The embedded discrete fracture model for two-phase flow involving the capillary pressure discontinuities in low-permeability reservoir

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Abstract. In low permeability reservoirs with conductive fractures, there is large capillary pressure contrast between the matrix and the fracture. It may cause very sharp changes in physical quantities near the matrix-fracture interface, which makes it difficult to calculate the matrix-fracture exchange flux accurately in the traditional embedded fracture model (EDFM). The strong capillary pressure contrast between the matrix and the fracture results in three flow patterns near the matrix-fracture interface and creates difficulty in calculation of the matrix-fracture exchange flux. In this article, a modified embedded discrete fracture model is proposed, where the analytical solutions for the three different patterns are employed to construct the numerical scheme for calculating the matrix-fracture exchange flux. The numerical tests show that the calculated results of the proposed model are in agreement with the reference model in which both the fracture cells and also the matrix cells near the fracture are subdivided sufficiently.

1. Introduction

Hydraulic fractures are manufactured flow paths by which hydrocarbons are efficiently extracted from low-permeability rocks. These fractures can accelerate oil or gas flow into production wells and thus improve the oil recovery efficiency. To accurately predict the fluid flow behaviors in reservoirs with high conductive fractures, it is necessary to develop an efficient numerical method, especially for the calculation of the exchange flow between the matrix and fracture.

In order to increase the efficiency of mesh generation, Lee et al. proposed the embedded fracture model (EDFM), where regular structural grids are inherited to divide the matrix and therefore unstructured meshing along fractures is avoided [1-3]. However, the original EDFM is unable to separately capture the fluxes on the two sides of the fracture [4]. So there are some obvious errors for the saturation calculation due to the discontinuity of phase saturation at the two sides of the fracture embedded in the matrix grid [5]. In low-permeability reservoirs, the strong capillary pressure contrast between the matrix and the fracture results in complex flow conditions at the interface and creates
difficulty in calculation of the matrix-fracture exchange flux. According to the research results of Wang et al. [6], there are three flow patterns for the two-phase flow across the matrix-fracture interface.

In this article, considering the three flow patterns across the matrix-fracture interface introduced above, a modified EDFM is proposed. This article is organized as follows. The mathematical model is described in section 2. A modified EDFM is proposed in section 3. In section 4, numerical tests are performed to illustrate the accuracy and efficiency of the proposed EDFM. Conclusions are drawn in Section 5.

2. Mathematical models
In this article, we consider incompressible two-phase water-oil flow in the 2D system. The flow in fracture is treated as one dimensional. The mass conservation equation in the matrix is written as:

\[
\phi \frac{\partial S_\alpha}{\partial t} + \nabla \cdot \mathbf{V}_\alpha \left[ \alpha \right] = -q^mf_\alpha \delta(r-r')
\]  
(1)

The mass conservation equation in the fracture is written as:

\[
\phi \frac{\partial S_\alpha}{\partial t} + \frac{\partial \mathbf{V}_\alpha}{\partial x} \left[ \alpha \right] = q^mf_\alpha
\]  
(2)

In these equations, superscripts m and f denote the physical quantities in matrix and fracture respectively. Subscript \( \alpha = w \) or \( \alpha = o \) denotes liquid or oil phase, respectively. \( \phi \) is the porosity. \( S_\alpha \) is the \( \alpha \) phase saturation. \( q^mf_\alpha \) represents the volume exchange flow rate of \( \alpha \) phase between matrix and the fracture. \( \mathbf{V}_\alpha \) is the seepage velocity for \( \alpha \) phase.

3. A modified embedded discrete fracture model
In the proposed EDFM, the original matrix control volume covering the fracture will be divided into two separated sub-ones at the two sides of fracture, whose discrete physical quantities will be defined for each sub-grid respectively. For example, the matrix control volume \( \text{im} \) in the Figure 1 will be divided into two matrix control volumes \( \text{im}_1 \) and \( \text{im}_2 \) (shown in Figure 1).

Figure 1. Schematic diagram of the refinement of the matrix grid where the fracture is embedded in

Another main characteristic of the proposed EDFM is that 1D analytical solution for two-phase steady flow is employed to estimate the exchange rate between the matrix and fracture. According to Wang et al.’s research results [6], there are three exchange flow patterns. When the pressure difference between the matrix and the fracture can overcome the capillary end effect, the two-phase fluids will both flow from the matrix to the fracture and the water phase saturation at the matrix side of interface is 1.
When the matrix pressure is higher than the fracture pressure, but the pressure difference cannot overcome the capillary force effect, only the oil phase will flow from the matrix to the fracture, and the water phase exchange flux is 0. When the matrix pressure is lower than the fracture pressure, both the two-phase fluids will flow from the fracture to the matrix and the ratio of water phase to oil phase exchange flux is just the water-oil mobility ratio at the fracture side. According these three exchange flow patterns, we can calculate the exchange flow rate $Q_{ex}$. 

4. Numerical example
Annclined fracture is embedded in the square region (see Figure 2). In reference model, each fracture grid cell and the matrix grid cell adjacent to the fracture are refined into 10 sub-cells. One injection well and one production well are located at the centre of the left and right reservoir boundaries. No-flow conditions are imposed to the outer boundary.

![Figure 2](image)

**Figure 2.** The sketch map of the simulation area and the meshing in the example 2

Figure 3 shows the evolution of the average saturation with time in the fracture. The traditional DFM still seriously underestimates the water phase saturation inside the fracture, since the interface exchange flux cannot be calculated accurately under the coarse grids when the pressure and saturation vary sharply near the matrix-fracture interface. The calculated saturation distributions in the matrix are compared in Figure 4. The calculations of the proposed EDFM are consistent with the reference one, while the traditional DFM cannot provide accurate results near the fracture.

![Figure 3](image)

**Figure 3.** Comparisons of average water saturation in the fracture in example 2
5. Conclusion
To deal with the difficulty caused by the capillary pressure contrast between the matrix and the fracture, a modified EDFM is proposed in this article. The modifications are composed of two aspects, compared with the traditional one. First, the matrix control volume covering the fracture is refined into two sub-matrix control volumes according to fractures embedded in it. This refinement guarantees the calculation accuracy for the matrix-fracture exchange flow on both two sides of the fracture. The second aspect of the proposed EDFM is that the analytical solution of one-dimensional steady two-phase flow near the matrix-fracture interface is employed to calculate the phase exchange flux rate, which can have a good calculation accuracy for the matrix-fracture exchange flow even on coarse grids.

Numerical tests show that the proposed EDFM can provide rather accurate results consistent with the reference one, even on coarse grids. In contrast, the traditional DFM may underestimate matrix-fracture phase exchange flow rate, therefore resulting in significant errors.

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