Design model for bending vibrations of single-stage tunnel fan rotor

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Abstract. Using of one-mass model of tunnel fan rotor is justified for estimation calculation of the natural bending vibrations frequency during the design stage. It’s shown that the evaluative computation of the main axial tunnel fan at the early design stage yields the acceptable accuracy. It is shown that after completion of the design, the mass of the stepped-type shaft differs from the mass of the calculated uniform-diameter shaft no more than by 40\%. Inclusion of this additional mass in the estimation calculation makes it possible to improve the calculation accuracy. The region of the dimensionless rotor design parameters at which the relative difference of frequency in the evaluative and verification calculations is not higher than 5 \% is determined.

A variety of tunnel fans employed in seven Russian subways tends to increase with the launch of new subway stations into service. Service life of some tunnel fans run down and their replacement or modernization with rotor module replacement is required \cite{1}. The design activity on perfection of tunnel fans is continuous in character. Scientific approach and computation methods consider the effect of ever variable dynamics of aerodynamic ventilation parameters in development and updating of fans to develop new more efficient facilities.

Advance in design of new aerodynamic systems permitted to gain the wanted performance parameters of single-stage fan facilities. The most popular is the driving actuator – transmission shaft – fan shaft structural rotor layout. VO-21 fan developed at Institute of Mining SB RAS is typical of this design structure. Rotor of VO-21 fan is shown in Figure 1.

![Figure 1. Rotor of VO-21 fan: 1 – electromotor; 2, 4 – compensating coupling; 3 – transmission shaft; 5 – front support; 6 – fan shaft; 7 – impeller; 8 – rear support.](image)
Of actual significance are the design techniques to develop tunnel fan rotors for ventilation networks subjected to essential disturbances in consumption and pressure of an air flow under piston–like influence of train traffic.

In design work an important issue is evaluation of the first critical frequency of fan rotor rotation, as resonance bending oscillations used to arise in close approximation to it. This operation mode leads to breakdown of a fan facility or accelerated wear of its parts. That is the reason why it is preferable to prevent operation mode in vicinity of resonance frequencies.

The accurate calculation of the critical frequency implies consideration of a variety of factors and is possible exclusively when all the design parameters of the shaft are known, so after the design of the fan is completed, the checking calculations are performed for verification purposes. It requires a complicated mathematical model including all the required parameters. It is a common practice to employ different numerical methods: finite element method [2], matrix methods (Miclestadt-Prol method) or integral equation methods [3].

Checking calculation of the critical frequency can be also performed by means of discrete single-, dual- or multimass models. This approach enables to obtain an analytical expression for frequency, but it is less precise as compared to numerical calculations. It makes difficult to take into account a number of factors, such as fillets, spline ways, grooves, etc. affecting a critical frequency value. Nevertheless, this calculation method can be sufficient for rigid rotors, viz., for rotors rotating at frequency less than critical, provided that frequency margin is enough large, in other words, the critical frequency is by more than 15% higher than fan rotation frequency [3].

The checking calculation—no matter whether numerical or analytical method is used—requires knowledge of all the design rotor parameters and can be realized only after the shaft design is completed. It can happen that the geometric shaft parameters approved in the design do not give satisfactory results, and the critical frequency appears too close to the operational frequency of the fan facility. In this case the re-design of the fan facility is imperative.

In order to prevent this situation it is preferable to perform an evaluative calculation of the critical frequency at the design stage when the final design parameters of the shaft are not known.

Before a designer gets down to work he is informed on inertia mass specifications of an impeller ($m_2$ is impeller mass, $I$ is axial moment of impeller inertia; $l_2$ is inertia moment of an impeller along axis normal to rotation axis), total length of fan shaft, a location of impeller hub web mounting, impeller supports, namely, lengths of fan shaft sections $l_2$, $l_3$, are known a priori (Figure 2).

The designer determines the minimal diameter of the impeller based on actuator motor capacity and general recommendations on shaft design. Usually the minimum shaft diameter matches bearing journal diameter or is close to it. Then bearings are selected. Next it is feasible to make evaluative calculation of the critical frequency.

In the evaluative calculations for a single–staged tunnel fan it is handy to use the simplest single-mass discrete model (Figure 2). In some cases when length of transmission shaft is large as compared to rotor shaft a two-mass model is applicable [4]. Let determine a range of rotor parameters at which evaluative calculation of the lowest frequency of eigen shaft vibrations on a single – mass model gives satisfactory results. The single – mass model considers impeller mass $m_2$, and a shaft is assumed as a shaft with a constant cross-section of the minimum diameter equal to bearing journal diameter. Besides it is possible to consider rigidity of the front $c_1$ and rear $c_2$ supports and gyroscopic moment on the part of the impeller.
The eigen vibration frequency of the system under consideration disregarding support rigidity and gyroscopic moment is determined from expression [5]:

$$ p_n = \sqrt{\frac{3EJ(l_2 + l_3)}{m_2l_2^2l_3^2}}. \quad (1) $$

To meet the static strength conditions the final design shaft is step–shaped with diameter increasing towards location of impeller hub mounting. As in the evaluative calculations the fan shaft diameter is assumed equal to the minimum diameter, viz., bore diameter of bearing, then rigidity of the ready-made shaft is actually much higher as compared to that of the shaft with the minimum constant diameter.

Analysis of available versions of axial tunnel fans with impeller diameter of 2.1 – 2.4 m reveals that in the final design version the stepwise shaft mass $m_2$ usually differs by no more than 40% from the mass of constant – diameter shaft assumed in the evaluative calculations. For example, since VO-21 fan shaft acquired a multi-staged shape, its mass increased by 23% as compared to the mass of the shaft of the uniform minimum diameter, assumed in the evaluative calculations.

Let assess the effect of shaft mass on a critical frequency value. Available publications report the well–known solution to account for a mass of uniform-cross-section beam under load vibrations after Rayleigh’s method [5]. The shaft mass multiplied by certain factors is added to a localized mass. Eigen vibration frequency in such system is determined from equation:

$$ p_n^* = \frac{\sqrt{3(l_2 + l_3)EJ}}{(m_2 + a\gamma l_2^2 + b\gamma l_3^2)l_2^2l_3^2}, \quad (2) $$

where

$$ a = \frac{1}{3} \frac{(l_2 + l_3)^2}{l_2^3} + \frac{23}{105} \frac{l_2^3}{l_3^3} - \frac{8}{15} \frac{l_2 l_3 (l_2 + l_3)}{l_3^3}, b = \frac{1}{12} \frac{(2l_2 + l_3)^2}{l_2^3} + \frac{1}{28} \frac{l_3^2}{l_2^3} - \frac{1}{10} \frac{l_3 (2l_2 + l_3)}{l_2^3} $$

factors determined from equation for kinetic energy of a vibrating beam; $\gamma$ is linear mass of a shaft, $EJ$ is bending rigidity of the shaft.

Introduce dimensionless parameters:

$$ n = l_3/l_2, \quad k = \frac{m_2}{m_1} = \frac{\gamma}{m_2} l_2 (1 + n), $$

where $m_1$ is a shaft of uniform cross-section.

Then expressions $a$ and $b$ are:

$$ a = \frac{2}{105} \frac{(1 + n)}{n^2} (1 + 29n), \quad b = \frac{2}{15} \frac{n^2}{n^2} + \frac{2}{15} + \frac{1}{3}. $$

**Figure 2.** Calculation scheme of the single-mass model of fan rotor considering moments of impeller inertia and rigidity of supports.
Introducing them into expression (2) gives:

$$p_s = \sqrt{\frac{3l^2(1+n)EJ}{m_s + \frac{2}{105} \left(1 + 29n\right) n^2 km + \left(\frac{2}{105} n^2 + n \frac{2}{15} + \frac{1}{3}\right) \frac{k m_n}{(1+n)}} n^2 l^4}.$$  \hspace{1cm} (3)

Transfer expression (1) with account for dimensionless values:

$$p_1 = \sqrt{\frac{3EJl^2(1+n)}{m_s n^2 l^4}}.$$  \hspace{1cm} (4)

Relative difference of solutions with/without account for shaft mass is:

$$\delta_s = \frac{p_1 - p_s}{p_1} = 1 - \left(1 + \frac{2}{105} \frac{(1 + 29n)}{n^2} k + \left(\frac{2}{105} n^2 + n \frac{2}{15} + \frac{1}{3}\right) \frac{k n}{(1+n)}\right)^{-1}.$$  \hspace{1cm} (5)

This expression is valid at $n > 1$.

Assume if difference $\delta_s$ is less than 5%, shaft mass insufficiently influences a value of the critical rotor frequency. In Figure 3 there is a shaded section in coordinates $n$–$k$ where $\delta_s < 5%$.

**Figure 3.** Rotor parameters domain when a relative difference of frequencies for evaluative and checking calculations does not exceed 5%.

The analysis of equation (3) revealed that to account for an increase in shaft mass it was enough to increase the uniform-cross-section shaft mass $m_s$ by 40% and the resultant frequency would be really lower than an actual one.

Thus, the evaluative calculations for VO-21 fan gave frequency of 36 Hz (2160 rpm). This value exceeds more than twice its operative rotation frequency (1000 rpm). The final design frequency of fan shaft determined in checking calculations was 64 Hz (3840 rpm).

As bearings used to yield to a certain extent, the account for them in calculations leads to reduction in the critical frequency. Flexibility of a discrete bearing can be calculated by the procedure described in [6]. The yielding values established empirically for bearings of certain diameters can be helpful in this case [3]. This process is especially convenient as it is possible to determine the rigidity based on the known bearing diameters, so this process offers undisputable advantages for evaluative calculations.
In earlier publication [4] it is established that yielding of bearings negligibly affects a calculated value of the critical frequency of an axial tunnel fan provided that inertia of impeller turning is disregarded. However in the case when the impeller turning inertia must be taken into account by all means, it is obligatory to account for yielding of supports.

Conclusions

It is demonstrated that at the initial design stage of development of a main axial fan it is feasible to make evaluative calculation of frequency of eigen rotor shaft vibrations based on a single-mass model with acceptable precision. The researchers established the domain of design rotor parameters where relative difference of frequencies of evaluative and checking calculations does not exceed 5%.

Given that in evaluative calculations for a rigid rotor the bending vibration frequency of the shaft differs by more than 15% from the driving force frequency, then the checking calculation can be omitted as the actual difference of frequencies should be higher than 15%.

References

[1] Krasyuk AM 2006 Tunnel Ventilation of Metro Novosibirsk: Nauka (in Russian)
[2] Genta G 2005 Dynamics of Rotating Systems New-York: Springer
[3] Maslov GS 1968 Calculating of Shafts Vibrations Reference book Moscow: Mashinostroyeniye (in Russian)
[4] Kosykh PV and Krasyuk AM 2016 To the method of calculating the critical rotational speed of the main ventilation fan rotor Mining Equipment and Electromechanics No 5 pp 36–42 (in Russian)
[5] Timoshenko SP, Young DH and Weaver W 1974 Vibration problems in engineering NY: Wiley
[6] Beyzelman RD, Tsypkin BV and Perel LY 1975 Rolling Bearings Handbook Moscow: Mashinostroyeniye (in Russian)