Augmented Quaternion MUSIC Method for Near-Field Noncircular Sources With a COLD Array

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ABSTRACT Based on the quaternion theory, a novel algorithm named non-circular augmented quaternion MUSIC (NCAQ-MUSIC) is proposed for DOA and range estimation of noncircular signals impinging on a concentred orthogonal loop and dipole (COLD) array. Firstly, based on the augmented quaternion, the proposed algorithm uses the noncircular characteristic of the signals to achieve the virtual array expansion; secondly, the DOA and range parameters can be completely separated in the principle of rank reduction, and finally, the parameters of DOA and range are estimated through one dimensional search. Compared with direct mutil-dimensional (M-D) searching algorithms, the proposed method merely requires several one-dimensional (1-D) spectral peak search which does not need parameter pairing. Simulation results verify the performance promotion of the proposed approach.

INDEX TERMS Quaternion, noncircular signals, near-field source, COLD array.

I. INTRODUCTION

In the field of array signal processing, many methods use different antenna arrays to estimate the parameters (angle, range, polarization, etc.) of the emission source \cite{1}--\cite{3}. Early direction of arrival (DOA) estimation algorithms for signals usually assume that the array is the scalar array composed of ideal array elements, and the DOA of the incident signals can be estimated by using the time delay information relative to different array elements. Compared with scalar antenna arrays, vector antenna arrays can extract the polarization information of incident electromagnetic waves to improve the performance of signal parameter estimation. Most of polarization array algorithms generally require the source to be located in the far-field (FF) region, then the spatial characteristics and polarization characteristics of the signals received by the array can be separated, such as the MUSIC method \cite{1}, the ESPRIT method \cite{2}.

In fact, there are many examples of near-field (NF) source localization, like microphone arrays localization \cite{4}--\cite{6}. The above-mentioned methods based on the FF assumption are not applicable to the NF situations, where a series of methods \cite{4}--\cite{9} have been developed for the NF source characterized by both range and DOA, such as maximum likelihood (ML) method \cite{7}, subspace methods \cite{8}--\cite{12}. Yet, all of the above NF localization methods do not use polarization information at the received array, and most methods require multi-dimensional (M-D) search or suffer from pairing problems.

Therefore, it is necessary to consider polarization in NF signal parameters to improve the estimation performance. By defining several cumulant matrices, the subspace-based methods were proposed in \cite{13}--\cite{15} to estimate the DOA and range of the NF signals, however, the computational issue occurs with the use of cumulant. Based on the symmetry structure of linear cross-dipole array, the covariance matrix of the array output was constructed to estimate the signal’s angle-range parameters by the
Note that all the aforementioned methods assume that the signals are circular. In modern wireless communications, non-circular signals are more widely used, such as amplitude modulation (AM) and binary phase shift keying (BPSK) signals. Therefore, the performance of DOA and range estimation can be improved by using the covariance matrix and conjugate covariance matrix of non-circular signals. And several excellent methods have been reported for the NF noncircular signals [18]–[23]. Further, to make use of both the polarization and noncircular characteristic of the NF signals, a two stages subspace method [22] was presented by transforming the array observations into the real-valued vectors. To enhance the orthogonality of MUSIC method with exploiting the additional constraint in quaternion domain, the quaternion dimension-reduced (QDR) algorithm and the quaternion non-circular (QNC) were proposed in [23] for NF signals, which unfortunately reduce the dimension of quaternion covariance matrix. As the fact that the parameter estimation accuracy of the subspace methods mainly depends on the dimension of the noise subspace of the covariance matrix, the method in [23] may bring about a degraded performance.

In the paper, we propose a novel non-circular augmented quaternion MUSIC (NCAQ-MUSIC) algorithm based on a centered orthogonal loop and dipole (COLD) uniform linear array (ULA) by constructing four quaternion models. Based on the model, the noise subspace is obtained by performing eigenvalue decomposition (EVD) of the adjoint matrix. Then the DOA and range parameters of NF noncircular sources are estimated by searching the one-dimension (1-D) spectra. Compared with the existing MUSIC-like algorithms, the superior performance of the proposed algorithm is shown through computer simulations.

**Notation:** $\cdot^*$, $[\cdot]^T$, $[\cdot]^H$ represent operations of conjugation, transpose, conjugate transpose, respectively; $E\{\}$ is the expectation operation; $\text{diag}\{\}$ stands for the diagonal operation; $\text{det}\{\}$ denote the trace and determinant of a matrix.

### II. QUATERNIONS AND POLARIZATION MODEL

#### A. Quaternions

In [24], Hamilton proposes a four dimensional hyper complex numbers system, namely quaternions, which can be regarded as the extension of complex numbers to four-dimensional (4-D) space. Basic properties about quaternions and their complete material can be found in [25], [26]. In this paper, we only focus on several useful basics about definitions and properties as follows.

A quaternion $q$ has four components with one real part and three imaginary parts, which can be represented in Cartesian form as

$$ q = r_0 + r_1i + r_2j + r_3k = (r_0 + r_1i) + (r_2 + r_3i)j = c_1 + c_2j \quad (1) $$

where $r_0, r_1, r_2, r_3$ are real numbers, $c_1 = r_0 + r_1i$, $c_2 = r_2 + r_3i$ are complex numbers, and $i, j, k$ are imaginary units obeying the following multiplication rules:

$$ ij = -ji = k, \quad jk = -kj = i $$

$$ k^2 = -1. \quad (2) $$

Then, given a quaternion matrix $B \in H^{M \times N}$, whose the $(p, q)$th entry noted with $B_{p,q} \in H$, the complex adjoint matrix $B^\sigma \in C^{2M \times 2N}$ can be expressed as

$$ B^\sigma = \begin{bmatrix} B_1 & B_2^* \\ -B_2 & B_1^* \end{bmatrix} \quad (3) $$

where $B_1 \in C^{M \times N}$, and $B_2 \in C^{M \times N}$ are the complex matrices, and the Cayley-Dickson notation of $B \in H^{M \times N}$ can be represented as $B_1 + B_2j$.

Specially, $B$ is a square quaternion matrix, the eigenvalue decomposition of the adjoint matrix $B^\sigma$ can be expressed as

$$ B^\sigma = \begin{bmatrix} U_1 & U_2 \\ -U_2 & U_1 \end{bmatrix} \text{diag}(\Sigma_p) \begin{bmatrix} U_1 & U_2^* \\ 0 & \Sigma_p^* \end{bmatrix} = U_c \Sigma \Sigma_c^H \quad (4) $$

where $\Sigma_p = \text{diag}(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_p)$ is the complex eigenvalues that appears in conjugated pairs in (4), $\Sigma_c$ and $U_c$ are the adjoint matrices of $\Sigma$ and $U = U_1 + U_2j$, respectively. Therefore, the eigenvalue decomposition of the quaternion matrix $B$ can be expressed as

$$ B = (U_1 + U_2j) \Sigma (U_1 + U_2j)^H \quad (5) $$

Further, it should be noted that when a quaternion square matrix $B$ is a Hermitian matrix satisfying $B = B^H$, its eigenvalues are real numbers, namely $\Sigma = \Sigma^*$, which are also the eigenvalues of the complex adjoint matrix $B^\sigma$.

#### B. Polarization Model

As shown in Fig.1, we consider an array model that $K$ NF narrowband completely polarized signals $s_k(t)$ ($k = 1, \ldots, K$) are impinging on a symmetric uniform linear array (ULA)
of $2M + 1$ pairs elements equipped with COLD antenna. The location of the NF source is parameterized by the tuple $(\theta_k, r_k)$, where $\theta_k \in (-\frac{\pi}{2}, \frac{\pi}{2})$ denotes the angle of the $k$th source measured from the $z$-axis and $r_k$ represents the range between the source and the origin of coordinates. The $d$ is the inter-element spacing of the ULA with $d \leq \lambda/4$, where $\lambda$ represents the wavelength of the incident signals.

The COLD array is equipped with one loop and one dipole antennas, which is aligned with the $X$ and $Y$ axes. And the loop measures the magnetic component of the completely polarized signal, the dipole measures the electric component. Hence, the above two components of the $k$th signal can be expressed a $2 \times 1$ vector as

$$
\xi_k = \begin{bmatrix} \xi_{k1} \\ \xi_{k2} \end{bmatrix} = \begin{bmatrix} -\sin \psi_k \cos \theta_k \cos \varphi_k \\ \cos \theta_k \cos \varphi_k \end{bmatrix} \begin{bmatrix} \cos \gamma_k \\ \sin \gamma_k e^{i\eta_k} \end{bmatrix}.
$$

With the assumption that all source are in $y$-$z$ plane, we have $\psi_k = 90^\circ$, thus $\xi_k$ can be rewritten as

$$
\xi_k = \begin{bmatrix} \xi_{k1} \\ \xi_{k2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \cos \gamma_k \\ \sin \gamma_k e^{i\eta_k} \end{bmatrix} = \begin{bmatrix} -\cos \gamma_k \\ -\sin \gamma_k e^{i\eta_k} \end{bmatrix}
$$

with $\gamma_k \in (0, \frac{\pi}{2})$ and $\eta_k \in (0, 2\pi)$ representing the polarization angle and polarization phase difference, respectively. Further, the $2 \times 1$ observed signal vector at time $t$, which is produced by the $m$th COLD component, can be given by

$$
x_m(t) = \begin{bmatrix} x_{1,m}(t) \\ x_{2,m}(t) \end{bmatrix} = \sum_{k=1}^{K} \alpha_m(\theta_k, r_k) \begin{bmatrix} -\cos \gamma_k \\ -\sin \gamma_k e^{i\eta_k} \end{bmatrix} s_k(t) + \begin{bmatrix} n_{1,m}(t) \\ n_{2,m}(t) \end{bmatrix}.
$$

In (8), we consider that the array center, viz. the $0$th sensor is the phase reference point. $x_{1,m}(t)$ represents the data received by the $m$th electric loop and $x_{2,m}(t)$ represents the data received by the $m$th magnetic dipole, which can be respectively expressed as

$$
x_{1,m}(t) = \sum_{k=1}^{K} \alpha_m(\theta_k, r_k) \xi_{k1} s_k(t) + n_{1,m}(t)
$$

$$
x_{2,m}(t) = \sum_{k=1}^{K} \alpha_m(\theta_k, r_k) \xi_{k2} s_k(t) + n_{2,m}(t)
$$

where $M = -M \ldots -1, 0, 1 \ldots M$, $\alpha_m(\theta_k, r_k) = \exp\left(j \frac{2\pi r \sin \gamma_k}{\lambda} \right)$ is the spatial phase factor of the NF incident signals, which can be simplified as $\alpha_m(\theta_k, r_k) = \exp\left(j \frac{m \gamma_k}{r_k} \right)$ with the Fresnel zone hypothesis, and $w_k = -2 \pi d \sin \psi_k / \lambda$ and $\phi_k = \pi \lambda^2 \cos^2 \theta_k / (\lambda r_k)$ are called electric angles. $n_{1,m}(t)$ and $n_{2,m}(t)$ are the additive zero-mean white Gaussian noise of the $m$th COLD component, respectively.

### III. THE PROPOSED ALGORITHM

By collecting a total of $T$ snapshots of the output vector, the key-point of the present problem is to estimate the angle $\theta_k$, and range $r_k$ and polarization parameters $\gamma_k$ and $\eta_k$ of the $k$th source. In this section, based on the strictly noncircularity, we propose a novel NCAQ-MUSIC algorithm for parameters (i.e., DOA, range) of the NF polarized sources with an ULA COLD array. The DOA and range parameters can be decoupled, and then estimated by twice 1-D RARE principle.

Firstly, the NCAQ model is constructed by introducing four quaternion polar models. By arranging the $2M + 1$ COLD pair components into two separate columns, then (9) can be rewritten into the following matrix form as,

$$
x_1(t) = AV_1s(t) + n_1(t)
$$

$$
x_2(t) = AV_2s(t) + n_2(t)
$$

where $A = [a_1, a_2, \ldots a_k]$ is the array manifold matrix with each column having $a_k = [a_{-M,k} \ldots a_{M,k}]^T$.

$s(t) = [s_1(t), s_2(t), \ldots, s_k(t)]^T$ denotes the incoming signal vector. $V_1(t) = \begin{bmatrix} 1, 1 \end{bmatrix}$, $V_2(t) = \begin{bmatrix} 1, 1 \end{bmatrix}$, \ldots denotes the diagonal matrix related to polar information, and $n_m(t) = \begin{bmatrix} n_{m,(-M)}(t), \ldots, n_{m,M}(t) \end{bmatrix}^T$ is the noise matrix.

Then four quaternion-based array output vector can be constructed as follows

$$
x_{11}(t) = x_1(t) + x_2(t) = \text{A}V_1s(t) + n_1(t) + n_2(t)
$$

$$
y_{11}(t) = AV_1s(t) + n_1(t)
$$

$$
x_{12}(t) = x_1(t) - x_2(t) = \text{A}V_2s(t) + n_1(t) - n_2(t)
$$

$$
y_{12}(t) = AV_2s(t) + n_2(t)
$$

$$
x_{1}(t) = x_1(t) + x_2(t) = \text{A}V_1s(t) + n_1(t) + n_2(t)
$$

$$
y_{1}(t) = AV_1s(t) + n_1(t)
$$

$$
x_{12}(t) = x_1(t) - x_2(t) = \text{A}V_2s(t) + n_1(t) - n_2(t)
$$

$$
y_{12}(t) = AV_2s(t) + n_2(t)
$$

where $V_1(t) = \text{diag}(\xi_{1,1} + \xi_{1,2}, \ldots, \xi_{1,k,1} + \xi_{1,k,2})$, $V_2(t) = \text{diag}(\xi_{2,1,1} + \xi_{2,1,2}, \ldots, \xi_{2,k,1} + \xi_{2,k,2})$ and $V_1 = \text{diag}(\xi_{1,1}^*, + \xi_{1,2}, \ldots, \xi_{1,k,1} + \xi_{1,k,2})$ are all $K \times K$ -dimensional diagonal quaternion matrices. $n_1, n_1, n_2, n_2$ are all $(2M+1) \times 1$ dimensional quaternion noise vector.

To proceed, we define a new column vector by concatenating $x_{11}(t)$, $x_{12}(t)$, $y_{11}(t)$ and $y_{12}(t)$, which can be represented as

$$
z(t) = \begin{bmatrix} x_{11}(t) \\ y_{11}(t) \\ x_{12}(t) \\ y_{12}(t) \end{bmatrix}.
$$

And the calculation of NCAQ covariance matrix $R$ is (13), as shown at the bottom of the next page.

For the strictly non-circular signal $s(t)$, it satisfies the relationship as follow

$$
E \left[ s^2(t) \right] = h_x e^{i\phi} E \left[ s(t) s^*(t) \right]
$$

$$
E \left[ s^2(t) \right] = h_x e^{i\phi} E \left[ s(t) s^*(t) \right]
$$

$$
E \left[ s^2(t) \right] = h_x e^{i\phi} E \left[ s(t) s^*(t) \right]
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$$
E \left[ s^2(t) \right] = h_x e^{i\phi} E \left[ s(t) s^*(t) \right]
$$

$$
E \left[ s^2(t) \right] = h_x e^{i\phi} E \left[ s(t) s^*(t) \right]
$$

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where $\psi_k$ is the non-circular phase and $h_k = 1$ is the maximal noncircular rate. For signal vector $s(t)$ composed of $K$ uncorrelated signals, its unconjugated covariance matrix can be expressed as

$$E \left[ s(t) s^T(t) \right] = \text{diag} \left\{ e^{i\psi_1} E \left[ s_1(t) s_1^*(t) \right], \ldots, e^{i\psi_K} E \left[ s_K(t) s_K^*(t) \right] \right\}$$

$$= \Psi R_s$$  \hspace{1cm} (15)

where $\Psi = diag (e^{i\psi_1}, e^{i\psi_2}, \ldots, e^{i\psi_K})$ is the diagonal matrix of non-circular phase, $R_s$ is the covariance matrix of incident signals. Substituting (15) into (13), and using the non-circular property, we can extend the covariance matrix $R$ as (16), as shown at the bottom of the page.

In actual situation, the theoretical array covariance matrix is unavailable. Assuming that the number of snapshots are $T$, the practical array covariance matrix can be approximated as

$$\hat{R} = \frac{1}{N} \left[ zz^H \right] = \frac{1}{N} \left[ (z_1 + z_{2j})^H (z_1 + z_{2j}) \right] = R_{s1} + R_{s2j}$$  \hspace{1cm} (17)

where $z_1$ and $z_2$ are $(8M + 4) \times T$-dimension snapshot data matrices. The eigenvalue decomposition of covariance matrix $R$ is shown as follows

$$R = U A U^H$$

$$= (U_1 + U_2) A (U_1 + U_2)^H$$

$$= U_a A_a U_a^H + U_n A_n U_n^H.$$  \hspace{1cm} (18)

Similar to the orthogonality in the complex-valued MUSIC method, the signal subspace $U_s$ and the noise $U_n$ subspace of the quaternion matrix are still orthogonal. Thus, we can construct the function as follow

$$\left[ \begin{array}{c} A V_{s1} \\ A V_{s1} \\ A^* V_{s2} \psi^* \\ A^* V_{s2} \psi^* \end{array} \right] U_n U_n^H \left[ \begin{array}{c} A V_{s1} \\ A V_{s1} \\ A^* V_{s2} \psi^* \\ A^* V_{s2} \psi^* \end{array} \right] = 0.$$  \hspace{1cm} (19)

By dividing the noise subspace $U_n$ into four block matrices, namely $U_{n1}, U_{n2}, U_{n3}$ and $U_{n4}$, the spectrum function of NCAQ-MUSIC can be simplified as (20) as shown at the bottom of the next page, where $M(\theta_k, r_k)$ is the function of the $k$ th source’s DOA and range parameters, the rank of $P(\gamma_k, \eta_k, \psi_k)$ is 1. Therefore the rank of $f_{\text{NCAQ}}(\theta_k, r_k, \gamma_k, \eta_k)$ and $M(\theta_k, r_k)$ are equal, the spectrum function can be changed to

$$f_{\text{NCAQ}}(\theta_k, r_k) = \det [M(\theta_k, r_k)].$$  \hspace{1cm} (21)

When $(\theta, r) = (\theta_k, r_k)$ are the true DOAs and ranges, $M(\theta_k, r_k)$ is not full rank. The estimation of DOA and range $(\hat{\theta}_k, \hat{r}_k)$ can be acquired by searching for the minima of $f_{\text{NCAQ}}(\theta_k, r_k)$. However, the parameter estimation in (21) still needs 2-D peak searching that involves huge computation. With the NF approximation model, $M(\theta_k, r_k)$ can be further simplified to reduce the computational effort. For the NF incident signals, $\alpha_m(\theta_k, r_k)$ can be approximated as

$$\alpha_m(\theta_k, r_k) = \zeta (\hat{\theta}_k) \nu (\hat{\theta}_k, \hat{r}_k)$$

$$= \begin{bmatrix} e^{i(M \omega_k)} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 1 \\ e^{iM \omega_k} & \ldots & 0 \end{bmatrix}.$$  \hspace{1cm} (22)
Substituting (22) into (20), $\mathbf{M}(\theta_k, r_k)$ can be rewritten as

$$
\mathbf{M}(\theta_k, r_k) = \begin{bmatrix}
\mathbf{v}(\theta_k, r_k) & 0 & 0 & 0 \\
0 & \mathbf{v}(\theta_k, r_k) & 0 & 0 \\
0 & 0 & \mathbf{v}^*(\theta_k, r_k) & 0 \\
0 & 0 & 0 & \mathbf{v}^*(\theta_k, r_k)
\end{bmatrix}
$$

where

$$
\mathbf{C}(\theta_k) = \begin{bmatrix}
\zeta(\theta_k) & 0 & 0 & 0 \\
0 & \zeta(\theta_k) & 0 & 0 \\
0 & 0 & \zeta^*(\theta_k) & 0 \\
0 & 0 & 0 & \zeta^*(\theta_k)
\end{bmatrix}^H
$$

$$
\times \begin{bmatrix}
\mathbf{U}_n & 0 & 0 & 0 \\
0 & \mathbf{U}_n & 0 & 0 \\
0 & 0 & \mathbf{U}_n & 0 \\
0 & 0 & 0 & \mathbf{U}_n
\end{bmatrix}
$$

$$
\times \begin{bmatrix}
\mathbf{v}(\theta_k, r_k) & 0 & 0 & 0 \\
0 & \mathbf{v}(\theta_k, r_k) & 0 & 0 \\
0 & 0 & \mathbf{v}^*(\theta_k, r_k) & 0 \\
0 & 0 & 0 & \mathbf{v}^*(\theta_k, r_k)
\end{bmatrix}
$$

$$
= \mathbf{Q}^H(\theta_k, r_k) \mathbf{C}(\theta_k) \mathbf{Q}(\theta_k, r_k) \quad (23)
$$

where

$$
\mathbf{f}_{\text{NCAQ}}(\theta_k) = \det[\mathbf{C}(\theta_k)]. \quad (25)
$$

The estimated DOAs $\theta_k$ can be obtained from $\mathbf{f}_{\text{NCAQ}}(\theta_k)$ through 1-D peak searching. After that, substituting (25) into (21), the range $\hat{r}_k$ parameters of all sources can be obtained through several 1-D peak searching.

Until now, the DOAs, ranges and polarization estimation have automatically paired and obtained through the NCAQ-MUSIC method, which can be summarized as in Table 1.

**IV. SIMULATION RESULTS**

In this section, the performance of the proposed NCAQ-MUSIC algorithm is compared with two existing algorithms (QDR-MUSIC, QNC-MUSIC). We use three examples to verify the performance of the proposed algorithm. The CRB
in [5] can be used as a performance benchmark. The simulation parameters are given as follow. We consider a ULA with 5 elements ($M = 2$) in the first experiment, and 9 elements ($M = 4$) in other two cases; the distance of each element is $d = \lambda/4$. The impinging sources are uncorrelated signals, and the noise is assumed to be additive white Gaussian noise.
Three algorithms decrease as SNR increases. Furthermore, the results, the RMSEs of DOA and range estimation of the are plotted from the Fig 5(a)-5(b). From the experimental of the 2-D parameters (DOA, range) of the three methods number of snapshots is set to 500. The averaged RMSE experiment. The SNR varies from 0 dB to 20 dB and the corresponding polarization parameters are the same with the first algorithm. The location of the two NF sources is charac-

$$\text{RMSE} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \left( \hat{\theta}_k - \theta_k \right)^2}$$  \hspace{1cm} (26)$$

where $\hat{\theta}_k$ is the estimate of the parameter, $\theta_k$ is the true value of the parameter. In the first experiment, the location of the two NF sources is characterized by $(-10^\circ, 0.6\lambda)$ and $(20^\circ, 0.8\lambda)$ respectively; the corresponding polarization parameters are $\gamma_1 = 30^\circ$, $\eta_1 = 85^\circ$, $\gamma_2 = 55^\circ$, $\eta_2 = 100^\circ$ respectively. The SNR is 0dB and the number of snapshots is 300. The estimations of DOA and range of three algorithm are shown from Fig. 2 to Fig. 4. From the experimental results, it can be observed that the parameters of DOA and range have been correctly estimated by the NCAQ-MUSIC and QNC-MUSIC, whose peaks of DOA and range are sharper than QDR-MUSIC.

In the second experiment, we investigate the effect of SNR (signal-to-noise ratio) on the performance of the proposed algorithm. The location of the two NF sources is characterized by $(5^\circ, 2\lambda)$ and $(15^\circ, 3\lambda)$ respectively. The corresponding polarization parameters are the same with the first experiment. The SNR varies from 0 dB to 20 dB and the number of snapshots is set to 500. The averaged RMSE of the 2-D parameters (DOA, range) of the three methods are plotted from the Fig 5(a)-5(b). From the experimental results, the RMSEs of DOA and range estimation of the three algorithms decrease as SNR increases. Furthermore, the proposed algorithm has higher performance accuracy than existing algorithms.

In the third experiment, the effect of snapshot number on the performance of the proposed algorithm is explored. The parameters of sources are the same with the second experiment. The snapshots varies from 10 to 1000 and the SNR is set to 10dB. The averaged RMSE of the 2-D parameters of the three methods are shown from the Fig 6(a)-6(b). From the figures, the RMSEs of the three algorithms decrease along with snapshots increases. Also, the proposed method is superior consistently over the other methods in estimation performance of both DOA and range, especially with the small snapshot case.

V. CONCLUSION

In this paper, based on noncircularity, we have proposed a localization method to estimate 2-D parameters of NF source with a ULA COLD array. By constructing the NCAQ model, the DOA and range parameters can be estimated through several 1-D searches, which are automatically paired. The analysis and simulation show that, compared with existing methods, the proposed NCAQ-MUSIC method has better estimation performance in both DOA and range.

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