Stability of strange dwarfs: a comparison with observations

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Abstract. We study the stability of strange dwarfs, superdense stars with a small self-confining core \( M_{\text{core}} < 0.02 M_\odot \) containing strange quark matter and an extended crust consisting of atomic nuclei and degenerate electron gas. The mass and the radius of these stars are of the same order of magnitude as those of ordinary white dwarfs. It is shown that any study of their stability must examine the dependence of the mass on two variables, which can, for convenience, be taken to be the rest mass (total baryon number) of the quark core and the energy density \( \rho_\text{tr} \) of the crust at the surface of the quark core. The range of variation of these quantities over which strange dwarfs are stable is determined. This region is referred to as the stability valley for strange dwarfs. The mass and radius obtained from theoretical models of strange dwarfs are compared with observational data obtained through the HIPPARCOS program and the most probable candidate strange dwarfs are identified.

1. Introduction
Strange quark matter may exist in a state that is more bound than the matter in atomic nuclei [1, 2]. If the MIT bag model equation of state [3] is used for the quarks, then for certain values of the phenomenological parameters of the model, a case may arise in which the average energy \( \epsilon_b \) per baryon at some baryon concentration \( n = n_{\text{min}} \) has a negative minimum \( (\epsilon_b(n_{\text{min}}) < 0) \). In this case, the quark matter may be in a self-bound state, so that the existence of self-confining cosmic objects, or so-called "strange stars" (ss), which can exist without gravitation, becomes possible [4]. Gravitation places a limit on the maximum mass of these configurations, which, as in the case of neutron stars, is of the order of \( 2 M_\odot \).

At the surface of a strange star the electron density \( n_e \) is several orders of magnitude below that of the quarks. Since the electrons are confined only by an electrostatic field, some of them can move away from the quark surface of the strange star by hundreds of fermis \( (l \sim n_e^{-1/3}) \) to form a thin charged layer where the electric field reaches \( 10^{17} - 10^{18} \ \text{V/cm} \) [5]. This field isolates the crust, which is made up of atomic nuclei and a degenerate electron gas ("Ae" matter), and is not in thermodynamic equilibrium with the strange quark matter; it is coupled to the quark core only by gravitation. Since free neutrons, with no electrical charge, can pass unimpeded through the electrostatic barrier and be absorbed by the strange quark matter, the maximum density of the crust is limited by the rate of escape of neutrons from nuclei, \( \rho_{\text{drip}} \approx 4.3 \cdot 10^{11} \ \text{g/cm}^3 \).

Models of strange stars with a crust have been examined [6] over the entire range of variation of the central density of a star. It was found that for strange stars with strange quark core...
masses $M_{\text{core}}/M_\odot > 0.1$, the thickness and mass of the crust are negligibly small compared to the star’s radius and mass. The situation is different for strange stars with low core masses $M_{\text{core}}/M_\odot < 0.02$. In configurations of this sort, the crust swells greatly, and their mass and radius are as large as found for white dwarfs ($wd$). They differ from $wd$, however, by the fact that they have a core in the form of a small-radius, low-mass strange star and a crust in which the density can be two orders of magnitude greater than the limiting density reached in white dwarfs. These models are referred to as strange dwarfs ($sd$) [7, 8].

2. The stability valley for strange dwarfs

Only stable equilibrium configurations of superdense stars are of physical interest. Glendenning, et al. [7, 8], have studied the stability of strange dwarfs ($sd$) by the method of small radial perturbations developed in the general theory of relativity by Chandrasekhar [9, 10]. They examined the dependence of the mass $M$ on the central energy density of the quark core, $\rho_c$, for a sequences of $sd$ in which the energy density of the nuclear-electron crust at the surface of the quark core $\rho_{tr}$ was set equal to its maximum value, i.e. on the curve $M(\rho_c, \rho_{tr} = \rho_{\text{drip}})$. It was found that, as opposed to ordinary white dwarfs and neutron stars, $sd$ configurations for which $dM/d\rho_c < 0$ have $\omega_0^2 > 0$ ( $\omega_0$ is the fundamental mode frequency for radial pulsations), i.e., they are stable, while configurations with $dM/d\rho_c > 0$ have $\omega_0^2 < 0$, i.e., are unstable.

This, however, is nothing surprising, since complete information on the stability of $sd$ can be obtained only by examining the entire range of variation of $\rho_c$ and $\rho_{tr}$, i.e., we have to consider the functional surface, i.e., the mass of the $sd$ as a function of these variables, $M(\rho_c, \rho_{tr})$. Here it is convenient to take $u$ and $\rho_{tr}$ as the independent variables rather than $\rho_c$ and $\rho_{tr}$, where $u$ is the total number of baryons in the quark core $N_{\text{core}}$ of the $sd$ expressed in solar masses, i.e., $u = mN_{\text{core}}/M_\odot$, where $m = M(^{56}\text{Fe})/56$ and $N_{\text{core}} = 4\pi \int_0^{R_0} r e^{\lambda/2} \rho_{tr} dr$, where $n$ is the baryon concentration, $R_0$ is the radius of the quark core, and $exp(\lambda)$ is the radial component of the metric tensor. In fact, as opposed to neutron stars, the $M(\rho_c)$ curve for bare (without a crust) strange dwarfs is not bounded from below; it approaches zero when the central density $\rho_c$ approaches the density of self-bound quark matter neglecting gravitation. The surface of strange stars without a crust, as usual, is determined by the radius over which the pressure goes to zero: $P(R_{ss} = 0)$. In the case of strange dwarfs, on the other hand, where there is an extended nuclear-electron crust over the surface of a small quark core ($u \leq 0.02M_\odot$), the energy density $\rho_{tr}$ and pressure $P_{tr}$ at the surface of a fixed quark core will increase as the mass of the crust increases. In the bag model used for the quark core, the energy density at the center of the quark core will increase extremely slowly. As noted above, the increase in $\rho_{tr}$ is limited by maximum value $\rho_{tr} \leq \rho_{\text{drip}}$.

Each small quark core becomes the basis of a family of $sd$ with different crust masses. Thus, in studying the stability of strange dwarfs it is necessary to study the stability of individual families of this sort [11, 12]. Here it becomes possible to use a static criterion for stability developed by Zel’dovich [13] and generalized by Bisnovatyi-Kogan [14] that is free of the cumbersome mathematical calculations characteristic of Chandrasekhar’s method. Since the electrostatic field at the surface of the quark core inhibits the penetration of matter from the crust into the quark core, for small radial oscillations the masses of both the crust and the core are unchanged. Thus, the static criterion is applicable to the sequences of this kind. At the maxima of the dependence of the $sd$ mass on $\rho_{tr}$ for a fixed value of $u$ (the $M_u(\rho_{tr})$ curves), the $sd$ loses stability.

Therefore, as opposed to neutron stars and white dwarfs, models of the stability of strange dwarfs in $M, u, \rho_{tr}$ space occupy the part of the $M(u, \rho_{tr})$ surface that is bounded from above by the curve joining the maxima of the $M(u, \rho_{tr})$ curves (see Fig. 1). We refer to this region of the $M(u, \rho_{tr})$ surface as the stability valley for strange dwarfs [15].
For the chosen equations of state of the quark core and crust, the integral parameters of an sd (mass, total baryon number, radius) are uniquely determined by the energy density $\rho_c$ at the center of the quark core and the energy density $\rho_{tr}$ of the nuclear-electron matter at the boundary between the quark core. The integral parameters are then obtained by integrating the relativistic Tolman-Oppenheimer-Volkov (TOV) equations of hydrostatic equilibrium [16, 17].

Many sequences of strange dwarfs with fixed values of $u$ within $10^{-4} \geq u \geq 0.1$ were studied. Figure 1. The mass $M$ of strange dwarfs as a function of the parameters $u$ and $\rho_{tr}$ (see the explanations in the text). The leading part of the surface $M(u, \rho_{tr})$ is bounded from above by the curve ecba, which is the stability valley for strange dwarfs.

Some of the results of the calculations for typical sequences are shown in Fig. 1. Here we examine the $M, u, \rho_{tr}$ space ($M$ is the sd mass), and plot the $M_u = M(u, \rho_{tr})$ curves (the mass as a function of $\rho_{tr}$) on the $M = M(u, \rho_{tr})$ surface for $u = const$. The curves are extended up to their intersection with the coordinate plane $\rho_{tr} = \rho_{drip}$. It is seen that the mass increases with increasing $\rho_{tr}$ for each individual sequence, and the stability is lost at $M_{max}$. For the sequences with $u = 0.013$, the sd mass reaches $M_{max}$ for $\rho_{tr} = \rho_{drip}$ (the point $c$ in Fig. 1). For sequences of strange dwarfs with $u > 0.013$, the curves intersect the $\rho_{tr} = \rho_{drip}$ plane when the mass has no longer reached the maximum (the segment $bc$ in Fig. 1). For the sequences that intersect the $\rho_{tr} = \rho_{drip}$ plane in the segment $ab$, where $u \geq 0.02$, the curves are horizontal, i.e., for these sequences the mass of the crust is negligible compared to the core and $M/M_\odot \approx u$.

The segments of the individual $M(u, \rho_{tr})$ curves for stable configurations of strange dwarfs in $M, u, \rho_{tr}$ space on the saddle surface $M = M(u, \rho_{tr})$, occupy the stability valley for the strange dwarfs. This region is bounded by the curves $cc$ (the maxima of the individual series) and $cb$ (the front portion of the surface in Fig. 1). The part of this saddle surface that applies to unstable configurations and is formed by the segments of the $M_u(\rho_{tr})$ curves after the maximum points, intersects the $\rho_{tr} = \rho_{drip}$ plane along the dc curve, where $\partial M/\partial u$ and, therefore, $\partial M/\partial \rho_c$ are greater than zero. Unstable configurations of this sort naturally have $\omega_0 > 0$ [7, 8]. But this does not mean that the quark cores of the individual sequences in the segment dc cannot form stable sd configurations.

Strange dwarf configurations for which the masses of the individual sequences $M_u(\rho_{tr})$ have not reached the maximum values, so that $\omega_0 > 0$ for them, lie in the $\rho_{tr} = \rho_{drip}$ plane along the curve bc. And there was no justification for the surprise of Glendenning, et al. [7, 8], that this is so, for, on going from one series to another along the bc curve, $\partial M/\partial u$ and, therefore, $\partial M/\partial \rho_c$ are negative. Similar results will be obtained for the intersection of the $M(u, \rho_{tr})$ surface with planes parallel to $\rho_{tr} = \rho_{drip}$ for smaller values of $\rho_{tr}$, as was shown by Glendenning, et al. [8].
Vartanyan, et al. [12] pointed out that, although $\omega_0^2 > 0$ for $sd$ on the segment $bc$, the fact that the condition $\rho_{tr} = \rho_{drip}$ holds for them makes them analogous to the configurations of series with $u < 0.013$, for which the mass is maximal and $\omega_0^2 = 0$ (these are the configurations lying on the curve $ec$ in Fig. 1). Thus, if the crust density is increased even slightly for these configurations (e.g., as a result of radial pulsations), $\rho_{tr}$ will become greater than $\rho_{drip}$ and free neutrons will be born at the surface of the quark core and will move into the quark core, increasing its mass (the total number of baryons). Since $\partial M/\partial u < 0$ on the segment $cb$ in this case, the new quark core with its larger mass cannot form $sd$ with the initial baryon number. Thus, this kind of configuration loses equilibrium. It undergoes a transition to a branch of strange stars $ss$ in a state with the same number of baryons, but with a thin nuclear-electron crust (an extension of the $ab$ branch in Fig. 1). The radius $R_{ss}$ of these configurations is of the order of 10 km, as in typical neutron stars. An energy $\Delta W_G \sim GM^2/R_{ss}$ is released during this transition; this is of the same order of magnitude as the energy released in supernova explosions. The error in the conclusion that strange dwarfs with $\omega_0^2 > 0$ and $\rho_{tr} = \rho_{drip}$ can be stable arises from the fact that Chandrasekhar’s method is applicable when no irreversible processes take place involving matter in the star during the pulsations. This condition is violated when neutrons pass into the quark core. These configurations are at the edge of loss of stability. In this case, for each fixed quark core with $u \geq 0.013$, the distance of the $sd$ from the critical state (the stability “margin”) is greater when the difference $\rho_{drip} - \rho_{tr} > 0$ is larger.

3. Comparison with observation
The European Space Agency’s HIPPARCOS satellite has played a major role in determining and refining the radii and masses of white dwarfs. HIPPARCOS data have been analyzed [18, 19] to obtain improved values for the masses and radii of 22 white dwarfs within the mass range of $0.4 \div 1.0 M_\odot$. In these papers, data on the radii and masses of the observed white dwarfs with the associated measurement errors were shown in a plot of the theoretical dependence [20] of the radius on $wd$ mass, obtained separately for four atomic nuclei, $^4$He, $^{12}$C, $^{24}$Mg, and $^{56}$Fe. These results are shown in Fig. 2. While the $R(M)$ curves are very similar for the first three nuclei (for which $Z/A = 1/2$), in the case of $^{56}$Fe, $Z/A = 0.46$ and this curve is significantly lower than the other and, for a given mass, the corresponding $wd$ radius is the smallest.

![Figure 2. The radius $R$ as a function of mass $M$ for a white dwarf and a strange dwarfs. The curves $^4$He, $^{12}$C, $^{24}$Mg and $^{56}$Fe are for white dwarfs. The data is taken from from [20]. The other notation are explained in the text.](image)
Vartanyan, et al. [6], have compared the data for observed \( wd \) with the \( R(M) \) curve for strange dwarfs that have \( \rho_{tr} = \rho_{drip} \). It was found that the two observed \( wd \), EG-50 and G238-44 lie very close to this curve and, therefore, may be candidates for strange dwarfs. The same problem has been examined in more detail in Ref. [21], where sequences of \( sd \) with \( \rho_{tr} = \rho_{drip} \) for a crust containing \( ^{12}\text{C} \) nuclei were examined.

For completeness of the comparison, it is necessary to examine \( R(M) \) not just for one sequence of strange dwarfs with \( \rho_{tr} = \rho_{drip} \), but also the curves for characteristic fixed values of \( u \) from the stability valley calculated for different elements. This is of special interest for \( ^{56}\text{Fe} \) nuclei, since only \( sd \) configurations can lie below the \( R(M) \) curve for \( wd \) of this element.

Under the \( R(M) \) curve for white dwarfs containing \( ^{56}\text{Fe} \) in Fig. 2 we have plotted \( R(M) \) for sequences of stable strange dwarfs with \( u = 0.005, 0.01, 0.013, \) and 0.016 based on data of Ref. [12]. The curves are lower for larger \( u \). The last sequence corresponds to the maximum sized quark core, for which an extended crust can develop, i.e., a strange dwarfs can be formed. The \( R(M) \) curve for the \( sd \) sequences with \( \rho_{tr} = \rho_{drip} \) is also shown in this figure.

Thus, under \( R(M) \) curve for iron \( wd \) in Fig. 2, there is a limiting strip on which only \( sd \) can lie. If the observed white dwarfs include candidates with masses and radii (with small enough measurement errors) that locate them below the \( R(M) \) curve for \( wd \) consisting of \( ^{56}\text{Fe} \), then these observed \( wd \) may be identified with a strange dwarf. Only EG-50 is very close to satisfying this requirement among the observed \( wd \) mentioned here.

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