Lattice results relevant to the CKM matrix determination

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A brief discussion of the recent lattice studies of the quantities that are relevant to the Standard Model description of CP-violation in the hadronic sector is presented. A comment on $B \to K^* \gamma$ decay is made too.

1. Introduction

The most popular way to estimate the amount of CP-violation in the Standard Model is to study the shape of the CKM unitarity triangle involving the third generation of quark flavors. The standard unitarity triangle analysis (UTA) \([1]\), which requires the least theoretical assumption, comprises the $K^0 - \bar{K}^0$ and $B_0^d - \bar{B}_0^d$ mixing amplitudes, as well as the SU(3) breaking ratio in the $B_0^d - \bar{B}_0^d$ system. Additional constraint comes from information on $|V_{ub}/V_{cb}|$ ratio, which will be discussed towards the end of this review. The three “golden” mixing amplitudes over-constrain the CKM-triangle through a hyperbola and circles in the $(\bar{\rho}, \bar{\eta})$-plane which emerge from the comparison of the theoretical predictions with the experimental determinations of $|\varepsilon_K|$, $\Delta m_d$ and the limit on $\Delta m_s/\Delta m_d$. Besides the CKM parameters, the theoretical expressions contain the Wilson coefficients and the hadronic matrix elements. The Wilson coefficients encode the information about physics at high energy scales, they are computed in perturbation theory and are known at NLO accuracy (for a review see e.g. [2]). The hadronic matrix elements, instead, describe the low energy QCD dynamics and their computation requires a theoretical control of non-perturbative QCD. To be specific, the following matrix elements need to be computed:

\begin{align}
\langle K^0 | O_{sd}(\mu) | \bar{K}^0 \rangle &= \frac{8}{3} m_K^2 f_K^2 B_K(\mu), \\
\langle B_0^d | O_{bd}(\mu) | \bar{B}_0^d \rangle &= \frac{8}{3} m_{B_d}^2 f_{B_d}^2 B_{B_d}(\mu), \\
\langle B_0^s | O_{bs}(\mu) | \bar{B}_0^s \rangle &= \frac{8}{3} m_{B_s}^2 f_{B_s}^2 B_{B_s}(\mu),
\end{align}

where the operator $O_{Qq} = (\bar{Q}q)_{V-A}(\bar{Q}q)_{V-A}$. On the r.h.s. of eq. (1) we recognize the pseudoscalar decay constants $f_K$ and $f_{B_{d(s)}}$, and the familiar “bag” parameters, $B_K(\mu)$ and $B_{B_{d(s)}}(\mu)$. That is where the lattice QCD enters the stage and provides us with a completely non-perturbative way to calculate these (and other) hadronic quantities. The following two points should be emphasized:\textsuperscript{1}

\begin{itemize}
  \item Lattice QCD is not a model but the approach allowing the computation of various Green functions from first principles of QCD: the only parameters entering the computation are those that appear in the QCD lagrangian, namely the quark masses and the (bare) strong coupling;
  \item QCD is solved numerically on the lattice by using the Monte-Carlo methods which induce the statistical errors in the results. The central limit theorem applies and ensures that those errors fall as $1/\sqrt{N_{\text{conf}}}$, with $N_{\text{conf}}$ being a number of independent SU(3) gauge field configurations on which a given Green function has been calculated. Thus, in principle, computation of the hadronic quantities on the lattice can be made to an arbitrary accuracy.
\end{itemize}

In practice, however, various approximations have to be made, each introducing some systematic uncertainty into the final results. Yet all those approximations are improvable by: (a) increasing the computing power so that, one by one, for details about formulating QCD on the lattice, please see e.g. [3].
all the approximations can be removed from the computation; (b) improving the approach theoretically so that the hadronic quantities, computed on the lattice, converge faster to their values in the continuum limit.

Now I return to the lattice determination of the “golden” hadronic quantities entering the standard UTA. The present situation is discussed at length in ref. [4], and the actual results are:

\[
\hat{B}_K = 0.86(6)(14), \\
\hat{B}_{B_d} = 1.34(12), \\
f_{B_d} = 203(27) \left( \frac{\pm 0.09}{20} \right) \text{MeV}, \\
f_{B_s}/f_{B_d} = 1.18(4) \left( \frac{\pm 12}{-0.06} \right), \\
\xi = (f_B, \sqrt{B_{B_s}})/(f_{B_d}, \sqrt{B_{B_d}}) = 1.18(4) \left( \frac{\pm 12}{-0.00} \right).
\]

The above quantities are completely dominated by the systematic uncertainties, some of which are discussed in what follows.

2. $K^0 - \bar{K}^0$ mixing

Results of the precision lattice computations are presented as $B_{K\overline{K}}^{\text{MS}}(2 \text{ GeV})$, in the quenched approximation, i.e. the quark loops in the background gauge field configurations are being neglected, $n_f = 0$. In the real physical world, however, one needs $n_f = 3$. Numerically, the conversion from the $\overline{\text{MS}}(\text{NDR})$ scheme to the renormalization group invariant (RGI) form, $\hat{B}_K$, is almost completely independent of $n_f$ (see discussion in ref. [5]). By using $\Lambda_{\overline{\text{MS}}}^{n_f=3} = 338(40) \text{MeV}$ [6], I obtain

\[
\hat{B}_K = 1.382(15) \cdot B_{K\overline{K}}^{\text{MS}}(2 \text{ GeV}).
\]

A map of the precision quenched lattice QCD determinations of $\hat{B}_K$ is shown above the dashed line in fig. 1, where the asterisks denote the results presented this year. The most accurate is the value obtained by using the so-called staggered fermions [7]: $\hat{B}_K = 0.87(6)$. The advantage of that action is that it preserves the chiral symmetry, but the drawback is that it breaks the flavor symmetry. With Wilson fermions, instead, the flavor symmetry is preserved but the chiral symmetry is sacrificed. The loss of the chirality induces the additive renormalization of the operator $O_{sd}$, which is more involved than the (usual) multiplicative one. The non-perturbative method to subtract these mixing has been first implemented in ref. [8] by using the quark Ward identities, and then in ref. [9] by using the so-called RI/MOM method [10]. The problem of spurious mixing has been alleviated by a judicious application of the Ward identity on the parity violating operator which does not suffer from the spurious mixing problem on the lattice [11]. That proposal has been implemented in the high statistics lattice simulation and the result, in the continuum limit, reads $\hat{B}_K = 0.88(11)$. The results below the horizontal line in fig. 1 do not refer to the continuum limit. Their major significance lies in the fact that they are obtained by using the actions that satisfy both the chiral and
the flavor symmetry. Computations with these actions are much more involved and require huge computational resources. This year, first results for $\hat{B}_K$, obtained by using the so-called overlap fermions, have been reported by the MILC \cite{12} and by the Boston-Marseille \cite{13} collaborations. Even though it is early to draw conclusions, the observed good agreement with the values above the horizontal line in fig. 1 is encouraging. Somewhat lower are the values obtained by using the so-called domain wall fermions (DWF). Although their statistical errors are very small, in my opinion, their systematic errors are not fully realistic (see discussion in ref. \cite{14}). Nevertheless, it should be stressed that the results for $\hat{B}_K$ obtained by using DWF agree –within the error bars– with other lattice determinations. The remaining systematics is almost entirely due to quenching. Although the first (low statistics) lattice studies suggest that the unquenching is not changing the value of $B_K$ \cite{10}, we follow ref. \cite{17} and assume that the quenching error is of the order of 15%. Thus it is of upmost importance, and challenging task for the lattice community, to unquench $\hat{B}_K$.

3. $B_{s(d)}^0 - \bar{B}_{s(d)}^0$ mixing

In this case, besides the light quark, one also has to deal with the heavy quark on the lattice. Since the Compton wavelength of the realistic $b$-quark is smaller than the presently attainable lattice resolution, one has to adopt some other strategy in order to compute the properties of the heavy-light hadrons. Four ways to deal with that problem have been implemented in the quenched computation of the decay constant $f_{B_{s(d)}}$, and/or the bag-parameter $\hat{B}_{B_{s(d)}}$:

\begin{itemize}
  \item Lattice QCD: one can afford to work with propagating heavy quarks of masses around the physical charm quark mass and then one has to extrapolate to the $b$-quark by means of standard heavy quark scaling laws \cite{17}:
    \begin{itemize}
      \item Heavy quark effective theory (HQET) on the lattice is very difficult to extend beyond the static limit ($m_b \to \infty$). The long standing problem to renormalize the axial current non-perturbatively has been solved recently (see eg. \cite{18}). The static results will therefore be possible to use in constraining the extrapolations of the results obtained in the full theory, i.e. with propagating heavy quarks;
      \item Non-relativistic QCD (NRQCD), is the approach in which the $1/m_b$-corrections are included in both the lagrangian and the operators. Besides difficulties in renormalizing the operators in such a theory on the lattice, a particularly problematic is the fact that the expansion is in $1/(am_b)$ ($a$ being the lattice cut-off, followed by a redefinition of the hadron masses and reinterpretation of the theory in terms of a $1/m_b$-expansion. In some cases the approach is believed to be plagued by “renormalon shadows” \cite{19}.
    \end{itemize}
  \end{itemize}

None of the enumerated approaches is fully satisfactory on its own and, at present, they all should be used to check the consistency of the obtained results. For example, the results for $f_B$, as obtained by using various approaches, agree quite impressively (see ref. \cite{11}). Nevertheless, the systematics (within the quenched approximation) cannot be further reduced until the fully QCD based lattice method is devised to compute the heavy-light quantities directly on the lattice. Otherwise, one should wait for the next generation of parallel computers to do the job. A candidate method for the precision computation of $f_B$ has been proposed this year in ref. \cite{20}, in which the step scaling function has been employed to make a controllable $1/(Lm_b)$-expansion, with $L$ being the side of the lattice box. From an exploratory study, $f_B = 170(11)(23)$ MeV has been quoted. The authors promise to reduce the error bars very soon and to respond to the questions raised in ref. \cite{21}.

The effect of unquenching has been extensively studied this year by the MILC collaboration \cite{22}. They conclude that $f_{B_{n=2}}^B$ is about 15% larger than $f_{B_{n=0}}^B$. Moreover, in their preliminary study with $n_f = 3$, they show that the ratio $f_{B_f}/f_{B_d}$ remains essentially unchanged when one switches from $n_f = 0 \to 2 \to 3$ \cite{23}. That brings me
to the important estimate of the hadronic parameter $\xi$ (see eq. (2)). The SU(3) light flavor breaking effect in the bag parameter is equal to $B_{B_s}/B_{B_d} = 1.00(4)$ \cite{22}, and the whole effect of $\xi$ being different from unity comes from $f_{B_s}/f_{B_d}$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Quenched and unquenched ($n_f = 2$) lattice estimates for the SU(3) breaking ratio $f_{B_s}/f_{B_d}$, as obtained by three different methods indicated on the left. For more details see refs. \cite{22} (MILC) and \cite{23} (CP-PACS). No shift due to the chiral log term is displayed.}
\end{figure}

3.1. Chiral logs in $f_{B_s}/f_{B_d}$

Even if one gets a good handle over the heavy quark, there is still an extrapolation in the light quark mass to be made. With respect to the physical strange quark mass, the light quarks that are directly accessible from the lattice simulations lie in the range $1/2 \lesssim m_q/m_s \lesssim 3/2$, in which one observes that the heavy-light decay constant changes nearly linearly under the variation of the light quark mass. In other words, $f_{B_s}$ is accessed directly, whereas an extrapolation in the light quark mass is necessary in order to reach $f_{B_d}$.

It has been argued recently that if in the standard (linear) extrapolation one also includes the chiral logarithmic terms, the SU(3) breaking ratio may get shifted from $f_{B_s}/f_{B_d} = 1.16$, by $+0.16$, which is a 100% error on the net SU(3) breaking \cite{21}. In ref. \cite{21} the shift can be as large as $+0.24$. There is an ambiguity while implementing the chiral logs in extrapolation. It is not clear at what mass scale the chiral logs dominate over the other, higher order, terms in the chiral expansion. If one sets it for the light meson masses around 600 MeV, the extrapolated value will be strongly shifted upwards (w.r.t. the result of linear extrapolation) \cite{20}. The shift is roughly halved if the chiral logs are introduced for the light pseudoscalar mesons $\sim$ 350 MeV. To avoid that ambiguity, ref. \cite{27} proposes to study the double ratio

$$ R = \frac{f_{B_s}/f_{B_d}}{f_{K}/f_{\pi}}, $$

in which almost all the chiral log corrections cancel, and one can make the linear extrapolations with a small error. From the available lattice data with $n_f = 2$, the NLO chiral correction in the ratio (4) seems to cancel too. From that consideration, ref. \cite{27} concludes that $f_{B_s}/f_{B_d} \simeq f_{K}/f_{\pi}$, where the latter ratio is known very well from the experiments, $f_{K}/f_{\pi} = 1.22(1)$. Therefore, the shift is indeed present but it is not as spectacular as previously thought. Similar conclusion has been reached by the MILC collaboration \cite{22}. They showed that if the chiral logarithms are consistently included in the entire analysis of the lattice data, the shift in $f_{B_s}/f_{B_d}$ is much smaller, namely $+0.04$.

Notice that the second errors in $f_{B_s}$, $f_{B_s}/f_{B_d}$ and $\xi$, quoted in eq. (2) (see also ref. \cite{23}), reflect a guesstimate of the impact of the chiral logs on extrapolation in the light quark mass. As we just explained, those errors cannot be too large, and I personally believe that through the strategies such as those proposed in refs. \cite{22} and \cite{27}, the lattice community will be able to cut down on those errors very soon.
4. $|V_{ub}|$ from $B \to D^{(*)}\ell\nu$

The extraction of $|V_{ub}|$ from the exclusive semileptonic modes, $B \to D^{(*)}\ell\nu$, requires the computation of the corresponding form factors. The cleanest method for their extraction has been proposed and implemented in ref. [28]. It has been discussed at this conference by J. Simone [29], and the reader is referred to those articles for more details. It would be very nice if the other lattice approaches adopt that strategy and compute the relevant form factors through the double ratios described in ref. [28].

5. $|V_{ub}|$ from $B \to \pi(\rho)\ell\nu$

In recent years several new lattice studies of the $B \to \pi$ form factors appeared [28,30-34,36]. The present situation with lattice results is depicted in fig. 3, where we plot the two form factors that parametrize the hadronic matrix element

$$\langle \pi(p')|V_\mu|B(p)\rangle = c_+ F_+(q^2) + c_0 F_0(q^2),$$

with $c_+/0$ being the known kinematic factors. The range of the transfer momenta available from this decay is huge: $0 \leq q^2 \leq (m_B - m_\pi)^2 = 26.4$ GeV$^2$. The lattice can be used only for large $q^2$'s (small recoil), and therefore the assumptions are necessary if we are to cover the entire $q^2$-range. In ref. [30] a simple parametrization has been proposed which includes various symmetry constraints onto the shapes of the form factors [30,31,33], as well as the kinematic condition $F_+(0) = F_0(0)$. It is often referred to as BK and it reads

$$F_+(q^2) = \frac{c (1 - \alpha)}{(1 - q^2/m_{B*}^2)(1 - \alpha q^2/m_{B*}^2)}, $$
$$F_0(q^2) = \frac{c (1 - \alpha)}{1 - q^2/\beta m_{B*}^2},$$

thus consisting of three parameters only: $c$, $\alpha$ and $\beta$. Results of the fit of the lattice data to this form are given in table I. We notice a pleasant agreement with the results obtained by LCSR [32], which is believed to be the method of choice in the low $q^2$-region. We also see a good consistency of the results obtained by using three different approaches. The agreement is especially good for

![Figure 3. Recent (quenched) lattice results for the two form factors are plotted head-to-head, since they satisfy the condition $F_+(0) = F_0(0)$. For illustration a fit to the BK-form is shown as well as the prediction obtained by using the light cone QCD sum rules (LCSR).](image)

| $F(0)$ | $\alpha$ | $\beta$ | ref. |
|--------|-----------|---------|-----|
| 0.30^{+0.3}_{-0.3}(+0.9) | 0.46^{+0.0}_{-0.0}(+0.37) | 1.27^{+0.4}_{-0.3}(+0.12) | UKQCD [33] |
| 0.28(6)(5) | 0.45(17)(+0.06) | 1.20(13)(+0.15) | APE-I [31] |
| 0.26(5)(4) | 0.40(15)(9) | 1.22(14)(+0.15) | APE-II [31] |
| 0.33^{+0.3}_{-0.3} | 0.34^{+0.3}_{-0.3} | 1.31^{+0.3}_{-0.3} | FNAL [32] |
| 0.23^{+0.3}_{-0.3} | 0.58^{+0.1}_{-0.0} | 1.28^{+0.1}_{-0.20} | JLQCD [32] |
| 0.28(5) | 0.32^{+0.2}_{-0.0} | – | KRWWY [33] |
particularly for the form factor $F_0(q^2)$. That conclusion has been reached by confronting the corresponding expressions derived in the quenched and in the full ChPT. The remaining step, in my opinion, is to unquench the $B \rightarrow \pi$ form factors.

In the case of $B \rightarrow \rho \ell \nu$, instead of 2 one has 4 form factors and less kinematic constraints, which makes the computation more complicated. Like in the case of $B \rightarrow \pi$ transition, the lattice results for $B \rightarrow \rho$ decay are accessible for relatively large values of $q^2$. The constrained extrapolations to the low $q^2$'s, guided by the symmetry relations [37,38], are however much more difficult to control in this case. To make the phenomenologically relevant predictions, the lattice results for $q^2 > 10 \text{ GeV}^2$ of ref. [40] have been interpolated and then combined with the light cone QCD sum rule results of ref. [50], which are again expected to be reliable in the region of low $q^2$'s. From that exercise and by using the experimental branching ratio $B(\bar{B}_0 \rightarrow \rho^+ \ell \nu)$, recently measured by CLEO, BaBar and Belle [51], we obtain

$$|V_{ub}| = 0.0034(6).$$

The reader interested in new results for $B \rightarrow \rho \ell \nu$ decay form factors is invited to consult ref. [40].

6. $B \rightarrow K^*\gamma$

$b \rightarrow s\gamma$ is a fascinating decay mode: it is mediated by the flavor changing neutral current and therefore completely dominated by the loop effects. The hope is that by confronting theory vs. experiment, one can probe the content in the loops and perhaps detect the non-Standard Model physics contributions. For that purpose the accurate Standard Model predictions are necessary. The NLO theoretical prediction for the inclusive $B \rightarrow X_s\gamma$ decay has been recently completed in ref. [41] (see also references therein). For the exclusive decays, instead, the situation is obscured by the necessity for the accurate computation of the hadronic form factors

$$\langle K^*(p')|\bar{s}\sigma_{\mu\nu}(1 + \gamma_5)b|B(p)\rangle = \sum_{i=1}^{3} c^{(i)}_{\mu\nu}T_i(q^2),$$

where the kinematic factors $c^{(i)}_{\mu\nu}$ are known (see e.g. [52]). When the photon is on-shell, i.e. at

$$q^2 = 0, c^{(3)}_{\mu\nu} = 0 \text{ and } T_1(0) = T_2(0).$$

Previous lattice results were hampered by the lack of information about the heavy mass scaling behavior in the range of low $q^2$'s. That generated a proliferation of the $q^2$-forms to which the data have been fitted at large $q^2$'s, and then extrapolated to $q^2 = 0$. The needed extra information comes from the symmetry constraints discussed in refs. [38,12], allowing one to write down a parameterisation similar to (6). The “pole/dipole” form discussed in refs. [34,13,16] is also consistent with symmetry constraints [38,12,13]. More details on the new lattice results by the SPQcdR collaboration can be found in ref. [48]. The new (still
preliminary!) result for \( T_1(0) = T_2(0) \equiv T(0) \) is
\[
T^{B \to K^*}(0) = 0.24(5)(^{+1.1}_{-1.2}) . \tag{8}
\]
In fig. 4 we compare the available results for the form factor \( T(0) \) computed on the lattice, with the values obtained by using the QCD sum rules \([43,44]\). We see that the lattice estimates are lower than the QCD sum rule ones. Although the QCD sum rule method is more suitable for the kinematic corresponding to \( q^2 = 0 \), it is not clear where the discrepancy with the lattice QCD results comes from. More research in that direction is needed (as emphasized at this conference in ref. \([53]\)). In addition, it should be reiterated that so far all the lattice studies are made in the quenched approximation.

7. Conclusion

In this talk I discussed the status of the lattice QCD computation of the quantities that are relevant to the standard determination of the shape of the CKM unitarity triangle (see ref. \([54]\) for the most recent update of the UTA). Quite a bit of progress has been made in gaining more control over the systematic uncertainties in the quenched lattice studies. A lot of work is still needed, especially in unquenching the current estimates. Besides the “golden” quantities for the CKM triangle analysis, other quantities, such as the quark masses, decay constants, form factors, light cone wave functions, ... are computed on the lattice. Accurate determinations of these quantities is very useful for theoretical studies of the non-leptonic heavy-to-light decays. Finally, I would like to mention the intensive activity, within the lattice community, in taming the \( K \to \pi \pi \) decay amplitudes which I did not have space nor time to cover in this talk. For a status report please see \([55]\).

Acknowledgments

I am grateful to all my collaborators from the SPQcdR collaboration for their advice in preparing this talk. The correspondence with S. Sharpe and N. Yamada, many discussions with L. Lellouch, as well as the help from C. Maynard, T. Onogi and J. Simone in preparing the table 1, are kindly acknowledged. Finally, I thank the organizers for the invitation to this exciting and enjoyable conference.

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