On reducing lift and drag forces of a stationary circular cylinder using side-by-side pair of inward rotating cylinders

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Abstract. Effect of side-by-side pair of inwardly rotating circular cylinders on the nature of drag and lift forces of the stationary circular cylinder is numerically investigated for Reynolds number $Re=100$. Non-dimensional rotation rates of cylinders in consideration are $\alpha=0, 1, 3, \text{ and } 5$. Center to center gap spacing between rotating cylinders is taken as 6D (D is cylinder diameter), and the stationary cylinder is centrally placed in the gap. For the case of two side-by-side inward rotating cylinders, above a certain critical rotation rate, Chan et al. \cite{1} reported the formation of a virtually elliptical body accompanied by vortex suppression. Drag on both the cylinders became almost zero after the formation of a virtually elliptical body. The present study aims to use the same concept of a virtually elliptical body to reduce the drag and lift forces on a stationary circular cylinder. At $\alpha=3$, complete suppression of vorticity is observed, followed by the formation of a virtual elliptical body at $\alpha=5$. With an increase in $\alpha$ from 0 onwards, the drag coefficient for the stationary cylinder reduces and becomes zero at $\alpha=5$.

1. Introduction

Investigation of flow past rotating cylinders is of great significance from the point of view of wake dynamics and applications in flow control. Rotation of the cylinder substantially modifies the wake’s behaviour compared to the stationary case due to the alteration in the boundary layers (before separation) and the detached shear layers. Numerous numerical and experimental studies have been carried out to document the flow behaviour for single and multiple rotating cylinders. Mittal and Kumar \cite{2} numerically investigated the flow past rotating cylinder for low Reynolds number equal to 200. In their finite element based simulation, they considered the range of rotation rate $\alpha$ from 0 to 5. $\alpha$ is the non-dimensional rotational rate and is defined as the ratio of the surface velocity of the cylinder to the free-stream velocity, $U_\infty$. For $\alpha < 1.9$, von Karman type vortex shedding was observed behind the cylinder. As the $\alpha$ was increased from 0, the wake became narrower, and the frequency of vortex shedding reduced. Finally, at $\alpha = 1.91$, vortex shedding was completely suppressed. Vortex shedding suppression prevailed for the range of $1.91 < \alpha < 4.34$, beyond which vortex shedding again resumed for $4.34 < \alpha < 4.7$. In this second range of instability, interestingly, only counter-clockwise vortices were observed compared to von Karman vortex street in the first range of $\alpha < 1.91$. Pralits et al. \cite{3} analyzed the mechanism for first and second shedding modes for the single rotating cylinder at $Re =100$. They attributed suppression of vortex shedding around $\alpha = 1.9$ to the shear layer weakening for flow in the wake. The core of instability for the second shedding mode (for the range of $4.34 < \alpha < 4.7$) was observed in the advection of vorticity of flow from the lower portion to the stagnation point,
where vorticity will gather and then shedding happens. Kumar et al. [4] studied experimentally the flow past a single rotating cylinder at $Re = 200, 300,$ and 400 in the range of $0 < \alpha < 5$. Their results agreed well with the numerical analysis of Mittal and Kumar [2], giving the first experimental evidence of second shedding mode with single signed vortices. The Strouhal number measurements in the second shedding mode showed a high dependence of vortex shedding frequency on rotation rates $\alpha$ compared to the Reynolds number. Munir et al. [5] carried out a three-dimensional numerical analysis below $Re = 500$ and defined different flow regimes for flow around a rotating cylinder. They observed that flow above $Re = 250$ is inherently three-dimensional, and hence vortex suppression cannot be seen above $Re = 250$. In three-dimensional flow, finger-shaped (elongated) and ring-shaped vortices were reported.

Many researchers have studied the side-by-side arrangement of rotating cylinders recently to check whether vortex suppression exists for such configuration or not. Yoon et al. [6] numerically studied flow past side-by-side rotating cylinders at $Re = 100$ for $0 < \alpha < 2$ and gap spacing of 0.2D to 3D. For inward rotation case (i.e., top cylinder rotating clockwise and bottom one rotating counter-clockwise), they found that vortex suppression occurs for all gap spacing. They also mentioned a decrease in the mean drag coefficient and an increase in the mean lift coefficient as the rotation rate $\alpha$ is increased. Chan et al. [1] extended the work of Yoon et al. [6] by considering the effect of outward rotation (i.e., top cylinder rotating counter-clockwise and bottom one rotating clockwise) and increasing the range of rotation to $0 < \alpha < 5$ for $Re = 100, 150$ and 200. For inward rotation, $\alpha_{crit}$ (critical value of $\alpha$) where vortex shedding gets completely suppressed, monotonically increased with an increase in gap spacing. The formation of a virtually elliptical body was reported beyond a certain critical rotation rate, wherein both cylinders get enveloped into elliptical shape fluid layers, and drag on both cylinders tend to zero. In-phase (i.e., asymmetric) vortex shedding mode was observed for inward rotation case before vortex suppression, whereas for outward rotation, anti-phase (i.e., symmetric) vortex shedding was observed. Kumar et al. [7] carried out experimental investigation for $Re = 100, 200, 300, 400$ and 500 in the range $0 < \alpha < 5$ for three different gap spacing. They found good agreement with the results of Yoon et al. [6] and Chan et al. [1]. Vortex shedding suppression was quite clearly visible for the inward rotation case for all the cases considered, but for outward rotation of cylinders, vortex shedding was clearly evident only for the case of $Re = 100$. Supradeepan and Roy [8] prepared a flow regime map for the side-by-side rotating cylinders for $Re = 100, \alpha = 0.5, 1.0, 1.5,$ and non-dimensional spacing of 1.1D to 3.5D. They defined eight different flow regimes based on vorticity contours, $\lambda 2$ criterion, and force coefficients for the considered range of parameters. Dou et al. [9] investigated the effect of the rate of cylinder rotation and Reynolds number ($Re$) on the stability regions of the flow. For stability analysis, they used the energy gradient theory by which energy gradient function $K$ is defined. Through their work, they concluded that $K$ is dominant in the separated shear layer region past two cylinders and hence is prone to instability. Schulmeister et al. [10] showed that drag on a stationary cylinder could be controlled using small rotating cylinders placed downstream to the cylinder surface.

Thus, we conclude that a side-by-side pair of rotating cylinders leads to the formation of a virtually elliptic body beyond a certain critical rotational speed. Whether such configuration can reduce the forces over a stationary body is the motivation behind the present study. The present study aims at finding the effect of the rotational rate of pair of cylinders on the force coefficients and nature of the wake of the single stationary cylinder.

2. Numerical details and methodology

The computational domain for the considered problem is as shown in figure 1. A stationary cylinder (C2) is centrally placed between the inward rotating pair of cylinders (C1 rotating clockwise and C3 rotating anticlockwise). All the cylinders are of identical diameter equal to D. Other dimensions of the computational domain are defined in terms of D and are shown in figure 1.
Reynolds number is set to $Re = 100$ for all considered cases. No-slip boundary condition is used on the surface of stationary cylinder C2, and rotational velocity of rotating cylinders C1 and C3 is defined in terms of $\alpha$ ($\alpha$ is the ratio of cylinder surface velocity to the free stream velocity, $U_\infty$). $\alpha$ is given values equal to 0, 1, 3, and 5. A uniform velocity $U_\infty$ is prescribed at the inlet. Free slip boundary conditions have been applied on the lateral walls of the computational domain. Zero velocity gradient and zero pressure boundary conditions are used at the outlet. A time step of 0.001 seconds is considered for all cases. The total number of mesh elements in the domain are 185K, with 200 nodes along the circumference of each cylinder and near-wall spacing of 0.001. Domain dimensions and mesh elements are selected based on the available literature.

Numerical simulation is performed by solving the unsteady incompressible Navier-Stokes equations. Finite volume-based open-source CFD tool kit OpenFOAM is used to solve the above governing equations. The PIMPLE algorithm, which is a combination of the SIMPLE (semi-implicit method for pressure-linked equations) algorithm and the PISO (pressure-implicit split-operator) algorithm, is incorporated to solve for velocity and pressure. Convection and diffusion terms are discretized using the second-order central differencing scheme.

3. Validation
For validation purpose, simulations were performed for two cases; the first case was a single rotating cylinder and the second case was a pair of inward rotating cylinders.

| $\alpha$ | $C_{l,\text{mean}}$ (Present) | $C_{l,\text{mean}}$ (Results from Mittal et al. [2]) |
|---------|-------------------------------|---------------------------------------------------|
| 0       | 0                             | 0                                                 |
| 1       | -2.55                         | -2.5                                             |
| 3       | -10.19                        | -10.08                                           |
| 5       | -27.02                        | -26.90                                           |

For a single rotating cylinder case, the value of the mean lift force coefficient $C_{l,\text{mean}}$ was compared for $Re = 200$ with the results given by Mittal et al. [2] for varying non-dimensional rotation rate, $\alpha$ (see table 1). Excellent agreement was found quantitatively as well as qualitatively.
Figure 2. Comparison of $C_{D,mean}$ and $C_{L,mean}$ of the top cylinder with the results by Chan et al. [1] for the case of inward rotating cylinders.

For the case of two inward rotating cylinders, mean lift and drag force coefficients ($C_{D,mean}$ and $C_{L,mean}$) of the top cylinder were compared at $Re=100$ and non-dimensional gap spacing of 1 for different values of $\alpha$ (see figure 2). Good agreement was found with the results by Chan et al. [1].

4. Results and discussion

Figure 3 shows the instantaneous streamlines and vorticity contours for $\alpha = 0, 1, 3, 5$ at $Re = 100$ and gap spacing of 3D between two consecutive cylinders. For $\alpha = 0$, all 3 cylinders are stationary.

The interaction of vortices is clearly evident in figure 3a. Due to asymmetry in the upstream velocity profile for the top (C1) and bottom cylinder (C3), vortex shedding starts prior to the middle cylinder (C2). Once the vortex shedding starts behind the C2, the merging of vortices of the same sign is observed. Merging follows periodic behaviour. First, the identical signed vortices of three cylinders merge, later merging happens for C2 and C3, followed by combining of vortices from three cylinders again. Lastly, the same signed vortices of C1 and C2 interact. While the vortices from any two
cylinders interact, the vortex shedding behind the third cylinder follows the von Karman type of pattern. This periodic behaviour of merging of the same signed vortices modulates the sinusoidal time series of force coefficients of C2, showing the presence of secondary frequency as shown in figure 4a.

Vortex shedding for $\alpha=1$ follows the same pattern (figure 3b) with a slight increase in the secondary frequency accompanied by a decrease in the amplitude of lift forces. Reduction in the lift forces is due to the suppression of vorticity resulted because of cylinder rotation.

![Figure 4. Time series of lift coefficients of middle cylinder C2. a) $\alpha = 0$ and 1 b) $\alpha = 3$ and 5](image)

The nature of vorticity and streamlines changes significantly at $\alpha = 3$ and 5, as shown in figures 3c and 3d. Increased rotation rate leads to suppression of vorticity behind the C1 and C3. Rotation causes the shifting of the wake of C1 and C3 towards the center, pumping the fluid towards C2. This shifting of wake compresses the shear layer originating from the middle cylinder. The extent of compression is sufficient to completely suppress the vortex shedding behind the C2 at $\alpha = 3$. Also, the amount of fluid passing over the C2 reduces as the streamlines start shifting towards the outer edges of C1 and C3. At $\alpha = 5$, the system of three cylinders is entirely enveloped by elliptical-shaped fluid layers, not allowing any amount of fluid to enter into the gap and hence confirms the formation of a virtually elliptic body. Also, the vortex shedding is completely suppressed. Bypassing of incoming fluid and suppression of vorticity dramatically reduces the forces acting on the C2. It was expected that, for $\alpha = 3$ and 5, suppression of vorticity should lead to a non-fluctuating time series of force coefficients. But exactly the opposite happens, as shown in figure 4b. Though the amplitude of the signals of the lift coefficient is small, the frequency seems to have increased hugely. Intentionally, we have kept the range of time small (i.e., from 118.4 seconds to 119.1 seconds) to improve the visibility of the nature of signals. Otherwise, with increasing time range, signals look like a straight line. Power spectra of signals of lift coefficient for $\alpha = 3$ and 5 are shown in figure 5.

![Figure 5. Power spectra of time series of lift coefficient for a) $\alpha = 3$, b) $\alpha = 5$](image)
The peak at around 205 cycles per second confirms that C2 is subjected to this high frequency. These vibrations are attributed to localized opposite signed vorticity being imposed on C2 by the rotation of C1 and C3.

![Figure 6. Variation of $C_{D,\text{mean}}$ and $C_{L,\text{rms}}$ of C2 with increasing $\alpha$](image)

Figure 6 shows the variation of force coefficients with increasing cylinder rotation rate. The mean coefficient of drag ($C_{D,\text{mean}}$) almost shows a linear reduction with $\alpha$. It is important to note that $C_{D,\text{mean}}$ becomes negative for $\alpha = 5$, which signifies pressure building up downstream of C2. The root-mean-square coefficient of lift ($C_{L,\text{rms}}$) also reduces with increasing $\alpha$ and becomes almost zero at $\alpha = 3$, remaining constant after that.

5. Conclusion

The numerical simulation was performed to understand the effect of side-by-side pair of inwardly rotating circular cylinders on the nature of drag and lift forces of the stationary circular cylinder placed at the center of pair of rotating cylinders. Reynolds number in consideration was 100, and non-dimensional rotation rates of cylinders were varied for $\alpha = 0, 1, 3,$ and $5$ with gap spacing of 6D between rotating cylinders. It is observed that both the mean drag coefficient ($C_{D,\text{mean}}$) and root-mean-square coefficient of lift ($C_{L,\text{rms}}$) for the middle cylinder (C2) reduce with increasing rotation rate. The virtually elliptic body is evident at $\alpha = 5$. $C_{D,\text{mean}}$ reduces linearly with $\alpha$ and tends to zero for $\alpha = 5$. $C_{L,\text{rms}}$ becomes almost zero for $\alpha = 3$ and remains constant after that. Time series of lift coefficients of C2 at $\alpha = 0$ and 1 shows the existence of two frequencies; primary frequency is associated with vortex shedding behind C2, whereas secondary frequency reflects the periodic merging of the same signed vortices of C2 and the rotating cylinders. Careful observation of time series of lift coefficients for $\alpha = 3$ and 5 reveals that C2 is subjected to periodic fluctuations of frequency as high as 210 cycles per second, which are highly undesirable.

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