Extended Minimal Flavour Violating MSSM

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Abstract: The recently reported measurements of the CP asymmetry \( a_{\psi K} \) by the BABAR and BELLE collaborations are in good agreement with the standard model prediction. With the anticipated precision in \( a_{\psi K} \) at the B factories and hadron colliders, one hopes to pin down any possible deviation from the SM. We discuss an extension of the MFV-supersymmetric models which comfortably accommodates the current measurements of the CP asymmetry \( a_{\psi K} \), but differs from the SM due to an additional flavour changing structure beyond the CKM matrix. We analyze the compatibility of this model with present data and suggest specific tests in forthcoming experiments in B physics. In addition to the CP-asymmetries in B-meson decays and the \( B^0 - \bar{B}^0 \) mass difference, we emphasize measurements of the radiative transition \( b \to d \gamma \) as sensitive probes of the postulated flavour changing structure. Interestingly, the CKM–unitarity analysis in these models also allows solutions with \( \gamma > \pi/2 \), where \( \gamma = - \arg V_{ub} \). Such large values of \( \gamma \) are hinted by the current measurements of the branchig ratios for the decays \( B \to \pi \pi \) and \( B \to K \pi \).

1. Outline of the model

This talk is based on the paper in Ref. [1]. The supersymmetric model that we consider is essentially based on the assumptions of heavy squarks (of the first two generations) and gluinos. The charged Higgs and the lightest chargino and stop masses are required to be heavier than 100 GeV in order to satisfy the lower bounds from direct searches. The rest of the SUSY spectrum is assumed to be almost degenerate and heavier than 1 TeV. In this framework the lightest stop is almost right–handed and the stop mixing angle (which parameterizes the amount of the left-handed stop \( \tilde{t}_L \) present in the lighter mass eigenstate) turns out to be of order \( O(M_W/M_{\tilde{t}}) \approx 10\% \); for definiteness we will take \( |\theta | \leq \pi/10 \).

The assumption of a heavy gluino totally suppresses any possible gluino–mediated SUSY contribution to low energy observables. On the other hand, the presence of only a
single light squark mass eigenstate has strong consequences. In the MIA-framework, all the FC effects which are not generated by the CKM mixing matrix are proportional to the properly normalized off-diagonal elements of the squark mass matrices. In order to take into account the effect of a light stop, we exactly diagonalize the $2 \times 2$ stop system and adopt the slightly different MIA implementation proposed in Ref. [3]. In this approach, a diagram can contribute sizably only if the inserted mass insertions involve the light stop. All other diagrams require necessarily a loop with at least two heavy squarks and are therefore suppressed. This leaves us with only two unsuppressed flavour changing sources other than the CKM matrix, namely the mixings $\tilde{u}_L - \tilde{t}_2$ (denoted by $\delta_{\tilde{u}_L \tilde{t}_2}$) and $\tilde{c}_L - \tilde{t}_2$ (denoted by $\delta_{\tilde{c}_L \tilde{t}_2}$).

In order to deal with a predictive model, we choose to set $\delta_{\tilde{u}_L \tilde{t}_2} = 0$. This assumption is partly justified by the remarkable agreement of the measured rate of the inclusive decay $B \to X_s \gamma$ ($B_{SM}^{b \to s \gamma} = (3.22 \pm 0.40) \times 10^{-4}$) with the SM prediction ($B_{SM}^{b \to s \gamma} = (3.29 \pm 0.33) \times 10^{-4}$). This hypothesis will be further tested in CP-asymmetries $A_{CP}^{B \to (X_s, K^*) \gamma}$ at the B-factories. Notice that the exclusion of $\delta_{\tilde{c}_L \tilde{t}_2}$ from our analysis introduces strong correlations between the physics that governs $b \to d$ and $b \to s$ transitions.

The free parameters of the model are the common mass of the heavy squarks and gluino ($M_{\tilde{q}}$), the mass of the lightest stop ($M_{\tilde{t}_2}$), the stop mixing angle ($\theta_t$), the ratio of the two Higgs vevs ($\tan \beta$), the two parameters of the chargino mass matrix ($\mu$ and $M_2$), the charged Higgs mass ($M_{H^\pm}$) and $\delta_{\tilde{u}_L \tilde{t}_2}$. All these parameters are assumed to be real with the only exception of the mass insertion whose phase in not restricted a priori.

2. SUSY contributions

The effective Hamiltonian that describes the $B_d - \bar{B}_d$ system is

$$H_{eff}^{\Delta B = 0} = -\frac{G_F M_W^2}{(2 \pi)^2} (V_{tb} V_{ts}^*)^2 (C_1 \bar{d}_L \gamma^\mu b_L \cdot \bar{d}_L \gamma_\mu b_L + C_2 \bar{d}_L b_R \cdot \bar{d}_L b_R$$

$$+ C_3 \bar{d}_R^\alpha b^\beta_R \cdot \bar{d}_R^\alpha b^\beta_R) \pm h.c.,$$

(2.1)

where $\alpha, \beta$ are colour indices. The $B_s$ and $K$ system cases are obtained respectively with the substitutions $d \to s$ and $b \to s$.

It is possible to show that, in the framework described in sec. 1, only $C_1$ receives sizable contributions. In models in which the split between the two stop mass eigenstates is not so marked, diagrams mediated by the exchange of both stops must be considered and it is possible to find regions of the parameter space (for large $\tan \beta$) in which SUSY contributions to $C_3$ are indeed dominant. We note that the $\tan^4 \beta$ enhanced contributions to the coefficients $C_{2,3}$ whose presence is pointed out in Ref. [4], do not impact significantly for the range of SUSY parameters that we consider ($\tan \beta < 35$ and $|\theta_t| < \pi/10$).

The structure of the SUSY contributions can be summarized as follows:

$$\Delta M_{B_s} : C^{SM}_1 \to C^SM_1 (1 + f)$$

$$\epsilon_K, \Delta M_{B_d}, a_{\psi K} : C^SM_1 \to C^SM_1 (1 + f + g)$$
where

\[ f \equiv (C_1^{H^\pm} + C_1^N)/C_1^{SM}, \]

\[ g \equiv g_R + ig_I \equiv \frac{C_{1}^{M1} \delta_{uL}^2}{C_1^{SM}}. \]

\( C_1^{H^\pm}, C_1^N \) and \( C_{1}^{M1} \delta_{uL}^2 \) are the contributions of the charged Higgs and of charginos without and with the mass insertion \([1]\). Note that the only complex phase enters through \( \delta_{uL}^2 \).

In order to understand the possible size of the above depicted SUSY contributions we varied the input parameters over a reasonable range \( (\mu, M_2, M_{H^\pm} \in [100, 1000]\text{GeV}, M_{\tilde{t}_2} \in [100, 600]\text{GeV}, \theta_t \in [-0.3, 0.3], \tan \beta \in [3, 35]) \) and included the constraints from \( B \rightarrow X_s \gamma \) \((2.41 \leq B^{B \rightarrow \tau \gamma} \times 10^4 \leq 4.02 \text{ at 95\% C.L.})\), the anomalous magnetic moment of the muon \((10 \leq \delta a_{\mu^+} \times 10^{10} \leq 74 \text{ at 95\% C.L.} [10])\) and \( B \rightarrow \rho \gamma \) \( (B^{B \rightarrow \rho \gamma}/B^{B \rightarrow K^* \gamma} < 0.28 \text{ at 90\% C.L.} [11])\). The result of this analysis is presented in Fig. 1. Clearly \( f \) and \( |g| \) are restricted to the range \( f < 0.4, |g| < 2.0 \).

3. Unitarity Triangle Analysis

In this section we analyze the implications of this parametrization on the standard analysis of the UT (see Ref. [1] for a more complete and extensive presentation).

Our first step is to investigate the regions of the parameter space spanned by \( f, g_R \) and \( g_I \) that are favoured by the present experimental data. The procedure consists in writing the \( \chi^2 \) of the selected observables and in accepting only values of \( f \) and \( g \) which satisfy the condition \( \min_{\rho, \eta}(\chi^2) \leq 2 \). The resulting allowed regions are presented in Fig. 2a. The shaded regions correspond to solutions in which the phase \( \theta_d \) (which enters the analysis through \( a_{\psi_K} = \sin 2(\beta + \theta_d) \)) and is given by \( \theta_d = 1/2 \arcsin(1 + f + g) \) is extremely large. In this case, large values of \(|g|\) are required and these regions will be, possibly, excluded once the lower bounds on sparticle masses and the experimental errors on the branching ratio of \( B \rightarrow X_s \gamma \) will become more stringent. In the computation of \( \chi^2 \) we use \( \epsilon_K = (2.271 \pm 0.017) \times 10^{-3}, \Delta M_{B_d} = 0.484 \pm 0.010 \text{ ps}^{-1}, |V_{ub}/V_{cb}| = 0.090 \pm 0.025, a_{\psi_K} = 0.79 \pm 0.12 \text{ [12,13]} \) and \( \Delta M_{B_s} \geq 14.9 \text{ ps}^{-1} \).

In order to illustrate the possible different impact of this parametrization on the unitarity triangle analysis, we focus on the \( f = 0 \) case and choose three generic points inside the...
allowed region in the plane \((g_R, g_I)\). In Fig. 2b we plot the 95% C.L. contours in the \((\bar{\rho}, \bar{\eta})\) plane that correspond to the points we explicitly show in Fig. 2a; the central values of the various observables are summarized in Table 1. Contour 3 is particularly interesting since it corresponds to a solution in which \(a_{\psi K_S}\) is larger than in the SM and the Wolfenstein parameter \(\bar{\rho}\) is negative implying a value of the inner angle \(\gamma\) in the domain \(\pi/2 < \gamma < \pi\). This is in contrast with the SM–based analyses which currently lead to \(\gamma < \pi/2\) at 2 standard deviations. We note that analyses \([5, 6, 7]\) of the measured two–body non–leptonic decays \(B \rightarrow \pi \pi\) and \(B \rightarrow K \pi\) have a tendency to yield a value of \(\gamma\) which lies in the range \(\gamma > \pi/2\). While present data, and more importantly the non–perturbative uncertainties in the underlying theoretical framework do not allow to draw quantitative conclusions at present, this may change in future. In case experimental and theoretical progress in exclusive decays force a value of \(\gamma\) in the domain \(\pi/2 < \gamma < \pi\), the extended–MFV model discussed here would be greatly constrained and assume the role of a viable candidate to the SM.

**Figure 2:** (a) Region of the \((g_R, g_I)\) plane for which \(\text{min}_{\rho, \eta}(\chi^2) \leq 2\) from the CKM-UT fits. The contours correspond to \(f = 0, 0.2\) and 0.4 and the constraints on \(|g|\), coming from Fig. 1, are \(|g| \leq 1, 2\) and 1.5 respectively. (b) Allowed 95% C.L. contours in the \((\bar{\rho}, \bar{\eta})\) plane. The solid contour is the SM case. The two semicircles represent the 2 \(\sigma\) region allowed by \(|V_{ub}/V_{cb}| = 0.090 \pm 0.025\). The contours numbered 1 to 3 correspond respectively to the points indicated in (a).

| Contour | \(g_R\) | \(g_I\) | \(|V_{ub}/V_{cb}|\) | \(\Delta M_{B_s}\) | \(a_{\psi K_S}\) | \(\alpha\) | \(\gamma\) |
|--------|--------|--------|----------------|---------------|--------------|--------|--------|
| 1      | 0.2    | 0.2    | 0.094          | 20 ps\(^{-1}\) | 0.78         | 119°   | 40°    |
| 2      | 0.0    | -0.2   | 0.110          | 20 ps\(^{-1}\) | 0.71         | 101°   | 51°    |
| 3      | -0.4   | 0.1    | 0.081          | 17 ps\(^{-1}\) | 0.73         | 64°    | 98°    |

**Table 1:** Central values of the CKM ratio \(|V_{ub}/V_{cb}|\), the \(B_s–\bar{B}_s\) mass difference, the CP asymmetry \(a_{\psi K_S}\) and the inner angles \(\alpha\) and \(\gamma\) of the unitarity triangle for the contours of Fig. 2b.
4. Conclusions

We investigated an extension of the so-called Minimal Flavour Violating MSSM and its implications on $B$ physics. The non–CKM structure in this Extended-MFV model reflects the two non–diagonal mass insertions which influence the transitions $b \to d$ and $b \to s$. In the present analysis, we have neglected the effect of the mass insertion in the $b \to s$ transition. We have shown that, as far as the analysis of the unitarity triangle is concerned, it is possible to encode, in this model, all SUSY effects in terms of three parameters $(f, g_R$ and $g_I$). We have worked out the allowed regions in the plane $(f, |g|)$ by means of a scatter plot scanning the underlying SUSY parameter space and taking into account constraints from $B \to (X_s, \rho)\gamma$ and $(g-2)\mu$. We have then used these informations and the requirement of compatibility with the fit of the unitarity triangle in order to single out the overall allowed ranges of $f$, $g_R$ and $g_I$. We have then chosen some generic points in order to show the impact of these models on the phenomenology of the unitarity triangle. Remarkably, we found solutions that admit $\gamma > \gamma_{SM}$ as it is suggested by analyses of the decays $B \to K\pi$ and $B \to \pi\pi$. It is also possible to show that the same SUSY parameter space leaves room for sizable contributions to observable related to the transition $B \to \rho\gamma$. In particular, we considered the ratio $B^{B \to \rho\gamma}/B^{B \to K^\ast\gamma}$, the isospin breaking ratio (present since $B^{B \to \rho\pm\gamma}/2B^{B \to \rho^0\gamma} \neq 1$) and the $CP$ asymmetry in the charged modes.

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