Numerical investigation of heat transfer and friction factor characteristics of circular tube fitted with an array of semi-elliptical vortex generator inserts

Manjunath M.S¹ and Dolfred Vijay Fernandes¹*

Abstract: This article presents the heat transfer and friction factor characteristics of circular tube fitted with an array of novel semi-elliptical vortex generator inserts using air as working fluid. The study makes use of computational fluid dynamics methodology and the influence of varying aspect ratio (AR = 1, 2, 4 and 6), flow attack angle (45°, 60°, 75° and 90°) and longitudinal pitch (P = 30 mm, 60 mm and 90 mm) of vortex generator is investigated for the turbulent flow Reynolds number regime of 8000–26000. The numerical results reveal that the presence of semi-elliptical vortex generator has significant influence on both Nusselt number and friction factor characteristics of circular tube. The vortex generators produce strong longitudinal vortices which intensifies fluid mixing near the tube wall region thereby augmenting the heat transfer to the flowing air stream. In addition, the presence of flow impingement effect on the upstream side and flow reattachment zones on the downstream side of vortex generator is found to contribute to enhanced heat transfer. The influence of aspect ratio on heat transfer is found to be more significant than that of flow attack angle. However, both parameters affect the friction factor significantly. On the other hand, the pitch distance of insert significantly affects both heat transfer and friction factor. The maximum rise in Nusselt number and friction factor is about 2.1 times and 6.34 times higher than plain tube,

ABOUT THE AUTHOR
Manjunath M.S is an academician with an experience of 12 years in teaching undergraduate and post graduate students in Mechanical engineering and has a Doctoral degree in Thermal Engineering. The author is specialized in thermal sciences and conducts research related to solar energy utilization and heat exchangers. The key research activities include experimental and computational investigations for heat transfer enhancement in solar air heaters, heat exchangers and solar distillation systems. The main focus of heat transfer enhancement studies carried out by the author generally involves the use of turbulence promoters such as ribs and vortex generators of different designs. The author is also working on multi-phase simulation studies in solar distillation system.

PUBLIC INTEREST STATEMENT
The world is driven by energy which is mostly derived from fossil fuels which are consumed at an increasing pace and are bound to deplete in future which makes it imperative to conserve the precious available energy. One way to conserve energy is to reduce wastage of energy in every possible applications. Wastage of energy is generally seen whenever it is transferred from one device to the other, such as an electricity generation plant, large air conditioning systems in malls and so on. The energy wastage occurs simply due to the fact that energy exchange within these devices does not happen at higher level. Therefore, there is a need to increase the energy exchange within these devices preferably by using passive techniques which will help to ultimately reduce the wastage of precious energy. This article offers one such passive technique that could help in achieving energy conservation.
respectively, for P = 30 mm. The thermal enhancement factor (TEF) is found to decrease with increasing flow attack angle and aspect ratio for all flow rates considered in the study. However, the TEF increases with decreasing pitch distance of inserts and the overall TEF is found to be in the range of 0.86–1.19.

**Subjects:** Heat Transfer; Heating Ventilation & Air Conditioning; Renewable Energy; Energy & Fuels

**Keywords:** Circular tube; CFD; Semi-elliptical vortex generator; TEF; Heat transfer enhancement; friction factor

1. **Introduction**

Energy security is at the core of every nation’s future energy strategy and are predominantly based on renewable energy utilization. However, the world is not yet completely ready in extracting renewable energy for all their energy needs owing to the technological limitations of existing renewable energy systems. Hence, most of the energy demands are still met through conventional energy sources such as oil and gas for applications related to transport, industry, residential and commercial sectors. All these sectors make use of energy transport systems such as heat exchangers which transfers energy in the form of heat from one side of the system to the other side to carry out the process such as in a refrigeration system, air-conditioning system, waste heat recovery device, chemical reactors, steam generators, power generation systems etc. Hence, the efficacy with which heat transfer takes place in such thermal systems plays a huge role in achieving precious energy savings. Moreover, increased operational efficiency of such systems also aid in the development of compact systems that in turn reduce its cost as well as installation space requirements (Mousa et al., 2021).

A wide variety of heat exchangers are in use depending on the type of application and heat loads. Tubular heat exchangers are among the commonly used heat exchangers which consists of several circular tubes across which the heat energy exchange is carried out. For the sake of convenience in carrying out investigations for performance improvement in tubular heat exchangers, the analysis is usually carried out by selecting only one circular tube as a common practice as seen in several investigations carried out in the past (Dang & Wang, 2021; Deshmukh & Vedula, 2014; Nakhchi & Esfahani, 2019; Salem et al., 2017; Sheikholeslami et al., 2016; Wijayanto et al., 2020). The focus of most of these investigations has been to evaluate the efficacy of different passive techniques such as twisted tape inserts (Wijayanto et al., 2020), coiled wire inserts (Dang & Wang, 2021), baffle inserts (Salem et al., 2017), ring inserts (Sheikholeslami et al., 2016), conical nozzle inserts (Nakhchi & Esfahani, 2019) and vortex generator inserts (Deshmukh & Vedula, 2014) in augmenting the heat transfer process in circular tube. Each of these techniques have been shown to provide different levels of heat transfer improvement and have different range of useful flow rates of heat transfer fluid. Hence, there are continuous efforts to develop better tubular heat exchanger systems using innovative designs.

Vortex generator (VG) is one of the commonly used device for heat transfer augmentation in a circular tube, which enhances fluid turbulence by generating longitudinal vortices that provide increased flow mixing. Vortex generators are inserted into the tube with the help of inserts using plates or rods that are placed centrally in the tube or by using ring inserts on which the vortex generators are attached circumferentially. Curved delta wings that are similar to conical strips in design appearance were initially used by Deshmukh and Vedula (Deshmukh & Vedula, 2014) using central rod inserts with air as working fluid. The Reynolds number was varied from 10,000 to 45000, and the analysis was carried out for fixed aspect ratio of the wing. The effect of non-dimensional pitch (1.4–7.9), flow attack angle (15°–45°) and non-dimensional height (0.09–0.25) on thermal performance was investigated using experimental methodology. The results show that the flow attack angle of 45°, non-dimensional pitch of 1.6, non-dimensional height of 0.25 produce best thermal performance. A similar design was used with perforations (Chamoli et al., 2017) with varying perforation index of 4%–16% for a fixed flow attack angle of 45° and non-dimensional
pitch of 0.25. The flow Reynolds number was varied between 3000 and 21,000 in their experimental study. The presence of hole was reported to create flow impingement effect on the immediate downstream side of the strip which disrupts the thermal boundary layer region thereby contributing to increased heat energy exchange. Longitudinal swirl flow generator (Wang et al., 2019) whose geometry is a slice of hollow conical body showed good performance in the laminar flow regime for air as working fluid with a maximum Nusselt number improvement of 7.1 times the smooth tube. The friction factor was found to increase by 11.3 times that of smooth tube. The vortex generator specifically generated rubbing effect of air flow at the tip clearance with the tube wall which increased the heat transfer. The presence of swirling movements of air stream in the core flow region also generated excellent flow mixing. In a similar study, bidirectional conical strips (Liu et al., 2018) were numerically evaluated for laminar flow conditions which show the formation of multiple longitudinal vortices. The flow streamlines revealed the presence of interaction of core flow and near wall flow regions which increased flow mixing and heat transfer. As a further improvement, twisted conical strips (Pouramezan & Ajam, 2016) have been shown to create additional swirling flows with a heat transfer improvement of about 3.5 times that of smooth tube.

Louvered strips (Eiamssa-ard et al., 2008) mounted on the central rod inserts were experimentally shown to perform well in augmenting heat transfer for the flow Reynolds number range of 6000–42000 with water as working fluid. The inclination of strip (15°–30°) and arrangement pattern (backward and forward) varied and the results show that the heat transfer improved by 2.84 times while the friction factor increased by 4.1 times for forward arrangement relative to smooth tube. A slightly modified geometry of louvered strip (Yaningsih et al., 2018) which appears to have a leaf shape with different slant angles of 15°–25° showed that the heat transfer increased by 77% with a friction factor increase of 3.35 times that of plain tube. It is also shown that the use of conical strip inserts enhance Nusselt number and friction factor by nearly 5 times and 10 times, respectively. Strip angle is shown to be more influential than the pitch distance. The analysis was carried out numerically for the flow Reynolds number range of 12,000–42000 and they made use of RNG k-ε turbulence model for solving the turbulence parameters. Larger slant angles were observed to generate greater fluid mixing near the tube wall region which complement the heat transfer process. Presence of perforations on these louvered strips (Nakhchi et al., 2020) has been shown to further improve mixing of air streams from the core and tube wall regions. The analysis was carried out numerically using RNG k-ε turbulence model for the flow Reynolds number range of 5000–14000. Double perforations performed better than single perforations with a maximum thermal enhancement factor (TEF) of 1.84 at Re = 14,000.

Bali and Sarac (Bali & Sarac, 2014) conducted an interesting experimental study using propeller type vortex generators (PTVG) using two arrangements for the flow Reynolds number of 5000–30000 using air as working fluid. In the first arrangement, only one PTVG was used at the tube entrance while in the other arrangement, an additional PTVG with slightly different design which has circumferential vanes was used. The number of vanes as well as the flow attack angle was varied using experimental methodology and the results show that the second vane enhances the heat transfer by augmenting the decaying swirl flows from the first PTVG. The results showed that the heat transfer increased by 190.1% relative to smooth tube. Twisted cross-baffle inserts (Nanan et al., 2017) exhibit higher thermal enhancement factor of 1.7 as compared to other configurations of straight cross-baffles, straight baffles, twisted and alternate twisted baffles for the flow Reynolds number of 6000–20000. The twisted cross-baffle insert was found to generate stronger longitudinal vortices causing greater flow mixing between the core and wall region flows which improves heat transfer. Recently, a novel parallelogram winglet (Lin et al., 2017) was used as insert that is twisted about its longitudinal axis in circular tube. Different axial spacing and winglet attack angle was tried in the numerical analysis for laminar flow regime of air flow. The thermal enhancement factor was found to be in the range of 1.25–1.85.

In another design, V-shaped winglets (Promvonge et al., 2020) were attached to plate inserts in V-up and V-down configuration for varying relative heights (0.1–0.2) and relative pitch (0.5–2) for a fixed flow attack angle of 30° with air as working fluid. The numerical results revealed the
formation of two counter-rotating vortices creating common flow down movements for V-up arrangement and common flow up movements for V-down arrangement both of which enhance flow mixing between the core and wall region flows and improve heat transfer. The V-down pattern proved to be better than V-up design by about 3.6% and 1.65% in heat transfer and thermal enhancement factor, respectively. Lei et al. (Lei et al., 2017) conducted three-dimensional CFD analysis using delta winglets on only one side of plate inserts and the numerical results showed that the Nusselt number increased to a maximum extent of 117% while the friction factor increased by 466% for the flow attack angle of 60°. Triangular winglets (Pourhedayat et al., 2020) were placed on both sides of the plate insert for varying latitudinal pitch (0–40 mm) and winglet inclination in forward and backward arrangement pattern. The results show that the latitudinal pitch should be such that the winglets are neither closer to the central region nor to the tube wall and the latitudinal pitch of 20 mm showed the best heat transfer enhancement. In another similar study, curved winglets were placed on the plate insert (Skullong et al., 2018) with an inclination of 45° to the flow for the turbulent flow regime (Re = 4150 to 25,400). Punched curved winglets have been shown to lower the pressure loss and the hole diameters were varied from 1 mm to 3 mm. The TEF for perforated winglets were better than normal winglets by 9%. Wijayanta et al. (Wijayanta et al., 2018) made use of delta wings on plate insert on which holes were created in front of wings. The wings deflect some of the core flow through these holes into the other side of the insert which interacts with the swirl flows emanating from the wake region of upstream winglet.

As an alternative, the delta winglets (Xu et al., 2017) were also placed directly on the tube wall with different flow attack angle (0°–45°) and blockage ratio (0.1–0.3) for Re = 6000–33000. The simulation results showed that the longitudinal vortices remain strong for a relatively longer distance in the flow direction which improves heat transfer. The blockage ratio of 0.1 and flow attack angle of 30° showed the best heat transfer performance. An array of winglet vortex generators (Liang et al., 2018) in a circular tube has been shown to provide considerable heat transfer performance with lower pressure drop penalty as the longitudinal vortices generated in the wake region is shown to persist for a longer distance on the tube wall. The highest improvement in Nusselt number is found to be about 1.36 times while the friction factor rise is about 2.43 times that of smooth tube. A similar study was conducted using ring inserts on which delta winglets (Xu et al., 2018) were placed with different attack angles (0–45°), blockage ratio (0.1–0.3) and relative pitch (1.6–4.8). The ring inserts were placed as an array along the tube length with four winglets placed at equal angles along the ring circumference and the highest TEF was found to be about 1.45. A similar such design in the form of delta winglet pairs (Zhai, Islam, Alam et al., 2019; Zhai, Islam, Simmons et al., 2019) were used in the circular tube for turbulent flow regime of air flow where the attack angle (10°, 20°, 30° and 40°), height of winglet (5–10 mm) and pitch (10–20 mm) were varied in the experimental analysis. The winglets were attached to the ring insert and were placed close to the tube wall. It showed significant improvement in heat transfer by about 73% as compared to smooth tube while raising the friction factor by 2.5 times. The highest thermal enhancement factor is found to be about 1.44.

In a very recent study conducted by Zhang et al. (Zhang et al., 2020), rectangular vortex generators (RVG) were used in a circular tube where the length of the RVG was varied between 5.1 and 15.3 mm (length ratio = 0.1 to 0.3). Different flow attack angles (45°, 60°, 75° and 90°) and RVG arrangement pattern were investigated for the flow Reynolds number range of 6000–20000. They conducted both experimental and numerical investigations where RNG k-ε turbulence model was used for solving the turbulence flow parameters. The results showed that the heat transfer improvement was better for V-shaped arrangement (V-RVG) as compared to parallel arrangement (P-RVG), However, the V-arrangement exhibited relatively higher increase in friction factor. The heat transfer increased by 54–118% and 60–118% for P-RVG and V-RVG, respectively. The heat transfer enhancement was primarily due to strong longitudinal vortices generated in the wake regions of RVG that extend to some distance along the tube wall. They also developed correlations
for Nusselt number and friction factor in terms of flow Reynolds number, length ratio and flow attack angle.

The above research findings of Zhang et al. (Zhang et al., 2020) which made use of rectangular vortex generators placed close to the heated tube wall surface using ring inserts served as motivation for the current study. From the above-outlined literature review, it is noted that the geometrical design of vortex generator significantly influences the flow and heat transfer characteristics of circular tube. This inspired the authors to investigate the influence of a semi-elliptical vortex generator (SEVG) on the thermal characteristics of circular tube in place of rectangular vortex generator. The semi-elliptical geometry has curved edges which could have different extents of influence on the flow structure depending on the size and orientation of SEVG. The present investigation aims to bring out the efficacy of SEVG as heat transfer enhancement device. To the best of authors knowledge which is based on the extensive literature review carried out, the proposed semi-elliptical vortex generator configuration is not reported in the literature for tubular heat exchanger applications. The present study makes use of three-dimensional Computational Fluid Dynamics (CFD) methodology with an objective of evaluating the effect of (i) aspect ratio (i.e., the size) of SEVG (ii) flow attack angle and (iii) longitudinal pitch (i.e., the number of SEVG inserts) on heat transfer and friction factor characteristics for air flow through the circular tube in turbulent flow regime.

2. Numerical analysis

2.1. Geometric configurations of semi-elliptical vortex generator

The study makes use of a circular tube provided with an array of semi-elliptical vortex generator inserts as shown in Figure 1. The dimensions of the tube and inserts are based on the study conducted by Zhang et al. (Zhang et al., 2020) who made use of rectangular vortex generator on ring inserts. The inner diameter (D) of copper tube is taken as 51 mm. Ring inserts having a width (w) of 2 mm and height (h) of 1 mm are used to attach the vortex generators inside the tube as shown in Figure 1. Eight vortex generators are placed at equal distances along the circumference of ring insert. The pitch distance (P) of inserts are varied as 30 mm, 60 mm and 90 mm. The size of the semi-elliptical vortex generator is varied using the aspect ratio (AR = A/B) and the flow attack angle (α) is varied as 45°, 60°, 75° and 90°. The height of the vortex generator is fixed at 2.5 mm for the entire analysis and is referred using the non-dimensional parameter called blockage ratio “BL = (B/R)” which is fixed as 0.098. The aspect ratio is varied as 1, 2, 4 and 6 to effect the size variation of semi-elliptical vortex generator by varying its base length (A) as 2.5 mm, 5 mm, 10 mm and 15 mm, respectively. The various geometric configurations used in the analysis are given in Table 1, which also depicts the resulting geometry of semi-elliptical vortex generator as per the variations of aspect ratio.

2.2. Governing equations and performance parameters

The equations that govern the flow and heat transfer in a steady, incompressible and turbulent flow are given below:

Continuity equation:

$$\frac{\partial}{\partial x_i} (\rho u_i) = 0$$

Momentum equation:

$$\frac{\partial}{\partial x_i} (\rho u_i u_j) = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu_{\text{eff}} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + \frac{\partial}{\partial x_i} \left( -\rho u_i u_j \right)$$
Energy equation:

$$\frac{\partial}{\partial x_i} (\rho u_i T) = \frac{\partial}{\partial x_j} \left[ (\Gamma + \Gamma_t) \frac{\partial T}{\partial x_j} \right]$$

(3)

The RNG k-ε turbulence model:

$$\frac{\partial}{\partial x_i} (\rho k u_i) = \frac{\partial}{\partial x_j} \left( \alpha_k \mu_{eff} \frac{\partial k}{\partial x_j} \right) + G_k - \rho \epsilon$$

(4)

$$\frac{\partial}{\partial x_i} (\rho \epsilon u_i) = \frac{\partial}{\partial x_j} \left( \alpha_\epsilon \mu_{eff} \frac{\partial \epsilon}{\partial x_j} \right) + C_{1\epsilon} \frac{\epsilon}{k} G_k - C_{2\epsilon} \rho \epsilon^2 - R_e$$

(5)

The effective viscosity is given by,

$$\mu_{eff} = \mu + \mu_t$$

The turbulent viscosity is given by,

$$\mu_t = \rho C_p \frac{k^2}{\epsilon}$$

(6)
Table 1. Various semi-elliptical vortex generator configurations used in the analysis

| Case   | Aspect ratio of vortex generator AR = (A/B) | Blockage ratio (B_L = B/R) | Pitch (P) mm | Flow attack angle ($\alpha$) | Corresponding geometric configurations of semi-elliptical vortex generator |
|--------|--------------------------------------------|-----------------------------|--------------|-------------------------------|--------------------------------------------------------------------------------|
| Case 1 | 1                                          | 0.098                       | 60           | 75°                           | ![Image of a semi-elliptical vortex generator](image1.png)                      |
|        | 2                                          |                             |              |                               |                                                                               |
|        | 4                                          |                             |              |                               |                                                                               |
|        | 6                                          |                             |              |                               |                                                                               |
| Case 2 | 4                                          |                             | 60           | 45°                           | ![Image of a semi-elliptical vortex generator](image2.png)                      |
|        | 60                                         |                             |              | 60°                           |                                                                               |
|        | 75°                                         |                             |              | 75°                           |                                                                               |
|        | 90°                                         |                             |              | 90°                           |                                                                               |
| Case 3 | 4                                          |                             | 30           | 75°                           | ![Image of a semi-elliptical vortex generator](image3.png)                      |
|        | 60                                         |                             |              | NA                            |                                                                               |
|        | 90                                         |                             |              | NA                            |                                                                               |
The value of the constants in the above equations are given by,

\[ C_{1e} = 1.42, \quad C_{2e} = 1.68 \quad \text{and} \quad C_{4} = 0.0845. \]

Also, \( G_{p} = -\rho C_{p} \frac{Nu}{T_{w}} \), \( C_{2e} = C_{2} + C_{4} \alpha (1 - \frac{\mu}{\mu_{w}}) \)

\[ \eta = \frac{Sk_{i}}{\rho_{w}} \beta = 0.012. \]
\[ \eta_{a} = 4.38. \quad C_{2e} = 1.68. \quad C_{4} = 1.42. \quad C_{p} = 0.0845 \quad \text{and} \quad \alpha_{e} = 1.393. \]

The important performance parameters used in the analysis are described below:

### 2.3. Nusselt number

The Nusselt number for plain tube is given by the well-known Dittus--Boelter equation and is given by,

\[ Nu_{p} = 0.023Re^{0.8}Pr^{0.4} \] \( (7) \)

The Nusselt number for circular tube in the presence of vortex generator is determined as follows:

\[ h = \frac{Q'}{(T_{w} - T_{a})} \] \( (8) \)

where \( T_{a} = \frac{T_{w} + T_{s}}{2} \) is the bulk mean temperature of air (K). The heat flux used in the above expression does not contain the surface area of vortex generator inserts as they are assumed to be adiabatic surfaces in the study.

\[ Nu = \frac{hD}{K} \] \( (9) \)

### 2.4. Friction factor

The friction factor for plain tube is determined from the Blasius equation given by,

\[ f_{p} = 0.316Re^{-0.25} \] \( (10) \)

The friction factor for circular tube in the presence of vortex generator is determined using the below equation:

\[ f = 2 \left( \frac{\Delta p}{L} \right) \frac{D}{\rho V^{2}} \] \( (11) \)

The well-known Dittus--Boelter and Blasius equations have been used to determine the Nusselt number and friction factor for smooth tube configuration under heating conditions and fully developed turbulent flow through the plain circular tube. These correlations have been widely used for smooth tube configurations owing to its simplicity for a wide range of Reynolds number for fully developed turbulent flow regime.

### 2.5. Thermal enhancement factor (TEF)

The effective thermal performance is evaluated by the commonly used thermal enhancement factor, which considers the relative gain of heat transfer and pressure drop penalty to determine if the proposed design is truly beneficial for heat transfer enhancement. The TEF is given by (Skullong et al., 2018, 2016; Webb & Eckert, 1972),

\[ TEF = \left( \frac{Nu}{Nu_{0}} \right) \left( \frac{f}{f_{p}} \right)^{1.3} \] \( (12) \)
This expression is based on the criteria of equal pumping power for rough and smooth surfaces and a detailed mathematical deduction can be found in Skullong et al. (Skullong et al., 2016) and Webb and Eckert (Webb & Eckert, 1972).

2.6. Boundary conditions, assumptions and numerical schemes

The circular tube is supplied with a uniform heat flux of 500 W/m² on its top surface as per Zhang et al. (Zhang et al., 2020). The simulation of flow and heat transfer for the entire length of circular tube which are usually several metres long is very computationally intensive and also involves unmanageable number of grids in the computational domain. Hence, many of the previous studies using inserts in circular tube have made use of periodic conditions (Nanan et al., 2017; Pourramezan & Ajam, 2016; Skullong et al., 2018) at the inlet and outlet boundaries so as to reduce the computational efforts owing to the periodic nature of velocity field and heat transfer that repeats itself after every periodic module under fully developed flow conditions. This periodic condition is valid as long as the geometry repeats itself periodically from one periodic module to other and the thermal conditions are uniform for every periodic module as described by Patankar et al. (Patankar et al., 1977). Since the geometry considered in the analysis for a given pitch distance repeats itself and the thermal conditions are also uniform at the boundaries, periodic flow conditions are adopted in the present investigation. Hence, one periodic module of the computational domain having the periodic length equal to the pitch distance is used for numerical analysis as highlighted in Figure 1. The inlet and outlet boundaries are therefore defined as periodic boundaries. The flow rate of air at the inlet is varied from 0.059 kg/s to 0.0191 kg/s, which corresponds to the flow Reynolds number of 8000–26000. The interface between the solid wall and fluid is specified with no-slip and impermeable wall conditions. The walls of the insert as well as vortex generator are defined as adiabatic walls and no heat transfer is assumed to take place through their surface as specified in the experimental study of Zhang et al (Zhang et al., 2020). The thermo-physical properties of air are assumed to remain constant at the average bulk temperature. The air flow through the tube is assumed to be steady, incompressible and fully developed. The heat lost by radiation and convection from the tube surface to atmosphere is also neglected in the analysis. These assumptions have been commonly used by several similar studies in the past (Nanan et al., 2017; Pourramezan & Ajam, 2016; Skullong et al., 2018; Zhang et al., 2020).

The analysis is carried out using the methodology of computational fluid dynamics (CFD) in which finite volume approach is used for discretizing the governing equations of flow and heat transfer. The simulation is carried out using Ansys Fluent software tool and RNG k-ε turbulence model is used in the analysis with enhanced wall treatment feature to resolve the near wall region which has been widely used in several earlier studies (Chamoli et al., 2017; Nakhchi et al., 2020; Yaningsih et al., 2018; Zhang et al., 2020) for heat transfer analysis in circular tubes owing to its capability to predict flows better in computational domains having curvatures and swirling flows. Moreover, the RNG k-ε turbulence model has been shown to predict results closer to that of experiments for rectangular vortex generators (Zhang et al., 2020). The coupled solver is used for solving the governing equations which offers faster convergence while the flow and energy equations were discretized using higher order QUICK scheme. The convergence of solution is considered when the residuals reach values lower than 10⁻⁶ for flow and turbulence equations and 10⁻⁸ for energy equation.

2.7. Grid independence study and CFD model validation

In order to carry out the validation of CFD model used in the analysis along with its associated numerical schemes and boundary conditions, the experimental results of Zhang et al (Zhang et al., 2020) are used as benchmark results, which involve rectangular vortex generator inserts in a circular tube as shown in Figure 2. The present study is based on this experimental work where the rectangular vortex generator is replaced with semi-elliptical vortex generator to numerically evaluate its influence on heat transfer and friction factor performance. Hence, the same configuration of rectangular vortex generator used in their experimental study is considered to carry out validation study using fully developed periodic flow condition. The height of rectangular
vortex generator is 2.55 mm, length is 5.1 mm and thickness is 1 mm and is arranged at equal distance (8 numbers) on a ring insert which has a width of 2 mm and height of 1 mm. The inner diameter of the tube is 51 mm and the longitudinal pitch of insert is 60 mm. A grid independence test is first carried out before the validation study to ensure grid independent solution. The computational domain is discretized using tetrahedron volumes to generate varying number of control volumes. Further, to ensure proper capturing of wall gradients, the inflation feature is adopted at the wall regions to resolve the boundary layer region by adding 20 inflation layers with a growth rate of 1.15 as shown in Figure 2(b).

The number of control volumes in the domain is varied as 191,380, 461,745, 865,475 and 1,398,538 and the Nusselt number as well as friction factor is noted for each mesh density as listed in Table 2. It is seen that the variation in Nusselt number as well as friction factor is marginal and is significantly less than 1% when the number of control volumes were increased from 865,475 to 1,398,538. Hence, all the models used in the analysis were ensured to contain a minimum of 865,475 control volumes to ensure grid independent solution. Further, the results
The results show that the CFD results obtained for rectangular vortex generator insert are in close agreement with that of experimental results with a maximum deviation of 11.7% at Re = 34,000 and a minimum deviation of 0.57% at Re = 10,000 for Nusselt number. The average deviation over the entire range of Reynolds number is found to be about 6.97%. The deviations between the CFD and experimental, however small, could be due to variations in the experimentally measured temperatures and CFD temperature values at the tube exit as a result of heat loss from the tube in actual experiments despite the insulation. Moreover, the CFD model assumes a perfect adiabatic surface for the vortex generator inserts which enforces zero heat flux from these surfaces. On the other hand, there is a possibility of heat transfer from the surface of inserts in actual experiments leading to deviations between experimental and CFD Nusselt number. For the friction factor, the error between the CFD and experimental values could be due to deviations in the experimental configuration and CFD model. In the experimental configuration, the ring insert which has vortex generator fixed on has a clearance gap of 0.5 mm with the tube wall in order to make it possible to push them inside the tube. In the CFD model, this clearance is not

| Number of control volumes | Nusselt number | Percentage change in Nusselt number | Friction factor | Percentage change in friction factor |
|---------------------------|---------------|------------------------------------|----------------|-------------------------------------|
| 191,380                   | 87.64         | —                                  | 0.1074         | —                                   |
| 461,745                   | 86.02         | −1.84%                             | 0.0975         | −9.19%                              |
| 865,475                   | 84.71         | −1.53%                             | 0.0965         | −0.99%                              |
| 1,398,538                 | 84.73         | 0.02%                              | 0.0960         | −0.53%                              |

Figure 3. Comparison of present CFD Nusselt number and friction factor results for rectangular vortex generator with the experimental results of Zhang et al (Zhang et al., 2020).
considered which could lead to deviations in the prediction of flow behaviour in the near wall region around the inserts where the boundary layer effects are predominant. As a result, the deviations are found to be relatively higher at lower Reynolds number of the flow. The CFD friction factor is found to have a deviation of about 19.03% at \( \text{Re} = 8000 \). However, as the Reynolds number increases, these deviations are found to reduce significantly which could be due to the effect of boundary layer thinning and the error is found to be only about 4.7% at \( \text{Re} = 34,000 \). The average deviation over the entire range of Reynolds number is found to be about 10.2%.

\[
Nu = 0.10182\text{Re}^{0.71412}\text{LR}^{0.12851}(1 + \cot \alpha)^{-0.12513}
\]  

(13)  

\[
f = 1.02554\text{Re}^{-0.11224}\text{LR}^{0.49995}(1 + \cot \alpha)^{-0.71158}
\]  

(14)  

Where, \( \text{LR} = \text{length ratio} = (\text{length of VG/tube diameter}) = 0.1 \).

3. Results and discussions

The analysis involves three-dimensional CFD study to evaluate the effect of aspect ratio, attack angle as well as longitudinal pitch of the semi-elliptical vortex generator on heat transfer and friction factor characteristics of circular tube. The results are presented in a systematic way using the following sections:

- Effect of aspect ratio
- Effect of flow attack angle
- Effect of pitch

3.1. Effect of aspect ratio

The effect of aspect ratio of semi-elliptical vortex generator on heat transfer and friction factor behaviour is investigated for different flow Reynolds number ranging from 8000 to 26000. The variation of Nusselt number for different aspect ratio \( (\text{AR} = 1, 2, 4, \text{&} 6) \) for a given blockage ratio of 0.098, longitudinal pitch of 60 mm and flow attack angle of \( \alpha = 75^\circ \) is shown in Figure 4.

The results show that the Nusselt number for all the vortex generator configurations considered in the analysis is considerably greater than that of the plain tube for all flow rates of air. This is due to the presence of complex flow structure variations induced by the vortex generators that provide enhanced flow mixing and heat transfer near the tube wall. The nature of the flow induced by the vortex generator is evident from the streamlines of velocity as depicted in Figure 5, which shows the formation of longitudinal vortices from each vortex generator in the region close to the heated tube wall. A closer look into the streamlines reveals that the flow structure is indeed complex which could be analysed by identifying the key regions of heat transfer on the tube wall that are influenced by the presence of semi-elliptical vortex generator. It is observed that the vortex generator augments heat transfer in three key regions on the tube wall accentuated by the numbers 1, 2 and 3 in square boxes in the enlarged view of Figure 5(a). The region ‘1’ is characterised by flow impingement of incoming air stream which are deflected from the upstream face of the vortex generator and hit the tube surface on its immediate upstream region as shown in Figure 5(b). As a result, relatively colder air from the core region is brought in contact with the hotter tube surface resulting in increased temperature gradient thereby improving the heat transfer. Subsequently, the air stream is seen to move spirally along the insert and eventually join the main flow as shown by the dotted line in Figure 5(b). The region ‘2’ is characterised by the presence of longitudinal vortices in the downstream region of vortex generator and are characterised by intense flow mixing. The vortex flow causes the relatively colder air stream from the core region to come in direct contact with the heated tube wall thereby increasing the heat energy exchange from the tube wall regions that are under the influence of these vortex flows. The tube
Figure 4. Effect of aspect ratio on Nusselt number and friction factor for $B_L = 0.098$, $P = 60$ mm and $\alpha = 75^\circ$.

Figure 5. Streamlines (coloured by velocity in m/s) of air flow over the vortex generator inserts depicting (a) key locations of heat transfer augmentation around the vortex generator and (b) air flow impingement on the upstream side of vortex generator.
wall region that is in contact with these vortex flows are significantly large thereby ensuring augmented heat transfer to the air stream. The streamlines also reveal that the longitudinal vortices move along the tube and eventually lose its strength in the flow direction.

Interestingly, the vortices are also seen to move circumferentially creating a sideways movement of air flow thereby bringing the air stream in greater contact with the heated tube wall which ensures increased heat transfer. The region ‘3’ consists of flow reattachment zones where the air stream separated from the tube wall on the upstream side of the insert reattaches with the tube surface on its immediate downstream region. The reattachment zones receive air streams from the main flow as well as from the vortex generator through flow deflection effect and the relative contribution depends on the aspect ratio as well as the flow attack angle of vortex generator. Thus, in essence, the combined effect of augmented flow turbulence due to intense longitudinal vortices, circumferential movement along the tube, flow impingement effect on the upstream side and flow reattachment on the downstream side of semi-elliptical vortex generator is found to collectively augment the heat transfer as compared to plain tube.

Further, it is interesting to note that the variation of aspect ratio has significant influence on heat transfer as shown in Figure 4. As the aspect ratio increases from AR = 1 to AR = 2, the Nusselt number is found to increase marginally. However, with further increase of aspect ratio from AR = 2 to AR = 6, the Nusselt number increases considerably for all flow rates of air. This is due the fact that the variation of aspect ratio of semi-elliptical vortex generator significantly alters its geometrical profile which in turn affects the extent of flow disturbance as well as heat transfer augmentation. This is clearly seen in Figure 6(a) which shows the streamlines of air flow (coloured by velocity in m/s) for AR = 1 for a fixed blockage ratio of 0.098. It is seen that the aspect ratio has significant influence on flow structure and heat transfer. For the aspect ratio AR = 1, the vortex generator geometry is a semi-ellipse with relatively smaller size which affects its influence on the flow. Owing to smaller geometric size, the flow impingement effect occurs on a relatively smaller tube wall region at which higher heat transfer takes place. This aspect is clearly seen in Figure 7 which shows the comparison of Nusselt number distribution on the tube walls fitted with vortex generators having AR = 1 and AR = 6. The region at which higher heat transfer takes place due to

Figure 6. Comparison of flow structure, turbulence intensity (%) and temperature distribution (K) at a sectional plane of Z/P = 0.58 from inlet for (a) AR = 1 and (b) AR = 6 for B_L = 0.098.
flow impingement on the immediate upstream region of vortex generator is seen to be relatively smaller for the configuration having AR = 1. Further, the streamlines for AR = 1 also reveal that there are larger flow reattachment regions due to the presence of larger circumferential gap between the two consecutive vortex generators as seen in Figure 6(a). Therefore, there are larger areas that exhibit increased heat transfer on the tube wall as indicated by the presence of higher Nusselt number in Figure 7(b).

Further, the streamlines also reveal that the longitudinal vortices generated by the vortex generator having AR = 1 is relatively weak owing to lower levels of flow disturbance as a result of its smaller size. Also, these longitudinal vortices are seen to diminish within a short distance in the flow direction. As a result, the extent of flow mixing induced by the vortex generator in its downstream region is also lower as seen by the turbulence intensity contour plots in Figure 6(a) for the configuration of AR = 1. The turbulence intensity contours are taken on a transverse plane which is at a non-dimensional distance of Z/P = 0.58, which corresponds to a distance of 35 mm from the inlet and is just behind the vortex generator insert. The turbulence intensity is found to be significant in the regions immediately behind the vortex generator due to the presence of vortices. These vortices associated with elevated turbulence intensity move along the tube to a shorter distance before it loses its strength. The tube wall region that comes in direct contact with these vortices experiences increased cooling effect as indicated by the presence of higher Nusselt number (dotted horizontal oval symbol) in Figure 7(b).

As the aspect ratio increases, the influence of vortex generator on flow structure also increases resulting in increased heat transfer. As the aspect ratio increases, the geometric profile of the vortex generator changes making it relatively bigger at AR = 6 as seen in Figure 6(b). Due to increased size, the vortex generator deflects more air stream to impact the tube wall thereby generating larger regions of flow impingement zones. Therefore, there are larger areas of tube wall with higher Nusselt number due to flow impingement effect as depicted in Figure 7(a). Further, due to increased size of vortex generator, the circumferential gap between them also reduces resulting in smaller regions (region “A” in Figure 7(a)) of heat transfer enhancement due to flow reattachment. However, the larger size of vortex generator exhibits the formation of larger and stronger longitudinal vortices and is found to have higher levels of turbulence intensity in the air stream as seen in Figure 6(b). The extent of flow disturbance at higher aspect ratio is evident from the fact that turbulence intensity is also at elevated level for most part of the air stream in the core flow region as well. As a result, there are larger areas of tube wall that are in direct contact with these stronger longitudinal vortices which exhibit elevated levels of heat transfer as seen in Figure 7(a). The vortices generated in the wake region of vortex generators brings in higher velocity air stream from the core flow region in contact with the heated tube wall.
Since, these air stream coming from the core region are relatively colder, the temperature gradient at the regions on tube wall increases considerably causing boundary layer thinning thereby leading to increased heat transfer from these regions as indicated by the contour plots of temperature (at \(Z/P = 0.58\)) in Figure 6(a) and Figure 6(b). It is to be noted that the thermal boundary layer thinning is more pronounced for \(AR = 6\) as compared to that of \(AR = 1\), which is the result of greater flow disturbances introduced in the air stream. The tube wall temperature variation along the circumferential line taken at a distance of \(Z/P = 0.58\) from the inlet for all the aspect ratio used in the analysis further reveals the nature of varying influence of vortex generator depending on the aspect ratio as shown in Figure 8. The temperature along the circumferential line is plotted with reference to angle “\(\theta\)”, which varies from \(0^\circ\) to \(180^\circ\) in clockwise direction and \(0^\circ\) to \(-180^\circ\) in the anti-clockwise direction along the circumferential line. It is observed that the wall temperature profiles are oscillatory in nature and repeat itself around each vortex generator. Also, the temperature variations are seen to depend on the aspect ratio to a considerable extent. The temperature variations are relatively smaller for the configuration of \(AR = 1\), whereas at \(AR = 6\), the temperature variations are relatively greater and has several locations with lower tube wall temperatures owing to increased heat transfer. Thus, higher aspect ratio is found to provide greater heat transfer and the maximum improvement in Nusselt number is found to be about 1.9 times as compared to plain tube for the configuration having \(AR = 6\).

The presence of vortex generator obstructs the flow and will lead to additional pressure drop across the tube which depends on the aspect ratio. Figure 4 shows that the friction factor for semi-elliptical vortex generator is always greater than plain tube for all flow rates used in the analysis. Further, the friction factor increases with increasing aspect ratio as a result of increased flow obstruction caused by larger sized vortex generator at higher aspect ratio. Figure 9 shows the comparison of pressure distribution on a central longitudinal plane passing through the tube which clearly shows that the pressure variations are considerably dependent on the aspect ratio. The air stream impacts the upstream side of the vortex generator and loses its kinetic energy which leads to a corresponding rise in pressure. This pressure rise depends on the size of vortex generator as
well as the extent of flow disturbance generated in its wake region. For the configuration having AR = 6, the frontal area impacting the incoming air stream is about 5 times larger than that of AR = 1 which obstructs the flow to a greater extent causing higher pressure rise on its upstream side as seen in Figure 9(b).

In addition, it also creates larger and stronger longitudinal vortices in its downstream which leads to increased pressure energy loss due to complex flow interactions. Therefore, the friction factor is found to be higher for AR = 6. The pressure distribution taken at a transverse sectional plane at Z/P = 0.48 and Z/P = 0.58 shows that the pressure difference is significant across the vortex generator as compared to other locations. The same trend is observed for the configuration having AR = 1 but the differential pressure is relatively low as compared to that of AR = 6 as seen in Figure 9(a). Thus, friction factor increases with aspect ratio and the maximum increase in friction factor is found to be about 5.62 times for the aspect ratio of 6 at the flow Reynolds number of 26,000. The lowest friction factor enhancement is found to be about 2.33 times that of plain tube for the aspect ratio of 1 at the flow Reynolds number of 8000.

3.2. Effect of flow attack angle
The influence of flow attack angle of semi-elliptical vortex generator on heat transfer for a given aspect ratio of AR = 4 for different flow Reynolds number is shown in Figure 10. The flow attack angle is varied as 45°, 60°, 75° and 90°. It is seen that the flow attack angle has limited effect on Nusselt number for a given aspect ratio of vortex generator. The heat transfer is seen to improve as the flow attack angle increases from 45° to 75°. However, the change in Nusselt number from 75° to 90° is found to be marginal, although, the Nusselt number for the flow attack angle of 90° is found to be slightly lower than that of 75°. The highest increase in Nusselt number for the flow attack angle of 45°, 60°, 75° and 90° is found to be about 1.68, 1.71, 1.79 and 1.76 times that of plain tube, respectively, for the flow Reynolds number of 8000. A closer look into the streamlines of air flow across the vortex generator (refer Figure 11(a)) reveals that the flow attack angle of 45°
Figure 10. Comparison of Nusselt number and friction factor for different flow attack angle at AR = 4, P = 60 mm and $B_L = 0.098$ for different flow Reynolds number.

has a relatively lower levels of flow disturbance as compared to that of 75° and 90°. The air stream passes over the vortex generator relatively smoothly with slightly lesser interference in its flow as seen in Figure 11(a). However, as the flow attack angle increases to 75°, the flow interference increases further leading to the formation of stronger and larger longitudinal vortices in its wake (refer Figure 11(b)). As a result, the flow turbulence is higher which aids in heat transfer enhancement. The contour plots of turbulence intensity (%) clearly show the existence of higher turbulence intensity in the air stream for higher flow attack angles. The turbulence intensity is plotted by taking a central longitudinal plane passing through the tube. However, for the flow attack angle of 90°, the flow structure is found to deviate as clearly seen in Figure 11(c) where the vortices formed in the wake region consists of two counter-rotating vortices. These vortices are seen to diminish relatively quickly as compared to longitudinal vortices of other angles. However, these vortices generate strong flow mixing effect as indicated by higher turbulence intensity in Figure 11(c). In addition, the flow deflection effect from vortex generator which causes some of the core flow to reach the tube wall in the flow reattachment region is seen to be absent as the vortex generator is at right angles to the incoming air stream. As a result, the flow reattachment is solely contributed by the main air flow stream that reattach with the tube wall after experiencing flow separation over the inserts in the region between the vortex generators as indicated in Figure 11(c). Further, the flow impingement effect is more significant for the flow attack angle of 90° owing to greater flow deflection by the vortex generator as it faces the incoming air stream in its normal direction. In effect, the vortex generator is able to completely bend the air flow towards its base making it to impinge on the tube wall which augments the heat transfer.

This is clearly seen in Figure 12 where a significant drop in tube wall temperature is observed on the immediate upstream of the vortex generator with $\alpha = 90^\circ$ and these are the locations that are associated with augmented heat transfer. It is also noted that the flow impingement effect becomes more dominant with increasing flow attack angle as seen in Figure 12, which clearly shows that the tube wall area affected by flow impingement becomes larger at higher attack angle and all these areas show significant temperature drop. Further, a comparison of temperature
distribution for different flow attack angles shows similar trend except for $\alpha = 90^\circ$. The temperature distribution is clearly different in the downstream side of the tube for $\alpha = 90^\circ$ due to the formation of two counter-rotating vortices. It is interesting to note that higher temperature drop occurs on the upstream side of vortex generator for $\alpha = 90^\circ$ due to greater flow impingement effect, whereas this effect is observed on both the upstream and downstream side for other angles.

Figure 13 shows the comparison of velocity distribution and velocity vectors taken on the transverse sectional plane at $Z/P = 0.58$ and $Z/P = 0.98$ to examine the flow behavior in its flow direction. The velocity contours show similar profiles for flow attack angles less than $90^\circ$ and
Figure 12. Comparison of temperature distribution of tube wall for different flow attack angles at AR = 4, P = 60 mm and $B_L = 0.098$ for Re = 26,000.

exhibit circumferential movement of air stream in anti-clockwise direction as seen in Figure 13(a) and Figure 13(b). While such circumferential movements are seen to diminish considerably for $\alpha = 45^\circ$ they continue for a longer tube length for $\alpha = 75^\circ$ thereby allowing the air stream to cover larger tube wall region, which improves heat transfer. These circumferential movements are created due to flow deflection from the inclined vortex generator which change the flow direction of core flow upon impact. However, for $\alpha = 90^\circ$, since the flow deflection effect is negligible, the circumferential movement of air stream in the tube is not seen as indicated in Figure 13(c) in its entire downstream side for $\alpha = 90^\circ$. However, this limitation is more than compensated by the presence of increased flow impingement effect as explained earlier as well as the presence of stronger counter-rotating vortices which have higher turbulence intensity. As a result, the heat transfer enhancement for $\alpha = 90^\circ$ drops only marginally as compared to $\alpha = 75^\circ$. Thus, the flow attack angle does not affect heat transfer as significantly as the aspect ratio. However, they are found to have considerable impact on friction factor enhancement as seen in Figure 10. Larger flow attack angle exhibits greater friction factor increase, which can be due to increased flow obstruction. At lower flow attack angle, the vortex generator is relatively more streamlined with the flow which offers lower flow obstruction. As a result, the pressure rise upon impact on the upstream face of vortex generator is also relatively lower as is seen in Figure 14(a) for $\alpha = 45^\circ$. In addition, since the vortices generated in the wake are also relatively lower in size and strength, the flow energy loss is also lower and hence the friction factor increase is lesser than all other flow attack angles used in the study. As the flow attack angle increases, the vortex generator gradually increases the flow obstruction on the upstream side and generates stronger and larger vortices in its downstream side. As a result, the pressure drop across the vortex generator increases with increasing flow attack angle for a given aspect ratio and blockage ratio. The maximum increase in friction factor is found to be about 4.77 times that of plain tube for $\alpha = 90^\circ$ at the flow Reynolds number of 26,000.

3.3. Effect of pitch

The effect of number of vortex generator inserts on heat transfer is shown in Figure 15. It is seen that the Nusselt number increases with decreasing longitudinal pitch distance of the insert. As the longitudinal pitch decreases, the number of inserts inside the tube also increases. As a result, the
number of locations where the inserts enhance heat transfer also increases. It is interesting to note that the Nusselt number increases monotonously with decreasing pitch as seen in Figure 15.

The maximum rise in Nusselt number is about 2.1 times higher than plain tube for the pitch value of 30 mm for a given aspect ratio, blockage ratio and flow attack angle at Re = 8000. The same trend is observed with the friction factor as well where lower pitch values cause greater rise in friction factor. At lower pitch, owing to the presence of larger number of inserts, the flow disturbances increase significantly in addition to increased flow obstruction. As a result, the pressure energy loss across the inserts increase causing a rise in the friction factor. The maximum rise in friction factor is about 6.34 times higher than plain tube for the pitch value of 30 mm for a given aspect ratio, blockage ratio and flow attack angle at Re = 26,000.
Figure 14. Comparison of pressure distribution (N/m²) for different flow attack angle on a central longitudinal plane at Re = 26,000 for AR = 4, P = 60 mm and $B_L = 0.098$.

(a) Angle=45°  
(b) Angle=75°  
(c) Angle=90°

Figure 15. Effect of longitudinal pitch on Nusselt number and friction factor for AR = 4, $B_L = 0.098$ and $\alpha = 75°$. 

![Graph showing Nusselt number and friction factor vs. Reynolds number](image-url)
### 3.4. Thermal enhancement factor

Heat transfer enhancement using vortex generators is always associated with friction factor enhancement. Hence it becomes important to evaluate the overall gain in thermal performance for the flow rates used in the analysis to determine if the vortex generators are effective in augmenting the heat transfer with acceptable friction factor rise. This is determined using the thermal enhancement factor commonly used in the literature which considers the relative gain of Nusselt number over friction factor under equal pumping power conditions. Vortex generator with lower aspect ratio for a given pitch, flow attack angle and blockage ratio show higher TEF at all flow rates used in the analysis as seen in Figure 16. However, the TEF falls below the value of 1 at Reynolds number greater than 22,000. This is due to the fact that the configuration of AR = 1 exhibits the lowest friction factor enhancement (i.e., f/f_p) among other aspect ratio which is in the range of 2.33–2.68 while the configuration having AR = 6 has a friction factor enhancement in the range of 4.84–5.61 which is significantly higher. On the other hand, the Nusselt number rise (i.e., Nu/Nu_p) for AR = 1 is between 1.35 and 1.56 while it is between 1.56 and 1.91 for AR = 6. Thus, although the heat transfer enhancement for AR = 1 is about 18% lower than AR = 6, the friction factor enhancement is about 52% lower than that of AR = 6 which is significant. Hence, its TEF is relatively higher and ranges between 1.18 and 0.97.

Figure 17 shows the effect of flow attack angle on TEF for a given pitch, aspect ratio and blockage ratio. The TEF is found to decrease with increasing flow attack angle for all flow rates considered in the study. This is due to relatively lower friction factor rise at lower flow attack angle as a result of more streamlined position of vortex generator with respect to the incoming flow. The Nusselt number rise for α = 45° is in the range of 1.68–1.44 while it is between 1.76 and 1.46 for α = 90°. On the other hand, the friction factor rise is between 2.85 and 3.18 for α = 45° while it is between 4.05 and 4.77 for α = 90°. Thus, although the Nusselt number enhancement is about 4.5% lower than that of α = 90°, the friction factor enhancement is about 29.6% lower which is
significant. Hence, the TEF is found to be higher for $\alpha = 45^\circ$ and varies between 1.19 and 0.98 and its TEF falls below 1 at all Reynolds number greater than 22,000.

The effect of longitudinal pitch on TEF is shown in Figure 18, which indicates that lower pitch value of 30 mm provides greater TEF and is in the range of 1.14–0.89. However, its TEF falls below 1 for all Reynolds number greater than 14,000 due to greater friction factor rise at higher flow rates. It is interesting to see that the TEF for all the pitch values are in close range and hence although lower pitch values exhibit a slightly higher TEF, it is prudent to use higher pitch value of 90 mm as it offers closer TEF performance with three times lesser number of inserts in the tube. Moreover, lesser number of inserts also reduce pressure energy losses in the air flow. Thus, use of higher pitch values, lower flow attack angle and lower aspect ratio for a given blockage ratio of 0.098 produce improved overall thermal performance.

3.5. Comparison with published works on vortex generator inserts

The results of present investigation are compared with that of past research findings that made use of vortex generators in circular tube for thermal performance enhancement and are listed in Table 3. The thermal performance of semi-elliptical vortex generator is seen to be better than that of twisted conical inserts (Pourramezan & Ajam, 2016), Array of winglet vortex generator inserts (Xu et al., 2018), Delta wings on plate insert (Eiamsa-ard & Promvonge, 2011), V-nozzle inserts (Eiamsa-ard & Promvonge, 2006), conical rings with holes (Nakchi & Esfahani, 2019) and conical rings (Ibrahim et al., 2019) for similar range of flow Reynolds number.

Moreover, the semi-elliptical vortex generator used in the present study which is based on the rectangular vortex generator insert of (Zhang et al., 2020) is found to perform better in terms of TEF as seen in Figure 19 for all the flow Reynolds number conditions. For the sake of comparison, both the vortex generators have the same flow attack angle of 75°, aspect ratio of 2 and longitudinal pitch value of 60 mm. The TEF of semi-elliptical vortex generator (SEVG) is better due to
Table 3. Comparison of present results with previous research findings

| Sl.No. | Vortex Generator Configuration          | Author                                         | TEF Range       | Operating and Geometric Parameters                                                                 |
|--------|----------------------------------------|-----------------------------------------------|-----------------|---------------------------------------------------------------------------------------------------|
| 1      | Discrete V-winglets on plate insert    | Promvonge et al. (Promvonge et al., 2020)     | 1.5–2.02        | Re = 4200 to 25,800 Relative winglet height = 0.1 to 0.2 Relative winglet pitch = 0.5 to 2       |
| 2      | Quadruple perforated-delta winglet pairs | Skullong et al. (Skullong et al., 2016)       | 1.57–1.9        | Re = 4180 to 26,000 α = 30° Pitch ratio = 0.5 to 2 Blockage ratio = 0.1 to 0.25                 |
| 3      | Curved winglets with perforations on plate insert | Skullong et al. (Skullong et al., 2018) | 1.21–1.76       | Re = 4150 to 25,400 α = 45° Blockage ratio = 0.1 to 0.3 Pitch ratio = 0.5 to 2                  |
| 4      | Delta winglet pair inserts             | Zhai et al. (Zhai, Islam, Simmons, 2019)       | 1.25–1.45       | Re = 5000–25000 α = 0°–40° Winglet length = 5, 7.5 and 10 mm Pitch ratio = 9.6                   |
| 5      | Perforated vortex generator insert     | Chamoli et al. (Chamoli et al., 2017)         | 1.13–1.65       | Re = 3000–21,000 α = 45° Relative pitch = 2 to 6 Perforation index = 4%–16%                  |
| 6      | Twisted cross baffle inserts           | Nanan et al. (Nanan et al., 2017)             | 1.5–1.4         | Re = 6000 to 20,000 Pitch ratio = 1 to 2 Twist ratio = 3 to 8                                  |
| 7      | Winglet vortex generator insert        | Chokphoemphun et al. (Chokphoemphun et al., 2015) | 0.94–1.59       | Re = 5300–24,000 α = 1030° Pitch11 ratio = 0.5 to 2 Non-dimensional height =0.1 to 0.2       |
| 8      | Delta winglet insert                   | Lei et al. (Lei et al., 2017)                 | 1.14–1.35       | Re = 6000–20,000 α = 15°–60° Non-dimensional Pitch = 1 to 4                                   |
| 9      | Delta wings on plate insert            | Eiamsa-ard and Promvonge (Eiamsa-ard & Promvange, 2011) | 0.88–1.29       | Re = 4000–20,000 Pitch-to-width ratio = 0.75–1.25                                             |
| 10     | Array of winglet vortex generator inserts | Liang et al. (Liang et al., 2018)            | 0.95–1.08       | Re = 6000 to 27,000 α = 0°–45° Winglet length = 10, 15 and 20 mm                             |
| 11     | Twisted conical strips                 | Pourramezan and Ajam (Pourramezan & Ajam, 2016) | 0.8–1.12        | Re = 5000–45000 Pitch = 10 – 30 mm Strip angle = 15°–35° Angle of twist = 5°–35°              |
| 12     | V-nozzle turbulators                   | Eiamsa-ard and Promvonge (Eiamsa-ard & Promvange, 2006) | 0.85–1.19       | Re = 8000–18000 Pitch ratio = 2–7                                                             |
| 13     | Conical rings with holes               | Nakhchi and Esfahani (Nakhchi & Esfahani, 2019) | 0.65–1.24       | Re = 4000–14000 Non-dimensional hole diameter = 0.4–0.6 Hole number = 0–10                    |
| 14     | Conical ring inserts                   | Ibrahim et al. (Ibrahim et al., 2019)         | 0.1–1.10        | Re = 6000–25000 Diameter ratio = 0.3–0.7 Pitch = 2–4                                           |
| 15     | Semi-elliptical vortex generator insert | Present study                                 | 0.86–1.19       | Re = 8000–26000 α = 45° to 90° Bi = 0.098 P = 30, 60 and 90 mm                                  |
Figure 18. Comparison of TEF for different longitudinal pitch of vortex generator at AR = 4, $B_L = 0.098$ and $\alpha = 75^\circ$.

Figure 19. Comparison of TEF for rectangular and semi-elliptical vortex generator of present study.
the presence of smooth curves on semi-elliptical vortex generator which create relatively smoother interaction with the incoming air stream as compared to that of sharp corners in rectangular vortex generator. Moreover, the frontal impact area is also relatively smaller for semi-elliptical geometry for the same width and blockage ratio of rectangular geometry which creates lower flow obstruction. Hence, the friction factor is found to be lower than rectangular vortex generator as seen in Figure 20 and the average reduction is about 12.7%. However, the Nusselt number for semi-elliptical geometry is lower due to relatively reduced flow disturbance as seen in Figure 20. However, the average reduction in Nusselt number is only about 6.3% which is offset by considerable reduction in friction factor. Hence, the TEF is found to be higher than rectangular VG for all the flow rates used in the analysis. The TEF for rectangular VG ranges from 0.89 to 1.13, whereas it is in the range of 0.92–1.16 for semi-elliptical VG.

Finally, there is still scope for further improvement of current thermal performance of present SEVG inserts which can further improve the range of flow rates as well as the TEF with further research. Although the present analysis does not show significant increase in TEF from rectangular geometry, the results establish that the semi-elliptical geometry can considerably reduce the pressure drop penalty to the extent of 12.7% which is the bright aspect. Further, in the present analysis, the blockage ratio is fixed and only the aspect ratio is varied for a given flow attack angle. A further analysis on blockage ratio variation, that is, the height variation could result in interesting outcomes in terms of Nusselt number and friction factor for Semi-elliptical design. In addition, the introduction of perforations (single or multiple) on the vortex generator surface can also influence the heat transfer and pressure drop as seen in other designs of vortex generators outlined in the introduction section.

4. Conclusions
A three-dimensional numerical analysis is carried out to determine the heat transfer and friction factor characteristics of a novel semi-elliptical vortex generator insert in a circular tube for the flow Reynolds number range of 8000–26000. The influence of aspect ratio (AR = 1.2, 4 and 6), flow attack angle (45°, 60°, 75° and 90°) and the number of inserts (P = 30 mm, 60 mm and 90 mm) is
investigated using air as the working fluid. The following major conclusions can be drawn from the analysis:

- The presence of semi-elliptical vortex generator has significant influence on both Nusselt number and friction factor characteristics of circular tube. The vortex generators produce strong longitudinal vortices which intensifies the fluid mixing near the tube wall region thereby augmenting the heat transfer to the flowing air stream. In addition, the presence of flow impingement effect on the upstream side and flow reattachment zones on the downstream side of vortex generator is found to contribute to enhanced heat transfer.

- Circumferential movements in the flow which allows the air stream to cover larger heated tube wall regions are observed for all flow attack angles less than 90°.

- The influence of aspect ratio on heat transfer is found to be more significant than that of flow attack angle. However, both parameters affect the friction factor significantly. On the other hand, the pitch distance of inserts significantly affects both heat transfer and friction factor.

- For the given flow attack angle of 75°, higher aspect ratio is found to provide greater heat transfer and the maximum improvement in Nusselt number is found to be about 1.9 times as compared to plain tube for the configuration having AR = 6, P = 60 mm, Bt = 0.098 and Re = 8000.

- For the given flow attack angle of 75°, the maximum increase in friction factor is found to be about 5.62 times higher than plain tube for AR = 6, P = 60 mm, Bt = 0.098 at Re = 26000. The lowest friction factor enhancement is found to be about 2.33 times that of plain tube for AR = 1 and Re = 8000.

- For a given aspect ratio, the pressure drop across the vortex generator increases with increasing flow attack angle. However, the Nusselt number is found to decrease for flow attack angles greater than 75°. The maximum increase in friction factor is found to be about 4.77 times that of plain tube for α = 90° at Re = 26000.

- Lower the pitch distance, greater the enhancements in heat transfer and friction factor. The maximum rise in Nusselt number is about 2.1 times higher than plain tube for the pitch value of 30 mm at Re = 8000. The maximum rise in friction factor is about 6.34 times higher than plain tube for the pitch value of 30 mm at Re = 26,000.

- TEF is found to decrease with increasing flow attack angle and aspect ratio for all flow rates considered in the study. However, the TEF increases with decreasing pitch distance of inserts. The overall TEF is found to be in the range of 0.86–1.19.

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### Author details
Manjunath M.S
Dolfred Vijay Fernandes
E-mail: dolfred.fernandes@manipal.edu
ORCID ID: http://orcid.org/0000-0003-4751-1855
Department of Mechanical and Manufacturing Engineering, Manipal Institute of Technology, Manipal Academy of Higher Education, Manipal, India.

### Nomenclature
- G: Generation of turbulence kinetic energy
- Nu: Nusselt number
- ΔP: Pressure drop, Pa
- Pr: Prandtl number
- S: Modulus of strain rate
- T: Temperature, K
- Q: Heat flux, Wm⁻²
- V: Mean air velocity, ms⁻¹
- f: Friction factor
- h: Convective heat transfer coefficient, Wm⁻²K⁻¹
- k: Turbulence kinetic energy, m²s⁻²
- u: Velocity component, ms⁻¹

### Greek Symbols
- μ: Turbulent viscosity, Ns⁻²
- Γ: Molecular thermal diffusivity, m²s⁻¹
- Γt: Turbulent thermal diffusivity, m²s⁻¹
- ε: Rate of dissipation, m²s⁻³

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