FLOW CHARACTERISTICS OF MHD OSCILLATORY TWO-PHASE BLOOD FLOW THROUGH A STENOSED ARTERY WITH HEAT AND MASS TRANSFER

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ABSTRACT

In blood, the concentration of red blood cells varies with the arterial diameter. In the case of narrow arteries, red blood cells concentrate around the center of the artery and there exists a cell-free plasma layer near the arterial wall due to Fahraeus-Lindqvist effect. Due to non-uniformity of the fluid in the narrow arteries, it is preferable to consider the two-phase model of the blood flow. The present article analyzes the heat and mass transfer effects on the two-phase model of the unsteady pulsatile blood flow when it flows through the stenosed artery under the effects of radiation and chemical reaction. The direction of the artery is assumed to be vertical and the magnetic field is applied along the radial direction of the artery. We assume that the value of the shear stress is high enough so that nature of blood can be modeled as Newtonian in both erythrocytes suspended core region as well as RBC-depleted plasma region. We derive a mathematical model for the mixed convection problem of two-phase blood flow as nonlinear partial differential equations and get the exact solutions for the velocity, temperature and concentration profiles. Further, a comparative study is carried out between the single-phase and two-phase model of the blood flow, and it is observed that the two-phase model fits the experimental data more accurately than the single phase model. Subsequently, to measure two-phase blood flow behavior under the effects of applied magnetic field and thermal radiation, we demonstrate the graphs of wall shear stress, impedance, and total flow rate under the effect of applied magnetic field and thermal radiation via simulations.

KEYWORDS:
Two-phase flow, stenosis, magnetohydrodynamical fluid (MHD), chemical reaction.

1. INTRODUCTION

To estimate the hemodynamic resistance and analyze the heat and mass transfer process in arterioles and venules a quantitative comprehension of the blood flow is necessary. A major characteristic of the blood depends upon the hematocrit level as it is the percentage of whole blood occupies by the red blood cells(RBC) [1]. In the artery, hematocrit level of the blood flow depends upon the diameter of the artery, and this relationship has important implications for physiological phenomena related to blood flow. Fahraeus effect explains this relationship which states that as the value of arterial diameter decreases, hematocrit level present in the
artery also decreases. Due to the Fahraeus effect in arteries having the diameter less than 500 \( \mu m \), erythrocytes move towards the center of the artery and thus forming the RBC depleted plasma layer near the wall \([2–4]\). In smaller arteries, due to the higher concentration of red blood cell near the center and existence of cell-free plasma layer near the arterial wall, blood flow is considered as two-phase fluid flow \([5]\).

Cokelet and Goldsmith \([6]\) through \emph{In vitro} analysis documented the phenomenon in more fully tube of 172-\( \mu m \) diameter and found that the two-phase flow of a suspension may lead to a decrease in hydrodynamic resistance. They suggested that in vertical tubes at very low shear stress, decreased hydrodynamic resistance depends upon the aggregation of red blood cells and magnitude of this effect increases as the degree of the aggregation cells increases. They also identified that in the core region with the effect of gravity hydrodynamic resistance is greater in upward than the downward flowing suspension. The importance of the two-phase flow come to light only with decreasing size of the vessel however for single phase flow the flow characteristic is independent of the vessel size. Further, assuming different viscosity of core region than the RBC depleted plasma region Saran and Popel \([7]\) presented a two-phase model of blood flow in narrow arteries. They solved the model by applying discrete model approach for multiple rigid particles considering single file movement of the red blood cell through the artery. Through their work, they found that due to the dissipation of energy the effective viscosity of the plasma region in a tube of fixed diameter increases as the level of the hematocrit increases. Above mentioned research basically describe the two-phase blood flow under the normal arterial condition, what will happen if in artery some hemodynamical perturbations induced.

In the human body cardiovascular system mainly functions in transport of nutrients and waste product from one body part to other. To supply proper oxygen-rich blood to all the tissues through arteries an adequate blood circulation is necessary \([8]\). Any type of intrusion of fat into the arterial wall, block the way of blood flow. Hard plaque formation in the artery is known as stenosis which causes a well-known disease in the human body named as atherosclerosis \([9]\), a major cause of heart attacks. To analyze the two-phase blood flow behavior under these hemodynamical disturbances Ponalagusamy \([10]\) stabilized the two fluid model for blood flow through a tapered stenotic artery considering core region as couple stress fluid and a peripheral region of plasma as a Newtonian fluid. The author reported that wall shear stress is high in the case of converging tapered stenosis and it is low in the case of non-tapered diverging stenosis. Further, for the case of symmetric and axisymmetric stenosis Sankar \([11]\) proposed a mathematical model for two-phase blood flow and investigated that presence of RBC-depleted peripheral layer near the wall helps in the functioning of the diseased arterial system. Further considering the extra effect of magnetic field many authors explored the two-phase blood flow modeling of the stenosed artery. In the artery to recognize the existence of the atherosclerosis disease, the presence of which alters the velocity field a non-invasive technique based on MRI devices is used \([12]\). These MRI devices use the strong magnetic field to work and when it performs over the particular area of our body it affects the velocity field presence in the area. Now it has gained serious attention by researches to study the blood flow through the stenosed artery under the influence of applied magnetic field.

Magnetohydrodynamics (MHD) is about to study the motion of highly conducting fluid under the influence of magnetic field. Blood is an electrically conducting fluid because the erythrocytes contain iron oxide molecules in its content. When magnetic field applies on blood flow it induces electric as well as magnetic fields, interactions of which generates a mechanical force on the body known as Lorentz force \([13–15]\). In their founding, many researchers have
been shown the effects of magnetization on the arterial vessel by considering the two-phase fluid model of the blood flow. In the presence of an arterial stenosis Ponalamasamy and Selvi [16] examined the effect of magnetic field on the two-phase model of oscillatory blood flow assuming both core and plasma regions as a Newtonian fluid. They solved the model for both core and plasma regions separately using boundary conditions at the arterial wall as well as for interface region. The authors reported that as the value of magnetic field increases, the flow resistance of the blood flow in the stenosed artery also increases. Further Ponalamasamy and Priyadharshini [17] examined the effect of magnetic field on two-phase blood flow through a tapered stenosed artery, assuming micropolar fluid in the core region and Newtonian fluid in the plasma region. A mathematical model presented by Mirza et al. [18] analyzes the effect of magnetic field on a transient laminar electromagneto-hydrodynamic two-phase blood flow using continuum approach. Solving the model analytically authors graphically displayed the effect of the magnetic field for both blood velocity and particles velocity separately and concluded that as the effects of the magnetic field increases, both blood and particles velocities decrease for electromagneto-hydrodynamic two-phase blood flow.

Above mentioned studies were focused on analyzing momentum and the heat transfer phenomenon for two-phase models of blood flow with artery assuming horizontal from the axis. To show the effect of total movement of the mass from one place to the other in the two-phase model of blood flow our study include mass transfer as an important part of the investigation, which has not been done earlier. The present article analyzes the effects of radiation, chemical reaction and the external applied magnetic field on the two-phase model of blood flow considering mild stenosis in the artery. We calculate and get the exact solutions for the velocity, temperature and concentration profiles of both core and plasma regions and plot their graphs against the radial distance for different values of parameters having used in the problem. With the aim of having adequate insight into the two-phase flow behavior of blood flow through a stenosed arterial segment flow resistance, total flow rate and wall shear stress have been estimated their respective graphs have been plotted with varying values of applied magnetic field, the ratio of the viscosity and radiation. To show the effectiveness of the two-phase model of blood flow a comparison result has been plotted with the experimental data which shows two phase model of blood flow fits more appropriately with the experimental data as compared to single phase data.

2. The Mathematical Model

Consider the continuum model of unsteady, incompressible, oscillatory two-phase blood flow through a vertically stenosed coronary artery of length $L$ in the presence of applies magnetic field $M$ as shown in Fig. 1. In the artery of radius $r$, the two-phase model of blood consists a core region of radius $r_c$ which contains erythrocytes a suspension of the uniform hematocrit of viscosity $\mu_c$ and the RBC-depleted plasma layer of radius $r_p$ having viscosity $\mu_p$. Artery is assumed as cylindrically shaped as $(\vec{u}_c, \vec{v}_c, \vec{w}_c)$ are the velocity vectors for core region and $(\vec{u}_p, \vec{v}_p, \vec{w}_p)$ represent the velocity vectors for plasma region in $(\vec{r}, \vec{\theta}, \vec{z})$ directions. Shear rates are assumed as high enough so that for both the regions blood is treated as Newtonian fluid [7]. The temperature of the outer wall of the artery is maintained as $T_w$, which is high enough to induce radiative heat transfer and concentration of the blood particles near the wall is assumed as $C_w$. To analyze the two-phase model of blood flow viscosity for core and plasma regions are
defined separately as

\[
\bar{\mu}(\bar{r}) = \begin{cases} \\
\bar{\mu}_c & \text{for} \quad 0 \leq \bar{r} \leq \bar{r}_c(\bar{z}) \\
\bar{\mu}_p & \text{for} \quad \bar{r}_c(\bar{z}) \leq \bar{r}_p(\bar{z}).
\end{cases}
\]

Geometry of the stenosis in plasma region, which is assumed to be symmetric is defined as [19],

\[
\bar{R}(\bar{z}) = \begin{cases} \\
1 - \eta^* (\bar{L}_0^{n-1}(\bar{z} - \bar{d}) - (\bar{z} - \bar{d})^n) & \text{for} \quad \bar{d} \leq \bar{z} \leq \bar{d} + \bar{L}_0, \\
1 & \text{otherwise,}
\end{cases}
\]

In core region geometry of the stenosis is defined as [20],

\[
\frac{\bar{R}_1(\bar{z})}{\bar{R}_0} = \begin{cases} \\
\beta - \eta^* (\bar{L}_0^{n-1}(\bar{z} - \bar{d}) - (\bar{z} - \bar{d})^n) & \text{for} \quad \bar{d} \leq \bar{z} \leq \bar{d} + \bar{L}_0, \\
\beta & \text{otherwise},
\end{cases}
\]

where \(\bar{L}_0\) is the length of the stenosis as shown in Figure 2 and \(\beta\) is the ratio of the central core radius to the normal artery radius, \(\beta \bar{R}_0\) is the radius of the core region of the normal artery. In which \(\eta^*\) is expressed as

\[
\eta^* = \frac{\delta_s n^{\frac{n}{n-1}}}{\bar{R}_0 \bar{L}_0^{\frac{n}{n-1}} (n - 1)}
\]

where \(\bar{R}_0\) is the radius of the artery and \(n\) determines the shape of the constriction profile [21] as symmetric stenosis occurs for value \(n=2\), \(\delta_s\) indicates the maximum height of the stenosis located at

\[
\bar{z} = \bar{d} + \frac{\bar{L}_0}{n(\frac{n}{n-1})}.
\]
So, for symmetric case maximum height of the stenosis occurs at mid point of the stenotic region 
\[ z = \bar{d} + \frac{L_0}{2}. \]

In mentioned two-phase flow it is assumed that flow in both the regions are in the axial direction while an applied magnetic field is working just perpendicular to the flow direction. Under these assumptions equations for momentum, heat and mass transfer for core region is as follows

\[
\frac{\partial \bar{u}_c}{\partial t} = -\frac{\partial \bar{p}}{\partial z} + \bar{\mu}_c \left( \frac{\partial^2 \bar{u}_c}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{u}_c}{\partial r} \right) - \bar{\sigma} \bar{B}_0^2 \bar{u}_c + \bar{g} \bar{\rho}_c \bar{\beta} (\bar{T}_c - \bar{T}_0) + \bar{g} \bar{\rho}_c \bar{\gamma} (\bar{C}_c - \bar{C}_0)
\]

\[
\bar{\rho}_c \bar{c}_c \frac{\partial \bar{T}_c}{\partial t} = \bar{K}_c \left( \frac{\partial^2 \bar{T}_c}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{T}_c}{\partial r} \right) - \frac{\partial \bar{q}_c}{\partial r},
\]

\[
\frac{\partial \bar{C}_c}{\partial t} = \bar{D}_c \left( \frac{\partial^2 \bar{C}_c}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{C}_c}{\partial r} \right) - \bar{E}_c' (\bar{C}_c - \bar{C}_0),
\]

where \( \bar{B}_0 \) is the magnetic field intensity, \( \bar{\sigma} \) represents the electrical conductivity of the blood and \( \frac{\partial \bar{\rho}_c}{\partial r} \) stands for pressure gradient, and it is assumed that magnitude of these physical quantities are same for both the regions. In core region \( \bar{D}_c \) is the coefficient of mass diffusivity, \( \bar{\rho}_c \) is the density, \( \bar{c}_c \) is the specific heat, \( \bar{K}_c \) is the chemical reaction parameter and \( \bar{\gamma}_c \) represents the thermal conductivity. In flow direction velocity vector is represented by \( \bar{u}_c \), \( \bar{T} \) is the temperature and \( \bar{C} \) denotes the concentration for blood flow in the core region.

Now, the governing equations of momentum, heat and mass transfer for plasma region are given as

\[
\frac{\partial \bar{u}_p}{\partial t} = -\frac{\partial \bar{p}}{\partial z} + \bar{\mu}_p \left( \frac{\partial^2 \bar{u}_p}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{u}_p}{\partial r} \right) - \bar{\sigma} \bar{B}_0^2 \bar{u}_p + \bar{g} \bar{\rho}_p \bar{\beta} (\bar{T}_p - \bar{T}_0) + \bar{g} \bar{\rho}_p \bar{\gamma} (\bar{C}_p - \bar{C}_0)
\]

\[
\bar{\rho}_p \bar{c}_p \frac{\partial \bar{T}_p}{\partial t} = \bar{K}_p \left( \frac{\partial^2 \bar{T}_p}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{T}_p}{\partial r} \right) - \frac{\partial \bar{q}_p}{\partial r},
\]

\[
\frac{\partial \bar{C}_p}{\partial t} = \bar{D}_p \left( \frac{\partial^2 \bar{C}_p}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{C}_p}{\partial r} \right) - \bar{E}_p' (\bar{C}_p - \bar{C}_0)
\]

where \( \bar{D}_p \) is the coefficient of mass diffusivity, \( \bar{\rho}_p \) is the density, \( \bar{c}_p \) is the specific heat, \( \bar{E}_p' \) is the chemical reaction parameter and \( \bar{K}_p \) is the thermal conductivity of the blood for plasma region. The last term in (2.6)-(2.9) is due to the radiation effect of heat transfer where \( \bar{q}_c \) and \( \bar{q}_p \) represent the radiative heat flux in core and plasma regions respectively.

In a unit volume the conservation equation of the radiative heat transfer for all wavelength is described as [22]

\[
\nabla \cdot \bar{q}_c = \int_0^\infty K_\lambda(\bar{T})(4\varepsilon_\lambda \bar{T} - G_\lambda) d\lambda,
\]

where \( \varepsilon_\lambda \) is the Plank’s function, \( G_\lambda \) is the incident radiation and \( \lambda \) represents the injection parameter, defined as [23]

\[
G_\lambda = \frac{1}{\pi} \int_{4\pi} \varepsilon_\lambda(\Omega) d\Omega.
\]
Now, for an optically thin fluid exchanging radiation at temperature $T_0$ and according to the Kirchhoff’s law, definition of the conserved radiative heat transfer from (2.11), incident radiation is given as $G_\lambda = 4\varepsilon_\lambda(T_0)$

\begin{equation}
\lambda = \frac{u_0 R_0}{\mu}
\end{equation}

(2.12)

\[ \nabla . \vec{q}_c = 4 \int_0^\infty K_\lambda (T) (\varepsilon_\lambda(T) - \varepsilon_\lambda(T_0)) \, d\lambda. \]

Now, by using Taylor series expansion formula for $K_\lambda(T)$ and $\varepsilon_\lambda(T_0)$, the expression of (2.12) can be written as

\begin{equation}
\nabla . \vec{q}_c = 4(\bar{T} - \bar{T}_0) \int_0^\infty K_{\lambda 0} \left( \frac{\partial \varepsilon_{\lambda 0}}{\partial T} \right)_0 d\lambda,
\end{equation}

where $K_{\lambda 0} = K_\lambda(T_0)$. Now by assuming $\alpha_c^2 = \int_0^\infty K_{\lambda 0} \left( \frac{\partial \varepsilon_{\lambda 0}}{\partial T} \right)_0 d\lambda$ the (2.13) becomes

\[ \nabla . \vec{q}_c = 4(\bar{T} - \bar{T}_0) \alpha_c^2, \]

Hence, the radiative heat fluxes for core and plasma regions can be expressed respectively as

\begin{equation}
\frac{\partial \vec{q}_c}{\partial F} = 4\alpha_c^2 (\bar{T}_c - \bar{T}_0), \quad \frac{\partial \vec{q}_p}{\partial F} = 4\alpha_p^2 (\bar{T}_p - \bar{T}_0)
\end{equation}

where $\vec{q}_c$ and $\vec{q}_p$ represent the radiative heat transfer coefficients and $\alpha_c$ and $\alpha_p$ are the mean radiation absorption coefficients for core and plasma regions respectively. Values of $\alpha_c$ and $\alpha_p$ consider here are less than unity ($\bar{\alpha} \ll 1$) because it is observed that, fluid like plasma and blood in the physiological conditions are optically thin with low density [24].

To solve the momentum, energy and concentration equations of the two-phase model of the blood flow no-slip boundary conditions are considered on the arterial wall. It is assumed that the functions of velocity, temperature, and concentration are continuous at the interface of core and plasma regions so their values of both core and plasma regions are equal at that point and due to symmetry their gradient vanishes along with the axis. It is believed that at the interface of core and plasma regions heat and mass transfer effects are same for both the regions [7]. So, the boundary conditions for the model under consideration are as follows:

\begin{equation}
\begin{cases}
\bar{u}_p = 0, & \bar{T}_p = \bar{T}_w, & \bar{C}_p = \bar{C}_w \quad \text{at} \quad \bar{r} = \bar{R}(\bar{z}) \\
\bar{u}_c = \bar{u}_p, & \bar{T}_c = \bar{T}_c, & \bar{C}_c = \bar{C}_c \quad \text{at} \quad \bar{r} = \bar{R}_1(\bar{z}) \\
\frac{\partial \bar{u}_c}{\partial \bar{r}} = 0, & \frac{\partial \bar{T}_c}{\partial \bar{r}} = 0, & \frac{\partial \bar{C}_c}{\partial \bar{r}} = 0 \quad \text{at} \quad \bar{r} = 0 \\
\bar{r}_c = \bar{r}_p, & \frac{\partial \bar{r}_c}{\partial \bar{r}} = \frac{\partial \bar{r}_p}{\partial \bar{r}}, & \frac{\partial \bar{C}_c}{\partial \bar{r}} = \frac{\partial \bar{C}_p}{\partial \bar{r}} \quad \text{at} \quad \bar{r} = \bar{R}_1(\bar{z})
\end{cases}
\end{equation}

To analyze the model more precisely we convert the model in nondimensional form by using following dimensionless parameters

\[\begin{align*}
u_c &= \frac{\bar{u}_c}{u_0}, & r &= \frac{\bar{r}}{R_0}, & z &= \frac{\bar{z}}{R_0}, & t &= \bar{t}, & \frac{\bar{R}(\bar{z})}{R_0} &= R(Z), & \frac{\bar{R}(\bar{z})}{R_0} &= R_1(Z), \\
u_p &= \frac{\bar{u}_p}{u_0}, & p &= \frac{\bar{p}}{u_0 \mu_0}, & R_e &= \frac{\bar{p}_0 \bar{R}_0}{\mu_p}, & \theta_c &= \frac{(\bar{T}_c - \bar{T}_0)}{\bar{T}_w - \bar{T}_0}, & \theta_p &= \frac{(\bar{T}_p - \bar{T}_0)}{\bar{T}_w - \bar{T}_0}, & \mu_p &= \frac{\bar{\mu}_p}{\mu_p}, & \sigma_c &= \frac{\bar{C}_c - \bar{C}_0}{\bar{C}_w - \bar{C}_0}, & \delta &= \frac{\bar{\delta}}{R_0}, & \sigma_p &= \frac{\bar{C}_p - \bar{C}_0}{\bar{C}_w - \bar{C}_0}, & N^2 &= \frac{4\bar{R}_0^2 \alpha_p^2}{K_p}, & M &= \frac{\bar{\sigma}_p \bar{R}_0^2}{\mu_p}, & \sigma_p &= \frac{\bar{C}_p - \bar{C}_0}{\bar{C}_w - \bar{C}_0}.
\end{align*}\]
\[ P_c = \frac{\bar{\rho}_p \bar{c}_p \bar{R}_0^2 \omega}{\bar{R}_0}, \quad S_c = \frac{\bar{\mu}_p}{\bar{D}_p \bar{\rho}_p}, \quad G_r = \frac{\bar{g} \bar{\rho}_p \beta \bar{R}_0^2}{\bar{u}_0 \bar{\mu}_p} \left( \bar{T}_w - \bar{T}_0 \right), \quad D_0 = \frac{\bar{D}_p}{\bar{D}_c}. \]

\[
\tau_c = \frac{\tau_c \bar{R}_0^2}{\bar{u}_0 \bar{\mu}_p}, \quad \tau_p = \frac{\tau_p \bar{R}_0^2}{\bar{u}_0 \bar{\mu}_p}, \quad \rho_0 = \frac{\bar{\rho}_p}{\bar{\rho}_c}, \quad \mu_0 = \frac{\bar{\mu}_p}{\bar{\mu}_c}, \quad E_p' = \frac{E \bar{\rho}_p}{\bar{\rho}_c \bar{R}_0^2}, \quad E_0' = \frac{E_p}{E_c}.
\]

So, the equations (2.5)-(2.7) of core region in terms of these dimensionless parameters can be written as

\[
(2.16) \quad \left( \frac{R_e}{\rho_0} \right) \frac{\partial u_c}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{\mu_0} \left( \frac{\partial^2 u_c}{\partial r^2} + \frac{1}{r} \frac{\partial u_c}{\partial r} \right) - M^2 u_c + \left( \frac{G_r}{\rho_0} \right) \theta_c + \left( \frac{G_m}{\rho_0} \right) \sigma_c,
\]

\[
(2.17) \quad \frac{P_r \bar{K}_0}{\rho_0 s_0} \left( \frac{\partial \theta_c}{\partial t} \right) = \left( \frac{\partial^2 \theta_c}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_c}{\partial r} \right) - K_0 \frac{\alpha_0}{\alpha_0} N^2 \theta_c,
\]

\[
(2.18) \quad R_e \left( \frac{\partial \sigma_c}{\partial t} \right) = \frac{1}{D_0} \left( \frac{1}{S_c} \right) \left( \frac{\partial^2 \sigma_c}{\partial r^2} + \frac{1}{r} \frac{\partial \sigma_c}{\partial r} \right) - \frac{E}{E_0} \sigma_c.
\]

Equations (2.8)-(2.10) for plasma region in dimensionless form are as follows

\[
(2.19) \quad R_{e_p} \frac{\partial u_p}{\partial t} = -\frac{\partial p}{\partial z} + \left( \frac{\partial^2 u_p}{\partial r^2} + \frac{1}{r} \frac{\partial u_p}{\partial r} \right) - M^2 u_p + G_r \theta_p + G_m \sigma_p
\]

\[
(2.20) \quad P_c \frac{\partial \theta_p}{\partial t} = \left( \frac{\partial^2 \theta_p}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_p}{\partial r} \right) - N^2 \theta_p
\]

\[
(2.21) \quad R_{e_p} \left( \frac{\partial \sigma_p}{\partial t} \right) = \left( \frac{1}{S_c} \right) \left( \frac{\partial^2 \sigma_p}{\partial r^2} + \frac{1}{r} \frac{\partial \sigma_p}{\partial r} \right) - \frac{E}{E_0} \sigma_p
\]

where \( \alpha_0 \) is the ratio of mean radiation absorption coefficient in the plasma region to mean radiation absorption coefficient in the core region, \( K_0 \) is the ratio of thermal conductivity of plasma and core region, \( s_0 \) is the specific heat ratio of plasma and core regions and \( E_0 \) is the ratio of chemical moles present in the plasma region to chemical moles in the core region. In non-dimensional form factors of thermal radiation, applied magnetic field and chemical reaction are represented by \( N \), \( M \) and \( E \) for both core and plasma regions.

Geometry of the stenosis in nondimensional form is defined in which we assume that the length of the stenosis \( L_0 \) is same as radius of the artery \( \bar{R}_0 \),

\[
(2.22) \quad R(z) = \begin{cases} 
1 - \eta ((z - l) - (z - l)^n) & \text{for } l \leq z \leq 1 + l, \\
1 & \text{otherwise,}
\end{cases}
\]

Stenosis in core region in non-dimensional form is as follows

\[
(2.23) \quad R_1(z) = \begin{cases} 
\beta - \eta ((z - l) - (z - l)^n) & \text{for } l \leq z \leq 1 + l, \\
\beta & \text{otherwise,}
\end{cases}
\]

where

\[
\eta = \frac{\delta n^{\frac{n-1}{n}}}{(n - 1)}, \quad l = \frac{\bar{d}}{L_0}, \quad \delta = \frac{\bar{\delta}}{\bar{R}_0}
\]
Corresponding boundary conditions in nondimensional form to solve the model for both core and plasma regions are given as

\[
\begin{align*}
\left\{ \begin{array}{l}
u_p = 0, \quad \theta_p = 1, \quad \sigma_p = 1 \quad \text{at} \quad r = R(z) \\
u_p = u_c, \quad \theta_p = \theta_c, \quad \sigma_p = \sigma_c \quad \text{at} \quad r = R_1(z) \\
\tau_c = \tau_p, \quad \frac{\partial \nu_c}{\partial r} = \frac{\partial u_p}{\partial r}, \quad \frac{\partial \theta_c}{\partial r} = \frac{\partial \theta_p}{\partial r}, \quad \text{at} \quad r = R_1(z) \\
\frac{\partial u_c}{\partial r} = 0, \quad \frac{\partial \theta_c}{\partial r} = 0, \quad \frac{\partial \sigma_c}{\partial r} = 0 \quad \text{at} \quad r = 0.
\end{array} \right.
\]

(2.24)

3. Solution

Since pumping action of heart results in a pulsatile blood flow, so the pressure gradient can be represented as

\[
-\frac{\partial p}{\partial z} = P_0 e^{i\omega t}
\]

In non-dimensional form pressure gradient can be written as

\[
-\frac{\partial p}{\partial z} = P_0 e^{it}
\]

and flow variables for core and plasma regions in nondimensional forms can be represented in terms of \(t\) as

\[
\left\{ \begin{array}{l}
u_c(r, t) = u_{c0}(r)e^{it}, \quad u_p(r, t) = u_{p0}(r)e^{it} \\
\theta_c(r, t) = \theta_{c0}(r)e^{it}, \quad \theta_p(r, t) = \theta_{p0}(r)e^{it}, \\
\sigma_c(r, t) = \sigma_{c0}(r)e^{it}, \quad \sigma_p(r, t) = \sigma_{p0}(r)e^{it}.
\end{array} \right.
\]

(3.1)

Substituting expressions from (3.1) in to (2.16)-(2.18), we get time independent momentum, energy and concentration equations of core region

\[
\left( \frac{\partial^2 u_{c0}}{\partial r^2} + \frac{1}{r} \frac{\partial u_{c0}}{\partial r} \right) - \left( M^2 + \frac{\mu_0 R e}{\rho_0} \right) u_{c0} = - \left( P_0 + \frac{G_r \theta_{c0}}{\rho_0} + \frac{G_m \sigma_{c0}}{\rho_0} \right) \mu_0
\]

(3.2)

\[
\frac{\partial^2 \theta_{c0}}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_{c0}}{\partial r} - \left( \frac{K_0 N^2}{\alpha_0} + i \frac{P_e}{\rho_0} \left( \frac{K_0}{s_0} \right) \right) \theta_{c0} = 0
\]

(3.3)

\[
\frac{\partial^2 \sigma_{c0}}{\partial r^2} + \frac{1}{r} \frac{\partial \sigma_{c0}}{\partial r} - \left( i R_e D_0 S_c + \frac{E}{E_0} D_0 S_c \right) \sigma_{c0} = 0
\]

(3.4)

Same, in (2.19)-(2.21) of plasma region substitute values from (3.1)

\[
\left( \frac{\partial^2 u_{p0}}{\partial r^2} + \frac{1}{r} \frac{\partial u_{p0}}{\partial r} \right) - \left( M^2 + R e i \right) u_{p0} = - \left( P_0 + \frac{G_r \theta_{p0}}{\rho_0} + \frac{G_m \sigma_{p0}}{\rho_0} \right)
\]

(3.5)

\[
\frac{\partial^2 \theta_{p0}}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_{p0}}{\partial r} - \left( N^2 + i P_e \right) \theta_{p0} = 0
\]

(3.6)

\[
\frac{\partial^2 \sigma_{p0}}{\partial r^2} + \frac{1}{r} \frac{\partial \sigma_{p0}}{\partial r} - \left( i R_e S_c + E S_c \right) \sigma_{p0} = 0
\]

(3.7)
To find the exact solution of heat transfer (3.3) and (3.6) of core and plasma regions under the given boundary conditions (2.24), we apply the definition of Bessel differential equation by assuming

\[ \beta_1 = -\left( \frac{K_0N^2}{\rho_0} + iP_e \left( \frac{K_0}{\delta_0} \right) \right), \]

\[ \beta_2 = -(N^2 + iP_e). \]

So, the equations (2.6) and (2.9) change in the form of

\[ \frac{\partial^2 \theta_c}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_c}{\partial r} + \beta_1 \theta_c = 0 \]  \hspace{1cm} (3.8)

\[ \frac{\partial^2 \theta_p}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_p}{\partial r} + \beta_2 \theta_p = 0 \]  \hspace{1cm} (3.9)

Now, the exact solution for temperature profile of core region under the given boundary conditions (2.24) is calculated as

\[ \theta_c = \left[ U_1 \left( \frac{\sqrt{\beta_2} Y_1(\sqrt{\beta_2} R_1)}{U_4 Y_0(\sqrt{\beta_2} R)} - \sqrt{\beta_1} U_2 J_1(\sqrt{\beta_1} R_1) \right) + U_2 \right] J_0(\sqrt{\beta_1}(r)), \]  \hspace{1cm} (3.10)

where values of \( U_1, U_2 \) and \( U_4 \) are expressed as

\[ U_1 = \left( \frac{J_0(\sqrt{\beta_2} R_1)}{J_0(\sqrt{\beta_1} R_1)} - \frac{J_0(\sqrt{\beta_2} R) Y_0(\sqrt{\beta_2} R_1)}{Y_0(\sqrt{\beta_2} R) J_0(\sqrt{\beta_1} R_1)} \right), \]

\[ U_2 = \frac{Y_0(\sqrt{\beta_2} R_1)}{J_0(\sqrt{\beta_1} R_1) Y_0(\sqrt{\beta_2} R)} \]

\[ U_3 = J_1(\sqrt{\beta_2} R_1) - \frac{J_0(\sqrt{\beta_2} R)}{Y_0(\sqrt{\beta_2} R)} Y_1(\sqrt{\beta_2} R_1), \]

\[ U_4 = \sqrt{\beta_1} U_1 J_1(\sqrt{\beta_1} R_1) - \sqrt{\beta_2} U_3. \]

Solution for the temperature profile of the plasma region under the given boundary conditions (2.24) is calculated as

\[ \theta_p = \left[ \left( \frac{\sqrt{\beta_2} Y_1(\sqrt{\beta_2} R_1)}{U_4 Y_0(\sqrt{\beta_2} R)} - \sqrt{\beta_1} U_2 J_1(\sqrt{\beta_1} R_1) \right) \left( J_0(\sqrt{\beta_2} R) - \frac{J_0(\sqrt{\beta_2} R Y_0(\sqrt{\beta_2} R)}{Y_0(\sqrt{\beta_2} R) \right) \right] + \frac{Y_0(\sqrt{\beta_2} R)}{Y_0(\sqrt{\beta_2} R)}, \]  \hspace{1cm} (3.11)

Now, the final expressions for temperature profile considering unsteady flow for core and plasma regions respectively are as follows

\[ \theta_c = \left[ U_1 \left( \frac{\sqrt{\beta_2} Y_1(\sqrt{\beta_2} R_1)}{U_4 Y_0(\sqrt{\beta_2} R)} - \sqrt{\beta_1} U_2 J_1(\sqrt{\beta_1} R_1) \right) + U_2 \right] J_0(\sqrt{\beta_1}(r)) e^{it}, \]  \hspace{1cm} (3.12)

\[ \theta_p = \left[ \left( \frac{\sqrt{\beta_2} Y_1(\sqrt{\beta_2} R_1)}{U_4 Y_0(\sqrt{\beta_2} R)} - \sqrt{\beta_1} U_2 J_1(\sqrt{\beta_1} R_1) \right) \left( J_0(\sqrt{\beta_2} R) - \frac{J_0(\sqrt{\beta_2} R Y_0(\sqrt{\beta_2} R)}{Y_0(\sqrt{\beta_2} R) \right) \right] e^{it}. \]  \hspace{1cm} (3.13)
(3.4) \[ \frac{Y_0(\sqrt{\gamma_2}r)}{Y_0(\sqrt{\gamma_2}R)} e^{it}, \]

where \( J_n(x) \) is simply the Bessel function of first kind and \( Y_n(x) \) represents the the Bessel function of second kind for integer value of \( n \).

To solve (3.4) and (3.7) subject to the boundary conditions (2.24), we assume

(3.15) \[ \gamma_1 = -\left( iR_c D_0 S_c + \frac{E}{E_0} D_0 S_c \right), \]

(3.16) \[ \gamma_2 = -\left( iR_c S_c + ES_c \right). \]

So the Eqs. (3.4) and (3.7) become

(3.17) \[ \frac{\partial^2 \sigma_{c_0}}{\partial r^2} + \frac{1}{r} \frac{\partial \sigma_{c_0}}{\partial r} + \gamma_1 \sigma_{c_0} = 0 \]

(3.18) \[ \frac{\partial^2 \sigma_{p_0}}{\partial r^2} + \frac{1}{r} \frac{\partial \sigma_{p_0}}{\partial r} + \gamma_2 \sigma_{p_0} = 0 \]

Now, apply the definition of Bessel differential equation to calculate the value of concentration profiles under the given boundary conditions (2.24) for both core as well as plasma regions. So, Solution for concentration profile for core region is calculated as

(3.19) \[ \sigma_{c_0} = \left[ U_5 \left( \sqrt{\frac{\gamma_2}{\gamma_1}} J_1(\sqrt{\frac{\gamma_2}{\gamma_1}} R_1) - \sqrt{\frac{\gamma_2}{\gamma_1}} U_6 J_1(\sqrt{\frac{\gamma_2}{\gamma_1}} R_1) \right) + U_6 \right] J_0(\sqrt{\gamma_1}(r)), \]

where the values of \( U_5, U_6 \) and \( U_8 \) are assumed as

\[ U_5 = \left( \frac{J_0(\sqrt{\gamma_2} R_1)}{J_0(\sqrt{\gamma_1} R_1)} - \frac{J_0(\sqrt{\gamma_2} R_0) Y_0(\sqrt{\gamma_2} R_1)}{Y_0(\sqrt{\gamma_2} R)} \right), \]

\[ U_6 = \frac{Y_0(\sqrt{\gamma_2} R_1))}{Y_0(\sqrt{\gamma_1} R_1) Y_0(\sqrt{\gamma_2} R)}, \]

\[ U_7 = J_1(\sqrt{\gamma_2} R_1) - \frac{J_0(\sqrt{\gamma_2} R_0) Y_0(\sqrt{\gamma_2} R_1)}{Y_0(\sqrt{\gamma_2} R)}, \]

\[ U_8 = \sqrt{\gamma_1} U_5 J_1(\sqrt{\gamma_1} R_1) - \sqrt{\gamma_2} U_7. \]

Solution for concentration profile of plasma region under the given boundary conditions is calculated as

(3.20) \[ \sigma_{p_0} = \left[ \left( \frac{\sqrt{\gamma_2} Y_1(\sqrt{\gamma_2} R_1)}{U_8 Y_0(\sqrt{\gamma_2} R)} - \sqrt{\gamma_1} U_6 J_1(\sqrt{\gamma_1} R_1) \right) \left( J_0(\sqrt{\gamma_2} r) - \frac{J_0(\sqrt{\gamma_2} R_0) Y_0(\sqrt{\gamma_2} R_1)}{Y_0(\sqrt{\gamma_2} R)} \right) \right] \]

(3.17) \[ \frac{\partial^2 \sigma_{c_0}}{\partial r^2} + \frac{1}{r} \frac{\partial \sigma_{c_0}}{\partial r} + \gamma_1 \sigma_{c_0} = 0 \]

So, the final solutions of concentration profile for unsteady flow in core and plasma regions respectively are as follows

(3.21) \[ \sigma_c = \left[ U_5 \left( \sqrt{\frac{\gamma_2}{\gamma_1}} Y_1(\sqrt{\gamma_2} R_1) - \sqrt{\frac{\gamma_2}{\gamma_1}} U_6 J_1(\sqrt{\gamma_1} R_1) \right) + U_6 \right] J_0(\sqrt{\gamma_1}(r)) e^{it}, \]
\[ \sigma_p = \left[ \left( \frac{\sqrt{\gamma_0 Y_1} (\sqrt{\gamma_2 R})}{U_8 Y_0 (\sqrt{\gamma_2 R})} - \frac{\sqrt{\gamma_1 U_6 J_1} (\sqrt{\gamma_1 R})}{U_8} \right) \left( J_0 (\sqrt{\gamma_2 r}) - \frac{J_0 (\sqrt{\gamma_2 R}) Y_0 (\sqrt{\gamma_2 R})}{Y_0 (\sqrt{\gamma_2 R})} \right) \right] e^{it} \]

\[ + \frac{Y_0 (\sqrt{\gamma_2 r})}{Y_0 (\sqrt{\gamma_2 R})} e^{it}. \]

To find the solution for velocity profile in core region, we put the values of \( \theta_c \) and \( \sigma_c \) from (3.10) and (3.19) in to (3.2) and apply the method variation of parameters for the given non-homogeneous differential equations by assuming

\[ \lambda_1 = -\left( M^2 + \frac{\mu_0 R_e}{\rho_0} i \right), \]

\[ \left( \frac{\partial^2 u_{c0}}{\partial r^2} + \frac{1}{r} \frac{\partial u_{c0}}{\partial r} \right) + \lambda_1 u_{c0} = -\left( P_0 + \frac{G_r \theta_{c0}}{\rho_0} + \frac{G_m \sigma_{c0}}{\rho_0} \right) \mu_0. \]

So, the momentum equation convert in the form of (3.24) which can be treated as an ordinary differential equation. In variation of parameters method first, we calculate the complementary solution for homogeneous differential equation by using the definition the Bessel differential equation as

\[ u_{c0c} = C_1 J_0 \sqrt{\lambda_1 r} + C_2 Y_0 \sqrt{\lambda_1 r}, \]

where we have

\[ u_{c01} = J_0 \sqrt{\lambda_1 r}, \quad u_{c02} = Y_0 \sqrt{\lambda_1 r}. \]

The Wronskian of these two functions is

\[ W_1 = \begin{vmatrix} J_0 (\sqrt{\lambda_1 r}) & Y_0 (\sqrt{\lambda_1 r}) \\ -\sqrt{\lambda_1} J_1 (\sqrt{\lambda_1 r}) & -\sqrt{\lambda_1} Y_1 (\sqrt{\lambda_1 r}) \end{vmatrix} = \frac{2}{\pi r}. \]

Now, for finding the complete solution of the non-homogeneous (3.24) we find

\[ A_1 = -\int \frac{Y_0 \sqrt{\lambda_1 r} \left( P_0 + \frac{G_r \theta_{c0}}{\rho_0} + \frac{G_m \sigma_{c0}}{\rho_0} \right) \mu_0}{W_1} dr, \]

\[ B_1 = \int \frac{J_0 \sqrt{\lambda_1 r} \left( P_0 + \frac{G_r \theta_{c0}}{\rho_0} + \frac{G_m \sigma_{c0}}{\rho_0} \right) \mu_0}{W_1} dr. \]

The complete solution of the velocity profile for the core region is of the form of

\[ u_{c0} = C_1 J_0 \sqrt{\lambda_1 r} + C_2 Y_0 \sqrt{\lambda_1 r} + A_1 J_0 \sqrt{\lambda_1 r} + B_1 Y_0 \sqrt{\lambda_1 r}. \]

Now for (3.5) of plasma region, we assume that

\[ \lambda_2 = -(M^2 + Re i). \]

So the (3.5) convert in terms \( \lambda_2 \) as

\[ \left( \frac{\partial^2 u_{p0}}{\partial r^2} + \frac{1}{r} \frac{\partial u_{p0}}{\partial r} \right) + \lambda_2 u_{p0} = -\left( P_0 + \frac{G_r \theta_{p0}}{\rho_0} + \frac{G_m \sigma_{p0}}{\rho_0} \right). \]
For given (3.28) complementary solution for homogeneous differential equation can be calculated as

\[(3.29)\]
\[u_{p_0c} = C_3 J_0(\sqrt{\lambda_2}r) + C_4 Y_0(\sqrt{\lambda_2}r)\]

So, we have
\[u_{p_01} = J_0(\sqrt{\lambda_2}r), \quad u_{p_02} = Y_0(\sqrt{\lambda_2}r).\]

The Wronskian of these two functions is
\[W_2 = \begin{vmatrix} J_0(\sqrt{\lambda_2}r) & Y_0(\sqrt{\lambda_2}r) \\ -\sqrt{\lambda_2}J_1(\sqrt{\lambda_2}r) & -\sqrt{\lambda_2}Y_1(\sqrt{\lambda_2}r) \end{vmatrix} = \frac{2}{\pi r} \]

For solution of the non-homogeneous equation (3.28) further, we calculate
\[A_2 = -\int \frac{Y_0(\sqrt{\lambda_2}r)}{W_2} \left( \frac{P_0 + G_r \theta_0}{\rho_0} + \frac{G_m \sigma_0}{\rho_0} \right) \mu_0 dr,\]
\[B_2 = \int \frac{J_0(\sqrt{\lambda_2}r)}{W_2} \left( \frac{P_0 + G_r \theta_0}{\rho_0} + \frac{G_m \sigma_0}{\rho_0} \right) \mu_0 dr.\]

The complete solution of the velocity profile for the plasma region is of the form of

\[(3.30)\]
\[u_{p_0} = C_3 J_0(\sqrt{\lambda_1}r) + C_4 Y_0(\sqrt{\lambda_1}r) + A_2 J_0(\sqrt{\lambda_1}r) + B_2 Y_0(\sqrt{\lambda_1}r).\]

Now, we calculate the values of unknowns \(C_1, C_2, C_3 \) and \(C_4\) by using boundary condition (2.24) in to (3.26) and (3.30).

First applying the boundary condition \(\frac{\partial u}{\partial r}\) at \(r = 0\), we get
\[C_2 = 0\]

Now, Eq.(3.26) become
\[(3.31)\]
\[u_{c_0} = C_1 J_0(\sqrt{\lambda_1}r) + A_1 J_0(\sqrt{\lambda_1}r) + B_1 Y_0(\sqrt{\lambda_1}r).\]

After applying all the boundary conditions in to the (3.30) and (3.31), we get the linear system of \(C_1, C_3\) and \(C_4\) in the form of

\[(3.32)\]
\[\begin{pmatrix} J_0(\sqrt{\lambda_1}r_1) & -J_0(\sqrt{\lambda_2}r_1) & -Y_0(\sqrt{\lambda_2}r_1) \\ -\sqrt{\lambda_1}J_1(\sqrt{\lambda_1}r_1) & \sqrt{\lambda_2}J_1(\sqrt{\lambda_2}r_1) & \sqrt{\lambda_2}Y_1(\sqrt{\lambda_2}r_1) \end{pmatrix} \begin{pmatrix} C_1 \\ C_3 \end{pmatrix} = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}\]

where \(D_1, D_2\) and \(D_3\) are expressed as
\[D_1 = -A_1(R_1) J_0(\sqrt{\lambda_1}r_1) - B_1(R_1) Y_0(\sqrt{\lambda_1}r_1) + A_2(R_1) J_0(\sqrt{\lambda_2}r_1) + B_2(R_1) Y_0(\sqrt{\lambda_2}r_1)\]
\[D_2 = -A_2(R) J_0(\sqrt{\lambda_2}r) - B_2(R) Y_0(\sqrt{\lambda_2}r)\]
\[D_3 = \frac{\partial A_1(R_1)}{\partial r} J_0(\sqrt{\lambda_1}r_1) + A_1(R_1) \sqrt{\lambda_1} J_1(\sqrt{\lambda_1}r_1) - \frac{\partial B_1(R_1)}{\partial r} Y_0(\sqrt{\lambda_1}r_1) + B_1(R_1) \sqrt{\lambda_1} Y_1(\sqrt{\lambda_1}r_1) + \frac{\partial A_2(R_1)}{\partial r} J_0(\sqrt{\lambda_2}r_1) + \frac{\partial B_2(R_1)}{\partial r} Y_0(\sqrt{\lambda_2}r_1)\]
It can be clearly seen from (3.32) that it is simply the linear system of order $3 \times 3$ with $C_1$, $C_3$ and $C_4$ unknowns and which has the unique solution. Final exact solutions for velocity profiles $u_{c0}$ and $u_{p0}$ for core and plasma regions obtain by putting the values of $C_1$, $C_3$ and $C_4$ in (3.30) and (3.31) respectively.

The total volumetric flow rate of the blood flow in the artery is calculated as

$$Q = 2\pi R^2 \int_0^{R_1} u_c(r, t) dr + 2\pi R^2 \int_{R_1}^R u_p(r, t) dr, \quad (3.33)$$

The shear stress at the interface wall of core and the plasma region is obtained as

$$\tau = \left( \frac{\partial u_c}{\partial r} \right), \quad (3.34)$$

At the outer wall of the artery shear stress is determined as

$$\tau' = \frac{1}{\mu_0} \left( \frac{\partial u_p}{\partial r} \right), \quad (3.35)$$

Flow resistance in two phase blood flow is defined as

$$\lambda = \int_0^z \frac{P_0 e^{i\omega t}}{Q} dz, \quad (3.36)$$

using the values of $Q$ from Eq.(3.33), we get the final expression for $\lambda$.

4. RESULTS AND DISCUSSION

For having adequate insight into the two-phase flow behavior of blood flow through a stenosed arterial segment flow resistance, total flow rate, and wall shear stress have been estimated assuming pulsatile, unsteady and Newtonian nature of the blood flow for both core and plasma regions. A computational study has been carried out to graphically show the effects of RBC-
Table 1. Values of the parameters

| Parameters                                      | Values (Unitfree) | Source |
|------------------------------------------------|-------------------|--------|
| Magnetic field Parameter \((M)\)              | 1.5-3             | [26], [27] |
| Schmidt Parameter \((S_c)\)                   | 0.5-1.5           | [28]   |
| Radiation Parameter \((N)\)                   | 2-5               | [24]   |
| Chemical reaction Parameter \((E)\)           | 0.5-2             | [29]   |
| Peclet Number \((P_e)\)                       | 0.87              | [30]   |
| Grashof Number \((G_r)\)                      | 2-3               | [29]   |
| Modified Grashof Number \((G_m)\)            | 2-3               | [29]   |
| Ratio of Thermal Conductivity in core and plasma regions \((K_0)\) | 0.4-0.8          | [16]   |
| Ratio of Specific heat in core and plasma regions \((s_0)\) | 1.0              | [1]    |
| Ratio of density in core and plasma regions \((\rho_0)\) | 1.05             | [1]    |
| Ratio of mean radiation absorption coefficients in core and plasma region \((\alpha_0)\) | 1.0              | [16]   |
| Reynold Number \((R_e)\)                      | 0.005             | [30, 31] |
| Ratio of viscosities in core and plasma region \((\mu_0)\) | 1.2              | [1, 7] |
| Pressure gradient \((P_0)\)                   | 10                | [16]   |

depleted plasma layer on blood flow with the variation of different quantities of interest. List of parameters with their values used to graphically analyze the effectiveness of the model are given in Table 1.

Figure 2 displays the comparative studies between single phase and two-phase model of blood flow with the experimental results of Bugliarello and Sevilla [25], who through their In Vitro experimental studies under the steady flow conditions measured the cell velocity distribution in a fine glass tube for 40% RBC containing blood. As in two phase of blood flow erythrocytes concentrate towards the center of the artery and there exist the RBC-depleted plasma layer near the arterial wall so in our study we assume that RBC are present in 70% part of the artery and remaining 30% part is plasma. Calculated mean squared errors (MSE) for the measured data of two phase and single phase blood flow as compared with the experimental data of \(n = 21\) no. of points are given below.

\[
\frac{1}{n}\sum_{i=1}^{n} (\hat{Y}_{EX,i} - Y_{T,i})^2 = 0.003094882,
\]

(4.1)

\[
\frac{1}{n}\sum_{i=1}^{n} (\hat{Y}_{EX,i} - Y_{S,i})^2 = 0.010313676,
\]

(4.2)

where \(\hat{Y}_{EX}\) represents the experimental data and \(Y_{T}, Y_{S}\) show data for two phase and single phase models respectively for \(n = 21\) data points. In (4.1)-(4.2) expression \((\hat{Y}_{EX,i} - Y_{T,i})^2\)
shows average of the squares of the deviations from experimental data to two phase model data and similarly the calculated value of \((\bar{Y}_{\text{EX}} - \bar{Y}_{\text{S}})^2\) gives average of the squares of the deviations from experimental data to single phase model data. So, from the results it can be clearly observe that two phase model data fits with the experimental data more appropriately as it has 0.3\% mean squared error than the single phase model data of 1\% mean squared error.

![Figure 3.](image1.png)  
**Figure 3.** Variation of velocity profile for two phase model of blood flow with the ratio of viscosity in plasma and core regions

![Figure 4.](image2.png)  
**Figure 4.** Effect of magnetic field parameter \((M)\) on velocity profile for two phase model blood flow

Now, the following figures are plotted to analyze the effects of various dimensionless parameters on velocity, temperature and concentration profiles. Figure 3 displays the variation of velocity profile for different values of \((\mu_0)\) which is the ratio of the viscosity in plasma region to the core region. In figure continuous and dotted lines show the velocity variations in core and plasma regions respectively, in which plasma region varies from 0.7 to 1 radius of the artery. From Figure 3 it is clear that as the value of viscosity ratio increases, the value of velocity profiles also increase for both core and plasma regions. Figure 4 shows the effect of varying applied magnetic field on velocity profiles as continuous lines in the figure show the magnetic field effect on velocity profile of core region and dotted lines display the effect of varying magnetic field on velocity profile of plasma region. It is clear from Figure 4 that as the value of uniform applied magnetic field increases from 0.5 to 3 velocity profiles for both core and plasma regions decrease respectively. This happens because the mature red blood cells contain high concentration hemoglobin molecules in its content which are oxides of iron. So, when MHD blood flows under the influence of uniform applied magnetic field erythrocytes orient with their own disk plane parallel to the direction of applied magnetic field. This action forms red blood cells and magnetic particles more suspended in blood plasma, so increased concentration of magnetic particles causes an increase in the internal blood viscosity [32]. This results in a decrease blood velocity as Lorentz force opposes the flow of blood and magnetic particles [33]. The magnetic field which is applied just perpendicular to the direction of the blood flow in the artery affects the velocity profile of blood plasma as we increase the value of magnetic field parameter from 1.5 to 3, velocity profile decreases from interface region towards the arterial wall [34].
Figure 5 and Figure 6 show the variation in axial velocity profiles with modified Grashof number and Grashof number respectively. In which dotted lines show the variation in velocity profile of plasma region and continuous lines display the variation velocity profile of core region, as we change the values of modified Grashof number and Grashof number. It can be clearly observed from the figures that as we increase the value of modified Grashof number and Grashof number velocity profiles increase respectively for both core and plasma regions.

Figure 7 and Figure 8 illustrate the effects on the temperature profile of two phase blood flow with increase in the values of $K_0$ and $P_e$ parameters respectively. Continuous lines in the plot
show the variation of the temperature profile in the core region and dotted lines display the variations of the temperature profile in the plasma region. As it could be seen from these figures that as the ratio of the thermal conductivity of plasma and core regions \((K_0)\) and value of the Peclet number \((P_e)\) increase, the temperature profile of the two-phase blood flow decrease in both core and plasma regions respectively. Figure 9 exhibits behavior of temperature profile of two phase blood flow as values of the radiation parameter vary. It can be clearly observed from the figure that for a particular value of radiation parameter \((N)\) temperature profile increases from mid of the artery to the interface region of core and plasma and continuously increases up to the arterial wall. As we increase the values of radiation parameter, temperature profile decreases respectively in both core and plasma regions. Comparison result for temperature profiles of single phase and two-phase model of blood flow have been displayed in Figure 10, by taking same numerical values of the parameters as given in Table.1. There is no RBC-depleted plasma layer exists in the artery as we consider the case \(R_1 = 1\), we assume that all components of the blood are uniformly distributed in the artery and for the case of \(R_1 = 0.7\) we consider plasma occupies the \(R = 0.3\) region of the artery. It is clear from Figure 10 that having same numerical values of different dimensionless parameters, temperature profile of the two-phase blood flow model has lower value than the temperature profile of the single phase blood flow model.

![Figure 9](image1.png)  
**Figure 9.** Effect of radiation parameter \((N)\) on temperature profile for two phase model of blood flow

![Figure 10](image2.png)  
**Figure 10.** Comparision of temperature profiles of single and two phase model of blood flow

Figure 11 and Figure 12 reveal that under the purview of the present computational study, concentration profiles for both core and plasma regions in two phase blood flow decrease as the values of the chemical reaction parameter and Schmidt number increase respectively. For any particular value of chemical reaction parameter and Schmidt number concentration profile increase, as we move further and further from mid of the artery to interface region and it increase up to the arterial wall. The variation of the concentration profile with radial distance for the different values of \(E_0\) is displayed in Figure 13. From the figure it can be clearly observed that as the value of \(E_0\), which is the ratio of chemical moles present in the plasma region and the core region increases, concentration profile of two phase blood flow also increases respectively.
for both core and plasma regions. Using same numerical values of the parameters as given Table.1, Figure 14 shows the comparison result of concentration profiles between single phase blood flow as $R_1 = 1$ and two phase blood flow for $R_1 = 0.7$ and it is clear from the figure that, concentration profile attains higher values in single phase blood flow model in which blood components uniformly distributed over the artery, than the two-phase model of core consisting of erythrocytes and existence of plasma layer near the wall.

It is found that in the artery vascular tissue shows histological and morphological alterations, when it is physically stressed then the role of wall shear stress comes into the picture. So
in the study of stenosed arterial blood flow, wall shear stress is a major flow component to measure [35]. For two phase model of blood flow Figure 15 displays the variation of wall shear stress with the time for different values of Reynolds number. It can be clearly observed from Figure 15 that as the of Reynolds number increases from 0.005 to 0.405 with time cycle, shear stress at the wall of the artery increases. Here we use the velocity profile of plasma region as it is assumed that RBC-depleted plasma layer exists near the arterial wall to evaluate the value of wall shear stress in the stenosed artery. Figure 16 depicts the variation of wall shear stress with time as the value of the uniform applied magnetic field increases. Throughout the time scale, oscillatory nature of the wall shear stress is observed in the figure. The flow impedance gives
the strong correlation between the localization of stenosis and arterial wall as it important to understand the development of arterial disease. Figure 17 and Figure 18 are plotted to show the impedance profile against the axial distance for two phase model of blood flow by taking different values of radiation parameter and magnetic field parameter respectively. It can be clearly observed from the figures that as the value of radiation parameter increases from 2 to 6, total impedance of two phase blood flow decreases and with varying values of applied magnetic field from 0.5 to 3, it increases.

Figure 19 and Figure 20 are displayed to show the effects on the flow rate of two phase model of blood flow as the value of both radiation and magnetic field parameter change respectively. A pulsatile nature of the total flow rate is observed for both the cases as time varies. Initially, total flow is at rest and under the influence of varying pressure gradient, it starts the motion. As it is clear from these figures that flow rate of MHD oscillatory two phase blood flow decreases as values of the radiation parameter as well as magnetic field parameter increase. Under the influence of applied magnetic field, it happens may be because of the rotational motion of the magnetic particle which flows within the blood [36]. The action of rotation make magnetic particles more suspended in the blood flow and it increases the viscosity of the blood flow and which decreases the total flow rate of the two-phase blood flow.

**Figure 19. Variation of total flow rate with time for different values of $N$**

**Figure 20. Variation of total flow rate with time for different values of $M$**

**Contour Plots**

Figure 21, Figure 22 and Figure 23 are the contour plots for the two-phase model of the blood flow, which show the distribution of velocity with time as the intensity of the uniform applied magnetic field varies. Where X, Y, and Z display the scale of the time, radius and velocity at that point respectively so it can be clearly noticed from the figures that as the value of applied magnetic field increases from 0.5 to 4 velocity of the two-phase blood flow decreases and slowly appearing trapping bolus shift toward the arterial wall. For different values of the Reynolds number contour plots of the velocity variations against time have been displayed with the help of Figure 24, Figure 25 and Figure 26. Following the sinusoidal behavior with time, the velocity
of the two-phase blood flow increases as the value of the Reynolds number increases and with more turbulence it makes flow locally unstable with respect to large-scale disturbances [37].
In the present study, we have theoretically analyzed the effects of heat and mass transfer on the two-phase model of blood flow through a vertical stenosed artery under the influence of applied magnetic field and thermal radiation with the effect of the chemical reaction. A mathematical model of coupled nonlinear differential equations have been developed for momentum, concentration, and energy of central core and RBC-depleted plasma layer considering Newtonian fluid in both the regions and the exact solution has been found. It is analyzed through our results that as the value of both thermal Radiation and chemical reaction parameter increase, temperature and concentration profiles in both core and plasma regions decrease respectively. It is evident from the graphical result that high intensity of the applied magnetic field influences the flow of blood in the artery as it slows down the velocity profiles of both central core region of suspended erythrocytes and cell-free plasma regions. To show the effectiveness of the two-phase model we carried out the comparative study between the single-phase and two-phase model of the blood flow and found that the two-phase model fits with the experimental data more accurately than the single phase model of blood flow. To get the proper understanding of the effects of two-phase blood flow on stenosed artery the graphs of the variations of wall shear stress, total impedance, and total flow rate have been plotted which show their sinusoidal behavior with time.

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