A companion document to
"Point Estimation with Exponentially Tilted Empirical Likelihood"

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This manuscript provides the calculational details involved in the proof of the equivalence between the $O(n^2)$ variance of the ETEL and EL estimator that are omitted from the appendix of "Point Estimation with Exponentially Tilted Empirical Likelihood" by Susanne M. Schennach.

Whenever possible, the following calculations were verified using the symbolic calculations capabilities of Maple.

1 Definitions

Let

\[ g = g(x_i; \cdot) \]
\[ g_0 = \theta g(x_i; \cdot) \]

To simplify the notation, the dependence of the above quantities on \( i \) and \( \cdot \) is implicit.

\[ G = E[G(x_i; \cdot)] \]
\[ = E[g(x_i; \cdot)g(x_i; \cdot)^0] \]
\[ = \sum_{i=1}^{n} X \]
\[ G = \sum_{i=1}^{n} X \]
\[ G = \sum_{i=1}^{n} X \]

Convention for transpose: $G^0_{\cdot \cdot}$ \( (G^0_{\cdot \cdot}) \)

Expectations evaluated at \( \cdot \) will be denoted by E \( [\cdot \cdot \cdot] \). For instance, \( E[G_0] = G \) and \( E[gg^0] = \).

1.1 Moment conditions for ETEL

In ETEL, \( \hat{ETEL}_i \) solves $\sum_{i=1}^{n} \hat{ETEL}_i = 0$, where
ETEL = \frac{6}{4} + \frac{7}{5} g_0 + \frac{2}{3} G_0 + \frac{1}{2} g_0 + G_0 + G_0

and where

_ = \exp(0 g)

Define

\begin{align*}
\text{ETEL}_{i j} &= \text{ETEL}_{i j} \\
\text{ETEL}_{i j k} &= \text{ETEL}_{i j k} \\
\text{ETEL}_{i j k h} &= \text{ETEL}_{i j k h}
\end{align*}

\begin{align*}
\text{ETEL}_{i} &= n_{1=2}^i \text{ETEL}_{i} \\
\text{ETEL}_{i j} &= n_{1=2}^{i j} \text{ETEL}_{i j} \\
\text{ETEL}_{i j k} &= n_{1=2}^{i j k} \text{ETEL}_{i j k} \\
\text{ETEL}_{i j k h} &= n_{1=2}^{i j k h} \text{ETEL}_{i j k h}
\end{align*}

where the dot replacing the subscripts denote all the elements of the matrix.

1.2 Moment conditions for EL

The moment conditions for the EL parameters $^0 \text{ETEL}$ and $^0 \text{ETEL}$ are

\begin{align*}
\sum_{i=1}^{n} X_{i} \text{ETEL}_{i} &= 0 \\
\sum_{i=1}^{n} \text{ETEL}_{i} &= 0
\end{align*}

where $^0 \text{ETEL}$ is the Lagrange multiplier of the moment constraints, which has been relabelled to simplify the comparison with ETEL. Furthermore, to again simplify the comparison with ETEL, we augment this...
vector by 1 + dim additional moment conditions and introduce the same number of additional parameters \((\cdot; \cdot)\)
where \(2 R \text{ and } 2 R^{1m}\):

\[
\begin{align*}
\mathbf{n}^\top \mathbf{X}_i &= \begin{bmatrix} \mathbf{g} \end{bmatrix}_i \\
&= 0
\end{align*}
\]

where \(\cdot = \exp(\begin{bmatrix} \mathbf{g} \end{bmatrix}_i)\). The additional moment conditions merely define the values of the new parameters \(^\pmb{i}; \cdot\) and do not change the values of \({}^\pmb{i}; \cdot\). Indeed, whenever \(\cdot^\pmb{i}; \cdot\) are such that the bottom two subvectors are zero, one can always find a value of \(\cdot^\pmb{i}; \cdot\) that will make the top two subvectors vanish as well. Since the origin is in the convex hull of \(\mathbf{g}_i\), there exists \(\cdot^\pmb{i}; \cdot\) such that \(n^\pmb{i} \cdot^\pmb{i} = 0\) w.p.a. Then, we can just set \(\cdot = n^\pmb{i} \cdot^\pmb{i}\).

Finally, since just-identified GMM is invariant under linear transformations of the vector of moment conditions, the moment conditions for EL can equivalently be written as \(n^\pmb{i} \cdot^\pmb{i} \cdot = 0\), where

\[
\begin{align*}
&\mathbf{E}_{\cdot^\pmb{i} \cdot} = \begin{bmatrix} 2 & 3 \\
&-\mathbf{g} \end{bmatrix}\\
&\mathbf{E}_{\cdot^\pmb{i} \cdot^\pmb{i} \cdot^\pmb{i}} = \begin{bmatrix} 2 & 3 & 1 & 0 & \mathbf{g} \end{bmatrix}
\end{align*}
\]

This particular version of the EL moment conditions will drastically simplify our calculations, due to the fact that the matrices of first derivatives for \(\mathbf{ETE}_{\cdot^\pmb{i} \cdot^\pmb{i}}\) and \(\mathbf{EL}_{\cdot^\pmb{i} \cdot^\pmb{i}}\) become nearly identical:

\[
\begin{align*}
\mathbf{EL}_{\cdot^\pmb{i} \cdot^\pmb{i} \cdot^\pmb{i}} &= \begin{bmatrix} 2 & 3 & 1 & 0 & \mathbf{g} \end{bmatrix}\\
&\mathbf{ETE}_{\cdot^\pmb{i} \cdot^\pmb{i} \cdot^\pmb{i}} = \begin{bmatrix} 2 & 3 & 1 & 0 & \mathbf{g} \end{bmatrix}
\end{align*}
\]

Note that

\[
\begin{align*}
\mathbf{ETE}_{\cdot^\pmb{i} \cdot} &= \mathbf{EL}_{\cdot^\pmb{i} \cdot} \\
\mathbf{ETE}_{\cdot^\pmb{i} \cdot^\pmb{i} \cdot^\pmb{i}} &= \mathbf{EL}_{\cdot^\pmb{i} \cdot^\pmb{i} \cdot^\pmb{i}}
\end{align*}
\]

while \(\mathbf{ETE}_{\cdot^\pmb{i} \cdot^\pmb{i}}\) and \(\mathbf{EL}_{\cdot^\pmb{i} \cdot^\pmb{i}}\) differ by a single element.

1.3 Other definitions and conventions

\[
\begin{align*}
\mathbf{ETE}_{\cdot^\pmb{i} \cdot^\pmb{i}} &= \begin{bmatrix} \cdot \end{bmatrix}_i \\
\mathbf{ETE}_{\cdot^\pmb{j} \cdot^\pmb{i} \cdot^\pmb{j}} &= \begin{bmatrix} \cdot \end{bmatrix}_j \\
\mathbf{ETE}_{\cdot^\pmb{i} \cdot^\pmb{j} \cdot^\pmb{k} \cdot^\pmb{i} \cdot^\pmb{j} \cdot^\pmb{k}} &= \begin{bmatrix} \cdot \end{bmatrix}_k
\end{align*}
\]

and similarly for \(\mathbf{EL}\).

Again, a dot replacing a given subscript denotes a vector of all the values taken for all values of that subscript.
Following Newey and Smith (2001), define

\[ P = \mathbf{1} \mathbf{1} \mathbf{G}^0 \mathbf{1} \mathbf{G}^0 \mathbf{1} \]
\[ H = \mathbf{G}^0 \mathbf{1} \mathbf{G}^0 \mathbf{1} \]
\[ = \mathbf{G}^0 \mathbf{1} \]

Identities:

\[ \mathbb{P} \mathbb{G} = 0 \]
\[ \mathbb{P}^0 = \mathbb{P} \]
\[ \mathbb{P} \mathbb{P} = \mathbb{P} \]
\[ \mathbb{P} \mathbb{H}^0 = 0 \]
\[ \mathbb{H} \mathbb{H}^0 = \]

Define

\[ l = 0 \]
\[ l = 1 \]
\[ l = 1 + \text{dim} \]
\[ l = 1 + 2 \text{dim} \]

These symbols will be used to isolate subvectors and submatrices. For instance,

\[ \mathbb{E}_{\text{EL}} \mathbb{h}_{li + j} = \mathbb{E} \frac{\partial \mathbb{E}_{\text{EL}}}{\partial j} \]

with \( j \) implicitly ranging from 1 to \( \text{dim} \).

2 Stochastic expansion

The following merely rewrites the conclusion of Lemma A.4 in Newey and Smith (2001) using our notation.

\[ \hat{\theta}^i = n \frac{1}{2} Q + n \frac{3}{2} R + O(n^2) \]

where

\[ Q_l = \frac{1}{2} \mathbb{Q}_{lj} + \frac{1}{2} \mathbb{Q}_{lj} \]
\[ R_l = \frac{1}{2} \mathbb{R}_{lj} + \frac{1}{2} \mathbb{R}_{lj} \]

using the convention that repeated indices are summed over, e.g. \( \mathbb{P}_{lj} \).

We also simplified Newey and Smith's result using the fact that \( \mathbb{Q}_{lj} = \mathbb{Q}_{lj} \).

The quantities associated with each estimator will be distinguished by an \( \text{ETEL} \) or \( \text{EL} \) superscript.

The variance of \( \hat{\theta}^i \) is:

\[ \text{Var}\hat{\theta}^i = n \text{Var}\hat{\theta}^i + n^2 \text{Var}\mathbb{Q} + n^2 \text{Covar}\mathbb{R}^0 + \mathbb{O}(n^2) \]
3 Partitioned Inverse of $\mathbf{P}$:

\[
\begin{align*}
\begin{bmatrix}
1 & 0 & 0 & 0 & 3 & 1 & 2 & 2 & 1 & 3 & 2 & 0 & 0 & 0 & 3 \\
6 & 0 & 0 & G & 7 & = & 6 & 2 & 0 & 0 & G & 3 & 1 & 7 \\
4 & 0 & 0 & 5 & = & 4 & 4 & 0 & 5 & 4 & 0 & 5 & 5 \\
0 & G^0 & 0 & 0 & 0 & G^0 & 0 & 0 \\
2 & 0 & G & = & 0 & G & 0 & 3 & 1 \\
\end{bmatrix}
\end{align*}
\]

where $4$ can be found using partitioned inverse formula:

\[
G^5 = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}^{-1}
\]

\[
\begin{align*}
A_{11} & = \begin{bmatrix}
1 & 1 \\
0 & 1
\end{bmatrix} \\
A_{12} & = \begin{bmatrix}
1 & 1 \\
0 & 1
\end{bmatrix} \\
A_{21} & = \begin{bmatrix}
1 & 1 \\
0 & 1
\end{bmatrix} \\
A_{22} & = \begin{bmatrix}
1 & 1 \\
0 & 1
\end{bmatrix}
\end{align*}
\]

\[
B_{11} = A_{11}^{-1} (I + A_{12}B_{22}A_{21})A_{11}^{-1} = \begin{bmatrix}
1 & 1 \\
0 & 1
\end{bmatrix}
\]

\[
B_{12} = A_{11}^{-1}A_{12}B_{22} = \begin{bmatrix}
1 & 1 \\
0 & 1
\end{bmatrix}
\]

\[
E_{TEL} = E_{EL} = \begin{bmatrix}
1 & 1 \\
0 & 1
\end{bmatrix}
\]

\[
\begin{align*}
&\text{Var} = \begin{bmatrix}
6 & 6 & 0 & 4 & 4 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} \\
&Q_1 = u_{jk} + u_{jk} k=2
\end{align*}
\]

5 Q TERM

5.1 Proof that $Q_1^{ETEL}$ and $Q_1^{EL}$ differ by less than $O_p n^1$. This result will imply that all the parameters $\gamma, \delta_0, \eta_0, \gamma$ for ETEL and EL differ by less than $O_p n^1$. This may come as a surprise since the Lagrange multipliers of EL and ET are known to differ by $O_p n^1$. However, we
have carefully defined the parameter vector of each estimator so that the Lagrange multiplier of EL (^) corresponds to the auxiliary parameter ^ of ETEL instead of the Lagrange multiplier ^ of ETEL.

\[ \text{Lagrange multiplier of EL} \]
\[ \text{Lagrange multiplier of ETEL} \]

Hence, our results do not imply that the Lagrange multiplier EL and ETEL are equivalent up to O_p n^1 but rather that the auxiliary parameter ^ of ETEL in fact plays the role of EL’s Lagrange multiplier in the ETEL estimator. Since the influence function of EL and ETEL are the same, we have that

\[ Q_{ETEL}^E = Q_{EL}^E \]

5.1 Calculation of

5.1.1 Calculation of

Thus, \( \text{ETEL}_{ij} = \text{EL}_{ij} \), where

\[ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \]

5.1.2 Calculation of

where we used the identities \( ^ {a} g = \frac{1}{a} \frac{1}{g} \) and \( ^ {a} g = \frac{1}{a} \frac{1}{g} \). To calculate \( ^ {h} g \) :
Then

\[
\begin{align*}
E + \theta^2 = 0.5
\end{align*}
\]

where \( G_j \).

Since \( j \) and \( k \) are zero for all \( h \), it follows that \( j \) and \( k \) are zero for all \( h \).

Conclusion: \( Q_1^{ETL} = Q_1^{EL} = 0 \).

5.2 Calculation of \( Q_1^{ETL} = Q_1^{EL} = Q_1 \)

\[ Q_1^{EL} = E L_{ij} + j E L_{ijk} k=2 \]

5.2.1 Calculation of \( E L_{ij} \)

\[ E L_{ij} = E L_{i+1} + E L_{i0} \]

5.2.2 Calculation of \( E L_{ijk} \)

\[ j E L_{ijk} k=2 = \frac{h}{h+1} \]

\[ j E L_{ijk} k=2 \]

\[ = 0 \]

\[ g^0 + 0 + 0 \]

\[ j E L_{ijk} k=2 \]

\[ = 0 \]

\[ g^0 + 0 + 0 \]

\[ j E L_{ijk} k=2 \]

\[ = 0 \]

\[ g^0 + 0 + 0 \]

\[ j E L_{ijk} k=2 \]

\[ = 0 \]
\[
\begin{align*}
\text{Collecting these 4 results into a single vector, we obtain:} \\
\mathbf{g}^P &= \frac{\mathbf{g}^P g}{\mathbf{g}^H} \\
&= \frac{\mathbf{g}^P E}{\mathbf{g}^H} \frac{\mathbf{g}^P g}{\mathbf{g}^H} + \frac{\mathbf{g}^P E}{\mathbf{g}^H} \frac{\mathbf{g}^P g}{\mathbf{g}^H} + \frac{\mathbf{g}^P E}{\mathbf{g}^H} \frac{\mathbf{g}^P g}{\mathbf{g}^H} + \frac{\mathbf{g}^P E}{\mathbf{g}^H} \frac{\mathbf{g}^P g}{\mathbf{g}^H} \\
&= \left( \frac{\mathbf{g}^P}{\mathbf{g}^H} \right) \frac{\mathbf{g}^P g}{\mathbf{g}^H} + \left( \frac{\mathbf{g}^P}{\mathbf{g}^H} \right) \frac{\mathbf{g}^P g}{\mathbf{g}^H} + \left( \frac{\mathbf{g}^P}{\mathbf{g}^H} \right) \frac{\mathbf{g}^P g}{\mathbf{g}^H} + \left( \frac{\mathbf{g}^P}{\mathbf{g}^H} \right) \frac{\mathbf{g}^P g}{\mathbf{g}^H}
\end{align*}
\]
\[ + (g^2 H)^j E \frac{\partial^2 g^j}{\partial \theta^2} H g + (g^2 P)^j E \frac{\partial^2 \theta}{\partial \theta^2} \]
\[ H^0 (g^2 P)^j E \frac{\partial g^j}{\partial \theta} P g + (g^2 P)^j E \frac{\partial g^j}{\partial \theta} H g + (g^2 H)^j E \frac{\partial^2 \theta^2}{\partial \theta^2} P g \]
\[ 2 = H (g^2 P)^j E \frac{\partial g^j}{\partial \theta} P g + (g^2 P)^j E \frac{\partial g^j}{\partial \theta} H g + (g^2 H)^j E \frac{\partial^2 \theta^2}{\partial \theta^2} P g + \]
\[ + (g^2 H)^j E \frac{\partial^2 g^j}{\partial \theta^2} H g + (g^2 P)^j E \frac{\partial^2 \theta}{\partial \theta^2} H g + (g^2 H)^j E \frac{\partial^2 \theta^2}{\partial \theta^2} P g \]

Finally,
\[ Q^{E L}_{ij} = \frac{\partial P g}{\partial \theta} Q + \frac{\partial \theta}{\partial \theta} \]
\[ g^2 P g = 2 \]
\[ = \frac{6}{4} + \frac{1}{2} \frac{3}{4} (g^2 P)^j E \frac{\partial g^j}{\partial \theta} P g + \frac{7}{5} \]

where
\[ 3 = \frac{1}{2} + P P g + H G P g + P g + \]
\[ 4 = \frac{1}{2} + H P g + G P g \]

6 Calculation of R

We will prove that \( R^{ETEL}_{1+1} R^{EL}_{1+1} 1+m = 0 \) \( n^2 \). This will be done by showing that most of the terms comprised in \( R^{ETEL}_{1+1} R^{EL}_{1+1} \) are zero. The remaining non-zero terms will be shown to be uncorrelated with \( 1+m \) (up to \( n^2 \)).

Since \( Q^{ETEL}_{1+1} Q^{EL}_{1+1} \)
\[ R^{ETEL}_{1+1} R^{EL}_{1+1} = \frac{\partial E T E L}{\partial \theta} \frac{\partial E L}{\partial \theta} \frac{\partial E T E L}{\partial \theta} + \frac{\partial E T E L}{\partial \theta} \frac{\partial E L}{\partial \theta} \]
\[ + \frac{1}{2} \frac{\partial E T E L}{\partial \theta} \frac{\partial E L}{\partial \theta} \frac{\partial E L}{\partial \theta} \]

6.1 Calculation of \( E T E L_{1+1+1} E L_{1+1+1} Q_{1} \)
\[ E T E L_{1+1+1} E L_{1+1+1} Q_{1} = \frac{\partial E T E L}{\partial \theta} \frac{\partial E L}{\partial \theta} \]
\[ = \frac{\partial E T E L}{\partial \theta} \frac{\partial E L}{\partial \theta} \frac{\partial E T E L}{\partial \theta} \]
\[ = \frac{\partial E T E L}{\partial \theta} \frac{\partial E L}{\partial \theta} \]

6.2 Calculation of \( E T E L_{1+1+1} E L_{1+1+1} kQ_{1} \)
\[ E T E L_{1+1+1} E L_{1+1+1} kQ_{1} = \frac{\partial E T E L}{\partial \theta} \frac{\partial E L}{\partial \theta} \]
\[ = \frac{\partial E T E L}{\partial \theta} \frac{\partial E L}{\partial \theta} \]

5.12: Since \( E T E L_{1+1} = \frac{6}{4} \frac{\partial \theta}{\partial \theta} g + \frac{7}{5} \frac{\partial \theta}{\partial \theta} \)
\[ \frac{\partial \theta}{\partial \theta} g + \frac{7}{5} \frac{\partial \theta}{\partial \theta} \]

9
Collecting the previous four quantities into a single vector, we have

Then, since

we obtain

Collecting the previous four quantities into a single vector, we have

The first term is exactly cancels the first term of the expansion of $R$ (see section 6.1).

The second term is uncorrelated with $1 + l$, which implies that it does not contribute to the $n^2$ variance. Indeed, all the random contributions to the second term are of the form $P_g$, while $1 + l$ is of the form $H_g$ and $E = F gg H = P H = 0$.

6.3 Calculation of
6.4 Calculation of $E_{klh}$

where \( \triangle_{Ef} \), \( \triangle_{gh} \), and \( \triangle_{ghf} \) are uncorrelated with \( \triangle_i \). The \( 0 \times 2 \) correlation of \( E_{klh} \) with \( \triangle_i \) can be written as

\[
E_{klh} \equiv \sum_{jk} \triangle_{gh} \epsilon_{jkh} \epsilon_{jkh}.
\]

Since \( \epsilon_{jkh} \) contributes to the \( 2 \times 2 \) variance only through its correlation with \( \triangle_i \), we can omit the terms that are uncorrelated with \( \triangle_i \). This follows that, in the same way, \( \epsilon_{jkh} \), \( \epsilon_{jkh} \), and \( \epsilon_{jkh} \) are uncorrelated with \( \triangle_i \).

Letting \( \epsilon_{jkh} = \epsilon_{jkh} = \epsilon_{jkh} = \epsilon_{jkh} \), we use the relation that \( \epsilon_{jkh} = \epsilon_{jkh} = \epsilon_{jkh} = \epsilon_{jkh} \). This means that \( \epsilon_{jkh} \) contributes only through its correlation with \( \triangle_i \), and making use of the fact that \( \epsilon_{jkh} = \epsilon_{jkh} = \epsilon_{jkh} = \epsilon_{jkh} \), we can omit the terms that are uncorrelated with \( \triangle_i \). This follows that, in the same way, \( \epsilon_{jkh} \), \( \epsilon_{jkh} \), and \( \epsilon_{jkh} \) are uncorrelated with \( \triangle_i \).
When writing the intermediate steps of the calculations, we omit the terms that will be multiplied by or after all the derivatives have been evaluated, since these terms will vanish when the true values = 0 and = 0 are substituted in.)

\[
\begin{align*}
\text{ETEL} & \quad (L+q) = 0 \\
\text{ETEL} & \quad (L+hljk) = 0 \\
\text{ETEL} & \quad (L+hljk) = E \\
\end{align*}
\]

\[
\begin{align*}
\frac{6}{4} & \quad 0 0 0 0 E \quad 0 0 0 0 \\
\frac{2}{4} & \quad 0 0 0 0 \quad 7 7 7 7 \\
\frac{1}{4} & \quad 0 0 0 0 0 5 \\
\end{align*}
\]
\[
\begin{align*}
2^0 & \quad h^0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
40 & \quad E \frac{\theta^3}{\theta_q} \quad (\ldots \cdot \frac{\theta (a \cdot \alpha q)}{\theta_q}) \quad 0 \quad 0 \\
20 & \quad 0 \quad 0 \quad 0 \quad h^0 \quad 0 \quad 0 \\
6 & \quad 0 \quad 2E \frac{\theta (a \cdot \alpha q)}{\theta_q} \quad E \frac{\theta (a \cdot \alpha q)}{\theta_q} \quad 0 \quad 0 \\
4 & \quad E \frac{\theta^3}{\theta_q} \quad (\ldots \cdot \frac{\theta (a \cdot \alpha q)}{\theta_q}) \quad 0 \quad 0 \\
\end{align*}
\]

\[
\begin{align*}
E \frac{\theta^3}{\theta_q} \quad (\ldots)G^0_h + \frac{\theta^3}{\theta_q} \quad -G^0_h \quad \frac{\theta^3}{\theta_q} \quad \frac{\theta^3}{\theta_q} \quad (\ldots)G^0_h = \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
E \frac{\theta^3}{\theta_q} \quad (\ldots)G^0_h + \frac{\theta^3}{\theta_q} \quad -G^0_h \quad \frac{\theta^3}{\theta_q} \quad \frac{\theta^3}{\theta_q} \quad (\ldots)G^0_h = \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
\end{align*}
\]

It follows that

\[
\begin{align*}
E \frac{\theta^3}{\theta_q} \quad (\ldots)G^0_h + \frac{\theta^3}{\theta_q} \quad -G^0_h \quad \frac{\theta^3}{\theta_q} \quad \frac{\theta^3}{\theta_q} \quad (\ldots)G^0_h = \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \\
\end{align*}
\]
In particular, the fourth term entering $R_{1+1}^{ETEL} R_{1+1}^{EL} l + m = 0 + o(n^2)$.

7 Conclusion

$ETEL = EL$

$Q_{ETEL} = Q_{EL}$

$R_{ETEL} \in R_{EL}$ but $E R_{ETEL} R_{ETEL}^{EL} l + m = 0 + o(n^2)$.

In particular, the four terms entering $R_{1+1}^{ETEL} R_{1+1}^{EL} l + m = 0 + o(n^2)$ have the following properties

$ETEL l + lj \quad EL l + lj \quad Q_{j} = \frac{1}{2} H_{j} h_{j} g_{j} p_{g} g_{j} (see \ Section 6.1)$.

$ETEL l + lj \quad EL l + lj \quad kQ_{j} = \frac{1}{2} H_{k} g_{k} g_{p} g_{j} \quad \gamma_{j} l$ where $\gamma_{j} l$ is such that $E \gamma_{j} l 1 + m = 0 + o(n^2)$

(see \ Section 6.2).

$ETEL \quad EL \quad l + lj \quad j k = 0$ (see \ Section 6.3).

$ETEL \quad EL \quad l + lj \quad j k h = \delta_{j} l$ such that $E \delta_{j} l 1 + m = 0 + o(n^2)$ (see \ Section 6.4).