\(\nu\text{DFSZ}:\) a technically natural non-supersymmetric model of neutrino masses, baryogenesis, the strong CP problem, and dark matter

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We describe a minimal extension of the standard model by three right-handed neutrinos, a scalar doublet, and a scalar singlet (the “\(\nu\text{DFSZ}\)”) which serves as an existence proof that weakly coupled high-scale physics can naturally explain phenomenological shortcomings of the SM. The \(\nu\text{DFSZ}\) can explain neutrino masses, baryogenesis, the strong CP problem, and dark matter, and remains calculably natural despite a hierarchy of scales up to \(\sim 10^{11}\) GeV. It predicts a SM-like Higgs boson, (maximally) TeV-scale scalar states, intermediate-scale hierarchical leptogenesis \((10^5 \text{ GeV} \lesssim M \lesssim 10^7 \text{ GeV})\), and axionic dark matter.

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I. Introduction: The standard model (SM) and the paradigm of spontaneous electroweak symmetry breaking, realised by a scalar potential

\[ V_{SM} = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \tag{1} \]

where \(\mu^2(m_Z) \approx -(88 \text{ GeV})^2\), has proven extremely successful in explaining low energy phenomena. Nevertheless, it fails to explain neutrino masses, the baryon asymmetry of the Universe (BAU), the smallness of the neutrino electric dipole moment (the strong CP problem), dark matter, and gravity. Whether nature realises these phenomena in a “natural” way, i.e., in such a way that \(\mu^2\) is (sufficiently) insensitive to physically meaningful quantum corrections, remains an open question. Still, motivated by aesthetics, the pursuit of a natural “theory of everything” has motivated much of modern particle physics.

In the same vein, this paper describes an extension of the SM by three right-handed neutrinos, a scalar doublet, and a scalar singlet. The model can be thought of as an extension of the Dine–Fischler–Srednicki–Zhitnitsky (DFSZ) invisible axion model [1,2] by right-handed neutrinos, and is thus henceforth referred to as the \(\nu\text{DFSZ}\). Notably, as we will describe in this paper, the \(\nu\text{DFSZ}\) can explain neutrino masses, the BAU, the strong CP problem, and dark matter in a calculably natural way, even despite a hierarchy of scales up to \(\sim 10^{11}\) GeV. This is achieved by a seesaw mechanism, hierarchical leptogenesis, the Peccei–Quinn (PQ) mechanism, an invisible axion, and a technically natural decoupling limit, respectively.

The paper is organised as follows. We first detail the \(\nu\text{DFSZ}\), its vacuum, and its scalar sector (and constraints). We then describe how it provides explanations for the strong CP problem, dark matter, neutrino masses, and the BAU. Penultimately, we describe our naturalness philosophy, identify the symmetries which protect each scale from quantum corrections, and study an example point in the parameter space. Finally we conclude.

II. The \(\nu\text{DFSZ} \) Lagrangian: The scalar content of the model is a complex singlet \(S\) and two complex doublets \(\Phi_{1,2}\) of hypercharge +1. The potential is

\[
V_{\nu\text{DFSZ}} = M^2_{11} \Phi_1^\dagger \Phi_1 + M^2_{22} \Phi_2^\dagger \Phi_2 + M^2_{SS} S^* S^\dagger
+ \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \frac{\lambda_3}{2} (S^* S)^2
+ \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)
+ \lambda_{1S} (\Phi_1^\dagger \Phi_1)(S^* S) + \lambda_{2S} (\Phi_2^\dagger \Phi_2)(S^* S)
- \epsilon (\Phi_1^\dagger \Phi_2 S^2 - \Phi_2^\dagger \Phi_1 S^2), \tag{2}
\]

where \(M^2_{SS} \sim -(10^{11}\text{ GeV})^2 \equiv -M^2_{PQ}\) sets the PQ scale. Additional terms otherwise allowed by gauge symmetry are forbidden by a global \(U(1)_{PQ}\) symmetry to be defined in Sec. \[\] which is essential in solving the strong CP problem. The \(\epsilon\) terms are necessary to assign a PQ charge to \(S\) and help to generate neutrino masses once \(S\) obtains a vacuum expectation value (vev).

The only addition to the SM fermionic content is three right-handed neutrinos. The strong CP solution dictates that \(\Phi_1, (\Phi_2)\) couple to \(u_R\) \((d_R)\), and our solution for natural neutrino masses and leptogenesis requires that \(\Phi_2\) couple to \(\nu_R\). The Yukawa Lagrangian is therefore

\[ -\mathcal{L}_Y = y_\nu \overline{\nu_L}\Phi_1 u_R + y_d \overline{d_L}\Phi_2 d_R
+ y_\tau \overline{\tau_L}\Phi_1 e_R + y_{\nu_L}\overline{\nu_L}\Phi_2 \nu_R \\
+ \frac{1}{2} y_N (\nu_R)^T S \nu_R + H.c., \tag{3} \]

where family indices are implied, \(\Phi_i \equiv i\tau_2 \Phi_i^\dagger\), and \(J = 2\ (1)\) is a Type II (Flipped) \(\nu\)-two-Higgs-doublet model

\[ \]

1 As far as we are aware, the \(\nu\text{DFSZ}\) was first discussed in Ref. [3]. See Refs. [3,4] and references therein for other minimal approaches to connecting the PQ mechanism with neutrino masses.

2 Another option is to have terms \(-\langle \Phi_1^\dagger \Phi_2 S + H.c. \rangle\) [7].
(ν2HDM) arrangement. We will work in the basis where $y_N$ is diagonal and real. Again, additional terms are
forbidden by the $U(1)_{PQ}$ symmetry.\footnote{In this model the right-handed neutrinos gain mass from the vev of $S$, but an alternative scenario with explicit Majorana masses is also possible.}

We note here that each of $\epsilon \to 0$, $y_N \to 0$, and $y_b \to 0$ is a technically natural limit, since they lead to an extra $U(1)$ symmetry which can be identified with lepton number. As well, there are two apparently technically natural decoupling limits associated with enhanced Poincaré symmetries $[8]$: $\epsilon, \lambda_1 S, \lambda_2 S, y_N \to 0$ decouples $S$, and $\epsilon, \lambda_1 S, \lambda_2 S, y_b \to 0$ decouples the $(\nu_R, S)$ subsystem. These limits will prove important in protecting the hierarchy of scales in the model.

**III. Vacuum:** The scalar fields acquire vevs $\langle S \rangle \equiv v_S/\sqrt{2}$, $\langle \Phi \rangle \equiv (0, v_i/\sqrt{2})$. If $v_S \gg v_i$, then

$$v_S \approx -2M^2_{\Phi S}/\lambda_S \sim 10^{11} \text{ GeV}. \quad (4)$$

Expanding around this vev, the right-handed neutrinos acquire Majorana masses $M_N = y_N \langle S \rangle$ and the scalar potential becomes

$$V_{\nu2HDM} \approx m^2_{\nu_1} \Phi_1^2 \Phi_1 + m^2_{\nu_2} \Phi_2^2 \Phi_2 + m^2_{\nu_3} \Phi_3^2 \Phi_2 + ...,$$

where $m^2_{\nu_i} = M^2_{\nu_i} + y_i \langle S \rangle^2$ and $m^2_{\nu_2} = \epsilon \langle S \rangle^2$. We will adopt the natural explanation of neutrino masses and baryogenesis detailed in Ref. [9]. This requires $v_2/\epsilon \sim O(10^2 - 10^4)$GeV achieved with $m^2_{\nu_1} < 0$, $m^2_{\nu_2} > 0$, and $m^2_{\nu_2}/m^2_{\nu_2} < 1\!\!\!1$. Anticipating $m^2_{\nu_2} \gg v_2^2 (\lambda_3 + \lambda_4)$, $\lambda_2 v_3^2$, the $\Phi_i$ vevs can be written

$$v_2 \equiv \frac{v_i}{\tan \beta} \approx \frac{m^2_{\nu_1}}{m^2_{\nu_2}}, \quad v_1 \approx \sqrt{\frac{2}{\lambda_1} \left(-m^2_{\nu_1} + \frac{m^2_{\nu_2}}{\tan^2 \beta}\right)}, \quad (6)$$

where $\sqrt{v_1^2 + v_2^2} = v \approx 246$ GeV and we have implicitly defined $\tan \beta$. There is an important consistency condition $2m^2_{\nu_2}/\tan^2 \beta \lesssim \lambda_1 v_1^2$ to ensure $m^2_{\nu_2} < 0$ and avoid a fine-tuning for $v$ (see Ref. [9]).

Typical values for the mass parameters are $m^2_{\nu_1} \approx -88$ GeV$^2$, $m^2_{\nu_2} \approx 10^{3}$ GeV, and $m^2_{\nu_2}/\tan^2 \beta \lesssim |m^2_{\nu_1}|$. Therefore, there is no fine-tuning between $M^2_{\nu_i}$ and $m^2_{\nu_i}$, we already expect $\lambda_1 S \lesssim 10^{-4}$, $\lambda_2 S^2 \lesssim 10^{-4}$, and $\epsilon \ll 10^{-18}$.

**IV. Scalar sector:** The scalar mass eigenstates are, up to $v_1/m_{\nu_2}$ and $m^2_{\nu_2}/m^2_{\nu_2}$ corrections (see Ref. [9] for expressions), a $CP$ even state $(h)$ with $m^2_h \approx \lambda_1 v_1^2$, three heavy scalar states $(H, A, H^\pm)$ with masses $\approx m_{\nu_2}$, a PQ-scale neutral scalar $(s)$ with $m^2_s = \lambda_3 v_1^2$, and a very light pseudo-Goldstone boson (the invisible axion).

Owing to the approximate $U(1)$ symmetry due to $m^2_{\nu_2}/m^2_{\nu_2} \ll 1$ and $v^2/m^2_{\nu_2} \ll 1$, the state $h$ closely resembles the SM Higgs.\footnote{Note that, like $\epsilon \to 0$ in the original Lagrangian, $m^2_{\nu_2}/m^2_{\nu_2} \to 0$ is a technically natural limit associated with $U(1)_L$ [10].}

In Fig. 1 we illustrate the various constraints on $m_{\nu_2}$. These are the aforementioned consistency condition, measurements of $B \to X_s \gamma$\footnote{Up to $v/v_S$ corrections, at low scale the model is just the ν2HDM discussed in Ref. [9] with a very weakly coupled axion.} [11, 12], $H/A \to \tau \tau$ LHC searches (for the Type II model) [13, 14], perturbativity up to the Planck scale [15], and naturalness [9]. The naturalness bound was derived in Ref. [9] subject to the naturalness condition we describe in Sec. IX and we refer the reader there for details.

**V. The strong CP problem:** Gauge invariance and renormalisability permit the physical $CP$ violating term, $\theta_{\alpha\beta} G^\mu_\alpha G^\nu_\beta$ where $G$ is the gluon field strength tensor and $G$ its dual, to be added to the SM Lagrangian.

Bounds on the neutron electric dipole moment constrain $\theta \lesssim 10^{-10}$ [16]. The strong $CP$ problem is: why is $\theta \approx 0\!\!\!1$\?\footnote{For reviews of the strong $CP$ problem and axions see e.g. Refs. [17–21].}

The PQ solution [22] to the strong $CP$ problem is to demand a global chiral $U(1)_{PQ}$ symmetry; if the sum of the $u_R$ and $d_R$ PQ charges is non-zero then, after PQ symmetry breaking, $\theta$ becomes dynamical, and the vacuum potential “selects” $\langle \theta \rangle = 0$. The $\nu$DFSZ model is one manifestation of this solution.

Let us now identify the global $U(1)_{PQ}$ symmetry. Defining the $U(1)_{PQ}$ charge numerals as in Table I we can (without loss of generality) set $X_q = 0$ and $X_u + X_d = 1$. Equations (2) and (3) set an additional six constraints on

FIG. 1. Constraints on $m_{\nu_2}$, as labelled. Solid black contours are $m^2_{\nu_2}/\text{GeV}^2 = -80^2, -70^2$, and so on. The $H/A \to \tau \tau$ bound is taken from the CMS search [14]. The naturalness bound is only illustrative (see Ref. [9]).
the charges, which brings the total to eight for nine unknown charges. They are completely defined by setting $X_1 = \alpha X_2$, as long as $\alpha \neq 1$: it is convenient to choose $\alpha = -\cot^2 \beta$ so that the PQ current does not couple to the field eaten by the Z boson. The resulting charge values are given in Table I.

A final comment before moving on. In the SM, if $\theta$ is set to zero by hand at some high scale, renormalisation implies $\theta \lesssim 10^{-17}$ [23, 24], well below the experimental bound. In this sense, in the SM, $\theta \approx 0$ is already natural. Yet this explanation remains unsatisfying, since the limit $\theta \to 0$ cannot be identified with a symmetry. The $\nu$DFSZ solution requires $\lambda_1 \ll 1$, and thus one could similarly ask: why are the $\lambda_i$ identified with an increased Poincaré symmetry. As well, in the presence of CP violating new physics (such as the right-handed neutrinos), this solution guarantees $\theta \approx 0$.

**VI. Dark matter:** The $\nu$DFSZ axion gains a mass

$$m_a \approx 6 \ \mu eV \left( \frac{10^{12} \text{ GeV}}{f_a} \right),$$

due to the chiral anomaly [18, 20], where $f_a \approx (S)$ is the axion decay constant, and inherits $v/f_a$ suppressed couplings to nucleons, photons, and electrons (for expressions see e.g. Ref. [18]). Stellar energy loss constrains $f_a \gtrsim 4 \times 10^9 \text{ GeV}$ [18, 7].

The axion provides a cold dark matter candidate via the misalignment mechanism [28, 30], wherein a significant amount of energy density is stored in coherent oscillations of the axion field, $10$

$$\Omega_a h^2 \approx 0.7 \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^7 \left( \frac{\theta}{\pi} \right)^2,$$

where $-\pi \leq \theta \leq \pi$ is the misalignment angle. The requirement that this quantity not exceed the measured cold dark matter energy density $\Omega_{CDM} h^2 \approx 0.12$ [31] implies

$$f_a \lesssim 2 \times 10^{11} \text{ GeV} \left( \frac{\pi}{\theta} \right)^7,$$

with equality reproducing the observed density. If the PQ symmetry is broken after inflation, then the misalignment angle is the average value taken over many distinct patches, $\theta^2 \approx \pi^2/3$, and one obtains $f_a \lesssim 6 \times 10^{11} \text{ GeV}$ [17] .

**V. Stellar energy loss:** Stellar energy loss constrains couplings to nucleons, photons, and electrons (for expressions see e.g. Ref. [18]). Stellar energy loss constrains $f_a \gtrsim 4 \times 10^9 \text{ GeV}$ [18, 20], well below the experimental limit $\theta \lesssim 10^{-17}$ [23, 24], well below the experimental bound. In this sense, in the SM, $\theta \approx 0$ is already natural. Yet this explanation remains unsatisfying, since the limit $\theta \to 0$ cannot be identified with a symmetry. The $\nu$DFSZ solution requires $\lambda_1 \ll 1$, and thus one could similarly ask: why are the $\lambda_i$ identified with an increased Poincaré symmetry. As well, in the presence of CP violating new physics (such as the right-handed neutrinos), this solution guarantees $\theta \approx 0$.

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**Future projections of the ADMX and CAPP resonant microwave cavity experiments promise to probe this interesting region of parameter space [33, 34].**

**VII. Neutrino masses:** The neutrino mass matrix is given by

$$m_\nu = \left( \frac{v_2}{2} D_1 y_\nu D_1 y_\nu^T \right),$$

where the bracketed quantity is the typical Type I seesaw formula [35, 38]. The mass matrix is diagonalised by a unitary matrix $U$, $U m_\nu U^T = \text{diag}(m_1, m_2, m_3) \equiv D_m$, where $m_i$ are the neutrino masses. It will be useful to express $y_\nu$ in the Casas–Ibarra [39] form,

$$U y_\nu = \sqrt{\frac{3}{v_2}} D_2^{-1} R D_2^{-1},$$

where $R$ is a (possibly complex) orthogonal matrix.

**VIII. The BAU:** The BAU is produced analogously to standard hierarchical thermal leptogenesis [40], via the out-of-equilibrium, CP violating decays of the lightest right-handed neutrino: $N_1 \to \Phi_2$ [9].

If only decays and inverse decays are considered, the evolution of the asymmetry is characterised (in the one-flavour approximation) by the decay parameter $K$,

$$K = \frac{\Gamma_D}{H_{T=M_1}} \equiv \frac{\bar{m}_1}{m_\nu},$$

where $\Gamma_D$ is the $N_1$ decay rate, $H$ is the Hubble rate, $\bar{m}_1$ is the effective neutrino mass, and $m_\nu$ is the equilibrium neutrino mass,

$$\Gamma_D = \frac{1}{8\pi} (y_{\nu} y_{\nu})_{11} M_1, \quad \bar{m}_1 \equiv \frac{(y_{\nu} y_{\nu})_{11} v_2^2}{2 M_1},$$

$$H \approx \frac{17 T^2}{M_{Pl}}, \quad m_\nu \approx 10^{-3} \text{ eV} \tan^2 \beta.$$
It still may be the case that quantum gravity effects at the Planck scale induce a naturalness problem. However, this cannot be rigorously computed in the absence of a cogent theory of quantum gravity, so an agnostic stance on this possible problem seems reasonable to us. In other words, we remain agnostic to the physics which sets the boundary condition $\mu^2(\mu_R \gg m_Z) \lesssim \text{TeV}^2$ at some high scale; this is a separate problem – a hierarchy-type problem – which may or may not have a natural solution (see Ref. [7], for example). In the limit $m_1 = 0$, this corresponds to a $U_y$ which has one zero row, but is otherwise approximately diagonal. We note that, in this configuration, $\tilde{m}_1$ and the $CP$ asymmetry are sufficiently stable under radiative corrections. The reader may check this claim against the renormalisation group equation (RGE) in Appendix A.

**IX. OUR NATURALNESS PHILOSOPHY:** A naturalness problem arises when a low mass scale is subject to large and physically meaningful quantum corrections from a high mass scale. In particular, the electroweak scale can receive such corrections from high-scale new physics.

The RGE formalism is a sensible way to quantify a naturalness problem in any perturbative quantum field theory. For example, in the SM, the $\mu^2(\mu_R)$ RGE is dominated by the top quark Yukawa,

$$\frac{d\mu^2}{d\ln \mu_R} \approx \frac{1}{(4\pi)^2} 6y_t^2 \mu^2,$$

where $\mu_R$ is a representative energy scale. From the RGE perspective, $\mu^2$ is not subject to any large physical corrections and can therefore be considered natural, in the sense that once it is set to be electroweak scale it stays as such within the region of validity of the model ($\mu_R < M_{Pl}$). It follows intuitively that $\mu^2(\mu_R \gg m_Z)$ is not finely tuned against $\mu^2(m_Z)$. In the following we demonstrate that there exists a region of $\nu$DFSZ parameter space where our phenomenological goals can be achieved and the heavy PQ scale induces no naturalness problem, i.e., all scales remain stable under RG evolution.

**X. NATURALNESS IN THE $\nu$DFSZ:** Defining $D \equiv (4\pi)^2 \frac{d}{d\ln \mu_R}$ and keeping only $y_{t,b,s,\nu}$ Yukawas, the one-loop RGEs for the Type II, Flipped $\nu$2HDM mass parameters can be written [51]
\[ Dm_{12}^2 = m_{12}^2 \left( \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 2\lambda_3 + 4\lambda_4 + 2\lambda_S + 4\lambda_{1S} + 4\lambda_{2S} + 3y^2_f + 3y^2 + y^2 + \text{Tr} \left( y'_{\nu}y_{\nu} \right) \right), \]

\[ Dm_{11}^2 = m_{11}^2 \left( \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 6\lambda_1 + 6y^2_f + \left[ 0, 2y^2_r \right] \right) + m_{22}^2 (4\lambda_3 + 2\lambda_4) + \langle S \rangle^2 \lambda_{1S} (4\lambda_{1S} + 4\lambda_S) + M_{SS}^2 2\lambda_{1S}, \]

\[ Dm_{22}^2 = m_{22}^2 \left( \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 6\lambda_2 + 6y^2_f + \left[ 2y^2_r, 0 \right] + 2\text{Tr} \left( y'_{\nu}y_{\nu} \right) \right) + m_{11}^2 (4\lambda_3 + 2\lambda_4) - 4\text{Tr} \left( y_{\nu}M_N^2 y_{\nu} \right) + \langle S \rangle^2 \lambda_{2S} (4\lambda_{2S} + 4\lambda_S) + M_{SS}^2 2\lambda_{2S}, \]

where \( M_N^2 = y_N^* y_N \langle S \rangle^2 \) is the (diagonalised) right-handed neutrino mass matrix. The \( \langle S \rangle^2 \) and \( M_{SS}^2 \) terms correspond to the contribution from the heavy real singlet \( s \) in the broken phase. We provide the full list of RGEs in Appendix A.

These RGEs make manifest the decoupling limit \( \lambda_{1S}, \lambda_{2S}, \text{Tr}(y'_{\nu}y_{\nu}y_N^* y_N) \rightarrow 0 \) which protects the scales from large corrections. First, corrections to \( m_{12}^2 \) are proportional to \( m_{12}^2 \), reflecting the fact that \( \epsilon \rightarrow 0 \) reinstates a \( U(1)_L \) symmetry. Second, because the parameters \( \lambda_{3,4} \) are reintroduced by gauge loops, \( m_{11}^2 \) can only be protected from \( m_{22}^2 \) by having \( m_{22}^2 \) not too much larger than \( m_{11}^2 \); it was argued in Ref. 9 that \( m_{22} \lesssim \text{few} \times 10^9 \text{ GeV} \) can accommodate a 10% fine-tuning measured at \( M_{Pl} \). Third, \( m_{22}^2 \) is protected from the \( M_N \) by (roughly) \( \text{Tr}(y'_{\nu}y_{\nu}y_N^* y_N)/(4\pi^2) \lesssim m_{22}^2/(\langle S \rangle^2) \); this translates to the sufficient condition \[ M_N \lesssim 3 \times 10^7 \text{ GeV} \frac{m_{22}^2}{\tan \frac{\beta}{2}} \left( \frac{\text{TeV}}{\text{TeV}} \right)^{\frac{2}{3}} , \]

for all the right-handed neutrinos, illustrated in Fig. 2.

Last, the \( m_{11}^2 \) are protected from the PQ scale by (again roughly) \( \lambda_{1S} \lesssim m_{11}^2/(\langle S \rangle^2) \). We note that there is a lepton box induced correction to \( \lambda_{2S} \); this correction is also bounded by \( m_{22}^2/(\langle S \rangle^2) \) through Eq. \[ (22) \].

\[ \lambda_{3}(m_{22}) = 0.02, \quad \lambda_{1S}(m_{22}) = 10^{-18}, \]

\[ \lambda_{1}(m_{22}) = 0.26, \quad \lambda_{2S}(m_{22}) = 10^{-16}, \]

\[ y_{\nu}(m_{22}) = 0.96/\sin \beta, \quad g_{1}^2(m_{22}) = 0.13, \]

\[ y_{\tau}(m_{22}) = 0.017/\cos \beta, \quad g_{2}^2(m_{22}) = 0.43, \]

\[ y_{\tau}(m_{22}) = 0.010/\cos \beta, \quad g_{3}^2(m_{22}) = 1.48. \]

Their evolution is shown in Appendix B.

To evolve the mass parameters we set \( m_{11}^2(m_{22}) = -(88 \text{ GeV})^2 \) and consider \( m_{22}(m_{22}) = 0.6, 0.8, 1.0, 2.0 \text{ TeV} \). The \( N_i \) and \( s \) are decoupled with step functions at their thresholds. Their RG evolution is shown in Fig. 3. It is plain that the mass parameters in this (viable) example remain relatively small up to high scales, and are therefore natural according to our philosophy.

\[ 11 \text{ For definiteness we take a Type II arrangement, but the Flipped arrangement gives very similar results.} \]

\[ 12 \text{ We note that the parameter } \lambda_1 \text{ tends to run negative, threatening the stability of the electroweak vacuum; nevertheless we expect the problem to be no worse than in the SM, i.e., we expect a metastable vacuum.} \]
mechanism, and a DFSZ invisible axion, respectively. It contains four scales: $|m_{11}| \approx 88 \text{ GeV}$, $m_{22} \sim 10^4 \text{ GeV}$, $M_N \sim 10^5$–$10^7 \text{ GeV}$, and $M_{PQ} \sim 10^{11} \text{ GeV}$, each protected from quantum corrections by a technically natural decoupling limit. The $\sim \text{TeV}$-scale scalars and the invisible axion of the model will be probed in upcoming experiments.

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A. RGEs: Following is the full list of RGEs in the [Type II, Flipped] model, found using PyR@TE [51]. Shown in blue/underlined are those parameters which, for simplicity, we did not evolve in our RGE analysis.

\[ DM_{11}^2 = M_{11}^2 \left( 6\lambda_1 - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 + 6 g_t^2 + [0, 2 y_t^2] \right) + M_{22}^2 (4\lambda_3 + 2\lambda_4) + M_{SS}^2 2\lambda_1 S, \]  
(\textit{A1})

\[ DM_{22}^2 = M_{22}^2 \left( 6\lambda_2 - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 + 6 g_t^2 + [2 y_t^2, 0] + 2\text{Tr} \left( y^\dagger y \right) \right) + M_{11}^2 (4\lambda_3 + 2\lambda_4) + M_{SS}^2 2\lambda_2 S, \]  
(\textit{A2})

\[ DM_{SS}^2 = M_{SS}^2 \left( 4\lambda_1 + \text{Tr} \left( y^\dagger y N \right) \right) + M_{11}^2 4\lambda_1 S + M_{SS}^2 4\lambda_2 S, \]  
(\textit{A3})

\[ D(S)^2 = - \text{Tr} \left( y_N^\dagger y_N \right) (S)^2 \text{ [i.e. the wave function renormalisation]}, \]  
(\textit{A4})

\[ Dg_{(1,2,3)} = \{7, -3, -7\} g_{(1,2,3)}, \]  
(\textit{A5})

\[ D\lambda_1 = \frac{3}{4} g_1^4 + \frac{3}{4} g_1^2 g_2^2 + \frac{9}{4} g_2^4 - \lambda_1 \left( 3g_1^2 + 9g_2^2 \right) + 12\lambda_1^2 + 4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_2^3 + 2\lambda_1^2 S \]  
+ 12\lambda_1 y_t^2 - 12 y_t^4 + [0, 4\lambda_1 y_t^2 - 4y_t^4], \]  
(\textit{A6})

\[ D\lambda_2 = \frac{3}{4} g_1^4 + \frac{3}{4} g_1^2 g_2^2 + \frac{9}{4} g_2^4 - \lambda_2 \left( 3g_1^2 + 9g_2^2 \right) + 12\lambda_2^2 + 4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_2^2 + 2\lambda_2 S \]  
+ 12\lambda_1 y_t^2 - 12 y_t^4 + [4\lambda_1 y_t^2 - 4y_t^4, 0] + 4\lambda_2 \text{Tr} \left( y_t^\dagger y_t \right) - 4\lambda_2 \left( y_t^\dagger y_t y_N^\dagger y_N \right), \]  
(\textit{A7})

\[ D\lambda_3 = \frac{3}{4} g_1^4 + \frac{3}{4} g_1^2 g_2^2 + \frac{9}{4} g_2^4 - \lambda_3 \left( 3g_1^2 + 9g_2^2 \right) + 6\lambda_3 + 2\lambda_4 \left( \lambda_1 + \lambda_2 \right) + 4\lambda_3^2 + 2\lambda_3 + 2\lambda_1 S \lambda_2 S \]  
+ \lambda_3 \left( 6y_t^2 + 6y_N^2 + 2y_t^2 + 2\text{Tr} \left( y_t^\dagger y_t \right) \right) - 12y_t^2 y_t^2 - [0, 4 \left( y_t^\dagger y_t \right)_{33} y_t^2], \]  
(\textit{A8})

\[ D\lambda_4 = 3g_1^2 g_2^2 - \lambda_4 \left( 3g_1^2 + 9g_2^2 \right) + 12\lambda_4 \left( \lambda_1 + \lambda_2 \right) + 8\lambda_4\lambda_3 + 4\lambda_4^2 \]  
+ \lambda_4 \left( 6y_t^2 + 6y_N^2 + 2y_t^2 + 2\text{Tr} \left( y_t^\dagger y_t \right) \right) + 12y_t^2 y_t^2 + [0, 4 \left( y_t^\dagger y_t \right)_{33} y_t^2], \]  
(\textit{A9})

\[ DL = 10\lambda_3^2 + 2\lambda_2 S \text{Tr} \left( y_N^\dagger y_N \right) + 4\lambda_1^2 S + 4\lambda_2^2 S - 2\text{Tr} \left( y_N^\dagger y_N y_N^\dagger y_N \right); \]  
(\textit{A10})

\[ DL_{1S} = \lambda_1 S \left( -\frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 + 4\lambda_1 S + 4\lambda_4 + 6\lambda_1 \right) + \lambda_2 S \left( 4\lambda_3 + 2\lambda_4 \right) \]  
+ \lambda_1 S \left( 6y_t^2 + [0, 2y_t^2] + \text{Tr} \left( y_N^\dagger y_N \right) \right), \]  
(\textit{A11})

\[ DL_{2S} = \lambda_2 S \left( -\frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 + 4\lambda_2 S + 4\lambda_4 + 6\lambda_2 \right) + \lambda_1 S \left( 4\lambda_3 + 2\lambda_4 \right) \]  
+ \lambda_2 S \left( 6y_t^2 + [2y_t^2, 0] + 2\text{Tr} \left( y_t^\dagger y_t \right) + \text{Tr} \left( y_N^\dagger y_N \right) \right) - 4\text{Tr} \left( y_t^\dagger y_t y_N^\dagger y_N \right), \]  
(\textit{A12})

\[ D\epsilon = \epsilon \left( -\frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 + 2\lambda_3 + 4\lambda_4 + 2\lambda_2 S + 4\lambda_1 S + 4\lambda_2 S + 3y_t^2 + 3y_t^2 + y_t^2 + \text{Tr} \left( y_t^\dagger y_t \right) + \text{Tr} \left( y_N^\dagger y_N \right) \right), \]  
(\textit{A13})

\[ D y_t = y_t \left( -\frac{17}{12} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 + \frac{9}{2} y_t^2 + \frac{1}{2} y_t^2 + [0, y_t^2] \right), \]  
(\textit{A14})

\[ D y_N = y_N \left( -\frac{5}{12} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 + \frac{9}{2} y_N^2 + \frac{1}{2} y_t^2 + [y_t^2, 0] \right), \]  
(\textit{A15})

\[ D y_t = y_t \left( -\frac{15}{4} g_1^2 - \frac{9}{2} g_2^2 + \frac{5}{2} y_t^2 + [3y_t^2, 3g_1^2] \right), \]  
(\textit{A16})

\[ D y_N = y_N \left( -\frac{3}{4} g_1^2 - \frac{9}{2} g_2^2 + 3y_t^2 + \text{Tr} \left( y_t^\dagger y_t \right) \right) + \left[ y_t^2, y_t^2 - \frac{3}{2} \text{Diag} (0, 0, y_t^2) y_t, 0 \right] + \frac{3}{2} y_t^2 y_t^2 y_N \]  
+ \frac{1}{2} y_t^2 y_N^\dagger y_N, \]  
(\textit{A17})

\[ D y_N = \frac{1}{2} \text{Tr} \left( y_N^\dagger y_N \right) y_N + y_N y_N^\dagger y_N + y_N y_t^\dagger y_t + y_t^\dagger y_N^\dagger y_N. \]  
(\textit{A18})
**B. Explicit example RG evolution:** Shown below is the RG evolution of dimensionless parameters in our explicit example (as a function of $\log_{10} \mu_R$).