NONPERTURBATIVE VERTICES IN SUPERSYMMETRIC QUANTUM ELECTRODYNAMICS

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Abstract
We derive the complete set of supersymmetric Ward identities involving only two- and three- point proper vertices in supersymmetric QED. We also present the most general form of the proper vertices consistent with both the supersymmetric and $U(1)$ gauge Ward identities. These vertices are the supersymmetric equivalent of the non supersymmetric Ball-Chiu vertices.

1 Introduction
While supersymmetry (SUSY) is generally agreed to be integral to any theory incorporating both gravity and gauge forces, techniques for investigating non-perturbative effects such as chiral symmetry breaking are still in their early stages. A small number of authors have employed Dyson Schwinger Equations (DSEs) to analyse various SUSY theories and small inroads into numerical solutions of the SUSY DSE (SDSE) in Supersymmetric Quantum Electrodynamics (SQED) in 2+1 dimensions (SQED$_3$) have been made.

Analyses of SUSY theories generally use the rainbow approximation to truncate the DSEs at a manageable level. One exception is Clark and Love who use the superfield formalism and derive a differential $U(1)$ gauge Ward Identity for the superfields. They find that the effective mass contains a prefactor which vanishes in Feynman gauge and conclude that there can be no spontaneous mass generation in SQED, even beyond the rainbow approximation. However the superfield approach suffers the disadvantage that each DSE contains an infinite number of terms. This is dealt with by truncating diagrams containing seagull and higher order $n$-point vertices.

The work of Clark and Love has been criticized by Kaiser and Selipsky on two grounds. Firstly they argue that the truncation of seagull diagrams is too severe as it ignores contributions even at the one-loop level. Secondly they point out that infinities arising from infrared divergences which plague the superfield formalism can counter the vanishing prefactor and allow spontaneous mass generation. These criticisms highlight some of the dangers of attempting
to extract phenomenological consequences of supersymmetric DSEs by working solely with the superfield formalism. In fact, analyses in the literature have generally found the component formalism to be the most efficient way to proceed.

Koopmans and Steringa, using the component formalism, also sought to be consistent with the differential $U(1)$ gauge WI in their analysis of SQED3 with two-component fermions. To this end they multiplied the bare vertices by $A(q^2)$ where the electron propagator is given by $S^{-1}(q) = i(\gamma \cdot q A(q^2) + B(q^2))$. This approach is questionable as it implicitly approximates the functions $A(p^2)$ and $B(p^2)$ as being flat. While this approximation is reasonable over most of the momentum range, it is not valid in the low momentum limit where the dynamics are largely determined.

Attempts to go beyond the rainbow approximation in non-SUSY theories began with the Ball and Chiu vertex ansätze for QED and QCD. These are the minimal vertices which “solve” the Ward Takahashi Identities (WTIs) while avoiding kinematic singularities. Ball and Chiu also gave the general form of the possible “transverse” pieces which may be added. Since then several authors have sought to construct ansätze which improve on the minimal Ball-Chiu vertex.

That analogous progress has not been made in SQED using the component formalism is not surprising. Not only must the gauge particle vertices be dressed but the gaugino vertices also. Indeed substituting the minimal Ball and Chiu vertex for photon interactions in SQED3 while leaving the other vertices bare exacerbates the SDSE’s gauge violating properties. The problem of going beyond the rainbow approximation in SUSY theories is the problem of finding the gaugino vertices corresponding to the improved photon vertex. Gaugino vertices are not constrained by the WTI since the gaugino is invariant to gauge transformations. However they are related to the gauge particle vertices by SUSY Ward Identities (SWIs). It is the purpose of this paper to derive and solve the SWIs for SQED and obtain the most general form of the three-point vertex functions consistent with both SUSY and $U(1)$ gauge Ward identities.

Sec. gives the SWIs between the various two-point functions of SQED and their solution which is unique once the electron propagator is known. Sec. shows how to treat proper functions of auxiliary fields. Sec. gives the SWIs constraining the three-point proper functions and finds that the rainbow approximation violates SUSY. The most general form of the vertices consistent with these identities is presented in Sec. and proven to be so in Appendix.
2 \textit{U(1) and Supersymmetric Ward Identities}

The conventions used in this paper are that $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$, $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

The Lagrangian of SQED,

$$ L = |f|^2 + |g|^2 + |\partial_\mu a|^2 + |\partial_\mu b|^2 - \bar{\psi} \not{\partial} \psi - m(a^* f + af^* + bg^* + i\bar{\psi}\psi) $$

$$ -ieA^\mu(a^* \not{\partial}_\mu a + b^* \not{\partial}_\mu b + \bar{\psi}\gamma_\mu\psi) $$

$$ -e[\bar{\lambda}(a^* + i\gamma_5 b^*)\psi - \bar{\psi}(a + i\gamma_5 b)\lambda] $$

$$ +ieD(a^* b - ab^*) + e^2 A_\mu A^\mu(|a|^2 + |b|^2) $$

$$ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} \bar{\lambda} \not{\partial} \lambda + \frac{1}{2} D^2, $$

(2.1)

is, by construction, invariant with respect to both $U(1)$ gauge transformations and SUSY transformations where the SUSY transformations are given by

$$ \delta_S a = -i\bar{\zeta}\psi, $$

$$ \delta_S b = \bar{\zeta}\gamma_5 \psi, $$

$$ \delta_S \psi = [f + i\gamma_5 g + i\gamma \cdot \partial(a + i\gamma_5 b) - e\gamma \cdot A(a - i\gamma_5 b)]\zeta, $$

(2.2)

$$ \delta_S f = \bar{\zeta}[\gamma \cdot \partial \psi + e[-a\lambda - ib\gamma_5 \lambda + i\gamma \cdot A\psi]], $$

$$ \delta_S g = i\bar{\zeta}[\gamma_5 \gamma \cdot \partial \psi + e[-\gamma_5 \lambda - ib\lambda - i\gamma \cdot A\gamma_5 \psi]], $$

for the chiral multiplet and

$$ \delta_S A_\mu = \bar{\zeta}\gamma_\mu \lambda, $$

$$ \delta_S \lambda = \sigma^{\nu\mu} \partial_\mu A_\nu \zeta + i\gamma_5 D\zeta, $$

$$ \delta_S D = i\bar{\zeta}\gamma_5 \gamma \cdot \partial \lambda, $$

for the vector multiplet. It is important to note that the transformations in Eqn. (2.2) are not true SUSY transformations but SUSY transformations plus a gauge transformation. This is a manifestation of the Wess-Zumino (WZ) gauge which is used to make the Lagrangian polynomial. A true SUSY transformation spoils the WZ gauge and must be followed by a gauge
transformation which restores it for the Lagrangian to be invariant. It is from this invariance that the SWIs arise.

The SWIs completely specify the selectron propagators in terms of the electron propagator \( \langle \psi \bar{\psi} \rangle = i \langle a^* f \rangle - i \gamma \cdot p \langle a^* a \rangle = i \langle b^* g \rangle - i \gamma \cdot p \langle b^* b \rangle \), \( \langle \psi \bar{\psi} \rangle = i \langle a^* f \rangle + i \gamma \cdot p \langle f^* a \rangle = -i \langle b^* g \rangle + i \gamma \cdot p \langle g^* b \rangle \). (2.3)

Substituting in the fermion propagator \( S(p) \equiv \langle \psi \bar{\psi} \rangle = -i \gamma \cdot p A(p^2) + B(p^2) \), (2.5)
gives the scalar propagators \( D(p^2) \equiv \langle a^* a \rangle = \langle b^* b \rangle = A(p^2) \), \( \langle a^* f \rangle = \langle b^* g \rangle = B(p^2) \). (2.6)

\( \langle a^* f \rangle = \langle b^* g \rangle = -i \Gamma_{a^* a}(p) + i \gamma \cdot p \Gamma_{f^* f}(p) = -i \Gamma_{b^* b}(p) + i \gamma \cdot p \Gamma_{g^* g}(p) \). (2.9)

SWIs hold between proper vertices too of course. Taking \( \Gamma \) to be the effective action we define \( \Gamma_{X...Z} \equiv \delta_{\Gamma} \delta X...\delta Z \). The two-point proper vertices are constrained by

\[ \gamma \cdot p \Gamma_{\psi \bar{\psi}}(p) = i \Gamma_{a^* a}(p) - i \gamma \cdot p \Gamma_{a^* f}(p) = i \Gamma_{b^* b}(p) - i \gamma \cdot p \Gamma_{b^* g}(p), \] (2.10)

to be

\[ \Gamma_{a^* a}(p) = \Gamma_{b^* b}(p) = p^2 A(p^2), \] (2.11)

\[ \Gamma_{a^* f}(p) = \Gamma_{f^* a}(p) = \Gamma_{b^* g}(p) = \Gamma_{g^* b}(p) = -B(p^2), \] (2.12)

\[ \Gamma_{f^* f}(p) = \Gamma_{g^* g}(p) = A(p^2). \] (2.13)

It is interesting that \( \Gamma_{a^* a}(p) = \Gamma_{b^* b}(p) \neq D(p^2)^{-1} \). This can be attributed to the presence of the auxiliary fields \( f \) and \( g \). The treatment of proper functions involving selectrons is discussed in the next section.
3 Handling the Proper Functions of Auxiliary Fields

One of the difficulties of the component notation in SQED is that of dealing with the auxiliary fields $f, g$ and $D$. The first two are particularly difficult as they contribute off-diagonal quadratic terms which give the scalar propagators an unfamiliar form. To make the free field theory manifestly Gaussian we define,

\[ [a] \equiv \begin{pmatrix} a \\ f \end{pmatrix}, \]
\[ [b] \equiv \begin{pmatrix} b \\ g \end{pmatrix}, \]
\[ [a]^\dagger \equiv \begin{pmatrix} a^* & f^* \end{pmatrix}, \]
\[ [b]^\dagger \equiv \begin{pmatrix} b^* & g^* \end{pmatrix}. \]

The Lagrangian becomes

\[ L = [a]^\dagger \begin{pmatrix} \partial^2 & -m \\ -m & 1 \end{pmatrix} [a] + [b]^\dagger \begin{pmatrix} \partial^2 & -m \\ -m & 1 \end{pmatrix} [b] - \bar{\psi}(\not\partial + im)\psi \]
\[ -ieA^\mu([a]^\dagger \begin{pmatrix} \partial^\mu & 0 \\ 0 & 0 \end{pmatrix} [a] + [b]^\dagger \begin{pmatrix} \partial^\mu & 0 \\ 0 & 0 \end{pmatrix} [b] + \bar{\psi}\gamma^\mu\psi) \]
\[ -e[\bar{\lambda}([a]^\dagger + i\gamma_5[b]^\dagger) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi - \bar{\psi}[1 0]([a] + i\gamma_5[b])\lambda] \]
\[ +ieD([a]^\dagger \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} [b] - [b]^\dagger \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} [a]) \]
\[ +e^2A_\mu A^\nu([a]^\dagger \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} [a] + [b]^\dagger \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} [b]) \]
\[ -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\bar{\lambda}\not\partial\lambda + \frac{1}{2}D^2, \]

and the problem of “interpreting” auxiliary fields is therefore side-stepped.

We shall denote the propagators or proper vertices involving $[a]$ or $[b]$ by enclosing them in square brackets to distinguish them from the propagators or vertices of the single component fields $a, b, f$ and $g$. Thus the $[a]$ and $[b]$ propagators are

\[ [D(p^2)] = \begin{pmatrix} \langle a^* a \rangle & \langle a^* f \rangle \\ \langle f^* a \rangle & \langle f^* f \rangle \end{pmatrix} = \begin{pmatrix} \langle b^* b \rangle & \langle b^* g \rangle \\ \langle g^* b \rangle & \langle g^* g \rangle \end{pmatrix}. \]
their photon interaction is
\[ [\Gamma_{(a,b)\mu} \Lambda_{\mu(a,b)}](p, q) = [\Gamma_{(a,b)\mu} \Lambda_{\mu(a,b)}(p, q)]_{\mu} \] ; \quad (3.7)

the photino interactions are
\[ [\Gamma_{\lambda(a,b)\psi}](p, q) = [\Gamma_{\lambda(a,b)\psi}(p, q)]_{\lambda} \] ; \quad (3.8)

and
\[ [\Gamma_{\bar{\psi}(a,b)\lambda}](p, q) = [\Gamma_{\bar{\psi}(a,b)\lambda}(p, q)]_{\bar{\psi}} \] ; \quad (3.9)

and their D interactions are
\[ [\Gamma_{(a,b)\mu} \lambda_{\mu}](p, q) = [\Gamma_{(a,b)\mu} \lambda_{\mu}(p, q)]_{\mu} \] ; \quad (3.10)

One readily checks that Eqs. (2.11) to (2.13) are consistent with
\[ \bar{\Gamma} = 0 \] corresponding to the following SWIs are given
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4 Supersymmetric Vertex Ward Identities

Before we can find the vertices to substitute into the SDSE, we need the SWIs which constrain them. These are found by taking functional derivatives of \( \delta S \Gamma = 0 \) where \( \Gamma \) is the effective action and \( \delta S \) is defined in Eqn. (2.2). The functional derivatives of \( \delta S \Gamma = 0 \) corresponding to the following SWIs are given in Table 1

\[ \gamma_\mu \Gamma_{\alpha \rho} \Lambda_{\rho \alpha}(p, q) \] \( \text{corresponding to the following SWIs are given in Table 1} \)

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Table 1: Each SWI is derived from a functional derivative of $\delta_2 \Gamma = 0$. The functional derivative leading to each SWI (indicated by its equation number) is given in this table.

| Functional Derivative of $\delta_2 \Gamma = 0$ | SWI | Functional Derivative of $\delta_2 \Gamma = 0$ | SWI |
|-----------------------------------------------|-----|-----------------------------------------------|-----|
| $\delta^3 \Gamma/(\delta a(y)\delta a^*(x)\delta \lambda(z))$ | 4.1 | $\delta^3 \Gamma/(\delta \psi(y)\delta D(z)\delta a^*(x))$ | 4.14 |
| $\delta^3 \Gamma/(\delta b(y)\delta b^*(x)\delta \lambda(z))$ | 4.2 | $\delta^3 \Gamma/(\delta \psi(y)\delta D(z)\delta b^*(x))$ | 4.13 |
| $\delta^3 \Gamma/(\delta f(y)\delta a^*(x)\delta \lambda(z))$ | 4.3 | $\delta^3 \Gamma/(\delta \psi(y)\delta D(z)\delta f^*(x))$ | 4.16 |
| $\delta^3 \Gamma/(\delta g(y)\delta g^*(x)\delta \lambda(z))$ | 4.4 | $\delta^3 \Gamma/(\delta \psi(y)\delta D(z)\delta g^*(x))$ | 4.17 |
| $\delta^3 \Gamma/\delta f(y)\delta f^*(x)\delta \lambda(z)$ | 4.5 | $\delta^3 \Gamma/(\delta b(y)\delta D(z)\delta a^*(x))$ | 4.18 |
| $\delta^3 \Gamma/\delta g(y)\delta g^*(x)\delta \lambda(z)$ | 4.6 | $\delta^3 \Gamma/(\delta a(y)\delta \lambda(z)\delta \lambda^*(x))$ | 4.19 |
| $\delta^3 \Gamma/(\delta \psi(y)\delta A_{\mu}(z)\delta f^*(x))$ | 4.7 | $\delta^3 \Gamma/(\delta g(y)\delta \lambda(z)\delta a^*(x))$ | 4.20 |
| $\delta^3 \Gamma/\delta g(y)\delta A_{\mu}(z)\delta g^*(x)$ | 4.8 | $\delta^3 \Gamma/(\delta f(y)\delta \lambda(z)\delta b^*(x))$ | 4.21 |
| $\delta^3 \Gamma/(\delta \psi(y)\delta A_{\mu}(z)\delta a^*(x))$ | 4.9 | $\delta^3 \Gamma/(\delta a(y)\delta \lambda(z)\delta g^*(x))$ | 4.22 |
| $\delta^3 \Gamma/(\delta \psi(y)\delta A_{\mu}(z)\delta b^*(x))$ | 4.10 | $\delta^3 \Gamma/(\delta \psi(y)\delta \lambda(z)\delta f^*(x))$ | 4.23 |
| $\delta^3 \Gamma/(\delta \psi(y)\delta \lambda(z)\delta g^*(x)$ | 4.11 | $\delta^3 \Gamma/(\delta g(y)\delta A_{\mu}(z)\delta f^*(x))$ | 4.24 |
| $\delta^3 \Gamma/(\delta f(y)\delta \lambda(z)\delta g^*(x))$ | 4.25 |

\[
\gamma_{\mu}^f_{\gamma^a_\mu}(p, q) + eA(p^2) = \Gamma_{\lambda\alpha^*\psi}(-q, -p) + \Gamma_{\lambda f^*\psi}(p, q)\gamma \cdot q, \quad (4.3)
\]
\[
\gamma_{\mu}^\Gamma_{\gamma^a_\mu}(p, q) - eA(p^2) = i\Gamma_{\lambda\beta^*\psi}(-q, -p)\gamma_5 + i\Gamma_{\lambda g^*\psi}(p, q)\gamma \cdot q\gamma_5. \quad (4.4)
\]

\[
\gamma_{\mu}^f_{\gamma^a_\mu}(p, q) = \Gamma_{\lambda f^*\psi}(-q, -p) - \Gamma_{\lambda f^*\psi}(p, q), \quad (4.5)
\]
\[
\gamma_{\mu}^\Gamma_{\gamma^a_\mu}(p, q) = i\Gamma_{\lambda\beta^*\psi}(-q, -p)\gamma_5 - i\Gamma_{\lambda g^*\psi}(p, q)\gamma_5, \quad (4.6)
\]

\[
i\sigma^\mu\nu(p - q)\psi^\nu\Gamma_{\lambda f^*\psi}(p, q) = \Gamma_{\psi^\mu\nu}(p, q) - i\gamma \cdot q\Gamma_{\gamma^a_\mu}(p, q) + i\Gamma_{\gamma^a_\mu}(p, q) - ie\gamma^\mu A(p^2), \quad (4.7)
\]
\[
i\sigma^\mu\nu(p - q)\psi^\nu\Gamma_{\lambda g^*\psi}(p, q) = i\gamma_5\Gamma_{\psi^\mu\nu}(p, q) + \gamma_5\gamma \cdot q\Gamma_{\gamma^a_\mu}(p, q) - \gamma_5\Gamma_{\gamma^a_\mu}(p, q) + e\gamma_5\gamma^\mu A(p^2), \quad (4.8)
\]
\[ i\sigma^{\mu\nu}(p-q)\gamma^{\lambda}(p) = i\gamma^{\lambda} \cdot q (\Gamma^{\mu}_{\alpha\beta}(p) - c\gamma^{\mu} S^{-1}(p) - \gamma \cdot p \Gamma^{\mu}_{\psi A\nu}(p) + ie\gamma^{\mu} B(p^{2})) \]

(4.9)

\[ i\sigma^{\mu\nu}(p-q)\gamma^{\lambda}(p) = -\gamma_{5}\Gamma^{\mu}_{\lambda\beta}(p) + \gamma_{5}\gamma \cdot q \Gamma^{\mu}_{\lambda\beta}(p) - i\gamma_{5}c\gamma^{\mu} S^{-1}(p) - i\gamma_{5}\gamma \cdot p \Gamma^{\mu}_{\psi A\nu}(p) - e\gamma_{5}\gamma^{\mu} B(p^{2}). \]

(4.10)

It follows from both (4.9) and (4.10) that the rainbow approximation, that is, dressed vertices replaced by bare vertices, violates SUSY in the same way that it violates \( U(1) \) gauge invariance.

From

\[ 0 = -i(\gamma \cdot q)_{\alpha}\Gamma_{\psi f\lambda}(p, q)_{\beta} + (\gamma_{5} \cdot q)_{\alpha}\Gamma_{\psi A\nu}(p, q)_{\beta} \]

(4.11)

\[ -i(\gamma \cdot p C)_{\beta\sigma} \frac{\delta}{\delta \psi_{\alpha}(q)}(\delta \Gamma_{\psi f\lambda}(p, q))_{\sigma} + \frac{\delta}{\delta \psi_{\alpha}(q)}(\delta \Gamma_{\psi A\nu}(p, q))_{\beta} \]

(4.11)

where \( C \) is the charge conjugation matrix, we obtain

\[ 0 = (\gamma \cdot p - \gamma \cdot q)\gamma_{5} \text{Tr}(\Gamma_{\bar{\psi} \bar{D}\psi}(p, q)) + \gamma_{\mu} \text{Tr}(\Gamma^{\mu}_{\psi A\nu}(p, q)) + i\Gamma_{\bar{\psi} \bar{D}\psi}(p, q) \]

(4.12)

\[ -\gamma_{5}\Gamma_{\bar{\psi} \bar{A}\lambda}(p, q) - i\Gamma_{\bar{\psi} \bar{A}\lambda}(q, -p) + \gamma_{5}\Gamma_{\bar{\psi} \bar{A}\lambda}(q, -p) \]

(4.12)

\[ -i\gamma \cdot \bar{q} \Gamma_{\bar{f}\lambda}(p, q) + \gamma_{5}\gamma \cdot \bar{q} \Gamma_{\bar{g}\lambda}(p, q) - i\gamma \cdot p \Gamma_{\bar{f}\lambda}(q, -p) + \gamma_{5}\gamma \cdot p \Gamma_{\bar{g}\lambda}(q, -p), \]

by setting \( \beta = \alpha \) and summing, and

\[ 0 = i\text{Tr}(\Gamma_{\bar{\psi} \bar{A}\lambda}(p, q)) - \gamma_{5}\text{Tr}(\Gamma_{\bar{\psi} \bar{A}\lambda}(p, q)) - i\gamma \cdot \bar{q} \text{Tr}(\Gamma_{\bar{f}\lambda}(p, q)) \]

(4.13)

\[ +\gamma_{5}\gamma \cdot \bar{q} \text{Tr}(\Gamma_{\bar{g}\lambda}(p, q)) - i\gamma_{5}\bar{h} \cdot \psi(p, q) + \gamma_{5}\Gamma_{\bar{h} \bar{g}\lambda}(p, q) - i\gamma \cdot p \Gamma_{\bar{h} \bar{f}\lambda}(p, q) \]

(4.13)

\[ -\gamma \cdot p \gamma_{5}\Gamma_{\bar{h} \bar{g}\lambda}(p, q) + \gamma_{5}\Gamma_{\bar{h} \bar{g}\lambda}(p, q) - \gamma_{5}(\gamma \cdot p - \gamma \cdot q) \Gamma_{\bar{D}\psi}(p, q), \]

by setting \( \beta = \kappa \) and summing.

Finally there are the SWIs governing the vertices of the \( D \) particle:

\[ i\gamma_{5}\Gamma_{\bar{A} \lambda}(p, q) \]

(4.14)

\[ = \gamma \cdot p \Gamma_{\bar{D}\psi}(p, q) + \gamma_{5}\Gamma_{\bar{A} \lambda}(p, q) - \gamma_{5}\gamma \cdot \bar{q} \Gamma_{\bar{D}\psi}(p, q), \]

(4.15)
\( \gamma_5 \Gamma_f \cdot D_b (p, q) \)  
\( = i \gamma_5 \Gamma_{\lambda_f^* \psi} (p, q) + \gamma_5 \gamma \cdot q \Gamma_f \cdot D_g (p, q) + \Gamma_{\overline{\psi} D_q} (p, q), \)  
\( \gamma_5 \Gamma_{g^* D_a} (p, q) \)  
\( = - \Gamma_{\lambda_g^* \psi} (p, q) + \gamma_5 \gamma \cdot q \Gamma_{g^* D_f} (p, q) - \Gamma_{\overline{\psi} D_q} (p, q), \)  
\( \gamma_5 (\gamma \cdot p - \gamma \cdot q) \Gamma_{a^* D_b} (p, q) \)  
\( = \Gamma_{\lambda_b^* \psi} (-q, -p) \gamma \cdot p + i \Gamma_{\lambda_a^* \psi} (p, q) \gamma \cdot q \gamma_5 + i e \gamma_5 (B(p^2) - B(q^2)), \)  
\( \gamma_5 (\gamma \cdot p - \gamma \cdot q) \Gamma_{b^* D_a} (p, q) \)  
\( = i \Gamma_{\lambda_a^* \psi} (-q, -p) \gamma \cdot p \gamma_5 + \Gamma_{\lambda_b^* \psi} (p, q) \gamma \cdot q + i e \gamma_5 (B(p^2) - B(q^2)), \)  
\( \gamma_5 (\gamma \cdot p - \gamma \cdot q) \Gamma_{f^* D_g} (p, q) \)  
\( = \Gamma_{\lambda_g^* \psi} (-q, -p) \gamma \cdot p - i \Gamma_{\lambda_a^* \psi} (p, q) \gamma_5 + i e \gamma_5 A(q^2), \)  
\( \gamma_5 (\gamma \cdot p - \gamma \cdot q) \Gamma_{b^* D_f} (p, q) \)  
\( = \Gamma_{\lambda_f^* \psi} (-q, -p) \gamma \cdot q - \Gamma_{\lambda_b^* \psi} (p, q) \gamma_5 + i e \gamma_5 A(p^2), \)  
\( \gamma_5 (\gamma \cdot p - \gamma \cdot q) \Gamma_{f^* D_b} (p, q) \)  
\( = i \Gamma_{\lambda_f^* \psi} (p, q) \gamma \cdot q \gamma_5 + \Gamma_{\lambda_b^* \psi} (-q, -p) - i e \gamma_5 A(p^2), \)  
\( \gamma_5 (\gamma \cdot p - \gamma \cdot q) \Gamma_{f^* D_g} (p, q) \)  
\( \gamma_5 (\gamma \cdot p - \gamma \cdot q) \Gamma_{f^* D_g} (p, q) = \Gamma_{\lambda_g^* \psi} (-q, -p) - i \Gamma_{\lambda_f^* \psi} (p, q) \gamma_5, \)  
\( \gamma_5 (\gamma \cdot p - \gamma \cdot q) \Gamma_{g^* D_f} (p, q) = i \Gamma_{\lambda_f^* \psi} (-q, -p) \gamma_5 - \Gamma_{\lambda_g^* \psi} (p, q). \)  

These make up the entire set of SWIs containing only three-or-fewer point proper functions, modulo charge conjugation. A suitable vertex ansatz must also be consistent with the WTIs:

\[
(p - q) \mu [\Gamma_{(a, b)^* A_{(a, b)}^*]} (p, q) = e [\Gamma_{(a, b)^* (a, b)}] (p) - e [\Gamma_{(a, b)^* (a, b)}] (q), \quad (4.26)
\]

\[
(p - q) \mu [\Gamma_{A_{(a, b)}^* \overline{\psi}_{(a, b)}}] (p, q) = e S^{-1} (p) - e S^{-1} (q). \quad (4.27)
\]

We also have from charge conjugation invariance that:

\[
[\Gamma_{(a, b)^* \lambda}] (p, q) = - C [\Gamma_{\lambda^* (a^*, b^*)}] (-q, -p)^T C^{-1},
\]

\[
[\Gamma_{(a^*, b^*) D_{(a, b)}}] (p, q) = - [\Gamma_{D_{(a, b)^*} (a^*, b^*)}] (-q, -p). \quad (4.28)
\]
5 Solution to SWIs and WTIs in SQED

Below is a solution for the SWIs and WTIs. It is the most general set of vertices consistent with both the WTIs and the SWIs and free of kinematic singularities if one assumes charge conjugation invariance and

\[ \Gamma_{a^* A_{a}}(p,q) = \Gamma_{b^* A_{a}}(p,q). \]  

(5.1)

Proof of this is presented in Appendix A. The assumption of Eqn.(5.1) is true to all orders in perturbation theory, and any nonperturbative violations of this assumption are restricted by the WTIs to lie completely within their transverse components.

Our general solution is as follows:

The scalar-photon vertices are

\[ \Gamma_{a^* A_{a}}(p,q) = \Gamma_{b^* A_{a}}(p,q) \]  

(5.2)

\[ \frac{e}{p^2 - q^2}(p^2 A(p^2) - q^2 A(q^2))(p + q)^\mu + [p^\mu(q^2 - p \cdot q) + q^\mu(p^2 - p \cdot q)]T_{aa}(p^2, q^2, p \cdot q), \]

(5.3)

\[ \frac{e}{p^2 - q^2}(B(p^2) - B(q^2))(p + q)^\mu + [p^\mu(q^2 - p \cdot q) + q^\mu(p^2 - p \cdot q)]T_{af}(p^2, q^2, p \cdot q), \]

(5.4)

where the three functions \( T_{aa}(p^2, q^2, p \cdot q), T_{af}(p^2, q^2, p \cdot q) \) and \( T_{ff}(p^2, q^2, p \cdot q) \), each satisfying \( T(p^2, q^2, p \cdot q) = T(q^2, p^2, p \cdot q) \), are free of kinematic singularities and represent the only degrees of freedom inherent in the solution. The forms \( (5.2) \) to \( (5.4) \) are equivalent to that given by Ball and Chiu in the context of non SUSY scalar QED. The photino vertices are

\[ \Gamma_{\lambda \alpha^* \psi}(p,q) = \frac{e}{p^2 - q^2}(p^2 A(p^2) - q^2 A(q^2)) + \frac{e}{p^2 - q^2}(B(p^2) - B(q^2))\gamma \cdot q \]

\[ + \frac{1}{2}e(p^2 - \gamma \cdot q \gamma \cdot p)T_{oo}(p^2, q^2, p \cdot q) \]

(5.5)

\[ + \frac{1}{2}e(p^2 - \gamma \cdot q \gamma \cdot p)T_{ff}(p^2, q^2, p \cdot q) \]

\[ + \frac{1}{2}e[p(p^2 - q^2) - 2 \gamma \cdot q(p^2 - p \cdot q)]T_{af}(p^2, q^2, p \cdot q), \]

and

\[ \Gamma_{\lambda f^* \psi}(p,q) = -\frac{e}{p^2 - q^2}(A(p^2) - A(q^2))\gamma \cdot q - \frac{e}{p^2 - q^2}(B(p^2) - B(q^2)) \]

10
The electron-photon vertex must be restricted at least to the form given by Ball and Chiu for non SUSY QED. For the SUSY case we find

\[ \Gamma_{\psi A}^{\mu}(p, q) = \Gamma_{BC}^{\mu}(p, q) + \frac{ie}{p^2 - q^2} (A(p^2) - A(q^2))\left[\frac{1}{2} T_3^{\mu} - T_8^{\mu}\right] + \frac{ie}{p^2 - q^2} (B(p^2) - B(q^2)) T_5^{\mu} + \frac{1}{2} ie T_{aa}(p^2, q^2, p \cdot q) T_3^{\mu} + \frac{1}{2} ie T_{af}(p^2, q^2, p \cdot q) [\frac{1}{2} (p - q)^2 T_5^{\mu} - T_1^{\mu}] + \frac{1}{2} ie T_{ff}(p^2, q^2, p \cdot q) [T_2^{\mu} - p \cdot q T_3^{\mu} - (p - q)^2 T_8^{\mu}], \]

where

\[ \Gamma_{BC}^{\mu}(p, q) = \frac{1}{2} \frac{ie}{p^2 - q^2} (\gamma \cdot p + \gamma \cdot q)(A(p^2) - A(q^2))(p + q)^\mu + \frac{ie}{p^2 - q^2} (B(p^2) - B(q^2))(p + q)^\mu, \]

\[ T_1^{\mu} = p^\mu (q^2 - p \cdot q) + q^\mu (p^2 - p \cdot q), \]
\[ T_2^{\mu} = (\gamma \cdot p + \gamma \cdot q) T_1^{\mu}, \]
\[ T_3^{\mu} = \gamma^\mu (p - q)^2 - (\gamma \cdot p - \gamma \cdot q)(p - q)^\mu, \]
\[ T_5^{\mu} = \sigma^{\mu\nu} (p - q)_\nu, \]
\[ T_8^{\mu} = \frac{1}{2} (\gamma \cdot p \gamma \cdot q)^\mu - \gamma^\mu \gamma \cdot q \gamma \cdot p. \]

Finally there are the vertices for the D-boson, namely,

\[ \Gamma_{a \cdot D}^{\mu}(p, q) = -\Gamma_{b \cdot D}^{\mu}(p, q) \]
\[ = \frac{ie}{p^2 - q^2} (p^2 A(p^2) - q^2 A(q^2)) - iep \cdot q T_{aa}(p^2, q^2, p \cdot q) + \frac{1}{2} iep^2 q^2 T_{ff}(p^2, q^2, p \cdot q). \]
\[ \Gamma_{f \cdot Dg}(p, q) = -\Gamma_{g \cdot Df}(p, q) \]  
\[ = \frac{ie}{p^2 - q^2}(A(p^2) - A(q^2)) + ieT_{a^*a}(p^2, q^2, p \cdot q) \]  
\[ - iep \cdot q T_{f \cdot f}(p^2, q^2, p \cdot q), \]  
\[ \Gamma_{g^*Da}(p, q) = \frac{ie}{p^2 - q^2}(B(p^2) - B(q^2)) \]  
\[ - iec(q^2 - p \cdot q) T_{a \cdot f}(p^2, q^2, p \cdot q), \]  
\[ \Gamma_{a^*Dg}(p, q) = \frac{-ie}{p^2 - q^2}(B(p^2) - B(q^2)) \]  
\[ + iec(p^2 - p \cdot q) T_{a \cdot f}(p^2, q^2, p \cdot q), \]  
\[ \Gamma_{f^*D6}(p, q) = \frac{-ie}{p^2 - q^2}(B(p^2) - B(q^2)) \]  
\[ + iec(q^2 - p \cdot q) T_{a \cdot f}(p^2, q^2, p \cdot q), \]  
\[ \Gamma_{b^*Df}(p, q) = \frac{ie}{p^2 - q^2}(B(p^2) - B(q^2)) \]  
\[ - iec(p^2 - p \cdot q) T_{a \cdot f}(p^2, q^2, p \cdot q), \]  
\[ \text{and} \]  
\[ \Gamma_{\bar{\psi}D\psi}(p, q) = \frac{1}{2}ie\gamma_5[(p^2 - q^2) T_{a \cdot f}(p^2, q^2, p \cdot q) \]  
\[ + (\gamma \cdot p + \gamma \cdot q) T_{a^*a}(p^2, q^2, p \cdot q) \]  
\[ - (\gamma \cdot q p^2 + \gamma \cdot q p^2) T_{f \cdot f}(p^2, q^2, p \cdot q)]. \]  

6 Conclusion

We have derived the three-point SWIs for SQED and found a solution, given in sections 2 and 5, which, under the reasonable assumptions of charge conjugation invariance and symmetry between \([a]\) and \([b]\) with respect to their photon interaction, comprises the most general set of vertices consistent with both the SWIs and WTIs and free of kinematic singularities. They are, in fact, the SUSY equivalent of the Ball-Chiu vertex. These SUSY Ball-Chiu vertices have only three degrees of freedom between them once the electron propagator is known, compared with non SUSY QED which has eight. The loss of degrees of freedom occurs entirely within the electron-photon vertex. The scalar-photon vertices remain unchanged from non SUSY scalar QED (with auxiliary fields).

We have given the form of the electron DSE. There is no need to consider also the DSE for scalar partners since SWIs ensure that the propagators of all
chiral multiplet fields can be written in terms of the same two scalar functions $A(p^2)$ and $B(p^2)$ (See Sec.(2)). Solving the DSE for any chiral multiplet field can therefore be accomplished by projecting from the electron DSE a pair of coupled integral equations for $A(p^2)$ and $B(p^2)$.

Numerical solutions of the analogous calculation in non SUSY QED\[12\],\[13\],\[14\],\[15\] and QED\[3\] using the minimal Ball-Chiu and Curtis-Pennington\[8\] vertex ansätze exist in the literature. The same task in SUSY is conceptually similar and the presence of extra terms in the DSE is not expected to reduce its feasibility. Indeed such numerical work has been done already in the rainbow approximation in SQED\[3\]. The way now lies open to transcend the rainbow approximation in the analysis of SQED and SQED\[3\] in the nonperturbative limit.

A Appendix: Derivation of the Nonperturbative Vertices

Below is a derivation of the most general form of the proper vertices consistent with both the SWIs and the WTIs. It is convenient to define the following notation:

The operator $\Omega$ performs the interchange $(p, q) \rightarrow (-q, -p)$.

A function $F(p, q)$, invariant to $\Omega$, is written as $F((p, q))$. If $F(p, q)$ is a scalar function $F(p^2, q^2, p \cdot q)$ then it is written as $F((p^2, q^2, p \cdot q))$.

Alternately, a function $G(p, q)$ which changes sign under $\Omega$ is written as $G((p, q))$, or $G((p^2, q^2, p \cdot q))$ if it is scalar.

Eqs. (5.2, 5.3, 5.4) follow, by the reasoning of Ball and Chiu\[7\], from the WTI for $[a]$ and $[b]$ (See Eqn.(4.26)).

Substituting Eqn.(5.1) into Eqn.(4.10) and comparing to Eqn.(4.9) gives

$$\bar{\Gamma}_{\lambda f \psi}(p, q) = i \gamma^5 \bar{\Gamma}_{\lambda a \psi}(p, q) \tag{A.1}$$

Similarly, from Eqs.(4.16, 4.17),

$$\bar{\Gamma}_{\lambda g \psi}(p, q) = i \gamma^5 \bar{\Gamma}_{\lambda f \psi}(p, q) \tag{A.2}$$

Any $\bar{\Gamma}_{\lambda f \psi}(p, q)$ consistent with Eqn.(4.9) can be put in the general form

$$\bar{\Gamma}_{\lambda f \psi}(p, q) = \frac{-e}{p^2 - q^2} (A(p^2) - A(q^2)) \gamma \cdot q + H((p, q)) \tag{A.3}$$

$$- \frac{1}{2} e [\gamma \cdot (p(q^2 - p \cdot q) + \gamma \cdot q(p^2 - p \cdot q)] T_{ff}((p^2, q^2, p \cdot q))$$

Using Eqn.(A.1) to equate Eqs.(4.16, 4.17), we find that

$$\Gamma_{f^* Db}(p, q) = -\Gamma_{g^* Da}(p, q) \tag{A.4}$$

$$\Gamma_{f^* Dg}(p, q) = -\Gamma_{g^* Df}(p, q) \tag{A.5}$$
We obtain, by substituting Eqs.\(^{(A.2, A.3)}\) into Eqn.\(^{(4.20)}\),
\[
\gamma_5(\gamma \cdot p - \gamma \cdot q)\Gamma_{\delta^*} D_f(p^2, q^2, p \cdot q)
\]
\[
= \frac{-ie}{p^2 - q^2}(A(p^2) - A(q^2))\gamma_5(\gamma \cdot p - \gamma \cdot q) + iH((p, q_5)\gamma_5 - i\gamma_5 H((p, q)).
\]
Dividing \(H((p, q))\) into its odd-numbered and even-numbered \(\gamma\)-matrix components, \(H^{\text{odd}}((p, q))\) and \(H^{\text{even}}((p, q))\) respectively, we see from Eqn.\(^{(A.6)}\) that \(H^{\text{odd}}((p, q))\) is of the form
\[
H^{\text{odd}}((p, q)) = (\gamma \cdot p - \gamma \cdot q) \hat{H} (p^2, q^2, p \cdot q),
\]
due to its anti-commutation with \(\gamma_5\) and its invariance under \(\Omega\). If we substitute Eqs.\(^{(A.2, A.4, 4.28)}\) into Eqn.\(^{(4.20)}\) we get
\[
\gamma_5(\gamma \cdot p - \gamma \cdot q)\Gamma_\delta D_f(p^2, q^2, p \cdot q)
\]
\[
= i\Gamma\lambda\psi(p, q)\gamma_5 - i\gamma_5 A(q^2) - i\gamma_5 \Gamma\lambda\psi(-q, -p)\gamma \cdot p,
\]
which, when added to Eqn.\(^{(1.24)}\), produces
\[
i\gamma_5 \Gamma\lambda\psi(p, q) + i\Gamma\lambda\psi(p, q)\gamma_5
\]
\[
= 2ieA(q^2) + i\gamma_5 \Gamma\lambda\psi(-q, -p)\gamma \cdot p - i\Gamma\lambda\psi(-q, -p)\gamma_5 \gamma \cdot p.
\]
Any \(\Gamma\lambda\psi(p, q)\) consistent with Eqs.\(^{(A.3, A.7, A.9)}\) must be of the form
\[
\Gamma\lambda\psi(p, q) = \frac{e}{p^2 - q^2}(p^2 A(p^2) - q^2 A(q^2))
\]
\[
+ \frac{1}{2}e[p^2(q^2 - p \cdot q) + \gamma \cdot q \gamma \cdot p(p^2 - p \cdot q)]T_{ff}((p^2, q^2, p \cdot q)
\]
\[
+ (p^2 - \gamma \cdot q \gamma \cdot p) \hat{H} (p^2, q^2, p \cdot q) + \Gamma^{\text{odd}}\lambda\psi(p, q),
\]
where the superscript “odd” on the last term denotes that it is the component of \(\Gamma\lambda\psi(p, q)\) with only odd numbers of \(\gamma\)-matrices. \(\Gamma^{\text{odd}}\lambda\psi(p, q)\) is unrestricted by Eqn.\(^{(A.9)}\) due to its anti-commutation with \(\gamma_5\).
Substituting Eqs.\(^{(5.2, A.10)}\) into Eqn.\(^{(4.1)}\) tells us that
\[
\hat{H}((p^2, q^2, p \cdot q) = \frac{1}{2}e(T_{aa}(p^2, q^2, p \cdot q) - p \cdot q T_{ff}((p^2, q^2, p \cdot q)).
\]
The even \(\gamma\)-matrix component of \(\Gamma\lambda\psi(p, q)\) is therefore
\[
\Gamma^{\text{even}}\lambda\psi(p, q) = \frac{e}{p^2 - q^2}(p^2 A(p^2) - q^2 A(q^2))
\]
\[ + \frac{1}{2} e(p^2 - \gamma \cdot q \gamma) T_{aa}(p^2, q_i^2, p \cdot q) \]
\[ + \frac{1}{2} e p^2 (q^2 - \gamma \cdot p \gamma \cdot q) T_f (p^2, q_i^2, p \cdot q), \]

and the odd \( \gamma \)-matrix component of \( \Gamma_{\chi f} (p, q) \) is

\[
\Gamma_{\chi f}^{\text{odd}} (p, q) = -\frac{e}{p^2 - q^2} (A(p^2) - A(q^2)) \gamma \cdot q \\
+ \frac{1}{2} e(\gamma \cdot p - \gamma \cdot q) T_{aa}(p^2, q_i^2, p \cdot q) \\
- \frac{1}{2} e \gamma \cdot q(p^2 - \gamma \cdot p \gamma \cdot q) T_f (p^2, q_i^2, p \cdot q).
\]

It now remains to find \( \Gamma_{\lambda \alpha}^{\text{odd}} (p, q) \) and \( H_{\text{even}}^{\text{even}} (p, q) \). Subtracting Eqn.\( \text{[A.8]} \) from Eqn.\( \text{[4.21]} \) we get

\[ (\gamma \cdot p - \gamma \cdot q) \Gamma_{\alpha \beta} (p^2, q_i^2, p \cdot q) = -i \Gamma_{\lambda \alpha}^{\text{odd}} (p, q) - i H_{\text{even}}^{\text{even}} (p, q) \gamma \cdot p. \] (A.14)

The result of substituting Eqs.\( \text{[A.12], [A.13]} \) into Eqn.\( \text{[4.3]} \) and operating with \( \Omega \) is

\[ 0 = \Gamma_{\lambda \alpha}^{\text{odd}} (p, q) - H_{\text{even}}^{\text{even}} (p, q) \gamma \cdot p \] (A.15)

Adding Eqn.\( \text{[A.15]} \) to \(-i \times \) Eqn.\( \text{[A.14]} \) produces

\[ -i(\gamma \cdot p - \gamma \cdot q) \Gamma_{\alpha \beta} (p^2, q_i^2, p \cdot q) \] (A.16)

\[ = -2 H_{\text{even}}^{\text{even}} (p, q) \gamma \cdot p - \frac{e}{p^2 - q^2} (B(p^2) - B(q^2)) (\gamma \cdot p + \gamma \cdot q) \\
+ e [\gamma \cdot p(q^2 - p \cdot q) + \gamma \cdot q(p^2 - p \cdot q)] T_{a \beta} (p^2, q_i^2, p \cdot q). \]

\( H_{\text{even}}^{\text{even}} (p, q) \) is of the general form,

\[ H_{\text{even}}^{\text{even}} (p, q) = H_{\text{scalar}}^{\text{scalar}} (p^2, q_i^2, p \cdot q) + \gamma_5 H_5^5 (p^2, q_i^2, p \cdot q) \] (A.17)

\[ + \frac{1}{2} (\gamma \cdot p \gamma \cdot q - \gamma \cdot q \gamma \cdot p) H_7^7 (p^2, q_i^2, p \cdot q) \]

\[ + \frac{1}{2} \gamma_5 (\gamma \cdot p \gamma \cdot q - \gamma \cdot q \gamma \cdot p) H_5^{10} (p^2, q_i^2, p \cdot q). \]
The symmetry properties of the scalar functions in Eqn.(A.17) follow from the invariance of $H(p,q)$ under $\Omega$. Remembering that $\Gamma_{\psi}D(p^2, q^2, p \cdot q)$ is scalar, and substituting Eqn.(A.17) into Eqn.(A.16), we find that

$$H^{5\sigma}(p^2, q^2, p \cdot q) = 0 = H^{5}(p^2, q^2, p \cdot q), \quad (A.18)$$

and

$$H^{\text{scalar}}(p^2, q^2, p \cdot q) = 0 \quad (A.19)$$

Finally, substituting Eqs.(A.17) to (A.20) into Eqn.(A.15),

$$\Gamma_{\psi}^{\text{odd}}(p,q) = e(p - q)^2 T_{af}(p^2, q^2, p \cdot q) - \frac{e}{p^2 - q^2}(B(p^2) - B(q^2)) \quad (A.20)$$

We now have the vertices $\Gamma_{\psi}^{\text{odd}}(p,q)$, given by Eqn.(A.17), and $\Gamma_{\psi}^{\text{odd}}(p,q)$, found by summing Eqs.(A.12) and (A.21) and given by Eqn.(A.13). $\Gamma_{\psi}^{\text{odd}}(p,q)$ is now determined by any one of the Eqs.(4.17) to (4.19), the scalar $D$-vertices are given by the Eqs.(4.18) through to (4.25), and the vertex $\Gamma_{\psi}D(p,q)$ is given by any one of the Eqs.(4.14) through to (4.17). It is simple to verify that the solution presented in section 5 is not further constrained by the SWIs not used in this derivation.

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