Percolation and jamming of random sequential adsorption samples of large linear $k$-mers on a square lattice

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The behavior of the percolation threshold and the jamming coverage for isotropic random sequential adsorption samples has been studied by means of numerical simulations. A parallel algorithm that is very efficient in terms of its speed and memory usage has been developed and applied to the model involving large linear $k$-mers on a square lattice with periodic boundary conditions. We have obtained the percolation thresholds and jamming concentrations for lengths of $k$-mers up to $2^{16}$. New large $k$ regime of the percolation threshold behavior has been identified. The structure of the percolating and jamming states has been investigated. The theorem of G. Kondrat, Z. Koza, and P. Brzeski [Phys. Rev. E 96, 022154 (2017)] has been generalized to the case of periodic boundary conditions. We have proved that any cluster at jamming is percolating cluster and that percolation occurs before jamming.

I. INTRODUCTION

Random sequential adsorption (RSA) is a standard method of modeling the adsorption of particles at a liquid-solid interface [1]. Such particles can be, for example, polymers, biomolecules or nanotubes, their aspect ratios typically being in the orders of $10^3$–$10^4$. During deposition, a spanning cluster may occur, i.e., a set of neighboring particles which span between the opposite borders of the substrate. Additional adsorption of particles leads the system into a jammed state where no additional particle can be deposited due to the absence of appropriate holes [2]. The resulting concentration of particles is known as the jamming coverage, $p_j$. The formation of spanning clusters is associated with the percolation phase transition, and the corresponding concentration of particles is known as the percolation threshold, $p_c$. Over the last several decades, percolation theory has been employed to study the properties of disordered media [3, 4]. The properties of the media are considerably different below and above the percolation threshold. Some models of percolation theory are simple, yet productive, models of phase transitions.

One of the possible ways to simulate the adsorption of such large elongated particles is based on the use of a discrete space, e.g., a square lattice and linear $k$-mers (also denoted as needles, linear segments, stiff-chains, rods, or sticks), i.e., rectangular “molecules”, which occupy $k$ successive faces of the lattice.

Becklehimer and Pandey reported that, for $k \leq 20$, the percolation threshold depends strongly on the value of $k$ with a power-law

$$p_c(k) \propto k^{-1/2},$$

whereas the jamming coverage decreases [3].

The percolation of $k$-mers up to $k = 40$ has been studied by Leroyer and Pommiers [6]. The investigators found that the percolation cluster is built of regions (stacks) of the same orientation, the typical size of which is $k$. Non-monotonic dependence of the percolation threshold on $k$ was found. For small values of $k$, the percolation threshold decreases as

$$p_c(k) = k^{-1} + \text{const}$$

but it increases for larger values of $k$. Minimal values of the percolation threshold correspond to $k \approx 15$. The authors argued this behavior in terms of the stack structure of the percolation cluster. The authors suggested that the percolation threshold can be reached for any (finite) size of the $k$-mers in contrast to the simulations with rectangles and squares where jamming saturation occurs before percolation [7]. Moreover, the structure of the percolation cluster was studied using the local order parameter

$$S = \left\langle \frac{N_y - N_x}{N_y + N_x} \right\rangle,$$

where $N_x$ ($N_y$) is the number of sites covered by the horizontal (vertical) needles in a given box. Note that the stack structure of the jamming state in terms of the order parameter was studied in [8], where it was found that the characteristic size of each stack is of the order of $k \times k$.

It is evident that Eqs. (1) and (2) describe quite different behaviors. This is due to the use of different kinds of RSA, viz., Leroyer and Pommiers used conventional RSA, while Becklehimer and Pandey applied a so-called “end-on” mechanism of RSA [2].

The percolation and jamming of needles and squares on a square lattice have been investigated by Vandewalle et al. [9]. The ratio $p_c/p_j$ was found to be a constant of $0.62 \pm 0.01$ up to $k = 20$. The authors suggested that...
both quantities scale as
\[ p(k) = C \left( 1 - \gamma \left( \frac{k - 1}{k} \right)^2 \right), \tag{3} \]
where \( \gamma = 0.31 \pm 0.01 \) (except for the case \( k = 1 \)). The presented results evidence that the percolation threshold is significantly smaller than the jamming coverage when \( k \to \infty \) if Eq. (3) is valid for any values of \( k \). Moreover, the existence of stacks, not only at the percolation threshold but also in the jammed state was reported.

The jamming and percolation of needles with lengths up to \( k = 2000 \) have been investigated by Kondrat and Pękalski \[10\]. For \( k > 15 \), an increase in the percolation threshold was found. For \( 15 \leq k \leq 45 \), the fitting formula
\[ p_c / p_j \propto 0.50 + 0.13 \log_{10} k \tag{4} \]
was proposed. A monotonic increase in \( p_c / p_j \) holds over a wide range of values of \( k \) even up to \( k = 2000 \). The dependence of the jamming concentration on the value of \( k \) was fitted by
\[ p_j(k) = p_j(\infty) + a/k^\alpha, \tag{5} \]
where \( p_j(\infty) = 0.66 \pm 0.01 \), \( a = 0.44 \), \( \alpha = 0.77 \). The parameters of the fitting formula were refined in Ref. \[11\] as \( p_j(\infty) = 0.655 \pm 0.009 \), \( a = 0.416 \), \( \alpha = 0.720 \pm 0.007 \) based on simulations and scaling analysis up to \( k = 256 \).

It was also found that
\[ p_c(k) = a_0/k^{\alpha_0} + b \log_{10} k + c, \tag{6} \]
where \( a_0 = 0.36 \pm 0.02 \), \( \alpha_0 = 0.81 \pm 0.12 \), \( b = 0.08 \pm 0.01 \), and \( c = 0.33 \pm 0.02 \). For the ratio \( p_c / p_j \), a fitting formula was proposed
\[ p_c / p_j = B \log_{10} k + C, \tag{7} \]
where \( B = 0.119 \pm 0.003 \) and \( C = 0.513 \pm 0.006 \). The monotonic behavior of the percolation threshold as a function of \( k \) for \( k \leq 15 \) was reported and the following fitting formula was proposed
\[ p_c(k) = p_c^* + \Omega \exp \left( -\frac{k}{\kappa} \right), \tag{8} \]
where \( p_c^* = 0.461 \pm 0.001 \), \( \Omega = 0.197 \pm 0.02 \), and \( \kappa = 2.775 \pm 0.02 \). Naturally, jamming and percolation produced by means of the RSA of \( k \)-mers has been studied on other kinds of substrates, e.g., on triangular lattices \[14, 15\]. A non-monotonic size dependence of the percolation threshold and decrease of the jamming coverage have also been observed on such triangular lattices \[14\].

In a recent paper \[16\], a proof was presented that there will be a percolating cluster in any jammed configuration of nonoverlapping fixed-length, horizontal, or vertical needles on a square lattice. The theorem disproves the recent conjecture \[12, 17, 18\] that in the random sequential adsorption of such needles on a square lattice, percolation does not occur if the needles are longer than some threshold value \( k^* \), estimated to be of the order of several thousand.

We note that percolation is absent for a jammed system in a continuous model \[10, 20\]. Hence, these results imply that the qualitative behavior of continuous percolation cannot be approximated using a finite lattice model, regardless of the lattice size.

The theorem ensures that the ratio of the percolation threshold and jamming coverage \( p_c / p_j \) is well defined for all needle lengths, but it does not predict the asymptotic behavior of this quantity for \( k \to \infty \), which has not been reported up to now to the best of our knowledge. The theorem was proved only for the case of rigid boundaries, but it is also interesting to consider periodic boundary conditions for this problem. The aim of this work is the direct numerical verification of the theorem and a determination of the character of the dependence of the percolation thresholds on \( k \) for large values of \( k \).

For this purpose, we developed a parallel algorithm that is very efficient in terms of speed and memory usage (Section II). This has allowed us to obtain the percolation and jamming concentrations for lengths of \( k \)-mer up to \( 2^{17} \) and to demonstrate the new large \( k \) regime of their behavior (Section III). The percolation threshold and the jamming coverage show very different behavior for \( k \geq 500 \) compared to the behaviors for \( k \leq 512 \) (Section IV). We also generalize the results of \[10\] for the case of periodic boundary conditions in Appendix A.

\[ \text{II. METHODS} \]

A. Details of simulations

Deposition of the \( k \)-mers onto a square lattice \( L \times L \) with periodic boundary conditions, i.e., onto a torus, was performed using the RSA mechanism. Calculations were performed with \( L = 100k \) for \( k \leq 2^{14} \) and \( L = 16384 \) for \( k > 2^{14} \). For \( k \leq 2^{14} \) a single calculation was performed, while for \( k > 2^{14} \) two independent runs were carried out.

To estimate statistical uncertainty, we performed additional test for particular values of \( k \) and \( L \) with up to 200 independent runs.

Scaling analysis was performed for some particular cases to estimate the finite-size effect, since the percolation threshold is expected to vary as
\[ p_c(\infty) - p_c(L) \propto \left( \frac{L}{k} \right)^{-1/\nu}, \tag{9} \]
where the critical exponent in 2D is \( \nu = 4/3 \). While the jamming coverage is expected to vary as \[21, 22\]
\[ |p_j(\infty) - p_j(L)| \propto L^{-1}. \tag{10} \]
The cluster size distribution was also studied. At the percolation threshold, the average number (per site) of clusters containing \( s \) sites each is expected to obey the following relation

\[
n_s \propto s^{-\tau}
\]

for large values of \( s \). Here, \( \tau \) is the Fisher exponent, \( \tau = 187/91 \) for a percolation in 2D [3, 23].

To characterize the connectedness of the system under consideration, the relative number of interspecific contacts was used

\[
n_{xy}^* = \frac{n_{xy}}{n_x + n_y + n_{xy}},
\]

where \( n_x \) (\( n_y \)) is the number of contacts between sites belonging to the horizontal (vertical) needles, and \( n_{xy} \) is the number of contacts between needles oriented in mutually perpendicular directions. In fact, this quantity shows how strong the stacks are connected with each other. A configuration with strongly connected stacks implies the existence of percolation, while poorly connected stacks imply that percolation may not occur [12].

### B. Deposition of \( k \)-mers onto the lattice

Straightforward realization of the RSA has two major drawbacks in the final stage of the algorithm, viz.,

1. due to the lack of space, many attempts will fail prior to each successful placement of an object;
2. the program would not know whether jamming had occurred.

Both these problems can be solved by generating two lists of viable sites (one for each orientation) when the density is large enough, e.g., after percolation has occurred [2, 24, 25]. “Viable” sites are those, where a \( k \)-mer can be placed, considering the orientation. After the lists are filled, the coordinates should be taken from a random position in these arrays. Regardless of these coordinates still being viable or not, the chosen element should always be removed from the list.

### C. Cluster labeling

Percolation transition occurs when a cluster wraps around the torus. Such a cluster is defined as a wrapping cluster. In order to check whether percolation is present, one should label all the clusters of \( k \)-mers. This task can be done using the Union-Find algorithm [21, 27]. The idea is to label each occupied site of the lattice with the cluster’s number (ID). The algorithm itself consists of two parts.

Find: when a new \( k \)-mer is added to the lattice we must find which clusters touch this \( k \)-mer.

Union: if a \( k \)-mer connects different clusters, they should be merged into a single cluster.

The union part can be achieved by reassigning the cluster ID for all the sites of one of the clusters. It is reasonable always to choose the smaller clusters when such a need to change their ID is required, meaning it is useful to have previously stored the size of every cluster.

### D. Checking for percolation

Due to the periodic boundary conditions, percolation occurs when a cluster wraps around the torus. We are looking for a cluster which wraps around the torus in one or both directions. Finding a wrapping cluster can be performed using the Machta algorithm [27, 28]. This requires choosing one site in each cluster (the root) and, for each site in this cluster, storing its \( x \) and \( y \) displacements from the root. These displacements can be negative and they should be updated each time different clusters are merged.

It is important to notice that a wrapping percolation can only occur if the new \( k \)-mer connects two parts of the same cluster (to wrap around the lattice). Each time this happens the algorithm should calculate \(|x_1 - x_2| + |y_1 - y_2|\), where \( x_1, y_1 \) are the displacements in the first site and \( x_2, y_2 \) are those in the second. If this expression is not greater than \( 2k \), then no percolation is present. Otherwise, percolation has occurred.

### E. Memory issues

In order for the algorithm to work properly, at least three integer numbers (12 bytes) need to be stored for each site, viz., the cluster ID and the displacements. Keeping in mind that the lattice can contain about \( 10^{12} \) sites, this leads to enormous memory consumption, hence, some serious optimization is required for large values of \( k \). The method is used only for \( k \geq 96 \).

The idea is to store the cluster ID and the displacements for the whole \( k \)-mer rather than for each site. Each site should store four bits:

1st bit: whether the site is occupied or not,
2nd bit: orientation of the \( k \)-mer the site belongs to,
3rd bit: whether the site is the first one (parent site) in the \( k \)-mer,
4th bit: information about the \( k \)-mer.

Using the first three bits it is easy to find the parent of the \( k \)-mer. Since the \( k \)-mer is long enough (\( k \geq 96 \)) it can be divided into several parts, as shown in Fig. 11. While writing information into the sites the appropriate number should be decomposed into 32 bits and put into the fourth bit in each of the 32 corresponding sites. While reading
information this same procedure has to be performed in reverse.

| cluster ID | x displ. | y displ. |
|------------|----------|----------|
| 32 sites   | 32 sites | 32 sites |
| k-96 sites |          |          |

![FIG. 1. Memory organization for storing the cluster ID and displacements. Each site of every k-mer corresponds to a single bit of information.](image)

This method of memory usage requires 0.5 byte for each site, which is 24 times less than the original one. The method is used only for \( k \geq 96 \). Figure 2(a) shows that there is additional speedup due to the memory compression.

F. Parallelization

All simulations were performed on a computer with two Intel Xeon Platinum 8164, 2 GHz processors and 1536 GB RAM. The speed of the algorithm is a matter of importance when it comes to modelling huge lattices. This can be improved by using parallel programming with OpenMP.

One of the most time consuming parts is relabeling the clusters. It is obvious that different sets of clusters can be merged simultaneously by parallel threads if they do not connect with each other. To prevent different threads from processing the same clusters an additional shared array is used, where each of the clusters can be marked as “busy” or “free”.

To prevent intersections only one thread should be placing a k-mer at any time. This can be achieved using a critical section in OpenMP, which contains the following:

- check whether a k-mer can be placed, (otherwise the algorithm quits the critical section and randomly chooses other lattice site),
- check that the surrounding clusters are “free”, (otherwise the algorithm quits the critical section and waits for the surrounding clusters to be processed)
- place the k-mer,
- mark all surrounding clusters as “busy”.

Prior to entering the critical section each thread makes sure that a k-mer can be placed, and that the surrounding clusters are not “busy”. The thread waits if the latter condition does not hold. Verification of these conditions inside the critical section is necessary for the correctness of the program, while the same checkup outside the critical section is performed in order to minimize the number of entries to the critical section, because extra entries can significantly slow down the program. The clusters are marked “free” after all the necessary processes of merging the clusters and checking for percolation have been completed.

Figure 2(b) shows that the approach allows to speed up the program approximately by a factor of 2.4. The speedup is limited to the condition that only one thread is placing a k-mer at any time, so the parallelization is efficient for a large number of CPU cores only in the final phase of the calculation, when a lot of cluster relabeling is being performed. Future improvements of the parallelization method may include dividing the lattice into several parts, and allowing several threads to place k-mers simultaneously into different parts of the lattice.

We have used the library RNGAVXLIB [29] for the efficient and safe generation of random numbers in parallel.
threads.

III. RESULTS AND DISCUSSION

A. Percolating threshold, jamming coverage and their ratios

The obtained percolation thresholds and jamming coverages and their ratios are shown in Fig. 3 and presented in Table I. Known approximations (3), (4), (6), and (8) are also shown in Fig. 3 for comparison. For $2^4 \leq k \leq 2^{17}$, the data can be fitted by

$$p_c = A + \frac{B}{C + \sqrt{k}}$$  \hspace{1cm} (13)

where $A = 0.615 \pm 0.001$, $B = -2.26 \pm 0.09$, $C = 10.2 \pm 0.6$, the adjusted coefficient of determination is $R^2 = 0.999$.

| $n$ | $p_c$ | $p_j$ |
|-----|------|------|
| 0   | 0.5933 | 1    |
| 1   | 0.5611 | 0.9062 |
| 2   | 0.5023 | 0.8094 |
| 3   | 0.4717 | 0.7479 |
| 4   | 0.4590 | 0.7103 |
| 5   | 0.4733 | 0.6892 |
| 6   | 0.4894 | 0.6755 |
| 7   | 0.5111 | 0.6686 |
| 8   | 0.5292 | 0.6628 |
| 9   | 0.5450 | 0.6618 |
| 10  | 0.5628 | 0.6592 |
| 11  | 0.5740 | 0.6596 |
| 12  | 0.5850 | 0.6575 |
| 13  | 0.5913 | 0.6571 |
| 14  | 0.6003 | 0.6561 |
| 15  | 0.6039 | 0.6548 |
| 16  | 0.6086 | 0.6545 |
| 17  | 0.6067 | 0.6487 |

The results are in agreement with the previously known values of $p_c/p_j$ for $k \leq 512$ taken from [11, 12], which are shown as closed symbols. Figure 3 shows the new large $k$ regime of the behavior of the percolation threshold. In particular, the ratio $p_c/p_j$ shows very different behavior for $k \geq 500$ compared to the behavior for $k \leq 512$. The slow increase of the ratio for large values of $k$ compared to Eq. (7) is in agreement with the results of [13]. We have generalized the results of [13] for the case of periodic boundary conditions in Appendix A and have proved that, in thermodynamic limit, percolation always occurs before jamming.

B. Estimations of the accuracy

Figure 3 shows the dependence of the standard deviations (STDs), $\sigma$, of $p_c$ and $p_j$ on $L/k$ for $k = 256$, calculated over 100 independent runs. The STDs in Fig. 3 decrease with $L$ approximately as $\sigma(p_c) \propto L^{-3/4}$ and $\sigma(p_j) \propto L^{-1}$, in accordance with Eqs. (9) and (10). The finite size effect is expected to be weak, except for the case when $k$ and $L$ are comparable [30]. The STDs of the ratio $p_c/p_j$ is approximately 0.0051 for $k = 256$ and $L = 100k$, hence, a single calculation provides reasonably high accuracy for the case $L = 100k$.

Figures 5 and 6 show that the statistical error primarily depends on $L$, and only weakly depends on $k$. 

![Figure 3](image-url)
FIG. 4. STDs, $\sigma$, of $p_c$ and $p_j$ vs $L/k$ for $k = 256$, estimated over 100 independent runs. The critical exponents are $0.74 \pm 0.05$ for the STD of the percolation threshold and $1.23 \pm 0.17$ for the STD of the jamming coverage.

C. Structure of the percolating and jamming states

Figure 7 shows the cluster size distribution at the percolation threshold for $k = 256$, $L = 100k$. $n_s$ is the average number per site of clusters consisting of $s$ sites each. The dashed line shows the function $s^{-187/91}$, so one can see that the distribution behaves approximately as $n_s \propto s^{-187/91}$ in accordance with Eq. (11).

We found that, in practice, the jamming state always contains only a single cluster. However, it is theoretically possible to construct a jamming state with several clusters. An example is shown in Fig. 8 for $k = 4$ and $L = 10k$.

Figure 9 demonstrates the stack structure of a wrapping cluster. The stacks of mutually perpendicular orientations are connected according to the behavior of the relative number of interspecific contacts. The edges of the clusters are “ragged”. Hence, the stacks look like...
FIG. 8. Example of a jamming state with two clusters for a square lattice with \(k = 4\), \(L = 10k\) and periodic boundary conditions. Courtesy of R.K. Akhunzhanov.

IV. CONCLUSION

We have studied the behavior of percolation and jamming thresholds for the isotropic random sequential adsorption of large linear \(k\)-mers onto a square lattice. We have presented a parallel algorithm which is very efficient in terms of speed and memory usage. Our results are in agreement with the previously obtained results of [11, 12, 16] and we have obtained the percolation and jamming concentrations for lengths of \(k\)-mer up to \(2^{17}\). The ratios of percolation and jamming densities show quite different behavior for \(k \geq 500\) compared to the behavior for \(k \leq 512\). We are not aware of any other studies of this problem that have considered values of \(k\) larger than 512 except [10] where the values of \(k\) were up to 2000 and the lattice size was only \(L = 2500\). We have also analyzed the structure of the percolating and jamming states in terms of the cluster size distribution and the relative number of interspecific contacts.

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Appendix A: Jammed systems and periodic boundary conditions

**Proposition.** Each cluster in every jammed configuration of fixed-length nonoverlapping horizontal or vertical needles on a finite square lattice with periodic boundary conditions is a percolating cluster.

**Proof.** Consider the sides of an arbitrary square of size \(L\) as the edges of the system, where \(L \times L\) is the lattice size. It has previously been proved, for rigid boundary conditions, that every cluster in a jammed configuration extends to one of two consecutive edges of the system (see the Lemma in Method I in [16]). This statement holds also for periodic boundary conditions, because its proof in [16] is directly applicable for this case. We will label the system edges using geographical notation (\(N, E, S, \) and \(W\) for the top, right, bottom, and left edge, respectively).

There are two cases: either the cluster extends to all four edges of the system, or it does not. In the latter case, it does not touch at least one system edge, say, \(N\). Then it follows from the Lemma that it extends to the two edges adjacent to \(N\), that is, to \(E\) and \(W\). Let us now shift the edges to the left by one column, while...
keeping the configuration unchanged. Then the cluster still does not touch the edge N, so it must still touch the new edge E in at least one lattice site. Proceeding with such shifts to the left, each time we find, in the leftmost column, at least one lattice site, that belongs to the cluster. We proceed with such shifts until the same lattice site is found twice during this process. This will happen eventually, because there are only a finite number of lattice sites. On the other hand, finding the same site twice during this process means that the cluster wraps all the way around the lattice, hence it is a percolating cluster.

In the former case, when the cluster touches all four edges of the system, we shift the edges upward by one row. We proceed with such shifts until the cluster does not touch the edge N. If the process completes and the cluster does not touch the edge N, then such a situation was considered in the previous paragraph, so it is a percolating cluster. If the process does not complete during L shifts, then we find at each step a lattice site where the cluster touches the edge N. We proceed with such shifts until the same lattice site is found twice during this process. This will happen eventually, because there are only a finite number of lattice sites. On the other hand, finding the same site twice during this process means that

**Consequence.** In a system of fixed-length non-overlapping horizontal or vertical needles on a square lattice with periodic boundary conditions (on a torus), in the thermodynamic limit, percolation always occurs before jamming, \( p_c < p_j \).

**Proof.** Pinson proved \cite{Pinson1} that the probability, \( R \), of finding a wrapping cluster on a torus at the percolation threshold is \( R(p_j) = R^* \), where \( 0 < R^* < 1 \). The value of \( R^* \) depends on the way how the wrapping cluster is found. In particular, in the thermodynamic limit, the probability that there exists a cluster that wraps around the boundary conditions in the horizontal direction is \( R = 0.521058290 \) \cite{Pickup1, Pickup2}. For \( L \to \infty \),

\[
R = \begin{cases} 
0, & \text{if } p < p_c, \\
R^*, & \text{if } p = p_c, \\
1, & \text{if } p > p_c.
\end{cases}
\]  

(A1)

Since \( R(p_j) = 1 \) therefore \( p_j > p_c \).

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[1] J. Talbot, G. Tarjus, P. R. Van Tassel, and P. Viot, “From car parking to protein adsorption: an overview of sequential adsorption processes,” Colloids Surf. A 165, 287–324 (2000)

[2] J. W. Evans, “Random and cooperative sequential adsorption,” Rev. Mod. Phys. 65, 1281–1329 (1993)

[3] D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor & Francis, London, 1992).

[4] M. Sahimi, *Applications Of Percolation Theory* (Taylor & Francis, 1994).

[5] Jeffrey Becklehimer and Ras B. Pandey, “A computer-simulation study of sticks percolation,” Physica A 187, 71–76 (1992)

[6] Y. Leroyer and E. Pommiers, “Monte Carlo analysis of percolation of line segments on a square lattice,” Phys. Rev. B 50, 2795–2799 (1994)

[7] Mitsunobu Nakamura, “Percolational and fractal property of random sequential packing patterns in square cellular structures,” Phys. Rev. A 36, 2384–2388 (1987).

[8] Yu. Yu. Tarasevich, A. V. Eserkepov, V. V. Chirkova, and V. A. Golteva, “Monte Carlo simulation of entropy-driven pattern formation in two-dimensional system of rectangular particles,” (2018), arXiv:1809.00790.

[9] N. Vandewalle, S. Galam, and M. Kramer, “A new universality for random sequential deposition of needles,” Eur. Phys. J. B 14, 407–410 (2000)

[10] Grzegorz Kondrat and Andrzej Pękalski, “Percolation and jamming in random sequential adsorption of line segments on a square lattice,” Phys. Rev. E 63, 051108 (2001)

[11] Nikolai I. Lebovka, Natalia N. Karmazina, Yuri Yu. Tarasevich, and Valeri V. Laptev, “Random sequential adsorption of partially oriented linear k-mers on a square lattice,” Phys. Rev. E 84, 061603 (2011)

[12] Yuri Yu. Tarasevich, Nikolai I. Lebovka, and Valeri V. Laptev, “Percolation of linear k-mers on a square lattice: From isotropic through partially ordered to completely aligned states,” Phys. Rev. E 86, 061116 (2012)

[13] V. Cornette, A. J. Ramirez-Pastor, and F. Nieto, “Percolation of polyatomic species on a square lattice,” Eur. Phys. J. B 36, 391–399 (2003)

[14] I. Budinski-Petkovic, I. Loncarevic, M. Petkovic, Z. M. Jakišić, and S. B. Vrhovac, “Percolation in random sequential adsorption of extended objects on a triangular lattice,” Phys. Rev. E 85, 061117 (2012)

[15] E. J. Perino, D. A. Matoz-Fernandez, P. M. Pasinetti, and A. J. Ramirez-Pastor, “Jamming and percolation in random sequential adsorption of straight rigid rods on a two-dimensional triangular lattice,” J. Stat. Mech. Theory Exp. 2017, 073206 (2017)

[16] Grzegorz Kondrat, Zbigniew Koza, and Piotr Brzeski, “Jammed systems of oriented needles always percolate on square lattices,” Phys. Rev. E 96, 022154 (2017)

[17] Yuri Yu. Tarasevich, Valeri V. Laptev, Nikolai V. Vygoroitiskii, and Nikolai I. Lebovka, “Impact of defects on percolation in random sequential adsorption of linear k-mers on square lattices,” Phys. Rev. E 91, 012109 (2015)

[18] P. M. Centres and A. J. Ramirez-Pastor, “Percolation and jamming in random sequential adsorption of linear k-mers on square lattices with the presence of impurities,” J. Stat. Mech. Theory Exp. 2015, P10011 (2015)

[19] R. M. Ziff and R. D. Vigil, “Kinetics and fractal properties of the random sequential adsorption of line seg-
ments," J. Phys. A 23, 5103–5108 (1990).
[20] P. Viot, G. Tarjus, S. M. Ricci, and J. Talbot, “Saturation coverage in random sequential adsorption of very elongated particles," Physica A 191, 248–252 (1992).
[21] Daniel Vågberg, Daniel Valdez-Balderas, M. A. Moore, Peter Olsson, and S. Teitel, “Finite-size scaling at the jamming transition: Corrections to scaling and the correlation-length critical exponent,” Phys. Rev. E 83, 030303 (2011).
[22] Pavel L. Krapivsky, Sidney Redner, and Eli Ben-Naim, “Adsorption,” in A Kinetic View of Statistical Physics (Cambridge University Press, 2010) pp. 199–232.
[23] Michael E. Fisher, “The theory of condensation and the critical point,” Physics Physique Fizika 3, 255–283 (1967).
[24] R. S. Nord, “Irreversible random sequential filling of lattices by Monte Carlo simulation,” J. Stat. Comput. Simul. 39, 231–240 (1991).
[25] Benjamin J. Brosilow, Robert M. Ziff, and R. Dennis Vigil, “Random sequential adsorption of parallel squares,” Phys. Rev. A 43, 631–638 (1991).
[26] M. E. J. Newman and R. M. Ziff, “Efficient Monte Carlo algorithm and high-precision results for percolation,” Phys. Rev. Lett. 85, 4104–4107 (2000).
[27] M. E. J. Newman and R. M. Ziff, “Fast Monte Carlo algorithm for site or bond percolation,” Phys. Rev. E 64, 016706 (2001).
[28] J. Machta, Y. S. Choi, A. Lucke, T. Schweizer, and L. M. Chayes, “Invaded cluster algorithm for Potts models,” Phys. Rev. E 54, 1332–1345 (1996).
[29] M. S. Guskova, L. Yu. Barash, and L. N. Shchur, “RNGAVXLIB: Program library for random number generation, AVX realization,” Comput. Phys. Commun. 200, 402–405 (2016).
[30] S. S. Manna and N. M. Švrakić, “Random sequential adsorption: line segments on the square lattice,” J. Phys. A: Math. Gen. 24, L671–L676 (1991).
[31] Haru T. Pinson, “Critical percolation on the torus,” J. Stat. Phys. 75, 1167–1177 (1994).