Deutsch-Jozsa algorithm via deterministic quantum computation with one qubit model

Márcio M. Santos and Eduardo I. Duzzioni

1 Instituto de Física, Universidade Federal de Uberlândia, 38400-902, Uberlândia, MG, Brazil and 2 Departamento de Física, Universidade Federal de Santa Catarina, 88040-900, Florianópolis, SC, Brazil

Quantum correlations have been pointed out as the most likely source of the speed-up in quantum computation. Here we study the presence of quantum correlations on the implementation of Deutsch-Jozsa algorithm running in the DQC1 model. We extend the analysis to a previous experimental realization of this computation presented in Phys. Rev. A 78, 022317 (2008). The Deutsch-Jozsa algorithm, an oracle based problem, can efficiently be solved in this model. Although there is no quantum correlations in the initial and final states of the computation, we find quantum correlations in intermediate steps of the algorithm. The presence of quantum correlations is not a sufficient property for computational gain in this case, since the performance of the classical probabilistic algorithm is better than the quantum one. The reason for this unexpected result relies on the fact that we have access to the outcomes of the control qubit only. A comparison between the efficiency of DQC1 and DQCp circuits to solve the Deutsch-Jozsa problem is also presented.

I. INTRODUCTION

Recently a lot of efforts have been made to discover the source of speed-up in quantum computing processing. For pure state computation, entanglement is seen as a necessary resource to observe advantage over classical computing [1, 2]. Such resource does not seem to be so essential for quantum computations with mixed states. It has been shown that the amount of entanglement present in the trace evaluation of a unitary matrix realized in the Deterministic Quantum Computation with one qubit (DQC1) model cannot explain the resulting speed-up [3]. Nevertheless, there are quantum correlations present in the system that can be quantified by quantum discord [4, 5]. Therefore quantum correlations, including entanglement, can be the source of the quantum computation advantage over the classical one.

Some computational problems have a more efficient solution through algorithms running in a quantum computation model than in the classical one. Among these algorithms are Shor factorization algorithm [7], Grover algorithm for search in an unsorted list [8], and Deutsch-Jozsa algorithm [9]. The last one, the most simple of them, tries to define the class of a function as balanced or constant. The realization of the deterministic version of the Deutsch-Jozsa algorithm is a simple way to show an exponential advantage of quantum computation over classical computation (see the details in the main text).

The Collins version of Deutsch-Jozsa algorithm was implemented in the DQC1 computation model with a NMR ensemble system [10]. Only one measurement over the ensemble was performed, showing, at first glance, an advance over the classical solution. Here we observe that there is no generation of quantum correlations at the end of the computation. When the computation is synthesized in a sequence of one and two qubit quantum gates, we find out that quantum correlations are generated and further eliminated in intermediate steps of such synthesis. We also compare the efficiency of the DQC1 and classical solutions and find that the quantum solution is, at most, equivalent to the classical probabilistic solution. This result points out that although there are quantum correlations in intermediate steps of the algorithm, there is no computational gain in the quantum case. In order to address the role played by mixed states in DQC1 model, we substitute them by pure states and verify that in this case the entanglement between two partitions scales with the system size as the efficiency of the algorithm remains unchanged. This indicates that although quantum correlations could have an important role in some tasks, it is crucial finding ways to benefit from them.

In Sec. II we present a brief review of the Deutsch-Jozsa problem via DQC1 model and analyze the efficiency of the algorithm, the correlations among the qubits, and the readout of the algorithm. These analyzes are also performed considering the experimental realization of the computation. A comparison between the efficiency of DQC1 and DQCp circuits to solve the Deutsch-Jozsa problem is made in Sec. III. We present our conclusions in Section IV.

II. DEUTSCH-JOSZA ALGORITHM VIA DQC1 MODEL

Let $f : \{0,1\}^n \rightarrow \{0,1\}$ be a constant or balanced function. The function is said to be constant if $f(j) = 0$ or $f(j) = 1$ for all $j' = 0, \ldots, 2^n - 1$ and is balanced if $f(j) = 0$ for half of the $j$ values and $f(j) = 1$ for the remaining $j$ values. Classically, it will take $2 \cdot 2^{n-1} + 1$ evaluations of $f$ to know the function class with certainty. In quantum computation the Deutsch-Jozsa algorithm solves the problem with just one collective measurement or $n$ individual qubit measurements. In the ideal case, this is the best solution to the problem and it will be used as benchmark solution. The Collins version of this algorithm uses a $n$ qubit register and is represented by the circuit in Fig. [1] where the unitary $U$ encodes the
function \( f[11] \). After the \( n \) first Hadamard gates the register is in an equal superposition of \( 2^n \) states \(|j\rangle\). The action of \( U \) on \(|j\rangle\) is \( U|j\rangle = (-1)^{f(j)}|j\rangle \), which means \( U \) is a matrix of the form

\[
U = \sum_j (-1)^{f(j)} |j\rangle \langle j|.
\]

The readout of the algorithm can be made by projecting the final state on the state \(|j = 0\rangle\), giving the result

\[
\frac{1}{2^n} \sum_{j=0}^{2^n-1} (-1)^{f(j)} = \begin{cases} 1 & \text{if } f \text{ is constant,} \\ 0 & \text{if } f \text{ is balanced.} \end{cases}
\]

Fig. 1: Collins version for the Deutsch-Jozsa algorithm.

The structure of \( U \) allows the definition of the function class of \( f \) by the evaluation of its normalized trace. An efficient way to evaluate the trace of a unitary matrix is given by the DQC1 model [12]. The DQC1 model is composed by \( n+1 \) qubits, where \( n \) qubits are in the fully mixed state \( I^\otimes n / 2^n \) and only one qubit presents a degree of purity controlled by \( \alpha \) (\( 0 < \alpha < 1 \)), as represented by the circuit in Fig. 2. The system initial state is \( \rho_f = (I_0 + \alpha Z_0) \otimes I^\otimes n \), where the index 0 refers to the semi-pure qubit (control qubit), \( I \) is the identity matrix, \( Z \) is the Pauli matrix \( \sigma_Z \), and \( I^\otimes n \) is the identity matrix of the \( n \) mixed qubits. Just before the measurement the state of the system is

\[
\rho = \frac{1}{2^{n+1}} (I_0 \otimes I_n + \alpha X_0 \otimes U_n),
\]

where \( X_0 \) is the Pauli matrix \( \sigma_x \). One way of implementing the Deutsch-Jozsa algorithm via DQC1 model is choosing the unitary matrix \( U_n \) as \( U \) defined in Eq. [4]. In this case, the final state of the control qubit is \( \rho_{\text{bal}} = I_0/2 \) for a balanced function and \( \rho_{\text{const}} = (I_0 \pm \alpha X_0)/2 \) for a constant function. Thus, if a \( \sigma_x \) measurement is performed on this qubit, the result for its expected value is \( \langle \sigma_x \rangle = 0 \) and variance \( \Delta \sigma_x = 1 \) for a balanced function and \( \langle \sigma_x \rangle = \pm \alpha \) with variance \( \Delta \sigma_x = \sqrt{1-\alpha^2} \) for a constant function.

Fig. 2: DQC1 circuit used to evaluate the normalized trace of a unitary matrix \( U \).

The natural states observed in ensemble of molecules in Nuclear Magnetic Resonance (NMR) experiments turn this system attractive for the realization of computations by DQC1. Indeed, some algorithms have already been implemented in this scenario [10] [13] [14]. It is worth noting that this algorithm was also realized with photons in the case with one maximally mixed qubit, although this fact was not mentioned in Ref. [15].

Efficiency of the algorithm According to Ref. [16], the best situation to distinguish between the states \( \rho_{\text{bal}} \) and \( \rho_{\text{const}} \) occurs when \( \alpha = 1 \), so that this will be the value assumed for alpha henceforth. If a sequence of measurements is performed, a balanced function will be identified when both values 1 and −1 are among the outcomes. If, in a fixed number of measurements procedure, all outcomes present the same value, then the function will be classified as constant with certain probability. The algorithm is efficient since it needs just a polynomial number of runs to identify the function class with high success of probability. This claim comes from the fact that this algorithm is equivalent to the classical probabilistic one presented by J. Preskill [17] [18]. In both cases, quantum and classical, a balanced function will be identified with certainty if two outcomes present two different values in a sequence of measurements. Otherwise, if after \( k \) measurements we obtain the same result, a guess is made that the function is constant assuming a probability \( P_{\text{err}} \) of an erroneous guess. In the classical algorithm the probability of two sequential measurements having the same outcome, given the function is balanced, is \( 1 \times (2^{n-1} - 1) / (2^n - 1) \), for three equal outcomes the probability is \( 1 \times (2^{n-1} - 1) \times (2^{n-2} - 1) / (2^n - 1) \), so, for \( k \) measurements the probability of the outcomes being all equal is \( g(k, n) = 1 \times (2^{n-1} - 1) \times (2^{n-2} - 1) \times ... \times (2^{n-k+1} - 1) / (2^n - 1) \). Let \( p \) be the probability of the function being balanced, so the probability of an erroneous guess is \( P_{\text{err}} = g(k, n)p \). In the quantum algorithm, every time we run the algorithm, we restart the system to perform a new measurement, so that in every new measurement the probability of the outcome being 1 or −1 is 1/2 and is independent of the number of qubits \( n \). Thus, the probability of an erroneous guess is \( P_{\text{err}} = p/2^{k-1} \). This result is recovered by the classical algorithm when \( 2^{n-1} \gg k \) and shows that the classical algorithm performance is an upper bound to the quantum algorithm implemented via DQC1 model, as can be seen for \( p = 1/2 \) in Fig. 3. Comparing the classical probabilistic solution with the quantum deterministic one [11], we observe that the former is also efficient, since one need just \( k \) measurements to obtain the solution with certain probability, while in the quantum deterministic case we need \( n \) measurements to obtain the exact solution. In Fig. 3 we observe that the probability of an erroneous guess \( P_{\text{err}} \) about the function class is very low, with just \( k = 6 \) measurements the chance of an erroneous guess is about 2%, whichever the number of qubits \( n \). Our result is in accordance with Ref. [18], which shows that the performance of the Deutsch-Jozsa algorithm in ensemble quantum computation is worst than the classical probabilistic algorithm presented above.

It might be desirable that, instead of using the same system at every new computation, to use a set with
computer (3) can be written as

\[ P_{err} = \sum_{j=0}^{2^n-1} (1/2^{n+1}) \left[ |0\rangle \langle 0| + \alpha(-1)^{f(j)} |0\rangle \langle 1| + \alpha(-1)^{f(j)} |1\rangle \langle 0| \right] \otimes |j\rangle \langle j| \]

where \(|a_j\rangle = \cos\phi |0\rangle + (-1)^{f(j)} \sin\phi |1\rangle\) and \(|b_j\rangle = \sin\phi |0\rangle + (-1)^{f(j)} \cos\phi |1\rangle\), and \(sin(2\phi) = \alpha\) [20]. Particularly for \(\alpha = 1\) the final state is

\[ \rho = \sum_{j=0}^{2^{n-1}} (1/2^n) |f(j)\rangle \langle f(j)| \otimes |j\rangle \langle j| , \]  

Figure 3: (Color online) Probability of error in the solution of Deutsch-Jozsa algorithm, \(P_{err}\), after successive \(k\) equal measurements outcomes, given the function is balanced, either for the realization in the DQC1 model (solid line) and the classical solution for 3 (squares), 5 (triangles) and 7 bits (circles). For the classical algorithm \(P_{err}\) increases with the number of bits and has the quantum algorithm as an upper bound.

Quantum correlations

It has been shown that the consumption of quantum correlations, such as quantum discord, is able to improve some computational tasks [19]. Based on that, we explore the role played by quantum correlations on the execution of the Deutsch-Jozsa algorithm via DQC1 model. The final state of the quantum computer [3] can be written as

\[ \rho = \sum_{j=0}^{2^{n-1}} (1/2^n) |f(j)\rangle \langle f(j)| \otimes |j\rangle \langle j| , \]

with \(|f(j)\rangle = (|0\rangle + (-1)^{f(j)} |1\rangle)/\sqrt{2}\). The states [4] and [9] represent classical probability distributions, therefore they have no quantum correlations [21]. States like these also have a null value of the global quantum discord defined in Ref. [22]. It is not obvious that a quantum algorithm is equivalent to the classical one provided that the initial and final states present no quantum correlations. Because it is possible that quantum correlations are present in intermediate states of the computation even when the initial and final states do not have any. These correlations may be related to the speedup of quantum computation since the quantum computer can evolve through states that have quantum correlations while a classical computer is restricted to a class of states that does not have them, which may result in the use of less gates in the quantum solution [23]. To investigate this possibility, we use the procedure presented by S. Bullock and L. Markov to decompose a diagonal unitary operator in a sequence of one qubit rotations and CNOTs [24]. Such synthesis is general so that it can describe any unitary applying any function related to the Deutsch-Jozsa algorithm, balanced or constant. The decomposition was done for the two and three mixed qubits cases, where the later is presented in Fig. 4. To determine the presence or absence of quantum correlations after each quantum gate in the synthesized algorithm is equivalent to observe the possibility of writing the system state as a classical probability distribution or not. For two mixed qubits case, corresponding to four values for the index \(j\) \((j = 00, 01, 10, 11)\), no quantum correlations are found in any point of the algorithm. In our decomposition for three mixed qubits, quantum correlations are found between the second and the last but one CNOT gates for some balanced functions. In this last case we found a null value of the negativity evaluated for all steps in the synthesized algorithm considering different splits for all kind of functions: i) a splitting that separates the control qubit from all the others and ii) another one that puts the top two qubits in one partition and the bottom two
in the other partition [25]. Collins, Kim and, Holton arrived at a similar conclusion for the Deutsch-Jozsa algorithm implemented through the conventional pure state quantum computation model [11]. They found that no entanglement is generated between two qubits, while for three or more qubits some balanced functions generate entanglement among them. Chaves and de Melo showed that there are functions for which it is possible to implement the Deutsch-Jozsa algorithm in the one-way quantum computation method with decoherence from a state that presents only classical correlations [26]. Arvind, Doria and, Kulmar implemented the Deutsch-Jozsa algorithm in a NMR experiment and observed the absence of entanglement for the one and two qubits cases and entanglement generation for some balanced functions in the three qubit case [27].

With the search for correlations above in mind, it may be valuable to look for quantum correlations in the experimental realization of the Deutsch-Jozsa algorithm in the DQC1 model with a NMR setup [10]. In this experiment the quantum computer is formed by an ensemble of molecules that form systems with five qubits. Through decoupling techniques the algorithm is performed with only four active qubits. Applying pulsed field gradients the state \( \rho_t = 2^{-4}(I_0 + \alpha Z_0) \otimes I^{\otimes 3} \) is prepared from the initial thermal equilibrium state. Particularly, the polarization is considered \( \alpha = 1 \). To apply the controlled unitary which encodes the function \( f_0 = x_1 x_2 \oplus x_3 \) (labels 1, 2 and 3 refer to the top, middle and bottom mixed qubits, respectively) an effective Hamiltonian is provided that reads \( H_{\text{eff}} = \frac{\tau}{4}\left\{ 2Z_1 Z_3 - 4Z_1 Z_2 Z_3 + 2Z_2 Z_3 + 2Z_1 Z_2 + 4Z_1 Z_4 - 3Z_1 - Z_2 - Z_3 - 2Z_4 \right\} \), where \( \tau \) is the duration of the evolution under \( H_{\text{eff}} \) and \( Z_i \) is related to the Pauli matrix by \( \sigma_i^Z/2 \). The effect of this Hamiltonian is the application of the desired controlled unitary except for a global phase. Since any pair of terms in \( H_{\text{eff}} \) commute, \( H_{\text{eff}} \) can be applied as a sequence of its terms, and the terms can be applied in any order. The order of the terms applied for the system evolution in the experiment is as written above, that is, the term proportional to \( Z_1 Z_3 \) is applied first, then the one proportional to \( Z_1 Z_2 Z_3 \) and so on. The linear terms correspond to one qubit rotations and are implemented via composite pulses. The bilinear terms proportional to \( Z_1 Z_3, Z_2 Z_3 \) and \( Z_1 Z_2 \) are implemented through the natural interaction between qubits in the molecule. The remaining terms are achieved with the aid of unitary transformations applied on the terms of direct interaction between qubits.

In order to detect quantum correlations in the system after the application of each term of \( H_{\text{eff}} \) the quantum discord is evaluated for every bipartition that separates one qubit from the other three. Quantum discord, restricted to these bipartitions, has non null values only for the bipartition that separates the control qubit from the completely mixed qubits (with measurements over the control qubit). Moreover, the quantum discord is found only for the first three terms, after which, no quantum discord is detected. This result, by no means implies that there is no quantum correlations after the application of the fourth term of \( H_{\text{eff}} \), once the procedure applied to detect quantum correlations is not general as only specific partitions are considered. Although present in the computation, the quantum correlations found do not improve the efficiency of the quantum solution since it is no better than the classical one.

Even though the analysis of correlations in the experimental realization is restricted to quantum discord in specific bipartitions, note that here, through a synthesis that is general for any balanced function, our analysis is not restricted to entanglement, but we consider all kind of quantum correlations which encompass some possible partitions of the system. Even though the implementation of the Deutsch-Jozsa algorithm via DQC1 model does not present advantage over its classical counterpart, there are other examples where solutions to problems achieved by quantum computation without entanglement are more efficient than their classical analogs [28–30].

The readout of the algorithm The final states of the Deutsch-Jozsa algorithm implemented via DQC1 model for the possible function classes do not have orthogonal support, which means they can not be distinguished with just one measurement as in the conventional pure state quantum computation [11]. This makes such algorithm probabilistic and equivalent to the classical probabilistic algorithm. The solution becomes probabilistic because we have access to the outcomes of just one qubit measurements, which means we obtain the average information processed by the quantum circuit on the control qubit.

### III. DEUTSCH-JOZSA ALGORITHM VIA DQCP

The basic idea of the deterministic quantum computation with pure states (DQCP) is to reproduce in the answer qubit the expectation values of DQC1 model for the control qubit [12]. The same results for the Deutsch-Jozsa algorithm in DQC1 model presented above, that is, the expected values and variances of \( \sigma_i^Z \) for the control qubit, can be achieved if \( \alpha = 1 \) and the \( n \) register qubits in the DQC1 circuit are initialized in the state \( |+\rangle^\otimes n = [(|0\rangle + |1\rangle)/\sqrt{2}]^\otimes n \). This result, by its turn, demonstrates that to solve oracle problems in a quantum computer running via DQC1 model can be as powerful as in a quantum computer running with pure state qubits (DQCP) [12]. Here, an important difference between DQC1 and DQCP models is that with the pure initial states the circuit can generate significant amounts of entanglement among the qubits at the end of the computation, while with mixed states no quantum correlation was generated. To verify this hypothesis, the algorithm was run 50 times with random balanced functions for a number of work qubits from 1 to 10, and the negativity was evaluated for two different splits: i) a split that separates the \((n + 1)/2\) top qubits and the \((n + 1)/2\) bottom qubits for \( n \) odd, and ii) the \( n/2 \) top qubits and \( n/2 + 1 \) bottom qubits for \( n \) even. The maximum value for the
Quantum Correlations

negativity achieved for each number of qubits is shown in Fig. 5. The resulting curve presents an overall increasing pattern, with the characteristic of sequential values of the negativity being approximately constant since it is limited by the dimension of the smaller partition [20]. Although entanglement increases with the system size, the presence of such quantum correlations does not improve the quantum algorithm when compared to the classical one. We observe that the rotation angles $\theta_j$ present in the synthesis of the algorithm may have, among other values, $\pm \pi/4$ for some balanced functions. In these situations, the operator $R_j$ becomes the $T$ (or $\pi/8$) gate, a unitary that lies outside the Clifford group. Despite of Gottesman-Knill theorem and Eastin result do not apply to these cases, the algorithm presented above can efficiently be simulated in a classical computer.

![Figure 5: Negativity for the final state in the realization of Deutsch-Jozsa algorithm via DQC1 model as a function of the number of qubits $n$.](image)

### IV. CONCLUSION

We have presented a brief review about the Deutsch-Jozsa algorithm and how to implement it using the DQC1 model and also expanded this idea to the DQCp model. Although the qubits in the initial and final states of the computation are classically correlated in DQC1 model, they can be quantum correlated at some point in the realization of the algorithm. We were able to decompose the Deutsch-Jozsa unitary and detected quantum correlations in intermediate steps of the computation, in despite
of such correlations the quantum solution shows no advantage over the classical one as the answer cannot be extracted from the system state in a more efficient way. Indeed, the performance of the classical algorithm is an upper bound to the quantum algorithm implemented via DQC1 model, so that the later falls on the former in the limit $2^{n-1} \gg k$, where $n$ is number of qubits and $k$ is the number of measurements. We have analyzed the efficiency and the presence of quantum correlations in the experimental realization of the Deutsch-Jozsa algorithm in the DQC1 model. The states of the system which contains quantum discord in a particular splitting were identified, considering the successive application of terms of the effective Hamiltonian. As the DQC1 computation on the ensemble can be seen as equivalent to multiple simultaneous computations we conclude that the computation realized in the experiment is not more efficient than the classical solution. In the DQC1 model original paper [12], the authors claim that such a model is inefficient to solve oracle problems provided that it needs to perform an exponential number of measurements to distinguish between two unitaries as the outcomes differ by a value much lower than unity. Our example contradicts this statement and shows that for the Deutsch-Josza algorithm the DQC1 and DQCp models present the same complexity.

Acknowledgments

The authors acknowledge the financial support from Brazilian agencies CAPES, FAPEMIG, CNPq, and Brazilian National Institute of Science and Technology for Quantum Information (INCT-IQ).

[1] R. Jozsa and N. Linden, Proc. R. Soc. A 459, 2011 (2003).
[2] For a recent result on the theme see M. V. den Nest, Phys. Rev. Lett. 110, 060504 (2013) and references therein.
[3] A. Datta, S. T. Flammia, and C. M. Caves, Phys. Rev. A 72, 042316 (2005).
[4] A. Datta, A. Shaji, and C. M. Caves, Phys. Rev. Lett. 100, 050502 (2008).
[5] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001).
[6] L. Henderson and V. Vedral, J. Phys. A: Math. Gen. 34, 6899 (2001).
[7] P. W. Shor, SIAM J. Comput. 26, 1484 (1997).
[8] L. K. Grover, Phys. Rev. Lett. 79, 325 (1997).
[9] D. Deutsch and R. Jozsa, Proc. R. Soc. London A 439, 553 (1992).
[10] A. F. Fahmy, R. Marx, W. Bermel, and S. J. Glaser, Phys. Rev. A 78, 022317 (2008).
[11] D. Collins, K. W. Kim, and W. C. Holton, Phys. Rev. A 58, R1633 (1998).
[12] E. Knill and R. Laflamme, Phys. Rev. Lett. 81, 5672 (1998).
[13] C. A. Ryan, J. Emerson, D. Poulin, C. Negrevergne, and R. Laflamme, Phys. Rev. Lett. 95, 250502 (2005).
[14] G. Passante, O. Moussa, C. A. Ryan, and R. Laflamme, Phys. Rev. Lett. 103, 250501 (2009).
[15] B. P. Lanyon, M. Barbieri, M. P. Almedia, and A. G. White, Phys. Rev. Lett. 101, 200501 (2008).
[16] John M. Myers, A. F. Fahmy, S. J. Glaser, and R. Marx, Phys. Rev. A 63, 032302 (2001).
[17] J. Preskill, Lecture Notes on Quantum Computation, Physics 219 (Available at [http://www.theory.caltech.edu/people/preskill/ph229/notes/chap6.pdf].
[18] Arvind and D. Collins, Phys. Rev. A 68, 052301 (2003).
[19] M. Gu, H. M. Chrzanowski, S. M. Assad, T. Symul, K. Modi, T. C. Ralph, V. Vedral, and P. K. Lam, Nat. Phys. 8, 671 (2012).
[20] A. Datta, Studies on the role of entanglement in mixed-state quantum computation, Ph.D. thesis, University of New Mexico, [arXiv:0807.4490] (2008).
[21] M. Piani, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. 100, 090502 (2008).
[22] C. C. Rulli and M. S. Sarandy, Phys. Rev. A 84, 042109 (2011).
[23] A. Datta and A. Shaji, Int. J. Quantum Inf. 9, 1878 (2011).
[24] S. S. Bullock and I. L. Markov, Quantum Inf. Comp. 4, 27 (2004).
[25] G. Vidal and R. F. Werner, Phys. Rev. A 65, 032314 (2002).
[26] R. Chaves and F. de Melo, Phys. Rev. A 84, 022324 (2011).
[27] Arvind, K. Dorai, and A. Kumar, Pram. J. Phys. 56, L705 (2001).
[28] S. L. Braunstein and A. K. Pati, Quantum Inf. Comp. 2, 3998 (2002).
[29] E. Biham, G. Brassard, D. Kenigsberg, and T. Mor, Theo. Comp. Sci. 320, 15 (2004).
[30] D. Kenigsberg, T. Mor, and G. Ratasby, Quantum Inf. Comp. 6, 606 (2006).