Thermal interaction of underground pipeline with freezing heaving soil

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Abstract. A mathematical model and a method for calculating the stress-strain state of a pipeline describing the heat-power interaction in the "underground pipeline - soil" system in the conditions of negative temperatures in the soils of soils are offered. Some results of computational-parametric research are presented.

1. Introduction
Reliability of pipelines in the underground method of laying is determined to a large extent by the thermal and moisture state of the surrounding soils subjected to frost heaving. In conditions of seasonal temperature fluctuations, under certain conditions, frost heave forces are formed in the soil base and, as a consequence, additional stresses appear in the walls of the pipeline. The magnitude of the cramped forces depends on the soil type and, to a large extent, on the moisture state, therefore, to predict the thermophysical and strength parameters, a complex physico-mathematical modeling of heat and mass transfer in the soil and calculation of stresses in the walls of the pipeline are necessary [3, 8, 9, 10].

2. Materials and methods
In the offered model, the soil is considered as a substance consisting of three phases: water, ice and a dry soil skeleton. The mass balance equations for each time interval \((t(n), t(n+1)=t(n)+\Delta t)\) in a fixed \(ij\)-th control volume (CV) \(V_{ij}\) are recorded separately for each phase [1,7]:

- for water

\[
\frac{\rho_{ijw}^{(n+1)} - \rho_{ijw}^{(n)}}{\Delta t} \cdot V_{ij} = - \sum_{k=1}^{4} \rho_{ijkw} \cdot V_{ijkw} \cdot \Delta S_{ijk} + I_{iw} \cdot V_{ij}
\]

(1)

- for ice

\[
\frac{\rho_{ij}^{(n+1)} - \rho_{ij}^{(n)}}{\Delta t} \cdot V_{ij} = I_{wI} \cdot V_{ij}
\]

(2)
where: $\rho_{ijw} = \frac{m_w}{V_{ij}}$ – reduced water density; $\rho_{ijI} = \frac{m_I}{V_{ij}}$ – reduced ice density; $V_{ij}^n$ – the velocity of the migrating moisture flux for the $k$-th bound CV $\Delta S_{ijk}$; $I_{Iw}$ – pore ice melting intensity; $I_{wI}$ – pore water crystallization intensity.

The equation of internal energy in the assumption of the same temperature of the components of a substance, and taking into account the neglect of the power of internal forces for the corresponding time $(t(n), t(n+1)=t(n)+\Delta t)$ in each $ij$-th CV, will be as follows:

$$
\left(\rho_{ij} \cdot U_{ij} + \rho_{ijI} \cdot U_{ijI} + \rho_{ijw} \cdot U_{ijw}\right)^{n+1} - \left(\rho_{ij} \cdot U_{ij} + \rho_{ijI} \cdot U_{ijI} + \rho_{ijw} \cdot U_{ijw}\right)^n \cdot V_{ij} = \\
- \sum_{k=1}^{4} \rho_{ijkw} \cdot V_{ijkw} \cdot U_{ijkw} \cdot \Delta S_{ijk} - \sum_{k=1}^{4} q_{ijk}^n \cdot \Delta S_{ijk},
$$

where: $\rho_{ij} = \frac{m_{ij}}{V_{ij}}$ – reduced density of dry soil; $U_{ij}$, $U_{ijI}$, $U_{ijw}$ – specific internal dry skeleton of soil, ice, water; $q_{ijk}^n$ – conductive specific heat flux through the $k$-th edge $\Delta S_{ijk}$ of the $ij$-th.

The specific internal energy of the substance component in the $ij$-th CV is defined as [6, 13]:

$$
U_{ij} = C_{ij} \cdot T_{ij} + U_{ijI} = C_I \cdot T_{ij}; U_{ijw} = C_w \cdot (T_{ijI} - T_o) + U_{wI},
$$

where: $T_{ij}$ – the temperature of the center of the $ij$-th control volume under consideration; $C_{ij}$, $C_I$, $C_w$ – specific heat for the dry skeleton of soil, ice, water; $U_{wI}$ – a constant determined from the normalization condition of the phase transition: $U_{wI} = l_w + C_W T_0 + P_o / \rho_L^{(0)} - P_o / \rho_I^{(0)}$.

where: $l_w$ – specific heat of ice melting; $P_o, T_0$ – pressure and corresponding temperature of phase transition; $\rho_L^{(0)}, \rho_I^{(0)}$ – density of ice, water.

In view of the smallness of the temperature gradient, the process of transfer of liquid moisture is described by the law of isothermal moisture conductivity [5, 14]:

$$
\bar{\nu} = -D(\theta, \theta_I) \cdot \nabla \theta,
$$

where: $D(\theta, \theta_I) = D_{L}(\theta) / (1 + n \cdot \theta_I)$ – the diffusion coefficient of moisture in the soil, determined by water content $\theta = \rho_w / \rho_w^{(0)}$ and the volume content of ice $\theta_I = \rho_I / \rho_I^{(0)}$; $D_L(\theta)$ – the diffusion coefficient of thawed soil; $n$ - experimental constant, the value of which depends on the soil.

Specific densities of heat fluxes $q_{ijk}^n$ are calculated according to the Fourier law. The coefficient of thermal conductivity of the soil is determined by taking into account the phase composition for each control volume at each time step. The thermophysical model of heat and mass transfer is closed by defining equations for transfer of liquid moisture and phase transitions, thermophysical and hydrodynamic characteristics of soils, and boundary conditions for heat and mass transfer.

The position of the pipeline in the vertical plane at an arbitrary time step can be found from the solution of the well-known longitudinal-transverse bending equation [4], written with regard to the boundary conditions corresponding to the rigid fixation at the ends of the section under consideration $[0;L]$:
\[
\begin{align*}
E \cdot J \cdot \frac{d^4W}{dz^4} + N \cdot \frac{d^2W}{dz^2} - q_n(z) &= 0 \\
W(0) &= W(L) = 0 \\
\frac{dW(0)}{dz} &= \frac{dW(L)}{dz} = 0 \\
\quad z \in [0; L] 
\end{align*}
\]  

where \( q_n(z) \) – the specific strength of the frost heaving of the soil acting per pipeline length unit; \( W(z) \) – cross-motion of the pipeline section with coordinate \( z \), function \( W(z) \) determines the altitude of the pipeline section; \( L \) – length of the pipeline section subjected to the action of frost heaving; \( E \cdot J \) – bending rigidity of the considered pipeline section; \( E \) – Young's modulus of the pipeline wall material; \( N \) – longitudinal force at the ends of the pipeline section. The additional longitudinal stresses arising as a result of the frost heave action in the upper and lower generatrix of the pipeline are calculated as:

\[
\sigma(z) = \pm E \cdot D_H \cdot \frac{d^2W}{dz^2},
\]

where: \( D_H \) – the outer diameter of the pipeline.

Function \( q_n(z) \) is constructed according to the quadratic law of constrained heave [4]:

\[
q_n(z) = D_H \cdot R \cdot \left( 1 - \frac{W(z)}{\Delta H^{(n)}} \right)^2,
\]

where: \( R \) is the design resistance of the soil; \( \Delta H^{(n)} \) – displacement of the ground averaged over the pipeline diameter. For the \( n \)-time step, \( \Delta H^{(n)} \) is determined by the increase in height \( \Delta h^{(n)}_{ij} \) of the CV under the pipeline (Fig. 1), assuming that the increase in the volume of the ground during heaving is realized only in the vertical direction:

\[
\Delta H^{(n)} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M(i)} \Delta h^{(n)}_{ij}}{N},
\]

Free piercing \( \Delta h^{(n)}_{ij} \) of CV, at each time step according to the calculated fields of liquid moisture and ice content is calculated as:

\[
\Delta h^{(n)}_{ij} = \begin{cases} 
0, & m_{ij} \geq \frac{\rho^{(n)}_{ijB}}{\rho^0_B} + \frac{\rho^{(n)}_{ijL}}{\rho^0_L} \\
\Delta y_{ij} \cdot \left( \frac{\rho^{(n)}_{ijB}}{\rho^0_B} + \frac{\rho^{(n)}_{ijL}}{\rho^0_L} - m_{ij} \right), & m_{ij} < \frac{\rho^{(n)}_{ijB}}{\rho^0_B} + \frac{\rho^{(n)}_{ijL}}{\rho^0_L},
\end{cases}
\]

where: \( m_{ij} \) - the porosity of the soil corresponding to the CV; \( \Delta y_{ij} \) - vertical size of CV.
The calculation and parametric study of the effect of pipe and outdoor air temperatures on the pipeline's altitude position and on the value of additional longitudinal stresses in its walls was performed. The study was carried out for an underground pipeline with a diameter of 820 mm, and laying depth of 600-1000 mm [11, 12].

The ambient air temperature was set by a potential function:

\[ T_B = A \cdot \sin\left( 2 \cdot \pi \cdot \frac{\tau}{365} \right) \quad (10) \]

where: \( A \) – maximum deviation of the temperature from 0°C;
\( \tau \) - time in days (\( \tau \in [0;180] \)).

The calculated change in the vertical deflections of the pipeline over time is shown in Fig. 2.

Fig. 3 shows the calculated distributions of additional longitudinal stresses of the lower generatrix of the pipeline. Fig. 4, 5 show calculated dependences of the vertical displacement of the central section of the pipeline.

Figure 2. High-altitude position of the pipeline at different times. The pipe temperature is -10 °C; the amplitude value of the air temperature is -20 °C
Figure 3. Distributions of additional longitudinal stresses under the pipeline over time. The wall temperature of the pipeline is -10°C; the value of the parameter is $A = -20^\circ C$.

Figure 4. Dependence of the vertical displacement of the central section of the pipeline on its temperature.

Figure 5. Dependence of the vertical displacement of the central section of the pipeline on its temperature, taking into account migration of moisture: 1 - taking into account migration of moisture; 2 - not taking into account the migration of moisture.
3. Conclusion
Based on the results of the computational and parametric study, a significant influence of the pipe wall and external air temperature on the additional longitudinal stresses arising in the walls was revealed. The maximum value of the additional voltage is formed at the ends of the section.

A significant effect of migration of moisture on the change in the altitude position of the pipeline is shown.

It has been established that at a pipe wall temperature of -4 °C, the deformation of the pipe in clayey soil increases 1.4-1.5 times in comparison with similar conditions in the absence of migration.

Based on the results of the study of the effect of pipe and outdoor air temperatures, a significant influence of the pipeline temperature on its altitude and the magnitude of the additional stresses arising in the walls of the pipe was established. The initial distribution of the reduced moisture density in depth was set at a constant value of 450 kg/m$^3$, which corresponded to the total moisture saturation of the soil. The greatest value of additional stress is observed at the ends of the pipeline section under consideration. It was noted that a decrease in the minimum air temperature from -10 °C to -20 °C with a given constant pipe temperature in the range from -5 °C to -15 °C leads to an increase in altitude position of only 1.01-1.03 times. Thus, under the conditions considered, there is a more noticeable effect of the pipe temperature than the ambient temperatures on the stresses.

The study of the influence of the moisture migration intensity made it possible to establish a significant role of migration in the pipeline deformation when the temperature of the pipe varies from -2 °C to -5 °C. At the initial moment, a linear increase in the reduced moisture density from 437 kg/m$^3$ on the surface to 450 kg/m$^3$ (full moisture saturation) was taken at a depth of 3 m or more. As the temperature of the pipe decreases, the role of migration decreases significantly, so at a constant pipe temperature of -10 °C, the ratio of vertical displacements calculated with and without migration is only 1.04, and at a temperature of -4 °C; the ratio is 1.54.

Investigations of the influence of snow cover thickness, carried out for a pipeline having a temperature equal to the local soil temperature, showed that under the considered conditions, a significant increase in the vertical pipeline movement to 0.012 m occurs with a decrease in the thickness of the snow cover from 0.2 m to zero. With a snow cover thickness more than 0.2 m, the pipeline deformations are insignificant.

Thus, the considered results of calculations show that the developed model and the calculation method are a convenient tool allowing determining the dynamics of changes in additional stresses in the pipe wall under real conditions of changing the parameters of heat and mass transfer in frozen soils.

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