There are many interesting topics at the intersection of physics and astrophysics we call Supernova Theory. A small subset of them include the origin of pulsar kicks, gravitational radiation signatures of core bounce, and the possible roles of neutrinos and rotation in the mechanism of explosion. In this brief communication we summarize various recent ideas and calculations that bear on these themes.

1.1 What is the Mechanism of Pulsar Kicks?

Radio pulsars are observed to have large proper motions that average $\sim 400$-$500$ km s$^{-1}$ (Lyne & Lorimer 1994) and whose velocity distribution might be bimodal (Fryer, Burrows, and Benz 1998; Arzoumanian, Chernoff, & Cordes 2002). If bimodal, the slow peak would have a mean speed near $\sim 100$ km s$^{-1}$ and the fast peak would have a mean speed near 500–600 km s$^{-1}$. A bimodal distribution implies different populations and different mechanisms, but what these populations could be remains highly speculative.

Many arguments suggest that pulsars are given “kicks” at birth (Lai 2000; Lai, Chernoff, and Cordes 2001), and are not accelerated over periods of years or centuries. The best explanation is that these kicks are imparted during the supernova explosion itself. We think that this view is compelling. The two suggested modes of acceleration and impulse are via net neutrino anisotropy during the neutrino emission phase (which lasts seconds) and anisotropic mass motions and aspherical explosion which impart momentum to the residual core. The former requires but a $\sim 1\%$ net anisotropy in the neutrino angular distribution to provide a $\sim 300$ km s$^{-1}$ kick. However, anisotropies in the neutrino radiation field are more easily smoothed than matter anisotropies due to convection, rotation, aspherical collapse, etc.
relativistic particles such as neutrinos are not as efficient as non-relativistic matter at converting a given amount of energy into recoil (momentum). To achieve the requisite neutrino anisotropies people have generally invoked large ($\sim 10^{15-16}$ gauss) magnetic fields, which may not obtain generically (see Lai 2000 for a summary). Furthermore, all multi-D calculations to date imply that convective motions between the inner core and the shock result in significant jostling of the protoneutron star. Velocities of $\sim 100-200$ km s$^{-1}$ arise quite naturally by dint of the basic hydrodynamics of the convective mantle of the iron core after bounce (“Brownian Motion”; Burrows, Hayes, and Fryxell 1995 (BHF); Burrows and Hayes 1996a; Janka and Müller 1994; Scheck et al. 2003). This process must be a stochastic contributor to pulsar proper motions. In addition, due to the associated torques, modest spins can be imparted (Burrows, Hayes, and Fryxell 1995).

However, the average recoil speeds obtained theoretically by Burrows, Hayes, and Fryxell (1995), Burrows and Hayes (1996a), and Scheck et al. (2003) due to the brownian motion of the core are only $\sim 200$ km s$^{-1}$; this is not sufficient to explain either the average pulsar speed or the high-speed peak of a bimodal distribution. To do that might require an initial mild anisotropy ($\sim$ percents) in the density or velocity profiles of the collapsing Chandrasekhar core (Lai 2000; Lai and Goldreich 2000). Such small anisotropies have been shown to result in significant impulses and implied kicks of 550 to 800 km s$^{-1}$ (Burrows and Hayes 1996b). It may be that whatever determines whether the initial core is anisotropic results in the high-speed peak of the bimodal distribution, while the low-speed peak is due to the natural jostling by convective plumes and the resultant brownian motion of the core. The latter is stochastic and not deterministic, but has been a robust prediction of the collapse theory for many years.

An instability in the pre-collapse structure that might result in aspherical collapse (particularly relevant in the supersonic region of the collapse since it can not smooth itself out by pressure forces) might be progenitor mass dependent; high-mass progenitors might result in high-velocity pulsars, while low-mass progenitors might result in low-velocity pulsars (on average) (or vice versa!). Whatever the origin of pulsar kicks and their apparent bimodality may be, new calculations are desperately needed. No hydrodynamic calculation to date has actually freed the very inner core to respond to the pressure impulses in a consistent fashion. All calculations have anchored the core and recoils have been inferred due to the integrated anisotropic pressure distributions seen. Freeing the core to respond to pressure and gravity effects and allowing the associated feedback processes will be crucial for obtaining self-consistent and credible results.
1.2 Gravitational Waves from Core Collapse

Gravitational radiation signatures can in principle provide a dramatic potential constraint on core-collapse supernovae. Massive stars (ZAMS mass $\gtrsim 8 \, M_\odot$) develop degenerate cores in the final stages of nuclear burning and achieve the Chandrasekhar mass. Gravitational collapse ensues, leading to dynamical compression to nuclear densities, subsequent core bounce, and hydrodynamical shock wave generation. These phenomena involve large masses at high velocities ($\sim c/4$) and great accelerations. Such dynamics, if only slightly aspherical, will lead to copious gravitational wave emission and, arguably, to one of the most distinctive features of core-collapse supernovae. The gravitational waveforms and associated spectra bear the direct stamp of the hydrodynamics and rotation of the core and speak volumes about internal supernova evolution. Furthermore, they provide data that complement (temporally and spectrally) those from the neutrino pulse (which also originates from the core), enhancing the diagnostic potential of each.

Most stars rotate and rotation can result in large asphericity at and around bounce. This provides hope that the emission of gravitational radiation from stellar core collapse can be significant. Furthermore, Rayleigh-Taylor-like convection in the protoneutron star, the aspherical emission of neutrinos, and post-bounce triaxial rotational instabilities are also potential sources of gravitational radiation. Together these phenomena, with their characteristic spectral and temporal signatures, make core-collapse supernovae promising and interesting generators of gravitational radiation.

Ott et al. (2003) use the 2D hydro code VULCAN/2D (Livne 1993) and follow Zwerger & Müller (1997) in forcing the one-dimensional initial models to rotate with constant angular velocity on cylinders according to the rotation law

$$\Omega(r) = \Omega_0 \left[ 1 + \left(\frac{r}{A}\right)^2 \right]^{-1},$$

(1.1)

where $\Omega(r)$ is the angular velocity, $r$ is the distance from the rotation axis, and $\Omega_0$ and $A$ are free parameters that determine the rotational speed/energy of the model and the scale of the distribution of angular momentum. The rotation parameter $\beta$ is defined by

$$\beta = \frac{E_{rot}}{|E_{grav}|},$$

(1.2)

where $E_{rot}$ is the total rotational kinetic energy and $E_{grav}$ is the total gravitational energy. We (Ott et al. 2003) name our runs according to the following convention: [initial model name]$\Lambda$[in km]$\beta$[in %]. For example,
s11A1000\(\beta_{0.3}\) is a Woosley and Weaver (1995) 11 \(M_\odot\) model with \(A=1000\) km and an initial \(\beta_i\) of 0.3%.

Representative results are those found for model s15A1000\(\beta_{0.2}\) (Ott et al. 2003). The spectrum of s15A1000\(\beta_{0.2}\) is dominated by frequencies between 300 Hz and 600 Hz and peaks at 460 Hz. Most of the smaller peaks are connected to the first spike in the waveform during which 94\% of the total gravitational wave energy of this model is radiated. There is, however, a contribution by the radial and non-radial ring-down pulsations that have characteristic periods of 2 - 2.5 ms in this model, translating into frequencies of 400-500 Hz. The peak is at 700 Hz and there are higher harmonics around 1400 Hz. With increasing \(\beta_i\) the spectrum shifts to lower frequencies and lower absolute values, peaking at 152 Hz (\(\beta_i=0.40\%\)), 91 Hz (\(\beta_i=0.60\%\)), and 38 Hz (\(\beta_i=0.80\%\)). Furthermore, a prominent peak at low frequencies can be directly associated with the oscillation frequency of the post bounce cycles.

The models of Ott et al. (2003) yield absolute values of the dimensionless maximum gravitational wave strain in the interval \(2.0 \times 10^{-23} \leq h_{TT}^{\text{max}} \leq 1.25 \times 10^{-20}\) at a distance of 10 kpc. The total energy radiated (\(E_{GW}\)) lies in the range \(1.4 \times 10^{-11} \ M_\odot c^2 \leq E_{GW} \leq 2.21 \times 10^{-8} \ M_\odot c^2\) and the energy spectra peak (with the exception of a very few models) in the frequency interval \(20 \ Hz \lesssim f_{\text{peak}} \lesssim 600 \ Hz\).

Ott et al. (2003) find that at a distance of 10 kpc, i.e. for galactic distances, the 1st-generation LIGO, once it has reached its design sensitivity level, will be able to detect more than 80\% of our core collapse models under optimal conditions and orientations. Assuming random polarizations and angles of incidence, this reduces to 10\%. Advanced LIGO, however, should be able to detect virtually all models at galactic distances. Figure \ref{fig1} presents peak \(h_{\text{char}}\) (the points), the maxima of the characteristic gravitational wave strain spectrum, but it also includes the actual \(h_{\text{char}}\) spectra of selected models (see Ott et al. 2003 for details). These \(h_{\text{char}}\) serve to put the issues of detectability in the LIGO detector into sharp relief.

### 1.3 Rotational Effects

The evolution of the rotation parameter \(\beta\) and of the angular velocity is of particular interest, since they are connected to two still unanswered questions in core-collapse supernovae physics: What are the periods of newborn neutron stars? What is the role of rotation in the mechanism of core-collapse supernovae? As a prelude, Ott et al. (2003) addressed two related points:

1) There exists a maximum value of \(\beta\) at bounce for a given progenitor
Fig. 1.1. LIGO sensitivity plot. Plotted are the optimal root-mean-square noise strain amplitudes $h_{\text{rms}} = \sqrt{\int S(f)}$ of the initial and advanced LIGO interferometer designs. Optimal means that the gravitational waves are incident at an optimal angle and optimal polarization for detection and that there are coincident measurements of gravitational waves by multiple detectors. For gravitational waves from burst sources incident at random times from a random direction and a signal-to-noise ratio (SNR) of 5, the rms noise level $h_{\text{rms}}$ is approximately a factor of 11 above the one plotted here (Abramovici et al. 1992). We have plotted solid squares at the maxima of the characteristic gravitational wave strain spectrum ($h_{\text{char}}(f)$) of our s11, s15, and s20 models from Woosley and Weaver (1995) that were artificially put into rotation. Our nonrotating models are marked with stars; diamonds stand for models from Heger et al. (2000, 2003). The distance to Earth was set to 10 kpc for all models. Most of our models lie above the optimal design sensitivity limit of LIGO I. Hence, the prospects for detection are good. Those models that are not detectable by the 1st-generation LIGO are those that rotate most slowly and those which are the fastest rotators. See Ott et al. (2003) for details.

model and value of $A$. Interestingly, the maximum $\beta$ is not reached by the model with the maximum $\beta_4$, but by a model with some intermediate value of $\beta_4$. $\beta$ at bounce is determined by the subtle interplay between initial angular momentum distribution, the equation of state, centrifugal forces...
and gravity. The “optimal” configuration leads to the overall maximum $\beta$ at bounce for a given $\beta_i$. Generally, $\beta$ increases during collapse by a factor of $\sim$10-40.

2) As with $\beta$, overall the angular velocity increases with increasing $\beta_i$ until a maximum is reached. It subsequently decreases with the further increase of $\beta_i$. The initially more rigidly rotating models actually yield larger post-bounce angular velocity gradients inside 30 km. The equatorial velocity profile peaks off center for moderate $\beta_i$ at radii between 6 and 10 km. An initially more differentially rotating model (at a given $\beta_i$) leads to the highest central values of the angular velocity, while its angular velocity profile quickly drops to low values and near rigid rotation for $\beta_i \geq 0.3\%$. Model s15A500/30.2 in Ott et al. (2003) (see §1.2) results in the shortest rotation period near the center ($\sim$1.5 ms). Model s15A50000/30.5 yields the shortest period of the A=50000 km model series ($\sim$1.85 ms).

In sum, the amplification of the angular velocity (frequency) due to collapse is generally large, from a factor of $\sim$25 to $\sim$1000. An initial period of 2 seconds in the iron core can translate into a period at bounce of $\sim$5 milliseconds, depending upon the initial rotational profile. The angular velocity shear exterior to the peak at 6-10 km exhibited by these models has also been identified in the one-dimensional study of Akiyama et al. (2003). These authors consider such shear a possible driver for the magneto-rotational instability (MRI), which could be a generator of strong magnetic fields.

**1.3.1 Rotation and Explosion**

The large amplification of the angular velocity during bounce implies that rotation may be a factor in core collapse phenomenology and in the explosion mechanism. Though the latter remains to be demonstrated, there are a few aspects of rotating collapse that bear mentioning and that distinguish it from spherical collapse: 1) Rotation lowers the effective gravity in the core, increasing the radius of the stalled shock and the size of the gain region. Since ejection is inhibited by the deep potential well, rotation might in this manner facilitate explosion. 2) Rotation generates vortices that might dredge up heat from below the neutrinospheres and thereby enhance the driving neutrino luminosities. 3) Rotation lowers (slightly) the optical depth of a given mass shell, thereby increasing the $\nu_e$ neutrino luminosity. (However, as Fryer and Heger (2000) have shown the $\bar{\nu}_e$ luminosity is at the same time decreased due to the lower temperatures achieved.) 4) Importantly, rotation results in a pronounced anisotropy in the mass accretion flux after bounce. In fact, rotation can create large pole-to-equator differ-
quences in the density profiles of the infalling matter, due to the centrifugal barrier along the poles. The actual magnitude and evolution of this barrier is a function of the degree of rotation and its profile, but can be quite pronounced. Very approximately, in the equatorial region the distance from the axis \( \rho \), in cylindrical coordinates) of the barrier is given by \( j^2/(GM) \), where \( M \) is the interior mass and \( j \) is the specific angular momentum at that mass. If the slope of \( j \) with \( r \) is positive, then as matter from further and further out accretes onto the protoneutron star the centrifugal barrier is expected to grow in extent. Even if the \( j \) profile is flat, \( j^2/(GM) \) might be an interesting \( (\sim 10-300 \text{ kilometers ?}) \) number (Heger et al. 2000,2003).

Figure 1.2 depicts a snapshot after bounce of various nested isodensity contours for a rapidly rotating initial progenitor \( (\Omega_0 \sim 2\pi/2 \text{ rad s}^{-1}; \text{ eq. 1.1}) \). The fact that outer (lower-density) contours pierce into the inner regions along the poles implies that a partially evacuated region is carved out along these poles. Since the total accretion rate is set by the initial mass density profile and since accretion powers the early post-bounce neutrino luminosity, this luminosity is not much changed. However, due to significant rotation the mass accretion flux at the poles is small after bounce. This suggests that the neutrino-driven mechanism is naturally facilitated along
the poles (Burrows & Goshy 1993). A bipolar explosion would result (Fryer & Heger 2000, though see Buras et al. 2003). Thus, such bipolarity (and the consequent optical polarization of the debris) is not an exclusive signature of MHD driven explosions and may be a natural consequence of the neutrino-driven mechanism with rotation. However, for rotation to be pivotal in the mechanism, it might need to be rapid. This would beg the question of how the excess angular momentum is ejected to leave only the modestly rotating pulsars observed. Clearly, much works remains to be done.

1.4 Reprise on Supernova Energetics, Made simple

The discussion concerning the rudiments of supernova energetics included in Burrows and Thompson (2002) summarizes our thoughts on the origin of the supernova energy scale and contains a useful perspective on the true efficiency of the neutrino-driven mechanism. It is reprised here without shame due to the continuing interest outside the supernova community in simple arguments that are not as opaque as numerical simulations can be (Thompson, Burrows, and Pinto 2003; Liebendörfer et al. 2001; Rampp & Janka 2000).

1.4.1 Supernova Energetics Made Simple (?)

"It is important to note that one is not obliged to unbind the inner core (~10 kilometers) as well; the explosion is a phenomenon of the outer mantle at ten times the radius (50-200 kilometers). One consequence of this goes to the heart of a general confusion concerning supernova physics. Though the binding energy of a cold neutron star is \( \sim 3 \times 10^{53} \) ergs and the supernova explosion energy is near \( 10^{51} \) ergs, a comparison of these two numbers and the large ratio that results are not very relevant. More germane are the binding energy of the mantle (interior to the shock or, perhaps, exterior to the neutrinospheres) and the neutrino energy radiated during the delayed phase. These are both at most a few\( \times 10^{52} \) ergs, not \( \sim 3 \times 10^{53} \) ergs, and the relevant ratio that illuminates the neutrino-driven supernova phenomenon is \( \sim 10^{51} \) ergs divided by a few\( \times 10^{52} \) ergs. This is \( \sim 5-10\% \), not the oft-quoted 1%, a number which tends to overemphasize the sensitivity of the neutrino mechanism to neutrino and numerical details.

Furthermore, there is general confusion concerning what determines the supernova explosion energy. While a detailed understanding of the supernova mechanism is required to answer this question, one can still proffer a few observations. First is the simple discussion above. Five to ten per-
cent of the neutrino energy coursing through the semi-transparent region is required, not one percent. Importantly, the optical depth to neutrino absorption in the gain region is of order $\sim 0.1$. The product of the sum of the $\nu_e$ and $\bar{\nu}_e$ neutrino energy emissions in the first 100’s of milliseconds and this optical depth gives a number near $10^{51}$ ergs. Furthermore, the binding energy of the progenitor mantle exterior to the iron core is of order a few $\times 10^{50}$ to a few $\times 10^{51}$ ergs and it is very approximately this binding energy, not that of a cold neutron star, that is relevant in setting the scale of the core-collapse supernova explosion energy. Given the power-law nature of the progenitor envelope structure, it is clear that this binding energy is related to the binding energy of the pre-collapse iron core (note that they both have a boundary given by the same $GM/R$), which at collapse is that of the Chandrasekhar core. The binding energy of the Chandrasekhar core is easily shown to be zero, modulo the rest mass of the electron times the number of baryons in a $\sim 1.4 \, M_\odot$ Chandrasekhar mass. (The Chandrasekhar mass/instability is tied to the onset of relativity for the electrons, itself contingent upon the electron rest mass). The result is $\sim 10^{51}$ ergs.

The core-collapse explosion energy is near the explosion energy for a Type Ia supernovae because in a thermonuclear explosion the total energy yield is approximately the 0.5 MeV/baryon derived from carbon/oxygen burning to iron times the number of baryons burned in the explosion. The latter is $\geq$ half the number of baryons in a Chandrasekhar mass. The result is $\sim 10^{51}$ ergs. This is the same number as for core-collapse supernovae because 1) in both cases we are dealing with the Chandrasekhar mass (corrected for electron captures, entropy, general relativity, and Coulomb effects) and 2) the electron mass and the per-baryon thermonuclear yield are each about 0.5 MeV.

While more detailed calculations are clearly necessary to do this correctly, the essential elements of supernova energetics are not terribly esoteric (if neutrino-driven), at least to within a factor of 5, and should not be viewed as such.”

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