Mono-static RCS of Chaff Simulated Based on MoM

Hongxia Wang, Honghui Sun, Aijun Li and Jiating Xu
Xi’an Research Inst. of Hi-Tech, Hongqing Town, Xi’an 710025, China
E-mail: redlightw@163.com

Abstract. Method of moment (MoM) is an effective numerical method for calculating electromagnetic scattering. In this paper, we derive the numerical formula for calculating the mono-static radar cross section (RCS) of chaff, and obtain the relationship between mono-static RCS of chaff with different lengths and the incident angle of radar wave. The parameter of average mono-static RCS of chaff per unit mass is introduced, and the relationships between the scattering parameter and the length and diameter of chaff are calculated. The simulation results show that the scattering parameter exhibits resonant peaks when the lengths of chaff are slightly less than integer multiples of half wavelength, and the peak values decrease with the increase of chaff’s lengths. The scattering parameter is also related to the chaff’s diameter. For 3cm incident wave, when the diameter of chaff is near 17μm, the scattering parameter reaches the maximum value.

1. Introduction
In recent years, with the development of electronic science and technology, electromagnetic scattering research has very important application value in many military and civilian fields, so the solution of electromagnetic scattering problem has attracted the attention of many scholars at home and abroad. Chaff is a kind of linear scatterer, which has strong electromagnetic scattering ability and can achieve a relatively large RCS[1-4] with a relatively light weight. Chaff has an important application in passive electronic countermeasures. The calculation of RCS of single chaff is the most basic problem in chaff cloud electromagnetic scattering theory. Therefore, the simulation calculation of chaff electromagnetic scattering characteristics has important reference value for studying the effect of chaff cloud interference technology.

When a target is illuminated by A radar and received by B radar, the bistatic scattering characteristics of chaff cloud will play a role. For mono-static radar, the backscattering characteristics of chaff cloud will play a role. RCS is an important parameter reflecting radar scattering characteristics of targets. Compared with bistatic RCS analysis, mono-static RCS analysis often requires continuous calculation of RCS with different incident angles[5-8]. Therefore, the calculation amount of mono-static RCS is larger than that of bistatic RCS, which is the reason why the research of mono-static RCS is relatively less. In the analysis of electromagnetic field problems, operator equations in integral or differential form are often encountered. Owing to the complexity of operators, analytical solutions are often not available. The method of moment[9-13] is an effective numerical method for solving electromagnetic field problems with high accuracy. The operator equation can be discretized into matrix equation by using the method of moment, which can be solved numerically by computer. This paper mainly calculates the average mono-static RCS of chaff based on the method of moment.
2. The numerical formula for Mono-static RCS of Chaff

The method of moments can be used to determine the induced current on the chaff, thus the RCS value of the chaff can be calculated. If the chaff length direction is z axis, the incident radar wave is TE plane electromagnetic wave of unit amplitude, the angle between incident direction and z axis is $\theta_i$, and azimuth $\varphi_i = 0$, then the component of electric field along z axis is:

$$E_{iz} = \sin \theta_i \cdot \exp(-jkz \cos \theta_i) \quad (1)$$

According to the continuous condition of tangential electric field on the conductor surface, the integral equation satisfied by the induced current is

$$j\omega \mu \int \left[1 + \frac{1}{k^2} \frac{\partial^2}{\partial z^2}\right] I(z')G(z, z')dz' = \sin \theta_i \cdot \exp(-jkz \cos \theta_i) \quad (2)$$

We solve the equation (2) by using the domain basis and point matching method in MoM. Firstly, the chaff with $L$ length and $a$ radius is evenly divided into $N$ segments, and the expression of induced current on chaff is expressed as follows:

$$I = \sum_{n=1}^{N} I_n p_n \quad n = 1, 2, \ldots N \quad (3)$$

For $n$ paragraph, $p_n = 1$, at other locations, $p_n = 0$. The following linear algebraic equations can be obtained by substituting equation (3) into equation (2), multiplying the two ends of the equation by weight function $\omega_m = \delta(z - z_m)$, and then integrating, the following linear algebraic equations can be obtained:

$$j\omega \mu \sum_{n=1}^{N} I_n \int_{L_n} \left[1 + \frac{1}{k^2} \frac{\partial^2}{\partial z^2}\right] G(z_m, z')dz' = \sin \theta_i \cdot \exp(-jkz_m \cos \theta_i) \quad m = 1, \ldots N \quad (4)$$

Where, $L_n$ denotes chaff in paragraph $n$. According to equation (4), the impedance matrix can be obtained as follows:

$$Z(m, n) = j\omega \mu \int_{L_n} \left[1 + \frac{1}{k^2} \frac{\partial^2}{\partial z^2}\right] G(z_m, z')dz' \quad m, n = 1, 2, \ldots N \quad (5)$$

Induced current can be obtained by inversion of impedance matrix:

$$[I_n] = \left[Z_{nn}\right]^{-1} \cdot [\sin \theta_i \cdot \exp(-jkz_m \cos \theta_i)] \quad (6)$$

After calculating the induced current, the numerical formula of scattering field $E_s$ of chaff can be derived according to the theory of electromagnetic field.

$$E_s(r) = \frac{\eta_0}{4\pi} \frac{\exp(-jkr)}{r} \sum_{n=1}^{N} I_n \left\{ \exp(jk\Delta l \cos \theta_s) - \exp[jk(n - 1)\Delta l \cos \theta_s] \right\} \quad (7)$$

Where, $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$, $\theta_s$ is the scattering angle, indicating the angle between the direction of scattering field propagation and $z$ axis.

The RCS of the target is represented by the letter $\sigma$, which is defined as follows:
\[ \sigma = \lim_{r \to \infty} 4\pi r^2 \frac{|E_i|^2}{|E|^2} \]

For mono-static radar, the source and receiver are in the same place, \( \theta_s = \theta_i \). If the radar wave is TE plane electromagnetic wave of unit amplitude, the following formula of mono-static RCS for chaff can be obtained:

\[ \sigma(\theta_i) = \frac{\eta_0^2}{4\pi} \tan^2 \theta_i \sum_{n=1}^{N} I_n \left\{ \exp(jk n \Delta l \cos \theta_i) - \exp[jk(n-1)\Delta l \cos \theta_i] \right\}^2 \]

For chaff clouds with uniform spatial orientation, the average mono-static RCS is

\[ \bar{\sigma} = \frac{1}{2} \int_{0}^{\pi} \sigma(\theta_i) \sin \theta_i d\theta_i \]

From the above theoretical analysis, it can be seen that for the average mono-static RCS, the excitation conditions shown in equation (1) will change once for every angle calculated. But according to the equations (5) and (6), it is found that when the angle of incident wave changes, the induced current changes, but the impedance matrix \( Z \) is unchanged. Therefore, for chaff with a certain length, only the impedance matrix is calculated once, which can save a lot of calculation time.

3. Calculation results

Let the radar wavelength \( \lambda=3\text{cm} \) and the chaff radius \( a=20\mu\text{m} \). The relationships between the mono-static RCS for chaff and the incident angle of radar wave are shown in Figure 1. The direction of peak value of mono-static RCS is also the strongest direction of chaff backscattering. Fig.1 shows the variation of mono-static RCS with incident angle is different for different chaff length \( L \). When the incident angle \( \theta_i = 0 \) and \( \theta_i = \pi \), the mono-static RCS values of the four lengths of chaff are all 0. For chaff with length \( L=0.5\lambda \), at \( \theta_i = \pi/2 \), mono-static RCS reaches its maximum, which indicates that half-wavelength chaff has strong backscattering when radar wave is irradiated vertically. For chaff with length \( L=\lambda \), the minimum Mono-static RCS is achieved at \( \theta_i = \pi/2 \), which indicates that the backscattering of full-wavelength chaff is very weak when radar wave is irradiated vertically. The mono-static RCS has two peaks near angle slightly less than \( \pi/3 \) and slightly greater than \( 2\pi/3 \). For chaff with length \( L=1.5\lambda \), mono-static RCS has two strong peaks and one sub-peak; for chaff with length \( L=2\lambda \), mono-static RCS has two strong peaks and two smaller sub-peaks.
The relationship between mono-static RCS of chaff and chaff length calculated from equation (9) is shown in Figure 2. The calculation results show that when the radar wave is incident at an angle of $\pi/4$, the mono-static RCS of chaff peak appears near the position of integer multiple of half wavelength ($L=0.48\lambda$, $0.98\lambda$, $1.45\lambda$, $1.9\lambda$). When the radar wave illuminates the chaff vertically, the mono-static RCS of chaff peak only appears at the position slightly less than the odd multiple of half wavelength ($L=0.48\lambda$, $1.45\lambda$), and there is no peak near the position of even multiple of half wavelength. That is to say, mono-static RCS has different peak distribution at different incident angles.

Because the resonance length of chaff is related to the incident angle, the average of mono-static RCS with different incident angles can better reflect the real interference performance of chaff cloud. Figure 3 (a) shows the relationship between the average mono-static RCS of chaff and chaff length calculated from equation (10). It can be seen that the corresponding positions of the four peaks are $L=0.48\lambda$, $0.98\lambda$, $1.46\lambda$ and $1.90\lambda$ respectively. That is to say, the average mono-static RCS of chaff peaks not only occur at odd multiple of half wavelength, but also at even multiple of half wavelength. The peak value increases with the increase of chaff length.

Because the total chaff quality of chaff bullet is certain, if the size of a single chaff is different, the total number of chaff is different, so the average mono-static RCS of chaff per unit mass ($\bar{\sigma}/m$) should be introduced to analyze the influence of chaff size on interference effect. When the mass density of chaff is $\rho = 2.7\times10^3 \text{kg/m}^3$, the relationship between the average mono-static RCS of chaff length and chaff length is shown in Figure 3 (b).
per unit mass and chaff length are calculated as shown in Fig.3(b). Obviously, the peak value still appears at the position which is slightly less than integer multiple of half-wavelength, but the peak value decreases with the increase of chaff length. It shows that the interference performance of half-wavelength chaff bomb is the strongest when the total mass is the same.

![Figure 3](image)

**Figure 3** The relationship between average mono-static RCS and chaff length

Fig. 4 (a) is the relationship between the average mono-static RCS of half-wavelength chaff ($L=0.48\lambda$) and the diameter D of chaff. Obviously, the diameter of chaff also affects the average mono-static RCS, especially when the diameter of chaff is in the range of 10µm-30µm. Fig.4(b) shows the relationship between the average mono-static RCS of half-wavelength chaff per unit mass and the diameter of chaff. It can be seen that the peak value appears near the position of $D=17\mu M$.

![Figure 4](image)

**Figure 4** The relationship between average mono-static RCS and chaff diameter ($L=0.48\lambda$)

4. Conclusions

The MoM can be used to determine the induced current on the chaff, and then the scattering field of the chaff can be calculated and its scattering characteristics can be analyzed. The calculation shows that the mono-static RCS of chaff is related to incident angle of radar wave, chaff’s length and diameter. If the incident angle is different, the peak value of mono-static RCS varies with the chaff’s length. When radar wave illuminates chaff vertically, the mono-static RCS of chaff peak only appears at the position less than odd times of half-wavelength, and there is no peak near even times of half-wavelength, while the average mono-static RCS shows resonance peak when chaff’s length is
slightly less than integer multiple of half wavelength. For chaff bomb, it is more reasonable to introduce the average mono-static RCS of chaff per unit mass ($\bar{\sigma}/m$) to analyze its interference performance. The calculation results in this paper have a certain reference value and guiding significance for choosing the size of chaff bomb jammer and designing chaff bomb, and have good engineering application value.

References
[1] Nagl A, Ashraft D and Uberall H 1991 IEEE Trans on Antennas Propagate 39 105
[2] Tang B, Li H M and Sheng X Q 2012 IET Microw Antenna P 6 1451
[3] Zaharis Z D and Sahalos J N 2003 Electr Eng 85 129
[4] Peebles P Z 1984 IEEE T Aero Elec Sys 20 128
[5] Schuh M J, Woo A C and Simon M P 1994 IEEE Antennas Propag M 36 76
[6] Marcus S W 2015 IEEE Trans on Antennas Propagate 63 4091
[7] Guo Y P and Uberall H 1992 IEEE Trans on Antennas Propagate 40 837
[8] Peebles P Z 1984 IEEE T Aero Elec Sys 20 798
[9] Rao S M, and Gothard G K2015 Microw.Opt.Techn Let 19 271
[10] He J, Yu T, Geng N and Carin L 2016 Radio Sci. 35 305
[11] Araújo M G, Taboada J M, Rivero J and Solís D M 2012 IEEE Antenn Propag M 54 81
[12] Wang H X, Zhang Q H and Sun H H 2013 Adv Mater Res 712-715 220