Neutrino Large Mixing in Universal Yukawa Coupling Model with Small Violation

T. Teshima,† T. Asai, and Y. Abe

Department of Applied Physics, Chubu University, Kasugai 487-8501, Japan

Abstract

We have analyzed the possibility that the universal Yukawa coupling (democratic mass matrix) with small violations of Dirac and Majorana neutrinos can induce the large mixing of neutrinos through the seesaw mechanism. The possibility can be achieved by the condition that the violation parameters of Majorana neutrinos are sufficiently smaller than the violation parameters of Dirac neutrinos. Allowed regions of the violation parameters producing the observed neutrino mass hierarchy and large neutrino mixing are not so restricted at present in contrast to the violation parameters for quark sector. In order to obtain the distinctive features in violation parameters for neutrino sector as quark sector, one should take more precise data concerned with the neutrino experiments, for example mass values of neutrinos, mixing parameters of $U_{\text{MNS}}$ and CP violation phases.

PACS numbers: 12.15.Ff, 13.15.+g, 14.60.Pq

†Electronic address: teshima@isc.chubu.ac.jp
I. INTRODUCTION

Recent neutrino experiment in Super-Kamiokande \cite{1} on atmospheric neutrinos has confirmed the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation being large mixing, $\sin^2 2\theta_{\text{atm}} > 0.88$, and found the range of the mass parameter $\Delta m^2_{\text{atm}}$ to be $(1.5 - 5) \times 10^{-3}$ eV$^2$. Solar neutrino experiments by the Super-Kamiokande collaboration \cite{2} favor the large mixing angle (LMA) MSW solution, although there can be four solutions, the small mixing angle (SMA) MSW solution, the low $\Delta m^2$ (LOW) solution and the vacuum (VAC) solution. The mass and mixing parameter ($\Delta m^2_{\odot}$, $\sin^2 2\theta_{\odot}$) for LMA-MSW solution is $(3.2 \times 10^{-5}\text{eV}^2, 0.75)$ \cite{2}. As we consider the three-flavor neutrinos, we can set $\Delta m^2_{\text{atm}}$ to $\Delta m^2_{23}$ and $\Delta m^2_{\odot}$ to $\Delta m^2_{12}$, and $\sin^2 2\theta_{\text{atm}} = \sin^2 2\theta_{23} \sim 1$ and $\sin^2 2\theta_{\odot} = \sin^2 2\theta_{12} \sim 1$. The remaining mixing angle $\theta_{13}$ is restricted to $\sin^2 2\theta_{13} < 0.10$ by the CHOOZ experiment \cite{3}.

To answer a question why the neutrino sector mixings expressed by a $U_{\text{MNS}}$ matrix \cite{4} are so large in contrast to the small mixings of the quark sector expressed by a $U_{\text{CKM}}$ matrix is one of the most challenging issues at present. Furthermore, the values of neutrino masses predicted in neutrino oscillation lead to the next question why the neutrino masses are so small compared to other Fermion masses. In order to explain the second question, almost authors assume the seesaw mechanism \cite{5} introducing the heavy right handed Majorana neutrinos. For the explanation of first questions, there are two scenarios; first \cite{3, 8} is to use the so-called Froggatt Nielsen mechanism \cite{8} in order to make the mass hierarchy of lepton masses, and second \cite{3, 10, 11} to use the democratic mass matrix with small violations in order to make the mass hierarchy of lepton masses. We call the second type of mass matrix "universal type mass matrix" hereafter. In the first scenario, the matrix elements except 33 element of mass matrix are suppressed by the powers of spontaneous breaking of some family symmetry. Then, we call the first type of mass matrix "family symmetry dependent type mass matrix".

If one assumes a standpoint of grand unified theory (GUT), the mass matrices of quarks and charged leptons should be the same type. Mass matrix for Dirac neutrino in models should have the same type of quark and charged lepton mass matrix. Furthermore, it is natural to assume that the mass matrix of the Majorana neutrino is the same type of mass matrix for other Dirac Fermion. In scenarios assuming the family symmetry dependent type mass matrix, a few models \cite{7} adopt the mass matrix of the Majorana neutrino similar to
the type of mass matrix for other Dirac Fermions. In models assuming the universal type mass matrix, literature \[10, 11\] adopts the mass matrix for the Majorana neutrino similar to the type of mass matrix for other Dirac Fermion.

Works in Refs. \[10, 11\] use the universal Yukawa coupling (democratic mass matrix) with small violations for Dirac neutrino and also for Majorana neutrino, and discuss the possibility producing the neutrino large mixing through the seesaw mechanism. We call this possibility “neutrino large mixing induced through seesaw mechanism”. Neutrino mixing matrix $U_{\text{MNS}}$ is given by a product of an unitary matrix $U_l$ diagonalizing the charged lepton mass and a unitary matrix $U_\nu$ diagonalizing the effective neutrino mass $M_D M_M^{-1} M_D^T$, where $M_D$ and $M_M$ are the Dirac neutrino and the Majorana neutrino mass matrix, respectively.

The charged lepton mass matrix is the democratic mass matrix $M_0 = m \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ in the limit neglecting small violations, and this mass matrix is diagonalized by the unitary matrix

$$U_0 = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}$$

in the limit neglecting small violations. For the neutrino masses, the effective neutrino mass $M_{\text{eff}}$ is produced through the seesaw mechanism as $M_{\text{eff}} = M_D M_M^{-1} M_D^T$. Though these neutrino mass matrices $M_D$ and $M_M$ are democratic type mass matrices, the effective neutrino mass matrix $M_{\text{eff}}$ could be almost diagonal if the small violations in $M_M$ satisfy a certain condition as discussed in section III. If the effective neutrino mass matrix $M_{\text{eff}}$ is almost diagonal, the transforming matrix $U_\nu$ for the neutrino is close to the unit matrix. Then the neutrino mixing matrix $U_{\text{MNS}}$ satisfies $U_{\text{MNS}} \sim U_0$, and large neutrino mixing is realized.

II. NEUTRINO LARGE MIXING AND VIOLATION OF UNIVERSAL YUKAWA COUPLING

In the universal Yukawa coupling scenario, the main mass hierarchy is produced by the universality of the Yukawa coupling (democratic mass matrix \[12\]), and another mass hierarchy is produced by small violations added to the democratic mass matrix. This violation is considered to be just like the $SU(3)$ violation in the hadron spectroscopy and hadron decay processes. This $SU(3)$ violation is considered to be produced by the quark mass difference
(violation from the $SU(3)$ symmetry) and quark dynamics. Similarly, the violations added to the democratic mass matrix are considered to be produced by some violation from a horizontal symmetry and some dynamics of the quarks and leptons. Because the origin of the violation is not clear at present, we treat these small violations as free parameters.

We assume the following mass matrices for charged leptons and Dirac neutrinos:

$$
M_l = \Gamma_l \begin{pmatrix} 1 & 1 - \delta_l^1 & 1 - \delta_l^2 \\ 1 - \delta_l^1 & 1 & 1 - \delta_l^3 \\ 1 - \delta_l^2 & 1 - \delta_l^3 & 1 \end{pmatrix}, \quad \delta_l^i \ll 1 \ (i = 1, 2, 3),
$$

(1)

$$
M_D = \Gamma_D \begin{pmatrix} 1 & 1 - \delta_D^1 & 1 - \delta_D^2 \\ 1 - \delta_D^1 & 1 & 1 - \delta_D^3 \\ 1 - \delta_D^2 & 1 - \delta_D^3 & 1 \end{pmatrix}, \quad \delta_D^i \ll 1 \ (i = 1, 2, 3).
$$

(2)

This violation pattern is similar to that of the quark sector (neglecting the phases)

$$
M_q = \Gamma_q \begin{pmatrix} 1 & 1 - \delta_q^1 & 1 - \delta_q^2 \\ 1 - \delta_q^1 & 1 & 1 - \delta_q^3 \\ 1 - \delta_q^2 & 1 - \delta_q^3 & 1 \end{pmatrix}, \quad \delta_q^i \ll 1 \ (q = u, d, i = 1, 2, 3).
$$

(3)

These mass matrices are diagonalized by a unitary matrix $U(\delta_1, \delta_2, \delta_3)$ close to $U_0$, i.e.,

$$
U(\delta_1, \delta_2, \delta_3) \sim U_0 = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}.
$$

(4)

Eigenvalues are obtained as

$$
m_1 \approx \left[ \frac{1}{3}(\delta_1 + \delta_2 + \delta_3) - \frac{1}{3}\xi \right] \Gamma \approx \delta_1 \Gamma,
$$

$$
m_2 \approx \left[ \frac{1}{3}(\delta_1 + \delta_2 + \delta_3) + \frac{1}{3}\xi \right] \Gamma \approx \frac{2}{3}(\delta_2 + \delta_3) \Gamma,
$$

$$
m_3 \approx \left[ 3 - \frac{2}{3}(\delta_1 + \delta_2 + \delta_3) \right] \Gamma \approx 3\Gamma,
$$

(5)

where

$$
\xi = \left[ (\delta_2 + \delta_3 - 2\delta_1)^2 + 3(\delta_2 - \delta_3)^2 \right]^{1/2}.
$$

(6)
In the same democratic mass matrix scenario, Branco et al. [10] assumed the following pattern of violations for charged leptons $l$ and Dirac neutrinos $D$.

$$
M_i = \Gamma_i \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 + \varepsilon_i^l & 1 \\
1 & 1 & 1 + \varepsilon^D_i
\end{pmatrix}, \quad \varepsilon_{i, l, D}^l \ll 1 \quad (i = l, D).
$$

(7)

For neutrino masses, we use the seesaw mechanism. Then the effective left-handed neutrino masses are given as

$$
M^\nu_{\text{eff}} = M_D^\nu M_M^\nu -1 (M_D^\nu)^t,
$$

(8)

using the right-handed Majorana neutrino mass matrix $M^\nu_M$. For this Majorana neutrino mass matrix, we assume the following democratic mass matrix with small violations in all matrix elements except for 33 element

$$
M^\nu_M = \Gamma^\nu_M \begin{pmatrix}
1 - \Delta^\nu_1 & 1 - \Delta^\nu_2 & 1 - \Delta^\nu_3 \\
1 - \Delta^\nu_2 & 1 - \Delta^\nu_4 & 1 - \Delta^\nu_5 \\
1 - \Delta^\nu_3 & 1 - \Delta^\nu_4 & 1
\end{pmatrix}, \quad \Delta^\nu_i \ll 1 \quad (i = 1, 2, 3, 4, 5),
$$

(9)

adding breaking terms to the $(1,1)$ and $(2,2)$ elements in order to keep the generality. This pattern of violation is most general form on the left-right symmetry scenario. Branco et al. [10] assumed the pattern of violations similar to their charged lepton and Dirac neutrino mass matrices for Majorana neutrinos $M^\nu_M$.

$$
M^\nu_M = \Gamma^\nu_M \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 + \varepsilon^M_1 & 1 \\
1 & 1 & 1 + \varepsilon^M_2
\end{pmatrix}, \quad \varepsilon_{i, l}^M \ll 1 \quad (i = 1, 2).
$$

(10)

The charged lepton mass matrix $M^l$ in Eq. (1) is diagonalized by the unitary matrix $U^l$ close to $U_0$ in Eq. (4). Thus if the effective left-handed Majorana neutrino mass matrix $M^\nu_{\text{eff}}$ is diagonalized by the unitary matrix $U^\nu$ close to unit matrix, the neutrino mixing matrix $M_{\text{MNS}}$ defined by $U_{\text{MNS}} = U^l U^\nu^t$ is close to the unitary matrix $U_0$ as

$$
U_{\text{MNS}} \sim U_0.
$$

(11)

This result gives the large $e-\mu$ and $\mu-\tau$ mixing, thus the neutrino bimaximal mixing is induced from seesaw mechanism. Main purpose of this work is to show that one can obtain the allowed values of violation parameters satisfying the condition that the unitary matrix $U^\nu$ diagonalizing the effective neutrino mass matrix $M^\nu_{\text{eff}}$ becomes nearly unit matrix.
III. VIOLATION PARAMETERS OF DIRAC AND MAJORANA NEUTRINO MASS MATRIX

A. Neutrino Large Mixing induced through Seesaw Mechanism

First, we study a general property of the \( \mathbf{M}^{\text{eff}} = \mathbf{M}_D \mathbf{M}_M^{-1} \mathbf{M}_D \mathbf{M}_M^T \), concerned with violation parameters in \( \mathbf{M}_D \) and \( \mathbf{M}_M \), where we suppressed the superscript \( \nu \) in \( \mathbf{M}_D \) and \( \mathbf{M}_M \). We represent the mass matrix \( \mathbf{M}_D \) and \( \mathbf{M}_M \) as

\[
\mathbf{M}_D = \Gamma_D (\Sigma + \mathbf{P}_D), \quad \mathbf{M}_M = \Gamma_M (\Sigma + \mathbf{P}_M),
\]

where

\[
\Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},
\]

and \( \mathbf{P}_D \) and \( \mathbf{P}_M \) are matrices of violations as

\[
P_D = \begin{pmatrix} 0 & -\delta_1 & -\delta_2 \\ -\delta_1 & 0 & -\delta_3 \\ -\delta_2 & -\delta_3 & 0 \end{pmatrix}, \quad P_M = \begin{pmatrix} -\Delta_1 & -\Delta_2 & -\Delta_3 \\ -\Delta_2 & -\Delta_4 & -\Delta_5 \\ -\Delta_3 & -\Delta_5 & 0 \end{pmatrix},
\]

for our analysis, and

\[
P_D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon_D^P & 0 \\ 0 & 0 & \varepsilon_D^P \end{pmatrix}, \quad P_M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon_M^P & 0 \\ 0 & 0 & \varepsilon_M^P \end{pmatrix},
\]

for Branco’s analysis [10].

Inverse matrix of \( \mathbf{M}_M \) in \( \mathbf{M}^{\text{eff}} \) is

\[
\mathbf{M}_M^{-1} = \Gamma_M^{-1} (\Sigma + \mathbf{P}_M)^{-1} = \frac{1}{\Gamma_M \mathbf{D}_2 + \mathbf{D}_3} (\mathbf{L} + \mathbf{Q}),
\]

where \( \mathbf{D}_2 \) and \( \mathbf{D}_3 \) are quadratic and cubic polynomials of violation parameters \( \Delta_i \) or \( \varepsilon_i^M \) in determinant of matrix \( \Sigma + \mathbf{P}_M \), respectively. \( \mathbf{L} \) and \( \mathbf{Q} \) are matrices with linear and quadratic elements of violation parameters \( \Delta_i \) or \( \varepsilon_i^M \) in \( \mathbf{P}_M \). Using the relations as shown in literature [10],

\[
\Sigma \mathbf{L} = \mathbf{L} \Sigma = 0, \quad \sum_{i,j} Q_{ij} = \mathbf{D}_2, \quad \Sigma \mathbf{Q} \Sigma = \mathbf{D}_2 \Sigma,
\]

(16)
we can obtain a general expression for $M_{\text{eff}}$

$$M_{\text{eff}} = \frac{\Gamma_2^2}{\Gamma_M D_2 + D_3} [D_2 \Sigma + P_D L P_D + \Sigma Q P_D + P_D Q \Sigma + P_D Q P_D].$$  \hspace{1cm} (17)$$

In Eq.(17), the first term is quadratic in $\Delta_i$ or $\varepsilon_i^M$ and the second term is linear in $\Delta_i$ or $\varepsilon_i^M$ and quadratic in $\delta_i$ or $\varepsilon_i^D$, and the third, forth and fifth terms are quadratic in $\Delta_i$ or $\varepsilon_i^M$ times linear, linear and quadratic in $\delta_i$ or $\varepsilon_i^D$, respectively. Thus, if the smallness of the small violation parameters $\Delta_i$ or $\varepsilon_i^M$ and $\delta_i$ or $\varepsilon_i^D$ are same order, the first term is dominant over other terms. In this case, $M_{\text{eff}}$ is close on the democratic type mass matrix, and then $U^\nu$ is close on $U_0$ and $U_{\text{MNS}}$ cannot give the large mixing. However, if small violation parameters $\delta_i$ and $\Delta_i$ or $\varepsilon_i^M$ and $\varepsilon_i^D$ are satisfy the relation

$$\Delta_i \ll \delta_i \ll 1, \quad \varepsilon_i^M \ll \varepsilon_i^D,$$  \hspace{1cm} (18)$$

the second term is dominant over other terms. In this case, $M_{\text{eff}}$ can be apart from the democratic mass matrix and can produce the unitary matrix diagonalizing the $M_{\text{eff}}$ close to the unit matrix. Thus we can induce the neutrino bi-maximal mixing through the seesaw mechanism adopting the violation parameters satisfying Eq. (18).

\textbf{B. Numerical Result}

Here, we would like to each allowed numerical values of violation parameters in Dirac and Majorana neutrino mass matrices producing the observed neutrino mass hierarchy and large neutrino mixing. Solar neutrino mass and atmospheric neutrino mass are observed as $\Delta m^2_{\odot} \sim 10^{-5} - 10^{-4}$ eV$^2$ and $\Delta m^2_{\text{atm}} \sim (1.5 - 5) \times 10^{-5}$ eV$^2$, respectively, then

$$\frac{m_{\nu_e}}{m_{\nu_\mu}} < 0.34, \quad 0.03 < \frac{m_{\nu_e}}{m_{\nu_\tau}} < 0.3.$$  \hspace{1cm} (19)$$

For $U^\nu$, we set the off diagonal matrix elements smaller than 0.15,

$$\text{off diagonal element of } U^\nu \leq 0.15.$$  \hspace{1cm} (20)$$

In our previous analysis \[\Box\], we searched the allowed numerical values of violation parameters fixing a condition $D_2 = 0$ in Eq. (17) in order to make a democratic part $D_2 \Sigma$ in Eq. (17) not affect. In present analysis, we would not impose such condition.
1. Non-diagonal Violation Parameter Case

First, we analyze the Dirac and Majorana mass matrices with non-diagonal violation parameters as

\[
P_D = \begin{pmatrix}
0 & -\delta_1 & -\delta_2 \\
-\delta_1 & 0 & -\delta_3 \\
-\delta_2 & -\delta_3 & 0
\end{pmatrix},
\]

\[
P_M = \begin{pmatrix}
-\Delta_1 & -\Delta_2 & -\Delta_3 \\
-\Delta_2 & -\Delta_4 & -\Delta_5 \\
-\Delta_3 & -\Delta_5 & 0
\end{pmatrix}.
\]

In Fig. 1, we showed the allowed region of violation paramours for \((\delta_1, \delta_2, \delta_3)\) limiting \(-0.2 < \delta_3 < 0.2\). In order to read the allowed values distinctly, we showed the allowed

![Diagram](image_url)

**FIG. 1**: Allowed region of violation parameters for \((\delta_1, \delta_2, \delta_3)\) limiting \(-0.2 < \delta_3 < 0.2\)

region of \((\delta_1, \delta_2)\) fixing \(\delta_3 = 0.2, 0.15, 0.1, 0.05\) in Fig. 2 (a), (b), (c), (d), respectively. For negative \(\delta_3\), allowed values of \(\delta_1\) and \(\delta_2\) are exchanged to opposite sign.

We showed the allowed region of violation parameters for \((\Delta_2, \Delta_{14}, \Delta_{35})\) where \(\Delta_{14} = \Delta_1 + \Delta_4\) and \(\Delta_{35} = \Delta_3 + \Delta_5\) in Fig. 3. In Fig. 4, projected regions of allowed \((\Delta_2, \Delta_{14}, \Delta_{35})\) to \((\Delta_{14}, \Delta_{35})\), \((\Delta_2, \Delta_{14})\) and \((\Delta_2, \Delta_{35})\) planes are shown.

From this result, we can recognize that (1) \(\Delta_i\)'s are smaller than \(\delta_i\)'s as discussed in previous subsection, (2) numerical values in rather large range of violation parameters \(\delta_i\)'s can produce the neutrino large mixing, (3) \(|\delta_3|\) is not smaller than 0.05 and \(\delta_1\) and \(\delta_2\) are mutually symmetric, (4) the values of violation parameters \(\Delta_i\)'s of the Majorana neutrinos
FIG. 2: Allowed region of \((\delta_1, \delta_2)\) fixing \(\delta_3 = 0.2, 0.15, 0.1, 0.05\).

FIG. 3: Allowed region of violation parameters for \((\Delta_2, \Delta_{14}, \Delta_{35})\). \(\Delta_{14} = \Delta_1 + \Delta_4, \Delta_{35} = \Delta_3 + \Delta_5\).
FIG. 4: Projected regions of allowed $(\Delta_2, \Delta_{14}, \Delta_{35})$ to $(\Delta_{14}, \Delta_{35})$, $(\Delta_2, \Delta_{14})$ and $(\Delta_2, \Delta_{35})$ planes are rather restricted compared to the violation parameters of $\delta_i'$ of Dirac neutrinos, (5) in parameters $\delta_i$’s, $\Delta_{14}$ and $\Delta_{35}$ seem independent parameters rather than $\Delta_1$, $\Delta_3$, $\Delta_4$ and $\Delta_5$.

2. Diagonal Violation Parameter Case

For this case, Branco et al. discussed precisely [11]. They used the Dirac and Majorana Mass matrices with diagonal violation parameters as

\[
P_D = \begin{pmatrix}
0 & 0 & 0 \\
0 & \varepsilon_1^D & 0 \\
0 & 0 & \varepsilon_2^D
\end{pmatrix}, \quad P_M = \begin{pmatrix}
0 & 0 & 0 \\
0 & \varepsilon_1^M & 0 \\
0 & 0 & \varepsilon_2^M
\end{pmatrix}.
\]
In this case, $M_{\text{eff}}$ is written as

$$M_{\text{eff}} = \frac{\Gamma^2_D}{\Gamma^2_M} \left( \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 + \frac{\varepsilon^D_1}{\varepsilon^D_2} & 1 \\ 1 & 1 & 1 + \frac{\varepsilon^D_2}{\varepsilon^D_2} \end{array} \right). \quad (21)$$

Then if $\frac{\varepsilon^D_1}{\varepsilon^D_1}$ and $\frac{\varepsilon^D_2}{\varepsilon^D_2}$ are sufficiently larger than 1, one can get the large mixing of $U_{\text{MNS}}$. Numerically, allowed regions of $\varepsilon^D_1$, $\varepsilon^M_1$, $\varepsilon^D_2$ and $\varepsilon^M_2$ satisfying the condition Eqs. (19) and (20) are restricted as follows:

$$\varepsilon^D_1 = 0.02 \sim 0.04 \sim -0.02 \sim -0.04, \quad \varepsilon^D_2 = 0.2(\text{fixed}),$$

$$\varepsilon^M_1 = 0.000001 \sim 0.00025 \sim -0.000001 \sim -0.00025,$$

$$\varepsilon^M_2 = 0.00001 \sim 0.0016 \sim -0.00001 \sim -0.0018, \quad (22a)$$

$$\varepsilon^D_1 = 0.01 \sim 0.02 \sim -0.01 \sim -0.02, \quad \varepsilon^D_2 = 0.1(\text{fixed}),$$

$$\varepsilon^M_1 = 0.000007 \sim 0.000063 \sim -0.000007 \sim -0.000063,$$

$$\varepsilon^M_2 = 0.000075 \sim 0.000375 \sim -0.000075 \sim -0.000045, \quad (22b)$$

$$\varepsilon^D_1 = 0.005 \sim 0.01 \sim -0.005 \sim -0.01, \quad \varepsilon^D_2 = 0.05(\text{fixed}),$$

$$\varepsilon^M_1 = 0.000002 \sim 0.000016 \sim -0.000002 \sim -0.000016,$$

$$\varepsilon^M_2 = 0.000015 \sim 0.000105 \sim -0.000015 \sim -0.000105. \quad (22c)$$

$\varepsilon^D_2$ can take arbitrary value, then we showed the cases fixed to 0.2, 0.1 and 0.05. Furthermore in the above allowed regions, we restricted the $\varepsilon^D_1$ to the ranges where the ratio $\varepsilon^D_1/\varepsilon^D_2$ is smaller than 0.2. From this result, we can say that (1) $\varepsilon^M_i$ are smaller than $\varepsilon^D_i$ as discussed in previous subsection, (2) numerical values in rather large range of violation parameters $\varepsilon^D_i$’s can produce the neutrino large mixing.

3. Comparison of the Violating Patterns on the Quark Sector and the Neutrino Sector

We present the case analyzed precisely in our work [13] where violation parameters are involved in nondiagonal element of the quark mass matrix. We use the following quark mass...
matrices Eq.(3) with small phases:

\[
M_q = \Gamma_q \begin{pmatrix}
1 & (1 - \delta_q^q)e^{i\phi_q^q} & (1 - \delta_2^q)e^{i\phi_2^q} \\
(1 - \delta_1^q)e^{-i\phi_1^q} & 1 & (1 - \delta_3^q)e^{i\phi_3^q} \\
(1 - \delta_2^q)e^{-i\phi_2^q} & (1 - \delta_3^q)e^{-i\phi_3^q} & 1
\end{pmatrix}, \ (q = u, \ d)
\]

\[
\delta_i^q \ll 1, \ \phi_i^q \ll 1, \ (i = 1, \ 2, \ 3).
\]

Diagonalising these matrices and fitting these eigenvalues to the numerical mass ratios and the CKM matrix \(U_{\text{CKM}}\) to the observed values,

\[
\frac{m_u}{m_c} = 0.0038 \pm 0.0025, \quad \frac{m_d}{m_s} = 0.050 \pm 0.035,
\]

\[
\frac{m_c}{m_t} = 0.0042 \pm 0.0013, \quad \frac{m_s}{m_b} = 0.038 \pm 0.019,
\]

\[
V_{\text{CKM}} = \begin{pmatrix}
0.9747-0.9759 & 0.218-0.224 & 0.002-0.005 \\
0.218-0.224 & 0.9738-0.9752 & 0.032-0.048 \\
0.004-0.015 & 0.030-0.048 & 0.9988-0.9995
\end{pmatrix},
\]

we were able to obtain a result for the violation parameters:

\[
\delta_i^u = 0.00001-0.0004, \quad \delta_+^u \equiv \delta_2^u + \delta_3^u = 0.0064-0.0125, \quad \delta_-^u \equiv \delta_2^u - \delta_3^u = \pm(0.0-0.0043),
\]

\[
\delta_i^d = 0.001-0.015, \quad \delta_+^d \equiv \delta_2^d + \delta_3^d = 0.040-0.129, \quad \delta_-^d \equiv \delta_2^d - \delta_3^d = \pm(-0.038--0.006),
\]

\[
\phi_+^d \equiv \frac{\phi_2^d + \phi_3^d}{2} = -4^\circ - 3^\circ, \quad \phi_-^d \equiv \phi_2^d - \phi_3^d = \pm(-1^\circ-0^\circ).
\]

Using these values for the parameters, we obtained the unitarity triangle parameters \(\rho\) and \(\eta\) characterizing the CP violation. The values for \(\rho\) and \(\eta\) are in close agreement with the results obtained from experiment (see Ref. [13]). These parameters display a power law behavior parameterized by only 2 parameters, \(\lambda\) and \(\phi\), as

\[
\delta_+^u = \lambda^8, \quad \delta_-^u = \lambda^6, \quad \delta_+^d = \lambda^4, \quad \delta_-^d = \lambda^3, \quad \delta_+^d = \lambda^2,
\]

\[
\lambda \approx 0.32, \quad \phi_+ \equiv \phi \approx -4^\circ.
\]

Violation parameters in quark sector have a remarkable character as power rule, and then the character should tell a hint for the flavor symmetry. We can take the allowed regions of the violation parameter in neutrino sector, but cannot take a remarkable character as quark sector, because one does not have the precise data of \(U_{\text{MNS}}\) and mass of neutrinos at present.
In order to obtain the more restrictive character for the violation parameters in neutrino sector, one has to take the more precise data on the neutrino experiments, for example mass values of neutrinos and mixing parameters of $U_{MNS}$ and CP violation phases.

**IV. DISCUSSIONS**

We have discussed the possibility that the universal Yukawa coupling (democratic mass matrix) with small violations of Dirac and Majorana neutrinos can induce the large mixing of neutrinos through the seesaw mechanism. That possibility can be achieved by the condition that the violation parameters of Majorana neutrinos are sufficiently smaller than the violation parameters of Dirac neutrinos. And then, we analyzed the numerical condition for the violation parameters in which the observed mass hierarchy of neutrinos and bi-maximal neutrino mixing is satisfied by the violation parameters. We gave the allowed numerical results of violation parameters in both cases where violation parameters are involved in nondiagonal elements of Dirac and Majorana mass matrices and in diagonal elements of those.

Obtained result for violation parameters has following characters: Numerical values in rather large range of violation parameters $\delta_i$’s can produce the neutrino large mixing, $|\delta_3|$ is not smaller than 0.05 and $\delta_1$ and $\delta_2$ are mutually symmetric. The values of violation parameters of the Majorana neutrinos are rather restricted compared to the parameters of Dirac neutrinos. $\Delta_{14}$ and $\Delta_{35}$ seem independent parameters rather than $\Delta_1, \Delta_3, \Delta_4$ and $\Delta_5$ in non-diagonal violation case. The violation parameters of neutrino mass matrices are not so restricted in contrast to the quark sector. In order to obtain the distinctive characters for violation parameters in neutrino sector, one should take the more precise data concerned with the neutrino experiments for example mass values of neutrinos and mixing parameters of $U_{MNS}$ and CP violation phases.

[1] Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 81, 1562(1998); 82, 2644(1999).

[2] M. C. Gonzalez-Garcia et al., Phys. Rev. D63, 033005(2001); M. C. Gonzalez-Garcia and C. Peña-Garay, Nucl. Phys. Proc. Suppl. 91, 80(2000); Super-Kamiokande Collaboration,
Y. Fukuda et al., Phys. Rev. Lett. 82, 1810(1999); ibid. 82, 2430(1999).

[3] M. Apollonio et al., Phys. Lett. B466, 415(1999).

[4] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28, 870(1962).

[5] T. Yanagida, in Proceedings of the Workshop on the Unified Theories and Baryon Number in the Universe Tsukuba, 1979, ed. O. Sawada and A. Sugamoto, KEK report No.79-18, Tsukuba (1979), p. 95; M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, Proceedings of the Workshop, Stony Brook, New York, 1979, ed. P. van Nieuwenhuizen and D. Freedmann (North-Holland, Amsterdam, 1979), p. 315.

[6] H. Dreiner et al., Nucl. Phys. B436, 461(1995); N. Haba, Phys. Rev. D59, 035011(1999); M. Bando and T. Kugo, Prog. Theor. Phys. 101, 1313 (1999); S. Lola and G. G. Ross, Nucl. Phys. B553, 81(1999); H. Fritzsch and Z. Z. Xing, Phys. Rev. D61, 073016(2000).

[7] S. F. King and G. G. Ross, Phys. Lett. B520, 243(2001).

[8] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B147, 277 (1979).

[9] J. Sato and T. Yanagida, Phys. Lett. B430, 127(1998); M. Fukugita, M. Tanimoto and T. Yanagida, Phys. Rev. D57, 4429(1998).

[10] E. Kh. Akhmedov, G. C. Branco, F. R. Joaquim and J. I. Silva-Marcos, Phys. Lett. B498, 237(2001); G. C. Branco and J. I. Silva-Marcos, Phys. Lett. B526, 104(2002).

[11] T. Teshima and T. Asai, Prog. Theor. Phys. 105, 763 (2001).

[12] H. Harari, H. Haut and J. Weyers, Phys. Lett. B78, 459(1978); Y. Koide, Phys. Rev. D28, 252(1983); M. Tanimoto, Phys. Rev. D41, 1586(1990); G. C. Branco, J. I. Silva-Marcos and M. N. Rebelo, Phys. Lett. B237, 446(1990); G. C. Branco and J. I. Silva-Marcos, Phys. Lett. B359, 166(1995).

[13] T. Teshima and T. Sakai, Prog. Theor. Phys. 97, 653 (1997).

[14] J. Gasser and H. Leutwyler, Phys. Rep. 87, 77(1982); S. Narison, Phys. Lett. B216, 191(1989).

[15] N. Haba, Y. Matsui, N. Okamura and M. Sugiura, Eur. Phys. J. C10, 677(1999).