Mechanisms of Decoherence at Low Temperatures

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Abstract

We briefly review the oscillator and spin bath models of quantum environments, which can be used to describe the low-energy dynamics of open quantum systems. We then use them to discuss both the mechanisms causing decoherence at low $T$, and the dynamics of this decoherence. This is done first for a central 2-level system coupled to these environments- the results can be applied to the dynamics of quantum nanomagnets and superconducting SQUIDs, where large-scale tunneling of magnetisation and flux take place. Decoherence in these systems is caused principally by coupling to electrons and nuclear spins- the spin bath couplings are particularly dangerous at low $T$.

We may also generalise these models to discuss quantum measurements in which the measured system, the measuring apparatus, and the environment are treated quantum mechanically. The results can be used to calculate the dynamics of coupled SQUIDs and/or nanomagnets, in which one acts as a measuring apparatus and the other exhibits large-scale quantum superpositions. The same model can be used to describe coupled qubits.

Key words: Decoherence, Qubits, Measurement

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1 Introduction

The physics of “open” quantum systems encompasses a vast array of phenomena, ranging from sub-nuclear to cosmological scales. Nevertheless, in this thematic kaleidoscope there are a few central problems, whose generality makes them relevant at almost any scale. Here we look at one such problem – focusing on low energy scales (ie., low-T) where clear experimental tests of the-
ory are possible. The problem is that of decoherence – where it comes from, and how to understand its effect on the dynamics of quantum effects involving many degrees of freedom, at both microscopic and macroscopic scales.

Discussions of decoherence have often been associated with rather ineffable questions about quantum measurements, quantum cosmology, and quantum computation and some of this literature has tended to wander around in circles, saying little that might be quantitatively testable in the real world. This has given the subject an eccentric reputation, which is unfortunate, since many experiments are now searching for (and in some cases finding) macroscopic quantum phenomena in magnets and superconductors. At the same time entanglement experiments in optical cavities are showing multi-atom entangled states. In both kinds of experiment the crucial stumbling block preventing progress is decoherence coming from, amongst other things, coupling to environmental modes. The decoherence problem is thus an “open quantum systems problem” par excellence; it provides an extremely severe test of our understanding of the quantum dynamics of complex systems. Since the same physical mechanisms responsible for decoherence are involved in many other processes of relaxation in the quantum regime (as well as in the crossover to the classical regime), a lot turns on these developments.

In what follows, we first explain the decoherence problem, and describe some models used both for this problem and for more general discussions of quantum relaxation. We also describe the connection to experiments, particularly those in magnetic and superconducting systems. In both cases we explain why the most severe source of decoherence in the low-$T$ limit will often be the “spin bath” environment of localised modes (defects, spin impurities, nuclear spins) coupling to the collective coordinates (magnetisation, flux) of interest.

It is then but a small step to a discussion of the quantum measurement problem, for the simple reason that one can also use superconducting SQUIDs and nanomagnets as measuring systems themselves. Thus we also grasp the nettle and consider models in which the entire set-up (quantum system, apparatus, and environment) is quantised. This not only allows us to calculate the quantum dynamics of such a set-up, but also to discuss, in a more quantitative way than usual, the measurement problem itself. These same models also describe a pair of coupled mesoscopic qubits- and we discuss why decoherence typically has a much worse effect on the mutual coherence between these than it would on each qubit by itself.

**Decoherence Puzzles:** The basic idea of decoherence as a solution to the measurement problem has been around for some 50 years [1,2]; and as a physical phenomenon it has been noted since before Quantum Mechanics. At first glance it seems almost trivial- most physicists and chemists are familiar with the way in which the phase interference in a “2-slit” kind of experiment is
destroyed. So, why care about decoherence now? The short answer is that we now need to know rather urgently what mechanisms cause decoherence, particularly at the mesoscopic and macroscopic scales, or in entangled systems like quantum computers— and we find that there are many basic things that we don’t understand. One way to introduce the problems is in the form of a set of “decoherence puzzles” or questions, which are clearly down-to-earth and within the purview of physicists and chemists. Here are a few (some of intense current interest):

(1) **Mechanisms of Decoherence**: What are the mechanisms of decoherence in nature and how do they relate to those governing quantum relaxation? This problem is very difficult because decoherence is extraordinarily sensitive to the different environmental couplings in nature. Thus ad hoc theoretical models, devised for their solvability rather than their realism of generality, will not do here. We are entering a particularly exciting time for this puzzle, since experiments are now directly addressing coherence phenomena at the mesoscopic and macroscopic levels.

(2) **Measurement Puzzle**: Does decoherence “solve” the measurement problem (as many [1–3] have advocated)? How then must we understand measurement operations in systems where macroscopic coherence exists?

(3) **Dynamics of Decoherence**: How does decoherence (in the relevant components of the relevant density matrix) evolve in time? How does decoherence affect N-particle entanglement (e.g., in an N-qubit quantum computer [4]) and how do N-particle decoherence rates depend on N? How can the dynamics be controlled/tested experimentally?

(4) **The $\tau_{\phi}$ puzzle**: Why, in defiance of very basic theory [5] does the decoherence rate $\tau_{\phi}^{-1}(T)$ for electron propagation in mesoscopic conductors appear to saturate [6] to a finite constant at low $T$? Does a similar saturation exists in superconductors or magnets? How can $\tau_{\phi}^{-1}(T \to 0)$ be controlled?

(5) **Quantum-Classical Crossover**: This is induced by raising either temperature, or external fields, or temperature. The environment inducing the transition is usually neither in internal equilibrium nor in equilibrium with the system making the crossover. Yet this departure from equilibrium is actually crucial (in, for example, many discussions of quantum measurements); how can it be treated properly?

(6) **High Energies**: What is the role of decoherence in the early universe [7] (another possible quantum-classical crossover), or in black hole physics [8]? Can high-energy sources of decoherence up to or beyond the Plank scale have an effect at low energies [9]? How can this be tested experimentally?

Note that these puzzles are all more or less related, and that they span many
different areas in physics (with obvious repercussions in physical chemistry). It is also clear that the first puzzle, about the mechanisms governing decoherence, is quite central, and is the sort of bread-and-butter question that physicists and chemists deal with every day. Perhaps the first thing to understand is why it is possible to approach this puzzle in a unified way (rather than by piecemeal discussion of many different systems), and so we start with this.

2 Spin Baths and Oscillator Baths

Theorists working on the quantum dynamics of open systems like to have models that are both realistic (realistic enough to survive experimental tests on particular physical systems) and also general (so that the results are generic, or at least apply to a large class of physical systems). One way of deriving such such models is to divide the quantum environment into extended and localised modes, and then proceed as follows:

(i) Extended Modes: In a box of volume $V$ containing $N_o \sim \mathcal{O}(V)$ extended modes below an ultraviolet cutoff $E_c$, the coupling between these modes and some “central system” (collective coordinate, etc) of interest is necessarily $\sim \mathcal{O}(N_o^{-1/2})$. Then for large $N_o$ one can treat perturbatively and map to an environment of oscillators [10,11]. The general model is then described by a Hamiltonian $H = H_o(P,Q) + H_{os}^{\text{int}} + H_{os}(\{x_q,p_q\})$ where $H_o(P,Q)$ is the Hamiltonian of the “central system” we are interested in, having canonical coordinates $P, Q$, and the oscillator terms are [10,12]

\[
H_{os} = \frac{1}{2} \sum_{q=1}^{N_o} m_q (\dot{x}^2_q + \omega^2_q x^2_q) \\
H_{os}^{\text{int}} = \sum_{q=1}^{N_o} [F_q(Q)x_q + G_q(P)p_q]
\]

where the $\{x_q, p_q\}$ are canonical coordinates for the extended environmental modes (with $q = 1, 2, \ldots N_o$). The linear couplings $F_q, G_q \sim \mathcal{O}(N_o^{-1/2})$, so that the bath modes have individual effects $\sim \mathcal{O}(1/N_o)$ on the central system - summation over these gives a well defined thermodynamic limit independent of $N_o$ as $N_o \to \infty$.

The derivation of oscillator bath models generally uses some kind of Born/Opffenheimer-like assumption that one can separate “slow” system modes $(P, Q)$ and “fast” environmental modes $\{x_q, p_q\}$; such derivations are of course very
familiar to quantum chemists. To be valid they require that

$$V_{qq'}(P, Q) \ll (\omega_q - \omega_{q'})$$  \hspace{1cm} (3)$$

(or an appropriate generalisation to finite temperature $T$) for all $q, q', P, Q$ of interest, where the matrix element at a given $P, Q$ is given, in terms of the full multi-dimensional coordinate space $X$ of the environment, by

$$V_{qq'}(P, Q) = \int dX \, \phi_q^*(X) \, \hat{H}_{\text{int}}^{os} \phi_{q'}(X)$$  \hspace{1cm} (4)$$

where the wave-functions $\phi_q(X)$ are the eigenstates of the environmental Hamiltonian (for more details see, eg., Caldeira and Leggett [11]). Note that this restriction is not nearly as bad as it seems since one has considerable freedom in the choice of the environmental modes- indeed the $\{\omega_q\}$, as well as the couplings in (2), typically depend on $T$ (because, eg., one chooses a $T$-dependent effective Hamiltonian for the environment).

However, there are obvious cases where the assumption (3) breaks down. In the case of extended environmental modes a well-known example is the Quantum Hall system, where the environmental Landau levels are all degenerate- and the effect of $H_{\text{int}}^{os}$ is typically to break this degeneracy [13]. However the breakdown is much more common in the case of localised environmental modes, to which we now turn.

(ii) **Localised Modes**: In the same volume $V$ there will also be be many localised environmental modes, associated with defects, impurities, paramagnetic spins and the nuclear spin bath. Each of these has a finite Hilbert space in the energy range $< E_c$, and can thus be mapped to a spin degree of freedom, very often a spin $-1/2$, or two-level system. Some set $\{\sigma_k\}$ of these with $k = 1, 2, \ldots N_s$, will also couple to the central system, giving extra terms $H_{\text{int}}^{sp} + H_{sp}$ in the total Hamiltonian, where [14]

$$H_{sp} = \sum_{k} h_k \cdot \sigma_k + \sum_{k,k'} V_{kk'}^{\alpha\beta} \sigma_k^\alpha \sigma_{k'}^\beta$$  \hspace{1cm} (5)$$

$$H_{\text{int}}^{sp} = \sum_{k} \left[ F_k^\parallel (Q) \sigma_k^z + (F_k^\perp (P, Q) \sigma_k^+ + h.c.) \right]$$  \hspace{1cm} (6)$$

The important point here is that there is no requirement that the couplings $\{F_k^\parallel, F_k^\perp\}$ be $O(N_s^{-1/2})$; indeed very often they are independent of $N_s$ ! A simple example is the hyperfine coupling, crucial in magnetic systems since it couples the macroscopic magnetisation to the nuclear spins- this is obviously independent of $N_s$. Note that the couplings are not necessarily small either;
for example, hyperfine couplings can be as large as $0.5K$ each, and individual exchange couplings to paramagnetic impurities can be much larger again. Now 2 crucial points arise, viz.,

(i) The intrinsic dynamics of the environmental modes, coming from $H_{sp}$ in (5) is now governed by energy scales $|\hbar_k|$, $|V_{\alpha\beta}^{\alpha\beta}|$ which are often much weaker than the couplings $\{F^\parallel_k, F^\perp_k\}$, putting us then in an “anti- Born Oppenheimer” limit, where the environmental modes are strongly slaved to the system coordinates $P, Q$. This is because the modes, having localised wave-functions, usually couple very weakly to each other (via, eg., residual dipolar couplings).

We may again take magnetic systems as an example, where the internuclear coupling $|V_{k\alpha\beta}^{\alpha\beta}| \sim 10^{-8} K$ typically, i.e., up to 8 orders of magnitude smaller than the hyperfine couplings $\{F^\parallel_k, F^\perp_k\}$.

(ii) The Hilbert space of an oscillator is infinite-dimensional and thus quite different from that of a spin (or any environmental mode with only a few discrete levels). This would not be important if the weak-coupling assumption (3) were satisfied; but in the common case where it is not, one has to use spin propagators in dealing with the environmental modes.

Readers working in different fields will recognise many of their favourite models in these general forms (quite how general they are has been discussed at some length in the literature [12,14–16]). The models are completely specified once (a) the Central system Hamiltonian has been given, and (b) the various couplings (or more usually, their distributions over $q$ and $k$) are known. At this point two simplifying features usually emerge, viz.;

(a) The distribution of couplings to the environmental modes shows a fairly typical pattern, (see Fig. 1). Coupling to oscillator systems is usually much weaker at low energies and temperatures, as the available phase space disappears, except for the case of “Ohmic baths” such as conducting electrons in a metal (and even this coupling rapidly disappears at low temperatures if the metal goes superconducting, so that the electronic spectrum becomes gapped). Phonons and photons in particular have very weak low frequency couplings to most central systems of interest. On the other hand, the coupling to spin baths is often much stronger at low energies – this is particularly true for coupling to nuclear spin systems, which becomes much stronger as one goes to energies $\sim E_o = [\sum_k (F^\parallel_k)^2]^{1/2}$. For this reason, spin baths are often much more serious sources of low-$T$ decoherence than oscillator baths.

(b) At low temperature the Hamiltonian of the central system is often almost trivial. In microscopic systems like 2-level atoms this may not come as a surprise (although an atom is of course already a very complex many-body system). In mesoscopic or macroscopic quantum systems the central Hamiltonian is typically describing a collective coordinate which may have a quite
large “mass” or “inertia”, but whose dynamics simplifies at low energy because it is restricted by some confining potential. Thus one ends up with a few “canonical models” [17] which have been applied to many different physical situations - these include a 1-dimensional harmonic oscillator [18], a simple tunneling potential [11,16], and a variety of problems in which the central Hamiltonian $H_o$ describes one or more 2-level systems. In fact the problem of a single “central spin” (or “qubit”) is so important that we pause to sketch a few details.

(iii) **Spin-Boson & Central Spin Models:** If we couple a central two-level system to either an oscillator or a spin environment, we get two very useful models. In the case of an oscillator bath we get the spin-boson Hamiltonian [15,16]

$$H_{SB} = \Delta \hat{\tau}_x + \xi \hat{\tau}_z + \sum_q \left[ c_q^\parallel \hat{\tau}_z + (c_q^\perp \hat{\tau}^+ + h.c) \right] + H_{os}$$

(7)

with $H_{os}$ given in Eq. (1). The spin $\tau$ is in a longitudinal bias field $\xi$, with tunneling matrix element $\Delta$ between states $|\uparrow\rangle$ and $|\downarrow\rangle$; the couplings $c_q^\parallel, c_q^\perp \sim \mathcal{O}(N_o^{-1/2})$. This model has been widely used [15,16,19] to discuss systems like SQUIDs, or nanomagnets, or other 2-state mesoscopic systems coupled to delocalised excitations like electrons, phonons, magnons, photons, etc (it was originally introduced to discuss problems like nucleons [20] or the Kondo problem [21], involving microscopic 2-level systems).

If we couple to a spin bath we get the central spin Hamiltonian

$$H_{CS} = \left( \Delta \hat{\tau}_+ \exp i \sum_k \alpha_k \cdot \sigma_k + H.c. \right) + \sum_k \omega_k^z \hat{\tau}_k \sigma_k^z + H_{sp}$$

(8)

with $H_{sp}$ given in Eq. (5). Here, as noted above, $\omega_k^z$ and $\alpha_k$ are often independent of $N_s$. This model has been used to describe systems like nanomagnets [19] or SQUIDs [14,22,23] coupled to nuclear and paramagnetic spins.

The non-diagonal couplings $c_q^\perp$ and $\alpha_k$ in these models are typically smaller than the diagonal couplings - for a complete discussion of this point, see Ref. [14]. However, one should be careful not to drop them when discussing decoherence (see below).

These 2 models can be combined (i.e., a central spin couples simultaneously to spin and oscillator baths - this often happens in reality [14,19]), and they can also be generalised to discuss 2 or more spins coupled both amongst each other and to an environment - we will use such models in sections 5 and 6.

(iv) **Averaging over the environment:** To complete the discussion of these
models for open quantum systems one must specify how to average over or “integrate out” the environmental modes. This has been extensively reviewed [16,14,24] so we simply recall the results here. For both spin and oscillator baths the best way is to use path integral methods (lending themselves particularly to strong-coupling and tunneling problems). For the oscillator bath a common method is to write the propogator for the density matrix as a functional integral over the “influence functional” $\mathcal{F}[q,q']$, in the form [16,24]:

$$
K(Q_f,Q'_f;Q_i,Q'_i;t) = \int Dq(\tau) \int Dq'(\tau) e^{i(S_o[q]-S_o[q'])} \mathcal{F}[q(\tau),q'(\tau)]
$$

where $S_o[q]$ is the action of the free system (no environment) and $\mathcal{F}[q,q']$ is a functional over all possible paths $q(\tau), q'(\tau)$ of the system. $\mathcal{F}[q,q']$ is usually a rather complex object, and the functional integration over it is not easy either– what makes it possible at all is the weak coupling to the oscillators. The only cases for which the dynamics of $\hat{\rho}(t)$ have been established so far are when the central system is either an oscillator itself, a central spin (the spin-boson model) a pair of spins (the PISCES model discussed below), or a particle moving either in zero potential or hopping on a lattice, in the presence of oscillators.

For the spin bath a nice simplifying feature appears, originating from the finite Hilbert space associated with each spin bath mode. One begins by quantizing the bath spins along a $\hat{z}$ axis, and classifying all the $2^N$ bath states according to their total polarisation $M$ along this axis. Instead of functional integrals over all possible paths of the system, one has just ordinary integrals over the density matrix $\hat{\rho}(o)(t)$ of the free system [14]:

$$
\rho(Q,Q';t) = \hat{A}^T(\phi)\hat{A}^O(y)\hat{A}^B(\epsilon,M) \rho^{(o)}(\phi,y,\epsilon,M;Q,Q';t)
$$

where the parameters in the free density matrix $\rho^{(o)}(t)$ now become simple functions of (i) a phase variable $\phi$ associated with rotations of bath spins induced by transitions of the central system (ii) a similar variable $y$ associated with spin bath precession in between central system transitions, and (iii) the energy bias $\epsilon$ acting on the bath spins, and the polarisation $M$ of the bath. The averages $\hat{A}^T(\phi)$, $\hat{A}^O(y)$, and $\hat{A}^B(\epsilon,M)$ are just simple weighted integrations over these variables (with weighting functions depending on the couplings to the bath and the bath temperature). The replacement of functional integrals by ordinary integrals makes the calculation of dynamics for the spin bath much simpler than for oscillator baths- in fact one can usually find analytic answers. One sees this clearly in the example of the central spin model (8) (see ref [14] and below).
3 Decoherence for single “qubits”: Experimental Implications

Perhaps the best way to understand a formalism is to look at some important application of it to a real system. We therefore begin with a very important one, to experiments on mesoscopic SQUIDs and magnets—both systems are strong candidates for qubits in some future quantum computer. In fact many such experiments have been done in the last few years, which we can somewhat arbitrarily divide up as follows:

(i) Experiments claiming observation of coherence, or at least interference, between mesoscopic quantum states, either in SQUIDs [25,26] or quantum magnetic systems [27–29].

(ii) Experiments on incoherent tunneling at the mesoscopic or macroscopic scales, also in superconductors [30–32] and magnets [33–37]), done in response to theory [11,15,38–42] which indicated that even in the presence of a vast number of environmental modes or “microstates”, such tunneling was possible.

In fact the incoherent tunneling experiments were very successful— and amongst other things they indicate that the theoretical treatment of the environment works pretty well. Thus, the SQUID tunneling experiments of Clarke et al. [31] gave amazingly good agreement with the Caldeira-Leggett [11] predictions for electronic “oscillator bath” dissipation in SQUIDs. Recent experiments on crystalline ensembles of molecular magnets [37,43] (including direct verification of the role of nuclear spins, by isotopic variation [43]) also agreed quantitatively with the theoretical predictions [19,42,44] for nuclear spin-mediated incoherent tunneling in these systems. Thus at least for incoherent tunneling, the theory seems to work.

However, as emphasized long ago in the measurement theory literature, interference between 2 “classical” (i.e., mesoscopically or macroscopically different) states is a much more sensitive test of our understanding of decoherence mechanisms. In the same way it severely tests our understanding of superconductors and magnets, and indeed of many-body physics as a whole. The coherence experiments in SQUIDs and magnets are usually analysed using the spin-boson and/or central spin models just given, because in both cases the collective coordinate dynamics is governed by a “2-well” potential at low energies when $k_B T < \omega_o/2\pi$, where $\omega_o$ is roughly the same as the gap from the 2 lowest states in this potential, to the higher levels. In mesoscopic magnets this “spin gap” may easily be 10 $K$; in SQUIDs it is the Josephson plasma frequency, which can be made as large as 1 – 2 $K$ (incidentally, $\Omega_o$ does not decrease exponentially with the size of the system, as has sometimes been argued [2]; in magnets it may hardly depend on system size at all).

In this section we discuss the mechanics of decoherence for problems like these,
returning at the end of the section to make some brief comments about the experiments themselves. Note what is meant by coherence in the case of these 2-level systems – if, say, we start with the 2-level system in the pure state $|\uparrow\rangle$, then a purely coherent time evolution thereafter is described by the “free” density matrix $\rho_{ij}^{(o)}(t)$, where (see Fig. 2):

$$
\rho^{(o)}(t) = \begin{pmatrix}
\frac{1}{2} (1 + \cos (2\Delta t)) & \frac{i}{2} \sin (2\Delta t) \\
-\frac{i}{2} \sin (2\Delta t) & \frac{1}{2} (1 - \cos (2\Delta t))
\end{pmatrix}
$$

The job of the theory is to predict the evolution of $\rho_{ij}(t)$ in the presence of the environment – coherence between the states $|\uparrow\rangle$ and $|\downarrow\rangle$ is then described by the off-diagonal matrix elements $\rho_{12}$ and $\rho_{21}$. So far the experiments have not directly tested any decoherence calculations, and indeed the magnetic coherence experiments are controversial- see below. However they soon will, and so in this section we discuss what theory says for the decoherence dynamics of these off-diagonal matrix elements.

(i) **Spin-Boson Model:** We include only the diagonal coupling $c_0^\parallel$ for simplicity, and assume to be specific that the coupling is Ohmic and weak- formally this means that the Caldeira-Leggett [11] spectral function $J(\omega) = \pi \alpha \omega$ with $\alpha \ll 1$. This model then applies directly to the electronic contribution to decoherence in SQUID tunneling between flux states $\pm \phi_m$ with charging energy $E_c$ and a given Q-factor [38]; where $\alpha = (16 \phi_m^2 \omega_0 / E_c) Q^{-1}$. The dynamics for this model is fairly well-known [15,16]; in the weak-coupling limit one has standard exponential damping of the oscillations, with diagonal and off-diagonal decay rates given respectively by [15,45,46]:

$$
\Gamma_{11}(\Delta_r, \xi) = (\Delta_r / E)^2 \gamma(E)
$$

$$
\Gamma_{12}(\Delta_r, \xi) = \frac{1}{2E^2}[2\xi^2\gamma(0) + \Delta^2\gamma(E)]
$$

where the rate function $\gamma(E)$ is

$$
\gamma(E) = \pi \alpha E \coth(E/k_B T)
$$

and $E = (\xi^2 + \Delta^2_r)^{1/2}$ with a renormalised tunneling matrix element $\Delta_r$. A somewhat peculiar point is that the size of the off-diagonal matrix elements is also reduced, by a factor $\Delta_r / \Delta \sim (\Delta/E_c)^{\alpha/1-\alpha}$, which explicitly contains the high-energy cut-off in the oscillator bath spectrum (Fig. 3); similar effects can also be seen for a free particle [47].
We can think of these results as calculations of the rates $T_1^{-1} \equiv \Gamma_{11}$ and $T_2^{-1} \equiv \Gamma_{12}$, with $T_2 = 2T_1$ for the symmetric (zero bias) case $\xi = 0$. Thus for a spin weakly coupled to an oscillator bath, the decoherence rate $T_2^{-1}$ is simply related to the longitudinal rate $T_1^{-1}$. This is partly a reflection of the simplicity of the decoherence mechanism here— the longitudinal coupling $c_\parallel$ couples $\tau_z$ to a “fluctuating bias field” (the oscillators).

Anyone familiar with the usual arguments for the destruction of off-diagonal matrix elements as a solution to the “measurement problem” ought to be suspicious of this simple relationship between relaxation and decoherence in the spin-boson model. In fact it is too simple, indeed rather exceptional— as a general rule one expects no particular connection between them. Results from the spin-boson model can thus be misleading, so we now turn to the central spin results.

(ii) **Central Spin model**: From Fig. 1 we learn that most candidates for a “qubit” will begin to feel the deleterious effects of the spin bath at sufficiently low $T$. By itself the coupling $\omega_\parallel$ causes no decoherence at all— except for a rather weak contribution coming from the internal dynamics of the spin bath (driven by $V_{kk'}$). The 2 main sources of decoherence are (i) the precession of the spins in the spin bath between transitions of the central spin, driven by the term $\omega_\perp$, and (ii) transitions in the spin bath caused by the flipping of the central spin (coming from the $\alpha_k$ term— the probability for $\sigma_k$ to flip during a single central spin transition is $|\alpha_k|^2$ when $|\alpha_k| \ll 1$. Both of these decoherence mechanisms [42,14] involve the accumulation of a random phase by the environment, which when integrated over to obtain the reduced density matrix of the central spin, lead to decoherence. The extent of this decoherence is quantified using the parameters

$$\lambda = \frac{1}{2} \sum_k |\alpha_k|^2$$

and

$$\kappa = \frac{1}{2} \sum_k (\omega_\perp^k / \omega_\parallel^k)^2$$

(we assume $|\alpha_k|, |\omega_\perp^k / \omega_\parallel^k| \ll 1$). Roughly speaking, these dimensionless numbers tell us the mean phase accumulated by each of these mechanisms whilst it operates. Thus we expect strong decoherence if $\lambda > 1$ and/or $\kappa > 1$. To substantiate this we give the results for the off-diagonal matrix element $\rho_{12}(t)$ of the central spin density matrix in each case (for the diagonal matrix elements see refs. [42,14]), since these matrix elements quantify decoherence effects:

(a) **Topological Decoherence limit**: In this limit the main source of decoherence is the transitions in the spin bath caused by a transition in the central system (“co-flip processes”); the time-dependent perturbation on the bath spins,
coming from the central spin transition, changes the spin bath state, and each such transition gives a random phase (mean value $\lambda$) to the bath. One then has

$$\rho_{12}(t) = \hat{A}^T(\phi)\rho_{12}^{(o)}(\phi; t) = \sum_{\nu=-\infty}^{\infty} e^{i\nu\phi - 4\lambda\nu^2} \rho_{12}^{(o)}(\phi; t)$$  \hspace{1cm} (17)

which is easily evaluated using $\rho_{12}^{(o)}(\phi; t) = (i/2) \sin(\Delta(\phi)t)$, where the phase-dependent tunneling matrix element is found [42] to be $\Delta(\phi) = \Delta \cos(\phi)$. Only terms with odd winding number $\nu$ contribute, and one finds that

$$\rho_{12}(t) = \sum_{\nu} e^{-4\lambda(2\nu+1)^2} J_{2\nu+1}(\Delta t)$$  \hspace{1cm} (18)

In the case $\lambda \to 0$ (ie., the bath decouples from the central spin) this just goes back to the free spin result in (11). If $\lambda \sim O(1)$ or greater, we have $|\rho_{12}^{(o)}(t)| \sim O(e^{-\lambda})$, ie., exponential suppression of the off-diagonal matrix element (cf. Fig. 4)!

(b) Orthogonality Blocking limit: In this limit “co-flip” processes are unimportant, but the bath spin precession between each central spin transition now adds a random phase to the bath part of the total wave-function- when integrated over this also leads to decoherence, in much the same way as with topological decoherence. This mechanism becomes particularly important when the couplings $\{\omega_{k}\}$ are large compared to $\Delta$, in which case it is important to distinguish the different polarisation groups in the spin bath. We then have

$$\rho_{12}^{(o)}(t) = \hat{A}^O(y)\rho_{12}^{(o)}(y; t) \to \sum_{M=-N}^{N} w_M(T) \langle \rho_{12}^{(o)}(t, \Delta_M(y)) \rangle$$  \hspace{1cm} (19)

where the weighting function $w_M(T)$ tells us the fraction of spin baths, in a thermally weighted ensemble, to be found in the $M$-th polarisation group, the phase-weighted matrix element is now $\Delta_M(y) = \Delta J_M(\sqrt{2\kappa y})$ and the average is

$$\langle \rho_{12}^{(o)}(t, \Delta_M(y)) \rangle = \delta_{M,0} \int_0^{\infty} dy e^{-y} \rho_{12}^{(o)}(t, \Delta_M(y))$$  \hspace{1cm} (20)

This result is rather remarkable- since the fraction of spin baths in an ensemble which find themselves in polarisation group $M = 0$ is $\sim (2\pi N)^{-1/2}$, we have only a tiny contribution to the off-diagonal matrix elements! The physical reason for this result is actually very simple- for coherence to proceed, the
state of the nuclear bath must not change between initial and final states. But it is easy to see that in the present case this can only happen if \( M = 0 \), because otherwise the system cannot make resonant transitions (recall that \( \{\omega^k \} > \Delta \)). Consider, eg., the case \( M = 1 \). If no bath spins flip during the transition, then afterwards the central spin sees \( M = -1 \), ie., the total coupling energy, acting here like a longitudinal bias, has changed by roughly \( 2\omega^k \) (for more detail on this physics see ref. [19], Appendix B).

(iii) Experimental Implications: Amongst other things these results demonstrate the well-known fact that there is no necessary connection between the decay of diagonal matrix elements and those of off-diagonal ones. This may seem obvious to those familiar with either measurement theory or NMR. In NMR one often sees a huge difference in behaviour between \( T_1 \) and \( T_2 \); but coherent dynamics exists only over timescales \( < T_2 \) (off-diagonal matrix elements), and not for \( T_2 < t < T_1 \) (even though oscillations in the diagonal density matrix elements may still be strong).

However as far as we are aware this point has not been noted up until now in the context of macroscopic coherence experiments. In fact all of these experiments have either looked at densities of states using thermodynamic measurements [29], or else at the absorption spectrum of the relevant system (SQUID or magnet) as a function of frequency and applied field [22,25,27,28]. We would like to emphasize that none of these experiments is then directly demonstrating coherence- one can easily envisage a situation in which resonant structure in either the thermodynamic density of states or the resonant absorption is coming entirely from the diagonal matrix elements. Certainly the SQUID experiments of van der Wal et al. [22], which look at “ground state tunneling” (unlike the excited state tunneling experiments of Friedman et al. [25]), give strong evidence that such coherence may soon be demonstrated directly- but they do not in themselves constitute such a demonstration. As for the magnetic experiments so far, two of these experiments (both highly controversial [48,49]) have been done on large ensembles of disordered nanoparticles, and although the specific heat experiment [29] has been done on an ordered set of nanomolecules, it does not probe the magnetic dynamics directly.

However what is certainly true is that there is nothing in the decoherence results described so far that rules out macroscopic coherence. Decoherence coming from the oscillator bath can easily be made very small for either a SQUID (where one can make \( \alpha \ll 1 \)) or an insulating magnet (where phonons give very little low-energy decoherence). The results for spin bath decoherence seem more worrying but only if \( \lambda \) and/or \( \kappa \) are large. In a SQUID the best strategy to avoid spin bath decoherence is to make \( \Delta \) large (as experimentalists are doing), control substrate decoherence, and possibly also to increase the size of the SQUID ring. In a magnetic system the best way to reduce decoherence effects turns out to be using strong transverse fields, which not only increases
Δ but also radically reduces κ; again, the theory indicates that coherence should be visible. We do not as yet have any reason to disbelieve this theory-in fact it has done rather well in predicting the tunneling characteristics of these large systems. Thus the short-term prognosis looks good-it would seem reasonable to bet that direct evidence for single mesoscopic solid-state qubit operation will be found in the next year or two. What then?

4 Coupled Qubits, Quantum Measurements, and all that

The next step up from single qubits is to couple pairs of them and look at their “mutual coherence” or “entanglement”, and how it is destroyed by decoherence processes. In the same way that one can write down general models for single qubits coupled to an environment (the spin-boson and central spin models) one can do the same for a pair of them-this is the “PISCES model”, to which we come below. However it is interesting to realise that the same model also describes a measurement system, itself operating in the quantum regime, and set up to probe a single qubit. This equivalence is not reflected in the literature, where 2 quite different approaches coexist (somewhat uneasily!).

(i) Quantum Measurements: The quantum measurement puzzle is perhaps the most controversial of all those associated with Quantum Mechanics. The first problem is of course to say what it is-this depends partly on one’s view of “physical reality”, as well as on how one formulates the theory. One simple approach begins by noting that in an “ideal” measurement, the coupled “system-apparatus” is supposed to evolve according to

\[ \Psi_{in} = \Phi_o \sum_j c_j \phi_j \rightarrow \Psi_f = \sum_j c_j \Phi_j \phi_j \]  

leaving a “post-measurement” state \( \Phi_j \) of the apparatus in 1 ↔ 1 correlation with an initial state \( \phi_j \) of the measured system (obviously we can also have \( \Psi_f = \sum_j c_j \chi_j \Phi_j \), with \( \chi_j = \sum_i \alpha_{ji} \phi_i \)). The puzzle is then simply that for the apparatus to function as intended, the states \( \{ \Phi_j \} \) must be macroscopically different from each other-meaning that (21) describes a superposition of macroscopically different states.

Answers to this puzzle vary widely. Many formulations of quantum mechanics (including the Copenhagen interpretation) make measurements a primitive and unanalyzable part of the theory, and insist that a measuring apparatus must by definition be classical. The superposition \( \Psi_f \) is then replaced by a mixture, the two being distinguished formally by their density matrices:

\[ \rho_{ij}^{\text{pure}} = c_i c_j^* \quad (\text{superposition}) \]  

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Various arguments have been given that, FAPP ("For All Practical Purposes" [50]), or even in principle, this manoeuvre is justified. Most of these start from the idea that the apparatus is extremely “complex”, and so has many microstates, in a very large Hilbert space, associated with each of the “macrostates” describing a given outcome of a measurement [2,3]. It is also often argued that large amplification and very strong irreversibility (and/or chaotic dynamics) are associated with the time evolution of the apparatus during measurement. Hence, it is argued, no conceivable measurement performed on “system + apparatus” could distinguish $\rho_{ij}^{\text{pure}}$ from $\rho_{ij}^{\text{mixt}}$ (it would require an operator simultaneously off-diagonal in all the relevant states!). Many of the early “decoherence” arguments concerning quantum measurements, going back some 50 years, make a similar point [1]. From now on we will refer to this set of arguments as the “orthodox argument” for the denial of macroscopic quantum superpositions.

At first glance it appears to be a very powerful argument – after all, the macroscopic world does appear to be classical, and the number of microstates to which a macroscopic “pointer state” couples is very large indeed. However the orthodox argument also suffers from a remarkable vagueness- no attempt is made to substantiate it by any detailed quantitative theory, susceptible to experimental test on a real measuring system. One is reminded of the 1960’s conceptual art movement (in which the point was not to create a work of art but rather to talk about what it would be like if completed). The problem with the argument is obvious- insofar as it denies the existence and/or observability of mesoscopic or macroscopic superpositions, it is in imminent danger of contradicting experiment (see previous section) and certainly conflicts with a lot of well-established theory! But what precisely is the flaw in the orthodox argument?

(ii) Coupled qubits: In the Quantum Computation literature [4], on the other hand, the above orthodox argument is usually ignored completely (except for the occasional reference to decoherence as “an important problem to be resolved”, occasionally accompanied by some rough calculation including a phenomenological “noise term”). It is very hard to understand this attitude, which entirely ignores the microstates so central to both the orthodox argument and to decoherence- after all, the whole point of quantum computation is to set up extremely delicate superpositions involving many qubits and a huge number of states. So how sensitive is the mutual coherence between qubits to the environment?

Theoretical analysis of this question is simplified by the fact that quantum computation algorithms can be set up by coupling pairs of qubits in vari-
ous sequences [4]. This allows us to discuss decoherence in the measurement process using the same model as we use for decoherence in the simplest quantum computations, a model involving 2 quantum systems plus the quantum environment. Let us now recall this model.

(iii) **Effective Hamiltonian:** To address the above questions we consider a SQUID magnetometer or a nanomagnetic field sensor, in the quantum regime ($kT < \Omega_o/2\pi$), set up so its 2 low-energy “apparatus pointer macrostates” $|\uparrow\rangle, |\downarrow\rangle$ couple diagonally to another 2-state system (see Fig. 5). Here we will assume the the measured system is also a mesoscopic or macroscopic SQUID or nanomagnet; the apparatus pointer states will be described by a spin-1/2 variable $\vec{\tau}_1$, and the system states by a spin $\vec{\tau}_2$. Obviously such a set-up can also be used as a coupled qubit system if the parameters are adjusted suitably, and decoherence is sufficiently small in both of them.

We may represent the low-energy dynamics of this coupled system by a Hamiltonian which generalises to a pair of spins the spin-boson and central spin Hamiltonians given above. For simplicity we will consider here only delocalised environmental excitations (ie., “microstates”); it is easy to generalise the following discussion, when necessary, to incorporate the coupling to a spin bath. Then we get the “PISCES” Hamiltonian [51] $H_{PISCES} = H_o^P(\tau_1, \tau_2) + H_{int} + H_{os}$, with $H_{os}$ given by (1), and

$$H_o^P = (\Delta_1 \hat{\tau}_1^x + \Delta_2 \hat{\tau}_2^x) + (\xi_1 \hat{\tau}_1^z + \xi_2 \hat{\tau}_2^z) + K \hat{\tau}_1^z \hat{\tau}_2^z$$

$$H_{int} = \sum_q \left[ c_q^{(1)} \hat{\tau}_1^z + c_q^{(2)} \hat{\tau}_2^z \right] x_q$$

To be specific in the following discussion we have made some restrictive assumptions, as follows:

(i) we assume only longitudinal couplings to the oscillators. A more general model would have a coupling $K_{\alpha\beta} \hat{\tau}_\alpha^z \hat{\tau}_\beta^z$ between the 2 spins $\tau_1$ and $\tau_2$, but if the systems are mesoscopic, the longitudinal couplings usually dominate the low-energy limit [51]. We assume only an oscillator bath- and to be even more specific we will assume a $T$- independent Ohmic coupling to the 2 spins.

(ii) We ignore time-dependence in the parameters- this is because we are interested here in environmental decoherence (as opposed to that caused by this time-dependence, which is of course crucial to the actual operation of a pair of qubits).

A microscopic derivation of this Hamiltonian for the present situation is basically just the derivation of the effective couplings, and this we know how to do. For a single isolated SQUID one has a coupling to electronic excitations [38], as well as less important couplings to phonons and photons- and one
has calculable couplings to a spin bath of nuclear and paramagnetic spins, as well as charge defects [23]. In a similar way one can treat the coupling of a single quantum nanomagnet to electrons, phonons, and nuclear spins in this low-energy limit [19,52]. When two such systems are coupled one has to do a little more work – the couplings to the various baths cause a renormalisation of the effective coupling between the two spins – but this can also be done for these systems [51].

Let us now consider the basis of the “orthodox argument” in the light of all this. The microscopic derivations actually do all the work for us – they give an explicit representation of the “microstates” associated with each apparatus pointer macrostate (these being electronic, photonic, nuclear spin, etc., states, coupled explicitly to the pointer states). Needless to say, each macrostate is associated with a vast number of microstates – however their number is not directly relevant to the problem. What counts is how they couple to the macrostate in particular, how each individual microstate is affected by the transition $|\uparrow\rangle \leftrightarrow |\downarrow\rangle$. Here several crucial physical points arise:

(i) The important oscillator bath states are either gapped, by an energy $E_G$ typically much greater than the frequency scale $\omega_o$ involved in the apparatus dynamics (e.g., the Josephson plasma frequency for a SQUID); this is the case for the electronic excitations in a SQUID or the magnons in a magnet. Any remaining gapless excitations (phonons, photons, etc.) have very little spectral weight and extremely weak coupling to the apparatus macro-coordinate at these frequencies (recall Fig. 1). If the electronic excitations are not gapped, then their effect is drastic- one gets the famous orthogonality catastrophe [53,14] which immediately renders either the SQUID dynamics [15] or the nanomagnet dynamics [19] both incoherent and strongly damped.

(ii) Gapped excitations are only very weakly disturbed by the slow apparatus dynamics- this is a typical example of quasi-adiabatic perturbation of the environmental microstates, which can be formulated to lowest approximation à la Born-Oppenheimer (and more rigourously using the effective Hamiltonian approach). We can think of the perturbation of the microstates as a slow readjustment to a changing underlying vacuum state (this analogy becomes precise when the apparatus state corresponds to that of a quantum soliton, as indeed it does both for SQUIDs (a tunneling fluxon) and when the magnetic transition corresponds to the tunneling of a magnetic domain wall [41].

(iii) The real danger comes from low-lying spin bath states, where this quasi-adiabatic argument no longer applies. In the case of nuclear spins we are often saved by an “anti-adiabatic” argument- the characteristic energy scale of the spin bath states (resulting from the interaction itself) is now much lower than $\omega_o$, and so again, they are hardly affected by the apparatus transitions (the quantitative measure of the effect is provided by the parameters $\lambda$ and/or $\kappa$,
whichever is applicable – see previous section). In the case where some of the spin bath couplings $\omega_k$ are similar to $\omega_o$, we have the case of “loose spins” [42], where these spins are strongly affected by an apparatus transition- the apparatus dynamics again becomes completely incoherent and highly damped.

From these remarks we see that the main flaw in the orthodox argument is simply that it never attempts a proper quantitative discussion of the microstates it invokes. However, to get further insight into the physics, and to cover the problem of coupled qubits, we must go to the dynamics of the PISCES model.

(iv) **Dynamics**: The dynamics of this model in the absence of external biases was solved by us a number of years ago [51]. It is controlled by the interaction $K$, the tunneling matrix elements $\Delta_1$ and $\Delta_2$, the temperature $T$ and three friction coefficients $\alpha_1$, $\alpha_2$ (describing the Ohmic coupling of each spin to the bath), and $\alpha_{12}$ (which describes the effective friction acting on the mutual correlations of the spins). Away from the perturbative limit $K \ll \Delta$, the spins can be either totally locked (if $K \gg T$, which results in a single spin-boson system with tunneling matrix element $\tilde{\Delta} \sim \Delta_1\Delta_2/K$) or, if $K \ll T$, still in a correlated phase but strongly disordered by temperature. Coherent behaviour of the 2-spin complex is however possible in the “Mutual Coherence Phase” ($T \gg \Delta_j/\alpha_j$, $K$, where $j = 1, 2$). This phase then allows situations in which the combined system-apparatus complex is in a a coherently entangled macroscopic state; or in the case of coupled qubits, where a computation can be done in time scales much shorter than the decoherence time for this state. At this point we note an important feature of this model- decoherence effects are massively amplified as soon as $K > \Delta_\beta$.

Let us first use the results to set up a measurement scheme treating both a measuring apparatus $\tau_A$: $\{\uparrow, \downarrow\}$ and a system $\tau_s$: $\{\uparrow, \downarrow\}$ in a fully quantum way and in presence of an environmental bath. To be specific, we consider the PISCES model, with bare Hamiltonian

$$H_0 = \Delta_A \tau_A^x + \Delta_s \tau_s^x - \xi_A \tau_A^z - K \tau_A^z \tau_s^z$$

such that $K \gg \xi_A \gg \Delta_A, T$ represents a strong ferromagnetic interaction and $\Delta_A \gg \Delta_s$; $\alpha_A \geq 1$. Without coupling, the apparatus is in state $| \uparrow \rangle$ and the restriction $\Delta_A \gg \Delta_s$ insures that the apparatus reacts quickly to any changes in the system.

With the apparatus initially in the state $| \uparrow \rangle$ and the coupling turned on at some time $t$, then a state $| \uparrow \uparrow \rangle$ remains due to bias $\xi_A$. On the other hand, if at time $t$ the coupled state is $| \uparrow \downarrow \rangle$, fast relaxation to the state $| \downarrow \downarrow \rangle$ then occurs (at a rate $\tau_A^{-1} \sim (\Delta_A^2/\omega_0)(K/\omega_0)^{2\alpha_A-1}$) and the combined apparatus-system is then essentially frozen. Relaxation to the state $| \uparrow \uparrow \rangle$ takes place at the extremely slow rate $\tau_c^{-1} \sim (\Delta_A^2/\omega_0)(T/\omega_0)^{2\alpha_A-1} \ll \tau_A^{-1}$. In effect, an
“ideal” measurement of the form \[54\]

\[
\begin{align*}
|\uparrow\rangle|\uparrow\rangle & \longrightarrow |\uparrow\rangle|\uparrow\rangle \\
|\uparrow\rangle|\downarrow\rangle & \longrightarrow |\downarrow\rangle|\downarrow\rangle
\end{align*}
\]

(27)

has been performed. The difference in time scales is what makes the measurement possible.

Now, let us look at another limit of this model, more appropriate to the discussion of decoherence in a pair of coupled qubits. All external biases are removed and we assume that (a) each system is identical, and (b) that the dissipative couplings \(\alpha = \alpha_1 = \alpha_2\) are very small. If we wish to operate the coupled system as a pair of qubits, what then will be the decoherence rates? let us consider the specific situation where the 2 qubits are in the pure state \(|\uparrow\uparrow\rangle\) at time \(t = 0\) and calculate the diagonal elements \(\rho_{\nu_1\nu_2}(t)\) with \(\nu_1 = \{\uparrow, \downarrow\}\) and \(\nu_2 = \{\uparrow, \downarrow\}\). Without going into details (which are complex- the time evolution of the density matrix involves three charactristic frequencies and 3 different relaxation times \[51\]) one can make the following general remarks. First, the general relaxation of the “ferromagnetic” state, as measured from the time behaviour of the combination \(\rho_{\uparrow\uparrow}(t) + \rho_{\downarrow\downarrow}(t)\), is controlled by combinations of the decay rates \(\Gamma_{11}(2\Delta_r, \mathcal{K})\) and \(\Gamma_{12}(2\Delta_r, \mathcal{K})\), using the same notation as we used above for the diagonal and off-diagonal matrix elements of the single spin-boson problem in Eqs. (12) and (13), and replacing \(\gamma(E)\) by \(\gamma(\sqrt{K^2 + \Delta^2})\). On the other hand, the “dephasing” to the states \(|\uparrow\downarrow\rangle\) and \(|\downarrow\uparrow\rangle\) occurs essentially at rates \(\Gamma_{11}(\Delta_r, \mathcal{K})\) and \(\Gamma_{12}(\Delta_r, \mathcal{K})\). These are also the decay rates for the antiferromagnetic global combination \(\rho_{\uparrow\downarrow}(t) + \rho_{\downarrow\uparrow}(t)\). Thus the principal result is that the mutual coherence or “entanglement” correlations are very fragile-they are rendered incoherent by a combination of environmental couplings and the inter-qubit coupling \(\mathcal{K}\) itself, in a way very much like the destruction of coherent oscillations for a single qubit once a bias is applied (note how \(\mathcal{K}\) plays much the same role in the 2-qubit entanglement relaxation as the bias \(\xi\) does in the single qubit relaxation).

However we also see that if \(\alpha\) is small enough (ie., that \(\Gamma/\Delta \ll 1\)) then decoherence will not be a problem for the operation of a pair of qubits. One can go into more details here, looking at the regions in the parameter space of variables \(\mathcal{K}, T, \alpha, \) and \(\Delta\) where a pair of coupled qubits can operate (the “mutual coherence regime” defined in Dubé and Stamp [51]), but this discussion becomes rather technical so we refrain from this here. We also note that any realistic analysis of this dynamics must also include the spin bath.

In any case our main point is clear - by fiddling with the parameters of this coupled system we can make it go from a coherent regime to an incoherent regime. In the latter the apparatus really acts as a measuring device, and in the former as a pair of qubits. We note also that a crucial feature of the
apparatus when in the conventional measurement mode is that it is sufficiently damped to settle into a single “macrostate”. This is of course hardly new-it is follows from the very definition of a ”measurement” that this must be so! But this does not affect our point- which is that the same system that can behave as a measuring apparatus can also, with only small changes in coupling constants or fields, show macroscopic superpositions. One may try to argue (like Peres [3,55]) that the word “macroscopic” by definition ought to imply that macroscopic superpositions are disallowed (so that coherent superpositions of SQUID flux states, or magnetic magnetisation states, must by definition be considered microscopic no matter how large the SQUIDs, magnets, or flux or magnetisation differences may be). But then the whole orthodox argument becomes a mere tautology, and explains nothing- as well as forcing a very peculiar notion of the term “macroscopic” upon us (one in which a system changes from macroscopic to microscopic by, eg., tuning of an external field!).

We finish by stressing that these considerations are not academic. Experimentalists searching for large-scale quantum effects in magnets and superconductors must push their SQUID magnetometers or nanomagnetic detectors into the quantum regime to attain increased sensitivity. They will eventually have to take account of quantum correlations in the functioning of the measuring apparatus. This promises to lead to all sorts of interesting discussions! And of course a world-wide effort is on to build solid-state qubits. Thus the decoherence story is only just beginning.

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[50] For a discussion of the limitations of “FAPP” discussions of quantum measurement, see J. S. Bell, Physics World (Aug 1990), p.33.

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Figure Captions

Fig. 1: Typical form for the dimensionless coupling $g(\omega, T)$, with $\omega = T$, between a mesoscopic system such as a superconductor or a nanomagnet, and various environmental modes. In this particular case we assume a 3-dimensional system with nuclear spin energy scale $E_o \sim 3 \times 10^3 \, K$, and a superconducting transition at $T \sim 2 \, K$.

Fig. 2: Diagonal and Off-diagonal of the density matrix for a "Free spin", uncoupled from the environment, and in zero bias field ($\xi = 0$).

Fig. 3: Diagonal and Off-diagonal elements of the density matrix in the spin-boson problem with parameter $\alpha = 0.05$, $T/\Delta = 0.5$ and $E_c = 50\Delta$. The value $\rho_{12}(t \to \infty) = (\Delta/\Delta_r) \tanh(\Delta_r/2T)$. The system is again unbiaised, i.e., $\xi = 0$.

Fig. 4: Diagonal and Off-diagonal elements of the density matrix in the central spin problem with zero bias, and with topological decoherence dominating. The value of the topological decoherence parameter is relatively weak: $\lambda = 1/8$.

Fig. 5: The general problem involved in dealing with a measuring system (collective coordinate $Q'$ coupled to some other quantum system (collective coordinate $Q$), with both coupled to a quantum environment; all variables are quantised. The same kind of model describes a pair of coupled qubits.
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.