Dynamic Analysis of High Building with Cracks in Column Subjected to Earthquake Loading

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Abstract: This paper is devoted to studying the dynamic response of a high building on an elastic foundation with cracks on the column of the building under earthquake loading using the finite element method (FEM). The authors used a 3D modeling frame - plate system and integration method developed by Newmark, along with construction calculation programs in the Matlab environment to analyze the problem. The results were reflected the effect of the cracks on the columns and other parameters.

Keywords: 3D Frame, Plate, Seismic Load

1. Introduction

When Ming Ming Yao [1], Mourad Nasser [2], and Sushma Pulikant [3] used the finite element method with a space frame model to simulate a high building under earthquake loading, including foundation replacement via contact connection, they analyzed the dynamic response of the earthquake effect-bearing system. Their findings showed the impacts of foundation parameters as well as structure on the displacement of stories’ floors. More recently, Aslan Sadeghi Hokmabadi [4], and Ahmed Abdelraheem Farghaly [5] used the finite element method to calculate the earthquake loading of a high building, including the use of a space frame model for simulating high building and viscous connection - foundation modeling elastic spring. This showed the impacts of height on the displacement of stories’ floors and shearing force in a high building frame. When D. K. Jain and M. S. Hora [6] used ANSYS software and BEAM4: 3-D elements to describe lateral displacement, SHELL63 to describe the floor surface, and SOLID45 to describe cushion course, they developed the model of a high building under seismic load. J. Sandhya Rani, P. Srinivas Reddy and M. Koti Reddy [7] analyzed high buildings having a water reservoir located in different positions of the overstory under earthquake loading, including situations of different numbers of stories. Their findings showed the effects of the location of the water reservoir on the displacement of the overstory floor and cutting force in the column base. As well, they initially provided some results of cutting force, moment in the footing, and seismic load converted following the equivalent standard. Unlike previous studies, the aim of this article is to study algorithm, calculation method and working conditions and more specifically to select suitable parameters (for structure) for a high building having cracks in the column base. This is an original approach that the authors want to mention in this research.

2. Finite Element Simulation and Dominant Equations

A high building is modeled using the frame-plate composite system in the general coordinate system (X, Y, Z) under earthquake loading following X and Y directions as per Figure 1.
Figure 1. Frame-plate composite system model to simulate high building.

Hypothesis: Materials have a structure of linear elasticity, and small deformation. Panel satisfies Reissner-Mindlin theoretical conditions. Only the horizontal acceleration of the foundation is considered. Construction work columns and foundation operate simultaneously.

2.1. Response Relation of Elements

2.1.1. Space Beam Element Describes the Frame

The beam element is shown in Fig. 2, and each node has 6 degrees of freedom, including \( u_i, v_i, w_i, \theta_{xi}, \theta_{yi}, \theta_{zi} \).

Displacement in any node of the bar with \((x, y)\) coordinates is identified as follows [8], [10]:

\[
\begin{align*}
\begin{align*}
  u &= u(x, y, z, t) = u_0(x, t) + z\theta_x(x, t) - y\theta_z(x, t), \\
  v &= v(x, y, z, t) = v_0(x, t) - z\theta_y(x, t), \\
  w &= w(x, y, z, t) = w_0(x, t) + y\theta_z(x, t),
\end{align*}
\end{align*}
\]  

where,

- the subscript “0” represents axis \( x (y = 0, z = 0) \)
- \( t \) represents time;
- \( U, v \) and \( w \) are the displacements along \( x, y \) and \( z \);
- \( \theta_x \) is the rotation of cross section about the longitudinal axis \( x \); and
- \( \theta_y \) and \( \theta_z \) denote rotations of the cross section about \( y \) and \( z \) axes.

The strain components:

\[
\begin{align*}
  \varepsilon_x &= \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} + z \frac{\partial \theta_x}{\partial x} - y \frac{\partial \theta_z}{\partial x}, \\
  \gamma_{zi} &= \frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} = \frac{\partial v_0}{\partial x} + y \frac{\partial \theta_y}{\partial x} + \theta_z, \\
  \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial v_0}{\partial x} - z \frac{\partial \theta_z}{\partial x} - \theta_x.
\end{align*}
\]  

\( \{q\}_x = [q_1 \ q_2 \ q_3 \ q_4 \ q_5 \ q_6 \ q_7 \ q_8 \ q_9 \ q_{10} \ q_{11} \ q_{12}]^T \) (3)

Element stiffness matrix is set up from 4 types of component stiffness matrices:

\[
\begin{array}{c}
K_{xx} \\
K_{yy} \\
K_{xy} \\
K_{xz}
\end{array}
\times
\begin{array}{c}
K_{xx} \\
K_{yy} \\
K_{xy} \\
K_{xz}
\end{array}
\times
\begin{array}{c}
K_{xx} \\
K_{yy} \\
K_{xy} \\
K_{xz}
\end{array}
\times
\begin{array}{c}
K_{xx} \\
K_{yy} \\
K_{xy} \\
K_{xz}
\end{array}
= \begin{array}{c}
K_x \\
K_y \\
K_{xy} \\
K_{xz}
\end{array}.
\]  

(4)

In the \((X, Y, Z)\) coordinate system

\[
\begin{align*}
\begin{align*}
  [K]_x^T &= [T]_x^T [K]_x^T [T]_x, \\
  [M]_x^T &= [T]_x^T [M]_x^T [T]_x
\end{align*}
\end{align*}
\]  

Figure 2. Three dimension 2-node beam element model.
node load vector of element in the natural coordinate system is:

\[
\{ f \}_c^b = \{ f \}_{12}^T = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 & f_8 & f_9 & f_{10} & f_{11} & f_{12} \end{bmatrix}^T
\]  \hspace{1cm} (8)

In the (X, Y, Z) coordinate system

\[
\{ f \}_c^b = [T]_c^T \{ f \}_c^b.
\]  \hspace{1cm} (9)

### 2.1.2. Plate Element Describes the Floors

The displacement fields are written as [8], [12]:

\[
\begin{align*}
\mathbf{u}(x, y, z, t) &= u_0(x, y, t) + z\theta_y(x, y, t), \\
\mathbf{v}(x, y, z, t) &= v_0(x, y, t) - z\theta_x(x, y, t), \\
\mathbf{w}(x, y, z, t) &= w_0(x, y, t),
\end{align*}
\]  \hspace{1cm} (11)

where, \(u_0, v_0, w_0\) correspond to displacements of plate average, and \(\theta_x, \theta_y\) correspond to rotation angle of normal section of the plate plane in the survey node.

Deformation vector:

\[
\{\mathbf{\varepsilon}\}_p = \left\{\begin{bmatrix} \varepsilon_x & \varepsilon_y & \gamma_{xy} \\ \gamma_{yx} & \gamma_{zz} \end{bmatrix} \right\}^T = \left\{\begin{bmatrix} \mathbf{\varepsilon}^b \end{bmatrix}^T \begin{bmatrix} \mathbf{\varepsilon}^s \end{bmatrix}^T \right\}^T,
\]  \hspace{1cm} (12)

where,

\[
\left\{\mathbf{\varepsilon}^b\right\} = \left\{\begin{bmatrix} \frac{\partial u_0}{\partial x} & \frac{\partial v_0}{\partial y} & \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right) \end{bmatrix}\right\}^T + z\left\{\begin{bmatrix} \frac{\partial \theta_y}{\partial x} - \frac{\partial \theta_x}{\partial y} & \frac{\partial \theta_y}{\partial x} - \frac{\partial \theta_x}{\partial y} \end{bmatrix}\right\}^T = \{\mathbf{\varepsilon}_0\} + z\{\kappa\},
\]  \hspace{1cm} (13)

\[
\left\{\mathbf{\varepsilon}^s\right\} = \left\{\begin{bmatrix} \gamma_{xz} & \gamma_{yz} \end{bmatrix}\right\}^T = \left\{\begin{bmatrix} \frac{\partial w_0}{\partial x} + \theta_y & \frac{\partial w_0}{\partial y} - \theta_x \end{bmatrix}\right\}^T
\]  \hspace{1cm} (14)

\[
\{\kappa\} = \left\{\begin{bmatrix} k_x & k_y & k_{xy} \end{bmatrix}\right\}^T = \left\{\begin{bmatrix} \frac{\partial \theta_x}{\partial x} & \frac{\partial \theta_x}{\partial y} & \frac{\partial \theta_x}{\partial y} \end{bmatrix}\right\}^T.
\]  \hspace{1cm} (15)

Stress-deformation relation:

The stiffness matrix \([K]_c^b\) of the cracked element can be written as:

\[
[K]_c^b = [K]_c^b - [K]_c^b,
\]  \hspace{1cm} (10)

where \([K]_c^b\) is the reduction in the stiffness matrix due to the crack according to Peng et al. [13].

Figure 3. Model of 4-node plate element and the coordinate system.
\[
\{\sigma^h\} = \begin{bmatrix}
\sigma_x^h \\
\sigma_y^h \\
\tau_{xy}^h
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \frac{1}{1-\nu^2}
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
D^b \\
0 \\
D^s
\end{bmatrix} \{\varepsilon^c\} = \begin{bmatrix}
\varepsilon_x^c \\
\varepsilon_y^c \\
\gamma_{xy}^c
\end{bmatrix} = \begin{bmatrix}
D^b \\
0 \\
D^s
\end{bmatrix} \{\varepsilon_0^c + z\{\kappa^c\}\},
\]

(16)

Where,
\{\sigma^b\} is stress vector excluding shearing force,
\{\sigma^s\} is stress vector caused by horizontal deformation,
\{\varepsilon^c\} = \begin{bmatrix}
\varepsilon_x^c \\
\varepsilon_y^c \\
\gamma_{xy}^c
\end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \frac{1}{2}
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
D^b \\
0 \\
D^s
\end{bmatrix} \{\varepsilon^c\},

(17)

E is Young’s modulus, and \(\nu\) is the Poisson coefficient of materials.

If using (17) and (18) relations, internal force vector \(\{\sigma^f\}\) is identified as follows:
\[\{\sigma^f\} = \begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix}^T = \int_{-\frac{h}{2}}^{\frac{h}{2}} z \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} dz = \begin{bmatrix}
h^3 \\
h^3 \\
h^3
\end{bmatrix} \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} \{\varepsilon^c\},
\]

(19)

\(\{\sigma^f\} = \begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix}^T = \int_{-\frac{h}{2}}^{\frac{h}{2}} z \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} dz = \begin{bmatrix}
h^3 \\
h^3 \\
h^3
\end{bmatrix} \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} \{\varepsilon^c\},
\]

(20)

Therefore:
\[\{\sigma^f\} = \begin{bmatrix}
D^c \\
D^s
\end{bmatrix} \{\varepsilon^c\},
\]

(21)

where,
\[\begin{bmatrix}
D^c \\
D^s
\end{bmatrix} = \begin{bmatrix}
h^3 [D^f] \\
0
\end{bmatrix}, \{\varepsilon^c\} = \begin{bmatrix}
k_x & k_y & k_{xy} \\
\gamma_x & \gamma_y & \gamma_{xy}
\end{bmatrix}^T
\]

- components of the bending deformation vector and shear deformation vector, and \(\alpha = 5/6\) is the coefficient including shear deformation.

Displacement components of any node of the element [8]:
\[w = \sum_{i=1}^{4} N_i w_i, \quad \theta_x = \sum_{i=1}^{4} N_i \theta_{xi}, \quad \theta_y = \sum_{i=1}^{4} N_i \theta_{yi},
\]

(22)

where, \(w_i, \theta_{xi}, \theta_{yi}\) are displacements of w, \(\theta_x, \theta_y\) at node ith of the element, and written as \(N_i\) is shape function of node i.

Therefore:
\[\{\varepsilon^c\} = [B]_e \{q\} = \sum_{i=1}^{4} [B]_e \{q\}_i.
\]

(23)

where,
\([B]_e\) is matrix of strain - displacement relation,
\[\begin{bmatrix}
\{q\}_i \\
\theta_{xi} \\
\theta_{yi}
\end{bmatrix} = \begin{bmatrix}
\{q_i\}_T \\
\theta_{xi} \\
\theta_{yi}
\end{bmatrix} \begin{bmatrix}
1 \\
2 \\
3
\end{bmatrix}, \quad (i = 1, 2, 3, 4).
\]

Replacing (23) with (21) leads to
\[\{\sigma^f\} = \sum_{i=1}^{4} [D^c B]_i \{q_i\}_e,
\]

(24)

where,
\([D^c B]_i\) - matrix for calculating bending moment and cutting force.

The total potential energy expression of the element is written as [8]:
\[ \Pi_e = \frac{1}{2} \int_{A_e} \left[ \sigma^T \right]_e \left[ D^e \right] \left[ \sigma^T \right]_e \, dA_e - \int_{A_e} w p dA_e = \frac{1}{2} \int_{A_e} \left[ q^T \right]_e \left[ K^p \right] \left[ q \right]_e - \left[ q^T \right]_e \left[ P \right]_e, \]  
(25)

where,

\[ \left[ K^p \right]_e = \int_{A_e} \left[ B \right]^T \left[ D^e \right] \left[ B \right] \, dA_e, \]

\[ \left[ P \right]_e = \int_{A_e} \left[ N \right]^T p dA_e \] are stiffness matrix and node load vector of the element,

\[ \left[ N \right] = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 & 0 \end{bmatrix} \]

\( p \) is uniform load.

Kinetic energy \( T_e \) of the element [11]:

\[ T_e = \frac{1}{2} \int _{t_1}^{t_2} \rho \left[ \dot{q} \right]^T_\varepsilon \left[ \dot{q} \right]_\varepsilon \, dV_e \frac{1}{2} \int _{t_1}^{t_2} \rho \left[ \dot{q} \right]^T_\varepsilon \left[ N \right] \, dV_e \left[ \ddot{q} \right]_\varepsilon = \frac{1}{2} \int _{t_1}^{t_2} \left[ \dot{q} \right]^T_\varepsilon \left[ M \right]_\varepsilon \left[ \dot{q} \right]_\varepsilon, \]  
(26)

Use of Hamilton principle for the element:

\[ \delta \int_{t_1}^{t_2} [T_e - \Pi_e] \, dt = 0 \]  
(30)
\[
[C] = \alpha_r [M] + \beta_r [K], \tag{35}
\]
where \(\alpha_r, \beta_r\) are Rayleigh damping coefficients [7], [11].

After amplitude suppression, (32) becomes:

\[
\left[J\right] \left[\ddot{U}\right] + \left[C\right] \left[\dot{U}\right] + \left[K\right] \left[U\right] = \left[R_y \right] + \left[R_o \right] = \left[R\right]. \tag{36}
\]

This is a linear equation; the correct position of such equation depends on time. This equation is solved using the Newmark direct integral method and programming in the Matlab environment, including the earthquake loading vector updated after each time step.

3. Finite Element Algorithm

Step 1. Assign initial conditions
Step 2. Calculate initial parameters, give foundation acceleration chart function \(\left(\Delta t, -\ddot{u}_g^X, \Delta t, -\ddot{u}_g^Y\right)\);
Step 3. Conduct integral cycle in each time step:
- Update node load vector caused by earthquake;
- Identify vectors of displacement, velocity, acceleration and internal force;
Step 4. Export results.

The calculation program was developed by the authors, and is called 3D_frame_earthquake.m.

4. Numerical Results and Discussion

The 9–story building is calculated with the model of a frame-plate system, including 20 vertical columns, 90 beams and 10 floors. They are described to be flat slabs, the general story has a height of \(h_0 = 5\) m. For the 9 remaining stories, each story has a height of \(h = 3.5\) m, total height level of the building \(H = 6\) m + 9\(h = 36.5\) m, width \(W = 4 \times 6\) m = 24 m, depth \(B = 3\times6\) m = 18 m. Cross section of columns is 0.55 m \(\times\) 0.55 m, beams are 0.3 m \(\times\) 0.5 m, and floors’ thickness is 0.15 m. Columns and floors are made of reinforced concrete with properties of \(E = 3.64 \times 10^{10}\) N/m\(^2\), \(\nu = 0.3\), \(\rho = 2.76 \times 10^3\) kg/m\(^3\). The middle column base of the building has a V- shaped crack with a depth of \(h_0 = 0.1\) m. Finite element model of the building is shown in Fig. 4. The soil acceleration influencing 2 directions, including \(\ddot{u}_g^X\), \(\ddot{u}_g^Y = 0\), \(\ddot{u}_g^{X,Y}\) is shown in Fig. 5.

Figure 5. Soil acceleration with \(\left(\ddot{u}_g^X\right)_{\text{max}} = 17085\) m/s\(^2\).

Results export: Central point of the 9\(^{th}\) story’s floor (point A) and column base in the middle and front sides of the building (point B).

The displacement \(u\) at the point A and bending moment \(M_y\) at the point B are shown in Fig. 6, 7 following time.

Figure 6. Time history of \(u\) displacement.
The maximum displacement ($u_{\text{max}}$) and bending moment ($M_{y}^{\text{max}}$) are shown in Table 1.

**Table 1.** The maximum of the displacement and bending moment (with crack and without crack).

| $u_{\text{max}}$ [m] | $M_{y}^{\text{max}}$ [Nm] |
|-----------------------|--------------------------|
| with crack 0.0684     | without crack 0.0598     |
| with crack 4.9535e+05  | without crack 6.1870e+05 |

Error: 14.38% for displacement, 19.94% for bending moment.

The crack affects the dynamics of the building accordingly in the event of an increase in the cracks in the column, and the displacement of the building.

### 4.1. Influences of Damping

Survey is conducted in two scenarios, with damping and without damping. The results in terms of horizontal displacement and bending moment at points A and B are as shown in Figs. 8, 9 and Table 2.

**Table 2.** The maximum of displacement and bending moment (with damping and without damping).

| $u_{\text{max}}$ [m] | $M_{y}^{\text{max}}$ [Nm] |
|-----------------------|--------------------------|
| with damping 0.0684   | without damping 0.1432   |
| with damping 4.9535e+05 | without damping 7.1462e+05 |

Error: 2.09 times for displacement, 44.27% for bending moment.

The damping of the structures affects the dynamic of the building and reduces the displacement and internal force.

### 4.2. Influences of the Building Height

The relationship between the height of the building and the displacement of the floors of each story is shown in Fig. 10.

Comment: Displacement of the building has a non-linear dependence on the height of the building; it increases quickly when $H \in [5 \div 22.5]$ m (2.98 times-with damping, 3.36 times-without damping) and slowly when $H \in [22.5 \div 36.5]$ m (1.28 times-with damping, 1.26 times-without damping). This shows that the solution of decreasing vibration in intermediary positions of the building is reasonable.
5. Discussion of Results

Our research arrived at several main results, including the development of an algorithm and dynamic analysis program for high buildings under earthquake loading, foundation acceleration following two orthogonal directions in the horizontal plane, and the use of a 3D modeling frame-plate system with cracks on the main column. The numerical analysis conducted in some Mathematics problems with changing parameters reflects the suitability of the calculation model; algorithm and calculation capacity of the program. The results contribute to the calculation, design and selection of reasonable solutions for new design, diagnosis of damages and reinforcement of modern high buildings.

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