Robust and Stable Flow Shop Scheduling Problem under Uncertain Processing Times and Machines’ Disruption

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1. INTRODUCTION

The flow Shop Scheduling Problem (FSSP) covers many real case studies in practical problems [1]. Some papers described the applications of the two-machine flow shop scheduling problem (FSSP) [2–4]. Total completion time minimizes Work in Process (WIP) costs and the rapid turnaround of jobs. The two-machine FSSP with the sum of jobs’ completion time as a primary objective, this paper’s focus, even in deterministic scheduling environments, is strongly NP-hard [5]. Some effective heuristics proposed to cope with the problem's complexity do not seem superior over the other [6]. Besides, the job or machine-related uncertainties that lead to an interruption in the flow of jobs and result in unwanted delays are commonly occurring in the production environment, enhancing the problem's complexity. Arriving of an unanticipated new job [7], due date uncertainty [8], breakdown occurrence [9], uncertainty in job processing times [9, 10], etc are the
likes of uncertainties and disruptions. In 70% of uncertainty oriented flow shop scheduling studies in past decades, the job processing time is uncertain, by 25%, the disruption is machine failure, and by 10%, both of these factors consider [11]. Machine failure and uncertain processing times discuss in this paper. Robust and stable scheduling is one of the policies in confronting uncertainty. The sensitivity of a schedule performance to its objective function is called robustness, but stability refers to the insensitivity of the start (or completion) time. Stability is a measure of changes in the sequence of jobs on a machine to the original. The concept of robustness is very close to flexibility: the ease of schedule reparation and the power of converting to new, high quality scheduling in the face of uncertainties. The expected realized total completion time has been implemented as a robustness measure by itself [8]. Here we take this definition as robustness. A function of the sum of deviation between each jobs' start/completion times in the initial and realized schedules are often the stability measure [8], and the same definition is accepted here. The value of the expected performance measure obtained by applying the righting shift policy on the initial schedule is a realized schedule. Additionally, a justified schedule with a small deviation from the initial one in the face of uncertainty and without significant degradation in the main objective is robust and stable. Simultaneous consideration of robustness and stability besides maintaining the schedule feasibility improves its flexibility against uncertainties.

Dealing with uncertainty-related deviations can be done with predictive, reactive, or predictive-reactive (hybrid) strategies [12]. In the predictive strategy, future uncertainties consider in the initial plan. Reactive or hybrid approaches are common strategies for dealing with machine breakdown. In almost all robustness-focused studies, dealing with machine failure disruption performs with reactive approaches or in the reactive phase of hybrid approaches [8]. In reactive strategies (e.g., rescheduling), especially in large-size problems, it takes a long time to deal with uncertainty. Predictive strategies can overcome this by actively preparing for any future uncertainties [12], so here we adopt a predictive approach to cope with machine breakdown and uncertainty of job processing times. The two-machine Flow Shop Scheduling Problem (FSSP) under uncertainty of processing time is referenced in many papers, commonly with makespan as a primary objective function [13, 14]. \( C_{\text{max}} \), is also a primary objective in most FSSP under machine breakdown disruption studies [15, 16]. In addition, it has been a primary objective in the case of simultaneously considering the uncertainty of processing time and machine breakdown [9, 17]. Therefore, we define the robustness measure based on another performance measure; i.e., the total completion times. With a glimpse at the previous attempts in this realm of research, we can state contributions of this paper:

- Although various cases of the robust and stable flow shop-scheduling problem previously raised in studies, this article discusses a (particular) case of \( F2 \) for the first time.
- Simultaneously considering robustness and stability to meet the requirements of producers and workers.
- Besides, predictively coping with the uncertainty of job processing times, dealing with machine breakdowns is also predictive.
- Proposing a novel robust and stable heuristics to cope with aforementioned-uncertainty conditions.
- The way of considering the uncertainty, the proposed solution method, and the primary objective function is different from the previous more related works.

The remainder of this paper is as follows; the related literature review is in section 2. In section 3, we define the problem and propose our solution method. In sections 4, 5, and 6, we presented computational results, managerial insight, and paper conclusions.

## 2. LITERATURE REVIEW

This paper presents robust and stable scheduling approaches for a permutation two-machine flow shop scheduling problem (PFSSP) under uncertainty with expected total completion times as a primary objective. According to the classification of Graham et al. [18], the problem denotes as \( F2/\text{prmu} / \sum_{j=1}^{n} C_{2j} \), in the deterministic version, which is strongly Np-Hard [5]. The solution methods of \( F2/\text{prmu} / \sum_{j=1}^{n} C_{2j} \) categorizing into exact and approximate methods. The high-performance problem-solving branch and bound algorithms and Lagrangian methods had proposed for \( F2/\sum_{j=1}^{n} C_{2j} \) able to solve up to 50 jobs [19]. Due to the high complexity of this problem, heuristic methods have been ever the researchers' focus, for example, MINITI heuristic [6, 20]. Rossi et al. [21] proposed a simple high-efficiency FF-RN heuristic method by modifying the NEH heuristic and compared it with the best simple heuristics of PFSSP reviewed [21]. In the face of uncertainty, applying iterative simulation-based methods or producing robust (and stable) schedules are conventional approaches to encounter system disruptions [9]. Ghezail et al. [22] proposed a graphical robust, proactive approach to deal with uncertainty in the FSSP. Kasperski et al. [23] propose a predictive regret-based robust schedule with interval processing times. Katrajeni et al. [24] propose a heuristic to minimize normalized makespan and instability in a dynamic flow shop under uncertainty of machine breakdown and job-ready time variability. Ying [25] applied Iterated Greedy (IG) and Simulated Annealing (SA) heuristics to produce a predictive regret-based robust schedule in a maximum
completion time two-machine FSSP, with interval processing times. Fazayeli et al. [26] applied the Genetic Algorithm (GA) and SA to produce a robust predictive schedule in an m-machine PFSP under uncertain repair time and machine breakdown. Rahmani [9] employed GA to propose a proactive-reactive robust, and stable schedule for a two-machine PFSP under uncertain job processing time and machine failure. Also, she applied scenarios to show the uncertainty of processing times, Cmax as an efficiency measure, maximum realized completion times of jobs as robustness, and the expected sum of square deviations between the completion time in actual and initial schedules stability measure. Cui et al. [16] used a simulation-based method to propose a robust predictive schedule for two-machine PFSP under machine breakdown with Cmax as an efficiency measure. Liao and Fu [7] exploit GA to propose a robust predictive schedule for an m-machine PFSP with interval processing times. Abtahi et al. [11] employed a robust optimization method to produce efficient, robust, and stable schedules in an m-machine FSSP under uncertainty. They applied scenarios to show the uncertainty of processing times, total completion times as an efficiency measure, total realized tardiness of jobs as robustness, and the expected sum of square deviations between the completion time in actual and initial schedules as a stability measure. Here we adopt total realization completion times as robustness and the expected sum of square deviations between the completion time in actual and initial schedules as a stability measure. We propose a modified Shifting bottleneck (SB) to produce robust partial solutions in a two-machine FSSP with the makespan criterion. Elyasi and Salmasi [32] proposed a decomposition-based heuristic for a total completion time FSSP with bounded processing time. As can be seen, few papers focused on

- Producing a robust and stable FSSP with total completion time as a primary objective.
- Predictively producing a robust and stable FSSP while considering the uncertainty of job processing time and machine breakdown simultaneously.

In this paper, we propose a heuristic method to produce a robust and stable schedule in a stochastic two-machine FSSP specified case with total completion time as a primary objective. Then we compare it to an exact solution method. The former (Our proposed heuristic) employs modified SB and a theorem of Abtahi et al. [33], and the latter uses a theorem of Pinedo [34] to hedge against job processing time uncertainty. We employ the Right-Shifting (RS) rescheduling method to obtain a realistic schedule after machine failures occurrence.

3. PROBLEM DEFINITION AND SOLUTION METHOD

In this paper, we considered a two-machine FSSP. The uncertain job processing time and random breakdowns of machines are the system disruptions. The processing time of job \( j \) on the first and second machines respectively follow the exponential distribution with rates \( \lambda_j \) and \( \mu_j \). The time between two consecutive failures follows an exponential distribution with the rate of \( \theta \) and at most one failure is expected on a machine in each interval \((1/\theta)\). After each breakdown, minimal repairs perform to restore machines to the operating condition (which does not affect the machine age and breakdown parameter).

The following assumptions considered:

- All jobs are available at the beginning of the schedule,
- Machines have availability restriction; i.e., random machine break down may occur during the processing of job \( j \) on machine \( i \),
- The time between two consecutive breakdowns follows an exponential distribution. Also, constant repair times allocate after each failure,
- The rest of the disrupted job will perform after machine repairing,
- Only non-delay schedules considered,
- The objective function is a minimization of schedules’ robustness and stability simultaneously.

3.1 Solution Method  According to a classification by Graham et al. [18], a problem of robust and stable two-machine FSSP under uncertainty of job processing time and machine breakdowns is represented as follows:

\[
F_2 \left[ p_{ij} - \exp(\lambda_j), p_{ij} - \exp(\mu_j), \right. \\
\left. \text{brkdwn} : U - \exp(\theta), D - G(r) \right] 
\]

\[ \alpha.RM + (1 -\alpha).SM \]

Pinedo [34] showed that sorting the jobs in descending order of \((\lambda_j - \mu_j)\) optimizes the expected total completion times (i.e., the intended robustness measure) in a two-machine FSSP particular case (when job processing times on the first (second) machine pursued the exponential distribution with the rate \( \lambda_j(\mu_j) \)). Here, we proposed two robust and stable methods, and for each one, two policies in the face of machine failure; reactive and predictive. To predictably deal with the machine failure, the buffer time insertion method, and to encounter with reactively, the right shift rescheduling (RSH) is implemented to the affected jobs [13] for details.
Two algorithms are proposed in this paper to handle such a problem:

- The optimal theorem based method (OBM).
- The decomposition-based method (DBM).

Based on a theorem, the OBM considering the uncertainty of processing times acquires the optimal robust solution (of Pinedo [34]) according to the decreasing order of $\alpha_j - \mu_j$.

The DBM method employs a modified shifting bottleneck heuristics and one-machine robustness and stability optimization theorem [33]; shifting bottleneck (SB) heuristics [5] decomposes a problem into sub-problems and solve each sub-problem optimally [25] using the shortest expected processing time (SEPT) first rule.

**Theorem.** SEPT rule solves optimally [33], where $\theta$ is the rate of machine breakdown and $R$ is the expected repair time, $RM = E \sum_{j=1}^{n} C_j^r$, is a robustness measure, and $SM = E[\sum_{j=1}^{n}(C_j - C_j^r)^2]$ is a stability measure. According to the above theorem, sorting the jobs in a no descending order of processing times over each machine seems an acceptable idea to give a robust and stable sequence for the intended uncertain two-machine FSSP. The steps of the DOM are as follows:

- Decompose the intended two-machine uncertain flow shop-scheduling problem into two one-machine sub-problems, with predefined conditions of uncertainty.
- Sequence the jobs according to the SEPT on the first machine.
- The first job on the first machine (M1) continued its process on the second machine regardless of the amount of its expected processing time (on M2).
- To determine the order of the remaining jobs on M2, do as follows.
- Whenever because of incomplete remaining (previous) jobs on M2, there is a queue with more than two jobs on M1, order the jobs queue according to the shortest expected value of their processing times on M2.
- Otherwise, the jobs keep their sequence on M1.

Figure 1 shows the flow chart of DBM. In the next section, we compared the proposed methods after the implementation of reactive as well as predictive policy. The job sequence on the two machines are the same in OBM; however, during the execution of DBM, the job sequence on M1 may not keep.

### 4. COMPUTATIONAL RESULTS

#### 4.1. Data Generation

The job processing times on the first and second machines are uncertain and respectively follow the exponential distribution with the rate of $\lambda_j$ and $\mu_j$, where $\lambda_j$ and $\mu_j$ are independently generated from $U[0.1, 1]$. We select the number of jobs from set $\{3, 5, 10, 30, 50, 60, 100\}$. Then 100 instances generate for each job number. Hence, we have 700 problems. For each test problem, we chose the rate of machine breakdown from a set $\theta = \{1/50, 1/60, 1/80\}$: a higher value of $\theta$ represents a higher probability of machine breakdown disruption. Like Nouri et al. [35], the repair times duration follows an exponential distribution based on the meantime to repair value (MTTR) at two-level. The repair times’ duration calculates via $r = \exp(\text{MTTR})$, and the $\text{MTTR}$ based on the machine busy time (MB); for low level, $\text{MTTR}_l = [0.01MB, 0.05MB]$; and for high level, $\text{MTTR}_h = [0.05MB, 0.1MB]$. Ultimately we have combination of 4200 problems. The methods compare to reach a comprehensive conclusion as follows:

1. Without considering machine breakdown,
2. Applying the reactive policy after failure,
3. Dealing with breakdown disruption predictively.

#### 4.2. Two Comparative Methods without Considering Machine Breakdown

Here, we examine the performance (objective function) of the two methods provide the managerial results for (the problem in question without assuming machine breakdown)
4. 3. Two Comparative Methods Based on Applying Reactive Policy

Here, we examine two proposed methods for (the problem in question)

\[ F_2 / p_{ij} \sim \exp(\lambda), p_{ij} \sim \exp(\mu_i) ; \]

\[ \alpha.RM + (1 - \alpha).SM \]

applying the reactive policy after machine breakdown. The problem coded in MATLAB R2013b, and outputs has been reported different problems’ sizes in Table 1.

**Figure 2.** The comparison the expected completion time for two methods without considering machine breakdown (small-size problem)

**Figure 3.** The related deviation between two methods without considering machine breakdown (small-size problem)

**Z_ROBM, n, and TIME, respectively represent the robustness of DBM and OBM, the stability of DBM and OBM, the objective function value of DBM and OBM, the number of jobs, and the problem-solving time by applying reactive policy after machine breakdown. Figures 6 to 10 illustrate the contents of Table 1.**

According to Figures 6 to 10, in all cases of the small-size problem (n<=10), regardless of the values of TETA and MTTR, there is no significant difference between the two proposed methods’ performance. Nevertheless, in medium to large-size problems (n>=30), OBM outperforms DBM by applying reactive policy after machine breakdown. These results had obtained by considering the same coefficients for robustness and stability (\( \alpha = (1 - \alpha) = 0.5 \)).

4. 4. Two Comparative Methods’ Based on Applying Predictive Policy

Here, we examine the two proposed methods for solving (the problem in question)

\[ F_2 / p_{ij} \sim \exp(\lambda), p_{ij} \sim \exp(\mu_i) ; \]

\[ \alpha.RM + (1 - \alpha).SM \]

by applying the predictive policy to encounter the machine breakdown. The problem coded in MATLAB R2013b, and the results were reported for different problems’ sizes in Table 2 and Figure 11.
| No | 1/TETA MTTR | n | RR_DBM | RR_OBM | SR_DBM | SR_OBM | Z_RDBM | Z_ROBM | T |
|----|-------------|---|--------|--------|--------|--------|--------|--------|----|
| 1  | 80          | low 3 | 0.01 | 0.05 | 0.05 | 0.02 | 0.03 | 0.036 | 1.58 |
| 2  | 80          | low 5 | 0.3  | 0.23 | 0.26 | 0.1  | 0.28 | 0.16  | 1.8  |
| 3  | 80          | low 10 | 0.27 | 0.52 | 0.47 | 0.49 | 0.36 | 0.5   | 6.48 |
| 4  | 80          | low 30 | 21.7 | 1.8  | 6.7  | 5.6  | 14.2 | 3.7   | 99   |
| 5  | 80          | low 50 | 237  | 3.5  | 43   | 23.8 | 136  | 7.52  | 308  |
| 6  | 80          | low 60 | 335  | 5    | 63   | 77   | 189  | 14.5  | 648  |
| 7  | 80          | low 100 | 5858.3 | 15.3 | 226.7 | 79 | 3042.5 | 47 | 2798 |
| 8  | 60          | low 3 | 0.018 | 0.001 | 0.02 | 0.001 | 0.01 | 0.009 | 27   |
| 9  | 60          | low 5 | 0.05 | 0.33 | 0.18 | 0.12 | 0.11 | 0.22  | 11   |
| 10 | 60          | low 10 | 0.82 | 0.32 | 0.67 | 0.19 | 0.75 | 0.26  | 11   |
| 11 | 60          | low 30 | 23  | 4    | 7.7  | 8.3  | 15.5 | 6     | 202  |
| 12 | 60          | low 50 | 574  | 4    | 44   | 15   | 310  | 10    | 608  |
| 13 | 60          | low 60 | 1733 | 4    | 73   | 12   | 903  | 9     | 380  |
| 14 | 60          | low 100 | 4279  | 15    | 223  | 119  | 1251 | 68    | 3187 |
| 15 | 50          | low 3 | 0.02 | 0.1  | 0.08 | 0.04 | 0.05 | 0.08  | 1.38 |
| 16 | 50          | low 5 | 0.49 | 0.18 | 0.23 | 0.07 | 0.36 | 0.12  | 1.94 |
| 17 | 50          | low 10 | 0.62 | 0.54 | 0.72 | 0.46 | 0.67 | 0.5   | 5.43 |
| 18 | 50          | low 30 | 18  | 2    | 7.5  | 3.4  | 12.8 | 2.8   | 51.3 |
| 19 | 50          | low 50 | 475  | 4    | 45   | 13   | 260  | 9     | 213.2 |
| 20 | 50          | low 60 | 1581 | 7.5  | 81   | 50   | 831  | 29    | 363  |
| 21 | 50          | low 100 | 4013  | 13    | 245  | 93   | 2128 | 54    | 1567 |
| 22 | 80          | high 3 | 0.07 | 0.17 | 0.11 | 0.17 | 0.09 | 0.17  | 1.4  |
| 23 | 80          | high 5 | 0.43 | 0.4  | 0.4  | 0.41 | 0.42 | 0.41  | 1.8  |
| 24 | 80          | high 10 | 3   | 1.08 | 1.56 | 1.48 | 2.28 | 1.28  | 6.3  |
| 25 | 80          | high 30 | 147 | 5    | 23   | 17   | 85.3 | 11.16 | 72.3 |
| 26 | 80          | high 50 | 42.5 | 46   | 329  | 8    | 186  | 27    | 294  |
| 27 | 80          | high 60 | 2663 | 27   | 125  | 343  | 1394 | 185   | 455  |
| 28 | 80          | high 100 | 12388  | 94.5 | 418  | 2055 | 6403 | 1075  | 2049 |
| 29 | 60          | high 3 | 0.21 | 0.21 | 0.17 | 0.24 | 0.19 | 0.23  | 1.41 |
| 30 | 60          | high 5 | 1.03 | 0.42 | 0.46 | 0.50 | 0.75 | 0.46  | 1.62 |
| 31 | 60          | high 10 | 1.18 | 1.9  | 1.1  | 2.70 | 1.5  | 1.9   | 4.6  |
| 32 | 60          | high 30 | 166.9 | 6.6  | 25.7 | 42.8 | 96.3 | 6.6   | 70.4 |
| 33 | 60          | high 50 | 378.7 | 7.8  | 50.7 | 58.5 | 219.2 | 33.2  | 313.7 |
| 34 | 60          | high 60 | 1863.8 | 16.2 | 212.5 | 113.4 | 988.6 | 114.3  | 511.6 |
| 35 | 60          | high 100 | 11129.2  | 40.65 | 402  | 543  | 5765.7 | 291.8  | 2947 |
| 36 | 50          | high 3 | 0.09 | 0.16 | 0.13 | 0.08 | 0.11 | 0.12  | 1.5  |
| 37 | 50          | high 5 | 0.38 | 0.39 | 0.35 | 0.35 | 0.367 | 0.37  | 2.11 |
| 38 | 50          | high 10 | 5.46 | 1.09 | 1.9  | 1.5  | 3.7  | 1.3   | 5.27 |
| 39 | 50          | high 30 | 103.6 | 12.27 | 17.1 | 12.27 | 60.36 | 8.2   | 53.98 |
| 40 | 50          | high 50 | 1384.3 | 19.8 | 97.6 | 170.3 | 740.99 | 95   | 250  |
| 41 | 50          | high 60 | 2708.8 | 14.9 | 146.65 | 151.99 | 1427.7 | 83.47  | 444.9 |
| 42 | 50          | high 100 | 29.6 | 6328 | 441.2 | 316.9 | 3384.7 | 173.2  | 3017 |

TABLE 1. Robustness, stability, and the objective function of the two methods by applying the reactive policy for different problem-parameters.
Figure 6. Comparing the objective function of the two methods by applying the reactive policy, Low expected failure, and short repair time.

Figure 7. Comparing the objective function of the two methods by applying the reactive policy, medium expected failure, and short repair time.

Figure 8. Comparing the objective function of the two methods by applying the reactive policy, low expected failure, and short repair time.

Figure 9. Comparing the objective function of the two methods by applying the reactive policy, low expected failure, and high repair time.

Figure 10. Comparing the objective function of the two methods by applying the reactive policy, medium expected failure, and high repair time.

and OBM. The stability of DBM and OBM, the objective function value of the DBM and OBM, the number of jobs and the problem-solving time incorporated by applying predictive policy to encounter with machine breakdown. Figure 11 illustrates the contents of Table 2. According to Figure 11 and Table 2, DBM is preferred to OBM in all cases regardless of the values of TETA, and MTTR, especially when the number of jobs increases. These results were obtained by considering the same coefficients for robustness and stability ($\alpha = (1-\alpha) = 0.5$).

In the sensitivity analysis section, we will analyze the effect of different values of $\alpha$ (robustness coefficient) on the performance measure of the two methods.

4.4. Sensitivity Analysis

This section provides additional tests on the methods' parameters to gauge their effects on the objective functions' values.

4.4.1. Testing on the Rate of Machine Breakdown and Mean Time to Repair

According to Tables 3 to 8 and Figures 12 to 14, regardless of the number of jobs, the rate of a machine breakdown and the meantime to repair, DBM outperforms OBM. Also, as expected, the higher the failure rate (TETA) and the meantime to repair (MTTR), the worse the value of the robustness, stability, and two methods' objective functions.

4.4.2. Testing on the Stability and Robustness Coefficients

In this section, different values of the robustness coefficients ($\alpha$) had applied to achieve both methods' objective values. The results depicted for the low level of MTTR and $\theta = 0.0125$ in Figures 15 to 18. The effects of the varying coefficients of the robustness on the two methods’ objective functions by applying the predictive policy showed in Table 9. According to Table 9 and Figure 17, OBM outperforms DBM when $\alpha \geq 0.7$, especially for many jobs. For values less than 0.7, DBM is superior to the OBM. In comparing two methods by applying the reactive policy, the effect of the robustness coefficient ignores. In this case, OBM is always almost outperformed DBM.
### TABLE 2. Robustness, stability, and two methods’ objective function by applying the predictive policy for different problem parameters

| No | 1/TETA | MTTR | n  | RP_DBM | RP_OBM | SP_DBM | SP_OBM | ZP_DBM | ZP_OBM | T   |
|----|--------|------|----|--------|--------|--------|--------|--------|--------|-----|
| 1  | 80     | low  | 3  | 0.01   | 0.069  | 0.067  | 0.015  | 0.039  | 0.042  | 1.58|
| 2  | 80     | low  | 5  | 0.27   | 0.2    | 0.27   | 0.07   | 0.27   | 0.13   | 1.8  |
| 3  | 80     | low  | 10 | 0.26   | 0.95   | 0.73   | 0.32   | 0.5    | 0.48   | 6.48|
| 4  | 80     | low  | 30 | 16.6   | 16.6   | 9.5    | 23.7   | 13     | 20     | 99  |
| 5  | 80     | low  | 50 | 221.25 | 85.23  | 44     | 291.7  | 133    | 188.5  | 308 |
| 6  | 80     | low  | 60 | 524    | 164    | 73     | 1313   | 299    | 739    | 648 |
| 7  | 80     | low  | 100| 6637   | 667    | 366    | 9029   | 3501   | 4848   | 2798|
| 8  | 60     | low  | 3  | 0.002  | 0.05   | 0.05   | 0.002  | 0.25   | 0.25   | 27  |
| 9  | 60     | low  | 5  | 0.04   | 0.26   | 0.25   | 0.08   | 0.17   | 0.15   | 11  |
| 10 | 60     | low  | 10 | 0.54   | 0.36   | 0.55   | 0.13   | 0.54   | 0.25   | 11  |
| 11 | 60     | low  | 30 | 19.5   | 28     | 13     | 64     | 16     | 46     | 202 |
| 12 | 60     | low  | 50 | 552    | 134    | 68     | 936    | 309    | 535    | 608 |
| 13 | 60     | low  | 60 | 1514   | 213    | 110    | 2086   | 812    | 1149   | 380 |
| 14 | 60     | low  | 100| 14350  | 1018   | 529    | 22181  | 7640   | 11600  | 3187|
| 15 | 50     | low  | 3  | 0.02   | 0.1    | 0.1    | 0.03   | 0.06   | 0.07   | 1.38|
| 16 | 50     | low  | 5  | 0.5    | 0.2    | 0.3    | 0.04   | 0.4    | 0.1    | 1.94|
| 17 | 50     | low  | 10 | 0.4    | 1.1    | 0.9    | 0.4    | 0.65   | 0.77   | 5.43|
| 18 | 50     | low  | 30 | 21     | 26     | 12.5   | 52     | 17     | 39     | 51.3|
| 19 | 50     | low  | 50 | 588    | 146    | 73.5   | 972    | 331    | 559    | 213.2|
| 20 | 50     | low  | 60 | 2280   | 265    | 149    | 1361   | 1214   | 1713   | 363 |
| 21 | 50     | low  | 100| 1241   | 20487  | 608    | 41905  | 10548  | 21573  | 1567|
| 22 | 80     | high | 3  | 0.11   | 0.2    | 0.19   | 0.14   | 0.15   | 0.17   | 1.4 |
| 23 | 80     | high | 5  | 0.35   | 0.36   | 0.41   | 0.29   | 0.38   | 0.33   | 1.8 |
| 24 | 80     | high | 10 | 2.8    | 1.28   | 1.8    | 1.01   | 2.3    | 1.1    | 6.3 |
| 25 | 80     | high | 30 | 74.8   | 29     | 19.9   | 96.5   | 47.4   | 49.25  | 72.3|
| 26 | 80     | high | 50 | 490    | 167    | 76     | 1399.8 | 283    | 783    | 294 |
| 27 | 80     | high | 60 | 4458   | 384    | 208.8  | 5710   | 2333.5 | 3047   | 455 |
| 28 | 80     | high | 100| 30377.5| 1569   | 705    | 61743  | 15541  | 31656  | 2049|
| 29 | 60     | high | 3  | 0.16   | 0.21   | 0.2    | 0.18   | 0.18   | 0.2    | 1.4 |
| 30 | 60     | high | 5  | 0.91   | 0.38   | 0.51   | 0.29   | 0.71   | 0.34   | 1.62|
| 31 | 60     | high | 10 | 2      | 1.9    | 1.8    | 1.9    | 1.9    | 1.9    | 4.6 |
| 32 | 60     | high | 30 | 159.6  | 54.2   | 28.1   | 244.5  | 93.8   | 149.4  | 70.4|
| 33 | 60     | high | 50 | 973.7  | 202.1  | 96.1   | 2032.4 | 535    | 1117.3 | 313.7|
| 34 | 60     | high | 60 | 2248.8 | 416    | 167.2  | 7344.1 | 1208   | 3880   | 511.6|
| 35 | 60     | high | 100| 39551  | 1658   | 815    | 76047  | 20183  | 38852  | 2948|
| 36 | 50     | high | 3  | 0.06   | 0.15   | 0.14   | 0.05   | 0.1    | 0.1    | 1.5 |
| 37 | 50     | high | 5  | 0.26   | 0.38   | 0.37   | 0.22   | 0.32   | 0.30   | 2.11|
| 38 | 50     | high | 10 | 8.6    | 2.16   | 2.46   | 2      | 5.5    | 2.08   | 5.27|
| 39 | 50     | high | 30 | 201    | 58.87  | 33.97  | 312.85 | 117.5  | 185.86 | 53.98|
| 40 | 50     | high | 50 | 4492.4 | 176.9  | 5718.1 | 95.01  | 2334.68| 3029.5 | 250 |
| 41 | 50     | high | 60 | 6660   | 523.8  | 264.3  | 12592  | 3462.3 | 6558   | 444.9|
| 42 | 50     | high | 100| 61755.5| 2149.5 | 1119.6 | 110545.6| 31437.6| 56347.5| 3017|
Figure 11. Comparison of two methods’ objective function by applying the predictive policy, low expected failure, and short repair time

**TABLE 3.** Robustness, stability, and the objective function of two methods by applying the predictive policy for n=100, short MTTR, and different values of TETA

| n     | TETA | ZP_ OBM | ZP_ DBM | SP_ OBM | SP_ DBM | RP_ OBM | RP_ DBM |
|-------|------|---------|---------|---------|---------|---------|---------|
| 100   | 0.0125 | 4443    | 2374    | 8202    | 401     | 684     | 4293    |
|       | MTTR  | 0.016   | 11600   | 22181   | 529     | 1018    | 14350   |
|       | 0.02MB | 0.02    | 21573   | 41905   | 608     | 20487   | 1241    |

**TABLE 4.** Robustness, stability, and the objective function of two methods by applying the predictive policy for n=100, high MTTR, and different values of TETA

| n     | TETA | ZP_ OBM | ZP_ DBM | SP_ OBM | SP_ DBM | RP_ OBM | RP_ DBM |
|-------|------|---------|---------|---------|---------|---------|---------|
| 100   | 0.0125 | 31656   | 15541   | 61743   | 705     | 1569    | 30373   |
|       | MTTR  | 0.016   | 38852   | 20183   | 76047   | 815     | 1658    |
|       | 0.05MB | 0.02    | 56347   | 110545  | 1119    | 2149    | 61755   |

Figure 12. Comparison of two methods’ objective function by applying the predictive policy for n=100, high MTTR, and different values of TETA

**TABLE 5.** Robustness, stability, and the objective function for the two methods by applying the predictive policy for n=50, low MTTR, and different values of TETA

| n     | TETA | ZP_ OBM | ZP_ DBM | SP_ OBM | SP_ DBM | RP_ OBM | RP_ DBM |
|-------|------|---------|---------|---------|---------|---------|---------|
| 50    | 0.0125 | 359     | 191     | 300     | 82      | 57      | 662     |
|       | MTTR  | 0.016   | 535     | 309     | 936     | 68      | 134     |
|       | 0.02MB | 0.02    | 559     | 331     | 972     | 73.5    | 146     |

Figure 13. Comparison of two methods’ objective function applying the predictive policy for n=50, high MTTR, and different values of TETA

**TABLE 6.** Robustness, stability, and the objective function of the two methods by applying the predictive policy for n=50, high MTTR, and different values of TETA

| n     | TETA | ZP_ OBM | ZP_ DBM | SP_ OBM | SP_ DBM | RP_ OBM | RP_ DBM |
|-------|------|---------|---------|---------|---------|---------|---------|
| 50    | 0.0125 | 783     | 283     | 1399    | 76      | 167     | 490     |
|       | MTTR  | 0.016   | 1117    | 535     | 2032    | 96       | 202     |
|       | 0.05MB | 0.02    | 3029    | 2334    | 95      | 571     | 177     |

**5. MANAGERIAL INSIGHT**

By increasing the importance of producer satisfaction level (robustness) to the satisfaction level of the
production environment (stability), i.e., \( \alpha \geq 0.7 \), OBM outperforms DBM. In other words, if the production system’s interior completely aligns with the producer’s goals and the producer is not worried about the reaction of the production staff, select OBM, and otherwise DBM.

If \( \theta < 0.0125 \), OBM outperforms DBM, i.e., if the wear of the machines is negligible, and the predictive method has a higher cost than the reactive, select OBM, and otherwise DBM.

![Figure 14. Comparison of two methods’ objective function by applying the predictive policy for n=30, low MTTR, and different values of TETA](image)

\[ n=30, 0.02MB, \text{Predictive} \]

![Figure 15. Comparison of two methods’ objective function by applying the predictive policy for ALPHA=0.1, low level of MTTR, TETA](image)

\[ \text{ALPHA}=0.1 \]

![Figure 16. Comparison of two methods’ objective function by applying the predictive policy, ALPHA=0.3, low level of MTTR, TETA](image)

![Figure 17. Comparison of two methods’ objective function by applying the predictive policy, ALPHA=0.7, low level of MTTR, TETA](image)

![Figure 18. Comparison of two methods’ objective function by applying the predictive policy, ALPHA=0.9, low level of MTTR, TETA](image)

**Table 9.** Comparison of two methods’ objective function applying the predictive policy for different values of the robustness coefficient

| \( \alpha \) | 0.1 | 0.1 | 0.3 | 0.3 | 0.7 | 0.7 | 0.9 | 0.9 |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|
| No          | \( Z_{DBM} \) | \( Z_{PDBM} \) | \( Z_{DBM} \) | \( Z_{PDBM} \) | \( Z_{DBM} \) | \( Z_{PDBM} \) | \( Z_{DBM} \) | \( Z_{PDBM} \) |
| 1           | 0.0613 | 0.0204 | 0.0499 | 0.0312 | 0.0271 | 0.0528 | 0.0157 | 0.064 |
| 2           | 0.27  | 0.083 | 0.27  | 0.109 | 0.27  | 0.161 | 0.27  | 0.187 |
| 3           | 0.683 | 0.383 | 0.589 | 0.509 | 0.401 | 0.761 | 0.307 | 0.887 |
| 4           | 10.21 | 22.99 | 11.63 | 21.57 | 14.47 | 18.73 | 15.89 | 17.31 |
| 5           | 61.725 | 271.05 | 97.175 | 229.75 | 168.07 | 147.17 | 203.5 | 105.9 |
| 6           | 118.1 | 1198.1 | 208.3 | 968.3 | 388.7 | 508.7 | 478.9 | 278.9 |
| 7           | 993.1 | 8192.8 | 2247.3 | 6520.4 | 4755.7 | 3175.6 | 6009.9 | 1503 |
| 8           | 0.0452 | 0.0068 | 0.0356 | 0.0164 | 0.0164 | 0.0356 | 0.0068 | 0.045 |
| 9           | 0.229 | 0.098 | 0.187 | 0.134 | 0.103 | 0.206 | 0.061 | 0.242 |
6. CONCLUSION

In this study, we simultaneously considered the uncertainty of processing time and machine breakdowns in a two-machine flow shop scheduling problem. Two methods were proposed and compared in three situations to deal with this problem; without considering machine failure disruption and considering machine breakdown applying with the reactive and predictive policy. In the first situation, decomposition-based methods have an acceptable performance compared with the optimal base one. In the second status, OBM had a higher performance than DBM except in small-size problems. In applying the predictive policy, DBM had a higher performance than OBM, except in cases where producer satisfaction is more important than stability in the production environment. In all considering situations, the problem-solving time was acceptable and almost close to each other. Finally, OBM applying with the reactive policy, due to its lower objective function and its lower cost to DBM, seems more appropriate to solve the problem.

In this paper, a general approach proposed that can be used for robustness and stability optimization in an m-machine flow shop or job shop scheduling problem, with other measures of robustness and stability, or in the construction of predictive-reactive methods.
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Persian Abstract

چکیده

در این مقاله یک رویکرد پیش‌بینانه مقاوم و پایدار برای مسأله زمان‌بندی چرخان کارگاهی در محیط کارگاهی با فرض زمان رأی‌گیری احتمالی کارها و اختلال خارجی ماسکین ارائه شده است. در واقع، یک روش کلی ارائه شده که می‌توان از آن برای بهینه‌سازی مقاوم و پایدار در مسأله زمان‌بندی محیط کارگاهی و تولید محصول استفاده کرد مقایسه مقاومت، مقادیر مورد انتظار مجموع زمان‌بندی و انتظار انتظار مقدار انتظار مجموع زمان‌بندی واقع است، مقایسه پایداری، مقادیر مورد انتظار مربع مجموع افتخارات زمان انجام کارها در برنامه‌ریزی اولیه و واقع است، به مقدار حالت مسأله، دو روش پیشنهاد شده و موردهای مقایسه قرار گرفته است، یک روش بهینه بر جزئی مسأله مورد نظر به دو زیر مسأله و حل هر زیر مسأله به صورت بهینه و روشهای دیگر، بر پایه بر یک فضایی راگی بر اساس مدل مسئله است، روش دوم از نظر مقایسه و پایداری به ویژه در مورد مسائل با اندازه‌های بزرگ، مشکل‌های بهتری دارد. به عبارت دیگر، روش دوم به دلیل بهبود مقوری مشکلی که به مشتری و افزایش رضایت کارکنان خص تولید، برای حالت این مسئله بهبود می‌شود.