Inflationary Reheating in Grand Unified Theories

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Grand unified theories may display multiply interacting fields with strong coupling dynamics. This poses two new problems: (1) what is the nature of chaotic reheating after inflation, and (2) how is reheating sensitive to the mass spectrum of these theories? We answer these questions in two interesting limiting cases and demonstrate an increased efficiency of reheating which strongly enhances non-thermal topological defect formation, including monopoles and domain walls. Nevertheless, the large fluctuations may resolve this monopole problem via a modified Dvali-Liu-Vachaspati mechanism in which non-thermal destabilisation of discrete symmetries occurs at reheating.

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An ideal inflationary scenario should arise naturally from within supergravity or a Grand Unified Theory (GUT) without fine-tuning. It should solve the plethora of problems of standard cosmology while simultaneously diluting the monopoles inevitably produced due to the homotopic content $\pi_2(G/U(1)) \simeq \pi_1(U(1)) \simeq \mathbb{Z}$ of the standard model. Significant progress has been made within GUT’s and supersymmetric theories towards this utopic vision. However, the issue of reheating after inflation in these theories, where the universe is revived, phoenix-like, from the frozen vacuum state, has remained relatively unexplored. This is precisely one area where the full symmetry and particle content of the underlying theory is likely to be crucial.

Indeed reheating poses a severe threat to the simple ideal presented above since non-perturbative effects are typically dominant. Reheating is therefore not a minor phase at the end of inflation, of little dynamical interest. The large quantum fluctuations allow for GUT baryogenesis and, as pointed out by Kofman et al., may cause the monopole and domain wall problems to reappear due to non-thermal symmetry restoration. This last possibility is actually rather difficult in simple models of reheating with only two fields. As we shall show, however, this situation changes dramatically in the case of multiple fields, relevant for GUT models.

The main motivation of this work then is to understand what new effects multiple fields have on reheating. This issue encompasses two particularly interesting unknowns. (i) The nature of reheating at strong coupling when the fields evolve chaotically. This is relevant for GUT’s with divergent UV fixed points and models such as softly broken Seiberg-Witten inflation where reheating occurs in the strongly coupled, confining, regime. Setting aside the subtle issue of the quantum behaviour of gauge theories at strong coupling, in the two and three scalar-field cases studied so far, (classical) chaotic motion has been typical, especially at reheating. This chaotic evolution parallels results in the Einstein-Yang-Mills equations, semi-classical QCD and lattice gauge theory. Thus a natural question is “what is the nature of chaotic reheating?”

The second issue is (ii) the sensitivity of reheating to the mass spectrum of the theory. For example, in $SO(10)$, the one-loop effective potential is built from four quadratic invariants. “How does reheating depend on the relative masses of these four fields?”

We find that both these factors can significantly enhance the efficiency and power of reheating. This in turn improves the possibilities for GUT baryogenesis and non-thermal symmetry restoration (NTSR), with the concomitant formation of topological defects.

Consider the inflaton condensate $\phi$ in the reheating phase with large couplings $\lambda_{\Psi}$ to the fields $\Psi$. Further, couple $\phi$ to the minimally-coupled scalar field $\chi$ via the interaction term $\frac{1}{2} g^2 \phi^2 \chi^2$, where $g$ is a dimensionless coupling
constant. Then the modes of $\chi$ obey \[^{10}\]
\[\frac{d^2(a^{3/2}\chi_k)}{dt^2} + \left(\frac{k^2}{a^2} + m_\chi^2 + \frac{3\kappa}{4} p + \tilde{g}^2 \phi^2\right)(a^{3/2}\chi_k) = 0\]  
\hspace{1cm} (1)

where $a(t)$ is the scale factor of the universe obeying the Hamiltonian constraint $H^2 \equiv (\dot{a}/a)^2 = \rho_{tot}/3$ with $\rho_{tot}$ the total energy density and $p$ the total pressure of the universe. $\kappa \equiv 8\pi G = 8\pi$. $M_{pl}$ is the Planck mass.

In the simple model where the $\Psi$ fields are absent, $p = \hat{\phi}^2/2 - V(\phi)$, and if the inflaton effective potential is $V(\phi) = m_\phi^2 \hat{\phi}^2/2$, the solution is $\Phi(\phi) \sim \Phi(t) \sin \delta \phi$, where $\Phi$ decays slowly with the mean expansion.

In this case eq. \(^{10}\) can be cast in the form of the Mathieu equation \(^{11}\): $(a^{3/2}\chi_k)^{'''} + [A + F](a^{3/2}\chi_k) = 0$ with $F = -2q \cos(2t)$. The adiabatically evolving parameters \(^{11}\) $A = k^2/(m_\chi^2 a^2) + m_\phi^2/m_\phi^2 + 2\tilde{g}^2 \Phi^2/m_\phi^2$ and $q = \Phi^2/(m_\phi^2 + 3\kappa/16)$, span a plane geometrically dissected into stability and instability regions (fig. 1b) \(^{11}\). The solutions to the Mathieu equation in the instability regions are periodic with envelope $y \sim \exp(\mu m_{pl} t)$, and Floquet index $\mu > 0$. This resonance continues until backreaction due to $\langle (\delta \chi)^2 \rangle$ shuts off the resonance by forcing all physical $(k \geq 0)$ modes out of the dominant first resonance band \(^{11}\).

Now switch on the couplings $\lambda_\Psi$ to the fields $\Psi$ and move into the strongly coupled, chaotic region of the parameter space. While we are unable to study chaotic reheating in full generality, we have full control over the region in which the chaotic fluctuations are extremely rapid. This is because of the following little-known theorem \(^{11}\): In the limit of rapid variations, chaotic flows become uniformly indistinguishable from white noise \(^{11}\). This is an extremely powerful result since the investigation of reheating becomes very pure and tractable.

In this limit, eq. \(^{10}\) again reduces to quasi-Mathieu form, but now with a new potential $F$ \(^{20,21}\):

\[F = -2q \cos(2t) + g^2 \xi(t)\]  
\hspace{1cm} (2)

Eq. \(^{2}\) corresponds to $\lambda_\Psi$-independent stochastic evolution ($\xi(t)$ Gaussian white noise) describing the effect of multiple fields in the strong coupling limit (in which case $q = 0$, as there is no periodic component). This form of the potential also models the backreaction of quantum fluctuations on the mean periodic evolution of $\phi$ ($q \neq 0$) in the case where the $\Psi$ are absent \(^{22,20}\).

Using spectral theory it was shown \(^{21}\) that the stability bands essentially disappear in the case of stochastic potentials $F$. In the strong-coupling limit, $g \to \infty$ we have a rather intriguing analytical result for $\mu_k$ \(^{21}\) which is rather intractable and strongly emphasises the need for numerical study of eq. \(^{10}\). A selection from the extensive simulations performed \(^{23}\) for this problem are shown in figs. (2a,b). The main generic results are the complete breakup of the stability bands of the Mathieu equations, with $\mu > 0 \forall k$, and the significant increase of $\mu$, over the purely periodic case. This is seen in all our simulations and is illustrated in figs. (2a) and (3a) which give $\mu$ along the physical separatrix ($k = 0$): $A \simeq 2q$.

A crucial result of these large $\mu$ is the possibility of non-thermal symmetry restoration (NTSR) \(^{11}\). Consider the simplest 2-field effective potential with symmetry breaking:

\[V(\phi, \chi) = \frac{\lambda}{4} (\phi^2 - \phi_0^2)^2 + \frac{\tilde{g}^2}{2} \phi^2 \chi^2\]  
\hspace{1cm} (3)

This gains the following quantum corrections: $\Delta V = 3\lambda \langle (\delta \phi)^2 \rangle \phi^2/2 + \tilde{g}^2 \langle (\delta \chi)^2 \rangle \phi^2/2 + \tilde{g}^2 \langle (\delta \phi)^2 \rangle \chi^2/2$ \(^{11}\), where the dominant variance is given in the Hartree approximation by \(^{11}\) $\langle (\delta \phi)^2 \rangle = (2\pi^2 a^3)^{-1} \int dk \langle \delta \phi_k \rangle^2 \simeq \int dk k^2 e^{4\mu_0 m_{pl} t}$. From energy conservation, one gets the bound \(^{11}\) $\langle (\delta \phi)^2 \rangle \leq C\bar{g}^{-1} \lambda M_{pl} \ln^{-2} \tilde{g}^{-2}$ with $C \sim 10^{-2}$. Saturing this bound implies that Eq. \(^{3}\) can have a positive effective mass, $m_{\phi, eff}^2 \equiv (V + \Delta V)^{''}$ even for $\phi_0 \sim 10^{10} GeV$.

Nevertheless, exhaust the available energy and realising these large variations in the simple Mathieu equations is highly non-trivial: (i) sufficiently large $\mu$ are required since $\langle (\delta \phi)^2 \rangle \sim \int dk k^2 e^{2\mu_0 m_{pl} t}$. (ii) The breadth (and very existence of) the first instability band controls how long the resonance continues before backreaction shuts it off (see fig 1b). (iii) The expansion of the universe is forced to be monotonically decreasing: $\dot{H} = -\kappa \phi^2/2$, which drives $(A, q) \to (0, 0)$, damping the resonance and reducing the variances \(^{21}\). (iv) Finally, for defect production we require $m_{\phi, eff}^2 > 0$ for time scales $\delta t \gg \omega^{-1}_e$, the period of $\phi$ oscillations.

These factors conspire against NTSR and defect production in the simplest models of preheating \(^{11}\). However, in the strong-coupling limit of GUT’s considered here we know that (i) we can easily achieve very large $\mu$ (see fig. 2a), (ii) since the stability bands are completely destroyed \(^{20,21}\), modes never stop growing, $\mu > 0$, even when backreaction becomes important. (iii) Finally, and very importantly, when there are multiple fields, the expansion need not be monotonic and indeed, in our case will increase and decrease stochastically \(^{11}\) until reheating and thermalisation are
completed. This means that the expansion need not only have a damping effect, but can also act as a pump, when \( \dot{H} > 0 \) \([13]\), enhancing the fluctuations \( \langle (\delta \phi)^2 \rangle \) and \( \langle (\delta \chi)^2 \rangle \).

From these robust considerations, we expect NTSR to be much more effective at strong-coupling, allowing us to approach the limiting variances set by energy conservation. Indeed this is borne out by direct simulation of \( m_{\phi, eff}^2 \times \delta (\delta \phi)^2 + \bar{g}^2 (\langle \delta \chi \rangle)^2 \) (see fig. 3).

Examining the statistics of \( m_{\phi, eff}^2 \) we see that while \( m_{\phi, eff}^2 \) fluctuates very rapidly, very large values of \( \langle (\delta \chi)^2 \rangle \) occur, with \( m_{\phi, eff}^2 \in (-\lambda \phi_0^2, g^2 M_{pl}^2) \). Assuming \( \lambda \phi_0^2 \ll g^2 M_{pl}^2 \), the statistics of \( m_{\phi, eff}^2 \) are highly skewed, yielding \( m_{\phi, eff}^2 > 0 \). This means that most of the time symmetry is restored and \( \phi \) is likely to diffuse across the origin, leading to defect production including monopoles in a full model including \( SU(3) \times SU(2)L \times U(1)_Y \).

We now study the effect of mass spectrum deformations on reheating \([23]\). A simplified toy model to study this issue is given by the 3-field effective potential: \( V(\phi, \varphi, \chi) = m_\phi^2 \phi^2/2 + m_\varphi^2 \varphi^2/2 + \bar{g}^2 \phi^2 \chi^2/2 + \lambda_\chi^2 \varphi^2 \chi^2/2 \). Eq. (1) is modified and leads to a new \( F(2,25) \):

\[
F_{ap} = -q(\cos 2t + \cos 2\pi t) .
\]

where we have chosen \( m_\phi/m_\varphi = \pi \), which implies that (1) is a quasi-periodic function \([21,23]\) since the masses are irrationally related.

In our case, spectral theory results guarantee that the stable bands generically form a Cantor set \([2]\), which are often of very small measure - hence physically unimportant - mimicking the stochastic case presented earlier. Again however, no estimates for \( \mu \) are available analytically. Our numerical results show the growth of \( \mu \) in this case and the significant widening and steepening of the instability bands relative to the Mathieu case (compare figs. 1 and 4). Also interesting is the development of “non-thermal edges” in \( \mu \) (fig. 4a and (23)).

From our discussion of the stochastic case, we expect \( \langle (\delta \phi)^2 \rangle \) to increase and again this is borne out numerically. In figs. (5a,b) we plot \( \log \langle (\delta \phi)^2 \rangle \) for the pure Mathieu and quasi-periodic cases as functions of \( q \). The maximum variance is \( \sim 1.5 \) orders of magnitude larger in the quasi-periodic case after the short time \( t = 23 m_\phi^{-1} \). The quasi-periodic model therefore is significantly more efficient at restoring symmetry. In general the NTSR strength of GUT theories will depend sensitively on the mass spectrum of the theory, here encoded by \( m_\phi/m_\varphi \).

The monopole problem has now again become a major concern in large (“chaotic or incommensurate”) regions of the coupling/mass parameter space, including the expansion of the universe. Due to the non-thermal, quench-like, nature of the symmetry breaking, the correlation length \( \Xi \) of the fields will be much smaller than in the equilibrium case, and therefore the defect density \( \propto \Xi^{-n} \) (\( n = 1 \) for domain walls, \( n = 3 \) for monopoles) will be correspondingly larger than the equilibrium Kibble prediction \([24]\).

If NTSR succeeds, a second stage of inflation will occur \([1,7]\) while the vacuum energy \( V(0) \) dominates over the energy of the \( \langle (\delta \chi)^2 \rangle \). During this time \( a(t) \) increases by a factor \( \sim (g^2/\lambda)^{1/4} \) for the potential (3), which cannot, therefore, supply the needed \( \sim 20 \) e-foldings to dilute the monopole density sufficiently \([1]\). In \( SO(10) \) or \( SU(6) \), however, the monopole transition is separated from the lower transitions and this may allow enough secondary inflation to dilute the monopole abundance sufficiently.

However, this is rather model dependent and the large corrections to the effective potential offer us an alternative escape route via defect-defect interactions. The full corrections to the effective potential, include not only the quadratic contributions affecting \( m_{\phi, eff}^2 \), but also odd powers which do not respect any previously existing discrete symmetries (required for example for successful D-term inflation \([23]\)). The odd-power corrections to eq. (3), with \( \bar{g} = 0 \), are:

\[
\Delta V_{odd} = \lambda \phi (\delta \phi^3 - \phi_0^2 \delta \phi) + \lambda \delta \phi \phi^3
\]

These terms softly break the \( \mathbb{Z}_2 \phi \to -\phi \) symmetry of eq. (3). This allows an implementation of the Dvali-Liu-Vachaspati mechanism \([22]\) for solving the monopole problem as follows: imagine that NTSR was successful enough to produce monopoles and domain walls during reheating. The \( \delta \phi \) terms in eq. (3) automatically cause the domain walls to be unstable \([24]\) since the minima at \( \pm \phi_0 \) are no longer degenerate, causing a pressure difference \( \sim 2 \Delta V_{odd}(\phi_0) \) across the walls.

Since after preheating \( \phi \ll \langle (\delta \phi)^2 \rangle \), the first term of (3) dominates. The constraint that the walls percolate gives us \( (\delta \phi) \leq 10^{-1} M_{pl} \). Monopoles are then swept up on the walls and dissipate because the full symmetry (e.g. \( SU(5) \)) is restored there \([1]\). Requiring that the pressure difference drives the domain walls to collapse and decay before dominating the energy density of the universe yields \( \Delta V_{odd} \geq 10^{-4} \lambda^{-1/3} M_{pl} \approx 10^{-3} M_{pl} \) if \( \lambda \sim 10^{-3} \). These bounds are exactly in the range of values expected if NTSR is successful. The advantage of this formulation is that it provides a specific implementation of the general Dvali-Liu-Vachaspati mechanism and uses the same large quantum fluctuations which produced the monopoles to remove them.
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\[ \int_{0}^{t} \sqrt{2} f(\Phi_{t} \phi_{0}) d\tau - \sigma \lambda B(\lambda t, \phi_{0}) = O(\frac{1}{\lambda}) . \]  

E(f) denoting the expectation value of f. Since white noise is the derivative of Brownian motion, this implies that chaotic motion is, in the rapid fluctuation limit λ → ∞, indistinguishable from white noise.

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FIG. 1. Mathieu eq. (a) Floquet index $\mu$ vs $q$ along $A = 2q$. (b) Contour plot of $\mu$ on the instability chart $(A, q)$.

FIG. 2. Stochastic eq. (2), $g = 5$: (a) Floquet index $\mu$ vs $q$ along $A = 2q$. (b) Contour plot of instability chart - note the breakup of Mathieu bands and the very large peaks in $\mu$.

FIG. 3. Three realisations of $\langle (\delta \chi)^2 \rangle$ vs $g$ at $t = 2m^{-1}$ for $q = 5$. At larger times, $\langle (\delta \chi)^2 \rangle$ becomes completely dominated by a single peak (c.f. fig. 2(b)).
FIG. 4. Quasi-periodic case, eq: (4) (a) $\mu$ vs $q$ on $A = 2q$. (b) Instability chart $(A, q)$. Note the larger $\mu$ and proliferation of instability bands compared with figs. (1 a,b).

FIG. 5. $\log \langle (\delta \phi)^2 \rangle$ vs $q$ for (a) the Mathieu eq., and (b) the quasi-periodic eq., both calculated at $t = 23 m^{-1}$. The maximum value of $\langle (\delta \phi)^2 \rangle$ is about 20 times larger in (b).