Zero-Field Satellites of a Zero-Bias Anomaly

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Spin-orbit (SO) splitting, $\pm \omega_{SO}$, of the electron Fermi surface in two-dimensional systems manifests itself in the interaction-induced corrections to the tunneling density of states, $\nu(\epsilon)$. Namely, in the case of a smooth disorder, it gives rise to the satellites of a zero-bias anomaly at energies $\epsilon = \pm 2 \omega_{SO}$. Zeeman splitting, $\pm \omega_{z}$, in a weak parallel magnetic field causes a narrow plateau of a width $\delta \epsilon = 2 \omega_{z}$ at the top of each sharp satellite peak. As $\omega_{z}$ exceeds $\omega_{SO}$, the SO satellites cross over to the conventional narrow maxima at $\epsilon = \pm 2 \omega_{z}$ with SO-induced plateaus $\delta \epsilon = 2 \omega_{SO}$ at the tops.

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Introduction. A zero-magnetic-field splitting [4] of the electron spectrum in two-dimensional systems has its origin in the spin-orbit (SO) coupling. The effect attracts a steady interest especially since the proposal [2] to utilize it in a spin-transistor device. This proposal was substantiated by the recent experimental demonstration [3,4] that the SO coupling causes anomalies in the tunneling conductance, $V(\epsilon)$, at a finite bias, $V$. The positions of anomalies $V = \pm 2 \omega_{SO}$ reveal directly the splitting magnitude. They emerge as satellites of the conventional zero-bias anomaly at $V = 0$. The latter is the result of electron-electron interactions modified by the disorder [3]. To the best of our knowledge, these SO-induced satellites represent the first case when a disorder-related effect in transport allows to infer the parameter of the intrinsic electronic spectrum. We note that, in the experiment, very fine features (on the scale of $\epsilon \sim 0.1$ $meV$ [4,10]) in the tunneling density of states, $\nu(\epsilon)$, can be resolved upon the analysis of $g(V)$ curves at low temperatures $T \ll V$.

In two dimensions, the interaction-induced correction, $\delta \nu(\epsilon)$, to the free-electron density of states, $\nu_{0}$, behaves as [1] $\delta \nu(\epsilon) \propto \ln(|\epsilon|/\tau)$, for $\epsilon \ll \tau^{-1}$ (diffusive regime), where $\tau$ is the scattering time. This simple logarithmic form applies in the presence of a tunneling electrode which causes a long-distance cut-off of the Coulomb interactions [5]. Zeeman splitting of the electron spectrum in a parallel magnetic field leads to additional anomalies $\delta \nu(\epsilon) \propto \ln(|\epsilon| \pm 2 \omega_{z})$ [6]. Remarkably, these anomalies emerge even in the weak-field limit, when $\omega_{z} \ll \tau^{-1}$. This is because the scattering by the impurity potential does not mix the spin-split subbands. Thus, the wave functions of two particles with opposite spins that differ in energy by $2 \omega_{z}$ are strictly identical. Naturally, the SO scattering from the impurities suppresses the anomalies at $\epsilon = \pm 2 \omega_{z}$.

The situation with SO coupling is somewhat opposite. In this case, the disorder potential causes a mixing of the SO subbands. If the disorder is short-ranged, each scattering act results in the momentum transfer of the order of the Fermi wave vector, $k_F$. Thus, for weak SO coupling, $\omega_{SO} \tau \ll 1$, the mixing is strong, and the SO satellites of the zero-bias anomaly are smeared out. If, however, the disorder is smooth, the momentum transfer is much smaller than $k_F$. In this generic regime of a small-angle scattering, the SO subbands are almost decoupled [20] (see Fig. 1). As a result, the same reasoning that leads to the Zeeman satellites applies, so that the SO satellites are pronounced even if $\omega_{SO}$ is much smaller than $\tau^{-1}$. Analogously, with a smooth disorder, in the presence of a perpendicular magnetic field, electronic states separated in energy by a cyclotron quantum turn out to be strongly correlated [21]. Consequently, the interaction-induced $\delta \nu(\epsilon)$ is an oscillatory function of energy [22] at low (compared to $\tau^{-1}$) cyclotron frequencies. Under the condition of a small-angle scattering, the dominant contribution to $\delta \nu$ comes from the Hartree correction [22].

Below we also study the evolution of the SO satellites with a parallel magnetic field. Due to a non-trivial interplay between SO coupling and Zeeman splitting [23-24], this evolution is rather peculiar. Namely, weak magnetic field causes a flat top of the SO satellite within a narrow interval $\delta \epsilon = 2 \omega_{z} \ll \omega_{SO}$. Conversely, in a strong magnetic field, the Zeeman satellite [19] acquires a flat top of a width $\delta \epsilon = 2 \omega_{SO} \ll \omega_{z}$.
Basic Equations. Microscopic origin of the SO coupling for two-dimensional electrons can be either inversion asymmetry of the host crystal \[20\] or the confinement potential asymmetry \[1\]. Unless two effects are comparable in strength \[27\], we can choose the form \(\alpha(k \times \hat{\sigma})n\) for the SO Hamiltonian \[1\], where \(\alpha\) is the coupling constant, \(\hat{\sigma}\) is the spin operator, and \(n\) is the unit vector normal to the two-dimensional plane. In a parallel magnetic field, that induces the Zeeman splitting \(2\omega_z\), the Hamiltonian of a free electron has a form

\[
\hat{H} = \frac{\hbar^2 k^2}{2m} + \alpha(k \times \hat{\sigma})n + \omega_z \sigma_x, \tag{1}
\]

where \(m\) is the electron mass. The spectrum of the Hamiltonian Eq. (1) represents two branches, so that in the vicinity of the Fermi surface we have

\[
E_\mu(k) = E_F + \epsilon_\mu(k), \tag{2}
\]

where

\[
\epsilon_\mu(k) = \hbar v_F (k - k_F) + \mu \Delta(k), \tag{3}
\]

and

\[
\Delta(k) = \sqrt{\omega_{SO}^2 + \omega^2 + 2\omega_{SO}\omega_z \sin \phi_k}. \tag{4}
\]

Here \(v_F = \hbar k_F/m\) is the Fermi velocity, \(\mu = \pm 1\) is the branch index, \(\omega_{SO} = 2\alpha k_F\) is the SO splitting, and \(\phi_k\) is the azimuthal angle of \(k\). It is convenient to rewrite the diagonalized Hamiltonian Eq. (3) in the form

\[
\hat{H} = \sum_\mu E_\mu(k) \hat{\Lambda}_\mu(k), \tag{5}
\]

where the projection operators \(\hat{\Lambda}_\mu\) are defined as \[23\]

\[
\hat{\Lambda}_\mu(k) = \frac{1}{2} \left(1 - \mu \exp(i\varphi_k) \right) \exp(-i\varphi_k). \tag{6}
\]

The angle \(\varphi_k\) is related to the azimuthal angle \(\phi_k\) as follows

\[
\tan \varphi_k = \tan \phi_k + \frac{\omega_z}{\omega_{SO} \cos \phi_k}. \tag{7}
\]

In the presence of the disorder, the electron scattering time is determined by two processes, namely, intra-subband scattering

\[
\frac{1}{\tau_{\mu\mu}(k)} = \int \frac{dp}{(2\pi)^2} \text{Tr} \left( \hat{\Lambda}_\mu(k) \hat{\Lambda}_\mu(p) \right) S(|k - p|) \delta(E_\mu(k) - E_\mu(p)), \tag{8}
\]

and inter-subband scattering

\[
\frac{1}{\tau_{\mu\mu^-}(k)} = \int \frac{dp}{(2\pi)^2} \text{Tr} \left( \hat{\Lambda}_\mu(k) \hat{\Lambda}_{\mu^-}(p) \right) S(|k - p|) \delta(E_\mu(k) - E_{\mu^-}(p)) = \frac{1}{\tau_{\text{int}}(k)}, \tag{9}
\]

where \(S(k)\) is the Fourier transform of the correlator of the random potential. Our assumption that the disorder is smooth can be quantitatively expressed as \(\lambda_{k,p} \ll 1\), where the parameter \(\lambda_{k,p}\) is defined as

\[
\lambda_{k,p} = \text{Tr} \left( \hat{\Lambda}_\mu(k) \hat{\Lambda}_{\mu^-}(p) \right) = 1 - \cos(\varphi_k - \varphi_p) \approx \frac{(\varphi_k - \varphi_p)^2}{4}. \tag{10}
\]

Correspondingly, \(\text{Tr} \left( \hat{\Lambda}_\mu(k) \hat{\Lambda}_\mu(p) \right) = 1 - \lambda_{k,p}\) is close to unity. Using Eq. (5), the parameter \(\lambda_{k,p}\) can be expressed through the angles \(\phi_k\) and \(\phi_p\) as

\[
\lambda_{k,p} \approx \frac{\omega_{SO}^2(\omega_{SO} + \omega_z \sin \phi_k)^2}{4\Delta^4(k)} \left(\phi_k - \phi_p\right)^2. \tag{11}
\]
From Eqs. (8), (9) we get the final expression for the scattering time

\[
\frac{1}{\tau} = \frac{1}{\tau_{\mu\mu}} + \frac{1}{\tau_{\mu,-\mu}} = \frac{m}{2\pi} \int d\phi_p \, S(k_p \phi_p),
\]

(12)

where we assumed that \( \omega_{\text{SO}} \ll E_F \).

As it was discussed above, the satellite anomaly in \( \delta \nu(\epsilon) \) originates from the Hartree correction. In the case of two subbands, the expression for the energy-dependent part of the Hartree correction has the form

\[
\phi_{\mu} = e^{i\phi_{p} - \phi_{p_1}} \left( 1 - \frac{1}{2} \lambda_{p+q,p_1+q} - \frac{1}{2} \lambda_{p,p_1} \right) e^{-i(\phi_{p+q} - \phi_{p_1+q})} \\
\times \langle G^R_1(\epsilon + \omega, \mu,p_1 + q)G^A_1(\epsilon, \mu, p_1) \rangle S(k_p |\phi_p - \phi_{p_1}|),
\]

(13)

where we assumed that \( \omega_{\text{SO}} \ll E_F \).

\[
\Gamma^{++}_{--}(p, p', q, \omega) = \sum_{\epsilon} \langle G^R_1(\epsilon + \omega, p_1 + q)G^A_1(\epsilon, p_1) \rangle S(k_p |\phi_p - \phi_{p_1}|),
\]

(15)

The two-particle vertex function, \( \Gamma^{++}_{--} \), that is responsible for satellites, is determined from the standard Dyson-type equation with a kernel

\[
K(p, p_1, q, \omega) = e^{i(\phi_{p} - \phi_{p_1})} \left( 1 - \frac{1}{2} \lambda_{p+q,p_1+q} - \frac{1}{2} \lambda_{p,p_1} \right) e^{-i(\phi_{p+q} - \phi_{p_1+q})} \\
\times \langle G^R_1(\epsilon + \omega, \mu,p_1 + q)G^A_1(\epsilon, \mu, p_1) \rangle S(k_p |\phi_p - \phi_{p_1}|),
\]

(16)

where \( \omega_{\text{SO}} \ll E_F \).

Small-angle scattering implies that a typical \( q \ll k_p \). This allows to set in Eq. (13) \( \Delta(p+q) \approx \Delta(p) \), \( \lambda_{p+q,p_1+q} \approx \lambda_{p,p_1} \), and \( \phi_{p+q} \approx \phi_p \). Then the kernel Eq. (15) simplifies to

\[
K(p, p_1, q, \omega) = \frac{m\tau}{2\pi} \frac{(1 - \lambda_{p,p_1})S(k_p |\phi_p - \phi_{p_1}|)}{1 - i(\omega - 2\Delta(p)) + \tau + i\hbar q v_F \cos(\phi_p - \phi_q)}. \]

(17)

At this point we note that, upon integration over \( \phi_{p_1} \), the product \( (1 - \lambda_{p,p_1})S(k_p |\phi_p - \phi_{p_1}|) \) is proportional to \( (\tau^{-1} - \tau_{\text{int}}^{-1}(p)) \), where \( \tau_{\text{int}} \) is the inter-subband scattering time (8). Then, in the diffusive regime, \( (\omega - 2\Delta(p)) \tau \ll 1 \), the solution of the Dyson equation for the vertex function \( \Gamma^{++}_{--}(p, p', q, \omega) \) reads

\[
\Gamma^{++}_{--}(p, p', q, \omega) = \frac{S(k_p |\phi_p - \phi_{p'}|)}{-i(\omega - 2\Delta(p)) + \tau + Dq^2 + \tau_{\text{int}}^{-1}(p)},
\]

(18)

where we have neglected a weak anisotropy of the diffusion coefficient \( D = v_F^2 \tau_{tr}/2 \).

In principle, the Dyson equation for \( \Gamma^{++}_{--} \) couples (weakly) this function to the other vertex functions, \( i.e. \) \( \Gamma^{--} \).

This coupling would be important in the domain \( \omega_{\text{SO}} \ll \tau_{\text{int}}^{-1} \). In our case, \( \omega_{\text{SO}} \tau_{\text{int}} \gg 1 \), this coupling can be neglected.

Substituting Eq. (15) into Eq. (13) and performing integration over \( p \) and \( p' \), we obtain

\[
\frac{\delta \nu(\epsilon)}{\nu_0} = \frac{\tau v_0}{4\pi} \text{Re} \int_0^{1/v_F \tau} dq \int_0^{2\pi} d\phi_p \int_0^{2\pi} d\phi_{p'} \frac{V(k_p |\phi_p - \phi_{p'}|)S(k_p |\phi_p - \phi_{p'}|)}{-i(\epsilon - 2\Delta(p)) + Dq^2 + \tau_{\text{int}}^{-1}(p)}.
\]

(19)

The fact that characteristic \( \phi_p - \phi_{p'} \) is small allows to set \( V(k_p |\phi_p - \phi_{p'}|) = V(0) \), where \( V(0) = 1/\nu_0 \) (static screening). Then integration over \( \phi_{p'} \) yields \( 1/(m\tau) \). Finally we obtain
\[
\frac{\delta \nu(\epsilon)}{\nu_0} = -\left(\frac{1}{16\pi E_F \tau_{tr}}\right) \mathcal{L}(\epsilon),
\]

where the energy-dependent factor, \(\mathcal{L}(\epsilon)\), is defined as
\[
\mathcal{L}(\epsilon) = \int_0^{2\pi} \frac{d\phi}{2\pi} \ln \left( (\epsilon - 2\Delta(\phi))^2 + \frac{\tau^2}{\tau_{int}^2(\phi)} \right).
\]

We see that the energy cut-off in Eq. (21) is determined by \(\tau_{int}^{-1} \ll \tau^{-1}\). Note, that inter-subband scattering time \(\tau_{int}\) can be conveniently expressed through the conventional transport relaxation time \(\tau_{tr}\). Using Eqs. (3), (11) we obtain
\[
\tau_{int}(\phi) = 2\tau_{tr}\left(\frac{\omega_2^2 + 2\omega_2 \omega_{\pm SO} \sin \phi}{\omega_{\pm SO}^2 (\omega_{SO} + \omega_2 \sin \phi)^2}\right).
\]

Equations (20)–(22) constitute our main result. In principle, the exchange correction to \(\nu(\epsilon)\) also yields the anomaly at \(\epsilon \approx 2\Delta\). However, the exchange term is suppressed since it contains an extra factor \(\lambda \sim \tau/\tau_{tr}\) which comes from overlap integrals between different branches. Note also, that Eqs. (21)–(22) can be easily modified to the case when the crystalline anisotropy term \(\beta \sigma_x k_x - \beta \sigma_y k_y\), is present in the Hamiltonian (1) alongside with SO-term \(-\beta_0 k_x\).

Modification reduces to the replacement of \(\Delta_z\) by \(\beta k_y\), and of \(\phi\) by \(2\phi\).

Analysis of the anomaly.

(i) Zero-field limit: \(\omega_2 \ll 1/\tau_{tr} \ll \omega_{\pm SO}\).

The shape of the satellite peak in \(\delta \nu(\epsilon)\) is given by
\[
\frac{\delta \nu(\epsilon)}{\nu_0} = -\left(\frac{1}{16\pi E_F \tau_{tr}}\right) \ln \left( (\epsilon - 2\omega_{\pm SO})^2 + \frac{\tau^2}{4\tau_{tr}^2} \right).
\]

The peak is well pronounced when \(\omega_{\pm SO} \tau_{tr} \gg 1\).

(ii) Intermediate fields: \(1/\tau_{tr} \ll \omega_2 \ll \omega_{SO}\).

The broadening of the SO-satellite peak is determined by the angular dependence of \(\Delta(\phi)\). The integral \(\mathcal{L}(\epsilon)\) can be evaluated in this limit, yielding
\[
\mathcal{L}(\epsilon) = 2\ln\left(\frac{\epsilon}{2} - \omega_{SO}\right) \tau + \ln\left(\frac{\epsilon}{2} - \omega_{\pm SO}\right)^2 - \omega_{SO}^2 \tau^2 \right).}
\]

Remarkably, within a domain \(|\epsilon - 2\omega_2| \leq \omega_2\), there is a plateau in \(\delta \nu(\epsilon)\), i.e. within this domain \(\mathcal{L}(\epsilon) \equiv \ln \omega_2 \tau\).

(iii) Strong fields: \(1/\tau_{tr} \ll \omega_{SO} \ll \omega_2\).

In this case, conversely, SO coupling determines the shape of the Zeeman satellite
\[
\mathcal{L}(\epsilon) = 2\ln\left(\frac{\epsilon}{2} - \omega_{\pm SO}\right) \tau + \ln\left(\frac{\epsilon}{2} - \omega_{\mp SO}\right)^2 - \omega_{SO}^2 \tau^2 \right).}
\]

Again a plateau at \(|\epsilon - 2\omega_2| \leq \omega_{SO}\) emerges at the top of the Zeeman satellite.

Typical examples of the energy dependence of \(\delta \nu(\epsilon)\), obtained by numerical integration of Eq. (21), are shown in Fig. 2. They illustrate the successive broadening and then narrowing of the satellite peak with increasing magnetic field.

Conclusion. The above consideration was restricted to the diffusive regime \(\epsilon \sim \omega_{SO} \ll 1/\tau\). It is known however \cite{28}, \cite{31} that the conventional diffusive zero-bias anomaly persists at high electron energies \(\epsilon \gg 1/\tau\) (ballistic regime). The question arises whether the SO anomaly survives in the ballistic regime. We will discuss this case qualitatively. Without SO coupling, the physical mechanism responsible for the formation of the zero-bias anomaly is combined scattering of a probe electron from an isolated impurity and a perturbation of the electron density caused by this impurity. The latter perturbation falls off with distance as \(\sin(2k_F r)/r^2\) (Friedel oscillation). Then the anomaly emerges as a result of the Bragg backscattering from this almost periodic potential profile. In the presence of SO coupling, a single impurity induces Friedel oscillation with three wave vectors \(\pm k\), namely \(2k_F, 2k_F \pm 2\omega_{SO}/(v_F)\). The wave vector for an electron with chirality +1 and energy \(2\omega_{SO}\) above the Fermi level will satisfy the Bragg condition for the Friedel oscillation created by electrons with chirality −1. This is illustrated in Fig. 1. The efficiency of the Bragg scattering, and, hence, the anomaly at \(\epsilon = 2\omega_{SO}\), is suppressed in the absence of magnetic field due to the fact that, for a given chirality, the spinors corresponding to the wave vectors \(k\) and \(-k\) are orthogonal to each other. A transparent underlying physics of the ballistic zero-bias anomaly suggests that electrons with both chiralities
can experience Bragg scattering from the SO-specific $2k_{\nu}$-oscillation \[31\]. These resonances give rise to yet additional weak anomalies at energies $\epsilon = \pm \omega_{SO}$, that are absent in the diffusive regime.

Note in conclusion, that experimental studies \[32\] indicate that even in moderate quality GaAs–based samples with mobility $\sim 2 \cdot 10^5$ cm$^2$/V·s (as in \[16\]) the typical value of the ratio $\tau/\tau_{tr}$ is 0.1. Then for $\omega_{SO} = 0.5 meV$ the parameter $\omega_{SO}/\tau_{tr} \approx 6.6$.

As a final remark, SO coupling for two-dimensional holes is much stronger that for electrons. Therefore, the satellites of the zero-bias anomaly can be expected in the hole samples too. However, due to the warp of the valence band spectrum caused by the crystalline anisotropy, the subband splitting depends on the direction of the hole wave vector. This would lead to the smearing of the satellites.

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FIG. 1. Schematic illustration of the processes responsible for satellite anomaly at $\epsilon = 2\omega_{SO}$ in the diffusive (a) and ballistic (b) regimes.

FIG. 2. Normalized correction, $\delta \nu(\epsilon)$, calculated from Eqs. (21), (22) for $\omega_{SO} \tau_{tr} = 10$ is plotted versus dimensionless energy $\epsilon/2(\omega_{SO}^2 + \omega_Z^2)^{1/2}$ for various ratios $\omega_Z/\omega_{SO}$. For convenience different curves are shifted along the vertical axis.