Alternative ideas in cosmology*

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Received Day Month Year
Revised Day Month Year
Communicated by Managing Editor

Some remarkable examples of alternative cosmological theories are reviewed here, ranging
from a compilation of variations on the Standard Model through the more distant quasi-
steady-state cosmology, plasma cosmology, or universe models as a hypersphere, to the
most exotic cases including static models.

The present-day standard model of cosmology, \(\Lambda\)CDM, gives us a representation
of a cosmos whose dynamics is dominated by gravity (Friedmann equations derived from
general relativity) with a finite lifetime, large scale homogeneity, expansion and a hot
initial state, together with other elements necessary to avoid certain inconsistencies with
observations (inflation, non-baryonic dark matter, dark energy, etc.). There are however
some models with characteristics that are close to those of the standard model but differ-
ing in some minor aspects; we call these ‘variations on the Standard Model’. Many
of these models are indeed investigated by some mainstream cosmologists: different con-
siderations on CP violation, inflation, number of neutrino species, quark-hadron phase
transition, baryonic or non-baryonic dark-matter, dark energy, nucleosynthesis scenarios,
large-scale structure formation scenarios; or major variations like a inhomogeneous uni-
verse, Cold Big Bang, varying physical constants or gravity law, zero-active mass (also
called ‘\(R_h = ct^3\)’), Milne, and cyclical models.

At the most extreme distance from the standard model, the static models, a non-
cosmological redshift includes ‘tired-light’ hypotheses, which assume that the photon
loses energy owing to an intrinsic property or an interaction with matter or light as it travels some distance, or other non-standard ideas.

Our impression is that none of the alternative models has acquired the same level of
development as \(\Lambda\)CDM in offering explanations of available cosmological observations.
One should not, however, judge any theory in terms of the number of observations that it
can successfully explain (ad hoc in many cases) given the much lower level of development of
the alternative ones; but by the plausibility of its principles and its potential to fit

\*Some parts of this text were taken from the book *Fundamental Ideas in Cosmology. Scientific, philosophical and sociological critical perspectives*, to be published in IOP-Science by the first author.\(^\text{\footnote{2}}\)
data with future improvements of the theories. A pluralist approach to cosmology is a reasonable option when the preferred theory is still under discussion.

**Keywords**: Cosmology — Origin and formation of the Universe — Dark Matter — Dark Energy — Modified theories of gravity — Particle theory and field theory models of the early Universe — Background radiations — Origin, formation and abundances of the elements

PACS: 98.80.-k, 98.80.Bp, 95.35.+d, 95.36.+x, 04.50.Kd, 98.80.Cq, 98.70.Vc, 98.80.Ft

1. Introduction

Standard ΛCDM cosmology is well known among professional cosmologists. Other alternative cosmological models are not so well known and remain ignored for most of the professionals of physics and astronomy dedicated to study the universe as a whole. One explanation for this cognitive bias is that most cosmologists are quite sure that ΛCDM is the correct theory, and they do not need to think about possible major flaws in the basic notions of their standard theory. They do not usually work within the framework of truly alternative cosmologies with different fundamentals because they feel that these do not at present seriously compete with the standard model. These alternative models are certainly less developed because cosmologists do not work on them. It is a vicious circle.

Nonetheless, there is a large number of tensions and problems in the standard model \(^1-3\) that may lead us to consider that it is not the definitive model. Therefore, one may consider other scenarios, which might solve, at least partially, some of these caveats.

There is a rich variety of alternative ideas with intelligent proposals that merit consideration. Here we will offer a sample of these alternative models; not all of them, since their number is huge and it is impossible to mention them all in a single article. We will not give a complete list of models, but this sample is large enough to give an idea of which theoretical approaches are being discussed in cosmology from heterodox points of view. Defending any particular theory against standard cosmology or globally criticizing it is not our purpose here, although we may mention some aspects that are being debated concerning them. Here, we just review part of the literature and offer a classification of the different models.

We will mention some of the most remarkable models in the scientific literature of recent decades and also some contributions of minor impact, mostly coming from professional physicists or astronomers. There is also a vast literature produced by non-professional amateurs who try to open new routes within the golden odyssey of cosmological model creation, but very few of them will be mentioned here. They are examples of curious ideas which, although not fully developed, could be the seeds of competitive models when they are further elaborated. To a greater or lesser extent, all of these alternative models suffer from a lack of development in comparison with the standard ΛCDM, so the first thing we must take into account when reading this review is that they cannot compete with the standard model in all aspects
because many of these alternative ideas are in the hands of very few individuals—occasionally only a single individual—who lack a collaborative environment and who cannot produce hypotheses and ad hoc refinement of them to fit the ever increasing deluge of observational data at the same speed as the thousands of researchers working on the standard model. In any case, it may happen that an alternative theory might explain certain aspects of some observational data better than the $\Lambda$CDM model; moreover, even if it fails to explain other types of observations, it might be only a matter of time and ad hoc speculation of the existence of new unknown/dark elements for it to be made to fit those data too.

All the models shown in sections 2 and 3 are variations based on the Lemaître–Gamow idea of the Big Bang idea, but they differ in details concerning the latter development of the theory. But there are however other models that challenge the notions of a state of the universe with a singularity, unlimited density of matter-energy, or other important tenets of the standard model as proposed in the 1920s–1940s, as we show in the sections 4–7. In section 8, we give a summary scheme of classification of theories according to their characteristics. Certainly, the proposals with the most extreme difference with respect to the standard model have also the most serious problems to explain the cosmological observations, as those that we summarily mention in section 9 but under some refinements possibly some of these ideas might be the seed of future competitive models.

2. Minor variations with respect to the standard model

The present-day standard model of cosmology gives us a representation of a cosmos whose dynamics is dominated by gravity, modelled by Friedmann-Lemaître-Robertson-Walker (FLRW) equations derived from general relativity, with a finite lifetime, large scales homogeneity, expansion and a hot initial state, together with other elements necessary to avoid certain inconsistencies with observations (inflation, non-baryonic dark matter, dark energy, etc.). In this section and the following one, we enumerate some types of variations on this model, that is, changes with respect to the orthodox formulation that preserve some of the most important features of Big Bang idea.

By ‘minor’ variations, exposed in this section, we understand the proposals that keep the fundamentals of the standard model as stated before the 1980s and look for variations in the type of dark matter, the different equations of state of dark energy or even without dark energy, or the hundreds of variations on the type of inflation or alternative proposals, or other minor details of the theory.

2.1. Antimatter and CP violation

The topic of antimatter constitutes one of the consistency problems of the standard cosmological model. There are two possibilities and both are at odds with our present observations and experiments:
(1) If there were a symmetry of matter and antimatter at the beginning of the universe, where is the antimatter corresponding to the matter that we see now? Other galaxies cannot be made of antimatter because that would create a matter–antimatter boundary within the intergalactic medium that would create gamma rays that are not observed.

(2) If there were an asymmetry between matter and antimatter, symmetry-breaking would be needed causing other consequences to ensue that so far have not been successfully tested: the violation of charge-conjugation and parity-reversal (CP) symmetry of fundamental particles. This violation of CP symmetry causes most antimatter to annihilate with matter, but leaves much residual matter.

Early models indicated that an asymmetry between matter and antimatter would also imply a finite lifetime for the proton. Indeed, a lifetime of around $10^{30}$ years was originally predicted by Grand Unified Theory (GUT) as a necessary condition for resolving the antimatter problem. As experiments ruled out longer and longer lifetimes, these GUT were modified to predict longer lifetimes. To date, however, observations have ruled out a lifetime up to some $10^{34}$ yr (see Fig. 1). There is no experimental evidence of proton decay. The lack of such a finite lifetime...
would rule out the baryon number non-conservation needed to overcome the $10^{11}$-fold gap between Big Bang baryon density predictions and observations. So this massive contradiction of prediction and observation still exists and will continue to exist until we are able to observe a proton decay.

Experimenters are searching for other evidences of this asymmetry. CP violation has been proposed for electroweak baryogenesis from the dynamical Cabibbo-Kobayashi-Maskawa matrix (CKM matrix, quark mixing matrix), CP violation in Higgs boson interactions, and in leptons instead of quarks, which could generate the matter–antimatter disparity through a process called leptogenesis, or other mechanisms. So far, none of these ideas has received confirmation, though they are still being actively researched.

### 2.2. Inflation

Cosmic inflation posits an exponential expansion of space in the early universe. The inflationary epoch lasts from $10^{-36}$ s after the Big Bang singularity to some time between $10^{-33}$ and $10^{-32}$ s after it. The inflation paradigm explains the apparent paradox of a superluminal velocity of expansion at these short times. Following the inflationary period, the universe continued to expand but the expansion was no longer accelerating. It explains why the universe appears to be the same in all directions (isotropic), why the cosmic microwave background radiation is distributed evenly; two opposite points of the sky at the time of recombination are separated from each other by more than 70 times the distance that light could have travelled till that time. This smooth microwave background would be explained because inflation proceeds far faster than the speed of light; regions at one time in contact with each other, and thus at the same temperature, are blown farther apart from each other than the distance light could have travelled in the duration of the universe. Moreover, inflation explains why the universe is flat and the geometry nearly Euclidean ($K = 0$), and why no magnetic monopoles have been observed. The flatness problem is solved because the universe blew up to such a huge size, far bigger than the part we can observe. It also explains the origin of the large-scale structure of the cosmos: Quantum fluctuations in the microscopic inflationary region, magnified to cosmic size, become the seeds for the growth of structure in the universe. Different models were also constructed to explain the origin of the initial conditions as seeds for the large scale structure of galaxies (e.g. the later-refuted baryon isocurvature model of Ref. [12]).

Some authors have argued that the inflation necessary to explain a flat universe is highly improbable. If we adopt the idea of a multiverse in an eternal inflation, it has no predictive power because anything may happen. A theory that can predict anything is a theory that predicts nothing. Many cosmologists see inflation as disconnected from the speculative multiverse hypothesis but even so, whether with or without multiverse, there are one thousand models of inflation which may lead us to think that almost anything is possible with inflation.
Fig. 2. CMBR angular correlation function measured with Planck (dark solid curve), and associated 1σ errors (grey), compared with the prediction of the conventional inflationary ΛCDM (blue). The red curve shows the prediction of a truncated power spectrum, or a non-inflationary cosmology, with an optimized lower limit of wave number for the fluctuation power spectrum $u_{\text{min}} = 4.34$ and quadrupole amplitude $C_2 = 235.14$. (Reprinted with permission from Fig. 2 of Ref. [18]).

Ref. [16] argued that the theory is no longer scientific since it had become so all-encompassing and complicated with so many pliable parameters that it could be tweaked and adjusted to fit any kind of observation, and one model of inflation could be replaced by another as new observations appear to contradict it. Ref. [17] responded to the article by claiming that inflation is untestable because its predictions can be changed. Neither the attack nor the defence puts forward inflation as a solid theory, but merely as a speculative patch without empirical or observational support. There are even observations that disprove predictions of slow-roll inflation: the lack of large-angular scale correlations in CMBR anisotropy observations is in considerable conflict with the basic inflationary paradigm [18] (see Fig. 2). There is no global consensus among cosmologists concerning the reality of inflation, and we are far from having irrefutable proofs that keep it in line with solid evidence. This motivates multiple variations of the types of inflation, or even proposals without inflation [19][20].
2.3. Dark energy variations

As an alternative to dark energy it is suggested that an observational bias explains the appearance of dark energy. Using a much bigger database of supernovae, statistical tests show the data are quite consistent with a constant rate of expansion. Observations reveal a “bulk flow” in the local Universe which is faster and extends to much larger scales than are expected around a typical observer in the standard ΛCDM cosmology. Thus the cosmic acceleration deduced from supernovae may be an artefact of our being non-Copernican observers, rather than evidence for a dominant component of ‘dark energy’ in the Universe. It was also suggested to be due to intergalactic dust or metallicity evolution of supernovae. There is indeed a significant correlation between SNe Ia luminosity and stellar population age, which causes a serious systematic bias with look-back time able to mimic the dark energy effect (see Fig. 3). Therefore, old supernovae might be intrinsically fainter than the local ones and the cosmological constant would not be needed.

Admitting the dark energy, there are also variations with respect to the interpretation as cosmological constant Λ: Quintessence models of dark energy propose that the observed acceleration of the scale factor is caused by a dynamical field. The equation of state of this inhomogeneous component is different from baryons, neutrinos, dark matter, or radiation. Unlike the cosmological constant Λ, it evolves
dynamically and develops fluctuations, leaving a distinctive imprint on the microwave background anisotropy and mass power spectrum. Other authors play with the dark energy equation of state parameter (instead of the standard value for a cosmological constant of $w = -1$) and its evolution.

2.4. Scenarios without non-baryonic cold dark matter with standard gravity

Some dynamical problems in which dark matter has been claimed as necessary can indeed be solved without non-baryonic dark matter: galactic stability or warp creation, for instance. Velocities in galaxy pairs and satellites might also measure the mass of the intergalactic medium filling the space between the members of the pairs rather than the mass of dark haloes supposedly associated with the galaxies.

Rotation curves in spiral galaxies can be explained within standard gravity with magnetic fields, non-circular orbits in the outer disc, or types of dark matter different from non-baryonic cold dark matter proposed in the standard model.

There are also proposals stating that the dark matter necessary to solve many problems may be baryonic by simply placing baryonic dark matter in the outer disc. Other, more exotic, proposals include positively charged, baryonic particles (protons and helium nuclei) which are massive and weakly interacting, but only when moving at relativistic velocities. Simple composite systems include nucleons but are still bound together by comparable electric and magnetic forces, making up a three-body system (‘tresinos’) or four-body system (‘quatinos’), antiparticles, which have negative gravitational charge. Then, there is the proposal, among others, of a pressure-free fluid in general relativity.

Nonetheless, these proposals have their own set of problems when trying to explain large-scale structure or the Cosmic Microwave Background Radiation (CMBR).

2.5. Nucleosynthesis variations

The core of Big Bang nucleosynthesis calculations is the Boltzmann equation in an expanding universe. There are several parameters on which the equation depends: the neutron half-life, the number of neutrinos and the baryon density.

The neutron half-life is well known, so there is no discussion on it. The Standard Model of particle physics predicts the existence of three species of neutrinos, but different numbers have also been proposed to solve the inconsistencies. Curiously, a much better fit between primordial nucleosynthesis and observations is obtained when the number of neutrino species is two instead of the three predicted by the standard model for particles.

The baryon density is perhaps the parameter with larger number of disputes. In order to get the observed abundances of light elements with primordial nucleosynthesis, a baryon fraction is predicted for the ΛCDM model. However, this number does not fit the observations—at least not directly. Less than 50% of the baryons
predicted at low redshift have been found in some way in galaxies or the intergalactic medium, and the galaxies are observed to have a significantly smaller baryon fraction relative to the cosmic average.\textsuperscript{41,42} To solve it, it was proposed that in the quark–hadron transition stage in the very early universe,\textsuperscript{43} there could emerge some regions which have more protons and with more neutrons than the near-equality assumed in the thermodynamic equilibrium value of the standard model. Nonetheless, the cosmological consequences of a quark-hadron phase transition no longer hold and it is still unclear how baryons (not hadrons) could form at that cosmological transition.\textsuperscript{44}

### 2.6. Large–scale structure formation variations

Fig. 4. Images of the gas density at $z = 5.4$ obtained from hydrodynamical simulations by Ref. \textsuperscript{47}. On the left, the simulation box along the $z$-axis in a projection of comoving dimensions $(20\times20\times4)\ h^{-1}\ Mpc$ is projected. The smaller panels zoom into a region centred on the most massive halo at this redshift in a window of size $(4\times4\times2)\ h^{-1}\ Mpc$ in Cold Dark Matter (CDM; upper right) and the equivalent region in strong dark acoustic oscillations (sDAO; lower right), a type of interacting dark matter, resulting in a smoother matter distribution than the CDM volume at the same epoch, although with differences hard to discern on these scales. (Reprinted by permission of the AAS from Fig. 2 of Ref. \textsuperscript{47}).

In the cold dark matter (CDM) theory, the structure of the large-scale distribution of galaxies grows hierarchically: galaxies being formed in continuous episodes of accretion and merging, with small objects collapsing under self-gravity first and then merging in a continuous hierarchy to form larger and more massive objects. These semi-analytical ΛCDM models claim that very massive galaxies were formed
much later than small galaxies. It is the opposite of the hot dark matter (HDM) paradigm, which was more commonly used in the early 1980s, where structure does not form hierarchically (bottom-up), but forms by fragmentation (top-down), with the largest superclusters forming first in flat pancake-like sheets and subsequently fragmenting into smaller pieces that constitute the galaxies. There are also models that are a mixture of cold and hot dark matter, called warm dark matter (WDM), or the interacting dark matter models, which both result in a cut-off in the linear power spectrum, that have been competing with CDM in recent years (see, for instance, Fig. 4).

A monolithic scenario within CDM opposite to the hierarchical scenario, with galaxies all forming at once, is one of the minor variations available in the literature. Speculative solutions have also been proposed in terms of either a ‘downsizing’ scenario of galaxy formation or a mass function variation with redshift.

Indeed, current CDM models predict the existence of dark matter haloes for each galaxy whose density profile falls approximately as $r^{-2}$, although the original idea concerning hierarchical structures with CDM that gave birth to the present models was that the dark matter was distributed without internal substructure, more like a halo with galaxies than galaxies with a halo, such that, for instance, the Milky Way, Andromeda, and other galaxies of the Local Group are all embedded in the same common halo. The intergalactic matter in this case is not empty but contains the greater part of the mass of groups or clusters of galaxies.

3. Major variations with respect to the standard model

As major variations, we will refer here to the changes with respect to the standard model that keep most of the essential points but alter at least one of them: either homogeneity at large-scale, the hypothesis of high temperature at the beginning of the universe, constants of physics that are rendered into varying parameters, some FLRW solutions away from the original proposals, and cyclical universes are some of these models.

3.1. Inhomogeneous universe

In this variation, the density distribution of the universe is not homogeneous on very large scales. It may obey a fractal distribution when the mass within a sphere of radius $R$ is not proportional to $R^3$ for large enough $R$ (in the regime in which there should be homogeneity), but proportional to $R^D$ with a fractal dimension $D < 3$. There is little theoretical background to support a cosmology of these characteristics, but some observations may point in this direction.

Another idea stems from timescape cosmology in which apparent cosmic acceleration is an effect related to the calibration of clocks and rods of observers in bound systems relative to volume-average observers in an inhomogeneous geometry in ordinary general relativity. Back reactions have caused time to run more slowly or, in voids, more quickly, thus producing the illusion that supernovae are farther away
than they are and also implying that the expansion of the universe is in fact slowing down. An inhomogeneous isotropic universe described by a Lemaître–Tolman–Bondi solution of Einstein’s field equations can also provide a positive acceleration of the expansion without dark energy.

3.2. Cold Big Bang

This theory evolved from the 1960s. Rather than a very high temperature at the beginning of the universe with later progressive cooling, the universe starts with \( T = 0 \) K. Explanations are offered for the origin of the light elements in primordial and/or stellar nucleosynthesis, the cosmic microwave background radiation in terms of thermalization by intergalactic particles—a mixture of carbon/silicate dust and iron or carbon whiskers—of stellar radiation originating in Population III (further details of this idea are given in \( §4 \)), and other phenomena explained by the standard Hot Big Bang.

3.3. Variations or oscillations of physical constants

Some models propose variations or oscillations of physical constants (\( c, G, h, \alpha \), or others) with time or distance. For instance, Ref. 57 proposes variable \( G \) with distance, with which they explain the apparent variation of \( \Omega \) with the scale with no need of dark matter; or a temporal variation of \( G \) based on the Large Number Hypothesis which states that large dimensionless numbers are connected with the present epoch and vary with time. Other cosmological models propose a variable speed of light \( c \) over time or other variables or several physical constants varying at the same time. The consequences differ according to the models. They preserve the basic aspects of the standard model while keeping expansion and finite time since the beginning of the universe, but they differ with regard to a number of characteristics.

3.4. Modifications of the gravity law

These theories not only change \( G \), but also the gravity force equation. There are several tens of these alternative theories which we will not mention here. They can be found in other reviews.

The most popular alternative gravity theory is the modification of gravity law proposed in ‘Modified Newtonian Dynamics’ (MOND) which modifies the Newtonian law for accelerations lower than \( \sim a_0 \approx 1 \times 10^{-10} \text{ m/s}^2 \). This acceleration scale \( a_0 \) defines the variation with respect to Newton’s law necessary to fit the rotation curves. Its value is very similar in all galaxies and it has been interpreted as a possible sign of confirmation of MOND.

MOND was in principle a phenomenological approach. Its proponents attempted to incorporate elements that make it compatible with more general gravitation theories; for example, A QUAdratic Lagrangian theory (AQUAL) or Quasi linear...
approximation of MOND (QMOND)\cite{22} which expanded MOND to preserve the conservation of momentum, angular momentum, and energy, and follow the weak equivalence principle.

Fig. 5. The linear matter power spectrum $P(k)$ for $\Lambda$CDM and for different models of relativistic MOND\cite{77}, showing excellent fits to the Sloan Digital Sky Survey (SDSS) data release 7 (DR7) luminous red galaxies. (Reprinted with permission of American Phys. Soc. from Fig. 2 of Ref. \cite{77}).

Several relativistic versions of MOND exist. A relativistic gravitation theory of MOND was developed under the name Tensor-Vector-Scalar (TeVeS)\cite{73} which also tried to provide consistency with certain cosmological observations, including gravitational lensing. In MOND/TeVeS, there are different cosmological views. Ref. \cite{74} does not accept that a MOND-cosmology might be possible, stating that a quasi-Newtonian calculation adapted from Newtonian cosmology suggests that a MOND universe will recollapse and/or fail to satisfy the cosmological principle of a homogeneous universe. Other authors claim that a MOND-cosmology can be built\cite{75,76} that results in a uniform expansion and homogeneity on the horizon scale consistent with MOND-dominated non-uniform expansion and the development of inhomogeneities on scales out to a substantial fraction of the Hubble radius. Primordial nucleosynthesis, with its concomitant thermal and dynamical history of the universe, is identical to that of the standard cosmological model until matter dominates the energy density of the universe, a moment in which the MOND cosmology diverges from that of the standard model. The most recent version RMOND\cite{77} claims to successfully reproduce key cosmological observables such as the CMBR and the matter power spectrum of the universe (see Fig. 5).

Tentative detections of a departure from Newtonian gravity as predicted by MOND was claimed, for example, for wide binaries (binary stars with distances of
several thousands of AU), similar to the expectations of modified gravity theories\(^{78}\), although this was done by comparing only the dispersion of radial heliocentric velocities, and the gravitational effects of the surrounding areas are not clear. Moreover, the distribution of separations of wide binaries and their evolution\(^{79,80}\) should also have an effect that should be considered carefully in this test. Nevertheless, the successes of MOND and its relativistic version are mostly limited to galactic scales, the model has a missing mass problem for clusters of galaxies\(^{67}\) and the relativistic extensions need ad-hoc assumptions to describe the necessary phenomenological facts.

A different alternative gravity theory with a certain impact is a scalar-tensor-vector one known as modified gravity (MOG, Ref. 81). Another family of theories is the \(f(R)\) gravity group, which modify general relativity by defining a different function of the Ricci scalar.\(^{82}\) Other proposals include the dependence of space-time on curvature in a non-metric theory of gravity,\(^{83}\) or an interpretation of Mach’s principle in which the rotational reference frames for stars in galactic orbits have a relationship to the rotating matter in the local galaxy and/or distant galaxies.\(^{84}\) There are many others: Einstein-ether theory, bimetric or general higher-order theories, Hořava-Lifshitz gravity, Galileons, Ghost Condensates, and models of extra dimensions, including Kaluza-Klein, Randall-Sundrum, Dvali-Gabadadze-Porrati model 4D gravity on a brane in 5D Minkowski space, or higher co-dimension braneworlds, Weyl conformal gravity (invariant under Weyl transformations), etc. The cosmological implications of these changes in gravity laws are important, but in most cases they have not yet been developed.

### 3.5. Other Friedmann-Lemaître-Robertson-Walker (FLRW) solutions

Instead of the particular solutions of ΛCDM with fixed parameters \(\Omega_m \approx 0.3, \Omega_\Lambda \approx 0.7\), the FLRW equations may have other solutions. Here we have a couple of examples:

One remarkable case that has generated a large number of papers in recent years is the Zero-active mass condition, also called \(R_h = c t\). This model was firstly proposed under the name of an ‘Ur theory’, which relates cosmology to particle physics and quantum theory,\(^{85}\) or a special (flat) case of an eternal coasting model\(^{86}\) consistent with a scale factor proportional to cosmic time \(a(t) \propto t\) or equivalently an active mass \(\rho + 3p/c^2\) equal to zero at all times, which, together with the ansatz \(\Lambda = 0\), makes the acceleration of the expansion equal to zero for all times. The density is fitted to keep the universe flat.

The most important researcher defending this model nowadays is Fulvio Melia, who has produced several tens of papers with theoretical and observational support.

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\(^{4}\)In string theory and related theories such as supergravity theories, a brane is a physical element that generalizes the notion of a point particle to higher dimensions. Branes are dynamical objects that can propagate through spacetime according to physical laws of quantum mechanics.
for the model. There is the coincidence that now the deceleration of the Hubble–Lemaître flow is compensated by the acceleration of the dark energy; the average acceleration throughout the history of the universe is almost null and the size of the universe is such as if there were constant expansion. This coincidence supports the constant expansion ratio posited as $R_h = c t$. According to Melia, this model fits the data pretty well where $\Lambda$CDM does. Moreover, it offers some further advantages at high $z$, where the standard model has some difficulties in concealing the existence of objects that usually need a long time to be formed in a very young universe that does not allow time for such evolution. $R_h = c t$ solves the problem because the age of the universe at redshift $z$, $t(z)$, is much greater in this model than with the standard $\Lambda$CDM. The Big Bang would have happened $H_0^{-1} = 14.57$ Gyr ago (for $H_0 = 67.4$ km/s/Mpc; Ref. 92), longer than the 13.79 Gyr for $\Lambda$CDM with the same Hubble–Lemaître constant. There would have been no inflation. Interestingly, an even more remarkable difference from the standard model is the prediction that CMBR is formed at $z \approx 16$ by dust rethermalization of Population III stellar light, although with a major difficulty in justifying a perfect blackbody shape for that radiation, since we do not know of any kind of dust that produces such a flux shape. This very different explanation of CMBR makes a major difference with the standard model.

Melia also points out that the $R_h = ct$ model has a higher probability of being correct than $\Lambda$CDM, even with the same quality of fits, based on the fact that $R_h = ct$ has no free parameters in the fits, whereas $\Lambda$CDM has. However, we must bear in mind that the election of the $\rho + 3\rho c^2 = 0 \forall t$ condition is posited because, a priori, we know it gives similar results to the standard models with $\Omega_m = 0.3$ in many cosmological tests at low and intermediate redshift, and non-acceleration of the expansion at present, so there is an implicit choice of parameters. Melia thinks that this choice is not arbitrary and derives it from general relativity for FLRW metrics, but this interpretation of general relativity is not supported by other specialists in gravitation (Ref. 94; a paper replied to by Melia in Ref. 90).

Another example of an FLRW model is Milne Cosmology, which is obtained by demanding that the energy density, pressure, and cosmological constant are all equal to zero, and that the spatial curvature is negative. Indeed, this can also be considered as a Zero active model, but with the particular case of $\rho = 0$. This strictly null density of matter is unrealistic since stars, gas and other components of the universe have mass and are real, but might possibly consider the density to be low enough to be considered close to zero, in comparison with the critical density in the standard model. Also, a symmetric Milne universe composed of matter with positive and negative mass in equal quantities would have net $\rho = 0$. From these assumptions and the FLRW equations, it follows that the scale factor must depend linearly on the time coordinate, $a(t) \propto t$, as in all zero-active mass cases. The model has no expansion of space, but the mathematical equivalence of the zero energy density version of the FLRW metric to Milne’s model implies that a full general
relativistic treatment using Milne’s assumptions would result in an increasing scale factor and an associated metric expansion of space. The Milne Cosmology does not need dark matter or dark energy; moreover, the model evades horizon, flatness, and cosmological constant problems affecting the standard cosmology, which requires inflation to solve it; it also fits some cosmological data.

3.6. Cyclical universes

In the ‘Conformal cyclic cosmology’ model based in the framework of general relativity, the universe iterates through infinite cycles, with the future timelike infinity of each previous iteration being identified with the Big Bang singularity of the next. So the Big Bang (although without inflation) applies to our present universe, but it is speculated that many other Big Bangs happened previously and will happen after ours. The Big Bang of each is taken to be a smooth conformal continuation of the remote future of the previous one via an infinite conformal rescaling; there is no collapsing phase. The second law of thermodynamics, with the curious nature of its origin, is automatically incorporated, where Hawking evaporation of black holes provides a key ingredient.

Another variation of the cyclical model proposed by Ref. is an endless universe without beginning or end, an endless sequence of epochs that starts with a ‘Big Bang’ (again without inflation) and ends in a ‘Big Crunch’ (collapse of the universe due to an excess of mass-energy of the critical density in a closed universe). Although the model is motivated by M-theory, branes, and extra dimensions, the scenario can be described almost entirely in terms of conventional 4D field theory and 4D cosmology, with a continual cycle of expansion and contraction as parallel universes (or ‘branes’) collide.

In the ‘Dynamic universe’ the universe is also described as a contraction-expansion cycle from infinity in the past to infinity in the future or in repeated cycles. In the contraction phase, mass in space gets its energy of motion from its own gravitation and releases it back to gravity in the ongoing expansion phase. Phenomena that general relativity theory explains in terms of modified space-time metrics are explained as consequences of their different energy states. The buildup of local structures in space converts part of the momentum in the fourth dimension into momentum in space via local bending of space. Galactic space in the DU appears in Euclidean geometry, and the magnitudes of high redshift supernovae are explained without assumptions of dark energy or accelerating expansion.

4. Quasi-Steady State Cosmology

All the previous models are variations based on the Lemaitre–Gamow idea. But there are however other models that challenge the notions of a state of the universe

\(^{M\text{-theory unifies all consistent versions of superstring theory.}}\)
with a singularity, unlimited density of matter-energy, or other important tenets of the standard model as proposed in the 1920s–1940s. Among the most developed models, with several professional researchers working on it over several decades, perhaps the alternative hypothesis of highest impact during the last century is the ‘Quasi-Steady State Cosmology’ (QSSC), which was indeed first called the ‘Steady State Cosmology’ when it was a cosmological model competing at the same level of importance and impact with the Big Bang hypothesis. It was developed and defended for more than sixty years, and even today we cannot declare it dead, although its main supporters have passed away or retired, and the new generations of cosmologists no longer work on it.

This model is indeed something beyond a small variation on Lemaître-Gamow ideas, it is a radically different view in which there is no beginning of the universe, but instead an eternal cycle of matter creation. The difference with the cyclical universes of §3.6 is that there is no singularity of state of infinite or unlimited density, and the moment of ‘creation’ is not unique, but continuous and constant (in the Steady State version) or oscillating (in the Quasi-Steady State version).

Fred Hoyle, and independently Hermann Bondi and Thomas Gold, proposed in 1948 the hypothesis of the Steady State\footnote{Indeed, prior to Hoyle, Bondi, or Gold, Albert Einstein attempted to construct a Steady-State model of the universe, as shown in one of his unpublished manuscripts\cite{Einstein1931}. The manuscript, which appears to have been written in early 1931, demonstrates that Einstein once explored a cosmic model in which the mean density of matter in an expanding universe is maintained constant by the continuous formation of matter from empty space. Einstein’s Steady-State model contained a fundamental flaw and this was possibly the reason why he abandoned this line of research.\cite{Einstein1931}.} in which, contrary to the Big Bang approach, there was no beginning of the universe. The universe is expanding, it is eternal, and the homogeneous distribution of matter is being created at a rate of $\sim 10^{-24}$ baryons/cm$^3$/s owing to the existence of a putative ubiquitous C-field of matter creation, instead of the unique moment of creation in the Big Bang. The perfect cosmological principle of a universe being observationally the same from anywhere in space and at any time is maintained in this model, whereas the standard model only gives a cosmological principle in space but not in time. There is no evolution. The universe remains always the same. The matter distribution is homogeneous and the redshift is caused by the expansion of the space with a scale factor $S(t) \propto e^{Ht}$. Newly created matter forms new galaxies that substitute those that are swept away by the expansion.

During the 1950s, both the Big Bang and Steady-State theories held their ground. While there were attempts to explain the abundances of the chemical elements with Gamow et al.’s theory, the Steady State Theory also provided plausible explanations. The abundances of the light elements (helium, lithium, deuterium, and others) were explained in terms of stellar nucleosynthesis and collision with cosmic rays in the remote past of the universe, as proposed by Margaret Burbidge and collaborators.\footnote{Indeed, prior to Hoyle, Bondi, or Gold, Albert Einstein attempted to construct a Steady-State model of the universe, as shown in one of his unpublished manuscripts\cite{Einstein1931}. The manuscript, which appears to have been written in early 1931, demonstrates that Einstein once explored a cosmic model in which the mean density of matter in an expanding universe is maintained constant by the continuous formation of matter from empty space. Einstein’s Steady-State model contained a fundamental flaw and this was possibly the reason why he abandoned this line of research.\cite{Einstein1931}.}
end also had to adopt the stellar nucleosynthesis of Burbidge et al. for the heavy elements.

Nonetheless, the Steady State theory would lose competitiveness by the mid-sixties, because it could not explain certain observational facts. It could not explain why the galaxies were younger at higher redshift. Neither could it explain the excess of radio sources at large distances, or the distribution of quasars. Most importantly, it did not explain the CMBR as interpreted in cosmological terms in 1965. This strongly favoured the Big Bang theory.

In 1993–94, Fred Hoyle, Geoffrey Burbidge, and Jayant Narlikar published a modification of the model that was called the ‘Quasi-Steady State’ theory. The main modification consisted in positing an oscillatory expansion apart from the exponential term (see Fig. 6):

\[ S(t) \propto e^{t/P}[1 + \eta \cos(2\pi \theta(t)/Q)], \tag{1} \]

with a long time scale of expansion of \( P \sim 10^{12} \) years, \( \theta(t) \sim t \). The exponential factor had already been introduced in the first version of the Steady State model to keep \( \frac{\dot{X}}{X} \) = constant and consequently maintain a constant density of matter by invoking the continuous creation of matter. The new term here is the sinusoidal oscillation with period \( Q \). The exact value of the parameter \( Q \) and \( \eta \) would be determined from Hubble–Lemaître’s constant, the age of globular clusters, and the maximum observed redshift in the galaxies. The time since the last maximum of \( S(t) \) is 0.85\( Q \). The time since the last minimum of \( S(t) \) is 14 Gyr. With the parameters given in the original version of the QSSC theory, the maximum observable redshift should be around 5, although it would be increased later as the parameters changed.
to adapt to new observations. The creation of matter is confined to epochs with minimum $S(t)$ rather than being continuous. These creation events involve Planck particles and eventually make hydrogen gas plus the lightest elements, deuterium and the two isotopes of helium. Since the overall time scale is very long, many generations of stars (and galaxies) will evolve and die.

With the introduction of the Quasi-Steady variation, some of the problems that affected the original theory of 1948 were solved. This explained why there are younger galaxies at higher redshift, the problem of the radio sources, the distribution of quasars (with lower density for redshifts lower than 2.5), and the formation of large-scale structure (clusters, voids, filaments). Ref. 110 complained that Hoyle et al. had not solved the problem of the radio sources completely, but Ref. 112 later replied that the question might be solved with a change of parameters.

The CMBR and its blackbody spectrum would be explained as the effect of the thermalization of radiation emitted by stars of the last cycle $P/3$ due to absorption and re-emission that produce iron needle-shaped particles (‘whiskers’) in the intergalactic medium. Because of the long distances travelled by the CMBR photons in the maxima of the oscillation and the thermalization that occurs at each minimum, there is no accumulation of anisotropies from one cycle to another. Only the fluctuations of the last minimum survive, which gives fluctuations of temperature comparable to the observed $\Delta T/T \sim 5 \times 10^{-6}$. First, the carbon needles thermalize the visible light from the stars, giving rise to far-infrared photons at $z \sim 5$, thus maintaining the isotropy of the radiation. Afterwards, iron needles dominate, degrading the infrared radiation to produce the observed microwave radiation. Within QSSC, on the other hand, the anisotropies of this radiation would also be explained in terms of interaction of the radiation with clusters of galaxies and other elements.

Extragalactic iron whiskers might be formed in a process similar to metallic vapours cooling slowly enough in the laboratory. Whiskers are formed by this type of process during the expansion of the envelopes of supernovae. As a matter of fact, some of the defenders of the QSSC model have claimed that iron whiskers are observed in the emission spectrum of the Crab pulsar PSR0531+21. Thermalization was also proposed to be due to the plasma of the intergalactic space, but in this case it takes about 450 Gyr for the starlight to get thermalized.

Concerning the origin of the redshift of galaxies, the proposers of this model admit a component due to the expansion $S(t)$, as in the standard model, but further posit the existence of intrinsic redshifts. This allows the solution of problems such as the periodicity of redshift in quasars and the existence of numerous cases with possible anomalous redshifts.

Summing up, QSSC competes with the standard cosmological model to explain many observations, at least in an approximate way, but with a very different description of the universe. According to its authors, QSSC can even explain some facts that the standard model cannot, such as possible anomalies in the redshifts of quasars. It also contains predictions that are different from those of the standard
model, although these are difficult to test. The predictions include: the existence of faint galaxies \((m > 27)\) with small blueshifts \((|\Delta z| < 0.1)\), the existence of stars and galaxies older than 14 Gyr, an abundance of baryonic matter in ratios above those predicted by ΛCDM, and gravitational radiation derived from the creation of matter.

5. Plasma Cosmology

Plasma Cosmology is another alternative model that has occupied two or three generations of researchers, some of them still active today. Its proponents include the physics Nobel laureate Hannes Alfvén, Oskar Klein, Anthony L. Peratt, and Eric J. Lerner. It has a strong argument against one of the main pillars of the standard model: It proposes an alternative to the belief that gravitation is the fundamental force that controls the dynamics of the universe. It assumes instead that most of the mass in the universe is plasma controlled mainly by electromagnetic forces (and also gravity, of course), rather than gravity alone. According to this theory the universe has always existed, it is always evolving, and it will continue to exist forever.

The plasma in the laboratory, through electric currents and magnetic fields, creates filaments similar to those observed in the large-scale filamentary structure of the universe. The plasma cosmology model predicts the observed morphological hierarchy: distances among stars, galaxies, cluster of galaxies, and filaments of huge sizes in the large-scale structure. The observed velocities of the streams of galaxies in regions close to the largest superclusters are coincident with those predicted by the model, without the need for dark matter. The formation of galaxies and their dynamics would also be governed by forces and interactions of electromagnetic fields.

Hubble–Lemaître expansion was admitted in the first version of plasma cosmology and was explained by means of the repulsion between matter and antimatter. A plasma mechanism can separate matter from antimatter and, when an antimatter cloud bumps into an ordinary-matter cloud, they will not totally annihilate each other; instead, only a thin layer will be annihilated generating a hot, low-density plasma layer which will push the clouds apart. Alfvén proposed his ‘fireworks’ model, in which a supercluster is repelled by other superclusters; within a supercluster each cluster is repelled by other clusters; and within a given cluster each galaxy is repelled by the other galaxies, and so on, obeying a distribution of matter and antimatter. In each local volume, a small explosion would impose its own local Hubble–Lemaître relationship, and this would explain the variations in the velocities of the Hubble–Lemaître law, i.e. the different values of the Hubble constant measured in the ’70s and ’80s when Alfvén posited his hypothesis, in different ranges of distances or looking in different directions, all without invoking dark matter. The energy derived from the annihilation of protons and electrons would produce a background radiation of X- and γ-rays. There are some objections against
the existence of antimatter based on the absence of \( \gamma \)-rays from annihilation, but they are model-dependent. Many of the objections against antimatter have been analysed\(^{129}\) and it has been shown that none of them is crucial. Nonetheless, some critics remarked that it is not consistent with the isotropy of the X-ray backgrounds\(^{130}\).

Instead of expansion caused by matter–antimatter repulsion, in more recent times, some proponents of plasma cosmology\(^{131}\) have stated that there is no expansion, that the universe is static, and that the redshift of the galaxies would be explained by some kind of tired-light effect of the interaction of photons with electrons in the plasma. (See \(\S\) 7)

With regard to the CMBR, Refs. \(^{132,133}\) explain it in terms of absorption and re-emission of radiation produced by stars. It is similar to the mechanism proposed by QSSC, but here the thermalization is due to interaction with electrons. The interaction of photons and electrons produces a loss of direction in the path of the light, giving rise to isotropic radiation.

6. Universe as a Hypersphere

Another category of models that have appeared in different versions is one that posits that our universe is a hypersphere of a higher-dimensionality geometrical entity; that is, a set of points at a constant distance from its centre, constituting a manifold with one dimension less than that of the ambient space.

One of the first models maintaining this idea is the ‘Chronometric Cosmology’ of Irving E. Segal\(^ {134,135}\). This model assumes that global space structure is a 3D-hypersphere (or 3D-hypersurface, as Segal calls it) in a universe of four dimensions. Events in the universe are ordered globally according to a temporal order. This model makes an application of general relativity different from the standard model, getting a relationship of the redshift with distance \( r \):

\[
\frac{z}{R} = \tan^2 \frac{z}{R}.
\]

His cosmology gives a good fit to the various curves versus redshift: magnitude, counts, angular size, etc., but, with data about redshifts of galaxies and distances, it is now known the proposed relationship between redshift and distance cannot be correct\(^ {136}\). Moreover, there is no explanation for the CMBR. Many other refutations of Segal’s claims have also been published\(^ {137-140}\).

More recent is the hypothesis of the existence of five combined spacetime dimensions. By making some peculiar assignments between coordinates and physical distances and time, a hyperspherical symmetry is made apparent by assigning the hypersphere radius to proper time and distances on the hypersphere to usual 3-dimensional distances in a Euclidean universe\(^ {141}\) which can explain the Hubble–Lemaître expansion law without appealing to dark matter; an empty universe will expand naturally at a flat rate in this way. Another variation is the Hypersphere World-universe Model\(^ {142,143}\) which claims the existence of a 3-dimensional hypersphere with respect to a 4-dimensional Nucleus of the World. Matter in this universe is of the ordinary kind with the addition of a multicomponent dark-matter (instead of Cold Dark Matter plus Dark Energy). This model has a number of peculiar char-
acteristics: the beginning of the universe, instead of originating from a singularity, stems from a 4-dimensional Nucleus of the World; the radius of this Nucleus, increasing with speed $c$, is what produces the expansion; the CMBR stems from the thermodynamic equilibrium of photons with intergalactic plasma; and the nucleosynthesis of light elements occurs inside dark matter cores of Macro-objects.

7. Static Models and non-cosmological redshifts

At the farthest extreme of alternative models with respect to the standard cosmology, we have proposals of universe that contradict the main interpretation upon which standard cosmology was created from the 1920s onwards: expansion of the universe and the interpretation of redshifts as cosmological. Some versions of plasma and hypersphere cosmologies figure among these models, but there exist plenty of other models that are characterized by the lack of an origin of time (a universe of indefinite age), no expansion, and in some cases no limit is imposed on space which is Euclidean.

Of the many cases of static and/or non-cosmological redshift models in the literature, we will mention just a few. Some of them are totally obsolete and with no researcher actively working on them today, but they deserve to be mentioned from a historical point of view. Most of them are proposed by single researchers, they are the author’s own theories, in some cases produced by non-active professional individuals within astrophysics, retired researchers or professionals of other fields; nonetheless, they serve to illustrate the range of possible speculations within cosmology.

7.1. Cosmological models motivated by tired-light redshifts

The redshift of galaxies given by Hubble–Lemaître’s law can be due to some mechanism different from the expansion or Doppler effect. A bibliographical catalogue of non-trivial redshifts collected by Reboul in 1981 reduces hundreds of references into 19 classes of alternative theories, of which the tired-light scenario is the largest.

A tired-light scenario assumes that the photon loses energy owing to some proposed photon–matter process, photon–photon interactions, or some dissipative property of the photon. The energy loss can be as a function of time or distance: the distance can be either long, spanning the intergalactic space between the object and the earth, or short, for instance taking into consideration only the corona enveloping the object. The scenario is sometimes presented as a possible phenomenological approach describing photon energy loss in a putative static universe, sometimes given as a theory for the energy loss mechanism.

There are several hypotheses that can produce this tired-light effect. The idea of loss of energy of the photon in the intergalactic medium was first suggested in 1929 by Zwicky and was defended by him for a long time. As late as the mid-twentieth century, he maintained that the hypothesis of tired-light was viable.
Other authors from this epoch also supported the idea. But there are two problems: 1) for a particle to lose energy in an interaction also implies a momentum transfer that smears out the coherence of the radiation from the source, and so all images of distant objects would look blurred if intergalactic space produced scattering, something that is incompatible with present-day observations; 2) the scattering effect and consequent loss of energy would be frequency dependent, which is again incompatible with what we observe in galaxies. Nonetheless, there are theories that propose solutions to mitigate these problems.

Fig. 7. Comparison of the model Hubble diagram for distant Type Ia supernovae in a static tired-light cosmology (line) with the Type Ia supernova (SN) data. The model has no free adjustable parameters. The squares represent the low-redshift SN data adopted from Riess et al., the triangles represent IIfA Deep Survey SN data adopted from Barris et al., and the circles represent the Hubble Space Telescope SN data adopted from Riess et al. The high-redshift SN data are subject to uncertain apparent magnitude corrections for host galaxy extinction. (Adapted from Ref. [157] under the Creative Commons Attribution 4.0 International License, https://creativecommons.org/licenses/by/4.0/legalcode.)

The strong appeal of tired-light redshift is that it could naturally explain many types of non-trivial redshift phenomenologies such as ‘redshifts on or by the Sun’, ‘general problems of redshifts’, ‘redshifts of stars’, ‘morphological redshifts’ and ‘Fingers of God’ in the large-scale-structure (usually explained as gravitational infall velocities that produce Doppler). By extension contemporary problems such as the ‘anisotropy of the Hubble Law’, ‘anomalous redshift of quasar-galaxy associations’ ‘galactic rotation curves’, the ‘Hubble tension’ and the ‘acceleration of
expansion \(^\text{(157)}\) (see Fig. 7) can also be easily interpreted as an effect of a tired-light redshift.

A ‘static cosmology’ common to these models may be described with the Einstein static cosmology \(^\text{(158)}\) or with other metrics different from general relativity, although there are also proposals away from relativistic approaches. Since in a static universe a cosmological redshift cannot be produced, the tired-light mechanism supplies a differential Hubble law in the form

\[
d\nu/\nu = -(c/H)dr_p,
\]

(2)

where \(r_p\) is the proper distance. From this metric follows these relations: luminosity distance \(d_L = (1 + z)^{1/2} r_p\), distance modulus \(m(z) = 2.5 \log(1 + z) + 5 \log(\ln(1 + z)) + 25\), angular size \(\theta \propto \ln(1 + z)\), number counts \(\log N(z) = 3 \log[\ln(1 + z)] + cte\), and surface brightness \(\log SB(z) = 2.5 \log(1 + z)\).

The hierarchical argument \(^\text{(159, 160)}\) proposes that the density of the largest structures observed in the universe remain well below the Schwarzschild limit over 40 orders of magnitudes (see Fig. 8). The hierarchical universe gives an easy solution to the gravitational paradox of Seeliger. It also implies a curvature radius that is extremely large \(R \gg c/H\).

As a result, proponents of a static universe consider that these observations forbid the concept where the singularity occurs everywhere at the same time \(^\text{(161)}\).

The CMBR is interpreted to be that of a thermalization of stellar radiation based on predictions made in the 1950s by Finlay-Freundlich and Max Born on the basis of a tired-light mechanism \(^\text{(161)}\).

Most of these considerations are used in the following static cosmologies, with some specific variations.

7.1.1. Curvature Cosmology

Proposed by David F. Crawford \(^\text{(152, 165)}\). This cosmology is based on a combination of general relativity and quantum mechanics. The concept of curvature pressure arises from the ideas that the density of particles produces curved spacetime and the velocity of these particles produces a reaction pressure that acts to decrease the local spacetime curvature. Although the geometry is similar to the original Einstein static model, this cosmology differs in that it is stable because of a feedback mechanism: the plasma produces curved space-time through its density entering the stress-energy tensor in Einstein’s field equations, and the velocities of the plasma particles decrease this curvature \(^\text{(155)}\). In the case of the photons that travel across matter, the curvature produces a tired-light effect as a product of the gravitational interaction between wave packets and curved spacetime which will follow geodesics and be subject to geodesic focusing, giving rise to the observed curvature redshift of galaxies.

Since this will alter the transverse properties of the wave, some of its properties such as angular momentum would be altered, which is forbidden by quantum me-
Fig. 8. The hierarchical argument uses the density of the largest structures observed in the universe which remain well below the Schwarzschild limit over 40 orders of magnitudes. The universal density-radius relation gives the maximum average density of matter \( \text{[g/cm}^3] \) in spherical volumes of radius \( R \) [cm] from neutron stars (dashed line at top) to the largest domain in which galaxies have been counted (asterisk at bottom). The range of densities by the virial theorem for stellar and galaxy clusters is shown (thin dashes). (Adapted with permission of the AAAS from Fig. 3 in Ref. [159].)

Mechanics. Instead, the result of the interaction of the photon is three new photons: one with almost identical energy and momentum to that of the original photon and two extremely low energy secondary photons. Anomalous redshift cases might be produced by the extra redshift being due to the photons’ passage through the cloud around the anomalous object.\textsuperscript{152,154} The CMBR comes from the curvature-redshift process acting on the high-energy electrons and ions in the cosmic plasma. The energy loss which gives rise to the spectrum of photons of the CMBR occurs when an electron that has been excited by the passage through curved spacetime interacts with a photon or charged particle and loses its excitation energy.
7.1.2. Plasma-Redshift

Proposed by Ari Brynjolfsson,\(^{166,167}\) the plasma-redshift is a result of a photon-electron interaction which follows strictly from the average electron density in intergalactic space. The magnitude-redshift relation also includes extinction caused by Compton and Rayleigh scattering on bound electrons in atoms. There is no time dilation and no expansion of the universe.

Plasma redshift decreases the photon energy and increases the kinetic energy of the electrons, which produces the steep temperature rise in the transition zone to million degrees K in the solar corona. Although the universe is infinite, the gravitational potential has a finite range due to the finite propagation speed of gravity. Random thermal motion reduce the ability of the gravitational field to transfer information about its direction and strength over large distances. Apart from small ripples (e.g. gravitational lensing) space is flat.

7.1.3. Subquantum Kinetics

Proposed by Paul A. LaViolette,\(^{168}\) This is a unified field theory with the foundations for a new wave–theory of matter. Its non-dispersing, periodic structures resolve the wave–particle duality and produce de Broglie wave diffraction effects. The Subquantum kinetics model proposes an open, order-generating universe, continuously creating matter and energy in the form of neutrons in the vicinity of existing particles.\(^{169}\) It predicts that gravitational potential should have a finite range. It uses tired-light redshift in a static universe, without radiating a secondary photon, angular deflection, or a strong wavelength dependence. It works as if intergalactic space were on average endowed with a negative gravitational mass density.

7.1.4. Scale Expanding Cosmos

Proposed by Carl Johan Masreliez,\(^{170,171}\) A universe is proposed in which not only space expands (therefore, it is not properly a static = non-expanding model) but time also expands: the relationship between space and time could remain constant during the cosmological expansion and all cosmological locations in time and space could be equivalent, if the metrics of both space and time expand. The scale expansion could be eternal, which would eliminate the creation event. Redshift is a tired-light effect. The theory is presented based on the proposition that all four metric coefficients of space and time change with cosmological expansion. Such a universal scale expansion would preserve the four-dimensional spacetime geometry and therefore, by general relativity, most physical relationships. In addition, if the scale expansion were exponential with time, all epochs would be equivalent. There are also other proposals on the reinterpretation of space in terms of time expansion or comoving system of units.\(^{172,173}\)
7.1.5. Dichotomous cosmology

Proposed by Yuri Heyman. Contrary to general relativity, here there is a dichotomy between light and matter dynamics: the luminous portion of the Universe is expanding at a constant rate as in the de Sitter cosmology in a flat Universe, whereas the matter component is static, and the emitted light wavelength gets stretched due to a tired-light process. As a consequence time-dilation of supernovae light curves is observed with a stretching factor of $1 + z$. In addition, the expanding luminous world is consistent with the radiation energy density factor $(1 + z)^4$ inferred from the CMBR.

7.1.6. Wave System Cosmology

Proposed by Thomas B. Andrews. The universe is a pure system of waves with mass density and tension parameters proportional to the local intensity of the modes of the waves. The peaks of the constructive interferences are the elementary particles. The energy input to a single proton from all of the interacting particles in the universe is found to be equal to the mass energy of the proton. Newton’s law of gravitation derives from the wave modes originating among protons. In this flat model of the universe, the redshift is produced by a tired-light mechanism.

7.1.7. Eternal Universe

Proposed by Gerald S. Hawkins. Based on the existence of a negative pressure in a cosmic fluid derived from general relativity (not very different from the role that the cosmological constant has acquired nowadays). The main point which differentiates this model from the standard theory is the proposal that the universe is static, infinite, without an instant of creation, and without expansion. The redshift of the galaxies is explained as a gravitational effect combined with a slight amount of intergalactic extinction produced by certain particles located in the space surrounding galaxies, which is $\sim 10^{-7}$ times the local interstellar absorption per unit distance. Ref. [180] argues that his model is not unstable, having no tendency to collapse or expand, and that the CMBR is due to the emission of galactic and intergalactic dust grains. Olbers’ paradox is solved by means of absorption in clouds of dust.

7.2. Other non-cosmological redshifts and other static models

A gravitational redshift in a fractal universe as a source of the redshift of the galaxies, or theories with the context of alternative views of gravitation can also produce a non-cosmological redshift without expansion. Instead of new ideas on gravitation, some researchers explored electromagnetism. Furthermore, just to give further examples from the huge literature on the topic, there are explanations of non-cosmological/non-Doppler redshifts in terms of the
local shrinkage of the quantum world; variation of the speed of light; variation of Planck’s constant; time acceleration or deceleration; proposals for a quantum long-time energy redshift in which the energy is smaller owing to a quantum effect when the photon travels over a very long time; chronometric cosmology (see §6); the variable mass hypothesis or secular mass variation of particles (see §4 and §7.2.1); etc. All these proposed mechanisms show us that it is quite possible to construct a cosmological scenario with non-expansion redshifts. Nonetheless, all these theories are at present just speculation without direct experimental or observational support.

7.2.1. **Self Creation Cosmology (SCC)**

Proposed by Garth Antony Barber and others, the self creation cosmology is an adaptation of the Brans Dicke theory in which the conservation requirement is relaxed in order to allow the scalar field to interact with matter. SCC can be thought of as general relativity combined with Mach’s principle and local conservation of energy. In SCC, energy is conserved but energy–momentum is not. Particle masses increase with gravitational potential energy and, as a consequence, cosmological redshift is caused by a secular, exponential increase of particle masses. The universe is static and eternal in its Jordan frame and linearly expanding in its Einstein frame. Furthermore, as the scalar field adapts the cosmological equations, these require the universe to have an overall density of only one third of the critical density while remaining spatially flat. The cosmological redshift is interpreted as a measurement of the cosmological increase of the atomic masses of the measuring apparatus rather than by a Doppler shift. The theory also posits a time-slip between atomic clock time on the one hand, and gravitational ephemeris and cosmological time on the other, which would result in the observed cosmic acceleration.

7.2.2. **Cellular Cosmology**

The cellular cosmology of Conrad Ranzan is based on a Dynamic Steady State Universe model. The universe is an assembly of gravity cells which have a nominal diameter of 60 Mpc. The change in velocity of the aether flow is what produces gravity. The mechanism that causes the gravitation effect and sustains the Universe’s gravity-cell structure is also the mechanism that causes the wavelength stretching manifesting as cosmic redshift. Intrinsic spectral shift occurs with a transit of the photon across/through any gravity well (sink). It is caused by the difference in propagation velocity between the axial ends of the photon or wave packet. The CMBR is the “now-and-forever” temperature of the steady state universe.

7.3. **Plausibility of static models**

Static models are usually rejected by most cosmologists. However, from a purely theoretical point of view, the representation of the Cosmos as Euclidean and static
is not excluded. Both expanding and static spaces are possible for the description of
the universe. Before Einstein and the entry of Riemannian and other non-Euclidean
geometries into physics, there were attempts to describe the known universe in
terms of Euclidean geometry, but these faced the problem of justifying a stable
equilibrium. Within a relativistic context, Einstein proposed a static model that
included a cosmological constant, his ‘biggest blunder’ according to himself. This
model still has problems in guaranteeing stability, but it might be amenable to some
kind of solution. Ref. 165 solves the stability problem with a feedback mechanism
resulting from the tired-light process itself. Ref. 198 solves it within a variation of
the Hoyle–Narlikar conformal theory of gravity, in which small perturbations of the
flat Minkowski spacetime would lead to small oscillations about the line element
rather than to a collapse. Ref. 206 analyses the stability of the Einstein static
universe by considering homogeneous scalar perturbations in the context of $f(R)$
modified theories of gravity, and it is found that a stable Einstein cosmos with a
positive cosmological constant is possible. Other authors solve the stability problem
through variation of fundamental constants. 207,208 Another idea from Ref. 209 is
that hypothetical gravitons responsible for gravitational interaction have a finite
cross-sectional area, so that they can only travel a finite distance, however great,
before colliding with another graviton. So the range of the force of gravity would
necessarily be limited in this way and collapse avoided.

Curved geometry (general relativity and its modifications) does not conserve the
energy–momentum of the gravity field. However, Minkowski space does follow the
conservation of energy–momentum of the gravitational field. One approach with a
material tensor field in Minkowski space is given in Feynman’s gravitation, in
which space is static but matter and fields can be expanding in that static space.
Also worth mentioning is a model related to modern relativistic and quantum field
theories of basic fundamental interactions (strong, weak, electromagnetic): the relativistic field gravity theory and fractal matter distribution in static Minkowski
space.

Olber’s paradox for a universe without limits is an old problem and also
needs subtle solutions, but extinction, absorption, and re-emission of light, fractal
distribution of density, and the mechanism which itself produces the redshift of the
galaxies might have something to do with its solution.

8. Classification according to characteristics

We have described a representative sample of alternative cosmological models, from
the minor variations with respect to $\Lambda$CDM to the most extreme heterodox propos-
als of static universes.

Attending to the different features of the models, the following differences can
be observed among them:

Gravity, forces: general relativity in the standard $\Lambda$CDM and its minor variations
or in the cases of inhomogeneous universe, Cold Big Bang, Zero Active Mass and
Milne, QSSC and even in some cases of static universe like Hawkins’s Eternal Cosmology, Crawford Curvature Cosmology (together with quantum mechanics) and Masreliez’s Space Expanding Cosmology. However, there are deviations from general relativity when varying physical constants, alternative gravity or cyclical universes are assumed as major variations. Plasma Cosmology takes electromagnetic forces as dominant dynamical element, and the plasma-redshift cosmology includes finite propagation speed of gravity. Hypersphere models use different hypersphere geometry. Dichotomous cosmology states a dichotomy in the dynamics of matter and light, against general relativity principles. Barber’s Self-Creation Cosmology uses Brans Dicke gravitation, and Andrews’ Wave System and LaViolette Subquantum’s kinematics posit the equivalence of Matter and Waves. In the Cellular Cosmology, gravity is produced by the change in velocity of an aether flow.

**Expansion:** included in all minor or major variations on the standard model and in QSSC. Plasma Cosmology and Hyperspheres may assume either expansion or non-expansion. The static models include non-expansion, except Masreliez’s Scale Expanding Model that assume expansion of space and time.

**Age of the universe:** finite (e.g., 13.8 Gyr for ΛCDM; 14.6 Gyr for Zero Active Mass or Milne cosmologies) in all minor or major variations on the standard model except in the Cyclical Universes that keep a eternal existence. The rest of the models keep also an infinite age of the universe, except Hyperspheres that keep a finite age and Masreliez’s Scale Expanding Model where it can be finite or infinite.

**Redshift:** cosmological in all minor or major variations on the standard model. QSSC assumes a combination of cosmological and variable mass hypothesis redshifts. Plasma Cosmology has two options: either Doppler effect due to repulsion among galaxies (due to matter-antimatter interaction), or tired-light hypothesis, a plasma-redshift also included in the static cosmology of that name. Hypersphere models have their own cosmological redshift in term of a geometric interpretation. Hawkins’s Eternal Cosmology assumes gravitation redshifts. Barber’s Self Creation Cosmology assumes a variable mass hypothesis too. Other static models cited here use a tired-light hypothesis. In the Cellular Cosmology is related to the same mechanism that produces gravity, a change of velocity in an aether flow.

**Dark elements:** CP violation, inflation (inflaton), dark matter and dark energy are the four dark nightmares of ΛCDM. The different variations within standard paradigm may include some of these elements, with some changes in the minor variations, but some cases do not include them or do not explicitly tell us about their existence. In the case of Conformal Cyclic Cosmology (one of the major variations with cyclical universes), there is the extra element of black holes evaporation. QSSC also keeps dark matter and dark energy, plus another mysterious element: the C-field of matter creation. LaViolette’s Subquantum kinetics also contains a continuous creation of matter. Plasma Cosmology gets...
rid of all these dark forces, provided that strong magnetic fields (undetected so far) exist. Hypersphere has a multicomponent of dark matter. Hawkins’ Eternal Universe contains some negative pressure (not very different from the concept of dark energy nowadays). Other models are not explicit about dark elements existence.

**CMBR origin:** decoupling of matter and radiation at \( z \approx 1100 \) in \( \Lambda CDM \) or its inhomogeneous variation. In some major variations like Cold Big Bang or Zero Active Mass, or totally different models like QSSC, Plasma Cosmology or Hypersphere Model, the origin is instead a thermalization of radiation of Population III of stars by particles in the intergalactic medium. Static models are not clear about it, although some of them point out to dust emission (Hawkins’ Eternal Cosmology), or redshifted high energy charged particles (Crawford’s Curvature Cosmology). In the Cellular Cosmology, CMBR is the temperature of a steady-state universe.

**Light Element Nucleosynthesis:** primordial in the standard model and its variations, except the Cold Big Bang, which admits a combination of primordial and stellar (with stars of population III) nucleosynthesis. For QSSC and Plasma Cosmology, it is pure stellar nucleosynthesis. In the version of hypersphere universe by Netchitailo, it is produced in the dark cores of Macro-objects.

**Homogeneity at large-scale:** in all cases except in the major variation of explicitly inhomogeneous universe, or in some cases with alternative gravity scenarios.

**Galaxy formation:** by evolution of density fluctuations with gravity as the only force when explicitly established. This evolution might be quicker in scenarios with alternative gravity. The exception is Plasma Cosmology, where electromagnetic forces govern the dynamics.

### 9. General problems of the alternative models

None of the alternative cosmological models is as competitive as the standard \( \Lambda CDM \) model, because they are not so developed and there are many observations pending to be explained with these models. This does not mean that \( \Lambda CDM \) is necessarily the correct model of the universe, it also has a bunch of problems pending to be solved and many dark elements from which we know nothing. However, the level of accuracy of the representation of the set of cosmological data is much higher and none of the caveats is conclusive so far as to falsify this model. Moreover, the alternative proposals have some problems too, and they are yet more severe than in the standard model (see, for instance, Edward L. Wright’s web-page), perhaps because these theories are not as developed and polished as the standard model.

A solution to Olbers’ paradox by dust absorption in a universe of no defined extent, for instance, is not clear. One may wonder, if energy does not disappear, whether the absorbing element (dust) should be heated and re-emit, and, if the en-

\(^4\)http://www.astro.ucla.edu/~wright/errors.html
ergy disappears how that can be consistent with known physical laws. This problem has no easy solution.

Expansion in universes without a time origin is either taken as a fact, in which case the models need speculative elements to argue that there was no beginning of the universe, or an alternative explanation must be given for the redshift of the galaxies which raises its own set of difficulties.

Light element abundances require in some cases very early stellar populations (Population III) that have not been observed yet. In an indefinitely old universe however, there is more than enough time to form all heavy elements and Pop. III stars does not need to be hypothesized. In that case their non-observation is not a problem.

The CMBR has alternative explanations to ΛCDM’s, but with ad hoc elements without direct proof, such as hypothetical particles to thermalize stellar radiation. A power spectrum with oscillations is a rather normal characteristic expected from any fluid with clouds of overdensities that emit/absorb radiation or interact gravitationally with photons, and with a finite range of sizes and distances for those clouds. The standard cosmological interpretation of ‘acoustic’ peaks, from the hypothesis of primaeval adiabatic perturbations in an expanding universe, is just a particular case: peaks in the power spectrum might be generated in scenarios that have nothing to do with oscillations owing to gravitational compression in a fluid. Nonetheless, all proposals to explain a CMBR produced in the intergalactic medium—even assuming that a perfect black body shape can be produced—have the problem that the integration along the line of sight gives a superposition of many layers of black body radiations, each with a different redshift, giving in total something different from a black body. Otherwise the CMBR can originate from the local universe (z ≈ 0; in such a case, the problem would be that space would be too opaque to allow the observations of distant radio sources and it could not explain the Sunyaev-Zel’dovich effect of the interaction with clusters of galaxies) or at a given high redshift z > 0 but within a layer with small Δz.

The most elaborate alternative models, such as QSSC, do indeed apply the same methodology as the standard model: each one has some basic tenets, a lot of free parameters and ad hoc elements that are introduced every time some observation does not fit the model. The modern version of QSSC, for example, is able to explain most of the difficulties of the previous (Steady State) version of the model. The authors introduced ad hoc elements without observational support in the same way that the standard model introduces ad hoc non-baryonic dark matter, dark energy, inflation, etc. The very idea of continuous creation of matter also necessitates some very exotic physics, and has no empirical support. But the authors continued ad hoc to skip over the inconsistencies; for instance, the maximum redshift of a galaxy was set to be 5 in the initial version of QSSC; however the model’s free

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The idea of continuous creation of matter as a result of a modification of Einstein’s equation of gravitational field is also independently explored by other alternative cosmological models.
parameters were conveniently changed when some new observation does not fit the initial predictions. In the end, then, the authors can introduce ad hoc corrections that render their theory compatible with any maximum redshift of a galaxy.

One should not, however, judge any theory according the number of observations that it can successfully explain, but by the plausibility of its principles and its potential to fit data (provided that we have an army of theoreticians able to correct the theory ad hoc every time new observations need to be accommodated). A pluralist approach to cosmology is a reasonable option when the preferred theory is still under discussion. Therefore, given the number of problems with the standard model, it is quite reasonable to keep a weather eye on alternative ideas that might at least provide better partial explanations or interpretations of certain observed phenomena. Nonetheless, a global cosmological theory that provides a satisfactory explanation of astronomical observations does not yet exist according to either standard or in alternative viewpoints.

Acknowledgments

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