Dynamical study of $QQ-\bar{u}\bar{d}$ mesons

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It has been recently conjectured by Selem and Wilczek [1] the existence of a $s\bar{s}-[\bar{u}\bar{d}]$ meson due to strong correlations between the two light antiquarks. We make a detailed study of this system within a dynamical quark model which has proven to be successful in reproducing the most important features of low-energy hadron phenomenology. Our results, obtained within a parameter-free calculation, show that the antidiquark component of the $ss\bar{a}\bar{d}$ system indeed entails the stronger attraction, and drives its energy much lower than the $N\Xi$ threshold, but still above the $\bar{K}^0K^{*-}$ or $\bar{K}^0K^-$ thresholds. We have also studied the $cc\bar{u}\bar{d}$ and $bb\bar{u}\bar{d}$ systems. Exotic mesons are only expected to exist in the limit of large mass for the two-quark subsystem, $bb\bar{u}\bar{d}$, since the calculated mass is below the $B^0B^{*-}$ or $B^0\bar{B}^-$ thresholds.

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It has been recently re-emphasized [1–3] the potential importance of strong diquark (antidiquark) correlations in hadronic physics [4,5]. Theoretically the idea of diquark (antidiquark) correlations inside hadrons is a consequence of color cancellation. The disturbance produced by the color charges of two quarks in empty space can be halved by bringing them together into a single 3 representation of the color SU(3) group. If this is joined with the more favorable spin-singlet state and Fermi statistics, the quarks must be in the antisymmetric 3 representation of flavor SU(3). Besides, one should expect that any effect disrupting the correlations will induce a repulsive force. Among such effects we may quote the presence of an additional diquark or a spectator quark. Such ideas suggest that the easiest way of constructing low-energy exotics could be based on strongly correlated diquarks (antidiquarks) as building-blocks.

However, these arguments are rather qualitative, and merely based on the group theoretical structure of QCD. To study quantitatively whether or not such strong correlations between the light quarks (antiquarks) are indeed present, one needs QCD-based dynamical studies such as lattice QCD, although this eventually must be checked by experiments. In view of present status of the lattice QCD simulations [6], it is still meaningful to use phenomenological models which contain the main features of QCD, once the model parameters are calibrated and constrained by as many observables as possible. Here, we perform such a consistent, parameter-free dynamical calculation for the study of the correlations between light antiquarks, and investigate the possible existence of the exotic meson, $ss-[\bar{u}\bar{d}]$, which has been recently conjectured by Selem and Wilczek [1]. For completeness, we have also analyzed the $cc\bar{u}\bar{d}$ and $bb\bar{u}\bar{d}$ systems.

The present study has been done within the framework of a constituent quark model which has been successfully applied to study the baryon spectra and the baryon-baryon interaction [7]. This model has been generalized to include also strange ($s$), charm ($c$), and beauty ($b$) flavors, and it has also been shown to give a reasonable description of the meson spectra [8]. The description of experimental data gets improved when four-quark ($qq\bar{q}\bar{q}$) components are also considered [9,10]. The model parameters have been strongly constrained by the study of different hadron observables, what represents an advancing feature compared to studies based on models designed ad hoc for a particular problem.

The model is based on the assumption that the $u$, $d$, and $s$ constituent quarks acquire their masses due to the spontaneous breaking of the original $SU(3)_L \otimes SU(3)_R$ chiral symmetry at some momentum scale, which is one of the most important nonperturbative phenomena for low-energy hadron structure. In this domain of momenta quarks are quasiparticles with constituent masses interacting through scalar (sigmas, OSE) and pseudoscalar (pions, OPE; kaons, OKE; and etas, OEE) boson-exchange potentials. Note that for the case of heavy quarks, $c$ and $b$, boson-exchange potentials are not present in the model [10], since chiral symmetry is badly broken already at the level of the current quark masses. Beyond the chiral symmetry breaking scale one expects the dynamics being governed by QCD perturbative effects. They are taken into account through the one-gluon-exchange (OGE) potential, a standard color Fermi-Breit interaction. Finally, any model imitating QCD should incorporate confinement (CON). Lattice calculations in the quenched approximation for heavy quarks show that the confining interaction is linearly dependent on the interquark distance. The presence of sea quarks, apart from valence quarks (unquenched approximation), suggests a screening effect on the potential when increasing the interquark distance. Creation of light quark-antiquark pairs out of vacuum in between the quarks becomes energetically preferable, resulting in a complete screening of quark color charges at large distances. String breaking has been definitively confirmed through lattice calculations [11] in coincidence with the quite rapid crossover from a linear rising to a flat potential well established in $SU(2)$ Yang-Mills theories [12]. Explicit expressions for
the interaction potentials derived from the nonrelativistic reduction of the Lagrangian density in the static approximation, and a more detailed discussion of the model can be found in Ref. [8].

For the description of the most general $QQ\bar{u}\bar{d}$ $(Q = s, c$ or $b)$ system we introduce the Jacobi coordinates,
\[ \mathbf{x} = \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{y} = \mathbf{r}_3 - \mathbf{r}_4, \]
\[ \mathbf{z} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} = \frac{m_3 \mathbf{r}_3 + m_4 \mathbf{r}_4}{m_3 + m_4}, \quad \mathbf{R} = \frac{\sum m_i \mathbf{r}_i}{\sum m_i}, \]
(1)
where $1$ and $2$ ($3$ and $4$) stand for quarks (antiquarks). The ground state energy of the four-body problem can be estimated by a variational method using a trial wave function that includes all possible color-flavor-spin components relevant to a given configuration. For each component, $| \phi_i \rangle$, such a basis wave function will be a tensor product of color ($c_i$), flavor ($f_i$), spin ($x_i$) and spatial ($R_i$) parts,
\[ | \phi_i \rangle = | c_i(1234) \rangle \otimes | f_i(1234) \rangle \otimes | x_i(1234) \rangle \otimes | R_i(1234) \rangle. \]
(2)
The most general spatial wave function can be expressed as a combination of six scalar components,
\[ | R_i(1234) \rangle = R_i(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{x} \cdot \mathbf{y}, \mathbf{x} \cdot \mathbf{z}, \mathbf{y} \cdot \mathbf{z}). \]
(3)
The variational spatial wave function is taken to be a linear combination of generalized Gaussians,
\[ | R_i(1234) \rangle = \sum_{j=1}^{n} \beta_i^{(j)} | R_i^{(j)} \rangle = \sum_{j=1}^{n} \beta_i^{(j)} e^{-a_i^{(j)} \mathbf{x}^2 - b_i^{(j)} \mathbf{y}^2 - c_i^{(j)} \mathbf{z}^2 - d_i^{(j)} \mathbf{x} \cdot \mathbf{y} - e_i^{(j)} \mathbf{x} \cdot \mathbf{z} - f_i^{(j)} \mathbf{y} \cdot \mathbf{z}}, \]
(4)
where $n$ is the number of terms to expand the spatial wave function of each color-flavor-spin component, and $a_i^{(j)}, b_i^{(j)}, \ldots, f_i^{(j)}$ are the variational parameters.

With respect to the color wave function, $| c_i(1234) \rangle$, one can couple the two quarks (1,2) and the two antiquarks (3,4) to a color singlet state in different ways:
\[ | 1_{13}, 1_{24} \rangle, \quad | 8_{13}, 8_{24} \rangle; \]
(5)
\[ | 1_{14}, 1_{23} \rangle, \quad | 8_{14}, 8_{23} \rangle; \]
(6)
\[ | 3_{12}, 3_{34} \rangle, \quad | 6_{12}, 6_{34} \rangle. \]
(7)
The couplings in Eqs. (5) and (6) are convenient for asymptotic meson-meson channels (or meson-meson molecules) while those in Eq. (7) are more appropriate for tetraquark bound states. With our choice of the Jacobi coordinates the color basis in Eq. (7) is more suitable to deal with the Pauli principle in an easier way.

The spin part can be written as,
\[ | x_1(1234) \rangle = \langle 12 \rangle s_{12} \langle 34 \rangle s_{34} | \delta_s \rangle, \]
(8)
where the spin of the two quarks is coupled to $S_{12}$ and that of the antiquarks to $S_{34}$.

Concerning the flavor part, $| f_i(1234) \rangle$, since the heavy quarks (those with flavor $s, c$ or $b$) have isospin zero, they do not contribute to the total isospin. Therefore one can classify the tetraquark wave function by the isospin of the light quarks $I = 0, 1$. Taking into account all degrees of freedom, the Pauli principle must be satisfied for each subsystem of identical quarks (antiquarks). It restricts the quantum numbers of the basis states, that justifies to use the $[\{QQ\}(\bar{u}\bar{d})]$ coupling.

Using the wave functions described above, we search for a variational solution for the Hamiltonian. The color, flavor and spin parts are integrated out and the coefficients $\beta_i^{(j)}$ of the spatial wave function are obtained by solving the system of linear equations,
\[ \sum_{i=1}^{n} \beta_i^{(j)} \langle R_i^{(j)} | H | R_i^{(j)} \rangle - E_i^{(j)} \delta_{i,j} = 0 \]
(9)
for all $k, i'$, once the eigenvalues $E_i^{(j)}$ are obtained by a minimization procedure with respect to the variational parameters. The stable tetraquark states are identified by comparing the obtained eigenvalues with the corresponding physical thresholds. If they are above the threshold they would be very broad objects, very hard to detect experimentally.

In a realistic model tetraquarks will not overpopulate the meson spectra, in fact they may complement two-quark components and, indeed, they seem to be necessary in order to understand the rich meson phenomenology [9,10]. This is due, on the one hand, to the constituent mass of the quarks, and on the other one, to the finite spectra generated by screened confining potentials [13]. Only positive parity tetraquark states, those that do not need internal orbital angular momentum between the constituents, may appear in the low-energy region of the meson spectra and they could mix with $qq$ states with the same quantum numbers. Negative parity four-quark states need a unit of orbital angular momentum what means an average excitation energy of 800–900 MeV [14]. These ideas have been recently used to explain the abnormal number of low-energy scalar-isoscalar mesons [9] and also the unexpected low masses of positive parity ($0^+$ and $1^+$) open-charm mesons [10]. They are perfect examples of the way how the enlargement of the Fock space may help in the understanding of meson phenomenology. As explained in these works, only those states with exotic quantum numbers may appear as pure four-quark resonances on the meson spectra. Unfortunately, the present uncertainties on the experimental data concerning exotic
Let us concentrate on the particular meson state, $ss-[\bar{u}\bar{d}]$, conjectured in Ref. [1]. It has the property of the $[\bar{u}\bar{d}]$ subsystem being an antidiquark state, which means the two antiquarks are in a color ($3_s$), flavor ($I_{[\bar{u}\bar{d}]} = 0$), and spin ($S_{[\bar{u}\bar{d}]} = 0$) antisymmetric state. It requires a completely symmetric radial wave function for the two antiquarks to satisfy the Fermi statistics. As the antidiquark component $[\bar{u}\bar{d}]$ should be in a relative $S$-wave, one can neglect the crossing terms in the trial radial wave function, those depending on the scalar product of different Jacobi coordinates in Eq. (4). In order to obtain a color singlet wave function, the two $s$ quarks must be in a color antisymmetric, $3_s$, state. Being flavor symmetric, the corresponding spin wave function may be either in (i) an antisymmetric, $S_{ss} = 0$, state that would require the antidiquark configuration mixing.

In Table I we present our results for the antidiquark configuration, nondiquark configuration, and full calculation for the $ss\bar{u}\bar{d}$ system. The same calculation has been repeated for the $cc\bar{u}\bar{d}$ and $bb\bar{u}\bar{d}$ systems, and the results are presented in Tables II and III, respectively.

The first important conclusion that can be extracted from the results of Tables I, II, and III is that the energy of the antidiquark configuration is always the lowest. It is interesting to note how the pseudoscalar force acting between the light quarks is responsible for that, since the results for the antidiquark and nondiquark configurations are almost degenerate if only the confinement and one-gluon exchange are retained. The reason for this stems on the strangeness $-2$ would require also to consider other vectors in the Hilbert space. In particular, the same state could also be constructed from a different vector, where the two antiquarks would not be an antidiquark state while it still has a completely symmetric radial wave function,

$$[[6_s, S = 1, I = 0]_{\bar{u}\bar{d}}, [[6_s, S = 0, I = 0]]_{ss}, (S = 1, I = 0)).$$

(11)

This vector, which will be referred to as the nondiquark component of the $ss\bar{u}\bar{d}$ system, should be considered in the calculation of the four-quark state without requiring the antidiquark configuration, whereas, it will not be included if only the antidiquark configuration is imposed. For the full calculation, both the antidiquark and nondiquark configurations will be included with the corresponding configuration mixing.

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strangeness $-2$ would require also to consider other vectors in the Hilbert space. In particular, the same state could also be constructed from a different vector, where the two antiquarks would not be an antidiquark state while it still has a completely symmetric radial wave function,
different symmetry for both components in color-spin and flavor-spin spaces. While both are symmetric in color-spin space, the antidiquark (nondiquark) component is symmetric (antisymmetric) in flavor-spin space. Therefore, if strong diquark correlations were dictated by QCD for light quarks, the dynamical explanation could not rely on the simple one-gluon exchange dynamics, but it would need meson-exchange forces between the constituent quarks. The similar effect has been also observed in the case of baryon spectra, where pseudoscalar meson exchanges between the constituent quarks are able to revert the relative position in the energy spectra of the nucleon Roper resonance, with a dominant flavor-spin symmetric wave function, with respect to negative parity states, with a flavor-spin antisymmetric wave function [16]. It is also interesting to notice that the antidiquark and nondiquark components are not exactly degenerate when only the confining interaction is taken into account. This can be easily understood by looking at Table IV, where we present the contribution of the interaction between $QQ, V_{12}, \bar{u}\bar{d}, V_{34}$, and $\bar{Q}n\ (n = u, d), V_{13}$, for the $QQ\bar{u}\bar{d}$ system as a function of the mass of $Q, m_Q$. The minimization procedure modifies the variational parameter $d_{ij}^{(n)}$ in Eq. (4) for the $x^2$ coordinate due to the smaller size of the $Q\bar{Q}$ subsystem when the mass $m_Q$ increases. As a consequence it gives a smaller contribution to the energy of the system. In other words, the dependence on the mass of the quark is introduced into the calculation through the variational parameters.

As predicted by Selem and Wilczek [1], the mixing between the antidiquark and nondiquark components of the wave function diminishes when increasing the mass of the heavy quarks (see Table V), in such a way that for the $b$ quark case the nondiquark component gives almost no contribution to the ground state energy of the system. This effect, interpreted as the less capacity of the spin of a heavy quark to disrupt the correlation of the diquark (antidiquark), comes from the $1/(m_{Q}\Lambda_{Q})$ (constituent quark masses) dependence of the one-gluon exchange interaction, which is responsible for the mixing between these components. Therefore, the mixing decreases with increasing the mass of the heavy quark. A similar evidence, the less capacity of the spin of the heavy quarks to disrupt the system, has also been observed in the spin-orbit splitting of the $\Lambda - \Lambda_{c}^{*}$ and $\Lambda_{b} - \Lambda_{b}$ hypernuclei [17], where $s$, $c$ and $b$ quarks exclusively carry the total spin of the $\Lambda$, $\Lambda_{c}$ and $\Lambda_{b}$, respectively, and $u$ and $d$ quarks are coupled to an isospin zero and spin zero diquark in each baryon.

Regarding the possibility of observing these systems, the results obtained are always far above their corresponding lowest two-meson thresholds, as indicated in Tables I, II, and III, being the only exception the $bb\bar{u}\bar{d}$ system.

Experimentally the possibility to detect a $QQ\bar{u}\bar{d}$ meson relies on two different aspects. First of all the rate of production of $QQ\bar{u}\bar{d}$, and second the existence of decay modes that can provide a unique signature. For the production at hadronic or $e^+e^-$ colliders one needs to produce two pairs of charm or bottom quarks. These pairs should be close spatially and the quarks within each quark-antiquark pair should have small relative momenta in order to combine in a two-quark, $cc$ or $bb$, state. Finally, these two-quark states should pick up an antidiquark $[\bar{u}\bar{d}]$ to form the desired $QQ - [\bar{u}\bar{d}]$ system. The production rates for the case of charm quarks have been estimated in Refs. [18,19].

TABLE IV. Expectation value, in MeV, of different contributions of the confining interaction for the different components of the $QQ\bar{u}\bar{d}$ system and for two different values of the mass of the quark in the two-quark subsystem.

| $V_{12}$ | $V_{34}$ | $V_{13}$ |
|---------|---------|---------|
| $\langle V_{12} \rangle$ | +458 | +527 | +259 |
| $\langle V_{34} \rangle$ | -263 | -284 | +633 |
| $\langle V_{13} \rangle$ | +241 | +513 | +225 |
| $\langle V_{15} \rangle$ | -176 | -264 | +536 |

TABLE V. Probability, in %, of the antidiquark component, $QQ - [\bar{u}\bar{d}]$, as a function of the mass of the quark, in MeV, in the two-quark subsystem. We also give the mass of the $QQ\bar{u}\bar{d}$ system in MeV.

| $m_Q$ | $M_{QQ\bar{u}\bar{d}}$ | $P(QQ - [\bar{u}\bar{d}])$ |
|-------|------------------|------------------|
| 313   | 1431             | 96.09            |
| 555   | 1696             | 97.59            |
| 755   | 1980             | 98.18            |
| 1255  | 2815             | 99.11            |
| 1555  | 3352             | 99.37            |
The signal of the strong antidiquark correlation would come from decay channels preferring to keep the antidiquark structure. So, instead decaying by splitting into a two-meson system, it would proceed through a two-baryon system as it would be $\bar{N}\Xi$, for the charm case and $\bar{N}\Xi_{bb}$ for the bottom case. The absence of a dynamical enhancement of the antidiquark component would open the decay into two mesons. For the $ss\bar{u}\bar{d}$ system, the antidiquark component is strong enough so as to force it to decay into a two-baryon system via $(ss - [\bar{u}\bar{d}]) \to \bar{N}\Xi$, one can expect a natural decay in an $S$-wave, which needs a $J^P = 1^-$ state for the system. As the energy excitation for a unit of orbital angular momentum costs about 800–900 MeV [14], this would make the system to be in the continuum above the $\bar{N}\Xi$ threshold, therefore being broad and difficult to detect. The dynamical enhancement of the diquark component is one of the possible reasons to explain the decay of the $\Lambda(3/2^-)(1520)$ 45% of the time to $NK$ channels in order to retain the diquark structure of the $\Lambda$ inside the $N$. Finally, let us remark that we only support the possible existence of $QQ\bar{u}\bar{d}$ states for $Q = b$ and with more uncertainty for $Q = c$, but never for $Q = s$ [20]. In the bottom case the predicted state should be very narrow and easy to observe (if produced) since it is far below the two-meson and two-baryon physical thresholds.

To summarize, we have made a dynamical, parameter-free, calculation for the $QQ\bar{u}\bar{d}$ system ($Q = s$, $c$, $b$) within a realistic constituent quark model. We have found that the antidiquark configuration of these systems always gives a lower energy than the nondiquark configuration. The mixing between the antidiquark and nondiquark states is due to the one-gluon exchange potential, and because of its $1/(m_m \gamma)$ dependence, it decreases when increasing the heavy quark mass. As a consequence one can expect that the conjectured mesons $QQ - [\bar{u}\bar{d}]$ could be stable for $Q = b$, but we do not find any reason why these systems should be bound for $Q = s$. Moreover, for light quarks, $Q = s$, there is no dynamical reason why the antidiquark component should be favored compared to the nondiquark one (within an uncorrelated quark model). If the existence of any such systems with two light quarks, $Q = s$, and therefore with a strongly correlated light antidiquark would be confirmed via the postulated baryon-antibaryon final channel, it would mean that a dynamical mechanism responsible for the correlations has not been considered in our simple realizations of models for QCD (based on uncorrelated quarks), and the question for the existence of exotic systems, such as the pentaquark, should be addressed in a corresponding manner. On the other hand, the present results may imply that the pentaquark with a heavy $b$ quark, $[ud][ud]b$ with a negative parity, may have a chance to be stable, if the predicted repulsion between diquarks is not strong enough to destroy the system. This has a merit of further study within the same model.

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