Quintessential Inflation in a thawing realization

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We study quintessential inflation with an inverse hyperbolic type potential \( V(\phi) = V_0/\cosh(\phi^n/\lambda^n) \), where \( V_0, \lambda \) and “n” are parameters of the theory. We obtain a bound on \( \lambda \) for different values of the parameter n. The spectral index and the tensor-to-scalar-ratio fall in the 1σ bound given by the Planck 2015 data for \( n \geq 5 \) for certain values of \( \lambda \). However, for \( 3 \leq n < 5 \) there exist values of \( \lambda \) for which the spectral index and the tensor-to-scalar-ratio fall only within the 2σ bound of the Planck data. Furthermore, we show that the scalar field with the given potential can also give rise to late time acceleration if we invoke the coupling to massive neutrino matter. We also consider the instant preheating mechanism with Yukawa interaction and put bounds on the coupling constants for our model using the nucleosynthesis constraint on relic gravity waves produced during inflation.

I. INTRODUCTION

The standard model of the Universe needs to be modified at early and late times in order to address the problems therein. It is interesting to note that both the early and late time shortcomings of the model can be successfully circumvented by introducing an early phase of accelerated expansion called inflation together with late time cosmic acceleration. The inflationary paradigm solves the horizon problem, the flatness problem and the monopole problem and provides a mechanism for generation of primordial perturbations \([1,7]\). Late time cosmic acceleration is a recent discovery \([8,9]\) backed up by observations of type Ia supernovae, CMB background and galaxy clustering. It requires the presence of an exotic matter of energy momentum tensor with a large negative pressure dubbed dark energy \([10–17]\). Cosmological constant is a recent discovery \([8,9]\) for which the spectral index and the tensor-to-scalar-ratio fall in the 1σ bound given by the Planck 2015 data for \( n \geq 5 \) for certain values of \( \lambda \). However, for \( 3 \leq n < 5 \) there exist values of \( \lambda \) for which the spectral index and the tensor-to-scalar-ratio fall only within the 2σ bound of the Planck data. Furthermore, we show that the scalar field with the given potential can also give rise to late time acceleration if we invoke the coupling to massive neutrino matter. We also consider the instant preheating mechanism with Yukawa interaction and put bounds on the coupling constants for our model using the nucleosynthesis constraint on relic gravity waves produced during inflation.
to the scalar fields becomes more prominent. The latter ensures the appearance of a minimum in the field potential at late times which is desirable for the commencement of late time cosmic acceleration. In this paper we follow in the footsteps of the above authors and discuss a model with inverse cosine-hyperbolic potential that might successfully give rise to quintessential inflation.

The inverse cosine-hyperbolic potential is a tachyonic type potential and is inspired from Sring theory. Some of the works related to tachyonic potential can be found in Refs. [28, 29]. The inverse cosine hyperbolic type potential is mentioned in the Refs. [28, 29]. However, it is not studied in detail by considering the cosmological data of Planck 2015. Further, the late time evolution of the Universe is not studied yet. In this paper we generalize the inverse cosine hyperbolic potential to describe quintessential inflation.

II. INFLATION AND OBSERVATIONAL CONSTRAINTS

As mentioned in the introduction, we are looking for a scenario that would facilitate slow roll at early times followed by scaling behavior in the post inflationary regime such that the exit to late time acceleration is caused by non-minimally coupled massive neutrinos. To this effect, we shall consider the following action,

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{1}{2} \partial^\mu \chi \partial_\mu \chi - V(\chi) \right] + S_m + S_\nu (C^2(\phi) g_{\mu\nu} : \psi_\nu) + S_R. \tag{1}
\]

with,

\[
V(\phi) = \frac{V_0}{\cosh \left( \frac{\phi}{\lambda}\right)} = \frac{V_0}{\cosh[\beta^n (\phi/M_p)^n]} \tag{2}
\]

\[
C \sim e^{\gamma \phi/M_p} \tag{3}
\]

where \( \lambda = \alpha M_p; \beta = \frac{1}{n} \) and \( \alpha, \beta \) are dimensionless parameters and \( n \) is a positive integer.

Eq. (1) includes the actions for standard matter \((S_m)\), massive neutrino \((S_\nu)\), and radiation \((S_R)\). We have assumed that the scalar field is directly coupled to massive neutrino matter whereas the dark matter is minimally coupled. As for inflation, a remark about the matter actions is in order. Since matter is generated during reheating, the said actions should be dropped while disussing inflation.

In what follows, we specialize to a flat FRW background to obtain the evolution equations for the action (1),

\[
3H^2 M_p^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi), \tag{4}
\]

\[
\left(2 \dot{H} + 3H^2 \right) M_p^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi) \tag{5}
\]

and

\[
\ddot{\phi} + 3H \dot{\phi} + \frac{dV}{d\phi} = 0. \tag{6}
\]

The slow roll parameters for a potential \( V(\phi) \) are defined as usual \( 23, 32 \),

\[
\epsilon = \frac{M_p^2}{2} \left( \frac{1}{V} \frac{dV}{d\phi} \right)^2, \tag{7}
\]

and

\[
\eta = \frac{M_p^2}{V} \frac{d^2V}{d\phi^2}. \tag{8}
\]

The end of inflation is marked by,

\[
\epsilon|_{\phi=\phi_{end}} = 1, \tag{9}
\]

where “end” represents the value at the end of inflation. Let us consider a period which begins when the modes cross the horizon and ends with the end of inflation. Then the number of e-foldings during this period is given by \( 23, 32 \),

\[
N = M_p^{-1} \int_{\phi_{end}}^{\phi} \frac{d\phi}{\sqrt{2\epsilon(\phi)}} \tag{10}
\]

\[
= \frac{1}{M_p} \int_{\phi_{end}}^{\phi} \frac{V(\phi')}{V'(\phi')} d\phi'. \tag{11}
\]

The tensor to scalar ratio \( r \) is given by,

\[
r = 16\epsilon, \tag{12}
\]

and the scalar spectral index \( n_s \), which is defined as,

\[
n_s - 1 = \frac{d(\log P_R)}{d(\log k)}, \tag{13}
\]

where \( P_R \) is the spectrum of curvature perturbations, is reduced to the form,

\[
n_s = 2\eta - 6\epsilon + 1. \tag{14}
\]

We now study a model based on a potential given by Eq. (2) for general \( n \) and derive expressions for the slow roll parameters and the spectral index in terms of \( n \). Introducing a dimensionless scalar field \( \chi = \frac{\phi}{M_p} \) and using Eq. (7), Eq. (8) and Eq. (2), we obtain,

\[
\epsilon = \frac{1}{n^2} \beta^2 \chi^{2n-2} \tanh^2 (\beta^n \chi^n), \tag{15}
\]

\[
r = 16\epsilon = 8n^2 \beta^2 \chi^{2n-2} \tanh^2 (\beta^n \chi^n), \tag{16}
\]

and
\eta = -n\beta^2 (\beta \chi)^{n-2} \left[ n(\beta \chi)^n + (n-1) \tanh (\beta^n \chi^n) - 2n(\beta \chi)^n \tanh^2 (\beta^n \chi^n) \right]. \tag{17}

From Eq. (14), Eq. (16) and Eq. (17) we also obtain

\begin{align*}
  n_s &= 1 - 2n^2 \beta^2 n^{2n-2} - 2n(n-1) \beta^n \chi^{n-2} \tanh (\beta^n \chi^n) + n^2 \beta^2 n^{2n-2} \tanh^2 (\beta^n \chi^n). \tag{18}
\end{align*}

Eq. (11) gives us

\begin{align*}
  \frac{n}{\sqrt{2}} \beta^n \chi^{n-1} \tanh (\beta^n \chi^n)|_{\chi = \chi_{\text{end}}} &= 1. \tag{19}
\end{align*}

From Eq. (11) we obtain,

\begin{align*}
  N_k &= \frac{1}{n\beta^n} \int^{\chi_{\text{end}}}_{\chi} \coth (\beta^n \chi^n) \frac{d\chi}{\chi^{n-1}}. \tag{20}
\end{align*}

We now set the number of e-foldings to be 60. Comparing the theoretical value of the power spectrum,

\begin{equation}
  P_R = \frac{1}{24\pi^2 M_{pl}^4} \frac{V}{\epsilon}, \tag{21}
\end{equation}

where \(M_{pl} = (8\pi G_N)^{-\frac{1}{2}} = 2.4 \times 10^{18}\) GeV, with its observational value \(P_R \approx 2.19 \times 10^{-9}\) and combining it with Eq. (2) and Eq. (15), one derives an estimate for \(V_0\),

\begin{equation}
  V_0 = (2.14916 \times 10^{68}) \beta^2 n \chi_i^{2n-2} \cosh (\beta^n \chi_i^n) \tanh^2 (\beta^n \chi_i^n). \tag{22}
\end{equation}

where, \(\chi_i\) is the value of \(\chi\) at the beginning of inflation.

According to the Planck 2015 results [33], \(n_s = 0.968 \pm 0.006\) at the 1\(\sigma\) confidence level. For \(n = 1\), Eqns. (19) and (18) reduce to

\begin{equation}
  \epsilon = \frac{\beta^2}{2} \left( \tanh (\beta \chi_{\text{end}}) \right)^2 = 1 \tag{23}
\end{equation}

and

\begin{equation}
  n_s = 1 - \beta^2 \left[ 2 - \frac{2e^{-2N\beta^2}}{\beta^2 - 2 + 2e^{-2N\beta^2}} \right], \tag{24}
\end{equation}

respectively, where the expression for \(n_s\) is derived analytically by inverting the equation,

\begin{equation}
  N(k) = \frac{1}{\beta^2} \left[ \ln \left( \sqrt{\frac{2}{\beta^2 - 2}} \right) - \ln |\sinh (\beta \chi)| \right] \tag{25}
\end{equation}

to obtain \(\sinh (\beta \chi)\) and hence \(\tanh (\beta \chi)\). Since \(\tanh (\beta \chi_{\text{end}})\) can take a maximum value of one, therefore, for inflation to end and hence for \(\epsilon\) to be unity, \(\beta \geq \sqrt{2}\). However, for \(\beta = \sqrt{2}\), \(n_s = -1\) and as the value of \(\beta\) increases further \(n_s\) becomes more and more negative. For \(n = 2\), we have a positive value of \(n_s\) but it still does not fall in the observational range as shown in Fig. (11). As we increase the value of \(n\), the theoretical value of \(n_s\) shifts towards the observed value. From Fig. (11) it is clear that for \(n = 3\) and \(4\), there exist no values of \(\beta\) for which \(n_s\) lies in this range at the 1\(\sigma\) confidence.
For $r \sim 10^{-6}$, $N = 60$, we find that $\delta \phi \gtrsim 0.2 M_{pl}$ and Lyth bound is satisfied as stated.

III. PREHEATING BASED UPON INSTANT PARTICLE PRODUCTION

In the paradigm of quintessential inflation, the underlying potential is typically of a run away type. Thereby the standard mechanism of (p)reheating\[32–38\] can not be implemented in this case. One of the possible alternatives is provided by gravitational particle production. However, this process is inefficient and as a result the field spends a long time in the kinetic regime ($\rho_\phi \sim 1/a^6$). The energy density of gravity waves which were produced at the end of inflation becomes sufficiently larger than $\rho_\phi$ during the kinetic regime and this leads to a violation of the nucleosynthesis constraint. The nucleosynthesis constraint restricts the duration of the kinetic regime and we can also put a lower bound on the reheating temperature using this constraint. Therefore, we require an efficient reheating mechanism to address the issue. The instant reheating mechanism turns out to be suitable.

In what follows, we shall consider a scenario where the inflaton field $\phi$ interacts with another scalar field $\chi$ and we assume that the scalar field $\chi$ interacts with the matter field $\psi$ such that\[39,\]

$$\phi \xrightarrow{g} \chi \xrightarrow{h} \psi \bar{\psi}. \quad (27)$$

Let us choose the following interaction terms in the Lagrangian,

$$\mathcal{L}_{int} = -\frac{1}{2}g^2 \phi^2 \chi^2 - h \bar{\psi} \psi \chi \quad (28)$$

in order to implement the aforesaid. Here $g$ and $h$ are positive coupling constants\[34, 40, 43\]. The form of the Lagrangian is chosen such that $\chi$ field has no bare mass and its effective mass is given by $m_\chi = g |\phi| \[44\]$. Particle production commences after inflation provided that $m_\chi$ changes non-adiabatically, $m_\chi \gtrsim m_\chi^2$. The mass of produced $\chi$ particles grows as the field evolves to larger values after the end of inflation and $\chi$ decays into various species of particles. Assuming that all the energy is instantaneously converted into radiation and thermalized, one can obtain the following estimate (see Appendix A for details),

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{end} \approx \frac{3}{2} \left(\frac{2\pi}{g^2}\right)^3 \quad (29)$$

As mentioned before, the energy density of gravity waves becomes significantly larger than $\rho_\phi$ during the kinetic regime and this might later lead to a violation of the nucleosynthesis constraint during the radiative regime. The said constraint,

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{eq} \lesssim 10^{-2}, \quad (30)$$

We should emphasize that the permitted range of parameters in the model leads to small numerical values of $r$, thereby, the Lyth bound \[34\] is easily satisfied in the sub-Planckian region. Indeed, using Eq. \[\text{(11)}\],

$$N = \frac{1}{M_{pl}} \int_{\phi_{in}}^{\phi_{end}} \frac{d\phi}{\sqrt{2\epsilon}} \rightarrow N \approx \frac{\phi_{in} - \phi_{end}}{M_{pl} \sqrt{2\epsilon_{min}}} \approx \frac{\delta \phi}{M_{pl} \sqrt{2\epsilon_{min}}}, \quad (26)$$

As $\epsilon$ is a monotonically increasing function in the model under consideration, $\epsilon_{min} = \epsilon_{in} = r/16$ and the range of inflation is given by, $\delta \phi \gtrsim N \sqrt{2\epsilon_{in}} M_{pl} = N \sqrt{r/8} M_{pl}$.
FIG. 3: $\omega_i$, $\Omega_i$ and $\ln(\rho/\rho_{c,0})$ are plotted as a function of $\ln(1 + z)$ in the sub-figures (a), (b) and (c) respectively for $n = 6$. The normalized potential $V_0 = \frac{\omega_0}{\cosh [\beta (\rho/\rho_r)^{n/3}]}$ is plotted for $n = 5$ and $n = 6$ using the blue and red-dashed lines respectively in the sub-figure (d).

where, \( \left( \frac{p_\phi}{p_r} \right)_{eq} = \frac{64}{3\pi} h_{GW}^2 \left( \frac{p_\phi}{p_r} \right)_{end} \) combined with the value of the dimensionless gravity wave amplitude $h_{GW}$ calculated for our model (see appendix A) puts an upper bound on $\rho_\phi/\rho_r$, 

\[
\left( \frac{\rho_\phi}{\rho_r} \right)_{end} \lesssim 2.53 \times 10^{11} \tag{31}
\]

at the end of inflation. This gives us a lower bound on $(\rho_r)_{end}$, $(\rho_r)_{end} \gtrsim 2.48 \times 10^{49} GeV^4$ and hence a bound on the reheating temperature, $(T_r)_{end} \gtrsim 2.2 \times 10^{12} GeV$, is obtained. Using Eq. (29) together with Eq. (31), we find out the lower limit on $g$. The limit on $h$ can be set by avoiding the back reaction of $\chi$ particles on the post inflationary dynamics. We find a comfortable range in the parameter space $(g, h)$ which can give rise to efficient preheating after inflation (see Appendix A for details).

IV. LATE TIME ACCELERATION WITH NON-MINIMAL COUPLING TO MASSIVE NEUTRINO MATTER

In the absence of non-minimal coupling, the scalar field with a potential given by Eq. (2) can not give rise to late time acceleration as the potential is steep. To obtain late time acceleration one needs to introduce a feature in the potential which can be achieved by coupling the scalar field non minimally to massive neutrino matter. The coupling of the scalar field with neutrinos is not prominent in the radiation era while it plays a crucial role during the late stages of evolution where it induces a minimum in the potential giving rise to a de Sitter solution. The result is a late time attractor. As mentioned before, it is desirable to leave the matter era intact and assume that the field couples to massive neutrino matter only.

The neutrinos here are relativistic $(p_\nu = \frac{E_\nu}{c})$ like radiation during early times but turn non relativistic $(p_\nu = 0)$ at late times. This behaviour can be mimicked by the
following ansatz for parametrization of $\omega_\nu(z)$,

$$\omega_\nu(z) = \frac{p_\nu}{\rho_\nu} = \frac{1}{6} \left( 1 + \tanh \left[ \frac{\ln(1+z) - z_{eq}}{z_{dur}} \right] \right). \quad (32)$$

The equation of state $\omega_\nu(z)$ is thus chosen keeping in mind the phase transition from the relativistic state to the non relativistic one. $z_{eq}$ and $z_{dur}$ give the time and duration of the transition. The equation of continuity for massive neutrinos, in presence of coupling, takes the form, \[33\] \[34\].

$$\dot{\rho}_\nu + 3H (\rho_\nu + p_\nu) = \gamma (\rho_\nu - 3p_\nu) \frac{\dot{\phi}}{M_{pl}}. \quad (33)$$

Varying the action \[1\] with respect to $\phi$ and following the Ref. \[34\], we obtain,

$$\ddot{\phi} + 3H \dot{\phi} = -\frac{\partial V}{\partial \phi} - \gamma (\rho_\nu - 3p_\nu). \quad (34)$$

We note that in the radiation era when the neutrino is relativistic, in Eqs \[33\] and \[34\], the coupling term of the scalar field and neutrinos vanishes since $\rho_\nu - 3p_\nu = 0$. Thus the coupling becomes effective only when the neutrinos become non-relativistic. We thus have a system of equations,

$$3H^2 M_{pl}^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi) + \rho_m + \rho_r + \rho_\nu, \quad (35)$$

$$(2\dot{H} + 3H^2)M_{pl}^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi) - \frac{1}{3} \rho_r - p_\nu, \quad (36)$$

$$\ddot{\phi} + 3H \dot{\phi} = -\frac{\partial V}{\partial \phi} - \gamma (\rho_\nu - 3p_\nu), \quad (37)$$

$$\dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) = \gamma (\rho_\nu - 3p_\nu) \frac{\dot{\phi}}{M_{pl}}. \quad (38)$$

$$\dot{\rho}_i + 3H (\rho_i + p_i) = 0; \quad i = \text{matter and rad.}, \quad (39)$$

which we can solve to obtain the evolution of the various energy densities. To solve this system of differential equations, we introduce the dimensionless variables $x$, $y$, $\omega_\nu$, $\Omega_m$, $\Omega_r$, and $\lambda$ where $x = \frac{\phi}{\sqrt{6}HM_{pl}}$, $y = \frac{\sqrt{V}}{\sqrt{6}HM_{pl}}$, $\Omega_m = \frac{\rho_m}{3H^2M_{pl}^2}$, $\Omega_r = \frac{\rho_r}{3H^2M_{pl}^2}$, $\Omega_\nu = \frac{\rho_\nu}{3H^2M_{pl}^2}$ and $\lambda = -M_{pl} \frac{V_{,\phi}}{V}$. The evolution equations can then be cast in a convenient form,

$$x' = \frac{x}{2} \left[ 3(\omega_\nu - 1) + (1 - 3\omega_\nu) \Omega_r - 3\omega_\nu \Omega_m \right. \right. \right.$$  

$$- 3\omega_\nu x^2 - (3\omega_\nu + 3) y^2 \right] + \frac{3}{2} x^3 + \sqrt{\frac{3}{2}} \frac{\lambda}{2} x y^2 \right] + \sqrt{\frac{3}{2}} (3\omega_\nu - 1) \gamma (1 - \Omega_m - \Omega_r - x^2 - y^2), \quad (40)$$

$$y' = \frac{y}{2} \left[ 3(\omega_\nu + 1) + (1 - 3\omega_\nu) \Omega_r - 3\omega_\nu \Omega_m \right. \right. \right.$$  

$$+ 3(1 - \omega_\nu) x^2 - 3\omega_\nu y^2 - \sqrt{6} x \lambda \right] - \frac{3}{2} x y^2, \quad (41)$$

$$\Omega'_m = \Omega_m \left[ 3\omega_\nu + (1 - 3\omega_\nu) \Omega_r - 3\omega_\nu \Omega_m \right. \right. \right.$$  

$$+ 3(1 - \omega_\nu) x^2 - 3(\omega_\nu + 1) y^2 \right], \quad (42)$$

$$\omega_\nu' = \frac{2\omega_\nu}{z_{dur}} (3\omega_\nu - 1), \quad (43)$$

$$z' = \sqrt{6} x. \quad (45)$$

Here, prime $'$ is derivative w.r. to $N$ and we have used the constraint, $\Omega_\nu = 1 - \Omega_r - \Omega_m - x^2 - y^2$. We now solve this dynamical system numerically for $\beta = 1$ and plot these variables taking the initial conditions at the end of inflation. The initial values of $x$, $y$, $z$, $\omega_\nu$, $\Omega_m$ and $\Omega_r$ are approximately $10^{-51}$, $10^{-51}$, 0.88, 0.3, $10^{-23}$ and 1 respectively. $z_{dur}$ and $\gamma$ are 6.9 and 100 respectively. The initial value of $\Omega_\nu$ is $10^{-7}$. From the plots of Fig. \[3\], it is clear that we can obtain late time acceleration with neutrino coupling using these initial conditions, since we can obtain the current values of $\Omega_\phi$, $\Omega_m$ and $\omega_\phi$ which are approximately 0.7, 0.3 and $-1$ respectively. Similar graphs were plotted for $\beta = 2, 3$ etc. and the same result was obtained. A comment regarding the thawing nature of evolution is in order. The potential \[2\] belongs to the class of tracker potentials. Indeed, One can easily check that our potential behaves as $V \sim \exp(-\beta^3 \phi^6/M_{pl}^6)$ \[15\] asymptotically for large values of $\phi$. However, on the contrary, we see a thawing behaviour. It is clear from subfigure (3d) that after inflation ends, the potential drops sharply resulting in a deep overshoot of $\rho_\phi$ with respect to the background energy density. Clearly, it takes long time for the field to recover from freezing which happens when the background energy density become comparable to $\rho_\phi$. When it happens, the field has already evolved to the slow roll regime near the minimum of the potential and consequently, tracker regime is missed out.

In this case nucleosynthesis poses no constraint on the slope of the potential unlike the case of standard exponential potential.
V. CONCLUSIONS

In this paper, we have considered a single scalar field model with a potential $V_0 / \cosh(\phi^n / \lambda^n)$ and demonstrated that the model can give rise to quintessential inflation. First, we have demonstrated the viability of the theory for inflation. In particular, we have shown that for lower values of $n$, the model is ruled out by observation. For instance if $n = 2$, the theoretical value of $n_s$ fails to lie in the observed range. However, situation improves for higher values of $n$: potential flattens significantly in this case. In particular, for $n = 3$ and 4, the considered potential provides a value of $n_s$ such that it can lie in the range of $2 - \sigma$ confidence level of Planck 2015 data. The value of $\beta$ is greater than 0.17 and 0.085 for $n = 3$ and 4 respectively. The most profound results are obtained for $n = 5$ and 6. The theoretical value of $n_s$ satisfies the Planck 2015 data at the 1$\sigma$ confidence level and its value becomes $\sim 0.9625$ and $\sim 0.963$ for $\beta \geq 0.16$ and $\beta \geq 0.09$ for $n = 5$ and 6 respectively. It should be noted that our model mimics the hill-top potential [32] for $\beta \sim 1$ as $\phi << M_p$. It is therefore not surprising that the tensor-to-scalar ratio $r$ is negligibly small in our case similar to that of the Starobinsky-model [40]. Nevertheless, the very small value of the tensor-to-scalar ratio is a salient feature of our model compared to other models discussed in the literature. If we take $\beta$ to be much less than unity, we find that the value of $r$ is significantly greater than $10^{-6}$. Further, we have plotted the evolution of normalized values of different components of the energy densities $\Omega_\rho, \omega_\nu, \omega_\phi$ and the energy density of the scalar field $\rho_\phi$. The plots explain that the universe undergoes late time acceleration, i.e., for today, $\Omega_\rho \sim 0.7, \Omega_m \sim 0.3$ and $\omega_\phi = -1$. Though the underlying potential belongs to the tracker class, the thawing behaviour manifests itself in the post-inflationary region. This behaviour depends on the specific initial conditions taken at the end of inflation. The latter then leads to a deep overshoot of the field and consequently misses the tracker region of the potential. Thus, in the model under consideration (in a thawing realization), the scalar field which drives inflation in the early Universe is also responsible for late time acceleration once we couple the scalar field to massive neutrino matter. Finally, we considered the instant preheating mechanism for particle production where we assumed that the scalar field couples to the matter field via Yukawa interaction. We have put bounds on the coupling constants $g$ and $h$ defined in the Yukawa interaction which are found to be $g \geq 3.8 \times 10^{-5}$ and $h \geq 2.5 \times 10^{-3}/\sqrt{g}$ such that $(T_r)_{\text{end}} \geq 2.2 \times 10^{12}\text{GeV}$. These values satisfy the nucleosynthesis constraint. We have also studied the spectrum of relic gravity waves in our model (see appendix B): the generic feature of the scenario includes the blue spectrum of gravity waves on scales smaller than comoving horizon scales. Fig. 4 shows the energy spectrum for $n = 6$ and $\beta = 1$ as a function of the wavelength $\lambda$ together with the sensitivity curves of advLIGO and LISA.

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Appendices

Appendix A: Instant preheating

In order to implement the instant particle production, we consider the following Lagrangian,

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}g^2 \dot{\phi}^2 \chi^2 - hv\psi \chi, \quad (46)$$

where $g$ and $h$ are positive coupling constants with a restriction, $g, h < 1$ such that a perturbative treatment is viable for the Lagrangian [40, 34, 40, 43]. The effective mass of $\chi$ can be read of from [40] as, $m_\chi = g |\phi|$. [44]. Production of $\chi$ particles takes place after inflation provided $m_\chi$ changes non adiabatically, [16, 34, 40, 43]

$$m_\chi \gtrsim m_\chi^2 \rightarrow \dot{\phi} \gtrsim g\phi^2. \quad (47)$$

Let us now confirm that the above condition is met in the model under consideration. To this effect, we estimate $\phi_{\text{end}}$ using the expression of slow roll parameter, $\epsilon = 4\pi G \dot{\phi}^2 / \Pi^2$,

$$\phi_{\text{end}}^2 \sim V_{\text{end}} \rightarrow |\phi_{\text{end}}| \gtrsim V_{\text{end}}^{1/2} \quad (48)$$
The condition for particle production then becomes,

\[ |\phi| \lesssim |\phi_{\text{prod}}| \]

\[ = \sqrt{\frac{\dot{\phi}_{\text{end}}}{g}} \]

\[ = \sqrt{\frac{V_{\text{end}}}{g}} \rightarrow g^2 \gtrsim M_{\text{pl}}^{-4} V_{\text{end}} (\phi_{\text{pd}} \leq M_{\text{pl}}), \quad (49) \]

Using Eq. (49), we have following expression,

\[ |\phi| \approx |\phi_{\text{prod}}| = g^{-\frac{1}{2}} |\phi_{\text{end}}|^{-\frac{1}{2}}. \]

The occupation number for \( \chi \) particles is given by the following expression,

\[ n_k \approx e^{-\frac{\pi \rho}{\sqrt{8} \sqrt{g}}} \]

\[ = e^{-\left(\frac{\pi \rho}{\sqrt{8} \sqrt{g}}\right)} \]

\[ = e^{-\frac{\pi \rho}{\sqrt{8} \sqrt{g}}} \]

\[ = e^{-\frac{\pi \rho}{\sqrt{8} \sqrt{g}}} \]

We can then find the number density and energy density of created particles,

\[ N_{\chi} = \frac{1}{(2\pi)^3} \int_0^\infty n_K d^3K \]

\[ \simeq \left(\frac{g |\phi_{\text{end}}|}{(2\pi)^3}\right)^{\frac{3}{2}} \]

\[ \rho_{\chi} = N_{\chi} m_{\chi} \simeq \left(\frac{g |\phi_{\text{end}}|}{(2\pi)^3}\right)^{\frac{3}{2}} |\phi_{\text{end}}|. \]

Now plugging \( |\phi_{\text{b}}| \approx g^{-\frac{1}{2}} |\phi_{\text{end}}|^{-\frac{1}{2}} \) in the above equation, one finds,

\[ \rho_{\chi} \simeq \frac{g^2 |\phi_{\text{end}}|^2}{(2\pi)^3}. \]

Substituting \( |\phi_{\text{end}}| \approx V_{\text{end}}^{\frac{3}{2}} \) in the above equation, \( \rho_{\chi} \) reduces to

\[ \rho_{\chi} \simeq \frac{g^2 V_{\text{end}}}{(2\pi)^3}. \]

Using Eq. (49) and \( \rho_{\phi} = \frac{g^2}{2} + V(\phi) \),

\[ (\rho_{\phi})_{\text{end}} \simeq \frac{3}{2} (\phi_{\text{end}}^2) \simeq \frac{3}{2} V_{\text{end}}. \]

Combining Eq. (55) and Eq. (57) yields,

\[ \left(\frac{\rho_{\phi}}{\rho_{\chi}}\right)_{\text{end}} \simeq \frac{3 (2\pi)^3}{2 g^2}. \]

Assuming that the \( \chi \) field gets converted into radiation and thermalization takes place instantaneously, we have,

\[ \rho_r \simeq \rho_{\chi}. \]

We thus obtain,

\[ \left(\frac{\rho_{\phi}}{\rho_r}\right)_{\text{end}} \simeq \frac{3 (2\pi)^3}{2 g^2}. \]

The nucleosynthesis constraint \[34\] dictates that

\[ \left(\frac{\rho_{\phi}}{\rho_r}\right)_{eq} \lesssim 10^{-2}, \]

where

\[ \left(\frac{\rho_{\phi}}{\rho_r}\right)_{eq} = \frac{64}{3\pi} h^2_{\text{GW}} \left(\frac{\rho_{\phi}}{\rho_r}\right)_{\text{end}} \]

Combining \( h^2_{\text{GW}} = \frac{H^2_{\text{in}}}{M_{\text{pl}}^2} \) and \( H^2_{\text{in}} = \frac{V_{\text{in}}}{3M_{\text{pl}}^2} \) with Eq. (60), Eq. (61) and Eq. (62), one obtains the following inequality,

\[ (\rho_{r})_{\text{end}} \gtrsim \frac{400 V_{\text{in}} V_{\text{end}}}{3\pi M_{\text{pl}}}. \]

Considering the form of our potential at the beginning and end of inflation, the above inequality reduces to,

\[ (\rho_{r})_{\text{end}} \gtrsim \frac{400}{3\pi} \frac{M_{\text{pl}}^4}{M_{\text{pl}}^4 \cosh (\beta n^2 \chi_{\text{end}}^2)} \]

Using \( M_{\text{pl}} = 2.4 \times 10^{18} \text{GeV} \) together with \( \chi_{\text{end}} = 0.88, \chi_{\text{in}} = 0.44 \) and \( V_0 = 4.64 \times 10^{60} \text{GeV}^4 \), where these values are obtained for \( n = 6 \) and \( \beta = 1 \), the following bound on \( (\rho_{r})_{\text{end}} \) is calculated,

\[ (\rho_{r})_{\text{end}} \gtrsim 2.48 \times 10^{49} \text{GeV}^4. \]

Using \( (T_r)_{\text{end}} \sim [(\rho_{r})_{\text{end}}]^{1/4} \), the bound on \( (T_r)_{\text{end}} \) can be written as

\[ (T_r)_{\text{end}} \gtrsim 2.2 \times 10^{12} \text{GeV}. \]

Using Eq. (67), the inequality Eq. (68) can be rewritten as

\[ \left(\frac{\rho_{\phi}}{\rho_r}\right)_{\text{end}} \lesssim \frac{9 n M_{\text{pl}}^4}{800 V_{\text{in}}} = \frac{9 n M_{\text{pl}}^4 \cosh (\beta n^2 \chi_{\text{in}}^2)}{800 V_0}. \]
Evaluating the expression on the right hand side for $\beta = 1$ and $n = 6$ as above, the above inequality is calculated as,

$$\left(\frac{\rho_{\phi}}{\rho_{\text{end}}}\right)_{\text{end}} \lesssim 2.53 \times 10^{11}. \quad (68)$$

Plugging Eq. (60) in Eq. (68), we obtain the bound on $g$,

$$g \gtrsim 3.8 \times 10^{-5}. \quad (69)$$

We now calculate the bound on $h$. For the reaction, $\chi \to \tilde{\psi} \psi$, the decay width $\Gamma_{\tilde{\psi} \psi}$ satisfies the following inequality [16, 34–44, 47],

$$\Gamma_{\tilde{\psi} \psi} \gg H_{\text{end}} \to h^2 \gtrsim \frac{8\pi H_{\text{end}}}{g|\phi|}. \quad (70)$$

Now using $H^2 = \frac{8\pi G}{3} \rho_{\phi}$, Eq. (67) and $|\phi| \leq M_{pl}$ for our potential one obtains

$$h^2 \gtrsim \frac{4\pi \sqrt{2} V_{\text{end}}}{gM_{pl}^2} = \frac{4\pi \sqrt{2} V_0 / (\cosh (\beta n \chi_{\text{end}}))}{gM_{pl}^2}, \quad (71)$$

which simplifies to

$$h \gtrsim \frac{2.5 \times 10^{-3}}{\sqrt{g}}, \quad (72)$$

for $n = 6$ and $\beta = 1$. From Eq. (69), $g$ can take any value greater than or equal to $3.8 \times 10^{-5}$ and based on the value taken by $g$, $h$ satisfies Eq. (72). The bounds on $g$ and $h$ that are calculated above are such that $(T_F)_{\text{end}}$ always satisfies the inequality (66).

**Appendix B: Relic Gravity Wave Spectrum**

One of the tests for inflationary models is the measurement of the spectrum of gravity waves. In this appendix, considering the given reheating temperature, we estimate the spectrum of the relic gravity wave. The gravitational wave equation in flat FRW space time can be written in its standard form as,

$$h''_k (\tau) + 2 \frac{d^2}{d\tau} h'_k (\tau) + k^2 h_k (\tau) = 0. \quad (73)$$

This equation describes how gravity wave evolves with time in a flat FRW Universe. The energy spectrum is defined as [48],

$$\Omega_{gw} (k, \tau) = \frac{1}{\rho_{\text{crit}}(\tau)} \frac{d ((|\rho_{gw}(\tau)| |0\rangle)}{d (\ln k)} \quad (74)$$

where $\rho_{\text{crit}} = \frac{3H^2(\tau)}{8\pi G}$ and the gravitational energy density $\rho_{gw}$ is given by

$$\rho_{gw} = -T^0_0 = \frac{1}{64\pi G} \frac{(h''^2_i) + (\nabla h_i)^2}{a^2}. \quad (75)$$

It can be shown that the spectrum of gravity waves depends on the equation of state of the dominant fluid comprising the universe [24, 48]. Therefore, the gravitational waves can be categorized according to three epochs of the universe - the matter dominated regime, the radiation dominated regime and the kinetic regime, and in these regimes the energy spectrum of relic gravity waves is given by the following expressions [24]:

$$\Omega_{gw}^{(M.D.)} (\lambda) = \frac{3}{8\pi^3} h_{G.W.}^{2} \Omega_{om} \left(\frac{\lambda}{\lambda_{h}}\right)^2, \quad \lambda_{M.D.} < \lambda \leq \lambda_{h} \quad (76)$$

$$\Omega_{gw}^{(R.D.)} (\lambda) = \frac{1}{6\pi} h_{G.W.}^{2} \Omega_{om}, \quad \lambda_{R.D.} < \lambda \leq \lambda_{M.D.} \quad (77)$$

$$\Omega_{gw}^{(kin)} (\lambda) = \Omega_{gw}^{(R.D.)} \left(\frac{\lambda_{R.D.}}{\lambda}\right), \quad \lambda_{kin} < \lambda \leq \lambda_{R.D.}, \quad (78)$$

where, $\lambda_h = 2cH_0^{-1} \approx 1.8 \times 10^{26} \text{h^{-1} cm} \approx 2.57 \times 10^{28} \text{cm}$. $\lambda_{M.D.}$, $\lambda_{R.D.}$ can be estimated using the boundary conditions: $\Omega_{gw}^{(M.D.)}|_{\lambda=\lambda_{M.D.}} = \Omega_{gw}^{(R.D.)}|_{\lambda=\lambda_{M.D.}}$ and $\Omega_{gw}^{(kin)}|_{\lambda=\lambda_{R.D.}} = \Omega_{gw}^{(R.D.)}|_{\lambda=\lambda_{R.D.}}$. So, we have,

$$\lambda_{M.D.} = 2\pi H_0 \left(\frac{\Omega_{om}}{\Omega_{om}}\right)^{1/2}, \quad (79)$$

$$\lambda_{R.D.} = 4\pi \left(\frac{\Omega_{om}}{T_{M.D.}}\right)^{1/2} \frac{T_{M.D.}}{T_{rh}}, \quad (80)$$

and $\lambda_{kin}$ is given by

$$\lambda_{kin} = cH_{kin}^{-1} \left(\frac{T_{rh}}{T_0}\right) \left(\frac{H_{kin}}{H_{rh}}\right)^{1/2}. \quad (81)$$

The gravity waves with wavelength $\lambda < \lambda_{R.D.}$ are generated during the kinetic regime ($\omega \sim 1$). The spectrum of these waves is inversely proportional to the wavelength. During the radiation phase, $\lambda_{R.D.} < \lambda < \lambda_{M.D.}$, it is constant. However, for $\lambda > \lambda_{M.D.}$, the waves are created in the matter phase and its spectrum increases according to Eq. (76). Now we calculate $\lambda_{M.D.}$, $\lambda_{R.D.}$ and $\lambda_{kin}$.

$$\lambda_{M.D.} = 3.10607 \times 10^{26} \text{cm}. \quad (82)$$

Here, we plugged $10^{-5}$ and 0.3 for $\Omega_{om}$ and $\Omega_{om}$ respectively.

$$\lambda_{R.D.} = 3.64851 \times 10^{5} \text{cm}, \quad (83)$$

where, we used $T_{M.D.} = 1.3524 \text{eV}$ and $T_{rh} = 2.2 \times 10^{12} \text{GeV}$.

$$\lambda_{kin} = cH_{kin}^{-1} \left(\frac{T_{rh}}{T_0}\right) \left(\frac{H_{kin}}{H_{rh}}\right)^{1/2}. \quad (84)$$
Using $H_{\text{kin}} \approx H_{\text{rh}} \approx H_{\text{end}}$ and $H_{\text{end}}^{-1} = (4\pi G V_{\text{end}})^{\frac{1}{2}}$ we get,
\[
\lambda_{\text{kin}} = \frac{c}{\sqrt{4\pi G V_{\text{end}}}} \times \left( \frac{T_{\text{rh}}}{T_0} \right) . \tag{85}
\]
We have $n = 6$, $\beta = 1$, $\chi_{\text{end}} = 0.88$, $V_0 = 4.64 \times 10^{60} \text{GeV}^4$,
\[
V_{\text{end}} = \frac{V_0}{\cosh(\beta n \chi_{\text{end}})} = 4.18 \times 10^{60} \text{GeV}^4. \tag{86}
\]
Substituting $V_{\text{end}}, T_0 = 2.34813 \times 10^{-15} \text{GeV}$ and $T_{\text{rh}} = 2.2 \times 10^{12} \text{GeV}$ in Eq. (85), $\lambda_{\text{kin}}$ turns out to be,
\[
\lambda_{\text{kin}} = 0.306992 \text{ cm}. \tag{87}
\]
Now using $h_{\text{G.W.}}^2 = \frac{h_{\text{kin}}^2}{4M_{\text{pl}}^2}$ and $H_{\text{in}}^2 = \frac{V_0}{4W_{\text{pl}}}$ one obtains,
\[
h_{\text{G.W.}}^2 = \frac{V_0}{24M_{\text{pl}}^4 \cosh(\beta n \chi_{\text{in}})}. \tag{88}
\]
Using $\beta = 1$, $\chi_{\text{in}} = 0.44$, $V_0 = 4.64 \times 10^{60} \text{GeV}^4$ and $M_{\text{pl}} = 2.4 \times 10^{18} \text{GeV}$, $h_{\text{G.W.}}^2$ can be written as
\[
\frac{h_{\text{G.W.}}^2}{5.82139 \times 10^{-15}}, \tag{89}
\]
which simplifies Eqs. (78), (86) and (88) as following,
\[
\Omega_g^{(M.D.)}(\lambda) = 3.1978 \times 10^{-74} \lambda^2,
\]
for $3.1 \times 10^{26} \text{ cm} < \lambda \leq 2.57 \times 10^{28} \text{ cm}, \tag{90}
\]
\[
\Omega_g^{(R.D.)}(\lambda) = 3.0983 \times 10^{-21},
\]
for $3.64851 \times 10^5 \text{ cm} < \lambda \leq 3.10 \times 10^{26} \text{ cm}, \tag{91}
\]
\[
\Omega_g^{(\text{kin})}(\lambda) = \frac{1.12678 \times 10^{-15}}{\lambda},
\]
for $0.306992 \text{ cm} < \lambda \leq 3.64851 \times 10^5 \text{ cm}. \tag{92}
\]

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