Systematic Research on Measuring Acceleration of Gravity by Laser Interferometry

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Abstract. Gravity acceleration is an important physical quantity, and its precise measurement results have important applications in practice. According to the principle of measuring gravity acceleration in college physics experiment, a scheme of measuring gravity acceleration by laser interferometry is designed, and the main sources and sizes of experimental errors are analyzed and calculated. Compared with the common simple pendulum method and falling body method, the theoretical results show that our measurement method has higher precision and is feasible to accurately measure the acceleration of gravity.

Keywords: Gravitational acceleration, Laser interference, Measurement accuracy.

1. Introduction
Laser interferometry is a non-contact measurement technology with high measurement sensitivity and accuracy. Because it is a non-contact measurement, the object to be measured does not constitute damage and interference, etc., so the laser interferometric measurement technology is widely used in the precision measurement of length, displacement, angle and vibration. In addition, it is also used in micro analysis of material surface and interference microscopy. Gravity acceleration is an important physical quantity, its size is related to the evolution of the earth's internal structure and shape, as well as the underground rocks and minerals. Its small changes play an important role in the process of earthquake prediction and prospecting. Therefore, in the process of measuring gravity acceleration with high precision, the absolute gravimeter also adopts laser interference measurement technology, and its measurement accuracy can reach [1]. However, the simple pendulum method and the falling body method are commonly used in college physics experiments to measure the acceleration of gravity, and the measurement accuracy is only in [2]. In order to measure the gravity acceleration more accurately, fully understand the principle of light interference and master the method of laser interference measurement, we designed a set of laboratory scheme of laser interference measurement of gravity acceleration, so that the laser interference measurement technology can be used in university physics experiments.
2. Principle of measuring gravity acceleration by laser interferometry and design of experimental device

When using the falling body method to measure the acceleration of gravity, by testing the movement of a free-falling object relative to a stationary object, the change rule of relative displacement between two objects with time is analyzed, and the acceleration of gravity of the object to be measured can be calculated. According to this principle, a scheme for measuring the acceleration of gravity by laser interferometry is designed, that is, using laser interferometry to accurately measure the falling displacement of the object to obtain the acceleration of gravity of the object to be measured. The principle is shown in Figure 1.

![Figure 1: Schematic diagram of laser interferometry for measuring acceleration of gravity](image)

Figure 1 Schematic diagram of laser interferometry for measuring acceleration of gravity

The experimental device is a Michelson interferometer with a photodetector e fixed at the bottom, a laser at the left end and a prism m at the right end. The middle mirror g is a beam splitter, which can make half of the light ray’s incident on its surface reflect and the other half transmit. The angular prism w of the object to be measured is suspended at the lower end of the bracket by a thin fuse.

After the parallel light emitted from the laser reaches the beam splitter, it is divided into two beams of light "1" and "2" by the beam splitter. The light "1" passes through g and hits m, then is reflected back by m and hits g, and reaches e after being reflected. The light "2" is reflected from the object w and transmitted by g to e. Because the two rays are separated by the same beam, they are coherent, so the interference fringes can be formed at e.

Fuse the fuse wire [3] hanging the object to be measured, and the object to be measured will make free falling motion. As a result, the optical path difference between the two rays’ changes, resulting in the movement of interference fringes at e. The interference fringe changes with the change of the optical path difference. The photodetector can input the information about the speed of the interference fringe over time to the spectrometer and record it. According to the speed of the interference fringe movement, the gravitational acceleration can be calculated.

3. Theoretical analysis of coherent light intensity

After the fuse of the object to be tested is blown, the object to be tested will fall in the vertical direction with the acceleration of gravity. If the moment of fusing is recorded as the zero moment of time, the relationship between the displacement of the object to be measured and the time t is as follows:

\[
\Delta d = \frac{1}{2} g t^2
\]  \hspace{1cm} (1)
When the object to be measured falls down, the optical path difference between the two beams at E changes constantly. Let the laser wavelength be, and when the number of interference fringes moves is \( N \), the change of optical path difference is:

\[
\Delta r = N\lambda
\]  

(2)

Since the change in optical path difference \( \Delta r \) is twice the relative displacement \( \Delta d \), there are:

\[
N\lambda = gt^2\quad N\alpha = gt^2
\]  

(3)

According to the interference theory of light, the light intensity of the superposition of two coherent beams is

\[
I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_0\right),
\]

where \( \Delta\phi_0 \) is the initial phase difference of the two beams. If two beams have the same intensity, namely \( I_1 = I_2 = I_0 \) and \( \Delta\phi_0 = 0 \), then:

\[
I = 4I_0\cos^2\left(\frac{\pi}{\lambda}\Delta r\right)
\]

(4)

According to the formula (2) – (4), the relationship between coherent light intensity and time is as follows:

\[
I = 4I_0\cos^2\left(\frac{\pi}{\lambda}gt^2\right)
\]

(5)

Equation (5) shows that the intensity of interference fringes changes periodically with time. If a He-Ne gas laser is used as the light source, the wavelength of the laser is \( \lambda = 6.328 \times 10^{-7} \) m. Assuming that the measured gravitational acceleration is \( 9.80 \text{ m/s}^2 \), the change of the light intensity at E with time \( t \) during the falling process is shown in Figure 2. It can be seen from Figure 2 that the moving rate of interference fringes increases with time. According to the change of fringe moving rate, the gravity acceleration can be obtained.

![Figure 2](image)

**Figure. 2** Variation curve of coherent light intensity with time

4. **Theoretical analysis of data processing**

During the experiment, the interference light intensity signal was converted into voltage signal by A/D conversion. Then, by collecting data, the light intensity change (voltage change) curve similar to that shown in Figure 2 can be obtained.

Analyze the data of a certain period of time during the fall of the system. At this time, the relationship between the relative displacement \( \Delta d \) of the two objects and time \( t \) will be:
\[ \Delta d = \Delta d_0 + \Delta v_0 t + \frac{1}{2} g t^2 \]  

(6)

In which \( \Delta d_0 \) and \( \Delta v_0 \) are the relative displacement and relative velocity of two objects at the initial time. Since the optical path difference \( \Delta r \) is twice the relative displacement \( \Delta d \), the coherent light intensity is:

\[
I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\left(\frac{2\pi}{\lambda} \Delta r + \Delta \phi_0\right) \\
= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\left(\frac{4\pi}{\lambda} \Delta d + \Delta \phi_0\right) \\
= A + B \cos\left(\frac{4\pi}{\lambda} \Delta d + \Delta \phi_0\right)
\]  

(7)

Among them \( A = I_1 + I_2 \), \( B = 2\sqrt{I_1 I_2} \). \( A \) is the DC component of light intensity, and \( B \) is the amplitude of the AC component, so the relationship between \( A \) and \( B \) and the maximum value \( I_{\text{max}} \) and minimum value \( I_{\text{min}} \) of light intensity is:

\[
\begin{align*}
A &= \frac{I_{\text{max}} + I_{\text{min}}}{2} \\
B &= \frac{I_{\text{max}} - I_{\text{min}}}{2}
\end{align*}
\]  

(8)

From equation (7), it can be concluded that:

\[
\Delta d = \frac{\lambda}{4\pi} \left[ 2N\pi + (-1)^k \arccos\frac{I - A}{B} - \Delta \phi_0 \right]
\]  

(9)

Where \( k \) is an integer. The relationship between it and the number of fringes moving \( N \) is: in the process of data processing, \( N = \frac{k + 1}{2} \) \( k \) Take odd numbers can appropriately select the starting point of the data, so that the initial phase difference \( \Delta \phi_0 = 0 \) at the beginning of recording is simplified as (9):

\[
\Delta d = \frac{\lambda}{4\pi} \left[ 2N\pi + (-1)^k \arccos\frac{I - A}{B} \right]
\]  

(10)

In each fluctuation period, the maximum value \( I_{\text{max}} \) and the minimum value \( I_{\text{min}} \) of light intensity are found out from the recorded light intensity data \( \{I_i\} \), and the values of \( A \) and \( B \) in this period are obtained by using formula (8), and then converted into the relationship \( \{t_i, \Delta d_i\} \) between displacement and time by using formula (10). The data of each period in a certain period of time are
transformed in this way, then connected, and then the quadratic term coefficient $\frac{g}{2}$ in the
displacement-time function (6) can be obtained by fitting these data, thus the magnitude of gravity
acceleration can be obtained.

5. Experimental error analysis
The error of the laser interferometry to measure the acceleration of gravity mainly comes from the non-
parallelism of the falling line and the measuring line and the influence of the time resolution of the
spectrometer.

5.1. The influence of parallelism between survey line and falling line

As shown in Figure 3, if the light reflected by the object to be measured is not parallel to the falling
line, there is a deflection angle $\theta$, and the optical path difference is $\Delta r = \frac{2 \Delta d}{\cos \theta}$. The measured
acceleration of gravity is as follows:

$$g' = \frac{2\Delta d}{t^2 \cos \theta}$$  \hspace{1cm} (11)

Because $\theta$ is very small, then:

$$g' \approx \frac{2\Delta d}{t^2}[1 + \frac{\theta^2}{2}]$$  \hspace{1cm} (12)

When the survey line is parallel to the falling line, the gravity acceleration is $g = \frac{2\Delta d}{t^2}$, and the
measurement error introduced when it is not parallel is:

$$\Delta g_1 = |g' - g| = \frac{\theta^2 g}{2}$$  \hspace{1cm} (13)

During the experiment, $\theta$ can be controlled within $2^6$ [4], and the resulting measurement error
$\Delta g_1$ is $6 \times 10^{-4} g$. 
5.2. Influence of time resolution on spectrum analyzer

The time relative accuracy of the spectrometer is generally 25ppm:

\[
\frac{\Delta g}{g} = 2 \frac{\Delta t}{t} = 5 \times 10^{-5}
\]  
(14)

That is, the error caused by the time resolution of the spectrometer to the measurement of gravity acceleration is \( \Delta g = 5 \times 10^{-5} g \).

The above experimental error analysis shows that the designed laser interferometer method for measuring the acceleration of gravity can reach the \( 6 \times 10^{-4} \) level of measurement accuracy.

6. Conclusions

According to the principle of gravity acceleration measurement by falling body method, a scheme of gravity acceleration measurement by laser interferometry is designed, and the experimental error is analyzed and calculated in detail. The measurement accuracy can reach \( 6 \times 10^{-4} g \). Compared with the common measuring techniques of simple pendulum method and falling body method, the precision of gravity acceleration measured by laser interferometry is obviously improved. Michelson interferometer is an important experimental instrument in university physics laboratory, so our research expands the application range of Michelson interferometer, enriches the measurement technology of gravity acceleration, and improves students' analysis and research ability.

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