\[ \theta_{BAO} \text{ estimates and the } H_0 \text{ tension} \]

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An observational tension on estimates of the Hubble parameter, \( H_0 \), using early and late Universe information, is being of intense discussion in the literature. Additionally, it is of great importance to measure \( H_0 \) independently of CMB data and local distance ladder method. In this sense, we analyze \( \sim 15 \) measurements of the transversal BAO scale, \( \theta_{BAO} \), obtained in a weakly model-dependent approach, in combination with other data sets obtained in a model-independent way, namely, Big Bang Nucleosynthesis (BBN) information, 6 gravitationally lensed quasars with measured time delays by the H0LiCOW team, and measures of cosmic chronometers (CC). We find \( H_0 = 74.88^{+1.9}_{-2.1} \text{ km s}^{-1} \text{ Mpc}^{-1} \) and \( H_0 = 72.06^{+1.2}_{-1.3} \text{ km s}^{-1} \text{ Mpc}^{-1} \) from \( \theta_{BAO}+\text{BBN}+\text{H0LiCOW} \) and \( \theta_{BAO}+\text{BBN}+\text{CC} \), respectively, in full accordance with local measurements. Moreover, we estimate the sound horizon at drag epoch, \( d_{\text{drag}} \), independent of CMB data, and find \( d_{\text{drag}} = 144.1^{+5.3}_{-5.5} \) Mpc (from \( \theta_{BAO}+\text{BBN}+\text{H0LiCOW} \)) and \( d_{\text{drag}} = 150.4^{+2.7}_{-3.3} \) Mpc (from \( \theta_{BAO}+\text{BBN}+\text{CC} \)). In a second round of analysis, we test how the presence of a possible spatial curvature, \( \Omega_k \), can influence the main results. We compare our constraints on \( H_0 \) and \( d_{\text{drag}} \) with other reported values. Our results show that it is possible to use a robust compilation of transversal BAO data, \( \theta_{BAO} \), jointly with model-independent measurements, in such a way that the tension on the Hubble parameter disappears.

I. INTRODUCTION

The standard cosmological model, the flat \( \Lambda \text{CDM} \), based on general relativity theory plus a positive cosmological constant and dark matter, has been able to explain accurately the most diverse observations made in the past two decades. Despite that, as new astronomical observations improve, in precision and in the diversity of cosmic tracers, arises a possible inability to explain within the standard paradigm quantitatively different measurements, and this is putting the \( \Lambda \text{CDM} \) cosmology in a crossroads. The most notable issue is the current tension on the Hubble parameter \( H_0 \). Assuming the \( \Lambda \text{CDM} \) scenario, Planck-CMB data analysis provides \( H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1} \) \cite{1}, while a model-independent local measurement from Hubble Space Telescope observations of 70 long-period Cepheids in the Large Magellanic Cloud results \( H_0 = 74.03 \pm 1.42 \) km s\(^{-1}\)Mpc\(^{-1} \) \cite{2}. These estimates are in \(4.4 \sigma \) tension. Additionally, a combination of time-delay cosmography from H0LiCOW lenses and the distance ladder results is at \(5.2 \sigma \) tension with CMB constraints \cite{3}. Another accurate independent measurement was carried out in \cite{4}, from Tip of the Red Giant Branch, showing \( H_0 = 69.8 \pm 1.1 \text{ km s}^{-1} \text{ Mpc}^{-1} \). Other recent analysis also put in crisis the \( \Lambda \text{CDM} \) model \cite{5–10}. In addition to this disagreement with diverse observations, it is important to remember that the cosmological constant suffers from some theoretical problems \cite{11,12} that motivates alternative scenarios that could, at the same time, explain the observational data and have some theoretical appeal. This stimulated recent discussions about whether a new physics beyond the standard cosmological model can solve the \( H_0 \) tension \cite{13–26}.

Other less noticed –but not less important– issue concerns the standard ruler measurement, that is, the comoving sound horizon scale at the end of drag epoch, \( r_{\text{drag}} = d_{\text{drag}} \). Assuming the flat \( \Lambda \text{CDM} \) cosmology, analyses of the CMB measurements from the Planck collaboration \cite{1} and the WMAP team \cite{27} give \( r_{\text{drag}} = 147.09 \pm 0.26 \) Mpc and \( r_{\text{drag}} = 152.99 \pm 0.97 \) Mpc \cite{4}, respectively. But there are also estimates of the sound horizon scale at low redshift combining data from large-scale structure: \( r_{\text{drag}} = 150.0 \pm 4.7 \) Mpc (CSB), \( r_{\text{drag}} = 143.9 \pm 3.1 \) Mpc (CSBH), where C-S-B-H indicate a combination of data from Cosmic Chronometers, SNe, BAO data, and local \( H_0 \) measurement (for details, see \cite{28}). An interesting information regarding the estimate of \( r_{\text{drag}} \) using CMB data is that this derivation can be somehow biased by model hypotheses \cite{29}. For this, the literature exhibits the efforts to obtain a model-independent estimate of \( r_{\text{drag}} \) \cite{28,30}. An estimate of this type obtains \( r_{\text{drag}} = 136.7 \pm 4.1 \) Mpc \cite{30}, which is in tension of \( \sim 2.5 \sigma \) and \( \sim 3.8 \sigma \) with the Planck and WMAP values, respectively (for other analyses see, e.g., \cite{29,31,32}). Recently, final measurements from the completed SDSS lineage of experiments in large-scale structure provide \( r_{\text{drag}} = 149.3 \pm 2.8 \) Mpc \cite{33}, in good agreement with Planck-CMB estimate.

The main aim of this work is to obtain constraints on some cosmological parameters of interest in current lit-
erature, namely $H_0$ and $r_d$, independent of CMB and local distance ladder data, using sets of data obtained following weakly model-dependent or model-independent approaches. Such analyses are important, and of great interest, because provides an alternative way to quantify the current observational tension on these parameters and will provide results that can shed light on this problem. To achieve these objectives, in this work we use measurements of the transversal BAO scale ($\theta_{\text{BAO}}$), data obtained following an approach that weakly depends on the assumption of a cosmological model, as described in ref. [34] (all these measurements were obtained following the same methodological approach, however, since the clustering analyses were performed with diverse cosmological tracers –blue galaxies, luminous red galaxies, quasars– one should be careful with the systematics of each dataset [for some tests to deal with systematics in data analyses see, e.g., [35–37]). See [38–41] for others recent discussions on the cosmological constraints investigations under the perspective of the BAO measurements by other independent groups.

In these combined analyses we also use the big bang nucleosynthesis (BBN) data, information from gravitationally lensed quasars with measured time delays (H0LiCOW data), and the cosmic chronometers (CC) data. We found that a robust analysis from these data sets is possible to get an accuracy up to $\sim 1.7\%$ on $H_0$, and this parameter lives in the range to be compatible with local measurements of $H_0$. To the authors’ knowledge, this is the first $H_0$ measurement using BAO data information plus others data sets obtained in a weakly model-dependent way, able to generate high $H_0$ values, in order to be compatible with local and model-independent measures, within the $\Lambda$CDM framework.

The paper is structured as follows. In the next section we present the data sets used in this work and the statistical methodology. In section II we discussed the main results of our analysis. In section IV we outline our final considerations and perspectives.

II. METHODOLOGY

We describe below the observational data sets and the statistical methods that we use to explore our parameter space.

**Transversal BAO**: Let us adopt 15 BAO measurements, $\theta_{\text{BAO}}(z)$, obtained in a weakly model-dependent approach, compiled in table I in [32]. These measurements were obtained using public data releases (DR) of the Sloan Digital Sky Survey (SDSS), namely: DR7, DR10, DR11, DR12, DR12Q (quasars) [12]. It is important to notice that due to the cosmological-independent methodology used to perform these transversal BAO measurements their errors are larger than the errors obtained using a fiducial cosmology approach. The reason for this fact is that, while in the former methodology the error is given by the measure of how large is the BAO bump, in the later approach the model-dependent best-fit of the BAO signal quantifies a smaller error. Typically, in the former methodology the error can be of the order of $\sim 10\%$, but in some cases it can arrive to $18\%$, and in the later approach it is of the order of few percent [34].

**BBN**: The deuterium abundance and the radiative capture of protons on deuterium to produce $3\text{He}$ is one the most widely used primordial elements for constraining the baryon density. The empirical value for the reaction rate is computed in [43], constraining the baryon density to $100\Omega_0 h^2 = 2.235 \pm 0.016$, where the dimensionless parameter $h = H_0/100$ is the reduced Hubble constant. We adopt this value as the Gaussian prior likelihood in our analysis.

**H0LiCOW**: A powerful geometric method to measure $H_0$ is offered by the gravitational lensing. The time delay between multiple images, produced by a massive object (lens) and the gravitational potential between a light-emitting source and an observer, can be measured by looking for flux variations that correspond to the same source event. This time delay depends on the mass distribution along the line of sight and in the lensing object, and it represents a complementary and independent approach with respect to the CMB and the distance ladder. Due to their variability and brightness, lensed quasars have been widely used to determine $H_0$ (see, e.g., [44–46] and references therein). The time delay is highly sensitive to $H_0$, but with a weak dependence on other cosmological parameters. In the present work, we use the six systems of strongly lensed quasars reported by the H0LiCOW Collaboration [3].

**CC**: The late expansion history of the Universe can be studied in a model-independent fashion by measuring the age difference of cosmic chronometers (CC), such as old and passively evolving galaxies that act as standard clocks [47, 48]. In our analysis we consider the measurements of CC as presented in [48].

We ran CLASS+MontePython code [49–51] using Metropolis-Hastings mode to derive constraints on cosmological parameters from the BAO+BBN, BAO+BBN+H0LiCOW and BAO+BBN+CC data combination. In a first round of analysis we consider that the background expansion framework is fix assuming a flat-$\Lambda$CDM scenario. Next, we also analyze the case $\Lambda$CDM + $\Omega_0$. All of our runs reached a Gelman-Rubin convergence criterion of $R - 1 < 10^{-3}$. In what follows, we discuss the main results of our analyses.
III. RESULTS

The left panel of figure 1 shows the parametric space in the plane $H_0 - \Omega_m$ from $\theta_{BAO}+BBN$, $\theta_{BAO}+BBN+H0LiCOW$ and $\theta_{BAO}+BBN+CC$ data combination. We find $H_0 = 74.88^{+1.9}_{-1.8}$ km s$^{-1}$ Mpc$^{-1}$ and $H_0 = 72.06^{+1.2}_{-1.3}$ km s$^{-1}$ Mpc$^{-1}$ at 68% confidence level (CL) from $\theta_{BAO}+BBN+H0LiCOW$ and $\theta_{BAO}+BBN+CC$, respectively. The total matter density (baryon + dark matter density) is fit to be $\Omega_m = 0.2763^{+0.027}_{-0.028}$ and $\Omega_m = 0.2515^{+0.016}_{-0.016}$ at 68% CL from $\theta_{BAO}+BBN+H0LiCOW$ and $\theta_{BAO}+BBN+CC$, respectively. Since the measurements of $\theta_{BAO}$ have error bars a bit larger than other BAO data compilations, one can notice that the $H_0$ parameter becomes more degenerate from $\theta_{BAO}+BBN$ constraints when compared to other BAO + BBN analyses performed in the literature [52, 53].

Interestingly to note that the $H_0 - \Omega_m$ plane, from $\theta_{BAO}$ data, also tends to be positively correlated, but generating high $H_0$ values. We add H0LiCOW lenses and CC data to better bounds the parameter space. In Figure 1 the horizontal light purple and light red bands correspond to $H_0$ values from the BAO + BBN analysis [52] and the SHOES measurement [2], respectively. We note that $H_0$ is at $\sim 2\sigma$ and $\sim 2.5\sigma$ tension from $\theta_{BAO}+BBN+H0LiCOW$ and $\theta_{BAO}+BBN+CC$, respectively, when compared to the measurements performed in [52]. In contrast, our $H_0$ estimates are in agreement with SHOES [2].

Therefore, combining $\theta_{BAO}$ with other data obtained in a model-independent way, and without using CMB and supernovae data, we see their concordance with local measurements of $H_0$. A direct interpretation of why the $\Lambda$CDM scenario is generating high $H_0$ values, is because our global fit predicts less dark matter today—in contrast, more dark energy—via the relation $\Omega_m + \Omega_{DE} = 1$, where $\Omega_m = \Omega_b + \Omega_{DM}$. Notice that $\Omega_b$ here is determined from BBN information. So, the change on $\Omega_m$ estimate is due to dark matter density only, once the radiation (photons + neutrinos) contribution is negligible at low-$z$. Because our joint analysis predicts more dark energy at late times, the Universe expands faster, generating a larger $H(z)$ evolution and high $H_0$ values. In [52], was analyzed CMB + $\theta_{BAO}$, where we report $H_0 = 69.23^{+0.50}_{-0.40}$ km s$^{-1}$ Mpc$^{-1}$, where we can see a displacement of $\sim +2$ km s$^{-1}$ Mpc$^{-1}$, in comparison with the Planck + BAO analysis made by the Planck Collaboration [1]. Again, it is clear that $\theta_{BAO}$ tends to generate higher $H_0$ values in comparison with other BAO compilation in literature. The $H_0$ value from CMB data is inferred analyzing the first acoustic peak position, which depends on the angular scale $\theta_\ast = d_\ast^A / D_A$, where $d_\ast$ is the sound horizon at decoupling (the distance a sound wave traveled from the big bang to the epoch of the CMB-baryons decoupling) and $D_A$ is the angular diameter distance at decoupling, which in turn depends on the expansion history, $H(z)$, after decoupling, controlled also by the ratio $\Omega_{DM} / \Omega_{DE}$ and $H_0$ mainly. Our joint fit is generating a larger $H(z)$

FIG. 1. Left panel: The 68% CL and 95% CL regions in the $H_0 - \Omega_m$ plane, inferred from $\theta_{BAO} + BBN$ analyses in combination with H0LiCOW and CC data. The vertical light-purple and light-red bands correspond to $H_0$ from BAO + BBN taken from [52] and the SHOES measurement [2], respectively. Right panel: The 68% CL and 95% CL regions in the $H_0 - r_d$ plane from $\theta_{BAO} + BBN + H0LiCOW$ and $\theta_{BAO} + BBN + CC$ analyses. The parameter $H_0$ is measured in units of km s$^{-1}$ Mpc$^{-1}$ and $r_d$ in Mpc.
and, at the same time, changing the slope of the Sachs-Wolfe plateau, that is, the late-time integrated Sachs-Wolfe effect (ISW). Thus, our joint fit (CMB + \(\theta_{BAO}\)) is changing primarily the \(D_A^*\) history, increasing the angular diameter distance to the last scattering surface, thus generating high estimates on the \(H_0\) parameter.

Figure 2 shows a compilation of \(H_0\) measurements taken from the recent literature for direct comparison with our results. We can notice that \(H_0\) obtained in this work is in agreement with SH0ES, H0LiCOW+STRIDES and CCHP. Our estimates start to have a significant tension when compared to measures involving other BAO data compilation and Planck data only.

The right panel of Figure 1 shows the parametric space in the \(H_0 - r_d\) plane. We find \(r_d = 144.1^{+5.3}_{-5.5}\) Mpc (from \(\theta_{BAO}+\text{BBN}+\text{H0LiCOW}\)) and \(r_d = 150.4^{+2.7}_{-3.3}\) Mpc (from \(\theta_{BAO}+\text{BBN}+\text{CC}\)) at 68% CL. Both measures are compatible with each other. This fit represents an \(r_d\) constraint obtained independently of CMB data. For a qualitative comparison, the Planck team reported the value \(r_d = 147.21 \pm 0.23\) Mpc from CMB + BAO joint analysis. We see that this estimate is in concordance with ours. Regarding analyses independent of the CMB data, we can mention, for instance, a model-independent reconstruction of \(H(z)\) done in ref. \[59\], where it is reported \(r_d = 148.48^{+3.73}_{-3.74} \pm 0.23\) Mpc. In ref. \[57\], the sound horizon at radiation drag is considered as a standard ruler, and it is found \(r_d = 142.8 \pm 3.7\) Mpc. Also, in ref. \[28\] the authors found \(r_d = 143.9 \pm 3.1\) Mpc using CC, SNe Ia, BAO, and a local measurement of \(H_0\). Using the inverse distance ladder method, the DES collaboration found \(r_d = 145.2 \pm 18.5\) Mpc from SNe Ia and BAO measurements \[58\]. Our estimates are consistent with these measurements too. We note that only the \(r_d\) from \(\theta_{BAO}+\text{BBN}+\text{CC}\) joint analysis is in \(\sim 1\sigma\) tension with ref. \[28\]. Other results independent of CMB data were obtained in \[59\] \[60\].

### A. Adding spatial curvature

Until now we have performed statistical analyses considering the flat \(\Lambda\)CDM model. Here we extend the parameter space to analyse these important quantities, \(H_0\) and \(r_d\), within a model beyond the flat \(\Lambda\)CDM. For this, we now consider the spatial curvature as a free parameter, i.e., \(\Omega_k \neq 0\). As we shall see below, our analyses show compatibility with \(\Omega_k = 0\), within 1\(\sigma\) error, although we observe an enlargement of the error bars (as expected because there is one more parameter in the analysis).

Analyzing \(\Lambda\)CDM + \(\Omega_k\) from BAO+BBN+H0LiCOW we find: \(H_0 = 75.08^{+3.6}_{-3.0}\) km s\(^{-1}\) Mpc\(^{-1}\) and \(\Omega_k = -0.0697^{+0.19}_{-0.26}\). As argued in \[5\], the time delay is highly sensitive to \(H_0\), but with a weak dependence on other parameters. Thus, we can note that when assuming \(\Omega_k\) as a free parameter, and considering the H0LiCOW sample, no significant changes are observed in the baseline of parameters. Only the effect of slightly increasing the error bars due to the presence of an extra parameter, \(\Omega_k\). This scenario can change the perspectives when considering BAO+BBN+CC; in fact, in this case we find \(H_0 = 66.54 \pm 3.76\) km s\(^{-1}\) Mpc\(^{-1}\) and \(\Omega_k = 0.276^{+0.17}_{-0.28}\). In this case, considering \(\Omega_k\) as a free parameter, this can significantly changes the evolution of the \(H(z)\) function, which depends directly on all physical species and geometrical effects. We note this effect by observing an enlargement and shift in the estimate and error bar of \(H_0\) to accommodate \(\Omega_k\) effects into the \(H(z)\) function. In this particular case we find \(\Omega_m = 0.2378^{+0.020}_{-0.019}\). Thus, to accommodate \(\Omega_k\) effects, looking through the relationship \(\Omega_k + \Omega_m + \Omega_\Lambda = 1\), and using for comparison the \(\Omega_m\) best fit derived in the previous section without \(\Omega_k\), we note that the presence of \(\Omega_k\) decreases mainly the value of \(\Omega_\Lambda\). The left panel in Figure 3 shows the constraints in the plane \(H_0 - \Omega_k\). We did not find significant deviations from the \(\Omega_k = 0\) case.

The right panel of Figure 3 shows the 68% CL and 95% CL regions in the \(H_0 - r_d\) plane, analyses done with and without the parameter \(\Omega_k\) for comparison.
Assuming ΛCDM + Ω_k, we find r_d = 146.9^{+4.2}_{-5.5} Mpc (from BAO+BBN+H0LiCOW) and r_d = 158.3^{+5.8}_{-7.1} Mpc (from BAO+BBN+CC). As previously commented, in the BAO+BBN+H0LiCOW joint analyses no significant deviations were observed, as compared to the flat case. In the BAO+BBN+CC analyses, we can clearly notice an enlargement for higher values in r_d, possibly due to the change in H_0 and the strong correlation of r_d with H_0. It is important to emphasize that all analyses investigated here agree with each other.

**B. SDSS final release**

During the final stage of preparation of this work, the SDSS collaboration released their BAO final measurements covering eight distinct redshift intervals, obtained and improved over the past 20 years [33]. Given the importance of these data for cosmology in recent years, here we perform a brief analysis for comparison with our measurements, using the D_V(z)/r_d, D_M(z)/r_d, and D_H(z)/r_d measurements compiled in Table 3 in [33], regarding BAO-only data. In what follows, we call this data compilation by SDSS (BAO). We assume that the uncertainties are Gaussian approximations to the likelihoods for each tracer ignoring the correlations between measurements (as suggested in the SDSS collaboration paper).

Figure 3 shows the 68% CL and 95% CL regions in the H_0−Ω_k plane from θ_{BAO} + BBN, SDSS (BAO) + BBN, and θ_{BAO} + SDSS (BAO) + BBN joint analysis. Evidently, the accumulation of accuracy and improvement in the measurements over the years make the analysis of SDSS (BAO) + BBN very robust in the errors determination, in a direct comparison with θ_{BAO} + BBN only (see the figure 1). We find H_0 = 68.32^{+0.98}_{-1.1} km s^{-1} Mpc^{-1}, r_d = 151.9^{+2.8}_{-2.8} Mpc, Ω_m = 0.27^{+0.015}_{-0.016} at 68% CL from θ_{BAO} + SDSS (BAO) + BBN joint analyses. This estimate of H_0, influenced by SDSS (BAO) data, is in agreement with the Planck-CMB data, and in ~4σ tension with the SHOES [2] value. There is no tension on the r_d parameter when compared to Planck-CMB data.

**IV. FINAL REMARKS**

We obtained accurate constraints on H_0 and r_d parameters, independently of CMB data and local distance ladder data. In this work we are motivated to look how recent transversal BAO measurements (that is, from θ_{BAO} estimates [32]), in combination with other model-independent data sets, can bound these parameters and what direction do they take in light of recent observational tensions, especially in the context of the H_0 tension. We find an accuracy of ~2.6% and ~1.7% on H_0 from θ_{BAO}+BBN+H0LiCOW and θ_{BAO}+BBN+CC, respectively. We observe that both values are compatible with local estimates of H_0, and in tension with Planck data only and some joint analyses in combination with other BAO compilations of the literature. Our results show that it is possible to use a robust compilation of BAO data, i.e., the θ_{BAO} compilation, in such a way that...
FIG. 4. Parametric space in the $H_0 - \Omega_m$ plane inferred from $\theta_{BAO} + $ BBN, SDSS (BAO) final release + BBN, and $\theta_{BAO} + $ SDSS (BAO) final release + BBN joint analyses. The $H_0$ parameter is measured in units of km s$^{-1}$ Mpc$^{-1}$.

the tension on the $H_0$ parameter is minimized or even not exist, when compared to local and model-independent measurements.

An interesting perspective regards the measurements of transversal BAO data. With arriving new data from ongoing astronomical surveys we expect new transversal BAO measurements, with both features: more precise estimates and performed at diverse redshifts. In fact, these data has shown potential to constrain better $r_d$ and $H_0$, important quantities in modern cosmology because they provide absolute scales to measure the Universe evolution at opposite sides.

ACKNOWLEDGMENTS

RCN would like to thank the agency FAPESP for financial support under the project No. 2018/18036-5. AB acknowledges a CNPq fellowship.

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