Variable mass theories in relativistic quantum mechanics as an explanation for anomalous low energy nuclear phenomena

Mark Davidson
Spectel Research Corporation, Palo Alto, CA USA
E-mail: mdavid@spectelresearch.com

Abstract. A recent theoretical explanation for observed anomalous low energy nuclear phenomena which have puzzled physicists for many years is expanded on. Based on covariant relativistic quantum mechanics and historical time wave equations, it explains a large number of observed anomalous effects by supposing that nuclear masses can vary in “nuclear active environments” in condensed matter settings. The modified quantum wave equation originally introduced by Fock and Stueckelberg in the 1930s with significant enhancements up to the present by Horwitz and others prove that variable masses are compatible with the principles of both quantum mechanics and relativity. They can explain all of these effects by modifying the kinematic constraints of the reaction, enhancing electron screening and quantum tunneling rates, and allowing for resonant tunneling. Some previous results are recounted, and experimental evidence based on variable radioactive decay rates and other evidence for variable masses is presented which adds some new potential support for this theory.

Keywords: hydrated palladium, deuterated palladium, LENR, fusion, radioactivity
PACS: 24.10.-i, 3.65.-w, 3.70.+k

1. Introduction

Experimental claims of anomalous nuclear reactions in condensed matter settings are increasing in number, and this poses both a problem and an opportunity for physics. These controversial observations today cover a wide spectrum of phenomena including unexplained heat production in deuterated Palladium with commensurate production of Helium-4 suggesting d-d fusion, excessive heat production in hydrated nickel and titanium as well, transmutation of elements in hydrated or deuterated metals, strong gamma ray and positron bursts from lightning storms, transmutation of elements possibly observed in certain bacteria, variable radioactive decay rates for radioactive isotopes implanted in metals in special circumstances, time varying nuclear decay rates of radioactive isotopes, anomalous radiations being emitted from LENR experiments, neutron bursts coming from deuterium gas in strong time-varying magnetic fields, anomalous Helium-3/Helium-4 ratios rising from the earth’s interior, piezo-nuclear phenomenon, x-ray bursts emitted from peeling Scotch tape in a near vacuum, etc. Although some of these might be explainable with conventional physics, it is the author’s opinion that either the experiments are mostly wrong or else they indicate new physics. The key ingredient that most of these experiments suggest is that in some special circumstances in condensed matter, where these
reactions are taking place, the nuclear rest masses of the particles involved in the reaction are not constant, but can vary. These special regions have been called the “nuclear active environment” by Storms [1]. Although other explanations have been proposed, my opinion is that none of them provide as satisfactory and complete an explanation as this. Variable rest masses can make forbidden nuclear processes possible, because the kinematics depend on them. Moreover, mass variation can lead to greatly enhanced quantum tunneling, and can also lead to resonant tunneling [2]. Although it is common in solid state physics to talk about effective masses, especially for electrons and holes, these masses depend on context and are typically used in phenomenological transport equations and have little or nothing to do with nuclear decays which in conventional physics depend only on the standard isolated particle’s rest mass. External electromagnetic fields will shift the 4-momentum of charged particles placed in that field by changing the 4-momentum stored in the total field around the particle, but this effect cancels out from the kinematic equations of nuclear reactions as explained in [2] due to charge conservation and therefore cannot change nuclear reaction rates either (of course external electromagnetic fields can change the kinetic energy of particles as well as the center of mass two-body collision energy, but there is no other influence than this on nuclear rates). The kind of rest mass variation we are talking about is completely ruled out by conventional nuclear and condensed matter physics, as was made clear in [2]. Nevertheless, there is a body of theoretical literature that proves that such variation is compatible with the fundamental principles of relativistic classical and quantum mechanics. The discrepancy between conventional physics and some experimental claims of d-d fusion amounts to 50 orders of magnitude for the deuterium fusion rate [3]. To most physicists this is absolute proof that these experiments are incorrect. It is my opinion too that conventional solid state physics and nuclear physics cannot explain these discrepancies. Variable masses, which would entail new physics, can make up for the discrepancy as was shown in [2]. The parent fields of both nuclear and condensed matter physics are particle physics and relativistic quantum theory, and we must look to these to find a way to incorporate variable masses. It must be made clear from the outset that what has been proposed in [2] is radical, and has never been directly observed, as was pointed out there. Nevertheless, the possibility that new physics may be required to describe such a huge discrepancy is simply too important for physics to ignore. There have been many attempts at explaining the observed LENR phenomena in terms of conventional theory. The subject is too vast to delve into here, but it is clear that none of these have gained any widespread acceptance by the physics community. I feel that the many approximate ad hoc attempts to explain this or that LENR effect with conventional nuclear physics models are hiding the true nature of the phenomenon, which requires radical new physics in the form of variable mass theories.

Most variable mass theories in modern physics are based on spontaneous symmetry breaking. These techniques are rooted in quantum field theory, and they do not seem to be appropriate for the origin of mass variation that is observed in LENR. This is because the condensed matter environment cannot be expected to modify in any significant way the Higgs mechanism that determines particle masses. Moreover, nuclear physics and condensed matter physics are described mainly with quantum wave mechanics as opposed to second quantized field theory, and it would thus be desirable to have a variable mass wave mechanical theory. It has been argued that relativistic quantum theory contains the option of having inherently variable mass behavior which is not caused by any sort of Higgs mechanism [4, 5, 6, 7]. Specific theories with variable masses were developed early on by Fock and Stueckelberg [8, 9, 10] which evolved into the modern historical time models described below in section [3].

We use units such that $\hbar = c = 1$, and a relativistic metric signature $(-,+,+,+)$, unless otherwise noted.
2. A list of nuclear anomalies that can likely be explained by variable nuclear rest masses in condensed matter settings

A listing of nuclear anomalies which have been observed in experiments are presented here. For the most part these phenomena are strongly rejected by the physics community. The reason is that they are believed to be obviously incompatible with existing theories of nuclear physics. I believe that all of these anomalies can be explained by variable masses, as we discuss below.

2.1. Unexplained excess heat in hydrated and deuterated metals.

Reviews of this topic, together with citations to the literature may be found in [1, 11, 12, 13, 14]. The most substantiated data is for the deuterium-Palladium system where deuterium loading is achieved through electrolysis with heavy water ($D_2O$). In highly deuterated palladium, with loading factors (ratio of the number densities of d and pd) greater than about 0.9, it is claimed that many electrolytic cells have produced heat in excess of what can be accounted for chemically by a number of independent laboratories and using different calorimetry and electrolysis equipment. They also claim to have observed Helium-4 production in the reactions in quantities which were approximately consistent with the excess heat if the reaction were nuclear fusion with two deuterons forming an alpha particle, and with the entire energy release going into heat. This is a surface phenomenon restricted to within a few thousand angstroms of the surface of the palladium.

Considering the possibility of d+d fusion in a $d_2$ molecule, Koonin and Nauenberg calculated the penetration factor through the Coulomb barrier at room temperature [3] which gave reaction rates 50 orders of magnitude smaller than the claimed experimental results of Fleischmann and Pons [15]. Leggett and Baym [16, 17] came to similar negative conclusions based on necessary but unobserved affinity enhancement for $^4$He in deuterated palladium. They and others concluded that either the experiments must be wrong, or that the effects were not due to fusion. Recent experiments on deuteron beam scattering off of deuterated metals of various kinds have exhibited much larger fusion cross sections than these calculations [18, 19, 20, 21, 22, 23]. This enhancement has been attributed to unexpectedly large electron screening effects. The other main problem is to explain the extreme distortion of the branching ratios as compared with plasma fusion which has the well-measured ratios [24].

\[
d + d \rightarrow t(1.01 \text{ MeV}) + p(3.02 \text{ MeV}), \ (Q = 4.04 \text{ MeV}), \ 51\% \quad (1)
\]

\[
d + d \rightarrow ^3He(0.82 \text{ MeV}) + n(2.45 \text{ MeV}), \ (Q = 3.27 \text{ MeV}), \ 49\% \quad (2)
\]

\[
d + d \rightarrow ^4He(0.076 \text{ MeV}) + \gamma(23.77 \text{ MeV}), \ (Q = 23.85 \text{ MeV}), \ (3.8 \times 10^{-3})\% \quad (3)
\]

where Q is the kinetic energy released by the reaction in the center of mass Lorentz frame. When excess heat is observed in LENR experiments, it is produced with practically no neutrons, tritium, or gamma rays.
As explained in [2], this effect can be explained by a combination of continuous electron mass increase along with continuous deuterium mass decrease in the nuclear active environment. The excess heat is produced by two tunneling effects acting together. The electron mass increase leads to enhanced electron screening and a commensurate increase in the d-d tunneling rate for neighboring deuterium nuclei. Then, if the deuteron masses drop to the point where the sum of the masses of these two equals the mass of Helium-4, resonant tunneling occurs, greatly enhancing the tunneling rate from the already enhanced rate due to electron mass increase. Because the mass of the deuterons is changing in this process, the usual branching ratios are greatly distorted in a time dependent way from the standard values above. If this process occurs in nature, then it is so radically different from ordinary on-mass-shell fusion that it deserves a different name. Perhaps the prosaic “variable mass fusion” is a reasonable term. If the electron masses increase with the deuteron masses staying on-shell, then one obtains ordinary “hot fusion” with approximately equal numbers of tritium and neutrons produced with the rates and kinetic energies given by [1] and [2].

It is possible that the enhanced electron screening is not due to the electron’s mass changing as there might be other effects in a solid to enhance it, as has been proposed in the literature. The modified ratio of fusion channels can only be explained by a deuterium mass change. It’s also possible that fusion is not occurring as in Widom-Larsen theory [25], but in that case the electron’s mass must also increase in order for the reaction to proceed (see section 4.1.1).

2.2. Transmutation of elements in hydrated and deuterated metals.

There are extensive experimental claims of a variety of elemental transmutations taking place inside or on deuterated palladium and also in other metal-hydrogen alloys [1, 11, 26, 27, 28, 29, 30, 31]. With varying particle masses, transmutations can conceivably in theory occur in a number of ways in a palladium-deuterium lattice. These include electron capture, beta and alpha decay of nuclei whose mass has increased, resonant fusion of protons, deuterons, alpha particles with other nuclei including palladium, and even fission of palladium and/or other impurity elements after a mass increase. Also, there is the possibility of neutron creation and subsequent capture as in the Widom-Larsen theory [25], leading to many possible reactions with no Coulomb barrier to be overcome. In short a world of possibilities exist. The fusion possibilities mentioned in this list have a much higher Coulomb barrier than the deuterium molecule. Nevertheless, if resonant tunneling occurs, there might be small numbers of such events occurring, as has been reported in [28, 31].

2.3. Neutron and tritium production.

In the following subsections only experiments that showed either neutrons or tritium significantly above background levels are included. The neutrons detected were all energetic and non-thermal, but the kinetic energy of the tritium was typically not measured. There were experiments which showed results consistent with background levels. The number of positive experiments is large enough that they represent a critical mass establishing a high probability of an effect. If nuclear rest mass variation is the cause that enables the anomalous production of neutrons and tritium, then the amounts will depend on how the electron and deuteron or hydrogen masses vary, and this may vary from experiment to experiment yielding different rates and branching ratios.
2.3.1. Neutron production at low levels in hydrated and deuterated metals. These papers have reported low levels of neutrons produced in various metals which have been loaded with hydrogen or deuterium or both [32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49]. In order to produce neutrons in our model, the electron mass must increase while the deuterium mass must remain close to the mass shell. The increased electron mass can account for the 20 order of magnitude enhancement required by the data as was shown in [3]. Notice that if the deuterium mass moves quickly off the mass shell to a lower value where neutrons and tritium can no longer be produced, the experiment will show nothing - no excess heat, no neutrons, and no tritium. Perhaps a few gamma rays. Nevertheless, the system can still be considered as nuclear active. Only when the adjacent deuteron masses combined are equal to the mass of helium-4 will resonant tunneling lead to excess heat and no radiation [2].

2.3.2. Neutron bursts produced in deuterium gas at liquid Nitrogen temperature in time-varying magnetic fields This is an intriguingly simple experiment performed by Tadahiko Mizuno [43]. He observed neutron emissions from pure deuterium gas after it was cooled in liquid nitrogen and placed in a magnetic field. Neutron emissions were observed in ten out of ten test cases. It is not clear whether the neutrons are produced in the bulk $D_2$ gas or at the interface between the pyrex container and the $D_2$. If the reaction occurs in the bulk, then it is extremely simple. Just a time-varying magnetic field and the deuterium gas cooled to liquid nitrogen temperature. Assuming that this is evidence for d-d fusion, then it would be explained by an increase in the electron mass in the gas. This experiment is tantalizingly simple. Mizuno has speculated that perhaps cosmic muons or neutrinos may be playing a role here because the neutrons come in bursts. I would like to see this experiment replicated, and redesigned to determine whether the neutrons are coming from the surface or bulk of the gas. This simple experiment eliminates the material preparation uncertainties of the metal in traditional LENR experiments, and so it should be easier to understand what is happening. The neutrons are produced sporadically, and this non-Poisson temporal distribution is quite surprising for a bulk reaction in the $D_2$ gas. In muon catalyzed fusion, a single muon can catalyze a number of fusions in a deuterium gas, limited by the alpha-sticking problem and the lifetime of the muon [50]. Not more than about 100 $d - d$ fusions can be catalyzed by a single muon. The mean energy of muons from cosmic rays at sea level is $\sim 4GeV$, and so these muons would be moving too fast to be captured into a deuterium molecule that could undergo muon catalyzed fusion. The average muon flux on a horizontal detector at sea level is about $1/cm^2/min$. It seems quite unlikely that muons could be playing a role in these neutron bursts through muon catalyzed fusion. There is no known physical mechanism by which neutrinos could catalyze fusion or produce neutrons in the quantity that has been reported in this experiment either. This suggests that the reaction is probably not occurring in the bulk $D_2$ gas, but rather at the pyrex-gas interface where sporadic effects related to the glass chemistry and structure can be envisioned that might lead to neutron bursts. But for this to happen, the electron mass would have to increase to reduce the Coulomb barrier.

2.3.3. Tritium production at low levels in hydrated and deuterated metals. Tritium is generally produced in larger amounts than neutrons, and has been observed in many experiments [51, 52, 53, 54, 55, 56, 57, 58]. There are two mechanisms for tritium production: $d+d \rightarrow t+p$ and $d+p+e^- \rightarrow t+\nu_e$. The production of tritium in $d+d \rightarrow t+p$ was discussed in [2].
2.4. Anomalous nuclear radiation emissions from hydrated and deuterated metals

A large number of uncategorized radiation has been observed in numerous experiments. See Storms [1] for a review.

2.5. Much larger than expected d-d fusion rates observed in deuterated Palladium in electrolysis experiments and low energy deuterium beams incident on Palladium surface.

There were early observations of neutrons, with energies characteristic of anomalous d-d fusion, being produced in electrolytic cells with heavy water and Palladium cathodes [32]. The magnitude of the discrepancy between theory and experiment was estimated to be about 20 orders of magnitude in [3], whereas the discrepancy between theory and the excess heat reports were 50 orders of magnitude [4]. There are now several experiments that show anomalous d-d fusion and neutron production in scattering experiments with low energy deuterons incident on a Palladium target [18, 19, 20, 21, 22, 23], and these show much higher d-d fusion rates than were expected as well, adding credence to the earlier results in [32].

2.6. Helium-4 produced in electrolysis of Palladium electrodes in heavy water where the amount is consistent with the excess heat produced if the reaction is deuterium fusion.

A number of experiments have looked for evidence of production of $^4$He in deuterated palladium systems [1, 59, 60, 61]. The excess heat in these experiments agreed approximately with the number of $^4$He produced if the reaction were $d + d \rightarrow ^4He + 23.85\text{MeV}$ of heat.

2.7. Reduction of tritium decay rate when adsorbed onto nanocrystal Titanium particles and heated as observed by Otto Reifenschweiler at Philips laboratories, Eindhoven.

These experiments were performed in the late 1950s at the Philips Research Laboratories, Eindhoven, The Netherlands. The results were not published until 1994 [62, 63, 64, 65, 66]. Quoting from [62]:

“By heating a $TiT_0.0035$ preparation [0.0035 is the number ratio of tritium atoms to Titanium atoms] consisting of extremely small monocrystalline particles ( $\phi \approx 15$ nm) a decrease of the radioactivity by 40% was observed [on heating the sample from $115^\circ C$ to $275^\circ C$]. In further experiments the concentration of tritium in such preparations was varied ($TiT_x$ experiments) showing that the radioactivity of the tritium increased less than proportionally to its concentration [which also indicates a reduction of decay rate]. Careful analysis of the experiments seems to rule out the possibility of trivial errors.”

“In our experiments the titanium preparation was made by evaporation of the metal in argon at a suitable pressure, e.g. 0.5 to 2 cm Hg. . . . It was deposited as a kind of soot, consisting of monocrystalline particles about 15 nm in diameter, arranged in chains, on the inner wall of the measuring vessel. . . . After the argon was pumped out the tritium was added and was completely absorbed within a few seconds. This was confirmed by hundreds of experiments. In one part of the experiments the electron current ( $\beta$-particles and secondaries) was measured by a vibrating reed electrometer via a cylindrical electrode placed inside the vessel. In several other experiments there was a thin steel window to enable the x-radiation accompanying the $\beta$-decay (internal and external bremsstrahlung and characteristic X-rays . . . ) to be measured by a
Various experiments established for both detection systems the linear relationship between the read-out and the activity."

These experiments have no explanation in conventional nuclear physics. The conclusions were criticized [67], but this criticism was addressed by Reifenschweiler [63]. The reduction in the decay rate has a very simple interpretation in terms of the tritium mass variation. If the tritium mass is decreasing during the heating phase, then the phase space of the tritium decay will be reduced, and consequently the decay rate will be reduced. Unfortunately there have been no attempts to replicate these experiments as far as I know.

Tritium decays 100% by beta decay in the reaction.

\[ ^3H \rightarrow ^3He + e^- + \nu_e, \quad Q = 18.59 \text{ keV} \] (4)

The Fermi form for the electron energy spectrum of tritium \(\beta\)-decay is [68]

\[ \frac{d\Gamma_F}{dE_e} = \frac{G_F^2}{2\pi^3} |M|^2 F(Z, R_e, E_e)p_e E_e (E_e^{max} - E_e)^2 \] (5)

In this formula the neutrino mass has been approximated by zero. where \(G_F\) is the Fermi constant \(p_e\), \(E_e\), and \(E_e^{max}\) are the momentum, energy, and maximum endpoint energy, respectively, of the electron, and \(|M|^2\) is the absolute square of the nuclear matrix element. The Fermi function, \(F(Z, R_e, E_e)\), provides a correction to the decay rate which is due to the Coulomb interaction of the decay electron with the charge 2e of the daughter nucleus \(^3He\). If the tritium mass can vary, then the maximum electron energy in the decay can vary by the formula

\[ E_e^{max} = (\text{mass of tritium}) - (\text{mass of } ^3He) = Q + M_e \] (6)

\[ Q = E_e^{max} - M_e \] (7)

The kinematic factor

\[ R = p_e E_e (E_e^{max} - E_e)^2 \] (8)

controls the decay rate as \(\frac{d\Gamma_F}{dE_e} \propto R\). But since \(Q \ll M_e\), we have the approximate result

\[ \frac{d\Gamma_F}{dE_e} \propto p_e (E_e^{max} - E_e)^2 \] (9)

the total decay rate can be obtained by integration

\[ \Gamma_F = \int_{M_e}^{E_e^{max}} p_e (E_e^{max} - E_e)^2 dE_e = \int_{M_e}^{E_e^{max}} \sqrt{2M_e (E_e - M_e)} (E_e^{max} - E_e)^2 dE_e \] (10)
Clearly, if the mass of the tritium drops then the decay rate will also. If the tritium mass reaches the mass of $M_{3\text{He}} + M_e$ or lower, then the decay rate will stop altogether. According to Reifenschweiler, the tritium decay rate fits a statistical model in which there are two states for the tritium in his experiments, either decaying with the usual expected rate or not decaying at all [69]. The tritium mass only has to drop 18.59 keV to cause the tritium decay rate to be zero. This is quite small compared to the approximate 12 MeV drop that is required for resonant tunneling of deuterium. So if the deuterium mass can drop 12 MeV in a deuterated Palladium undergoing vigorous electrolysis, then it seems quite reasonable that tritium, another isotope of Hydrogen after all, can experience a mass drop of just 18.59 keV. Thus tritium might be useful as a tracer element that can be placed into hydrated metals to test if its mass is varying in a candidate nuclear active location.

2.8. Radiogenic and nucleogenic isotope geology

If LENR effects occur in a laboratory, then undoubtedly they will occur naturally in planetary matter, and this means that they could have an impact on isotope geology.

2.8.1. Helium-3 to Helium-4 ratio

The atmospheric ratio of the natural abundances of $^3\text{He}$ over $^4\text{He}$ is $R_a = 1.37 \times 10^{-6}$. In different locations in the earth this ratio can vary and differ considerably from $R_a$. The conventional explanation for this is complicated [70]. Whereas $^4\text{He}$ is readily formed in alpha decay of many radioactive isotopes, $^3\text{He}$ is harder to form and therefore rarer.

(i) $^3\text{He}$ can be trapped in the earth at formation or added by meteoric dust.
(ii) It can result from cosmic ray scattering, and this is called cosmogenic.
(iii) Lithium spallation is the process by which a high-energy neutron (typically from Uranium or Thorium decay in the earth) bombards a lithium atom, creating a $^3\text{He}$ and a $^4\text{He}$ ion.
(iv) The $^3\text{He}$ that reaches the atmosphere is continually lost into the vacuum of space at a relatively high rate due to its low mass.
(v) It has been observed that the ratio of $^3\text{He}$ over $^4\text{He}$ in thermally active locations on the earth often tends to be larger than $R_a$. Such sites include volcanic emissions, geothermal hot springs, oceanic ridges, and deep lakes. These sources of high ratio are assumed to indicate that the $^3\text{He}$ comes from the mantle and is largely primordial in nature.

It has been suggested that part of the reason for higher ratios of $^3\text{He}$ over $^4\text{He}$ coming from volcanoes and deep lakes are LENR reactions occurring in the earth’s mantle or crust [71, 72, 73]. This is absolutely not accepted by the geophysical community, but we cannot ignore it here because if masses can vary then the prospect that LENR is occurring in the earth is quite possible.

2.8.2. Deuterium to hydrogen ratio in the oceans and the origins of water on earth

It is thought today that most water in earth’s hydrosphere probably comes from Chondritic meteorites [74, 75, 76]. Comets tend to have too high a value of D/H (by about a factor of two) compared with the earth’s oceans, and analysis of the ratio $^{15}\text{N}/^{14}\text{N}$ is the earth’s atmosphere and in comets limits the contribution of comets to the earth’s water to a small fraction [75]. The ratio of deuterium to hydrogen can vary from one comet to the next, and the average is not known. The
The total amount of water in the earth’s hydrosphere has been estimated to be about $1.4 \times 10^9 \text{km}^3$ as estimated by Igor Shiklomanov for the United Nations [77]. The annual hydrologic cycle of evaporated water amounts to about $5.05 \times 10^5 \text{km}^3/\text{yr}$. Some of this evaporation ends up flowing through the earth, and there it will come in contact with various metals (like Palladium, Nickel, and Titanium) which might allow for some LENR to occur. If deuterium fusion is possible, as many LENR experiments have suggested and which could be possible with variable masses as we are discussing here, then some deuterium could be removed permanently in each cycle, and over time the reduction in deuterium might be measurable. The energy released in these reactions could also add to the energy balance of the earth. If the D/H ratio is declining in time on the earth, then this factor may enter into the debate over origins of water on the earth.

The ratio of deuterium to hydrogen on the moon might shed light on this issue. The age of the moon is about 4.5 billion years. Since both would receive the same flux of similar incoming meteorites and comets, they should have about the same ratio of deuterium to hydrogen incrementally added. But the moon would not experience LENR because it doesn’t have a hydrological cycle like earth. Recent evidence suggests that the amount of deuterium vs hydrogen on the moon is higher than on the earth [78]. But this could be due to preferential loss of light hydrogen to the vacuum of space rather than to the fact that comets were the source of water on the moon.

2.9. Unexpected gamma ray and positron bursts from lightning storms.

Scientists using NASA’s Fermi Gamma-ray Space Telescope have detected beams of antimatter produced above thunderstorms on Earth, a phenomenon never seen before [79]. These are produced in addition to gamma ray bursts [80, 81, 82]. These events are not occurring in metals, and they probably are not of LENR origin, but they are as yet unexplained and so we include them in this list of anomalies.

2.10. Variation in beta and alpha decay radionuclides when implanted into metals and other materials

The decay rates of various radioactive isotopes embedded into various materials have been observed to change from their natural standard values [83, 84]. The standard explanation for this effect is that electron screening lengths and electron densities of materials vary, and these will affect nuclear decays which proceed through tunneling through a Coloumb barrier or by electron capture. Our LENR theory suggests that in a nuclear active environment the electrons may gain mass. This will have an effect similar to enhanced electron screening. However, the electron mass gain effect, if it exists at all, can be expected to occur only in small localized volumes of the material, and even then sporadically. In addition to the electron mass change, our theory could even support the possibility that the radioactive isotope’s mass can change. This would affect the decay rate by changing the available phase space for the decay. An extreme example of this was the Reifenschweiler experiment with tritium decay mentioned above. There have been experiments in which the change in the decay rate was very large, but which have not been repeatable. For example, $^{22}\text{Na}$ decay was measured in palladium [85] at room temperature and at 12K, and the half-life was 1.2% shorter at the lower temperature. A similar but smaller result was observed in [86] where the half-life at 15K was 0.46% shorter than at room temperature. A third experiment showed no change with temperature at all [87], but the embedding metal was gold instead of palladium. Perhaps this variability is due to an electron mass variation in nuclear active environments in the Palladium which does not occur in gold, and as in LENR, the
properties of the material are important and different conditions can produce different results. Moreover, the expected nuclear active zone is expected to occur on the surface of the palladium only, and so the depth of embedding of the radionuclide would be a critical factor. The biggest effect should occur near the surface.

It would be extremely interesting to measure radioactive decay rates in highly deuterated palladium where any mass change effects, if present, could make a large impact on the decay rates. Perhaps an interesting experiment would be to perform the $^{22}$Na decay experiment in highly deuterated palladium which is undergoing gaseous diffusion. It is expected that this decay rate will depend on the diffusion rate of deuterium through the palladium, and also on the loading factor. The temperature might also be varied. The half-life should get shorter as the mass of the electrons increase which would be expected to happen with increased loading. Many experiments along these lines suggest themselves. One might vary the depth of the radionuclide into the surface of the palladium, or try other metals, try alpha decays as well as beta decays, study the effect of temperature, pressure, magnetic fields, etc.

2.11. Unaccounted Temporal variation of radionuclide decay rates and LENR without any apparent variation in the environmental conditions.

There are a number of experiments that show radioactive decay rates changing cyclically with time for no apparent cause [88, 89, 90, 91, 92, 93, 94]. This has been attributed to an extraterrestrial source, possibly solar neutrinos [93, 89, 88]. Periods of 1 year, 27 days, and 1 day have been reported. Other experiments do not show such variations, and so there is again experimental uncertainty. But small oscillations in the rest masses of electrons or the radioactive isotopes themselves can clearly account for such varying decay rates. Temporal variation of LENR experiments have also been reported [95, 96], and the experimenters have felt they were hard to explain as due to the variation in temperature and pressure in the environment. Another feature of LENR experiments is their sporadic nature. Periods of nuclear activity tend to appear suddenly and unpredictably, like earthquakes, and then last an unpredictable amount of time with varying intensity, and then nothing again for a while, and then perhaps again activity appears unpredictably again.

2.12. Sonoluminescence and sonofusion

Sonoluminescence, the production of light by collapsing bubbles in a fluid, is a completely reproducible and accepted experimental phenomenon [97, 98, 99]. Sonofusion, on the other hand, the claimed $d-d$ fusion of deuterium inside collapsing bubbles undergoing forced cavitation is quite controversial [100]. The temperature rise inside the collapsing bubble is quite high $\sim 15,000$K, but this is not high enough to produce significant hot fusion in deuterium. If electron masses could increase inside the bubble, then the tunneling rate could increase, and it might be possible to produce neutrons and tritium in measurable quantities, and if the deuterium mass could change, then it could changed the branching ratios of the neutron and tritium channels.

2.13. X ray production from peeling scotch tape - triboelectric X ray source

Researchers at UCLA found X rays being produced by peeling transparent tape in a vacuum of about $10^{-3}$ Torr of air [101, 102, 103, 104]. The X ray photons range in energy from zero to 100keV with a mean about 20 keV. The photons are produced in nanosecond bursts.
with many photons per burst suggesting an avalanche discharge effect. The usual explanation for triboluminescence, which normally produces visible wavelengths, is that charges build up between two different materials that are being rubbed together, and then electron currents can arc though air and produce radiation through bremsstrahlung. Such theories look promising to describe this phenomenon, provided the necessary large charging of the peeling tape and roll can be explained. The angular distribution is more sharply peaked than simple bremsstrahlung however [104], and so we tentatively list it as an anomaly. At the very least, these experiments definitely show that unexpectedly high energy radiation can be produced by very mild conditions at room temperature in everyday objects. These experiments are also 100% reproducible and they work on demand. Products have already been developed which use this technology in commercial x-ray imaging systems. They illustrate beyond any doubt that nuclear scale energies are possible in seemingly innocuous condensed matter settings at room temperature. It turns out that the output of x-rays depend critically on the tape speed, and on the pressure in the atmosphere surrounding the tape. Outside of a range in both of these parameters there are no x-rays produced. This was not apparent at the start of the experiment, and so some experimenters could miss the effect altogether if they were not persistent. LENR is similar in that with knowledge and experience more things that can affect the performance are being found.

3. Theoretical basis for variable mass particle theories.

Fixed particle rest masses are not a requirement of special relativity, general relativity, quantum mechanics, or quantum field theory, but experiments show that all elementary particles of a given type, in isolation, always have the same mass. Quantum field theory tells us that particles in continuous interaction must have variable masses though. Historical time models, introduced by Fock and Steuckelberg [8, 9, 10] are one prominent and simple example of a variable mass theory. They were introduced in order to solve some of the problems encountered early on in developing a relativistic wave mechanics. Modern variants of this theory offer the most well-developed variable mass theories. Other possibilities exist though. An analysis of the possibility of variable mass particle theories, in the context of general relativity and the principal of equivalence, was carried out by Greenberger [4, 5, 6, 7]. He treated the proper time of a particle as a quantum operator with coordinate time serving the role of describing the causal evolution. This is the reverse of the historical time approach where the coordinate time is an operator, but the Lorentz-invariant historical time (which is formally similar to proper time) is simply a real-valued numerical parameter which parametrizes the causal evolution of the system. His conclusion is that variable mass theories are compatible with the requirements of general relativity. For the case of particles with spin, another approach for variable mass particles was developed by Corben [105, 106]. The standard model of particle physics incorporates spontaneous symmetry breaking and the Higgs mechanism to produce masses for the bosons of the weak interaction, and these masses are dependent on the Higgs field, and therefore somewhat variable as well.

Consider the classical relativistic particle equation of motion for particle orbit in Minkowski space.

$$\frac{dP^\mu(\tau)}{d\tau} = F^\mu(\tau)$$

(11)

where $\tau$ is proper time and $P = MU(\tau)$ with $u(\tau)$ being the proper 4-velocity $U(\tau) = dx(\tau)/d\tau$. In general, the mass of the particle will change by the formula.
\[
\frac{dP^\mu P_\mu}{d\tau} = 2P^\mu F_\mu
\]  

(12)

There is no requirement that the mass be any particular fixed value in general. If it so happens that \( P^\mu F_\mu = 0 \), then the mass will be fixed on an orbit, but not constrained to any particular value initially. The electromagnetic Lorentz force \( \frac{dP^\mu(x)}{d\tau} = qU_\nu F^{\mu\nu}(x) \) satisfies this because the Faraday tensor \( F^{\mu\nu} \) is antisymmetric in its indices, and thus conserves the mass. So relativity alone does not require fixed mass particles. In conventional relativistic quantum mechanics for a single particle, time is taken to be a real number which parametrizes the evolution of the state as it is in non-relativistic quantum mechanics, but the position operators and their conjugate momenta are operators in a Hilbert space that do not commute. Therefore, ab initio, time and space are fundamentally different in quantum wave mechanics, and this contravenes relativity which asserts that they must reside in a 4D manifold and can mix under Lorentz transformations. If we mix a real-numbered time with an operator valued position to get the time in a new frame, then it will become a non-commuting operator. Thus time is required to play two roles simultaneously, first as a c-number parametrization of causally related events unfolding in space-time, and second as an operator in a Hilbert space that does not commute with the energy operator. Thus, combining relativity and quantum mechanics is fundamentally difficult. Field theory is different, because in a field the position 4-vector is a c-number vector for both time and space components, and the dynamical variables are the fields and not the particle coordinates. Thus manifest covariance is easier to achieve. As a consequence, the modern approach in physics has been to give up on trying to develop a relativistically covariant multi-particle wave function approach and appeal to second-quantized field theory as the only acceptable relativistic particle theory. Attempts at relativistic wave mechanics, such as the Dirac equation or the Klein-Gordon equation, have unresolved problems with negative energy states or the impossibility of defining a positive definite probability density. However, in historical time models the rest masses of particles can vary and manifest covariance can be achieved, and this is why they were and are still studied.

### 3.1. Historical time models

In quantum field theory, the standard method of introducing variable masses is via spontaneous symmetry breaking and the Higgs mechanism, but in nuclear physics or condensed matter physics, the standard theoretical analysis is done in terms of multi particle wave functions in the Schrödinger representation. To introduce variable masses in this context, one naturally turns to the historical time models which are the most well-known and developed versions of wave-function-based variable mass theories.

The application of historical time models by Feynman to the relativistic path integral problem is illustrative [107](see Appendix A). In order to put space and time on a manifestly covariant footing, he chose to introduce a second time variable, call it \( \tau \). The usual Klein-Gordon equation is

\[
(i\partial - A)^\mu (i\partial - A)_\mu \Psi = -M^2 \Psi
\]

(13)

where \( M \) is the particle’s rest mass, and where \( A \) is the vector potential for an external electromagnetic field. There are well known problems with the probabilistic interpretation of this equation because the conserved current is \( j_\mu = i [\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*] \) and \( j_0 \) takes on both positive and negative values. Moreover, a localizable Schrödinger type position operator cannot
be defined for this equation \[108\], \[109\]. Feynman wished to apply his path integration method of quantization to this. In his words “... we try to represent the amplitude for a particle to get from one point to another as a sum over all trajectories of an amplitude \(\exp(iS)\) where \(S\) is the classical action for a given trajectory. To maintain the relativistic invariance in evidence the idea suggests itself of describing a trajectory in space-time by giving the four variables \(x^\mu(\mu)\) as functions of some fifth parameter \(\tau\) (rather than expressing \(x^1, x^2, x^3\) in terms of \(x^4\)”). So Feynman replaces \[13\] with (we use \(x^0 = t\), and \(\tau\) instead of \(\mu\) and we use spacelike metric, whereas Feynman used timelike)

\[
\frac{i}{\sqrt{\mu}} \frac{\partial \varphi(x, \tau)}{\partial \tau} = \frac{1}{2} (i\partial - A)^\mu (i\partial - A)^\mu \varphi(x, \tau), \text{ where } x = \{x^0, x^1, x^2, x^3\} \tag{14}
\]

It is very similar to the time dependent Schrödinger equation, but with the “historical time” \(\tau\) replacing the usual time variable \(t\), and with the four coordinates of space-time \(x^\mu\) replacing the usual three coordinates of space. Because of this, it is quite easy to formulate the path integration method for this system by following the same steps as used for the Schrödinger equation. Feynman points out that if \(A^\mu\) does not depend on \(\tau\), then separable solutions exist so that \(\varphi(x, \tau) = \Psi(x) \exp(i \frac{1}{2} M^2 \tau)\) and \(\psi\) is the solution to the usual Klein-Gordon equation. Equations of this type were first studied by Fock \[8\] and Stueckelberg \[9, 10, 110\]. The path integral solution includes all paths in space-time connecting two space-time points and parametrized by \(\tau\), with no restrictions on the path, so that off-mass-shell paths are included in the path integral solutions. This reflects that fact that Feynman diagrams contain virtual particles which are not on the mass shell. The reformulation of quantum mechanics along these lines was initially called the “proper time formalism”, and was also studied by Nambu \[111, 112\], Cooke \[113\], and others. A good theoretical justification for studying this type of theory as a replacement for fixed-mass relativistic wave mechanics was given by Pearle \[114\] who also addressed the two-body scattering problem. A general multi-body theory was developed by Horwitz and Piron \[115\]. A 5D generalization of electrodynamics and quantum electrodynamics called “Pre Maxwell theory” was introduced by Saad, Horwitz, Arshansky, and Land \[116, 117, 118\]. The literature on historical time models is now quite extensive. A good source for much of the subject is Fanchi \[119, 120\].

### 3.2. Horwitz-Piron theory

Horwitz and Piron \[115, 121\] supposed that a single historical time \(\tau\) simultaneously parametrizes multiple particles, with their motion determined by a Hamiltonian \(K\), conjugate to \(\tau\), that had only simultaneous (in \(\tau\)) interactions. This simplifies the calculation of many-particle systems considerably. They postulated a Hamiltonian \(K\), conjugate to the historical time \(\tau\). They chose \(K\) as follows

\[
K = \sum_{i=1}^{n} \left[ \frac{1}{2M_i} (p_i - eA(x_i))^\mu (p_i - eA(x_i))^\mu \right] + \sum_{i<j} V_{ij}(|x_i - x_j|) \tag{15}
\]

where \(A^\mu\) is an external electromagnetic four vector potential, and \(V\) is an inter-particle potential energy. The “dot” notation denotes \(\tau\) derivatives. Hamilton’s equations of motion become
\[ \dot{x}_i^\mu = \frac{\partial K}{\partial p_{i\mu}} = (p_i - eA(x_i))/M_i, \quad \dot{p}_i^\mu = -\frac{\partial K}{\partial x_{i\mu}} \quad (16) \]

\[ \frac{dK}{d\tau} = 0 \quad (17) \]

This leads to the Lorentz force equation

\[ M_i \ddot{x}_i^\mu = eF_{\mu\nu} \dot{x}_i^\nu - \sum_{j \neq i} \frac{\partial V_{ij}(|x_i - x_j|)}{\partial x_{i\mu}}, \quad \text{where} \quad M_i \dot{x}_i^\mu = p_i^\mu - eA^\mu \quad (18) \]

Because of the similarity to Newtonian mechanics, the solution of these equations (the Cauchy problem) is no more difficult, except for the extra dimension. The mass of a particle is given by

\[ m_i^2 = -M_i^2 \dot{x}_i^\mu \dot{x}_i^\mu = -(p_i - eA(x_i))^\mu (p_i - eA(x_i))_\mu \quad (19) \]

The particle’s mass \( m_i \) is not necessarily equal to \( M_i \), and is not even necessarily a constant of the motion if \( V \) is not zero. Particles can scatter and end up off the mass shell (i.e. so that \( m_i \neq M_i \)), and then move apart into isolated regions where there are no fields, and they can remain off the mass shell there. There is no obvious restorative force that returns particles to the mass shell in this theory, however, since the Hamiltonian \( K \) is independent of historical time \( \tau \), whenever the particles are far apart and well separated so that the \( V_{ij} \) can be ignored, and if the fields vanish there too, we have

\[ K = -\sum_{i=1}^{n} \frac{m_i^2}{2M_i} \quad (20) \]

Therefore, if some particles increase their mass, then others will have to decrease their mass in order to keep \( K \) constant. In the absence of an external force, the total 4-momentum \( P = \sum_i p_i \) is conserved as well.

### 3.3. Pre-Maxwell 5D theory

Off-shell electrodynamics was generalized to a five dimensional gauge invariant theory by Saad, Horwitz, Arshansky, and Land [116, 117, 118]. Here we follow the notation in [118]. In this theory, \( \tau \)-dependent gauge invariance was assumed so that the wave equation is invariant under local gauge transformations of the form

\[ \psi(x, \tau) \rightarrow e^{i\sigma_0 \Lambda(x, \tau)} \psi(x, \tau) \quad (21) \]

\[ \left( i \frac{\partial}{\partial \tau} + e_0 a_5(x, \tau) \right) \psi(x, \tau) = \frac{1}{2M} (p - e_0 a(x, \tau))^\mu (p - e_0 a(x, \tau))_\mu \psi(x, \tau) \quad (22) \]

This leads to a non-trivial generalization of electromagnetism. The parameter \( e_0 \) has units of length and is proportional to the electric charge \( e \)

\[ e = \frac{e_0}{\lambda} \quad (23) \]
where the constant parameter $\lambda$ is new. The fields transform as

$$a^\mu(x, \tau) \rightarrow a^\mu(x, \tau) + \partial^\mu \Lambda(x, \tau)$$

$$a_5(x, \tau) \rightarrow a_5(x, \tau) + \partial_\tau \Lambda(x, \tau)$$  \hspace{1cm} (24)

Non-vanishing values of $a_5$ leads to mass variation. The Schrödinger equation (23) leads to a five dimensional conserved current

$$\partial_\mu j^\mu + \partial_\tau j^5 = 0$$  \hspace{1cm} (25)

$$j^5 = \rho = |\psi(x, \tau)|^2$$  \hspace{1cm} $j^\mu = -\frac{i}{2M} \left\{ \psi^* (\partial^\mu - ie_0a^\mu) \psi - \psi (\partial^\mu + ie_0a^\mu) \psi^* \right\}$$  \hspace{1cm} (26)

The usual Maxwell theory is recovered by integrating over $\tau$, a process termed “concatenation”.

$$J^\mu(x) = \int_{-\infty}^{+\infty} j^\mu(x, \tau) d\tau$$  \hspace{1cm} $A^\mu(x) = \int_{-\infty}^{+\infty} a^\mu(x, \tau) d\tau$  \hspace{1cm} (27)

where $A^\mu$ and $J^\mu$ are the usual Maxwell potential fields and 4-current respectively. Saad et al [116] suggested an action which had higher 5-dimensional spacetime symmetry so that the Lorentz group O(3,1) is a subgroup. This requires either O(4,1) or O(3,2) symmetry. Both are considered in the literature, and to handle this, a 5D metric tensor is introduced $g^{\alpha\beta} = \text{diag}(-1, 1, 1, 1, \sigma)$, where $\sigma = +1$ for O(4,1) and $\sigma = -1$ for O(3,2).

This theory has been studied extensively by Horwitz in particular along with a number of co-authors [116, 122, 123, 124, 125, 118, 126, 117, 127, 128]. Path integral quantization has been analyzed [129] as has more canonical second quantization approaches to off-shell quantum electrodynamics for spinless charged particles [OSQED][118]. A theory of relativistic wave functions generalizable to arbitrary spin and based on Wigner’s induced representations of the homogeneous Lorentz group has been described by Horwitz in [130] and references therein. This theory produces relativistic wave functions which satisfy all the desired requirements by introducing an arbitrary timelike vector $n^\mu$ for massive particles, and which is then used to define the induced representation and Wigner’s little group for the particle.

Land and Horwitz [118] have applied perturbative quantum field theory methods to off-shell pre-Maxwell electromagnetism in interaction with spinless charged particles. They have developed Feynman rules and applied them to various scattering processes. This is a 5D theory, and there are two possibilities for the 5D symmetry group that contains the Lorentz group as a subset. The Lorentz group O(3,1) is a subgroup of either O(4,1) or O(3,2), and these two possibilities lead to two different theories with different physical properties. The results for particle-particle scattering show that in general the masses will change for particles in a scattering process. They present detailed cross-section calculations for both Miller and Compton scattering, and both calculations show mass changes after scattering. This is an extremely interesting paper as it provides ample off-shell behavior. The mass of a particle in this theory is a function of the past history of the particle.

The self interaction of a classical charged particle in pre-Maxwell theory has also been studied. The solutions are more complicated than the Lorentz-Dirac equation [128, 131, 132].
runaway solutions are replaced by chaotic nonlinear equations which include variation of the
classical particle’s mass with time.

There are no rigorous bounds on how far off-shell the massive particles can wander in the various
historical time theories for realistic condensed matter systems, especially in non-equilibrium
situations.

3.4. How to perhaps reconcile these ideas with the standard model

The established mechanism for mass fixing in the standard model of particle physics is
spontaneous symmetry breaking and the Higgs mechanism. It is probably unrealistic to
imagine that the vacuum Higgs field could be modified in a condensed matter setting. But,
it is conceivable that these fixed mass values are the on-shell parameters that appear in the
Horwitz-Piron or pre-Maxwell theories which might be effective Lagrangian approximations to
the standard model in some circumstances.

In relativistic field theory all of the loop momenta in a Feynman diagram are off the mass shell.
Since all matter is interacting with other matter all the time through long range forces, we can
deduce that all particles in our universe are very slightly virtual all of the time, and are never
exactly on the mass shell. The question is how far off the mass shell can they wander. The
LENR experiments suggest that there exist situations in condensed matter settings where this
mass variation can be much larger than expected.

The nuclear active environment for LENR in metals is known to occur near the surface of
palladium or other metals, and in areas where the lattice has been deformed. The surface
morphology and chemistry are also known to be critical factors - rough surfaces on the nano-
scale are much more active, and of course high loading of deuterium or hydrogen is also
a requirement. In general, they also require statistical non-equilibrium induced by outside
influences like continuous electrolytic loading, pulsing electrical signals injected into the material,
optical stimulation of the surface, thermal cycling, magnetic cycling, acoustic waves, incoming
particle beams, etc.

Effective Lagrangian approximations are routinely employed in nuclear physics. Many of these
methods predate the standard model. They often exploit the low values for mass of the u and d
quarks which when taken to zero lead to chiral-isospin symmetry which can then justify utilizing
group theoretical methods to derive general forms for interacting potentials [133, 134, 135].
Most of theoretical nuclear physics calculations are based on the Schrödinger equation with
effective nuclear and electromagnetic potential functions which do not allow any off-mass-shell
behavior, even though the relativistic field theory perturbation does. Up until LENR effects
were discovered, there was no need for such behavior in nuclear physics. It is my opinion that
there is such a need now since conventional approaches have failed by more than 50 orders of
magnitude [3] to explain what is being seen in LENR experiments, and it seems that off-mass-
shell behaviour of one type or another can explain all of the experimental anomalies.

In multi-dimensional tunneling for many-body systems, many tunneling paths compete for the
“path of least barrier” to reach the true ground state from a metastable one [136]. Condensed
matter systems are always in fact quasi-stable systems. The true ground state - after allowing
for all nuclear reactions - is at a much lower energy level, but large Coulomb barriers prevent
their decay. Of course, it is usually observed that tunneling to the true ground state through
nuclear reactions happens at such a slow rate that it can be ignored, and this agrees with
calculations of tunneling rates using quantum mechanics. LENR phenomenon suggest that in
some special unique situations a nuclear active environment can arise in which the tunneling rate is much larger. Mass deviation, chaos\cite{137}, resonances\cite{138,139}, and driving forces\cite{140} are all mechanisms for enhanced tunneling rates. Given all these possibilities, there is certainly no rigorous theorem based on the standard model that says the experimental claims of LENR cannot occur. There may exist effective off-mass-shell Lagrangians for such systems which facilitate calculations of multi-dimensional tunneling, which give the correct S-matrix when all particles are separated spatially in the initial and final state, and which allow for LENR behavior.

So in this paper, as in\cite{2}, it is proposed that in a condensed matter setting in special regions or domains (nuclear active domains), the electromagnetic interaction of charged particles can be described by an effective off-mass-shell Lagrangian. The best choice seems to be the historical time models, but a completed derivation of this from the standard model in the future may reveal extra terms in the effective Lagrangian that have not yet been studied or anticipated. In the absence of such a complete derivation, experiments can help to guide the way to implementing this approach.

### 3.5. A Phenomenological working model

The historical time approach to relativistic quantum mechanics provides an excellent starting point for attempting to develop an acceptable theory of variable masses in LENR. Extra terms in the Lagrangian or some other modification or understanding will need to be found to clarify how particles get back onto the mass shell after they have departed form it, and also to describe both the nuclear active environment and ordinary non-nuclear-active environments with the same theory. Obviously this will involve one or several parameters which control the degree of nuclear activity in a given condensed matter setting. I think that experiments should lead the way in this search. To have a starting point we have proposed a phenomenological approach. In order to describe the experiments in LENR, the following assumptions were made about mass variation in\cite{2}

(i) Mass variation is rare and it happens at best at very localized micro-regions of a material which have been termed “nuclear active environments” in the LENR literature. The precise requirements for such regions to exist are not known. The assumption here is that the only thing that distinguishes a nuclear active environment is that it is a region where masses can change. The Pre-Maxwell theory is an electromagnetic phenomenon, and it seems quite reasonable to assume that all charged particles can experience mass variation, and in general the mass can increase or decrease.

(ii) The mass distribution of a particular particle type can be a time varying probability distribution. The mean value and the variance of this distribution can change with time. For fusion to occur, it was assumed in\cite{2} that as the two neighboring deuterons approach resonance in the Helium-4 channel from above, their combined mass is sharply peaked and changes slowly with time. Such adiabatic behavior ensures that the combined mass will pass very close to the alpha particle’s mass, which is a requirement for resonant tunneling to occur.

(iii) Immediately after a nuclear reaction involving strong or weak processes, the resulting particles are assumed to be exactly on the mass shell. This assumption was deemed necessary to make a sensible theory. The Horwitz-Piron theory and the Pre-Maxwell theories, which describe electromagnetic interactions only, do not have this property. They allow off-mass-shell particles in the final scattering state as well as the initial state of the scattering process. A general problem with these theories is that if a particle has gotten off
the mass shell by a succession of scattering processes, it is not clear how that particle will get back onto the mass shell. Nuclear decays involving strong and weak forces are assumed to co-exist with a Horwitz-Piron or pre-Maxwell type model for the electromagnetic interaction. But, for electromagnetic decays, this assumption still is a bit problematical. What is needed is a mechanism to allow particles to be forced back onto the mass shell if they are off of it, and this effect must probably be mediated by some parameter that interpolates continuously between a pure historical time model and conventional on-mass-shell quantum mechanics. Such a parameter was suggested in [2].

Ultimately I hope that the historical-time models can be modified to exhibit these assumptions rigorously, to clarify the return to the mass-shell after excursions, and to characterize the unique distinguishing parameters that determine the degree of nuclear activity in a region of condensed matter.

4. Possible mechanisms for anomalous nuclear behavior

Here we enumerate a variety of effects that can lead to nuclear reactions in nuclear active environments.

4.1. Electron mass change

4.1.1. Widom-Larsen effect enabling the reaction $p + e \Rightarrow n + \nu_e$  Low energy neutrons can be produced if the electron mass changes which can cause neutron activated changes in the isotopic mix [25]. This reaction was apparently first proposed and discussed by Swartz [141] to explain experimental results observed in nickel-hydrogen electrolysis experiments. In conventional nuclear physics, the velocity of electrons needs to be 92% of the speed of light in order to be able to form a neutron in collision with a proton. The presence of electromagnetic fields inside of a solid can change the speed of the electrons, but not their nuclear mass [2]. If electrons of this velocity were inside a solid, they would produce bremsstrahlung x-rays and gamma-rays. These are not typically observed in LENR experiments in sufficient quantities to explain excess heat production. In a historical time model, however, the rest mass of a particle depends on its past history, and the possibility of off-mass-shell electrons or protons cannot be ruled out a priori. In these theories the electron’s nuclear rest mass might be large (the threshold mass for neutron production is about $2.5 \times electron\ mass$), and yet its velocity small so that there could be no x-ray bremsstrahlung. Thus I accept the Widom-Larsen process as possible. As it is mediated by the weak interaction, it must compete with other processes which may have higher reaction rates in selected instances. The historical time model must be such that inside the bulk metal, there is no significant off-mass-shell behavior. Only in a rare surface state - the nuclear active environment - can mass variation be occurring. Ongoing experiments are gaining knowledge about the requirements for a nuclear active environment or domain.

4.1.2. Enhanced electron screening leading to greatly enhanced tunneling rates for fusion reactions In the Born-Oppenheimer approximation to molecules and solids, the size scale is determined by the inverse of the electron mass [2]. Therefore, an electron mass increase will reduce the inter-nuclear distance, and greatly enhance quantum tunneling and fusion rates [3,2]. This is a form of electron screening. The enhancement of deuterium fusion in a $d_2$ molecule was rigorously quantified in [3], where it was shown that a mass of ten electron masses was necessary to explain the reaction rates measured by Fleischmann and Pons [15], and a mass of
five electron masses was needed to explain Jones’ results [32]. With simply a large electron mass, one cannot explain the discrepancy in the branching ratio, but with a simultaneous reduction in the deuterium mass, one can explain that as well [2], and in this case the electron mass increase factor can be much smaller than 10 since the resonant tunneling effect greatly enhances the fusion rate. Electron mass increase factor of 5 are sufficient to explain the results of Jones [32][3], and this could be adequate, when combined with resonant tunneling, to explain the excess heat experiments that have been observed [2].

4.1.3. Enhanced electron screening leading to modified radioactive decay rates

Alpha decay rates of radioisotopes are increased (half lives reduced) by enhanced electron screening because the potential barrier that the alpha particle must overcome to get out of the nucleus is reduced by the more effective electron screening cloud surrounding the nucleus[142]. Beta$^+$ decay rates are enhanced by electron screening, whereas beta$^-$ decay rates are reduced [143].

4.1.4. Electron capture by nuclei not normally allowed.
The modified electron mass could combine with a nucleus (whose mass might also be off shell) to form a different nuclear isotope or isomer which would be forbidden by energy conservation requirements for on-shell electrons.

4.2. Deuterium mass change

4.2.1. Decrease in the mass leads to a change of branching ratios in d+d fusion reactions as described in [2]. If the deuterium nucleus experiences a decreased mass, then the first result in a d+d fusion reaction is that the available energy released decreases for each of the reaction channels. The various reaction products have reduced kinetic energies. If the deuterium mass decreases enough, there will no longer be enough center of mass energy to produce neutrons, and the main reaction becomes the tritium channel. If the mass decreases still more, eventually there is not enough energy to produce tritium and the only channel that is open is the gamma-ray channel. As the deuterium mass decreases still further, eventually it reaches the resonant point where the center of mass energy is exactly equal to $^4He$, and resonant tunneling occurs.

4.3. Tritium mass decrease can lead to a reduction of the tritium decay rate as has been observed by Reifenschweiler

As described in [2.7] if the tritium mass decreases, the result is a reduction on the available phase space for the three particle beta decay.

4.4. Alpha particle mass change could lead to resonant fusion involving alpha particles and other nuclei

The same tunneling effects that can allow d-d fusion [2] might apply to fusion of nuclei, and the easiest such reactions would occur with lighter elements for which the Coulomb barriers are lower. The combination of increases electron mass - leading to enhanced electron screening - with modified alpha particle or nucleus mass resulting in resonant tunneling can in theory at least allow for fusion reactions to occur which would normally be almost unimaginably unlikely because of the extremely large Coulomb barriers that must be overcome. These types of reactions can also occur through neutron capture as well, as in the Widom-Larsen model.
4.5. *Radioactive isotope mass variation causing decay rates to change since available phase space depends on the mass of the parent radio-isotope.*

The decay rate of any particle into a final state of decay product particles is given by the general formula

\[ d\Gamma = \frac{S|M|^2}{2M}d\Phi_n(P, p_1, p_2, ..., p_n) \]  

(28)

where \( M = \sqrt{P\mu P\mu} \) is the mass of the decaying particle, \( n \) is the number of decay particles, \( S \) a combinatoric factor accounting for identical particles, \( M \) is the quantum transition matrix element, and \( d\Phi_n \) is the phase space of the decay given by

\[ d\Phi_n(P, p_1, p_2, ..., p_n) = (2\pi)^4 \delta^4(P - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \frac{d^3p_i}{(2\pi)^3 E_i} \]  

(29)

where \( P \) is the 4-momenta of the decaying particle, \( p_i \) the 4-momenta of the decay products, and where \( E_i \) is the total energy of particle \( i \). Examples of phase space calculations can be found in [144]. The phase space increases monotonically as a function of the decaying particles mass \( M \). Thus, any increase or decrease in this mass will directly increase or decrease the decay rate, however the decay matrix element \( M \) may also depend on this mass, and so the phase space may not give the whole picture, but I would expect the phase space to be the dominant factor. So, observations of varying radioactive decay rates in condensed matter settings may indicate mass variation if no other explanations seem plausible.

5. Conclusion

We have listed here a number of anomalous nuclear reactions which have not been explained by conventional nuclear physics. Many of these are probably impossible to explain without something new. But, if nuclear rest masses can vary in special “nuclear active environments”, then all of these reactions can be allowed with the right mass variation. After surveying the literature on mass variation in relativistic physics, the historical time models pioneered by Fock and Stueckelberg are prominent. The modern refinements of this theory especially by Horwitz and co-workers stand out as the most likely candidate theories around which to construct a model for LENR. The task before us is to try and quantify what physical effects cause a nuclear active environment to come into existence. The theories for mass variation are not yet complete. In particular, we don’t have a theory which interpolates between theories in which the rest mass is quite freely variable, like for instance pre-Maxwell theory, and ones that have fixed mass, like conventional QED or the standard model (at least as they are normally understood). I have in mind a new phenomenological parameter that would depend on position inside a material and control how easy or hard it is for particles to drift off the mass shell. In ordinary matter or in vacuum, this parameter would make deviation from standard rest masses exceedingly small, but in the nuclear active environments it would allow significant deviation of mass from the expected value. It is my opinion that this is the most likely way to reach an understanding of the LENR experiments in total.

Once again, as in [2], I remind the reader that there is no direct evidence that masses can change as has been suggested here. This is a radical proposal which deserves to be treated with great
skepticism. The only reason I am proposing it is because it is the only way I can see to explain all these anomalous experiments, and I feel that they must be explained. In the words of Sherlock Holmes “When you have eliminated the impossible, whatever remains, however improbable, must be the truth” [145] (in chapter 6).

Many of the objections to LENR experimental claims in the past were based on the fact that neutrons and tritium were not observed in experiments involving deuterated palladium in the proper amounts, energies, and ratios as deuterium fusion would require [146, 147]. But as we have seen, if masses of electrons and deuterons can vary, then this not a valid justification for rejecting these results out of hand. So, the rejection of LENR has been based largely on a theoretical bias. Since all particles in condensed matter are slightly virtual (in the relativistic Feynman diagram sense) all the time, they are slightly off the mass shell all the time. Therefore we know that assuming the particles in condensed matter to be exactly on the mass shell cannot be exactly correct. What we don’t know is how far off-shell they may wander. It is routinely found (or at least assumed) in most situations that they don’t wander off shell enough to make a difference for nuclear kinematics, or for chemistry and solid state physics for that matter, but relativistic quantum mechanics permits the possible existence of anomalous regions in condensed matter where masses can vary significantly. So, if one is to rule out a theory like the present one, the change in the ratios of tritium to neutrons or their energies cannot be used to dismiss experiments. Rather, they might be viewed as experimental evidence in support of variable masses. Experiments like the x-ray producing scotch tape [101, 102, 103, 104] prove with certainty that unexpected energetic excursions can occur in ordinary condensed matter. Perhaps mass excursions can occur as well to explain other anomalies.

Much of modern physics has separated from close laboratory experimental feedback. Fields like quantum gravity, string theory, interpretations of quantum mechanics, emergent theories of gravity, etc. are not dealing with specific discrepancies with laboratory experiments. They have become to some extent excursions in pure mathematics, beautiful and interesting in their own right, but lacking the compass of frequent experimental guidance. Here we have a large discrepancy which may be resolved by a fundamental modification of our theories for nuclear physics and condensed matter, and we also have lots of laboratory feedback to aid us. It seems that physicists ought to re-examine this subject in the light of these new developments, and determine with certainty whether or not the nuclear anomalies are real.

Acknowledgements

I thank Larry Horwitz for extensive valuable correspondence and Martin Land, Philip Mannheim, Peter Hagelstein, Michael McKubre, Michael Melich, David Nagel, Mitchell Swartz, Fran Tanzella, Vladimir Kresin, Bob Perlmutter, Peter Schattner, and Andrew Davidson for informative discussions and correspondence.

References

[1] Storms E 2007 Science of Low Energy Nuclear Reaction: A Comprehensive Compilation of Evidence and Explanations about Cold Fusion (World Scientific Publishing Company) ISBN 9812706208
[2] Davidson M 2014 Found. Phys. 44 144–174
[3] Koonin S E and Nauenberg M 1989 Nature 339 690–691
[4] Greenberger D M 1970 Journal of Mathematical Physics 11 2329–2340 ISSN 00222488
[5] Greenberger D M 1970 Journal of Mathematical Physics 11 2341–2347 ISSN 00222488
[6] Greenberger D M 1974 Journal of Mathematical Physics 15 395 ISSN 00222488
[7] Greenberger D M 1974 Journal of Mathematical Physics 15 406 ISSN 0022-2488
[8] Fock V 1937 Phys. Z. Sowjetunion 12 404–425
[9] Stueckelberg E 1941 Helv. Phys. Acta 14 588–594
[10] Stueckelberg E 1941 Helv. Phys. Acta 14 322–323
[11] Srinivasan M, Miley G and Storms E 2011 Low-energy nuclear reactions: Transmutations Nuclear Energy Encyclopedia ed Krivit S B, Lehr J H and Kingery T B (John Wiley & Sons, Inc.) pp 503–539 ISBN 9781118043493
[12] Hagelstein P L, McKubre M C H, Nagel D J, Chubb T A and Hekman R J 2006 Condensed matter nuclear science: Proceedings of the 11th international conference on cold fusion
[13] Nagel D J 1998 Radiat. Phys. Chem. 51 653–668
[14] Hubler G K 2007 Surface and Coatings Technology 201 8568–8573 ISSN 0257-8972
[15] Fleischmann M, Pons S and Hawkins M 1989 Journal of Electroanalytical Chemistry 261 301–308
[16] Leggett A J and Baym G 1989 Nature 340 45–46 ISSN 0028-8889
[17] Czerski K, Huke A, Biller A, Hoefe M and Ruprecht G 2001 Europhysics Letters (EPL) 54 449–455 ISSN 0295-5075, 1286-4854
[18] Czerski K, Huke A, Heide P and Ruprecht G 2004 EPL (Europhysics Letters) 68 365–369
[19] Czerski K, Huke A, Heide P and Ruprecht G 2006 Eur. Phys. J. A 27 83–88
[20] Czerski K, Huke A, Martin L, Targoss N, Blauth D, GórKa A, Heide P and Winter H 2008 Journal of Physics G: Nuclear and Particle Physics 35 014012 ISSN 0954-3899, 1361-6471
[21] Czerski K 2009 Enhanced electron screening and nuclear mechanism of cold fusion ICCF-15 vol 15 (Rome, Italy: ENEA) pp 197–202
[22] Czerski K 2010 Physics of Atomic Nuclei 75 153–159 ISSN 1063-7788, 1562-692X
[23] Atzeni S and Meyer-ter Vehn J 2004 The Physics of Inertial Fusion: Beam-Plasma Interaction, Hydrodynamics, Hot Dense Matter: Beam-Plasma Interaction, Hydrodynamics, Hot Dense Matter (Oxford University Press) ISBN 9780198562641
[24] Widom A and Larsen L 2006 The European Physical Journal C - Particles and Fields 46 107–111 ISSN 1434-6044, 1434-6052
[25] Mizuno T 1997 Nuclear Transmutation: The Reality of Cold Fusion (Infinite Energy Press) ISBN 1-892925-00-1
[26] Iwamura Y, Itoh T, Sakano M, Yamazaki N, Kuribayashi S, Terada Y, Ishikawa T and Kasagi J 2006 Observation of nuclear transmutation reactions induced by d2 gas permeation through pd complexes ICCF-11 vol 11 (Marseilles, France) pp 339–350
[27] Iwamura Y, Itoh T, Terada Y and Ishikawa T 2012 Transmutation reactions induced by deuterium permeation through nano-structured pd multilayer thin film Transactions of the American Nuclear Society, Vol. 107 (San Diego, CA USA) pp 422–425
[28] Takahashi A, Ohta M and Mizuno T 2001 Japanese Journal of Applied Physics 40 7031–7046
[29] Zhang W S and Dash J 2007 Excess heat reproducibility and evidence of anomalous elements after electrolysis in pd+2SO4 electrolytic cells Proceedings, ICCF-13 (Sochi, Russia) p 202
[30] Jones S E, Palmer E P, Czirr J B, Decker D L, Jensen G L, Thorne J M, Taylor S F and Rafelski J 1989 Nature 338 737–740
[31] Jones S E and a 2003 Neutron emissions from metal deuterides in Tenth International Conference on Cold Fusion (Cambridge, MA: LENR-CANR.org)
[32] Claytor T N, Tuggle D G and Menlove H 1991 Tritium generation and neutron measurements in pd-si under high deuterium gas pressure in Second Annual Conference on Cold Fusion (Como, Italy: Societa Italiana di Fisica, Bologna, Italy)
[33] Jorne J 1991 Fusion Technol. 19 371–374
[34] Jorne J 1994 Fusion Technol. 26 244–247
[35] Jorne J 1996 Fusion Technol. 29 83–89
[36] Ciriillo D, Germano R, Tontodonato V, Widom A, Srivastava Y, Del Giudice E and Vitiello G 2011 Key Engineering Materials 495 104–107 ISSN 1662-9795
[37] Ciriillo D, Germano R, Tontodonato V, Widom A, Srivastava Y N, Giudice E D and Vitiello G 2012 Key Eng. Mater. 495 104–107
[38] Domenico C 2012 Transactions of the American Nuclear Society 107 418–421
[39] Lipson A G, Lyakhov B F, Rousetskii A S, Akimoto T, Mizuno T, Asami N, Shimada R, Miyashita S and Takahashi A 2000 Fusion Technol. 38 238–252
[40] Mizuno T, Akimoto T and Sato N 1989 Denki Kagaku 57 742–743
[41] Mizuno T, Akimoto T, Takahashi A and Celani F 2004 Neutron emission from d<sub>2</sub> gas in
magnetic fields under low temperature

Eleventh International Conference on Condensed Matter Nuclear Science

Mosier-Boss P A, Gordon S S n F E and Forsley L P G 2008 Detection of energetic particles and neutrons emitted during pd/d co-deposition Low-Energy Nuclear Reactions Sourcebook (Vol. 1) vol 1 ed Marwan J (Washington, USA: Oxford University Press) pp 311–334 ISBN 9780841269668, 0841269661

Shyam A e a 1995 Observation of high multiplicity bursts of neutrons during electrolysis of heavy water with palladium cathode using the dead-time filtering technique. in 5th international conference on cold fusion in 5th International Conference on Cold Fusion. (Monte-Carlo, Monaco: IMRA Europe, Sophia Antipolis Cedex, France)

Oya Y e a 1998 Material conditions to replicate the generation of excess energy and the emission of excess neutrons in The Seventh International Conference on Cold Fusion (Vancouver, Canada: ENECO, Inc., Salt Lake City, UT.)

Srinivasan M 2009 J. Sci. Explor. 23 477–482

Iyengar P K, Srinivasan M, Sikka S K, Shyam A, Chitra V, Kulkarni L V, Rout R K, Krishnan M S, Malhotra S K, Gaonkar D G, Sathishkumar H K, Nagvenkar V B, Nayar M G, Mitra S K, Raghunathan P, Degwekar S B, Radhakrishnan T P, Sundaresan R, Arunachalam J, Raju V S, Kalyanaraman R, Gangadhara N, Venkateswaran G, Moorthy P N, Venkatasewarlu K S, Yuvaraju B, Kishore K, Gula S N, Panajakar M S, Rao K A, Raj P, Suryanarayana P, Sathyanamoorthy A, Datta T, Bose H, Prabhun I H, Sankaranarayanan S, Shetye R S, Veeraraghavan N, Murthy T S, Sen B K, Joshi P V, Sharma K G B, Joseph T B, Iyengar T S, Shrihinda V K, Mittal K C, Misa C S, Lal M and Rao P S 1990 Fusion Technol. 18 32–94

Prelas M and Lukosi E 2014 J. Condensed Matter Nucl. Sci. 51 49a

Packham N J C, Wolf K L, Wiss J C, Kainthla R C and Bockris J O M 1989 J. Electroanal. Chem. 270 451–458

SANKARANARAYANAN T K, SRINIVASAN M, BAJPAYI M B and GUPTA D S 1995 Evidence for tritium generation in self-heated nickel wires subjected to hydrogen gas absorption/desorption cycles 5th International Conference on Cold Fusion. 1995. (Monte-Carlo, Monaco: IMRA Europe, Sophia Antipolis Cedex, France.)

Sankaranarayanan T K, Srinivasan M, Baijai M B and Gupta D S 1996 Fusion Technol. 30 349–354

Claytor T N, Schwab M J, Thoma D J, Teter D F and Tuggle D G 1996 TRITIUM PRODUCTION FROM PALLADIUM ALLOYS (Vancouver, Canada: ENECO, Inc., Salt Lake City, UT) p 88

Packham N J C, Wolf K L, Wiss J C, Kainthla R C and Bockris J O M 1989 J. Electroanal. Chem. 51 419–153

Reifenschweiler O 1994 Lett. A 149–153

Reifenschweiler O 1995 Z. Naturforsch 50a 973–974

Reifenschweiler O J A 1996 Fusion technology 30 261–272 ISSN 0748-1896

Reifenschweiler O 1997 Fusion technology 31 291–299 ISSN 0748-1896

Reifenschweiler O 1998 Radiation Physics Chemistry 51 327–334

Wicke E 1994 Z. Naturforsch 49a 1259–1261

Robertson R G H and Knapp D A 1988 Annual Review of Nuclear and Particle Science 38 185–215

Reifenschweiler O 2004 Infnite Energy 14–22

Dickin A P 2005 Radiogenic Isotope Geology 2nd ed (Cambridge, UK ; New York: Cambridge University Press) ISBN 9780521530170

Jiang S, Liu J and He M 2010 Naturwissenschaften 97 655–662 ISSN 0028-1042, 1432-1904

Jiang S, He M, Yue W and Liu J 2008 Journal of Fusion Energy 27 346–354 ISSN 0164-0313, 1572-9591

Jiang S and He M 2008 Chinese Science Bulletin 53 540–547 ISSN 1001-6538, 1861-9541

Alexander C M O, Bowden R, Fogel M L, Howard K T, Herd C D K and Nittler L R 2012 Science 337 721–723 ISSN 0036-8075, 1095-9203

Hutsemékers D, Manfroid J, Jehin E and Arpigny C 2009 Icarus 204 346–348 ISSN 0019-1035

Cleeves L I, Bergin E A, Alexander C M O, Du F, Graninger D, Öberg K I and Harries T J 2014 Science 345 1590–1593 ISSN 0036-8075, 1095-9203

Villiers M D 2003 Water: The Fate of Our Most Precious Resource (Toronto: M&S) ISBN 9780771026416

23
Horwitz L P and Piron C 1973 *Helv. Phys. Acta* 46 316–326

Saad D, Horwitz L P and Arshansky R I 1989 *Foundations of Physics* 19 1125–1149

Land M C 1998 *Foundations of Physics* 28 1479–1487 ISSN 0015-9018, 1572-9516

Land M C and Horwitz L P 1996 *arXiv:hep-th/9601021*

Fanchi J R 1993 *Parametrized Relativistic Quantum Theory* (Springer-Verlag GmbH) ISBN 9780792323761

Fanchi J R 1993 *Foundations of Physics* 19 1125–1149

Land M C 1998 *Foundations of Physics* 28 1479–1487 ISSN 0015-9018, 1572-9516

Land M C and Horwitz L P 1991 *Foundations of Physics Letters* 4 61–71 ISSN 0894-9875, 1572-9524

Land M C 1997 *Foundations of Physics* 27 19–41 ISSN 0015-9018, 1572-9516

Land M 2011 *Journal of Physics: Conference Series* 330 012015 ISSN 1742-6596

Aharonovich I and Horwitz L P 2012 *Journal of Mathematical Physics* 53 032902–032902–29 ISSN 00222488

Seidewitz E 2009 *Annals of Physics* 324 309–331 ISSN 0003-4916

Horwitz L 2013 *Journal of Physics A: Mathematical and Theoretical* 46 035305 ISSN 1751-8121

Horwitz L P, Katz N and Oron O 2004 *Discrete Dynamics in Nature and Society* 2004 179–204 ISSN 1026-0226, 1607-887X

Horwitz L and Shnerb N 1998 *Foundations of Physics* 28 1509–1519 ISSN 0015-9018

Horwitz L and Shnerb N 1998 *Foundations of Physics* 21 299–310 ISSN 0015-9018, 1572-9516

Horwitz L C and Horwitz L P 1991 *Foundations of Physics Letters* 4 61–71 ISSN 0894-9875, 1572-9524

Horwitz L C and Horwitz L P 1991 *Foundations of Physics Letters* 4 61–71 ISSN 0894-9875, 1572-9524

Horwitz L C and Horwitz L P 1991 *Foundations of Physics Letters* 4 61–71 ISSN 0894-9875, 1572-9524

Horwitz L C and Horwitz L P 1991 *Foundations of Physics Letters* 4 61–71 ISSN 0894-9875, 1572-9524

Horwitz L C and Horwitz L P 1991 *Foundations of Physics Letters* 4 61–71 ISSN 0894-9875, 1572-9524

Horwitz L and Shnerb N 1998 *Foundations of Physics* 28 1509–1519 ISSN 0015-9018

Horwitz L and Shnerb N 1998 *Foundations of Physics* 28 1509–1519 ISSN 0015-9018

Horwitz L and Shnerb N 1998 *Foundations of Physics* 28 1509–1519 ISSN 0015-9018

Horwitz L and Shnerb N 1998 *Foundations of Physics* 28 1509–1519 ISSN 0015-9018

Horwitz L and Shnerb N 1998 *Foundations of Physics* 28 1509–1519 ISSN 0015-9018