Energy-Efficient Optimization for Physical Layer Security in Multi-Antenna Downlink Networks with QoS Guarantee

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Abstract—In this letter, we consider a multi-antenna downlink network where a secure user (SU) coexists with a passive eavesdropper. There are two design requirements for such a network. First, the information should be transferred in a secret and efficient manner. Second, the quality of service (QoS), i.e. delay sensitivity, should be taken into consideration to satisfy the demands of real-time wireless services. In order to fulfill the two requirements, we combine the physical layer security technique based on switched beam beamforming with an energy-efficient power allocation. The problem is formulated as the maximization of the secrecy energy efficiency subject to delay and power constraints. By solving the optimization problem, we derive an energy-efficient power allocation scheme. Numerical results validate the effectiveness of the proposed scheme.

Index Terms—Physical layer security, energy-efficient power allocation, switched beam beamforming, QoS guarantee.

I. INTRODUCTION

Without doubt, information security is a critical issue of wireless communications due to the open nature of wireless channel. Traditionally, information security is realized by using cryptography technology. In fact, information theory has proven that if the eavesdropper channel is degraded, secure communication can be guaranteed by only using physical layer technology, namely physical layer security [1] [2], even if the eavesdropper has strong computation capabilities.

The essence of physical layer security is to maximize the secrecy rate. If there are multiple antennas at the information source, transmit beamforming can be developed to improve the legitimate channel capacity and to degrade the eavesdropper channel capacity, so the achievable secrecy rate is increased [3] [4]. In [5], the problem of optimal beamforming was addressed by maximizing the secrecy rate. A potential drawback of the above approach lies in that the source must have full channel state information (CSI) to design the transmit beam. To alleviate the assumption, a joint power allocation and beamforming scheme was proposed based on the full CSI of the legitimate channel and the partial CSI of the eavesdropper channel [6]. Yet, the CSI is difficult to obtain for the source, especially the CSI of the eavesdropper channel. It is proved that if there is no CSI of the eavesdropper channel, the beamforming along the direction of the legitimate channel is optimal [7]. Thus, the authors in [7] proposed to convey the quantized CSI of the legitimate channel for beamforming, and derive a performance upper bound as a function of the feedback amount. Since the source has no knowledge of the eavesdropper channel that varies randomly, it is impossible to provide a steady secrecy rate. In this case, the secrecy outage capacity is adopted as a useful and intuitive metric to evaluate security, which is defined as the maximum rate under the constraint that the outage probability that the real transmission rate is greater than secrecy capacity is equal to a given value [8]. In multi-antenna systems, the switched beam beamforming is a popular limited feedback beamforming scheme because of its low complexity, small overhead and good performance [9]. In this study, we propose to adopt the switched beam beamforming to increase the capacity of the legitimate channel, and success in deriving the closed-formed expression of the outage probability in terms of the secrecy outage capacity.

Recently, the increasing interest in various advanced wireless services results in an urgent demand for the communications with quality of service (QoS) guarantee, such as delay sensitivity for the video transmission [10]. Many previous analogous works focus on the maximization of secrecy outage capacity with the QoS requirement, in which it is optimal to use the maximum available power. Lately, energy-efficient wireless communication [11], namely green communications receives considerable attentions due to energy shortage and greenhouse effect. In [12], energy-efficient resource allocation for secure OFDMA downlink network was studied. Therefore, we also expect to maximize the secrecy energy efficiency, namely the number of transmission bits per Joule, while meeting delay and power constraints. By solving this problem, we derive an energy-efficient adaptive power allocation scheme according to the channel condition and performance requirement.

The rest of this letter is organized as follows. We first give an overview of the secure multi-antenna network in Section II, and then derive an energy-efficient power allocation scheme by maximizing the secrecy energy efficiency while satisfying the delay and power constraints in Section III. In Section IV, we present some numerical results to validate the effectiveness of the proposed scheme. Finally, we conclude the whole paper in Section V.
II. System Model

We consider a multi-antenna downlink network, where a base station (BS) with $N_t$ antennas communicates with a single antenna secure user (SU), while a single antenna eavesdropper also receives the signal from the BS and tries to detect it. We use $\alpha_t \textbf{h}$ to denote the $N_t$ dimensional legitimate channel vector from the BS to the SU, where $\alpha_t$ is the channel large-scale fading component, including path loss and shadow fading, and $\textbf{h}$ represents the channel small-scale fading component, which is a circularly symmetric complex Gaussian (CSCG) random vector with zero mean and unit variance. Similarly, we use $\alpha_e \textbf{g}$ to denote the $N_t$ dimensional eavesdropper channel vector from the BS to the eavesdropper, where $\alpha_e$ and $\textbf{g}$ are the large-scale and small-scale channel fading components, respectively. The network is operated in time slots. We assume that $\alpha_t$ and $\alpha_e$ remain constant during a relatively long time period due to their slow fading, while $\textbf{h}$ and $\textbf{g}$ remain constant in a time slot and independently fade slot by slot. At the beginning of each time slot, the SU selects an optimal column vector from a predetermined $N_t \times N_t$ unitary matrix $\mathbb{W} = \{\textbf{w}_1, \textbf{w}_2, \cdots, \textbf{w}_{N_t}\}$, where $\textbf{w}_i$ is the $i$-th column vector, according to the following criteria:

$$i^* = \arg \max_{1 \leq i \leq N_t} |\textbf{h}^H \textbf{w}_i|^2 \quad (1)$$

Then, the SU conveys the index $i^*$ to the BS via the feedback link, and the BS performs beamforming to the predetermined signal by using $\textbf{w}_{i^*}$, namely switched beam beamforming. Thus, the receive signals at the SU and the eavesdropper are given by

$$y_s = \sqrt{P_\alpha} \textbf{h}^H \textbf{w}_{i^*} x + n_s \quad (2)$$

and

$$y_e = \sqrt{P_\alpha} \textbf{g}^H \textbf{w}_{i^*} x + n_e \quad (3)$$

respectively, where $x$ is the Gaussian distributed transmit signal with unit variance, $P$ is the transmit power, $n_s$ and $n_e$ are the additive Gaussian white noises with unit variance at the SU and the eavesdropper, respectively. Hence, the capacities of the legitimate and eavesdropper channels can be expressed as

$$C_s = W \log_2(1 + \gamma_s) \quad (4)$$

and

$$C_e = W \log_2(1 + \gamma_e) \quad (5)$$

where $W$ is the spectrum bandwidth, $\gamma_s = P_{\alpha_s} |\textbf{h}^H \textbf{w}_{i^*}|^2$ and $\gamma_e = P_{\alpha_e} |\textbf{g}^H \textbf{w}_{i^*}|^2$ are the signal-to-noise ratio (SNR) at the SU and the eavesdropper, respectively. Therefore, from the perspective of information theory, the secrecy capacity is given by $C_{sec} = [C_s - C_e]^+$, where $[x]^+ = \max(x, 0)$. Since there are no knowledge of the eavesdropper channel at the BS, it is impossible to provide a steady secrecy capacity. In this letter, we take the secrecy outage capacity $R_{sec}$ as the performance metric, which is defined as the maximum rate under the condition that the outage probability that the transmission rate surpasses the secrecy capacity is equal to a given value $\varepsilon$, namely

$$P_r \left( R_{sec} > C_s - C_e \right) = \varepsilon \quad (6)$$

Substituting (4) and (5) into (6), the outage probability can be transformed as

$$\varepsilon = P_r \left( \gamma_s < 2^{R_{sec}/W} (1 + \gamma_e) - 1 \right) = \int_0^\infty \int_0^{2^{R_{sec}/W} (1 + \gamma_e) - 1} f_{\gamma_s}(x) f_{\gamma_e}(y) dy dx$$

$$= \int_0^\infty F_{\gamma_e} \left( 2^{R_{sec}/W} (1 + y) - 1 \right) f_{\gamma_e}(y) dy \quad (7)$$

where $f_{\gamma_s}(y)$ is the probability density function (pdf) of $\gamma_s$, $f_{\gamma_e}(x)$ and $F_{\gamma_e}(x)$ are the pdf and cumulative distribution function (cdf) of $\gamma_e$, respectively. Since $\textbf{w}_{i^*}$ is independent of $\textbf{g}$, $|\textbf{g}^H \textbf{w}_{i^*}|^2$ is exponentially distributed. Thus, we have

$$f_{\gamma_e}(y) = \frac{1}{P_{\alpha_e}^2} \exp \left( -\frac{y}{P_{\alpha_e}^2} \right) \quad (8)$$

Similarly, $|\textbf{h}^H \textbf{w}_{i^*}|^2$ can be considered as the maximum one of $N_t$ independent exponentially distributed random variables caused by beam selection, so we have

$$F_{\gamma_s}(x) = \left( 1 - \exp \left( -\frac{x}{P_{\alpha_s}^2} \right) \right)^{N_t} \quad (9)$$

Substituting (8) and (9) into (7), it is obtained that

$$\varepsilon = 1 + \sum_{n=1}^{N_t} \frac{N_t}{n} (-1)^n \left( \frac{1}{1 + n 2^{R_{sec}/W} P_{\alpha_e}^2 / P_{\alpha_s}^2} \right) \exp \left( \frac{n (2^{R_{sec}/W} - 1)}{P_{\alpha_s}^2} \right) = G(R_{sec}, P) \quad (10)$$

Intuitively, $G(R_{sec}, P)$ is a monotonically increasing function of $R_{sec}$ and a monotonically decreasing function of $P$. Thus, given transmit power $P$ and the requirement of outage probability $\varepsilon$, the secrecy outage capacity can be derived by computing $R_{sec} = G^{-1}(\varepsilon, P)$, where $G^{-1}(\varepsilon, P)$ is the inverse function of $G$.
function of $G(R_{sec}, P)$. Moreover, from (10), we can obtain the probability of positive secrecy probability as 

$$P_r(C_{sec} > 0) = 1 - G(0, P)$$

$$= 1 - \frac{\alpha_r^2}{\alpha_e^2} B \left( \frac{\alpha_r^2}{\alpha_e^2} N_t + 1 \right)$$ (11)

where $B(x, y)$ is the Beta function. Interestingly, it is found that the probability $P_r(C_{sec} > 0)$ is independent of $P$. As $N_t$ increases, the probability increases accordingly, because more array gains can be obtained from the switched beam beamforming. Furthermore, (11) reveals that the short accessing distance of the SU is benefit to enhance the information secrecy.

Since most of wireless services are delay sensitive, we take the delay constraint into consideration. It is assumed that the data for the SU is arrived in the form of packet of fixed length $N_0$ bits with average arrival rate $\lambda$ (packets per slot), and it has the minimum average delay requirements $D$ (slots) related to its service style. Following [13], in order to satisfy the delay constraint, the secrecy outage capacity must meet the following condition:

$$R_{sec} \geq \frac{2D\lambda + 2 + \sqrt{(2D\lambda + 2)^2 - 8D\lambda N_0}}{4D} T = C_{min}$$ (12)

where $T$ is the length of a time slot.

III. ENERGY-EFFICIENT POWER ALLOCATION

Considering the limitation of energy resource and the requirement of green communication, energy efficiency becomes an important performance metric in wireless communications. In this section, we attempt to derive a power allocation scheme to maximize the secrecy energy efficiency while satisfying the delay and power constraints, which is equivalent to the following optimization problem:

$$J_1 : \max \frac{R_{sec}}{P_0 + P}$$ (13)

s.t. $G(R_{sec}, P) \leq \varepsilon$  \hspace{0.5cm} (14)

$R_{sec} \geq C_{min}$  \hspace{0.5cm} (15)

$P \leq P_{max}$  \hspace{0.5cm} (16)

where $P$ is the power consumption in the power amplifier, and $P_0$ is the power for the work regardless of information transmission, such as the circuit power. (13) is the so called energy efficiency, defined as the number of transmission bits per Joule. (14) is used to fulfill the secrecy constraint based on physical layer security, and (15) is the delay constraint, where $C_{min}$ is determined by both the data arrival rate $\lambda$ and the delay requirement $D$ as shown in (12). $P_{max}$ is the constraint on maximum transmit power. Since $G(R_{sec}, P)$ is a monotonically increasing function of $R_{sec}$ and a decreasing function of $P$, the condition of $G(R_{sec}, P) = \varepsilon$ is optimal in the sense of maximizing the secrecy energy efficiency. Thus, (14) can be canceled and $R_{sec}$ can be replaced by $G^{-1}(\varepsilon, P)$. Notice that there may be no feasible solution for $J_1$, due to the stringent secrecy and delay constraints. Under such a condition, in order to obtain a solution, we have to relax the constraint on the outage probability, average delay or transmit power.

The objective function (13) in a fractional program is a ratio of two functions of the optimization variable $P$, resulting in $J_1$ is a fractional programming problem, which is in general nonconvex. Following [12], the objective function is equivalent to $G^{-1}(\varepsilon, P) - q^*(P_0 + P)$ by exploiting the properties of fractional programming, where $q^*$ is the secrecy energy efficiency when $P$ is equal to the optimal power $P^*$ of $J_1$, namely $q^* = G^{-1}(\varepsilon, P^*)/(P_0 + P^*)$. Thus, $J_1$ is transformed as 

$$J_2 : \max \frac{G^{-1}(\varepsilon, P) - q^*(P_0 + P)}{\min_{\mu, \nu}}$$ (17)

s.t. $G^{-1}(\varepsilon, P) \geq C_{min}$  \hspace{0.5cm} (18)

$P \leq P_{max}$  \hspace{0.5cm} (19)

$J_2$, as a convex optimization problem, can be solved by the Lagrange multiplier method. By some arrangement, its Lagrange dual function can be written as

$$\mathcal{L}(\mu, \nu, P) = G^{-1}(\varepsilon, P) - q^*(P_0 + P) + \mu G^{-1}(\varepsilon, P) - \mu C_{min} - \nu P + \nu P_{max}$$ (20)

where $\mu \geq 0$ and $\nu \geq 0$ are the Lagrange multipliers corresponding to the constraint (13) and (19), respectively. Therefore, the dual problem of $J_2$ is given by

$$\min_{\mu, \nu} \max_{P} \mathcal{L}(\mu, \nu, P)$$ (21)

For the given $\mu$ and $\nu$, the optimal power $P^*$ can be derived by solving the following KKT condition

$$\frac{\partial \mathcal{L}(\mu, \nu, P)}{\partial P} = (1 + \mu) \frac{\partial G^{-1}(\varepsilon, P)}{\partial P} - q^* - \nu = 0$$ (22)

Note that if $P^*$ is negative, we should let $P^* = 0$. Moreover, $\mu$ and $\nu$ can be updated by the gradient method, which are given by

$$\mu(n + 1) = [\mu(n) - \Delta_{\mu}(G^{-1}(\varepsilon, P) - C_{min})]^+$$ (23)

$$\nu(n + 1) = [\nu(n) - \Delta_{\nu}(G_{max} - P)]^+$$ (24)

where $n$ is the iteration index, and $\Delta_{\mu}$ and $\Delta_{\nu}$ are the positive iteration steps. Inspired by the Dinkelbach method [14], we propose an iterative algorithm as follows

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**Algorithm 1: Energy-Efficient Power Allocation**

1) Initialization: Given $N_t$, $T$, $W$, $\alpha_s$, $\alpha_e$, $C_{min}$, $P_0$, $P_{max}$, $\Delta_{\mu}$, $\Delta_{\nu}$ and $\varepsilon$. Let $\mu = 0$, $\nu = 0$, $P = 0$ and $q^* = G^{-1}(\varepsilon, P)/(P_0 + P)$. $\varepsilon$ is a sufficiently small positive real number.

2) Update $\mu$ and $\nu$ according to (23) and (24), respectively.

3) Computing the optimal $P^*$ by solving the equation (22) using some math tools, such as *Mathematics* and *Matlab*.

4) If $G^{-1}(\varepsilon, P^*) - q^*(P_0 + P^*) > \varepsilon$, then set $q^* = G^{-1}(\varepsilon, P^*)/(P_0 + P^*)$, and go to 2). Otherwise, $P^*$ is the optimal transmit power.
IV. Numerical Results

To examine the effectiveness of the proposed energy-efficient power allocation scheme, we present several numerical results in the following scenarios: we set $N_t = 4$, $W = 10$KHz, $C_{\min} = 0.8$Kb/s, $P_0 = 0.5$Watt and $P_{\max} = 10$Watt. $\alpha^2$ is normalized to 1, and we use $\rho = \alpha^2 e / \alpha^2$ to denote the relative large-scale fading of the eavesdropper channel. It is found that the proposed energy-efficient power allocation scheme converges after no more than 20 times iterative computation in all simulation scenarios.

Fig. 2 compares the secrecy energy efficiency of the proposed adaptive power allocation scheme and the fixed power allocation scheme with $\epsilon = 0.05$. Intuitively, it is optimal to use $P_{\max}$ as the transmit power in the sense of maximizing the secrecy outage capacity, so we set $P = P_{\max}$ for the fixed power allocation scheme. As seen in Fig. 2, the proposed scheme performs better than the fixed one, especially when $\rho$ is small. For example, when $\rho = 0.10$, there is about 2Kb/J gain. Therefore, the proposed scheme is more suitable for the future green and secure communications. It is found that when $\rho = 0.25$, the secrecy energy efficiency reduces to zero. This is because there is no nonzero secrecy outage capacity under such constraint conditions. In order to support the case with the large $\rho$, we need to relax the constraint conditions or deploy more antennas at the BS to obtain more array gain.

Fig. 3 investigates the effect of the requirements of outage probability on the secrecy power efficiency of the proposed scheme. For a given $\rho$, as $\epsilon$ decreases, the secrecy energy efficiency reduces accordingly, this is because more power is used to decrease the outage probability. On the other hand, for a given requirement of the outage probability, the increase of $\rho$ leads to the decrease of the secrecy energy efficiency, since the eavesdropper has a strong eavesdropping ability.

V. Conclusion

A major contribution of this paper is the introduction of an energy-efficient power allocation scheme into a multi-antenna downlink network employing physical layer security with delay guarantee. Considering the importance of the CSI in multi-antenna networks, the switched beam beamforming is adopted to realize the adaptive transmission. Numerical results confirm the effectiveness of the proposed scheme. In future works, we will further study the cases with multi-antenna eavesdropper, imperfect CSI, robust beamforming, etc.

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