Evaluation of the performance of the proposed algorithm in the presence of channel estimation error. Each point in this figure is averaged over 200 random networks.

Fig. 11. Construction of the network for clause $C_q = X_1 \lor X_2 \lor X_3$ for CAPA resource allocation strategy.