Response of a ferrofluid to traveling-stripe forcing

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Abstract
We observe the dynamics of waves propagating on the surface of a ferrofluid under the influence of a spatially and temporally modulated field. In particular, we excite plane waves by applying a traveling lamellar modulation of the magnetization. By means of this external driving, both the wavelength and the propagation velocity of the waves can be controlled. The amplitude of the excited waves exhibits a resonance phenomenon similar to that of a forced harmonic oscillator. Its analysis reveals the dispersion relation of the free surface waves, from which the critical magnetic field for the onset of the Rosensweig instability can be extrapolated.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
When a critical value of the vertical magnetic induction is surpassed, the surface of a ferrofluid exhibits an array of liquid crests. This so called Rosensweig instability (Cowley and Rosensweig 1967) has been investigated in a static field (see e.g. Bacri and Salin 1984, Richter and Barashenkov 2005, Gollwitzer et al 2007) and under temporal modulation of either the magnetic field (see e.g. Mahr and Rehberg 1998) or the gravitational acceleration (Ko et al 2003). For a recent survey see Richter and Lange (2008). While these excitations are homogeneous in space, a combined spatiotemporal forcing of the plane surface has been implemented by Kikura et al (1990) using an array of solenoids. They investigate the surface waves for small magnetic fields far below the Rosensweig threshold and measure the resulting volume flow rate. In contrast, we apply a traveling-stripe forcing to the surface of magnetic liquids at the advent of the Rosensweig instability and uncover a resonance phenomenon of the wave amplitude with respect to the lateral driving velocity. So far, such a spatiotemporal driving of a pattern forming system has only been realized in a chemical experiment by Miguez et al (2003). A review by Rüdiger et al (2007) calls for further experiments, and the present paper provides one.

2. Experimental details
Our experimental setup is sketched in figure 1. A rectangular vessel machined from Perspex™ is placed in the center. The inner dimensions are 100 mm (x), 120 mm (y), 25 mm (z). It is filled with ferrofluid up to \( z = 3 \) mm. A Helmholtz pair of coils generates a constant magnetic induction which is homogeneous with a deviation of 1% over the size of the container. In addition we apply a small (5%) spatiotemporal modulation of the magnetic field with the spatial periodicity of the critical wavelength \( \lambda_c = 2\pi \sqrt{\sigma/(\rho g)} \), which propagates parallel to the surface with constant velocity \( v \). Then traveling waves at the surface of the fluid are formed with the same wavelength and speed as the driving. The field modulation
is realized by a ‘conveyor belt’, made from a textile band which harbors periodically placed iron rods. The lateral distance between neighboring rods was selected to be as close as possible to the critical wavelength $\lambda_c = 9.98 \text{ mm}$. We achieved $9.3 \pm 1 \text{ mm}$. The rods have a length of $70 \text{ mm}$ and are made of welding wire with a diameter of $1.0 \text{ mm}$. The vertical distance between the symmetry axis of the rods and the surface of the fluid at rest is $8 \pm 0.5 \text{ mm}$. The band is driven by an electric motor which allows us to vary the velocity up to $30 \text{ cm s}^{-1}$. Figure 2(a) shows the bare belt and (b) its location and its effect within the setup. The iron rods amplify the magnetic field locally due to their higher susceptibility; thus the magnetic field strength varies along the driving direction. As demonstrated in figure 3, the excitation profile is approximately sinusoidal. We investigate the response of the ferrofluids APG 512a and EMG 909 (Ferrotec Co.). The parameters of those fluids are listed in table 1.

For measuring the amplitudes we direct a beam of a helium–neon laser onto the surface of the magnetic fluid in the middle of the container, from where it is reflected to a screen. The beam position on the screen is acquired via a charge-coupled-device (CCD) camera and reveals the slope of the surface at the incident point of the laser beam. We extract the height of the undulations $A$ by assuming that the surface modulation is sinusoidal, which is valid in this case of only small deformations. When the conveyor belt moves under the vessel, the fluid ridges travel with the iron rods and the position of the reflected beam oscillates.

Like in figure 3, the surface undulations are approximately sinusoidal. Deviations stem from inaccuracies in the spacing of the iron rod lattice, and fluctuations in the driving velocity of about $0.1\%$, which results in a phase jitter of $\pm \pi/2$. The noise spectrum extends above and below the modulation frequency, and can be removed by applying a bandpass filter centered around the mean frequency of the moving iron rods.

The amplitude of the surface waves is determined from the Fourier spectrum from the intensities of the mode corresponding to the mean passage time of one rod and the two neighboring modes. The spectrum is calculated within a passage time of 314 rods, i.e. one complete cycle of the conveyor belt. Because the data can be continued periodically, a window function for the elimination of artifacts is not needed.

### 3. Experimental results

We have measured the amplitude of the fluid waves below the critical field of the Rosensweig instability as a function of the driving velocity and the applied magnetic field for two different magnetic fluids. For each fluid we selected about ten different magnetic inductions in a range where visible undulations occur. The results for three representative values of $B$ are displayed in figure 4. For slow driving velocities the amplitude of the undulations resembles that of the static case. For increasing velocity, the amplitude of the traveling waves passes through a maximum and decays for high driving velocities. When increasing the magnetic field the undulations become remarkably higher and the maximum shifts to lower driving speeds. When the critical field is reached, the undulations are replaced by Rosensweig spikes. Note the different responses of the highly viscous (a) and the less viscous fluid (b).

Below the Rosensweig threshold $B_c$, the amplitude response to this spatiotemporal driving can be modeled as a damped forced harmonic oscillator

$$A(v) = \frac{A_0 v_p^2}{\sqrt{(v_p^2 - v)^2 + 4\gamma^2 v^2}}$$  \hspace{1cm} (1)

Here $A(v)$ denotes the amplitude dependent on the driving velocity $v$, where $A_0$ is the amplitude at zero velocity, $v_p$ is the resonant frequency, and $\gamma$ is the damping coefficient.
Figure 4. Amplitude of the waves versus the driving velocity for varying magnetic induction for the ferrofluids APG 512a (a) and EMG 909 (b). The triangles mark the highest induction, the circles an interim value and the boxes the lowest field. The data have been captured at the following magnetic inductions: \( \triangle: 15.0 \text{ mT} \), \( \odot: 13.4 \text{ mT} \), \( \square: 10.4 \text{ mT} \), \( \blacktriangle: 20.9 \text{ mT} \), \( \bullet: 14.9 \text{ mT} \), \( \blacksquare: 10.4 \text{ mT} \). The solid lines display a fit with (1).

denotes the phase velocity of the unforced surface waves and \( \gamma \) the damping constant. As displayed in figure 4, the experimental data are well captured by fits to (1).

The viscosity determines the damping constant in our model (1). This is corroborated by figure 4(a), where the curves of the highly viscous APG 512a show an overdamped behavior. In contrast, the fluid EMG 909, with a 25 times smaller viscosity, displays a clearly visible maximum for all magnetic inductions (see (b)).

The resonant propagation speed \( v_p \) can be calculated from the dispersion relation. For infinite layer thickness and inviscid fluids the surface waves on ferrofluid are described by the plane dispersion relation

\[
\omega^2 = gk + \frac{\sigma}{\rho} k^3 - \frac{1}{\rho \mu_0 \mu_r} \frac{(\mu_r - 1)^2}{(\mu_r + 1)} B^2 k^2 \tag{2}
\]

put forward by Cowley and Rosensweig (1967). In our experiment we intentionally constrain the wavenumber of the surface waves by means of the iron rods to \( k_c \). From (2), we get a phase velocity \( v_p = \frac{\omega}{k} \) which depends on the magnetic induction:

\[
v_p = \sqrt{\frac{2 g \sigma}{\rho} - \frac{1}{\rho \mu_0 \mu_r} \frac{(\mu_r - 1)^2}{(\mu_r + 1)} B^2} = \sqrt{\frac{B_c^2 - B^2}{B_c^2}} \frac{B^2}{B_c^2}, \text{ with } \alpha = 2 \sqrt{\frac{g \sigma}{\rho}}. \tag{3}
\]

If the driving velocity coincides with this velocity, the system is in resonance and the amplitude exhibits its maximum. The phase velocity \( v_p \) for different magnetic fields is experimentally obtained by fitting (1) to the data. The results are shown in figure 5. With increasing \( B \) the velocity \( v_p \) decreases and eventually becomes zero at the measured critical induction \( B_{c,m} \). At this induction the surface exhibits non-propagating undulations, which is a manifestation of the Rosensweig instability.

Fitting the data with (3), where \( \alpha \) is computed from the material parameters from table 1 and only \( B_c \) is adjusted, yields the curves marked by dashed lines. Fitting both \( B_c \) and \( \alpha \) gives the solid lines. The values corresponding to the latter procedure are included as \( B_{c,m} \) in table 1.

4. Discussion and conclusion

Applying a novel type of magnetic traveling-stripe forcing with \( k = k_c \) to the subcritical regime of the Rosensweig instability we measured the response of surface waves at different driving velocities \( v \). For a driving at the phase velocity of free surface waves a resonance phenomenon is observed, which can quantitatively be described as that of a damped harmonic oscillator. The resonant velocity \( v_p \) depends on the applied magnetic induction and decreases to zero when the critical induction \( B_c \) is approached. The functional dependence of \( v_p \) is essentially captured by the dispersion relation for an inviscid magnetic layer of infinite depth (Cowley and Rosensweig 1967).
When comparing the values $B_c$ as computed from the material parameters and the fitted values $B_{c,m}$, the latter are shifted by 8% and 9% to lower inductions. This deviation is larger than the discrepancy of 3% obtained from amplitude measurements in the supercritical regime of the Rosensweig instability (Gollwitzer et al. 2007). This may partly be explained by the limited resolution in the immediate vicinity of $B_c$. More importantly the true value for $B_c$ can only be obtained in the limiting case for vanishing modulation amplitudes $\Delta B$, while $\Delta B$ in our case is approximately as large as the deviation $B_c - B_{c,m}$. In addition, ansatz $B = \Delta B$ in our case is approximately as large as the deviation $B_c - B_{c,m}$. Moreover our magnetic forcing may be applied to the supercritical regime of the Rosensweig instability, where new resonances between hexagonal, square, or stripe-like patterns and the traveling-stripe forcing are predicted (Rüdiger et al. 2007).

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