A novel GIS-based decision-making framework for the school bus routing problem

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The school bus routing problem (SBRP) is a central issue in transportation planning and optimization systems. SBRP seeks to plan an efficient schedule for a fleet of school buses where each bus picks up students from various bus stops and delivers them to their designated schools while satisfying various constraints such as the maximum capacity of a bus, and the time window of a school. Due to its inherent complexity, many heuristics have been proposed to solve this combinatorial problem in an effective way. In this paper, a novel geographic information systems (GIS)-based decision-making framework that combines GIS, clustering techniques, network cutting techniques, and a hybrid ant colony optimization metaheuristic with the iterated Lin–Kernighan local improvement heuristic is proposed for solving the SBRP as a split delivery vehicle routing problem (SDVRP). Experiments were conducted for evaluating the proposed framework by comparing the results for solving 11 routing problems using both the proposed decision-making framework and ArcGIS 9.2 Network Analyst which uses the greedy Dijkstra’s algorithm. The reported results of the proposed framework generally outperform that of the ArcGIS Network Analyst. In addition, the proposed decision-making framework was applied to solve a real life SBRP to demonstrate its application.

Keywords: GIS; vehicle routing problem; school bus route; ant colony optimization; Lin–Kernighan

1. Introduction

The effectiveness of many transportation systems is often measured by the efficiency of vehicle fleet planning, as efficient vehicle routing and utilization can yield shorter delivery routes and thus a lower cost. How to transport students to and from their schools in the safest, most economical, equitable, and convenient manner is the important question facing many school boards. In solving the school bus routing problem (SBRP) one attempts to find routes that serve all eligible students, economically, equitably, and safely (1). SBRP seeks to plan an efficient schedule for a fleet of school buses where each bus picks up students from various bus stops and delivers them to their designated schools while satisfying various constraints such as the maximum capacity of a bus, the maximum riding time of a student in a bus, and the time window of a school (2). SBRP consists of smaller subproblems (1, 2): data preparation, bus selection from available bus fleet (homogeneous or heterogeneous), bus stop selection, students assignment to stops, bus stop assignments for buses, bus route generation, school bell time adjustment, and route scheduling. Although these subproblems are highly interrelated, they are treated separately and sequentially in most existing approaches due to the complexity and size of the problem. Moreover, in most literature, only some parts of SBRP are considered. Specifically, since the locations of bus stops and the opening and closing hours of schools are highly related to the policies of board of education, many studies assume that this information is a given and concentrate on the routing and scheduling of vehicles (2).

Although SBRP itself is a unique and independent problem, a single subproblem or a combination of its subproblems can be classified as a variant of existing optimization problems (2). In a large number of studies SBRP, the bus route generation subproblem, is referred to as an important real-world application of the vehicle routing problem (VRP) (3, 4). VRP seeks to generate efficient routes for a fleet of vehicles in order to deliver (or collect) products from depots to a set of customers. Finding optimal routes for a fleet of vehicles performing assigned tasks on a number of spatially distributed customers can be formulated as a combinatorial optimization problem: the VRP. A solution of this problem is the best route serving all customers using a fleet of vehicles, respecting all operational constraints, such as vehicle capacity and the driver’s maximum working time, and minimizing the total transportation cost. There are various variants of VRP, such as VRP with time windows, capacitated VRP, split delivery VRP (SDVRP), etc. Detailed descriptions of VRP and its variants are reported elsewhere (5, 6).

Since all the VRP variants including SBRP are NP-hard, their combinatorial complexity makes them intractable as soon as the search space becomes too large, and
in vehicle routing this happens in practice when there are a few dozens of customers to serve. Thus, heuristic and metaheuristics methods are the only feasible way to provide near optimal solutions for industrial-scale problems (7).

Various metaheuristics have been successfully applied to the VRP and its variants such as simulated annealing (8), tabu search (9, 10), granular tabu search (11), genetic algorithms (12, 13), guided local search (14), variable neighborhood search (15), and ant colony optimization (ACO) (6, 7).

Spatial decision-making problems such as transportation planning, routing, and management are multifaceted challenges. Not only they often involve numerous technical requirements, but may also contain economical, social, environmental, and political dimensions that may have conflicting values. Solutions for these problems involve highly complex spatial data analysis processes and frequently require advanced means to address physical suitability conditions while considering the multiple socioeconomic variables (16). Geographic information systems (GIS) have increasingly been used for solving spatial decision problems such as transportation planning, routing, and management. However, current GIS lack the mathematical modeling, iterative equation solving, routing, and management. Thus, GIS cannot adequately support decision-making. Recent technological advances in operational research, artificial intelligence, and information technology have enabled the development of high quality spatial decision support systems (SDSS). These systems are explicitly designed to overcome the limited capabilities of GIS in the spatial decision-making process and to help decision-makers solve complex spatial problems (16). The applications of these techniques to GIS-based transportation planning, routing, and management have gained popularity in recent years. Notable examples include (1, 17–21).

In addition to the plentiful studies addressing the VRP, since the appearance of the first publication on it by Newton and Thomas (22), SBRP has been constantly studied. However, there is no single dominant approach for the study of SBRP and many of the available approaches are problem dependent (2).

In this paper, a novel GIS-based decision-making framework that combines GIS, clustering techniques, network cutting techniques, and a hybrid ACO metaheuristic with the iterated Lin–Kernighan (ILK) local improvement heuristic is proposed for solving the SBRP as a SDVRP. The rest of this paper is arranged in following sections: Section 2 outlines the SBRP formulation. Section 3 describes the proposed decision-making framework while Section 4 presents its evaluation process. Section 5 illustrates the application of the proposed decision-making framework in solving a real SBRP while Section 6 concludes the paper.

2. Problem formulation

The solution structure of the single-school problem is similar to that obtained from traditional VRP, in which, a route starts from a depot, traverses a number of customers, and finally returns to its starting depot (2). In this paper, the SBRP is formulated as a SDVRP which was introduced in the literature nearly 20 years ago by Dror and Trudeau (23) who motivated the study of the SDVRP by showing that there can be savings generated by allowing split deliveries (24). By formulating the SBRP as a SDVRP, a fleet of capacitated homogeneous vehicles (school buses) is available to serve a set of customers (bus stops). Each customer (bus stop) can be visited more than once, and the demand (students’ number) of each customer (bus stop) can be greater than the capacity of the vehicles (buses). No constraint on the number of available vehicles (buses) is considered. There is a single depot (school) for the vehicles (buses) and each vehicle (bus) has to start and end its tour at the depot (school). The objective is to find a set of vehicle (bus) routes that serve all the customers such that the sum of the quantity (students) delivered in each tour does not exceed the capacity of the vehicles (buses) and the total distance, or time travelled is minimized (25). Hence, the main goal is to design a set of routes that minimizes total cost (distance or time) according to the following assumptions:

- each bus performs exactly one route,
- each route begins and arrives at the school,
- each bus stop can be visited by one or more than one bus, and
- the total number of students must be satisfied and the number of students on each bus must not exceed the bus capacity.

Thus, the problem can be defined over a network $N = (V; E)$ with vertex set $V = \{0, 1, \ldots, n\}$, where 0 denotes the depot (school) and the other vertices contain bus stops and $E$ is the edge set that connect among these vertices. The traversal cost ($c_{ij}$) of an edge $(i, j) \in E$ is assumed to be nonnegative. This cost may be distance or time. An integer number of students $s_i$ is associated with each bus stop $i \in V \setminus \{0\}$. Number of buses ($m$) = $\sum_{i=1}^{n} \lfloor \frac{s_i}{q} \rfloor$, where $q$ represents bus capacity. For each edge $(i, j)$, $x_{ij}$ is a decision variable that can be defined as:

$$
    x_{ij} = \begin{cases} 
    1 & \text{if the bus } b \text{ drives directly from vertex } i \text{ to vertex } j \\
    0, & \text{otherwise} 
    \end{cases} \quad (1)
$$

Also, $y_{ib}$ is a variable which is the number of students at stop $i$ that are travelled by the bus $b$.

The following equations demonstrate the problem mathematical formulations (24):

\begin{align*}
    \text{Minimize} & \sum_{i=1}^{n} \sum_{j=i+1}^{n} c_{ij} x_{ij} \\
    \text{Subject to} & \sum_{j=1}^{n} x_{ij} = 1 & \text{for all } i \in V \setminus \{0\} \\
    & \sum_{i=1}^{n} x_{ij} = 1 & \text{for all } j \in V \setminus \{0\} \\
    & \sum_{j=1}^{n} y_{ij} = s_i & \text{for all } i \in V \setminus \{0\} \\
    & \sum_{i=1}^{n} y_{ib} \leq q & \text{for all } b \in \{1, \ldots, m\} \\
    & y_{ib} \geq 0 & \text{for all } i \in V \setminus \{0\}, b \in \{1, \ldots, m\}
\end{align*}
The objective Equation (2) minimizes the total cost (distance or time). Constraints (3)–(5) are the classical routing constraints. Constraint (3) imposes that each vertex is visited at least once, (4) is the flow conservation constraint, and (5) is the subtours elimination constraint. Constraints (6)–(8) concern the allocation of the total number of students among the buses. Constraint (6) imposes that bus stop \( i \) is served by bus \( b \) only if \( b \) passes through \( i \), constraint (7) ensures that the entire number of students of each vertex is satisfied, while constraint (8) guarantees that the quantity delivered by each bus does not exceed the bus capacity. Constraint (9) determines whether each edge \((i, j)\) has direct connection or not. Finally, constraint (10) ensures that the bus \( b \) travels with at least one student.

3. Proposed decision-making framework

In this paper, a novel GIS-based decision-making framework for solving the SBRP as a SDVRP is presented. The proposed framework integrates the capabilities of GIS, clustering techniques, network cutting techniques, and a hybrid ACO metaheuristic with ILK local improvement heuristic as shown in Figure 1.

To implement the proposed decision-making framework, a prototype GIS-based decision support system for SBR was developed as an extension to ArcGIS 9.2 using component object modeling technology. Detailed description of the development process is reported elsewhere (26). The proposed system consists of five modules: GIS module, clustering module, network cutting module, routing module, and the user interface. Figure 2 depicts the process model of the proposed system.

3.1. GIS module

ESRI ArcGIS Desktop was used as the GIS module in the proposed system to manage the spatial data, to conduct the required spatial analysis operations and to visualize the results. ArcGIS Desktop is a scalable set of state-of-the-art software for geographic data creation, management, integration, analysis, and presentation (27).

3.2. Clustering module

Clustering is a process of partitioning a set of data (or objects) into a set of meaningful subclasses called clusters (28). The main objective of the proposed clustering algorithm is dividing pickup locations into groups based on the Euclidean distance and bus capacity. First, it sorts the pickup locations based on their locations to the destination location, then it clusters the pickup locations into groups based on bus capacity as shown in Figure 3.

3.3. Network cutting module

Limiting the search space is an efficient technique for obtaining efficient feasible solutions in an appropriate time when dealing with large-scale problems with huge search spaces such as transportation network routing problems (29). The main objective of the proposed network cutting algorithm is cutting the whole searched network into customized networks based on each cluster’s boundary, in order to limit the search space. First, the whole network and all clusters are loaded. Then, for each

![Figure 1. The proposed decision-making framework.](image-url)
cluster, the algorithm gets $X_s$ and $Y_s$ boundaries. Next, the algorithm cuts the whole network based on these $X_s$ and $Y_s$ boundaries, so each cluster has specific and limited network. Finally, these clusters’ networks are saved and inserted in the routing module as shown in Figure 4.

3.4. Routing module

The main objective of this module is solving the routing problem using an ACO metaheuristic hybridized with the ILK local improvement heuristic as shown in Figure 5.

3.4.1. ACO metaheuristic

ACO is a relatively new metaheuristic technique that uses artificial ants to find solutions for large numbers of combinatorial optimization problems. It simulates the behavior of ant colonies in nature as they forage for food and find the most efficient routes from their nests to food sources (30). Insects like ants are social. That means that ants live in colonies and their behavior is directed more to the survival of the colony as a whole, rather than to that of a single individual. In nature, an individual ant is unable to communicate or effectively hunt for food, but as a group, ants possess the ability to solve complex problems and successfully find and collect food for their colony. Ants communicate using a chemical substance called pheromone. As an ant travels, it deposits a constant amount of pheromone that other ants can follow. Pheromone reinforcement is autocatalytic, since the shortest the path, the least time will be taken to travel back and forth, and therefore, while ants on longer paths are still in transit, the ants on the shortest path can restart the route again, reinforcing the pheromone trail on the shortest path. Over time, the majority of the ants will travel on that path, while a minority will still choose alternative paths. The behavior of this minority is important,
since it allows exploring the environment to find even better solutions, which initially were not considered (6, 7). This cooperative work of the colony determines the insects' intelligent behavior and has captured the attention of many computer scientists. The first ACO algorithm was developed by Clorini et al. (31) and successfully applied to the traveling salesman problem (TSP) based on the path-finding abilities of real ants (32). Bulleneimer et al. (33) have designed the first ant system for the VRP and subsequently proposed an improved ant system algorithm.

The ACO algorithm replicates this behavior, adding some features to make it more efficient in the computer implementation. Ants are implemented as a set of concurrent and asynchronous agents. They construct a solution visiting a series of nodes on a graph. They select the move along an edge to the next node to visit according to two parameters: trails and attractiveness. As real ants, also artificial ants will prefer in most cases a deterministic choice of the path, based on the selection of the path with the strongest pheromone and on the highest attractiveness. Yet, in a fraction of cases, the choice will be made probabilistically, though guided by attractiveness and trails. Detailed descriptions of ACO are reported elsewhere (7,34,35). Figure 6 depicts the ACO algorithm used.

### 3.4.2. ILK local improvement heuristic

The Lin–Kernighan (LK) algorithm belongs to the class of so-called local search algorithms. A local search algorithm starts at some location in the search space and subsequently moves from the present location to a neighboring location. The LK heuristic is generally considered to be one of the most effective methods for generating optimal or near-optimal solutions for the symmetric TSP. The algorithm is specified in exchanges (or moves) that can convert one candidate solution (tour) into another. Given a feasible tour, the algorithm repeatedly performs exchanges that reduce the length of the current tour, until a tour is reached for which no exchange yields an improvement. This process may be repeated many times from initial tours generated in some randomized way. The LK performs so-called $k$-opt moves on tours. A $k$-opt move changes a tour by replacing $k$ edges from the tour by $k$ edges in such a way that a shorter tour is achieved (36). The ILK is an enhanced version of LK that tries to improve the tour at each step. It is started by the ACO tour, at each step it improves and adapts the previous tour until there is no tour adaptation. In this paper, this method is used to reduce the distance traveled by a traveler route, by continually removing and reconnecting the tour until there is no improvement to be found, as all exchanges of $k$ edges are tested until there is no feasible exchange that improves the current solution. The resulting tour

![Figure 6. ACO algorithm (adapted from Dorigo and Stützle (35)).](image)

![Figure 7. ILK local improvement heuristic (adapted from Helsgaun (37)).](image)
represents a local optimum which is called \( k \)-optimal. This optimality is called intra-route due to optimizing a single route (route by route). Detailed descriptions of ILK are reported elsewhere \((36, 37)\). Figure 7 depicts the used ILK algorithm.

### 3.5. User interface

The user interface of the proposed system was developed using C# 2005. The main purpose of this interface is to control and integrate the software modules of the proposed system using .NET technology. This interface contains two tabs. The first one is used to specify the user inputs, while the other one is used to specify the output options.

### 4. Proposed system evaluation

To evaluate the proposed system, 11 real routing problems (shortest path) were solved using both the proposed system and ArcGIS 9.2 network analyst route solver that uses the greedy Dijkstra’s algorithm. The results are presented and compared. The reported results of the proposed framework generally outperform that of the ArcGIS Network Analyst as shown in Table 1. Detailed descriptions of ArcGIS 9.2 network analyst route solver can be found on http://www.esri.com(2009).

### 5. Case study

This section presents implementation results from the proposed system on a test regional area. The test problem utilized a real regional site (Zagazig, Sharqia Governorate, Egypt). The goal of the test problem is to solve a SBRP. The following subsections describe the study area, the data used for case study, and the results.

#### 5.1. Study area

The ABC school is located in Zagazig city, a town of Lower Egypt, in the eastern part of the Nile delta. Zagazig is the capital of the province of Sharqia Governorate. ABC school has 120 students distributed on 13 locations all over Zagazig city and the capacity of each bus is 28 students. Zagazig includes 8858 streets with 6560 junctions as shown in Figure 8. The case study data input is shown in Figure 9.

#### Table 1. Comparison of the results.

| Test problem | Number of points | ArcGIS 9.2 network analyst output (m) | Proposed system’s output (m) | The difference (m) |
|--------------|------------------|--------------------------------------|------------------------------|-------------------|
| First        | 2                | 1054.1                               | 1054.1                       | 0                 |
| Second       | 4                | 2563.6                               | 2515.1                       | 48.5              |
| Third        | 5                | 2274.6                               | 2066.9                       | 207.7             |
| Fourth       | 8                | 1657.9                               | 1596.9                       | 61                |
| Fifth        | 12               | 4817.5                               | 3041.1                       | 1776.4            |
| Sixth        | 20               | 11,176.8                             | 7651.5                       | 3525.3            |
| Seventh      | 25               | 7603.6                               | 6888.4                       | 715.2             |
| Eighth       | 30               | 8210.6                               | 7179.7                       | 1030.9            |
| Ninth        | 40               | 12,081.1                             | 10,898.5                     | 1182.6            |
| 10th         | 50               | 21,836.3                             | 14,184.2                     | 7652.1            |
| 11th         | 60               | 24,507.3                             | 16,015.6                     | 8491.7            |

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Figure 8. Data of the case study.

Figure 9. Case study data input.
5.2. Results

Figure 10 shows the entire case study results while the resulting clusters and routes are shown individually in Figures 11–15. The final system output is shown in Figure 16.

6. Conclusion and future work

In this paper, the SBRP was formulated as a SDVRP. A novel GIS-based decision-making framework that combines GIS, clustering techniques, network cutting techniques, and a hybrid ACO metaheuristic with the ILK local improvement heuristic was proposed for this routing problem. Evaluation experiments indicated that the reported results from the proposed framework generally outperform that of the ArcGIS Network Analyst route solver. This work is intended as the first step toward a fully integrated SDSS for SBR that could be used to solve the different SBR subproblems such as data preparation, bus stop selection, bus route generation, school bell time adjustment, and bus scheduling.

Notes on contributor

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