Signals of the QCD Critical Point in Hydrodynamic Evolutions

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Abstract

The presence of a critical point in the QCD phase diagram can deform the trajectories describing the evolution of the expanding fireball in the $\mu_B$-$T$ phase diagram. The deformation of the hydrodynamic trajectories will change the transverse velocity ($\beta_T$) dependence of the proton-antiproton ratio when the fireball passes in the vicinity of the critical point. An unusual $\beta_T$ dependence of the $\bar{p}/p$ ratio in a narrow beam energy window would thus signal the presence of the critical point.

1. Towards quantitative analyses of QCP

The existence and location of the QCD critical point (QCP) in the QCD phase diagram have been attracting many physicists' interests in heavy ion collision physics. However, recent studies based on effective theories show many possible locations of the QCP in the QCD phase diagram. In addition, the latest finite temperature and imaginary chemical potential lattice QCD calculation shows that even the existence of the QCD critical point is uncertain \cite{1}. At present, experiments and quantitative phenomenological analyses for the QCP are needed, because it seems to be very difficult to reach a solid conclusion about the QCP just from lattice QCD and effective theories. Here we discuss the consequences of the QCP from the point of view of the quantitative phenomenological analyses in heavy ion collisions. Towards quantitative analyses of the QCP, we need the following three steps: a realistic dynamical model which describes expansion of hot and dense matter after collisions, an equation of state (EoS) with QCP and appropriate physical observables which show signals of QCP clearly.

For a realistic dynamical model, we use a combined fully three-dimensional macroscopic / microscopic transport approach employing relativistic 3d-hydrodynamics for the early, dense, deconfined stage of the reaction and a microscopic non-equilibrium model for the later hadronic stage where the equilibrium assumptions are not valid anymore \cite{2}. Within this approach we study the dynamics of hot, bulk QCD matter, which is being created in ultra-relativistic heavy ion collisions at RHIC. Our approach is capable of self-consistently calculating the freezeout of the hadronic system, while accounting for the collective flow on the hadronization hypersurface generated by the QGP expansion. We succeed in explaining a lot of experimental data consistently at RHIC: $P_T$ spectra of various particles including multistrangeness particles, rapidity distributions, mean $P_T$ as a function of particle mass, freezeout time distribution of particles, and elliptic flow. In this calculation, we adopt the EoS with the 1st order phase transition without QCP which is used in most hydrodynamic models. Now we replace the EoS in 3d hydro + UrQMD model by an EoS with QCP.
The EoS of QCP which we construct here is composed of two parts: one is a singular part around QCP and another part is non-singular part which is described by usual QGP phase and hadron phase [3]. For the singular part of the EoS, we assume that QCD has the same universality class as 3d Ising model. First we construct the EoS of 3d Ising model as a function of reduced temperature \((r)\) and external magnetic field \((h)\) [4] and map it on the QCD phase diagram, \(\mu_B-T\) plane. The magnetization as a functions of \(r\) and \(h\) in 3d Ising model shows different behavior of phase transition between negative \(r\) (1st order) and positive \(r\) (crossover). In mapping the EoS with 3d Ising model as a function of \(r\) and \(h\) on the \(\mu_B-T\) plane, however, there is no universality between QCD and 3d Ising model. We can not determine the direction of \(h\) axis to \(r\) axis, though we can fix \(r\) axis as it is parallel to tangential line at QCP to the phase boundary. Here we choose the direction of \(h\) to be perpendicular to \(r\) axis. Besides not only the size of critical region around the QCD and location of it on the QCD phase diagram are input parameters in our model. The EoS with the QCP has a very interesting feature in isentropic trajectories on \(\mu_B-T\) plane: the QCP works as an attractor of isentropic trajectories on the QCD phase diagram [5, 3], which gives us a clue of finding the QCP.

2. Signals of QCP

Next we discuss how to find clear consequences of QCP in heavy ion collisions. The promising signals of QCP should survive not only in expanding fireball but also even after the freezeout process. Here we investigate two candidates of them: fluctuations and hadron ratios. For fluctuation, naively, we have to pick up fluctuations of conserved values such as charge, baryon number and strangeness during whole process of collisions. Hadron ratios are fixed at chemical freezeout temperature and hold the same value during freezeout process and final state interactions.

Figure 1 shows behavior of static and dynamical fluctuations in 1d hydrodynamic expansion along isentropic trajectories on the \(\mu_B-T\) plane [3]. In both cases of static and dynamic fluctuations, we can see the effect of QCP, i.e enhancement of fluctuation around QCP. For static case fluctuation becomes maximum just at QCP. However for dynamic case maximum value of fluctuation appears after passing in the vicinity of QCP because of the critical slowing down and the maximum value itself is not so large as the static case. There is possibility that fluctuations which are induced by QCP do not become so large as a signal of QCP, if the expansion of fireball is fast.

For hadron ratios key issue from the point of view of QCP is that a chemical freezeout temperature depends on transverse velocity of hadrons. Figure 2 shows that \(\bar{p}/p\) ratio has transverse velocity dependence in a microscopic transport model, UrQMD in which the QCP does not exist. We find that on isentropic trajectories the freezeout process occurs gradually [6]: particles with higher transverse velocity are emitted at earlier time of expansion and those with lower transverse velocity are produced at later time. This suggests that hadron ratios may change on isentropic trajectory between a hadronization point on the QCD phase boundary on the \(\mu_B-T\) plane and chemical freezeout point. The hadron ratio, especially \(\bar{p}/p\) ratio as a function of transverse velocity (momentum) is sensitive to behavior of isentropic trajectories on \(\mu_B-T\) plane and may show a consequence of QCP clearly.

Next we do a demonstrative calculation to show how the signal of QCP appears through \(\bar{p}/p\) ratio in heavy ion collisions. Here we focus on SPS energy region and put the QCP which is parameter in our model and the chemical freezeout point which is obtained in a statistical model [7] to \((\mu_B, T) = (550, 159)\) MeV and \((406, 105)\) MeV on the \(\mu_B-T\) plane, respectively. Figure 3 shows that hydrodynamical trajectories in the QCD phase diagram with and without the presence
of a critical point. Possible trajectories in the plane in the absence of a critical point are shown as solid line (for a crossover transition (CO)) or dash-dotted line (for a first-order transition (FO)); the trajectory in the presence of a critical point is shown as dashed line (QCP). All trajectories meet at the bulk chemical freezeout point. This dotted line in which clear focusing effect appears stands for isentropic trajectory of the QCP. Hadronization occurs between the phase boundary and chemical freezeout point. And between them we can see clear differences in three cases. In the case of QCD critical point the ratio of $\bar{p}/p$ decreases along the line or almost the same. On the other hand, in the case of 1st order phase transition and crossover this value increases along isentropic trajectory.

Antiproton-to-proton ratio along the trajectories is shown in Fig. 4 as a function of the entropy density which is proportion to transverse momentum. The curves start at the phase boundary 160 MeV and continue down to chemical freezeout temperature (145 MeV). The location of the chemical freezeout point deduced from experimental data is indicated by the open and solid squares. Note that the ratio only rises for the trajectory deformed by the critical point. In actual experimental data this evidence should appear as steeper $\bar{p}$ spectra at high $P_T$.

We find interesting experimental data which may suggest a signal of QCP: $\bar{p}$ spectra obtained
by NA49 [8]. They show $\bar{p}$ and $p$ spectra on collision energies from 20 GeV to 158 GeV. Only at 40 GeV collision energy slope of $P_T$ spectra of $\bar{p}$ seems to be steeper compared to other collision energies, which would require a trajectory of the type expected in the vicinity of the QCP (Fig. 4). The size of the statistical errors of the measurement does not permit a firm conclusion about this anomaly, but it is certainly compatible with the arguments presented here.

Finally we show an example of realistic calculation by 3d hydro + UrQMD model with EoS including QCP. For initial conditions of our hybrid model we set maximum value of energy density and baryon number density to be 2.0 GeV/fm$^3$ and 0.15 fm$^{-3}$, respectively, which corresponds to SPS energy region. In the following calculations we use the same initial conditions for the both cases of EoS with and without QCP and set a switching temperature from hydrodynamic model to UrQMD to be 150 MeV. Because of different behavior in isentropic trajectories between EoS with and without QCP, chemical potential at the switching temperature is 430 MeV in presence of QCP which is larger than one in absence of QCP (250 MeV). This difference appears in hadron ratio. Figure 5 shows $P_T$ spectra of $\pi$, $K$, $p$ with QCP (left) and without QCP (right). We can see the effect of different isentropic trajectories in hadron ratios.

In summary, we have shown that the evolution of the $\bar{p}/p$ ratio along isentropic curves between the phase boundary in the QCD phase diagram and the chemical freeze-out point is strongly dependent on the presence or absence of a critical point. When a critical point exists, the isentropic trajectory approximately corresponding to hydrodynamical expansion is deformed, and the $\bar{p}/p$ ratio grows during the approach to chemical freeze-out. Depending on the actual size of the attractive region around the critical point, the search for an anomaly in the $P_T$ dependence of the $\bar{p}/p$ ratio may require small beam energy steps.

Figure 5: $P_T$ spectra for $\pi$ (solid circles), $K$ (solid square) and $p$ (solid triangle) with QCP (left) and without QCP (right).

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