Modified method of detecting dynamic buckling

Shoko ARITA* and Yasuyuki MIYAZAKI**
* Department of Mechanical Engineering, Faculty of Engineering, Shizuoka University
3-5-1 Johoku, Naka-ku, Hamamatsu-shi, Shizuoka, 432-8011, Japan
E-mail: arita.shoko@shizuoka.ac.jp
** Department of Aerospace Engineering, College of Science and Technology, Nihon University
7-24-1 Narashinodai, Funabashi-shi, Chiba, 274-8501, Japan

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Abstract
Deployable structures are necessary for spacecraft to challenge advanced missions. It is important in designing the deployable space structures that they are easily deployable and reliably repeatable. Conventional study for improving the repeatability was conducted by investigating errors and its effect to the deployment. However, there was no general numerical method to evaluate the reliability efficiently and quantitatively, not estimating any errors. Therefore, the authors proposed an original numerical method to evaluate quantitatively the reliability of the deployment by detecting buckling in a dynamic analysis of FEM in 2016. In the recent study, we found a problem of false detection of the buckling by the conventional method, and we also found a method to solve the problem. The conventional method to detect the buckling in the dynamic analysis uses mode orthogonality to the rigid-body modal space and the eigenvalue. However, it was confirmed that the rigid-body mode is not discriminated correctly for the mixed modes of the rigid-body motion and the deformation. The cause of the problem is using the eigenvalue for the judgement of the buckling because it cannot be evaluate that the sign of the eigenvalue derives whether from the rigid-body motion or from the deformation. Therefore, this study proposes a modified method, not using the eigenvalue but using the work of the deformation for the judgement of the buckling. Finally, it was confirmed that the modified method enabled proper detection of the buckling for the mixed modes of the rigid-body motion and the deformation.

Keywords: Deployable structures, Repeatability, Buckling, Dynamics, FEM, Stiffness matrix

1. Background
Deployable structures are necessary for spacecraft to challenge advanced missions such as a Solar Sail, a large antenna, Space Solar Power Systems and so on. It is important in designing the deployable space structures that they are easily deployable and reliably repeatable. Conventional study for improving the repeatability was conducted by investigating errors and its effect to the deployment (Yamasaki and Furuya, 2013, Saito and Tanaka, 2012). However, there was no general numerical method to evaluate the reliability efficiently and quantitatively, not estimating any errors. Therefore, the authors proposed an original numerical method to evaluate quantitatively the reliability of the deployment by detecting buckling in a dynamic analysis of FEM (Arita and Miyazaki, 2016). Because the reliability of the deployment decreases by the presence of the buckling, the method is available for development of the spacecraft. The proposed method does not require the estimation of the errors, and the method enables efficient evaluation of instability of the deployment. As for stability, much research has been carried out since Lagrange, Dirichlet. In particular, the stability evaluation methods based on Lyapunov stability theory have been studied extensively, and the relation of the energy and deformation was studied in detail (Thompson and Hunt, 1973, Dinkler, 1989, Dinkler and Pontow, 2006). The buckling analysis used in this study is also based on Lyapunov stability theory, and additionally, this study newly defined two index values of the instability specialized on the deployable structure, that is, a disturbance amount and a buckling displacement amount. It is because the two index values are more available to grasp design parameters and the unexpected displacement magnitude which can be caused by disturbance at the time of actual design.
Regarding gossamer space structures, the shape changes significantly every moment, and various buckling appears and disappears. Therefore, this study sets out to develop methods of buckling analysis in the dynamic FEM so as to detect appearance and disappearance of the buckling in the entire structure from start to finish of the deployment. In terms of dynamic buckling, it was a problem that the experimental results of shell intensity was much lower than the theoretical value in 1960s. Hence, several methods for determining the buckling load and modes when impulse load or step load is added by formulating the characteristic equation taking the inertia term into consideration have been proposed and evaluated (Huang, 1969, Svalbonas and Kalnins, 1977). Later Thompson and Hunt has been codified as general theory of elastic discrete system using perturbation theory (Thompson and Hunt, 1973). Therefore, buckling detection in FEM is generally performed by eigenvalue analysis of a stiffness matrix, but some mathematical models that do not directly solve the eigenvalue problem have been proposed to reduce computational cost (Noguchi and Fujii, 2003, Takakura, 1992). However, in the dynamic FEM, it is necessary to distinguish the rigid-body motion from the buckling because the non-positive eigenvalue appears not only the buckling but also for the rigid-body motion, but since this method was not proposed, the authors solved this problem (Arita and Miyazaki, 2016).

In the recent study, we found a problem of false detection of the buckling by the conventional method, and we also found a method to solve the problem. This paper reports them. The conventional method and the problem of false detection of the buckling are explained in Section 2. The modified method to solve the problem are proposed in Section 3. Results of the validation of the modified method are reported in Section 4. The conclusions are given in Section 5.

2. Conventional method of the dynamic buckling detection and the problem of the false detection

2.1 Conventional method of the dynamic buckling detection

The authors formulated the definition of the rigid-body motion in the FEM by truss element (Arita and Miyazaki, 2016). Using the element stiffness matrix \( K^{ab} \) of the truss element given by Eq.(1), the buckling detection is conducted according to the following three procedures. In this regard, \( E \) is Young’s modulus, \( A \) is cross-sectional area of the element, \( L \) is length of the element before deformation, \( l \) is the length of the element after deformation, \( e \) is the axial strain, \( e \) is unit vector from a node to another node, and \( I_{33} \) is 3 by 3 identity matrix.

\[
K^{ab} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \text{ with } k = EA \left[ 1 - \varepsilon \right] \left[ 1 - \varepsilon \right] \left( I_{33} - e \otimes e \right) \tag{1}
\]

**Procedure1:** When a structure is moving and deforming, let us assume each element of the structure has no strain at the moment. Then, the stiffness matrix, which is given by Eq.(1), is represented as Eq.(2) plugging in 0 for \( \varepsilon \) and \( l \) for \( L \).

\[
K^{ab} \big|_{\varepsilon=0} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \text{ (with } k = EA \left[ 1 - \varepsilon \right] \left[ 1 - \varepsilon \right] e \otimes e \text{)} \tag{2}
\]

We define \( 'K \) as a matrix merged \( 'K^{ab} \) over the entire structure. The structure is in a state that all elements are free length and no internal force exists. Therefore, only the constant rigid-body motion can displace the structure without external force. Hence, the rigid-body mode is defined as an eigenvector of \( 'K \) of which eigenvalue \( \chi \) is zero. The rigid-body mode is defined by the following equation:

\[
\hat{\eta}_i = [w \mid \lambda w = 'Kw = 0] \tag{3}
\]

Furthermore, we define rigid-body modal space \( \hat{Q} \) by the following equation:

\[
\hat{Q} = [\hat{q}_1, ..., \hat{q}_n] \tag{4}
\]

**Procedure2:** We define \( K \) as a matrix merged \( K^{ab} \) over the entire structure. Modes of the structure in motion are the eigenvectors of \( K \). The number of the modes \( n \) is the number of generalized coordinates of the structure. In order to detect the rigid-body mode, we check if each eigenvector of \( K \) is included in the rigid-body modal space \( \hat{Q} \). If an eigenvector \( \eta_i \) is not included in \( \hat{Q} \), the eigenvector can be so determined that it is not a rigid-body mode but a deformational mode. We calculate the orthogonal component \( \eta_i' \) of each eigenvector \( \eta_i \) to the rigid-body modal space \( \hat{Q} \) by Eq.(5) in order to check if \( \eta_i \) is included in \( \hat{Q} \). Figure 1 shows the conceptual diagram of calculation of the orthogonal component.

\[
\eta_i' = \eta_i - \sum_{k=1}^{n} \left( \eta_i \cdot \hat{q}_k \right) \hat{q}_k \tag{5}
\]

Furthermore, we define degree of orthogonality \( \kappa_i \) as below:
If \( \kappa = 0 \), \( \eta \) is the rigid-body mode.

**Procedure 3:** An eigenvector of \( K \) with non-positive eigenvalue is either the rigid-body mode or the buckling mode. Therefore, when the eigenvector is deformational mode, we check its eigenvalue \( \lambda \). If the eigenvalue is less than or equal to zero, the eigenvector is the buckling mode.

Figure 2 shows the flowchart of judging a mode. Each eigenvector is categorized into six types, as shown in Fig. 2.

**2.2 Problem of the false detection**

In the case of the model 1 given as Fig. 3 and Table 1, it was confirmed that only the snap through buckling is correctly detected as shown in Table 2 using the proposed method explained in Section 2.2 (Arita and Miyazaki, 2016). Table 2 shows that the degree of the orthogonality \( \kappa \) of each mode is perfectly 0 or 1 for the model 1 shown in Fig. 3. However, in the case of more complex models presumed actual space structures, \( \kappa \) is rarely absolute 0 or 1 (Arita and Miyazaki, 2015). The modes, of which \( \kappa \) are not perfectly 0 or 1, mean mixture of the rigid-body motion and the deformation. Because the eigenvalues of the stable deformation and the rotation with tensile strain are positive, and the eigenvalues of the buckling, instable deformation, translation, the rotation with compressive strain and the rotation without strain are non-positive, the existence of the mixed modes indicates possibility of the false detection as below:

1. When a mode consists of stable deformation and rotational rigid-body motion with compressive strain, the mode may be wrongly determined as the unstable deformation mode.
2. When a mode consists of unstable deformation and rotational rigid-body motion with tensile strain, the mode may be wrongly determined as the stable deformation mode.

From the above background, we confirmed the false detection by the model 2 shown in Fig. 4 and Table 3, whose degree of freedom is larger than the model 1 of Fig. 3. The transient response was same as the model 1 of Fig. 3. After the snap through buckling occurs at Step 1924 for the first time, the periodic snap through buckling occurs. The transient response is shown as Fig. 5. Figure 6 shows the number of the buckling modes detected at each time step. Several modes otherwise than the snap through buckling are judged as the buckling from the first step to the last step. Table 4 shows the results of the mode and the judge of Step 1 and Step 100 as an example. Four modes at Step 1 and three modes at Step 100 are judged as the buckling (Type 13) because their degree of orthogonality are not 0 and their eigenvalues are negative value. The degree of orthogonality means the fraction of the deformation. Thus, the model 1-5 are almost rigid-body motion. Figure 7 describes each mode according as the eigenvector in Table 4. Note that mode 6 and mode 7 are not considered here because they indicate the boundary condition, which is \( y \) constraint for the \( \text{mode}^1 \) and \( \text{mode}^2 \), and because they are automatically distinguished by absolute 1 of the eigenvalue. According to Fig. 7, each mode shows the following motion:

**mode 1:** \( y \) axis rotation  
**mode 2:** \( z \) axis rotation  
**mode 3:** \( x \) axis rotation  
**mode 4:** \( z \) axis translation  
**mode 5:** \( x \) axis translation  
**mode 8:** Deformation of the snap through buckling with two compressed elements  
**mode 9:** Deformation with a compressed element and a tensioned element

In the case of the model 1 of Fig. 3, only the snap through buckling of the mode 8 is correctly detected as shown in Table 2. We considered the reason that several buckling modes appear in the case of the model 2 of Fig. 4 as below:

According as Table 4, the model 1-5 are judged by their eigenvalues if buckling or not according to Fig. 2. It is
because each degree of orthogonality of the model1-5 is not absolute 0. On the other hand, the eigenvalue of the rotational rigid-body mode with the compressive strain is negative value. Thus, it can be considered that the eigenvalues of the rotational modes are negative value because all elements are compressed. The eigenvalues of the model1-3 at Step100 are less than at Step1 respectively because the more time step goes, the more compressive load increases. That is to say, the possibility is confirmed that the rigid-body mode is not discriminated correctly when the degree of orthogonality is not absolute 0.

Table 1  Description of the truss arch model 1

| Parameter                  | Symbol | Value | Unit |
|----------------------------|--------|-------|------|
| time step size             | $dt$   | $5\times 10^{-3}$ | [s]  |
| stiffness of an element    | $EA$   | $2.8\times 10^7$  | [N]  |
| density of an element      | $\rho$ | 2.7   | [g/cm$^3$] |
| increment of external force parameter | $\Delta f$ | $2.5\times 10^7$ | [-] |
| condition of constraint:   | $x, y, z$ of node$^1$, $x, y, z$ of node$^2$ | | |
| condition of loading:      | Increasing $f$ step-by-step by $\Delta f$, node$^3$ is subjected to -$EAf$ in the direction of $x$ axis. | | |

Table 2  Result of the eigenvalue, the eigenvector, the degree of orthogonality and the type. The degree of orthogonality and the type are obtained by the method proposed in Section 2.2. Comparing the types and the eigenvectors, we can confirm that the types are categorized correctly, and that the buckling mode is detected correctly by the proposed method.

![Fig.3 Overview of the truss arch model 1](image1)

Table 3  Description of the truss arch model 2

| Parameter                  | Symbol | Value | Unit |
|----------------------------|--------|-------|------|
| time step size             | $dt$   | $5\times 10^{-5}$ | [s]  |
| stiffness of an element    | $EA$   | $2.8\times 10^7$  | [N]  |
| density of an element      | $\rho$ | 2.7   | [g/cm$^3$] |
| increment of external force parameter | $\Delta f$ | $2.5\times 10^5$ | [-] |
| condition of constraint:   | $y$ of node$^1$ and node$^2$ | | |
| condition of loading:      | Increasing $f$ step-by-step by $\Delta f$, node$^3$ is subjected to -$EAf/2$ in the direction of $x$ axis, and node$^1$ and node$^2$ are subjected to $-EAf/2$ respectively in the direction of $x$ axis. | | |

![Fig.4 Overview of the truss arch model 2](image2)

![Fig.5 The transient response. After the snap through buckling occurs for the first time, the periodic snap through buckling occurs.](image3)

![Fig.6 The number of the buckling modes. Several modes otherwise than the snap through buckling are judged as the buckling.](image4)
The modified method also uses the same procedures from Procedure1 to Procedure2, and Procedure3 is modified as below:

**Procedure3:**

The work of the deformation is written as below:

---

**Table 4**  Result of the eigenvalue, the eigenvector, the degree of orthogonality and the type. The orthogonality and the type are obtained by the method introduced in Section 2.2. Four modes at Step1 and three modes at Step100 are judged as the buckling (Type13). Comparing the types and the eigenvectors, the possibility is confirmed that the rigid-body modes are not discriminated correctly when the degree of orthogonality is not absolute 0.

![Fig.7](image-url)  Each mode according as the eigenvector in Table4. The mode1-3 are the rotational rigid-body modes, the mode4-5 are the translational rigid-body modes the mode8 is the snap through buckling mode and the mode9 is the deformation mode.

3. Proposal of a modified method to solve the problem of the conventional method

When the degree of orthogonality is not absolute 0 or 1, the mode is mixture of the rigid-body motion and the deformation. Hence, a new evaluation index is required for such a mixed mode because it cannot be evaluate that the sign of the eigenvalue derives whether from the rigid-body motion or from the deformation. Therefore, the authors propose using work of the deformation \( w_d \) for the judgement of the buckling. The method does not use the eigenvalue for the judgement of the buckling. That is to say, when the degree of orthogonality \( \kappa \), is not absolute 0, we calculate the work of the deformational component \( \eta \). If the work \( w_d \) is positive, the deformation is stable. If the work \( w_d \) is non-positive, the deformation is buckling and instable. It is because the negative eigenvalue due to the buckling in the static buckling detection means that the work of the deformation is negative. The modified method also uses the same procedures from Procedure1 to Procedure2, and Procedure3 is modified as below:

**Procedure3:**

The work of the deformation is written as below:
\[ w_d = \eta^* \cdot (K \cdot \eta^* \cdot \eta^* \cdot (K \cdot \eta^*) ) \]  

If \( w_d \) is negative, the mode is judged as a mode containing instable deformation. In the same way, if \( w_d \) is 0, the mode is judged as a mode containing critical buckling point. Figure 8 shows the modified flowchart of judging a mode. Each eigenvector is categorized into six types, as shown in Fig.8.

4. Validation results of the modified method

The results using the modified method are given below. Treatment of numerical error must be noted here. As for a minute value of \( \kappa_i \), it is difficult to discriminate whether it is a numerical error or a truly minute displacement. Therefore, in this calculation, when \( \kappa_i \) shows a minute value, it is calculated on the premise that it involves a corresponding amount of deformation. Incidentally, it is possible to consider that deformation amount of modes of the fixed node can be regarded as the numerical error. In the case of this calculation as an example, the values of \( y \) of the node 1 and node 2 in Table 4, 5, that is 10^-11 or less is regarded as numerical error. Additionally, we note the handling guidelines of the numerical error regarding to \( w_d \). It is because the correct buckling cannot be detected correctly when the sign of work reverses due to numerical error. When a numerical error that reverses the sign of \( w_d \) occurs, either the residual within the convergence radius in the eigenvalue analysis or the residual within the convergence radius for finding the stiffness matrix is inverted between positive and negative. Therefore, this problem can be avoided by regarding the residuals as 0 before calculating \( w_d \). Although this method was not set in this calculation, \( w_d \) with minute \( \kappa_i \) were investigated and it was confirmed that it was all positive, that is, the expected buckling detection was performed correctly in this calculation.

Firstly, we investigated the detailed judgement of each mode at a time step. Table 5 shows the results of the mode and the judge of Step 1 and Step 100 in order to compare with Table 4 shown in Section 2.2. The mode 1-4, which are judged as the buckling modes (Type 13) by the conventional method, are judged as modes containing stable deformation (Type 23) by the modified method. So that means the negative eigenvalue of each mode 1-5 derive from the rigid-body motion.

Secondly, we investigated the judgements of whole time steps and the adequacy of the detection of the snap through buckling. Figure 9 shows the number of the buckling modes detected at each time step. Figure 10 shows the number of the buckling modes except mode 8, which is the snap through buckling mode. Figure 11 shows comparison of the conventional method with the modified method about the buckling detection for the mode 8. Figure 11 shows only after Step 1900 because the buckling of the mode 8 is detected at Step 1924 for the first time by both methods. Additionally, it is confirmed from Table 6 that the sign of \( w_d \) of mode 8 changes between Step 1923 and 1924, and the snap-through buckling is detected at Step 1924.

According to Fig. 9, the maximum number of the buckling mode is 1, and according to Fig. 10, the buckling except the mode 8 is not detected for whole time steps. Thus, we can confirm that only the snap through buckling was detected by the modified method. Comparing Fig. 9 with Fig. 6, we can confirm that the false detections are improved. According to Fig. 11, results of the detection of the snap through buckling by both methods are all the same. The buckling occurs periodic, and it is in consistency with the transient response.

As the facts above, we confirmed that the modified method enabled proper detection of the buckling for the mixed modes of the rigid-body motion and the deformation.
Table 5  Result of the degree of orthogonality, the eigenvalue, the work of deformation, the type, and the eigenvector.

The work and the type are obtained by the modified method proposed in Section 3. Comparing the types and the eigenvectors, we can confirm that the types are categorized correctly, and that the rigid-body modes are discriminated correctly by the modified method.

| time step | mode No. | degree of orthogonality | eigenvalue | work of deformation | Type | eigenvector | node① | node② | node③ |
|-----------|----------|-------------------------|------------|---------------------|------|-------------|-------|-------|-------|
| 1         | 1        | 5.44×10⁻¹⁰             | -6.94×10⁻¹⁶ | 1.67×10⁻¹⁶         | 23   | 0           | 0     | -0.408 | 0.817 |
| 2         | 1        | 1.84×10⁻¹⁰             | -2.29×10⁻¹⁶ | 1.16×10⁻¹⁶         | 23   | 0           | 0     | 0.707  | -5.707|
| 3         | 1        | 3.05×10⁻¹⁰             | -2.9×10⁻¹⁶  | 2.45×10⁻¹⁷         | 23   | 0           | 0     | -0.707 | -5.707|
| 4         | 1        | 6.60×10⁻¹⁰             | -4.89×10⁻¹⁶ | 2.43×10⁻¹⁰         | 23   | 0           | 0     | 0.577  | 6.23×10⁻¹⁰|
| 5         | 1        | 5.80×10⁻¹⁰             | -1.80×10⁻¹⁶ | 6.68×10⁻¹⁰         | 23   | 0           | 0     | -0.577 | 6.23×10⁻¹⁰|
| 6         | 1        | 1.35×10⁻⁰              | -4.16×10⁻¹³ | 3.16×10⁻¹³         | 23   | 0           | 0     | -5.707 | 6.23×10⁻¹³|
| 7         | 1        | 1.39×10⁻⁰              | -8.96×10⁻¹⁵ | 1.39×10⁻¹⁵         | 23   | 0           | 0     | -5.707 | 6.23×10⁻¹⁵|
| 8         | 1        | 2.09×10⁻¹³             | -2.00×10⁻¹⁰ | 0.577              | 23   | 0.408       | 0     | 0.577  | 0.577 |
| 9         | 1        | 5.59×10⁻¹⁰             | -0.0250     | -5.55×10⁻¹⁰        | 23   | 0           | 0     | -0.0250| -0.0250|

Step1

Table 6  Result of the first snap through buckling detection. The sign of \( w_d \) of mode 8 changes between Step1923 and 1924, and the snap-through buckling is detected at Step1924.

| time step | mode No. | degree of orthogonality | eigenvalue | work of deformation | Type | eigenvector | node① | node② | node③ |
|-----------|----------|-------------------------|------------|---------------------|------|-------------|-------|-------|-------|
| 1         | 1        | 5.44×10⁻¹⁰             | -2.11×10⁻¹⁶ | 1.65×10⁻¹⁰         | 23   | 5.38×10⁻¹⁰ | 0     | 0.408  | -0.408|
| 2         | 1        | 4.39×10⁻¹⁰             | -7.09×10⁻¹⁰ | 1.18×10⁻¹⁰         | 23   | 0.701       | 0     | -0.701 | -0.701|
| 3         | 1        | 7.66×10⁻¹⁰             | -7.09×10⁻¹⁰ | 1.18×10⁻¹⁰         | 23   | -1.70×10⁻¹⁰| 0     | 0.701  | -0.701|
| 4         | 1        | 7.61×10⁻¹⁰             | -1.65×10⁻¹⁰ | 5.23×10⁻¹⁰         | 23   | 1.51×10⁻¹⁰ | 0     | -0.577 | -0.577|
| 5         | 1        | 2.51×10⁻¹⁰             | 1.19×10⁻¹⁰  | 2.22×10⁻¹⁰         | 23   | 0.577       | 0     | 1.06×10⁻¹⁰ | 0.577|
| 6         | 1        | 1.15×10⁻¹⁰             | -1.45×10⁻¹⁰ | 1.70×10⁻¹⁰         | 23   | 1.15×10⁻¹⁰ | 0     | -1.45×10⁻¹⁰| 1.15×10⁻¹⁰|
| 7         | 1        | 1.15×10⁻¹⁰             | -1.45×10⁻¹⁰ | 1.70×10⁻¹⁰         | 23   | 1.15×10⁻¹⁰ | 0     | -1.45×10⁻¹⁰| 1.15×10⁻¹⁰|
| 8         | 1        | 2.03×10⁻¹⁰             | 2.03×10⁻¹⁰  | 0.577              | 23   | -0.408      | 0     | 0.577  | 0.577 |
| 9         | 1        | 5.59×10⁻¹⁰             | -0.0247     | -1.17×10⁻¹⁰        | 23   | -0.0247     | 0     | 0.0247 | 0.0247|

Step 100

The number of the buckling modes detected at each time step. Comparing with Fig.6, we can confirm that the false detections are improved.

Fig.9

Fig.10  The number of the buckling modes except mode8. We can confirm that no buckling is detected except the snap through buckling.
Conclusion

The conventional method to detect the buckling in the dynamic analysis uses mode orthogonality to the rigid-body modal space and the eigenvalue. This study investigated the following problem of the conventional method, and proposed the following modified method to solve the problem:

**Problem of the conventional method:**
1. The rigid-body mode is not discriminated correctly for the mixed modes of the rigid-body motion and the deformation.
2. The cause of the problem is using the eigenvalue for the judgement of the buckling.

**Modified method:**
1. The authors proposed not using the eigenvalue but using the work of the deformation for the judgement of the buckling.
2. We confirmed that the modified method enabled proper detection of the buckling for the mixed modes of the rigid-body motion and the deformation.

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