Doubly Lopsided Mass Matrices from Supersymmetric SU(N) Unification

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Abstract

It is shown that in supersymmetric SU(N) models with $N > 5$ the so-called “doubly lopsided” mass matrix structure can emerge in a natural way. The non-trivial flavor structure is entirely accounted for by the SU(N) gauge symmetry and supersymmetry, without any “flavor symmetry”. The hierarchy among the families results directly from a hierarchy of scales in the chain of breaking from SU(N) to the Standard Model group. A simple SU(7) example is presented.
1 Introduction

In a recent paper, it was shown that grand unified theories (GUTs) based on the groups $SU(N)$, with $N > 5$, can lead to a non-trivial flavor structure for the known quarks and leptons even in the absence of flavor symmetries [1]. The central point is that the different families, which all transform in the same way under either the standard model group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ or $SU(5)$, can transform differently under $SU(N)$ [2]. Since they transform differently, the mixing of the families is inhibited by $SU(N)$. Thus it is the “vertical” unification group rather than a “horizontal” symmetry that distinguishes the families and produces a non-trivial flavor structure. Moreover, the flavor structure that typically arises for the most economical sets of quarks and leptons is a “doubly lopsided” one [3], which is known to reproduce many of the qualitative features of the observed pattern of quark and lepton masses and mixings.

In Ref. 1, a non-supersymmetric $SU(8)$ model was presented that illustrated these ideas. An appealing feature of non-supersymmetric models of this type is that the family hierarchy can be a “radiative” one, with the heaviest quarks and leptons (maybe only the top quark) getting mass from renormalizable tree-level terms and the lighter quarks and leptons getting mass from higher-dimension operators induced by loops. However, if one supersymmetrizes such models, these loops are suppressed by factors of $M_{SU SY}/M_{GUT}$ due to the non-renormalization theorems. In this paper we look at supersymmetric $SU(N)$ models in which there is a non-radiative fermion mass hierarchy. In these models the smaller elements of the quark and lepton mass matrices are suppressed by powers of $M_N/M_{Pl}$, where $M_N$ is a symmetry-breaking scale associated with the breaking of $SU(N)$ down to $G_{SM}$. There can be several such scales. In fact, in the $SU(7)$ example that will be described, one gauge-symmetry-breaking scale controls the masses of the second family and another controls the masses of the first family. An interesting feature, then, of the kinds of models we are proposing is that a hierarchy in the scales of breaking of the “vertical” unification group is directly reflected in the “horizontal” mass hierarchy among the families.

The paper is organized as follows. In section 2, a very brief review will be given of the basic idea of doubly lopsided models and of how $SU(N)$ GUTs naturally produce a hierarchical and doubly lopsided structure. In section 3, a simple supersymmetric $SU(7)$ model is described and the roles played
by the grand unified symmetry and supersymmetry in restricting the form of the mass matrices is explained. In section 4, a different version of the $SU(7)$ model is briefly discussed, in which only the top quark obtains mass from renormalizable Yukawa terms and the other fermions get mass from higher-dimension, Planck-scale-suppressed operators. The final section gives a summary and conclusions.

2 Lopsided models and $SU(N)$ unification

The basic idea of doubly lopsided models [3] is that there is large mixing among the three $\mathbf{5}$ multiplets of quarks and leptons and small mixing among the three $\mathbf{10}$ multiplets. (We use $SU(5)$ language in discussing the fermion multiplets for convenience. The chain of breaking of the grand unified group need not actually go through $SU(5)$.) For example, suppose the mixing between the $\mathbf{10}_2$ and $\mathbf{10}_3$ is of order $\epsilon \ll 1$, the mixing between $\mathbf{10}_1$ and $\mathbf{10}_2$ is of order $\delta \ll 1$, and the mixings among the $\mathbf{5}$'s are all of order 1. Then, since the mass matrices of the up-type quarks, down-type quarks, charged leptons, and light neutrinos appear in terms respectively of the form, $\mathbf{10}_i(M_U)_{ij}\mathbf{10}_j$, $\mathbf{10}_i(M_D)_{ij}\mathbf{5}_j$, $\mathbf{5}_i(M_L)_{ij}\mathbf{10}_j$, and $\mathbf{5}_i(M_\nu)_{ij}\mathbf{5}_j$, these matrices have the form

$$M_U \sim \begin{pmatrix} \delta^2 \epsilon^2 & \delta \epsilon^2 & \delta \\ \delta \epsilon^2 & \epsilon^2 & \epsilon \\ \delta \epsilon & \epsilon & 1 \end{pmatrix} m,$$

$$M_D \sim \begin{pmatrix} \delta \epsilon & \delta \epsilon & \delta \epsilon \\ \epsilon & \epsilon & \epsilon \\ 1 & 1 & 1 \end{pmatrix} m',$$

$$M_\nu \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} m_\nu,$$

$$M_L \sim \begin{pmatrix} \delta \epsilon & \epsilon & 1 \\ \delta \epsilon & \epsilon & 1 \\ \delta \epsilon & \epsilon & 1 \end{pmatrix} m'.$$

Note that the mass matrices in Eq. (1) are written in the convention that they are multiplied from the left by the left-handed fermions and from the right by right-handed fermions. The $\sim$ symbol means that only the order in $\delta$ and $\epsilon$ of the matrix elements is given. Inspection of these matrices shows that there is a relatively steep mass hierarchy among the up-type-quark masses ($m_u/m_c \sim \delta^2$, $m_c/m_t \sim \epsilon^2$), a less steep hierarchy among the down-type-quark masses and among the charged-lepton masses ($m_d/m_s \sim m_e/m_{\mu} \sim \delta$, $m_s/m_b \sim m_{\mu}/m_{\tau} \sim \epsilon$), and a very weak hierarchy among the neutrino masses. These predictions correspond to what is observed. Inspection of the
mass matrices also reveals that there are small mixing angles for the left-handed quarks, since they are in the 10’s ($V_{cb} \sim \epsilon$, $V_{us} \sim \delta$, $V_{ub} \sim \delta \epsilon$) and large ($O(1)$) mixing angles for the left-handed leptons, since they are in the $\overline{5}$’s. This pattern of angles also corresponds to what is observed.

As Eq. (1) shows, the mass matrices of the down-type quarks and charged leptons are highly asymmetrical, which is the reason for the name “lopsided”. In (singly) lopsided models [4], [5] only the 23 and 32 elements of these mass matrices are highly asymmetrical, explaining the fact that the large atmospheric neutrino mixing angle is large while the corresponding quark mixing angle $V_{cb}$ is small. ($U_{\mu 3} \equiv \sin \theta_{atm} \sim 1$, $V_{cb} \sim \epsilon$.) In the doubly lopsided models, there is assumed also to exist a large asymmetry in the 13 and 31 elements, as in Eq. (1). This asymmetry explains the fact that the solar neutrino angle is large while the corresponding quark angle $\sin \theta_{C} = V_{us}$ is small. ($U_{e2} \equiv \sin \theta_{sol} \sim 1$, $V_{us} \sim \delta$.)

As explained in Ref. 1, the doubly lopsided pattern emerges naturally in $SU(N)$ grand unification, even without any flavor symmetry. The reason has to do with the embedding of the three families in the multiplets of $SU(N)$. If the three families of light quarks and leptons come from antisymmetric tensor representations of $SU(N)$, then the most economical way to cancel $SU(N)$ anomalies with a specified number of families in the low-energy spectrum is to have a few larger tensors (i.e. rank $> 1$) plus many anti-fundamental representations. Some examples will be given later. The 10’s of $SU(5)$ are contained in the larger tensors and typically transform differently under the $SU(N)$, whereas typically all the $\overline{5}$’s of $SU(5)$ are contained in the anti-fundamentals of $SU(N)$ and therefore transform in exactly the same way under $SU(N)$. Since the $\overline{5}$’s are not distinguished from each other by the symmetries of the theory, they tend naturally to mix strongly with each other, whereas the 10’s can only mix with each other to the extent that the symmetries of $SU(N)$ that distinguish them are broken, and therefore their mixing is suppressed.

In the model of Ref. 1, for example, the quarks and leptons are contained in the anomaly free set of $SU(8)$ representations $56 + 28 + 9(\overline{5}) = \psi^{[ABC]} + \psi^{[AB]} + \psi_{(m)A}$, where the indices $A, B, C (= 1, ..., 8)$ are $SU(8)$ indices, while $m (= 1, ..., 9)$ just labels the nine anti-fundamental representations. In $SU(8)$, this is the most economical anomaly-free set of fermions that gives three families. When this set is decomposed under $SU(5)$ it gives $4(10) + \overline{10} + 9(\overline{5}) + 6(5) + 31(1)$, which leaves a low-energy residue of three ($10 + \overline{5}$).
families after vectorlike pairs get superlarge mass. The 10’s are \( \psi^{\alpha\beta} \), \( \psi^{\alpha\beta6} \), \( \psi^{\alpha\beta7} \), and \( \psi^{\alpha\beta8} \) (where \( \alpha, \beta = 1, ..., 5 \) are \( SU(5) \) indices), which obviously transform differently under \( SU(8) \). On the other hand, the \( 5 \)'s are all of the form \( \psi_{(m)\alpha} \) and are not distinguished in any way by the gauge symmetries.

3 An illustrative supersymmetric model

The model we will study in this paper has gauge group \( SU(7) \) and quarks and leptons in the following multiplets: \( 35 + 2(21) + 8(7) = \psi^{[ABC]} + \psi^{[AB]} + \psi_{(m)A} \), where \( A, B, C (= 1, ..., 7) \) are \( SU(7) \) indices, and \( a (= 1, 2) \) and \( m (= 1, ..., 8) \) are labels distinguishing multiplets of the same type. When decomposed under \( SU(5) \) (which it is convenient to use to classify the fermions, even if the chain of symmetry breaking does not go through \( SU(5) \)), this gives

\[
\begin{align*}
\psi^{[ABC]} &= 35 &\rightarrow &\bar{10} + 2(10) + 5 \\
&= \psi_{\alpha\beta\gamma} + \psi_{\alpha\beta I} + \psi_{\alpha67}, \\
\psi^{[AB]} &= 2(21) &\rightarrow &2(10) + 4(5) + 2(1) \\
&= \psi_{(a)\beta} + \psi_{(a)I} + \psi_{67}, \\
\psi_{(m)A} &= 8(7) &\rightarrow &8(\bar{5}) + 16(1) \\
&= \psi_{(m)\alpha} + \psi_{(m)I},
\end{align*}
\]

where \( \alpha, \beta, \gamma = 1, ..., 5 \) are \( SU(5) \) indices and \( I = 6, 7 \) are \( SU(2)' \) indices of the \( SU(5) \times SU(2)' \times U(1)' \) subgroup of \( SU(7) \). (This is one of the most economical three-family sets of \( SU(7) \) fermions, having a total of 11 multiplets with 133 components. Another economical set, which gives a similar model, is \( 2(35) + 21 + 7(7) \), which has 10 multiplets and 140 components. The set with the fewest components is \( 3(21) + 9(7) \), which has 12 multiplets with 126 components. The set with the fewest multiplets is \( 3(35) + 6(7) \), which has 9 multiplets with 147 components. However, in these last two possibilities there is simply a triplication of multiplets, so that \( SU(7) \) does not distinguish among the families. These four sets are the most economical by far, the next simplest sets having 189 and 196 components. So of the four simplest possibilities in \( SU(7) \), two give models of the type being proposed.)

The Higgs content of the model consists of the following types of Higgs
superfields (there can be several of each type): adjoint multiplets $\Omega^A$, plus the totally antisymmetric tensor multiplets $H^A$, $\overline{H}^A$, $H^{[AB]}$, $\overline{H}^{[AB]}$, $H^{[ABC]}$, and $\overline{H}^{[ABC]}$. All the $G_{SM}$-singlet components of these Higgs multiplets are assumed to have superlarge VEVs, namely $H^I$ and $\overline{H}_I$ ($I = 6, 7$), $H^{67}$, $\overline{H}^{67}$, and $\Omega^A$. All the components that transform under $G_{SM}$ in the same way as the neutral components of $H_u$ and $H_d$ of the MSSM are assumed to get weak-scale VEVs, namely $H_2$, $H_2^I$ ($I = 6, 7$), $H_2^{67}$, $\overline{H}_2$, $\overline{H}_2^{67}$, and $\overline{H}_2^{67}$. (We use the convention that $\alpha = 1, 2$ are $SU(2)_L$ indices, and $\alpha = 3, 4, 5$ are $SU(3)_c$ indices.) It should be noted that there must be a “matter parity” symmetry to distinguish “Higgs” multiplets from “matter” (i.e. quark and lepton) multiplets. This is typical of supersymmetric models. However, no “flavor” symmetry exists that distinguishes among the matter multiplets or among the Higgs multiplets.

The most general renormalizable Yukawa superpotential contains the following couplings (where we use the obvious notation that $[p]$ refers to a rank-$p$ antisymmetric tensor and $[\bar{p}]$ refers to its conjugate, the subscript $L$ refers to a left-handed supermultiplet of quarks and leptons, and the subscript $H$ refers to a Higgs supermultiplet): the

\begin{align}
([3]_L[2]_L)[2]_H &= a_a(\psi^{ABC}\psi^{DE}_{(a)})H^{FG}\epsilon_{ABCDEFG}, \\
([3]_L[1]_L)[2]_H &= b_m(\psi^{ABC}\psi_{(m)A})\overline{H}_{BC}, \\
([2]_L[2]_L)[3]_H &= c_{ab}(\psi^{AB}_{(a)}\psi^{CD}_{(b)})H^{EFG}\epsilon_{ABCDEFG}, \\
([2]_L[1]_L)[1]_H &= d_{am}(\psi^{AB}_{(a)}\psi_{(m)A})\overline{H}_B, \\
([1]_L[1]_L)[2]_H &= e_{mn}(\psi_{(m)A}\psi_{(n)B})H^{AB}.
\end{align}

Even though it is assumed that there may several copies of Higgs multiplets of the same type, no index has been used to distinguished among them in Eq. (3). Note that there is no $([3]_L[3]_L)[1]_H$ term listed in Eq. (3), since such a term vanishes identically by the antisymmetry of $[3]_L$.

Of the terms in Eq. (3), only the first contributes to a superlarge mass term for the $SU(5)$ $\mathbf{10}$ of fermions that sits in the $[3]_L$ multiplet. In particular, this term contains $a_a(\psi^{\alpha\beta\gamma}_{(a)}\psi^{\delta\epsilon}_{(a)})H^{67}\epsilon_{\alpha\beta\gamma\delta\epsilon67}$, which “mates” the $\mathbf{10} = \psi^{\alpha\beta\gamma}$
to one linear combination of the two $\psi^{(\delta)}_{\alpha\beta}$, leaving the other linear combination light. Without loss of generality, one can define the superheavy linear combination to be $\psi^{(\delta)}_{(2)}$. Altogether, there remain three light $10$'s, which it will be convenient to denote by $10_1 \equiv \psi^{\delta\gamma\delta}_{(1)}$, $10_2 \equiv \psi^{\delta\epsilon\delta}_{(2)}$, and $10_3 \equiv \psi^{\delta\delta}_{(1)}$.

Turning now to the superlarge masses of the $\mathbf{5}$'s and $\mathbf{5}'$'s, one sees that the term $([3]_L[1]_L)[2]_H$ in Eq. (3) contains $b_{m}(\psi^{\alpha\beta\gamma\delta}_{(2)} H^{267}) \epsilon^{\alpha\beta\gamma\delta}$, which couples the $\mathbf{5}$ in the rank-3 tensor to a $\mathbf{5}$, and the term $([2]_L[1]_L)[1]_H$ contains $d_{am}(\psi^{\alpha\beta}_{(a)} \psi^{(m)\delta}_{(m)}) H_{111}$, which couples the $\mathbf{5}$'s in the rank-2 tensor to $\mathbf{5}'$'s. (If there is only one $\mathbf{1}^H_H$ multiplet, then not all of the $\mathbf{5}$'s in the rank-2 tensor get mass from this term; however, we will assume in the model presented this section that there are at least two $\mathbf{1}^H_H$ multiplets.) What remains light are three $\mathbf{5}'$'s, all of which come, of course, from the anti-fundamentals of $SU(N)$ and have the form $\psi^{(\alpha)}_{(m)\alpha}$. Without loss of generality one can define the three light $\mathbf{5}'$'s to be $\mathbf{5}'_1 \equiv \psi^{(1)}_{(1)\alpha}$, $\mathbf{5}'_2 \equiv \psi^{(2)}_{(1)\alpha}$, and $\mathbf{5}'_3 \equiv \psi^{(3)}_{(1)\alpha}$.

One is now in a position to understand how the weak-scale masses of the light quarks and leptons arise in this model. First consider the $10$ to $10$ couplings that give mass to the up-type quarks. The third term in Eq. (3), namely $([3]_L[1]_L)[1]_H$, contains $c_{11}(\psi^{\alpha\beta\gamma\delta}_{(1)} H^{267}) \epsilon^{\alpha\beta\gamma\delta}$. Since $\psi^{(1)}_{(1)} \equiv \mathbf{10}_3$, this term gives a $33$ element to $M_U$, the mass matrix of the up-type quarks.

The first term in Eq. (3), namely $([3]_L[2]_L)[2]_H$, contains $a_{11}(\psi^{\alpha\beta\gamma\delta}_{(1)} H^{27}) - (\psi^{\alpha\beta\gamma\delta}_{(1)} H^{267}) \epsilon^{\alpha\beta\gamma\delta}$, which give contributions to the $13$, $31$, $23$, and $32$ elements of $M_U$. (It should be noted that if there were only one $H^{AB}$ multiplet, then $([3]_L[2]_L)[2]_H$ would only involve the superheavy field $\psi^{(4)}_{(2)}$; not the light field $\psi^{(3)}_{(1)}$. This is not so, however, if more than one $H^{AB}$ multiplet exists, as is assumed in the model described in this section.)

There is no renormalizable Yukawa term that couples $\mathbf{10}_1 \equiv \psi^{\delta\gamma\delta}_{(1)}$ and $\mathbf{10}_2 \equiv \psi^{\delta\epsilon\delta}_{(2)}$ to themselves or to each other. The only renormalizable term that could do so would be a $([3]_L[3]_L)[1]_H$ coupling $(\psi^{\alpha\beta\gamma\delta}_{(1)} H^{267}) \epsilon^{\alpha\beta\gamma\delta}$. However, as already noted, this vanishes identically by the antisymmetry of the indices. Thus, the most general set of renormalizable terms consistent with $SU(7)$ and supersymmetry gives a $M_U$ of the form

$$M_U \sim \begin{pmatrix} 0 & 0 & \delta \epsilon \\ 0 & 0 & \epsilon \\ \delta \epsilon & \epsilon & 1 \end{pmatrix} v_u,$$  \hspace{1cm} (4)
This matrix has been written in a way that suggests that the 13 and 31 elements of $M_U$ are much smaller than the 23 and 32 elements, and that they in turn are much smaller than the 33 element, as in Eq. (1). This would not generally be the case, of course, but would be if there were the following hierarchy among the VEVs of the Higgs fields:

$$
\langle H^2 \rangle, \langle H^{267} \rangle \sim v_u \\
\gg \langle H^{27} \rangle \sim \epsilon v_u \\
\gg \langle H^{26} \rangle \sim \delta \epsilon v_u,
$$

(5)

It will be assumed that this hierarchy holds, as well as the related hierarchies

$$
\langle \overline{H}_2 \rangle, \langle \overline{H}^{267} \rangle \sim v_d \\
\gg \langle \overline{H}^{27} \rangle \sim \epsilon v_d \\
\gg \langle \overline{H}^{26} \rangle \sim \delta \epsilon v_d,
$$

(6)

and

$$
\langle H^{67} \rangle, \langle \overline{H}^{67} \rangle \sim M_{P\ell} \\
\gg \langle H^7 \rangle, \langle \overline{H}_6 \rangle \sim \epsilon M_{P\ell} \\
\gg \langle H^6 \rangle, \langle \overline{H}_7 \rangle \sim \delta \epsilon M_{P\ell}.
$$

(7)

Such a hierarchy can be understood in a group-theoretical way. The group $SU(7)$ contains the subgroup $SU(5) \times SU(2)' \times U(1)'$, where $SU(5)$ contains the Standard Model group $G_{SM}$ and $SU(2)'$ acts on the indices 6 and 7. Denote the diagonal generator of $SU(2)'$ (namely diag(0,0,0,0,0,1/2, −1/2)) by $I'_3$. Then the hierarchies in Eqs. (5) - (7) can be succinctly stated by saying that components of left-handed superfields that have $I'_3 = 1/2$ are suppressed by a factor $\delta \epsilon$, those with $I'_3 = −1/2$ are suppressed by a factor $\epsilon$, and those with $I'_3 = 0$ are unsuppressed. Later it will be seen that such a pattern can naturally arise in the context of supersymmetry. (The $SU(2)'$ subgroup of $SU(7)$ is playing the role of a flavor group, under which the 10's of $SU(5)$ of the lightest two families transform as doublets. It is interesting that many models have been proposed in which the three families transform as $2 + 1$ under a flavor symmetry that is either $SU(2)$ or a discrete subgroup of $SU(2)$ [6].)

Note that the form of the matrix in Eq. (4) has rank = 2, implying that the renormalizable Yukawa terms leave the $u$ quark massless. In fact, this is the only quark or lepton (besides the light neutrinos) that must obtain mass
from higher-dimension operators. This corresponds to the fact that the ratio $m_u/m_t$ is by far the smallest of all the interfamily mass ratios of the quarks and leptons.

The elements of the 12 block of $M_U$ (and thus $m_u$) can be induced by higher-dimension operators such as $(\psi^{ABC}\psi^{DEK})(H^{FG}\overline{H}_K/M_{Pl})\epsilon_{ABCDEF}$. This operator gives, in particular, the following terms:

(a) $(\psi^{\alpha\beta}\psi^{\gamma\delta})(H^{27}\overline{H}_6/M_{Pl})\epsilon_{\alpha\beta\gamma\delta267}$, which is a contribution to $(M_U)_{22}$ and is of order $\epsilon^2 v_u$;
(b) $(\psi^{\alpha\beta}\psi^{\gamma\delta})(H^{26}\overline{H}_7/M_{Pl})\epsilon_{\alpha\beta\gamma\delta267}$, which is a contribution to $(M_U)_{11}$ and is of order $\delta^2 \epsilon^2 v_u$; and
(c) $(\psi^{\alpha\beta}\psi^{\gamma\delta})([H^{27}\overline{H}_7 - H^{26}\overline{H}_6]/M_{Pl})\epsilon_{\alpha\beta\gamma\delta267}$, which is a contribution to $(M_U)_{22}$ and is of order $\delta \epsilon^2 v_u$. Thus, the matrix $M_U$ has the form

$$M_U \sim \begin{pmatrix} \delta^2 \epsilon^2 & \delta \epsilon & \delta \epsilon \\ \delta \epsilon & \epsilon & \epsilon \\ \delta \epsilon & \epsilon & 1 \end{pmatrix} v_u,$$

as in Eq. (1).

The masses for the down-type quarks and charged leptons come from 10 to 5 terms contained in the second and fourth terms of Eq. (3). The term $(2L \overline{1} L \overline{2} H)$ contains $d_{1m}(\psi^{26}_(1)\psi^{m}_a)\overline{H}_2$. This produces a mass coupling 10 to 5, $m = 1, 2, 3$, and thus 3m elements of $M_D$, the mass matrix of the down-type quarks, and $m$3 elements of $M_L$, the mass matrix of the charged leptons. These elements are all of order $v_d$.

The term $(3L \overline{1} L \overline{2} H)$ contains $b_m([\psi^{\alpha\beta\gamma\delta}_a\psi^{m}_a]^{-26} - \psi^{27}_a\psi^{m}_a)\overline{H}_2$. This gives mass terms coupling 10 to 5 that are of order $\epsilon v_d$ and mass terms coupling 10 to 5 that are of order $\delta \epsilon v_d$.

The resulting mass matrices have the form

$$M_D \sim \begin{pmatrix} \delta \epsilon & \delta \epsilon & \delta \epsilon \\ \delta \epsilon & \epsilon & \epsilon \\ \delta \epsilon & \epsilon & 1 \end{pmatrix} v_d, \quad M_L \sim \begin{pmatrix} \delta \epsilon & \epsilon & 1 \\ \delta \epsilon & \epsilon & 1 \\ \delta \epsilon & \epsilon & 1 \end{pmatrix} v_d.$$

as in Eq. (1). Actually, the operators considered above give the “minimal $SU(5)$” relations $M_D = M_L^T$. A breaking of this relation can result from higher-dimension terms involving the adjoint Higgs fields. However, such terms, if induced by Planck-scale effects, would be suppressed by $M_5/M_{Pl}$, where $M_5$ is the scale at which $SU(5)$ breaks. This is too small to give realistic
“Georgi-Jarlskog” factors \[7\]. However, such terms can easily be induced by integrating out fields that have mass of order \(M_5\) rather than \(M_{Pl}\). Another possibility is that there are Higgs multiplets in the representation \(H_{ABCD}\), which would allow renormalizable terms of the form \((\psi^{(a\beta)} \psi^{(m')\alpha}) H_{2\gamma}^{\alpha\gamma}\). This is a generalization of having a \(45_H\) contribute to fermion masses in \(SU(5)\) models, as in the original model of Georgi and Jarlskog \[7\].

The question now arises whether the hierarchies among the VEVs shown in Eqs. (5) - (7) are natural. First consider the superlarge VEVs given in Eq. (7). Rather than doing a complete minimization with the entire superpotential, it will suffice to consider the terms in the superpotential that couple Higgs superfields of different ranks. For example, consider the terms of the form \(\alpha H_{AB} H_A H_B + M H_A \bar{H}_A\). (If there were only one anti-fundamental Higgs multiplet, then the term with the coefficient \(\alpha\) would vanish identically, because of the antisymmetry of the indices \(A\) and \(B\). However, it is being assumed that there are at least two such Higgs multiplets, so that \(\alpha\) and \(M\) are really matrices. There should be an index on \(H_A\) to distinguish among the different copies, and indices on \(\alpha\) and \(M\), but these indices have been suppressed.) We assume that \(\alpha\) is of order 1 and \(M\) is of order \(M_{Pl}\). Minimizing the superpotential with respect to \(H_A\) gives the equation \(H_A = \frac{-1}{M^T \alpha} H_{AB} H_B\). Thus, if \(\langle H^{i67} \rangle \sim M_{Pl}\), then \(\langle H^6 \rangle \sim \langle \bar{H}_7 \rangle\) and \(\langle H^7 \rangle \sim \langle \bar{H}_6 \rangle\), as in Eq. (7).

Note that supersymmetry plays a crucial role. The desired pattern of VEVs would not be natural in a non-supersymmetric model, since in such a model the conjugate field \((\bar{H})^7\) could be substituted anywhere for \(H^7\), and thus minimization would tend to give \(\langle H^7 \rangle \sim \langle \bar{H}_7 \rangle\), and similarly \(\langle H^6 \rangle \sim \langle \bar{H}_6 \rangle\).

Turning to the \(SU(2)_L\)-doublets Higgs fields, one sees that there is a four-by-four Higgs mass matrix \(M\)

\[
H M \bar{H} = \left( H^i, H^{i67}, H^{i6}, H^{i7} \right) \left( \begin{array}{cccc} M_{Pl} & \langle H^{i67} \rangle & \langle H^6 \rangle & \langle H^7 \rangle \\ \langle \bar{H}^{i67} \rangle & M_{Pl} & \langle \bar{H}_7 \rangle & \langle \bar{H}_6 \rangle \\ \langle \bar{H}^i \rangle & \langle H^7 \rangle & M_{Pl} & \langle (H^7)_{i6} \rangle \\ \langle \bar{H}_i \rangle & \langle H^6 \rangle & \langle (H^6)_{6i} \rangle & M_{Pl} \end{array} \right) \left( \begin{array}{c} \bar{H}_i \\ \bar{H}^{i67} \\ \bar{H}^i \rangle \\ \bar{H}_i \rangle \end{array} \right)
\]

(10)

where \(i\) is the \(SU(2)_L\) index. In Eq. (10) we have not shown dimensionless coefficients of order 1. From the hierarchy in Eq. (7), it follows that
M \sim \begin{pmatrix}
1 & 1 & \delta \epsilon & \epsilon \\
1 & 1 & \delta \epsilon & \epsilon \\
\epsilon & \epsilon & 1 & \epsilon^2 \\
\delta \epsilon & \delta \epsilon & \delta^2 \epsilon^2 & 1
\end{pmatrix} M_{\ell \ell}. \quad (11)

In this paper we do not address the question of a “technically natural” solution of the gauge hierarchy and doublet-triplet splitting problem. Rather, we simply assume that $M$ is fine-tuned so that it has one weak-scale eigenvalue. (This could be justified “anthropically”, for example in a landscape scenario [8].) First, consider the limit $\delta, \epsilon \to 0$. In that limit, one form of $M$ that has a weak-scale eigenvalue is

$$M = \begin{pmatrix}
AA' & AB' & 0 & 0 \\
BA' & BB' & 0 & 0 \\
0 & 0 & C & 0 \\
0 & 0 & 0 & D
\end{pmatrix} \frac{M_{\ell \ell} + O(M_{weak})}{\epsilon^2}, \quad (12)$$

where $A, A', B, B', C, D \sim 1$. ($M$ could also have a weak-scale eigenvalue if the 12 block did not have this factorized form, but either $C$ or $D$ were of order the weak scale. However, this possibility is not of interest for present purposes.) If $M$ has the form in Eq. (12), then the light Higgs doublets are $(H_u) = (-B^* H^i + A^* H^{i67})/\sqrt{|A|^2 + |B|^2}$ and $(H_d) = (-B^* H_i + A^* H_{i67})/\sqrt{|A|^2 + |B|^2}$. Now, taking $\delta, \epsilon$ to be non-zero but much less than 1, the form of Eq. (12) becomes

$$M = \begin{pmatrix}
AA' + O(\delta \epsilon^2) & AB' + O(\delta \epsilon^2) & 0 & 0 \\
BA' + O(\delta \epsilon^2) & BB' + O(\delta \epsilon^2) & 0 & 0 \\
0 & 0 & C + O(\delta \epsilon^2) & 0 \\
0 & 0 & 0 & D + O(\delta \epsilon^2)
\end{pmatrix} \frac{M_{\ell \ell} + O(M_{weak})}{\epsilon^2}, \quad (13)$$

from which it is easy to see that the light doublets are

$$(H_u)^i = (-B^* H^i + A^* H^{i67} + O(\delta \epsilon) H^{i6} + O(\epsilon) H^{i7})/\sqrt{|A|^2 + |B|^2},$$

$$(H_d)_i = (-B^* H_i + A^* H_{i67} + O(\epsilon) H_{i6} + O(\delta \epsilon) H_{i7})/\sqrt{|A|^2 + |B|^2}. \quad (14)$$
If $\langle (H_u)^2 \rangle \equiv v_u$ and $\langle (H_d)^2 \rangle \equiv v_d$, then

\[
\begin{align*}
\langle (H^2), (H^{267}), (H^{26}), (H^{27}) \rangle &= \left( -\frac{B}{\sqrt{|A|^2+|B|^2}}, \frac{A}{\sqrt{|A|^2+|B|^2}}, O(\delta\epsilon), O(\epsilon) \right)v_u, \\
\langle (H^2), (H_{267}), (H_{26}), (H_{27}) \rangle &= \left( -\frac{B'}{\sqrt{|A'|^2+|B'|^2}}, \frac{A'}{\sqrt{|A'|^2+|B'|^2}}, O(\epsilon), O(\delta\epsilon) \right)v_d,
\end{align*}
\]

which is just the pattern assumed in Eqs. (5) and (6).

Since the model just outlined is supersymmetric and has a lopsided mass matrix structure, the question of flavor violation arises. It is well-known that supersymmetric lopsided models give larger flavor violation than non-lopsided models, due to the large off-diagonal elements in the quark and lepton mass matrices [5]. However, if supersymmetry is broken in a flavor-blind way, as in models with gauge-mediated SUSY-breaking, excessive flavor violation due to the lopsided is avoided.

## 4 An alternative version of the model

In the $SU(7)$ model described in the previous section, it was assumed that there were several copies of certain Higgs multiplets. But it is interesting to consider also the possibility that there is just one copy of each antisymmetric tensor multiplet of Higgs fields. This has several consequences. First, not all of the vectorlike pairs of $SU(5)$ multiplets would then get superheavy mass from renormalizable Yukawa operators. In a non-supersymmetric model, they would get superheavy mass from loops, as in the model described in Ref. 1. In a model with low-energy supersymmetry such loops would be suppressed, but those fermions can nonetheless get superheavy mass from non-renormalizable operators induced by Planck-scale physics. A second consequence of the more limited set of Higgs multiplets is that most of the light quarks and leptons would not get weak-scale masses from renormalizable Yukawa operators. Here again, operators induced by Planck-scale physics can generate these masses.

First, consider the masses of the superheavy $10$'s and $\bar{10}$'s. The only renormalizable term that contributes to these is the first term in Eq. (3). If there are several $H^{AB}$ in the model— denote them $H^{AB}_{(\lambda)}$ — as assumed in the previous section, then several linear combinations of the $\psi^{AB}_{(a)}$ (namely $\lambda^a_a \psi^{AB}_{(a)}$) appear in the first term in Eq. (3), so that in general both $\psi^{AB}_{(1)}$ and
\( \psi_{(2)}^{AB} \) appear. However, if there is only a single \( H^{AB} \), then only one linear combination of \( \psi_{(a)}^{AB} \) appears in the first term of Eq. (3) (namely \( a_{a} \psi_{(a)}^{AB} \)). Without loss of generality, one can call this \( \psi_{(2)}^{AB} \). Therefore, the first term of Eq. (3) contains \( a_{2} \psi_{(2)}^{\alpha\beta\gamma} \psi_{(2)}^{'\delta\epsilon} H^{67} \epsilon_{\alpha\beta\gamma\delta\epsilon 67} \), which gives \( \psi_{(2)}^{AB} \) superheavy mass. If one assumes that this is the only contribution to the superheavy masses of the \( 10 \)'s (i.e. if one neglects contributions from higher-dimension operators), it follows that \( \psi_{(2)}^{\alpha\beta} \) is superheavy and \( \psi_{(1)}^{\alpha\beta} \) is light. As in the last section, denote the light multiplet \( \psi_{(1)}^{\alpha\beta} \) by \( 10_{3} \). Since only \( \psi_{(2)}^{AB} \) appears in the first term of Eq. (3), that term makes no contribution to the masses of the light \( 10 \)'s (if one neglects higher-dimension operators).

In the model of the previous section, it was precisely the first term in Eq. (3) that gave rise to the mass terms coupling \( 10_{3} \) to \( 10_{1} \) and \( 10_{2} \), terms such as \( a_{1} \psi_{(1)}^{\alpha\beta\gamma} \psi_{(1)}^{\gamma\delta} H^{27} \epsilon_{\alpha\beta\gamma\delta 267} \). Here, as just argued, this cannot happen, and so the 13, 31, 23, and 32 elements of \( M_{U} \) vanish if higher-dimension operators are neglected. They do receive non-vanishing contributions, however, from various higher-dimension operators, such as \( (\psi_{(a)}^{\alpha\beta\gamma} \psi_{(a)}^{\gamma\delta}) (H^{2K} \Omega_{K}/M_{Pl}) \epsilon_{\alpha\beta\gamma\delta 21J} \) and \( (\psi_{(a)}^{\alpha\beta\gamma} \psi_{(a)}^{\gamma\delta}) (H^{2K} \Omega_{K}/M_{Pl}) \epsilon_{\alpha\beta\gamma\delta 21J} \). The elements of the 12 block of \( M_{U} \) also arise from higher-dimension operators, though different ones.

By analogous reasoning, one can show that all of the elements of \( M_{D} \) and \( M_{L} \), the mass matrices of the down-type quarks and charged leptons, vanish if higher-dimension operators are neglected. The point is that the linear combinations of the \( \Phi \)'s \( \psi_{(m)\alpha} \) that appear in the terms of Eq. (3), get superheavy masses, and so none of the light \( \Phi \)'s would get weak-scale masses from the terms in Eq. (3) alone.

One sees, then, an interesting feature of the version of the model we are considering here, in which only a single copy of each Higgs multiplet exists: only the top quark gets mass from a renormalizable term; all the other light fermions get mass from higher-dimension operators. This is what happens in the model of Ref. 1 also, except that there the higher-dimension operators came from loops and here they come from Planck-scale physics. On the other hand, in the model of Ref. 1, since it is not supersymmetric, a hierarchy among the VEVs analogous to that given in Eqs. (5)-(7) is not natural. Here, however, it can hold; and so, as in the version of the model described in the previous section, the mass matrices have the form given in Eqs. (8) and (9).
5 Conclusions

The group $SO(10)$ is often regarded as the most elegant for grand unification. Its good features are that an entire family fits so neatly into one of its irreducible multiplets and that it requires that right-handed neutrinos exist, unlike $SU(5)$. However, if the families are simply placed in spinors of $SO(10)$, they transform identically under the unification group, and one must introduce flavor symmetries ad hoc in order to explain the non-trivial structure of the quark and lepton mass matrices. Virtually all published models of quark and lepton masses have flavor symmetries in addition to the “vertical” gauge group.

One of the beauties of $SU(N)$ unification, pointed out long ago [2], is that the families do not in general transform in the same way under $SU(N)$ if $N > 5$. This creates the possibility, pointed out in [1], of eliminating flavor symmetries entirely. Nor does $SU(N)$ lack the supposed advantages of $SO(10)$. $SU(N)$ unification gives an abundance of Standard Model-singlet fermions that play the role of right-handed neutrinos. Moreover, if it is assumed that the quarks and leptons are in totally antisymmetric tensor multiplets of $SU(N)$, then upon breaking to $SU(5)$ or $SU(3)_C \times SU(2)_L \times U(1)_Y$, one automatically obtains a set of families just like those observed, with quantum numbers that make them appear to come from $SO(10)$.

As noted in [1], the smallest anomaly-free sets of fermions that give a fixed number of families typically have a few larger tensors (rank $> 1$) and a many anti-fundamental multiplets. The $10$'s of $SU(5)$ are in the larger tensors and tend to transform differently under $SU(N)$, which suppresses their mixing, whereas the $5$'s of $SU(5)$ are typically all in the anti-fundamentals and transform in exactly the same way under $SU(N)$, which allows them to have large mixings. This gives exactly the “doubly lopsided” structure that is known to reproduce well many of the features of the quark and lepton spectrum.

In this paper, a supersymmetric model has been constructed using the group $SU(7)$. The quarks and leptons are in one of the most economical sets that gives three families. No symmetry has been assumed except $SU(7)$, supersymmetry, and a matter parity that distinguishes Higgs multiplets from matter (i.e. quark and lepton) multiplets. No flavor symmetry is needed. An interesting feature of this model is that the hierarchy among the families comes directly from a hierarchy of breaking scales of the unified group. The
hierarchy of $SU(7)$-breaking VEVs is of an interesting type that requires supersymmetry to achieve. $SU(7)$ breaks at near the Planck scale to $SU(5) \times SU(2)'$. The $SU(2)'$ breaks in two stages, with VEVs of components having $I'_3 = +1/2$ being smaller than VEVs of components having $I'_3 = -1/2$. These two “small” scales (small compared to $M_{Pl}$) control the mass scales of the first and second families.

It is rather remarkable that simple grand unified models, of a kind proposed very long ago [2], tend automatically to give a lopsided mass matrix structure that yields small quark mixings, large neutrino mixings, and other features of the quark and lepton spectrum that were unknown at that time.

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