Comment on ""Forbidden" transitions between quantum Hall and insulating phases in p-SiGe heterostructures"

S. S. Murzin
Institute of Solid State Physics RAS, 142432, Chernogolovka, Moscow District, Russia

It is shown that recent [1] and earlier [2,3,4] experiments, which claimed to observe a disagreement with the global phase diagram (GPD) [5] of the quantum Hall effect, do not, in fact, contradict the GPD. Two aspects should be taken into account: (i) insulating phases between quantum Hall phases are possible owing to the Shubnikov-de Haas oscillations of the "bare" diagonal resistivity $\rho_{xx}$; (ii) according to the two-parameter scaling theory [6,7], the filling factor $\nu$ does not determine directly the positions of the quantum Hall phases on the magnetic field axis at $\omega_c \tau \lesssim 1$.

PACS numbers: 71.30.1+h, 73.43.2-f

In a recent paper [1] Sakr et al. reported that they observed multiple quantum Hall-insulator-quantum Hall transitions (MQH-I-QHT) in p-SiGe heterostructures, where insulating phases occurred between quantum Hall (QH) phases at filling factors $\nu = 1$ and 2, 2 and 3, and 4 and 6. Previously it was reported that MQH-I-QHT had been observed in silicon MOSFETs [2,3], and the insulating phase had been detected between QH phases at $\nu \approx 1.5$ in p-SiGe heterostructures [4]. The authors of the papers [1,2,3,4] claimed that their results are in a glaring contradiction with the global phase diagram (GPD) for the QH effect [5], which follows from the scaling theory [6,7], and a new type of the phase diagram was suggested [1,3].

In this comment I argue that, in fact, the experimental results [1,2,3,4] do not contradict the GPD if two circumstances are taken into account:

(i) The insulating phase between the QH phases is possible owing to the Shubnikov-de Haas oscillations of the "bare" diagonal resistivity $\rho_{xx}$, which corresponds to diffusive motion of electrons without interference effects over a distance larger than diffusion length.

(ii) According to the two-parameter scaling theory [6,7], the filling factor $\nu = nh/eB$ does not determine directly positions of the QH phases on the magnetic field axis at $\omega_c \tau \lesssim 1$ ($\omega_c = eB/m$ is the cyclotron frequency, $\tau$ is the transport relaxation time).

The scaling theory presented graphically by the flow diagram [6,7] deals with the Hall $\sigma_{xy}$ and the diagonal $\sigma_{xx}$ conductivity components, and it does not require the Landau quantization of the electron spectrum. The Landau quantization is incorporated into the theory through the starting values for the renormalization (i.e. the change in the conductivity due to diffusive interference effects) which are the "bare" conductivities $\sigma_{xy}^0$ and $\sigma_{xx}^0$.

For a totally spin polarized electron system, maxima of the Shubnikov-de Haas oscillations of the "bare" resistivity $\rho_{xx}^0$ occur when the centers of the Landau levels $E_L^i = (i + 1/2)\hbar \omega_c \equiv (i + 1/2)E_F/\nu$ cross the Fermi level $E_F^T = E_F$ (as is shown in Fig.1 for the case $E_F = const$) at half-integer filling factors $\nu = i + 1/2$. Here $i$ is an integer. The

$$E_L^i = (i + 1/2)\hbar \omega_c \equiv (i + 1/2)\frac{E_F}{\nu}$$

FIG. 1. Sketch of the magnetic-field dependence (a) of energies of the Landau levels $E_L^i$ (dashed line) and of critical states $E_{c_i}^i$ (solid lines), and (b) of the "bare" diagonal resistivity $\rho_{xx}^0$ (dashed lines) and low temperature resistivity of $\rho_{xx}$ corresponding to the quantum-Hall-effect regime (solid lines) for the case of the totally spin-polarized 2D electron system. The horizontal solid line in the top figure plots the Fermi level $E_F$. 

1
renormalization leads to a transformation of the magnetic-field dependence of $\rho_{xx}^0$ with the Shubnikov-de Haas oscillations to the QH effect picture with peaks in the resistivity $\rho_{xx}$ (and conductivity $\sigma_{xx}$) separating different QH phases at magnetic fields where

$$\sigma_{xy}^0 = (i + 1/2)e^2/h.$$ \hspace{1cm} (2)

The Hall conductivity $\sigma_{xy} = \sigma_{xy}^0$ is not renormalized at these fields. The nonzero $\sigma_{xx}$ implies that the extended states are situated at the Fermi level.

The energies of the extended states, $E_i^c$, as functions of magnetic field can be calculated by Eq. (4) with the classical expression for $\sigma_{xy}^0$

$$\sigma_{xy}^0(E) = \frac{n(E)e^2\tau}{m} \left[ 1 + \frac{\omega_c\tau}{1 + (\omega_c\tau)^2} \right] = \frac{2E\tau}{\hbar} - \frac{\omega_c\tau}{1 + (\omega_c\tau)^2}. \hspace{1cm} (3)$$

Here the conduction band is occupied up to energy $E$, $n(E) = E\bar{n}/\hbar$ is the electron density, $\hbar = h/2\pi$. As a result

$$E_i^c = (i + 1/2)\hbar\omega_c \left[ 1 + \frac{1}{(\omega_c\tau)^2} \right]. \hspace{1cm} (4)$$

(see Fig.1). The positions of the $\rho_{xx}$ peaks are determined by equation $E_i^c = E_F$. It is assumed here that $\tau$ is independent on the energy $E$. This assumption, however, is not important for finding the peaks positions since only the value of $\tau$ at the Fermi level is essential for this calculation.

For a pure 2D system at high magnetic field, when $\omega_c\tau \gg 1$, the filling factor $\nu = nh/eB$ is equal to $\sigma_{xy}^0\hbar/e^2$. Therefore, peaks of $\rho_{xx}$ should occur at the same fields where the Shubnikov-de Haas oscillations have maxima. For $\omega_c\tau \lesssim 1$ the value $\sigma_{xy}^0\hbar/e^2$ is different from $\nu$ and the positions of the $\rho_{xx}$ peaks are very different from the positions of the maxima of the Shubnikov-de Haas oscillations.

In the plane $\rho_{xy}^0 - \rho_{xx}^0$ the phase boundaries are semicircles (see Fig.2) described by the equation

$$\sigma_{xy}^0 = \frac{\rho_{xy}^0}{(\rho_{xx}^0)^2 + (\rho_{xy}^0)^2} = (i + 1/2)e^2/h, \hspace{1cm} (5)$$

following from Eq. (2). At a given magnetic field, the quantized value of the Hall resistance $\rho_{xy} = h/ie\nu$ is determined by the position of point $(\rho_{xx}^0, \rho_{xy}^0)$ on the phase diagram. For example, for $(\rho_{xx}^0, \rho_{xy}^0)$ located between the two upper semicircles, $\rho_{xy} = 1$. As the magnetic field increases, $\rho_{xxxx}^0(\rho_{xy}^0)$ follows a straight horizontal line which crosses each phase boundary two times because $\rho_{xy}^0 = m/ne^2\tau$ is independent of the magnetic field. The QH phases are situated between low- and high-field insulators in this model.

The classical expression for $\sigma_{xy}^0$ is quite adequate at low magnetic fields, $\omega_c\tau \ll 1$, and at high magnetic field, $\omega_c\tau \gg 1$. At $\omega_c\tau \sim 1$ the Shubnikov-de Haas oscillations of $\sigma_{xy}^0$ should be taken into account in accurate calculations of $E_i^c$. This could give rise to additional crossing points between lines $E_i^c(\omega_c\tau)$ and the Fermi level, hence we have more crossing points between the curve of $\rho_{xx}^0(\rho_{xy}^0)$ and the phase boundary (see Fig.2), as compared with the case of the classical expression for $\sigma_{xy}^0$.

The insulating phase occurs between the QH phases at $\rho_{xy}^0(C) < \rho_{xy}^0 < \rho_{xy}^0(D)$, in addition to low- $\rho_{xy}^0 < \rho_{xy}^0(A)$ and high-field $\rho_{xy}^0 > \rho_{xy}^0(E)$ insulators. Note that the case illustrated by Fig.2, $i = 1$ at filling factor $\nu = 2$ and $i = 0$ at $\nu = 4, 6, ...$

Thus, the filling factors $\nu$ do not directly determine the positions of the QH phases on the magnetic field axis, so $i \neq \nu$ at integer values of $\nu$ if $\omega_cT \lesssim 1$. Sakr et al. [3] treat the shallow minima at $\nu = 2, 3, 4, 6$ with values of $\rho_{xx} > 0.3h/e^2$ as QH minima of the diagonal resistivity $\rho_{xx}$ with $i = \nu$. This interpretation of the experimental data seems questionable.

Note that equation (4) does not describe exactly an electron system with two different spin projections [10], and the phase boundaries are different from those plotted in Fig. 2. Even so, in the case of the spin splitting energy smaller than $\hbar\omega_c$, the topology of the phase diagram should remain the same.

In summary, the existence of the insulating phase between the quantum Hall phases observed in Ref. [1,2,3,4] is quite consistent with the global phase diagram [3] for the quantum Hall effect. It is possible owing to the Shubnikov-de Haas oscillations of “bare” diagonal resistivity $\rho_{xx}^0$. The experimental results do not produce evidence in favor of direct transitions from the insulating to quantum Hall phases with large $i$.

I would like to thank S. I. Dorozhkin, V. M. Pudalov and V. N. Zverev for helpful discussions. This work is supported by RFBR, PICS-RFBR and INTAS.
[1] M. R. Sakr, Maryam Rahimi, S. V. Kravchenko, P. T. Coleridge, R. L. Williams, and J. Lapointe, Phys. Rev. B 64, 161308 (2001).

[2] M. D’Iorio et al., Phys. Lett. A 150, 422 (1990); Phys. Rev. 46, 15992 (1992); S. V. Kravchenko et al., Phys. Rev. B 44, 13513 (1991); V. M. Pudalov et al., Pisma Zh. Eksp. Teor. Fiz. 57, 592 (1993) [JETP Lett. 57, 608 (1993)]; Surf. Sci. 305, 107 (1994).

[3] S. V. Kravchenko et al., Phys. Rev. Lett. 75, 910 (1995).

[4] F. F. Fang et al., Surface Science 263, 175 (1992); S. I. Dorozhkin et al., Phys. Rev. B 52, 11638 (1995); Surface Science 361/362, 933 (1996); M. D’Iorio et al., Surf. Sci. 362, 937 (1996); R. B. Dunford et al., Surface Science 361/362, 550 (1996), J. Phys.: Condens. Matter 9, 565 (1997); P. T. Coleridge et al., Solid State Communications 102, 755 (1997); T. Y. Lin et al., J. Phys.: Condens. Matter 10, 9691 (1998).

[5] S. Kivelson, D. H. Lee, and S. C. Zhang, Phys. Rev. B 46, 2223 (1992).

[6] D. E. Khmelnitskii, Pis’ma Zh. Eksp. Teor. Fiz. 38, 454 (1983) [JETP Lett. 38, 552 (1984)].

[7] A. M. M. Pruisken, in The Quantum Hall Effect, edited by R. E. Prange and S. M. Girven, Springer-Verlag, 1990

[8] D. E. Khmelnitskii, Phys. Lett. A 106, 182 (1984);

[9] R. B. Laughlin, Phys. Rev. Lett. 52, 2304 (1984).

[10] D. E. Khmelnitskii, Helvetica Phys. Acta 65, 164 (1992).