Construction and Application of Groundwater Environmental Health Assessment Model Based on Entropy Weight and Fuzzy Matter Element

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Abstract. Groundwater overdraft has become a major challenge, most serious in northwestern and northern China. A series of environmental geological problems caused by groundwater overdraft will be more extreme, which restrict the development of society. In this study, matter-element theory is introduced. Moreover, a matter-element evaluation model is established, and a matter-element evaluation algorithm based on fuzzy entropy weight is proposed. The algorithm takes nature, environment, economy and society as the characteristic indexes of the evaluation category. Four kinds of typical regions were chosen including Core Economic Zone (A), Bearing Industry and Trade Economic Zone (B), Agricultural Machinery and Trade Economic Zone (C) and Ecological Agricultural Economic Zone (D) as the study region. The risk assessment of groundwater is evaluated using the groundwater environmental health assessment model in this study. The results showed that the constructed model can be used for the assessment of environment health of groundwater successfully. The simulation results showed that the groundwater was in a medium health state in Linqing City. The results showed that groundwater environment of ‘healthy’ state of nearness concentrated between 0.2 and 0.4. The descending order for typical regions groundwater environment ‘healthy’ state of nearness from high to low are B, D, A, and C.

1. Introduction
Environmental health of groundwater is one of the foundations and guarantees for ecosystem health[1]. With the continuous growing of industry and cities, the level of groundwater continues to fall. Moreover, many unreasonable mining has increased the lack of water resources, and seriously affected the environmental health in China. Serious groundwater pollution not only cause infectious diseases, but also severely restricts urban economic and social development[2]. Therefore, it is urgent to evaluate the environmental health of groundwater. At present, domestic and international research methods for groundwater environmental health assessment mainly include functional fuzzy evaluation method[3], grey relational model based on entropy weight[4], grey relevance model[5], grey clustering model[6], based on Fuzzy optimization method for maximum entropy, and so on[7-8]. Although the above methods have achieved certain results in the theoretical analysis, the evaluation methods are relatively simple and are not suitable for multi-objective decision-making and multi-index evaluation. This paper adopts the groundwater environmental health assessment model based on entropy weight-fuzzy matter element, and takes the groundwater system of Linqing in Shandong Province as an example to evaluate the groundwater environment health status. The results provide a basis for sustainable development and utilization of groundwater resources and virtuous cycle of groundwater system.
2. Groundwater environment health assessment model based on entropy weight-fuzzy matter element

2.1 Entropy weight method to determine index weights

2.1.1 Initial matrix standardization

The groundwater environmental health assessment index system has m evaluation samples and n evaluation indicators, it constitutes the initial matrix $(X_{ij})_{m\times n}$, and it is standardized, that is the standardized matrix; $R=(r_{ij})_{m\times n}$.

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}$$  

(1)

2.1.2 Calculate the entropy value $e_j$ of the j-th evaluation index

$$e_j = -k \sum_{i=1}^{m} p_{ij} \cdot \ln p_{ij}$$  

(2)

Where, $k = \frac{1}{\ln m}$, $p_{ij} = r_{ij} / \sum_{i=1}^{m} r_{ij}$

2.1.3 Calculate the weight of the j-th evaluation indicator $w_j$

$$w_j = (1-e_j) / \sum_{j=1}^{n} (1-e_j)$$  

(3)

2.2 Fuzzy matter element evaluation model

2.2.1 Compound fuzzy matter element

In the groundwater environmental health assessment index system, the corresponding fuzzy values of the n evaluation indicators $C_1, C_2, ..., C_n$ are $u(x_1), u(x_2), ..., u(x_n)$, respectively, then the compound fuzzy matter elements composed of m evaluation samples are:

$$R = \begin{bmatrix} M_1 & M_2 & \cdots & M_n \\ C_1 & u(x_1) & \cdots & u(x_n) \\ C_2 & u(x_1) & \cdots & u(x_n) \\ \vdots & \vdots & \ddots & \vdots \\ C_n & u(x_1) & \cdots & u(x_n) \end{bmatrix}$$  

(4)

Where, $M_i$ is the i-th evaluation sample, $i=1,2,..m$; $C_j$ is the j-th evaluation index, $j=1,2,..n$; $u(x_{ij})$ is the fuzzy value corresponding to the j-th indicator of the i-th sample.

2.2.2 Fuzzy matter element of optimal subordinate degree

The corresponding fuzzy value of each individual assessment index is subject to the corresponding fuzzy value of each assessment index in standard samples. That subordinate degree is called the optimal subordinate degree which can be calculated by using Eqs (5) and (6). Some eigenvalues of the evaluation indicators are the maximum optimum type, but others are the minimum optimum type in different projects. Therefore, it is necessary to adopt different formulate according to respective assessment index, this paper adopts two forms as follows:

The bigger-more-optimal type assessment index can be transformed according to

$$u_y = \frac{x_y - \min x_y}{\max x_y - \min x_y}$$  

(5)
The less-more-optimal type assessment index can be transformed according to
\[
u_{ij} = \frac{\text{max } x_{ij} - x_{ij}}{\text{max } x_{ij} - \text{min } x_{ij}}
\]
(6)

Where: \(x_{ij}\) is the magnitude corresponding to the j-th evaluation index of the i-th sample, and \(\text{max } x_{ij}\), \(\text{min } x_{ij}\) are the maximum value and the minimum value, respectively.

2.2.3 Standard fuzzy matter elements and difference square fuzzy matter elements.
The n-dimensional standard fuzzy matter element, which is the optimal fuzzy matter element, is constructed by selecting the maximum optimal subordinate degree in the evaluation sample:

\[
R_{si} = \begin{bmatrix}
    M_1 & M_2 & \ldots & M_n \\
    C_i & A_i & \ldots & A_i \\
    \vdots & \vdots & \ddots & \vdots \\
    C_e & A_e & \ldots & A_e
\end{bmatrix}
\]
(7)

The difference square compound fuzzy matter element \(R_{ij}\) can be obtained by using \(\Delta_{ij}\) to represent the square of the difference between the standard fuzzy matter element and the optimal subordinate degree fuzzy matter element, that is to say:

\[
\Delta_{ij} = (u_{ij} - u_{ij})^2, \quad i=1,2,\ldots,m; \quad j=1,2,\ldots,n.
\]

\[
R_{ij} = \begin{bmatrix}
    M_1 & M_2 & \ldots & M_n \\
    C_i & A_i & \ldots & A_i \\
    \vdots & \vdots & \ddots & \vdots \\
    C_e & A_e & \ldots & A_e
\end{bmatrix}
\]
(8)

2.2.4 Euclid approach degree compound fuzzy matter element
Approach is the degree to which the evaluated sample are evaluated close to standard things. According to the approach, the groundwater environment health status of the evaluation sample can be sorted, classified and given the health level. The larger the Euclid approach value \(\rho_{hi}\) is, the better the groundwater environment health condition will be in the region. The smaller the Euclid approach value indicates the worse the groundwater environment health condition in the region.

\[
\rho_{hi} = 1 - \frac{\sum w_i A_{ij}}{\text{max } w_i A_{ij}}, \quad j=1,2,\ldots,m
\]
(9)

Then the compound fuzzy matter element \(R_{ij}\) of the Euclid approach degree is constituted as:

\[
R_{pi} = \begin{bmatrix}
    M_1 & M_2 & \ldots & M_n \\
    \rho_{1i} & \rho_{2i} & \ldots & \rho_{ni}
\end{bmatrix}
\]
(10)

3. Groundwater environmental health assessment in Linqing
The groundwater in the area is Quaternary pore diving, and the main aquifer is fine sand and medium fine sand, followed by sandy loam and fissure clay. The permeability coefficient of the aquifer is \(5.4 \times 10^{-3} \sim 1.9 \times 10^{-4}\) cm/s, which is a medium permeable layer. The water table is relatively large, with a depth of 2.00~11.00m and an elevation of 25.46~30.25 m. It mainly accepts atmospheric precipitation and surface water replenishment, with evaporation, runoff and artificial water as its main excretion method.

3.1 Assessment index selection and assessment criteria
This study combines the actual situation of Linqing and draws on the existing academic research\[9-11\], based on the easy-to-acquire, comprehensive and scientific selection principle of indexes, 10 evaluation indexes are selected from natural, social, environmental and economic attributes (Table 1). Based on the assessment criteria of domestic groundwater environmental health research and index classification criteria, a set of reference standards for groundwater environmental health status in Linqing was established to classify groundwater environmental health, which was classified into five grades: healthy,
relatively healthy, medium healthy, sub-healthy and unhealthy. The specific standards are shown in Table 2.

Table 1 Groundwater environmental health assessment indexes in Linqing

| Assessment object | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 |
|-------------------|----|----|----|----|----|----|----|----|----|-----|
| Healthy           | >80| >30| >2000| >80| I | <40 | <40 | 1 | <40 | <50 |
| Relatively healthy| 40~80| 20~30| 1200~2000| 60~80| II | 40~60 | 40~50 | 2 | 40~55 | 50~100 |
| Medium healthy    | 17~40| 10~20| 800~1200| 40~60| III | 60~80 | 50~60 | 3 | 55~73 | 100~500 |
| Sub-healthy       | 5~17| 5~10| 400~800| 20~40| IV | 80~90 | 60~75 | 4 | 73~90 | 500~1000 |
| Unhealthy         | <5 | <5 | <400 | <20 | V | >90 | >75 | 5 | >90 | >1000 |

3.2 Groundwater environmental health assessment

According to industrial development layout plan and urban system plan in Linqing, the groundwater environmental health assessment is divided into four parts: Core Economic Zone A, Bearing Industry and Trade Economic Zone B, Agricultural Machinery and Trade Economic Zone C and Ecological Agricultural Economic Zone D (Figure 1).

![Figure 1 Groundwater environmental health assessment zoning map of Linqing](image)

3.2.1 Entropy weight method to calculate weights

① Initial matrix standardization

Referring to the relevant data of Linqing, the value of each evaluation index is shown in Table 3.

Table 2 Groundwater environmental health assessment standards

| Assessment index | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 |
|------------------|----|----|----|----|----|----|----|----|----|-----|
| Healthy          | >80| >30| >2000| >80| I | <40 | <40 | 1 | <40 | <50 |
| Relatively healthy| 40~80| 20~30| 1200~2000| 60~80| II | 40~60 | 40~50 | 2 | 40~55 | 50~100 |
| Medium healthy   | 17~40| 10~20| 800~1200| 40~60| III | 60~80 | 50~60 | 3 | 55~73 | 100~500 |
| Sub-healthy      | 5~17| 5~10| 400~800| 20~40| IV | 80~90 | 60~75 | 4 | 73~90 | 500~1000 |
| Unhealthy        | <5 | <5 | <400 | <20 | V | >90 | >75 | 5 | >90 | >1000 |

Four assessment objects were constructed from the assessment standard values, and the initial matrix of 10 assessment features:
The initial index value has positive and negative indicators. In order to make the positive and negative indicators comparable, this paper first uses the membership function method to process the original data. The formula is as follows:

$$R_j = \frac{x_{ij} - x_{\min}}{x_{\max} - x_{\min}}\quad\text{(Positive indicator)}$$

$$R_j = \frac{x_{\max} - x_{ij}}{x_{\max} - x_{\min}}\quad\text{(Negative indicator)}$$

In the formula, \(x_{\min}\) and \(x_{\max}\) are the upper and lower bounds of the function. In this paper, the maximum value of the index is increased by 10% and the minimum value is reduced by 10% as the upper and lower bounds. \(C_1\sim C_4\) are positive indicators, and \(C_5\sim C_{10}\) are negative indicators. According to the formula, the normalized matrix \(R=(r_{ij})_{m\times n}\) is calculated as follows:

$$R = \begin{bmatrix}
A & B & C & D \\
C_1 & 0.415 & 0.8293 & 0.0976 & 0.0976 \\
C_2 & 0.0387 & 0.4541 & 0.6151 & 0.8406 \\
C_3 & 0.1519 & 0.4053 & 0.7215 & 0.7848 \\
C_4 & 0.8600 & 0.4600 & 0.9600 & 0.2600 \\
C_5 & 0.1539 & 0.5385 & 0.9231 & 0.9231 \\
C_6 & 0.2152 & 0.5094 & 0.8481 & 0.8481 \\
C_7 & 0.8188 & 0.6377 & 0.4565 & 0.2391 \\
C_8 & 0.1539 & 0.5385 & 0.9231 & 0.9231 \\
C_9 & 0.8994 & 0.4469 & 0.2257 & 0.1814 \\
C_{10} & 0.8148 & 0.9074 & 0.5370 & 0.1667 \\
\end{bmatrix}$$

② Calculate the entropy value \(e_j\):

Calculate the entropy value \(e_j\) according to the Eqs (2), and the result is shown in Table 4.

| Healthy | Relatively healthy | Medium healthy | Sub-healthy | Unhealthy |
|---------|-------------------|----------------|-------------|-----------|
| Healthy | 80                | 0              | 17          | 5         |
| Relatively healthy | 104            | 170            | 198         | 30        |
| Medium healthy | 70              | 30             | 20          | 5         |
| Sub-healthy | 50              | 60             | 40          | 20        |
| Unhealthy | 20              | 28             | 36          | 10        |

③ Calculate weight \(w_j\):

Weight \(w_j\) is calculated according to Eqs (3)

$$w_j = (0.2016, 0.1060, 0.0782, 0.1559, 0.0851, 0.0564, 0.0478, 0.085, 0.1087, 0.0752)$$

3.2.2 Fuzzy matter element evaluation model

① Construction of a compound fuzzy matter element

Based on the tabular data and grading standards, a compound fuzzy matter element \(R\) of 10 evaluation indicators is established:

$$R = \begin{bmatrix}
A & B & C & D \\
C_1 & 10 & 14 & 8 & 8 & 80 & 40 & 17 & 5 \\
C_2 & 104 & 150 & 170 & 198 & 30 & 20 & 10 & 5 \\
C_3 & 120 & 140 & 165 & 170 & 2000 & 1200 & 800 & 400 \\
C_4 & 70 & 50 & 30 & 40 & 80 & 60 & 40 & 20 \\
C_5 & 4 & 3 & 2 & 2 & 1 & 2 & 3 & 4 & 5 \\
C_6 & 85 & 70 & 60 & 60 & 40 & 40 & 60 & 20 \\
C_7 & 50 & 55 & 60 & 66 & 40 & 40 & 50 & 60 & 70 \\
C_8 & 4 & 3 & 2 & 2 & 1 & 2 & 3 & 4 & 5 \\
C_9 & 50 & 70 & 80 & 82 & 40 & 40 & 55 & 73 & 90 \\
C_{10} & 22 & 20 & 28 & 36 & 50 & 50 & 100 & 500 & 1000 \\
\end{bmatrix}$$
2. Construction of the fuzzy matter element of optimal subordinate degree
Eqs (5), (6) is used to calculate the $R_{mn}$ of 10 indicators in 9 samples composed of 4 districts and 5 grade assessment standards in Linqing.

$$R_{mn} = \begin{bmatrix} A & B & C & D & \text{Healthy} & \text{Relatively healthy} & \text{Medium healthy} & \text{Sub-healthy} & \text{Unhealthy} \\ C_1 & 0.1250 & 0.1750 & 0.1000 & 0.1000 & 1.0000 & 0.5000 & 0.2125 & 0.0625 \\ C_2 & 0.5252 & 0.7576 & 0.6586 & 1.0000 & 0.1515 & 0.1515 & 0.1010 & 0.0916 \\ C_3 & 0.0600 & 0.0700 & 0.0825 & 0.0850 & 1.0000 & 1.0000 & 0.6000 & 0.4000 \\ C_4 & 0.8750 & 0.6250 & 0.3750 & 0.5000 & 1.0000 & 1.0000 & 0.7500 & 0.5000 \\ C_5 & 0.2500 & 0.3333 & 0.5000 & 0.5000 & 1.0000 & 0.5000 & 0.3333 & 0.2500 \\ C_6 & 0.4706 & 0.5714 & 0.6667 & 0.6667 & 1.0000 & 1.0000 & 0.6667 & 0.5000 \\ C_7 & 0.8000 & 0.7272 & 0.6667 & 0.6667 & 1.0000 & 1.0000 & 0.8000 & 0.6667 \\ C_8 & 0.2500 & 0.3333 & 0.5000 & 0.5000 & 1.0000 & 0.5000 & 0.3333 & 0.2500 \\ C_9 & 0.8000 & 0.5714 & 0.5000 & 0.4878 & 1.0000 & 1.0000 & 0.7273 & 0.5479 \\ C_{10} & 0.9911 & 1.0000 & 0.7143 & 0.5556 & 0.4000 & 0.4000 & 0.2000 & 0.0400 \\ \end{bmatrix}$$

3. The difference square fuzzy matter element are constructed.
The difference square fuzzy matter element $R_{ij}$ is calculated by the Eqs (8).

$$R_{ij} = \begin{bmatrix} A & B & C & D & \text{Healthy} & \text{Relatively healthy} & \text{Medium healthy} & \text{Sub-healthy} & \text{Unhealthy} \\ C_1 & 0.7555 & 0.6006 & 0.8100 & 0.8100 & 0.0000 & 0.0000 & 0.2500 & 0.6202 \\ C_2 & 0.2254 & 0.0598 & 0.0200 & 0.0000 & 0.7200 & 0.7200 & 0.0802 & 0.9166 \\ C_3 & 0.1856 & 0.6969 & 0.8418 & 0.8372 & 0.0000 & 0.0000 & 0.1600 & 0.3600 \\ C_4 & 0.0156 & 0.2406 & 0.3906 & 0.2500 & 0.0000 & 0.0000 & 0.0625 & 0.5250 \\ C_5 & 0.5625 & 0.4445 & 0.2500 & 0.2500 & 0.0000 & 0.0000 & 0.0445 & 0.5525 \\ C_6 & 0.2803 & 0.1837 & 0.1111 & 0.1111 & 0.0000 & 0.0000 & 0.0111 & 0.2900 \\ C_7 & 0.6400 & 0.0744 & 0.1111 & 0.1552 & 0.0000 & 0.0000 & 0.0400 & 0.1111 \\ C_8 & 0.5625 & 0.4445 & 0.2500 & 0.2500 & 0.0000 & 0.0000 & 0.0445 & 0.5525 \\ C_9 & 0.0400 & 0.1837 & 0.2500 & 0.2623 & 0.0000 & 0.0000 & 0.0744 & 0.2044 \\ C_{10} & 0.0003 & 0.0000 & 0.0185 & 0.1975 & 0.5600 & 0.3600 & 0.0400 & 0.9216 \\ \end{bmatrix}$$

4. Calculate the Euclid approach degree compound fuzzy matter element
The Euclid approach of each evaluation sample is calculated by Eqs (10). The Euclid approach degree compound fuzzy matter element is shown as:

$$R_{ai} = \begin{bmatrix} A & B & C & D & \text{Healthy} & \text{Relatively healthy} & \text{Medium healthy} & \text{Sub-healthy} & \text{Unhealthy} \\ 0.3932 & 0.4148 & 0.3839 & 0.3939 & 0.6784 & 0.6380 & 0.4537 & 0.2969 & 0.1887 \end{bmatrix}$$

3.2.3 Analysis of results
According to the Euclid approach degree, the groundwater environment health of four areas in Linqing was evaluated, and the health level of each area was obtained (see Table 5 for details).

| Assessment area | Assessment results in four areas of Linqing |
|-----------------|------------------------------------------|
| Core Economic Zone A | Medium healthy 3 |
| Bearing Industry and Trade Economic Zone B | Medium healthy 1 |
| Agricultural Machinery and Trade Economic Zone C | Medium healthy 4 |
| Ecological Agricultural Economic Zone D | Medium healthy 2 |

The results of groundwater environmental health assessment in Linqing showed that the groundwater is in a Medium healthy state. The simulation results showed that the groundwater was in a medium healthy state. The results showed that groundwater environment of ‘healthy’ state of nearness concentrated between 0.2 and 0.4. The descending order for typical regions groundwater environment ‘healthy’ state of nearness from high to low are B, D, A, and C.

4. Conclusion
(1) The example results show that the entropy weight-fuzzy matter element evaluation model is effective in the comprehensive evaluation of groundwater environment health with multiple indicators and multiple points.

(2) The groundwater environment health status in Linqing is Medium healthy. The approach of the four sections of the groundwater environment to the ‘healthy’ state is basically concentrated between...
0.3 and 0.5. The order from the largest to the smallest is: Bearing Industry and Trade Economic Zone B, Ecological Agricultural Economic Zone D, Core Economic Zone A, Agricultural Machinery and Trade Economic Zone C.

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