Scale-free model for governing the universe dynamics

O. LUONGO1,2,3, G. IANNONE4 and C. AUTIERI4

1 Dipartimento di Fisica, Università di Roma “La Sapienza” - Piazzale Aldo Moro 5, I-00185 Roma, Italy, EU
2 ICRANet and ICRA (International Center of Relativistic Astrophysics Networks) - Piazzale della Repubblica 10, I-65122 Pescara, Italy, EU
3 Dipartimento di Scienze Fisiche, Università di Napoli “Federico II”, Complesso Universitario di Monte S. Angelo, Edificio G - Via Cinthia, I-80126 Napoli, Italy, EU
4 Dipartimento di Fisica “E. R. Caianiello”, Università di Salerno - I-84081 Baronissi (Salerno), Italy, EU

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Abstract – We investigate the effects of scale-free statistics in Friedman-Robertson-Walker cosmology, in a toy model which considers a matter distribution $P(k) \sim k^{-\gamma}$, with $2 < \gamma < 3$. Then we address the consequences on dynamics by using the kinematical parameters, namely the cosmographic parameters (VISSE M., Phys. Rev. D, 78 (2008) 063501), i.e. $H(t)$ (Hubble parameter), $q(t)$ (deceleration parameter), $j(t)$ (jerk parameter) and $s(t)$ (snap parameter) for the so-called cosmographic test (see the paper by Visser). We show that the scale-free additive background seems able to explain the origin of dark energy giving a statistical interpretation of the cosmological overcoming the fine-tuning and the coincidence problems. After this, a consistent comparison with data by a Supernovae Ia test shows that the model behaves well at $z \ll 1$.

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Introduction. – The recent impressive amount of astrophysical data has shown that the universe is passing through an accelerated phase [1–5]. In the framework of Einstein picture this would mean that an exotic component must be added. One way to overcome this challenge is to put into Einstein equations a cosmological constant $\Lambda$ which can drive the accelerated scenario. This approach suffers from the so-called fine-tuning problem that concerns the huge difference between the magnitude order of $\Lambda$ in particle physics and its observed value. In addition it is not clear why at low redshift the $\Lambda$ fraction density $\Omega_\Lambda$ would be so closed to the matter fraction density $\Omega_m$, which is usually referred as the coincidence problem [6,7].

Inspite of these difficulties the above picture, being the $\Lambda$CDM model [8], seems the most viable one; the model fits well the Supernovae Ia data, better than all the other different approaches.

From these considerations it would be possible to think that the $\Lambda$CDM model should emerge from a different mechanism which deals with some properties of matter distribution and allows us to mimic the $\Lambda$CDM behavior at low redshift. We propose an alternative possible explanation of dark-energy effects only recoiling the $\Lambda$ effects by the use of statistics. Indeed, if $\Lambda$ were a result of a particular statistical law for the universe matter content, then the coincidence problem would be overcome. At the same time the fine-tuning problem does not have reason to exist if we discard the common vacuum interpretation of $\Lambda$. Among all the possibilities, an intriguing and relevant statistical model, which has increased its importance in the last decade, would be the scale-free model [9,10].

It has been first studied in the analysis of topological features of networks and graphs in various contexts, just spanning from condensed-matter physics to molecular biology [11] or social science [9,12–17].

The theoretical features deal with considering complex networks whose vertices are the elements of the system and edges represent the interaction between them. The idea is to consider at each vertex the constituents of the physical system under analysis; for example the vertices can be the atoms in a crystal lattice with the edges being the interactions among them, people in a social network and the friendship relations or a paper with all its citations...
and so on (see [9,18,19] for reviews). Recently Barabási et al. [20] have discovered a surprising feature which characterizes the self-organization of large-scale complex networks, by analyzing several data. They found that, (independently of the nature of the system and of the identities of its constituents), the probability that a certain vertex has, \( k \) connection with other \( k \) vertices decays as the power law
\[
P(k) \sim k^{-\gamma},
\]
with \( 2 < \gamma < 3 \). By starting from the ansatz that the matter distributes in the universe following a scale-free statistics, we try to describe the possible consequences on dynamics.

Following what has been done in [21], we try to investigate the consequences of the degree of heterogeneity on the dynamics properties of networks, isolating a region of universe of correlated networks with a completely random organization [22], defined by the scale-free distribution, of eq. (1), where \( \gamma \) must be considered as an integer. Every particle of the distribution \( \rho \) should be thought to be connected by each other through a network or a family of networks, while the number of particles \( N \) is just fixed at the beginning. In order to mimic the low interaction of standard density matter in the universe no interactions are supposed, following a mean-field regime for the description of the system.

Hence the first quantity introduced to analyze the matter distribution is \( \rho_k(t) \), which represents the average density of particles in nodes of degree \( k \); it reads
\[
\dot{\rho}_k(t) = -\rho_k + k\rho z_1 + \Lambda[\rho_k],
\]
which is a possible example of Langevin equation for nodes, i.e. a continuity equation for the universe matter nodes distribution.

The above formula suggests that the first two terms on the right are due to the diffusion in uncorrelated random graphs and the third could be considered as a reaction kernel that depends only on \( \rho_k \). From this equation we derive the dynamics of a particular choice of \( k \). Immediately the total density matter is given by \( \rho(t) = \sum_k P(k)\rho_k(t) \). In the limit in which nodes are a large number, the above sum becomes an integral over the \( k \) space, with a cut-off \( k^* \) \( \sim z_1/\rho(t) \), necessary to avoid divergences.

A consequent matter distribution is then [10,21]
\[
\int P(k)\rho_k^2dk \propto \begin{cases} 
\rho\gamma^{-1} + c k^{2-\gamma} \rho, & \text{if } k_c > k^*, \\
k_0^{3-\gamma} \rho^2, & \text{if } k_c < k^*,
\end{cases}
\]
where one takes the validity of the mean-field approach only considering a critical point \( k_c \) for which the mean-field regime holds if \( k^* > k_c \). For this purpose the continuity equation reads
\[
\dot{\rho}(t) \simeq a \rho - b \rho^2 + \sqrt{(a \rho + 2b \rho^2)}/N \eta(t),
\]
where \( a \) and \( b \) are two integration constants and, in particular \( k_c \propto N^{1/\omega} \). The last expression describes the behavior in the scale-free toy model that we have considered. It represents the equation we must take into account in the Friedman equations in order to study the effects of statistics.

The article is organized as follows: in the next section we find the exact solutions of dynamics by adding the scale-free information to a Friedman-Robertson-Walker (FRW) universe. The third section is devoted to the observable limit, fixing constraints through the use of the so-called cosmography (which will be well described afterwards). The last part of the third section is devoted to a cosmological analysis by SNeIa and to a comparison with the data.

The paper ends with conclusions and future perspectives.

The model dynamics. – In this paragraph we discuss the consequences of the scale-free model on dynamics. In order to make it possible we now introduce the concept of cosmography as a tool to perform a first cosmological test only by requiring a FRW universe [23]. Indeed, it appears evident that a surprising amount of data in cosmology is pure kinematics and, following the Visser proposal [24,25], we refer to cosmography as a completely model-independent procedure for the underlying dynamics governing the evolution of the universe. Hence cosmography would encounter kinematical quantities as coefficients of the Taylor expansion of \( a(t) \) (see [24]). Therefore the constraints of cosmography are represented by a set of quantities evaluated at \( z \approx 0 \), i.e. \( (H_0,q_0,j_0,s_0) \), related to the derivatives of the scale factor [26–28] at our time.

Since considerable high-\( z \) data are available (at \( z \approx 1.75 \)) for example, it would be necessary to ask if the series of \( a(t) \) converges for large \( z \) or not. It has been shown that the radius of convergence of any series expansion is \( z < 1 \).

To include observed data which have \( z > 1 \) it is possible to follow the Visser suggestion\(^1\), that is to consider an expansion in terms of \( y = \frac{a^2}{a_0^2} < 1 \) [29,30].

We refer to the cosmographic parameters as \( H(t) = \frac{\dot{a}}{a}, q(t) = -\frac{\ddot{a}}{a^2} \) and \( j(t) = \frac{a^3\dot{a}^2}{a^2}, s(t) = \frac{a^4\dot{a}^2}{a^2} \).

The above parameters are denoted with a subscript 0, as mentioned above, when they are tested at \( z = 0 \).

Hence, assuming a homogeneous and isotropic universe described by the FRW metrics \( ds^2 = c^2dt^2 - a(t)^2dl^2 \), with \( dl^2 = dr^2 + r^2 \sin^2 \theta d\theta^2 \), and an energy-momentum tensor for a perfect fluid with equation of state \( P = w \rho \), the first Friedman equation reads \( H^2 = a^2 + \beta \), where \( a = \frac{4\pi}{3} \) and \( \beta = \frac{\Lambda c^2}{H_0^2} \). Note that \( \beta \) is the cosmological term due to a cosmological constant put by hand into Einstein equations which explains (in \( \Lambda \)CDM) the acceleration. The purpose of our work is to deeply analyze the case \( \beta = 0 \) and then what consequences the use of scale-free statistics on \( \rho(z) \) only leads to.

\(^1\)Anyway it will not be our case because we perform at the end of the work a Supernovae, \( z \ll 1 \), test.

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Taking $\rho(t)$ as expressed in the scale-free model we can put the first Friedman equation into eq. (4), in the simplest case in which $\eta(t)$ is taken and is equal to zero for $t \neq 0$. Then we have in general\(^2\), $H = -\left(\frac{\beta^3}{2} + \frac{b \delta^2}{\alpha}\right) \frac{1}{t} + \left(\frac{\beta}{2} + \frac{q^2}{\alpha}\right) H - \frac{b}{2\alpha} \frac{d}{2} H^{2} + \sqrt{\frac{f(t)}{N}} \delta(t)$, and by positions $a \equiv \frac{\beta^3}{2} + \frac{b \delta^2}{\alpha}$, $b \equiv \frac{\beta}{2} + \frac{q^2}{\alpha}$ and $c \equiv \frac{b}{2\alpha}$ and $A = \frac{1}{\sqrt{2\alpha}}$ we get (see fig. 1)

$$H(t-t^*) = A \sqrt{b + \sqrt{d} \tanh \sqrt{d}(t-t^*)}, \quad (5)$$

while if $\beta = 0$

$$H(t-t^*) = \sqrt{\frac{a t}{1 + b e^{a(t-t^*)}}}. \quad (6)$$

In the first expression we consider the position $d = b^2 - 4ac$ and the integration constant $t^*$. Note that, depending on the $d$ sign, we could have different evolutions of $H(t)$; i.e. $d < 0$ shows us a definite time interval in which $H(t)$ (and so $a(t)$) loses its physical meaning. Hence, for describing a variable $a(t)$, for all the time, we must have $d > 0$. On the other hand, this is provided by calculations, by considering directly the numerical values for $d$, i.e. $d = \frac{e^2}{4} + pq\beta \left(\frac{3}{\pi \alpha} - 2\right)$, which is always positive for all the possible values assumed by $p$ and $q$ if we require that $pq > 0$, which are "phenomenological" values defining the type of reaction and process. According to other works [21] we may assume $p < 0$ and $q < 0$.

After evaluating the $H(t)$ dynamics we naturally succeed in finding an exact and most general solution for $a(t)$ (see fig. 2) of the form

$$a(t) = a(0)e^{\frac{a(t-t^*)}{\sqrt{2\alpha}}}, \quad (7)$$

\(^2\)Considering first $\beta \neq 0.$

where the function $G(t-t^*)$ is expressed by

$$G(t-t^*) - \tilde{G} =$$

$$-\sqrt{|b - \sqrt{d}|} \tanh^{-1} \left\{ \frac{\sqrt{b} + \sqrt{d} \tanh \sqrt{d}(t-t^*)}{\sqrt{|b - \sqrt{d}|}} \right\}$$

$$+ \sqrt{b + \sqrt{d}} \tanh^{-1} \left\{ \frac{\sqrt{b} + \sqrt{d} \tanh \sqrt{d}(t-t^*)}{\sqrt{b + \sqrt{d}}} \right\} \quad (8)$$

with the integration constant $\tilde{G}$ of the form

$$\tilde{G} = \sqrt{|b - \sqrt{d}|} \tanh^{-1} \left\{ \frac{\sqrt{b}}{\sqrt{b - \sqrt{d}}} \right\}$$

$$- \sqrt{b + \sqrt{d}} \tanh^{-1} \left\{ \frac{\sqrt{b}}{\sqrt{b + \sqrt{d}}} \right\}. \quad (9)$$

Finally the expression for the deceleration parameter [27,28] $q(t)$ (see fig. 3) reads

$$q(t) = \frac{\sqrt{c/2d} \sqrt{b + \sqrt{d} \tanh \sqrt{d}(t-t^*)}}{(b \cosh \sqrt{d}(t-t^*) + \sqrt{d} \sinh \sqrt{d}(t-t^*) )^2}. \quad (10)$$

It appears that $q(t) < 0$ suggesting then that the model is compatible with the accelerating-universe picture. The other expressions for $j(t)$ and $s(t)$, which discriminate the type of universe dynamics, are described as a function of time in the appendix. We only underline that the sign of $j(t)$ and $s(t)$ is at $z = 0$ the same of $\Lambda$CDM\(^3\).

The observable limit and the comparison with WMAP data. – We proposed a scenario which does

\(^3\)For the meaning of this see [24].
not take into account \textit{directly} the cosmological constant term \( \Lambda \) related to a vacuum energy. As a consequence of statistics, the \( \Lambda \)CDM model would be mimicked by the constraint \( \Omega_\Lambda + \Omega_{\text{SC}} = 1 \) which has the particularity that the cosmological constant density, \( \Omega_\Lambda \), is replaced by an emergent dark-energy density \( \Omega_{\text{SC}} \) derived from our scale-free toy model.

In order to implement a Supernovae Ia (SNeIa) test based on the Union data, it would be convenient to rewrite the observable quantities in terms of the redshift \( z \). Soon, rewriting eq. (4), (assuming \( \beta = 0 \)), after straightforward calculations we get for the Hubble rate

\[
H(z) = H_0 \sqrt{\Omega_m (1 + z)^3 + \Omega_{\text{SF}} \tanh^2[E + |F| \ln(1 + z)]},
\]

where, for convenience, we write \( \Omega_{\text{SF}} \equiv \tilde{\Omega}_{\text{SF}} \tanh^2 E \). A remarkable property is that, at low redshift, we reobtain a \( \Lambda \)CDM-like \( H[z] \), without using a cosmological constant put by hand into Einstein equations.

At our time, in this picture, the deceleration parameter reads

\[
q_0 = -1 + \frac{3}{2} \Omega_m - \tilde{\Omega}_{\text{SF}} \left( \cosh^{-2} E + \cosh^{-4} E \right). \tag{12}
\]

Note that in the \( \Lambda \)CDM case we have \( q_0 = -1 + \frac{3}{2} \Omega_m \); the scale-free corrective term is the last part of eq. (12). The jerk parameter at \( z = 0 \) reads

\[
j_0 = 1 + \tilde{\Omega}_{\text{SF}} |F| \left( \frac{|F|}{\cosh^2 E} - 3 \tanh E + 2 |F| \tanh^2 E \right), \tag{13}
\]

again the scale-free correction is the part different from unity in the right-hand side of eq. (13). By varying \( q_0 \in [-1, -0.59] \), we expect [31] that the correction due to the scale-free model should be small, which implies \( j_0 \approx 1 \). Hence, approximatively we infer the numerical intervals, for the free constants

\[
\tilde{\Omega}_{\text{SF}} \in [0.74, 1], \quad E \in [1.2, 2.2],
\]

and \( F \sim 10^{-2} \). Soon, in order to implement a SNeIa test, we rewrite the luminosity distance \( D_L \equiv c (1 + z) \int \frac{dz}{H(z)} \), expanding in series for \( z \ll 1 \) (at the second order)

\[
D_L \sim z \left( H_0 \sqrt{\Omega_m + \Omega_{\text{SF}} \tanh E} \right)^{-1}
\]

\[
+ z^2 \Omega_m + F \Omega_{\text{SF}} \cosh^{-2} E + 4 \Omega_{\text{SF}} \tanh E \frac{H(z)}{4 H_0 (\Omega_m + \Omega_{\text{SF}} \tanh E)^2}. \tag{15}
\]

Hence we follow the procedure adopted by [32], observing the SNeIa apparent magnitudes \( m_i \) and redshift \( z_i \), obtaining the absolute magnitude \( M \), relating to the luminosity \( L \). Only by using the relation between the luminosity distance \( D_L \) and magnitudes, \( \mu \equiv m - M = 5 \log_{10} D_L + M \), where conventionally \( D_L = H_0 d_L \) and (in general)

\[
d_L(z) = (1 + z) \frac{c}{H_0} \frac{1}{\sqrt{|\Omega_{h,0}|}} F \left( H_0 \sqrt{|\Omega_{h,0}|} \int_0^z \frac{dz'}{H(z')} \right), \tag{16}
\]

we find the numerical values of the quantities of eq. (18).

The likelihood function \( L \) from chi-square statistic \( L \propto \exp(-\chi^2/2) \) reads

\[
\chi^2 = \sum_i \frac{(m_i^{\text{theor}} - m_i^{\text{obs}})^2}{\sigma_i^2}. \tag{17}
\]

Bayes’ theorem provides the cosmological parameters probabilities, then a standard Gaussian reduced \( \chi^2 \) minimization procedure has been achieved. The results of our fits are

\[
\Omega_m = 0.300 \pm 0.024, \quad E = 2.10 \pm 0.47, \quad F = 0.020 \pm 0.009, \tag{18}\tag{19}\tag{20}
\]

where the best fit for the Hubble constant is \( H_0 = 69.50 \pm 0.47 \) and \( \Omega_{\text{SF}} = 1 - \Omega_m \), with the reduced chi-squared \( \chi = 1.03119 \). The results of eq. (18) are in agreement with the cosmographic intervals already discussed.

All the previous results and the SNeIa test have been evaluated in the case of \( k = 0 \), which seems to be the observed best result [7]. Without neglecting the curvature term \( k \neq 0 \) it would be possible to achieve an expression for \( a(t) \), which is not analytically defined.

The expression for \( a(t) \) would be easily obtained numerically, introducing a curvature term \( \Omega_k \sim 0.009 \) [32]. In fig. 4 we can see the differences between the case with and without curvature.

Finally, recalling the definition of scale-free parameters, postulated at the beginning, the critical transition parameter is given by \( k_c \sim 1.429 H_0^2 \). The effects of dark energy are modeled by the presence of a cut-off \( k_c \); for our epoch \( k > k_c \). The result of the statistics is, then, to replace the cosmological constant by the effects derived from the statistics \( P(k) \sim k^{-\gamma} \), with \( k > k_c \).
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Future perspectives may deal with a generalization of the use of scale-free dynamics, with particular interest in understanding whether dark matter could be as an effect of a particular statistics. The consequences on structure formations and dark-matter distribution would then be objects of study. The proposed approach indeed develops a possible toy model for a given set of free constants, whose results depend on changing the initial choice of networks and nodes, for example invoking an alternative scale-free conservation equation. In principle it would be interesting to test other different relevant possibilities in order to underline the possible consequences on the universe dynamics.

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Appendix: cosmographic parameters. – The jerk parameter \( j(t) \) is defined as follows:

\[
\frac{j(t)}{f(t)} = \beta_1 + \beta_2 \cosh(4\sqrt{dt}) + 4 \cosh(2\sqrt{dt}) (\beta_3 + g_4) + \beta_5 \sinh(4\sqrt{dt}) + g_6 + 2\sqrt{d} \sinh(2\sqrt{dt}) (\beta_7 + g_8),
\]

where

\[
f(t) = \frac{\cosh^{-4}(\sqrt{dt})}{8\sqrt{2}} \left[ b + \sqrt{d} \tanh(\sqrt{dt}) \right]^3 \quad \text{and} \quad g_n \equiv \frac{\beta_n H(t)}{A}.
\]

The snap parameter \( s(t) \) is

\[
\frac{s(t)}{h(t)} = g_{11} - g_{12} + g_{13} + g_{14} + \cosh(6\sqrt{dt})(g_{15} + g_{16} + g_{17}) + \sinh(2\sqrt{dt})(\beta_9 + g_{18} - g_{19} - g_{20} + g_{21}) + \sinh(4\sqrt{dt})(\beta_{10} + g_{22} - g_{23} - g_{24}) + \sinh(6\sqrt{dt})(g_{25} + g_{26}) + 2 \cosh(4\sqrt{dt})(\beta_{27} + g_{28} + g_{29} - g_{30} - g_{31}) - \cosh(2\sqrt{dt})(\beta_{32} - g_{33} + g_{34} + g_{35} + g_{36}).
\]

The complete list of constants involved in the calculations is

\[
\begin{align*}
\beta_1 &= \sqrt{2}(3b^3 - 3bd + 4cd^2), & \beta_2 &= \sqrt{2}(b^2 + 3d), \\
\beta_3 &= \sqrt{2}(b^3 - 2cd^2), & \beta_4 &= 3\sqrt{cd}, & \beta_5 &= \sqrt{2d}(3b^2 + d), \\
\beta_6 &= 12b\sqrt{cd}, & \beta_7 &= -\sqrt{2}(-3b^2 + d + 4bcd), & \beta_8 &= 6\sqrt{cd}, \\
\beta_9 &= -8\sqrt{2d}b^3/2(-9b^2 + 3d + 10bcd).
\end{align*}
\]

Fig. 4: In this graphic \( a(t) \) is plotted with \( k \neq 0 \). The set of parameters is \( a = 1, \ b = 3 \) and \( c = 1 \), with normalization \( a(0) = 1 \).

Conclusion and perspectives. – We investigated in this paper some effects derived from the introduction of the scale-free model into the Einstein picture. Assuming as a toy model what has been proposed in [10,21], we define a mean-field regime which mimics the \( \Lambda \)CDM paradigm, giving a cosmological constant density which emerges from the scale-free dynamics. The only ansatz is to use the scale-free statistics for the matter distribution.

All the previous results refer to a finite number of galaxies. The case of an infinite number of networks by using the diverging cut-off \( k_c \) would be a next step. Notice that, depending on \( \gamma - 1 \) and \( q \) quantities, different results may be found.
\[ \beta_{10} = 4\sqrt{2}cd^{5/2}(9b^2 + 3d + 8bcd), \]
\[ \beta_{11} = 10b^4, \quad \beta_{12} = -12b^2d, \]
\[ \beta_{13} = 2d^2, \quad \beta_{14} = 56bcd^2, \quad \beta_{15} = b^4, \]
\[ \beta_{16} = 6b^2d, \quad \beta_{17} = d^2, \]
\[ \beta_{18} = 20b^3\sqrt{d}, \quad \beta_{19} = -12b^2d^{3/2}, \quad \beta_{20} = -64b^2cd^{3/2}, \]
\[ \beta_{21} = 56cd^{2/2}, \quad \beta_{22} = 16b^3\sqrt{d}, \quad \beta_{23} = -32b^2d^{3/2}, \]
\[ \beta_{24} = -32cd^{3/2}, \quad \beta_{25} = 4b^4\sqrt{d}, \quad \beta_{26} = 4bd^{3/2}, \]
\[ \beta_{27} = 2\sqrt{2}cd(3b^3 + b(9 + 4bc)d + 4cd^2), \quad \beta_{28} = 3b^4, \]
\[ \beta_{29} = 6b^2d, \quad \beta_{30} = -d^2, \quad \beta_{31} = -32bcd^2, \]
\[ \beta_{32} = 16\sqrt{2}cd(-3b^3 + 2b^2cd + 3cd^2), \quad \beta_{33} = -15b^4, \]
\[ \beta_{34} = 6b^2d, \quad \beta_{35} = d^2, \quad \beta_{36} = 8bcd^2. \]

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