RESILIENCE ANALYSIS FOR PROJECT SCHEDULING WITH RENEWABLE RESOURCE CONSTRAINT AND UNCERTAIN ACTIVITY DURATIONS

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Abstract. In the real world, project construction usually suffers various uncertain factors. Traditionally, uncertainty is modelled as either random variables or dynamic events. This paper addresses the case that the information about uncertainty is partially known when developing project schedules. The concept of resilience is introduced into project scheduling problems with resource constraint to measure a schedule’s ability to absorb possible perturbation. The definition of resilience is given based on project equilibriums. Since the calculation of resilience is time intensive, a new surrogate measure is proposed to indicate schedule resilience. Correlation analysis between resilience and proposed surrogate measure is carried out. The experimental results suggest that the proposed surrogate measure is more appropriate to indicate resilience than makespan or total free slack.

1. Introduction. Project scheduling problems (PSPs) have been attached much attention from both researchers and practitioners because of their practical importance. Since 1950s, network techniques, such as Project Evaluation and Review Technique (PERT) and Critical Path Method (CPM), have been widely used to manage schedules of projects. However, these techniques neglect resource restriction, which is a crucial matter in PSPs [26]. Given limited resources, the problems can be modeled as resource-constrained project scheduling problems (RCPSPs), which are one of the most well studied branches of PSPs. In RCPSPs, project managers need to decide when activities should be started with the consideration of scare resources. According to existing literature, a large number of exact and heuristic or metaheuristic approaches have been proposed to solve deterministic RCPSP. Several comprehensive surveys can be found in [29, 13, 5, 17, 16, 8]. Note that resources can be generally classified into two categories: renewable resources such as manpower, equipment, machine and non-renewable resources such as cash budget,
material. We restrict our research in renewable resources. In the remainder of this paper, the term “resource” is referred to “renewable resource”.

In deterministic RCPSPs, activity durations are assumed to be constant during the execution of a project. However, in the real world, project is usually confronted with uncertain factors, such as requirement change, bad weather condition or delay of material supply, which may cause adverse effects on the whole project schedule. In the presence of uncertainty, methodology of robust project scheduling is usually used to construct a baseline schedule with good makespan performance or stability[33, 2].

Robust scheduling approaches provide project managers an avenue to take into account uncertainty at schedule design stage. However, most of the researches assume that activity durations are followed specific distributions and associated parameters are known beforehand. The assumption usually cannot be held in real-world projects. Sometimes, project managers are aware of that project execution environment is uncertain. But they cannot precisely estimate the uncertainty level due to the lack of data and information. For instance, in a R&D project, the activity of developing a new product may be perturbed by several factors, such as technological risk and organizational maturity. Due to the fact that the activity is new rather than simply repetitive and the change of internal or external uncertain factors, the adverse effect resultant from uncertainty is not available during schedule construction. In such a case, project managers cannot access to the analysis of performance under uncertainty, e.g., expected makespan or deviation. Instead, project managers would be interested into the question that how much uncertainty the schedule can absorb while important aspects of a project, such as deadline and delay amount, can still be satisfied? The benefits of such analysis are twofold: On the one hand, project managers can be provided with a new criterion to evaluate schedules at early stage. On the other hand, once the maximum uncertainty level can be obtained for a given schedule, project managers can decide when and how to react to possible perturbations during project execution with the gradual reveal of more information and data about uncertainty. Thus, we introduce the concept of “resilience” into project scheduling problems with resource constraint and uncertain activity durations to analyze the schedule’s ability to deal with perturbations.

The remainder of this paper is organized as follows. Section 2 gives a brief review on project scheduling under uncertainty and concept of resilience. The definition and calculation of resilience for RCPSP with uncertain durations are presented in Section 3. A new surrogate measure for resilience is proposed in Section 4. Correlation analysis between resilience and the proposed surrogate measure is described in Section 5. Finally, Section 6 concludes the main contribution of this paper.

2. Literature review. In this section, we briefly review the recent relevant work in project scheduling under uncertainty(Section 2.1) and concept of resilience(Section 2.2).

2.1. Project scheduling under uncertainty. It is common that the construction or development of a project is usually affected by uncertain factors, such as perturbation on durations or uncertain resource[42]. Thus, project scheduling under uncertainty has been attached much attention by researchers and practitioners. The approaches for project scheduling under uncertainty can be mainly classified as proactive and reactive[11, 12, 33]. Proactive scheduling is aim at building robust schedules which can deal with the uncertainty as well as possible[38, 39]. The robustness can be categorized as quality and solution robustness. The former refers
to the realized project duration, while the latter is measured by the deviation between the planned and realized start times of the individual activities\[35, 36, 2\]. In proactive scheduling, activity durations under uncertainty are usually modelled as random variables following specific distributions, such as uniform distribution, beta distribution and normal distribution\[34, 3\]. Then, realized performances, e.g., expected makespan, service level\[3\] or start times deviation\[20\], are approximated by simulation methods. However, the approximation of expected performance in the presence of uncertainty is time intensive. Thus, several surrogate measures for robustness are proposed and optimized in proactive scheduling. The representative surrogate measures are slack-based functions, e.g., total free slack\[1\], weighted free slack\[6\], minimum free slack or ration of free slack/duration\[15\], weighted total slack\[10\]. When project due date is predefined, including time buffers between activities is usually used to improve schedules’ stability (solution robustness)\[34, 21\]. What should be noted is that no matter how robust a schedule is designed, it almost impossible that the schedule can absorb all perturbations. In such a case, a reactive scheduling procedure is need to deal with disturbances that cannot be absorbed by the proactive schedule. Several possible reactive procedures are defined and reviewed in \[33\]. In this research, we focus on designing resilient schedules that can absorb possible perturbations as much as possible. Thus, reactive procedures during schedule execution are out of the scope of this paper. Furthermore, we restrict our research in non-buffer approach due to the following two reasons: The first is that makespan minimization is still one of main objectives. The second is that the decision of time and amount of buffer insertion is not a trivial task and we leave this issue in the future research.

2.2. Concept of resilience. The term “resilience” is firstly introduced to measure ecological systems’ properties in the presence of disturbances\[14\]. Resilience of a system has been defined in two different ways in the ecological literature\[7\]. The first definition concentrates on the ability to return to an equilibrium following a perturbation and speed of return to the equilibrium is used to measure the property\[32\]. In the second definition, the measurement of resilience is the magnitude of disturbance that can be absorbed before the system changes its structure by changing the variables and processes that control behavior\[14\]. The first view is more traditional and provides one of the foundations for economic theory as well and may be termed engineering resilience.

In recent years, resilience has gained prominence as a topic in other fields. \[31\] analyzes resilience in the field of disaster research and measures resilience by the functionality of an system attacked by a disaster and also by the time needed to return to the performance level before disaster. \[22\] studies the resilience of ground transportation system using network analysis. In their research, resilience is interpreted as the ability of addressing the challenge of optimizing the system as a whole to deal with change, shocks, and interruptions. \[37\] studies resilience in the context that enterprise information system which is partially damaged. The authors considered the resilience as the recovery ability of the system after a partial damage. The proposed measurement of resilience in that research is mathematically formulated as the function of time within which all function are recovered. \[4\] review the concept of resilience in various fields, e.g., physical systems, ecological systems, disaster management and organization. To the best of our knowledge, there is no literature addressing resilience in project management, especially in project scheduling area.
Table 1. List of notations.

| Symbol  | Description                      |
|---------|----------------------------------|
| $N_i$   | the $i$th activity               |
| $Succ_i$| successors of activity $N_i$     |
| $Pred_i$| predecessors of activity $N_i$   |
| $d_i$   | duration of activity $N_i$       |
| $s_i$   | planned start time of activity $N_i$ |
| $s_i^*$ | actual start time of activity $N_i$ |
| $ft_i$  | planned finish time of activity $N_i$ |
| $fs_i$  | the free slack of activity $N_i$ |
| $K$     | index of resource type           |
| $R_k (k = 1, \cdots, K)$ | the availability of each resource type |
| $r_{ik}$| the required amount of resource $R_k$ by activity $N_i$ |
| $s$     | an activity schedule             |
| $M(s)$  | the makespan of schedule $s$     |
| $D$     | the due date of the project      |

3. Resilience in RCPSP. For the convenience of understanding, we first list the notation used in this section, shown as in Table 1.

3.1. RCPSP with uncertain durations. We consider RCPSP in single project context and describe the problem as follows. The projects consist of a set of activities $N = \{N_0, N_1, \cdots, N_n, N_{n+1}\}$ that need to be executed without interruption. $N_0$ and $N_{n+1}$ are dummy activities which are used to, respectively, represent the start and the end states of the project. There are precedence relationships among activities. In other words, some activities can only be stated until other activities are finished. We represent these precedence relationships as an activity-on-node direct acyclic graph, denoted as $G(N, A)$, where $N$ is the set of nodes (activities) and $A$ is the set of edges. The direct successors and predecessors of an activity $N_i (i = 0, 1, \cdots, n, n+1)$ are denoted as $Succ_i$ and $Pred_i$, respectively. There are $K$ types of renewable resources with availabilities $R = \{R_1, \cdots, R_k\}$ during project execution. Each activity need a specific amount of each type of resources to be performed, denoted as $r_{ik} (i = 0, 1, \cdots, n, n+1, k = 1, \cdots, K)$. Note that $R_k$ and $r_{ik}$ are both non-negative integers. For dummy activities $N_0$ and $N_{n+1}$, the required resources $r_{ik} (i = 0, n + 1)$ are zero.

Each activity is associated with a duration, $d_i$, which is a non-negative real. Different from deterministic RCPSP, $d_i$ is uncertain and cannot be estimated precisely in project planning phase. The uncertainty of activity durations can be explained as the result of perturbation on activities. In the presence of perturbation, activity durations fluctuate around the estimated values. Thus, $d_i$ is modeled as follows:

$$d_i = d_i^0 (1 + \varepsilon_i)$$  \hspace{1cm} (1)

where $d_i^0$ is the most possible duration estimated at planning phase, $\varepsilon_i$ represents the uncertain level for each activity. Note that $\varepsilon_i$ is a stochastic parameter whose mean is 0 and variance is unknown. The addressed RCPSP with uncertain durations is denoted as RCPSP-UD.

We consider the simplified situation that there is only single perturbation on activities and all activities have the same sensitivity to the perturbation. Then, we
have:

\[ \varepsilon_1 = \varepsilon_2 = \cdots = \varepsilon_n = \varepsilon \]  

(2)

3.2. Definition of resilience. As aforementioned, resilience can be defined in two different ways. By the first way, resilience is usually quantified in terms of time which is needed to return to an equilibrium. In the second definition, resilience is measured by how much perturbation a system can absorb before the change of its structure or functions. The second views of resilience is usually employed in the case that multiple stable equilibriums are assumed to be existed\cite{7}.

Before we define the resilience in project scheduling problems, an issue needed to be addressed is that whether it exists more than one stable equilibrium for a project or not. During planning phase, project managers or decision makers need to take into account different factors, such as finish time(or time-to-market), impacts on ecology and environment, social responsibility, economy benefit or risk. Each aspect may represent views of different stakeholder. A final schedule reflects negotiation and compromise among different stakeholders to certain degree. In this sense, a baseline project schedule embodying trade-off among various aspects can be considered as an equilibrium or stable state, which is characterized by makespan, cost, risk, net present value, etc. In the presence of perturbations, such an equilibrium may be broken, resulting in cost overrun, makespan tardiness, performance degradation, or even project failure. For most projects, especially large scale projects, the construction is unrepeatable. Once a project is disturbed and departs from the equilibrium, it cannot return to the same stable state. Instead, constraints, variables and structure may change. Also, decision makers need to adapt their preference or expectation on objectives to arrive at another stable state, which can be accepted by different stakeholders. Such a process is described in Figure 1.

From the above discussion, we state that their is no single equilibrium for project scheduling problems. By changing the structure of a project schedule, the stable state moves from one to another when facing uncertainties and disturbances. Thus, this research adopts the second view of resilience for project scheduling problems. The resilience is considered as the magnitude of perturbations that can be absorbed.

\[ \text{Figure 1. Conceptual example of equilibriums in project scheduling problems.} \]
before a schedule departs from an equilibrium. Figure 2 shows the process of performance degradation of systems resulted from the increase of perturbation magnitude. Different from the definition in [31], the “resilience triangle” in the figure stands for the perturbation magnitude that can be absorbed until the system performance degrade to the threshold, which is the minimum level of the system performance stakeholders can accept. In project scheduling problems, the “system” can be considered as a schedule of activities. Project planners are aiming at maximizing the resilience triangle size by searching for optimal sequence of activities.

One can notice that the measurement of resilience in project scheduling problems is associated with two elements: equilibrium definition and perturbation magnitude. In real-world project management problems, the identification of a stable state may involve numerous factors. Since this research focuses on activity duration perturbations, for the sake of simplicity, we investigate an equilibrium of project schedule from two duration-related elements: makespan measurement and stable performance. When a project is to be executed in the presence of perturbation, especially on activity durations, the expected makespan is a logical and appropriate objective for project managers[3]. Apart from makespan minimization, project managers may be interested in stable schedules, especially in the case that activities are subcontracted and changes of activity start times or movement of resources may result in highly additional cost.

We first describe the definition of equilibrium associated with makespan measurement. For a project, the due date \( D \) is imposed to avoid infinite increase of makespan. For given probability \( \text{prob} \), possible risk averseness can be presented as follows[3]:

\[
Pr\{M(s) > D\} \leq \text{prob}
\]

where \( M(s) \) is the makespan of a schedule \( s \). We refer to the probability \( \text{prob} \) as tardiness risk level, which stands for the maximum allowed likelihood of makespan delay. Another issue needed to be addressed is tardiness amount. We represent the
magnitude of tardiness by relative delay amount (RDA), denoted as follows:

\[ RDA = \frac{\sum_{i=1}^{P} \max\{0, M_i(s) - D\}}{P^* D} \]  

where \( P \) is the sample space size, \( P^* \) is the number of tardiness occurrence, \( M_i(s) \) is the makespan of the \( i \)th sample. The maximum allowed value of \( RDA \) is referred to as tardiness risk degree, denoted as \( \delta \).

Then, we describe an equilibrium of a project by two parameters: tardiness risk level \( \text{prob} \) and tardiness risk degree \( \delta \). The equilibrium is denoted as \( E_1 \) and can be formally given as follows:

\[ E_1 = \Pr\{M(s) > D\} \leq \text{prob} \land RDA \leq \delta \]  

Equilibrium \( E_1 \) is described from the view that makespan is expected to meet specific level imposed by project managers. To address the stability of a schedule under duration perturbation, we investigate the expected deviation of start times, denoted as \( EDS \), which is calculated as follows.

\[ EDS = \frac{\sum_{p=1}^{P} \sum_{i=1}^{n} |st_i - st_i^*(p)|}{n} \]  

where \( P \) is sample size and \( n \) is the number of activities, \( st_i \) is the planned start time of activity \( N_i \) and \( st_i^*(p) \) is the actual start time in each simulation sample \( p \).

The magnitude of \( EDS \) may be different from various individual projects. Thus, we use the relative value of \( EDS \), represented as \( EDS \), to indicate the stable performance of a schedule.

\[ \overline{EDS} = \frac{EDS}{D} \]  

\( D \) is the due date of the project. Similarly, there is a maximum allowed value of \( \overline{EDS} \), referred to as variation risk degree \( \theta \). Then, the equilibrium associated with both makespan and stability is denoted as \( E_2 \) and formally given as follows.

\[ E_2 = \Pr\{M(s) > D\} \leq \text{prob} \land RDA \leq \delta \land EDS \leq \theta \]  

For a project schedule, the resilience \( R \), is defined as the maximum uncertainty level which can be absorbed by the schedule before it departs from the equilibrium \( E \). \( R \) can be represented as follows:

\[ R = \max\{\varepsilon | \text{The equilibrium } E(E_1 \text{ or } E_2) \text{ holds} \} \]  

3.3. **Resilience calculation.** The calculation of resilience is a process of finding the maximum uncertainty level which causes a schedule departs from an equilibrium state. Note that the equilibrium is a predefined state given by stakeholders and captured by \( \text{prob} \), \( \delta \) and \( \theta \). Since there is no enough information about uncertainty level, we have to trial to toward the maximum level. To avoid infinite computation, the resilience is accurate to 3 decimal places. Figure 3 shows the procedure of resilience calculation. In the procedure, \( \text{WhetherDepartFromEquilibrium()} \) is the function to indicate whether the schedule departs from the equilibrium under specific uncertainty level. Monte Carlo approximation with sample size \( P \) is used to \( \text{WhetherDepartFromEquilibrium()} \) to calculate the probability of \( \Pr\{M(s) > D\} \) as well as the values of \( RDA \) and \( EDS \). \( t \) is the parameter of calculation iteration times. In this paper, \( t \) is set as 6. Then, the length of interval where resilience may exist is tightened, with maximum value of approximate 0.0078(1/27). Since the accuracy of resilience is to thousandth, the maximum calculation times is 15, until the nearest resilience approximation is obtained.
\[ R_{\text{low}} = 0, \quad R_{\text{up}} = 1, \quad R = 0.5; \]
\[
\text{bool} \ \text{BetweenInterval} = \text{false};
\]
\[
\text{while} \ \text{BetweenInterval} = \text{false} \ \text{do}
\]
\[
\text{bool} \ \text{Violate};
\]
\[
\text{for} \ i = 0 \ \text{to} \ t \ \text{do}
\]
\[
\text{Violate} = \text{WhetherDepartFromEquilibrium}();
\]
\[
\text{if} \ \text{Violate} = \text{false} \ \text{then}
\]
\[
R_{\text{low}} = R;
\]
\[
R = (R + R_{\text{up}})/2;
\]
\[
\text{else}
\]
\[
R_{\text{up}} = R;
\]
\[
R = (R + R_{\text{low}})/2;
\]
\[
\text{BetweenInterval} = \text{true};
\]
\[
\text{end if}
\]
\[
\text{end for}
\]
\[
\text{if} \ \text{BetweenInterval} = \text{false} \ \text{then}
\]
\[
R_{\text{low}} = R;
\]
\[
R_{\text{up}} = R + 1;
\]
\[
\text{end if}
\]
\[
\text{end while}
\]
\[
\text{Violate} = \text{WhetherDepartFromEquilibrium}();
\]
\[
\text{if} \ \text{Violate} = \text{true} \ \text{then}
\]
\[
\text{while} \ \text{Violate} = \text{true} \ \text{and} \ R \geq 0 \ \text{do}
\]
\[
R = R - 0.001;
\]
\[
\text{Violate} = \text{WhetherDepartFromEquilibrium}();
\]
\[
\text{end while}
\]
\[
\text{else}
\]
\[
\text{while} \ \text{Violate} = \text{false} \ \text{and} \ R \geq 0 \ \text{do}
\]
\[
R = R + 0.001;
\]
\[
\text{Violate} = \text{WhetherDepartFromEquilibrium}();
\]
\[
\text{if} \ \text{Violate} = \text{true} \ \text{then}
\]
\[
R = R - 0.001;
\]
\[
\text{end if}
\]
\[
\text{end while}
\]
\[
\text{end if}
\]

**Figure 3.** Pseudocode of resilience calculation.

4. **Surrogate measure for resilience in RCPSP-UD.** In the presence of uncertainty, simulation-based approximation of expected objective has a better precision on the performance analysis\cite{40}. However, simulation is usually time intensive. In the calculation of resilience, the iterative process augments the computational requirement. Thus, instead of calculating expected values, surrogate measures are often employed to indicate a schedule’s various abilities in uncertain environment. Usually, slack-based surrogate models are used to measure a schedule’s robustness.
or stability. In this part, we propose a modified surrogate measure to indicate a schedule’s resilience.

4.1. Example illustrating the deficiency of some existing slack-based surrogate measures. Let us consider a project consists of 8 non-dummy activities with durations and required resources presented in Figure 4. It is assumed that only one resource type is used, of which 7 units are available during the whole duration of the project.

Figure 5 to 7 show three different feasible schedules, which are based on the same activity priority list (1,3,2,5,8,4,6,7). The only difference among these schedules is the start time of activity $N_4$. In schedule $-I$, activity $N_4$ is started at its earliest start time, while in schedule $-II$, it is started at its latest start time. In schedule $-III$, activity $N_4$ is started after 2 time units when $N_2$ is finished.
The free slacks of each activity (if exit) are represented by gray blocks. It is clear that these three schedules have the same values on makespan(22) and total free slack(18). According to the robustness measure proposed in [1], which is calculated as total free slack, three schedules would have the same ability to deal with duration perturbation. Now we suppose activities $N_2$ and $N_4$ are perturbed and each activity needs 1 unit of additional duration. We can find the three schedules have different performances in the presence of perturbation. In schedule - I the start time of $N_4$ will be postponed due to the delay of completion of $N_2$. The situation is worse in schedule - II, both start time of $N_7$ and project makespan will be delayed. While in schedule - III, perturbations can be absorbed without any delay on start time.
or makespan. This can be contributed to the appropriate spread of free slacks in the schedule.

From the above analysis, we may state that total free slack is not appropriate for indicating resilience. Similar comments were given by [15], where two measures were defined to calculate a schedule’s robustness. The one is the minimum of all free slacks, the another is the minimum of ratios free slack/duration. However, for schedules generated by non-buffering approaches, their minimum of free slacks or free slack/duration ratios are possibly equal to 0, despite the fact that these schedules may perform differently in the presence of perturbation. Furthermore, the minimum of both free slacks and free slack/duration ratios are problematic for resilience measuring. Figure 8 shows another schedule, denoted as schedule – IV, which is different from schedule – III at the start times of $N_2$ and $N_5$. In the figure, the dashed block following $N_1$ is not a free slack because any delay of $N_1$ will result in the postpone of its successor $N_5$. One can see from the figure that the schedule’s total free slack is decreased. Regarding the minimum of all free slacks or free slack/duration ratios, both of them remain as 0. If we only consider the free slacks which are greater than 0, we can see that the free slacks of $N_4$ and $N_8$ decrease to 1 and 6 units, respectively. According to the measures defined in [15], schedule – IV is non-robust compared with schedule – III. However, schedule – IV still can absorb the perturbations on $N_2$ and $N_4$. Now, we suppose that activity $N_1$ is also perturbed and 1 units of additional duration are required. Then, the performance of schedule – III and schedule – IV with the perturbations on $N_1$, $N_2$ and $N_4$ are investigated, shown as in Figure 9 and 10, respectively. Additional duration of perturbed activities are represented by black blocks. We can see from the figures that the makespans of both schedule – III and schedule – IV are delayed to 23. However, in schedule – III, the start times of $N_5$, $N_6$, $N_7$ and $N_8$ are delayed, while only $N_5$, $N_6$ and $N_7$ are delayed in schedule – IV. In other words, schedule – III is more non-robust than schedule – IV, although the former schedule has a better slack-based robustness measure.

**Figure 8.** The Gantt chart of schedule – IV.
4.2. Proposed surrogate measure for resilience. Since the definition of resilience indicate the performance of both solution and quality of a schedule in the presence of perturbation, existing surrogate measures such as total free slack or minimum slack cannot reflect a schedule’s such ability very well. We propose to modify the surrogate measure to indicate a schedule’s resilience. In the presence of uncertainty, perturbed durations are modelled to be proportional to deterministic durations. Thus, we focus on ratios to activity durations. By comparing schedule – III and schedule – IV, one may find that the main difference is the intervals between activity $N_1$ and its successors. In schedule – III, $N_2$ and $N_8$
are very close to their predecessor $N_1$, while in schedule $- IV$, their are larger lags between $N_1$ to its successors. Intuitively, a larger interval between two activities with precedence relation gives a higher opportunity to absorb delay on upstream activities. Then, a resilient schedule is defined as follows.

**Definition 4.1.** A resilient schedule is a schedule which has following characteristics:
1) have enough free slacks, 2) free slacks are distributed evenly in the schedule, 3) have enough interval between the finish time of an activity and the start time(s) of its successor(s), 4) intervals are distributed evenly in the schedule.

For each activity $N_i$, the ratio free slack/duration is denoted as $r_i^1$, which is calculated as follows:

$$r_i^1 = \frac{fs_i}{d_i}$$

where $fs_i$ is the free slack of $N_i$.

For each activity $N_i$ whose successor is not dummy activity, the interval between $N_i$ and $N_j \in Succ_i$ is denoted as $\alpha_{ji}^i$, which is calculated as follows:

$$\alpha_{ji}^i = st_j - ft_i$$

where $ft_i$ is the finish time of the activity $N_i$.

The sum of intervals can be obtained as follows:

$$\alpha_i = \sum_{j=1, N_j \in Succ_i}^{n} \alpha_{ji}^i$$

(12)

The ratio average interval/duration is denoted as $r_i^2$ and is calculated as follows:

$$r_i^2 = \frac{\alpha_i}{|succ_i|d_i^0}$$

where $|succ_i|$ is the number of the successors of $N_i$.

Then, we have two set of ratios, $r_1 = (r_1^1, \cdots, r_n^1)$ and $r_2 = (r_1^2, \cdots, r_n^2)$, where $n_1$ is the number of activities whose successor is not dummy activity. To address two issues in the definition of resilient schedule, i.e., enough amount and even distribution, mean-variance model[24, 25, 41] is employed in the calculation. We use $f_1$ and $f_2$ to represent mean-variance models of $r_1$ and $r_2$, respectively.

$$f_1 = mean(r_1) - var(r_1)$$

(14)

$$f_2 = mean(r_2) - var(r_2)$$

(15)

where $mean(\cdot)$ and $var(\cdot)$ respectively indicate the values of mean and variance.

The proposed surrogate measure, denoted as $SM$, is the linear combination of the two mean-variance models.

$$SM = \gamma f_1 + (1 - \gamma) f_2$$

(16)

where $\gamma$ is the weight factor. For the sake of calculation, $\gamma$ is set as 0.5 in our research.

According to the proposed surrogate measure, the calculation of $SM$ for schedule $- III$ and schedule $- IV$ is shown in Table 2. A larger value on $SM$ indicates the schedule is more resilient.
Table 2. Calculation of resilience surrogate measure for schedule – III and schedule – IV.

| Schedule    | Mean($r_1$) | Var($r_1$) | $f_1$ | Mean($r_2$) | Var($r_2$) | $f_2$ | SM  |
|-------------|-------------|------------|-------|-------------|------------|-------|-----|
| Schedule – III | 1.0836      | 5.0790     | -3.9954 | 0.5646      | 0.3592     | 0.2054 | -1.8950 |
| Schedule – IV  | 0.6925      | 1.1467     | -0.4542 | 0.5800      | 0.3130     | 0.2670 | -0.0936 |

Generate a schedule by SSGS

\[ i = n \]

\[ \text{while } i \geq 1 \text{ do} \]

\[ \text{if } f s_i > 1 \text{ unit then} \]

\[ s t_i = s t_i + \text{Round}(f s_i/2) \]

\[ f t_i = f t_i + \text{Round}(f s_i/2) \]

Update $f s_i$

Update $f s_j (j = 1, \cdots, i - 1)$

\[ \text{end if} \]

\[ i = i - 1 \]

\[ \text{end while} \]

Figure 11. Pseudocode of the heuristic procedure to improve resilience surrogate measure.

4.3. Improving surrogate measure by heuristic. In our research, shortening makespan is one of the primary objectives of projects and non-buffering approach is employed for schedule construction. Then, we first use Serial Sequence Generation Scheme (SSGS) [19, 9] procedure to transform an activity priority list into a feasible schedule. In the precedence, resource feasible activities are started at their earliest possible time. After a schedule is constructed, a heuristic is used to improve the value of surrogate measure. The procedure of heuristic is shown in Figure 11, where Round is the function to obtain the integer part of a number. The heuristic is simple and works as follows: If an activity has more than one unit of free slack, then the free slack is separated as two parts and the first part is moved before the activity.

5. Correlations analysis. Since uncertainty parameters are unknown in RCPSP-UD, statistic performances (e.g., expected makespan or expected tardiness) are unavailable for predesigned schedules. However, for each schedule, deterministic performance or measure can still be obtained. Among others, makespan and total free slack are widely considered as objectives to be optimized. In this section, we investigate how resilience is related to makespan and total free slack, as well as proposed surrogate measure. Then, the necessity of studying schedules’ resilience, besides makespan and total free slack, can be revealed to certain extent.

A sample of solutions need to be generated to approximate the correlations. Minimization of makespan is usually considered as an objective for scheduling problems [23]. Similar with [3], to avoid large makespan, solutions were generated by a genetic algorithm (GA), which is usually employed as an optimizer for project
Table 3. Parameter settings of 10 different equilibriums.

| equilibriums | prob | δ   | θ  |
|--------------|------|-----|----|
| \(E_1^{-1}\) | 0.4  | 0.25| NA |
| \(E_1^{-2}\) | 0.55 | 0.35| NA |
| \(E_2^{-1}\) | 0.4  | 0.25| 0.06|
| \(E_2^{-2}\) | 0.55 | 0.35| 0.18|

Table 4. Coefficients for the relationship between resilience and makespan (M), total free slack (TSF) and proposed surrogate measure (SM) for 4 equilibriums, with due date coefficient 1.1 and different sample sizes (average ± variance).

| P  | \(E_1^{-1}\)     | \(E_1^{-2}\)     | \(E_2^{-1}\)     | \(E_2^{-2}\)     |
|----|------------------|------------------|------------------|------------------|
| 30 | \(M\)            | -0.5843±0.0874   | -0.3108±0.0763   | -0.1088±0.0321   | -0.1764±0.0132   |
|    | \(TSF\)          | -0.1158±0.1002   | -0.1342±0.0982   | 0.0976±0.0809    | 0.1243±0.1100    |
|    | \(SM\)           | 0.2401±0.0204    | 0.2104±0.1012    | 0.4321±0.1308    | 0.5304±0.1200    |
| 100| \(M\)            | -0.3981±0.1324   | -0.4774±0.1003   | -0.1124±0.0843   | -0.2034±0.1022   |
|    | \(TSF\)          | 0.1079±0.1011    | -0.0876±0.1350   | 0.2214±0.0886    | 0.2011±0.1224    |
|    | \(SM\)           | 0.1208±0.1031    | 0.2308±0.0971    | 0.6534±0.1031    | 0.7013±0.1099    |
| 200| \(M\)            | -0.5164±0.0453   | -0.4789±0.0873   | -0.1254±0.0349   | 0.0201±0.0721    |
|    | \(TSF\)          | 0.0682±0.0335    | -0.1031±0.0459   | -0.0988±0.0498   | 0.0246±0.0578    |
|    | \(SM\)           | -0.1345±0.0984   | 0.3105±0.1542    | 0.7390±0.1104    | 0.6804±0.1300    |

scheduling problems[43]. Standard GA was employed, in which two-point position-based crossover[27, 28] was used and mutation operation was taken from [30]. The sample size was set as 500. The first 500 solutions generated after crossover and mutation were selected. Note that for schedules whose makespan are greater than due date, the related resilience are 0. Thus, these solutions were excluded in the selection.

The correlation analysis was based on 48 instances randomly taken from the benchmark problem set j30 generated with PROGEN[18]. Since the calculation of resilience is dependent with the definition of equilibrium state, to avoid loss of generalization, we investigated the correlations with different equilibriums. Table 3 shows the parameter settings of 4 different equilibriums. The experimental results with different due date coefficients are reported in Table 4 to 6. For each experiment, three different sample sizes (30, 100 and 200) were used for resilience calculation, shown in the first column of the tables. The coefficients for the relationship between resilience and proposed surrogate measure (SM) were compared with makespan (M) and total free slack (TSF), respectively.

The coefficients were calculated by Excel and the values were range from -1 to 1. A positive value of coefficient indicates the two sets of data have positive relation. Similarly, a negative value indicates negative relation. The absolute value of coefficient reflects the degree of relation between two sets of data. A higher absolute value represent stronger relation and vice versa.

By looking at the relation between resilience and makespan, we can find that the coefficients are negative for equilibrium \(E_1^{-1}\) and \(E_1^{-2}\). For instance, in Table 4, the average coefficient between resilience and makespan with sample size 30 and
Table 5. Coefficients for the relationship between resilience and makespan (M), total free slack (TFS) and proposed surrogate measure (SM) for 4 equilibriums, with duedate coefficient 1.2 and different sample sizes (average ± variance).

| P    | $E_{1-1}$         | $E_{1-2}$         | $E_{2-1}$         | $E_{2-2}$         |
|------|-------------------|-------------------|-------------------|-------------------|
| 30   | $M$ -0.5343±0.0544 | -0.4032±0.0643    | -0.1743±0.0542    | -0.1437±0.0711    |
|      | $TFS$ -0.0931±0.0782 | -0.1452±0.0883    | 0.1022±0.0433     | 0.0945±0.0788     |
|      | $SM$ 0.1502±0.0812 | 0.1011±0.1004     | 0.5120±0.0430     | 0.6021±0.0866     |
| 100  | $M$ -0.3038±0.0477 | -0.3404±0.0633    | 0.0904±0.0492     | -0.1287±0.0441    |
|      | $TFS$ -0.1581±0.0632 | -0.1105±0.0387    | 0.1443±0.0733     | 0.1104±0.0885     |
|      | $SM$ 0.2401±0.0677 | 0.1942±0.0553     | 0.7801±0.0793     | 0.5099±0.0773     |
| 200  | $M$ -0.4322±0.0846 | -0.3932±0.0447    | -0.0967±0.0498    | -0.1204±0.0844    |
|      | $TFS$ -0.1045±0.0711 | -0.0699±0.0712    | 0.2301±0.0901     | 0.0992±0.0673     |
|      | $SM$ 0.1409±0.0432 | 0.2508±0.1022     | 0.7099±0.1344     | 0.6604±0.0779     |

Table 6. Coefficients for the relationship between resilience and makespan (M), total free slack (TFS) and proposed surrogate measure (SM) for 4 equilibriums, with duedate coefficient 1.3 and different sample sizes (average ± variance).

| P    | $E_{1-1}$         | $E_{1-2}$         | $E_{2-1}$         | $E_{2-2}$         |
|------|-------------------|-------------------|-------------------|-------------------|
| 30   | $M$ -0.3342±0.0678 | -0.4504±0.0801    | -0.1138±0.0701    | -0.1002±0.0442    |
|      | $TFS$ -0.1033±0.0721 | 0.0804±0.0558     | 0.1908±0.0883     | 0.1329±0.0413     |
|      | $SM$ 0.1702±0.0904 | 0.2011±0.0732     | 0.701±0.0453      | 0.6608±0.0732     |
| 100  | $M$ -0.5009±0.0881 | -0.4894±0.0720    | -0.1033±0.0883    | -0.1104±0.0447    |
|      | $TFS$ -0.2035±0.0954 | -0.1174±0.0745    | 0.1903±0.0779     | 0.1045±0.0674     |
|      | $SM$ 0.2432±0.0904 | 0.2021±0.0883     | 0.701±0.0476      | 0.5048±0.0794     |
| 200  | $M$ -0.3776±0.0458 | -0.4221±0.0709    | -0.0994±0.0582    | -0.1120±0.0831    |
|      | $TFS$ -0.1021±0.0682 | -0.1104±0.0834    | 0.2205±0.0774     | 0.2301±0.0874     |
|      | $SM$ 0.2012±0.0432 | 0.1334±0.0883     | 0.7204±0.0840     | 0.6988±0.0443     |

equilibrium $E_{1-1}$ is -0.5843. It indicates that makespan is negatively related to resilience for equilibrium $E_1$. Compared with the first two equilibriums ($E_{1-1}$ and $E_{1-2}$), for the equilibriums $E_{2-1}$ and $E_{2-2}$, the absolute values of most coefficients between resilience and makespan are much lower. Some values are even positive, such as the coefficient between resilience and makespan with sample size 200 and equilibrium $E_{2-2}$ is 0.0201. This is because in the definition of $E_2$, shown in Equation 8, the deviation of start times is considered. Thus, the activities of schedules with small makespan are prone to deviate from the planned start times.

In robust project scheduling, minimization of makespan and maximization of total free slack are in conflict with each to certain degree [1]. This conflict is still can be found according to the figures reported in the tables. One can see that for equilibriums $E_{1-1}$ and $E_{1-2}$, most average values of coefficients between resilience and total free slack are negative. It indicates that for most cases, schedules with large total free slack are non-resilient compared with schedules with small total free slack. However, the absolute value are much lower than coefficients between resilience and makespan, which represents a weak relation between resilience and total free slack.
with equilibriums $E_{1-1}$ and $E_{1-2}$. When deviation of start times were taken into account in the definition of equilibrium, situation was slight different. For equilibriums $E_{2-1}$ and $E_{2-2}$, total free slack is positively related to resilience for most instances, although the relation was not very strong. This shows that total free slack has contribution to improve schedules’ stability. Then, schedules’ resilience is improved to certain extent.

The coefficients between resilience and proposed surrogate measure (SM) with equilibriums $E_{1-1}$ and $E_{1-2}$ suggests that resilience is positively related to SM with weak relation. This may be counterintuitive and can partially demonstrate our previous statement that schedules with evenly distributed free slack and well designed structure have better ability to absorb perturbations. However, the average values of coefficients between resilience and SM is lower than coefficients between resilience and makespan. Thus, we may state that for equilibriums only considering makespan performance, minimization of makespan has stronger relation with resilience, compared with total free slack and SM. But we should be aware of that such a relation is not so tight that we can neglect the resilience of schedules. For equilibriums $E_{2-1}$ and $E_{2-2}$, the coefficients between resilience and SM are positive and the values are much higher. This represents that when both makespan performance and stability are considered in equilibriums, the proposed surrogate measure is the best one to indicate resilience, among the three measures.

6. Conclusion. In the real world, project construction is usually in the presence of uncertain environment, which requires project schedules have the ability to deal with or absorb possible perturbations. Traditionally, uncertainty is categorized as “known” and “unknown”, and is modelled in different ways, such as random variables and dynamic events. This paper addresses the uncertainty which is partially “known”. In concrete, uncertainty is modelled as a random variable whose distribution parameters are unknown. In such uncertain environment, the expectation performance of a schedule is unavailable. Thus, this paper studies the magnitude of uncertainty which can be absorbed by a schedule given specific performance levels. The main contribution of this paper are twofold:

(1) Introducing the concept of resilience to project scheduling and giving a definition and calculation of resilience for RCPSP with uncertain durations.

(2) Proposing a new surrogate measure for resilience. In robust scheduling, surrogate measures are usually used to indicate a schedule’s robustness. Similarly, the calculation of resilience is time consuming. Thus, this paper proposes a mean-variance model based surrogate measure to indicate a schedule’s resilience.

This paper studies uncertain project scheduling from a new aspect. Schedules’ resilience is highly related to the definition of project equilibriums, which could vary according to different problem domains. Thus, appropriate modification may be necessary when the approach is applied in specific projects. Moreover, surrogate measure for uncertain project scheduling is still an open issue. More efficient surrogate measures for resilience are worthwhile to be addressed in future research.

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