Feedback-controlled transport in an interacting colloidal system

K. Lichtner and S. H. L. Klapp

Institute of Theoretical Physics, Secr. EW 7-1, Technical University Berlin - Hardenbergstr. 36, D-10623 Berlin, Germany

received 23 August 2010; accepted in final form 29 October 2010
published online 6 December 2010

PACS 05.40.Jc – Brownian motion
PACS 82.70.Dd – Colloids
PACS 05.60.Cd – Classical transport

Abstract – Based on dynamical density functional theory (DDFT) we consider a non-equilibrium system of interacting colloidal particles driven by a constant tilting force through a periodic, symmetric “washboard” potential. We demonstrate that, despite pronounced spatio-temporal correlations, the particle current can be reversed by adding suitable feedback control terms to the DDFT equation of motion. We explore two distinct control protocols with time delay, focussing on either the particle positions or the density profile. Our study shows that the DDFT is an appropriate framework to implement time-delayed feedback control strategies widely used in other fields of non-linear physics.

Introduction. – Transport phenomena of Brownian particles in complex geometries are a topic that has received intense and continuous attention for decades [1–3]. A large number of studies has been devoted to transport in structured 1D systems such as colloids or biomolecules in microchannels [4], colloids in optical potentials [5,6], or cold atoms in optical lattices [7]. Theoretical studies of such systems have predicted spectacular effects such as ratchet mechanisms in systems with asymmetric spatial potential [1], giant diffusion [5,6,8,9] and dispersionless transport [10] in symmetric systems under constant external bias (“tilted washboards”), and the negative mobility effect [4,11]. Many of these effects have also been observed experimentally (see, e.g., [4,5,11,12]), often involving colloidal systems. A related topic is how these non-equilibrium phenomena can be manipulated by control forces [13]. Particularly promising are feedback control schemes which depend on the state of the system. A special case is the time-delayed feedback control method suggested by Pyragas [14], where the control term involves the difference between an output variable (the control target) at time \( t \) and its value at time \( t - \tau \), with \( \tau \) being the delay time. This method is particularly suitable to stabilize certain, otherwise unstable (periodic) states. Moreover, a time delay naturally occurs in experiments involving feedback control due to the lag between the collection of information and the feedback. Indeed, time-delayed feedback control is nowadays used in a broad variety of non-linear systems such as lasers, neural dynamics, and excitable macroscopic media [15]. In the area of 1D transport, the method has already been applied, on a theoretical level, to Brownian motors (driven by an unbiased, time-periodic force) [16], systems rectified by delayed correlated noise [17], and flashing ratchets (involving asymmetric, time-dependent potentials) [13,18]. A first experimental realization of a feedback-controlled flashing ratchet already exists [19]. Very recently, feedback strategies with delay have also been explored, on the basis of Langevin equations [20,21], as a tool to manipulate the particle current in tilted washboard potentials. However, despite intense research we are still far away from a full understanding of the usefulness of control schemes in transport processes and its potential applications in biology and nanotechnology.

One open, yet very important point is the role of particle interactions, investigations of which have only started recently (see, e.g., [22]). In the present letter we show, for the first time, that control in an interacting, driven, overdamped system can be well implemented within the framework of dynamical density functional theory (DDFT) [23–25]. In the DDFT the basic dynamic variable is the time-dependent, continuous density field, where the microscopic interactions determining the underlying system of discrete Brownian particles enter via a free energy functional. Therefore, DDFT has a bridging position between microscopic (Langevin equation based) models, on one hand, and mesoscopic, hydrodynamic models for transport of continuous phases (see, e.g., [26]),

\(^{(a)}\)E-mail: klapp@physik.tu-berlin.de
on the other hand; a connection, which is also highlighted by a recent derivation of the DDFT via projector operator techniques [27]. In the last years, DDFT has been applied to a variety of driven systems such as colloids in unstable traps [28] and sedimenting colloids [29]. Moreover, in a very recent work [30], DDFT is employed to study attracting colloidal particles in 1D (time-dependent) ratchet potentials. Here we apply the method to a system of soft, repulsive colloids in a static, tilted washboard potential. We focus on the impact of time-delayed feedback control. Indeed, we find that the current in our strongly correlated system can be efficiently controlled via time-delayed feedback schemes focusing either on the average particle position or directly on the density profile.

Model and results. – Our model system consists of $N$ colloidal particles in an one-dimensional channel of length $L$. Particles at positions $z_1$ and $z_2$ in the channel interact via a repulsive Gaussian potential $V^{GCM}(z_1 - z_2) = \varepsilon_0 \exp[-(z_1 - z_2)^2/\sigma^2]$ (with $\varepsilon_0 > 0$), a typical coarse-grained potential modeling a wide class of soft, partially penetrable macroparticles (e.g., polymer coils) with effective (gyration) radius $\sigma$ [31, 32]. The particles are subject to a spatially periodic, symmetric (“washboard”) potential $V^{wb}(z) = U_0 \sin^2(kz)$, where $k$ defines the wavelength and $U_0$ is the amplitude.

Typical distributions of the system in the absence of an external drive are illustrated in fig. 1(a) and fig. 1(b). Specifically, we consider the space-dependent one-particle density $\rho(z) = \langle \sum_{i=1}^{N} \delta(z - z_i) \rangle$ (with $\langle \ldots \rangle$ being an ensemble average) at two values of the interaction strength $\varepsilon = \varepsilon_0/k_B T$, where $k_B$ and $T$ are Boltzmann’s constant and temperature, respectively. The particle number is fixed to $N = \int_{-L/2}^{L/2} dz \rho(z,t) = 4$. The data have been obtained numerically using the DDFT formalism described below, with the initial configuration ($t = 0$) being a single density peak centered in one potential well. The data in fig. 1(a) pertain to a washboard potential of strength $U = U_0/k_B T = 15$. Here, thermal fluctuations are too weak to allow for barrier crossing within the observation time and, consequently, the particles remain localized in the potential well. This is true both for strong interactions ($\varepsilon = 4$) and in a small-coupling case ($\varepsilon = 1$), where the narrower peak of $\rho(z)$ suggests an even more pronounced localization. Figure 1(b) shows, for comparison, a profile corresponding to $U = 3$ (and $\varepsilon = 4, F = 0$), yielding a fluid-like situation with the density becoming non-zero everywhere. Within our calculations, we did not identify a threshold value of $U$ separating the two regimes, consistent with the continuous dependence of the equilibrium mobility on $U$ in the corresponding (exactly solvable) single-particle problem [3].

From now on we focus on the more interesting localized situation depicted in fig. 1. Here, an effective motion of the particles can be induced by a constant tilting force $F^{bias} = F_0 \hat{z}$ (with $\hat{z}$ being the unit vector in $z$-direction), corresponding to a linear potential $V^{bias}(z) = -F_0 z$. We choose $F_0 > 0$ such that the particles move preferentially to the right. The non-equilibrium dynamics of the driven system is investigated via dynamical density functional theory (DDFT) [23–25, 27], where the central quantity is the time-dependent, one-particle density of the particles, $\rho(z,t)$. By construction, DDFT assumes the dynamics to be overdamped, i.e., inertial effects are neglected. The exact Smoluchowski equation for $\rho(z,t)$ is then approximated such that non-equilibrium two-particle correlations at time $t$ are set to those of an equilibrium system with density $\rho(z,t)$. Neglecting, moreover, hydrodynamic interactions, one obtains the key DDFT equation [23–25, 27]

$$\Gamma^{-1} \frac{\partial \rho(z,t)}{\partial t} = \nabla \left[ \rho(z,t) \nabla \frac{\delta \mathcal{F}[\rho(z,t)]}{\delta \rho(z,t)} \right],$$

where $\Gamma$ is a mobility coefficient (i.e., $\Gamma = D_0/k_B T$ with $D_0$ being the short-term diffusion coefficient), and $\mathcal{F}[\rho]$ is a free energy density functional determining the effective “current” $j = -\Gamma \nabla \left( k_B T \frac{\delta \mathcal{F}[\rho]}{\delta \rho} \right)$. Specifically, $\mathcal{F} = \mathcal{F}^{id} + \mathcal{F}^{int} + \mathcal{F}^{ext}$, where $\mathcal{F}^{int} = k_B T \int dz \rho(z,t) \ln \rho(z,t)$ is the ideal part (with $\Lambda$ being the thermal wavelength), $\mathcal{F}^{ext} = \int dz \rho(z,t) \left( V^{wb}(z) + U^{bias}(z) \right)$ is the external field contribution, and $\mathcal{F}^{ext}$ accounts for the colloidal interactions. Here we employ the mean-field (MF) approximation, that is, $\mathcal{F}^{int} = (1/2) \int dz_1 \int dz_2 \rho(z_1,t) V^{GCM}(z_1 - z_2) \rho(z_2,t)$. Due to the penetrable nature of the Gaussian potential (which allows an, in principle, infinite number of neighbors) the MF approximation is known to become quasi-exact in the high-density limit and yields reliable results even at low and moderate densities [32].

The impact of the external drive on the density profile is illustrated in fig. 1(c) and fig. 1(d), where we consider two values of the driving strength $F = F_0 \sigma/k_B T$. K. Lichtner and S. H. L. Klapp
The numerical calculations have been performed in channels with closed boundaries, such that any drift motion is, strictly speaking, of transient character. However, by choosing channels of sufficient lengths \( L \geq 60\sigma \) we have ensured that all shown profiles are free of boundary effects.

In the small-drive situation depicted in fig. 1(c) \( (F = 0.5) \), the peak barely moves within the time range considered, indicating that the probability for the particles to jump over the barriers is still small. This changes at \( F = 3 \) (fig. 1(d)) where the drive causes a shift of the entire density peak to the right (here and in the following, time is measured in units of the Brownian time scale \( \tau_B = \sigma^2 / (\Gamma k_B T) \), which is of the order of \( 10^{-9} \) s for typical colloids). At the same time, the distribution broadens in the sense that now several potential wells are (potentially) occupied by particles. The influence of \( F \) is also reflected by the time dependence of the mean-squared displacements (MSD), \( w(t) = N^{-1} \sum_{i=1}^{N} \langle (z_i(t) - z_i(0))^2 \rangle \). Since, within the DDFT formalism, we do not have direct access to the particle positions, we obtain the MSD rather via the relation \( w(t) = \int_{-\infty}^{\infty} dz z^2 G_d(z,t) \). Here, \( G_d(z,t) \) is the self-part of the van Hove correlation function measuring the probability that a particle moves over a distance \( z \) during time \( t \). We calculate this function, as well as its distinct counterpart \( G_s(z,t) \) measuring the time dependence of two-particle correlations, within the DDFT formalism via the test particle method [33]. Some numerical results for two-particle correlations, within the DDFT formalism, are given here is still smaller than the "critical" force \( F_{\text{crit}} = \text{max}(dF / dwb / dz) \approx 5.9 \), beyond which the potential barriers are eliminated, and a single particle can slide freely. Therefore, we do not see the maximum and subsequent decrease of \( D \) expected in the vicinity of \( F_{\text{crit}} [8,9] \) (indeed, investigation of that range is hindered by the finite length of our system which eventually yields boundary effects). We recall in this context that the particles in our system are interacting such that one may expect deviations of the behavior of \( D \) from the uncorrelated case [22]. Indeed, as indicated in fig. 2(b), a decrease of the coupling strength yields a shift of the curve \( D(F) \) towards larger driving strength. Thus, the repulsive interactions in our model support the particles in crossing the barriers. Finally, fig. 2(d) shows the distinct van Hove function, \( G_d(z,t) \), as calculated by the test particle method [33]. At \( t = 0 \), where \( G_d \) is proportional to the usual pair distribution function, the correlations are restricted to the first potential well in which the particle where confined initially (note the correlation hole at \( t = 0 \)). At later times \( G_d(z,t) \) develops additional peaks in the neighboring potential valleys at \( z > 0 \), indicating pronounced spatio-temporal correlations in the driven system.

We now aim at manipulating the dynamics by time-delayed feedback control, which involves the difference between an appropriate system variable (the control target) at time \( t \) and its value at time \( t - \tau \). The use of such closed-loop strategies in 1D systems subject to tilted washboard potentials has been previously explored, e.g., in refs. [20,21]. These studies investigated non-interacting Brownian particles by direct numerical solution of the corresponding Langevin equations. Here we consider an interacting colloidal system and the dynamics is described by the DDFT equation (1).

We explore two distinct control strategies. Within the first protocol, which is similar in spirit to that
in [20], the control target is the first moment of the density corresponding to the average particle position \( \bar{z} = \int_{-\infty}^{\infty} \bar{z} \rho(z, t) \, dz \). We note that particle positions and thus, their average, are measurable variables in experiments of colloidal transport (see, e.g., refs. [5,6]).

To implement the feedback control we supplement the external potential entering in the free energy functional \( \mathcal{F}^{\text{ext}} \) (see text below eq. (1)) by a linear control potential \( U^c(z, t; \tau) = -\varepsilon F^c(t; \tau) \), where

\[
F^c(t; \tau) = -K_0 (1 - \tanh[\bar{z}(t) - \bar{z}(t - \tau)])
\]

(2)
is a spatially homogeneous control force. In the DDFT equation (1) this force yields a current \( J^c = -\Gamma \rho(z, t) F^c(t, \tau) \). Note the restriction \(-2K_0 \leq F^c \leq 0\), where \( K_0 \) is assumed to be positive. In the numerical calculations we switch on the control force at \( t_{\text{start}} = 1.5 \tau_B \) (i.e., \( F^c(t < t_{\text{start}}) = 0 \)).

The impact of the control force \( F^c \) on the normalized first moment \( \langle z \rangle = \bar{z}(t)/N \) is illustrated in fig. 3(a), where we have chosen \( F = 3 \) and \( \tau = \tau_B \). This delay time corresponds roughly to an intrinsic time scale of the system. Indeed, as seen from fig. 2(a), \( t = \tau_B \) is within the time range where the crossover from sub-diffusive into diffusive behavior of the MSD of the uncontrolled system occurs.

In the absence of control \( K = K_0 \bar{\rho}/k_B T = 0 \) the first moment plotted in fig. 3(a) just increases with \( t \), reflecting the rightward motion expected at \( F = 3 \) (see fig. 1(d)).

Fig. 3: (a) Normalized first moment as a function of time for various control amplitudes \( K \) (see eq. (2)). (b) First moment (solid line) and effective bias (dashed) at \( K = 3 \). (c) Density profiles for various times \( (F = 3, \tau = \tau_B, t_{\text{start}} = 1.5 \tau_B) \).

The slope of the function \( \langle z \rangle \), at large \( t \) may be interpreted as an average velocity \( v = \lim_{t \to \infty} \frac{d\langle z \rangle_1}{dt} \). Increasing \( K \) from zero, the velocity first decreases until the peak motion stops (i.e., the time average of \( \langle z \rangle_1 \) becomes constant) at \( K = 3 \). This value corresponds to a balance between control force and bias. Even larger control amplitudes then result in a significant backward motion, i.e., \( \langle z \rangle_1 \) and \( v \) become negative.

We now consider in more detail the time dependence of the feedback control. First, a significant influence on \( \langle z \rangle_1 \) appears only at relatively large times \( t \gg t_{\text{start}} \) (which depend, in turn, on the actual value of \( K \)). At earlier times, the density peak moves so quickly that \( \bar{z}(t) \gg \bar{z}(t - \tau) \) and the tanh-function in eq. (2) approaches 1, yielding \( F^c(t; \tau) \approx 0 \). With the “slow-down” of \( \langle z \rangle_1 \) at somewhat later times (visible also at \( K = 0 \)), the argument of the tanh decreases. Thus, the control sets in, yielding an effective biasing force \( F_{\text{eff}}(t) = F^0 + F^c(t; \tau) \leq F_0 \). The behavior of \( F_{\text{eff}}(t) \) and the function \( \langle z \rangle_1 \) is shown in fig. 3(b) for the “balanced” case \( K = 3 \). One sees that the control becomes effective at \( t \approx 10 \tau_B \). After that, the first moment displays an oscillating behavior changing between small backward motion and forward motion. These oscillations yield, in turn, oscillations of \( F_{\text{eff}}/k_B T \) between its maximum value, \( F = 3 \), and its minimum \( F - 2K = -3 \). As a consequence, the overall motion stops. Interestingly, the oscillations (which seem to persist in the long-time limit) have a period of about \( 5 \tau_B \), that is, much larger than the delay time \( (\tau = \tau_B) \). Density profiles related to one “cycle” of \( \langle z \rangle_1 \) and \( F_{\text{eff}}(t) \) are plotted in fig. 3(c), where \( t/\tau_B = 25 \) and 30 correspond roughly to the minimum and maximum of \( F_{\text{eff}} \), respectively (see fig. 3(b)). It is seen that the entire particle distribution is shifted with the periodic changes of \( F_{\text{eff}} \). These shifts are accompanied by changes in the peak shapes, which become most asymmetric when \( F_{\text{eff}} \) is minimal, i.e., the control is maximal \( (t/\tau_B = 25) \). To complete the picture, we plot in fig. 4 the long-time velocity \( v \) (averaged over the oscillations of \( \langle z \rangle_1 \), if present) as function of the control amplitude. We have included data for different delay times \( \tau \) and different interaction (i.e., repulsion) strengths \( \varepsilon \). All systems considered display a clear current reversal at \( K = 3 \), where the velocity \( v \) changes from positive to

Fig. 4: Long-time velocity as a function of \( K \) for several values of \( \tau \) and \( \varepsilon \) \( (F = 3, t_{\text{start}} = 1.5 \tau_B) \). Lines are guides for the eye.

40007-p4
The impact of this scheme on the function $\langle z \rangle_t$ as a function of time and various control amplitudes $K$ for the second control loop (see eq. (3)) and $\varepsilon = 4$. The inset in (a) contains data for $\varepsilon = 1$ at otherwise same parameters $F = 3$, $\tau = \tau_B$, and $t_{\text{start}} = 1.5\tau_B$. (b) Density profiles at some characteristic times for $K = 1.4$. The washboard potential is indicated by the dotted lines.

negative values irrespective of $\varepsilon$ and $\tau$. These parameters, however, do have an impact on the magnitude of the velocities in the two regimes $K < 3$ and $K > 3$. Specifically, reduction of $\varepsilon$ (at fixed $\tau$) yields a decrease of $v$ as compared to the case $\varepsilon = 4$. Similarly, $v$ decreases in magnitude when the delay time decreases (at fixed $\varepsilon$) from $\tau = \tau_B$ towards $\tau = 0.2\tau_B$. In other words, the time delay supports the current reversal in the parameter range considered. We also note that all of these results are robust, on a qualitative level, against slight changes of the control protocol, such as a reversal of the argument of the transcendental function in eq. (2).

The control scheme considered so far focuses on the average particle position. However, given that the basic dynamical variable in DDFT is the density profile $\rho(z,t)$, it is interesting to briefly discuss a control loop based on that quantity. Specifically, we consider the potential

$$U_2^c(z, [\rho]) = -K_0 z \left(1 - \tanh[\rho(z, t - \tau) - \rho(z, t)]\right). \quad (3)$$

The impact of this scheme on the function $\langle z \rangle_t$ is shown in fig. 5(a), where the inset contains data for the weakly interacting case $\varepsilon = 1$. For small control amplitudes ($K < 1$), the behavior of both systems, $\varepsilon = 4$ and $\varepsilon = 1$, is similar to what is observed with the previous control loop (eq. (2)) in the sense that the long-time velocity decreases with increasing $K$. However, contrary to this previous loop, further increase of $K$ then yields an abrupt reversal of the motion, that is, the particles “bounce backwards” in the sense that $v \to -\infty$. Moreover, this abrupt change occurs at values of $K$ much smaller than the biasing force ($F = 3$). The spatio-temporal behavior of the microscopic density profile $\rho(z,t)$ related to the sudden reversal of motion is illustrated in fig. 5(b). At $t = 3.5\tau_B$, that is, before the reversal, the initial density peak centered at $z = 0$ has extensions to the next potential wells at positive $z$, indicating that the particles move rightwards. However, already at this time, there is a small bump at positions $z < 0$ not present within the first control scheme (see fig. 3(c)). At time $t = 5\tau_B$ after the reversal this additional bump has increased significantly along with a left-ward shift of the center of mass. In a future publication we will analyze in more detail to which extent the sudden reversal and the associated complex behavior of $\rho(z,t)$ is a true instability. Nevertheless, already the results in fig. 5 indicate that a local control scheme could be more efficient, in the sense that such smaller perturbations $K$ are required to yield current reversal, compared to control focussing on a space-averaged quantity.

Concluding remarks. – In conclusion, we have demonstrated that the transport in an interacting, driven colloidal system can be efficiently manipulated by time-delayed feedback control. Our control goal in the present context was to stop or reverse the motion in a tilted washboard potential below the critical driving strength $F_{\text{crit}}$ beyond which the washboard becomes ineffective. We have shown that this goal can be achieved with different control schemes that involve the same delay time $\tau$ (chosen equal to the intrinsic, Brownian time scale) but different control targets (average particle position $\langle z \rangle_t$ vs. density profile $\rho(z,t)$). The fact that the schemes produce comparable results indicates a certain robustness of the feedback control method for transport phenomena in 1D systems, consistent with earlier theoretical findings for ratchet systems [13,17–19] and for non-interacting particles in tilted washboards [20,21]. In that sense, our study also supports the more general perspective that time-delayed feedback control can be extremely useful for the manipulation of non-linear systems [15].

An experimental realization of the present results seems possible with micron-sized colloidal particles in 1D tilted washboard potentials created by optical (laser) fields [6,12,19]. For instance, the study [12] employs particles of size $\sigma = 1.5 \mu m$ in potentials with periodicity $\lambda = 3.14 \mu m \approx 2.1\sigma$, and similar dimensions occur in [19]. The barrier heights in [12] are $U = 4.5–11.5$, and the biasing forces are in the range $0 \leq F \lesssim 100$, suggesting that our parameters for the uncontrolled system ($\lambda = 8\sigma$, $U = 15$, $F = 3$) are not unrealistic. With respect to control, we stress that in colloidal systems the trajectories of the individual particles can be monitored with a video camera [12,19] yielding, in principle, both the average particle position and the full density field as possible control target. Indeed, the average colloid position is also
targeted in a recent experimental realization of a flashing ratchet with time-delayed feedback control [19]. Moreover, in these experiments the delay of \( \tau = 5\text{ ms} \) (arising from the finite time required to locate the colloids), is much smaller than the time scale for diffusion \((\approx 300\text{ ms})\), consistent with our assumption \( \tau \ll \tau_0 \).

Beyond the actual physical behavior, our study also shows that delayed feedback control can be conveniently implemented within the framework of DDFT, a recently established method to describe colloidal dynamics based on the microscopic interactions, yet with close relations to mesoscopic models of continuous phases [27]. As a side-result of our analysis, we note that the DDFT appears to correctly describe also the non-controlled case, which has already been intensively investigated by other methods [2,3]. Since DDFT is an approximate theory, the present results for the impact of control remain to be tested against quasi-exact data from Brownian dynamics computer simulations. However, given the good performance of the DDFT in other contexts [28,29] we expect our findings to be at least qualitatively right.

Clearly, there is a number of questions prompted by our study. First, one needs to explore more systematically the precise role of the time delay in the various control schemes, and of its interplay with the intrinsic time scales of the system. Second, given the complex behavior of non-controlled systems at driving strength \( F \approx F_{\text{crit}} \) (related to “giant diffusion”) [8,9,22], it would be interesting to extend the present analysis (where \( F < F_{\text{crit}} \)) accordingly. Third, we need to better understand the role of the nature of the interactions in the context of control. Here we have focused on soft, repulsive interactions characterizing, e.g., polymer coils [31], but also paramagnetic particles oriented parallel to a magnetic field (in 1D). What would happen in the presence of additional attractive contributions such as van der Waals or depletion interactions (arising from solvent particles in the colloidal suspension)? Indeed, as recently shown for colloids in 1D ratchet potentials (without control) [30], attractive interactions and the resulting clustering tendency strongly influences the transport behavior. Moreover, on a macroscopic scale, attractive interactions determine the wettability and as a consequence, the transport behavior of drops on (heterogeneous) substrates; a topic which is immediately relevant in the context of microfluidics [26] and may be accessible within the present framework after a proper reinterpretation of the density fields. Finally, the present methodology may also be used to investigate systems consisting of several species, a question relevant in the context of particle sorting effects [4,11]. Work in these directions is under way.

REFERENCES

[1] Reimann P., Phys. Rep., 361 (2002) 57.
[2] Marchesoni F. and Hänggi P., Rev. Mod. Phys., 81 (2009) 387.
[3] Risken H., The Fokker-Planck Equation (Springer Verlag, New York) 1989.
[4] Ros A., Eichhorn R., Regtmeier J., Duong T. T., Reimann P. and Anselmetti D., Nature, 436 (2005) 928.
[5] Lee S. H. and Grier D. G., Phys. Rev. Lett., 96 (2006) 190601.
[6] Blickle V., Speck T., Seifert U. and Bechinger C., Phys. Rev. E, 75 (2007) 060101(R).
[7] Gommer R., Lebedev V., Brown M. and Renzoni F., Phys. Rev. Lett., 100 (2008) 040603.
[8] Costantini G. and Marchesoni F., Europhys. Lett., 48 (1999) 491.
[9] Reimann P., Van den Broeck C., Linke H., Hänggi P., Rubi J. M. and Perez-Madrid A., Phys. Rev. Lett., 87 (2001) 010602.
[10] Lindenberg K., Sancho J. M., Lacasta A. M. and Sokolov I. M., Phys. Rev. Lett., 98 (2007) 020602.
[11] Eichhorn R., Regtmeier J., Anselmetti D. and Reimann P., Soft Matter, 6 (2010) 1858.
[12] Evstigneev M., Zyvagolskaya O., Bleil S., Eichhorn R., Bechinger C. and Reimann P., Phys. Rev. E, 77 (2008) 041107.
[13] Cao F. J., Dinis L. and Parrondo J. M. R., Phys. Rev. Lett., 93 (2004) 040603.
[14] Pyragas K., Phys. Lett. A, 170 (1992) 421.
[15] Schöll E. and Schuster H. G., Handbook of Chaos Control (Wiley VCH, Weinheim) 2008.
[16] Wu D. and Zhu S., Phys. Rev. E, 73 (2006) 051107.
[17] Borromeo M., Giusepponi S. and Marchesoni F., Phys. Rev. E, 74 (2006) 031121.
[18] Craig E. M., Long B. R., Parrondo J. M. R. and Linke H., EPL, 81 (2008) 1002.
[19] Lopez B. J., Kuwada N. J., Craig E. M., Long B. R. and Linke H., Phys. Rev. Lett., 101 (2008) 220601.
[20] Hennig D., Phys. Rev. E, 79 (2009) 041114.
[21] Hennig D., Schimansky-Geier L. and Hänggi P., Phys. Rev. E, 79 (2009) 041117.
[22] Evstigneev M., von Gehlen S. and Reimann P., Phys. Rev. E, 79 (2009) 011116.
[23] Marconi U. M. B. and Tarazona P., J. Chem. Phys., 110 (1999) 8032.
[24] Marconi U. M. B. and Tarazona P., J. Phys.: Condens. Matter, 12 (2000) 413.
[25] Archer A. J. and Evans R., J. Chem. Phys., 121 (2004) 4246.
[26] John K. and Thiele U., Appl. Phys. Lett., 90 (2007) 264102.
[27] Espanol P. and Löwen H., J. Chem. Phys., 131 (2009) 244101.
[28] Rex M. and Löwen H., Phys. Rev. Lett., 101 (2008) 148302.
[29] Royall C. P., Dzubiella J., Schmidt M. and van Blaaderen A., Phys. Rev. Lett., 98 (2007) 188304.
[30] Pototsky A., Archer A. J., Bestehorn M., Merkt D., Savelev S. and Marchesoni F., Phys. Rev. E, 82 (2010) 030401(R).
[31] Likoś C. N., Phys. Rep., 348 (2001) 267.
[32] Louis A. A., Bolhuis P. G. and Hansen J. P., Phys. Rev. E, 62 (2000) 7961.
[33] Archer A. J., Hopkins P. and Schmidt M., Phys. Rev. E, 75 (2007) 040501(R).