Dyonic configurations in nonlinear electrodynamics coupled to general relativity

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We consider static, spherically symmetric configurations in general relativity, supported by nonlinear electromagnetic fields with gauge-invariant Lagrangians depending on the single invariant \( f = F_{\mu\nu}F^{\mu\nu} \). After a brief review on black hole (BH) and solitonic solutions, obtained so far with pure electric or magnetic fields, an attempt is made to obtain dyonic solutions, those with both electric and magnetic charges. A general scheme is suggested, leading to solutions in quadratures for an arbitrary Lagrangian function \( L(f) \) (up to some monotonicity restrictions); such solutions are expressed in terms of \( f \) as a new radial coordinate instead of the usual coordinate \( r \). For the truncated Born-Infeld theory (depending on the invariant \( f \) only), a general dyonic solution is obtained in terms of \( r \). A feature of interest in this solution is the existence of a special case with a self-dual electromagnetic field, \( f = 0 \) and the Reissner-Nordström metric.

1 Introduction. A brief review

In the framework of general relativity (GR) and its extensions, nonlinear electrodynamics (NED) as a possible material source of gravity attracts much attention since, among other reasons, it leads to many space-time geometries of interest, in particular, regular black holes and starlike or solitonlike configurations.

The history of NED apparently begins with Born and Infeld’s effort to remove the central singularity of a point charge by generalizing Maxwell’s theory [1], the version of NED of Heisenberg and Euler motivated by particle physics [2], and their further extension by Plebanski [3] in the framework of special relativity.

In this paper, we consider static, spherically symmetric configurations in NED coupled to GR and begin with a brief overview of the results obtained thus far in this area. For such systems, one usually considers the action

\[
S = \frac{1}{2} \int \sqrt{-g} d^4x [R - L(f)], \quad f = F_{\mu\nu}F^{\mu\nu}, \tag{1}
\]

with an arbitrary function \( L(f) \) (\( F_{\mu\nu} \) is the Maxwell tensor, and units with \( c = 8\pi G = 1 \) are used). Then, assuming static spherical symmetry, the stress-energy tensor (SET) satisfies the condition \( T^t_t = T^r_r \), hence, due to the Einstein equations, the metric can be written as

\[
ds^2 = A(r) dt^2 - \frac{dr^2}{A(r)} - r^2(d\theta^2 + \sin^2 \theta d\phi^2). \tag{2}
\]

The only nonzero components of \( F_{\mu\nu} \) are \( F_{tr} = -F_{rt} \) (a radial electric field) and \( F_{\theta\phi} = -F_{\phi\theta} \) (a radial magnetic field). The Maxwell-like equations \( \nabla_\mu (L f F^{\mu\nu}) = 0 \) and the Bianchi identities \( \nabla_\mu \ast F^{\mu\nu} = 0 \) for the dual field \( \ast F^{\mu\nu} \) lead to

\[
r^2 L f F^{tr} = q_e, \quad F_{\theta\phi} = q_m \sin \theta, \tag{3}
\]

where \( q_e \) and \( q_m \) are the electric and magnetic charges, respectively, and \( L_f \equiv dL/df \). Accordingly, the nonzero SET components are

\[
T^t_t = T^r_r = \frac{1}{2} L + f_e L_f, \quad T^\theta_\theta = T^\phi_\phi = \frac{1}{2} L - f_m L_f,
\]

\[
f_e = 2 F_{tr} F^{rt} = \frac{2 q_e^2}{L^2 r^4}, \quad f_m = 2 F_{\theta\phi} F^{\theta\phi} = \frac{2 q_m^2}{r^4},
\]

thus the invariant \( f \) is \( f = f_m - f_e \). The metric function \( A(r) \) is determined from the Einstein equations as

\[
A(r) = 1 - \frac{2M(r)}{r}, \quad M(r) = \frac{1}{2} \int \mathcal{E}(r)r^2 dr, \tag{5}
\]

where \( \mathcal{E}(r) \equiv T^t_t \) is the energy density, and \( M(r) \) is the mass function. It is a general relation [4], but it is only a part of a possible solution: the latter
requires knowledge of $L(f)$ and both electric and magnetic fields.

For configurations with an electric field only, the general solution was obtained in 1969 by Pellicer and Torrence [5]. They found, under some reasonable assumptions, that the solution can have well-behaved electromagnetic and metric tensors at the center $r = 0$. However, a no-go theorem proved in [6] showed that there is no such function $L(f)$ having a Maxwell weak-field limit ($L \sim f$ as $f \to 0$) that the electric solution described by [2], [3], [4] has a regular center. The reason is that at such a center the electric field should be zero but the field equations then imply $fL_f^2 \to \infty$, hence $L_f \to \infty$ as $r \to 0$.

In 1998 and 1999 appeared a few papers by Ayon-Beato and Garcia (see, e.g., [7]), where some special cases of the Pellicer-Torrence solution were presented, describing configurations with or without horizons (that is, BH or solitonic ones), having a regular center and a Reissner-Nordström asymptotic behavior at large $r$. Each of these examples seemed to violate the above no-go theorem. Trying to clarify the situation, I found the following explanation [8]: in all such cases, in a “regular” solution there are different functions $L(F)$ at large and small $r$: at large $r$ we have $L \sim f$ whereas at small $r$ the theory is strongly non-Maxwell ($f \to 0$ but $L_f \to \infty$, in agreement with the no-go theorem). An inspection showed that it is indeed the case in all examples [4].

It was further shown in [4] that a regular center is also impossible in dyonic configurations, with both $q_e \neq 0$ and $q_m \neq 0$, if $L(f) \sim f$ as $f \to 0$. However, purely magnetic configurations, both BH and solitonic ones, are possible and are readily obtained under the condition $L(f) \to L_\infty < \infty$ as $f \to \infty$ [4]. Electric models with the same regular metrics can be obtained from the magnetic ones using the so-called FP duality [4] (not to be confused with the familiar electric-magnetic duality in Maxwell’s theory) that connects solutions with the same metric corresponding to different NED theories. However, unlike the magnetic solutions, the electric ones suffer serious problems connected with multivaluedness of $L(f)$ and a singular behavior of the electromagnetic fields on the corresponding branching surfaces [4].

Many results of interest were obtained since then.

Burinskii and Hildebrandt [9] showed that the above no-go theorem for electric solutions may be circumvented by considering a kind of phase transition on a certain sphere, outside which there is a purely electric field $F_{\mu\nu}$ but inside which it is purely magnetic. An external observer then sees an electrically charged BH or soliton. They also pointed out links between these regular GR/NED solutions and the superconductivity and confinement phenomena [9] [10].

Some authors considered GR/NED solutions with NED Lagrangians depending on both electromagnetic invariants $f = F_{\mu\nu}F^{\mu\nu}$ and $f = \epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$, where $\epsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita unit completely antisymmetric tensor [11] [12] [13]. It should be mentioned that the most famous versions of NED, the Born-Infeld [1] and Euler-Heisenberg [2] theories belong to this class.

A straightforward extension of static, spherically symmetric NED solutions to GR with a nonzero cosmological constant $\Lambda$, leading to their (anti-)de Sitter (dS and AdS) asymptotic behavior, was considered, in particular, in [14] [15] [16]. If we add $-2\Lambda$ to $R$ in the action (1), the only change in the expression (5) for the metric is that the term $-\Lambda r^2/3$ is added to $A(r)$.

Thermodynamic properties of regular NED BHs were discussed in [17] [18] [19] [20] [21] [22] [23] [24], in particular, the first law of thermodynamics, heat capacity, the validity of Smarr’s formula for the BH mass, etc.

Cylindrically symmetric counterparts of the spherical solutions were considered in [25], with different directions of the electric or magnetic fields: radial, longitudinal and azimuthal ones. There are no BH solutions, while the field behavior on a possible regular axis (especially in the case of radial fields) is somewhat similar to that at a regular center in spherical symmetry, and special conditions appear if asymptotic flatness is required.

More complicated axially symmetric regular GR/NED configurations were discussed in [26] [27] [28]. It was claimed, in particular, that in these models the Kerr ring singularity is replaced by a ring with superconducting current that serves as a nondissipative source of the asymptotically Kerr-Newman geometry [28]. It seems that this area of utmost interest is only beginning to be studied.

It has been shown that NED can be a source of evolving wormholes [29] [30] [31], while static worm-
holes are impossible since the SET (1) (though marginally) satisfies the weak and null energy conditions.

Many authors studied the stability properties of NED BHs [32, 33, 34, 35, 36, 20] and their quasinormal modes related to different kinds of perturbations [37, 35, 36, 38, 16] in cases where these BHs are stable. In particular, simple conditions on the NED Lagrangian have been derived, which imply linear stability in the domain of outer communication [32], and linear stability has been verified for a number of particular examples.

Quantum effects in the fields of NED BHs were studied in [39, 15].

This list is certainly incomplete: it does not include numerous papers devoted to studies of special cases of electric and magnetic solutions, to say nothing of such subjects as gravitational lensing, particle motion and matter accretion in the fields of NED BHs, their counterparts in scalar-tensor, f(R) and multidimensional theories of gravity, inclusion of dilaton-like interactions, non-Abelian fields, constructions with thin shells, etc.

A substantial gap in these studies is the absence of dyonic solutions (at least to our knowledge), and we here try to consider this challenging problem. Before that, for completeness and comparison, we recall the methods of obtaining pure magnetic and electric solutions.

2 Pure magnetic and electric solutions

Pure magnetic solutions (q_e = 0) are obtained from Eqs. (1) and (5) quite easily. Indeed, if L(f) is specified, then, since now f = 2q_m/r^4, the function E(r) = L/2 is known from (1), and the metric function A(r) is found by integration in (5). If, on the contrary, A(r) is known (or chosen at will), then E(r) = L(f)/2 is found from (5), and L(f) is restored since f = 2q_m/r^4. Solutions with a regular center are obtained if we require A(r) = 1 + O(r^2) at small r (and this in turn requires L → L_0 < ∞ as f → ∞ [4]), asymptotically flat configurations require A(r) = 1 - 2m/r + o(1/r), where m is the Schwarzschild mass, and, in the case Λ ≠ 0, asymptotically (A)dS solutions are obtained by adding -Ar^2/3 to A(r) in [5]. This is an easy way of constructing regular magnetic BH and solitonic solutions, used by many authors, probably beginning with [4].

Pure electric solutions (q_e ≠ 0, q_m = 0) are obtained in a similar way by using a Hamiltonian form of NED [5, 7], obtained from the original, Lagrangian one by a Legendre transformation: one introduces the tensor P_{µν} = L_f F_{µν} with its invariant p = -P_{µν}P^{µν} and considers the Hamiltonian-like quantity H = 2fL_f - L = 2T_f^t as a function of p; then H(p) can be used to specify the whole theory. One has then

L = 2pH_p - H, \quad L_f H_p = 1, \quad f = pH_f^2. (6)

with H_p ≡ dH/dp. Thus we have simply p = 2q_e^2/r^4, and specifying H(p) = 2E(r), we directly find M(r) and A(r) from (5). On the contrary, specifying A(r), from (5) we determine E(r) = H(p)/2.

A regular center r = 0 requires a finite limit of H as p → ∞, then the integral in (5) gives the mass function for given q_e that provides regularity. However, in any regular asymptotically flat (or (A)dS) solution f = 0 at both r = 0 and r = ∞, so f inevitably has at least one maximum at some p = p^*, violating the monotonicity of f(p), which is necessary for equivalence of the f and p frameworks. It has been shown [4] that at an extremum of f(p) the Lagrangian function L(f) suffers branching, its plot forming a cusp, and different functions L(f) correspond to p < p^* and p > p^*. Another kind of branching occurs at extrema of H(p), if any, and the number of Lagrangians L(f) on the way from infinity to the center equals the number of monotonicity ranges of f(p) [4].

It might seem that the Hamiltonian' framework is not worse than the Lagrangian one, even though the latter is directly related to the least action principle. However, as shown in [4], at p = p^* the electromagnetic field exhibits a singular behavior, well revealed using the effective metric [10, 11] in which NED photons move along null geodesics. This metric is singular at extrema of f(p), and the effective potential for geodesics exhibits infinite wells at which NED photons are infinitely blueshifted [4, 11] and can thus create a curvature singularity by their back reaction on the metric. Thus any regular electric solution not only fails to correspond to a fixed Lagrangian L(f) but has other important undesired features.

Each choice of A(r) thus gives rise to both electric and magnetic solutions related by FP duality,
which connects the $F_{\mu\nu}$ and $P_{\mu\nu}$ frameworks \[1\]. Though, the choice of $A(r)$ can be restricted by imposing the weak and dominant energy conditions (see, e.g., \[22\]), and individual features of numerous special solutions also deserve discussion.

## 3 Dyonic configurations

Let us now assume that both $q_e$ and $q_m$ are nonzero. The difficulty with considering Eqs. (\[1\]) and (\[5\]) is that $f(r)$ (or alternatively $p(r)$) is now not known explicitly. Using, for definiteness, the Lagrangian formulation of the theory, we have the expression

$$f(r) = \frac{2}{r^2} \left( q_m^2 - \frac{q_e^2}{L_f^2} \right).$$

(7)

Comparing the expressions for $E(r)$ from (\[4\]) and from (\[5\]), we can write ($M' \equiv dM/dr$)

$$\frac{1}{2} L_f(r) + \frac{2q_e^2}{L_f r^4} = \frac{2M'(r)}{r^2} = E(r).$$

(8)

To obtain a solution, we can specify one function, it is either $L(f)$ or, for example, $A(r)$. If $A(r)$ is known, or equivalently $M(r)$ or $E(r)$, then Eq. (7) expresses $r$ in terms of $f$, $L_f$ and the two charges, and substituting it into Eq. (8), we arrive at a highly nonlinear first-order differential equation with respect to $L(f)$. This method does not seem fruitful since there is very little hope to solve such an equation, and moreover, a choice of $A(r)$ or $M(r)$ is not physically evident: it is only clear (due to the above-mentioned no-go theorem) that assuming the metric with a regular center, we cannot obtain $L(f)$ with a Maxwell weak-field limit.

A better way is to specify the theory by choosing $L(f)$. Then Eq. (7) can be considered as either (A) an equation (in general, transcendental) for the function $f(r)$ or (B) an expression of $r$ as a function of $f$.

In case A, if $f(r)$ can be found explicitly, integration of Eq. (8) (equivalent to (5)) gives the metric function $A(r)$.

The scheme B leads to a solution in quadratures in terms of $f$ which can then be chosen as a new radial coordinate. Indeed, assuming that $L(f)$ and $r(f)$ are known and monotonic, so that $L_f \neq 0$ and $r_f \neq 0$, Eq. (8) can be rewritten as

$$M_f = \frac{r^2 r_f}{2} \left[ \frac{L}{2} + \frac{q_e^2}{L_f r^4} \right].$$

(9)

(10)

Examples of interest of using method B are yet to be found, therefore let us restrict ourselves to considering two examples in which method A can work.

The first example is trivial: the Maxwell theory, $L = f$, it is used to verify the method. Substituting $L = f$ and $L_f = 1$ to Eq. (8), we obtain $2M' = (q_e^2 + q_m^2)/r^2$, whence $2M(r) = 2m - (q_e^2 + q_m^2)/r$ and

$$A(r) = 1 - \frac{2m}{r} + \frac{q_e^2 + q_m^2}{r^2}, \quad m = \text{const},$$

(10)

that is, the dyonic Reissner-Nordström solution, as should be the case.

In the second example we assume that Eq. (7) is linear with respect to $f$, which unambiguously leads to the truncated Born-Infeld Lagrangian,

$$L(f) = b^2 (1 + \sqrt{1 + 2f/b^2}), \quad b = \text{const}$$

(11)

(it differs from the full Born-Infeld Lagrangian by the absence of the invariant $(F_{\mu\nu} F^{\mu\nu})^2$).

Indeed, for Eq. (7) to be linear in $f$, we have to assume $L_f = c_1 f + c_2$ with $c_{1,2} = \text{const}$. Integrating this condition, we get $L = L_0 + (2/c_1) \sqrt{c_1 f + c_2}$. The requirement of a Maxwell behavior, $L \approx f$, at small $f$ leads to $c_2 = 1$, $L_0 = -2/c_1$. Then, denoting $2/c_1 = b^2$, we arrive at Eq. (11).

With (11), we obtain

$$f(r) = \frac{2b^2 (q_m^2 - q_e^2)}{4q_e^2 + b^2 r^4},$$

$$E(r) = -\frac{b^2}{2} + \left( \frac{b^2}{2} + \frac{2q_e^2}{r^4} \right) \sqrt{\frac{4q_e^2 + b^2 r^4}{4q_m^2 + b^2 r^4}}.$$  

(12)

The simplest solution is obtained in the special case of a self-dual electromagnetic field, $q_e^2 = q_m^2$, leading to $f = 0$ and $L_f = 1$, the same as for the Maxwell theory. This leads to $E = 2q_e^2/r^4$ and the dyonic Reissner-Nordström metric with $A(r)$ given by (10).

In the more general case, $q_e^2 \neq q_m^2$, $A(r)$ is a bulky expression in terms of the Appel hypergeometric function $F_1$, not to be presented here. There are, however, some features of the solution to be
noticed. First of all, as expected, at large $r$ the quantities $f_e$, $f_m$ and the energy density $\mathcal{E}$ decay as $r^{-4}$, and the solution is asymptotically flat. At small $r$, from (12) it follows that $f(r)$ tends to a finite limit while $\mathcal{E} \approx 2|q_e q_m|/r^4$. However, this relation does not hold in the cases of pure electric or magnetic solutions: in the electric case we also have $\mathcal{E} \sim r^{-4}$ whereas in the magnetic case $\mathcal{E} \sim r^{-2}$. This confirms that $r = 0$ is a singular center in all dyonic solutions and that the magnetic solution must also be singular since the function (11) does not tend to a finite limit as $f \to \infty$. However, in this theory, the energy density in the pure magnetic solution blows up at the center slower than in all other cases, so the curvature singularity must be also milder.

4 Conclusion

The main results of this study are a scheme of finding dyonic solutions in NED coupled to GR by quadratures for an arbitrary Lagrangian function $L(f)$ (up to some monotonicity restrictions) and a dyonic solution for the truncated Born-Infeld theory. A feature of interest in this solution is the existence of a special case with a self-dual electromagnetic field and the Reissner-Nordström metric.

We did not discuss here the question of horizons and global causal structure of the resulting configurations. It is clear, however, that since the positive energy density $\mathcal{E}(r)$ makes a positive contribution to the metric function $A(r)$ (and even leads to a repulsive central singularity like the Reissner-Nordström one) while the Schwarzschild mass $m$ appearing there as an integration constant can raise or lower the plot of $A(r)$ at intermediate $r$ (its influence at small $r$ is weaker than that of the electromagnetic field), the situation with horizons as zeros of $A(r)$ in dyonic solutions is the same as is known in numerous electric and magnetic ones: assuming fixed $q_e$ and $q_m$, solutions with small $m$ have no horizons, their central singularities are naked; at some critical value of $m$, depending on the charge values, a double (extremal) horizon appears, and at still larger $m$ there are two horizons. The space-time causal structure is then quite similar to that of the Reissner-Nordström solution.

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