Operational control algorithm of parameters of high-pressure sodium lamps based on a statistical time series model

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Abstract. The developed operational control algorithm parameters of a high pressure sodium lamp is considered in this work. We used a mathematical model to develop the algorithm. An analytical method was used to describe the operation of a sodium lamp based on differential equations as well as an algorithm for evaluating time series. The mathematical expectation and dispersion were also evaluated. The described models for quality control of sodium lamps have been tested in production. The obtained simulation results coincide with the results of the experiment. Using a series of tests based on the proposed control method we confirmed the adequacy of the developed model.

1. Introduction
The development of the lighting industry in Russia is mainly associated with the improvement of production and research enterprises in this industry. A significant share of the production volume is made up of high-pressure sodium lamps which are one of the most effective light sources. As a rule, until now manual monitoring of a limited number of controlled lamp parameters is realized. This leads to control errors of the first and second kind. In this regard, the automation control problems solution, reasonable selection of controlled parameters, the use of processing modern methods of controlled information is relevant.

2. Problem statement
The difficulty of solving the above problems is related to the complexity of physical processes occurring in the lamp, the non-linearity and mathematical models approximation that display these processes, and the controlling complexity some physical variables. Overcoming control difficulties related to approximation of mathematical models and poor observability of physical variables is currently successfully implemented in the framework of intellectual control and management tasks.

Increasing the reliability of the output control of high-pressure sodium lamps is achieved by automating the control process and making a reasonable choice of controlled parameters based on statistical analysis methods using time series. As it was previously stated, the algorithm for controlling lamp parameters is based on the analysis of time series parameters generated by the control system. It inheres in smoothing the time series, building time series models, determining the parameters of these models, setting tolerances (confidence limits) for the defined parameters of models, and tolerance
control. The developed algorithm for monitoring the parameters of high-pressure sodium lamps includes operational and long-term monitoring. Operational control is carried out directly on the test bench based on the measurements results. Long-term control is performed based on historical trends have been received from the archiving station.

3. Theory
Determine the dimension of the controlled parameters of the conductivity model. To do this, we will randomly change the parameters of this model on the state space model (1) and in each experiment we will calculate the parameters of the conductivity model.

\[
\begin{align*}
\frac{dx_1}{dt} &= \left(\frac{1}{L} \left[ U_0 - \left( \frac{1}{x_2^2 x_3^3} + R \right) x_1 \right] \right).
\end{align*}
\]

\[
\begin{align*}
\frac{dx_2}{dt} &= A_1 U_0^2 x_2^2 \left( \frac{x_1}{U_0 x_2 x_3^3} \right)^2 \frac{1}{1 + k_1 \left( \frac{|x_1|}{U_0 x_2 x_3^3} - 1 \right)}.
\end{align*}
\]

\[
\begin{align*}
\frac{dx_3}{dt} &= \left[ k_2 + k_3 \left( \frac{|x_1|}{U_0 x_2 x_3^3} \right)^4 \right] \left[ 1 + k_1 \left( \frac{|x_1|}{U_0 x_2 x_3^3} - 1 \right) - x_3 \right],
\end{align*}
\]

where L, R are the inductance and active resistance of the limiting throttle, respectively; x1 is a current flowing through the lamp; x2 is reduced conductance of the lamp taking into account the average electron concentration; x3 is a nondimensional coefficient that changes over time and takes into account the mobility of electrons; U0 is a rated voltage on the lamp; A1 is a coefficient determined by the construction design of the lamp; k1-k4 are electrical coefficients determined for a specific type of lamp.

The obtained parameters are subjected to tolerance control with the boundaries of the tolerance field equal to [+σ – σ], where σ is the standard deviation.

The practice of control for this parameter showed insufficient reliability of the test results. For the purpose of increase the reliability of the control, a test of statistical hypothesis was proposed. The hypothesis was whether the measured values of the voltage drop of two rows of lamps installed on the test bench belong to the same general population. The Wilcoxon test was used for this checking. This is a nonparametric statistical criterion used to evaluate differences between two small samples taken from a distribution law other than normal. The Wilcoxon test is "ideal" for the monitoring system in question since two samples are controlled – each with a volume of 15 units with a distribution law different from the normal one [1].

Suppose that the statistical hypothesis being tested is accepted if the criterion value is greater than 0.5, and the tolerance field boundaries are U\text{max}=130 \text{ V}, U\text{min}=95 \text{ V}, then the figures show that there are cases when the tolerance control is positive and the Wilcoxon test is negative and opposite – the tolerance control is negative and the Wilcoxon test is positive. However, the number of such cases for the first option does not exceed several hundred and for the second option is 6 and they can be ignored with the total sample size exceeding 200 thousand [1].

The operational control algorithm is implemented on the basis of a time series statistical model generated by the monitoring system. The monitoring system generates data such as:
- registering a time series;
- the measured voltage drop across the lamps;
- smoothing;
- forming samples of two numerical series from the measured voltage drops;
- tolerance control;
- checking of the statistical hypothesis about the statistical correspondence of two time series using the nonparametric Wilcoxon test;
- making a decision about the product's suitability when the results of the tolerance control and the results of the Wilcoxon test are positive.
3. checking the statistical hypothesis about the distribution law by the nonparametric Wilcoxon test [2, 4].

4. recurring calculation of estimates of mathematical expectation and dispersion.

\[ m(i) = m(i-1) + \frac{1}{i} [x(i) - m(i-1)]; \]  

\[ D(i) = \frac{i-1}{i} D(i-1) + \frac{1}{i-1} [x(i) - m(i)]^2, \]  

where \( m(i) \), \( D(i) \) are estimation of mathematical expectation and dispersion at step \( i \), \( x(i) \) is current value of the variable, for which mathematical expectation estimates and dispersion are calculated, \( i \)-step number.

5. Tolerance control of parameters of the original variable, its mathematical expectation and dispersion.

6. Making a decision on the product's shelf life on a majority principle. Analysis of majority control of device states consists in analyzing the results of majority elements in order to occurrence:

- malfunctions,
- intermittent failures;
- stable failures.

A recurrent calculation of the expectation value, dispersion, and histogram of the input signal is shown in the figure 1 (a–d).

Figure 1. Input value of the variable, recursive calculation of estimates of mathematical expectation, dispersion and histogram of the input signal.

The model of the time series random component is represented as ARMA. It is a model of a random component whose parameters are calculated using the recursion ordinary least squares [5].
The recursion ordinary least squares can also be used for estimating the parameters of random signal models. We assume that the random signal is represented by a stationary autoregression process with a running average [4]:

\[ y(k) + cl_{1}y(k-l) + \ldots + cly(k-p) = v(k) + dl_{1}v(k-l) + \ldots + dpv(k-p). \]  

(4)

In this equation, the immeasurable interference is replaced by a measurable signal \( y(k) \). Converting this equation (4), we get:

\[ y(k) = \psi^{T}(k)\theta(k-1) + v(k), \]  

(5)

where

\[ \psi^{T}(k) = [-y(k-l)\ldots -y(k-p) v(k-l) \ldots v(k-p)]. \]  

(6)

\[ \theta^{T} = [c_{1} \ldots c_{p} d_{1} \ldots d_{p}]. \]  

(7)

Applying the recursion ordinary least squares to equation (4) would be possible for known values \( v(k-1), \ldots, v(k-p) \). In this case, the value \( v(k) \) in (5) could be considered as an error of the equation which by definition should be a random statistically independent variable.

By the time the next measurement of \( y(k) \) is made the values \( y(k-1), \ldots, y(k-p) \) are already known. Assuming that the estimates \( v(k-1), \ldots, v(k-p) \) and \( \theta(k-1) \), have also been obtained by this point, the estimate of the current input signal \( v(k) \) can be determined from the equation (5):

\[ v(k) = y(k) - \psi^{T}(k)\theta(k-1). \]  

(8)

When processing statistical data it is of practical interest to use discrete models of random processes and calculate the statistical characteristics of these processes using the parameters of discrete models. In this case the estimation of the correlation function and spectral density is reduced to the estimation of the coefficients of the discrete transfer function and the dispersion of unobservable flat noise. In other words, it is reduced to identifying the ARMA- model parameters that generates a stationary ergodic random process or ARMA-process. ARMA-process allows us to get estimates of spectral density directly from observations without calculating their statistical characteristics. Due to this, the using of ARMA-models has slightly displaced methods based on the fast Fourier transform. In addition to them, we can use simpler autoregressive and random component models to simulate random processes [3].

As is known, the spectral density of the process resulting from the passage of flat noise through a linear system is equal to the product of the input noise intensity and the module square of the complex frequency characteristic of the system. The complex frequency response of the system is a module of the discrete transfer function of the system obtained by substituting in the discrete transfer function

\[ z = e^{j\alpha}, \]  

where \( j = \sqrt{-1} \).

We define the spectral density of the ARMA- process if the sampled data transfer function corresponds to it

\[ W(z) = \frac{D(z)}{C(z)} = \frac{1 + \sum_{k=1}^{p} d_{k} z^{-k}}{1 - \sum_{i=1}^{p} c_{i} z^{-i}}. \]  

(9)

Making a substitution \( z = e^{j\alpha} \) in the transfer function (7) we get:
\[ W(e^{j\omega}) = \frac{1 + \sum_{k=1}^{q} d_k e^{-j\omega k}}{1 - \sum_{i=1}^{p} c_i e^{-j\omega i}}. \]  \hspace{1cm} (10)

Then the square of the module of the complex frequency response is calculated by the formula

\[ |W(e^{j\omega})|^2 = \left( \frac{1 + \sum_{k=1}^{q} d_k \cos k\omega}{1 - \sum_{i=1}^{p} c_i \cos i\omega} \right)^2 + \left( \frac{\sum_{k=1}^{q} d_k \sin k\omega}{\sum_{i=1}^{p} c_i \sin i\omega} \right)^2. \]  \hspace{1cm} (11)

It follows that the spectral density of the ARMA-models described by the transfer function is equal to:

\[ S(\omega) = \frac{1}{\sigma^2} \left[ \frac{1 + \sum_{k=1}^{q} d_k \cos k\omega}{1 - \sum_{i=1}^{p} c_i \cos i\omega} \right]^2 + \left( \sum_{k=1}^{q} d_k \sin k\omega \right)^2, \]  \hspace{1cm} (12)

where \( \sigma \) – the dispersion of process.

It is worth noting that the estimation of the spectral density (9) is reduced to the estimation of the transfer function coefficients (7) and the dispersion of unobservable flat noise, i.e., to the identification of the ARMA-process.

To form a given spectral density \( S(\omega) \) we can also use models of the random component. A forming filter based on the ARMA-model is called a recursive filter or infinite impulse response filter. A formative filter based on the MA-model is called a non-recursive or finite impulse response filter. For more information about designing digital filters with specified characteristics see [7, 8].

Even easier to calculate the correlation functions of the ARMA-process. To calculate the mutual correlation function of a random process defined by ARMA-model it is sufficient to calculate the weight function of this model and multiply it by the dispersion of the process. The correlation function of the random process itself is somewhat more difficult to calculate. To do this, we need to find the convolution from the weight function of the ARMA-model which in the discrete version is set by the expression [6] and then the resulting numerical values of the convolution are also multiplied by the variance of the process.

\[ R_{\text{mut}}(k) = \sum_{j=\max(1, k-n+1)}^{\min(k, n)} \omega(j)\omega(k-j-1), \]  \hspace{1cm} (13)

4. Results of experiments

Taking into account the above the algorithm for building a random component model is formed from the stages that are presented below [2].

1. Selection of a random component from the initial time series of the controlled parameter.
2. centering the time series
3. setting the dimension of ARMA-model
4. Calculating the parameters of the ARMA-model using the recurrent least squares method
5. Analysis of the modeling error by its statistical characteristics.
The algorithm is based on a recurrent estimation of the shaping filter parameters that generates this time series. Then the initial values of the model parameter estimation vector and the data vector are set:

\[ \hat{\theta}(0) = 0; P(0) = aI \quad \] (14)

Measurements of the object's output signals are made and the original vector of data \( \Psi(k) \) is formed.

The current value of the input signal is calculated

\[ \hat{\nu}(k) = y(k) - \Psi^T(k)\theta(k-1) \quad \] (15)

A new data vector is formed

\[ \Psi'(k + 1) = [-y(k), ..., -y(k - n), \hat{\nu}(k), ..., \hat{\nu}(k - m)] \quad \] (16)

The correction vector is calculated

\[ \gamma(k) = \frac{P(k)\Psi(k + 1)}{\Psi'(k + 1)P(k)\Psi(k + 1) + 1} \quad \] (17)

A new estimate of the model parameters is found

\[ \hat{\theta}(k + 1) = \hat{\theta}(k) + \gamma(k)[y(k + 1) - \Psi'(k + 1)\hat{\theta}(k)] \quad \] (18)

A new data vector is calculated

\[ P(k + 1) = [I - \gamma(k)\Psi'(k + 1)]P(k) \quad \] (19)

Then the calculation process is repeated

Transfer function of a discrete-time forming filter is presented in [1].

\[ W_{od} = \frac{0.2095z^2 - 0.2201z}{z^2 - 0.77z - 0.1502} \quad \] (20)

5. Conclusion

The features analysis of control of high-pressure sodium lamps showed an increase in the customer's risks. It is associated with emergency failures of these lighting devices that endanger consumers. This circumstance requires increasing the reliability of control using more modern methods of statistical analysis.

The review of statistical methods of data analysis allowed us to focus on the analysis of time series parameters generated by the monitoring system with using statistical and dynamic time series models. Based on the maximum error of parametric identification a mathematical model was selected for quality control of high pressure sodium lamps.

Such algorithms for monitoring high-pressure sodium lamps as smoothing, control based on a statistical model of a time series, selection of trend and periodic components, and construction of a random component model of a time series have been developed.

Control of lamp parameters using the developed algorithms allowed excluding the influence of the human factor on its results. Also control increased accuracy and reliability by an average of 12%, and it provided archiving and registration of control results, and improved production culture.

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