Can relativity be considered complete? 
From Newtonian nonlocality to quantum nonlocality and beyond

Nicolas Gisin  
Group of Applied Physics, University of Geneva, 1211 Geneva 4, Switzerland  
(Dated: October 31, 2018)

We review the long history of nonlocality in physics with special emphasis on the conceptual breakthroughs over the last few years. For the first time it is possible to study "nonlocality without signaling" from the outside, that is without all the quantum physics Hilbert space artillery. We emphasize that physics has always given a nonlocal description of Nature, except during a short 10 years gap. We note that the very concept of "nonlocality without signaling" is totally foreign to the spirit of relativity, the only strictly local theory.

PACS numbers:

I. INTRODUCTION

100 years after Einstein miraculous year and 70 years after the EPR paper 1, I like to think that Einstein would have appreciated the somewhat provocative title of this contribution. However, Einstein would probably not have liked its conclusion. But who can doubt that relativity is incomplete? and likewise that quantum mechanics is incomplete! Indeed, these are two scientific theories and Science is nowhere near its end (as a matter of fact, I do believe that there is no end 2). Well, actually, I am, of course, not writing for Einstein, but for those readers interested in a (necessarily somewhat subjective) account of the peaceful co-existence 3 between relativity and quantum physics in the light of the conceptual and experimental progresses that happened during the last ten years, set in the broad perspective of physics and nonlocality since Newton 4.

II. NON-LOCALITY ACCORDING TO NEWTON

Isaac Newton, the great Newton of Universal Gravitation, was not entirely happy with his theory. Indeed, he was well aware of an awkward consequence of his theory: if a stone is moved on the moon, then our weight, of all of us, here on earth, is immediately modified. What troubled so much Newton was this immediate effect, i.e. the nonlocal prediction of his theory. Let's read how Newton described it himself 5:

That Gravity should be innate, inherent and essential to Matter, so that one Body may act upon another at a Distance thro a Vacuum, without the mediation of any thing else, by and through which their Action and Force may be conveyed from one to another, is to me so great an Absurdity, that I believe no Man who has in philosophical Matters a competent Faculty of thinking, can ever fall into it. Gravity must be caused by an Agent acting constantly according to certain Laws, but whether this Agent be material or immaterial, I have left to the Consideration of my Readers.

It would have been hard for Newton to be more explicit in his rejection of nonlocality! However, most physicists didn’t pay much attention to this aspect of Newtonian physics. By lack of alternative, physics remained nonlocal until about 1915 when Einstein introduced the world to General Relativity. But let’s start ten years earlier, in 1905.

III. EINSTEIN, THE GREATEST MECHANICAL ENGINEER

In 1905 Einstein introduced three radically new theories or models in physics. Special relativity of course, but more relevant to this section are his descriptions of Brownian motion and of the photo-electric effect. Indeed, both descriptions show Einstein's deep intuition about mechanics. Brownian motion is explained as a complex series of billiard-ball-like-collisions between a visible molecule - the particle undergoing Brownian motion - and invisible smaller particles. The random collisions of the latter explaining the erratic motion of the former. Likewise, the photo-electric effect is given a mechanistic explanation. Light beams contain little billiard-balls whose energy depends on the color, i.e. wavelength, of the light. These light-billiard-balls (today called photons and recognized as not at all billiard-ball-like) hit the electrons on metallic surfaces and mechanically kick them out of the metal, provided they have enough energy.

General relativity can also be seen as a mechanical description of gravitation. When a stone is moved on the moon, a bunch of gravitons (in modern terminology) fly off in all directions at a finite speed, the speed of light. Hence, about a second later, the earth is informed and only then is our weight affected. This is, I believe, the greatest achievement of Einstein, the greatest mechanical engineer 1 of all times: Einstein turned physics into

---

1 My friends know well that in my mouth "engineer" has no negative connotation, quite the opposite. For me, a physicist must be a good theorist and a good engineer! Well, I warned you, dear reader, this is a somewhat subjective article.
a local theory!

IV. QUANTUM MECHANICS IS NOT MECHANICAL

Only about ten years after general relativity came quantum mechanics. This was quite an extraordinary revolution. Until then, greatly thanks to Newton and Einstein’s genius, Nature was seen as made out of billiards. Until then, greatly thanks to Newton and Einstein’s genius, Nature was seen as made out of many little billiard-balls that mechanically bang into each other. Yet, quantum mechanics is characterized by the very fact that it no longer gives a mechanical description of Nature. The terminology quantum mechanics is just a historical mistake, it should be called Quantum Physics as it is a radically new sort of physical description of Nature.

But this new description let nonlocality back into Physics! And this was unacceptable for Einstein.

It is remarkable and little noticed that since Newton, physics gave a local description of Nature only during some 10 years, between about 1915 and 1925. All the rest of the time, it was nonlocal, though, with quantum physics, in quite a different sense as with Newton gravitation. Indeed, the latter implies the possibility of arbitrarily fast signaling, while the former prohibits it.

V. NON-LOCALITY ACCORDING TO EINSTEIN

In 1935 two celebrated papers appeared in respectable journals, both with famous authors, both stressing the - unacceptable in their authors view - nonlocal prediction of quantum physics. A lot has been written on the EPR “paradox” and I won’t add to this. I believe that Einstein’s reaction is easy to understand. Here is the man who turned physics local, centuries after Newton wrote his alarming text, he is proud of his achievement and certainly deserves to be. Now, only a few years latter, nonlocality reappears! Today one should add that quantum nonlocality is quite a different concept from Newtonian nonlocality, but Einstein did not fully realize this.

What Einstein and his colleagues saw is that quantum physics describes spatially separated particles as one system in which the two particles are not logically separated. What they did not fully realize is that this does not allow for signaling, hence it is not in direct conflict with relativity. In the next section I’ll try to present this using modern terminology.

Most physicists didn’t pay much attention to this aspect of quantum physics. A kind of consensus established that this was to be left for future examination, once the technology would be more advanced. The general feeling was that quantum nonlocality was nothing but a laboratory curiosity, not serious physics.

Young physicists may have a hard time to believe that such an important concept, like quantum nonlocality, was, during many decades, not considered as serious. But this was indeed the real state of affairs: ask any older professors, a vast majority of them still believes that it is unimportant. Let me add two little stories that illustrate what the situation was like. John Bell, the famous John Bell of the Bell inequalities and of the Bell states, never had any quantum physics student. When a young physicist would approach him and talk about nonlocality, John’s first question was: “Do you have a permanent position?”... Indeed, without such a permanent position it was unwise to dare talking about nonlocality! Notice that John Bell almost never published any of his remarkable and nowadays famous papers in serious journals: the battle with referees were too ... time wasting (not to use a more direct terminology). Further, if you went to CERN where John Bell held a permanent position in the theory department and asked at random about John’s contributions to physics, his work on the foundation of quantum physics would barely be mentioned (true enough, he had so many other great contributions!)^2.

Anyway, so quantum nonlocality remained for decades in the curiosity lab and no one paid much attention. But in the 1990’s two things changed. First, a conceptual breakthrough happened thanks to Artur Ekert and to his adviser David Deutsch. They showed that quantum nonlocality could be exploited to establish a cryptographic key between two distant partners and that the confidentiality of the key could be tested by means of Bell’s inequality. What a revolution! This is the first time that someone suggested that quantum nonlocality is not only real, but that it could even be of some use. A second contribution came from the progress in technology. Optical fibers had been developed and installed all over the world. And Mandel’s group at the University of Rochester (where I held a one-year post-doc position and first met with optics) applied parametric down-conversion to produce entangled photon pairs. This was enough (up to the detectors) to demonstrate quantum nonlocality outside the curiosity laboratory. In 1997 my group at Geneva University demonstrated the violation of Bell inequalities between two villages around Geneva, see Fig. 1, separated by a little more than 10 km and linked by a 15km long standard telecom fiber (since then, we have achieved 50km). So quantum nonlocality became politically acceptable! But what is it? Let me introduce the concept using students undergoing “quantum exams”.

---

^2 Another story happened to me while I was a young post-doc eager to publish some work. In a paper I wrote “A quantum particle may disappear from a location A and simultaneously reappear in B, without any flow in-between”. The referee accepted the paper under the condition that this outrageous sentence is removed. This referee considered his paternalist attitude so constructive that he declared himself to me: “look how helpful I am to you” (admittedly, he was politically correct).
VI. QUANTUM EXAMS: ENTANGLEMENT

Assume that two students, Alice and Bob, have to pass some exams. As always for exams, the situation is arranged in such a way that the students can’t communicate during the exam. Clearly however, they are allowed, and even encouraged, to communicate beforehand. Alice and Bob know in advance the list of possible questions, they also know that this is a kind of exam allowing only a very limited number of possible answers, often only a binary choice between yes and no. During the exam Alice receives one question out of the list, let’s denote it by $x$; Bob receives question $y$. Finally, denote $a$ and $b$ Alice and Bob’s answers, respectively. Hence, an exam is a realization of a random process described by a conditional probability function, often merely called a correlation:

$$ P(a, b|x, y) \quad (1) $$

Clearly, the choice of questions $x$ and $y$ are under the professor’s control. However, as all professors know, the students’ answers $a$ and $b$ are not! This is similar to experiments: the choice as to which experiment to perform is under the physicists control, but not the answer given by Nature.

In the following, we shall consider three kinds of exams, in order to understand what kind of constraints they set on the correlation $P(a, b|x, y)$.

A. Quantum exam #1

In this first kind of quantum exam Alice is asked to tell which question is given to Bob, and vice-versa. This is clearly an unfair exam! Why? Because Alice and Bob are not supposed to communicate. How could they then succeed with a probability greater than mere chance? This simple example shows that prohibiting signaling already limits the set of possible correlation $P(a, b|x, y)$. For example $P(a, b|x, y) = \delta(a = y)\delta(b = x)$ is excluded. Notice that a correlation $P(a, b|x, y)$ is non-signaling if and only if its marginal probabilities are independent of the other side input: $\sum_b P(a, b|x, y)$ is independent of $y$ and $\sum_a P(a, b|x, y)$ is independent of $x$.

B. Quantum exam #2

The second kind of quantum exam is closer to standard exams. Alice and Bob are simply requested to provide the same answer whenever they receive the same question. This is clearly feasible: we all expect that good students give the same answer to the same question. It suffices that they prepare for the exam well enough. Note that the quantum exam #2 under consideration here is even easier than standard exams, as there is no notion of correct or incorrect answers. All that is required is that Alice and Bob give consistent answers: it suffices that they jointly decide in advance which answer to give for each of the possible questions.

Now, a central problem: Could Alice and Bob succeed with certainty for such an exam #2 by other means, that is without jointly deciding the answers in advance? Think about it. If you found an alternative trick, then, if you are a student, you should use your trick to pass the next examination: just apply your trick together with the best student, you’ll get the same mark as him/her. And if you are a professor and found a trick, then you should stop testing your student with standard exams! Well, of course, there is no other trick, at least none applicable to classical students.

Correlations that satisfy $P(a = b|x = y) = 1$ are necessarily of the form

$$ P(a, b|x, y) = \sum_\lambda \rho(\lambda)Q(a|x, \lambda)Q(b|y, \lambda) \quad (2) $$

for some probability function $Q$ and some distribution $\rho$ of common strategy $\lambda$. Historically, the $\lambda$ were called "local hidden variables", computer scientist call them "shared randomness"; here the $\lambda$ denote common strategies.

$P(a = b|x = y) = 1$ is but one example of a local correlation, among infinitely many others. Relation (2) characterizes all local correlations.

In summary, some exams require common strategies; in other words, some observed correlations can’t be explained except by common causes.

C. Quantum exam #3

The third kind of quantum exam is the most tricky and interesting. For (apparent) simplicity let’s restrict the set of questions and answers to binary sets and let us label them by bits, "0" and "1". In this exam Alice and Bob are required to always output the same answer, except when they both receive the question labelled "1" in which case they should output different answers. Note that formally this exam requires that Alice and Bob’s data satisfy the following equality, modulo 2: $a + b = x \cdot y$. This time it is not immediately obvious whether they can prepare a strategy that guaranties success.

Assume first that the strategy forces Alice to output an answer that depends only on her input, i.e. Alice’s strategy is deterministic. But in such a case, whenever

---

3 Somewhat surprisingly there is a strategy such that the probability that both players succeed is 50%.

4 Admittedly, the danger is that both student that get the bad mark! But, on average, the poor student improves.
Bob receives the question 1, he can’t decide on his output since it should depend on Alice’s question. Next, if Alice’s output is random, this is clearly of no help to Bob. Consequently there is no way for Alice and Bob to succeed with certainty.

Let us emphasize that successfully completing this exam does not necessarily imply communication between Alice and Bob. Indeed, assume that somehow Alice and Bob’s data would always satisfy \( a + b = x \cdot y \). Would this allow Alice to communicate to Bob, or vice-versa? Well, it depends! If Alice’s output \( a \) is known to Bob, for instance they decide on \( a = 0 \) always, then whenever Bob receives \( y = 1 \), he can deduce Alice’s question from \( a + b = x \cdot y \) and from his output: \( x = b \) in the example. But if Alice’s outcome is unknown to Bob, for instance if Alice outcome is merely a random bit, then the relation \( a + b = x \cdot y \) is of no help to Bob. We shall come back to this concept of a non-signaling correlation satisfying \( a + b = x \cdot y \) in section VII.

Let us define the mark \( M \) of this quantum exam #3 as the sum of the success probabilities \( M = P(a + b = xy|x = 0, y = 0) + P(a + b = xy|x = 0, y = 1) + P(a + b = xy|x = 1, y = 0) + P(a + b = xy|x = 1, y = 1) \) (3)

It is not difficult to realize that the optimal strategy for Alice and Bob consists in deciding in advance on a common answer, independent of the questions they receive. With such a strategy they are able to achieve the mark \( M = 3 \). This is indeed the optimal mark achievable by common strategies:

\[ M \leq 3 \] (4)

This is an example of a Bell inequality: a constraint on correlations arising from common strategies. Interesting Bell inequalities are those that can be violated by quantum physics. In the case of \( M_{QP} = 2 + \sqrt{2} \approx 3.41 \), Tsirelson proved that this is the highest mark achievable using quantum correlations \( |x \rangle \).

Accordingly, quantum theory predicts that some tasks can be achieved that can’t be predicted by any local mechanical model, i.e. some exams are passed with higher marks than classically possible. The fact that such tasks were invented for the purpose of showing the superiority of quantum physics doesn’t affect the conclusion. Still, it is only once some useful and natural tasks were found, concretizing the superior power of quantum physics over all possible local strategies, that quantum nonlocality became accepted by the physics community\(^5\).

\( \text{FIG. 1: Bernex and Bellevue are the two villages north and south of Geneva between which our long-distance test of Bell inequality outside the lab was performed in 1997, section VII.} \)

\( \text{The inset represent two player that toss coins, as explained in section VII. In the real experiment the coins were replaced by photons, the players by interferometers, their right and left hands by phase modulators and head/tail by two detectors. The experimental results are similar to that of the game, with weaker but still nonlocal correlations.} \)

\( \text{VII. COIN TOSSING AT A DISTANCE} \)

Another way to present nonlocality to non-physicist friends is the following. Imagine two hypothetical players that toss coins. The players are separated in space and toss their coin once per minute. They use their free-will to decide, for each toss, whether to use their right hand or their left hand, independently of each other. And they mark all results (time, hand and head/tail) in a big black laboratory notebook, see Fig. 1.

After thousands of tosses, they get bored. Especially, given that nothing interesting happens: for each of the two players, heads and tails occur with a frequency of 50%, independently of which hand they use. Hence, the players decide to go for a beer. There, in the bar, they compare their notes and get very excited. Indeed, quickly they notice that whenever at least one of them happened to have chosen his right hand for tossing his coin, both players always obtained the same result: either both head or both tails. But whenever, by mere chance, they both chose the left hand, then they always obtained opposite results: head/tail or tail/head. A very remarkable correlation!

The observation of correlations and the development view Letters. I bet that a phase transition happen in the early 1990’s, after Ekert’s paper on quantum cryptography. In 1997 I started a PRL with the sentence: "Quantum theory is nonlocal." and got considerable reactions to what was felt as a provocative statement; today the same statement can be found in many papers, not provoking any reaction.

\( ^5 \) I wish someone establishes the statistics of the occurrences of the words "Bell inequality" and "nonlocality" in Physical Re-
VIII. EXPERIMENTS: GOD DOES PLAY DICE, HE EVEN PLAYS WITH NONLOCAL DICE

Physics is an experimental science and experiments have again and again supported the nonlocal predictions of quantum theory. All kind of experiments have been performed, in laboratories \[11, 12, 19\] and outside \[11, 12, 19\], with photons and with massive particles \[20\], with independent observers to close the locality loophole \[11, 12, 19\], with photons and with massive particles \[20\], with quasi-perfect detectors \[20\] to close the detection loophole, with high precision timing to bound the speed of hypothetical hidden communication \[22\], with moving observers to test alternative models \[23\] (multi-simultaneity \[24\] and Bohm’s pilot wave \[25\]). All these results proclaim loudly: God plays dice. Note how ironic the situation is: the conclusion “God plays dice” is imposed on us by the experimental evidence supporting quantum nonlocality and by Einstein’s postulate that no information can travel faster than light. Indeed, as mentioned in sub-section \[V.\]C a violation of \[4\] with deterministic outputs leads to signaling. Consequently, the experimental violation of \[4\] and the no-signaling principle imply randomness \[26, 27\].

Actually, the situation is even more interesting: Not only does God play dice, but he plays with nonlocal dice! The same randomness manifests itself at several locations, approximating \(a + b \approx x \cdot y\) better than possible with any local classical physics model. A very small minority of physicists still refuse to accept quantum nonlocality. They ask (sometimes with anger) How can these two space-time locations, out there, know about what happens in each other without any sort of communication? I believe that this is an excellent question! I have slept with it for years \[28\]. I summarize my conclusion in the next section.

IX. ENTANGLEMENT AS A CAUSE OF CORRELATION

Quantum physics predicts the existence of a totally new kind of correlation that will never have any kind of mechanical explanation. And experiments confirm this: Nature is able to produce the same randomness at several locations, possibly space-like separated. The standard explanation is “entanglement”, but this is just a word, with a precise technical definition \[29, 30\]. Still words are useful to name objects and concepts. However, it remains to understand the concept. Entanglement is a new explanation for correlations. Quantum correlations simply happen, as other things happen (fire burns, hitting a wall hurts, etc). Entanglement appears at the same conceptual level as local causes and effects. It is a primitive concept, not reducible to local causes and effects. Entanglement describes correlations without correlata \[31\] in a holistic view \[32\]. In other words, a quantum correlation is not a correlation between 2 events, but a single event that manifests itself at 2 locations.

Are you satisfied with my explanation of what entanglement is? Well, I am not entirely! But what is clear is that entanglement exists. Moreover, entanglement is incredible robust! The last point might come as a surprise, since it is still often claimed that entanglement is as elusive as a dream: as soon as you try to talk about it, it evaporates! Historically this was part of the suspicion that entanglement was not really real, nothing more

---

\[6\] The conclusion that follows from all these experiments is so important for the physicist’s world-view, that an experiment closing simultaneously both the locality and the detection loophole is greatly needed.
than some exotic particles that live for merely a tiny fraction of a second. But today we see a growing number of remarkable experiments mastering entanglement. Entanglement over long distances \[11, 12, 13, 14, 15, 16\], entanglement between many photons \[17\] and many ions \[18\], entanglement of an ion and a photon \[19, 20\], entanglement of mesoscopic systems (more precisely entanglement between a few collective modes carried by many particles) \[21, 22, 23\], entanglement swapping \[11, 12, 13\], the transfer of entanglement between different carriers \[14\], etc.

The correlation (5) is often referred to as a PR-box, to recall the seminal work by Popescu and Rohrlich \[24\], etc.

In summary: entanglement exists and is going to affect future technology. It is a radically new concept, requiring new words and a new conceptual category.

X. FROM QUANTUM NONLOCALITY TO MERE NONLOCALITY

So far we have seen that quantum physics produces nonlocal correlations. And so what? Ok, this can be used for Quantum Key Distribution and other Quantum Information processes, but that doesn’t help much to understand non-locality. Conceptually, one would like to study non-locality without all the quantum physics infrastructure: Hilbert spaces, observables and tensor products. Not too surprisingly, once the existence of non-locality was accepted, the conceptual tools to study it came very naturally. Actually, the tools were already there, in the mathematics \[25\] and even the physics \[26, 27\] literature, waiting for a community to wake up! The basic tool is simple, doesn’t require any knowledge of quantum physics and allows one, so to say, to study quantum nonlocality ”from the outside”, i.e. from outside the quantum physics infrastructure.

Let us go back to the quantum exam #3 (subsection VI C). Assume that Alice and Bob are not restricted by quantum physics, but only restricted by no-signaling. Consequently, they would fail the quantum exam #1. But under this mild no-signaling condition they could perfectly succeed in the quantum exam #3: Alice and Bob would each output a bit which locally looks perfectly random and independent from their inputs - hence there would be no signaling - yet their 2 bits would satisfy

\[
\delta (a + b = x \cdot y)
\]

Theoretical "machine" that produces precisely this correlation is a basic example of the kind of conceptual tools we need to study nonlocality without quantum physics. Formally, the correlation function is defined by:

\[
P(a, b|x, y) = \frac{1}{2} \delta (a + b = x \cdot y)
\]

where the \( \delta (z_1 = z_2) \) function takes value 1 for \( z_1 = z_2 \) and value 0 otherwise.

The correlation (5) is often referred to as a PR-box, to recall the seminal work by Popescu and Rohrlich \[24, 25\], or as a NL-machine (a Non-Local machine\(^7\)). The idea of these terminologies is to emphasize the similarities between quantum measurements on 2 maximally entangled qubits and the correlation (5): in both cases the outcome is available as soon as the corresponding input is given (Alice knows \( a \) as soon as she inputs \( x \) into her part of the machine and similarly Bob knows \( b \) as soon as he inputs \( y \), there is no need to wait for the other’s input) and in both the quantum and the PR-box cases the ”machine” can’t be used more than once (once Alice has input \( x \), she can’t change her mind and give another input). Notice a third nice analogy, neither the quantum nor the NL machines allow for signaling. Indeed, in all cases the marginals are pure noise, independently of any input.

Note that quantum physics is unable to produce the PR correlation (5). Indeed, this correlation violates the Bell inequality \[28\] up to its algebraic maximum, \( M = 4 \), while Tsirelson’s theorem \[29\] states that quantum correlations are restricted to \( M \leq 2 + \sqrt{2} \). However, the correlation (5) is much simpler than quantum correlations, while sharing many of their essential features. In particular (5) is nonlocal but non-signaling.

In order to get some deeper understanding of the power of this hypothetical machine (5) as a conceptual tool, let us consider 3 properties of quantum correlations (many further nice aspects can be found in \[30, 31, 32\]). First we shall consider the so-called quantum no-cloning theorem and see that it is actually not a quantum theorem, but a no-signaling theorem. The next natural step is to analyze quantum cryptography, whose security is often said to be based on the no-cloning theorem, and as we would expect by now, we shall find ”non-signaling cryptography”. Finally, we consider the question of the communication cost to simulate maximal quantum correlation. But before all this we need to recall some facts about non-signaling correlations.

A. The set of non-signaling correlations

Let us consider the set of all possible bi-partite correlations \( P(a, b|x, y) \), where the inputs are taken from finite alphabets \( \{x\} \) and \( \{y\} \) and similarly for the outputs \( \{a\} \) and \( \{b\} \), and which are non-signaling:

\[
\sum_b P(a, b|x, y) = P(a|x) \text{ is independent of } y
\]

\[
\sum_a P(a, b|x, y) = P(b|y) \text{ is independent of } x
\]

\(^7\) A ”machine” is a physicist’s terminology for an input-output black-box that is not necessarily mechanical. I believe that this terminology appeared in the quantum physics context with the ”optimal cloning machines” introduced by Buzek and Hillery \[33\].
A priori this set looks huge. But it has a nice structure. First, it is a convex set: convex combinations of non-signaling correlations are still non-signaling. Second, there are only a finite number of extremal points (mathematician call such sets polytopes and the extremal point vertices); accordingly every non-signaling correlation can be decomposed into a (not necessarily unique) convex combination of extremal points. This is analog to quantum mixed states that can be decomposed into convex mixtures of pure states.

Among the non-signaling correlations are the local ones, i.e., those of the form \( a, b, x, y \). These correspond to separable quantum states. The set of local correlations also forms a polytope, a sub-polytope of the non-signaling one. Moreover, all vertices of the local polytope are also vertices of the non-signaling polytope, see Fig. 2. The facets of the local polytope are in one-to-one correspondence with all tight Bell inequality.

Let us illustrate this for the simple binary case (which is in any case the only one we need in this article), i.e., \( a, b, x, y \) are 4 bits. In this case, it is known that there are only 8 non-trivial Bell inequalities (i.e., not counting the trivial inequalities of the form \( P(a, b, x, y) \geq 0 \)), i.e., only 8 relevant facets of the local polytope. Interestingly, Barret and co-workers demonstrated that the non-signaling polytope has only 8 vertices more than the local polytope, exactly one per Bell inequality! Each of these 8 vertices is equivalent to the PR correlation, up to an elementary symmetry (flip an input and/or an output). Although these polytopes live in an 8-dimensional space, their essential properties can be recalled from the simple geometry of figure 2.

### B. No-cloning theorem

Details can be found in, as here we would merely like to present the intuition. Let us assume that Alice (input and output bits \( x \) and \( a \), respectively) shares the correlation both with Bob (bits \( y \) and \( b \)) and with Charly (bits \( z \) and \( c \)): \( a + b = xy \) and \( a + c = xz \). Note that this situation is different from the case where Alice would share one "machine" with Bob and share another independent "machine" with Charly: in the situation under investigation Alice holds a single input bit \( x \) and a single output bit \( a \). We shall see that the assumption that Alice’s input and output bits \( x \) and \( a \) are correlated both to Bob and to Charly leads to signaling. Hence in a Universe without signaling, Alice can’t share the correlation with more than one partner: the correlation can’t be cloned.

In order to understand this, assume that Bob and Charly come together, input \( y = 1 \) and \( z = 0 \), and add their output bits \( b + c \). According to the assumed correlations and using the modulo 2 arithmetic \( a + a = 0 \), one gets: \( b + c = a + b + a + c = xy + xz = x \). Hence, they could determine from their data that Alice’s input bit is \( x \), i.e., Alice could signal to them!

A natural question is how noisy should the correlation be to allow cloning? The answer is interesting: as long as the Alice-Bob correlation violates the Bell inequality, the Alice-Charly correlation can’t violate it; if not there is signaling.

We have just seen that the CHSH-Bell inequality is monogamous, like well kept secrets. Let’s now see that this is not a coincidence!

### C. Non-signaling cryptography

In 1991 Artur Ekert’s discovery of quantum cryptography based on the violation of Bell’s inequality changed the (physicist’s) world: entanglement and quantum non-locality became respectable. Now, as we shall see in this subsection, the essence of the security of quantum cryptography does not come from the Hilbert space structure of quantum physics (i.e., not from entanglement), but is due to no-signaling nonlocal correlation! The fact that quantum physics offers a way to realize such correlation makes the idea practical. However, if one would find any

---

8 More precisely, 8 is the dimension of the space of non-signaling correlations.
other way to establish such no-signaling nonlocal correlations (a way totally unknown today), then this would equally well serve as a mean to establish cryptographic keys\(^9\). Let us emphasize that the goal is to assume no restriction on the adversary’s power, i.e. on Eve, except no-signaling\(^9\). Obviously, if one assumes additional restrictions on Eve, like restricting her to quantum physics, then Alice and Bob can distill more secret bits from their data. But qualitatively, the situation would remain unchanged.

Assume that two partners, Alice and Bob, hold devices that allow them to each input a bit (make a binary choice of what to do, e.g. which experiment to perform) and each receives an output bit (e.g. a measurement result). This can be cast into the form of an arbitrary correlation: \(P(a, b | x, y)\), with \(a, b, x, y\) four bits. Assume furthermore that the devices held by Alice and Bob do not allow signaling. This simple and very natural assumption suffices to give a nice structure to the set of correlations \(P(a, b | x, y)\): as we recall in subsection \(\text{XXA}\) this set is convex and has a finite number of extreme points, called vertices. The nice property is that any correlation \(P(a, b | x, y)\) can be decomposed into a convex combination of vertices, hence once one knows the vertices one knows essentially everything. If the correlation is local, i.e. of the form \(\text{(2)}\), then it is not useful for cryptography; indeed the adversary Eve may know the strategy \(\lambda\). Hence, let’s assume that \(P(a, b | x, y)\) violates the Bell inequality \(\text{(4)}\). Consequently \(P(a, b | x, y)\) lies in a well defined corner of the general polytope, a sub-polytope. Barrett and co-workers found that this sub-polytope has only 9 vertices \(\text{(58)}\), 8 local ones for which \(M = 3\) and only one nonlocal vertex, that corresponding to our conceptual tool, i.e. to \(a + b = xy\), for which \(M = 4\), see Fig. 2.

In the case that Alice and Bob are maximally correlated (maximally but non-signaling!), i.e. their correlation correspond to the nonlocal vertex of Fig. 2, it is intuitively clear that the adversary Eve can’t be correlated neither to Alice, nor to Bob, by the no-cloning argument sketched in the previous subsection. Hence, in such a case Alice and Bob receive from their apparatuses perfectly secret bits. However, these bits are not always correlated: when \(x = y = 1\) they are anti-correlated. But this can be easily fixed by the following protocol. After Alice and Bob received their output bits, Alice announces publicly her input bit \(x\) and Bob changes his output bit to \(b' = b + xy\). Now Alice and Bob are perfectly correlated and Eve still knows nothing about \(a\) and \(b'\).

Consider now that Alice and Bob are non maximally correlated:

\[
P(a, b | x, y) = \frac{1 + p_{NL}}{2} \delta(a + b = x \cdot y) + \frac{1 - p_{NL}}{4} \quad (8)
\]

For \(p_{NL} > 0\) this correlation violates the inequality \(\text{(4)}\), for \(p_{NL} \leq \sqrt{2} - 1\), it can be realized by quantum physics. Can Alice and Bob exploit such a correlation for cryptographic usage secure against an arbitrary adversary who is not restricted by quantum physics, but only restricted by the no-signaling physics? The full answer to this fascinating question is still unknown. However, there is an optimistic answer if one assumes that Eve attacks each realization independently of the others, the so-called individual attacks. In such a case, one may assume that Eve does actually distribute the apparatuses to Alice and Bob. Some apparatuses are ordinary local ones, for these Eve knows exactly the relation between the input and output bits, both for Alice and for Bob. For example, Eve sends to Alice an apparatus that always outputs a 0, and to Bob an apparatus that outputs the input bit: \(b = y\). In this example Eve knows Alice’s bit \(a\), but doesn’t know Bob’s bit. For some local pairs of apparatuses Eve knows both \(a\) and \(b\), or \(b\) but not \(a\). But, if the Alice-Bob correlation \(\text{(8)}\) violates the Bell inequality \(\text{(4)}\), i.e. if \(p_{NL} > 0\), then Eve must sometimes send to Alice and Bob the apparatuses that produce the maximal nonlocal correlation \(a + b = xy\), in which case she knows nothing about Alice and Bob’s output bits \(a\) and \(b\). A detailed analysis can be found in \(\text{[51]}\). Here we merely recall the result. For \(p_{NL} > 0.318\) the Shannon mutual information between Alice and Bob is larger than the Eve-Bob mutual information \(\text{[51]}\). Hence for \(p_{NL} > 0.318\) Alice and Bob can distill a cryptographic secret key out of their data, secure even against an hypothetical post-quantum adversary, provided this adversary is still subject to no-signaling.

Actually, in \(\text{[51]}\) we worked out a 2-way protocol for key distillation valid down to \(p_{NL} > 0.09\). There, it is also proven that the intrinsic information is positive for all positive \(p_{NL}\). It is thus tempting to conjecture that secret key distillation is possible if and only if the Bell inequality is violated\(\text{[11]}\).

---

\(^9\) No-signaling should be understood here as in the previous subsection on the no-cloning theorem. That is, even if two parties joint, for example Eve and Bob come together, then they should not be able to infer any information about the third party’s input, e.g. Eve and Bob should not have access no Alice’s input.

\(^{10}\) One may think that Eve should sometimes send a weakly non-local machine. But all such correlations are convex combinations of local and fully non-local NL-machines. Hence, it is equivalent for Eve to always send either a local or a NL-machine, with appropriate probabilities.

\(^{11}\) In \(\text{[57]}\) we proved that a correlation \(P(a, b | x, y)\) is nonlocal iff any possible non-signaling extensions \(P(a, b, e | x, y, z)\) has positive Alice-Bob condition mutual information, conditioned on Eve, \(I(A, B | E)\), i.e. has positive intrinsic information. This nicely complements the similar result that holds for entangled quantum states and purifications \(\text{[55]}\). In \(\text{[51]}\) we proved that the same relation between nonlocality and positive intrinsic information does also hold when Alice announces her input and Bob adapts his output in such a way as to maximize his mutual information with Alice. Proving this in full generality would be a marvellous result.
Another beautiful result is the observation of an information gain versus disturbance relationship, very similar to that of quantum physics, based on Heisenberg’s uncertainty relations \[^5\]. Let us analyze separately the cases where Alice announces \( x = 0 \) and \( x = 1 \), and denote the respective Alice-Bob error rates \( QBER_x \) and the Eve-Bob mutual informations \( I_x(B, E) \), i.e. \( QBER_x = \sum_y P(a \neq b) |x, y \rangle \) and \( I_x(B, E) = H(B|x) - H(B|E, x) \). Remarkably, \( I_0(B, E) \) is a function of only \( QBER_1 \) and \( I_1(B, E) \) of \( QBER_0 \) \[^{12}\]: information gain for one input necessarily produces errors for the other input, in analogy with the quantum case where information gain on one basis necessarily perturbs information encoded in a conjugated basis!

To conclude this subsection, let us emphasize that the distribution of the correlation \[^3\] by quantum means requires a protocol that differs from the famous BB84 protocol \[^5\]. Indeed, the data obtained by Alice and Bob following the BB84 protocol do not violate any Bell inequality, hence the BB84 protocol is not secure against a non-signaling post-quantum adversary. Indeed, even the noise-free BB84 data can be obtained from quantum measurements on a separable state in higher dimension. The additional dimension could, for the example of polarization coding, be side-channels due to accidental additional wavelength coding. Consequently, standard security proofs \[^{18},^{32}\] must make assumptions about the dimension of the relevant Hilbert spaces (accordingly, no security proof of quantum key distribution is unconditional, contrary to widespread claims). But it is easy to adapt the BB84 protocol, it suffices that Alice measures the physical quantities corresponding to the Pauli matrices \( \sigma_z \) or \( \sigma_y \), depending on her input bit value 0 or 1, respectively, exactly as in BB84, and Bob measures in the diagonal bases: \( \sigma_{+45^\circ} \) and \( \sigma_{-45^\circ} \) for \( y = 0 \) and \( y = 1 \), respectively. In this way Alice and Bob’s data are never perfectly correlated, but they can violate the Bell inequality and be thus exploited to distil a secret key valid even against post-quantum adversaries. Note that the violation of a Bell inequality guarantees that no side channels accidentally leak out information. Furthermore, in this protocol, that we like to call the CHSH-protocol, in honor of the 4 inventors \[^{14}\] of the most useful Bell inequality (actually equivalent to \[^{40}\]), Alice announces her input bit \( x \), i.e. her basis as in BB84, but Bob doesn’t speak, he always accepts and merely flips his bit in case \( x = y = 1 \). In summary, in the CHSH protocol Alice and Bob use all the raw bits, however their data are initially noisier than in the BB84 protocol.

\[^{12}\] Precisely one has: \( I_0(B, E) = 2 \cdot QBER_1 \) and \( I_1(B, E) = 2 \cdot QBER_0 \).

D. Cost of simulating quantum correlations

Among the many contributions of computer science to quantum information is the beautiful simple question (actually anticipated by Maudlin \[^{58}\]): what is the cost of simulating quantum correlations? More precisely, Gilles Brassard, Richard Cleve and their student Alain Tapp \[^{59}\], and independently Michael Steiner \[^{60}\], asked the question: \textit{How many bits must Alice and Bob exchange in order to simulate (projective) measurement outcomes performed on quantum systems?} The question concerns the communication during the measurement simulation, clearly there must have been prior agreement on a common strategy. If the systems are in a separable state, no communication at all is needed. On the contrary, if the state allows measurements that violate a Bell inequality, i.e. if the state has quantum nonlocality, then it is impossible to simulate it without some communication or some other nonlocal resources.

For the simplest case of two 2-level systems (2 qubits), this game assumes that Alice and Bob receive as input any possible observable, i.e. any vector \( \vec{a} \) and \( \vec{b} \) of the Poincaré sphere. And they should output one bit, corresponding to the binary measurement outcome ”up” or ”down” in the physicist’s spin \( \frac{1}{2} \) language. A simple way to simulate the quantum measurements is that Alice communicates her input \( \vec{a} \) to Bob and outputs a predetermined bit (predetermined by Alice and Bob’s common strategy). But communicating a vector corresponds to infinitely many bits! My first intuition was that there is no way to do any better, after all the input space is a continuum, quite the contrary to the case of Bell inequalities where the input space is finite, usually even limited to a binary choice. Yet, Brassard and co-workers came out with a model using only 8 bits of communication! What a surprise: is entanglement that cheap? But this was only a start. Steiner published a model valid only for vectors lying on the equator of the sphere, but this model was easy to generalize to the entire sphere \[^{61}\]: it uses only 2 bits! 2 bits, like in dense coding and teleportation: that should be the end, I thought! But, yet again, I was wrong. Bacon and Toner produced a model using one single bit of communication \[^{62}\]. Well, by now we should be at the limit, isn’t it? But actually, not quite!

Let’s come back to the real central question: How does Nature manage to produce random data at space-like separated locations that can’t be explained by common causes? The idea that Nature might be exploiting some hidden communication (hidden to us, humans) is interesting. With my group at Geneva University we spent quite some time trying to explore this idea, both experimentally and theoretically. We could set experimental bounds of the speed of this hypothetical hidden communication \[^{22}\]. We also investigated the idea that each observer sends out hidden information about his result at arbitrary large speeds as defined in its own inertial reference frame \[^{23}\]. The measured bounds on the speed of the hypothetical hidden communication were very high.
and the latter assumption contradicted by experiments. Also our theoretical investigation cast serious doubts on the existence of hidden communication. Analyzing scenarios involving 3 parties we could prove that if all quantum correlations would be due to hidden communication, then one should be able to signal (i.e., the hidden communication do not remain hidden) [63, 64]. Hence, the only remaining alternative is that Nature exploits both hidden communication and hidden variables: each one separately contradicts quantum theory, but both together could explain quantum physics. However, this seems quite an artificial construction. Hence, let’s face the situation: Nature is able to produce nonlocal data without any sort of communication. But is she doing so using all the quantum physics artillery? Aren’t there logical building blocks of nonlocality? A partial answer follows.

Let us come back to the problem of simulating quantum measurements, but instead of a few bits of communication let us give Alice and Bob a weaker resource: one instance of the nonlocal machine \( a + b = xy \). That is indeed a weaker resource follows from the observation that the correlation \( a + b = xy \) can’t be used to communicate any bit, but that by sending a single bit one can easily simulate the nonlocal correlation (since Alice’s input is only a bit \( x \), it suffices that she communicates it to Bob). The nice surprise is that this elementary resource is sufficient to simulate any pair of projective measurements on any maximally entangled state of two qubits! For the proof the reader is referred to the original article [62] and to the beautiful account in [63] where the relations between all these models are presented.

The above results are very encouraging. One can get the feeling that, at last, one can start understanding nonlocality without the Hilbert space machinery, that, at last, one can study quantum physics from the outside, i.e., from the perspective of future physical theories (assuming these will keep Einstein’s no-signaling constraint) and no longer from the perspective of the old classical mechanical physics. But there is still a lot to be done! For instance, it is surprising (and annoying in my opinion) that one is still unable to simulate measurements on partially entangled states using the nonlocal correlation (actually we could prove that this is impossible with a single instance of the correlation, but there is hope that one can simulate partially entangled qubit pairs with 2 instances [62]). Let me emphasize that all of today’s known simulation models for partially entangled qubits include some sort of communication, let’s say from Alice to Bob. Consequently, in all these models Bob can’t output his results before Alice was given her input. This contrasts with the situation in quantum measurements where Bob doesn’t need to wait for Alice (he does not even need to know about the existence of Alice) and with the simulation model for maximally entangled qubits using the PR-box. It would be astonishing if partially entangled state could not be simulated in a time-symmetric way.

XI. CONCLUSION

The history of non-locality in physics is fascinating. It goes back to Newton (section II) It first accelerated around 1935 with Einstein’s EPR and Schrödinger cat’s papers. Next, it slowly evolved, with the works of John Bell, John Clauser and Alain Aspect among many others, from a mere philosophical debate to an experimental physics question, or even to experimental meta-physics as Abner Shimony nicely put it [70]. Now, during the last decade, it has run at full speed. Conceptually the two major breakthroughs were, first Artur Ekert’s 1991 PRL which strongly suggests a deep link between non-locality and cryptography, section X A. The second breakthrough, in my opinion, is the PR-box, section X B. the understanding that non-signaling correlations can be analyzed for themselves, without the need of the usual Hilbert space artillery, thus providing a simple conceptual tool for the unravelling of quantum non-locality. We have reviewed that the no-cloning theorem, the uncertainty relation, the monogamy of extreme correlation and the security of key distribution, all properties usually associated to quantum physics are actually properties of any theory without signaling, section X. In particular we emphasized that the second breakthrough, the PR-box, allows one to confirm the first breakthrough: there is an intimate connection between violation of a Bell inequality and security of quantum cryptography.

And relativity, can it be considered complete? Well, if nonlocality is really real, as widely supported by the accounts summaries in this article, then all complete theories should have a place for it. Hence, the question is: “Does relativity hold a place for non-signaling nonlocal correlations?”.

Acknowledgment

This article has been inspired by talks I gave in 2005 at the IOP conference on Einstein in Warwick, the QUPON conference in Vienna, the Annuus Mirabilis Symposium in Zurich, le séminaire de l’Observatoire de Paris and the Ehrenfest Colloquium in Leiden. This work has been supported by the EC under projects RESQ and QAP (contract n. IST-2001-37559 and IST-015848) and by the Swiss NCCR Quantum Photonics.
[1] A. Einstein, B. Podolsky & N. Rosen, Can quantum-mechanical description of physical reality be considered complete?, Phys. Rev. 47, 777-780 (1935).

[2] In contrast to S. Weinberg, Dreams of a final theory, Vintage/Randome House, 1994.

[3] A. Shimony, in Foundations of Quantum Mechanics in the Light of New Technology, ed. S. Kamefuchi, Phys. Soc. Japan, Tokyo, 1983.

[4] For a lively account of the history of quantum nonlocality and of the people who made it happen, see: The Age of Entanglement, Louisa L. Gilder, Knopf publishing, New-York, 2006

[5] Isaac Newton, Papers & Letters on Natural Philosophy and related documents, page 302, Edited, with a general introduction, by Bernard Cohen, assisted by Robert E. Schofield Harvard University Press, Cambridge, Massachusetts, 1958

[6] E. Schrödinger, Naturwissenschaften 23, 807 (1935).

[7] J. S. Bell, Speakable and Unspeakeable in Quantum Mechanics: Collected papers on quantum philosophy (Cambridge University Press, Cambridge, 1987, revised edition 2004).

[8] N. Gisin, J. Math. Phys. 24, 1779-1782 (1983).

[9] A.K. Ekert, Phys. Rev. Lett. 67, 661 (1991).

[10] L. Mandel, Optical coherent & quantum optics, Cambridge University Press, 1995.

[11] W. Tittel, J. Brendel, N. Gisin and H. Zbinden, Phys. Rev. Lett. 81, 3563-3566 (1998).

[12] Tittel W., Brendel J., Gisin N. & H. Zbinden, Long-distance Bell-type tests using energy-time entangled photons, Phys. Rev. A, 59, 4150-4163 (1999).

[13] Ivan Marcikic, Hugues de Riedmatten, Wolfgang Tittel, Hugo Zbinden, Matthieu Legeré and Nicolas Gisin, Phys. Rev. Lett. 93, 180502 (2004).

[14] M ≤ 3 is equivalent to the famous CHSH-Bell inequality: J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969).

[15] B.S. Cirel'son, Lett. Math. Phys. 4, 93 (1980).

[16] J. S. Bell, Speakable and unspeakable in quantum mechanics, page 152, Cambridge: University Press, 1987.

[17] A. Aspect, J. Daëmbard and Roger, Phys. Rev. Lett. 49, 1804 (1982).

[18] J. Freedman, and J. F. Clauser, Phys. Rev. Lett., 28, 938-941 (1972); A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett., 47, 460-463 (1981); Z. Y. Ou and L. Mandel, Phys. Rev. Lett., 61, 50-53 (1988); Shihi and Alley, Phys. Rev. Lett. 61, 2921 (1988); P. G. Kwait, K. Mattle, H. Weinfurter, A.Zeilinger, A. V. Sergienko, and Y. H. Shihi, Phys. Rev. Lett., 75 4337 (1995); J. G. Rarity and P. R. Tapster, Phys. Rev. Lett., 64 2495-2498 (1990); J. Brendel, E. Mohler, and W. Martienssen, Europhys. Lett., 20, 575-580 (1992); P. R. Tapster, J. G. Rarity, and P. C. M. Owens, Phys. Rev. Lett., 73, 1923-1926 (1994).

[19] G. Wechs, M. Reck, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett., 81, 5039 (1998).

[20] M.A. Rowe et al., Nature 149 791-794 (2001).

[21] N. Gisin and H. Zbinden, Phys. Lett. A 264 103-107 (1999).

[22] H. Zbinden, J. Brendel, W. Tittel, and N. Gisin, Phys. Rev. A, 63, 022111 (2001); H. Zbinden, J. Brendel, N. Gisin and W. Tittel, J. Phys. A : Math. Gen., 34, 7103-7109 (2001).

[23] N. Gisin, V. Scarani, W. Tittel and H. Zbinden, 100 years of Q theory, Proceedings, Annal. Phys. 9, 831-842 (2000); quant-ph/0009055; André Stefanov, Hugo Zbinden and Nicolas Gisin, Physical Review Letters 88, 120404 (2002); N. Gisin, Sundays in a Quantum Engineer’s Life, Proceedings of the Conference in Commemoration of John S. Bell, Vienna 10-14 November 2000; V. Scarani, W. Tittel, H. Zbinden and N. Gisin, Phys. Lett. A 276 1-7 (2000).

[24] A. Suarez & V. Scarani, Phys. Lett. A 232, 9 (1997).

[25] D. Bohm, Phys. Rev., 85, 166-193 (1952); D. Bohm and B.J. Hilley, The Undivided Universe, New York: Routledge, 1993.

[26] S. Popescu and D. Rohrlich, Found. Phys. 24, 379 (1994).

[27] D. Rohrlich and S. Popescu, quant-ph/9508009 and quant-ph/9709026.

[28] N. Gisin, quant-ph/0503007.

[29] R.F. Werner, Phys. Rev. A 40, 4277 (1989).

[30] B.M. Terhal, M.M. Wolf and A.C. Doherty, Physics Today, pp 46-52, April 2003.

[31] N.D. Mermin, quant-ph/9609013 and quant-ph/9801057.

[32] M. Esfeld, Studies in History and Philosophy of Modern Physics 35B, 601-617, 2004.

[33] Cheng-Zhi Peng et al., Phys. Rev. Lett. 94, 150501 (2005).

[34] Zhi Zhao et al., Nature 430, 54-58 (2004).

[35] H. Haefliger et al., Appl. Phys. B 81, 151 (2005).

[36] B.B. Blinov, D.L. Moehring, L.M. Duan and C. Monroe, Nature 428, 153-157 (2004).

[37] J. Volz et al., quant-ph/0511183.

[38] B. Julsgaard, J. Sherson, J.I. Cirac and E.S. Polzik, Nature 432, 482-486 (2004).

[39] E. Altwiescher et al., Nature 418, 304 (2002); S. Fasel et al., Phys. Rev. Lett. 94 110501, 2005 and quant-ph/0512022

[40] C.W. Chou et al., quant-ph/0510055.

[41] J.W.Pan, D. Bouwmeester, H.Weinfurter and A. Zeilinger, Phys. Rev. Lett. 80, 3891 (1998).

[42] T. Jennewein, G. Weihs, J.-W. Pan, and A. Zeilinger, Phys.Rev.Lett. 88, 017903 (2002).

[43] H. de Riedmatten et al., Phys. Rev. A 71, 050302 (2005).

[44] S. Tanzilli et al., Nature 437, 116-120 (2005).

[45] B.S. Tsurelson, Hadronic J. Supplement 8, 329 (1993).

[46] V. Buzek and M. Hillery, Phys. Rev. A 54, 1844 (1996)

[47] L. Masanes, A. Acin and N. Gisin, quant-ph/0508016

[48] J. Barrett, N. Linden, S. Massar, S. Pironio, S. Popescu and D. Roberts, Phys. Rev. A 71, 022101 (2005).

[49] W. van Dam, quant-ph/0501159; S. Wolf and J. Wullschleger, quant-ph/0502030; H. Buhrman, M. Christandl, F. Unger, S. Wehner and A. Winter, quant-ph/0504133; T. Short, N. Gisin and S. Popescu, quant-ph/0504134; J. Barrett and S. Pironio Phys. Rev. Lett. 95, 140401 (2005); N.S. Jones and L. Masanes, quant-ph/0506182; J. Barrett, quant-ph/0508211.

[50] D. Collins and N. Gisin, J. Phys. A: Math. Gen. 37, 1775 (2004).

[51] A. Acin, N. Gisin and L. Masanes, quant-ph/0510094.

[52] For an independent but related work see: J. Barrett, L. Hardy and A. Kent, Phys. Rev. Lett. 95, 010503 (2005).

[53] N. Gisin and S. Wolf, Phys. Rev. Lett. 83, 4200 (1999);
N. Gisin and S. Wolf, *Proceedings of CRYPTO 2000*, Lecture Notes in Computer Science **1880**, 482, Springer-Verlag, 2000, quant-ph/0005042; A. Acin and N. Gisin, Phys. Rev. Lett. **94**, 020501 (2005); quant-ph/0310054.

[54] V. Scarani, private communication.

[55] C. H. Bennett and G. Brassard, Proc. IEEE Int. Conf. Computers, Systems and Signal Processing, New York, 175 (1984).

[56] P. W. Shor and J. Preskill, Phys. Rev. Lett. **85**, 441 (2000).

[57] B. Kraus, N. Gisin and R. Renner, Phys. Rev. Lett. **95**, 080501 (2005); quant-ph/0410215.

[58] T. Maudlin, Philosophy of Science Association **1**, 404-417, 1992.

[59] G. Brassard, R. Cleve and A. Tapp, Phys. Rev. Lett. **83**, 1874 (1999).

[60] M. Steiner, Phys. Lett. A **270**, 239 (2000).

[61] B. Gisin, N. Gisin, Phys. Lett. A **260**, 323 (1999).

[62] B. F. Toner, D. Bacon, Phys. Rev. Lett. **91**, 187904 (2003).

[63] V. Scarani and N. Gisin, Physics Letters A **295**, 167-174 (2002); Quant-ph/0110074.

[64] V. Scarani and Nicolas Gisin, Brazilian Journal of Physics **35**, 328-332 (2005); quant-ph/0410025.

[65] N. J. Cerf, N. Gisin, S. Massar and S. Popescu Phys. Rev. Lett. **94**, 220403 (2005).

[66] J. Degorre, S. Laplante and J. Roland, quant-ph/0507120.

[67] N. Brunner, N. Gisin and V. Scarani, New Journal of Physics **7**, 1-14 (2005); quant-ph/0412109.

[68] S. Wolf and J. Wullschleger, quant-ph/0502030.

[69] For another recent result sustaining the conjecture the partially entangled state are more nonlocal than maximally entangled states see: A. Acin, R. Gill and N. Gisin, Phys. Rev. Lett. **95**, 210402 (2005); quant-ph/0506225; and for a recent review read A. Methot and V. Scarani, quant-ph/05XXX???

[70] *Experimental Metaphysics*, eds R.S. Cohen, M. Horne and J. Stachel, Kluwer Acad. Press, 1997.