The Perils of ‘Soft’ SUSY Breaking

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Abstract

We consider a two dimensional SU(N) gauge theory coupled to an adjoint Majorana fermion, which is known to be supersymmetric for a particular value of fermion mass. We investigate the ‘soft’ supersymmetry breaking of the discrete light cone quantization (DLCQ) of this theory. There are several DLCQ formulations of this theory currently in the literature and they naively appear to behave differently under ‘soft’ supersymmetry breaking at finite resolution. We show that all these formulations nevertheless yield identical bound state masses in the decompactification limit of the light-like circle. Moreover, we are able to show that the supersymmetry-inspired version of DLCQ (so called ‘SDLCQ’) provides the best rate of convergence of DLCQ bound state masses towards the actual continuum values, except possibly near or at the critical fermion mass. In this last case, we discuss improved extrapolation schemes that must supplement the DLCQ algorithm in order to obtain correct continuum bound state masses. Interestingly, when we truncate the Fock space to two particles, the SDLCQ prescription presented here provides a scheme for improving the rate of convergence of the massive t’Hooft model. Thus the supersymmetry-inspired SDLCQ prescription is applicable to theories without supersymmetry.

1
1 Introduction

Over the last several years we have learned a great deal about supersymmetric gauge theories following the discovery of dualities between string/M-theory and supersymmetric gauge theories [1, 2, 3]. Recently this has been extended to conformal field theories without supersymmetry [4]. Evidently, it would be desirable to have a deeper understanding of supersymmetry breaking in order to bridge the gap between the formulation of physics in a supersymmetric world, and its more realistic counterpart, where no such symmetry is manifest. One straightforward approach is to start with a supersymmetric formulation, and then proceed to break supersymmetry ‘softly’ by adding appropriate mass terms.

The context within which we will consider this is a theory that has been well studied before: two dimensional SU($N$) gauge theory coupled to an adjoint Majorana fermion [5]. Interestingly, this theory is known to exhibit supersymmetry at a particular value of the fermion mass, $m = m_{SUSY}$ [7]. This is believed to be a theory with two parameters $g$ and $m$, both of which have the dimensions of mass. Since the only $g$ dependence is an overall $g^2$ factor in the Hamiltonian the theory depends on one dimensionless parameter $X = \frac{m^2 \pi}{g^2 N}$ and therefore all the bound state masses, in units of $\frac{g^2 N}{\pi}$, must be determined in terms of the one parameter $X$. In this work, we provide evidence that, while this viewpoint is still correct, there is still scope for an additional operator (and associated coupling constant) that may be introduced to improve convergence of the DLCQ bound state masses towards their actual continuum values. Of course, these continuum masses will be unaffected by the presence of such an operator, but a judicious choice of coupling will serve to improve the rate of convergence of our numerical results.

Naively, when one adds a ‘soft’ breaking term to the two DLCQ formulations of the theory, we appear to arrive at different spectra. The two formulations we are alluding to are the ‘Principle Value’ (PV) and ‘Supersymmetric Discrete light Cone Quantization’ (SDLCQ), and are discussed in detail below. The question is whether these two spectra are truly different or simply rescalings of the same spectrum which become identical in the continuum limit. It would be very good news if they were in fact the same because the PV prescription is generally accepted as correct ([8, 9]), while the SDLCQ approach is known to converge more rapidly in general.

Actually, to understand the relation between these two schemes, it is helpful to present a formulation that interpolates between the PV and SDLCQ prescriptions by introducing
an additional operator and associated coupling constant that we will call $Y$. In particular, $Y = 0$ will correspond to the PV prescription, while $Y = 1$ will imply the SDLCQ prescription. Intermediate values for $Y$ will correspond to a ‘mixture’ of the two schemes. By diagonalizing the DLCQ Hamiltonian matrix, and extrapolating to the continuum limit, we are able to solve for bound state masses and wave functions at different values of the fermion mass parameter $X$ and coupling constant $Y$.

We shall show that the continuum bound state masses are independent of the coupling $Y$, as expected from a scheme independent prescription, although the rate of convergence towards the actual continuum mass will be significantly affected by our choice for $Y$. In fact, it will turn out that the value for $Y$ that arises naturally in the regularization of supersymmetric theories (i.e. $Y = 1$) provides the best convergence towards actual continuum masses. Thus, the supersymmetric formulation of DLCQ (SDLCQ), corresponding to $Y = 1$ – first highlighted in the work – yields a method for improving numerical convergence of DLCQ bound state masses even for theories without supersymmetry.

To show this, we study the DLCQ bound state integral equations at high resolution, which is made possible by truncating the Fock space to two particles. In particular, we show that the SDLCQ approach converges more uniformly and rapidly for all values of $X$ that are sufficiently far from the critical value $X = 0$.

We remark that at the supersymmetric point $X = 1$, the SDLCQ prescription preserves supersymmetry even in the discretized theory. The advantages of such an approach have been exploited in a study of a wide class of supersymmetric gauge theories in two \cite{12, 13, 14, 15, 6} and three dimensions \cite{16}.

2 Formulations of The Theories

In this section we will consider the formulations of $1 + 1$ dimensional QCD coupled to adjoint Majorana fermions having arbitrary mass (see for example \cite{5}) in the light cone gauge $A^+ = 0$. After eliminating non-physical degrees of freedom by solving constraint equations, the light–cone components of total momentum are found to be:

\begin{equation}
\begin{align*}
P^+ &= \int d^x Tr(i\psi \partial_- \psi), \\
P^- &= \int d^x Tr \left( \frac{-im^2}{2} \psi \frac{1}{\partial_+} \psi - \frac{g^2}{2} J^+ \frac{1}{\partial^-} J^+ \right)
\end{align*}
\end{equation}
\[
\begin{align*}
\frac{m^2}{2} \int_0^\infty \frac{dk}{k} b^\dagger_{ij}(k)b_{ij}(k) + \frac{g^2N}{\pi} \int_0^\infty \frac{dk}{k} \int_0^k dp \frac{k}{(p-k)^2} b^\dagger_{ij}(k)b_{ij}(k) + \\
g^2 \frac{2\pi}{2} \int_0^\infty dk_1dk_2dk_3dk_4 \left( \delta(k_1 + k_2 - k_3 - k_4)A(k)b^\dagger_{kj}(k_3)b^\dagger_{ji}(k_4)b_{kl}(k_1)b_{lj}(k_2) + \right.
\delta(k_1 + k_2 + k_3 - k_4)B(k)(b^\dagger_{kj}(k_4)b_{kl}(k_1)b_{lj}(k_2)b_{ji}(k_3) - b^\dagger_{kj}(k_4)b^\dagger_{ji}(k_1)b_{lj}(k_2)b_{kl}(k_3)) \right)
\end{align*}
\]

with
\[
\begin{align*}
A(k) &= \frac{1}{(k_4 - k_2)^2} - \frac{1}{(k_1 + k_2)^2}, \\
B(k) &= \frac{1}{(k_3 + k_2)^2} - \frac{1}{(k_1 + k_2)^2}.
\end{align*}
\]

Here \(x^\pm = (x^+ \pm x^-)/\sqrt{2}\) and \(J^+_ij = 2\psi_{ik}\psi_{kj}\) is the longitudinal component of the fermion current. To avoid introducing an additional mass scale in the theory we will write this in terms of mass operators: \(M^2 = 2P^+P^-\). It is well known that at the special value of fermionic mass (namely \(m^2_{SU SY} = g^2N/\pi\)) this system is supersymmetric \([7]\). We will use a dimensionless mass parameter \(X = \frac{m^2_{SU SY}}{g^2N}\), and the supersymmetric point is \(X = 1\) and the masses of all bound states will be quoted in units of \(g^2N/\pi\). The supercharge is given by
\[
Q^- = 2^{1/4} \int dx^- tr(2\psi \frac{1}{\partial^-}\psi).
\]
\[
= \frac{i2^{-1/4}N}{\sqrt{\pi}} \int_0^\infty dk_1dk_2dk_3 \delta(k_1 + k_2 - k_3) \left( \frac{1}{k_1} + \frac{1}{k_2} - \frac{1}{k_3} \right) \times
\]
\[
( b^\dagger_{ik}(k_1)b^\dagger_{kj}(k_2)b_{ij}(k_3) + b^\dagger_{ij}(k_3)b_{ik}(k_1)b_{kj}(k_2) ) \right),
\]

Using the anticommutator at equal \(x^+\):
\[
\{ \psi_{ij}(x^-), \psi_{kl}(y^-) \} = \frac{1}{2}\delta(x^- - y^-)
\]

it can be checked that at \(m = m^2_{SU SY}\) the SUSY algebra \(\{Q^-, Q^-\} = 2\sqrt{2}P^-\) is satisfied. In the DLCQ approximation the system lives in a \(x^-\) box of length \(L\) and one has to sums over discrete variables \(k^+ \neq 0\) instead of integrations in the above formulas. For periodic boundary conditions (BC), \(k^+ = n\pi/L\) where \(n = 1, 2, \ldots, K\) and \(K\) is called the resolution.

One formulation of DLCQ which we will denote as the principal value (PV) prescription \([8]\), treats the singularities of the Hamiltonian using a PV prescription and can be
formulated using either anti-periodic or periodic BC. The anti-periodic boundary condition must break the supersymmetry at finite resolution because the fermions and bosons are in different Fock sectors. The PV prescription with periodic BC could in principle give supersymmetric results at finite resolution, although this is not the case. In the PV prescription the supersymmetry at \( X = 1 \) is restored only in the decompactification limit \( (K \to \infty) \). This restoration was shown in [5]. The Hamiltonian for this formulation will be referred to as \( P_{PV} \).

The prescription that preserves supersymmetry at finite resolution will be called SDLCQ. In SDLCQ one simply uses DLCQ to calculate the supercharges and then uses the supercharges to calculate the Hamiltonian and longitudinal momentum operator [11]. Here we must use periodic BC because the supercharge \( Q^- \) is cubic in the fields, while the supercharge \( Q^+ \) is quadratic.

The SUSY algebra is reproduced at a special value of fermion mass and at every finite resolution the supercharge matrices give a representation of the super algebra. Both SDLCQ and \( P_{PV} \) at \( X = 1 \) give the same results as the resolution goes to infinity [13].

We now want to add identical `soft' SUSY breaking terms (mass terms) to these theories and study the resulting non-supersymmetric theory. Since we already have a mass term in \( P_{PV} \) this only requires varying \( X \), but for SDLCQ this means explicitly adding a mass term.

It is very instructive to actually do the numerical calculation differently and introduce a third formulation, \( P_{SUSY} \) which includes both SDLCQ and \( P_{PV} \). We have found the operator which is the difference between the SDLCQ and the PV formulation [6]. Thus if we add this operator to the PV Hamiltonian it is now supersymmetric at every resolution and produces exactly the same mass and wave functions as SDLCQ. In the large \( N \) approximation the operator take the form.

\[
\frac{g^2NK}{\pi} \sum_n \frac{1}{n^2} B^\dagger_{ij}(n) B_{ij}(n).
\]  

(6)

Numerically, this operator does not alter the actual continuum values observed in the PV approach when \( X = 1 \). In our numerical formulation of \( P_{SUSY} \) we included this operator with an adjustable coupling \( Y \). We can now think of \( P_{SUSY}^- \) as a single theory.

\[\text{They find that convergence is slower for period BC}\]
\[\text{However SDLCQ converges much faster}\]
\[\text{To date we have only found this operator for this particularly simple theory but it should be possible to find it for other theories as well. The calculation of this operator involves a careful study of the intermediate zero modes that contribute to the square of the supercharge}\]
in the coupling constant space \((X,Y)\). The formulation we called PV corresponds to setting \(Y = 0\) and allowing \(X\) to be arbitrary, while the prescription we call SDLCQ corresponds to setting \(Y = 1\). In the following, we will present results for the lightest bosonic bound states as a function of \(X\) and \(Y\). For a few values of \(X\) and \(Y\) we will truncate the Fock space to allow only two particles Fock states, which will permit us to investigate the t’Hooft equation for higher resolutions than would otherwise be possible.

3 ‘Soft’ SUSY Breaking

Our investigation of this theory indicates that at \(X = 1\) (the supersymmetric value of the fermion mass) the lightest fermionic and bosonic bound states are degenerate with continuum masses approximately \(M^2 = 26\) \([5, 13]\). Using \(P_{\text{SUSY}}\) we arrive at the same conclusion for any value of \(Y\). Boorstein and Kutasov \([17]\) have investigated ‘soft’ supersymmetry breaking for small values of this difference, \(X − 1\) and they found that the degeneracy between the fermion and boson bound state masses is broken according to

\[
M_F^2(X) − M_B^2(X) = (1 − X)M_B(1) + O((X − 1)^3). 
\]  

(7)

They calculated these masses using the PV prescription \((Y = 0)\) with anti-periodic BC and found very good agreement with the theoretical prediction. We have compared this theoretical prediction at \(Y = 1\) and we find that eq (7) is very well satisfied. At resolution \(K = 5\), for example, the slope is 4.76 and the predicted slope \(M_B(1)\) is 4.76. The indication is that this result is true for any value of \(Y\).

In Fig. 1 we show the contour plots of the mass squared \(M^2\) of the two lightest bosonic bound states as a function of \(X\) and \(Y\) at resolution \(K = 10\). These contours are lines of constant mass squared. Selecting a particular value of the mass of the first bound state then fixes a particular contour in Fig. 1a as a contour of fixed mass, which we can write as \(Y = Y_p(X)\).

Interestingly, constructing the same contour plot for the next to lightest bosonic bound state – see Fig. 1b – yields contours that have approximately the same functional dependence implied by Fig. 1a. In fact, one obtains approximately the same contour plots for the next twenty bound states (which is as far as we checked). The simple conclusion is that the coupling \(Y\) which represents the strength of the additional operator affects all bound state masses more or less equally. This in turn suggests that at finite resolution,
Figure 1: (a) The contour plots of $Y = Y(X)$ for the mass squared of the lowest bound state in units of $g^2N/\pi$ as a function of $X = m\pi/g^2N$ and $Y$ (b) The contour plots of $Y = Y(X)$ for the mass squared of the second lowest bound state in units of $g^2N/\pi$ as a function of $X = m\pi/g^2N$ and $Y$.

we can smoothly interpolate between different values of fermion mass $X$, and different prescriptions specified by the coupling $Y$, without affecting too much the actual numerical spectrum. Of course, in the decompactification limit $K \to \infty$, such a dependence on $Y$ disappears, due to scheme independence.

Since the lightest bosonic bound state is primarily a two particle state it is reasonable to truncate the Fock basis to two particle states. This will permit very high resolutions, which will be needed to carefully scrutinize any possible discrepancies between the two versions of ‘soft’ symmetry breaking presented here. In fact, we are able to study the theory for $K$ up to 800. The mass of the lowest state as a function of the resolution for various values of $X$ and $Y$ are shown in Fig. 2. Each converging pair of lines – which extrapolate the actual data points – in Fig. 2 corresponds to different values of fermion mass $X$. The top upper curve in each pair runs through data points that were calculated via SDLCQ (i.e. $Y = 1$), while the lower corresponds to the PV (i.e. $Y = 0$) prescription commonly adopted in the literature. We find that each pair of curves converge to the same point at infinite resolution, although this may not be completely obvious for the lowest pair in the figure (corresponding to the critical mass $X = 0$).

Away from $X = 0$, the SDLCQ formulation is fitted with a linear function of $1/K$, while the PV formulation is fit with a polynomial of $1/K^{2\beta}$, where $\beta$ is the solution of
1 - X/2 = \pi \beta \cot(\pi \beta) [21]. It now appears that SDLCQ not only provides more rapid convergence for supersymmetric models, but also for the massive t'Hooft model, which is not supersymmetric. For the massless case, the situation is reversed; the SDLCQ formulation converges slower. It is fit by a polynomial in 1/Log(K) and gives the same mass at infinite resolution as the PV formulation. This behavior may be understood from the observation that the wave function of this state does not vanish at x = 0. We have looked closely at ‘small’ masses, such as X = .1, and one finds that both PV and SDLCQ vary as a polynomial in 1/K^{2\beta} at large resolution. Thus careful extrapolation schemes must be adopted at small masses.

We therefore conclude that the continuum of regularization schemes that interpolate smoothly between the SDLCQ and PV prescriptions – which we characterized by the parameter Y – yield the same continuum bound state masses, although the rate of convergence of the DLCQ spectrum may be altered significantly. This implies that the contour plots observed in Fig. 1 eventually approach lines parallel to the Y axis, and the sole dependence on the parameter X is recovered.

![Figure 2: Mass of the of the lowest bound state in units of g^2 N/\pi calculated in the t'Hooft model. The top pair is at X = 1, the second is at X = .5, and the bottom pair is at X = 0](image)

4 Discussion

The two dimensional gauge theory of adjoint Majorana fermions has been studied extensively [3, 7, 13, 17, 18] and is known to be a theory with an overall mass scale g^2, and one
real coupling – the mass of the fermion – which we write as $X$ in our notation. When one adds a ‘soft’ supersymmetry breaking term, the supersymmetric (SDLCQ) and principle value (PV) prescriptions for regulating the Coulomb singularity appear to give different bound state masses at finite resolution.

We observed at finite resolution that these different bound state masses may be smoothly connected – in an approximate sense – by introducing a new operator, and an associated coupling $Y$, and then varying the couplings $X$ and $Y$ along an appropriately chosen contour.

By truncating the Fock space to two particles, we were able to study the DLCQ bound state equations up to $K = 800$, which we summarized in Fig. 2. We concluded that after carefully extrapolating the data, the different prescriptions yielded identical continuum bound state masses. Moreover, we observed that the SDLCQ prescription improved convergence for sufficiently large values of fermion mass.

Interestingly, since the two-body equation studied here for the adjoint fermion model is simply the t’Hooft equation with a rescaling of coupling constant, we have arrived at an alternative prescription for regulating the Coulomb singularity in the massive t’Hooft model that improves the rate of convergence towards the actual continuum mass. Thus, a prescription that arises naturally in the study of supersymmetric theories is also applicable in the study of a theory without supersymmetry. We believe that this idea deserves to be exploited further in a wider context of theories. In particular, it is an open question whether this procedure could provide a sensible approach to regularizing softly broken gauge theories with bosonic degrees of freedom, and in higher dimensions.

In any case, it appears that the special cancellations afforded by supersymmetry – especially in the context of DLCQ bound state calculations – might have scope beyond the domain of supersymmetric field theory. This would be a crucial first step towards a serious non-perturbative study of theories with broken supersymmetry.

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References
[1] T. Banks, W. Fischler, S. Shenker, L. Susskind, Phys. Rev. D55 (1997) 5112, hep-th/9610043.
[2] L.Susskind, *Another Conjecture About Matrix Theory*, [hep-th/9704080](http://arxiv.org/abs/hep-th/9704080).

[3] Juan M. Maldacena, Adv.Theor.Math.Phys.2:231-252,1998 [hep-th/9711200](http://arxiv.org/abs/hep-th/9711200).

[4] I. Klebanov and A. Tseytlin, “A Non-Supersymmetric Large N CFT from Type 0 String Theory” [hep-th/9901101](http://arxiv.org/abs/hep-th/9901101).

[5] G. Bhanot, K. Demeterfi, I.R. Klebanov, *Phys. Rev.* D48 (1993) 4980, [hep-th/9307111](http://arxiv.org/abs/hep-th/9307111).

[6] F. Antonuccio, O. Lunin, and S. Pinsky, Phys.Lett.B442:173-179,1998; [hep-th/9809165](http://arxiv.org/abs/hep-th/9809165).

[7] D. Kutasov, *Phys. Rev.* D48 (1993) 4980, [hep-th/9306013](http://arxiv.org/abs/hep-th/9306013).

[8] G. t’Hooft, Nuc. Phys. B72 (1974), 461.

[9] H.-C. Pauli and S.J. Brodsky, *Phys. Lett.* D32 (1985) 1993, 2001.

[10] S.J. Brodsky, H.C. Pauli, and S.S. Pinsky, Phys.Rept.301:299-486,1998 [hep-ph/9705477](http://arxiv.org/abs/hep-ph/9705477).

[11] Y. Matsumura, N. Sakai, and T. Sakai, *Phys.Rev.* D52:2446-2461,1995 [hep-th/9504150](http://arxiv.org/abs/hep-th/9504150).

[12] F. Antonuccio, O. Lunin, S. Pinsky, *Phys.Lett.* B429 (1998) 327-335; [hep-th/9803027](http://arxiv.org/abs/hep-th/9803027).

[13] F. Antonuccio, O. Lunin, S. Pinsky, *Phys.Rev.* D58 (1998) 085009; [hep-th/9803170](http://arxiv.org/abs/hep-th/9803170).

[14] F. Antonuccio, O. Lunin, H.C. Pauli, S. Pinsky, and S. Tsujimaru, *Phys.Rev.* D58(1998) 105024; [hep-th/9806133](http://arxiv.org/abs/hep-th/9806133).

[15] F. Antonuccio, H.C. Pauli, S. Pinsky, Phys.Rev.D58:125006,1998 ); [hep-th/9808120](http://arxiv.org/abs/hep-th/9808120).

[16] F. Antonuccio, O. Lunin, and S. Pinsky, “Super Yang-Mills at Weak, Intermediate and Strong Coupling”, (to appear in *Phys.Rev. D* ); [hep-th/98011083](http://arxiv.org/abs/hep-th/98011083).

[17] J. Boorstein and D. Kutasov Nucl.Phys.B421:263-277,1994 [hep-th/9401044](http://arxiv.org/abs/hep-th/9401044).

[18] F. Antonuccio, and S. Pinsky Phys.Lett.B439:142-149,1998 [hep-th/9805188](http://arxiv.org/abs/hep-th/9805188).

[19] S. Pinsky, *The Analog of the t’Hooft Pion with Adjoint Fermions*” Invited talk at New Nonperturbative Methods and Quantization of the Light Cone, Les Houches, France, 24 Feb - 7 Mar 1997. [hep-th/9705242](http://arxiv.org/abs/hep-th/9705242) Abstract and Postscript from Los Alamos (or from France or Italy or U.K.)
[20] *Review of Matrix Theory* D. Bigatti, L. Susskind SU-ITP-97-51, hep-th/9712072

[21] B. van de Sande, Phys.Rev.D54:6347-6350,1996 hep-ph/9605409

[22] M.J. Strassler “Manifolds of fixed Points and Duality in Supersymmetric Gauge theories” Proceedings of the Yukawa International Seminar (YKIS95), Prog.Theor.Phys.Suppl.123:373-380,1996 hep-th/960202.