Study of the ionizing gas flow in the channel of plasma accelerator with different ways of gas inflow at the inlet

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Abstract. Numerical studies of the ionization process are presented for various ways of the gas supplying at the inlet of the coaxial channel of the quasi-stationary plasma accelerator. Simulation of two-dimensional axisymmetric flows of ionizing gas was carried out on the basis of the equations of radiation magnetic gas dynamics taking into account radiation transport. The approximation of local thermodynamic equilibrium in three-component medium consisting of atoms, ions, and electrons was used in the MHD model. It was determined that non-uniform gas inflow at the channel inlet leads to even greater instability of the pulsating unsteady flows of the ionizing gas in the plasma accelerator. The method of long characteristics was used in the 3D model of radiation transport, including the basic mechanisms of emission and absorption for different portions of the spectrum. The radiation field in the ionizing gas flow was determined, including the integral radiation characteristics and the spectral radiation intensities in the narrow ranges of frequencies or photon energies.

1. Introduction
New cycle of theoretical and experimental research in the quasi-stationary plasma accelerators (QSPA) (see e.g. [1-5]) is conducted in order to study the effect of various ways of the gas supplying to the QSPA channel on the gas ionization process. The simplest plasma accelerator, shown schematically in figure 1, consists of two coaxial electrodes connected to the electrical circuit. Breakdown in gas between the electrodes leads to the formation of an ionization front corresponding to the phase transition from the gaseous state of matter to the plasma. In classical installations, the processes occur in the presence of the azimuthal magnetic field \(H_a\), which is generated by the electric current flowing along the inner electrode. Beyond the front, the ionized plasma is accelerated in the longitudinal direction due to the Ampere force \(F_A = \frac{1}{c} j \times H\), where \(j\) is the current in the plasma, which has predominantly radial direction. These processes take place, in particular, in small accelerators of the first stage of the two-stage QSPA [1]. The QSPA installations are intended for technological applications and thermonuclear research (see e.g. [1, 6, 7]), and are also of interest for the development of the promising powerful electrojet plasma engines, since they are flow-type systems in which gas is continuously injected at the inlet. The latter circumstance becomes especially relevant in view of the developed small-sized atomic reactors.
Theoretical and numerical studies of the ionization process in the QSPA are carried out using models of various levels of complexity. The ionizing gas flow in the framework of the quasi-one-dimensional approximation was considered in [8, 9] for narrow cylindrical channel. The foundations of the theory of processes at the ionization front for stationary flows have been developed in [10]. Three-dimensional numerical model of radiation transport in streams of ionizing gas is presented in [11] taking into account a number of factors related to the accuracy of description of the geometry of the radiating volume, the exclusion of shadow regions from the calculations, the details of description of the radiation spectrum and the main mechanisms of emission and absorption.

This paper presents the results of numerical studies of two-dimensional axisymmetric flows of ionizing gas in the QSPA-T installation modernized at the Troitsk Institute for Innovation and Fusion Research to study the possibility of different ways of the gas supplying at the inlet to the accelerator channel.

2. Equations of radiation magnetic gas dynamics
The MHD model of two-dimensional axisymmetric flows of an ionizing gas is based on the transfer equations for three-component medium [12], consisting of atoms, ions and electrons, as well as the diffusion equation of the magnetic field, which follows from the Maxwell's equations and the Ohm's law, if the inertia of electrons and the displacement current are neglected. The ionization process is being investigated for the hydrogen used in the experiments at this stage. The masses of atoms and ions are equal $m_a = m_i = m$. The temperature at the ionization front increases up to a level $1 \div 3 \, eV$. Concentration of gas entering into the channel has fairly high values. We can assume that the medium is quasi-neutral $n_i = n_e$, and the velocities of components of medium are equal to each other $V_i = V_e = V_a = V$. Estimates and experiments also show that we can confine ourselves to the case of single temperature mixture.

Transformation of the initial equations with regard to the above assumptions leads to the following set of the modified MHD equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \, V) = 0,$$
$$\frac{\partial V}{\partial t} + \nabla P = \frac{1}{c} J \times H,$$
$$\frac{\partial H}{\partial t} = \nabla \times (V \times H) - c \, \text{rot}_\sigma \, j,$$

$$P = P_a + P_i + P_e = (1 + \alpha) (c_p - c_v) \rho \, T, \quad \alpha = n_e / (n_a + n_i), \quad q = - \, \kappa_{e-a} \, \nabla T,$$

$$\alpha = n_e / (n_a + n_i), \quad q = - \, \kappa_{e-a} \, \nabla T.$$

The equations contain the density of heavy particles $\rho = m (n_a + n_i)$, the degree of ionization $\alpha$, the heat flux $q = - \, \kappa_{e-a} \, \nabla T$ caused by electron-atomic heat conduction $\kappa_{e-a}$, and the density of the radiation energy flux $W$. The equation for internal energy includes the Joule heating $Q_{ei} = J^2 / \sigma$, which considerably exceeds the heat generated by friction with other components. The concept of internal energy per unit mass $\varepsilon = (1 + \alpha) c_v T + \varepsilon_i$ includes an additional term $\varepsilon_i = \zeta \alpha l / m_i$ that corresponds to the energy losses due to

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**Figure 1.** Scheme of the simplest coaxial plasma accelerator
ionization, where value \( I \) is the ionization energy of atom. The electrical conductivity of three-component medium in equations (1) is equal to \( \sigma = e^2 n_e / m_e v_e \), where the average frequency of electron collisions with other particles is composed of the frequencies of collisions with atoms and ions \( \nu_e = \nu_{ea} + \nu_{ei} \), where \( \nu_{ea} = n_a \langle V_e \rangle S_{ea} \), \( \nu_{ei} = n_i \langle V_e \rangle S_{ei} \), \( S_{ea} \) and \( S_{ei} \) are the effective collision cross sections. Heat transfer at high degrees of ionization is determined by electron thermal conductivity across the magnetic field including dependence on parameter \( \omega_e \tau_e \) [12], and at low degrees of ionization, atomic thermal conductivity plays a certain role.

In the case of local thermodynamic equilibrium (LTE), the concentrations of all three components of the medium and the electron temperature are connected by the Saha relation

\[
\frac{n_i n_e}{n_a} = K_1(T) = \frac{2 \Sigma_i}{\Sigma_e} \left( \frac{m_e k_B T}{2 \pi \hbar^2} \right)^{3/2} \exp \left( -\frac{I}{k_B T} \right),
\]

(2)

where the value \( K_1 \) is the constant of the ionization balance, \( \Sigma_e \) and \( \Sigma_i \) are the statistical sums of atom and ion. The Saha relation (2) corresponds to the direct process of ionization of atoms from the ground state as a result of the electron impact and the reverse process of recombination in the collision of ions with electrons: \( A + e \leftrightarrow A^+ + e + e \). The equation (2) under the condition of quasi-neutrality determines the degree of ionization

\[
\alpha = -K_1(T) / 2 n + \sqrt{(K_1(T) / 2 n)^2 + K_1(T) / n}.
\]

(3)

The heat transfer due to the radiation energy flux, which can play a certain role in the redistribution of energy, is taken into account in the framework of radiation magnetic gas dynamics. The density of the radiation energy flux \( W \) and the radiation energy density \( U \), which is substantially less than the thermal or internal energy of the medium \( U < \rho \epsilon \), are determined by radiation intensity \( I_{\nu}(r, \Omega) \) along the chosen direction and for a certain part of spectrum using the following relations

\[
U(r) = \frac{1}{c} \int_0^\infty \int_0^{4\pi} I_{\nu}(r, \Omega) \, d\Omega \, dv, \quad W(r) = \int_0^\infty \int_0^{4\pi} I_{\nu}(r, \Omega) \, d\Omega \, dv.
\]

(4)

The radiation intensity is calculated by means of the stationary equation of the radiation transport

\[
\frac{\partial}{\partial r} \left( \rho \frac{\partial}{\partial r} I_{\nu}(r, \Omega) \right) = \frac{\partial}{\partial \Omega} \left( \eta_{\nu}(r) - \kappa_{\nu}(r) \cdot I_{\nu}(r, \Omega) \right),
\]

(5)

in which the absorption coefficient \( \kappa_{\nu}(r) \) and emissivity \( \eta_{\nu}(r) \) depend on the state of the medium, its density and temperature, as well as the spectral parameter \( \nu \) associated with the photon energy \( h\nu \). The whole spectrum is divided into more than 300 spectral groups. The absorption coefficient and emissivity are determined using relations in [11] that contain three parts corresponding to absorption and emission in lines, photo-ionization and photo-recombination, and also scattering (see e.g. [13–17]). The coefficients in equation (5) also depend on the relative concentration of \( k \)-th state of atom \( x_k = n_k / n \), the oscillator strength \( f_{kj} \) for \( k \to j \) transition in atom, the line profile \( \phi_{kj}(\nu) \) corresponding to the bound-bound transition, the cross sections of photo-ionization and bremsstrahlung. The line profile, which takes into account the different mechanisms of broadening of spectral lines, is determined using the Voigt formula. The populations of states under the LTE condition are related by the Boltzmann formula \( x_k = \frac{n_a g_k}{n} \exp \left( -\frac{E_k}{k_B T} \right) \), where \( g_k \) are the statistical weights of \( k \)-th state of atom, \( E_k = Z^2 e^2 / 2 a_o k^2 \) is the energy levels of hydrogen atom and \( a_o \) is the Bohr radius.

3. Formulation of problem and numerical solution methods

To find the numerical solution of the problem with equations (1) and (3), dimensionless variables are used. The units of measurement are the characteristic gas concentration \( n_o \) and temperature \( T_o \) at the inlet to the accelerator channel, the length of the channel or its part \( L \), and the characteristic value of azimuthal magnetic field \( H_o \) at the inlet, which is determined by the discharge current \( I_d \) in the system so that \( H_o = 2 I_d / c R_o \), where \( R_o \) is the characteristic channel radius. The units of measurement of other variables are formed using the specified parameters, including units of pressure \( P_o = H_o^2 / 4 \pi \), velocity \( V_o = H_o / \sqrt{\pi} \rho_o \), time \( t_o = L / V_o \), electric field \( E_o = H_o V_o / c \), current in the plasma
$j_0 = c H_0 / 4\pi L$, and density of the radiation energy flux $W_0 = V_0 H_0^2 / 4 \pi$. As a result, the set of the MHD equations in dimensionless variables contains the dimensionless parameters presented in [8–11].

The gas at the inlet to the channel for $z = 0$ is supplied with the known values of density $\rho(r) = f_1(r)$ and temperature $T(r) = f_2(r)$. The discharge duration is significantly greater than the particle transit time. Therefore, we believe in the numerical model that the current in the system is kept constant and flows only through the electrodes. Hence we have $j_z = 0$ or $r H_0 = r_0 = \text{const}$ ($r_0 = R_0 / L$) at $z = 0$. Coaxial electrodes $r = r_a(z)$ and $r = r_i(z)$, forming the channel walls, are equipotential ($V_0 = 0$) and impermeable ($V_0 = 0$) surfaces.

The algorithm for the numerical solution of the problem with equations (1) in the profiled channel, shown in figure 2 in the plane of variables $(z, r)$, includes the mapping of the computational domain onto a rectangle in the plane $(y, z)$ using the relation

$$r = (1 - y) r_i(z) + y r_a(z).$$

The numerical model of two-dimensional axisymmetric flows of ionizing gas and plasma suggests the splitting in the coordinate directions and physical factors. The calculation of the hyperbolic part of the MHD equations is based on the difference scheme with correction of flows (see e.g. [18]). The effect of electrical conductivity and heat conduction in the parabolic part of the set of the MHD equations is taken into account by means of the flux variant of the back-substitution method [19].

The three-dimensional formulation of the problem of radiation transport for the axisymmetric flow of ionizing gas and plasma in the accelerator channel is necessary within the framework of the integral relations (4), on the basis of which the density and flux of radiation energy are calculated at each node or cell of the coordinate grid. Three-dimensional grid is built by rotating the initial grid in variable plane $(z, r)$ by 360 degrees around the axis of the channel with a certain step. An additional angular grid along the azimuthal and polar angles is built at the nodes of the coordinate grid, providing the uniform distribution of the rays along the directions in the complete spatial angle $\Omega = 4 \pi$. Ray tracing is carried out in accordance with the method of long characteristics [16, 17] in order to determine the points of intersection of the rays with the faces of the cells of the 3D grid. We assume that the absorption coefficient $\kappa_\nu$ and emissivity $\eta_\nu$ in equation (5) are constant within a single cell. The method of characteristics allows to take into account the geometry of the accelerator channel in details. The invisible shadow regions are excluded from the calculation of the radiation energy flux. The matching of the known solutions of equation (5), corresponding to constant coefficients $\kappa_\nu$ and $\eta_\nu$ within each cell, is performed at their boundary. As a result, we obtain a solution on a characteristic passing through an arbitrary number of homogeneous regions. Details of the algorithm for solving the problem of radiation transport in the accelerator channel are presented in [11]. The developed algorithm makes it possible to determine the emission spectrum in any spectral range for any ray emerging from the plasma volume.

The heated walls of the accelerator channel can serve as an additional source of radiation. To evaluate the effect of radiation coming from the channel walls, we used the data of [20] on the degree of blackness of various materials, including copper. Calculations taking into account this factor showed that even in the case of an increase in wall temperature, the radiation of the electrodes does not significantly affect the radiation intensity inside the channel.

4. Results of numerical experiments

Calculations of two-dimensional axisymmetric flows of ionizing gas in the channel of the modernized plasma accelerator QSPA-T were carried out on the basis of the presented modified MHD model for various ways of the gas supplying at the inlet. One of the calculations shown in figure 2 for the following characteristic parameters of the problem: $n_0 = 4 \cdot 10^{17} \text{cm}^{-3}$, $T_0 = 750 \text{K}$, $I_d = 50 \text{kA}$, and $L = 10 \text{cm}$ corresponds to the non-uniform supply of gas at the inlet in accordance with curve 1 in figure 3. Possible ways of specifying the density at the channel inlet are shown in figure 3 at $z = 0$. Here, line 2 corresponds to uniform inflow of gas at the inlet, curves 1 and 3 correspond to non-uniform inflow. Concentration of gas for curve 1 in figure 3 is less in the vicinity of the inner electrode.
Figure 2. Distributions of (a) temperature and plasma current, (b) density and velocity vector field in the ionizing gas flow under condition of non-uniform inflow at the inlet.

The characteristic temperature distribution $T(z, r)$ in axisymmetric plasma flow is given in figure 2(a) in the plane $(z, r)$. Level dashed lines complement the temperature distribution. The solid curves or level lines for function $r H_\phi = \text{const}$ in figure 2(a) correspond to the current in the plasma. We have $r H_\phi = r_0$ for $z = 0$. Direction of the arrows for plasma current in figure 2(a) corresponds to the selected polarity of electrodes. We assume that external electrode is anode.

It can be seen that the ionization front is formed in the narrow part of the channel at $z \approx 0.8$. The sharp increase in temperature occurs in the vicinity of the front. Plasma current mainly flows in the volume of the ionized gas behind the ionization front. The sharp acceleration of the plasma occurs behind the front due to the Ampere force. Figure 2(b) shows the characteristic density distribution and the velocity vector field in the ionizing gas flow under condition of non-uniform inflow of gas at the inlet. The scale of the velocity vectors is determined by the vector $V_\phi = \text{const} \cdot V_0$ indicated in figure 2(b). Here, the unit of velocity is equal to $V_0 = 0.7 \cdot 10^6 \text{ cm / s}$ for the initial parameters indicated above.

Figure 3. Possible distributions of gas density at the inlet to the channel

Figure 4. Temperature change over time behind the front
It is known that the ionizing gas flow can be pulsating or non-stationary at certain values of discharge current $I_d$ and mass flux $\dot{m}$. The empirical condition for stationarity of the gas ionization process was formulated and presented in [9] within the framework of the quasi-one-dimensional flow model by means of the inequality $I_d^2 / J_m > K$, where $J_m = em/\dot{m}$ (kA) is the mass flux expressed in current units, and the constant $K$ generally depends on the geometry of the accelerator channel.

The calculation variant presented in figure 2 corresponds to the unsteady flow of ionizing gas. In this case, the most noticeable changes over time are observed for temperature and degree of ionization. Curve 1 in figure 4 characterizes the temperature change with time at $z = 0.9$ on the middle coordinate line for $\alpha = 0.5$ in accordance with (6) and for the above parameters corresponding to figure 2.

The characteristic one-dimensional distributions of the MHD variables corresponding to figure 2 are presented on figure 5 along the middle coordinate line at $y = 0.5$ taking into account relation (6). Figure 5(a) shows that a sharp increase in temperature at the ionization front at $z \approx 0.8$ is accompanied by a sharp increase in the degree of ionization (solid curve). Figure 5(b) illustrates the change in the flow velocity (solid curve), the gas-dynamic sound velocity $C_g = \sqrt{\gamma P/\rho}$ (dashed curve) and the speed of the fast magnetosonic wave $C_s$ (dash-dotted curve) along the middle coordinate line. The speed of the fast magnetosonic wave in the absence of a longitudinal magnetic field or in the presence of a single azimuthal field component corresponds to the signal speed [1], which is equal to $C_s = \sqrt{C_g^2 + C_A^2}$, where $C_A^2 = H^2/\rho$. This figure shows that the acceleration process is accompanied by a sequential transition of the flow velocity values through the values of the gas-dynamic sound velocity and the signal velocity.

Numerical studies have been conducted for various ways of gas supply at the inlet. In some cases, there is a breakdown of the ionization process. This corresponds to curve 3 in figure 3, for example. Uniform gas supply at the inlet, corresponding to curve 2 in figure 3, also led to the unsteady flow of ionizing gas (see curve 2 in figure 4) for the above parameters of the problem. Accordingly, the transition to the non-uniform gas supply at the inlet does not contribute to the stabilization of the ionizing gas flow for the specified parameters and the discharge current $I_d = 50$ kA. On the contrary, the comparison of curves 1 and 2 in figure 4 shows that the non-uniform supply of gas at the inlet leads to an even greater destabilization of the pulsating flows of ionizing gas, obtained under the condition of the uniform supply of gas.
The integral flow characteristics, which are calculated in the outlet section of the channel, are also of some interest. In this case, mass flux is equal to \( \dot{m} \approx 2 \ g/s \), and thrust of installation is \( P \approx 173 \ n \).

5. Integral and spectral characteristics of radiation

The radiation field in the ionizing gas flow is determined on the basis of the solution of the radiation transport equation in the 3D formulation of the problem. The two-dimensional distribution of the radiation energy density \( U \) and the vector field of the radiation energy flux density \( W \) in the axisymmetric ionizing gas flow are presented in figure 6. This radiation field corresponds to the density and temperature distributions shown in figure 1 under the condition of the non-uniform gas supply at the inlet to the channel.

![Figure 6](image)

**Figure 6.** Distribution of the radiation energy density and vector field of the radiation energy flux density in the ionizing gas flow.

The figure 6 shows the distribution of the magnitude \( \tilde{U} \), which is related to the radiation energy density by means of relation \( \tilde{U} = 10^{-8} \cdot c \cdot U \ \text{erg/cm}^2 \text{s} \). The scale of the vectors \( W \) is determined by the modulus of the vector \( W = 7 \cdot 10^7 \ \text{erg/cm}^2 \text{s} \), which is indicated in the figure.

There is a region with relatively high values of the radiation energy density \( U \) in the vicinity of the ionization front. The radiation energy flux \( W \) is directed in all directions from this region, including the direction of the incoming flux of weakly ionized gas. This leads to its preliminary ionization directly ahead of the front, as well as in the depth of volume of the incoming gas.

![Figure 7](image)

**Figure 7.** Spectral intensities of radiation (a) in direction of ray 1 and (b) for ray 2, presented in figure 6.

The 3D model of radiation transport allows determining the spectral radiation intensity in any frequency range or photon energy for any ray emerging from the plasma volume in the direction of the detector. This circumstance opens up new opportunities for carrying out the complex theoretical and
experimental studies based on flux spectroscopy. For example, figure 7 shows the spectral intensities of radiation in the frequency range, which contains the \( \alpha \) and \( \beta \) Lyman lines for radial rays 1 and 2, presented in figure 6 and exiting the plasma volume in the direction of the detectors.

6. Conclusion
New cycle of theoretical and experimental research for the modernized quasi-stationary plasma accelerator QSPA-T, developed in the Troitsk Institute for Innovation and Fusion Research, is being conducted to study the influence of various factors on the gas ionization process. Computational research of the phase transition in the flow of the ionizing gas and plasma in the coaxial channel of the plasma accelerator have been carried out on the basis of the MHD equations in the approximation of local thermodynamic equilibrium. The results of numerical studies of two-dimensional axisymmetric flows of the ionizing gas under various conditions of gas supply at the inlet to the accelerator channel were presented. It is shown that non-uniform gas inflow at the inlet leads to the still greater destabilization of the pulsating unsteady flows of ionizing gas, obtained under the condition of uniform supply of gas for the considered parameters of the problem.

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