The Influence of Dusty Size Distribution Model on Propagation of Electromagnetic Waves in Inhomogeneous Plasma Sheath

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Abstract. In this paper, a linear distribution model, a double exponential distribution model and a hyperbolic number distribution model are developed, and the effect of dust particle size distribution on its attenuation coefficient is considered. On this basis, the transmission characteristics of electromagnetic waves in dusty plasma under different electron density distribution models are investigated. The attenuation coefficients of electromagnetic waves are derived in detail using analytical methods, and the effects of electromagnetic wave frequency, particle collision frequency and dust particle density on the transmission characteristics of electromagnetic waves are discussed. The results show that the choice of terahertz wave as the carrier wave and the appropriate change of particle collision frequency can improve the attenuation of electromagnetic waves. In summary, the relevant research can provide some theoretical references for propagation science.

Keywords: Dusty plasma; Collision frequency; Electron density; Attenuation coefficient.

1. Introduction

Dust particles are commonly found in the universe and are a form of existence of the solid matter of the universe[1-2]. When any uncharged ordinary gas is subjected to high external energy, some of the electrons become free from the nucleus, while the atoms become positively charged ions due to the loss of electrons, so that the matter consisting of a large number of free electrons, ions and some neutral particles is called plasma. The plasma and the dust particles floating in it form a dust plasma, which is widely found in nature[3-5]. Therefore, the study of plasma is necessary. For example, the effect of plasma on the absorption rate of electromagnetic waves was analysed by Chen Cong et al. using the WKB method[6-7], and Yuan Cheng-xun et al. studied the effect of collision frequency on electromagnetic wave propagation[8-9]. Since most studies on dusty plasma do not consider the dust particle size distribution, which makes the theoretical analysis results have a large error with the actual experimental results. In recent years, the influence of plasma electron density distribution models on electromagnetic wave transmission has received more and more attention. Zhao Han-zhang et al. proposed an inhomogeneous plasma electron density distribution model and established the Bassel function to analyse the inhomogeneous plasma model[10], many scholars have studied the propagation characteristics of terahertz waves in plasma[11-12], and Chen Yun-yun et al. proposed a hyperbolic number distribution model[13-17]. In order to understand more clearly the effect of dust plasma on the attenuation coefficient of electromagnetic wave transmission, a linear distribution model, a double
exponential distribution model and a hyperbolic number distribution model are developed in this paper, and the effect of dust particle size distribution on its attenuation coefficient is considered. Based on this, computational simulations are carried out and the attenuation coefficients of gigahertz and terahertz waves are compared.

2. Derivation of the Analytic Method

2.1. Attenuation Coefficient of Dusty Plasma

The potential influence factors have a significant effect on the attenuation coefficient of the electromagnetic wave, which is expressed as \( e^2 \frac{2}{2 e m} \)

\[ \eta_{\text{el}} = \frac{N_e e^2 N_d \gamma_s^2}{m_e}, \]  
(1)

The influence factor of the potential of the dusty plasma can be expressed as:

\[ \eta_{\text{pl}} = \frac{Z N_e e^4 N_d \gamma_s^3}{2 \varepsilon_0 m_e K_B T_e}, \]  
(2)

Electrical conductivity can be expressed as:

\[ \sigma_{\text{pl}} = \frac{(\eta_{\text{el}} + \eta_{\text{pl}})(\omega^2 - v_{\text{ch}} v_m)}{\omega^2 (\omega^2 + v_{\text{ch}}^2)(\omega^2 + v_m^2)} \]  
(3)

The expression for the relative permittivity is

\[ \varepsilon_{\text{pl}} = \frac{\varepsilon_0}{\varepsilon_0 (\omega^2 + v_{\text{ch}}^2)(\omega^2 + v_m^2)} \]  
(4)

The Boltzmann equation and the dust particle charging equation show that the attenuation constant of a weakly ionised dusty plasma for microwaves can be expressed as:

\[ \vartheta = \frac{\omega}{\sqrt{2 \varepsilon_c e^2}} \left( -\left( 1 - \frac{\omega_m^2}{\omega^2 + v_m^2} + \varepsilon_{\text{pl}} \right) + \sqrt{\left( 1 - \frac{\omega_m^2}{\omega^2 + v_m^2} + \varepsilon_{\text{pl}} \right)^2 + \left( \frac{\omega_m^2}{\omega^2 + v_m^2} \varepsilon_{\text{el}} + \frac{\varepsilon_{\text{pl}}}{\varepsilon_0 \omega} \right)^2} \right)^{1/2} \]  
(5)

Where \( \omega_{\text{pe}} = \sqrt{\frac{\varepsilon_c e^2}{\varepsilon_0}} \) is the angular frequency of the plasma electron oscillation, \( \varepsilon_0 \) is the vacuum dielectric constant and \( \varepsilon_{\text{pl}} \) is the relative permittivity of the dusty plasma.

Assume that \( \frac{\omega}{k} = c \), \( v_{\text{ch}} \) is the plasma charging frequency, \( v_m \) is the collision frequency, \( r \) is the dust particle radius, \( e \) is the electron charge. \( N_e \) is the electron density, \( m_e \) is the electron mass, \( T_e \) is the electron temperature, \( N_p \) is the dust particle density and \( k_B = 1.38 \times 10^{-23} \text{J/K} \) is the Boltzmann constant.

2.2. Dust Particle Size Distribution Model

Since the dust particle size distribution affects the charge response factor as well as the potential influence factor of the dusty plasma, here we consider dust particles satisfying a polynomial distribution, assuming that the dust particle radius size distribution satisfies the following conditions\(^{[16]}\):

\[ n(r) = a_0 + a_1 r + a_2 r^2 + a_3 r^3 + \ldots + a_n r^n \]  
(6)

(1) If \( n(r) = a_0, r = r + \delta, r = r - \delta, r = \frac{r + r - \delta}{2} \), \( n(r) \) is the dust particle size, then the charge response factor can be expressed as:
The expression for the relationship between $r_d$ and $\tilde{r}$ is $r_d = \sqrt{\frac{3}{2} r + \delta^2}$, The potential influence factor can be expressed as:

$$\eta_{ed} = \frac{N_e \pi \delta^2 N_{ed} (r + \delta^2)}{m_e},$$

(7)

The expression for the charging response factor is:

$$\eta_{ed} = \frac{N_e \pi \delta^2 N_{ed}}{m_e (1 + \lambda r r)}.$$

(8)

The expression for the relationship between $r_d$ and $\tilde{r}$ is:

$$r_d = \left( \frac{a_i (r + \frac{\delta^2}{3}) + a_i \tilde{r} (r + \frac{\delta^2}{3})}{1 + \lambda \tilde{r}} \right)^{1/2}.$$

(9)

Then the potential influence factor can be expressed as:

$$\eta_{ed} = \frac{Z N_e \pi \delta^2 N_{ed}}{2 e_0 m_e K u r e} \sqrt{\frac{a_i (r + \frac{\delta^2}{3}) + a_i \tilde{r} (r + \frac{\delta^2}{3})}{1 + \lambda \tilde{r}}}.$$

(10)

2.3. Electron Density Distribution Model

2.3.1. Linear distribution model. In this study, the expression for the linear distribution model is:\[14]\:

$$N_e(z) = n_{e1} + (n_{e2} - n_{e1}) \frac{z}{z_2 - z_1}.$$

(12)

where $z_1$ is the left boundary of the dust plasma and $z_2$ is its right boundary, $n_{e1}$ is the linear electron distribution model left boundary electron density and $n_{e2}$ is the right boundary electron density.

2.3.2. Double exponential distribution model. As shown in Figure. 1, the expression for the double exponential distribution model is\[10]\:

$$N_e(z) = \begin{cases} n_e \exp\left(\frac{z}{z_{10}}\right) & z_1 \leq z \leq 0 \\ n_e \exp\left(-\frac{z}{z_{20}}\right) & 0 \leq z \leq z_2 \end{cases}.$$

(13)

2.3.3. Hyperbolic number distribution model. the expression for the Hyperbolic number distribution model is\[17]\:
Where $Ne(z)$ is the electron density, $z_{10}$ and $z_{20}$ represent steepness, $z_2 - z_1$ is the thickness of the plasma. Suppose that the dusty plasma is stratified. Then each layer of plasma can be determined by the coordinates of $z_1$ and $z_2$, assuming that each layer has the same degree of steepness.

$$Ne(z) = \begin{cases} 
    n_e \left( \frac{z_2^2}{(z-z_1)^2} \right) & z_1 < z \leq 0 \\
    n_e \left( \frac{z_2^2}{(z-z_1^2)} \right) & 0 \leq z < z_2
\end{cases}$$

(14)

Figure 1. Electron density distribution model

3. Analysis and Discussion

Figure 2. Effect of electromagnetic wave frequency on attenuation coefficient

In order to compare the trend of the attenuation coefficient with electromagnetic wave frequency in different electron density distribution models, Figure. 2 is supplied, which shows that the attenuation coefficient increases and then decreases as the electromagnetic wave increases, with a clear inflection point when $f$ is equal to $1.45 \times 10^6 \text{ Hz}$. The inflection point indicates that there are factors other than the frequency of the electromagnetic wave that affect the attenuation coefficient.

Figure 3. Effect of particle collision frequency on attenuation coefficient

In order to compare the trend of the attenuation coefficient with particle collision frequency in different electron density distribution models, Figure. 3 is supplied. As shown in the figure, the
attenuation coefficient of electromagnetic waves decreases with increasing collision frequency, with more pronounced changes at low frequencies. In addition, the attenuation coefficient increases with increasing dust particle radius.

![Figure 4. Effect of dust particle density on attenuation coefficient](image)

Figure 4. Effect of dust particle density on attenuation coefficient

Figure 4 shows the trend of the attenuation coefficient with dust particle density. As shown, the attenuation coefficient increases with increasing dust particle density and, in addition, the effect of dust particle radius on the attenuation coefficient is more pronounced.

![Figure 5. Effect of peak electron density on attenuation coefficient](image)

Figure 5. Effect of peak electron density on attenuation coefficient

To investigate the effect of electron density on the attenuation coefficient, Figure. 5 is supplied. As shown in the figure, the attenuation coefficient increases with increasing electron density. In addition, it can be found that the attenuation coefficient of terahertz waves is larger than that of gigahertz waves, which verifies that dusty plasma has a stronger suppression effect on electromagnetic waves with smaller incident frequencies\[^{17}\].

4. Conclusion

In this paper, a linear distribution model, a double exponential distribution model and a hyperbolic number distribution model are developed, and the effect of dust particle size distribution on its attenuation coefficient is considered. On this basis, the transmission characteristics of electromagnetic waves in dusty plasma under different electron density distribution models are investigated. The analytical method is used to derive the attenuation coefficients of electromagnetic waves in detail, and the effects of electromagnetic wave frequency, particle collision frequency and dust particle density on the transmission characteristics of electromagnetic waves are discussed. The results show that the attenuation coefficient increases with increasing dust particle density and decreases with increasing particle collision frequency, and that changing the incident frequency of electromagnetic waves can effectively improve the attenuation of electromagnetic waves. Therefore, choosing terahertz waves as the carrier wave and appropriately increasing the particle collision frequency can improve the attenuation of electromagnetic waves in dusty plasma. In summary, the relevant research can provide some theoretical references for propagation science.

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