Emergence of the Gaia Phase Space Spirals from Bending Waves

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ABSTRACT

We discuss the physical mechanism by which pure vertical bending waves in a stellar disc evolve to form phase space spirals similar to those discovered by Antoja et al. in Gaia Data Release 2. These spirals were found by projecting Solar Neighbourhood stars onto the \( z - v_z \) plane. Faint spirals appear in the number density of stars projected onto the \( z - v_z \) plane, which can be explained by a simple model for phase wrapping. More prominent spirals are seen when bins across the \( z - v_z \) plane are coloured by \( v_R \) or \( v_\phi \). We use both toy model and fully self-consistent simulations to show that the spirals develop naturally from vertical bending oscillations of a stellar disc. The underlying physics follows from the observation that the vertical energy of a star (essentially, its “radius” in the \( z - v_z \) plane) correlates with its angular momentum or, alternatively, guiding radius. Moreover, at fixed physical radius, the guiding radius determines the azimuthal velocity. Together, these properties imply the link between in-plane and vertical motion that lead directly to the Gaia spirals. We show that the cubic \( R - z \) coupling term in the effective potential is crucial for understanding the morphology of the spirals. This suggests that phase space spirals might be a powerful probe of the Galactic potential. In addition, we argue that self-gravity is necessary to properly model the evolution of the bending waves and their attendant phase space spirals.

Key words: Galaxy: kinematics and dynamics – Galaxy: structure – Galaxy: disc

1 INTRODUCTION

Gaia Data Release 2 (GDR2) (Gaia Collaboration et al. 2018a), which includes measurements of the six-dimensional phase space coordinates for some seven million stars, has afforded astronomers an unprecedented picture of the local stellar kinematics of our Galaxy Gaia Collaboration et al. (2018b). Arguably, the most intriguing result from the nascent analysis of this data set has been the discovery of spiral patterns in certain phase space projections of Solar Neighbourhood stars by Antoja et al. (2018). They selected stars in a circular arc that spanned 8° in Galactic azimuth \( \phi \) and a range in Galactocentric radius \( R \) from 8.24 kpc to 8.44 kpc. They then computed the number density distribution of the \( \sim 900 \) kpc within this region as a function \( z \) and \( v_z \), the position and velocity in the direction perpendicular to the Galactic midplane. The resulting plot showed a spiral pattern of 1-2 complete wraps within a region of the \( z - v_z \) plane extending to \( \sim 600 \) pc in \( |z| \) and \( \sim 40 \) km s\(^{-1}\) in \( |v_z| \). More prominent spirals appeared when they “coloured” the \( z - v_z \) plane by median \( v_R \) and \( v_\phi \). Antoja et al. (2018) interpreted the spirals as a telltale sign of phase wrapping and were able to explain their general morphology by a simple model for the local gravitational potential.

In this paper, we delve deeper into the physics of the “Gaia spirals”. Our hypothesis is that a pure bending wave naturally evolves to form spiral patterns in the \((z, v_z, v_\phi, v_R)\) phase space. (For a different take on some of the same ideas, see Binney & Schoenrich (2018).) Bending waves are a natural feature of disc galaxies. The most conspicuous examples are the warps seen in edge-on galaxies (See Binney (1992) and Sellwood (2013) for reviews of galactic warps.). These warps amount to a bending of both HI and stellar discs from the midplane by \( \sim 1 - 3 \) kpc. Recently, Xu et al. (2015) found asymmetries in the number counts of stars above and below the disc at Galactocentric radii between 12 kpc and 18 kpc, which they interpreted as evidence for ripples or corrugations in the disc. On the other hand, Schönrich & Dehnen (2018) found wavelike patterns in the mean vertical motion of Solar Neighbourhood stars as a function of \( L_z \), the angular momentum about the rotation axis of the Galaxy. Since \( L_z \) is roughly proportional to guiding radius (at least, where the rotation curve is approx-
imimately flat) these velocity ripples may well be the velocity counterpart to those seen in number counts (Chequers et al. 2018). Further evidence for bending and breathing modes has been seen in both number counts and bulk motions of Solar Neighbourhood stars (see, for example, Widrow et al. (2012); Williams et al. (2013); Carlin et al. (2013); Yanny & Gardner (2013); Gaia Collaboration et al. 2018a)). They can be excited by a passing satellite (Widrow et al. 2012; Feldmann & Spolyar 2015; Gómez et al. 2013, 2017), spiral structure or the bar (Debattista 2014; Monari et al. 2015), or even shot noise in an N-body simulation (Chequers & Widrow 2017).

In this paper we sidestep the issue of what perturbs the disc by imposing an initial ad hoc vertical perturbation. The excitation of bending and breathing waves due to a passing satellite is discussed in detail by Widrow et al. (2014) and demonstrated in N-body simulations of both isolated discs in discs in fully cosmological simulations by Gómez et al. (2013); Feldmann & Spolyar (2015); Gómez et al. (2017); Chequers et al. (2018). In essence, these excitations are the early stages of the disc-heating events discussed in Toth & Ostriker (1992) and Sellwood et al. (1998). The excitation of perturbations in the context of phase space spirals using the impulse approximation is discussed in Binney & Schoenrich (2018).

In Section 2 we consider a series of toy-model simulations where test particles are evolved in time-dependent, axisymmetric potentials. These simulations allow us to illustrate the essential physics of phase space spirals and, in particular, highlight the importance of the non-separable nature of the effective potential for disc stars. In Section 3, we then demonstrate the link between bending waves and phase space spirals in a fully self-consistent disc-bulge-halo model of the Milky Way using standard N-body methods. We conclude in Section 4 with a summary of our results and suggestions for further investigations of these ideas.

2 KINEMATIC SPIRALS

2.1 Distribution Function

Consider a distribution of test particles in a time-independent, axisymmetric potential \( \Phi (R, z) \). All particle orbits admit two integrals of motion: the angular momentum about the symmetry axis \( L_z = R \sigma \alpha \) and the total energy \( E = \frac{1}{2} c^2 + \Phi (R, z) \). If the potential is an additively separable function, that is, if it can be written in form

\[
\Phi (R, z) = \psi (R) + \chi (z)
\]

then the vertical energy

\[
E_z = \frac{1}{2} c^2 + \Phi (R, z) - \Phi (R, 0)
\]

is also an integral of motion. In this case, it is convenient to define the planar energy \( E_p = E - E_z \). Even if the potential cannot be written in the form given by Eq. 1, the vertical energy is approximately conserved for particles on nearly circular orbits (Binney & Tremaine 2008).

Analytic functions of \( E_p, E_z, \) and \( L_z \) can be used as the building blocks for a disc that is close to equilibrium.

Following Kuijken & Dubinski (1995) we consider the quasi-isothermal distribution function (DF)

\[
f(E_z, E_p, L_z) = \frac{g(L_z)}{\sigma_z(R) \sigma_R(R)} \exp \left[ \frac{-E_p - E_0(L_z)}{\sigma_R^2(L_z)} - \frac{E_z}{\sigma_z^2(L_z)} \right]
\]

where \( E_r \) is the energy of a particle on a circular orbit with angular momentum \( L_z \).

The function \( g(L_z) \) determines the radial profile of the surface density, which is obtained by integrating the DF over velocities and \( z \). Furthermore, the guiding radius \( R_e \) for a particle on a circular orbit is an implicit function of \( L_z \). Thus, we can use \( R_e \), which is an integral of motion, as a proxy for radius \( R \) when constructing the DF. For example, an exponential disc is obtained by choosing \( g \propto \exp (-R_e/R_d) \).

Our interest here is in stars in the Solar Neighbourhood. We are therefore led to consider the idealized case where

\[
g(L_z) \propto \delta (L_z - L_{z0}).
\]

The density is then given by

\[
\rho (R, z) \propto e^{-(\Phi_{\text{eff}}(R, 0) - E_0)/\sigma_R^2(R_0) + \Phi(R, z)/\sigma_z^2(R_0)}
\]

where \( \Phi_{\text{eff}}(R) = \Phi (R, 0) + L_{z0}^2/2R^2 \) is the effective potential for a particle with angular momentum \( L_{z0} \) and \( \Phi (R, z) \equiv \Phi (R, z) - \Phi (R, 0) \). Note that the peak of the radial distribution of particles coincides with the minimum of the effective potential, that is, the guiding radius of a particle with angular momentum \( L_{z0} \). The main advantage of this DF is that it easily sampled. It is, however, too simplistic for our purposes as we now describe.

In the DF given by Eq. 3, the angular momentum determines a particle’s guiding radius and hence the probability distribution functions for a particle’s phase space coordinates \( R, z, v_R, v_z \), and \( v_z \). If we introduce a bending perturbation to this DF, that is, if we displace the DF in either \( R \) or \( v_z \), then the distribution will phase wrap in the \( z - v_z \) plane producing a spiral pattern similar to the one seen in the number counts. However, since the DF is separable in \( z - v_z \) and \( R - v_R - v_\phi \), spiral patterns will not appear when the \( z - v_z \) plane is coloured by median \( v_R \) or \( v_\phi \).

Of course, in a real galaxy one has a distribution of stars in \( L_z \) at a given \( R \). To mimic this effect while retaining the simplicity of Eq. 4 we consider a superposition of two delta-function distributions,

\[
g(L_z) = \alpha_1 \delta (L_z - L_{z1}) + \alpha_2 \delta (L_z - L_{z2}),
\]

which we refer to as populations 1 and 2. In what follows, we assume that \( \alpha_1 = 0.5 \) and choose \( L_{z1} \) and \( L_{z2} \) so that the two guiding radii are 8.16 kpc and 8.64 kpc, respectively. Furthermore, we assume that \( \sigma_R \) and \( \sigma_z \) are given by

\[
\sigma_R(R_e) = \sigma_{R,0} e^{-R_e/R_d}
\]

\[
\sigma_z(R_e) = \sigma_{z,0} e^{-R_e/R_d}
\]

with \( R_d = 2.1 \) kpc, \( \sigma_{R,0} = 26 \) km s\(^{-1}\), and \( \sigma_{z,0} = 16 \) km s\(^{-1}\).

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2.2 $R - z$ Coupling in the Potential

In this section we consider a toy model potential that allows us to isolate the effect $R - z$ coupling terms in the effective potential on phase space spirals. Recall that in the epicycle approximation (Binney & Tremaine 2008) one expands the effective potential to terms quadratic in $R - R_e$ and $z$. To the extent that the approximation holds, stars execute simple harmonic motion in $R$ and $z$. Of course, without the higher order terms in the potential, all stars would orbit in the $z - v_z$ plane at the same frequency and phase space spirals would never develop.

As discussed above, the vertical energy $E_z$ is conserved so long as the potential is additively separable. We first consider a model for the potential that is additively separable but includes higher order terms in $z$ and $R - R_e$. As we’ll see, spirals appear in this model but lack many of the qualitative features seen in the data. We then augment this model with an $(R - R_e)z^2$ term. This term, along with an $(R - R_e)^3$ term, are the leading corrections to the epicycle approximation. The spirals in this case have a richer phenomenology than the ones in our separable model and share many of the features with the spirals seen in Gaia DR2.

2.2.1 kinematic spirals in an additively separable potential

Consider an additively separable potential given by Eq. 1 with

$$
\chi(z) = ab^2 \sqrt{\frac{z^2}{b^2} + 1 + cz^2},
$$

(8)

and

$$
\psi(R) = \frac{v^2}{2} \ln \left(R^2 + R_o^2\right)
$$

(9)

The form of the vertical potential $\chi$ is similar to that from the Oort problem analysis of Kuijken & Gilmore (1989) while $\psi$ is the logarithmic potential (see, for example, Binney & Tremaine (2008)), which gives a flat rotation curve for $R \gg R_o$. In what follows we choose $a = 0.07 \ \text{Myr}^{-2}, \ b = 23 \ \text{pc}$ and $c = 0.006 \ \text{Myr}^{-2}$, which give a vertical potential consistent with models for the vertical potential in the Solar Neighborhood (See Widrow et al. (2014)). We also choose $v_o = 217 \ \text{kms}^{-1}$ and $R_o = 6.1 \ \text{kpc}$ to give a radial potential consistent with the realistic Milky Way model considered in Section 3.3.

We next sample the DF for 500k particles. Properties of the initial conditions are exhibited in Fig. 1 and 2. The top panel of Fig. 1 shows the number density of stars as a function of Galactocentric radius for the two distributions. Evidently, the model has the desired property that the distribution of stars at a given radius comprises an admixture from the two $L_z$ populations. The middle panel shows the $v_o$ distribution of stars again as a function of $R$. Since $v_o = L_z/R$, the distribution at a given radius is essentially the sum of two delta functions with population 1 stars having a lower azimuthal velocity than the population 2 stars at the same $R$. Finally, in the lower panel, we show the $v_z$ distribution. We note that stars at smaller $R$ preferentially come from the higher $L_z$ distribution and have a higher $\sigma_z$. Very similar features are found in the $v_R$ distribution.

Fig. 2 shows the $z - v_z$ plane coloured by mean $v_o$. Stars at large radii in the $z - v_z$ plane, that is, stars with high $E_z$ preferentially come from stars in population 1, which have lower $v_o$, as seen in the middle panel of Fig. 1.

The initial DF shown in Fig. 2 is perturbed by displacing it by $30 \ \text{kms}^{-1}$ in $v_z$. The particles are then evolved using a standard leap-frog algorithm for 300 Myr and the results are shown in Fig. 3. The number density shows the spiral pattern typical of phase wrapping. Stars orbit the $z - v_z$ plane in a clockwise sense and since those at large $E_z$ have a lower vertical epicycle frequency, the DF quickly develops a trailing spiral. Also shown is the $z - v_z$ plane coloured by mean $v_R$ and $v_o$. Here, the pattern is essentially featureless, apart from the imprint of the number density distribution: With our separable potential the in-plane and vertical epicyclic motions completely decouple.
2.2.2 Kinematic Spirals in a Coupled Potential

Next we consider the addition of a $R - z$ term to the toy model potential:

$$\Phi(R, z) = \psi(R) + \chi(z) + \xi(R - R_s)z^2 .$$  (10)

$R_s$ is a constant radius characteristic of the distribution (If a Taylor expansion of the effective potential is performed for each $L_z$ population, $R_s$ is replaced by $R_c(L_z)$). For illustrative purposes, we choose $\xi = -0.24 \text{ Myr}^{-2}\text{kpc}^{-1}$ and $R_s = 8 \text{ kpc}$. These values are consistent with the cubic terms that arise in realistic Milky Way potentials. The main effect of the new term is to introduce a linear dependence in $R$ to the vertical epicyclic frequency. In particular the difference between the vertical frequency for a population 1 star with $R_c = 8.16 \text{ kpc}$ and a population 2 star with $R_c = 8.64 \text{ kpc}$ is 1.85 Myr$^{-1}$.

As before we sample an initial distribution (Eq. 3) with 500k particles, perturb it and integrate the perturbed distribution for 300 Myr. The results are shown in Figure 4. Not surprisingly, the spirals that appear in the $z - v_z$ plane for the toy model considered in Section 2.2.1 (toy model with additively separable potential). Top panel shows the number density. Bins in the middle and lower panels are coloured by $v_R$ and $v_\phi$, respectively. Bin sizes and particle number threshold are the same as in Fig. 2.

2.3 Realistic Milky Way Potential

We conclude this section on test-particle simulations by considering a realistic Milky Way potential that comprises an exponential disc, an NFW halo (Navarro et al. 1997), and a Hernquist bulge (Hernquist 1990). We approximate the disc as a superposition of three Miyamoto-Nagia potentials (Miyamoto & Nagai 1975; Smith et al. 2015). The potential is similar to the MWPotential2014 model included with the GALPY code (Bovy 2015), but the specific model we used can be defined in GALPY using HernquistPotential(a=0.035, amp=0.5), MN3ExponentialDiskPotential(amp=7, hr=2.8/8, hz=0.3/8, sech=True) and NFWPotential(amp=5, a=1.4). The decomposition of the circular speed curve into the three components is shown in Fig. 5.

The results are shown in Fig. 6. The number density spiral is a bit more tightly wound than our previous example implying a stronger gradient in the vertical frequency with $E_z$. More striking are the differences between the $v_R$ and $v_\phi$ spirals seen here and in our previous examples. In particular, there are now prominent gradients along the edges of the spirals. For example, along the inner edge of the $v_\phi$ spiral, the stars alternate between high azimuthal velocity (red) and low azimuthal velocity (blue) five times in a little over 360°. The fact that the variations in $v_\phi$ are strongest along the edges of the spirals makes sense since these stars have the largest $E_z$ where the effect of the coupling term is the largest.
3 PHASE SPACE SPIRALS WITH SELF-GRAVITY

In the previous section, we considered test particles in a fixed axisymmetric potential. The evolution of their DF was therefore governed by kinematic phase mixing. Test particles were used by de la Vega et al. (2015) in their study of bending and breathing modes and by Antoja et al. (2018) and Binney & Schoenrich (2018) to explain the Gaia spirals. However, as stressed by Hunter & Toomre (1969) (see also Sparke & Casertano (1988)) there are two effects associated with a bending perturbation of a disc: the restoring force of the unperturbed disc on the perturbation and the potential associated with the perturbation acting on the unperturbed disc. In effect, the test-particle models treat the former and ignore the latter. In linear perturbation theory, both effects enter at the same order. It follows that self-gravity will be important for the evolution of bending waves and phase space spirals.

In this Section we present results from simulations of a fully self-consistent disc-bulge-halo system. The initial conditions are generated with GALACTICS, which uses a disc DF given by Eq. 3 with \( g(L_z) \) chosen to yield an exponential profile for the surface density (See Kuijken & Dubinski (1995); Widrow et al. (2008)). The model parameters are chosen so that the rotation curve decomposition into the three components matches the realistic Milky Way model introduced in the previous section, as shown in Fig. 5. We use 1M particles for the halo and 200K particles for the bulge. To boost mass resolution near the solar circle, we sample the region of the disc between 6.5 kpc and 9.5 kpc with 5M particles and the remainder with 1M particles. The particles in the ring then have a mass of about 1300 \( M_\odot \) or about 28 times less massive than the particles in the rest of the disc. Over time heavy disc particles will infiltrate the ring region and introduce noise to the system via two-body relaxation effects. These will be discussed below.

We introduce an ad hoc bend to the disc by applying a velocity perturbation to the disc stars of the form

\[
\delta v_z = v_0 e^{-(R-R_0)^2/\delta R^2} \cos \phi
\]

where we choose \( v_0 = 30 \text{ km s}^{-1} \), \( R_0 = 8 \text{ kpc} \), and \( \delta R = 500 \text{ pc} \). Thus, initially one side of the disc will bend to the North while the other will bend to South.

The system is evolved for 1 Gyr. Face-on maps of the mean vertical velocity for the 250 Myr, 500 Myr, and 1 Gyr snapshots are also shown in Fig. 7. Evidently, once bending waves are introduced to the disc they persist for many dynamical times.

To gain a handle on the effect self-gravity has for the evolution of bending waves and the accompanying phase space spirals, we also follow the evolution of our ring particles under the assumption that the background potential is fixed and given by the unperturbed potential of the model. The face-on maps of the mean vertical velocity are also
shown in Fig. 7. We see that the kinematic bending waves rapidly damp due to phase mixing. Clearly, self-gravity is crucial to the persistence of bending waves in discs.

We next search for phase space spirals in a manner similar to what was done with GDR2 stars. In particular, we select stars in a circular arc between 7.5 kpc and 8.5 kpc and over a range in \( \phi \) that spans 1 rad. This region typically contains 250k particles, or about 1/20 of the total number of particles in our high-resolution ring. The number of stars is thus a factor of 3−4 less than the number of stars in the arcs considered by Antoja et al. (2018). On the other hand, our arc is about five times wider in \( R \) and seven times wider in \( \phi \) than the one they chose. Thus, even with our high-resolution ring our simulations have poorer particle and spatial resolution in comparison with the data.

Results from our simulations for the \( z - v_z \) plane are shown in Fig. 8 and Fig. 9 for the same snapshots used in Fig. 7. Though there is broad qualitative agreement between the self-consistent and test-particle simulations a detailed comparison reveals a number of differences. Most notably, the test-particle run shows sharper features and narrower spirals in all of the \( z - v_z \) maps. As noted above, the low-mass particles in our high-resolution ring will undergo two-body scattering with the more massive particles from the halo as well as the rest of the disc. Of course real discs have noisy potentials due to molecular clouds, globular clusters, and the like so these effects may not be completely unphysical. Indeed, the spirals in the data are not as sharp as those in our test particle simulation. Nevertheless, in future work we intend to explore the best ways of implementing multimass models for disc simulations.

The spirals in the self-gravity simulation appear to be more tightly wound than those in the test-particle run. The difference is most evident in the 500 Myr snapshot. The phase of the \( v_R \) spiral at this time is also different. The differences could prove important if one is attempting to constrain the gravitational potential and/or the initial perturbation using the \( z - v_z \) spirals.

4 CONCLUSIONS

In this paper we have focused on three facets of phase space spirals. First, spirals similar to those found in \( Gaia \) DR2 naturally arise when the disc experiences a bending perturbation. In general, a passing satellite will cause the disc to bend (Toth & Ostriker 1992; Sellwood et al. 1998; Widrow et al. 2014; Binney & Schoenrich 2018), and within a few dynamical times these bends lead to spirals in the \( z - v_z - v_R - v_\phi \) space. Second, the non-separable nature of the effective potential is crucial for understanding the morphology of the spirals. This effect, which appears at cubic and higher order in the Taylor expansion of the effective potential, implies a coupling of the in-plane and vertical epicyclic motions and leads to variations in \( v_R \) and \( v_\phi \) across and along the edges of the spirals. Finally, self gravity appears to be an essential ingredient in any study of bending waves in discs.

The results from our toy-model simulations suggest that by studying the morphology of phase space spirals, we can probe the detailed structure of the Galactic potential. In particular, one might gain a handle on the coefficients of cubic and quartic terms in the Taylor expansion of the effective potential near the Sun and beyond, once future data releases from \( Gaia \) become available.

The natural way forward is to test these ideas with different models for the Galactic potential. Studies of this type will allow us to determine just how sensitive the phase space spirals are to the potential. Such a program should be straightforward to accomplish with kinematic models where test particles are used as probes. However, our self-consistent simulations suggest that these models may miss the essential physics of self-gravity in the perturbed disc. Unfortunately, self-consistent simulations are computationally expensive, and the mass and spatial resolution that can be achieved, especially if one aims to study a large parameter space, are well below the resolution available in the data.

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