Optimal Retail Tariff Design With Prosumers: Pursuing Equity at the Expenses of Economic Efficiencies?

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Abstract—Distributed renewable resources owned by prosumers can be an effective way to strengthen the resilience of the grid and enhance sustainability. However, prosumers serve their own interests, and their objectives are unlikely to align with that of society. This article develops a bilevel model to study the optimal design of retail electricity tariffs considering the balance between economic efficiency and energy equity. The retail tariff entails a fixed charge and a volumetric charge tied to electricity usage to recover utilities’ fixed costs. We analyze solution properties of the bilevel problem and prove an optimal rate design, which is to use fixed charge to recover fixed costs and to balance energy equity among different income groups. That is, the first-best policy is to leave the wholesale power market intact; any recovery based on a volumetric principle is likely to be inefficient. This suggests that programs similar to CARE (California Alternative Rate of Energy), which offer lower retail rates to low-income households, are unlikely to be efficient, even if they are politically appealing.

Index Terms—Energy expenditure incidence, fixed cost recovery, retail and wholesale market, net-billing, net-metering, mathematical program with equilibrium constraints, prosumers.

I. INTRODUCTION

Concerns about climate change, resilience to hazardous events, and sustainability have shifted the electric power sector in the U.S. and elsewhere toward more involvement on the demand side to harness flexible distributed energy resources (DERs). More recently, the U.S. Federal Energy and Regulatory Commission (FERC) issued Order 2222, which removes barriers to the integration of DERs into wholesale electricity markets.[1] More specifically, the ruling allows the integration of multiple DERs owned by different entities of different sizes and diverse technologies to participate in regional organized wholesale capacity, energy and ancillary services markets alongside traditional resources. This ruling offers incentives for households that own DERs to “value-stack” their assets to provide various types of energy-related commodities to the grid. Already, we are witnessing some activities in the marketplace in response to or anticipation of the order. For instance, OhmConnect, a clean tech company, recently announced a plan to link homes spread in California to form a 550 MW virtual power plant (VPP) of distributed energy resources.

Naturally, prosumers with concurrent power generation and consumption are self-served for their own interests, and their behavior resulting from optimizing their private benefits is unlikely to be in the best interests of the energy market as a whole. Not surprisingly, how to compensate for the energy produced by prosumers has emerged as a critical issue that can facilitate or impair the deployment of DER generation and is currently subject to contentious debates[2,3,4].

The situation is also complicated by the existing formation of a retail tariff. In general, a retail tariff consists of four core elements: (i) costs of electric energy; that is, wholesale locational marginal prices (LMPs), (ii) costs of other energy-related services, such as operating reserves or capacity costs, (iii) costs for network-related services, including investment and maintenance costs of transmission and distribution network assets, and (iv) charges to recover policy costs, such as procurement costs to support state’s renewable portfolio standards (RPS) [5]. The last two items, (iii) and (iv), are generally lumpy and non-convex as they are not directly tied to the level of energy consumption. These two elements are termed “residual costs” in [6]. Breakup of these four elements depends on specific markets; for example, the energy or LMP component can be as low as 10% in the Netherlands or as high as 60% in New Jersey [5].

Recovering residual costs is a thorny political-economic endeavor, which may facilitate or impair the deployment of residential DER generation such as photovoltaic (PV), and has been subject to contentious debates[2,4]. Two systems are of great interest. The first is referred to as net-metering; that is, prosumers are only billed for their “net” energy use. The energy they sell back to the grid will be paid at the same rate as buying from the grid. The second system is called net-billing, under which two meters are installed, recording two quantities:
energy withdrawn from and energy injected into the grid. The withdrawal and injection can be subject to different prices. While the net-metering is the most common approach and has provided strong incentives for DER investment, it is also causing serious equity issues, as the more prosumers take advantage of net metering, the fewer residual costs are paid into the system, resulting in higher rates for non-net metering customers, likely those of low-income.

The California Public Utility Commission (CPUC) recently engaged in a regulatory process to revamp its net-metering policy, as the CPUC is fully aware of its drawbacks. Such issues have also been vetted by the academics, which have been described as “revenue erosion” or “network defection”; that is, utilities are forced to increase the retail tariff to compensate for the revenue deficiency, further exacerbating the situation and leading to the so-called “death spiral” [7], [8], [9], [10], [11]. Some empirical evidence emerges: for instance, using data from three investor-owned utilities in California, Wolak [12] finds that two-thirds of the increases in residential distribution prices can be attributed to the growth of solar capacity.

A number of recent studies have also addressed the issues of pricing energy produced by prosumers. Clastres et al. [13] estimate the extent of cross-subsidies between prosumers and conventional consumers in France. The authors also conclude that a demand charge may alleviate the network defection or death spiral problem facing distributed system operators. Using stylized models, Gautier et al. [14] concludes that net-metering decreases the payment from prosumers, which is cross-subsidized by the higher bills of conventional consumers. More recently, Gorman et al. [15] compare grid costs to off-grid costs of more than 2,000 utilities in the U.S. and find that network defection could increase from 1% to 7%, with 3% in the Southwest region and California and 7% in Hawaii. In Brown and Sappington [16], they show theoretically that for a vertically-integrated utility, net-metering is inferior to policies that compensate prosumers based on the “value of solar”; namely, the marginal value to reduce the utility’s generation, transmission and distribution cost, as well as the reduction of air pollutant emissions. However, this article does not address the equity issue.

The efficiency versus equity in the design of electricity tariffs is not new and can be dated back to [17]. The advent of prosumers could exacerbate equity issues. A review article [18] provides a good summary of current state of knowledge. However, the reviewed studies in the article are either simulation-based numerical case studies [19], with focus on a single utility company [20], or merely policy discussions without any mathematical expositions, such as [21], [22], [23]. Moreover, to our best knowledge, all the papers do not provide an “operational” definition of or a quantitative measurement of energy equity within an optimization framework that also considers an organized wholesale market and transmission networks.

Our earlier work [24] fills this gap by proposing a quantitative measure of equity on energy expenditure and integrates it within a wholesale energy market model. The article concludes that net-metering is more regressive than net-billing under the volumetric tariff and contemplates that a hybrid policy, which also features an income-based fixed charge, may potentially improve energy equity. The current study extends our previous work to offer a policy prescription on optimal retail tariff design in the face of a growing presence of prosumers. The problem is formulated as a bilevel optimization problem: the upper level represents a public utility commission (PUC)’s decision-making problem that has to decide a certain retail tariff structure to guarantee the recover utility’s fixed costs as well as maintaining energy equity. The lower level represents an market equilibrium that consists of prosumers, consumers, producers, and an independent system operator (ISO), with the prosumer/consumers’ retail rates set by the upper-level PUC. Although each prosumer may be relatively small, possessing limited ability to engage in the bulk energy market, we assume that an entity integrates a large number of prosumers and participates in the bulk energy market on their behalf; this is consistent with FERC Order 2222’s requirements. The prosumers are endowed with renewables and decide on the amounts of self-consumption, dispatchable energy to produce, such as from back-up generators or energy storage, and energy to sell to or buy from the bulk energy market to maximize their net benefit. The independent system operator (ISO) minimizes the generation costs while treating sales or purchases by the prosumers as exogenous.

Similar to the earlier work by Woo [25], we also explicitly consider the problem facing a PUC and distinguish the retail rate from the wholesale rate. However, unlike [25], we extend the analysis to consider the energy expenditure incidence among income groups to address equity issues. Our analysis of the theoretical properties of the bilevel model is also worth noting. More specifically, we prove that even with a revenue adequacy constraint for utility companies, laissez-faire is still socially optimal; that is, zero volumetric charge will maximize the social surplus, as it will not distort the equilibrium price in the wholesale market, and energy equity can be achieved through different fixed charges among different income groups. While a fixed-charge-only tariff is implausible in reality, our formulation is amenable to computing a second-best solution when a proportion of the utility’s fixed cost is required to be recovered from volumetric charges. While our lower-level problem is related to [13], [14], it is different in significant ways. In particular, we consider the transmission network and market details, e.g., pool-type market settlement, capacity ownership, generation capacity constraints, retail-wholesale market linkage, which are crucial in determining realistic electricity market outcomes.

The policy indications from our work are very similar to a recent article by Boreinstein et al. [26]; that is, volumetric cost recovery is regressive, as it can cause market inefficiency, but fixed monthly charges that are the same for all residential customers are also highly regressive, as they may disproportionately take a larger share of lower-income household’s expenditure. Their conclusions are based on numerical results from the Avoided Cost Calculator [27] developed by Energy and Environmental Economics, Inc (E3), using real data in California; while our conclusions are based on energy market equilibrium models with

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1[Online]. Available: https://www.cpuc.ca.gov/-/media/cpuc-website/divisions/energy-division/documents/net-energy-metering-nem/nemrevisit/430903088.pdf.
transmission constraints and rigorous theoretical proofs. Our work and [26] should be viewed as perfectly complementary to each other. In addition, our modeling and computational approach can calculate the optimal tariff for prosumers energy sales, and our numerical results show that net-metering policy is indeed preferred over net-metering in terms of efficiency and energy equity maximization.

The remainder of this article is organized as follows. Section II presents the lower- and upper-level models of the bilevel problem. Solution properties and theoretical results are shown in Section III. A numerical case study is presented in Section IV. Finally, concluding remarks are provided in Section V.

II. MODEL

We present the complete model in this section, starting with the lower-level market equilibrium formulation for consumers, prosumers, power plants, and the ISO, followed by the upper-level problem for a government agency to maximize social surplus and energy equity while ensuring revenue adequacy. The resulting problem can written as either a mathematical program with equilibrium constraints (MPECs) or a bilevel problem (BLP), with the former formulation amenable to computation and the latter one easier for theoretical analysis.

A. Lower-Level Problem

The lower-level problem consists of problems faced by the consumers, prosumers, power plants, and the ISO. As a starting point, we make the blanket assumption that the market is perfectly competitive; that is, all market participants are price-takers of the market prices, without contemplating on how to manipulate the equilibrium prices through their unilateral actions. Models of imperfect competition, such as Cournot competition [28], can be used here as well without difficulty, but is left for future research.

1) Consumers: Consider an energy market that has \( N \) nodes and \( K \) transmission lines that connect the nodes. Consumers at each node \( i = 1, \ldots, N \) are grouped into two types, including conventional consumers and prosumers, whose marginal benefit functions (that is, their willingness-to-pay functions), denoted by \( p_{i}^{\text{con}} \) and \( p_{i}^{\text{pro}} \), respectively, are represented by the following linear inverse demand functions:

\[
p_{i}^{\text{con}}(d_i) = P_i^{0} - (P_i^{0}/((1 - \alpha_i)Q_i^{0})) d_i, \quad \forall i \tag{1}
p_{i}^{\text{pro}}(l_i) = P_i^{0} - (P_i^{0}/(\alpha_i Q_i^{0})) l_i, \quad \forall i, \tag{2}
\]

where \( P_i^{0} > 0 \) and \( Q_i^{0} > 0 \) respectively represent the vertical and horizontal intercepts of the “horizontally aggregated” retail inverse demand function: \( p_i^r(d_i + l_i) = P_i^{0} - (P_i^{0}/(\alpha_i Q_i^{0})) (d_i + l_i) \), as illustrated in Fig. 1.\(^{2}\) The quantities demanded by conventional consumers and prosumers are denoted by \( d_i \) and \( l_i \), respectively. The parameter \( \alpha_i \) is the fraction of prosumers at node \( i \). Note that while \( \alpha_i \) varies between 0 and 1, the aggregated demand does not change.

Let \( p_i \) denote the wholesale energy price at node \( i \), and \( \tau^b \) be the volumetric charge of energy purchase for all consumers/prosumers, which is a part of consumers’ retail rates.\(^{3}\) The other part of the retail rates is the fixed charge. Since the fixed-charge rate will serve as a main tool to realize energy equity, we assume that conventional consumers and prosumers can be subject to different fixed-charge rates, and denote them as \( \phi_i^{\text{con}} \) and \( \phi_i^{\text{pro}} \), respectively.

With the marginal benefits and costs defined, conventional consumers at each node \( i = 1, \ldots, N \) maximize their net benefits (also referred to as surplus) by solving:

\[
\text{maximize } \int_{0}^{d_i} p_i^{\text{con}} (m_i) \, dm_i - (p_i + \tau^b) d_i - \phi_i^{\text{con}}. \tag{3}
\]

Since \( p_i \), \( \tau^b \), and \( \phi_i^{\text{con}} \) are all exogenous to consumers, the optimization problem is easily seen to be a strongly convex problem with the given linear inverse demand function as in (1). Hence, an optimal solution always exists with respect to any \( (p_i, \tau^b, \phi_i^{\text{con}}) \), and the first-order optimality conditions, aka the KKT conditions, are both necessary and sufficient for optimality. The collection of conventional consumers’ KKT conditions are that for \( i = 1, \ldots, N \):

\[
0 \leq d_i \perp P_i^{0} - (P_i^{0}/((1 - \alpha_i)Q_i^{0})) d_i - (p_i + \tau^b) \leq 0, \tag{4}
\]

where the ‘\( \perp \)’ sign means that the product of the scalars or vectors is 0, and such a constraint is referred to as a complementarity constraint. The KKT conditions have intuitive economic interpretation: at an optimal solution (denoted with a ‘\(^*\)’ superscript), if \( d_i^* > 0 \), then consumers choose to purchase energy at the level where the marginal benefit, \( P_i^{0} - (P_i^{0}/((1 - \alpha_i)Q_i^{0})) d_i^* \), equals the marginal cost, which is the retail price \( p_i^r := p_i + \tau^b \).

2) Prosumers: For prosumers, they pay the same volumetric charge \( \tau^b \) when buying from the grid. However, when they sell to the grid, we assume that the rate they receive is \( p_i + \tau^s \). If \( \tau^s = \tau^b \), then it is the net-metering policy; otherwise, it is net billing. Note that while \( \tau^b \) is always non-negative, \( \tau^s \) can be positive or negative. When \( \tau^s > 0 \), the prosumers effectively receive a “subsidy” in addition to the wholesale price \( p_i \). In the

\(^{2}\)Empirical studies suggest that the price elasticities of the residential demand for electricity in the U.S. range from \(-0.1\) to \(-0.4\), which means a 1% increase in the price leads to decreases in the quantities demanded by 0.1 to 0.4% [29].

The concept of the inverse demand function is detailed in [30].

\(^{3}\)Note that retail rates are usually the same covering a broad area of customers, and hence, we do not have a node index of \( \tau \), but can certainly do so.
case where \( \tau^a < 0 \), it means that the prosumers are subject to a “tax” when selling power to the grid.

With the exogenous volumetric rates \( \tau^b \), \( \tau^a \) and fixed rate \( \phi^{pro} \), we posit that a prosumer maximizes its surplus by deciding i) energy to buy \( (z_i^b) \) from or sell \( (z_i^b) \) to the grid at node \( i \), ii) consumption level \( l_i \), given renewable output \( R_i > 0 \), and iii) generation \( g_i \) from the backup dispatchable technology with a cost \( C_i^p(g_i) \). The prosumer’s problem at node \( i \) is:

\[
\text{maximize } \quad z_i^b + l_i \quad \text{s.t. } \begin{cases}
(p_i + \tau^a)z_i^b - (p_i + \tau^b)z_i^b + \sum_{i=1}^{T} \phi_i^{pro}(m_i)\delta_i - C_i^p(g_i) - \phi^{pro} & (5a) \\
0 & (5b) \\
l_i + (z_i^b - z_i^b) - g_i - R_i = 0 & (5c) \\
g_i \leq G_i & (5d)
\end{cases}
\]

In the constraint set, \( (5b) \) ensures that the net demand, \( l_i + z_i^b - z_i^b \), is balanced with the prosumer’s own renewable and backup generation \( (R_i + g_i) \). \( (5c) \) limits the backup output to be less than its capacity \( G_i \).

Two things to note about the above optimization problem. First, if \( \tau^a > \tau^b \), the optimization problem is clearly unbounded. This is intuitive: if \( p_i + \tau^a > p_i + \tau^b \), meaning that selling electricity back to the grid earns more than buying from the grid. Then theoretically speaking, a prosumer could simply arbitrage and earn an infinite amount of profit. To rule out this case, we make the no-arbitrage assumption that \( \tau^a \leq \tau^b \).

Second, under the no-arbitrage assumption, for the buy and sell decisions, \( z_i^b \) and \( z_i^b \) should not both be positive in an optimal solution, from a common sense perspective. Mathematically, however, this is not guaranteed, unless we have some explicit constraints such as \( z_i^b - \frac{1}{\delta_i} = 0 \). In the following, we present the simple fact that such an explicit constraint is not necessary. To do so, we first write down the KKT conditions of the prosumers optimization problems (assuming \( C_i^p(\cdot) \) is differentiable).

\[
\begin{align}
0 & \leq l_i - P_i^b - \frac{P_i^b}{Q_i}l_i - \delta_i \leq 0, \quad \forall i \quad \text{(6a)} \\
0 & \leq g_i - C_i^p(g_i) + \delta_i - \kappa_i \leq 0, \quad \forall i \quad \text{(6b)} \\
0 & \leq \kappa_i + g_i - G_i \leq 0, \quad \forall i \quad \text{(6c)} \\
0 & \leq z_i^b - (p_i + \tau^a) - \delta_i \leq 0, \quad \forall i \quad \text{(6d)} \\
0 & \leq z_i^b - (p_i + \tau^b) + \delta_i \leq 0, \quad \forall i \quad \text{(6e)} \\
\delta_i \text{ free}, \quad l_i + z_i^b - z_i^b - g_i - R_i = 0, \quad \forall i. \quad \text{(6f)}
\end{align}
\]

Since the constraints \( (5b)-(5c) \) are all linear, and with a given tuple \((p_i, \tau^a, \tau^b, \phi_i^{pro})\), the objective function is concave (under the assumption that \( C_i^p(\cdot) \) is a convex function), the KKT conditions are again necessary and sufficient optimality conditions. Then we have the following result.

**Lemma 1:** Assume that \( C_i^p(\cdot) \) is continuously differentiable and convex. With a given tuple \((p_i, \tau^a, \tau^b, \phi_i^{pro}) \in \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+ \times \mathbb{R} \) in which \( \tau^a \leq \tau^b \), an optimal solution of the problem \((5a)-(5c)\), denoted by \( (l_i^*, g_i^*, z_i^b, z_i^b) \), exists. In addition, if \( \tau^a \neq \tau^b \), then \( z_i^b \cdot z_i^b = 0 \).

**Proof:** Let \( X_i^{pro} \in \mathbb{R}^4 \) denote the feasible region of the prosumers problem at node \( i \). It is easy to see that \( X_i^{pro} \neq \emptyset \) since the zero vector is always in \( X_i^{pro} \). \( X_i^{pro} \) is also clearly a closed set. When \( \tau^a \leq \tau^b \), the objective function \( (5a) \) goes to \(-\infty \) for any \( (l_i, g_i, z_i^b, z_i^b) \in X_i^{pro} \) with \( \| (l_i, g_i, z_i^b, z_i^b) \| \to \infty \), which means that \( (5a) \) is coercive on \( X_i^{pro} \). Since \( (5a) \) is also continuous, an optimal solution exists by a variant of the well-known Weierstrass’ Theorem (such as Proposition A.8 in [31]).

For the second part, when \( \tau^a \neq \tau^b \), if \( z_i^b \cdot z_i^b > 0 \), the conditions \((6d)\) and \((6e)\) imply that \( p_i + \tau^a = \delta_i = p_i + \tau^b \), a contradiction.

When \( \tau^a = \tau^b \), clearly the optimal solutions \( z_i^b \) and \( z_i^b \) are not unique and can be both positive. In this case, we can simply define \( z_i^b := \max\{z_i^b - z_i^b, 0\} \) and \( z_i^b := \max\{z_i^b - z_i^b, 0\} \); then at most one of them is nonzero in an optimal solution.

In the following sections, we will see that it is always the net of a prosumer’s decision, namely, \( z_i^b - z_i^b \), that appears in the other part of the market equilibrium model. Hence, not including an explicit constraint such as \( z_i^b \cdot z_i^b = 0 \) will not affect the outcomes of a market equilibrium in any way, and omitting such combinatorial constraints will considerably simplify both theoretical analysis and computation.

3) **Power Producers and the ISO:** We assume that an ISO collects offers from electric power producers to minimize the total generation cost, while treating the demand and prosumers’ buy and sell decisions as exogenous. The optimization problem is as follows:

\[
\begin{align}
\text{minimize } & \sum_{i=1}^{N} \sum_{h \in H_i} C_{ih}(g_{ih}) & (7a) \\
\text{subject to } & g_{ih} - G_{ih} \leq 0, & \forall i, h \in H_i & (7b) \\
& \sum_{i=1}^{N} y_i = 0 & (7c) \\
& \sum_{i=1}^{N} PTDF_{ki}y_i \leq T_{k}^+ (\lambda_k^+), \quad \forall k & (7d) \\
& -\sum_{i=1}^{N} PTDF_{ki}y_i \leq T_{k}^- (\lambda_k^-), \quad \forall k & (7e) \\
& y_i - \sum_{h \in H_i} g_{ih} - (z_i^b - z_i^b) + d_i = 0(p_i), \quad \forall i. & (7f)
\end{align}
\]

In the above problem, \( (7b) \) is the generation capacity constraint. The set \( H_i \) represents all power plants at node \( i \); therefore, we do not need to assume that there is only one power plant at each node \( i \). \( (7e) \) ensures that the total net injection/withdrawal in the system is equal to zero, where \( y_i \) represents the energy flow from an arbitrarily assigned hub node to node \( i \). We represent the transmission network as a hub-spoke system; that is, the energy flows from node \( i \) to \( j \) are considered as from \( i \) to the hub, and from the hub to \( j \). Equations \((7d)-(7e)\) describe that
the flow in link \( k \) is less than its transmission limit \( T_k \). The term \( PTDF_{ki} \) represents the power transfer distribution factors based on linearized DC flows. Equation (7f) is the mass balance constraint, whose shadow price, denoted by \( p_i \), is exactly the wholesale energy price at node \( i \) (that is, the LMP at \( i \)).

To aid in model development and analysis, we make the following blanket assumption throughout the article.

**Assumption 1:** The generation cost function \( C_{ih} (\cdot) \) is a convex quadratic function (including a linear function or a constant) for all \( i = 1, \ldots, N \) and \( h \in H_i \).

According to (7f), with a given \( d_i \), the variable \( y_i \) is implicitly bounded for \( i = 1, \ldots, N \), since \( g_{ih} \)'s are bounded, and so is the quantity \( z^s_i - z^b_i \) based on (5b). Hence, with Assumption 1, if the feasible region is not empty with respect to a given demand vector \( (d_i)_{i=1}^N \) and \( (z^s_i, z^b_i)_{i=1}^N \), then an optimal solution of the above optimization problem exists by the well-known Weierstrass extreme value theorem, and the KKT conditions are necessary and sufficient for optimality, due to the all-linear constraints and the convex objective function. The detailed KKT conditions are:

\[
\begin{align*}
0 & \leq g_{ih} \quad -C_{ih}^*(g_{ih}) - \rho_{ih} + p_i \leq 0, \ \forall i, h \in H_i \quad (8a) \\
-\theta + \sum_{k=1}^{K} PTDF_{ki}(\lambda^k - \lambda^k_h) - p_i = 0, \ \forall i \quad (8b) \\
0 & \leq \rho_{ih} \quad g_{ih} - G_{ih} \leq 0, \ \forall i, h \in H_i \quad (8c) \\
\theta & \text{ free, } \sum_{i=1}^{N} y_i = 0 \quad (8d) \\
0 & \leq \lambda^k - \sum_{i=1}^{N} PTDF_{ki} y_i - T_k \leq 0, \ \forall k \quad (8e) \\
0 & \leq \lambda^k_h - \sum_{i=1}^{N} PTDF_{ki} y_i - T_k \leq 0, \ \forall k \quad (8f) \\
p_i & \text{ free, } y_i - \sum_{h \in H_i} g_{ih} - (z^s_i - z^b_i) + d_i = 0, \ \forall i. \quad (8g)
\end{align*}
\]

**B. Upper-Level Problem**

1) **Fixed Cost Recovery:** First and foremost, revenue accruing through retail electricity rates (both volumetric and fixed rates) must cover utilities’ fixed costs, which is ensured by the following constraint in the upper-level problem:

\[
\sum_{i=1}^{N} \left( -z^s_i \tau^s + z^b_i \tau^b + d_i \tau^b + \sum_{j \in \text{pro,con}} \phi^i_j \right) = T, \quad (9)
\]

where \( T \) represents the fixed cost to be recovered, which is exogenous to the model, and can be decided by utilities and approved by an energy commissioner.\(^4\)

2) **Equity and Energy Expenditure Incidence:** The upper-level decision-maker aims to strike the balance between maximizing the social surplus and maintaining energy equity when deciding on the retail electricity rates. To establish a model to aid decision making, we need quantitative measures of energy equity. In this work, we use the same measures (but with a small update) as developed in our earlier work [24]. To make this article self-contained, we reintroduce the measures here. The key concept is the energy expenditure incidence (EEI), defined for conventional consumers \( (inc^\text{con}_i) \) and prosumers \( (inc^\text{pro}_i) \) at node \( i \) as follows:

\[
\begin{align*}
inc^\text{con}_i &= \frac{(p_i + \tau^b)d_i + \phi^\text{con}_i}{I^\text{con}_i}, \quad (10) \\
inc^\text{pro}_i &= \frac{(p_i + \tau^b)z^b_i + \phi^\text{pro}_i}{I^\text{pro}_i} + C_i(g_i) + SC_i. \quad (11)
\end{align*}
\]

The EEI measures the proportion of consumers’ spending on electricity compared to total household income (denoted by \( I^\text{con}_i \) and \( I^\text{pro}_i \) for conventional consumers and prosumers, respectively). The conventional consumers’ energy spending is easy to understand, which is the sum of the fixed charge \( \phi^\text{con}_i \) and the cost of purchasing \( d_i \) energy at the rate of \( p_i + \tau^b \); the prosumers’ spending includes the cost of backup generation \( (C_i(g_i)) \) and the sunk costs \( (SC_i) \) spent on purchasing or renting renewable energy equipment.\(^5\)

With the EEI defined, our idea of energy equity is to minimize the differences in EEI between conventional consumers and prosumers, that is, to minimize \( \sum_i (inc^\text{con}_i - inc^\text{pro}_i)^2 \). The definition of EEI in (10) and (11) involves both lower-level variables \( (d_i, z^s_i, z^b_i, g_i, p_i)_{i=1}^{N} \) and upper-level variables \( (\tau^b, \phi^\text{con}, \phi^\text{pro}) \), where \( \phi^\text{con} := (\phi^\text{con}_1, \ldots, \phi^\text{con}_N) \in \mathbb{R}^N \) and \( \phi^\text{pro} := (\phi^\text{pro}_1, \ldots, \phi^\text{pro}_N) \in \mathbb{R}^N \). To simplify the argument, we define a vector \( \zeta \equiv (\tau^b, \tau^s, \phi^\text{con}, \phi^\text{pro}) \in \mathbb{R}^{N+2} \), and use the notation \( d(\zeta) \) to denote the optimal solution mapping with a given \( \zeta \); that is, \( d(\zeta) \) is a set of all optimal solutions of the consumers’ optimization problem (3) with a given \( \zeta \). The notation for all other optimal solution mappings is the same. Then we define the difference-of-incidence function as:

\[
B[\zeta; d(\zeta), z^b(\zeta), g(\zeta), p(\zeta)] := \sum_{i=1}^{N} (inc^\text{con}_i - inc^\text{pro}_i)^2. \quad (12)
\]

3) **The Complete Model:** The upper-level decision-maker’s problem is to choose volumetric rates \( (\tau^b, \tau^s) \) and fixed charges \( (\phi^\text{con}, \phi^\text{pro}) \) to minimize the EEI difference (aka the \( B \) function in (12)) while ensuring revenue adequacy for the utility company through (9). A global minimizer of the EEI difference function, while feasible to (9), may not be unique (we will discuss the existence in the following); hence, the upper-level decision-maker wants to choose among the feasible minimizers to maximize the go through rate-making proceedings to finalize what rate will be charged to its customers [32].

---

\(^4\)In the U.S., the retail rates are under the purview of the state’s Public Utility Commission. For example, CPUC approves all the rates that each electric utility charges its customers. When a utility’s revenue requirement is determined, mostly to recover the costs of completed or ongoing projects, a utility must

\(^5\)Note that we do not subtract the prosumers’ earnings (i.e., \((p_i + \tau^s)z^s_i\)) from the energy spending in the numerator in (11) because it could lead to a negative EEI, which makes equating the incidence among groups impossible.
social surplus of the electricity wholesale market, which is defined as the sum of all the market participants’ surplus. Let \( x \) represent the collection of all lower-level variables \((d, l, z^s, z^b, g, y)\) and \( \Pi(x; (\tau^b, \tau^s)) \) denote social surplus corresponding to given volumetric rates \((\tau^b, \tau^s)\). Then its formulation is as follows:

\[
\Pi[x; (\tau^b, \tau^s)] := \sum_{i=1}^{N} \left[ \int_{0}^{d_i} p_i^{con}(m_i) \, dm_i - \tau^b d_i \right] + \sum_{i=1}^{N} \left[ \tau^s z_i^s - \tau^b z_i^b + \int_{0}^{d_i} p_i^{pro}(m_i) \, dm_i - C_i^p(g_i) \right] - \sum_{i=1}^{N} \sum_{h \in H_i} C_{ih}(g_{ih}).
\]

(13)

By following the convention in economics, we do not include fixed charges \( \phi^{con} \) and \( \phi^{pro} \) in the social surplus, as the behavior of wholesale market participants is not affected by fixed charges in anyway.

With the social surplus and EEI-difference function defined, the complete two-level problem can be written as follows:

\[
\begin{align*}
\text{maximize} & \quad \Pi[x; (\tau^b, \tau^s)] - M \cdot B(\zeta; x) \\
\text{subject to} & \quad (4), (6a)–(6f), (8a)–(8g), \text{and} (9), \\
& \quad \tau^s \leq \tau^b, \tau^b, \phi^{con}, \phi^{pro} \geq 0, \tau^s \text{ free},
\end{align*}
\]

(14)

where the parameter \( M > 0 \) in the objective function is to balance between the two (possibly conflicting) objectives of maximizing the social surplus and minimizing the difference of energy expenditure incidence. Note that the collection of KKT conditions (4), (6a)–(6f), (8a)–(8g) appear in the constraints, instead of the primal constraints of each market player (5b)–(5c), (7b)–(7f) because the upper-level decision maker, usually a state-level energy commission, does not run the wholesale energy market. With a chosen volumetric rate \((\tau^b, \tau^s)\), consumers, prosumers and electric power producers each solve their own surplus-maximization problem, with the total supply and demand cleared by the ISO. The corresponding market outcomes, including the nodal electricity prices \( p_i \) needed in the EEI difference function, are then captured by the KKT conditions.

Problem (14) is an MPEC, which can be solved by nonlinear programming (NLP) solvers capable of dealing with complementarity constraints, such as KNITRO [33] or FILTER [34]. Granted that (14) is a nonconvex problem, only a locally optimal (or stationary) solution can be computed using an NLP solver. However, in the next section, we introduce a BLP formulation and establish the relationship between the MPEC and the BLP. The BLP will provide an approach to find a globally optimal solution of (14) by solving only convex problems. We also present some key theoretical results regarding the optimal solutions in the following section.

III. THEORETICAL RESULTS

A. Bilevel Reformulation

The idea of reformulating the MPEC into a BLP is simple: to replace the lower-level market equilibrium conditions with a centralized optimization problem. This is based on the well-known first fundamental theorem in welfare economics, which states that a competitive equilibrium leads to a Pareto efficient market outcome (see, for example, Proposition 3 in [35]). More specifically, let the set \( X \) denote the feasible region of the lower-level variables \( x \); that is, \( X = \{ x : (5b)–(5c), (7b)–(7f) \} \). We can define the following optimal value function:

\[
V(\tau^b, \tau^s) := \max_{x \in X} \Pi[x; (\tau^b, \tau^s)].
\]

(15)

Let \( x(\tau^b, \tau^s) \) denote the optimal solution of the above optimization problem with respect to a given \((\tau^b, \tau^s)\), which is a set-valued mapping in general. However, we state below that under mild conditions, the mapping is a singleton.

\textbf{Lemma 2:} With the inverse demand functions as in (1) and (2) and under Assumption 1, for any given \((\tau^b, \tau^s)\) with \( \tau^b \geq 0 \) and \( \tau^s \leq \tau^b \), an optimal solution of the optimization problem in (15) exists. In addition, the vectors \( d, l \) and the quantity \( z^s - z^b \) in such an optimal solution are all unique.

\[ \square \]

The proof is relatively straightforward, with the existence following from the coerciveness of the objective function over the feasible region \( X \), and the uniqueness as the result of strict convexity of the objective function. Details are omitted here. With the above notation, the complete BLP model can be written as follows.

\[
\begin{align*}
\text{Minimize} & \quad B \left[ \bar{\tau}^b, \bar{\tau}^s, \phi^{con}, \phi^{pro}, \right. \\
\text{subject to} & \quad \sum_{i=1}^{N} \left[ -z_i^s(\bar{\tau}^b, \bar{\tau}^s)\tilde{\tau}^s + z_i^b(\bar{\tau}^b, \bar{\tau}^s)\tilde{\tau}^b \\
& \quad + d_i(\bar{\tau}^b, \bar{\tau}^s)\tilde{\tau}^b + \sum_{j \in \text{pro,con}} \phi^i_j \right] = T, \\
& \quad \tilde{\tau}^s \leq \bar{\tau}^b, \tilde{\tau}^s, \phi^{con}, \phi^{pro} \geq 0, \\
& \quad (\tilde{\tau}^b, \tilde{\tau}^s) \in \text{arg max } V(\tau^b, \tau^s), \\
& \quad (\bar{\tau}^b, \bar{\tau}^s) \in X_{KKT}(\tilde{\tau}^b, \tilde{\tau}^s),
\end{align*}
\]

(16a)

(16b)

(16c)

(16d)

(16e)

where the set \( X_{KKT}(\tilde{\tau}^b, \tilde{\tau}^s) \) represents the KKT system of the lower-level optimization problem (15) with respect to \((\tilde{\tau}^b, \tilde{\tau}^s)\). Under the assumptions of Lemma 2, the lower-level optimization problem is clearly a convex optimization problem with all linear constraints. Hence, the KKT conditions are necessary and sufficient optimality conditions. By Lemma 2, with the uniqueness of optimal solutions corresponding to a given \((\tilde{\tau}^b, \tilde{\tau}^s)\), constraint (16b) is well defined. However, the objective function (16a) may not be. The issue is \( p(\tilde{\tau}^b, \tilde{\tau}^s) \), which is the Lagrangian multiplier (i.e., the LMP) associated with the flow balancing constraint (7f)
may not be unique, even when the primal variables are unique. If the set of multipliers is not a singleton, it is understood that the optimization problem chooses a \( p(\tau^b, \tau^s) \) to minimize the objective function (16a).

To further ensure the validity of the BLP, we need to ensure the attainability of maximum of the optimal value function \( V(\tau^b, \tau^s) \). To do so, we first argue that \( (\tau^b, \tau^s) \) should be in a bounded polyhedral set. We observe that, based on the KKT conditions (4), (6a), and (6e), if \( \tau^b > \max_{i=1,...,N} \{ P^b_i \} \), that is, if the volumetric charge is greater than the largest willingness-to-pay of any consumers/prosumers, then all the \( d_i \)'s and \( l^i \)'s will be 0 in an optimal solution, indicating no energy consumption at all due to the high costs. To avoid this, \( \tau^b \) should be bounded above by \( \max_{i=1,...,N} \{ P^b_i \} \), which we denote by \( \bar{\tau}^b \). Similarly, if \( \tau^s \) is too negative, no prosumers would sell their self-produced energy, and all \( z^b_i \) would be 0 in an optimal solution. Hence, we assume that \( \tau^s \) is implicitly bounded below by \( \underline{\tau}^s < 0 \). Define the following set:

\[
\mathcal{T} := \{ (\tau^b, \tau^s) : 0 \leq \tau^b \leq \bar{\tau}^b, \underline{\tau}^s \leq \tau^s \leq \bar{\tau}^b \} \subseteq \mathbb{R}^2,
\]

which is a bounded polyhedron. For the optimal value function \( V(\tau^s, \tau^b) \), we want to show that it is continuous over \( \mathcal{T} \).

Under Assumption 1, \( V(\tau^b, \tau^s) \) is the optimal value function of a convex quadratic program with parameterization only on the linear term in the objective function. Properties of such functions are well established, and in this specific case, it is known that \( V(\tau^b, \tau^s) \) is a continuous function. (See Theorem 47 in [36].) Therefore, an optimal solution of \( \max_{(\tau^b, \tau^s) \in \mathcal{T}} V(\tau^b, \tau^s) \) exists and is attainable, again by the Weierstrass Extreme Value Theorem.

**B. Relationship Between the MPEC and the BLP Model**

Note that the relationship between the MPEC model and the BLP model is different than their usual relationship. In a typical BLP, writing out the lower-level problem’s KKT conditions will lead to an MPEC formulation. There, the upper-level decision maker does not optimize the lower-level objective function; that is, there is no optimization of the optimal value function \( V(\tau^b, \tau^s) \) in (16d). It is needed here since the upper-level decision maker wants to maximize social surplus as well as maintaining equity among all energy consumers. Because of (16d), the BLP formulation (16a)–(16e) is not equivalent to the MPEC formulation (14) in general, due to the multi-objective decision-making in (14); that is, there can be a solution that leads to a lower EEI-difference function \( B \), but does not maximize the social surplus \( \Pi \). In the following, we show that if an optimal solution exists for each model, the MPEC and the BLP formulations are indeed equivalent when there are no additional constraints on the volumetric and fixed charges. We then show that an optimal solution of the BLP does exist, with the optimal \( \tau^b = \tau^s = 0 \).

**Proposition 3:** Assume that both the MPEC model (14) (for an arbitrary \( M > 0 \)) and the bilevel optimization model (16a)–(16e) have an optimal solution, which are denoted as \( (\bar{x}^{MPEC}, \bar{\zeta}^{MPEC}) \) and \( (\bar{x}^{BLP}, \bar{\zeta}^{BLP}) \), respectively. Under the assumptions in Lemma 2 and if the LICQ holds at \( x(\bar{x}^{BLP}) \) in the set \( \mathcal{X} \), then \( \bar{\zeta}^{MPEC} \) is a globally optimal solution of the BLP model (16a)–(16e); conversely, for each \( \bar{\zeta}^{BLP} \) and a corresponding \( x(\bar{\zeta}^{BLP}) \in \mathcal{X} \), the pair \( (x(\bar{\zeta}^{BLP}), \bar{\zeta}^{BLP}) \) is a globally optimal solution of the MPEC (14).

**Proof:** Let \( (\bar{\tau}^{BLP}, \bar{\tau}^{BLP}) \) be the optimal \( \tau \)'s in an optimal solution of the BLP. First, note that since \( (\bar{\tau}^{BLP}, \bar{\tau}^{BLP}) \) maximizes \( V(\tau^b, \tau^s) \) over all \( (\tau^b, \tau^s) \in \mathcal{T} \), we have that

\[
\Pi [x(\bar{\zeta}^{BLP}), \bar{\zeta}^{BLP}] \geq \Pi [x^{MPEC}, \bar{\zeta}^{MPEC}].
\]

If \( B(\bar{\zeta}^{BLP}) < B(\bar{\zeta}^{MPEC}) \), which means that the bilevel program can yield a lower energy index, then we have

\[
\Pi [x(\bar{\zeta}^{BLP}), \bar{\zeta}^{BLP}] - M \cdot B(\bar{\zeta}^{BLP}) > \Pi [x^{MPEC}, \bar{\zeta}^{MPEC}] - M \cdot B(\bar{\zeta}^{MPEC}).
\]

According to the discussion following the bilevel model (16a)–(16e), the KKT conditions are both necessary and sufficient for an optimal solution of the lower-level problem; hence, \( (x(\bar{\zeta}^{BLP}), \bar{\zeta}^{BLP}) \) is feasible for the MPEC model, which contradicts the assumption that \( (x^{MPEC}, \bar{\zeta}^{MPEC}) \) is an optimal solution of the MPEC model.

Now suppose that \( B(\bar{\zeta}^{BLP}) : x(\bar{\zeta}^{BLP})) > B(\bar{\zeta}^{MPEC}) : x^{MPEC}) \). For the given \( \tau^{BLP} := (\tau^{BLP}_1, \tau^{BLP}_2) \), by Lemma 2 and the assumption of LICQ, we know that the corresponding quantities \( d(\tau^{BLP}), p_i(\tau^{BLP}), \) and \( z^b(\tau^{BLP}) \) are all unique (given that \( z^b(\tau^{BLP}) \) is defined as \( \max \{ z^b_i (\tau^{BLP}_1), \tau^{BLP}_2 \} \)). Then the following linear system of equations with respect to \( (\phi^{con}, \phi^{pro}) \) is well defined:

\[
\begin{bmatrix}
\frac{p_i + \bar{\tau}^b d_i + \phi^{con}_i}{\bar{I}^{con}_i} \\
\frac{p_i + \bar{\tau}^b d_i + \phi^{pro}_i + C_i(g_i) + SC_i}{\bar{I}^{pro}_i}
\end{bmatrix} = T.
\]

The system of equations has \( N \) equations but \( 2N \) variables. The coefficient matrix clearly has a full row rank. Hence, for any given \( (\bar{\tau}^b, \bar{\tau}^s) \), there exists a solution \( (\phi^{con}, \phi^{pro}) \) to the system of equations. A solution of the system of equations in (19) can lead to a 0 value of the EEI difference function \( B(\bar{\zeta}^{BLP}) \), which is the absolute minimum of the function based on its definition. Hence, the assumption that \( B(\bar{\zeta}^{BLP}) : x(\bar{\zeta}^{BLP})) > B(\bar{\zeta}^{MPEC}) : x^{MPEC}) \) is invalid, and we obtain that \( B(\bar{\zeta}^{BLP}) : x(\bar{\zeta}^{BLP})) = B(\bar{\zeta}^{MPEC}) : x^{MPEC}) = 0 \). Coupled with the fact that \( (x(\bar{\zeta}^{BLP}), \bar{\zeta}^{BLP}) \) is feasible for the MPEC model and by the inequality (18), we derive that \( (x(\bar{\zeta}^{BLP}), \bar{\zeta}^{BLP}) \) is a globally optimal solution of the MPEC model.

The reverse of the statement, namely, an optimal solution of the MPEC must be optimal to the BLP model, can be shown in a similar fashion.

**Proposition 4** is established under the assumption that an optimal solution exists for both the MPEC and the BLP. In the
following, we show that the BLP indeed has an optimal solution and in such an optimal solution, \( \tau^b = \tau^s = 0 \). This solution is trivially optimal for the MPEC model as well.

**Proposition 4:** Under the assumptions of Lemma 2 and Assumption 1, we have \( \max_{(\tau^b, \tau^s) \in T} V(\tau^b, \tau^s) = V(0,0) \).

**Proof:** See appendix. \( \square \)

**Remark 1:** With \( \tau^b = \tau^s = 0 \), as argued in the proof of Proposition 3, the system of equations (19) always has a solution, which will make the B function equal to zero; hence, they form an optimal solution of the BLP formulation, which can be easily seen to be a globally optimal solution of the MPEC formulation. Obtaining such a solution can be done in two steps: first, solve the lower-level market equilibrium (either by solving a complementarity problem or an optimization problem as in (15), with \( (\tau^b, \tau^s) = (0,0) \); second, solve the linear system equations (19). Both steps can now be done with efficient algorithms since no non-convex problems are involved. One thing to note is that since the system equations have fewer equations than the variables when \( N > 1 \), the solution \( \phi_i \)'s are not unique. In this case, we can minimize \( ||\phi||_2^2 \) over all solutions, which is a convex quadratic program and can be solved efficiently.

**Remark 2:** Proposition 4 is consistent with a well-known fact in economics: under perfect competition, laissez-faire is the first-best equilibrium. Any type of tax or subsidy can only reduce economic efficiency and social surplus. A fixed-charge-only tariff, however, would likely encourage excessive energy use, as customers’ energy bills are not tied to energy usage at all. Hence, in reality, almost all tariffs consist of both volumetric and fixed charges. In Section IV, we numerically study the impact of different levels of volumetric charges on market outcomes and energy equity. However, one thing to note is that with additional requirements (aka constraints) on the fixed and volumetric charges, the system of equations (19) (together with the additional constraints) may no longer have a solution, which means that the EEI difference function can no longer be zero. (This is indeed what we observe in certain numeric cases.) In this case, the equivalence between the MPEC and the BLP model (in terms of optimal solutions) may also break down.

**C. Model and Theoretical Results Under Stochasticity**

The above analyses are performed based on deterministic models. In this section, we show that the main result in Proposition 4 still holds under uncertainties, regardless of the probability distributions. To do so, however, we need to present a formulation under stochasticity. Let \( \xi \in \mathbb{R}^w \) denote the collection of all the random variables with a generic dimension of \( w \) (such as prosumers’ solar output \( R_{i0} \), generation capacity \( G_{ik} \), etc.). It is defined in the probability space \( (\Omega, \mathcal{F}, \mathbb{P}) \), with \( \Omega \) being the sample space of all the uncertainties, \( \mathcal{F} \) the sigma field of \( \Omega \) and \( \mathbb{P} \) a probability measure in \( (\Omega, \mathcal{F}) \); that is, \( \xi \) is a measurable mapping from \( \Omega \) to a set \( \Xi \in \mathbb{R}^w \). Let \( \mathbf{x}^\xi \) and \( \mathbf{X}(\xi) \) respectively denote the lower-level variables and feasible region corresponding to a realization of the random vector \( \xi \). Furthermore, let \( \Pi(\mathbf{x}^\xi; (\tau^b, \tau^s), \xi) \) represent the lower-level objective function under uncertainty, with a given volumetric rate \( (\tau^b, \tau^s) \). Then we can define the expected optimal value function of the lower-level problem as follows:

\[
EV(\tau^b, \tau^s) := \mathbb{E}_\xi \left\{ \max_{\mathbf{x}^\xi \in \mathbf{X}(\xi)} \Pi(\mathbf{x}^\xi; (\tau^b, \tau^s), \xi) \right\}.
\]  (20)

To ensure that the above expectation is well-defined, we need the following assumption.

**Assumption 2:** All the random functions in \( \Pi(\mathbf{x}^\xi; (\tau^b, \tau^s), \xi) \) and \( \mathbf{X}(\xi) \) have finite moments.

Under Assumption 2, for any \( (\tau^b, \tau^s) \in \mathcal{T} \), with \( \mathcal{T} \) defined in (17), the expectation in (20) is well defined following the same argument as in [37]. With a finite expected value in (20), since expectation preserves convexity (see, for example, Section III-B1 in [38]), we have that \( EV(\tau^b, \tau^s) \) is also a convex function with respect to \( (\tau^b, \tau^s) \in \mathcal{T} \). Hence, the extension of Proposition 4 to the stochastic case is straightforward, and we only state the result below, omitting the proof.

**Proposition 5:** Under the assumptions of Lemma 2, and Assumption 1 and 2, we have \( \max_{(\tau^b, \tau^s) \in \mathcal{T}} EV(\tau^b, \tau^s) = EV(0,0) \). \( \square \)

With the lower-level optimal value function defined, we can write out the stochastic version of the upper-level problem now. Note that the upper-level decisions on the retail rates have to be made before lower-level uncertainties are realized; that is, they are the here-and-now type of decisions. Since the lower-level optimal solutions \( d, z^b, \) and \( p \) depend on the random vector \( \xi \), if we simply require that the revenue adequacy constraint (9) is held for all \( \xi \) (almost surely), it will likely to be always infeasible. To remedy this, a natural idea is to make constraint (9) a chance constraint as follows:

\[
\min_{\tau, \phi} \mathbb{E}_\xi \left\{ B \left( d_i^\xi (\tilde{\tau}^b, \tilde{\tau}^s), \tilde{z}^b, \phi_i \right) \right\}
\]

subject to

\[
\mathcal{P} \left\{ \sum_i \left[ -z_i^\xi (\tilde{\tau}^b, \tilde{\tau}^s) \tilde{z}^b + z_i^B (\tilde{\tau}^b, \tilde{\tau}^s) z^b + d_i^\xi (\tilde{\tau}^b, \tilde{\tau}^s) \tau^b + \sum_{j \in \text{pro,con}} \phi_i^j \right] \geq T \right\} \geq 1 - \epsilon
\]

\((\tilde{\tau}^s, \tilde{\tau}^b) \in \arg \max_{(\tau^s, \tau^b)} EV(\tau^s, \tau^b), \quad (21)\)

where \( \epsilon \) is a pre-specified parameter. Solving the chance-constrained stochastic program (SP) (21) is much more involved than its deterministic counterpart, even when Proposition 5 holds. In this case, sample average approximation (SAA) methods developed for chance-constrained SPs, such as [39], can be directly applicable here. However, if there are other requirements on the volumetric charges such that they cannot be zero, then specialized (and likely iterative) algorithms need to be developed to solve (21), since it also includes an optimal value function in its constraints. Developing such algorithms (and presenting numerical results under stochasticity) is deferred to future research.

**IV. Numerical Case Study**

**A. Setup**

To illustrate the effects of optimal pricing schemes, we apply the models developed in Section II to a case study considering
a three-node network with ten generating units, and three transmission lines. This setup is sufficiently general because it allows firms to compete across different locations subject to transmission constraints. The three-node network is the simplest that allows for looped flows, which is important in modeling a power grid. We assume that a daily fixed cost of $80 to be reimbursed to a utility.

Consumers are grouped into three income levels: high, medium, and low, residing at nodes A, B, and C, respectively. The baseline daily demand of the low-income group is assumed to be 20 kW. The daily demand of the medium- and high-income groups are assumed to be 25% and 50% larger than the low-income group, broadly consistent with the data from the 2015 RECS survey [40]. Given the assumed demand in each node, we then recover the number of households in each income group. (Proportion of households among income groups are also compatible with the 2015 RECS results.) The income level is obtained by assuming that electricity expenditure is 1.5% of the income in each group. Finally, based on RECS 2015, we assume that 20% of the households in high-income group (or 3,067 households) own rooftop solar energy with a capacity of 8 kW each household or 25 MW in total. Thus, in the extreme case during a sunny summer day, we consider a daily solar output from prosumers to be $R = 150$ MWh; while in a cloudy/raining winter day, it generates only $R = 25$ MWh. The numbers are carefully chosen with one corresponding to insufficient DER generation for prosumers, while the other with excess DER generation from prosumers. For the sunk cost $SC$ to be included in prosumers’ energy expenditure in (11), we assume a daily cost $5/day. A 25 MW backup generator or energy storage is assumed for the prosumers. The MPEC formulation is used for computation, and it is written in AMPL and solved by Knitro solver version 12.4 on a Mac Book Pro with 2.8 GHz Quad-Core and Intel Core i7.

### B. Main Results

First, we add additional constraints to the upper-level problem and require that a certain percentage of utility’s fixed cost $T$ to be recovered through volumetric charge. More specifically, we consider two cases: 10% and 90% of $T$ to be recovered by volumetric charges. Later we will compare such results with the case without the additional constraints (i.e., the original formulation as in (14)). The results of the volumetric charge $10\% T$ and $90\% T$ for the cases with a renewable output equal to 25 MWh and 150 MWh are reported in Tables I and II, respectively.

---

Table I indicates a significant increase in volumetric charge when a higher percentage of $T$ is required to be recovered from the actual energy use. Specifically, $r$ increases from $5.25$/MWh under $10\% T$ to $49.18$/MWH under $90\% T$. In the case of 25 MWH of DER output, prosumers in both cases are in a net-buying position. The prosumers benefit from the case of requiring a higher percentage of volumetric charge, with an increase of surplus from $73.47$ K to $78.23$ K and a decline of energy expenditure incidence from 1.34% to 1.29%. Note that prosumers’ energy incidence is less than that of consumers under the $90\% T$ case; meaning that with this requirement, the system of (19) no longer has a solution of $(\phi_{\text{pro}}, \phi_{\text{con}})$ that can lead to zero value of the EEl difference function $B$. This indicates that energy equity can no longer be maintained across different income groups. The generation from the wholesale market is $1,519.17$ MWh and $1,464.09$ MWh, under the $10\%-$ and $90\%-$volumetric-charge requirement, respectively. This is so because the higher energy prices faced by consumers in the $90\% T$ case suppress demand, including that of prosumers, by a margin of 3.6%.

We now turn to discuss Table II, the case when the DER output equals 150 MWh. In this case, prosumers have excess output and are in a net-selling position, while subject to $r$ when selling to the grid. The fact that $r < 0$ (−$3.99$/MWH and −$13.97$/MWH under the $10\% T$ and $90\% T$ cases, resp.) indicates that prosumers is excess prosumer’s DER capacity (i.e., under the 150 MWH case, they can sell electricity to the grid), the fixed charge can be different from that in the case when prosumer’s DER capacity is insufficient to cover their own load under the 25 MWH case.

---

| Variable/Volumetric charge | 10% | 90% |
|-----------------------------|-----|-----|
| Volumetric charge | $3.91$/MWh | $49.18$/MWh |
| Prosumer’s sale (old) [MWh] | $78.70$ | $78.70$ |
| Prosumer’s load [MWh] | $96.30$ | $96.30$ |
| Backup generation [MWh] | $23.0$ | $23.0$ |
| Prosumer surplus [SK] | $79.75$ | $80.93$ |
| Prosumer incidence [%] | $1.30$ | $1.06$ |
| Prosumer fixed charge [Household] | $3.33$ | $2.41$ |

| Variable/Volumetric charge | 10% | 90% |
|-----------------------------|-----|-----|
| Volumetric charge | $3.41$/MWh | $49.93$/MWh |
| Prosumer’s sale (new) [MWh] | $78.70$ | $78.70$ |
| Prosumer’s load [MWh] | $96.30$ | $96.30$ |
| Backup generation [MWh] | $23.0$ | $23.0$ |
| Prosumer surplus [SK] | $79.75$ | $80.93$ |
| Prosumer incidence [%] | $1.30$ | $1.06$ |
| Prosumer fixed charge [Household] | $3.33$ | $2.41$ |

---

This represents the “energy equity” case at the baseline. The 1.5% is at the lower end based on 2015 RECS. However, our interest lies on the relative changes of the energy incidence when the power sales by prosumers are subject to different tariff designs. This assumption is not essential. This is calculated approximately based on $830$ K initial investment on solar panels (including installation) and an assumed break-even period of 15 years. This is also in the ballpark of rental costs of solar panels.

We want to clarify that the different levels of fixed charge $\phi_i$ reported in this section do not mean that $\phi_i$ would vary within a fixed period of time. The two cases herein do not refer to different realizations of a random variable. They simply mean two separate scenarios, and our intention is to show that when there

---

**Table I**

| Variable/Volumetric charge | 10% | 90% |
|-----------------------------|-----|-----|
| Volumetric charge | $3.91$/MWh | $49.18$/MWh |
| Prosumer’s sale (old) [MWh] | $78.70$ | $78.70$ |
| Prosumer’s load [MWh] | $96.30$ | $96.30$ |
| Backup generation [MWh] | $23.0$ | $23.0$ |
| Prosumer surplus [SK] | $79.75$ | $80.93$ |
| Prosumer incidence [%] | $1.30$ | $1.06$ |
| Prosumer fixed charge [Household] | $3.33$ | $2.41$ |

| Variable/Volumetric charge | 10% | 90% |
|-----------------------------|-----|-----|
| Volumetric charge | $3.41$/MWh | $49.93$/MWh |
| Prosumer’s sale (new) [MWh] | $78.70$ | $78.70$ |
| Prosumer’s load [MWh] | $96.30$ | $96.30$ |
| Backup generation [MWh] | $23.0$ | $23.0$ |
| Prosumer surplus [SK] | $79.75$ | $80.93$ |
| Prosumer incidence [%] | $1.30$ | $1.06$ |
| Prosumer fixed charge [Household] | $3.33$ | $2.41$ |

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**Table II**

| Variable/Volumetric charge | 10% | 90% |
|-----------------------------|-----|-----|
| Volumetric charge | $3.41$/MWh | $49.93$/MWh |
| Prosumer’s sale (new) [MWh] | $78.70$ | $78.70$ |
| Prosumer’s load [MWh] | $96.30$ | $96.30$ |
| Backup generation [MWh] | $23.0$ | $23.0$ |
| Prosumer surplus [SK] | $79.75$ | $80.93$ |
| Prosumer incidence [%] | $1.30$ | $1.06$ |
| Prosumer fixed charge [Household] | $3.33$ | $2.41$ |

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TABLE III

| Volumetric Rate | 25 MWh | 150 MWh |
|-----------------|--------|---------|
| Volumetric charge [$/MWh] | 0.10 | 0.10 |
| Prosumer’s sale (cost) [$/MWh] | 45.97 | 32.00 |
| Prosumer’s load [$/MWh] | 95.97 | 96.96 |
| Battery generation [$/MWh] | 25.0 | 28.0 |
| Prosumer surplus [$] | 73.24 | 79.62 |
| Prosumer income [%] | 1.38 | 1.33 |
| Prosumer fixed charge [$/kWh] | 2.33 | 3.40 |

Fig. 2. Distribution of surplus.

On the contrary, prosumers sell energy to the grid when $R = 150$ MWh. Its surplus in this situation is affected by two countering forces. On the one hand, its increased contribution to fixed cost through $\tau^b$ reduces its surplus until $\tau^s$ drops to approximately $-10$/MWh; beyond which its surplus is adequately offset by a decline in fixed charged (or an increase in the volumetric charge fraction), leading its surplus to be leveled around $80$ k until the volumetric charge fraction is greater than $90\%$. When the volumetric charge fraction is $100\%$, the prosumer can avoid the fixed cost entirely, leading to a surge in its surplus as shown in Fig. 2(e). With regard to consumers, their surplus is affected by retailed power prices and allotted fixed costs. As alluded to earlier, forgo consumption is one way to minimize the impact of fixed cost recovery. This strategy effectively enhances consumer surplus until the allotted fraction of volumetric charge equal to $40\%$. However, beyond this level, the retail power prices become too high, leading to a drop in the surplus.

Overall, we observe that the wholesale surplus in Fig. 2(b) continues to decline with the increase in the volumetric fraction under $R = 25$ MWh, forming a concave curve due to the impacts on consumer surplus in 2(c). Finally, the changes in consumer surplus is “neutralized” by the changes of prosumer surplus in Fig. 2(e) under $R = 150$ MWh, leading to the total surplus in Fig. 2(a).

Additionally, one of key outcomes from the numerical case study is the optimal rates $p_i + \tau^s$ paid to prosumers when they sell energy back to the grid. Fig. 3 shows the optimal $\tau^s$ corresponding to the different percentages of fixed cost recovery from volumetric charges when $R = 150$ MWh. Recall that when $\tau^b = \tau^s$, it is the net-metering policy, while $\tau^b \neq \tau^s$ leads to the broad class of net-billing policies. The results show that, other than the case where $\tau^b = \tau^s = 0$, net metering is never an

should contribute to recovering fixed costs when selling power to the grid (i.e., net billing with sales payment less than the utility retail rate is more optimal than net metering). This is consistent with the recommendation by the CPUC concerning the recent debate to revamp the net-metering policies in California [41]. Overall, the same observations about the surplus distribution as in Table I emerge in Table II. More importantly, the $90\%$T requirement leaves the PUC with no adequate fund to maintain energy equity, leading to a divergence of energy incidence across different income groups.

For comparison purposes, Table III provides the results without any requirements on volumetric or fixed charges. As proven in Proposition 4, the optimal $\tau^b = \tau^s = 0$, and the total social surplus in the cases of $R = 25$ MWh and $R = 150$ MWh are higher than their counterparts in Tables I and II, consistent with the theoretical results in Section III. This suggests that the first-best policy is not to rely on volumetric charge but to leverage the fixed charge to maintain the energy equity. Furthermore, under the first-best policy, the fixed charge of prosumers increases with respect to their installed DER capacity, while that of conventional consumers declines accordingly. This is mainly because the possession of larger DERs effectively lowers prosumers’ EEI; consequently, it subjects them to a higher fixed charge in order to maintain the same EEI as conventional consumers at the optimum.

Fig. 2 plots the rent distribution among various entities in the market against the fraction of fixed costs assigned to the volumetric rate. The palpable difference of prosumer surplus in Fig. 2(e) of $R = 25$ MWh cf. $R = 150$ MWh cases affects the wholesale market surplus in Fig. 2(b) and deserves some explanation. When $R = 25$ MWh, the prosumers do not have enough DER generation and need to buy energy from the grid. An increase in the volumetric charge in x-axis provides an opportunity for the prosumer to “avoid” fixed charged via increasing self reliance or reducing consumption. As a result, its surplus steadily increases until the volumetric charge becomes $100\%$, where $\tau^b = 80$/MWh or nearly $50\%$ of the retail price at node $A$, an unbearably high level.

9The total social surplus is defined as the sum of prosumer surplus and the wholesale surplus, which includes surplus from producers, consumers, and the ISO. Our intention to separate prosumer surplus from all the other entities is to highlight the fact that they differ from other conventional entities in the wholesale market.
would become shows the EEI by (b) continues to increase. Consumers can reduce their means that prosumers pay to cover a portion of is affected by i) retailed $R = 25$ MWh cases.

Fig. 3. Optimal $\tau^b$ corresponding to the fraction of volumetric charge under 150 MWh cases.

A similar trend is also seen for ii) quantities demanded, and iii) fixed charge. When the fixed charge continues to decline from left to right in Fig. 4, volumetric charge $\tau^b$ continues to increase. Consumers can reduce their energy incidence through demand response by lowering their power consumption when facing higher retail power prices. The strategy is effective when the fraction of volumetric is small or the retail rate is low. When the retail rate is high, corresponding to the segment of the inverse demand curve where it is more price responsive, reducing consumption becomes less effective in lowering the retail prices, leading to an increase in the incidence. A similar trend is also seen for $R = 150$ MW. EEI continues to decline with increases in volumetric charge fraction and bottoms at the x-axes roughly equal to 65% and 70% in Fig. 4(a) and (b), respectively. Thus, if the government’s policy goal is to achieve the lowest possible and equitable EEI, those are the fractions of volumetric charges that should be aimed for. However, while a lower EEI is preferred by consumers and prosumers, it is at the expense of the total social surplus, especially of producers, as shown in Fig. 2(a) and (d). Finally, we demonstrate that with the allotted volumetric charge fraction less than or equal to 80% and 70% in the 25 MWh and 150 MWh cases, respectively, it is possible to maintain energy equity. Beyond 80% and 70% for these two cases, respectively, the available fund is inadequate to effectively compensate low-income groups, leading to energy inequity.

V. CONCLUSION

Recovering utilities’ fixed costs has presented a significant regulatory challenge in designing electricity tariffs. While emphasis has typically been placed on economic efficiency and incentive to conserve energy, equally important are their impacts on the energy equity among different income groups. The situation is further exacerbated by the presence of prosumers who are typically among the most affluent income groups, taking advantage of the electricity tariff, adopting new technologies and optimizing their self-interests.

This study examines the optimal retail tariff in the presence of prosumers. We demonstrate that a volumetric approach to recover fixed costs based on energy consumption is likely to be less efficient. Our analysis concludes that the first-best policy is to leave the wholesale market intact and rely on fixed charge to recover fixed costs based on the comparison of the total social surplus in Table III and Tables I and II. Fig. 2(a) also illustrates the total surplus is highest when the volumetric charge equals 0%. This conclusion is valid and robust supported by the proposition in section III while various numeric results to illustrate the findings could be different, depending on the setting. However, such a policy prescription is likely implausible: lower retail prices will likely provide a disincentive for energy conservation, against the effort to decarbonizing economy. In addition, a lumpy fixed charge on low-income households can be challenging to those families who already face harsh economic situations. Therefore, a policy that provides directly financial compensation to low-income households is economically efficient. Our analysis therefore suggests that programs, such as California Alternate Rates for Energy (CARE), designed to mitigate low-income households’ energy expenditure via lower retail rates, a volumetric approach, are unlikely to be efficient. That is, while it alleviates the energy burden of low-income households, it negatively impacts the total social surplus, which is a measurement of economic efficiency. This is because the CARE program entails volumetric subsidies that lower the retail power price facing low-income households, thereby inflating their demand or inducing load which otherwise will not be consumed under the 100% fixed charge case, a similar finding as in [42]. Moreover, as prosumers can effectively lower their maximal load by relying on DERs, a monthly demand charge, based on the highest kW use during the month and commonly implemented by utilities to recover fixed costs, is likely to be subject to the same concern as the volumetric charge. Finally, while the federal IRS (Internal Revenue Service) and state agencies, such as the California Department of Tax &

11A similar observation is also concluded in [42].
Fee Administration, have household income information, they cannot share it with CPUC or other state energy agencies. Thus, the actual implementation of the income-based fixed charge examined in this article will need a careful maneuver over the political and legal landscape.

Because our analysis is short-run based, which does not consider the interaction between power-system operations and expansion decisions, it is subject to a number of long-run implications. In particular, while the policy is expected to improve conventional consumers’ energy expenditure incidence, it may offset their economic incentive to invest in DERs, slowing the development of non-utility-scale DERs. Moreover, the impact on incidence can also be affected by demand elasticity. When demand is less price-responsive, consumers cannot forgo consumption in response to higher power prices, leading to higher consumption and a lower volumetric tariff. Last but not least, the equilibrium modeling framework assumes full rationality of consumers and prosumers. To account for bounded rationality, such as the lack of knowledge of others’ (or even their own) utility functions, completely different frameworks, such as a learning-based approach [43], are needed and worth exploring in future research.

APPENDIX

Proof of Proposition 4

As discussed earlier, $V(\tau^b, \tau^s)$ is the optimal value function of a convex quadratic program with parameters only in the objective function. It is well known that in this case, $V(\tau^b, \tau^s)$ is a convex function (such as Theorem 47 in [36]). By the fact regarding maximizing a convex function over a bounded polyhedral set (see Theorem 3.4.7 in [44]), at least one of the extreme points of $T$ must be an optimal solution of $\text{max}_{(\tau^b, \tau^s) \in T} V(\tau^b, \tau^s)$. The set $T$, as defined in (17), has four extreme points: $\tau^I := (\tau^b_0 = 0, \tau^s = 0)$, $\tau^{II} := (\tau^b = \tau^b_0, \tau^s = \tau^b_0)$, $\tau^{III} := (\tau^b = \tau^s = \tau^s_0)$, and $\tau^{IV} := (\tau^b = 0, \tau^s = \tau^s_0)$. In the following, we use $x(\tau^e)$ (or an element in the vector of $x$), $e = I, II, III$ or IV to denote the optimal solution corresponding to one of the four extreme points. Note that since the feasible region $X$ is not perturbed by $\tau$, any optimal solution $x(\tau^e)$ will also be feasible for problem $\text{max}_{x \in X} \Pi(x(\tau^e))$.

First, we compare $\Pi(x(\tau^I); \tau^I)$ and $\Pi(x(\tau^{IV}); \tau^{IV})$. Note that since $\tau^{IV}$ is defined to be a sufficiently negative number such that $z_i^b$ will be in an optimal solution for any $i = 1, \ldots, N$, at an optimal solution $x(\tau^{IV})$, $z_i^b = 0$, for all $i$. In addition, when $\tau^b = 0$ in $\tau^{IV}$, we have that $\Pi(x(\tau^{IV}); \tau^I) = \Pi(x(\tau^{IV}); \tau^I) \leq \text{max}_{x \in X} \Pi(x(\tau^I); \tau^I)$. Consider $\tau^{II} := (\tau^b = \tau^b_0, \tau^s = \tau^b_0)$. Next, we compare $\Pi(x(\tau^{II}); \tau^{II})$ and $\Pi(x(\tau^{III}); \tau^{III})$. Since $\tau^{IV} < \tau^b_0$, it is easy to see that $\Pi(x(\tau^{II}); \tau^{III}) \leq \Pi(x(\tau^{III}); \tau^{III})$.

Finally, we compare $\Pi(x(\tau^I); \tau^I)$ and $\Pi(x(\tau^{II}); \tau^I)$. Consider the objective function parameterized at $\tau^{II}$: $\Pi(x; \tau^I)$. After rearranging, we have

\[ \Pi(x; \tau^{II}) := \sum_i \left[ \int_0^{d_i} p^{con}(m_i) \, dm_i + \int_0^{l_i} p^{pro}(m_i) \, dm_i - C_i^g(g_i) - \sum_{h \in H_i} C_{ih}(gh) \right] + \sum_i \left[ \tilde{z}_i^b (z_i^b - z_i^b - d_i) \right]. \]

Using the constraint (7c) and (7e), we can rewrite the last term above as follows:

\[ \tilde{z}_i^b \sum_i \left( z_i^b - z_i^b - d_i \right) = \tilde{z}_i^b \sum_i \left( y_i - \sum_{h \in H_i} g_{ih} \right) = \tilde{z}_i^b \left( \sum_i y_i - \left( \sum_{h \in H_i} g_{ih} \right) \right) = \tilde{z}_i^b \left( \sum_i g_{ih} \right). \]

Since $\tilde{z}_i^b > 0$ and $g_{ih} \geq 0$ for all $i$ and $h \in H_i$, by replacing $\tilde{z}_i^b$ with 0, we can obtain a no smaller objective function value. As a result, we have

\[ \Pi(x(\tau^{II}); \tau^{I}) \leq \Pi(x(\tau^{II}); \tau^{I}) = \tilde{z}_i^b \left( \sum_i g_{ih} \right). \]

Hence, we have that $\tau^I = (0, 0) \in \arg \text{max}_{\tau \in T} V(\tau)$. □

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