CAN SUPERMASSIVE BLACK HOLES FORM IN METAL-ENRICHED HIGH-REDSHIFT PROTOPHALOS?

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ABSTRACT

Primordial gas in protogalactic DM halos with virial temperatures \( T_{\text{vir}} \gtrsim 10^4 \) K begins to cool and condense via atomic hydrogen. Provided that this gas is irradiated by a strong UV flux and remains free of \( \text{H}_2 \) and other molecules, it has been proposed that the halo with \( T_{\text{vir}} \sim 10^4 \) K may avoid fragmentation and lead to the rapid formation of an SMBH as massive as \( M \approx 10^{5} - 10^{6} M_{\odot} \). This “head start” would help explain the presence of SMBHs with inferred masses of several times \( 10^9 M_{\odot} \), powering the bright quasars discovered in the SDSS at redshift \( z \gtrsim 6 \). However, high-redshift DM halos with \( T_{\text{vir}} \sim 10^4 \) K are likely already enriched with at least trace amounts of metals and dust produced by prior star formation in their progenitors. Here we study the thermal and chemical evolution of low-metallicity gas exposed to extremely strong UV radiation fields. Our results, obtained in one-zone models, suggest that gas fragmentation is inevitable above a critical metallicity, whose value is between \( Z_{\text{cr}} \approx 3 \times 10^{-4} Z_\odot \) (in the absence of dust) and as low as \( Z_{\text{cr}} \approx 5 \times 10^{-6} Z_\odot \) (with a dust-to-gas mass ratio of about 0.01 Z/\( Z_\odot \)). We propose that when the metallicity exceeds these critical values, dense clusters of low-mass stars may form at the halo nucleus. Relatively massive stars in such a cluster can then rapidly coalesce into a single more massive object, which may produce an intermediate-mass BH remnant with a mass up to \( M \lesssim 10^2 - 10^3 M_{\odot} \).

Subject headings: cosmology: theory — galaxies: formation — stars: formation

Online material: color figures

1. INTRODUCTION

The discovery of bright quasars at redshifts \( z \gtrsim 6 \) in the Sloan Digital Sky Survey (SDSS) implies that black holes (BHs) as massive as several times \( 10^9 M_{\odot} \) were already assembled when the age of the universe was less than \( \approx 1 \) Gyr (see the recent review by Fan 2006). The BH masses are inferred from the quasars’ luminosities, assuming that these sources shine near their Eddington limit. Strong gravitational lensing or beaming could, in principle, mean that the inferred BH masses are overestimated; however, there is no obvious sign of either effect in the images and spectra of these quasars (Willott et al. 2003; Richards et al. 2004). Indeed, their relatively “normal” line-to-continuum ratio, consistent with those in lower redshift quasars, makes it unlikely that the apparent flux of these sources was significantly boosted by beaming (Haiman & Cen 2002). Likewise, the lack of a second detectable image on Hubble Space Telescope images (Richards et al. 2004) essentially rules out the hypothesis that most of the sources experienced strong magnification by lensing (Comerford et al. 2002; Keeton et al. 2005).

Relatively little time is available for the growth of several times \( 10^9 M_{\odot} \) supermassive black holes (SMBHs) prior to \( z \sim 6 \), and their seed BHs must be present as early as \( z \sim 10 \) (e.g., Haiman & Loeb 2001). As the SMBHs grow from high-redshift seed BHs by accretion, they are expected to encounter frequent mergers. A coalescing BH binary experiences a strong recoil due to gravitational waves (GWs) emitted during the final stages of their merger. The typical recoil speed is expected to be \( v_{\text{recoll}} \approx 100 \) km s\(^{-1}\) (and may be as large as \( 4000 \) km s\(^{-1}\) for special BH spin configurations; see, e.g., Campanelli et al. 2007 and references therein), significantly exceeding the escape velocity \( (\lesssim 10 \) km s\(^{-1}\) ) from typical dark matter (DM) halos that exist at \( z \approx 10 \). As a result, SMBHs are often ejected from their host halos at high redshift. The repeated loss of the growing seeds makes it especially challenging to account for the several times \( 10^9 M_{\odot} \) SMBHs at \( z \gtrsim 6 \) without at least a brief phase of super-Eddington accretion, or some equivalent “head start” (Haiman 2004; Yoo & Miralda-Escudé 2004; Shapiro 2005; Volonteri & Rees 2006).

There have been several recent proposals that such a head start may occur in metal-free gas in high-redshift DM halos with virial temperatures exceeding \( T_{\text{vir}} \gtrsim 10^4 \) K, leading to the rapid formation of SMBHs with a mass of \( M \approx 10^5 - 10^6 M_{\odot} \). As primordial gas falls into these halos, it initially cools via the emission of hydrogen Ly\( \alpha \) photons. Provided that the gas is free of \( \text{H}_2 \) molecules, its temperature will remain near \( T_{\text{vir}} \sim 10^4 \) K. Bromm & Loeb (2003a, hereafter BL03) performed hydrodynamical simulations of a metal- and \( \text{H}_2 \)-free halo, with a mass of \( \sim 10^8 M_{\odot} \) collapsing at \( z \approx 10 \), corresponding to a \( 2 \sigma \) Gaussian overdensity and to \( T_{\text{vir}} \sim 10^4 \) K. Under these conditions, which may apply to some dwarf galaxies collapsing close to the epoch of reionization, the primordial gas is marginally able to collapse and remains nearly isothermal. BL03 found that during the evolution, fragmentation of the gas cloud is very inefficient, leading at most to binary formation even with some degree of rotation. Thus, a supermassive star is expected to form and evolve into an SMBH with a mass as high as \( M \approx 10^5 - 10^6 M_{\odot} \). Oh & Haiman (2002) and Lodato & Natarajan (2006) have also showed that if \( \text{H}_2 \) formation is inhibited, a primordial gas disk is stable to fragmentation and a single massive object is formed in accordance with BL03’s conclusion. Volonteri & Rees (2005) arrived at similar conclusions, by considering Bondi accretion onto a stellar seed BH, which can significantly exceed the Eddington rate at the gas density and temperature in a similar halo. Finally, Begelman et al. (2006) and Spaans & Silk (2006) proposed different mechanisms.
to form similarly massive BHs by the direct collapse of primordial, atomic gas. For reference, we note that the total (DM+gas) mass of halos with $T_{\text{vir}} = (1-5) \times 10^4$ K at $z = 10$ is $M_{\text{tot}} \approx 10^6 - 10^9 M_\odot$, so that such SMBHs would represent $\approx 0.2\% - 20\%$ of the gas mass in these halos. We also note that in the WMAP5 cosmology, the age of the universe at $z = 10$ and 6.5 is $\approx 0.5$ and $\approx 0.9$ Gyr, respectively. At the $\epsilon$-folding timescale of $4 \times 10^7$ yr (assuming Eddington accretion and a radiative efficiency of 10%; see, e.g., Haiman & Loeb 2001), a seed BH of $M \approx 10^6 M_\odot$ at $z \approx 10$ could easily grow to an SMBH of $M \approx 2 \times 10^9 M_\odot$ at $z \approx 6.5$, if unfed uninterrupted.

A crucial assumption in all of the above proposals is that H$_2$ molecules cannot form as the gas cools and condenses in the DM halo. This assumption can be justified in the presence of a sufficiently strong far-ultraviolet (FUV) radiation, so that molecular hydrogen (or the intermediary H$^+$ necessary to form H$_2$) is photodissociated. The relevant criterion is that the photodissociation timescale is shorter than the H$_2$ formation timescale; since generically $t_{\text{diss}} \propto J$ and $t_{\text{form}} \propto \rho$, the condition $t_{\text{diss}} \approx t_{\text{form}}$ yields a critical flux $J \propto \rho$. In DM halos with $T_{\text{vir}} \leq 10^4$ K, whose gas cannot cool in the absence of H$_2$, the densities remain low and H$_2$ can be dissociated even when background flux is as low as $J_{21} \approx 10^{-2}$ (e.g., Haiman et al. 1997; Mesinger et al. 2006; here $J_{21}$ is the flux just below 13.6 eV, in the usual units of $10^{-21}$ ergs cm$^{-2}$ sr$^{-1}$ s$^{-1}$ Hz$^{-1}$). However, if a gas cloud is massive enough and has a virial temperature higher than $\approx 8000$ K, it is able to cool and start its collapse via atomic hydrogen Ly$\alpha$ cooling. Even if the FUV field is initially above the critical value, molecular hydrogen can form and dominate the gas cooling at a later stage during the collapse (Oh & Haiman 2002); the H$_2$ formation rate is furthermore strongly boosted by the large out-of-equilibrium abundance of free electrons in the collisionally ionized gas in these halos (Shapiro & Kang 1987; Susa et al. 1998; Oh & Haiman 2002). The critical flux required to keep the gas H$_2$-free as it collapses by several orders of magnitude therefore increases significantly; for halos with $T_{\text{vir}} \approx 10^4$ K the value has been found to be $J_{21} \approx 10^{-3} - 10^{-2}$, depending on the assumed spectral shape (Omukai 2001, hereafter O01; BL03). In halos exposed to such extremely intense UV fields, the gas cloud is still able to collapse only via atomic hydrogen line cooling, namely, Ly$\alpha$ and H$^+$ free-bound (f-b) emission (O01).

One possible source of such an intense UV field is the intergalactic UV background just before the epoch of cosmic reionization (BL03). The ionizing photon flux $J_{21}$ can be evaluated from the number density of hydrogen atoms in the intergalactic medium (IGM) and the average number of photons needed to ionize a hydrogen atom $N_\gamma$, which, in general, is $>1$, owing to recombinations in an inhomogeneous IGM. Using the escape fraction of ionizing radiation $f_{\text{esc}}$, the flux $J_{21}$ just below the Lyman limit is given by

$$J_{21} \approx \frac{1}{f_{\text{esc}}} \frac{\hbar c N_\gamma Y_{\text{ni}} \rho_b}{4\pi m_\text{H}} \approx 4 \times 10^8 \left( \frac{N_\gamma}{10} \right) \left( \frac{f_{\text{esc}}}{0.01} \right)^{-1} \left( \frac{1+z}{11} \right)^{3},$$

(1)

where $Y_{\text{ni}} = 0.76$ is the mass fraction of hydrogen, $m_\text{H}$ is the proton mass, and $\rho_b$ is the baryon density (assumed here to correspond to $\Omega_b h^2 = 0.023$; Dunkley et al. 2008). Equation (1) shows that $J_{21}$ can approach the critical value at $z \approx 10$, provided that $f_{\text{esc}}$ is small and $N_\gamma$ is large: $f_{\text{esc}}/N_\gamma \approx 10^{-3}$. Although the value of $f_{\text{esc}}$ is quite uncertain, in low-mass minihalos, the expectation is $f_{\text{esc}} \approx 1$, so that these halos are easily self-ionized, and most of their ionizing radiation escapes into the IGM (Kitayama et al. 2004; Whalen et al. 2004). Observations of nearby star-forming galaxies indicate lower values $f_{\text{esc}} < 0.1$ (Leitherer et al. 1995; Inoue et al. 2005); nevertheless, it appears unlikely that the mean cosmic background will reach the critical values. A few halos that have close and bright neighbors may still see a sufficiently increased flux (M. Dijkstra et al. 2008, in preparation). Alternatively, the critical value could be established by sources internal to the halo, e.g., by a vigorous phase of starburst (Omukai & Yoshii 2003) or by the accreting seed BH itself (Volonteri & Rees 2005).

So far, the process of supermassive star (and SMBH) formation in dwarf galaxies in the presence of a strong UV background has been investigated only under the hypothesis that the gas is metal-free. However, the $T_{\text{vir}} \approx 10^4$ K, $M \approx 10^8 M_\odot$ halos forming close to the epoch of reionization are built from lower mass progenitors that had collapsed earlier. Many, and perhaps all, of these halos should therefore be enriched with at least some trace amount of metals. Furthermore, it is unlikely that the strong critical FUV flux could be generated and maintained without efficient star formation at higher redshifts (if $J_{21} \gtrsim 10^{-6}$ was produced by accreting BHs, this would significantly overpredict the present-day soft X-ray background; Dijkstra et al. 2004). It is well known that adding metals and dust into a primordial gas, even at trace amounts as low as $Z \approx 10^{-6} Z_\odot$, can significantly affect its cooling properties (Schneider et al. 2003, 2006; Omukai et al. 2005). Two independent hydrodynamic simulations (Tsuribe & Omukai 2006; Clark et al. 2008) recently studied the fragmentation of metal-enriched collapsing protogalactic clouds, in the absence of an external FUV field, and found efficient fragmentation for $Z \approx 10^{-6} Z_\odot$.

In the present paper, our goal is to answer the following question: can cooling and fragmentation be avoided in metal-enriched $T_{\text{vir}} \approx 10^4$ K halos, irradiated by a strong FUV flux? If so, this would suggest that SMBHs may form, similar to the metal-free case, in the more likely case of metal-enriched high-redshift protogalaxies. To investigate this possibility, we here study the thermal and chemical evolution of low-metallicity gas, exposed to extremely strong UV radiation fields. We evaluate the critical metallicity, above which fragmentation becomes unavoidable in the presence of a strong FUV flux.

In § 2 we describe our one-zone modeling procedure. Our results are presented and discussed in § 3, first for the metal-free case (§ 3.1) and then for the metal-enriched case (§ 3.2). The fragmentation and subsequent evolution of the metal-enriched clouds are then discussed in §§ 3.3 and 3.4, respectively. In § 4 we summarize our results and offer our conclusions.

2. MODEL

2.1. Basics

We use the one-zone model described in O01 to follow the gravitational collapse of gas clouds. The model includes a detailed description of gas-phase chemistry and radiative processes and the effect of DM on the dynamics in a simplified fashion. In addition, in the present version of the model we have implemented the contribution of metal lines and dust to gas cooling.

In what follows, all physical quantities are evaluated at the center of the cloud. The gas density increases as

$$\frac{d\rho_{\text{gas}}}{dt} = \rho_{\text{gas}}/t_{\text{cool}},$$

(2)
where the collapse timescale, $t_{\text{col}}$, is taken to be equal to the free-fall time,

$$t_{\text{col}} = t_{\text{ff}} \equiv \sqrt{\frac{3\pi}{32G\rho}}$$

(3)

and $\rho$ is the sum of the gas and DM density. The DM density follows the evolution of a top-hat overdensity,

$$\rho_{\text{DM}} = \frac{9\pi^2}{2} \left(\frac{1 + z_{\text{vir}}}{1 - \cos \theta}\right)^3 \Omega_{\text{DM}} \rho_{\text{crit}}.$$  

(4)

with

$$1 + z_{\text{vir}} = (1 + z_{\text{vir}}) \left(\frac{\theta - \sin \theta}{\pi}\right)^{-2/3}$$

(5)

(e.g., Padmanabhan 1993, p. 276), where the turnaround and the virialization correspond to $\theta = \pi$ and $2\pi$, respectively. Although, strictly speaking, this is correct only in the Einstein–de Sitter universe ($\Omega_0 = 1$), it does not cause a significant error in the high-$z$ universe ($z \gtrsim 10$) we consider.

The initial epoch of calculation is taken at the turnaround at redshift $z_{\text{vir}} = 17$. From equation (5), the virialization and turnaround redshifts have the relation $1 + z_{\text{vir}} = 2^{-2/3}(1 + z_{\text{vir}})$; thus, $z_{\text{vir}} \approx 10$. In our calculation, the DM density is kept constant after reaching its virialization value $\rho_{\text{DM}}(z_{\text{vir}})$. The initial values of the gas number density, temperature, ionization degree, and H$_2$ fraction have been assumed to be $n_{\text{H}} = 4.5 \times 10^{-3}$ cm$^{-3}$, $T = 21$ K, $\gamma(e) = 3.7 \times 10^{-4}$, and $\gamma(H_2) = 2 \times 10^{-6}$, respectively, to reflect conditions at the turnaround at $z_{\text{vir}} = 17$. Some runs with initial temperature $10$ times higher ($210$ K) are also performed to confirm independence of our main results from the initial temperature. The cosmological parameters are $\Omega_{\text{DM}} = 0.24$, $\Omega_b = 0.04$, and $h = 0.7$.

Our calculation does not include the virialization shock. Owing to fast cooling by Ly$\alpha$ emission, the central region whose evolution we intend to follow does not experience the virialization shock in the spherically symmetric case (Birnboim & Dekel 2003). In more realistic calculations, the outer regions can experience shocks and the temperature and electron fraction become higher than in our case. In addition, recent numerical calculations (e.g., Keres et al. 2005) show that low-mass galaxies, especially at high redshifts, obtain their gas through accretion predominantly along the large-scale filaments. Three-dimensional effects such as asymmetric accretion might affect the evolution at low densities. However, since we are considering halos with $T_{\text{vir}} \approx 10^4$ K, which can marginally collapse by Ly$\alpha$ cooling, the shock is not strong: the temperature increase is modest and the electron fraction reaches at most $\approx 10^{-2}$ (see Figs. 5a and 5c in BL03). These additional electrons alter the early evolution for the $J = 0$ case. However, in the irradiated clouds, where H$_2$ formation is suppressed, during the collapse by the Ly$\alpha$ cooling recombination proceeds until the free electron fraction reaches $x_e \approx 1.2 \times 10^{-3}$ at $t_{\text{col}}$, the value set by the balance between the recombination and the collapse time $t_{\text{col}} \sim t_{\text{col}}$ at 8000 K. Thus, our results for molecule formation and cooling are hardly affected.

We adopt $t_{\text{ff}}$ as the collapse timescale just because it has been widely used in other studies (e.g., Palli et al. 1983). Note that the free-fall time (eq. [3]) is the time for the density of an initially static cloud to reach infinity, while the dynamical timescale $t_{\text{col}} = \mu(d\rho/dt)$ in the free-fall collapse is

$$t_{\text{col,ff}} = \frac{1}{\sqrt{24\pi G \rho}}$$

in the limit where the density has become sufficiently larger than the initial value. Thus, the rate we adopted (eq. [3]) is $3\pi/2 = 4.7$ times slower collapse than the genuine free-fall one. In fact, pressure gradients oppose gravity and the collapse becomes slower than the free-fall one within a factor of a few (e.g., Foster & Chevalier 1993). Adoption of $t_{\text{ff}}$ as the e-folding time for density increase mimics the pressure effect. The assumption of nearly free-fall collapse is violated, and the collapse is slowed down, once the cloud becomes optically thick to continuum radiation. However, our result on the thermal evolution is not altered: with little radiative cooling, the temperature is now determined by the adiabatic compression and the chemical cooling by dissociation and ionization, both of which are independent of the collapse timescale. Moreover, the evolution after the cloud becomes optically thick is not relevant to our argument on fragmentation, which occurs at much lower density, in the optically thin regime.

The overall size of the collapsing gas cloud (or of the roughly uniform density central region) determines its optical depth and is therefore important for its thermal evolution. Here we assume that the size equals the Jeans length,

$$\lambda_J = \sqrt{\frac{\pi k T_{\text{gas}}}{G \rho_{\text{gas}} \mu_{\text{H}1}}}$$

(7)

where $T_{\text{gas}}$ is the gas temperature and $\mu$ is the mean molecular weight. Similarly, its mass is given by the Jeans mass

$$M_J = \rho_{\text{gas}} \lambda_J^3.$$  

(8)

Specifically, we assume that the radius of the cloud is $R_c = \lambda_J/2$ and the optical depth is

$$\tau_{\nu} = \kappa_{\nu} R_c = \kappa_{\nu} \left(\frac{\lambda_J}{2}\right).$$  

(9)

In addition to dust absorption (see § 2.2), we include the following primordial gas processes as sources of the opacity $\kappa_{\nu}$ (Table 1 of O01): the bound-free absorption of H, He, H$^-$, and H$_2$; free-free absorption of H$^-$ and H; collision-induced absorption of H$_2$-H$_2$ and H$_2$-He; Rayleigh scattering of H; and Thomson scattering of electrons.

The temperature evolution is followed by solving the energy equation:

$$\frac{de}{dt} = -p \frac{d}{dt} \left(\frac{1}{\rho_{\text{gas}}} - \frac{\Lambda_{\text{net}}}{\rho_{\text{gas}}} \right),$$  

(10)

where $e$ is the internal energy per unit mass

$$e = \frac{1}{\gamma_{\text{ad}} - 1} \frac{k T_{\text{gas}}}{\mu_{\text{H}1}},$$  

(11)

$p$ is the pressure, $\gamma_{\text{ad}}$ is the adiabatic exponent, and $\Lambda_{\text{net}}$ is the net cooling rate per unit volume. In addition to cooling and heating processes for the primordial gas, which include continuum emission, as well as emission by H and H$_2$ lines, and chemical heating/cooling, the net cooling rate includes emission by C
and O fine-structure lines $\Lambda_{\text{metal}}$, by dust grains $\Lambda_{\text{gr}}$, and heating by photoelectric emission of dust grains $J_{\text{pe}}$. Cooling by fine-structure lines of [C ii] and [O i] is included as in Omukai (2000). Dust processes are described below in § 2.2.

Primordial gas chemical reactions are solved for the nine species of H, H$_2$, e, H$^+$, H$_2^+$, H$^-$, He, He$^+$, and He$^{-}$. We do not explicitly include the chemical reactions involving metals. Instead, all the carbon and oxygen is assumed to be in the form of C or O, respectively. Having a lower ionization energy (11.26 eV) than hydrogen, carbon remains in the form of C in the atomic medium owing to photoionization by the background radiation. We maintained this assumption even in $J = 0$ runs, although carbon is expected to recombine and become neutral in these cases. The cooling rates by C or C$^+$ fine-structure lines are within a factor of $\approx 2$ difference for $T \gtrsim 30$ K; therefore, this assumption does not significantly affect the results. On the other hand, the ionization potential of oxygen (13.61 eV) is very similar to that of hydrogen (13.60 eV), and the charge exchange reaction

$$O^+ + H \leftrightarrow H^+ + O$$

keeps its ionization degree equal to that of hydrogen. In fact, the coefficient of the rightward reaction being $6.8 \times 10^{-10}$ cm$^3$ s$^{-1}$, these reactions reach equilibrium only in $\sim 50n_H^{-1}$ yr.

In a cold (less than or approximately a few hundred kelvin) and dense ($\gtrsim 10^3$–$10^4$ cm$^{-3}$) environment, molecular coolants such as CO and H$_2$O may become important (Omukai et al. 2005). Since we are interested here in metal effects on warm (greater than or approximately a few thousand kelvin) atomic clouds, we neglect the contribution to cooling of metals in molecules. This simplification does not affect the early evolution of gas clouds, when the effects of metals induce a deviation from the primordial evolutionary track at several thousand kelvin. It is true that it may alter the predicted thermal behavior at later stages, when the gas has cooled significantly ($\lesssim 1000$ K). However, even in such cold environments, the error in the temperature caused by neglect of metal molecular coolants is very small (see Fig. 10 of Omukai et al. 2005), and the thermal evolution is well reproduced when only dust processes and fine-structure line cooling of C and O are considered.

### 2.2. Dust Processes

Dust in the local interstellar medium (ISM) originates mainly from the asymptotic giant branch (AGB) stars, whose age is $\gtrsim 1$ Gyr, longer than the Hubble time at $z \gtrsim 6$. At higher redshifts, supernovae (SNe) are considered to be the major dust factories. Indeed, the observed extinction law of high-$\zeta$ quasars and gamma-ray bursts can be well reproduced by this scenario (Maiolino et al. 2004; Stratta et al. 2007). Dust grains produced in SN ejecta are more effective in cooling and H$_2$ formation because of their smaller size and larger area per unit mass (Schneider et al. 2006). However, their composition and size distribution are still affected by many uncertainties, such as the degree of mixing in the ejecta and the efficiency of grain condensation and their destruction by the reverse shock (Nozawa et al. 2007; Bianchi & Schneider 2007).

To be conservative, in this work the properties of dust, such as grain composition and size distribution, are assumed to be similar to those in the solar neighborhood, and its amount is reduced in proportion to the assumed metallicity of the gas clouds. Specifically, we adopt the dust opacity model developed by Semenov et al. (2003). This model partly follows the scheme proposed by Pollack et al. (1994), which was used in Omukai et al. (2005), assuming the same dust composition, size distribution, and evaporation temperatures, but uses a new set of dust optical constants. Overall, the opacity curves of the two models are in good agreement, the largest difference being at most a factor of 2 (for a thorough discussion see Semenov et al. 2003). The main dust constituents include amorphous pyroxene ([Fe, Mg]SiO$_3$), olivine ([Fe, Mg$_2$]SiO$_4$), volatile and refractory organics, amorphous water ice, troilite (FeS), and iron. The grains are assumed to follow a size distribution modified from that by Mathis et al. (1977) with the inclusion of large (0.5–5 $\mu$m) grains.

At each density and gas temperature, the dust is assumed to be in thermal equilibrium, and its temperature $T_{\text{gr}}$, which is followed separately from the gas temperature, is determined by the energy balance equation

$$4\pi \int \kappa_{\nu,\text{gr}}B_\nu(T_{\text{gr}})\,d\nu = \Lambda_{\text{gas-dust}} + 4\pi \int \kappa_{\nu,\text{dust}}J^\text{m}_\nu\,d\nu.$$  

Here $\Lambda_{\text{gas--dust}}$ is the energy transfer rate per unit mass from gas to dust due to gas-dust collisions, which we take from Hollenbach & McKee (1979); $\kappa_{\nu,\text{gr}}$ is the absorption opacity of dust; and $J^\text{m}_\nu$ is the mean intensity of the radiation field inside the cloud. Note that $\Lambda_{\text{gas--dust}}$ also represents the net cooling rate of the gas, caused by the presence of dust grains at temperature $T_{\text{gr}}$. We model the external radiation field assuming a diluted thermal spectrum (i.e., a blackbody spectrum, scaled by an overall constant representing a mean geometrical dilution). Its shape is then fully described by only two free parameters: $J_{\nu,0}$, the mean intensity at the Lyman limit ($\nu_{\text{Ly}}$), and $T_{\nu}$, the color temperature,

$$J^\text{m}_\nu = J_{\nu,0} \frac{T^\text{ex}_\nu + \xi_{\nu}S_{\nu,\text{Gr}}}{1 + \xi_{\nu}x_{\nu}},$$  

where $1 - \xi_{\nu}$ is the single-scattering albedo, $S_{\nu,\text{Gr}}$ is the source function,

$$x_{\nu} = \max\left(r_{\nu}^2, \tau_\nu\right),$$

and the optical depth $\tau_{\nu}$ includes both dust and gas opacities.

The gas is heated by photoelectrons ejected from dust grains after absorption of FUV photons. Following Bakes & Tielens (1994), we compute the photoelectric heating rate as

$$\Gamma_{\text{pe}} = \Gamma_{\text{pe}} - \Lambda_{\text{pe}}$$

$$= \left\{10^{-24}G_0 n_H - 4.65 \times 10^{-30}T^{0.94}\right\} \times \left\{\frac{G_0 T^{1/2}}{n(e)}\right\} n(e) n_H \frac{Z}{Z_\odot},$$
The thermal evolution of metal-free clouds irradiated by an UV flux. The temperature evolution at different initial temperatures is shown, with solid and dashed lines, bottom to top (see the legend in the panel). Diagonal dotted lines correspond to different thermal evolutionary tracks. At higher initial temperatures, the temperature evolution at $n_\text{H} \geq 10^{4.9} \text{ cm}^{-3}$ is delayed because higher densities and temperatures are required for H$_2$ formation to compensate for the photodissociation. If the UV intensity is below a threshold value, $J_{21}$, there is always a density at which H$_2$ cooling starts to become effective. The temperature then decreases and eventually reaches the non-radiative evolution track, along which it evolves thereafter. The temperature where this condition is met does not contradict previous results (BL03). In fact, the predicted temperature of each fluid element in the simulation of BL03 shows a large scatter at low densities. This scatter reflects the radial temperature gradient, and the central value, which we calculate here, corresponds to the lower boundary of the scattered points and is in agreement with our result. We expect that the central temperature of the gas cloud does not reach the virial temperature of the host DM halo since the innermost region starts to cool and collapse during the adiabatic compression and does not experience the virialization shock.

As the external radiation intensity $J_{21}$ increases, the onset of H$_2$ cooling is delayed because higher densities and temperatures are required for H$_2$ formation to compensate for the photodissociation. If the UV intensity is below a threshold value, $J_{21,\text{thr}}$, which we find to be in the range $10^2$–$10^3$ for $T_\nu = 10^4$ K and $(1-3) \times 10^3$ for $T_\nu = 10^5$ K, there is always a density at which H$_2$ cooling starts to become effective. The temperature then decreases and eventually reaches the non-radiative evolution track, along which it evolves thereafter. On the other hand, if the radiation is stronger than the threshold value, H$_2$ cooling never becomes important. In this case, atomic hydrogen cooling by H excitation (for $\gtrsim 10^2$ cm$^{-3}$) and H$^-$ f-b emission (for $\gtrsim 10^5$ cm$^{-3}$) are the main cooling channels (see Fig. 3).

In Figure 1, runs with higher initial temperature (210 K) are also shown (dotted lines). During the initial adiabatic phase, the temperature at a given density is proportional to its initial value and thus higher in runs with higher initial temperature. However, after the onset of efficient radiative cooling, these initially different thermal evolution tracks soon converge. At higher densities, the results are independent of the initial temperature (see Fig. 1).

As it can be inferred from Figures 1 and 2, we find that the threshold value, $J_{21,\text{thr}}$, is lower for a radiation temperature of $T_\nu = 10^4$ K than for $T_\nu = 10^5$ K. Thus, for comparable radiation intensities, $J_{21}$, the lower $T_\nu$ radiation has a stronger impact on...
the cloud evolution. To understand why this is the case, in Figure 4 we show the H2 and H+ photodissociation rates, for the same intensity \(J_{21} = 1\). The dilution factor \(W\), defined by \(J_{\nu} \equiv WB_{\nu}(T_*)\), which was used in Omukai & Yoshii (2003), is also shown for reference. As the figure shows, the H+ photodissociation rate decreases steeply with \(T_*\), while the H2 photodissociation rate remains nearly constant. The H2 and H+ photodissociation rate coefficients are

\[
k_{H_2,ph} = 1.4 \times 10^2 J_\nu (12.4 \text{ eV})
\]

in the unattenuated case and

\[
k_{H^+,ph} = \int_{0.755 \text{ eV}} 4 \pi J_\nu \sigma_\nu d\nu,
\]

where the lower limit on the latter integral, 0.755 eV, is the threshold energy for H+ photodissociation. Since the radiation field is normalized at the Lyman limit (13.6 eV), \(k_{H_2,ph}\) is not sensitive to \(T_*\), whereas \(k_{H^+,ph}\), which reflects the radiation field above 0.755 eV, depends significantly on the adopted \(T_*\). H2 formation proceeds via a two-step process (H+ channel),

\[
H + e \rightarrow H^- + \gamma
\]

and

\[
H^- + H \rightarrow H_2 + e.
\]

If H+ is photodissociated, it cannot activate the second step (eq. [26]). This is the reason why a lower \(T_*\), radiation field leads to a less efficient H2 formation rate and to a lower H2 fractional abundance than a higher \(T_*\), radiation field.

Cooling via H+ f-b emission occurs through the radiative association reaction (eq. [25]). Subsequently, H+ is collisionally dissociated through the reaction

\[
H^- + H \rightarrow 2H + e,
\]

and the whole process results in a net cooling by the emitted photon. Thus, as long as the collisional dissociation rate exceeds the photodissociation rate, the H+ f-b emission is hardly quenched, even in the presence of a strong FUV radiation field. Due to the small opacity, H+ f-b cooling becomes important only at high densities (\(\gtrsim 10^6 \text{ cm}^{-3}\)) and temperatures (roughly several times 10^4 K), where the above condition is always satisfied. Then, H+ f-b cooling is not affected by photodissociation.

As shown in Figure 3, radiative cooling and compressional heating rates almost balance for a wide range of densities. Namely, in equation (10), the two terms on the right-hand side almost cancel and the left-hand side is much smaller than those terms. Suppose that the radiative cooling rate per unit volume depends on the density and temperature as

\[
\dot{\Lambda}_{rad}/\rho_{gas} \propto \rho_{gas}^{-1} T^{3/2}.
\]

Since the compressional heating rate

\[
-p \frac{d}{dt} \frac{1}{\rho_{gas}} \frac{d}{dt} = \frac{p}{\rho_{gas} \rho_{gas, col}} \propto \rho_{gas}^{1/2} T^{3/2},
\]

the thermal balance of those two terms results in the following temperature evolution:

\[
T \propto \rho_{gas}^{(3/2-\alpha)/(\beta-1)}.
\]

Both the H line and H+ f-b emissions are very sensitive to temperature, and thus \(\beta > 1\). For those collisional processes, \(\alpha = 2\) for fixed chemical abundances. With chemical evolution, it deviates from 2 but remains \(>3/2\). Therefore, the exponent in equation (30) is negative for the atomic cooling track as long as the cloud is optically thin: the temperature decreases with density as observed in Figures 1 and 2. On the other hand, on the molecular cooling track, \(\alpha = 1\) for densities higher than the critical value for the LTE. Thus, the temperature increases with density for \(n_H \gtrsim 10^4 \text{ cm}^{-3}\).

The existence of a threshold UV background and the discontinuity of thermal evolution at this value are due to the presence...
of a non–local thermodynamic equilibrium (non-LTE) to LTE transition of the H2 rovibrational level population at $\sim 10^4$ cm$^{-3}$. When the gas density is higher than this value, the cooling rate saturates and more H$_2$ is needed to compensate for compressional heating. In addition, after the LTE is reached, the collisional dissociation rate is enhanced owing to a large H$_2$ level population in the excited levels. Thus, if a strong FUV radiation delays H$_2$ formation and cooling until the critical density for LTE is reached, a fraction of the remaining H$_2$ is collisionally dissociated. Thus, the gas cloud is no longer able to cool by H$_2$ even at a later phase of the evolution. On the other hand, if the UV background is slightly smaller than the threshold, the cloud begins to cool by H$_2$ and the temperature begins to fall before the collisional dissociation effect becomes significant (see Fig. 5b of OO1 for cooling rates by each process in such a case). The lower temperature allows further H$_2$ formation and resultant cooling. The cooling proceeds in this accelerated fashion and the temperature eventually reaches the molecular cooling track. This is the origin of the dichotomy between the atomic and molecular cooling tracks. To summarize, the main effect of the FUV radiation is to photodissociate H$_2$ directly and to decrease the H$_2$ formation rate through photodissociating H$^-$. If these two processes inhibit H$_2$ formation and cooling until the critical density for LTE is reached, the gas remains warm (greater than or approximately several thousand kelvin) and H$_2$ is collisionally dissociated at higher densities. Thus, the high density evolution is not affected by the presence of the FUV field and depends only on the temperature at the H$_2$ critical density.

3.2. Metallicity Effects on Irradiated Clouds

In this section we show the effects induced by the presence of metals and dust grains on the thermal evolution of gas clouds irradiated by an FUV field with a mean intensity larger than $J_{21,thr}$. In what follows, the total metallicity is expressed relative to the solar value, as $[M/H] \equiv \log (Z/Z_\odot)$. Unless specified otherwise, the fractions of metals in the gas phase and in dust grains are assumed to be the same as in the ISM of the Galaxy. Specifically, the number fractions of C and O nuclei in the gas phase with respect to H nuclei are $y_{C,\text{gas}} = 0.927 \times 10^{-4}(Z/Z_\odot)$ and $y_{O,\text{gas}} = 3.568 \times 10^{-4}(Z/Z_\odot)$. The mass fraction of dust grains relative to the mass in gas is $0.939 \times 10^{-2}(Z/Z_\odot)$ below the ice vaporization temperature ($T_{\text{vap}} \leq 100$ K).

In Figure 5 we present the thermal evolution of clouds with metallicity in the range $-6 \leq [M/H] \leq -3$ irradiated by extremely strong FUV radiation fields. The parameters of the radiation fields are (a) $T_r = 10^4$ K, $J_{21} = 10^3$, and (b) $T_r = 10^5$ K, $J_{21} = 3 \times 10^5$, respectively. Under these conditions, the clouds would collapse only via atomic cooling in the absence of metals or dust grains (see Figs. 1 and 2). For a metallicity as low as $[M/H] \leq -6$, the predicted thermal evolution follows the metal-free track. In both panels of Figure 5, deviations from the metal-free tracks start to appear at a density $\sim 10^{11}$ cm$^{-3}$ when the metallicity is $[M/H] \simeq -5.3$. For the sake of comparison, thin lines show the expected evolution in the absence of radiation for the same initial values of metallicity. At metallicity $[M/H] = -5.3$, although the temperature drops and eventually reaches the molecular cooling track at $\sim 10^{10}$ cm$^{-3}$, this arrival is after the minimum in the molecular cooling at $\sim 10^{14}$ cm$^{-3}$. With a slightly higher metallicity of $[M/H] = -5$, this arrival takes place at $\sim 10^{11}$–$10^{12}$ cm$^{-3}$, and the temperature subsequently decreases to the minimum in the no-radiation case. For higher metallicities, the temperature drop occurs at lower density and the temperature minima become lower. In Figure 6 we show the cooling and heating rates contributed by each process during the evolution of the cloud with $T_r = 10^4$ K, $J_{21} = 10^3$, and $[M/H] = -5$. Up to $10^{10}$ cm$^{-3}$, cooling is dominated by the H line emission (denoted as “H” in the figure; $\leq 10^7$ cm$^{-3}$) and H$^-$ f-b emission (“H$^-$/f-b”; $\leq 10^2$ cm$^{-3}$), and the cloud collapses along the atomic cooling track (see Fig. 5a). However, at a density $\sim 10^{10}$ cm$^{-3}$, cooling by the dust grain (“grain”) becomes dominant and causes the sudden temperature drop. Now the temperature is lower than that in the atomic cooling track, and the H$_2$ collisional dissociation rate is also reduced, which causes a high equilibrium value of the H$_2$ fraction. As a result of H$_2$ cooling, the temperature decreases further, although this effect is almost completely balanced by heating due to H$_2$ formation (“H$_2$ form”). Note that fine-structure line cooling (“CII, OI”) is not important at such low metallicities (see the discussion below). Eventually, the thermal evolutionary tracks reach those of the corresponding metallicity in the no-radiation case (shown as thin lines in Fig. 5) and evolve along them thereafter.

We also run models with an external UV field with parameters $T_r = 10^4$ K, $J_{21} = 10^4$, which is 10 times stronger than that considered in Figure 5a, and we found that the critical value of the metallicity at which deviations from the metal-free evolution appear, $Z_{\text{crit}} \sim 5 \times 10^{-6} Z_\odot$, does not depend on the intensity of the FUV radiation as long as $J_{21} > J_{21,thr}$. Furthermore, for the same value of the metallicity, the evolutionary tracks at high densities ($\gtrsim 10^4$ cm$^{-3}$) are independent of the type of external radiation, as can be seen comparing the results in Figures 5a and 5b. In fact, once the evolution has reached the density at which molecular and atomic cooling tracks bifurcate ($\gtrsim 10^3$ cm$^{-3}$), collisional processes rather than radiative ones dominate the energy balance (see also § 3.1).

It is interesting to stress that the physical processes responsible for the origin of a critical metallicity and its numerical value are the same as those found in the absence of FUV radiation (Schneider et al. 2003; Omukai et al. 2005). This is because, despite the higher gas temperatures induced by the presence of a strong FUV field (several thousands of degrees), the dust temperature remains at a few tens of degrees until the energy transfer rate from gas to dust by collisions becomes important and the dust and gas temperatures approach each other (with the associated gas cooling). Nevertheless, the ultimate fate of protogalaxies at low metallicity can be significantly affected by the presence of the UV flux (see discussion in § 3.4). In Figure 7, the evolution of dust and gas temperatures is shown for the lowest metallicity tracks presented in Figure 5a. The disappearance of the dust temperature curve for $[M/H] = -6$ at a density of $\sim 10^{14}$ cm$^{-3}$ is due to complete vaporization of grains, which occurs at $\sim 1300$ K. At dust temperatures higher than this value, grains are no longer present in the cloud. Other smaller discontinuities in the dust temperature also reflect vaporization of some dust compounds. For example, those at $\sim 130$ K at $\sim 10^9$ cm$^{-3}$ for $[M/H] = -6$ and $-5$; at $\sim 10^{11}$ cm$^{-3}$ for $[M/H] = -4$ are due to vaporization of water ice. This figure indeed shows that despite the high gas temperature, the dust temperature remains low at a few tens of kelvin, which allows survival of grains until very high densities.

As discussed above, O i and C ii line emission contributes negligibly to gas cooling in the metallicity and density range where the effects of dust grains start to become relevant ($Z_{\text{crit}} \sim 5 \times 10^{-6} Z_\odot$, $\eta_{\text{th}} \sim 10^{10}$ cm$^{-3}$). Metal-line cooling causes a deviation from the metal-free atomic track only when the metallicity reaches $[M/H] \sim -3$. In this case, since the temperature track converges to the $J = 0$ track before the dust-cooling phase, two temperature minima appear at $10^5$ and $10^{10}$ cm$^{-3}$ (see Figs. 5a and 5b). In the absence of dust grains, a higher fraction of metals is required to cool the gas at a rate such that the thermal evolution
deviates from the atomic cooling metal-free tracks. To demonstrate this, we have performed a numerical experiment where we have suppressed the contribution of dust grains to the energy balance of the collapsing clouds. The results for models with radiation field parameters of $T_\star = 10^4$ K and $J_{21} = 10^3$ are shown in Figure 8. When the metallicity is below $[\text{M/H}] \approx -3.5$, the temperature evolution is exactly the same as the metal-free one (shown by the $2 \times 10^{-4} \, \text{Z}_\odot$ track in the figure). For higher metallicity, fine-structure line cooling becomes dominant when $n_{\text{H}} \lesssim 10^4$ cm$^{-3}$ and the temperature drops abruptly by more than

![Figure 5a](image1.png)

**Fig. 5a**

**Fig. 5b**
2 orders of magnitude. Therefore, we find that the critical metallicity \( \frac{M}{H} \approx -3.5 \) \((\approx 3 \times 10^{-4} Z_{\odot})\) required to modify the thermal evolution is almost 2 orders of magnitude higher than in models with dust. This level of metallicity is approximately the same as the value at which the metal cooling rate exceeds the \( \text{H}_2 \) cooling rate in clouds that are already collapsing by molecular cooling (Bromm & Loeb 2003b; Santoro & Shull 2006; Frebel et al. 2007).

3.3. Fragmentation Properties

The thermal properties of star-forming clouds have an important influence on how they fragment into stars (Larson 2005). There is observational evidence that protostellar cores have a mass spectrum that resembles the stellar initial mass function (IMF), indicating that cloud fragmentation must be responsible for setting some fundamental properties of the star formation process (e.g., Motte et al. 1998; Lada et al. 2008; for recent reviews see Bonnell et al. 2007; Elmegreen 2008).

In the absence of an external FUV radiation field, the temperature of metal-free clouds decreases with density in the range \( 0.1 \text{ cm}^{-3} \leq n_{\text{HI}} \leq 10^5 \text{ cm}^{-3} \) and increases at higher densities, after the major coolant \( \text{H}_2 \) has reached the LTE. Dense cores form around this density with typical masses of \( 10^4 M_{\odot} \), which is close to the Bonnor-Ebert mass at this thermal state (Bromm et al. 1999, 2002; Abel et al. 2002). As the metallicity increases to \( Z_{\odot} = 10^{-6} Z_{\odot} \), dust-induced fragmentation leads to solar or subsolar fragments (Schneider et al. 2006), making a fundamental transition in the characteristic mass scales of protostellar cores.

It is important to stress that the presence of a temperature dip in the thermal evolution and the softening of the equation of state \( \gamma < 1 \) imply only the possibility of fragmentation. For example, fragmentation depends also on the initial conditions and requires the existence of sufficiently large initial density perturbations. In the first cosmological objects, which are barely able to cool and collapse, fragmentation can be less efficient. Still, self-gravitating cores form comparable with that predicted by the above criterion are observed to form in high-resolution three-dimensional (3D) simulations (Abel et al. 2002; Yoshida et al. 2006). Even for turbulent molecular clouds of solar metallicity, 3D simulations show that fragmentation is efficient when \( \gamma \approx 0.7 \) and is suppressed after \( \gamma \) increases to \( \approx 1.1 \) (Jappsen et al. 2005).

The evolution with density of the effective adiabatic index, \( \gamma \), is presented in Figure 9 for clouds with initial metallicities \( \frac{M}{H} = -\infty, -6, -5, -4, -3 \), and \( -2 \), irradiated by a field with parameters \( T_{\odot} = 10^4 \text{ K} \) and \( J_{\odot} = 10^3 \), whose temperature evolution is shown in Figure 5. The application of the above
arguments to predict the typical fragment mass from the thermal evolution of the clouds is not straightforward because, along the metal-free atomic cooling tracks and over a broad density range $10^1 - 10^{16} \text{ cm}^{-3}$, the effective adiabatic index remains $\gamma \approx 1$, although slightly below unity ($0.95 - 1$; see Fig. 9, top panel). If we adopt $\gamma_{\text{frag}} = 1$ as the threshold value of the effective adiabatic index for fragmentation, in this case fragmentation would be expected to occur up to densities of $\sim 10^{12} \text{ cm}^{-3}$, leading to solar-mass fragments, as discussed by O01 and Omukai & Yoshii (2003).

In contrast, the numerical simulations by BL03 show that down to the highest density reached by the simulations ($\lesssim 10^{7} \text{ cm}^{-3}$) fragmentation is very inefficient. Even with some degree of rotation, the cloud fragments at most into two pieces, resulting in a binary system. Although fragmentation might occur at higher densities, in BL03’s calculations neither efficient fragmentation leading to the formation of a star cluster nor the growth of elongation of the clouds is observed. We speculate that this result is due to the following reasons. The objects considered by BL03 are those only marginally able to collapse by atomic cooling and thus are initially close to the hydrostatic equilibrium. During this initial epoch, the Jeans mass is large, and density and velocity perturbations are erased by pressure forces. In addition to this little initial seed perturbation, since $\gamma$ is only slightly below unity, the growth of perturbation would be very slow. Thus, the perturbation might not grow enough to cause fragmentation.

Note however that, although we find that along the atomic cooling tracks $\text{H}^{-}$ cooling is the dominant cooling agent at high densities, $\gtrsim 10^{7} \text{ cm}^{-3}$, this process is not considered in the simulation of BL03, which implements only $\text{H Ly}()$ cooling. To check whether this omission might cause the lack of fragmentation, we have followed the evolution of a metal-free cloud under the influence of an external FUV radiation field with $T_r = 10^4 \text{ K}$ and $J_{\text{21}} = 10^3$ but turning off the $\text{H}^{-}$ cooling by hand. The result is shown in Figure 10. With no $\text{H}^{-}$ cooling, the cloud follows a slightly higher temperature track when the density is $\gtrsim 10^{7} \text{ cm}^{-3}$. However, below $\sim 10^{12} \text{ cm}^{-3}$, the difference is small and it has a weak effect on the cloud dynamics. Therefore, the inclusion of $\text{H}^{-}$ cooling would not affect the results of BL03’s simulation, which is limited to densities $\lesssim 10^{9} \text{ cm}^{-3}$.

On the basis of these considerations, we assume that for metal-free clouds irradiated by a strong FUV background, fragmentation does not occur during the atomic cooling phase, where
which corresponds to two dips in the temperature (or $\gamma$) evolutionary track. The outcome of this kind of track is not clear without any numerical work studying their effect. Here we speculate that the first dip produces clumps as a result of the fragmentation of clouds. Then the clumps fragment again into cores owing to the second dip.

In the absence of dust, the temperature minimum appears at a lower density, $n_H \sim 10^3$ cm$^{-3}$, and higher metallicity [M/H] $\approx -3.5$ (see Fig. 8). Therefore, the corresponding fragment mass remains as high as 10–100 $M_\odot$, and the formation of subsolar mass fragments is not possible in this case. This property of prestellar clouds enriched only by gas-phase metals has been already proven to hold in the absence of external FUV fields (Schneider et al. 2006).

To summarize, our results show that in the presence of a sufficiently strong FUV radiation field the collapse of metal-free clouds by molecular cooling is inhibited and it can proceed only via atomic cooling. Under these conditions, cloud fragmentation is highly inefficient, leading at most to the formation of a binary system. The typical mass of prestellar clouds is therefore $10^2$–$10^5$ $M_\odot$, and the formation of a supermassive star, seed of an SMBH, is the likely outcome of the evolution (BL03). However, this scenario is altered as soon as trace amounts of metals and dust grains are present in the collapsing clouds: dust cooling leads to fragmentation of the clouds into subclumps with mass as low as $\sim 0.1$ $M_\odot$ already at a floor metallicity of $Z_{\text{tr}, f} \sim 5 \times 10^{-6} Z_\odot$. This conclusion holds independently of the intensity and spectrum of the FUV radiation field. In the absence of dust, an enrichment level of $Z_{\text{tr}} \sim 3 \times 10^{-4} Z_\odot$ is required for O i and C ii line cooling to fragment the cloud; the fragments in this case are predicted to be relatively more massive, $\sim 10$–100 $M_\odot$.

3.4. Dynamical Interactions and Accretion

Since dust-induced fragmentation takes place at high densities, a dense protostellar cluster is expected to form (Oomukai et al. 2005; Schneider et al. 2006; Clark et al. 2008). As an example, when the initial metallicity of the collapsing cloud is [M/H] $= -5$, the sudden temperature drop, where $\gamma < \gamma_{\text{frag}} = 0.8$, begins at $T_{\text{drop}} \sim 3500$ K and $n_{\text{drop}} \sim 10^{10.5}$ cm$^{-3}$. At this stage, the size and mass of the cooling region, or protocluster, are given by the corresponding Jeans length, $\lambda_J \approx 4 \times 10^{-3}$ pc, and mass $M_{\text{cl}} \approx 70 M_\odot$. When a different threshold value $\gamma_{\text{frag}}$ is adopted, these quantities change, e.g., to $\lambda_J \approx 3 \times 10^{-2}$ (1 $\times 10^{-2}$) pc and $M_{\text{cl}} \approx 40$ (200) $M_\odot$ for $\gamma_{\text{frag}} = 0.7$ (0.9). In the following order-of-magnitude estimation, we use $\lambda_J \sim 1 \times 10^{-2}$ pc and $M_{\text{cl}} \approx 100 M_\odot$ as typical values. After virialization, the protostellar cluster has a size half of this. Since each ultimate fragment has a typical mass of $M_{\text{frag}} \sim 0.1 M_\odot$, which is set by the Jeans mass at the end of the fragmentation process ($n_{\text{H}} \sim 10^{14}$ cm$^{-3}$), we expect that up to $N_c \sim M_{\text{cl}}/M_{\text{frag}} \sim 1000$ low-mass stars can be formed and confined into a small region of size $\sim 0.01$ pc. The difference between the formation epochs of each protostar is of the order of the free-fall time of the protocluster gas. Since the cluster begins to form in a dense cloud with density $\sim 10^{10}$ cm$^{-3}$, protostar formation is synchronized on a time-scale of $\sim 300$ yr.

The fate of dense, compact star clusters has been discussed extensively in the literature (see, e.g., Rasio et al. [2004] for a recent review, focusing on the possibility of intermediate BH [IMBH] formation through a runaway collapse that is relevant in our case). It is important to stress that, even assuming a star formation efficiency of order unity (which seems likely when the density exceeds $\geq 10^5$ cm$^{-3}$; Alves et al. 2007), the stellar IMF will be strongly affected by gravitational interactions, collisions,
and mergers. In fact, observed properties of present-day star-forming regions, as well as numerical simulations, suggest that gravitational fragmentation is probably responsible for setting a characteristic stellar mass, but the full mass spectrum and the Salpeter-like slope of the IMF are most likely formed through continued accretion and dynamical interactions in a clustered environment (see Bonnell et al. 2007 and references therein). Furthermore, in young and compact cluster supermassive stars may form through repeated collisions (e.g., Portegies Zwart et al. 1999, 2004; Ebisuzaki et al. 2001).

We can therefore ask, what is the expected fate of the dense star cluster forming in our clouds? The evolution of a star cluster with half-mass radius \( R_{\text{cl}} \) and mass \( M_{\text{cl}} \) proceeds on the dynamical friction timescale (Binney & Tremaine 1987),

\[
t_{\text{fric}} \simeq \frac{1.2 \times 10^5}{\ln \Lambda} \, \text{yr} \left( \frac{r}{0.01 \, \text{pc}} \right)^2 \left( \frac{R_{\text{cl}}}{0.01 \, \text{pc}} \right)^{-1/2} \times \left( \frac{M_{\text{cl}}}{10^2 M_\odot} \right)^{1/2} \left( \frac{m_s}{1 M_\odot} \right)^{-1},
\]

(32)

which is the time required for a star with mass \( m_s \), which is on the massive side of the spectrum, to sink from the radius \( r \) to the cluster center by gravitational interactions with background, less massive stars. In the following, the Coulomb logarithm \( \ln \Lambda \) is taken to be \( \simeq 7 \), a value typical for open clusters. In the above equation, the density profile is assumed to be isothermal, \( \rho \propto 1/r^2 \). The dynamical friction timescale was originally derived for a fixed background. What actually occurs for a cluster on this timescale is the equipartition of kinetic energy among the member stars. Heavy stars move slowly and then drop deeper in the potential well, leading to mass segregation in the cluster. The stellar merger rate is greatly enhanced if higher mass stars reach the cluster center within the lifetime of a very massive star, i.e., if \( t_{\text{fric}} \) is less than a few megayears.

Using the estimated size and mass of the protocluster at the onset of dust-induced fragmentation for the \( [M/H] = -5 \) track, the dynamical friction timescale for a star initially at radius \( r \) can be rewritten more generally as

\[
t_{\text{fric}} \simeq 1.6 \times 10^3 \text{ yr} \left( \frac{M_\odot}{M_s} \right)^2 \left( \frac{\mu}{1.22} \right)^{3/2} \left( \frac{T_{\text{drop}}}{5000 \, \text{K}} \right)^{-3/2} \left( \frac{m_s}{1 M_\odot} \right)^{-1},
\]

(33)

where \( M_s \) is the mass enclosed inside radius \( r \) and we have expressed \( M_{\text{cl}} \) and \( R_{\text{cl}} \) in terms of \( n_{\text{drop}} \) and \( T_{\text{drop}} \) (note that \( n_{\text{drop}} \) drops out of the equation). Following Portegies Zwart et al. (2004), we assume that a very massive star can be formed by stellar mergers if

\[
t_{\text{fric}} < 4 \, \text{Myr}.
\]

(34)

From equation (33), we infer that the inner region of mass

\[
M_{\text{fric}} = 5 \times 10^3 M_\odot \left( \frac{\mu}{1.22} \right)^{-3/4} \left( \frac{T_{\text{drop}}}{5000 \, \text{K}} \right)^{3/4} \left( \frac{m_s}{1 M_\odot} \right)^{1/2}
\]

(35)

satisfies the condition given by equation (34). The mass fraction of sinking stars relative to the cluster background stars, \( f_{\text{sink}} \), is uncertain and probably depends on the stellar mass spectrum, but it can be safely assumed to be less than a half. In addition, not all the stars that sink to the center are incorporated into the runaway merging object. Since merging events usually proceed via three- or more body interactions, where the runaway object and a star coalesce by kicking the lightest star (Portegies Zwart et al. 1999), the merging efficiency, \( f_{\text{merg}} \), with the runaway object among stars fallen to the cluster center can be assumed to be about a half. Thus, the mass of the central object resulting from this process can be estimated as

\[
M_{\text{cen}} = f_{\text{sink}} f_{\text{merg}} M_{\text{fric}}
\]

\[
\approx 3.5 \times 10^2 M_\odot \left( \frac{f_{\text{sink}}}{0.25} \right) \left( \frac{f_{\text{merg}}}{0.5} \right) \times \left( \frac{T_{\text{drop}}}{5000 \, \text{K}} \right)^{3/4} \left( \frac{m_s}{1 M_\odot} \right)^{1/2}.
\]

(36)

As can be seen in Figures 5 and 9, dust-induced fragmentation in clouds with a metallicity \( [M/H] \leq -4 \) starts at \( T_{\text{drop}} \sim 5000 \, \text{K} \), \( n_{\text{drop}} \gtrsim 10^8 \, \text{cm}^{-3} \), and the corresponding protocluster mass is \( M_{\text{fric}} < 10^3 M_\odot \). That is, \( M_{\text{fric}} < M_{\text{fric}} \) and the entire cluster satisfies the condition given by equation (34). In this case, the mass of the central object is limited by the mass of the cluster rather than by \( M_{\text{fric}} \) and its final mass can be estimated as

\[
M_{\text{cen}} = f_{\text{sink}} f_{\text{merg}} M_{\text{fric}} \lesssim 100 M_\odot.
\]

On the other hand, dust-induced fragmentation of clouds with metallicity, \( -4 \leq [M/H] \leq -3 \), leads to protostellar clusters with masses \( M_{\text{fric}} \gtrsim M_{\text{fric}} \), and the mass of the central object is given by equation (37). Thus, in this case a very massive star can form with \( M_{\text{fric}} \lesssim 350 M_\odot \). In either case, we note that the metal-poor, massive star ultimately forming at the center of the halo is likely to leave behind a seed BH remnant, except in a narrow range of metallicity, where they produce a pair instability supernova, either by direct collapse or by fallback (Heger et al. 2003). For higher metallicity, there are two episodes of fragmentation: a metal-induced one at low density and a dust-induced one at high density. In this case, the mass scale of the star cluster is relatively low and a massive BH seed is not formed.

As pointed out above, the critical metallicity levels we find in the case of strong FUV irradiation, at which the cloud behavior is modified from the metal-free case, are very similar to the critical metallicity found in earlier work for \( J = 0 \). It is therefore important to ask whether the presence of the flux will, in fact, make any difference to the ultimate fate of the cloud. At metallicities above \( [M/H] \simeq -3 \), the flux has essentially no impact on the evolutionary track of clouds at high densities. For example, comparing the thin and thick solid lines of Figure 5a, it is clear that when \( [M/H] = -3 \), the flux has no effect at \( n_{\text{H}} \gtrsim 10^5 \, \text{cm}^{-3} \). At lower densities, we expect that the first fragmentation phase will occur as the Jeans mass drops from \( M_J \sim 10^6 \) to \( 10^5 M_\odot \) at \( n_{\text{H}} \sim 10^5 \, \text{cm}^{-3} \) in the \( J = 0 \) case, and to \( \sim 10^6 M_\odot \) at \( 10^5 \, \text{cm}^{-3} \) in the strong UV background case. Therefore, the size of the molecular clumps that form is larger in the \( J = 0 \) case. However, the thermal evolution thereafter is the same: the molecular clumps will experience a second phase of fragmentation when the Jeans mass falls further, from \( M_J \sim 10^6 \) to \( \sim 0.1 M_\odot \). Thus, for protostellar gas clouds with \( [M/H] \simeq -3 \), the presence of an external UV field determines only the amount of gas in the envelope in which the \( \sim 10 M_\odot \) star cluster is embedded.

In contrast, the presence of an FUV background significantly affects the evolution of protostellar clouds with lower values of metallicity, \( [M/H] < -3 \). In these models, the atomic gas experiences only the second phase of fragmentation induced by dust cooling; therefore, more massive star clusters (\( 10^5-10^7 M_\odot \)) increasing with metallicity) are formed compared to those in the \( J = 0 \) limit (a few to \( 10 M_\odot \), depending weakly on metallicity). In addition to the size of the star clusters, the mass and physical
conditions of the corresponding envelopes are different: when \( J = 0 \), the star cluster is embedded in a molecular envelope of \( 10^3 - 10^4 M_\odot \) with temperature of a few hundred kelvin, while, in the presence of an external UV field, the surrounding envelope is more massive \( (10^5 - 10^6 M_\odot) \) and it is made by atomic gas at several thousand kelvin. As a consequence, the presence of a strong UV background case favors the formation of a more massive central star by stellar merger (larger stellar cluster mass and higher \( T_{\text{env}} \) in eq. [37]).

Finally, our results and the discussion above suggest that the direct formation of a supermassive star or SMBH as massive as \( 10^5 - 10^6 M_\odot \), as envisioned in the metal-free case, is not possible when the metallicity is above a critical value, and the gas fragments into smaller pieces. However, we note that if fragmentation of the inner regions of the collapsing protogalaxy is not fully efficient, and if radiative and mechanical feedback from the stars does not expel the leftover gas from the nucleus, then the star cluster at the center of the halo, as well as its coalesced massive remnant star, can be embedded within a thin residual gaseous envelope with temperature \( T_{\text{env}} \) of roughly several thousand kelvin. Since this gas envelope is self-gravitating, with a temperature below the virial temperature, it can undergo dynamical collapse and may still produce an SMBH either directly or by accretion onto the central stellar-mass BH at the Bondi rate (see, e.g., Begelman et al. 2006 and discussion therein). In particular, in the UV irradiated case, the accretion rate onto the star cluster, and possibly to the central coalesced star, is high; this is because of the high temperature in the envelope, and since the accretion rate of the self-gravitating gas is given by (Shu 1977)

\[
\dot{M} \simeq \frac{c_s^3}{G} \left( \frac{T_{\text{env}}}{5000 \text{ K}} \right)^{3/2}.
\]

In conclusion, higher initial mass, higher accretion rate, and larger amount of reservoir gas are more favorable for BH growth in the strong UV case than the case without radiation.

4. SUMMARY AND CONCLUSIONS

In this paper we have investigated the thermal evolution and fate of protostellar gas clouds in \( T_{\text{vir}} \gtrsim 10^4 \) K halos irradiated by a strong FUV background. Under these conditions, which may apply to some dwarf galaxies collapsing close to the epoch of reionization, we find the following:

1. The effect of an external UV background is to photodissociate H\(_2\) directly and to decrease the H\(_2\) formation rate through photodissociation of H\(^-\). When the UV background reaches a critical threshold value, \( J_{21,\text{thr}} \), these two processes inhibit H\(_2\) formation and cooling until the critical density for LTE is reached. Thereafter, the gas cloud can cool only via atomic hydrogen transitions.

2. For gas clouds of primordial composition, an external UV background with intensity \( J_{21} < J_{21,\text{thr}} \) only delays the onset of H\(_2\) formation and H\(_2\) cooling becomes important at some (higher) density: fragmentation occurs at densities \( 10^3 - 10^4 \text{ cm}^{-3} \), leading to average fragment masses in the range \( 10^2 - 10^3 M_\odot \), similarly to the case with \( J_{21} = 0 \).

3. For \( J_{21} > J_{21,\text{thr}} \), not enough H\(_2\) is formed to activate cooling and the evolution of primordial clouds is controlled by atomic (H and H\(^-\)) cooling. The clouds collapse nearly isothermally with a temperature of several thousand kelvin (“atomic track”) up to very high densities \( \sim 10^{16} \text{ cm}^{-3} \). According to previous numerical calculation by BL03, the clouds collapse directly into a single \( 10^5 - 10^6 M_\odot \) object, leading to supermassive star and SMBH formation. A core envelope structure inevitably develops under these circumstances, and the star grows by accretion onto an initially small inner core. During the accretion phase, radiative and mechanical feedback effects might become important and eventually halt the accretion at some phase (e.g., Omukai & Palla 2003; McKee & Tan 2008). If so, the mass of the central object can remain far below \( 10^5 - 10^6 M_\odot \).

4. Independently of the values and properties of the external FUV field (as long as it is \( J_{21} > J_{21,\text{thr}} \)), deviations from the metal-free atomic track start to appear when the gas is enriched by even trace amounts of metals and dust. When \( Z > Z_{\text{cr}} \), dust cooling induces fragmentation at \( n_{\text{H}} \sim 10^{10} \text{ cm}^{-3} \) and a protostellar cluster is expected to form with average protostellar (fragment) mass of \( \sim 0.1 M_\odot \). If only gas-phase metals are present, a 2 orders of magnitude larger value of metallicity is needed, \( Z_{\text{cr}} \sim 3 \times 10^{-4} Z_\odot \), before C \( ii \) and O \( i \) line cooling induces a deviation from the atomic track, leading to fragmentation at \( n_{\text{H}} \sim 10^9 \text{ cm}^{-3} \) and to protostellar clusters with average protostellar (fragment) mass of \( 10 - 100 M_\odot \).

5. The physical processes responsible for the origin of a critical metallicity and of its numerical value are the same as those found in the absence of an external FUV field. However, due to the higher gas temperature, the final outcome of the protostellar cloud collapse can be significantly affected. Namely, if we assume that the size of the protostellar cluster formed by dust-induced fragmentation is set by the Jeans mass at the onset of the rapid temperature drop, it depends on the intensity of the FUV background field: when \( J_{21} < J_{21,\text{thr}} \), relatively small clusters are formed (a few to 10 \( M_\odot \)), whereas when \( J_{21} > J_{21,\text{thr}} \), very dense star clusters with masses \( 100 - 1000 M_\odot \) are formed, at the center of which stellar coalescences are expected to occur. The central merger object might grow to a very massive star of a few hundred solar masses.

6. In addition, the presence of an external FUV background affects the physical conditions of the envelope surrounding the protostellar clusters: when \( J_{21} < J_{21,\text{thr}} \), the envelope is fully molecular, with a mass of \( 10^2 - 10^3 M_\odot \) and a temperature of a few hundred kelvin. Conversely, when \( J_{21} > J_{21,\text{thr}} \), the envelope is made of atomic gas and reaches a mass of \( 10^2 - 10^3 M_\odot \) and temperature of several thousand kelvin. The higher temperature and larger gas reservoir favor BH growth by accretion, which can be as high as \( 10^{-2} M_\odot \text{ yr}^{-1} \).

According to the above, the conditions that would allow the formation of the direct formation of an SMBH are (1) to be hosted within a \( T_{\text{vir}} \sim 10^5 \) K halo, which is (2) irradiated by a strong UV field with \( J_{21} > J_{21,\text{thr}} \), and (3) to be still metal- and dust-free, with \( Z < Z_{\text{cr}} \). The main new result of the present paper is that if the metallicity is too high, so that condition 3 does not hold, then instead of an SMBH, a dense cluster of low-mass stars forms at the halo nucleus. The stars in such a cluster may still rapidly coalesce into a single massive star, which may produce an IMBH remnant, but with a smaller mass of \( M \sim 10^5 - 10^6 M_\odot \).

While the above conclusion that even trace amounts of dust enable cooling and fragmentation of the protostellar clouds appears to be robust, the exact value of the threshold metallicity is vulnerable to the uncertain nature of dust in early protogalaxies (see also Schneider et al. 2006). However, it is interesting to note that when \( Z \geq Z_{\text{cr}} \), the characteristic fragment mass, which is related to the characteristic stellar mass, is highly insensitive to environmental conditions, such as the presence of an external
FUV radiation field, as also recently discussed by Elmegreen et al. (2008).

We warn the reader that our discussion on the nature and evolution of the resulting protostellar cluster is still speculative as it is based on a few numerical experiments that apply to dense stellar systems in present-day star-forming regions (see the discussion and references in §3.4). For example, we assume that the size of the cluster is set at the onset of the efficient cooling phase, which appears plausible but has not yet been confirmed. In particular, in models with $J_{21} > J_{21, \text{th}}$ and $Z > 10^{-4} Z_\odot$ (when both dust grains and gas-phase metals are present), the thermal evolution curves appear to have two separate temperature minima, which correspond to metal- and dust-induced cooling and fragmentation. The fate of these collapsing clouds is at present unknown, and dedicated numerical simulations would be highly desirable. If protostellar clusters are indeed formed under the conditions that we suggest, the formation and the nature of a central object by repeated collisions and accretion are highly uncertain. For example, the fraction of stars on the massive side of the spectrum that falls to the cluster center (i.e., $f_{\text{sink}}$ in eq. [37]) by dynamical friction is unknown and may vary significantly depending on the IMF.

Despite the above uncertainties, our results suggest that even trace amounts of metals preclude the rapid formation of SMBHs as massive as $M \approx 10^5-10^6 M_\odot$ in protogalactic halos. While such promptly appearing SMBHs would help solve the puzzle of the $M \approx 10^7 M_\odot$ quasar black holes at $z \approx 6$, our results suggest that the low-metallicity halos may instead produce dense stellar clusters: the cluster may coalesce to produce an IMBH, but still with a much lower mass of $M \approx 10^2-10^3 M_\odot$.

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