BI-LEVEL MULTIPLE MODE RESOURCE-CONSTRAINED PROJECT SCHEDULING PROBLEMS UNDER HYBRID UNCERTAINTY

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ABSTRACT. This study focuses on the multi-mode resource-constrained projects scheduling problem (MRCSP), which considers the complex hierarchical organization structure and hybrid uncertainty environment in the decision making process. A bi-level multi-objective MRCSP model with fuzzy random coefficients and bi-random coefficients is developed for the MRCSP. In the model, construction contractor, the upper level decision maker (ULDM), aims to minimize the consumption of resources and maximize the quality level of project. Meanwhile, outsourcing partner, the lower level decision maker (LLDM), tries to schedule the activities under resource allocation with the objective of minimizing the total tardiness penalty cost. To deal with the uncertainty variables, the fuzzy random parameters are transformed into the trapezoidal fuzzy variables, which are de-fuzzified by the expected value subsequently. For the bi-random parameters, the expected value operator is employed. After obtaining the equivalent crisp model, the passive congregation-based bi-level multiple objective particle swarm optimization algorithm (PC-based BL-MOPSO) is designed to obtain the Pareto solutions. Finally, a practical application is presented to verify the practicability of the proposed bi-level multi-objective MRCSP model and the efficiency of algorithm.

1. Introduction. Clean and renewable energy, such as hydropower resources, has become more important since the demand of energy in China is increasing according to the rapid development. Chinese government has emphasized the renewable energy development, especially in the areas of water conservancy and hydropower [1]. In this situation, the southwestern China, which has abundant hydropower resources (ranking in the first place in China), is the key areas. The Bureau VIII of China Water Conservancy and Hydropower Engineering (IV-CWCHE) is appointed to supply clean energy to the Sichuan-Chongqing region and the east-west electricity transmission in China through the development of hydropower resources on the Jinsha
River. The Xiluodu Hydropower Project, which is a large-scale water conservancy and hydropower construction project located on the downstream of Jinsha River, is one of the IV-CWCE’s projects being construction. Taking Subsidiary Dam Construction (contract No. XLD/0889) (a part of Xiluodu Hydropower Project) as a case, this study focuses on the practical bi-level multi-mode resource-constrained project scheduling problems (MRCPSP) in the Xiluodu Hydropower Project.

For the real-life resource-constrained project scheduling problems (RCPSP), it is possible to perform the activities in alternative ways (multiple modes), so lots of RCPSP can be modeled as multi-mode resource-constrained project scheduling problems (MRCPSP). In MRCPSP, each mode of an activity represents an alternative relation between the resource requirements and its corresponding duration (Zhang and Tam, 2006 [44]). Actually, since the first study of Elmaghraby (1977) [12] for multiple modes in networks-project planning, MRCPSP has attracted increasing attention from theory and practice. Basically, there is an implicit assumption that the decision-maker is unique in the most MRCPSP literatures. However, since the scale of project is usually large, complex hierarchical organization structure and more related elements should be considered in the scheduling process of MRCPSP, especially for a practical one. Unfortunately, the studies about applying multi-level programming to project scheduling problems are very few. Kopanos et al. (2009) [23] presented a two-layered decomposition methodology to the resolution of difficult batch scheduling problems arising in chemical production facilities. Peng et al. (2010) [32] developed a bi-level model to maintenance fund allocation and project prioritization, and designed genetic algorithm-based dynamic programming as the solution method. Kovács and Kis (2011) [22] considered the bi-level scheduling problem where the manager of the company (the leader) was responsible for order acceptance and the workshop foreman (the follower) decided on the execution sequence of the tasks corresponding to accepted orders. Xu and Gang (2013) [39] investigated a transportation scheduling problem by multi-objective bi-level programming. Gan and Xu (2013) [14] proposed a bi-level decision framework based on a risk assessment program and demonstrated how this could be extended to enhance multimode resource-constrained project-scheduling problem to realize risk control and effective scheduling. In this paper, we will take up a challenge to depict the characteristic hierarchical organization structure in MRCPSP by bi-level programming.

In MRCPSP, however, uncertainty is an inevitable issue for the project managers when developing the robust schedule in practice. Construction projects, for instance, are amongst others subject to disruptions caused by accidents, resource breakdowns, bad weather conditions, unreliable deliveries and unreliable subcontractors (Lambrechts et al., 2008 [27]). Traditionally, the uncertainty in project scheduling is assumed to be stochastic. In [13], Freeman first applied probability theory in the project scheduling problem. After that, many scholars considered randomness in the scheduling process, such as activity durations (Ke and Liu, 2005 [18]; Zhu et al., 2007 [49]; Klerides and Hadjiconstantinou, 2010 [21]; Deblaere et al., 2011 [11]), task costs (Choi and et al. 2004, [10]; Aytug et al. 2005, [5]), resource consumption and availabilities (Lambrechts et al., 2008 [26]; Keller and Bayraksan, 2009 [19]) and so on. Expect for the stochastic factor mentioned above (objective uncertainty) which is from external cause (such as treacherous weather and equipment failure), the other one is from internal (such as the perception and dissension of decision makers) in the scheduling process, i.e., subjective uncertainty, and it can
be cope with fuzzy theory. Following the study of Prade (1979) [34], which is first applied fuzzy theory to the project scheduling problems in 1979, many researchers considered the fuzzy factors in MRCPSP, such as Chen et al. (2011) [9], Bhaskar et al. (2011) [7], Atli and Kahraman (2012) [4]. However, the objective and subjective uncertainty are considered as separate aspects in the above literatures. Actually, we may face a hybrid uncertain environment in the practical MRCPSP. Hence, the fuzzy random variable and bi-random variable, which was initialized by Kwakernaak (1978) [25] and Peng and Liu (2007) [33], can be the useful tools which are able to deal with the complex uncertainties. So far, no attempt has been made on considering complex hierarchical organization structure and hybrid uncertainty environment in the MRCPSP, hence, there is a strong signification for this study.

This paper will solve MRCPSP under hybrid uncertainty by the bi-level programming. The remainder of the paper is organized as follows. In section 2, problem statement is presented, including the explanation of motivation for employing the fuzzy random variables and bi-random variables. Then a bi-level multi-objective MRCPSP model is developed in section 3. The details of dealing with the uncertain variables are presented in section 4 to obtain the equivalent crisp model. In section 5, passive congregation-based bi-level multiple objective particle swarm optimization algorithm (PC-based BL-MOPSO) is designed to resolve the bi-level MRCPSP model. By the practical application, the practicability and efficiency of proposed model and algorithm are verified in section 6. Concluding remarks and the discussion about further research are made in section 7.

2. Research problem and statement. The problem considered in this study is from the Xiluodu Hydropower Project, which is a large-scale water conservancy and hydropower construction project located on the downstream of Jinsha River. The location and general layout of Xiluodu Hydropower Project is shown in Fig. 1 and 2.

![Figure 1. The location of Xiluodu Hydropower Station](image)

2.1. Bi-level problem description. In the Xiluodu Hydropower Project, Bureau VIII of China Water Conservancy and Hydropower Engineering (IV-CWCHE) is the construction contractor for Subsidiary Dam Construction (contract No. XLD/0889). IV-CWCHE is in charge of the operation and management such as exploration, construction site design, and so on. However, IV-CWCHE can not undertake all the
activities because typically the activities are hidden and outsourced in the large-scale water conservancy and hydropower construction project. In this case, the practical MRCPSP discussed in this study considers the construction contractor as the upper level decision maker (ULDM), while the outsourcing partner as the lower level decision maker (LLDM). Furthermore, ULDM aims to minimize the consumption of resources while maximize the quality level of project, and LLDM tries to schedule the activity under resource allocation with the objective of minimizing the total tardiness penalty cost.

At the beginning of scheduling design process, the ULDM get the relevant data of Subsidiary Dam Construction first, including quantity and type of all resources, preliminary exploration data, and so on. After that, the ULDM proposes the initial resource allocation of whole project, and the LLDM assigns the resources to determine the duration and executive mode of activities, and then feedbacks the results to the ULDM. Subsequently, according to the upper objective functions, ULDM adjust that whether the results are satisfactory or not. Repeat this process, until the optimal schedule is obtained finally. The decision making process is shown in Fig. 3 in detail, where $S$ and $T$ are the start activity and final activity, $d_{ij}$ is the duration of activity $i$ select mode $j$, $m_i$ is the total number of modes for each activity $i$, $r_{ijk}$ is the resource $k$ required to execute activity $i$ using mode $j$.

2.2. Motivation for employing fuzzy random variables and bi-random variables in MRCPSP. The uncertainty factors are inevitable in the practical MRCPSP. Taking Xiluodu Hydropower Station as an example: Xiluodu dam is built in the mountain region, and the basic intensity of earthquake in dam area is VIII, so it is hard to design or construct with many technologies that surpass the world level. In this situation, the complex uncertainty environment should be considered in the decision making process. Actually, the requirements of applying uncertainty theory in project scheduling problems have been widely recognized (Herroelen and Leus, 2005 [17]). Although the possibility and probability theory has been employed to project scheduling problems successfully, sometimes it may
not be suitable for depicting the complex uncertainty in the MRCPS because the preliminary and other relevant information are often imprecise. For instance, in order to collect the data for the scheduling design of Xihoudu Hydropower Station, the investigation and survey should be made first to get professional suggestions...
from the experienced engineers. Suppose that there are $E$ experienced engineers employed by the IV-CWCHE, and the index of engineers is $e$ (i.e., $e = 1, 2, \cdots, E$). Since the imprecise preliminary and relevant data and the complexity of construction environment, the engineer $e$ describes the parameter as an interval $[l_e, r_e]$ with the most possible value $m_e$ instead of the exact parameter. For example, the requirement of cement is described as “between 26.50 and 29.00 million kg, and the most possible quantity is 28.25 million kg”. Since different engineers have different suggestions, the project manager of Subsidiary Dam Construction obtain a series of the requirement of cement. In practice, he could select the minimum value of all $l_e$ (denoted as $a$) and the maximal value of all $r_e$ (denoted as $b$) as the left and right border for the requirement of cement, respectively. Meanwhile, using the maximum likelihood method, the fluctuation of all the $m_e$ can be characterized as a stochastic normal distribution $\phi(\omega) \sim N(\mu, \sigma^2)$. In this case, the requirement of cement is a fuzzy variable taking a random parameter, i.e., a fuzzy random variable $(a, \phi(\omega), b)$, where $\phi(\omega) \sim N(\mu, \sigma^2)$. This process above is depicted in Fig. 4. Similarly, other subjective uncertainties, such as the total tardiness penalty cost coefficient, also could be represented as a triangular fuzzy random variable while modeling the MRCPSP.

In addition, there are the objective uncertainty in the MRCPSP besides the subjective uncertainty which are described by the fuzzy random variables above. Taking the duration of “Pilot hole drilling” as an example. At first, the duration is a random variable $\xi \sim N(\lambda, \gamma^2)$ attribute to the complex hydro-geological conditions. But in the construction process, the duration is actually determined by the outsourcing partner, and the proactive scheduling plan will be disturbed by some unpredictable stochastic factors such as treacherous weather, resource breakdowns, and so on. Hence, sometimes the random variable is not able to cope with the complicated situation to obtain a robust scheduling plan. To react the destabilization caused by the stochastic factors, the project manager of Subsidiary Dam Construction revises the mean of duration as a random variable following uniform distribution $\tilde{\lambda} \sim U[a, b]$ based on the past experience. Hence, the duration of “Pilot hole drilling” is a random variable taking a random parameter, i.e., a bi-random variable $\tilde{\xi} \sim N(\tilde{\lambda}, \varrho^2), \tilde{\lambda} \sim U[a, b]$ (see Fig. 5).

In fact, there are some uncertainties in the practical construction period besides the factors mentioned above, such as equipment failure, complex construction environment, and so on. Therefore, the fuzzy random variable and bi-random variable should be employed in MRCPSP to cope with the hybrid uncertain environment. As the case stands, the fuzzy random variable and bi-random variable has been successfully applied in many research areas, such as project scheduling problems (Xu and Zhou, 2009 [42]; Nematian et al., 2010 [31], Xu and Zhang, 2012 [41]), portfolio selection problems (Ammar, 2008 [3]; Yan, 2009 [43]; Li and Xu, 2009 [28]), vendor selection (Xu and Ding, 2011 [38]), inventory problems (Bag et al., 2009 [6]; Xu and Zeng, 2012 [40]), navigation coordinated scheduling (Zhang and Xu, 2013 [47]) and so on. In this study, motivated by the practical situation, we characterize the subjective uncertainty by fuzzy random variables and objective uncertainty by bi-random variables in the scheduling process, respectively. Some details about the fuzzy random variable and bi-random variable will be discussed further.

3. Bi-level MRCPSP modelling under hybrid uncertainty. In order to model the MRCPSP under hybrid uncertain environment, the descriptions are as follows:
(1) A single project consists of a number of activities with several known execution modes, and each activity-mode combination requires a constant amount of one or more types of resources;

(2) The start time of each activity is dependent upon the completion of its precedence activities;

(3) Activities can not be interrupted, and each activity must be performed in one mode. Mode switching is not allowed;

(4) Expect for the shared resources, the activities also require separate resource. To simplify the problem, independent resource allocation is not considered in this paper, and all the independent requirement could be satisfied;

(5) For activity $i$, the quantity of renewable resources is $\tilde{r}_{ijk}$ when it is execute by mode $j$, and the quantity of no-renewable resources is $\tilde{r}_{ijn}$. $\tilde{r}_{ijk}$ and $\tilde{r}_{ijn}$ are fuzzy random variables;

(6) The duration of activity $i$ select mode $j$ is $\tilde{d}_{ij}$, which requires the shared resources, is proportional to its workload, and inversely proportional to the allocation of shared resources. $\tilde{d}_{ij}$ is a bi-random variable;
The duration of "Pilot hole drilling" i.e. Bi-random variable:
Complex hydro-geological conditions
Random variable: \( \xi \sim N(\lambda, \gamma^2) \)
Unpredictable stochastic factors
Treacherous weather, resource breakdowns, accidents, unreliable deliveries and so on
How to react this situation?
The duration is revised following the stochastic factors above
The the mean of duration is also a random variable following uniform distribution based on the past experience

\[ \xi \sim N(\hat{\lambda}, \hat{\gamma}^2), \hat{\lambda} \sim U[a,b] \]

\[ 2 \sim (\alpha, \beta), \sim [\alpha, \beta] \]

\[ \xi \sim \lambda \gamma \lambda \%

**Figure 5.** Flowchart of why the duration of “Pilot hole drilling” is a bi-random variable

(7) The tardiness penalty cost coefficient of activity \( i \) is also a fuzzy random variable, which denoted as \( \hat{c}_i \);

(8) The objectives of ULDM are to minimize the consumption of resources while maximize the quality level of project, and LLDM is to minimize the total tardiness penalty cost.

MRCPSP in this study can be stated as follows: there are a set of activities \( i = \{0, 1, \cdots, I+1\} \) need to be scheduled under the precedence and resource constraints. Activities 0 and \( I+1 \) are dummies, and they have no duration (just represent the initial and final activity). The modes of activities 0 and \( I+1 \) are denoted as 1. There are \( m_i \) modes for each activity \( i \), and no activity may be started before all its predecessors are finished. Resources in MRCPSP can be renewable \( K = \{1, 2, \cdots, K\} \) which are available in limited quantities but renewable from time to time, such as equipment and manpower) or nonrenewable \( N = \{1, 2, \cdots, N\} \) (which are available in limited quantities and non-renewable in the scheduling process, such as material and capital).

3.1. **Notation.** Following notation are used in the MRCPSP with hierarchical decision structures under hybrid uncertain environment (see Appendix A).

3.2. **Modelling.** In view of the requirements of ULDM and LLDM, a bi-level multi-objective MRCPSP model under hybrid uncertainty is proposed.
3.2.1. **Upper level model.** For the ULDM, the first objective is to minimize the consumption of resources,

\[
\min R = \sum_{i=1}^{I} \sum_{j=1}^{m_i} \left( \sum_{k=1}^{K} \tilde{r}_{ijk} + \sum_{n=1}^{N} \tilde{r}_{ijn} \right)
\]

Furthermore, the second objective of ULDM is to maximize the quality level of project. To simplify the problem, let the slope of duration-quality

\[
s_i = q_i^w + s_i (d_{ij}^b - d_{ij}^w),
\]

where \(d_{ij}^b\) and \(d_{ij}^w\) are durations following best and smallest acceptable quality level respectively. Thus,

\[
\max Q = \sum_{i=1}^{I} \sum_{j=1}^{m_i} \omega_i \left[ q_i^w + \frac{q_i^b - q_i^w}{d_{ij}^b - d_{ij}^w} (\tilde{d}_{ij} - d_{ij}^w) \right]
\]

where \(\omega_i > 0, \sum_{i=1}^{I} \omega_i = 1\). According the descriptions (6) above, i.e., the duration of activity \(i\) is proportional to its workload, and inversely proportional to the allocation of shared resources, we have

\[
\tilde{d}_{ij} = \begin{cases} \frac{R}{r_{ij}}, & \text{if } i \text{ consume the shared resources } r \\ \frac{D}{r_{ij}}, & \text{otherwise} \end{cases}
\]

The constraints for the upper level in MRCPSP are divided into resource constraints and other logical constraints. Since renewable and nonrenewable resources are available in limited quantities,

\[
\begin{align*}
\sum_{i=1}^{I} \sum_{j=1}^{m_i} \tilde{r}_{ijk} \sum_{t=1}^{t_{ij}} x_{ijt} & \leq r_M^k, \forall i, j, k \\
\sum_{i=1}^{I} \sum_{j=1}^{m_i} \tilde{r}_{ijn} \sum_{t=1}^{t_{ij}} x_{ijt} & \leq r_M^n, \forall j, k, n
\end{align*}
\]

can be employed. In addition, for describing non-negative variables and 0-1 variables for the practical significance, we get

\[
\begin{align*}
d_{ij}^w & \geq \tilde{d}_{ij} \geq d_{ij}^b \geq 0, \\
q_i^b & \geq q_i \geq q_i^w \geq 0, \\
t_{ij}^{LF} & \geq 0, \quad t_{ij}^{EF} \geq 0, \\
r_k^M & \geq 0, \quad r_n^M \geq 0 \\
\omega_i & > 0, \quad \sum_{i=1}^{I} \omega_i = 1, \\
x_{ijt} & = 0 \text{ or } 1, \\
\sum_{j=1}^{m_i} x_{ijt} & = 1.
\end{align*}
\]

Thus, the upper level model can be obtained as Eq. (16), see Appendix B.

3.2.2. **Lower level model.** For LLDM, the objective is to minimize the total tardiness penalty, which is the sum of penalty costs for all activities,

\[
\min C = \sum_{i=1}^{I} \sum_{j=1}^{m_i} \tilde{c}_i |t_{ij}^{EF} - t_{ij}^{LF}|
\]
Therefore, the fuzzy random variable $\hat{\xi}$ has been completed already, therefore constraints. Since the successive activities must be scheduled after all the predecessors have been completed already, therefore

$$x_{ijt}d_{ij} \leq x_{ijt}t_{ij} - x_{ejt}t_{ej}$$

(6)

where $e \in \text{Pre}_i$. Moreover

$$t_{ij}^{EF} \leq t_{ij}^f \leq t_{ij}^{LF}$$

$$t_i^f \geq 0$$

(7)

Hence, we have the lower level model as as Eq. (17), see Appendix B.

3.2.3. Global model. By integrating the equations mentioned above, the following global model for the bi-level multi-objective MRCPSP model is formulated in Eq. (18), see Appendix B.

Since some coefficients in the proposed bi-level multi-objective MRCPSP model are fuzzy random variable and bi-random variable, so it is very hard to be solved. In this case, we will discuss the equivalent crisp model of the uncertain one as follow.

4. Equivalent crisp model. To better understand this paper, the basic knowledge of fuzzy random variable and bi-random variable is stated in Appendix C.

**Theorem 1.** Let $\hat{\xi} = (\xi_L, \phi(\omega), \xi_R), \phi(\omega) \sim N(\mu, \sigma^2)$ be a fuzzy random variable defined on a domain $U$, $\alpha \in (0, \sup p_\phi(x)]$ be the given probability level of the random variable, and $\beta \in \left[\frac{\xi_L - \hat{\xi}}{\phi(\omega) - \hat{\xi}} + (\xi_R - \xi_L), 1\right]$ be the possibility level of the fuzzy variable (where $\phi_L$ and $\phi_R$ are left and right border of $\alpha$-level set for $\phi(\omega)$, respectively). If $\phi(\omega) \geq \alpha$ and $\mu(\omega) \geq \beta$, the fuzzy random variable $\hat{\xi}$ can be transformed into a $(\alpha, \beta)$-level trapezoidal fuzzy variable, i.e., $\hat{\xi} \rightarrow \hat{\xi}_{(\alpha, \beta)} = (\xi_L, \hat{\xi}, \hat{\xi}, \xi_R)$.

**Proof.** See Appendix D.

Hence, the fuzzy random variables $\hat{c}_i, \hat{r}_{ij}, \hat{r}_{ijn}$ in the proposed model (18) can be transformed into the $(\alpha, \beta)$-level trapezoidal fuzzy variables as:

$$\hat{c}_i \rightarrow \hat{c}_{i(a_1, a_2)} = ([c_i]_L, \hat{c}_i, [c_i]_R)$$

$$\hat{r}_{ij} \rightarrow \hat{r}_{ij(a_2, a_2)} = ([r_{ij}]_L, \hat{r}_{ij}, [r_{ij}]_R)$$

$$\hat{r}_{ijn} \rightarrow \hat{r}_{ijn(a_3, a_3)} = ([r_{ijn}]_L, \hat{r}_{ijn}, [r_{ijn}]_R)$$

(8)

In addition, denote the expected value of trapezoidal fuzzy variable $\hat{\xi}_{(\alpha, \beta)} = (\xi_L, \hat{\xi}, \hat{\xi}, \xi_R)$ as $E[\hat{\xi}_{(\alpha, \beta)}]$, then (Liu and Liu, 2002 [29])

$$E[\hat{\xi}_{(\alpha, \beta)}] = \frac{\xi_L + \hat{\xi} + \xi_R}{4}$$

Therefore, the fuzzy random variable $\hat{\xi} = (\xi_L, \phi(\omega), \xi_R)$ can be calculated as $\frac{\xi_L + \hat{\bar{\xi}} + \xi_R}{4}$, where

$$\xi = \xi_R - \beta \left[\xi_R - \left(\mu - \sqrt{-2\sigma^2\ln(\alpha\sqrt{2\pi} \sigma)}\right)\right]$$

(9)

and

$$\hat{\xi} = \xi_L + \beta \left[\mu + \sqrt{-2\sigma^2\ln(\alpha\sqrt{2\pi} \sigma)}\right] - \xi_L$$

(10)
Now, we have
\[
E[\tilde{c}_{i(\alpha_1, \beta_1)}] = \frac{([c_d]_L + [c_d]_R + [c_d]_R)}{4}
\]
\[
E[\tilde{r}_{ij(k(\alpha_2, \beta_2)]} = \frac{([r_{ijk}]_L + [r_{ijk}]_R + [r_{ijk}]_R)}{4}
\]
\[
E[\tilde{r}_{ijn(\alpha_3, \beta_3)]} = \frac{([r_{ijn}]_L + [r_{ijn}]_R + [r_{ijn}]_R)}{4}
\]
where the parameters \([\cdot]_L, [\cdot]_R, \mu, \sigma, \alpha, \beta\) can be obtained by the statistical and group decision making methods previously, and \([\cdot]_L\) and \([\cdot]_R\) can be calculated by Eqs. (9) and (10).

**Theorem 2.** Let \(\tilde{\xi}\) is a bi-random variable and \(\tilde{\xi} \sim N(\bar{\mu}, \sigma^2)\) with \(\bar{\mu} \sim U[a, b]\), then,
\[
E[\tilde{\xi}] = \frac{a + b}{2}
\]

**Proof.** See Appendix D.

Thus, if \(\tilde{d}_{ij} \sim N(\tilde{\lambda}_{d_{ij}}, \tilde{\sigma}_{d_{ij}}), \tilde{\lambda}_{d_{ij}} \sim U[a_{d_{ij}}, b_{d_{ij}}]\), then \(E[\tilde{d}_{ij}] = \frac{a_{d_{ij}} + b_{d_{ij}}}{2}\) by theorem 2.

Based on the above hybrid method, all the uncertain coefficients in the proposed bi-level multi-objective MRCPSP model, including fuzzy random variable and bi-random variable, can be calculated. In this case, the proposed multi-objective bi-level MRCPSP model can be transformed into an equivalent crisp one.

5. Passive congregation-based bi-level multiple objective particle swarm optimization algorithm (PC-based BL-MOPSO). It is well known that the project-scheduling problem without resource constraints can be solved efficiently with the critical-path method (CPM) but becomes NP-hard when only a single resource is present (Zhu et al., 2006 [48]). In this case, a hybrid evolutionary algorithm, named passive congregation-based bi-level multiple objective particle swarm optimization algorithm (PC-based BL-MOPSO), is designed to solve the equivalent crisp bi-level MRCPSP model in this section. In fact, due to the superior search performance and fast convergence, particle swarm optimization (PSO) algorithm (first proposed by Kennedy and Eberhart in 1995 [20]) is considered as an effective tool for solving bi-level optimization problems (Gao et al., 2011 [15]; Kuo and Huang, 2009 [24], Zhang et al., 2012 [45]). In PSO, the particles fly through the problem space following the optimum particles, and the solutions are represented by the \(n\)-dimensional positions of a particle. The updating mechanism of particle is:
\[
v_d^i(\tau + 1) = w(\tau)v_d^i(\tau) + c_p r_1[p_d^i, best(\tau) - p_d^i(\tau)] + c_g r_2[g_d^i, best(\tau) - p_d^i(\tau)]
\]
\[
p_d^i(\tau + 1) = p_d^i(\tau) + v_d^i(\tau + 1)
\]
where \(v_d^i(\tau)\) is the velocity of \(i^{th}\) particle at the \(d^{th}\) dimension in the \(\tau^{th}\) iteration, \(w\) is an inertia weight, \(p_d^i(\tau)\) is the position of \(i^{th}\) particle, \(r_1\) and \(r_2\) are random numbers in the range \([0, 1]\), \(c_p\) and \(c_g\) are personal and global best position acceleration constant, \(p_d^i, best\) and \(g_d^i, best\) are personal and global best position of \(i^{th}\) particle at the \(d^{th}\) dimension, the inertia weight (Shi and Eberhart, 1998 [36])
\[
w(\tau) = w(T) + \frac{t - T}{1 - T}[w(1) - w(T)]
\]
Instead of global best position \( g^u_{d, \text{best}}(\tau) \), the external repository is utilized to the upper level to store several equally good non-dominated solutions in the multiple objective particle swarm optimization algorithm (MOPSO). All the particles of this swarm are compared to each other and the non-dominated particles are stored in the repository, and the positions of particles is updated by:

\[
v^u_d(\tau_u+1) = \tau_u^v(\tau_u) + c_{p_u} r_1[p^u_{d, \text{best}}(\tau_u) - p^u_d(\tau_u)] + c_{g_u} r_2[\text{REP}(\tau_u) - p^u_d(\tau_u)]
\]

\[
p^u_d(\tau_u+1) = p^u_d(\tau_u) + v^u_d(\tau_u + 1)
\]

where \( u \) is the particle index of upper level, \( \tau_u \) is iteration index \( \tau_u = 1, 2, \cdots, T_u \), \( c_{p_u} \) and \( c_{g_u} \) are personal and global best position acceleration constant, \( \text{REP} \) is the positions of the particles that represent non-dominated vectors in the repository.

In the lower level, the passive congregation is employed to avoid the premature convergence. This passive congregation coefficient can help the algorithm jump out of the local optimal solution in the running process, and improve the global search ability (He et al. (2004) [16]). Let \( c_{pc} \) be the passive congregation coefficient, \( p^l_{pc}(\tau_l) \) be a particle selected randomly from the swarm, and \( r_3, r_4, r_5 \) be the uniform distributed random numbers, thus the velocity is updated by:

\[
v^l_d(\tau_l+1) = \tau_l^v(\tau_l) + c_{p_l} r_3[p^l_{d, \text{best}}(\tau_l) - p^l_d(\tau_l)] + c_{g_l} r_4[p^l_{d, \text{best}}(\tau_l) - p^l_d(\tau_l)] + c_{pc} r_5[p^l_{pc}(\tau_l) - p^l_d(\tau_l)]
\]

\[
p^l_d(\tau_l+1) = p^l_d(\tau_l) + v^l_d(\tau_l + 1)
\]

where \( l \) is the particle index of lower level, \( \tau_l \) is iteration index \( \tau_l = 1, 2, \cdots, T_l \), \( c_{p_l} \) and \( c_{g_l} \) are personal and global best position acceleration constant.

In PC-based BL-MOPSO, the feasible particles \((x_{ijl})\) of upper level are generated first, then the solutions are set into the lower level model. For the \( u^{th} \) particle, after checking the feasibility of LLDM’s initial particles, the particles are evaluated and the personal best position is gained. Then, the position and velocity of the LLDM’s particles are updated by Eq. (15). By evaluating the updated particles, the new personal best position and global best position are obtained. Thus, the response from LLDM, i.e., the vector global best position \((t^l_{ij})\), is generated. After that, evaluate the ULDM’s particles and get the personal best position. Similarly, update the position and velocity by Eq. (14), and then store several equally good non-dominated solutions by the external repository. Repeat the process above, until output the final Pareto solutions. The overall procedure is outlined in Fig. 6.

6. Practical application. The bi-level MRCPSP considered in this paper is from Subsidiary Dam Construction (contract No. XLD/0889) of Xiluodu Hydropower Station, which are constructed by Bureau VIII of China Water Conservancy and Hydropower Engineering (IV-CWCHE). The representation of case problem, data collection, case problem results and analysis are presented as follows.

6.1. Representation of the case problem. Subsidiary Dam Construction in Xiluodu Hydropower Station contains twenty activities, see Table 1.

There are three types of resources in Subsidiary Dam Construction project, i.e., manpower \((r_1, \text{renewable resources})\), equipment \((r_2, \text{renewable resources})\), and material \((r_3, \text{nonrenewable resources})\). Manpower is consist of drill irrigation worker, electric welder, inclinometer worker, installer, plumber, electrician, repairman, ordinary worker, chauffeur, and manager. Equipment is composed of geological drill,
construction drill, rock drill, grouting pump, injection pump, double-axle blade agitator, high speed mixer, wet grinder, inclinometer, and truck. Material involves cement (32.5MPa, 42.5MPa), concrete, reinforcement, admixture, channel steel, angle iron, deformed steel bar, steel grating, fir rod. In order to calculate different resources expediently, we transfer the quantities of all resources consumption into a cash value (ten thousand CNY per unit). Manpower \( r_1 = \text{number of workers} \times \text{working time} \times \text{wage} \). Equipment \( r_2 = \text{number of equipment} \times \text{working time} \times [\text{cost of equipment purchases (or rental cost)} + \text{loss of equipment}] \). Material \( r_3 = \text{the demand} \times \text{material price} \). The project uses month as the measure of duration, i.e., one month per unit. The multiple modes, duration and resources requirement of each activity are shown in Fig. 7.
Figure 7. Activity nodes of Subsidiary Dam Construction project
Table 1. The activities in Subsidiary Dam Construction

| No. | Activity                                                   |
|-----|------------------------------------------------------------|
| A1  | Pilot hole drilling                                       |
| A2  | Hydraulic pressure test of pilot hole                     |
| A3  | Lift observation hole drilling                            |
| A4  | Curtain inspection hole drilling                           |
| A5  | Hydraulic pressure test of curtain inspection hole         |
| A6  | Curtain grouting                                          |
| A7  | Foundation consolidation grouting                         |
| A8  | Cavern consolidation grouting                              |
| A9  | Slope contact grouting                                     |
| A10 | Backfill grouting                                         |
| A11 | Drainage water hole drilling, Φ = 50 mm, L = 4 mm          |
| A12 | Drainage water hole drilling, Φ = 110 mm, L = 17 mm        |
| A13 | Drainage water hole drilling, Φ = 110 mm, L = 34 mm        |
| A14 | Cooling pipe plugging grouting                            |
| A15 | Core-drilling and hole sealing Φ = 216 mm                 |
| A16 | Single-point Hydraulic pressure test                      |
| A17 | Hole sonic test hole drilling and sealing                  |
| A18 | Cross-hole sonic test hole drilling and sealing            |
| A19 | Backfill grouting of reserved construction channel         |
| A20 | Control-joint grouting of reserved construction channel    |

6.2. Data collection. All the data of Subsidiary Dam Construction project in Xihuodu Hydropower Station are collected from Bureau VIII of China Water Conservancy and Hydropower Engineering (IV-CWCHE). The fuzzy random and bi-random data, including the quantity of resources $\tilde{r}_{ijk}$ and $\tilde{r}_{ijn}$, the tardiness penalty cost coefficient $\tilde{c}_i$, and the duration $\tilde{d}_{ij}$ are obtained based on the previous data and experts’ experience. The quantity of resources for each mode and other detailed information is shown in Table 3 and Table 4 (see Appendix E).

6.3. Results and analysis. The optimal scheduling scheme will be generated by PC-based BL-MOPSO. The parameters of PC-based BL-MOPSO for this practical case problem are: swarm size popsize $L = 50$, iteration max $T = 100$, personal and global best position acceleration constant of upper and lower level $c_{pu} = c_{pl} = c_{gu} = c_{gl} = 2$, passive congregation coefficient $c_{pc} = 1$, inertia weight of upper and lower level $w_u(1) = w_l(1) = 0.1, w_u(\tau_u) = w_l(\tau_l) = 0.9$. Using Matlab 7.0 and Visual C++ language on a Inter Core I7 M370, 2.40 GHz, with 2048 MB memory, the case problem is solved by the proposed algorithm within 36 minutes on average. Fig. 8 are the Pareto optimal solutions of upper level programming, which is the most satisfied solution accepted by the ULDM and LLDM.

Following Fig. 8, the ULDM could choose the scheduling scheme according to the actual situation in practice. For example, if ULDM determines that the objective of the quality level is the more important factor, they may allow an increased resource consumptions. Thus, they would choose the far right solutions, such as $R = 906.6717, Q = 3.1069$, and the duration is $T = 39.03$ months. In addition, the project scheduling scheme is shown in Fig. 9. In practice, the DMs can change the relevant optimistic-pessimistic parameters to obtain the different solutions.

6.4. Sensitivity analysis. In order to evaluate the effectiveness of variations in model parameters, sensitivity analysis is performed in this subsection. For illuminating the sensitivity clearly, the probability level of the random variable $\alpha_1$ and the possibility level of the fuzzy variable $\beta_1$ (it is similar to $\alpha_2$ and $\beta_2$, $\alpha_3$ and $\beta_3$) are changed in turn, then compared the corresponding results to analyze the parameter. This sensitivity analysis can provide the necessary information to the project manager for choosing each parameter in the scheduling process.
Figure 8. Pareto optimal solutions of upper level programming for Subsidiary Dam Construction project

First, let $\alpha_1$ still be 0.4 as before, and $\beta_1 = 0.8, 0.7, 0.6$, respectively. Then, we get Fig. 10. Subsequently, let $\beta_1$ still be 0.8, and $\alpha_1 = 0.4, 0.3, 0.2$, respectively. Then we obtain Fig. 11. Based on the sensitivity analysis of the results, the proposed bi-level model for MRCPSp is proved to be sensitive to probability level of the random variable $\alpha_1$.

In practice, the project manager can change the parameters $\alpha$ and $\beta$ to obtain the different solutions. The solutions reflect different predictions of possibility levels and different optimistic-pessimistic attitudes for uncertainty.

6.5. Algorithm evaluation. Optimization results for the bi-level MRCPSp using PC-based BL-MOPSO are compared with standard PSO and standard GA in this subsection. In order to carry out the comparisons under a similar environment, the parameters stated in the last subsection are also adopted for standard PSO, and
the crossover possibility and the mutation possibility for the standard GA are set as $p_c = 0.75$ and $p_m = 0.05$.

Table 2 shows the comparison results obtained from 50 runs, including the average and sample standard deviation for the convergence iteration number and the elapsed time. The comparison results and convergence histories are based on the averages of the optimal results from 50 computational runs (for PC-based BL-MOPSO, standard PSO and standard GA, respectively) that are able to obtain the Pareto optimal solutions, excluding the locally trapped ones.

| Approach            | Sample size | Convergence iteration number | Elapsed Time (minutes) |
|---------------------|-------------|------------------------------|------------------------|
|                      |             | Average | Standard deviation | Average | Standard deviation |
| PC-based BL-MOPSO   | 50          | 58      | 2.1497               | 36.2842 | 11.3425            |
| Standard PSO        | 50          | 84      | 4.5246               | 40.2466 | 16.2451            |
| Standard GA         | 50          | 71      | 2.9452               | 62.5431 | 13.5289            |

It can be seen that the search time of PC-based BL-MOPSO is faster than standard PSO and standard GA in 50 computational runs (requires fewer iterations and elapsed time), and PC-based BL-MOPSO also has a better computational stability compared to the standard PSO and the standard GA. Summarizing, PC-based BL-MOPSO shows its improved search performance when compared with standard PSO and standard GA in a similar environment.

7. Conclusions and further research. In this paper, considering the complex hierarchical organization structure and the hybrid uncertain environment, a bi-level multi-objective MRCPS model is developed for the practical resource-constrained multiple project scheduling problems. The proposed model employs the fuzzy random variables and bi-random variables to characterize the hybrid uncertain environment, and this application makes the model more suitable for describing the vague situation in the real world. This work is original. Motivated by the particular nature of model, passive congregation-based bi-level multiple objective particle swarm optimization algorithm (PC-based BL-MOPSO) is designed to obtain the
optimal solutions. For illustrating the effectiveness, the proposed model and algorithm are successfully applied to the Subsidiary Dam Construction project in Xiluodu Hydropower Station, and a brief comparison among the PC-based BL-MOPSO, standard PSO and standard GA is made to demonstrate the significance of proposed algorithm, along with the sensitivity analysis.

One of the most important follow-up researches should be focused on the software development that is based on the proposed bi-level multi-objective model and algorithm in this study. Besides, more realistic factors and constraints for the MRCPSP with complex hierarchical organization structure should be considered. Another area for the further research includes characterizing other practical uncertainty by different forms of complex uncertain variables in the decision making process. All of three areas are very important and worth our equal concern.

Appendix A. Notation.

\begin{itemize}
  \item $t$ time index, $t = 1, \cdots, T$
  \item $i$ activity index, $i = 1, \cdots, I$
  \item $j$ mode index, $j = 1, 2, \cdots, m_i$ ($m_i$ is the maximum number of possible modes for activity $i$)
  \item $k$ renewable resource, $k = 1, 2, \cdots, K$
  \item $n$ nonrenewable resource, $n = 1, 2, \cdots, N$
  \item $\text{Pre}(i)$ the set of immediate predecessors of activity $i$
  \item $U_i$ the workload of activity $i$ which use the shared resources
  \item $\tilde{d}_{ij}$ duration of activity $i$ select mode $j$ using shared resources
  \item $d^D_i$ the duration of activity $i$ which use the independent resources;
  \item $t_{ij}^{EF}$ early finish time of activity $i$ select mode $j$
  \item $t_{ij}^{LF}$ lately finish time of activity $i$ select mode $j$
  \item $t_{ij}$ finish time of activity $i$ select mode $j$
  \item $t_i^{E}$ expected finish time of activity $i$
  \item $q_i$ the quality level of activity $i$
  \item $q_i^{w}$ the smallest acceptable quality level of activity $i$
  \item $q_i^{b}$ the best acceptable quality level of activity $i$
  \item $\omega_i$ the weight of the quality level of activities $i$ for the quality level of the whole project
  \item $r_k^M$ maximal limited renewable resource $k$
  \item $r_n^M$ maximal limited nonrenewable resource $n$
  \item $\tilde{r}_{ijk}$ renewable resource $k$ required to execute activity $i$ using mode $j$
  \item $\tilde{r}_{ijn}$ nonrenewable resource $n$ required to execute activity $i$ using mode $j$
  \item $\hat{c}_i$ tardiness penalty cost coefficient of activity $i$
\end{itemize}
\[ x_{ijt} = \begin{cases} 
1, & \text{if activity } i \text{ executed in mode } j \text{ scheduled to be finished in time } t \\
0, & \text{otherwise} 
\end{cases} \]

Appendix B. The mathematical model of MRCPSP.

The upper level model:

\[ \begin{align*} 
\min R &= \sum_{i=1}^{I} \sum_{j=1}^{m_i} \left( \sum_{k=1}^{K} \hat{r}_{ijk} + \sum_{n=1}^{N} \hat{r}_{ijn} \right) \\
\max Q &= \sum_{i=1}^{I} \sum_{j=1}^{m_i} \omega_i \left[ q_i^w + \frac{q_i^b - q_i^w}{d_i^j - d_i^w} (\hat{d}_{ij} - d_i^w) \right] \\
\text{s.t.} & \begin{align*} 
\sum_{i=1}^{I} \sum_{j=1}^{m_i} \hat{r}_{ijk} \sum_{\tau=t}^{t+\hat{d}_{ij}-1} x_{ij\tau} & \leq r_k^M \\
\sum_{i=1}^{I} \sum_{j=1}^{m_i} \hat{r}_{ijn} \sum_{t=t_{ij}^F}^{t_{ij}^E} x_{ijt} & \leq r_n^M \\
d_i^w & \geq \hat{d}_{ij} \geq d_i^b \geq 0 \\
qu_i^b & \geq q_i \geq q_i^w \geq 0 \\
t_i^{EF} & \geq 0, t_i^{LF} \geq 0 \\
r_k^M & \geq 0, r_n^M \geq 0 \\
\omega_i > 0, \sum_{i=1}^{I} \omega_i & = 1 \\
x_{ijt} & = 0 \text{ or } 1 \\
\sum_{j=1}^{m_i} x_{ijt} & = 1 \\
U_i & \geq 0 
\end{align*} \] (16)

The lower level model:

\[ \begin{align*} 
\min C &= \sum_{i=1}^{I} \sum_{j=1}^{m_i} \hat{c}_i \left| t_{ij}^f - t_i^E \right| \\
x_{ijt} \hat{d}_{ij} & \leq x_{ijt} t_{ij}^f - x_{ejt} t_{ej}^f \\
t_i^{EF} & \leq t_{ij}^f \leq t_i^{LF} \\
t_i^E & \geq 0 \\
\text{s.t.} & \begin{align*} 
e \in \text{Pre}_i \\
i & = 1, 2, \ldots, I \\
j & = 1, 2, \ldots, m_i \\
k & = 1, 2, \ldots, K \\
n & = 1, 2, \ldots, N 
\end{align*} \] (17)
The global model:

\[
\begin{align*}
\min R &= \sum_{i=1}^{I} \sum_{j=1}^{m_i} \left( \sum_{k=1}^{K} \hat{r}_{ijk} + \sum_{n=1}^{N} \hat{r}_{ijn} \right) \\
\max Q &= \sum_{i=1}^{I} \sum_{j=1}^{m_i} \omega_i \left[ q_i^w + \frac{q_i^b - q_i^w}{\theta_i^b - \theta_i^w} (d_{ij} - a_{ij}) \right] \\
\text{s.t.} & \sum_{i=1}^{I} \sum_{j=1}^{m_i} \hat{r}_{ij} \leq y_k^M \\
& \sum_{i=1}^{I} \sum_{j=1}^{m_i} \hat{r}_{ijn} \sum_{t=t_i^L}^{t_i^U} x_{ijt} \leq r_n^M \\
& d_{ij}^w \geq \hat{d}_{ij} \geq d_{ij}^b \geq 0 \\
& q_i^b \geq q_i^w \geq 0 \\
& t_{ij}^E \geq 0, t_{ij}^L \geq 0 \\
& r_k^M \geq 0, r_n^M \geq 0 \\
& \omega_i > 0, \sum_{i=1}^{I} \omega_i = 1 \\
& x_{ijt} = 0 \text{ or } 1 \\
& \sum_{j=1}^{m_i} x_{ijt} = 1 \\
& U_i \geq 0 \\
& \min C = \sum_{i=1}^{I} \sum_{j=1}^{m_i} \hat{c}_i \left| t_{ij}^f - t_{ij}^E \right| \\
& x_{ij} \hat{d}_{ij} \leq x_{ij} t_{ij}^f - x_{ej} t_{ej}^f \\
& t_{ij}^E \leq t_{ij}^f \leq t_{ij}^L \\
& t_{ij}^E \geq 0 \\
& e \in \text{Pre}_i \\
& i = 1, 2, \cdots, I \\
& j = 1, 2, \cdots, m_i \\
& k = 1, 2, \cdots, K \\
& n = 1, 2, \cdots, N 
\end{align*}
\]

Appendix C. The basic knowledge of fuzzy random variable and bi-random variable.

Definition 1. (Puri and Ralescu, 1986 [35]) Let \((\Omega, A, Pr)\) be a probability space, and \(F\) a collection of fuzzy variables defined on the possibility space. A fuzzy random variable is a function \(\hat{\xi}: \Omega \to F\) such that for any \(r \in (0, 1]\)

\[
\hat{\xi}_r(\omega) = \{ x | \mu_{\hat{\xi}_r}(x) \geq r, x \in R \}, \quad \forall \omega \in \Omega,
\]

is a measurable function of \(\omega\).

Example 1. Let \((\Omega, A, Pr)\) be a probability space. If \(\Omega = \{ \omega_1, \omega_2, \cdots, \omega_n \}\) and
\[ u_1, u_2, \cdots, u_n \text{ are fuzzy variables in } F, \text{ then the function} \]
\[
\hat{\xi}(\omega) = \begin{cases} 
\hat{\xi}_1(\omega_1), & \text{if } \omega = \omega_1 \\
\hat{\xi}_2(\omega_2), & \text{if } \omega = \omega_2 \\
\cdots \\
\hat{\xi}_n(\omega_n), & \text{if } \omega = \omega_n 
\end{cases}
\]

where
\[ \mu_{\hat{\xi}_i}(\omega_i) = \begin{cases} 
L\left(\frac{m_i - x}{a_i}\right), & x \leq m_i, a_i > 0 \\
R\left(\frac{x - m_i}{b_i}\right), & x > m_i, b_i > 0 \\
0, & \text{otherwise} 
\end{cases} \]

Thus, \( \hat{\xi}(\omega) \) is a fuzzy random variable apparently, see Fig. 7.

**Figure 12.** The fuzzy random variable in example 1

**Definition 2.** (Peng and Liu, 2007 [33]) A bi-random variable \( \tilde{\xi} \) is a mapping from a probability space \( (\Omega, A, Pr) \) to a collection of random variables \( S \) such that for any Borel subset \( B \) of the real line \( R \), the induced function \( Pr\{\tilde{\xi}(\omega) \in B\} \) is a measurable function with respect to \( \omega \) (see Fig. 13). **Example 2.** Assume that \( \tilde{\xi} \) is a bi-random variable on \( (\Omega, A, Pr) \) as \( \tilde{\xi} \sim N(\tilde{\mu}, 1), \tilde{\mu} \sim U[2.25, 2.62] \), then according to definition 2, \( \tilde{\xi} \) is clearly a bi-random variable (see Fig. 14).

**Definition 3.** (Peng and Liu, 2007 [33]) Let \( \tilde{\xi} \) be a bi-random variable defined on the probability space \( (\Omega, A, Pr) \). Then the expected value of bi-random variable \( \tilde{\xi} \) is defined as
\[ E[\tilde{\xi}] = \int_0^{+\infty} Pr\{\omega \in \Omega | E[\tilde{\xi}(\omega)] \geq t\} dt - \int_{-\infty}^{0} Pr\{\omega \in \Omega | E[\tilde{\xi}(\omega)] \leq t\} dt \quad (19) \]
proved that at least one of the above two integrals is finite.
Appendix D. The Proof of Theorems.

The proof of Theorem 1.

Proof. Let \( p_\phi(x) \) be the probability density function of \( \phi(\omega) \), then the \( \alpha \)-level set of \( \phi(\omega) \) is \( \phi_\alpha = \{ \phi_\omega \in U | p_\phi(x) \geq \alpha \} \), where \( \alpha \in (0, \sup p_\phi(x)] \), \( \phi_\alpha^R \) and \( \phi_\alpha^L \) are left and right border of \( \alpha \)-level set for \( \phi(\omega) \), respectively (see Fig. 15).

Let \( p_\phi(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \alpha \), then we can get

\[
x = \mu \pm \sqrt{-2\sigma^2\ln(\alpha\sqrt{2\pi\sigma})}
\]

Thus,

\[
\phi_\alpha^L = \mu - \sqrt{-2\sigma^2\ln(\alpha\sqrt{2\pi\sigma})} \\
\phi_\alpha^R = \mu + \sqrt{-2\sigma^2\ln(\alpha\sqrt{2\pi\sigma})}
\]  

Hence, \( \hat{\xi} = (\xi_L, \phi(\omega), \xi_R) \) is transferred as a class of triangular fuzzy numbers, see Fig. 16.

Consider the left and right border of \( \hat{\xi} = (\xi_L, \phi(\omega), \xi_R) \), i.e., \( \hat{\xi} = (\xi_L, \phi_\alpha^L, \xi_R) \) and
\[ \hat{\xi} = (\xi_L, \phi^L_{\alpha}, \xi_R), \]

we can easily get the intersection point in Fig. 16 as

\[
x = \frac{\xi_R\phi^R_{\alpha} - \xi_L\phi^L_{\alpha}}{\phi^R_{\alpha} - \phi^L_{\alpha} + (\xi_R - \xi_L)}
\]

\[
\mu_{\hat{\xi}}(x) = \frac{\xi_R - \xi_L}{(\phi^R_{\alpha} - \phi^L_{\alpha}) + (\xi_R - \xi_L)}
\]

For the given possibility level \(\beta \in \left[\frac{\xi_R - \xi_L}{\phi^R_{\alpha} - \phi^L_{\alpha}} + (\xi_R - \xi_L), 1\right]\), the \(\beta\)-level set of these triangular fuzzy numbers can be obtained as Fig. 17. From Fig. 17, we can see that the \(\beta\)-level is a trapezoidal fuzzy variable (the bold one) \(\hat{\xi}_{(\alpha, \beta)} = (\xi_L, \xi, \xi_R, \xi_R)\) (where \(\xi = \xi_R - \beta(\xi_R - \phi^L_{\alpha})\) and \(\xi = \xi_L + \beta(\phi^R_{\alpha} - \xi_L)\)), and the membership function is

\[
\mu_{\hat{\xi}_{(\alpha, \beta)}}(x) = \begin{cases} 
0, & x < \xi_L, x > \xi_R \\
\frac{x - \xi_L}{\xi - \xi_L}, & \xi_L \leq x < \xi \\
1, & \xi \leq x \leq \xi \\
\frac{x - \xi_R}{\xi - \xi_R}, & \xi \leq x \leq \xi_R \\
\frac{\xi - x}{\xi - \xi_R}, & \xi < x \leq \xi_R 
\end{cases}
\]  

(21)
Based on the discussion above, the fuzzy random variable \( \hat{\xi} = (\xi_L, \phi(\omega), \xi_R) \) is finally transferred as a trapezoidal fuzzy variable \( \hat{\xi}(\alpha, \beta) = (\xi_L, \xi, \xi, \xi_R) \). Theorem 1 is proven.

The proof of Theorem 2.

Proof. By definition 3, we can get

\[
E[\hat{\xi}] = \int_{-\infty}^{+\infty} \Pr\{E[\hat{\xi}(\omega)] \geq t\} - \int_{-\infty}^{0} \Pr\{E[\hat{\xi}(\omega)] \leq t\} dt \\
= \int_{0}^{+\infty} \Pr\{\hat{\mu} \geq t\} - \int_{-\infty}^{0} \Pr\{\hat{\mu} \leq t\} dt
\]

(22)

Since \( \hat{\mu} \sim U[a, b] \), we have

\[
Pr\{\hat{\mu} \geq t\} = \begin{cases} 
1, & t \leq a \\
\frac{b-t}{b-a}, & a < t \leq b \\
0, & t > b 
\end{cases}
\]

\[
Pr\{\hat{\mu} \leq t\} = \begin{cases} 
0, & t \leq a \\
\frac{t-a}{b-a}, & a < t \leq b \\
1, & t > b 
\end{cases}
\]

Then Eq. (22) can be transferred as

\[
E[\hat{\xi}] = \left( \int_{0}^{a} Pr\{\hat{\mu} \geq t\} dt + \int_{a}^{b} Pr\{\hat{\mu} \geq t\} dt + \int_{b}^{+\infty} Pr\{\hat{\mu} \geq t\} dt \right) \\
- \left( \int_{-\infty}^{a} Pr\{\hat{\mu} \leq t\} dt + \int_{a}^{b} Pr\{\hat{\mu} \leq t\} dt + \int_{b}^{0} Pr\{\hat{\mu} \leq t\} dt \right)
\]

(23)

(1) \( 0 \leq a < b \)
Obviously, \( \int_{-\infty}^{a} Pr\{\hat{\mu} \leq t\}dt = 0 \), then

\[
E[\hat{\xi}] = \left( \int_{0}^{a} Pr\{\hat{\mu} \geq t\}dt + \int_{a}^{b} Pr\{\hat{\mu} \geq t\}dt + \int_{b}^{+\infty} Pr\{\hat{\mu} \geq t\}dt \right) = 0
\]

\[
= \int_{0}^{a} 1dt + \int_{a}^{b} \frac{b-t}{b-a} dt + \int_{b}^{+\infty} 0dt
\]

\[
= a + \frac{(b-t)^2}{2(b-a)} \bigg|_{a}^{b} = a + \frac{b-a}{2} = \frac{a+b}{2}
\]

\[
(2) \ a \leq 0 < b
\]

\[
E[\hat{\xi}] = \left( \int_{0}^{b} Pr\{\hat{\mu} \geq t\}dt + \int_{b}^{+\infty} Pr\{\hat{\mu} \geq t\}dt \right)
\]

\[
- \left( \int_{-\infty}^{a} Pr\{\hat{\mu} \leq t\}dt + \int_{a}^{0} Pr\{\hat{\mu} \leq t\}dt \right)
\]

\[
= \left( \int_{0}^{b} \frac{b-t}{b-a} dt + \int_{b}^{+\infty} 0dt \right) - \left( \int_{-\infty}^{a} 0dt + \int_{a}^{0} \frac{t-a}{b-a} dt \right)
\]

\[
= - \frac{(b-t)^2}{2(b-a)} \bigg|_{0}^{b} - \frac{(t-a)^2}{2(b-a)} \bigg|_{a}^{b}
\]

\[
= \frac{b^2-a^2}{2} - \frac{a^2}{2} = \frac{a+b}{2}
\]

\[
(3) \ a < b \leq 0
\]

Since \( \int_{0}^{+\infty} Pr\{\hat{\mu} \geq t\} = 0 \), we have

\[
E[\hat{\xi}] = 0 - \left( \int_{-\infty}^{a} Pr\{\hat{\mu} \leq t\}dt + \int_{a}^{b} Pr\{\hat{\mu} \leq t\}dt + \int_{b}^{0} Pr\{\hat{\mu} \leq t\}dt \right)
\]

\[
= - \left( \int_{-\infty}^{a} 0dt + \int_{a}^{b} \frac{t-a}{b-a} dt + \int_{b}^{0} 1dt \right)
\]

\[
= - \frac{(t-a)^2}{2(b-a)} \bigg|_{a}^{b} + b = -\frac{b-a}{2} + b = \frac{a+b}{2}
\]

Integrating Eqs. (24) ~ (26), theorem 2 is proved.

\[
\text{Appendix E. Quantity of resources and the data.}
\]
Table 3. Quantity of resources

| $f_{111}$ | $f_{112}$ | $f_{113}$ | $f_{114}$ | $f_{115}$ | $f_{116}$ | $f_{117}$ | $f_{118}$ | $f_{119}$ | $f_{120}$ |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| (32.61, $\omega_{111}$) | (33.64, $\omega_{112}$) | (31.46, $\omega_{113}$) | (9.44, $\omega_{114}$) | (22.12, $\omega_{115}$) | (39.84, $\omega_{116}$) | (23.10, $\omega_{117}$) | (21.06, $\omega_{118}$) | (21.06, $\omega_{119}$) | (21.06, $\omega_{120}$) |

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| Penalty co-efficient | $\theta_1 = (0.99, \omega_{11}, 1.21)$, $\omega_{10} \sim N(1.25, 0.09)$, $\theta_2 = (0.54, \omega_{12}, 0.71)$, $\omega_{13} \sim N(1.13, 0.00)$ | $\theta_3 = (0.85, \omega_{14}, 1.68)$, $\omega_{15} \sim N(1.38, 0.49)$, $\theta_4 = (0.46, \omega_{16}, 0.66)$, $\omega_{17} \sim N(1.43, 0.81)$ | $\theta_5 = (2.62, \omega_{18}, 2.31)$, $\omega_{19} \sim N(1.32, 0.36)$, $\theta_6 = (1.03, \omega_{20}, 1.42)$, $\omega_{21} \sim N(1.25, 0.09)$ | $\theta_7 = (0.99, \omega_{22}, 1.39)$, $\omega_{23} \sim N(1.17, 1.00)$, $\theta_8 = (2.01, \omega_{24}, 2.85)$, $\omega_{25} \sim N(2.62, 0.64)$ |
|---------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| $\gamma_1 = (2.68, \omega_{26}, 2.99)$, $\omega_{27} \sim N(1.85, 0.49)$, $\gamma_2 = (0.68, \omega_{28}, 1.33)$, $\omega_{29} \sim N(1.64, 0.60)$ | $\gamma_3 = (1.66, \omega_{30}, 1.97)$, $\omega_{31} \sim N(1.50, 0.40)$, $\gamma_4 = (2.13, \omega_{32}, 2.79)$, $\omega_{33} \sim N(1.25, 0.01)$ | $\gamma_5 = (0.46, \omega_{34}, 0.83)$, $\omega_{35} \sim N(2.15, 0.91)$, $\gamma_6 = (0.54, \omega_{36}, 0.96)$, $\omega_{37} \sim N(0.98, 0.64)$ | $\gamma_7 = (1.95, \omega_{38}, 2.21)$, $\omega_{39} \sim N(2.25, 0.49)$, $\gamma_8 = (1.24, \omega_{40}, 1.75)$, $\omega_{41} \sim N(2.14, 1.00)$ |
| $\textbf{Duration}$ | $\omega_{42} = (1.96, \omega_{43}, 1.97)$, $\omega_{44} \sim N(1.95, 0.81)$, $\omega_{45} = (0.96, \omega_{46}, 1.32)$, $\omega_{47} \sim N(1.24, 1.00)$ | $\omega_{48} = (0.96, \omega_{49}, 1.32)$, $\omega_{50} \sim N(1.24, 1.00)$ | $\omega_{51} = (0.96, \omega_{52}, 1.32)$, $\omega_{53} \sim N(1.24, 1.00)$ | $\omega_{54} = (0.96, \omega_{55}, 1.32)$, $\omega_{56} \sim N(1.24, 1.00)$ |

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