The Quantum Theory of Conductivity of Spatially - Heterogeneous Systems

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The quantum theory of conductivity of semiconductor objects, to which the quantum wells, wires and dots concern, is constructed. Average values of current and charge densities, induced by a weak electromagnetic field, are calculated. It is shown, that in both cases average current and charge densities contain two contributions, first of which is expressed through electric field, and second - through a spatial derivative of electric field. Appropriate expressions for the conductivity tensor, dependent on coordinates and applicable to any spatially-heterogeneous systems, are deduced. The results may be used in the theory of secondary light radiation from low-dimensional objects in cases of monochromatic light and light pulses.

I. INTRODUCTION

In connection with the increased interest to experiment and theory concerning light reflection and absorption by low-dimensional semiconductor objects - quantum wells, wires and dots - at light pulse excitation (see, for example, [1, 2]) there appears a question on what kind of electromagnetic wave-electron interaction is more convenient to use, - containing vector potential \( \mathbf{A}(\mathbf{r}, t) \) or electric field \( \mathbf{E}(\mathbf{r}, t) \). The work [3] is devoted to the same question with reference to calculation of a differential section of inelastic light scattering by infinite crystals. In [3] it is shown, that exact expressions for scattering sections (with use sets of exact electron wave functions in a crystal), obtained with use of two various kinds of electromagnetic wave-electron interaction, coincide. But the set of exact electron wave functions in a crystal \( \mathbf{r} \) taking into account precisely, for example, electron-phonon interaction is unknown, therefore at calculation of sections the approached methods are used. Namely, it is taken into account electron-phonon interaction in the lowest order, not all intermediate states of an electronic system are considered. If to use these approximations, various kinds of light-electron interaction (containing \( \mathbf{A} \) or \( \mathbf{E} \)) will result in distinguished results. The authors of [3] approve, that in a case of non-resonant scattering the use of interaction containing \( \mathbf{E} \), gives the best results.

We assume to construct the general theory of secondary light radiation by low-dimensional semiconductor objects, to which number quantum wells, quantum wires and quantum dots concern. First of all, the theory should describe light reflection and absorption by such objects, and also various kinds of light scattering (Raman scattering, Releigh scattering). The theory should be applicable in a case of monochromatic irradiation, as well as in a case of a light pulse irradiation. We are limited to linear approximation on intensity of exciting light.

From told above follows, that others tasks are decided, than in [3], where light scattering in bulk crystals is considered. Therefore the problem of an interaction choice (through \( \mathbf{A} \) or \( \mathbf{E} \)) is necessary to be solved anew.

In the present work we shall calculate average values of current and charge densities induced by an electromagnetic field in the case of spatially-heterogeneous matter. To this case concern semiconductor objects of the lowered dimension. Having calculated linear on electric and magnetic fields the contributions to average values of current and charge densities, we can then to determine these fields inside and outside of semiconductor objects, solving the Maxwell equations. Expressions for fields appropriate reflected and past through object light thus can be obtained. Such procedure, taking into account all orders light-electron interaction, is done in [4], where intensity of reflected and absorbed light at monochromatic irradiation quantum well of final thickness are calculated. In the present work we deduce expression for average induced current density used in [4].

The operator of interaction of the charged particles with electric and magnetic fields is expressed through vector \( \mathbf{A}(\mathbf{r}, t) \) and scalar \( \varphi(\mathbf{r}, t) \) potentials. Therefore average values of the induced of current and charge densities also are expressed through these potentials. However to use these expressions is inconvenient because of the contribution

\[
-\frac{e}{mc} \langle 0 | \rho(\mathbf{r}) | 0 \rangle \mathbf{A}_\alpha(\mathbf{r}, t).
\]

In average of a current density, where \( e, m \) is the charge and mass of a particle respectively, \( \rho(\mathbf{r}) \) is the operator of a charge density (see section IV lower). Therefore we shall express average values of current and charge densities through electric \( \mathbf{E}(\mathbf{r}, t) \) and magnetic \( \mathbf{H}(\mathbf{r}, t) \) fields. Our task will consist
in transition from expressions for physical values, containing potentials, to expressions containing fields $E(r, t)$ and $H(r, t)$.

In the present work the case of temperature $T = 0$ is considered. Further we assume to calculate the average induced current and charge densities in a case spatial - heterogeneous systems and final temperatures. We shall use some results of [1,2].

The article is organized as follows. In sections II-IV the statement of a task is stated, operators of current and charge densities and their average values on ground state of system are entered. In sections V - IX the task about expression of average values through an electric field and its spatial derivatives is solved. Section X is devoted to exception of average values of diagonal matrix elements of operators $r_i$. In section XI the general expression for conductivity tensor of heterogeneous system is given. In sections XII and XIII the case of zero electric and constant magnetic fields is considered.

II. STATEMENT OF A TASK.

Let us consider system of $N$ particles with the mass $m$ and charge $e$ in a weak electromagnetic field characterized by intensities $E(r, t)$ and $H(r, t)$. Let us introduce the vector $A(r, t)$ and scalar $\varphi(r, t)$ potentials

$$E(r, t) = -\frac{1}{c} \frac{\partial A}{\partial t} - \nabla \varphi,$$

$$H(r, t) = \nabla \times A. \quad (1)$$

We shall consider fields as classical. The gage of $A(r, t)$ and $\varphi(r, t)$ is arbitral. For completeness of a task we shall consider, that the system of particles can be placed in a constant magnetic field $H_c$, which may be strong (SMF). This field is described by the vector potential $\mathbf{A}(r)$, so

$$H_c = \text{rot} \mathbf{A}(r).$$

The total Hamiltonian $H_{total}$ is written as

$$H_{total} = \frac{1}{2m} \sum_i \left( P_i - e \frac{A(r_i)}{c} + e A(r_i, t) \right)^2 + V(r_1 \ldots r_N) + e \sum_i \varphi(r_i, t), \quad (2)$$

where $P_i = -i\hbar(\partial/\partial r_i)$ is the generalized momentum operator (see, for example, [5,6]), $V(r_1 \ldots r_N)$ is the potential energy including interaction between particles and the external potential. In Eq. (2) It is necessary to take into account non-commutativity of $P_i$ and $A(r_i), A(r_i, t)$. Let us allocate in Eq. (2) energy $U$ interaction of particles with the electromagnetic field, having included interaction with a strong magnetic field in the main Hamiltonian $H_{total} = H + U.$

$$H = \frac{1}{2m} \sum_i p_i^2 + V(r_1 \ldots r_N), \quad p_i = P_i - \frac{e}{c} A(r_i), \quad (4)$$

$$U = U_1 + U_2, \quad U_1 = -\frac{1}{c} \int d^3r j(r) A(r, t) + \int d^3r \varphi(r, t),$$

$$U_2 = \frac{e}{2mc} \int d^3r \rho(r) A^2(t, r), \quad (5)$$

where the operators of current and charge densities are introduced

$$j(r, t) = \sum_i j_i(r, t),$$

$$j_i(r, t) = \frac{e}{2} \left( \delta(r - r_i) \mathbf{v}_i + \mathbf{v}_i \delta(r - r_i) \right), \quad \mathbf{v}_i = \mathbf{p}_i/m, $$

$$\rho(r) = \sum_i \rho_i(r), \quad \rho_i(r) = e\delta(r - r_i). \quad (6)$$

Our task is to calculate in linear approximation on external fields $E(r, t)$ and $H(r, t)$ the induced current and charge densities averaged on the system ground state.

III. THE DEFINITION OF OPERATORS

The charge density operator $\rho(r)$ in the Schrödinger representation does not contain additives proportional to fields, but the current density operator in fields looks like

$$j(r, t) + \Delta j(r),$$

where

$$\Delta j(r) = e \frac{1}{2} \sum_i \{ \Delta \mathbf{v}_i \delta(r - r_i) + \delta(r - r_i) \Delta \mathbf{v}_i \}, \quad (7)$$

$$\Delta \mathbf{v}_i = (i/\hbar)[U, r_i] = -\frac{e}{mc} A(r_i, t), \quad (8)$$

$[F, Q]$ is the commutator of operators $F$ and $Q$, and hence

$$\Delta j(r) = -\frac{e}{mc} \rho(r) A(r, t). \quad (9)$$

In the interaction representation we have

$$\rho(r, t) = e^{i\mathcal{H}t/\hbar} \rho(r) e^{-i\mathcal{H}t/\hbar},$$

$$j(r, t) = e^{i\mathcal{H}t/\hbar} j(r) e^{-i\mathcal{H}t/\hbar},$$

$$\Delta j(r, t) = -\frac{e}{mc} \rho(r, t) A(r, t). \quad (10)$$

Now we shall determine the current and charge density operators in the Heisenberg representation. In [2] (page 82) it is shown, that connection between the operator
\( F(t) \) in the interaction representation and the operator \( F_G(t) \) in the Heisenberg representation is expressed as

\[
F_G(t) = S^{-1}(t)F(t)S(t),
\]

where \( S \) is the S-matrix. It is determined as

\[
S(t) = S(t, -\infty) = 1 - \frac{i}{\hbar} \int_{-\infty}^{t} dt_1 U(t_1) + \ldots,
\]

(12)

Using Eq. (12), we find, that linear additives on potentials \( A(r, t) \) and \( \varphi(r, t) \) to the current and charge density operators in the Heisenberg representation are equal

\[
j_{1\alpha}(r, t) = \Delta j_{1\alpha}(r, t) - \frac{i}{\hbar} \int_{-\infty}^{t} dt' [j_{1\alpha}(r, t'), U_1(t')],
\]

(13a)

\[
\rho_1(r, t) = -\frac{i}{\hbar} \int_{-\infty}^{t} dt'[\rho(r, t), U_1(t')].
\]

(13b)

The index 1 means the first order on potentials \( A(r, t) \) and \( \varphi(r, t) \). Substituting Eqs. (9) and (5) in last expressions, we obtain

\[
j_{1\alpha}(r, t) = \frac{e}{mc} \rho(r, t) A_{\alpha}(r, t)
\]

\[
+ \frac{i}{\hbar c} \int d^3r' \int_{-\infty}^{t} dt'[j_{1\alpha}(r, t), j_{\beta}(r', t')]A_{\beta}(r', t'),
\]

\[
- \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{t} dt'[j_{1\alpha}(r, t), \rho(r', t')]\varphi(r', t'),
\]

(14)

\[
\rho_1(r, t) = \frac{i}{\hbar c} \int d^3r' \int_{-\infty}^{t} dt'[\rho(r, t), j_{\beta}(r', t')]A_{\beta}(r', t')
\]

\[
- \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{t} dt'[\rho(r, t), \rho(r', t')]\varphi(r', t').
\]

(15)

### IV. AVERAGING ON THE SYSTEM GROUND STATE

Let us consider a zero temperature case and average Eqs. (14), (15) on the ground state of our system. In all further calculations we shall assume, that on the indefinitely removed distances there are no charges and currents, and also that on times \( t \to -\infty \) \( E(r, t) \) and \( H(r, t) \) are equal \( 0 \), what corresponds to an adiabatic switching-on of these fields. In [8] (page 84) it is shown, that at averaging it is necessary to use wave functions \( \langle 0 | \) of the ground state without taking into account the interaction \( U \). For averaged values of current and charge densities we shall enter designations \( \langle 0 | j_{1\alpha}(r, t) | 0 \rangle \) and \( \langle 0 | \rho_1(r, t) | 0 \rangle \).

In Eqs. (14) and (15) we shall make replacement of \( t' \) by \( t'' = t' - t \). At averaging \( \langle 0 | \ldots | 0 \rangle \) we shall take into account, that

\[
\langle 0 | \hat{F}(t) | 0 \rangle = \langle 0 | e^{i\hat{H}t/\hbar} \hat{F} e^{-i\hat{H}t/\hbar} | 0 \rangle = \langle 0 | \hat{F} | 0 \rangle,
\]

(16)

where \( \hat{F} \) is any operator. Then we shall obtain

\[
\langle 0 | j_{1\alpha}(r, t) | 0 \rangle = -\frac{e}{mc} \langle 0 | \rho(r) | 0 \rangle A_{\alpha}(r, t)
\]

\[
+ \frac{i}{\hbar c} \int d^3r' \int_{-\infty}^{0} dt'[\langle 0 | j_{\alpha}(r, j_{\beta}(r', t')) | 0 \rangle A_{\beta}(r', t' + t')
\]

\[
- \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{0} dt'[\langle 0 | j_{\alpha}(r), \rho(r', t') | 0 \rangle \varphi(r', t' + t') \times A_{\beta}(r', t' + t')
\]

(17)

\[
\langle 0 | \rho_1(r, t) | 0 \rangle = \frac{i}{\hbar c} \int d^3r' \int_{-\infty}^{0} dt'[\langle 0 | \rho(r, j_{\beta}(r', t')) | 0 \rangle \times A_{\beta}(r', t' + t')
\]

\[
- \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{0} dt'[\langle 0 | \rho(r), \rho(r', t') | 0 \rangle \times \varphi(r', t' + t').
\]

(18)

Thus, we have obtained expressions for current and charge densities averaged on the ground state through vector and scalar potentials. But averaged values should be expressed through measured values - the fields \( E(r, t) \) and \( H(r, t) \) and their derivative. Let us make transition to expressions of \( \langle 0 | j_{1}(r, t) | 0 \rangle \) and \( \langle 0 | \rho_1(r, t) | 0 \rangle \) through fields.

### V. TIME DERIVATIVES FROM AVERAGED CURRENT AND CHARGE DENSITIES

Let us apply the following reception: let us calculate time derivatives from values (17) and (18):

\[
\frac{\partial}{\partial t} \langle 0 | j_{1\alpha}(r, t) | 0 \rangle = \frac{\partial}{\partial t} \langle 0 | j_{1\alpha}(r, t) | 0 \rangle_A
\]

\[
+ \frac{\partial}{\partial t} \langle 0 | j_{1\alpha}(r, t) | 0 \rangle_\varphi,
\]

(19)

where indexes \( A \) and \( \varphi \) designate contributions of vector and scalar potentials, respectively, equal

\[
\frac{\partial}{\partial t} \langle 0 | j_{1\alpha}(r, t) | 0 \rangle_A = -\frac{e}{mc} \langle 0 | \rho(r) | 0 \rangle \frac{\partial A_{\alpha}(r, t)}{\partial t}
\]

\[
+ \frac{i}{\hbar c} \int d^3r' \int_{-\infty}^{0} dt'[\langle 0 | j_{\alpha}(r, j_{\beta}(r', t')) | 0 \rangle \times A_{\beta}(r', t' + t')
\]

(20)
\[
\frac{\partial}{\partial t} \langle 0 | j_\alpha(r, t) | 0 \rangle_A = \frac{i}{\hbar} \int d^3r' \int_{-\infty}^0 dt' \times \langle 0 | \rho(r), j_\beta(r', t') \rangle | 0 \rangle \frac{\partial A_\beta(r', t + t')}{\partial t},
\]
(21)
\[
\frac{\partial}{\partial t} \langle 0 | j_\alpha(r, t) | 0 \rangle_\varphi = -\frac{i}{\hbar} \int d^3r' \int_{-\infty}^0 dt' \times \langle 0 | \rho(r), j_\beta(r', t') \rangle | 0 \rangle \frac{\partial \varphi(r', t + t')}{\partial t},
\]
(22)
\[
\frac{\partial}{\partial t} \langle 0 | \rho_\alpha(r, t) | 0 \rangle = -\frac{i}{\hbar} \int d^3r' \int_{-\infty}^0 dt' \times \langle 0 | j_\alpha(r), \rho(r') \rangle | 0 \rangle \frac{\partial \varphi(r', t + t')}{\partial t},
\]
(23)

Let us transform Eqs. (22) and (23), containing scalar potential \( \varphi \). We use identity
\[
\frac{\partial \varphi(r, t + t')}{\partial t} = \frac{\partial \varphi(r, t + t')}{\partial t'},
\]
then we integrate on \( t' \) in parts. We obtain
\[
\frac{\partial}{\partial t} \langle 0 | j_\alpha(r, t) | 0 \rangle_\varphi = -\frac{i}{\hbar} \int d^3r' \int_{-\infty}^0 dt' \times \varphi(r', t)
+ \frac{i}{\hbar} \int d^3r' \int_{-\infty}^0 dt' \langle 0 | \left[ j_\alpha(r), \frac{\partial \rho(r', t')}{\partial t'} \right] \rangle 0 \rangle \times \varphi(r', t + t').
\]
(24)

In the first term of the RHS Eq. (24) we execute integration on \( r' \) and we use Eq. (6) for operators \( j(r) \) and \( \rho(r) \). To calculate the second term we use the continuity equation
\[
\nabla j(r, t) + \frac{\partial \rho(r, t)}{\partial t} = 0,
\]
(25)
which is true for operators, determined in Eq. (10) with taking into account the constant magnetic field \( \mathbf{H}_c \). Further in this term we make integration on \( r'_\beta \) in parts, transferring derivative on the scalar potential \( \varphi(r', t + t') \).
It results in
\[
\frac{\partial}{\partial t} \langle 0 | j_\alpha(r, t) | 0 \rangle_\varphi = -\frac{e}{m} \langle 0 | \rho(r) | 0 \rangle \frac{\partial \varphi(r, t)}{\partial r_\alpha}
+ \frac{i}{\hbar} \int d^3r' \int_{-\infty}^0 dt' \langle 0 | j_\alpha(r), j_\beta(r', t') \rangle | 0 \rangle \times \frac{\partial \varphi(r', t + t')}{\partial r'_\beta}.
\]
(26)

Summing Eqs. (20) and (26) and using Eq. (1), we obtain
\[
\frac{\partial}{\partial t} \langle 0 | j_\alpha(r, t) | 0 \rangle = e \langle 0 | \rho(r) | 0 \rangle E_\alpha(r, t)
- \frac{i}{\hbar} \int d^3r' \int_{-\infty}^0 dt' \langle 0 | j_\alpha(r), j_\beta(r', t') \rangle | 0 \rangle \times E_\alpha(r', t + t').
\]
(27)

We obtain completely similarly
\[
\frac{\partial}{\partial t} \langle 0 | \rho_\alpha(r, t) | 0 \rangle = -\frac{e}{m} \langle 0 | \rho(r) | 0 \rangle \frac{\partial \varphi(r, t)}{\partial r_\alpha}
- \frac{i}{\hbar} \int d^3r' \int_{-\infty}^0 dt' \langle 0 | \rho(r), j_\beta(r', t') \rangle | 0 \rangle E_\alpha(r', t + t').
\]
(28)

So, we managed to express time derivatives from averaged current and charge densities through electric fields, having got rid of vector and scalar potentials.

VI. AVERAGE VALUES OF CURRENT AND CHARGE DENSITIES EXPRESSED THROUGH ELECTRIC FIELD

Integrating Eqs.(27) and (28) on time, we obtain expressions for average current and charge densities
\[
\langle 0 | j_\alpha(r, t) | 0 \rangle = \int_{-\infty}^{t} dt' \frac{\partial}{\partial t'} \langle 0 | j_\alpha(r, t') | 0 \rangle + C_\alpha,
\]
(30)
\[
\langle 0 | \rho_\alpha(r, t) | 0 \rangle = \int_{-\infty}^{t} dt' \frac{\partial}{\partial t'} \langle 0 | \rho_\alpha(r, t') | 0 \rangle + C'.
\]
(31)

We believe \( C_\alpha = C' = 0 \), what corresponds to absence of induced currents and charges on times, indefinitely removed in the past.

Let us enter a designation
\[
a(r, t) = -c \int_{-\infty}^{t} dt' \mathbf{E}(r, t').
\]
(32)

Then with the help Eqs. (27) - (29) we obtain
\[
\langle 0 | j_\alpha(r, t) | 0 \rangle = -\frac{e}{mc} \langle 0 | \rho(r, t) | 0 \rangle a_\alpha(r, t)
+ \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{t} dt' \langle 0 | j_\alpha(r, t), j_\beta(r', t') \rangle | 0 \rangle a_\beta(r', t'),
\]
(33)
\[
\langle 0 | \rho_\alpha(r, t) | 0 \rangle = \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{t} dt' \langle 0 | \rho(r, t), j_\beta(r', t') \rangle | 0 \rangle a_\beta(r', t').
\]
(34)

Comparing expressions Eqs. (33) and (34) with Eqs. (17) and (18), we see, that the second differ from the first by absence of the scalar potential \( \varphi \) and replacement of the vector potential \( \mathbf{A}(r, t) \) by the vector \( \mathbf{a}(r, t) \), determined in Eq. (30). Thus, we have managed to express average values of the induced current and charge densities only through electric fields. However, without the
further transformations Eqs. (31) and (32) are inapplicable at transition to electric field, independent on time, since at integration on \( t \) in Eq. (30) there appears some uncertainty, - the frequency \( \omega \) in the denominator turns in zero. It concerns and to the case \( E = 0, H \neq 0 \). We shall return to this question in section XII.

VII. TRANSFORMATION OF EXPRESSIONS FOR AVERAGE CURRENT AND CHARGE DENSITIES

After we managed to express average current and charge densities through an electric field, the task is to pass to expressions, obviously appropriate to electron-field interaction of a kind

\[
\tilde{U}_1 = -e \sum_i r_i \beta E_\beta(t),
\]

which, for example, is used in [8] for electric field, \( E(t) \), independent on coordinates.

To achieve this purpose we shall make some transformations of Eqs. (31) and (32). We shall enter fictitious operators \( j'_1(r, t) \) and \( \rho'_1(r, t) \), for which we have

\[
\langle 0 | j'_{1\alpha}(r, t) | 0 \rangle = \langle 0 | j^{f}_{1\alpha}(r, t) | 0 \rangle,
\]

\[
\langle 0 | \rho_1(r, t) | 0 \rangle = \langle 0 | \rho^{f}_1(r, t) | 0 \rangle.
\]

It follows from Eqs. (31), (32)

\[
\begin{align*}
  j'_{1\alpha}(r, t) &= -\frac{e}{mc} \rho(r, t) a_\alpha(r, t) \\
  &+ \frac{i}{\hbar c} \int d^3r' \int_{-\infty}^{t} dt'[j_{\alpha}(r, t), j_\beta(r', t')] a_\beta(r', t')
\end{align*}
\]

\[
\rho'_1(r, t) = \frac{i}{\hbar c} \int d^3r' \int_{-\infty}^{t} dt'[\rho(r, t), j_\beta(r', t')] a_\beta(r', t').
\]

Comparing the fictitious operators Eqs. (35) - (36) with the genuine operators Eqs. (14) - (15), we find, that for transition from genuine to the fictitious operators it is necessary to put \( \varphi(r, t) = 0 \) and to replace the vector potential \( \mathbf{A}(r, t) \) by \( \mathbf{a}(r, t) \), determined in Eq. (30).

Let us enter also a fictitious operator of interaction of particles with a field

\[
U^{f}_1 = -\frac{1}{c} \int d^3r j(r) a(r, t),
\]

which differs from the true operator \( U_1 \), determined in Eq. (5) by condition \( \varphi(r, t) = 0 \) and by replacement \( \mathbf{A}(r, t) \) on \( \mathbf{a}(r, t) \). To the interaction Eq. (37) there corresponds a linear on a field the additive to the current density operator

\[
\Delta j^{f}_\alpha = -\frac{e}{mc} \rho(r) a_\alpha(r, t).
\]

It is easy to see, that Eqs. (35) and (36) may be written as

\[
\begin{align*}
  j'_{1\alpha}(r, t) &= \Delta j^{f}_{1\alpha}(r, t) - \frac{i}{\hbar} \int_{-\infty}^{t} dt'[j_{\alpha}(r, t), U^{f}_{1}(t')],\\
  \rho'_1(r, t) &= -\frac{i}{\hbar} \int_{-\infty}^{t} dt'[\rho(r, t), U^{f}_{1}(t')],
\end{align*}
\]

what is similar to Eqs. (13a) and (13b) with replacement of the genuine operators by fictitious. Let us transform Eqs. (39) - (40) so that to remove the first term in the RHS Eq. (39). It is possible to rewrite the integral of a kind

\[
\int_{-\infty}^{t} dt'[F(r, t), U^{f}_{1}(t')]
\]

from the RHSs Eqs. (39) and (40) as

\[
\int dt'[F(r, t), \tilde{U}_1(t')] - [F(r, t), R(t)],
\]

if

\[
\tilde{U}_1 = U^{f}_{1}(t) + \frac{dR(t)}{dt},
\]

where \( F(r, t) \) is the operator, in case Eq. (39) equal \( j_{\alpha}(r, t) \), and in case Eq. (40) - \( \rho(r, t) \), and \( R(t) \) is any operator in the interaction representation

\[
R(t) = e^{\mathcal{H}t/\hbar} R_{\text{Sch}}(r) e^{-\mathcal{H}t/\hbar},
\]

\( R_{\text{Sch}} \) is the operator in the Schrödinger representation. It is possible to show, that if

\[
R_{\text{Sch}} = R_{\text{Sch}}(r_1 \ldots r_N, t),
\]

i.e. the operator \( R_{\text{Sch}} \) does not contain momentums, the relation is carried out

\[
\frac{i}{\hbar} [j_{\alpha}(r, t), R(t)] = \Delta j_{R\alpha}(r, t),
\]

where

\[
\Delta j_{R\alpha}(r, t) = \frac{e}{2} e^{i\mathcal{H}t/\hbar} \sum_i [\Delta v_{iR\alpha} \delta(r - r_i) + \delta(r - r_i) \Delta v_{iR\alpha}] e^{-i\mathcal{H}t/\hbar},
\]

\[
\Delta v_{iR\alpha} = \frac{i}{\hbar} [\dot{R}_{\text{Sch}}, r_{i\alpha}],
\]

\[
\dot{R}_{\text{Sch}} = \frac{i}{\hbar} [\mathcal{H}, R_{\text{Sch}}] + \frac{\partial R_{\text{Sch}}}{\partial t}.
\]
Also it is obvious, that under condition of Eq. (44),
\[ [\rho(r,t), R(t)] = e^{i\mathcal{H}_t/\hbar}[\rho(r), R_{Sch}]e^{-i\mathcal{H}_t/\hbar} = 0. \] (49)

Thus, it is proved, that instead of \( U_f^l(t) \) it is possible to choose any operator, determined in Eq. (42), if \( R_{Sch} \) does not contain momentums, and instead of Eqs. (39) and (40) to write down
\[ j_{\alpha}^l(r,t) = \Delta \tilde{j}_{\alpha}(r,t) - \frac{i}{\hbar} \int_{-\infty}^{t} dt'[\tilde{j}_{\alpha}(r,t), \tilde{U}_1(t')], \] (50)

\[ \rho_{\alpha}^l(r,t) = -\frac{i}{\hbar} \int_{-\infty}^{t} dt'[\rho(r,t), \tilde{U}_1(t')], \] (51)

where
\[ \Delta \tilde{j}_{\alpha}(r) = \frac{e}{2} \sum_i [\Delta \tilde{v}_{i\alpha}\delta(r - r_i) + \delta(r - r_i)\Delta \tilde{v}_{i\alpha}], \] (52)

\[ \Delta \tilde{v}_{i\alpha} = \frac{i}{\hbar} [\tilde{U}_1, r_{i\alpha}]. \] (53)

Having substituted Eqs. (42) and (52) in Eqs. (50) and (51), we obtain
\[ j_{\alpha}^l(r,t) = -\frac{e}{mc}\rho(r,t)a_\alpha(r,t) + \frac{i}{\hbar}[j_{\alpha}(r,t), R(t)] \]
\[ + \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{t} dt'[j_{\alpha}(r,t), j_{\beta}(r', t')]a_{\beta}(r', t') \]
\[ - \frac{i}{\hbar} \int_{-\infty}^{t} dt'[j_{\alpha}(r,t), \frac{d\rho(r,t)}{dt}] , \] (54)

\[ \rho_{\alpha}^l(r,t) = \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{t} dt'\rho(r,t), j_{\beta}(r', t')a_{\beta}(r', t') \]
\[ - \frac{i}{\hbar} \int_{-\infty}^{t} dt'\rho(r,t), \frac{d\rho(r,t)}{dt} \] (55)

VIII. CHOICE OF THE OPERATOR \( R_{Sch} \)

Let us choose the operator \( R_{Sch} \) as
\[ R_{Sch} = \frac{e}{c} \sum_i r_{i\beta}a_{\beta}(r_i, t). \] (56)

Then
\[ R(t) = e^{i\mathcal{H}_t/\hbar}R_{Sch}(r)e^{-i\mathcal{H}_t/\hbar} = \frac{1}{c} \int d^3rd_{\beta}(r, t)a_{\beta}(r, t), \] (57)

where the designation is entered
\[ d(r) = e \sum_i r_i\delta(r - r_i) = er \sum_i \delta(r - r_i) = r\rho(r). \] (58)

Let us calculate the contributions in the RHS Eqs. (54) and (55), containing \( R(t) \) and \( dR(t)/dt \). It is possible to show, that
\[ \frac{i}{\hbar}[j_{\alpha}(r,t), R(t)] = \frac{e}{mc}\rho(r,t)a_{\alpha}(r,t) \]
\[ + \frac{e}{mc}d_{\beta}(r,t)\frac{\partial a_{\beta}(r,t)}{\partial r_{\alpha}}. \] (59)

Also it is possible to obtain the result
\[ \frac{dR(t)}{dt} = \frac{1}{c} \int d^3rd_{\beta}(r, t)\frac{\partial a_{\beta}(r, t)}{\partial t} \]
\[ + \frac{1}{c} \int d^3rj_{\beta}(r, t)a_{\beta}(r, t) \]
\[ + \frac{1}{c} \int d^3rY_{\beta\gamma}(r, t)\frac{\partial a_{\beta}(r, t)}{\partial r_{\gamma}}, \] (60)

where
\[ Y_{\beta\gamma}(r) = r_{\beta}j_{\gamma}(r). \] (61)

Substituting Eqs. (59) and (60) in the RHS Eqs. (54) and (55), we find, that the terms, not containing derivative \( \partial a_{\beta}(r,t)/\partial t \) or \( \partial a_{\beta}(r,t)/\partial r_{\alpha} \), are reduced. In a result we have
\[ j_{\alpha}^l = \frac{e}{mc}d_{\beta}(r, t)\frac{\partial a_{\beta}(r, t)}{\partial r_{\alpha}} \]
\[ - \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{t} dt'[j_{\alpha}(r,t), j_{\beta}(r', t')]\frac{\partial a_{\beta}(r', t')}{\partial t'} \]
\[ - \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{t} dt'[j_{\alpha}(r,t), Y_{\beta\gamma}(r', t')]\frac{\partial a_{\beta}(r', t')}{\partial r_{\gamma}}, \] (62)

\[ \rho_{\alpha}^l = \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{t} dt'[\rho(r,t), j_{\beta}(r', t')]\frac{\partial a_{\beta}(r', t')}{\partial t'} \]
\[ - \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{t} dt'\rho(r,t), Y_{\beta\gamma}(r', t')]\frac{\partial a_{\beta}(r', t')}{\partial r_{\gamma}}. \] (63)

Averaging the fictitious operators Eqs. (62) and (63) on the ground state, we shall obtain required by us expressions for averaged values of the induced current and charge densities.

Taking into account the definition Eq. (30), we find, that terms in the RHS Eqs. (62) and (63) are divided on two categories: to the first concern what contain an electric field, to the second what contain derivative from this field on coordinates. Therefore it is convenient to write down average values from Eqs. (62) and (63) as
\[ \langle 0|j_{1\alpha}(r,t)|0 \rangle = \langle 0|j_{1\alpha}(r,t)|0 \rangle_E + \langle 0|j_{1\alpha}(r,t)|0 \rangle_{\partial E/\partial \mathbf{r}'} \]
(64)
\[ \langle 0 | \rho_1(r, t) | 0 \rangle = \langle 0 | \rho_1(r, t) | 0 \rangle_E + \langle 0 | \rho_1(r, t) | 0 \rangle_{\partial E / \partial r}, \quad (65) \]

where

\[ \langle 0 | j_{1\alpha}(r, t) | 0 \rangle_E = \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{t} dt' \langle 0 | [j_{\alpha}(r, t), d_{\beta}(r', t')] | 0 \rangle E_{\beta}(r', t'), \quad (66) \]

\[ \langle 0 | \rho_1(r, t) | 0 \rangle = \frac{i}{\hbar} \int d^3r' \times \int_{-\infty}^{t} dt' \langle 0 | \rho(r, t), d_{\beta}(r', t') | 0 \rangle E_{\beta}(r', t'), \quad (67) \]

\[ \langle 0 | j_{1\alpha}(r, t) | 0 \rangle_{\partial E / \partial r} = - \frac{e}{m} \langle 0 | d_{\beta}(r) | 0 \rangle \int_{-\infty}^{t} dt' \frac{\partial E_{\beta}(r', t')}{\partial r_\alpha} + \frac{i}{\hbar} \int d^3r' \int_{-\infty}^{t} dt' \langle 0 | [j_{\alpha}(r, t), Y_{\beta\gamma}(r', t')] | 0 \rangle \times \int_{-\infty}^{t'} dt'' \frac{\partial E_{\beta}(r', t'')}{\partial r_\gamma}. \quad (68) \]

So, the task of expression of averaged over the ground state current and charge densities in linear approximation on electric field and its derivatives on coordinates is solved.

**IX. THE TRANSFORMED FICTITIOUS OPERATOR OF INTERACTION**

For completeness of a picture we shall determine a kind of the fictitious operator of interaction \( \hat{U}_1 = \hat{U}_1^S + \hat{R}_{Sch} \). Using Eqs. (37) and (60) we obtain in the Schrödinger representation

\[ \hat{U}_1 = \hat{U}_{1E} + \hat{U}_{1\partial E / \partial r}, \quad (70) \]

\[ \hat{U}_{1E} = - \int d^3r d_{\beta}(r) E_{\beta}(r, t), \quad (71) \]

\[ \hat{U}_{1\partial E / \partial r} = - \int d^3r Y_{\beta\gamma}(r) \int_{-\infty}^{t} dt' \frac{\partial E_{\beta}(r, t')}{\partial r_\gamma}. \quad (72) \]

Having executed integration on \( r \) in the RHS Eqs. (71) and (72), we obtain

\[ \hat{U}_{1E} = -e \sum_i r_{i\beta} E_{\beta}(r_i, t), \quad (73) \]

\[ \hat{U}_{1\partial E / \partial r} = - \frac{e^2}{4} \int_{-\infty}^{t} dt' \sum_i \left\{ \left[ \frac{v_{i\gamma}}{\partial r_\gamma} \frac{\partial E_{\beta}(r_i, t')}{\partial r_\gamma} + \frac{\partial E_{\beta}(r_i, t')}{\partial r_\gamma} v_{i\gamma} \right] r_{i\beta} \right. \]

\[ + r_{i\beta} \left( v_{i\gamma} \frac{\partial E_{\beta}(r_i, t')}{\partial r_\gamma} + \frac{\partial E_{\beta}(r_i, t')}{\partial r_\gamma} v_{i\gamma} \right) \} \times \int_{-\infty}^{t'} dt'' \frac{\partial E_{\beta}(r_i, t'')}{\partial r_\gamma} r_{i\beta}. \quad (74) \]

Thus, \( \hat{U}_{1E} \) contains an electric field, and \( \hat{U}_{1\partial E / \partial r} \) contains derivatives from electric field on coordinates and an integral on time. In the case, when terms, containing derivatives from electric field on coordinates, on what of reasons give the small contribution \( \mathbb{I} \), it is possible to use Eq. (73). It coincides with the formula for interaction of charged particles in \( \mathbb{I} \). Let us notice, that Eq. (70) for fictitious interaction \( \hat{U}_1 \) may be written compactly

\[ \hat{U}_1 = \frac{e}{2c} \sum_i \left\{ \frac{da_{\beta}(r_i, t)}{dt} r_{i\beta} + r_{i\beta} \frac{da_{\beta}(r_i, t)}{dt} \right\}, \quad (75) \]

where \( da_{\beta}(r_i, t)/dt \) is the full time derivative:

\[ \frac{da_{\beta}(r_i, t)}{dt} = \frac{\partial a_{\beta}(r_i, t)}{\partial t} + \frac{i}{\hbar} [\mathcal{H}, a_{\beta}(r_i, t)], \quad (76) \]

and

\[ \frac{i}{\hbar} [\mathcal{H}, a_{\beta}(r_i, t)] = \frac{1}{2} \left\{ \frac{\partial a_{\beta}(r_i, t)}{\partial r_{i\alpha}} \right. \]

\[ + \frac{\partial a_{\beta}(r_i, t)}{\partial r_{i\gamma}} v_{i\gamma} \left\}. \quad (77) \]

The following linear on a field additive to speed corresponds to interaction Eq. (70)

\[ \Delta \tilde{v}_{i\alpha} = \frac{e}{mc} r_{i\beta} \frac{\partial a_{\beta}(r_i, t)}{\partial r_{i\alpha}}, \quad (78) \]

what corresponds, according to Eq. (52), to an additive to the current density

\[ \Delta \tilde{j}_{\alpha}(r) = \frac{e}{mc} d_{\beta}(r) \frac{\partial a_{\beta}(r, t)}{\partial r_\alpha}. \quad (79) \]

Passing to the interaction representation, we obtain the first term in the RHS Eq. (62).

*Strictly speaking, it follows from the Maxwell equations, that if the electric field depends on time, it also depends on coordinate, i.e. derivatives of field components on coordinates are distinct from zero.
X. EXCEPTION OF DIAGONAL ELEMENTS OF OPERATORS OF PARTICLE COORDINATES

Let us return to Eqs. (64) - (69) for average induced current and charge densities at $T = 0$, obtained in section VIII. Taking into account definition (58) of operators $\mathbf{d}(\mathbf{r})$, and also definition (61) of operators $Y_{\beta \gamma}(\mathbf{r})$, which may be rewritten as

$$Y_{\beta \gamma}(\mathbf{r}) = \frac{1}{2} \sum_i (j_{i \beta} r_{i \beta} + r_{i \beta} j_{i \gamma}), \quad (80)$$

we see, that these operators contain coordinates $\mathbf{r}_i$ of particles. But averaged values $\langle 0| j_1(\mathbf{r}, t)|0 \rangle$ and $\langle 0| \rho_1(\mathbf{r}, t)|0 \rangle$ do not owe to depend on an initial point of readout of coordinates $\mathbf{r}_i$. Let’s transform expressions (64) - (69) so that last property became obvious. Let us represent a vector $\mathbf{r}_i$ as two parts

$$\mathbf{r}_i = \bar{\mathbf{r}}_i + \langle 0| \mathbf{r}_i |0 \rangle. \quad (81)$$

It is obvious, that matrix elements of the operator $\bar{\mathbf{r}}_i$, as diagonal, and not diagonal, do not vary at change points of readout of coordinates $\mathbf{r}_i$. We shall show, that in expressions for average values $\langle 0| j_1(\mathbf{r}, t)|0 \rangle$ and $\langle 0| \rho_1(\mathbf{r}, t)|0 \rangle$ the operators $\mathbf{r}_i$ may be replaced by $\bar{\mathbf{r}}_i$. Let’s write down average $\langle 0| j_1(\mathbf{r}, t)|0 \rangle$ as

$$\langle 0| j_1(\mathbf{r}, t)|0 \rangle = \langle 0| j_1(\mathbf{r}, t)|0 \rangle_{\bar{\mathbf{r}}_i} + \sum_i \langle 0| \mathbf{r}_i |0 \rangle x_{i \alpha \beta}(\mathbf{r}, t), \quad (82)$$

where $\langle 0| j_1(\mathbf{r}, t)|0 \rangle_{\bar{\mathbf{r}}_i}$ - is the contribution of the operators $\bar{\mathbf{r}}_i$,

$$x_{i \alpha \beta}(\mathbf{r}, t) = \frac{i}{\hbar} \int d^3 r' \int_{-\infty}^{t} dt' \langle 0| j_1(\mathbf{r}, t), \rho_i(\mathbf{r}', t')| 0 \rangle \times E_\beta(\mathbf{r}', t') + \frac{e}{mc} \langle 0| \rho_i(\mathbf{r})| 0 \rangle \frac{\partial a_\alpha(\mathbf{r}, t)}{\partial r_\alpha}$$

$$- \frac{i}{\hbar c} \int d^3 r' \int_{-\infty}^{t} dt' \langle 0| j_1(\mathbf{r}, t), j_i \gamma(\mathbf{r}', t')| 0 \rangle \times \frac{\partial a_\beta(\mathbf{r}', t')}{\partial r_\gamma}. \quad (83)$$

Having executed in the first and third terms of the RHS Eq. (82) integration on $\mathbf{r}'$, we obtain

$$x_{i \alpha \beta}(\mathbf{r}, t) = - \frac{ie}{\hbar c} \int_{-\infty}^{t} dt' \times \langle 0| \left[ j_1(\mathbf{r}, t), \frac{\partial a_\beta(\mathbf{r}, t')}{\partial t'} \right]| 0 \rangle$$

$$+ \frac{e}{mc} \langle 0| \rho_1(\mathbf{r})| 0 \rangle \frac{\partial a_\alpha(\mathbf{r}, t)}{\partial r_\alpha}, \quad (84)$$

where the designation for the full time derivative is used

$$\frac{da_\beta(\mathbf{r}_i, t)}{dt} = \frac{d}{dt} \left( e^{iHt/\hbar} a_\beta(\mathbf{r}_i, t) e^{-iHt/\hbar} \right)$$

$$= e^{iHt/\hbar} \frac{da_\beta(\mathbf{r}_i, t)}{dt} e^{-iHt/\hbar}, \quad (85)$$

and the derivative $da_\beta(\mathbf{r}_i, t)/dt$ is determined in Eq. (76). Having executed in the first term of the RHS Eq. (83) integration on $t'$ and having calculated the commutator $[j_1(\mathbf{r}), a_\beta(\mathbf{r}, t)]$, we obtain the result

$$\langle 0| j_1(\mathbf{r}, t)|0 \rangle = \langle 0| j_1(\mathbf{r}, t)|0 \rangle_{\bar{\mathbf{r}}_i} = \langle 0| \mathbf{d}(\mathbf{r})|0 \rangle. \quad (87)$$

It is similarly possible to obtain

$$\langle 0| \rho_1(\mathbf{r}, t)|0 \rangle = \langle 0| \rho_1(\mathbf{r}, t)|0 \rangle_{\bar{\mathbf{r}}_i} = \langle 0| \mathbf{f}(\mathbf{r})|0 \rangle. \quad (88)$$

Let us represent Eqs. (87) and (88) as two parts

$$\langle 0| j_1(\mathbf{r}, t)|0 \rangle = \langle 0| j_1(\mathbf{r}, t)|0 \rangle_{\bar{\mathbf{r}}_i} + \langle 0| j_1(\mathbf{r}, t)|0 \rangle_{\mathbf{r}_i}, \quad (89)$$

$$\langle 0| \rho_1(\mathbf{r}, t)|0 \rangle = \langle 0| \rho_1(\mathbf{r}, t)|0 \rangle_{\bar{\mathbf{r}}_i} + \langle 0| \rho_1(\mathbf{r}, t)|0 \rangle_{\mathbf{r}_i}, \quad (90)$$

where

$$\langle 0| j_1(\mathbf{r}, t)|0 \rangle_{\bar{\mathbf{r}}_i} = \frac{i}{\hbar} \int d^3 r' \int_{-\infty}^{t} dt' \langle 0| [j_1(\mathbf{r}, t), \tilde{d}_\beta(\mathbf{r}', t')]|0 \rangle E_\beta(\mathbf{r}', t')$$

$$- \frac{i}{\hbar c} \int d^3 r' \int_{-\infty}^{t} dt' \langle 0| [j_1(\mathbf{r}, t), \tilde{Y}_{\beta \gamma}(\mathbf{r}', t')]|0 \rangle \times \frac{\partial a_\beta(\mathbf{r}', t')}{\partial r_\gamma}, \quad (91)$$

$$\langle 0| \rho_1(\mathbf{r}, t)|0 \rangle_{\bar{\mathbf{r}}_i} = \frac{i}{\hbar c} \int d^3 r' \int_{-\infty}^{t} dt' \langle 0| \rho(\mathbf{r}, t), \tilde{d}_\beta(\mathbf{r}', t')|0 \rangle E_\beta(\mathbf{r}', t')$$

$$\times \langle 0| [\rho(\mathbf{r}, t), \tilde{Y}_{\beta \gamma}(\mathbf{r}', t')]|0 \rangle \frac{\partial a_\beta(\mathbf{r}', t')}{\partial r_\gamma}, \quad (92)$$

$$\langle 0| \rho_1(\mathbf{r}, t)|0 \rangle_{\bar{\mathbf{r}}_i} = \frac{i}{\hbar c} \int d^3 r' \int_{-\infty}^{t} dt' \langle 0| \rho(\mathbf{r}, t), \tilde{Y}_{\beta \gamma}(\mathbf{r}', t')|0 \rangle E_\beta(\mathbf{r}', t')$$

$$\times \langle 0| [\rho(\mathbf{r}, t), \tilde{Y}_{\beta \gamma}(\mathbf{r}', t')]|0 \rangle \frac{\partial a_\beta(\mathbf{r}', t')}{\partial r_\gamma}, \quad (93)$$

$$\langle 0| \rho_1(\mathbf{r}, t)|0 \rangle_{\bar{\mathbf{r}}_i} = \frac{i}{\hbar c} \int d^3 r' \int_{-\infty}^{t} dt'$$

$$\times \langle 0| [\rho(\mathbf{r}, t), \tilde{Y}_{\beta \gamma}(\mathbf{r}', t')]|0 \rangle \frac{\partial a_\beta(\mathbf{r}', t')}{\partial r_\gamma}, \quad (94)$$

and

$$\mathbf{d}(\mathbf{r}) = e \sum_i \tilde{\mathbf{r}}_i \rho(\mathbf{r}), \quad (95)$$

$$\tilde{Y}_{\beta \gamma}(\mathbf{r}) = \frac{1}{2} \sum_i (j_{i \beta} \tilde{r}_{i \beta} + \tilde{r}_{i \beta} j_{i \gamma}). \quad (96)$$
Results (89) - (94) are basic in the present work. Let’s emphasize, that splittings Eq. (89) and Eq. (90) average sizes on two parts do not coincide with splittings Eq. (64) and Eq. (65).

The contributions with an index $I$ we shall name basic, as they do not disappear in case of an electrical field independent from coordinates $r$. Contributions $II$ contain derivative from an electrical field on coordinates.

XI. THE CONDUCTIVITY TENSOR, DEPENDENT ON COORDINATES

Let us consider at first only basic part of the induced current density, designated in Eq. (90) by index $I$. We shall write down Eq. (91) as

$$
(0|j_{I\alpha}(r,t)|0)_{I} = \int d^{3}r \int_{-\infty}^{\infty} dt' l_{\alpha\beta}(r,t,r',t')E_{\beta}(r',t'),
$$

(97)

where

$$
l_{\alpha\beta}(r,t,r',t') = \frac{i}{\hbar} \Theta(t-t')\langle 0| [j_{\alpha}(r,t), \bar{d}_{\beta}(r',t')]|0\rangle.
$$

(98)

Let us enter a tensor

$$
s_{I\alpha\beta}(r',t'|r,t) = l_{\alpha\beta}(r,t,r'-r',t-t').
$$

(99)

Designation Eq. (99) with a partition we have borrowed from [11]. Then it is possible to rewrite Eq. (97) as

$$
(0|j_{I\alpha}(r,t)|0)_{I} = \int d^{3}r' \int_{-\infty}^{\infty} dt' s_{I\alpha\beta}(r',t'|r,t)E_{\beta}(r'-r',t-t'),
$$

(100)

where

$$
s_{I\alpha\beta}(r',t'|r,t) = \frac{i}{\hbar} \Theta(t'-t)\langle 0| [j_{I\alpha}(r,t), \bar{d}_{\beta}(r'-r',t-t')]|0\rangle.
$$

(101)

It is visible from Eq. (101), that the tensor $s_{I\alpha\beta}(r',t'|r,t)$ does not depend on $t$.

Now we shall make the Fourier transformation. Let us write down electric field as

$$
E_{\alpha}(r,t) = E_{\alpha}^{(+)}(r,t) + E_{\alpha}^{(-)}(r,t),
$$

(102)

where

$$
E_{\alpha}^{(+)}(r,t) = \frac{1}{(2\pi)^{3}} \int d^{3}k \int_{0}^{\infty} d\omega E_{\alpha}(k,\omega) e^{ikr-\omega t},
$$

(103)

$$
E_{\alpha}^{(-)}(r,t) = (E_{\alpha}^{(+)}(r,t))^*,
$$

(104)

$$
E_{\alpha}(k,\omega) = \int d^{3}r \int_{-\infty}^{\infty} dt E_{\alpha}(r,t)e^{-i\omega t}. 
$$

(105)

Let us enter a Fourier-image of the tensor $\sigma_{I\alpha\beta}(r',t'|r,0)$ on variables $r', t'$

$$
\sigma_{I\alpha\beta}(k,\omega|r) = \int d^{3}r' \int_{-\infty}^{\infty} \sigma_{I\alpha\beta}(r',t'|r,0) e^{i\omega t'-i kr'}. 
$$

(106)

Then

$$
\langle 0|j_{I\alpha}(r,t)|0\rangle = \langle 0|j_{I\alpha}(r,t)|0\rangle_{I}^{(+)} + \langle 0|j_{I\alpha}(r,t)|0\rangle_{I}^{(-)} 
$$

(107)

Substituting Eq. (101) in Eq. (106), we obtain

$$
\sigma_{I\alpha\beta}(k,\omega|r) = \frac{i}{\hbar} \int d^{3}r' \int_{-\infty}^{\infty} \Theta(t')e^{-i kr'+i\omega t'}
$$

$$
\times \langle 0|[j_{\alpha}(r), \bar{d}_{\beta}(r'-r',t-t')]|0\rangle. 
$$

(108)

By similar way we find the contribution in conductivity with an index $II$. Finally we obtain

$$
\langle 0|j_{II\alpha}(r,t)|0\rangle_{II}^{(+)} = \frac{1}{(2\pi)^{3}} \int d^{3}k \int_{0}^{\infty} d\omega \sigma_{II\alpha\beta}(k,\omega|r)E_{\beta}(k,\omega) e^{i kr - i \omega t},
$$

(111)

$$
\sigma_{II\alpha\beta}(k,\omega|r) = \frac{e}{\hbar c} \langle 0| \bar{d}_{\beta}(\bar{r})|0\rangle
$$

$$
- \frac{i k_{\gamma}}{m \omega} \int d^{3}r' \int_{-\infty}^{\infty} dt' \Theta(t') e^{-i kr' + i \omega t'}
$$

$$
\times \langle 0|[j_{\alpha}(r), \bar{d}_{\beta}(r'-r',-t')]|0\rangle. 
$$

(113)

The conductivity tensor $\sigma_{I\alpha\beta}(k,\omega|r)$ does not depend from coordinates $r$ only in case of spatially - homogeneous system. At consideration of semiconductor objects of the lowered dimension dependence of the conductivity tensor from $r$ is rather essential. Dependence of the conductivity tensor of the low dimensional semiconductor objects on $r$ is rather essential.

In all our previous works (see, for example, [12,13,14]) Eq. (91) was used for calculations of an induced density of a current.
XII. TRANSITION TO EXPRESSIONS CONTAINING MAGNETIC FIELD

Till now we left behind frameworks of our consideration a case, when electric field $\mathbf{E}$ does not depend on time, in particular the case $\mathbf{E} = 0$, $\mathbf{H}(r, t) = \text{const}$. To consider the last case we shall transform expressions for the average induced current and charge densities obtained at the end of section VIII, having entered in them magnetic field $\mathbf{H}(r, t)$. For this purpose each of values $<0|j_{1\alpha}(r, t)|0>$ and $<0|\rho_1(r, t)|0>$ determined, respectively, in Eqs. (68) and (69), we shall break on two parts as follows:

\[
<0|j_{1\alpha}(r, t)|0>_{\partial E/\partial r} = <0|j_{1\alpha}(r, t)|0>^{(+)} + <0|j_{1\alpha}(r, t)|0>^{(-)},
\]

\[
<0|\rho_1(r, t)|0>_{\partial E/\partial r} = <0|\rho_1(r, t)|0>^{(+)} + <0|\rho_1(r, t)|0>^{(-)},
\]

where

\[
<0|j_{1\alpha}(r, t)|0>^{(\pm)} = \frac{e}{2mc}<0|d_{\beta}(r)|0> \times \left( \frac{\partial a_\beta(r, t)}{\partial r_\alpha} \pm \frac{\partial a_\alpha(r, t)}{\partial r_\beta} \right).
\]

\[
- \frac{i}{2\hbar c} \int d\mathbf{r}' \int_{-\infty}^{t} dt' \langle 0|j_\alpha(r, t), Y_{\beta\gamma}(r', t')|0 \rangle \times \left( \frac{\partial \alpha_\beta(r', t')}{\partial r'_\gamma} \pm \frac{\partial \alpha_\gamma(r', t')}{\partial r'_\beta} \right),
\]

\[
<0|\rho_1(r, t)|0>^{(\pm)} = - \frac{i}{2\hbar c} \int d\mathbf{r}' \int_{-\infty}^{t} dt' \langle 0|\rho(r, t), Y_{\beta\gamma}(r', t')|0 \rangle \\
\times \left( \frac{\partial \alpha_\beta(r', t')}{\partial r'_\gamma} \pm \frac{\partial \alpha_\gamma(r', t')}{\partial r'_\beta} \right),
\]

At first we shall consider contributions with index (-).

Let us return to vector $\mathbf{A}(r, t)$ and scalar $\varphi(r, t)$ potentials. Taking into account definition Eq. (30) for the vector $\mathbf{a}(r, t)$ and first of the formulas Eq. (1), we obtain

\[
\mathbf{a}(r, t) = \mathbf{A}(r, t) + c \int_{-\infty}^{t} dt' \partial \varphi(r', t')/\partial \mathbf{r}.
\]

Having substituted Eq. (119) in the RHSs of expressions $<0|j_{1\alpha}(r, t)|0>^{(-)}$ and $<0|\rho_1(r, t)|0>^{(-)}$, we find, that contributions from scalar potential $\varphi$ become zero, and

\[
\langle 0|j_{1\alpha}(r, t)|0>^{(-)} = \frac{e}{2mc} \langle 0|d_{\beta}(r)|0 >
\times \left( \frac{\partial A_\beta(r, t)}{\partial r_\alpha} - \frac{\partial A_\alpha(r, t)}{\partial r_\beta} \right) - \frac{i}{2\hbar c} \int d\mathbf{r}' \int_{-\infty}^{t} dt' \langle 0|j_\alpha(r, t), Y_{\beta\gamma}(r', t')|0 \rangle \\
\times \left( \frac{\partial A_\beta(r', t')}{\partial r'_\gamma} - \frac{\partial A_\gamma(r', t')}{\partial r'_\beta} \right),
\]

\[
<0|\rho_1(r, t)|0>^{(-)} = - \frac{i}{2\hbar c} \int d\mathbf{r}' \int_{-\infty}^{t} dt' \langle 0|\rho(r, t), Y_{\beta\gamma}(r', t')|0 \rangle \\
\times \left( \frac{\partial A_\beta(r', t')}{\partial r'_\gamma} - \frac{\partial A_\gamma(r', t')}{\partial r'_\beta} \right).
\]

Taking into account, that $\mathbf{H}(r, t) = \text{rot} \mathbf{A}(r, t)$ (second equality from Eq. (1)), we obtain easily

\[
<0|j_{1\alpha}(r, t)|0>^{(-)} = - \frac{e}{2mc} (\mathbf{H}(r, t) \times \mathbf{r})_\alpha <0|\rho(r)|0>
\times \left( \frac{\partial A_\beta(r, t)}{\partial r_\alpha} - \frac{\partial A_\alpha(r, t)}{\partial r_\beta} \right).
\]

\[
<0|\rho_1(r, t)|0>^{(-)} = - \frac{i}{2\hbar c} \int d\mathbf{r}' \int_{-\infty}^{t} dt' \langle 0|\rho(r, t), Y_{\beta\gamma}(r', t')|0 \rangle \\
\times \left( \frac{\partial A_\beta(r', t')}{\partial r'_\gamma} - \frac{\partial A_\gamma(r', t')}{\partial r'_\beta} \right).
\]

Thus, the values $<0|j_{1\alpha}(r, t)|0>^{(-)}$ and $<0|\rho_1(r, t)|0>^{(-)}$ are expressed through magnetic field.

Now we shall transform Eqs. (117) and (118) for the contributions with an index (+). We work under the following circuit. Let us consider, for example, Eq. (117). In the RHS part

\[
Y_{\beta\gamma}(r') = r'_{\beta} j_{\gamma}(r')
\]

enters; for $j_\gamma(r)$ it is easy to deduce a ratio

\[
j_{\gamma}(r) = \hat{d}_{\gamma}(r) + \partial Y_{\gamma\beta}(r)/\partial r_\beta.
\]

Let us substitute Eqs. (124) and (125) in the second term of the RHS Eq. (117), which in result breaks up on two contributions - occurring from $\partial Y_{\gamma\beta}(r)/\partial r_\beta$ and from $\hat{d}_{\gamma}(r)$. In the first contribution we integrate in parts on $r_\beta$, in second - on variable $t'$ also in parts. It results in

\[
<0|j_{1\alpha}(r, t)|0>^{(+)} = - <0|j_{1\alpha}(r, t)|0>^{(+)}
\times \frac{e}{mc} <0|\rho(r)|0> r_\beta r_\gamma \frac{\partial^2 a_\beta(r, t)}{\partial r_\alpha \partial r_\gamma}
\times \left( \frac{\partial A_\beta(r, t)}{\partial r_\alpha} - \frac{\partial A_\alpha(r, t)}{\partial r_\beta} \right) - \frac{i}{\hbar c} \int d\mathbf{r}' r'_{\beta} r'_{\gamma} \int_{-\infty}^{t} dt' \langle 0|j_\alpha(r, t), j_\beta(r', t')|0 \rangle.
\]

\footnote{Here top indexes (+) and (-) have no any relation to the same indexes in section XI.}
Using a ratio

\[ -c^{-1} \partial a_{\beta}(r, t)/\partial t = E_\beta(r, t), \]

we obtain finally from Eq. (125):

\[
\langle 0 | j_{1\alpha}(r, t) | 0 \rangle^{(+)} = -\frac{e}{2mc} \langle 0 | \rho(r) | 0 \rangle r_\alpha r_\gamma \int dt' \langle 0 | j_{\alpha}(r, t), j_\beta(r', t') | 0 \rangle \\
\times \frac{\partial a_{\beta}(r', t')}{\partial r_\gamma} \\
+ \frac{i}{2\hbar c} \int dt' \langle 0 | j_{\alpha}(r, t), j_\beta(r', t') | 0 \rangle \\
\times \frac{\partial a_{\beta}(r', t')}{\partial r_\gamma} \\
\times \frac{\partial a_{\beta}(r', t')}{\partial t}.
\]

and, similarly,

\[
\langle 0 | \rho(r) | 0 \rangle^{(+)} = \frac{i}{2\hbar c} \int dt' \langle 0 | j_{\alpha}(r, t), j_\beta(r', t') | 0 \rangle \\
\times \frac{\partial a_{\beta}(r', t')}{\partial r_\gamma} \\
\times \frac{\partial a_{\beta}(r', t')}{\partial t}.
\]

Let us notice, that unlike Eqs. (68) and (69), Eqs. (127) and (128) contain only second derivative from a vector \( \mathbf{a}(r, t) \). The obtained results can be written down in the symmetric form:

\[
\langle 0 | j_{1\alpha}(r, t) | 0 \rangle^{(+)} = -\frac{e}{4mc} \langle 0 | \rho(r) | 0 \rangle r_\alpha r_\gamma \\
\times \frac{\partial}{\partial r_\alpha} (\partial a_{\beta}(r, t)/\partial r_\gamma + \partial a_{\gamma}(r, t)/\partial r_\beta) \\
+ \frac{i}{4\hbar c} \int dt' \langle 0 | j_{\alpha}(r, t), \Omega_{\beta\gamma}(r', t') | 0 \rangle,
\]

\[
\langle 0 | \rho(r) | 0 \rangle^{(+)} = \frac{i}{4\hbar c} \\
\times \int dt' \langle 0 | \rho(r, t), \Omega_{\beta\gamma}(r', t') | 0 \rangle
\]

where the symmetric tensor

\[
\Omega_{\beta\gamma}(r, t) = j_\gamma(r, t) \frac{\partial}{\partial r_\beta} (\partial a_{\beta}(r, t)/\partial r_\gamma + \partial a_{\gamma}(r, t)/\partial r_\beta) \\
+ \rho(r, t) \frac{\partial}{\partial t} (\partial a_{\beta}(r, t)/\partial r_\gamma + \partial a_{\gamma}(r, t)/\partial r_\beta).
\]

XIII. CONSTANT MAGNETIC FIELD

Let us consider a case of a constant in time and space magnetic fields \( \mathbf{H} = \text{const}, \mathbf{E} = 0 \). Let us remind, that we have included constant magnetic field in unperturbed Hamiltonian \( \mathcal{H} \) (see section II). But in the present section we do not include a field in our Hamiltonian and consider it so weak, that it is possible to be limited by linear on field contributions to induced current and charge densities. As \( \mathbf{E} = 0 \), contributions to the average induced by \( \mathbf{E} \) densities are equal 0 (See Eqs. (66) and (67)). The contributions with indexes \( \partial \mathbf{E}/\partial r \) are broken on two parts, which we have designated by the top indexes (+) and (-). Parts with indexes (+) are equal 0. It is easy to be convinced of it if to choose vector and scalar potentials, for example, in gage \( \mathbf{A}(r) = (1/2) \times [r \times \mathbf{H}], \quad \varphi = 0 \)

and to use Eqs. (118), (126) and (127).

So, there are only contributions with indexes (-), determined in Eqs. (122) and (123). Having put \( \mathbf{H}(r, t) = \mathbf{H} \) and having made replacement variable \( t' \) by \( t'' = t' - t \), we obtain time-independent results:

\[
\langle 0 | j_{1\alpha}(r, t) | 0 \rangle_H = -\frac{e}{2mc} \langle 0 | \rho(r) | 0 \rangle + \frac{i}{2\hbar c} \\
\times \int dr' \int_0^0 dt'' (\mathbf{H} \times r')_\beta \langle 0 | [j_{\alpha}(r), j_\beta(r', t'')] | 0 \rangle,
\]

\[
\langle 0 | \rho_1(r) | 0 \rangle_H = \frac{i}{2\hbar c} \\
\times \int dr' \int_0^0 dt'' (\mathbf{H} \times r')_\beta \langle 0 | [\rho(r), j_\beta(r', t'')] | 0 \rangle.
\]

The indexes \( H \) in the LHSs mean \( \mathbf{H} = \text{const}, \mathbf{E} = 0 \).

In obtained expressions we shall pass from operators \( \mathbf{r}_i \) to the operators \( \bar{\mathbf{r}}_i \), determined in Eq. (81). By a way, similar to a way stated in section X, we obtain

\[
\langle 0 | j_{1\alpha}(r, t) | 0 \rangle_H = -\frac{e}{2mc} \langle 0 | \mathbf{H} \times \bar{\mathbf{r}}_\alpha | 0 \rangle \\
+ \frac{ie}{2\hbar c} \int dr' \int_0^0 dt' \sum_i \langle 0 | j_{\alpha}(r, -t'), (\mathbf{H} \times \bar{\mathbf{r}}_i)_\beta v_{\beta i} | 0 \rangle,
\]

\[
\langle 0 | \rho_1(r) | 0 \rangle_H = \frac{ie}{2\hbar c} \\
\times \int_0^0 dt' \sum_i \langle 0 | [\rho(r, -t'), (\mathbf{H} \times \bar{\mathbf{r}}_i)_\beta v_{\beta i}] | 0 \rangle.
\]

Let us note that the relations are performed

\[
\text{div}(\mathbf{0}) j_1(r, t) | 0 \rangle_H = 0, \quad \int_0^0 \langle 0 | j_1(r, t) | 0 \rangle_H dr = 0.
\]
XIV. CONCLUSION

Let us consider the basic results. Since operator of interaction of charged particles with electromagnetic field is expressed through potentials $\mathbf{A}(\mathbf{r}, t)$ and $\varphi(\mathbf{r}, t)$, but not through electric $\mathbf{E}(\mathbf{r}, t)$ and magnetic $\mathbf{H}(\mathbf{r}, t)$ fields, initial expressions for average values of induced current and charge densities also are expressed through potentials (see Eqs. (17) and (18)). But Eq. (17) for a current density is inconvenient, since it contains the contribution proportional to average $\langle 0|\rho(\mathbf{r}, t)|0 \rangle$ of a charge density, and this contribution is not completely small.

Therefore we have put a task to express induced densities through fields $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{H}(\mathbf{r}, t)$, that is certainly feasible, since the average current and charge densities are observable values.

In result Eqs. (31) and (32) were obtained, containing only electric field. However, Eq. (31) has the same lack, as Eq. (17), - it contains the contribution proportional $\langle 0|\rho(\mathbf{r}, t)|0 \rangle$.

If to use an approximation, in which the electric field $\mathbf{E}(\mathbf{r}, t)$ does not depend on coordinates, but only on time, the contribution proportional $\langle 0|\rho(\mathbf{r}, t)|0 \rangle$, may be removed, if to enter in expression for average current density the operators $\mathbf{r}_i$ of $i$th particle coordinates , having taken a ratio $v_i = (i / \hbar)[\mathcal{H}, \mathbf{r}_i]$. This reception in essence is used in $\mathbb{R}$, where the theory of light scattering in bulk crystals is discussed. In the appendix A it is shown, how the same reception allows to exclude the charged particles concentration from the formula for conductivity of intrinsic bulk semiconductors at $T = 0$.

The same result turns out, if in approximation $\mathbf{E}(\mathbf{r}, t) \simeq \mathbf{E}(t)$ to write down interaction of particles with a field as $-e \sum \mathbf{r}_i \mathbf{E}_i(t)$, as it is made in $\mathbb{R}$.

However, the task becomes complicated if to take into account dependence of an electric field on coordinates. We have put to ourselves by the purpose to obtain such expression for induced current density, which would pass in the Kubo-formula at transition from $\mathbf{E}(\mathbf{r}, t)$ to $\mathbf{E}(t)$, and similar expressions for induced charge density. For a case $T = 0$ this task is solved in sections VII - IX.

Essence of used reception consists in the following. The operator of interaction of particles with a field, expressed through electric and magnetic fields, does not exist. But we enter a fictitious operator of interaction, which results into correct results for average on the ground state of system values of the induced current and charge densities. This fictitious operator $U_f^t$ Eq. (37) is expressed only through an electric field $\mathbf{E}(\mathbf{r}, t)$, in an integral on time. Then such transformation of interactions $U_f^t \rightarrow \tilde{U}_f$ is made, which does not change values of average densities, but eliminate the contribution containing $\langle 0|\rho(\mathbf{r}, t)|0 \rangle$ from expression for average induced current density. In result Eqs (64) - (69) for $\langle 0|\mathbf{j}_1(\mathbf{r}, t)|0 \rangle$ and $\langle 0|\rho_1(\mathbf{r}, t)|0 \rangle$ are obtained, in which the basic contributions contain an electric field, and additional - the derivatives from $\mathbf{E}(\mathbf{r}, t)$ on coordinates. The transformed fictitious interaction $\tilde{U}_f$ also is divided on basic and additional parts. The first is equal $-e \sum \mathbf{r}_i \mathbf{E}_i(t)$, the second contains derivative $\partial \mathbf{E}_i(\mathbf{r}, t)/\partial t$ in an integral on time.

In section X Eqs. (64) - (69) for average densities of induced currents and charges are transformed so, that obviously do not depend on an origin of coordinates $\mathbf{r}_1$ of particles. It is the basic result of the present work. In section X from Eqs. (64) - (69) for average induced currents and charges densities the diagonal elements $\langle 0|\rho_1|0 \rangle$ are excluded. The results Eqs. (90) - (96), containing only non-diagonal elements of the operators $\mathbf{r}_i$, are obtained.

This is the main result of the present work. In section XI the expressions for the conductivity tensor $\sigma(\mathbf{k}, \omega|\mathbf{r})$ are obtained, dependent from coordinates, in a case of spatially-heterogeneous systems.

Further in section XII from expressions for $\langle 0|\mathbf{j}_1(\mathbf{r}, t)|0 \rangle$ and $\langle 0|\rho_1(\mathbf{r}, t)|0 \rangle$, containing only electric field and derivative from $i$ on coordinates, we pass to expressions containing also magnetic field $\mathbf{H}(\mathbf{r}, t)$. It is necessary for reception of results in case $\mathbf{H} = \text{const}, \mathbf{E} = 0$, which is considered in section XIII.

In the Appendix B the expression for the operator of total acceleration of system of the charged particles is obtained. The acceleration is caused by an external weak electromagnetic field. It is shown, that averaged on to the basic state of system the total acceleration can be expressed through electrical and magnetic fields. In the case of free particles the obtained result passes to a correct limit containing a force caused by an electric field, and the Lorentz force. Further it is supposed to use obtained results for construction of the general theory of secondary radiation from low-dimensional semiconductor objects.

The work has obtained the partial financial support from the Russian Fund of basic researches (00-02-16904), Program MNTK "Physics of semiconductor nano-structures” and the Federal Program "Integration".

APPENDIX A

Three expressions for the conductivity tensor in a case of a spatially - homogeneous matter and an electric field independent on coordinates.

Proceeding from Eq. (31) for the average induced current density and taking into account a following connection between the Fourier-components

$$ a(\mathbf{k}, \omega) = (i\epsilon/\omega)|\mathbf{E}(\mathbf{k}, \omega), $$

(A1)

following from definition Eq. (30), we obtain

$$
\sigma_{\alpha\beta}(\mathbf{k}, \omega|\mathbf{r}) = \frac{i\epsilon}{m\omega} \langle 0|\rho(\mathbf{r})|0 \rangle \delta_{\alpha\beta} + \frac{1}{\hbar\omega} \int dr' \int_{-\infty}^{\infty} dt' \\
\times e^{-i\mathbf{k}\cdot\mathbf{r}'} + i\omega t' \Theta(t') \langle 0|\mathbf{j}_\alpha(\mathbf{r}), \mathbf{j}_\beta(\mathbf{r} - \mathbf{r}', \mathbf{r} - t')|0 \rangle.
$$

(A2)
In a case of spatially - homogeneous system the tensor 
\( \sigma_{\alpha \beta}(k, \omega | r) \) does not depend on \( r \). In the first term we use a ratio
\[
\langle 0 | \rho(r) | 0 \rangle = cn, \tag{A3}
\]
where \( n \) is the concentration of charged particles, in the second term we integrate on \( r \) and \( r' \), also we shall divide the result on the normalized volume \( V_0 \). We obtain
\[
\sigma_{\alpha \beta}(k, \omega) = \frac{ie^2 n}{m \omega} \delta_{\alpha \beta} + \frac{e^2}{4m^2 \hbar \omega V_0} \sum_{i,j} \int_{-\infty}^{\infty} dt \Theta(t) e^{i \omega t}
\times \left\{ \left[ e^{i H t / \hbar} (e^{-i k r_i} p_i) + p_i e^{-i k r_i} \right], (e^{i k r_j} p_j + p_j e^{i k r_j}) \right\} \langle 0 \rangle,
\tag{A4}
\]
where \( p_i \) is the momentum operator determined in Eq. (4) with the account of a constant strong magnetic field.

In case of electric field \( E(t) \), independent on coordinates, we enter also conductivity \( \sigma_{\alpha \beta}(\omega | r) \), included in definition of an average current density
\[
\langle 0 | j_{\alpha}(r, t) | 0 \rangle^+ = \frac{1}{2\pi} \int_0^\infty d\omega \sigma_{\alpha \beta}(\omega | r) E_\alpha(\omega) e^{-i \omega t},
\tag{A7}
\]
where the index \( h \) means spatially-homogeneous electric field.

Using section XI, it is easy to show, that
\[
\sigma_{\alpha \beta}(\omega | r) = \sigma_{\alpha \beta}(k = 0, \omega | r).
\tag{A8}
\]

From (A4) and (A8) in a case of spatially - homogeneous matter and the field \( E(t) \) we obtain
\[
\sigma_{\alpha \beta}^I(\omega) = \frac{ie^2 n}{m \omega} \delta_{\alpha \beta} + \frac{e^2}{h \omega V_0} \int_{-\infty}^{\infty} dt \Theta(t) e^{i \omega t} \langle 0 | [V_\alpha(t), V_\beta] \rangle,
\tag{A9}
\]
where \( V_\alpha = (1/m) \sum p_\alpha \) is the operator of total speed of charged particles. (A9) is the first formula for the conductivity tensor. For reception two others we use a ratio
\[
V_\alpha = (i/\hbar)[H, R_\alpha],
\tag{A10}
\]
where \( R_\alpha = \sum r_\alpha \). Having substituted (A10) in (A8), we obtain
\[
\sigma_{\alpha \beta}(\omega) = \frac{ie^2 n}{m \omega} \delta_{\alpha \beta} + \frac{e^2}{h \omega V_0} \int_{-\infty}^{\infty} dt \Theta(t)
\times e^{i \omega t} \frac{d}{dt} \langle 0 | [R_\alpha(t), V_\beta] | 0 \rangle.
\tag{A11}
\]

Let us execute integration on \( t \) in parts. At \( t \to \infty \) function \( \Theta(t) e^{i \omega t} \frac{d}{dt} \langle 0 | [R_\alpha(t), V_\beta] | 0 \rangle \to 0 \) at replacement \( \omega \to i \delta, \delta \to 0 \). Let us take into account also, that \( d\Theta(t)/dt = \delta(t) \) and
\[
(1/V_0)[R_\alpha, V_\beta] = (i \hbar n/m) \delta_{\alpha \beta},
\tag{A12}
\]
where we shall obtain
\[
\sigma_{\alpha \beta}^{III}(\omega) = \frac{e^2 n}{h \omega} \int_{-\infty}^{\infty} dt \Theta(t) e^{i \omega t} \langle 0 | [D_\alpha(t), V_\beta] | 0 \rangle,
\tag{A13}
\]
where the designation is entered
\[
\mathbf{D} = e \mathbf{R}.
\tag{A14}
\]

Eq. (A13) is the second expression for the conductivity tensor. Let us emphasize, that at transition from (A9) to (A13) there was a reduction of the first term from the RHS (A9), containing concentration \( n \).

At transition to the third expression (A10) for the operator \( V_\beta \) from the RHS (A13) is used. Further we work under the same circuit, as at transition from (A9) to (A13). Taking into account, that the commutator \([D_\alpha, D_\beta] = 0\), we obtain the third formula
\[
\sigma_{\alpha \beta}^{III}(\omega) = \frac{\omega}{h V_0} \int_{-\infty}^{\infty} dt \Theta(t) e^{i \omega t} \langle 0 | [D_\alpha(t), D_\beta] | 0 \rangle.
\tag{A15}
\]

It is convenient to apply (A9), (A13) and (A15) to different systems.

For free particles, when \( V(r_1...r_N) = 0 \), \( \mathbf{H} = 0 \), the velocity operator \( \mathbf{V} \) commutes with the Hamiltonian \( \mathcal{H}_{free} = m \sum v_i^2 \). Let us use (A9), taking into account, that \( v_\alpha(t) = v_\alpha, [v_\alpha, v_\beta] = 0 \), and we obtain the known result
\[
\sigma_{\alpha \beta}^{free}(\omega) = \frac{ie^2 n}{m \omega} \delta_{\alpha \beta}.
\tag{A16}
\]

In a case, when exited states are separated from the ground state by an energy gap, i.e. the energy of these states \( E_n = \hbar \omega_n \neq 0 \), it is more convenient to use (A13) or (A15). Using exact wave functions \( |n> \) of the exited states and having calculated integral on time, we shall express (A15) through matrix elements of the operator \( \mathbf{D} \):
\[
\sigma_{\alpha \beta}^{III}(\omega) = \frac{\omega}{h V_0} \sum_n \left\{ \begin{array}{l} <0|D_\alpha |n> <n|D_\beta |0> \\ \omega - \omega_n \end{array} \right\}.
\]
and scalar potentials. Having executed integration on of system of particles we use Eq. (14), containing vector state 
\[ \chi_{\alpha\beta}(\omega) = (i/\omega)\sigma_{\alpha\beta}(\omega), \]
so from (A17) we obtain
\[ \chi_{\alpha\beta}(\omega) = \frac{e^2}{\hbar V_0} \sum_{n} \left\{ \frac{<0|D_{\alpha}|n><n|D_{\beta}|0>}{\omega_n - \omega} \right\}, \]
(A18)

Using the ratio between matrix elements
\[ <0|D_{\alpha}|n> = (i e/\omega_n) <0|\nu_{\alpha}|n>, \]
\[ <n|D_{\alpha}|0> = -(i e/\omega_n) <n|\nu_{\alpha}|0>, \]
from (And 18) we find
\[ \chi_{\alpha\beta}(\omega) = \frac{e^2}{\hbar V_0} \sum_{n} \left\{ \frac{<0|\nu_{\alpha}|n><n|\nu_{\beta}|0>}{\omega_n - \omega} \right\}. \]
(A19)

If \( \omega \ll \omega_n \), \( \chi_{\alpha\beta} \) is real and also does not depend on frequency \( \omega \), but at \( \omega \simeq \omega_n \) this dependence becomes strong and there appears an imaginary part, distinct from zero, of the tensor \( \chi_{\alpha\beta} \), determining the resonant light absorption on frequencies \( \omega \simeq \omega_n \). For calculation of an imaginary part it is necessary to replace frequency \( \omega \) by \( \omega + i\gamma/2 \), \( \gamma \to 0 \), when the field is switched on adiabatically, or to replace frequency \( \omega_n \) by \( \omega_n - i\gamma/2 \), i. e. to take into account a final life time of a system in a state \( n \).

**APPENDIX B**

**Acceleration of particles system**

To obtain of the operator \( J_1(t) \) of the induced current of system of particles we use Eq. (14), containing vector and scalar potentials. Having executed integration on \( r \) and \( r' \), we obtain
\[ J_{1\alpha}(t) = -\frac{e^2}{mc} \sum_i A_{\alpha}(r_i(t), t) + \]
\[ + \frac{ie}{\hbar} \int_{-\infty}^{\infty} dt' U_1(t'), \sum_i \nu_{\alpha}(t), \]
(B1)

where the designation is used
\[ A(r_i(t), t) = e^{i\hat{H}t/\hbar} A(r_i, t) e^{-i\hat{H}t/\hbar}. \]

Differentiating (B1) on time and having divided on e, we shall obtain the operator \( W_1(t) \) of the induced total acceleration:
\[ W_{1\alpha}(t) = -\frac{e}{mc} \sum_i \frac{\partial A_{\alpha}(r_i(t), t)}{\partial t} - \frac{e}{m} \sum_i \frac{\partial e}{\partial r_{\alpha}(t)}
\]
\[ + \frac{e}{2mc} \sum_i \left\{ \nu_{\alpha}(t) \left( \frac{\partial A_{\beta}(r_i(t), t)}{\partial r_{\alpha}(t)} - \frac{\partial A_{\alpha}(r_i(t), t)}{\partial r_{\beta}(t)} \right) \right\}
\]
\[ - (\nu/\omega)^2 \sum_i [A(r_i(t), t) \times \mathbf{H}_c]_{\alpha} \]
\[ + (i/\hbar) \int_{-\infty}^{t} dt' [U_1(t'), W_{\alpha}(t)]. \]
(B3)

where the designations are used:
\[ \frac{\partial A_{\alpha}(r_i(t), t)}{\partial t} = e^{i\hat{H}t/\hbar} A_{\alpha}(r_i, t) e^{-i\hat{H}t/\hbar}, \]
\[ \frac{\partial e}{\partial r_{\alpha}(t)} = e^{i\hat{H}t/\hbar} \frac{\partial e}{\partial r_{\alpha}(t)} e^{-i\hat{H}t/\hbar}, \]
\( \mathbf{H}_c \) is the constant strong magnetic field included in the basic Hamiltonian \( \hat{H} \). At transition from (B2) to (B3) the ratio is used \( \nu_i \times \nu_i = (i\hbar e/m^2c) H_{cz} \), and the LHS \( \neq 0 \) because of non-commutativity of various projections of the velocity, for example
\[ [\nu_i \times \nu_i]_{z} = [\nu_{ix}, \nu_{iy}] = (i\hbar c/m^2c) H_{cz}. \]
(B4)

The operator \( w_{\alpha}(t) \) of acceleration \( \alpha \)th particle is equal
\[ w_{\alpha}(t) = \frac{i}{\hbar} e^{i\hat{H}t/\hbar} \left[ \mathbf{H}, \nu_{\alpha}(t) \right] e^{-i\hat{H}t/\hbar}. \]
Therefore (B3) will be transformed to
\[ e^{i\alpha t/\hbar} \left( (v_i \times H(r_i, t))_\alpha - (H(r_i, t) \times v_i)_\alpha \right) e^{-i\alpha t/\hbar}. \]

(B5)

With the help Eq. (1) it is easy to see, that the expression in braces in (B3) is equal
\[ e^{i\alpha t/\hbar} \left( (v_i \times H(r_i, t))_\alpha - (H(r_i, t) \times v_i)_\alpha \right) e^{-i\alpha t/\hbar}. \]

Therefore (B3) will be transformed to
\[ W_1(t) = \frac{e}{m} \sum_i E(r_i(t), t) + \frac{e}{2mc} \sum_i \{v_i(t) \times H(r_i(t), t) - H_i(r_i(t), t) \times v_i(t)\} \]
\[ - \left( \frac{e}{mc} \right)^2 \sum_i A(r_i(t), t) \times H_c \]
\[ + \frac{i}{\hbar} \int_{-\infty}^{t} dt' [U_1(t'), W(t)]. \]

(B6)

It is obvious, that the second term corresponds to the Lorentz force, at which the non-commutativity of the operators \(v_i\) and \(H(r_i, t)\) is taken into account. The third term caused by \(H_c\), contains the additive to speed \(\Delta v_i\), determined in Eq. (8) and induced by a weak electromagnetic field.

In the case of free particles
\[ H_c = 0, \quad V(r_1, \ldots, r_N) = 0, \quad W = 0, \]

and in the RHS (B6) only two first terms containing weak electric and magnetic fields are kept.

But if particles are not free, the operator (B6) fails to be expressed only through fields, since the last two terms contain vector and scalar potentials. Average value \(\langle 0|W_1(t)|0 \rangle\) of the induced acceleration should be expressed only through fields, that we now shall prove. For this purpose we shall calculate the value \(\langle 0|W_1(t)|0 \rangle\) in another way, and then we shall check up, that both ways give coinciding results. Let us use Eq. (27) and integrate its both parts on \(r\). We obtain
\[ \langle 0|W_{1\alpha}(t)|0 \rangle = \frac{e}{m} \sum_i \langle 0|E_{\alpha}(r_i, t)|0 \rangle \]
\[ - \frac{ie}{2\hbar} \int_{-\infty}^{t} dt' \left( \sum_j v_{ij} \sum_i \{v_j E_\beta(r_i(t'), t + t') \right) \]
\[ + E_\beta(r_i(t'), t + t') v_{j\beta}(t') \right) \left| 0 \right\rangle. \]

(B8)

This expression contains only electric fields. On the other hand, having averaged the operator (B6), we obtain
\[ \langle 0|W_{1\alpha}(t)|0 \rangle = \frac{e}{m} \sum_i \langle 0|E_{\alpha}(r_i, t)|0 \rangle \]
\[ + \frac{e}{2mc} \sum_i \langle 0|(v_i \times H(r_i, t))_\alpha - (H(r_i, t) \times v_i)_\alpha|0 \rangle \]
\[ - \frac{e}{mc} \sum_i \langle 0|(A(r_i, t) \times H_c)_\alpha|0 \rangle + C_\alpha(t), \]

(B9)

where
\[ C_\alpha(t) = (i/\hbar) \int_{-\infty}^{t} dt' \langle 0|U_1(t'), W_\alpha(t)|0 \rangle. \]

(B10)

Let us transform (B10). Integrating in parts, we obtain
\[ C_\alpha(t) = C_\alpha^1(t) + C_\alpha^2(t), \]

(B11)

\[ C_\alpha^1(t) = -(i/\hbar) \left\langle 0 \left| \frac{\partial}{\partial r_{\alpha i}} \right| 0 \right\rangle, \]

(B12)

\[ C_\alpha^2(t) = \frac{i}{\hbar} \frac{d}{dt} \int_{-\infty}^{t} dt' \left\langle 0 \left| \sum_i v_{\alpha i}(t) \right| 0 \right\rangle. \]

(B13)

In the RHS (B12) we shall substitute Eq. (5) for \(U_1\) and we shall calculate the commutator. We obtain
\[ C_\alpha^1(t) = \frac{e}{m} \sum_i \left\langle 0 \left| \frac{\partial \varphi(r_i, t)}{\partial r_{\alpha i}} \right| 0 \right\rangle \]
\[ + \left( \frac{e}{mc} \right)^2 \sum_i \langle 0|(A(r_i, t) \times H_c)_\alpha|0 \rangle \]
\[ - \frac{e}{2mc} \sum_i \left\langle 0 \left| v_{i\beta} \frac{\partial A_\beta(r_i, t)}{\partial r_{\alpha i}} + \frac{\partial A_\beta(r_i, t)}{\partial r_{\alpha i}} v_{i\beta} \right| 0 \right\rangle. \]

(B14)

In (B13) in integral we pass to \(t'' = t - t'\) and we break the integral on two parts, first of which contains vector, and second - scalar potentials:
\[ C_\alpha^2(t) = C_{A\alpha}(t) + C_{\varphi\alpha}(t), \]

(B15)

\[ C_{A\alpha}(t) = \frac{ie}{2c\hbar} \int_{-\infty}^{t} dt' \left\langle 0 \left| \sum_j v_{j\alpha} \right| 0 \right\rangle. \]
\begin{align*}
\times \sum_i \left\{ v_{i\beta}(t') \frac{\partial A_{\beta}(r_i(t'), t + t')}{\partial t} + \frac{\partial A_{\beta}(r_i(t'), t + t')}{\partial t} v_{i\beta}(t') \right\} \right|_0 = 0, \quad (B16)
\end{align*}

\[ C^2_{\phi\alpha}(t) = -\frac{ie}{\hbar} \int_0^\infty dt' \]

\[ \times \left\langle 0 \left| \left[ \sum_j v_{j\alpha}, \sum_i \frac{\partial \varphi(r_i(t'), t + t')}{\partial t} \right] \right| 0 \right\rangle. \quad (B17) \]

We shall leave (B16) without changes, and in (B17) we use a ratio

\[ \frac{\partial \varphi(r_i(t'), t + t')}{\partial t} = \exp(iHt'/\hbar)(\partial \varphi(r_i, t + t')/\partial t) \exp(-iHt'/\hbar) = \exp(iHt'/\hbar)(\partial \varphi(r_i, t + t')/\partial t') \exp(-iHt'/\hbar). \]

Then we integrate in parts on \( t' \) and obtain

\[ C^2_{\phi\alpha}(t) = -\frac{ie}{\hbar} \sum_i \langle 0|v_{i\alpha}, \varphi_i(t)\rangle|0 \rangle \]

\[ -\frac{e}{\hbar^2} \int_0^\infty dt' \left\langle 0 \left| \left[ \sum_j v_{j\alpha}, e^{iHt'/\hbar} \right] \right| 0 \right\rangle \]

\[ \times \sum_i \left[ \mathcal{H}, \varphi_i(t, t + t') \right] e^{-iHt'/\hbar} \right| 0 \rangle. \quad (B18) \]

The commutator in the first term is calculated, and for transformations of the second term we notice, that

\[ \frac{i}{\hbar} [\mathcal{H}, \varphi(r_i, t)] = \frac{1}{2} \left( v_{i\beta}(t) \frac{\partial \varphi(r_i, t)}{\partial r_{i\beta}} + \frac{\partial \varphi(r_i, t)}{\partial r_{i\beta}} v_{i\beta}(t) \right). \quad (B19) \]

Having substituted (B19) in (B18), we obtain:

\[ C^2_{\phi\alpha}(t) = -\frac{e}{m} \sum_i \langle 0|\partial \varphi(r_i, t)/\partial r_{i\alpha}\rangle|0 \rangle + \]

\[ + i\frac{e}{2\hbar} \int_0^\infty dt' \left\langle 0 \left[ \sum_j v_{j\alpha}, \sum_i \frac{\partial \varphi(r_i(t'), t + t')}{\partial r_{i\beta}(t')} \right] \right| 0 \right\rangle \quad (B20) \]

Summing (B14), (B16) and (B20), we obtain finally

\[ C_{\alpha}(t) = -\frac{e}{2mc} \left\langle 0 \left| v_{i\beta} \frac{\partial A_{\beta}(r_i, t)}{\partial r_{i\alpha}} + \frac{\partial A_{\beta}(r_i, t)}{\partial r_{i\alpha}} v_{i\beta} \right| 0 \right\rangle \]

\[ + (e/mc)^2 \sum_i \langle 0|(A(r_i, t) \times \mathbf{H}_{\alpha})|0 \rangle - \frac{ie}{2\hbar} \int_0^\infty dt' \]

\[ \times \left\langle 0 \left| \left[ \sum_j v_{j\alpha}, \sum_i \{v_{i\beta}(t')E_{\beta}(r_i(t'), t + t') \right] \right| 0 \right\rangle. \quad (B21) \]

Having substituted (B21) in (B9), we obtain

\[ \langle 0|W_{i\alpha}|0 \rangle = \frac{e}{m} \sum_i \langle 0|E_{\alpha}(r_i, t)|0 \rangle - \frac{ie}{2\hbar} \int_0^\infty dt' \]

\[ \left\langle 0 \left| \left[ \sum_j v_{j\alpha}, \sum_i \{v_{i\beta}(t')E_{\beta}(r_i(t'), t + t') \right] \right| 0 \right\rangle \]

\[ - \frac{e}{2mc} \sum_i \left\langle 0 \left| v_{i\beta} \frac{\partial A_{\alpha}(r_i, t)}{\partial r_{i\beta}} + \frac{\partial A_{\alpha}(r_i, t)}{\partial r_{i\beta}} v_{i\beta} \right| 0 \right\rangle. \quad (B22) \]

This expression coincides with (B8) except for last term from the RHS (B22). But it is possible to show, that this term is equal 0. Really,

\[ (1/2) \left\langle 0 \left| v_{i\beta} \frac{\partial A_{\alpha}(r_i, t)}{\partial r_{i\beta}} + \frac{\partial A_{\alpha}(r_i, t)}{\partial r_{i\beta}} v_{i\beta} \right| 0 \right\rangle = \]

\[ = (i/\hbar) \langle 0|\mathcal{H}, A_{\alpha}(r_i, t)\rangle|0 \rangle = 0, \quad (B23) \]

since the operator \( \mathcal{H} \) has only diagonal matrix elements \( \langle 0|\mathcal{H}|0 \rangle \). So, we have checked up, that results (B8) and (B9), obtained by different ways, coincide.

Let us consider a case of free particles, when the conditions (B7) are carried out. Then \( C_{\alpha}(t) = 0 \), the penultimate term from the RHS (B9) also is equal to zero, and comparing (B8) and (B4), we find, that for the average Lorentz force a ratio owes to be carried out

\[ \frac{e}{2e} \langle 0|v_i \times \mathbf{H}(r_i, t)\rangle|0 \rangle - \langle 0|\mathbf{H}(r_i, t) \times v_{i\alpha}|0 \rangle_{free} = \]

\[ = \frac{iem}{2\hbar} \int_0^\infty dt' \left\langle 0 \left[ \sum_j v_{j\alpha}, \sum_i \{v_{i\beta}(t')E_{\beta}(r_i(t'), t + t') \right. \right] \right\rangle \]

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\[ +E_\beta(\mathbf{r}_i(t'), t + t')v_{i\beta}(t') \right] \bigg|_{t_{\text{free}}} \]  

(B24)

where the index "free" means performance of conditions (B7). Let us check up (B24) by direct account, transforming the RHS. Since \( \mathcal{H}_{\text{free}} = (m/2) \sum v_i^2 \), it is possible to write down

\[ v_{i\alpha} = e^{-i\mathcal{H}_{\text{free}}t'/\hbar} v_{i\alpha} e^{-i\mathcal{H}_{\text{free}}t'/\hbar} = v_{i\alpha}(t'), \]

then a bordering \( \exp(i\mathcal{H}_{\text{free}}t'/\hbar) \ldots - \exp(-i\mathcal{H}_{\text{free}}t'/\hbar) \) from the RHS (B24) leaves. We shall designate the RHS (B24) as \( \Psi_\alpha(t) \), and after calculation of the commutator it appears equal

\[
\Psi_\alpha(t) = -\frac{e}{2} \int_{-\infty}^{0} dt' \sum_i \left\langle 0 \bigg| v_{i\beta} \frac{\partial E_\beta(\mathbf{r}_i, t + t')}{\partial r_{i\alpha}} \right. + \left. \frac{\partial E_\alpha(\mathbf{r}_i, t)}{\partial r_{i\beta}} v_{i\beta} \bigg| 0 \right\rangle.
\]

We shall notice, that the equality

\[
\frac{1}{2} \left\langle 0 \bigg| v_{i\beta} \frac{\partial E_\alpha(\mathbf{r}_i, t)}{\partial r_{i\beta}} + \frac{\partial E_\beta(\mathbf{r}_i, t + t')}{\partial r_{i\alpha}} v_{i\beta} \bigg| 0 \right\rangle = -(i/\hbar)(0|H, E_\alpha(\mathbf{r}_i, t))|0\rangle = 0
\]

is carried out similar (B23), therefore \( \Psi_\alpha(t) \) it is possible to write down as

\[
\Psi_\alpha(t) = -\frac{e}{2} \int_{-\infty}^{0} dt' \sum_i \langle 0 | v_i \times \text{rot} \mathbf{E}(\mathbf{r}_i, t + t') \rangle_{\alpha}
\]

(B26)

that is equal

\[
\Psi_\alpha(t) = -\frac{e}{2} \int_{-\infty}^{0} dt' \sum_i \langle 0 | (v_i \times \text{rot} \mathbf{E}(\mathbf{r}_i, t + t'))_\alpha
\]

(B27)

Using the Maxwell equation \( \text{rot} \mathbf{E}(\mathbf{r}, t) = -(1/c)(\partial \mathbf{H}(\mathbf{r}, t)/\partial t) \) and having calculated integral on \( t' \), we obtain, that \( \Psi_\alpha(t) \) is equal to the LHS (B24), as it was required to prove.

Let us notice, that in a case \( \mathbf{E} = 0, \mathbf{H} = \text{const} \), both sides of (B24) address in 0. It is obvious to the RHS, and in the LHS the matrix elements appear \( \langle 0 | v_{i\beta} | 0 \rangle = (i/\hbar)\langle 0 | [\mathcal{H}, r_{i\beta}] | 0 \rangle = 0 \) because the operator \( \mathcal{H} \) is diagonal.