Experimental result on the propagation of Coulomb fields

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Abstract.
In planetary systems the problem arises whether gravity attracting the planets towards the central star has an instantaneous action or propagates with finite velocity. Laplace noticed that, if gravity propagated with finite velocity, planets motion would become unstable due to a torque originating from time lag of the gravitational interactions. Given that actions describing gravitational interaction are formally the same as that describing electrostatic interactions, we have performed an experiment meant to measure the time/space evolution of the electric field generated by an uniformly moving set of electrons. The results we obtain seem compatible with an electric field rigidly carried by the beam itself.

1. Introduction
The idea of performing an experiment to measure Coulomb fields propagation speed bears its rationale from the puzzling phenomenon of the gravitational interaction for planetary systems. In the solar system, for instance, the force attracting Earth towards the Sun is not directed where we see the Sun but where the Sun really is, that is where we shall see it about eight minutes later. In other words it is as the gravitational force propagated with infinite velocity, or, at least, with a speed much greater than the speed of light; The reason being that a time lag between actual position and position where we see the Sun would generate a torque which would not allow a stable orbit. This fact is well known since the Newton’s time [1]. Laplace [2] calculated that the propagation velocity of the Newtonian force should have been more than a million times the velocity of light.

Such an intriguing behavior occurs in electromagnetism when the field of an electric charge moving with constant velocity is computed. One finds that in such a case the electric field at a given point \( \mathbf{P}(x, y, z, t) \) evaluated with the Liénard-Wiechert (L.W.) potentials is identical to that calculated by assuming that the Coulomb field travels with infinite velocity. Feynman [3] points out that a straight uniform motion continues indefinitely and that the uniform speed can be used to determine times and positions.

To verify if the Feynman interpretation of the L.W. potentials holds in case of a charge moving with constant velocity for a finite time, we have performed an experiment to measure the time evolution of the electric field produced by an electron beam in our laboratory, covering a wide range of transverse distances with respect to the beam line (up to 55 cm). The result have been published in [4] and might be interpreted as compatible with an instantaneous propagation of the Coulomb field.
In this paper I want to discuss in more detail the experimental results and interpretation.

2. **Theoretical Considerations**

The electric field at \( \mathbf{r}(x, y, z) \) from a charge \( e \) traveling with constant velocity \( \mathbf{v} \), at a time \( t \) can be written, using the Liénard-Wiechert retarded potentials as [5, 6, 7]:

\[
E(\mathbf{r}, t) = \frac{e}{4\pi\varepsilon_0} \frac{1 - \frac{v^2}{c^2}}{R(t') - \frac{\mathbf{R}(t') \cdot \mathbf{v}}{c} - \frac{\mathbf{R}(t') \cdot \mathbf{v}}{c} \frac{\mathbf{R}(t')}{c}}, 
\]

where

\[
\mathbf{R}(t') = \mathbf{r} - \mathbf{v}t'
\]

is the distance between the moving charge and the space point where one measures the field at time \( t \), and

\[
t' = t - \frac{R(t')}{c}.
\]

The field from a steadily moving charge can also be written (as easily deducible from eqn 1 in case of constant velocity) [3, 5, 6, 7] as

\[
E(t) = \frac{e}{4\pi\varepsilon_0} \frac{\mathbf{R}(t)}{R(t)^3} \frac{1 - \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}\sin^2(\theta(t))}^2
\]

where \( \mathbf{R}(t) \) is the vector joining the charge position and the point at which we evaluate the e.m. field at time \( t \) (eqns. 38.8 and 38.9 of [5]) and \( \theta(t) \) is the angle between \( \mathbf{v} \) and \( \mathbf{R}(t) \).

A pictorial view of the various quantities mentioned in eqns. 1 and 4 can be seen in Fig.1.

If we indicate with \( y \) the generic transverse coordinate, using eqn.1 we can compute the maximum transverse electric field w.r.t. the direction of motion, given by \( \gamma \equiv 1/\sqrt{1 - v^2/c^2} \):

\[
E_{\text{max}} = \frac{e}{4\pi\varepsilon_0} \frac{\gamma}{y^2},
\]

a value obtained when the charge is at a distance \( \gamma y \) at a time

\[
t' = t - \frac{\gamma y}{c}
\]

Using the L.W. potentials as given by eqn.1 we have calculated the field, normalized to \( E_{\text{max}} \), generated by a relativistic electron (\( E = 500 \text{ MeV} \)) moving along the \( z \) axis, at a transverse distance \( y = 30 \text{ cm} \), as shown in Fig. 2.

The sensor is located at the zero abscissa and measures the field launched by the charge when it is at the \( z \) location. We observe that the maximum value of the field, in this particular case, is obtained if the field is launched by the charge at a distance of \( z = 300 \text{ meters} \) away from the sensor. By the time it takes to the field to reach the sensor, traveling with velocity \( c \), the electric charge, traveling with its own velocity smaller than \( c \), reaches a point just under the sensor located at a transverse coordinate \( y = 30 \text{ cm} \). Thus, from the point of view of the observer, he does not know whether the signal is due to the field launched by the charge at a distance of 300 m or to an instantaneous field due to the charge when it has reached the closest distance, as calculated with eqn.4.

Should one evaluate the field, assuming that just the last 10 m of the beam path would be active in launching the field itself, one then would be in the situation depicted in the lower graph of Fig.2: the response of the sensor would be smaller by a factor \( \approx 10^{-4} \).
3. The Experiment

In our experiment we measure the electric field generated by the electron beam produced at the DAΦNE Beam Test Facility (BTF) [8], a beam line built and operated at the Frascati National Laboratory to produce a well-defined number of electrons (or positrons) with energies between 50 and 800 MeV. At maximum intensity the facility yields, at a 50 Hz repetition rate, 10 nsec long beams with a total charge up to several hundreds pCoulomb. The electron beam is delivered to the 7 m long experimental hall in a beam pipe of about 10 cm diameter, closed by a 40 µm Kapton window. Tests were carried out shielding the exit window with a thin copper layer, but we did not observe any change in the experimental situation. At the end of the hall a lead beam dump absorbs the beam particles. In our measurements we used 500 MeV beams of \(0.5 \div 5.0 \times 10^8\) electrons/pulse.\((\gamma \simeq 10^3)\).

A schematic view of the experimental setup is shown in Fig. 3. At the beam pipe exit flange, electrons go through a fast toroidal transformer measuring total charge and providing redundancy on our LINAC-RF based trigger.

To measure the electric field we used as sensors 14.5 cm long, 0.5 cm diameter Copper round bars, connected to our Data Acquisition System by means of fast, terminated coax cables. Signals out of the sensors were stored on a Switched Capacitor Array (SCA) working at 5 GHz sampling frequency.

The Coulomb field acts on the sensor quasi-free electrons, generating a current. An example of the recorded signals is shown in Fig. 4. The pulse shape depends on the inductance, capacitance and resistance (L, C, R) of the detectors.

Figure 1. A pictorial view of various quantities mentioned in eqns 1 and 4
Figure 2. The electric field from eqn. 1 normalized to its maximum value, $E_y(R)/E_{max}$, generated by 500 MeV electrons as a function of $z'$ (or $t'$, lower abscissa scales), expected at $(z = 0 \text{ cm}, y = 30 \text{ cm})$. $z'$ and $t'$ are defined in eqns. 2 and 3. The horizontal scale of the upper graph (a) is such to include the point where $E_y(R) = E_{max}$; the lower graph (b) is a close-up of the region $z \in [-10, 0] \text{ m}$ typical of our experiment (note the different vertical scales).

The sensor response $V(t)$ for a step excitation $V_0$ can be written as:

$$V(t) = V_0 \cdot e^{-\frac{R^2}{L^2}} \sin(\omega t).$$

(7)

The natural frequency of the detectors is $\approx 250 \text{ MHz}$. The voltage difference between the bar ends for the maximum value of the Coulomb field, obtained suitably modifying eqn. 5 for a finite longitudinal extent of the charge distribution, (in our case, the electron beam is $\approx 3 \text{ m. long}$) is:

$$V_{max}^t = \eta \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{y + 14.5 \text{ cm}}{y}\right),$$

(8)

where $\lambda$ is the charge per unit length of the incoming beam and $\eta$ is the sensor calibration constant. In the electric field calculations, the image charges appearing on the flange as the beam exits the pipe have also been included. However, as their effect decreases rapidly with the distance from the flange, it is completely negligible in our experiment (minimum distance $\approx 1 \text{ m}$). The sensor calibration has been carried out using a known field generated by a parallel plate capacitor. We find experimentally $\eta = 7.5 \times 10^{-2} \pm 3\%$, however due to various systematic effects we believe our calibration to be good to $\approx 20\%$, in absolute terms.

Assuming that the L.W. formula, (eqn 8), holds (which should apply only if the uniform charge motion would last indefinitely and the charges generating the field would not be shielded by conductors) we expect, in our typical beam operating conditions, pulse heights of the order of 10 mV out of our sensors.
Figure 3. Schematic side of our experimental apparatus. Sensors A2, A4, orthogonal to the figure plane are not shown.

In the more realistic hypothesis that because of the beam pipe shielding and the finite lifetime for the charges uniform motion, as it is in our experiment, the expected amplitude due to the field launched from short distances, cfr. fig.2, would be of the order of few nanoVolt and hence unmeasurable.

We used six sensors: four of them, A1,A2,A3 and A4 in the following, were located at a (longitudinal) distance of 92 cm from the beam exit flange (cfr. fig 5), in a cross configuration, each at a transverse distance of 5 cm from the beam line. The main purpose of these four sensors is to provide reference for the other two detectors A5 and A6 located, through out the measurements, at various longitudinal and transverse coordinates along the beam trajectory.

4. Measurements and data base
Electron beams were delivered by BTF operators at a rate of few Hertz; data were collected in different runs, identified by given longitudinal and transverse position of the movable detectors (A5 and A6).

We collected a total of eighteen runs, spanning six transverse positions and three longitudinal positions of A5 and A6 for a total of about 15,000 triggers. Through out the data taking, the references sensors (A1,A2,A3,A4) were left at the same location (92 cm. from the beam exit flange) in order to extract a timing and amplitude reference. As mentioned before, we collected data with the movable sensor at 172 cm, 329.5 cm and 552.5 cm longitudinal distance from the beam exit flange. For each of the longitudinal positions we collected data on six transverse positions: 3, 5, 10, 20, 40 and 55 cm from the nominal beam line. For each run, A5 and A6 were positioned symmetrically with respect to the nominal beam line; spatial precision in the
Figure 4. Typical A5 (left) and A6 (right) sensor responses. The lower plots show in detail the granularity of our time measurements. (200 psec./bin)

Figure 5. A photograph of the beam pipe exit window and of the four reference sensors in the cross configuration.
sensor positioning was of the order few mm in the longitudinal coordinate and about 1 mm in the transverse one.

We define:

$$S_n = \frac{V_{\text{max}} \times 10^8}{N_{\text{elec}}}, \quad (9)$$

where $V_{\text{max}}$ is the peak signal recorded by the SCA and $N_{\text{elec}}$ is the total number of electrons in the beam, as measured by the fast toroid. The factor $10^8$ in eqn.9 takes into account the typical beam charge. The details of the analysis have been described at length in our paper [4], so we will just show our final results.

We have made several measurements, keeping the reference sensors A1...A4 at the same location and the moving sensors A5 and A6 at different $y$ and $z$ locations in order to verify the response of the reference sensors in different environmental situations.

The measured values of $S_n$ for the reference sensors 1 and 3 are $S_n(1) = (8.98 \pm 0.54)$ mV and $S_n(3) = (10.25 \pm 0.59)$ mV and do not depend on the different environmental situations, as expected. These figures are consistent with the potential calculated with the L.W. formula of eqn.8 on the same reference sensors, located at $y=5$ cm: $V_{\text{max}}^{t} = 9.78$ mV, the calculated value for a charge indefinitely moving with constant speed. It is important to remark that no normalization is applied to equalize the absolute scales of the measured and calculated values.

We find the agreement between measurements and prediction very remarkable.

We stress again that the amplitude we measure is many orders of magnitude higher than the one would expect from the unshielded beam charge. Were we sensitive only to fields generated by the electron beam once they exited the beam pipe, our pulse height would have been, as mentioned in the previous section, in the few nanoVolt range, much less than the noise of our apparatus, and then undetectable. Instead we do detect large signals in the milliVolt region, so they cannot be due to the unshielded beam charges.

In figs. 6, 7 and 8 we show the amplitude ratios between sensors A5 and A1 (A6 and A3) as a function of transverse distance from the beam line. Also in this case, data are completely consistent with the logarithmic behavior of eqn 8.

We remark that the amplitudes measured at all transverse positions of the detectors do not depend on the different longitudinal distances, showing, once more, that the field does not decreases moving far away from the beam exit pipe.

4.1. Timing measurements

Our 200 psec/chn SCA provides timing information for detector outputs, so that it is possible to measure both longitudinal and transverse position-time correlations. As a reminder we stress again that, in the hypothesis of stationary constant speed motion, no time difference is expected as a function of transverse distance, while different longitudinal positions should exhibit delays consistent with particles traveling at $\gamma \approx 1000$.

The timing performance of our detectors are shown in fig.10 where the time distribution of each sensor with respect to the beam trigger is depicted. Also for the time measurements we will have to rescale the errors yielded by standard procedures extracting central values from quasi Gaussian distributions; in this case we impose that, by symmetry, the time difference between A5 and A6 be independent of the transverse distance between detector and nominal beam line.

The upper graph of fig 10 shows the time difference relative to 172 cm longitudinal1 distance data. The amount of rescaling, in this case, is about a factor of 10 and the overall resolution on time difference measurements is of the order of 50 psec.

The data show no time dependence of the sensor signal on transverse distance: the reduced $\chi^2$ for the hypothesis of a constant delay as function of $y$ is always below 2 at each longitudinal positions. Furthermore, adding a linear term depending on transverse distance for the sensor
Figure 6. Upper graph: the points show the ratio \( \frac{V_{\text{max}(A_5)}}{V_{\text{max}(A_1)}} \) at \( z_{A_5,A_6} = 172.0 \text{ cm} \) versus the transverse distance. Lower graph: \( \frac{V_{\text{max}(A_6)}}{V_{\text{max}(A_3)}} \). The continuous lines represent eqn. 8 for the depicted ratios. The two reduced \( \chi^2 \) are respectively 1.82 and 1.06. No fit has been performed on the data: the reduced \( \chi^2 \) has been evaluated from eqn 8 and the experimental data.

We summarize the time distance correlations in Table 1, where the data obtained at the three different longitudinal positions are shown.

Table 1. Timing measurements. The expected differences are calculated for 500 MeV electrons. In the last column the average timing value between A5 and A6 (see text).

| Longitudinal distances between two sensors [cm] | Expected [ns] | Experimental \( \overline{A_5, A_6} \) [ns] |
|-----------------------------------------------|--------------|------------------------------------------|
| \( (552.5-329.5) \) 223.0±1.5                 | 7.43 ± 0.05  | 7.40 ± 0.06                              |
| \( (552.5-172.0) \) 380.5±1.5                  | 12.68 ± 0.05 | 12.73 ± 0.09                             |
| \( (329.5-172.0) \) 157.5±15.                   | 5.19 ± 0.05  | 5.19 ± 0.07                              |

The time measurement reported on our original paper were criticized by A. Shabad [12]: actually one single measurement was discussed at length. The author speculated that it might infringe the speed of light boundary. The effect amounts to a bit less than three standard deviations, usually considered in the realm of fluctuations; however, if one analyzes ALL the measurements reported in table 1 of ref [4] one finds that measurements concerning sensor 5 come early, while the measurements concerning sensor 6 tend to be late. There is probably a
Figure 7. The same plot as in fig.6 at $z_{A5,A6}=329.5$ cm. The two reduced $\chi^2$ are respectively 1.36 and 0.66.

Figure 8. The same plot as in fig.6 at $z_{A5,A6}=552.5$ cm. The two reduced $\chi^2$ are respectively 0.91 and 2.48.
Figure 9. Time measurements accuracy. On the left hand side the four stationary sensors are shown. The time difference for the different pair is due to different cabling: a 10 nsec. delay was added to the radial sensors for calibration purposes. On the right hand side the movable sensors are shown. The widths of the distributions are remarkably similar with a $\sigma$ of $\approx 0.5$ nsec. With the available statistics the accuracy with which we can determine the centroid of the distribution is $\approx 50$ psec.

systematic effect having to do with a less than perfect flatness of the experimental hall floor which translates in a non vertical support for the sensors: $\approx 35$ mrad angle would cause the reported difference$^1$.

4.2. Do our sensors see E.M. radiation?

We performed different tests in order to ascertain that E.M. radiation coming from the interaction of the electron beam with its environment was not the original cause of our sensors’ response.

With the beam steering system, we changed the launch angle in the experimental hall; varying the current of the beam line magnet(s) one can predict the amplitude ratio of two detectors located right and left of the beam line, according to the calculated beam position at the sensors’ longitudinal coordinates. Special runs were taken to this purpose and the results are completely consistent with the expected horizontal beam displacement w.r.t. the nominal position.

Other E.M. phenomena are related to boundary crossings: as the beam travels between different media (e.g. the beam exit flange) E.M. radiation can be generated which, in turn, might mimic pulses we assume due to the interaction of the beam itself with our sensors. The experimental situation can be schematized as a Tamm [9] problem: a beam of particles traveling inside the LINAC vacuum pipe, suddenly appears out of the end flange of the accelerator, moves with uniform velocity through out the experimental hall ($\approx 7$ m.) and disappears in the concrete wall of the hall. A calculation of the expected effect, using the formulae reported in [10] lent us confidence that this background was not relevant; however, in order to demonstrate that such a phenomenon does not contribute (or contributes very little) to our sensors’ signal, we had a dedicated run during April 2014.

We collected data in two different modes:

(i) *Calibration* runs in order to match the data collected during the 2012 campaign to the latest (2014) runs.

(ii) *Beam dump* runs in which the electron beam was stopped in a 40 $X_0$ lead dump before reaching the vertical detectors A5 and A6.

$^1$ A detailed confutation of the Shabad criticism is reported in ref.[13].
Figure 10. Top graph: time difference between A5 and A6 versus the transverse distance. Errors have been rescaled according to the procedure described in the text. Middle and bottom plots: time difference between the movable sensors A5 and A6 (respectively) and one of the fixed sensors, A1. \( z_{A5,A6} = 172 \text{ cm.}, z_{A1} = 92 \text{ cm.} \) The line at \( 1.549 \pm 0.036 \text{ nsec.} \) (middle) and \( 1.500 \pm 0.036 \text{ nsec} \) (lower) indicate the weighted average of our measurements, once the reduced \( \chi^2 \) is rescaled according to the procedure described in the text (par. 4.2). The values obtained for the A5-A1, A6-A1 absolute delays have to be corrected for the cables of different lengths (1.1 nsec.).

The underlying idea was that runs taken with the beam dump, would yield the response of the A5,A6 detectors in a no-beam situation thus allowing us to map the pulse height of our detectors when just backgrounds were present in the experimental hall. From fig.11 one can infer that the main features of the previous (2012) measurements are retained in the (2014) latest run; small difference in the absolute values for the given ratios can be attributed to a less than perfect alignment of the two sets of detector on the beam line.

Fig 12 shows the measurements when the 40 X\_0\text{ lead absorber is inserted between the A1...A4 and the A5,A6 sensors. The dot shows the average measurement of the two vertical sensors in the cross configuration before the beam stopper. The triangles show the average measurements made with the movable detectors A5 and A6 for different vertical positions, after the beam has been stopped. The vertical sensors responses are, with the beam dump in place, reduced by a factor \( \approx 10 \), at 5 cm (transverse) distance, with practically no dependence on (transverse) distance from the beam line. Such behavior lends itself to the interpretation that the overall amount
Figure 11. Amplitude ratios vs transverse distance of sensors A5, A6 for calibration runs: open circles: A1 A3 ratio, triangles: A5 A6 ratio, full dots: A5 A1 ratio.

Figure 12. Measurements with the beam stopper. The open circle shows the average measurement of the two vertical sensors in the cross configuration before the beam stopper. The triangles show the average measurements made with the movable detectors A5 and A6 for different vertical positions, after the beam has been stopped.
E.M. background originating either at the transition flange or at the beam dump entrance is indeed small w.r.t. the response obtained when beams unimpeded go through the experimental hall.

Speculations have been made [12] that signals out of our sensors might come from electron beams radiation in the last bend of the experimental hall. On this count we have to point out that:

(i) no dependence has been seen of the sensor signals on the longitudinal distance from the last bend.

(ii) A complete calculation has been carried out of the total energy electron beams radiate in the last bend and, in the frequency range our sensors cover, it has been found to be completely negligible with respect to the response our detectors provide as the electron beams go through the experimental hall. A detailed report in in preparation soon to be published soon.[13].

5. Discussion
With reference to table 1, we notice that the longitudinal time differences are completely consistent with the hypothesis of a beam traveling along the $z$ axis with a Lorentz factor $\gamma \approx 1000$.

Such a situation agrees with the Liénard-Weichert model. Retarded potentials, however, predict that most of the virtual photons[11] responsible for the field detected at coordinates $z$ and $y$ are emitted several hundred meters before the sensor positions and at different times according to the detectors transverse distances. Conversely, assuming that such virtual photons are emitted in a physically meaningful region (between the beam exit window and our detectors), the amplitude response of the sensors should be several order of magnitude smaller than what is being measured (cfr. Fig.2). Our result, obtained with a well definite set of boundary conditions (longitudinal and transverse distance between beam line and sensors, details of the beam delivery to the experimental hall etc.) matches precisely (within the experimental uncertainties) the expected value of the maximum field calculated according to L.W. theory, that is also the value calculated with Eq.4 when the beam is at the minimum distance from the sensor.

We again point out that the consistency of our measurement with eqn 8 has been obtained without any adjustment of normalization.

6. Conclusions
Assuming that the electric field of the electron beams we used would act on our sensor only after the beam itself has exited the beam pipe and given that Cerenkov and/or transition radiation effects are negligible, the L.W. model would predict sensors responses orders of magnitudes smaller than the ones we measure. The Feynman interpretation of the Liénard-Weichert formula for uniformly moving charges does not show consistency with our experimental data. Even if the steady state charge motion in our experiment lasted few tens of nanoseconds, our measurements indicate that everything behaves as if this state lasted indefinitely.

Apart from complete consistency of the sensors’s amplitude with L.W. calculation for an unshielded beam, it is extremely difficult, in our opinion, to reconcile any ballistic theory with the results of the beam dump measurement (cfr. section 4.2). To summarize our finding in few words, one might say that our measurements are consistent with the hypothesis that the Coulomb field is carried rigidly by the electron beam.

We welcome new experimental work on the subject as well as new theoretical interpretations of the reported measurements.
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