Minimally flavored colored scalar in $B \to D(\ast)\tau\bar{\nu}$ and the mass matrices constraints

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Abstract: The presence of a colored scalar that is a weak doublet with fractional electric charges of $|Q| = 2/3$ and $|Q| = 5/3$ with mass below 1 TeV can provide an explanation of the observed branching ratios in $B \to D(\ast)\tau\bar{\nu}$ decays. The required combination of scalar and tensor operators in the effective Hamiltonian for $b \to c\tau\bar{\nu}$ is generated through the $t$-channel exchange. We focus on a scenario with a minimal set of Yukawa couplings that can address a semitauonic puzzle and show that its resolution puts a nontrivial bound on the product of the scalar couplings to $\bar{\tau}b$ and $\bar{\epsilon}\nu$. We also derive additional constraints posed by $Z \to b\bar{b}$, muon magnetic moment, lepton flavor violating decays $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to e\gamma$, and $\tau$ electric dipole moment. The minimal set of Yukawa couplings is not only compatible with the mass generation in an $SU(5)$ unification framework, a natural environment for colored scalars, but specifies all matter mixing parameters except for one angle in the up-type quark sector. We accordingly spell out predictions for the proton decay signatures through gauge boson exchange and show that $p \to \tau^0\epsilon^+$ is suppressed with respect to $p \to K^+\bar{\nu}$ and even $p \to K^0\epsilon^+$ in some parts of available parameter space. Impact of the colored scalar embedding in 45-dimensional representation of $SU(5)$ on low-energy phenomenology is also presented. Finally, we make predictions for rare top and charm decays where presence of this scalar can be tested independently.
1 Introduction

Results from LHC and B-factories have narrowed down the possibilities for New Physics (NP) close to the electroweak scale. After precise measurements of many observables in $\Delta B = 2$ and $\Delta B = 1$ transitions hopes for the presence of NP persist only in the sector of leptonic and/or semileptonic decays. Namely, there have been experimental indications of enhanced branching ratios in the $B \rightarrow D(D^*)\tau\bar{\nu}_\tau$ decays with respect to the Standard
Model (SM) predictions. In particular, BaBar collaboration has presented the following ratios [1]

\[ R_{\tau/\ell}^* = \frac{B(B \to D^* \tau\nu)}{B(B \to D^* \ell\nu)} = 0.332 \pm 0.030, \]  
\[ R_{\ell/\tau} = \frac{B(B \to D\tau\nu)}{B(B \to D\ell\nu)} = 0.440 \pm 0.072. \]

Both results are consistent with previous measurements performed by Belle [2] but are larger than the SM values of \( R_{\tau/\ell}^{*\text{SM}} = 0.252(3) \) and \( R_{\ell/\tau}^{\text{SM}} = 0.296(16) \) with 3.4 \( \sigma \) significance when the two observables are combined (see Ref. [3]). If confirmed these results might point to NP in (semi)tauonic \( b \to c \) transitions. We accordingly consider a plausible scenario where the presence of one light color triplet, weak doublet, scalar leptoquark (LQ) state addresses the aforementioned discrepancy through a minimal set of couplings to matter. The minimality is motivated by the fact that the presence of this state was shown to be too severely constrained by rare decays of charm and strange mesons and of tau lepton to be able to affect \( c \to s\ell^+\nu \) significantly [4, 5]. It also renders our analysis more manageable.

The colored scalar we consider appears in various studies of possible extensions of the SM [5–10]. In particular, it can be embedded in 45-dimensional scalar representation of \( SU(5) \) [11–15] to help provide viable unification of the SM interactions. Inclusion of additional scalar representations can result in two important improvements over the original \( SU(5) \) setup [16]. Namely, the mass relations between down-type quarks and charged leptons can be corrected [11] and strong, weak and electromagnetic interactions can unify [12–15]. We thus investigate whether the minimal set of Yukawa couplings we posit can be made compatible with the mass generation mechanism within a simple \( SU(5) \) setup that relies on the use of 45-dimensional representation.

In this paper we systematically study impact of the colored scalar on the exclusive \( B \to D^{(*)}\tau\bar{\nu} \) decays, \( Z \to b\bar{b} \) decay, muon anomalous magnetic moment, \( \tau \) electric dipole moment and LFV decays \( \mu \to e\gamma \), \( \tau \to \mu\gamma, e\gamma \). We also point out additional observables that can reveal the presence of this state. These are \( t \to c\tau^+\tau^- \) and \( D^0 \to \tau^- e^+ \). We finally discuss viability of having the colored scalar be part of 45-dimensional representation of \( SU(5) \) and impact of such an embedding on relevant low-energy phenomenology.

Our work is organized as follows. In Section 2 we list all color triplet candidates that can have an impact on \( b \to c\tau\bar{\nu} \) transitions. There we single out the color triplet we focus on in the rest of our study and specify the minimal set of Yukawa couplings that is required to address the \( B \to D^{(*)}\tau\bar{\nu} \) puzzle. The individual LQ contributions to \( B \to D\tau\bar{\nu} \) and \( B \to D^*\tau\bar{\nu} \) are discussed in Section 3, where we present numerical fit to data. Sections 4, 5 and 6 are devoted to the LQ impact on \( Z \to b\bar{b} \), lepton electromagnetic moments and \( \ell \to \ell' \gamma \) decays, respectively. We then investigate the possibility to have a viable embedding of LQ setup in an \( SU(5) \) scenario in Section 7. The connection between \( \ell \to \ell' \gamma \) and \( B \to D^{(*)}\tau\bar{\nu} \) that follows from the proposed embedding is discussed in Section 8. We proceed to offer predictions on \( B_c \to \tau^+\nu \) and rare decays in Section 9 and provide conclusions in Section 10.
2 Color triplet candidates

The $b \to c \ell \bar{\nu}$ decay can be mediated, among other proposals, by color triplet bosons with renormalizable leptoquark couplings to the SM fermions. These bosons can be either scalars or vectors with electric charges of $|Q| = 1/3$ and $|Q| = 2/3$. We list quantum numbers of all states with potential contributions to $b \to c \ell \bar{\nu}$ decay in Tab. 1 where we specify their properties under $SU(3)$ and $SU(2)$ gauge groups as well as hypercharge $Y$, where hypercharge is defined in terms of electric charge and weak isospin as $Y = Q - T_3$.

We also show possible scalar and vector contractions of the SM fermions, omitting the generation and color indices, and present associated baryon $(B)$ and lepton $(L)$ numbers, where applicable.

The vector states are also listed in Tab. 1 for completeness but we do not consider them further since it seems difficult to implement light colored vectors in realistic scenarios. We also discard two particular scalars—$(3, 1)_{-1/3}$ and $(3, 3)_{-1/3}$—since they destabilize proton [17]. Of the two remaining scalar states the one with $Y = 1/6$ couples to the right handed neutrino and would not interfere with the SM amplitudes. This does not pose any problem since the two observables require enhancement of the semi-tauonic decays. However, introduction of a light RH neutrino requires an explanation of its origin and we choose not to pursue this option in the following. Therefore we dismiss this state as a suitable candidate for modification of $b \to c \tau \bar{\nu}$. The only scalar left, on the other hand, couples to the left-handed neutrino and interferes with the SM semileptonic amplitude. We denote that scalar with $\Delta \equiv (3, 2)_{7/6}$ in what follows.

| $(SU(3), SU(2))_Y$ | spin | LQ couplings | $3B$ | $L$ |
|---------------------|------|--------------|------|-----|
| $(3, 2)_{1/6}$      | 0    | $\overline{Q}_{\ell R}, \overline{d}_R L$ | +1   | −1  |
| $(3, 2)_{7/6}$      | 0    | $\overline{Q}_{\ell R}, \overline{u}_R L$ | +1   | −1  |
| $(3, 1)_{-1/3}$     | 0    | $\overline{Q}_{\nu \ell^2} L^C, \overline{d}_R \nu_{\ell^2} C, \overline{u}_R \ell^2_L C$ | +1   | −1  |
| $(3, 3)_{-1/3}$     | 0    | $\overline{Q}_{\tau i \overline{\tau}^2} L^C$ | +1   | −1  |
| $(3, 1)_{2/3}$      | 1    | $\overline{u}_R \gamma_{\mu} \nu_{\ell R}, \overline{Q}_{\tau} \gamma_{\mu} L$ | +1   | −1  |
| $(3, 3)_{2/3}$      | 1    | $\overline{Q}_{\tau} \gamma_{\mu} L$ | +1   | −1  |
| $(3, 2)_{1/6}$      | 1    | $\overline{\nu}_R \gamma_{\mu} \nu_{\ell R}, \overline{Q}_{\tau} \gamma_{\mu} L$ | +1   | −1  |
| $(3, 3)_{2/3}$      | 1    | $\overline{Q}_{\tau} \gamma_{\mu} L$ | +1   | −1  |
| $(3, 2)_{5/6}$      | 1    | $\overline{Q}_{\tau} \gamma_{\mu} L$ | +1   | −1  |

Table 1. Scalar and vector leptoquarks that trigger $b \to c \ell \bar{\nu}$ via renormalizable couplings.

Yukawa couplings of $\Delta$ to the SM fermions are

$$\mathcal{L} = \overline{\ell}_R Y \Delta^\dagger Q + \overline{u}_R Z \Delta^\dagger L + \text{H.c.},$$

where we have used $\Delta = i \tau_2 \Delta^*$ for the conjugated state. Transition to the mass basis splits Yukawa couplings of the weak doublets to two sets of couplings relevant for the upper and the lower doublet components. $Y$ and $Z$ in Eq. (2.1) are Yukawa matrices that differ by a relative rotation when applied to respective components. $Y$ represents couplings between charged leptons and down-type quarks while $Z$ connects up-type quarks with charged...
leptons. We use the basis where all relative rotations are assigned to neutrinos and up-type quarks and the transition to such basis is achieved by substituting $\nu_L \rightarrow V_{\text{PMNS}}^\dagger \nu_L$ and $u_L \rightarrow V_{\text{CKM}}^\dagger u_L$, where $V_{\text{PMNS}}$ and $V_{\text{CKM}}$ represent Pontecorvo-Maki-Nakagawa-Sakata (PMNS) and Cabibbo-Kobayashi-Maskawa (CKM) mixing matrices, respectively. It is in this basis that we unambiguously define $Y$ and $Z$. The two components of colored scalar, i.e., $\Delta^{(2/3)}$ and $\Delta^{(5/3)}$, then couple as

$$L^{(2/3)} = (\bar{\ell} R Y d_L) \Delta^{(2/3)*} + (\bar{u}_R Z \nu_L) \Delta^{(2/3)} + \text{H.c.}, \quad (2.2)$$

$$L^{(5/3)} = (\bar{\ell} R Y^\dagger_{\text{CKM}} u_L) \Delta^{(5/3)*} - (\bar{u}_R Z \ell_L) \Delta^{(5/3)} + \text{H.c.}. \quad (2.3)$$

It is non-existence of a diquark bilinear with hypercharge $Y = 7/6$ that makes it possible to assign lepton and baryon number consistently to $\Delta$.

In the first two generations of quarks and leptons the flavor changing processes are well fitted with CKM and PMNS parameters. This agreement is not disturbed by the leptoquarks if we introduce the minimal set of couplings needed to explain the $b \rightarrow c\tau\bar{\nu}$ branching fraction. Hence, we only require nonzero coupling of $\Delta^{(2/3)}$ to $\bar{\tau}b$ but not to $\bar{b}\mu$ or $\bar{b}\ell$ as suggested by nonobservation of anomalies in $b \rightarrow c\ell\bar{\nu}$, with $\ell = e, \mu$. We also require that only $c$ quark but not $u$ or $t$ couples to neutrinos. These requirements yield

$$Y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \gamma_{33} \end{pmatrix}, \quad Z_{\text{PMNS}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2_{21} & 2_{22} \\ 0 & 0 & 0 \end{pmatrix}. \quad (2.4)$$

The $\Delta^{(5/3)}$ Yukawa couplings are related to the above ones through CKM and PMNS rotations that induce CKM-suppressed couplings of $\tau$ to up-type quarks and PMNS-rotated couplings of $c$ quarks to charged leptons. We have

$$Y V_{\text{CKM}}^\dagger = \gamma_{33} \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{array} \right), \quad Z = \left( \begin{array}{ccc} 0 & 0 & 0 \\ \tilde{2}_{21} & \tilde{2}_{22} & \tilde{2}_{23} \\ 0 & 0 & 0 \end{array} \right), \quad (2.5)$$

where $\tilde{2}_{2j}$ are linear combinations of $z_{2j}$ with $O(1)$ coefficients related to the PMNS matrix elements. One can, at this stage, regard null-entries in $Y$ and $Z$ to be sufficiently small numbers that can thus be neglected in subsequent analyses. We will show later, in Section 7, that this ansatz is indeed compatible with an $SU(5)$ framework. Also, if the Yukawa couplings at the low energy scale are not too large, the ansatz will be preserved at the high energy scale associated with the scale where gauge couplings unify. This argument, of course, works both ways.

3 The $(3,2)_{7/6}$ contribution in $b \rightarrow c\tau\nu$

The observed anomaly in $R_{\tau/\ell}$ and $R'_{\tau/\ell}$ ratios has been subject of many studies \cite{18–26}. Variety of considered NP scenarios reduces to the effective Lagrangians in which either new vector/axial-vector and tensor currents, or (pseudo)scalar density operators are responsible for the measured discrepancy. There are additional observables that could
help single out the class of NP operators preferred by the data [19]. In all these analyses the effective operator contribution was included into decay amplitude on individual basis. The model of LQ mediation we consider here results in scalar/pseudoscalar and tensor contributions simultaneously. Namely, the relevant effective Hamiltonian for semileptonic $b \to c$ transition induced by the $(3, 2)_{7/6}$ state is

$$
\mathcal{H}^{(2/3)} = \frac{y_{33} \gamma_{2b}}{2 m_{\Delta}^2} \left[ (\bar{\tau}_R \nu_L)(\bar{\epsilon}_R b_L) + \frac{1}{4} (\bar{\tau}_R \sigma_{\mu \nu} \nu_L)(\bar{\epsilon}_R \sigma_{\mu \nu} b_L) \right], 
$$

(3.1)

where $m_{\Delta}$ is the mass of the LQ component with charge $|Q| = 2/3$ and is also defined as a matching scale for the above Hamiltonian. (In what follows we assume that $\Delta^{(2/3)}$ and $\Delta^{(5/3)}$ are degenerate in mass.) This means that the appropriate Wilson coefficients of scalar and tensor operators, $g_S$ and $g_T$, are uniquely determined and correlated. The above leptoquark effective Hamiltonian will affect semileptonic decays with the tau lepton, but contrary to the SM the final state neutrino is not necessarily a $\bar{\nu}_\tau$. The most natural mechanism to enhance $b \to c\tau\bar{\nu}_\tau$ is to have a constructive interference between the SM and the LQ amplitudes of $b \to c\tau\bar{\nu}_e$, whereas pure leptoquark contributions, producing $\bar{\nu}_e$ and $\bar{\nu}_\mu$ are negligible. This implies that we employ a Hamiltonian that includes the SM as well as the LQ contribution to $b \to c\tau\bar{\nu}_\tau$ decay

$$
\mathcal{H} = \frac{4 G_F}{\sqrt{2}} V_{cb} \left[ (\bar{\tau}_L \gamma^\mu \nu_L)(\bar{\epsilon}_L \gamma^\mu \mu b_L) + g_S (\bar{\tau}_R \nu_L)(\bar{\epsilon}_R b_L) + g_T (\bar{\tau}_R \sigma_{\mu \nu} \nu_L)(\bar{\epsilon}_R \sigma_{\mu \nu} b_L) \right], 
$$

(3.2)

where the scalar and tensor effective couplings are related to the underlying Yukawa couplings at the matching scale

$$
g_S(m_{\Delta}) = 4 g_T(m_{\Delta}) \equiv \frac{1}{4} \frac{y_{33} \gamma_{23}}{2 m_{\Delta}^2} \frac{\sqrt{2}}{G_F V_{cb}}. 
$$

(3.3)

Hadronic (pseudo)scalar and tensor operators in Eq. (3.1) have anomalous dimensions in QCD and dependence of their matrix elements on the renormalization scale is canceled by the scale dependence of the Wilson coefficients at the leading logarithm approximation,

$$
g_S(m_b) = \left( \frac{\alpha_S(m_b)}{\alpha_S(m_t)} \right)^{-\frac{\gamma_S}{2 \log(\mu/m_b)}} \left( \frac{\alpha_S(m_t)}{\alpha_S(m_{\Delta})} \right)^{-\frac{\gamma_S}{2 \log(\mu/m_t)}} g_S(m_{\Delta}),
$$

$$
g_T(m_b) = \left( \frac{\alpha_S(m_b)}{\alpha_S(m_t)} \right)^{-\frac{\gamma_T}{2 \log(\mu/m_b)}} \left( \frac{\alpha_S(m_t)}{\alpha_S(m_{\Delta})} \right)^{-\frac{\gamma_T}{2 \log(\mu/m_t)}} g_S(m_{\Delta}).
$$

(3.4)

Anomalous dimension coefficients are $\gamma_S = -8$, $\gamma_T = 8/3$ and coefficient $\beta^{(f)}_0 = 11 - 2/3 n_f$, where $n_f$ is a number of active quark flavours [27, 28]. The relation between the Wilson coefficients, given in Eq. (3.3), is valid at matching scale $m_{\Delta}$ which we set to the reference mass of $m_{\Delta} = 500$ GeV. The coefficients are then run to the beauty quark scale, i.e., $\mu = m_b = 4.2$ GeV, at which the matrix elements of hadronic currents are calculated. Difference between running of $g_S$ and $g_T$ modifies the matching scale relation (3.3) to

$$
g_T(m_b) \simeq 0.14 g_S(m_b)
$$

(3.5)
3.1 $B \to D\tau\nu$

The exclusive decay amplitudes for $B \to D\tau\nu$ transition contain the hadronic matrix element of the vector current, conventionally parametrized by $f_+(q^2)$ and $f_0(q^2)$ form factors

$$
\langle D(p_D)|\bar{c}\gamma^\mu b|B(p_B)\rangle = \left(p_B^\mu + p_D^\mu - \frac{m_B^2 - m_D^2}{q^2}q^\mu\right)f_+(q^2) + \frac{m_B^2 - m_D^2}{q^2}q^\mu f_0(q^2),
$$

(3.6)

where $p_B^\mu, p_D^\mu$ are four vectors of momenta of $B$ and $D$ mesons and $q^\mu = p_B^\mu - p_D^\mu$. The presence of the tensor operator in Eq. (3.2) requires inclusion of an additional form factor $f_T(q^2)$, defined as

$$
\langle D(p_D)|\bar{c}\sigma^{\mu\nu}b|B(p_B)\rangle = -i(p_B^\mu p_D^\nu - p_D^\mu p_B^\nu)f_T(q^2)\frac{2f_T(q^2)}{m_B + m_D}.
$$

(3.7)

As usual, the scalar matrix element is related to $f_0(q^2)$ form factor $\langle D|\bar{c}b|B\rangle = \frac{m_B^2 - m_D^2}{m_b - m_c}f_0(q^2)$, where $m_b$ and $m_c$ are masses of $b$ and $c$ quarks in $\overline{MS}$ scheme at the scale $\mu = m_b$ [22]. The differential branching ratio can be calculated from the formula

$$
\frac{d\mathcal{B}}{dq^2}(B \to D\tau\nu) = \frac{\tau_B G^2_F|V_{cb}|^2}{192\pi^3 m_B^3} f_+(q^2) \left(1 - \frac{m_B^2}{q^2}\right) \frac{\lambda^{1/2}}{\lambda^2} \left[1 + \frac{m_B^2}{q^2} \left(1 + \frac{m_B^2}{2q^2}\right) - \lambda^2 \frac{6m_B}{m_B + m_D} R(q^2) f_T(q^2) \right]
$$

where $\lambda$ denotes the function $\lambda(m_B^2, m_D^2, q^2) = (m_B^2 - m_D^2 - q^2)^2 - 4m_B^2 q^2$. The constant value of the ratio $f_T(q^2)/f_+(q^2) = 1.03(1)$, used in the branching ratio in Eq. (3.8), is evaluated in the model of Ref. [29]. In the heavy quark limit this ratio is $f_T(q^2)/f_+(q^2) = 1$, as the form factors are equally related to the Isgur-Wise function, $f_+(q^2) = f_T(q^2) = \frac{m_B^2 + m_D^2}{2\sqrt{m_B m_D}} \xi(w)$. We employ the following parametrization of vector form factors [30–32]

$$
\begin{align*}
\frac{f_+(q^2)}{R_D} &= G_1(w)|_{w(q^2)}, \\
\frac{f_0(q^2)}{R_D} &= R_D \frac{1 + w}{2} G_1(w) \frac{1 + r_D}{1 - r_D} \Delta(w)|_{w(q^2)},
\end{align*}
$$

(3.9)

where the constant $R_D$ and the new kinematic variable $w$ are given as

$$
R_D = \frac{2\sqrt{m_B m_D}}{m_B + m_D}, \quad w = \frac{m_B^2 + m_D^2 - q^2}{2m_B m_D},
$$

(3.10)

and $r_D$ denotes the ratio of masses of $D$ and $B$ mesons. Lattice estimate of the function $\Delta(w)$ is consistent with constant value $\Delta(w) = 0.46 \pm 0.02$ [33]. Function $G_1(w)$ can be found in Appendix A.
Figure 1. Values of the scalar Wilson coefficient $g_S(m_b)$ ($g_T(m_b) \simeq 0.14 g_S(m_b)$) consistent at 2$\sigma$ with BaBar Collaboration’s measurements of ratios $R(D)$ (bright ring) and $R(D^*)$ (darker ring). The 1$\sigma$ (2$\sigma$) region, fitted to the two constraints, is doubly (singly) hatched.

3.2 $B \to D^*\tau\bar{\nu}$

The decay $B \to D^*\tau\bar{\nu}$ offers additional tests of the SM and NP due to the richer spin structure of the final state particles [19]. Namely, in the case of light lepton in the final state, one vector and two axial form factors are present in the decay amplitude, while with $\tau$ in the final state an additional form factor $A_0(q^2)$ appears. The mediation of the $(3,2)_{7/6}$ leptoquark induces effective Lagrangian containing the tensor operator that requires knowledge of tensor form factors. Following notation of Ref. [19], the polarization four-vectors of the final state leptons and $D^*$ vector meson are denoted by $\tilde{\epsilon}_\mu(\lambda)$ and $\epsilon_\mu(\lambda_{D^*})$, respectively. Polarizations take the following values: $\lambda = 0, \pm, t$ and $\lambda_{D^*} = \pm, 0$. Here $t$ denotes the time-like polarization vector. Standard parametrization of vector and axial hadronic matrix elements for the $B \to D^*$ transition is given by

$$\langle D^*(p_{D^*}, \epsilon) | \bar{c} \gamma_\mu b | B(p_B) \rangle = \frac{2iV(q^2)}{m_B + m_{D^*}} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_B^\alpha p_{D^*}^\beta, \quad (3.11a)$$

$$\langle D^*(p_{D^*}, \epsilon) | \bar{c} \gamma_\mu \gamma_5 b | B(p_B) \rangle = 2m_{D^*} A_0(q^2) \frac{\epsilon^* \cdot q}{q^2} q_\mu + (m_B + m_{D^*}) A_1(q^2) \left( \epsilon^*_\mu - \frac{\epsilon^* \cdot q}{q^2} q_\mu \right)$$

$$- A_2(q^2) \frac{\epsilon^* \cdot q}{m_B + m_{D^*}} \left( (p_B + p_{D^*})_\mu - \frac{m_B^2 - m_{D^*}^2}{q^2} q_\mu \right). \quad (3.11b)$$

For the parametrization of the tensor hadronic matrix elements, we adopt the form found
in Ref. \[18\] that reads

\[
(D^*(p_{D*}), \epsilon^*|\bar{c}\sigma_{\mu\nu}(1 - \gamma_5)b|\bar{B}(p_B)) = T_0(q^2) + \frac{\epsilon^* \cdot q}{(m_B + m_{D*})^2} \epsilon_{\mu\nu\alpha\beta} p_B^\alpha p_{D*}^\beta + T_1(q^2) \epsilon_{\mu\nu\alpha\beta} p_B^\alpha \epsilon^*_{\mu
u} + i \left[ T_3(q^2)(\epsilon^*_\mu p_B_{\nu,\mu} - \epsilon^*_\nu p_B_{\mu,\mu}) + T_4(q^2)(\epsilon^*_\mu p_{D*\nu,\nu} - \epsilon^*_\nu p_{D*\mu,\mu}) + T_5(q^2) \frac{\epsilon^* \cdot q}{(m_B + m_{D*})^2} (p_B_{\mu,\mu} p_{D*\nu,\nu} - p_{B,\nu} p_{D*\mu,\mu}) \right].
\]

Vector and axial form factors in the heavy quark (HQ) limit are related to function \(h_{A_1}(w)\) \[34\]

\[
A_0(q^2) = \frac{R_0(w)}{R_{D*}} h_{A_1}(w)|_{w(q^2)}, \quad A_1(q^2) = \frac{w + 1}{2} R_{D*} h_{A_1}(w)|_{w(q^2)},
\]

\[
A_2(q^2) = \frac{R_2(w)}{R_{D*}} h_{A_1}(w)|_{w(q^2)}, \quad V(q^2) = \frac{R_1(w)}{R_{D*}} h_{A_1}(w)|_{w(q^2)}.
\]

Functions \(R_i(w)\) and \(h_{A_1}(w)\) can be found in Appendix A. The tensorial form factors from the parametrization \((3.12)\) in the HQ limit are related to the function \(h_{A_1}(w)\) \[18, 35\]

\[
T_0(q^2) = T_5(q^2) = 0,
\]

\[
T_1(q^2) = T_3(q^2) = \sqrt{\frac{m_{D*}}{m_B}} h_{A_1}(w)|_{w(q^2)},
\]

\[
T_2(q^2) = T_4(q^2) = \sqrt{\frac{m_B}{m_{D*}}} h_{A_1}(w)|_{w(q^2)}.
\]

Using the effective Hamiltonian \((3.2)\), one finds that the contributing vector and axial-vector, pseudo-scalar and tensor amplitudes are given in terms of corresponding hadronic and leptonic helicity amplitudes \[18, 19, 25\]:

\[
A_{V, A, A, A}^{\lambda, \lambda, \alpha, \lambda} = \sum_{\lambda, \lambda'} \eta_{\lambda, \lambda'} H_{V, A, A, A}^{\lambda, \lambda, \alpha, \lambda},
\]

\[
A_P^{\lambda, \lambda, \alpha, \lambda} = g_P H_P^{\lambda, \lambda, \alpha, \lambda},
\]

\[
A^{\lambda, \lambda, \alpha, \lambda} = gT \sum_{\lambda, \lambda'} \eta_{\lambda, \lambda'} H^\lambda_{T, \lambda, \lambda}, L^\lambda_{T, \lambda, \lambda}.
\]

The metric factor \(\eta\) has values \(\eta_{\pm, 0} = 1\) and \(\eta_T = -1\) \[36\]. Hadronic and leptonic helicity amplitudes are defined in Appendix A.

The branching ratio is calculated after integrating the following formula over \(q^2\) and angle \(\theta_l\) \[36\]:

\[
dB = \frac{\tau_B G_F^2 |V_{cb}|^2}{(8\pi)^3 m_B^2 |p_{D*}|} \left( 1 - \frac{m_B^2}{q^2} \right) \sum_{\lambda, \lambda', \lambda''} |A^{\lambda, \lambda, \alpha, \lambda''}|^2 dq^2 d\cos\theta_l.
\]

We constrain the allowed values of tensor and scalar Wilson’s coefficients using BaBar’s measurements of the ratios \(R_{\tau/\ell} = B(B \to D\tau\nu)/B(B \to D\ell\nu)\) \(\left( R_{\tau/\ell} = B(B \to D^*\tau\nu)/B(B \to D^*\ell\nu) \right)\) as shown in Fig. 1, where also the result of the fit to both ratios is shown. We derive 1\(\sigma\) range for the Wilson coefficient \(g_S\) at the low scale

\[
g_S(m_b) = -0.37^{+0.10}_{-0.07},
\]
where we have assumed $g_S$ to be real in estimating the error bars. The coupling $g_S$ at the matching scale, defined in Eq. (3.3), is rescaled by factor 0.64 with respect to the above value due to QCD corrections (3.4).

4 $Z \to b\bar{b}$

The LEP experiment measured precisely decay modes of $Z$ bosons to $f\bar{f}$ pairs. Standard parameterization of the $Zb\bar{b}$ renormalizable coupling is adopted

$$L_{Zb\bar{b}} = \frac{g}{c_W} Z^\mu \gamma_\mu \left[(g_L^b + \delta g_L^b) P_L + (g_R^b + \delta g_R^b) P_R\right] b.$$  (4.1)

$SU(2)$ coupling is denoted by $g$, $c_W$ is the cosine of the Weinberg angle and $P_{L,R} = (1 \pm \gamma_5)/2$ are the chiral projectors. At the SM tree-level, the couplings are $g^0_L = -1/2 + s_W^2/3$ and $g^0_R = s_W^2/3$. Higher-order electroweak corrections that are contained within $g^0_{L,R}$ get largest contributions from top quark in loops. A recent electroweak fit that includes updated theoretical predictions and new results from LHC points to tensions in the $Z \to b\bar{b}$ observables reaching above 2σ significance in $R_b$ and $A_{FB}^b$ [37] (see also [38–43]). The shifts with respect to the SM values of couplings are found to be

$$\delta g^b_L = 0.001 \pm 0.001, \quad \delta g^b_R = (0.016 \pm 0.005) \cup (-0.17 \pm 0.005).$$  (4.2)

$\Delta^{(2/3)}$ component possesses a possibly large coupling $y_{33}$ to $b\tau$ pair that contributes to $Z \to b\bar{b}$ amplitude at order $|y_{33}|^2$ and thus allows one to constrain it directly. The LQ correction to the left-handed coupling is

$$\delta g^b_L(y_{33}) = \frac{|y_{33}|^2}{384\pi^2} \left[g_0(x) + s_W^2 g_2(x)\right],$$  (4.3)

where $x = m_\Delta^2/m_Z^2$, and we take $m_Z = 91.2$ GeV and $s_W^2 = 0.231$. Asymptotic expansion of functions $g_0$ and $g_2$ at large $x$ is

$$g_0(x) \simeq -\frac{2}{3x},$$
$$g_2(x) \simeq \frac{8}{x}(\log x + 2/9 + i\pi),$$

while their full expressions can be found in Appendix B. Although $\delta g^b_L(y_{33})$ develops an imaginary part due to on-shell $\tau$ leptons, the constraint from the electroweak fit (4.2) is sensitive to the interference term between the approximately real $g^b_L$ and complex $\delta g^b_L(y_{33})$, and therefore only the real part of $\delta g^b_L(y_{33})$ enters the prediction. The constraint

$$\text{Re}[\delta g^b_L(y_{33})] = 0.001 \pm 0.001,$$  (4.4)

is shown in the $m_\Delta - |y_{33}|$ plane on Fig. 2. In the range of $m_\Delta$ shown there we approximate the central value (thick line) by a linear function that is accurate to within 5%:

$$|y_{33}|_{\text{central}} = 1.57 + 2.86 \frac{m_\Delta}{500 \text{ GeV}},$$  (4.5)
For $m_\Delta$ above 300 GeV large portion of preferred $|y_{33}|$ range lies within the nonperturbative regime that is situated above a bright dashed line in Fig. 2. In order to maintain a predictable setup we assume that coupling $y_{33}$ is perturbative, i.e., $|y_{33}| < \sqrt{4\pi}$. Contributions to $\delta g_R^b$ in this scenario are further suppressed by $m_b^2/m_\Delta^2$ and can be safely neglected. Later on, when we discuss the grand unified theory (GUT) embedding, we will find that the perturbativity of Yukawa couplings all the way to the scale of unification puts more stringent upper bound on $|y_{33}|$ at the low energy scale.

On the other hand, corrections to the self-energies of electroweak gauge bosons that are measured by the oblique parameters $S$, $T$, and $U$ do not enforce relevant constraints on the LQ Yukawa couplings, as long as the two mass eigenstates are approximately degenerate [44].

We have also checked what further information on the coupling $y_{33}$ could be extracted from atomic parity violation experiments. In our model the ansatz for couplings ensures there are no tree-level contribution to parity-odd dimension-6 operators with flavor $\bar{u}ue^+e^-$ that can be generated with the leptoquark only through loop-induced $Z\bar{u}u$ vertex. One can see from Eq. (2.5) that loop diagrams with $\Delta(5/3)$ and $\tau$ leading to $Z \to \bar{u}u$ are suppressed by $|V_{ub}|^2$ and it is due to this suppression that the constraints on $Z\bar{u}u$ vertex, presented in [45], play no relevant role in constraining the considered model.

## 5 Lepton electromagnetic moments

Three form factors encompass the structure of $\ell\ell\gamma$ vertex and generalize the tree-level QED vertex (see, for example, Eq. (17) of Ref. [46] and Eq. (2.2) in Ref. [47])

$$-ie\bar{u}_\ell(p+q)\gamma^\mu u_\ell(p) \to -ie\bar{u}_\ell(p+q) \left[ F_{E}(q^2)\gamma^\mu + \frac{F_{M}(q^2)}{2m_\ell}q_\mu + F_{d}(q^2)\sigma_{\mu\nu}q_\nu\gamma_5 \right] u_\ell(p).$$

(5.1)
The lepton vertex with electromagnetic field will be modified by penguin diagrams involving virtual exchanges of $\Delta^{(5/3)}$ and charm quark. For the muon the $Z$ couplings in Lagrangian (Eqs. (2.3) and (2.5)) will contribute to the magnetic moment $F_M(q^2)$, but not to the electric dipole moment $F_\ell_d(q^2)$ which is a CP violating quantity and requires two different couplings with different complex phases. For the $\tau$, however, there are two distinct couplings—$Y_{\text{CKM}}$ and $Z$—at our disposal that are sufficient to generate the electric dipole moment (EDM).

### 5.1 Muon ($g - 2$)

The two penguin diagrams and field renormalization factors are calculated employing the Feynman rules listed in Tab. 2. We decompose the amplitude as in Eq. (5.1) and identify $F_M(q^2)$. Then we set $q^2 = 0$ and expand $F_M(0)$ to first order in $m_\mu^2$. The muon anomalous moment gets shifted due to new contribution of $\Delta^{(5/3)}$ by

$$
\delta a_\mu \equiv F_M^\mu(q^2 = 0) = -\frac{N_c |\tilde{z}_{22}|^2 m_\mu^2}{16\pi^2 m_\Delta} [Q_c F_q(x) + Q_\Delta F_\Delta(x)] ,
$$

where $x = m_c^2/m_\Delta^2$ and $N_c = 3$. Functions $F_q(x)$ and $F_\Delta(x)$ are

$$
F_q(x) = \frac{x^3 - 6x^2 + 3x + 6x \log x + 2}{6(x - 1)^4} ,
$$

$$
F_\Delta(x) = \frac{2x^3 + 3x^2 - 6x^2 \log x - 6x + 1}{6(x - 1)^4} .
$$

The above formulas agree with Ref. [48]. Comparing the prediction of $\delta a_\mu$ with the value reported by the PDG [49],

$$
\delta a_\mu^{\text{exp-SM}} = (287 \pm 80) \times 10^{-11} ,
$$

we observe that $a_\mu$ is pulled further away from the experimental value. The best fit point corresponds to the SM limit, and is 3.6 $\sigma$ below the measured value ($\chi^2_{\text{SM}} = 12.87$). We determine the allowed 1 $\sigma$ range from the condition $\chi^2 - \chi^2_{\text{SM}} \leq 1$ that translates to a constraint

$$
|\delta a_\mu| < 10.9 \times 10^{-11} ,
$$

or put in terms of $\tilde{z}_{22}$ (see Fig. 3)

$$
|\tilde{z}_{22}| < 0.51 \frac{m_\Delta}{500 \text{ GeV}} .
$$

This bound will also need to be reconsidered in view of the GUT embedding we present in Section 7. It can, however, be considered as a correct upper limit if one discusses simple extension of the SM with additional LQ with relevant couplings given in Eqs. (2.1) and (2.4).
5.2 τ electric dipole moment

Penguin diagrams where $\Delta(5/3)$ scalar couples to fermions with different couplings may generate EDM. In this section we focus exclusively on contributions proportional to product of $Y V_{CKM}^\dagger$ and $Z$. The two couplings have opposite chiralities and at least one helicity flip, proportional to mass, is needed on the internal charm propagator. As a result both penguin diagrams are finite and result in the following expression for the $\tau$ EDM

$$d_\tau \equiv e F_\tau^c (q^2 = 0) = e \frac{m_c \text{Im}[V_{cb} y_{23}^* z_{23}]}{32 \pi^2 m_\Delta^2} \left[ 1 + 4 \log \frac{m_\Delta^2}{m_\tau^2} \right]. \quad (5.6)$$

At the moment, the best bounds from Belle experiment are orders of magnitude too weak to directly probe the parameter range, preferred by $B \rightarrow D(\ast)\tau\bar{\nu}$ [50]. From the 90% confidence level range $-1.5 \times 10^3 < \text{Im}[V_{cb} z_{23} y_{33}^*] < 3.0 \times 10^3$, valid at a typical mass $m_\Delta = 500$ GeV. Of course, couplings of these magnitudes are meaningless. However, we can turn around the reasoning and require that all of them stay perturbative. Then we find from $|z_{23} y_{33}^*| < 4\pi$ that the upper bound on tau EDM in the perturbative setting is $|d_\tau| < 2.6 \times 10^{-21} e$ cm.

6 $\ell \rightarrow \ell'\gamma$ decays

The decay $\tau \rightarrow \ell\gamma$ is mediated by a loop diagram with $\Delta(5/3)$ scalar and a charm quark. The electromagnetic dipole transition amplitude can be written as a sum over two photon polarization amplitudes

$$A_{\tau \rightarrow \ell\gamma} = \ell(p')\sigma^{\mu\nu} \epsilon_\mu(q) q_\nu \left( A_{\tau\ell} P_R + B_{\tau\ell} P_L \right) \tau(p), \quad (6.1)$$

with $q = p - p'$ and $\epsilon$ the polarization of the photon. Polarization-averaged branching ratio is then

$$B(\tau \rightarrow \ell\gamma) = \frac{\tau_\tau}{16\pi} \left( \frac{m_\tau^2 - m_\ell^2}{m_\tau^3} \right)^3 \left( |A_{\tau\ell}|^2 + |B_{\tau\ell}|^2 \right).$$
Expressed in terms of the underlying couplings and masses, and to leading order in $m_c$ and $m_\tau$, the polarization amplitudes are

\begin{align}
A_{\tau\ell} &= \frac{-N_c e}{48\pi^2 m_\Delta} \left[ m_c V_{cb} y_{33}^\ast \tilde{z}_2^\ast \Delta \left(1 + 4 \log x_c\right) + \frac{m_\tau}{2} \tilde{z}_{22}^\ast \tilde{z}_{23}^\ast (3 + 4 x_c \log x_c) \right], \quad (6.2) \\
B_{\tau\ell} &= 0, \quad (6.3)
\end{align}

where $x_c = m_c^2 / m_\Delta$. To connect the two vertices with opposite chiralities of quarks (couplings $V_{cb} y_{33}$ and $\tilde{z}_{2\ell}$) a helicity flip on the charm quark propagator is required. If one considers a contribution proportional to $\tilde{z}_{22} \tilde{z}_{23}$ we need a helicity flip on the lepton legs, where the $\tau$ mass insertion contributes to $A_{\tau\ell}$ while the contribution of $\mu$ mass insertion to $B_{\tau\ell}$ is negligible. The decay $\mu \to e\gamma$ proceeds exclusively through the second term in (6.2) and is relatively suppressed by a factor $m_\mu / m_\tau$.

\begin{align}
A_{\mu e} &= \frac{-N_c e m_\mu}{96\pi^2 m_\Delta} \tilde{z}_{22}^\ast \tilde{z}_{21}^\ast (3 + 4 x_c \log x_c), \quad (6.4)
\end{align}

whereas $B_{\mu e}$ is rendered negligible due to helicity flip on the electron leg.

The best experimental bounds on LFV radiative decays of $\tau$ were presented by BaBar collaboration in Ref. [51]. At 90% confidence level they read

\begin{align}
B(\tau \to e\gamma) < 3.3 \times 10^{-8}, \quad B(\tau \to \mu\gamma) < 4.4 \times 10^{-9}, \quad (6.5)
\end{align}

and severely constrain the combination of couplings, present in Eq. (6.2). The experimental upper limits for $\mu \to e\gamma$ branching ratio are orders of magnitude more stringent and in conjunction with the small width of the muon they compensate the for the $m_\mu$ suppression in sensitivity. We rely on the latest result from the MEG experiment obtained from data collected in years 2009–2011 [52],

\begin{align}
B(\mu \to e\gamma) < 5.7 \times 10^{-13}, \quad \text{at 90\% C.L.} \quad (6.6)
\end{align}

These constraints and our Yukawa ansatz will both be interpreted in the GUT framework we introduce next.

### 7 (3, 2)_{7/6} in GUT framework

The most natural setting for the scalar leptoquark $\Delta \equiv (3, 2)_{7/6}$ is within a framework of matter unification [16, 53]. For example, if we resort to a language of $SU(5)$ we can show that $\Delta$ with couplings to the SM fermions can be found in 45- and 50-dimensional representations of that group [54]. However, only 45-dimensional representation can simultaneously generate both types of couplings in Eq. (2.1). If, on the other hand, $\Delta$ originates solely from 50-dimensional representation we could reproduce only those couplings of $\Delta$ to matter that are proportional to the $Y$ matrix entries. It is the former scenario that we will thoroughly study later on.

One might prefer to discuss the origin of $\Delta$ within an $SO(10)$ framework. In the $SO(10)$ setup the relevant representations that couple to matter and contain $\Delta$ are 120- or 126-dimensional ones. In particular, there is one scalar found in both 120- and 126-dimensional
representations of $SO(10)$ that couples to the matter as if it is $\Delta$ from 45-dimensional representation of $SU(5)$. $\Delta$ from 50-dimensional representation of $SU(5)$, on the other hand, can only be embedded into 126-dimensional representation of $SO(10)$. Either way, the $SO(10)$ origin of $\Delta$ would be imprinted on its couplings to matter in a flavor basis on top of the $SU(5)$-like behavior. Namely, it is well-known that 120- and 126-dimensional representations of $SO(10)$ couple to the matter antisymmetrically and symmetrically, respectively. The $\Delta$ couplings to matter would simply inherit these properties.

Let us now look in detail whether a particular low-energy ansatz with Yukawa couplings, as given in Eq. (2.1), is compatible with the idea of grand unification. In what follows we exclusively use the language of $SU(5)$ to specify relevant operators. In view of the comments put forth in the previous paragraph it is easy to see that a switch to the $SO(10)$ framework is straightforward, if and when needed.

We take $\Delta$ to originate from 45-dimensional scalar representation. Again, this is the only option available to generate the LQ interactions proportional to the $Z$ coupling matrix. The same scalar representation is actually needed to generate viable fermion masses. It is this fact that allows one to relate the LQ couplings to fermion masses as we show later in this section. Realistic charged fermion masses require presence of an additional 5-dimensional scalar representation $[11]$. We accordingly assume presence of both scalar representations and refer to them as $5$ and $45$ using their dimensionality. We furthermore denoted vacuum expectation values (VEVs) of neutral components of Higgs doublets in $5(\equiv 5^{\alpha})$ and $45(\equiv 45_{\alpha\beta\gamma})$ with $v_5$ and $v_{45}$, respectively. Here, $\alpha, \beta, \gamma(=1, \ldots, 5)$ represent $SU(5)$ indices. Of course, to address observed masses of charged leptons and down-type quarks both VEVs are needed. Our normalization is such that $|v_5|^2/2 + 12|v_{45}|^2 = v^2$, where $v(=246$ GeV$)$ is the electroweak VEV. In other words, we take $\langle 5^5 \rangle = v_5/\sqrt2$ and $\langle 45^{15} \rangle = \langle 45^{25} \rangle = \langle 45^{35} \rangle = v_{45}/\sqrt2$ $[55]$.

We turn our attention to the $\Delta$ couplings to matter that are proportional to $Z$. The $SU(5)$ contractions relevant for these Yukawa couplings are

\begin{align}
(Y_1)_{ij}10,5,45, \quad (Y_2)_{ij}10,5,5, \quad (Y_3)_{ij}10,5,5, \quad (Y_4)_{ij}10,5,5
\end{align}

where $10$ and $5$ together comprise an entire generation of fermions $[16]$. $Y_1$ and $Y_3$ are, at this stage, arbitrary $3 \times 3$ matrices in flavor space with $i,j (=1,2,3)$ being corresponding generation indices.

We denote unitary transformations of the down-type quark fields to be $D_L$ and $D_R$, where subscripts $L$ and $R$, from now on, correspond to redefinitions of the left- and right-handed fields, respectively. These rotations take the down-type quark fields from a flavor into a mass eigenstate basis. For the up-type quark (charged lepton) sector we similarly adopt $U_L$ and $U_R$ ($E_L$ and $E_R$) to be appropriate unitary matrices. We further assume that neutrinos are Majorana particles and accordingly denote unitary matrix that defines the neutrino mass eigenstates with $N$. Exact mechanism of the neutrino mass generation is not important for our discussion.

The low-energy ansatz we want to implement in the $SU(5)$ framework is completely specified through the following set of transformations: $\nu_L \rightarrow V_{PMNS}\nu_L$ and $u_L \rightarrow V_{\text{CKM}}^T u_L$. 

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This implies that the unitary rotations of the left-handed down-type quarks \((D_L)\) and charged leptons \((E_L)\) are both diagonal unitary matrices at the \(SU(5)\) symmetry breaking scale. The same ansatz also fixes \(U_L\) and \(N\) in terms of low-energy observables \(V^{\dagger}_{\text{CKM}}\) and \(V_{\text{PMNS}}\), respectively. The entries of \(V^{\dagger}_{\text{CKM}}\) and \(V_{\text{PMNS}}\) are, on the other hand, well-measured observables. What is not \(a \text{ priori}\) known, however, are rotations in the right-handed fermion sector, i.e., \(U_R, D_R\) and \(E_R\). We demonstrate in what follows that all angles in \(U_R\), \(D_R\) and \(E_R\) are specified through the ansatz of Eq. (2.4), except for one angle in \(U_R\) matrix, within the proposed \(SU(5)\) framework. This behavior can be traced back to a restrictive form of \(Z\) couplings. Moreover, we show that the entries of the \(Z\) coupling matrix exhibit hierarchy that is similar to the mass hierarchy present in the charged lepton and the down-type quark sectors. To simplify our discussion, we take both these sectors to be real. This we do because low-energy phenomenology requires no phases whatsoever.

What we find, after we impose the ansatz of Eq. (2.1) on contractions in Eqs. (7.1) and (7.2) at the GUT scale, are the following two relations that connect fermion mass matrices of down-type quarks and charged leptons with the original Yukawa couplings:

\[
2M_D^{\text{diag}} D_R^T = -2Y_1 v_{45} - Y_3 v_5, \tag{7.3}
\]
\[
2E_R M_E^{\text{diag}} = 6Y_1 v_{45} - Y_3 v_5. \tag{7.4}
\]

Here, \(M_D^{\text{diag}}\) (\(M_E^{\text{diag}}\)) is a diagonal mass matrix for down-type quarks (charged leptons) and we take both \(v_5\) and \(v_{45}\) to be real. Note that the relations in question contain only the right-handed unitary transformations \(D_R\) and \(E_R\). We proceed by identifying a connection between \(Y_1\) and \(Z\) to be \(Y_1 = -U_R Z\). This, then, leads us to the following matrix equation

\[
M_D^{\text{diag}} D_R^T - E_R M_E^{\text{diag}} = 4U_R Z v_{45}. \tag{7.5}
\]

If \(Z\) that at the GUT scale has a form given in Eq. (2.5) is to satisfy this set of equations our ansatz would be compatible with the idea of \(SU(5)\) grand unification.

Let us count the number of relevant parameters in Eq. (7.5). Clearly, since \(Z\) has only one row of elements different from zero we have a situation where elements of only one column of \(U_R\) enter the matrix equation. The entries of that column can be parametrized with two angles we denote with \(\phi\) and \(\theta\). More specifically, we take \((U_R)_{21} = \sin \theta \cos \phi\), \((U_R)_{22} = \sin \theta \sin \phi\) and \((U_R)_{23} = \cos \theta\). As for \(D_R\) and \(E_R\) matrices, each one contains three angles. We accordingly parametrize them via angles \(\theta_D^i\) and \(\theta_E^i\), \(i = 1, 2, 3\), where we define generic orthogonal matrix \(O(\theta_1, \theta_2, \theta_3) = O_3(\theta_3) O_2(\theta_2) O_1(\theta_1)\) through

\[
O_3(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix}, \quad O_2(\theta) = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}, \quad O_1(\theta) = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{7.6}
\]

Of course, the elements of \(Z\) matrix, i.e., \(z_{21}, z_{22}\) and \(z_{23}\), are not fixed and we consider them to be free parameters as well. Note that they scale with a common factor \(v_{45}\). If \(\Delta\) is a mixture of states that reside in both 45- and 50-dimensional representations the common factor would then also contain relevant parameter that describes a level of that mixing. All in all, the number of real parameters is eleven whereas the number of equations is nine.
Eq. (7.5) should be satisfied at the scale of \( SU(5) \) unification. This scale depends on the details of the underlying model. To provide relevant input parameters we resort to a particular \( SU(5) \) scenario [13] that besides \( 10, 5, 45 \) and \( 5 \) encompasses two 24-dimensional representations. One of these is a scalar representation needed to break \( SU(5) \) symmetry in the usual manner. The other one is fermionic in nature and is required to generate realistic masses and mixing angles in the neutrino sector [56]. Unification within this setup has been extensively studied in Refs. [15, 57]. Our numerical input for the masses of down-type quarks and charged leptons at the scale of unification reads
\[
\begin{align*}
    m_d &= 0.00105 \text{ GeV}, \\
    m_s &= 0.0187 \text{ GeV}, \\
    m_b &= 0.782 \text{ GeV}, \\
    m_e &= 0.000435 \text{ GeV}, \\
    m_\mu &= 0.0918 \text{ GeV} \\
    m_\tau &= 1.56 \text{ GeV}.
\end{align*}
\]
(See Tab. VIII of Ref. [55] for details on how these input parameters are generated.) We stress that these masses do not vary significantly with respect to small changes in the scale of unification within this scenario. In fact, we obtain similar values for the model where fermionic 24-dimensional representation is replaced with 15-dimensional scalar representation [14].

What we find numerically is that there exists only one satisfactory solution to Eq. (7.5) that reads
\[
\begin{align*}
    v_{45}\tilde{z}_{21} &= 0.012 \text{ GeV}, \\
    v_{45}\tilde{z}_{22} &= 0.16 \text{ GeV}, \\
    v_{45}\tilde{z}_{23} &= 0.50 \text{ GeV}.
\end{align*}
\]
This solution corresponds to the following values of angles:
\[
\begin{align*}
    \theta^D_1 &= 2.807, \\
    \theta^D_2 &= 0.062, \\
    \theta^D_3 &= 0.942, \\
    \theta^E_1 &= 0.198, \\
    \theta^E_2 &= 0.038, \\
    \theta^E_3 &= -2.988, \\
    \theta &= 0.129 \quad \text{and} \quad \phi = -1.342.
\end{align*}
\]
The very existence of this fit implies that the simplest mechanism to generate viable fermion masses in the down-type quark and the charged lepton sectors in \( SU(5) \) is compatible with our ansatz. Moreover, the ansatz yields ratios between \( \tilde{z}_{21}, \tilde{z}_{22} \) and \( \tilde{z}_{23} \) that mimic mass hierarchy in the down-type quark and the charged lepton sectors with \( \tilde{z}_{23} \) being a dominant element. Namely, we find that
\[
\tilde{z}_{21} : \tilde{z}_{22} : \tilde{z}_{23} = 0.024 : 0.32 : 1.
\]
We stress that these results are completely independent from the up-type quark and the neutrino sectors, where CKM and PMNS mixing parameters reside, respectively.

The fact that the numerical solution is unique even though available parameters outnumber equations can be traced back to the hierarchical property of the down-type quark and the charged lepton sectors. Namely, to address substantial mass splittings for three generations one needs at least three non-zero entries in the \( Z \) matrix. We could thus argue that our ansatz belongs to a class of the most minimal ones that are still compatible with the \( SU(5) \) unification.

Note that we did neglect the influence of running on null entries in matrix \( Z \). The main reason we do that is because we do not \textit{a priori} know the strength of the Yukawa coupling entries in \( Z \) at the low energy scale. The running of Yukawa couplings and subsequent numerical fit would require iterative approach within very precisely defined model that is beyond the scope of this work. One should accordingly view our numerical analysis as a first approximation of the full-fledged model dependent study. However, we do find that for the values of \(|\tilde{z}_{2i}| \lesssim 0.5, i = 1, 2, 3\), at the low energy scale we preserve the perturbativity of Yukawa couplings all the way to the GUT scale while preserving the ansatz given in Eq. (2.4). Self-consistency of our analysis would thus require that we consider regime in which \( v_{45} > O(1) \) GeV. The same study yields that \(|y_{33}|\), at the low energy scale, should
be below 0.8 in order to preserve perturbativity. In fact, in the regime we advocate, the radiative corrections on null entries in $Z$ can indeed be neglected. Note that the estimate of the radiative correction effects is done under simplifying assumption that the only light degrees of freedom are the SM particles and the leptoquark $\Delta$.

The angle $\xi$ that enters $U_R = (O_2(\xi) O_3(\phi) O_1(\theta))^T$ cannot be deduced using the constraints imposed by the form of $Z$. We thus resort to proton decay signatures to see whether it could be fixed if and when proton decay is observed. We assume that proton decay is dominated by processes that involve exchange of gauge boson leptoquarks.

To predict gauge boson mediated proton decay signatures for two-body final states with good accuracy one needs to know all unitary transformations in the charged fermion sector, masses of all proton decay mediating gauge bosons and a gauge coupling constant. In $SU(5)$ the masses of LQ gauge bosons coincide with the scale of unification, i.e., GUT scale $m_{GUT}$ and the gauge coupling at that scale is $\alpha_{GUT} = g_{GUT}^2/(4\pi)$. Proton decay predictions in our setup hence depend on $m_{GUT}$, $\alpha_{GUT}$ and angle $\xi$. (What actually enters decay widths is ratio $\alpha_{GUT}/m_{GUT}^2$ and $\xi$.)

We can turn the argument around and use experimental limits on partial proton decay lifetimes to constrain the scale of unification. We present limits on $m_{GUT}$ as a function of $\xi$ in Fig. 4. The region below each curve is excluded by current experimental limit on corresponding process. To generate results shown in Fig. 4 we use the following experimental input on partial proton decay lifetimes: $\tau_{p \rightarrow \pi^0 e^+} > 1.3 \times 10^{34}$ years [58], $\tau_{p \rightarrow K^+ \bar{\nu}} > 4.0 \times 10^{33}$ years [59], $\tau_{p \rightarrow K^0 e^+} > 1.0 \times 10^{33}$ years [60], $\tau_{p \rightarrow \pi^0 \mu^+} > 1.1 \times 10^{24}$ years [58], $\tau_{p \rightarrow K^0 \mu^+} > 1.6 \times 10^{33}$ years [61] and $\tau_{p \rightarrow \pi^+ \bar{\nu}} > 3.9 \times 10^{32}$ years [62]. We take $\hat{\alpha} = -0.0112 \text{GeV}^3$ [63], where $\hat{\alpha}$ is the relevant nucleon matrix element. For the strength of unified gauge coupling we take $\alpha_{GUT} = 0.033$ and for the leading-log renormalization corrections of the relevant $d = 6$ operator coefficients we use $A_{SL} = 2.6$ and $A_{SR} = 2.4$ [15]. These values vary very slightly with a change in the field content of the particular $SU(5)$ scenario in a framework without supersymmetry. For example, the model with 15-dimensional scalar representation yields $\alpha_{GUT} = 0.031$, $A_{SL} = 2.8$ and $A_{SR} = 2.6$. The dependence of the decay widths on unitary transformations is taken from Ref. [64] and the setup on how to propagate $A_{SL}$ and $A_{SR}$ coefficients from the GUT scale down to the low-energy scale is described in Ref. [15]. Note that the lifetimes scale with $m_{GUT}^2$ to the fourth (second) power.

The reasons we present these predictions for proton decay are twofold. Firstly, this clearly demonstrates the level of predictivity of our ansatz. Secondly, and more importantly, this demonstrates that even the simplest possible ansatz can predict that for some portions of parameter space it is not $p \rightarrow \pi^0 e^+$ channel that dominates if one assume that the main contribution to proton instability comes from the gauge boson exchanges. It is clear from Fig. 4 that $p \rightarrow \pi^0 e^+$ is suppressed with respect to $p \rightarrow K^+ \bar{\nu}$ and even $p \rightarrow K^0 e^+$ in some parts of available parameter space, contrary to what is commonly stated in the literature. We can also see in Fig. 4 that the proton decay signature for $p \rightarrow \pi^+ \bar{\nu}$ process is completely rotated away for one particular value of $\xi$. The possibility that this sort of suppression of individual decay modes could take place has been discussed before [65, 66]. These findings only confirm that proton decay represents fertile, yet treacherous, ground to test Yukawa
Figure 4. Limits on the unification scale $m_{\text{GUT}}$ as a function of angle $\xi$ as inferred from the latest experimental constraints on two-body proton decay. The region below each curve represents parameter space that is excluded by corresponding experimental limit.

sector of the theory [67].

Our analysis is performed for the proton decay signatures due to a tree-level exchange of gauge bosons. We are potentially in a position to present similar analysis for the two-body proton decay signatures due to scalar exchange. For example, we know how proton mediating leptoquarks in 45-dimensional representation couple to the down-type quark and the charged lepton sectors, up to angle $\xi$ and the overall scale parameter $v_{45}$, i.e., the VEV of 45. We accordingly point out one particular property of our ansatz with regard to the proton decay signatures through scalar LQ exchange. Namely, there exists a color triplet leptoquark $(3, 1)_{-1/3}$ in 45 that is synonymous with matter instability as can be seen from Tab. 1. But, in this particular setup, all $d = 6$ proton decay operators due to exchange of that triplet are completely suppressed. (Relevant coefficients of $d = 6$ operators for the color triplet are given in Eqs. (10), (11) and (12) of Ref. [17].)

To complete our numerical study we spell out products $v_{45} \tilde{z}_{2i}$, $i = 1, 2, 3$, at the GUT scale: $v_{45} \tilde{z}_{21} = 0.14$ GeV, $v_{45} \tilde{z}_{22} = 0.17$ GeV and $v_{45} \tilde{z}_{23} = 0.48$ GeV. $V_{\text{PMNS}}$ entries that are needed to convert $\tilde{z}_{2i}$ into $z_{2i}$ are adopted from Ref. [68]. Of course, the values we present should be run down from the GUT scale to the electroweak scale where we generate constraints on $\tilde{z}_{2i}$, $i = 1, 2, 3$, coefficients. We estimate that the low-energy values will be increased by a scaling factor $f_{\text{RGE}}$ that is within the following range: $f_{\text{RGE}} \in [1.1, 3.7]$ [55]. This holds as long as the initial value at the low energy scale for $|\tilde{z}_{2i}|$, $i = 1, 2, 3$, is below 0.5. The ratio between $\tilde{z}_{2i}$, $i = 1, 2, 3$, coefficients, however, should be constant with regard to the running effects.
8 GUT connection between \( \ell \to \ell'\gamma \) and \( B \to D^{(*)}\tau\bar{\nu} \)

We may take advantage of the known hierarchy in how \( \Delta^{(5/3)} \) and charm quark couple to charged leptons. Recall that the CKM induced couplings to \( u_L, c_L \) and \( t_L \) are hierarchically suppressed, due to our ansatz of Eq. (2.4), with factors \( V_{ub}, V_{cb} \) and \( V_{tb} \), respectively. Similar hierarchy in the \( Z \) matrix originates, on the other hand, from the requirement of realistic fermion masses in the GUT setting. In turn, this reduces the parameter set of the model to two independent Yukawa couplings. One of these must be \( y_{33} \) while the other can be any one of \( z_{2j} \) or \( \tilde{z}_{2j}, \ j = 1, 2, 3 \). We choose \( \tilde{z}_{22} \), the coupling of \( \Delta^{(5/3)} \) scalar to \( c\mu \) pair, to be the other independent parameter while remaining \( z_{2j} \) and \( \tilde{z}_{2j} \) are determined either from the hierarchical pattern (7.7) or via PMNS relation (2.4).

The PMNS rotation connecting \( \tilde{z}_{2j} \) and \( z_{2k} \) couplings reduces to a simple linear relation between \( \tilde{z}_{22} \) and \( z_{23} \) when Eq. (7.7) is applied:

\[
z_{23} = \tilde{z}_{22} V_{k3} \approx \tilde{z}_{22} c_{13} (s_{23} + 3.22 c_{23}) .
\]

(8.1)

The numerical factor 3.22 in Eq. (8.1) comes from the hierarchy between \( \tilde{z}_{2k} \) couplings. \( V_{ij}, s_{ij} \) and \( c_{ij} \) denote the PMNS matrix elements and the mixing angles that parameterize it, respectively. Using the 3 \( \sigma \) ranges for mixing angles from a recent PMNS fit [68] we find

\[
z_{23} = \omega \tilde{z}_{22}, \quad 2.63 < \omega < 3.17 .
\]

(8.2)

Effect of the aforementioned experimental constraints on \( (3,2)\gamma/6 \) with the minimal Yukawa texture (2.4), additionally restricted by the pattern of fermion masses, is shown on Fig. 5. As expected, the central role is played by the constraint on \( g_S \), although \( \tau \to \mu\gamma \) reduces the parameter space remarkably. Due to suppressed coupling to \( e \), sensitivity of \( \tau \to e\gamma \) and \( \mu \to e\gamma \) observables is reduced, however, the latter overcomes this suppression by a very stringent experimental upper bound and therefore has the most important role next to constraint on \( g_S \). An order of magnitude improvement on the experimental bound on \( \mu \to e\gamma \) would cause tension with the \( R(D^{(*)}) \) observables, and smaller values of the \( g_S \) coupling would be preferred. Note that only the 2 \( \sigma \) region (hatched in Fig. 5) is overlapping with the region where \( \Delta y_{33} \) is perturbative all the way to the GUT scale. We mention in passing that for mass 200 GeV < \( m_\Delta \) < 1 TeV all predictions are approximately invariant under rescaling of couplings and \( m_\Delta \) by same factor, as indicated on the axes of Fig. 5.

The 2\( \sigma \) region at \( m_\Delta = 500 \text{ GeV} \) in the \( y_{33} - \tilde{z}_{22} \) plane implies the following bounds

\[
|y_{33}| > 0.74, \quad |\tilde{z}_{22}| < 0.037 .
\]

(8.3)

Region that satisfies perturbativity of the couplings all the way to the GUT scale, outlined by dashed frame in Fig. 6, restricts the above 2\( \sigma \) ranges to

\[
0.74 < |y_{33}| < 0.80, \quad 0.021 < |\tilde{z}_{22}| < 0.032 .
\]

(8.4)

These bounds can be further put to use to generate limits on allowed values of \( v_{45} \). We find, using the GUT deduced values for the product \( v_{45}\tilde{z}_{2j}, \ j = 1, 2, 3 \), that

\[
f_{\text{RGE}} \ 5.0 \text{ GeV} < v_{45} < f_{\text{RGE}} \ 7.6 \text{ GeV} .
\]

(8.5)

This, in turn, confirms that the GUT embedding is self-consistent as the required values for \( v_{45} \) lead to small radiative corrections to Yukawa couplings.
Figure 5. Constraints on the couplings to $b\tau (y_{33})$ and to $c\mu (\tilde{z}_{22})$ coming from the $1\sigma$ region of $\mathcal{R}_{\tau/\ell}^{(*)}$ (thin hyperbolic region), $90\%$ CL upper bounds on $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$. Dashed frame represents the region where couplings remain perturbative all the way to the GUT scale, as explained in the text. Doubly (singly) hatched area is allowed at $1\sigma$ ($2\sigma$).

9 Predictions

9.1 $B_c \to \tau\nu$

The leptonic decay is governed by the same effective Hamiltonian as semileptonic decay and is therefore directly related to the latter, remaining insensitive to the underlying correlations between the LQ couplings. The only difference with respect to the semileptonic decay is that the decay $B_c \to \tau\nu\tau$ probes only the (pseudo)scalar operator of effective Hamiltonian (3.2) and is insensitive to tensor one due to vanishing matrix element $\langle 0|\bar{c}\sigma_{\mu\nu}b|B\rangle$. The value of decay constant of $B_c$ meson, $f_{B_c} = 0.427(6)(2)$ GeV has been recently calculated by HPQCD Collaboration in lattice QCD with fully relativistic formalism [69]. The branching ratio of the process in the SM is calculated using the formula

$$B(B_c \to \ell\nu) = \frac{m_{B_c}}{8\pi} f_{B_c}^2 |G_F V_{cb} m_\ell|^2 \left(1 - \frac{m_\ell^2}{m_{B_c}^2}\right)^2 r^2,$$

(9.1)

where factor $r$ given by

$$r = \left| 1 + \frac{m_{B_c}^2}{m_\tau(m_b + m_c)} g_S \right|.$$

(9.2)
Here $\tau_{B_c}$ is lifetime of $B_c$ meson and $g_S$ is the Wilson coefficient defined in Eq. (3.3). The HPQCD Collaboration gives SM prediction $\mathcal{B}(B_c \to \tau \nu) = 0.0194(18)$ [69]. For the best fit value $g_S = -0.37$ the branching ratio is suppressed by the factor $r^2 \simeq 0.36$ with respect to SM, while also large enhancement ($r^2 \simeq 84$) is allowed for $g_S \simeq 1.8 \pm 0.4 i$. The possible large enhancement by the same operator, in the context of Two Higgs Doublet models was also recently reported in [70]. One should note that the production rate of the $B_c$ meson at high energy colliders is several orders of magnitude smaller than for other $B$ mesons [71], and the observation prospects for $B_c$ leptonic channels are not very promising.

9.2 $t \to c\tau^+\tau^-$

Experiments at LHC have already established upper bounds for events with rare decays $t \to qZ$ with $q = u, c$ and with $Z$ reconstructed from light leptons. In the model with colored scalar a t-channel exchange of $\Delta^{(5/3)}$ contributes to the $t \to c\tau^+\tau^-$ decay. Tau leptons may further decay to light leptons and feed into the SM signal of $t \to Zc \to \ell^+\ell^-c$, or they could be reconstructed in a study dedicated to the $\tau^+\tau^-$ mode. The decay $t \to c\tau^+\tau^-$ can be treated in an effective approach, using a Lagrangian that encompasses the couplings induced by the $\Delta^{(5/3)}$ exchange (2.3),

$$
\mathcal{L}_{\text{eff}} = A(\bar{\tau}_R \gamma^\mu \tau_R)(\bar{c}_L \gamma_\mu t_L) + B \left( (\bar{\tau}_R \tau_L)(\bar{c}_R t_L) + \frac{1}{4} (\bar{\tau}_R \sigma^{\mu\nu} \tau_L)(\bar{c}_R \sigma_{\mu\nu} t_L) \right). \tag{9.3}
$$

Production mechanism of $t$ quarks may also be affected by colored scalars and can be treated independently of the decay branching fractions. The differential decay width of the $t \to c\tau^+\tau^-$ decay expressed in terms of normalized tau-pair invariant mass, $\hat{s} \equiv m^2_{\tau\tau}/m^2_t$, reads

$$
\frac{d\Gamma}{d\hat{s}} = \frac{m^5_t}{3072\pi^3} \left( 48|A|^2 \hat{s} (1 - \hat{s}) + |B|^2 \left( 11 + 20\hat{s} - 13\hat{s}^2 \right) \right). \tag{9.4}
$$

A very small SM contribution to this decay has been neglected altogether [72]. The two Wilson coefficients, given in terms of the couplings in the renormalizable Lagrangian (2.3), are

$$
A = -\frac{|y_{33}|^2 V_{cb} V^*_{tb}}{2m^2_\Delta}, \quad B = \frac{y_{33} \tilde{Z}_{26} V^*_{tb}}{2m^2_\Delta}. \tag{9.5}
$$

The values of the above couplings are constrained by $B \to D^{(+)}\tau\bar{\nu}$, $\tau \to \mu\gamma$, and perturbativity requirement that are mapped to the plane of predictions of $t \to c\tau^+\tau^-$ and $D^0 \to \tau^- e^+$ in Fig. 6, where the branching fraction of top decay below the $Z$ peak, $|m^2_{\tau\tau} - m^2_Z| < m_Z\Gamma_Z$, is expected to lie in the range of few times $10^{-9}$. Relaxing the perturbativity constraint can give an order of magnitude enhancement. We have also checked that the $t \to c\tau^+\tau^-$ total branching fraction, that is not limited to the region where $m^2_{\tau\tau} \approx m^2_Z$, is further enhanced by one order of magnitude.

9.3 $D^0 \to \tau^- e^+$

This is the only allowed lepton flavor violating decay of the neutral charm meson since the decay to $\tau^\pm \mu^\mp$ is kinematically closed. The effective Lagrangian is a combination of scalar
and tensor parts due to Fierz identities

\[ \mathcal{L}_{\text{eff}} = -\frac{V_{ub}^* y_{33} z_{21}}{2m^2_\Delta} \left[ (\bar{\tau}_R e_L)(\bar{e}_R u_L) + \frac{1}{4} (\bar{\tau}_R \sigma^{\mu\nu} e_L)(\bar{e}_R \sigma_{\mu\nu} u_L) \right], \quad (9.6) \]

and only the scalar operator contributes to the decay width. The decay width is suppressed by small coupling to \( e \), tiny phase space (\( m_\tau / m_{D^0} = 0.95 \)), and \( V_{ub} \):

\[ B(D^0 \to \tau^- e^+) = \frac{\tau_{D^0}}{256\pi} \frac{m^2_{D^0}}{m^2_e} \left( 1 - \frac{m^2_\tau}{m^2_{D^0}} \right)^2 m^3_D f^2_D |V_{ub}| \left( \frac{|z_{21} y_{33}|}{m^2_\Delta} \right)^2. \quad (9.7) \]

The combined effect of all suppression factors renders the branching fraction of this decay in the ballpark of \( 10^{-15} \) if perturbativity to the GUT scale is required (see Fig. 6). Relaxing this criterion can increase branching fraction up to 1 order of magnitude.

10 Conclusions

In order to explain the observed deviation from the Standard Model prediction of the ratios \( R(D) \) and \( R(D^*) \) we have explored the possibility of introducing a single light leptoquark state. Of all possible scalar and vector states with renormalizable couplings we have identified a scalar leptoquark with the Standard Model quantum numbers \( (3, 2)_{7/6} \) as the most suitable one.

In the framework of effective theory for semileptonic decays the tree-level exchange of charge-2/3 component of this leptoquark introduces a new operator, a particular combination of scalar and tensor currents, that interferes constructively with the vector current...
of the Standard Model. We have endowed the leptoquark with a minimal set of flavor couplings that are adequate to explain the discrepancy in $b \to c\tau\bar{\nu}$ processes and do not disturb flavor changing processes involving first two generation of quarks and leptons. A combination of $bt$ and $cv$ Yukawa couplings suffices to explain the anomalies in $B \to D\tau\bar{\nu}$ as well as in $B \to D^*\tau\bar{\nu}$ decays. As it turns out, the latter decay has dominant sensitivity to the tensor operator, although the scalar operator is almost an order of magnitude larger. The overlap of the two observables prefers small effective coupling that interferes positively with the Standard Model.

Regardless of the careful choice of the Yukawa couplings for the charge-2/3 component of this leptoquark, the charge-5/3 component will nevertheless induce lepton and quark flavor changing processes. We analyze in detail the constraints imposed by $Z \to b\bar{b}$ decay, value of the muon magnetic moment, lepton flavor violating decays $\mu \to e\gamma$, $\tau \to \mu\gamma, e\gamma$, and $\tau$ electric dipole moment. All these occur at one-loop level. The long standing anomaly in $Z \to b\bar{b}$ requires deviation of both couplings $g_R$ and $g_L$. Since the leptoquark $(3,2)_{7/6}$ connects in vertices to $b_L$, one can modify only the left handed coupling. This bound gives us a constraint on the coupling $y_{b33}$, i.e., to $bt$ pair, that is weaker than perturbativity limit on $y_{b33}$. Presence of this leptoquark pushes the Standard Model prediction of muonic $g - 2$ further away from the measured value. The experimental result puts a quite strong constraint on the coupling to $c\mu$, $\tilde{z}_{22} \leq 0.51$, for the typical mass of $m_\Delta = 500$ GeV in a simple extension of the SM with the aforementioned LQ. In the case of electric dipole moment, CP violating product of two different couplings can arise only in the case of $\tau$ lepton. Current experimental bound from Belle experiment is orders of magnitude too weak to address the problem of phase of the product $V_{cb}y_{b33}^*\tilde{z}_{23}^*$, at least in the perturbative setting. On the other hand, current experimental bound on the lepton flavor violating decays $\tau \to \ell\gamma$ with $\ell = \mu, \tau$, and especially $\mu \to e\gamma$, provide a stringent constraint on the underlying leptoquark couplings to $\tau\bar{e}$, $\mu\bar{e}$, and $e\bar{e}$.

After completing the analysis of phenomenological constraints on relevant leptoquark couplings we consider interplay between these constraints and the mass generation mechanism for charged leptons and down-type quarks in a GUT framework. We find that the minimal set of Yukawa couplings is not only compatible with the $SU(5)$ unification, a natural environment for colored scalars, but specifies all matter mixing parameters except for one angle in the up-type quark sector. We present predictions for the proton decay signatures through gauge boson exchange, as a function of that angle, and show that $p \to \pi^0e^+$ process is suppressed with respect to $p \to K^+\bar{\nu}$ and even $p \to K^0e^+$ in some parts of available parameter space. This goes against the expectations commonly encountered in the literature. The ansatz yields ratios between the leptoquark couplings that mimics mass hierarchy in the down-type quark and the charged lepton sectors. Namely, we find that $\tilde{z}_{21} : \tilde{z}_{22} : \tilde{z}_{23} = 0.024 : 0.32 : 1$. In order to preserve perturbativity at the GUT scale we require that $|y_{b33}| < 0.8$ and $|\tilde{z}_{23}| < 0.5$ at the low energy scale. In this regime the radiative corrections to our ansatz for Yukawa structure are under control.

Both the approaches from the low energy phenomenology point of view and the constraints coming from the GUT setup and proton decay result in a number of predictions. An obvious independent test of the $b \to c\tau\bar{\nu}$ process would be the decay $B_c \to \tau\bar{\nu}_\tau$ that
probes the same couplings as the semileptonic decay. Presence of scalar operator in the effective Lagrangian entails enhancements of 1 to 2 orders of magnitude of $B_c \to \tau \bar{\nu}_\tau$ decay width, although an experimental search for this decay is notoriously difficult. When the scalar state $(3, 2)_{7/6}$ is a part of 45-dimensional GUT representation there are only two independent couplings to consider. Then it turns out that $\tau \to \mu \gamma$ decay should lie within one or two orders of magnitude below the current experimental bound, and that $\mu \to e\gamma$ should be virtually one step beyond the reach of the MEG experiment bound, if this leptoquark is to explain the measured values of $R(\tau\ell)$. We make definite predictions, within the $SU(5)$ setup, of the lepton flavor violating process $\bar{D}^0 \to \tau^- e^+$ and of the rare process $t \to c\tau^+ \tau^-$. The former decay is very suppressed. The top decay with $\tau^+ \tau^-$ emulating the $Z \to \tau^+ \tau^-$ is also expected to proceed with branching fraction less than $10^{-8}$. However, a dedicated search for $t \to c\tau^+ \tau^-$ with integrated ditau spectrum could find a branching fraction of up to $10^{-7}$. We have also found allowed values for the vacuum expectation value of the 45-dimensional representation to be $f_{\text{RGE}} 5.0\text{ GeV} < v_{45} < f_{\text{RGE}} 7.6\text{ GeV}$, where $f_{\text{RGE}} \in [1.1, 3.7]$.

Due to the peculiar couplings of the $(3, 2)_{7/6}$ leptoquark to $\tau b$ pair and allowing it to couple to charm quark and all three neutrinos we can successfully explain the anomalies observed in semileptonic $b \to c$ transitions. If the presence of this state indeed solves these anomalies then it should also be observed eventually in lepton flavor violating decays of the tau leptons and muons and rare decays of top quark to charm quark and a pair of tau leptons.

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A Parameterization of the $B \to D^{(*)}$ form factors

For the $B \to D$ vector form factor we use expansion of the function $G_1(w)$ [34]

$$G_1(w) = G_1(1)[1 - 8\rho_D^2 z(w) + (51\rho_D^2 - 10)z(w)^2 - (252\rho_D^2 - 84)z(w)^3],$$  \hspace{1cm} (A.1)

in terms of variable $z(w) = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$. The slope parameter has been determined by Heavy Flavour Averaging Group [73], and has value $\rho_D^2 = 1.186 \pm 0.055$.

For the amplitudes of the $B \to D^\ast \bar{\ell}\bar{\nu}$ decay, Eq. (3.15), the hadronic helicity amplitudes are defined as contractions of matrix elements of corresponding lepton and hadronic currents with the helicity four-vectors $\tilde{c}_\mu(\lambda)$:

$$H_{\nu, A, A'}^{\mu, \rho, \sigma}(q^2, \cos \theta_\ell) = \tilde{c}_\mu(\lambda) \langle D^\ast(p_{D\ell}), \epsilon(\lambda_{D\ell})|\bar{c}\gamma^\rho (1 - \gamma_5)|b B(p_B)\rangle,$$  \hspace{1cm} (A.2a)

$$H_{\nu}^{\mu, \rho, \lambda}(q^2, \cos \theta_\ell) = \langle D^\ast(p_{D\ell}), \epsilon(\lambda_{D\ell})|\bar{c}\gamma^\rho (1 - \gamma_5)|b B(p_B)\rangle,$$  \hspace{1cm} (A.2b)

$$H_{T, \lambda, \lambda'}^{\mu, \rho, \sigma}(q^2, \cos \theta_\ell) = \tilde{c}_\mu(\lambda) \tilde{c}_{\nu}(\lambda') \langle D^\ast(p_{D\ell}), \epsilon(\lambda_{D\ell})|\bar{c}\sigma^\mu\nu (1 - \gamma_5)|b B(p_B)\rangle.$$  \hspace{1cm} (A.2c)
The explicit form of these functions is found in the literature, see e.g. [19, 25, 74, 75]. Leptonic helicity amplitudes are defined in analogous fashion.

Functions $R_i(w)$ entering form factors $A_{1,2,3}(w)$ and $V(w)$ are expanded around point $w = 1$ [19, 34]

$$h_{A_1}(w) = h_{A_1}(1)[1 - 8\rho^2 z(w) + (53\rho^2 - 15)z(w)^2 - (231 - 91)z(w)^3],$$

$$R_0(w) = R_0(1) - 0.11(w - 1) + 0.01(w - 1)^2,$$

$$R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2,$$

$$R_2(w) = R_2(1) + 0.11(w - 1) - 0.06(w - 1)^2,$$

$$R_3(w) = R_3(1) - 0.052(w - 1) + 0.026(w - 1)^2.$$

(A.3)

The numerical values of $R_i(w)$ used in calculations are $h_{A_1}(1) = 0.919\pm0.035$, $R_0(1) = 1.14$, $R_1(1) = 1.403 \pm 0.033$, $R_2(1) = 0.854 \pm 0.020$ and $R_3(1) = 1.22$ [19, 76].

B EW gauge couplings of the scalar $(3, 2)_{7/6}$

Consistently with Eq. (4.1) we define the covariant derivative as $D_\mu = \partial_\mu - igW^3_\mu Z^3 + ig'B_\mu Y$ and the weak mixing angle as $W^3_\mu = \cos \theta_W Z_\mu + \sin \theta_W A_\mu$. The Feynman rules for colored scalar-gauge boson interactions are listed in Tab. 2.

| $Z_\mu(q)$ | $\Delta^{(Q)}(p + q)$ |
| $\rightarrow$ | $\Delta^{(Q)}(p)$ |
| $-ig\left(-Qc_W + \frac{Y}{c_W}\right)(2p + q)_\mu \epsilon^\mu_Z$ |

| $A_\mu(q)$ | $\Delta^{(Q)}(p + q)$ |
| $\rightarrow$ | $\Delta^{(Q)}(p)$ |
| $ieQ(2p + q)_\mu \epsilon^\mu_A$ |

Table 2. Feynman rules for scalar couplings to neutral gauge bosons.
C Loop function of $Z \to b \bar{b}$

$$g_0(x) = -12x^2 \left[ - \text{Li}_2 \left( \frac{1}{x} - 1 - x^2 \right) \left( x - i \sqrt{x - 1/4} - 1/2 \right) \right] - \text{Li}_2 \left( \frac{1}{x} - 1 - x^2 \right) \left( x + i \sqrt{x - 1/4} + 1/2 \right)$$

$$+ \text{Li}_2 \left( \frac{1}{x} - 1 \right) + \text{Li}_2 \left( x - i \sqrt{x - 1/4} - 1/2 \right) + \text{Li}_2 \left( x + i \sqrt{x - 1/4} - 1/2 \right)$$

$$+ 2 \left[ \pi^2 x^2 + 3i \pi \sqrt{1 - 4x} (2x - 1) + \sqrt{1 - 4x} (2x - 1) \log(8) \right]$$

$$+ 3(4x - 5) + 6 \sqrt{1 - 4x} (1 - 2x) \log \left( \frac{-2x + \sqrt{1 - 4x} + 1}{x} \right),$$

$$g_2(x) = 16x^2 \left[ \frac{3}{2} \text{Li}_2 \left[ 1 + \frac{1}{x} \right] \right]$$

$$+ \text{Li}_2 \left( \frac{1}{x} - 1 - x^2 \right) \left( x - i \sqrt{x - 1/4} - 1/2 \right) + \text{Li}_2 \left( \frac{1}{x} - 1 - x^2 \right) \left( x + i \sqrt{x - 1/4} - 1/2 \right)$$

$$- \text{Li}_2 \left( \frac{x - 1}{x} \right) - \text{Li}_2 \left( x - i \sqrt{x - 1/4} - 1/2 \right) - \text{Li}_2 \left( x + i \sqrt{x - 1/4} - 1/2 \right)$$

$$- 4 \pi^2 x^2 / 3 + 48i \pi x^2 \log \left( \frac{1}{x} + 1 \right) + 4i \pi \left( 2 \sqrt{1 - 4x} - 3 \right) (2x - 1) - 2(4x + 7)$$

$$+ 8 \sqrt{1 - 4x} (1 - 2x) \log \left( \frac{-2x + \sqrt{1 - 4x} + 1}{x} \right) + 12(1 - 2x) \log(x) + 8 \sqrt{1 - 4x} (2x - 1) \log(2).$$

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