Research on the formation mechanism of surface morphology in three-excitation ultrasonic spatial vibration-assisted turning

Jingwei Duan1 · Ping Zou1 · Shiyu Wei1 · Rui Fang1 · Liting Fang1

Received: 7 January 2022 / Accepted: 10 July 2022 / Published online: 30 July 2022
© The Author(s), under exclusive licence to Springer-Verlag London Ltd., part of Springer Nature 2022

Abstract
To improve the machining performance of different processing materials, a three-excitation ultrasonic spatial vibration-assisted turning system is proposed, which realizes the non-unity of the plane where the cutting trajectory of the tool is located. The influence and formation law of three-excitation ultrasonic spatial vibration-assisted turning on the surface roughness of the workpiece under different vibration parameters (amplitude) and machining parameters (cutting speed, cutting depth, and feed) were analyzed by response surface methodology. The results show that in terms of vibration parameters, the influence of ultrasonic vibration applied in the horizontal direction on surface roughness is significantly greater than that of ultrasonic vibration applied in the vertical direction, while the feed has the greatest influence on surface roughness, followed by cutting speed. The surface roughness of common turning, one-dimensional ultrasonic vibration-assisted turning, ultrasonic elliptical vibration-assisted turning, and three-excitation ultrasonic spatial vibration-assisted turning was theoretically analyzed and experimentally compared. The results show that compared with the other three turning methods, the three-excitation ultrasonic spatial vibration-assisted turning can obtain a lower surface roughness and have good machinability.

Keywords Three-excitation ultrasonic spatial vibration-assisted turning · Response surface methodology · Surface roughness · Formation mechanism · Influencing factors

1 Introduction
Austenitic stainless steel is widely used in many fields such as industry and commerce because of its good corrosion resistance [1]. Due to its poor thermal conductivity, severe work hardening, easy generation of built-up edge, high toughness, and ductility, it is classified as a difficult-to-machine material [2–4]. Studies have shown that the application of ultrasonic vibration to traditional machining methods can effectively improve surface quality and machining accuracy, especially in difficult-to-machine materials. Ultrasonic vibration-assisted turning (UVT) can be divided into one-dimensional ultrasonic vibration-assisted turning (1D-UVT), ultrasonic elliptical vibration-assisted turning (UEVT), and three-dimensional elliptical vibration-assisted turning (3D-EVT) according to the vibration trajectory of the tool nose.

In terms of 1D-UVT, numerous studies [5–9] show that the surface roughness obtained by 1D-UVT is significantly lower than that of CT. Among them, Celaya et al. [10] cut AISI1045 by 1D-UVT with ultrasonic vibration applied in the feeding direction, and the obtained surface quality was improved by more than 60% compared with CT. Celaya et al. [11] studied the effect of 1D-UVT in different vibration directions on the surface quality, and the results showed that the surface quality of 1D-UVT can be improved by 35% compared to CT. They also observed that the ultrasonic vibration in the main direction of motion had a higher effect on the surface roughness than the ultrasonic vibration applied in the feed direction. Nath and Rahman [12] found that the surface quality obtained by 1D-UVT with ultrasonic vibration applied in the main motion direction can be improved by 75 ~ 85%. Puga et al. [13] studied the effect of 1D-UVT on the surface quality of aluminum alloys and found that compared with CT, the average roughness obtained by 1D-UVT was reduced by 55 ~ 82%, and the maximum height of the surface profile was reduced by 59 ~ 76%. He et al. [14] found in thread turning that the average roughness (1.70 µm) and maximum surface profile...
height (5.75 µm) obtained by 1D-UVT were much lower than those obtained by CT (3.90 µm) and surface profile maximum height (19.51 µm).

In terms of UEVT, Shamoto and Moriwaki [15] first proposed the UEVT machining method, which realized the elliptical vibration trajectory of the cutting edge and could change the vibration trajectory by changing parameters such as vibration amplitude, tool geometry, cutting speed, and phase difference [16]. UEVT can not only reduce the chip thickness and width, and improve the shape accuracy [17] and surface integrity of the workpiece [18], but also suppress burr generation [19], reduce tool wear [20], and increase the critical cutting thickness [21], to improve the machining accuracy of parts and the stability of the workpiece system [22]. After processing AISI 304 stainless steel, He et al. [23] found that the surface roughness obtained by UEVT was only 19.23~62.35% of CT, and the axial force and radial force generated were almost equal to 50% of CT. From the perspective of tribological properties, Usman et al. [24] studied the surface functional parameters of CT and UEVT cutting processes and believed that the surface functional parameters of UEVT can be improved to 19.44%. In addition, Ahn et al. [25] believed that the non-resonant UEVT also had a good cutting performance. Kim and Loh [26–28] found that the cutting force obtained by non-resonant UEVT was only about 10% of that of CT, and they believed that the reduction of frictional force was the main reason for reducing the cutting force. They also found that the non-resonant UEVT can effectively improve the surface quality of the workpiece; this phenomenon corresponds to the literature [15]. Pan et al. [29] found that the surface roughness (49.792 nm) obtained by UEVT was much smaller than that of CT (485.59 nm). Tan et al. [30] found that the micro-groove width deviation of Ti-6Al-4 V alloy processed by UEVT could be reduced to less than 25% of that of CT, and attributed it to the reduction of plastic lateral flow and material spring back; they further observed that the surface roughness of the groove bottom processed by UEVT can be reduced to less than 50% of CT.

In terms of 3D-EVT, compared with UEVT, 3D-EVT can realize any elliptical trajectory of the vibrating tool in three-dimensional space [31], which is more suitable for machining free-form surfaces [32]. Sajjadi et al. [33] compared four cutting methods, CT, 1D-UVT, UEVT, and 3D-EVT, and the results showed that compared with CT, the surface roughness of 3D-EVT could be reduced by 41.7%, but it was bigger than 1D-UVT and UEVT. Lin et al. [34] also discussed the non-resonant 3D-UVT and showed that the surface roughness of the non-resonant 3D-EVT was reduced by 53.46% relative to CT. In addition, Lin et al. [35, 36] combined the non-resonant 3D-EVT system with the robust adaptive fuzzy control technology to improve the stability of the system; compared with the ordinary non-resonant 3D-EVT, the surface roughness of the improved 3D-EVT is reduced by 20~32%.

The current research on the output shape of the 3D-EVT tool nose vibration trajectory is mainly based on the spatial ellipse, and there is almost no research on the other vibration output shapes of the tool. To better explain the impact of the tooltip vibration trajectory on the surface quality of the workpiece, the present paper proposes a novel resonant three-excitation ultrasonic vibration-assisted turning (3D-USVT) system driven by three excitations. The nose of the tool can generate complex, stable, and closed vibration trajectories in a single cycle. The vibration trajectory keeps the chip flow angle in a changing state during the cutting process, which facilitates the discharge of chips and improves the surface quality of the workpiece.

2 3D-USVT cutting system

Simultaneously applying excitation vibration with a certain phase difference and ratio in the three vertical directions of the tool can make the vibration trajectory of the tool nose in the three-dimensional space a stable closed curve. Some studies [37–40] have shown that different processing materials lead to different processing properties. The difficulty of the cutting edge of the tool entering the workpiece is not only related to the workpiece material but also affected by the geometric parameters of the tool. Therefore, to improve the machining performance of different workpiece materials under the same processing method, it is necessary to change the geometric parameters of the tool. Due to the diversity of processing materials, it is impossible to match the tool geometric parameters for each processing material. Therefore, by changing the motion trajectory of the tool during cutting, the tool’s geometric parameters other than the fillet radius can be changed. Figure 1 shows the tool nose vibration trajectory under different vibration frequency ratios. It can be seen that the geometric angle of the tool after applying ultrasonic vibration changes continuously with the vibration trajectory, which helps to improve the processing performance of different materials. It can be seen from Fig. 1d to f that when the vibration frequencies in the three directions are not equal, as the vibration frequency increases, the vibration trajectory of the tool nose in a single vibration period is more complicated. Due to the complex shape of the vibration track, the excessive vibration of the tool will not only generate large alternating stress and aggravate tool wear, but also cause tool chatter and deteriorate the surface quality of the workpiece. Studies have shown [41] that smaller vibration frequencies can reduce material accumulation and improve machined surface quality. In addition, with the increase of excitation frequency, not only the stability of the entire processing system is reduced, but also the
existing manufacturing technology is difficult to guarantee the output power of the sensor. Therefore, a relatively small vibration frequency should be selected as far as possible while meeting the processing requirements. When the vibration frequencies in the three directions are equal or equal in pairs, and the ratio of any two vibration frequencies is a rational number, the vibration trajectory of the tool is a stable and relatively simple closed curve (Fig. 1a–c). Compared with Fig. 1a, b, the variation range of the tool geometry angle in Fig. 1c is relatively large, which expands the selection range of machining materials to a certain extent. When vibration is applied in three different directions at the same time, the resulting vibration displacement will lead to a certain coupling phenomenon between each other. Literature [42] shows that the orthogonal output structure can not only effectively reduce the shock caused by the vibration displacement in different directions, but also make it easier to combine and output the vibration displacement in different directions. To better adapt to and improve the processing performance of the workpiece material, the frequency ratio of ultrasonic vibration applied in the three mutually perpendicular directions of X–Y–Z is set to 3:2:3, and the tool nose vibration trajectory in Fig. 1c is used as the output trajectory of 3D-USVT.

Therefore, the 3D-USVT studied in this paper adopts a quadrature output structure. The three excitations work together in two vertical directions so that the cutting edge of the tool produces a simple, stable, and closed vibration trajectory in three-dimensional space (Fig. 2).

The 3D-USVT studied in this paper directly applies the vibration displacement to the tool holder through the luffing system so that the tool nose produces a three-dimensional trajectory. To study the effect of ultrasonic vibration on the surface roughness of the workpiece, the tool holder coordinates need to be established. In the horizontal direction of 3D-USVT, the ultrasonic vibration generated by the two excitation devices does not directly act on the cutting depth and the feeding direction; when the O2Y3 axis of the main coordinate system rotates π/4 clockwise, the ultrasonic vibration is applied to the tool along the horizontal axis of the main coordinate system. First, set the tool fillet center as the origin of the assumed working plane reference system OXYZ, and define the intersection line formed by the back plane, base plane, and the assumed working plane, which are perpendicular to each other; they are defined as the axis of OX, OY, OZ, converting the assumed work plane reference system OXYZ to the tool-holder coordinate system O1X1Y1Z1 by coordinate transformation. When the 3D-USVT

---

**Fig. 1** Tool nose vibration trajectory of 3D-USVT under different frequency ratios. a Frequency ratio 1:1:1. b Frequency ratio 2:3:2. c Frequency ratio 3:2:3. d Frequency ratio 3:2:4. e Frequency ratio 4:3:4. f Frequency ratio 4:3:5

**Fig. 2** Schematic diagram of 3D-USVT
system is working, the coordinate of any point of the tool can be expressed in the tool holder coordinate system as

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = R_x(-\lambda_x) \cdot R_y(-k_y) \cdot R_z(-\alpha_0) \begin{bmatrix}
X_1 \\
Y_1 \\
Z_1
\end{bmatrix} + \begin{bmatrix}
a \cdot \sin(2\pi f_1 t) \\
b \cdot \sin(2\pi f_2 t + \varphi_1) \\
c \cdot \sin(2\pi f_4 t + \varphi_2)
\end{bmatrix}
\]

(1)

where \(\lambda_x, k_y,\) and \(\alpha_0\) respectively represent the cutting edge inclination angle, entering angle, and rake angle of the tool. \(a, b,\) and \(c\) respectively represent the vibration amplitudes in the three vibration directions; \(f_1, f_2\) represent the vibration frequencies in the three directions, and \(f_1/f_2\) are rational numbers; \(\varphi_1\) represents the phase difference between the excitation of the OY axis and the excitation of the OX axis, and \(\varphi_2\) represents the phase difference between the excitation of the OZ axis and the excitation of the OX axis.

Assuming that the origin of the machine’s main coordinate system is located on the axis of the processed bar, the motion path of the tool in the machine’s main coordinate system \(O_xO_yO_z\) can be expressed as

\[
\begin{bmatrix}
X_2 \\
Y_2 \\
Z_2
\end{bmatrix} = R_y\left(\frac{\pi}{4}\right) \cdot \begin{bmatrix}
S - DOC \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
0 \\
vvt \\
0
\end{bmatrix} + \begin{bmatrix}
0 \\
-L \cdot f_4 \cdot t \\
0
\end{bmatrix}
\]

(2)

where \(L\) is the total length of the bar, \(f\) is the feed during normal cutting, \(f_4\) is the rotation frequency of the workpiece, \(DOC\) is the depth of cut, \(S = r_e + R\) represents the distance between the center of the tool nose arc and the axis of the workpiece after tool setting, \(r_e\) is the corner radius, \(R\) is the initial radius of the workpiece, and \(v\) is the cutting speed during processing.

Since the tool vibration trajectory of 3D-USVT is a three-dimensional space vibration trajectory, it is difficult to test it in reality. In order to better test the actual vibration trajectory of 3D-USVT, the excitation sine wave in three directions is tested; according to the sine wave, the phase difference of the waves and the vibration amplitudes in the three directions can predict the actual vibration trajectory of the 3D-USVT. The transient response points of the vibrating tool were collected by a KathMatic laser vibrometer with a test frequency of 19.8 kHz and a sampling rate of 312.5 kHz, and the collected data were processed by KathMatic dual-pipeline software.

Through analysis, it is found that the phase difference between two sine waves of the same frequency in the XOZ plane is 86.34° (Fig. 3a), and the phase difference between two sine waves of different vibration frequencies in the XOY plane is 72.74° (Fig. 3b). Through testing, it is found that the maximum vibration displacement along the OX direction is 21.306854 μm, the maximum vibration displacement along the OZ direction is 24.602176 μm, and the vibration displacement along the OY direction is 31.072936 μm. Figures 3d and 4a represent the actual vibration trajectory of 3D-USVT. It can be seen from Fig. 4a, e, f that the vibration trajectory of 3D-USVT is related to the ultrasonic amplitude.

3 Experiment setup and steps

The machine tool selected for the experimental platform in the present paper is the general lathe CA6140, with AISI 304 stainless steel as the cutting object, and the diameter is 35 mm. The tool is a CCMT120404-HMP cemented carbide tool with a tip fillet radius of 0.4 mm. In order to produce a closed Lissajous vibration trajectory of the tool in 3D spatial, the system vibration generator is adjusted to realize the 3D-USVT driven at different vibration frequencies. The 3D-USVT is fixedly installed on the slide box of the lathe by bolts. During the machining process, the movement and cutting of the tool in the direction of cutting speed and feed are realized by spindle rotation (Fig. 5).

To verify the processing feasibility of the 3D-USVT system, the vibration textures of CT, 1D-UVT, UEVT, and 3D-USVT were compared. Under the same cutting parameters (cutting speed, depth of the cut, and feed), CT, 1D-UVT, UEVT, and 3D-USVT are used to cut AISI 304 stainless steel, respectively. After the processing is completed, a micromap 3D profiler and digital optical microscope were used to observe the macroscopic and microscopic topography of the workpiece surface, as shown in Figs. 6 and 7.
Under the same cutting conditions, when cutting with the CT method, brittle fracture occurs on the surface of the workpiece due to the mutual extrusion and friction between the tool and the workpiece during the cutting process, and the processed surface is composed of linear grooves and ridges of varying depths, which are caused by cutting feed and tool profile. During this period, plastic deformation of the workpiece and tool friction is caused by the long-term non-separation and extrusion between the tool and the workpiece material, and the surface of the workpiece also shows irregular cracks, material peeling, scratches, and build-up of debris sticking, even scaly thorns. The machined surface produced by 1D-UVT is composed of vibration tracks and tool feed marks that are similar in depth and uniformly distributed like tire indentation. Due to the cutting characteristics of 1D-UVT, the machined surface is prone to scratches, which destroy the integrity of the machined surface. However, UEVT produces a regularly distributed vibration texture with intermittent concave characteristics on the machined surface. It can be seen from Fig. 6c that the machined surface produced by this cutting method is prone to “feeding ridges” in the feed direction. The machined surface produced by 3D-USVT forms an intermittent and evenly distributed vibration texture like tooth marks in the direction of the tool path. Compared with UEVT, this cutting method is also prone to a “feeding ridge” in the feed direction, but the “feed ridge” is not distinct. Regardless of the linear grooves and ridges produced by common turning (CT) or the vibration lines of different shapes engender by ultrasonic vibration-assisted turning (UVT), these will cause the surface quality of the workpiece to deteriorate.

It can be seen from Fig. 7 that under the same cutting parameters, the residual geometric height of the machined surface produced by the CT method is significantly higher.
than that of other methods, indicating that the ultrasonic vibration-assisted cutting method can significantly improve the quality of the machined surface of the workpiece. Compared with 1D-UVT and UEVT, the residual geometric height of the workpiece surface obtained by 3D-USVT is the smallest, indicating that the 3D-USVT method has good processing performance.

It can be seen from Fig. 8 that as the amplitude continues to increase, the processed surface becomes increasingly rough. With the increase of X direction vibration amplitude and Z direction vibration amplitude, the intermittent pits produced by the cutter in the depth of cut direction are more obvious, and the plow slip marks produced by the cutter in the feed direction become more and more obvious.

3D-USVT can obtain different machined surface qualities under different machining parameters (Fig. 9). It can be seen from Fig. 9a–d that increasing the vibration amplitude can significantly improve the flatness of the workpiece surface. However, as the vibration amplitude continues to increase, the “concave-convex peak” on the machined surface becomes obvious again, and even the localized distribution of the “concave-convex peak” appears. From this, it can be seen that by reasonably selecting the vibration amplitude of 3D-USVT, a machined surface with uniform distribution of “concave-convex peaks” and relatively small peak-to-valley heights can be obtained. From Fig. 9e–j, it can be seen that the distribution of the “concave-convex peak” is also affected by the cutting parameters. It can be seen that the machined surface quality obtained by 3D-USVT is not only affected by ultrasonic vibration but also related to the three machining parameters of cutting speed, feed [43], and depth of cut.
3.1 Response surface analysis of vibration amplitude

To better analyze the impact of vibration amplitude in three directions on the surface roughness of 3D-USVT during the cutting process, this paper adopts the BBD method and takes the amplitude in the three directions as the research factors, obtaining the influence of different amplitudes on surface roughness. Table 1 shows the range of amplitude in three directions.

To describe accurately the surface roughness of the workpiece, this paper uses the three-dimensional surface roughness parameter Sa as the standard for evaluating turning processing parameters [44, 45]. At the same time, the root mean square height Sq is also analyzed. Take the amplitude of 3D-USVT in the X, Y, and Z directions as independent variables, and Sa as the response value for response surface experiments. The experimental plan and results are shown in Table 2. To ensure the accuracy of the experimental data, three repeatability tests were done for each group of data. Three points are selected on the surface of the workpiece obtained in each experiment to measure the three-dimensional profile of the surface, and the average of the measurement results is taken as the test data of each set of the Sa value.

Through the analysis and comparison of three different models of Sa, it can be seen from Table 2 that the model and lack-of-fit of the three models only meet the requirements of the Quadratic model. The R-Sq value and R-Sq (adj) of the Quadratic model are relatively high, indicating that the model has a high degree of fit with the experimental data, and the accuracy of predicting the response value is also high; the R-Sq value of the Quadratic model is close to the R-Sq (adj) value, indicating that the reliability of the model is higher; the signal-to-noise ratio and CV% value of the Quadratic model are the maximum and minimum values in the three models, respectively, which shows the accuracy and reliability of the experimental data.

Figure 10a shows that the experimental values are concentrated on both sides of the predicted value, reflecting the consistency between the experimental values and the predicted values. Figure 10b shows that the residual points are concentrated in the horizontal band formed by −3 and +3, it shows the validity of the selected Quadratic model. Figure 10c shows that the experimental points obey the normal distribution, which verifies the reliability of the Quadratic model.

To better analyze the relationship between vibration amplitude in three directions (a, b, c) and machined surface roughness (Sa, Sq), the influence of ultrasonic amplitude on surface roughness was analyzed by the response surface method. When analyzing the influence of ultrasonic vibration in one direction (Fig. 11) or two directions (Figs. 12 and 13) on surface roughness, the
The ultrasonic amplitude in other directions is defaulted to the intermediate value of the selected amplitude range (as shown in the panel at the upper right corner of Figs. 11–13).

The three mutually perpendicular excitations work together to realize the complete separation between the tool, the chip, and the workpiece, which not only shortens the net cutting time for the tool to participate in the cutting but also causes cracks on the workpiece to be processed due to the extreme instantaneous acceleration of the tool. The cutting force is reduced, the tool wear is reduced, and the system stability is improved to reduce $S_a$. Figure 11a shows that when the Y direction amplitude is greater than 11 $\mu$m, the decreasing trend of $S_a$ gradually becomes slower. When the Y direction vibration amplitude is equal to 15 $\mu$m, the $S_a$ value appears to rise. The reason for this phenomenon is probably because as the amplitude increases, the mechanical impact of the tool becomes larger, and the excessive impact force causes additional vibration of the tool, which reduces the stability of the 3D-USVT system; the material removal area of the workpiece increases, the cutting temperature increases, and the momentary impact on the tool nose becomes larger. This not only aggravates tool wear, but also reduces the stability of the 3D-USVT and eventually leads to the gradual deterioration of workpiece surface quality.

With the increase of the amplitude in the X direction, the tool produces a great instantaneous component acceleration in the feed direction, and the workpiece material is subjected to mechanical impact to produce cracks, which reduces the cutting force and the $S_a$ value. At the same time, with the increase of the amplitude in the X direction, the tool generates deeper vibrations along the cutting path in the depth-of-cut direction, which increases the $S_a$ value. The changing trends of the two types of $S_a$ restrict each other. Therefore, when the X direction amplitude is less than 5 $\mu$m, $S_a$ changes slowly under the constraints of these two changing trends, as shown in Fig. 11b. When the X direction amplitude is greater than 5 $\mu$m, the cutting path of the tool along the feed direction becomes wider, which improves the cutting accuracy and processing efficiency to a certain extent, but the mechanical impact of the tool becomes larger; the excessive impact force not only causes the tool to produce additional vibration but will aggravate the wear of the tool, reduce the stability of the system, and increase the value of $S_a$.

It can be seen from Fig. 11c that when the amplitude in the Z direction is less than 6 $\mu$m, the $S_a$ value increases with the increase of the amplitude in the Z direction. This may be because with the increase of the amplitude in the Z direction, the vibration texture generated by the tool in the depth direction is more obvious, and the surface quality of the workpiece is worse. When the amplitude of the Z direction is greater than 6 $\mu$m, the increase of $S_a$ begins to become flat. This may be because of the continuous increase in the amplitude of Z direction; the continuous overlap rate of the vibration trajectories of adjacent cutting paths is increased, which to a certain extent the height of
Fig. 9 Microstructure of workpiece surface under different machining conditions. 

- **a** $b = 5 \mu m$, $c = 3 \mu m$, $a = 6 \mu m$.  
- **b** $b = 13 \mu m$, $c = 5 \mu m$, $a = 4 \mu m$.  
- **c** $b = 13 \mu m$, $c = 6 \mu m$, $a = 8 \mu m$.  
- **d** $b = 15 \mu m$, $c = 9 \mu m$, $a = 9 \mu m$.  
- **e** $v = 200 \text{r/min}$.  
- **f** $v = 320 \text{r/min}$.  
- **g** $f = 0.1 \text{mm}$.  
- **h** $f = 0.2 \text{mm}$.  
- **i** $a_p = 0.05 \text{mm}$.  
- **j** $a_p = 0.2 \text{mm}$.  

(a) $b = 5 \mu m$, $c = 3 \mu m$, $a = 6 \mu m$  
(b) $b = 13 \mu m$, $c = 5 \mu m$, $a = 4 \mu m$  
(c) $b = 13 \mu m$, $c = 6 \mu m$, $a = 8 \mu m$  
(d) $b = 15 \mu m$, $c = 9 \mu m$, $a = 9 \mu m$  
(e) $v = 200 \text{r/min}$  
(f) $v = 320 \text{r/min}$  
(g) $f = 0.1 \text{mm}$  
(h) $f = 0.2 \text{mm}$  
(i) $a_p = 0.05 \text{mm}$  
(j) $a_p = 0.2 \text{mm}$
the ridge generated on the machined surface of the workpiece in the previous cutting is reduced, and the increase of Sa is delayed. Figure 12 is the response surface plot and contour plot of Sa under different amplitudes. Figure 12a can be seen that when the Y direction amplitude takes different values, the value of Sa increases nonlinearly with the increase of the X direction amplitude. Conversely, when the amplitude in the X direction takes different values, the value of Sa decreases nonlinearly with the increase in the amplitude in the Y direction. This shows that the interaction between the Y direction amplitude and the X direction amplitude is not significant. The change rate of Sa along with the X direction amplitude enlarge with the Y direction amplitude increase, and when the Y direction amplitude is equal to 14–15 μm, the change rate of Sa reaches the maximum, and when the Y direction amplitude is equal to 5–6 μm, the change rate of Sa reaches the minimum; the change rate of Sa along with the Y direction amplitude enlarge with the X direction amplitude increase, and when the X direction amplitude is equal to 3–4 μm, the change rate of Sa reaches the maximum, and when the Z direction amplitude is equal to 8–9 μm, the change rate of Sa reaches the minimum; the change rate of Sa along with the Z direction amplitude decreases as the X direction amplitude increases, and when the Z direction amplitude is equal to 3–5 μm, the change rate of Sa reaches the maximum, and when the Z direction amplitude is equal to 8–9 μm, the change rate of Sa reaches the minimum.

It can be seen from Fig. 12a–c that when the X direction amplitude and the Z direction amplitude are smaller, and the Y direction amplitude is larger, smaller surface roughness can be obtained.

It can be seen from Fig. 12d, f that, compared to the Y direction amplitude and the Z direction amplitude, the X direction amplitude has a more significant impact on Sa; Fig. 12e shows that the amplitude of Z direction has a more significant effect on Sa than the amplitude of Y direction. It can be seen that the influence order of the amplitude in the three directions of 3D-USVT on the Sa value is X direction amplitude > Z direction amplitude > Y direction amplitude.

In order to obtain the best response under the interaction of different factors, the target value of Sa is set to the minimum, and the factor is set to the unconstrained state. The obtained optimal vibration amplitude combination is shown in Table 3.

The variation trend of the root mean square height (Sq) of the machined surface with a single vibration amplitude and the interactive response surface graph is shown in Fig. 13. It can be seen that under the combined action of different amplitudes, the variation trends of Sq and Sa of the machined surface are basically the same. This indicates that there is an obvious correlation between Sa and Sq obtained by 3D-USVT; when analyzing the effect of ultrasonic vibration of 3D-USVT on surface roughness, only Sa needs to be analyzed and discussed.

| Table 1 | Selection of the 3D-USVT amplitude parameters |
|---------|-----------------------------------------------|
| Level   | Y direction vibration amplitude (μm) | X direction vibration amplitude (μm) | Z direction vibration amplitude (μm) |
| -1      | 5                               | 3                               | 3                           |
| 0       | 10                              | 6                               | 6                           |
| 1       | 15                              | 9                               | 9                           |

| Table 2 | Analysis of variance of different Sa models |
|---------|---------------------------------------------|
| Process order | Model     | Lack-of-fit | R-Squared | R-Squared (adj) | Pred R-Square | Adeq precision | C.V.% |
| Linear     | Significant | Significant | 0.7772   | 0.7257     | 0.5537     | 12.218     | 4.81  |
| 2FI        | Significant | Significant | 0.7797   | 0.6475     | −0.0481    | 8.147      | 5.45  |
| Quadratic  | Significant | Not significant | 0.9813   | 0.9571     | 0.8439     | 24.289     | 1.90  |
3.2 Response surface analysis of cutting parameters

In order to better explain the effect of the 3D-USVT on surface roughness, based on the optimized amplitude, the experiment also studied the influence of cutting parameters on Sa. The cutting speed (v), depth of cut (ap), and feed (f) of 3D-USVT are used as independent variables, and Sa is the response value for the response surface experiment. The level selection of cutting factors is shown in Table 4.

According to the selection principle and applicability of the model, the Quadratic model is selected as the research object, and the influence of cutting parameters on the surface roughness formed by the 3D-USVT method is analyzed (Fig. 14).
Sa obtained by the 3D-USVT cutting method decreases first and then increases with the increase of the \( v \), and reaches the minimum value at \( v = 320 \text{ r/min} \) (Fig. 15a). With the increase of the \( a_p \), Sa increases nonlinearly and slowly (Fig. 15b). Sa increases linearly with the increase of the feed, and the changing trend is faster than the \( v \) and \( a_p \) (Fig. 15c). The reasons for the above trends will be discussed in Sect. 3.2 of this article, so we will not go into too much detail here. It can be seen that the effect of cutting parameters on the surface roughness of the workpiece obtained by the 3D-USVT cutting method is the same as that of the CT cutting method (Table 5).

To analyze the variation trend of surface roughness (Sa, \( S_q \)) under different cutting parameters (\( v \), \( a_p \), \( f \)), the same analysis method as Sect. 3.1 is used. When analyzing the influence of one cutting parameter (Fig. 15) or two cutting parameters (Figs. 16 and 17) on surface roughness, other cutting parameters are defaulted to the intermediate value of the selected parameter range (as shown in the panel at the upper right corner of Figs. 15–17).

Figure 16a shows that as the \( a_p \) increases, Sa increases. When the \( v \) increases, as the \( a_p \) increases, the rate of change of Sa gradually becomes slower. It can be thought that the greater the \( v \), the smaller the effect of \( a_p \) on Sa. As the \( v \) increases, Sa first decreases and then increases. When the \( a_p \) increases, as the \( v \) increases, the rate of change of Sa slightly increases. It can be thought that the greater the \( a_p \), the more obvious the effect of the \( v \) on Sa.

Figure 16b shows that as the \( f \) increases, Sa becomes larger. When the \( v \) increases, with the increase in the \( f \), the rate of change of Sa gradually becomes slower. It can be observed that the greater the \( v \), the smaller the influence of the \( f \) on Sa. As the \( v \) increases, Sa first decreases and then increases. When the \( f \) increases, as the \( v \) increases, the rate of change of Sa slightly increases. It can be seen that the greater the \( f \), the more obvious influence of the \( v \) on Sa.

Figure 16c shows that as the \( a_p \) increases, Sa becomes larger. When the \( f \) increases, as the \( a_p \) increases, the rate of change of Sa gradually becomes obvious. It can be seen that the larger the \( f \), the more obvious influence of the \( a_p \) on Sa. As the \( f \) increases, Sa becomes larger. When the \( a_p \) increases, with the increase of the \( f \), the rate of change of Sa slightly increases. It can be seen that the greater the \( a_p \), the more obvious influence of the \( f \) on Sa.

It can be seen that there is obvious interaction among \( v \), \( a_p \), and \( f \). Therefore, when selecting the cutting parameters of 3D-USVT, the effects of \( v \), \( a_p \), and \( f \) on surface roughness should be comprehensively considered.
Fig. 12  Response surface plot and contour plot of SA under the action of vibration amplitude
Figure 16d shows that the influence of $v$ on $S_a$ is more significant than that of $a_p$. From Fig. 16e, f, it can be seen that the influence of $f$ on $S_a$ is more significant than that of $v$ and $a_p$. It can be concluded that the order of influence of changing cutting speed ($v$), depth of cut ($a_p$), and feed ($f$) on $S_a$ is $f > v > a_p$.

In order to obtain the best response under the interaction of different factors, set the target value of $S_a$ to the minimum, and set the cutting speed ($v$), depth of cut ($a_p$), and feed ($f$) to the unconstrained state. The obtained cutting parameters obtained are shown in Table 6.

Similar to the effect of ultrasonic vibration on $S_a$ and $S_q$, the effect of cutting speed, depth of cut, and feed on $S_q$ and $S_a$ of the 3D-USVT machined surface is the same (Figs. 15, 16 and 17);

| Y direction amplitude (μm) | X direction amplitude (μm) | Z direction amplitude (μm) |
|---------------------------|---------------------------|---------------------------|
| 14.13                     | 3.74                      | 3                         |

Table 3 Optimal amplitude parameter values
it shows that the influence of three processing parameters on Sa and Sq has obvious correlation. Therefore, when analyzing the influence of cutting speed, depth of cut, and feed on surface roughness of 3D-USVT, only Sa can be analyzed and discussed.

### 3.3 Effect of ultrasonic vibration on turning force and cutting temperature of 3D-USVT

Due to the existence of ultrasonic vibration, the intermittent contact between the vibrating tool and the workpiece causes obvious fluctuations in the instantaneous cutting force. Large turning force fluctuations not only lead to reduced stability of the vibrating tool but also destroy the integrity of the machined workpiece surface. In addition to this, the cutting temperature also has an effect on the quality of the machined surface. In order to obtain better workpiece surface quality, this section discusses the effect of 3D-USVT on turning force and cutting temperature by changing the ultrasonic amplitude. The turning force is collected by the Kistler5070A three-way force measuring instrument, Kistler5070B signal amplifier, Kistler5697A1, and post-processed by DynoWare software. The cutting temperature generated during turning is tested by infrared thermography FLIR A40.

The influence of different ultrasonic vibrations on turning force and cutting temperature was analyzed by the Taguchi test method, and it was found that the ultrasonic vibration of 3D-USVT in three directions had different effects on turning force and cutting temperature. It can be seen from Tables 7 and 8 that the ultrasonic vibration in the Y direction has the greatest influence on the turning force and cutting temperature, followed by the ultrasonic vibration in the X direction.

Increasing the ultrasonic vibration can effectively reduce the turning force and the average cutting temperature, and the change trends of the two are basically the same (Fig. 18a–c). The increase in Y direction vibration reduces the tool-chip force, thereby reducing the cutting temperature. However, excessive ultrasonic vibration will not only increase the instantaneous impact force between the tool and the workpiece but also increase the difficulty of cutting the tool due to the reduction of the softening effect of the workpiece material, which ultimately weakens the reduction in turning force and cutting temperature. Since the X-direction vibration and Z direction vibration act directly on the cutting depth direction, when the X direction vibration and Z direction vibration are increased, the material removed by the tool in the cutting depth direction will increase, which eventually leads to an increase in the plowing effect, and the turning force appears to have a rising trend. Since the X direction vibration and Z direction vibration act directly on the depth-of-cut direction, when the X direction vibration and Z direction vibration are increased, the material removed by the tool in the depth of cut direction will increase, which eventually leads to an increase in the plowing effect and a recovery of the turning force. In addition, the larger plowing effect also aggravates the friction and extrusion between the tool and the chip. When the ultrasonic vibration increases to a certain value, the friction and extrusion between the tool and the chip become obvious, which eventually leads to the resulting cutting heat beginning to rise sharply. Increasing Y direction ultrasonic vibration can simultaneously obtain smaller turning force, cutting temperature, and surface roughness, the effect of X direction ultrasonic vibration and Z direction ultrasonic vibration on turning force and cutting temperature, and the effect on surface roughness are almost opposite (Fig. 18). It can be seen from this that in order to obtain better-machined surface quality, a larger Y direction amplitude, a smaller X direction amplitude, and a smaller Z-direction amplitude should be selected (Fig. 19).

### 4 Comparison and discussion of experimental results

This section discusses the processing performance of the researched equipment through theoretical analysis and experimental results. First, the influence of the tool tip on the residual geometric height of the machined surface when cutting the workpiece under four different cutting methods of CT, 1D-UVT, UEVT, and 3D-USVT was discussed, and the residual geometric height of the four machining processes was compared. Then, under the same cutting conditions, by changing the cutting parameters (v, ap, and f), the variation trend of workpiece surface roughness under four different machining processes is discussed.

### 4.1 Theoretical analysis of surface roughness

When cutting the workpiece, the surface roughness of the machined surface of the workpiece can be expressed as

\[
R_{th} = R_{new} - s + w + \varepsilon_i
\]  

where \(R_{new}\) represents the contribution of tool copying to the surface roughness, \(s\) indicates the contribution of the elastic recovery of the workpiece material to the surface roughness, \(w\) represents the contribution of the plastic side flow of the workpiece material to the surface roughness, and \(\varepsilon_i\) is the contribution of surface defects in the workpiece material to the surface roughness.

| Table 4 Selection of 3D-USVT cutting parameters |
|-----------------------------------------------|
| Level | v (r/min) | ap (mm) | f (mm) |
|-------|-----------|---------|--------|
| -1    | 100       | 0.05    | 0.08   |
| 0     | 250       | 0.1     | 0.14   |
| 1     | 400       | 0.15    | 0.2    |
The contribution of the tool to the surface roughness of the workpiece can be expressed as

\[ R_{tew} = r_e - \sqrt{r_e^2 - \frac{f^2}{4}} \approx \frac{f^2}{8r_e} \]  

(4)

The height of the plastic side flow convex peak of the machined surface of the workpiece can be expressed as [46]

\[ w = 1.32 \sqrt{0.46 \rho r_e \cos^{-1} \left( 1 - \frac{R_{tew}}{r_e} \right)} \]  

(5)

where \( \rho \) indicates the fillet radius of the cutting edge, and \( r_e \) represents the tool tip fillet radius.

The elastic rebound height of the processed surface material of the workpiece can be expressed as [47]

\[ s = cr_e \left( 1 - \epsilon_p \right) \]  

(6)

where \( c \) represents dimensionless coefficient, and \( \epsilon_p \) is the plastic strain of the workpiece material.

From Fig. 20a, it can be concluded that the contribution of tool copy to the residual geometric height of the workpiece when CT cutting the workpiece is

| Table 5 Analysis of variance of different Sa models |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Process order   | Model           | Lack-of-fit     | R-Squared       | R-Squared (adj) | Pred R-Square   | Adeq precision  | C.V.%           |
| Linear          | Significant     | Significant     | 0.8126          | 0.7694          | 0.6731          | 13.898          | 8.59            |
| 2FI             | Significant     | Significant     | 0.8533          | 0.7653          | 0.5182          | 10.874          | 8.67            |
| Quadratic       | Significant     | Not significant | 0.9967          | 0.9923          | 0.9639          | 55.699          | 1.56            |

Fig. 14 Applicability of Sa model. a Experimental results and predicted values. b Internal standard residual. c Normal probability.
The workpiece was processed in CT, and the tool is always in contact with the chips and the workpiece. The cutting heat generated during the cutting process causes the tool nose and the workpiece material to have a thermal expansion effect, and the tool nose fillet radius and the workpiece material are thermally expanded. According to Eq. (4), it can be seen that the part of the machined surface \( R_{tew} \) formed by CT cutting decreases with the increase of the fillet radius, but the thermal expansion effect will not only produce gradient expansion of the tool nose and cause the expansion coefficient of the tool in the depth-of-cut direction to be greater than the feed direction, but also increased tool wear. Under the same cutting conditions, the amount of workpiece material removed by CT in the depth of cut will increase, and the residual peak and valley height of the machined surface of the workpiece is significantly greater than the theoretical value. According to Eqs. (5) and (6), it can be seen that the plastic side flow and elastic recovery in the turning process both increase significantly with the increase of the tool fillet radius. Due to its own cutting characteristics, the cutting heat and cutting force generated in the cutting process of 1D-UVT are significantly lower than those of CT. The plastic side flow, elastic recovery phenomenon, and lathe vibration phenomenon of the workpiece material will all be reduced, the separation characteristics of the tool and the chips increase the overlap of adjacent cutting paths, the overlapping area of adjacent cutting paths is cut repeatedly, and the residual height is reduced. However, the ironing effect of the tool flank on the machined surface reduces the elastic recovery of the machined surface material, and to a certain extent prevents the reduction of the residual geometric height of the workpiece surface. Overall, compared with CT, the surface roughness of the machined surface of 1D-UVT cutting is significantly smaller and closer to the theoretical value. Zong et al. [48] believe that material expansion will increase with the increase of polishing and friction, thereby increasing the surface roughness of the workpiece. The high-frequency repeated ironing effect when 1D-UVT is cutting the workpiece will not only increase the surface roughness, but the flank of the tool will also produce micro-cutting of the processed surface material during the ironing process.

\[
2\sqrt{r_r^2 - (r_r - H_{CT})^2} = f 
\]  

(7)

Fig. 15 Single-factor effect diagram of Sa
resulting in surface wear and surface fatigue on the processed surface, thus reducing the machining accuracy of the workpiece.

When using ultrasonic ellipse vibration-assisted turning (UEVT) to process workpieces, due to the difference in the phase difference between the two vibration frequencies, or at the initial point of the adjacent tool trajectories, the difference in the tool vibration trajectory points will lead to differences in the surface geometry of the workpiece. For this purpose, the coefficient $M$ is introduced.

| $v$ (r/min) | $a_p$ (mm) | $f$ (mm) |
|-------------|-------------|-----------|
| 277.79      | 0.069       | 0.084     |
where $S$ represents the total cutting stroke of the tool when the workpiece rotates once, $v$ indicates the rotation speed of the workpiece, $p$ is an integer, and $q$ is a decimal.

When $p \neq 0$ and $q=0$, the residual height of the machined surface formed by UEVT is the largest. At this time, the contribution of the tool copy to the machined surface of the workpiece can be expressed as

$$M = 2\omega S/v = p + q$$  \hspace{1cm} (8)$$

where $A_y$ represents the vibration texture height of the tool in the depth of cut direction, $H_{CT}$ indicates the residual height of the tool in the feed direction, that is, the geometric height of the machined surface formed by CT cutting.

When $p \neq 0$ and $q=0.5$, the residual height of the machined surface formed by UEVT is small. At this time,

$$R_{rew} = H_{UEVC} = A_y + H_{CT}$$  \hspace{1cm} (9)$$
the contribution of cutting tool scoring to the machined surface of the workpiece can be expressed as

\[
\sqrt{r^2_e - (r_e - H_{UEVC} - A_{x1} - A_{x2})^2} + \sqrt{r^2_e - (r_e - H_{UEVC} - A_{x2})^2} = f
\]

(10)

where \(A_{x1}\) represents the distance between the tool tip fillet center and the centerline of the track when the tool is at the lowest point of a single vibration trajectory, and \(A_{x2}\) indicates the distance between the center of the tool nose fillet and the centerline of the track when the tool nose is cut to half of the net cutting time of a single cycle.

During the cutting process of UEVT, the tool is completely separated from the workpiece and chip, and the generated cutting heat and cutting force are significantly reduced compared with CT and 1D-UVT. Due to less cutting heat and cutting force, the thermal expansion effect of the tool and workpiece is reduced, and the vibration of the lathe is also greatly reduced, thereby reducing the waviness of the machined surface. According to Eq. (9), it can be seen that when the instantaneous vibration form of the tool is the same at the starting point of adjacent tool paths, the surface geometric residual height of the machined surface obtained by UEVT is significantly larger than that of CT. When the instantaneous vibration form of the tool is different at the starting point of each tool pass, there is a phase difference between the different tool passes of the vibrating tool. According to Eq. (8), it can be seen that only when \(n = 0.5\), the surface geometric residual height obtained by UEVT is the lowest. According to Eqs. (7) and (10), it can be seen that when \(n = 0.5\), the geometric residual height of the workpiece surface obtained by UEVT is significantly lower than that of CT and 1D-UVT.

When \(p \neq 0\), \(q \in [(0,0.5) \cup (0.5,1)]\), the contribution of UEVT tool scoring to the residual geometric height of the workpiece surface is between Eqs. (9) and (10).

From Fig. 20d, it can be concluded that the contribution of 3D-USVT tool scoring to the residual geometric height of the workpiece surface can be expressed as

\[
2\sqrt{r^2_e - (r_e - H_{3D-UVC} - A_{x})^2} = f
\]

(11)

where \(A_{x}\) represents the distance between the center of the tool nose fillet and the centerline of the path when the tool is at the lowest point of the vibration path.

Similar to UEVT, when 3D-USVT cuts the workpiece, the tool, and the workpiece are completely separated, the cutting heat and cutting force are reduced, and the thermal expansion effect of the tool and the workpiece and the vibration of the lathe has basically disappeared. Comparing Eqs. (10) and (11), it can be found that \(H_{3D-USVT}\) is slightly smaller than \(H_{UEVT}\), and it can be seen that the residual geometric height of the machined surface produced by 3D-USVT cutting is smaller than the minimum residual geometric height produced by UEVT cutting. It can be seen from Fig. 20d that when cutting the workpiece in 3D-USVT, the tip of the tool vibrates not only in the direction of the depth of cut but also in the direction of the feed. This increases the width of the cutting trajectory in the feed direction, thereby increasing the coincidence of adjacent two cuts; the residual height generated by the tool on the adjacent cutting trajectory is partially cut, and the residual height is reduced. There is also vibration in the cutting speed direction of the tool, and the ultrasonic vibration in the three directions is coupled with each other. According to Fig. 5d, it can be seen that the depth of the concave surface formed by the tool tip in the depth of cut direction is significantly reduced, and in the same cycle, the number of grooves formed by the tool tip in the cutting track direction will also increase. Comparing with UEVT, this not only reduces the surface roughness of the machined surface, but also increases the lubricating oil contact area of the workpiece in later use, and extends the service life of the workpiece.

### 4.2 Analysis of test results

Relative to CT, UV with separation properties can achieve small cutting temperatures (Fig. 21). In UV, UEVT and 3D-USVT achieve complete unloading of the tool, which can effectively reduce the cutting force (Fig. 22) and cutting temperature. Compared with UEVT, the plowing effect of 3D-USVT in the depth of cut direction is relatively small (Fig. 7c–d), which helps to generate a smaller cutting temperature. However, the complex tool vibration trajectory makes the 3D-USVT have a large instantaneous impact force, which increases the cutting temperature of the workpiece.

### Table 7 Response value table of turning force

| Level | b   | a   | c   |
|-------|-----|-----|-----|
| 1     | 19.28| 17.44| 17.18|
| 2     | 16.09| 16.03| 16.15|
| 3     | 14.31| 16.21| 16.35|
| Delta | 4.97 | 1.41 | 1.02 |
| arranged order | 1 | 2 | 3 |

### Table 8 Response value table of average cutting temperature

| Level | b   | a   | c   |
|-------|-----|-----|-----|
| 1     | 94.67| 89.00| 89.17|
| 2     | 86.83| 88.17| 88.17|
| 3     | 85.67| 90.00| 89.83|
| Delta | 9.00 | 1.83 | 1.67 |
| arranged order | 1 | 2 | 3 |
3D-USVT. Compared with CT, the cutting temperature was reduced by 18.6% for 1D-UVT, 28.9% for UEVT, and 25.2% for 3D-USVT.

As the tool motion path is different, the cutting force generated by different cutting methods is also different (Fig. 22). Compared with CT, the cutting force generated by UVT is relatively small, so it can be seen that applying ultrasonic vibration to the tool can effectively reduce the cutting force. Among them, the radial force ($F_z$) generated by 1D-UVT is relatively large because the flank of 1D-UVT does not separate and there is an ironing effect. In UVT, since UEVT and 3D-USVT achieve complete unloading of the tool, the total cutting force generated is significantly smaller than that of 1D-USVT. Compared with 1D-UVT, UEVT and 3D-USVT have ultrasonic vibration in the depth-of-cut direction, which increases the plowing force and causes the radial force ($F_z$) to decrease more slowly. The radial force ($F_z$) generated by UEVT is larger than that of 3D-USVT, which may be
due to the existence of ultrasonic vibration in the feeding direction of 3D-USVT relative to UEVT; this results in the cross-arrangement of the vibrating textures generated by the tool on the tool path (Fig. 7d), which reduces the material removal of the tool in the depth-of-cut direction to a certain extent. The axial force \( F_x \) of 1D-UVT and UEVT is almost equal, while the axial force \( F_x \) of 3D-USVT is relatively large because the axial ultrasonic vibration of 3D-USVT makes the transient axial force larger. Compared with CT, the total cutting force of 1D-UVT decreased by 25.36 ~ 38.51%, UEVT decreased by 32.43 ~ 46.72%, and 3D-USVT decreased by 36.15 ~ 48.5%.

The tools of CT, 1D-UVT, UEVT, and 3D-USVT all have hard point wear, and the hard point wear and coating peeling phenomenon of 1D-UVT are more serious. This may be caused by the high-frequency reciprocating ironing of the tool flank on the machined surface. The coating peeling phenomenon also appeared in the UEVT, which may be due to the relatively large plowing force of the UEVT (Fig. 23c). In addition, the 3D-USVT also showed obvious thermal fatigue cracks and boundary wear (Fig. 23d). Under the same machining conditions, compared with CT, the tool wear can be reduced by 3.1% for 1D-UVT, 16.4% for UEVT, and 18.3% for 3D-USVT.

In conventional turning (CT) and ultrasonic vibration-assisted turning (1D-UVT, EUVT, and 3D-USVT), the evolution of surface roughness with machining parameters \( (v, a_p, \text{ and } f) \) was obtained by a single factor analysis method, as shown in Fig. 24.

Figure 24a shows that with the increase of the cutting speed \( v \), the Sa values obtained by the four cutting methods all decrease non-linearly. This is because, with the increase of the cutting speed \( v \), the increase of the deformation speed shortens the contact time between the tool and the workpiece, reduces the plastic deformation of the workpiece material, and helps to reduce the plastic side flow [5], and improved workpiece surface quality. From Eqs. (3), (5), (6) and (11), it can be seen that the geometric residual height of the workpiece surface is mainly affected by the corner radius of the tool nose, the feed rate, and the ultrasonic vibration. Liu and Melkote [49] believed that the plastic lateral flow was the biggest factor affecting the roughness height of peaks and valleys, and the research results of Chen et al. [50] also show from
the side that the influence of other factors such as cutting temperature and lathe vibration on the geometric residual height can be ignored when the corner radius of the tool nose is small. Since it is difficult to accurately detect the height of plastic lateral flow, it can only be explained according to the phenomenon of plastic lateral flow [51, 52]. According to the formation law of the geometric residual height [47, 53], it can be seen that when the workpiece is cut with a small feed and other cutting parameters remain unchanged, the effect of cutting speed on plastic lateral flow can be directly explained by the height difference between surface peaks and valleys. By comparing the peak-to-valley height (Sz) of the workpiece surface under different cutting speeds (Fig. 24d), it is found that with the increase in cutting speed, the Sz under the four cutting methods shows a downward trend. Among them, CT can be reduced by up to 3.3 μm, 1D-UVT can be reduced by up to 2.9 μm, UEVT can be reduced by up to 1.6 μm, and 3D-USVT can be reduced by up to 1.4 μm. Figure 24a shows that with the increase of v, the surface roughness of the machined surface obtained by CT is much greater than that of the other three cutting methods. This is because the tool with high-frequency vibration has high acceleration, which makes the tool perform micro elastic deformation and ductility processing [44] and reduce the waviness of the machined surface grooves, and can also inhibit the plastic side flow of the workpiece surface [54] and reduce the surface roughness. When v is between 50r/min and 200r/min, the value of Sa shows an upward trend. This may be because the built-up edge produced by CT processing 304 stainless steel at low speed falls off and adheres to the processed surface, thereby increasing the surface roughness of the workpiece. The Sa value of the machined surface under the three cutting modes of 1D-UVT, UEVT, and 3D-USVT changes relatively smoothly. During 1D-UVT cutting, the flank face of the tool does not separate from the workpiece, and the high-frequency repeated fretting ironing effect reduces the surface roughness of the machined surface of the workpiece, it will also cause the machined surface to wear and even produce some scratches, which leads to the deterioration of the accuracy of the machined surface. Due to its cutting characteristics, UEVT not only reduces tool wear but also improves the integrity of the machined surface. Figure 24a shows that when v is greater than 400r/min, the Sa value of 3D-USVT is the smallest, which verifies the correctness of the conclusion in 3.1; when v is greater than 320r/min, the Sa value of 3D-USVT began to rise sharply. This may be because the higher v not only causes the instantaneous impact force of the tool to increase and aggravate the wear of the tool but also caused the tool to generate additional vibration, which reduced the stability of the cutting system.

By changing the ap to cut the workpiece, it can be seen from Fig. 24b that with the increase of the ap, the Sa under the four different cutting methods shows an upward trend. This is because as the ap increases, the plastic side flow phenomenon of the material intensifies, and the residual geometric height of the workpiece surface increases [48].

\[
F = \frac{\tau A_s}{\sin\cos(\varphi - \alpha)} = \frac{\tau b_D h_D}{\sin\cos(\varphi + \beta - \alpha)}
\]

where \(\tau\) represents the shear stress, \(A_s\) represents the cutting area, \(b_D\) represents the cutting width, and \(h_D\) represents the cutting thickness.

According to Eq. (12), it can be concluded that when other cutting conditions remain unchanged and the \(a_p\) is increased, the cutting area will increase accordingly, resulting in greater deformation resistance and friction, higher cutting force, which will eventually lead to vibrations in the entire turning system, the surface roughness becomes larger. Figure 24c shows that with the increase of the \(f\), the Sa under the four different cutting methods shows an upward trend, but the Sa change rate is slightly different. According to Eqs. (4) and (12), it can be seen that when the feed amount is small, the residual geometric height and cutting force generated when cutting the workpiece are small. The cutting vibration is not obvious, and the Sa is small.
When the $f$ is increased, it will not only lead to a larger cutting force, the stability of the turning system will be reduced, but also a significant spiral groove will be generated on the surface of the workpiece, reducing the surface finish of the workpiece. Figure 24c shows that when the feed per revolution of the tool exceeds 0.1 mm, the surface roughness of 1D-UVT increases faster than the other three cutting methods which may be since the overlapping area of two adjacent cutting trajectories decreases with the increase of feed, and the ironing effect of the tool is not significant.

By comparing Fig. 24, it can be found that the effect of changing the $a_p$ on $S_a$ is significantly lower than the effect of $v$ and $f$ on $S_a$. The changing trend of this phenomenon is the same in traditional turning and ultrasonic-assisted turning. In general, for the four different cutting methods (CT, 1D-UVT, UEVT, and 3D-USVT), the maximum surface roughness appears in CT cutting, and the surface roughness value becomes smaller with the increase of the applied vibration direction. Compared with 1D-UVT and UEVT, the surface roughness value obtained by 3D-USVT is the smallest, and the reduction degree can be up to 45.5% compared with CT.

### 5 Conclusion

Cutting 304 stainless steel by the 3D-USVT, the influence of vibration parameters and cutting parameters on the 3D-USVT was studied, and the best parameter combination was established by the surface corresponding method. The surface roughness of CT, 1D-UVT, UEVT, and 3D-USVT cutting methods was analyzed theoretically, and the theoretical analysis results were checked by experiments. The experiments show that 3D-USVT has excellent cutting performance. The main conclusions are as follows:

1. In terms of vibration parameters, the amplitude in three directions has an impact on the surface roughness obtained by the 3D-USVT. The horizontal vibration along the OX direction has the greatest impact on $S_a$ and $S_q$, and the vertical vibration along the OY direction has the least impact on $S_a$ and $S_q$. In terms of cutting parameters, feed ($f$) has the greatest impact on $S_a$ and $S_q$, and depth of cut ($a_p$) has the least impact on $S_a$ and $S_q$. Optimal vibration parameters and optimal machining parameters of the 3D-USVT are determined.
2. The causes of surface roughness are analyzed, and according to the influence of UEVT on surface topography, the coefficient $M$ is introduced. When there is fractional part in the $M$, the surface roughness obtained by the CT method is the largest, followed by 1D-UVT, and 3D-USVT is the smallest. When there is no fractional part in the $M$, the surface roughness obtained by the CT method is the largest, followed by UEVT, and 3D-USVT is the smallest.

3. The experimental results show that the surface roughness obtained by the four cutting methods will increase with the increase of feed ($f$) and depth of cut ($a_p$), and decrease with the increase in cutting speed ($v$). Among them, the cutting force and surface roughness obtained by 3D-USVT are the smallest, and the degree of tool wear is relatively insignificant, indicating that the method has good machining performance.

Author contribution J.D.: methodology, experiment, validation, editing, and writing—original draft. P.Z.: resources and supervision. S.W., R.F., and L.F.: investigation. All authors read and approved the final manuscript.

Funding This project is supported by National Natural Science Foundation of China (51875097) and the Fundamental Research Funds for the Central Universities (N2103006 and N2203001).

Availability of data and materials All data used in the manuscript are available as submitted.

Code availability Not applicable.

Declarations

Ethics approval The authors declare that there is no ethical issue applied to this article.

Consent to participate The authors declare that all authors have read and approved to submit this manuscript to IJAMT.

Consent for publication The authors declare that all authors agree to sign the transfer of copyright for the publisher to publish this article upon on acceptance.

Conflict of interest The authors declare no competing interests.

References

1. Bedi SS, Prasad Sahoo S, Vikas B, Datta S (2021) Influence of cutting speed on dry machinability of AISI 304 stainless steel. Mater Today Proc 38:2174–2180. https://doi.org/10.1016/j.matpr.2020.05.554

2. Hong H, Riga AT, Gahoon JM, Scott CG (1993) Machinability of steels and titanium alloys under lubrication. Wear 162–164:34–39. https://doi.org/10.1016/0043-1648(93)90481-Z

3. Tekiner Z, Yeşilyurt S (2004) Investigation of the cutting parameters depending on process sound during turning of AISI 304 austenitic stainless steel. Mater Des 25:507–513. https://doi.org/10.1016/j.matdes.2003.12.011

4. Xavior MA (2012) Evaluating the machinability of AISI 304 stainless steel using alumina inserts. J Achiev Mater Manuf Eng 55:841–847

5. Skelton RC (1969) Effect of ultrasonic vibration on the turning process. Int J Mach Tool Des Res 9:363–374. https://doi.org/10.1016/0020-7357(69)90020-1

6. Babitsky VI, Kalashnikov AN, Meadows A, Wijesundara AAH (2003) Ultrasonically assisted turning of aviation materials. J Mater Process Technol 132:157–167. https://doi.org/10.1016/S0924-0136(02)00844-0

7. Babitsky VI, Mitrofanov AV, Silberschmidt VV (2004) Ultrasonically assisted turning of aviation materials: simulations and experimental study. Ultrasonics 42:81–86. https://doi.org/10.1016/j.ultras.2004.02.001

8. Zou P, Xu Y, He Y, Chen M, Wu H (2015) Experimental investigation of ultrasonic vibration assisted turning of 304 austenitic stainless steel. Shock Vib

9. Xu Y, Gao F, Zou P, Zhang Q, Fan F (2020) Theoretical and experimental investigations of surface roughness, surface topography, and chip shape in ultrasonic vibration-assisted turning of Inconel 718. J Mech Sci Technol 34:3791–3806. https://doi.org/10.1007/s12206-020-0830-x

10. Celaya A, Campa FJ, Lamikiz A (2016) Application of ultrasonics as assistance in machining operations. University of the Basque Country, pp 159–172

11. Celaya A, Lopez de Lacalle LN, Campa FJ, Lamikiz A (2010) Ultrasonic assisted turning of mild steels. Int J Mater Prod Technol 37:60–70

12. Nath C, Rahman M (2008) Effect of machining parameters in ultrasonic vibration cutting. Int J Mach Tools Manuf 48:965–974. https://doi.org/10.1016/j.ijmachtools.2008.04.008

13. Puga H, Grilo J, Carneiro VH (2019) Ultrasonic assisted turning of Al alloys: influence of material processing to improve surface roughness. Surfaces 2:326–335

14. He Y, Zhou Z, Zou P, Gao X, Ehmann KF (2019) Study of ultrasonic vibration–assisted thread turning of Inconel 718 superalloy. Adv Mech Eng 11:1687814019883772. https://doi.org/10.1177/1687814019883772

15. Shamote E, Moriwaki T (1994) Study on elliptical vibration cutting. CIRP Ann 43:35–38. https://doi.org/10.1016/S0007-8506(07)62158-1

16. Zhang J, Cui T, Ge C, Sui Y, Yang H (2016) Review of micro/nano machining by utilizing elliptical vibration cutting. Int J Mach Tools Manuf 106:109–126. https://doi.org/10.1016/j.ijmachtools.2016.04.008

17. Moriwaki T, Shamote E (1995) Ultrasonic elliptical vibration cutting. CIRP Ann 44:31–34. https://doi.org/10.1016/S0007-8506(07)62269-0

18. Zhang X, Senthil Kumar A, Rahman M, Nath C, Liu K (2011) Experimental study on ultrasonic elliptical vibration cutting of hardened steel using PCD tools. J Mater Process Technol 211:1701–1709. https://doi.org/10.1016/j.jmatprotec.2011.05.015

19. Ma C, Shamote E, Moriwaki T, Zhang Y, Wang L (2005) Suppression of burrs in turning with ultrasonic elliptical vibration cutting. Int J Mach Tools Manuf 45:1295–1300. https://doi.org/10.1016/j.ijmachtools.2005.01.011

20. Khajehzadeh M, Boostanipour O, Amiri S, Razfar MR (2020) The influence of ultrasonic elliptical vibration amplitude on cutting tool flank wear. Proc Inst Mech Eng Part B J Eng Manuf 234:1499–1512. https://doi.org/10.1017/S0954405420929782
21. Suzuki N, Masuda S, Haritani M, Shamoto E (2004) Ultraprecision micromachining of brittle materials by applying ultrasonic elliptical vibration cutting. In: Micro-Nanomechatronics and Human Science, 2004 and The Fourth Symposium Micro-Nanomechatronics for Information-Based Society, pp 133–138

22. Ma C, Shamoto E, Moriwaki T, Wang L (2004) Study of machining accuracy in ultrasonic elliptical vibration cutting. Int J Mach Tools Manuf 44:1305–1310. https://doi.org/10.1016/j.ijmachtools.2004.04.014

23. He Y, Zou P, Zhu W-L, Ehmann KE (2017) Ultrasonic elliptical vibration cutting of hard materials: simulation and experimental study. Int J Adv Manuf Technol 91:363–374. https://doi.org/10.1007/s00170-016-9716-8

24. Usman MM, Zou P, Tian Y, Wang W (2020) Experimental investigation on surface functional indices in ultrasonic elliptical vibration cutting of C45 carbon steel. Int J Adv Manuf Technol 109:187–200. https://doi.org/10.1007/s00170-020-05661-8

25. Ahn J-H, Lim H-S, Son S-M (1999) Improvement of micromachining accuracy by 2-dimensional vibration cutting

26. Kim GD, Loh BG (2007) Characteristics of chip formation in micro V-grooving using elliptical vibration cutting. J Micromech Microeng 17:1458

27. Kim GD, Loh BG (2007) An ultrasonic elliptical vibration cutting device for micro V-groove machining: kinematical analysis and micro V-groove machining characteristics. J Mater Process Technol 190:181–188. https://doi.org/10.1016/j.jmatprocess.2007.02.047

28. Kim GD, Loh BG (2007) Characteristics of elliptical vibration cutting in micro-V-grooving with variations in the elliptical cutting locus and excitation frequency. J Micromech Microeng 18(2):25002

29. Pan Y, Kang R, Bao Y, Du W, Yin S, Dong Z (2022) Surface formation mechanism in ultrasonic elliptical vibration cutting of tungsten alloy. In: International Conference on Nanomanufacturing. Springer, pp 109–122

30. Tan R, Zhao X, Sun T, Zou X, Hu Z (2019) Experimental investigation on micro-groove machining of Ti-6Al-4V alloy by using ultrasonic elliptical vibration assisted cutting. Mater Des 129:3086

31. Jieqiong L, Yingchun L, Xiaqin Z (2013) Tool path generation for fabricating optical freeform surfaces by non-resonant three-dimensional elliptical vibration cutting. Proc Inst Mech Eng Part C J Mech Eng Sci 228:1208–1222. https://doi.org/10.1177/0954406213502448

32. Shamoto E, Suzuki N, Tsuchiya E, Horii Y, Yoshino K (2005) Development of 3 DOF ultrasonic vibration tool for elliptical vibration cutting of sculptured surfaces. CIRP Ann 54:321–324. https://doi.org/10.1016/S0007-8506(07)60113-9

33. Sajjady SA, Abadi HNH, Amini S, Nosouhi R (2016) Analytical and experimental study of topography of surface texture in ultrasonic vibration assisted turning. Mater Des 93:311–323. https://doi.org/10.1016/j.matdes.2015.12.119

34. Lin J, Lu M, Zhou X (2016) Development of a non-resonant 3D elliptical vibration cutting apparatus for diamond turning. Exp Tech 40:173–183. https://doi.org/10.1007/s40799-016-0021-0

35. Lin J, Zhou J, Lu M, Wang H, Yi A (2020) Design of robust adaptive fuzzy controller for a class of single-input single-output (siso) uncertain nonlinear systems. Math Probl Eng

36. Du Y, Lu M, Wang H, Zhou J, Lin J (2021) Parameter tuning of robust adaptive fuzzy controller for 3D elliptical vibration-assisted cutting. Mech Sci 12:433–442

37. Curti R, Marcon B, Furferi R, Denaud L (2019) Specific cutting coefficients at different grain orientations determined during real machining operations. Proc Int Wood Mach Sci 53–62

38. Furukawa Y, Moronuki N (1988) Effect of material properties on ultra precise cutting processes. CIRP Ann 37:113–116

39. Olovsjö S, Nyborg L (2012) Influence of microstructure on wear behaviour of uncoated WC tools in turning of Alloy 718 and Waspaloy. Wear 282–283:12–21. https://doi.org/10.1016/j.wear.2012.01.004

40. Lee WB, To S, Sze YK, Cheung CF (2003) Effect of material anisotropy on shear angle prediction in metal cutting—a meso- plasticity approach. Int J Mech Sci 45:1739–1749. https://doi.org/10.1016/j.ijmecsci.2003.09.024

41. Doan D-Q, Fang T-H, Chen T-H (2021) Machining mechanism and deformation behavior of high-entropy alloy under elliptical vibration cutting. Intermetallics 131:107079. https://doi.org/10.1016/j.intermetal.2020.107079

42. Zhang C, Song Y (2019) Design and kinematic analysis of a novel decoupled 3D ultrasonic elliptical vibration assisted cutting mechanism. Ultrasonics 95:79–94

43. Sharma V, Pandey PM (2017) Experimental investigations and statistical modelling of surface roughness during ultrasonic-assisted turning with self-lubricating cutting inserts. Proc Inst Mech Eng Part E J Process Mech Eng 232:709–722. https://doi.org/10.1080/0954408917738127

44. Krizbergs J, Kromanis A (2006) Methods for prediction of the surface roughness 3D parameters according to technological parameters. In: 5th International DAAAM Baltic Conference, Tallinn

45. Shuang Y, John M, Songlin D (2019) Experimental investigation on the performance and mechanism of graphene oxide nanofluids in turning Ti-6Al-4V. J Manuf Process 43:164–174. https://doi.org/10.1016/j.jmapro.2019.05.005

46. Chen GJ, Liu XL, Yue CX (2010) Study on causes of material plastic side flow in precision hard cutting process. In Advanced materials research. Trans Tech Publ 1875–1878

47. He C, Zong W, Sun T (2016) Origins for the size effect of surface roughness in diamond turning. Int J Mach Tools Manuf 106:22–42

48. Zong W, Huang Y, Zhang YL, Sun T (2014) Conservation law of surface roughness in single point diamond turning. Int J Mach Tools Manuf 84:58–63. https://doi.org/10.1016/j.ijmachtools.2014.04.006

49. Liu K, Melkote SN (2006) Effect of plastic side flow on surface roughness in micro-turning process. Int J Mach Tools Manuf 46:1778–1785

50. Chen G, Liu X, Wang L, Zhou X (2016) The research and modeling about plastic flow measurement of machined surface in precision turning of hardened steel GCr15. Int J Nanomanuf 12:154–166

51. Ishawiy HA, Elbestawi MA (1999) Effects of process parameters on material side flow during hard turning. Int J Mach Tools Manuf 39:1017–1030. https://doi.org/10.1016/S0890-6955(98)00084-4

52. Liang X, Liu Z, Yao G, Wang B, Ren X (2019) Investigation of surface topography and its deterioration resulting from tool wear evolution when dry turning of titanium alloy Ti-6Al-4V. Tribol Int 135:130–142. https://doi.org/10.1016/j.triboint.2019.02.049

53. Kong MC, Lee WB, Cheung CF, To S (2006) A study of materials swelling and recovery in single-point diamond turning of ductile materials. J Mater Process Technol 180:210–215. https://doi.org/10.1016/j.jmatprocess.2006.06.006

54. Kim J-D, Choi I-H (1997) Micro surface phenomenon of ductile cutting in the ultrasonic vibration cutting of optical plastics. J Mater Process Technol 68:89–98

Publisher’s note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.