On the optimization of the asymmetric spur gears fillet geometry using Bézier curves

Gh Plesu¹, S Cazan²
¹ Faculty of Machines Construction and Industrial Management, Technical University, Iasi, Romania, 700050
² Faculty of Mechanical Engineering, Technical University, Iasi, Romania, 700050

E-mail: cazan.stelian@yahoo.com

Abstract. Recent research on asymmetric spur gears are focused on the geometry of the gear fillet. This happens because it is very important to avoid the high level of bending stress of the teeth. There are some authors that show how bending stress level can get lower using a proper geometry of the fillet curve. In this paper, there is presented a new concept to optimizing the geometry of the fillet curve. There is used the reasoning of finding a regular curve with minimum length that doesn’t affect the functionality of the gear. Because the procedure of finding this curve is a difficult one, there is presented a special kind of curves that can accomplish all the technical requirements. The procedure is based on the considerations that are illustrated below.

1. Introduction
To maintain a proper level of stress, the base section of the gear tooth must increase. The authors in [2] proved that substituting the trochoide with a circle, the bending stress level is reduced. In this paper, the main topic is to find a better geometry for the fillet to increase the base strength of the tooth [1]. The presentation is based on the following considerations:
- there is necessary a regular curve, with minimum length (the minimum length condition is required for a larger base section of the tooth), and that curve must have at least 1-degree contact with the involute curves, (for a proper manufacture; if there is not a 1-degree contact between the curve and the involute curves, there will appear angular points and such points involve the impossibility of manufacturing the gear);
- the proposed geometry does not have to affect the functionality of the gears.

In the following, there is presented the benefits brought by the substitution of the circle with a regular curve and the resulting advantages using Bézier curves for the gear fillet geometry.

2. Different geometry
2.1. Mathematical approach
As it was mentioned before, there must exist at least 1-degree contact between the gear fillet and the involute curves. The mathematical procedure is presented below [3], [5].

Let us consider the coordinate system Sd as being the main coordinate system (fig. 1) and the equations of the involute curves. There will be presented the possibility of finding the “best” circle as a gear fillet curve and then there will be presented the proper geometry of the required curve.
The equations of the driving side involute curve are parametrically presented below in $S_d$:

\[
\begin{align*}
    x_d(\phi_d) &= r_{rd}(\cos \phi_d + \phi_d \sin \phi_d) \\
    y_d(\phi_d) &= r_{rd}(\sin \phi_d - \phi_d \cos \phi_d)
\end{align*}
\]  

(1)

and the equations of the coast side involute curve in $S_c$:

\[
\begin{align*}
    x_c(\phi_c) &= r_{rc}(\cos \phi_c + \phi_c \sin \phi_c) \\
    y_c(\phi_c) &= r_{rc}(\sin \phi_c - \phi_c \cos \phi_c)
\end{align*}
\]

(2)

according to [6].

**Figure 1.** The coordinate systems $S_d$ and $S_c$

Here, $\sigma = \text{inv}_{d} + \frac{\lambda}{r_{rd}} + \text{inv}_{c}$. $\alpha_d$ and $\alpha_c$ represents the pressure angles of the driving side involute curve and coast side involute curve on the points where $\phi_d$ and $\phi_c$ are 0, respectively. $\phi_d$ and $\phi_c$ represents the parameters for these two curves and they can be defined as:

$\phi_d^2 = \sqrt{\left(\frac{2\pi}{r_{rd}}\right)^2} - 1$, where $r_{M}$ represents the module of the vector position of any point that belongs to the driving side involute curve. Similar:

$\phi_c^2 = \sqrt{\left(\frac{2\pi}{r_{rc}}\right)^2} - 1$, where $r_{N}$ represents the module of the vector position of any point that belongs to the coast side involute curve.

The authors of [2] presented a methodology of substitution of the trochoid with a circle. The circle and the curve that must be found has that 1-degree contact mentioned above with the driving side involute curve, because this is the side of the tooth that meshes with the mating gear. To find the right circle and the fillet curve, it is necessary the mathematical approach presented below.

Let there be $\phi(\phi_d)$ the contact function. For a 1-degree contact, the contact function must satisfy the following equations:

\[
\begin{align*}
    \phi(\phi_d) &= 0 \\
    \phi'(\phi_d) &= 0
\end{align*}
\]  

(3)

To demonstrate the relation (3), the following mathematical proposition must be used:

**Proposition:** Let there be two simple curves $c \in \mathbb{C}^n$, defined as:

\[
(\gamma_1): F(x,y) = 0; (\gamma_2): r = r(t) = x(t)i + y(t)j, t \in I.
\]

These curves have a $m$-degree contact $(m<n)$, in their common point $M_0(x_0,y_0)$, $x_0 = x(t_0), y_0 = y(t_0)$ if and only if

$\phi(t_0) = 0, \phi'(t_0) = 0, ..., \phi^{(m)}(t_0) = 0, \phi^{(m+1)}(t_0) \neq 0$, where $\phi : I \to R, \phi(t) = (F \circ r)(t) = F(x(t), y(t))$ is called the contact function between these two curves [5].
In our case, the curve parameter is $\phi_d$ (6), the explicit-defined curve is the circle (or the fillet curve) and the parametrically defined curve is the involute curve.

2.2 The substitution of a trochoid with a circle

Using the methodology presented above, the contact function has the following expression in this case:

$$\phi(\phi_d) = [x_d(\phi_d) - x_0]^2 + [y_d(\phi_d) - y_0]^2 - r^2 = 0$$

After the calculations, we obtain:

$$\phi(\phi_d) = (r_{bd} - x_0)^2 + y_0^2 - r^2 = 0$$

We determine now the mathematical expression of the first derivative of the contact function according to (3).

$$\phi'(\phi_d) = [(r_{bd} - x_0)^2 + y_0^2 - r^2]'$$

But if

$$\phi'(\phi_d) = 0$$

and we take into consideration the relation (4), we obtain:

$$x_0 = r_{bd}$$ and $$y_0 = -r.$$ 

Here, $x_0$ and $y_0$ represent the coordinates of the center of the circle, and $r$ is the radius of the circle. That means that the circle and the driving side involute curve have at least a 1-degree contact and have the same tangent and the same normal in the point $M(r_{bd}, 0)$, which means that the $X_d$ axis is tangent to the involute curve of the driving side. This is the best variant of a circle to maintain a good bending stress level, because the base section of the tooth has maximum value. To improve this geometry by substituting the circle with a regular curve, the following conditions must be accepted:

- the curve has minimum length and at least a 1-degree contact with the involute curves of the tooth;
- the proper functionality of the gears must be maintained.

These conditions involve finding the solutions of a differential system (for finding the required curve), which is not a simple work. But there exists a type of curves that can satisfy both conditions. These curves are called Bézier curves [3], [4] and their geometry and the generation principle are presented below.

2.3 The substitution of a trochoid with a Bézier curve

2.3.1. Bézier curves

Let there be $P_0, P_1, \ldots, P_{n+1}$ distinct points $\epsilon \mathbb{R}^2$ called the control points. The polygon that can be formed with these points is called the Bézier polygon. This polygon is not unique.

The $n$-degree Bézier curve can be written as:

$$B(t) = \sum_{k=0}^{n} b_{kn} P_k(t)$$

where $b_{kn}$ represents the Bernstein polynomials:

$$b_{kn} = \frac{n!}{k!(n-k)!} t^k (1-t)^{n-k}, t \epsilon [0,1].$$

Bézier curves are widely used in computer graphics to model smooth surfaces. There are several ways to define a Bézier curve such as parametric definition, polynomial definition, recursive definition, explicit definition, etc. In this paper, we present the parametric definition. Any Bézier curve has the following properties:

- the first point of the curve is $P_0$ and the last one is $P_n$;
- the curve is a straight line if and only if all the control points are collinear;
- the start of the curve and the end of the curve are tangent to the first and the last section of the Bézier polygon;
- if we split a Bézier curve in two pieces, those pieces are also Bézier curves;
- some curves that seem simple cannot be described exactly by a Bézier curve; these curves can be approximated by a Bézier curve;
- Bézier curves have the variation diminishing property, which means that a Bézier curve does not "undulate" more than the polygon of its control points and may actually "undulate" less;
- moving a control point of a Bézier curve affects the aspect of the entire curve.

In the following, there is presented a generation method of the fillet curve using a cubic Bézier curve. There may exist better ways to creating the required curve, but in this paper there is presented the benefits brought by using this kind of curves. The possibility of using a Bézier curve with a bigger-than-3-degree will be analyzed in further papers.

2.3.2. The substitution of a trochoide with a cubic Bézier curve
A cubic Bézier curve (n=3, 4 control points), can be written as:

\[ B(t) = (1-t)^3P_0 + 3t(1-t)^2P_1 + 3t^2(1-t)P_2 + t^3P_3 \]

\[ p_{01}, i = 0 \ldots n, \text{represents the control points.} \]

Let us take a look again at the fig.1 and let us consider \( S_d \) the main coordinate system. Using this system, there will be written the equations of the Bézier curve. So it is necessary to transform the coordinates of the coast side involute curve from \( S_c \) to \( S_d \). To accomplish this, it is necessary a counterclockwise rotation with the angle \( \sigma + \frac{2\pi}{z} \). (fig.1) [1]

**Figure 2.** The representation in the same coordinate system (\( S_d \)) of the curves: AM- coast side involute curve, MN- the Bézier curve and NP- the driving side involute curve

The relations (9) can also be written as:

\[ r_{dc}(\phi_c) = M_{dc} r_c \]

\[ \begin{pmatrix} x_c(\phi_c) \\ y_c(\phi_c) \end{pmatrix} = \begin{pmatrix} \cos(\sigma + \frac{2\pi}{z}) & \sin(\sigma + \frac{2\pi}{z}) \\ -\sin(\sigma + \frac{2\pi}{z}) & \cos(\sigma + \frac{2\pi}{z}) \end{pmatrix} \begin{pmatrix} x_c(\phi_c) \\ y_c(\phi_c) \end{pmatrix} \]

\[ \Rightarrow \begin{pmatrix} x_{dc}(\phi_c) \\ y_{dc}(\phi_c) \end{pmatrix} = \begin{pmatrix} x_c(\phi_c)\cos(\sigma + \frac{2\pi}{z}) + y_c(\phi_c)\sin(\sigma + \frac{2\pi}{z}) \\ -x_c(\phi_c)\sin(\sigma + \frac{2\pi}{z}) + y_c(\phi_c)\cos(\sigma + \frac{2\pi}{z}) \end{pmatrix} \]

\[ z = \text{number of teeth of the gear and } M_{dc} \text{ is the rotational coordinate transformation matrix from } S_c \text{ to } S_d. \]

The relations (9) can also be written as:
Using a cubic Bézier curve, there are necessary 4 control points: \( P_0, P_1, P_2 \) and \( P_3 \). The segment \( P_0P_1 \) is tangent to the curve and also to the driving side involute curve and the segment \( P_2P_3 \) is tangent to the coast side involute curve and also to the Bézier fillet curve (see above the Bézier curves properties). The first point will be situated on the driving side involute curve where \( \phi_d = 0 \), the point \( P_1 \) will be situated on the driving side involute curve tangent or on the \( X_d \) axis (because \( X_d \) axis is tangent at the involute and \( P_0P_1 \) is the segment that belongs to Bézier polygon). The same thing will happen with the other two points: \( P_3 \) will be situated on the coast side involute curve where \( \phi_c = 0 \) and \( P_2 \) will be situated on the tangent coast side involute curve tangent. The coordinates of \( P_0 \) and \( P_3 \) are known:

The parametric equations of the cubic Bézier curve are:

\[
\begin{align*}
(x(t)) &= (1-t)^3x_0 + 3t(1-t)^2x_1 + 3t^2(1-t)x_2 + t^3x_3 \\
y(t) &= 3t(1-t)^2y_1 + 3t^2(1-t)y_2 + t^3y_3
\end{align*}
\]

We will determine below the coordinates of the other two points taking into consideration the functionality conditions of the gear.

The main conditions to determining the coordinates of the points are the condition of the radial backlash existence and the avoiding interference condition. To avoid interference and maintain the required backlash, the following condition must be accepted:

\[ x_f = x_{2c} - 0.25m \]

where \( x_i \) is the \( X_d \) coordinate of \( r_i \), \( m \) is the gear module and 0.25m represents the expression of the radial backlash.

Assuming this, we have:

\[ x_f = r_c \cos \alpha \]

where \( \alpha \) is the angle between \( X_d \) axis and \( r_i \) ( \( r_i \) being the root radius or the radius of the asymmetric gear dedendum circle ) and

\[ \alpha = \arccos \frac{x_2c - 0.25m}{r_f} = \arccos \frac{x_f}{r_f} \]

In the same way, we can obtain the expression of \( y_i \):

\[ y_f = -r_c \sin \alpha \]

The conditions of existence of Bézier curves leads to the following result: point \( P_1 \) must be positioned on \( X_d \) axis (because \( X_d \) is tangent at the driving side involute curve ) and point \( P_2 \) must be positioned on the coast side involute curve tangent. The coast side involute tangent pass through the origin of \( S_c \) (due to the counterclockwise rotation of \( S_c \) with \( \sigma + \frac{2\pi}{x} \) ) and we cannot know precisely where \( P_2 \) will be situated on that tangent. Assuming this, there will be used the following reasoning.

Let us consider a different Bézier polygon. This polygon will be formed using 4 points situated like this:

- The first point (\( P_0 \)) will be situated on \( X_d \) axis, where \( \phi_d = 0 \) for the driving side involute curve.
- The second point (\( P_1 \)) will be situated lower, with an unknown position on the \( X_d \) axis (let’s called that distance \( x_1 \), which represents the distance from the origin along the \( X_d \) axis). This distance will be determined further.
- The third point (\( P_2 \)) will be situated on a straight line which passes through the first point of the coast side involute curve (where \( \phi_c = 0 \) ) and this straight line will be parallel with \( X_d \) axis. This point will have the same \( X_d \) axis coordinate like the second (\( x_1 \)), because it is necessary to keep the lowest point of the
Bézier curve as being the point which corresponds with the root diameter. The $Y_d$ coordinate will be $-r_{bc} \sin \frac{2\pi}{z}$ (because this is the $Y_d$ coordinate of the first point of the coast side involute curve in $S_d$).

The fourth point ($P_3$) will be situated on the first point of the coast side involute curve, where $\phi_c = 0$.

**Remark:** Whether we move the third point along the $Y_d$ axis, the lowest point of the Bézier curve will have the same $X_d$ coordinate, which means that the position of the root diameter will not be changed. The movement of the third point is necessary for building the proper Bézier curve and to maintain the same conditions like those illustrated above. There is used this reasoning to find out the position of the third point on the coast side involute curve tangent. This thing involves that, knowing the $Y_d$ coordinate of the third point (for the current case), we can determine its proper position on the tangent to the coast side involute curve. This happens if we translate the current $Y_d$ coordinate of the third point until this point intersects the tangent of the coast side involute curve (where $\phi_c = 0$). The initial Bézier polygon is represented below (fig. 3).

![Figure 3](image.png)

**Figure 3.** The initial Bézier polygon: 1- the driving side involute curve, 2- the coast side involute curve, 3- the initial Bézier polygon (red), 4- the straight line which passes through the first point of the coast side involute curve (parallel with $X_d$) and 5- the resulting Bézier curve (orange)

As it was said before, there will be a translation along the $Y_d$ axis of the $Y_d$ coordinate of the third point (the translation of $P_2$ can be seen below; the new point is $P_2'$). This thing can be easily seen in fig. 4.

Let us called this new point $P_2'$ and let there be $r_{P_2}$, its position vector. Assuming that the distance along the $X_d$ axis from the origin to $P_1$ is $x_1$, the equation of $r_{P_2}$ can be written as:

$$ r_{P_2} = \frac{x_2}{\cos \phi_c \cos \theta_c - \cos \phi_c} \frac{x_3}{\sin \theta_c \cos \phi_c - \sin \phi_c} \frac{x_4}{\sin \phi_c - \sin \theta_c} \frac{x_5}{\cos \phi_c} $$

After this factor is determined and also knowing $x_1$, we can determine the $Y_d$ coordinate of $P_2$:

$$ y_{d_2} = -r_{P_2} \sin \frac{x_2}{\cos \phi_c - \cos \theta_c} \frac{x_3}{\sin \theta_c \cos \phi_c - \sin \phi_c} \frac{x_4}{\sin \phi_c - \sin \theta_c} \frac{x_5}{\cos \phi_c} $$

To determine all the necessary coordinates of the four control points, there is necessary also to determine $x_1$. To determine $x_1$, there is used the following reasoning.
Figure 4. The proper Bézier polygon: 1- the driving side involute curve, 2- the coast side involute curve, 3- the coast side involute curve tangent, 4- the straight line parallel with Xd axis, 5- the proper Bézier polygon, 6- the resulting cubic Bézier curve

Let us consider the point of dedendum, whose coordinates are known \((x_f, y_f)\). The parametric equations of the cubic Bézier curve are presented above (see relation 10). All the terms in the second equation \(y\) equation) are known \((y_{d1}, y_{d2})\) and these are the \(Y_d\) coordinates of the points \(P_2\) and \(P_3\). The \(Y_d\) coordinates of \(P_0\) and \(P_1\) are 0. Using this equation, there will be determined the parameter \(t\) in this point.

The following equation is the equation of \(y_f\):

\[
y(t) = y_f = -3t^2(1-t)y_{d2} - t^3r_{bc}\sin\frac{2\pi}{r_f}
\]

which is a 3-degree equation.

Determining \(t\) in the dedendum point, we can substitute its numerical value in the first equation of \(x(t_k)\):

\[
x(t_k) = (1-t_k)^3x_{d1} + 3t_k(1-t_k)^2x_{d2} + 3t_k^2(1-t_k)x_1 + t_k^3r_{bc}\cos\frac{2\pi}{r_f}
\]

where \(t_k\) is the found numerical value from the equation (13) and substituted in eq. (14). In (14), there appears only \(x_1\) coordinate because we have mentioned above that \(X_d\)coordinates of \(P_1\) and \(P_2\) are equals (which means that \(x_1 = x_2\)). Having the coordinates of the four control points, the cubic Bézier curve can be defined properly. So:

\[
R_0(0,0), R_1(x_f,0), R_2(y_{d2}), R_3(r_{bc}\cos\frac{2\pi}{r_f}, -r_{bc}\sin\frac{2\pi}{r_f}).
\]

2.4 Particular case

Using the methodology described above, in the following will be presented a comparison between the use of circle and the use of a cubic Bézier curve as a gear fillet. There will be presented also a graphical proof that is better to use a Bézier curve instead a circle, because the base thickness of the tooth increases which means that the tooth strength is higher. The graphical proof is based on the computational generation of these 2 curves using Matlab programming language.

Input data: \(z_1 = 27, z_2 = 49, m = 3, \alpha_p = 32^\circ, \alpha_c = 18^\circ\).

Geometrical calculations:

- \(d_1 = mz_1 = 81\,\text{mm} \Rightarrow r_1 = 40.5\,\text{mm}\) – pitch diameter circle of the pinion.
- \(d_2 = mz_2 = 147\,\text{mm} \Rightarrow r_2 = 73.5\,\text{mm}\) – pitch diameter circle of the gear.
- \(d_{1bd} = d_1 \cos \alpha_d = 68.69\,\text{mm} \Rightarrow r_{bd} = 34.345\,\text{mm}\) – base diameter circle for driving side involute curve.
- \(d_{1bc} = d_1 \cos \alpha_c = 77.03\,\text{mm} \Rightarrow r_{bc} = 38.517\,\text{mm}\) – base diameter circle for coast side
involute curve.
- \( d_{a2} = 2(\pi a_2 + m(h_{pa} - x_1)) - d_1 = 153 \text{ mm} \) - addendum circle diameter of the gear.
- \( d_f = 2(a_{12} - r_{a2} - 0.25m) = 73.5 \text{ mm} \Rightarrow r_f = 36.75 \text{ mm} \) - addendum circle diameter of the pinion.

\[
x_{3c} = x_{3c} = \frac{2\pi}{z_1} = 37.48 \text{ mm}, x_f = x_{3c} - 0.25m = 37.48 - 0.75 = 36.73 \text{ mm}.
\]

\[
y_{f3} = -r_{bd} \sin \theta \cos \frac{x_f}{r_f} = -1.27 \text{ mm} \) - this is the \( Y_d \) coordinate of the lowest point for the first Bézier curve (see fig. 3).

If \( x_1 \) is determined, there can be determined also \( y_{d1} \), using (11) and (12).

\[
(11) \Rightarrow \quad y_{d1} = 31.65 \text{ mm} \quad \text{and} \quad (22) \Rightarrow \quad y_{d2} = -1.044 \text{ mm}.
\]

The coordinates of the four control points are:

\[
P_0(34.345, 0), \quad P_1(31.627, 0), \quad P_2(31.627, -1.044), \quad P_3(37.48, -8.88).
\]

The curve was generated in Matlab programming language and it looks like below (fig. 5).

\[\textbf{Figure 5.} \quad \text{The resulting cubic Bézier curve}\]

Let us analyse the situation when there is a circle arc instead of a Bézier curve. It is known that the radius of the circle must be maximum 1/3 \( h_f \), where \( h_f \) is the length of the pinion tooth. The length of the pinion tooth, in our case, is 13.5 mm which means that the radius is 3.375 mm. The parametric equation of the circle is:

\[
x = x_0 + r \cos \theta \quad \text{and} \quad y = y_0 + r \sin \theta \quad (16).
\]

Here, \((x_0, y_0)\) are the coordinates of the center of the circle and \( \theta \) is the angle between \( Y_d \) axis and the radius of circle in clockwise rotation.

\[
(16) \Rightarrow x = 34.345 + 3.375 \cos \theta \quad \text{and} \quad y = -3.375 - 3.375 \sin \theta \quad (28), \quad \theta \in [0, \frac{\pi}{2}].
\]

The generated circle looks like below:
There can be seen the difference between the use of circle and the use of a Bézier curve: using a Bézier curve involves the increase of the base tooth thickness. In fig.5, we can see that the $Y_d$ coordinate of the dedendum point is -1.044. In fig.6, we can see that the $Y_d$ coordinate of the dedendum point is -3.375, which is the numerical value of the circle radius. Calculating the tooth strength as a built-in beam, the bending moment at the base of the tooth will decrease because the base section (when there is used a Bézier curve) is bigger than the base section of the tooth when we use a circle. That’s why using a Bézier curve instead using a circle brings more benefits regarding tooth strength.

3. Further research
We are focused on the determination of the optimal Bézier curve to satisfy the required functional conditions. This paper presents only the possibility to improve the gear fillet geometry substituting the circle with a Bézier curve. There was also presented a variant of gear fillet using a cubic Bézier curve. Finding the optimal curve may be a difficult task. There is not only finding the number of control points that can define the curve but also (the most difficult) to determine the control points coordinates. This task will be approached in the next paper where there will be also presented a methodology of optimization of gear fillet geometry using these curves.

4. Conclusions
This paper illustrates a better way to increase the strength of the gear tooth improving the gear fillet geometry. Recent studies [2] have shown that using a circle instead a trochoid can reduce the bending stress level of the tooth. The improvements presented in this paper are brought by using a special curve that accomplishes all the manufacturing and functionality requirements. Determining a regular curve that can accomplish all the requirements was not a simple task. That is why there were presented Bézier curves, because these curves can be generated easier and their equations can be determined immediately if there are known the coordinates of control points. There was illustrated the case of using a cubic Bézier curve and the determination of its control points generally. Also, there is presented the fact that if it used a Bézier curve instead a circle, the base of the tooth gets bigger which means the bending stress moment gets lower. In further research, there will be presented the
possibility of finding the optimum Bézier curve and all the control points coordinates. The use of Bézier curves as asymmetric gear fillet may represent the best option for any asymmetric gear fillet geometry.

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