THE NEUTRINO CROSS SECTION AND UPWARD GOING MUONS

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Abstract

The charged current cross section for neutrinos with energy of a few GeV is reanalysed. In this energy range the cross section for the lowest multiplicity exclusive channels is an important fraction of $\sigma_{CC}$ and the approximation of describing the cross section with deep inelastic scattering formulae may be inaccurate. Possible consequences of our reanalysis of the cross section in the interpretation of the data obtained by deep underground detectors on $\nu$-induced upward going muons (both stopping and passing) are discussed.

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It is well known that measurements of the atmospheric neutrino fluxes allow to perform sensitive studies on neutrino oscillations. In some cases an anomaly has been observed, and interpreted as positive evidence for neutrino oscillations \[1,2\]. Other experiments have instead obtained results compatible with the no–oscillation hypothesis, and gave therefore exclusion plots in the neutrino oscillation parameter space \((\Delta m^2, \sin^2 2\theta)\) \[3,4\]. In these studies it is often necessary to compare experimental results with theoretical calculations that depend on assumptions about the neutrino fluxes and interactions.

In this letter we want to reanalyse the charged current (CC) neutrino and antineutrino cross sections with particular attention to the energy range \(E_\nu \lesssim \sim 10\) GeV, and discuss possible consequences for the interpretation of the measurements of the atmospheric neutrino fluxes obtained by deep underground detectors. We will argue that it is possible to improve on the description of the cross sections used in several recent analyses of neutrino-induced upward going muons \[4–6\] including a more careful treatment of the lowest multiplicity channels (quasi–elastic scattering and single pion production). The main effect of this more careful description of the neutrino cross sections is an increase of the flux of low energy upward going muons. Information on the energy spectrum of upward going muons can be obtained from the measurement of the ratio of event numbers with muons stopping in the apparatus or passing through \[4\]. Since with respect to previous calculations \[5,6\] we obtain a larger rate of stopping muon events and the predicted rate of passing events is increased by a smaller amount, the excluded region in the neutrino oscillation parameter space obtained from the measured ratio stopping/passing could be significantly reduced. The interpretation of integral measurements of the muon flux with a low \((E_{\mu}^{\text{min}} \sim 1\) GeV) energy threshold could also be modified.

The flux of upward going muons of energy \(E_\mu \geq E_{\mu}^{\text{min}}\) and direction \(\Omega\) can be calculated as

\[
\Phi_\mu^\pm(E_{\mu}^{\text{min}}; \Omega) = \int_{E_{\mu}^{\text{min}}}^{\infty} dE_\nu \ \phi_{\nu_\mu}(\nu_\mu)(E_\nu; \Omega) \ n_{\nu_\mu(\sigma_{\nu_\mu}) \rightarrow \mu^\pm}(E_{\mu}^{\text{min}}; E_\nu) ,
\]

(1)

where \(\phi_{\nu_\mu}(\nu_\mu)(E_\nu; \Omega)\) is the differential flux of \(\nu_\mu\) \((\nu_\mu)\) and \(n_{\nu_\mu(\sigma_{\nu_\mu}) \rightarrow \mu^\pm}(E_{\mu}^{\text{min}}; E_\nu)\) is the average number of muons above a threshold energy \(E_{\mu}^{\text{min}}\) produced by a neutrino of energy \(E_\nu\). It is given by

\[
n_{\nu_\mu(\sigma_{\nu_\mu}) \rightarrow \mu^\pm}(E_{\mu}^{\text{min}}; E_\nu) = N_A \int_{E_{\mu}^{\text{min}}}^{E_\nu} dE_0 \ \frac{d\sigma_{\nu_\mu(\sigma_{\nu_\mu})}}{dE_0}(E_0; E_\nu) \ [R(E_0) - R(E_{\mu}^{\text{min}})] .
\]

(2)

In equation (2), \(N_A\) is Avogadro’s number and the cross section refers to neutrino-nucleon CC scattering. The energy \(E_0\) of the muon at the production point is weighted by a factor \(d\sigma/dE_0\), the relevant cross section, and by a factor \(R(E_0) - R(E_{\mu}^{\text{min}})\) \((R(E)\) is the range in rock of a muon of energy \(E)\) that takes into account the larger effective target available with increasing muon energy. Note that \(n_{\nu_\mu(\sigma_{\nu_\mu}) \rightarrow \mu^\pm}\) depends not only on the total CC cross–section but also on the shape of the muon energy spectrum. In equation (1) we are implicitly assuming that the observed muons are collinear with the parent neutrinos \((\Omega_\mu \simeq \Omega_\nu)\) and in eq.(2) we are neglecting fluctuations in the muon energy losses \[7\].
The calculated muon flux (eq. 1) depends on the inclusive cross-section for muon production. In the literature [4–6] this cross section has been evaluated using the deep inelastic scattering formalism. The deep inelastic scattering (DIS) cross section, expressed in terms of the usual kinematical variables 

\[ y = \frac{1 - E_\mu/E_\nu}{1 + m_N/(2E_\nu)} \]

must be integrated over \( x \) and \( y \) up to the kinematical limits which, neglecting the lepton mass \( m_\nu \), are 0 \( \leq x \leq 1 \) and 0 \( \leq y \leq (1 + x m_N/(2 E_\nu))^2 \). We will denote this method of calculation as Method I.

We observe that the DIS formulae are expected to be valid only for \( Q^2 \) sufficiently large, and that using them for calculating the cross section of low energy neutrinos implies an extrapolation into a region where nonperturbative effects may become important. Most of the parametrizations of the parton distribution functions (PDF) [9] are valid only above a minimum \( Q^2 \) of several GeV$^2$. The parton distributions for \( Q^2 \leq Q^2_\circ \) have been considered to be the same as at \( Q^2_\circ \), neglecting further evolution [5,6]. More recently new sets of parton distributions that consider the evolution down to a lower value \( Q^2_\circ \approx 0.3 \) GeV$^2$ have been made available [10].

As a possible improvement we suggest to consider separately the contributions of the exclusive channels of lowest multiplicity, i.e. quasi–elastic scattering and single pion production, and describe the additional channels collectively using the DIS formulae (Method II). We decompose therefore the CC neutrino cross section as the sum of three contributions:

\[ \sigma_{\nu(\bar{\nu})}^{\text{CC}} = \sigma_{\text{QEL}} + \sigma_{1\pi} + \sigma_{\text{DIS}}. \]  (3)

The quasi–elastic cross section in eq.3 is calculated following [11]. The main uncertainty is in the axial–vector form factor, for which we follow [12] assuming \( F_A(Q^2) = -1.25(1 + Q^2/M_A^2)^{-2} \), with \( M_A = 1.0 \) GeV. Inclusion of nuclear effects would decrease \( \sigma_{\text{QEL}} \) by \( \sim 5\% \).

To determine the energy distribution of the muon produced together with a single pion we make the simplifying assumption of dominance of \( \Delta(1232) \) production. The cross section is normalized to the results of the more complete calculation by Fogli and Nardulli [13], for a maximum mass of the pion-nucleon system \( W_c = 1.4 \) GeV. To avoid double counting, the deep inelastic scattering contribution \( \sigma_{\text{DIS}} \) is limited to the kinematic region where the mass of the hadronic system in the final state is \( W \geq W_c \) [14]. This corresponds in the \((x, y)\) plane to the condition \( 2 m_N E_\nu y(1 - x) \leq W_c^2 - m_N^2 \).

We observe that for neutrino energies not much larger than the nucleon mass, a large fraction of the phase space corresponds to \( W \leq W_c \). The deep–inelastic formula in this region does not take into account the detailed features of the dynamics, with consequences both on the absolute value of the cross section and on the shape on the muon spectrum. Moreover, the kinematical region \( m_N^2 < W^2 < (m_N + m_\pi)^2 \) is unphysical. When it is included in \( \sigma_{\text{DIS}} \), as in Method I, it takes (but only roughly) into account the quasi–elastic contribution.

The neutrino CC cross section and its components calculated according to equation (3), using the PDF of Owens for \( \sigma_{\text{DIS}} \), are plotted in fig.1 and compared with large statistics data for the inclusive process at high [15] and low [16] energy, and with data on one–pion
production \[17\] and quasi–elastic scattering \[18\]. The exclusive channels contribute 89%
(12%) of the cross section for \(E_\nu = 1 \text{ (10) GeV}\). Our model is explicitly non–scaling,
the ratio \(\sigma_\nu/E_\nu\) is not a constant. In the energy range \(E_\nu \sim 1–10 \text{ GeV}\) the ratio \(\sigma_\nu/E_\nu\)
is \(\sim 20\%\) higher than the value measured at higher energy. At lower energies, closer
to the threshold for muon production, \(\sigma_\nu/E_\nu\) drops to zero. The comparison with data \[19\]
of the neutrino CC cross sections is rather encouraging, notwithstanding the large
experimental errors. The situation is much less satisfactory for antineutrinos, in that our
recipe yields results systematically larger than the sparse available data \[20\]. Note that
the contribution of antineutrinos to the total flux of positive and negative muons is \(\sim 1/3\)
of the total.

We have calculated the flux of upward going muons according to equation (1) and
its differential version \[5\] using different models of the neutrino fluxes \[21\] and different
parametrizations \[9,10\] for the nucleon structure functions in \(\sigma_{DIS}\). In fig.2 we show the
result of a differential muon flux calculation, in which the neutrino flux from Butkevich
et al. \[21\], and the parton distributions given by Owens \[9\] have been used. The solid
curve is obtained using Method II for the cross section (the dotted curve is the partial
contribution of \(\sigma_{DIS}\)), while the dashed curve is obtained with Method I (with results in
excellent agreement with other authors \[3\]). The fluxes obtained with the two methods
are essentially equal for \(E_\mu \gtrsim 10 \text{ GeV}\), but at lower muon energies Method II gives a result
that is considerably larger, by \(\sim 12\%\) for \(E_\mu = 3 \text{ GeV}\) and by \(\sim 25\%\) for \(E_\mu = 1 \text{ GeV}\).
The contribution of the exclusive channels in Method II amounts to \(\sim 60\% \text{ (32\%)}\) for
\(E_\mu = 1 \text{ GeV} \text{ (3 GeV)}\).

We now discuss possible consequences of this correction to the muon flux estimate.
The IMB collaboration \[4\] has measured a ratio of rates of stopping and passing upward
going muons \((N_s/N_p) = 0.16 \pm 0.019\). They have compared this result with a detailed
Montecarlo calculation based on Method I, using the structure functions of EHLQ \[3\]
and the neutrino flux of Volkova, corrected at low energy with the results of Lee and
Koh \[21\], obtaining 0.163 for the stopping/passing ratio in the absence of oscillations. It
has been noted that in performing the ratio stopping/passing the uncertainty due to the
absolute normalization of the neutrino flux is greatly reduced and this has been verified
in independent calculations \[3,3\].

To calculate a prediction for the rates of stopping and passing upward going muons in
a specific detector, one needs of course a detailed knowledge of acceptances and detection
efficiencies. As an approximation to the real experimental situation, following \[3\], we
define stopping muons those in the energy interval \(1.25 \leq E_\mu \leq 2.5 \text{ GeV}\) and passing
muons those with \(E_\mu \geq 2.5 \text{ GeV}\), assuming moreover that the detector acceptance and
efficiency for muons above 1.25 GeV are approximately independent from the direction
and energy of the particles. The fluxes \(\Phi_s(\Phi_p)\) are obtained integrating eq. 1, with the
appropriate \(E_{min}\), on the entire downward hemisphere.

We report in Table I the results of calculations of \(\Phi_s, \Phi_p\) and their ratio obtained
with different models for the neutrino fluxes and different sets of leading–order parton
distributions. Both Method I (only DIS) and Method II – our preferred one – have been
used. We observe that the results of Method II are larger than those of Method I. For
those parton distributions that have a high $Q^2_\circ$ the ratio $\Phi_s/\Phi_p$ is increased by $\sim 10\%$. The variation is much less for parton distributions having low $Q^2_\circ$, but it is interesting to note that Method II calculations using different structure functions are in better agreement with each other. The neutrino fluxes of Volkova and Butkevitch (and Mitsui too) predict very similar $\Phi_s/\Phi_p$, notwithstanding the differences of $\sim 10\%$ in normalization, while the Bartol flux is flatter and predicts a ratio somewhat smaller (by $\sim 5\%$).

In order to discuss the possible effects of neutrino oscillations, we define the quantity

$$r_s = \frac{(\Phi_s/\Phi_p)_{\circ}}{(\Phi_s/\Phi_p)}, \quad (4)$$

where $(\Phi_s/\Phi_p)_{\circ}$ is the stopping/passing ratio in absence of neutrino oscillations. The ratio $r_s$ depends on the neutrino oscillation parameters $(\Delta m^2, \sin^2 2\theta)$. We will only consider here the case of $\nu_\mu \leftrightarrow \nu_\tau$ oscillations.

To take into account the effects of oscillations we have to make in eq. (1) the substitution $\phi_{\nu_\mu} \rightarrow [1 - P_{\nu_\mu \rightarrow \nu_\tau}] \phi_{\nu_\mu}$, and analogously for antineutrinos. The double ratio $r_s$ may take the values $r_{\min} \leq r_s \leq 1$. With our definitions of $\Phi_s$ and $\Phi_p$, the maximum suppression of the stopping to passing ratio is $r_{\min} \simeq 0.66$, corresponding to $\sin^2 2\theta = 1$ and $\Delta m^2 \simeq 1.3 \times 10^{-3}$ eV$^2$. The ratio $r_s$ becomes unity in the limit $\sin^2 2\theta \rightarrow 0$ and/or $\Delta m^2 \rightarrow 0$, that correspond to the no oscillation case, and also for $\Delta m^2 \rightarrow \infty$. In this limit (in practice for $\Delta m^2 \gtrsim 1$ eV$^2$) neutrinos of all significant energies oscillate many times, so that the spectrum is suppressed without distortions by a constant factor $1 - \frac{1}{2} \sin^2 2\theta$, and the stopping/passing ratio remains unchanged.

In fig.3 we have drawn lines of constant $r_s$ in the $\Delta m^2, \sin^2 2\theta$ plane. The region favoured at 90% c.l. by the recent Kamiokande-III analysis [2] is also shown, limited by the dashed line. As one can see, parameters in this region imply measurable effects in observations of upward going muons: in fact the region to the right of the curve $r_s = 0.8$ corresponds approximately to the region excluded by the (Method I) analysis of the IMB collaboration [4].

Let us assume that $r_{s^*}$ be the ratio of an experimental result with the theoretical no-oscillation value and its error $\Delta r_{s^*}$ includes both the experimental error and the systematic uncertainty in the theoretical calculation. At 90% c.l. one could exclude in the $(\Delta m^2, \sin^2 2\theta)$ plane the region corresponding to values $|r_s - r_{s^*}| \geq 1.64 \Delta r_{s^*}$ [22]. Use of Method II for the neutrino cross section increases the theoretical prediction by $\sim 10\%$, therefore $r_{s^*}$ would be lowered by the same amount, with obvious consequences on the excluded region. We urge therefore the experimental groups that have collected and are collecting data on upward going muons to reanalyze them going beyond the DIS approximation for the $\nu$ cross section.

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[22] This is true for gaussian errors and far from the boundaries of the physical region (for a discussion, see [19]). Note that assuming a 12% relative error on experiment, a
much smaller theoretical error, and \( r_s^* = 1 \), one obtains for the 90% c.l. lower limit \( r_s \geq 0.8 \).
TABLE I. Fluxes (in units $10^{-13}$ (cm$^2$ s sr$^{-1}$)) of ‘stopping’ ($1.25 \leq E_\mu \leq 2.5$ GeV) and ‘passing’ ($E_\mu \geq 2.5$ GeV) upward going muons calculated with different models of the neutrino fluxes, different choices of the parton distribution functions and using Method I and Method II to describe the neutrino cross section.

| $\nu$–flux | EHLQ–2 | Owens | MRS–LO |
|-------------|--------|-------|--------|
|             | $\Phi_s$ | $\Phi_p$ | $\Phi_s/\Phi_p$ | $\Phi_s$ | $\Phi_p/\Phi_p$ | $\Phi_s$ | $\Phi_p$ | $\Phi_s/\Phi_p$ |
| Volkova (II) | 0.453 | 2.140 | 0.212 | 0.484 | 2.361 | 0.205 | 0.492 | 2.409 | 0.204 |
| Volkova (I)  | 0.383 | 2.053 | 0.187 | 0.432 | 2.295 | 0.188 | 0.476 | 2.387 | 0.199 |
| (II)/(I)     | 1.18  | 1.04  | 1.13  | 1.12  | 1.03  | 1.09  | 1.03  | 1.01  | 1.03  |
| Butkevich (II) | 0.507 | 2.378 | 0.213 | 0.544 | 2.622 | 0.207 | 0.553 | 2.678 | 0.206 |
| Butkevich (I) | 0.433 | 2.275 | 0.190 | 0.488 | 2.544 | 0.192 | 0.536 | 2.652 | 0.202 |
| (II)/(I)     | 1.17  | 1.05  | 1.12  | 1.11  | 1.03  | 1.08  | 1.03  | 1.01  | 1.02  |
| Bartol (II)  | 0.454 | 2.308 | 0.197 | 0.487 | 2.547 | 0.191 | 0.496 | 2.601 | 0.191 |
| Bartol (I)   | 0.389 | 2.214 | 0.176 | 0.439 | 2.475 | 0.177 | 0.482 | 2.578 | 0.187 |
| (II)/(I)     | 1.17  | 1.04  | 1.12  | 1.11  | 1.03  | 1.08  | 1.03  | 1.01  | 1.02  |
FIGURES

FIG. 1. $\nu_\mu$ CC cross sections plotted as a function of energy.

FIG. 2. Differential flux of upward going muons averaged in angle over one hemisphere and plotted as a function of muon energy.

FIG. 3. Curves in the $(\Delta m^2, \sin^2 2\theta)$ plane that correspond to constant values for the double ratio $r_s$. Also shown (dashed) is the 90% c.l. curve obtained by the Kamiokande-III combined analysis of contained and semi-contained events [4].
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Figure 1
Figure 2
Figure 3