Superconductor-Metal-Insulator Crossover in Disordered Thin Films *

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Superconductor-Metal-Insulator crossover in disordered thin films is discussed on the basis of a random junction-network model.

Key Words: superconductor-metal-insulator crossover, disordered thin films, random resister network, Josephson junction network

The superconductor-insulator (S-I) transition in thin films is one of the typical phenomena of quantum phase transition. [1] Recently a metallic phase intervening between superconducting and insulating phases has been drawing attention. [2] Although several theories [2] modifying the idealized theory [1] for the S-I transition have been proposed, present understanding of the metallic phase is far from conclusive. In this Short Note we also propose a simple phenomenological model and try to understand the temperature dependence of the resistance from which we judge the phase at zero temperature, superconductor (S) or metal (M) or insulator (I).

Our phenomenological model is a combination of two models. One is the random Josephson-junction-network model [3] for disordered superconductors and the other is the random resister-network model [4] for disordered metals or insulators. We consider a network consisting of superconducting islands linked by two types of junctions, J (Josephson) and N (normal). At the type-J junction the current is carried by Cooper pairs, while at the type-N junction by normal electrons. The type-N junction in our model is expected for the case where the distance between islands is so large that the Cooper pairs are broken in the junction. In real materials the type-N junction is also expected for the case where the size of the island is so small that the Cooper pairs are not formed there. In our model only the resistance caused by the junctions is taken into account and the resistance drop at around the superconducting transition temperature $T_c$ of the islands is neglected. In the following we set $\hbar = k_B = 1$. At a type-J junction [3] the resistance $R(T)$ as a function of the temperature $T$ is assumed to be given by the Ambegaokar-Halperin formula

$$R(T) = R_N/[I_0(E_J(T)/T)]^2,$$

where $I_0(x)$ is the modified Bessel function of order 0. The Josephson-coupling energy $E_J(T)$ is given by the Ambegaokar-Baratoff form

$$E_J(T) = \frac{1}{4} \frac{R_0}{R_N} \Delta(T) \tanh \frac{\Delta(T)}{2T},$$

where $R_0$ is the quantum unit of the resistance, $R_0 = \pi/e^2$, and $\Delta(T)$ is the BCS gap function. This model is appropriate for relatively thick films where contacts among islands are relatively strong. Since the normal resistance $R_N$ between islands saturates at low temperatures and exhibits only weak temperature dependence for such a strong contact, we neglect the temperature dependence of $R_N$ in accordance with the former study. [3] In contrast to this good metallic behavior a poor metallic junction discussed below exhibits some temperature dependence.

At a type-N junction [4] the resistance is assumed to be given by the Landauer formula for quantum point contact as

$$R(T) = R_0 \cdot [1 + \exp(E_c/T)],$$

where $E_c$ is the threshold energy for the electron transmission measured from the chemical potential. The junction is insulating for $E_c > 0$ and metallic for $E_c < 0$. This model is appropriate for relatively thin films where contacts among islands are relatively weak. For such a weak contact the resistance exhibits poor metallic or insulating temperature dependence. In Fig. 1 the temperature dependence of the resistance of a type-N junction is shown for a poor metallic case. While the resistance saturates at low temperatures, it exhibits almost exponential temperature dependence in some temperature range.

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In the following we consider the circuit consisting of 3 conductors connected in parallel. Each conductor consists of 10 composites of resistors connected in series. Each composite consists of 3 resistors connected in parallel. Each resistor is modeled by either type-J or type-N junction. Although circuits in general cannot be reduced to series-parallel combinations of resistors, we adopt this model, 3-10-3 circuit, for simplicity to discuss qualitative aspects of the resistance of a random network.

In Fig. 2 the temperature dependence of the resistance for 3-10-3 circuit is shown in the case where every resistor in the circuit is type-J junction. The values of $R_N$ for junctions are randomly chosen. The resistance vanishes faster than single exponential function of $T$ as $T$ is decreased. This temperature dependence is similar to that obtained in the former study. [3]

In Fig. 3 the temperature dependence of the resistance for 3-10-3 circuit is shown in the case where every resistor in the circuit is type-N junction. The values of $E_c$ are randomly chosen. A crossover from metallic to insulating behavior is seen as $E_c$ is increased. This crossover is similar to that obtained in the former study. [4]

In Fig. 4 the temperature dependence of the resistance for 3-10-3 circuit is shown in the case where a resistor is either superconducting or metallic. Almost exponential temperature dependence is seen reflecting the behavior of metallic junctions. This behavior is consistent with experiments, while it was unexplained in the former study [3] where only superconducting junctions were taken into accounts.

In experiments a S-M-I crossover [5] is observed when the film thickness is tuned. Such a crossover is also seen in Figs. 2-4. In our phenomenological analysis it is reduced to the nature of each junction and has nothing to do with a macroscopic phase transition. It should be also noted that the present metallic state is irrelevant to the recent issue [6] of the presence of a metallic state in two-dimensional disordered systems. The issue corresponds to the presence of a coherence at macroscopic scale in uniform system after averaging. On the other hand, a metallic state is possible at mesoscopic scale. Our system is a mesoscopic one in terms of junctions, while superconducting islands are macroscopic objects. The system is highly heterogeneous where the coherence is maintained within the islands and the loss of the coherence occurs only at the junctions.

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