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Phys. Rev. B 72, 180503R (2005)
Manifestations of the oscillations of the order parameter

\[ I_0 \sin(\varphi) \]

\[ S \quad F \quad S \]

V. V. Ryazanov et al., PRL 86, 2427–2430 (2001)

\[ I_0(\mu A) \]

T. Kontos et al., PRL 89, 137007 (2002)

\[ d=75 \text{ Å} \]

T. Kontos et al., PRL 86, 304 (2001)

\[ d/\xi_F \]

A. Bauer et al., PRL 92, 217001 (2004)
Boundary conditions for a diffusive S/F interface

Following D. Huertas-Hernando, Y. Nazarov and W. Belzig (2002)

\[ 2g_F \hat{G}_F \frac{\partial \hat{G}_F}{\partial x} = G_T [ \hat{G}_S, \hat{G}_F ] + \frac{iG_\phi \hat{D}_F^+ + G_{MR}}{2} [ \hat{D}^+, \hat{G}_F ] \]

\[ + iG_\chi [ \hat{G}_S \hat{D}^-, \hat{G}_F ] + iG_\xi [ \hat{D}^- \hat{G}_F, \hat{G}_F ] \]

- \( G_T = G_Q \sum_n T_n \) Tunnel conductance
- \( G_\phi = 2G_Q \text{Im} \left( \sum_n r_{n,\uparrow}^F r_{n,\downarrow}^F * - 4(t_{n,\uparrow}^S t_{n,\downarrow}^S *) / T_n \right) \) Phase-shifting conductance
- \( G_{MR} = G_Q \sum_n \left( |t_{n,\uparrow}^F|^2 - |t_{n,\downarrow}^F|^2 \right) \) Magnetoresistance term
- \( G_\chi = -G_Q \text{Im} \left( \sum_n t_{n,\uparrow}^S t_{n,\downarrow}^S * \right) \)
- \( G_\xi = G_Q \text{Im} \left( \sum_n T_n (r_{n,\uparrow}^F r_{n,\downarrow}^F * + t_{n,\uparrow}^S t_{n,\downarrow}^S *) / 4 \right) \)

\( F \) is weakly polarized \( T_n \ll 1 \)
Boundary conditions for a diffusive S/F interface

Following D. Huertas-Hernando, Y. Nazarov and W. Belzig (2002)

V. V. Kuprianov et Lukichev (1988)

\[
2g_F \frac{\partial \hat{G}_F}{\partial x} = G_T [\hat{G}_S, \hat{G}_F] + iG_\phi \hat{D}_F^+ + \frac{G_{MR}}{2} [\hat{D}_+^+, \hat{G}_F] + iG_\chi [\hat{G}_S \hat{D}_+, \hat{G}_F] + iG_\xi [\hat{D}_-^-, \hat{G}_F, \hat{G}_F]
\]

- \( G_T = G_Q \sum_n T_n \) \quad Tunnel conductance
- \( G_\phi = 2G_Q \text{Im} \left( \sum_n r_{n,\uparrow}^F r_{n,\downarrow}^F - 4(t_{n,\uparrow}^S t_{n,\downarrow}^S)^*/T_n \right) \) \quad Phase-shifting conductance
- \( G_{MR} = G_Q \sum_n \left( |t_{n,\uparrow}^F|^2 - |t_{n,\downarrow}^F|^2 \right) \) \quad Magnetoresistance term
- \( G_\chi = -G_Q \text{Im} \left( \sum_n t_{n,\uparrow}^S t_{n,\downarrow}^S \right) \)
- \( G_\xi = G_Q \text{Im} \left( \sum_n T_n (t_{n,\uparrow}^F t_{n,\downarrow}^F + t_{n,\uparrow}^S t_{n,\downarrow}^S)^*/4 \right) \)

\( \hat{D}^- = [\bar{m} \sigma \hat{t}_3, \hat{G}_S] \)
\( \hat{D}^+ = \{\bar{m} \sigma \hat{t}_3, \hat{G}_S\} \)

F is weakly polarized
\( T_n << 1 \)
Josephson current in a SFS junction

\[ I_S = \pi g_F k_B T \sum_{n, \sigma} Q_\sigma(\omega_n) / e \]

Asymmetric case: \( \gamma_t^R \gg \gamma_t^L \)

Asymmetric case:

\[
\begin{array}{ccc}
S & F & S \\
\Delta_L & \gamma^L & \gamma^R \\
\phi_S / 2 & \gamma^L & -\phi_S / 2 \\
0 & d & x \\
\end{array}
\]

\[
\theta^{\sigma}_{SFS}(x) = \theta^{\sigma}_{SFI}(x) + \delta \theta^{\sigma}(x)
\]

\[
\phi^{\sigma}_{SFS}(x) = \phi^{\sigma}_{SFI}(x) + \delta \phi^{\sigma}(x)
\]

d << \xi_F, T=0 \Rightarrow

\[
\left. \begin{array}{l}
e l_0 \\
\gamma_t^L G_t^R \Delta \\
\end{array} \right. = \pi \sin \left( \frac{d}{\xi_F} + \lambda(\gamma_\phi) \right) - \frac{d}{\xi_F}
\]

\[
\lambda(\gamma_\phi) = \arg(i - (1 + \gamma_\phi))
\]

\[
\gamma_\phi = G_\phi / G_t
\]
Josephson current in a SFS junction
very asymmetric case

\[ \frac{eI_o}{\Delta \gamma_t G_t} \]

\[ \Delta L = \Delta_R = \Delta \]
\[ k_B T / \Delta = 0.15 \]
\[ \gamma_t^R - \gamma_t^L \]

\[ \gamma_{\phi}^R = +2 \]
\[ \gamma_{\phi}^R = -0.5 \]
\[ \gamma_{\phi}^R = -1 \]

\[ d / \xi_F \]

\[ \gamma_{\phi}^R \]

\[ 0 \]

\[ \pi \]
Theoretical interpretation of Proximity effect measurements in PdNi

$\alpha_t^0 = 0.4$

$\xi = 36 \text{ Å}$

$\gamma^0 = 0$

$\gamma^0 = -1.3$

$\xi = 50 \text{ Å}$

$\gamma^0 = 0$

$\gamma^0 = -1.6$

Theoretical interpretation of Proximity effect measurements in PdNi
CONCLUSIONS

A new framework for describing the superconducting proximity effect in diffusive ferromagnets:

- The interfaces are characterized by $G_T$ and $G_\phi$

- $G_\phi$ modifies the phase and amplitude of the spatial oscillations of physical signals

- New interpretation of proximity effect measurements in PdNi

- Identification of experimental signatures of spin-dependent interfacial phase shifting
Superconducting proximity effect at a superconducting/normal interface (S/N)

Andreev reflection

\[ \langle \psi_\uparrow \psi_\downarrow \rangle \]

\[ \exp(-x/\xi_N) \]

\[ \xi_N = \sqrt{\frac{\hbar D}{k_B T}} \]

\[ D \text{ diffusion constant in N} \]
PROGRAM:

1) Proximity effect in diffusive S/F circuits

2) New boundary conditions for describing S/F interfaces

3) Predictions for S/F, S/F/I and S/F/S geometries

4) Interpretation for the proximity effect measurements in Nb/PdNi

5) Conclusions
Superconducting proximity effect at a superconducting/ferromagnetic interface (S/F)

\[ \langle \psi_\uparrow \psi_\downarrow \rangle \]

\[ \exp\left(-\frac{x}{\xi_F}\right) \]

\[ \xi_F = \sqrt{\frac{\hbar D}{E_{ex}}} \]

\[ Q = \frac{E_{ex}}{\hbar v_F} \]

\[ D \quad \text{diffusion constant in F} \]
Theoretical description of proximity effect from Gorkov to Usadel equations

**BCS description**
\[
\Gamma \left( \vec{P}, \vec{r}, E \right) \langle \vec{\rho} \rangle |_{\vec{P}}
\]
\[
\vec{r} = (\vec{r}_1 + \vec{r}_2) / 2
\]
\[
\vec{R} = (\vec{r}_1 - \vec{r}_2) / 2
\]

**Quasi-classical limit**
\[
\vec{v}_F
\]
\[
\hat{g}(\vec{v}_F, \vec{r}, E)
\]

**Diffusive limit**
\[
\hat{G}(\vec{r}, \omega_n)
\]
\[
\langle \vec{v}_F / \vec{v}_F \rangle
\]
\[
D = v_F l / 3
\]
\[
\omega_n = \pi (2n + 1) k_B T
\]

**Usadel Eqs.**
\[
D \partial_r \left( \hat{G} \partial_r \hat{G} \right) = \left[ (\omega_n + iE_{ex} \hat{\sigma}) \hat{\tau}_3 + \hat{\Delta}(\vec{r}) \right] \hat{G}
\]

**Gorkov Eqs.**
**Eilenberger Eqs.**
Spin degenerate boundary conditions for a diffusive S/N interface

V. V. Kuprianov et Lukichev (1988)

Barrier described in a scattering approach

\[ S_\sigma = \begin{bmatrix} r_{n,\sigma}^S & t_{n,\sigma}^N \\ t_{n,\sigma}^S & r_{n,\sigma}^N \end{bmatrix} \]

\[ S_\uparrow = S_\downarrow \]

Tunnel limit:

\[ T_n = \sum_{\sigma} |t_{n,\sigma}|^2 \ll 1 \implies 2g_N \hat{G}_N \frac{\partial \hat{G}_N}{\partial \chi} = G_T [\hat{G}_S, \hat{G}_N] \]

\[ G_T = G_Q \sum_n T_n \]

\[ G_Q = e^2 / \hbar \]

\[ g_N : \text{Conductance per unit length of N} \]
Theoretical interpretation of Proximity effect measurements in PdNi

\[ \xi_F = \left( \frac{\hbar D}{E_{ex}} \right)^{1/2} \]

\[ a_t^0 = \left. \frac{G_i^L d}{g_F} \right|_{d \text{ small}} = 0.4 \]

Data of T. Kontos et al. (2001, 2002)
Theoretical interpretation of Proximity effect measurements in PdNi

Usadel Equations
+ spin degenerate boundary conditions of V. V. Kuprianov et Lukichev (1988)

\[ \xi_F = \left( \frac{\hbar D}{E_{ex}} \right)^{1/2} \]

\[ a_t^0 = \left. \frac{G_t^L d}{g_F} \right|_{d \text{ small}} = 0.4 \]

Data of T. Kontos et al. (2001, 2002)
Spin-dependent interfacial scattering

Following D. Huertas-Hernando, Y. Nazarov and W. Belzig (2002)

\[ t_{n,\sigma}^{1(2)} = |t_{n,\sigma}^{1(2)}| e^{i\varphi_{t,\sigma}^{1(2)}} \]

\[ r_{n,\sigma}^{1(2)} = |r_{n,\sigma}^{1(2)}| e^{i\varphi_{r,\sigma}^{1(2)}} \]

Spin-dependence of the phases!
Spin dependence of the interfacial diffusion phase shifts
Effects in the F/N/F diffusive case

\[ G_{\text{mix}} = G_Q \sum_n \left( 1 - r_{n,\uparrow}^N r_{n,\downarrow}^N \right) \]

\[ \text{Im}(G_{\text{mix}}) = 0 \text{ if } \varphi_{r,\uparrow}^N = \varphi_{r,\downarrow}^N \]

A. Braatas et al., (2001)

S. Urazdin et al.,
Phys. Rev. B 71, 100401 (2005)

Spin accumulation suppressed by interfacial precession effects for \( \theta \neq 0, \pi \)
S/F/N ballistic point contact

points contact

lines $T_s = 0.05$
symbols $T_s = 0.1$

$D_{\varphi} = \pi$

$D_{\varphi} = \pi/2$

$D_{\varphi} = 0$

Andreev resonances:

$$2 \varphi_A - \sigma \left[ D_{\varphi} \right] = 0 \ [2\pi]$$

$$D_{\varphi} = \varphi_{r,\uparrow} - \varphi_{r,\downarrow}$$

$$\varphi_A = 2 \arccos \left( \frac{eV}{\Delta} \right)$$

E. Zhao et al. PRB 70, 134510 (2004)
Angular parametrisation of the problem

- Pairing angle: $\theta_\sigma(\omega_n)$
- Superconducting phase: $\varphi_\sigma(\omega_n)$

Bulk solutions:
- In S: $\theta_\sigma(\omega_n) = \pi / 2$
- In F: $\theta_\sigma(\omega_n) = 0$

Usadel equations in the limit of a weak proximity effect:

\[
\frac{\partial^2 \theta_\sigma}{\partial x^2} - \frac{k_\sigma^2}{\xi_F^2} \theta_\sigma - \frac{Q_\sigma^2}{\theta_\sigma^3} = 0
\]

with

\[
k_\sigma = \sqrt{2\left(i\sigma \text{sgn}(\omega_n) + \frac{\omega_n}{E_{ex}}\right)}
\]

\[
\xi_F = \left(\frac{\hbar D}{E_{ex}}\right)^{1/2}
\]
Description of S/F diffusive hybrid circuits in the limit of weak proximity effect

Boundary conditions in F for a S/F interface:

\[
\begin{align*}
\frac{\partial \theta_\sigma}{\partial x} &= G_T \left( \cos(\theta_S) \theta_\sigma - \sin(\theta_S) \cos(\varphi_\sigma - \varphi_S) \right) + iG_\phi \theta_\sigma \\
\frac{\partial \varphi_\sigma}{\partial x} &\theta_\sigma = G_T \sin(\theta_S) \sin(\varphi_\sigma - \varphi_S)
\end{align*}
\]

In this seminar:

- Rigid boundary conditions: \( \theta_S = \arctan(\frac{\Delta}{\omega_n}) \)
- \( \Delta \quad \text{E}_{ex} \quad \Rightarrow \quad k_\sigma = 1 + i\sigma \text{sgn}(\omega_n) \)
Semi-infinite S/F structure

\[
Q_\sigma = 0
\]

\[
\theta_{SF}^\sigma(x) = \frac{\gamma_t \sin(\theta_S)}{k_\sigma + i\gamma_\phi \sigma \text{sgn}(\omega_n)} e^{-k_\sigma \frac{x}{\xi_F}}
\]

\[
\gamma_t(\phi) = \frac{G_{t(\phi)\xi_F}}{g_F}
\]

![Graphs showing calculations and relations for semi-infinite S/F structure]
Experimental parameters for Nb/PdNi structures

S/F/I = Nb/PdNi/Alox/Al
S/F/S = Nb/PdNi/Alox/Al/Nb \implies \gamma^R_t / \gamma^L_t \sim 1

\Delta_{Nb}=1.35 \text{ meV}, \Delta_{Al/Nb}=0.6 \text{ meV} \implies E_{ex} \sim 10 \text{ meV} \implies k_\sigma = 1 + i\sigma \text{sgn}(\omega_n)

T=1.5K \sim T_c / 6

\begin{align*}
E_{ex} &\sim d \\
\Delta_{Nb} &\sim 1.35 \text{ meV} \\
\Delta_{Al/Nb} &\sim 0.6 \text{ meV} \\
k_\sigma &\sim 1 + i\sigma \text{sgn}(\omega_n) \\
T &\sim T_c / 6
\end{align*}
### Experimental parameters for Nb/PdNi structures

| \( d < d_{\text{sat}} \) | \( \frac{a_t^0 \xi_F^0}{d} \) | \( \gamma \phi \) | \( \xi_F \) |
|----------------|----------------|--------|--------|
| \( d > d_{\text{sat}} \) | \( \frac{a_t^0 \xi_F^0}{\sqrt{d_{\text{sat}} d}} \) | \( \gamma \phi \sqrt{\frac{d}{d_{\text{sat}}}} \) | \( \xi_F \sqrt{\frac{d_{\text{sat}}}{d}} \) |

- \( \gamma_t^L \)
- \( \gamma \phi \)
- \( \xi_F \)

\[ \sigma_F = 2e^2 N_0 D \]

\[ D = \nu_F l_e / 3 \]

\[ \xi_F = (\hbar D / E_{\text{ex}})^{1/2} \]

\[ \gamma_t(\phi) = \frac{G_t(\phi) \xi}{\sigma_F \eta} \]

\[ E_{\text{ex}} \propto d \]

\[ G_{\phi}^L \propto E_{\text{ex}} \]

3 fitting parameters: \( a_t^0, \gamma \phi, \xi_F \)

Constraints:

- Nb/Pd: \( a_t^0 = 0.2 \)
- \( \xi_F \approx 50 \text{ Å} \)
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\[ \alpha^0_t = 0.4 \]

\[ \xi^0 = 36 \text{ Å} \]

\[ \xi^0 = 50 \text{ Å} \]
Theoretical interpretation of Proximity effect measurements in PdNi

\[ \alpha^0_t = 0.4 \]

\[ \xi^0_F = 36 \text{ Å} \]

\[ \gamma^0_\phi = 0 \]

\[ \frac{e_d}{G^R} (\mu\text{eV}) \]

\[ N(0)/N_0 \]

\[ d(\text{Å}) \]