Measurability of the non-minimal scalar coupling constant

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Abstract

The "measurability" of the non-minimal coupling is discussed in the context of the effective field theory of gravity. Although there is no obvious motive for excluding a non-minimal scalar coupling from the theory, we conclude that for reasonable values of the coupling constant it makes only a very small correction.

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Recently the study of perturbative quantum gravity - i.e. gravity treated as a theory of small quantum fluctuations around a flat Minkowski background spacetime - has found a novel rejuvenation [1], [2], [3]. According to this view, gravity is an effective field theory (for a discussion of the effective field theory approach to quantum gravity see [2]) which can be quantized in a standard way if we restrict its region of applicability to low enough energies and small curvatures. (Of course this approach fails when the energy reaches the Planck scale where new degrees of freedom become important.) The interesting thing raised in [1] is that this framework provides a basis to make quantum predictions [2], [3].

On the other hand, it has been suggested that the action for gravity should contain, in addition to the Einstein-Hilbert term, certain non-minimal functionals of the scalar field. The only possible local term involving a dimensionless coupling between the curvature and the scalar field is of the form $\xi R \phi^2$, with $\xi$ a constant [4]. Such a term was used in [3] to soften the divergences of the stress tensor. Other reasons to justify the presence of this term are the inclusion of a symmetry breaking mechanism into gravity [8], the construction of non-singular models for the universe [7], the investigation of inflationary models with a non-minimally coupled scalar field [8], the inclusion of Mach theory in Jordan-Brans-Dicke theory of gravitation [4], the analysis of oscillating universes [10], the reconciliation of cosmic strings with inflation [11], the low energy limit of superstring theories [12], the Kaluza-Klein compactification scheme [13], and others [14], [15].

Usually the value of the coupling $\xi$ is chosen to be zero (minimal coupling) or for massless scalars $\xi = \frac{n-2}{4(n-1)}$ (conformal coupling in $n$-dimensional spacetime). Even though these values are widely used in the literature it is not possible to fix a priori the value of $\xi$. The only way to gain a feel for $\xi$ is to compare some experimental result with a theoretical prediction, but presently no experiment can reveal such coupling. In all the studies of effective quantum gravity the non-minimal coupling between the scalar curvature and the matter fields has been neglected. Despite this view, there is no first principle we are aware of that can be invoked to get rid of this term from the beginning.

Even though the region of applicability of the effective approach is restricted and therefore does not constitute a definite answer to the quantum gravity problem, it is interesting to clarify this issue within this context. As we shall see in what follows the main problem is that the effect of the non-minimal coupling is tiny, but the smallness of this term is spoiled by its presence in the scattering amplitudes. So the question that naturally arises is whether or not we are allowed to discard the $R \phi^2$ term from our initial theory.

To answer this question several authors focused their attention on $2 \rightarrow 2$ scattering processes with external or exchanged gravitons. In general, only a few scattering processes can reveal such a coupling, namely

$$gs \rightarrow gs$$

$$ss \rightarrow ss$$

$$sf \rightarrow sf$$

$$s\gamma \rightarrow s\gamma$$
\( gs \rightarrow \gamma s \) \hspace{1cm} (5)

(Here \( g \) denotes a graviton, \( s \) a scalar, \( f \) a fermion and \( \gamma \) a photon.) Therefore using the following Lagrangian density

\[
\mathcal{L} = \mathcal{L}_E + \mathcal{L}_{KG} + \mathcal{L}_D + \mathcal{L}_{EM},
\]

(6)

\[
\mathcal{L}_E = \frac{2}{\kappa^2} \sqrt{-g} R,
\]

\[
\mathcal{L}_{KG} = \frac{\sqrt{-g}}{2} \left( g^{\mu\nu} D_\mu \phi D_\nu \phi - m^2 \phi^2 + \xi R \phi^2 \right),
\]

\[
\mathcal{L}_{EM} = -\frac{1}{4} \sqrt{-g} g^{\mu\nu} g^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta},
\]

\[
\mathcal{L}_D = \sqrt{-g} \left( \frac{i}{2} (\bar{\psi} \gamma^\mu (\nabla_\mu + ieA_\mu) \psi - \bar{\psi} (\nabla_\mu + ieA_\mu) \gamma^\mu \psi - m \bar{\psi} \psi) \right),
\]

as a starting point the processes (1)-(5) have been computed [16], [17], [18], [19]. Instead of reporting in any detail the computation of the scattering amplitudes we have performed

\[\text{The fact that the } \xi \text{-dependence of any scattering amplitude which stems from } \mathcal{L}_E + \mathcal{L}_{KG} \text{ is related to the presence of a massive particle can easily be explained. Using}
\]

\[
R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = 8\pi G T^{\mu\nu},
\]

we obtain

\[
\xi R \phi^2 = -8\pi G \xi \phi^2 T_\mu^\mu,
\]

so the non-minimally coupled term is proportional to the trace of the stress-energy tensor. Now by using the expression given in [20] for \( T^{\mu\nu} \) it is straightforward to see that in the massless case:

\[
T_\mu^\mu = \mathcal{O}(\text{fields}^5),
\]

whereas

\[
T_\mu^\mu = \mathcal{O}(\text{fields}^4),
\]

in the massive case. This tells us that in the massless case we should expect no dependence of the scattering amplitude on \( \xi \). This fact can be shown directly from the Lagrangian density reparametrising the graviton field by means of \( h_{\mu\nu} \rightarrow (1 + \Omega(\phi^2, \xi))h_{\mu\nu} \). It is possible to prove that the \( \Omega \) is of the form \( \Omega(\phi^2, \xi) = -\frac{\xi^2}{4} \phi^2 \) and is unique [21], [22].
as a check of previous results, we prefer to analyze another phenomenon, which provides an alternative way of measuring the non-minimal coupling: the helicity flip of a fermion in a gravitational field generated by a scalar mass.

The behavior of a spinning particle in a gravitational field has been studied semiclassically \[23\] and in the linearized approach for \( \xi = 0 \) \[24\], in which the helicity flip appears as a dynamical effect, coming from the local coupling of spin to gravity. In \[24\] it is shown that only massive spin \( 1/2 \) particles can have their helicity flipped, whereas no change in the polarization is expected for massless fermions. (Of course in non-minimally coupled theories this result is still valid since for massless fermions the non-minimally coupled part of the scattering amplitude is zero.) In the context of linearized quantum gravity with a non-minimal coupling it is interesting to re-examine this problem. The interest lies in the possibility that this method offers to measure \( \xi \), apart from the fact that it generalizes the results of \[24\].

The helicity flip rate of a fermion interacting with a scalar via graviton exchange is

\[
P = \frac{M^2_{LL} - M^2_{LR}}{M^2_{LL} + M^2_{LR}}
\]

where \( M \) is the fermion-scalar elastic scattering amplitude given by:

\[
M'_{sf \to sf} = -i\kappa^2 \bar{u}(\lambda', p_2) \times \\
\times \left( (m^2 + \xi t)(\hat{p}_1 + \hat{p}_2) + \frac{u - s}{2}(\hat{p}_1 - \hat{p}_2 + 2\hat{q}) \right) u(\lambda, p_1),
\]

where \( p_1 \) (\( p_2 \)) is the initial (final) fermion momentum and \( u(\lambda, p_1) \) \((\bar{u}(\lambda', p_2))\) is, respectively, the spinor and \( \bar{a} = a_{\mu} \gamma^{\mu} \) and \( \lambda \) (\( \lambda' \)) is the helicity of the initial (final) fermion of mass \( m_f \).

For our purposes is sufficient to plot \( P \) as a function of \( \xi , m_f \). We used Mathematica to perform the calculation of \( P \) and the dependence of \( P \) on \( m_f \) is plotted for several values of \( \theta \) and \( \xi \), fixing the values of \( E \), of the scalar mass \( m_s \) as indicated in the figures’ footnotes. The results are shown in Fig 1 - Fig 2. The result of Fig 1 shows the agreement with \[24\] (the larger the scattering amplitude, the larger the helicity flip). From Fig 2 one can argue that no drastic changes, with respect to the minimally coupled case, arise for \( \xi \) non-zero (Our numerical study is restricted to values, \(-100 \leq \xi \leq 100\)). Even though the non minimal coupling appears explicity in the scattering amplitudes its effect seems to be irrelevant at ordinary energies.

We note, in passing, that a massive spinor field is not a representative of all non-conformal matter in the effective theory, i.e. other matter could be capable of producing observable interactions with these scalars. In any case as far as the Lagrangian \[8\] is concerned the effect of the non-minimally coupled term at leading orders in scattering amplitudes gives raise to a tiny effect \[17\], \[18\], \[22\]. A problem which is important to mention is related to the

\footnote{There is no particular reason to restrict \( \xi \) to the range \([-100, +100]\), but, as follows from cosmological applications, we expect the value to be small.}
quantum conformal anomalies, which make scalar couplings to Yang-Mills fields and spinor fields incapable of being eliminated, even in classically conformally coupled theories. This last problem requires a more careful investigation within the effective field theory framework and we hope to address this in a future work.

At this point we might ask what kind of principle we can invoke to get rid of the $R\phi^2$ term in the starting lagrangian. We can gain some feel by looking at the contribution of this term to the static gravitational potential. For clarity, let us first consider the contribution to the static potential due to $R$ terms.

In [1], [2] the rationale for choosing the gravitational action proportional to $R$ is explained. The starting point is the following action, ordered in a derivative expansion,

$$S = \int d^4x \sqrt{-g} (\Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots + \mathcal{L}_{\text{matter}}),$$

in which all infinitely many terms allowed by general coordinate invariance are included. Cosmological bounds ($|\Lambda| < 10^{-46}\text{Gev}^4$) allow us to neglect, at ordinary energies, the cosmological constant term. Higher derivative terms are negligible as shown in [1], [25]. The main argument given is that the potential which stems from the Lagrangian density

$$\mathcal{L} = \frac{2}{\kappa^2} R + cR^2,$$

is

$$V(r) = -\frac{Gm^2}{r}(1 - e^{-r/\sqrt{\kappa^2c}}). \quad (8)$$

Experimental bounds on $c_i$'s, given in [25], are

$$c_1, \ c_2 < 10^{74}.$$  

This means that higher derivative terms are irrelevant at ordinary scales ($c = 1$ implies $\sqrt{\kappa^2c} \simeq 10^{-35}m$). In [1] it is stressed that in an effective field theory the $R^2$ terms need not be treated to all orders, but must only include the first corrections in $\kappa^2c$. This is because at higher orders we should include other terms in the action ($R^3, R^4, \ldots$) - note that this argument cannot be extended to the $R\phi^2$ term. At first order, the potential (8) becomes a representation of the delta function, i.e. the low energy potential has the form

$$V(r) = -Gm^2\left(\frac{1}{r} + 128\pi^2G(c_1 - c_2)\delta^3(\vec{r})\right). \quad (9)$$

As seen from (8) $R^2$ terms lead to a very weak and short-ranged modification of the gravitational interaction.

Similar arguments can be easily extended to the $R\phi^2$ term. Notice that this result is not obvious since we could not exclude the $R\phi^2$ term on the basis of an energy expansion or any symmetry consideration or on the basis of renormalizability. The interaction potential between two massive spinless bosons is defined via the relation
\[ V(r) = \frac{1}{4m^2} \mathcal{F}(M_{ss\rightarrow ss}) , \]  
\[ (10) \]

where \( \mathcal{F}(M_{ss\rightarrow ss}) \) is the Fourier transform of the elastic scattering amplitude \( M_{ss\rightarrow ss} \) of two scalar particles of mass \( m \). This leads to \( V'(r) \) as non-minimal correction to the Newtonian potential, where

\[ V'(r) = -\frac{\kappa^2}{4} \xi m^2 (6\xi - 5) \delta^3(\vec{r}) . \]  
\[ (11) \]

Therefore, the correction induced on the Newton potential due to the presence of the non-minimal coupling is similar in form to the one which stems from higher derivative terms, thus leading to a weak and short-ranged effect. The previous formula tells us that it is not possible to measure \( \xi \) at tree level in the region of energy/curvature where the effective approach is valid.

At this point we might ask whether this is true at one-loop. The calculation is in this case more involved and the use of a symbolic manipulation program is unavoidable \[26]. Here we simply mention that the one-loop correction to the static gravitational potential is proportional to \( \frac{2}{m^2} \log(q^2) \), thus giving a contribution which is seen to be subleading with respect to the leading power correction computed in \[3, 4\].

Therefore we come to the conclusion that it is possible to exclude naturally the \( R\phi^2 \) term in the starting action, or, in other words, that we can start with the more general Lagrangian density, thus including the non-minimal coupling, finding that the effect of the latter is not visible. Therefore, the effective field theory of gravity is not disturbed by the presence of this term.

**Acknowledgements**

We would like to thank the referee for raising the issue of the anomalies and suggesting a number of clarifications. A.F. acknowledges the Ridley Foundation for financial support.
FIG. 1. Helicity flip rate as a function of $m_f$ (MeV) for $E = 100 \text{MeV}$, $m_s = 10^3 \text{MeV}$ $\theta = \pi/9$ (dashed line), $\theta = 2/9\pi$ (thick line), $\theta = 3/9\pi$ (straight line).

FIG. 2. $P$ vs $m_f$ for $E = 100 \text{MeV}$, $m_s = 10^3 \text{MeV}$, $\theta = \pi/9$, $\xi = -100$ (dot-dashed line), $\xi = 0$ (straight line), $\xi = 100$ (dashed line).
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