Breakdown of the Fermi liquid theory in heavy fermion compounds

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Abstract

We review the anomalous properties of heavy fermion compounds like CeCu6−xAu x or CeMIn5 close to a zero temperature phase transition called a quantum critical point. Anomalous behavior of the resistivity, specific heat and magnetic properties observed in various compounds suggests a fundamental breakdown of the Fermi liquid theory. Recent measurements on YbRh2Si2, field-tuned through the quantum critical point, provide interesting insights on the evolution of the Fermi liquid close to criticality. We discuss the possibility of a kind of spin-charge separation at the quantum critical point and make some remarks about its possible link with the underscreened Kondo model.

Key words: heavy fermions; quantum critical points; Fermi liquid theory.

1. Experimental overview and theoretical insights

1.1. experiments

Fermi liquid theory [1], developed by Landau in the mid 1950’s, provides the foundation for much of our current understanding of electron fluids. According to the Landau Fermi Liquid theory (LFL), thermodynamic and transport properties of a metal at very low temperatures are described in terms of weakly interacting fermions, or “Landau quasiparticles”. Landau quasiparticles consist of electrons, surrounded by a cloud of spin and charge polarization. They share the same quantum numbers as free electrons but their masses can be strongly renormalized by the back flow of the surrounding fluid. A striking example of the robustness of the LFL is provided by heavy fermion behavior, where the effective mass of the quasiparticles can be hundreds of times greater than that of a bare electron. The physical properties of this fluid follow certain characteristic power-law dependences on temperature. For example, the specific heat has a linear temperature dependence, the resistivity a quadratic one.

A wide body of experimental results show that when heavy fermion materials are tuned to a zero temperature antiferromagnetic transition, their physical properties depart qualitatively from LFL theory. There is a growing list of heavy electron materials that can be tuned into the quantum critical point by, either alloying, such as CeCu6−xAu x [2] or the layered materials CeMIn5[3], through the direct application of pressure, as in case of CeIn3[4] or CePd2Si2[5], or via the application of a magnetic field, as in the case of YbRh2Si2[6]. At the quantum critical point (QCP) the key properties suggesting a breakdown of the Fermi liquid are:

– the specific heat coefficients diverge at the QCP[7,8,9]. For most cases the divergence displays a logarithmic temperature dependence

\[ \gamma(T) = \frac{C_v}{T} = \gamma_0 \log\left[\frac{T_0}{T}\right]. \] (1)

This implies that the the quasi-particle mass diverges and the Fermi temperature goes to zero at a QCP.
\[
\frac{m^*}{m} \to \infty \quad T_\rho^* \to 0 \, .
\]

- a quasi-linear temperature dependence of the resistivity, contrasting with the quadratic dependence of the FL [5,6,10].

\[
\rho \propto T^{1+\epsilon}
\]

with \(\epsilon\) in the range of \(0 - 0.6\). Several compounds including YbRh\(_2\)Si\(_2\) [11] and CeCu\(_{6-x}\)Au\(_x\) [9] have perfectly linear resistivity, a property reminiscent of the normal phase of high temperature superconductors.

- the spin susceptibilities acquire anomalous exponents

\[
\chi^{-1}(T) = \chi_0^{-1} + T^a
\]

with \(a < 1\) for CeCu\(_{5.9}\)Au\(_{0.1}\), YbRh\(_2\)(Si\(_{1-x}\)Ge\(_x\))\(_2\) \((x = 0.05)\) and CeNi\(_2\)Ge\(_2\).[5] In the case of CeCu\(_{6-x}\)Au\(_x\) [9] magnetization measurements reveal that at finite fields \(H\), the differential magnetic susceptibility exhibits exhibits \(H/T\) scaling. Neutron scattering measurements on the same material [9] reveal the presence of \(\omega/T\) scaling in the dynamic spin susceptibility, which may be cast into the form

\[
\chi^{-1}(q,\omega) = f(q) + (i\omega + T)^a.
\]

where the function \(f(q)\) vanishes in the vicinity of the ordering wave vector. The \(q\)-independence of second term suggests a local origin to the damping of the critical spin fluctuations, an observation that has stimulated recent efforts to develop a self-consistent locally quantum-critical model of the heavy electron QCP.[12] Remarkably, a single exponent \(a \sim 0.75\) governs both the \(H/T\) and \(\omega/T\) scaling. The presence of the anomalous exponent \(a\) suggests that the interaction amongst the quantum critical modes has renormalized to strong coupling. This fact is made clear by the scaling laws in \(\omega/T\) and \(H/T\). This type of “naïve” scaling property (quantities scale according to their dimensions) is the hallmark of a system lying below its upper critical dimension.[13]

1.2. Key theoretical insights

The experimental observations at criticality suggest an underlying universality in the physics. [13]. The ability to express the dynamical susceptibility of Cu\(_{5.9}\)Au\(_{0.1}\) in terms of a scaling function \(F(\omega/T, B/T)\) and the observation that the effective Fermi temperature \(T_F\) renormalizes to zero at the QCP, both suggest that underlying physics at the QCP is scale invariant. In this situation, we expect that the physical properties will be independent of the microscopic scales of the material- they are universal.

Moreover, the observation of anomalous exponents in the magnetic susceptibility, the transport and thermodynamic properties are evidence that the quantum critical physics may be described by an effective theory whose coupling constants flow to strong coupling. In particular the effective critical lagrangian must lie below its upper critical dimension.

1.3. Spin density wave scenarios

The constraints at criticality enable us to rule out the 3D spin density wave (SDW) theoretical scenario [14,15] for the heavy electron quantum critical point (figure 1,left). In this scenario, magnetism develops by the spin polarization of the Fermi surface and non-Fermi liquid (NFL) behavior results from Bragg diffraction of electrons off quantum critical spin fluctuations in the magnetization. As shown in figure 1, three dimensional spin fluctuations only couple strongly to electrons along “hot lines” of the Fermi surface, separated by the wave vector \(\mathbf{Q}\) of the antiferromagnetic order. The term “hot lines” refers to the \(\mathbf{k}\) points on the Fermi surface which are coupled the magnetic fluctuations at zero temperature. The width of the hot lines increases with temperature like \(\sqrt{T}\). On the hot lines, the electrons are strongly perturbed by the magnetic fluctuations and their life time is strongly suppressed. More precisely in three dimensions the imaginary part of the self-energy varies like \(\Sigma_H \sim \omega\). Their effective mass experiences a logarithmical increase with temperature. Away from the hot lines however, the scattering of the electrons off the critical modes is strongly suppressed - the magnetic fluctuations getting massive. Within the 3D SDW scenario, the hot lines are not wide enough to destabilize the FL theory. In this situation, the critical contributions to the free energy and the specific heat coefficient are given by

\[
F \sim T^{3/2} \quad \text{and} \quad C/T = \gamma_0 - \sqrt{T}
\]

When the temperature is driven to zero at criticality, the system eventually flows towards the FL fixed point, keeping the universal exponents of the FL theory.

In order to account for the properties at criticality, one can imagine that the spin fluid is magnetically frustrated, dividing into layers of decoupled plans. In
2.1. The upturn in the specific heat

Several unusual features are revealed at criticality. In addition to the previously observed logarithmic increase when lowering the temperature, the specific heat coefficient $\gamma$ shows a strong upturn below 30 mK (figure 2). The electronic nature the upturn has been checked first by carefully removing the nuclear contribution, second by the observation that under a small magnetic field, the electronic contribution in the FL phase gradually saturates the upturns, as one gets closer to the QCP. Such an upturn of electronic nature reveals a strong departure from any SDW scenario, where the maximum possible increase of the specific heat coefficient is logarithmic. More specifically, the behavior of $\gamma$ can be cast into the form

$$\gamma(T, B) = B^{-\alpha} \Phi\left(\frac{T}{B^\beta}\right)$$

(7)

with $\alpha \approx 0.33$ and $\eta \approx 1$. $\Phi$ is a scaling function of the temperature and the magnetic field. The form of (7) is remarkable in many respects. It reveals that two energy scales govern the behavior of the electrons in the FL phase. A first energy scale $T_F$ is associated to the inverse of the density of states. The second energy scale $T_\gamma(B)$, characteristic of the excitations of the system appears in the scaling function $\Phi$. These two energies vary with the applied magnetic field as follows:

$$T_F \sim B^\alpha \quad \text{and} \quad T_\gamma(B) \sim B$$

(8)

The scaling form (7) of $\gamma$ ensures that at zero magnetic field $\gamma(T, B = 0) \propto T^{-\alpha/\eta}$. Since the exponent $\alpha$ is smaller than one, no residual entropy is sitting at the QCP.

2.2. Kadowaki-Woods ratio is weakly field dependent

An alternative way of investigate the scattering behavior at the QCP is to approach it from the FL phase, gradually reducing the applied magnetic field towards its critical value $B_c$. Away from criticality, the resistivity is quadratic $\Delta \rho(T) = \rho(T) - \rho_0 = A(B)T^2$ ($\rho_0$...
is the residual resistivity), below a characteristic temperature \( T_F \). The temperature at which the \( T^2 \) behavior, characteristic of the Landau FL theory appears, sets the crossover between the heavy fermion regime and the quantum critical regime. Inside the FL phase, even though the quantum fluctuations are not strong enough to destabilize the FL, the electron undergoes virtual scattering off the critical magnetic modes. The variation of \( A(B) \) close to the QCP provides insight into these virtual excitations. Initial measurements on germinium doped YbRh\(_2\)Si\(_{2-x}\)Ge\(_x\) show that \( A(B) \sim 1/(B - B_c) \) in the approach to the QCP. Over a wide range of field, the Kadowaki Woods ratio is found to be approximately constant, with a value

\[
K = A/(\gamma_0)^2 \approx 5.8\mu\Omega cm K^2 m\Omega^2/J \tag{9}
\]

is found to agree within 40% with the empirical Kadowaki Woods ratio. More recent measurements closer to the QCP show a slow upturn in \( K \) at very small values of \( B - B_c \). [19] Constancy of the Kadowaki woods ratio implies that the transport scattering rate scales strictly with the square of the renormalized \( T_F^*(B) \) of the heavy electron fluid. A truly field independent Kadowaki Woods ratio would indicate that the momentum dependence of the scattering amplitude does not renormalize with \( T_F^*(B) \).

### 2.3. The 2D SDW scenario is ruled out

This set of data allows us to rule out the 2D SDW scenario. While the 2D SDW scenario leads to the result \( A \propto 1/(B - B_c) \) as observed experimentally, the soft 2D spin fluctuations produce only a small renormalization in the heavy electron density of states with \( \gamma \propto \ln 1/(B - B_c) \). This would give a strongly divergent Kadowaki Woods ratio in the approach to the QCP: \( K_{SDW} \propto 1/((B - B_c) \ln^2(B - B_c)) \). The weak field dependence of the Kadowaki Woods ratio suggests that the most singular quasiparticle scattering amplitudes have a far weaker momentum dependence than expected in a spin density wave scenario.

### 3. The breakup of the heavy electron- is there a preformed Kondo effect at the QCP?

The study of YbRh\(_2\)(Si\(_{2-x}\)Ge\(_x\)) reveals a very unusual property. While the specific heat coefficient experiences a strong upturn below \( T = 30mK \), there is no reflection of this sudden loss of entropy in the transport properties: the resistivity stays linear in \( T \) for the whole temperature regime. This suggests that the degrees of freedom associated with the upturn in the specific heat are not involved in the charge transport. What is the origin of this separation?

One of the challenges here is to understand the clear separation between the single ion Kondo temperature \( T_K \), which clearly remains finite at the QCP, and the renormalized \( T_F^*(B) \) which is driven continuously to zero. Burdin et al.,[20] have suggested the analogy with pre-formed pairs in a superconductor, according to which \( T_K \) is the temperature at which pre-formed local Kondo singlets develop and \( T_F^* \) is the scale at which phase coherence sets in to form mobile quasi-particles. By analogy one can consider the QCP in heavy fermions as a kind of preformed scenario for the Kondo effect. In such a picture, the presence of very strong antiferromagnetic fluctuations close to a QCP dephases the heavy fermion composite quasi-particles at increasingly low temperatures, as one gets closer to the QCP. Eventually at the QCP the coherence temperature is driven to zero, leading to the disintegration of the heavy composite electron.

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