Charged Higgs contributions to CP violation in $\tau^- \to K^-\pi^0\nu_\tau$

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Abstract

In this paper we derive the new contributions to the effective Hamiltonian governs $|\Delta S| = 1$ semileptonic tau decays in the framework of two Higgs doublet model with generic Yukawa structure. We list all operators, in the effective Hamiltonian, generated from the charged Higgs exchange up to one loop-level and provide analytical expression for their corresponding Wilson coefficients. Moreover, we analyze the role of the different contributions, originating from the scalar, vector and tensor hadronic currents, in generating direct CP asymmetry in the decay rate of $\tau^- \to K^-\pi^0\nu_\tau$. We show that non vanishing direct CP asymmetry in the decay rate of $\tau^- \to K^-\pi^0\nu_\tau$ can be generated due to the presence of both, the weak phase in the Wilson coefficient corresponding to the tensor operator and the strong phase difference resulting from the interference between the form factors expressing the matrix elements of the vector and tensor hadronic currents. After taking into account all relevant constraints, we find that the generated CP asymmetry can be enhanced 6 orders of magnitude larger than the standard model prediction.

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I. INTRODUCTION

Symmetries play an important role in particle physics. Among these symmetries, Parity (P) and the Charge-conjugation (C) symmetries are of particular interest. The combined Charge-Conjugation parity (CP) symmetry violation are needed to explain many phenomena in physics. These phenomena include the electric dipole moment, the baryon number asymmetry of the universe and the observed mixing in the neutral mesons.

In the meson sector, the observation of the decaying of the short-lived and the long-lived components of the neutral kaon into $\pi^+\pi^-$ was the first clear evidence of the violation of the CP symmetry in the K meson [1]. Moreover, BABAR[2] and Belle [3] observed the first hints of CP violation in the neutral $B_d$ meson. Regarding the D meson, a combined results from BABAR [2], Belle [3] and CDF [4] showed an evidence of the $D^0 - \bar{D}^0$ mixing. This evidence has been confirmed later at LHCb after the first experimental observations of the slow rate of the $D^0 - \bar{D}^0$ mixing [5].

In the lepton sector, namely in $\tau$ decays, earlier searches by CLEO Collaboration [6] and Belle Collaboration [7] for CP-violation in the decay-rate of $\tau \rightarrow K_S\pi\nu_\tau$ pointed out no signals had been observed. In contrast to these findings, recently BaBar collaboration pointed out a signal for CP-violation in the decay-rate of $\tau \rightarrow K_S\pi\nu_\tau$ [8]. The measured CP-violating asymmetry is determined to be $(-0.36 \pm 0.23 \pm 0.11)$% [8] which correspond to Standard Model prediction using the Quantum Field Formalism to describe the $K - \bar{K}$ mixing as shown in ref.[9].

Theoretically, semileptonic $|\Delta S| = 1$ tau decays can be generated via $\tau^- \rightarrow s\bar{u}\nu_\tau$ transition. CP violation in the modes $\tau \rightarrow K_S\pi\nu_\tau$ and $\tau \rightarrow K\pi\nu_\tau$ have been studied in the literature in Refs.[9, 14] and Refs.[15-24] respectively. Regarding CP violation in $\tau \rightarrow K_S\pi\nu_\tau$ decay and within SM, an estimated CP asymmetry of $O(10^{-3})$ has been pointed out in Ref.[12]. This CP asymmetry is induced indirectly from $K^0 - \bar{K}_0$ mixing. In Ref.[10], it was shown that the CP-violating asymmetry measured by the BaBar collaboration can not be generated with a simple charged scalar non-standard interactions due to the lack of the required strong phase needed to produce a non-vanishing CP asymmetry. Moreover, as shown in Ref.[10], a possible mechanism to produce the required strong phase and thus generating the measured asymmetry is through interference between vector and new nonstandard tensor interactions. In a recent study, it is shown that this interference is suppressed by at least
two orders of magnitude due to Watson’s final-state-interaction theorem [11]. In addition, the bounds from the neutron electric dipole moment and $D$–$\bar{D}$ mixing severely constraint the strength of the relevant $CP$-violating tensor interaction [11].

Turning now to the decay mode $\tau \rightarrow K\pi\nu_\tau$, an estimation of the direct $CP$ asymmetry in the decay rate of this process within SM framework showed that the calculated asymmetry is negligibly small of order $10^{-12}$ [18]. This result motivated further studies of $CP$ violation in this decay mode within the framework of supersymmetric extension of the SM [21, 22]. In minimal supersymmetric extension of the SM with $R$ parity conservation, direct $CP$ asymmetry of order $O(10^{-7})$ can be generated through the interference between the vector and tensor interactions [21]. On the other hand, within supersymmetric extensions of SM with allowed $R$ parity violating terms, no direct $CP$ asymmetry in the decay rate can be generated at tree-level due to the absence of tensor interactions [22]. In Refs. [17, 19, 23], it was pointed out that $CP$ violation in $\tau^- \rightarrow K^-\pi^0\nu_\tau$ can arise in multi Higgs models with complex couplings in the quark sector due to the interference of the vector and scalar quark currents.

Two Higgs doublet models (2HDM) are simple extensions of the SM in which the scalar sector of the SM is enlarged to contain new scalars [25, 26]. Based on the couplings of the new scalars to quarks and leptons we can classify these models to several types such as type I, II or III [27]. In the two Higgs doublet model with generic Yukawa structure or simply type III (2HDM III), the couplings of the new scalars to quarks and leptons can be complex [28–30]. As a consequence, these couplings can serve as the source of the weak $CP$ violating phases essential for generating non vanishing $CP$ asymmetries. The effect of these new weak phases on the $CP$ asymmetry in $D$ meson sector have been investigated in Refs. [31–33]. The resultant direct $CP$ asymmetry in this model can be enhanced several orders of magnitudes larger than SM predictions for the all investigated modes of the $D$ meson decays. The aim of this paper is to derive the new contributions to the effective Hamiltonian governing the semileptonic $|\Delta S| = 1$ tau decays in the framework of 2HDM III up to one loop level. With the presence of new weak phases and new tensor operator, we analyze the direct $CP$ violating effects in the decay rate of $\tau^- \rightarrow K^-\pi^0\nu_\tau$.

This paper is organized as follows. In Sec. [11] we derive the effective Hamiltonian describing semileptonic $|\Delta S| = 1$ tau decays, $\tau^- \rightarrow s\bar{u}\nu_\tau$ transition, in the presence of new physics (NP) beyond SM decay. Based on this Hamiltonian, we derive the general expression of the
differential decay width of the decay process \( \tau^- \rightarrow K^-\pi^0\nu_\tau \). Switching off NP contributions to the differential decay width, we show in Sec. III that no direct CP asymmetry in the decay rate of \( \tau^- \rightarrow K^-\pi^0\nu_\tau \) can be generated in the SM at tree-level. In Sec. IV, we derive the analytic expressions of the Wilson coefficients up to one loop level originating from the charged Higgs mediation in 2HDM III. In addition, we give our estimation of the direct CP asymmetry in the decay rate of \( \tau^- \rightarrow K^-\pi^0\nu_\tau \). Finally, in Sec. V, we give our conclusion.

II. EFFECTIVE HAMILTONIAN AND THE DIFFERENTIAL DECAY WIDTH OF \( |\Delta S| = 1 \) \( \tau \) DECAYS

In the presence of NP beyond SM, the effective Hamiltonian governs \( |\Delta S| = 1 \) \( \tau \) decays transition can be expressed as

\[
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{us}^\ast \sum_{i=1}^{6} C_i(\mu)Q_i(\mu),
\]

where \( V_{us} \) is the \( us \) Cabibbo-Kobayashi-Maskawa (CKM) matrix element and \( Q_i \) represent the four-fermion local operators at low energy scale \( \mu \sim m_\tau \). The operators can be written as

\[
\begin{align*}
Q_1 &= (\bar{\nu}_\tau \gamma^\mu L\tau)(\bar{s}\gamma_\mu Lu), \\
Q_2 &= (\bar{\nu}_\tau \gamma^\mu L\tau)(\bar{s}\gamma_\mu Ru), \\
Q_3 &= (\bar{\nu}_\tau R\tau)(\bar{s}Ru), \\
Q_4 &= (\bar{\nu}_\tau R\tau)(\bar{s}Lu), \\
Q_5 &= (\bar{\nu}_\tau \sigma^{\mu\nu} R\tau)(\bar{s}\sigma_{\mu\nu} Ru), \\
Q_6 &= (\bar{\nu}_\tau \sigma^{\mu\nu} R\tau)(\bar{s}\sigma_{\mu\nu} Lu),
\end{align*}
\]

where \( \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \) and \( L, R = 1 \mp \gamma_5 \). The Wilson coefficients, \( C_i \), corresponding to the operators \( Q_i \) can be expressed as

\[
C_i = C_i^{SM} + C_i^{NP}
\]

where \( C_i^{SM} \) and \( C_i^{NP} \) represent SM and NP contributions to the Wilson coefficients respectively. In order to proceed to write the amplitude we need first to calculate the matrix
elements of the operators in the effective Hamiltonian. For this, we need to assign the momenta of the particles involved in the decay process. We express the momenta as

$$\tau^-(p_\tau) \to K^-(p_K) + \pi^0(p_\pi) + \nu_\tau(p_\nu),$$  

(4)

Second step, we need to estimate the matrix elements of the local operators $Q_i$ appear in Eq.(2). The matrix elements of the hadronic currents in the $Q_i$ operators are usually parameterized in terms of particles momenta and form factors. Due to parity conservation in the $K \to \pi$ matrix elements we need only to calculate the matrix element of the vector, scalar and tensor quarks currents only. The matrix element of the vector quark current can be expressed as

$$\langle K^- \pi^0 | \bar{s} \gamma^\mu u | 0 \rangle = \frac{1}{\sqrt{2}} \left( (p_K - p_\pi)^\mu f_+(s) + (p_K + p_\pi)^\mu f_-(s) \right),$$  

(5)

and

$$f_-(s) = \frac{M_K^2 - M_\pi^2}{s} \left( f_0(s) - f_+(s) \right).$$  

(6)

here $s$ is the invariant mass defined as $s = (p_K + p_\pi)^2$ of the $\pi K$ system. The matrix element of the scalar quark current can be obtained from Eq.(5) by taking the divergence in the usual form and hence we get

$$\langle K^- \pi^0 | \bar{s} u | 0 \rangle = \frac{(M_K^2 - M_\pi^2)}{\sqrt{2}(m_s - m_u)} f_0(s) = \frac{\Delta}{\sqrt{2}(m_s - m_u)} f_0(s),$$  

(7)

where we have defined $\Delta = M_K^2 - M_\pi^2$ and $m_{s,u}$ denote $s,u$ current quark masses. Finally, the matrix element of the tensor quarks current, $\langle K^- \pi^0 | \bar{s} \sigma_{\mu\nu} u | 0 \rangle$, can be expressed as

$$\langle K^- \pi^0 | \bar{s} \sigma_{\mu\nu} u | 0 \rangle = \frac{i(p_K^\mu p_\pi^\nu - p_\pi^\mu p_K^\nu)}{\sqrt{2} M_K} B_T(s),$$  

(8)

The total amplitude, $\mathcal{A}$, of $\tau^- \to K^- \pi^0 \nu_\tau$ decay can be expressed as

$$\mathcal{A} = \frac{G_F V_{us}(C_1 + C_2)}{\sqrt{2}} \left\{ \left( (p_K - p_\pi)^\mu f_+(s) + (p_K + p_\pi)^\mu f_-(s) \right) \left( \bar{u}(p_\nu) \gamma_\mu L u(p_\tau) \right) \right. + \frac{(C_3 + C_4) \Delta}{(m_s - m_u)(C_1 + C_2)} f_0(s) \left( \bar{u}(p_\nu) R u(p_\tau) \right) + i \frac{(p_K^\mu p_\pi^\nu - p_\pi^\mu p_K^\nu)(C_5 + C_6)}{M_K(C_1 + C_2)} B_T(s) \left( \bar{u}(p_\nu) \sigma_{\mu\nu} R u(p_\tau) \right) \right\}.$$  

5
The differential decay width is given as
\[
\frac{d\Gamma}{ds} = G_F^2 |V_{us}|^2 |C_1 + C_2|^2 S_{\text{EW}} \frac{\lambda^{1/2}(s, M_{\pi}^2, M_K^2)(m_\tau^2 - s)^2 \Delta^2}{2048\pi^3 m_\tau s^3}
\times \left[ \frac{(m_\tau^2 + 2s)\lambda(s, M_{\pi}^2, M_K^2)}{3m_\tau^2 \Delta^2} \left( |f_+(s) - T(s)|^2 + \frac{2(m_\tau^2 - s)^2}{9sm_\tau^2} |T(s)|^2 \right) + |S(s)|^2 \right],
\]
(9)
where \(\lambda(x, y, z)\) is given by
\[
\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz,
\]
and
\[
S(s) = f_0(s) \left( 1 + \frac{s(C_3 + C_4)}{m_\tau (m_s - m_u) (C_1 + C_2)} \right),
\]
\[
T(s) = \frac{3s}{m_\tau^2 + 2s} \frac{m_\tau (C_5 + C_6)}{M_K (C_1 + C_2)} B_T(s).
\]
(10)

Non vanishing direct CP asymmetry in the decay rate requires the presence of two types of phases, the weak CP violating phases and the strong CP conserving phases. The weak CP violating phases can be generated in the Wilson coefficients upon existence of complex couplings. On the other hand, the strong CP conserving phases originate from the phases in the form factors expressing the matrix elements of the hadronic currents.

Breit-Wigner forms are used to parameterize the contributions of the different resonances dominating the scalar and vector hadronic currents. As a consequence, form factors originating from these currents can be expressed as a summation of Breit-Wigner forms. Previous studies of CP asymmetries in \(\tau \to K\pi\nu_\tau\) decays, for instances Refs. [10, 21], adopted the assumption that the form factor \(B_T(s)\) has no strong phases. However, this assumption is incorrect as argued in Ref. [11]. As shown in Refs. [37, 38], spin-1 resonances can be described equivalently by vector or antisymmetric tensor fields. Hence the same resonances \(K^*(892)\) and the \(K^*(1410)\) that dominate the Breit-Wigner forms in \(f_+(s)\) will appear in \(B_T(s)\) as well [11]. It should be noted that, this conclusion can be derived by analyzing the unitarity relation for the form factors as shown in details in Ref. [11]. Thus, we conclude that \(B_T(s)\) has a strong phase that should be taken into account in the calculations of the CP asymmetry.

### III. CP ASYMMETRY OF \(\tau^+ \to K^- \pi^0 \nu_\tau\) IN THE SM

In the SM, at tree-level, the Wilson coefficients \(C_i\) reduces to
\[
C_1^{SM} = 1, \quad C_j^{SM} = 0 \text{ for } j = 2, 3, ..., 6
\]
(11)
This accounts for the fact that $\tau^\rightarrow K^-\pi^0\nu_\tau$, at tree-level, can be generated as a result of exchanging single $W^-$ boson which contributes only to $Q_1$ operator. The differential decay width given in Eq.(9) in this case reduces to

$$\frac{d\Gamma}{ds}\bigg|_{\text{SM}} = \frac{G_F^2|V_{us}|^2S_{\text{EW}}}{1024\pi^3m_\tau s^3} \left(\frac{\lambda^{1/2}(s, M_\pi^2, M_K^2)(m_\tau^2 - s)^2\Delta^2}{3m_\tau^2\Delta^2} \right) \times \left[ \frac{(m_\tau^2 + 2s)\lambda(s, M_\pi^2, M_K^2)}{3m_\tau^2\Delta^2} |f_+(s)|^2 + |f_0(s)|^2 \right],$$

(12)

The decay rate of the process $\tau^\rightarrow K^-\pi^0\nu_\tau$ in the SM, $\Gamma_{SM}$, can be then obtained upon integrating the previous equation with respect to the kinematic variable $s$. Thus we get

$$\Gamma_{SM} = \frac{G_F^2|V_{us}|^2S_{\text{EW}}\Delta^2}{1024\pi^3m_\tau} \int_{(M_K+M_\pi)^2}^{m_\tau^2} ds \left(\frac{\lambda^{1/2}(s, M_\pi^2, M_K^2)(m_\tau^2 - s)^2\Delta^2}{3m_\tau^2\Delta^2} \right) \times \left[ \frac{(m_\tau^2 + 2s)\lambda(s, M_\pi^2, M_K^2)}{3m_\tau^2\Delta^2} |f_+(s)|^2 + |f_0(s)|^2 \right],$$

(13)

The CP asymmetry in total decay rate of $\tau^\rightarrow K^-\pi^0\nu_\tau$ is given by:

$$a_{CP} = \frac{\Gamma(\tau^\rightarrow K^-\pi^0\nu_\tau) - \Gamma(\tau^+\rightarrow K^+\pi^0\nu_\tau)}{\Gamma(\tau^\rightarrow K^-\pi^0\nu_\tau) + \Gamma(\tau^+\rightarrow K^+\pi^0\nu_\tau)}$$

(14)

Clearly, from the expression of $\Gamma_{SM}$, direct CP asymmetry in the decay rate will vanish due to the absence of the weak phase, $C_1^{SM}$ is real, and also due to the remark that the form factors $f_+(s)$ and $f_0(s)$ do not interfere and hence the relative strong phase essential for CP asymmetry vanishes. Thus, to generate no vanishing CP asymmetry in the decay rate within SM, it is essential to consider higher order terms contributing to the amplitude as done in Ref.[18]. These terms can be generated from diagrams with exchanging two $W$ bosons. Thus, the generated asymmetry is expected to be very small. As shown in Ref.[18], the resulting CP asymmetry is suppressed by the CKM factor $V_{td}\simeq 10^{-3}$ and also by a higher order suppression factor $g^2/4\pi M_W^2 \simeq 10^{-8}$. As a consequence, the resulting CP rate asymmetry is expected to be negligible, as confirmed in Ref.[18].

IV. CP ASYMMETRY OF $\tau^\rightarrow K^-\pi^0\nu_\tau$ IN 2HDM III.

The scalar sector of the 2HDM III consists of two Higgs doublets. The mass eigenstates constructed from these doublets are $H_0$ (heavy CP-even Higgs), $h_0$ (light CP-even Higgs)
FIG. 1. Diagrams contributing to semileptonic $|\Delta S| = 1$ $\tau$ decays up to one loop-level due to charged Higgs mediation.

and $A_0$ (CP-odd Higgs) and $H^\pm$. In 2HDM III, the charged Higgs couplings to quarks and leptons can be expressed as [28, 29]

$$L_{eff}^{H^\pm} = \bar{u} \Gamma_{us}^{LR} P_R s + \bar{u} \Gamma_{us}^{RL} P_L s + \frac{m_\tau \tan \beta}{v} (\bar{\nu}_\tau P_R \tau),$$

where

$$\Gamma_{us}^{LR} = \sin \beta \left( V_{12} \frac{m_s}{v_d} - \sum_{j=1}^{3} V_{ij} d_j V^*_{j2} \tan \beta \right),$$
$$\Gamma_{us}^{RL} = \cos \beta \left( \frac{m_u}{v_u} V_{12} - \sum_{j=1}^{3} V_j u_j V^*_{j1} \tan \beta \right)$$

Here $v_u$ and $v_d$ stand for the vacuum expectations values of the neutral component of the Higgs doublets, $\tan \beta = v_u/v_d$ and finally $V$ is the CKM matrix. Using the Feynman-rules given in Eq.(15), we can proceed to derive the Wilson coefficients $C_i^H$, corresponding to the operators $Q_i$ in Eq.(2), after integrating out the charged Higgs in the diagrams in Fig.1. We can classify these coefficients into three classes namely, $C_{1,2}^H$, $C_{3,4}^H$ and $C_{5,6}^H$ corresponding to the vector, scalar and tensor hadronic currents respectively. At $\mu = m_H$ scale, they are given as

$$C_1^{H^\pm} = \frac{8 m_W^2 m_s m_\tau^2 \tan \beta \sin \beta}{v V^*_{us} c_w^2} \left( -\frac{1}{2} + s_w^2 \right) \left( -\frac{1}{2} + \frac{1}{3} s_w^2 \right) g(x_s, x_\tau, x_Z) + \frac{4 v^2 \pi \alpha m_u c_w^2}{3 m_W^2 m_s} k(x_u, x_\tau)$$
$$\times \left( \frac{m_s V^*_{us}}{v_d} - \sum_{j=1}^{3} V_{ij} d_j \epsilon_{j2} \tan \beta \right),$$
$$C_2^{H^\pm} = \frac{8 m_W^2 m_s m_\tau^2 \sin \beta s_w^2}{3 v V^*_{us} c_w^2} \left( -\frac{1}{2} + s_w^2 \right) g(x_s, x_\tau, x_Z) + \frac{4 v^2 \pi \alpha m_u c_w^2}{m_W^2 m_s s_w^2} k(x_u, x_\tau)$$
$$\times \left( \frac{m_u V^*_{us}}{v_u} - \sum_{j=1}^{3} V^*_{j2} u_j \tan \beta \right) ,$$

(17)
\[ C_{3}^{H^{\pm}} = -\frac{4v m_{\tau} \sin \beta}{V_{us} m_{H}^{2}} \left( 1 - \frac{16\pi m_{H}^{2}}{3} f(x_{u}, x_{\tau}) + \frac{2 m_{W}^{2} m_{H}^{2}}{3 v^{2} c_{w}^{2}} f(x_{u}, x_{Z}) + \frac{4\pi m_{H}^{2}}{3} f(x_{s}, x_{\tau}) \right) \]

\[ - \frac{2 m_{W}^{2} m_{H}^{2}}{v^{2} c_{w}^{2}} \left( -\frac{1}{2} + \frac{1}{3} s_{w}^{2} \right) h(x_{s}, x_{\tau}, x_{Z}) + \frac{8\pi m_{H}^{2}}{3} f(x_{u}, x_{\tau}) \)

\[ \left( m_{u} V_{us}^{*} - \sum_{j=1}^{3} V_{j2}^{*} c_{j1} \tan \beta \right) \]

\[ C_{4}^{H^{\pm}} = -\frac{4v m_{\tau} \sin \beta \tan \beta}{V_{us} m_{H}^{2}} \left( 1 - \frac{16\pi m_{H}^{2}}{3} f(x_{u}, x_{\tau}) - \frac{m_{W}^{2} m_{H}^{2}}{v^{2} c_{w}^{2}} \left( \frac{1}{2} - \frac{2}{3} s_{w}^{2} \right) f(x_{u}, x_{Z}) + \frac{4\pi m_{H}^{2}}{3} f(x_{s}, x_{\tau}) \right) \]

\[ - \frac{2 m_{W}^{2} m_{H}^{2}}{3 v^{2} c_{w}^{2}} h(x_{s}, x_{\tau}, x_{Z}) + \frac{8\pi m_{H}^{2}}{3} f(x_{u}, x_{\tau}) \)

\[ \left( m_{s} V_{us}^{*} - \sum_{j=1}^{3} V_{1j}^{*} \epsilon_{j2} \tan \beta \right) \]

(18)

\[ C_{5}^{H^{\pm}} = -\frac{4v m_{\tau} \sin \beta}{V_{us} m_{H}^{2}} \left( \frac{2 m_{W}^{2} m_{H}^{2}}{3 v^{2} c_{w}^{2}} f(x_{u}, x_{Z}) + \frac{4\pi m_{H}^{2}}{3} f(x_{s}, x_{\tau}) - \frac{2 m_{W}^{2} m_{H}^{2}}{3 v^{2} c_{w}^{2}} \left( -\frac{1}{2} + \frac{1}{3} s_{w}^{2} \right) h(x_{s}, x_{\tau}, x_{Z}) \right) \]

\[ + \frac{8\pi m_{H}^{2}}{3} f(x_{u}, x_{\tau}) \]

\[ \left( m_{u} V_{us}^{*} - \sum_{j=1}^{3} V_{j2}^{*} c_{j1} \tan \beta \right) \]

\[ C_{6}^{H^{\pm}} = \frac{4v m_{\tau} \sin \beta}{V_{us} m_{H}^{2}} \left( \frac{m_{W}^{2} m_{H}^{2}}{v^{2} c_{w}^{2}} \left( \frac{1}{2} + \frac{2}{3} s_{w}^{2} \right) f(x_{u}, x_{Z}) - \frac{4\pi m_{H}^{2}}{3} f(x_{s}, x_{\tau}) + \frac{2 m_{W}^{2} m_{H}^{2}}{3 v^{2} c_{w}^{2}} h(x_{s}, x_{\tau}, x_{Z}) \right) \]

\[ - \frac{8\pi m_{H}^{2}}{3} f(x_{u}, x_{\tau}) \]

\[ \left( m_{s} V_{us}^{*} - \sum_{j=1}^{3} V_{1j}^{*} \epsilon_{j2} \tan \beta \right) \]

(19)

where \( \alpha \simeq 1/137, v = 174 \text{ GeV} \) and the integration loop functions are given as

\[ g(x_{i}, x_{j}, x_{k}) = \frac{1}{16\pi^{2} m_{H}^{4}} \left( \frac{x_{i} \log x_{i}}{(x_{i} - 1)(x_{i} - x_{j})(x_{i} - x_{k})} + \frac{x_{j} \log x_{j}}{(x_{j} - 1)(x_{j} - x_{i})(x_{j} - x_{k})} \right) \]

\[ + \frac{x_{k} \log x_{k}}{(x_{k} - 1)(x_{k} - x_{i})(x_{k} - x_{j})} \]

\[ k(x_{i}, x_{j}) = \frac{-1}{16\pi^{2} m_{H}^{4}} \left( \frac{1}{x_{i} - x_{j}} \log x_{i} - (x_{i} \leftrightarrow x_{j}) \right) \]

\[ f(x_{i}, x_{j}) = \frac{1}{16\pi^{2} m_{H}^{4}} \left( \frac{1}{x_{i} - x_{j}} \log x_{i} - (x_{i} \leftrightarrow x_{j}) \right) \]

\[ h(x_{i}, x_{j}, x_{k}) = \frac{1}{16\pi^{2} m_{H}^{4}} \left( \frac{x_{i}^{2} \log x_{i}}{(x_{i} - 1)(x_{i} - x_{j})(x_{i} - x_{k})} + \frac{x_{j}^{2} \log x_{j}}{(x_{j} - 1)(x_{j} - x_{i})(x_{j} - x_{k})} \right) \]

\[ + \frac{x_{k}^{2} \log x_{k}}{(x_{k} - 1)(x_{k} - x_{i})(x_{k} - x_{j})} \]

(20)

with \( x_{i} = \frac{m_{i}^{2}}{m_{H}^{2}} \). In order to estimate the contributions of the charged Higgs to the amplitude of the decay process under consideration we need to discuss the constraints imposed on the couplings \( \epsilon_{ij}^{u,d} \) appear in the expressions of \( C_{i}^{H^{\pm}} \) above. The couplings \( \epsilon_{12}^{d} \) and \( \epsilon_{32}^{d} \) are stringently constrained from flavor changing neutral current processes, in the down quark
sector, due to the tree-level neutral Higgs exchange \[^{29,30}\]. On the other hand, the coupling \( \epsilon_{22}^d \) can be strongly constrained upon applying the naturalness criterion of 't Hooft to the quark masses that reads \[^{29}\]

\[
| \epsilon_{ij}^d | \leq \left| V_{ij} \right| \frac{\max \left[ m_{d_i(u_i)}, m_{d_j(u_j)} \right]}{|v_{u(d)}|}.
\]

(21)

Clearly, from this bound, \( \epsilon_{22}^d \) is severely constrained by the smallness of the \( s \) quark mass. As a result, we can safely neglect the contributions of the couplings \( \epsilon_{ij}^d \) to \( C_{H^\pm}^1, C_{H^\pm}^4 \) and \( C_{H^\pm}^6 \). Other terms, in these Wilson coefficients are real and thus are not relevant for generating \( CP \) asymmetries. Thus to a good approximation we can set \( C_{H^\pm}^1 \simeq C_{H^\pm}^4 \simeq C_{H^\pm}^6 \simeq 0 \) and thus we are left with

\[
C_{H^\pm}^2 \simeq \frac{8m_W^2 m_s^2 m_{H^\pm}^2 \sin^2 \beta s_w^2 \epsilon_{21}^u}{3vV_{us}^* c_w^2 \cos \beta} \left( \left( -\frac{1}{2} + s_w^2 \right) g(x_s, x_r, x_Z) + \frac{4\pi \alpha m_u c_w^2}{m_W^2 m_s^2 s_w^2} h(x_u, x_r) \right)
\]

\[
C_{H^\pm}^3 \simeq \frac{4v m_{\tau} \sin^2 \beta \epsilon_{21}^u}{V_{us}^* m_{H^\pm}^2 \cos \beta}
\]

\[
C_{H^\pm}^5 \simeq \frac{4v m_{\tau} \sin^2 \beta \epsilon_{21}^u}{V_{us}^* m_{H^\pm}^2 \cos \beta} \left( \frac{2m_W^2 m_{H^\pm}^2 s_w^2}{3v^2 c_w^2} f(x_u, x_Z) - \frac{2m_W^2 m_{H^\pm}^2 s_w^2}{v^2 c_w^2} \left( -\frac{1}{2} + \frac{1}{3} s_w^2 \right) h(x_s, x_r, x_Z) \right)
\]

\[
+ \frac{8\pi \alpha m_{H^\pm}^2}{3} f(x_s, x_r) + \frac{4\pi \alpha m_{H^\pm}^2}{3} f(x_s, x_r)
\]

(22)

It should be noted that, in the above expressions, we neglected non relevant real terms. In addition we neglected terms proportional to \( \epsilon_{11}^u \) which is severely constrained from the bound in Eq.(21) due to the smallness of the up quark mass. Finally we also neglected terms proportional to \( \epsilon_{31}^u \) as they are suppressed by the CKM factor \( V_{ts}^* \).

Recently, a lower bound \( m_{H^\pm} \gtrsim 600 \text{ GeV} \), independent of \( \tan \beta \), has been obtained in 2HDM II after taking into account all relevant results from direct charged and neutral Higgs boson searches at LEP and the LHC, as well as the most recent constraints from flavour physics \[^{39}\]. This bound should be also respected in 2HDM III \[^{29}\]. Thus, for \( m_{H^\pm} = 600 \) GeV and \( \tan \beta = 50 \) we find that

\[
C_{H^\pm}^2 \simeq 4.76 \times 10^{-6} \epsilon_{21}^u
\]

\[
C_{H^\pm}^3 \simeq 0.74 \epsilon_{21}^u
\]

\[
C_{H^\pm}^5 \simeq -4.82 \times 10^{-3} \epsilon_{21}^u,
\]

(23)
Clearly, to a good approximation, we can neglect contribution of \( C_2^{H^\pm} \) to the amplitude as it is very small. The previous equation shows that \( C_3^{H^\pm} \) is much larger than \( C_5^{H^\pm} \). However, as we will see later, only \( C_5^{H^\pm} \) can generate non-vanishing direct CP asymmetry.

The total differential decay width, including charged Higgs contributions, can be obtained from that one given in Eq. (9) by setting \( C_1 = C_1^{SM} \simeq 1, \ C_2 = C_4 = C_6 \simeq 0, \ C_3 = C_3^{H^\pm} \) and \( C_5 = C_5^{H^\pm} \). Thus we get

\[
\frac{d\Gamma}{ds}\bigg|_{SM+H^\pm} = G_F^2 |V_{us}|^2 S_{EW} \frac{\lambda^{1/2}(s, M_{\pi}^2, M_K^2)(m_\tau^2 - s)^2 \Delta^2}{1024\pi^3 m_\tau s^3} \\
\times \left[ \frac{(m_\tau^2 + 2s)\lambda(s, M_{\pi}^2, M_K^2)}{3m_\tau^2 \Delta^2} \left( |f_+(s) - T'(s)|^2 + \frac{2(m_\tau^2 - s)^2}{9sm_\tau^2} |T'(s)|^2 \right) + |S'(s)|^2 \right],
\]

(24)

where

\[
S'(s) = f_0(s) \left( 1 + \frac{s C_3^{H^\pm}}{m_\tau(m_s - m_u)} \right),
\]

\[
T'(s) = \frac{3s}{m_\tau^2 + 2s} \frac{m_\tau C_5^{H^\pm}}{M_K} B_T(s).
\]

(25)

As can be seen from the last two equation, the hadronic form factor \( f_0(s) \), in \( S'(s) \), does not interfere with the form factor \( f_+(s) \). The absence of this interference leads to the absence of the strong phase difference between their contributions to the decay rate. This phase difference is essential for generating non-vanishing direct CP asymmetry. As a consequence, and for having non-vanishing direct CP asymmetry in the decay rate, we are left only with the interference between \( B_T(s) \), in \( T'(s) \), and \( f_0(s) \) as a possible source for the required strong phase difference. However, this interference was estimated to be small due to Watson’s final-state-interaction theorem \([11, 40]\). The CP asymmetry in total decay rate of \( \tau^- \to K^-\pi^0\nu_\tau \) in this case is given by:

\[
a_{CP} = - \frac{G_F^2 |V_{us}|^2 S_{EW} Im(C_5^{H^\pm})}{512\pi^3 m_\tau M_K \Gamma_T BR(\tau \to K\pi\nu_\tau)} \\
\times \int_{(M_\tau + M_K)^2}^{m_\tau^2} \frac{\lambda^{3/2}(s, M_{\pi}^2, M_K^2)(m_\tau^2 - s)^2}{s^2} |f_+(s)| |B_T(s)| \sin(\delta_+(s) - \delta_T(s))
\]

(26)

where \( \delta_+(s), \delta_T(s) \) are the phases of \( f_+(s) \) and \( B_T(s) \). Using \( BR(\tau \to K^-\pi^0\nu_\tau) = (4.33 \pm 0.15) \times 10^{-3} \) \([41]\), \( f_+(0) |V_{us}| = 0.2165(4) \) \([41]\), \( B_T(0)/f_+(0) = 0.676(27) \) from lattice QCD \([42]\), particle masses and couplings from \([41]\) and the estimations of the form factors in Ref. \([11]\), we find that the CP asymmetry can be estimated as

\[
|A_{CP}| \lesssim 6.7 \times 10^{-5} |\varepsilon_{u21}^w|,
\]

(27)
With the bound $|\epsilon_{21}^{u}| \leq 3.0 \times 10^{-2}$ obtained from $D^0 \rightarrow \mu^+ \mu^-$, we finally found that

$$|A_{CP}| \lesssim 2 \times 10^{-6},$$ (28)

Although the estimated CP asymmetry is still small however, the charged Higgs contributions can enhance the the CP asymmetry 6 orders of magnitude larger than the standard model prediction.

V. CONCLUSION

In this paper we have derived the contributions to the effective Hamiltonian governs the semileptonic $|\Delta S| = 1$ tau decays in 2HDM III up to one loop-level. We have discussed the imposed constraints on the elements in the parameter space of the model relevant to the decay channel $\tau^- \rightarrow K^- \pi^0 \nu_\tau$. In addition, we have analyzed the role of the different contributions, originating from the scalar, vector and tensor hadronic currents, in generating direct CP asymmetry in the decay rate of $\tau^- \rightarrow K^- \pi^0 \nu_\tau$.

We have shown that non vanishing direct CP asymmetry in the decay rate of $\tau^- \rightarrow K^- \pi^0 \nu_\tau$ can be generated in the model due to the presence of the weak phase in the Wilson coefficient $C_5^{H^\pm}$ and due to the strong phase difference resulting from the interference between the form factors $B_T(s)$, and $f_0(s)$. After taking into account all relevant constraints, we have found that the CP asymmetry can be enhanced 6 orders of magnitude larger than the standard model prediction.

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