Effects of Family Nonuniversal $Z'$ Boson on Leptonic Decays of Higgs and Weak Bosons

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Abstract

Though not completely a surprise according to the standard model and existing indirect constraints, the Higgs-like particle, $h$, of mass around 125 GeV recently observed at the LHC may offer an additional window to physics beyond the standard model. In particular, its decay pattern can be modified by the existence of new particles. One of the popular scenarios involves a $Z'$ boson associated with an extra Abelian gauge group. In this study, we explore the potential effects of such a boson with family-nonuniversal couplings on the leptonic decays of $h$, both flavor-conserving and flavor-changing. For current constraints, we take into account leptonic decays at the $Z$ pole, LEP II scattering data, limits on various flavor-changing lepton transitions, and lepton magnetic dipole moments. Adopting a model-independent approach and assuming that the $Z'$ has negligible mixing with the $Z$ boson, we find that present data allow the $Z'$ effects to reach a few percent or higher on $h$ decays into a pair of leptons. Future measurements on $h$ at the LHC or a linear collider can therefore detect the $Z'$ contributions or impose constraints on its couplings. We also consider $Z'$-mediated four-lepton decays of the $Z$ and $W$ bosons.
I. INTRODUCTION

The recent discovery at the LHC [1] of a new particle having mass about 125 GeV and other properties compatible with those of the standard model (SM) Higgs boson undoubtedly has far-reaching implications for efforts to search for new physics beyond the SM. Particularly, all new-physics models would have to include such a particle, hereafter denoted by $h$, as one of their ingredients. In general, different models would have different production rates and decay patterns for $h$ because of contributions from and/or mixing with other new particles. It is therefore important to have a detailed study of the characteristic quantum numbers of $h$ and its interactions with SM particles. It is hoped that through such an analysis, the newly discovered particle will provide us with hints of new physics.

One of the possible scenarios for new physics is the existence of an extra U(1) gauge group involving a massive gauge particle, the $Z'$ boson. Such a gauge symmetry may have its origin from various grand unified theories, string-inspired models, dynamical symmetry breaking models, and little Higgs models [2], just to name a few. The $Z'$ boson in different representative models has been directly searched for at colliders as well as indirectly probed via a variety of precision data [3, 4], putting limits on its gauge coupling and/or mass. Generally speaking, the $Z'$ couplings to SM fermions can be family universal or nonuniversal. The latter case has especially attracted a lot of interest in recent years due to its many interesting phenomenological implications [5, 6].

In this work, we focus on family-nonuniversal interactions of the $Z'$ with the neutrinos and charged leptons and explore constraints on its relevant couplings from a number of experiments on transitions involving leptons in the initial and final states, plus possibly a photon. These processes suffer less from QCD corrections and hadronic uncertainties than processes involving hadrons. Moreover, many of such experiments have been performed at a high precision, imposing relatively stringent constraints on any possible new interactions.

More specifically, we assume that the $Z'$ boson arises from a new U(1) gauge symmetry, interacts with leptons in a family-nonuniversal way, and has negligible mixing with the $Z$ boson for simplicity, but otherwise adopt a model-independent approach to make the analysis as general as possible. Due to the family nonuniversality, such a $Z'$ boson would feature flavor-changing leptonic couplings at tree level, leading to the distinctive signature of lepton-flavor violation. However, lepton-flavor violating processes have been searched for at colliders with null results. We therefore examine a number of flavor-conserving and flavor-changing processes to evaluate constraints on the leptonic $Z'$ couplings. The results are then used to estimate $Z'$ contributions to both flavor-conserving and flavor-changing decays of the Higgs boson into a pair of leptons, $h \rightarrow l^+l^-$, at the one-loop level. As $h$ will be probed with increasing precision at the LHC in coming years, and even more so if a Higgs factory is built in the future, the acquired data could reveal the signals of the $Z'$ boson which we consider, assuming that $h$ is a SM-like Higgs boson. Last but not least, we will compute the $Z'$ effects on several four-lepton decays of the $W$ and $Z$ bosons and make comparison with the data if available. The LHC may also be sensitive to such indirect indications of the $Z'$ presence. The information on the $Z'$ gained from the $h$, $W$, and $Z$ measurements would be complementary to that from the $Z'$ direct searches.

This paper is organized as follows. We present the interactions of the $Z'$ boson with the leptons in Section II, allowing for in particular flavor-nonnuniversal couplings. In Section III, we study
constraints on the couplings of the $Z'$ from $Z$-pole data, cross sections of $e^+ e^-$ scattering into lepton-antilepton pairs measured at LEP II, various experimental limits on low-energy flavor-changing processes involving charged leptons, and measurements of their anomalous magnetic moments. In Section IV, we proceed to make predictions on both flavor-conserving and flavor-changing decays of $h$ into a pair of charged leptons. We also explore the $Z'$ contributions to $W$ and $Z$ decays into four leptons. We summarize our findings in Section V.

II. INTERACTIONS

The Lagrangian describing the interactions of the $Z'$ boson with neutrinos $\nu'_j$ and charged leptons $\ell'_j$ can be expressed as

$$\mathcal{L} = -g'_{L_j} \bar{\nu}'_j \gamma^\lambda P_L \nu'_j Z'_\lambda - \bar{\ell}'_j \gamma^\lambda (g'_{L_j} P_L + g'_{R_j} P_R) \ell'_j Z'_\lambda,$$

where summation over $j = 1, 2, 3$ is implied, the primes of the lepton fields refer to their interaction eigenstates, $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$, and the parameters $g'_{L_j,R_j}$ are generally different from one another, reflecting the family nonuniversality. The Hermiticity of $\mathcal{L}$ requires these coupling constants to be real. Since each of the left-handed neutrinos and its charged counterpart form a SM weak doublet, they share the same real. Since we are concerned with processes below the electroweak scale, we do not consider right-handed neutrinos in the low-energy spectrum. The $Z'$ may also have couplings to quarks and other nonstandard fermions, but we do not address them in this analysis.

The interaction states are related to the mass eigenstates $\nu_j$ and $\ell_j$ by

$$\nu'_{iL} = (V_\nu)_{ij} \nu_j L, \quad \ell'_{iL} = (V_L)_{ij} \ell_j L, \quad \ell'_{iR} = (V_R)_{ij} \ell_j R,$$

where $f_{L,R} = P_{L,R} f$ for fermion $f$ and the $3 \times 3$ matrices $V_{\nu,L,R}$ are unitary. In terms of the mass eigenstates, one can then write

$$\mathcal{L} = -b'^{\nu}_{ij} \bar{\nu}_i \gamma^\lambda P_L \nu_j Z'_\lambda - \bar{\ell}_j \gamma^\lambda \left(b'^{\nu}_{L_j} P_L + b'^{\nu}_{R_j} P_R\right) \ell_j Z'_\lambda,$$

where summation over $i, j = 1, 2, 3$ is implied and

$$b'^{\nu}_{ij} = (V_\nu)_{rj} g'_{L_j} (V_\nu)_{is}, \quad b'^{\nu}_{L_j} = (V_L)_{rj} g'_{L_j} (V_L)_{is}, \quad b'^{\nu}_{R_j} = (V_R)_{rj} g'_{R_j} (V_R)_{is}.$$

It follows that

$$b'^{\nu}_{ij} = (b'^{\nu}_{ij})^*, \quad b'^{\nu}_{L,R} = (b'^{\nu}_{L,R})^*, \quad b'^{\nu}_{i} = U^\dagger_{rj} b'^{\nu}_{L_j} U_{js},$$

where $U = V_L^\dagger V_\nu$. Hence family nonuniversality implies that the $Z'$ interactions with the leptons can be flavor violating at tree level. Furthermore, $b'^{\nu}_{ij}$ and $b'^{\nu}_{L_j}$ are generally unequal.

1 Note that throughout the paper, we use $\ell$ to denote the triplet of charged leptons, $(\ell_1, \ell_2, \ell_3) = (e, \mu, \tau)$, and $l$ to refer to an individual charged lepton in general.
III. CONSTRAINTS

A. High-energy observables

We begin with the determination of constraints from the existing data on the $Z$-boson decays $Z \rightarrow l^+l^-$ and $Z \rightarrow \nu\bar{\nu}$. For the former, the amplitude takes the form

$$\mathcal{M}_{Z \rightarrow l^+l^-} = \bar{t}_\lambda (L_{\ell l} P_L + R_{\ell l} P_R) l \varepsilon^\lambda_Z,$$  \hspace{1cm} (6)

where $L_{\ell l}$ and $R_{\ell l}$ contain both SM and $Z'$ contributions and are given by

$$L_{\ell l} = g_{\ell}^{sm} (1 + \epsilon_{\ell}^{UZ}), \quad R_{\ell l} = g_{R}^{sm} (1 + \epsilon_{R}^{UZ}),$$  \hspace{1cm} (7)

\begin{align*}
g_{\ell}^{sm} &= \frac{g}{2c_w} (2s_w^2 - 1), \quad g_{R}^{sm} = \frac{g s_w^2}{c_w}, \quad c_w = \sqrt{1 - s_w^2} \hspace{1cm} (8)
\end{align*}

with as usual the weak coupling constant $g$ and $s_w^2 = \sin^2 \theta_W$ involving the Weinberg angle.

In the absence of $Z-Z'$ mixing, the $Z'$ effects modify the $Zl^+l^-$ vertex and leptonic self-energy diagrams at the one-loop level. We obtain

$$\epsilon_{C}^{UZ} = \mathcal{F}_Z(\delta) \sum_{f=e,\mu,\tau} |b_{C}^{l}|^2, \quad \delta = \frac{m_{Z'}^2}{m_{Z}^2}, \quad C = L, R,$$

$$\mathcal{F}_Z(\delta) = \frac{1}{16\pi^2} \left\{ -\frac{7}{2} - 2\delta - (3 + 2\delta) \ln \delta - 2(1 + \delta)^2 \left[ \ln \delta \ln \frac{\delta}{1 + \delta} + \text{Li}_2 \left( -\frac{1}{\delta} \right) \right] \ight.$$ \hspace{1cm} (9)

$$- i\pi \left[ 3 + 2\delta + 2(1 + \delta)^2 \ln \frac{\delta}{1 + \delta} \right] \right\},$$

where $\text{Li}_2$ is the dilogarithm. The expression for the real part of $\mathcal{F}_Z$ has been derived previously [7].

The relevant observables here are the forward-backward asymmetry at the $Z$ pole and decay rate

$$A_{FB}^{(0, l)} = \frac{3}{4} A_{l} A_{l}, \quad \Gamma_{Z \rightarrow l^+l^-} = \frac{\sqrt{m_{Z}^2 - 4m_{l}^2}}{16\pi m_{Z}^2} |\mathcal{M}_{Z \rightarrow l^+l^-}|^2,$$  \hspace{1cm} (10)

where

\begin{align*}
A_{l} &= \frac{|L_{\ell l}|^2 - |R_{\ell l}|^2}{|L_{\ell l}|^2 + |R_{\ell l}|^2}, \quad |\mathcal{M}_{Z \rightarrow l^+l^-}|^2 = \frac{2}{3} (|L_{\ell l}|^2 + |R_{\ell l}|^2) (m_{Z}^2 - m_{l}^2) + 4 m_{l}^2 \Re (L_{\ell l}^* R_{\ell l}). \hspace{1cm} (11)
\end{align*}

In $Z \rightarrow \nu\bar{\nu}$, the $Z'$-loop contributions are analogous to those in the charged-lepton case, but without the right-handed couplings. Since the unobserved neutrinos in the final state may belong to different mass eigenstates, we can express the amplitude as

$$\mathcal{M}_{Z \rightarrow \nu_{r} \bar{\nu}_{s}} = \frac{g}{2c_w} \bar{\nu}_{r} \gamma_{\lambda} N_{rs} P_{L} \nu_{s} \varepsilon_{Z}^{\lambda}, \quad N_{rs} = \delta_{rs} + \mathcal{F}_Z(\delta) \sum_{j} b_{l}^{rj} b_{l}^{js}. \hspace{1cm} (12)$$

Since $b_{l}^{ij}$ and $b_{l}^{i tj}$ are related according to Eq. (5), summing over the final neutrinos then results in the decay rate

$$\Gamma_{Z \rightarrow \nu\bar{\nu}} = \sum_{r,s} \Gamma_{Z \rightarrow \nu_{r} \bar{\nu}_{s}} = \frac{g^2 m_{Z}^2}{96\pi c_w^2} \sum_{r,s} |N_{rs}|^2 = \frac{g^2 m_{Z}^2}{96\pi c_w^2} \sum_{l=e,\mu,\tau} |1 + \epsilon_{l}^{UZ}|^2. \hspace{1cm} (13)$$

\hspace{1cm} (13)
Accordingly, this channel may offer a complementary probe for $b_L^{\ell C}$. From the formula for $c_C^{\ell Z}$ in Eq. (9), we can then see that these $Z$ decay modes are potentially sensitive to not only the flavor-conserving $Z'$ couplings, but also the flavor-changing ones.

These $Z$-pole observables have been measured with good precision. The experimental and SM values of $\Gamma_{Z \rightarrow l^+ l^-}$ are [8], in MeV,

$$
\Gamma_{Z \rightarrow e^+ e^-}^{\text{exp}} = 83.91 \pm 0.12 \text{ MeV,} \quad \Gamma_{Z \rightarrow \mu^+ \mu^-}^{\text{exp}} = 83.99 \pm 0.18 \text{ MeV,} \quad \Gamma_{Z \rightarrow \tau^+ \tau^-}^{\text{exp}} = 84.08 \pm 0.22 \text{ MeV,}
$$

$$
\Gamma_{Z \rightarrow e^+ e^-}^{\text{SM}} = \Gamma_{Z \rightarrow \mu^+ \mu^-}^{\text{SM}} = 84.01 \pm 0.07 \text{ MeV,} \quad \Gamma_{Z \rightarrow \tau^+ \tau^-}^{\text{SM}} = 83.82 \pm 0.07 \text{ MeV}, \quad (14)
$$

while those of $A_l$ are [8]

$$
A_{e}^{\text{exp}} = 0.1515 \pm 0.0019 \text{ MeV,} \quad A_{\mu}^{\text{exp}} = 0.142 \pm 0.015 \text{ MeV,} \quad A_{\tau}^{\text{exp}} = 0.143 \pm 0.004 \text{ MeV,}
$$

$$
A_{e}^{\text{SM}} = A_{\mu}^{\text{SM}} = A_{\tau}^{\text{SM}} = 0.1475 \pm 0.0010 \text{ MeV.} \quad (15)
$$

For $Z \rightarrow \nu \bar{\nu}$, we have [8], also in MeV,

$$
\Gamma_{Z \rightarrow \text{invisible}}^{\text{exp}} = 499.0 \pm 1.5 \text{ MeV,} \quad \Gamma_{Z \rightarrow \text{invisible}}^{\text{SM}} = 501.69 \pm 0.06 \text{ MeV.} \quad (16)
$$

Numerically, for the SM contributions we employ the tree-level formulas in Eqs. (6)-(8) and (12) along with the effective values

$$
geff = 0.6517, \quad s_{w, \text{eff}}^2 = 0.23146 \quad (17)
$$

which allow us to reproduce the SM numbers in Eqs. (14) and (15) within their errors and obtain $\Gamma_{Z \rightarrow \nu \bar{\nu}}^{\text{SM}} = 501.26 \text{MeV}$ in agreement with $\Gamma_{Z \rightarrow \text{invisible}}^{\text{SM}}$, indicating that other SM invisible modes are negligible. To extract the upper limit on $|b_L^{\ell C}|$, we assume it to be the only nonvanishing coupling subject to the 90% confidence-level (CL) ranges

$$
83.71 \leq \Gamma_{Z \rightarrow e^+ e^-} \leq 84.11, \quad 83.69 \leq \Gamma_{Z \rightarrow \mu^+ \mu^-} \leq 84.29, \quad 83.72 \leq \Gamma_{Z \rightarrow \tau^+ \tau^-} \leq 84.44, \quad 0.1459 \leq A_e \leq 0.1546, \quad 0.117 \leq A_{\mu} \leq 0.167, \quad 0.136 \leq A_{\tau} \leq 0.150, \quad 497 \leq \Gamma_{Z \rightarrow \nu \bar{\nu}} \leq 502, \quad (18)
$$

the rate numbers being in MeV.\(^2\)

We find that the constraint on $b_L^{\ell C}$ from $Z \rightarrow \nu \bar{\nu}$ is weaker (stronger) than that from $Z \rightarrow l^+ l^-$ for $l = e$ ($l = \mu, \tau$). Assuming that only one flavor-conserving coupling is nonzero at a time, we show the results for $10 \text{GeV} \leq m_{Z'} \leq 3 \text{TeV}$ in Fig. 1(a), where the displayed curves represent the stronger limit in each $l$ case. The blue dotted (dash-dotted) curve refers to $b_L^{\ell C}(R)$, the green short-dashed (long-dashed) curve $b_L^{\ell C}(R)$, and the red solid curve $b_R^{\ell C}$. The curve for $b_L^{\ell C}$ coincides with that for $b_L^{\mu \mu}$ because of the $Z \rightarrow \nu \bar{\nu}$ constraint. The horizontal solid straight line for $m_{Z'}$ above 500 GeV marks the perturbativity limit, $|b_L^{\ell C}(R)| < \sqrt{4\pi}$, which we have imposed as an extra requirement on the couplings. We present another view on the results in Fig. 1(b), which has the corresponding limits on the couplings divided by the $Z'$ mass.

\(^2\) We have taken the lower (upper) bound of $A_e$ ($\Gamma_{Z \rightarrow \nu \bar{\nu}}$) to be its SM lower (upper) value because $A_e^{\text{SM}}$ is above $A_e^{\text{exp}}$ ($\Gamma_{Z \rightarrow \nu \bar{\nu}}^{\text{SM}}$ is below $\Gamma_{Z \rightarrow \nu \bar{\nu}}^{\text{exp}}$) by $\sim 2$ sigmas.
FIG. 1: Upper limits on (a) $|b^{ll}_{L,R}|$ and (b) $|\hat{b}^{ll}_{L,R}| = |b^{ll}_{L,R}|/m_{Z'}$ for $l = e, \mu, \tau$ at 90% CL from $Z$ decay data if only one of the couplings is not zero at a time. The blue dotted (dash-dotted) curve refers to $b^{e\mu}_{L(R)}$, the green short-dashed (long-dashed) curve $b^{\mu\mu}_{L(R)}$, and the red solid curve $b^{\tau\tau}_{L}$. The curves for $b^{\tau\tau}_{L}$ and $b^{\mu\mu}_{L}$ coincide due to the $Z \rightarrow \nu \bar{\nu}$ constraint. The straight portions of the solid curves denote the perturbativity requirement, $|b^{ll}_{L,R}| < \sqrt{4\pi}$. These results also serve as limits on the flavor-changing couplings $|b^{ll'}_{L,R}|$, as explained in the text.

If instead only one of the flavor-changing couplings is nonvanishing at a time, we can get its upper limit also from Fig. 1, in view of $|\epsilon^{lZ}_{C}|$ in Eq. (9). Accordingly, the limits on $|b_{C}^{\mu,\mu e}|$, $|b_{C}^{\tau,\tau e}|$, and $|b_{C}^{\mu,\tau e}|$ are the same as those on $|b_{C}^{e}|$, $|b_{C}^{\mu}|$, and $|b_{C}^{\mu\mu}|$, respectively, for $C = L$ or $R$. As we will see later, there may be stronger bounds on some of these individual flavor-changing couplings from other measurements, depending on the $Z'$ mass.

Since $Z'$-mediated diagrams can contribute to $e^+e^- \rightarrow l^+l^-$ scattering, its data can provide additional restrictions on the $Z'$ couplings. Here we will use LEP-II measurements at various center-of-mass energies above the $Z$ pole, from 130 to 207 GeV [9]. In the absence of $Z-Z'$ mixing, the amplitude if $l \neq \bar{e}$ is

$$
\mathcal{M}_{e\bar{e} \rightarrow ll} = -4\pi \alpha \frac{\bar{\gamma}_\rho e_l}{s} \frac{\bar{\gamma}_\rho (g_L P_L + g_R P_R) e_l}{s - m^2_Z + \Gamma_Z m_Z} + \frac{\bar{\gamma}_\rho (b^{e\mu}_{L} P_L + b^{e\mu}_{R} P_R) e_l}{s - m^2_{Z'} + \Gamma_{Z'} m_{Z'}}
$$

where $\alpha$ is the electromagnetic fine-structure constant, the lepton masses have been neglected, $\Gamma_{Z,Z'}$ denote the total widths, the plus sign of the $t$-channel term follows from Fermi statistics, $s = (p_{e^+} + p_{e^-})^2$, and $t = (p_{e^-} - p_{l^-})^2$.

In the absence of flavor-changing couplings, thus the last line of Eq. (19), the resulting cross-section $\sigma_{e\bar{e} \rightarrow ll}$ and forward-backward asymmetry $A_{FB} = \sigma^{F}_{e\bar{e} \rightarrow ll}/\sigma^{B}_{e\bar{e} \rightarrow ll}$, with $\sigma^{F}_{e\bar{e} \rightarrow ll} = \sigma^{F}_{e\bar{e} \rightarrow il} - \sigma^{B}_{e\bar{e} \rightarrow il}$, are known in the literature (see, e.g., [6, 10]). As mentioned in Ref. [6] (which also has the more general formulas in the presence of $Z-Z'$ mixing), the expressions for these observables imply that in a model-independent study their experimental values cannot lead to restrictions on the individual
flavor-conserving $Z'$ couplings, assumed to be free parameters, but can nevertheless still restrain their products, $b_C^{e e} b_{C'}^{ll}$.

Since we have left the $Z'$ total width, $\Gamma_{Z'}$, unspecified in concentrating on its couplings to leptons and since it would be needed to compute the cross sections if $s \sim m_Z^2$, we evaluate the limits on $b_C^{e e} b_{C'}^{ll}$ from the LEP II data only for $m_{Z'}$ values starting from 210 GeV up to 3 TeV. Using as before Eq. (17), along with the effective value $\alpha_{\text{eff}} = 1/132.4$, and the 90% CL ranges of the experimental numbers [9], we draw Fig. 2(a) for the upper limits on the products $b^{ee} b^{\mu\mu}_L$ (green long-dashed curve), $b^{ee} b^{\mu\mu}_R$ (green short-dashed curve), $b^{ee} b^{\tau\tau}_L$ (red dotted curve), $b^{ee} b^{\tau\tau}_R$ (red solid curve). Since the positive and negative limits on these coupling products are not generally symmetric with respect to zero, we present Fig. 2(b) for the negative limits. The plots on the right (c and d) depict the corresponding limits for $\pm b^{ee} b_{C'}^{ll} = \pm b^{ee} b_{C'}^{ll} / m_{Z'}^2$.

![Graphs](image_url)

**FIG. 2:** Upper limits on (a) $b^{ee} b_{C'}^{ll}$ and (b) $-b^{ee} b_{C'}^{ll}$ for $l = \mu, \tau$ and $C, C' = L, R$ at 90% CL from LEP II data if only one of the coupling products is nonvanishing at a time. The different curves are described in the text. The right plots (c and d) show the corresponding limits on $\pm b^{ee} b_{C'}^{ll} = \pm b^{ee} b_{C'}^{ll} / m_{Z'}^2$.

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3 A few of the $\sigma_{e e \rightarrow l \bar{l}}$ and $A_{FB}$ measurements disagree with their SM predictions by about 2 sigmas or more. In each of those cases, we take the lower (upper) bound of the required range of the relevant observable to be its SM lower (upper) value if the measurement is above (below) the SM prediction.
If one of the flavor-changing couplings $b_{L,R}^{e\mu, e\tau}$ in Eq. (19) does not vanish, we can constrain it separately, assuming $b_{L,R}^{e\mu, e\tau} = 0$. In that case, the $e^+e^- \to l^+l^-$ cross-sections have the expressions given in the Appendix, and so $\Gamma_{Z'}$ is not required in the calculation. Using the LEP II data again, we obtain the upper limits on $|b_{L,R}^{e\mu, e\tau}|$ for $10 \text{ GeV} \leq m_{Z'} \leq 3 \text{ TeV}$ in Fig. 3. The second plot displays the corresponding limits on $|\hat{b}_{L,R}^{e\mu, e\tau}| = |b_{L,R}^{e\mu, e\tau}|/m_{Z'}$. These results are evidently more stringent than the bounds on $|b_{L,R}^{e\mu, e\tau}|$ inferred from Fig. 1.

![Fig. 3](image)

**FIG. 3**: Upper limits on (a) $|b_{L,R}^{e\mu, e\tau}|$ and (b) $|\hat{b}_{L,R}^{e\mu, e\tau}| = |b_{L,R}^{e\mu, e\tau}|/m_{Z'}$ for $l = \mu, \tau$ at 90\% CL from LEP II data. The green long-dashed (short-dashed) curves refer to $|b_{L,R}^{e\mu}|$ and the red dotted (solid) curves $|b_{L(R)}^{e\tau}|$.

**B. Low-energy processes**

Turning our attention now to constraints on the $Z'$ couplings from low-energy data, we will consider a number of lepton flavor-violating processes and leptonic anomalous magnetic moments. The relevant formulas are available from the expressions given in Ref. [6] in the more general case with $Z-Z'$ mixing.

We begin by mentioning an additional process that can restrict $b_{L,R}^{e\mu}$ separately, namely the muonium-antimuonium conversion, $\mu^+e^- \to \mu^-e^+$. In this case, the constraints are the same as those determined in Ref. [6],

$$|\hat{b}_{L,R}^{e\mu}| \leq 4.4 \times 10^{-4} \text{ GeV}^{-1}.$$ (20)

These are stricter than their counterparts in Fig. 3 if $m_{Z'}$ goes below 30 GeV.

We next look at various constraints on the products of a pair of different couplings of the $Z'$, at least one of them being flavor changing, from the experimental limits [8] for flavor-changing leptonic 3-body and radiative 2-body decays of the $\mu$ and $\tau$ leptons. The leptonic decays arise from tree-level $Z'$-mediated diagrams, whereas the radiative decays proceed from loop diagrams containing the $Z'$ and an internal lepton. Assuming that only the two couplings in each of the products are present at a time, we collect the results in Table I.

The anomalous magnetic moments $a_e$ and $a_\mu$ of the electron and muon have been measured very precisely and therefore can provide extra constraints. In contrast, the experimental information on
TABLE I: Limits on products of $Z'$ couplings determined from low-energy data. The second column contains 90% CL experimental upper-limits on the branching ratios of the listed decay modes.

| Decay mode | Measured limits [8] | Derived limits on products of $b = b/m_{Z'}$, in GeV$^{-2}$ |
|------------|---------------------|-------------------------------------------------|
| $\mu \to e\bar{e}e$ | $1.0 \times 10^{-12}$ | $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\mu} | \leq 2.3 \times 10^{-11}$, $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\mu} | \leq 3.3 \times 10^{-11}$ |
| $\tau \to e\bar{e}e$ | $2.7 \times 10^{-8}$ | $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\tau} | \leq 9.1 \times 10^{-9}$, $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\tau} | \leq 1.3 \times 10^{-8}$ |
| $\tau \to \mu \bar{e} \bar{\mu}$ | $2.1 \times 10^{-8}$ | $|\hat{b}_{\mu}^{\mu} \hat{b}_{e}^{\tau} | \leq 8.0 \times 10^{-9}$, $|\hat{b}_{\mu}^{\mu} \hat{b}_{e}^{\tau} | \leq 1.1 \times 10^{-8}$ |
| $\tau \to \mu \bar{e} \bar{\mu}$ | $1.8 \times 10^{-8}$ | $|\hat{b}_{\mu}^{\mu} \hat{b}_{e}^{\tau} | \leq 1.0 \times 10^{-8}$, $|\hat{b}_{\mu}^{\mu} \hat{b}_{e}^{\tau} | \leq 1.0 \times 10^{-8}$ |
| $\tau \to e\mu \bar{\mu}$ | $2.7 \times 10^{-8}$ | $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\tau} | \leq 1.3 \times 10^{-8}$, $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\tau} | \leq 1.3 \times 10^{-8}$ |
| $\tau \to e\bar{e} \bar{\mu}$ | $1.5 \times 10^{-8}$ | $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\tau} | \leq 6.8 \times 10^{-9}$, $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\tau} | \leq 9.6 \times 10^{-9}$ |
| $\tau \to \mu \mu \bar{\mu}$ | $1.7 \times 10^{-8}$ | $|\hat{b}_{\mu}^{\mu} \hat{b}_{e}^{\tau} | \leq 7.2 \times 10^{-9}$, $|\hat{b}_{\mu}^{\mu} \hat{b}_{e}^{\tau} | \leq 1.0 \times 10^{-8}$ |
| $\mu \to e\gamma$ | $2.4 \times 10^{-12}$ | $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\mu} | \leq 1.3 \times 10^{-9}$, $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\mu} | \leq 4.3 \times 10^{-10}$, $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\mu} | \leq 2.6 \times 10^{-11}$ |
| $\tau \to e\gamma$ | $3.3 \times 10^{-8}$ | $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\tau} | \leq 3.6 \times 10^{-7}$, $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\tau} | \leq 3.6 \times 10^{-7}$, $|\hat{b}_{e}^{\mu} \hat{b}_{e}^{\tau} | \leq 1.2 \times 10^{-7}$ |
| $\tau \to \mu \gamma$ | $4.4 \times 10^{-8}$ | $|\hat{b}_{\mu}^{\mu} \hat{b}_{e}^{\tau} | \leq 4.2 \times 10^{-7}$, $|\hat{b}_{\mu}^{\mu} \hat{b}_{e}^{\tau} | \leq 4.2 \times 10^{-7}$, $|\hat{b}_{\mu}^{\mu} \hat{b}_{e}^{\tau} | \leq 1.4 \times 10^{-7}$ |

$a_\tau$ is still too limited to provide significant bounds [8]. For the $Z'$ contributions to $a_e$ and $a_\mu$, we will retain only the terms induced by the $\tau$ lepton in the loop, as they are enhanced by the $\tau$ mass compared to the other lepton terms [6],

$$a_\tau^{Z'} = \frac{m_\tau m_\tau \text{Re}(\hat{b}_{e}^{\tau} \hat{b}_{\tau}^{\tau})}{4\pi^2 m_{Z'}^2}, \quad a_\mu^{Z'} = \frac{m_\mu m_\tau \text{Re}(\hat{b}_{e}^{\mu} \hat{b}_{\tau}^{\mu})}{4\pi^2 m_{Z'}^2}.$$  \hspace{1cm} (21)

The SM prediction for $a_e$ is compatible with its most recent measurement, the difference between them being $a_e^{\text{exp}} - a_e^{\text{SM}} = (-105 \pm 82) \times 10^{-14}$ [11]. Consequently, we can impose the 90% CL range $-2.4 \times 10^{-12} \leq a_e^{Z'} \leq 0.3 \times 10^{-12}$. In contrast, the SM and experimental values of $a_\mu$ presently differ by nearly 3 sigmas, $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (249 \pm 87) \times 10^{-11}$ [12]. This suggests that we may require $0 \leq a_\mu^{Z'} \leq 3.3 \times 10^{-9}$. It follows that

$$-1.0 \times 10^{-7} \leq \text{Re}(\hat{b}_{e}^{\tau} \hat{b}_{\tau}^{\tau}) \text{GeV}^2 \leq 0.1 \times 10^{-7}, \quad 0 \leq \text{Re}(\hat{b}_{e}^{\mu} \hat{b}_{\tau}^{\mu}) \text{GeV}^2 \leq 6.9 \times 10^{-7}.$$  \hspace{1cm} (22)

The $\hat{b}_{e}^{\tau} \hat{b}_{\tau}^{\tau}$ limits are complementary to the individual bounds on $\hat{b}_{e}^{\tau}$ illustrated in Fig.3.

**IV. PREDICTIONS**

The results above allow us to explore how the $Z'$ effects may modify the leptonic decays of the newly found particle, $h$, assumed to be a SM-like Higgs boson, and also those of the weak bosons ($Z$ and $W$). We will deal with both flavor-conserving and -violating channels involving two and four leptons in the final states.
A. Two-body decays

We first look at the flavor-violating decay $h\to l^+l^\mp$. It proceeds from a $Z'$-loop diagram having an $hl^+l^-$ and two lepton-$Z'$ vertices, at least one of the latter being flavor changing, and one-loop $l\to l'$ diagrams with flavor-changing couplings. We calculate the amplitude to be

$$
\mathcal{M}_{h\to l^+l^-} = \frac{\sqrt{m_l m_{l'}}}{v} \left( \epsilon_L^{ll'} P_L + \epsilon_R^{ll'} P_R \right) l',
$$

(23)

where $v = 246$ GeV is the Higgs vacuum expectation value,

$$
\epsilon_L^{ll'} = \mathcal{F}_h(r) \left( \frac{m_f}{\sqrt{m_l m_{l'}^'}} \right) b_{RL}^f b_{L,R}^{ll'} , \quad r = \frac{m_h^2}{m_{Z'}^2} ,
$$

$$
\mathcal{F}_h(r) = -\frac{1}{8\pi^2} \left[ \ln r \ln(1+r) + \text{Li}_2(-r) - i\pi \ln(1+r) \right],
$$

(24)

with $f$ being the fermion in the loop. The rate for $m_{l,l'}^2 \ll m_h^2$ is then

$$
\Gamma_{h\to l^+l^-} = \frac{m_h m_l m_{l'}}{16\pi v^2} \left( |\epsilon_L^{ll'}|^2 + |\epsilon_R^{ll'}|^2 \right).
$$

(25)

To predict the largest rates, we observe from the bounds derived in the last section and Eq. (24) that the most important contribution comes from the internal lepton $f = \tau$ and that the rates are maximized for final states with one $\tau$. Thus, we take $|b_{RL}^{\tau\tau} b_{RL}^{e\tau}| = 1.2 \times 10^{-7} m_{Z'}^2$/GeV$^2$ and $|b_{RL}^{\tau\tau} b_{RL}^{\mu\tau}| = 1.4 \times 10^{-7} m_{Z'}^2$/GeV$^2$ from the $\tau \to e\gamma, \mu\gamma$ bounds in Table I. For definiteness, we take $m_h = 125.5$ GeV, compatible with the average $h$ mass of 125.7 $\pm$ 0.4 GeV from the LHC measurements [1]. With only one nonzero product of couplings being present in each case and $\Gamma_h = 4.14$ MeV [13], we obtain for 10 GeV $\leq m_{Z'} \leq 3$ TeV

$$
\mathcal{B}(h\to \mu\tau) = \mathcal{B}(h\to \mu^+\tau^-) + \mathcal{B}(h\to \mu^-\tau^+) \lesssim 3 \times 10^{-9},
$$

(26)

the upper bound occurring at $m_{Z'} = 3$ TeV, and a somewhat smaller number for $\mathcal{B}(h\to e\tau)$. Clearly these $Z'$-induced flavor-violating Higgs decays will not be observable in the near future.

We next consider the $Z'$ impact on the flavor-conserving decay $h\to l^+l^-$. The $Z'$ contribution follows from Eq. (23) after setting $l' = l$. Combining the result with the SM tree-level contribution, we then get

$$
\mathcal{M}_{h\to l^+l^-} = \frac{m_l}{v} \left[ \left( 1 + \epsilon_L^{lh} \right) P_L + \left( 1 + \epsilon_R^{lh} \right) P_R \right] l
$$

(27)

leading to

$$
\Gamma_{h\to l^+l^-} = \frac{m_h m_l^2}{16\pi v^2} \left( |1 + \epsilon_L^{lh}|^2 + |1 + \epsilon_R^{lh}|^2 \right).
$$

(28)

We expect again that the internal lepton $f = \tau$ in the loop yields the maximal impact. Accordingly, for $h \to e^+e^-$ and $h \to \mu^+\mu^-$ we take, respectively, the $b_{RL}^{e\tau} b_{RL}^{e\tau}$ and $b_{RL}^{\mu\tau} b_{RL}^{\mu\tau}$ ranges
in Eq. (22), assuming that the coupling products are purely real or dominated by the real part. The graphs in Fig. 4 depict the resulting fractional change

$$\Delta_l = \frac{\Gamma_{h \to l^+l^-}}{\Gamma_{h \to l^+l^-}^{\text{SM}}} - 1$$

in the rate due to the presence of $Z'$ exclusively via these flavor-changing couplings. Although the $Z'$ contribution can reduce the $h \to e^+e^-$ rate sizeably [blue-shaded areas in Fig. 4(a)], this decay mode, with a branching ratio of $\sim 5 \times 10^{-9}$ in the SM, may be beyond reach for a long time. Much more interesting is $h \to \mu^+\mu^-$, which has a SM branching ratio of $\sim 2 \times 10^{-4}$ and therefore may be measurable in the not-so-distant future with precision possibly sensitive to the $Z'$ effect, indicated by the green-shaded areas in Fig. 4(b).

As it turns out, potentially more considerable modifications to the $h \to l^+l^-$ rate can be induced by the flavor-conserving couplings $b_{lL,R}^l$, which enter $\Gamma_{h \to l^+l^-}$ as the product $b_{lL}^l b_{lR}^l$. To estimate their maximal impact, we focus on the $l = \mu, \tau$ cases. In each of them, we set all the other couplings to zero and scan the values of $b_{lL,R}^l$ satisfying the requirements in Eq. (18) as well as the perturbativity condition $|b_{lL,R}^l| < \sqrt{4\pi}$. We find that the coupling values allowed by these constraints can translate into substantial $\Delta_{\mu,\tau}$ that are positive or negative. Especially, in the $10 \text{ GeV} \leq m_{Z'} < 50 \text{ GeV}$ region, the decrease in the rate could reach a few tens percent, whereas the increase could exceed 100%, even up to $\sim 300\%$, as can be seen in Fig. 5.

The $h \to \tau^+\tau^-$ decay has begun to be observable at the LHC. The latest signal strengths for this channel reported by the ATLAS and CMS Collaborations are $\sigma/\sigma_{\text{sm}} = 0.8 \pm 0.7$ and $0.7 \pm 0.5$, respectively, at $m_h = 125 \text{ GeV}$ [14]. Obviously, these early findings already disfavor some parts of the $Z'$ parameter space implied by Fig. 5(b), although it is still too soon to be quantitative about the exclusion zones in view of the sizable uncertainties of these current data. As their precision continues to improve, the upcoming experiments can either uncover a $Z'$ signal or restrain its $\tau$ couplings further. A similar expectation can be stated regarding the $\mu$ couplings from future measurements of the $h \to \mu^+\mu^-$ mode.

FIG. 4: Range of fractional change to SM rate of $h \to l^+l^-$ for (a) $l = e$ and (b) $l = \mu$ due to only the coupling products $b_{R,L}^{e\tau} b_{L,R}^{e\tau}$ and $b_{R,L}^{\mu\tau} b_{L,R}^{\mu\tau}$, respectively.
FIG. 5: Upper limits on fractional changes $\Delta l$ and $-\Delta l$ to SM rate of $h \to l^+ l^-$ for (a) $l = \mu$ and (b) $l = \tau$ if only the flavor-conserving couplings $b^H_{L,R}$ are nonzero.

The flavor-violating decays $Z \to \bar{l}l$ are not yet observed, but have been searched for, the experimental upper-limits on their branching ratios being [8]

$$B(Z \to e\mu) \leq 1.7 \times 10^{-6}, \quad B(Z \to e\tau) \leq 9.8 \times 10^{-6}, \quad B(Z \to \mu\tau) \leq 1.2 \times 10^{-5},$$

(30)
at 95% CL, where $B(Z \to l_1 l_2) = B(Z \to l_1^+ l_2^-) + B(Z \to l_1^- l_2^+)$. These modes get $Z'$-loop contributions analogously to those in $Z \to l^+ l^-$ and hence may provide further limits on the $Z'$ couplings if the experimental limits are less than the predicted values derived from the upper limits on the couplings. To make the predictions for the $e\mu$, $e\tau$, and $\mu\tau$ final states, we take the biggest values

$$|\hat{b}_C^{e\mu} \hat{b}_C^{e\mu}| = 1.3 \times 10^{-9}, \quad |\hat{b}_C^{e\tau} \hat{b}_C^{e\tau}| = 3.6 \times 10^{-7}, \quad |\hat{b}_C^{\mu\tau} \hat{b}_C^{\mu\tau}| = 4.2 \times 10^{-7}$$

(31)
from the $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ bounds, respectively, in Table I. These translate into

$$B(Z \to e\mu)_{Z'} < 7.1 \times 10^{-12}, \quad B(Z \to e\tau)_{Z'} < 5.4 \times 10^{-7}, \quad B(Z \to \mu\tau)_{Z'} < 7.4 \times 10^{-7}.$$  

(32)

Hence, although the $e\mu$ mode is likely to be undetectable, the $e\tau$ and $\mu\tau$ predictions are only 18 and 16 times below their respective experimental bounds. It is worth noting that these results are comparable to those found in Ref. [6] in the presence of $Z-Z'$ mixing, in accord with the expectation mentioned therein.

B. Four-body decays

The presence of the $Z'$ may also manifest itself in the decays of $h$ and the weak bosons into four leptons. The CMS Collaboration [15] has recently reported the first observation of $Z$ decays into four charged leptons ($e$ and $\mu$) consistent with the SM expectations. This suggests that such decays can be measured well in the future at the LHC, thereby providing another means to look for new-physics
Applying the constraints given in Fig. 1, we estimate the possible ranges of the fractional change \( \Delta \) specified in flavor-conserving decays. The flavor-violating four-lepton decays of \( Z \) incorporating the new vertices in the model file and assuming \( M_{Z'} \geq 210 \) GeV, since we have not specified the \( Z' \) total width.

In the flavor-conserving decays \( Z \to l_i^+l_j^-l_k^+l_l^- \), the \( Z' \) contributions depend on the \( b_{C}^{ll} \) couplings. Applying the constraints given in Fig. 1, we estimate the possible ranges of the fractional change \( \Delta_{Z}^{2\mu \nu} = \frac{\Gamma_{Z \to l_i^+l_j^-l_k^+l_l^-}}{\Gamma_{Z \to l_i^+l_j^-l_k^+l_l^-}} - 1 \) for \( l_1 = l_2 = l = e, \mu, \tau \), taking only one of \( b_{C}^{ll} \) to be nonzero at a time. Thus, we obtain

\[
-0.0032 \leq \Delta_{Z}^{2\mu \nu} \leq 0 , \quad -0.016 \leq \Delta_{Z}^{4\mu} \leq 0 , \quad -0.033 \leq \Delta_{Z}^{4\tau} \leq 0 ,
\]

where the lowest numbers all belong to \( m_{Z'} = 210 \) GeV and we have imposed the kinematical cut \( M_{R} > 4 \) GeV in estimating the rates. Since the SM predicts \( B(Z \to l_i^+l_j^-l_k^+l_l^-)_{sm} \simeq 1.2 \times 10^{-6} \) for \( l = e, \mu \) [15], a large luminosity is required to observe the changes. We also compute the possible ranges of the fractional change \( \Delta_{Z}^{2\mu \nu} = \frac{\Gamma_{Z \to l_i^+l_j^-l_k^+l_l^-}}{\Gamma_{Z \to l_i^+l_j^-l_k^+l_l^-}} - 1 \) in the \( l \neq l' \) cases where \( l'^{0} = e, \mu, \tau \). For \( (l, l') = (e, \mu) \) and \( (e, \tau) \) we should take into account the constraints on the products of a pair of different couplings in Fig. 2, whereas for \( (l, l') = (\mu, \tau) \) we apply the upper limits on \( |b_{C}^{\mu \mu}| \) and \( |b_{C}^{\tau \tau}| \) from Fig. 1. We find that the magnitudes of \( \Delta_{Z}^{2\mu \nu} \) and \( \Delta_{Z}^{2\mu \tau} \) are smaller than \( O(10^{-5}) \) because of the constraints on \( |b_{C}^{e \mu}b_{C}^{e \mu}| \) and \( |b_{C}^{e \tau}b_{C}^{e \tau}| \) in Fig. 2, while the magnitude of \( \Delta_{Z}^{2\mu \tau} \) can be comparatively greater,

\[
-0.013 \leq \Delta_{Z}^{2\mu \tau} \leq 0.014 ,
\]

the positive (negative) sign of \( \Delta_{Z}^{2\mu \tau} \) corresponding to the negative (positive) sign of \( b_{C}^{\mu \mu}b_{R}^{\tau \tau} \). One can see that the upper bound of \( |\Delta_{Z}^{2\mu \tau}| \) is roughly similar in order of magnitude to the lower bounds of \( \Delta_{Z}^{4\mu} \) and \( \Delta_{Z}^{4\tau} \).

We also consider the flavor-violating 4-lepton decays of the \( Z \), concentrating on the modes \( Z \to \tau^+\tau^-\tau^\pm e^\mp \), \( Z \to \tau^+\tau^-\tau^\pm \mu^\mp \), and \( Z \to \tau^+\tau^-\mu^\mp \mu^\pm \), as their amplitudes are determined by \( |b_{C}^{\tau \tau}b_{C}^{\tau \tau}| \), \( |b_{C}^{\mu \tau}b_{C}^{\mu \tau}| \), and \( |b_{C}^{e \tau}b_{C}^{e \tau}| \), respectively. The constraints on the first two products of couplings are given in Table I, and the upper limits on \( |b_{C}^{\mu \tau}| \) come from Fig. 1. Applying the biggest values from Eq. (31) and the upper limit on \( |b_{C}^{\mu \tau}| \) in Fig. 1, we find that the branching ratios are of \( O(10^{-13}) \) for the first two modes and \( O(10^{-9}) \) for the third one, which will be unobservable in the near future.

The 4-lepton decays of the \( W \) are treated similarly. In the flavor-conserving case, the fractional change \( \Delta_{W}^{4\mu} = \frac{\Gamma_{W \to l_i^+l_j^-l_k^+l_l^-}}{\Gamma_{W \to l_i^+l_j^-l_k^+l_l^-}} - 1 \) for \( l = e, \mu, \tau \) depend on \( |b_{L}^{ll}| \), in light of the relation between \( b_{L}^{ll} \) and \( b_{R}^{ll} \) in Eq. (5). We obtain

\[
-0.0083 \leq \Delta_{W}^{4\mu} \leq 0 , \quad -0.012 \leq \Delta_{W}^{4\mu,3\tau} \leq 0 ,
\]

where the lowest value corresponds to \( m_{Z'} = 210 \) GeV and we have employed the \( M_{R} \geq 4 \) GeV cut as before in estimating the rates. The \( \Delta_{W}^{4\mu,3\tau} \) numbers are almost the same because the upper limits of \( |b_{L}^{\mu \mu,\tau \tau}| \) are the same. Since the SM prediction is \( \mathcal{B}(W^{+(-)} \to l_i^+l_j^-l_k^\mp l_k^\pm) \simeq 1.1 \times 10^{-6} \), again a large luminosity is required to measure the changes, as in the \( Z \)-decay case.
For the flavor-violating 4-lepton decays of the $W$, we consider the $W^{\pm(-)} \rightarrow \tau^{\pm} \tau^{\pm} e^{-}\nu(e^{+}\bar{\nu})$ and $W^{(+)} \rightarrow \tau^{+} \tau^{+} \mu^{-}\nu(\mu^{+}\bar{\nu})$ channels. We obtain the biggest branching ratios of these modes from the upper limits of $|\hat{b}^L_{\tau}\hat{b}^R_{e}|$ and $|b^\mu_{L,R}|$, respectively. Using Eq. (31) and Fig. 1, we find that the branching ratio of the first mode is at most of $\mathcal{O}(10^{-12})$ and that of the second mode $\mathcal{O}(10^{-9})$. Hence they will likely be undetectable for a long time, as the flavor-violating 4-lepton $Z$ decays.

V. CONCLUSIONS

We have explored a $Z'$ boson originating from a new U(1) gauge symmetry and interacting with leptons in a family-nonuniversal way. The $Z'$ is assumed to have no mixing with the $Z$ boson. We have studied the effects of the $Z'$ as a virtual particle in various processes with leptons in the initial and/or final states, especially the leptonic decays of the newly discovered $h$, putatively a standard model-like Higgs boson.

We first study bounds on the leptonic couplings of the $Z'$ from available experimental data. We employ the $Z$-pole observables, including the $Z \rightarrow l^+l^-$ rates, the associated forward-backward asymmetries, and the invisible $Z$ decay rate, to put constraints on $|b_{L,R}^{ll}|$. The cross sections of $e^+e^- \rightarrow ll$ for $l = \mu, \tau$ from LEP-II experiments are used to place bounds on the products of chiral couplings $b_{L,R}^{ee}b_{L,R}^{ll}$ for $C = L, R$. Further restrictions are found from various low-energy processes, including the muonium-antimuonium conversion for $|b_{L,R}^{e\mu}|$ in particular, several flavor-changing leptonic 3-body and radiative 2-body decays of the muon and tauon, and the anomalous magnetic moments of the electron and muon.

We then apply the constraints on the $Z'$ couplings and mass to make predictions for the flavor-conserving and -violating 2-body leptonic decays of $h$ as well as 4-body decays of the $W$ and $Z$. For the flavor-violating $h \rightarrow ll$ decays, the most important contribution comes from the case with one $\tau$ in the final state and an internal $\tau$ running in the loop due to mass enhancement. Unfortunately, such $Z'$-mediated flavor-violating Higgs decays are too small to be observable in the near future. On the other hand, the flavor-conserving $h \rightarrow l^+l^-$ decay can be significantly affected by the contribution of an internal $\tau$. With flavor-changing couplings alone, the branching ratio of $h \rightarrow \mu^+\mu^-$ decay may be enhanced by a mere few percent. In contrast, with only flavor-conserving couplings, both the $h \rightarrow \mu^+\mu^-$ and $h \rightarrow \tau^+\tau^-$ channels can be considerably modified by a few tens to a few hundreds percent. Consequently, upcoming measurements of the latter mode with better precision than its current data will either uncover or constrain the $Z'$ boson.

We have found that the flavor-violating $Z \rightarrow l^l$ decays, mediated by the $Z'$ at the loop level, are consistent with current experimental upper bounds at 95% CL. The 4-body leptonic decays of the $Z$ have started to be measured by the LHC. Our calculations show that a larger luminosity is required to observe modifications in flavor-conserving $Z \rightarrow 4l$ decays due to the $Z'$, the maximal changes being of $\mathcal{O}(10^{-2})$. A similar situation holds for flavor-conserving $W \rightarrow 3l + \nu$ decays. We have also found that the changes in the flavor-violating $Z \rightarrow 4l$ and $W \rightarrow 3l + \nu$ decays are unlikely to be observable soon.
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Appendix A: Cross sections of $e^+e^- \rightarrow l^+l^-$

If the flavor-changing couplings in Eq. (19) are absent, the cross-sections are known in the literature [6, 10]. If instead $b_{L,R}^{e,l} = 0$, one derives from Eq. (19)

$$
\sigma_{ee\rightarrow ll} = \frac{4\pi\alpha^2}{3s} + \frac{\alpha}{6} \frac{(g_L + g_R)^2 (s - m_Z^2)}{(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2} + \frac{(g_L^2 + g_R^2)^2 s}{48\pi \left[ (s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2 \right]} + \frac{\left[ (b_L^{el})^2 + (b_R^{el})^2 \right]^2}{4s^3} + \frac{\left[ g_L^2 (b_L^{el})^2 + g_R^2 (b_R^{el})^2 \right] (s - m_Z^2)}{16\pi \left[ (s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2 \right] s^2} \times \left[ 2m_{Z'}^2 s + 3s^2 + 2(m_{Z'}^2 + s)^2 \ln \frac{m_{Z'}^2}{m_{Z'}^2 + s} \right]
+ \frac{(b_L^{el})^4 + (b_R^{el})^4}{16\pi m_{Z'}^2 s^2} \left[ 2m_{Z'}^2 s + s^2 + 2m_{Z'}^2 (m_{Z'}^2 + s) \ln \frac{m_{Z'}^2}{m_{Z'}^2 + s} \right] + \frac{(b_L^{el})^2 (b_R^{el})^2 s}{8\pi m_{Z'}^2 (m_{Z'}^2 + s)}, \quad (A1)
$$

$$
\sigma_{ee\rightarrow ll}^{FB} = \frac{\alpha}{8} \frac{(g_L - g_R)^2 (s - m_Z^2)}{(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2} + \frac{(g_L^2 - g_R^2)^2 s}{64\pi \left[ (s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2 \right]} + \frac{\left[ (b_L^{el})^2 + (b_R^{el})^2 \right]^2}{8s^3} + \frac{\left[ g_L^2 (b_L^{el})^2 + g_R^2 (b_R^{el})^2 \right] (s - m_Z^2)}{32\pi \left[ (s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2 \right] s^2} \times \left[ s^2 + 4(m_{Z'}^2 + s)^2 \ln \frac{4m_{Z'}^2 (m_{Z'}^2 + s)}{(2m_{Z'}^2 + s)^2} \right]
+ \frac{(b_L^{el})^4 + (b_R^{el})^4 (m_{Z'}^2 + s)}{16\pi m_{Z'}^2 (2m_{Z'}^2 + s)s^2} \left[ s^2 + 2m_{Z'}^2 (2m_{Z'}^2 + s) \ln \frac{4m_{Z'}^2 (m_{Z'}^2 + s)}{(2m_{Z'}^2 + s)^2} \right]
+ \frac{(b_L^{el})^2 (b_R^{el})^2 s^2}{8\pi m_{Z'}^2 (m_{Z'}^2 + s)(2m_{Z'}^2 + s)}, \quad (A2)
$$

where we have assumed $g_{L,R}^{el}$ to be real and neglected the lepton masses.

[1] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716, 1 (2012) [arXiv:1207.7214 [hep-ex]]; S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716, 30 (2012) [arXiv:1207.7235 [hep-ex]].

[2] For the case of heavy Z' bosons, see, for example, P. Langacker, Rev. Mod. Phys. 81, 1199 (2009) [arXiv:0801.1345 [hep-ph]] and references therein. For the lighter case, see, for example, M. Williams,
C.P. Burgess, A. Maharana, and F. Quevedo, JHEP 1108, 106 (2011) [arXiv:1103.4556 [hep-ph]] and references therein.

[3] G. Brooijmans, M.C. Chen, and B.A. Dobrescu, in Ref. [8].

[4] F. del Aguila, J. de Blas, P. Langacker and M. Perez-Victoria, Phys. Rev. D 84, 015015 (2011) [arXiv:1104.5512 [hep-ph]].

[5] V. Barger, C.W. Chiang, J. Jiang, and P. Langacker, Phys. Lett. B 596, 229 (2004) [arXiv:hep-ph/0405108]; V. Barger, C.W. Chiang, P. Langacker, and H.S. Lee, Phys. Lett. B 598, 218 (2004) [arXiv:hep-ph/0406126]; A. Arhrib, K. Cheung, C.W. Chiang, and T.C. Yuan, Phys. Rev. D 73, 075015 (2006) [arXiv:hep-ph/0602175]; K. Cheung, C.W. Chiang, N.G. Deshpande, and J. Jiang, Phys. Lett. B 652, 285 (2007) [arXiv:hep-ph/0604223]; X.G. He and G. Valencia, Phys. Rev. D 74, 013011 (2006) [arXiv:hep-ph/0605202]; C.W. Chiang, N.G. Deshpande, and J. Jiang, JHEP 0608, 075 (2006) [arXiv:hep-ph/0606122]; X.G. He and G. Valencia, Phys. Lett. B 651, 135 (2007) [arXiv:hep-ph/0703270]; V. Barger, L.L. Everett, J. Jiang, P. Langacker, T. Liu, and C.E.M. Wagner, JHEP 0912, 048 (2009) [arXiv:0906.3745 [hep-ph]]; X.G. He and G. Valencia, Phys. Lett. B 680, 72 (2009) [arXiv:0907.4034 [hep-ph]]; Q. Chang, X.Q. Li, and Y.D. Yang, JHEP 1002, 082 (2010) [arXiv:0907.4408 [hep-ph]]; N.G. Deshpande, X.G. He, and G. Valencia, Phys. Rev. D 82, 056013 (2010) [arXiv:1006.1682 [hep-ph]]; A.K. Alok, S. Baek, and D. London, JHEP 1107, 111 (2011) [arXiv:1101.1333 [hep-ph]]; F. Yu, Phys. Rev. D 83, 094028 (2011) [arXiv:1104.0243 [hep-ph]]; K. Cheung and J. Song, Phys. Rev. Lett. 106, 211803 (2011) [arXiv:1104.1375 [hep-ph]]; P. Ko, Y. Omura, and C. Yu, Phys. Lett. B 710, 197 (2012) [arXiv:1104.4066 [hep-ph]]; P.J. Fox, J. Liu, D. Tucker-Smith, and N. Weiner, Phys. Rev. D 84, 115006 (2011) [arXiv:1104.4127 [hep-ph]]; S. Chang, K.Y. Lee, and J. Song, arXiv:1104.4560 [hep-ph]; Z. Liu, P. Nath, and G. Peim, Phys. Lett. B 701, 601 (2011) [arXiv:1105.4371 [hep-ph]]; J.L. Hewett and T.G. Rizzo, arXiv:1106.0294 [hep-ph]; J. Fan, D. Krohn, P. Langacker, and I. Yavin, arXiv:1106.1682 [hep-ph]; P. Ko, Y. Omura, and C. Yu, arXiv:1108.0350 [hep-ph]; J.Y. Liu, Y. Tang, and Y.L. Wu, J. Phys. G 39, 055003 (2012) [arXiv:1108.5012 [hep-ph]]; C.X. Yue, Y.M. Zhang, and L.J. Liu, Phys. Lett. B 547, 252 (2002) [hep-ph/0209291]; J.I. Aranda, J. Montano, F. Ramirez-Zavaleta, J.J. Toscano, and E.S. Tututti, Phys. Rev. D 86, 035008 (2012) [arXiv:1202.6288 [hep-ph]].

[6] C.W. Chiang, Y.F. Lin, and J. Tandean, JHEP 1111, 083 (2011) [arXiv:1108.3969 [hep-ph]].

[7] C.D. Carone and H. Murayama, Phys. Rev. Lett. 74, 3122 (1995) [hep-ph/9411256]; Phys. Rev. D 52, 484 (1995) [hep-ph/9501220]; E. Ma and D.P. Roy, Phys. Rev. D 58, 095005 (1998) [hep-ph/9806210].

[8] J. Beringer et al. [Particle Data Group Collaboration], Phys. Rev. D 86, 010001 (2012).

[9] J. Alcaraz et al. [ALEPH and DELPHI and L3 and OPAL and LEP Electroweak Working Group Collaborations], hep-ex/0612034.

[10] B. Kors and P. Nath, JHEP 0507, 069 (2005) [arXiv:hep-ph/0503208].

[11] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. Lett. 109, 111807 (2012) [arXiv:1205.5368 [hep-ph]].

[12] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. Lett. 109, 111808 (2012) [arXiv:1205.5370 [hep-ph]].

[13] https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CERNYellowReportPageBR2.

[14] ATLAS Collaboration, Report No. ATLAS-CONF-2012-170, 2012; CMS Collaboration, Report No. CMS-PAS-HIG-12-043, 2012.

[15] S. Chatrchyan et al. [CMS Collaboration], JHEP 1212, 034 (2012) [arXiv:1210.3844 [hep-ex]].

[16] A. Pukhov et al., arXiv:hep-ph/9908288.