Fifth order semi analytical solution of exact Korteweg-de Vries equation

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Abstract. This study concerns on the solution of exact Korteweg de Vries (KdV) equation in its application in generating extreme waves. The method of asymptotic expansion is employed up to the fifth order. In the previous research, the same method was applied up to the third order and fifth order but it only considered the side band solutions. Here, solutions at each order will be analyzed. The existence of resonance terms at the odd orders and side band terms are interesting to observe considering the importance of these quantities in analyzing the wave deformation which link to the phenomenon of wave’s amplitude amplification. Bichromatic signal is used as the initial wave signal as it experiences instability during its propagation which results the amplitude amplification. The amplitude amplification is presented as Maximal Temporal Amplitude (MTA) which is a quantity measuring the highest elevation at every spatial position during the observation time.

1. Introduction

Study of surface waves on liquid medium in an ideal condition have been commonly conducted by using the Laplace equation or full water equation. An ideal fluid is assumed to be homogenous, irrotational, viscous, and inviscid. An extreme behavior happened on the surface of water body is sometimes linked to the soliton phenomenon which occurs due to the nonlinearity of dispersive waves [1]. For technical purposes or offshore structures (like ships or oil/gas platform) test, a wave is generated in a wave tank through the movement of wave makers. The movement of the wave makers is assigned based on the wave characteristics which can be mathematically modeled. There are some mathematical model that describes the propagation of surface wave, e.g. Boussinesq equation, Kadomtsev-Petviashvili (KP) equation, Benjamin Bona Mahony (BBM) equation, Korteweg de Vries (KdV) equation. These equations are simplified equations derived from the full water equation (Laplace equation with the complete boundary conditions) [2].

Many studies have been conducted experimentally [3-5], analytically [6-8], and numerically [9-11] to study the behavior of waves especially extreme waves. One way to study the extreme wave is to apply the asymptotic method to the wave model. Marwan [7] applied the asymptotic series to generate an extreme wave solution from Boussinesq model. In another research, the same method was applied to KdV equation to study the deformation of initially bichromatic waves [12]. In this study, the asymptotic series was expanded up to the third order. In [13], asymptotic expansion up to the fifth
order was applied to study the deformation of wave which was generated by bichromatic signal following KdV equation. It was obtained that the fifth order solution gave higher peaking than one of the third order. Nonlinear evolution of wave group with three frequencies according to KdV model was examined in [6]. The Extreme waves generation using BBM model was also studied in [14,15]; bichromatic and trichromatic signals were assigned as the initial signal due to their propagation which experiences instability triggering an extreme wave. The study of extreme wave through envelope evolution was also carried out not only in water wave realm but also in optical and plasma wave field (see [16-20]).

In this paper, a solution of KdV equation will be developed by using an asymptotic expansion of the elevation \( \eta \) and wavenumber \( k \) up to the fifth order. This study is an extension of the study conducted by Marwan [12] which applied the asymptotic series only up to the third order and the study in [13] which applied the series up to the fifth order but only consider the effect of side bands solution. Here, all solution in each order, the resonance, and side band terms will be further analyzed. Bichromatic waves will be assigned as the input signal. The odd order solution which gives resonance terms will be examined to obtain the maximum position \( (x_{\text{max}}) \) where the maximum peaking happens. The maximum peaking will be observed through the Maximal Temporal Amplitude (MTA) which presents the maximum wave elevation at each spatial position during the observation time [7,12,13]. The amplitude amplification can be measured by comparing the amplitude at the maximum position \( (x_{\text{max}}) \) to the initial amplitude at the initial position \((x = 0)\).

The next section will present the determination of KdV equation’s solution by using asymptotic expansion. Section 3 will presents the explanation of the resonance terms which occur in the odd order solution. In section 4, the derivation of formula to determine the maximum position will be introduced. The last section will conclude the results which have been obtained.

2. Asymptotic solution of KdV equation

Perturbation parameter explains a mathematical model which employs expansion technique concerning perturbation parameter to solve a problem. The physical influence of this parameter can be further studied in Nayfeh [21]. Let \( \varepsilon \) be the perturbation parameter, the asymptotic expansion of wave elevation \( \eta \) concerning the perturbation parameter can be stated as

\[
\eta = \varepsilon \eta_1 + \varepsilon^2 \eta_2 + \varepsilon^3 \eta_3 + \varepsilon^4 \eta_4 + \varepsilon^5 \eta_5,
\]

where \( \varepsilon = a/h \), \( a \) is wave amplitude, \( h \) is water depth, and \( \eta_1 = 2a[\cos \alpha + \cos \beta] \) which represents bichromatic waves with \( \alpha = (k_{\pm}x - \omega t), \beta = (k_{\pm}x - \omega t) \), where \( k \) and \( \omega \) are wavenumber and frequency, respectively. Here, the wavenumber \( (k_{\pm}) \) is also expanded as follow,

\[
k_{\pm} = k_{\pm 0} + \varepsilon k_{\pm 1} + \varepsilon^2 k_{\pm 2} + \varepsilon^3 k_{\pm 3} + \varepsilon^4 k_{\pm 4}.
\]

In this study, we consider the wave propagation based on the exact KdV equation [12,22] :

\[
\partial_t \eta + i\Omega(-i\partial_x)\eta + \mu \eta \partial_x \eta = 0,
\]

where \( \Omega \) is dispersion relation reading \( \Omega(k) = k \sqrt{\tanh \frac{h}{k}} \). The relation dispersion \( \Omega(k_{\pm}) \) itself is expanded in Taylor series.

Equation (1) and (2) are substituted in to the equation (3) and then separated based on the order of \( \varepsilon \). The solutions of each order are called as bound wave and are represented as follow,

\[
\eta_2 = 2a^2[A_1 \cos 2\alpha + A_2 \cos 2\beta + A_3 \cos(\alpha + \beta) + A_4 \cos(\alpha - \beta)],
\]

\[
\eta_3 = 2a^3[B_1 \cos 3\alpha + B_2 \cos 3\beta + B_3 \cos(2\alpha + \beta) + B_4 \cos(\alpha + 2\beta) + B_5 \cos(2\alpha - \beta) + B_6 \cos(\alpha - 2\beta)],
\]

\[
\eta_4 = 2a^4[C_1 \cos 4\alpha + C_2 \cos 2\alpha + C_3 \cos 4\beta + C_4 \cos 2\beta + C_5 \cos(3\alpha + \beta) + C_6 \cos(2\alpha + 2\beta) + C_7 \cos(\alpha + 3\beta) + C_8 \cos(\alpha + \beta) + C_9 \cos(3\alpha - \beta) + C_{10} \cos(2\alpha - 2\beta) + C_{11} \cos(3\alpha - 3\beta) + C_{12} \cos(\alpha - \beta)],
\]

\[
\eta_5 = 2a^5[D_1 \cos 5\alpha + D_2 \cos 3\alpha + D_3 \cos 5\beta + D_4 \cos 3\beta + D_5 \cos(4\alpha + \beta) + D_6 \cos(3\alpha + 2\beta) + D_7 \cos(2\alpha + 3\beta) + D_8 \cos(\alpha + 4\beta) + D_9 \cos(4\alpha - \beta) + D_{10} \cos(3\alpha - 2\beta) + D_{11} \cos(2\alpha - 3\beta) + D_{12} \cos(\alpha - 3\beta) + D_{13} \cos(2\alpha - \beta) + D_{14} \cos(2\alpha + \beta) + D_{15} \cos(\alpha - \beta)].
\]
\[ \eta_5 = 2a^5 \left[ D_1 \cos 5\alpha + D_2 \cos 3\alpha + D_3 \cos 5\beta + D_4 \cos 3\beta + D_5 \cos (4\alpha + \beta) + D_6 \cos (3\alpha + 2\beta) + D_7 \cos (2\alpha + 3\beta) + D_8 \cos (2\alpha + \beta) + D_9 \cos (\alpha + 4\beta) + D_{10} \cos (\alpha + 2\beta) + D_{11} \cos (4\alpha - \beta) + D_{12} \cos (3\alpha - 2\beta) + D_{13} \cos (2\alpha - 3\beta) + D_{14} \cos (2\alpha - \beta) + D_{15} \cos (\alpha - 4\beta) + D_{16} \cos (\alpha - 2\beta) \right] , \]

where

\[ A_1 = \mu \left( \frac{k_{4\alpha}}{2\omega_1 - \Omega (2\kappa_{4\alpha})} \right), \]

\[ A_2 = \mu \left( \frac{k_{4\beta}}{2\omega_2 - \Omega (2\kappa_{4\beta})} \right), \]

\[ B_1 = \mu \left( \frac{A_1 [3\kappa_{4\alpha}]}{3\omega_1 - \Omega (3 \kappa_{4\alpha})} \right), \]

\[ B_2 = \mu \left( \frac{A_2 [3\kappa_{4\beta}]}{3\omega_2 - \Omega (3 \kappa_{4\beta})} \right), \]

\[ B_3 = \mu \left( \frac{[A_1 + A_3] [2k_{4\alpha} + k_{4\alpha}]}{(2\omega_1 + \omega_2) - \Omega (2k_{4\alpha} + \kappa_{4\alpha})} \right), \]

\[ B_4 = \mu \left( \frac{[A_1 + A_3] [2k_{4\beta} + k_{4\beta}]}{(2\omega_1 + \omega_2) - \Omega (2k_{4\beta} + \kappa_{4\beta})} \right), \]

\[ B_5 = \mu \left( \frac{[A_1 + A_3] [2k_{4\alpha} + k_{4\beta}]}{(2\omega_1 + \omega_2) - \Omega (2k_{4\alpha} + \kappa_{4\beta})} \right), \]

\[ C_1 = \mu \left( \frac{B_1 [4k_{4\alpha}] + [A_2^2 [2k_{4\alpha}]]}{4\omega_1 - \Omega (4 \kappa_{4\alpha})} \right), \]

\[ C_2 = \mu \left( \frac{B_2 [4k_{4\beta}] + [A_1^2 [2k_{4\beta}]]}{4\omega_2 - \Omega (4 \kappa_{4\beta})} \right), \]

\[ C_3 = \mu \left( \frac{B_3 [4k_{4\alpha} + 2k_{4\beta}]}{2\omega_1 - \Omega (2 \kappa_{4\alpha} + \kappa_{4\beta})} \right), \]

\[ C_4 = \mu \left( \frac{B_4 [4k_{4\beta} + 2k_{4\alpha}]}{2\omega_2 - \Omega (2 \kappa_{4\alpha} + \kappa_{4\beta})} \right), \]

\[ C_5 = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3] [3k_{4\alpha} + k_{4\alpha}]}{(3\omega_1 + \omega_2) - \Omega (3 \kappa_{4\alpha} + \kappa_{4\alpha})} \right), \]

\[ C_6 = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3] [2k_{4\alpha} + 2k_{4\beta}]}{(2\omega_1 + 2\omega_2) - \Omega (2 \kappa_{4\alpha} + 2 \kappa_{4\beta})} \right), \]

\[ C_7 = \mu \left( \frac{[A_1 + A_3] [2k_{4\alpha} + k_{4\alpha}]}{(2\omega_1 + \omega_2) - \Omega (2 \kappa_{4\alpha} + \kappa_{4\beta})} \right), \]

\[ C_8 = \mu \left( \frac{[A_1 + A_3] [2k_{4\alpha} + k_{4\beta}]}{(2\omega_1 + \omega_2) - \Omega (2 \kappa_{4\alpha} + 2 \kappa_{4\beta})} \right), \]

\[ C_9 = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3] [3k_{4\alpha} - k_{4\alpha}]}{(3\omega_1 + \omega_2) - \Omega (3 \kappa_{4\alpha} - \kappa_{4\alpha})} \right), \]

\[ C_{10} = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3] [2k_{4\alpha} - 2k_{4\beta}]}{(2\omega_1 - 2\omega_2) - \Omega (2 \kappa_{4\alpha} - 2 \kappa_{4\beta})} \right), \]

\[ C_{11} = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3] [3k_{4\alpha} - 3k_{4\alpha}]}{(3\omega_1 - 3\omega_2) - \Omega (3 \kappa_{4\alpha} - 3 \kappa_{4\alpha})} \right), \]

\[ C_{12} = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3] [3k_{4\alpha} - k_{4\alpha}]}{(3\omega_1 - \omega_2) - \Omega (3 \kappa_{4\alpha} - \kappa_{4\alpha})} \right), \]

\[ D_1 = \mu \left( \frac{[A_1 + A_3] [5k_{4\alpha}]}{5\omega_1 - \Omega (5 \kappa_{4\alpha})} \right), \]

\[ D_2 = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3 + C_1 + C_3 + C_{11}][3k_{4\alpha}]}{3\omega_1 - \Omega (3 \kappa_{4\alpha})} \right), \]

\[ D_3 = \mu \left( \frac{[A_1 + A_3] [5k_{4\beta}]}{5\omega_2 - \Omega (5 \kappa_{4\beta})} \right), \]

\[ D_4 = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3 + C_1 + C_3 + C_{11}][3k_{4\beta}]}{3\omega_2 - \Omega (3 \kappa_{4\beta})} \right), \]

\[ D_5 = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3 + C_1 + C_3][4k_{4\alpha} + \kappa_{4\alpha}]}{(4\omega_1 + \omega_2) - \Omega (4 \kappa_{4\alpha} + 2 \kappa_{4\alpha})} \right), \]

\[ D_6 = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3 + C_1 + C_3 + C_{11}][3k_{4\alpha} + 2k_{4\alpha}]}{(3\omega_1 + 2\omega_2) - \Omega (3 \kappa_{4\alpha} + 2 \kappa_{4\alpha})} \right), \]

\[ D_7 = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3 + C_1 + C_3][4k_{4\alpha} + 3k_{4\alpha}]}{(4\omega_1 + 3\omega_2) - \Omega (4 \kappa_{4\alpha} + 3 \kappa_{4\alpha})} \right), \]

\[ D_8 = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3 + C_1 + C_3][2k_{4\alpha} + 2k_{4\alpha}]}{(2\omega_1 + 3\omega_2) - \Omega (2 \kappa_{4\alpha} + 3 \kappa_{4\alpha})} \right), \]

\[ D_9 = \mu \left( \frac{[A_1 + A_3] [4k_{4\beta}]}{(3\omega_1 + \omega_2) - \Omega (2 \kappa_{4\alpha} + 2 \kappa_{4\beta})} \right), \]

\[ D_{10} = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3 + C_1 + C_3][4k_{4\alpha} + 2k_{4\alpha}]}{(3\omega_1 + 2\omega_2) - \Omega (3 \kappa_{4\alpha} + 2 \kappa_{4\alpha})} \right), \]

\[ D_{11} = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3 + C_1 + C_3][4k_{4\alpha} - k_{4\alpha}]}{(4\omega_1 - \omega_2) - \Omega (4 \kappa_{4\alpha} - \kappa_{4\alpha})} \right), \]

\[ D_{12} = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3 + C_1 + C_3][4k_{4\alpha} - 2k_{4\alpha}]}{(3\omega_1 - 2\omega_2) - \Omega (3 \kappa_{4\alpha} - 2 \kappa_{4\alpha})} \right), \]

\[ D_{13} = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3 + C_1 + C_3][4k_{4\alpha}]}{(4\omega_1 - \omega_2) - \Omega (4 \kappa_{4\alpha} - \kappa_{4\alpha})} \right), \]

\[ D_{14} = \mu \left( \frac{[A_1 + A_3 + B_1 + B_3 + C_1 + C_3][4k_{4\alpha}]}{(4\omega_1 - \omega_2) - \Omega (4 \kappa_{4\alpha} - \kappa_{4\alpha})} \right), \]
\[ D_{15} = \mu \left( \frac{[A_2 B_6 + A_3 B_2 + C_1 + C_{11}][k_{\pm 0} - 4k_{\pm 0}]}{(\omega_1 - \omega_2) - \Omega(k_{\pm 0} - 2k_{\pm 0})} \right), \]

\[ D_{16} = \mu \left( \frac{[A_1 B_4 + A_4 B_2 + A_3 B_5 + C_4 + C_{10} + C_11 + C_{12}][k_{\pm 0} - 2k_{\pm 0}]}{(\omega_1 - 2\omega_2) - \Omega(k_{\pm 0} - 2k_{\pm 0})} \right). \]

It is obtained that \( \Omega(k_{\pm 0}) = \omega_1 \) and \( \Omega(k_{\pm 0}) = \omega_2 \). Moreover, the correction wavenumbers are obtained as follow,

\[
k_{\pm 1} = k_{\pm 3} = 0, \quad k_{\pm 2} = -\frac{\alpha^2 \mu (A_{1,2} + A_{3} + A_{4}) k_{\pm 0}}{\Omega'(k_{\pm 0})},
\]

\[
k_{\pm 4} = -\alpha^4 \mu \frac{[A_1 B_{2,3} + A_2 B_{3,2} + A_2 B_{4,3} + A_3 B_{3,4} + A_{41} B_5 + C_{2,4} + C_7 + C_{12}][k_{\pm 0} + k_{\pm 2} + k_{\pm 4}]}{\Omega''(k_{\pm 0})},
\]

To retain the bichromatic wave form at \( x = 0 \), it is defined free waves \((\eta_{f})\) at the second to the fifth order. Free waves have the same amplitudes and frequencies as the corresponding bound waves, but the wave numbers are obtained from the inverse of the dispersion relation with respect to the frequencies at each term in each order, \( \Omega^{-1}(\omega) \). Therefore, the semi analytical solution of KdV equation up to the fifth order reads

\[
\eta = \epsilon \eta_1 + \epsilon^2 (\eta_2 - \eta_{2,f}) + \epsilon^3 (\eta_3 - \eta_{3,f}) + \epsilon^4 (\eta_4 - \eta_{4,f}) + \epsilon^5 (\eta_5 - \eta_{5,f}).
\]

In order to investigate the amplification of the wave during its propagation, a quantity called Maximum Temporal Amplitude (MTA) is introduced [12]. MTA measures the wave’s highest elevation at every position during the observation time. MTA can be stated as \( MTA(x) = \max \eta(x, t) \). Assigning parameter values \( h = 5 \) m, \( a = 0.08 \) m, \( \omega_1 = 3.30 \) rad/s, \( \omega_2 = 2.99 \) rad/s, and the KdV equation’s nonlinear coefficient \( \mu = 3/2 \), the MTA plot is presented in Figure 1. The wave signals at some position are exhibited in Figure 2. It can be observed that the waves experience amplification during its propagation, reach its maximum peaking at \( x = 116,57 \) m, and after the maximum peaking, the wave start decreasing.

**Figure 1.** The MTA plot of bichromatic wave signal for \( h = 5 \) m, \( a = 0.08 \) m, \( \omega_1 = 3.30 \) rad/s, \( \omega_2 = 2.99 \) rad/s.
3. Resonance and Side Band Terms

The interactions of waves which happen at odd orders, i.e. third and fifth order, yield certain solutions which contain same frequencies as the ones at the first order. These same frequencies may cause a resonance among the input waves and the nonlinear waves. This resonance deforms the generation of the extreme waves. A way to avoid the resonance is to expand the wave number as presented in the equation (3). Through this way, the values of correction wavenumbers \( k_2 \), \( k_4 \), \( k_6 \), and \( k_8 \) are obtained such that the values of \( k_4 \) and \( k_6 \) can be determined.

On the third order solution, the wave’s phase angle \( 2(\alpha - \beta) \) has a frequency which is close to the term \( \alpha \) (frequency \( \omega_1 \)) as much as \( \Delta \omega = \omega_1 - \omega_2 \), while the term \( 2(\beta - \alpha) \) is close to the term \( \beta \) (frequency \( \omega_2 \)) as much as \( \Delta \omega \). On the fifth order, the term \( 3(\alpha - \beta) \) has a frequency which is close to term with phase angle \( \alpha - 2\beta \) at the third order as much as \( \Delta \omega \) and to the term with phase angle \( \alpha \) at the first order as much as 2\( \Delta \omega \), while the term \( 2(\beta - \alpha) \) resonates with the term with phase angle \( 2(\beta - \alpha) \) at the third order and has close frequency to the term \( \alpha \) at the first order as much as \( \Delta \omega \). On the fifth order, the term with phase angle \( 3(\beta - 2\alpha) \) has frequency which is close to the term with phase angle \( 2(\beta - \alpha) \) at the third order as much as \( \Delta \omega \) and to the term with phase angle \( \beta \) at the first order as much as 2\( \Delta \omega \), while the term \( 2(\beta - \alpha) \) resonates with the term \( 2(\beta - \alpha) \) at the third order and has close frequency to the term \( \beta \) at the first order as much as \( \Delta \omega \). The existence of these side band terms will yield a unification of waves at various order and energy focusing. The energy focusing is a strong cause of the forming of the soliton wave.

4. Maximum peaking position

The maximum peaking position is obtained from the side band terms \( (\eta_{3,SB}) \) at the odd orders including the first order. The elevation of the odd orders is presented as

\[
\eta = \eta_1 + \eta_{3,SB} - \eta_{3,SB,f} + \eta_{5,SB} - \eta_{5,SB,f}.
\]

Hence,
\[ \eta = \cos \left[ 2 \cos(kx - vt) \left( a + 4e^{iQ}(D_{12} \cos 4 \left( k - \frac{1}{2}Q \right)x + 2\cos(2kx - vt) \right) \right] - 2D_{12}e^{iQ} \cos 4 \left( k - \frac{1}{2}Q \right)x - vt \cos 3(kx - vt) \\
+ \cos \left( [R + Q]x - 3vt \right) + 4 \cos \left( [R - Q]x + 3vt \right) \cos 2\sigma t \\
+ 4 \cos 3 \left( k - \frac{1}{3}Q \right)x - vt \cos 2\bar{k}x \right] \]

\[ - 2e^{-iR} (D_{12} \cos \left( [R' + R]x - 5vt \right) + 2D_{13} \cos \left( [R' - R]x + 5vt \right) \cos 2\bar{o}t) \]

(4)

with \( P = (kx - \sigma vt) \), \( Q = \frac{1}{2}([R - \bar{k}]x \), dan \( R = \frac{1}{2}([R' - \bar{k}]x \). In the equation (4), there are terms containing waves with terms \( e^{iQ} \) and \( e^{-iR} \) which only depends on the position. The two terms will affect the carrier modulation and determine the maximum peaking position, \( x_{\text{max}} \). Based on this criterion the position of the maximum amplification is

\[ x_{\text{max}} = \frac{\pi}{k_{\text{MTA}}} \]

where \( k_{\text{MTA}} = \frac{1}{2} \left( \frac{[R - \bar{k}] + ([R' - \bar{k}])}{2} \right) \), \( \bar{R} = \frac{1}{2} \left( \omega^{-1}(2\omega_1 - \omega_2) + \omega^{-1}(\omega_1 - 2\omega_2) \right) \), \( \omega \), \( \omega_1 \) and \( \omega_2 \). The extreme position is affected by the half of the given frequency different \( v = \frac{1}{2} \left( \omega_1 - \omega_2 \right) \).

5. Conclusion

We have derived the semi analytical solution of KdV equation up to the fifth order. It has been observed that the higher orders affect the first maximum peaking position. Based on the simulation conducted, the first peaking occurs at \( x = 116.57 \) m. This result is slightly closer to the initial position where the wave is generated compared to the result of the third order solution, which is 118 m, and side band fifth order solution, which is 118.22 m. It is obtained that the amplitude amplification factor is 2.75. The value of \( k_{\pm 4} \) depends on the values of \( k_{\pm 1} \) and \( k_{\pm 2} \) while the value of \( k_{\pm 2} \) only depends on \( k_{\pm 0} \). This describes the number of waves which are formed at each position unit. The increase at the value of \( k_{\pm 4} \) obtained in this study is different from the previous result. This result is caused by the expansion of \( \Omega \) to the higher order.

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