TRANSPORT IN QUANTUM WIRES WITH IMPURITIES AT FINITE TEMPERATURE

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The temperature dependence of Coulomb blockade peaks of a one dimensional quantum dot is calculated. The Coulomb interaction is treated microscopically using the Luttinger liquid model. The electron interaction is assumed to be non-homogeneous with a maximum strength near the quantum dot. The conductance peaks show non-analytic power law behaviour induced by the interaction. It is shown that there is a crossover in the power law which is related to the inhomogeneity of the interaction.

1 Introduction

One dimensional quantum dots can be formed between two impurities in a semiconductor quantum wire when the electron density is sufficiently low. Transport measurements at such dots reflect the properties of the interaction between the electrons. Especially, the temperature power-law dependence of the Coulomb blockade conductance peaks is given by the interaction strength. In the present paper, we show that if the (short-ranged) interaction is non-homogeneous, i.e.

\[ V(x, y) = [V_0 + \varphi(x)] \delta(x - y), \]

there is a crossover in the temperature-power law between the interaction strength near the quantum dot \((|x| \ll b, \text{maximum of } \varphi(x))\) for high temperature, and the interaction strength far away from the dot \((|x| \gg b, \varphi(x) \approx 0)\) at low temperature.

2 Inhomogeneous Luttinger Liquid

Using the Bosonisation method the excitations in the spinless one dimensional (1D) interacting electron system are described by charge density waves in terms of the conjugate Boson fields \( [\varphi(x), \Pi(x')] = i \delta(x - x') \). The Hamiltonian is

\[ H = \frac{h v_F}{2} \int dx \left\{ \Pi^2(x) + \frac{1}{g_0} [\partial_x \varphi(x)]^2 \right\} + \frac{1}{2\pi} \int dx \varphi(x) [\partial_x \varphi(x)]^2 \]

with \( v_F \) the Fermi velocity. The fields are related to the electron density \( \rho \) via \( \partial_x \varphi(x) = \sqrt{\pi} [\rho(x) - \rho_0] \) where \( \rho_0 = k_F/\pi \) is the mean density. The parameter \( g_0 = [1 + V_0/\pi \hbar v_F]^{-1/2} \) is the interaction constant that arises from the asymptotic constant part of the interaction for \(|x| \to \infty\). The second term is due to the inhomogeneity treated as a perturbation. It is considered here only in lowest order.
The temperature Green function $G(x, x'; \tau, \tau') = \langle T_\tau [\vartheta(x, \tau) \vartheta(x', \tau')] \rangle$, where $T_\tau$ stands for imaginary time ordering, is

$$G_{\omega_n}(x, x') = G_{\omega_n}^0(x - x') + \frac{1}{\pi} \int dy \ G_{\omega_n}^0(x - y) \left[ \frac{\partial_y \varphi(y) \partial_y}{\partial_y \varphi(y) \partial_y - \omega_n} \right] G_{\omega_n}^0(y - x'). \quad (3)$$

The unperturbed Green function is $G_{\omega_n}^0 = (g_0/2\hbar \omega_n) \exp \{-g_0|\omega_n(x - x')|/\nu F\}$ at the Matsubara frequencies $\omega_n = 2\pi n/\beta \hbar$ with the inverse temperature $\beta = 1/k_B T$.

The transport properties are related to the propagator $G$. For instance, the nonlocal DC-conductivity is $\sigma_\omega_{\omega_n\to 0}(x, x') = \lim_{\omega_n \to 0} e^2 \omega_n G_{\omega_n}(x, x')/\pi$. This leads to the conductance quantum $g_0 e^2/h$. Hence, the conductance of a pure quantum wire is given by the interaction parameter for $|x| \to \infty$, $g_0$. It does not depend on the interaction within the wire.\[\]

3 Double Barrier

In the experiment, the electron density of the 1D electron system is lowered such that two maxima of the impurity potential are higher than the Fermi energy. Thus, a 1D quantum dot is defined between the maxima. We model this by assuming $V_d(x) = V_0 \delta(x - x_i)$ where $x_i = x_d, x_d + a$. The corresponding Hamiltonian is

$$H_d = V_d \sum_i \cos[2k_F x_i + 2\sqrt{\pi} \vartheta(x_i)].$$

For the conductance we need to calculate the effective action corresponding to the Hamiltonian. This is done with the imaginary time path integral method. The variables at $x \neq x_i$ are traced out. Eventually one obtains the effective action in the variables $\theta_\pm = (\theta(x_d) \pm \theta(x_d + a))/2$

$$S_{\text{eff}} = \frac{1}{2\hbar \beta} \sum_{n, \pm} \theta_\pm(\omega_n) K_\pm(\omega_n) \theta_\pm(\omega_n) + S_d[\theta_+, \theta_-], \quad (4)$$

where $S_d$ corresponds to $H_d$. The kernels depend directly on the one- and two-point functions of the inhomogeneous Luttinger system and can be written as $K_\pm(\omega_n) = \pi/2[G_{\omega_n}(x_d, x_d) \pm G_{\omega_n}(x_d, x_d + a)]$. We assumed the distance between the barriers, $a$, being much smaller than the extension $b$ of the inhomogeneity potential, $a \ll b$ such that the interaction varies only very slowly within the dot. The interaction parameter at the position of the double barrier is $g(x_d) = [1 + (V_0 + \varphi(x_d))/\pi \hbar v_F]^{-1}/2$. The charging energy of the dot then reads

$$E_c = K_-(\omega_n \downarrow 0) = \frac{\pi \hbar v_F}{2ag^2(x_d)}.$$\[\]

Thus, the charging energy is a local quantity which depends on the length of the dot and the repulsion strength in the dot region through $g(x_d)$.

The sequential tunneling rates are obtained in the limit of high barriers\[\]

$$\Gamma(E) = \left( \frac{\Delta}{2} \right)^2 \int_{-\infty}^{\infty} dt \ e^{iEt/\hbar - W(t)}. \quad (6)$$

The energies $E$ correspond to forward and backward tunneling through the left and right barrier respectively. The tunneling amplitude $\Delta$ is related to $V_d$ via the
WKB-method. The kernel $W(t)$ contains the information about the excitations in the leads and the excited states in the dot. By using the above Green function,

$$W(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\hbar\omega^2} \operatorname{Im} \frac{1}{G_{\omega, \omega, -\omega(x_d, x_d)}} \left[ 1 + \epsilon \sum_n \delta(\hbar\omega - n\epsilon) \right] \frac{1 - e^{-i\omega t}}{1 - e^{-\hbar\omega/\beta}} e^{-\omega/\omega_c}$$

(7)

where $\omega_c$ is the high frequency cutoff. The energy $\epsilon \equiv \pi \hbar v_F / a g(x_d) = 2g(x_d)E_c$ is the discrete level spacing of the plasmon states in the quantum dot. As the charging energy, this depends only on the interaction near the quantum dot, i.e. the local interaction strength $g(x_d)$. This holds for arbitrary shapes of $\varphi(x)$.

### 4 Linear Transport

In the limit of linear transport the chemical potentials in the left and right leads and the dot are aligned. Then Coulomb blockade is relaxed and the conductance versus the gate voltage shows a peak. For sequential tunneling we can use the master equation method for calculating the conductance for $k_B T \ll \epsilon$

$$G = \frac{e^2}{4} \frac{\beta - \beta \mu/2}{\cosh \beta \mu/2} \Gamma(\mu).$$

(8)

Here $\mu$ is the distance from the resonance energy. In the limit $\varphi(x) \equiv 0$ one can obtain the temperature behaviour of the conductance maximum analytically: $G_{\text{max}} \propto (\beta \hbar \omega_c)^{2-1}/g_0$. In general one needs to calculate $G$ numerically. Figure 1 shows the peak shape and the temperature behaviour of the maximum for $\varphi(x) = V_1/(1 + 4x^2/b^2)$. Contrary to the naïve guess, namely that the exponent contained simply the local interaction parameter $g(x_d)$, we observe a crossover from an effective $g_{\text{eff}}$ resembling $g(x_d)$ at high temperatures towards $g_0$ at low temperatures. The crossover temperature is related to the extension of the inhomogeneity $b$. Thus, the temperature power law can be governed by any value of $g_{\text{eff}}$ between the two asymptotic limits. Our finding leads to a better understanding of the controversy between the experimentally determined interaction parameter in nonlinear and linear transport through a 1D quantum dot device. The nonlinear transport experiment probes the local interaction at the position of the dot ($g(x_d) < g_0$) in terms of level spacing and charging energy. However, the examination of the conductance peaks involves the entire Luttinger liquid system with its excitations and the inhomogeneity. Hence an effective $g$-parameter depending on the temperature range of the experiment can in principle be extracted from the power law behaviour of the peak maximum.

Preliminary results suggest that for a considerable inhomogeneity the temperature behaviour could even be reversed during the crossover from $g(x_d)$ to $g_0$ and thus indicating almost arbitrary effective interaction parameters extracted from experiments in such temperature regimes.

### 5 Conclusion

For the example of the temperature dependence of the Coulomb blockade conductance peaks of a 1D quantum dot, we have shown that inhomogeneity effects in a
Figure 1. Logarithmic dependence of the conductance maximum on temperature $T$ for $g_0 = 1.0$ and $g(x_d) = 0.6$ (full curve). Parameters are $\omega_c = v_F/g_0b$ crossover frequency; dot position in the center of the inhomogeneity $x_d = 0$; ratio between dot length and extension of the inhomogeneity $a/b = 0.1$. Straight dotted lines: homogeneous limits for $g(x) = g_0$ and $g(x) = g(x_d)$. Inset: Coulomb blockade conductance peaks for $k_B T/\hbar \omega_c = 0.05, 0.1, 0.4$ (top to bottom at $\mu = 0$); $E_C$ charging energy; peaks corresponding to $g_0$ and $g(x_d)$ are out of the scale of the inset.

Luttinger system can lead to considerable changes in the non-analytic power law. Depending on the temperature region, different exponents can occur that reflect the strength of the interaction in different parts of the system. Our results indicate that deducing Luttinger liquid parameters from different sets of experimental data might lead to contradicting numerical values. In particular, the above model opens a possibility to explain certain discrepancies in the parameters obtained from cleaved-edge overgrowth quantum dots.

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