On the interpretation of the multicolour disc model for black hole candidates

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ABSTRACT
We present a critical analysis of the usual interpretation of the \textit{multicolour disc} model parameters for black hole candidates in terms of the inner radius and temperature of the accretion disc. Using a self-consistent model for the radiative transfer and the vertical temperature structure in a Shakura-Sunyaev disc, we simulate the observed disc spectra, taking into account doppler blurring and gravitational redshift, and fit them with multicolour models. We show not only that such a model systematically underestimates the value of the inner disc radius, but that when the accretion rate and/or the energy dissipated in the corona are allowed to change the inner edge of the disc, as inferred from the multicolour model, appears to move even when it is in fact fixed at the innermost stable orbit.

Key words: accretion, accretion discs - black hole physics - radiative transfer - X-rays: stars

1 INTRODUCTION
X-ray spectra of galactic black hole candidates (GBHC) are becoming an increasingly important tool in determining both the physical properties of the source surroundings and, as the ultimate goal, the parameters (mass and specific angular momentum) of the black hole. Two main spectral states, as defined by their spectral components and flux level (typically in the $1 - 10$ keV band), have been observed in GBHC. In the \textit{Hard/Low state}, the sources emit most of their power in a hard power law tail with photon index $\Gamma \sim 1.3 - 1.7$ and exponential cutoff at about 100 keV. In the generally more luminous \textit{Soft/High state}, most of the energy comes from a thermal component with characteristic temperature $kT \lesssim 1$ keV, while a power law component (with $\Gamma \sim 2.0 - 2.5$) dominates above a few keV. In addition, sources have been reported to be in a \textit{Very high state}, spectrally intermediate between the Soft/High and the Hard/Low states, when the power law component has a flux comparable to the thermal one and no sign of a cutoff is seen in the high energy tail.

We will focus our discussion on the Soft/High state. According to the common paradigm in interpreting the observed spectra, the ultra-soft thermal component is the product of the emission from an optically thick accretion disk. On the other hand, the hard power law component is probably due to multiple Compton scattering of soft photons by a population of hot electrons which reside in an active (or flaring) coronal region surrounding the accretion disc.

In the zeroth order approximation for describing emission from the accretion disc, not only is the disc assumed to be geometrically thin and optically thick (Shakura & Sunyaev 1973), but also the vertical temperature structure is neglected, as is the effect of Comptonization on the emergent spectrum. Each point of the disc is then assumed to radiate like a blackbody at an effective temperature which scales with the radius as $r^{-3/4}$. This is the so-called \textit{multicolour disc model} (MCD; Mitsuda et al. 1984). Such a simple model has the advantage of being easy to use in trying to fit spectral data. It has only two adjustable parameters: a \textit{(colour)} temperature, independent on the source distance, and a normalization factor, which in turn depends on the inner radius of the disc, on the distance of the source and on the inclination of the disc from the line of sight. When these two latter quantities are independently known, or can be reasonably inferred, fitting the ultra-soft component of a GBHC spectrum with the MCD model immediately gives estimates of the temperature in the inner region of the disc and its inner radius.

The aim of this letter is to demonstrate that the values one can obtain with this fitting procedure, which is now a standard one, are not in general directly related (or at least not in a reliable way) to the actual disc parameters. We will use a self-consistent model for the radiative transfer in Shakura-Sunyaev accretion discs about a compact source, including first-order corrections for gravitational redshift and Doppler blurring, as developed by Ross & Fabian (1996). The observable spectrum is computed for a number of different physical situations, each determined by the values of the viscosity parameter $\alpha$, the accretion rate and
the fraction of the accreted power dissipated in the corona (Svensson & Zdziarski 1994). In all the cases we consider, the value of the inner radius of the disc will be kept fixed. Nevertheless, when trying to fit the computed spectra with the MCD model, we will obtain results at variance with this assumption. That is because changes in the normalization factor of the MCD model are produced, in a very complicated way, by the variations in the disc structure (and hence in the Comptonized spectrum) induced by variability of the accretion rate and coronal activity.

2 STRUCTURE OF THE DISC

We assume the basic structure for spatially thin accretion discs around Schwarzschild black holes given by the standard theory of Shakura & Sunyaev (1973). The disc is assumed to have a fixed inner boundary at the innermost stable orbit at radius $R_{\text{in}} = 3R_S = 6GM/c^2$. We consider both an inner, radiation pressure dominated region and an outer, gas pressure dominated region of the disc. In determining the structure of the disc, the opacity is assumed to be dominated by Thomson scattering,

$$\kappa \approx \kappa_T = 0.2(1 + X) \text{cm}^2 \text{g}^{-1},$$

where $X$ is the mass fraction of hydrogen. Following Svensson & Zdziarski (1994), we slightly modify the standard set of disc equations, allowing a fraction $f$ of the disc accretion power to be dissipated in the corona instead of in the cold disc itself. Furthermore, at every given radius $R$, the density is taken to be uniform in the vertical direction (which is the correct solution for the disc structure when radiation pressure dominates).

Choosing dimensionless parameters

$$m = \frac{M}{M_\odot}, \quad \dot{m} = \frac{\dot{M}}{M_{\text{Edd}}}, \quad r = \frac{R}{R_S},$$

where $\dot{M}_{\text{Edd}} = 3.1 \times 10^{-8}M_\odot \text{yr}^{-1}$ is the Eddington accretion rate for a disk efficiency $\eta = 0.083$, the flux emerging from the surface of the disc is given by

$$F_0 = 1.2 \times 10^{27}m^{-1}r^{-3}[\dot{m}\bar{J}(r)](1 - f),$$

with $\bar{J}(r) = (1 - \sqrt{3/r})$. The radial structure of the disc is summarized by the values of the half-thickness of the disc (in units of the Schwarzschild radius) $h$ and the uniform gas density $\rho_0$:

(i) Radiation pressure dominated region

$$h = \frac{H}{R_S} = 5.3(1 + X)[\dot{m}\bar{J}(r)](1 - f)$$

$$\rho_0 = 3.4 \times 10^{-7}(1 + X)^{1/10}\alpha^{-1/10}m^{-1/10}r^{3/2}$$

$$\times[\dot{m}\bar{J}(r)]^{-2}(1 - f)^{-3/2} \text{g cm}^{-3},$$

(ii) Gas pressure dominated region

$$h = 3.4 \times 10^{-2}(1 + X)^{1/10}\alpha^{-1/10}m^{-1/10}r^{21/20}$$

$$\times[\dot{m}\bar{J}(r)]^{1/3}(1 - f)^{1/3}$$

$$\rho_0 = 4(1 + X)^{-3/10}\alpha^{-7/10}m^{-7/10}r^{-33/20}$$

$$\times[\dot{m}\bar{J}(r)]^{-2/3}(1 - f)^{-3/10} \text{g cm}^{-3}.$$ 

The radiative transfer and the vertical temperature structure are treated self-consistently at each radius, as described in Ross & Fabian (1996), using the Fokker-Planck/diffusion equation of Ross, Weaver & McCray (1978) in plane-parallel geometry. The local temperature profile, $T(z)$, is found by balancing the heating rate due to dynamic heating, Compton scattering and free-free absorption with the cooling rate due to inverse Compton scattering and free-free emission.

Incoherent Compton scattering within the accretion disc must be treated properly whenever it has an important effect on the emergent spectrum. The competition between Compton scattering of higher-energy photons and inverse Compton scattering of lower energy photons often results in the emergence of a Wien-law hard tail. The effective optical depth for absorption is given by

$$\tau_c(\nu) = \sqrt{3\tau_\nu(\nu_T + \tau_\nu(\nu))},$$

where $\tau_\nu$ is the Thomson depth below the surface, and $\tau_\nu(\nu)$ is the optical depth due to free-free absorption. For high energy photons the thermalization depth (where $\tau_c(\nu) = 1$) can be reached at very high Thomson depths, where the temperature is considerably higher than near the surface. Compton scattering in the outer layers of the disc downscatters these photons to lower energies and produces a Wien-law tail.

Decreasing the flux emerging from the disc itself, either by lowering the accretion rate ($\dot{m}$) or increasing the fraction ($f$) of the accretion power dissipated in the corona, results in even larger regions of very high density, both in the outermost portions of the disc and, for very low fluxes, in the very innermost portions as well. In those cases Comptonization is not complete or 'saturated' because

$$y = \frac{4kT}{m_e c^2}r_T^2 < 1$$

at the thermalization depth. In these regions we treat Compton scattering as coherent, dropping the Fokker-Planck term in the radiative transfer equation (see discussion in Ross & Fabian 1996).

In our calculations we fix the value of the black hole mass $m = 10$ and assume a composition $X = 0.71$. Following Ross & Fabian (1996) we divide the region $3 < r < 200$ of the disc into 20 annuli that make comparable contributions to the total luminosity. For each set of the parameters $\alpha$, $\dot{m}$ and $f$, we find the radius at which radiation pressure equals gas pressure, which is the root of the equation (Shakura & Sunyaev 1973)

$$J(r_{AB})^{16/21} \approx 370(m\alpha)^2/21\dot{m}^{16/21}(1 - f)^{6/7}.$$ 

We use equations (3) and (4) for all the annuli for which $3 < r < r_{AB}$, and equations (5) and (6) for $r > r_{AB}$. The typical emergent spectrum from each annulus is calculated and then multiplied by the area of the annulus to find the contribution to the spectral luminosity. The resulting spectra are added together, taking into account blurring due to gravitational redshift and transverse Doppler effect using the method described by Chen, Halpern & Filippenko (1983). Finally, in order to allow direct comparison with observations in which a dramatic change in the inner radius has been reported (GRS 1915+105, see e.g. Belloni et al. (1997c)), we choose an inclination angle $i = 70^\circ$ and a distance $D = 12.5 \text{kpc}$. 

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3 THE MULTICOLOUR DISC MODEL

For each set of parameters the emergent disc spectrum is used as a table model to create a fake set of data (with RXTE response matrix p012_{LR1}p070804.rsp, integration time 1000 s). These data are then fitted with the standard multicolour blackbody model (DISKBB in XSPEC) in the 2–20 keV band. The model assumes that the local emission from the disc is Planckian, with a temperature profile \( T(r) \propto r^{-3/4} \). Therefore the observed flux from the disc is approximated by

\[
F_d(E) = \frac{8\pi m_i}{3D^2} \int^{R_{\text{out}}}_{R_{\text{in}}} \left( \frac{T}{T_{\text{col}}} \right)^{-11/3} B(E, T) \frac{dT}{T_{\text{col}}},
\]

where \( B(E, T) \) is the Planck function and \( T_{\text{out}} \) is the temperature at the outer radius of the disc (and it is assumed that \( T_{\text{out}} < T_{\text{col}} \)). The two fit parameters of the model are the colour temperature of the inner accretion disc (\( T_{\text{col}} \)) in keV and the normalization factor

\[
n = \frac{R_{\text{col}}^2 \cos i}{D^2},
\]

where \( R_{\text{col}} \) is expressed in km and the distance \( D \) in units of 10 kpc.

The parameter \( R_{\text{col}} \) is not the effective inner disc radius (i.e. the radius at which the temperature of the disc is the highest). The assumption behind using the MCD model is that the Comptonized emergent spectrum can be approximated by a diluted blackbody spectrum with a temperature \( (T_{\text{col}}) \) higher than the effective temperature \( (T_{\text{eff}}) \). The ratio \( f_{\text{col}} = T_{\text{col}}/T_{\text{eff}} \), the spectral hardening factor (Shimura & Takahara 1995), is then assumed to be constant throughout the disc and for varying physical parameters (\( \dot{m} \), \( \alpha \), \( f \), etc.). Thus the actual inner edge of the disc would be given by

\[
R_{\text{in}} = \eta R_{\text{eff}} = \eta g(i) R_{\text{col}} (T_{\text{col}}/T_{\text{eff}})^2,
\]

where \( \eta \) is the ratio of the inner radius of the disc to the radius at which the emissivity actually peaks, typically \( \eta \approx 0.6 \) – 0.7 (Shakura & Sunyaev 1973; Shimura & Takahara 1995). The factor \((T_{\text{col}}/T_{\text{eff}})^2\) comes from the assumed dilution of the blackbody spectrum, while \( g(i) \) takes into account general relativistic corrections and is of the order \( g \approx 0.7 \) – 0.8 (Ebisawa et al. 1994).

In the next section we will show that these assumptions are not justified in general due to the differences in Compton processes induced by changes in \( \dot{m} \), \( f \), and \( \alpha \).

4 RESULTS

We have simulated disc spectra in eleven different physical situations represented by the ten sets of parameters S1–S11 (see Table 1). The sets are listed in order of decreasing value of the radius \( \dot{m} \), the boundary between the radiation pressure and gas pressure dominated regions. Table 1 also lists the value of the radial coordinate \( r_{\text{coh}} \) for the boundary of the region(s) where Compton scattering has been treated as coherent (see section 5). As the power dissipated in the disc decreases and the density increases, from the combined effects of changes in \( \dot{m} \) and \( f \), this boundary moves inwards. That is, for an increasing portion of the disc Comptonization is incomplete and the local spectrum will be somewhat harder. For the S10 model, the density in the innermost region of the disc is so high (due to the \( J(r)^{-4} \) term) that we have to consider incomplete Compton scattering also in the first inner annuli, where a significant fraction of the X-ray power is emitted. Consequently, for this set of parameters, two values of \( r_{\text{coh}} \) are listed. Finally, for the model S11, when the disc is entirely gas pressure dominated, we considered incomplete Compton scattering throughout all the disc (but see next section for a caveat on the applicability of our model to this extreme case).

Table 1 lists the values of the fit parameters \( T_{\text{col}} \) and \( R_{\text{col}} \) for the ten cases, as well as the corresponding reduced \( \chi^2 \) of the fits and the spectral hardening factors obtained from eq. (11) by assuming \( \eta g(70°) = 0.5 \) and \( R_{\text{in}} = 88.6 \) km, as appropriate for the \( 10 M_\odot \) Schwarzschild black hole we are considering. The results are also shown in Figures 2a and 2b, where we plot the spectral hardening factor and the colour radius as functions of \( \dot{m}(1-f) \), which is a measure of the flux emergent from the disc itself.

To check the sensitivity of our results to the energy range fitted, we also fitted the spectra in the 3–10 keV range. For comparison, Fig. 1 also shows the blurred spectra that would be observed if each point on the surface of the accretion disc emitted a blackbody spectrum at the local effective temperature. The observed spectra are harder than such effective blackbody spectra, and the discrepancy increases as we increase the density of the disc and decrease the power released in the disc itself (going from S1 to S10).
Figure 1. The disc spectra (solid lines) computed for the parameter sets S1 ($\dot{m} = 0.3$, $f = 0$, $\alpha = 0.1$); S4 ($\dot{m} = 0.2$, $f = 0.5$, $\alpha = 0.3$); S7 ($\dot{m} = 0.1$, $f = 0.4$, $\alpha = 0.1$) and S10 ($\dot{m} = 0.05$, $f = 0.5$, $\alpha = 0.1$). For each of the plots also shown are the spectra that would be observed for a blackbody-emitting disc (dashed lines).

band. The values obtained for the colour radii were, at most, just one or two kilometers smaller than those obtained in the 2-20 keV band (i.e. a change of about 3 – 4%).

It is clear that the multicolour model gives systematically lower values for the “inner radius” than the actual ones, a point which is well known. More interestingly, our results show that the hardening factor is by no means constant when we change the physical parameters of the disc. For example, assuming the conventional value $f_{\text{col}} = 1.7 \pm 0.2$ (Shimura & Takahara 1995; Sobczak et al. 1999) and taking Doppler blurring and gravitational redshift into account, we would have underestimated the value for the inner disc radius by up to a factor of 2 (S11) using the fits with a multicolour model. On the other hand the model gives quite stable and acceptable results for high accretion rates and/or lower values of the fraction of the power eventually dissipated in the corona. This in turn can help to understand the cases in which a nearly constant value of $R_{\text{col}}$ is observed even when $T_{\text{col}}$ varies (Tanaka & Lewin 1995).

5 DISCUSSION

We have shown that MCD models systematically underestimate the value of the inner radius of the accretion disc for a black hole candidate. We also have shown that the spectral hardening factor $f_{\text{col}}$, which is needed to correct the results of the fits with a multicolour model, is not, as is usually assumed, constant when the accretion rate and/or fractional coronal activity change.

Recent observations of galactic black hole candidates seem to point towards an extreme dependence of the observed colour radius on $f$.

In the work of Sobczak et al. (1999), where the authors report on the RXTE spectral observations of the 1996-97 outbursts of the microquasar GRO J1655-40, a sudden jump inward of the color radius (see their Fig. 7) occur for the five very high state observations of the source during which the power-law flux was extremely high and the blackbody–to–total flux ratio was less than 0.5 (see their Fig. 5).

A similar trend has been reported by Muno et al. (1999) in the case of the microquasar GRS 1915+105. In their observations, the smallest values of the inner disc radius obtained by fitting the spectrum with the MCD model are associated with the highest values of the power-law–to--
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Figure 2. (a): the points are the values of $R_{\text{col}}$ (in km) plotted as functions of $\dot{n}(1-f)$, which is a measure of the flux emergent from the disc, while the solid line marks the actual inner boundary of the disc, fixed in all our ten simulations. (b): triangles represent the value of the spectral hardening factor needed to rescale the computed $R_{\text{col}}$ to $R_{\text{in}}$.

blackbody flux ratio, which in turn should be directly related to the value of $f$.

In both these cases the inferred inner disc radii can change by more than a factor of four.

Belloni et al. (1997a,1997b), analyzing RXTE data of GRS 1915+105, interpreted the sudden change measured in the inner colour radius of roughly a factor of four in terms of the rapid disappearance of the whole inner part of the geometrically thin accretion disc. Once again, in that case the inner edge of the disc appears to shrink to its smallest values during an outburst in which the power law flux dominates the blackbody, and the blackbody–to–total flux ratio is less than 0.5.

This behaviour is exactly what our results predict we should expect from the MCD model when we increase $f$ and/or reduce the accretion rate. In this case, every time we the observations imply that the coronal activity is dominant, the multicolour fits should be corrected with a varying hardening factor $1.7 < f_{\text{col}} < 3$ in order to recover the actual value of the inner disc radius.

It should be clear, however, that the issue of determining the exact shape of accretion disc spectra is very complicated and is very sensitive to the actual physical models used to describe it. In particular, key elements are the vertical structure of the disc, its density profile and, in particular, the surface density.

It is quite possible, and perhaps easier to conceive, that dramatic changes in the disc spectrum are produced by changes in the surface properties of the accretion disc rather than by the disappearance of the entire inner regions. This, unfortunately, is hard to model theoretically. It would be interesting, for example, to assess the problem in the extreme case when almost all the accretion power is dissipated in the corona (i.e. in the limit $f \rightarrow 1$). As we believe that even in our most extreme simulation (S11) with $f = 0.8$ the simple treatment of the vertical disc structure we used here can be inadequate, and as this must be true a fortiori for larger values of $f$, we do not push our study further.

To do that we would need to take into account the X-ray reflection spectra of discs illuminated by a hot corona and to model carefully the heating of the disc atmosphere (e.g., see Sincell & Krolik [1997], Ross & Fabian [1993], who show how a hard tail in the disc spectrum is produced in this case). It is natural to believe, given the trend we observe in the simulations presented above and the observational results we referred to, that in such a situation the oversimplified multicolour model will give even smaller values of the colour radius.

A detailed treatment of this kind requires further numerical work which is beyond the aim of this Letter.

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