Superconductivity In Disordered Sr$_2$RuO$_4$

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Abstract

We discuss the influence of disorder on the critical temperature $T_C$ of a $p$-wave superconductor. To describe disordered Sr$_2$RuO$_4$ we use extended Hubbard model with random site energies treated in the Coherent Potential Approximation.

I. INTRODUCTION

Recent experimental evidence suggests that the Cooper pairs in superconducting Sr$_2$RuO$_4$ are triplets with $p$-wave internal symmetry, as in the case of superfluid $^3$He. One of characteristic features of this exotic state is the strong influence of impurities on its superconducting properties. Studies of the electronic structure in Sr$_2$RuO$_4$ have identified an extended van Hove singularity close to the Fermi energy $E_F$. In this note we investigate the interplay between the van Hove singularity and disorder in a model $p$-wave superconductor.

II. THE MODEL

We consider a simple extended Hubbard Hamiltonian.
\[
H = \sum_{i\sigma} (\varepsilon_i - \mu) \hat{n}_{i\sigma} + \sum_{ij\sigma} t_{ij} c_{i\sigma}^+ c_{j\sigma} + \frac{1}{2} \sum_{ij\sigma\sigma'} U_{ij} \hat{n}_{i\sigma} \hat{n}_{i\sigma'} \tag{1}
\]

where as usual \( c_{i\sigma}^+ \) and \( c_{i\sigma} \) are the Fermion creation and annihilation operators for an electron on site \( i \) with spin \( \sigma \), \( \hat{n}_{i\sigma} \) is the number operator and \( \mu \) is the chemical potential. Disorder is introduced into the problem by allowing the local site energy \( \varepsilon_i = \pm \delta/2 \) to vary randomly from site to site with equal probability. \( U_{ij} \) is the attractive interaction \((i \neq j)\) between nearest sites and \( t_{ij} \) is the hopping integral from site \( j \) to site \( i \) which takes nonzero values between nearest and next nearest sites. In \( k \)-space: \( \varepsilon_k = \sum_j t_{ij} \exp (i R_{ij} \cdot k) = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y \), where the hopping parameter \( t' = 0.45t \) as well as the band filling \( n_\gamma/2 = 0.66 \) were fitted to the experimental cyclotron masses and corresponding carriers occupations for the \( \gamma \) band of Sr\(_2\)RuO\(_4\), the interaction \( U_{ij}/t = 0.446 \) was chosen to get \( T_C = 1.5K \).

## III. DENSITY OF STATES AND \( T_C \)

The linearized gap equation for the critical temperature \( T_C \) of p-wave pairing from the Hamiltonian (1) reads:

\[
1 = \frac{|U|}{\pi} \int_{-\infty}^{\infty} dE \text{Tanh} \frac{E}{2k_BT_C} \text{Im} \frac{\overline{G}^p_{11}(E)}{2E - \text{Tr} \Sigma(E)}. \tag{2}
\]

where \( \overline{G}^p_{11}(E) \) is an averaged electron Green function which defines the weighted density of states (DOS) of p-wave electron states \( \overline{N}_p(E) \):

\[
\overline{N}_p(E) = -\frac{1}{\pi} \overline{G}^p_{11}(E) = -\frac{1}{\pi N} \sum_k \text{Im} \frac{2\sin^2 k_x}{E - \Sigma_{11}(E) - \varepsilon_k + \mu}, \tag{3}
\]

where \( \Sigma_{11}(E) \) is a Coherent Potential which describe the electron self energy in the disordered system.

In case of a clean system \( (\text{Im} \Sigma_{11}(E) = 0) \) we get a conventional gap equation with the DOS \( N(E) \) substituted by \( \overline{N}_p(E) \) under the integral (Eq. 2). In Fig 1a we show \( N(E) \) and \( \overline{N}_p(E) \) for the clean system. Note that the van Hove singularity in \( N(E) \) produces the maximum in \( \overline{N}_p(E) \). The singularity in \( \overline{N}_p(E) \) is smeared by the presence of the term \( \sin^2 k_x \)
in Eq. 3. This leads to a maximum in $T_C$ with changing $n$ (Fig. 1a). Equations 2 and 3 are influenced by disorder by different effects. Firstly, the peak in $\overline{N}_p(E)$ is smeared (Fig. 1b) leading to small decrease of $T_C$. The second and more interesting effect arises from Eq. 2, where $\Sigma_{11}(E)$ acts as a pair breaker. Using the arguments of Ref. 5, Eq. 2 can be evaluated to yield:

$$\ln \left( \frac{T_C}{T_{C0}} \right) = \psi \left( \frac{1}{2} \right) - \psi \left( \frac{1}{2} + \frac{|\text{Im}\Sigma_{11}(0)|}{2\pi T_C} \right),$$

(4)

where $T_{C0}$ denotes the critical temperature in a clean system. The full influence of disorder on $\overline{N}_p(E)$ is illustrated in Fig. 1b. Note that the position of the maximum value is not affected by small disorder. On the other hand the critical temperature $T_C$, plotted in Fig. 2a is degraded strongly with disorder. This is due to the pair-breaking term $-\text{Im}\Sigma_{11}(E)$, which is shown in Fig. 2b.

IV. SUMMARY

When the Fermi energy is close to a Van Hove singularity relatively weak disorder can cause very rapid $T_C$ degradation in a $p$-wave superconductor. The case of Sr$_2$RuO$_4$ may be an example of this phenomenon.

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FIG. 1. (a) $N(E)$ (1) and $N_p(E)$ (2) in a clean system, (b) $\overline{N}_p(E)$ for disordered system with $\delta/t = 0.0, 0.3, 0.6$ starting from the top line ($\mu = 0$).
FIG. 2. (a) $T_C$ as a function of $n$ for various values of disorder potential ($\delta/t = 0.0, 0.10, 0.13$ for curves 1, 2, 3 respectively, (b) Imaginary part of self energy for various values of disordered potential $\delta/t = 0.10, 0.13, 0.15$ starting form the bottom line ($\mu = 0$).