Adjoint based noise minimization of a round supersonic jet

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Abstract. Jets with complex shock-cell structures appear in numerous technological applications. The shock/shear-layer interaction emanates a broadband noise component. This may trigger the thin shear layer at the nozzle exit, forming a feedback loop which results in a discrete noise component called screech. Both components are undesirable from structural and environmental (cabin noise) points of view. Screech tones produce sound pressure levels of 160 dB and beyond.

The focus of the present research project lies in the minimization of supersonic jet-noise and in particular in the minimization of jet-screech. Since screech – a phenomenon which is not yet understood in all details – seems to be affected by the presence of the jet-nozzle, a porous material will be added to the nozzle exit to suppress the feedback mechanism. Thus, to minimize the emanated noise. It is by no means clear how the shape an characteristic properties of the porous material should be. To this end, an optimization technique, based on adjoint methods will be applied to optimize the material with respect to the emanated noise.

1. INTRODUCTION

Supersonic jets, as found in civil- or military aircraft, are responsible for loud noise sources, polluting the environment. One can distinguish between three physical flow phenomena responsible for supersonic jet noise. The first and usually quietest noise sources are of rather low frequency, and are caused by the turbulent mixing in the shear layers surrounding the jet core. Especially vortex pairing and its so related vortex deformation contribute to these sources. A second phenomenon, the so called shock induced noise, is caused by an interaction of these turbulent eddies with the oblique shocks in the jet core. Shock induced noise is of high frequency and distributed over a vast range of frequencies in the power spectrum (Seiner, 1984). In contrast to mixing noise which is mainly directed in downstream direction, shock induced noise is directed mainly in the upstream direction.

Considering a subsonic co-flow of the supersonic jet, the upstream traveling shock induced acoustic waves can reach the nozzle exit. At this point, these acoustic pressure fluctuations trigger Kelvin–Helmholtz instabilities in the mixing layers. On their way downstream the jet, the instabilities are growing to larger coherent structures and interact with the oblique shocks in the jet core. Again, shock induced noise is emanated upstream and closing a feedback loop with a certain frequency (cf. Fig. 1(a,b)). In the literature this effect is referred to as screech and can produce sound pressure levels of up to 160 dB (Tam & Block, 1978).
Figure 1. (a) Sketch of a typical spectrum for a supersonic jet-noise application with screech. The peak of the screech signifies the dominant noise source and can be several dB louder than all other noise sources (Seiner, 1984). (b) Schematic view of an under-expanded supersonic jet including the nozzle. In the jet core oblique shocks are interacting with the turbulent mixing layers and emanating noise traveling upstream. At the nozzle exit, the acoustic waves are interacting with the mixing layer and triggering instabilities. Filled gray area: Solid part of the nozzle, Shaded area: Porous part of the nozzle; can be optimized.

As explained, the screech feedback loop is closed by upstream traveling acoustics interacting with the nozzle exit lip. These acoustics are impinging at the nozzle lip, being reflected, and cause a pressure gradient at the nozzle exit, forcing the instabilities in the thin mixing layer. Thus, by a modification of the nozzle lip, the screech tone can be influenced. Generally speaking, a thick nozzle lip is producing louder screech tones than a small lip (Raman, 1999).

The focus of the present research project lies in the minimization of jet screech by a modification of the nozzle exit. In contrast of changing the nozzle geometry, we will follow a different approach by adding a porous medium at the nozzle exit to suppress screech tones. The idea is to damp impinging acoustic waves in the porous medium at the nozzle exit to suppress their sensitivity to forced instabilities in the mixing layers. The porous medium investigated, is modeled by a volume force, similar to Darcy’s law for incompressible flow. For the present aeroacoustic application the volume force is implemented in the compressible Navier–Stokes equations leading to a new set of porous flow equations.

The porous medium is characterized by only two parameters, the porosity $\phi$ and the permeability $K$. In which $\phi$ describes the volume ratio of void space to the whole volume of the porous material. Thus, a porosity equal to one represents void space only, and a porosity of zero a solid material, where no fluid could penetrate. The latter extreme causes a singularity in the porous flow equations, thus only values of $0 < \phi \leq 1$ are feasible.

The second parameter $K$ stands for the permeability of the material and is a symmetric and positive definite tensor,

$$K = \begin{pmatrix}
\kappa_{11} & \kappa_{12} & \kappa_{13} \\
\kappa_{21} & \kappa_{22} & \kappa_{23} \\
\kappa_{31} & \kappa_{32} & \kappa_{33}
\end{pmatrix}. \quad (1)$$

The entries in that tensor can reach values of zero for a solid material which is not permeable and infinity for a material with no influence on the fluid (zero drag, void material). To keep the freedom in the design process of the porous medium, the two coefficients $\phi$ and $K$ are supposed to be functions of space and time. Anyway, since the realization of a time dependent structure of a porous medium for technical applications is hard to implement, only the space dependence will be considered ($\phi = \phi(x), \ K = K(x)$).
Since it is by no means clear how to choose the two parameters to reduce screech tones, an optimization technique will be used to obtain the optimal distribution of the porosity and permeability. Due to the space-dependence of the design parameters, the number of parameters to control can easily be in the order of several thousand. To handle this amount of degrees of freedom in the optimization algorithm, adjoint methods will be used in an iterative design process to obtain the gradient information of the objective function (noise). The adjoint equations are based on the compressible porous Navier–Stokes equations and are derived in a continuous manner without any further simplifications. Since the control parameters are constraint \((0 < \phi \leq 1\) and \(K < 0\)), due to their physical properties, a method based on slack variables will be used.

In the present paper we will present and demonstrate the mathematical framework to combining an adjoint-based optimization algorithm with porous media to reduce supersonic jet screech. In particular the influence of the porous medium to an adjoint–based code will be discussed. The organization of the paper is as follows: in the section §2, we will first give a general description of the porous flow equations to implement a porous material in an existing flow solver. This is followed by deriving the governing equations of the iterative optimization scheme with porous adjoint equations. In section §3 results of a supersonic jet with an optimized porous nozzle are shown. In section §4 we conclude with a summary of the main results of our study.

2. Mathematical framework

To include a porous medium in the computational domain, the equations of continuity, momentum and energy have to be modified. The main idea is based on a relation found by Darcy in 1856 to relate the flow velocities and the pressure gradient with the permeability of the porous medium in a linear way. This relation, called Darcy’s law, has been validated in several experiments and reads:

\[
v = -\frac{K}{\mu} \nabla p, \tag{2}\]

with the so called Darcy velocity \(v = \phi u\). This volume force is added to the momentum equation and acts like a source term, damping all velocities in the porous medium.

2.1. Governing porous flow equations

The compressible porous Navier–Stokes equations read in pressure, velocity and entropy formulation:

\[
\frac{\partial p}{\partial t} + u_i \frac{\partial p}{\partial x_i} = -\phi \rho e \left( \frac{\partial u_i}{\partial x_i} C_v \left( \frac{\partial s}{\partial t} + u_i \frac{\partial s}{\partial x_i} \right) \right) \tag{3a}
\]

\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\phi \rho} \frac{\partial p}{\partial x_i} + \frac{1}{\phi \rho} \frac{\partial \tau_{ij}}{\partial x_j} - \frac{\mu}{\rho} (K^{-1})_{ij} u_j \tag{3b}
\]

\[
\frac{\partial s}{\partial t} + u_j \frac{\partial s}{\partial x_j} = \frac{1}{\rho e T} \left( \frac{\partial}{\partial x_j} \left( \lambda \frac{\partial T}{\partial x_j} + \Phi \right) \right) + \frac{\mu}{\rho T} (K^{-1})_{ij} u_i u_j - \frac{\dot{h}_s}{\phi} (T - T_s) \tag{3c}
\]

These modified Navier–Stokes equations for porous media are closed with the thermodynamic relation for an ideal gas and Sutherland’s law to compute the temperature dependent viscosity. A detailed derivation of the porous equations can be found in Schulze & Sesterhenn (2011).

The equations are decomposed into characteristic waves (see Sesterhenn, 2000) and solved on a Cartesian grid, stretched in transverse direction to refine the grid in the area of the jet core. To capture the sound generation and propagation processes, spatial discretization is performed using a finite difference compact scheme of sixth order (Lele, 1992). A Sketch of the computational
domain including the boundary conditions can be found in Fig. 2(a). It contains $1024 \times 512 \times 512$ grid points.

The porous flow equations (3) can not only be used to implement a porous medium but also a solid body. The distinction between a solid body and a porous medium is only depending on the choice of the value of the permeability. A small value of $K$ corresponds to a solid body whereas $K \to \infty$ is void space. Values of $0 \ll K \ll \infty$ can be regarded as a porous medium. To this end it is easy to implement complex geometries in a Cartesian domain by setting $K$ close to zero inside the solid geometry. This method is also known as volume penalization and related to immersed boundary methods. The solid part of the nozzle in Fig. 1(b) is implemented using this technique.

Depending on the value of $K$ of the source term in the momentum equation (3b) for porous media, additional highly damped eigenvalues appear in the spectrum of the evolution operator. This eigenvalue constitutes a restriction to the time-step for standard explicit time integration methods, like Runge–Kutta. To this end, the time advancement is performed using an exponential integration based on Krylov subspaces. This integration is a low-dimensional approximation of the matrix exponential which represents the exact evolution operator for an autonomous linear system, and shows excellent stability properties for explicit schemes (cf. Schulze et al., 2009).

A visualization of an instantaneous snapshot of the supersonic jet including the nozzle is presented in Fig. 2(b). It shows a shadowgraph of a slice through a center plane of the jet where the shocks in the jet core and the turbulence in the mixing layers is clearly visible.

2.2. Optimization Framework

The optimization algorithm is based on adjoint equations. These equations are obtained out of the porous flow equations of section §2.1. We follow the continuous adjoint approach to derive the equations without any further simplifications, like a constant viscosity ($\mu = \text{const.}$) or setting $\Phi = 0$, which is a usual fashion (see e. g. Wei & Freund (2006)). A full derivation of the compressible adjoint equations would extend the scope of this paper and can be found in detail in Schulze et al. (2011). The derivation of the additional terms arising from the porosity are straightforward, since they are self-adjoint and it can be performed in the same way as the derivation of an adjoint sponge (damping) term (see Wei & Freund, 2006).

A common way of obtaining the optimization framework, is to set up a Lagrange functional
to enforce the constraints:

$$\mathcal{L} = J - \langle N(q, K), q^* \rangle - [H(q), h^*] - (G(q), g^*) \quad (4)$$

The objective function $J$ is the measure of the noise emanated by the jet in a certain region $\sigma(x)$ in the acoustic near field, and reads:

$$J = \frac{1}{2} \int_0^{t_1} \int_{\Omega} (p - p_\infty)^2 \sigma(x) \, dt \, d\Omega, \quad (5)$$

where the ambient pressure $p_\infty$ has to be chosen carefully to distinguish between hydrodynamic and aeroacoustic fluctuations. The noise is measured on a cylindrical surface placed concentrically around the centerline of the jet with a radius of $r = 7.5D$ ($\sigma(x)$). To avoid numerical instabilities for the adjoint computation of the optimization algorithm, the surface is smoothed out with a Gaussian distribution. $N(q, K) = 0$ is a symbolic way of stating the governing equations for the state variable $q$ and control $K$, given in the present study by the compressible Navier-Stokes equations (Eq. (3)). In a similar symbolic form, the boundary conditions and initial conditions can be formulated as $H(q) = 0$ and $G(q) = 0$, respectively. We note that the governing equations, boundary and initial conditions are added to the objective functional by the Lagrange multipliers $q^*, h^*$ and $g^*$. Since the constraints have to be satisfied locally, the Lagrange multipliers are fields, and the three different scalar products in (4) are thus given as

$$\langle p, q \rangle = \int_0^{t_1} \int_{\Omega} pq \, d\Omega \, dt; \quad \langle p, q \rangle = \int_{\Omega} pq \, d\Omega; \quad [p, q] = \int_{t_0}^{t_2} pq \, dt. \quad (6)$$

The variables $q^*, h^*$ and $g^*$ are also known as the adjoint variables which enforce the governing equations, the boundary conditions and the initial conditions. In what follows, adjoint variables will be indicated by the superscript $(\cdot)^*$. A detailed and recent review of adjoint optimization using a Lagrange functional can be found in Bewley (2001).

The Lagrange functional depends on the independent variables $q(x, t)$, $q^*(x, t)$, $h^*(t)$, $g^*(x)$ and $K(x, t)$. A minimum (or stationary point) of $\mathcal{L}$ requires the first variation with respect to all independent variables to vanish. This yields a system of equations that can be solved iteratively to determine the minimum of the objective functional. A detailed description of the iterative optimization loop can be found in Schulze et al. (2011).

3. Results

In the following section §3.1 results of the computation of a supersonic jet with a solid nozzle are presented. This is followed by the results of the supersonic jet including the optimized porous nozzle in Sec. 3.2.

3.1. Computation of a supersonic jet

In the present three-dimensional case, the Mach number in the jet core is $M = 1.55$, the underexpanded pressure ratio is $p_1/p_\infty = 2.0$. Based on the jet Mach number and the diameter $D$ of the nozzle exit, the Reynolds number is $Re = 5000$. Figure 3(a) shows the dominant screech Strouhal number for round jets versus the fully expanded jet Mach number together with experimental data. For the present computation, the screech frequency is in good agreement with the experimental data. The corresponding shock cell spacing is presented in Fig. 3(b). It appears to be, that the computational result is slightly larger than the analytical result found by Tam & Tanna (1982).
3.2. Optimized jet

The optimized porous nozzle is presented in Fig. 4. In the optimization region (shaded area of Fig. 1(b)), the algorithm created the optimal distribution of the porous medium. Visualized is the inverse of the first index of the permeability tensor ($1/\kappa_{11}$). Hence, a large value (red) stands for a solid body (mainly the solid part of the nozzle), whereas blue areas represent for a rather porous medium. The void areas in the illustration are the pores of the porous medium ($1/\kappa_{11} \to 0$). One can identify a complex structure of the porous medium with elongated pores.

In Fig. 5(b) the performance of the optimization algorithm is presented by measuring the noise reduction in dB. The noise is measured along a line placed parallel to the jet core in a distance of $7.5D$ to the centerline of the jet. One can see a reduction of up to 4 dB. In Fig. 5(a) the noise reduction is measured in frequency space. A reduction of the dominant screech tone ($f_s D_j/U_j \approx 0.25$) of 5 dB can be identified. The reduction in the low frequency band (mixing
Figure 5. Performance of the optimization algorithm. (a) Comparison of the noise of the controlled (---) and uncontrolled (----) jet in frequency space. (b) Noise reduction measured along a line placed parallel to the jet core in a distance of $7.5D$ to the centerline of the jet.

noise) is even up to 10 dB, whereas the broadband shock induced noise component seems to be unaffected by the influence of the porous medium.

In Fig. 6 the mean local Mach number and the RMS value of the Lighthill source are presented in a $x$-$y$-cross-section through the jet core. It seems that the porous medium has an influence on the flow of the jet even though the porous medium does not "touch" the jet flow (cf. Fig. 1(b)). For the mean Mach number of the uncontrolled case (a), the jet core is about one shock cell shorter than in the controlled case (c). The porous medium tends to stabilize the jet core and delays the instability of the jet. This phenomenon seems to be important for the reduction of screech tones. The source of screech tones is usually located around the third to fifth shock cell, depending on the jet Mach number (see e.g. Berland et al., 2007; Raman, 1999). In the present case, the mean profiles of the controlled an uncontrolled computation start to differ at the fifth shock cell. This phenomenon is more clearly visible in Fig. 7(a, b). The extension of the shock cell structure seems to be responsible for the screech tone reduction. Glass (1968) had been the first who observed a decay of the jet velocity of almost 50% on the jet axis at $x/D = 15$ for a jet with screech compared to a jet without screech. For the current computation, a reduction of 40% can be observed at $x/D = 15$. Krothapalli et al. (1986), Raman & Taghavi (1996), Raman (1998), Zaman (1999) and others observed similar results for round and rectangular jet where the screech phenomenon is responsible for an enhanced jet mixing.

Another difference in the flow structure can also be identified in the RMS value of the Lighthill source. The area of the dominant sources is delayed from the fifth to the seventh shock cell. In addition to that the maximum source amplitude of the controlled jet is reduced and the total area of the source region is scaled down. The reduction and the shift of the aeroacoustic sources are also visible in Fig. 7(d).

In Fig. 7 the mean values along the centerline of the jet are presented for different variables. Again, the main difference arises in the delay of the shock cell structure for the controlled jet. While the shock cell structure dissipated after the fifth shock cell for the uncontrolled jet it remains visible up to the seventh shock cell for the controlled jet (cf. panel (a) and (b)). A similar result was obtained by Taghavi & Raman (1994) where an enhanced mixing for a screeching jet disintegrated the shock cell structure compared to a jet with suppressed screech. The delay can be also observed in the turbulence kinetic energy (TKE) of panel (c) where a shift of the peak values of the length of one shock cell in the downstream direction is visible. Besides
Figure 6. (a, c) Mean Mach number in a cross section of the jet. Color range from $0 < M < 2.5$. Nozzle exit at $x/D = 0$. Sonic line, $M = 1$ (-- -- -- --). (a) no control, (c) control. (b, d) RMS value of the Lighthill source in a cross section of the jet. Nozzle exit at $x/D = 0$. (b) no control, (d) control

of the shift, the peak values of the controlled jet are reduced by a factor of 20% compared to the uncontrolled jet. This result confirms the phenomenon of an enhanced mixing of a screeching jet.

4. Conclusion

In the present paper the method of optimizing a porous material to reduce supersonic jet noise has been presented and applied to a three-dimensional round jet. Together with the porous flow equations for a compressible fluid, the corresponding adjoint porous flow equations are used in an iterative optimization loop to obtain the gradient of the objective function (noise). This gradient is used to update the control parameter (permeability of the porous material) and finally to find the optimal spatial distribution of a porous material to minimize jet noise.

As a showcase, to demonstrate the proposed optimization method, a three-dimensional supersonic jet ($M = 1.55, Re = 5000$) with a mixed, solid and porous nozzle, is investigated. The computed jet noise and shock cell spacing without the porous material is in good agreement with experimental data. Only the porous part of the nozzle, placed on the outside of the nozzle surface can be controlled by the optimization algorithm. Our numerical experiments achieved a noise reduction of up to $4 \text{dB}$ after the first iteration, by placing an optimized porous medium around the nozzle. The porous medium tends to stabilize the jet core and delays the instability of the jet. This phenomenon seems to be important for the reduction of screech tones. These discrete tones force the instability of the jet and enhance mixing. The reduction of screech tones and the associated reduction of mixing also minimizes mixing noise of low frequency. Reductions of up to $10 \text{dB}$ can be observed.

Adjoint optimization techniques with porous media thus establish a powerful tool to optimize not only aeroacoustic applications, like jet noise, but can be also applied for any further fluid-mechanical optimization purpose, which we will leave for a future effort.
Figure 7. Mean profiles of different flow variables along the centerline of the jet also including the inside of the nozzle which ends at $x/D = 3$. (----) controlled jet; (-----) uncontrolled jet. (a) Mach number, (b) pressure, (c) turbulence kinetic energy, (d) RMS Lighthill source (average in y-direction)

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