Effect of Aharonov-Bohm Phase on Spin Tunneling

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Abstract

The role of Aharonov-Bohm effect in quantum tunneling is examined when a potential is defined on the S1 and has N-fold symmetry. We show that the low-lying energy levels split from the N-fold degenerate ground state oscillate as a function of the Aharonov-Bohm phase, from which general degeneracy conditions depending on the magnetic flux is obtained. We apply these results to the spin tunneling in a spin system with N-fold rotational symmetry around a hard axis.

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1 Introduction

Quantum tunneling in mesoscopic spin systems with large spin has attracted much attention over a decade, because of its fundamental and practical interests[1]. From a fundamental point of view, these systems reveal many interesting quantum mechanical phenomena such as the tunneling of a large spin out of a metastable potential minimum, termed as the macroscopic quantum tunneling, and coherent spin tunneling between classically degenerate potential minima, the macroscopic quantum coherence[2]. A more surprising phenomenon in these systems is the perfect quenching of the tunnel splitting between two degenerate ground states. This phenomenon was predicted theoretically[3,4] and observed in recent experiment[5] in the spin system which has two-fold degenerate classical ground states.

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The quenching of spin tunneling in a spin system is based on the quantum interference of the topological phase, known as Berry phase. More explicitly, consider a biaxial spin system which has two-fold symmetry about a hard axis so that there are two degenerate ground states along each direction of an easy axis. In the semiclassical approach, the topological phase formed by two symmetric instanton paths which connect the two degenerate minima in opposite directions gives rise to a destructive interference, and hence leads to a vanishing tunnel splitting for half-integer spin (i.e., the Kramers degeneracy)\[3\]. When a magnetic field is applied along the hard axis the two instanton paths are still symmetric and wind the hard axis in opposite directions to give the topological phase interference which depends on the applied field. The tunnel splitting in this case oscillates as the field varies, which is responsible for the field dependent quenching effect\[4\].

In this letter we show that the quenching of the spin tunneling can be understood from the viewpoint of Aharonov-Bohm(AB) effect\[6\] when the effective potential for the spin tunneling can be considered as a two-fold symmetric potential on $S^1$. More generally, when a potential is defined on the $S^1$ and has an $N$-fold symmetry, we investigate the effect of the AB-phase on level splittings from the $N$-fold degenerate ground state due to the tunneling. Using the semiclassical methods (i.e., the instanton approach), we develop an AB tunneling amplitude for the tunneling of a particle in the $N$-fold symmetric potential on the $S^1$, from which the level splittings are found to oscillate as a function of the AB-phase. Furthermore, general degeneracy conditions depending on the AB-phase are also found from the oscillating level splittings. By applying these results to the spin tunneling in a spin system with $N$-fold rotational symmetry around the hard anisotropy axis we derive general degeneracy conditions depending on both the magnetic field and spin. When $N = 2$ our results reproduce the same quenching conditions found in references \[3,4\], which confirms that the Aharonov-Bohm effect leads to the quenching of the spin tunneling. In the following Section, starting from a one-dimensional periodic potential, we construct the AB tunneling amplitude in $S^1$. The energy eigenvalues depending on the AB-phase and degeneracy conditions will be derived from this amplitude. In Section 3, we present the application of these results to the spin tunneling problem. Finally, there will be a summary in Section 4.

2 AB tunneling amplitude in $S^1$

Consider a particle in a one-dimensional periodic potential $V(\phi)$ with period $2\pi$ and global minima at $\phi = 0, \pm 2\pi, \pm 4\pi, \cdots$, so that the Hamiltonian is $\hat{H} = \frac{\hat{p}^2}{2M} + V(\phi)$, where $\hat{p} = -i\partial/\partial\phi$ is the momentum operator, and $M$ is the particle mass. The transition amplitude for a tunneling between two adjacent
potential minima can be obtained from Euclidean time formalism. Introducing an Euclidean time $\tau = -it$, the Euclidean amplitude for the tunneling from $\phi = 0$ at $\tau' = -\tau/2$ to $\phi = 2\pi$ at $\tau' = \tau/2$ is expressed as

$$K^{PP}(2\pi, \tau) \equiv \langle 2\pi \mid e^{-\hat{H} \tau} \mid 0 \rangle^{PP} = \int D\phi(\tau') \exp \left[-S_E[\phi(\tau')] \right],$$

where “PP” stands for the periodic potential, and

$$S_E[\phi(\tau')] = \int_{-\tau/2}^{\tau/2} d\tau' \left[ \frac{M}{2} \left( \frac{d\phi}{d\tau'} \right)^2 + V(\phi) \right]$$

is the Euclidean action for the periodic potential $V(\phi)$. Since we are interested in the tunneling between two neighboring ground states the limit $\tau \to \infty$ is assumed. In the semiclassical approximation $K^{PP}(2\pi, \tau)$ can be calculated using the dilute gas approximation and expressed by

$$K^{PP}(2\pi, \tau) = \sum_{n_1, n_2} \delta_{n_1-n_2,1} G_{n_1, n_2}(2\pi, \tau),$$

where $G_{n_1, n_2}(2\pi, \tau)$ is the contribution of $n_1$ instantons and $n_2$ anti-instantons to the Euclidean amplitude. The general form of this is given by [7]

$$G_{n_1, n_2}(2\pi, \tau) = \frac{e^{-(n_1+n_2)S_{cl}}}{n_1! n_2!} (D\tau)^{n_1+n_2} e^{-\frac{\omega\tau}{2}} \sqrt{\frac{\omega}{\pi}},$$

where $S_{cl}$ is the Euclidean classical instanton action, $\omega$ is equal to $\sqrt{d^2 V(\phi)/d\phi^2}$ evaluated at the potential minima $\phi = 0$ or $2\pi$, and $D$ is some real number which arises from the collective coordinate treatment of time translational symmetry [8]. The explicit form of $D$, however, is not necessary for further consideration.

Now, using the identity

$$\delta_{n_1-n_2,N} \equiv \int_{-\pi}^{\pi} \frac{d\vartheta}{2\pi} e^{-i(n_1-n_2-N)\vartheta}$$

with $N = 1$, we can show that

$$K^{PP}(2\pi, \tau) = \int_{-\pi}^{\pi} \frac{d\vartheta}{2\pi} e^{i\vartheta} \sqrt{\frac{\omega}{\pi}} \exp \left[-\left( \frac{\omega}{2} - 2D e^{-S_{cl} \cos \vartheta} \right) \tau \right].$$
Since $\phi$ runs from $-\infty$ to $\infty$ the lowest energy levels are infinitely degenerate when the tunneling is absent. However, when the tunneling exists, these levels will be split and form a continuous band of energy levels. Introducing $\hat{H} \mid \psi_\theta > = E_\theta \mid \psi_\theta >$, where $E_\theta$ is an energy level in the lowest energy band, and $\mid \psi_\theta >$ is the corresponding eigenfunction, the Euclidean transition amplitude for the tunneling, in the limit $\tau \to \infty$, can also be written as

$$K^{PP}(2\pi, \tau) = \int_{-\pi}^{\pi} d\vartheta < 2\pi \mid \psi_\theta < < \psi_\theta \mid 0 > e^{-E_\theta \tau}. \quad (7)$$

Comparing this with Eq. (6) we can see that the spectrum of the lowest energy band is

$$E_\theta = \frac{\omega}{2} - 2De^{S_{cl}} \cos \vartheta, \quad (8)$$

and the wavefunction $\psi_\theta(\phi)$ satisfies the relation

$$\psi_\theta(2\pi) = e^{i\vartheta} \psi_\theta(0). \quad (9)$$

As we can anticipate from the periodic property of the potential $V(\phi)$, Eqs. (8) and (9) represent the well-known band structure and Bloch theorem for an electron in one-dimensional periodic lattice.

If we impose a periodic boundary condition on the wavefunction such that

$$\psi_\theta(2\pi) = \psi_\theta(0) \quad (10)$$

the above potential can be considered as a particle moving on a circle. In this case, since $\phi$ is restricted in the range $0 \leq \phi \leq 2\pi$, the periodic potential $V(\phi)$ is reduced to a 1-fold symmetric potential on the $S^1$ geometry. When the periodic boundary condition (10) holds the phase factor $\vartheta$ in Eq. (9) becomes zero, so that we can insert $\delta(\vartheta)$ into the integral in Eq. (6). The Euclidean amplitude for the tunneling in the $S^1$ geometry is then obtained to be

$$K^{S1}(2\pi, \tau) = \frac{1}{2\pi} \sqrt{\frac{2\omega}{\pi}} \exp \left[ -\left( \frac{\omega}{2} - 2De^{-S_{cl}} \right) \tau \right], \quad (11)$$

where we have changed the superscript from “$PP$” to “$S^1$” to denote the $S^1$ geometry. Notice that, since there are no degenerate levels in the 1-fold symmetric potential, the Euclidean amplitude provides only the lowest energy level $E = \frac{\omega}{2} - 2De^{-S_{cl}}$ instead of giving an energy band.
We now consider the effect of the Aharonov-Bohm phase on the tunneling of a particle in the 1-fold symmetric potential on the $S^1$ geometry. To this end we assume that an external magnetic field $H$ is applied along the axis of the $S^1$. The wavefunction then acquires a phase factor $\Phi$ (i.e., the magnetic flux) per winding due to the AB effect. In this case, the ordinary winding number representation of the Euclidean amplitude can be constructed by inserting $\sum_{m=-\infty}^{\infty} \delta(\vartheta - m\Phi)$ to the integrand in Eq. (6). In the problem of tunneling, however, we need only one phase factor $\Phi$, which means only $m = 1$ sector contributes to the transition amplitude for the tunneling. The Euclidean amplitude for this case, which we call AB tunneling amplitude, is then

$$K_{S^1_{AB}}(2\pi, \tau) = \frac{e^{i\Phi}}{2\pi} \sqrt{\frac{\omega}{\pi}} \exp \left[ - \left( \frac{\omega}{2} - 2De^{-S_{cl}} \cos \Phi \right) \tau \right],$$

(12)

where we have inserted a subscript “AB” to specify the inclusion of the Aharonove-Bohm effect. From this it can be seen that the energy eigenvalue becomes

$$E(\Phi) = \frac{\omega}{2} - 2De^{-S_{cl}} \cos \Phi$$

(13)

which is a periodic function of the flux $\Phi$ with period $\Phi = 2\pi$. This implies that the energy level oscillates by varying the magnetic field (i.e., the AB-phase). For a given value of $\Phi$ the energy level is shifted up by $2De^{-S_{cl}}(1 - \cos \Phi)$, and the shift becomes zero whenever $\Phi = 2n\pi$, where $n$ is a non-negative integer.

Let us generalize the above argument to the case of $N$-fold ($N \geq 2$) symmetric potential $V_N(\phi)$ in the $S^1$ geometry. The corresponding one-dimensional periodic potential has a period of $2\pi/N$ and minima at $\phi = 0, \pm 2\pi/N, \ldots$. In this case, the transition amplitude for the tunneling between adjacent minima is represented by $K_{PP}(2\pi/N, \tau)$, and the contribution of $n_1$ instantons and $n_2$ anti-instantons is $G_{n_1, n_2}(2\pi/N, \tau)$ which has the same form as in Eq. (4). Our task is to find a general AB tunneling amplitude $K_{S^1_{AB}}(2\pi/N, \tau)$. Before we do this, let us first calculate $K_{S^1_{AB}}(2\pi, \tau)$ to see how the instanton and anti-instanton contribute to the AB tunneling amplitude for the $N$-fold symmetric potential. Following the same analysis as in the 1-fold case we can calculate this amplitude within the dilute gas approximation. The only difference from the previous case is to replace $\delta_{n_1-n_2}$ in Eq. (3) by $\delta_{n_1-n_2,N}$. Using the identity in Eq. (5) and inserting $\delta(\vartheta - \Phi/N)$ into the integrand in Eq. (6) we can obtain

$$K_{S^1_{AB}}(2\pi, \tau) = \frac{1}{2\pi} \sum_{n_1, n_2} e^{-i \frac{\Phi}{N}(n_1-n_2-N)} G_{n_1, n_2}(\frac{2\pi}{N}, \tau)$$

$$= \frac{e^{i\Phi}}{2\pi} \sqrt{\frac{\omega}{\pi}} \exp \left[ - \left( \frac{\omega}{2} - 2De^{-S_{cl}} \cos \frac{\Phi}{N} \right) \tau \right].$$

(14)
This shows that, through the tunneling from $\phi = 0$ to $\phi = 2\pi$ in the $N$-fold symmetric potential, the instanton and anti-instanton behave as if they carry phase factors $\frac{\Phi}{N}$ and $-\frac{\Phi}{N}$, respectively, up to overall phase factor in spite of their classical nature.

We are now in position to find the AB tunneling amplitude for the $N$-fold symmetric potential. The observation mentioned above makes it possible to compute this quantity without overall phase factor which is irrelevant for the present problem, especially for the computation of the level splitting due to the tunneling. Since the instanton and anti-instanton carry their own phases, the multi-instanton contribution to the AB tunneling amplitude for the $N$-fold symmetric potential can be described by

$$ K_{\text{AB}}^{S^1}(2\pi N, \tau) = \sum_{n_1} \sum_{n_2} P_{n_1,n_2}^{N} G_{n_1,n_2} \left( \frac{2\pi}{N}, \tau \right) e^{-i \frac{\Phi}{N} (n_1 - n_2)} $$

where $P_{n_1,n_2}^N$ represents all possible sequence of instantons and anti-instantons. For $N \geq 2$ this can be expressed as

$$ P_{n_1,n_2}^N = \sum_{n,m=0}^{\infty} \left[ \delta_{n_1,Nm} \delta_{n_2,Nn+1} + \delta_{n_1,Nm+1} \delta_{n_2,Nn} + \sum_{\eta \geq 2} \delta_{n_1,Nm+\eta} \delta_{n_2,Nn+\eta-1} \right], $$

where the summation over $\eta$ exists only when $N \geq 3$. Using this relation and the expression of $G_{n_1,n_2} (2\pi/N, \tau)$ in Eq. (4) we can write the Eq. (15) as

$$ K_{\text{AB}}^{S^1}(2\pi N, \tau) = \sqrt{\frac{\omega}{\pi}} e^{-\frac{\omega}{2} \tau} \left[ f_{N,0}(x_1) f_{N,N-1}(x_2) + f_{N,1}(x_1) f_{N,0}(x_2) + \sum_{\eta \geq 2} f_{N,\eta}(x_1) f_{N,\eta-1}(x_2) \right], $$

where

$$ x_1 = x_2^* \equiv D e^{-S_{el}} \left( \cos \frac{\Phi}{N} + i \sin \frac{\Phi}{N} \right) \tau, \quad (18) $$

and

$$ f_{N,\sigma}(x) = \sum_{l=0}^{\infty} \frac{x^{Nl+\sigma}}{(Nl+\sigma)!} $$

which satisfies following relations;

$$ f_{N,\sigma}(x) = \frac{d^{N-\sigma} f_{N,0}(x)}{dx^{N-\sigma}}, \quad \frac{d^{N} f_{N,0}(x)}{dx^{N}} = f_{N,0}(x). \quad (20) $$
The boundary conditions for the differential equations are given by

\[ f_{N,0}(0) = 1, \quad f_{N,\sigma \geq 1}(0) = 0. \]  

(21)

Using these boundary conditions we can solve the differential equation and find

\[
f_{N,0}(x) = \begin{cases} 
\frac{2}{N} \left[ \cosh x + \sum_{r=1}^{\frac{N}{2}-1} \cos \left( x \sin \frac{2\pi r}{N} \right) \exp \left( x \cos \frac{2\pi r}{N} \right) \right], & \text{if } N=\text{even}; \\
\frac{1}{N} \left[ e^x + 2 \sum_{r=1}^{\frac{N-1}{2}} \cos \left( x \sin \frac{2\pi r}{N} \right) \exp \left( x \cos \frac{2\pi r}{N} \right) \right], & \text{if } N=\text{odd}.
\end{cases}
\]

(22)

Substituting these solutions into the Eq. (17) and performing the summation over \( \eta \) for each case of \( N \) we obtain

\[ K_{S1}^{S_{\text{AB}}} \left( \frac{2\pi}{N}, \tau \right) = \frac{1}{N} \sqrt{\frac{\omega}{\pi}} \sum_k \exp \left[ -\left( \frac{\omega}{2} - 2De^{-S_{\text{cl}}} \cos \frac{\Phi - 2\pi k}{N} \right) \tau - i\frac{2\pi k}{N} \right]. \]

(23)

Thus, the energy eigenvalues due to the tunneling in the \( N \)-fold symmetric potential on the \( S^1 \)-geometry are

\[ E_{Nk}(\Phi) = \frac{\omega}{2} - 2De^{-S_{\text{cl}}} \cos \frac{\Phi - 2\pi k}{N}, \]  

(24)

where \( k \) is an integer allowed in the range

\[ -\frac{N}{2} < k \leq \frac{N}{2}. \]

(25)

Unlike the 1-fold symmetric potential, there are now \( N \) eigenvalues for a given \( N \), of which all depend on the AB phase \( \Phi \). The reason for the existence of the several eigenvalues is obvious from the fact that the potential \( V_N(\phi) \) has \( N \) minima within \( 2\pi \), and hence there are \( N \)-fold degenerate lowest levels which are split by the tunneling. Note, however, that, since each level oscillates as a function of the AB-phase, the levels may intersect each other to become degenerate states at certain values of \( \Phi \). To see how this happens we consider the difference between any two levels corresponding to \( k \) and \( k' \) \( (k \neq k') \), respectively:

\[ \Delta E_N = E_{Nk}(\Phi) - E_{Nk'}(\Phi) = 4De^{-S_{\text{cl}}} \sin \frac{\Phi - (k + k')\pi}{N} \sin \frac{\pi(k - k')}{N}. \]

(26)
From this and the Eq. (25) we observe following general features for the degeneracy depending on $\Phi$:

\[
\begin{align*}
\frac{N}{2} & \quad \text{double degeneracies at } \Phi = (2n + 1)\pi \\
\frac{N}{2} - 1 & \quad \text{double degeneracies at } \Phi = 2n\pi \\
\frac{N-1}{2} & \quad \text{double degeneracies at } \Phi = n\pi,
\end{align*}
\]

when $N$ is even;

\[
\begin{align*}
\frac{N}{2} & \quad \text{double degeneracies at } \Phi = (2n + 1)\pi \\
\frac{N}{2} - 1 & \quad \text{double degeneracies at } \Phi = 2n\pi \\
\frac{N-1}{2} & \quad \text{double degeneracies at } \Phi = n\pi,
\end{align*}
\]

when $N$ is odd,

where $n$ is a non-negative integer. Note that, in all cases, the degree of the degeneracy is at most double. This implies that a complete suppression of the tunnel splitting (i.e., the quenching) is possible only when $N = 2$. From above conditions, this quenching effect occurs whenever the AB-phase becomes an odd-integer multiple of $\pi$.

### 3 Application to spin tunneling

As an application of above results, let us now consider the spin tunneling in a spin system which has an $N$-fold symmetry about a hard axis. Especially, we will look into the cases of $N = 2, 3, 4, 6$ which can be realized in orthorhombic, trigonal, tetragonal, and hexagonal structures, respectively. We choose $z$ as the hard axis so that $xy$ is the easy plane. We also assume that an external magnetic field is applied along the hard axis. With this choice the spin Hamiltonian which displays the $N$-fold symmetry can be written as

\[
\hat{H}_N = AS_z^2 - C(S_z^N + S_z^{N-2}) - g\mu_B H_z S_z,
\]

where $N = 2, 3, 4, 6$, $g = 2$, $\mu_B$ is the Bohr magneton, and $A > C > 0$. For simplicity, we have neglected the higher order uniaxial anisotropy terms $S_z^4$ and $S_z^6$. To study the spin tunneling within the semiclassical methods we use the spin coherent state path integral approach.

In the spin coherent state representation the anisotropy energy corresponding to the Hamiltonian (28) is given by

\[
\mathcal{E}_N(\theta, \phi) = \langle \Omega | \hat{H}_N | \Omega \rangle = AS^2 \left( \cos^2 \theta - \lambda S^{N-2} \sin^N \theta \cos N\phi - h \cos \theta \right),
\]

where $| \Omega \rangle = | \theta, \phi \rangle$ is the spin coherent state with $\theta, \phi$ being the polar and azimuthal angles, respectively, $\lambda \equiv 2C/A$, $h \equiv g\mu_B H_z/AS$, and $S$ is the spin which is assumed to be large so that the semiclassical methods can be applied.
Since we have chosen the $z$ axis as the hard axis $\lambda S^{N-2} < 1$ should also be assumed. The energy $\mathcal{E}_N(\theta, \phi)$ exhibits $N$-fold degenerate classical minima at $\theta = \theta_0, \phi = 0, 2\pi/N, \ldots, 2(N-1)\pi/N$, where $\theta_0 = \pi/2$ for $h = 0$, and decreases smoothly to 0 as $h$ is increased. Since $\phi$ is defined in the range $0 \leq \phi \leq 2\pi$, the spin tunneling from $(\theta_0, 0)$ to $(\theta_0, 2\pi/N)$ is essentially same as the tunneling in the $N$-fold symmetric potential on the $S^1$ geometry.

To see this explicitly we write the $\text{AB}$ tunneling amplitude for the spin tunneling in the spin coherent state representation:

$$
\mathcal{K}_{\text{AB}}^{S^1} \left( \frac{2\pi}{N}, \tau \right) = < \Omega_f \mid e^{-\hat{H}_{\tau}} \mid \Omega_i >
= \int \mathcal{D}[\phi(\tau')]\mathcal{D}[\cos \theta(\tau')]e^{-S_E[\theta(\tau'), \phi(\tau')]} \tag{30}
$$

where

$$
S_E[\theta(\tau'), \phi(\tau')] = \int_{-\tau/2}^{\tau/2} [\mathcal{E}_N(\theta, \phi) - iS\dot{\phi}(\cos \theta - 1)]d\tau' \tag{31}
$$

is the Euclidean action including the Berry-phase term. Here, the boundary conditions are $| \Omega(-\tau) > = | \Omega_i >, | \Omega(\tau) > = | \Omega_f > = | \theta_0, 2\pi/N >$. The analogy of the spin tunneling with the tunneling in $S^1$ geometry can be revealed by making the action (31) similar to Eq. (2), which can be done by performing the integral over $\cos \theta$ in Eq. (30). Note, however, that the integral is not Gaussian for $N \geq 3$. Since we are interested in the general feature of the effect of the $\text{AB}$ phase on the spin tunneling, we look for an analytical result. To do this, we assume that $\lambda$ and $h$ are small enough so that an approximation

$$
\sin^N \theta \approx 1 - \frac{N}{2} \cos^2 \theta \tag{32}
$$

can hold for $N \geq 3$. By using this approximation and performing the $\theta$ integral we obtain an effective Euclidean action

$$
S_{\text{eff}} = \int_{-\tau/2}^{\tau/2} d\tau \left[ \frac{M_N(\phi)}{2} \dot{\phi}^2 + V_N(\phi) \right] + i \int_{-\tau/2}^{\tau/2} d\tau \mathcal{A}_N(\phi) \dot{\phi}, \tag{33}
$$

where

$$
M_N(\phi) = \frac{1}{2A \left( 1 + \frac{\lambda N\lambda S^{N-2}}{2} \cos N\phi \right)}. 
$$
\[ A_N(\phi) = S \left[ 1 - \frac{h}{2 (1 + \frac{\lambda N S^{N-2}}{2} \cos N\phi)} \right], \]  
(34)

\[ V_N(\phi) = -A S^2 \left[ \lambda S^{N-2} \cos N\phi + \frac{h^2}{4 (1 + \frac{\lambda N S^{N-2}}{2} \cos N\phi)} \right]. \]

Note that the action is complex. Comparing the real part with Eq. (2), \( M_N(\phi) \) can be understood as the position dependent mass, and hence \( V_N(\phi) \) become the potential that has an \( N \)-fold symmetry and minima at \( \phi = 0, 2\pi/N, \cdots, 2(N-1)\pi/2 \) on the \( S^1 \) because \( \phi \) is restricted in the range \( 0 \leq \phi \leq 2\pi \). The imaginary part contributes to the AB tunneling amplitude (30) as a phase factor which plays the same role as the phases carried by the instanton and anti-instanton: For the tunneling from \( \phi = 0 \) to \( \phi = 2\pi/N \), the instanton picks up a phase factor \( \Phi_N = \int_0^{2\pi/N} d\phi A_N(\phi) \), while the anti-instanton acquires a phase \(-\Phi_N\). When these two paths interfere each other and make a closed loop around the hard axis along which the magnetic field is applied, the integral in the imaginary part become the AB-phase. From this notion the AB-phase can be calculated by integrating the imaginary part along the \( S^1 \):

\[ \Phi = \int_0^{2\pi} A_N(\phi) d\phi = 2\pi S \left[ 1 - \frac{h}{\sqrt{4 - \lambda^2 N^2 S^{2(N-2)}}} \right], \]  
(35)

where \( N = 2, 3, 4, 6 \). As we have mentioned below Eq. (14), it can be seen that the phase carried by the instanton \( \Phi_N \) is equivalent to \( \Phi/N \).

Substituting this into Eqs. (24) and (26) we observe several interesting features for the level splittings due to the spin tunneling in the \( N \)-fold symmetric potential. First, we see that the energy levels split from the ground state oscillate as the field varies, and thus intersect each other at certain values of \( h \) as expected. From the degeneracy conditions in Eq. (27), we note that not all levels are simultaneously degenerate in the cases of \( N = 3, 4, 6 \) although there can be more than one double degeneracy for a given value of \( \Phi \). The \( N = 2 \) case, which corresponds to the 2-fold symmetric potential on the \( S^1 \) geometry, is more interesting. In this case we do not need the approximation (32) to calculate the \( \theta \) integral in Eq. (30) and are able to find exact condition for the degeneracy. Using the condition (27) and Eq. (35) with \( N = 2 \) we find that the two ground states are degenerate whenever

\[ h = \frac{2\sqrt{1 - \lambda^2}}{S} \left( S - n - \frac{1}{2} \right) \]  
(36)

which coincides with the previously found result[4] noticing that \( A + 2C = k_1 \) and \( 4C = k_2 \). Thus, the field dependent quenching of the spin tunneling in a 2-
fold symmetric spin system can be understood from the viewpoint of Ahronov-Bohm effect.

Second, since we included the Berry phase term in Eq. (31) the AB-phase in Eq. (35) provides more information about the degeneracy. To see this we consider the field free case, i.e., \( h = 0 \). Then, the AB-phase becomes \( \Phi = 2\pi S \).

Using this and Eq. (26) we find following degeneracy conditions depending on the spin:

\[
\begin{align*}
\frac{N}{2} & \text{ double degeneracies for half-integer } S & \text{ when } N = \text{ even;} \\
\frac{N}{2} - 1 & \text{ double degeneracies for integer } S \\
\frac{N-1}{2} & \text{ double degeneracies for half-integer and integer } S, & \text{ when } N = \text{ odd.}
\end{align*}
\]

Notice that, for \( N \geq 3 \), levels can be doubly degenerate for both half-integer and integer spin and the number of double degeneracies increases as the symmetry increases. When \( N = 2 \), the above results show that there exists one double degeneracy for half-integer spin. This means that, since there are two degenerate ground states in this case, the level splitting due to the spin tunnelling disappears for half-integer spin. Thus, the spin parity effect (i.e., the Kramers degeneracy) discussed in reference [3] can be explained by the argument based on the Aharonov-Bohm effect.

4 Summary

We have studied the Aharonov-Bohm effect on the tunneling in an \( N \)-fold symmetric potential on the \( S^1 \) geometry. Using the semiclassical methods we found that the low-lying levels split from the \( N \)-fold degenerate ground state oscillate as a function of the Aharonov-Bohm phase, from which general degeneracy conditions depending on the phase are also obtained. By applying these observations to the spin tunneling in the spin system which has an \( N \)-fold rotational symmetry around the hard axis we found oscillating tunnel splittings with varying magnetic field applied along the hard axis and degeneracy conditions which are dependent on the spin. From this, the quenching of the spin tunneling in the 2-fold symmetric spin system can be interpreted as the Aharonov-Bohm effect.
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