AN APPROACH FOR SOLVING FUZZY ASSIGNMENT PROBLEM USING BRANCH AND BOUND TECHNIQUE

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Abstract
This paper aims to obtain a minimum fuzzy assignment cost for a fuzzy assignment problem by using fuzzy branch and bound technique. Without transforming the fuzzy assignment problem into a corresponding crisp assignment problem a solution procedure to assign different resources to different activities is proposed to attain the fuzzy optimal cost. In this paper for comparing fuzzy numbers a ranking function in terms of parametric form is used. To showcase the proposed method an example is discussed and a comparison is made with an existing method.

Key words: Trapezoidal Fuzzy numbers, Fuzzy assignment problem, Fuzzy branch and bound technique, Fuzzy number ranking

1. Introduction
An assignment problem is a linear programming problem which is solved using Hungarian method where allocation is done on one – one way in which different activities are assigned to different resources so that the total time or cost involved is minimized and the total profit or sale is maximized. Many real life problems can be solved by assignment problem technique. The main idea of assignment problem is to determine the optimum solution for a given problem. In order to solve assignment problem the decision parameters must be crisp one. But in real life problems we deal with vagueness and imprecise parameters. This imprecision occurs due to measurement inaccuracy, variation of parameters, computational errors etc. In 1965, Lotfi Zadeh\textsuperscript{[12]} introduced fuzzy sets that give a procedure to deal with uncertainty.

Pandian et al\textsuperscript{[7]} proposed parallel moving method to solve Fuzzy assignment problem and an optimal solution is found. Nagoor Gani et al\textsuperscript{[5]} suggested a ranking method that converts fuzzy assignment problem to crisp assignment problem in LPP form by LINGO 9.0. Dhanasekar et al\textsuperscript{[2]} has solved fuzzy assignment problem without changing the fuzzy cost into crisp cost using Yagers ranking. Anchal Choudhary\textsuperscript{[1]} solved fuzzy assignment problem using trapezoidal fuzzy numbers. Soland et al\textsuperscript{[9]} solved generalized assignment problem using branch and bound algorithm by resolving a series of binary knapsack problem to decide the bounds. Narayananmooorthy et al\textsuperscript{[6]} suggested branch and bound technique with trapezoidal fuzzy number to solve fuzzy assignment problem. Srinivas et al\textsuperscript{[8]} proposed bound and bound algorithm to solve triangular and trapezoidal fuzzy number and Robust ranking technique is applied to transform fuzzy numbers to crisp values. Hema et al\textsuperscript{[4]} an optimal solution is obtained without converting fuzzy numbers to crisp numbers by using branch and bound method. Hosseinzadeh et al\textsuperscript{[3]} proposed a method for solving fuzzy linear programming by lexicography method. Thorani et al\textsuperscript{[10]} proposed a new algorithm is proposed for solving fuzzy assignment problem with fuzzy cost. Thangavelu et al\textsuperscript{[11]} proposed a method of converting Generalized trapezoidal fuzzy number to crisp one using a new ranking method.
This paper aims at finding a fuzzy optimal assignment cost to a fuzzy assignment problem with assignment cost represented as trapezoidal fuzzy number (TrFN). This paper is structured as follows. In section 2, the basic definition, ranking function and arithmetic operation of TrFN are reviewed. In section 3, we introduce the fuzzy assignment problem with TrFN and the fuzzy branch and bound algorithm. A numerical example is provided in section 4 to compare the suggested method with an existing method.

2. Preliminaries
In this section we review certain definition, ranking function and arithmetic operation of TrFN.

2.1 Fuzzy set
A fuzzy set is characterized by a unique membership function which maps each element in the universe of discourse $X$ to the unit interval $[0,1]$.

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$$

The degree of membership function of $\tilde{A}$ is defined by $\mu_{\tilde{A}}(x) : X \to [0,1]$. In a fuzzy set $\tilde{A}$, $\mu_{\tilde{A}}(x)$ is called as the membership degree of $x \in X$.

2.2 Trapezoidal Fuzzy number (TrFN)
A TrFN is a four valued argument $A(a_1,a_2,a_3,a_4)$ and is described by the following membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\
1 & a_2 \leq x \leq a_3 \\
\frac{a_4-x}{a_4-a_3} & a_3 \leq x \leq a_4 \\
0 & \text{elsewhere}
\end{cases}$$

2.3 Parametric representation of TrFN
A fuzzy number can also be illustrated in parametric form as $(m,\alpha,\beta)$ where $m$ is the mean value and $\alpha, \beta$ are left and right spreads respectively. Hence any TrFN $\tilde{A}=(a_1,a_2,a_3,a_4)$ is represented as $\tilde{A}=(m,\alpha,\beta)$ where $m = \frac{a_2+a_3}{2}$; $\alpha = a_2-a_1$ and $\beta = a_4-a_3$.

2.4 Ranking Function for TrFN:
A function $\mathfrak{R} : F(R) \to R$ is defined as a ranking function where $F(R)$ represents the set of all TrFN. A natural order is provided by the ranking function in which each of the trapezoidal fuzzy number is mapped into a real number in $R$. Let $\tilde{A} = (m,\alpha,\beta)$, then the ranking function of $\tilde{A}$ is defined as

$$\mathfrak{R}(\tilde{A}) = m + \frac{\beta - \alpha}{4}$$
2.5 Arithmetic operation for TrFN

Let \( \tilde{A}_1 = (m, \alpha, \beta) \) and \( \tilde{A}_2 = (n, \gamma, \delta) \) be any two TrFNs. Based on the parametric representation of TrFN the arithmetic operations are given by

(i) \( \tilde{A}_1 + \tilde{A}_2 = (m + n, \alpha + \gamma, \beta + \delta) \)

(ii) \( -\tilde{A}_1 = (-m, \beta, \alpha) \)

(iii) \( \tilde{A}_1 - \tilde{A}_2 = (m - n, \alpha + \delta, \beta + \gamma) \)

3. Mathematical Expression of Fuzzy Assignment Problem

Mathematical model of fuzzy assignment problem is given by,

\[
\min Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_{ij}
\]

subject to \( \sum_{i=1}^{n} \tilde{x}_{ij} = 1 \)

\( \sum_{j=1}^{n} \tilde{x}_{ij} = 1 \)

Here \( \tilde{x}_{ij} \) denotes the assignment of worker \( i \) to job \( j \) such that

\[
\tilde{x}_{ij} = \begin{cases} 
1 & \text{if } i^\text{th} \text{ worker is assigned to } j^\text{th} \text{ job} \\
0 & \text{otherwise}
\end{cases}
\]

and \( \tilde{a}_{ij} \) is the assigning cost of \( j^\text{th} \) job \( i^\text{th} \) worker.

| Table 1 Data Matrix |
|---------------------|
| Job1 | Job 2 | Job 3 | … | Job j | Job n |
|-------|-------|-------|---|-------|-------|
| Worker 1 | \( \tilde{a}_{11} \) | \( \tilde{a}_{12} \) | \( \tilde{a}_{13} \) | … | \( \tilde{a}_{1j} \) | \( \tilde{a}_{1n} \) |
| Worker 2 | \( \tilde{a}_{21} \) | \( \tilde{a}_{22} \) | \( \tilde{a}_{23} \) | … | \( \tilde{a}_{2j} \) | \( \tilde{a}_{2n} \) |
| Worker i | \( \tilde{a}_{i1} \) | \( \tilde{a}_{i2} \) | \( \tilde{a}_{i3} \) | … | \( \tilde{a}_{ij} \) | \( \tilde{a}_{in} \) |
| Worker n | \( \tilde{a}_{n1} \) | \( \tilde{a}_{n2} \) | \( \tilde{a}_{n3} \) | … | \( \tilde{a}_{nj} \) | \( \tilde{a}_{nn} \) |

3.1 Fuzzy Branch and Bound Algorithm

The fuzzy branch and bound procedure is utilized for resolving fuzzy assignment problem. The first step in the algorithm is to determine the fuzzy upper and the lower bound of the problem solving. The fuzzy lower and upper bound are used to close up the region of solution value. When the bound value is found the value is used to determine which of the sub problem should be eliminated. The feasible solution is represented by leaves of the tree.

There are two ways for computing the fuzzy cost function:

1. For each worker, we select a job with least fuzzy cost from list of non-allocated jobs.
2. For each job, we select a worker with least fuzzy cost for that job from list of non-allocated workers.

Stepwise procedure of the fuzzy branch and bound algorithm is given below:

Step:1 Minimum fuzzy cost is added from each row and fix the fuzzy lower bound for the given problem.
Step: 2 Eliminate the fuzzy least cost for assigning first worker to n jobs and form the sub problem for worker 2.
Step: 3 Continue by eliminating the worker 2, worker 3, …… worker n by assigning the job with fuzzy minimum cost and obtain the fuzzy feasible solution.

4. Numerical example
Let us consider an example discussed by Srinivas [7]. A fuzzy assignment problem involving four jobs and four workers is given below whose cost are represented by trapezoidal fuzzy number.

| Workers | Jobs         |
|---------|-------------|
|         | 1           | 2           | 3           | 4           |
| a       | (3, 5, 6, 7)| (5, 8, 11, 12)| (9, 10, 11, 15)| (5, 8, 10, 11)|
| b       | (7, 8, 10, 11)| (3, 5, 6, 7)| (6, 8, 10, 12)| (5, 8, 9, 10)|
| c       | (2, 4, 5, 6)| (5, 7, 10, 11)| (8, 11, 13, 15)| (4, 6, 7, 10)|
| d       | (6, 8, 10, 12)| (2, 5, 6, 7)| (5, 7, 10, 11)| (2, 4, 5, 7)|

Min \((3, 5, 6, 7)x_{11} + (5, 8, 11, 12)x_{12} + (9, 10, 11, 15)x_{13} + (5, 8, 10, 11)x_{14} + (7, 8, 10, 11)x_{21} + (3, 5, 6, 7)x_{22} + (6, 8, 10, 12)x_{23} + (5, 8, 9, 10)x_{24} + (2, 4, 5, 6)x_{31} + (5, 7, 10, 11)x_{32} + (8, 11, 13, 15)x_{33} + (4, 6, 7, 10)x_{34} + (6, 8, 10, 12)x_{41} + (2, 5, 6, 7)x_{42} + (5, 7, 10, 11)x_{43} + (2, 4, 5, 7)x_{44}\)

Subject to
\(x_{11} + x_{12} + x_{13} + x_{14} = 1; x_{21} + x_{22} + x_{23} + x_{24} = 1; x_{31} + x_{32} + x_{33} + x_{34} = 1; x_{41} + x_{42} + x_{43} + x_{44} = 1\)

\(x_{11} + x_{21} + x_{31} + x_{41} = 1; x_{12} + x_{22} + x_{32} + x_{42} = 1; x_{13} + x_{23} + x_{33} + x_{43} = 1; x_{14} + x_{24} + x_{34} + x_{44} = 1\)

**Figure 1.** Fuzzy optimal assignment
The fuzzy optimal assignment is given by \(a \rightarrow 1; \ b \rightarrow 2; \ c \rightarrow 4; \ d \rightarrow 3\). The minimized fuzzy assignment cost is obtained as \((15,23,29,35)\).

Table - 3

| Method              | Fuzzy assignment cost |
|---------------------|-----------------------|
| Proposed method     | \((15,23,29,35)\)     |
| Srinivas et al method | \((19,28,35,40)\)     |

Hence we obtained an improved minimized cost.

5. Conclusion

In this paper, a fuzzy branch and bound technique is introduced to resolve fuzzy assignment problem. Without defuzzification procedure we directly solve the fuzzy assignment problem and obtained the optimal assignment cost. A comparison study is made with the existing method to show case the improvement of solution procedure.

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