Modelling compact pulsators

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Abstract. We currently know of five distinct classes of compact pulsators, loosely defined as oscillating stars with a surface gravity above log $g = 5$. Three of these fall into the white dwarf regime (GW Vir, V777 Her and ZZ Ceti stars), while the other two are identified with hot B subdwarfs (EC 14026 and PG 1716 stars). In all cases, the instabilities are thought to be associated with the partial ionisation of the envelope constituents. We discuss, for each type of pulsator, our current theoretical understanding of the observed instability strips as well as the details of the driving mechanism at work. In terms of attempts at a quantitative exploitation of the observed period spectra through asteroseismology, we focus on the particularly successful case of the rapidly pulsating subdwarf B (EC 14026) stars. So far, we have been able to constrain the fundamental stellar parameters for 12 targets. This is sufficient for first comparisons with different evolutionary scenarios, and will likely play a crucial part in an eventual understanding of the formation of these objects.

1. Introduction to compact pulsators

Of the five known classes of compact oscillators, three are identified with white dwarfs, while the other two fall into the category of hot B subdwarfs. White dwarfs are the post-asymptotic giant branch (AGB) remnants of low- and intermediate mass stars comprising a degenerate C/O core and masses between 0.2 and 1.3 $M_{\odot}$, the majority clustering around 0.5–0.7 $M_{\odot}$. They can be divided into two main groups, one of which retains its hydrogen-rich envelope during the post-AGB phase (corresponding to 75–80\% of white dwarfs) and one of which does not (the remaining 20–25\%; see Figure 1 for typical evolutionary tracks). As they evolve along the cooling sequence, members of the first class - spectroscopically identified with DA white dwarfs - see the hydrogen dominating their envelope recombine at \( \sim 12,000 \) K. This greatly increases the envelope opacity, hinders the flow of radiation, and ultimately causes the driving of pulsations in a well-defined region of the H-R diagram known as the ZZ Ceti instability strip [1; 2]. Members of the second group are subject to a violent mixing event induced by a late helium flash in the post-AGB phase that produces an envelope devoid of hydrogen, but instead made up of helium, carbon and oxygen in roughly equal parts (see [3] for a review). During the early stages of white dwarf evolution associated with very high temperatures (\( \sim 120,000 \) K), the ionisation of carbon and oxygen in the envelope can lead to pulsational instability via a classical opacity mechanism, producing the GW Vir stars [4]. As these stars cool and the heavy envelope constituents settle, pulsations cease and the atmosphere is increasingly dominated by helium, corresponding to a DB
white dwarf in spectral type. When the temperature in the envelope has decreased to \( \sim 25,000 \) K, the helium recombines and causes pulsational instability in the so-called V777 Her stars in a similar way as the hydrogen does in the ZZ Ceti stars [1]. All types of white dwarf pulsar excite low-degree, low-order \( g \)-modes with periods on the order of hundreds to thousands of seconds. Note that white dwarfs have a fundamentally different mechanical structure to main sequence stars - they have thin atmospheres, thin superficial convection zones (in the ZZ Ceti and V777 Her stars) and thin composition transition zones - and \( g \)-mode propagation is not limited to the deep interior as in "normal" stars, but extends well into the outer envelope.

![Figure 1. Region of the \( \log g - T_{\text{eff}} \) plane where the compact pulsators are found. Each of the five distinct classes is identified by its IAU name (the V361 Hya stars are more commonly referred to as EC 14026 stars), except for the more recently discovered Betsy stars (also known as PG 1716 stars), and the year of discovery of the prototype is also indicated in each case. Typical evolutionary tracks are plotted showing a) the path followed by a 0.6 \( M_\odot \) post-AGB, H-rich star which becomes a H-atmosphere white dwarf (red curve), b) the evolution of a 0.6 \( M_\odot \) post-AGB, H-deficient star which becomes a He-atmosphere white dwarf (blue curve) and c) the track followed by a 0.478 \( M_\odot \) post-EHB model which passes through the sdB phase and ends as a low-mass H-atmosphere white dwarf (black curve).](image)

In contrast to the post-AGB remnants that are white dwarfs, subdwarf B (sdB) stars correspond to extreme horizontal branch (EHB) stars that at the time of the helium flash lost too much of their hydrogen envelope to ascend the AGB. Composed of helium-burning cores surrounded by very thin hydrogen-dominated envelopes, they have masses tightly constrained to \( \sim 0.47 \ M_\odot \), and are thought to eventually contribute to a small fraction of the white dwarf population in the form of low-mass objects. They lie somewhere in between main sequence stars and white dwarfs in terms of their mechanical structure, and hence their pulsational properties.
For instance, the pulsation modes are not totally dominated by temperature effects as they are in white dwarfs, but nor are radial effects as important as in main sequence stars. Similarly, their $g$-mode propagation zone does not extend out to the stellar surface, while not being confined to the innermost regions either. Thus, it is not surprising that we know of both $p$- and $g$-mode pulsators among hot subdwarfs. Referred to as the EC 14026 stars after the prototype [5], the former show typical luminosity variations on a timescale of 100-200 s and correspond to the hotter subdwarfs at around 33,000 K. The latter are known as the PG 1716 or Betsy stars (after their discoverer Betsy Green, see [6]), exhibit longer periods of $\sim 3000\text{–}8000$ s and lie at the cooler end of the sdB distribution ($\sim 26,000$ K). While the short- and long-period variables are found in distinct regions of the H-R diagram, it is apparent from Figure 1 that the two domains touch. Indeed, some of the coolest previously known EC 14026 stars have been discovered to also exhibit PG 1716-like slow oscillations, making them hybrid objects with the potential for probing both the envelope and the outer core using the $p$ and $g$-modes respectively [7]. For the sake of completeness, we should mention that pulsations have also recently been discovered in one He-rich sdB [8] and in one apparent sdO [9], however since these are not yet firmly established as a class we do not consider them further here.

In what follows, we give a very brief overview of the current understanding of the driving mechanism at work in each of the 5 classes of compact pulsators. We invite the interested reader to consult the recent extensive review by Fontaine & Brassard (ASP Conf. Ser., submitted) for further information on pulsating white dwarfs, and the shorter article by Fontaine et al. [10] for more details on pulsating B subdwarfs.

![Figure 2](image_url)

**Figure 2.** Instability domain in the log $g$ – $T_{\text{eff}}$ diagram for the ZZ Ceti stars (filled circles), compared to the constant stars (indicated by open circles). Typical uncertainties on the atmospheric parameters are given by the cross in the lower left-hand part of the diagram. The dotted curves show evolutionary tracks for H-envelope white dwarfs at different masses, ranging from 0.4 to $1.1 M_\odot$ in steps of $0.1 M_\odot$. The solid lines indicate the theoretical blue edge assuming two convective efficiencies as indicated.
Figure 3. Details of the driving/damping process for a typical $g$-mode excited in a representative ZZ Ceti star model. The dotted line represents the Rosseland opacity (RHS ordinate axis), the solid curve shows the work integral of the mode, and the dashed line indicates the running work integral from the center (left) to the surface (right) of the model. The vertical dotted line on the left (right) gives the location of the base of the atmosphere (photosphere). The yellow strip indicates the convection zone.

2. Instability strips and pulsation driving

2.1. The ZZ Ceti stars

We begin our discussion of the empirical instability strips and their theoretical modelling with the ZZ Ceti stars, for which we have a large and homogeneous sample of both pulsators and non-pulsators, as shown in Figure 2 [11]. It is clear that the latter is entirely consistent with the idea of a pure instability strip, implying that all DA white dwarfs, with no exception, are to become ZZ Ceti pulsators as they cool. The blue edge of the empirical instability strip is well reproduced by nonadiabatic calculations of complete stellar models under the assumption that the outer convection zone (a result of H recombination) reacts instantaneously to the perturbations caused by the oscillatory motions in the star [12]. Its exact location depends on the version of the mixing-length theory appropriate at the base of the convection zone, where the contribution to the driving of pulsations is highest (see Figure 3). A relatively high convective efficiency of $ML2/\alpha=1.0$ seems to reproduce the empirical blue edge almost perfectly. This is in contrast to the less efficient calibration of $ML2/\alpha=0.6$ determined for the atmospheric layers on the basis of spectroscopy [13], and indicates a depth-variable convective efficiency for these stars, an idea that should also be incorporated into models. The convection zone plays an important part in the driving of pulsations in ZZ Ceti stars, as can be appreciated from Figure 3. We see a very pronounced bump in the Rosseland opacity due to the partial ionisation of HI, which in turn produces a super-adiabatic temperature gradient and convective instability, the latter
accounting for around 99.9% of the flux transported in this region. Given that the peak in the work integral (indicating the layers of the model that contribute most to pulsation driving) is more intimately related to the base of the convection zone than the opacity peak, it is clear that oscillations in ZZ Ceti stars are not excited by a pure $\kappa$-mechanism, which operates only in radiative environments, but instead are subject to a mechanism known as "convective driving".

2.2. The V777 Her stars

Figure 4. Instability domain for the V777 Her stars. Only 3 objects have known H/He abundances; for all the other stars the left position of a pair corresponds to zero H, and the right position to the maximum possible amount just below the detection limit. This is also the case for the theoretical blue edge.

The observational characterisation of the V777 Her instability strip shown in Figure 4 [14] is more challenging than for the case of the ZZ Ceti stars for two main reasons: firstly, the V777 Her stars are much less numerous (13 known vs. 136 ZZ Ceti stars), and secondly, the exact atmospheric parameters are hard to pin down due to their sensitivity to small traces of atmospheric H generally spectroscopically invisible in the optical domain. Making allowances for this, the data are consistent with the idea of a pure instability strip, however a lot of work remains to be done in order to confirm this and establish clear blue and red edges on the observational side. As with the ZZ Ceti stars, non-adiabatic calculations are able to qualitatively reproduce the blue edge of the empirical instability strip while the red edge remains elusive (however, see Dupret et al., these proceedings, for promising new results). It is clear from Figure 5 that the excitation mechanism operating in the two types of pulsator is very similar, with the important difference that the opacity bump is due to the partial ionisation of He, rather than H, in the V777 Her stars. Moreover, it is rather weaker, which results in a slightly less important convection zone in terms of the percentage flux carried and its impact on pulsation driving. This can be appreciated from the fact that the peak in the work integral now lies between the base of the
Figure 5. Similar to Figure 3, but for a representative model of a V777 Her star. In this case, the bump in the Rosseland opacity corresponds to the fusion of the He I and the deeper He II partial ionisation zones. Note the change on the RHS ordinate axis compared to Figure 3.

Figure 6. Instability domain for the GW Vir stars. The modelled blue edge is dependent on atmospheric chemical composition, and hence drawn fuzzy. The red edge is computed from a specific wind mass law calibrated to match the empirical red edge.
convection zone and the opacity bump. Nevertheless, the pulsations in V777 Her stars can be described as excited by "convective driving" in direct analogy with the ZZ Ceti stars.

2.3. The GW Vir stars

The GW Vir instability strip shown in Figure 6 differs markedly from those of the ZZ Ceti and V777 Her stars in that it is clearly not pure, instead harbouring a mix of pulsators and non-pulsators at very similar atmospheric parameters [3]. It was demonstrated only recently [15] that this could be naturally understood in terms of a spread in the chemical composition from one star to the next. Since convection is negligible in the envelopes of these very hot stars, the excitation of pulsations is the result of a classical \( \kappa \) mechanism associated with the partial ionisation of C and O (see Figure 7). Therefore, it makes sense that only those stars with a sufficient amount of C and O in their envelopes will be able to pulsate. The envelope composition of individual GW Vir stars is determined by the exact details of their formation via the so-called born again scenario as well as the degree of mass loss experienced via stellar winds. These play a key role in preventing the quick settling of the heavier envelope constituents due to the high gravity of the white dwarf; in their absence, diffusion would very quickly turn the envelope of a GW Vir star into a He-dominated region, with no possibility for driving. It is therefore the competition between the stellar wind and the gravitational settling of C and O that determines the red edge of the instability strip. When the star cools below \( \sim 75,000 \) K, the stellar wind weakens sufficiently to allow the settling of the heavy elements, and the driving of pulsations ceases until the now dominant He in the envelope recombines at \( \sim 25,000 \) K and enables the driving of V777 Her type pulsations.

![Figure 7](image_url). Similar to Figure 3, but for a representative GW Vir star. The deeper C/O opacity bump is associated with pulsation driving; the shallower peak in the opacity profile is the usual "Z-bump". The thin convection zone carries negligible flux.
2.4. The EC 14026 & Betsy stars

![Figure 8](image_url)

Figure 8. The empirical instability strips for the EC 14026 and Betsy stars. Note that non-pulsating sdB stars co-exist with the two types of pulsator, but are not shown in the plot. Two of the three known hybrid pulsators are also indicated by arrows and are located at the boundary between the two domains.

In contrast to the white dwarf oscillators, the domains of instability for the sdB stars touch each other, to the point where hybrid objects exist, exhibiting both rapid and slow pulsations. The driving mechanism in both types of pulsator is a classical $\kappa$ mechanism (convection is negligible in the envelope) associated with the ionisation of heavy elements, in particular iron. It excites $p$-modes in the hotter EC 14026 stars and $g$-modes in the cooler Betsy stars, in direct analogy to the case of the $\beta$ Cephei and SPB pulsators found on the main sequence. Modelling of the instability strip has been very successful for the case of the EC 14026 stars, to the point where not only the red and blue edges can be recovered perfectly, but where also the period ranges observed are in agreement with non-adiabatic predictions in almost all cases (see [16] for a review). The key to this is the incorporation of diffusion into the models, since it is the competitive action of gravitational settling and radiative levitation that creates the local iron overabundance necessary to produce the Z-bump at the origin of the pulsational instability. One challenge that remains however is the co-existence of pulsators and non-pulsators in the instability strip. While it is likely that this is related to the fraction of heavy elements in the envelope, abundance analyses on the basis of FUSE spectra have so far failed to find systematic differences between oscillators and constant sdBs at very similar atmospheric parameters [17], unlike the case of the GW Vir stars where a distinct signature was detected. It seems probable that this is the result of the atmospheric abundances measured not accurately reflecting the conditions deeper in the envelope where the pulsations are driven, however this idea remains to be investigated in more detail.
Figure 9. Similar to Figure 3, but for a representative EC 14026 star. The plot for a representative Betsy star would look comparable, and is hence not illustrated. The opacity bump responsible for pulsation driving is the deeper Z-bump, while the shallower peak is associated with the partial ionisation of He.

For the Betsy stars, the situation is more complicated since current models are not able to reproduce the empirical blue edge, the theoretical blue edge for low-degree $\ell = 1, 2$ modes being some 5000 K too cool [18]. It was recently suggested that improved models also taking into account other heavy elements apart from iron, in particular nickel, and employing the newer OP rather than the old OPAL opacities would be able to overcome this problem [19], but this has yet to be implemented for sdB models also incorporating radiative levitation.

3. Asteroseismology of EC 14026 stars
The rapidly pulsating EC 14026 stars have become one of the success stories in asteroseismology, 12 of 34 known targets now having been submitted to such analyses. These are based on the well-known ”forward method”, where the period spectra computed for a large grid of models (characterised by $T_{\text{eff}}$, $\log g$, the total mass $M_*$ and the fractional depth of the transition zone between the He-core and the H-rich envelope $\log q(H)$) are quantitatively compared to those observed, and models that present the best fits are objectively isolated using a $\chi^2$ double-optimisation technique (see e.g. [20; 21] for details). The so-called ”optimal” models must not only be able to account well for all observed periodicities simultaneously (the period dispersion is typically well below 1 %), but also have to comply with non-adiabatic theory and independent spectroscopic estimates of $\log g$ and $T_{\text{eff}}$. For the sdB stars, a lot of the motivation behind carrying out systematic asteroseismic analyses for a large fraction of the known pulsators lies in the fact that the evolution of these objects is still a mystery. While it seems clear that hot subdwarfs are stars that failed to ascend the AGB because of the extreme thinness of their H-envelope, the details surrounding the strong and sudden mass loss that must have occurred are
Figure 10. Observed mass distribution for the 12 EC 14026 stars so far submitted to asteroseismology (histogram) together with those predicted from single and double star evolutionary scenarios.

Figure 11. Theoretical and observed (points with error bars) correlations between the total mass, the hydrogen envelope mass and the effective temperature on the zero-age EHB. The black curves show the computed relations between $M_{\text{env}}$ and $T_{\text{eff}}$ for different total masses, onto which the asteroseismologically determined masses are projected (thick bars).
poorly understood. A number of different formation channels, including binary evolution with both stable and unstable Roche lobe overflow as well as the merger of two He-white dwarfs have been proposed [22], but have remained largely untested due to a lack of empirical data on the total stellar mass and the H-envelope thickness of these stars. The opening up of sdB stars to asteroseismology therefore presents a unique opportunity to constrain evolutionary scenarios on the basis of an observed mass- and H-envelope-thickness distribution. Figures 10 and 11 show the first attempts in this direction; it is clear that more asteroseismic targets are needed in order start discriminating and assessing the relative importance of the different formation channels, however the results obtained so far look very promising. Hopefully, the asteroseismology of sdB stars will not only shed light on their formation, but also motivate and further similar analyses of other oscillating stars.

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