Variable Neighborhood Search for the University Lecturer-Student Assignment Problem

Martin Josef Geiger, Wolf Wenger

Lehrstuhl für Industriebetriebslehre,
Universität Hohenheim,
D-70593 Stuttgart, Germany
{mjgeiger,w-wenger}@uni-hohenheim.de

Abstract

The paper presents a study of local search heuristics in general and variable neighborhood search in particular for the resolution of an assignment problem studied in the practical work of universities. Here, students have to be assigned to scientific topics which are proposed and supported by members of staff. The problem involves the optimization under given preferences of students which may be expressed when applying for certain topics.

It is possible to observe that variable neighborhood search leads to superior results for the tested problem instances. One instance is taken from an actual case, while others have been generated based on the real world data to support the analysis with a deeper analysis.

An extension of the problem has been formulated by integrating a second objective function that simultaneously balances the workload of the members of staff while maximizing utility of the students. The algorithmic approach has been prototypically implemented in a computer system. One important aspect in this context is the application of the research work to problems of other scientific institutions, and therefore the provision of decision support functionalities.

Keywords: Variable Neighborhood Search, University Lecturer-Student Assignment Problem, Multi Objective Optimization
1 Introduction

An integral part of a successful completion of most university studies is the participation in seminars within which a written report on a given scientific subject has to be produced and handed in by a student. The topics are usually proposed by members of staff that also support the work carried out by the students. For many departments the assignment of students to topics and therefore to lecturers is a recurring task that usually has to be carried out once per term. In practice, the assignment is done with respect to a set of aspects combining preferences of students/lecturers as well as balancing the workload of staff.

Although the precise situation studied in the work presented here has not been tackled in literature before, similar problems are available in the context of course assignment and faculty assignment. The general problem there lies in assigning teachers to courses with respect to a set of side constraints and personal preferences [8]. Most quantitative formulations are based on integer linear programming models [1] and solved using commercially available software packages [6]. Other resolution methods used are based on genetic algorithms [7] or model explicit knowledge taken from human experts [5] in order to solve the problem.

Problems in the mentioned area have already quite early been considered to employ multiple criteria [3] as preferences of a set of persons have to be met at once and therefore require the identification of a compromise solution. While goal programming here plays an important role, another methodology that has been successfully applied in the past is the Analytic Hierarchy Process AHP [4].

The paper is organized as follows. In the following section 2, the problem of assigning students to topics/lecturers is explained and formulated as studied in the real world case at the University of Hohenheim. Section 3 presents a study of local search approaches used to solve the problem at hand. Experiments are carried out on a set of problem instances, one of them taken from the real world situation and others generated on the basis of the practical case. Conclusions are derived in section 4.
2 The university lecturer-student assignment problem

The studied problem of assigning students to lecturers has to be solved by the Department Production and Operations Management of the University of Hohenheim on a recurring basis. Within a seminar, organized twice a year, each member of staff proposes a number of scientific topics for which students may apply. Depending on the number of applications, identical topics may be assigned to different students to increase the capacity of the seminar. It can be experienced that the acceptance of a particular topic and the corresponding motivation to work in it depends on the personal preferences students may have for particular subjects.

In the past the assignment of students to topics has been done on a first-come-first-served basis, leading to a potential unsatisfying situation for some late enrolling participants. Students have already before been given the possibility of articulating their preferences by ranking topics, but when assigning topics priority was given with respect to the sequence in which the applications have been received.

The concept currently under investigation allows each student $i$ to articulate his/her preferences by assigning nonnegative weights $w_{ij}$ to each topic $j$ which correspond to a measure of utility associated with the assignment of a certain topic. For practical reasons, a total weight of $\sum_j w_{ij} = w_{\text{max}}$ has been chosen and communicated to the applicants, a potential alternative however would be the normalization of the weight values. Applications are collected up to a certain deadline, and an optimization problem of maximizing the realized utility values is formulated and solved as follows.

$$\text{max } \text{UTILITY} = \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij} x_{ij}$$ (1)

subject to

$$\sum_{i=1}^{n} x_{ij} \geq a_j \quad \forall j = 1, \ldots, m$$ (2)

$$\sum_{i=1}^{n} x_{ij} \leq b_j \quad \forall j = 1, \ldots, m$$ (3)

$$\sum_{j=1}^{m} x_{ij} = 1 \quad \forall i = 1, \ldots, n$$ (4)

$$x_{ij} \in \{0, 1\} \quad \forall i = 1, \ldots, n, \quad j = 1, \ldots, m$$ (5)

Expression (1) maximizes the total utility associated with an assignment $x_{ij}$ of students $i$ to
topics \( j \). The assignment is done with respect to some side constraints, namely the minimum number \( a_j \) and maximum number \( b_j \) of students per topics given in expressions (2) and (3), and (4) and (5) make sure that every student is assigned to exactly one topic. In the practical case, values of \( a_j = \left\lfloor \frac{n}{m} \right\rfloor \forall j = 1, \ldots, m \) and \( b_j = \left\lceil \frac{n}{m} \right\rceil \forall j = 1, \ldots, m \) have been chosen for \( n \) students applying for \( m \) topics.

In addition to the preferences articulated by the students, some applicants sometimes express the wish to be assigned together as a team to a certain topic. The current procedure of defining an assignment allows this, but only if such an alternative is also optimal with respect to expression (1).

3 Experimental investigation

3.1 Neighborhood search

Although most practical instances of the model described in section 2 may be solved using e.g. CPLEX, a drawback lies in the identification of a single optimal alternative only. In order to allow an identification of an optimal alternative that bears certain characteristics as the mentioned simultaneous assignment of students to a particular topic, a procedure that identifies a whole set of equally optimal alternatives is necessary. We therefore investigated the effectiveness of local search techniques for the problem at hand. Starting from an initial feasible alternative, applicable neighborhoods to the problem are in detail:

- ‘swap2’ neighborhood \( nh_{\text{swap2}} \), randomly selecting two variables \( x_{ij}, x_{kl} \) \( \forall j \neq l \wedge x_{ij} = x_{kl} = 1 \) and changing the values to \( x_{ij} = x_{kl} = 0, x_{il} = x_{kj} = 1 \).

- ‘swap3’ neighborhood \( nh_{\text{swap3}} \), randomly selecting three variables \( x_{ij}, x_{kl}, x_{op} \) \( j \neq l \wedge l \neq p \wedge p \neq j \wedge x_{ij} = x_{kl} = x_{op} = 1 \) and changing the values to \( x_{ij} = x_{kl} = x_{op} = 0, x_{il} = x_{kp} = x_{oj} = 1 \).

- ‘shift’ neighborhood \( nh_{\text{shift}} \), randomly selecting a variable \( x_{ij} \) \( \forall j \leq 1 \wedge \sum_{k=1}^{n} x_{kj} > a_j \) and a topic \( l \) \( \forall l \wedge \sum_{k=1}^{n} x_{kl} < b_j \) and changing the values to \( x_{ij} = 0, x_{il} = 1 \).

- ‘shift+swap2’ neighborhood \( nh_{\text{shift+swap2}} \), combining the two neighborhoods \( nh_{\text{shift}} \) and \( nh_{\text{swap2}} \) by applying both changes to the variables.

In addition to the application of a single neighborhood only, a variable neighborhood search concept that includes all neighborhoods in a set \( \mathcal{NH} \) is used to solve the problem. The general idea...
is sketched in algorithm 1.

**Algorithm 1** Reduced variable neighborhood search (VNS)

1: Generate initial solution $S$, set $P_{\text{approx}} = \{S\}$
2: repeat
3: Randomly select some $S \in P_{\text{approx}}$
4: Randomly select some neighborhood $nh_i$ from the set of neighborhoods $NH = \{nh_1, \ldots, nh_q\}$
5: Compute neighboring solution $S'$ by applying $nh_i$ to $S$
6: Update $P_{\text{approx}}$ with $S'$
7: until termination criterion is met
8: Return $P_{\text{approx}}$

In contrast to established methods of variable neighborhood search [2], algorithm 1 maintains a set $P_{\text{approx}}$ containing alternatives of equal quality with the aim of allowing the identification of a whole set of optimal solutions.

### 3.2 Maximizing the total utility

Experiments have been carried out for a real world problem instance ‘n34’ with $n = 34$ students applying for $m = 15$ topics. In order to study the behavior of the neighborhoods when facing problems with differing characteristics, instances from $n = 30$ to $n = 45$ students have been generated by randomly discarding or adding students from the instance n34.

Each neighborhood has been tested on each problem instance starting from an initial randomly generated solution, and average values of the total utility as given in expression (1) have been computed over a total of 25 test runs for each problem instance/neighborhood structure combination. The termination criterion has been set to 100,000 evaluations. On an Intel Pentium IV processor running at 3 GHz each test run takes less than 3 seconds. The running time is independent from the size of the problem instance as evaluation of alternatives is possible in constant time.

The average results of the total utility are given in table 1 with the real world instance n34 marked ⋆. It can be seen that VNS outperforms the simple local search approaches in almost all problem instances with the only exception being n30. For the problem instance taken from the real world case, an optimal solution is known with a total utility of 282, and the VNS successfully identifies one or several optimal alternatives in almost every test run. The single neighborhood with
the best results is consistently $nh_{shift+swap2}$, and while the results are close to the ones of VNS, they still remain inferior.

An interesting resolution pattern can be recognized depending on the structure of the problem instances. Figure 1 plots and analyzes the difference of the results achieved by simple local search to the results of VNS. The gap in performance clearly shows that instances with little possibilities to shift assignments are not successfully solved with $nh_{shift}$. On the other hand, the relative performance of the swap neighborhoods $nh_{swap2}$ and $nh_{swap3}$ decreases with increasing potential of applying shift moves.

### 3.3 Balancing utility and distribution of workload

The encouraging results of the VNS including the neighborhoods described in section 3.1 led to the application of the search concept to a multi objective extension of the problem. While the first
objective function, given in expression 1, maximizes the total utility of the students, assignments may lead to an imbalanced workload for the members of staff as some topics tend to be significantly more popular than others and therefore receive higher weight assignments $w_{ij}$.

In the studied case of the real world problem, four members of staff $B = \{B_1, \ldots, B_4\}$ are involved, with three persons supporting three topics each ($B_1 = \{1, 2, 3\}$, $B_1 = \{4, 5, 6\}$, $B_1 = \{7, 8, 9\}$) and one person being responsible for six topics ($B_4 = \{10, \ldots, 15\}$). In order to balance the assignment of students, the difference between the lecturer with the most load and the member of staff with the lowest load is minimized. The load is here expressed as the ratio between the number of assigned students to the number of available spaces of the specific lecturer, given in expression (6). The problem is then solved by identifying all optimal alternatives in the sense of Pareto-optimality.

$$\min \text{IMBALANCE} = \left( \max_{B_k \in B} \left( \frac{\sum_{i=1}^{n} \sum_{j \in B_k} x_{ij}}{\sum_{j \in B_k} b_j} \right) \right) - \left( \min_{B_k \in B} \left( \frac{\sum_{i=1}^{n} \sum_{j \in B_k} x_{ij}}{\sum_{j \in B_k} b_j} \right) \right)$$

(6)

The efficient solutions are plotted in outcome space in figure 2. It can be shown that in addition to the optimal alternatives for the optimality criterion of maximizing the total utility, other efficient points exist that are better balanced while being slightly worse in terms of their utility. A total of 139 distinct alternatives possessing values of UTILITY = 279 and IMBALANCE = 0.055 have been identified, 29 alternatives for the outcome of 281/0.222, and 13 for the values 282/0.333.
4 Conclusions and further research

The paper presented a study of a real world assignment problem, consisting of the assignment of student to scientific topics and therefore members of staff. The problem has to be solved on a recurring basis, and the implications of the assignments are of high practical and personal value to the parties involved.

A quantitative model to describe the situation at hand has been formulated and solved using a local search heuristic. Variable neighborhood search led to superior results, independent from the underlying problem characteristics. In addition to the maximization of the student’s utility, a second criterion has been introduced balancing the workload of the lecturers. Pareto optimal alternatives could have been identified, showing a tradeoff between imbalance and utility in outcome space.

The study conducted demonstrates the applicability of local search heuristics in general and variable neighborhood search in particular, but is so far only based on the experiences gathered at the University of Hohenheim. To generalize the conclusions, we are currently evaluating the status quo of the assignment process within seminars in universities throughout Germany with the final goal of proposing a more general methodology that supports the problem found in practice. The investigation is carried out involving a questionnaire and personal communication with staff being involved in organizing seminars.
To allow the application of the optimization procedure to real world problems, a prototypical implementation within a computer system has been made available. Figure 3 shows the current user interface with which the user may interact and which visualizes the alternatives computed by the system. The names of the students are however here blurred due to privacy restrictions. As the problem also has to be solved by institutions with a low affinity to the use of computer programs, an integrated system providing decision support may increase acceptance in practice.

Acknowledgements

The authors would like to thank PD Dr Andreas Kleine for providing an optimal alternative for the real world instance n34 of the data sets using CPLEX.
References

[1] Jon A. Breslaw. A linear programming solution to the faculty assignment problem. *Socio-Economic Planning Sciences*, 10:227–230, 1976.

[2] Pierre Hansen and Nenad Mladenović. Variable neighborhood search. In Fred Glover and Gary A. Kochenberger, editors, *Handbook of Metaheuristics*, volume 57 of *International Series in Operations Research & Management Science*, chapter 6, pages 145–184. Kluwer Academic Publishers, Boston, Dordrecht, London, 2003.

[3] Richard H. McClure and Charles E. Wells. Modeling multiple criteria in the faculty assignment problem. *Socio-Economic Planning Sciences*, 21(6):389–394, 1987.

[4] Mujgan S. Ozdemir and Rafail N. Gasimov. The analytic hierarchy process and multiobjective 0–1 faculty course assignment. *European Journal of Operational Research*, 157:398–408, 2004.

[5] Fariborz Y. Partovi and Bay Arinze. A knowledge based approach to the faculty-course assignment problem. *Socio-Economic Planning Sciences*, 29(3):245–256, 1995.

[6] Hussein M. Saber and Jay B. Gosh. Assigning students to academic majors. *Omega*, 29:513–523, 2001.

[7] Yen-Zen Wang. An application of genetic algorithm methods for teacher assignment problems. *Expert Systems with Applications*, 22:295–302, 2002.

[8] Chin W. Yang and Charles J. Pineno. An improved approach to solution of the faculty assignment problem. *Socio-Economic Planning Sciences*, 23(3):169–177, 1989.