Three-Loop Neutrino Masses via New Massive Gauge Bosons from $E_6$ GUT

Bhaskar Dutta$^1$, Sumit Ghosh$^1$, Ilia Gogoladze$^2$, Tianjun Li$^3$.$^4$

$^1$Mitchell Institute for Fundamental Physics and Astronomy, Department of Physics and Astronomy, Texas A&M University, College Station, TX 77843
$^2$Bartol Research Institute, Department of Physics and Astronomy, University of Delaware, Newark, DE 19716, USA
$^3$CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, 100190, P. R. China
$^4$School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, P. R. China

We propose a $SU(3)_C \times SU(2)_L \times SU(2)_X \times U(1)_Y$ model arising from $E_6$ Grand Unified Theory (GUT). We show that the tiny neutrino masses in this model can be generated at three-loop involving the $SU(2)_X$ gauge bosons. With Yukawa couplings around 0.01 or larger and TeV-scale $SU(2)_X$ gauge bosons, we show that the neutrino oscillation data can be explained naturally by presenting a concrete benchmark set of input parameters. All new particles are around the TeV scale. Thus our model can be tested at the ongoing/future collider experiments.

I. INTRODUCTION

One of the great achievements in particle physics during the last few decades is the discovery of the neutrino oscillations [1, 2], which can be explained by assuming nonzero masses of neutrinos. However, neutrinos are massless in the Standard Model (SM). Therefore, the neutrino oscillations provide a solid evidence for new physics beyond the SM.

The lightest charged particle in the SM is the electron, and its mass is at least six orders of magnitude larger than the predicted neutrino mass [3]. Thus, any new physics theory beyond the SM should explain why the neutrino masses are so tiny. Several attempts have been made in last a few decades. In the minimal SM extension, there is a unique Weinberg’s dimension five operator [4]

$$\mathcal{L}_5 = f_{ijmn} \bar{l}^C_i \gamma^\alpha l^C_j \phi^{(m)}_{\alpha\beta} \epsilon_{\gamma\delta} + f'_{ijmn} \bar{l}^C_i \gamma^\alpha l^C_j \phi^{(n)}_{\alpha\beta} \epsilon_{\gamma\delta}, \quad (1)$$

where $\phi^{(m)}$ is scalar field and can be one or more; $l$ is the lepton doublet; $\alpha$, $\beta$, $\gamma$ and $\delta$ are the $SU(2)_L$ indices; $i$ and $j$ are the generation indices. The $f$ and $f'$ are roughly of the order $1/M$, where $M$ is the mass scale of new physics. At the tree level, there exist only three different mechanisms to realize this operator [5]: Type-I [6–9], Type-II [10–15], and Type-III [16] see-saw mechanisms involving singlet fermion, scalar triplet, and Majorana triplet fermion, respectively, as heavy intermediate particles with the mass of the order of $M$. We can obtain a tiny neutrino mass by integrating out the heavy fields, which is roughly given by $\langle v^{(m)}_0 \rangle^2 / M$, where $\langle v^{(m)}_0 \rangle$ is the Vacuum Expectation Value (VEV) of the scalar $\phi^{(m)}$. The neutrino mass is suppressed by the heavy mass scale $M$, which is generally close to the unification scale in Grand Unified Theory (GUT) for standard high energy seesaw models where not all the Yukawa couplings are very small. Such a high energy scale is inaccessible at experiments like LHC.

In order to get a testable new physics scale, we need a suppression mechanism different from the usual see-saw mechanisms. One such mechanism could be the radiatively generated neutrino masses [17–33]. The suppression arises from the loop integrals and the new physics scale $M$ is usually the TeV scale. At the $n$-loop order, a dimension $d$ diagram estimates the neutrino mass as

$$m_\nu \sim c \times \left( \frac{1}{16\pi^2} \right)^n \times \frac{\langle v^{(m)}_0 \rangle^2}{M}, \quad (2)$$

---

$^a$ dutta@physics.tamu.edu
$^b$ ghosh@tamu.edu
$^c$ iliag@udel.edu
$^d$ tli@itp.ac.cn
where \( c \) is a dimensionless quantity contains all the coupling constants and other mass ratios.

The existing works on 3 loop masses, for example, the KNT model \([25]\), the AKS model \([29]\) and the Cocktail model \([34]\), involve new particles assuming SM gauge symmetry extended by an additional discrete symmetry. In this work, we present a model with an additional \( SU(2)_N \) gauge symmetry \([35, 36]\) where the gauge symmetry group \( SU(2)_N \) can arise as a subgroup in the decomposition of the \( E_6 \) GUT model \([37–43]\). The particle content of the model restricts the Majorana neutrino masses to be generated below the three-loop level. The \( SU(2)_N \) gauge bosons play an important role in the determination of the neutrino masses at three loops. Due to a large suppression factor, \((\frac{\sigma}{\mathrm{TeV}})^3 \sim 10^{-7}\), arising from the loop integrals, the TeV mass scale can be the new physics scale of our model. The new gauge symmetry \( SU(2)_N \) can be broken around the TeV scale, so our model can be tested at the ongoing LHC and/or HE-LHC, FCC, and SpPc, etc.

This paper is organized as follows: in Section II, we present the model in details and discuss the possible Yukawa coupling terms. We study the Higgs potential and its minimization in Section III. In section IV, we calculate the masses for different scalar particles and obtain the physical states. Section V includes the details about the gauge sector of the model, such as gauge boson masses and their couplings with scalars and fermions. We obtain an analytical expression for the neutrino mass matrix in Section VI. A numerical analysis, to show the consistence of the analytical expression with the experimental data, is given in section VII and we conclude in Section VIII.

## II. MODEL BUILDING

Our model can arise from the \( E_6 \) GUT. One possible maximal subgroup of \( E_6 \) is \( SU(6) \times SU(2)_N \). The \( SU(6) \) group has maximal subgroup \( SU(5) \times U(1)' \). We assume that the \( U(1)' \) gauge symmetry is broken around the GUT scale. Because \( SU(5) \) group contains the SM gauge symmetry, the low energy gauge symmetry of our model is \( SU(3)_C \times SU(2)_L \times SU(2)_N \times U(1)_Y \). The \( SU(2)_N \) has no component to the electric charge operator in our model, so the charge operator is defined as \( Q = T_{3L} + Y \). We assume that the \( SU(2)_L \) doublet assignments are vertical while the \( SU(2)_N \) doublets are horizontal.

Under the gauge symmetry \( SU(3)_C \times SU(2)_L \times SU(2)_N \times U(1)_Y \), the quantum numbers for the fermions are

\[
\begin{align*}
Q_i & \sim \left( \begin{array}{c} u_i \\ d_i \end{array} \right) \sim (3, 2, 1, \frac{1}{6}), \\
U_i & \sim (3, 1, 1, -\frac{2}{3}), \\
D_i & \sim (1, 1, -\frac{1}{3}), \\
L_i & \sim \left( \begin{array}{c} E_i^\alpha \\ \nu_i \\ \epsilon_i \end{array} \right) \sim (1, 2, 2, -\frac{1}{2}), \\
E_i^c & \sim (1, 1, 1, 1), \\
L_i' & \sim \left( \begin{array}{c} E_i^+ \\ E_i^0 \end{array} \right) \sim (1, 2, 1, \frac{1}{2}), \\
N_i & \sim \left( n_i^c, n_i^d \right) \sim (1, 1, 2, 0), \\
XE_i & \sim \left( X_{3i}, X_{2i}, -X_{1i} \right) \sim (1, 1, 3, -1), \\
XE_i^c & \sim \left( X_{3i}^c, X_{2i}^c, -X_{1i}^c \right) \sim (1, 1, 3, 1),
\end{align*}
\]

where \( i = 1, 2, 3 \). \( Q_i, U_i^c, D_i, L_i, E_i^c, L_i' \), and \( N_i^c \) arise from \( 27 \) representation of \( E_6 \), while \( XE_i \) and \( XE_i^c \) come from the \( 351 \) and \( 351^\dagger \) representations of \( E_6 \), respectively. Because \( E_6 \) is a real group while \( XE_i \) and \( XE_i^c \) are vector-like, the gauge anomalies are canceled.

The scalar sector of the model consists of the following particles

\[
\begin{align*}
H_d & \sim \left( \begin{array}{c} \phi_1^0 \\ d_1^- \end{array} \right) \sim (1, 2, 2, -\frac{1}{2}), \\
H_u & \sim \left( \begin{array}{c} \phi_2^+ \\ \phi_2^0 \end{array} \right) \sim (1, 2, 1, \frac{1}{2}), \\
S^0 & \sim \left( S^0_1, S^0_2 \right) \sim (1, 1, 2, 0), \\
T & \sim \left( \begin{array}{cc} T_{11}^{++} & T_{12}^{++} \\ T_{12}^{++} & T_{22}^{++} \end{array} \right) \sim (1, 2, 2, \frac{3}{2}).
\end{align*}
\]

The \( H_d, H_u \), and \( S^0 \) come from \( 27 \) representation, while the bidual scalar \( T \) arises from the \( 650 \) representation of \( E_6 \). The \( SU(2)_N \) gauge symmetry is broken when \( S^0 \) acquires a VEV, and the electroweak gauge symmetry is broken by the VEVs of \( H_d \) and \( H_u \).

The Lagrangian for the Yukawa sector and vector-like mass terms are

\[
\begin{align*}
-\mathcal{L}_{\text{Yukawa}} &= y_{i1j} L_{1i\alpha} T_{j\beta} X E_{j\delta} \epsilon_{\alpha\gamma} \epsilon_{\beta\delta} \epsilon_{\gamma\delta} + y_{21j} L_{2i\alpha} H_{d\beta} X E_{j\delta} \epsilon_{\alpha\gamma} \epsilon_{\beta\delta} \epsilon_{\gamma\delta} + y_{31j} Q_{1i\alpha} H_{d\beta} D_{j\delta} \epsilon_{\alpha\gamma} \epsilon_{\beta\delta} \epsilon_{\gamma\delta} + y_{31j} Q_{2i\alpha} H_{d\beta} L_{j\delta} \epsilon_{\alpha\gamma} \epsilon_{\beta\delta} \epsilon_{\gamma\delta} + y_{31j} Q_{3i\alpha} H_{d\beta} N_{j\delta} \epsilon_{\alpha\gamma} \epsilon_{\beta\delta} \epsilon_{\gamma\delta} + y_{31j} L_{1i\alpha} H_{d\beta} E_{j\delta} \epsilon_{\alpha\gamma} \epsilon_{\beta\delta} \epsilon_{\gamma\delta} + y_{31j} L_{2i\alpha} H_{d\beta} N_{j\delta} \epsilon_{\alpha\gamma} \epsilon_{\beta\delta} \epsilon_{\gamma\delta} + y_{31j} L_{3i\alpha} H_{d\beta} N_{j\delta} \epsilon_{\alpha\gamma} \epsilon_{\beta\delta} \epsilon_{\gamma\delta} + \frac{1}{2} M_{ij} X E_i X E_i^c + \mu_{ij} X E_i E_j^c + m_{Nij} N_{i} N_{j}^c,
\end{align*}
\]

where \( \alpha, \beta, \gamma \) and \( \delta \) are \( SU(2) \) indices; \( i \) and \( j \) are generation indices; and \( \epsilon_{i\alpha} \) is the totally antisymmetric \( SU(2) \) tensor with \( \epsilon_{12} = +1 \). For simplicity, we assume \( M_{ij} = M_{i\delta}, \) and \( \mu_{ij} = 0 \). \( N_{i}^c \)s are needed (and the related terms
in the above Lagrangian) only if the SU(3)_C × SU(2)_L × SU(2)_N × U(1)_Y symmetry of our model has an E_6 origin. However, if we choose to work with the E_6 GUT model then we introduce a discrete Z_2 symmetry to forbid the Type I seesaw mechanism in this model. Under this Z_2 symmetry, only N^c_i is odd, while all the other particles are even. In such a situation, the y_{7ij} and y_{9ij} terms in Eq. (4) are forbidden and the lightest fermion of N^c_i can be a dark matter candidate.

Using the explicit components of the fields, we get

\[
-\mathcal{L}_{\text{Yukawa}} = y_{ij}(E_i^0 T_j^+ X E_3 + \nu_i T_j^+ X E_3 + E_j^0 T_i^+ X E_3 - e_i^+ T_2^+ X E_j) \\
+ y_{ij}(E_i^0 T_j^+ X E_3 + \nu_i T_j^+ X E_3 + E_j^0 T_i^+ X E_3 - e_i^+ T_2^+ X E_j) \\
+ y_{ij}(E_i^0 T_j^+ X E_3 + \nu_i T_j^+ X E_3 + E_j^0 T_i^+ X E_3 - e_i^+ T_2^+ X E_j) \\
+ y_{ij}(E_i^0 T_j^+ X E_3 + \nu_i T_j^+ X E_3 + E_j^0 T_i^+ X E_3 - e_i^+ T_2^+ X E_j) \\
+ y_{ij}(E_i^0 T_j^+ X E_3 + \nu_i T_j^+ X E_3 + E_j^0 T_i^+ X E_3 - e_i^+ T_2^+ X E_j)
\]

(4)

We consider three nonzero VEVs \langle \phi^0_i \rangle = \frac{v_i}{\sqrt{2}}$, \langle \phi^0_j \rangle = \frac{v_j}{\sqrt{2}}$ and \langle S^0 \rangle = \frac{v}{\sqrt{2}}. From Eq. (4) we get that \frac{v_i}{\sqrt{2}} gives the down-type quark masses and charged lepton masses, \frac{v_j}{\sqrt{2}} gives the up-type quark masses, and \frac{v}{\sqrt{2}} gives masses to the vector-like particle (D^c_i, D_i, (E_i^c, E_i^0)) and (E^c_i, E^0_i). However, there is no neutrino mass term at tree level, a dark matter candidate.

III. THE HIGGS POTENTIAL

We need the complete Higgs potential to get the physical scalar states and their masses. The most general renormalizable scalar potential for the Higgs scalars of our model is

\[
V_{\text{potential}} = m_1^2 H_{d_{\alpha\beta}}^\dagger H_{d_{\beta\alpha}} + m_2^2 H_u^\dagger H_u + m_3^2 S^0_\alpha \phi^0_\alpha + m_4^2 T_{\alpha\beta}^\dagger T_{\beta\alpha} + \lambda_1 \frac{1}{2} H_{d_{\alpha\beta}}^\dagger H_{d_{\beta\alpha}} + \lambda_2 \frac{1}{2} H_u^\dagger H_u + \lambda_3 \frac{1}{2} S^0_\alpha S^0_\beta + \lambda_4 \frac{1}{2} T_{\alpha\beta}^\dagger T_{\beta\alpha} + \lambda_5 \frac{1}{2} D^c_\alpha \phi^0_\alpha + \lambda_6 \frac{1}{2} D_\alpha \phi^0_\alpha + \lambda_7 \frac{1}{2} S^0_\alpha \phi^0_\beta + \lambda_8 \frac{1}{2} T_{\alpha\beta} S^0_\alpha \phi^0_\beta + \lambda_9 \frac{1}{2} H_{d_{\alpha\beta}} \phi^0_\alpha + \lambda_{10} \frac{1}{2} H_u \phi^0_\alpha + \lambda_{11} \frac{1}{2} S^0_\alpha \phi^0_\alpha + \lambda_{12} \frac{1}{2} T_{\alpha\beta} \phi^0_\alpha + \lambda_{13} \frac{1}{2} D^c_\alpha \phi^0_\alpha + \lambda_{14} \frac{1}{2} D_\alpha \phi^0_\alpha + \lambda_{15} \frac{1}{2} S^0_\alpha \phi^0_\alpha + \lambda_{16} \frac{1}{2} T_{\alpha\beta} \phi^0_\alpha + \lambda_{17} \frac{1}{2} H_{d_{\alpha\beta}} \phi^0_\alpha + \lambda_{18} \frac{1}{2} H_u \phi^0_\alpha + \lambda_{19} \frac{1}{2} S^0_\alpha \phi^0_\alpha + \lambda_{20} \frac{1}{2} T_{\alpha\beta} \phi^0_\alpha
\]

(5)

where all the parameters are real. Here \alpha, \beta, \gamma, \delta, \rho, \sigma, \mu and \nu are the SU(2) indices and \epsilon_{\alpha\beta} is the totally antisymmetric SU(2) tensor with \epsilon_{12} = +1.

The minimum of the potential is given by

\[
V_0^{\text{potential}} = \frac{1}{2} m_1^2 v_1^2 + \frac{1}{2} m_2^2 v_2^2 + \frac{1}{2} m_3^2 v_3^2 + \frac{1}{2} m_4^2 v_4^2 + \frac{1}{8} (\lambda_1 + \lambda_3) v_1^4 + \frac{1}{8} \lambda_2 v_2^2
\]

(6)

The minimisation conditions are

\[
m_1^2 + \frac{1}{2} (\lambda_1 + \lambda_3) v_1^2 + \frac{1}{2} \lambda_2 v_2^2 + \frac{1}{2} \lambda_3 v_3^2 - \frac{1}{\sqrt{2}} \lambda' v_1 v_2 = 0
\]

(7)

\[
m_2^2 + \frac{1}{2} \lambda_4 v_1^2 + \frac{1}{2} \lambda_5 v_2^2 + \frac{1}{2} \lambda_6 v_3^2 - \frac{1}{\sqrt{2}} \lambda' v_1 v_2 = 0
\]

(8)

\[
m_3^2 + \frac{1}{2} \lambda_7 v_1^2 + \frac{1}{2} \lambda_8 v_2^2 + \frac{1}{2} \lambda_9 v_3^2 - \frac{1}{\sqrt{2}} \lambda' v_1 v_2 = 0
\]

(9)
After \( H_d, H_u \) and \( S^0 \) acquire VEVs, we can write them as
\[
H_d \sim \left( \frac{1}{\sqrt{2}} (v_3 + \rho_3 + i \eta_3) \right) \frac{1}{\sqrt{2}} (\rho_3 + i \eta_3), \tag{10}\]
\[
H_u \sim \left( \frac{1}{\sqrt{2}} (v_2 + \rho_2 + i \eta_2) \right) \frac{1}{\sqrt{2}} (v_3 + \rho_3 + i \eta_3). \tag{11}\]

### IV. SCALAR MASSES

With the scalars in Eqs. (10) and (11), we can now obtain the terms in the Lagrangian density which gives masses to the different scalars from Eq. (5). The mass terms for the single charged scalars are
\[
V_{\text{mass}} = \left( \frac{\lambda_v v_1 v_2}{2} + \frac{\lambda_v v_3}{\sqrt{2}} \right) (\phi_1^+ - \phi_2^-) \left( \frac{\lambda_{v_1}}{1} + \frac{1}{\sqrt{2}} \right) \left( \frac{\phi_1^+}{\sqrt{2}} \phi_2^- \right) + \text{chiral terms} \tag{12}\]

First, we get a mixing between \( \phi_1^+ \) and \( \phi_2^+ \). That mixing gives four scalars \( h_1^\pm \) and \( h_2^\pm \) with mass squared zero and \( \sqrt{\lambda_{v_1} \lambda_{v_2}} \). The states are
\[
h_1^+ = \cos \beta \phi_1^+ + \sin \beta \phi_2^+, \tag{13}\]
\[
h_2^+ = -\sin \beta \phi_1^+ + \cos \beta \phi_2^+, \tag{14}\]

where the mixing angle is given by \( \tan \beta = \frac{\lambda_{v_1}}{\lambda_{v_2}} \). The two massless states \( h_1^\pm \) are corresponding to two charged Goldstone modes, and the other two states \( h_2^\pm \) are two single charged physical scalars.

The scalars \( \phi_3^+ \) and \( T_2^\pm \) will mix and give the following four mass eigenstates
\[
H_1^\pm = \cos \theta \phi_3^+ + \sin \theta T_2^\pm, \tag{15}\]
\[
H_2^\pm = -\sin \theta \phi_3^+ + \cos \theta T_2^\pm, \tag{16}\]

with mass squared
\[
m^2_{H_1^\pm} = \frac{1}{2} (m_2^2 + m_3^2) + \frac{1}{2} \sqrt{(m_2^2 - m_3^2)^2 + 144 \lambda^2 v_1^4}, \tag{17}\]

and
\[
m^2_{H_2^\pm} = \frac{1}{2} (m_2^2 + m_3^2) - \frac{1}{2} \sqrt{(m_2^2 - m_3^2)^2 + 144 \lambda^2 v_1^4}, \tag{18}\]

respectively. The mixing angle is given by \( \tan 2\theta = \frac{12 \lambda v_2}{m_2^2 - m_3^2} \). The definition of \( m_2^2 \) and \( m_3^2 \) are
\[
m_2^2 = m_T^2 + \frac{(\lambda_{v_1} + \lambda_{v_2}) v_1^2}{2} + \frac{(\lambda_{v_1} + \lambda_{v_2}) v_2^2}{2} + \frac{(\lambda_{v_1} + \lambda_{v_2}) v_2^2}{2} + \frac{\lambda v_2 v_3}{\sqrt{v_1}}, \tag{19}\]

and
\[
m_3^2 = -\frac{\lambda v_1^2}{2} + \frac{\lambda v_2^2}{2} + \frac{\lambda v_2^2}{2} + \frac{\lambda v_2 v_3}{\sqrt{v_1}}. \tag{20}\]

The four states \( H_1^\pm \) and \( H_2^\pm \) are identified as four single charged physical scalar. From Eq. (12) we get two more single charged physical scalar \( T_1^\pm \) with mass squared
\[
m^2_{T_1^\pm} = m_T^2 + \frac{(\lambda_{v_1} + \lambda_{v_2}) v_1^2}{2} + \frac{(\lambda_{v_1} + \lambda_{v_2}) v_2^2}{2} + \frac{\lambda v_2 v_3}{2}. \tag{21}\]
The following term of the Lagrangian density gives the masses of the double charged scalars

\[
V_{\text{mass}}^{\pm \pm} = (T_1^{-} - T_2^{-}) \left( m^2_T + \frac{(\lambda_{15} + \lambda_{16} + \lambda_{17} + \lambda_{18})v^2_2}{2} + \frac{\lambda_{14}v^2_2}{2} + \frac{\lambda_{11}v^2_2}{2} \right) \left( T_2^{\pm} \right). \tag{22}
\]

The mass matrix is already diagonalized and gives the mass squared of the four doubly charged physical scalar \(T_1^{\pm \pm}\) and \(T_2^{\pm \pm}\)

\[
m^2_{T_1^{\pm \pm}} = m^2_T + \frac{\lambda_{15}v^2_2}{2} + \frac{\lambda_{14}v^2_2}{2} + \frac{(\lambda_{11} + \lambda_{12})v^2_2}{2},
\]

and

\[
m^2_{T_2^{\pm \pm}} = m^2_T + \frac{\lambda_{15}v^2_2}{2} + \frac{\lambda_{13}v^2_2}{2} + \frac{(\lambda_{11} + \lambda_{12})v^2_2}{2},
\]

respectively. Next we consider the mass terms for the five neutral scalars

\[
V_{\text{mass}}^{\rho} = (\rho_1 \rho_2 \rho_2 s) \left( \frac{\lambda_{11}v^2_2}{2} + \frac{\lambda_{11}v^2_2}{2} - \frac{\lambda_{11}v^2_2}{2} + \frac{\lambda_{11}v^2_2}{2} \right) \left( \rho_1 \rho_2 \rho_2 s \right)
\]

\[
+ \left( \frac{\lambda_{10}v^2_2}{2} + \frac{\lambda_{10}v^2_2}{2} \right) \left( \rho_3 \rho_1 s \right) \left( \frac{v_1}{v_1} \frac{v_1}{v_1} \right) \left( \rho_3 \rho_1 s \right).
\]

Here, \(\rho_3\) and \(\rho_1 s\) are the states to mix and give one neutral scalar Goldstone mode and one neutral physical scalar with mass squared equal to \(\frac{\lambda_{11}v^2_2}{2}\). We get three more neutral physical scalars from the mixing of \(\rho_1, \rho_2\) and \(\rho_2 s\). The term below gives the masses of pseudoscalars

\[
V_{\text{mass}}^{\eta} = \frac{\lambda^2}{2v^2_2} \left( \eta_1 \eta_2 \eta_2 s \right) \left( \frac{v_1}{v_1} \frac{v_1}{v_1} \frac{v_2}{v_2} \frac{v_2}{v_2} \right) \left( \eta_1 \eta_2 \eta_2 s \right) \left( \frac{\lambda_{10}v^2_2}{2} + \frac{\lambda_{10}v^2_2}{2} \right) \left( \eta_3 \eta_1 \eta_1 s \right) \left( \frac{v_1}{v_1} - 1 \right) \left( \eta_3 \eta_1 \eta_1 s \right),
\]

where \(\eta_1, \eta_2,\) and \(\eta_2 s\) will mix and give two neutral pseudoscalars Goldstone mode and one physical neutral pseudoscalar with mass squared \(\frac{\lambda^2}{2v^2_2}\). \(\eta_3\) and \(\eta_1\) will mix and give another neutral pseudoscalar Goldstone mode and another physical neutral pseudoscalar with mass squared equal to \(\frac{\lambda^2}{2v^2_2}\).

We start with 24 scalar degrees of freedom and end up with 18 physical scalars. The other six degrees of freedom correspond to the six Goldstone mode are eaten by the massless gauge bosons. The Goldstone modes will become the longitudinal modes of gauge bosons, which will become massive. So there will be six massive gauge bosons and one massless gauge boson.

V. GAUGE BOSONS

In this Section, we discuss the gauge boson masses and physical gauge boson states, as well as their interactions with the physical scalars and fermions. The Lagrangian density, which gives the gauge boson masses and their interactions with the scalars, is

\[
L_{\text{gauge-scalar}} = (D_\mu H_\alpha)^{\dagger}_\alpha (D^\mu H_\alpha) + (D_\mu H_\alpha^T)^{\dagger}_{\alpha \beta} (D^\mu H_\alpha^T)_{\beta \alpha} + (D_\mu S_\alpha G^T)^{\dagger}_{\alpha} (D^\mu S_\alpha G^T)_{\alpha} + (D_\mu T_\alpha)^{\dagger}_{\alpha \beta} (D^\mu T_\alpha)_{\beta \alpha},
\]

where \(\alpha\) and \(\beta\) are the SU(2) indices. The covariant derivative is defined as

\[
D_\mu = \partial_\mu + i g W_\mu + i g' Y B_\mu ,
\]

where \(g, g'_2\) and \(g'\) are the coupling constant corresponding to the SU(2)_L, SU(2)_N, and U(1)_Y groups respectively. W_μ, W'_μ, and B_μ are the gauge bosons of the SU(2)_L, SU(2)_N, and U(1)_Y groups respectively.
V.I. Gauge Boson Masses

We define \( \sqrt{2} W_{\mu}^\pm = W_{1\mu} \pm iW_{2\mu} \) and \( \sqrt{2} X_{1,2\mu} = W'_{1\mu} \pm iW'_{2\mu} \). After the spontaneous symmetry breaking of the gauge groups the massless gauge boson will become massive. We write the part of Eq. (27) that gives the masses of the gauge bosons

\[
\mathcal{L}_{\text{mass}} = \frac{1}{4} g^2 (v_1^2 + v_2^2) W_\mu W^- + \frac{1}{4} g'^2 (v_1^2 + v_2^2) X_{2\mu} X_1^\mu + \frac{1}{8} (B_\mu W_{3\mu} W_{3\mu}') \left( \begin{array}{ccc}
ger_W^2 (v_1^2 + v_2^2) & -g' g_1^2 v_1 & -g' g_2^2 v_1 \\ -gg' (v_1^2 + v_2^2) & g^2 (v_1^2 + v_2^2) & gg_1^2 v_1 \\ -g' g_2^2 v_1 & gg_1^2 v_1 & g^2 (v_1^2 + v_2^2) \end{array} \right) \left( \begin{array}{c} B_\mu \\ W_{3\mu}' \\ W_{3\mu}'' \end{array} \right). \tag{29}
\]

After the spontaneous symmetry breaking, \( B_\mu, W_{3\mu} \) and \( W_{3\mu}' \) will mix and give three physical gauge bosons, which can be written as

\[
A_\mu = \sin \theta_W W_{3\mu} + \cos \theta_W B_\mu 
\]

\[
Z_\mu = \cos \theta_N \cos \theta_W W_{3\mu} - \cos \theta_N \sin \theta_W B_\mu + \sin \theta_W W_{3\mu}'. \tag{31}
\]

\[
X_{3\mu} = -\sin \theta_N \cos \theta_W W_{3\mu} + \sin \theta_N \sin \theta_W B_\mu + \cos \theta_W W_{3\mu}', \tag{32}
\]

where the mixing angles are given by, \( \tan \theta_W = \frac{g'}{g} \) and \( \tan 2\theta_N = \frac{b}{a_\mp} \). The definitions of \( b \) and \( a_\pm \) are

\[
b \equiv \frac{1}{8} g_2' \sqrt{g^2 + g'^2 v_1^2}, \tag{33}
\]

\[
a_\pm \equiv \frac{1}{16} \left( g_2'^2 (v_1^2 + v_2^2) \pm (g^2 + g'^2) (v_1^2 + v_2^2) \right). \tag{34}
\]

There are four other physical gauge bosons, which are \( W_\mu^\pm \) and \( X_{1,2\mu} \). The mass squared of all the physical gauge bosons are then given by,

\[
m_{W_\mu}^2 = \frac{1}{4} g^2 (v_1^2 + v_2^2), \tag{35}
\]

\[
m_{X_{1,2}}^2 = \frac{1}{4} g'^2 (v_1^2 + v_2^2), \tag{36}
\]

\[
m_A^2 = 0, \tag{37}
\]

\[
m_Z^2 = a_+ - \sqrt{a_+^2 + b^2}, \tag{38}
\]

\[
m_X^2 = a_+ + \sqrt{a_+^2 + b^2}. \tag{39}
\]

Also, there are exactly six massive gauge bosons corresponding to six Goldstone modes.

V.II. Gauge Bosons Interactions

Next, we study the interactions between the physical gauge bosons and scalars. A few important terms in Eq. (27) are

\[
\mathcal{L}_{\text{int}} = \frac{1}{2} g_2^2 X_{2\mu} X_1^\mu (\dot{\phi}_3^+ \phi_3^- + i \sqrt{2} g_2' X_{2\mu} (\partial^\mu \phi_3^+) \dot{\phi}_3^- - i \sqrt{2} g_2 X_{1\mu} \phi_3^+ (\partial^\mu \phi_3^-)) + \frac{i}{\sqrt{2}} g_2' X_{2\mu} (\partial^\mu \phi_3^+) \dot{\phi}_3^- - \frac{i}{\sqrt{2}} g_2' X_{1\mu} (\partial^\mu \phi_3^-) T_2^+ \tag{40}
\]

\[
- \frac{i}{\sqrt{2}} g_2' X_{1\mu} T_1^- (\partial^\mu T_2^+) - \frac{i}{\sqrt{2}} g_2' X_{2\mu} T_1^+ (\partial^\mu T_2^-) - \frac{i}{\sqrt{2}} g_2' X_{2\mu} (\partial^\mu T_1^+) T_2^- + \ldots.
\]
We rewrite Eq. (40) in terms of the physical scalars using Eqs. (13)-(16), and then derive the necessary Feynman rules for the interactions in the following

\[
\mathcal{L}_{gs}^{\text{int}} = \frac{1}{2} g_2' X_{2\mu} X_1^{\mu} \left[ H_1^+ H_1^- + H_2^+ H_2^- \right] + \frac{i}{\sqrt{2}} g_2' X_{2\mu} \cos \theta \sin \beta \left[ H_1^+ (\partial^\mu h_2^-) - (\partial^\mu H_1^-) h_2^- \right] + \frac{i}{\sqrt{2}} g_2' X_{2\mu} \sin \theta \cos \beta \left[ (\partial^\mu H_2^+) h_2^- - H_2^+ (\partial^\mu h_2^-) \right] + \frac{i}{\sqrt{2}} g_2' X_{2\mu} \sin \theta \left[ T_1^+ (\partial^\mu H_1^-) - (\partial^\mu T_1^-) H_1^- \right] + \frac{i}{\sqrt{2}} g_2' X_{2\mu} \cos \theta \left[ T_1^+ (\partial^\mu H_2^-) - (\partial^\mu T_1^-) H_2^- \right] + h.c + \ldots .
\]

Next, we consider the kinetic energy terms of the \(L_i\) leptons. These terms give us the interactions of the leptons with the gauge bosons. Let us first write down the kinetic term for the interactions in the following rules for the interactions in the following:

\[
\mathcal{L}_{\text{kinetic}}^L = (\bar{L}_i)_{\alpha\beta} i\gamma^\mu (\partial^\mu L_i)_{\beta\alpha} + (\bar{L}_i)_{\alpha\beta} i\gamma^\mu \left( \frac{1}{2} ig_s W_{\mu\alpha} L_i \right)_{\beta\alpha} + (\bar{L}_i)_{\alpha\beta} i\gamma^\mu \left( \frac{1}{2} ig_s W_{\mu\alpha} L_i \right)_{\beta\alpha} + (\bar{L}_i)_{\alpha\beta} i\gamma^\mu \left( \frac{1}{2} ig_s B_{\mu\alpha} L_i \right)_{\beta\alpha} ,
\]

where \(i\) is the generation index; \(\alpha\) and \(\beta\) are \(SU(2)\) index; and \(a = 1,2,3\). The Eq. (42) will give us the important interaction term between the neutral leptons and gauge bosons \(X_{1,2}\) as below

\[
\mathcal{L}_{\text{kinetic}}^L = -\frac{1}{\sqrt{2}} g_2' X_{2\mu} \bar{\nu}_i \gamma^\mu E_i^0 + h.c + \ldots .
\]

We need one more interaction term which will play an important in the next section. We write the Yukawa sector, given by Eq. (4), in terms of the physical scalar and fermions. We write all the relevant terms here:

\[
\mathcal{L}_{\text{Yukawa}} = -y_{1ij} E_i^0 T_1^+ X E_{3j} + y_{2ij} \sin \phi \ E_i^0 h_2^- X E_{3j}^c + \ldots .
\]

**VI. NEUTRINO MASSES**

We obtain an analytical expression for the neutrino mass matrix elements in this Section. As mentioned before, the particle content of our model does not allow us to generate the neutrino masses below three loop, thus, the leading contributions to neutrino masses arise from the three loop diagrams shown in Fig. 1. The new gauge bosons, \(X_1\) and \(X_2\), are responsible for these three loop diagrams\(^1\).

![Diagram](attachment:image.png)

FIG. 1: Three loop diagrams responsible for the Majorana neutrino masses. We have two similar diagrams for \(X_1\) gauge boson.

We have all the necessary physical particle masses and the interaction terms to calculate the three loop diagram in Fig. 1. In unitary gauge, the Majorana mass matrix elements are given by

\[
(M_\nu)_{ji} = \frac{1}{4} g_2' y_{1ji} y_{2lj} \sin 2\theta \ \sin^2 \beta \times I_{3\text{loop}} ,
\]

\(^1\) We have used the package TikZ-Feynman [50] to draw the diagram.
where \( i, j, l = 1,2,3 \). And \( I_{\text{loop}} \) is the three-loop integral given by

\[
I_{\text{loop}} = \frac{1}{(16\pi^2)^3} \left( m_X^2 - m_0^2 \right) \left( m_X^2 - m_0^2 \right) m_X^2 \int_0^\infty \frac{dr}{r + M^2} \left[ \frac{1}{r + m_{H_1}^2} + \frac{1}{r + m_{H_2}^2} \right] \\
\times \left\{ 4M_1m_0/m_0 \{ f_h(r, m_X^2, m_0^2, m_{H_2}^2)g_{2T}(r, m_X^2, m_0^2, m_{T_1}^2) + f_T(r, m_X^2, m_0^2, m_{H_2}^2)g_{2h}(r, m_X^2, m_0^2, m_{H_2}^2) \right\} \\
+ 2m_0f_T(r, m_X^2, m_0^2, m_{T_1}^2) \{ g_{4T}(r, m_X^2, m_0^2, m_{T_1}^2) - m_X^2g_{2h}(r, m_X^2, m_0^2, m_{T_1}^2) \} \\
- 2m_0f_h(r, m_X^2, m_0^2, m_{H_2}^2) \{ g_{4T}(r, m_X^2, m_0^2, m_{T_1}^2) - m_X^2g_{2h}(r, m_X^2, m_0^2, m_{H_2}^2) \} \right\}.
\]

(46)

The definitions of the integral functions appeared in Eq. (46) are

\[
f_h(r, m_X^2, m_0^2, m_{H_2}^2) = \int_0^1 dx \ln \frac{x(1-x)r + (1-x)m_X^2 + x m_{H_2}^2}{x(1-x)r + (1-x)m_0^2 + x m_{H_2}^2} ,
\]

(47)

\[
f_T(r, m_X^2, m_0^2, m_{T_1}^2) = \int_0^1 dx \ln \frac{x(1-x)r + (1-x)m_X^2 + x m_{T_1}^2}{x(1-x)r + (1-x)m_0^2 + x m_{T_1}^2} ,
\]

(48)

\[
g_{2h}(r, m_X^2, m_0^2, m_{H_2}^2) = m_X^2 \int_0^1 dx \ln \frac{x(1-x)r + (1-x)m_X^2 + x m_{H_2}^2}{m_X^2} \\
- m_0^2 \int_0^1 dx \ln \frac{x(1-x)r + (1-x)m_0^2 + x m_{H_2}^2}{m_X^2} ,
\]

(49)

\[
g_{2T}(r, m_X^2, m_0^2, m_{T_1}^2) = m_X^2 \int_0^1 dx \ln \frac{x(1-x)r + (1-x)m_X^2 + x m_{T_1}^2}{m_X^2} \\
- m_0^2 \int_0^1 dx \ln \frac{x(1-x)r + (1-x)m_0^2 + x m_{T_1}^2}{m_X^2} ,
\]

(50)

\[
g_{4h}(r, m_X^2, m_0^2, m_{H_2}^2) = m_X^4 \int_0^1 dx \ln \frac{x(1-x)r + (1-x)m_X^2 + x m_{H_2}^2}{m_X^2} \\
- m_0^4 \int_0^1 dx \ln \frac{x(1-x)r + (1-x)m_0^2 + x m_{H_2}^2}{m_X^2} ,
\]

(51)

\[
g_{4T}(r, m_X^2, m_0^2, m_{T_1}^2) = m_X^4 \int_0^1 dx \ln \frac{x(1-x)r + (1-x)m_X^2 + x m_{T_1}^2}{m_X^2} \\
- m_0^4 \int_0^1 dx \ln \frac{x(1-x)r + (1-x)m_0^2 + x m_{T_1}^2}{m_X^2} .
\]

(52)

The mass matrix elements get large suppression from \( \frac{g^4}{(16\pi^2)^2} \sim 10^{-11} \), which pushes the new scale to TeV. The numerical analysis depends on the choice of various parameters in the model, particularly the contribution from the gauge bosons \( X_{1,2} \) will be very crucial.

\[\text{---}\]

2 A large part of the loop integral calculation is done by using the FeynCalc package [44, 45].
In this Section, we show that the neutrino mass matrix given by the Eq. (45) can fit the neutrino oscillation data. We only consider the normal hierarchy of the neutrino masses. The discussion of the inverted hierarchy case will be similar. The best fit of the neutrino oscillation data for normal hierarchy at 3σ range [46] are

\[
\begin{align*}
\sin^2 \theta_{12} & = 0.271 - 0.345; \quad \sin^2 \theta_{23} = 0.385 - 0.635; \quad \sin^2 \theta_{13} = 0.01934 - 0.02392; \quad \delta_{CP} = 0^\circ - 360^\circ; \\
\Delta m_{21}^2 & = 7.03 \times 10^{-5}\text{eV} - 8.09 \times 10^{-5}\text{eV}; \quad \Delta m_{31}^2 = 2.407 \times 10^{-3}\text{eV} - 2.643 \times 10^{-3}\text{eV}.
\end{align*}
\]

We define the matrix \(M_{\nu} = \text{diag}(m_1, m_2, m_3)\) as the diagonalised neutrino mass matrix. In the normal hierarchy scenario, the oscillation data correspond to \(m_1 < m_2 < m_3\). In the simplest scenario, the lightest neutrino can be assumed to be massless. We take the neutrino mass eigenvalues as follows,

\[
m_1 \simeq 0\text{eV}; \quad m_2 \simeq 8.66 \times 10^{-3}\text{eV}; \quad m_3 \simeq 4.98 \times 10^{-2}\text{eV}.
\]

We then obtain the Majorana mass matrix from the \(M_{\nu}\) matrix as

\[
M_{\nu} = U^{-1} M_{\nu} U,
\]

where \(U\) is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [47, 48] parametrized by

\[
U = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\
s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13}
\end{pmatrix} \times P,
\]

where \(c_{ab} \equiv \cos \theta_{ab}\) and \(s_{ab} \equiv \sin \theta_{ab}\). \(P\) is the unit matrix for Dirac neutrinos or a diagonal matrix with two phase angles for Majorana neutrinos. We take both the Majorana phase angles to be zero, and the central values of the parameters from Eq. (53). Without loss of generality, we choose \(\delta_{CP}\) to be zero as well. We can now obtain the mass matrix from Eq. (55)

\[
M_{\nu} = \begin{pmatrix}
6.35989 \times 10^{-12} & 1.18618 \times 10^{-11} & 1.32647 \times 10^{-11} \\
1.18618 \times 10^{-11} & 2.3611 \times 10^{-11} & 2.59738 \times 10^{-11} \\
1.32647 \times 10^{-11} & 2.59738 \times 10^{-11} & 2.86893 \times 10^{-11}
\end{pmatrix}\text{GeV}.
\]

We have the constraint \(v_1^2 + v_2^2 \simeq 246^2\text{GeV}^2\) and \(\tan \beta = \frac{v_2}{v_1}\). The \(v_1\) and \(v_2\) are also constrained by the top and bottom quark masses. The small values of \(\tan \beta\) will give large value of the top quark Yukawa coupling \((y_t)_{33}\), which will make the model non-perturbative. To avoid that, we take \(\tan \beta = 2\), which gives us the mixing angle \(\beta\) to be equal to 63°. We then get \(\frac{v_2}{v_1}\) and \(\frac{v_2}{v_1}\) to be 78 GeV and 156 GeV, respectively. We choose the VEV \(\frac{v_2}{v_1}\) at the TeV scale, to be 17 TeV. Now choosing \(\frac{v_2}{v_1}\) to be equal to 0.35, we get the mass of the gauge boson \(X\) to be 5 TeV. By choosing appropriate values for different \(\lambda\) parameters in Eqs. (19) and (20), we can take \(m_T^2\) and \(m_N^2\) to be \(2.5 \times 10^7\text{GeV}^2\) and \(2.5 \times 10^9\text{GeV}^2\) respectively. Now choosing \(\lambda \approx 0.03\), we obtain \(m_{T_1}\) and \(m_{T_2}\) to be 5 TeV and 500 GeV respectively from Eqs. (17) and (18) using the mixing angle, \(\theta = 0.005^\circ\). Similarly, Eq. (21) gives \(m_{T_3}\) to be 500 GeV. The other mass parameters needed are the vector-like particle masses. The lower bound on the vector-like lepton mass is 101 GeV, which comes from the LEP experiment [49]. We take \(m_0\) to be 115 GeV, 125 GeV, and 135 GeV respectively for the first, second, and third generations. We use \(M\), the mass for the \(XE\) particle to be 110 GeV, 120 GeV, and 130 GeV respectively for the three generations. All parameters are as follows,

\[
m_{T_1} = 5\text{ TeV}; \quad m_{T_2} = 500\text{ GeV}; \quad m_X = 5\text{ TeV}; \quad m_{h_2} = 268\text{ GeV}; \quad m_{T_1} = 500\text{ GeV} , \\
\beta = 63^\circ; \quad \theta = 0.005^\circ; \quad M = (110, 120, 130)\text{ GeV}; \quad m_0 = (115, 125, 135)\text{ GeV}.
\]

We can use these parameters in Eq. (45) to fit the neutrino mass matrix given in Eq. (57) for Yukawa couplings that satisfy \(y_1 \times y_2\) to be of the order of 0.001 to 0.0001.

We have presented one set of viable input parameters. There exist many other possible sets as well. The Yukawa coupling constants can be made even larger by taking larger values of \(\frac{v_2}{v_1}\). The mass \(m_X\) controls the value of the numerical integration. The other mass parameters do not play an important role in the calculations. The mass gap between \(m_{H_1}\) and \(m_{H_2}\) can be small, which does not affect the numerical result in any significant way. Another important factor, which affects the value of the numerical result, is the loop suppression factor. The value of \(\tan \beta\) can change the numerical results as well.

VII. NUMERICAL ANALYSIS

We can use these parameters in Eq. (45) to fit the neutrino mass matrix given in Eq. (57) for Yukawa couplings that satisfy \(y_1 \times y_2\) to be of the order of 0.001 to 0.0001.
VIII. CONCLUSION

To construct a natural radiative neutrino mass model which can be tested at the future collider experiments, we have extended the SM gauge symmetry by the $SU(2)_N$ gauge group, which comes from the decomposition of the $E_6$ GUT. We have presented the particle content and all the possible Yukawa interactions and studied the scalar and gauge sectors in details. Interestingly, the tiny neutrino masses are found to be only generated at three loops where the $SU(2)_N$ gauge bosons play an important role. The new gauge bosons $X_{1,2}$ and vector-like fermions enter into the three loop diagrams. Because of the large suppression from the loop integral, the new physics scale can be around TeV, which is testable, unlike the high-scale tree-level see-saw mechanism, as well as the one-loop and two-loop neutrino mass models.

We have obtained an analytical expression for the Majorana neutrino masses. This mass expression depends on the spontaneous symmetry breaking scale of the $SU(2)_N$ gauge group. From the three loop calculation, we have shown that the analytical expression, in our radiative neutrino mass model, is consistent with the neutrino oscillation data. For example, for $\Delta m_{\nu_2}^2$ to be 17 TeV and the new gauge boson mass to be 5 TeV. The other mass parameters are chosen to be between the electroweak and TeV scale, which is consistent with our goal of obtaining neutrino mass at experimentally testable scale. Using these input parameters along with the neutrino mass matrix obtained from the oscillation data, we found the Yukawa couplings to be 0.01 or larger. For larger values of $\Delta m_{\nu_2}^2$ the Yukawa couplings will be larger.

Acknowledgments

BD and SG are supported in part by the DOE grant DE-SC0010813. IG is supported in part by Bartol Research Institute. TL is supported in part by the Projects 11475238 and 11647601 supported by National Natural Science Foundation of China and by Key Research Program of Frontier Science, CAS.

[1] The Super-Kamiokande Collaboration (Y. Fukuda et al.), Phys. Rev. Lett. 81 (1998) 1562.
[2] The SNO Collaboration (Q. R. Ahmad et al.), Phys. Rev. Lett. 89 (2002) 011301.
[3] The KamLAND-Zen Collaboration (A. Gando et al.), Phys. Rev. Lett. 117 (2016) 082503.
[4] S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566.
[5] E. Ma, Phys. Rev. Lett. 81 (1998) 1171.
[6] P. Minkowski, Phys. Lett. B67 (1977) 421.
[7] T. Yanagida, Conf. Proc. C7902131 (1979) 95.
[8] M. Gell-Mann, P. Ramond and R. Slansky, Conf. Proc. C790927 (1979) 315.
[9] R. N. Mahapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
[10] M. Magg and C. Wetterich, Phys. Lett. B34 (1980) 61.
[11] J. Schechter and J. Valle, Phys. Rev. D22 (1980) 2227.
[12] C. Wetterich, Nucl. Phys. B187 (1981) 343.
[13] G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B181 (1981) 287.
[14] R. N. Mahapatra and G. Senjanovic, Phys. Rev. D23 (1981) 165.
[15] T. P. Cheng and L. F. Li, Phys. Rev. D22 (1980) 2860.
[16] R. Foot, H. Lew, X. He and G. C. Joshi, Z. Phys. C44 (1989) 441.
[17] A. Zee, Phys. Lett. B93, (1980) 389.
[18] L. Wolfenstein, Nucl. Phys. B175 (1980) 93.
[19] A. Zee, Phys. Lett. B161 (1985) 141.
[20] K. Babu, Phys. Lett. B203 (1985) 132.
[21] E. Ma, Phys. Rev. Lett. 81 (1998) 1171.
[22] P. Fileviez Perez and M.B. Wise, Phys. Rev. D80 (2009) 053006.
[23] S. Choubey, M. Duerr, M. Mitra and W. Rodejohann, JHEP 1205 (2012) 017.
[24] K. Babu and C. Macesanu, Phys. Rev. D67 (2003) 073010.
[25] L. M. Krauss, S. Nasri and M. Trodden, Phys. Rev. D67 (2003) 085002.
[26] K. Cheung and O. Seto, Phys. Rev. D69 (2004) 113009.
[27] E. Ma, Phys. Rev. D73 (2006) 077301.
[28] E. Ma and U. Sarkar, Phys. Lett. B653 (2007) 288.
[29] M. Aoki, S. Kanemura and O. Seto, Phys. Rev. Lett. 102 (2009) 051805.
[30] M. Aoki, S. Kanemura and O. Seto, Phys. Rev. D80 (2009) 033007.
[31] D. Aristizabal Sierras and M. Hirsch, JHEP 0612 (2006) 052.
[32] L. G. Jin, R. Tang and F. Zhang, Phys. Lett. B741 (2015) 163.
[33] Y. Cai, J. Herrero-Garcia, M. A. Schmidt, A. Vicente and R. R. Volkas, Front. in Phys. 5 (2017) 63.
[34] M. Gustafson, J.M. No and M. A. Rivera, Phys. Rev. Lett. 110 (2013) 211802.
[35] D. London and J. L. Rosner, Phys. Rev. D34 (1986) 1530.
[36] E. Ma and J. Wudka, Phys. Lett. B712 (2012) 391.
[37] R. Slansky, Phys. Rep. 79 (1981) 1.
[38] F. Gursey, P. Ramond and P. Sikivie, Phys. Lett. B60 (1976) 177.
[39] Y. Achiman and B. Stech, Phys. Lett. B77 (1978) 389.
[40] R. Barbieri and D.V. Nanopoulos, Phys. Lett. B91 (1980) 369.
[41] T. Kugo and J.Sato, Prog. Theor. Phys. 91 (1994) 1217.
[42] M. Bando and T. Kugo, Prog. Theor. Phys. 101 (1999) 1313.
[43] J. L. Hewett and T. G. Rizzo, Phys. Rep. 183 (1989) 193.
[44] V. Shtabovenko, R. Mertig and F. Orellana, Comput. Phys. Commun. 207 (2016) 432.
[45] R. Mertig, M. Böhm and A. Denner, Comput. Phys. Commun. 64 (1991) 345.
[46] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler and T. Schwertz, JHEP. 1701 (2017) 087.
[47] B. Pontecorvo, Sov. Phys. JETP 7 (1958) 172.
[48] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962) 870.
[49] Particle Data Group ( C.Patrignani et al.), Chin. Phys. C40 (2016) 100001.
[50] E. Joshua, Comp. Phys. Comm. 210 (2017) 103.