Bottomonium production associated with a photon at a high luminosity $e^+e^-$ collider with next-to-leading order QCD corrections

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We make a detailed discussion on the bottomonium production associated with a photon, i.e. via the channel $e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow |H_{bb}\rangle + \gamma$, up to next-to-leading order (NLO) QCD corrections, where $|H_{bb}\rangle$ stands for the color-singlet bottomonium state such as $\eta_b$, $\Upsilon$, $h_b$ or $\chi_{bJ}$ ($J=0$, 1 or 2), respectively. At the super $Z$ factory with the collision energy $E_{cm} \sim m_Z$, by summing up the cross sections for all bottomonium states, we obtain a large NLO correction, i.e. $|R| \sim 30\%$. This ensures the necessity and importance of the NLO corrections for the present process. Further more, for the $\eta_b$, $h_b$ and $\chi_{bJ}$ production, their cross sections are dominated by the $s$-channel diagrams and are enhanced by the $Z^0$ boson resonance effect when $E_{cm} \sim m_Z$. While, for the $\Upsilon$ production, such resonance effect shall be smeared by a large $t(u)$-channel contribution that dominant over the $s$-channel one. Theoretical uncertainties caused by the slight change of collision energy $E_{cm}$, the $b$-quark mass, the renormalization scale and etc. have been presented. At the super $Z$ factory with a high luminosity up to $\mathcal{L} = 10^{36}\text{cm}^{-2}\text{s}^{-1}$, the bottomonium plus one photon events are sizable, especially for $\eta_b$ and $\Upsilon$, which have large signal significance. Summing up all bottomonium states, we shall totally have $\sim 3.8 \times 10^9$ bottomonium events in one operation year. So, the super $Z$ factory shall provide a good platform for studying the bottomonium properties.

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I. INTRODUCTION

Within the framework of the non-relativistic QCD factorization theory (NRQCD), the heavy quarkonium production or decay can be factorized into a sum of products of short-distance coefficients and the non-perturbative but universal long-distance matrix elements [1]. The short-distance coefficients are perturbatively calculable, which can be expanded in a combined power series of the strong coupling constant ($\alpha_s$) and the relative velocity of the constituent quarks in the quarkonium rest frame ($v$). The top quark is too heavy to form a steady bound state before it decays, thus, the bottomonium is the heaviest and most compact bound state of quark-antiquark system within the standard model. Both its typical coupling constant and relative velocity are smaller than those of charmonium. In general, the perturbative results for bottomonium will be more convergent over the expansion of $\alpha_s$ and $v^2$ than the charmonium cases. Thus, if an experimental platform enough bottomonium events could be generated, it shall provide a relatively more confident test of NRQCD.

At the $B$ factory, the bottomonium production via the channel $e^+e^- \rightarrow \gamma^* \rightarrow |H_{bb}\rangle + \gamma$ has been studied up to next-to-leading order (NLO) accuracy [2,3], where $|H_{bb}\rangle$ equals to $\eta_b$ or $\chi_{bJ}$ ($J=1,2,3$) that has the charge-conjugation parity $C = +1$. Because the bottomonium mass is close to the threshold of the $B$ factories as Belle and BABAR, the emitted photons could be soft and even non-perturbative. Thus, the pQCD estimations on the bottomonium production at the $B$ factories is questionable. Some extra treatments have to be introduced to make a reliable pQCD estimation. For example, in Ref. [3], an artificial phase-space factor \(\left(1 - \frac{M^2_{H_{bb}}}{s}\right)^2\) has been suggested to suppress the singular contributions from the end-point region. On the other hand, the super $Z$ factory, which runs at a much higher collision energy and with a high luminosity up to $10^{34-36}\text{cm}^{-2}\text{s}^{-1}$ [5], provides a more reliable platform for studying the bottomonium properties. The bottomonium production via semi-exclusive channels as $e^+e^- \rightarrow |H_{bb}\rangle + X$ with $X = b + \bar{b}$ or $g + g$, has shown one of such examples [6]. Moreover, in Refs. [7, 8], the authors concluded that a large amount of charmonium events can be produced via the charmonium plus photon channel at the super $Z$ factory. It is natural to assume that we shall also observe enough bottomonium events at this platform. One can treat the bottomonium in a similar way as that of charmonium, however, it has its own properties and specific points, deserving a separate discussion.

For the purpose, we shall calculate the production channel $e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow |H_{bb}\rangle + \gamma$ up to NLO level. Different to previous treatment [2,4], where only the $s$-channel diagrams have been calculated, we shall discuss both the $s$-channel and $t(u)$-channel diagrams, including their cross terms, so as to achieve a sound estimation. We shall show that the $t(u)$-channel diagrams (or roughly, the initial state radiation diagrams [4]) can also provide sizable contributions. Due to angular momentum conservation and Bose statistics (known as the Landau-Pomeranchuk-Yang theorem [10]), those channels via $\gamma^*$ propagator are forbidden, i.e. the $s$-channels of $e^+e^- \rightarrow \gamma^* \rightarrow \Upsilon (h_b) + \gamma$ and the $t(u)$-channels of...
e^+ e^- \rightarrow \gamma^* \rightarrow \eta_b (h_b, \chi_{bJ}) + \gamma. Moreover, we shall not take the color-octet quarkonium states into consideration. Since due to the color suppression for the hard-scattering amplitude and also the suppression from the nonperturbative color-octet matrix element, the color-octet charmonium states shall give negligible contributions to those production processes.

The remaining parts of the paper are organized as follows. In Sec.II, we present the calculation technology for dealing with the bottomonium plus one photon production at the LO and NLO levels, respectively. In Sec.III, we present our numerical results. Sec.IV is reserved for a summary.

II. CALCULATION TECHNOLOGY

The cross section for $e^+ e^- \rightarrow H_{bb}(p_3) + \gamma(p_4)$ can be factorized as:

$$d\sigma = \sum_n d\hat{\sigma}(e^+ e^- \rightarrow (b\bar{b})[n] + \gamma)\langle O^H(n)\rangle, \tag{1}$$

where the matrix element $\langle O^H(n)\rangle$ stands for the probability of the perturbative state $(b\bar{b})[n]$ into the bound state $|H_{bb}\rangle$ with the same quantum number $|n\rangle$. The matrix elements are non-perturbative and universal, the color-singlet ones can be directly related to the wavefunction at the origin for the S-wave states or the first derivative of the wavefunction at the origin for the P-wave states; while the color-octet ones can be determined by comparing with the data.

The differential cross section $d\hat{\sigma}(e^+ e^- \rightarrow (b\bar{b})[n] + \gamma)$ stands for the $2 \rightarrow 2$ short-distance cross section, i.e.

$$d\hat{\sigma}(e^+ e^- \rightarrow (b\bar{b})[n] + \gamma) = \frac{1}{4\sqrt{(p_1 \cdot p_2)^2 - m_{\epsilon e}^2 m_{\epsilon e}^2}} |\mathcal{M}|^2 d\Phi_2, \tag{2}$$

where $d\Phi_2$ refers to the two-body phase space and the symbol $\sum$ means we need to average over the initial degrees of freedom and sum over the final ones.

A. Leading-order calculation

As shown in Fig.1, at the LO level, there are four Feynman diagrams for $e^+ e^- \rightarrow H_{bb} + \gamma$, where $|H_{bb}\rangle$ denotes for $\eta_b$, $\Upsilon$, $h_b$ and $\chi_{bJ}$ $(J=0, 1, 2)$, respectively. Two are $s$-channel diagrams and two are $t(\bar{u})$-channel ones. For those diagrams, the propagator in between can be either a virtual photon or a $Z^0$ boson. Since the behaviors of the hard scattering matrix elements for the $s$-channel and $t(\bar{u})$-channel are quite different, we shall treat them separately.

Firstly, the hard scattering matrix elements for the $s$-channel diagrams can be written as

$$iM = C L_{rr}^\mu D_{\mu\nu} \sum_{j=1}^{j_{\text{max}}} A_j^\nu, \tag{3}$$

where $C$ is an overall factor and the leptonic current is

$$L_{rr}^\mu = \bar{v}_r(p_2) \Gamma^\mu u_r(p_1), \tag{4}$$

with the indices $r$ and $r'$ standing for the spin projections of the initial electron and positron. $j_{\text{max}}$ equals to 2 for the tree-level, and it equals to 8 for the one-loop QCD correction. For the production via the $Z^0$ propagator, we have $\Gamma^\mu = \gamma^\mu (1 - 4 \sin^2 \theta_w^Z - \gamma^5)$ and $D_{\mu\nu} = -i (g_{\mu\nu} - k_{\mu} k_{\nu}/k^2) / (k^2 - m_Z^2 + i m_Z v Z), \tag{5}$ where $\Gamma_z$ stands for the total decay width of $Z^0$ boson. While, for the production via the virtual photon propagator, we have $\Gamma^\mu = \gamma^\mu$ and $D_{\mu\nu} = -i g_{\mu\nu}/k^2$.

For specific bottomonium production, the expressions for the $s$-channel $A_j^\nu$ are

\begin{align}
A_1^{(S=0,L=0)}(\eta_b) &= i \text{ Tr} \left[ \Pi^{\nu}_{p_3}(-p_3^{\nu} - p_4^{\nu} + m_b \gamma(p_3^{\nu} + p_4^{\nu}) \right]_{q=0}, \\
A_2^{(S=0,L=0)}(\eta_b) &= i \text{ Tr} \left[ \Pi^{\nu}_{p_3}(-p_3^{\nu} - p_4^{\nu} + m_b \gamma(p_3^{\nu} + p_4^{\nu}) \right]_{q=0}, \\
A_1^{(S=1,L=0)}(\Upsilon) &= i \varepsilon_{s,\beta}(p_3) \text{ Tr} \left[ \Pi^{\nu}_{p_3}(-p_3^{\nu} - p_4^{\nu} + m_b \gamma(p_3^{\nu} + p_4^{\nu}) \right]_{q=0}, \\
A_2^{(S=1,L=0)}(\Upsilon) &= i \varepsilon_{s,\beta}(p_3) \text{ Tr} \left[ \Pi^{\nu}_{p_3}(-p_3^{\nu} - p_4^{\nu} + m_b \gamma(p_3^{\nu} + p_4^{\nu}) \right]_{q=0},
\end{align}

and
FIG. 1: Tree-level Feynman diagrams for $e^+e^- \to \gamma^*/Z^0 \to H_{bb}$ + $\gamma$, where $|H_{bb}|$ denotes for $\eta_b$, $\Upsilon$, $h_b$ and $\chi_{bJ}$ ($J=0,1,2$), respectively. The left two diagrams are $t(u)$-channel, and the leaving ones are $s$-channel.

The interaction vertex

$$\Gamma^\nu_{bb} = \gamma^\nu (\xi_1 P_L + \xi_2 P_R),$$

where $P_L = (1 - \gamma^5)/2$ and $P_R = (1 + \gamma^5)/2$. Here $\xi_1 = 2 - \frac{4}{3} \sin^2 \theta_w$ and $\xi_2 = -\frac{4}{3} \sin^2 \theta_w$ for $(Zbb)$-vertex, $\xi_1 = 1 \text{ and } \xi_2 = 1 \text{ for } (\gamma^*bb)$-vertex. The momenta of the constituent quarks are

$$p_{31} = \frac{m_b}{M_{bb}} p_3 + q \text{ and } p_{32} = \frac{m_b}{M_{bb}} p_3 - q,$$

where the bottomonium mass $M_{bb} = m_b + m_b$, $q$ is the relative momentum between the two constituent quarks inside the bottomonium.

The covariant form of the projectors are [12 13],

$$\Pi^0_{\beta_3}(q) = \frac{1}{\sqrt{8m_b^2}} \left( \frac{p_{\beta 3}}{2} - q - m_b \right) \gamma^5 \left( \frac{p_{\beta 3}}{2} + q + m_b \right),$$

and

$$\Pi^\beta_{\beta_3}(q) = \frac{1}{\sqrt{8m_b^2}} \left( \frac{p_{\beta 3}}{2} - q - m_b \right) \gamma^\beta \left( \frac{p_{\beta 3}}{2} + q + m_b \right).$$

The sum over the polarization for a spin-triplet $S$-wave state or a spin-singlet $P$-wave state is given by,

$$\Pi_{\alpha\beta} = \sum_{J_z} \varepsilon_\alpha \varepsilon_\beta^* = -g_{\alpha\beta} + \frac{p_{3\alpha} p_{3\beta}}{p_3^2},$$

where $\varepsilon$ stands for the polarization vector $\varepsilon_1$ or $\varepsilon_s$ respectively. The sum over the polarization for the spin-triplet $P$-wave states ($^3P_J$ with $J = 0, 1, 2$) is given by [11 14],

$$\varepsilon_{(0)\alpha\beta}^\gamma \varepsilon_{(0)*\alpha'\beta'} = \frac{1}{3} \Pi_{\alpha\beta} \Pi_{\alpha'\beta'},$$

and

$$\sum_{J_z} \varepsilon_{\alpha\beta}(1) \varepsilon_{\alpha'(1)*} = \frac{1}{2} (\Pi_{\alpha\alpha'} \Pi_{\beta\beta'} - \Pi_{\alpha\beta} \Pi_{\alpha'\beta'}),$$

$$\sum_{J_z} \varepsilon_{\alpha\beta}(2) \varepsilon_{\alpha'(2)*} = \frac{1}{2} (\Pi_{\alpha\alpha'} \Pi_{\beta\beta'} + \Pi_{\alpha\beta} \Pi_{\alpha'\beta'}) - \frac{1}{3} \Pi_{\alpha\beta} \Pi_{\alpha'\beta'}. $$

Secondly, the hard scattering matrix elements for the $t(u)$-channel diagrams can be written as

$$iM = C \left[ L_{\mu}(1) + L_{\mu}(2) \right] D_{\mu\nu} A^\nu,$$

where the leptonic currents are,

$$L_{\mu}(1) = \bar{\nu}_r(p_2) \gamma^\mu \left( \frac{p_\beta - p_\alpha + m_e}{(p_4 - p_2)^2 - m_e^2} \right) u_r(p_1)$$

and

$$L_{\mu}(2) = \bar{\nu}_r(p_2) \gamma^\mu \left( \frac{p_\beta - p_\alpha + m_e}{(p_4 - p_2)^2 - m_e^2} \right) u_r(p_1).$$

The reduced amplitude $A^\nu$ for each bottomonium states takes the form

$$A^\nu(S=0,L=0)(\eta_b) = i \text{ Tr } [\Pi_{\beta_3}(q) \Gamma^\nu_{bb}]_{q=0},$$

$$A^\nu(S=1,L=0)(\Upsilon) = i \varepsilon_{\alpha_s\beta}(p_3) \text{ Tr } [\Pi_{\beta_3}(q) \Gamma^\nu_{bb}]_{q=0},$$

$$A^\nu(S=0,L=1)(h_b) = i \varepsilon_{\alpha_1}(p_3) \frac{d}{dq_\alpha} \text{ Tr } [\Pi_{\beta_3}(q) \Gamma^\nu_{bb}]_{q=0},$$

$$A^\nu(S=1,L=1)(\chi_{bJ}) = i \varepsilon_{\alpha_J}(p_3) \frac{d}{dq_\alpha} \text{ Tr } [\Pi_{\beta_3}(q) \Gamma^\nu_{bb}]_{q=0}.$$
we shall meet the virtual corrections. amplitudes and calculation technologies adopted here have been described in detail in Ref. \[8\]. For self-consistency, we present the main points here. The interesting reader may turn to Ref. \[8\] for detailed technology. More explicitly, we adopt FeynArts \[21, 22\] to generate Feynman diagrams and the analytical amplitudes up to NLO level. Then the package FeynCalc \[23\] is applied to simplify the trace of the \(\gamma\)-matrixes for the close fermion quark-line loop such that the hard scattering amplitudes are transformed into expressions with the tensor integrals over the loop momentum \(t\). By means of the Mathematica function TIDL \[23\], those tensor integrals can be further reduced to expressions of the scalar products of all independent momenta, such as \(l \cdot p_3, l \cdot p_4\) and \(l^2\). Finally, with the help of the Mathematica package \$\text{Apart} \[24, 25\] together with the Feynman integral reduction algorithm FIRE \[26, 27\], those complicated scalar integrals can be reduced to the simple/conventional irreducible master integrals \[28\]. For convenience, we put the necessary analytical expressions of all the master integrals in Appendix A.

### B. Next-to-leading order calculation

We proceed to calculate the one-loop QCD corrections for the process \(e^+e^- \rightarrow |H_{bk}\rangle + \gamma\). It is noted that the \(t(u)\)-channel diagrams also have sizable contributions, especially at the super Z factory. Thus, it is necessary to take both the \(s\)- and \(t(u)\)-channels into consideration so as to achieve a sound estimation.

Due to vanishing of the color factor, there is no real corrections for \(e^+e^- \rightarrow |H_{bk}\rangle + \gamma\) at the NLO level, and the cross section can be schematically written as:

\[
\frac{d\sigma}{d t} = \frac{d\sigma_{\text{Born}}}{d t} + d\sigma_{\text{virtual}} + O(\alpha_s^2)
\]

with

\[
\frac{d\sigma_{\text{Born}}}{d t} = \frac{1}{2 \pi} \sum |M_{\text{Born}}|^2 d\Phi_2
\]

and

\[
\frac{d\sigma_{\text{virtual}}}{d t} = \frac{1}{8} \sum |M_{\text{Born}}| M_{\text{virtual}}^* d\Phi_2,
\]

where \(s = (p_1 + p_2)^2\). \(M_{\text{Born}}\) refers to the tree-level amplitudes and \(M_{\text{virtual}}\) stands for the amplitudes of the virtual corrections.

The one-loop Feynman diagrams for \(e^+e^- \rightarrow |H_{bk}\rangle + \gamma\) are shown in Figs. (2,3). At the one-loop level, there are eight virtual diagrams for the \(s\)-channel and two more virtual diagrams for the \(t(u)\)-channel. Among these diagrams, the self-energy diagrams (N5, N6) contain ultraviolet (UV) divergence, the vertex diagrams (N1-N4 and the two \(t(u)\)-channel diagrams) contain both UV and IR divergences, while the box diagrams (N7, N8) have infrared (IR) divergences. Moreover, in the box diagrams (N7, N8) and the two \(t(u)\)-channel diagrams, the gluon exchange between the constituent quark and antiquark of bottomonium leads to Coulomb singularities, which can be absorbed into the redefinition of the bottomonium matrix elements. In our calculation, the usual dimensional renormalization scheme is adopted (\(D = 4 - 2\epsilon\)), which can be further performed in on-mass-shell (OS) scheme such that the external quark lines do not receive any QCD corrections. As for the channels via the \(Z^0\) boson, we shall meet the \(\gamma^*\) problem, which can be treated following the idea of Refs. \[15-20\]. The subtle points for the treatment of \(\gamma^*\) have been listed in Ref. \[8\]. After canceling all those UV and IR divergences and absorbing the Coulomb singularities, we can get the finite results for the mentioned production processes.

All calculations are done automatically via proper using of Fortran or Mathematica packages. The loop calculation technologies adopted here have been described in detail in Ref. \[8\]. For self-consistency, we present the main points here. The interesting reader may turn to Ref. \[8\] for detailed technology. More explicitly, we adopt FeynArts \[21, 22\] to generate Feynman diagrams and the analytical amplitudes up to NLO level. Then the package FeynCalc \[23\] is applied to simplify the trace of the \(\gamma\)-matrixes for the close fermion quark-line loop such that the hard scattering amplitudes are transformed into expressions with the tensor integrals over the loop momentum \(t\). By means of the Mathematica function TIDL \[23\], those tensor integrals can be further reduced to expressions of the scalar products of all independent momenta, such as \(l \cdot p_3, l \cdot p_4\) and \(l^2\). Finally, with the help of the Mathematica package \$\text{Apart} \[24, 25\] together with the Feynman integral reduction algorithm FIRE \[26, 27\], those complicated scalar integrals can be reduced to the simple/conventional irreducible master integrals \[28\]. For convenience, we put the necessary analytical expressions of all the master integrals in Appendix A.

## III. NUMERICAL RESULTS

In doing numerical calculation, the \(b\)-quark mass is taken as, \(m_b = 4.90_{-0.10}^{+0.10}\) GeV. This choice of \(m_b\) is consistent with the suggestion of the so-called 1S-mass \[29\] and the potential-model estimations \[30, 31\]. The bottomonium wavefunctions at the origin (for \(S\)-wave states) and their first derivatives (for \(P\)-wave states) from the potential model calculations are \(|R_{1S}(0)|^2 = 6.477\) GeV\(^3\) and \(|R_{1P}(0)|^2 = 1.417\) GeV\(^3\) \[30\], respectively. As for the renormalization scale \(\mu_R\), we choose \(2m_b\) to be its central value. We adopt the two-loop strong coupling constant to do our calculation, i.e.

\[
\alpha_s(\mu_R) = \frac{4\pi}{\beta_0 L} - \frac{4\pi \beta_1 \ln L}{\beta_0^2 L^2},
\]

where \(L = \ln(\mu_R^2/\Lambda_{\text{QCD}}^2)\), \(\beta_0 = 11 - \frac{2}{3}n_f\), and \(\beta_1 = \frac{2}{3}(153 - 19n_f)\) with the active flavor number \(n_f = 5\) and \(\Lambda_{\text{QCD}}^2 = 0.231\). Thus, we have \(\alpha_s(2m_b) = 0.18\). The fine-structure constant \(\alpha = 1/137\). Other input parameters are taken from the Particle Data Group \[32\]: \(m_c = 0.50 \times 10^{-3}\) GeV, \(\Gamma_Z = 2.4952\) GeV, \(m_Z = 91.1876\) GeV and\(\sin^2 \theta_w = 0.2312\).

As a cross check of our calculation, when taking the same input parameters as those of Ref. \[3\], we obtain the same LO and NLO cross sections at the \(B\) factory, in which the channel \(e^+e^- \rightarrow \gamma^* \rightarrow |H_{bk}\rangle + \gamma\) provides dominant contribution and the channel \(e^+e^- \rightarrow Z^0 \rightarrow |H_{bk}\rangle + \gamma\) is negligible.

### A. Total and differential cross sections for the bottomonium production at the super Z factory

We put the total cross sections for the bottomonium production at the LO and NLO levels via \(e^+e^- \rightarrow \)
By adding all 1

FIG. 3: One-loop corrections to the \(t(u)\)-channel diagrams of \(e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow |H_{bb} \rangle + \gamma\), where \(|H_{bb} \rangle\) denotes \(\eta_b, \Upsilon, h_b\) and \(\chi_{bJ}\) \((J = 0, 1, 2)\), respectively.

\[
\begin{array}{cccccc}
\eta_b & \Upsilon & h_b & \chi_{b0} & \chi_{b1} & \chi_{b2} \\
\hline
\sigma_{LO} \text{ (fb)} & 1.27 & 51.41 & 7.34 \times 10^{-2} & 1.10 \times 10^{-2} & 7.19 \times 10^{-2} & 2.45 \times 10^{-2} \\
\sigma_{NLO} \text{ (fb)} & 1.25 & 36.51 & 4.14 \times 10^{-2} & 1.15 \times 10^{-2} & 6.83 \times 10^{-2} & 1.17 \times 10^{-2} \\
\end{array}
\]

TABLE I: Total cross sections for the bottomonium production via \(e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow |H_{bb} \rangle + \gamma\) up to NLO level at the super \(Z\) factory. \(\sqrt{s} = m_Z, m_b = 4.9\) GeV and \(\mu_R = 2m_b\).

\(\gamma^*/Z^0 \rightarrow |H_{bb} \rangle + \gamma\) at the super \(Z\) factory in Table II where the channels via a virtual photon or \(Z^0\), including their cross-terms, have been taken into consideration simultaneously. We have set \(\mu_R = 2m_b\) and \(\sqrt{s} = m_Z\). By adding all 1S-wave bottomonium states together, we obtain

\[
\sigma_{LO} \left( e^+e^- \rightarrow |H_{bb} \rangle (1S) + \gamma \right) = 52.68 \text{ fb} \quad (30)
\]

\[
\sigma_{NLO} \left( e^+e^- \rightarrow |H_{bb} \rangle (1S) + \gamma \right) = 37.76 \text{ fb} \quad (31)
\]

And by adding all 1P-wave bottomonium states together, we obtain

\[
\sigma_{LO} \left( e^+e^- \rightarrow |H_{bb} \rangle (1P) + \gamma \right) = 2.01 \text{ fb} \quad (32)
\]

\[
\sigma_{NLO} \left( e^+e^- \rightarrow |H_{bb} \rangle (1P) + \gamma \right) = 1.33 \text{ fb} \quad (33)
\]

It is noted that the bottomonium cross sections are dominated by \(\Upsilon\) production, which is due to the \(t(u)\)-channel diagrams and has already been pointed out by Ref. [9] at the LO level. By adding all the mentioned color-singlet bottomonium states together, we observe that the NLO QCD correction for all bottomonium states is large and negative, \(R \approx -30\%\), where the ratio \(R\) is defined as \((\sigma_{NLO} - \sigma_{LO})/\sigma_{LO}\). This indicates the necessity and importance of the NLO corrections. More specifically, for the cases of \(\Upsilon, h_b\) and \(\chi_{b2}\) production, the NLO corrections are moderate, \(R_{\eta_b} = -1.6\%, R_{\chi_{b0}} = 4.5\%\) and \(R_{\chi_{b1}} = -5.0\%\). As for the usually analyzed s-channel only, for the bottomonium production via the channel \(e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow |H_{bb} \rangle + \gamma\), it is found that only \(\eta_b\) and \(\chi_{bJ}\) can provide non-zero contributions at the super \(Z\) factory. And even those non-zero cross sections are very small, i.e.

\[
\sigma_{NLO}^{\gamma^*/Z^0} \left( \eta_b + \gamma \right) = 1.74 \times 10^{-3} \text{ fb},
\]

\[
\sigma_{NLO}^{\gamma^*/Z^0} \left( \chi_{b0} + \gamma \right) = 1.61 \times 10^{-5} \text{ fb},
\]

\[
\sigma_{NLO}^{\gamma^*/Z^0} \left( \chi_{b1} + \gamma \right) = 9.54 \times 10^{-5} \text{ fb},
\]
where the superscripts s and γ* indicate the cross section is for the process via the s-channel with virtual photon. Comparing with the total cross sections listed in Table II, one can conclude that the total cross sections via $e^+e^- \rightarrow Z^0 \rightarrow |H_{bb}| + \gamma$ are dominant over those via $e^+e^- \rightarrow \gamma^* \rightarrow |H_{bb}| + \gamma$ by about three orders. This is caused by the $Z^0$ boson resonance effect, as is one of the advantage of the super $Z$ factory.

| s-channel | t(u)-channel | Total |
|-----------|-------------|-------|
| $\sigma_{LO}$ (fb) | 2.69 | 49.35 | 51.41 |
| $\sigma_{NLO}$ (fb) | 2.79 | 34.27 | 36.51 |

TABLE II: Total cross sections for the $\Upsilon$ production via $e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow \Upsilon + \gamma$ up to NLO level, $\sqrt{s} = m_Z$, $m_b=4.9$ GeV and $\mu_R = 2m_b$. ‘Total’ refers to the sum of s- and t(u)-channels including their cross terms.

Moreover, it is found that for the t(u)-channels via $\gamma^*$ propagator, the conditions |cos $\theta$| ≤ 1 can not be detected, since under such condition, the produced bottomonium moves very close to the beam direction. Here $\theta$ stands for the angle between the three-vector momentums of the bottomonium and the electron. Considering the detector’s abilities and in order to offer experimental references, we try various cuts on |cos $\theta$|. The results are listed in Table III. As for $\eta_b$ and P-wave bottomonium production, the changes of the cross section over the |cos $\theta$| cut are moderate: setting |cos $\theta$| ≤ 0.9, the cross sections are changed by ∼ 14%; setting |cos $\theta$| ≤ 0.8, the cross sections are further changed by ∼ 15%. While for $\Upsilon$ production, when setting |cos $\theta$| ≤ 0.9, the cross section is changed by 86%; when setting |cos $\theta$| ≤ 0.8, the cross section is further changed by 25%. Such great differences can be explained by comparing with their differential cross sections, which are presented in Fig. 5.

At the super $Z$ factory, with the high luminosity $L = 10^{36}$ cm$^{-2}$s$^{-1}$ (or $\sim 10^8$ fb$^{-1}$ in one operation year), large amount of bottomonium can be generated. Thus, the super $Z$ factory could be a useful platform for studying bottomonium properties, especially for $\eta_b$ and $\Upsilon$. More explicitly, without applying any kinematical cut, we ob-

\[
\sigma_{NLO}^{s,\gamma^*}(\chi_{bb} + \gamma) = 1.63 \times 10^{-5} \text{ fb},
\]

\[
\sigma_{LO}^{s,\gamma^*}(\chi_{bb} + \gamma) = 1.30 \times 10^{-3} \text{ fb},
\]

\[
\sigma_{NLO}^{t,\gamma^*}(\chi_{bb} + \gamma) = 5.27 \times 10^{-5} \text{ fb}.
\]

For the t(u)-channels via $\gamma^*$ propagator, the conditions are similar. However, there is an important exception, it is noted that the t(u)-channel diagrams for $e^+e^- \rightarrow \gamma^* \rightarrow \Upsilon + \gamma$ provide quite large contribution. To show this point more clearly, we show the cross sections for s- and t(u)-channels of $e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow \Upsilon + \gamma$ in Table II. It indicates that the t(u)-channel are dominant for the $\Upsilon$ production, i.e. it increases the s-channel cross section by about eighteen times for LO level and about twelve times for NLO level. The cross-terms between s- and t(u)-channels are sizable, whose magnitudes are about 20% of s-channel cross sections. The ‘Total’ cross sections in Table II are the sum of s- and t(u)-channels including their cross terms.

To show how the total cross sections change with the $e^+e^-$ collision energy $E_{cm}(=\sqrt{s})$, we present the total cross sections versus $E_{cm}$ up to NLO level in Fig. 4. As for $\eta_b$ and P-wave bottomonium states production, in most of the $e^+e^-$ collision energies, they are dominated by the s-channel diagrams of $e^+e^- \rightarrow Z^0 \rightarrow |H_{bb}| + \gamma$, and the $Z^0$ boson resonance effect are significant at $E_{cm} = m_Z$. While for the $\Upsilon$ production, its $Z^0$ boson resonance effect is smeared by the large contributions from the t(u)-channel diagrams of $e^+e^- \rightarrow \gamma^* \rightarrow \Upsilon + \gamma$.

Experimentally, no detector can cover all the kinematics of the events, thus, only some of them can be observed. For example, the bottomonium events with |cos $\theta$| → 1 can not be detected, since under such condition, the produced bottomonium moves very close to the beam direction. Here $\theta$ stands for the angle between the three-vector momentums of the bottomonium and the electron. Considering the detector’s abilities and in order to offer experimental references, we try various cuts on |cos $\theta$|. The results are listed in Table III. As for $\eta_b$ and P-wave bottomonium production, the changes of the cross section over the |cos $\theta$| cut are moderate: setting |cos $\theta$| ≤ 0.9, the cross sections are changed by ∼ 14%; setting |cos $\theta$| ≤ 0.8, the cross sections are further changed by ∼ 15%. While for $\Upsilon$ production, when setting |cos $\theta$| ≤ 0.9, the cross section is changed by 86%; when setting |cos $\theta$| ≤ 0.8, the cross section is further changed by 25%. Such great differences can be explained by comparing with their differential cross sections, which are presented in Fig. 5.

At the super $Z$ factory, with the high luminosity $L = 10^{36}$ cm$^{-2}$s$^{-1}$ (or $\sim 10^8$ fb$^{-1}$ in one operation year), large amount of bottomonium can be generated. Thus, the super $Z$ factory could be a useful platform for studying bottomonium properties, especially for $\eta_b$ and $\Upsilon$. More explicitly, without applying any kinematical cut, we ob-

\[
\sigma_{NLO}^{s,\gamma^*}(\chi_{bb} + \gamma) = 1.63 \times 10^{-5} \text{ fb},
\]
FIG. 5: Differential cross sections for $e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow |H_{bs}| + \gamma$ versus $\theta$. Here, all $P$-wave bottomonium states’ cross sections have similar behaviors and have been summed up. For the $\eta_b$ production, its LO and NLO curves are almost coincide with each other. $\sqrt{s} = m_Z$, $m_b = 4.9$ GeV and $\mu_R = 2m_b$.

$$N_{\eta_b} = 1.25 \times 10^4, \quad N_T = 3.65 \times 10^5,$$
$$N_{\chi_b^0} = 4.14 \times 10^2, \quad N_{\chi_b^0} = 1.15 \times 10^2,$$
$$N_{\chi_b^1} = 6.83 \times 10^2, \quad N_{\chi_b^2} = 1.17 \times 10^2.$$

(34)

If setting $|\cos \theta| \leq 0.9$, we obtain

$$N_{\eta_b} = 1.07 \times 10^4, \quad N_T = 4.95 \times 10^4,$$
$$N_{\chi_b^0} = 3.55 \times 10^2, \quad N_{\chi_b^0} = 9.84 \times 10^1,$$
$$N_{\chi_b^1} = 5.86 \times 10^2, \quad N_{\chi_b^2} = 1.00 \times 10^2.$$

(35)

If setting $|\cos \theta| \leq 0.8$, we obtain

$$N_{\eta_b} = 9.09 \times 10^3, \quad N_T = 3.69 \times 10^4,$$
$$N_{\chi_b^0} = 3.02 \times 10^2, \quad N_{\chi_b^0} = 8.36 \times 10^1,$$
$$N_{\chi_b^1} = 4.99 \times 10^2, \quad N_{\chi_b^2} = 8.55 \times 10^1.$$

(36)

B. Theoretical uncertainties

To show the sensitivity of the total cross sections to the collision energy, we calculate them by taking $E_{cm} = m_Z \pm 0.5$ GeV. The results are presented in Table IV. Consistent with the above observations from Fig. 5, i.e. the $Z^0$ boson resonance effect is sizable for the $\eta_b$ and $P$-wave bottomonium production, their total cross sections drop down by $\sim 14\%$ from their central values when varying $E_{cm}$ by merely 0.5 GeV. While under the same energy changes, the total cross section for the $Y$ production changes only by less than 2%.

Different choice of the effective $b$ quark mass shall also lead to sizable changes to the total cross section. We put the results for $m_b = (4.9 \pm 0.1)$ GeV. $\sqrt{s} = m_Z$ and $\mu_R = 2m_b$.

| $m_b$  | 4.8 GeV | 4.9 GeV | 5.0 GeV |
|-------|---------|---------|---------|
| $\sigma_{\eta_b + \gamma}$ | 1.28 | 1.25 | 1.22 |
| $\sigma_{\tau + \gamma}$ | 38.66 | 36.51 | 34.53 |
| $\sigma_{\chi_b^0 + \gamma}$ | $3.50 \times 10^{-2}$ | $4.14 \times 10^{-2}$ | $3.91 \times 10^{-2}$ |
| $\sigma_{\chi_b^0 + \gamma}$ | $1.22 \times 10^{-2}$ | $1.15 \times 10^{-2}$ | $1.08 \times 10^{-2}$ |
| $\sigma_{\chi_b^1 + \gamma}$ | $7.27 \times 10^{-2}$ | $6.83 \times 10^{-2}$ | $6.41 \times 10^{-2}$ |
| $\sigma_{\chi_b^2 + \gamma}$ | $1.23 \times 10^{-2}$ | $1.17 \times 10^{-2}$ | $1.11 \times 10^{-2}$ |

TABLE V: Total NLO cross sections (in fb) for the bottomonium production channel $e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow |H_{bs}| + \gamma$ for $m_b = (4.9 \pm 0.1)$ GeV. $\sqrt{s} = m_Z$ and $\mu_R = 2m_b$.

| $\mu_R = \sqrt{s}/2$ |
|-------------------|
| $\sigma_{\eta_b + \gamma}$ | $2.24$ |
| $\sigma_{\tau + \gamma}$ | $33.53$ |
| $\sigma_{\chi_b^0 + \gamma}$ | $3.50 \times 10^{-2}$ | $4.14 \times 10^{-2}$ | $3.91 \times 10^{-2}$ |
| $\sigma_{\chi_b^0 + \gamma}$ | $1.16 \times 10^{-2}$ | $1.15 \times 10^{-2}$ | $1.14 \times 10^{-2}$ |
| $\sigma_{\chi_b^1 + \gamma}$ | $7.27 \times 10^{-2}$ | $6.83 \times 10^{-2}$ | $6.41 \times 10^{-2}$ |
| $\sigma_{\chi_b^2 + \gamma}$ | $1.23 \times 10^{-2}$ | $1.17 \times 10^{-2}$ | $1.11 \times 10^{-2}$ |

TABLE VI: Conventional scale uncertainties for the total cross sections (in fb) for $e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow |H_{bs}| + \gamma$ at the NLO level by adopting three typical renormalization scales. $\sqrt{s} = m_Z$ and $m_b=4.9$ GeV.

As an estimation of the physical observable, one can take a typical momentum flow of the process as the scale and vary it within certain range, or directly take several typical scales, to study the renormalization scale uncertainty. At present, we take three frequently used scales, $m_b, 2m_b$ and $\sqrt{s}/2$, to study the conventional scale uncertainty. The results are presented in Table VI. At the NLO level, the renormalization scale uncertainty can be up to 30% (for the case of $\chi_b^2$).
As has been suggested in Ref.[33], the principle of maximum conformity (PMC) provides a systematic way to eliminate the renormalization scale ambiguity. It however depends on how well we know the $\beta$-terms in pQCD series. Those $\beta$-terms rightly determine the running coupling behavior via the renormalization group equation and fix the renormalization scale to a certain degree[33]. Strictly, we need to finish a next-to-next-to-leading order (NNLO) calculation for the present considered bottomonium production process such that we can find out the $\{\beta_i\}$-terms and set the PMC scale for the NLO terms. Such a NNLO calculation is not available at the present. As a compromise, we can use an improved way suggested in Ref.[34] to reestimate the renormalization scale uncertainty, in which the one-higher order $\beta$-terms are directly determined from the renormalization group equation. A comparison with those two approaches is presented in Fig.(6). Fig.(6) shows that a more reliable scale uncertainty can indeed be achieved by using the improved scale setting approach.

| $m_e \times 10^{-3}$GeV | 0.40 | 0.45 | 0.50 | 0.55 | 0.60 |
|------------------------|------|------|------|------|------|
| $\sigma_{LO}$          | 50.30| 49.80| 49.35| 48.95| 48.58|
| $\sigma_{NLO}$         | 34.93| 34.58| 34.27| 33.99| 33.73|

TABLE VII: Total cross sections (in fb) for $\Upsilon$ production $e^+e^- \to \gamma^*/Z^0 \to \Upsilon + \gamma$ via $t(u)$-channels only. $\sqrt{s} = m_\Upsilon$, $m_b = 4.9$ GeV and $\mu_R = 2m_b$. 

As has already been observed, the $t(u)$-channel diagrams provide a dominant contribution to $\Upsilon$ production. Such large $t(u)$-channel contributions for the $\Upsilon$ production are reasonable, since there are large contributions from the kinematic region when the photons are almost collinear to the incident electron line. The numerical singularity has been cured by taking a (small) electron mass, i.e., $m_e = 0.50 \times 10^{-3}$ GeV. As a final remark on theoretical uncertainty, we discuss the sensitivity of the considered $t(u)$-channel results on $m_e$. The results for $m_e \in [0.40, 0.60] \times 10^{-3}$ GeV are presented in Table VII. A steady cross section over $m_e$, i.e., the cross sections at both LO and NLO change by less than ±2% for $m_e \in [0.40, 0.60] \times 10^{-3}$ GeV, shows that our treatment is reasonable and effective.

C. An estimation of physical background

Experimentally, one may observe the process $e^+e^- \to |H_{bb}| + \gamma$ by analyzing the energy spectrum of the radiated photons. The dominant background subprocesses are $e^+e^- \to q + \bar{q} + \gamma$[2], the Feynman diagrams are shown in Fig.(7), where $u, d, s, c$ or $b$ quark, respectively. To estimate the background, we take: $m_u = 2.3$ MeV, $m_d = 4.8$ MeV, $m_s = 95$ MeV and $m_c = 0.5 \times 10^{-3}$ GeV[32]. We restrict the energy range of the radiated photons as $45.067 \pm 0.005$ GeV, where the central value is determined by the process $e^+e^- \to |H_{bb}| + \gamma$, since its radiated photon right has a fixed energy $E_\gamma = (s - m^2_{H_{bb}})/(2\sqrt{s}) = 45.067$ GeV by taking $m_{H_{bb}} = 2m_b$ and $\sqrt{s} = m_\Upsilon$.

The total cross sections for the background process $e^+e^- \to q + \bar{q} + \gamma$ are $\sigma_{BG} \approx 130$ fb for $|\cos \theta| \leq 1$, $\sigma_{BG} \approx 59$ fb for $|\cos \theta| \leq 0.9$ and $\sigma_{BG} \approx 50$ fb for $|\cos \theta| \leq 0.8$. These background cross sections are greater than the wanted signal cross sections shown in Tables I and III. However, the high luminosity super $Z$ factory might still allow one to measure the bottomonium production associated with a photon. As a rough estimation, we can compute the signal significance $S(H)$ for the bottomonium production plus one photon, which is defined as $S(H) = N_H/\sqrt{N_{BG}}$. At the super $Z$ factory, taking the luminosity $\mathcal{L} = 10^{36}$cm$^{-2}$s$^{-1}$, then, if setting $|\cos \theta| \leq 1$, we have $S(\eta_b) = 11$, $S(\Upsilon) = 320$ and $S(1P) = 1.2$, where $S(1P)$ stands for the sum of
the mentioned $P$-wave bottomonium states; if setting $|\cos \theta| \leq 0.9$, we have $S(y_b) = 14$, $S(\Upsilon) = 64$ and $S(1P) = 1.5$, where $S(1P)$ stands for the sum of the mentioned $P$-wave bottomonium states.

**IV. SUMMARY**

At the super $Z$ factory, we can obtain a more reliable pQCD estimation for the bottomonium properties and a more confidential test of NRQCD factorization theorem than the $B$ factory. In the present paper, we have studied the bottomonium production via $e^+e^- \to \gamma^*/Z^0 \to |H_{bJ}| + \gamma$ up to NLO level at the super $Z$ factory. By adding all the mentioned color-singlet bottomonium states together, we observe that the NLO QCD correction for all bottomonium states is large and negative, $R = (\sigma_{\text{NLO}} - \sigma_{\text{LO}})/\sigma_{\text{LO}} \sim -30\%$. This indicates the necessity and importance of the NLO corrections. Especially, for the cases of $\Upsilon$, $h_b$ and $\chi_{bJ}$ production, whose NLO corrections are large, i.e. $R_{\Upsilon} = -29.0\%$, $R_{h_b} = -43.6\%$ and $R_{\chi_{bJ}} = -52.2\%$.

In low $e^+e^-$ collision energy, e.g. at the $B$ factory, the channel $e^+e^- \to \gamma^*/Z^0 \to |H_{bJ}| + \gamma$ provides dominant contribution and the channel $e^+e^- \to Z^0 \to |H_{bJ}| + \gamma$ is negligible. Because the bottomonium mass is close to the threshold of the $B$ factories as Belle and BABAR, the emitted photons could be soft and even non-perturbative. Thus, the pQCD estimations on the bottomonium production at the $B$ factories is questionable.

On the other hand, a more confidential estimation can be achieved at the super $Z$ factory. Due to $Z^0$ boson resonance effect, the process $e^+e^- \to Z^0 \to |H_{bJ}| + \gamma$ shall dominate over the process $e^+e^- \to \gamma^*/Z^0 \to |H_{bJ}| + \gamma$ for the bottomonium states such as $y_b$, $h_b$ and $\chi_{bJ}$. In fact, such a $Z^0$ boson resonance effect is very important for measuring these states, a slight change of $E_{\text{cm}}$ from $m_Z$ by merely 0.5 GeV shall reduce their total cross sections by $\sim 14\%$ from their central values. The only exception is the $\Upsilon$ production, whose $t(u)$-channel diagrams provide dominant contribution to the total cross section as shown by Table [I].

More specifically, Fig. [S] shows the relative importance of the $s$-channel and $t(u)$-channel contributions. It shows that the $t(u)$-channel can provide significant contributions to the $\Upsilon$ production at the whole kinematic region.

At the super $Z$ factory with a high luminosity up to $\mathcal{L} = 10^{36}$ cm$^{-2}$ s$^{-1}$, the bottomonium production events are sizable. By summing all bottomonium states together, in one operation year, we shall have $3.8 \times 10^5$ bottomonium events for $|\cos \theta| \leq 0.9, 6.1 \times 10^4$ bottomonium events for $|\cos \theta| \leq 0.8$ and $4.7 \times 10^4$ bottomonium events for $|\cos \theta| \leq 0.6$. Thus, the high luminosity super $Z$ factory may allow one to measure the bottomonium properties, even though one needs to deal with the background processes carefully.

As a final remark, we put a comparison of $s$- and $t(u)$-channel distributions versus $\cos \theta$ for the charmonium production in Fig. [I]. The charmonium is produced via the channel $e^+e^- \to \gamma^*/Z^0 \to J/\psi + \gamma$. E$_{\text{cm}} = m_Z, m_c = 1.5$ GeV and $\mu_R = 2m_c$.

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![Graph](image-url)

**FIG. 8:** A comparison of $s$- and $t(u)$-channel differential NLO cross sections for $\Upsilon$ production via $e^+e^- \to \gamma^*/Z^0 \to \Upsilon + \gamma$. E$_{\text{cm}} = m_Z, m_b = 4.9$ GeV and $\mu_R = 2m_b$.

**FIG. 9:** A comparison of $s$- and $t(u)$-channel differential NLO cross sections for $J/\psi$ production via $e^+e^- \to \gamma^*/Z^0 \to J/\psi + \gamma$. E$_{\text{cm}} = m_Z, m_c = 1.5$ GeV and $\mu_R = 2m_c$. 

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Appendix A: Analytical results for the one-loop integrals

For simplicity, we take $\mathcal{I}^{(n)}$ to denote the master integrals, where $n = 1, 2$ and 3 for one-point, two-point and three-point scalar integrals, respectively.

$$\mathcal{I}^{(1)} = N \int \frac{d^D l}{(l + \frac{m_a}{2})^2 - m_b^2} = N \int \frac{d^D l}{(l + \frac{m_a}{2} + p_4)^2 - m_b^2} = N_1 m_b^2 \left[ \ln \left( \frac{\mu^2}{m_b^2} \right) + \frac{1}{\epsilon} - \gamma_E + 1 \right],$$

$$\mathcal{I}^{(2)}_1 = N \int \frac{d^D l}{l^2 \left[ (l + \frac{m_a}{2})^2 - m_b^2 \right]} = N_1 \left[ \ln \left( \frac{\mu^2}{m_b^2} \right) - \frac{2r - 2 - 2r - 1}{2r - 1} \ln(2r - 2) + \frac{1}{\epsilon} - \gamma_E + 2 \right],$$

$$\mathcal{I}^{(2)}_2 = N \int \frac{d^D l}{(l^2 - m_b^2) \left[ (l - p_3 - p_4)^2 - m_b^2 \right]} = N_1 \left[ \ln \left( \frac{\mu^2}{m_b^2} \right) - \ln(ab) + \frac{1}{\epsilon} - \gamma_E + 2 \right],$$

$$\mathcal{I}^{(3)}_1 = N \int \frac{d^D l}{l^2 \left[ (l + \frac{m_a}{2})^2 - m_b^2 \right] \left[ (l - \frac{m_a}{2} - p_4)^2 - m_b^2 \right]} = \frac{N_1}{s - 4m_b^2} \left[ 2\text{Li}_2(a) + 2\text{Li}_2(b) - 2\text{Li}_2 \left( \frac{1}{2r - 1} \right) + \ln^2(a) + \ln^2(b) - \ln^2(2r - 1) \right],$$

$$\mathcal{I}^{(3)}_2 = N \int \frac{d^D l}{(l^2 - m_b^2) \left[ (l - p_3)^2 - m_b^2 \right] \left[ (l - p_3 - p_4)^2 - m_b^2 \right]} = \frac{N_1}{s - 4m_b^2} \left[ \text{Li}_2(a) + \text{Li}_2(b) + \frac{\ln^2(a)}{2} + \frac{\ln^2(b)}{2} - \frac{\pi^2}{6} \right],$$

(A1)

where

$$N = \frac{\mu^{2r}\Gamma(\epsilon)}{(4\pi)^{2-\epsilon}}, \quad N_1 = \frac{1}{(4\pi)^{2-\epsilon}}, \quad r = \frac{s}{4m_b^2}, \quad a = \frac{1}{2} \left( 1 + \sqrt{\frac{r - 1}{r}} \right), \quad b = \frac{1}{2} \left( 1 - \sqrt{\frac{r - 1}{r}} \right).$$

(A2)
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