Supersymmetry Breaking and Restoration in the Interval\(^a\)

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ABSTRACT

We study fermions, such as gravitinos and gauginos in supersymmetric theories, propagating in a five-dimensional bulk where the fifth dimension is an interval. We show the mass spectrum becomes independent from the Scherk-Schwarz parameter if the boundary mass terms obey a relation of alignment with the bulk supersymmetry breaking.

1. The method: Fermions in the interval

In a manifold \( M \) with a boundary the dynamics is determined by two equally important ingredients: the bulk equations of motion and the boundary conditions (BC’s). An economical way to determine a set of consistent BC’s together with the bulk equations of motion is the action principle \(^b\): under a variation of the dynamical fields the action must be stationary.

Since we are mainly interested in supersymmetric theories, we will take the fermions to be symplectic-Majorana spinors. In particular we will consider the gaugino case, the treatment of gravitinos being completely analogous. The 5D spinors \( \Psi^i \) satisfy the symplectic-Majorana reality condition and we can represent them in terms of two chiral 4D spinors according to \(^c\)

\[
\Psi^i = \left( \eta^i_{\alpha} \chi^i_{\dot{\alpha}} \right), \quad \chi^i_{\dot{\alpha}} \equiv \epsilon^{ij} \eta^j_{\beta} \epsilon^{\dot{\alpha}\dot{\beta}}.
\]

Consider thus the bulk Lagrangian

\[
\mathcal{L}_{\text{bulk}} = i \bar{\Psi} \gamma^M D_M \Psi = \frac{i}{2} \bar{\Psi} \gamma^M D_M \Psi - \frac{i}{2} D_M \bar{\Psi} \gamma^M \Psi.
\]

where the last equation is not due to partial integration but holds because of the symplectic-Majorana property, Eq. (1). The derivative is covariant with respect to the \( SU(2)_R \) automorphism symmetry and thus contains the auxiliary gauge connection \( V_M \). The field \( V_M \) is non propagating and appears in the off-shell formulation of 5D supergravity \(^d\). A vacuum expectation value (VEV) \(^d\)

\[
V_M = \delta_M^5 \frac{\omega}{R} \hat{q} \cdot \hat{\sigma}, \quad \hat{q}^2 = 1
\]

implements a Scherk-Schwarz (SS) supersymmetry breaking mechanism \(^5\) in the Hosotani basis \(^6\). The unitary vector \( \hat{q} \) points toward the direction of SS breaking. We supplement the

\(^a\)Talk based on Ref.\(^1\)

\(^b\)For an alternative approach see \(^2\).

\(^c\)We use the Wess-Bagger convention \(^3\) for the contraction of spinor indices.

\(^d\)Consistent with the bulk equation of motion \( d(\hat{q} \cdot \hat{V}) = 0 \) \(^4\).
bulk action by the following boundary terms at \( y = y_f \) (\( f = 0, \pi \)) with \( y_0 = 0 \) and \( y_\pi = \pi R \):

\[
\mathcal{L}_f = \frac{1}{2} \bar{\Psi} \left( T^{(f)} + \gamma^5 V^{(f)} \right) \Psi = \frac{1}{2} \eta^i M^{(f)}_{ij} \eta^j + \text{h.c.},
\]

(4)

The variation of the bulk action gives

\[
\delta S_{\text{bulk}} = \int d^5x i \left( \delta \bar{\Psi} \gamma^M D_M \Psi - D_M \bar{\Psi} \gamma^M \delta \Psi \right) - \int d^4x \left[ \delta \eta^i \epsilon_{ij} \eta^j + \text{h.c.} \right]_{\pi R},
\]

(5)

where the boundary piece comes from partial integration. One now has to add the variation of the boundary action. Enforcing that the total action \( S = S_{\text{bulk}} + S_{\text{boundary}} \) has zero variation we get the standard Dirac equation in the bulk provided that all the boundary pieces vanish. The latter are given by

\[
\left[ \delta \eta^i \left( \epsilon_{ij} + M^{(f)}_{ij} \right) \eta^j + \text{h.c.} \right] \bigg|_{y = y_f} = 0.
\]

(6)

Since we are considering unconstrained variations of the fields, the BC’s we obtain from Eqs. (6) are given by

\[
\left( \epsilon_{ij} + M^{(f)}_{ij} \right) \eta^j \bigg|_{y = y_f} = 0.
\]

(7)

These equations only have trivial solutions (are overconstrained) unless

\[
\det \left( \epsilon_{ij} + M^{(f)}_{ij} \right) = \det \left( \epsilon_{ij} + M^{(\pi)}_{ij} \right) = 0.
\]

(8)

Imposing these conditions, we get the two complex BC’s which are needed for a system of two first order equations. Note that this means that an arbitrary brane mass matrix does not yield viable BC’s; in particular a vanishing brane action is inconsistent.

The BC’s resulting from Eqs. (7) are of the form

\[
\left( \eta^2 - z_f \eta^1 \right) \bigg|_{y = y_f} = 0, \quad z_f = -\frac{M^{(f)}_{11}}{1 + M^{(f)}_{12}} = \frac{1 - M^{(f)}_{11}}{M^{(f)}_{22}}.
\]

(9)

The mass spectrum is found by solving the EOM with the boundary conditions (9). To simplify the bulk equations of motion it is convenient to go from the Hosotani basis \( \Psi^i \) to the SS one \( \Phi^i \), related by the transformation

\[
\Psi = U \Phi, \quad U = \exp \left( -i \vec{q} \cdot \vec{\sigma} \omega \frac{y}{R} \right) \Rightarrow i \gamma^M \partial^M \Phi = 0.
\]

(10)

We now decompose the chiral spinor \( \eta^i (x, y) \) in the Hosotani basis as \( \eta^i (x, y) = \varphi^i (y) \psi^i (x) \), with \( \psi^i (x) \) a 4D chiral spinor. As a consequence of the transformation (10) the SS parameter \( \omega \) manifests itself only in the BC at \( y = \pi R \) :

\[
\zeta_0 \equiv \frac{\partial^2 \psi^i}{\partial y^2} \bigg|_{y = 0} = z_0, \quad \zeta_\pi \equiv \frac{\partial^2 \psi^i}{\partial y^2} \bigg|_{y = \pi R} = \frac{\tan (\pi \omega) (iq_1 - q_2 - iq_3 z_\pi) + z_\pi}{\tan (\pi \omega) (iq_1 z_\pi + q_2 z_\pi + iq_3) + 1},
\]

(11)

\(^e\)In the sense that the action principle does not provide a consistent set of BC’s as boundary equations of motion.

\(^f\)Notice that this agrees with the methods recently used in Ref. [8].

\(^g\)Notice that \( U(y = 0) = 1 \). The roles of the branes and hence of \( z_\pi \) and \( z_0 \) can be interchanged by considering the SS transformation \( U'(y) \equiv U(y - \pi R) \).
where $\zeta_f$ are the BC’s in the SS basis. In particular the boundary condition $\zeta_\pi$ is a function of $\omega, \vec{q}$ and $z_\pi$. From this it follows that we can always gauge away the SS parameter $\omega$ in the bulk Lagrangian going into the SS basis through (10). However now in the new basis $\omega$ reappears in one of the BC’s.

The bulk equations have the following generic solution

$$\phi(y) = \left( \bar{a} \cos(my) + z_0 \bar{a} \sin(my) \right), \quad a = \frac{z_0 - \zeta_\pi}{|z_0|} + \frac{1 + z_0 \bar{\zeta}_\pi}{1 + z_0 |\zeta_\pi|}. \quad (12)$$

The solution (12) satisfies the BC’s Eq. (11) for the following mass eigenvalues

$$m_n = \frac{n}{R} + \frac{1}{\pi R} \arctan \left( \frac{z_0 - \zeta_\pi}{1 + z_0 |\zeta_\pi|} \right). \quad (13)$$

2. Supersymmetry breaking: independence on the Scherk-Schwarz breaking scale

Physically inequivalent BC’s span a complex projective space $\mathbb{C}P^1$ homeomorphic to the Riemann sphere. In particular, $z_f = 0$ leads to a Dirichlet BC for $\eta_2$, and the point at infinity $z_f = \infty$ leads to a Dirichlet BC for $\eta_1$. Notice that these BC’s come from $SU(2)_R$ breaking mass terms. Special values of $z_f$ correspond to cases when these terms preserve part of the symmetry of the original bulk Lagrangian. In particular when both the SS and the preserved symmetry are aligned those cases can lead to a persistent supersymmetry as we will see. Generically,

when $z_0 = \zeta_\pi$ there is a zero mode and supersymmetry remains unbroken.

i.) When the only sources of supersymmetry breaking reside on the branes, setting them to cancel each other, $z_0 = z_\pi$, preserves supersymmetry [9].

ii.) Once supersymmetry is further broken in the bulk, an obvious way to restore it is by determining $z_\pi$ as a function of $z_0$ and $\omega$ using the relation (11) with $\zeta_\pi = z_0$. This will lead to an $\omega$-dependent brane-Lagrangian at $y = \pi R$. In this case we could say that supersymmetry, that was broken by BC’s (SS twist) is restored by the given SS twist (BC’s) [10].

iii.) There is however a more interesting case: suppose the brane Lagrangian determines $z_\pi$ to be

$$z_\pi = z(\vec{q}) \equiv \frac{\lambda - q_3}{q_1 - iq_2}. \quad (14)$$

with $\lambda = \pm 1$. This special value of $z_\pi$ is a fixed point of the SS transformation, i.e. $\zeta_f = z_f$. For $z_\pi = z(\vec{q})$ the spectrum becomes independent on $\omega$. In other words, for this special subset of boundary Lagrangians, the VEV for the field $\vec{q} \cdot \vec{V}_5$ does not influence the spectrum. The reason for this can be understood by going back to the Lagrangian which we used to derive the BC’s. From the relation [9] one can see that condition (14) is satisfied by the mass matrix

$$M^{(\pi)}_{12} = \lambda q_3$$
$$M^{(\pi)}_{11} = -\lambda (q_1 + iq_2)$$
$$M^{(\pi)}_{22} = \lambda (q_1 - iq_2) \quad (15)$$

which can be translated into a mass term at the boundary $y = y_\pi$ along the direction of the SS term, i.e. $V^{(\pi)} = 0$ and $T^{(\pi)} = -\lambda \vec{q} \cdot \vec{\sigma}$ in the notation of Eq. [14]. In particular this brane
mass term preserves a residual $U(1)_R$ aligned along the SS direction $\vec{q}$. In other words, the SS-transformation $U$ leaves both brane Lagrangians invariant and $\omega$ can be gauged away. When we further impose $z_0 = z(\pm \vec{q})$, i.e. $V(0) = 0$ and $T(0) = \pm T(\pi)$ the $U(1)_R$ symmetry is preserved by the bulk. In particular if $z_0 = z(-\vec{q})$ supersymmetry remains unbroken, although the VEV of $\vec{q} \cdot \vec{V}_5$ is nonzero. One could say that in this case the theory is persistently supersymmetric even in the presence of the SS twist, with mass spectrum $m_n = n/R$. On the other hand if $z_0 = z(\vec{q})$ the theory is (persistently) non-supersymmetric and independent on the SS twist: the mass spectrum is given by $m_n = (n+1)/R$. In this case supersymmetry breaking amounts to an extra $Z_2'$ orbifolding [11].

Something similar happens in the warped case: when bulk cosmological constant and brane tensions are turned on, invariance of the action under local supersymmetry requires gravitino mass terms on the brane. In the tuned case, – i.e. in the Randall-Sundrum (RS) model – those brane mass terms precisely give rise to the BC $z_0 = z_\pi = z(\vec{q})$ [12]. Note that there $\vec{q} \cdot \vec{V}_5$ is replaced by $A_5$, the fifth component of the graviphoton. In fact, it has been shown that in this case there always exists a Killing spinor and supersymmetry remains unbroken [13,14], consistent with the result that in RS supersymmetry can not be spontaneously broken by the SS mechanism [16].

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4. References

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\[\text{h}\] A discrete supersymmetry breaking by BC’s, $z_0 = z(-\vec{q})$, $z_\pi = z(\vec{q})$, was performed in Ref. [15].