Improved Similarity Coefficient Method for Cell Formation with Considering Operation Sequences and Times

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Abstract: Cell formation is one of the important problems in the design of a cellular manufacturing system. In recent years, many works have been carried out in this active research field. Specially, similarity coefficient methods have been studied widely to form part-machine cells so as to minimize intercellular part movements, but few of them had considered influence of operation sequences and times on cell formation simultaneously. In this paper, we improve an existing similarity coefficient method based on considering the factors of operation sequences and operation times. A feasible and effective operation sequences and times-based similarity coefficient method is presented to solve cell formation problems. The improved method is applied to several examples from literatures and a numerical example with some repeated production processes. The results demonstrate that it is necessary to consider the operation sequences and times into similarity measure between each pair of parts or machines, and the proposed method is valid and available for cell formation in a cellular manufacturing system with some repeated operations.

Key Words: Similarity coefficient method, Cell formation, Operation sequences, Operation times

1. Introduction

With the competitions in the global market and the diversification of user requirements, enterprises have to produce a large variety of products in small batch sizes to fit in with the changing of market. The cellular manufacturing system (CMS) has emerged as an important scientific principle in improving the productivity of multi-type and small batch manufacturing systems. The CMS which is based on the concept of group technology (GT) philosophy aims at increasing productivity and production efficiency by reducing throughput times. Crucial objective of GT in CMS is to group parts and machines for cell formation.

1.1 Part-machine Cell Formation

Cell formation (CF) is the most fundamental step for implementation of CMS. In general, three main objectives exist in the cell formation problem: part-family formation, machine-cell formation and allocation of part-families to the machines. The CF techniques have been categorized into four kinds of methods, namely, similarity based methods, graph-theoretic methods, evolutionary approaches, and neural network methods.

Among various CF methodologies, similarity coefficient method is one of the most powerful methods. Since similarity coefficient method was presented by Reisman et al., Selim et al., Mansouri et al., Yin and Yasuda, and so on [5–8]. Defersha et al. [9] developed a comprehensive mathematical model for designing cellular manufacturing systems and used a commercial optimization software to solve models for small size problems. Wu et al. [10] have applied a genetic algorithm for cellular manufacturing design and layout. They proposed a new dynamic selection strategy to deal with concurrent decisions that involve highly correlated objectives. Ameli et al. [11] focused on the configuration of machine cells considered production volumes and process sequences of parts to develop a pure linear integer program for CF problem. Caux et al. [12] presented an iterative method, in which two problems including the route selection using branch and bound method and cell formation using a simulated annealing algorithm are solved iteratively, and one problem is solved based on the results of the other problem. Heragu et al. [13] proposed a two-stage heuristic for the cell design problem. First, the number of each machine type for processing parts is determined according to demand requirements, then part families and machine sets are identified. Yin et al. [14] designed manufacturing cells while minimizing overall material flows by a nonlinear mathematical model and a heuristic methodology. Bajestani [15] presented a multi-objective dynamic cell formation problem by considering the minimum of total cell load variation and sum of the miscellaneous costs.

Among various CF methodologies, similarity coefficient method has been extensively used in the field of cell formation. However, few of them have considered operation sequences and times in cell formation simultaneously. In this paper, we improve an existing similarity coefficient method based on considering the factors of operation sequences and operation times. A feasible and effective operation sequences and times-based similarity coefficient method is presented to solve cell formation problems. The improved method is applied to several examples from literatures and a numerical example with some repeated production processes. The results demonstrate that it is necessary to consider the operation sequences and times into similarity measure between each pair of parts or machines, and the proposed method is valid and available for cell formation in a cellular manufacturing system with some repeated operations.
method (SCM) is more flexible in incorporating various types of manufacturing data into the cell formation problem [16]. This paper describes an improved SCM which identifies part-families and their associated machine groups simultaneously in the design process of CF. In recent years, numerous researchers have applied SCM to solve CF problem, and achieved great success in this field. Next section introduces review of the previous research on application of SCM in CF problem.

1.2 SCM-based Algorithm in Cell Formation

McAuley is the first researcher to use the SCM to cluster machine cells. McAuley [17] presented the Jaccard similarity coefficient to measure the similarity for each pair of machines, and then grouped the machines into cells based on their similarity measurements.

Generally, the SCM-based algorithm is used to calculate similarity between each pair of machines or parts. According to the result of similarity matrix, group parts and machines with application of clustering technique, and then part-family can be manufactured within corresponding machine-group. The steps of SCM are prescribed as follows [18].

Step 1: Form the initial part-machine incidence matrix. In general CF process, the requirement data between part and machine is organized in a binary matrix called ‘part-machine incident matrix’ in which all the elements are 0 or 1. The element 1 in row i and column k of the matrix indicates that machine i has an operation for corresponding part k; the element 0 indicates it does not. This proposed definition looks like the transpose of classical incidence matrix.

\[
A = \begin{pmatrix}
  a_{11} & \ldots & a_{1k} \\
  a_{21} & \ldots & a_{2k} \\
  \vdots & \ddots & \vdots \\
  a_{i1} & \ldots & a_{ik}
\end{pmatrix}
\]

The element \(a_{ik}\) is defined as follows.

\[
a_{ik} = \begin{cases}
  1 & \text{if part } k \text{ visit machine } i \\
  0 & \text{otherwise}
\end{cases}
\]

\(i\) is the machine index \((i = 1, 2, \ldots M)\);

\(k\) is the part index \((k = 1, 2, \ldots P)\);

\(M\) is the number of machines;

\(P\) is the number of parts.

Step 2: Select a similarity coefficient method and compute similarity values between machine (part) pairs and construct a similarity matrix. Each element of the matrix represents the sameness between any two machines (parts).

Step 3: Use clustering algorithm to process the values in the similarity matrix, which results in a diagram called a tree, or dendrogram, that shows the hierarchy of similarities among all pairs of machines (parts). Find machines groups (part families) from the tree or dendrogram, check all predefined constraints such as the number of cells, cell size, etc.[18].

In the first step, the initial part-machine incidence matrix can be composed based on actual conditions of manufacturing. The step 2 and 3 are crucial points for the application of SCM in CF. There are a few models to calculate similarity coefficients in the reported literatures. Some well-known similarity coefficients are summarized in Table 1 [8],[18].

Like other CF approaches, SCM also involves some production factors, such as production volume, machine requirement, machine setup times, utilization, workload, alternative routings, machine capacities, operation sequences, setup cost and cell layout. Gupta et al.[19] suggested a methodology for evaluating alternative solutions from different algorithms on a quantitative basis using modified version of an existing coefficient. Their proposed SCM incorporates relevant production data such as part type production volume, routing sequence and unit operation time. Gunasingh et al.[21] suggested an index of similarity between any two machines is defined in terms of the tooling requirements of the parts, processing requirements and tools available on the machine capabilities. Vakharia et al.[3] proposed a new coefficient by considering the within-cell machine sequence and machine loads in the CF process. Gupta [22] suggested a new similarity coefficient which required that alternative routing of parts should be considered while calculating the pairwise similarity coefficient between machines.

Remarkably, more and more researchers pay attention to operation sequence when SCM would be used to solve CF problem. Moussa et al.[24] also proposed a new similarity coefficient which took into consideration the operation sequences and process time during the assignment process. Akturk et al.[25] presented a new multi-objectives mathematical model for the CF problem. Both of manufacturing attributes and operation sequences were considered in this method. Sarkar et al.[26] reviewed the state-of-the-art in researching on operation sequence-based cell formation. A number of operation sequence-based similarity/dissimilarity coefficients were discussed in their research. They classified the methods of cell formation based on the operation sequences into four kinds: mathematical programming, network analysis, materials flow analysis method, and heuristics. Won et al.[2] proposed a new approach that two types of production data-based part-machine incidence matrices from real-field such as operation sequences and production volumes of parts were incorporated. Yin et al.[27] presented an definition of operation sequence ratio (OSR) and modified existing similarity coefficients based on the OSR to solve cell formation problems.

Though there are some drawbacks like chaining and problems of bottleneck machines in SCM, numerous advantages also are obvious. Some of the major advantages are generation of alternate solutions, flexibility and intrinsic determination of similarity level. Therefore, abundant SCMs are existing in the literatures of CF.

1.3 Research Objective

Although many factors had been considered into SCM in the former research, few of them paid attention to the CF problem based on both operation sequences and operation times. In some practical cellular manufacturing systems, it is normal that part visits machine not just once but twice or more. Operation sequences and times can respect not only the machine requirements but also materials flow of the parts effectively.

The main objective of this study is to propose an improved SCM based on consideration of operation sequences and times for solving CF problem in the CMS with some repeated operations. For this purpose, this paper presents operation sequences ratio to modify an existing SCM firstly, and then involves operation times into the similarity coefficients for getting opera-
tion times-based SCM. Finally, to achieve operation sequences and times-based SCM by considering the two factors simultaneously. We deem that the consideration of operation sequences and times is necessary and feasible for CF in complex CMS. And the proposed operation sequences and times-based SCM is valid and effective to achieve similarity coefficient matrix for part-machine cells formation. In order to prove the propositions, a number of examples are adopted to compare and evaluate performance of the improved SCM.

The remainder of the paper is organized as follows. Improved model for SCM is presented in Section 2. Section 3 describes numerical examples and computational results. Conclusion and direction of future research are summarized in Section 4.

2. Description of the Improved Model for Similarity Coefficient Method

In this chapter, we firstly introduce a commonly used SCM named relative matching coefficient which had been proposed by Islam et al. [29]. And then, operation sequences ratio is presented to modify the relative matching coefficient model for designing operation sequences-based SCM. In addition, consider operation times into similarity coefficients to present operation times-based SCM. Finally, combine the factors of operation sequences and operation times together to attain a rigorous and comprehensive approach of operation sequences and times-based SCM.

2.1 Relative Matching Coefficient

Islam et al. presented that most of coefficients fail to reflect the actual impact of relational values in the incident matrix on the conformance and non-conformance to the properties of similarity coefficients. Therefore, they proposed a new SCM for cell formation which is named relative matching coefficient. Some researchers pointed out it is the one of most commonly used similarity coefficients for CF problem [27].

As the most commonly known SCM for cell formation, Islam’s relative matching coefficient model is defined between two machines or parts in terms of the incident 0-1 matrix. It is expressed as follows:

\[
S_{ij} = \frac{a + (ad)^{1/2}}{a + b + c + d + (ad)^{1/2}}
\]

where:
- \(S_{ij}\) is similarity coefficient between machines \(i\) and \(j\);
- \(a\) is number of machines process only part \(i\), or parts visit both machines \(i\) and \(j\);
- \(b\) is number of machines process only part \(i\), or parts visit only machine \(i\);
- \(c\) is number of machines process only part \(i^\prime\), or parts visit only machine \(j\);
- \(d\) is number of machines process neither part \(i\) nor \(i^\prime\), or parts visit neither machine \(j\) nor \(j^\prime\);
- \(i, j\) is index of machines;
- \(i^\prime, j^\prime\) is index of parts.

Islam et al. [29] considered most of well known similarity coefficients did not involve the interaction of \(a\) and \(d\) together in the numerator and fail to follow the properties of similarity coefficients. Hence, they were sure that the relative matching coefficient has the capability of conforming to all five properties: no mismatch, minimum match, no match, complete match, and maximum match. In their study, relative matching coefficient had been tested in term of a certain set of data and the test results supported model satisfactorily passed the qualifications for the similarity coefficients.

The relative matching coefficient is easy to understand and commonly applied in the area of CF. Nevertheless, it does not reach the case of diversified and iterative processes in the CMS. The part-machine incidence matrix in Islam’s model only tells us whether there is an operation between part and machine or not, but does not take operation sequences and times into account. We speculate as to if it might have some different results when operation sequences and times are considered into relative matching coefficient. In next section, it is an attempt to verify the speculation.

2.2 Improved Similarity Coefficient Method

Notation and variable:
is number of parts;
\(m\) is number of machines;
\(i', i\) is index of parts, \(i', i = 1, \ldots, p;\)
\(j', j\) is index of machines, \(j', j = 1, \ldots, m;\)
\(n_{i'}\) is the number of times machine \(j\) processes part \(i;\)
\(n_{i'}\) is the number of times machine \(j\) processes part \(i';\)
\(n_{i'}\) is the number of times part \(i\) visits machine \(j;\)
\(n_{i'}\) is the number of times part \(i\) visits machine \(j';\)
\(b\) is index of times part \(i\) visits machine \(j, b = 1, \ldots, n_{i'};\)
\(b'\) is index of times part \(i\) visits machine \(j', b' = 1, \ldots, n_{i'};\)
\(t\) is index of times machine \(j\) processes part \(i, t = 1, \ldots, n_{i'};\)
\(t'\) is index of times machine \(j\) processes part \(i', t' = 1, \ldots, n_{i'};\)
\(r_{ji}\) is the sequence number of machine \(j\) processes part \(i\) in the \((t)\)th time;
\(r_{ji'}\) is the sequence number of machine \(j\) processes part \(i'\) in the \((t')\)th time;
\(r_{ij}^b\) is the sequence number of part \(i\) visits machine \(j\) in the \((b)\)th time;
\(r_{ij'}^b\) is the sequence number of part \(i\) visits machine \(j'\) in the \((b)\)th time;
\(a_{ji}^1\) equates 1, if machine \(j\) processes both part \(i\) and \(i';\)
equates 0, otherwise;
\(b_{ji}^1\) equates 1, if machine \(j\) processes part \(i\), but does not process part \(i';\)
equates 0, otherwise;
\(c_{ji}^1\) equates 1, if machine \(j\) processes part \(i'\), but does not process part \(i;\)
equates 0, otherwise;
\(d_{ji}^1\) equates 1, if machine \(j\) processes neither part \(i\) nor \(i';\)
equates 0, otherwise;
\(d_{ji}^1\) equates 1, if part \(i\) visits both machine \(j\) and machine \(j';\)
equates 0, otherwise;
\(b_{ji}^1\) equates 1, if part \(i\) visits machine \(j\), but does not visit machine \(j';\)
equates 0, otherwise;
\(c_{ji}^1\) equates 1, if part \(i\) visits machine \(j'\), but does not visit machine \(j;\)
equates 0, otherwise;
\(d_{ji}^1\) equates 1, if part \(i\) visits neither machine \(j\) nor \(j';\)
equates 0, otherwise;
\(S_{ji}^{\text{OS}}\) is operation sequences-based similarity coefficient between part \(i\) and \(i';\)
\(S_{ji}^{\text{OS}}\) is operation sequences-based similarity coefficient between machine \(j\) and \(j';\)
\(S_{ji}^{\text{OT}}\) is operation times-based similarity coefficient between part \(i\) and \(i';\)
\(S_{ji}^{\text{OT}}\) is operation times-based similarity coefficient between machine \(j\) and \(j';\)
\(S_{ji}^{\text{ST}}\) is operation sequences and times-based similarity coefficient between part \(i\) and \(i';\)
\(S_{ji}^{\text{ST}}\) is operation sequences and times-based similarity coefficient between machine \(j\) and \(j'.\)

### 2.2.1 Operation Sequences-based Similarity Coefficient Method

Tam [30] presented three weights to develop an operation sequences similarity measure. But it is too subjective for judging similarity of parts or machines. Ho et al.[31] presented an operation sequence similarity coefficient that depends on two kinds of forward and backward indexes of compliant sequences of product. Yin et al.[27] extended the existing similarity coefficients to solve cell formation problems with proposing a definition of operation sequence ratio to modify the Jaccard similarity coefficients. However, according to Yin’s modified SCM, if one of parts visits both machines \(j\) and \(j',\) but the visit is a non-sequential manner, the similarity coefficient between machine \(j\) and \(j'\) will be 0. We think the similarity coefficient should be lower but more than 0, even if the non-sequential visits happen between two machines. Consequently, we trace a new method to overcome the shortcoming in the following model.

The designated and exclusive operation sequence for a given part is a set of ordered machines or processes. In order to improve SCM to solve cell formation problems with operation sequences, a new operation sequence ratio between part or machine pairs will be defined.

We deem that part \(i\) would be highly correlated with part \(i'\) if they visit machine \(j\) at the same sequences. If not, the correlation would be lower relatively. Therefore, the operation sequence ratio \(OSR_{ij}\) between parts \(i\) and \(i'\) is defined as follows:

\[
OSR_{ij'} = \frac{\sum_{j=1}^{m}(a_{ji}^1 \cdot K_{ji}^j)}{2 \cdot \sum_{j=1}^{m}[a_{ji}^1 \cdot \max(n_{i}^j, n_{i'}^j)]} \tag{2}
\]

where

if \(a_{ji}^1 = 0, K_{ji}^j = 0\)

if \(a_{ji}^1 = 1 \text{ and } n_{i}^j \geq n_{i'}^j, K_{ji}^j = \sum_{j=1}^{n_{i}^j} r_{ji}^j r_{ji'}^j\). where,

\[
r_{ji'}^j \begin{cases} 2 & \text{if } r_{ji'}^j = \forall r_{ji}, t' = 1, \ldots, n_{i'}^j \\ 1 & \text{otherwise} \end{cases}
\]

if \(a_{ji}^1 = 1 \text{ and } n_{i'}^j \geq n_{i}^j, K_{ji}^j = \sum_{j=1}^{n_{i'}^j} r_{ji}^j r_{ji'}^j\). where,

\[
r_{ji}^j \begin{cases} 2 & \text{if } r_{ji}^j = \forall r_{ji}, t = 1, \ldots, n_{i}^j \\ 1 & \text{otherwise} \end{cases}
\]

This paper proposes that the number of produced movements by operation sequences must be directly involved in the similarity coefficients measure between machines. That is, if machine \(j\) and \(j'\) process part \(i\) orderly, the movements caused by them would be minimum and the similarity coefficients between machine \(j\) and \(j'\) should be high. Hence, we sort the operated sequences of part \(i\) by machine \(j\) and \(j'\), and calculate movements based on the sorting to define the operation sequence ratio \(OSR_{ij'}\) between machine \(j\) and \(j'\) as follows:

\[
OSR_{ij'} = \frac{1}{\sum_{i=1}^{p} a_{ji}^1} \cdot \sum_{i=1}^{p} \left(\frac{H_{ji}^j}{n_{i}^j + n_{i'}^j - 1}\right) \tag{3}
\]

where

\(d_{ji}^1 \neq 0\)

\(H_{ji}^j = 1/(\alpha - \beta) + 1/(\beta - \gamma) + \cdots + 1/(\delta - \epsilon)\);

\(\alpha, \beta, \ldots, \epsilon \in r_{ji}^j, r_{ji'}^j\), and order by \(\alpha > \beta > \cdots > \epsilon\)

The relative matching coefficient showed in formula (1) is modified as following.
The operation times-based similarity coefficient $S^T_{jj}$ between machines $j$ and $j'$ is stated as follows:

$$S^T_{jj} = \frac{a_{jj} + \sqrt{a_{jj} \cdot d_{jj}}}{a_{jj} + b_{jj} + c_{jj} + d_{jj} + \sqrt{a_{jj} \cdot d_{jj}}}$$

(7)

where

$$a_{jj} = \sum_{p=1}^{n} a_{ij} \cdot n_{j} \cdot n_{j'}$$

$$b_{jj} = \sum_{p=1}^{n} b_{ij} \cdot n_{j} \cdot n_{j'}$$

$$c_{jj} = \sum_{p=1}^{n} c_{ij} \cdot n_{j} \cdot n_{j'}$$

$$d_{jj} = \sum_{p=1}^{n} d_{ij}$$

2.2.3 Operation Sequences and Times-based Similarity Coefficient Method

Now, from the elaboration of OSR and operation times-based SCM above, the finally operation sequences and times-based similarity coefficient for parts and machines are produced.

$$S^{ST}_{ii} = S^T_{ii} \cdot OSR_{ii}$$

(8)

Operation sequences and times in a given process are sure to be reflected into similarities and relationships of pairs of parts or machines. This paper designed them into modified SCM showed in the equation (8) and (9). In order to explain and evaluate the performance of improved SCM, we present some numerical examples in the next section.

3. Numerical Example and Computational Performance

To illustrate the feasibility and validity of the improved similarity coefficient, we find some examples from previous researches [27],[36],[37]. These previous researches had involved operation sequences and included some repeated operations in their examples. Hence, it is fit for utilizing the improved similarity coefficient measure to form cells production system. In order to compare with previous researches, the similarity coefficients of part pairs and machine pairs of the examples are attained with applications of the operation sequences-based SCM, operation times-based SCM, and operation sequences and times-based SCM. And then, we use SLINK, which is a classical and robust clustering algorithm for CF problems, to obtain part families and machine groups. However, we get the same results with adopted examples. After analysis, the reason is repeated operations in the examples are so few that the operation times are failed to influence cell formation in performance of the modified SCM. In view of this state, for manifesting some advantages of the improved SCM, an example with more repeated operations is designed.

3.1 Numerical Example

A numerical example is devoted to illustrate the mechanics of the improved SCM. Assume there are five parts and five machines in the production system, and the initial part-machine incidence matrix is given in Table 2.

|     | p1 | p2 | p3 | p4 | p5 |
|-----|----|----|----|----|----|
| m1  | 1.3| 1  | 1  | 1  | 1  |
| m2  | 1  | 2  | 2  | 2  | 4  |
| m3  | 3.5| 4  | 5  | 3  | 7  |
| m4  | 2  | 4  | 3  | 2.4| 6  |
| m5  | 7  | 4  | 5  |    |    |

In the Table 2, p1 means part 1 and m1 means machine 1. The elements in the matrix indicate the operation sequences of parts. Some of parts visit a machine more than once, such as part 1 visits machine 3 twice and machine 4 three times.

3.2 Computational Performance

Based on the initial part-machine incidence matrix and the algorithm above, we respectively use the methods of relative matching coefficient, operation sequences-based SCM, operation times-based SCM, and operation sequences and times-based SCM to calculate the similarity coefficients between each pair of parts and machines in this subdivision.

3.2.1 For Relative Matching Coefficient

The method of relative matching coefficient which has been introduced in section 2.1 is used to calculate similarity coefficients of part and machine pairs firstly. From the initial part-machine incidence matrix in Table 2, a triangular part similarity

$$S^{ST}_{jj} = S^T_{jj} \cdot OSR_{jj}$$

(9)
matrix \( S_{ij} \) and machine similarity matrix \( S_{ij} \) are constructed by using equation (1). For example, calculating procedure of similarity coefficient between parts 1 and 4: \( a=2, b=2, c=1, d=0 \), and the result is \( S_{14} = 0.4 \); also, similarity coefficient between machines 2 and 3: \( a=3, b=1, c=1, d=0 \), and the result is \( S_{23} = 0.6 \). Likewise, the similarity coefficients of part pairs and machine pairs could be computed and shown in Table 3 and 4.

### Table 3  Relative matching coefficients of parts

| \( S_{ij} \) | \( p1 \) | \( p2 \) | \( p3 \) | \( p4 \) | \( p5 \) |
|-------------|-------|-------|-------|-------|-------|
| \( p1 \)    | 0.40  | 0.40  | 0.60  | 0.40  | 0.60  |
| \( p2 \)    | 0.40  | 0.40  | 0.73  | 0.40  |       |
| \( p3 \)    | 0     | 0.40  | 0.60  |       |       |
| \( p4 \)    | 0     | 0.40  |       |       |       |
| \( p5 \)    |       |       |       |       | 0     |

### Table 4  Relative matching coefficients of machines

| \( S_{ij} \) | \( m1 \) | \( m2 \) | \( m3 \) | \( m4 \) | \( m5 \) |
|-------------|-------|-------|-------|-------|-------|
| \( m1 \)    | 0     | 0.60  | 0.60  | 0.40  | 0.40  |
| \( m2 \)    | 0     | 0.60  | 0.40  | 0.40  |       |
| \( m3 \)    | 0     | 0.40  | 0.40  |       |       |
| \( m4 \)    | 0     | 0.73  |       |       |       |
| \( m5 \)    |       |       |       |       | 0     |

3.2.2 For Operation Sequences-based Similarity Coefficient Method

The calculation manner of operation sequences-based SCM is the most complicated point in this study. Therefore, the following calculating example is quoted to illustrate its performance in detail. The examples are also about \( OSR_{14} \) between parts 1 and 4 and \( OSR_{23} \) between machines 2 and 3.

From the initial part-machine incidence matrix and equation (2), we can get operation sequence ratio between parts 1 and 4.

\[
a_{14}^1 = 0, a_{14}^2 = 1, a_{14}^3 = 1, a_{14}^4 = 0, a_{14}^5 = 0
\]

\[
K_{14}^1 = 0, K_{14}^2 = 2, K_{14}^3 = 3, K_{14}^4 = 0, K_{14}^5 = 0
\]

\[
\max (n_1^1, n_2^1) = n_1^1 = 2, \max (n_1^2, n_2^2) = n_2^2 = 2
\]

\[
\max (n_1^3, n_2^3) = n_1^3 = 3
\]

\[
\max (n_1^4, n_2^4) = n_1^4 = 1
\]

Finally, \( OSR_{14} = 0.63 \).

From the initial part-machine incidence matrix and equation (3), we can get operation sequence ratio between machines 2 and 3.

\[
a_{23}^1 = 1, a_{23}^2 = 1, a_{23}^3 = 1
\]

\[
H_{23}^1 = 1, H_{23}^2 = 0.5, H_{23}^3 = 1.5
\]

\[
n_1^2 = 1, n_2^2 = 2, n_2^3 = 1, n_2^4 = 1, n_2^5 = 1
\]

Finally, \( OSR_{23} = 0.58 \).

On the basis of equation (4) and (5), operation sequences-based similarity coefficients between parts 1 and 4, and between machines 2 and 3 can be achieved.

\[
S_{14}^S = S_{14} \cdot OSR_{14} = 0.25
\]

\[
S_{23}^S = S_{23} \cdot OSR_{23} = 0.35
\]

The same procedures are implemented for the part and machine pairs. All the results are showing in Table 5 and 6.

### Table 5  Operation sequences-based similarity coefficients of parts

| \( S_{ij}^S \) | \( p1 \) | \( p2 \) | \( p3 \) | \( p4 \) | \( p5 \) |
|-------------|-------|-------|-------|-------|-------|
| \( p1 \)    | 0.20  | 0.30  | 0.25  | 0.50  |       |
| \( p2 \)    | 0.33  | 0.58  | 0.25  |       |       |
| \( p3 \)    | 0     | 0.30  | 0.36  |       |       |
| \( p4 \)    | 0     | 0.25  |       |       |       |
| \( p5 \)    |       |       |       |       | 0     |

### Table 6  Operation sequences-based similarity coefficients of machines

| \( S_{ij}^S \) | \( m1 \) | \( m2 \) | \( m3 \) | \( m4 \) | \( m5 \) |
|-------------|-------|-------|-------|-------|-------|
| \( m1 \)    | 0.60  | 0.33  | 0.23  | 0.12  |       |
| \( m2 \)    | 0.35  | 0.33  | 0.13  |       |       |
| \( m3 \)    | 0.38  | 0.20  |       |       |       |
| \( m4 \)    | 0     | 0.61  |       |       |       |
| \( m5 \)    |       |       |       |       | 0     |

3.2.3 For Operation Times-based Similarity Coefficient Method

As well as the above computing examples, parts 1 and 4, machines 2 and 3 are applied to demonstrate the usage of operation times-based SCM again. The related data between parts 1 and 4 are given as follows.

\[
a_{14}^1 = 0, a_{14}^2 = 1, a_{14}^3 = 1, a_{14}^4 = 0, a_{14}^5 = 0
\]

\[
b_{14}^1 = 0, b_{14}^2 = 0, b_{14}^3 = 0, b_{14}^4 = 1, b_{14}^5 = 1
\]

\[
c_{14}^1 = 1, c_{14}^2 = 0, c_{14}^3 = 0, c_{14}^4 = 0, c_{14}^5 = 0
\]

\[
d_{14}^1 = 0, d_{14}^2 = 0, d_{14}^3 = 0, d_{14}^4 = 0, d_{14}^5 = 0
\]

\[
n_1^1 = 0, n_1^2 = 2, n_1^3 = 1, n_1^4 = 2, n_1^5 = 1
\]

\[
n_1^4 = 3, n_1^5 = 0, n_1^1 = 1, n_1^2 = 0
\]

Finally, the result \( S_{14}^T = 0.4 \) is calculated by using equation (6).

Also, the related data between machines 2 and 3 are given as follows.

\[
a_{23}^1 = 1, a_{23}^2 = 1, a_{23}^3 = 0, a_{23}^4 = 1, a_{23}^5 = 0
\]

\[
b_{23}^1 = 0, b_{23}^2 = 0, b_{23}^3 = 1, b_{23}^4 = 0, b_{23}^5 = 0
\]

\[
c_{23}^1 = 0, c_{23}^2 = 0, c_{23}^3 = 0, c_{23}^4 = 0, c_{23}^5 = 1
\]

\[
d_{23}^1 = 0, d_{23}^2 = 0, d_{23}^3 = 0, d_{23}^4 = 0, d_{23}^5 = 0
\]

\[
n_2^1 = 1, n_2^2 = 2, n_2^3 = 1, n_2^4 = 1, n_2^5 = 0
\]

\[
n_2^4 = 2, n_2^5 = 1, n_2^1 = 0, n_2^3 = 2
\]

Finally, the result \( S_{23}^T = 0.63 \) is calculated by using equation (7).

All operation times-based similarity coefficients of part and machine pairs are showing in Table 7 and 8.
3.2.4 For Operation Sequences and Times-based Similarity Coefficient Method

The final computational performance is achieved by operation sequences and times-based SCM which is blending relative matching coefficient, operation sequences and operation times together as equation (8) and (9). The more reasonable similarity coefficients of part and machine pairs are showing in Table 9 and 10.

Table 7 Operation times-based similarity coefficients of parts

| $S_{ij}$ | p1  | p2  | p3  | p4  | p5  |
|---------|-----|-----|-----|-----|-----|
| p1      | 0.33| 0.63| 0.40| 0.88|
| p2      | 0.50| 0.86| 0.44|
| p3      | 0.57| 0.63|
| p4      | 0   | 0.40|
| p5      | 0   |

Table 8 Operation times-based similarity coefficients of machines

| $S_{ij}$ | m1  | m2  | m3  | m4  | m5  |
|---------|-----|-----|-----|-----|-----|
| m1      | 0.76| 0.67| 0.36| 0.29|
| m2      | 0   | 0.63| 0.40| 0.33|
| m3      | 0   | 0.80| 0.57|
| m4      | 0   | 0.86|
| m5      | 0   |

The purpose of similarity coefficients acquisition is to group parts and machines for CF. Consequently, all the similarity coefficients in the above tables are employed to identify part families and machine groups by cluster analysis in the next part.

3.3 Clustering Analysis for Grouping

For solution of cell formation problem by SCM, existing two different solution methodologies - the optimal solution methodology and the heuristic approach [29]. The optimal solution methodology contains various mathematical programming models, network analysis models, material flow analysis models, and so on. These diversified models involved many production factors, e.g., machine capacities, setup time, cost, throughput. Heuristic approach is more brilliant in the previous literatures, such as McAuley’s single linkage clustering (SLINK) [17], Seifoddini and Wolfe’s average linkage clustering (ALINK) [32], Wei and Kern’s linear cell clustering (LCC) [33]. The SLINK clustering algorithm will be used to form part-families and machine-groups in this research. The classical SLINK was first presented by McAuley to form machines cells for CMS. The clustering results are shown in Table 11-14.

Table 11 Part-machine cells formed by relative matching coefficient

| m1  | p2  | p4  | p3  | p1  | p5  |
|-----|-----|-----|-----|-----|-----|
| 1.3 | 1   | 1   | 1   | 1   |
| 2   | 2.4 | 2   | 1   |
| 4   | 5   | 3.5 | 3.7 |
| 3   | 2.4,6 | 2.4,6 |
| 4   | 7   | 5   |

Table 12 Part-machine cells formed by operation sequences-based SCM

| m1  | p2  | p4  | p3  | p1  | p5  |
|-----|-----|-----|-----|-----|-----|
| 1.3 | 1   | 1   | 1   | 1   |
| 2   | 2.4 | 2   | 1   |
| 4   | 5   | 3.5 | 3.7 |
| 3   | 2.4,6 | 2.4,6 |
| 4   | 7   | 5   |

Table 13 Part-machine cells formed by operation times-based SCM

| m1  | p2  | p4  | p3  | p1  | p5  |
|-----|-----|-----|-----|-----|-----|
| 1.3 | 1   | 1   | 1   | 1   |
| 2   | 2.4 | 2   | 1   |
| 4   | 5   | 3.5 | 3.7 |
| 3   | 2.4,6 | 2.4,6 |
| 4   | 7   | 5   |

The matrix tables for grouping parts and machines are the desired block-diagonal matrix like all the CF problems solved by SCM. From the cell formation results in the above tables 11-14, two cells as the bold fonts marking have been obtained by all the four types of different similarity coefficients measure. However, each of them contains exceptional elements what create interactions between two cells. The next is to evaluate the results for identifying which one is the best cell-formation. Hence, for evaluating the performance of a block-diagonal matrix, some evaluation indicators and approaches are presented below.

3.4 Result Analysis and Evaluation

Four similarity coefficient methods - relative matching coefficient, operation sequences-based SCM, operation times-based SCM, and operation sequences and times-based SCM, will be evaluated by measuring quality of the formed cells. Several evaluation criteria have been developed in literatures, such as grouping efficiency, grouping efficacy, the sum of intercellular and intracellular material handling costs. In this paper, we use three measure criterions to evaluate and compare the performances of part-machine cells formed by the four SCMs. They are grouping efficiency, grouping efficacy and the number of intercell movements.

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Grouping efficiency had been proposed by Chandrasekharan and Rajagopalan first [34].

Grouping efficiency,

\[
\eta = q \cdot \eta_1 + (1 - q) \cdot \eta_2
\]

Where

\[
\eta_1 = \frac{\text{Number of entries '1' in diagonal blocks}}{\text{Total number of elements in diagonal blocks}}
\]

\[
\eta_2 = \frac{\text{Number of entries '0' in off diagonal blocks}}{\text{Total number of elements in off diagonal blocks}}
\]

\[
q \in (0, 1) \text{ is a weighting factor, and a value of } q = 0.5 \text{ is commonly used.}
\]

In this case, we develop the grouping efficiency as follows.

Grouping efficiency,

\[
\eta' = q \cdot \eta_1' + (1 - q) \cdot \eta_2'
\]

Where

\[
\eta_1' = \frac{(\sum_{i=1}^{p} \sum_{j=1}^{m} E_{ij})/((\sum_{i=1}^{p} \sum_{j=1}^{m} E_{ij} + E_d))}{e}
\]

\[
\eta_2' = \frac{e_o/((\sum_{i=1}^{p} \sum_{j=1}^{m} e_{ij} + e_o))}{e}
\]

\[
q \in (0, 1) \text{ is a weighting factor, and a value of } q = 0.5 \text{ is commonly used.}
\]

Kumar et al.[35] developed the grouping efficiency and proposed other new criterion called grouping efficacy. This expression has the requisite properties like non-negativity and zero to one range. Moreover, it is not affected by the size of matrix.

Grouping efficacy,

\[
\tau = 1 - \frac{\psi}{1 + \phi}
\]

Where

\[
\psi = (\sum_{i=1}^{p} \sum_{j=1}^{m} e_{ij})/e
\]

\[
\phi = E_d/e
\]

The third evaluation criterion is the number of intercell movements. Number of intercell movements is one of quite important and original purposes for CF. In this paper, although operation sequences and times are considered conditions in the procedure of CF, intercell movements are also generated obviously in term of production processing. However, the number of intercell movements is different in the four results.

In accordance with elaborations, the evaluation results of 4 similarity coefficient measures - relative matching coefficient, operation sequences-based SCM, operation times-based SCM, and operation sequences and times-based SCM are calculated and shown in following Table 15.

As can be seen from Table 15, it is distinct that the comprehensive evaluation results of operation sequences and times-based SCM are better than the others. From the comparison of results, operation sequences and times could influence the performance of CF obviously. There are two major problems with the different results - the number of exceptional operations in off-diagonal blocks and the number of voids in diagonal blocks. In this case, the exceptional operations in Table 11 are more than the other CF performances, and void in diagonal blocks generated by operation sequences-based SCM and operation times-based SCM. However, total number of exceptional operations and voids in diagonal blocks in Table 14 is minimum in the four CF results. Consequently, it could be said that operation sequences and times-based SCM possesses more advantages. It is necessary and feasible to consider operation sequences and times simultaneously into similarity measurement of part pairs and machine pairs for CF.

4. Conclusion

In this paper, a new improved SCM is presented in part-machine cell formation. This research formulates a calculation method for getting more valid similarity measurement of each pair of parts or machines in CMS with some repeated operations. Unlike previous methods, the proposed method provides consideration of operation sequences and times simultaneously to improve a traditional SCM. In actual CMS, the repeated operations are so frequent that the operation sequences and times are quite important factors in the design of part-machine cells. Obviously, similarity between a pair of parts will be prominent, if the two parts visit machine at the same sequences. In addition, it is the same case, if a pair of machines process part in order. Furthermore, if repeated operations are caused in parts process routing, corresponding similarity of parts and machines should be different to the cases without repeated operations. In these senses, we propose operation sequences-based SCM, operation times-based SCM and operation sequences and times-based SCM respectively to modify the existing relative matching coefficients and compare which one is the best for CF.

Several numerical examples from previous researches show that the improved method can form part-machine cells effectively but the results can not manifest advantages of consideration both operation sequences and times into CF. Consequently, in order to illustrate superiority of the operation sequences and times-based SCM, other numerical example with more repeated operations is provided. And SLINK is applied to cluster part-machine cells. The computational results indicate that the only consideration of operation sequences or times is not enough to reflect the true similarity of parts or machines in CMS with diversified and repeated operations. Under such circumstances, the performance of operation sequences and times-based SCM is comparatively more efficient and available. And it is necessary and feasible to consider both operation sequences and times into SCM for attaining reasonable and effective part-machine cells.

The proposed SCM can also consider other factors such as production volume, movement cost, machine capacity, process time. And a dedicated clustering algorithm also needs further research in the future.
Table 15 Summary of evaluation results

| Evaluation criterion | RMC | OSSCM | OTSCM | OTSCM* |
|----------------------|-----|-------|-------|--------|
| Grouping efficiency  | 0.73| 0.73  | 0.73  | 0.77   |
| Grouping efficacy    | 0.70| 0.75  | 0.75  | 0.78   |
| Number of intercell movements | 10  | 5     | 5     | 5      |

1: Relative Matching Coefficient
2: Operation Sequences-based SCM
3: Operation Times-based SCM
4: Operation Sequences and Times-based SCM

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