\[ B \rightarrow D(D^*) \text{ Form Factors in a Bethe-Salpeter model} \]

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Abstract

We calculate the form factors for the semileptonic decays of the $B$ meson to $D$ and $D^*$ mesons in a Bethe-Salpeter model. We show that our model is consistent with the constraints of Heavy Quark Effective Theory (HQET) and we extract the matrix elements that represent the $1/m_Q$ corrections to the form factors in HQET. With available data, we obtain $V_{cb}=(31.9 \pm 1.4) \times 10^{-3}$. 

\section{INTRODUCTION}

In previous papers \cite{1} we have developed a model for mesons based on the Bethe-Salpeter equation. Recently \cite{2}, we calculated the Isgur-Wise function $\xi(\omega)$ and extracted the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{cb}$ from data. In this paper we improve upon the work of Ref. \cite{2} by evaluating all the form factors relevant to the semileptonic decays of the $B$ mesons to the $D(D^*)$ meson. By including additional $1/m_Q$ effects, we obtain an improved value for $V_{cb}$.

The discovery of Heavy Quark Symmetry (HQS) in recent years \cite{3,4,5,6,7} has generated considerable interest in the study of systems containing heavy quark(s). It has been shown that, in the heavy quark limit, the properties of systems containing a heavy quark are greatly simplified. HQS results in relations between non-perturbative quantities, such as form factors, for different processes involving transitions of a heavy quark to another quark. The development of Heavy Quark Effective Theory (HQET) \cite{5} allows one to systematically calculate corrections to the results of the HQS limit in inverse powers of the heavy quark mass $m_Q$. Among other consequences, this has allowed a precision method for determining the CKM matrix elements. An accurate determination of the CKM matrix is crucial for testing the validity of the Standard Model.

In spite of impressive results obtained in HQET it has not solved the problem of calculating the transition form factors in QCD. In particular, HQS reveals relations between form factors but does not provide a determination of the form factors. Furthermore, the systematic expansion of the form factors in $1/m_Q$ in HQET involves additional non-perturbative matrix elements which are not calculable. We are thus forced to rely on models for the non-perturbative quantities. However, the constraints of HQET, which are based on QCD, allows one to construct models which are consistent with HQET and hence QCD. In this paper we first calculate the form factors in the semileptonic decays of the $B$ meson to $D$ and $D^*$ mesons in our Bethe-Salpeter model. The parameters of the model are fixed by fitting the meson spectrum so that the evaluations of the form factors do not involve any additional free parameters. The calculated form factors are, therefore, viewed as predictions of this model. We then obtain $V_{cb}$ from the measured differential decay rate of $B \to D^*\ell\nu$. We also show that our model is consistent with the requirements of HQET and that we are able to extract the unknown matrix elements that appear in the $1/m_Q$ corrections in HQET.

The paper is organized as follows: In Section 2, we discuss the general formalism for the calculation of the form factors. In Section 3, we present and discuss the results of our
2 THE GENERAL FORMALISM

The Lagrangian for the semileptonic decays involving the $b \rightarrow c$ transition has the standard current-current form after the $W$ boson is integrated out in the effective theory.

$$H_W = \frac{G_F}{2\sqrt{2}} V_{cb} \bar{c} \gamma_\mu (1 - \gamma_5) b \bar{\nu} \gamma_\mu (1 - \gamma_5) l$$

The lepton current in the effective interaction is completely known and the matrix element of the vector ($V_\mu$) and the axial vector ($A_\mu$) hadronic currents between the meson states are represented in terms of form factors which are defined in the equations below.

$$\langle D(P_D)|V_\mu|B(P_B)\rangle = f_+(P_B + P_D)_\mu + f_-(P_B - P_D)_\mu$$
$$\langle D^*(P_{D^*}, \bar{\varepsilon})|V_\mu|B(P_B)\rangle = ig\epsilon_{\mu\alpha\beta\gamma}(P_B + P_{D^*})^\alpha (P_B - P_{D^*})^\beta$$
$$\langle D^*(P_{D^*}, \bar{\varepsilon})|A_\mu|B(P_B)\rangle = f \bar{\varepsilon}_\mu^* + (\varepsilon^*.P_B)[a_+(P_B + P_{D^*})_\mu + a_-(P_B - P_{D^*})_\mu]$$

$f_+, f_-, g, f, a_+$ and $a_-$ are Lorentz invariant form factors which are scalar functions of the momentum transfer $q^2 = (P_B - P_{D^*})^2$.

where $P_B$, $P_D$ and $P_{D^*}$ are the four-momenta of the $B$, $D$ and $D^*$ mesons respectively. The calculation of the the form factors proceeds in two steps. In the first step, the full current from QCD is matched to the current in the effective theory (HQET) at the heavy quark mass scale $[8]$. Renormalization group equations are then used to run down to a low energy scale $\mu \sim 1$ GeV where the constraints of HQET operate and where it is reasonable to calculate matrix elements in a valence constituent quark model like the one we employ here $[11]$.

In the HQS limit, the heavy quark inside the meson acts as a color source and its velocity remains unchanged due to interactions with the light degrees of freedom. In the leading order the velocity of the heavy quark is the same as the velocity of the hadron. The effect of the external weak current is to instantaneously change the velocity of the color source from $v$ to a new velocity $v'$. In Heavy Quark Effective Theory, therefore, it is more appropriate to work with the velocities of the heavy mesons instead of their momenta and to consider the form factors as functions of $\omega = v \cdot v'$. The variable $\omega$ is related to the momentum transfer variable $q^2$ through

$$\omega = \frac{m_B^2 + m_{D(D^*)}^2 - q^2}{2m_B m_{D(D^*)}}$$
In terms of a new set of more convenient form factors we can write the matrix elements of
the vector and the axial currents as
\[
\langle D(v_D)|V_\mu|B(v_B)\rangle = \xi_+(\omega)(v_B + v_D)_\mu + \xi_-(\omega)(v_B - v_D)_\mu
\]
\[
\langle D^*(v_D, \varepsilon)|V_\mu|B(v_B)\rangle = i\xi_V(\omega)\epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu v_D^\alpha v_B^\beta
\]
\[
\langle D^*(v_D, \varepsilon)|A_\mu|B(v_B)\rangle = \xi_{A1}(\omega)(v_D^*\cdot v_B + 1)\varepsilon^*_{\mu\nu} - (\varepsilon^*\cdot v_B)[\xi_{A2}(\omega)v_{B\mu} + \xi_{A3}v_{D\mu}]
\]

The normalization of the meson states in Eq. (2) and Eq. (3) are
\[
\langle M(p')|M(p)\rangle = 2p^0(2\pi)^3\delta^3(p - p')
\]
and
\[
\langle M(v')|M(v)\rangle = \frac{2p^0}{m_M}(2\pi)^3\delta^3(p - p')
\]

The two sets of form factors defined above are related to each other through
\[
\xi_+ = \frac{1}{2}\left(\sqrt{m_B/m_D} + \sqrt{m_D/m_B}\right)f_+ + \frac{1}{2}\left(\sqrt{m_B/m_D} - \sqrt{m_D/m_B}\right)f_-
\]
\[
\xi_- = \frac{1}{2}\left(\sqrt{m_B/m_D} - \sqrt{m_D/m_B}\right)f_+ + \frac{1}{2}\left(\sqrt{m_B/m_D} + \sqrt{m_D/m_B}\right)f_-
\]
\[
\xi_V = 2\sqrt{m_D^3m_Bg}
\]
\[
\xi_{A1} = \frac{f}{\sqrt{m_D^3m_B(v_D^*\cdot v_B + 1)}}
\]
\[
\xi_{A2} = \frac{-m_B^2(a_+ + a_-)}{\sqrt{m_D^3m_B}}
\]
\[
\xi_{A3} = \frac{-m_B^*m_B(a_+ - a_-)}{\sqrt{m_D^3m_B}}
\]

In the limit that the mass of the heavy quark \(m_Q \to \infty\) four of the six form factors
defined in Eq. (3) can be expressed in terms of a single form factor, the Isgur-Wise function
\[
\xi_+(\omega) = \xi_-(\omega) = \xi_V(\omega) = \xi_{A1}(\omega) = \xi_{A3}(\omega) = \xi(\omega)
\]
and the other two simplify to
\[
\xi_{A2}(\omega) = \xi_-(\omega) = 0
\]
Furthermore, because of current conservation in the full QCD Lagrangian, the Isgur-Wise
function is normalized to unity at zero recoil \(i.e.\ \xi(\omega = 1) = 1\)
As already noted in the introduction, HQET allows one to systematically calculate $1/m_Q$ corrections to this zeroth order result given above. The two sources of the $1/m_Q$ corrections are from the expansion of the effective Lagrangian and the quark fields in the weak currents. Let us now look into the two sources of $1/m_Q$ corrections.

In the limit $m_Q \to \infty$, the heavy quark field $Q(x)$ in the full QCD Lagrangian is replaced by the effective field $h_v(x)$

$$h_v(x) = e^{im_Qv.x} P_+ Q(x)$$

where $P_+ = \frac{1 + \not{v}}{2}$ is the positive energy projection operator. The effective Lagrangian with $m_Q \to \infty$ can be written as

$$\mathcal{L}_{HQET} = \bar{h} v \cdot D h_v$$

where $D^\alpha = \partial^\alpha - ig_s t^a A^\alpha_a$ is the gauge covariant derivative. Corrections to the effective Lagrangian come from higher dimensional operators suppressed by inverse powers of $m_Q$. Including $1/m_Q$ corrections the effective Lagrangian is

$$\mathcal{L} = \mathcal{L} + \delta \mathcal{L}_1/2m_Q + ...$$

$$\delta \mathcal{L}_1 = \bar{h} (iD)^2 h + \frac{g_s}{2} \bar{h} \sigma_{\alpha\beta} G^{\alpha\beta} h$$

where $G^{\alpha\beta} = [iD^\alpha, iD^\beta] = ig_s t^a G^{a\alpha\beta}$ is the gluon field strength. The equation of motion for the heavy quark is

$$v.D h_v = 0$$

Next, one has to express the currents that mediate the weak decays of hadrons in terms of the effective field $h_v$. In our case we are interested in currents of the form $\bar{q} \Gamma Q$. At the tree level the expansion of the current in the HQET takes the form

$$\bar{q} \Gamma Q \to \bar{q} \Gamma h + \frac{1}{2m_Q} \bar{q} \Gamma i D h + \cdots$$

where $\Gamma$ is any arbitrary Dirac structure. Furthermore, the hadron mass ($M_H$) (which appears in the normalization of the states) can be expanded in inverse powers of the heavy quark mass in the following manner

$$M_H = m_Q + \bar{\Lambda} + O(1/m_Q)$$

The mass parameter $\bar{\Lambda}$ plays a crucial role in the description of $1/m_Q$ corrections to heavy meson and heavy baryon form factors and our calculation of the form factors will enable us to extract this quantity.
In order to calculate the form factors appearing in Eq. (3) we first match the currents of the full theory, \( J_{bc}^\mu \), to the current of the effective theory \( J_{bc}^\mu \). We can write the matching condition as

\[
J_{bc}^\mu = C_{bc} J_{bc}^\mu + \frac{\alpha_s}{\pi} \Delta J_{bc}^\mu + \sum_j \left( \frac{B_j}{m_b} + \frac{B_j'}{m_c} \right) O_j + O(1/m_Q^2) \tag{10}
\]

The second term on the RHS is a new current operator generated because of operator mixing during the process of matching. This term has a weak dependence on \( \omega \) and so we will approximate it with its value at \( \omega = 1 \). The operators \( O_j \) represent the \( 1/m_Q \) corrections. Ignoring \( O(\alpha_s/m_Q) \) terms, the connection between the QCD corrected form factors \( \xi_i \) and the form factors calculated in the quark model (i.e. in the effective theory) \( \xi_i^0 \) is

\[
\xi_i = [C_{bc} - 1] \xi(\omega) + \frac{\alpha(\sqrt{m_b m_c})}{\pi} C_{bc} \beta^\alpha \xi(\omega) + \xi_i^0(\omega) \tag{11}
\]

where

\[
C_{bc}(\omega) = \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{a_I} \left[ \frac{\alpha_s(m_c)}{\alpha_s(\mu)} \right]^{a_L(\omega)} \tag{12}
\]

with

\[
a_I = -\frac{6}{33 - 2N_f}
\]

\[
a_L(\omega) = \frac{8}{33 - 2N_f} [\omega r(\omega) - 1]
\]

\[
r(\omega) = \frac{1}{\sqrt{\omega^2 - 1}} \ln(\omega + \sqrt{\omega^2 - 1})
\]

\( \mu \) is the scale of the effective theory and \( N_f = 4, N_f' = 3 \). The expressions for \( \beta^\alpha(\omega) \sim \beta^\alpha(\omega = 1) \)

are given by

\[
\beta_+ + \beta_- = \gamma - \frac{2}{3} \chi
\]

\[
\beta_+ - \beta_- = \gamma + \frac{2}{3} \chi
\]

\[
\beta_V = \frac{2}{3} + \gamma
\]

\[
\beta_{A_1} = -\frac{2}{3} + \gamma
\]
\[ \beta_{A_2} = -1 - \chi + \frac{4}{3} \frac{1}{1 - z} + \frac{2}{3} \frac{1 + z}{(1 - z)^2} \gamma \]
\[ \beta_{A_3} = -\frac{4}{3} \frac{1}{1 - z} - \chi + \left[ 1 - \frac{2}{3} \frac{1 + z}{(1 - z)^2} \right] \gamma \]
\[ z = \frac{m_c}{m_b} \]
\[ \chi = -1 - \frac{\gamma}{1 - z} \]
\[ \gamma = \frac{2z}{1 - z} \ln \frac{1}{z} - 2 \]  

(13)

The next step is the calculation of the matrix elements of the currents in the effective theory or, in other words, the calculation of \( \xi_i^0(\omega) \). Such a calculation requires the knowledge of the meson wave functions. In our formalism the mesons are taken as bound states of a quark and an antiquark and the meson state is constructed from the constituent quark states. The wavefunctions for the mesons are calculated by solving the Bethe-Salpeter equation [1] and include \( 1/m_Q \) corrections to all orders (in this particular model). That is, the integral equation carries full dependence on the finite value of \( m_Q \). We represent the meson states as [10]

\[ |M(P_M, m_J)\rangle = \sqrt{2M_H} \int d^3p \langle Lm_LSm_S|Jm_J\rangle \langle sm_s\bar{s}m_s|Sm_S\rangle \Phi_{Lm_L}(p) |q(\frac{m_q}{M}P_M - p, m_s)\rangle |q(\frac{m_q}{M}P_M + p, m_s)\rangle \]  

(14)

where

\[ |q(p, m_s)\rangle = \sqrt{\frac{(E_q + m_q)}{2m_q}} \left( \frac{\sigma \cdot p}{(E_q + m_q)} \right)^{m_s} \]  

(15)

and \( M = m_q + m_\bar{q} \) and \( M_H \) is the meson mass. The meson and the constituent quark states are normalized as

\[ \langle M(P'_M, m'_{J})|M(P_M, m_J)\rangle = 2E \delta^3(P'_M - P_M) \delta_{m'_{J},m_J} \]  

(16)

\[ \langle q(p', m'_{s})|q(p, m_s)\rangle = \frac{E_q}{m_q} \delta^3(p' - p) \delta_{m'_{s},m_s} \]  

(17)

In constructing the meson states we maintain constituent quark model approach as we do not include \( q\bar{q} \) sea quark states nor the explicit gluonic degrees of freedom. We also assume
the validity of the weak binding approximation \[10, 11\]. In the weak binding limit our meson state forms a representation of the Lorentz group, as discussed in Ref. \[10\], if the quark momenta are small compared to their masses. Assuming that the quark fields in the current create and annihilate the constituent quark states appearing in the meson state, the calculation then reduces to the calculation of a free quark matrix element. In the rest frame of the $B$ meson with a suitable choice of the four-vector indices in Eq. (3) we can construct six independent equations which we can solve to extract the six form factors.

We now turn to the question of whether our model is consistent with the requirements of HQET. We will check consistency with HQET up to the subleading order in $1/m_Q$. We will therefore ignore $O(1/m_Q^2)$ and higher power corrections even though the wavefunction from the BSE equation includes power corrections to all order.

Following Neubert and Rieckert \[12\] we can expand, the form factors $\xi_i(\omega)$ including only $1/m_Q$ corrections as

$$\xi_i(\omega) = [\alpha_i + \gamma_i(\omega) + O(\alpha_i^2, 1/m_Q^2, \alpha_i/m_Q)]\xi(\omega)$$ (18)

$\alpha_i$ takes the values 0 or 1. The corrections $\gamma_i(\omega)$ represent the $1/m_Q$ corrections. These corrections are expressed in terms of matrix elements $\rho_i(\omega)$ given below

$$\gamma_+ = \left( \frac{1}{m_c} + \frac{1}{m_b} \right) \rho_1(\omega)$$
$$\gamma_- = \left( \frac{1}{m_c} - \frac{1}{m_b} \right) \left[ \rho_4(\omega) - \frac{1}{2} \bar{\Lambda} \right]$$
$$\gamma_V = \frac{1}{2} \bar{\Lambda} \left( \frac{1}{m_c} + \frac{1}{m_b} \right) + \frac{\rho_2(\omega)}{m_c} + \frac{\rho_1(\omega) - \rho_4(\omega)}{m_b}$$
$$\gamma_{A_1} = \frac{1}{2} \bar{\Lambda} \left( \frac{1}{m_c} + \frac{1}{m_b} \right) \frac{\omega - 1}{\omega + 1} + \frac{\rho_2(\omega)}{m_c} + \frac{\rho_1(\omega) - \rho_4(\omega)}{m_b}$$
$$\gamma_{A_2} = \frac{1}{\omega + 1} \left[ -\bar{\Lambda} + (\omega + 1) \rho_3(\omega) - \rho_4(\omega) \right]$$
$$\gamma_{A_3} = \frac{\bar{\Lambda}}{2} \left( \frac{1}{m_c} \omega - 1 + \frac{1}{m_b} \right) + \left[ \frac{\rho_2(\omega) - \rho_3(\omega) - \frac{1}{\omega + 1} \rho_4(\omega)}{m_c} \right] + \left[ \frac{\rho_1(\omega) - \rho_3(\omega)}{m_b} \right]$$ (19)

The form factors $\rho_i(\omega)$ can be related to the form factors $\chi_1(\omega)$, $\chi_2(\omega)$, $\chi_3(\omega)$ and $\xi_3(\omega)$ \[13, 14\] via

$$\rho_1(\omega)\xi(\omega) = \chi_1(\omega) - 2(\omega - 1)\chi_2(\omega) + 6\chi_3(\omega)$$
$$\rho_2(\omega)\xi(\omega) = \chi_1(\omega) - 2\chi_3(\omega)$$
$$\rho_3(\omega)\xi(\omega) = 2\chi_2(\omega)$$
$$\rho_4(\omega)\xi(\omega) = \xi_3(\omega)$$ (20)
The normalization conditions at maximum recoil which follows from the conservation of the vector current in the $m_b = m_c$ limit are [13]

$$\xi(\omega = 1) = 1$$

and

$$\rho_1(\omega = 1) = \rho_2(\omega = 1) = 0$$

which is the same as

$$\chi_1(\omega = 1) = \chi_3(\omega = 1) = 0$$

Using the calculated form factors $\xi_i(\omega)$ from the BSE model we can use Eqs. (18) - Eqs. (20) to extract the HQET parameters $\bar{\Lambda}$ and $\rho_i(\omega)$ or $(\chi_i(\omega)$ and $\xi_3(\omega)$). We note that Eqs. (18) involve 6 equations for 6 form factors. However, the equations are not linearly independent, so we utilize our model $\xi(\omega)$ [2] and reduce to 5 the number of form factors to be determined by these equations. We use the least squares method to solve this system of equations, Eqs.(18), to obtain the HQET parameters. One should keep in mind that the calculated form factors contain $1/m_Q^2$ and higher order power corrections and so the extracted HQET parameters also contain effects of $1/m_Q^2$ and higher order corrections.

3 Results and Discussions

In previous papers [1, 2] a covariant reduction of the Bethe-Salpeter equation (BSE) was used to calculate the Isgur-Wise function. The BSE was solved numerically and the parameters appearing in it (the quark masses, string tension and the running coupling strength for the one gluon exchange) were determined by fitting the calculated spectrum to the observed masses of more than 40 mesons. The resulting mass spectrum of the analysis was found to agree very well with the experimental data. Once the parameters of the model were fixed, the meson wavefunction could be calculated from the BSE. This wavefunction was used to calculate the Isgur-Wise function and determine $V_{cb}$ [2]. We now present the results of our present calculations. A number of similar calculations can be found in the literature [3, 4]. For the sake of brevity and whenever appropriate we will only compare our results with Ref. [11] and the QCD sum rule calculations [7].

In Fig.1 we show the calculated form factors $\xi_i$ as function of $\omega$. We also plot the Isgur-Wise function $\xi(\omega)$ for comparison. The size of the $1/m_Q$ corrections or corrections to the HQS limit is reflected in the deviations of the form factors from $\xi(\omega)$ or from 0 (in the case
\(\xi_-(\omega)\) and \(\xi_{A_2}(\omega)\). For \(\xi_v(\omega), \xi_+(\omega), \xi_{A1}(\omega)\) and \(\xi_{A3}(\omega)\) the \(1/m_Q\) corrections are positive and can have a maximum effect of 30\% without QCD corrections and about 40\% with QCD corrections (See Table 1). Based on the size of the \(1/m_Q\) corrections, we might naively argue that the total \(1/m_Q^2\) effects can be expected to be about 10 – 15\%. For \(\xi_-(\omega)\) and \(\xi_{A_2}(\omega)\) the power corrections are negative and negligible for \(\xi_{A_2}(\omega)\). We could naively expect the neglected power corrections to follow the same trend as that for the other form factors. In this calculation we have not included the perturbative QCD corrections.

In Fig.2 we show a plot of the HQET mass parameter \(\bar{\Lambda}\) versus \(\omega\) and we see that \(\bar{\Lambda}\) is almost independent of \(\omega\) indicating that our model is consistent with HQET where the mass parameter \(\bar{\Lambda}\) is independent of \(\omega\). However, as noted before, our extracted \(\bar{\Lambda}\) is modified due to higher power corrections which are present in the calculated form factors. The value of \(\bar{\Lambda} \sim 0.55 GeV\) is comparable to the value of this quantity extracted by other methods, such as QCD sum rules [13, 7]. This contrasts with typical values of \(\bar{\Lambda}\) extracted in quark models which are of the order of the constituent mass of the light quark in the heavy meson (See Ref. [6] and Ref. [11]). We believe that relativistic and spin effects, as treated in our approach, are responsible for the significant differences from the traditional quark models.

In Fig.3 we show the functions that represent the corrections to the form factors coming from the expansion of the Lagrangian in \(1/m_Q\) viz. \(\chi_1(\omega), \chi_2(\omega)\) and \(\chi_3(\omega)\) and the correction coming from the expansion of the heavy quark field in the weak current viz. \(\xi_3(\omega)\). We have plotted the dimensionless quantities \(\chi_1(\omega)/\bar{\Lambda}, \chi_2(\omega)/\bar{\Lambda}, \chi_3(\omega)/\bar{\Lambda}\) and \(\xi_3(\omega)/\bar{\Lambda}\) in the figure.

The function \(\chi_1(\omega)\) represents the correction coming from the kinetic energy operator while \(\chi_2(\omega), \chi_3(\omega)\) represent the chromomagnetic corrections that violate the spin symmetry of the effective theory. At \(\omega = 1\), we know from HQET that both \(\chi_1, \chi_3\) are zero. We expect \(\chi_1\) and \(\chi_3\) to deviate from 0 at \(\omega = 1\) because the functions \(\chi_i(\omega)\) includes corrections of order \(1/m_Q^2\) and higher. In fact the deviation of \(\chi_1\) and \(\chi_3\) from zero at maximum recoil is an indication of the size of the \(1/m_Q^n(n \geq 2)\) corrections. Our results for \(\chi_1(\omega), \chi_3(\omega)\) are close to what one expects in HQET. There are no constraints on \(\chi_2\) at \(\omega = 1\). We find \(\chi_2\) to be small, positive and slowly decreasing with \(\omega\). \(\chi_3\) is almost completely flat and remains close to zero for the entire range of \(\omega\) that we have considered.

Our results are consistent with QCD sum rule calculations [4] which find the chromomagnetic corrections to be a few percent. The calculated \(\chi_1\) shows a quadratic behaviour with \(\omega\). It peaks around \(\omega = 1.3\) with a maximum value of around 0.22 which is similar to QCD sum rule predictions. In HQET the function \(\xi_3(\omega)\) is expected to have a \(\omega\) dependence
similar to the Isgur-Wise $\xi(\omega)$. In fact it is customary to write

$$\xi_3(\omega) = \bar{\Lambda}\xi(\omega)\eta(\omega)$$

where $\eta(\omega)$ is expected to be a slowly varying function of $\omega$. Our calculated $\eta$ shows a mild variation with $\omega$ though the value of $\eta$ is smaller than estimated in QCD sum rules. In Ref. [11] $\chi_2(\omega) = 0$ and $\xi_3(\omega) = 0$.

The form factors calculated in our model have to be corrected by taking into account perturbative QCD corrections which are given in Eqs. (10-13). However, there is an uncertainty in our choice of the scale $\mu$, the scale of the effective theory. We have chosen $\mu \sim p_{av} \sim 0.6$ GeV where $p_{av}$ is the average value of the internal momentum inside the mesons [11]. In Fig.4 we show the form factors $\xi_i(\omega)$ including perturbative QCD corrections.

In Table 1 we give the values of the form factors at $\omega = 1$ with and without perturbative QCD corrections. We also show the numbers calculated in Ref. [11] for comparison. Even though we find agreement between our results and those of Ref. [11] for the sign of the power corrections the magnitude of the corrections are different. This is probably due to the wavefunction used in our calculation which includes $1/m_Q$ corrections of all order. It is important to note from Table. 1 that the magnitudes of the $1/m_Q$ corrections are comparable to the QCD corrections indicating the need to retain both.

Having obtained the form factors we calculate the decay rate for $B \to D^* l \nu$ [16] and fit it to the experimental measurements [17] to extract $V_{cb}$. The relation for the decay rate in terms of the form factors is given in Ref. [16]. The $\chi^2$ per degree of freedom of the fit is calculated to be 1.4 without QCD corrections and 0.95 with QCD corrections.

In Fig.5 we show the decay rate calculated using the form factors with and without QCD corrections. In our most complete approach (including QCD corrections) we extract a value for $V_{cb} = (31.9 \pm 1.4) \times 10^{-3}$, close to the lower limit quoted in the Particle Data Group [18]. As we have indicated before we might expect about 10 – 15% corrections to the form factors from the neglected higher order power corrections. Since the effects of the $1/m_Q$ corrections is to bring down the value of $V_{cb}$ from $(34.7 \pm 2.5) \times 10^{-3}$ calculated in Ref. [2] to $(31.9 \pm 1.4) \times 10^{-3}$ we could expect a few percent corrections to our value of $V_{cb}$ from the neglected $1/m_Q^2$ and higher power corrections. Since the $1/m_Q^2$ corrections to the form factors relevant to $B \to D^* l \nu$ are expected to be negative [3] we would expect a higher value for $V_{cb}$ if we were to include all the $1/m_Q^2$ corrections in our calculations.

In conclusion, we have presented the form factors in the semileptonic decays of $B \to D(D^*)$ in a Bethe-Salpeter model for mesons. The parameters of the model are fixed from
Table 1: The values of the form factors at $\omega = 1$

| $\xi_i$ | without QCD corrections | with QCD corrections | ISGW2
|---|---|---|---|
| $\xi_+^+$ | 1.048 | 1.086 | 1.00 |
| $\xi_-^-$ | $-9.37 \times 10^{-2}$ | $-6.94 \times 10^{-2}$ | $-9.0 \times 10^{-2}$ |
| $\xi_V$ | 1.27 | 1.37 | 1.17 |
| $\xi_{A_1}$ | 1.02 | 0.99 | 0.91 |
| $\xi_{A_2}$ | $-0.233$ | $-0.120$ | $-0.180$ |
| $\xi_{A_3}$ | 1.07 | 1.09 | 1.01 |

spectroscopy of the hadrons and the calculation of the form factors do not involve any new parameters. We have shown that our model is consistent with the requirements of HQET and have extracted the non-perturbative matrix elements that characterize the $1/m_Q$ corrections in HQET. We have also obtained $V_{cb}$ from the available data.

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4 Figure Captions

Fig.1: The calculated form factors $\xi_i(\omega)$ without perturbative QCD corrections and the calculated Isgur-Wise function $\xi(\omega)$.

Fig.2: The mass parameter $\bar{\Lambda}$ extracted from the form factors $\xi_i(\omega)$.

Fig.3: The $1/m_Q$ correction functions $\chi_1(\omega)/\bar{\Lambda}$, $\chi_2(\omega)/\bar{\Lambda}$, $\chi_3(\omega)/\bar{\Lambda}$ and $\xi_3(\omega)/\bar{\Lambda}\xi(\omega)$ extracted from the form factors $\xi_i(\omega)$. The calculated $\chi_i(\omega)$ and $\xi_3(\omega)$ include some $1/m_Q^n(n \geq 2)$ corrections coming from the meson wavefunctions.

Fig.4: The calculated form factors $\xi_i(\omega)$ with perturbative QCD corrections with $\mu = 0.6$ GeV.

Fig.5: The differential decay rate with and without the QCD correction, together with the corresponding values of $V_{cb}$. 


Fig. 1
Fig. 2
Fig. 5

The graph shows a comparison between two curves, one with QCD correction and the other without. The QCD-corrected curve includes a parameter $V_{cb}=31.9 \times 10^{-3}$, while the uncorrected curve uses $V_{cb}=30.5 \times 10^{-3}$. The data points with error bars represent experimental measurements.