Abstract

In a recent article we were considering a possibility that inside a proton and, more generally, inside hadrons there could be additional partons - tensor-gluons, which carry a part of the proton momentum. Tensor-gluons have zero electric charge, like gluons, but have a larger spin. Therefore we call them tensor-gluons. The nonzero density of tensor-gluons can be generated by the emission of tensor-gluons by gluons. Tensor-gluons can further split into the pairs of tensor-gluons through a different channels. To describe all these processes one should know the general splitting probabilities for tensor-gluons. These probabilities should fulfill very general symmetry relations, which we were able to resolve by introducing a splitting index. This approach allows to find out the general form of the splitting functions, to derive corresponding DGLAP evolution equations and to calculate the one-loop Callan-Simanzik beta function for tensor-gluons of a given spin. Our results provide a nontrivial consistency check of the theory and of the Callan-Simanzik beta function calculations, because the theory has a unique coupling constant and its high energy behavior should be universal for all particles of the spectrum. We argue that the contribution of all spins into the beta function vanishes leading to a conformal invariant theory at very high energies.
1 Introduction

In the recent article [1] we were considering a possibility that inside a proton and, more generally, inside hadrons there could be additional partons - tensor-gluons, which could carry a part of the proton momentum. Tensor-gluons have zero electric charge, like gluons, but have a larger spin. Therefore we call them tensor-gluons. The nonzero density of tensor-gluons can be generated by the emission of tensor-gluons by gluons [15, 16, 17, 18, 19, 20]. The process of gluon splitting into tensor-gluons suggests that part of the proton momentum which was carried by neutral partons is shared between vector and tensor gluons.

To describe this process one should know the splitting amplitudes of gluon into tensor-gluons. The tree level scattering amplitudes describing a fusion of gluons into tensor-gluons were found in [19]. They are generalizations of the Parke-Taylor scattering amplitude to the case of two tensor gauge bosons of spin $s$ and $(n - 2)$ gluons of spin 1 and allow to extract the splitting amplitudes [20]. With these splitting amplitudes in hand one can derive the generalization of the DGLAP evolution equations [5, 6, 7, 8, 9] for the parton distribution functions, which take into account the processes of emission of tensor-gluons by gluons [1]. In this approach the momentum sum rule allows to find the contribution of tensor-gluons of spin $s$ into the one-loop Callan-Simanzik beta function of spin 1 gluons, which takes the following form [1]:

$$\alpha(t) = \frac{\alpha}{1 + \beta_0 \alpha t}, \quad b_{11} = \frac{\sum_s (12s^2 - 1)C_2(G) - 4n_f T(R)}{12\pi}, \quad s = 1, 2, ...\tag{1.1}$$

where $s$ is the spin of the tensor-gluons running in the loop. The spin-dependent term $12s^2 - 1$ in the Callan-Simanzik beta function coefficient (1.1) makes the asymptotic freedom [2, 3, 4] even stronger and influence the high energy behavior ($t = \ln(Q^2/Q_0^2)$) of the structure functions. It also influences the unification scale at which the coupling constants of the Standard Model merge [35, 36], shifting its value to lower energies [1].

The consistency of the theory requires that the contribution of the tensor-gluons into the one-loop Callan-Simanzik beta function $b_{11}$ for spin 1 gluons should be equal to the contribution of high spin tensor-gluons to the one-loop Callan-Simanzik beta function $b_{rr}$ for tensor-gluons of spin $r = 2, 3, ...$. This nontrivial consistency requirement follows from the fact that the theory has unique coupling constant and its high energy behavior should be universal for all particles of the spectrum.

In order to approach this problem one should know the full set of splitting probabilities $P_{BA}^C$ which describe the decay of tensor-gluon of any spin $A = r$ into two tensor-gluons of arbitrary spins $B$ and $C$. These probabilities were not available [1]. The splitting probabilities $P_{BA}^C$ should fulfill very general symmetry relations [5, 6, 7, 8, 9], which, as we shall demonstrate in this article, allow to find the required

\<\textsuperscript{†}\> This notation for the coefficient $b_{11}$ aims to show that external particles are spin 1 gluons.
functions. The important input information which allows to resolve the symmetry equations are the splitting functions $P_{11}^1$ and $P_{s1}^s$ of spin 1 gluon into the tensor-gluons of spins $s$ which were found earlier in [1, 20].

The present paper is organized as follows. In section two the basic formulae for splitting functions and their symmetry relations are recalled, definitions and notations are specified. In section three we introduce the indexes of the splitting functions and extract them from the known splitting functions $P_{11}^1$ and $P_{s1}^s$. This approach allows to identify the unknown parameters and to find out the general structure of the splitting probabilities $P_{s-r\pm 1 r}^s$ of a tensor-gluon of spin $r$ into two tensor-gluons of spins $s - r \pm 1$ and $s$ (3.37), (3.38). In section four the generalized DGLAP evolution equations that include all tensor-gluon splitting probabilities are derived and the consistency of the one-loop Callan-Simanzik beta function calculation is demonstrated. We argue that the contribution of all spins into the beta function vanishes leading to a conformal invariant theory at very high energies. In section five we summarise the results. The Appendix provides the details of the regularization scheme.

2 Interaction Vertices and Splitting Functions

In the generalized Yang-Mills theory [15, 16, 17, 18] all interaction vertices between high-spin particles have dimensionless coupling constants, which means that the helicities $h_i, i = 1, 2, 3$ of the interacting particles in the vertex are constrained by the relation

$$h_1 + h_2 + h_3 = \pm 1 .$$

(2.2)

Therefore on-mass-shell interaction vertex between massless tensor-gluons, the TTT-vertex, has the following form [19]:

$$M_3(1^{h_1}, 2^{h_2}, 3^{h_3}) = gf^{abc} <1, 2> ^{-2h_1-2h_2-1} <2, 3> ^{2h_1+1} <3, 1> ^{2h_2+1}, \quad h_3 = -1 - h_1 - h_2,$$

$$M_3(1^{h_1}, 2^{h_2}, 3^{h_3}) = gf^{abc}[1, 2]^{2h_1+2h_2-1}[2, 3]^{-2h_1+1}[3, 1]^{-2h_2+1}, \quad h_3 = 1 - h_1 - h_2, (2.3)$$

where $g$ is the YM coupling constant and $f^{abc}$ are the structure constants of the internal gauge group $G$. In particular, considering the interaction between a gluon of spin $A = 1$ (helicities $h_2 = \pm 1$) and a tensor-gluon of spin $C = s$ (helicities $h_1 = \pm s$), the gluon-tensor-tensor GTT-vertex, one can find from (2.3) three solutions for the helicities of the third particle

$$h_3 = \pm |s - 2|, \quad \pm s, \quad \pm |s + 2| ,$$

(2.4)
Thus its spin can take three different values $B = s - 2, s, s + 2$. A simple analysis shows that all possible vertices can be counted from (2.4), if one considers two cases: $B = s - 2, s \geq 3$ and $B = s, s \geq 1$. The corresponding interaction vertices GTT have therefore the following form:

$$M_3(1^{-s}, 2^{++}, 3^{--}) \propto \begin{cases} <1, 3>^4 <1, 2> <2, 3> <3, 1> <1, 2> <2, 3> <2s-2> <3, 1> <1, 2> \end{cases}, \quad s \geq 3$$

$$M_3(1^{+s}, 2^{--}, 3^{--}) \propto \begin{cases} <2, 3>^4 <1, 2> <2, 3> <3, 1> <1, 2> <2s-2> <3, 1> <1, 2> \end{cases}, \quad s \geq 1. \quad (2.5)$$

The next type of interaction vertex involves tensor-gluons of spin $A = 2$ (helicities $h_2 = \pm 2$) and of spin $C = s$ (helicities $h_1 = \pm s$). In this case one can find from (2.3) four solutions:

$$h_3 = \pm s - 3|, \pm s - 1|, \pm s + 1|, \pm s + 3|. \quad (2.6)$$

All possible vertices can be obtained, if one considers only two cases: $B = s - 3, s \geq 5$ and $B = s - 1, s \geq 3$. The corresponding tensor-tensor-tensor interaction vertices $TTT$ have the following form:

$$M_3(1^{-s}, 2^{+2}, 3^{+s-3}) \propto \begin{cases} <3, 1>^4 <1, 2> <2, 3> <3, 1> <1, 2> <2, 3> <2s-2> <3, 1> <1, 2> \end{cases},$$

$$M_3(1^{+s}, 2^{-2}, 3^{-(s-1)}) \propto \begin{cases} <2, 3>^4 <1, 2> <2, 3> <3, 1> <1, 2> <2s-2> <3, 1> <1, 2> \end{cases}. \quad (2.7)$$

In the general case the interaction vertex, which involves the tensor-gluons of spin $A = r$ (helicities $h_2 = \pm r$) and of spin $C = s$ (helicities $h_1 = \pm s$), can be of four types (2.3):

$$h_3 = \pm s - r - 1|, \pm s - r + 1|, \pm s + r - 1|, \pm s + r + 1|. \quad (2.8)$$

In order to count all possible types of vertices it is sufficient to consider two cases of spin: $B = s - r - 1, s \geq 2r + 1$ and $B = s - r + 1, s \geq 2r - 1$, while $r = 1, 2, 3,...$

$$M_3(1^{-s}, 2^{+r}, 3^{+s-r-1}) \propto \begin{cases} <3, 1>^4 <1, 2> <2, 3> <3, 1> <1, 2> <2, 3> <2s-2> <3, 1> <1, 2> \end{cases},$$

$$M_3(1^{+s}, 2^{-r}, 3^{-(s-r+1)}) \propto \begin{cases} <2, 3>^4 <1, 2> <2, 3> <3, 1> <1, 2> <2s-2> <3, 1> <1, 2> \end{cases}. \quad (2.9)$$

At $s = 1$ and $r = 1$ they all reduce to the YM vertex [1].

Using these vertices one can compute the scattering amplitudes involving tensor-gluons [19]. They reduce to the famous Parke-Taylor formula [22] when $s = 1$ and can be used to extract splitting amplitudes of a gluon into two tensor-gluons [20]. Considering the amplitudes in the limit when two neighboring particles become collinear, $k_B \parallel k_C$, that is, $k_B = z k_A$, $k_C = (1 - z) k_A$, $k_A^2 \to 0$ and $z$ describes the longitudinal momentum sharing with the corresponding behavior of spinors $\lambda_B = \sqrt{z} \lambda_A$, $\lambda_C = \sqrt{1 - z} \lambda_A$, one can deduce that the amplitude takes the factorization form [21, 22, 28, 29, 20]:

$$M_n^{tree}(\ldots, B^{h_B}, C^{h_C}, \ldots) \xrightarrow{B \parallel C} \sum_{h_A} SPLIT^{tree}_{h_A}(B^{h_B}, C^{h_C}) \times M^{tree}_{n-1}(\ldots, A^{h_A}, \ldots), \quad (2.10)$$
Figure 1: The decay of the tensor-gluon of spin A into tensor-gluons of spins B and C. The arrows show the directions of the helicities. The corresponding splitting function is defined as $P_{B \rightarrow A}^{C^-}$.

where $\text{Split}_{h_A}^{-}(B^{h_B}, C^{h_C})$ denotes the splitting amplitude $A \rightarrow B + C$ and the intermediate state A has momentum $k_A = k_B + k_C$ and helicity $h_A$. Since the collinear limits of the scattering amplitudes are responsible for parton evolution \cite{5} one can extract from (2.10) the Altarelli-Parisi splitting probabilities for tensor-gluons. Indeed, the residue of the collinear pole in the square (of the factorized amplitude (2.10)) gives Altarelli-Parisi splitting probability $P(z)$ \cite{21, 22, 28, 29, 20}:

$$P_{BA}^{C}(z) = C^2_G \sum_{h_A, h_B, h_C} \left| \text{Split}_{h_A}^{-}(B^{h_B}, C^{h_C}) \right|^2 s_{BC},$$

where $s_{BC} = 2k_B \cdot k_C = \langle B, C \rangle$. The invariant operator $C_2$ for the representation R is defined by the equation $t^a t^a = C_2(R)$ \cite{1} and $tr(t^a t^b) = T(R) \delta^{ab}$. The same splitting probabilities can be extracted directly by considering of-mass-shell decay of the particle A. It describes the probability of finding a particle B inside a particle A with fraction $z$ of the longitudinal momentum of A and radiation of the third particle C with fraction $(1 - z)$ of the longitudinal momentum of A \cite{5}

$$P_{BA}^{C}(z) = \frac{1}{2} z(1 - z) \sum_{\text{helicities}} \frac{|M_{A \rightarrow B+C}|^2}{p_{\perp}^2},$$

where a sum is over the helicities of B and C and an average over the helicity of A.

It was found that the full set of splitting amplitudes describing the decay of gluons and tensor-gluons is \cite{20}:

$$\text{Split}_+(B^{+s}, C^{-s}) = \frac{(1 - z)^{s+1/2}}{z^{s-1/2}} \frac{1}{\langle B, C \rangle}, \quad \text{Split}_+(B^{-s}, C^{+s}) = \frac{z^{s+1/2}}{(1 - z)^{s-1/2}} \frac{1}{\langle B, C \rangle},$$

$$\text{Split}_+(B^{+s}, C^{-s}) = \frac{(1 - z)^{s+1}}{\sqrt{z(1 - z)}} \frac{1}{\langle B, C \rangle}, \quad \text{Split}_+(B^{-s}, C^{+s}) = \frac{z^{s+1}}{\sqrt{z(1 - z)}} \frac{1}{\langle B, C \rangle},$$

$$\text{Split}_-(B^{+s}, C^{+s}) = \frac{z^{-s+1}}{\sqrt{z(1 - z)}} \frac{1}{\langle B, C \rangle}, \quad \text{Split}_-(B^{-s}, C^{+s}) = \frac{(1 - z)^{-s+1}}{\sqrt{z(1 - z)}} \frac{1}{\langle B, C \rangle}.$$

This set of splitting amplitudes (2.13) $G \rightarrow TT$, $T \rightarrow GT$ and $T \rightarrow TG$ reduces to the full set of gluon splitting amplitudes when $s = 1$. Substituting the splitting amplitudes (2.13) into (2.11) we are getting
\[ P_{TG}(z) = C_2(G) \left[ \frac{z^{2s+1}}{(1-z)^{2s-1}} + \frac{(1-z)^{2s+1}}{z^{2s-1}} \right], \]
\[ P_{GT}(z) = C_2(G) \left[ \frac{1}{z(1-z)^{2s-1}} + \frac{(1-z)^{2s+1}}{z} \right], \]
\[ P_{TT}(z) = C_2(G) \left[ \frac{1}{(1-z)z^{2s-1}} + \frac{z^{2s+1}}{(1-z)} \right]. \]  

(2.14)

In the leading order the kernel \( P_{TG}(z) \) has a meaning of variation of the probability density of finding a tensor-gluon inside the gluon, \( P_{GT}(z) \) - of finding gluon inside the tensor-gluon and \( P_{TT}(z) \) - of finding tensor-gluon inside the tensor-gluon.

The important properties of the splitting functions are the symmetries \([5, 6, 7, 8, 9]\) over exchange of the particles \( B \leftrightarrow C \) (see Fig. 1) with complementary momenta fraction

\[ P_{BA}(z) = P_{CA}(1-z) \]  

(2.15)

and a crossing relation

\[ P_{AB}(z) = (-1)^{2h_a+2h_B+1}zP_{BA}(\frac{1}{z}) \], \n
(2.16)

which emerge because two splitting processes are connected by time reversal \( A \leftrightarrow B \). \textit{We shall postulate their validity for interacting particles of any spins.} It is easy to see that these relations fulfill in the case of higher spins \([2.14]\):

\[ P_{TG}(z) = P_{TG}(1-z), \quad P_{GT}(z) = P_{TT}(1-z), \quad 0 < z < 1, \]
\[ P_{TG}(z) = -zP_{GT}(\frac{1}{z}), \quad P_{TT}(z) = -zP_{TT}(\frac{1}{z}). \]  

(2.17)

For completeness we shall present also quark and gluon splitting probabilities \([5]\):

\[ p_{qq}(z) = C_2(R) \frac{1+z^2}{1-z}, \]
\[ p_{Gq}(z) = C_2(R) \left[ \frac{1}{z} + \frac{(1-z)^2}{z} \right], \]
\[ p_{qG}(z) = T(R) [z^2 + (1-z)^2], \]
\[ p_{GG}(z) = C_2(G) \left[ \frac{1}{z(1-z)} + \frac{z^4}{z(1-z)} + \frac{(1-z)^4}{z(1-z)} \right], \]

where \( C_2(G) = N, C_2(R) = \frac{N^2-1}{2N}, T(R) = \frac{1}{N} \) for the SU(N) groups.

Let us consider the splitting probabilities \([2.14]\) in the limit of the half-integer spin \( s \rightarrow 1/2 \). One can see that they reduce to the quark-gluon splitting probabilities \([2.18]\)

\[ p_{TT}(z) \rightarrow p_{qq}(z), \quad p_{GT}(z) \rightarrow p_{Gq}(z), \quad p_{TG}(z) \rightarrow p_{qG}(z) \]  

(2.19)
and that in the limit \( s \to 1 \) to the gluon-gluon splitting probability

\[
\frac{1}{2}(P_{TG}(z) + P_{GT}(z) + P_{TT}(z)) \to P_{GG}(z).
\] (2.20)

The set of splitting probabilities \((2.14)\) and \((2.18)\) provides an important data for the extension of the results to more general cases of transitions between high spin tensor, which we shall consider in the next section.

### 3 General Structure of Splitting Probabilities

As one can observe from the previous examples the splitting probabilities are polynomial functions of a simple form:

\[
P_{BA}^C \propto z^n(1 - z)^m.
\] (3.21)

Using the crossing symmetry relation \((2.16)\) for the interchange \( A \leftrightarrow B \) and the fact that we are considering integer spins

\[
P_{AB}^C(z) = (-1)^{2h_A + 2h_B + 1} z P_{BA}^C(\frac{1}{z}) = -z P_{BA}^C(\frac{1}{z}),
\] (3.22)

one can find

\[
P_{AB}^C(z) \propto -z \frac{1}{z^n}(1 - \frac{1}{z})^m = \frac{(z - 1)^m}{z^{n+m-1}},
\] (3.23)

and because the probabilities should be positive it follows that the integer \( m \) should be odd, \( m \to 2m + 1: \)

\[
P_{BA}^C \propto z^n(1 - z)^{2m+1}, \quad P_{AB}^C \propto \frac{(1 - z)^{2m+1}}{z^{2m+n}}
\] (3.24)

The interchange \((2.15)\) of particles \( A \leftrightarrow C \) and then of the \( C \leftrightarrow B \) gives

\[
P_{CB}^A \propto \frac{z^{2m+1}}{(1 - z)^{2m+n}} \Rightarrow P_{BC}^A \propto -z \frac{z^{2m+n}}{(z - 1)^{2m+n}} = -\frac{z^n}{(z - 1)^{2m+n}}.
\] (3.25)

It follows then that the integer \( n \) also should be odd, \( n \to 2n + 1 \), thus the splitting probabilities should have the following general form:

\[
P_{BA}^C \propto z^{2n+1}(1 - z)^{2m+1}.
\] (3.26)

The full set of splitting probabilities can be found by using symmetries \((2.15), (2.19)\) and has the following form:

\[
\begin{align*}
P_{BA}^C &\propto z^{2n+1}(1 - z)^{2m+1}, & P_{CB}^A &\propto \frac{(1 - z)^{2m+1}}{z^{2n+2m+1}}, \\
P_{AB}^C &\propto \frac{(1 - z)^{2m+1}}{z^{2m+2n+1}}, & P_{BC}^A &\propto \frac{(1 - z)^{2n+1}}{z^{2n+2m+1}}, \\
P_{CB}^A &\propto \frac{z^{2m+1}}{(1 - z)^{2n+2m+1}}, & P_{AC}^B &\propto \frac{z^{2n+1}}{(1 - z)^{2n+2m+1}}.
\end{align*}
\] (3.27)
Figure 2: The helicity diagrams corresponding to the splitting functions $P^{C\rightarrow\rightarrow}_{B_{\rightarrow\rightarrow}}$ represent the decay of a parton of spin $A$ into the partons of spin $B$ and $C$ in (3.30). The short arrows display spin 1 partons, the long arrows display spin s partons.

In order to identify the above parameters $n$ and $m$ with the helicity content of the interacting particles let us consider the analytical continuation of these functions to the full complex plane $C^2$ and take the limit $|z| \rightarrow \infty$. We shall define the ratio

$$\nu = \lim_{|z| \rightarrow \infty} \frac{\ln P}{\ln |z|}$$

as the index of a given splitting function $P$, thus from (3.27) we have

$$\nu = \lim_{|z| \rightarrow \infty} \left\{ \begin{array}{ll}
\frac{\ln P_A^C}{\ln |z|} & \propto +2n + 2m + 2,
\frac{\ln P_A^B}{\ln |z|} & \propto +2n + 2m + 2,
\frac{\ln P_A}{\ln |z|} & \propto -2n,
\frac{\ln P_A^A}{\ln |z|} & \propto -2n,
\frac{\ln P_A^A}{\ln |z|} & \propto -2m,
\end{array} \right.$$ (3.29)

Let us now compare these indexes with the indexes of the known GTT polarized splitting functions (2.13), (2.14):

$$P_{s^{-1}+}^s(z) = C_2(G) \frac{z^{2s+1}}{(1-z)^{2s-1}}, \quad P_{s^{-1}+}^s(z) = C_2(G) \frac{(1-z)^{2s+1}}{z^{2s-1}},$$

$$P_{s^{-1}+}^s(z) = C_2(G) \frac{1}{z(1-z)^{2s-1}}, \quad P_{s^{-1}+}^s(z) = C_2(G) \frac{(1-z)^{2s+1}}{z},$$

$$P_{s^{-1}+}^s(z) = C_2(G) \frac{1}{z(1-z)^{2s-1}}, \quad P_{s^{-1}+}^s(z) = C_2(G) \frac{z^{2s+1}}{(1-z)};$$

where $s \geq 1$. We shall get the following identification of the parameters $n$ and $m$ in the general expression (3.27):

$$2n + 2m + 2 = 2, \quad -2n = -2s, \quad -2m = 2s,$$

and the parameters corresponding to all possible polarizations of the interacting particles therefore are

$$A = 1, \quad B = s, \quad C = s \quad \Rightarrow \quad n = +s, \quad m = -s.$$ (3.31)
The splitting probabilities are:

\[ P_{(s-1)\rightarrow 2}^+ (z) = C_2(G) \frac{z^{2s-1}}{(1-z)^{2s-3}} , \quad P_{s+2}^{-} (z) = C_2(G) \frac{(1-z)^{2s-1}}{2s-3} , \]

\[ P_{(s-1)\rightarrow s}^+ (z) = C_2(G) \frac{1}{z(1-z)^{2s-3}} , \quad P_{2s}^{+(s-1)} (z) = C_2(G) \frac{(1-z)^{2s-1}}{z} , \quad P_{s+1}^{+(s-1)} (z) = C_2(G) \frac{z^{2s-1}}{1-z} , \]

where \( s \geq 3 \). In the second basic case of spin \( B = s - 3 \) given in (2.6),

\[ A = 2, \quad B = s - 3, \quad C = s \quad \Rightarrow \quad n = (s - 1), \quad m = -(s - 1) \]

and

\[ P_{(s-3)\rightarrow 2}^+ (z) = C_2(G) \frac{z^{2s-5}}{(1-z)^{2s-7}} , \quad P_{s+2}^{-} (z) = C_2(G) \frac{(1-z)^{2s-5}}{2s-7} , \]

\[ P_{(s-3)\rightarrow s}^+ (z) = C_2(G) \frac{1}{z(1-z)^{2s-3}} , \quad P_{2s}^{+(s-3)} (z) = C_2(G) \frac{(1-z)^{2s-5}}{z} , \quad P_{s+1}^{+(s-3)} (z) = C_2(G) \frac{z^{2s-5}}{1-z} , \]

where \( s \geq 5 \). For general spins \( A = r \) and \( C = s \) the third spin takes the values (2.8)

\[ B = s - r + 1, \quad s \geq 2r - 1, \]

\[ B = s - r - 1, \quad s \geq 2r + 1, \quad r = 1, 2, 3, \ldots \]

In these cases we shall get that

\[ A = r, \quad B = s - r + 1, \quad C = s \quad \Rightarrow \quad n = (s - r + 1), \quad m = -(s - r + 1), \]
and the splitting probabilities are

\[ P_{s-r+1}^+ - r^+(z) = C_2(G) \frac{z^{2s-2r+3}}{(1-z)^{2s-2r+1}}, \quad P_{s+r+1}^-(z) = C_2(G) \frac{1-z^{2s-2r+3}}{z^{2s-2r+1}}, \]

\[ P_{s-r+1}^+ + (z) = C_2(G) \frac{1}{z(1-z)^{2s-2r+1}}, \quad P_{s+r+1}^+(z) = C_2(G) \frac{(1-z)^{2s-2r+3}}{z}, \]

\[ P_{s-r+1}^- + (z) = C_2(G) \frac{1}{(1-z)^{2s-2r+1}}, \quad P_{s+r+1}^-(z) = C_2(G) \frac{z^{2s-2r+3}}{(1-z)}, \]

where \( s \geq 2r - 1 \) and finally for

\[ A = r, \quad B = s - r - 1, \quad C = s \quad \Rightarrow \quad n = (s - r - 1), \quad m = -(s - r - 1) \]

we get

\[ P_{s-r+1}^+ - r^+(z) = C_2(G) \frac{z^{2s-2r-1}}{(1-z)^{2s-2r-3}}, \quad P_{s+r+1}^-(z) = C_2(G) \frac{1-z^{2s-2r-1}}{z^{2s-2r-3}}, \]

\[ P_{s-r+1}^+ + (z) = C_2(G) \frac{1}{z^{2s-2r-3}}, \quad P_{s+r+1}^+(z) = C_2(G) \frac{(1-z)^{2s-2r-1}}{z}, \]

\[ P_{s-r+1}^- + (z) = C_2(G) \frac{1}{(1-z)^{2s-2r-3}}, \quad P_{s+r+1}^-(z) = C_2(G) \frac{z^{2s-2r-1}}{(1-z)}, \]

where \( s \geq 2r + 1 \). These general expressions (3.37), (3.38) describe all possible splitting probabilities corresponding to the interaction vertices with one space-time derivative and therefore to the dimensionless coupling constant, as it is in the generalized YM theory [11, 15].

4 Generalization of DGLAP Equation

Having in hand the new set of splitting probabilities for tensor-gluons (3.37), (3.38), one can calculate a possible emission of tensor-gluons which appears in addition to the quark-anti-quark and gluon "clouds' in a proton. Our goal is to derive DGLAP equations [5, 7, 8, 9, 10, 11, 12, 6] which will take into account these new emission processes.

In accordance with our hypothesis there is an additional emission of tensor-gluons in a proton, therefore one should introduce the corresponding densities \( T_i(x, t) \) of tensor-gluons (summed over colors) inside a proton in the \( P_\infty \) frame [11]. We can derive therefore the integro-differential equations that describe the \( Q^2 \) dependence of parton densities in this general case. They are:

\[ \frac{dG(x, t)}{dt} = \frac{\alpha(t)}{2\pi} \int_1 \frac{dy}{y} \sum_{j=1}^{2n_y} q^j(y, t) P_{qG}^j(x, y) + G(y, t) P_{G}G(x, y), \]

\[ \frac{dT_i(x, t)}{dt} = \frac{\alpha(t)}{2\pi} \int_1 \frac{dy}{y} \sum_{j=1}^{2n_y} q^j(y, t) P_{Gq}^j(x, y) + G(y, t) P_{G}G(x, y) + \sum_s T_s(y, t) P_{GT_i}^s(x, y), \]

\[ \frac{dq^i(x, t)}{dt} = \frac{\alpha(t)}{2\pi} \int_1 \frac{dy}{y} \sum_{j=1}^{2n_y} q^j(y, t) P_{qG}^j(x, y) + G(y, t) P_{q}q^i(x, y) + \sum_s T_s(y, t) P_{T_iq}^s(x, y). \]
The $\alpha(t)$ is the running coupling constant ($\alpha = g^2/4\pi$). In the leading logarithmic approximation $\alpha(t)$ is of the form

$$\frac{\alpha}{\alpha(t)} = 1 + b \, \alpha \, t \, ,$$

(4.40)

where $\alpha = \alpha(0)$ and $b$ is the one-loop Callan-Simanzik coefficient, which receives an additional contribution from the tensor-gluons running in the loop. Here the indices $i$ and $j$ run over quarks and antiquarks of all flavors and $s$ and $r$ run over all spins of the tensor-gluons. The number of quarks of a given fraction of momentum changes when a quark looses momentum by radiating a gluon or a gluon inside the proton produces a quark-antiquark pair [5]. Similarly the number of gluons changes because a quark may radiate a gluon or because a gluon may split into a quark-antiquark pair or into two gluons or into two tensor-gluons. The density of tensor-gluons also changes because there are transitions between them through the splittings which are described by the probabilities (3.37) and (3.38).

In order to guarantee that the total momentum of the proton, that is, of all partons is unchanged, one should impose the following constraint:

$$\frac{d}{dt} \int_0^1 dz z \left[ \sum_{i=1}^{2n_f} q_i(z,t) + G(z,t) + \sum_s T_s(z,t) \right] = 0 .$$

(4.41)

Using the evolution equations (4.39) one can express the derivatives of the densities in (4.41) in terms of splitting probabilities $P_C$ (kernels of the evolution equations) and derive that the following momentum sum rules should be fulfilled:

$$\int_0^1 dz z [P_{qq}(z) + P_{Gq}(z)] = 0 ,$$

$$\int_0^1 dz z [2n_f P_{qG}(z) + P_{GG}(z) + \sum_s P_{T_s G}(z)] = 0 ,$$

$$\int_0^1 dz z [P_{GT}(z) + \sum_s P_{T_s T_r}(z)] = 0 .$$

(4.42)

Before analyzing these momentum sum rules let us first inspect the behavior of the tensor-gluon kernels (3.30), (3.37), (3.38) at the end points $z = 0, 1$. As one can see, they are singular at the boundary values similarly to the case of the standard kernels (2.18). Though there is a difference here: the singularities are of higher order compared to the standard case [5]. Therefore one should define the regularization procedure for the singular factors $(1 - z)^{-2s+1}$ and $z^{-2s+1}$ reinterpreting them as the distributions $(1 - z)^{-2s+1}$ and $z^{-2s+1}$ similarly to the Altarelli-Parisi regularization. The details are given in the Appendix.

The first equation in the momentum sum rule (4.42) remains unchanged because there is no tensor-gluon contribution into the quark evolution. The second equation in the momentum sum rule (4.42) will
take the following form (see [1] and Appendix for details):

\[
\int_0^1 dz z [2n_f P_G(z) + P_{GG}(z) + \sum_s P_{T,s}G(z) + b_G \delta(z - 1)] = \\
= \int_0^1 dz z [2n_f T(R)z^2 + (1 - z)^2] + C_2(G) \left[ \frac{1}{z(1 - z)} + \frac{z^4}{(1 - z)z(1 - z)} + \frac{(1 - z)^4}{z(1 - z)z(1 - z)} \right] + C_2(G) \sum_s \left[ \frac{z^{2s+1}}{(1 - z)^{2s+1}} + \frac{(1 - z)^{2s+1}}{z^{2s-1}} \right] + b_{11} = \\
= \frac{2}{3} n_f T(R) - \frac{11}{6} C_2(G) - \sum_s \frac{12s^2 - 1}{6} C_2(G) + b_{11} = 0. \tag{4.43}
\]

We can extract now the additional contribution to the one-loop Callan-Symanzik beta function for gluons

\[b_{11}\text{ arising from the tensor-gluon loop of spin } s [1]:\]

\[b_{11} = \sum_s \frac{12s^2 - 1}{6} C_2(G), \quad s = 1, 2, 3, 4, \ldots. \tag{4.44}\]

At s=1 we are rediscovering the asymptotic freedom result [2, 3, 4]. For larger spins the tensor-gluon contribution into the Callan-Simanzik beta function has the same signature as the standard gluons, which means that tensor-gluons "accelerate" the asymptotic freedom (4.40) of the strong interaction coupling constant \(\alpha(t)\). At large transfer momentum the strong coupling constant tends to zero faster compared to the standard case [1].

One can confirm the above result by using effective action approach developed in the Yang-Mills theory [30, 31, 32, 33] extended to the higher spin gauge bosons. With the spectrum of the tensor-gluons in the external chromomagnetic field \(k_0^2 = (2n + 1 + 2s)g_H^2 + k_1^2\) one can perform a summation of the modes and get an exact result for the one-loop effective action similar to [30, 33]:

\[\epsilon = \frac{H^2}{2} + \frac{(g_H)^2}{4\pi} b_{11} \left[ \ln \frac{g_H}{\mu^2} - \frac{1}{2} \right], \tag{4.45}\]

where

\[b_{11} = -\frac{2C_2(G)}{\pi} \zeta(-1, 2s + 1, 2) = \frac{12s^2 - 1}{12\pi} C_2(G), \tag{4.46}\]

and \(\zeta(-1, q) = -\frac{1}{2}(q^2 - q + \frac{1}{6})\) is the generalized zeta function\(\|\). Because the coefficient in front of the logarithm defines the beta function [30, 31], one can see that (4.46) is in agreement with the result (4.44).

The third equation in the momentum sum rule (4.42) will take the following form:

\[\int_0^1 dz z [P_{G,T_r}(z) + \sum_s P_{T_r,T_s}(z) + b_r \delta(z - 1)] = 0, \tag{4.47}\]

\(\text{In (4.44) one should take into account the multiplicity of higher spin states in the theory under consideration.}\)

\(\|\text{The generalized zeta function is defined as } \zeta(p, q) = \sum_{k=0}^{\infty} \frac{1}{(k+q)^p} = \frac{1}{\Gamma(p)} \int_0^\infty dt t^{p-1} e^{-t} \left( \frac{-t}{1-e^{-t}} \right)^q.\)
and we shall get

$$C_2(G) \int_0^1 dz \sum_s \left[ \frac{z^{2s-2r+3}}{(1-z)^{2s-2r+1}} + \frac{(1-z)^{2s-2r+3}}{z^{2s-2r+1}} \right] + b_{rr} =$$

$$= - \sum_s \frac{12(s-r+1)^2 - 1}{6} C_2(G) + b_{rr} = 0. \quad (4.48)$$

We can extract the one-loop coefficient of the Callan-Symanzik beta function now for tensor-gluon of spin

\( r \), which has the form

$$b_{rr} = \sum_s \frac{12(s-r+1)^2 - 1}{6} C_2(G), \quad r = 1, 2, 3, 4, \ldots, \quad (4.49)$$

and it has the identical, quadratic, dependence on the particle spins running in the loop, as it is for gluons

(4.44). The consistency relation now will take the following form:

$$b_{11} = \sum_s \frac{12s^2 - 1}{6} C_2(G) = \sum_s \frac{12(s'-r+1)^2 - 1}{6} C_2(G) = b_{rr}. \quad (4.50)$$

In order to perform the summation over spins in the above equation one should take into account the

multiplicity of higher spin states in the theory under consideration. In the generalized YM theory the

values of the tensor-gluon spins run to infinity and the multiplicity is given in \[18\]:

$$\pm (s-1), \quad \pm (s-3), \ldots$$

$$\pm (s+1), \quad (4.51)$$

$$\pm (s-1), \quad \pm (s-3), \ldots$$

Therefore the sums on the both sides of the equation (4.50) are diverging. One can suggest two scenarios

[1]. In the first one the high spin gluons, let us say of \( s \geq 3 \), will get large mass and therefore can be ignored

at a given energy scale. In the second case, when all of them are massless or what is the same the energy

is much larges than all the masses of the tensor-gluons, then one can suggest the Reimann zeta function

regularization, similar to the Brink-Nielsen regularization [34]. The spectrum (4.51) is represented in the

following way:

$$\pm 1$$

$$\pm 2, \quad 0$$

$$\pm 3, \quad \pm 1, \quad \pm 1$$

$$\pm 4, \quad \pm 2, \quad \pm 2, \quad 0$$

$$\pm 5, \quad \pm 3, \quad \pm 3, \quad \pm 1, \quad \pm 1$$

$$\pm 6, \quad \pm 4, \quad \pm 4, \quad \pm 2, \quad \pm 2, \quad 0$$

............................................................................. \quad (4.52)
and the summation over columns gives:

\[
b_{11} = C_2(G)[\sum_{s=1}^{\infty} \frac{(12s^2 - 1)}{12\pi} + \sum_{s=0}^{\infty} \frac{(12s^2 - 1)}{12\pi} + \sum_{s=1}^{\infty} \frac{(12s^2 - 1)}{12\pi} + \ldots] = 4.53 = C_2(G)[\pi \zeta(-2) - \frac{1}{12\pi} \zeta(0) - \frac{1}{12\pi} \zeta(-2) - \frac{1}{12\pi} \zeta(0) + \ldots] = C_2(G)\left[\frac{1}{24\pi} - \frac{1}{12\pi} + \frac{1}{24\pi} + \ldots\right] = 0,
\]

where \(\zeta(-2) = 0\), \(\zeta(0) = -1/2\), leading to the theory which is \textit{conformally invariant} at very high energies. The above summation requires explicit regularisation and further justification.

## 5 Conclusion

In this article we continue to analyze a possibility that inside a proton and, more generally, inside hadrons there are additional partons - tensor-gluons, which can carry a part of the proton momentum [1]. A nonzero density of the tensor-gluons can be generated by the emission of tensor-gluons by gluons. The tensor-gluons themselves further split into the pairs of tensor-gluons through different channels. Therefore the density of neutral partons in a proton is given by the sum:

\[
G(x, t) + \sum_s T_s(x, t),
\]

where \(T_s(x, t)\) is the density of the tensor-gluons of spin \(s\). To describe these processes between gluons and tensor-gluons one should know the general splitting probabilities for tensor-gluons \(P_{BA}^C\). These probabilities fulfill very general symmetry relations, which we were able to resolve by introducing a splitting index (3.28). This approach allows to find out the general form of the splitting functions (3.37), (3.38), to derive corresponding DGLAP evolution equations (4.39) and to calculate the one-loop Callan-Simanzik beta function for tensor-gluons of different spins (4.49). Our results provide a nontrivial consistency check of the theory and of the Callan-Simanzik beta function calculations, because the theory has a unique coupling constant and its high energy behavior should be universal for all particles of the spectrum. We argue that the contribution of all spins into the beta function vanishes leading to a conformal invariant theory at very high energies.

This work was supported in part by the General Secretariat for Research and Technology of Greece and from the European Regional Development Fund MIS-448332-ORASY (NSRF 2007-13 ACTION, KRIKIS).

## 6 Appendix

The regularisation is defined in the following way [1]:

\[
\int_0^1 dz \frac{f(z)}{(1 - z)^{2s - 1}} = \int_0^1 dz \frac{f(z) - \sum_{k=0}^{2s-2} \frac{(-1)^k}{k!} f^{(k)}(1)(1 - z)^k}{(1 - z)^{2s-1}},
\]
\[
\int_0^1 dz \frac{f(z)}{z^{2s+1}} = \int_0^1 dz \frac{f(z) - \sum_{k=0}^{2s-2} \frac{1}{k!} f^{(k)}(0) z^k}{z^{2s+1}}, \quad (6.54)
\]

\[
\int_0^1 dz \frac{f(z)}{z(1-z)^s} = \int_0^1 dz \frac{f(z) - (1-z)f(0) - zf(1)}{z(1-z)}.
\]

where \(f(z)\) is any test function which is sufficiently regular at the end points and, as one can see, the defined substraction guarantees the convergence of the integrals. Using the same arguments as in the standard case \([5]\) we should add the delta function terms into the definition of the kernels.

The momentum sum rule integrals \((4.42), (4.43)\) and \((4.47)\) can be calculated in two different ways: with direct regularization of the integrals near \(z = 1\) and near \(z = 0\), as it is defined in \((6.54)\), or by using the substitution \(w = 1 - z\) in the second term of the integrand and then regularizing the resulting integrand near \(z = 1\). As one can get convinced both methods give identical results. Therefore the integral over \(P_{TG}(z)\) in \((4.43)\) gives:

\[
\int_0^1 dz \left[ \frac{z^{2s+2}}{(1-z)^{2s-1} + \frac{1}{2s-2}} \right] = \int_0^1 dz \left[ \sum_{k=2s-1}^{2s+2} \frac{(-1)^k (2s+2)!}{k!(2s+2-k)!} \frac{(1-z)^k}{(1-z)^{2s+1}} + \sum_{k=2s-2}^{2s+1} \frac{(-1)^k (2s+1)!}{k!(2s+1-k)!} \frac{z^k}{z^{2s-2}} \right] = -\frac{12s^2-1}{6}.
\] (6.55)

The same integral can be calculated by the substitution \(w = 1 - z\) in the second term of the integrand

\[
\int_0^1 dz \frac{z^{2s+1}}{(1-z)^{2s-1}} = \int_0^1 dz \sum_{k=2s-1}^{2s+1} \left[ \frac{(-1)^k (2s+1)!}{k!(2s+1-k)!} \frac{(1-z)^k}{(1-z)^{2s-1}} \right] = -\frac{12s^2-1}{6},
\]

which is identical to the previous result.

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