Elastic models application for thorax image registration

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Abstract. This work consist of the implementation and evaluation of elastic alignment algorithms of biomedical images, which were taken at thorax level and simulated with the 4D NCAT digital phantom. Radial Basis Functions spatial transformations (RBF), a kind of spline, which allows carrying out not only global rigid deformations but also local elastic ones were applied, using a point-matching method. The applied functions were: Thin Plate Spline (TPS), Multiquadric (MQ) Gaussian and B-Spline, which were evaluated and compared by means of calculating the Target Registration Error and similarity measures between the registered images (the squared sum of intensity differences (SSD) and correlation coefficient (CC)). In order to value the user incurred error in the point-matching and segmentation tasks, two algorithms were also designed that calculate the Fiduciary Localization Error. TPS and MQ were demonstrated to have better performance than the others. It was proved RBF represent an adequate model for approximating the thorax deformable behaviour. Validation algorithms showed the user error was not significant.

1. Introduction
During medical examination it is a common practice for patients to undergo a series of independent medical imaging studies that provide different kinds of information, not only anatomical but also physiological, about specific regions of the body. However, in many clinical and surgical applications, this information could be more useful if it were somehow integrated, which will help and provide valuable additional information to the professional in diseases detection, interpretation and prediction, as likewise in the planning and carrying out of the treatment. In order to integrate medical imaging studies it is necessary first to correlate their inner information by means of the proper association of the present structures.

This process, in which images are referenced over a same spatial coordinates system, is known as Registration. It takes into account spatial differences between corresponding anatomical structures, caused by different image acquisition procedures, changes in the patient position, or movement, either voluntary or involuntary, like cardiac contraction, respiration and digestion, which are physiological processes that induce movements. [4], [11]

In view of these problems, the advantages of using geometric transformations of the elastic type were studied with the purpose of being able to align images taken intrinsically in human body deformable regions, such as thorax or abdomen, in which the rigid body hypothesis (frequently used in brain studies) is not valid. A diversity of this type of transformations exists, among which the Radial Basis Functions were studied (RBF), that provide a no-rigid model of mapping that can be applied to registration problems in any dimension. The polynomials splines have been used in diverse
applications: facial expressions [1], fingerprint deformations [2], radiotherapy planning [10], breast images [11], among others.

2. Materials and Methods

2.1. Computational Set

The registration algorithms were carried out and proven in an intermediate technology personal computer, with AMD athlon 1GHz processor and 512 MBytes of RAM. The registration algorithms were programmed in Matlab © 7.0.1. R14 Service Pack 1 [9]. The images were simulated with the 4D NCAT 2.0 phantom. [7]

2.2. RBF Registration

An elastic registration algorithm was implemented, in search of approaching the local and global thoracic region deformation, through the use of RBF. These are continuous, differentiable and soft interpolation functions, and they require the application of a methodology based on control points (also called fiduciary or homologous points). A control point describes in an unique way the position of a characteristic point in the images to be registered. In general a space transformation in d-dimensions maps the control points group \( p_i \) of the floating image to the corresponding group of points \( q_i \) in the reference image, where \( i = 1,2,...,n \).

RBF map each one of the control points in an image to their corresponding ones in the other image and they provide an interpolation between them. A spatial RBF transformation in d dimensions for any point \( p \), called \( T(x,y) \), is made up of \( k \) map functions with \( k=1...d \), such that: \( T(p) = [f_1(p)...f_k(p)..., f_d(p)] \). To carry out the registration procedure a methodology based on intrinsic and manually matched control points was used.

\( N \) pairs of corresponding control points being given, each one of the \( k \) RBF mapping functions has the general following form:

\[
\hat{f}_k(p) = P_{mk}(p) + \sum_{i=1}^{n} A_{ik} g(r_i) \quad k = 1,...,d
\]

where the first component is an \( m \) grade polynomial that may not be present and represents the global adjustment of the transformation. The general form of a linear polynomial \( (m=1) \) makes the global component an affine transformation. The second component of the equation (1) is the weighted sum of the radial base function \( g(r_i) \) where \( r_i \) denotes the Euclidean distance among the points. In this way the radial base function consists of a radial symmetrical linear combination of a base function and a low grade polynomial.

In the bi-dimensional case the transformation of a control point \( p_i = (x,y) \) to its homologous \( q_j = (u,v) \) is determined by \( n+3 \) coefficients in each dimension. The function coefficients \( f_k(p) \) are determined by fulfilling the following interpolation conditions: \( f_1(p) = u_j, f_2(p) = v_j \) for \( j = 1...n \). What gives place to \( n \) linear equations.

The following compatibility restrictions should also be completed:

\[
\sum_{i=1}^{n} A_{ik} = \sum_{i=1}^{n} A_{ik} x_i = \sum_{i=1}^{n} A_{ik} y_i = 0
\]

(2)

They guarantee that RBF is reducible to an affine transformation when it is necessary. These conditions outline a linear system \( W = L^{-1} Y \), from which the coefficients that determines the RBF can be found.

Being \( L \) a composed matrix made up of four sub matrixes:

\[
L = \begin{bmatrix}
G & P \\
P^T & O
\end{bmatrix}
\]

(3)
Where $G$ is obtained from the application of the chosen radial base function to the Euclidean distances among $n$ control points of the floating image (the one to be transformed):

$$G = \begin{bmatrix}
g(r_{11}) & g(r_{12}) & \cdots & g(r_{1n}) \\
g(r_{21}) & g(r_{22}) & \cdots & g(r_{2n}) \\
\vdots & \vdots & \ddots & \vdots \\
g(r_{n1}) & g(r_{n2}) & \cdots & g(r_{nn})
\end{bmatrix} \tag{4}$$

Intuitively $g(r_{ij})$ measures the effect from the $j^{th}$ control point to the $i^{th}$ control point of the transformation.

In turn $P$ and its transposed ($P^T$) contain the $n$ control points’ coordinates in the floating image:

$$P^T = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
x_1 & x_2 & \cdots & x_n \\
y_1 & y_2 & \cdots & y_n
\end{bmatrix} \tag{5}$$

The $W$ matrix contains all the coefficients that are looked for; it has as many columns as dimensions of the transformation:

$$W^T = \begin{bmatrix}
A_{11} & A_{21} & A_{n1} & a_{01} & a_{11} & a_{21} \\
A_{12} & A_{22} & A_{n2} & a_{02} & a_{12} & a_{22}
\end{bmatrix} \tag{6}$$

Finally, the $Y$ matrix contains the $n$ control points coordinates in the reference image $q_i = (u, v)$ and a null submatrix. Its transposed matrix is the following:

$$Y^T = \begin{bmatrix}
u_1 & u_2 & \cdots & u_n & 0 & 0 & 0 \\
v_1 & v_2 & \cdots & v_n & 0 & 0 & 0
\end{bmatrix} \tag{7}$$

Singular Value Decomposition (SVD) method was used to obtain the inverse matrix of $L$.

Once the geometric transformation with RBF has been obtained an inverse mapping is carried out to obtain the gray levels. Nearest neighbor or bilinear interpolation were used with that purpose.

In table 1 different RBF used in this application can be observed. There can also be visualized the locality parameters and their permitted values that allow controlling the influence area of the base function. These parameters determine the visual interpolation smoothness for a given group of control points.

| Base Function | $g(r_i)$ | Parameters |
|---------------|----------|------------|
| TPS           | $r_i^2 \log r_i$ | -          |
| MQ            | $(r_i^2 + \delta)^{\mu}$ | $\delta > 0 \ y \ \mu \neq 0$ |
| Ga            | $\exp(-r_i^2 / \sigma)$ | $\sigma > 0$ |
| BS$^3$        | $2 \cdot \left(1 - \frac{r_i}{\beta}\right)^3 - \left(1 - \frac{2r_i}{\beta}\right)^3$ | $\beta \neq 0$ |

$^3$ The B-Spline + sign means that these terms are forced to zero when negative
Some of them grow as the distance of any image point to a control point, \( r \), increases (\textit{Thin Plate Spline (TPS)}, \textit{Multiquadric (MQ)}), some others, such as \textit{Gaussian (Ga)}, descend as \( r \) increases. The choice of a \textit{RBF} is determined by the dimension of the registration problem, the interpolation conditions and the wanted properties of the interpolator. Some \textit{RBF}, as \textit{TPS}, although they take into account some global behavior, they are not so sensitive to the control points’ distribution like those that contain some locality parameter (as \textit{MQ}, \textit{Ga} and \textit{B-Spline (BS)}). The base function influence range can be controlled through these parameters adjustment. [3], [4], [10], [12]

2.3. Phantoms

Digital phantoms provide a subject anatomy and/or physiology model. Given a physic model of the image acquisition process, the data acquired by a digital phantom can be generated using computational methods. An advantage is that different kind of anatomies or physiological situations could be simulated in order to value this investigations performance, without appealing to patients under critical conditions, which would be practically and ethically inappropriate. The phantoms used in this application were: -\textit{Improved 4D NCAT Phantom} which allows simulating radiation emission and attenuation studies from any part of the human body, including respiratory and cardiac movements. -\textit{Thorax Phantom}, which represents diverse principal thorax structures, usually visualized in tomographic studies [7], [8]

2.4. Validation

Registration accuracy was assessed by means of calculating among the registered images the following similarity measures: \textit{squared sum of intensity differences (SSD)} whose ideal value is zero and \textit{correlation coefficient (CC)}: \( CC \) values near to 1 mean a high level of correspondence. [4], [11], [12]

The measure of registration success is a statistical estimation of some alignment error measurement. To assess this error, a point to point manual segmentation procedure was developed, by using linear interpolation, to delimitate the structures of interest and calculate their centroids, through which the \textit{Target Registration Error (TRE)} was then evaluated. This determines the displacement (\( \text{TRE} \)) between two corresponding points of interest among the image obtained with the registration process and that of reference. As an assessment of the error made in the registration the root mean square (RMS) of these \( N \) displacements was taken.

\[
TRE_{\text{RMS}} = \left( \frac{1}{N} \sum_{i=1}^{N} \text{TRE}_i^2 \right)^{\frac{1}{2}} \quad (8)
\]

Finally, a measure of the \textit{Fiduciary Localization Error (FLE)} was carried out with the purpose of representing the error made by the user or an automatic algorithm when matching a point, since the marked point will inevitably differ of its exact value.

\[
FLE_i = \left[ (x_r - x_m)^2 + (y_r - y_m)^2 \right]^{\frac{1}{2}} \quad (9)
\]

Where the subscripts \( r \) and \( m \) represent respectively the real and matched point and \( x, y \) represent the spatial coordinates of this point. [4], [5], [6]

3. Results

3.1. Experience I: RBF Comparison

An algorithm was carried out divided in two stages. In stage I, \textit{Heart case}, it was worked on thorax images representing 2 frames of the same section level, where the deformation effects caused by the
heart movement can be observed (figure 1a and 1b). In stage II, Breathing case, images of another level were used, where the effects of the breathing movement could be observed (figure 2a and 2b). The images were generated with the NCAT 2.0 phantom.

Selected matched control points at each stage are shown in figures 3 and 4; through which the transformation is carried out.

**Figure 1:** Stage I: (a) Floating Image (the one to be transformed): Diastole End, (b) Reference Image: Systole End. Rows indicate left ventricle cavity.

**Figure 2:** Stage II: (a) Floating Image (the one to be transformed): 100% Inspiration (b) Reference Image: 50% Inspiration

**Figure 3:** Stage I: (a) Floating Image (the one to be transformed): Diastole End, (b) Reference Image: Systole End. In blue can be observed the matched control points, \(n=88\)

**Figure 4:** Stage II: (a) Floating Image (the one to be transformed): 100% Inspiration (b) Reference Image: 50% Inspiration. In blue can be observed the matched control points, \(n=61\)
In figures 5 and 6 are shown the results obtained with different transformations for both stages. In the first stage you must compare the results with the corresponding Reference Image in figure 1b. In the same way, compare the results of the second stage with the corresponding Reference Image in figure 2b.

**Figure 5: Stage I:**
Cardiac Movement: Transformed Images with: (a) TPS; (b) MQ, \( \delta = \text{mean}(r_i), \mu = 1 \); (c) Ga \( \sigma = 200 \); (d) B-Spline \( \beta = 35 \)

**Figure 6: Stage II:**
Breathing Movement: Transformed Images with: (a) TPS, (b) MQ, \( \delta = \text{media } r_i, \mu = 1 \) (c) Ga \( \sigma = 300 \) (d) B-Spline \( \beta = 35 \)

In Table 2 there is a comparison of the different splines used. Similarity measures and the errors calculated in this experience can be observed.
3.2. Experience II: Error Measurements

An evaluation algorithm was designed in two stages: the first one evaluates the error made when matching the control points, the second evaluates the error made in the segmentation stage. 10 volunteers participated who should repeat each stage several times so that the data were statistically significant. An affine transformation was used to deform the original image, having the following parameters: rotation. = 20º clockwise, translation, tx = 10 pixels, ty = 10 pixels, scaling sx = 1,5 sy = 1,0 and shear c = tg (10º).

Stage I: Points Matching. In this stage a simple image generated by computer was used, the same can be observed in the superior part of figure 7. Each user should mark in alternative way 9 couples of points and repeat this operation 10 times. Once obtained all the data were processed through an especially designed statistic algorithm.

Stage II: Segmentation. In this stage an image was used generated by the Thorax phantom, which is visualized in the inferior part of figure 7. The user should segment in both images the present structures and repeat this operation in the same order 5 times.

| Measures | TPS | MQ | Ga | BS |
|----------|-----|----|----|----|
| **Case: Cardiac Movement** | | | | |
| SSD      | 133,4344 | 113,3905 | 151,2025 | 152,4145 |
| CC       | 0,9730  | 0,9769 | 0,9693 | 0,9690 |
| TRE mean | 0,9471  | 0,6018 | 0,6427 | 0,6007 |
| TRE RMS  | 1,0208  | 0,6726 | 0,6703 | 0,7084 |
| **Case: Breathing Movement** | | | | |
| SSD      | 207,1381 | 195,0696 | 204,9189 | 206,3778 |
| CC       | 0,9575  | 0,9602 | 0,9582 | 0,9579 |
| TRE mean | 1,2745  | 0,9341 | 1,5492 | 1,0815 |
| TRE RMS  | 1,3267  | 1,0498 | 1,7413 | 1,3807 |

Figure 7: Points Matching: (a) Original Image, (b) Deformed Image with Affine Transformation (in red: matched points). Segmentation: (c) Original Image, (d) Deformed Image with Affine (the cross indicates the cursor position, where the user is segmenting) Transformation
In table 3 mean FLE can be appreciated in the 5 repetitions as much in the reference image as in the deformed one for each user, in both stages. The last line indicates the RMS value of FLE for the 10 users.

**Table 3: User Error Assessment**

| User | μFLE R² | μFLE D² | μFLE R² | μFLE D² |
|------|---------|---------|---------|---------|
| 1    | 0.7585  | 1.3700  | 0.7334  | 1.3520  |
| 2    | 0.6631  | 2.1664  | 0.5156  | 1.3277  |
| 3    | 1.2302  | 1.7451  | 0.5694  | 1.1244  |
| 4    | 1.7882  | 3.5475  | 0.6683  | 1.3951  |
| 5    | 0.6759  | 1.3306  | 0.6776  | 1.1109  |
| 6    | 0.7283  | 1.0930  | 0.4754  | 1.3645  |
| 7    | 0.6750  | 1.2242  | 0.3673  | 1.0576  |
| 8    | 1.2132  | 2.6788  | 0.7968  | 1.4331  |
| 9    | 0.5321  | 1.7664  | 0.5644  | 1.4994  |
| 10   | 0.9874  | 0.9505  | 0.6807  | 1.6968  |
| RMS  | 1.4071  | 2.7519  | 0.8731  | 1.9077  |

μ: mean; R: Reference Image; D: Deformed Image with an Affine Transformation

4. Conclusions

In the first experience it could be proven that all the registrations improve the original situation. All the measures indicate that the results were better in the Heart case than in the Breathing one; however, the last was a more complex case.

As regard to the comparison among RBF, a visual analysis indicates that as much TPS as MQ give very good adjustments and they are very similar between each other. Gaussian and B-Spline transformations give acceptable results but clearly of smaller quality than the other ones. On the other hand, similarity measures and TRE values indicate that indeed the function MQ is the best in general terms.

With the results of the second experience it can be stood out that little variability was observed, which indicates that the study is reproducible. Comparing the image data with the deformed one it was demonstrated that the worst visual conditions the biggest error the user makes at points matching and segmentation stages.

In general, a good accuracy can be appreciated in the application with errors on the order of the pixel; therefore the error made by user's intervention can be rejected in this application.

5. Discussion

The advantage of using RBF transformations over rigid ones is that certain located base functions allow strong deformations. On the other hand, the effectiveness of RBF depends on the election of the base function chosen \( g(r) \). Several parameters can be chosen in the application of RBF to control the localization and deformations strength. Besides, control points selection, as much in quantity as in distribution, as well as locality parameters selection, play an important role in the obtained results.

Additionally, it must be kept in mind that the present deformations in the images to be registered, the application type and of course, the computational allowed time, since the last increases as the number of control points grows.

Some advantages of this model can be pointed out: control points can be arbitrarily chosen, in a spaced and irregular form, the transformations incorporate as much rigid global deformations as non-rigid located ones, and they also have an easily controllable behavior.
6. Future Works
The implemented algorithms require user's intervention, what limits their application, since it depends on its qualification and dexterity. As far as processing time, user's dependent tasks are time consuming, at least 5 minutes in the homologous points matching task, in comparison with the computational processing times, which fall in the order of seconds or some few minutes, depending on the number of control points and on the grey levels interpolation chosen.

As improvements, it is suggested: to avoid user's intervention, first the use of points detection automatic algorithms (using fiduciary extrinsic markers) and the use of automatic algorithms of segmentation. Another improvement that can be carried out is the implementation expansion to images of more dimensions, since any tomographic study is of 3 dimensions.

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