A Privacy-Preserving Energy Management System for Cooperative Multi-Microgrid Networks

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Abstract—This paper presents an Energy Management System (EMS) that considers power exchanges between a set of interconnected microgrids (MGs) and the main grid, in the context of Multi-MG (MMG) systems. The model is first formulated as a centralized optimization problem, which is then decomposed into subproblems corresponding to each MG, using Lagrangian relaxation, and solved through a distributed approach using a subgradient method. The proposed model determines the power exchanges minimizing the overall operation cost of each MG, considering grid constraints and preserving the privacy of each MG by not revealing their generation cost and demand information. The distributed approach is validated with respect to the centralized problem, and various case studies are presented to demonstrate the performance of the proposed approach, comparing the costs of the MGs operating individually and cooperatively. The results show that all MGs in the MMG system improve their cost as consequence of the power exchanges, thus demonstrating the advantages of interconnecting MGs.

Index Terms—Distributed optimization, dual decomposition, energy management, multiple microgrids, subgradient method.

NOMENCLATURE

Sets and Indices

\( M \) \hspace{1cm} Set of microgrids
\( m, n \) \hspace{1cm} Index for each microgrid

Parameters

\( \alpha \) \hspace{1cm} Update step size for the subgradient method
\( \beta \) \hspace{1cm} Scaling factor of the transfer cost function
\( \frac{P_{m}}{P_{m,\text{PCC}}} \) \hspace{1cm} Maximum power transfer capacity through the point of common coupling of microgrid \( m \) [kW]
\( P_{m} \) \hspace{1cm} Maximum total power generation of microgrid \( m \) [kW]
\( a_{m} \) \hspace{1cm} Coefficient of the quadratic term of the cost function of microgrid \( m \) [$/kW^2]
\( b_{m} \) \hspace{1cm} Coefficient of the linear term of the cost function of microgrid \( m \) [$/kW]
\( c_{d,m} \) \hspace{1cm} Cost of buying electricity from the main grid for microgrid \( m \) [$/kW]
\( c_{m,d} \) \hspace{1cm} Cost of buying electricity from microgrid \( m \) for the main grid [$/kW]
\( c_{m} \) \hspace{1cm} Coefficient of the constant term of the cost function of microgrid \( m \) [$/]
\( D_{m} \) \hspace{1cm} Power demand in microgrid \( m \) [kW]

Variables

\( \lambda_{m} \) \hspace{1cm} Selling price of microgrid \( m \) to other microgrids [$/kW]
\( \varepsilon_{m} \) \hspace{1cm} Total power sold by microgrid \( m \) [kW]
\( P_{d,m} \) \hspace{1cm} Power sold by the main grid to microgrid \( m \) [kW]
\( P_{d,m} \) \hspace{1cm} Power bought by the main grid from microgrid \( m \) [kW]
\( P_{m,n} \) \hspace{1cm} Power exchange between microgrid \( m \) and microgrid \( n \) [kW]
\( P_{m} \) \hspace{1cm} Power generation in microgrid \( m \) [kW]

Functions

\( \gamma \) \hspace{1cm} Power transfer cost function [$/kW]
\( C_{m} \) \hspace{1cm} Generation cost function of microgrid \( m \) [$/]

I. INTRODUCTION

In recent years, environmental concerns have motivated a gradual transformation of power systems, shifting from fossil fuel-based energy sources, such as coal and natural gas, to Renewable Energy Sources (RES), such as wind and solar energy. In this context, microgrids (MGs) have been proposed as a solution to integrate these RES into existing power systems in a reliable and efficient manner. An MG can be described as a “cluster of loads, distributed generation (DG) units and energy storage systems (ESSs) operated in coordination to reliably supply electricity, connected to the host power system at the distribution level at a single point of connection, the point of common coupling (PCC)” [1]. MGs should be capable of operating both in isolated and grid-connected modes, thus enhancing the system resilience and reducing power transmission losses.

Connected MGs to form a multi-MG (MMG) system, working in coordination to further improve the operation of power systems, have recently attracted significant attention, with ongoing research targeting different aspects of the problem. A comprehensive discussion on the potential of networked MGs to support the resiliency of power systems in the case of extreme events is presented in [2]. The resiliency is enhanced by connecting geographically close MGs to form an MMG system, and coordinating power transfers to exploit the heterogeneity of each MG and ride through extreme events with the support of all MGs. A detailed survey focusing...
on distributed control and communication strategies in networked MGs is presented in [3], which reiterates the benefits of networked MGs and describes characteristics and issues with the communication network in MMG systems. Multi-agent system (MAS)-based distributed coordination control strategies are reviewed in [4], and topology and mathematical models and techniques are described, such as graph theory, non-cooperative game models, genetic algorithms, and particle swarm optimization; furthermore, distributed consensus protocols and techniques to mitigate communication delays are discussed. In [5], a discussion on the architecture, control, and communication techniques proposed in the literature for networked MGs is presented, where benefits and challenges in the implementation of such systems are outlined. An overview of proposed control solutions is provided in [6], where popular techniques such as model predictive control (MPC) and reinforcement learning (RL) are explained in detail. These references constitute a comprehensive review of the current state of research and development in MMG systems, and clearly establish their key features of interest to evolving power systems, i.e., enabling the safe and reliable inclusion of renewable energy sources with high penetration levels, enhancing the resilience of power systems, and exploiting the heterogeneity of local generation and demand to attain technical and economic benefits for the overall operation of the system.

In general, the existing literature falls into the categories of system planning, voltage and frequency control, communication in power sharing, service restoration, stability enhancement, and optimal energy management. The present work is in the last category, where most of the literature deals with mechanisms of coordination for operating the MMG system optimally, the modeling of the energy management system (EMS) as centralized or distributed optimization problems, the solution technique of such problems, and the characteristics of different system configurations [7]. In this context, in [8], a distributed and robust EMS for networked hybrid ac/dc MGs is proposed, coordinating the energy sharing through the dc network, to minimize the power transmission losses. A distributed solution to preserve the privacy of each MG is implemented using the Alternating-Direction Method of Multipliers (ADMM) algorithm, which is applicable to this model because all the energy exchanges occur through the dc network, i.e., MGs only need to update their energy exchange schedule with the network, and not other MGs.

A two-layer system to enable peer-to-peer electricity sharing is proposed in [9], where the first layer corresponds to a multi-agent coalition mechanism that allows “prosumers” to negotiate electricity trading; the second layer consists of a blockchain-based mechanism that ensures the secure settlement of the transactions established in the first layer. A peer-to-peer trading mechanism that uses locational marginal prices to compute network usage charges is presented in [10], with a decentralized configuration that focuses on preferences of each individual peer. The proposed mechanism is an iterative price-adjusting process that matches the seller and buyer prices for all energy trades. In [11], a “nested” day-ahead scheduling model for networked MGs is proposed, in which MGs are nested based on load priority. The layered structure enhances the system resiliency, and the operation costs are comparable to a centralized approach. However, this nested configuration significantly increases the complexity of the problem, which might deter its use in practical systems.

A decentralized energy management framework formulated as a bi-level quadratic optimization problem is developed in [12]. The upper level corresponds to the distribution system, and the lower level to each individual MG; the two levels are linked through the clearing price determined in the upper level optimization. A distributionally robust optimization algorithm is developed, which performs better than a stochastic programming approach, with results showing that the operation cost of the distribution system is greatly reduced when compared with a centralized solution. A similar technique is presented in [14], where a distributed MPC dispatch process is implemented, with upper and lower layers for the energy exchanges and supply and demand balance, respectively. Results show once again that using the coordinated MPC approach reduces the daily operational costs.

In [15], a coordination strategy for networked MGs formulated as a stochastic bi-level problem is proposed, where the first stage determines base generation setpoints according to the load and forecasts of non-dispatchable units, while the second stage adjusts the generation output based on realized scenarios. Simulations show that the stochastic model can lead to higher profit for entities with renewable generation, as it considers the corresponding uncertainties more accurately. However, this is not a distributed solution to the energy management problem. A decentralized framework based on ADMM to coordinate power exchanges between the distribution system and MGs is presented in [16], where each entity is modeled as a two-stage robust optimization problem to account for uncertainties in renewable generation and demand, assuming that only one MG is connected to the distribution system. For this reason, the information exchange just involves the distribution system and a single MG, which makes it possible to apply the ADMM algorithm. Similar approaches, with different models and modified versions of ADMM algorithms are presented in [17] and [18].

A distributed optimization framework for energy trading between MGs is presented in [19], which considers a simple MMG model, and implements a subgradient-based algorithm that requires minimal information exchange, thus preserving
the privacy of each MG. In [20], a mechanism based on Nash bargaining theory to incentivize energy trading among MMGs is presented. The bargaining problem is decomposed into two sequential problems, one for social cost minimization and one for trading benefit sharing, and a decentralized solution using an ADMM algorithm is implemented. Results indicate that the overall costs of the MGs participating in energy trading are reduced.

From the reviewed literature, it is clear that the topic of networked MGs is relevant for the future development of smart grids. However, most studies formulate a particular model suitable for the proposed solution approach, with results reported on diverse test systems, making the comparison between techniques very difficult. Hence, the contributions and objectives of this paper are as follows:

- Develop a centralized, decomposable, and more realistic model for the optimal and coordinated operation of MMGs, exchanging power among each other and with the main grid, while considering their physical interconnections and their limits, which most existing papers on this topic ignore, even though this is the case in practice.
- Implement a distributed model for optimal scheduling of the participating MGs, while preserving their privacy, and coordinating their cooperative operation.
- Test and validate the proposed model in an MMG system, considering its physical grid layer and associated constraints, while demonstrating the advantages of the MGs interconnection.

The rest of the paper is organized as follows: Section II provides a brief overview of MMG systems and distributed optimization techniques. Section III presents the proposed centralized model for the MMG system and the distributed approach, including the required modification to reach convergence and the decomposition procedure. In Section IV, case studies are presented and discussed, testing and validating the proposed model. Finally, Section V summarizes the conclusions and contributions of this paper, and identifies future research objectives.

II. BACKGROUND

An MMG system can be described as a network formed by multiple individual MGs that are geographically close and connected to the same distribution bus, with the ability to exchange power among MGs and/or the distribution grid at the PCC [5]. MMG systems offer several advantages, such as more economic operation in grid-connected mode, mitigation of congestion in distribution lines, and the capability to isolate from compromised sections of the grid when faults or natural disasters occur [7]. All MGs in an MMG system should be able to exchange energy with the main power grid, under the supervision of the distribution system operator, assuming three common topologies of MMGs: radial topology, in which each MG is connected directly to the main grid, with energy exchanges taking place through the distribution bus; daisy chain topology, in which energy and information is exchanged bidirectionally between adjacent MGs; and mesh topology, in which all MGs are interconnected to each other and the main grid directly, to exchange energy and information. Of these topologies, the radial topology shown in Fig. 1 is the most realistic, considering the physical connection of MGs to the distribution system.

The operation of an MMG system with a mesh topology is commonly studied in the literature, for example in [21] and [22], which provide a valuable mathematical insight into the MMG EMS problem. While this configuration is more general in theory, and supports the convergence of the distributed optimization algorithm in a more straightforward fashion [19], such topologies are not common in practice. Therefore, the radial topology is implemented in our proposed distributed approach to enhance the practicality of our model.

Recent developments in communication technologies have resulted in emergence of networked systems, in which interconnected subsystems cooperate to achieve a global objective beneficial to all participants. The concept of MMG networks is a good example of such a system, with an inherent decentralized structure that can be advantageously exploited. Several solution methods and algorithms have been reported in the literature, based on distributed optimization techniques [23], suitable for these types of problems.

In this paper, a procedure known as dual decomposition is used to solve the MMG operation model. This procedure has been widely applied to various optimization problems that benefit from a distributed solution. A generic description of dual decomposition, as presented in [24], can be explained based on the following mathematical model:

\[
\begin{align*}
\min_{x} & \quad \sum_{i=1}^{l} f_i(x_i) \quad \text{(1a)} \\
\text{s.t.} & \quad g_i(x_i) \leq A_i \quad \forall i \quad \text{(1b)} \\
& \quad \sum_{i=1}^{l} h_i(x_i) \leq B \quad \text{(1c)}
\end{align*}
\]

where the objective function \( f_i(x_i) \) is a convex, quadratic, and separable function with respect to variables \( x_i \). Constraints (1b) are also separable (one constraint for each \( i \)), but constraints (1c) are complicating constraints because they are not separable and prevent the decomposition of the problem. In order to relax the complicating constraints, a dual decomposition
procedure is used by dualizing (1c), as follows [25]:

\[
\max_{\lambda} \min_{x_i} \sum_{i=1}^{l} f_i(x_i) + \lambda \left[ B - \sum_{i=1}^{l} h_i(x_i) \right] \tag{2a}
\]

subject to

\[
g_i(x_i) \leq A_i \quad \forall i \tag{2b}
\]

This problem is not decomposable, due to the Lagrange multiplier (or dual variable) \( \lambda \) in the objective function. To solve this issue, the problem is relaxed by fixing \( \lambda \) to a constant value \( \bar{\lambda} \), i.e., turning it into a parameter, with the relaxed problem being decomposable into the following \( i \) subproblems:

\[
\min_{x_i} f_i(x_i) - \bar{\lambda} h_i(x_i) \tag{3a}
\]

subject to

\[
g_i(x_i) \leq A_i \quad \forall i \tag{3b}
\]

In dual decomposition, the maximization over \( \lambda \) is carried out through the iterative solution of the obtained subproblems, updating the fixed dual variable in each iteration with an appropriate technique, such as the subgradient or cutting plane methods [26]. The subgradient method is the most straightforward approach, based on the following update step:

\[
\lambda^{k+1} = \lambda^k + \alpha^k s^k \tag{4}
\]

where \( \alpha \) is the step size, \( s \) is the subgradient of (3), and \( k \) is the iteration counter. The step size can be chosen according to different rules, as described in [27]. For a constant step size, the subgradient algorithm is guaranteed to converge within some range of the optimal value in a finite number of steps, depending on the step size. The iterative procedure starts by initializing the dual variable at some fixed value, solving each subproblem independently with the dual variable as parameter, and updating the dual variable with the obtained solutions using (4). The process is then repeated by solving the subproblems with the updated dual variables, until convergence is reached. This distributed solution approach is used here.

III. METHODOLOGY

The MMG system considered in this work is comprised of \( m \) MGs connected to the main grid, as shown in Fig. 1. It is assumed that each MG can buy or sell power from and to the main grid, as well as exchange power with all other MGs through the main grid, but not directly, as per the most realistic MMG topology, which is not the case in most publications on this topic. In this system, it is important to consider the capacity limits of the connection of each MG; furthermore, information exchange is necessary to coordinate the power exchanges among MGs.

Two approaches, namely, centralized and distributed, can be used to solve the EMS problem within MMGs. This paper focuses on the formulation of an MMG EMS which is suitable for decomposition and the subsequent application of a distributed algorithm, with the goal of finding optimal or near-optimal solutions that preserve the privacy of each MG. Hence, the distribution and communication networks, and other components in an MMG system, such as power electronics that interface each MG with the main grid, are not directly considered. Furthermore, a simplified model is adopted, in which the only components within each MG are a single thermal generation unit and a given demand. Additional components of an MG, such as renewable energy generators and energy storage devices, can be readily included in the formulation, but are not considered here since the paper focuses on developing an appropriate decomposition approach suitable for MMG system operation that considers the physical grid interconnection and limitations, unlike other papers in the literature, particularly [19], which is the basis for the proposed decomposition technique. For the same reason, the model is developed for a single scheduling period (assumed to be one hour), but can be readily extended to consider several time periods. It is important to note that the proposed MMG EMS decomposition procedure and distributed algorithm are applicable even if the aforementioned modifications are introduced in the model.

A. Centralized Problem

Assuming that there is only one thermal generator such as a diesel unit in each MG, a single generation cost function \( C_m \) for the entire MG can be considered:

\[
C_m(P_m) = a_m P_m^2 + b_m P_m + c_m \quad \forall m \tag{5}
\]

where \( a_m \), \( b_m \), and \( c_m \) are the quadratic, linear, and constant coefficients of the cost function, and \( P_m \) is the power output of the generator, as defined in the Nomenclature section for this and the next equations. Then, the cost minimization (primal problem) for the operation of a system comprised of \( m \) MGs can be formulated as follows:

\[
\begin{align*}
\min_{P_m} \sum_m & [C_m(P_m) + c_{d,m} P_{d,m} - c_{m,d} P_{m,d}] \tag{6a} \\
\text{s.t.} \quad P_m + \sum_n P_{n,m}^e + P_{d,m} = D_m + \sum_n P_{m,n}^e + P_{m,d} \quad \forall m \tag{6b} \\
0 \leq P_m \leq P_m^f \quad \forall m \tag{6c} \\
\sum_n P_{m,n}^e + P_{m,d} \leq P_{m,d}^\text{PCC} \quad \forall m \tag{6d} \\
\sum_n P_{n,m}^e + P_{d,m} \leq P_{m,d}^\text{PCC} \quad \forall m \tag{6e} \\
P_{m,n}^e P_{n,m}^e = 0 \quad \forall m,n \tag{6f} \\
P_{m,n}^e, P_{d,m}, P_{m,d} \geq 0 \quad \forall m,n \tag{6g}
\end{align*}
\]

The selling and buying prices of the main grid to and from MG \( m \) are denoted by \( c_{d,m} \) and \( c_{m,d} \), respectively. Variables \( P_{d,m} \) and \( P_{m,d} \) represent the power sold and bought by the main grid to and from MG \( m \), respectively, and \( P_{m,n}^e \) correspond to the variable indicating power sold by MG \( m \) to MG \( n \) in the set of MGs \( M \), assuming that \( m \neq n \). \( D_m \) is the demand in MG \( m \), and \( P_m^f \) and \( P_{m,d}^\text{PCC} \) represent the maximum power generation and power transfer capacity through the PCC of MG \( m \).
The objective function (6a) considers the total generation cost, cost of buying from the main grid, and profit from selling to the main grid for all MGs. The supply and demand balance is defined by (6b) for each MG, considering exchanges between MGs and imports/exports from and to the main grid, with dual variables $\lambda_m$, which represent the selling prices of electricity for each MG. The generation limits of each MG are set by (6c), and the upper and lower limits of power transfer capacity through the PCC of each MG are enforced by (6d) and (6e). The simultaneous power exchanges in opposite directions between MGs are prevented by (6f), and all power exchange variables are defined as non-negative variables by (6g). This centralized optimization problem requires a central entity to have access to all the information of each MG (cost functions and demands, in this case). Such setup is in general not feasible if each MG is owned by independent entities, as expected in practice; hence, there is a need for a distributed solution method to avoid the need for exchange of sensitive information through a central operator.

### B. Distributed Approach

To solve problem (6) in a distributed manner, with minimum information exchange through a central entity, a decomposition approach is used to separate the complete problem into subproblems that can be solved locally by each MG. While the decomposition method in this paper is based on the model and procedure proposed in [19], the model is modified here to reflect the realistic operation of MMGs by including PCC capacity constraints and avoiding simultaneous transactions in opposite directions among MGs.

According to [25], the objective function of the problem must be convex with respect to the coupling variables, which are the power exchange variables. However, (6a) is not convex with respect to the $P^e$ variables, causing convergence issues when solving the dual problem with the iterative procedure described in Section II. Therefore, a power transfer cost function is added in the objective function to resolve this issue. This new term, which is assumed quadratic to convexify the function, is added in the objective function to resolve this issue. This new term, which is assumed quadratic to convexify the function, is added in the objective function to resolve this issue.

The objective function prevents simultaneous power exchanges in opposite directions between MGs. This important observation allows decomposing the problem by MGs, as (6f) is a complicating constraint that prevents decomposing the problem.

The coupling variables $P^e_{n,m}$ in (6b) and (6e) do not allow the problem to be separable. Thus, to decompose the problem, an auxiliary variable $\varepsilon_m$ is introduced, to represent the total power sold by MG $m$ to all other MGs, defined as follows:

$$\varepsilon_m = \sum_n P_m^e \forall m$$  (9)

Then, problem (8) can be modified by adding (9) as a constraint and changing (6b) and (6d) to the following constraints:

$$P_m + \sum_n P^e_{n,m} + P_{d,m} = D_m + \varepsilon_m + P_{m,d} \forall m$$  (10)

$$\varepsilon_m + P_{m,d} \leq P^e_{PCC} \forall m$$  (11)

With this modification, (9) is now the only complicating constraint, and therefore a decomposition method such as the dual decomposition method described in Section II can be applied. Thus, after dualizing (9), the Lagrangian dual problem can be expressed as follows:

$$\max_{\lambda_m} \left\{ \min_{P_m, P^e_{n,m}, P_{m,d}, P^e_{PCC}} \sum_m \left[ C_m(P_m) + \sum_n \gamma(P^e_{n,m}) + c_{d,m}P_{d,m} \right] \right. $$

$$- \varepsilon_m P^e_{PCC}$$  (12a)

$$\text{s.t. } (6c), (6e), (6g), (10), (11)$$  (12b)

This minimization problem can now be separated into subproblems corresponding to each MG, as follows:

$$\min_{P_m, P^e_{n,m}, P_{m,d}} \left\{ C_m(P_m) + \sum_n \gamma(P^e_{n,m}) + c_{d,m}P_{d,m} \right. $$

$$- \varepsilon_m P^e_{PCC} + P_{m,d}$$  (13a)

$$\text{s.t. } P_m + \sum_n P^e_{n,m} + P_{d,m} = D_m + \varepsilon_m + P_{m,d} \geq \lambda_m$$  (13b)

$$0 \leq P_m \leq P^e_{PCC}$$  (13c)

$$\varepsilon_m + P_{m,d} \leq P^e_{PCC}$$  (13d)

$$\sum_n P^e_{n,m} + P_{d,m} \leq P^e_{PCC}$$  (13e)

$$P_m^e, P_{n,m}, P_{m,d}, \varepsilon_m \geq 0 \forall n$$  (13f)

Note that (13) is now a local problem solved by each MG $m$ independently, with the dual variable $\lambda_m$ of its supply-demand balance constraint representing the selling price of each MG, and variable $P^e_m$ belonging to the subproblems of the other $n$ MGs. For example, assuming there are three MGs in the system, the variables associated with power exchanges among MGs in the subproblem of MG1 would be $P^e_{1,2}$, $P^e_{1,3}$, and $\varepsilon_1$, while the variables associated with power exchanges among
MGs for MG2 would be $P_{e,2}^m$, $P_{e,2}$, and $\bar{e}_2$; a similar pattern would be observed for the third MG. Thus, by separating the variables this way, each MG is optimizing its own problem in terms of the individual power bought from all other MGs, and the total power sold to all other MGs. The coupling constraint, which contains terms associated to all MGs, is now separated into terms appearing in each subproblem.

To solve the dual problem defined in (12), the subgradient method [27] is used, which is implemented through the iterative procedure described in Algorithm 1. The multiplier update step performed in (14) corresponds to the basic subgradient method. This algorithm is very sensitive to the chosen step size $\alpha$. Other methods proposed in the literature to update the multipliers, such as cutting plane or bundle methods, may converge faster than the subgradient method, but are more cumbersome to implement. Therefore, for the sake of clarity, the subgradient method was chosen in this paper. The other issue with the proposed method in Algorithm 1, is choosing the initial value of $\lambda_m$, which may be determined from existing and/or expected exchange prices.

**IV. CASE STUDIES**

The solution of the centralized problem (6) is used as reference to validate the proposed distributed model (13) for an MMG test system comprised of three MGs. All simulations were performed on a personal computer with a 1.6 GHz processor, and implemented using the Pyomo optimization modeling language. The IPOPT 3.11.1 solver was used for the nonlinear centralized problem [28], and the Gurobi 9.5 solver was used for the distributed problem [29].

Two different cases were simulated, namely, a base case, corresponding to the data presented in Table I, and a stressed case in which larger power exchanges were enforced by making the generation of one MG very expensive. The data in Table I was collected and adapted from [8] and [12]. The selling electricity price from the main grid to MGs was assumed $c_{d,m} = 0.082 \$/kW$, as per [30], and the selling electricity price from MGs to the main grid was assumed $c_{m,d} = 0.05 \$/kW$, which is lower than $c_{d,m}$ to incentivize the MG exchanges. Note that the units of these parameters are given in $\$/kW because a single scheduling period of one hour was assumed throughout the formulation.

**Algorithm 1 Dual Problem Solution**

1. Initialize selling prices $\lambda_m$
2. repeat
3. MGs share $\lambda_m$ with each other
4. MGs solve (13) independently
5. MGs share $P_{e,m}^n$ at the current $\lambda_m$
6. MGs update their selling prices:

   \[
   \lambda_m \leftarrow \lambda_m + \alpha \left( \sum_n P_{e,m}^n - e_m \right) \tag{14}
   \]
7. until Convergence criterion is satisfied

In order to have a fair comparison between the centralized and distributed solutions, results of the original centralized problem defined in (6), the modified centralized problem including the transfer cost defined in (8), and the MGs decomposed subproblems defined in (13) solved with the distributed algorithm, are analyzed next.

**A. Base Case**

Results of the centralized problem, modified centralized problem, and corresponding distributed solution for the base case are presented in Table II, for an update step $\alpha$ for the subgradient method of 0.0009 and a scaling factor $\beta$ for the power transfer cost function of 0.1, chosen through trial and error. The results show that the distributed algorithm converges to the solution of the modified centralized problem with sufficient precision. Figs. 2 to 4 show the convergence of the selling prices, dual objective function, and duality gap through the iterations, respectively. Observe in Fig. 4 that the duality gap, which is the difference between the primal and dual objective function values, converges towards zero, indicating that the distributed algorithm is finding the optimal solution of the centralized problem. In this particular case, there are no power exchanges among MGs in the modified centralized and distributed solutions, as shown in Table II. For this reason, the objective function values of the centralized, modified centralized, and distributed problems are all equal.

The iterative algorithm is relatively fast, requiring 29.84 seconds to complete the predefined 1,000 iterations, which is reasonable, since the scheduling period is assumed to be one hour. A fixed number of iterations was chosen to visualize the convergence more clearly. However, using a different convergence criterion, such as the duality gap expressed in percentage, will require less iterations, as can be seen in Fig. 4. The selling prices could also be initialized at different values, if a previous solution is known, which improves the convergence rate of the algorithm. However, since these values may not always be available, it is important to verify the convergence of the algorithm with arbitrary initial values; hence, the initial values of $\lambda_m$ were assumed to be zero.

**B. Stressed Case**

This case was designed to ensure that power exchanges among MGs occur, to study the performance of the distributed algorithm under such conditions. Thus, the following modifications to the data were made: increased the demand of MG2...
TABLE II
BASE CASE RESULTS

| Parameter MG1 | Centralized | Modified Centralized | Distributed |
|---------------|------------|----------------------|-------------|
| Objective [\$] | 70.54      | 70.54                | 70.54       |
| Generation Cost [\$] | 47.99      | 47.99                | 47.99       |
| Transfer Cost [\$] | –          | 0.00                 | 0.00        |
| \( P_1 \)       | 110.00     | 110.00               | 110.00      |
| \( P_2 \)       | 25.00      | 25.00                | 25.00       |
| \( P_3 \)       | 0.00       | 0.00                 | 0.00        |
| \( P_1^{d} \)   | 0.00       | 0.00                 | 0.00        |
| \( P_2^{d} \)   | 0.00       | 0.00                 | 0.00        |
| \( P_3^{d} \)   | 8.05       | 0.00                 | 0.00        |
| \( P_1^{m} \)   | 91.95      | 99.99                | 99.99       |
| \( P_2^{m} \)   | 91.10      | 75.00                | 75.00       |
| \( P_3^{m} \)   | 0.00       | 0.00                 | 0.00        |
| \( \lambda_1 \) | 0.22       | 0.22                 | 0.082       |
| \( \lambda_2 \) | 0.31       | 0.31                 | 0.082       |
| \( \lambda_3 \) | 0.082      | 0.082                | 0.082       |

Fig. 2. Convergence of the selling prices for the base case.

Fig. 3. Convergence of the dual objective for the base case.

from 125 kW to 325 kW; decreased the maximum generation of MG2 from 300 kW to 150 kW; increased the maximum generation of MG1 and MG3 from 150 kW to 200 kW; made generation of MG2 extremely expensive by increasing its linear cost coefficient from 0.3 to 30; increased the PCC capacity limit of MG2 from 100 kW to 200 kW (necessary for feasibility); and made the main grid selling prices extremely high, i.e., changed from 0.082 \$/kW to 82 \$/kW, to ensure that MG2 buys power from other MGs.

Table III shows the corresponding data considering the aforementioned modifications, with \( \alpha = 0.009 \) and \( \beta = 0.1 \).

In this case, there are now power exchanges among MGs, as shown in Table IV; for example, there is a power transfer of 26.67 kW between MG1 and MG2. The power transfer cost in the objective functions of the modified centralized and distributed problems represents around 5.2% of the objective function value of the centralized problem, and could be regarded as the tradeoff for attaining a decomposable problem suitable for solution with the distributed algorithm.

Once again, the distributed algorithm converges towards the solution of the centralized modified problem. Thus, Figs. 5 to 7 show the convergence of the selling prices, dual objective, and duality gap through the iterations, respectively. The duality gap converges towards zero as in the previous case, with 1,000 iterations being completed in 23.92 seconds, which is again a reasonable execution time.

TABLE III
MMG SYSTEM DATA FOR THE STRESSED CASE

| Parameter | MG1 | MG2 | MG3 |
|-----------|-----|-----|-----|
| \( a \) [\$/kW²] | 0.000132 | 0.0003 | 0.0001 |
| \( b \) [\$/kW] | 0.196 | 30 | 0.224 |
| \( c \) [\$] | 3.548 | 6.105 | 7.5 |
| \( D \) [kW] | 210 | 325 | 75 |
| \( P^{m}_{PCC} \) [kW] | 100 | 200 | 100 |
| \( P_{m} \) [kW] | 200 | 150 | 200 |
| \( c_{f,m} \) [\$/kW] | 82 | 82 | 82 |
| \( c_{m,d} \) [\$/kW] | 0.05 | 0.05 | 0.05 |

C. Individual and Cooperative Operation

A final experiment was carried out to compare the costs of MGs operating on their own, with no power exchanges, against...
the cost when there are power exchanges. For this test, the
stressed case data was used, with the modification of increasing
the maximum generation capacity of MG1 and MG2, from
200 kW and 150 kW to 250 kW and 350 kW, respectively,
to ensure feasibility when there are no power exchanges, and
the PCC limits of all MGs were set to zero to restrict the
power exchanges. The results are shown in Table V, where the
first column shows the operation costs of each MG and the
complete system, when operating individually, and the second
column shows the operation costs when power exchanges are
enabled. The third column shows the difference between the
individual and cooperative operation modes, which represents
the improvement in costs achieved by exchanging power. Note
that all MGs improve their individual operation costs as a
consequence of the exchanges, which reduces the total cost of
the system from $9,863.17 to $6,716.93 (around 32%). The
degree of improvement will depend on the generation costs of
each MG, their demand, the costs of buying and selling
electricity from and to the main grid, and the PCC limits. The
possibility of power exchanges allows MGs to benefit from
higher or lower costs to meet their demand.

### TABLE V

|                | Individual | Cooperative | Difference |
|----------------|------------|-------------|------------|
| MG1 [$]        | 50.52      | -980.68     | 1,031.20   |
| MG2 [$]        | 9,787.79   | 7,673.75    | 2,114.04   |
| MG3 [$]        | 24.86      | 23.86       | 1.00       |
| Total Cost [$] | 9,863.17   | 6,716.93    | 3,146.24   |

### V. Conclusion

This paper presented a model for the optimal operation of
an MMG system that considers power exchanges among a set
of MGs and the main grid. It was assumed that the interaction
among MGs occur through the main grid, and therefore
the MGs PCC could limit the amount of power exchange.
The model was decomposed using Lagrangian relaxation and
solved through an iterative distributed procedure using the
subgradient method to find the solution of the corresponding
dual problem, thus preserving the generation cost and demand
information of each MG. Results showed that the distributed
algorithm converges to the optimal solution with high precision within a reasonable execution time, making the proposed model a viable alternative for implementing a distributed EMS in an MMG system. Furthermore, the operation costs of all MGs improved when engaging in power exchanges, as opposed to their individual operation.

The developed formulation was based on a simplified model, designed to focus on the decomposition method and the benefits of the distributed approach, to facilitate their understanding. Future work will include more detailed modeling to consider relevant components in the MG, as well as uncertainties in renewable generation and demand. Furthermore, the convergence performance of the subgradient method could be improved by applying parallel processing techniques, which may significantly reduce execution times. Other methods for updating the multipliers could be also explored.

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