A new proof for non-occurrence of trapped surfaces and information paradox

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Abstract

We present here a very simple, short and new proof which shows that no trapped surface is ever formed in spherical gravitational collapse of isolated bodies. Although this derivation is of purely mathematical nature and without any assumption, it is shown, in the Appendix, that, physically, trapped surfaces do not form in order that the 3 speed of the fluid as measured by an observer at a fixed circumference coordinate $R$ (a scalar), is less than the speed of light $c$. The consequence of this result is that, mathematically, even if there would be Schwarzschild Black Holes, they would have unique gravitational mass $M = 0$. Recall that Schwarzschild BHs may be considered as a special case of rotating Kerr BHs with rotation parameter $a = 0$. If one would derive the Boyer-Lindquist metric [1] in a straightforward manner by using the Backlund transformation[2], one would obtain $a = M \sin \phi$ where $\phi$ is the azimuth angle. This relation demands that BHs have unique mass $M = 0$ (along with $a = 0$) which in turn confirms that there cannot be any trapped surface in realistic gravitational collapse where the fluid has real pressure and density. Since there is no trapped surface and horizon, there is no Information Paradox in the first place.

When a self-gravitating fluid undergoes gravitational contraction, by virtue of Virial Theorem, part of the gravitational energy must be radiated out. Thus the total mass energy, $M$, ($c = 1$) of a body decreases as its radius $R$ decreases. But in Newtonian regime ($2M/R \ll 1$, $G = 1$), $M$ is almost fixed and the evolution of the ratio, $2M/R$, is practically dictated entirely by $R$. If it is assumed that even in the extreme general relativistic case $2M/R$ would behave in the same Newtonian manner, then for sufficiently small $R$, it would be possible to have $2M/R > 1$, i.e, trapped surfaces would form[3,4].
Unfortunately, even when we use General Relativity (GR), our intuition is often governed by Newtonian concepts, and thus, intuitively, it appears that, as a fluid would collapse, its gravitational mass would remain more or less constant so that for continued collapse, sooner or later, one would have $2M/R > 1$, i.e., a “trapped surface” must form. The singularity theorems thus start with the assumption of formation of trapped surfaces\cite{3,4}. However, many readers (from experimental astronomy, particle physics, quantum-gravity and quantum information fields) may not be even aware that occurrence of trapped surface is a conjecture and the Singularity Theorems are based on this conjecture. Further, in the literature, there have been attempts to find out “sufficient and necessary condition” for realization of this conjecture of trapped surfaces for spherical collapse\cite{3} without showing that such “necessary and sufficient conditions” are fulfilled. Nonetheless, we have found that, by superficially going through such papers, many readers (erroneously) think that the conjecture of trapped surface has been proved. The actual situation regarding this has been succinctly brought out in the following statement\cite{5}:

“it is necessary to point out that a demonstration of the trapped surface conjecture remains elusive.”

In fact, there are specific examples that trapped surfaces do not form. For example, in the cosmological context, Nariai Metric\cite{6} is the first specific example of non-occurrence of trapped surfaces. In this context, Dadhich’s comment about this “assumption” of trapped surfaces is\cite{7}:

“It is noteworthy that violation of this assumption entailed no unphysical features. This assumption seriously compromises, as is demonstrated by this example, the generality of the theorems (i.e, singularity theorems, author). It has always been looked upon as suspect.”

Another work on spherical collapse using premeditated specific metric finds that collapse would not produce any Horizon because of heat flow, i.e., because of decrease of $M$ during collapse\cite{8}. This specific example focussed attention only at the boundary of the fluid and had it treated the inner mass shells, it might have found absence of trapped surfaces too.

In 1990, Senovilla constructed a specific model of a cylindrically symmetric universe without any trapped surface\cite{9}. In 2002, Goncalves, in a more general manner, showed the absence of trapped surface and singularities in cylindrical collapse\cite{10}.

In the cosmological context, $M$ remains fixed (because radiation cannot leave the universe) while for isolated bodies, $M$ necessarily decreases due to emission of radiation, and thus, it is more likely that trapped surfaces do not form.

In any case, once the formation of a “trapped surface” is assumed, then, a set of very reasonable assumptions would show that the collapse would become irreversible. Physically, once trapped surface formation is assumed,
it would appear that the sign of the pressure gradient force would reverse and thus pressure would aid the collapse rather than hinder the same. Furthermore, since outgoing radiation too would turn inward, no mass energy would escape and the mass energy enclosed within a shell $M(r)$, where $r$ is a comoving coordinate, would not decrease any longer. Hence if the collapse would lead to a BH with an all encompassing Event Horizon (EH), it would naturally appear that the mass of the BHs must be finite. Thus the intuitive and apparently correct notion of finite mass BHs rests on the assumption of formation of trapped surfaces, which, in turn, rests on our intuitive (but incorrect) idea that even when gravity would be so strong as to trap even light $M$ would remain more or less unchanged as in the Newtonian case. However, the following exact treatment would unequivocally show that, trapped surfaces do not form in any spherical collapse.

If we take the signature of spacetime as $(+,-,-,-)$ the spherically symmetrical metric for an arbitrary fluid, in terms of comoving coordinates $t$ and $r$ is[3,4,12]

$$ds^2 = g_{00}dt^2 + g_{rr}dr^2 - R^2(d\theta^2 + \sin^2 \theta d\phi^2)$$ (1)

where $R = R(r,t)$ is the Invariant Circumference coordinate and happens to be a scalar. Further, for radial motion with $d\theta = d\phi = 0$, the metric becomes

$$ds^2 = g_{00}dt^2(1 - x^2)$$ (2)

or,

$$(1 - x^2) = \frac{1}{g_{00}} \frac{ds^2}{dt^2}$$ (3)

where the auxiliary parameter

$$x = \frac{\sqrt{-g_{rr}}}{\sqrt{g_{00}}} \frac{dr}{dt}$$ (4)

For a fluid element at $r = fixed$, obviously $x = 0$ because $dr = 0$. However, $dr$, in general, is not zero; otherwise the metric in comoving coordinates would be

$$ds^2 = g_{00}dt^2 - R^2(d\theta^2 + \sin^2 \theta d\phi^2)$$ (5)

and which is not the case. The comoving observer at $r = r$ is free to do measurements of not only the fluid element at $r = r$ but also of other objects: If the comoving observer is compared with a static floating boat in a flowing river, the boat can monitor the motion of other boats or the pebbles fixed on the river bed. Here the fixed markers on the river bed are like the background $R = constant$ markers against which the river flows. If we intend to find the parameter $x$ for such a $R = constant$ marker, i.e, for a pebble lying on the river bed at a a fixed $R$, we will have
\[ dR(r, t) = 0 = \dot{R}dt + R' dr \quad (6) \]

where an overdot denotes a partial derivative w.r.t. \( t \) and a prime denotes a partial derivative w.r.t. \( r \). Therefore for the \( R = \text{constant} \) marker, we find that

\[ \frac{dr}{dt} = -\frac{\dot{R}}{R'} \quad (7) \]

and the corresponding \( x = x_c \) is

\[ x = x_c = \frac{\sqrt{-g_{rr}}}{\sqrt{g_{00}}} dt = -\frac{\sqrt{-g_{rr}}}{\sqrt{g_{00}}} \dot{R} \quad (8) \]

Using Eq.(3), we also have

\[ (1 - x_c^2) = \frac{1}{g_{00}} \frac{ds^2}{dt^2} \quad (9) \]

Now let us define[4]

\[ \Gamma = \frac{R'}{\sqrt{-g_{rr}}} \quad (10) \]

and[4]

\[ U = \frac{\dot{R}}{\sqrt{g_{00}}} \quad (11) \]

so that the combined Eqs. (8), (10) and (11) yield

\[ x_c = \frac{U}{\Gamma}; \quad U = -x_c \Gamma \quad (12) \]

As is well known, the gravitational mass of the collapsing (or expanding) fluid is defined through the equation[4,12]

\[ \Gamma^2 = 1 + U^2 - \frac{2M(r, t)}{R} \quad (13) \]

Then the two foregoing equations can be combined and transposed to obtain

\[ \Gamma^2(1 - x_c^2) = 1 - \frac{2M(r, t)}{R} \quad (14) \]

By using Eqs.(9) and (10) in the foregoing Eq., we have

\[ \frac{R'^2}{-g_{rr} g_{00}} \frac{ds^2}{dt^2} = 1 - \frac{2M(r, t)}{R} \quad (15) \]

Recall that the determinant of the metric tensor is always negative[11]:

\[ g = R^4 \sin^2 \theta \ g_{00} \ g_{rr} \leq 0 \quad (16) \]
so that we must always have

\[- g_{rr} \, g_{00} \geq 0 \]  \hspace{1cm} (17)

Further for the signature chosen here, \( ds^2 \geq 0 \) for all material particles or photons. Then noting Eq.(17), it follows that the LHS of Eq. (15) is *always positive*. So must then be the RHS of the same Eq. which implies that

\[ \frac{2M(r,t)}{R} \leq 1 \]  \hspace{1cm} (18)

This shows in the utmost general fashion that trapped surfaces are not formed in spherical collapse or expansion. As such this result is already known[12,13,14]. However, several readers have found the papers[12,13] too long and the central derivation to be rather complicated. Further when we wrote the papers [12,14], we failed to bring out the precise physical implication of the parameter "\( v \)" appearing there. We have removed this shortcoming in the present derivation. The Appendix I. will show, in a very transparent manner, that \( v \) is the 3-speed of the fluid with respect to an observer sitting at a fixed \( R \). Nevertheless, it may be noted that to arrive at the no trapped surface condition, we do not require this physical interpretation of \( v \).

In the absence of any trapped surface, a collapsing fluid will always keep on radiating and \( M(r) \) would keep on decreasing. In case it would be assumed that, the collapse would continue all the way upto \( R = 0 \), then the constraint (18) demands that \( M(R = 0) = 0 \) too. And this is the reason that all BHs (even if they would be assumed to exist) must have \( M = 0 \).

Once there would be no trapped surface, virial theorem would ensure that the collapse process causes not only emission of radiation but there is automatic increase of internal energy and attendant pressure gradient. Thus even if there may not be any stable equilibrium, there could be approximate quasistable states which could be supported by the collapse generated internal pressure gradient. Eventhough the gravitational mass would go on decreasing, the field strength would ever increase. This would keep on enlarging the proper radial depth (stretching of spacetime membrane) and collapse would continue indefinitely in the inner (proper) spacetime eventhough, externally, the mouth of the potential well \( R \rightarrow 0 \)[12,13,15, 16]. The equality of Eq.(18) can be satisfied only at this final stage, i.e, an apparent horizon can form as \( R \rightarrow R_H \rightarrow 0 \) and \( M \rightarrow 0 \)[12,13,15,16]. This picture is corroborated by a recent work which shows that for spherical gravitational collapse, it is possible to have a situation where[5]:

"the proper distance in the transformed metric from points ‘near’ the horizon to the horizon itself becomes infinite. However, the area of the spheres \( (4\pi R^2, R \rightarrow R_H \rightarrow 0, \text{author}) \) does not change \( (R \) hovering around
0) because the angular metric components are unaffected. This means that the manifold ‘near’ the horizon gets transformed into an infinitely long cylinder (inner spacetime, author) whose crosssection asymptotes to the original area of the horizon and the three-scalar-curvature along the cylinder is a positive constant ($\sim 1/R_H^2$).

As $R \to R_H \to 0$, the scalar curvature blows up. And as the proper radial depth $l \to \infty$, the local observer’s proper time of collapse $\tau \to \infty$ and the collapse becomes Eternal.

However in a finite proper time, there would always be a finite $M$ and finite $R$ associated with an Eternally Collapsing Object (ECO). It is quite likely that the so-called Black Hole Candidates are actually ECOs. Since the ECOs are expected to be

(i) Hot, i.e, supported largely by either trapped radiation (photons and neutrinos) pressure or freshly generated radiation pressure due to contraction at unimaginable slow rate and

(ii) Not in strict hydrostatic equilibrium, i.e, they may be collapsing in a strict sense in the same way as primordial clouds or pre-main sequence stars are collapsing

the conventional mass upper limit ($M_{\text{max}} \sim 3 - 4M_\odot$) of cold objects is not applicable to them. On the intergalactic scale, the BHCs could simply be something like hot Suppermassive Stars, which, however, are contracting at incredibly slow rate and generating radiation pressure even in the absence of nuclear fuel [12,13,14,15,16].

All astrophysical plasma is neutral on a macroscopic scale and thus even if one would imagine existence of finite mass BHs, they would be chargeless and without any intrinsic magnetic field. On the other hand since even neutral astrophysical plasma is expected to generate intrinsic magnetic field, in the absence of any EH, the ECOs are expected to have strong intrinsic magnetic field. This prediction that the BHCs should have strong intrinsic magnetic field has also been verified[17,18,19].

And when there is no BH or no EH, there is never any trapping of Quantum Information let alone vanishing of the same from the universe[11, 12]. Thus the entire debate on the supposed Quantum Information Paradox and its supposed resolution is meaningless. Since there is no singularity (in a finite proper time), neither will there be any “White Hole”, “Baby Universe”, “Worm Hole”, “Time Machine” or any other science fiction item.

Finally recall that, the Schwarzschild BHs can be considered as a special case of rotating Kerr BHs with the rotation parameter $a = 0$. And if one would derive the Boyer-Lindquist metric for rotating BHs[1] in a straight forward way by using the Backlund transformation[2], it would be found that $a$ and $M$ are interlinked through

$$a = M \sin \phi$$

(19)
Since $a$ and $M$ are constants while $\phi$ is not so, in order to satisfy this relationship, we must have

$$a = 0$$  \hspace{1cm} (20)$$

as well as

$$M = 0$$  \hspace{1cm} (21)$$

The last equation demands that there is no trapped surface as already shown. Any reader not having access to the ref.\cite{2} containing Eq.\,(19) is welcome to contact the author for a scanned image of the same.

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0.1 Appendix I: Physical significance of $x$ and $x_c$

Although it is not necessary, yet, it would be worthwhile to understand the physical reason for non-occurrence of trapped surfaces. To this effect, first note the most general form of a metric[11]

$$ds^2 = g_{00}(dx^0)^2 + g_{\alpha\beta}dx^\alpha dx^\beta + 2g_{0\alpha}dx^0 dx^\alpha$$

(22)

Here $\alpha, \beta = 1, 2, 3$ represent the 3 spatial coordinates and 0 represents the temporal coordinate. Note that this general metric(22) can be separated into a spatial and temporal part[11]:

$$ds^2 = d\tau^2 - dl^2$$

(23)
where
\[ d\tau_s^2 = g_{00}(dx^0 - g_\alpha dx^\alpha)^2; \quad g_\alpha = -\frac{g_{0\alpha}}{g_{00}} \] (24)

and
\[ dl^2 = \left(-g_{\alpha\beta} + \frac{g_{0\alpha}g_{0\beta}}{g_{00}}\right) dx^\alpha dx^\beta \] (25)

Here \( d\tau_s \) is the element of synchronized proper time and \( dl \) is element of proper distance[11]. The arbitrary metric can also be rewritten as
\[ ds^2 = d\tau_s^2 (1 - v^2) \] (26)

whence the 3-speed \( v \) gets defined as[11]
\[ v^2 = \frac{dl^2}{d\tau_s^2} \] (27)

If for a specific case, such as a static metric or the present spherical case (which is a non-stationary metric) \( g_{0\alpha} = 0 \), we will have
\[ d\tau_s^2 = d\tau^2 = g_{00} dt^2 \] (28)

and
\[ dl^2 = -g_{\alpha\beta} dx^\alpha dx^\beta \] (29)

where \( d\tau \) is the usual proper time interval. Further, when all cross terms are zero (as in the present case), i.e., when \( g_{\alpha\beta} \) too is diagonal
\[ dl^2 = -g_{rr} dr^2 \] (30)

Then
\[ v = \frac{\sqrt{-g_{rr}} dr}{\sqrt{g_{00}} dt} \] (31)

Now going back to Eqs.(2) and (3), we quickly indentify \( x \) as the 3-speed of an object (not necessarily fluid element) measured by the comoving observer at \( r = r \). Obviously, the 3 speed of the fluid element itself, at \( r = r \) is \( x = 0 \). But here \( x_c \) is the 3-speed of the \( R = constant \) marker, i.e., the pebble fixed on the river bed, and is non-zero.

Since \( x_c \) is the speed of the \( R = constant \) marker w.r.t. the \( r = constant \) observer, the speed of the \( r = constant \) observer, i.e, fluid itself, w.r.t. the \( R = constant \) marker is the negative of \( x_c \):
\[ v = -x_c = -\frac{\sqrt{-g_{rr}} dr}{\sqrt{g_{00}} dt} = +\frac{\sqrt{-g_{rr}} \dot{R}}{\sqrt{g_{00}} R'} \] (32)

In terms of \( v = -x_c \), let us rewrite Eqs. (12) and (14) as
\[ U = +v\Gamma \quad (33) \]

and

\[ \Gamma^2(1 - v^2) = 1 - \frac{2M(r,t)}{R} \quad (34) \]

Now using Eq.(18) in (34), we find that both sides of it are positive and hence

\[ v^2 \leq 1 \quad (35) \]

Thus, in retrospect, we see that non-occurrence of trapped surface is a direct consequence of this cornerstone of relativity: \( v^2 \leq 1 \) for material particles and photons. If \( \gamma \) is the corresponding Lorentz factor, the master equation (34) of general relativistic fluid motion acquires the appealing form:

\[ \frac{\Gamma^2}{\gamma^2} = 1 - \frac{2M(r,t)}{R} \quad (36) \]