Cosmological Perturbations and the Running
Cosmological Constant Model

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ABSTRACT: We study the matter density fluctuations in the running cosmological constant (RCC) model using linear perturbations in the longitudinal gauge. Using this observable we calculate the growth rate of structures and the matter power spectrum, and compare them with the SDSS data and other available data of the linear growth rate. The distribution of collapsed structures may also constraints models of dark energy. It is shown that RCC model enhances departures from the $\Lambda$CDM model for both cluster number and cumulative cluster number predicted. In general increasing the characteristic parameter $\nu$ leads to significant growth of the cluster number. In general, we found that the theory of perturbations provides a good tool to distinguish the new model RCC of the standard cosmological model $\Lambda$CDM.

KEYWORDS: Cosmology, dark energy, perturbation theory, number counts.

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1. Introduction

Recent results from supernova Ia and Wilkison Microwave Anisotropy Probe (WMAP), especially the five-year data indicate that the present Universe is accelerating and that has negligible space curvature.

In this context, the non-relativistic matter contributes about 30% (dark matter plus baryonic matter) of the critical density of the Universe and 70% of the energy density is not known and is called of dark energy. The dark energy is generally attributed to a cosmological constant (CC) and is physically equivalent to vacuum energy. This component breaks the strong energy condition, but is the simplest model that one can build. This model although satisfactory from an observational point of view, is anyway theoretically disfavored, since there is a huge different between the predicted and observed values of CC. Other possibilities have been considered, among these the most popular are based on scalar fields, known as quintessence models and models based on scalar fields with non-canonical kinetic energy, called of k-essence models. Unfortunately, these models are difficult of to distinguish from a CC, because the change of the scalar field may be extremely slow and there are an degeneration of the potential of quintessence. We may also mention other models of dark energy as are: alternative gravity scalar-tensor type, F(R) theories, and models that introduce extra-dimensions as the DGP model, among other. For a recent review of models of dark energy see.
In practice the dark energy can be seen into of the Friedmann equations through its effective energy density and pressure. The ratio of the energy density to pressure, known as the equation of state (\( EOS \)), ratio \( w(z) = P(z)/\rho(z) \), has important within the equations regardless of its physical origin. The function \( w(z) \), where \( z \) is the redshift, is key to understanding the dynamics of the cosmological expansion. Another quantity important in the background cosmological is the deceleration parameter \( q(z) \). Both are the major functions used in studies of constraints of parameters using background data. In particular, the deceleration parameter can be used to investigate the kinematics of the accelerated expansion using luminosity distances data (and other data, see [10] and [11]).

The next step, in the study of a cosmological model, is to use first order cosmological perturbations. In fact, the behaviour of linear perturbations in a scalar field and its effect on large scale structure formation has been investigated by a number of authors, see [12]. Also the behaviour of the nonlinear gravitational collapse has been investigated [13]. These studies are fundamentals to understanding and discrimination between competing models.

In this paper, we investigate the cosmological consequences of an model motivated in quantum field theory \( QFT \), more specifically from the renormalization group method. The idea to study the renormalization group effects as a way to solve the \( CC \) (energy of vacuum) has been explored in the papers [14, 15, 16, 17, 18, 19, 20]. In particular, a consistent model was presented in [21, 22] and is called of Running Cosmological Constant (RCC), where the \( CC \) is an evolving parameter. In general, in this approach the energy density of the vacuum \( \rho_\Lambda(z) \) can be deduced as a quadratic function of the expansion rate (see equation 2.3 in section II), which allows unambiguously resolve the equation of conservation of energy and determine the hubble parameter \( H(z) \) as function of the redshift. This model is equivalent to models of dark energy in the fluid approximation (as are, parameterizations of the \( EOS \) and quintessence models [23]). The background cosmology of this model has been well investigated together with data from supernovae Ia, restricting the values of the parameter \( \nu \), that represents the "run" of the \( CC \) in the RCC model. Also, in [25] was determined the matter power spectrum in the synchronous gauge.

In this paper we focus the analyze of the matter density perturbations using the longitudinal gauge, in which Bardeen gauge-invariant variables are identical to the remaining metric variables. This approach was pioneered in references [24, 27]. The formalism is applied to our RCC model to determine the linear growth rate and the matter power spectrum as functions of the \( \nu \) parameter. We compared the results with observational data from the SDSS (Sloan Digital Sky Survey).

On the other hand, has long been recognized in various theoretical work [28] and in simulations that the evolution of the clusters number (or number counts) can determine properties of the dark energy. Therefore, we use our results of linear density perturbations and the Press-Schechter formalism for determining the mass function, the number counts and the cumulative number counts as function of redshift, and study the sensitivity on the parameter \( \nu \). We use the \( \Lambda CDM \) model to compare our predictions. Also we investigate how the number counts and cumulative number counts, depends on the parameters \( \Omega_m \) and \( h \) when the value of \( \nu \) is fixed.

Our paper is organised as follows. In section II we introduce the RCC model and
discuss the behaviour of the comoving energy density and deceleration parameter. In section III we discuss the linear perturbations of matter. Section IV is devoted to the computation of the number counts in the Press-Schechter formalism. In section V we present our conclusion. In the appendix we display results for the DGP model and scalar-tensor theory, which are useful to calculate the linear growth rate.

2. Running Cosmological Constant

In this section we shall introduce our cosmological model by considering a FLRW metric and a Universe consists of matter (baryonic + dark) and a dark energy component or vacuum energy; thus the cosmological evolution is governed by the following equation of Friedmann

$$H^2 = \frac{8\pi G}{3} (\rho_m + \rho_\Lambda) - \frac{k}{a^2},$$  \hspace{1cm} (2.1)

where $\rho_m$ and $\rho_\Lambda$ are densities of matter and vacuum energies respectively, $a$ is the scale factor, $H = \frac{\dot{a}}{a}$ is the Hubble parameter and $k$ is a constant, which can be chosen to have the value $+1, 0, -1$. In this investigation we only restrict us to the flat case. Using the Bianchi identity, the densities of energies have the conservation law

$$\frac{d\rho_m}{dz} + \frac{d\rho_\Lambda}{dz} = 3\frac{\rho_m}{1 + z},$$ \hspace{1cm} (2.2)

Of the above equations we can see that there is an exchange of energy between the two components of the universe, so both are completely tied. However, we assume that the baryonic component no energy exchange with the variable $CC$, since that it would not be compatible with constraints of particle physics, but otherwise the dark matter component can in principle be subject to an energy exchange resulting in a time-dependent mass for the dark matter particles and a modification in its equation of state. Therefore, in this model, the dynamic of the Universe is dominated by the evolution of the $CC$ and its interaction with dark matter.

On the other hand, an important question to be noted is that the equations (2.1-2.2) do not form a complete set of equations, since that there are three free variables, $\rho_m(z), \rho_\Lambda(z), H(z)$ and two equations. We need a third equation to have a complete system. In this paper, we introduce an additional equation by considering the effects of applying the renormalization group to the $CC$. In this framework the energy density of $RCC$ is

$$\frac{d\rho_\Lambda}{d\ln H} = \frac{\sigma H^2 M^2}{(4\pi)^2},$$ \hspace{1cm} (2.3)

The above equation was proposed assuming that the renormalization group scale is identified with the square root of the curvature scalar $\sqrt{R}$, which in the FLRW cosmological context is equivalent to identify it with the $H(z)$ at any given redshift (or cosmological time). $M$ is an effective mass parameter representing the average mass of the heavy particles of the Grand Unified Theory (GUT) near the Planck scale, after taking into account their multiplicities. The coefficient $\sigma$ can be positive or negative, this sign depends on whether bosons ($\sigma = +1$) or fermions ($\sigma = -1$) dominate in the loop contribution, this is,
Figure 1: Comoving background matter density as a function of redshift for the RCC model. We can see that the decrease of density is a characteristic signature of the coupling between dark matter and vacuum energy. Notice that in this plot $\Lambda\text{CDM}$ correspond to a constant line equal to one. From bottom to top: $\nu = 0.25$ (orange), $= 0.065$ (red), $= 10^{-3}$ (blue). In all model we used $\Omega_{m0} = 0.24$ and $h = 0.72$.

depending on whether the fermions or bosons dominate at the highest energies. Without going into the details on the model, let us recall that the equation (2.3) is interpreted in the renormalization group framework as a "$\beta$-function" of QFT in curve-space time, which determines the running of the CC, for other details see [24]. Therefore, for background (cosmology of the zeroth order) these above three equations us allow a full description of the evolution of the RCC cosmological model. Now using the equation (2.3) we can obtain one explicit solution for $\rho_\Lambda$

$$\rho_\Lambda = \rho_{\Lambda 0} + \frac{3\nu}{8\pi} M_p^2 (H^2 - H_0^2), \quad (2.4)$$

where $\rho_{\Lambda 0}$ and $H_0$ are the current values of these parameters and, additionally, in this model appears a parameter new $\nu$ (dimensionless) that is given by

$$\nu \equiv \frac{\sigma}{12\pi} \frac{M^2}{M_p^2}, \quad (2.5)$$

thus, as we can see, from the equation (2.4), if $\nu$ is zero then the effects of a running are canceled and recover the standard model $\Lambda\text{CDM}$. For the case of the Planck mass, that is, $M = M_P$, the parameter $\nu$ takes the value (positive or negative, depending on $\sigma$),

$$\nu_0 = \frac{1}{12\pi} = \pm 2.6 \times 10^{-2}. \quad (2.6)$$
Figure 2: The deceleration parameter \(q(z)\) in the RCC model. For higher values of \(\nu\) faster we move away from a universe dominated by matter in the past. We used \(\Omega_{m0} = 0.24\) and \(h = 0.72\). For \(\nu = 10^{-3}\) (blue line) the RCC model is indistinguishable of \(\Lambda CDM\) (red line).

Based on the above equations the equation \((2.1)\) now can be written as

\[
\frac{H^2}{H_0^2} = 1 + \left( \Omega_{m0} - \frac{2\nu \Omega_{k0}}{1 - 3\nu} \right) \left( \frac{(1 + z)^{3-3\nu} - 1}{1 - \nu} \right) + \frac{\Omega_{k0}(z^2 + 2z)}{1 - 3\nu},
\]

(2.7)

where \(H_0^2\Omega_{k0}(1 + z)^2 = -\frac{k}{a^2}\) and \(\Omega_{m0} = \Omega_{MD} + \Omega_{B0}\), being \(\Omega_{DM}\) dark matter and \(\Omega_{B0}\) baryonic matter. This expression for the Hubble parameter is valid for Universes with positive or negative curvature. Therefore, the equations \((2.1), (2.4)\) and \((2.7)\) define our cosmological model. One of the first quantities that we can calculate is the matter density. Thus we defined the comoving matter density function as \(\Pi(z) = \rho_m(z) (1 + z)^3\). In the figure 1, we displayed it. There is a decrease of the density due to the coupling between dark matter and dark energy. Increasing the coupling leads to faster decreasing of the density. In the figure the line constant correspond to \(\Lambda CDM\) (\(\nu = 0\)). Note that this behavior is similar to coupled quintessence models [29] and we use only positive values of the parameter \(\nu\). To negative values the density \(\Pi\) is greater than one.

Another important parameter in cosmology of background is the deceleration parameter \(q(z) = -1 - H(1 + z)\frac{d}{dz}\left(\frac{1}{H}\right)\). In Figure 2 we show this parameter for our RCC model. Here we observe that if \(\nu\) takes large values (positive) then it is not phenomenologically compatible with a matter dominated phase in the past, that is, necessary for the formation of structures. In this sense, numerically, when \(\nu \leq 10^{-3}\) \((M \approx 0.38M_p)\) we can see that the value of \(q(z)\) tends rapidly to \(1/2\) and is indistinguishable from \(\Lambda CDM\). For the negative case \(q(z)\) is indistinguishable from \(\Lambda CDM\) for values around \(\nu > -10^{-3}\) in other
cases the value of $q(z)$ is greater than 1/2 in the past. In what follows we consider linear perturbations and structures formation.

3. The Linear Perturbations Equations

The theory of the cosmological perturbations is based on the expansion of Einstein’s equations to linear order about the background metric. The first step in the analysis of the metric fluctuations is to classify it according to their transformations properties under spatial rotations. There are, scalar, vector and tensor, perturbations. In this paper, we only study scalar perturbations, specifically density perturbations in the RCC model, since that we are interested in the matter power spectrum. Other fundamental question is the choice of gauge or choice of a coordinate system. The General Relativity leads to the issue of gauge freedom. This is, metric and matter fluctuations take on different values in different coordinate system. A way to deal with the gauge problem is to eliminate the gauge dependence entirely. This approach is called of gauge-invariant variables and was pioneered by Bardeen [30]. However, in the literature, many other gauges have been used [33]. The more commonly used is the synchronous gauge. For example, in this gauge, the dynamics of density perturbations for the RCC model was investigated in [25]. In the present work we extending the analysis of [25] by compute the matter power spectrum in the longitudinal gauge and to investigate the cluster number counts.

The longitudinal gauge is from the physical point of view much more intuitive, because the metric perturbations are similar to the Newtonian perturbations. This gauge is commonly chosen for work in CMB and gravitational lensing. On the other hand, conceptually this gauge fixed all freedom degree and the two scalar potentials $\Psi$ and $\Phi$ that appear in the line element correspond to the variable of Bardeen gauge-invariant [27, 30]; this is not the case of the synchronous gauge where exist a residual transformations and leads to the appearance of unphysical gauge. However its use is justified, since these spurious modes are canceled of some manner when calculating a physical observable, which by definition can not depend of a given system of coordinates. The metric in the longitudinal gauge is given by [26]

$$ds^2 = -(1 + 2\Psi(\vec{x}, t))dt^2 + (1 + 2\Phi(\vec{x}, t))dx^jdx^j.$$ (3.1)

It should be said that the longitudinal gauge is restricted for scalar modes; nonetheless, it can be easily generalized to include the vector and tensor degrees of freedom [31]. Further, in the absence of anisotropic stress one of the Einstein’s equations gives $\Psi = -\Phi$; the two gravitational potentials are equal and opposite [32]. Therefore, there remains only one free metric perturbations variable which is a generalization of the Newtonian gravitational potential. This justifies the name of Newtonian gauge.

In order to derive the equations for the density perturbations we follow the standard formalism [31, 33, 32]. We consider the entropy perturbation as negligible and a energy-momentum tensor free of anisotropic stresses. Thus, in these conditions the energy-
momentum tensor have the form of a perfect fluid

\[ T^\mu_\nu = pg^\mu_\nu + (\rho + p) U^\mu U_\nu, \quad (3.2) \]

where \( U^\mu = dx^\mu /(-ds^2)^{1/2} \) is the four-velocity of fluid, \( p \) is the pressure and \( \rho \) is the energy density of a perfect fluid. For a fluid moving with a small velocity \( v^i \equiv dx^i/d\tau \) (peculiar velocity), where \( v^i \) can be treated as a perturbation of the same order as \( \delta \rho = \rho - \bar{\rho} \) or \( \delta p = p - \bar{p} \). The quantities \( \bar{\rho} \) and \( \bar{p} \) are with respect at the background. In linear order the perturbations of the energy-momentum tensor that we used are given by

\[ T^0_0 = -(\bar{\rho} + \delta \rho), \]
\[ T^0_i = (\bar{\rho} + \bar{p}) v^i = -T^i_0, \]
\[ T^i_j = (\bar{p} + \delta \bar{p}) \delta^i_j, \quad (3.3) \]

the perturbed four-velocity is

\[ U^\alpha = ((1 - \Psi), v^j), \quad (3.4) \]

while one can directly work with the Einstein’s equations, it turns out to be convenient to use the equations of motion for the matter variables, since that, we are eventually interested in the matter perturbations. We consider the conservation of the energy-momentum tensor as

\[ T^\alpha_\beta ; \beta = \partial \Gamma^\alpha_\beta /\partial x^\alpha + \Gamma^\alpha_\delta T^\delta_\beta + \Gamma^\delta_\alpha T^\alpha_\beta = 0, \quad (3.5) \]

this expression gives us two equations for \( \beta = 0 \) and other for \( \beta = i \). To determine these equations we consider the perturbations for the two densities and the metric

\[ \rho_m \rightarrow \rho_m (1 + \delta_m), \]
\[ \rho_\Lambda \rightarrow \rho_\Lambda (1 + \delta_\Lambda), \]
\[ g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu}. \quad (3.6) \]

Indeed, in order to simplify our work we want to make full use of the symmetry under translations in space, which can best be exploited by working with the Fourier components of perturbations. Our convention, for all quantities, will be

\[ f(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}.\vec{x}} F(\vec{k}), \quad (3.7) \]

with these considerations, we use the line element (equation 3.1), the equations (3.3) and (3.6) into equation (3.5) and we get

\[ \dot{\delta \rho}_m + \delta \dot{\rho}_\Lambda + \rho_m (\theta + 3\Psi, \theta) + 3H \delta \rho_m = 0, \quad (3.8) \]
\[ \dot{\rho}_m \theta + \rho_m \dot{\theta} - \frac{k^2 \Psi}{2} + 3H \rho_m \theta = -\frac{(k^2)}{a^2} \delta \rho_\Lambda, \quad (3.9) \]

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where the dot is the derivative with respect to cosmic time and \( \theta = \partial_t v_i \). The energy density of the variable \( \CC \), \( \rho_\Lambda \), can be written as

\[
\rho_\Lambda = A + B(\nabla_\mu U^\mu)^2, \tag{3.10}
\]

where we have used the fact that \( \nabla_\mu U^\mu = 3H \) and defined that \( A = \rho_{\Lambda 0} - \frac{3\nu}{8\pi} M_0^2 H_0^2 \) and \( B = \frac{\nu M_0^2}{24\pi} \). Now using the perturbated four-velocity, equation (3.14), the Christoffel symbols perturbed and keeping only the linear order term, we find

\[
\delta_\Lambda = \frac{2H\nu[\theta - 3(\dot{\Psi} + H\Psi)]}{\rho_\Lambda(z)}. \tag{3.11}
\]

On the other hand, the Einstein’s equations in the longitudinal gauge are

\[
k^2 \Psi + 3H(z)(\dot{\Psi} + H(z)\Psi) = -4\pi G a^2 (\delta_m \rho_m(z) + \delta_\Lambda \rho_\Lambda(z)), \tag{3.12}
\]

\[
\dot{\Psi} + H(z)\Psi = -4\pi G a \rho_m \theta, \tag{3.13}
\]

substituting the equation (3.13) in (3.12) we obtain an new equation for the scalar potential in term of variables of matter

\[
k^2 \ddot{\Psi} = \frac{3H \rho_m \theta}{2k^2(1 + z)^3} - \frac{(\delta_m \rho_m + \delta_\Lambda \rho_\Lambda)}{2(1 + z)^2}. \tag{3.14}
\]

It is convenient for our numerical calculations to write the equations (3.8), (3.9), (3.11) and (3.14) in terms of the redshift, i.e., using \( dt = \frac{dz}{H(1+z)} \), and also is advantageous to use the following two ratios [25]

\[
f_1(z) = \frac{\rho_m}{\rho_t} = \frac{(1 + z)2H(H') - 2H_0^2 \Omega_k(1 + z)^2}{3(H^2 - H_0^2 \Omega_k(1 + z)^2)},
\]

\[
f_2(z) = \frac{\rho_\Lambda}{\rho_t} = \frac{3H^2 - 2H(1 + z)H' - H_0^2 \Omega_k(1 + z)^2}{3(H^2 - H_0^2 \Omega_k(1 + z)^2)}, \tag{3.15}
\]

and also

\[
\dot{\Psi} - H\Psi = \frac{\rho_t v}{2k^2(1 + z)}, \tag{3.16}
\]

\[
\theta - 3\dot{\Psi} = \frac{v}{f_1} + 3H \left( \Psi + \frac{\rho_t v}{2(1 + z)Hk^2} \right), \tag{3.17}
\]
where $\rho_t(z) = 3H^2(z) - 3H_0^2\Omega_k(1 + z)^2$. Using the equations (3.15-3.17), the equations system for the perturbations can be written as

\begin{equation}
\delta_\Lambda(z) = \frac{\nu v H}{\rho_t f_2} \left[ \frac{1}{f_1} - \frac{3 \rho_t}{2k^2(1 + z)} \right], \tag{3.18a}
\end{equation}

\begin{align}
\delta'_m(z) + \delta_m(z) &+ \left[ \frac{f'_1}{f_1} + \frac{3 f_1}{1 + z} - \frac{3}{(1 + z)} \right] \delta_\Lambda \\
&+ \frac{3 \Psi}{(1 + z)} + \frac{v}{H(1 + z)} \left( \frac{1}{f_1} + \frac{3 \rho_t}{k^2(1 + z)} \right) \\
&+ \frac{2 \nu}{f_1} \left( vf'_2 + f_2 v' \right) \left( K(z) + \frac{M(z)}{2} \right) + \frac{2 \nu f_2 v}{f_1} \left( K'(z) + \frac{M'(z)}{2} \right) = 0, \tag{3.18b}
\end{align}

\begin{equation}
v'(z) + \frac{3(f_1 - 1)v}{1 + z} = \frac{k^2(1 + z)}{H} \left( \delta_\Lambda f_2 - \frac{\Psi}{9H^2} \right), \tag{3.18c}
\end{equation}

\begin{equation}
k^2 \Psi = \frac{\rho_t}{2(1 + z)^2} \left[ \frac{3v}{k^2(1 + z)} - (\delta_m(z)f_1 + \delta_\Lambda(z)f_2) \right], \tag{3.18d}
\end{equation}

where the prime is derivative with respect to redshift, $v = f_1 \theta$ and the terms of the equation (3.18b) $M(z)$ and $K(z)$ are given by

\begin{equation}
M(z) = \frac{1}{f_2 k^2(1 + z)}, \tag{3.19}
\end{equation}

\begin{equation}
K(z) = \frac{1}{3H f_2 f_1}, \tag{3.20}
\end{equation}

in these equations $\nu$ is the parameter defined in (2.5) and is of great importance to us, since when $\nu = 0$ we can see that the perturbation on the vacuum energy is canceled. In this way, we recaptured the scenario $\Lambda CDM$ as a particular case.

### 3.1 The Linear Growth Rate

The solution of the system (3.18) allows us to determine the density contrast of matter, $\delta_m$, which is necessary to determine the growth factor, defined as

\begin{equation}
D(a) = \frac{\delta_m(a)}{\delta_m(a = 1)}. \tag{3.21}
\end{equation}

In the figure 3 (left) we show the growth factor for the RCC model for different values of $\nu$ and we can see that for $\nu = 10^{-4}$ there is concordance with a $\Lambda CDM$ model. However, observationally is more important the linear growth rate that measures how rapidly structure is being assembled in the Universe as a function of cosmic time (scale factor or redshift) and is defined by

\begin{equation}
g(a) = \frac{d \ln D(a)}{d \ln a}, \tag{3.22}
\end{equation}

this quantity has been measured using different catalogs. In general, the redshift maps of galaxies are distorted by the peculiar velocities of galaxies along the line of sight. For
Figure 3: The left figure is the growth factor and in the right figure we show the linear growth rate for the \textit{RCC} model. In all case we use $\Omega_{m0} = 0.24$ and $h = 0.72$.

Large scale such distortion can be expressed through the redshift distortion parameter $\beta$ and may be show to be related to the linear growth rate as \cite{34}

$$
\beta = \frac{g(z)}{b_L},
$$

where $b_L$ is the linear bias value ($b_L = \sigma_{R}^{\text{gal}}/\sigma_{R}^{\text{mass}}$), that is, the ratio between the root-mean-squared ($\text{rms}$) density contrasts in the galaxy and mass distributions on scale $R$ where linear theory applies. Therefore, a measure of $g(z)$ can be obtained by these two parameters. The $\beta$ parameter may be measured from redshift surveys by measuring the power spectrum of the galaxies \cite{34} and the $b_L$ can be obtained from the skewness induced in the bispectrum of a given survey. Using this technique a value of $\beta$ and $b_L$ has been measured using the 2dFGRS sample of 220 000 galaxies \cite{35}. Recently, another measure was made using the spectroscopic data from the VIMOS-LT Deep Survey \cite{36}; and finally there is another measure of growth rate using the 2dF-SDSS LRG and QSO survey \cite{37}; However, in this last case the value of $\beta$ and $b_L$ are not fully independent, because they have been obtained by imposing simultaneous consistency with the clustering measured at $z = 0$. All these data are compiled in the table 1. In figure 3 (right) we show this estimates of the growth rate compared to predictions from various theoretical models. For plotting the linear growth rate for the \textit{DGP} and scalar-tensor models, we use the results shown in the appendix. Despite the large error bars, the measurements indicate the need of a small value of the parameter $\nu$, and hence very close to the $\Lambda CDM$ model.

3.2 The Matter Power Spectrum

Other important amount that we can determine from the set of equations (3.18) is the
Table 1: In this table we compile the different values of the linear growth rate that there are in the literature.

| Survey                               | \( \beta \pm \delta \beta \) | \( b_L \pm \delta b_L \) | \( g \pm \delta g \) | \( z \) |
|--------------------------------------|-------------------------------|---------------------------|------------------------|-------|
| 2dFGRS [35]                          | 0.49 ± 0.09                   | 1.04 ± 0.10               | 0.49 ± 0.14            | 0.15  |
| 2dF-SDSS LRG and QSO [37]           | 0.45 ± 0.05                   | 1.66 ± 0.35               | 0.75 ± 0.35            | 0.55  |
| VVDS [36]                            | 0.70 ± 0.26                   | 1.30 ± 0.10               | 0.91 ± 0.36            | 0.80  |

Matter power spectrum defined as

\[ P(k, z) = \delta^2(k, z). \] (3.24)

To set the initial conditions we shall use the BBKS approximation for the transfer function [38]

\[ T(k) = \frac{\ln(1 + 2.34q(k))}{(2.34q^2)(1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4)^{1/4}}, \] (3.25)

in the presence of CC, we have that \( q = k/(\Sigma hMpc^{-1}) \), where the shape parameter is \( \Sigma = \Omega_m h^{-\Omega_0} \) and \( h = H_0/100 \). Since observable wavenumbers are in units \( hMpc^{-1} \). We assume than only 4% of the cosmic density is provided by conventional baryonic matter. For the present time \( a = 1 \), then the matter power spectrum is usually written using the transfer function in the form

\[ P(k) = Ak^n T^2(k), \] (3.26)

where \( n \) measures the slope of the primordial power spectrum (we will assume \( n = 1 \) [2, 17]) and \( A \) is a normalization constant. In order to obtain power spectra for the our RCC model we evolved equations (3.18) from \( z = 500 \) to \( z = 0 \). A value of 500 ensures that at that time the matter is dominant. In normalizing the power spectrum followed the methodology presented in [25].

In Figure 4 we present the matter power spectrum of our model and the power spectrum estimated by Percival et al., using data of the SDSS Project for \( \Omega_{m0} = 0.24 \) [39]. The impact that has the RCC model on linear power spectrum is dominated by equation (3.18a), which depends of the value of the parameter \( \nu \). Thus, if the parameter \( \nu \) is zero, then, we recover the \( \Lambda \)CDM model, since that, \( \delta_\Lambda = 0 \). Consequently, a value large \( \nu \) entails a more damping of the power spectrum for the case of \( \nu \) positive. Meanwhile, for \( \nu \) negative the deviation of the power spectrum with respect at the observational data is even stronger. In principle, this feature also can be seen from the inspection of the figure 4. Theoretically we can understand this behavior, because in the equation (3.9) we can observe that the parameter \( \nu \), appears as a factor of the term \( k^2 \) proportional which is derived from the pressure gradient. When \( \nu \) is negative this terms change of sign. Thus, it is expected that for small scales \((k > 0.15)\) and \( \nu < 0 \), the pressure terms become important. In general, for large scales \( k < 0.06 \) (for example) the \( k^2 \) terms may be negligible.

We can determine the mass in terms of the Planck mass for \( a \nu \approx 10^{-5} \) as \( M \approx 0.6 \times 10^{-2}M_p \), that is, corresponding approximately to the GUT particle spectrum. Therefore, the RCC model is viable for smaller masses than the Planck mass.
Figure 4: (left) The matter power spectrum for the RCC model. The curve blue is the $\Lambda CDM$ and is very close from the RCC model with the best fit $\nu = (1.29 \pm 2.39) \times 10^{-5}$. We used $\Omega_{m0} = 0.24$ and $h = 0.72$.

To get a more accurate value of $\nu$ and without using the rigorous theory of parameter estimation, we use the $\chi^2$ statistic, that is, a quantity that qualifies the fitting of the observational data for a given theoretical model, and is defined as: $\chi^2 = \sum_i \frac{(P_{ob} - P_{the})^2}{\sigma_{ob}^2}$, where and $P_{ob}$ is the observational value for the power spectrum, $P_{the}$ is the corresponding theoretical result and $\sigma_{ob}$ denotes the error bar. The index $i$ refers to a measurement corresponding to given wavenumber. Thus, the probability distribution function can be defined as $P(\nu) = F_0 e^{-\chi^2/2}$, where $F_0$ is a normalization constant. Minimizing $\chi^2$ we can determine the most probable value for the parameter $\nu$. This value is $\nu = (1.29 \pm 2.39) \times 10^{-5}$, where we have assumed a $\Omega_{m0} = 0.24$ and $h = 0.72$.

Finally, in the next section we consider only $\nu > 0$, and we leave for future work a detailed study for values $\nu < 0$, since that in the present paper our aim is to understand the overall behavior of the RCC model, rather than determine observational constraints, which requires a rigorous application of the theory of estimation of parameters [40].

4. The Number Counts

It has long been recognized that cluster of galaxies provide a useful probe of the fundamental cosmological parameters. The total abundance of cluster $N$ and their distribution in redshift $\frac{dN}{dz}$ should be determined by geometry of the universe and the power spectrum of initial density fluctuations. One of the first cosmological parameters to be constrained
was $\sigma_8$, the amplitude of mass density fluctuations on a scale of $8\ h^{-1}Mpc$. For example, recently Komatsu et al., have determined a value of $\sigma_8 = 0.812 \pm 0.026$ using the WMAP data for the $\Lambda CDM$ model. However, in general, the value of $\sigma_8$ is even inaccurate, since there is the implicit uncertainty on the value of $\sigma_8$ as a function of $w$ (see [41]).

Our objective in the present section is to determine the clusters number and its evolution with the redshift and investigate whether these quantities depend significantly from $\nu$ parameter and what value should be used for have a concordance with the $\Lambda CDM$ model. For this we used the Press-Schechter (PS) [42, 43] formalism that gave a prescription for estimating the mass function for a hierarchical gaussian density field.

In the PS model the comoving number density of collapsed dark matter haloes of mass $M$ in the interval $dM$, is given by

$$\frac{dn}{dM} = -\sqrt{\frac{2}{\pi}} \frac{\delta c}{\sigma(M,z)} \frac{d\ln(\sigma(M,z))}{dM} \exp(-\frac{\delta c^2}{2(\sigma(M,z))^2}), \quad (4.1)$$

where $\Pi$ is the comoving matter mean density of the universe and $\delta c$ is the linearly extrapolated density threshold above which structures collapse, i.e. $\delta c = \delta_L(z = z_{col})$. In an Einstein-de Sitter (EdS) model, an overdensity region collapses with a linear contrast $\delta c = 1.686$ and is the value that we adopt for our calculations. As a first approximation, we use this value of EdS, and we postponed for future work the use of spherical collapse model to calculate $\delta c = \delta_L(z_{col})$. In principle, this election is not far from reality because for homogeneous quintessence models coupled $\delta c$ does not separate too far from the EdS value (see [54]). Let us recall that the RCC model is equivalent with quintessence models [23]. The quantity $\sigma(M,z) = D(z)\sigma_M$ are the $rms$ linear fluctuations of density in spheres of radius $R$ containing a mass $M$ and $D(z)$ is the growth factor. In our analysis the $rms$
of the smoothed overdensity is given by

$$\sigma_M = \sigma_8 \left( \frac{M}{M_8} \right)^{-\gamma/3}, \quad (4.2)$$

where \(M_8 = 6 \times 10^{14} \Omega_m h^{-1} M_\odot\), the mass inside a sphere of radius \(R_8 = 8 h^{-1} \text{Mpc}\), \(M_\odot\) is the solar mass, and \(\sigma_8\) is the mentioned above variance of the overdensity field smoothed on a scale of size \(R_8\). The index \(\gamma\) is a function of the mass scale and shape parameter \(\Gamma\)

$$\gamma = (0.3 \Gamma + 0.2) [2.92 + \frac{1}{3} \log \left( \frac{M}{M_8} \right)]. \quad (4.3)$$

we use \(\Gamma = 0.167\) and we associate galaxy clusters with dark matter haloes of the same mass. Our analysis of the effect of a running cosmological constant on the number of dark matter haloes is done by computing two quantities. The first is the number of haloes per unit of redshift in a given interval of mass

$$\frac{dN}{dz} = \int_{4\pi} d\Omega \int^{M_{\text{Sup}}}_{M_{\text{inf}}} \frac{dn}{dM} \frac{dV}{dz d\Omega} dM,$$  \quad (4.4)

where \(\frac{dV}{dz d\Omega}\) is the comoving volume element and is given by \(r^2(z)/H(z)\), beging \(r(z) = \int_0^z dz/H(z)\). In the figure 5 (left panel) we display the comoving volume element evolution with redshift for the RCC model. We can see that there is a strong dependence on the value \(\nu\). Thus, large values of \(\nu\) comoving volume element increases. The concordance \(\Lambda\text{CDM}\) model is plotted for comparison. In the right panel we plot the comoving volume compared to Einstein-de Sitter volume for all case.
Figure 7: The figure shows the integrated number counts up to redshift \( z \) for objects with mass \( M > 10^{13} h^{-1} M_\odot \) (left) and \( M > 10^{14} h^{-1} M_\odot \) (right). For both case solid line represent \( \nu = 10^{-5} \) and represents \( \Lambda CDM \).

The other quantity that we compute is the all sky integrated number counts above a given mass threshold, \( M_{\text{inf}} \), and up to redshift \( z \)

\[
N(z, M > M_{\text{inf}}) = \int_{4\pi} d\Omega \int_{M_{\text{inf}}}^{\infty} \int_0^z \frac{dn}{dM} \frac{dV}{dz d\Omega} dzd\Omega,
\]

(4.5)

this result is also called cumulative mass function. To compute the two quantities, we must choose a normalization for the number density of haloes \( n(M) \). This is commonly expressed by \( \sigma_8 \). We choose to normalize all models by fixing the number density of haloes at redshift zero. This is at redshift zero all models have the same comoving background density \( \Pi \) and growth factor \( D \). Our model fiducial is \( \Lambda CDM \) (\( \Omega_{m0} = 0.24, h = 0.72 \)) with \( \sigma_8 = 0.9 \) [47].

Now we can determine the dependence of these quantities when the value \( \nu \) changes. In the figure 6, we displayed the number counts as function of the redshift, obtained from equation (4.4) for which there is a strong dependence on the value assumed by the parameter \( \nu \). An increase in the value of \( \nu \) produces an increase in the value of \( \frac{dN}{dz} \). Comparing the left panel to the right, we can see that there is a larger variation for greater values of mass. The results for the total number of collapsed structures above a given mass are displayed in the figura 7. In the left panel we plot the integration in the mass range \( M > 10^{13} M_\odot \) and in the right panel for \( M > 10^{14} M_\odot \), in both case \( M_{\text{sup}} = 10^{15} \). We do not use strictly infinite, since \( N(z, M > M_{\text{inf}}) \) is dominated by the contribution of the lower bound of the mass integration range.

The parameter \( \nu \) can be viewed as the coupling between dark energy (vacuum energy) and dark matter. Models with more coupling have higher values of \( dN/dz \). This can be understood by the behavior of other observables. Seeing the expressions (4.4) and (4.5) we note that number counts have a dependence on the growth factor \( D(z) \), comoving energy
density and the comoving volume element. An increase in the comoving volume translates into an increase in the number counts. This effect is compensates for the decrease in average density and $D(z)$. Both effects produce the observed results in the figures 6 and 7. This behavior is similar to models of quintessence homogeneous [54].

$$10^{13} < M/(h^{-1} M_\odot) < 10^{14}$$

$$10^{14} < M/(h^{-1} M_\odot) < 10^{15}$$

Figure 8: The figures show the evolution of number counts with redshift and the Integrated number counts, when changing the value of the parameter $\Omega_{m0}$, for objects with mass within the range $10^{13} < M/(h^{-1} M_\odot) < 10^{14}$ (top) and for objects with mass within the range $10^{14} < M/(h^{-1} M_\odot) < 10^{15}$ (bottom).

4.1 Cosmological Sensitivity on $\Omega_{m0}$ and $h$

In present section, we describe how variations of the parameters $\Omega_{m0}$ and $h$, affecting the cluster abundance. The dependence of the number counts on the cosmological parameters as $\Omega_{m0}$ and $h$ (and also $\nu$ in our case) is because the equations (4.1),(4.4) and (4.5) depend on these parameters through $\rho$, the factor growth and comoving volumen. The dependence cosmological is implicit, but is very strong. For example: increasing in the value of the matter parameter directly translates into decreased of the comoving volume, this can be seen in the reference [49]. On the other hand, a decrease of the latter amount, leads to a decrease in cluster number counts, and of the studied quantity $dN/dz$ that is directly proportional to the comoving volume.
First we consider the effects of changing $\Omega_{m0}$, which are displayed in figure 8. In left panel are showed results for $dN/dz$ for mass between $10^{13} < M/(h^{-1}M_{\odot}) < 10^{14}$ (top left) and $10^{14} < M/(h^{-1}M_{\odot}) < 10^{15}$ (bottom left). The curves correspond to a flat $RCC$ universe with $h = 0.7, \nu = 10^{-5}$ for all case, and $\Omega_{m0} = 0.20$ (light blue solid), $\Omega_{m0} = 0.24$ (dark blue dashed) and $\Omega_{m0} = 0.30$ (red solid). In right panel shows the total number of clusters $N(z)$ for the same values of parameters. Several conclusions can be drawn from figure 8. Overall, for a decrease in $\Omega_{m0}$ increases the number of cluster at all redshift (and vice versa). The curve closest to $\Lambda CDM$ is the central curve. Note that the dependence on $\Omega_{m0}$ is strong, for instance, a 16.7% decrease in $\Omega_{m0}$ increase the total number of cluster $N(z)$ by 21% for more massive structures (the bottom right panel).

The figura 9 also demonstrates the effects of changing $h$. Comparing the figures 8 and 9, the quantitative behavior of the observables ($N(z)$ and $dN/dz$) under changes in $h$ and $\Omega_{m0}$ are similar: decreasing $h$ increases the total number of clusters, but does not considerably change their redshift distribution for objects with mass within the range $10^{13} < M/(h^{-1}M_{\odot}) < 10^{14}$, but for mass between $10^{14} < M/(h^{-1}M_{\odot}) < 10^{15}$ the change
is greater.

5. Summary and Discussions

In this paper we have determinated the cosmological implications of the RCC model using linear perturbations. We use the longitudinal gauge to determine the density perturbations of matter. In analysing these perturbations we have found, similarly as in the reference [25], that the perturbation of vacuum energy density is proportional at the \( \nu \) parameter (equation 3.18a); thus when \( \nu \) is zero, then is recovered the standard scenario \( \Lambda CDM \).

The linear perturbations theory allows us to calculate the linear growth rate \( g(z) \) and compare our result with other models frequently studied in the literature and with data of growth rate. In left figure 3 we have seen that for values of \( \nu \leq 10^{-4} \) our model predicts a overdensity similar to other models (\( \Lambda CDM \), DGP and scalar-tensor theory). In right figure 3 we observed that for large values of \( \nu \) the predicted \( g(z) \) is well off the data. These three data of \( g(z) \) are not sufficient to rule out a given model. In Figure 4, we compared the matter power spectrum with the SDSS data obtained by Percival et al. [39] and showed that a RCC model with \( \nu \approx 10^{-5} \) is compatible with \( \Lambda CDM \). The best-fit for our free parameter is \( \nu = (1.29 \pm 2.39) \times 10^{-5} \).

We investigated the expected evolution of galaxy cluster abundance in the RCC cosmology. The number counts of collapsed structures are frequently proposed as a tool to probe dark energy models. In the present paper we have investigated, using the \( PS \) mass function, the modifications introduced by a running cosmological constant. Our model predicts a excess of sources when compared with the \( \Lambda CDM \) model. This difference is greater for more massive structures. The number counts predicted by our model is very close from \( \Lambda CDM \) for \( \nu \leq 10^{-5} \).

Therefore, we have shown that there is a dependence significant of clusters number counts in the RCC model through of the amount dark matter coupled to vacuum energy. Increasing the coupling between dark matter to vacuum energy, that is, increasing the value of \( \nu \) (see figures 6 and 7) increase the clusters number counts. This feature is compatible with the clustering properties of dark energy models of quintessence [29]. In our model, this effect is due to the decrease of comoving matter density and increase of comoving volume element when \( \nu \) increases.

In general our results on power spectrum and the linear growth rate are compatible with a large amount of other models. For example, for the coupled quintessence model presented in reference [48] the interaction must be very weak in the order of \( 10^{-3} \) to be compatible with data from 2dFGRS. Recently in reference [51] has been studied the perturbations of the called model \( \Lambda XCDM \), which includes two free parameters, the parameter \( \nu \) that has the same meaning as our case, and a equation of state parameter \( w_X \), the cosmon component. Both components can interact. In reference [51] was done an analysis of linear perturbations of these model, which includes perturbations of the dark component and the results were consistent with data from the 2dFGRS.

In the reference [52] is considered a model of holographic dark energy with infrared decay in CDM. They use three types of cut-off (Hubble, particle and future event horizons)
and for all cases is displayed that there are modes of growth for the density contrast, when the effective equation of state is in the range $-1 < w_{\text{eff}} < -1/3$. The authors, have used a Newtonian approximation, and the dark component is not perturbated, being only for the cut-off of the Hubble horizon that the model implies a dark energy density proportional to the square of the Hubble parameter, similar to our case. We expect that a full relativistic approach of density contrast, whatever in the synchronous or longitudinal gauge, determine results that should deviate little from the cosmological constant.

In reference [53] is considered a model with a cosmological term that decays linearly with the Hubble parameter. In that paper, the authors, consider a relativistic treatment of perturbations in the synchronous gauge and the dark component is also perturbed. They calculate the matter power spectrum and show that their results are inconsistent with 2dFGRS data, since the matter parameter resulting is very large ($\Omega_m \approx 0.7$).

On the other hand, the references [54, 55] explores changes in the number counts predicted for models of homogeneous and inhomogeneous dark energy, that is, two extreme limits for the evolution of dark energy in the overdensity region. In the case of the homogeneous dark energy density the value of overdensity inside is the same as in the background. In the inhomogeneous dark energy there is a collapse with dark matter. The authors show that there is a deviation of up to 15% with respect to $\Lambda CDM$ in the inhomogeneous case and for both types of models the largest deviations are observed for massive structures $M > 10^{16}$. These results are consistent with our calculations.

Finally, in the near future, we plan to further investigate other astrophysical implications of the $RCC$ model. It would be interesting to consider a modification for spherical collapse, where the $\delta_c$ is function of the redshift of collapse. Also can be evaluate the profile of density contrast around the cluster, and supercluster of voids matter, using as first approximation a $NFW$ (Navarro, Finken and While) profile [56]. Another important point is to study the concentration of haloes. For example, following the prescription given in reference [57] we can investigate whether the concentration of haloes in the $RCC$ model decrease with increasing of the mass as in the case of the $\Lambda CDM$ model. In principle, this also could provide limits on the model.

### A. Equations used to model DGP and Scalar-Tensor Theory

In this appendix we write the equations used to determine the growth factor in the DGP model and the scalar-tensor theory. The growth factor is defined in equation (30) and obeys the equation [58, 59]

$$D''(k, a) + \left(\frac{3}{a} + \frac{H'(a)}{H(a)}\right)D'(k, a) - \frac{3\Omega_m a^2}{2a^5 H^2(a)} f(k, a) D(k, a) = 0,$$

(A.1)

with the initial condition $D(a) \approx a$ for $a \approx 0$ (in the matter-dominated era). This equation does not take into account perturbations of dark energy and the anisotropic stress. The function $f(k, a)$ expresses the connection between the metric perturbations with the matter density perturbations. It therefore depends on the particular gravity theory.
A.1 DGP Model

In the DGP model for the case of flatness and when we only have matter on the brane the \( H(z) \) is given by \[ H^{2}_{\text{DGP}} = \sqrt{\Omega_{r}} + \sqrt{\Omega_{m0}a^{-3} + \Omega_{\tau c}}, \] (A.2) where \( \Omega_{\tau c} = \frac{1}{4}(1 - \Omega_{m0})^2 \) and for this theory the \( f(k, a) \) function is given by \[ f(k, a) = \left(1 + \frac{1}{3\beta}\right), \] (A.3) with \[ \beta = 1 - \frac{H_{\text{DGP}}(a)}{H_{0}\sqrt{\Omega_{\tau c}}(1 + \frac{aH'_{\text{DGP}}(a)}{3H_{\text{DGP}}(a)})}. \] (A.4)

A.2 Scalar-Tensor Theory

Scalar-tensor theory is simplest generalization of the General Relativity where the fundamental constants can be variable. The function \( f(k, a) \) in this case is given by \[ f(k, a) = \frac{G_{\text{eff}}(a)}{G_{\text{eff}}(a = 1)} \left(1 + \frac{1}{1 + (k/ma)}\right), \] (A.5) where \( G_{\text{eff}} \) is the effective Newton’s constant when the scalar factor is \( a \) and \( a = 1 \) is the present value, while \( m \) is the mass of the scalar field \( \Phi \) inducing a Yukawa cut-off to the gravitational field. We used a simple ansatz \[ \frac{G_{\text{eff}}(a)}{G_{\text{eff}}(a = 1)} = 1 + \xi(1 - a)^2. \] (A.6)

In the case of the DGP model and scalar-tensor theory the detection of an \( f(k, a) \neq 1 \) would be a signature of alternative theories of gravity. For the numerical calculations we used \( \xi = -0.2 \).

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