Intersecting D-branes in Type IIB Plane Wave Background

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ABSTRACT

We study intersecting D-branes in a type IIB plane wave background using Green-Schwarz worldsheet formulation. We consider all possible $D_{\pm}$-branes intersecting at angles in the plane wave background and identify their residual supersymmetries. We find, in particular, that $D_+ - D_-$ brane intersections preserve no supersymmetry. We also present the explicit worldsheet expressions of conserved supercharges and their supersymmetry algebras.

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1 Introduction

The Penrose limit of the $AdS_5 \times S^5$ background in type IIB supergravity corresponds to a plane wave solution \cite{1},

\[ ds^2 = -2dx^+dx^- - \mu^2 x^2(dx^+)^2 + dx_I^2, \]

\[ F_{+1234} = F_{+5678} = 2\mu. \]

This implies a correspondence between type IIB string theory in the plane wave background (1.1) and ${\cal N} = 4$ supersymmetric Yang-Mills theory with large R-charge. Since the background (1.1) is one of the very few Ramond-Ramond backgrounds on which string theory is exactly solvable \cite{2, 3}, one may have a genuine hope to explicitly check the conjectured AdS/CFT correspondence beyond the supergravity approximation on the string theory side. Indeed Berenstein, Maldacena, and Nastase \cite{4} succeeded in reproducing the string spectrum from perturbative super Yang-Mills theory, thereby putting the duality on a firm ground at the free theory level. Subsequent developments using the super Yang-Mills theory and the light-cone string field theory have accumulated strong evidences that the duality is still valid even after the interactions both on the super Yang-Mills theory side and on the string theory side are introduced.

D-branes can be described by boundary states of closed string state. The symmetries that the boundary state preserves are generically the combinations of the closed string symmetries that leave the boundary state invariant. However, in the plane wave background (1.1), it was shown by Skenderis and Taylor in \cite{5, 6} that an open string on a $D_+\bar{D}_-$-brane has a different kind of kinematical supersymmetry not descending from the closed string. Furthermore it was shown in \cite{7, 8} that oblique D-branes exist in the background (1.1) whose isometry is a subgroup of the diagonal $SO(4)$ symmetry of the background. Recently possible D-branes in plane wave backgrounds have been identified and their supersymmetries have also been classified \cite{5}-\cite{28}.

The classification of D-branes in plane wave backgrounds is still incomplete. In particular possible intersecting D-branes are not completely studied even in the type IIB plane wave background (1.1). $Dp - Dq$ brane systems have an important role in string theory since they probe the nonperturbative dynamics of the string theory and they have been used to study various duality aspects of the string theory. Furthermore intersecting D-branes have received intense interests since it has been known that chiral fermions can appear on the intersection of D-branes \cite{29} and so they are promising tools to construct a phenomenological model like the Standard Model in string theory context. Some aspects have been reviewed in \cite{30, 31}. Also recently there have been many works on intersecting D-branes in other plane wave and AdS backgrounds \cite{32}-\cite{39}.

In this paper we consider all possible $D_{\pm}$-branes intersecting at angles in the plane wave background (1.1) and identify their residual supersymmetries. It turns out that in some cases intersecting D-branes are similar to the flat spacetime case but in other cases they are quite
different from the flat spacetime case. To study the intersecting D-branes in the background (1.1), we develop the systematic method using the Green-Schwarz superstring action in light-cone gauge for an open string stretched between a $Dp$-brane and a $Dq$-brane. Our results consistently recover those in flat spacetime [30] when the limit $\mu \to 0$, namely, the flat spacetime, is taken. Up to our best knowledge, our methodology for studying intersecting D-branes, namely, the worldsheet formalism using Green-Schwarz superstring action, is new even in the flat spacetime case. Most relevant works may be [40, 41]. A merit of this formalism is that the spacetime supersymmetry of intersecting D-branes is manifest.

This paper is organized as follows. In Sec. 2, we present a worldsheet formulation using the Green-Schwarz superstring action in light-cone gauge for an open string stretched between a $Dp$-brane and a $Dq$-brane which are either parallel or intersecting at right angles. We get the mode expansion of open strings consistent with open string boundary conditions in the Green-Schwarz superstring theory context. We find that $D_\pm - D_\pm$ brane intersections preserve no supersymmetry. In Sec. 3, the analysis is generalized to the case of intersecting D-branes at general angles [29]. Since the rotational symmetry is reduced to $SO(4) \times SO(4)'$, there are only two kinds of supersymmetric intersection at general angles. One is generated by $SU(2) \subset SO(4)$ or $SO(4)'$ and the other is generated by $SU(2) \times SU(2) \subset SO(4) \times SO(4)'$. It turns out that the former case further breaks the supersymmetry by half after rotation while the latter does by three quarter like in the flat spacetime. In Sec. 4, remaining supersymmetries of intersecting D-branes are identified by finding conserved worldsheet supercurrents consistent with open string boundary conditions [23]. We summarize our results for the unbroken supersymmetries in Tables 1-2. In section 5, the explicit worldsheet expressions of conserved supercharges and their supersymmetry algebras are presented. In section 6, we briefly review our results and address some other issues. In Appendix A, we present calculational details of the supersymmetry algebra for a specific example, $D_3 - D_5$ intersection, to illustrate nontrivial aspects of the derivation.

2 $p - q$ Strings in A Plane Wave Background

The Green-Schwarz light-cone action in the plane wave background (1.1) describes eight free massive bosons and fermions. In the light-cone gauge, $X^+ = \tau$, the action is given by

$$S = \frac{1}{2\pi\alpha'p^+} \int d\tau d\sigma \left[ \frac{1}{2} \partial_+ X_I \partial_- X_I - \frac{1}{2} \mu^2 X_I^2 - i \bar{S}(\rho^A \partial_A - \mu \Pi) S \right]$$

(2.1)

where $\partial_\pm = \partial_+ \pm \partial_\sigma$. In this paper we will use the notation and the convention in [23]. The equations of motion following from the action (2.1) take the form

$$\partial_+ \partial_- X^I + \mu^2 X^I = 0,$$

(2.2)

$$\partial_+ S^1 - \mu \Pi S^2 = 0,$$

(2.3)

$$\partial_- S^2 + \mu \Pi S^1 = 0.$$
We use the following form for $\gamma^I$

$$\gamma^I = \begin{pmatrix} 0 & \gamma_{aa}^I \\ 0 & 0 \end{pmatrix}$$  \hspace{2.0cm} (2.4)

where $\gamma_{aa}^I = (\gamma^I)^a_a$ and take the $SO(8)$ chirality matrix as

$$\gamma = \begin{pmatrix} 1_8 & 0 \\ 0 & -1_8 \end{pmatrix}.$$  \hspace{2.0cm} (2.5)

In what follows, we assume that the spinors $S^A(\tau, \sigma), A=1,2$, are positive chiral fermions, $\gamma S^A = S^A$, of the form

$$S^A_\alpha = \begin{pmatrix} S^A_a \\ 0 \end{pmatrix},$$  \hspace{2.0cm} (2.6)

where $\alpha = 1, \cdots, 16$ and $a = 1, \cdots, 8$.

Consider an open string stretched between $D_p$-brane and $D_q$-brane in the plane wave background (1.1). The open string action is just defined by the action (2.1) with string length $\alpha = 2\alpha'p^+$ imposed with appropriate boundary conditions on each end of the open string. For longitudinal coordinates $X^r$ on D-branes, we impose the Neumann boundary condition

$$\partial_\sigma X^r|_{\partial \Sigma} = 0,$$  \hspace{2.0cm} (2.7)

while for transverse coordinates $X^{r'}$ we have the Dirichlet boundary condition

$$\partial_\tau X^{r'}|_{\partial \Sigma} = 0.$$  \hspace{2.0cm} (2.8)

The fermionic coordinates also have to satisfy the following boundary condition at each end of the open string [41]

$$(S^1 - \Omega_0 S^2)|_{\sigma=0} = 0, \quad (S^1 - \Omega_\pi S^2)|_{\sigma=\pi\alpha} = 0,$$  \hspace{2.0cm} (2.9)

where the matrix $\Omega_\theta = (\Omega_0, \Omega_\pi)$ is the products of $\gamma$-matrices along worldvolume directions and, depending on the type of D-branes, $D_{\pm}$-branes, satisfies

$$D_- : \Pi \Omega_\theta \Pi \Omega_\theta = -1, \quad D_+ : \Pi \Omega_\theta \Pi \Omega_\theta = 1.$$  \hspace{2.0cm} (2.10)

The condition (2.10) fairly restricts the possible D-branes and their polarization [10, 12, 13, 5, 20]. We use the notation $(+, -, m, n)$ [12] to indicate a brane wrapping the light-cone coordinates $X^\pm$, $m$ coordinates that used to be $AdS_5$ coordinates before the Penrose limit, and $n$ coordinates that used to be $S^5$ coordinates. Here we list the allowed choices:

$$D3 : (+, -, m, n) = (+, -, 2, 0), \quad (+, -, 0, 2),$$

$$D5 : (+, -, m, n) = (+, -, 3, 1), \quad (+, -, 1, 3),$$

$$D7 : (+, -, m, n) = (+, -, 4, 2), \quad (+, -, 2, 4).$$  \hspace{2.0cm} (2.11)
for $D_-$-branes and

\begin{align}
D1 : (+, -, m, n) &= (+, -, 0, 0), \\
D3 : (+, -, m, n) &= (+, -, 1, 1), \\
D5 : (+, -, m, n) &= (+, -, 4, 0), (+, -, 2, 2), (+, -, 0, 4), \\
D7 : (+, -, m, n) &= (+, -, 3, 3), \\
D9 : (+, -, m, n) &= (+, -, 4, 4)
\end{align}

for $D_+$-branes.

Now we will analyze the detailed statics of an open string stretched between $D_p$-brane and $D_q$-brane in the plane wave background (1.1) from which we will determine the residual supersymmetries of the D-brane configurations. In this section we will first consider parallel D-branes and intersecting D-branes at right angles. More general supersymmetric intersections will be discussed in section 3.

2.1 $D_- - D_-$ Brane Configurations

We consider an open string intervened between parallel or orthogonally intersecting D-branes. The coordinates $X^I(\tau, \sigma)$ of a $p-q$ string can be partitioned into four sets, NN, DD, ND, and DN, according to whether the coordinate $X^I$ has Neumann (N) or Dirichlet (D) boundary condition at each end. We first present the mode expansion of the bosonic coordinates $X^I(\tau, \sigma)$ for the four possible boundary conditions:

\begin{align}
\text{NN} &: X^r(\tau, \sigma) = \cos \mu \tau x_0^r + \sin \mu \tau \frac{p_0^r}{\mu} + i \sum_{n \neq 0} \frac{\alpha_n^r e^{-i\omega_n \tau}}{\omega_n} \cos \frac{n \sigma}{|\alpha|}, \\
\text{DD} &: X^r(\tau, \sigma) = x_1^r \cos \mu \sigma + \frac{x_2^r - x_1^r \cosh \pi \mu |\alpha|}{\sinh \pi \mu |\alpha|} \sinh \mu \sigma + \sum_{n \neq 0} \frac{\alpha_n^r e^{-i\omega_n \tau}}{\omega_n} \sin \frac{n \sigma}{|\alpha|}, \\
\text{ND} &: X^i(\tau, \sigma) = x_1^i \cosh \mu \sigma + i \sum_{\kappa \in \mathbb{Z}^{+\frac{1}{2}}} \frac{\alpha_{\kappa}^i e^{-i\omega_\kappa \tau}}{\omega_\kappa} \cos \frac{\kappa \sigma}{|\alpha|}, \\
\text{DN} &: X^i(\tau, \sigma) = \frac{x_1^i \cosh \mu (\sigma - \pi |\alpha|)}{\cosh \pi \mu |\alpha|} + \sum_{\kappa \in \mathbb{Z}^{+\frac{1}{2}}} \frac{\alpha_{\kappa}^i e^{-i\omega_\kappa \tau}}{\omega_\kappa} \sin \frac{\kappa \sigma}{|\alpha|},
\end{align}

(2.13)

where $x_1^r$ and $x_2^r$ denote the transverse positions of $D_p$-brane and $D_q$-brane, respectively and

$$\omega_\nu = \text{sgn}(\nu) \sqrt{\mu^2 + \nu^2 / \alpha^2}, \quad \text{for} \ \nu = n, \kappa.$$

(2.14)

\footnote{In the following we will use indices $(r, s, \cdots), (r', s', \cdots), (i, j, \cdots)$, and $(i', j', \cdots)$ for NN, DD, ND, and DN coordinates, respectively. We will also use the subscripts $(n, m, \cdots) \in \mathbb{Z}$ for integer modes of string oscillators while $(\kappa, \lambda, \cdots) \in \mathbb{R}$ for general real number modes.}
The commutation relations for the modes in Eq. (2.13) are given by

\[
[x^r_0, p^s_0] = i\delta^{rs}, \\
[a^I_n, \alpha^J_m] = \omega_n\delta_{m+n,0}\delta^{IJ}, \quad [\alpha^I_n, \alpha^J_n] = \omega_n\delta_{n+\lambda,0}\delta^{IJ}.
\] (2.15)

We are mainly interested in a supersymmetric intersection. In particular, the dynamical supersymmetry transformation is given by

\[
\delta_t X^I = \frac{1}{\sqrt{2p^+}}i\gamma^I S^A.
\] (2.16)

Hence, we take an appropriate combination of spinor fields \(\xi^A(\tau, \sigma)\) with integer modes and \(\eta^A(\tau, \sigma)\) with half-integer modes to be compatible with the bosonic case (2.13):

\[
S^1(\tau, \sigma) = \begin{cases} 
I_+\xi^1(\tau, \sigma) + I_-\eta^1(\tau, \sigma), & \text{for A-type;} \\
I_-\xi^1(\tau, \sigma) + I_+\eta^1(\tau, \sigma), & \text{for B-type,}
\end{cases}
\]

\[
S^2(\tau, \sigma) = I_+\xi^2(\tau, \sigma) + I_-\eta^2(\tau, \sigma),
\] (2.17)

where \(I_+\) and \(I_-\) are \(16 \times 16\) matrices satisfying

\[
I_+ + I_- = 1, \quad I_+I_- = 0, \quad I_+^2 = I_+, \quad I_-^2 = I_-.
\] (2.18)

The condition (2.18) simply states that one has to pick up only eight components from the two \(SO(8)\) chiral spinors \(\xi^A(\tau, \sigma)\) and \(\eta^A(\tau, \sigma)\) to give an \(SO(8)\) chiral spinor \(S^A(\tau, \sigma)\). For the reason explained later, the A-type solution is for \(|p-q| = 0, 4, 8\) in \(Dp-Dq\) brane configurations while the B-type solution for \(|p-q| = 2, 6\). We take the spinors \(\xi^A(\tau, \sigma)\) and \(\eta^A(\tau, \sigma)\) as the solution of the equation of motion (2.3) satisfying the boundary condition (2.9) at \(\sigma = 0\) [23]:

\[
\xi^1(\tau, \sigma) = \cos \mu \tau S_0 - \sin \mu \tau \Omega_0 \Pi S_0 + \sum_{n \neq 0} c_n(\varphi^1_n(\tau, \sigma)\Omega_0 S_n + i\rho_n\varphi^2_n(\tau, \sigma)\Pi S_n),
\]

\[
\xi^2(\tau, \sigma) = \cos \mu \tau \Omega_0^T S_0 - \sin \mu \tau \Pi S_0 + \sum_{n \neq 0} c_n(\varphi^2_n(\tau, \sigma)S_n - i\rho_n\varphi^1_n(\tau, \sigma)\Pi S_n),
\]

\[
\eta^1(\tau, \sigma) = \sum_{\kappa \in \mathbb{Z}+\frac{1}{2}} c_\kappa(\varphi^1_\kappa(\tau, \sigma)\Omega_0 S_\kappa + i\rho_\kappa\varphi^2_\kappa(\tau, \sigma)\Pi S_\kappa),
\]

\[
\eta^2(\tau, \sigma) = \sum_{\kappa \in \mathbb{Z}+\frac{1}{2}} c_\kappa(\varphi^2_\kappa(\tau, \sigma)S_\kappa - i\rho_\kappa\varphi^1_\kappa(\tau, \sigma)\Pi S_\kappa),
\] (2.19)

where the basis functions \(\varphi^{1,2}_\nu(\tau, \sigma)\) are defined by

\[
\varphi^{1}_\nu(\tau, \sigma) = e^{-i(\omega_\nu\tau - \frac{\omega_\nu}{\mu}\sigma)}, \quad \varphi^{2}_\nu(\tau, \sigma) = e^{-i(\omega_\nu\tau + \frac{\omega_\nu}{\mu}\sigma)}
\] (2.20)

and

\[
\omega_\nu = \text{sgn}(\nu)\sqrt{\mu^2 + \nu^2/\alpha^2}, \quad \rho_\nu = \frac{\omega_\nu - \nu/|\alpha|}{\mu}, \quad c_\nu = \frac{1}{\sqrt{1 + \rho_\nu^2}}.
\] (2.21)
Here $\nu$ is either integer $n$ or half-integer $\kappa$. The commutation relations for the modes in (2.19) are given by
\[
\{S^a_n, S^b_m\} = \frac{1}{4} \delta_{n+m,0} \delta^{ab}, \quad \{S^a_\kappa, S^b_\lambda\} = \frac{1}{4} \delta_{\kappa+\lambda,0} \delta^{ab}.
\] (2.22)

At $\sigma = \pi|\alpha|$, the spinors satisfy the following relations
\[
\xi^1(\tau, \sigma = \pi|\alpha|) = \Omega_0 \xi^2(\tau, \sigma = \pi|\alpha|), \quad \eta^1(\tau, \sigma = \pi|\alpha|) = -\Omega_0 \eta^2(\tau, \sigma = \pi|\alpha|).
\] (2.23)

The boundary conditions (2.9) for the spinors $S^A(\tau, \sigma)$ in Eq. (2.17) require the following property for the projection matrices $I_{\pm}$:
\[
\Omega_0 I_{\pm} = \begin{cases} 
I_{\pm} \Omega_0, & \text{for A-type;} \\
I_{\pm} \Omega_0, & \text{for B-type, at } \sigma = 0;
\end{cases}
\]
\[
\Omega_0^T \Omega_\pi I_{\pm} = \pm I_{\pm}, \quad \text{at } \sigma = \pi|\alpha|.
\] (2.24)

Eq. (2.24) can be satisfied only if the matrix $\Omega_0^T \Omega_\pi$ is symmetric, i.e.,
\[
(\Omega_0^T \Omega_\pi)^T = \Omega_0^T \Omega_\pi.
\] (2.25)

The matrices $I_+$ and $I_-$ can be solved to give
\[
I_+ = \frac{1}{2} (1 + \Omega_0^T \Omega_\pi), \quad I_- = \frac{1}{2} (1 - \Omega_0^T \Omega_\pi).
\] (2.26)

Note that $\Omega_\theta^T = -\Omega_\theta$ for $D3$- and $D7$-branes, but $\Omega_\theta^T = \Omega_\theta$ for $D5$-branes and thus
\[
\Omega_0^T \Omega_\pi = \begin{cases} 
\Omega_0 \Omega_\theta^T, & \text{for A-type;} \\
-\Omega_0 \Omega_\theta^T, & \text{for B-type.}
\end{cases}
\] (2.27)

Using Eq. (2.27), we get useful identities:
\[
\Pi I_{\pm} = I_{\pm} \Pi, \quad \Omega_\theta I_{\pm} = I_{\pm} \Omega_\theta, \quad I_{\pm} \Omega_0 = \pm I_{\pm} \Omega_\pi, \quad \text{for A-type;}
\]
\[
\Pi I_{\pm} = I_{\mp} \Pi, \quad \Omega_\theta I_{\pm} = I_{\mp} \Omega_\theta, \quad I_{\pm} \Omega_0 = \mp I_{\pm} \Omega_\pi, \quad \text{for B-type.}
\] (2.28) (2.29)

One can easily see that the spinors in Eq. (2.17) satisfy the equations of motion (2.3) and the boundary conditions (2.9).

Note that $\Omega_0^T \Omega_\pi$ consists of products of $\gamma$-matrices along the ND and DN directions. Since $(\Omega_0^T \Omega_\pi)^2 = 1$ and $\text{Tr}(\Omega_0^T \Omega_\pi) = 0$ for $\Omega_0^T \Omega_\pi \neq \pm 1$, there can be only three kinds of possibility:
\[
\Omega_0^T \Omega_\pi = \begin{cases} 
\pm 1, & \sharp_{ND} = 0, \\
\pm \gamma, & \sharp_{ND} = 8, \\
\pm \begin{pmatrix} \Xi & 0 \\ 0 & \pm \Xi \end{pmatrix}, & \sharp_{ND} = 4,
\end{cases}
\] (2.30)
where
\[ \Xi = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \] (2.31)
and \( \sharp_{ND} \) denotes the total number of ND and DN directions.

The case \( \Omega_0^T \Omega_\pi = 1 \) corresponds to parallel \( Dp \)-branes while the case \( \Omega_0^T \Omega_\pi = -1 \) corresponds to \( Dp \)-anti-\( Dp \) branes, but the cases \( \Omega_0^T \Omega_\pi = \pm \gamma \) and \( \Omega_0^T \Omega_\pi = \pm \Xi \) correspond to \( Dp - Dq \) or \( Dp \)-anti-\( Dq \) branes with \( \sharp_{ND} = 8 \) and \( \sharp_{ND} = 4 \), respectively. Note that the B-type branes allow only the \( \sharp_{ND} = 4 \) case as seen from the list in (2.11). We will show in section 4 by deriving conserved worldsheet supercurrents that the brane configurations in (2.30) preserve the same amount of supersymmetries as in flat spacetime as long as two branes are at origin.

### 2.2 \( D_+ \) \(- D_+ \) Brane Configurations

We first analyze the mode expansions of bosons. \( D_+ - D_+ \) brane configurations where both branes have no worldvolume fluxes have the same mode expansion as the \( D_- - D_- \) case. We now consider the cases in which at least one of two \( D_+ \)-branes is a \( D5 \)-brane with a worldvolume flux. In this case the \( D5 \)-branes of type \((+, -, 4, 0)\) or \((+, -, 0, 4)\) with a Born-Infeld flux satisfy the modified Neumann boundary condition [20, 23]:
\[ (\partial_\tau X^r - \mu X^r)|_{\partial \Sigma} = 0, \quad \forall \ r \in \mathbb{N}. \] (2.32)

First we consider the brane configuration consisting of a \( Dp \)-brane with no worldvolume flux at \( \sigma = 0 \) and a \( D5 \)-brane with a worldvolume flux at \( \sigma = \pi|\alpha| \). In this brane configuration the bosonic coordinates with NN, DD, ND, and DN boundary conditions have the following mode expansions, respectively:

\[ \text{NN} : \quad X^r(\tau, \sigma) = i \sum_{\kappa} \frac{\alpha^r_\kappa}{\omega_\kappa} e^{-i\omega_\kappa \tau} \cos \frac{\kappa \sigma}{|\alpha|}, \]
\[ \text{DD} : \quad X'^r(\tau, \sigma) = x'^r_1 \cosh \mu \sigma + \frac{x'^r_2 - x'^r_1 \cosh \pi \mu |\alpha|}{\sinh \pi \mu |\alpha|} \sinh \mu \sigma + \sum_{n \neq 0} \frac{\alpha'_n}{\omega_n} e^{-2i\omega_n \tau} \sin \frac{n \sigma}{|\alpha|}, \]
\[ \text{ND} : \quad X^i(\tau, \sigma) = x^i_2 \cosh \mu \sigma + i \sum_{\kappa \in \mathbb{Z} + \frac{1}{2}} \frac{\alpha^i_\kappa}{\omega_\kappa} e^{-i\omega_\kappa \tau} \cos \frac{\kappa \sigma}{|\alpha|}, \]
\[ \text{DN} : \quad X'^i(\tau, \sigma) = x'^i_1 e^{\mu \sigma} + \sum_{\lambda} \frac{\alpha'^i_\lambda}{\omega_\lambda} e^{-2i\omega_\lambda \tau} \sin \frac{\lambda \sigma}{|\alpha|}, \] (2.33)

where \( \kappa \) and \( \lambda \) in NN and DN coordinates are generically irrational numbers determined by the equations
\[ e^{2\pi i \kappa} = \frac{\kappa - i \mu |\alpha|}{\kappa + i \mu |\alpha|}, \quad e^{2\pi i \lambda} = \frac{\lambda - i \mu |\alpha|}{\lambda + i \mu |\alpha|}, \] (2.34)
respectively. The frequencies \( \omega_\kappa, \omega_\lambda \) in the NN and DN coordinates are defined as in Eq. (2.21) with \( \kappa \) and \( \lambda \) satisfying the equations (2.34). It is obvious that there are infinitely many
solutions in Eq. (2.34) for \( \kappa \) and \( \lambda \). Also, if \( \kappa (\lambda) \) is a solution to the first (second) equation, then \(-\kappa (-\lambda)\) is also a solution. The reality condition for NN and DN coordinates is thus satisfied only if \((\alpha^{i}_{\kappa})^{\dagger} = \alpha_{-\kappa}^{i}\) and \((\alpha^{i}_{\lambda})^{\dagger} = \alpha_{-\lambda}^{i}\). Note that even NN coordinates do not have zero modes since \( \kappa = 0 \) is not a solution of the first equation in Eq. (2.34). This is due to different behaviors of zero modes on \( D_{+}\)-branes because the behavior on \( D_{+}\)-brane without flux has an oscillatory behavior around some fixed position while that on \( D_{+}\) brane with flux tends to move with constant velocity [23]. DN coordinates do not contain any zero modes either since a zero mode solution with \( \lambda = 0 \) in the DN directions is identically zero although it is a trivial solution of Eq. (2.34). The commutation relations for the modes in Eq. (2.33) are identical to the equations (2.38).

Next we consider the configurations of \((+,-,4,0)\) and \((+,-,0,4)\) branes with worldvolume fluxes altogether [23]. In order to find the mode expansion in this case, it is convenient to introduce new coordinates \( \tilde{X}^{I} \equiv e^{-\mu \sigma} X^{I} \). The new coordinates \( \tilde{X}^{I}(\tau, \sigma) \) then satisfy the modified equation of motion

\[
(\partial_{\tau}^{2} - \partial_{\sigma}^{2} - 2\mu \partial_{\sigma}) \tilde{X}^{I} = 0
\]

and the usual boundary conditions

\[
\partial_{\sigma} \tilde{X}^{r}|_{\partial \Sigma} = 0, \quad \partial_{\tau} \tilde{X}^{r}|_{\partial \Sigma} = 0.
\]

The mode expansions of the bosonic coordinates \( \tilde{X}^{I}(\tau, \sigma) \) and thus \( X^{I}(\tau, \sigma) \) are solved conveniently by a method called separation of variables for NN, DD, ND, and DN boundary conditions:

\[
\text{NN : } X^{r}(\tau, \sigma) = \sqrt{\frac{2\pi \mu |\alpha|}{e^{2\pi \mu |\alpha|} - 1}} \left( x^{r}_{0} + \frac{p_{0}^{r}}{\mu} \right) e^{\mu \sigma} + \frac{i}{2} \sum_{n \neq 0} \frac{\alpha^{r}_{n}}{\omega_{n}} \left( \varphi^{1}_{n}(\tau, \sigma) + \frac{n + i\mu |\alpha|}{n - i\mu |\alpha|} \varphi^{2}_{n}(\tau, \sigma) \right),
\]

\[
\text{DD : } X^{r}(\tau, \sigma) = x^{r}_{1} \cosh \mu \sigma + \frac{x^{r}_{2} - x^{r}_{1}}{\sinh \pi \mu |\alpha|} \cosh \mu \sigma + \frac{\alpha^{r}_{n}}{\omega_{n}} e^{-i\omega_{n} \tau} \sin \frac{n \sigma}{|\alpha|},
\]

\[
\text{ND : } X^{i}(\tau, \sigma) = x^{i}_{2} e^{\mu (\sigma - \pi |\alpha|)} + \frac{i}{2} \sum_{n \neq 0} \frac{\alpha^{i}_{n}}{\omega_{n}} \left( \varphi^{1}_{n}(\tau, \sigma) - e^{2\pi i \kappa} \varphi^{2}_{n}(\tau, \sigma) \right),
\]

\[
\text{DN : } X^{i}(\tau, \sigma) = x^{i}_{1} e^{\mu \sigma} + \frac{1}{2i} \sum_{\lambda} \frac{\alpha^{i}_{\lambda}}{\omega_{\lambda}} \left( \varphi^{1}_{\lambda}(\tau, \sigma) - \varphi^{2}_{\lambda}(\tau, \sigma) \right),
\]

where the mode numbers \( \kappa \) and \( \lambda \) are determined by the equations

\[
e^{2\pi i \kappa} = \frac{\kappa + i\mu |\alpha|}{\kappa - i\mu |\alpha|}, \quad e^{2\pi i \lambda} = \frac{\lambda - i\mu |\alpha|}{\lambda + i\mu |\alpha|},
\]

respectively. The basis functions \( \varphi^{1,2}_{\kappa,\lambda}(\tau, \sigma) \) are defined as in Eq. (2.20) with \( \kappa \) and \( \lambda \) satisfying the equations (2.38).
The mode expansion of the spinor field can be determined by following exactly the same procedure as in the previous subsection. We take a combination of spinor fields \( \xi^A(\tau, \sigma) \) with integer modes and spinor fields \( \eta^A(\tau, \sigma) \) with \( R \)-modes:

\[
S^1(\tau, \sigma) = \begin{cases} 
I_+\xi^1(\tau, \sigma) + I_-\eta^1(\tau, \sigma), & \text{for A-type;} \\
I_-\xi^1(\tau, \sigma) + I_+\eta^1(\tau, \sigma), & \text{for B-type,}
\end{cases}
\]

\[
S^2(\tau, \sigma) = I_+\xi^2(\tau, \sigma) + I_-\eta^2(\tau, \sigma),
\]

where

\[
\xi^1(\tau, \sigma) = \cosh \mu \sigma S_0 + \sinh \mu \sigma \Omega_0 \Pi S_0 + \sum_{n \neq 0} c_n(\varphi^1_n(\tau, \sigma)\Omega_0 \bar{S}_n + i\rho_n \varphi^2_n(\tau, \sigma)\Pi S_n),
\]

\[
\xi^2(\tau, \sigma) = \cosh \mu \sigma \Omega_0^2 S_0 + \sinh \mu \sigma \Pi S_0 + \sum_{n \neq 0} c_n(\varphi^2_n(\tau, \sigma)S_n - i\rho_n \varphi^1_n(\tau, \sigma)\Pi \Omega_0 \bar{S}_n),
\]

\[
\eta^1(\tau, \sigma) = \sum_\kappa c_\kappa(\varphi^1_\kappa(\tau, \sigma)\Omega_0 \bar{S}_\kappa + i\rho_\kappa \varphi^2_\kappa(\tau, \sigma)\Pi \Omega_0 \bar{S}_\kappa),
\]

\[
\eta^2(\tau, \sigma) = \sum_\kappa c_\kappa(\varphi^2_\kappa(\tau, \sigma)S_\kappa - i\rho_\kappa \varphi^1_\kappa(\tau, \sigma)\Pi \Omega_0 \bar{S}_\kappa).
\]

Similarly, we call A-type branes for \(|p - q| = 0, 4, 8\) in \( Dp - Dq \) brane configurations while B-type branes for \(|p - q| = 2, 6\). The projection matrices \( I_{\pm} \) are equally given by Eq. (2.26) and thus satisfy the identities, Eqs. (2.28) and (2.29). Therefore only three kinds of possibility in (2.30) are allowed.

In Eq. (2.40), we introduced \( c_\kappa \) and \( \rho_\kappa \) defined as in Eq. (2.21) with \( \nu \) replaced by \( \kappa \) being a solution of either the first or the second equation in Eq. (2.38) and

\[
\bar{S}_n = \frac{1}{\omega_n}\left(\frac{n}{|\alpha|} - i\mu \Pi \Omega_0\right)S_n, \quad \bar{S}_\kappa = \frac{1}{\omega_\kappa}\left(\frac{\kappa}{|\alpha|} - i\mu \Pi \Omega_0\right)S_\kappa.
\]

One can see that a zero mode solution in \( \eta^A(\tau, \sigma) \) with \( \kappa = 0 \) is identically cancelled as expected.

In order to see that Eq. (2.39) satisfies the boundary condition (2.9), it is more convenient to decompose the spinors \( S_\kappa^A \) into eigenspinors of \( \Pi \Omega_0 \) by defining

\[
S_\kappa^\pm = \frac{1}{2}(1 \pm \Pi \Omega_0)S_\kappa.
\]

The spinors then have the following property

\[
\Pi \Omega_0 S_\kappa^\pm = \pm S_\kappa^\pm, \quad \Pi S_\kappa^\pm = \pm \Omega_0 S_\kappa^\pm.
\]

Using this property, \( \eta^A(\tau, \sigma) \) in Eq. (2.40) can be rewritten as

\[
\eta^1(\tau, \sigma) = \sum_\kappa c_\kappa\left(\frac{\kappa}{|\alpha|} - \frac{i\mu}{\omega_\kappa} \varphi^1_\kappa(\tau, \sigma) + i\rho_\kappa \varphi^2_\kappa(\tau, \sigma)\right)\Omega_0 S_\kappa^+.
\]
The mode expansion of bosonic coordinates $X^I(\tau, \sigma)$ in $D_\mp - D_\pm$ brane configurations without a worldvolume flux is exactly the same as the previous $D_+ - D_-$ case, which is given by Eq. (2.13). And, for $D_\mp - D_\pm$ brane configurations where $D_\mp$-brane is a $D5$-brane with the worldvolume flux, the mode expansion of bosonic coordinates is exactly the same as Eq. (2.23).

From the previous analysis, we have seen that it is crucial that $\Omega_0^T \Omega_\pi$ is a symmetric matrix for the spinors $S^A(\tau, \sigma)$ to satisfy the boundary conditions (2.9). If $\Omega_0^T \Omega_\pi$ were an antisymmetric matrix and thus $(\Omega_0^T \Omega_\pi)^2 = -1$, it would have all eigenvalues $\pm i$ and so the boundary conditions (2.9) could not simultaneously be satisfied. Furthermore the matrices $I_\pm$ would have complex eigenvalues, so they could no longer be projection matrices. This leads to an intriguing consequence: $(+, -, 4, 0)$- and $(+, -, 0, 4)$-branes have no supersymmetric intersections with $D_-$-branes since $\Omega_0^T \Omega_\pi$ in this case is always an antisymmetric matrix and thus they cannot satisfy the boundary conditions (2.9) and $D_\pm$-brane cannot have a supersymmetric intersection with $D_3$- and $D_7$-branes. We will consider only the cases satisfying $(\Omega_0^T \Omega_\pi)^T = \Omega_0^T \Omega_\pi$.\footnote{This is the same reason that we excluded nonsupersymmetric brane configurations with $\sharp_{ND} = 2, 6$. Note that the boundary state formalism also meets a similar situation since the guiding principle in the construction of boundary states is the preservation of various supersymmetries [40].} Since the symmetric condition for the matrix $\Omega_0^T \Omega_\pi$ excludes $D5$-branes with flux, the bosonic mode expansion for the $D_\mp - D_\pm$ case becomes exactly equal to the $D_- - D_-$ case.
There are the following identities for A,B-type D-branes:

\[
\Pi I_{\pm} = I_{\mp} \Pi, \quad \Omega_\theta I_{\pm} = I_{\pm} \Omega_\theta, \quad I_{\pm} \Omega_0 = \pm I_{\pm} \Omega_\pi, \quad \text{for A-type}; \\
\Pi I_{\pm} = I_{\pm} \Pi, \quad \Omega_\theta I_{\pm} = I_{\mp} \Omega_\theta, \quad I_{\pm} \Omega_0 = \mp I_{\pm} \Omega_\pi, \quad \text{for B-type}.
\]  

(2.48)  

(2.49)

Note that a new feature arises in this case. Unlike as the \( D_+ - D_- \) cases, the matrices \( I_{\pm} \) change in different way passing the matrices \( \Pi \) and \( \Omega_\theta \) as seen in Eqs. (2.48) and (2.49). Thus, any solution of the equations of motion (2.3) for spinors cannot be simultaneously compatible with the boundary condition (2.9). This immediately implies that the \( D_+ - D_- \) brane intersections preserve no supersymmetry.\(^3\)

Let us briefly explain why it should be. First take a spinor of the following form as usual:

\[
S^1(\tau, \sigma) = \begin{cases} 
I_+ \xi^1(\tau, \sigma) + I_- \eta^1(\tau, \sigma), & \text{for A-type;} \\
I_- \xi^1(\tau, \sigma) + I_+ \eta^1(\tau, \sigma), & \text{for B-type,}
\end{cases}
\]

(2.50)

where the spinors \( \xi^1(\tau, \sigma) \) and \( \eta^1(\tau, \sigma) \) are supposed to satisfy the equations of motion (2.3) and the boundary condition (2.9) at \( \sigma = 0 \). In order for the spinor \( S^1(\tau, \sigma) \) to satisfy the equation of motion, \( \partial_+ S^1 - \mu \Pi S^2 = 0 \), the spinor \( S^2(\tau, \sigma) \) is necessarily of the form

\[
S^2(\tau, \sigma) = I_- \xi^2(\tau, \sigma) + I_+ \eta^2(\tau, \sigma).
\]

(2.51)

However, the spinors \( S^A(\tau, \sigma) \) in Eqs. (2.50)-(2.51) do not satisfy the boundary condition at \( \sigma = 0 \):

\[
\Omega_0 S^2(\tau, \sigma = 0) = \begin{cases} 
I_- \xi^1(\tau, \sigma = 0) + I_+ \eta^1(\tau, \sigma = 0), & \text{for A-type;} \\
I_+ \xi^1(\tau, \sigma = 0) + I_- \eta^1(\tau, \sigma = 0), & \text{for B-type,}
\end{cases}
\]

\( \neq S^1(\tau, \sigma = 0) \).

(2.52)

It is easily shown that the solution in Eqs. (2.50)-(2.51) cannot be compatible with supersymmetries. As we mentioned at the beginning of this subsection, NN and DD coordinates in the \( D_+ - D_- \) brane intersection have integer modes while ND and DN coordinates have half-integer modes. Thus, if the dynamical supersymmetry were strictly preserved, the supersymmetry transformation, for example, for an NN coordinate \( X_r \) and an ND coordinate \( X^i \) would be of the form

\[
\delta_\epsilon X^r = \frac{1}{\sqrt{2p^+}} \bar{\epsilon} A^r \gamma^r \xi^A, \quad \delta_\epsilon X^i = \frac{1}{\sqrt{2p^+}} \bar{\epsilon} A^i \gamma^i \eta^A,
\]

(2.53)

because the spinors \( \xi^A \) and \( \eta^A \) are supposed to be described by integer modes and half-integer modes, respectively. In order to satisfy the supersymmetric transformation (2.53), the spinor \( S^2(\tau, \sigma) \) should be of the form

\[
S^2(\tau, \sigma) = I_+ \xi^2(\tau, \sigma) + I_- \eta^2(\tau, \sigma).
\]

(2.54)

\(^3\)We thank a referee of Physical Review D for drawing our attention to clarify this problem.
To see this, first note that the constant spinors $\epsilon^4$ in the supersymmetry transformation (2.53) satisfy

$$\epsilon^1 = \Omega_0 \epsilon^2, \quad \epsilon^1 = \Omega_\pi \epsilon^2$$

and thus do

$$\epsilon^1 = \begin{cases} \Omega_0^T \Omega_\pi \epsilon^1, & \text{for A-type;} \\ -\Omega_0^T \Omega_\pi \epsilon^1, & \text{for B-type;} \end{cases}$$

$$\epsilon^2 = \Omega_0^T \Omega_\pi \epsilon^2.$$  

(2.56)

Then it is easy to show, using Eqs. (2.56), (4.11), and (4.12), that the supersymmetry transformation (2.16) reduces to Eq. (2.53). Indeed this explains why the mode expansion for supersymmetric intersecting D-branes necessarily takes the form such as Eqs. (2.17) and (2.39). The solution (2.51) therefore is contradictory to the dynamical supersymmetry. The similar thing happens for the kinematical supersymmetry since the preserved kinematical supersymmetry has to satisfy both the equation of motion and the boundary condition. Thus all $D_+ - D_-$ brane intersections preserve no supersymmetry.

### 3 D-branes at Angles

In the previous section we considered only either parallel D-branes or intersecting D-branes at right angles. In this section we will consider D-branes intersecting at general angles [29].

In flat spacetime, it was shown in [29] (see also [42]) that the condition for D-branes intersecting at angles to preserve supersymmetry is that the two branes should be related by an $SU(N)$ subgroup in a space rotation group $SO(d)$. The type IIB plane wave background (1.1) has the space rotation group $SO(4) \times SO(4)'$. Under any rotation $R$, the brane characterized by $\Omega$ is mapped to a brane described by $\hat{\Omega} = R^T \Omega R$. Indeed, one can consider two kinds of rotation in the plane wave background as recently emphasized in [8]. One is that $R$ is an element in $SO(4) \times SO(4)'$. The other is that $R$ is an element in $SO(8)$ that is not in $SO(4) \times SO(4)'$. We call the latter an oblique rotation. Since $SO(4) \times SO(4)'$ rotation commutes with $\Pi$, and hence,

$$\Pi \hat{\Omega} \Pi \hat{\Omega} = \begin{cases} -1, & \text{for } \Omega \in D_--; \\ +1, & \text{for } \Omega \in D_+, \end{cases}$$

(3.1)

the image of any supersymmetric brane under a rotation $R \in SO(4) \times SO(4)'$ therefore describes another supersymmetric D-brane of the same kind. On the other hand it can be shown [8] that an oblique rotation $R$ has to satisfy

$$R^4 = 1.$$  

(3.2)

We will not discuss the oblique D-branes in detail in this paper.
We are interested in D-branes intersecting at general angle, but preserving some fraction of supersymmetry. We will consider only intersecting D-branes generated by a subgroup of $SO(4) \times SO(4)'$ rotation. As shown in [29], the condition that two branes intersecting at angles preserve a common supersymmetry is that they should be related by an $SU(N)$ rotation in the $SO(4) \times SO(4)'$ group. Since $SO(4) \times SO(4)' = (SU(2)_L \times SU(2)_R)^2$, the $SU(N)$ factors in $SO(4) \times SO(4)'$ group are only $SU(2)_L$ and $SU(2)_R$, which are self-dual and anti-self-dual rotations in $SO(4)$’s, respectively. Since rotations that preserve the worldvolume of a brane do not change the resulting orientation of the brane, we restrict our attention to those rotating a plane which lies in the Neumann and Dirichlet directions of a brane.

One can see from the list in Eqs. (2.11) and (2.12) that the supersymmetric rotation is possible only for $D$- and $D7$-branes among the $D_+$-branes and $(+, -, 2, 2)$-brane in $D_+$-branes since for other branes a nontrivial rotation is $U(1)$ rotation, so completely breaks the supersymmetry. We first consider a $D3$-brane at first oriented along the $X_1, 3, 4$-axes, for example, and then rotated by the angle $\phi_1$ in the $X^1X^2$ plane and $\phi_2$ in the $X^3X^4$ plane. (A rotated $D7$-brane can also be treated similarly.) We define the complex coordinates $Z^1 = X^1 + i X^2$ and $Z^2 = X^3 + i X^4$. There can be two kinds of boundary conditions on stretched open strings. The first one is of the NN type given by

\[
\begin{align*}
\text{Re} \, \partial_\tau Z^i|_{\sigma=0} &= 0 = \text{Im} \, Z^i|_{\sigma=0}, \\
\text{Re} \, e^{i \phi_1} \partial_\sigma Z^i|_{\sigma=\pi|\alpha|} &= 0 = \text{Im} \, e^{i \phi_1} Z^i|_{\sigma=\pi|\alpha|},
\end{align*}
\]

where $i = 1, 2$ and the mode expansion of complex bosonic coordinates is

\[
Z^i(\tau, \sigma) = i \sum_{n_i \in \mathbb{Z}} \left( \frac{\alpha_{\kappa_i}^i}{\omega_{\kappa_i}} \varphi^1_{\kappa_i}(\tau, \sigma) + \frac{\tilde{\alpha}_{\lambda_i}^i}{\omega_{\lambda_i}} \varphi^2_{\lambda_i}(\tau, \sigma) \right),
\]

where $\kappa_i = n_i - \delta_i$ and $\lambda_i = n_i + \delta_i$ with $0 \leq \delta_i = \phi_i/\pi \leq \frac{1}{2}$. The second one is of the DN-ND type given by

\[
\begin{align*}
\text{Im} \, \partial_\tau Z^i|_{\sigma=0} &= 0 = \text{Re} \, Z^i|_{\sigma=0}, \\
\text{Im} \, e^{i \phi_1} \partial_\sigma Z^i|_{\sigma=\pi|\alpha|} &= 0 = \text{Re} \, e^{i \phi_1} Z^i|_{\sigma=\pi|\alpha|}
\end{align*}
\]

and the mode expansion is

\[
Z^i(\tau, \sigma) = \sum_{n_i \in \mathbb{Z}} \left( \frac{\alpha_{\kappa_i}^i}{\omega_{\kappa_i}} \varphi^1_{\kappa_i}(\tau, \sigma) + \frac{\tilde{\alpha}_{\lambda_i}^i}{\omega_{\lambda_i}} \varphi^2_{\lambda_i}(\tau, \sigma) \right),
\]

where $\kappa_i = n_i + \frac{1}{2} - \delta_i$ and $\lambda_i = n_i - \frac{1}{2} + \delta_i$. The basis functions $\varphi_{\kappa_i, \lambda_i}^{1, 2}(\tau, \sigma)$ and the frequencies $\omega_{\kappa_i, \lambda_i}$ are defined as in Eqs. (2.20) and (2.21) with $\kappa_i$ and $\lambda_i$. The oscillators have the reality conditions $\alpha_{-\kappa_i}^i = \tilde{\alpha}_{\kappa_i}^i$ and $\alpha_{-\lambda_i}^i = \tilde{\alpha}_{\lambda_i}^i$ and the commutation relation is

\[
[\alpha_{\kappa_i}^i, \alpha_{\kappa_j}^j] = \frac{1}{2} \omega_{\kappa_i, \kappa_j} \delta_{n_i, n_j} \delta^{ij}, \quad [\tilde{\alpha}_{\lambda_i}^i, \tilde{\alpha}_{\lambda_j}^j] = \frac{1}{2} \omega_{\lambda_i, \lambda_j} \delta_{n_i, n_j} \delta^{ij}.
\]
First we consider intersecting $D_\perp - D'_\perp$ branes at angles, especially $D_\perp p - D'3$ as a representative example, where a rotated brane is indicated with the prime. The boundary conditions for spinor fields $S^A(\tau, \sigma)$ are now given by

$$ (S^1 - \Omega_0 S^2)|_{\sigma=0} = 0, \quad (S^1 - \hat{\Omega}_\pi S^2)|_{\sigma=\pi \alpha} = 0, \quad (3.8) $$

where

$$ \hat{\Omega}_\pi = \Omega_\pi R^2, \quad \Omega_\pi = \gamma^{13} $$

and

$$ R = e^{i\sum_1^2 s_i \phi_i}. \quad (3.10) $$

When $[R, \Omega_0] = 0$, e.g. $(+, -, 4, 2)$-brane, the boundary condition (3.8) can be rewritten as the ordinary boundary condition (2.9) with respect to the rotated spinors $S'^A = R S^A$. Since the action (2.1) is invariant under $SO(4) \times SO(4)'$ rotations, the resulting supersymmetry is never changed. Thus we will not consider such cases either.

One may take explicit eigenvalues of the rotation $R$ in (3.10) as follows:

$$ R = e^{i\sum_1^2 s_i \phi_i}. \quad (3.11) $$

where $s_i = \pm \frac{1}{2}$ are eigenvalues of $\frac{i}{2} \gamma^{12}$ and $\frac{i}{2} \gamma^{34}$ acting on spinor fields. Since the second boundary condition can be rewritten as the form $(S^1 - \Omega_0 (\gamma^2 \Omega_\pi R^2 S^2)|_{\sigma=\pi \alpha} = 0$, one can see that the boundary condition becomes identical to the original boundary condition before the rotation if $R^2 = 1$, namely, $\phi_1 = \phi_2$, self-dual rotation, and $\phi_1 = -\phi_2$, anti-self-dual rotation. However the latter condition essentially reduces the number of spinors compatible with the boundary condition by half compared to the original brane configuration before rotation. Thus we expect that, when $R^2 = 1$, the supersymmetry is also further reduced by half, otherwise, the supersymmetry is completely broken. We will prove this claim in section 4.

The spinors $S^A(\tau, \sigma)$ have the same form as Eq. (2.17) where the projection matrices $I_{\pm}$ are still given by Eq. (2.26), but the mode numbers are quite different from Eq. (2.19) due to the second boundary condition in Eq. (3.8):

$$ \xi^1(\tau, \sigma) = \xi^1_0(\tau, \sigma) + \sum_\kappa c_\kappa (\varphi^1_\kappa(\tau, \sigma) \Omega_{0\kappa} S_\kappa + i \rho_\kappa \varphi^2_\kappa(\tau, \sigma) \Pi S_\kappa), $$

$$ \xi^2(\tau, \sigma) = \xi^2_0(\tau, \sigma) + \sum_\kappa c_\kappa (\varphi^2_\kappa(\tau, \sigma) S_\kappa - i \rho_\kappa \varphi^1_\kappa(\tau, \sigma) \Pi \Omega_{0\kappa} S_\kappa), $$

$$ \eta^1(\tau, \sigma) = \sum_\lambda c_\lambda (\varphi^1_\lambda(\tau, \sigma) \Omega_{0\lambda} S_\lambda + i \rho_\lambda \varphi^2_\lambda(\tau, \sigma) \Pi S_\lambda), $$

$$ \eta^2(\tau, \sigma) = \sum_\lambda c_\lambda (\varphi^2_\lambda(\tau, \sigma) S_\lambda - i \rho_\lambda \varphi^1_\lambda(\tau, \sigma) \Pi \Omega_{0\lambda} S_\lambda), \quad (3.12) $$

where $\xi^A_0(\tau, \sigma)$ are possible zero modes to be determined later. The first boundary condition in Eq. (3.8) is automatically satisfied due to the properties in Eqs. (2.28) and (2.29). Let us now
briefly explain what condition arises and how to determine the mode numbers \( \kappa \) and \( \lambda \) from the second boundary condition in Eq. (3.8). A nontrivial requirement is that \([R, I_\pm] = 0\) or equivalently, \(R\Omega_0 = \Omega_0 R^T\), which means that 

\[
\{\gamma^{12}, \Omega_0\} = \{\gamma^{34}, \Omega_0\} = 0, \quad (3.13)
\]

since we already excluded the case \([R, \Omega_0] = 0\) for the reason explained above. This requires that the brane characterized by \(\Omega_0\) has to span \(X^1 - X^3\) plane or \(X^2 - X^4\) plane. Then the second boundary condition in Eq. (3.8) reduces to the following equations which determine the mode numbers \(\kappa\) and \(\lambda\):

\[
e^{2\pi i x} S_\kappa - i \rho_\kappa \Pi \Omega_0 S_\kappa = R^2 (S_\kappa - i e^{2\pi i x} \rho_\kappa \Pi \Omega_0 S_\kappa),
\]

\[
e^{2\pi i \lambda} S_\lambda - i \rho_\lambda \Pi \Omega_0 S_\lambda = - R^2 (S_\lambda - i e^{2\pi i \lambda} \rho_\lambda \Pi \Omega_0 S_\lambda). \tag{3.14}
\]

The equations (3.14) can be easily solved and the result is given by

\[
\kappa = n + \nu_a, \quad \lambda = n - \frac{1}{2} + \nu_a, \quad n \in \mathbb{Z}, \tag{3.15}
\]

where \(\nu_a = \sum_{i=1}^{2} s_i \delta_i\). Of course, the phases \(\nu_a (a = 1, \cdots, 8)\) depend on the eigenvalues \(s_i\) of the spinors \(S_\kappa\) and \(S_\lambda\) for given angles \(\phi_i\), but the details are not important in our context.

For the case \(\phi_1 = \phi_2\) and \(\phi_1 = -\phi_2\), zero modes with \(\kappa = 0\) exist for \((2s_1, 2s_2) = (\pm 1, \mp 1)\) and \((\pm 1, \pm 1)\), respectively, and they are of the same form as the zero modes in (2.19), but with \(S_0\) satisfying \(R^2 S_0 = S_0\), i.e., \(\nu_0 = 0\). Thus the final number of zero modes is further reduced by half after rotation. For example, in the cases of \(D3\)-brane with \(\Omega_0 = \Omega_\pi\) and \(D7\)-brane with \(\Omega_0 = \gamma^{245678}\), there are zero modes while, in the cases of \(D3\)-brane with \(\Omega_0 = \pm \gamma^{24}\), \(D5\)-brane with \(\Omega_0 = \pm \gamma^{2345}\), and \(D7\)-brane with \(\Omega_0 = \pm \gamma^{135678}\), there exist two zero modes.

Now we consider \(D_+ p - (+, - 2, 2)\) brane intersections. We assume \((+, -, 2, 2)\)-brane is oriented along the \(X^{1,3}\)-axes and \(X^{5,7}\)-axes. In this case there are two possibilities rotating the \((+, -, 2, 2)\)-brane while preserving supersymmetry. One is an \(SU(2)\) rotation as in Eq. (3.10). The other is an \(SU(2) \times SU(2)\) rotation described by

\[
R = R_1 R_2 = e^{\frac{1}{2} (\gamma^{12} \phi_1 + \gamma^{34} \phi_2)} e^{\frac{1}{2} (\gamma^{56} \phi_3 + \gamma^{78} \phi_4)} \tag{3.16}
\]

whose eigenvalues are

\[
R = e^{i \sum_{i=1}^{4} s_i \phi_i}. \tag{3.17}
\]

Since the single \(SU(2)\) rotation is almost the same as the previous \(SU(2)\) case, we will analyze the double \(SU(2)\) case only. The mode expansion of complex bosonic coordinates \(Z^i (i = 1, \cdots, 4)\) where \(Z^3 = X^5 + i X^6\) and \(Z^4 = X^7 + i X^8\) has the same form as Eqs. (3.4) and (3.6) with the angles \(0 \leq \delta_i = \phi_i / \pi \leq \frac{1}{2}\).
Since $R$ is a trivial rotation in the case $[R, \Omega_0] = 0$, we restrict attention to the case satisfying the following condition

$$\{\gamma^{12}, \Omega_0\} = \{\gamma^{34}, \Omega_0\} = \{\gamma^{56}, \Omega_0\} = \{\gamma^{78}, \Omega_0\} = 0. \quad (3.18)$$

Only the $D_{+5} = (+, -2, 2)$-brane belongs to the nontrivial case of Eq. (3.18). Note that $(+, -1, 1)$- and $(+, -3, 3)$-branes are excluded even in the single $SU(2)$ rotation since a non-trivial rotation which is $U(1)$ or $U(1) \times U(1)$ completely breaks the supersymmetry. The mode expansion is given by Eq. (2.39) with

$$\xi^1(\tau, \sigma) = \xi^1_0(\tau, \sigma) + \sum_\kappa c_\kappa (\varphi^1_\kappa(\tau, \sigma)\Omega_0 \tilde{S}_\kappa + i\rho_\kappa \varphi^2(\tau, \sigma)\Pi S_\kappa),$$

$$\xi^2(\tau, \sigma) = \xi^2_0(\tau, \sigma) + \sum_\kappa c_\kappa (\varphi^2_\kappa(\tau, \sigma)S_\kappa - i\rho_\kappa \varphi^1(\tau, \sigma)\Pi \tilde{S}_\kappa),$$

$$\eta^1(\tau, \sigma) = \sum_\lambda c_\lambda (\varphi^1_\lambda(\tau, \sigma)\Omega_0 \tilde{S}_\lambda + i\rho_\lambda \varphi^2(\tau, \sigma)\Pi S_\lambda),$$

$$\eta^2(\tau, \sigma) = \sum_\lambda c_\lambda (\varphi^2_\lambda(\tau, \sigma)S_\lambda - i\rho_\lambda \varphi^1(\tau, \sigma)\Pi \Omega_0 \tilde{S}_\lambda), \quad (3.19)$$

where the mode numbers $\kappa$ and $\lambda$ are now determined by the following equations

$$e^{2\pi i\nu} = \frac{e^{2\pi i\nu_a} + i\rho_\kappa \kappa \pm i\mu}{1 \pm ie^{2\pi i\nu_a} \rho_\kappa \omega_\kappa}, \quad \text{for } S^\pm_\kappa, \quad (3.20)$$

$$e^{2\pi i\lambda} = \frac{e^{2\pi i\nu} \pm i\rho_\lambda \lambda \pm i\mu}{1 \mp ie^{2\pi i\nu_a} \rho_\lambda \omega_\lambda}, \quad \text{for } S^\pm_\lambda \quad (3.21)$$

with $\nu_a = \sum_{i=1}^4 s_i\delta_i$. We decomposed the spinors $S_\kappa$ and $S_\lambda$ as in Eq. (2.42):

$$S^\pm_\kappa = \frac{1}{2}(1 \pm \Pi \Omega_0)S_\kappa, \quad S^\pm_\lambda = \frac{1}{2}(1 \pm \Pi \Omega_0)S_\lambda. \quad (3.22)$$

If $\nu_a = 0$, one can see that there can be zero modes since $\kappa = 0$ is a solution of Eq. (3.20). They are of the same form as the zero modes in Eq. (2.39), but with $S_0$ satisfying $R^2 S_0 = S_0$. Thus the final number of zero modes is further reduced by quarter after the $SU(2) \times SU(2)$ rotation, which is the same situation as the flat spacetime [29, 30]. For example, in the cases with $\sharp_{ND} = 0, 8$, namely, $\Omega_0 = \Omega_7$ and $\Omega_0 = \Omega_\pi \gamma$, there are two zero modes while, in the case with $\sharp_{ND} = 4$, there exist only one zero mode.

4 Supersymmetry of Intersecting D-branes

In a light-cone gauge, the 32 components of the supersymmetries for closed strings decompose into kinematical supercharges, $Q^{+A}_a$, and dynamical supercharges, $Q^{-A}_a$. For superstrings in
the plane wave background with the action (2.1), the conserved super-Nöther charges were identified by Metsaev [2]:

\[ Q^{+1} = \frac{\sqrt{2p^+}}{2\pi\alpha'p^+} \int_0^{2\pi\alpha'|p^+|} d\sigma (\cos \mu \tau S^1 - \sin \mu \tau \Pi S^2), \tag{4.1} \]

\[ Q^{+2} = \frac{\sqrt{2p^+}}{2\pi\alpha'p^+} \int_0^{2\pi\alpha'|p^+|} d\sigma (\cos \mu \tau S^2 + \sin \mu \tau \Pi S^1), \tag{4.2} \]

\[ \sqrt{2p^+}Q^{-1} = \frac{1}{2\pi\alpha'p^+} \int_0^{2\pi\alpha'|p^+|} d\sigma (\partial_- X^I \gamma^I S^1 - \mu X^I \gamma^I \Pi S^2), \tag{4.3} \]

\[ \sqrt{2p^+}Q^{-2} = \frac{1}{2\pi\alpha'p^+} \int_0^{2\pi\alpha'|p^+|} d\sigma (\partial_+ X^I \gamma^I S^2 + \mu X^I \gamma^I \Pi S^1). \tag{4.4} \]

The super-Nöther charges of an open string are given by a subset of the symmetries of the closed string action which are compatible with the open string boundary conditions. Due to the boundary condition (2.9), it turns out that the conserved dynamical supercharge is given by the combination

\[ q^- = \frac{1}{2\pi|\alpha|} \int_0^{\pi|\alpha|} d\sigma q^- = I_+ (Q^{-2} - \Omega_\theta^T Q^{-1}). \tag{4.5} \]

It was shown that all half BPS D-branes in the type IIB plane wave background have to satisfy

\[ X^{r'}|_{\partial\Sigma} = 0, \quad \forall r' \tag{4.6} \]

for the Dirichlet coordinates of \( D_- \)-branes [5], and

\[ (\partial_\sigma X^r \Omega_\theta^T S^1 - \mu X^r \Pi S^1)|_{\partial\Sigma} = 0, \quad \forall r \tag{4.7} \]

for the Neumann coordinates of \( D_+ \)-branes [23].

Although \( D_- \)-branes located at a constant transverse position \( x_0' \neq 0 \) superficially appear to break all dynamical supersymmetries, the broken dynamical supersymmetries can be restored by incorporating a worldsheet symmetry [5]. The superstring action (2.1) is invariant under an arbitrary shift of the field by a parameter that satisfies the same field equation and open string boundary condition as the original field. Using this fact, one can find modified transformation rules by using the worldsheet symmetry, which now lead to a conserved charge. On the other hand, one cannot use the worldsheet symmetry to restore some apparently broken dynamical supersymmetry for \( D_+ \)-branes since the symmetry breaking terms involve Neumann coordinates as shown in Eq. (4.7). Only special classes of \( D_+ \)-branes allow the condition (4.7). These are \( D1 \)-branes in which, by definition, \( X^r = 0 \) for all \( r \) and \( D5 \)-branes of type \((+, -, 4, 0)\) or \((+, -, 0, 4)\), thus \( \Omega_\theta^T S^1 = \Pi S^1 \), with a Born-Infeld flux satisfying the modified Neumann
boundary condition (2.32). Another $D_+$-branes cannot satisfy the condition (4.7) and thus the dynamical supersymmetry is not conserved.

Keeping these facts in mind, one may deduce some conditions for supersymmetric intersection of D-branes. The dynamical supersymmetry of intersecting $D_- - D_-$ branes can be preserved only when they have coincident transverse positions, since the worldsheet shift symmetry must be simultaneously applied to both branes. Even parallel but separated $D_- - D_-$ branes of the same type preserve no dynamical supersymmetry. However, the dynamical supersymmetry of intersecting $D_+$-branes does not depend on their transverse locations.

We now give a rigorous worldsheet derivation on conserved supercharges for each case of intersecting D-branes. The conserved kinematical supersymmetry depends on the type of D-branes as shown in [5, 23]. Let us first consider $D_- - D_-$ brane configurations. The conserved kinematical supercharge is given by the form

\[
q^+ = \frac{1}{\pi |\alpha|} \int_0^{\pi |\alpha|} d\sigma q^\tau_+ \\
= \begin{cases} 
I_+(Q^+ + \Omega_0 Q^+), & \text{for A-type,} \\
I_-(Q^+ + \Omega_0 Q^+), & \text{for B-type.}
\end{cases}
\]

Using the equations of motion, Eqs. (2.2) and (2.3), it is easy to show that the kinematical supercharge density $q_+^\tau$ in Eq. (4.8) satisfies the following conservation law

\[
\frac{\partial q^+}{\partial \tau} + \frac{\partial q^+}{\partial \sigma} = 0,
\]

with

\[
q_+^\sigma = \sqrt{2p^+} \left( e^{i\Omega_0 \Pi} I_\pm (S^1 - \Omega_0 S^2) \right),
\]

(4.10)

where we used the fact that $I_\pm$ commute with $\Omega_0 \Pi$. Then one can see that $q^+\tau$ is conserved by observing that $q_+^\sigma$ vanishes at boundaries for A-type branes with $I_+$ and for B-type branes with $I_-$. Now one can easily count the remaining kinematical supersymmetries for each case in (2.30) and the components of conserved supersymmetries can be easily identified using the projection matrices $I_\pm$. For $\Omega_0 = \Omega_\pi$ and $\Omega_0 = \Omega_\pi \gamma$, $I_+$ becomes identity and we have 8 kinematical supersymmetries. For $\Omega_0 = -\Omega_\pi$ and $\Omega_0 = -\Omega_\pi \gamma$, $I_+$ becomes identically zero and so no kinematical supersymmetry is preserved. In the case of $\Omega_0 = \pm \Omega_\pi \Xi$, we have 4 kinematical supersymmetries since $I_+ = \frac{1}{2}(I_8 \pm \Xi)$. The conserved kinematical supersymmetry of intersecting $D_- - D_-$ branes is independent of their transverse locations.

It is useful to recall the (anti-)commutation relations between $\gamma^I = \{\gamma^r, \gamma^r, \gamma^i, \gamma^i\}$, $\Omega_0$ and $\Omega_\pi$ to find conserved dynamical supersymmetries:

\[
\{\gamma^r, \Omega_0\} = \{\gamma^i, \Omega_0\} = [\gamma^r, \Omega_0] = [\gamma^i, \Omega_0] = 0,
\]

(4.11)

\[
\{\gamma^r, \Omega_\pi\} = \{\gamma^i, \Omega_\pi\} = [\gamma^r, \Omega_\pi] = [\gamma^i, \Omega_\pi] = 0.
\]

(4.12)
Using the similar recipe used in the above kinematical supersymmetry, it is not difficult to show that the dynamical supercharge density \( q^- \) in Eq. (4.5) also satisfies the conservation law

\[
\frac{\partial q^-}{\partial \tau} + \frac{\partial q^-}{\partial \sigma} = 0,
\]

where

\[
q^- = \sqrt{\frac{1}{2p^+}} \left( (\partial_\tau X^\gamma \gamma^\gamma \Omega^\gamma_\theta + \mu X^\gamma \gamma^\gamma \Pi) I_{+,-}(S^1 - \Omega^\theta_\phi S^2) - \partial_\sigma X^\gamma \gamma^\gamma \Omega^\gamma_\theta I_{+,-}(S^1 + \Omega^\theta_\phi S^2) - (\partial_\tau X^\gamma \gamma^\gamma \Omega^\gamma_\theta - \mu X^\gamma \gamma^\gamma \Pi) I_{+,-}(S^1 + \Omega^\theta_\phi S^2) + \partial_\sigma X^\gamma \gamma^\gamma \Omega^\gamma_\theta I_{+,-}(S^1 - \Omega^\theta_\phi S^2) \right) + e^{i\theta}(\partial_\tau X^i \gamma^i \Omega^i_\theta + e^{i\theta} \mu X^i \gamma^i \Pi) I_{-,+}(S^1 - e^{i\theta} \Omega^\theta_\phi S^2) - e^{i\theta} \partial_\sigma X^i \gamma^i \Omega^i_\theta I_{-,+}(S^1 + e^{i\theta} \Omega^\theta_\phi S^2) \right) - e^{i\theta}(\partial_\tau X^i \gamma^i \Omega^i_\theta - e^{i\theta} \mu X^i \gamma^i \Pi) I_{-,+}(S^1 + e^{i\theta} \Omega^\theta_\phi S^2) + e^{i\theta} \partial_\sigma X^i \gamma^i \Omega^i_\theta I_{-,+}(S^1 - e^{i\theta} \Omega^\theta_\phi S^2) \right)
\]

with the first subscript in \( I_{+,-} \) or \( I_{-,+} \) for A-type branes while with the second one for B-type branes. Here we presented the general expression for \( \Omega^i_\phi \) with the first subscript in \( I_{+,-} \) or \( I_{-,+} \) for A-type branes while with the second one for B-type branes. Here we assume without loss of generality that the two branes are placed at origin, viz.,

\[
X^\gamma |_{\partial \Sigma} = X^i |_{\sigma = \pi \alpha} = X^\gamma |_{\sigma = 0} = 0.
\]

Using the crucial identities (2.28) and (2.29) as well as the open string boundary conditions, (2.7)-(2.9), and (4.15), it is easy to see that

\[
q^- |_{\partial \Sigma} = 0.
\]

Now it is simple to identify the conserved dynamical supersymmetry for each case in Eq. (2.30). Obviously the \( \Omega_0 = \Omega_\pi \) case preserves 8 dynamical supersymmetries whereas the \( \Omega_0 = -\Omega_\pi \) case does not preserve any dynamical supersymmetry as expected. This should be the case since the former corresponds to a coincident \( Dp-Dp \) brane configuration while the latter does to a \( Dp-anti-Dp \) brane configuration. Similarly, when \( \Omega_0 = -\Omega_\pi \gamma \), 8 dynamical supersymmetries are preserved while the dynamical supersymmetries are completely broken in the case of \( \Omega_0 = \Omega_\pi \gamma \). Finally, the cases of \( \Omega^T_0 \Omega_\pi = \pm \Xi \) preserve 4 dynamical supersymmetries. The results are summarized in Table 1.

We next consider \( D_+ - D_+ \) brane configurations. The kinematical supercharge density

\[
q^+ = \sqrt{2p^+} \phi \Omega^i_\theta \pi \mu |\alpha| e^{i\theta |\alpha|} \Omega^i_\theta \phi S^2 I_+(S^1 + \Omega^\theta_\phi S^2),
\]

satisfies the conservation law [23]

\[
\frac{\partial q^+}{\partial \tau} + \frac{\partial q^+}{\partial \sigma} = 0
\]
with
\[ q^+_\sigma = \sqrt{2p^+} \left[ \frac{\pi \mu |\alpha|}{\sinh \pi \mu |\alpha|} e^{\mu(\sigma - \frac{1}{2}|\alpha|)\Omega_\alpha} I_\pm(S^1 - \Omega_\sigma S^2) \right]. \]  

(4.19)

The corresponding kinematical supersymmetry can be easily seen to be conserved with \( I_+ \) for A-type branes and \( I_- \) for B-type branes, using Eqs. (2.28) and (2.29). One can see that the cases \( \Omega_0 = \Omega_\pi \) and \( \Omega_0 = \Omega_\pi \gamma \) preserve the 8 kinematical supersymmetries irrespective of their transverse locations while 4 supersymmetries for the cases \( \Omega_0 = \pm \Omega_\pi \Xi \). The other cases totally break the kinematical supersymmetry. This is the same as those in \( D_- - D_- \).

To find the conserved dynamical supersymmetry one can apply the same procedure as in the \( D_- \)-brane case. As already mentioned, only \( D1 \)-branes and \( D5 \)-branes with flux can preserve the dynamical supersymmetry. Hence it is sufficient to consider A-type branes only. After some calculation, one gets the result:
\[ \frac{\partial q^-}{\partial \tau} + \frac{\partial q^-}{\partial \sigma} = 0, \]  

(4.20)

where
\[ q^- = \sqrt{2p^+} \left( (\partial_\sigma X^r \gamma^r \Omega_\sigma^T - \mu X^r \gamma^r \Pi) I_+(S^1 + \Omega_\sigma S^2) + \partial_\tau X^r \gamma^r \Omega_\tau^T I_+(S^1 - \Omega_\sigma S^2) \right) \]
\[ + \frac{1}{2p^+} \left( (\partial_\sigma X^r \gamma^r \Omega_\sigma^T + \mu X^r \gamma^r \Pi) I_+(S^1 - \Omega_\sigma S^2) - \partial_\tau X^r \gamma^r \Omega_\tau^T I_+(S^1 + \Omega_\sigma S^2) \right) \]
\[ - e^{i\theta} (\partial_\sigma X^i \gamma^i \Omega_\sigma^T - e^{i\theta} \mu X^i \gamma^i \Pi) I_- (S^1 + e^{i\theta} \Omega_\sigma S^2) + e^{i\theta} \partial_\tau X^i \gamma^i \Omega_\tau^T I_- (S^1 - e^{i\theta} \Omega_\sigma S^2) \]
\[ + e^{i\theta} (\partial_\sigma X^i \gamma^i \Omega_\sigma^T + e^{i\theta} \mu X^i \gamma^i \Pi) I_- (S^1 - e^{i\theta} \Omega_\sigma S^2) - e^{i\theta} \partial_\tau X^i \gamma^i \Omega_\tau^T I_- (S^1 + e^{i\theta} \Omega_\sigma S^2) \). \]

(4.21)

In the case of \( \Omega_0 = \Omega_\pi \), that is, parallel \( D1 - D1 \) and \( D5 - D5 \) branes, one can see that \( q^- |_{\partial \Sigma} = 0 \), so 8 dynamical supersymmetries are preserved. In the case of \( \Omega_0^T \Omega_\pi = \pm \Xi \), however, \( q^- \) at the boundary \( \partial \Sigma \) reduces to
\[ q^- |_{\partial \Sigma} = \sqrt{2p^+} (\partial_\sigma X^i \gamma^i \Omega_\sigma^T + \mu X^i \gamma^i \Pi) I_- (S^1 - \Omega_\sigma S^2) \]  

(4.22)

for \( \Omega_\sigma = \Omega_0 \), while for \( \Omega_\sigma = \Omega_\pi \),
\[ q^- |_{\partial \Sigma} = \sqrt{2p^+} (\partial_\sigma X^i \gamma^i \Omega_\sigma^T + \mu X^i \gamma^i \Pi) I_- (S^1 - \Omega_\pi S^2). \]  

(4.23)

Therefore one can find the following results for the dynamical supersymmetries of \( D1 - D5 \) branes:
\[ q^- = I_+ (Q^-2 - \Omega_0^T Q^{-1}), \quad \text{for } D5 - D1, \]
\[ q^- = I_+ (Q^-2 - \Omega_\pi^T Q^{-1}), \quad \text{for } D1 - D5, \]

(4.24)

where only 4 dynamical supersymmetries are conserved for the two cases since \( I_+ = \frac{1}{2}(1 \pm \Xi) \).
When $\Omega_0 = -\Omega_\pi \gamma$ corresponding to $(+, -, 4, 0) - (+, -, 0, 4)$ brane configurations, only the last two lines in Eq. (4.21) are relevant and we meet an intriguing situation. Some parts in Eq. (4.21) (explicitly, (4.22) for $\Omega_0 = \Omega_0$ and (4.23) for $\Omega_0 = \Omega_\pi$) no longer vanish at the boundary $\partial \Sigma$. The dynamical supersymmetry of $(+, -, 4, 0) - (+, -, 0, 4)$ brane intersections is thus completely broken. This is certainly caused by worldvolume fluxes in $D5$-branes. The results are summarized in Table 2.

In section 3 we analyzed D-branes intersecting at general angles. The supersymmetric intersecting D-branes at general angles are possible only for specific branes such as $D3$- and $D7$-branes among $D_-$-branes and $(+, -, 2, 2)$-branes among $D_+$-branes. It turned out that under $SU(2)$ rotations the half of spinors have the same mode expansion as before the rotation while under $SU(2) \times SU(2)$ rotations the quarter of spinors have the same mode expansion. Thus we naturally expect that the conserved supersymmetry is further reduced by half in the case of single $SU(2)$ rotation and by quarter in the case of double $SU(2)$ rotation [29, 30, 31]. This should be the case since the analysis about conserved supersymmetries for intersecting D-branes at general angles is identical to parallel D-branes or D-branes intersecting at right angles except that the boundary condition at $\sigma = \pi |\alpha|$ is instead given by the second equation in (3.8) and thus a further constraint $(R^2 S^2 - S^2)_{\sigma = \pi |\alpha|} = 0$ is needed to satisfy the boundary condition at $\sigma = \pi |\alpha|$. This additional condition reduces the number of spinors satisfying $q^\pm_\sigma |\partial \Sigma = 0$ by half in the case of single $SU(2)$ rotation while by quarter in the case of double $SU(2)$ rotation as mentioned above. This is the same situation encountered in flat spacetime. We summarized our results for the remaining supersymmetries of all possible supersymmetric intersecting D-branes in Tables 1-2.

In the plane wave background (1.1), $D_{-9}$-brane is not supersymmetric and $D_{+9}$-brane is at most a quarter BPS. Thus it is not trivial to say about type I superstring theory in the plane wave background since it is not obvious how to introduce $D9$-branes carrying $SO(32)$ Chan-Paton factors. Furthermore T-duality transformation in the plane wave background is quite different from flat spacetime case since there is a RR 4-form and the geometry is curved. (It should be interesting to clarify these open problems in the type IIB plane wave background.) Nevertheless, we would like to use the flat spacetime analogue to plainly explain our results in Tables 1-2. The type I string theory contains 16 $D9$-branes carrying $SO(32)$ Chan-Paton factors, and $D1$-brane ($\Omega_0^T \Omega_\pi = \gamma$) and anti-$D1$-brane ($\Omega_0^T \Omega_\pi = -\gamma$) are both half-BPS states, preserving 8 supersymmetries [43]. Of course, if $D1$-brane and anti-$D1$-brane coexist in type I or type IIB string theory, they totally break the supersymmetry. Similarly, $D5$-$D1$ ($\Omega_0^T \Omega_\pi = \Xi$) corresponds to instantons while $D5$-anti-$D1$ ($\Omega_0^T \Omega_\pi = -\Xi$) does to anti-instantons in type IIB string theory, but both instantons and anti-instantons are BPS states preserving 8 supersymmetries in Yang-Mills gauge theory [44]. And other supersymmetric intersecting D-branes with $\sharp_{ND} = 4, 8$ in Tables 1-2 are plane wave analogues of the D-branes generated by T-duality from the previous examples in flat spacetime.
| $\hat{\#}_{ND}$ | $\Omega_0^\top \Omega_\pi$ | $q^+$ | $q^-$ |
|-----------------|----------------|--------|--------|
| 0               | 1              | 8      | 0      |
|                 | $-1$           | 0      | 0      |
| 4               | $\Xi$          | 4      | 0      |
|                 | $-\Xi$         | 4      | 0      |
| 8               | $\gamma$       | 8      | 0      |
|                 | $-\gamma$      | 0      | 0      |
| 0               | 1              | 4      | 0      |
|                 | $-1$           | 0      | 0      |
| 4               | $\Xi$          | 2      | 0      |
|                 | $-\Xi$         | 2      | 0      |
| 8               | $\gamma$       | 4      | 0      |
|                 | $-\gamma$      | 0      | 0      |
| 0               | 1              | 8      | 8      |
|                 | $-1$           | 0      | 0      |
| 4               | $\Xi$          | 4      | 4      |
|                 | $-\Xi$         | 4      | 4      |
| 8               | $\gamma$       | 8      | 0      |
|                 | $-\gamma$      | 0      | 8      |
| 0               | 1              | 4      | 4      |
|                 | $-1$           | 0      | 0      |
| 4               | $\Xi$          | 2      | 2      |
|                 | $-\Xi$         | 2      | 2      |
| 8               | $\gamma$       | 4      | 0      |
|                 | $-\gamma$      | 0      | 4      |

Table 1: Supersymmetry in $D_- - D_-$ intersections. Separated and coincident D-branes are denoted by the bi-arrow and the dash, respectively. A rotated D-brane by $SU(2)$ is indicated by the prime.
| $\mathbf{z}_{ND}$ | $\Omega_{\Omega_0}^T$ | $q^+$ | $q^-$ |
|------------------|--------------------|-------|-------|
| 0                | 1                  | 0     | 0     |
| 4                | $\Xi$              | 0     | 0     |
| 0                | $-\Xi$             | 0     | 0     |
| 8                | $\gamma$           | 0     | 0     |
| 8                | $-\gamma$          | 0     | 0     |

$D_p \leftrightarrow D_q$

$p \neq 1, q \neq 1$

$(+, -, 2, 2) \leftrightarrow (+, -, 2, 2)'$

$(+, -, 2, 2) \leftrightarrow (+, -, 2, 2)''$

$D1 \leftrightarrow D1$

$D1 \leftrightarrow \tilde{D5}$

$\tilde{D5} \leftrightarrow \tilde{D5}$

Table 2: Supersymmetry in $D_+ - D_-$ intersections. We used the bi-arrow to emphasize that the supersymmetry is independent of tranverse locations. $(+, -, 2, 2)'$ is rotated by $SU(2)$ while $(+, -, 2, 2)''$ is rotated by $SU(2) \times SU(2)$. A $D5$-brane with flux is indicated by the widetilde.
5 Supersymmetry Algebra of Intersecting D-branes

In this section we will present the explicit mode expansions and their supersymmetry algebra of conserved supercharges $q^\pm$ only for parallel and orthogonally intersecting D-branes for simplicity. The generalization to intersecting D-branes at general angles should be straightforward.

Let us explain how to easily calculate the mode expansion of dynamical supercharge $q^-$ without a little tedious manipulation. In section 2 we determined the mode expansions of an open string stretched between $D_p$-brane and $D_q$-brane to satisfy the equations of motion and boundary conditions. And it was shown in section 4 that dynamical supercharges satisfy conservation law such as Eqs. (4.13), (4.20), and the spatial worldsheet current $q^-\sigma$ vanishes at worldsheet boundaries. This immediately implies that the charge $q^-$ in Eq. (4.5) is conserved, in other words, it should be $\tau$-independent. From the open string mode expansion, one can see that the time dependent factor is always of the form $e^{-i(\omega_\kappa + \omega_\lambda)\tau}$ where $\omega_\kappa$ is from, say, a bosonic mode and $\omega_\lambda$ is from a fermionic mode. If $\kappa + \lambda \neq 0$, we have $\tau$-dependent terms and thus these terms are necessarily cancelled with another terms.

5.1 $D_- - D_-$

From the previous analysis, we have learned that $D_- - D_-$ brane configurations are mostly close to the flat spacetime case. The kinematical and the dynamical supercharges in this case are given by

$$q^+ = 2\sqrt{2}p^+ I_\pm S_0,$$

$$\sqrt{2}p^+ q^- = 2I_+ \left( p_0^\gamma' \Omega_0^T S_0 + \mu x_0^{\gamma'\gamma} \Pi S_0 + \sum_\kappa c_\kappa \alpha_\kappa^I \gamma' S_\kappa - \frac{i\mu}{2c_\kappa \omega_\kappa} \alpha_\kappa^I \Omega_0^T \gamma' \Pi S_\kappa \right),$$

where the index $I$ collectively denotes NN, ND, DN and DD indices; $I = \{r, i, i', r'\}$ and the matrix $I_+$ in $q^+$ is for A-type branes while $I_-$ for B-type branes. Here the mode number $\kappa$ should be understood as nonzero integers for $I = \{r, r'\}$ and half-integers for $I = \{i, i'\}$. Then the supersymmetry algebra reads as

$$\{q^+_a, q^+_b\} = 2p^+(I_+)_a b,$$

$$\{q^+_a, q^-_a\} = (I_+ \Omega_0 \gamma')_{aa} p^r - \frac{\mu}{p^r} (I_+ \Pi \gamma')_{aa} J^{++},$$

$$\{q^-_a, q^-_b\} = 2H (I_+)_a b + \frac{\mu}{2p^+} \left( (I_+ \gamma^J K \Pi \Omega_0)_{ab} J^{JK} - (I_+ \gamma^J K \Pi \Omega_0)_{ab} J^{J K} \right),$$

where $p^r = p_0^\gamma$ and $J^{++} = -x_0^{\gamma\gamma} p^+$ are translational and boost generators along NN directions, respectively. The hamiltonian $H$ is given by

$$2p^+ H = \frac{1}{2}(p_0^2 + \mu^2 x_0^2) - 2\mu i S_0 \Omega_0 \Pi I_\pm S_0 + \sum_{n \neq 0} \sum_{I = \{r, r'\}} \frac{1}{2} \alpha^-_{-n} \alpha^I_{n} + 2\omega_n S_{-n} I_+ S_n$$

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\[ + \sum_\kappa \left( \sum_{I=(i,i')} \frac{1}{2} \alpha^I_{-\kappa} \alpha^I_{\kappa} + 2\omega_\kappa S_{-\kappa} I_- S_\kappa \right) \tag{5.5} \]

and \( J^I_{iK} \) and \( J^I_{i'i'K} \) are rotational generators in NN, ND, DN and DD directions in the first \( SO(4) \) directions and the second \( SO(4)' \) directions, respectively, whose expressions in terms of modes are given by

\[ J^I_{iK} = x_{i0}^r \rho_0^s - x_{i0}^s \rho_0^r - iS_0 \gamma^I_{iK} I_\pm S_0 - i \sum_\kappa S_{-\kappa} \gamma^I_{iK} I_- S_\kappa \]

\[ -i \sum_{n \neq 0} \left( \frac{1}{2\omega_n} (\alpha^I_{-n} \alpha^K_n - \alpha^K_n \alpha^I_n) + S_{-n} \gamma^I_{iK} I_+ S_n \right) \tag{5.6} \]

for \( J^I_{iK} = \{ J^{rs}, J^{r's'} \} \) and

\[ J^I_{iK} = -iS_0 \gamma^I_{iK} I_\pm S_0 - i \sum_{n \neq 0} S_{-n} \gamma^I_{iK} I_+ S_n \]

\[ -i \sum_\kappa \left( \frac{1}{2\omega_\kappa} (\alpha^I_{-\kappa} \alpha^K_\kappa - \alpha^K_\kappa \alpha^I_\kappa) + S_{-\kappa} \gamma^I_{iK} I_- S_\kappa \right) \tag{5.7} \]

for \( J^I_{iK} = \{ J^{ij}, J^{i'j'} \} \). In Appendix A, we illustrate the superalgebra (5.4) for a specific example, \( D_3 - D_5 \), since some nontrivial identities have been used to derive it.

### 5.2 \( D_+ - D_+ \)

We concentrate only on the \( D_1 - D_5 \) brane intersection since parallel \( D1-D1 \) and \( D5-D5 \) branes satisfy the same supersymmetry algebra as the single \( D1 \)- and \( D5 \)-brane which had already been given in [23]. Therefore only DN and DD coordinates are involved in this brane configuration.

The conserved kinematical supersymmetry is given by

\[ q^+ = 2\sqrt{2p^+ I_+ \tilde{S}_0} \tag{5.8} \]

where

\[ \tilde{S}_0 = \sqrt{\frac{\sinh \pi \mu |\alpha|}{\pi \mu |\alpha|}} e^{i \pi \mu |\alpha| \Omega_0 I} S_0. \tag{5.9} \]

And the dynamical supercharge takes the form

\[ \sqrt{2p^+ q^-} = 2I_+ \sum_\kappa \sum_{I=(i',i')} \left( c_\kappa \alpha^I_{-\kappa} \gamma^I S_\kappa - \frac{i\mu}{2c_\kappa \omega_\kappa} \alpha^I_{-\kappa} \Omega_0 \gamma^I I S_\kappa \right) \tag{5.10} \]

where we assumed that the \( D1, D5 \)-branes are placed at origin for simplicity. Here the mode number \( \kappa \) should be understood as nonzero integers for \( I = r' \) and real numbers satisfying the second equation in Eq. (2.38) for \( I = i' \).
The supersymmetry algebra in this case is of the form:

\[
\{q_a^+, q_b^+\} = 2p^+(I_+)_{ab}, \quad (5.11)
\]
\[
\{q_a^+, q_b^-\} = 0, \quad (5.12)
\]
\[
\{q_a^-, q_b^-\} = 2H(I_+)_{\dot{a}\dot{b}}, \quad (5.13)
\]

where the Hamiltonian is given by

\[
2p^+ H = \sum_{n \neq 0} \left( \frac{1}{2} \alpha_-^{\nu_n} \alpha_-^{\nu_n} + 2 \omega_n S_n I_+ I_+ S_n \right) + \sum_{\kappa} \left( \frac{1}{2} \alpha_-^{\nu_\kappa} \alpha_-^{\nu_\kappa} + 2 \omega_\kappa S_\kappa I_+ I_+ S_\kappa \right). \quad (5.14)
\]

### 6 Discussion

In this paper we studied intersecting D-branes in the type IIB plane wave background using Green-Schwarz superstring action. It turned out that this method is quite elegant and sufficiently powerful since the spacetime supersymmetry of intersecting D-branes and stretched open strings between them are manifest without any GSO projection. However, we should confess that this method also has a similar defect appearing in the boundary state formalism [40].

First of all, it is not obvious how to treat intersecting D-branes with \( z_{ND} = 2, 6 \) which are the cases completely breaking supersymmetry. In addition, \( D3-D1 \) intersection with \( z_{ND} = 4 \) is invisible since the light-like coordinate \( X^- \) always has to satisfy Neumann boundary condition in the light-cone gauge [10, 23]. Since we have used the light-cone gauge in the plane wave background, \( D3-D1 \) intersection is missed in the Tables 1-2 although it is not obvious whether they preserve the supersymmetry.

In this work we have not considered oblique D-branes which was recently discussed in [7, 8]. It was argued [45] that the isometry in the plane wave background (1.1) is indeed \( SO(4) \times SO(4) \times Z_2 \) where the \( Z_2 \) symmetry interchanges simultaneously the two \( SO(4) \) directions

\[
Z_2 : (x^1, x^2, x^3, x^4) \leftrightarrow (x^5, x^6, x^7, x^8). \quad (6.1)
\]

The boundary condition of the oblique D-branes is invariant under the \( Z_2 \) involution (6.1). Thus it should be interesting to know whether the oblique D-branes can be understood by appropriately utilizing the \( Z_2 \) symmetry and to classify the complete list of the oblique D-branes and their intersections.

The most interesting feature of intersecting D-branes is the appearance of chiral fermions on the intersection of D-branes [29]. Since we have used spacetime fermions to study intersecting D-branes, the appearance of the chiral fermions should be more directly seen compared to the NSR formulation which relies on vertex operator construction and GSO projection. An interesting question is whether the chiral fermions can also appear on intersecting D-branes in a plane wave background. We have seen that fermion zero modes on \( D_- \)-branes become massive
in the plane wave background and even for the cases with massless fermion zero modes, e.g., $D_+ - D_+$ intersections, D-brane intersection to realize $\mathcal{N} = 1$ chiral multiplet seems to be impossible, as illustrated in Table 2. Thus the chiral fermion seems to be highly implausible at least in the type IIB plane wave background.

It was known [46, 47] that supersymmetric intersections with $\sharp_{ND} = 2,6$ can be allowed when a suitable B-field is turned on. In addition the BPS bound states of D-branes in the presence of Neveu-Schwarz B-field is T-dual to D-branes intersecting at angles [29]. It will be interesting to study how to generalize Green-Schwarz worldsheet formulation for intersecting D-branes to the case of the presence of the Neveu-Schwarz B-field. We hope to report our progress along this direction elsewhere.

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Appendix

A Supersymmetry Algebra of $D_{-3} - D_{-5}$ Intersection

To illustrate the superalgebra (5.4) explicitly, let us consider a $D_{-3}$-brane oriented along $X^{1,2}$ directions and intersecting with a $D_{-5}$-brane oriented along $X^{2,3,4,5}$. In this case, we have

$$\Pi \Omega_0 = -\gamma^{34}, \quad \Omega_0^T \Omega_\pi = -\gamma^{1345}. \quad (A.1)$$

We will use the indices $i', j' = 3,4,5$ for the DN coordinates and the indices $r', s' = 6,7,8$ for the DD coordinates. The following Fierz identity is useful to compute the supersymmetry algebra (5.4):

$$S^a_{12} = \frac{1}{8} \delta_{ab} S^a_{12} + \frac{1}{16} \gamma^{IJ} S^a_{12} \gamma^{IJ} S^a_{12} + \frac{1}{384} \gamma^{IJKL} S^a_{12} \gamma^{IJKL} S^a_{12}, \quad (A.2)$$

where $S^a_A = (I_+ S^A)^a$ and $S^A$ are the spinors with positive chirality.

For the brane configuration in Eq. (A.1), the algebra of dynamical supersymmetry is given by

$$\{q_a, q_b^-\} = \frac{2}{p^+} \{\sqrt{2p^+ Q_a^2}, \sqrt{2p^+ Q_b^-^2}\} \quad (A.3)$$
where
\[ \sqrt{2p^0 Q_a^\perp} = (p_0^2 \gamma^2 \Omega_0^T + \mu x_0^2 \gamma^2 \Pi) I - S_0 \] (A.4)
\[ + \sum_{n \neq 0} (c_n (\alpha_{-n}^2 \gamma^2 + \alpha_{-n}^r \gamma^r) I + \alpha_{-n}^r \gamma^r I - S_n - \frac{i\mu}{2c_n \omega_n} (\alpha_{-n}^2 \gamma^2 - \alpha_{-n}^r \gamma^r) \Pi \Omega_0^T S_n) \]
\[ + \sum_{\kappa} (c_\kappa (\alpha_{-\kappa}^1 \gamma^1 + \alpha_{-\kappa}^r \gamma^r) I - S_\kappa - \frac{i\mu}{2c_\kappa \omega_\kappa} (\alpha_{-\kappa}^1 \gamma^1 - \alpha_{-\kappa}^r \gamma^r) \Pi \Omega_0^T S_\kappa) \].

It is straightforward to calculate the anticommutator (A.3) for the supercharge (A.4) using the rule
\[ \{B_1 F_1, B_2 F_2\} = [B_1, B_2] F_1 F_2 + B_2 B_1 \{F_1, F_2\} \] (A.5)
for bosonic modes \(B_1, B_2\) and fermionic modes \(F_1, F_2\). The overall calculation is similar to the single D-brane case [23]. However, one should pay attention to the fact that the spinors \(S^{\pm}_\nu (\nu = n, \kappa)\) are now used instead of \(S_\nu\).

Let us explain the useful relations used in the calculation of Eq. (A.3):
\[ f^{IJ}_\perp \gamma^{IJ} I = (2 f^{1i'1'i'} + 2 f^{2r'2r'} + f^{i'j'i'j'} + f^{r's'r's'}) I, \] (A.6)
\[ \frac{1}{48} \sum_{\nu} f^{ijkl} S_{-\nu}^\pm \gamma^{ijkl} S_{-\nu}^{\pm} = \sum_{\nu} f^{1345} S_{-\nu}^\pm S_{-\nu}^{\pm}, \] (A.7)
\[ (\gamma^{15} \Pi \Omega_0 I_+)_{ab} S_{-\nu}^{\pm} = \pm (\gamma^{34} \Pi \Omega_0 I_+)_{ab} S_{-\nu}^{\pm} \] (A.8)
\[ (\gamma^{2r'} \Pi \Omega_0 I_+)_{ab} S_{-\nu}^{\pm} = \pm \frac{1}{2} (\gamma^{r's'} \Pi \Omega_0 I_+)_{ab} S_{-\nu}^{\pm} \] (A.9)
where \(f^{IJ}\) is a fermion bilinear and \(f^{ijkl}\) is a product of gamma matrices. In Eqs. (A.7) and (A.9), \(\gamma_{ab} = -\delta_{ab}\) was used according to Eq. (2.5). In addition, one needs to use the following zeta function regularization to get the algebra (5.4) correctly:
\[ \sum_{n \in \mathbb{Z}} 1 = \sum_{\kappa \in \mathbb{Z}^{\frac{1}{2}}} 1 = 0. \] (A.10)
After carefully using these facts, one can finally get the supersymmetry algebra (5.4) with the rotation generators \(J^{34}_I \in SO(2)\) and \(J^{r's'}_{II} \in SO(3)\).
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