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Effect of the Elastic Deformation of a Point-Sharp Indenter on Nanoindentation Behavior

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Abstract: The effect of the elastic deformation of a point-sharp indenter on the relationship between the indentation load P and penetration depth h (P-h curve) is examined through the numerical analysis of conical indentations simulated with the finite element method. The elastic deformation appears as a decrease in the inclined face angle β, which is determined as a function of the elastic modulus of the indenter, the parabolic coefficient of the P-h loading curve and relative residual depth, regardless of h. This indicates that nominal indentations made using an elastic indenter are physically equivalent to indentations made using a rigid indenter with the decreased β. The P-h curves for a rigid indenter with the decreased β can be estimated from the nominal P-h curves obtained with an elastic indenter by using a procedure proposed in this study. The elastic modulus, yield stress, and indentation hardness can be correctly derived from the estimated P-h curves.

Keywords: nanoindentation; elastic deformation; finite element method; numerical analysis

1. Introduction

Nanoindentation is a form of mechanical testing characterized as a depth-sensing indentation [1] to evaluate local mechanical properties through the analysis of the indentation load P versus the penetration depth h (P-h curve, hereafter). The analysis is principally based on a geometrical definition in which the indentation is carried out on a flat surface using an indenter geometrically defined such as flat-ended, spherical, ellipsoidal, point-sharp (e.g., conical, Berkovich, Vickers, cube corner, etc.). The point-sharp indentation has an advantage in local mechanical testing owes to the analytical simplicity for the geometrical similarity [2].

The bluntness of the indenter tip is one of the inevitable problems of undesirable tip geometry, especially for the point-sharp indentations, because it is impossible to make an ideally sharp indenter. The degree of the bluntness of a point-sharp indenter has been expressed in terms of the radius of curvature at the tip [3–5], but the actual geometry of a blunt tip is not guaranteed to be spherical. An area function [6,7] which gives the projected contact area at the maximum indentation load is another approach to express the bluntness of a point-sharp indenter, but the area function is theoretically valid only for hardness evaluation. A truncated tip which represents a blunt tip in an extremely poor situation [8] is a suitable model for a strict discussion on the effect of the tip bluntness on indentation behavior. According to the appendix of this paper, where a truncated tip is considered, the undesirable effect of the bluntness of a point-sharp indenter can be removed out simply if the P-h curve is shifted with Δhtip in the h direction for indentations deeper than 2Δhtip, where Δhtip is the distance between ideally sharp and blunt tips (see Figures A1–A4). In addition, Δhtip can be estimated...
through an extrapolation of the linear relationship between \( h \) and \( P^{***} \) observed in the large \( P \) and \( h \) region to \( P = 0 \) (see Figure A5).

The elastic deformation of an actual point-sharp indenter, which has been conventionally taken into account on the basis of Hertzian contact [6]; Hertzian contact was basically used for spherical indentations as a modification of the elastic modulus evaluation. It is also an inevitable problem of undesirable tip geometry, especially for indentations on a very hard material, and there is still some controversy whether the modification based on the Hertzian contact can be applied to point-sharp indentations. Moreover, there are no reports on the modifications of the indenter elastic deformation for other mechanical properties such as the indentation hardness or yield stress. The geometrical changes of a point-sharp indenter due to elastic deformation should be considered when evaluating local mechanical properties with the nanoindentation technique.

In this paper, the effect of the elastic deformation of a point-sharp indenter on a \( P-h \) curve is quantified in a numerical analysis of conical nanoindentation behaviors simulated with the finite element method (FEM). In addition, a procedure of deriving physically meaningful \( P-h \) curves, which should be utilized for mechanical property evaluation. Finally, the validity and accuracy of this method is examined.

2. FEM Simulation of Nanoindentation

A conical indentation on a cylindrical elastoplastic solid was modeled in order to avoid the difficulty of modeling a pyramidal indenter widely used for actual nanoindentations. The FEM simulation exploited the large strain elastoplastic capability of ABAQUS code (Version 5.8.1) in the same way as reported in the literature [9,10]. Indentation contact was simulated by the use of elastic cone indenters with two different inclined face angles \( \beta \) (19.7° and 30°). Young’s modulus of the elastic indenter was in the range of 300–1140 GPa. The finite-element mesh in the elastic indenter with \( \beta \) of 19.7° was composed of 775 4-node quadrilateral axisymmetric elements with 2443 nodes. The elastic indenter with \( \beta \) of 30° had 704 elements with 2258 nodes.

The FEM simulation used elastoplastic linear strain hardening rules, i.e., \( \sigma = E\varepsilon \) for \( \sigma < Y \), and \( \sigma = Y + E_p\varepsilon_p \) for \( \sigma \geq Y \), where \( \sigma \) is the stress, \( E \) the Young’s modulus and \( \varepsilon \) the strain. Here, \( Y \) is the yield stress and \( E_p \) \((\equiv d\sigma/d\varepsilon_p)\) is the plastic strain hardening modulus, where \( d\sigma \), \( d\varepsilon \), \( d\varepsilon_e \), and \( d\varepsilon_p \) are, respectively, the incremental values of stress, total, elastic, and plastic strains. Indentations were simulated for \( E \), \( Y \) and \( E_p \) ranges of 50–1000 GPa, 0.1–60 GPa, and 0–200 GPa, respectively. The von Mises criterion with isotropic hardening was used to determine the onset of yielding flow.

3. Results and Discussion

A quadratic relationship between \( P \) and \( h \) on loading is theoretically guaranteed for a point-sharp indentation on the flat surface of a homogeneous elastoplastic solid [11,12]. The quadratic relationship was also observed in simulated \( P-h \) curves made with an elastic cone indenter. This indicates that the elastic deformation of a cone indenter can be described as a decrease in \( \beta \) determined regardless of \( h \). Therefore, nominal indentations made with an elastic cone indenter with an original inclined face angle \( \beta_0 \) should be physically equivalent to indentations made with a rigid cone indenter with the decreased inclined face angle \( \beta_d \).

A nominal quadratic \( P-h \) relationship for an elastic cone indenter can be depicted as follows:

\[
P = k_{1n} h^2 \text{ for loading,}
\]

where \( k_{1n} \) is the nominal indentation loading parameter. Here, \( h \) in Equation (1) is the nominal penetration depth because the decrease in \( \beta \) from \( \beta_0 \) to \( \beta_d \), due to the elastic deformation of a cone indenter, gives a decrease in real penetration depth. Thus, a physically meaningful \( P-h \) relationship can be written with a true indentation loading parameter \( k_1 \), which should be observed in a \( P-h \) loading curve using a rigid cone indenter with \( \beta_d \) as
\[ P_{\text{max}} = k_1(h_{\text{max}} - \Delta h_d)^2, \]  

where \( \Delta h_d \) is the decrease in \( h \) at the maximum penetration depth \( h_{\text{max}} \) due to the elastic deformation of a cone indenter (see Figure 1). The combination of Equations (1) and (2) leads to the equation:

\[ k_1 = k_{1n} \left( 1 - \frac{\Delta h_d}{h_{\text{max}}} \right)^{-2}. \]  

This means that \( \Delta h_d/h_{\text{max}} \) is a key parameter to estimating \( k_1 \) from the nominal \( k_{1n} \). In other words, \( \Delta h_d/h_{\text{max}} \) can be simulated as

\[ \frac{\Delta h_d}{h_{\text{max}}} = 1 - \sqrt{k_{1n}/k_1}, \]  

where \( k_{1n} \) in Equation (3') is observed in a simulated \( P-h \) loading curve with an elastic cone indenter and \( k_1 \) in Equation (3') is evaluated with the mechanical properties inputted into the FEM model [9,10]. In the following paragraph, the effect of \( \Delta h_d/h_{\text{max}} \) on a \( P-h \) curve is examined quantitatively through numerical analysis.

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Figure 1. Schematic illustration of the effect of indenter elastic deformation on a \( P-h \) curve.

In addition to \( k_1 \), the relative residual penetration depth \( \xi \), defined as \( h_r/h_{\text{max}} \), where \( h_r \) is the residual penetration depth, characterizes a \( P-h \) curve and nominally decreased by the elastic deformation of a cone indenter to be \( \xi_n \). A true \( \xi \)-value, which should be observed in a \( P-h \) curve using a rigid cone indenter with \( h_d \), can also be evaluated with the mechanical properties inputted into the FEM model [9,10]. The numerical analysis revealed that the evaluated \( \xi \) can be correlated with the nominal \( \xi_n \) as a function of \( \Delta h_d/h_{\text{max}} \)

\[ \xi = \xi_n \left\{ 1 - \left( \frac{\Delta h_d}{h_{\text{max}}} \right)^{0.85} \right\}^{-0.50}. \]  

Figure 2 plots \( \xi \) estimated with Equation (4) and \( \xi_n \) against the true \( \xi \) evaluated with the mechanical properties inputted into the FEM model [9,10]. The results indicate the validity of using Equation (4) to estimate \( \xi \) from the nominal \( \xi_n \) and \( \Delta h_d/h_{\text{max}} \). In addition, it is confirmed that \( \xi_n \) is smaller than \( \xi \) because of the overestimation of the penetration depth \( h \) due to the elastic deformation of the indenter. Moreover, a true indentation unloading parameter \( k_2 \) defined as \( P_{\text{max}}/(h_{\text{max}} - h_r)^2 \),
which should be observed in an $P-h$ unloading curve using a rigid cone indenter with $\beta_d$, can be estimated from a simulated $P-h$ curve with an elastic cone indenter characterized by $k_{1n}$ and $\xi_n$ using Equations (3) and (4) as

$$
k_2 = \frac{k_1}{(1 - \xi)^2} = k_{1n} \left(1 - \frac{\Delta h_d}{h_{\text{max}}}\right)^{-2} \left[1 - \xi_n \left(1 - \left(\frac{\Delta h_d}{h_{\text{max}}}\right)^{0.85}\right)^{-0.50}\right]^{-2}.
$$

(5)

![Relative residual depth $\xi$ estimated with Equation (4) and $\xi_n$ nominally observed plotted against $\xi$ evaluated with mechanical properties inputted into the FEM model.](image1)

**Figure 2.** Relative residual depth $\xi$ estimated with Equation (4) and $\xi_n$ nominally observed plotted against $\xi$ evaluated with mechanical properties inputted into the FEM model.

Figure 3 plots the estimated $k_2$ with Equation (5) as well as the nominal indentation unloading parameter $k_{2n}$ determined from a simulated $P-h$ curve with an elastic cone indenter against the true $k_2$ evaluated with the mechanical properties inputted into the FEM model [9,10]. Figure 3 indicates that $k_2$ can be estimated correctly by using Equation (5) with $\Delta h_d/h_{\text{max}}$, and that the nominal $k_{2n}$ is quite far from $k_2$ owing to the overestimation of $h$.

![Indentation unloading parameter $k_2$ simulated and $k_{2n}$ nominally observed plotted against $k_2$ evaluated with mechanical properties inputted into the FEM model.](image2)

**Figure 3.** Indentation unloading parameter $k_2$ simulated and $k_{2n}$ nominally observed plotted against $k_2$ evaluated with mechanical properties inputted into the FEM model.

The numerical analysis also revealed that $\Delta h_d/h_{\text{max}}$ is determined to be

$$
\frac{\Delta h_d}{h_{\text{max}}} = 0.616 \left\{ \frac{k_{1n}}{E_i' \left(1 + \frac{\xi_n}{1.5}\right)^{1.5}} \right\}^{0.84},
$$

(6)

where $E_i'$ is defined as $E_i/(1 - \nu_i^*)$ and $E_i$ and $\nu_i$ are Young’s modulus and Poisson’s ratio of an elastic indenter, respectively. Figure 4 plots $\Delta h_d/h_{\text{max}}$ estimated with Equation (6) against $\Delta h_d/h_{\text{max}}$.
evaluated with Equation (3’). Figure 3 indicates that \( \Delta h_d/h_{\text{max}} \) can be estimated by using Equation (6) with a nominally observed \( P-h \) curve characterized by \( k_{1n} \) and \( \xi_n \), and with the elastic properties of an elastic indenter characterized by \( E_i \) and \( \nu_i \).

In order to estimate mechanical properties from a \( P-h \) curve characterized with \( k_1, k_2 \) and \( \xi \), we should know the inclined face angle \( \beta_d \) of the elastically deformed indenter. Numerical analysis revealed that \( \beta_d \) is given as a function of \( \beta_{o}, \xi_n \) and \( \Delta h_d/h_{\text{max}} \)

\[
\tan \frac{\beta_d}{\tan \beta_{o}} = 1 - \left( 1 - \xi_n^{0.8} \right)^{0.83} \left( \frac{\Delta h_d}{h_{\text{max}}} \right)^{0.90}.
\]  

(7)

Figure 5 plots \( \tan \frac{\beta_d}{\tan \beta_{o}} \) estimated with Equation (7) against \( \tan \frac{\beta_d}{\tan \beta_{o}} \) observed in a simulated nanoindentation, and indicates the validity to estimate the inclined face angle \( \beta_d \) of the elastically deformed indenter with \( \Delta h_d/h_{\text{max}} \).

The representative indentation elastic modulus \( E^* \), defined as \( E^* = \frac{E}{1-(\nu-0.225 \tan^{1.05} \beta_d)} \) in terms of \( \beta_d \) [9], can be estimated from the simulated \( P-h \) curve using Equations (3)–(7) if we know \( E_i \) and \( \nu_i \), whereas it is evaluated with \( E \) and \( \nu \) inputted into the FEM model and with \( \beta_d \) observed in simulated nanoindentions. Figure 6 plots the estimated \( E^* \) (black circles) against the evaluated \( E^* \). The white circles are \( E^* \) estimated with the nominal values of \( k_{2n} \) and \( \xi_n \) [9], which means that the elastic deformation of an indenter is not modified for the estimation of \( E^* \). Figure 6 indicates the modification of the elastic deformation of an indenter can determine \( E^* \) correctly. The underestimation of \( E^* \) without the modification (the white circles in Figure 6) is caused by the overestimation of the elastic rebound.
during the unloading process because the extrinsic elastic deformation of the indenter is added to the intrinsic elastic deformation of the indented material.

![Figure 6](image_url)

**Figure 6.** Representative indentation elastic modulus $E^*$ estimated with simulated $P$-$h$ curves plotted against $E^*$ evaluated with mechanical properties inputted into the FEM model.

The representative indentation yield stress $Y^*$, defined as $Y^* = \frac{Y + 0.25 E_p \tan \beta_d}{1 - (\nu - 0.225 \tan 10^\circ \beta_d)}$ in terms of $\beta_d$ [10], can also be estimated using a simulated $P$-$h$ curve and Equations (3)–(7) if we know $E_i$ and $\nu_i$. Moreover, it can be evaluated with $Y$, $E_p$ and $\nu$ inputted into the FEM model and with $\beta_d$ of the simulated indentation. Figure 7 plots the estimated $Y^*$ (black circles) against the evaluated $Y^*$. $Y^*_{in}$ estimated with $E^*_n$, $\xi_n$ and $\beta_0$ is plotted for comparison. This figure shows that the modification of the elastic deformation of an indenter more or less correctly estimates $Y^*$ although the difference between the modified and unmodified $Y^*$ is not so large with respect to the difference observed in $E^*$ (see Figure 6). A relatively large difference in $Y^*$ is typically found in the range of $\xi$ less than 0.1, where plastic deformation is not dominant. The small difference observed in Figure 7 is attributed to the decrease in $k_{1n}$ and $\xi_n$ due to elastic deformation of an elastic indenter, where the former decreases $Y^*$ nominally while the latter increases $Y^*$ apparently.

![Figure 7](image_url)

**Figure 7.** Representative indentation yield stress $Y^*$ estimated with simulated $P$-$h$ curves plotted against $Y^*$ evaluated with mechanical properties inputted into the FEM model.

A previous study on the indentation hardness $H_M$ found that it can be evaluated with the mechanical properties inputted into the FEM model and with the simulated $\beta_d$ [9,10]. On the other hand, $H_M$ can be estimated from a true $P$-$h$ curve characterized with $k_1$, $k_2$ and $\xi$. Figure 8 plots the estimated $H_M$ (black circles) against the evaluated $H_M$. The nominal $H_{Mn}$ estimated with the nominal $P$-$h$ curve is plotted as white circles in Figure 8, and a comparison reveals that the modification more or less correctly estimates $H_M$, although the difference between the estimated $H_M$ and the nominal $H_{Mn}$ is not so large with respect to the difference observed in $E^*$ (see Figure 6). The difference is rather high in the large $H_M$ region, where elastic deformation of the indenter is most severe. The small
difference observed in Figure 8 owes to the decrease in $k_{1n}$ and $\xi_n$ due to elastic deformation of an elastic indenter, where the former decreases $H_M$ nominally while the latter increases $H_M$ apparently through the decrease of nominal contact depth.

Figure 8. Indentation hardness $H_M$ estimated with simulated $P$-$h$ curves plotted against $H_M$ evaluated with mechanical properties inputted into the FEM model and with simulated $\beta_d$. We conducted nanoindentation experiments and reported $E^*$, $Y^*$ and $H_M$ for several materials [9,10]. These values were evaluated with modified $P$-$h$ curves (see Figure 9) considering elastic deformation of a diamond indenter with Young’s modulus and a Poisson’s ratio of 1140 GPa and 0.07, respectively. Table 1 shows these mechanical properties as well as those evaluated with a nominal $P$-$h$ curve made without considering any elastic deformation of the indenter. $\Delta h_d/h_{max}$ and $\beta_d$ estimated with the numerical analysis developed in this study are shown in order to examine the degree of the elastic deformation of the indenter. Even in that case, the changes in $Y^*$ and $H_M$ due to the elastic deformation of the indenter are not so large. In contrast, the change in $E^*$ is so large that it cannot be ignored. The underestimation of $E^*$ without the modification is caused by the overestimation of the elastic deformation during the unloading process because the extrinsic elastic deformation of the indenter is added to the intrinsic elastic deformation of the indented material.

![Flow chart of the procedure to evaluate mechanical properties.](image_url)
Table 1. Effect of elastic deformation of diamond indenter on mechanical property evaluations.

| Materials                  | Δh_d/h_max | θ_d/θ_o (deg.) | k_2/k_2n (10^3 GPa) | ξ/ξ_n | E* (GPa) | Y* (MPa) | H_M (GPa) |
|---------------------------|------------|----------------|----------------------|-------|---------|---------|---------|
|                           |            |                |                      |       |         |         |         |
| Brass                     | 0.020      | 19.6/19.7      | 7.65/4.78            | 0.930/0.913 | 102/81 | 584/597 | 1.26/1.29 |
| Duralumin                 | 0.020      | 19.6/19.7      | 4.37/3.03            | 0.909/0.893 | 77/64  | 605/618 | 1.31/1.33 |
| Beryllium copper alloy    | 0.027      | 19.6/19.7      | 9.49/5.44            | 0.925/0.904 | 136/103 | 841/863 | 1.82/1.86 |
| Fused silica              | 0.065      | 19.0/19.7      | 0.615/0.477          | 0.550/0.522 | 74/65  | 5.21 × 10^3/5.35 × 10^3 | 8.56/8.40 |
| Alumina                   | 0.158      | 18.3/19.7      | 4.90/2.25            | 0.690/0.614 | 340/225 | 12.6 × 10^3/12.9 × 10^3 | 24.3/22.8 |
According to Equation (6), the following equation can be derived

\[
\frac{H_M}{E' \nu} = \frac{\gamma^2}{8} \left(1 + \xi_n 1.5\right) \left(\frac{\Delta h_d}{0.616 h_{\text{max}}}\right)^{1/0.84}
\]  

(8)

When indentation hardness is not affected much by the indenter elastic deformation, where \(\gamma\) is the surface profile parameter defined as \(\gamma = h_{\text{max}}/h_c\), \(h_c\) is the contact depth, and \(g\) is the geometrical factor of a point-sharp indenter to be 24.5 for \(\beta = 19.7°\). \(E'\) is required to be about 250 times larger than \(H_M\) for \(\Delta h_d/h_{\text{max}}\) smaller than 0.05, where the effect of the indenter elastic deformation on a \(P-h\) curve may be ignored for indentations with Berkovich-type indenter.

4. Conclusions

The effect of the geometrical changes due to the elastic deformation of a point-sharp indenter was examined by conducting a numerical analysis of \(P-h\) curves simulated with FEM. The effect appears as a decrease in the inclined face angle \(\beta\). The key parameter \(\Delta h_d/h_{\text{max}}\), which can be utilized to derive the physically meaningful \(P-h\) curve and the decreased \(\beta\), can be estimated with an equation derived by numerical analysis. The mechanical properties of indented materials, such as \(E^*, Y^*\) and \(H_M\), can be estimated by using the \(P-h\) curve and \(\beta\) characterized by \(k_1, k_2\) and \(\xi\) estimated with the key parameter \(\Delta h_d/h_{\text{max}}\). The modification of a \(P-h\) curve and \(\beta\) with \(\Delta h_d/h_{\text{max}}\) is most effective for the estimation of the accurate \(E^*\) with respect to \(Y^*\) and \(H_M\).

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Author Contributions: Takashi Akatsu conceived of the ideas for this research, performed numerical analysis on the simulated nanoindentations and wrote the paper. Shingo Numata and Yutaka Shinoda simulated nanoindentation. Fumihiro Wakai advised on the numerical analysis and reviewed the manuscript of the paper.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix. Effect of the Tip Bluntness of a Point-Sharp Indenter on Nanoindentation Behavior

An indentation on a linearly elastic solid with a Young’s modulus \(E\) and Poisson’s ratio \(\nu\) of 100 GPa and 0.499, respectively, was simulated using a rigid indenter with a truncated tip [8] in order to represent a tip in an extremely poor situation. The simulation was basically carried out in the same way described in the Section 2 of this paper. The inclined face angle \(\beta\) of the indenter was set to be 19.7°, which is the Vickers–Berkovich equivalent angle. The distance \(\Delta h_{\text{tip}}\) between ideally sharp and truncated tips (see Figure A1) was set to be 716 or 1790 nm with respect to the maximum penetration depth of 10 \(\mu\)m.

![Figure A1. Schematic illustration of a truncated conical indenter tip.](image-url)

The \(P-h\) curves shown in Figure A2 were obtained by simulating an indentation on a perfectly elastic body with a Poisson’s ratio \(\nu\) of \(\approx 0.5\) for a series of \(\Delta h_{\text{tip}}\). We only examined a perfectly elastic...
body because the effect of bluntness would be most severe in this case. As shown in Figure A3, the curve deviates from that for an ideally sharp indenter ($\Delta h_{\text{tip}} = 0$) as $\Delta h_{\text{tip}}$ increases and the amount of the shift in the $h$ direction almost coincides with $\Delta h_{\text{tip}}$. The results in this figure indicate the effect of a blunt tip of a conical indenter on the $P$-$h$ curve can be simply described as a change in the penetration depth with $\Delta h_{\text{tip}}$, especially in the large $P$ and $h$ region. The error in Young’s modulus is evaluated as $\left| \frac{E_{P,h} - E_{\text{tip}}}{E_{\text{tip}}} \right|$ (expressed as a percentage), where $E_{P,h}$ and $E_{\text{tip}}$ are Young’s modulus values obtained from the simulated $P$-$h$ curve and inputted into an FEM model, respectively. Figure A4 plots this error as a function of penetration depth normalized by $\Delta h_{\text{tip}}$. It indicates that a reliable Young’s modulus can be obtained with an error less than 10% from the $P$-$h$ curve in the $h$ region larger than $2\Delta h_{\text{tip}}$. In other words, $P$-$h$ data for $h$-values shallower than $2\Delta h_{\text{tip}}$ should be omitted in order to obtain reliable mechanical properties. It may be possible to get more reliable $P$-$h$ data in the shallow $h$ region for other geometries, but the $P$-$h$ data are actually obscure and depend on the unknown geometry of the indenter tip. We simulated a truncated tip of a conical indenter because it represents the worst case of a blunt point-sharp indenter; consequently, the $P$-$h$ data obtained in the $h$ region larger than $2\Delta h_{\text{tip}}$ guarantees accuracy in all cases. Figure A5, where $h$ is plotted as a function of the square root of $P$, is an extrapolation of the linear relationship between $h$ and $P$ observed in the large $P$ and $h$ region to $P = 0$. It gives $\Delta h_{\text{tip}}$ as an absolute value on the $h$-axis and is a way to estimate $\Delta h_{\text{tip}}$ from the $P$-$h$ curve. We have already reported an example of estimating $\Delta h_{\text{tip}}$ from an experimental $P$-$h$ curve [13]; $\Delta h_{\text{tip}}$ is estimated to be about 50 nm for the Berkovich indenter that we actually used for the nanoindentation experiment.

![Figure A2](image1.png)

*Figure A2. Indentation load $P$ vs. penetration depth $h$ curves obtained with truncated conical indenters.*

![Figure A3](image2.png)

*Figure A3. Indentation load $P$ vs. modified penetration depth $h + \Delta h_{\text{tip}}$ curves obtained with truncated conical indenters.*
Figure A4. Error in Young’s modulus evaluation with a truncated conical indenter as a function of normalized penetration depth \( (h + \Delta h_{\text{tip}})/\Delta h_{\text{tip}} \).

Figure A5. Penetration depth \( h \) vs. square root of indentation load \( P^{1/2} \) obtained with truncated indenters.

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