\[ d_{x^2-y^2} \] superconductivity in a generalized Hubbard model.

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We consider an extended Hubbard model with nearest-neighbor correlated hopping and next nearest-neighbor hopping \( t' \) obtained as an effective model for cuprate superconductors. Using a generalized Hartree-Fock BCS approximation, we find that for high enough \( t' \) and doping, antiferromagnetism is destroyed and the system exhibits \( d \)-wave superconductivity. Near optimal doping we consider the effect of antiferromagnetic spin fluctuations on the normal self-energy using a phenomenological susceptibility. The resulting superconducting critical temperature as a function of doping is in good agreement with experiment.

I. INTRODUCTION

One of the most challenging properties that any effective one-band model for the high superconducting critical temperature (high-\( T_c \)) superconducting materials should explain is the experimentally observed dependence of \( T_c \) with doping (\( x \)). Extended \( t-J \) models seem to be successful to interpret several important properties of these materials \([1,3]\). They were derived as effective Hamiltonians in the strong-coupling limit (very large \( U \)). While numerical studies in \( t-J \)-like models unambiguously indicate that they exhibit sizable superconducting correlations with \( d \)-wave symmetry, in agreement with experiments \([2]\), this is not the case of the usual Hubbard model \([10]\). Apart from quantitative reasons concerning the strength of the local Coulomb repulsion, it is of interest to study an accurate enough effective model amenable to weak coupling many-body treatments. In particular, the observed pseudogap behavior for temperatures \( T_c < T < T^* \) in angle-resolved photoemission spectroscopy (ARPES) experiments \([11]\) has been interpreted as a precursor effect of the antiferromagnetic (AF) \([12]\) as well as the superconducting state \([13,14]\) between other possible scenarios \([15,16]\). For the attractive (negative \( U \)) Hubbard model, a good deal of research helped to elucidate the nature of the superconducting transition \([17]\) and most of this work is based on one appealing feature of this model: its explicit attractive interaction leading to a superconducting state in the Bardeen-Cooper-Schrieffer (BCS) approximation. Thus, even though the symmetry of the BCS gap is \( s \)-wave, this model has been used for a qualitative understanding of ARPES experiments \([13]\). Due to the lack of a genuine microscopic \( d \)-wave superconducting analogues of the of the negative-\( U \) Hubbard model, some phenomenological models with BCS-like interactions were studied \([14,15,19]\). While the model proposed in Ref. \([18]\) is built up by pure phenomenological interactions with several free parameters, in order to fit the experimentally observed \( T_c \) vs \( x \) curve, the AF and van Hove picture \([14]\) has been proposed on the basis of a simplification of the \( t-J \) model in which holes are constrained to move only in one sublattice (up or down) in a Neel spin background. Nevertheless, this is an oversimplification for realistic \( J < t \) \([20]\). The assumption of pairing coexisting with long-range AF order is shared with the strong-coupling polaron picture \([3]\), which also reproduces qualitatively the observed \( T_c \) vs \( x \) curve. However, nuclear magnetic resonance experiments indicate that the coherence length of the AF correlations \( \xi \) is of only a few lattice sites in the optimally doped materials \([21]\). An effective attractive separable potential has also been considered to study the \( d \)-wave superconducting pseudogap evolution \([14]\). In this case, however, the AF correlations are not taken into account at all, while they play a relevant role for underdoped and optimally doped materials \([12,22]\).

In this work, we calculate \( T_c \) vs \( x \) of an effective one-band model for the high \( T_c \) cuprates using weak-coupling techniques. We obtain beyond a certain doping, a stable paramagnetic phase with \( d \)-wave superconductivity (DWS). The mechanism for superconductivity is the electron-hole symmetric correlated hopping studied previously in one and two dimensions (2D) \([23,24]\). However, previously in 2D, the next-nearest-neighbor hopping \( t' \) was neglected and only \( s \)-wave superconductivity was found for doping high enough to inhibit long-range antiferromagnetism. In simplified terms, this can be understood as follows: on-site Coulomb repulsion \( U \) inhibits on-site pairs, but nearest-neighbor singlet pairs are favored by the correlated hopping. In mean field, the dependence of this effective attraction in reciprocal space is proportional to \((\cos k_x - \cos k_y)^2\) for \( d \) symmetry and to
where $<ij>$ ($<ij'>$) denotes nearest-neighbor (next-nearest-neighbor) positions of the lattice. Note that $U$ is not directly related with a Coulomb repulsion, but represents the cost in energy of constructing a Zhang-Rice singlet from two singly occupied cells. It is lower than the difference between $p$ and $d$ on-site energies of $H_{3b}$ (usually called charge-transfer energy $\Delta$). $t_{AA}$ represents the hopping of a Zhang-Rice singlet to a singly occupied nearest-neighbor cell. The terms with amplitude $t_{AB}$ correspond to destruction of a Zhang-Rice singlet and a nearest-neighbor cell without holes, creating two singly occupied cells and vice versa. $t_{BB}$ describes the movement of an isolated hole in $H_{3b}$. Clearly, the amplitude of these three correlated hopping processes should differ in general. The dependence of the next-nearest-neighbor hopping on the occupation of the sites involved in the hopping is neglected. Note that in the simplest strong-coupling derivation leading to the $t-J$ model $\[4\]$, $t_{AA}$ amounts to the only kinetic term in the model, the one with $t_{BB}$ is mapped out of the relevant Hilbert-space while the one with $t_{AB}$ is treated in second order of perturbation theory to define the exchange term $\[5\]$. More systematic strong-coupling derivations contain other terms as well as higher-order corrections $\[6\]$. For $t' = 0$, the occurrence of metal-insulator transitions and superconductivity have been investigated for particular cases of this model $\[23\]$, but a stable DWS phase has not been found before. We concentrate on the parameter regime $t_{AB} > t_{AA} = t_{BB} = t$, which seems more adequate for the cuprates $\[24\]$. The effect of $t'$ is crucial to stabilize the $d$-wave phase, because it removes the perfect nesting at half-filling and shifts the energy of the van Hove singularity (VHS) in the unper- turbed density of states (originated by the saddle points at $X$ in the dispersion relation) away from the Fermi energy at half-filling.

III. MEAN-FIELD APPROACH.

The correlated hopping terms of the Hamiltonian $\[4\]$ can be separated in one- two- and three-body contributions, with coefficients $t = t_{AA}, t_2 = t_{AA} - t_{AB}$ and $t_3 = 2t_{AB} - t_{AA} - t_{BB}$ respectively $\[24\]$. We treat Eq. $\[4\]$ within the generalized HF BCS approximation $\[24\]$. The term in $t_3$ contributes to the BCS solution in the $d$-wave as well as in the $s$-wave channels. The self-consistent parameters considered in the decoupling are $\langle n_{i\uparrow} \rangle - \langle n_{i\downarrow} \rangle = ne^{Q}\mathbf{R}_i, \tau = \langle c_{i+\delta\uparrow\sigma}^{\dagger} c_{i\sigma} \rangle, \psi = \langle c_{i\uparrow\downarrow}^{\dagger} c_{i\downarrow}^{\dagger} \rangle$ and $\varphi_{s} = \langle c_{i+\delta\uparrow\sigma}^{\dagger} c_{i\sigma} \rangle$, with $\delta = x, y$ being the movement of $s$ and $\varphi_{s}$ being vectors connecting nearest neighbors and $\varphi_{s} = \pm \varphi_{y}$ $\[24\]$. $n = n_{\uparrow} + n_{\downarrow} = 1 - x$ is the particle density. $L$ is the number of lattice-sites, $Q = (\pi, \pi)$ and $\mathbf{R}_i$ indicates the lattice-position. The problem is reduced to a one-particle one with three possibilities for the symmetry breaking perturbation: (i) $AF$
spin density wave (SDW) (with \( m \neq 0 \) and \( \psi = \varphi_x = 0 \)), (ii) DWS (with \( m = \psi = 0 \) and \( \varphi_x = -\varphi_y \neq 0 \)), and (iii) extended \( s \)-wave superconductivity (SWS) (with \( m = 0 \), \( \psi \neq 0 \) and \( \varphi_x = \varphi_y \neq 0 \)). The possibility of coexistence of SDW and superconductivity is left out here, since a previous study indicated that a sizeable superconducting gap is not possible within our model in presence of long-range antiferromagnetism \[26\]. For the three cases, the renormalized dispersion relation, effective hopping and effective chemical potential can be written in the form:

\[
\epsilon_k = -2t_{\text{eff}}(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y, \\
t_{\text{eff}} = t - t_2n + t_3[3\tau^2 + \psi^2 + \varphi_x^2 - (n^2 - m^2)/4] \\
\mu_{\text{eff}} = \mu - (Un/2 + 8t_2\tau + 4t_3[\tau n + \psi(\varphi_x + \varphi_y)]).
\]

The SDW, \( d \)-wave, and \( s \)-wave BCS order parameters are:

\[
\Delta^{SDW} = (U/2 + 4t_3\tau) m \\
\Delta^{BCS} = 4t_3\tau \varphi_x (\cos k_x - \cos k_y), \\
\Delta^{SWS} = (8t_3\tau - U) \psi - 4(2t_2 + nt_3) \varphi_x \\
+ [4t_3\tau \varphi_x - 2(2t_2 + nt_3)\psi] \\
(\cos k_x + \cos k_y), \tag{3}
\]

respectively. For a given wave vector \( \mathbf{k} \), they coincide with half of the corresponding energy gap. For both, \( d \)-and \( s \)-wave superconducting solutions, the dependence of \( T_c \) vs \( x \) is obtained from the linearized BCS gap equations. For the \( d \)-wave case, \( T_c \) is given by:

\[
1 = \left( \frac{d^2k}{2\pi^2} \right) \frac{\tanh(\frac{\epsilon_k}{2T})}{2\epsilon_k} \frac{4t_3\tau (\cos k_x - \cos k_y)}{2\epsilon_k} \cos k_x, \tag{4}
\]

where \( \epsilon_k = \epsilon_k - \mu_{\text{eff}} \), and \( T \) is the temperature. For the \( s \)-wave solution, \( T_c \) is the temperature at which \( \lambda_{\text{max}} \) equals one, being \( \lambda_{\text{max}} \) the largest eigenvalue of the matrix

\[
\begin{pmatrix}
\int \frac{d^2k}{(2\pi)^2} \tan(\frac{\epsilon_k}{2T}) \alpha_k \cos k_x / 2\epsilon_k & \int \frac{d^2k}{(2\pi)^2} \tan(\frac{\epsilon_k}{2T}) \beta_k \cos k_x / 2\epsilon_k \\
\int \frac{d^2k}{(2\pi)^2} \tan(\frac{\epsilon_k}{2T}) \alpha_k \cos k_y / 2\epsilon_k & \int \frac{d^2k}{(2\pi)^2} \tan(\frac{\epsilon_k}{2T}) \beta_k \cos k_y / 2\epsilon_k
\end{pmatrix}, \tag{5}
\]

where

\[
\alpha_k = 8t_3\tau - U - 2(2t_2 + nt_3)(\cos k_x + \cos k_y), \\
\beta_k = -4(2t_2 + nt_3) + 4t_3\tau (\cos k_x + \cos k_y), \tag{6}
\]

are the coefficients of \( \psi \) and \( \varphi_x \) in \( \Delta^{BCS} \). This method of obtaining \( T_c \) in second-order phase transitions when the thermodynamic potential \( \Omega \) depends on more than one parameter (here \( \psi \) and \( \varphi_x \)), has been used before \[27\], and is equivalent to the usual one of finding the first instability (as \( T \) is lowered) of the Hessian matrix formed by the second derivatives of \( \Omega \) with respect to the independent variables \[28\].

**FIG. 1.** \( d \)-wave BCS- critical temperature \( T_c \) in units of \( t \) as a function of the doping \( x \) (for \( x < 0.5 \)), for \( t_{AB} = 2t \) (open symbols) and \( t_{AB} = 1.5t \) (filled symbols). Circles, squares and triangles correspond to \( t' = 0 \), \(-0.2t \), \(-0.45t \), respectively. The solid symbols for \( x > 0.5 \) show \( T_c \) of the \( s \)-wave solution for \( t_{AB} = 1.5t \) and the same values of \( t' \) as those considered for the \( d \)-wave. The dot (dot-dashed) line indicates the boundary of the SDW at \( T = 0 \) for \( U = 4t \) \( (U = 6t) \), \( t_{AB} = 1.5t \) \( (t_{AB} = 2t) \) and \( t' = -0.45t \).

In Fig. 1, we show the superconducting critical temperature as a function of doping \( x = 1 - n \), where \( n \) is the number of electrons per site for both, \( d \)-wave and \( s \)-wave solutions. Most of the pairing terms, as well as the additional contribution to the usual \( U/2 \) term in the expression of the SDW gap, can be generated from the mean-field decoupling of an effective two-body interaction of the form \[25\]

\[
V_{\text{eff}} = t_3\tau \sum_{<ij>} (4s_i \cdot s_j + n_in_j). \tag{7}
\]

Paring in both \( d \)-wave and extended \( s \)-wave channels, is originated by the spin-flip terms of \( V_{\text{eff}} \). For the \( s \)-wave case, there are some additional contributions, as can be
observed in (3). From mere inspection of the three possible order parameters (3), it can be seen that \( \Delta^{SDW} \) and \( \Delta^{BCS} \) depend on \( U \) in such a way that the antiferromagnetic (s-wave-BCS) solution is enhanced (weakened) as \( U \) increases, whereas \( \Delta^{BCS} \) does not depend on \( U \). Thus, the doping region where the \( d \)-wave BCS solution exists is limited only by the difference in energy with the two other competing instabilities of the Fermi liquid. As mentioned in Section I, due to the wave-vector dependence of the superconducting order parameters, the shape of the Fermi-surface plays an important role in defining the doping regions at which each of the two BCS solutions are possible. In Ref. 24 we have shown that for \( t' = 0 \) and assuming a constant density of states, SWS exists for high enough \( x \) and low enough \( U \), while DWS exists near half-filling. Within this region, however, the SDW solution has lower energy than the DWS one, being the difference between the energy of both solutions small for \( U \sim 0 \) (as can be inferred from the form of the gaps, Eqs (3)). When \( t' \) is turned on, the VHS is displaced, lying at the Fermi energy for a finite doping. The increment in the density of states at finite doping enhances the critical temperature as well as the doping range associated to the s-wave solution. However, as a consequence of the vector dependence of the effective interaction, the optimal doping for SWS is always high. In Fig. 1, it is shown that for \( t_{AB} = 1.5 \) and \( U = 4t \), the doping region with SWS is \( x > 0.6 \), i.e. beyond the range of doping accessible experimentally in the cuprates. For the \( d \)-wave solution, the maximum of \( T_c \) occurs when the chemical potential coincides with the energy of the VHS. Due to the loss of perfect nesting and the closing of the indirect SDW gap for this doping, the SDW is weakened. Thus, even for large \( U \), for sizeable \( t' \) there exists a doping range for which the \( d \)-wave BCS solution is stable and robust. We indicate in the figure the doping region for the existence of the SDW solution \((m \neq 0)\) at \( T = 0 \) for the particular values \( U = 4t \) \((U = 6t)\), \( t_{AB} = 1.5t \), \( t_{AB} = 2t \) and \( t' = -0.45t \). The Neel temperature at which \( m \) becomes zero is not calculated here since: it is determined by spin fluctuations, and the mean-field value is too large. For nearly realistic bare parameters, the doping range for an stable DWS (beyond the dot-dashed line in Fig. 1) corresponds to the overdoped regime of the cuprates, in agreement with experiments on B2212 [11] which indicate that the superconducting state takes place in an otherwise ordinary Fermi liquid for overdoped compounds.

In the optimally doped regime, spin fluctuations not only affect directly the SDW order parameter, destroying the AF long-range order, but also affect indirectly the superconducting order parameters. Their effect should be included in a more realistic calculation of the superconducting critical temperature. This is the goal of the next section.

IV. THE EFFECT OF SPIN FLUCTUATIONS

Long-range antiferromagnetism is not expected to survive within the whole range of doping predicted by the mean-field approach. For doping larger than \( x \sim 0.05 - 0.1 \), the spin fluctuations are important and a Fermi liquid with strong AFSF is a more appropriate picture for this regime [21][23]. AFSF of any Hubbard-like microscopic model as the one considered here can be characterized by a spin susceptibility of the form

\[
\chi(q, \omega) = \frac{\chi(Q, 0)}{1 + (q - Q)^2 \xi^2 - i\omega/\omega_{sf}},
\]

where the parameters scale with the AF correlation length \( \xi \) as \( \chi(Q, 0) \sim \xi^2 \), \( \omega_{sf} = \Gamma_0 \xi^2 \), with \( \Gamma_0 \sim 40\text{meV} \) [21][29]. Assuming a phenomenological spin-fermion coupling, AFSF has been suggested to mediate the pairing in the cuprates [23]. This picture is supported by the observed correlation between \( \xi \) and \( T_c \) in several superconducting cuprates. Within the weak-coupling formalism, DWS due to correlated hopping is inhibited in a background with long-range AF correlations [24]. However, the effective interaction Eq. (6) provides an explicit channel for the coupling with collective AFSF within the doping region without AF long-range order, which has the same form as the phenomenological coupling used in Ref. 21. Thus, it might be expected that the AFSF would renormalize the bare value of \( t_{AB} \) (< 1.5t) for realistic values of the three-band parameters [11] to higher ones [22][23]. In what follows, we investigate how the BCS- \( T_c \) vs \( x \) dependence is modified by the effect of the AFSF for the \( d \)-wave solution.

For finite \( t' \), the Fermi surface contains hot spots (for which \( \epsilon_k = \epsilon_{k+Q} \)). Fermions located in the neighborhood of these points are the most affected by AF correlations and exhibit a peculiar T-dependence in the one-particle spectral properties, which is mainly determined by the magnitude of \( \xi/\xi_{th} \), and \( \omega_{sf}/T \) [22][29], with \( \xi_{th} = v_F/\xi \), being \( v_F \) the Fermi velocity. Hot spots are located at \( k_h \) near \( X \) ((0, \pi) and symmetry related points), i.e. near the antinodes of the DWS gap and with energies close to the VHS (for which \( v_F \sim 0 \)). As \( T_c \) vs \( x \) predicted by the BCS approximation, as well as the value of the maximum \( T_c \) itself, depend on the behavior of the density of states, AFSF are expected to play some further important role apart from the eventual renormalization of the effective pairing interaction. The self-energy obtained from (6) is

\[
\Sigma(k, i\omega_n) = T \sum_m \int \frac{d^2q}{(2\pi)^2} g(q) \tilde{g}(q) \chi(q, i\omega_n) G^S(k + q, i\omega_n + i\nu_m),
\]

\[
\chi(q, i\nu_m) = -\int_{-\omega_c}^{\omega_c} \frac{d\omega}{\pi} \text{Im} \chi(q, \omega) \frac{\omega_m - \omega}{\nu_m - \omega},
\]

where \([G^S(k, i\omega_n)]^{-1} = i\omega_n - \epsilon_k\), \( g(q) \) is an effective interaction between fermions and spin fluctuations, \( \omega_0 \)}
is a frequency cut off, $\nu_m = 2m\pi T$ and $\omega_n = (2n + 1)\pi T$. As usual, an effective coupling constant $g' \sim g(q)\delta(q)\chi(Q, 0)/\xi^2$ is defined, which in the present case should be proportional to $(U + 4t_3\tau)$ \cite{20}. The ensuing spectral function is $A(k, \omega) = -\text{Im}G(k, \omega)/\pi$, with $[G(k, \omega)]^{-1} = \omega - \epsilon_k - \Sigma(k, \omega)$. To examine how the changes in the behavior of $A(k, \omega)$ affect the $T_c$ vs $x$ dependence, we consider the effect of AF SF using the BCS form of the anomalous self energy $\Delta_{BCS}$ given by Eq. (3) and calculating the normal self energy in the one-loop approximation (Eq. 5). The resulting linearized gap equation,

$$\varphi_x = T \int \frac{d^2k}{(2\pi)^2} \Delta_{BCS} \cos k_x \sum_n e^{i\omega_n 0^+} G(k, i\omega_n) G(-k, -i\omega_n),$$  \hspace{1cm} (10)

can be cast in real frequency as

$$\varphi_x = - \int \frac{d^2k}{(2\pi)^2} \Delta_{BCS} \cos k_x \int d\omega d\omega' A(k, \omega)A(-k, \omega') \frac{\tanh(\omega T/2)}{\omega + \omega'},$$  \hspace{1cm} (11)

where $n_F(\omega) = 1/(1 + \exp((\omega - \mu)/T))$. Eq. (11) reduces to the linearized version of the usual BCS Eq. (2) when $A(k, \omega) = \delta(\omega - \epsilon_k)$. As discussed in previous works, assuming $\xi$ approximately constant or with a weak $T$-dependence, two different regimes due to the AFSF can be distinguished as a function of $T$ in the behavior of $A(k, \omega)$ \cite{23,24}: (i) For $T << \omega_{sf}$, quantum contributions dominate the behavior of the self-energy and $A(k, \omega)$ exhibits Fermi liquid-like quasiparticle peaks, even for Fermi points near $k_{hs}$. Within this regime, no relevant qualitative changes in comparison with the BCS description are expected in the solution of the gap Eq. (11). (ii) For $T >> \omega_{sf}$, and for $k$-points satisfying $\xi >> \xi_{th}$, classical effects (introduced by the $m = 0$-Matsubara frequency in (3)) dominate. The AF SF can be considered as quasistatic and $A(k_{hs}, \omega)$ exhibits a shadow-band structure for large enough values of $g' \geq 2\pi 23$. This implies a transfer of spectral weight from low to high frequencies, and we expect an effective blurring of the large density of states near the VHS with a concomitant decrease of $T_c$.

The above qualitative issues are confirmed by our numerical calculations, as illustrated in Figure 2. We restrict ourselves to the case with finite $t' = -0.45t$, which reproduces the observed Fermi surface of YBCO and B2212 and to the case of $t_{AB} = 2t$, for which the optimal doping and the maximum $T_c$ predicted by the BCS approximation are in good agreement with experiments (for these parameters, the ratio $t'/t_{eff} \sim 0.27$). To compute $T_c$ we evaluate $\mu_{eff}$ and $\tau$ from

$$n = 1 + T \int \frac{d^2k}{(2\pi)^2} \sum_n e^{i\omega_n 0^+} \text{Re}G(k, i\omega_n),$$  \hspace{1cm} (12)

and afterwards check if there exists (or not) a solution of the linearized gap Eq. (10). A cut-off $\Omega_m = 432\omega_{sf}$ was chosen for the $\nu_m$-Matsubara summation in (10) while another one $\Omega_m = 46t$, assuming $t = 250meV$, was used for the $\omega_m$-Matsubara summation in (10) and (11). The sum over the tails was approximated by an Euler-McLaurin formula. The $k$-integral in Eq. (3) is calculated with a relative precision of $10^{-6}$, while a fixed finite mesh was used to evaluate the $k$-integrals in Eqs. (10) and (12).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Critical temperatures $T_c$ in units of $t$ as a function of the doping $x$, for $t_{AB} = 2t$ and $t' = -0.45t$, considering AF SF. The dashed line (crosses) corresponds to the BCS-solution in the thermodynamic limit (using $k$-discretization). Circles correspond to $g' = 3t$, and $\xi = 2.5$ (open ones) and $\xi = 10$ (filled ones). Squares correspond to $g' = 10t$ and $\xi = 2.5$ (open ones) and $\xi = 4$ (filled ones).}
\end{figure}

In the latter step, some precision is lost and the BCS-$T_c$ vs $x$, calculated in the thermodynamic limit, is not exactly recovered as indicated in Fig. 2. We considered $\xi = 2.5$, $(\omega_{sf} = 0.0256t)$ and $\xi = 4$, $(\omega_{sf} = 0.011t)$, which are supposed to be representative of YBCO and LaSrCuO, respectively \cite{21} near optimal doping, and $\xi = 10$, $(\omega_{sf} = 0.0016t)$ which could be realistic only for strongly underdoped materials. Taking $g' = 3t$ a
weak effect on $T_c$ is observed. For $g' = 10t$, which is expected to be representative of the cuprates [25,29], a decrease in $T_c$ is observed as $\xi$ increases. For the case of $\xi = 10, g' = 10t$ (not shown in the figure) the maximum $T_c$ is below 0.012t.

V. DISCUSSION

We have shown that for high enough next-nearest-neighbor hopping $t'$, d-wave superconductivity is stabilized for doping $x \sim 0.35$ at the mean-field level in the correlated hopping model [1]. The corresponding critical temperature $T_c$ has the right order of magnitude ($\sim 70 - 100 K$). Our calculations are in 2D and this result may be, of course, affected by the fluctuations of the superconducting state, which has not been taken into account in the present work. However, within the optimally doped to overdoped regime, the superfluid density is large and the critical temperature corrected by these latter fluctuations is expected to be close to the mean-field BCS one, even when the transition is Kosterlitz-Thouless rather than BCS-like [15]. For dopings accessible experimentally in the cuprates ($x < 0.5$) the s-wave superconducting instability is suppressed. Including the effect of antiferromagnetic spin fluctuations (AFSF), we find that three different regimes remain as a function of doping in the realistic range: (i) long-range antiferromagnetism near half filling, (ii) d-wave superconductivity in a pure Fermi liquid (usual BCS) scenario within the overdoped regime; (iii) d-wave superconductivity in the presence of AF SF within the underdoped to optimally doped regime. The maximum $T_c$ within our BCS treatment depends on the strength of the interaction and the position of the VHS. The effect of the spin-fluctuations, however, modifies the BCS picture within this regime. The effective pairing interaction of (1) has the form of an exchange coupling, being ineffective in the Neel state [26], but providing an explicit coupling in the regime of doping for which the AF correlations are short ranged. We considered particular values of the model parameters for which optimal doping, $T_c$ and shape of the Fermi surface are in agreement with experiments and further examined the effect of the non-trivial temperature behavior induced by the existence of hot spots in the Fermi surface in the presence of AFSF. We found that $T_c$ decreases as the correlation length increases in correspondence with experiments. For optimally doped materials, we expect that the magnitude of $T_c$ is dominated by the effect of spin fluctuations rather than by phase fluctuations of the superconducting order parameter. Details of the coupling with AFSF, renormalizations of the bare interactions, and eventual consequences upon the pseudogap behavior requires a treatment of the full T-matrix of the effective pairing interaction on equal footing with the spin susceptibility and is left for future studies.

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