Abstract

Recently, Nishiura and the author have proposed a unified quark-lepton mass matrix model under a family symmetry $U(3) \times U(3)'$. The model can give excellent parameter-fitting to the observed quark and neutrino data. The model has a reasonable basis as far as the quark sector, but the form of the right-handed neutrino mass matrix $M_R$ does not have a theoretical grand, that is, it was nothing but a phenomenological assumption. In this paper, it is pointed out that the form of $M_R$ is originated in structure of neutrino mass matrix for $(\nu_i, N_\alpha)$ where $\nu_i$ ($i = 1, 2, 3$) and $N_\alpha$ ($\alpha = 1, 2, 3$) are $U(3)$-family and $U(3)'$-family triplets, respectively.

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1 Introduction

Recently, Nishiura and the author [1, 2] have proposed a unified mass matrix model under a family symmetry $U(3) \times U(3)'$:

\[
(f_L^i, F_L^\alpha) \begin{pmatrix}
\langle 0 \rangle_i^j \\
\langle \Phi_f \rangle_i^j \\
-\langle \hat{S}_f \rangle_i^j
\end{pmatrix}
\begin{pmatrix}
\langle \Phi_f \rangle^{-1}_i \alpha \\
\langle \hat{S}_f \rangle^{-1}_i \alpha \\
\langle \hat{A}_f \rangle^{-1}_i \alpha
\end{pmatrix}
\begin{pmatrix}
f_R^j \\
F_R^\beta \\
F_R^\beta
\end{pmatrix},
\]

where $f_i$ ($i = 1, 2, 3$) and $F_\alpha$ ($\alpha = 1, 2, 3$) are $U(3)$-family and $U(3)'$-family triplets, respectively, so that we obtain a Dirac mass matrix of $f$-sector as follows,

\[
(\hat{M}_f)_i^j = \langle \Phi_f \rangle_i^\alpha \langle \hat{S}_f \rangle^{-1}_i \alpha \langle \hat{A}_f \rangle^j, \]

under a seesaw approximation. (Hereafter, we denote $U(3)$-family nonet scalars as a notation $(\hat{A})_i^j$ and anti-6-plet scalars as a notation $(\hat{A})^{ij}$.) Here, the VEV matrices $\langle \Phi_f \rangle$ are given by

\[
\langle \Phi_e \rangle = m_{0e} \text{diag}(z_1, z_2, z_3),
\langle \Phi_d \rangle = m_{0d} \text{diag}(z_1, z_2, z_3),
\langle \Phi_u \rangle = m_{0u} \text{diag}(z_1 e^{i\phi_1}, z_2 e^{i\phi_2}, z_3 e^{i\phi_3}).
\]

Since we assume that the $U(3)'$ symmetry is broken into a discrete symmetry $S_3$, the vacuum expectation value (VEV) of $\hat{S}_f$ has, in general, to take a VEV form

\[
\langle \hat{S}_f \rangle = m_{0f} (1 + b_f X_3),
\]
where $\mathbf{1}$ and $X_3$ are defined by

$$
\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},
$$

(1.5)

and $b_f$ are complex parameters. We take $b_e$ as $b_e = 0$, so that the parameters $z_i$ are fixed as

$$
z_i = \frac{\sqrt{m_{e_i}}}{\sqrt{m_{e_1} + m_{e_2} + m_{e_3}}},
$$

(1.6)

where $(m_{e_1}, m_{e_2}, m_{e_3}) = (m_e, m_\mu, m_\tau)$. We may approximately regard $m_{e_i}$ in Eq.(1.6) as the observed charged lepton masses $m_{e_i}^{\text{obs}}$. However, note that the vales $m_{e_i}$ in Eq.(1.6) are not always the eigenvalues $(\hat{M}_e)_i^i$ ($i = 1, 2, 3$) of $\hat{M}_e$ given in Eq.(1.2), and $m_{e_i}$ are, in general, given by $\hat{M}_i^i = k_0 m_{e_i}$ ($i = 1, 2, 3$) with an arbitrary family-number-independent constant $k_0$.

The model [1] can successfully describe the observed quark masses and Cabibbo-Maskawa-Kobayashi (CKM) [3] mixings, especially, not only ratios among $m_{u_i} = (m_u, m_c, m_t)$ and among $m_{d_i} = (m_d, m_s, m_b)$, but also ratios $m_{u_i}/m_{d_j}$ when we take $m_{0_u} = m_{0_d}$. (The quark mass matrix structure has first been proposed by Fusaoka and the author [4] from the phenomenological point of view.)

In the neutrino sector, according to the conventional neutrino seesaw model [5], we consider that the Majorana mass matrix of the left-handed neutrino is given by

$$
(M_\nu)_{ij} = (\hat{M}_\nu)_i^k (M_R^{-1})_{kl} (\hat{M}_\nu^T)_l^j,
$$

(1.7)

under the Majorana mass matrix $M_R$ of the right-handed neutrino $\nu_R$ with a large mass scale, where $\hat{M}_\nu$ is a Dirac mass matrix of neutrinos defined as $\langle \bar{\nu}_L \rangle_i \langle \hat{M}_\nu \rangle_i^j \langle \nu_R \rangle_j$. However, in the $U(3) \times U(3)'$ model, the structure of $M_R$ has been given by a somewhat strange form

$$
M_R \propto \Phi_\nu \hat{M}_u + \hat{M}_u^T \Phi_\nu + \xi_R \hat{M}_\nu (\hat{M}_\nu)^T,
$$

(1.8)

where

$$
\hat{M}_\nu = \Phi_\nu \Phi_\nu^T,
$$

(1.9)

$$
\Phi_\nu = m_{0\nu} \text{diag}(z_1, z_2, z_3),
$$

(1.10)

similar to Eq.(1.3). Note that $M_R$ in Eq.(1.8) includes the up-quark mass matrix $\hat{M}_u$. When we use the VEV values of $\hat{M}_u$ fitted in the quark sector, the neutrino mass matrix $M_R$ is described by only one parameter $\xi_R$, and we can obtain excellent fitting [1] for the observed neutrino masses and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [6] mixings. (A form $M_R$ which is related up-quark mass matrix $M_u$ has first been proposed by the author [7] in somewhat different context $M_R \propto (M_u^D)^{1/2} M_e^D + \cdots$.)
The model has a reasonable basis as far as the quark sector, while the form $M_R$ (1.8) does not have a theoretical grand, and it was nothing but a phenomenological assumption. In this paper, it is pointed out the structure of $M_R$ is originated in the structure of neutrino mass matrix for $(\nu_i, N_\alpha)$ where $\nu_i$ and $N_\alpha$ are U(3)-family and U(3)$'$-family triplets, respectively.

2 Basic idea

Correspondingly to the seesaw mass matrix (1.6), we consider a seesaw mass matrix

$$(M_\nu)_{ij} = (\Phi_\nu)^\alpha(M_R^{-1})_{\alpha\beta}(\Phi_\nu^T)^\beta_j. \quad (2.1)$$

(Hereafter, in order to make the transformation property in U(3) and U(3)$'$ symmetry visual, we use symbols $\circ$ and $\bullet$ instead of indexes $i, j, \cdots$ and $\alpha, \beta, \cdots$) From the definition (1.1) of the Majorana neutrino mass matrix $(\bar{M}_R)^{\circ\circ}$, we introduce a Majorana mass matrix, $(M_R)^{\bullet\bullet}$, for $(N_R)^\bullet$, as follows:

$$(\bar{M}_R)^{\circ\circ} = (\Phi_\nu^T)^\circ(\bar{M}_R)^{\bullet\bullet}(\Phi_\nu)^\circ. \quad (2.2)$$

When we neglect U(3)$\times$U(3)$'$ indexes, from the relation (1.8), i.e.

$$(M_R)^{\circ\circ} = \xi_R(\Phi_\nu)^4 + \left\{\Phi_\nu\bar{M}_u + (\bar{M}_u\Phi_\nu)^T\right\}, \quad (2.3)$$

we can write $(M_R)^{\bullet\bullet}$ as follows:

$$(M_R)^{\bullet\bullet} = \xi_R(\Phi_\nu)^2 + \left\{\Phi_\nu^{-1}\bar{M}_u + (\Phi_\nu^{-1}\bar{M}_u)^T\right\}$$

$$= \xi_R(\Phi_\nu)^2 + \left\{P_u\tilde{S}_u\Phi_\nu + \text{(transposed)}\right\}, \quad (2.4)$$

where $\Phi_u \propto \Phi_\nu P_u$ and $P_u$ is a scalar with VEV values

$$P_u = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}). \quad (2.5)$$

For example, by considering U(3)$\times$U(3)$'$ transformation, we may consider $(M_R)^{\bullet\bullet}$ as

$$(M_R)^{\bullet\bullet} = (M_R^{1st})^{\bullet\bullet} + (M_R^{2nd})^{\bullet\bullet}, \quad (2.6)$$

$$(M_R^{1st})^{\bullet\bullet} = (\Phi_\nu^T)^\circ(E^{-1})^{\circ\circ}(\Phi_\nu)^\circ, \quad (2.7)$$

$$(M_R^{2nd})^{\bullet\bullet} = \left\{(\tilde{E}^T)^{\circ\circ}(P_u)_{\circ\bullet}(\tilde{S}_u^T)^\circ(P_u^{-1})^{\circ\circ}(\Phi_\nu)^\circ + \text{(transposed)}\right\}, \quad (2.8)$$

where we have neglected family-independent parameters.

In the next section, we will discuss a model which leads to the VEV relation (2.6).

3 Mass matrix of $(\nu_L, \nu_R, N_L, N_R)$

For convenience, in this section, we neglect family-independent parameters.
The first term (2.7) suggests a seesaw-like scenario. Therefore, we would like to consider that the second term is also derived from a seesaw-like scenario in the neutrino mass matrix for \((\nu_L, \nu_R, N_L, N_R)\):

\[
(M^{2nd}_R)^{\bullet\bullet} = \left\{ (E^T)_{\circ\circ} \langle \hat{S}^T_u \rangle_\circ \langle \Phi_\nu \rangle_\circ + (\text{transposed}) \right\}, \tag{3.1}
\]

where \(\langle \hat{S}^T_u \rangle_\circ\) should be given by

\[
\langle \hat{S}^T_u \rangle_\circ = \langle P_u \rangle_\circ \langle \hat{S}^T_u \rangle_\circ \langle P_u^{-1} \rangle_{\circ\circ}. \tag{3.2}
\]

In this model, \(\langle \bar{M}^{1st}_R \rangle\) and \(\langle \bar{M}^{2nd}_R \rangle\) can be understood by seesaw scenarios (2.7) and (3.1), while the relation (3.2) cannot be understood by seesaw scenario. Therefore, we consider that the form (3.2) is obtained from a SUSY vacuum condition \(\partial W/\partial \Theta = 0\) for the following superpotential:

\[
W = \text{Tr} \left\{ \left\{ (\hat{S}^T_u)_\circ \langle P_u \rangle_\circ + (P_u \rangle_\circ (\hat{S}^T_u)_{\circ\circ} \langle \Theta \rangle_\circ \right\} + (\text{transposed}) \right\}, \tag{3.3}
\]

where \(\Theta\) is a flavon with \((\Theta) = 0\).

The structures \((M^{1st}_R)^{\bullet\bullet}\) and \((M^{2nd}_R)^{\bullet\bullet}\) suggest the following mass matrix for \((\nu_L)_\circ, (\nu_R)_c, (N_L)_\circ, (N_R)_c)\):

\[
\begin{pmatrix}
\langle \bar{\nu}_L \rangle_\circ & \langle \bar{\nu}_R \rangle_\circ
\end{pmatrix}
\begin{pmatrix}
\langle \bar{N}_L \rangle_\circ & \langle \bar{N}_R \rangle_\circ
\end{pmatrix}
\begin{pmatrix}
\langle \bar{E} \rangle_\circ
\end{pmatrix}
\begin{pmatrix}
\langle \hat{S}^T_u \rangle_\circ
\end{pmatrix}
\begin{pmatrix}
\langle \Phi^T_\nu \rangle_\circ
\end{pmatrix}
\begin{pmatrix}
\langle \bar{\nu}_L \rangle_\circ
\end{pmatrix}
\begin{pmatrix}
\langle \hat{S}^T_u \rangle_\circ
\end{pmatrix}
\begin{pmatrix}
\langle \bar{\nu}_R \rangle_\circ
\end{pmatrix}
\begin{pmatrix}
\langle \bar{N}_L \rangle_\circ
\end{pmatrix}
\begin{pmatrix}
\langle \bar{N}_R \rangle_\circ
\end{pmatrix}
\end{pmatrix}. \tag{3.4}
\]

Here, \(\langle \ \rangle^{\bullet\bullet}\) is a room for would-be \((M_R)^{\bullet\bullet}\). Thus, we can assign all scalars (flavons) in this mass matrix (3.4) without duplication.

Finally, we would like to comment on \(R\) charge assignment. We adopt \(R\) charge assignment for flavons (scalars) \(A\) and fermions \(\psi\) as follows

\[
R(\bar{A}) = R(A), \quad R(\bar{\psi}_{L/R}) \neq R(\psi_{L/R}). \tag{3.5}
\]

For example, in Eq.(3.4), we have defined the flavon \(\hat{S}_u\) as \((\bar{N}_L)^{\bullet\bullet}(\hat{S}_u)^{\bullet\bullet}(N_R)_c\). This does not always mean \(R(U_L) = R(N_L)\) and \(R(U_R) = R(N_R)\) where \((U_L, U_R)\) are components of \((F_L, F_R)\) in the sector \(f = u\). It means only

\[
R(N_L) + R(N_R) = R(U_L) + R(U_R). \tag{3.6}
\]

Thus, we can put the flavon \(\hat{S}_u\) on the desirable position in the neutrino mass matrix (3.4). As we already stressed, it has an important meaning that we could assign all scalars (flavons) in
this mass matrix (3.4) without duplication. It means that we can uniquely assign those flavons without mixing under suitable $R$-charge assignment for $(\nu_{L/R}, N_{L/R})$.

Also, note that, in the mass matrix (3.4), there is no $(E)_{o\cdot}$ and $(\bar{P}_u)_{o\cdot}$ in spite of the existence $(\bar{E})_{o\cdot}$ and $(\bar{P}_u)_{o\cdot}$. This is possible only under the selection rule (3.5). For example, note that a conjugate term of the term $(\bar{\nu}_L)_{o\cdot}(N^c_L)_{o\cdot}$ is not $(\bar{N}^c_R)_{o\cdot}(\bar{P})_{o\cdot}(\nu_R)_{o\cdot}$, but $(\bar{N}^c_L)_{o\cdot}(\bar{P}_u)_{o\cdot}(\nu_L)_{o\cdot}$. Since

$$R((\bar{E})_{o\cdot}) = 2 - R((\nu_R)_{o\cdot}) - R((\bar{N}^c_R)_{o\cdot}),$$
$$R((\bar{P}_u)_{o\cdot}) = 2 - R((\nu_L)_{o\cdot}) - R((\bar{N}^c_L)_{o\cdot}),$$

if we take

$$R((\nu_R)_{o\cdot}) + R((\bar{N}^c_R)_{o\cdot}) \neq R((\nu_L)_{o\cdot}) + R((\bar{N}^c_L)_{o\cdot}),$$

we can regard $(\bar{P}_u)_{o\cdot}$ and $E_{o\cdot}$ as separate flavons.

### 4 Scales of VEV matrices

In the recent study [8] in the $U(3) \times U(3)'$ model, it has been concluded that flavon VEVs with $U(3) \times U(3)'$ indexes $A_{o\cdot\cdot}$, $B_{o\cdot}$ and $C_{o\cdot}$ take the following scales

$$\langle A_{o\cdot\cdot} \rangle \sim \Lambda_1 \sim 3 \times 10^7 \text{TeV}, \quad \langle B_{o\cdot} \rangle \sim \Lambda_2 \sim 3 \times 10^4 \text{TeV}, \quad \langle C_{o\cdot} \rangle \sim \Lambda_3 \sim 9 \text{TeV}. \quad (4.1)$$

In order that the seesaw scenario $M^{1st}_{R}$, Eq.(2.7), holds, the flavon scales have to satisfy the relation

$$\langle E_{o\cdot} \rangle \gg \langle (\Phi_{\nu})_{o\cdot} \rangle. \quad (4.2)$$

As we have proposed in Ref.[8], we adopt such the mechanism $\langle \Phi_{\nu} \rangle = \xi_{\nu} \langle \Phi_e \rangle$ with $\xi_{\nu} \ll 1$.

For the seesaw scenario $M^{2nd}_{R}$, Eq.(2.8), a VEV relation

$$\langle E^c_{o\cdot} \rangle \sim \langle (\Phi_{\nu})_{o\cdot} \rangle \ll \langle (\hat{S}^{' u}_{o\cdot})_{o\cdot} \rangle, \quad (4.3)$$

is required. We consider that a scale of the flavon $(\hat{S}^{' u}_{o\cdot})$ is an exceptional case against the general rule (4.1) in spite of its indexes $( )_{o\cdot}$, because the VEV relation has to be

$$\langle (\hat{S}^{' u}_{o\cdot})_{o\cdot} \rangle \sim \langle (\hat{S}_u)_{o\cdot} \rangle \sim \Lambda_1. \quad (4.4)$$

from the consistency among the scales in Eq.(3.3).

Here, we would like to comment on a scale of $SU(2)_L$. Flavons $\Phi_f$ and $\hat{S}_F$ given in (1.1) are singlets in the vertical symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$, so that they have only indexes of horizontal symmetry (family symmetry). The mass matrix (1.1) does not correspond to the real masses of quarks and leptons, but it represents the Yukawa coupling constant. Therefore,
note that $f_L$ does not mean $f_L = (u, d, e)_L$, and that $f_L$ has to be SU(2)$_L$ singlet. In Ref.[8], the fermions $f_L$ have been defined as follows:

$$f_L \equiv (f_u, f_d, f_\nu, f_e)_L \equiv \left( \frac{1}{\Lambda_H} H_u^0 q_L, \frac{1}{\Lambda_H} H_d^0 q_L, \frac{1}{\Lambda_H} H_u^0 \ell_L, \frac{1}{\Lambda_H} H_d^0 \ell_L \right)$$  \hspace{1cm} (4.5)

where

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad H_u = \begin{pmatrix} H_u^0 \\ H_u^- \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^+ \\ H_d^0 \end{pmatrix}. \hspace{1cm} (4.6)$$

Note that $f_L$ is singlet in SU(2)$_L$, but it has U(1)$_Y$ charge. Therefore, correspondingly, $f_R$, $F_L$ and $F_R$ are SU(2)$_L$ singlets, while they have U(1)$_Y$ charge. (We consider that $(F_u, F_d)_{L/R}$ have SU(3)$_c$ indexes.) For example, the selection of $\tilde{S}_f$ in $\Phi_f\tilde{S}_f^{-1}\Phi_f$ (for example, $\tilde{S}_u$ in $\Phi_u\tilde{S}_u^{-1}\Phi_u$) is done by $R$ charge, not by flavor symmetry and/or U(1)$_Y$ charge.

Our purpose in this paper was to discuss the neutrino mass matrix. The neutrino Dirac mass matrix $(\tilde{M}_\nu)_0^\circ$ given in Eq.(1.7) comes from the term $(1/\Lambda_H)\bar{\ell}H_u^0 = (1/\Lambda_H)(\bar{\nu}_L H_u^0 + \bar{e}_L H_u^-)$ with $\langle H_u^- \rangle = 0$, not from Eq.(3.4).

5 Concluding remarks

Since the previous U(3)$\times$U(3)$'$ model could give successful predictions for the observed quark masses and CKM mixings under a reasonable theoretical scenario, while the success in the neutrino sector was still phenomenological level. We have investigated a possible neutrino mass matrix structure in context of the U(3)$\times$U(3)$'$ model. As we stressed in Sec.3, it is essential that flavons are uniquely assigned in the neutrino mass matrix (3.4) without duplication. Under suitable $R$ charge assignment, especially under assumption $R(\bar{\psi}) \neq R(\psi)$, we can put $\hat{S}_u$, which was defined as $\hat{U}_L\hat{S}_u\hat{U}_R$ in the up-quark sector, into the neutrino sector without confusion.

In conclusion, we have succeeded in giving a theoretical basis to the semi-empirical part (the structure of the right-handed neutrino mass matrix $M_R$) in the previous U(3)$\times$U(3)$'$ model. As a result, the U(3)$\times$U(3)$'$ model has been able to become more realistic as a unified mass matrix model of quarks and neutrinos.
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