Neutrino nucleosynthesis: An overview

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Neutrinos produced during a supernova explosion induce reactions on abundant nuclei in the outer stellar shells and contribute in this way to the synthesis of the elements in the Universe. This neutrino nucleosynthesis process has been identified as an important contributor to the origin of 7Li, 11B, 19F, 138La, and 180Ta, but also to the long-lived radionuclides 22Na and 26Al, which are both key isotopes for γ-ray astronomy. The manuscript summarizes the recent progress achieved in simulations of neutrino nucleosynthesis.

I. INTRODUCTION

Core-collapse supernovae play a crucial role for the origin of the elements in the Universe [1]. First, and most importantly, the explosion frees nuclides, which have been produced by nuclear fusion reactions in the stellar interior during the star’s long hydrostatic life, and mixes them into the Interstellar Medium. Second, the explosion, related to its hot and neutron-rich environment, can give rise to dedicated nucleosynthesis processes like the γ-process, which produces the heavy neutron-deficient isotopes [2], or the rapid neutron capture process (r-process) where modern supernova simulations imply conditions supporting only the production of r-process nuclei up to the A ∼130 mass region (weak r-process [3], while the heavy r-process nuclei are likely to be produced in neutron-star mergers [4]. A unique feature of core-collapse supernovae is the essential role played by neutrinos, not only during the collapse phase (e.g. [5, 6]), but also for the explosion mechanism. The observation of neutrinos from SN1987a [7, 8] has confirmed that neutrinos are produced in overwhelmingly large numbers. Although the cross-sections for neutrino interactions with nuclei are tiny, they can therefore induce specific nucleosynthesis processes: The rp-process, that operates in the neutrino-driven wind just above a new-born neutron star [9–11], and the ν-process, which is the focus of this review.

The ν-process is initiated by neutrinos of all flavors produced in the hot and dense interior of the collapsed stellar core that pass through the outer shells of the star before and after the shock wave reaches this region. The neutrinos interact with nuclei present in these shells by charged- (c.c.) and neutral-current (n.c.) reactions. Here, it is important that the energies of the neutrinos involved (up to about 20 MeV) restrict c.c. reactions to electron neutrinos and electron antineutrinos, while they are high enough to excite nuclei by n.c. reactions to states above particle thresholds so that deexcitation proceeds via light particle emission. Thus, both c.c. and n.c. reactions produce new nuclides and, if these survive the fast reaction network associated with the subsequent passage of the shock wave, contribute to nucleosynthesis. A handful of isotopes have been identified as being entirely or to a significant portion created due to neutrino nucleosynthesis. Two important radionuclides, 22Na and 26Al, are also affected. As all neutrino flavors contribute to the production of these nuclei, neutrino nucleosynthesis serves as a test case for predicted supernova neutrino spectra and luminosities.

Our review is organized as follows: in the next section we present the input, strategies and outcomes of the neutrino nucleosynthesis studies that were performed in the last three decades. § 3 is devoted to a discussion regarding the sensitivity of the ν process to the neutrino spectra and luminosities, while an outlook on future challenges and opportunities is given in § 4.

II. NEUTRINO-NUCLEOSYNTHESIS STUDIES

The essential ingredients of neutrino nucleosynthesis studies are: a) neutrino spectra and luminosities, b) neutrino-induced nuclear cross-sections, and c) the supernova model, including the stellar structure of the progenitor model as well as effects of the shock wave. We briefly review these three items in turn.

a) Following the pioneering work by Woosley et al. [12] and until recently, neutrino spectra have been described by Fermi-Dirac distributions with vanishing chemical potentials and time-independent temperatures Tν for the various neutrino flavors individually taken from supernova simulations reflecting the cooling of the freshly born neutron star mostly by neutrino pair emission. On general grounds, simulations predict a hierarchy for the average neutrino energies: \( \langle E_\nu_\mu \rangle > \langle E_\nu_\tau \rangle > \langle E_\nu_e \rangle \). (Here \( x \) refers to mu and tau neutrinos and their antiparticles and the temperature is related to the average energy by \( T_\nu \approx 3.15 \langle E_\nu \rangle_x \).) With advanced sophistication of supernova modeling, in particular by considering additional processes affecting neutrino opacities, the values assumed for the various neutrino temperatures (i.e. average energies) dropped from 8 MeV to 4 MeV for μ and τ neutrinos, 6 MeV to 4 MeV for electron antineutrinos.
and 5 MeV to 2.8 MeV for electron neutrinos, where the higher values were adopted in the pioneering work [12] and the lower values in the recent study of Sieverding et al. [13], taken from the supernova simulation [14, 15]. Heger et al. [16, 17] used intermediate values, which were appropriate at the time they performed their studies: \( T_{\nu_e} = 6 \) MeV, \( T_{\bar{\nu}_e} = 5 \) MeV and \( T_{\nu_x} = 4 \) MeV.

Again following [12], the neutrino luminosity has been assumed to decrease exponentially with time, described by

\[
L_\nu(t) = L_0 e^{-t/\tau},
\]

where \( L_0 = 3 \times 10^{53} \) erg/\( \tau \) and \( \tau = 3 \) s, consistent with simulations and observations.

b) Simulations of neutrino nucleosynthesis require neutrino-induced partial nuclear cross-sections that take into account the decay by particle emission (protons, neutrons, alpha). These cross-sections are calculated applying a two-step procedure [18]. First, the neutrino-induced cross-section leading to an excited nuclear state, via charged- or neutral current, is calculated using a microscopic model, such as the Random Phase Approximation (RPA). At the typical neutrino energies involved, Gamow-Teller and giant dipole resonances dominate the cross-sections. Therefore, the RPA is adequate because it describes the centroid and the total strength of giant resonances quite well. The decay of an excited state into different channels is calculated employing the statistical model. RPA-based partial neutrino-nucleus cross-sections have been presented in [18–21], often already folded with a neutrino energy distribution assumed appropriate for supernova neutrinos. Ref. [13] has calculated partial neutrino-nucleus cross-sections spanning the entire nuclear chart and as a function of neutrino energy, which allows us to study the influence of the dynamically changing neutrino spectra on the neutrino nucleosynthesis process (see below). For light nuclei, like \( ^4 \)He [22–24] or \( ^{12} \)C [25, 26], cross-sections calculated with the hyperspherical model or the interacting shell model exist, which both are superior to RPA, because they also describe the fragmentation of the giant resonance strength well. Cross-section relevant for the nucleosynthesis of \( ^{138} \)La and \( ^{180} \)Ta can be determined, at least to a large extent, from experimentally determined Gamow-Teller strengths [27].

c) As a secondary process, neutrino nucleosynthesis predominantly operates on abundant seed nuclei and their presence in the star determines where the process can operate. Thus, the final (one-dimensional) model of a star, calculated with a stellar evolution code, such as KEPLER [17, 28], until just before collapse, is the starting point of a neutrino nucleosynthesis study. Since self-consistent explosion models require large scale simulations [29–31], which are computationally too expensive for nucleosynthesis studies, the stellar model is subjected to an artificial explosion that usually employs a piston model tuned to a total energy of order \( 10^{51} \) erg [32]. The associated nucleosynthesis is studied with an extensive nuclear reaction network, including neutrino-induced reactions. The neutrino spectra and luminosities are modeled as described above.

Starting with the pioneering work of Woosley et al. [12], various studies have identified a set of nuclei whose galactic abundance is being produced dominantly or in significant portion by neutrino nucleosynthesis: \( ^7 \)Li, \( ^{11} \)B, \( ^{15} \)N, \( ^{19} \)F, \( ^{138} \)La, \( ^{180} \)Ta, and the radionuclides \( ^{22} \)Na and \( ^{26} \)Al.

Table I gives the production factors for these nuclei relative to solar abundance [33] and normalized to \( ^{16} \)O obtained by neutrino nucleosynthesis calculations for a star with an initial mass of \( 15 \) M\( \odot \), representative for the nucleosynthesis of massive stars as explored by [13] for a range of stellar models between \( 13 \) M\( \odot \) and \( 30 \) M\( \odot \). This study, which for the first time considered a complete set of partial nuclear reaction cross-sections, did not give evidence for other nuclei being produced by neutrino nucleosynthesis. Two important conclusions can be derived from Table I. First, neutrino nucleosynthesis produces a large fraction of the solar yields for \( ^{11} \)B, \( ^{138} \)La and \( ^{180} \)Ta, and also contributes noticeably to the solar \( ^{15} \)N and \( ^{19} \)F abundances. But for all of these nuclides other nucleosynthesis processes are required to reproduce the full solar yields. Secondly, the effectiveness of neutrino nucleosynthesis is severely reduced if modern supernova neutrino spectra with lower average energies are considered rather than those spectra appropriate some years ago, e.g. those used by Heger et al. (2005) [16]. The modern spectra reduce predominantly the neutral-current contributions to the calculated yields as significantly less high-energy neutrinos are available to excite the nucleus above particle thresholds. This is particularly important for tightly bound nuclei like \( ^{12} \)C, \( ^{16} \)O and \( ^{20} \)Ne which have rather high thresholds for proton and neutron emission, that - as we will see below - are important for the neutrino nucleosynthesis production mechanisms for \( ^{11} \)B, \( ^{15} \)N and \( ^{19} \)F.

| Nucleus | High energies | Low energies | Simulation |
|---------|---------------|--------------|------------|
| \(^{7} \)Li | – | 0.083 | 0.187 |
| \(^{11} \)B | 1.884 | 0.280 | 0.516 |
| \(^{15} \)N | 0.487 | 0.116 | 0.141 |
| \(^{19} \)F | 0.602 | 0.180 | 0.209 |
| \(^{138} \)La | 0.974 | 0.487 | 0.824 |
| \(^{180} \)Ta | 0.964 | 0.484 | 0.636 |

Heger et al. (2005) Sieverding et al (2018) Sieverding et al (2019)
respectively. As the c.c. cross-sections are, in relative terms, less affected by the modifications of the supernova neutrino spectra, the relative weight to the total yields is shifted towards c.c. contributions.

In the following sections we will discuss the production sites and reactions for the different nuclides produced by the $\nu$-process.

For illustration, we use again the results from a calculation of a 15 solar mass star. Fig. 1 shows the mass fractions of the selected nuclides as function of enclosed mass. The different compositional shells of the star that result from the hydrostatic burning stages (He, C/O, O/C, O/Ne and Si) are indicated by the background colors. To unravel different neutrino contributions, nucleosynthesis calculations considering only n.c. or c.c. reactions were performed in addition to calculations that considered all reactions.

In the following paragraphs, the results for the six nuclides $^7$Li,$^{11}$B,$^{15}$N,$^{19}$F, $^{138}$La and $^{180}$Ta are discussed.

Without neutrino interactions, the stellar yield of $^7$Li is tiny (0.2 % of the solar content). Neutrino nucleosynthesis enhances it to 8 %, which still implies, that the solar $^7$Li abundance is produced by a different process, i.e. by cosmic ray irradiation [34, 35]. The main neutrino mechanism is triggered by n.c. spallation of protons and neutrons from $^4$He in the stellar He-shell, followed by an alpha-capture on the remaining $^3$H and $^3$He fragments.

The bulk of $^{11}$B is produced in the thin carbon shell by either neutrino-induced spallation of protons and neutrons on $^{12}$C or by $^{12}$C($\nu_e, e^- p)^{11}$C and $^{12}$C($\nu_e, e^- n)^{11}$B reactions, where the $^{11}$C fragments decay with a half-life of about 20 minutes to $^{11}$B. With modern neutrino spectra both n.c. and c.c. reactions contribute about the same amount. A minor amount of $^{11}$B is also produced in the O/Ne layer by the $^{16}$O($\nu, \nu' \alpha p$) reaction, which shows the relative importance of multi-particle emission reactions. We note that neutrino nucleosynthesis does not produce $^{10}$B in noticeable amounts. To reproduce the solar $^{11}$B/$^{10}$B ratio of about 4, Austin et al. [35] have argued that neutrino nucleosynthesis must produce about $(42 \pm 4)$% of the solar $^{11}$B abundance, with the rest stemming from cosmic ray irradiation [34].

Neutrino nucleosynthesis contributes only a small amount to the solar $^{15}$N abundance. While n.c. reactions (neutrino-induced spallation of protons and neutrons from $^{16}$O in the O/C shell) contributed nearly 10 % of the solar $^{15}$N yield when the previous neutrino spectra with higher energies were employed, the modern spectra reduced this amount significantly. With these spectra, both neutral- and charged-current reactions contribute about the same amount. Neutrino nucleosynthesis increases the solar $^{15}$N abundance by about 3 %. Massive stars and novae are production sites of $^{15}$N [36–38].

The $\nu$-process enhances the core-collapse supernova contribution to the solar $^{19}$F abundance by about 30%. In contrast to the other nuclides, $^{19}$F shows a distinct sensitivity to the progenitor mass. For smaller mass stars the hot environment associated with the passage of the shock wave triggers a thermonuclear sequence, $^{18}$O($p, \alpha)^{15}$N($\alpha, \gamma)^{19}$F, in the inner He-shell, which boosts the pre-supernova amount of $^{19}$F by about 30%, without inclusion of neutrino reactions. Neutrino nucleosynthesis by neutral- and charged-current reactions on $^{20}$Ne in the O/Ne layer adds another 25%. The thermonuclear component to the $^{19}$F production is quite sensitive to various nuclear reaction rates, the explosion energy, and to the compositional shell interfaces. In particular, the mass contained in the O/Ne layer increases with progenitor mass, and this, in turn, increases the effectiveness of neutrino nucleosynthesis to $^{19}$F production. For stars more massive than 17 solar mass, the $\nu$-process can increase the production of $^{19}$F by factors of 1.5–2. The main galactic sources of $^{19}$F are Asymptotic Giant Branch stars and Wolf-Rayet stars [39–41].

$^{138}$La is a $p$-nucleus, that is bypassed by the $s$-process moving along the chain of stable barium isotopes. In core-collapse supernovae the production site is the O/Ne layer where two processes can occur, both, with and without neutrinos. At the bottom of the layer, where temperatures are sufficiently high, $^{138}$La is synthesized by the $\gamma$-process. When the peak temperature drops below 2 GK, photodissociation is less effective and in this

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**FIG. 1.** Mass fraction profiles for the 15 M$\odot$ progenitor model with low neutrino energies as chosen in Ref. [13] for the six isotopes most affected by the $\nu$-process. Red dashed and blue dotted lines indicate results with only charged current (c.c.) and only neutral current (n.c.) reactions. Note that the scale is different in each panel. The background colors indicate the different compositional layers of the stellar model, indicated in the bottom right panel.
cooler region of the O/Ne layer, neutrino nucleosynthesis dominates the production of $^{138}$La, mainly by c.c. ($\nu_e, e^{-}$) reaction on pre-supernova $^{138}$Ba. Note that the $\nu_e$-induced reaction on $^{138}$Ba has also a branch towards excited states in $^{138}$La above the neutron threshold, that finally lead to $^{137}$La, whose pre-supernova abundance is tiny. Neutron capture on $^{137}$La adds then to the production of $^{138}$La.

$^{180}$Ta is the rarest isotope in Nature. Its production by photodissociation and neutrino nucleosynthesis is similar to the production of $^{138}$La, but with important additional twists. At the high temperatures in the inner part of the O/Ne layer the $\gamma$-process produces $^{180}$Ta from pre-supernova $^{181}$Ta. In the cooler outer region of the O/Ne shell, neutrino nucleosynthesis again dominates the production of $^{180}$Ta by ($\nu_e, e^{-}$) reaction on pre-supernova $^{180}$Hf. Neutral-current reactions contribute in an indirect way as neutron spallation from $^{16}$O, $^{20}$Ne and $^{24}$Mg generate free neutrons which are captured on $^{179}$Ta, which is more abundant than $^{180}$Ta in a low-mass progenitors because of the operation of the $\gamma$-process during the final burning stages. This is the case for the model shown in Fig. 1, where n.c. and c.c. reactions contribute similarly to the production of $^{180}$Ta.

$^{180}$Ta is the only isotope in the solar abundance which exists due to a long-lived isomeric state (with excitation energy of 75 keV), while the ground state decays with a half-life of about 8 hours. In the hot environment of the O/Ne layer, the $^{180}$Ta ground and isomeric states are in thermal equilibrium. It has been estimated that about 35 – 39% of $^{180}$Ta survives in the isomeric state [42, 43]. The production factors of $^{180}$Ta given in Table 1 have been corrected using this estimate.

Two observations are commonly made regarding the neutrino nucleosynthesis production of $^{138}$La and $^{180}$Ta. Both isotopes are mainly produced by ($\nu_e, e^{-}$) reactions and hence are a tempting way to explore the supernova $\nu_e$ spectrum (see below). Furthermore, the neutrino production of both isotopes increases with the size of the O/Ne layer which grows with progenitor mass. However, this is partly counterbalanced by decreasing abundances from photodissociation.

III. SENSITIVITY TO NEUTRINO LUMINOSITY AND SPECTRA

Until very recently it has been assumed that the neutrinos, which trigger the neutrino nucleosynthesis, are generated by cooling of the proto-neutron star. For all neutrino flavors, their luminosities and spectra have been approximated by an exponentially decreasing function and by Fermi-Dirac distributions with zero chemical potential. The neutrino temperatures (or average energies) have been individually adjusted to supernova simulations. As we have discussed in the last section, steadily improved descriptions of neutrino opacities within the simulations led to lower values of the neutrino temperatures that in turn reduced the abundances of isotopes produced by the $\nu$-process.

Fig. 2 shows the neutrino luminosities and average neutrino energies from a modern (one-dimensional) supernova simulation [15]. The spectra adopted in neutrino nucleosynthesis studies approximate the cooling phase, which describes the neutrino emission at a time about 1 second after bounce. In this phase, the luminosities drop approximately exponentially, but also the individual neutrino average energies decrease with time. However, before the cooling phase dominates the neutrino emission, there is a distinct burst in $\nu_e$ neutrinos and a so-called accretion phase with distinct neutrino emission. The burst neutrinos are generated from the fast capture of electrons on free protons that are created after matter has been dissociated by the shock wave. Before shock revival, matter falls through the stalled shock, accompanied by the emission of neutrinos, where in particular $\nu_e$ and $\bar{\nu}_e$ neutrinos have enlarged energies (with respect to the assumed average value) and luminosities. As stated above, the burst is in electron neutrinos only. Although
it lasts only for about 10 ms, its luminosity is so large that it carries about 10% of the total luminosity carried away by electron neutrinos.

Ref. [44] has explored which effect the explicit time dependence of the neutrino luminosities and spectra have on the neutrino nucleosynthesis yields. The luminosities and average energies were taken from a recent supernova simulation for a 27 M⊙ progenitor model. For the most important neutrino-nucleus reactions on 4He, 12C, 20Ne as well as on 138Ba and 180Ta, that are discussed in the previous section, the deviations of the spectrum from a Fermi-Dirac distribution with vanishing chemical potential, the pinching of the spectra [45–48], has also been taken into account, applying the α-fit description of Ref. [49].

The calculation considering full time dependence of the neutrino emission has been compared with calculations assuming constant average neutrino energies, derived as the appropriate mean of the emission spectrum, conserving total number and energy of each neutrino flavor. The results for $\bar{\nu}_e$ and $\nu_x$ neutrinos ($T_{\bar{\nu}_e} = 4.01$ MeV, $T_{\nu_x} = 3.72$ MeV and $T_{\bar{\nu}_e} = 3.96$ MeV) are quite similar to the values adopted in the neutrino nucleosynthesis study discussed in the previous section. Due to the contributions of the burst and accretion phases, the mean temperature for electron neutrinos ($T_{\bar{\nu}_e} = 3.46$ MeV) is somewhat larger than the value adopted in §II.

The impact of the time dependence of the neutrino spectra and luminosities on the neutrino nucleosynthesis yields is significant (see last column of Table I). The effect is largest for nuclides like 138La, 180Ta and also 11B, which are strongly produced by charged-current reactions and hence are affected by the modifications of the electron neutrino emissions. In the calculation that considers the full time dependence, the enhancement is larger than in calculations with constant average neutrino energies. This has two reasons. First, the energy dependence of the nuclear cross-sections gives stronger weight to the early high-temperature neutrinos. Second, in the calculation with full time dependent spectra, the fluence of neutrinos through particular stellar mass shells is larger for the late neutrinos, which have lower temperatures than the average value. Hence, a larger number of neutrinos is required to reach the same luminosity value.

The calculation of Ref. [44] clearly indicates that neutrino nucleosynthesis studies should consider the full time dependence of neutrino emission, extended to the full range of stellar masses of core-collapse supernovae. Furthermore, the duration of the accretion phase is still rather uncertain. This can also have a significant effect on the $\nu$-process yields, in particular on nuclei produced by charged-current reactions.

### IV. SUMMARY AND OUTLOOK

It is by now well established that the solar abundances of $^7$Li, $^{11}$B, $^{19}$F, $^{138}$La and $^{180}$Ta are, to an important or even dominating part, being produced by the $\nu$-process. Nevertheless, increasingly realistic studies with improved stellar models, better nuclear cross-sections and also improved neutrino emission data from advanced supernova simulations show that the individual abundances of these nuclides depend sensitively on astrophysical and nuclear modeling. The largest impact on the $\nu$-process yields has been due to changes in the neutrino spectra that are predicted with significantly lower average energies by modern supernova simulations than anticipated in earlier studies. This spectra change is reflected in the noticeable reduction of the various abundances. However, a recent study put in question the treatment of neutrino emission that assumes time-independent neutrino spectra. In fact, larger yields are found if the time dependence of neutrino emission is explicitly accounted for. This is particularly true for the charged-current contributions arising mainly from electron neutrinos from the early burst and accretion phases of the neutrino emission. Realizing this sensitivity, two steps are recommended to encourage advancement in this field: the study, performed for a 27 M⊙ progenitor, must be extended to the full range of core-collapse supernova progenitor masses and neutrino nucleosynthesis should be extended from one-dimensional supernova models, to which they were restricted recently, to multi-dimensional models.

Early on it has been realized that neutrino nucleosynthesis is sensitive to those neutrino flavors ($\nu_e$, $\nu_x$) which have not been observed from supernova SN1987A (whose signal was likely due to electron anti-neutrinos). Here 138La and 180Ba, which are dominantly made by charged-current reactions, serve as a constraint for the $\nu_e$ spectra and luminosities, while the other $\nu$-process nuclides are also sensitive to neutral-current reactions and thus to the other neutrino flavor spectra. It has been even pointed out that neutrino nucleosynthesis may be critical in determining the neutrino mass hierarchy or the famous $\theta_{13}$ angle of the neutrino matrix [26, 50–54]. The clue here is always that neutrino nucleosynthesis is expected to be sensitive to neutrino oscillations that shuffle the lower-energy $\nu_e$ spectrum with the higher-energy spectra of muon or tau neutrinos. The more complex effects of collective neutrino oscillations can also have an impact [55]. While this observation is in principle correct, a deduction of the important neutrino properties from $\nu$-process yields certainly becomes more difficult as the energy hierarchy of the supernova neutrinos is less pronounced in modern supernova simulations and further difficulties arise when the time dependence of the neutrino emission is considered.

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