Can GJ 876 host four planets in resonance?

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Abstract Prior to the detection of its outermost Uranus-mass object, it had been suggested that GJ 876 could host an Earth-sized planet in a 15-day orbit. Observation, however, did not support this idea, but instead revealed evidence for the existence of a larger body in a \( \sim 125 \) -day orbit, near a three-body resonance with the two giant planets of this system. In this paper, we present a detailed analysis of the dynamics of the four-planet system of GJ 876, and examine the possibility of the existence of other planetary objects interior to its outermost body. We have developed a numerical scheme that enables us to search the orbital parameter-space very effectively and, in a short time, identify regions where an object may be stable. We present details of this integration method and discuss its application to the GJ 876 four-planet system. The results of our initial analysis suggested possible stable orbits at regions exterior to the orbit of the outermost planet and also indicated that an island of stability may exist in and around the 15-day orbit. However, examining the long-term stability of an object in that region by direct integration revealed that the 15-day orbit becomes unstable and that the system of GJ 876 is most likely dynamically full. We present the results of our study and discuss their implications for the formation and final orbital architecture of this system.

Keywords Stability · Resonance · Hamiltonian Systems · Numerical Methods · Planetary Systems

1 Introduction

Since the announcement of its two resonant planets by Marcy et al. (2001), the planetary system of GJ 876 has had a special place in exoplanetary science. As the first system detected with two planets in a mean-motion resonance (MMR), GJ
876 b and c, with minimum masses of 2.27 and 0.71 Jupiter-masses and orbital periods of 61.11 and 30.08 days, respectively (Rivera et al., 2010), are in a 2:1 mean-motion resonance. GJ 876 has been the subject of extensive research and has had major contributions to the development of the models of planet migration and resonance capture (Lee & Peale, 2002, Kley et al., 2005). Because of the short period gravitational interactions between its two giant planets, the dynamics of this system and the possibility of its hosting additional planetary bodies have been subjects of intense studies. Soon after the announcement of its 30-day planet (Marcy et al., 2001), Kinoshita & Nakai (2001) studied the stability of this system and showed that its 2:1 MMR presents the most stable orbital configuration for different values of its planetary masses and orbital elements at different epochs. Rivera & Lissauer (2001) and Laughlin & Chambers (2001) also studied the dynamics of this system. These authors argued that because of the relatively small separation between the two giant planets and the possibility of their close approaches, when fitting the system’s radial velocities, the interactions between these two planets have to be taken into account. By presenting new fits to the radial velocities of GJ 876, Rivera & Lissauer (2001) showed that the system will be stable for long times when the planets’ orbits are co-planar and have low inclinations with respect to the plane of the sky. Laughlin & Chambers (2001) limited this inclination to a range of $30^\circ$ to $53^\circ$. The results by Rivera & Lissauer and Laughlin & Chambers were later confirmed by the analytical models of Ji et al. (2002) and in a thorough study of the global dynamics of GJ 876 by Goździewski et al. (2002).

Being a dynamically interesting system, GJ 876 has also been the target of observation for many years. In 2005, Laughlin et al. re-analyzed the new radial velocity data of this star and showed that when planet-planet interaction, stellar jitter, and instrumental uncertainties are taken into account, a 2:1 resonant co-planar orbit with an inclination less than $20^\circ$ would present the most viable and stable planetary configuration for the two giant planets of this system. These authors also showed that in addition to being in a 2:1 MMR, the two planets are locked in a secular resonance where they librate around apsidal alignment with an amplitude of $34^\circ$, and their joint line of apsides precesses at a rate of $41^\circ$ every year. The existence of this secular resonance had also been presented in the works of Laughlin & Chambers (2001), Goździewski et al. (2002), and Lee & Peale (2002) where these authors analyzed the dynamics of GJ 876 and presented a model for the migration and resonance capture of its two giant planets.

The continuous observation of GJ 876 resulted in even more fundamental discoveries. In 2005, Rivera et al. announced that a 6.83 Earth-masses planet exists in a 1.94-day orbit around this star. This discovery that marked the detection of the first super-Earth planet, prompted many astronomers to examine the possibility of the existence of other Earth-like bodies in this system (Ji & Liu, 2006, 2007; Rivera & Haghighipour, 2007). In that direction, Rivera & Haghighipour (2007) studied the dynamics of fictitious planets in the system of GJ 876 and showed that in addition to a small region interior to the orbit of planet c where the super-Earth planet of this system has a stable orbit, a region of stability exists beyond the orbit of the outer giant planet (i.e., planet b) corresponding to an exterior 2:1 resonance with this object. In 2010, the prediction by Rivera & Haghighipour materialized and a new Uranus-mass planet was discovered in a $\sim 125$-day orbit around GJ 876, making this system the first planetary system with three planets in a Laplace resonance.

Prior to the detection of its fourth planet, the three-planet system of GJ 876 was studied by Bean & Seifahrt (2003) and Correia et al. (2013). Bean & Seifahrt studied the architecture of the system and showed that the assumption of co-planarity is
valid, and the mutual inclination of the two giant planets are within 1° to 7°. In a very thorough dynamical analysis, Correia et al. (2010) used the combined data from HARPS and Keck, and while confirming the existence of planet d in a 1.94-day orbit, showed that the orbits of the two giant planets of the system are co-planar with inclinations of approximately $i_b = 48.9°$ and $i_c = 48.1°$. These authors also suggested that an Earth-sized planet in a 15-day orbit at ~0.08 AU could have a stable orbit for a long time. This finding was very interesting since the existence of such an additional planet would help to explain the anomalous high eccentricity of planet d. We would like to mention that in a recent article, Baluev (2011) re-analyzed the HARPS data and suggested that the non-zero eccentricity of planet d may be due to the lack of proper interpretation for red-noise in the data.

With their new four-planet system, Rivera et al. (2010) examined the possibility of a stable solution for the planet proposed by Correia et al. (2010). However, they were unable to find a signal corresponding to the 15-day planet in their radial velocity observations. Placing test particles in the region around 0.08 AU and integrating the entire system numerically, these authors found that only a small fraction of particles remained stable for the 10 Myr duration of the integration.

Given the resonant state of the three planets of the system, and that these planets have most probably captured each other in resonance while migrating inwards from outer regions, and also given the fact that GJ 876 is an M star and planet formation around M stars favors the formation of low-mass objects, it would be natural to examine whether GJ 876 can host additional (low-mass) planets, in particular an Earth-like object in a 15-day orbit. This paper addresses this and several other questions regarding the dynamics of the planetary system of GJ 876.

In the rest of this paper, we present our methodology and the results of our extensive numerical analysis of the dynamics of the system. We are particularly interested in the dynamical surrounding of the fitted orbit of planet e as given in Table 3 of Rivera et al. (2010), and the possibility of additional planets in the system including the Earth-sized planet proposed by Correia et al. (2010). We discuss our numerical method in section 2 and present the results in section 3. Section 4 summarizes our analysis where we also discuss their implications for the formation of this planetary system.

2 Numerical tools - Stability maps from variational equations

As mentioned above, the goal of our study is to examine the possibility of the existence of additional bodies in the four-planet system of GJ 876. Our approach is to use stability analysis to identify regions where an object can have a long-term stable orbit. Since there is no analytical solution to a general N-body problem with $N > 2$, the orbital evolution of the planetary system of GJ 876 has to be computed numerically. As explained below, we will use a symplectic integrator for this purpose. Due to their reliability and stability properties, especially for long-term integrations, these integrators have found a special place in celestial mechanics and planetary dynamics. We refer the reader to Hairer et al. (2002) for a general overview of symplectic integrators and to, for instance, Gladman et al. (1991), and Eggl & Dvorak (2010) for applications of these integrators to celestial mechanics.

To study the dynamical stability of an object, one has to identify (and exclude) conditions that result in chaotic motion of the body. Several methods exist for this purpose that discriminate between regular and chaotic motions. One can either analyze...
the orbit of a particular object [using e.g., the frequency analysis technique (Laskar, 1993)], or study the evolution of deviation vectors \( \delta \) from a given orbit. Examples of the latter include the maximal Lyapunov Characteristic Exponent \( \gamma \) [for a review, see e.g. Skokos (2010)], the Fast Lyapunov Indicator technique (FLI) (Froeschlé et al., 1997), the Smaller Alignment Index (SALI) method (Skokos, 2001) and its generalization GALI (Skokos et al., 2007), and the MEGNO (Mean Exponential Growth of Nearby Orbits) chaos indicator (Cincotta & Simó, 2000).

In this paper we concentrate on MEGNO as a fast and reliable method to study the dynamics of single orbits (Maffione et al., 2011). For this chaos indicator, the variation of a quantity \( Y \) and its mean value \( \bar{Y} \) during time \( t \) are given by

\[
Y(t) = \frac{2}{t} \int_0^t \delta \cdot \dot{\delta} \, ds \quad \text{and} \quad \bar{Y}(t) = \frac{1}{t} \int_0^t Y(s) \, ds.
\]

(1)

The quantity \( \dot{\delta} \) in equations (1) denotes the derivative of the deviation vector with respect to the independent variable \( s \). To determine the stability of an orbit, one has to analyze the time evolution of \( \bar{Y} \), which is connected to the maximal Lyapunov Characteristic Exponent of the orbit. For stable, periodic orbits, \( \bar{Y} \) approaches 0 asymptotically, while for quasi-periodic ones it will tend to 2. For chaotic initial conditions, \( \bar{Y} \) will grow with time as \( \gamma t/2 \). The Lyapunov time, defined as \( T_L = \gamma^{-1} \), can therefore be obtained from \( \bar{Y} \) using

\[
T_L = \frac{t}{2\bar{Y}}.
\]

(2)

In this equation, \( t \) is the total integration time.

The above-mentioned variational equations have to be integrated together with the equations of motion of the system. To increase the speed of these computations, we used a symplectic integrator, and integrated the entire set of equations of the system simultaneously. Skokos & Gerlach (2010) discuss different methods of employing these integrators for advancing the variational equations of a Hamiltonian system. The most efficient of these methods, the so called Tangent Map (TM) technique, was applied by Skokos & Gerlach (2010) and Gerlach & Skokos (2011) mainly to low-dimensional Hamiltonian systems with 2 and 3 degrees of freedom. Gerlach et al. (2011) have shown that this method can also be applied to multi-dimensional Hamiltonian systems and is very efficient and superior to other commonly used numerical schemes, both in accuracy and speed. We note that the TM method was first discussed in the context of celestial mechanics by Mikkola & Innanen (1999).

In this paper, we explore the dynamical stability of hypothetical planets in the system of GJ 876 for a wide range of their orbital elements. To make this task computationally feasible, we use the SABA/ SBAB integrators developed by Laskar & Robutel (2001), which proved to be efficient and reliable. These symplectic splitting integrators have been designed specifically for integrating Hamiltonian systems of the form \( H = A + \epsilon B \), where \( A \) and \( B \) are separately integrable, and \( \epsilon \ll 1 \), as in the case of hierarchical N-body systems. For more details on this technique and on different methods of splitting the Hamiltonian into two integrable parts, we refer the reader to e.g. Duncan et al. (1998), Chambers (1999), Goździewski et al. (2008) and references therein.

As mentioned above, we used the TM method to compute the variational equations.

The formulas for advancing variational equations using a symplectic integrator can be found in e.g. Mikkola & Innanen (1999) and Goździewski et al. (2008). We used
these formalisms, specifically the latter, where also a time-discrete approximation of equations (1) is given. Let us finally note that symplectic methods cannot be used with a trivial automated step-size control. Therefore, they are usually implemented with a fixed integration step, which is denoted by $\tau$ in this paper.

3 Stability analysis

In this section, we present the results of our stability analysis for the GJ 876 planetary system. We used the SABA$_4$ symplectic integrator [Laskar & Robutel, 2001] to integrate the equations of motion and the variational equations of the system. The latter are computed only for the test particles, which we define in this study to be always massless and used to determine the stability in the specific regions. Note that all orbital elements are given with respect to the central body and integrations were carried out incorporating only Newtonian gravity. No relativistic and tidal effects were included.

3.1 Global stability in the inner three-planet system of GJ 876

One of the exciting results of the dynamical analysis of GJ 876 by Correia et al. (2010) is the suggestion that a stable region exists for an Earth-sized planet at $\sim 0.083$ AU. This island of stability that corresponds to an orbital period of 15 days, would allow a terrestrial planet to be in a three-body 1:2:4 resonance with the two giant planets of the system. The interesting fact is that if this 15-day planet exists, it may be possible to use the interaction between this object and the innermost planet of the system (the 6.5 Earth-masses super-Earth that orbits the central star in a 1.94-day orbit) as a way to account for the relatively high eccentricity of the latter body.

Despite the predictions of Correia et al. (2010), radial velocity observations have not been able to detect a planet in the 15-day orbit. Instead, observations by Rivera et al. (2010) revealed that an additional planet does exist in the system, but it is Uranus-mass and in an orbit with a period of approximately 125 days. This body is currently the outermost planet of GJ 876 system, and as shown by Rivera et al. (2010), forms a three-body Laplace resonance with the two inner giant planets.

The purpose of our study is to determine how the discovery of this new planet affects the stability of the 15-day orbit. In other words, we would like to examine whether an Earth-sized planet can maintain a stable orbit at 0.083 AU in this new four-planet configuration? A few simulations by Rivera et al. (2010) suggested that stable motion may be possible around $\sim 0.083$ AU. However, those simulations had been carried out only for short integration times. Our goal is to study also the long-term stability of orbits in this region.

As we mentioned in section 2, our approach is numerical. In order to verify that our numerical method produces reliable results, we first used our code and integrated planets b, c and d together with a large battery of test particles in the region interior to the orbit of the 30-day giant planet. We carried out two sets of simulations: one with the orbital elements for the planets as given in Table 2 of Correia et al. (2010) (hereafter model I), and one with the data from Rivera et al. (2010, Table 2) (hereafter model II). The integrations were done for $t = 500$ years using a time-step of $\tau = 0.05$. 
Fig. 1 Stability graphs of test particles in the three-planet system of GJ 876. The color-coding represents the mean value of a particle’s MEGNO. Red corresponds to chaotic motions and black/blue denote stable regions. The integrations were carried out for 500 years using a step-size of $\tau = 0.05$ days. The initial orbital elements of the planets are given in Table 1. Graph (a) shows the results for model I and graph (b) depicts those of model II. A grid of 600 x 100 test particles was used, for which the initial inclinations were set to $i = 50^\circ$ in the simulations of graph (a) and they were considered to be co-planar with the known planets of the system, i.e. $i = 59^\circ$, in the simulations of graph (b). All other orbital angles were initially set to zero. The white lines in both graphs mark the boundaries of orbit-crossing with the 30-day and 1.94-day planets of the system.

Table 1 Orbital parameters for the different models used in sections 3.1 and 3.2.

| source | model I | model II |
|--------|---------|----------|
| $M_\text{source}$ [$M_\odot$] | | |
| Correia et al. (2010, Table 2) | | Rivera et al. (2010, Table 2) |
| planets | | |
| $a$ [AU] | 0.211 | 0.208 |
| $e$ | 0.029 | 0.029 |
| $i$ [$^\circ$] | 48.93 | 48.07 |
| $\Omega$ [$^\circ$] | 275.52 | 275.26 |
| $\omega$ [$^\circ$] | 35.61 | 50.70 |
| $M$ [$M_\text{Jup}$] | 2.64 | 2.276 |
taken from [Correia et al. (2010)], the stability does not include orbits with eccentricities close to zero. In Fig. 1(b), however, we find that the most stable test particles in the region of 0.08 AU are the ones in circular orbits. The apparent instability for zero-eccentricity orbits in Fig. 1(a) could be attributed to the differences between the orbital elements used in the two models. For instance, while in model I a full three-dimensional solution for the planets of the system is used, in model II, it is assumed that all planets are on the same plane. These differences could cause slight variations in their regions of stability.

We recall that the purpose of carrying out the above-mentioned simulations and generating Fig. 1(a) was to test the reliability of our numerical method. A comparison between this figure and Fig. 10 of Correia et al. (2010) indicates that our integrator is in fact reliable as it has been able to correctly reproduce the main features (i.e., the width and location of the island of stability at 0.08 AU) of the figure published by these authors. The slight differences between the two results, such as the stochastic pattern at \( a \approx 0.05 \) AU, are primarily due to the intrinsic chaotic characteristic of the problem and the differences in the used integrating schemes. As shown by Gerlach (2008), already existing small differences, as those resulting from the use of different chaos indication methods, or different numerical schemes, etc., could lead to very different stability results in areas where the phase-space is highly structured.

3.2 Orbital stability in the outer three-planet system of GJ 876

As indicated by Rivera et al. (2010), the orbit of the recently detected Uranus-mass planet of the system is stable for at least 400 Myr. Test particle simulations led these authors conclude that in order for this planet to maintain orbital stability, its orbit has to be in a Laplacian resonance with the two giant planets of the system where its period will be in a 4:2:1 ratio with the orbital periods of these bodies.

To verify the resonant state of the new planet (hereafter, planet e), and to quantify the size of its stable region, we integrated together with the massive planets of model II, the orbits of a large number of test particles in the region exterior to the orbit of planet b (orbital period of \( \sim 61 \) days) for different values of their semimajor axes (\( a \)) and mean longitudes (\( \lambda \)). We considered the test particles to be initially in circular orbits, and co-planar with the 3 planets (i.e., \( i = 59^\circ \)). We set all other orbital angular elements equal to zero. We integrate the system for \( 10^5 \) orbital periods of the proposed planet e (\( \approx 3400 \) years) using a step-size of \( \tau = 0.1 \) days.

Figure 2 shows the results. The \((a, \lambda)\) stability map in this figure clearly shows the boundary between regular and chaotic orbits. Using equation (2), one can estimate that the stable test particles in this figure have Lyapunov times of \( T_L > 340 \) years. An interesting feature of Fig. 2 is the small island of stability around \( a = 0.33 \) AU where planet e exists. The value of \( \lambda \) for this region is \( \sim 220^\circ \) which places it in a Laplace resonance with planets b and c. Given the small size of this area, it is clear that this resonance protects planet e against close encounters with the other planets of the system.

Although in their analysis, Correia et al. (2010) did not discuss the possibility of an island of stability at 0.33 AU, one can see from their Fig. 10 that the simulations by these authors also show a small region of stability around that area. The values of orbital eccentricity corresponding to this stable region is close to zero which agrees with the orbital eccentricity of planet e (0.055) as reported by Rivera et al. (2010).
Fig. 2 Graph of the orbital stability of test particles exterior to the orbit of planet b. Simulations were carried out for different values of the particles’ semimajor axes and mean longitudes. The color-coding corresponds to the particles’ mean MEGNO as described in Fig. 1. Integrations were done for ∼ 3400 years using a step-size of τ = 0.1 days and a grid of 400 x 150 test particles. The initial orbital inclinations of test particles were set to i = 59°, i.e., co-planar with the orbits of the three planets. However, all other orbital angular elements were initially set to zero. Results show a small stable region at λ ∼ 220° in a Laplace resonance with planets b and c, and in the nominal position of planet e (white cross) as given in Table 3 of Rivera et al. (2010).

This agreement between our results and the results of Correia et al. (2010) could serve as another confirmation of the validity of our integration method and the reliability of our results.

3.3 Could an additional small planet exist in the GJ 876 four-planet system?

As shown by our stability analysis of test particles in the three-planet system of GJ 876 (Fig. 1), an island of stability seems to exist in and around 0.083 AU, even when we use the orbital elements given by Rivera et al. (2010). A planet in this region will be in or near a 2:1 MMR with planet c, and as a result, will be protected against close encounters with other bodies. Given the orbital distance of planet e from this stable region, and the fact that the orbit of this object is in a mean-motion resonance with planets b and c, it would seem natural to assume that the addition of planet e to the system would not alter the stability of a planet at 0.083 AU. To examine this assumption, we carried out similar simulations as in section 3.1 but with planet e included. We used the planets’ orbital elements as given in Table 3 of Rivera et al. (2010) (hereafter called model III, see Table 2), and calculated the mean value of MEGNO for a suite of test particles in the region between 0.02 AU and 0.1 AU.
| source                  | model III                  |
|------------------------|----------------------------|
| $M_\star \ [M_{\odot}]$ | 0.32                      |
| planets                |                            |
| $a \ [\text{AU}]$      | 0.210 0.130 0.021 0.328    |
| $e$                    | 0.037 0.256 0.207 0.045    |
| $i \ [^\circ]$        | 59 59 59 59               |
| $\Omega \ [^\circ]$   | 0 0 0 0                  |
| $\omega \ [^\circ]$   | 43.27 48.74 234.07 251.36 |
| $\lambda \ [^\circ]$  | 15.88 343.33 229.38 214.06|
| $M \ [M_{\text{Jup}}]$| 2.276 0.714 0.022 0.046   |

Table 2 Orbital parameters for the model used in section 3.3.

Figure 3 shows the results. From this figure, it can be seen that the stable island at 0.083 AU still exists. However, its size on the eccentricity and semimajor axes has become smaller. This figure also shows that the region corresponding to the most stable orbits (the darkest area) is now exclusively for test particles in circular motion. Figure 3(b) shows that the stability also crucially depends on the initial value of the mean longitude, which should be zero in order to place the body inside the 2:1 MMR with planet c.

Although our calculations of MEGNO suggest stability at 0.083 AU, it is important to emphasize that this result has been obtained only from 500 years of simulations. This time span is sufficient to identify unstable regions at reasonable CPU costs, especially when the grids in semimajor axis, eccentricity, and other orbital elements are very dense. However, it is not long enough to allow us to make conclusions about the long-term stability of the identified stable regions. For the latter, integrations have to be carried out for longer time spans.

Fig. 3 Graphs of the orbital stability of test particles in the four-planet system of GJ 876. Simulations were carried out for different values of the semimajor axis, eccentricity, and mean longitude of test particles on a grid of 600 x 100. The color-coding represent the mean value of MEGNO as described in Fig. 1. Integrations were carried out for 500 years using a step-size of $\tau = 0.05$ days. The initial orbital elements of the planets are given in Table 2. All test particles were placed in co-planar orbits with planets, i.e. $i = 59^\circ$. In graph (a), the initial values of other angular variables of test particles were also set to zero. However, in graph (b), the mean anomalies of the test particles were kept at non-zero values, but instead, their initial eccentricities were set to zero. In (a) the white lines mark the boundary, above which collisions with planets b and d (crosses) are possible.
carried out for longer times. In that respect, Fig. 3 can be used to make the search for long-term stability very efficient by allowing to focus only on the areas of the orbital parameter-space where an object may have a chance to be stable. For the four-planet system of GJ 876, this area is the 15-day orbit. We, therefore, considered a test particle with orbital elements corresponding to the most stable zone around 0.083 AU (the darkest spot in Fig. 3), and integrated its orbit along with all other planets of the system, for longer times. The results are shown in Fig. 4 by red color. As can be seen in this figure, the orbit of the test particle was stable for only $9 \times 10^4$ years. During this time, the object was trapped in a 2:1 mean-motion resonance with planet c and its corresponding resonant angle, $\theta = -\lambda + 2\lambda_c - \omega_c$, was librating around 310°. The difference between the longitude of the periastron of the particle ($\omega$) and that of planet c ($\omega_c$) at that state was $\Delta \omega = \omega - \omega_c \sim 250°$.

Figure 4 also shows that during the course of integration, strong perturbations from planet c caused the eccentricity of the test particle to rapidly increase. At this state, the variations in $\Delta \omega$ (and $\theta$) developed large amplitudes until shortly before the ejection of the particle from the system when $\Delta \omega$ and $\theta$ began to circulate. The latter indicates that the orbit of the particle was no longer stabilized by the 2:1 MMR.

As mentioned before, the main purpose for carrying out these simulations was to determine the extent to which the stability of an object in the 15-day orbit would be affected by the newly discovered planet of the system. In that respect, and in order to be able to make a comparison with the stability analysis of the system prior to the detection of planet e, it is necessary to carry out similar N-body integrations for the

![Fig. 4](image-url)

**Fig. 4** Results of the long-term integration of the orbit of a test particle from the most stable part of the island of stability in Fig. 3. The semimajor axis of the particle is initially $a = 0.081420$ AU. Its other orbital elements were initially set to zero. The initial orbital elements of the four known planets of the system were taken from model III (Table 2). The integration was carried out using a step-size of $\tau = 0.05$ days. The graph on the left shows the orbits of the planets and that of the test particle (shown in black) in the $x-y$ plane. The graph on the right shows the time evolution of semimajor axis $a$, eccentricity $e$, difference in periastron longitudes $\Delta \omega = \omega - \omega_c$, and resonance angle $\theta = -\lambda + 2\lambda_c - \omega_c$ of the test particle. Shown in black color in the background is the orbital evolution for a test particle with the same initial conditions, but integrated in a system with only 3 planets as in the right panel of Fig. 4.
15-day orbit test particle with only planets b, c, and d included. We used the orbital elements of these planets as in model II and integrated the orbit of the same test particle as in Fig. 3 for 1 Myr. These results are presented as black dots in the right panel of Fig. 4. We found that for this configuration, the orbit of the test particle was stable for the entire duration of the integration. The latter indicates that the addition of planet e had a profound effect on the stability of this object. Given that the orbit of our test particle was the least chaotic one in the area around 0.083 AU, this result further suggests that the region between planets b, c, and e in the system of GJ 876 is naturally unstable. In other words, the system of GJ 876 seems to be dynamically full.

3.4 Stability outside planet e

We also examined the stability of an object exterior to the orbit of planet e. We used the orbital elements of the planets from Table 2 (model III) and carried out similar simulations for test particles in the region of $0.2 \leq a \leq 2.0$ AU. The results are given in Fig. 5. As shown there, a large stable region exists outside the orbit of planet e where the system can host additional planets. The inner edge of this region is determined mainly by interaction with planet e. The fine structures in this region, such as the gaps and islands of stability, are due to various mean-motion resonances. For instance, the large gap at 0.53 AU corresponds to a 4:1 mean motion resonance with planet b. Figure 5 also shows that as the semimajor axis of an object increases, stability extends to orbits with larger eccentricities. However, the upper limit of the eccentricity is set by the close approach of the object to planet e.

An interesting feature of Fig. 5 is the small islands of stability that appear at high eccentricities outside the gaps of instability that are due to mean-motion resonances. Whether these islands of stability are long-term stable is a topic that requires N-body integrations of an actual object in those locations. Such a study is, however, beyond the scope of this paper.

4 Summary and Conclusions

We presented the results of a detailed study of the stability of the planetary system of GJ 876. Our goal was to determine whether an Earth-sized planet could maintain a stable orbit in a four-body resonance configuration with the currently known planets of the system. We computed the value of the chaos indicator MEGNO for a large number of massless test particles to map the system’s orbital parameter-space, and identified regions where the orbit of an additional object could be stable. Results suggested an island of stability in and around the 15-day orbit. However, direct integration of test particles showed that even orbits that were initially very stable in that region became unstable in later times.

We would like to emphasize that although when comparing to Jovian-type planets, a test particle is a good approximation for a terrestrial-class object, the dynamical state of a test particle may not be a true representative of the dynamics of an actual Earth-sized body. The mutual interactions between this object and other bodies in the system play an important role in its dynamical state and may result in a stable orbital configuration in systems where test particle simulations indicate instability. In the case of GJ 876, however, we believe that the possibility of the existence of such an
Earth-sized planet is very small. Given the current masses of the three giant planets of the system and their orbital configuration, in order for a fifth planet in the 15-day orbit to be stable, the mass of this object has to be larger than Earth-mass so that its mutual interactions with other planets would become significant to allow its (in)stability. Given the long history of the observation of GJ 876 (over 12 years), such a planet is expected to have been discovered by now. Results of our study suggest that the planetary system of GJ 876 is most likely dynamically full.

It is also worth noting that from Hill radius calculations, one may argue that an additional, not too massive planet with a small eccentricity can maintain stability in the 15-day orbit. Although this argument has been shown to be valid in many multiple planet system, our numerical integrations indicate that in the system of GJ 876, Hill radius analysis may not be a good approach for identifying stability. The latter implies that this widely used technique has to be used with caution as it may not yield correct results at all times.

The orbital architecture of the planets around GJ 876, with a super-Earth in a close-in orbit and three planets in a Laplace resonance, combined with the fact that this system is dynamically full, raises an important and interesting question: How did this planetary system form? Given that GJ 876 is an M star, it is very unlikely that its outer three planets were formed in their current orbits. In fact, as shown by [Laughlin et al. (2004)], the core-accretion model of giant planet formation fails to produce Jovian-type planets around M stars. The fact that many M stars host giant planets suggests that
these planets must have formed in outer regions of the circumstellar disk and migrated to their current orbits. As shown by Lee & Peale (2002) and Kley et al. (2003), migrating planets may capture other bodies in their paths into mean-motion resonances (mainly the 1:2 resonance) where the two (or multiple) resonant bodies continue their migration until they either collide with the central star or reside in a stable orbit. For a more detailed and comprehensive review on the formation, migration, and resonance trapping of giant planets around M stars, we refer the reader to Haghighipour (2011).

While migrating inward, the resonant planets excite the orbits of smaller bodies causing many of them to be scattered out of the system. Some of these bodies may also collide with one another and form larger objects (e.g., super-Earths). The interactions between these objects and the migrating planets may result in their scattering to stable close-in orbits.

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