The $qqqq\bar{q}$ components and the magnetic moments of the charmed and the bottomed baryons

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Abstract

We give the explicit wave functions of the $qqqq\bar{q}$ components of the $C = +1, J = 1/2$ charmed baryons, $\Sigma_c, \Lambda_c$ and $\Xi^a_c$, and calculate the magnetic moments by adding the 5q components contributions, and we also compute the magnetic moments of the $\Sigma_b$ and $\Sigma^*_b$ baryons. The influence of the additional light and strange $q\bar{q}$ pairs is investigated. As we know, the constituent quark masses of the charm and beauty quarks are much heavier than that of the light and strange quarks, consequently, the hidden flavor contributions to the baryons magnetic moments may be significant.

What is interesting is that the inclusion of the $qqqq\bar{q}$ components contributions leads to different $\Lambda^+_c, \Xi^{a+}_c$ and $\Xi^{a0}_c$ magnetic moments, all of which are predicted to be the same value $0.38\mu_N$ on the basis of the classical $qqq$ quark model. And it’s shown that the differences of these magnetic moments are independent of the constituent mass of the charm quark.
I. INTRODUCTION

Recently, the measurements of the $\bar{d}/\bar{u}$ asymmetry in the nucleon sea indicate a considerable isospin symmetry breaking in the light quark sea of the nucleon. This indicates that the nucleon contains notable $qqq\bar{q}$ components, if no more exotic components, besides the conventional $qqq$ component\([1, 2, 3, 4]\). On the other hand, some experiments on parity violation in electron-proton scattering moreover indicate that the $s\bar{s}$ quark pairs lead to nonzero contributions to the magnetic moment of the proton\([5, 6, 7, 8, 9, 10, 11]\). The experimental results on the strangeness magnetic moments can be described, at least qualitatively, by $uudss\bar{s}$ configurations in the proton with the anti-quark is in its S-state and the four quarks subsystem in its P-state\([12, 13, 14]\). In Ref.\([15]\), taking large admixture of the pentaquark components to the conventional $qqq$ components, a natural explanation for the mass ordering of the $N^*(1440)1/2^+, N^*(1535)1/2^-$ and $\Lambda^*(1405)1/2^-$ resonances has been given, which is very puzzling in the classic $qqq$ quark model.

In Ref.\([16]\), the hidden flavor contributions to the baryon octet and decuplet magnetic moments have been investigated, and the results show that the average deviation from the empirical magnetic moment values drops from $\sim 9\%$ of the static quark model predictions to $\sim 5\%$ when the contributions of the $qqqq\bar{q}$ components are taken into account. The improvement is most notable in the case of the $\Sigma^-(\sim 14\%$ to $\sim 4\%)$, $\Xi^0(\sim 21\%$ to $\sim 3\%)$ and $\Lambda(\sim 10\%$ to $\sim 0\%)$. All these facts above suggest that there are significant $qqqq\bar{q}$ components in baryons.

In the case of the charmed and bottomed baryons, the masses are much heavier than that of the $SU(3)$ octet and decuplet, such as that the mass of the $\Lambda_c(J^{(P)} = \frac{1}{2}^{(+)})$ baryon is about 2286 MeV, the $\Sigma_c(J^{(P)} = \frac{1}{2}^{(+)})$ is about 2455 MeV\([17]\), and the new measured bottomed baryons $m_{\Sigma^+_b} = 5816$ MeV, $m_{\Sigma^0} = 5808$ MeV, $m_{\Sigma^-} = 5837$ MeV and $m_{\Sigma^{*+}} = 5829$ MeV\([18]\), which may result in that the $qqqq\bar{q}$ components play a more important role in the charmed and bottomed baryons. Because of that the charm and beauty quarks are much heavier than the light and strange quarks, the $qqqq\bar{q}$ components, i.e. the hidden light and strange flavors may have much more significant contributions to the baryon magnetic moments. Here the contributions to the $\Lambda_c^+, \Sigma_c$ and $\Xi_c^0$ as well as the $\Sigma_b$ and $\Sigma_b^*$ baryons magnetic moments from the $qqqq\bar{q}$ wave function components with at most one $u\bar{u}, d\bar{d}$ or $s\bar{s}$ sea quark pair are calculated in the non-relativistic constituent quark model. For the sake
of the much heavier masses of the charm and beauty quarks, we neglect the contributions of the $c\bar{c}$ and $b\bar{b}$ components.

What is interesting is that because the light and strange degrees of freedom are in spin zero configuration for the $\Lambda_c$, $\Xi_c^{a+}$ and $\Xi_c^{a0}$ in the classic $qqq$ quark model, all the magnetic moments of these baryons come from the contributions of the charm quark, if the constituent mass of the charm quark is set to be $m_c = 1.7$ GeV, these magnetic moments will be $0.38 \mu_N$. But the magnetic moments of these baryons should be different when the contributions of the $qqqq\bar{q}$ components in these baryons are taken into account, for the different flavor-spin configurations of the 5q components in these baryons. In this manuscript, it’s shown that the differences are independent of the constituent mass of the charm quark.

The present manuscript is organized in the following way. In Section II the explicit flavor wave functions of the $qqqq\bar{q}$ components in the charmed and bottomed baryons are given. Section III contains the expressions and the corresponding numerical results for the charmed baryons. The corresponding expressions and numerical results for the $\Sigma_b$ and $\Sigma_b^*$ magnetic moments are given in section IV. Finally section V contains a concluding discussion of this manuscript.

II. THE WAVE FUNCTIONS OF THE CHARMED AND BOTTOMED BARYONS

A baryon wave function that includes $qqqq\bar{q}$ components in addition to the conventional $qqq$ components may be written in the following general form:

$$|B\rangle = \sqrt{P_{3q}}|qqq\rangle + \sqrt{P_{5q}} \sum_i A_i|qqqq\bar{q}_i\rangle.$$  \hspace{1cm} (1)

Here $P_{3q}$ and $P_{5q}$ are the probabilities of the $qqq$ and $qqqq\bar{q}$ components respectively; the sum over $i$ runs over all the possible $qqqq\bar{q}_i$ components, and $A_i$ denotes the amplitude of the corresponding 5q component. The flavor (and spin) wave functions of every $qqqq\bar{q}_i$ is constructed along with a calculation of the corresponding amplitudes $A_i$.

The wave functions of the $qqq$ components in the baryons are the conventional ones, which are formed as combinations of the color, space, flavor and spin wave functions with appropriate Clebsch-Gordan coefficients. The ground states of the $C = +1$ baryons can be in the spin states $[21]_S$ and $[3]_S$, i.e. the flavor-spin states are $[3]_FS[21]_F[21]_S$ and $[3]_FS[3]_F[3]_S$.
in the static $qqq$ model, respectively. Here we consider the previous ones, $J^p = 1/2^+, \Sigma_c, \Lambda_c$ and $\Xi^a_c$ baryons. The spectra of the $J = 1/2$ charmed baryons, each with one charm quark, in the classic $qqq$ quark model, is just like that of the ones which have one or more strange quarks in the octet. For instance, the $\Lambda_c$ and $\Sigma_c$ spectra ought to look much like that of the $\Lambda$ and $\Sigma$, since a $\Lambda_c$ or a $\Sigma_c$ is obtained from a $\Lambda$ or a $\Sigma$ by changing the $s$ quark to a $c$ quark. But the $\Xi$ baryons have more than one strange quarks, then the $\Xi_c$ spectra is some more complicated than the $\Xi$ spectra. One may group the $\Xi_c$ baryons as that the strange and the light quarks are symmetric or the strange and the light quark are anti-symmetric, i.e. $\Xi^s_c$ and $\Xi^a_c$. We only consider the $\Xi^a_c$ baryons in this manuscript.

As we have discussed above, the wave functions of the $qqq$ components in the charmed baryons are the conventional ones, with flavor-spin symmetry $[^3]_{FS}[21]_F[21]_S$, where $[f]_i$ denotes the Young Pattern with $f$ being the sequence of the integers that indicates the number of boxes in the successive rows of the Young patterns. And this should be combined with the totally symmetric orbital state $[^3]_X$ and the antisymmetric color state $[^111]_C$ to form the completely antisymmetric state which is required by the Pauli exclusion principle.

If the hyperfine interaction between the quarks depends on spin and flavor are employed \[19\], the $qqqq$ subsystem with the mixed spatial symmetry $[^31]_X$ are expected to be the configurations with the lowest energy, and therefore most likely to form appreciable $qqqqb$ components of the charm baryons. Consequently the flavor-spin state of the $qqqq$ subsystem is most likely totally symmetric: $[^4]_{FS}$. Moreover in the case of the $C = +1, J = 1/2$ charmed baryons, the flavor-spin configurations of the four quark subsystem $[^4]_{FS}[22]_F[22]_S$, with one quark in its first orbitally excited state, and the anti-quark in its ground state, is likely to have the lowest energy \[20\]. The flavor wave functions of the $qqqqb$ components in the charmed baryons are listed in Table I. The weight method \[21\] has been employed here for the explicit construction of the flavor wave functions in $qqqqb$ configurations.

Actually, these flavor wave functions can be obtained directly from that in Table III in Ref. \[16\], with the substitution of the charm quark for the valence strange quark. Notice that there is a typo in Table 3 in Ref. \[16\]: the flavor wave function in the fifth row with flavor symmetry $[^22]_F$ should be: $-\sqrt{3}(ddsu\bar{u}) - \sqrt{2}(ddss\bar{s})$. The new feature is the appearance of the states $\Xi^a_c$ and $\Xi^s_c$, here we give the flavor wave functions of $\Xi^a_c$ baryons employing the weight diagram method, where the superscripts `a` on the $\Xi_c$ states indicate that the valence light and strange quarks are in antisymmetric states, the results are shown in Table II.
The flavor wave functions of the $qqqq$ components in the charmed baryons.

| Baryons wave functions                                      |                                                                 |
|-------------------------------------------------------------|------------------------------------------------------------------|
| $\Lambda^+_c$                                              | $-\sqrt{2}\left(\left[udcu\right]_{[22]}_{\rho} \otimes \bar{u}\right) + \left[udcd\right]_{[22]}_{\rho} \otimes \bar{d}\right)$ |
| $\Sigma^{++}_c$                                            | $\sqrt{\frac{2}{3}}\left[uscd\right]_{[22]}_{\rho} \otimes \bar{d} + \sqrt{\frac{1}{3}}\left[uscs\right]_{[22]}_{\rho} \otimes \bar{s}\right)$ |
| $\Sigma^{+}_c$                                             | $-\left(\sqrt{\frac{2}{3}}\left[udcu\right]_{[22]}_{\rho} \otimes \bar{u}\right) - \sqrt{\frac{1}{3}}\left[udcd\right]_{[22]}_{\rho} \otimes \bar{d}\right) + \sqrt{\frac{1}{3}}\left[uuds\right]_{[22]}_{\rho} \otimes \bar{s}\right)$ |
| $\Sigma^{0}_c$                                             | $-\left(\sqrt{\frac{2}{3}}\left[ddcu\right]_{[22]}_{\rho} \otimes \bar{u}\right) - \sqrt{\frac{1}{3}}\left[ddcs\right]_{[22]}_{\rho} \otimes \bar{s}\right)$ |
| $\Xi^{0+}_c$                                               | $-\sqrt{\frac{2}{3}}\left[uusd\right]_{[22]}_{\rho} \otimes \bar{u} + \sqrt{\frac{1}{3}}\left[uuds\right]_{[22]}_{\rho} \otimes \bar{d}\right) - \sqrt{\frac{1}{3}}\left[uuscs\right]_{[22]}_{\rho} \otimes \bar{s}\right)$ |
| $\Xi^{0}_c$                                                | $-\sqrt{\frac{2}{3}}\left[ddcs\right]_{[22]}_{\rho} \otimes \bar{d} + \sqrt{\frac{1}{3}}\left[ddcs\right]_{[22]}_{\rho} \otimes \bar{s}\right)$ |

In the case of the bottomed baryons, $\Sigma_b$ and $\Sigma^*_b$, we can construct the flavor wave functions by the similar way. For the $\Sigma_b$ baryons, $J^p = \frac{1}{2}^+$, the flavor-spin configurations of the $qqq$ components have the symmetry $[3]_{FS}[21]_{FS}[21]_{S}$, while that of the $\Sigma^*_b$, $J^P = \frac{3}{2}^+$, are $[3]_{FS}[3]_{FS}[3]_{S}$, and of course the orbital and color states of all these baryons have the totally symmetric $[3]_X$ and anti-symmetric $[111]_C$ configurations, respectively. For the $qqqq\bar{q}$ components in these baryons, the $qqqq$ subsystem may have the flavor-spin symmetry $[4]_{FS}[22]_{FS}[22]_S$ and $[4]_{FS}[31]_{FS}[31]_S$, respectively, which are expected to have the lowest energy $[20]$, therefore most likely to form notable $qqqq\bar{q}$ components in the baryons. Then we can construct the flavor wave functions of the $qqqq\bar{q}$ components in the $\Sigma_b$ and $\Sigma^*_b$ baryons by employing the weight method $[21]$ or some substitutions of the beauty quark for the valence strange quark in the flavor wave functions of the $\Sigma$ and $\Sigma^*$ baryons in Table I and Table II in Ref. [16], respectively. The results are shown in Table III.

| Baryons wave functions                                      |                                                                 |
|-------------------------------------------------------------|------------------------------------------------------------------|
| $\Sigma_b^+$                                               | $\sqrt{\frac{2}{3}}\left[uuud\right]_{[22]}_{\rho} \otimes \bar{u}\right) + \sqrt{\frac{1}{3}}\left[uuds\right]_{[22]}_{\rho} \otimes \bar{s}\right)$ |
| $\Sigma_b^-$                                               | $-\left(\sqrt{\frac{2}{3}}\left[uudd\right]_{[22]}_{\rho} \otimes \bar{u}\right) - \sqrt{\frac{1}{3}}\left[uuds\right]_{[22]}_{\rho} \otimes \bar{s}\right)$ |
| $\Sigma^*_b^+$                                             | $\frac{\sqrt{2}}{3}\left[uuud\right]_{[31]}_{\rho} \otimes \bar{u}\right) - \frac{\sqrt{1}}{3}\left[uuud\right]_{[31]}_{\rho} \otimes \bar{s}\right) - \frac{\sqrt{1}}{3}\left[uuds\right]_{[31]}_{\rho} \otimes \bar{s}\right)$ |
| $\Sigma^*_b^-$                                             | $\frac{\sqrt{1}}{3}\left[uuud\right]_{[31]}_{\rho} \otimes \bar{u}\right) + \frac{\sqrt{1}}{3}\left[uudd\right]_{[31]}_{\rho} \otimes \bar{s}\right) - \frac{\sqrt{1}}{3}\left[uuds\right]_{[31]}_{\rho} \otimes \bar{s}\right)$ |

Here the flavor-spin decomposition of the states with the mixed symmetry $[4]_{FS}[22]_{FS}[22]_S$.
and \([4]_{FS}[31]_{F}[31]_{S}\) are:

\[
|4 \rangle_{FS}[22]_{F}[22]_{S} = \frac{1}{\sqrt{2}} \left\{ |22 \rangle_{F_{1}}[22]_{S_{1}} + |22 \rangle_{F_{2}}[22]_{S_{2}} \right\}, \tag{2}
\]

\[
|4 \rangle_{FS}[31]_{F}[31]_{S} = \frac{1}{\sqrt{3}} \left\{ |31 \rangle_{F_{1}}[31]_{S_{1}} + |31 \rangle_{F_{2}}[31]_{S_{2}} + |31 \rangle_{F_{3}}[31]_{S_{3}} \right\}. \tag{3}
\]

And the notation of \([4]_{FS}[22]_{F}[22]_{S}\) is a shorthand for the Young tableaux decomposition:

\[
[4]_{FS}[22]_{F}[22]_{S} : \begin{array}{c|c|c|c} \hline & \hline \end{array}_{FS} \begin{array}{c|c} \hline & \hline \end{array}_{F} \begin{array}{c} \hline \end{array}_{S}, \tag{4}
\]

and the notation of the \([4]_{FS}[31]_{F}[31]_{S}\) is:

\[
[4]_{FS}[31]_{F}[31]_{S} : \begin{array}{c|c|c|c} \hline & \hline \end{array}_{FS} \begin{array}{c|c} \hline & \hline \end{array}_{F} \begin{array}{c} \hline \end{array}_{S}. \tag{5}
\]

Above the wave functions of the charmed and bottomed baryons have been given. Furthermore, the explicit forms of the flavor and spin configurations \([22]_{F}, [31]_{F}, [22]_{S}\) and \([31]_{S}\) can be found in Ref. [13]. The orbital and color states of the four quarks subsystem of all the \(qqqq\bar{q}\) components are the mixed symmetry \([31]_{X}\) and \([211]_{c}\), respectively, which are restricted by the Pauli exclusive principle to form a completely anti-symmetric state. The explicit form of the \([31]\) and \([211]\) configurations can be found in Ref. [22]. For instance, we give the explicit wave function of the \(uucd\bar{d}\) component in the \(\Sigma^{++}_{c}\) baryon.

### III. THE MAGNETIC MOMENTS OF THE CHARMED BARYONS

In the non-relativistic quark model, the magnetic moment of a baryon is defined as the expectation value of the following magnetic moment operator in the corresponding baryon state (in units of nuclear magnetons):

\[
\hat{\mu} = \sum_{i} \frac{Q_{i}M_{N}}{e m_{i}} (\hat{l}_{iz} + \hat{s}_{iz}). \tag{6}
\]

Here the sum over \(i\) runs over the quark content of the baryon, and \(Q_{i}\) denotes the corresponding electric charge of the \(ith\) quark and \(m_{i}\) is the constituent quark mass, \(e\) is the electric charge of the proton.

On the other side, with the combination of \(qqq\) and \(qqqq\bar{q}\) state wave functions \([1]\) the magnetic moment will have contributions from the diagonal matrix elements of the operator \([6]\) between the \(qqq\) component and the \(qqqq\bar{q}\) components, respectively, and also from
the off-diagonal terms, i.e. the transition matrix elements between the $qqq$ and the $qqqq\bar{q}$ components. These will depend both on the explicit wave function model and the model for the $q\bar{q} - \gamma$ vertices. If these vertices are taken to have the elementary forms $\bar{v}(p')\gamma u(p)$ and no account is taken of the interaction between the annihilating $q\bar{q}$ pair and the baryon, in the non-relativistic approximation, the transition operator which involve $q\bar{q}$ pair annihilation and creation, may take form in momentum space as follow:

$$\hat{T} = \sum_i Q_i \hat{\sigma}_i , \tag{7}$$

Here the sum over $i$ runs over all the possible $qqqq\bar{q}_i$ components, and $Q_i$ denotes the electric charge of the corresponding quark $q_i$. Note that the annihilation matrix elements of the current operator above just contribute to the non-diagonal term of the magnetic form factor, which will contribute to the magnetic moments in the $q^2 = 0$ limit, for instance, we give the calculations of the transition element between the $uuc$ and $uuc\bar{d}\bar{d}$ components in the $\Sigma^{++}$ baryons in appendix C.

The calculations of these non-diagonal contributions to the magnetic moments call for a specific orbital wave functions model. Here, for simplicity, harmonic oscillator constituent quark model wave functions are employed:

$$\varphi_{00}(\vec{p};\omega) = \frac{1}{(\omega^2\pi)^{3/4}} \exp\{-p^2/2\omega^2\} , \tag{8}$$

$$\varphi_{1m}(\vec{p};\omega) = \sqrt{2} \frac{p_m}{\omega} \varphi_{00}(\vec{p};\omega) . \tag{9}$$

Here $\varphi_{00}(\vec{p};\omega)$ and $\varphi_{1m}(\vec{p};\omega)$ are the s-wave and p-wave orbital wave functions of the constituent quarks, respectively. The oscillator parameters of the $qqq$ and $qqqq\bar{q}$ components, $\omega_3$ and $\omega_5$, will in general be different. The parameter $\omega_3$ may be determined by the baryon radius as $\omega_3 = 1/\sqrt{<r^2>}$. Here we employ the value $\omega_3 \sim 250$ MeV. The parameter $\omega_5$ may be treated as a free phenomenological parameter. In ref.[14] it was noted that the best description of the extant empirical strangeness form factors is obtained with $\omega_5 \sim 1$ GeV, which would imply that the $qqqq\bar{q}$ component is very compact.

With the wave functions and the magnetic moment operators above, the charmed baryons magnetic moments can be expressed as the sum of the diagonal matrix elements in the $qqq$ and $qqqq\bar{q}$ subspaces and the off-diagonal transition matrix elements of the form $qqq \rightarrow qqqq\bar{q}$
and $qqqq \rightarrow qqq$. The former only depend on the group theoretical factors, while the latter also depend on the spatial wave function model. The diagonal contributions to the charmed baryons magnetic moments are listed in table \[III\]

**TABLE III:** The diagonal contributions to the magnetic moments of the charmed baryons.

| Baryons | Magnetic moments |
|---------|-----------------|
| $\Lambda^+_c$ | $P_{3q} \frac{2M_N}{3M_m} + P_{(\Lambda^+_c)u\bar{u}}(\frac{7M_N}{18m} + \frac{M_N}{9m_c}) - P_{(\Lambda^+_c)d\bar{d}}(\frac{M_N}{9m} - \frac{M_N}{9m_c})$ |
| $\Sigma^{++}$ | $P_{3q}(\frac{8M_N}{9m} - \frac{2M_N}{9m_c}) + P_{(\Sigma^{++})u\bar{u}}(\frac{M_N}{18m} + \frac{M_N}{9m_c}) + P_{(\Sigma^{++})s\bar{s}}(\frac{2M_N}{9m} - \frac{M_N}{6m_c} + \frac{M_N}{9m_c})$ |
| $\Sigma^{+}$ | $P_{3q}(\frac{M_N}{9m} - \frac{M_N}{9m_c}) + P_{(\Sigma^+)u\bar{u}}(\frac{7M_N}{18m} + \frac{M_N}{9m_c}) - P_{(\Sigma^+)d\bar{d}}(\frac{M_N}{9m} - \frac{M_N}{9m_c}) + P_{(\Sigma^+)s\bar{s}}(\frac{M_N}{18m} - \frac{M_N}{6m_c} + \frac{M_N}{9m_c})$ |
| $\Sigma^0$ | $-P_{3q}(\frac{4M_N}{9m} + \frac{2M_N}{9m_c}) + P_{(\Sigma^0)u\bar{u}}(\frac{2M_N}{9m} - \frac{M_N}{9m_c}) - P_{(\Sigma^0)s\bar{s}}(\frac{M_N}{9m} + \frac{M_N}{6m_c} - \frac{M_N}{9m_c})$ |
| $\Xi^{++}$ | $P_{3q}(\frac{2M_N}{3M_m} + P_{(\Xi^{++})u\bar{u}}(\frac{4M_N}{9m} - \frac{M_N}{18m_c} + \frac{M_N}{9m_c}) + P_{(\Xi^{++})d\bar{d}}(\frac{M_N}{18m} - \frac{M_N}{18m_c} + \frac{M_N}{9m_c})$ |
| $\Xi^0$ | $+P_{(\Xi^{0})s\bar{s}}(\frac{M_N}{9m} - \frac{2M_N}{9m_c} + \frac{M_N}{9m_c})$ |
| $\Xi^0 \rightarrow \Lambda^+_c - P_{3q} \frac{M_N}{9m} + \frac{1}{4\sqrt{3}} P_{3q} \frac{M_N}{9m}$ |

Here the factors $P_{(B)q\bar{q}_i}$ are the probabilities of the $qqqq\bar{q}_i$ components in the baryon $B$. These are related to the corresponding amplitudes $A_i$ and the probability of the $qqqq\bar{q}$ components \[II\] as:

$$P_{(B)q\bar{q}_i} = P_{5q} A_i^2.$$ (10)

The contributions of the non-diagonal matrix elements are listed in table \[IV\].

The functions $F_{35}(P_{(B)qq})$ in table \[IV\] are defined as

$$F_{35}(P_{(B)qq}) = C_{35} \frac{M_N}{\omega_5} \sqrt{P_{3q}P_{(B)qq}},$$ (11)

where the factor $C_{35}$,

$$C_{35} = \left(\frac{2\omega_3 \omega_5}{\omega_3^2 + \omega_5^2}\right)^{9/2},$$ (12)

is the overlap integral of the s-wave wave functions of the quarks in the $qqq$ and $qqqq\bar{q}$ configurations.

Note that here we have taken all the phase factors for the non-diagonal matrix element between the $qqq$ and $qqqq\bar{q}$ components to be +1.
TABLE IV: The off-diagonal contributions to the magnetic moments of the charmed baryons.

| Baryons   | Magnetic moments                                                                 |
|-----------|----------------------------------------------------------------------------------|
| $\Lambda^+_c$ | $-\frac{1}{4} F^{35}(P_{5q})$                                                   |
| $\Sigma^+_c$ | $-2\sqrt{3} F^{35}(P_{(\Sigma^+_c)_{dd}}) + \frac{\sqrt{2}}{3} F^{35}(P_{(\Sigma^+_c)_{ss}})$ |
| $\Sigma^0_c$ | $\frac{4\sqrt{2}}{3} F^{35}(P_{(\Sigma^0_c)_{uu}}) - \frac{2\sqrt{3}}{3} F^{35}(P_{(\Sigma^0_c)_{ss}})$ |
| $\Xi^{a+}_c$ | $-\frac{2\sqrt{2}}{3} F^{35}(P_{(\Xi^{a+}_c)_{u\bar{d}}}) + \frac{\sqrt{2}}{3} F^{35}(P_{(\Xi^{a+}_c)_{s\bar{s}}})$ |
| $\Xi^{a0}_c$ | $\frac{2}{3} F^{35}(P_{(\Xi^{a0}_c)_{u\bar{u}}}) + \frac{\sqrt{2}}{3} F^{35}(P_{(\Xi^{a0}_c)_{d\bar{d}}}) - \frac{\sqrt{2}}{3} F^{35}(P_{(\Xi^{a0}_c)_{s\bar{s}}})$ |
| $\Sigma^{a+}_c \to \Lambda^+_c$ | $-\frac{2\sqrt{2}}{3} F^{35}(P_{5q})$                                           |

As in the above expressions, we can divided the 5q contributions into the oscillator model independent table III and dependent terms table IV. Furthermore, we can also extract the hidden light and strange flavors contributions to the charmed baryons magnetic moments in the previous expressions, respectively. We can find that in table III the hidden flavor contributions to the charmed baryon magnetic moments are proportional to $P_{(B)q\bar{q}i}$ and inversely proportional to the constituent quark mass, and in table IV they are proportional to the factor $A_{3q} A_{(B)q\bar{q}i}$, which are the amplitudes of the $qqq$ and $qqqq\bar{q}$ components respectively. From Ref. [12, 13, 14, 16], we know that the possibilities of the $qqqq\bar{q}$ components may be $10 - 40\%$, therefore, the hidden flavor contributions to the charmed baryons should be significant.

To obtain the numerical results of the charmed magnetic moments, here the parameters are the masses of the constituent quarks, the oscillator model parameter $\omega_5$, and the probability of the $qqqq\bar{q}$ component. We set the constituent quark masses to be $m_u = m_d = m = 280$ MeV, $m_s = 420$ MeV and $m_c = 1600$ MeV, respectively. The oscillator parameter $\omega_5$ is taken to have the value $0.57 \text{GeV}$ and $1 \text{GeV}$ respectively, which corresponds to a compact extension of the $qqqq\bar{q}$. The value $0.57 \text{ GeV}$ may be the best value for explaining the magnetic moments of the baryon octet, which has been shown in Ref. [16]. The probability of the $qqqq\bar{q}$ components are taken as the tentative value $P_{5q} = 20\%$. The numerical results are shown in Table V comparing with the results from the conventional static quark model, the chiral perturbation theory and the values predicted on the basis of the bound state soliton model.
TABLE V: Magnetic moments of the charmed baryons, the results are in units of nuclear magnetons. The column \( qqq \) contains the results of the conventional quark model from Refs. [24] and column C the results from the chiral perturbation theory [25], in column S the values predicted in Ref. [26] on the basis of the bound state soliton model are given. Column \( P_{ls} \) are the present results from the light and strange quarks contributions, which are the diagonal matrix elements, and \( P_c \) the charm quark contributions. The non-diagonal contributions are listed in column \( P_{n1} \) and \( P_{n2} \) with \( \omega_5 = 0.57 \) GeV and \( \omega_5 = 1 \) GeV, respectively. Finally, the present results are listed in column \( P_1 \) with \( \omega_5 = 0.57 \) GeV and \( P_2 \) with \( \omega_5 = 1 \) GeV.

| Baryons | \( qqq \) | \( P_{ls} \) | \( P_c \) | \( P_{n1} \) | \( P_{n2} \) | \( P_1 \) | \( P_2 \) | \( C \) | \( S \) |
|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| \( \Lambda_c^+ \) | 0.38 | 0.09 | 0.33 | -0.06 | -0.004 | 0.36 | 0.41 | 0.37 | 0.28 |
| \( \Xi_c^{++} \) | 0.38 | 0.06 | 0.33 | -0.08 | -0.006 | 0.46 | 0.39 | 0.42 | 0.28 |
| \( \Xi_c^{0} \) | 0.38 | -0.05 | 0.33 | -0.06 | -0.005 | 0.34 | 0.28 | 0.32 | 0.28 |
| \( \Sigma_c^{++} \) | 2.33 | 2.44 | -0.09 | -0.09 | -0.007 | 2.26 | 2.35 | 2.76 |
| \( \Sigma_c^{+} \) | 0.49 | 0.60 | -0.09 | -0.03 | -0.003 | 0.48 | 0.51 | 0.59 |
| \( \Sigma_c^{0} \) | -1.35 | -1.24 | -0.09 | -0.02 | 0.002 | -1.31 | -1.33 | -1.35 |
| \( \Sigma_c^{+} \rightarrow \Lambda_c^+ \) | -1.59 | -1.45 | 0 | -0.19 | -0.014 | -1.64 | -1.47 |

As we can see in column \( P_1 \) in table V, the magnetic moments of the \( \Xi_c^{a+(0)} \) are different from that of the \( \Lambda_c^+ \) by including the \( qqqq \bar{q} \) components contributions. Especially for that of the \( \Xi_c^{a+} \) baryon, which differ from that of the \( \Lambda_c^+ \) baryon by about 28%. It’s interesting that the average value of the \( \Xi_c^{a+} \), \( \Xi_c^{0} \) and \( \Lambda_c^+ \) baryons magnetic moments is about 0.39\( \mu_N \), it’s consistent with the value 0.38\( \mu_N \) of the classic static quark model, and it’s also in excellent agreement with the chiral perturbation theory value 0.37\( \mu_N \) [25]. In Ref. [25], the leading long-distance contributions to the magnetic moments of the \( \Lambda_c^+ \), \( \Xi_c^{a+} \) and \( \Xi_c^{0} \) charmed baryons from the spin symmetry breaking \( \Sigma_c^+ - \Sigma_c \) mass splitting in chiral perturbation theory are computed, these are nonanalytic in the pion mass and arise from calculable one-loop graphs. From that, the differences of the \( \Lambda_c^+ \), \( \Xi_c^{a+} \) and \( \Xi_c^{0} \) magnetic moments are independent of the charm quark mass. As shown in table III and IV, the charm quarks contributions from the diagonal terms to the \( \Lambda_c^+ \), \( \Xi_c^+ \) and \( \Xi_c^0 \) magnetic moments are same in our results, which means that the difference of these magnetic moments only comes from the contributions of the light and strange quark. Consequently, the differences of these magnetic moments
are also independent of the charm quark mass for that all the off-diagonal terms are quark masses independent. The similar conclusion may come from that both of the two method are based on the contributions of the sea quark. In column $S$, the results of Ref. [26] are given, which are similar to that of the non-relativistic constituent $qqq$ quark model.

The magnetic moment of $\Lambda_c^+$ is not very sensitive to the proportion of the $qqqqq$ components, the others vary a bit with $P_{5q}$ changed. For instance, if $P_{5q}$ increase to the value $P_{5q} = 0.50$, the parameter $\omega_5$ is set to be 0.57 GeV, these magnetic moments will be $\mu_{\Lambda_c^+} = 0.35$ n.m. , $\mu_{\Xi_c^{a+}} = 0.54$ n. m. and $\mu_{\Xi_c^{a0}} = 0.23$ n. m..

These magnetic moments have also been calculated in the relativistic quark model in Ref. [27], a bit corrections to the results of the non-relativistic constituent quark model have been given. As we can see in Ref. [27], a bit difference for the magnetic moments of the $\Lambda_c^+$, $\Xi_c^{a+}$ and $\Xi_c^{a0}$ can be also obtained in the relativistic quark model. The bound state approach is employed to calculate these magnetic moments in Ref. [28]. In that paper, they give a smaller value for the $\Lambda_c$ baryon, $\mu_{\Lambda_c^+} = 0.12$ n. m..

IV. THE MAGNETIC MOMENTS OF THE $\Sigma_b$ AND $\Sigma_b^*$ BARYONS

In the case of the bottomed baryons $\Sigma_b$, the lowest $qqqqq\bar{q}$ configuration are also expected to be that the $qqqq$ subsystem is assumed to have [4] $F_S$ [22] $F_S$ mixed symmetry, the flavor wave functions are shown in Table [II]. The matrix elements of the magnetic moment operator [6] in these wave functions, and the corresponding $qqq$ wave functions lead to the following diagonal contributions to the magnetic moments are listed in table [VI].

| TABLE VI: The diagonal contributions of the magnetic moments of the bottomed moments. |
|---------------------------------------------------------------|
| Baryons magnetic moments                                      |
| $\Sigma_b^+$                                                 |
| $P_{3q}(\frac{4M_N}{3M_s} + \frac{M_s}{3M_N}) + P_{(\Sigma_b^+)}dd(\frac{M_N}{15m} - \frac{M_s}{15m}) + P_{(\Sigma_b^+)}ss(\frac{2M_N}{9m} - \frac{M_s}{9m} - \frac{M_N}{15m})$ |
| $\Sigma_b^-$                                                 |
| $-P_{3q}(\frac{4M_N}{3M_s} - \frac{M_s}{3M_N}) + P_{(\Sigma_b^-)}uu(\frac{2M_N}{9m} - \frac{M_s}{9m}) - P_{(\Sigma_b^-)}ss(\frac{M_N}{9m} + \frac{M_s}{9m} + \frac{M_N}{15m})$ |
| $\Sigma_b^+$                                                 |
| $P_{3q}(\frac{4M_N}{3M_s} - \frac{M_s}{3M_N}) + P_{(\Sigma_b^+)}uu(\frac{2M_N}{9m} + \frac{M_s}{24m_N}) + P_{(\Sigma_b^+)}dd(\frac{2M_N}{9m} + \frac{M_s}{24m_N})$ |
| $+P_{(\Sigma_b^+)}ss(\frac{11M_N}{18m} + \frac{11M_s}{24m_N} + \frac{11M_N}{24m_N})$ |
| $\Sigma_b^-$                                                 |
| $P_{3q}(-\frac{2M_N}{3m} - \frac{M_s}{3M_N}) + P_{(\Sigma_b^-)}uu(-\frac{2M_N}{3m} + \frac{M_s}{24m_N}) + P_{(\Sigma_b^-)}dd(-\frac{M_N}{24m_N} + \frac{M_s}{24m_N})$ |
| $+P_{(\Sigma_b^-)}ss(-\frac{11M_N}{18m} + \frac{11M_s}{24m_N} + \frac{11M_N}{24m_N})$ |
And the non-diagonal matrix elements are listed in table VII.

**TABLE VII:** The off-diagonal contributions of the magnetic moments of the bottomed moments.

| Baryons magnetic moments |  |
|-------------------------|--|
| $\Sigma^+_b$           | $-\frac{2\sqrt{3}}{9}F_{35}(P_{(\Sigma^+_b)dd}) - \frac{2\sqrt{3}}{9}F_{35}(P_{(\Sigma^+_b)ss})$ |
| $\Sigma^-_b$           | $\frac{4\sqrt{3}}{9}F_{35}(P_{(\Sigma^-_b)u\bar{u}}) - \frac{2\sqrt{3}}{9}F_{35}(P_{(\Sigma^-_b)ss})$ |
| $\Sigma^{*-}_b$        | $\frac{\sqrt{3}}{6}F_{35}(P_{(\Sigma^{*-}_b)u\bar{u}}) + \frac{1}{12}F_{35}(P_{(\Sigma^{*-}_b)dd}) + \frac{1}{12}F_{35}(P_{(\Sigma^{*-}_b)ss})$ |
| $\Sigma^{*+}_b$        | $\frac{\sqrt{3}}{6}F_{35}(P_{(\Sigma^{*+}_b)u\bar{u}}) - \frac{\sqrt{3}}{12}F_{35}(P_{(\Sigma^{*+}_b)dd}) + \frac{1}{12}F_{35}(P_{(\Sigma^{*+}_b)ss})$ |

For the $\Sigma^*_b$ baryons, the lowest energy $qqqq\bar{q}$ configurations are expected to be that the $qqqq$ subsystem is assumed to have the $[4]_{FS}[31]_F[31]_S$ mixed symmetry, the flavor wave functions are also shown in Table II. And the diagonal matrix elements are listed in table VI and the non-diagonal elements are listed in table VII.

Note that the $qqqq$ subsystem of the $qqqq\bar{q}$ components in the $\Sigma^*_b$ baryons can have both total angular momentum $J = 1$ and $J = 2$, which are required by the spin 3/2 of the baryons. Here have assumed that $J = 1$. With 20% proportion of the $qqqq\bar{q}$ components assumed in the $\Sigma_b$ and $\Sigma^*_b$ baryons, the above expressions lead to the numerical results in Table VIII compared with the results from only the $qqq$ components. Here we set the constituent mass of the beauty quark to be $m_b = 5.0$ GeV. As it’s shown in Table VIII, the influence of the light and strange $q\bar{q}$ pairs is significant.

The numerical results will vary a bit with the proportion of the $qqqq\bar{q}$ components changed. For instance, if we increase it to the value $P_{3q} = 0.50$, these magnetic moments will be $\mu_{\Sigma^+_b} = 1.54$ n. m. , $\mu_{\Sigma^-_b} = -0.84$ n. m., $\mu_{\Sigma^{*-}_b} = 3.91$ n. m. and $\mu_{\Sigma^{*+}_b} = -1.90$ n. m., which means that the increasing of the $P_{3q}$ will make the absolute value of these magnetic moments decreased.

As in Ref. 25, they give the prediction that the magnetic moments of the b-baryons in the 3 of $SU(3)$ can be obtained by the same analysis as the c-baryons in 3, which means that some correction will be added to the value $-\frac{1}{3}m_b$ from the constituent quark model , which will lead to three different magnetic moments. Here if we take into account the contributions of the $qqqq\bar{q}$ components, we can also get the same prediction, and the difference of the three magnetic moments should be independent with constituent mass of the bottom quark.
TABLE VIII: Magnetic moments of the Σ_b and Σ_b^* baryons, the results are in units of the nuclear magnetons. Column qqq contains the results of that from only the qqq components. Column P_{ls} are the present results from the light and strange quarks contributions, which are the diagonal matrix elements, and P_b the beauty quark contributions. The non-diagonal contributions are listed in column P_{n1} and P_{n2} with ω_5 = 0.57 GeV and ω_5 = 1 GeV, respectively. Finally, column A contains the results including the contributions of the qqqq̄q components with ω_5 = 0.57 GeV and column B the values with ω_5 = 1 GeV. Here we set the constituent mass of the beauty quark to be m_b = 5.0 GeV.

| baryons | qqq | P_{ls} | P_b | P_{n1} | P_{n2} | A   | B   |
|---------|-----|--------|-----|--------|--------|-----|-----|
| Σ^+_b  | 3.00| 2.44   | 0.01| -0.09  | -0.007 | 2.37| 2.45|
| Σ^-_b  | -1.47| -1.24  | 0.01| 0.02   | 0.002  | -1.20| -1.22|
| Σ^+_b^*| 4.40| 4.23   | -0.05| 0.05   | 0.004  | 4.23| 4.18|
| Σ^-_b^*| -2.30| -2.10  | -0.05| 0.01   | 0.001  | -2.13| -2.14|

V. CONCLUSIONS

Here the charmed and bottomed baryons magnetic moments have been computed within the framework of the non-relativistic constituent quark model, and the influence of the light and strange q̄q pairs has been investigated. In addition explicit expressions of the wave functions for all the possible qqqq̄q components in the charmed and bottomed baryons are given. The calculated magnetic moments include the contributions from the qqq components and the qqqq̄q components with both of light and strange q̄q pairs. The magnetic moment expressions readily allow separation of the strangeness components from s̄s pairs as well as individual components from ūu and d̄d pairs.

Our main conclusion is that, the qqqq̄q components, i.e. the hidden light and strange flavors, have notable contributions to the charmed baryons magnetic moments. Of course, the additional qqqq̄q components contributions also lead to notable corrections to the bottomed baryons magnetic moments. The numerical results are shown in Table V and VIII with a ∼ 20% admixtures of the qqqq̄q components in the baryons.

As we have discussed in Ref. [16], the restriction in the calculation of the charmed baryon and Σ_b magnetic moments to the configuration with flavor-spin symmetry [4]_F_S [22]_F [22]_S given in Table II and III is motivated by its expected low energy. The configuration with
[4] F S [31] F [31] S symmetry is expected to have the next lowest energy and to give an at most very insignificant contribution to the magnetic moments. In the case of the Σ∗ b this configuration is, however, expected to have the lowest energy as the configuration [4] F S [22] F [22] S cannot contribute [16].

It is of course not in any way obvious that those \( q\bar{q}q\bar{q} \) configurations that have the lowest energy for a given hyperfine interaction model should have the largest probability in the baryons. The main terms should be expected to be those with the strongest coupling to the \( q\bar{q}q \) component. This coupling depends both on the confining interaction and (inversely) on the difference in energy from the rest energy of the baryons.

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APPENDIX A: THE EXPLICIT WAVE FUNCTIONS OF THE \( uuc \) AND \( uucd\bar{d} \) COMPONENTS IN THE \( \Sigma^{++}_c \) BARYON

The wave function of the \( uuc \) component is the traditional one, and that of the \( uucd\bar{d} \) component may be expressed as

\[
|\Sigma^{++}_c, +1/2\rangle_{5q} = \sqrt{\frac{1}{2}} \sum_{ab} \sum_{ms} C^{1/2}_{1m, \frac{1}{2}s} C^{[1]}_{[31]a[211]a} \left[ [31]_X, m(a) \right] [22]_F(b)[22]_S(b)[211]_C(a) \chi_s \psi(\{\vec{p}_i\}).
\]

(A1)

Here the color, space and flavor-spin wave functions of the \( qqqq \) subsystem have been denoted by their Young patterns, respectively, the sum over \( a \) runs over the three configurations of the \( [211]_C \) and \( [31]_X \) representations of \( S_4 \), and the sum over \( b \) runs over the two configurations of the \( [22] \) representation of \( S_4 \) respectively, and \( C^{[1]}_{[31]a[211]a} \) denotes the \( S_4 \) Clebsch-Gordan
coefficients of the representations $[1111][31][211]$, $\chi_{s_i}$ is the spin state of the anti-quark.

The explicit color-space part of the wave function (A1) can be expressed in the form

$$\psi_C(\{\vec{p}_i\}) = \frac{1}{\sqrt{3}} \{[211]c_3[31]x_1 - [211]c_2[31]x_2 + [211]c_1[31]x_3\} \varphi_0(\vec{p}_5)$$  \hspace{1cm} (A2)

where the explicit forms of the $[31]_x_i$ are defined as

$$| [31]_x_1 > = \frac{1}{\sqrt{12}} [3|0001\rangle - |0010\rangle - |0100\rangle - |1000\rangle] , \hspace{1cm} (A3)$$

$$| [31]_x_2 > = \frac{1}{\sqrt{6}} [2|0010\rangle - |0100\rangle - |1000\rangle] , \hspace{1cm} (A4)$$

$$| [31]_s_3 > = \frac{1}{\sqrt{2}} [0100\rangle - |1000\rangle] . \hspace{1cm} (A5)$$

Here the numbers '0' and '1' denote the ground and excited orbital states of the corresponding quark, for instance,

$$|0001\rangle = \varphi_0(\vec{p}_1) \varphi_0(\vec{p}_2) \varphi_0(\vec{p}_3) \varphi_{1m}(\vec{p}_4) \hspace{1cm} (A6)$$

The flavor-spin configuration of the $uuc$ component in the $\Sigma^{++}_c$ baryon has the symmetry $[3]_{FS}[21]_F[21]_S$, which can be expressed as:

$$| [3]_{FS}[21]_F[21]_S \rangle = \frac{1}{\sqrt{2}} ([|21]_F[1]|[21]_s_1) + [|21]_F[2]|[21]_s_2) \hspace{1cm} (A7)$$

The explicit forms of $[21]_F[1]$, $[21]_s_1$, $[21]_F[2]$ and $[21]_s_2$ are:

$$| [21]_F[1] \rangle = \sqrt{\frac{1}{6}} (|ucu\rangle + |cuu\rangle - 2|uuc\rangle) \hspace{1cm} (A8)$$

$$| [21]_s_1 \rangle = \sqrt{\frac{1}{6}} (|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle) \hspace{1cm} (A9)$$

$$| [21]_F[2] \rangle = \sqrt{\frac{1}{2}} (|ucu\rangle - |cuu\rangle) \hspace{1cm} (A10)$$

$$| [21]_s_2 \rangle = \sqrt{\frac{1}{2}} (|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle) \hspace{1cm} (A11)$$

The flavor-spin configuration of the $uucd$ subsystem of the $uucdd\bar{d}$ component is $[4]_{FS}[22]_F[22]_S$, which can be expressed as:

$$| [4]_{FS}[22]_F[22]_S \rangle = \frac{1}{\sqrt{2}} ([|22]_F[1]|[22]_s_1) + [|22]_F[2]|[22]_s_2) \hspace{1cm} (A12)$$

here the CG coefficient $\frac{1}{\sqrt{2}}$ is just the same one in equation (A1). The explicit forms of $[22]_F[1]$, $[22]_s_1$, $[22]_F[2]$ and $[22]_s_2$ are:

$$| [22]_F[1] \rangle = \frac{1}{\sqrt{24}} (|2uucd\rangle + 2|uucd\rangle + 2|dcuu\rangle + 2|cduu\rangle - |duuc\rangle - |uduc\rangle$$
\[ -|cudu\rangle - |ucdu\rangle - |cuud\rangle - |ducu\rangle - |ucud\rangle - |udcu\rangle \]  \hspace{1cm} (A13)

\[ |[22]_{S_{1}}\rangle = \frac{1}{\sqrt{12}} |[^2 \uparrow \uparrow \downarrow \downarrow] + 2[^2 \downarrow \downarrow \uparrow \uparrow] - [^2 \downarrow \uparrow \uparrow \downarrow] - [^2 \uparrow \downarrow \uparrow \downarrow] - [^2 \uparrow \uparrow \uparrow \downarrow] - [^2 \uparrow \uparrow \downarrow \uparrow] \]  \hspace{1cm} (A14)

\[ |[22]_{F_{2}}\rangle = \frac{1}{\sqrt{8}} [uduc\rangle + |cudu\rangle + |ducu\rangle - |duuc\rangle - |ucdu\rangle - |cuud\rangle] \]  \hspace{1cm} (A15)

\[ |[22]_{S_{2}}\rangle = \frac{1}{2} ([^1 \uparrow \downarrow \uparrow \downarrow] + [^1 \downarrow \uparrow \downarrow \uparrow] - [^1 \downarrow \uparrow \uparrow \downarrow] - [^1 \uparrow \downarrow \uparrow \uparrow]) \]  \hspace{1cm} (A16)

**APPENDIX B: THE DIAGONAL CONTRIBUTION OF THE \textit{uu}cc\overline{d} COMPO-ONENT**

Here we have given the explicit wave functions of the \textit{uu}c and \textit{uu}c\overline{d} components. In equation (6), the sum runs over all the quark and anti-quark content in the \textit{uu}c\overline{d} component. The four-quark subsystem has no contribution to the expectation value of the operator \(\sigma_{iz}\) since the total spin is \(S = 0\) for the \([22]_{S}\) symmetry. Consequently, it only contribute to the expectation value of the operator \(l_{iz}\). For the fourth quark, the contribution is:

\[
\frac{2}{3} < \Psi_{C} |l_{4z}| \Psi_{C} > < [4]_{FS} \frac{Q_{4} M_{N}}{em_{4}} |[4]_{FS} >
\]  \hspace{1cm} (B1)

Here the factor \(\frac{2}{3}\) is the square of the Clebsch-Gordan coefficient \(C_{11,\frac{1}{2},\overline{1}}^{1,\frac{1}{2},\overline{1}}\), where \(-1/2\) is the z-component of the anti-quark spin. After some calculations, we can get

\[
< \Psi_{C} |l_{4z}| \Psi_{C} > = \frac{1}{4}
\]

\[
< [4]_{FS} \frac{Q_{4} M_{N}}{em_{4}} |[4]_{FS} > = \frac{1}{4} \left( \frac{2M_{N}}{3m_{c}} + \frac{M_{N}}{m} \right)
\]  \hspace{1cm} (B2)

so the contributions of the four-quark subsystem should be:

\[
\mu_{uuu} = \frac{1}{6} \left( \frac{2M_{N}}{3m_{c}} + \frac{M_{N}}{m} \right)
\]  \hspace{1cm} (B3)

The contribution of the fifth quark, which refers to the anti-quark, is

\[
\mu_{\overline{d}} = \frac{Q_{4} M_{N}}{em_{4}} \left( [C_{11,\frac{1}{2},\overline{1}}^{1,\frac{1}{2},\overline{1}}]^{2} \left( \frac{1}{2}, - \frac{1}{2}, \frac{1}{2}, - \frac{1}{2} \right) + [C_{10,\frac{3}{2},\overline{1}}^{1,\frac{3}{2},\overline{1}}]^{2} \left( \frac{1}{2}, + \frac{1}{2}, \frac{1}{2}, - \frac{1}{2} \right) \right)
\]

\[
= - \frac{M_{N}}{9m}
\]  \hspace{1cm} (B4)

At last, the magnetic moment of the \textit{uu}c\overline{d} component should be

\[
\mu = \frac{1}{6} \left( \frac{2M_{N}}{3m_{c}} + \frac{M_{N}}{m} \right) - \frac{M_{N}}{9m}
\]

\[
= \left( \frac{M_{N}}{18m} + \frac{M_{N}}{9m_{c}} \right)
\]  \hspace{1cm} (B5)
APPENDIX C: THE TRANSITION ELEMENT BETWEEN THE \( uuc \) AND \( uucd \bar{d} \) COMPONENTS

As we know, on the hadron level, the baryon magnetic form factor can be expressed as the matrix element of the vector current operator in the following way (in Breit frame) \[29\]:

\[
\langle P' | \vec{J}^{em} | P \rangle = \bar{u}(\vec{P}') \vec{\gamma} u(\vec{P}) G_M(q^2)
\]

\[
\langle P' | J_{0}^{em} | P \rangle = \bar{u}(\vec{P}') \gamma_0 u(\vec{P}) G_M(q^2)
\]

(C1)

Here we get the relations between the electro-magnetic form factors of the baryons and the matrix elements of the current operators. In the case of the magnetic form factor, we only need to consider the first one. Assuming that both of the initial and final spin states are \( |1/2, \uparrow \rangle \), and \( \vec{q} = \vec{P}' - \vec{P} = q\vec{j} \), in the non-relativistic approximation, the matrix element \( \langle P' | \vec{J}^{em} | P \rangle \) will be

\[
\langle P' | \vec{J}^{em} | P \rangle = -\frac{i}{2M_B} q \hat{x} G_M(q^2)
\]

(C2)

here \( M_B \) denotes the mass of the baryon. Note that the vector factor will be canceled in the final result. On the quark level, taking the \( qqqq \bar{q} \) components into account, the transition matrix element \( qqqq\bar{q}_i \rightarrow qqq\gamma \) will also contribute to the baryon magnetic form factor. The transition operator takes the following form:

\[
\hat{T}_i = Q_i \langle \vec{p}'_i | \vec{J_i} | \vec{p}_i \rangle_{anni},
\]

\[
= Q_i \hat{\sigma}_i.
\]

(C3)

Analogous situation of the roper resonance decays to \( N\gamma \) has been considered in Ref. \[30\]. As in equation (C2), we only need to calculate the x-component of the matrix elements of the operator (C3). Note that the transition operator here applies for all frames.

For the case of the \( uucd \bar{d} \rightarrow uuc\gamma \) transition, the calculation of the matrix element of the operator (C3) involves calculation of the overlap of the \( uuc \) component with the residual \( uuc \) component that is left in the \( uucd \bar{d} \) component after the annihilation of a \( d\bar{d} \) pair into a photon.

First, from equations (A7)-(A16), we can obtain that the flavor-spin overlap factor may be

\[
C_{FS} = \frac{\sqrt{2}}{4}
\]

(C4)
The following step is the calculation of orbital matrix element. This may be cast in the form

$$C_O = \langle \varphi_{00}(\vec{p}_1')\varphi_{00}(\vec{p}_2')\varphi_{00}(\vec{p}_3')|111]C|\delta(\vec{p}_1 - \vec{p}_1)\delta(\vec{p}_2 - \vec{p}_2)\delta(\vec{p}_3 - \vec{p}_3)\delta(\vec{p}_4 + \vec{p}_5 - \vec{q})|\psi_C(\{\vec{p}_i\})\rangle,$$

(C5)

The operator (C3) confines that the anti-quark should be in the spin state $|1/2, \downarrow\rangle$, to lead to a nonzero matrix element: $\langle \bar{s}_z|\hat{\sigma}_x|1/2, \downarrow\rangle = -1$, consequently, the CG coefficient $C_{1/2, 1/2}^{1/2, 1/2}$ in equation (A1) can only be the one $C_{11, 1/2, -1/2}^{11, 1/2, -1/2} = \sqrt{\frac{2}{3}}$.

Note that only the orbital symmetry configuration of the $uuc\bar{d}$ component that is described by [31] in equation (A2) leads to a nonzero matrix element, when multiplied with the totally symmetric orbital state of the $uuc$ component upon annihilation of the fourth quark with the fifth anti-quark in the $uuc\bar{d}$ component. Consequently, we can obtain

$$C_O = \frac{3}{\sqrt{12}}\varphi_{00}(\vec{p}_1')\varphi_{00}(\vec{p}_2')\varphi_{00}(\vec{p}_3')|111]C|\delta(\vec{p}_1 - \vec{p}_1)\delta(\vec{p}_2 - \vec{p}_2)\delta(\vec{p}_3 - \vec{p}_3)\delta(\vec{p}_4 + \vec{p}_5 - \vec{q})|\frac{1}{\sqrt{3}}\times 14  
= \frac{1}{4}C_{35}\frac{i q}{\omega_5}\exp[-\frac{q^2}{4\omega_5^2}]$$

(C6)

Here we have obtained all the overlap factors, but we must notice that the $d$ quark which annihilates with the anti-quark may also be one of the first three quarks, for the $S_4$ symmetry, we only need to multiply a factor 4 to the overlap factors we have obtained. On the other side, the process $uuc\gamma \rightarrow uuc\bar{d}$ may contribute a same value as we have obtained, so there is another factor 2. Taking all the factors into account, and eliminating the factor $-\frac{i}{2M_{\Sigma^+}}q\hat{x}$ which has been shown in equation (C2), we can obtain

$$C_{anni}^{\Sigma^+} = -\frac{2\sqrt{3}}{9}C_{35}\sqrt{P_{3q}P_{(\Sigma^+^{++})dd}}\frac{M_{\Sigma^+}}{\omega_5}\exp[-\frac{q^2}{4\omega_5^2}]$$

(C7)

The result is in units of the $\frac{1}{2M_{\Sigma^+}}$. In the $q^2 \rightarrow 0$ limit, it will contribute to the baryon magnetic moment. In the traditional way, we can express the result in units of the nuclear magnetons as follow:

$$\mu_{\Sigma^+} = -\frac{2\sqrt{3}}{9}F_{35}(P_{(\Sigma^+^{++})dd})$$

(C8)