Transactive Energy Framework in Multi-Carrier Energy Hubs: A Fully Decentralized Model

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Abstract: This paper investigates a fully decentralized model for electricity trading within a transactive energy market. The proposed model presents a peer-to-peer (P2P) trading framework between the clients. The model is incorporated for industrial, commercial, and residential energy hubs to serve their associated demands in a least-cost paradigm. The alternating direction method of multipliers (ADMM) is implemented to address the decentralized power flow in this study. The optimal operation of the energy hubs is modeled as a standard mixed-integer linear programming (MILP) optimization problem. The corresponding decision variables of the energy hubs operation are transferred to the peer-to-peer (P2P) market, and ADMM is applied to ensure the minimum data exchange and address the data privacy issue. Two different scenarios have been studied in this paper to show the effectiveness of the electricity trading model between peers, called integrated and coordinated operation modes. In the integration mode, there is no P2P energy trading while in the coordinated framework, the P2P transactive energy market is taken into account. The proposed model is simulated on the modified IEEE 33-bus distribution network. The obtained results confirm that the coordinated model can efficiently handle the P2P transactive energy trading for different energy hubs, addressing the minimum data exchange issue, and achieving the least-cost operation of the energy hubs in the system. The obtained results show that the total operating cost of the hubs in the coordinated model is lower than that of the integrated model by $590.319, i.e. 11.75% saving in the costs. In this regard, the contributions of the industrial, commercial, and residential hubs in the total
cost using the integrated model are $3441.895, $596.600, and $988.789, respectively. On the other hand, these energy hubs contribute to the total operating cost in the coordinated model by $2932.645, $590.155, and $914.165 respectively. The highest decrease relates to the industrial hub by 14.8% while the smallest decrease relates to the residential hub by 1%. Furthermore, the load demand in the integrated and coordinated models is mitigated by 13% and 17%, respectively. These results indicate that the presented framework could effectively and significantly reduce the total load demand which in turn leads to reducing the total cost and power losses.

**Keywords**: Alternating Direction Method of Multipliers, Peer-to-Peer, Transactive Energy, Multi-Carrier Energy Hubs.

1- Introduction

A. Motivation

Emerging local generation technologies at load centers can provide considerable flexibility for prosumers and grid operators [1]. In this paradigm, serving the local electrical, heating, and cooling loads at the minimum cost is achievable through multi-carrier energy hubs. The main functionality of the energy hubs is to serve the consumer’s loads within the utilization area. Various technologies are used for converting, storing, and generating energy in the hubs to serve the end-users’ demands. In this regard, the energy hub operator is responsible for determining the optimal operating points of the installed assets in the hub. Unlike classic consumers, the energy hub operators can sell their surplus electricity to the upstream grid and as a result, they become prosumers in this paradigm since they can act as a producer as well as a consumer in the market [2]. This possibility for the prosumers can increase the flexibility of the energy hub operator in the market. On the other hand, the possibility of energy tradings between the clients in the P2P framework opens a new window for the end-users to trade electricity in a secure and least-cost model. In this environment, the clients can participate in a transactive energy market and benefit from this possibility to reduce their electricity cost by satisfying their energy needs from other energy hubs and increasing their benefits by selling their surplus power to other clients while avoiding paying extra tax or grid service costs. This paper investigates the
energy transactions between the energy hubs considering a transparent market mechanism for energy trading while addressing the minimum data exchange between the clients using the alternating direction method of multipliers (ADMM) based optimal power flow (OPF), known as the ADMM-OPF approach [3].

B. Literature Review

The problem of optimal operation of energy hubs has recently captured significant attention. There has been much research using a centralized framework to minimize the total operating costs while still supplying the required electrical, heating, and cooling demands. Also, multi-carrier energy systems using electricity and natural gas (NG) can be utilized. In this respect, combined heat and power (CHP) units could significantly improve the efficiency of the energy hub. This generation technology, besides the multi-carrier feature, would help provide the required flexibility to meet the load demand. In general, the studies carried out on energy hubs are categorized into two groups, either optimal management of energy hubs [4] or the optimal sizing of the hubs’ assets [5]. The problem of optimal operation of hubs is characterized mainly by using matrix-based or energy flow models. The energy flow models enable the system operator to efficiently apply the dynamic behavior of energy storage systems, either thermal or electrical, to the hub. The optimal operation of multiple interconnected energy hubs has been investigated in the context of microgrid (MG) operation. The resulting problem will be large if the number of hubs is considerable.

Ref. [6] proposed a new steady-state methodology for the optimal energy management of energy hubs, helping overcome the existing restrictions of prevailing energy hub management models. In this respect, a graph theory-based technique was used for the steady-state study of the system for certain operational scenarios of the MG.

A centralized operation framework has been developed in [7], with the aim to minimize the operating cost of the system while addressing the constraints of the energy hub. The system has been studied using an ACOPF tool to take into consideration the reactive power and voltage security of the system.
The problem has been formulated as a mixed-integer linear programming (MILP) problem and presented to promote the efficiency and robustness of the system’s day-ahead operation. The robust scheduling of combined cooling, heat, and power energy hubs in the presence of uncertainties of demands, photovoltaic (PV) systems, and wind power generation, as well as electricity price, has been addressed in [8]. The risk-aware scheduling model has been developed based on the information gap decision theory approach. The reported results illustrate that the sensitivity of the energy hub operating cost may be very different with respect to different sets of uncertain input data. The findings also show the significant effect of risk-awareness on the schedule of the hub components and its operation cost.

Another centralized model for the day-ahead operation of multi-carrier energy systems has been introduced in [9] aimed at minimizing the daily operating cost of the entire network. The nodal energy flow model for multi-energy carries has been investigated for MGs. The impacts of electric vehicles and parking lots on the day-ahead scheduling of a specific energy hub have been considered in [10] while the benefits of demand response programs have been demonstrated.

The optimal operation of a system with multiple MGs, including generation and storage technologies, has been addressed in [11] which uses a distributed stochastic framework with the least data exchange possible. In this regard, the resources of every MG are individually dispatched and the operation of each MG is coordinated with the operation of other MGs to make the most of the capabilities of such systems and maximize their social benefits. It is noteworthy that the Lagrangian dual method has been deployed for decentralizing the above-mentioned problem to enhance the system operation.

Another piece of research investigated the reserve allocation for multi-area energy networks by implementing a chance-constrained optimization framework, handling the risks caused by electricity and NG systems [12]. By using the presented model, the first problem with a centralized structure is transformed into various sub-problems for each area to solve individually. This technique would help promote the protection of information for each area. Moreover, the distributed problem is tackled by
utilizing the ADMM method which is further improved by the use of self-adaptive penalty parameters. By setting a pre-defined confidence level, a probabilistic tool was employed to satisfy the electricity and NG systems’ required reserve limitations.

The problem of optimal operation of multiple meshed MGs with interconnected energy hubs has been formulated as a distributed robust optimization problem in [13]. The developed framework is subject to the constraints of the MGs and gives a decentralized operational strategy while taking into account the ownership issues of energy suppliers. In this relation, the ADMM technique is deployed to enhance the data protection of the individual energy management systems.

Regarding the ADMM implementation for addressing data-privacy concerns, a comprehensive model has been introduced in [14] investigating a carbon emission trading system for power systems with considerable wind power generation. The proposed ADMM model has been implemented based on the finite-time average consensus algorithm reported in [3] dealing with sharing consensus variables between nodes. Furthermore, different graph topologies have been examined in [15] to validate the effectiveness and robustness of the algorithm.

A similar approach has been presented in [16] to address the optimal power flow with carbon emission trading based on the DCOPF framework in a centralized fashion. It is worth mentioning that transactive energy trading between end-users and small-scale prosumers not only occupies the distribution assets capacity but also introduces unknowns to utilities from a financial viewpoint leading to numerous issues. The first concern is that the income of the electrical utility decreases as a result of reduced energy delivered to its customers due to transactive energy trading amongst the customers. In addition, all energy transactions should respect the constraints of the distribution system and in this respect, two different strategies have been suggested in [17] as peer-centric and centralized strategies. By using the centralized framework, the system operation and transactive transactions are coordinated. The peer-centric methodology would not take the utility into consideration. It is also
noteworthy that the locational marginal prices are used to determine the network costs to pay to the utility.

Ref. [18] proposes a multi-bilateral economic dispatch model for a transactive market mechanism, providing the opportunity for multi-bilateral trading with product differentiation. An innovative relaxed consensus technique is proposed to tackle the problem within the decentralized framework. The framework resulted in the optimal solution while addressing the requirements of the consumers to maximize social welfare. A new optimization method based on the primal-dual gradient has been suggested in [19] for decentralized market clearing while disregarding any intervention by a central entity. The model takes into account the network constraints to allow for valid bidirectional energy trading while satisfying the consumers’ requirements. The mechanisms of transactive energy trading have been extensively reviewed in [20] and the practical prospects of transactive energy trading have been investigated in [21]. Regarding the implementation of a P2P energy trading mechanism in energy hubs, preliminary research has been conducted in [22]. An effective transactive energy trading mechanism has been developed which allows for multi-hub coordinated operation with enhanced performance through the use of an efficient self-adjusted trading mechanism for the hubs. A general scheme has also been designed for the transactive energy prices, initially incorporated into an enhanced ADMM technique in order to reach the consensus of the P2P transaction between energy hubs. However, this paper neglected the network constraints, solution feasibility, and data privacy issue.

Energy hubs within networked microgrids were investigated by [23] to meet the demand for various energy carriers. The authors used the ADMM to optimally schedule power exchanges between the various microgrids and the distribution network. The authors considered a robust optimization model to account for the various uncertainties in the model and compared the results to those reported by heuristic algorithms. The optimal solution from the ADMM model was superior compared to those obtained from the heuristic algorithms. An ADMM-based planning model considering both energy
hubs using electricity and natural gas was developed in [24]. The authors incorporated a unit commitment model into the planning model to better reflect the operational strategies. The energy flows between the energy hubs and the networks were defined as continuous coupling variables for updating the Lagrange multipliers in the ADMM model. A distributed auction mechanism for energy hubs was developed in [25]. The energy hubs served a variety of building users and used their expected demand profiles as the basis for the auction mechanism. The auction mechanism was shown to achieve the incentive compatibility in a Nash equilibrium and also to be robust against any manipulations by the users. The paper only considered residential users in the model. A model predictive control (MPC)-based model was presented in [26] for the optimal energy management of hybrid electric vehicles. The authors used the ADMM and compared its performance with the dynamic programming (DP). Results showed that the ADMM had a lower solution time with comparable accuracy in comparison with the DP. A decentralized transactive energy management framework was developed in [22]. This framework allowed for the coordination between multiple energy hubs to improve the financial performance of the system. The authors considered the peer-to-peer trading possibility within the energy hubs. This was carried out by using the ADMM to achieve consensus regarding the trades within the energy hubs. Results showed that consumers are encouraged to engage in the energy hubs activities. The demand response programs were not considered in this model. A large-scale energy hub was developed by [27]. This model was designed to coordinate energy flows between nations or large regions. The authors used the Bender’s decomposition and a convex ACOPF model to optimize the operation of the energy hubs. Results showed that the decentralized model performed better than other centralized operation models. A network of cooperative energy hubs was modeled by [28] with the objective to reduce operating costs. The energy hubs considered electrical, heating, and cooling loads as well as ice storage within the model. The authors modeled the problem by using a combination of mixed-integer linear programming and coalitional game theory to coordinate the energy hubs. The authors did not consider
demand response programs in the model. A centralized framework for solving the coordinated
operation of energy hubs in the presence of industrial, commercial, and residential hubs has been
proposed in [29] while the ACOPF problem was considered in a sequential and iterative sub-problem
in the optimization problem. However, the main challenging issue in the distribution networks are the
voltage drops [30] and congestion or overloading of the distribution feeders [31] and it must be
considered in the embedded models during both normal and contingent conditions.

This section has reviewed and critically evaluated the existing literature. Several gaps in the literature
were identified and in the next section, these gaps will be addressed through the contributions of the
present paper.

C. Main Contributions

The optimality and feasibility issues should be addressed in the optimization problem to more
precisely investigate the multi-hub operation in a distribution system. Accordingly, different types of
energy hubs including industrial, commercial, and residential hubs, have been modeled and studied.
The decentralized optimization model has been used to solve the OPF problem. In this regard, the
OPF problem has been studied using both integrated and coordinated models. Moreover, the MILP
framework has been employed to analyze the optimal operation problem of the energy hub.

Accordingly, the main contributions of this paper compared to the existing literature are as follows:

1- Developing an energy trading mechanism between different hubs in the distribution network
utilizing an ADMM based OPF (ADMM-OPF) model.

2- Addressing the P2P transactions for electrical energy trading between the grid participants.

3- Increasing the ability of power exchange between the energy hubs.

4- Presenting a MILP model for multi-carrier energy systems.
D. Paper Organization

This paper is organized as follows: The ADMM framework for solving the standard optimization problem is presented in Section 2. This decentralized framework addresses the data privacy of the clients while solving the optimization problem with the minimum data exchange. The mathematical problem formulation is introduced in Section 3, focusing on the application of the ADMM for the OPF application as well as a generalized optimization model for operating multi-carrier energy hubs.

For the sake of simplicity, the energy hub operation problem is investigated in the MILP framework. The mathematical formulation of this problem is presented for two different market mechanisms. The first mechanism, named “integrated operation”, relates to the grid-hub power transactions while in the second mechanism, called “coordinated operation”, the P2P electrical energy trading is applicable with a certain pricing mechanism for all stakeholders. The simulation results for the different case studies are analyzed in Section 4, and the concluding remarks and future works are presented in the last section of this paper.

2- Decentralized Optimization Framework based on ADMM

The problem in a decentralized optimization framework, using the ADMM would be expressed as follows:

\[
\min_{x,z} f(x) + g(z)
\]

subject to:

\[
Ax + Bz = c
\]

The objective should be minimized to determine the optimal values of the variables, \( x \) and \( z \). The principle of this method is based on updating the decision variables as well as Lagrangian multipliers of the problem. The augmented Lagrangian relaxation can be represented as:

\[
L_\rho(x, z, \lambda) := f(x) + g(z) + \lambda^T (Ax + Bz - c) + \frac{\rho}{2} \|Ax + Bz - c\|_2^2
\]

It is noteworthy that the Lagrangian multipliers of expression (1a) are denoted by \( \lambda^k \) and the positive parameter \( \rho \) is pre-determined, while \( \|\cdot\|_2 \) indicates the \( \ell_2 \)-norm of a vector. The ADMM uses an iterative process, where \( k \) shows the index of iterations of the ADMM. The decision variables would
be individually considered using the ADMM technique and the problem is tackled through several iterations to achieve the optimal solution with sufficient accuracy.

\[
x^{k+1} := \arg \min_x L_p(x, z^k, \lambda^k)
\]
(1d)

\[
z^{k+1} := \arg \min_z L_p(x^{k+1}, z, \lambda^k)
\]
(1e)

\[
\lambda^{k+1} := \lambda^k + \rho \left( Ax^{k+1} + Bz^{k+1} - c \right)
\]
(1f)

The primal residual of the balance constraint is as follows:

\[
r^{k+1} := Ax^{k+1} + Bz^{k+1} - c
\]
(1g)

The termination criterion for the iterative optimization problem according to the primal residual is as follows:

\[
\|r^k\| \leq \varepsilon
\]
(1h)

### 3. Problem Formulation

#### 3.1. Decentralized ACOPF model

The general problem formulation for an ACOPF model is as follows. A two-bus test system for the OPF problem definition using the ADMM technique concept has been depicted in Fig. 1:

\[
\text{Min} \sum_{i=1}^{NG} \sum_{t=1}^{NT} f(PG_{i,t})
\]
(2a)

Subject to:

\[
PG_i \leq PG_{i,j} \leq \bar{PG}_i \quad \forall i \in NG
\]
(2b)

\[
QG_i \leq QG_{i,j} \leq \bar{QG}_i \quad \forall i \in NG
\]
(2c)

\[
|V_i|^2 \leq |V_{i,j}|^2 \leq |V_j|^2 \quad \forall i \in NB
\]
(2d)

\[
\Delta \theta_{ij} \leq \theta_{i,j} - \theta_{j,i} \leq \Delta \bar{\theta}_{ij} \quad \forall \{i, j\} \in NB
\]
(2e)

\[
PG_{i,j} - PD_{i,j} = PN_{i,j} \quad \forall i \in NB
\]
(2f)

\[
QG_{i,j} - QD_{i,j} = QN_{i,j} \quad \forall i \in NB
\]
(2g)

\[
PN_{i,j} + jQN_{i,j} = V_{i,j} \sum_{j \in \mathcal{N}_i} Y_{i,j}^* V_{j,j}^* \quad \forall i \in NB
\]
(2h)
The objective is to minimize the cost of serving electrical energy, while in this case, the energy cost is assumed to be a predefined tariff, i.e. time-of-use (TOU) tariff. The OPF constraints are stated in (2b)-(2h). Inequalities (2b) and (2c) show the limitations of the active and reactive power, respectively. The constraints (2d) and (2e) indicate the limitations of the bus voltage magnitude and voltage angle, respectively. Eqs. (2f) and (2g) relate to the net active and reactive power, respectively. Also, Eq. (2h) states the power injected into transmission lines.

The decentralized OPF problem using the ADMM approach can be written as follows:

\[
    x := \left[ PG_{i,j}, QG_{i,j} \right]_{i \in NG}, \left[ PL_{q_{i,j}}, QL_{q_{i,j}}, V_{i,j}, \theta_{i,j} \right]_{i,j \in NL} \tag{3a}
\]

\[
    z := \left[ PG_{i,j}, QG_{i,j} \right]_{i \in NG}, \left[ PL_{q_{i,j}}, QL_{q_{i,j}} \right]_{i,j \in NL}, \left[ V_{i,j}, \theta_{i,j} \right]_{i,j \in NL} \tag{3b}
\]

\[
    \lambda := \left[ \lambda_{i,j}^{PG}, \lambda_{i,j}^{QG} \right]_{i \in NG}, \left[ \lambda_{i,j}^{PL}, \lambda_{i,j}^{QL}, \lambda_{V,i,j}, \lambda_{\theta,i,j} \right]_{i,j \in NL} \tag{3c}
\]

According to the ADMM technique, the corresponding decision variables are addressed in (3a) and (3b). The dual variables, corresponding to the problem constraint, are addressed as a set of associated Lagrange multipliers in (3c). The ADMM technique is applied to update the generators decision variables:

\[
    x_{i,j}^{k+1,G} = \arg\min_{x_{i,j}^{G}} \sum_{i=1}^{NG} \sum_{j=1}^{NT} f \left( PG_{i,j} \right) + \left( \lambda_{i,j}^{G} - \lambda_{i,j}^{k,G} \right)^T \left( PG_{i,j} - \lambda_{i,j}^{G} \right) + \frac{\rho}{2} \left\| PG_{i,j} - \lambda_{i,j}^{k,G} \right\|_2^2 + \frac{\rho}{2} \left\| QG_{i,j} - \lambda_{i,j}^{k,G} \right\|_2^2 \tag{3d}
\]

where \( x_{i,j}^{G} := \left[ PG_{i,j}, QG_{i,j} \right] \) and \( \lambda_{i,j}^{G} := \left[ \lambda_{i,j}^{PG}, \lambda_{i,j}^{QG} \right] \).

For the given transmission lines, the corresponding decision variables can be stated as below:
\[ x_{ij,j}^{k+1} = \arg \min_{x_{ij,j}} \left[ \left\{ \lambda_{ij,j}^{k,L}, \lambda_{ij,j}^{k,L} \right\}, x_{ij,j}^{k,L} \right] + \right. \\
\left. \sum_{(l,k) \in NL} \frac{\rho}{2} \left( \left\| V_{ij,j}^k \right\|_2^2 - V_{ij,j}^l V_{ij,j}^l \right) + \left\| \theta_{ij,j} - \theta_{lm,j} \right\|_2^2 + \left\| P_{L_{ij,j}} - P_{L_{ij,j}}^k \right\|_2 + \left\| Q_{L_{ij,j}} - Q_{L_{ij,j}}^k \right\|_2 \right] \right) \] (3e)

The local decision variables to be transferred to the other section can be written as:

\[ z_{ij,j} = \arg \min_{z_{ij,j}} \sum_{j \in B} \sum_{i \in l} \left( \lambda_{ij,j}^{k} \left[ P_{G_{ij,j}}^k - P_{G_{ij,j}}^l \right] + \frac{\rho}{2} \left( \left\| P_{L_{ij,j}}^k - P_{L_{ij,j}}^l \right\|_2 \right) + \left\| Q_{L_{ij,j}}^k - Q_{L_{ij,j}}^l \right\|_2 \right) \] (3f)

\[ + \left\| V_{ij,j}^k - V_{ij,j}^l \right\|_2 + \left\| \theta_{ij,j} - \theta_{ij,j}^k \right\|_2 \]

where \( z_{ij,j} = \left[ P_{G_{ij,j}}^k - P_{G_{ij,j}}^l, V_{ij,j}^k - V_{ij,j}^l, \theta_{ij,j} - \theta_{ij,j}^k \right] \) and \( \lambda_{ij,j}^{V,k} = \lambda_{ij,j}^{V,L}, \lambda_{ij,j}^{\theta,k} \)

### 3.2. Multi-Carrier Energy Hubs

For each peer in the MG, the minimization of the total cost is taken into consideration. The mathematical representation of the energy hub transactions is as follows, while the conceptual scheme of the studied hub is illustrated in Fig. 2.

![Fig. 2. The conceptual scheme of the studied hub [32].](image-url)
\[ \text{Min} \]

\[
OF_k = \sum_{t=1}^{NT} f(CHP_{k,t}) + f(Boiler_{k,t}) + \sum_{t=1}^{NT} \left[ f\left(P_{k,t}^{G\rightarrow H}\right) - f\left(P_{k,t}^{H\rightarrow G}\right) \right] + \sum_{m \neq k} \sum_{t=1}^{NT} \left[ g\left(P_{k,t}^{R\rightarrow P_m}\right) - g\left(P_{k,t}^{P_m\rightarrow R}\right) \right] 
\]

(4a)

\[ f(CHP_{k,t}) = \left( \frac{p_{k,t}^{\text{CHP}}}{\eta_{CHP}} + \frac{H_{k,t}^{\text{CHP}}}{\eta_h} \right) \text{NG}_{k,t} \]

(4b)

\[ f(Boiler_{k,t}) = \left( \frac{H_{k,t}^{\text{Boiler}}}{\eta_h} \right) \text{NG}_{k,t} \]

(4c)

\[ f\left(P_{k,t}^{G\rightarrow H}\right) = P_{k,t}^{G\rightarrow H} \eta_{k,t}^{G\rightarrow H} \]

(4d)

\[ f\left(P_{k,t}^{H\rightarrow G}\right) = P_{k,t}^{H\rightarrow G} \eta_{k,t}^{H\rightarrow G} \]

(4e)

\[ g\left(P_{k,t}^{R\rightarrow P_m}\right) = P_{k,t}^{R\rightarrow P_m} \eta_{k,t}^{R\rightarrow P_m} \quad m \neq k \]

(4f)

\[ g\left(P_{k,t}^{P_m\rightarrow R}\right) = P_{k,t}^{P_m\rightarrow R} \eta_{k,t}^{P_m\rightarrow R} \quad m \neq k \]

(4g)

The objective function defined for each peer is to minimize the total cost, which is comprised of three items. The first item indicates the cost due to the NG procurement through the NG system. It is worth noting that the CHP unit and boilers use NG while an hourly tariff is applied to every sector, i.e. industrial, commercial and residential sectors. The costs due to the centralized and P2P power transactions with the utility grid and other peers are modeled in the second and third items, respectively. Each peer uses the market hourly prices. Accordingly, the system operator should execute a regulated market and bills will be calculated for the values collected by smart meters in each sector. The costs, imposed by employing the CHP and boiler, are shown by functions (4b) and (4c), respectively. Expressions (4d)-(4g) state the cost/income due to purchasing/selling electricity in the centralized and P2P modes. A convex feasible operating region (FOR) using binary variables, has been used to characterize the CHP operation as follows [33].

\[
P_{k,t}^{\text{CHP}} I_{k,t}^{\text{CHP}} \leq P_{k,t}^{\text{CHP}} \leq P_{k,t}^{\text{CHP}} I_{k,t}^{\text{CHP}} 
\]

(5a)

\[
H_{k,t}^{\text{CHP}} I_{k,t}^{\text{CHP}} \leq H_{k,t}^{\text{CHP}} \leq H_{k,t}^{\text{CHP}} I_{k,t}^{\text{CHP}} 
\]

(5b)

\[
S_{k,t}^{\text{CHP}} I_{k,t}^{\text{CHP}} \leq P_{k,t}^{\text{CHP}} + H_{k,t}^{\text{CHP}} \leq S_{k,t}^{\text{CHP}} I_{k,t}^{\text{CHP}} 
\]

(5c)

\[
P_{k,t}^{\text{CHP}} = P_{k,t}^{\text{CHP} \rightarrow EL} + P_{k,t}^{\text{CHP} \rightarrow EH} + P_{k,t}^{\text{CHP} \rightarrow EES} + P_{k,t}^{\text{CHP} \rightarrow EHP} + \sum_{m \neq k} P_{k,t}^{\text{CHP} \rightarrow P_m} 
\]

(5d)
Equations (5a)-(5c) deal with the lower and upper bounds of the electrical power generation, heat generation, and the total capacity of CHP units. The hourly binary variable, $I_{k,t}^{CHP}$, states that if the CHP unit is running, the power and heat generated should be within the limits of the unit. The power generated by the CHP unit can be delivered to the load, the utility grid, the electric heater (EH), the battery to charge, or it can be sold to other hubs. The heat generated is also delivered to heating load demands or the absorption chiller (AC).

The boiler should operate within the permitted operating interval (6a). The heat generated by the boiler follows the same behavior as that of the CHP system and it is stated in (6b).

\[
H_{k,t}^{Boiler} I_{k,t}^{Boiler} \leq H_{k,t}^{Boiler} \leq H_{k,t}^{Boiler} I_{k,t}^{Boiler}
\]  

(6a)

\[
H_{k,t}^{Boiler} = H_{k,t}^{Boiler \rightarrow HL} + H_{k,t}^{Boiler \rightarrow AC}
\]  

(6b)

The heat generated by the boiler is delivered to the heating load or AC.

**Electric Heater Model:**

Inequality (7a) states the operating limits of the electric heater. Relationship (7b) shows the heat generation equation of the EH indicated as the product of the input power and the system efficiency, denoted by $I_{k,t}^{EH}$ and $\eta_{h}^{EH}$, respectively [4]. It is also noteworthy that the CHP unit, the electrical energy storage (EES) system, the utility grid, or the power trading with other energy hubs would provide the power needed by the EH. Besides, the expression (7d) shows that the heat output of the EH would supply the heating load.

\[
H_{k,t}^{EH} I_{k,t}^{EH} \leq H_{k,t}^{EH} \leq H_{k,t}^{EH} I_{k,t}^{EH}
\]  

(7a)

\[
H_{k,t}^{EH} = P_{k,t}^{EH} \eta_{h}^{EH}
\]  

(7b)

\[
P_{k,t}^{EH} = P_{k,t}^{G \rightarrow EH} + P_{k,t}^{CHP \rightarrow EH} + P_{k,t}^{EES \rightarrow EH} + \sum_{m \neq k} P_{m,t}^{EH \rightarrow EH}
\]  

(7c)

\[
H_{k,t}^{EH} = H_{k,t}^{EH \rightarrow HL}
\]  

(7d)
Electrical Heat Pump Model:

Inequalities (8a) and (8b) indicate that the heating power, $H_{EHP,k,j}^H$, and cooling power, $C_{EHP,k,j}^C$, produced by the electric heat pump (EHP) is constrained within the feasible operating interval, respectively. Furthermore, the operation modes of the EHP, i.e. heating and cooling operation modes, are determined by $I_{EHP,H}^{k,j}$ and $I_{EHP,C}^{k,j}$, respectively. It should be noted that the EHP is not capable of operating in the two modes simultaneously as stated in (8c). Eqs. (8d) and (8e) show that the heating power and cooling power outputs of the EHP are the product of the input electrical power and the associated system efficiency, respectively. The electrical power balance equation is stated in Eq. (8f) while Eq. (8g) and Eq. (8h) indicate the heating and cooling power balance equations respectively.

$$H_{EHP}^{k,j} I_{EHP,H}^{k,j} \leq H_{EHP}^{k,j} \leq H_{EHP}^{k,j} I_{EHP,H}^{k,j}$$  (8a)

$$C_{EHP}^{k,j} I_{EHP,C}^{k,j} \leq C_{EHP}^{k,j} \leq C_{EHP}^{k,j} I_{EHP,C}^{k,j}$$  (8b)

$$0 \leq I_{EHP,H}^{k,j} + I_{EHP,C}^{k,j} \leq 1$$  (8c)

$$H_{EHP}^{k,j} = P_{EHP}^{k,j} \cdot \text{COP}_{EHP}^H$$  (8d)

$$C_{EHP}^{k,j} = P_{EHP}^{k,j} \cdot \text{COP}_{EHP}^C$$  (8e)

$$P_{EHP}^{k,j} = P_{G \rightarrow EHP}^{k,j} + P_{CHP \rightarrow EHP}^{k,j} + P_{EES \rightarrow EHP}^{k,j} + \sum_{m=k}^n P_{m \rightarrow EHP}^{k,j}$$  (8f)

$$H_{EHP}^{k,j} = H_{EHP \rightarrow HL}^{k,j}$$  (8g)

$$C_{EHP}^{k,j} = C_{EHP \rightarrow CL}^{k,j}$$  (8h)

Absorption Chiller Model:

The absorption chiller (AC) should supply the cooling load demand by using the generation equation (9a) and its output is constrained as shown in inequality (9b). Eq. (9c) states that the power required by the AC would be provided by the CHP unit and the boiler. Besides, the cooling power flow of the AC is shown in expression (9d).

$$C_{AC}^{k,j} = H_{AC}^{k,j} \eta_c^AC$$  (9a)

$$C_{AC}^{k,j} I_{AC}^{k,j} \leq C_{AC}^{k,j} \leq C_{AC}^{k,j} I_{AC}^{k,j}$$  (9b)

$$H_{AC}^{k,j} = H_{CHP \rightarrow AC}^{k,j} + H_{Boiler \rightarrow AC}^{k,j}$$  (9c)

$$C_{AC}^{k,j} = C_{AC \rightarrow CL}^{k,j}$$  (9d)
Electrical Energy Storage Model:

The hourly energy balance equation of the EES system has been shown in Eq. (10a) while the upper and lower bounds of the energy stored in the system are applied through inequality (10b) [34]. The energy available at hour $t$ depends upon the amount of energy available in the previous hour as well as the charging/discharging power at time $t$ taking into consideration the system efficiency in each of the two modes. Furthermore, the charging and discharging rates are constrained as stated in (10c) and (10d), respectively. The binary variable, $I_{k,t}^{\text{ESS,Ch}}$, specifies the charging mode, while the discharging mode is specified by using the binary variable $I_{k,t}^{\text{ESS,Dis}}$. As constraint (10e) shows, the EES system can only operate in either discharging mode or charging mode at a given time [35]. To meet the operation requirements of the subsequent scheduling period, the energy that should be available at the final interval of the scheduling period must be equal to that of the initial time interval as indicated by constraint (10f), while the amount of energy available at the beginning of the scheduling period is determined by Eq. (10g). The energy flows in the charging and discharging modes are expressed in (10h) and (10i), respectively.

$$E_{k,t}^{\text{ESS}} = E_{k,t-1}^{\text{ESS}} + \left( P_{k,t}^{\text{ESS,Ch}} \eta_{k,t}^{\text{ESS,Ch}} \right) - \left( P_{k,t}^{\text{ESS,Dis}} \eta_{k,t}^{\text{ESS,Dis}} \right)$$  \hspace{1cm} (10a)

$$E_k^{\text{ESS}} \leq E_{k,t}^{\text{ESS}} \leq E_k^{\text{ESS}}$$  \hspace{1cm} (10b)

$$0 \leq P_{k,t}^{\text{ESS,Ch}} \leq P_k^{\text{ESS,Ch}} I_{k,t}^{\text{ESS,Ch}}$$  \hspace{1cm} (10c)

$$0 \leq P_{k,t}^{\text{ESS,Dis}} \leq P_k^{\text{ESS,Dis}} I_{k,t}^{\text{ESS,Dis}}$$  \hspace{1cm} (10d)

$$0 \leq I_{k,t}^{\text{ESS,Ch}} + I_{k,t}^{\text{ESS,Dis}} \leq 1$$  \hspace{1cm} (10e)

$$E_{k,t=NT}^{\text{ESS}} = E_{k,t=0}^{\text{ESS}}$$  \hspace{1cm} (10f)

$$E_{k,t=0}^{\text{ESS}} = \alpha_k E_{k,t}^{\text{ESS}}$$  \hspace{1cm} (10g)

$$P_{k,t}^{\text{ESS,Ch}} = P_{k,t}^{G \rightarrow \text{ESS}} + P_{k,t}^{\text{CHP} \rightarrow \text{ESS}} + P_{k,t}^{\text{PV} \rightarrow \text{ESS}} + \sum_{m=1}^{n} P_{k,t}^{P_m \rightarrow \text{ESS}}$$  \hspace{1cm} (10h)

$$P_{k,t}^{\text{ESS,Dis}} = P_{k,t}^{\text{ESS} \rightarrow \text{EL}} + P_{k,t}^{\text{ESS} \rightarrow G} + P_{k,t}^{\text{ESS} \rightarrow \text{EH}} + P_{k,t}^{\text{ESS} \rightarrow \text{EHF}} + \sum_{m=1}^{n} P_{k,t}^{\text{ESS} \rightarrow P_m}$$  \hspace{1cm} (10i)
PV Power Generation Model:

The hourly power generation of the PV system is dependent upon the solar irradiance, the temperature, and the characteristics of the panel. Equation (11a) presents the power generated by the PV panel [36], and the energy flow equation of the PV system is indicated by expression (11b). The parameters of the PV system generation are extracted from [36] while the same irradiance values have been considered in this study for each hub. It is evident that the PV power generation is equal or less than the rated capacity of the installed PV system. As this equation shows, the power produced by the PV panel can be consumed by the electrical load, the EHP, and the EH, or even sold to the utility grid or other peers.

\[
P_{k,t}^{PV} = \frac{G_{k,t}}{G_0} \left[ P_0^{PV} + \mu_T \left( T_{k,t}^{NOCT} - \frac{20}{800} - T_0^M \right) \right] (11a)
\]

\[
P_{k,t}^{PV} = P_{k,t}^{PV\rightarrow EL} + P_{k,t}^{PV\rightarrow G} + P_{k,t}^{PV\rightarrow EES} + P_{k,t}^{PV\rightarrow EHP} + P_{k,t}^{PV\rightarrow EH} + \sum_{m=k} P_{m,t}^{PV\rightarrow P_m} (11b)
\]

Power Transaction with the Distribution Network:

Equations (12a)-(12e) show the power transacted between the hub and the utility grid. The amount of power sold to the upstream system can be provided by the CHP unit, the EES system, or the PV panel as shown in (12a). Besides, the power purchased from the upstream system can be delivered to the EES system, the EH, the EHP, or directly consumed by the electrical loads. The maximum power that can be transacted between the energy hub and the utility grid is limited to the capacity of the transformer connecting these two systems as expressed in (12c) and (12d). It is noteworthy that the power transaction is a positive variable to specify the difference between the power purchased and sold from/to the grid. The simultaneous selling and purchasing of power are not allowed as shown by constraint (12e).

\[
P_{k,t}^{H\rightarrow G} = P_{k,t}^{CHP\rightarrow G} + P_{k,t}^{EES\rightarrow G} + P_{k,t}^{PV\rightarrow G} (12a)
\]

\[
P_{k,t}^{G\rightarrow H} = P_{k,t}^{EES\rightarrow G} + P_{k,t}^{EH\rightarrow G} + P_{k,t}^{EHP\rightarrow G} + P_{k,t}^{EL\rightarrow G} (12b)
\]

\[
0 \leq P_{k,t}^{H\rightarrow G} \leq P_{k,t}^{PV\rightarrow H\rightarrow G} (12c)
\]
Balance Equations:
The most important constraint of the presented problem is the power balance equation, expressed by constraints (13a)-(13c). As constraint (13a) indicates, the electrical load demand of the consumer can be supplied by using the upstream system, the EES system, the PV panel, and the CHP unit. It should be noted that all these variables are of positive type. Likewise, the heating and cooling power balance equations are stated by (13b) and (13c), respectively. The heating load demand would be supplied by the CHP unit, the boiler, the EHP in the heating mode, and the EH. The cooling load demand is also supplied by using the AC, and the EHP in the cooling mode.

\[ P_{k,j}^{\text{G}\to\text{EL}} + P_{k,j}^{\text{ESS}\to\text{EL}} + P_{k,j}^{\text{PV}\to\text{EL}} + P_{k,j}^{\text{CHP}\to\text{EL}} + \sum_{k=0}^{n} P_{k,j}^{\text{H}\to\text{H}} = P_{k,j}^{\text{EL}} \]  
\[ H_{k,j}^{\text{CHP}\to\text{HL}} + H_{k,j}^{\text{Boiler}\to\text{HL}} + H_{k,j}^{\text{EHP}\to\text{HL}} + H_{k,j}^{\text{EH}\to\text{HL}} = H_{k,j}^{\text{HL}} \]  
\[ C_{k,j}^{\text{EHP}\to\text{CL}} + C_{k,j}^{\text{AC}\to\text{CL}} = C_{k,j}^{\text{CL}} \]

3.3. P2P Market Mechanism

Expressions (14a)-(14f) show the power transaction mechanism between the hubs. As (14a) shows, the net power delivered by the hub \( k \), is determined by the surplus power generation of the PV panel, the EES system, and the CHP unit. The power, purchased from other generators can be consumed by the electrical load, the battery in the charging mode, the EHP, or the EH. Constraints (14a) and (14d) force the unidirectional power transaction between the peers.

The maximum power that can be transacted between two hubs is shown by these two constraints and constraint (14d) ensures the unidirectional power flow at each time interval. It is worth mentioning that it is not possible to simultaneously purchase power from the upstream grid and sell it to other hubs as stated in (14e). As the energy tariffs for the industrial, commercial, and residential sectors are different, it is not permitted to purchase power from the upstream grid at a lower price and sell it to
other peers at a higher price. However, according to the P2P energy transaction regulations, it is possible to purchase power over the off-peak hours and store it in the EES system and sell it over peak-load hours at higher prices.

\[
P_{k,d}^{H \rightarrow P_n} = P_{k,d}^{PV \rightarrow P_n} + P_{k,d}^{EES \rightarrow P_n} + P_{k,d}^{CHP \rightarrow P_n} \quad \forall m \neq k \tag{14a}
\]

\[
P_{k,d}^{P_n \rightarrow H} = P_{k,d}^{P_n \rightarrow EX} + P_{k,d}^{P_n \rightarrow EES} + P_{k,d}^{P_n \rightarrow EHP} + P_{k,d}^{P_n \rightarrow EH} \quad \forall m \neq k \tag{14b}
\]

\[
0 \leq P_{k,d}^{H \rightarrow P_n} \leq P_{k,d}^{P_n \rightarrow H} I_{k,d}^{H \rightarrow P_n} \quad \forall m \neq k \tag{14c}
\]

\[
0 \leq P_{k,d}^{P_n \rightarrow H} \leq P_{k,d}^{P_n \rightarrow H} I_{k,d}^{P_n \rightarrow H} \quad \forall m \neq k \tag{14d}
\]

\[
0 \leq I_{k,d}^{P_n \rightarrow H} + I_{k,d}^{H \rightarrow P_n} \leq 1 \quad \forall m \neq k \tag{14e}
\]

\[
0 \leq I_{k,d}^{H \rightarrow P_n} + I_{k,d}^{H \rightarrow P_n} \leq 1 \quad \forall m \neq k \tag{14f}
\]

### 3.4. P2P Implementation Model

Fig. 3 depicts the flow of data and energy within the energy hub. Both integrated and coordinated operation models have been illustrated in this figure. The main core of the proposed model consists of an energy trading center with the corresponding market manager. The market manager is the main responsible for handling the interactions within the local energy market as well as the centralized energy tradings. In this framework, the electrical energy can be traded in the local market between the clients and upstream grid, while the NG will be fed to the end-users and there is no way to buy the NG from the entities. In the integrated operation strategy, there is no possibility of trading energy between the peers while in the coordinated operation strategy, each peer has this opportunity to sell or buy electrical energy. The traded electrical energy will be transferred throughout the distribution company infrastructures and in this framework, the distribution system operator (DSO) is the responsible entity to address the energy losses as well as degradation cost of the infrastructures. The overall benefits of energy trading between the clients will cover the mentioned costs and it promotes the energy trading between the energy hubs within the local energy community.
4- Simulation Results
The modified 33-bus distribution system has been used to simulate the problem and evaluate the developed framework. The system comprises of three energy hubs of industrial, commercial, and residential types connected to the system at nodes 23, 20, and 27, respectively. In the base case, i.e. without any heating and cooling load demands and without any local power generation, the optimal electrical energy flow has been investigated in the decentralized case by using both the ADMM technique as well as the centralized case. Then, the proposed model is utilized for both integrated operation and coordinated operation of the distribution network in the presence of energy hubs and their contributions in the daily operation are studied. Fig. 4 illustrates the integrated electricity and NG systems under consideration. Technical parameters of the hub assets are presented in Table 1.
Fig. 4. The electric and NG systems used for the study.

Table 1. Technical parameters of hub assets.

| Asset | Parameter | Unit | Industrial | Commercial | Residential |
|-------|-----------|------|------------|------------|-------------|
| AC    | $C_k^{AC}$ | kW   | 750        | 200        | 400         |
|       | $C_k^{AC}$ | kW   | 0          | 0          | 0           |
|       | $\eta_e^{AC}$ | -    | 0.85      | 0.85      | 0.85        |
| Boiler| $H_k^{Boiler}$ | kW  | 400       | 150        | 150         |
|       | $H_k^{Boiler}$ | kW  | 100       | 50         | 50          |
|       | $\eta_h^{Boiler}$ | -    | 0.40      | 0.40      | 0.40        |
| CHP   | $S_k^{CHP}$ | kW   | 1400      | 1400       | 1400        |
|       | $S_k^{CHP}$ | kW   | 200       | 200        | 200         |
|       | $P_k^{CHP}$ | kW   | 800       | 800        | 800         |
|       | $P_k^{CHP}$ | kW   | 0         | 0          | 0           |
|       | $H_k^{CHP}$ | kW   | 1000      | 1000       | 1000        |
|       | $H_k^{CHP}$ | kW   | 200       | 200        | 200         |
|       | $\eta_e^{CHP}$ | -    | 0.45      | 0.45      | 0.45        |
|       | $\eta_h^{CHP}$ | -    | 0.35      | 0.35      | 0.35        |
| EES   | $E_k^{EES}$ | kWh  | 600       | 300        | 300         |
|       | $E_k^{EES}$ | kWh  | 100       | 50         | 50          |
|       | $P_k^{EES,Ch.}$ | kW | 50        | 25         | 25          |
|                | \( P_{k}^{EES,Dis.} \) | kW | 50  | 25  | 25  |
|----------------|--------------------------|----|-----|-----|-----|
| \( \alpha_k \) | -                        |    | 0.333 | 0.333 | 0.333 |
| \( \eta_{k,Ch.}^{EES} \) | -                        |    | 0.90 | 0.90 | 0.90 |
| \( \eta_{k,Dis.}^{EES} \) | -                        |    | 0.90 | 0.90 | 0.90 |
| **EH**         | \( H_k^{EH} \)          | kW | 300 | 100 | 100 |
|                | \( H_k^{EH} \)          | kW | 0   | 0   | 0   |
| \( \eta_h^{EH} \) | -                        |    | 0.85 | 0.90 | 0.90 |
| **EHP**        | \( H_k^{EHP} \)         | kW | 200 | 150 | 300 |
|                | \( H_k^{EHP} \)         | kW | 10  | 20  | 50  |
| \( C_k^{EHP} \) | kW                       | 200 | 150 | 300 |
| \( C_k^{EHP} \) | kW                       | 10  | 20  | 50  |
| \( COP_k^{EHP} \) | -                       |    | 1.25 | 1.25 | 1.25 |
| \( COP_k^{EHP} \) | -                       |    | 1.25 | 1.25 | 1.25 |
| **PV**         | \( P_{k}^{PV} \)        | kW | 200 | 100 | 100 |

**Fig. 5. The energy purchase price from the utility grid.**

Fig. 5 represents the energy purchase tariffs. The tariffs for selling surplus energy for the industrial, commercial, and residential hubs are 70%, 80%, and 80% of the energy purchase tariff. Fig. 6
demonstrates the active load demand of the energy hubs. The power factor of the industrial, commercial, and residential loads are considered to be 0.8, 0.85, and 0.85, respectively which are constant over the scheduling period.

Fig. 6. The hourly active power demand of the studied energy hubs in the integrated operation mode.

4.1 Base Case Operation Results

To show the functionality of the proposed model, the application of ADMM in solving the optimal power flow is examined in this section. In the base case, the daily operation of the distribution network is studied while there is no power injection from the energy hubs. In other words, the distribution system feeds the total loads in the network. Therefore, comparing the results for the base case can be made through the classic power flow models. In this case, the voltage at the injection bus is fixed at 1.0 p.u.

The optimal power flow results are the same in the two cases as the distribution system can adequately supply the load demand. Accordingly, it can be concluded that the ACOPF results derived using the ADMM technique are valid and reliable. Table 2 provides the nodal active and reactive power...
consumption. It can be seen that the total power consumption at the peak hour is $19.8737 \text{ MW}+j12.0654 \text{ MVAr}$. The industrial, commercial, and residential loads are $360 \text{ MW}+j200 \text{ MVAr}$, $525 \text{ MW}+j233.33 \text{ MVAr}$, and $1450 \text{ MW}+j604.17 \text{ MVAr}$, respectively. It is noted that the power consumption of the commercial load at this hour is substantial and the industrial load consumes the baseload according to Fig. 6.

Table 2. Active and reactive power demand at peak hour (18:00h)

| Bus  | P (kW) | Q (kVAr) | Node  | P (kW) | Q (kVAr) |
|------|--------|----------|-------|--------|----------|
| Node 02 | 504.71 | 302.83 | Node 18 | 454.24 | 201.88 |
| Node 03 | 454.24 | 201.88 | Node 19 | 454.24 | 201.88 |
| Node 04 | 605.65 | 403.77 | Node 20 | 525.00 | 233.33 |
| Node 05 | 302.83 | 151.41 | Node 21 | 454.24 | 201.88 |
| Node 06 | 302.83 | 100.94 | Node 22 | 454.24 | 201.88 |
| Node 07 | 1009.42 | 504.71 | Node 23 | 360.00 | 200.00 |
| Node 08 | 1009.42 | 504.71 | Node 24 | 2119.78 | 1009.42 |
| Node 09 | 302.83 | 100.94 | Node 25 | 2119.78 | 1009.42 |
| Node 10 | 302.83 | 100.94 | Node 26 | 302.83 | 126.18 |
| Node 11 | 227.12 | 151.41 | Node 27 | 1450.00 | 604.17 |
| Node 12 | 302.83 | 176.65 | Node 28 | 302.83 | 100.94 |
| Node 13 | 302.83 | 176.65 | Node 29 | 605.65 | 353.30 |
| Node 14 | 605.65 | 403.77 | Node 30 | 1009.42 | 3028.26 |
| Node 15 | 302.83 | 50.47 | Node 31 | 757.07 | 353.30 |
| Node 16 | 302.83 | 100.94 | Node 32 | 1059.89 | 504.71 |
| Node 17 | 302.83 | 100.94 | Node 33 | 302.83 | 201.88 |

The power flow results for the base case at the peak hour is presented in Table 3. In this table, the first and second columns are the sending and receiving ends, columns 3 and 4 are the sending ends’ active and reactive power, while columns 5 and 6 provide the active and reactive power at the receiving ends. The feeders’ active and reactive power losses are provided in the last two columns. The simulations results in both centralized and decentralized operation using ADMM are identical. It is noted that the centralized operation is performed by MATPOWER. The total amount of power losses at this hour is $1571.3761 \text{ kW}+1057.3551 \text{ kVAR}$. 
Table 3. Power transmitted through the feeders at the peak hour (18:00h)

| i       | j       | $P_{ij}$ (MW) | $Q_{ij}$ (MVar) | $P_{ji}$ (MW) | $Q_{ji}$ (MVar) | $P_{loss}$ (kW) | $Q_{loss}$ (kVar) |
|---------|---------|--------------|----------------|--------------|----------------|----------------|------------------|
| Node 01 | Node 02 | 21.4451      | 13.1228        | -21.3549     | -13.0761       | 90.1538        | 46.6414          |
| Node 02 | Node 03 | 18.9546      | 11.9269        | -18.5690     | -11.7305       | 385.5812       | 196.3883         |
| Node 03 | Node 04 | 13.4542      | 9.2637         | -13.2959     | -9.1831        | 158.3316       | 80.6366          |
| Node 04 | Node 05 | 12.6902      | 8.7793         | -12.5399     | -8.7027        | 150.3472       | 76.5741          |
| Node 05 | Node 06 | 12.237       | 8.5513         | -11.9280     | -8.2845        | 309.0744       | 266.8078         |
| Node 06 | Node 07 | 5.6122       | 2.7439         | -5.5992      | -2.7007        | 13.0721        | 43.2106          |
| Node 07 | Node 08 | 4.5897       | 2.1960         | -4.5097      | -2.1382        | 80.0581        | 57.7772          |
| Node 08 | Node 09 | 3.5003       | 1.6335         | -3.4714      | -1.6128        | 28.8606        | 20.7348          |
| Node 09 | Node 10 | 3.1686       | 1.5118         | -3.144       | -1.4944        | 24.6086        | 17.4429          |
| Node 10 | Node 11 | 2.8411       | 1.3934         | -2.8373      | -1.3922        | 3.8295         | 1.2661           |
| Node 11 | Node 12 | 2.6102       | 1.2408         | -2.6041      | -1.2387        | 6.0985         | 2.0165           |
| Node 12 | Node 13 | 2.3013       | 1.0621         | -2.2828      | -1.0476        | 18.4716        | 14.5332          |
| Node 13 | Node 14 | 1.9800       | 0.8709         | -1.9749      | -0.8643        | 5.0543         | 6.6529           |
| Node 14 | Node 15 | 1.3693       | 0.4605         | -1.3668      | -0.4583        | 2.4768         | 2.2044           |
| Node 15 | Node 16 | 1.0640       | 0.4078         | -1.062       | -0.4064        | 1.9540         | 1.4269           |
| Node 16 | Node 17 | 0.7592       | 0.3054         | -0.7574      | -0.3031        | 1.7479         | 2.3336           |
| Node 17 | Node 18 | 0.4546       | 0.2022         | -0.4542      | -0.2019        | 0.3692         | 0.2895           |
| Node 18 | Node 19 | 1.8956       | 0.8464         | -1.8945      | -0.8454        | 1.1022         | 1.0518           |
| Node 19 | Node 20 | 1.4402       | 0.6435         | -1.4344      | -0.6382        | 5.8451         | 5.2669           |
| Node 20 | Node 21 | 0.9094       | 0.4049         | -0.9088      | -0.4041        | 0.6397         | 0.7474           |
| Node 21 | Node 22 | 0.4545       | 0.2023         | -0.4542      | -0.2019        | 0.2771         | 0.3664           |
| Node 22 | Node 23 | 4.6606       | 2.2649         | -4.6410      | -2.2515        | 19.6417        | 13.4210          |
| Node 23 | Node 24 | 4.2810       | 2.0515         | -4.2479      | -2.0253        | 33.1087        | 26.1440          |
| Node 24 | Node 25 | 2.1281       | 1.0159         | -2.1198      | -1.0094        | 8.2949         | 6.4905           |
| Node 25 | Node 26 | 6.0129       | 5.4397         | -5.9890      | -5.4275        | 23.8804        | 12.1637          |
| Node 26 | Node 27 | 5.6862       | 5.3013         | -5.6553      | -5.2856        | 30.9303        | 15.7481          |
| Node 27 | Node 28 | 4.2053       | 4.6814         | -4.1291      | -4.6143        | 76.1696        | 67.1573          |
| Node 28 | Node 29 | 3.8263       | 4.5133         | -3.7734      | -4.4673        | 52.8184        | 46.0141          |
| Node 29 | Node 30 | 3.1678       | 4.114          | -3.1415      | -4.1006        | 26.2788        | 13.3853          |
| Node 30 | Node 31 | 2.1321       | 1.0724         | -2.1213      | -1.0617        | 10.7697        | 10.6437          |
| Node 31 | Node 32 | 1.3642       | 0.7084         | -1.3628      | -0.7067        | 1.4411         | 1.6797           |
| Node 32 | Node 33 | 0.3029       | 0.2020         | -0.3028      | -0.2019        | 0.0890         | 0.1384           |

It is should be noted that the simulation step time is supposed to be 1 hour in this study and the loads are supposed to be constant at each hour. The amount of daily energy losses is 24144.4362 kWh+j16239.5971kVarh. Table 4 provides the hourly active and reactive power, and energy losses in the base case scenario. The voltage profile of the base scenario is depicted in the next section.
| Hour | Ploss(kW) | Qloss(kVAr) | Hour | Ploss(kW) | Qloss(kVAr) |
|------|-----------|-------------|------|-----------|-------------|
| T01  | 523.5026  | 353.9953    | T13  | 1247.0473 | 834.1745    |
| T02  | 479.6929  | 324.2182    | T14  | 1238.7669 | 831.5321    |
| T03  | 454.6957  | 307.2324    | T15  | 1314.0399 | 882.5659    |
| T04  | 438.6580  | 296.3366    | T16  | 1423.2936 | 956.6733    |
| T05  | 471.7179  | 318.7987    | T17  | 1453.0826 | 983.6319    |
| T06  | 541.9198  | 366.5162    | T18  | 1571.3761 | 1057.3551   |
| T07  | 667.3331  | 451.8185    | T19  | 1540.0185 | 1036.2450   |
| T08  | 985.2178  | 659.5816    | T20  | 1408.8562 | 947.2502    |
| T09  | 1119.1774 | 750.2810    | T21  | 1255.5290 | 843.2869    |
| T10  | 1223.8077 | 821.3925    | T22  | 1043.8493 | 699.9003    |
| T11  | 1267.8470 | 851.2454    | T23  | 671.5076  | 454.8767    |
| T12  | 1276.2115 | 853.9063    | T24  | 527.2878  | 356.7825    |

### 4.2 Integrated and Coordinated Operation Results

In this section, two scenarios were investigated for the optimal operation of the industrial, commercial, and residential hubs by utilizing the ADMM approach. In the first scenario, namely ‘integrated operation’, the operation of each hub’s assets was studied separately and only power transaction with the upstream grid is allowed and this takes place at the connection nodes of the hubs. The results obtained from simulating the integrated scenario to supply the electrical, heating, and cooling load demands of each hub show that the local energy resources of each hub would be adequate to thoroughly supply the demand. Accordingly, the daily operating costs of the industrial, commercial, and residential hubs in the integrated scenario are $3441.895, $596.600, and $988.789 respectively. The total operating cost of the hubs would be $5027.284. In the coordinated scenario with the possibility of P2P energy transactions, the daily operating costs of the industrial, commercial, and residential hubs are $2932.645, $590.155, and $914.165 respectively. The total operating cost of the energy hubs would be $4436.965, which is lower than the integrated scenario by 11.75%. The largest decrease occurs in the industrial hub with 14.8% reduction while the smallest decrease occurs in the residential hub where costs are reduced by 1%. The industrial hub can make the largest profit by purchasing the surplus power generation from the commercial and residential hubs during hours 1-16 and at hour 24, and also by selling its surplus power generation to the other two hubs during
hours 17-23 in the coordinated scenario. Fig. 7 depicts the power transactions between the three hubs in the coordinated scenario.

![Fig. 7. Power transactions between the three hubs in the coordinated scenario.](image)

The energy stored in the EES systems of the three hubs in the integrated and coordinated scenarios is shown in Fig. 8. Figs. 9-11 illustrate the power flow results, obtained by using the ADMM technique in the base case, the integrated scenario, and the coordinated scenario, respectively. In the base case, it is assumed that the hubs cannot actively participate in the market and thus transact power; so they are pure consumers. In this respect, the active power demands in the three cases are shown in Fig. 9. It is also noteworthy that the load demands of the industrial, commercial, and residential hubs are substantial. As a result, the local power generation in the hubs not only reduces the operating costs but also mitigates the total energy losses of the distribution system and improves the voltage profile. The net energy purchased from the utility grid in the integrated scenario has considerably decreased as hubs can locally generate power to supply their load demands, independently. Consequently, the amount of energy purchased from the utility grid has dramatically reduced. The decrease in the power required in the coordinated model is considerably lower than that of the integrated model.
The reductions in the load demand over the peak hour, i.e. hour 18, are 1.76 MW and 2.489 MW in the integrated and coordinated models, respectively. The highest reductions in the load demand in the integrated and coordinated modes are 2.458 MW and 2.792 MW, respectively. The lowest reduction occurs during hours 2-7 by 0.24 MW and 0.7 MW in the integrated and coordinated modes, respectively. As expected, the coordinated model is more effective than the integrated model in mitigating the dependency on the upstream electrical grid.

Fig. 8. The energy stored in the EES system of the three hubs, (a): integrated operation, (b): coordinated operation
Fig. 9. The hourly active power demands of the distribution network for the base case, integrated case, and coordinated case.

The total power losses of the distribution system in the three operation cases are shown in Fig. 10. As can be observed, the losses are lower in the coordinated and integrated cases and as it is expected, the amount of power losses has been considerably reduced by using the local power generation in energy hubs. The voltage profiles of the industrial, commercial, and residential hubs are shown in Fig. 11. As this figure depicts, the voltage profiles of industrial, commercial, and residential loads are proportional to the peak load demand on that bus and the total load demand of the system. For instance, as Fig. 11(a) indicates the voltage profile significantly dropped during hours 8-16 at which the industrial peak load demand occurs. This trend in the subsequent hours is due to the indirect load demand increase of the commercial and residential energy hubs impacting the voltage profile of the industrial hub. As Fig. 11(b) illustrates, the highest voltage drop occurs at hour 19 with the load demand increase at this hour. However, the voltage profile has been enhanced through injecting power by the industrial hub in the coordinated and integrated models. It is noteworthy that batteries should charge to meet the constraint on the final state-of-charge (SoC) which in turn leads to the voltage drop during hours 20-21. Also, the voltage drop can be observed in
the residential hub during hours 17-22 due to the increase in the load demand. The voltage profile is improved over the final hours of the day with the decrease in the load demand.

Fig. 10. The total hourly losses in the three operation cases.
The power flow results derived by using the ADMM technique at hour 18, i.e. the peak hour, are illustrated in Figs. 12 and 13. The net values of power injected to the utility grid by the industrial, commercial, and residential hubs at this hour are 300.1444 kW, 184.4375 kW, and -205.693 kW at nodes 23, 20, and 27, respectively. Accordingly, the industrial and commercial hubs act as energy sellers and the residential hub is the buyer. Also, the power transaction between hubs is done as shown in Fig. 3. The residential hub has purchased 57.6375 kWh at hour 18 from the industrial and
commercial hubs, out of which 35.7 kWh has been provided by the industrial hub and 21.9375 kWh has been supplied by the commercial hub. Figs. 12 and 13 also show the power mismatch to better represent the convergence rate of the operation problem at hour 18. It is noted that the solution has converged after 98 iterations.

![Graph](image1)

**Fig. 12.** The net active power of the studied hubs by using the ADMM technique.

![Graph](image2)

**Fig. 13.** The net reactive power of the studied hubs by using the ADMM technique.

Table 5 represents the operating costs of the three hubs and the total operating cost in the integrated and coordinated operation scenarios.
Table 5. Comparative results of the operation of the three energy hubs.

| Hub           | Integrated ($) | Coordinated ($) |
|---------------|----------------|-----------------|
| Industrial    | 3441.895       | 2932.645        |
| Commercial    | 596.600        | 590.155         |
| Residential   | 988.789        | 914.165         |
| Total Cost    | 5027.284       | 4436.965        |

5-Conclusion

This paper investigated the problem of the operation of energy hubs, including industrial, commercial, and residential hubs. In this respect, the hubs’ assets were characterized using linear models. Besides, the operation problem of the hubs was investigated using different case studies. By employing the integrated model, the hub was only allowed to transact power with the utility grid based on a predetermined tariff, which included the tax and grid cost. The coordinated operation model allowed the hubs to transact power with other hubs using the P2P energy trading mechanisms. As a result, the priority of the hubs would be to trade energy through P2P transactions rather than transacting power with the utility grid. In addition to effectively addressing the data privacy issue of clients and hub operators, the ADMM-OPF implemented in this paper ensures the minimum data exchange between the clients. The results obtained from simulating the problem for a typical day on an hourly basis verified that the coordinated operation model could substantially reduce the consumers’ costs compared to the integrated operation model. Moreover, the energy losses were significantly reduced in comparison with both the base case and the integrated operation model, while the voltage profile of the system was considerably improved. As the presented model was based on the energy hubs participation in the P2P energy trading framework, future research would be conducted on the following items:

- Providing the capability of making bilateral contracts with price agreement and two degrees of freedom, i.e. the amount of power and the price.
• Extending the proposed model using the model predictive control in the presence of renewable energies, such as wind power generation, and modifying the energy trading strategy to include the intraday market.

The results, derived from simulating the presented modes showed that the coordinated model is more effective in reducing the total costs where the reduction in the costs was 11.75% more than the integrated scenario. For instance, the total costs reported for the industrial energy hub in the integrated and coordinated models were $3441.895 and $2932.645, respectively showing that by using the coordinated model, this cost could reduce by $509.25. The total costs of the commercial and residential hubs in the coordinated model with respect to the integrated model were mitigated by $6.450 and $74.524, respectively. It was observed that employing the coordinated model led to 17% drop in the total load demand while the integrated model resulted in 13% drop in the total load demand. In this regard, the industrial hub’s load demand reduced by 14.8% which was the largest rate among the three hubs. On the other hand, the residential hub’s load demand reduced by 1% which was the smallest rate compared to other hubs. The obtained results also depicted that it is possible for the industrial hub to make the largest profit by purchasing the surplus power generation from the commercial and residential hubs during hours 1-16 and at hour 24, and also by selling its surplus power generation to the other two hubs during hours 17-23 in the coordinated scenario.

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