Gravitational spin-orbit coupling through third-subleading post-Newtonian order: from first-order self-force to arbitrary mass ratios

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Exploiting simple yet remarkable properties of relativistic gravitational scattering, we use first-order self-force (linear-in-mass-ratio) results to obtain arbitrary-mass-ratio results for the complete third-subleading post-Newtonian (4.5PN) corrections to the spin-orbit sector of spinning-binary conservative dynamics, for generic (bound or unbound) orbits and spin orientations. We thereby improve important ingredients of models of gravitational waves from spinning binaries, and we demonstrate the improvement in accuracy by comparing against numerical simulations of binary black holes.

Introduction. — The success of gravitational-wave (GW) astronomy in the next decades relies on significantly improved theoretical predictions of GW signals from coalescing binaries of spinning compact objects such as black holes (BHs). A growing network of GW detectors [1–4] has now observed dozens of signals from binary BHs, measuring distributions of the BHs’ masses and spins and extrinsic properties, enabling diverse applications in astro- and fundamental physics [5–8]: e.g., discerning binary BH formation channels [6], measurement of the Hubble constant [8], and tests of general relativity (GR) [7]. The search for and parameter estimation of GW signals require accurate predictions, from the inspiral (treated by analytic approximations) to the last orbits and merger of the binary (treated by numerical relativity, NR). The current accuracy of theoretical predictions, from combined analytic and numerical methods, will likely become insufficient when current detectors reach design sensitivity around 2022 [9]. In particular, a more accurate modeling of spin effects is key for certain physics applications of future detectors.

The primary relevant analytic approximation is the post-Newtonian (PN, weak-field and slow-motion) approximation. The conservative orbital dynamics is known for nonspinning binaries to the fourth-subleading PN order [10–13] (with partial results at the fifth [14–16]), but only to second-subleading order (or next-to-next-to-leading order, N^2LO) in the spin-orbit sector [17–19]. The gravitational spin-orbit couplings, linear in the component bodies’ spins, are analogous to those in atomic physics. Recently, the three-loop Feynman integrals at N^3LO in the spin-orbit case were calculated [20], leaving however plenty of tensorial lower-loop integrals as a comparably large computational task. Innovations that complement these massive algebraic manipulations are thus of great potential value.

In this paper, we follow a line of reasoning which leads to a complete result for the sought-after N^3LO-PN spin-orbit dynamics (at 4.5PN order for rapidly spinning binaries), requiring relatively little computational effort by building on a diverse array of previous results. We extend to the spinning case a novel approach based on special properties of the gauge-invariant scattering-angle function [16, 21, 22], which encodes the complete binary dynamics (both bound and unbound). The weak-field approximation of the scattering angle is strongly constrained by results in the small-mass-ratio approximation, as treated in the gravitational self-force paradigm [23]. The scattering-angle constraints imply that known first-order (linear-in-mass-ratio) self-force results with spin [24–26] uniquely fix the full N^3LO-PN spin-orbit dynamics for arbitrary mass ratios. This result completes the 4.5PN conservative dynamics of (rapidly) spinning binaries, together with the NLO cubic-in-spin couplings [27] (see also [28]).

As applications, we compute quantities which can be employed to improve waveform models for GW astronomy: the circular-orbit aligned-spin binding energy and the effective gyro-gravitomagnetic ratios. The former is a crucial ingredient in the construction of faithful models (together with the GW energy flux), for which we quantify the accuracy gain due to the present results by comparing to NR simulations. The latter parameterize spin effects in the SEOBNR waveform codes [29–31] used in LIGO-Virgo searches and inference analyses [5], and are analogous to the famous “g-factor” describing the anomalous magnetic dipole moment of the electron, where contributions at the fifth subleading order were obtained [32] and lead to spectacular agreement with experiment [33].

Regarding the gravitational analog, experimental constraints on the gyro-gravitomagnetic ratios are so far seemingly out of reach. In fact, only two GW events were observed to contain nonvanishing spin effects with 90% confidence [5, 34]. However, this will change, e.g., when systems with precessing spins are observed in the future, since the precession of the orbital plane leads to a characteristic modulation of the emitted GWs. This may allow improved tests of GR and inference of spins. Measuring BH spins and their orientations is also important for discriminating binary formation channels.

We begin by extending the link between weak-field scattering and the self-force approximation [16, 21, 22]
to the spin-orbit sector. Using existing self-force results, we are then able to uniquely determine the N^3LO spin-orbit dynamics, as encoded in the gauge-invariant scattering angle. We continue by calculating the gyro-gravitomagnetic ratios and circular-orbit aligned-spin binding energy. We compare to NR simulations to quantify the accuracy improvement and present our conclusions. G denotes Newton’s constant, and c the speed of light.

The mass dependence of the scattering angle. — The local-in-time conservative dynamics of a two-massive-body system (without spin or higher multipoles) is fully encoded in the system’s gauge-invariant scattering-angle function $\chi(m_1, m_2, v, b)$ [35, 36]. This gives the angle $\chi$ by which both bodies are deflected in the center-of-mass frame, as a function of the masses $m_a$ ($a = 1, 2$), the asymptotic relative velocity $v$, and the impact parameter $b$. Based on the structure of iterative solutions in the weak-field (post-Minkowskian) approximation, it has been argued in Sec. II of Ref. [21] that this function exhibits the following simple dependence on the masses (at fixed $v$ and $b$), through the total mass $M = m_1 + m_2$ and the symmetric mass ratio $\nu = m_1 m_2 / M^2$,

$$\chi = \frac{GM}{b} X_{G^0}^\alpha(v) + \left(\frac{GM}{b}\right)^2 X_{G^2}^\alpha(v) + \left(\frac{GM}{b}\right)^3 \left[ X_{G^4}^\alpha(v) + \nu X_{G^4}^\nu(v) \right] + \mathcal{O}\left(\frac{GM}{b}\right)^5,$$

where $\Gamma = E / Mc^2$, with $E^2 = (m_1^2 + m_2^2 + 2m_1 m_2 \gamma)c^4$ being the squared total energy, and $\gamma = (1 - v^2/c^2)^{-1/2}$ the asymptotic relative Lorentz factor. The remarkable fact to be noted here is that the $\mathcal{O}(GM/b)^{1,2}$ terms are independent of $\nu$, while the $\mathcal{O}(GM/b)^{3,4}$ terms depend linearly on $\nu$.

As will be argued in detail in future work, this result generalizes straightforwardly to the case of spinning bodies in the aligned-spin configuration, i.e., spins pointing in the direction of the orbital angular momentum (as shown in Fig. 1). The aligned-spin dynamics is fully described by the aligned-spin scattering-angle function $\chi(m_a, S_a, v, b)$ [22]. Here, $S_a = m_a c a_a$ are the signed spin magnitudes, positive if aligned as in Fig. 1, negative if anti-aligned. At the spin-orbit (linear-in-spin) level, the form of Eq. (1a) holds, with the $X$ functions acquiring additional (linear) dependence on the spins only through the dimensionless ratios $a_a/b = S_a/m_a c b$, as follows:

$$X_{G^0}^{\nu_m} \rightarrow X_{G^0}^{\nu_m}(v) + \frac{a_\pm}{b} X_{G^0}^{a_\pm}(v) + \delta \frac{a}{b} X_{G^0}^{a_\pm}(v),$$

where $a_\pm = a_2 \pm a_1$ and $\delta = (m_2 - m_1) / M$, with the special constraints $X_{G^0}^{a_\pm} = 0 = X_{G^2}^{a_\pm}$; cf. Eq. (4.32) of Ref. [22], where this is seen to hold through N^2LO in the PN expansion. It is crucial to note that the impact parameter $b$ in Eq. (1), is the (“covariant”) one orthogonally separating the asymptotic worldlines defined by the Tulczyjew-Dixon condition [37, 38] for each spinning body [22, 39].

Now, the fourth order in $GM/b$ encodes the complete spin-orbit dynamics at N^3LO in the PN expansion, and according to Eq. (1) only terms up to linear order in the mass ratio $\nu$ appear on the right-hand side (noting $\delta \rightarrow \pm 1$ as $\nu \rightarrow 0$)—that is, first-order self-force (linear-in-$\nu$) results can be employed to fix the functions $X_{G^\nu}^{\nu_m}(v)$ for $n \leq 4$.

Scattering angle, Hamiltonian, and binding energy. — We now connect the scattering angle to an ansatz for a local-in-time binary Hamiltonian including spin-orbit interactions. If nonlocal-in-time (tail) effects are present, this step requires extra care [16], but this is not the case at the N^3LO spin-orbit level. Crucially, the Hamiltonian describes the dynamics for both unbound and bound orbits. The latter are not only most relevant for GW astronomy, but are also where the vast majority of self-force results are available. Hence, a gauge-dependent Hamiltonian allows us to connect the scattering angle (1) with known self-force results.

Let us parametrize our binary Hamiltonian $H$ in the effective-one-body (EOB) [40] form,

$$H = Mc^2 \left[ 1 + 2\nu \left( \frac{H_{\text{eff}}}{\mu c^2} - 1 \right) \right],$$

where $H_{\text{eff}} = H_{\text{eff}}(r, p_r, L)$ is the aligned-spin effective Hamiltonian and $\mu = M\nu$ is the reduced mass. The pairs of conjugate canonical variables are radial distance and momentum $(r, p_r)$, and orbital phase and angular momentum $(\varphi, L)$. (The Hamiltonian (2) is however independent of $\varphi$ due to rotation invariance.) For instance, the orbital angular velocity is $\omega = \dot{\varphi} = \partial H / \partial L$. For the aligned-spin effective Hamiltonian, we make the ansatz [41]

$$H_{\text{eff}} = H_{\text{eff}}^{\text{no spin}} + \frac{1}{c^4 r^3} L (g_S S + g_S S^*),$$

where $S = S_1 + S_2$, $S^* = \frac{m_1}{m_2} S_1 + \frac{m_2}{m_1} S_2$, and $H_{\text{eff}}^{\text{no spin}}$ is given to 4PN order by Eqs. (5.1) and (8.1) in Ref. [42]. The undetermined gyro-gravitomagnetic ratios are $g_S(r, p_r)$ and $g_S(r, p_r)$, for which we choose a
gauge such that they are independent of \( L \) [43–45]. Importantly, if one obtains \( g_S \) and \( g_{S*} \) for the aligned case (3), this fixes the generic-spin effective Hamiltonian as

\[
H_{\text{eff}} = H_{\text{eff}}^\text{no spin} + \frac{1}{c^2 r^4} L \cdot (g_S S + g_{S*} S^*), \quad (\text{generic}) \quad (4)
\]
on the enlarged phase space. The aligned-spin scattering angle is therefore sufficient to construct the generic-spin Hamiltonian, up to the spin-orbit level. Each term in a PN-expanded ansatz for \( g_S \) and \( g_{S*} \) carries a certain power in \( c \), from which the PN order can be read off; we include terms up to \( c^{-6} \) here. \( (c^{-2} \) corresponds to one PN order and \( c \to \infty \) to the Newtonian limit.\)

To ascribe physical significance to the spin-orbit Hamiltonian, we point to the striking similarity between the gravitational spin-orbit Hamiltonian and electromagnetic spin-orbit interactions in atomic physics, which makes \( g_S \) and \( g_{S*} \) analogous to the “g-factor” of the electron (except that \( g_S \) and \( g_{S*} \) depend on dynamical variables). This is no accident, since the gravito-magnetic field generated, e.g., by a rotating mass, can be interpreted to exert a Lorentz-like force. The relativistically preferred geometrical interpretation is that gravito-magnetic fields are dragging inertial/free-falling reference frames, as impressively demonstrated by the Gravity Probe B satellite experiment [46].

We constrain the ansatz for the Hamiltonian by requiring that it reproduces (i) the mass dependence of the scattering angle (1), (ii) the \( \nu \to 0 \) limit of the scattering angle, for a spinning test particle in a Kerr background, as obtained, e.g., by integrating Eq. (65) of Ref. [47], and (iii) certain gauge-invariant self-force observables, namely, the Detweiler-Barack-Sago redshift [24, 25, 48–53] and the spin-precession frequency [26, 53–59] for bound eccentric aligned-spin orbits, to linear order in the mass ratio. The scattering angle \( \chi \) is obtained from the Hamiltonian (2) via Eq. (4.10) of Ref. [22], with the translation from the total energy \( E = H \) and canonical orbital angular momentum \( L \) to the asymptotic relative velocity \( v \) and “covariant” impact parameter \( b \) accomplished by Eqs. (4.13) and (4.17) of Ref. [22]. The redshifts \( z_{1,2} \) and spin-precession frequencies \( \Omega_{1,2} \) are given by

\[
z_{1,2} = \left\langle \frac{\partial H}{\partial m_{1,2}} \right\rangle, \quad \Omega_{1,2} = \left\langle \frac{\partial H}{\partial S_{1,2}} \right\rangle, \quad (5)
\]
where \( \langle \cdots \rangle \) denotes an average over one period of the radial motion, following from a first law of binary mechanics for eccentric aligned-spin orbits [60–63]. The procedure for expressing these quantities, in the small-mass-ratio limit, in terms of variables used in self-force calculations is detailed in Ref. [64]. In this process, to reach the \( N^3 \)LO-PN accuracy in the spin-orbit sector, it is necessary to include the nonspinning 4PN part of the Hamiltonian, including the nonlocal tail part [10], given as an expansion in the orbital eccentricity as in Ref. [42]. After lengthy calculation, working consistently in the small-mass-ratio and PN approximations, we obtain, from our Hamiltonian ansatz, expressions for the redshifts and precession frequency, which can be directly compared with the self-force results in Eq. (4.1) of Ref. [25], Eq. (23) of Ref. [24] and Eq. (20) of Ref. [64] for the redshift, and Eq. (3.33) of Ref. [26] for the precession frequency. The resultant constraints uniquely fix \( g_S(r,p_r) \) and \( g_{S*}(r,p_r) \) at \( N^3 \)LO, via an overdetermined system of equations.

From the Hamiltonian, we can finally calculate the binding energy \( E_b = H - Mc^2 \) for circular orbits as a function of the circular-orbit frequency \( \omega \). This is a gauge-invariant relation that can be compared to NR. We decompose \( E_b \) into nonspinning and spin-orbit (SO) parts, and further into PN orders, as in

\[
E_b^{\text{SO}} = E_b^{\text{SO}_{b,LO}} + E_b^{\text{SO}_{b,NO}} + E_b^{\text{SO}_{b,N^2LO}} + E_b^{\text{SO}_{b,N^3LO}} + \ldots \quad (6)
\]
We can decompose the \( g_S \), \( g_{S*} \), and \( \chi_{SO} \) results from the previous discussion in the same way. The \( N^3 \)LO pieces of all these quantities are the main results of this paper:

\[
\frac{\chi_{SO}^{N^3LO}}{\Gamma} = \frac{v}{c b} \delta a_+ \left[ 1 + \frac{1}{4} \left( \frac{177 \nu}{v^6} - \frac{5}{v^6} \right) \left( \frac{GM}{v^2 b} \right)^3 + \pi \left[ \frac{3}{4} \left( \frac{1365 - 777 \nu}{315 - 45 \nu} \right) \frac{v^2}{c^2} - \frac{1}{32} \left( \frac{27371}{2737} - \frac{233 \nu^2}{8} \right) \nu \frac{v^6}{c^6} \left( \frac{GM}{v^2 b} \right)^4 \right] \right], \quad (7)
\]

\[
e^0 g_S^{N^3LO} = \frac{\nu}{1152} \left( -80399 + 1446 \pi^2 + 13644 \nu - 63 \nu^2 \right) \left( \frac{GM}{r^3} \right)^3 + \frac{3}{64} \left( -1761 + 2076 \nu + 23 \nu^2 \right) \frac{p_r^2 (GM)^2}{r^2} \\
+ \frac{\nu}{128} \left( 781 + 3232 \nu - 771 \nu^2 \right) \frac{p_r^4 GM}{r^3} + \frac{7 \nu}{128} \left( 1 - 36 \nu - 95 \nu^2 \right) \frac{p_r^6}{r^6}, \quad (8)
\]

\[
e^0 g_{S*}^{N^3LO} = -\frac{1}{384} \left[ 1215 + 2(7627 - 246 \pi^2) \nu - 4266 \nu^2 + 36 \nu^3 \right] \left( \frac{GM}{r^3} \right)^3 - \frac{3}{64} \left( 15 + 558 \nu - 1574 \nu^2 - 36 \nu^3 \right) \frac{p_r^2 (GM)^2}{r^2}
\]
where \( v_\omega = (GM\omega)^{1/3} = x^{1/2}c \). One needs to add our Eq. (7) to Eq. (4.32b) in Ref. [22] to obtain the complete spin-orbit scattering-angle contribution through N^3LO-PN and and \( \mathcal{O}(\frac{GM}{c^2})^4 \). The lower-order corrections to \( E_{b,\,N^3\text{LO}}^{\text{SO}} \) can be found in Eq. (5.4) of Ref. [18], and the lower-order gyro-gravitomagnetic ratios in Eqs. (55) and (56) of Ref. [44] (see also Ref. [43, 45]). Through the results for \( g_3 \) and \( g_8^\text{S} \), presented above, one can straightforwardly improve the SEOBNR waveform models [29–31] used in LIGO-Virgo searches and inference analyses [5]. Likewise, one can use them to improve the upcoming TEOBResumS waveform models [65, 66]. The other main waveform model used by LIGO-Virgo data analysis [5] is the IMRPhenom family [67–70], which can also be improved using our results, though less directly.

**Comparison to NR.** — We now quantify the improvement in accuracy from the new N^3LO spin-orbit correction. The circular-orbit aligned-spin binding energy is a particularly good diagnostic for this, since it encapsulates the conservative dynamics of analytical models, and it can also be obtained from accurate NR simulations [71, 72]. Of particular interest for us is the possibility to isolate the linear-in-spin (spin-orbit) contribution by combining results from simulations with different spin magnitudes (and orientations) [73, 74]. The result, based on recent NR simulations [74, 75], is shown in Fig. 2. Besides the analytic PN-expanded result (6), the figure also shows the spin-orbit binding energy extracted from the EOB Hamiltonian (2) numerically, combining binding energies for different spin values in the same way as in the NR case (see also Ref. [76] for details). The EOB resummation of the PN spin-orbit results shows a clear advantage over the pure PN one in the high-frequency regime, and similarly the N^3LO EOB result over the N^2LO one. This indicates that an inclusion of the N^3LO into existing waveform models leads to improvements even in the strong-field regime, otherwise accessible by computationally-expensive NR simulations only. Recall that gravitational waves are observed from low frequencies (where approximation methods are applicable) to high frequencies (where PN theory is expected to break down).

**Conclusions.** — Currently-operating (second-generation) gravitational-wave detectors require accuracy improvements for GW predictions by the time they reach design sensitivity around 2022, which become even more stringent for future upgrades and the upcoming third generation of detectors [9]. The detector upgrades in the coming years also imply an increased number of detections, making it overall more likely to observe binaries oriented “edge on” instead of “face on”, which allows measuring precession and extracting spin values with higher accuracy. The accurate modeling of GW modulations caused by precession, but also the phase-accuracy in the aligned-spin case and the contingent improvement in the estimation of spin parameters, motivate us to push predictions for gravitational spin effects to higher orders.

For this purpose, we extended to spin-orbit couplings a link between the weak-field and small-mass-ratio approximations, via the scattering-angle function, as proposed in the nonspinning case in Ref. [16, 21] (see also Ref. [22]). We employed existing self-force results [24–26] to uniquely determine a N^3LO PN spin-orbit binary Hamiltonian. We calculated the effective gyro-gravitomagnetic ratios as they would enter the SEOBNR and TEOBResumS waveform models, and we obtained the gauge-invariant scattering angle and circular-orbit binding energy for aligned spins. Since the spin-orbit interaction is universal, our results are applicable to generic spinning binaries, e.g., binaries containing neutron stars.

In Fig. 2 we compared the binding energy in the PN and EOB-resummed cases against NR results. The EOB resummation considerably improves on the convergence towards and agreement with NR compared to the
“vanilla” PN result. More importantly, the new contribution obtained in this paper roughly halves the gap to NR in the high-frequency regime compared to earlier N^2LO results. This means that improved analytical predictions based on our result can be trusted to higher frequencies, which may alleviate the need for longer and computationally very expensive NR waveforms. Hence, it is of particular value and urgency to improve the accuracy of the PN-approximate analytic part of GW models.

A clear avenue for future work is to consider higher orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles). In particular, in a forthcoming publication, we fix the orders in spin (and higher multipoles).

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