Entanglement of scattered single photon with atom

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Single–photon which is initially uncorrelated with atom, will evolve to be entangled with the atom on their continuous kinetic variables in the process of resonant scattering. We find the relations between the entanglement and their physical control parameters, which indicates that high entanglement can be reached by broadening the scale of the atomic wave or squeezing the linewidth of the incident single–photon pulse.

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I. INTRODUCTION

Quantum entanglement is of fundamental importance in the theory of quantum nonlocality[1] as well as in quantum information[2]. Recently, photon–atom entanglement is frequently discussed in their finite Hilbert spaces[2], such as, the polarizations of photon or the internal states of atom. With the progress of micro–cavity quantum electrodynamics[4] and high coupling artificial atom[5], single photon raises its ability to affect considerably not only the atom's internal state but also its external motion. As a result, it gives rise to some basic questions related to the photon–atom entanglement on their infinite kinetic degree of freedom.

In recent studies[6][7], entanglement in the continuous kinetic variables between single–photon and atom is mostly discussed in the process of single–photon emission with atomic recoil, where the atom is initially pumped to its excited level and the single–photon is prepared “intrinsically” by the atomic spontaneous emission. In our work, however, the resonant single–photon is initially injected from a tuneable single–photon generator[8], whereas an artificial atom is placed freely in vacuum on its steady state (“artificial” indicates that the atomic coupling to the single–photon is stronger than usual, which ensures the interaction observable[9]). We find that, after the interaction, the scattered single–photon will be entangled to the atom at a higher degree compared with the case of solely spontaneous emission. We explain this phenomena as the coherent pumping of the incident photon and evaluate it with a defined “entanglement pumping coefficient”.

To describe the degree of entanglement, firstly, we use the ratio \( R \) between the conditional and unconditional variance in momentum to evaluate the two particles’ correlation in the probability amplitude of their wave function, which is experimentally accessible and can be seen as the “amplitude entanglement” in momentum space[10][11]; secondly, we use the standard Schmidt decomposition[12] and treat Schmidt number \( K \)[13] as a criterion for the full entanglement contained both in amplitude and phase. For both criterions \( R \) and \( K \), we revealed their dependencies on the physical control parameters \( \tau \) and \( \eta \), and compare them in some region of interests, from which it is shown that: higher entanglement can be achieved by either broadening the scale of the atomic wave or squeezing the linewidth of the incident single–photon. Transmitted photon is also considered, which is different to the scattered photon, and exhibits little entanglement with the atom due to its interference with the transparent wave (initially incident photon wave profile).

II. THEORETICAL ANALYSIS

As shown in Fig. 1 (a), the two–level atom with transition frequency \( \omega_a \) and mass \( m \) is placed freely in vacuum, the ground and excited states of which are denoted by \( |1\rangle \) and \( |2\rangle \), respectively. The incident single–photon from some generator is resonant with the atom and exhibits a superposed state of different fock states due to its linewidth. For the realistic consideration in some experiments[14], we fix the photon detector and atom detector in opposite directions and make them both in the \( x–z \) plane for simplicity as in Fig. 1 (b), the angle \( \theta \) can be chosen to observe the scattering in needed directions.

Under the rotating wave approximation (RWA) the
therefore be expanded as:
atomic internal state. At time
wave vector of the atom, and of the photon, and the
where the arguments in the kets denote, respectively, the
of the Hilbert space can be denoted as
polarization to detect the photon.
photon state, since we can always choose a particular
the polarization index in the summation as well as in
and creation operators for the light mode with photonic
σ
direction as
(b) The incident photon is scattered by the atom, angle θ is
fixed to determine the direction of the detection.
(c) Schematic diagram for the absorption–emission process.
The process of emission with atomic recoil will generate entan-
glement between the recoiled atom and the scattered pho-
ton due to the momentum conservation.

Hamiltonian can be written in Schrödinger picture as:

$$\hat{H} = \frac{(\hbar \hat{p})^2}{2m} + \sum_k \hbar \omega_k \hat{a}_k^\dagger \hat{a}_k + \hbar \omega_a \hat{\sigma}_{22}$$

$$+ \hbar \sum_k \left[ g(\vec{k}) \hat{a}_k \hat{a}_k^\dagger \hat{c}_k e^{-i \vec{k} \cdot \vec{r}} + \text{H.c.} \right], \quad (1)$$

where $\hbar \hat{p}$ and $\vec{r}$ denote atomic center–of–mass
momentum and position operators, $\hat{\sigma}_{ij}$ denotes the atomic
operator $|i\rangle \langle j|$ ($i, j = 1, 2$), $\hat{a}_k$ and $\hat{a}_k^\dagger$ are the annihilation
and creation operators for the light mode with photonic
wave vector $\vec{k}$ and frequency $\omega_k = ck$, respectively. Note
the summation is performed over all coupled modes in the
continuous Hilbert space. We also suppress the polarization index in the summation as well as in
photon state, since we can always choose a particular
polarization to detect the photon. $g(\vec{k})$ is the dipole
coupling coefficient.

As there is only one photon in the interaction, the basis
of the Hilbert space can be denoted as $|\vec{q}, 1\vec{k}, i\rangle$ ($i = 1, 2)$,
where the arguments in the kets denote, respectively, the
wave vector of the atom, and of the photon, and
the atomic internal state. At time $t$ the state vector can
therefore be expanded as:

$$|\psi\rangle = \sum_{\vec{q}, \vec{k}} C_1(\vec{q}, \vec{k}, t)|\vec{q}, 1\vec{k}, 1\rangle + \sum_{\vec{q}} C_2(\vec{q}, t)|\vec{q}, 0, 2\rangle. \quad (2)$$

Substituting Eqs. (1) and (2) into Schrödinger equa-
tion yields:

$$i\hat{A}(\vec{q}, \vec{k}, t) = g(\vec{k}) B(\vec{q} + \vec{k}, t) e^{i(\omega_a - c k + \frac{\hbar}{2m}(2\vec{q} - \vec{k}) \cdot \vec{k}) t} \quad \text{(3)}$$

$$i\hat{B}(\vec{q}, t) = \sum_k g^*(\vec{k}) A(\vec{q} - \vec{k}, \vec{k}, t) e^{i\omega_a - c k + \frac{\hbar}{2m}(2\vec{q} - \vec{k}) \cdot \vec{k}) t} \quad \text{(4)}$$

where $A, B$ are the slowly varying parts of $C_1$ and $C_2$, i.e.:

$$A(\vec{q}, \vec{k}, t) = C_1(\vec{q}, \vec{k}, t) e^{i(\frac{\hbar^2}{2m} + c k) t} \quad \text{(5)}$$

$$B(\vec{q}, t) = C_2(\vec{q}, t) e^{i(\frac{\hbar^2}{2m} + \omega_a) t} \quad \text{(6)}$$

Suppose the atom is initially in the ground state and
has zero average velocity, the initial condition can be set as:

$$A(\vec{q}, \vec{k}, t = 0) = \chi_0 G(\vec{q}) P(\vec{k} - \vec{k}_0) \quad \text{(7)}$$

$$B(\vec{q}, t = 0) = 0 \quad \text{(8)}$$

where $G(\vec{q}) = G_x(q_x) G_y(q_y) G_z(q_z)$ and $P(\vec{k} - \vec{k}_0) =
\int (k_x P_x(k_x) P_y(k_y) P_z(k_z - k_0)).$ In this case, functions $G_i(q_i)$
and $P_i(k_i)$ ($i = x, y, z$) have zero center value and
bandwidths $\delta q_i$ and $\delta k_i$ separately. The coordinates are
chosen as in Fig. 1 (b), where we make the incident
direction as $z$–axis. $\chi_0$ is the normalized factor and
$\vec{k}_0 = (0, 0, \omega_a c)$ is the resonant wave
dvector.

We proceed to solve the equations with Laplace transforma-
tion and single pole approximation and yield:

$$B(\vec{q}, t) = -i\chi_0 \sum_k \{ g^*(\vec{k}) \times \frac{G(\vec{q} - \vec{k}) P(\vec{k} - \vec{k}_0)}{i L + \Gamma + i(\omega_a - c k + \frac{\hbar}{2m}(2\vec{q} - \vec{k}) \cdot \vec{k})} \}$$

where the frequency shift $L$ and atomic linewidth $\Gamma$ are
given as:

$$L = \sum_k \frac{|g(\vec{k})|^2}{\omega_a - c k + \frac{\hbar}{2m}(2\vec{q} - \vec{k}) \cdot \vec{k}},$$

$$\Gamma = \pi \sum_k |g(\vec{k})|^2 \delta(\omega_a - c k).$$

We can simplify Eq. (9), by replacing the term $G(\vec{q} - \vec{k})$
with $G(\vec{q} - \vec{k}_0)$ since the momenta bandwidth $\delta q_i$, due to
the recoil is normally much larger than the photon
linewidth $\delta k_i$; also, we can replace $\vec{k}$ with $\vec{k}_0$ in the term
$\frac{\hbar}{2m}(2\vec{q} - \vec{k}) \cdot \vec{k}$. With these approximations, the first
term in the curly bracket can be seen as the antifourier
transform of the product of the photonic shape and Lorentzian shape, and will cause a decay at a time scale
max$(\frac{1}{L}, \frac{1}{\Gamma})$; the second decay term $e^{-\frac{\hbar}{2m}(2\vec{q} - \vec{k}) \cdot \vec{k}}$ is due
to the spontaneous emission. Then, one can directly
find that \( B(\vec{q}, t \to \infty) \to 0 \). In the further calculations, we ignore the frequency shift since it can be treated as a modification of the atomic transition frequency, and regard the slowly varying function \( g(\vec{k}) \) as a constant.

With the approximations mentioned above, from Eqs. (3) and (9), we obtain the steady solution of \( A(\vec{q}, \vec{k}, t \to \infty) \):

\[
A(\vec{q}, \vec{k}, t \to \infty) = \chi_0 G(\vec{q}) P(\vec{k} - \vec{k}_0) + \frac{\chi_0 g^2 G(\vec{q} + \vec{k} - \vec{k}_0)}{\Gamma - i \left[ c k - a - [h(\vec{q} + \vec{k})]^2 / 2m + (h\vec{q})^2 / 2m \right]} \times \\
\sum_{\vec{k}_1} i \left[ c k - c k_1 - \frac{\hbar}{2m} (2\vec{q} + \vec{k}) \cdot \vec{k} + \frac{\hbar}{m} (\vec{q} + \vec{k}) \cdot \vec{k}_0 - \frac{\hbar}{2m} \vec{k}_0 \right] \right]
\tag{10}
\]

From Eq. (10), one sees that the final state is a superposition of the transparent wave (initially incident photon wave profile, depicted by the first term on the r.h.s.) and scattering wave (second term on the r.h.s.). In the scattering part, the atom and the photon are entangled due to the process of photon absorption and emission with atomic recoil, which is sketched in Fig. 1 (c). One may find that the Lorentzian–Gaussian factor in the scattering part is very similar to that in the case of spontaneous emission with recoil [8], where the Gaussian term is a reflection of momentum conservation and the Lorentzian term indicates the energy conservation.

The general formula (10) can be used to analyze the photon scattered in different directions. Without loss of physical generality, we choose the initial conditions for the atom as \( G_1(q_i) = e^{-(q_i/\theta q_i)^2} \), and for the photon \( P_i(k_i) = 1/(k_i/\delta k_i + 1) \) which is exactly the case if the incident single-photon is generated by spontaneous emission. As a remark, we point out that all the conclusions in the following keep available when the incident photon is chosen to be other shapes such as Gaussian or whatever.

III. AMPLITUDE ENTANGLEMENT IN SCATTERED PHOTON

To make the physical results more evident and avoid unnecessary mathematical complexity, we focus our attention on the photon scattered perpendicular to the incident direction, i.e., \( \theta = \frac{\pi}{2} \). Then we project Eq. (10) into the subspace \( |(q_x, 0, 0)\rangle \otimes |(k_x, 0, 0)\rangle \), with the same approximations used in Eq. (9), and yield:

\[
A_{2x} = N \cdot \exp \left[ - (\Delta q_x - \frac{\hbar k}{m \tau_x})^2 / 4 \tau_x^2 \right] / (\Delta k_x + \Delta q_x + i) \left[ (\Delta k_x + \Delta q_x) / \tau_x + i \right],
\]

\[
\approx N \cdot \exp \left[ - (\Delta q_x / \eta_x)^2 \right] / (\Delta k_x + \Delta q_x + i) \left[ (\Delta k_x + \Delta q_x) / \tau_x + i \right].
\tag{11}
\]

where \( \Delta k_i = \frac{k_i - k_i}{\tau_i} \), \( \Delta q_i = \frac{h k_p(q_i - k_0)}{m \tau_i} \), \( \eta_i = \frac{\delta k_p}{m \tau_i} \), \( \tau_i = \frac{\delta k_i}{\Gamma / \tau} \) \( i = x, y, z \) are all defined dimensionless parameters. Note that \( \eta_x \) and \( \tau_x \) contain all the physical parameters that determine the nature of the atom–photon system, thus can be treated as physical control parameters for the atom and the photon, respectively. We neglect tiny terms in Eq. (11) due to \( \hbar k_0^2 \ll m \Gamma \) and \( \hbar k_0 \ll mc \) in realistic conditions. \( N \) is the normalization factor where \( N^2 = \sqrt{2} (1 + \tau_x) / \pi \tau_x \eta_x \).

From Eq. (11) and Fig. 2, one sees that, variables \( \Delta q_x \) and \( \Delta k_x \) play the symmetric role in the two Lorentzian functions. It makes the probability amplitude \( |A_{2x}|^2 \) localized along the diagonal of the momentum space, which implies the nonfactorization of the photon–atom wave.
function, and then will generate entanglement between the two particles. In fact, we can treat the ratio \( R \) of the conditional and unconditional variances for \( \Delta q_x \) or \( \Delta k_x \) as an evaluation of entanglement \[10\]. This ratio, compared to the Schmidt number \( K \), reveals more obvious analytic dependence for the entanglement on its control parameters \( \eta_x \) and \( \tau_z \), and is also experimentally directly accessible \[14\].

We proceed to calculate the ratio for variable \( \Delta q_x \), i.e.,
\[
R \equiv \frac{\delta \Delta q_x^{\text{single}}}{\delta \Delta q_x^{\text{coine}}} \quad \text{where the unconditional variance is obtained from the single–particle observation as:}
\]
\[
\delta^2 \Delta q_x^{\text{single}} = \langle \Delta q_x^2 \rangle - \langle \Delta q_x \rangle^2 \quad \text{(12)}
\]
\[
= \int d\Delta k_x d\Delta q_x \Delta q_x^2 |A_{\Delta q}|^2 - \left( \int d\Delta k_x d\Delta q_x \Delta q_x |A_{\Delta q}|^2 \right)^2, \quad \text{(13)}
\]
and coincidence measurement gives the conditional variance at some specified \( \Delta k_x \):
\[
\delta^2 \Delta q_x^{\text{coine}} = \langle \Delta q_x^2 \rangle_{\Delta k_x} - \langle \Delta q_x \rangle_{\Delta k_x}^2
\]
\[
= \frac{\int d\Delta k_x d\Delta q_x \Delta q_x^2 |A_{\Delta q}|^2}{\int d\Delta q_x |A_{\Delta q}|^2} - \left( \frac{\int d\Delta k_x d\Delta q_x \Delta q_x |A_{\Delta q}|^2}{\int d\Delta q_x |A_{\Delta q}|^2} \right)^2.
\]

Substituting Eqs. (11)–(13) into the definition of \( R \), we yield \( R(\eta_x, \tau_z) \) as a function of parameters \( \eta_x \) and \( \tau_z \), the result of which is illustrated in Fig. 3 with \( \Delta k_x \) fixed at the origin. From that, one can see that the entanglement increases monotonously when \( \eta_x \) increases or \( \tau_z \) decreases, which indicates that higher entanglement can be achieved by squeezing the linewidth of the incident photon or broadening the wave packet of the atom. In particular, when \( \eta_x > 1 \), we have:
\[
R \approx \frac{\eta_x + \sqrt{\tau_z} (1 + \tau_z)}{2\sqrt{\tau_z}}, \quad \text{(14)}
\]
from which it is found that the entanglement increases linearly with \( \eta_x \) and will be abruptly enhanced when \( \tau_z \) tends to zero. As a remark, we emphasize that all the conclusions above hold qualitatively the same either if \( \Delta k_x \) is specified otherwise or one calculate the ratio \( R \) from the other variable \( \Delta k_x \).

The ratio \( R \), which can be obtained experimentally by comparing the momentum dispersion variance, is an appropriate quantification for the entanglement contained in the probability amplitude correlation (thus can be seen as an evaluation of the “amplitude entanglement”). Next, we can see that it reveals a correct varying tendency for the entanglement with its control parameters. However, the definition of \( R \) is dependent on its representation space and different choices for the basis of Hilbert space will cause distinct values of \( R \). This is because we only use the amplitude of the wavefunction to construct \( R \), and then all entanglements included in phase \[11\] is lost.

To obtain the “total entanglement”, we calculate the Schmidt number \[13\] and compare it with the entanglement ratio \( R \) in the following section.

IV. FULL ENTANGLEMENT IN SCATTERED PHOTON

Mathematically, for a bipartite system in pure state, the entanglement of an unfactorable wavefunction can be completely characterized by the Schmidt number, which is denoted by \( K \equiv (\sum_{n=0}^{\infty} \lambda_n)^{-1} \), where \( \lambda_n \)'s are eigenvalues of the integral equation \[12\]:
\[
\int d\Delta k_x' \rho^P(\Delta k_x, \Delta k_x') \phi_n(\Delta k_x') = \lambda_n \phi_n(\Delta k_x), \quad \text{(15)}
\]
the density matrix for photon is defined as:
\[
\rho^P(\Delta k_x, \Delta k_x') \equiv \int d\Delta q_x A_{\Delta q}(\Delta q_x, \Delta k_x) A_{\Delta q}^\dagger(\Delta q_x, \Delta k_x'), \quad \text{(16)}
\]
where, note that we have taken away the time–dependent phase in the density matrix since it does not contribute to entanglement. Although we do it with the photon, Schmidt number can be equally obtained through the atomic density matrix, and the eigenfunctions of atom \[\psi_n(\Delta q_x)\] can be related to those of photon through:
\[
\psi_n(\Delta q_x) = \frac{1}{\sqrt{\lambda_n}} \int d\Delta k_x A_{\Delta q}(\Delta q_x, \Delta k_x) \phi_n^*(\Delta k_x), \quad \text{(17)}
\]
where \( \phi_n(\Delta k_x) \) and \( \psi_n(\Delta q_x) \) \((n = 1, 2, \cdots)\) form complete orthonormal sets for the photon and atom respectively. With these discrete modes, the unfactorable wavefunction can be expanded into a sum of factored products uniquely:
\[
A_{\Delta q}(\Delta q_x, \Delta k_x) = \sum_n \sqrt{\lambda_n} \psi_n(\Delta q_x) \phi_n(\Delta k_x). \quad \text{(18)}
\]
Then, the Schmidt number $K$, which is an estimation of the number of modes that are “important” in making up the expansion of Eq. (18), serves as a quantitative measurement of entanglement. Note $K$ is independent from representation since all $X$’s keep the same in different representations, thus can be seen as a quantity of the full entanglement information (both amplitude and phase entanglement) kept in the collective wavefunction.

Since Eq. (15) is not analytically solvable, we use a discrete eigenvalue equation to approximate the integral equation. Up to a reliable precision, we use 1000 discrete eigenvalue equation to approximate the integral wavefunction.

From the numerical results, we find that, similar to the ratio $R$, $K$ rises linearly with parameter $\eta_z$ and will increase rapidly when the linewidth of incident photon is squeezed narrower to the atomic linewidth $\Gamma$, i.e., $\tau_z < 1$; secondly, when $\tau_z$ is fixed, the slope of $K(\eta_z)$ is always larger than that of $R(\eta_z)$, which means that more entanglement information will transfer to phase when $\eta_z$ becomes larger, and this phenomena will become more evident when $\tau_z$ is reduced, e.g., when $\tau_z = 0.1$, $R \approx 1.58\eta_z + 1.39$ whereas $K \approx 3.44\eta_z + 0.08$, which indicates that more than half of the entanglement information will be unavailable by momentum dispersion observation when $\eta_z$ goes large on this condition.

Another phenomena is notable, when $\tau_z = 1$, i.e., the linewidth of the incident photon is not squeezed and can be prepared directly by spontaneous emission from the same atom, we find $K \approx 0.75\eta_z + 0.16$ ($\eta_z \gg 1$) whereas in the case of spontaneous emission $K \approx 0.28\eta + 0.72$.

This difference indicates that, although in both cases, entanglement is generated from momentum conservation in the process of photon emission with atomic recoil, the absorption of the incident photon will add some entanglement due to its coherent pumping effect. As $K$ is linear with $\eta$ (or $\eta_z$), we define the “entanglement pumping coefficient” as:

$$\text{EPC} = \frac{\text{slope of } K(\eta_z) \text{ in scattering}}{\text{slope of } K(\eta) \text{ in spontaneous emission}},$$

since the constant term in $K(\eta)$ plays a minor role when entanglement is large. The defined coefficient EPC shows the times that entanglement is increased by the coherent pumping of an incident photon. As it is independent on the atomic parameter, it reflects the ability of entanglement of the photon separately. We collect some numerical results in Fig. 5 and fit it with EPC $\approx 1.1/\tau_z + 1.5$ within $\tau_z \in (0, 1)$, from which, one sees that EPC increases rapidly when $\tau_z$ diminishes, which also implies that, if the incident photon is prepared monochromatically on its limit condition, i.e., $\tau_z \to 0$, the scattered photon will be highly entangled to the recoiled atom.

We plot the amplitude of the first three Schmidt modes for the photon with $\eta_z = 10$ and $\tau_z = 1$ in Fig. 6. We find that their number of peaks in momentum space is proportional to the Schmidt mode index, but the separations of different peaks are more distinct than in the case of spontaneous emission.

\section{V. TRANSMITTED PHOTON}

To consider the transmitted photon, we make the observation angle $\theta = 0$, and yield the collective wavefunction from Eq. (10):

$$A_0 = -\chi_0 G_z(q_z) P_z(k_z) + \chi_0 \frac{\pi}{4} \left( \frac{\Gamma}{ck_0} \right)^2 \frac{\tau_z \tau_0 G_z(q_z) P_z(k_z)}{1 - i(\Delta k_z + \Delta q_z + \frac{\hbar k_z^2}{2m^2})} \quad (19)$$
FIG. 6: First three Schmidt modes for the scattered and transmitted photon. Left column is for the scattered photon with $\tau_z = 1$ and $\eta_x = 10$; right column is for the transmitted photon with $\tau_z = 1$, $\eta_x = 10$, and $\frac{\Delta k^2}{k_c^2} \tau_x \tau_y = 1$ for illustration.

One can see that, in Eq. (19), the first term describes that the two particles are free of interaction and keep their initial factorable wave form; the second term reflects the entanglement. Usually, the second term is much smaller than the first one since $\frac{\Delta k^2}{k_c^2} \ll 1$, but one can enlarge it by choosing some special physical system, such as the artificial atom with low excited level and high coupling to its resonant modes. However, this improvement can add few entanglement between the transmitted photon and recoiled atom, because interference between the two terms in Eq. (19) will weaken the correlation of the two particles at a great deal. To make it clear, we show the contour and density plots for the probability amplitude of $A_0$ in Fig. 2 on an artificial condition $\frac{\Delta k^2}{k_c^2} \tau_x \tau_y = 1$, and yield $R \approx K < 2$ in this situation.

The eigenfunctions of transmitted photon for the first three modes with $\eta_z = 10$ and $\tau_z = 1$ are collected in Fig. 6, from which one can see that, due to the interference, the corresponding modes of the transmitted photon exhibit one peak less than that of the scattered photon.

VI. CONCLUSION

We analyze the physically fundamental interaction between a single photon and a free artificial atom in vacuum. With a few physical approximations, the general solution of the photon–atom wave function is obtained, from which, it is found that the initially uncorrelated particles will evolve to be entangled due to momentum conservation in scattering. To evaluate the entanglement in the scattering, firstly, we use an experimentally accessible parameter $R$, which denotes the ratio between momentum variance in single-particle and in coincidence observations, and yield its simple dependences on the two physical control parameters $\eta_x \equiv \frac{\delta q}{\hbar k_0}$ and $\tau_z \equiv \frac{\delta k}{\Gamma_c}$; secondly, we use standard Schmidt decomposition to reveal the full entanglement information and find out its varying tendency similar to that of $R$, which indicates that high entanglement can be achieved by either squeezing the linewidth of the incident photon or broadening the scale of atomic wave packet. Furthermore, compared with spontaneous emission, we defined a parameter EPC to evaluate the entanglement enhancement due to the coherent pumping effect of the resonant incident photon. In the end, we found out that, for the transmitted photon, one can expect little entanglement due to the interference between the transparent and scattered wave.

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I. INTRODUCTION

Quantum entanglement is of fundamental importance in the theory of quantum nonlocality[1] as well as in quantum information[2]. Recently, photon–atom entanglement is frequently discussed in their finite Hilbert spaces[3], such as, the polarizations of photon or the internal states of atom. With the progress of micro–cavity quantum electrodynamics[4] and high coupling artificial atom[5], single photon raises its ability to affect considerably not only the atom’s internal state but also its external motion. As a result, it gives rise to some basic questions related to the photon–atom entanglement on their infinite kinetic degree of freedom.

In recent studies[6][7], entanglement in the continuous kinetic variables between single–photon and atom is mostly discussed in the process of single–photon emission with atomic recoil, where the atom is initially pumped to its excited level and the single–photon is prepared “intrinsically” by the atomic spontaneous emission. In our work, however, the resonant single–photon is initially injected from a tuneable single–photon generator[8], whereas an artificial atom is placed freely in vacuum on its steady state (“artificial” indicates that the atomic coupling to the single–photon is stronger than usual, which ensures the interaction observable). We find that, after the interaction, the scattered single–photon will be entangled to the atom at a higher degree compared with the case of solely spontaneous emission. We explain this phenomena as the coherent pumping of the incident photon and evaluate it with a defined “entanglement pumping coefficient”.

II. THEORETICAL ANALYSIS

As shown in Fig. 1(a), the two–level atom with transition frequency \( \omega_a \) and mass \( m \) is placed freely in vacuum, the ground and excited states of which are denoted by \( |1\rangle \) and \( |2\rangle \), respectively. The incident single–photon from some generator is resonant with the atom and exhibits a superposed state of different fock states due to its interference with the transparent wave.

To describe the degree of entanglement, firstly, we use the ratio \( (R) \) between the conditional and unconditional variance in momentum to evaluate the two particles’ correlation in the probability amplitude of their wave function, which is experimentally accessible and can be seen as the “amplitude entanglement” in momentum space[10][11]; secondly, we use the standard Schmidt decomposition[12] and treat Schmidt number \( K \)[13] as a criterion for the full entanglement contained both in amplitude and phase. For both criterions \( R \) and \( K \), we revealed their dependencies on the physical control parameters \( \tau \) and \( \eta \), and compare them in some region of interests, from which it is shown that: higher entanglement can be achieved by either broadening the scale of the atomic wave or squeezing the linewidth of the incident single–photon. Transmitted photon is also considered, which is different to the scattered photon, and exhibits little entanglement with the atom due to its interference with the transparent wave.

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therefore be expanded as:

$$|\psi_t\rangle = \sum_{\vec{q}, \vec{k}} C_1(\vec{q}, \vec{k}, t)|\vec{q}, \vec{k}, 1\rangle + \sum_{\vec{q}} C_2(\vec{q}, t)|\vec{q}, 0, 2\rangle. \quad (2)$$

Substituting Eqs. (1) and (2) into Schrödinger equation yields:

$$i\hat{A}(\vec{q}, \vec{k}, t) = g(\vec{k})B(\vec{q}, \vec{k}, t)e^{i(\omega_a - \frac{\hbar}{2m}(2\vec{q} - \vec{k}) \cdot \vec{k})t}, \quad (3)$$

$$i\hat{B}(\vec{q}, t) = \sum_{\vec{k}} g^*(\vec{k})A(\vec{q}, \vec{k}, t)e^{i(\omega_a - c\vec{k} + \frac{\hbar}{2m}(2\vec{q} - \vec{k}) \cdot \vec{k})t}, \quad (4)$$

where $A, B$ are the slowly varying parts of $C_1$ and $C_2$, i.e.:

$$A(\vec{q}, \vec{k}, t) = C_1(\vec{q}, \vec{k}, t)e^{i(\omega_a + \frac{\hbar}{2m}(2\vec{q} - \vec{k}) \cdot \vec{k})t}, \quad (5)$$

$$B(\vec{q}, t) = C_2(\vec{q}, t)e^{i(\delta\omega_a + \omega_a)\frac{t}{2}}. \quad (6)$$

Suppose the atom is initially in the ground state and has zero average velocity, the initial condition can be set as:

$$A(\vec{q}, \vec{k}, t = 0) = \chi_0 G(\vec{q})P(\vec{k} - \vec{k}_0), \quad (7)$$

$$B(\vec{q}, t = 0) = 0, \quad (8)$$

where $G(\vec{q}) = G_x(q_x)G_y(q_y)G_z(q_z)$ and $P(\vec{k} - \vec{k}_0) = P_x(k_x)P_y(k_y)P_z(k_z - k_0)$. In this case, functions $G_i(q_i)$ and $P_i(k_i)$ ($i = x, y, z$) have zero center value and bandwidths $\delta q_i$ and $\delta k_i$ separately. The coordinates are chosen as in Fig. 1 (b), where we make the incident direction as $z$-axis. $\chi_0$ is the normalized factor and $\vec{k}_0 = (0, 0, \omega_a)\frac{\hbar}{c}$ is the resonant wave vector. We proceed to solve the equations with Laplace transformation and single pole approximation and yield:

$$B(\vec{q}, t) = -i\chi_0 \sum_{\vec{k}} g^*(\vec{k}) \times$$

$$\left\{ \frac{G(\vec{q} - \vec{k})P(\vec{k} - \vec{k}_0)e^{i(\delta\omega_a - \omega_a)(2\vec{q} - \vec{k}) \cdot \vec{k})t - e^{-iLt - \Gamma t}}{iL + \frac{\hbar}{2m}(2\vec{q} - \vec{k}) \cdot \vec{k}} \right\}, \quad (9)$$

where the frequency shift $L$ and atomic linewidth $\Gamma$ are given as:

$$L = \sum_{\vec{k}} \frac{|g(\vec{k})|^2}{\omega_a - \omega_a - c\vec{k} + \frac{\hbar}{2m}(2\vec{q} - \vec{k}) \cdot \vec{k}},$$

$$\Gamma = \frac{\pi}{\hbar\omega_a} \sum_{\vec{k}} |g(\vec{k})|^2 \delta(\omega_a - c\vec{k}).$$

We can simplify Eq. (9), by replacing the term $G(\vec{q} - \vec{k})$ with $G(\vec{q} - \vec{k}_0)$ since the momentum bandwidth $\delta q_i$ due to the recoil is normally much larger than the photon linewidth $\delta k_i$; also, we can replace $\vec{k}$ with $\vec{k}_0$ in the term $\frac{\hbar}{2m}(2\vec{q} - \vec{k}) \cdot \vec{k}$, for the very sharp decay resonance of the denominator. With these approximations, the first term in the curly bracket can be seen as an antifourier transformation on the production of the photonic shape and Lorentzian shape, and will cause a decay at a time scale $\max\left\{ \frac{1}{\Gamma}, \frac{1}{\delta k_i} \right\}$; the second decay term $e^{-\frac{iLt}{2}}$ is
due to the spontaneous emission. Then, one can directly find that \( B(\vec{q}, t \to \infty) \to 0 \). In the further calculations, we ignore the frequency shift since it can be treated as a modification of the atomic transition frequency, and regard the slowly varying function \( g(\vec{k}) \) as a constant.

With the approximations mentioned above, from Eqs. (3) and (9), we obtain the steady solution of \( A(\vec{q}, \vec{k}, t \to \infty) \):

\[
A(\vec{q}, \vec{k}, t \to \infty) = \chi_0 G(\vec{q})P(\vec{k} - \vec{k}_0) + \frac{\chi_0 |\vec{q}|^2 G(\vec{q} + \vec{k} - \vec{k}_0)}{\Gamma - i \left[ c\vec{k} - \omega_a - \left( \frac{\hbar}{\chi_0} \right)^2/2m\hbar + (\hbar \vec{q})^2/2m\hbar \right]} \times \\
\sum_{\vec{k}_1} i \left[ c\vec{k} - c\vec{k}_1 - \frac{\hbar}{2m} (2\vec{q} + \vec{k}) \cdot \vec{k}_1 + \frac{\hbar}{m} (\vec{q} + \vec{k}) \cdot \vec{k}_0 - \frac{\hbar}{2m} k_0^2 \right]. \tag{10}
\]

III. AMPLITUDE ENTANGLEMENT IN SCATTERED PHOTON

To make the physical results more evident and avoid unnecessary mathematical complexity, we focus our attention on the photon scattered perpendicular to the incident direction, i.e., \( \theta = \vec{q} \). Then we project Eq. (10) into the subspace \( |(\eta, 0, 0)\rangle \otimes |(k_x, 0, 0)\rangle \), with the same approximations used in Eq. (9), and yield:

\[
A_{\eta}^{k_x} = N \cdot \exp \left[ -\left( \Delta q_x \cdot \frac{h\eta}{m\Gamma} \right)^2 \right] \frac{\exp \left[ -\left( \Delta k_x \cdot \frac{\hbar}{m \Gamma} \right)^2 \right]}{(\Delta k_x + \Delta q_x + i \left[ (\Delta k_x + \Delta q_x) / \tau_z + i \right]} \times \\
\frac{N \cdot \exp \left[ -\left( \Delta q_x / \eta \right)^2 \right]}{(\Delta k_x + \Delta q_x + i \left[ (\Delta k_x + \Delta q_x) / \tau_z + i \right]} \right], \tag{11}
\]

where \( \Delta k_i \equiv \frac{k - k_i}{\sqrt{mc}} \), \( \Delta q_i \equiv \frac{\hbar k_i}{2m} (q_i - k_0) \), \( \eta_i \equiv \frac{\hbar k_i}{m \Gamma} \), \( \tau_i \equiv \frac{\delta k_i}{1/\tau} \) \( (i = x, y, z) \) are all defined dimensionless parameters. Note that \( \eta_z \) and \( \tau_z \) contain all the physical parameters that determine the nature of the atom–photon system, thus can be treated as physical control parameters for the atom and the photon, respectively. We neglect tiny terms in Eq. (11) due to \( \hbar k_i^2 \ll mc \) in realistic conditions. \( N \) is the normalization factor where \( N^2 = \sqrt{2(1 + \tau_z) / \pi \tau_z \eta_z} \).

From Eq. (11) and Fig. 2, one sees that, variables \( \Delta q_x \) and \( \Delta k_x \) play the symmetric role in the two Lorentzian functions. It makes the probability amplitude \( |A_{\eta}^{k_x}|^2 \) localized along the diagonal of the momentum space, which

FIG. 2: (a) and (b) are contour and density plots of \( |A_{\eta}^{k_x}|^2 \) with the condition \( \tau_z = 1, \eta_z = 10 \); (c) and (d) are contour and density plots of \( |A_{0}^{k_x}|^2 \) with the condition \( \tau_z = 1, \eta_z = 10 \).
implies the nonfactorization of the photon–atom wave function, and then will generate entanglement between the two particles. In fact, we can treat the ratio \( R \) of the conditional and unconditional variances for \( \Delta q_x \) or \( \Delta k_x \) as an evaluation of entanglement\([10]\). This ratio, compared to the Schmidt number \( K \), reveals more obvious analytic dependence for the entanglement on its control parameters \( \eta_x \) and \( \tau_z \), and is also experimentally directly accessible\([10]\).

We proceed to calculate the ratio for variable \( \Delta q_x \), i.e.,

\[
R \equiv \frac{\delta^2 \Delta q_x^{\text{single}}}{\delta^2 \Delta q_x^{\text{coinc}}},
\]

where the unconditional variance is obtained from the single–particle observation as:

\[
\delta^2 \Delta q_x^{\text{single}} = \langle \Delta q_x^2 \rangle - \langle \Delta q_x \rangle^2 = \int d\Delta k_x d\Delta q_x |A_{\frac{\phi}{\Delta q_x}}|^2 - \left( \int d\Delta k_x d\Delta q_x |A_{\frac{\phi}{\Delta q_x}}|^2 \right)^2,
\]

and coincidence measurement gives the conditional variance at some specified \( \Delta k_x \):

\[
\delta^2 \Delta q_x^{\text{coinc}} = \langle \Delta q_x^2 \rangle_{\Delta k_x} - \langle \Delta q_x \rangle_{\Delta k_x}^2 = \frac{\int d\Delta q_x d\Delta q_x^2 |A_{\frac{\phi}{\Delta q_x}}|^2}{\int d\Delta q_x |A_{\frac{\phi}{\Delta q_x}}|^2} - \left( \frac{\int d\Delta q_x d\Delta q_x |A_{\frac{\phi}{\Delta q_x}}|^2}{\int d\Delta q_x |A_{\frac{\phi}{\Delta q_x}}|^2} \right)^2.
\]

Substituting Eqs. (11)–(13) into the definition of \( R \), we yield \( R(\eta_x, \tau_z) \) as a function of parameters \( \eta_x \) and \( \tau_z \), the result of which is illustrated in Fig. 3 with \( \Delta k_x \) fixed at the origin. From that, one can see that the entanglement increases monotonously when \( \eta_x \) increases or \( \tau_z \) decreases, which indicates that higher entanglement can be achieved by squeezing the linewidth of the incident photon or broadening the wave packet of the atom. In particular, when \( \eta_x > 1 \), we have:

\[
R \approx \eta_x + \frac{1}{2\sqrt{\tau_z}},
\]

from which it is found that the entanglement increases linearly with \( \eta_x \) and will be abruptly enhanced when \( \tau_z \) tends to zero. As a remark, we emphasize that all the conclusions above hold qualitatively the same either if \( \Delta k_x \) is specified otherwise or one calculate the ratio \( R \) from the other variable \( \Delta k_x \).

The ratio \( R \), which can be obtained experimentally by comparing the momentum dispersion variance, is an appropriate quantification for the entanglement contained in the probability amplitude correlation (thus can be seen as an evaluation of the “amplitude entanglement”). Next, we can see that it reveals a correct varying tendency for the entanglement with its control parameters. However, the definition of \( R \) is dependent on its representation space and different choices for the basis of Hilbert space will cause distinct values of \( R \). This is because we only use the amplitude of the wavefunction to construct \( R \), and then all entanglements included in phase\([11]\) is lost.

To obtain the “total entanglement”, we calculate the Schmidt number\([12]\) and compare it with the entanglement ratio \( R \) in the following section.

**IV. FULL ENTANGLEMENT IN SCATTERED PHOTON**

Mathematically, for a bipartite system in pure state, the entanglement of an unfactorable wavefunction can be completely characterized by the Schmidt number, which is denoted by \( K \equiv \left( \sum_{n=0}^{\infty} \lambda_n^2 \right)^{-1} \), where \( \lambda_n \)'s are eigenvalues of the integral equation\([12]\):

\[
\psi_{n}(\Delta q_x, \Delta k_x) = \frac{1}{\sqrt{\lambda_n}} \phi_n(\Delta k_x),
\]

where, note that we have taken away the time–dependent phase in the density matrix since it does not contribute to entanglement. Although we do it with the photon, Schmidt number can be equally obtained through the atomic density matrix, and the eigenfunctions of atom \( |\psi_{n}(\Delta q_x)\rangle \) can be related to those of photon through:

\[
\psi_{n}(\Delta q_x) = \frac{1}{\sqrt{\lambda_n}} \int d\Delta k_x A_{\frac{\phi}{\Delta q_x}}(\Delta q_x, \Delta k_x) \phi_n^*(\Delta k_x),
\]

where \( \phi_n(\Delta k_x) \) and \( \psi_{n}(\Delta q_x) \) \((n = 1, 2, \ldots)\) form complete orthonormal sets for the photon and atom respectively. With these discrete modes, the unfactorable wavefunction can be expanded into a sum of factored products uniquely:

\[
A_{\frac{\phi}{\Delta q_x}}(\Delta q_x, \Delta k_x) = \sum_{n} \sqrt{\lambda_n} \psi_{n}(\Delta q_x) \phi_n(\Delta k_x).
\]
Then, the Schmidt number $K$, which is an estimation of the number of modes that are “important” in making up the expansion of Eq. (18), serves as a quantitative measurement of entanglement. Note $K$ is independent from representation since all $\lambda$’s keep the same in different representations, thus can be seen as a quantity of the full entanglement information (both amplitude and phase entanglement) kept in the collective wavefunction.

Since Eq. (15) is not analytically solvable, we use a discrete eigenvalue equation to approximate the integral equation. Up to a reliable precision, we use 1000 discrete eigenvalue equation to approximate the integral wavefunction.

The numerical results, we find that, similar to the ratio $R$, $K$ rises linearly with parameter $\eta_z$ and will increase rapidly when the linewidth of incident photon is squeezed narrower to the atomic linewidth $\Gamma$, i.e., $\tau_z < 1$; secondly, when $\tau_z$ is fixed, the slope of $K(\eta_z)$ is always larger than that of $R(\eta_z)$, which means that more entanglement information will transfer to phase when $\eta_z$ becomes larger, and this phenomena will become more evident when $\tau_z$ is reduced, e.g., when $\tau_z = 0.1$, $R \approx 1.58\eta_z + 1.39$ whereas $K \approx 3.44\eta_z + 0.08$, which indicates that more than half of the entanglement information will be unavailable by momentum dispersion observation when $\eta_z$ goes large on this condition.

Another phenomena is notable, when $\tau_z = 1$, i.e., the linewidth of the incident photon is not squeezed and can be prepared directly by spontaneous emission from the same atom, we find $K \approx 0.75\eta_z + 0.16$ ($\eta_z \gg 1$) whereas in the case of spontaneous emission $K \approx 0.28\eta + 0.72$ ($\eta \gg 1$). This difference indicates that, although in both cases, entanglement is generated from momentum conservation in the process of photon emission with atomic recoil, the absorption of the incident photon will add some entanglement due to its coherent pumping effect. As $K$ is linear with $\eta$ (or $\eta_z$), we define the “entanglement pumping coefficient” as:

$$EPC = \frac{\text{slope of } K(\eta_z) \text{ in scattering}}{\text{slope of } K(\eta) \text{ in spontaneous emission}},$$

since the constant term in $K(\eta)$ plays a minor role when entanglement is large. The defined coefficient $EPC$ shows the times that entanglement is increased by the coherent pumping of an incident photon. As it is independent on the atomic parameter, it reflects the ability of entanglement of the photon separately. We collect some numerical results in Fig. 5 and fit it with $EPC \approx 1.1/\tau_z + 1.5$ within $\tau_z \in (0, 1)$, from which, one sees that $EPC$ increases rapidly when $\tau_z$ diminishes, which also implies that, if the incident photon is prepared monochromatically on its limit condition, i.e., $\tau_z \to 0$, the scattered photon will be highly entangled to the recoiled atom.

We plot the amplitude of the first three Schmidt modes for the photon with $\eta_z = 10$ and $\tau_z = 1$ in Fig. 6. We find that their number of peaks in momentum space is proportional to the Schmidt mode index, but the separations of different peaks are more distinct than in the case of spontaneous emission.\[\text{Fig. 6}\]

V. TRANSMITTED PHOTON

To consider the transmitted photon, we make the observation angle $\theta = 0$, and yield the collective wavefunction from Eq. (10):

$$A_0 = -\chi_0 G_z(q_z)P_z(k_z)$$

$$+ \chi_0 \frac{\pi}{4} \left( \frac{\Gamma}{ck_0} \right)^2 \frac{\tau_z \Gamma G_z(q_z)P_z(k_z)}{1 - i(\Delta k_z + \Delta q_z + \frac{kk_z}{2m\Gamma})}.$$

$$\text{Fig. 5: Entanglement pumping coefficient EPC as a function of } \tau_z. \text{ The solid line is its fitted function } 1.1/\tau_z + 1.5.$$
FIG. 6: First three Schmidt modes for the scattered and transmitted photon. Left column is for the scattered photon with $\tau_z = 1$ and $\eta_x = 10$; right column is for the transmitted photon with $\tau_z = 1$, $\eta_x = 10$, and $\frac{\Gamma(\Delta k_0)}{\pi^2} \tau_x \tau_y = 1$ for illustration.

One can see that, in Eq. (19), the first term describes that the two particles are free of interaction and keep their initial factorable wave form; the second term reflects the entanglement. Usually, the second term is much smaller than the first one since $\left(\frac{\Gamma(\Delta k_0)}{\pi^2}\right)^2 \ll 1$, but one can enlarge it by choosing some special physical system, such as the artificial atom with low excited level and high coupling to its resonant modes. However, this improvement can add few entanglement between the transmitted photon and recoiled atom, because interference between the two terms in Eq. (19) will weaken the correlation of the two particles at a great deal. To make it clear, we show the contour and density plots for the probability amplitude of $A_0$ in Fig. 2 on an artificial condition $\frac{\Gamma(\Delta k_0)}{\pi^2} \tau_x \tau_y = 1$, and yield $R \approx K < 2$ in this situation.

The eigenfunctions of transmitted photon for the first three modes with $\eta_z = 10$ and $\tau_z = 1$ are collected in Fig. 6, from which one can see that, due to the interference, the corresponding modes of the transmitted photon exhibit one peak less than that of the scattered photon.

VI. CONCLUSION

We analyze the physically fundamental interaction between a single photon and a free artificial atom in vacuum. With a few physical approximations, the general solution of the photon–atom wave function is obtained, from which, it is found that the initially uncorrelated particles will evolve to be entangled due to momentum conservation in scattering. To evaluate the entanglement in the scattering, firstly, we use an experimentally accessible parameter $R$, which denotes the ratio between momentum variance in single-particle and in coincidence observations, and yield its simple dependences on the two physical control parameters $\eta_x \equiv \frac{\delta q_x \hbar k_0}{\Delta k_0}$ and $\tau_z \equiv \frac{\delta k_z}{\Gamma c}$; secondly, we use standard Schmidt decomposition to reveal the full entanglement information and find out its varying tendency similar to that of $R$, which indicates that high entanglement can be achieved by either squeezing the linewidth of the incident photon or broadening the scale of atomic wave packet. Furthermore, compared with spontaneous emission, we defined a parameter EPC to evaluate the entanglement enhancement due to the coherent pumping effect of the resonant incident photon. In the end, we found out that, for the transmitted photon, one can expect little entanglement due to the interference between the transparent and scattered wave.

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