Effective value of gA in β and ββ decays

Suhonen, Jouni (2018). Effective value of gA in β and ββ decays. In C. Agodi, & F. Cappuzzello (Eds.), CNNP2017 : Conference on Neutrino and Nuclear Physics (Article 012056). IOP Publishing. Journal of Physics : Conference Series, 1056. https://doi.org/10.1088/1742-6596/1056/1/012056
Effective value of $g_A$ in $\beta$ and $\beta\beta$ decays

To cite this article: Jouni Suhonen 2018 J. Phys.: Conf. Ser. 1056 012056

View the article online for updates and enhancements.

Related content

- Investigating decay with the NEMO-3 and SuperNEMO experiments
  S Blot and NEMO-3 and SuperNEMO experiments

- Nuclear matrix elements for the decay of $^{76}$Ge
  B. A. Brown, D. L. Fang and M. Horoi

- Is it possible to study neutrinoless decay by measuring double Gamow-Teller transitions?
  Javier Menéndez, Noritaka Shimizu and Kentaro Yako
Effective value of $g_A$ in $\beta$ and $\beta\beta$ decays

Jouni Suhonen
University of Jyväskylä, Department of Physics, P.O. Box 35, FI-40014, Jyväskylä, Finland
E-mail: jouni.suhonen@phys.jyu.fi

Abstract. Effective value of the weak axial-vector coupling strength, $g_A$, in nuclear $\beta$ and $\beta\beta$ (double beta) decays is discussed. Both Gamow-Teller and first-forbidden $\beta$ decays are included in the analyses. Quenching of $g_A$ in $\beta$ and two-neutrino $\beta\beta$ decays is reviewed and impact of this quenching on neutrinoless $\beta\beta$ decays is addressed. New measurements of $\beta$ spectra of first-forbidden non-unique $\beta$ decays are encouraged, to learn about the mesonic enhancement of the weak axial charge in these decays.

1. Introduction
The impact of the quenching of $g_A$ on the half-lives of neutrinoless double beta ($0\nu\beta\beta$) decay has recently been discussed in Ref. [1]. The related decay rates are affected by the available phase space ($Q$ values), the nuclear matrix elements (NMEs) and the value of $g_A$ in its fourth power [2, 3, 4, 5]. The $0\nu\beta\beta$ decay is mediated by Majorana neutrinos and the measurements of the related half-lives offer access to the absolute mass scale of the neutrinos [2]. A number of nuclear models, including configuration-interaction based models like the interacting shell model (ISM) and the proton-neutron quasiparticle random-phase approximation (pnQRPA), and various mean field models, have been adopted for the calculations [5, 6, 7].

Surprisingly little attention has been paid, at least in the theory community, to the possible (large) quenching of $g_A$ and its possibly strong impact on the sensitivities of the present and planned $0\nu\beta\beta$-decay experiments [1]. This deviation (quenching or sometimes enhancement) from the free-nucleon value $g_A = 1.27$ can arise from the nuclear medium effects and the nuclear many-body effects. The former contain quenching related to the presence of spin-multipole giant resonances [8], non-nucleonic degrees of freedom (like the $\Delta$ isobar [9, 10]) and meson-exchange-driven two-body weak currents [11, 12, 13]. The latter relates to deficiencies of the nuclear many-body approaches used to compute the wave functions involved in the decay transitions. The effective value of $g_A$ can also depend on the energy scale of the process in question: the effective value can be different for $\beta$ decays (zero-momentum-exchange limit) and $0\nu\beta\beta$ decays (high momentum exchanges, $\sim 100$ MeV).

The effective value of $g_A$ relates to the renormalization factor $q$ (in case of quenching it is called quenching factor and in case of enhancement enhancement factor):

$$q = \frac{g_A}{g_A^{\text{free}}},$$

where $g_A^{\text{free}} = 1.2723(23)$ [14] is the free-nucleon value of the axial-vector coupling as measured in neutron beta decay. Here $g_A$ is the value of the axial-vector coupling derived from a given
theoretical or experimental analysis. From (1) one can derive the effective value of $g_A$ as

$$g^\text{eff}_A = q g^\text{free}_A.$$  

(2)

2. Quenching of $g_A$ in Gamow-Teller $\beta$ decays

Gamow-Teller decays are mediated by the Pauli spin operator $\sigma$ [15] and they thus change the initial nuclear spin $J_i$ by at most one unit. The renormalization of $g_A$ has long been studied for the Gamow-Teller $\beta$ decays in the framework of the interacting shell model (ISM). In these calculations, reviewed in Table 1, it appears that the value of $g_A$ is quenched, and the stronger the larger the nuclear mass $A$.

| Mass range                          | $g^\text{eff}_A$       | Reference |
|-------------------------------------|------------------------|-----------|
| Full $0p$ shell                     | $1.03^{+0.03}_{-0.02}$ | [16]      |
| $0p$ - low $1s0d$ shell             | $1.18 \pm 0.05$        | [17]      |
| Full $1s0d$ shell                   | $0.96^{+0.03}_{-0.02}$ | [18] (see also [19]) |
| $A = 41 - 50$ ($1p0f$ shell)        | $0.937^{+0.019}_{-0.018}$ | [21] (see also [19]) |
| $A = 52 - 67$ ($1p0f$ shell)        | $0.838^{+0.021}_{-0.020}$ | [22]      |
| $A = 67 - 80$ ($0f_5/21p0g_9/2$ shell) | $0.869 \pm 0.019$    | [22]      |
| $A = 63 - 96$ ($1p0f0g1d2s$ shell)  | $0.8$                   | [23]      |
| $A = 76 - 82$ ($1p0f0g9/2$ shell)   | $0.76$                  | [24]      |
| $A = 90 - 97$ ($1p0f0g1d2s$ shell)  | $0.60$                  | [25]      |
| $^{100}$Sn                          | $0.52$                  | [20]      |
| $A = 128 - 130$ ($0g_7/21d2s0h_{11/2}$ shell) | $0.72$ | [24] |
| $A = 130 - 136$ ($0g_7/21d2s0h_{11/2}$ shell) | $0.94$ | [26] |
| $A = 136$ ($0g_7/21d2s0h_{11/2}$ shell) | $0.57$ | [24] |

In Fig. 1 the ISM results of Caurier et al. [24] (red horizontal bars indicating the mass range) are contrasted against those obtained by the use of the proton-neutron quasiparticle random-phase approximation (pnQRPA) in the works [27, 28, 29] (see also [30]). The pnQRPA results constitute the light-hatched regions in the background of the ISM results. As can be seen in the figure, the ISM results and the pnQRPA results are commensurate with each other, which is non-trivial considering the large differences in their many-body philosophy.

3. Quenching of $g_A$ in two-neutrino $\beta\beta$ decays

Recently the possibly decisive role of $g_A$ in the half-life and discovery potential of the $0\nu\beta\beta$ experiments has surfaced [1, 31]. In Barea et al. [31] a comparison of the experimental and computed $2\nu\beta\beta$ half-lives of a number of nuclei yielded the rather surprising result

$$g^\text{eff}_A (\text{IBM-2}) = 1.269 A^{-0.18}; \quad g^\text{eff}_A (\text{ISM}) = 1.269 A^{-0.12},$$  

(3)

where $A$ is the mass number and IBM-2 stands for the microscopic interacting boson model. The results (3), depicted in Fig. 1 as red (ISM) and blue (IBM-2) dotted curves, imply that strongly
Figure 1. Effective values of $g_A$ in different theoretical approaches for the nuclear mass range $A = 62 - 136$. The quoted references are $Suhonen2017$ [1], $Caurier2012$ [24], $Faessler2007$ [32] and $Suhonen2014$ [34]. For more information see the text.

quenched effective values of $g_A$ are possible, thus being a threat to the discovery potential of the $0\nu\beta\beta$ experiments.

Although the study [31] was the first to draw considerable attention in the experimental $0\nu\beta\beta$ community, it was not the first one to point to a possible strong quenching of $g_A$. Already the pnQRPA study of Faessler et al. [32] gave indications of a heavily quenched effective $g_A$, in the range $g_A^{\text{eff}} = 0.39 - 0.84$. These results, along with their $1\sigma$ errors, are shown in Fig. 1 as black vertical bars. Later a similar study was carried out in [33, 34], with results comparable with those of [32] and depicted in Fig. 1 as green vertical bars.

In Suhonen [1] a two-stage fit of the particle-particle parameter $g_{pp}$ of the pnQRPA to the data on two-neutrino $\beta\beta$ decays was performed along the lines first introduced in Šimkovic et al. [35] and later used in Hyvärinen et al. [36]. The works [35, 36] were extended in [1] to include also strongly quenched values of $g_A$. In this analysis it turned out that there is a minimum value of $g_A$ for which the maximum NME can fit the $2\nu\beta\beta$-decay half-life. This lower limit of the possible $g_A$ values is presented in Fig. 1 as a solid black line and it is seen that it is consistent with the thick green vertical bars of $g_A$ ranges obtained in [33, 34] and also commensurate with the thin black vertical bars obtained in [32]. However, the main message of Suhonen [1] is that no matter how quenched the value of $g_A$ is, the half-lives of the present and future neutrinoless $\beta\beta$-decay measurements are only affected by factors of 6 or less.

4. Nuclear-structure effects in forbidden unique $\beta$ decays
Forbidden unique $\beta$ decays are the simplest beyond the allowed $\beta$ decays since, in the leading order, only one nuclear matrix element is involved in the process and thus the spectrum shape is a universal one, essentially independent of nuclear structure. The relation between the
corresponding half-life and the NME is

\[ f_{Ku} t^{1/2} = \frac{\kappa}{B_{Ku}} ; \quad B_{Ku} = \frac{g^2_{\pi}}{2J_i + 1} |M_{Ku}|^2, \tag{4} \]

where \( J_i \) is the angular momentum of the mother nucleus and \( \kappa \) is a constant with value \( \kappa = 6147 \) s [37]. The involved NME is given by

\[ M_{Ku} = \sum_{ab} M^{(Ku)}(ab)(\psi_f || [c^+_u \bar{c}_b]_{K+1} || \psi_i), \tag{5} \]

where the factors \( M^{(Ku)}(ab) \) are the single-particle matrix elements and the quantities \( (\psi_f || [c^+_u \bar{c}_b]_{K+1} || \psi_i) \) are the one-body transition densities, \( \psi_i \) being the initial-state and \( \psi_f \) the final-state wave function. The operator \( c^+_u \) is a creation operator for a nucleon in the orbital \( a \) and the operator \( \bar{c}_b \) is the corresponding annihilation operator. The single-particle matrix elements are given by

\[ M_{Ku}(ab) = \sqrt{4\pi} (a||r^K[Y_K \sigma]_{K+1} i^K || b), \tag{6} \]

where \( Y_K \) is a spherical harmonic of rank \( K \), \( r \) the radial coordinate, and \( a \) and \( b \) stand for the single-particle orbital quantum numbers. The NME (6) is given explicitly in [15].

In [38] a systematic study of the \( 2^{\pi}_g \leftrightarrow 0^{\pi}_g \) transitions between an odd-odd ground state \( 2^- \) and an even-even ground state \( 0^+ \) was performed in cases where experimental data on the log \( ft \) values of Eq. (4) are available. The associated NME \( M_{1u} \) of (5) was calculated by using a simple two-quasiparticle model (\( M_{qp} \)) and the pnQRPA (\( M_{pnQRPA} \)). Here \( \bar{M} = \sqrt{M_L M_R} \) represents the geometric mean of the NMEs corresponding to decay transitions between the left-side (\( M_L \)) or right-side (\( M_R \)) nucleus and the central one. Use of the geometric mean makes the analysis more stable and very weakly dependent on the value of the particle-particle interaction parameter \( g_{pp} \) of the pnQRPA. Comparing the \( M_{qp} \) NME, the \( M_{pnQRPA} \) NME and the experimental NME \( M_{exp} \) (extracted by using the free value \( g_A = 1.27 \)) the following overall ratios for the mass range \( A = 72 - 132 \)

\[ k_{NM} = \frac{M_{exp}}{M_{pnQRPA}} \approx 0.45; \quad k = \frac{M_{pnQRPA}}{M_{qp}} \approx 0.4, \tag{7} \]

were obtained. Here “NM” denotes the nuclear-medium effects (isobaric resonances, mesonic two-body currents, etc.) combined with the nuclear-model effect related to the imperfection of the nuclear model used in the calculations.

The work of [38] was followed by the study of Ref. [39] where the comparison of the NMEs \( M_{qp} \) and \( M_{pnQRPA} \) was made for 148 higher-forbidden unique \( \beta \) decays to access the ratio \( k \) of Eq. (7). Here lack of experimental data led only to an extrapolation, based on the results of Refs. [27] and [38], for the value of \( k_{NM} \).

5. Enhancement of the axial charge in first-forbidden \( J^- \leftrightarrow J^\beta \) decays

The enhancement of the axial-charge nuclear matrix element (NME) \( \gamma_5 \) due to meson-exchange currents was first suggested nearly four decades ago [40, 41, 42]. An enhancement of 40–70% over the impulse-approximation value was predicted based on chiral-symmetry arguments and soft-pion theorems. This enhancement is fundamental in nature and insensitive to nuclear-structure aspects [43, 44]. Since the \( \gamma_5 \) NME is one of the two rank-zero matrix elements contributing to first-forbidden \( \Delta J = 0 \) transitions it plays an important role in many of these transitions. Therefore, a significant enhancement of this matrix element can also affect the shapes of the corresponding beta spectra.
The half-life of a first-forbidden non-unique beta decay can be written as

$$t_{1/2} = \frac{\kappa}{\tilde{C}}, \quad (8)$$

where $\kappa$ was defined in connection with Eq. (4) and $\tilde{C}$ is the dimensionless integrated shape function, given by

$$\tilde{C} = \int_{w_e}^{w_0} C(w_e) pw_e (w_0 - w_e)^2 F_0(Z_f, w_e) dw_e, \quad (9)$$

where $w_e$ is the total energy of the emitted electron, $w_0$ its maximum, $p_e$ is the electron momentum, $Z_f$ is the charge number of the daughter nucleus and $F_0(Z_f, w_e)$ is the Fermi function taking into account the coulombic attraction of the electron and the daughter nucleus. The shape factor $C(w_e)$ of Eq. (9) contains complicated combinations of both (universal) kinematic factors and nuclear form factors. The nuclear form factors can be related to the corresponding NMEs using the impulse approximation. For the first-forbidden non-unique decays with $J_i = J_f$, considered in this review, the relevant NMEs are those of the transition operators denoted here by $\mathcal{O}(0^-)$, $\mathcal{O}(1^-)$, and $\mathcal{O}(2^-)$ [45] and given in the leading order by

$$\mathcal{O}(0^-) : g_A(\gamma 5)(\sigma \cdot p_e), \quad g_A(\sigma \cdot r), \quad (10)$$

$$\mathcal{O}(1^-) : g_V p_e, \quad g_A(\sigma \times r), \quad g_V r, \quad (11)$$

$$\mathcal{O}(2^-) : g_A[\sigma r]_2, \quad (12)$$

where $r$ is the radial coordinate and $p_e$ is the electron momentum. The enhancement of the $\gamma_5$ NME ($\sigma \cdot p_e$ in non-relativistic form) is included in the coupling strength $g_A(\gamma 5)$.

![Figure 2](image_url)

**Figure 2.** Electron spectrum of the ground-state-to-ground-state decay of $^{142}$Pr. The value of $g_A(\gamma 5)$ is marked by the line type. The values 1.27, 1.78, and 2.55 correspond to enhancements of 0 %, 40 %, and 100 % of the axial-charge matrix element. The color coding gives the effective value of $g_A$ used for the matrix elements $[\sigma r]_{0,1,2}$.

The enhancement of the axial-charge NME affects not only the partial half-lives of the $J^− \rightarrow J^+$ first-forbidden $\beta$ transitions but also the shapes of the corresponding electron spectra. An example is given in Fig. 2. A clear effect is induced by the values of both the $g_A(\gamma 5)$ and $g_A$, multiplying the matrix elements $[\sigma r]_{0,1,2}$ of Eqs. (10)–(12). Experimental measurements of the $\beta$ spectra would thus be welcome in order to access the $\gamma_5$ enhancement independent of the half-life considerations.
6. Final remark
The discussion above gives only scattered examples in relation to the effective values of the axial-vector coupling strength, as also to the nuclear-structure effects involved in model calculations. A more exhaustive discussion of these subjects is provided in the review [46].

Acknowledgments
This work has been partially supported by the Academy of Finland under the Finnish Centre of Excellence Programme 2012-2017 (Nuclear and Accelerator Based Programme at JYFL).

References
[1] Suhonen J 2017 Phys. Rev. C 96 055501
[2] Suhonen J and Civitarese O 1998 Phys. Rep. 300 123
[3] Maaalampi J and Suhonen J 2013 Adv. High Energy Phys. 2013 505874
[4] Vergados J D, Ejiri H and Simkovic F 2016 Int. J. Mod. Phys. E 25 1630007
[5] Engel J and Menéndez J 2017 Rep. Prog. Phys. 80 046301
[6] Suhonen J and Civitarese O 2012 J. Phys. G: Nucl. Part. Phys. 39 085105
[7] Suhonen J and Civitarese O 2012 J. Phys. G: Nucl. Part. Phys. 39 124005
[8] Jokinen L and Suhonen J 2017 Phys. Rev. C 96 034308
[9] Oset E and Rho M 1979 Phys. Rev. Lett. 42 47
[10] Bohr A and Mottelson B R 1981 Phys. Lett. 100B 10
[11] Towner L S 1997 Phys. Rep. 155 263
[12] Menéndez J, Gazit D and Schwenk A 2011 Phys. Rev. Lett. 107 062501
[13] Ekström A et al. 2014 Phys. Rev. Lett. 113 262504
[14] Patrignani C et al. (Particle Data Group) 2016 Chin. Phys. C 40 100001
[15] Suhonen J 2007 From Nucleons to Nucleus: Concepts of Microscopic Nuclear Theory (Berlin: Springer)
[16] Chou W T, Warburton E K and Brown B A 1993 Phys. Rev. C 47 163
[17] Wilkinson D H 1974 Nucl. Phys. A 225 365
[18] Wildenthal B H, Curtis M S and Brown B A 1983 Phys. Rev. C 28 1343
[19] Konieczka M, Baczyk P and Satula W 2016 Phys. Rev. C 93 042501(R)
[20] Siiskonen T, Hjorth-Jensen M and Suhonen J 2001 Phys. Rev. C 63 055501
[21] Martínez-Pinedo G, Poves A, Caurier E and Zuker A P 1996 Phys. Rev. C 53 R2602
[22] Kumar V, Srivastava P C and Samour C 1998 J. Phys. G: Nucl. Part. Phys. 24 105104
[23] Honma M, Otsuka T, Misuzaki T and Hjorth-Jensen M 2006 J. Phys.: Conf. Series 49 45
[24] Caurier E, Nowacki F and Poves A 2012 Phys. Lett. B 711 62
[25] Juodagalvis A and Dean D J 2005 Phys. Rev. C 72 024306
[26] Horoi M and Nencu A 2016 Phys. Rev. C 93 024308
[27] Ejiri H and Suhonen J 2015 J. Phys. G: Nucl. Part. Phys. 42 055201
[28] Pirinen P and Suhonen J 2013 Phys. Rev. C 91 054309
[29] Deppisch F P and Suhonen J 2015 Phys. Rev. C 94 055501
[30] Delion D S and Suhonen J 2014 Europhys. Lett. 107 52001
[31] Barea J, Kotila J and Iachello F 2013 Phys. Rev. C 87 014315
[32] Faessler A et al. 2008 J. Phys. G: Nucl. Part. Phys. 35 075104 ; arXiv 0711.3996v1 [nucl-th] 26 Nov 2007
[33] Suhonen J and Civitarese O 2013 Phys. Lett. B 725 153
[34] Suhonen J and Civitarese O 2014 Nucl. Phys. A 924 1
[35] Simkovic F, Rodin V, Faessler A and Vogel P 2013 Phys. Rev. C 87 045501
[36] Hyvärinen J and Suhonen J 2015 Phys. Rev. C 91 024308
[37] Hardy J C, Towner I S, Koslowsky V, Hagerberg E and Schmeig H 1990 Nucl. Phys. A 509 429
[38] Ejiri H, Soukouti N and Suhonen J 2014 Phys. Lett. B 729 27
[39] Kostensalo J and Suhonen J 2017 Phys. Rev. C 95 014322
[40] Kubodera K, Delorne J and Rho M 1978 Phys. Rev. Lett. 40 755
[41] Guichon P, Giffon M, Joseph J, Laverrriere R and Samour C 1998 Z. Phys. A 285 183
[42] Guichon P, Giffon M and Samour C 1978 Phys. Lett. 74B 15
[43] Towner I S and Khanna F C 1981 Nucl. Phys. A 372 331
[44] Delonne J 1982 Nucl. Phys. A 374 541c
[45] Behrens H and Bühring W 1982 Electron Radial Wave Functions and Nuclear Beta-decay (Oxford: Clarendon)
[46] Suhonen J 2017 Frontiers in Physics 5 55