Structure of the exotic $^9$He nucleus from the no-core shell model with continuum

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Background: The exotic $^9$He nucleus, which presents one of the most extreme neutron-to-proton ratios, belongs to the $N = 7$ isotonic chain famous for the phenomenon of ground-state parity inversion with decreasing number of protons. Consequently, it would be expected to have an unnatural (positive) parity ground state similar to $^{11}$Be and $^{10}$Li. Despite many experimental and theoretical investigations, its structure remains uncertain. Apart from the fact that it is unbound, other properties including the spin and parity of its ground state and the very existence of additional low-lying resonances are still a matter of debate.

Purpose: In this work we study the properties of $^9$He by analyzing the $n^+^9$He continuum in the context of the $ab$ initio no-core shell model with continuum (NCSMC) formalism with chiral interactions as the only input.

Methods: The NCSMC is a state-of-the-art approach for the $ab$ initio description of light nuclei. With its capability to predict properties of bound states, resonances, and scattering states in a unified framework, the method is particularly well suited for the study of unbound nuclei such as $^9$He.

Results: Our analysis produces an unbound $^9$He nucleus. Two resonant states are found at the energies of $\sim$1 and $\sim$3.5 MeV, respectively, above the $n^+^9$He breakup threshold. The first state has a spin-parity assignment of $J^+ = 1/2^-$ and can be associated with the ground state of $^9$He, while the second, broader state has a spin-parity of $3/2^-$. No resonance is found in the $1/2^+$ channel, only a very weak attraction.

Conclusions: We find that the $^9$He ground-state resonance has a negative parity and thus breaks the parity-inversion mechanism found in the $^{11}$Be and $^{10}$Li nuclei of the same $N = 7$ isotonic chain.

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I. INTRODUCTION

The study of neutron-rich nuclei, located far from the line of stability, is one of modern nuclear physics frontiers. From a theoretical perspective, these nuclei open new questions into the importance of many-body forces at extreme neutron excesses, and challenge our current computational techniques. From an experimental perspective, these nuclei are difficult to produce in sufficient quantities and are also challenging to analyze. Nevertheless, much interest has been generated by past experiments and theoretical calculations; an interest that will be further renewed once the next generation of rare-isotope facilities such as FRIB (USA) $^1$ become available.

The Helium isotopes chain, $^3$--$^9$He, is one of the few accessible to both detailed theoretical and experimental studies. In the case of $^9$He, the neutron to proton ratio is $N/Z = 3.5$, making it one of the most neutron extreme systems studied so far. The $^9$He system is particularly interesting theoretically since it is part of a series of $N = 7$ isotones in which it is believed that intruder states from the $1s0d$ shell are pushed down in energy into the $0p$ shell, promoting the possibility of a positive parity ground state. $^{11}$Be is the most famous example having an un-natural parity assignment for the ground state, which has been calculated theoretically $^2$--$^{20}$ as well as observed experimentally $^{21}$--$^{29}$. The same phenomenon is found in $^{10}$Li $^{30}$--$^{34}$, making it quite natural to hypothesize that the same trend continues for $^9$He. Both experimental $^{35}$--$^{47}$ and theoretical $^{48}$--$^{61}$ efforts have been dedicated in the past to probe this hypothesis. Experimentally the situation is still under debate and a detailed history of the experimental studies on $^9$He can be found in Ref. $^{[46]}$. Here we provide a brief summary of the experimental and theoretical results concerning the still open questions of the spin-parity of the ground state, and the existence of excited states.

The first experiment on $^9$He was performed by Seth et al. $^{[35]}$ in 1987, who found an unbound ground state at $1.13 \pm 0.10$ MeV above the neutron decay threshold with
spin-parity assignment of $J^π = 1/2^-$. Successively, other experiments were performed [36–39] confirming the same $1/2^-$ bound ground state while revising its energy to $1.27 \pm 0.10$ MeV; in particular, in Ref. [38] this state was identified as a narrow resonance with a width of $\Gamma = 100 \pm 60$ keV. The conclusion was that $^9$He breaks the trend of parity inversion observed in $^{11}$Be and $^{10}$Li. This was also supported by some theoretical results [48, 52] while contradicting other calculations [4, 50, 51, 53].

Then in 2001 Chen et al. [45] observed a $1/2^+$ ground state corresponding to a virtual state of energy less than 0.2 MeV above the neutron decay threshold characterized by an S-wave scattering length of $a_0 \lesssim -10$ fm, indicating for the first time parity inversion in the $^9$He nucleus. These results were also consistent with shell model calculations [49]. The presence of a $1/2^+$ state was also supported by some theoretical results [48, 52] directly accessing $^9$He by studying the intrinsic Hamiltonian.

In this paper we study the $^9$He nucleus by analyzing the $n + ^8$He continuum in the framework of the $ab\text{\ initio}$ no-core shell model with continuum (NCSMC) [62–64] that treats bound and unbound states in a unified way. This approach is based on a basis expansion with two key components: one describing all nucleons close together, forming the $^9$He nucleus, and a second one describing the neutron and $^8$He apart. The former part is built from an expansion over square-integrable many-body states treating all nine nucleons on the same footing. The latter part factorizes the wave function into products of $^8$He and neutron components and their relative motion with proper bound-state or scattering boundary conditions.

As the nuclear interaction input to our calculations we adopt nucleon-nucleon (plus three-nucleon) forces from chiral EFT [65, 66].

The paper is organized as follows: in Section II we outline the formalism of our calculation, giving a brief description of the NCSMC. We also detail our selection of input chiral interactions. In Section III we first present our results for the binding energies of $^4,^6,^8$He and then those obtained for $n + ^8$He scattering in the NCSMC formalism. Finally, in Section IV we summarize our findings and draw our conclusions.

II. THEORETICAL FRAMEWORK

A. NCSM

The no-core shell model (NCSM) [67–69] treats nuclei as systems of A non-relativistic point-like nucleons interacting through realistic inter-nucleon interactions. All nucleons are active degrees of freedom. The many-body wave function is cast into an expansion over a complete set of antisymmetric A-nucleon harmonic-oscillator (HO) basis states containing up to $N_{\text{max}}$ HO excitations above the lowest Pauli-principle-allowed configuration:

$$|\Psi_A^{J^πT}\rangle = \sum_{N=0}^{N_{\text{max}}} \sum_i c_{Ni}^{J^πT} |AN_i^{J^πT}\rangle.$$  (1)

Here, $N$ denotes the total number of HO excitations of all nucleons above the minimum configuration, $J^πT$ are the total angular momentum, parity and isospin, and $i$ additional quantum numbers. The sum over $N$ is restricted by parity to either an even or odd sequence. The basis is further characterized by the frequency $\Omega$ of the HO well. Square-integrable energy eigenstates expanded over the $N_{\text{max}} \cdot \hbar \Omega$ basis, $|AN_i^{J^πT}\rangle$, are obtained by diagonalizing the intrinsic Hamiltonian.

B. NCSMC

In most experiments, the properties of $^9$He are inferred from coincidence measurements involving a neutron and a $^8$He fragment being simultaneously detected. Thus, we can model the $^9$He continuum as a state of a neutron plus a $^8$He in relative motion. In this regard, the binary
cluster formulation of the NCSMC is well suited in particular at energies below the $^9\text{He}$ breakup threshold of $\sim 2.14$ MeV [70].

The $^9\text{He}$ wave function is represented as the generalized clustered expansion

$$|\Psi_{A=N}^{J^T}\rangle = \sum_{\lambda} c_{\lambda}^{JT} |^9\text{He}, \lambda J^T T\rangle + \sum_{\nu} \int dr r^2 \gamma_{\nu}^{J^T T}(r) A_\nu |\Phi^r_{\nu}^{J^T T}\rangle . \quad (2)$$

The first term consists of an expansion over NCSM eigenstates of the aggregate system ($^9\text{He}$) indexed by $\lambda$. These states are well suited to explain the localized correlations of the 9-body system, but are inadequate to describe clustering and scattering properties. The latter properties are addressed by the second term corresponding to an expansion over the antisymmetrized channel states

$$|\Phi^r_{\nu}^{J^T T}\rangle = \left[\left( ^9\text{He}, \lambda_1 J_1^T T_1\right) |n \; \frac{1}{2} \frac{1}{2} \right]^{(sT)} Y_\ell(\vec{r}_{8,1}) \right]^{(J^T T)} \times \frac{\delta(r-r_{8,1})}{r r_{8,1}} , \quad (3)$$

in the spirit of the resonating group method [71–75], which describe the $^9\text{He} + n$ in relative motion. Here, $r_{8,1}$ is the separation between the center-of-mass of $^9\text{He}$ and the neutron and $\nu$ is a collective index for the relevant quantum numbers. The $^9\text{He}$ wave function is also obtained within the NCSM with the same Hamiltonian adopted for the whole system.

The discrete expansion coefficients $c_{\lambda}^{JT}$ and the continuous relative-motion amplitudes $\gamma_{\nu}^{J^T T}(r)$ are the solution of the generalized eigenvalue problem derived by representing the Schrödinger equation in the model space of expansion (2) [64]. The resulting NCSMC equations are solved by the coupled-channel R-matrix method on a Lagrange mesh [76–78].

**C. Interaction input**

The microscopic Hamiltonian can be written as

$$\hat{H} = \frac{1}{A} \sum_{i<j=1}^A \left( \frac{\vec{p}_i - \vec{p}_j}{2m} \right)^2 + \sum_{i<j=1}^A \hat{V}_{ij}^{NN} + \sum_{i<j<k=1}^A \hat{V}_{ijk}^{3N} + \ldots , \quad (4)$$

with the interaction consisting of realistic nucleon-nucleon ($NN$) and typically also three-nucleon ($3N$) and even higher-body contributions that accurately reproduce few-nucleon properties. In the NCSM and NCSMC calculations we typically employ interactions derived in the framework of chiral effective field theory (EFT) [65, 66]. Chiral EFT uses a low-energy expansion in terms of $(Q/A)^n$ that allows for a systematic improvement of the potential by an increase of the chiral order $n$. Here $Q$ relates to the nucleon momentum/pion mass and $A_\chi$ corresponds to the break down scale of the chiral expansion that is typically on the order of 1 GeV. The chiral expansion provides a hierarchy of $NN$, $3N$, and many-nucleon interactions in a consistent scheme [79–82].

To accelerate convergence of the NCSM and NCSMC calculations, one can employ the similarity renormalization group (SRG) technique [83–87] to soften the chiral interaction and, in the standard scheme, keep two- and three-body SRG induced terms in all calculations, even in the case when the initial chiral $3N$ force is not included.

We performed exploratory calculations with several chiral interactions including the next-to-next-to-next-to-leading order ($N^3\text{LO}$) $NN$ of Ref. [88] combined with the $3N$ interaction at next-to next-to leading order ($N^2\text{LO}$) [89] as well as the $N^2\text{LO}_{\text{sat}} N^N+3N$ interaction [90]. However, due to the technically complex task of including the $3N$ interaction in the NCSMC we were able to perform $^9\text{He}$ calculations only up to $N_{\text{max}} = 7$ or 9. Such basis spaces turned out to be insufficient to obtain conclusive results about the behavior of the $S$-wave scattering in particular. Consequently, we decided to limit ourselves to the two-body component of the SRG-evolved $NN$ interaction.

In particular, we employed the new $NN$ chiral potential at $N^4\text{LO}$ developed by Entem, Machleidt, and Nosyk [91, 92] with a cutoff $\Lambda = 500$ MeV in the regulator function introduced to deal with the infinities in the Lippmann-Schwinger equation. We did not use the bare interaction as the convergence would require a basis size well beyond our computational capabilities, but we softened the $NN$ potential via the SRG and discarded the induced three-nucleon forces. As the chiral $3N$ interaction is typically attractive in light nuclei while the SRG induced $3N$ interaction is repulsive, it is possible to find an SRG resolution scale for which the net effect of the $3N$ forces tends to be suppressed, and disregarding them leads to binding energies close to experiment. In any case, with this interaction, we were able to reach $N_{\text{max}} = 11$ (with the m-scheme dimension of $\sim 350$ million for $^9\text{He}$) and understand the phase shift behavior in all partial waves as demonstrated in the next section. We note that in the basis spaces we could reach with the $NN+3N$ interactions, our results were qualitatively consistent at a given $N_{\text{max}}$ with those obtained with the $NN$-only interaction presented in the next section.

**III. RESULTS**

As stated in subsection II C, in the present work we used the new $NN$ chiral potential at $N^4\text{LO}$ [91, 92] with a cutoff $\Lambda = 500$ MeV that we evolved via the SRG, discarding both the induced and the initial chiral three-nucleon forces. In general, the more the potential is evolved the faster the many-body calculations converge, but induced $3N$ forces become larger and larger, such that the net effect of the $3N$ forces (initial plus induced) is no longer negligible. Our strategy to select the value
of the SRG evolution parameter $\lambda_{\text{SRG}}$ was to reproduce closely the binding energy of $^4\text{He}$ and obtain realistic ones for $^6\text{He}$ with the least amount of evolution. For our purposes it is important to show that our interaction predicts $^8\text{He}$ bound with respect to $^6\text{He}+2n$ and $^6\text{He}$ bound with respect to $^4\text{He}+2n$. For this reason, all our calculations have been performed with an SRG evolved $NN$ potential with $\lambda_{\text{SRG}} = 2.4 \text{ fm}^{-1}$, which has been identified as a satisfactory value.

A. NCSM calculations for $\text{He}$ isotopes

We begin the discussion of our calculations with the NCSM results for $^4\text{He}$ and $^8\text{He}$, with the latter very important for the subsequent NCSMC study of $^9\text{He}$. Let us note that we have developed a three-cluster version of the NCSM applicable to $^6\text{He}$ in particular [93, 94] that provides a superior description of this nucleus compared to a simple NCSM calculation. However, since we are interested here only in the ground-state energy of $^6\text{He}$ the NCSM, upon extrapolation to the infinite model space, is sufficient.

In Fig. 1 and in Fig. 2 we present the $^6\text{He}$ and $^8\text{He}$
ground-state energies as functions of $\hbar\Omega$ and for different values of $N_{\text{max}}$. In both cases, the computed energies display a convergence toward the experimental value represented by the dashed line; a rapid convergence is particularly evident for $^6\text{He}$ that can be calculated up to $N_{\text{max}} = 12$. For both these nuclei, the variational NCSM calculations obtained with the largest $N_{\text{max}}$ values exhibit a minimum in correspondence of $\hbar\Omega = 20$ MeV, which was then chosen for our subsequent $^9\text{He}$ investigation. Due to the convergence pattern, it is possible to extrapolate the energies for the higher $N_{\text{max}}$ values using the exponential function

$$E(N_{\text{max}}) = E_{\infty} + a e^{-b N_{\text{max}}},$$

(5)

where $a$, $b$, and $E_{\infty}$ are free parameters and $E_{\infty}$ represent the extrapolated energy in the limit of $N_{\text{max}} \to \infty$. The extrapolated energies at $\hbar\Omega = 20$ MeV are displayed with shaded bands because they include the theoretical error that was obtained as the difference between the fit done using the three points obtained with the last three $N_{\text{max}}$ values and a second fit in which we used the last four values, and it was estimated of the order of 0.3 MeV.

In Fig. 3 and in Fig. 4 we show the calculated and extrapolated energies as functions of $N_{\text{max}}$ computed with $\hbar\Omega = 20$ MeV, where the point at infinity corresponds to the $E_{\infty}$ parameter of Eq. (5). These results are summarized in Tab. I, where we also report the ground-state energy of $^4\text{He}$ computed within the NCSM. In this case the calculation was done up to $N_{\text{max}} = 20$ and the result is fully converged to a keV precision and close to the experimental value.

In Fig. 4, we also present the $^9\text{He}$ NCSM eigenergies of the $1/2^-$ and $1/2^+_1$ states up to $N_{\text{max}} = 10$ and 11, respectively, that serve as inputs into our NCSMC calculations of $^9\text{He}$.

We also investigated the convergence of the $^8\text{He}$ $2^+$ excited-state energy relevant for our NCSMC study of $^9\text{He}$. Using $\hbar\Omega = 20$ MeV, we find a change of the $E_x(2^+_2)$ from 4.67 MeV at $N_{\text{max}} = 6$ to 4.22 MeV at $N_{\text{max}} = 10$. This is a reasonable yet somewhat slower convergence rate compared to a typical well-bound-state calculation that can be attributed to the fact that the calculated $2^+$ state corresponds to an experimentally unbound state.

Table I. Ground-state energies of $^4,^6,^8\text{He}$ in MeV. NCSM calculations are performed using the SRG-evolved $^4\text{LO} N N$ potential $[91, 92]$ with $\lambda_{\text{SRG}} = 2.4$ fm$^{-1}$. All results are obtained using the HO frequency $\hbar\Omega = 20$ MeV. The $^4\text{He}$ energy is computed up to $N_{\text{max}} = 20$ and is converged to a keV precision, while the energies for $^6,^8\text{He}$ are extrapolated using Eq. (5).

| $E_{\text{g.s.}}$, MeV | $^4\text{He}$ | $^6\text{He}$ | $^8\text{He}$ |
|------------------------|--------------|--------------|--------------|
| NCSM                   | -28.36       | -28.94(20)   | -30.23(30)   |
| Exp.                   | -28.30       | -29.27       | -31.41       |

Figure 5. (Color online) Dependence of the NCSMC results from the $N_{\text{max}}$ basis size of the $^2S_{1/2}$ phase shift as a function of the kinetic energy in the center of mass. The SRG-evolved $^4\text{LO} N N$ potential $[91, 92]$ with $\lambda_{\text{SRG}} = 2.4$ fm$^{-1}$ and the HO frequency of $\hbar\Omega = 20$ MeV were used.

Figure 6. (Color online) The $^2P_{1/2}$ phase shift in analogy to Fig. 5.

### B. $^9\text{He}$ NCSMC calculations

We now present the NCSMC results for the $^9\text{He}$ nucleus. As discussed in subsection IIIA and according to Eq. (2), we first computed the $^9\text{He}$ and $^8\text{He}$ eigenergies and wave functions within the NCSM by diagonalizing the Hamiltonian of Eq. (4). The NCSMC calculation of the $^9\text{He}$ system was performed within a model space up to $N_{\text{max}} = 11$ (10) for positive (negative) parity and including the six lowest positive-parity ($1/2^+, 5/2^+, 3/2^+, 5/2^+_2, 1/2^+_2, 3/2^+_2$) and the four lowest negative-parity ($1/2^-, 3/2^-, 3/2^-_2, 3/2^-_3$) NCSM eigenstates of $^9\text{He}$, while the binary-cluster sector was computed including the two lowest eigenstates of $^8\text{He}$, i.e. ($0^+, 2^+$).
approximately 0 shift is positive with a maximum in correspondence of N phase shift as a function of the kinetic energy for different the experimentally debated 1 and n 2 and the total angular momentum, respectively, of s quantum numbers 8 He, in agreement with the findings of the experiments of Refs. \cite{35–39}.

In Fig. 6 we show the convergence pattern obtained for the 2P_{1/2} phase shift. In this case we do not show the N_{max} = 5 results, because we obtain a bound state in this very small basis space for this channel. For higher values of N_{max}, the phase shifts present a good convergence and display a fairly narrow resonance, which bears the quantum numbers corresponding to the experimentally observed 1/2^- state. In Fig. 7 we show the convergence pattern for the 6P_{3/2} phase shift. Here the increase of the N_{max} value produces a shift of the curves towards smaller kinetic energies corresponding in part to the fall-off of the 8He 2^+ state excitation energy, which presents a somewhat slower convergence than the 8He ground state. The 6P_{3/2} phase shifts are also resonant, corresponding to a 3/2^- state, which is thus taken as the first excited state of 9He. It is important to notice that this state is built on the first 2^+ excited state of 8He; a simpler calculation with only the 8He ground state would not produce this resonance. As a final comment we mention that our calculations do not include the 6He + 2n channel that opens at 2.14 MeV excitation energy of 8He \cite{70}. This introduces some uncertainties in our results especially for the 3/2^- resonance, which appears at energy where this channel is open. We note that while we are able to perform NCSMC calculations with three-body cluster states \cite{93, 94}, the 9He investigation with the 6He + 2n channel open corresponds to a four-body cluster (6He+3n) that is beyond our computational capability at present.

A qualitative idea of the energy spectrum of the 9He nucleus, which summarizes the current analysis, can be inferred from Fig. 8 where we display the phase shifts including higher partial waves computed within the NC-SMC at N_{max} = 11. We only found two resonances in the 2P_{1/2} and 6P_{3/2} channels, corresponding to the 1/2^+ and 3/2^- states, respectively. In all other channels we did not find any resonance, especially in the 2S_{1/2} channel, which represents the experimentally much debated 1/2^- state.

Finally, in Fig. 9 we present the results for the eigenphase shifts obtained by diagonalizing the scattering matrix. While the phase shifts allow us to have an insight of the physics in the partial wave channels and identify the resonances, the eigenphase shifts take into account the coupling of different partial waves that are used for a quantitative analysis of these resonances. For example, they are used to compute the resonance centroid and width. In panel (a) of Fig. 9 we show the eigen-
E (which is an input to the NCSMC) to the experimentally obtained NCSM eigenenergy of the $^2\alpha$ state in $^8\text{He}$ at the experimental energy of 3.1 MeV. In both cases the results were obtained at $N_{\text{max}} = 11$ using the SRG-evolved $^4\text{LO} \, \text{NN}$ potential [91, 92] with $\lambda_{\text{SRG}} = 2.4 \text{ fm}^{-1}$ and the HO frequency of $\hbar \Omega = 20 \text{ MeV}$ were used.

\begin{table}[h]
\centering
\begin{tabular}{c|cc|cc}
\hline
$J^\pi$ & $E_R$ (MeV) & $\Gamma$ & $E_R$ (MeV) & $\Gamma$ \\
\hline
$1/2^+$ & 0.69 & 0.83 & 0.68 & 0.37 \\
$3/2^-$ & 4.70 & 0.74 & 3.72 & 0.95 \\
\hline
\end{tabular}
\caption{Theoretical values for the resonance centroids and widths in MeV for the $1/2^+$ ground state and the $3/2^-$ excited state of $^9\text{He}$. Calculations are carried out as described in Fig. 9 and in the text.}
\end{table}


\section{IV. CONCLUSIONS}

In this work we used the \textit{ab initio} NCSMC approach to study the $^9\text{He}$ resonances by analyzing the $n+^8\text{He}$ scattering process. The exotic $^9\text{He}$ is a very interesting nucleus due to its extreme $N/Z$ ratio and because it is part of a series of $N = 7$ isotones where, given the systematics, the ground state could be expected to be a positive-parity state. Despite having been extensively experimentally investigated, its structure is currently a matter of debate.

The NCSMC is a method capable of describing bound and unbound states in a unified way by combining an $A$-body square-integrable (which contains the many-body correlations) and a continuous basis (which enables the description of long-range interactions between cluster-type states). The NCSMC calculations do not involve any adjustable parameters except for those used to generate the $\text{NN}$ interaction, which is the input of our approach.

Our calculations were performed with the SRG-evolved NN interaction derived from the new chiral potential at $^4\text{LO}$ [91, 92] discarding the three-body terms. This choice was motivated by the large basis needed to obtain reliable results for the $^9\text{He}$ system, that makes the calculation with three-body forces computationally prohibitive at present. We softened the NN potential via the SRG transformation using $\lambda_{\text{SRG}} = 2.4 \text{ fm}^{-1}$ for the evolution parameter. With this choice, the predicted binding energies of $^4.6.8^\text{He}$ are close to the experimental values.

Our analysis identified two resonances corresponding to spin-parity states of $1/2^-$ and $3/2^-$ respectively. The former is identified as the ground state of $^9\text{He}$, while the latter is built on the $2^+$ state of $^8\text{He}$ and represents the first excited state of $^9\text{He}$. In particular we did not find any resonance corresponding to a $1/2^+$ state; according to our calculations $^9\text{He}$ breaks the parity inversion observed in $^{11}\text{Be}$ and in $^{10}\text{Li}$.

In the future we plan to study the $^9\text{He}$ nucleus including the three-body interactions. We note that we already performed exploratory calculations with the chiral $^2\text{LO}$ potential...
three-body forces for smaller values of the $N_{\text{max}}$ parameters with results qualitatively consistent with those obtained with the SRG-evolved N$^3$LO $NN$ at the corresponding $N_{\text{max}}$. Unfortunately, it is not currently possible to perform a calculation with the complete $3N$ force at $N_{\text{max}} \geq 10$ due to the tremendous computational effort. The only possibility to achieve this goal is to adopt a truncation scheme, such as the normal ordering \cite{97}, which aims to introduce a controlled approximation for the $3N$ terms and it is currently under development. Finally, we also plan to study the $p+^9\text{He}$ scattering with $^9\text{Li}$ as the composite system including the $n+^8\text{Li}$ charge exchange channel, which permits access to the $T = 5/2$ isobaric analog states of $^9\text{He}$.

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