CP Violation in $e^+e^- \rightarrow t\bar{t}$: Energy Asymmetries and Optimal Observables of $b$ and $\bar{b}$ Quarks

A. Bartl
_Institut für Theoretische Physik, Universität Wien, A-1090 Vienna, Austria_

E. Christova
_Institute of Nuclear Research and Nuclear Energy, Boul. Tzarigradsko Chaussee 72, Sofia 1784, Bulgaria_

T. Gajdosik, W. Majerotto
_Institut für Hochenergiephysik der Österreichischen Akademie der Wissenschaften, A-1050 Vienna, Austria_

Abstract
The energy spectra of the $b$ and $\bar{b}$ quarks in $e^+e^- \rightarrow t\bar{t}$ with $t \rightarrow bW$ ($\bar{t} \rightarrow \bar{b}W$) are calculated assuming CP violation in the production process. Expressions for the corresponding CP violating asymmetries and optimal observables that determine the imaginary parts of the electric and weak dipole moment form factors of the top quark are derived. It is shown that polarization of the initial lepton beams is essential to disentangle $\Im m d^{\gamma}(s)$ and $\Im m d^{Z}(s)$. Numerical results are given in the Minimal Supersymmetric Standard Model with complex phases.

PACS: 14.65.Ha, 11.30.Er, 11.30.Pb, 12.60.Jv
There has been considerable interest in testing physics beyond the Standard Model (SM) through possible observation of CP violation in processes with top quarks [1, 2, 3]. The reason is that the SM effects are strongly suppressed, while there are many models in which CP violation arises naturally at one–loop level. The most widely discussed examples are supersymmetric models [4] and models with more than one Higgs doublet [5]. In [3] we have studied CP violating angular asymmetries of the $b$ and $\bar{b}$ quarks in the processes:

$$e^+ + e^- \rightarrow t + \bar{t} \rightarrow b + \bar{t}W^+, \quad (1)$$
$$e^+ + e^- \rightarrow t + \bar{t} \rightarrow \bar{b} + tW^-. \quad (2)$$

In this article we shall discuss the possibility of measuring CP non conservation through the energy distributions of $b$ and $\bar{b}$ quarks from the top quark decays. We also consider the case of longitudinally polarized initial beams.

Various CP violating energy asymmetries of the final leptons from $t$ and $\bar{t}$ decays have been considered in [2]. We show that measurements of the energies of $b$ and $\bar{b}$ quarks provide another possibility of obtaining information about CP violation in the $t\bar{t}$ production process. CP non conservation in the $t\bar{t}$ production process is induced by the electric $d^\gamma(s)$ and weak $d^Z(s)$ dipole moment form factors in the $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices

$$ (V_\gamma)^\mu = \frac{2}{3} \gamma^\mu - i(P^\mu/m_t)d^\gamma(s)\gamma^5, \quad (3)$$
$$ (V_Z)^\mu = \gamma^\mu (g_V + g_A \gamma^5) - i(P^\mu/m_t)d^Z(s)\gamma^5, \quad (4)$$

where $g_V = (1/2) - (4/3) \sin^2 \Theta_W$, $g_A = -(1/2)$ are the SM couplings of $Z$ to $t\bar{t}$, $P_\mu = p_{t\mu} - p_{\bar{t}\mu}$, and $\Theta_W$ is the Weinberg angle.

General expressions for determining the real and imaginary parts of $d^\gamma(s)$ and $d^Z(s)$ separately have been obtained in [3]. We continue this study considering the energy distributions of $b$ and $\bar{b}$ that provide additional measurements of $\Im m d^\gamma(s)$ and $\Im m d^Z(s)$. We get analytic expressions for the energy distributions of $b$ and $\bar{b}$ and define model independent CP violating observables, in particular optimal observables. We give numerical results in the Minimal Supersymmetric Standard Model (MSSM) with complex parameters, using the results for $d^\gamma,Z(s)$ of ref. [6].

The energy distributions of the $b$ and $\bar{b}$ quarks can be derived from the differential cross section for (1) and (2) obtained in [3]. Choosing a frame where the $z$–axis
points into the direction of the top quark one obtains

\[ \frac{d\sigma^{b(b)}}{dx_{b(b)}} = \frac{\pi\alpha^2_{em}}{s} \frac{m_t^2}{m_t^2 - m_W^2} (c_0^{b(b)} + c_1^{b(b)} x_{b(b)}) , \] (5)

where \( \lambda \) and \( \lambda' \) denote the degree of longitudinal beam polarizations of \( e^- \) and \( e^+ \), respectively, and

\[ c_0^{b(b)} = N_{tot} + 4\alpha_b (G_3 (\pm) 2 \Im \, H_1) , \]
\[ c_1^{b(b)} = -4\alpha_b \frac{2m_t^2}{m_t^2 - m_W^2} (G_3 (\pm) 2 \Im \, H_1) . \] (6)

We have used the conventional dimensionless energy variables \( x_{b(b)} = \frac{2E_{b(b)}}{\sqrt{s}} \) and the notation

\[ \alpha_b = \frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2} , \] (7)
\[ N_{tot} = (3 + \beta^2) F_1 + 3(1 - \beta^2) F_2 . \] (8)

The CP violating contribution is determined by the function \( H_1 \)

\[ H_1 = (1 - \lambda \lambda') H_1^0 + (\lambda - \lambda') D_1^0 , \] (9)

where \( H_1^0 \) and \( D_1^0 \) are two independent linear combinations of the dipole moment form factors \( d'(s) \) and \( d''(s) \),

\[ H_1^0 = (\frac{2}{3} - c_V g_V h_Z) d'(s) - (\frac{2}{3} c_V h_Z - (c_V^2 + c_A^2) g_V h_Z^2) d''(s) , \]
\[ D_1^0 = -c_A g_V h_Z d'(s) - (\frac{2}{3} c_A h_Z - 2c_V c_A g_V h_Z^2) d''(s) . \] (10)

The SM contribution is determined by the functions \[ F_i \]

\[ F_i = (1 - \lambda \lambda') F_i^0 + (\lambda - \lambda') G_i^0 \quad i = 1, 2, 3 , \]
\[ G_i = (1 - \lambda \lambda') G_i^0 + (\lambda - \lambda') F_i^0 , \] (11)

where

\[ F_{1,2}^0 = \frac{4}{3} - \frac{4}{3} c_V g_V h_Z + (c_V^2 + c_A^2)(g_V^2 \pm g_A^2) h_Z^2 , \]
\[ F_3^0 = -\frac{4}{3} c_A g_A h_Z + 4 c_V c_A g_V g_A h_Z^2 , \]
\[ G_{1,2}^0 = -\frac{4}{3} c_A g_V h_Z + 2 c_V c_A(g_V^2 \pm g_A^2) h_Z^2 , \]
\[ G_3^0 = -\frac{4}{3} c_V g_A h_Z + 2(c_V^2 + c_A^2) g_V g_A h_Z^2 . \] (12)
The quantities $c_V = -(1/2) + 2 \sin^2 \Theta_W$, $c_A = (1/2)$ are the SM couplings of $Z$ to the electron and $h_Z = [s/(s - m_Z^2)]/\sin^2 2\Theta_W$.

The energy distributions of $b$ and $\bar{b}$ are linear functions of $x_{b(\bar{b})}$, eq.(5). If CP invariance holds the energy distributions of $b$ and $\bar{b}$ are equal. The sensitivity to CP violation is determined by two factors: $\alpha_b$ measuring the sensitivity of the $b$ quarks to the top quark polarization, and $\Im m d^\tau Z(s)$ entering the top quark polarization in the production plane.

In general CP invariance implies

$$\frac{d\sigma_{\lambda \lambda'}}{dx_b} = \frac{d\sigma_{-\lambda'(\lambda')}}{dx_b}. \quad (13)$$

The corresponding integrated energy observable $A_{\lambda \lambda'}^E$ indicating CP violation is the difference between the asymmetries in the energy distributions

$$A_{\lambda \lambda'}^E = R_{\lambda \lambda'}^b - R_{-\lambda' \lambda}^b = -4 \alpha_b \beta \Im m H_1 / N_{tot}, \quad (14)$$

where

$$R_{\lambda \lambda'}^b = \Delta N^{b(\bar{b})}(\lambda, \lambda') / N_{tot}^{b(\bar{b})}(\lambda, \lambda'), \quad (15)$$

$$\Delta N^{b(\bar{b})}(\lambda, \lambda') = N^{b(\bar{b})}(x > x_0, \lambda, \lambda') - N^{b(\bar{b})}(x < x_0, \lambda, \lambda'), \quad (16)$$

$$x_0 = \frac{m_t^2 - m_W^2}{2m_t^2}. \quad (17)$$

$x_0$ is the mean value of the energy, $x_{min} = x_0(1-\beta)$, and $x_{max} = x_0(1+\beta)$. $N^{b(\bar{b})}(x > x_0, \lambda, \lambda')$ is the number of $b(\bar{b})$ quarks with $x > x_0$ for the beam polarizations $\lambda$ and $\lambda'$, respectively.

Using polarized electron beams we can define the following polarization asymmetry:

$$P^E = R_P^b - R_P^{\bar{b}} = -4 \alpha_b \beta \Im m D_1^0 / N_{tot}^0, \quad (18)$$

where

$$N_{tot}^0 = N_{tot}(\lambda = \lambda' = 0), \quad (19)$$

$$R_P^{b(\bar{b})} = \frac{(1 - \lambda \lambda')}{(\lambda - \lambda')} \times \frac{\Delta N^{b(\bar{b})}(\lambda, \lambda') - \Delta N^{b(\bar{b})}(-\lambda, \lambda')}{N_{tot}^{b(\bar{b})}(\lambda, \lambda') + N_{tot}^{b(\bar{b})}(-\lambda, \lambda').} \quad (20)$$
Notice that by measuring the polarization asymmetry eq.(18) one obtains information on \( \Im D_1^0 \), whereas by measuring the asymmetry eq.(14) information on the combination eq.(9) of \( \Im H_1^0 \) and \( \Im D_1^0 \) can be obtained. Only by performing experiments with different beam polarizations can one determine both \( \Im H_1^0 \) and \( \Im D_1^0 \).

Optimal observables for extracting information in a statistically optimal way were introduced in [10, 11]. We will apply this method for extracting CP violating quantities in the reactions (1) and (2). We rewrite eq.(5) in the form

\[
d\sigma_{b(\bar{b})} = (S_0 (\pm) S_1 \Im H_1) d x_{b(\bar{b})}.
\]

The coefficients \( S_0 \) and \( S_1 \) follow from eqs.(5)–(11):

\[
S_0 = A \left[ N_{\text{tot}} + 4\alpha_b G_3 (1 - x/x_0) \right],
\]

\[
S_1 = 8A \alpha_b G_3 (1 - x/x_0),
\]

where \( A = \pi \alpha_{em}^2 / (s 2x_0) \). We choose the optimal observable \( O = S_1/S_0 \). The measurable expectation values are

\[
\langle O \rangle_{\lambda\lambda'} = \langle O \rangle_{b(\bar{b})} = \int d\sigma_{b(\bar{b})} O = \langle \pm \rangle c_{\lambda\lambda'} \Im H_1,
\]

where “+” is for the \( b \) and “–” for the \( \bar{b} \). Since \( \int dx_{b(\bar{b})} S_1 = 0 \), one has

\[
c_{\lambda\lambda'} = \int dx_{b(\bar{b})} S_0 O^2.
\]

With \( a = 4\alpha_b \beta G_3 / N_{\text{tot}} \) we have

\[
c_{\lambda\lambda'} = -\frac{4}{(G_3)^2} \left( 1 + \frac{1}{2a} \log \left| \frac{1-a}{1+a} \right| \right). \tag{25}
\]

Since \( \langle O \rangle_{\lambda\lambda'}^b = -\langle O \rangle_{\lambda\lambda'}^b \) we define

\[
\langle \overline{O} \rangle_{\lambda\lambda'} = \langle O \rangle_{\lambda\lambda'}^b - \langle O \rangle_{\lambda\lambda'}^b. \tag{26}
\]

In order to disentangle \( \Im H_1^0 \) and \( \Im D_1^0 \) we use the beam polarization. We define

\[
O_{\lambda\lambda'} = c_{\lambda\lambda'}^{-1} \langle \overline{O} \rangle_{\lambda\lambda'}.
\]

Measuring \( O_{\lambda\lambda'} \) and \( O_{-\lambda-\lambda'} \) we get two equations for \( \Im H_1^0 \) and \( \Im D_1^0 \) and obtain

\[
4 \Im H_1^0 = (O_{\lambda\lambda'} + O_{-\lambda-\lambda'}) / (1 - \lambda \lambda'),
\]

\[
4 \Im D_1^0 = (O_{\lambda\lambda'} - O_{-\lambda-\lambda'}) / (\lambda - \lambda'). \tag{28}
\]
So far our expressions for the asymmetries and the optimal observables are general and model independent. As an example we present in the following results in the MSSM with complex parameters. We use the results for the electroweak dipole moment form factors of the top quark as obtained in [6], where we assumed that $\mu$, $A_t$ and $A_b$ are complex. In [3] a complete study of the gluino, chargino and neutralino exchanges in the loops of $\gamma t\bar{t}$ and $Z t\bar{t}$ was performed. In Fig. 1a,b we show the dependence of $A_{E,\lambda\lambda'}^λ$ and $D_{E,\lambda\lambda'}^λ$ on the c.m.s. energy $\sqrt{s}$ for the beam polarizations $\lambda = -\lambda' = \{-0.8, 0., 0.8\}$. We take the following values for the mass parameters: $M = 230$ GeV, $|\mu| = 250$ GeV, $m_{\tilde{t}_1} = 150$ GeV, $m_{\tilde{t}_2} = 400$ GeV, $m_{\tilde{b}_1} = 270$ GeV, and $m_{\tilde{b}_2} = 280$ GeV, and for the phases we take $\varphi_{\mu} = \frac{4\pi}{3}$ and $\varphi_{\tilde{t}} = \frac{\pi}{6}$. We assume the GUT relation between the gaugino mass parameters: $m_{\tilde{g}} = \left(\frac{\alpha_s}{\alpha_2}\right)M \approx 3M$ and $M' = \frac{5}{3}\tan^2\Theta_W M$. For the same parameter set we show in Fig. 2 the optimal observable $\langle D \rangle_{\lambda\lambda'}$, eq.(26), as a function of the c.m.s. energy $\sqrt{s}$ for the beam polarizations $\lambda = -\lambda' = \{-0.8, 0., 0.8\}$.

The spikes are due to thresholds of the intermediate particles. As can be seen the dependence on the beam polarization can be strong in certain regions of $\sqrt{s}$. As already stated in [3], to observe an effect of $10^{-3}$ at a future $e^+e^-$ linear collider, an integrated luminosity of 300 fb$^{-1}$ is desired.

To conclude, we have worked out model independent analytic expressions for CP violating energy asymmetries as well as for optimal observables of $b$ and $\bar{b}$ quarks in $t\bar{t}$ production at an $e^+e^-$ linear collider. They give information on the electric and weak dipole moment form factors of the top quark. In order to disentangle $\Im m d^E(s)$ and $\Im m d^E(s)$ through the energy distribution of the $b$ and $\bar{b}$ quarks beam polarization is necessary.

This work has been supported by the 'Fonds zur Förderung der wissenschaftlichen Forschung' of Austria, project no. P10843–PHY. E.C.’s work has been supported by the Bulgarian National Science Foundation, Grant Ph–510. E.C. thanks the organizers of the winter school ‘37. Internationale Universitätswochen für Kern– und Teilchenphysik’ for the financial support where this work was finished.

References

[1] W. Bernreuther, O. Nachtmann, P. Overmann, and T. Schröder, Nucl. Phys. B388, 53 (1992); G. L. Kane, G. A. Ladinsky, and C.–P. Yuan, Phys. Rev.
D 45, 124 (1992); B. Grzadkowski and J. F. Gunion, Phys. Lett. B 287, 237 (1992); E. Christova and M. Fabbrichesi, Phys. Lett. B 315, 113 (1993); B320, 299 (1994); W. Bernreuther and P. Overmann, Z. Phys. C 61, 599 (1994); 72, 461 (1996); A. Bartl, E. Christova, and W. Majerotto, Nucl. Phys. B460, 235 (1996); B465, 365(E) (1996); M. S. Baek, S. Y. Choi, and C. S. Kim, Phys. Rev. D 56, 6835 (1997).

[2] C. R. Schmidt and M. E. Peskin, Phys. Rev. Lett. 69, 410 (1992); C. R. Schmidt, Phys. Lett. B 293, 111 (1992); D. Chang, W. Keung, and I. Phillips, Nucl. Phys. B408, 286 (1993); T. Arens and L. M. Seghal, Phys. Rev. D 50, 4372 (1994); P. Poulose and S. Rindani, Phys. Lett. B 349, 379 (1995); W. Bernreuther, A. Brandenburg, P. Overmann, Proc. of the Workshop "$e^+e^-$ Collisions at TeV Energies: The Physics Potential", Annecy, Gran Sasso, Hamburg, 1995, ed. P. M. Zerwas, DESY 96-123D, p. 49; B. Grzadkowski and Z. Hioki, Nucl. Phys. B484, 17 (1997).

[3] A. Bartl, E. Christova, T. Gajdosik, and W. Majerotto, Phys. Rev. D 58, 074007 (1998); hep-ph / 9802352

[4] M. Dugan, B. Grinstein, and L. Hall, Nucl. Phys. B255, 413 (1985).

[5] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, The Higgs Hunters Guide, Addison Wesley (1992).

[6] A. Bartl, E. Christova, T. Gajdosik, and W. Majerotto, Nucl. Phys. B507, 35 (1997).

[7] E. Christova and D. Draganov, Phys. Lett. B 434, 373 (1998); hep-ph / 9710225

[8] C. J.–C. Im, G. L. Kane, and P. J. Malde, Phys. Lett. B 317, 454 (1993); D. Atwood, S. Bar–Shalom, G. Eilam, and A. Soni, Phys. Rev. D 54, 5412 (1996); S. Bar–Shalom, D. Atwood, and A. Soni, Phys. Rev. D 57, 1495 (1998).

[9] H. E. Haber and G. L. Kane, Phys. Rep. 177, 75 (1985); J. F. Gunion and H. E. Haber, Nucl. Phys. B272, 1 (1986).

[10] D. Atwood and A. Soni, Phys. Rev. D 45, 2405 (1992).

[11] M. Diehl and O. Nachtmann, Z. Phys. C 62, 397 (1994).
Figure 1: (a) the energy asymmetry $A_{\lambda\lambda'}^E$, eq.(14), as functions of $\sqrt{s}$ [GeV] for a longitudinal electron-beam polarization of -80% (dashed line), 0% (full line), and 80% (dotted line) and (b) the polarization asymmetry $P^E$, eq.(18), depending on $\sqrt{s}$ [GeV] (dashed–dotted line).
Figure 2: The optimal observable $\langle \mathcal{O} \rangle_{\lambda \lambda'}$, eq.(20), as a function of $\sqrt{s}$ [GeV] for a longitudinal electron-beam polarization of -80% (dashed line), 0% (full line), and 80% (dotted line).