Modelling nuclei far from stability with a multichannel approach

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Abstract. Recently, new high-precision data reconfirmed the existence of narrow resonances in the \( p^{+}\)\(^{14}\)O cross section, first predicted by the multi-channel algebraic scattering (MCAS) theory of light-mass, low-energy scattering and later found by other experimental means. Prompted by this and a decade of extension to the method, we have updated the original MCAS study. Additionally, we use MCAS to interpret the low-energy spectrum of \( ^{19}\)F, which has an interesting structure that appears to stem from clusterisation, and we compare the results of several types. As only early, small-basis shell model investigations of this spectrum exist, we also perform a complete \((0+2)\sqrt{\hbar}\omega\) calculation.

MCAS is well suited to these studies, having the advantage of accounting for the Pauli principle between the nucleons of the clusters, and so spurious states of the compound nucleus are removed.

1. The MCAS method

The multi-channel algebraic scattering (MCAS) method \cite{1, 2} solves coupled-channel Lippmann-Schwinger equations, in momentum space, by using finite-rank expansions of interactions between two clusters. These are:

\[
T_{cc'}^{J\pi}(p, q; E) = V_{cc'}^{J\pi}(p, q) + \mu \sum_{c''=1}^{\text{open}} \int_{0}^{\infty} V_{cc''}^{J\pi}(p, x) \frac{x^2}{k_{c''}^2 - x^2 + i \epsilon} T_{c''c'}^{J\pi}(x, q; E) dx \\
- \sum_{c''=1}^{\text{closed}} \int_{0}^{\infty} V_{cc''}^{J\pi}(p, x) \frac{x^2}{\hbar_{c''}^2 + x^2} T_{c''c'}^{J\pi}(x, q; E) dx
\]

\text{(1)}

where \( c \) are indices denoting the channels, \( V_{cc'} \) are the potentials, the the wave numbers are

\[
k_c = \sqrt{\mu(E - \epsilon_c)} \quad \text{and} \quad \hbar_c = \sqrt{\mu(\epsilon_c - E)}.
\]

\text{(2)}

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Sturmian functions are used for the expansion basis, and this choice allows for the location of all compound-system resonance energies and widths, regardless of how narrow or wide they may be. It further allows for negative-energy Sturmians to be used to find sub-threshold bound states of the compound nucleus. The principle used is that separable $T$-matrices lead to separable potentials, and this being true in reverse potentials are expanded as

$$ V_{cc'} = -\sum_p |\chi_{cp}\rangle \frac{1}{\eta_p} \langle \chi_{cp}'| . $$  

The $|\chi_{cp}\rangle$ are determined from Sturmians [3], and the $\eta_p$ are the Sturmian eigenvalues. This leads to solutions to the Lippmann-Schwinger equations having the form

$$ T_{cc'} = -\sum_p |\chi_{cp}\rangle \frac{1}{1-\eta_p} \langle \chi_{cp}'| . $$  

Orthogonalizing pseudopotentials (OPP) are used to remove the influence of occupied orbitals from the potentials before the generation of Sturmians. That is,

$$ V_{cc'}(r,r') = V_{cc'}(r)\delta(r-r') + \lambda A_c(r) A_{c'}(r') \delta_{c,c'} , $$  

where $A_c(r)$ are the radial wave functions of occupied orbitals, and $\lambda$ denotes parameters that determine the magnitude of the blocking or the corresponding orbitals. This procedure ensures that the Pauli principle is not violated [4, 5], even when interaction potentials are defined from collective model descriptions of the clusters. In the absence of this approach, some compound nucleus wave functions will possess spurious components [6]. In addition, MCAS incorporates a method of accounting for states of the clusters which are themselves particle-unstable resonances [7, 8].

The potentials used in this investigation are derived from a Tamura collective model of vibrational character. The basic potential is:

$$ V_{cc'}(r) = f(r) \left\{ V_0 \delta_{cc'} + V_{ll}[\ell \cdot \ell]_{cc'} + V_{ss}[s \cdot I]_{cc'} \right\} + g(r) V_{ls}[\ell \cdot s]_{cc'} , $$

where $f(r)$ is the deformed Woods-Saxon potential and $g(r)$ is derived from its derivative:

$$ f(r) = \left[ 1 + e^{(r-R)/\epsilon} \right]^{-1} ; \quad g(r) = \frac{1}{r} \frac{df(r)}{dr} . $$  

If one of the scattering clusters is taken to be an incompressible drop of nuclear matter undergoing vibration, its surface can be defined as:

$$ R(\theta, \phi) = R_0 [1 + \epsilon] = R_0 \left[ 1 + \sum_{\lambda \mu} \alpha_{\lambda \mu} Y_{\lambda \mu}(\theta, \phi) \right] . $$  

where $Y_{\lambda \mu}(\theta, \phi)$ are spherical harmonics, and $\alpha_{\lambda \mu}$ are the phonon creation and annihilation operators:

$$ \alpha_{\lambda \mu} \Rightarrow \sqrt{\frac{\hbar}{2B\omega_\lambda}} \left[ b_{\lambda \mu} + (-)^\mu b_{\lambda \mu}^\dagger \right] . $$

To give the basic potentials defined in Eq. 6 a vibrational character, the Woods-Saxon potentials of Eq. 7 are expanded, to second order, in terms of the $\epsilon$ of Eq. 8, i.e.

$$ f(r) = f_0(r) + \epsilon \left( \frac{\partial f(r)}{\partial \epsilon} \right)_0 + \frac{1}{2} \epsilon^2 \left( \frac{\partial^2 f(r)}{\partial \epsilon^2} \right)_0 . $$

Full details of the potential are available in Ref. [9].
2. The \( p^{+14}\text{O} \) elastic scattering cross-section and the spectrum of \( ^{15}\text{F} \)

The first \( ^{14}\text{O}(p, p)^{14}\text{O} \) cross section data appeared in the literature in 2004 [10]. This spanned the two then-known states of the nucleus, of \( J^\pi = \frac{1}{2}^+ \) and \( \frac{5}{2}^+ \), both of which are unbound. It further provided fits to the data found from Woods-Saxon potentials. This paper stimulated much research effort. Notable amongst this, a microscopic cluster model was used in 2005 to analyse the data [11], and further data was published by another group [12]. In 2006, Ref. [13] used a microscopic model to study the \( \frac{1}{2}^+ \) state, which itself has prompted new research [14].

Also in 2006, we used the multichannel algebraic scattering method (MCAS) to analyse the data, with potentials between \( ^{14}\text{O} \) and protons defined with a Tamura collective model with rotor character (rather than the vibrator character detailed in Section 1). This study obtained a very good fit with the data, but also predicted narrow resonances at higher energies: a \( \frac{1}{2}^- \) state with energy (width) of 5.49 (0.005) MeV, a \( \frac{5}{2}^- \) state at 6.88 (0.01) MeV, and a \( \frac{3}{2}^- \) state at 7.25 (0.04) MeV. When resonances have such a small width, it indicates that they have little overlap with the ground state. In the case of \( ^{15}\text{F} \), this is because one state decays by single-proton emission and the other by two-proton emission. This effect can be explained by Pauli hindrance [15]. Also predicted in Ref. [16] were a broad \( \frac{1}{2}^+ \) state with energy (width) of 7.21 (1.2) MeV, and two further narrow states of \( J^\pi = \frac{5}{2}^+ \) and \( \frac{3}{2}^+ \) with energies 7.75 (0.4) and 7.99 (3.6) MeV, respectively.

The \( \frac{1}{2}^- \) state was subsequently found experimentally, as was evidence of a \( \frac{5}{2}^- \) state and/or \( \frac{3}{2}^- \) state, with such narrow widths [17, 18, 19]. A state which may correspond to the MCAS \( \frac{3}{2}^+ \) state was also found. (Note that in Table I of Ref. [19], the labels for results of Ref. [16] were switched with those of another calculation.) Most recently, Ref. [20] presented higher-precision data over a larger energy range, which provided details of the shape of the \( \frac{1}{2}^- \) resonance. This indicated that it is a dip in cross section, as was predicted by MCAS.

Given the availability of this new data, the MCAS calculation has recently been updated [21], using instead the vibrational description of the \( ^{14}\text{O} \). Further, channels have been defined using five states of \( ^{14}\text{O} \) rather than the three in Ref. [16], exact masses of the nucleons and nuclei have been used rather than exact multiples of the mass numbers, and the Coulomb interaction has been derived from a three parameter Fermi (3pF) form for the charge distribution in \( ^{14}\text{O} \), rather than a uniform charged sphere as in Ref. [16].

Fig. 1 shows the newly-calculated MCAS \( ^{14}\text{O}(p, p)^{14}\text{O} \) cross section against the results of Ref. [20], along with the energies and widths of \( ^{15}\text{F} \) states found by Ref. [19], where Table 1 lists the energies and widths of the \( ^{15}\text{F} \) states. Amongst other small changes stemming from the differences between this and the 2006 calculation, the order of the \( \frac{3}{2}^- \) and \( \frac{5}{2}^- \) states has been swapped.

3. The spectrum of \( ^{19}\text{F} \)

The spectrum of \( ^{19}\text{F} \) (as well as that of its mirror, \( ^{19}\text{Ne} \)) is challenging to interpret, as it begins with a near-triplet of states, above which there is an energy gap of \( \sim 1.5 \) MeV, followed by another near-triplet of states, another gap of \( \sim 1.2 \) MeV, a single state, and then another gap of \( \sim 1.2 \) MeV. Historically this has been best described by various clusterisations [23].

Fig. 2 shows the experimentally-known low-lying spectrum of \( ^{19}\text{F} \) [24], compared with several MCAS calculations where \( ^{19}\text{F} \) is modelled as \( p^{+18}\text{O} \), \( ^3\text{H}^+^{16}\text{O} \), and \( \alpha^{+15}\text{N} \). Full details of these calculations will be provided in a forthcoming paper. Results are plotted with zero energy being the ground state, rather than the respective scattering thresholds. Below each calculation the calculated threshold energy is given, and in all cases these agree with those measured. All MCAS calculations recreate the lowest-energy triplet of states with spin-parities in the observed order and with good agreement in the energy spacing. At higher energies, however, the calculated
Figure 1. $p^{+}^{14}O$ elastic scattering cross-section data taken at 180° from Ref. [20], compared with the result of an MCAS calculation [21]. The centroid and upper limit on widths as found by Ref. [19] are shown by the solid and dashed vertical lines, respectively.

Table 1. $^{15}F$ spectra. Theoretical energies [21] within $\approx 300$ keV of data shown in bold face. For the $\frac{1}{2}^+$ ground state, the latest measured width and its statistical error are shown, as well as the NNDC value. Note, Ref. [19] Table I lists widths for the $(\frac{5}{2}^-, \frac{3}{2}^-)$ state as 0.2(2), but text on page 10 explains that 0.2 MeV is rather the experimental resolution.

| Ref. | $J^\pi$ | $E_r$ | $\Gamma$ | $J^\pi$ | $E_r$ | $\Gamma$ |
|------|---------|-------|-------|---------|-------|-------|
| [20] | $\frac{1}{2}^+$ | 1.270 | 0.376(0.070) | $\frac{1}{2}^+$ | 1.282 | 0.715 |
| [20] | $\frac{5}{2}^+$ | 2.794 | 0.30±0.010 | $\frac{5}{2}^+$ | 2.757 | 0.337 |
| [20] | $\frac{1}{2}^-$ | 4.757 | 0.036±0.014 | $\frac{1}{2}^-$ | 4.765 | 0.107 |
| [20] | $\frac{3}{2}^-$ | 6.391 | 0.096 | $\frac{3}{2}^-$ | 6.726 | 0.074 |
| [20] | 180° | 6.4 | $\leq 0.2$ | 180° | 7.030 | 0.279 |
| [20] | $\frac{3}{2}^-$ | 7.069 | 0.273 | $\frac{3}{2}^-$ | 7.365 | 0.515 |
| [20] | $\frac{3}{2}^-$ | 7.379 | 0.487 | $\frac{3}{2}^-$ | 7.577 | 0.093 |
| [19] | $\frac{1}{2}^+$ | 7.836 | 3.229 | $\frac{1}{2}^+$ | 8.076 | 7.484 |
spectra differ from each other and that known. Notably, the $\frac{3}{2}^+$ state differs significantly in each case, and is found in agreement with the known energy using the $\alpha+^{15}N$ and $p+^{18}O$ models. The $\frac{5}{2}^-$ state is matched only by the $\alpha+^{15}N$ calculation.

The final column shows the $^{19}F$ positive parity states found from a $(0 + 2)\hbar\omega$ shell model calculation made using a complete basis with the $G$-matrix interaction of Zheng et al. [25]. This calculation was performed as all previous shell model studies were made some time ago, and encountered problems resulting from basis size.

![Figure 2](image)

**Figure 2.** The known low-lying spectrum of $^{19}F$ [24] denotes ‘exp.’, compared with MCAS coupled-channels calculations of the clusterisations $p+^{18}O$, $^3H+^{16}O$, and $\alpha+^{15}N$. The column ‘S.M.’ shows results of a $(0 + 2)\hbar\omega$ shell model calculation. States are labelled with $2J$.

Ref. [26] made a calculation restricted to the $sd$ shell, which located the first negative parity states at approximately $1\hbar\omega$, or $\sim 14$ MeV. It was concluded that a much expanded shell model basis space was required to obtain the small energies observed for negative parity states, especially the $\frac{1}{2}^-$ states, but that such a space would involve prohibitively large-dimension matrices. Ref. [27] had truncated such severely, and while they found a low energy $\frac{3}{2}^-$ state, such truncation can lead to spurious center of mass motion effects [28]. It was estimated that this was not significant enough to influence calculated results for states below $\sim 2$ MeV in the spectrum, and thus the two lowest-lying states calculated were sufficiently non-spurious to be used to evaluate parity non-conservation properties important in this nucleus. However, for the higher spectrum, spuriosity would be a problem. Ref. [29] took the approach of using a $2\hbar\omega$ space limited to the inclusion of components with seniority less than five, and with no occupation of single-particle states above the $1f_{\frac{7}{2}}$ orbit. This investigation reported no negative parity states, acknowledging the impact of spurious center of mass motion.

Owing to the increase in available computing power, the above limitations are to some extent lifted. We see from the current calculation that a full $(0 + 2)\hbar\omega$ space can place the spin-parities
of the first triplet in the correct order, though with energy spacings that are too large. It was also found that this basis is still not sufficient to credibly reproduce the energies of negative parity states, and so they are not shown.

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