Cosmic ray driven outflows in global galaxy disc models

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ABSTRACT

Galactic-scale winds are a generic feature of massive galaxies with high star formation rates across a broad range of redshifts. Despite their importance, a detailed physical understanding of what drives these mass loaded global flows has remained elusive. In this paper, we explore the dynamical impact of cosmic rays (CRs) by performing the first three-dimensional, adaptive mesh refinement simulations of an isolated starbursting galaxy that includes a basic model for the production, dynamics and diffusion of galactic CRs. We find that including CRs naturally leads to robust, massive, bipolar outflows from our 10^{12} M_⊙ halo, with a mass loading factor \( M / SFR = 0.3 \) for our fiducial run. Other reasonable parameter choices led to mass loading factors above unity. The wind is multiphase and is accelerated to velocities well in excess of the escape velocity. We employ a two-fluid model for the thermal gas and relativistic CR plasma and model a range of physics relevant to galaxy formation, including radiative cooling, shocks, self-gravity, star formation, supernovae feedback into both the thermal and CR gas and isotropic CR diffusion. Injecting CRs into star-forming regions can provide significant pressure support for the interstellar medium (ISM), suppressing star formation and thickening the disc. We find that CR diffusion plays a central role in driving superwinds, rapidly transferring long-lived CRs from the highest density regions of the disc to the ISM at large, where their pressure gradient can smoothly accelerate the gas out of the disc.

Key words: methods: numerical – cosmic rays – galaxies: formation.

1 INTRODUCTION

Galaxy formation, in the broadest strokes, is a story of cooling, collapse and infall, as baryons settle into dark matter haloes and form stars (e.g. Rees & Ostriker 1977; White & Rees 1978). But a more precise account of this process reveals an important role for heating, expansion and outflow, as stars and black holes release energy, momentum and material back into the interstellar medium (ISM) and beyond, strongly impacting the resulting structure. Galactic-scale winds are among the most important of these latter processes, as they can remove appreciable mass from dense, star-forming regions. This feedback can substantially alter the distribution of luminous matter: studies that match the observed bright galaxies with their required dark matter haloes find that typically only 20 per cent of the baryons are accounted for in \( L_\star \) galaxies, with the fraction decreasing for both larger and smaller systems (Vale & Ostriker 2004; Conroy, Wechsler & Kravtsov 2006; Behroozi, Conroy & Wechsler 2010; Guo et al. 2010). These estimates also agree reasonably well with the dark matter content of individual galaxies estimated either with rotation curves (e.g. McGaugh et al. 2000, 2010; Stark, McGaugh & Swaters 2009) or weak lensing (e.g. Mandelbaum et al. 2006). Generally, simulations have predicted much higher baryon fractions, producing an offset in the Tully–Fisher relation (e.g. Steinmetz & Navarro 2002), although very strong feedback appears to be capable of solving this issue (e.g. Brook et al. 2012). Beyond aiding in this mass displacement, galactic winds also carry energy and metals beyond their host halo, polluting the intergalactic medium (IGM; e.g. Cowie et al. 1995; Porciani & Madau 2005).

Most actively star-forming galaxies with high specific star formation rates (specific SFRs) host galactic-scale outflows (see Veilleux, Cecil & Bland-Hawthorn 2005, for a recent review of galactic winds and an account of these observations). Both locally and at high redshift, these highly productive systems can often direct more mass into the outflowing winds than into newly formed stars, i.e. their mass loading factor (the ratio of mass loss from the system to the SFR) is above unity (Martin 1999; Steidel et al. 2010). Multiple gas phases comprise these flows, with pockets of neutral, warm-ionized and soft X-ray gas observed travelling at hundreds of km s^{-1} relative to their host galaxies (Heckman, Armus & Miley 1990; Pettini et al. 2001; Chen et al. 2010; Rubin et al. 2010).

Despite the importance and ubiquity of galactic winds, the driving mechanisms are not well understood. Many models have assumed hot evacuated gas from repeated supernovae (SNe) drives the wind (Larson 1974; Chevalier & Clegg 1985; Dekel & Silk 1986), though more detailed simulations able to resolve interacting SNe have failed to produce large mass loading, particularly for gas-rich discs.
propagating shock fronts, with implications for diffuse shock acceleration of CRs. These 1D models make a persuasive case for the dynamical importance of CRs but can only treat aspects of the flow inherent to a uniform, coherent wind along ordered field lines. In reality, the dynamics are likely to involve multiphase, turbulent gas flows and field lines with far richer topologies. In addition, all these models treat the flow of gas and rays beyond the disc as an inner boundary condition, and do not explore how gas and rays are produced within and rise out of the patchwork star-forming regions of a real disc.

CRs have only recently been incorporated into three-dimensional (3D), global hydrodynamic simulations for preliminary explorations into their dynamical role in galaxy evolution. Enßlin et al. (2007) and Jubelgas et al. (2008) modified the smoothed particle hydrodynamics (SPH) code GADGET to include CRs and used it to examine both idealized and cosmological simulations of galaxies. They found CRs can significantly suppress the SFRs and other properties of galaxies with circular velocities less than 80 km s$^{-1}$. Wadepuhl & Springel (2011) applied this model to the formation of a MW-sized halo, studying how CR feedback can suppress star formation in luminous satellites, perhaps patching the discrepancy between the observed distribution of MW satellites and the predictions of many cosmological simulations. These pioneering studies demonstrated the importance of CRs, but had a number of shortcomings: most production runs did not include CR diffusion or streaming; most cosmological runs were at high redshift, and the simulations included a ‘stiff’ thermal equation of state from the Springel & Hernquist (2003) subgrid model, which already builds in feedback to suppress disc fragmentation. CRs have also been added to cluster scale simulations (e.g. Miniati et al. 2000; Pfrommer et al. 2007; Vazza et al. 2012).

Very recently, Ublig et al. (2012) built on this earlier work by including CR streaming and found the production of significant outflows, although again the simulations did not include CR diffusion, used the Springel & Hernquist (2003) subgrid model, and were not cosmological.

In this paper, we adopt a simple two-fluid model for the CRs and thermal plasma to explore the dynamical impact of the CR pressure on high resolution, global simulations of an idealized $10^2$ M$_\odot$ disc galaxy. We assume the rays can be treated as a relativistic plasma of negligible inertia that is tied to the thermal plasma except for an isotropic diffusion term. We also include source terms for the CRs under the assumption that they are mostly produced in strong shock waves generated by SNe. As we will show, this model – although simple – can drive significant outflows. In Section 2 we describe the CR model and initial conditions; in Section 3, we describe the results of our numerical experiments, and finally, in Section 4, we describe a simple picture to understand our results, and discuss both implications and shortcomings of this work.

2 METHODOLOGY

2.1 The two-fluid model for gas and CRs

We begin with a two-fluid approach to modelling CRs (Drury 1985; Drury & Falle 1986; Jun, Clarke & Norman 1994). The model assumes an ultrarelativistic gas of protons which we treat as an ideal gas with $y = 4/3$ that is tied to the thermal plasma except for a diffusion term. While a detailed treatment of the high-energy particles involving both a distribution in momentum space and a treatment of magnetic fields could lead to an anisotropic CR pressure on the gas,
the two-fluid approach assumes a scalar pressure to make the model tractable. Observations are consistent with an isotropic distribution of particles, particularly in the GeV range which contributes primarily to the pressure. We neglect diffusion of the CRs in energy, as well as energy loss terms due to direct collisions or to interactions with the magnetic field. We further assume the large-scale magnetic field to be dynamically subdominant. These assumptions lead to the following set of equations (Drury 1985):

\[
\begin{align*}
\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) &= 0, \\
\rho (\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) &= -\nabla (P_{TH} + P_{CR}), \\
\partial_t \epsilon_{TH} + \nabla \cdot (\epsilon_{TH} \mathbf{u}) &= -P_{TH} (\nabla \cdot \mathbf{u}) + \Gamma + \Lambda, \\
\partial_t \epsilon_{CR} + \nabla \cdot (\epsilon_{CR} \mathbf{u}) &= -P_{CR} (\nabla \cdot \mathbf{u}) + \nabla \cdot (\kappa_{CR} \nabla \epsilon_{CR}) + \Gamma_{CR},
\end{align*}
\]

along with the state equations:

\[
P_{TH} = (\gamma_{TH} - 1)\epsilon_{TH},
\]

\[
P_{CR} = (\gamma_{TH} - 1)\epsilon_{CR},
\]

where \(\rho\) is the gas density, \(\mathbf{u}\) is the gas velocity, \(P\) is the pressure, \(\epsilon\) is the energy density and \(\gamma\) is the adiabatic index. The constant \(\kappa_{CR}\) is the CR diffusion coefficient, which we treat as isotropic and independent of any of our state quantities. \(\Gamma\) and \(\Lambda\) represent source and loss terms for the fluid. In our galaxy simulations both the CR and thermal fluids receive energy injections within star-forming regions. In these runs the thermal gas is also subject to radiative cooling. We ignore CR loss terms in our present work.

These equations represent the standard Euler equations of an ideal fluid, with a second (diffusive) CR fluid that interacts with the gas only via the momentum equation. Note that the CR mass density is negligible, allowing us to ignore it in equation (1). Here we implicitly ignore the motion of scatterers relative to the fluid, while still accounting for this process as diffusion of the CR energy density, \(\epsilon_{CR}\). We will discuss some possible ramifications of these assumptions later in this paper; however, we are interested in first exploring a simple model that both captures the key effect and allows us to carry out a relatively large number of simulations.

### 2.2 Implementation

Our CR model was integrated into the well-tested Eulerian hydrodynamics code ENZO, described in Bryan et al. (2013), Bryan & Norman (1997), Bryan (1999), Norman & Bryan (1999) and O’Shea et al. (2004). One of ENZO’s main strengths is adaptive mesh refinement (AMR), which uniformly resolves the entire simulation region on a coarse grid but provides higher resolution, ‘refined’ subgrids as needed in regions where the dynamics grow complex.

With our new two-fluid model, the list of conserved quantities in the code grows to include the CR energy density, \(\epsilon_{CR}\). For our preliminary investigations, we opted to work with the simple and robust ZEUS hydro method (Stone & Norman 1992), where the equations are broken into source and transport steps. The CR modifications leave the transport step unaltered. Passing this new quantity to the transport solver automatically implements the right-hand side of equation (4), where the new CR energy density is advected with the thermal fluid.

Next, within the source step, we have implemented the pressure gradient term in equation (2) and the first left-hand side term in equation (4). These terms represent work done by the rays on the thermal gas, and the loss in CR energy density due to that work, respectively. As in the case of the hydro quantities, simple, explicit, centred-difference derivatives were used.\(^1\)

The fastest information can propagate across a fluid is the sound speed (important for subsonic flows) compounded with the bulk speed (important for supersonic flows). For accuracy and stability, our time step must remain smaller than the time it takes information to cross an entire grid cell. In a standard fluid, the sound speed can be derived from the thermal pressure. For our new two-fluid code, we may define an effective sound speed:

\[
c_{\text{eff}} = \sqrt{\frac{\gamma_{TH} P_{TH}}{\rho}},
\]

where the total pressure \(P_T = P_{TH} + P_{CR}\), and \(\gamma_{eff} = (\gamma_{TH} P_{TH} + \gamma_{CR} P_{CR})/P_T\). In practice, we found small, low-density, CR-dominated cavities can develop above our disc during bursts of star formation. Within these pockets, the effective sound speed becomes enormous since

\[
c_{s,\text{eff}} = \sqrt{\frac{\gamma_{CR} P_{CR}}{\rho}} \\approx \frac{\gamma_{CR} P_{CR}}{\rho}.
\]

where \(P_{CR}/P_{TH} \gg 1\). A high sound speed within a low-density pocket can cause our computations to grind to a halt, since the time step will be limited by the Courant condition:

\[
\Delta t \propto \frac{\Delta x}{c_{\text{eff}} + u} \approx \Delta x \frac{1}{c_{\text{eff}}} \propto \Delta x \left(\frac{\rho}{P_{CR}}\right)^{1/2},
\]

where \(u\) is the magnitude of the fluid speed, which is appreciably smaller than \(c_{\text{eff}}\) in the regions of interest. This relationship suggests that raising the density within these regions can speed the pace of our runs, and since these regions are very much CR dominated the artificially enhanced density should not substantially change the dynamics. We place an upper limit on the allowed effective sound speed by increasing the gas density in cells that exceed this limit so that \(c_{\text{eff}} < c_{s,\text{max}}\). We explore the implications of this artificial ceiling in our parameter study below, where we find the choice does not substantially affect our results.

The diffusive term in equation (4) is likewise implemented with an explicit finite-difference scheme. To ensure stability, the time step of our diffusion scheme should remain smaller than the time it takes information to propagate beyond our differencing scheme’s domain of dependence. To ensure this, multiple time steps may be taken within our CR-diffusion scheme for every source time step, no larger than

\[
\Delta t_{\text{CR diffusion}} \leq \frac{1}{2N} \frac{\Delta x^2}{\kappa_{CR}},
\]

where \(N\) is the dimensionality of our simulation. This subcycling is limited by the number of ghost cells buffering each subgrid, since otherwise the fluid would diffuse beyond a subgrid before its parent grid could be made aware.

\(^1\) Following the original ZEUS implementation, ENZO stores vector quantities on cell faces, and scalar quantities at the centre of cells. Thus the spatial components of material derivatives are automatically centred-difference (second order).
2.3 Two-fluid model tests

The two-fluid model admits an analytic solution to the Riemann problem for non-diffusive CRs ($\kappa_{\text{CR}} = 0.0$), an extension of the classic Sod shock-tube problem described in Sod (1978). The analytic solution for the two-fluid case is derived in Pfrommer et al. (2006, hereafter P06). The resulting evolution is qualitatively similar to the classic case: a shock front and contact discontinuity (CD) propagate forward and a rarefaction fan spreads back as characteristics send word of the initial disequilibrium through the domain. The pressure and density profiles are significantly modified, as the disparity in $\epsilon_{\text{CR}}$ causes a jump in thermal pressure at the CD (where the
totalpressure remains identical on either side for all time). This leads to a significant enhancement of the density between the shock front and CD.

We reproduce this result in a 1D ENZO simulation using our ZEUS hydro scheme. Our initial conditions, described in Table 1, were chosen to produce a shock with Mach Number, $M = 10.0$. We follow P06 and define an effective Mach number:

$$M \equiv \sqrt{\frac{(P_{t,1} - P_{t,5}) x_s}{\rho_s c_{d,5}^2 (x_s - 1)}},$$

where Region 5 is the rightmost low-density, pre-shock region (right) and Region 1 is the leftmost, high-density downstream region. Regions 2–4 appear as the system evolves, as described in P06. Here $x_s = \rho_1/\rho_s$ and $c_{d,5}$ and $\gamma_{d,5}$ are given by equation (7).

The top row of Fig. 1 shows the results for a modest resolution of $N = 80$. At higher resolution or with an adaptive mesh the solution converges nicely to the analytic case, largely devoid of any spurious oscillations or overshoot. As seen here, low-resolution runs can be quite diffusive, smearing out the result at non-smooth points in the solution, particularly at the CD. ENZO also agreed with this solution when we ran the problem in 3D with plane–parallel initial conditions.

Diffusion of the CR fluid will prove central to our investigation of CR-driven outflows. The bottom row of Fig. 1 shows a solution to the same Riemann problem, but now for $\kappa_{\text{CR}} = 0.1$. Convergence at high resolution and the absence of any spurious oscillations leads us to conclude the diffusion scheme is stable, even in the presence of shocks, CDs, sonic points and local extrema in both fluid quantities. Beyond the obvious effects of diffusion, a noticeable spike in gas density occurs behind the shock front. This density spike is a classic feature of diffusive shock acceleration (e.g. Jun et al. 1994; Jun & Jones 1997).

We can test the diffusion scheme by itself in a more quantitative manner for a simpler test problem: we fill a 1D domain with high-density gas ($\rho = 10000$ in code units) at rest in a region of uniform pressure. We then place a small amplitude, local enhancement of CRs at the centre of this domain. The thermal gas here has too much inertia to be altered by the CR pressure over the relevant diffusion time-scale. Thus the two-fluid model reduces to a simple, linear diffusion equation for constant $\kappa_{\text{CR}}$: $\partial_t \epsilon_{\text{CR}} = \rho_{\text{CR}} \delta^2 \epsilon_{\text{CR}}$. This equation admits a classic analytic solution for $\epsilon_{\text{CR}}(t = 0) = \delta(x - x_0)$, given by

$$\epsilon_{\text{CR}}(x, t) = \frac{1}{\sqrt{4\pi \kappa_{\text{CR}} t}} \exp\left(-\frac{x^2}{4\kappa_{\text{CR}} t}\right).$$

We evolve this solution, beginning at $t = 1.0$ for the case where $\kappa_{\text{CR}} = 0.1$, as shown in Fig. 2. The simple finite difference scheme does a great job matching the solution, here for $N = 50$ grid cells.

| Downstream (left) | Pre-shock (right) |
|-------------------|-------------------|
| $\rho$            | 1.0               | 0.2               |
| $P_{\text{TH}}$   | $2/3 \times 10^5$ | 267.2             |
| $\epsilon_{\text{CR}}$ | $4.0 \times 10^5$ | 801.6             |
| $u$               | 0.0               | 0.0               |

Table 1. Riemann problem initial conditions.

Figure 1. The top row shows the CR-modified shock tube problem of P06, plotting (from left to right) the density, velocity and pressure (both CR and thermal). Dotted lines show the $t = 0$ initial conditions, while solid lines indicate the analytic result. No CR diffusion is used. The bottom row shows the same quantities for a run with diffusion, where $\kappa_{\text{CR}} = 0.1$. The line shade indicates the resolution used, from 50 cells (light blue) to 800 cells (dark blue).

2.4 Initial conditions

Our work follows that of Tasker & Bryan (2006), whose Initial Conditions (ICs), summarized in Table 2, provided the point of
departure for our simulations. These runs include radiative cooling of the thermal gas but no cooling of the CR component (see Section 4.3).

Our runs begin with an isothermal gas disc at $10^5$ K whose density follows:

$$\rho_D(r, z) = \rho_0 e^{-\frac{r^2}{2 z_0} - \frac{x}{1+z_0}},$$  \hspace{1cm} (13)

where we set the vertical scale height to $z_0 = 325$ pc and the radial scale height to $r_t = 3.5$ kpc. We set the total disc gas mass to $6 \times 10^{10} M_\odot$ which gives us a $\rho_0 \sim 10^{-20}$ kg m$^{-3}$. This total mass is roughly that of the MW total disc components (stars and gas – e.g. Klypin, Zhao & Somerville 2002).

In addition to the disc’s self gravity, it sits in a static dark matter potential with the standard form (Navarro, Frenk & White 1997), given (in spherical coordinates) by

$$M_{DM}(r) = \frac{M_{200}}{f(c)} \left[ \ln (1 + x) - \frac{x}{1 + x} \right].$$  \hspace{1cm} (14)

We set the virial mass, $M_{200}$, to $10^{12} M_\odot$. The dimensionless radius $x = R_c/R_{200}$, where $c$ is the concentration parameter, which we set to $c = 12$. $f(c)$ is given by

$$f(c) = \ln (1 + c) - \frac{c}{1 + c}. \hspace{1cm} (15)$$

To begin our runs in mechanical equilibrium given our gaseous and dark matter mass, $M_{tot}$, we set the orbital speed within the disc to

$$V_{circ}(R) = \left( GM_{tot}/R \right)^{1/2}.$$  \hspace{1cm}

For runs including CRs, we begin by adopting a simple prescription that maps CR energy density, $\epsilon_{CR}$, to gas density, $\rho$, by

$$\epsilon_{CR} (r, z) = \alpha_{CR} \rho (r, z).$$ \hspace{1cm} (16)

in dimensionless code units. For standard runs, we set $\alpha_{CR} = 0.1$, which corresponds to $\epsilon_{CR} \approx 3 \times 10^{-12}$ erg cm$^{-3}$ in the solar neighbourhood, in line with laboratory results. This corresponds to a constant $F_{TH}/P_{CR} = 3$ throughout our domain. Although this set-up is not realistic, the generation and diffusion of CR rays quickly dominates the CR distribution and the choice of CR initial conditions has only a tiny effect on our results.

Our galaxy sits at the centre of a (500 kpc)$^3$ box, partitioned into 128$^3$ cells. Within regions where density exceeds the background density by a factor of 4, ENZO instantiates a higher resolution ‘sub-grid’, rebuilt at each time step, that increases resolution by a factor of 2. This refinement occurs recursively, and we allow up to six levels of refinement in our fiducial run, for an effective spatial resolution of 61 pc in the highest density regions (the majority of the galactic disc).

### 2.5 Star formation and feedback

For star formation, we follow the prescription of Cen & Ostriker (1992), with updates first described in O’Shea et al. (2004). A cell in ENZO will produce a ‘star particle’ if (1) the gas density exceeds a threshold density; (2) the gas mass of the cell exceeds the local Jeans mass; (3) the flow converges, i.e. $\nabla \cdot \mathbf{v} < 0$ and (4) the dynamical time exceeds the gas cooling time, $t_{cool} < t_{dyn}$, or the temperature is at the minimum allowed value. Pursuant to these conditions, ENZO siphons gas from the grid cell into a star particle of mass

$$m_* = \epsilon_{SF} \frac{\Delta t}{t_{dyn}} \rho_{gas} \Delta x^3,$$  \hspace{1cm} (17)

where $\epsilon_{SF}$ is the star formation efficiency. Tasker & Bryan (2006) found that $\epsilon_{SF} = 0.05$ does a good job reproducing the global Schmidt–Kennicutt law, and we adopt this value for our fiducial run. To prevent an excess of small star particles bogging down our computation, we set a minimum $m_{* min} = 10^3 M_\odot$. For cells where $m_0 < m_{* min}$ is the only obstacle to forming a star particle, a particle may still be created with a probability $m_0/m_{* min}$ whose mass is either the minimum mass or 80 per cent of the cell mass, whichever is smaller. The particle’s creation occurs over a dynamical time, its mass grows following:

$$m_*(t) = m_* \int_{t_0}^t \frac{t - t_0}{\tau^2} \exp \left( -\frac{t - t_0}{\tau} \right) \, dt,$$ \hspace{1cm} (18)

where $m_0$ on the right-hand side is the final mass of the particle from equation (17), $t_0$ is when the particle’s formation began and $\tau = \max(t_{dyn}, 10$ Myr). This equation provides a simple yet smooth model for the conversion of gas into stars over a few dynamical times and is taken from Cen & Ostriker (1992).

We also include stellar feedback from Type II SNe, which deposits a fraction of the star’s mass and energy back into the fluid quantities of the occupied cell over a dynamical time. The prescription is given by

$$\Delta M_{gas} = f_c m_*, \hspace{1cm} (19)$$

$$\Delta E_{gas} = (1 - f_{CR}) \epsilon_{SNe} m_* c^2, \hspace{1cm} (20)$$

$$\Delta E_{CR} = f_{CR} \epsilon_{SNe} m_* c^2, \hspace{1cm} (21)$$

where $f_c = 0.25$ is the mass fraction of the star ejected as winds and SNe ejecta, $\epsilon_{SNe}$ is the Type II SNe efficiency and $f_{CR}$ is the fraction of this energy feedback donated to the relativistic CR fluid. For our fiducial run we set $\epsilon_{SNe} = 3 \times 10^{-6}$, corresponding to $10^{51}$ erg for every 185 $M_\odot$ of stars formed. We also typically set $f_{CR} = 0.3$.
Table 3 summarizes the various uncertain parameters in our model. The central row describes the fiducial values used in our standard run. Columns denote the range of values investigated in 20 additional simulations. Entries in resolution columns denoted with * were run both with and without the CR fluid. The parameter values in Table 3 are not known with precision—in fact, we deliberately chose a wide range of values because they are uncertain. This not only helps elucidate under what conditions we can launch a mass loaded wind, but also determines how the wind scales with each parameter. We now briefly discuss in more detail four of these parameters explicitly related to the CR physics.

We begin with $\epsilon_{\text{SF}}$, the amount of energy generated by SNe in terms of the SFR. This parameter depends on the SNe mass cut-off, the energy produced as a function of progenitor mass and the initial mass function (IMF). The IMF is probably the most uncertain (e.g. Cappellari et al. 2012), and our choice is relatively conservative (for instance Guedes et al. 2011, who adopt an effective $\epsilon_{\text{SN}}$ 1.8 times larger than this). A closely related parameter is $f_{\text{CR}}$, the fraction of SNe energy fed to the CRs. This parameter is also very uncertain and may depend on details of the environment in which the SNe explosion occurs. Our fiducial choice of $f_{\text{CR}} = 0.3$ is within the range often quoted (e.g. Wefel 1987; Kang & Jones 2006; Ellison et al. 2010); however, the precise value is not well known. For this reason, we explore a range of 0–1 in our tests. The CR loss time-scale for mildly relativistic rays is much shorter than that of ultrarelativistic CRs, thus the energy given to these lower momentum rays quickly winds up back in the thermal gas. Our present work ignores these loss processes, and thus $f_{\text{CR}}$ represents the fraction of energy donated to long lived, ultrarelativistic rays. Note also that for CR wind-driven feedback, the parameter combination of consequence is actually $\epsilon_{\text{SN}}/f_{\text{CR}}$, as this controls how much of the CR energy is produced per unit star formation.

Our fiducial choice of diffusion parameter $\kappa_{\text{CR}}$ is consistent both with recent GALPROP models (e.g. Pitskin et al. 2006; Ackermann et al. 2012) and with observational measurements (e.g. Strong & Moskalenko 1998; Tabataa et al. 2013). As discussed later, this choice also results in CR diffusion velocities consistent with the Alfvén velocity. Finally, we expect that $\gamma_{\text{CR}}$ is very close to 4/3, i.e. ultrarelativistic CRs dominate the ray fluid. Our goal in exploring different values for $\gamma_{\text{CR}}$ is thus largely pedagogical: we want to better understand the mechanism in play and whether or not the ‘stiffness’ of this second fluid is central to our model’s results.

## 3 RESULTS

We now present the results of our simulations, first describing the outcome of our run with all parameters set to their fiducial values, and then exploring how the results depend on the parameter values. This will allow us to gauge how robust our results are to small changes in the model, and will help us gain intuition into what role the various physical processes play. A majority of the analysis presented here was facilitated by the simulation data analysis and visualization tool VT described in Turk et al. (2011).

Table 3 provides a description of the parameters varied in this work. The central row describes all parameter choices for our fiducial run. The other entries of the table represent single-parameter deviations from the fiducial case in our 20 additional simulations. The varied parameters are (from left to right) spatial resolution, $\Delta x_{\text{min}}$, in parsecs; mass resolution, or resolution of the base grid ‘size’; maximum CR sound speed, $c_{\text{s,max}}$, in km s$^{-1}$ (see Section 2.2); star formation efficiency, $\epsilon_{\text{SF}}$; SN feedback efficiency, $\epsilon_{\text{SN}}$; the fraction of energy feedback diverted into the CR fluid, $f_{\text{CR}}$; the CR diffusion constant, $\kappa_{\text{CR}}$ in cm$^2$ s$^{-1}$ and the power index for the CR equation of state, $\gamma_{\text{CR}}$.

### 3.1 Fiducial run

We now describe the results of our fiducial run, which has CR diffusion and parameters set to observationally or physically motivated values. We will begin with a visual examination of the results, before moving on to 1D profiles and finally the evolution of global values.

#### 3.1.1 Visual evolution

The top half of Fig. 3 shows a ‘face-on’ view of the gas surface density as our run progresses. Though somewhat altered by the presence of CRs, the disc evolves in a manner quite similar to that described in Tasker & Bryan (2006): it cools down to 300 K, and the gas quickly slips to less than a kiloparsec in thickness, beginning in the galactic centre where the dynamical time is smallest. The collapse then ripples outward as spiral filaments funnel gas into knots. These knots exceed their surroundings in density by over two orders of magnitude, and act like softened point-sources of gravity, scattering off one another and making small excursions from the disc. At late times their number and size stabilize within the unstable portion of the disc.

The bottom half of Fig. 3 shows an ‘edge-on’ view of gas surface density, where immediately evident are robust, filamentary flows of gas out of the disc and into the galaxy’s halo, beginning about 50 Myr after the start of the simulation (coincident with the start of a strong starburst, which will be discussed in more detail below). In the innermost regions of the halo, the highest surface densities can be found just above where the collapse and fragmentation of the galactic disc proceeds radially outward. However, the projected density appears more homogenous far from the disc, especially at later times, where it fills spherical lobes above and below the plane of the disc with densities around $10^{-26}$ g cm$^{-3}$. These lobes grow continuously, meeting the boundary of our 500-kpc-cubed simulation box by roughly 500 Myr, implying an average speed for the shock front of $\approx$500 km s$^{-1}$.  

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Table 3. Varied parameters.

| CR physics | Star formation and feedback | Numerics |
|------------|----------------------------|----------|
| $\kappa_{\text{CR}}$ | $\gamma_{\text{CR}}$ | $\epsilon_{\text{SF}}$ | $\epsilon_{\text{SN}}$ | $f_{\text{CR}}$ | $c_{\text{s, max}}$ | $\Delta x_{\text{min}}$* | Size* |
| High | $3 \times 10^{28}$ | $3/2$ | 0.05 | $1 \times 10^{-6}$ | 1.0 | 3518 km s$^{-1}$ | 31.0 pc | 256$^1$ |
| Fiducial | $1 \times 10^{28}$ | $4/3$ | 0.01 | $3 \times 10^{-6}$ | 0.3 | 1550 km s$^{-1}$ | 61.0 pc | 128$^1$ |
| Low | $3 \times 10^{27}$ | – | $1 \times 10^{-5}$ | 0.0 | – | – | 122.0 pc | – |
| Very low | 0 | – | – | – | – | – | – | – |

---
Fig. 3 shows projections through the simulation box of CR energy density, which we will refer to as CR surface energy density, or simply CR surface density. Recall $\epsilon_{\text{CR}} = (1 - \gamma_{\text{CR}})p_{\text{CR}}$, and thus we can regard bright regions of these plots as areas of high CR pressure, where the rays may work to liberate gas from bound structures. The rays initially populate the simulation box in a profile identical to the thermal gas; however, the CR distribution is quickly dominated by the ongoing injection from star formation. As described earlier, we add CRs into cells where stars form, and thus the face-on projections show bright clumps that trace recent star formation. These clumps coincide with the dense knots in the thermal gas plots, home to the most vigorous star formation. The rays advect along with the thermal gas, but unlike the thermal gas the rays are also highly diffusive; over time these clumps dissolve to fill their surrounding
Figure 4. Face on (top) and edge on (bottom) projection of CR energy density for our fiducial run. In all but one panel, we show just one quadrant of the disc.

regions, both within and above the disc. From the edge-on view in Fig. 4, we see rays flow out into the halo in a manner similar to the thermal gas, but with a smoother distribution.

Fig. 5 shows a face-on projection of stars in the disc compared to both CRs and gas density. Bright clumps of stars and CRs show a one-to-one correspondence across the projections, although some of the largest central star clumps do not figure as prominently in the CR surface density because they are older and so generating few CRs. The CR energy density in a clump is set by a competition between injection from star formation and diffusion/advection. This results in the CR energy density having a lower contrast than the gas; however, a net pressure gradient in the CR component still persists, which – as we will show – can drive significant outflows. Many star clusters have interacted, producing tidal tails. These gravitational features are absent from the CR plots. These dense clumps also appear in the thermal gas plot (rightmost panel), although this fluid
Figure 5. The surface density of stars (left), CRs (centre) and gas (right) at $t = 302$ Myr. Although there exists a one-to-one correspondence between clumps in all three quantities, many of the brightest star clusters are much fainter in CR surface density, implying that these clumps are older, and producing fewer new stars (and thus fewer CRs). The projection of the diffusive CRs shows less structure than the gas plot or even the stellar plot. Bright patches highlight only the most recent star formation.

Figure 6. Slices of mass flux, thermal gas pressure, CR pressure and $\epsilon = P_{\text{CR}}/P_{\text{T}}$ at $t = 37.7$ Myr during our fiducial run. This snapshot displays the most violent burst of star formation in the fiducial run, and thus an ideal study of the anatomy of our winds. Shows far more filamentary/cavity structure than either the stellar or CR distributions. The CR fluid thus appears to be a good tracer of recent star formation.

We can better understand these flows by plotting mass flux and both relevant pressures (thermal and CR). Fig. 6 does so at $t = 37.7$ Myr, during an early burst of particularly intense star formation. Here we show an edge-on slice through the galaxy, in four different quantities. Since these flows exhibit noticeable asymmetries, Fig. 6 shows only the upper left-hand quadrant of the slice in each quantity, flipped horizontally and vertically to appear as a complete picture. An indicated in the figure caption, the quadrants represent (1) pressure of the thermal gas; (2) pressure of the CR fluid; (3) vertical mass flux and (4) a ratio of CR pressure to combined pressure, $\epsilon = P_{\text{CR}}/(P_{\text{TH}} + P_{\text{CR}})$. In this last quadrant, deep
red implies strongly CR-dominated dynamics while blue implies strongly gas dominated. From these plots we see the gas accelerates close to the disc itself, in pockets of strong CR pressure mostly devoid of gas pressure. Ahead of these fast, evacuated flows are a denser, slower component that carries more mass flux. Beyond the current reach of the diffusive rays the halo sits dormant, in hydrostatic equilibrium.

Fig. 6 can be regarded as a caricature of the run at large: at $t = 37.7$ Myr the initial conditions are collapsing into a cooler disc, and the SFR is $\sim 400 M_{\odot}$ yr$^{-1}$, a very large burst. Much later in the run, the SFR has fallen to $\sim 50 M_{\odot}$ yr$^{-1}$, and the acceleration of gas out of the plane has likewise fallen, but the qualitative features of this scene persist, and mass flux falls off less rapidly than the SFR. At later times, the acceleration region (where CRs dominate the pressure) has grown tremendously, providing a gentler acceleration that nevertheless persists to high altitude, which we describe in a more quantitative fashion next.

### 3.1.2 One-dimensional profiles

To better understand the dominant role CRs play in these flows, we turn to 1D profiles of key quantities, shown in Fig. 7. Here we plot the time evolution as a colour gradient, over 300 Myr in $\sim 40$ Myr intervals, with lighter colours representing earlier times. For these plots, we construct a cylinder of radius 50 kpc, centred on the galactic centre, aligned with the galaxy’s angular momentum vector. We then average the quantity of interest in a volume-weighted sense at a given height above the plane within this cylinder. Thus a data point on these plots represents an average of the quantity of interest within a wide disc, a distance $z$ from the galactic mid plane (both above and below).

The leftmost panel of Fig. 7 plots both CR and gas pressures over our 300 Myr period of interest. Although our initial conditions place CRs in a secondary role throughout the simulation domain, they rapidly assert themselves as the dominant pressure source beyond the disc. Similar mass-weighted profiles verify this dominance even in denser pockets of the multiphase wind. As the winds push outward, to hundreds of kpc in height above the disc, the rays continue to dominate the dynamics, except in a swept up shell of gas at the forefront of the flow, where thermal gas pressure spikes. The slope of this pressure profile beyond 20 kpc goes roughly as $z^{-4}$, consistent with adiabatic expansion of our $\gamma_{CR} = 4/3$ ultrarelativistic CR fluid for spherical outflows.

The central panel of Fig. 7 plots vertical mass flux away from the disc: $\rho v \cdot z(z/|z|)$. Close to the disc (within $\sim 50$ kpc) this quantity falls as $z^{-1/2}$, suggesting the flow is rather collimated and the majority of mass continues to rise once it has left the disc. Its normalization rises rapidly at early times (consistent with the peak in the SFR, described below), before falling at late times, as star formation declines. Far from the disc, the flux drops off as $z^{-3}$, consistent with a constant spherical outflow. These and similar mass-weighted profiles appear to rule out a primarily fountain-like flow of even our densest halo gas, at least in this extreme starburst setting.

In the rightmost panel of Fig. 7, we plot the mass-weighted average vertical velocity, $v = u_z$, of the gas, normalized by the escape velocity at that height. Gas above the dotted line, if free to follow a ballistic trajectory, would leave our $10^{12} M_{\odot}$ halo. In contrast to standard energy- and momentum-driven feedback models, our outflowing gas does not obtain its full velocity near the disc – instead the acceleration process is smoother, with CRs in the halo powering flows with increasing velocity tens of kpc above the disc. This gentle mechanism shows no sign of abatement at late times, even as the SFR has fallen far below the exotic ultraluminous infrared galaxies (ULIRG) values of our early evolution.

### 3.1.3 Global quantities

The outflows observed in these CR galaxy simulations should have meaningful implications for the global properties of our galaxy. The top left-hand panel of Fig. 8 shows the SFR for two runs as a function of time – one is our fiducial run (with CRs and diffusion), and the other a run without any CR component. Both simulations

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**Figure 7.** Profiles of fluid quantities as a function of height above the disc plane, $z$. Here we ‘bin’ the data, averaging over all cells inside a cylinder of radius 50 kpc at a given height. Lighter colours represent earlier times, plotted in $\sim 38$ Myr increments for roughly 300 Myr. The leftmost panel plots pressure of the thermal gas (blue) and CRs (orange), in volume-weighted profile. The central panel plots vertical mass flux away from the disc, $\rho v \cdot z(z/|z|)$, with the azimuthal and radial averaging again volume weighted. The final panel plots the ratio of gas velocity to the escape velocity at that height, now a mass-weighted average of all gas at a given height above the plane. In this last panel, a value of unity (indicated by a dashed line) implies this gas parcel would escape the galaxy’s halo, barring any subsequent hydrodynamic interactions.
Figure 8. Top row: SFR as our run progresses. Bottom row: total baryonic mass within the galactic disc over time, also broken down into stars and gas. The leftmost column compares our fiducial run, described in the central row of Table 3, against an identical run devoid of CRs. The central column compares runs where we have varied the CR diffusion coefficient, $\kappa_{\text{CR}}$. Finally, the rightmost column compares runs with various $\gamma_{\text{CR}}$.

show an immediate burst of star formation, but the run with CRs has a lower SFR at almost all times. Here we emphasize that both runs contain an equal amount of energetic feedback from SNe: in the CR run, 30 per cent is injected into the CR fluid, whereas in the non-CR run, this energy is injected into the thermal gas.

The key result of these simulations lies in the bottom left-hand panel of Fig. 8, where we plot the amount of gas and stars in the disc. Within 500 Myr, our CR-laced disc loses roughly 20 per cent of its baryonic mass while converting roughly two-thirds of its gas to stars – a mass loading factor of $\sim 0.3$. No equivalent mass loss occurs in the non-CR run despite its SNe feedback.

This result is important, as Tasker & Bryan (2006) demonstrated that, regardless of parameter choice, it is very difficult to generate significant outflows with purely thermal feedback. We will next explore how this result changes when we modify our numerical and physical parameters.

3.2 Impact of the CR diffusion coefficient

To better understand what role our choice of diffusion coefficient $\kappa_{\text{CR}}$ plays in these dynamics, we ran three additional runs with higher and lower diffusion coefficients, one below the fiducial value of $10^{28}$ cm$^2$ s$^{-1}$ and two above, in half-decade increments. We also performed a run with CRs but no diffusion. The central column of Fig. 8 shows the SFR and the baryonic mass in the disc over time for these runs.

We begin by looking at the case with no diffusion, described by the blue lines in both panels of Fig. 8, an obviously unrealistic scenario that nonetheless provides insight into the mechanism behind our outflows. In this case, the rays are completely tied to the thermal fluid, and the combined two-fluid acts almost like an adiabatic gas (since there is no cooling of the CRs). This strongly suppresses the star formation but does not lead to any significant gas outflows. From the top-central panel of the figure, we see star formation drops by roughly a factor of 4 compared to the non-CR run. Despite this effective feedback, the CRs cannot drive flows since SNe deposit them only into the densest regions of the disc where star formation occurs. Bound to this gas, they remain dynamically subdominant. This leads to a thickening of the disc in what is essentially a convective process. And since the rays cannot diffuse beyond these dense regions, they cannot assert their presence in the lower density regions of the disc where they effectively drive outflows in the runs with diffusion.

A remarkable thing happens when we turn on CR diffusion. For our lowest diffusion coefficient run ($\kappa_{\text{CR}} = 3 \times 10^{27}$ cm$^2$ s$^{-1}$), the SFR rises somewhat compared to the no-diffusion case, but the bottom-central panel of Fig. 8 reveals a qualitative shift in the dynamics: we immediately see strong outflows, leading to a mass loading factor of nearly unity. We therefore conclude that diffusion plays a crucial role in this process, moving CRs from pockets of very high density, where star formation occurs, to areas of the disc.

2 The mass within our cylinder increases slightly in the no-CR run since radiative cooling allows outlying disc gas to come within 5 kpc of the central plane.
with lower density. Once out in the diffuse ISM, the rays dominate the dynamics, and the gradient in the CR fluid pressure works to accelerate disc material away from dense clumps and ultimately beyond the disc. We will discuss a simple model which captures these dynamics below.

However, if this diffusion occurs too quickly, the rays do not linger long enough to accelerate much gas: from Fig. 8, we see the mass loading factor drops steadily as $\kappa_{CR}$ rises. Thus the shorter the mean free path for the rays, the more important this mechanism will prove in the disc’s evolution. As we increase the CR diffusion coefficient further, the SFR increases, approaching an evolution very similar to that seen in the case of no CRs. As $\kappa_{CR}$ rises beyond $10^{29}$ cm$^2$ s$^{-1}$, the SFR approaches the no-CR case and the mass loading drops towards zero. This picture is consistent with the naive expectation that for very high diffusion coefficients CR-enhanced regions rapidly wash out, eliminating any CR pressure gradients and rendering the rays dynamically irrelevant.

### 3.2.1 CR diffusion velocity

In this paper, we test a range of diffusion coefficients, as the actual value is uncertain and likely depends on details of the local ISM. One concern is that the diffusion velocity implied by our choice of diffusion coefficient should not exceed the Alfvén speed, otherwise CRs streaming along magnetic fields would excite Alfvén waves and non-adiabatically transfer energy from the CR fluid to the gas (e.g. Kulsrud 2005). For typical ISM conditions in the MW, the Alfvén speed ($v_A = B^2/4\pi \rho$) is thought to be of order $10$ km s$^{-1}$, although for strongly star-forming discs, the actual value may be considerably larger. In the top panel of Fig. 9, we show the distribution of diffusion velocities that we find in and near the disc, for runs with $\kappa_{CR} \in [3 \times 10^{27}, 10^{29}]$ cm$^2$ s$^{-1}$. We estimate the diffusion velocity with $v_{Diff} = \kappa_{CR} \nabla P_{CR}/P_{CR}$. As this plot shows, for each run, there is a range of diffusion velocities with a systematic trend towards larger diffusion velocities as a function of $\kappa_{CR}$. The bottom panel of Fig. 9 shows this more quantitatively, also clearly demonstrating that $v_{Diff}$ increases sublinearly with $\kappa_{CR}$, despite the fact that the definition is linearly dependent on $\kappa_{CR}$. This scaling occurs because increasing diffusion leads to smaller CR gradients, which decreases the implied CR diffusive velocities.

We also note that, for our fiducial choice of $\kappa_{CR} = 10^{28}$ cm$^2$ s$^{-1}$, the implied diffusion velocity is comparable to but perhaps slightly larger than the typical Alfvén velocities of the local ISM in the MW. This may imply that our standard $\kappa_{CR}$ is too large (as we show later in the paper, a smaller value would imply stronger winds); however, this is hard to say with certainty, both because the ISM is multiphase and diffusion may occur preferentially in the high-temperature, low-density phase which may have a larger Alfvén velocity, and also because the system we are simulating has a higher SFR than the present-day MW, and so the Alfvén velocity may well be larger.

### 3.3 Impact of $\gamma_{CR}$

Our CR model as implemented in this present work is too simplistic to capture many subtleties of a real population of galactic CRs. We assume an ultrarelativistic gas of CRs, and thus our second fluid’s equation of state has an index $\gamma_{CR} = 4/3$. In reality, this index depends on the distribution of CRs in momentum space. When lower momentum, moderately relativistic rays dominate the population, gamma rises towards that of a thermal gas (5/3) and the ray fluid exerts a stronger pressure response (see Section 4.3 for details).

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**Figure 9.** The top panel shows the volume-weighted (smoothed) histogram of the CR diffusion velocity magnitude, $v_{Diff} = \kappa_{CR} \nabla P_{CR}/P_{CR}$, for the four different simulations with differing $\kappa_{CR}$, as labelled. The bottom panel shows the peak of the distribution in each case, along with a linear relation for comparison.

The rightmost column of Fig. 8 explores how changes in this index affect our outflows. From the bottom right-hand panel, we find that as the CR fluid becomes less relativistic, the outflows strengthen, presumably since the CR fluid becomes ‘stiffer’, responding to compression with a more dramatic rise in pressure for the same energy injection. Thus our simplistic model’s choice of $\gamma_{CR}$ seems unlikely to exaggerate CR-driven winds (but see the discussion for other caveats to these results).

### 3.4 Impact of feedback prescription

We continue our parameter study by investigating the role of our star formation and feedback parameter choices. Fig. 10 again plots the SFR and disc mass for these runs. In the leftmost column, we have varied $\epsilon_{SN}$, the SN efficiency, above and below the fiducial case by a half decade; the other parameters are left unchanged. As demonstrated by the SFR plot, increasing the SN efficiency can suppress star formation by a noticeable fraction, though lowering the efficiency does not have as strong an effect. For all cases, the SFR tends towards a comparable, low, residual value at late times. The disc mass falls most dramatically for runs with high SN feedback efficiency: the traditional choice of $3 \times 10^{-6}$ manages to liberate roughly half the disc mass with otherwise fiducial parameter choices. The mass loading of our outflows seems to depend strongly on our choice of $\epsilon_{SN}$. Although this parameter strongly
affects how many stars we form and thus the gas fraction, as we saw earlier, it does not have a strong effect on the residual gas mass in our disc.

We next varied the star formation efficiency $\epsilon_{\text{SF}}$, related to how much mass in a thermally unstable gas parcel is converted into stars. The SFR plot in the central column of Fig. 10 demonstrates that lowering this efficiency can strongly suppress star formation at the beginning of our run, when the rapidly cooling disc of pure gas first collapses. However, the SFR outpaces the fiducial run roughly halfway through the simulation, presumably as feedback becomes more important in regulating the dynamics. From the mass plot, we find the total mass ejected from the disc is roughly independent of this efficiency; however, the total residual mass of gas in the disc is approximately a factor of 2 larger for the low-efficiency case. This occurs because a lower efficiency means that gas must collapse to higher densities to match the same SFR as in the fiducial run (since the SFR $\propto \rho^{3/2}$).

We also investigated the role that $f_{\text{CR}}$ – the fraction of SN feedback given to the relativistic CR fluid – plays in our model. The fiducial case sets $f_{\text{CR}} = 0.3$, but we also investigated no CR feedback, $f_{\text{CR}} = 0.0$, and complete CR feedback $f_{\text{CR}} = 1.0$. As seen in the rightmost column of Fig. 10, enhancing the fraction of feedback in the form of CRs allows them to more effectively suppress the SFR throughout our run. For the case where feedback is entirely thermal, the SFR is comparable to a run devoid of any CRs. The mass plot shows that the mass loading of CR-driven outflows strongly depends on $f_{\text{CR}}$, with higher CR feedback liberating more disc mass. A run with full CR feedback and otherwise fiducial parameters can remove roughly half the disc mass. On the other hand, when all of the energy is in thermal form ($f_{\text{CR}} = 0$), no outflows are generated. As in the case of $\epsilon_{\text{SF}}$, the choice of $f_{\text{CR}}$ does not much affect the evolution of the residual gas mass in the disc. The $f_{\text{CR}} = 0$ case features global SFR and disc mass evolution nearly identical to our non-CR run. This suggests our choice of CR initial conditions is unimportant, since CR diffusion quickly erases this information and the presence of CRs in the disc over long times is entirely regulated by star formation. From phase plots (see Fig. 11), we find the ratio of CR-to-gas energy density in the disc is consistent with observations in the solar neighbourhood for runs with $f_{\text{CR}} = 0.3$.

### 3.5 Impact of numerical parameters

We wrap up our parameter study by exploring the impact of the primary numerical parameters important in these simulations.

#### 3.5.1 CR sound speed ceiling

As CRs diffuse into the galactic halo they can evacuate cavities near the disc which have low levels of thermal gas, and the CR pressure can strongly dominate over the thermal pressure. This can be seen in Fig. 6, and tends to happen in the early evolution of the system. As discussed earlier, these high sound speed regions can dramatically slow the pace of our runs, and thus we elected to implement a maximum sound speed (via the gas density). For our fiducial run, we chose a $c_{s, \text{max}} = 1055$ km s$^{-1}$. 

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Figure 10. The SFR rate (top row) and disc mass (bottom row) for a variety of runs which vary the SN efficiency (left-hand column), the SF efficiency (middle column) and the CR feedback fraction (right-hand column).
4 DISCUSSION

4.1 A simple model for CR-diffusion-driven outflows

As our parameter study demonstrates, including the CR fluid in our simulations will only launch mass loaded, galaxy-scale outflows when the CR fluid is diffusive. The stronger the CR feedback and the longer CRs linger in the disc (i.e. the smaller the diffusion coefficient), the more gas can be ejected. This evidence points to a basic model for the outflows, illustrated in Fig. 13. The top panel of the figure illustrates a clump of dense gas, formed as the unstable portion of the disc begins to fragment, where both the matter and CR energy density become enhanced. The enhancement in CRs occurs for two related reasons: the first is CR acceleration through SN resulting from star formation in the core, and the second is the compression of the CR fluid as the clump collapses gravitationally.

At first, the CR fluid is everywhere secondary in strength to the thermal pressure, and does not dominate the dynamics. However, very quickly, the diffusive CR fluid begins to spread out, as illustrated in the bottom panel of Fig. 13, and its width grows beyond that of the thermal gas. Now the lower density wings of the thermal gas feel the presence of an enhanced CR pressure gradient sloping outward, away from the clump’s centre. This pressure gradient exceeds the local self-gravity of the gas and accelerates the double-fluid away from the clump. In a thin disc, the flow continues unimpeded out of the plane, and many clumps throughout the disc conspire to drive a galactic scale flow away from the mid-plane.

The time-scale for CR diffusion for our choice of parameters is comparable to the dynamical time of the clump, and so these steps are not cleanly separated as shown in the cartoon. In our simulations, the rays actually provide an immediate form of feedback, even before star formation commences due to the compression of pre-existing CRs. But without replenishment of the CR population via SN shock acceleration, the local CR peak can only persist a few Myr before diffusion and advection of the accelerated material depletes the CR reservoir. We find that star formation accelerated CRs are crucial for driving extended winds.

This model shows why diffusion is essential for driving outflows. In our model, without diffusion, the CR fluid never gains pressure dominance, particularly in the lower density regions of the cloud for which a given pressure gradient will cause a larger acceleration. This explains why in simulations without diffusion the CRs act to puff up the disc but cannot drive significant outflows. Uhlig et al. (2012) came to a similar conclusion for their SPH runs with CR feedback, though they modelled CR streaming rather than CR diffusion (discussed below). They found that turning on this streaming, so rays could rapidly escape the densest star-forming regions, was likewise necessary to drive outflows.

It also helps us to understand the observed dependence of outflow strength as we vary the physical parameters. As we saw in the...
In our model as depicted in Fig. 13, the time-scale for spreading out of the CR profile is directly proportional to the diffusion coefficient, and if we make the simplifying assumption that the gas does not move significantly during this process, we see that a given parcel of low-density gas in the wings will only feel the CR pressure gradient for this time period. Therefore, the resulting velocity of the gas is proportional to $\kappa_{\text{CR}}$, and more of the gas will exceed the escape speed, exactly as observed.

Most of the other parameters are even more straightforward – a higher SN energy, or a higher CR fraction will result in larger pressure gradients for a given diffusion strength, and so stronger outflows. The star formation efficiency is less obvious, although qualitatively we see that for a lower efficiency a given SFR (and likewise CR generation rate) will be delayed until the central clump density is higher; however, for the other parameters held constant, the CR acceleration is unaffected, as observed.

Finally, we note that the model indicates that the dense gas in the centre is not accelerated by the CR fluid. This is also observed in the simulations, with star-forming clumps (molecular clouds) lasting for tens of Myr (or longer). This indicates that CR feedback is not an efficient way to disperse molecular clouds, which is not surprising – as we discuss in more detail below, another physical mechanism (e.g. radiation pressure, stellar winds) is required. This also limits the amount of gas ejected since in our simulations the highest mass loading we achieve (the ratio of mass ejected to mass of stars formed) is roughly unity. But this is not a fundamental limit for this mechanism: higher mass loading could be achieved if molecular clouds were dispersed into the ISM with another feedback mechanism.
4.2 Implications of our model

The standard picture of SNe-driven galactic-scale winds places the thermal gas in a starring role: the star-forming region endows the gas with enough energy and/or momentum to rise out of the galaxy’s potential while entraining the denser ambient medium in its path. In this model, diffuse, hard X-ray emitting gas at $\sim10^8$ K fills the majority of the volume, acting like a piston to sweep up a shell of denser gas (Chevalier & Clegg 1985; Strickland & Stevens 2000). This dense forerunner may succumb to Rayleigh–Taylor instabilities and the hot wind escapes as denser clumps fall back towards the disc in a ballistic ‘fountain’ fashion. As the evacuated gas mixes with cooler components of the halo and climbs beyond the galaxy’s gravitational potential, it continually decelerates before it manages to escape the galaxy, delivering heavier elements to the IGM (Strickland & Heckman 2007). With reasonable parameters, high-resolution simulations of this model fail to launch appreciable mass into the IGM (e.g. Mac Low & Ferrara 1999; Melioli, de Gouveia Dal Pino & Geraissate 2013), although they may expel a significant amount of energy and metals.

The CR-diffusion driven winds we find here depart from this traditional picture in many fundamental ways. Here we identify two key differences with observational consequences.

(i) Beyond our galaxy’s disc plane CR pressure dominates the dynamics, launching slower, more massive winds where all but the densest clumps continue to accelerate throughout the flow. In stark contrast to a ballistic, momentum-driven feedback approach, our winds start slow, climbing towards the escape velocity $\sim10$ kpc away from the disc. Recent observations may favour this gentler, extended acceleration mechanism; Steidel et al. (2010) investigated the kinematics of 89 Lyman-break galaxies with survey-quality far-UV spectra and found their features were well matched by a scenario in which the gas velocity rises with distance, out to at least 100 kpc.

(ii) In our runs, while evacuated, $10^7$ K gas exists in pockets, particularly during the most extreme burst of star formation, the wind mostly comprises denser material below $10^6$ K, with an appreciable warm-ionized, $10^8$ K component. These winds deliver more disc mass to both the gaseous halo of the galaxy and the IGM beyond. Thus our model may help explain why star-forming galaxies show evidence for substantial amounts of multiphase gas in their haloes (e.g. Chen et al. 2010; Rubin et al. 2010; Tumlinson et al. 2011).

4.3 Missing physics

Although our simulations feature relatively high resolution, and by including CRs we help push forward the science of modelling galaxies, we are well aware that our simulations are a cheap substitute for the turbulent, multiphase, magnetized ISM, rife with molecules, dust and radiation from massive stars. In this section, we briefly discuss many of the limitations of our work and even more briefly touch on their likely impacts.

We begin with resolution. As in Tasker & Bryan (2006), we find that the SFR throughout our run depends monotonically on resolution, with higher resolution runs producing more stars at a given time. Higher resolution runs can track collapsed gas into scales, and thus higher densities, where the SFR rises, as indicated by the Schmidt-type star formation law we adopt. However, these higher resolution runs may not produce more accurate SFRs in the disc, since at these smaller scales feedback mechanisms we do not attempt to capture become important, as we noted earlier. This makes it difficult to do a proper convergence study, although we did attempt it (see Section 3.5). However, it is likely that convergence will only come with a mechanism for dispersing molecular clouds. Ironically, it may be that the lower resolution runs, which better match the Kennicutt relation, are more realistic models.

Our simulations make no attempt at a self-consistent evolution for the magnetic fields. For this two-fluid picture to strictly hold as implemented, we require a stochastic, tangled field throughout our simulation region. These inhomogeneities cause CRs to random walk through the fluid, thus obeying our simple model with advection and isotropic diffusion. Observations of both the MW and other local galaxies indicate the magnetic fields within a galactic disc are roughly equally divided in energy between such a stochastic component and a large-scale coherent field that traces the spiral structure (e.g. Beck & Wielebinski 2013). Thus CRs may preferentially stream within the plane of the disc, since the diffusion coefficient along fields lines is larger than perpendicular to them. More work is required to better understand how diffusion depends on field topology, strength and gas density (e.g. Xu & Yan 2013); however, from our parameter study, we find our qualitative result does not change when we vary the diffusion coefficient by orders of magnitude and therefore we suspect that the basic picture of CR-driven winds does not depend strongly on how diffusion works.

We note that we also do not include the impact of CR streaming. In our model, the rays are tied to the field which is assumed to be frozen to the gas. In reality, CR pressure gradients cause the rays to stream along field lines which can excite Alfvén waves leading to heating of the gas (Skillling 1975). In the halo, a more ordered, open topology may exist, perhaps providing a larger role for MHD waves excited by the streaming rays. At the disc–halo interface, the magnetic field structure is very uncertain, altered by bulk motions of the ISM that circulates above the disc. A better treatment of MHD and star formation at significantly higher resolution would be necessary to include all these processes in an accurate, self-consistent fashion.

Our CR fluid undergoes only adiabatic changes, except when bolstered by injections within star-forming regions and diminished by isotropic diffusion. In reality, diffuse shock acceleration on galactic scales and baryonic activity near the galaxy’s supermassive black hole may also contribute to the CR fluid. In these non-cosmological runs, shock fronts do not play a central role in creating our galactic CRs. And for the purposes of this study, we wish to isolate star formation feedback from active galactic nucleus (AGN) feedback. Likewise, a more realistic model would capture CR loss processes, the most important being Coulomb losses and ‘catastrophic losses’. In the former process, the charged rays slowly lose energy irreversibly to the surrounding plasma at large, heating the thermal gas while diminishing the CR energy density. The latter process involves the production of pions which decay into photons, electrons and neutrinos, resulting in a net loss of energy from the entire plasma via radiation. The relative importance of these two mechanisms depends on the distribution of CRs in energy: for CR fluids dominated by highly relativistic rays, catastrophic losses prove more important, and vice versa. Both processes scale inversely with the density of the thermal gas, $\rho$. An accurate calculation of these cooling rates also involves knowing the detailed momentum–space distribution of the CRs, since lower momentum rays lose energy much faster than higher momentum, ultrarelativistic particles. Our present work makes no attempt at modelling the CR spectrum. In fact, we implicitly assume the CR fluid is composed exclusively of ultrarelativistic, $\gamma_{CR} = 4/3$ rays, with a spectral index in momentum space of $\alpha = 2$. In their work, Jubelgas et al. (2008), building on the work of Enßlin et al. (2007), capture key aspects of the CR spectral distribution by assuming a CR distribution $d^2N/dP dV \propto P^{-\theta(p-q)}$
with constant spectral index $\alpha \in (2, 3)$ and low-momentum cut-off $q$ (here $\theta$ is the Heaviside function). Within this framework, they calculate loss rates for the CR fluid, modelling the process as simply a rising cut-off, $q$, as low-$p$ rays lose their energy. They present cooling time-scale curves as a function of the cut-off, $q$, for Coulomb and catastrophic losses. These time-scales scale roughly with the inverse of gas density. If we assume a low-momentum cut-off of approximately $m_{\text{H}}c_0$, or higher, we find a lifetime for CRs of $\approx 1.2$ Myr in the densest star-forming regions, $\approx 0.5$ Gyr in the disc at large, and $\approx 10$–1000 Gyr within our outflowing halo gas. A lower momentum cut-off can pull down these times scales an order of magnitude. Thus rays appear to be long lived, compared to the time-scales of our dynamics, and thus our decision to ignore loss processes seems justified.

By forgoing a detailed description of the CR energy distribution, we also forfeit accurate knowledge of $\gamma_{\text{CR}}$, a function primarily of spectral index, $\alpha$. Our fiducial choice of $\gamma_{\text{CR}} = 4/3$ implicitly assumes $\alpha \to 2$, and thus the ‘softest’ possible equation of state, where an adiabatic compression of the CR fluid produces a steeper rise in pressure than a thermal gas with $\gamma_{\text{TH}} = 5/3$. Observations motivate a choice of $\alpha$ closer to 2.5, and thus a somewhat stiffer CR fluid. Our parameter study has shown that a stiffer pressure response in the CR fluid enhances our outflows, so our fiducial runs are conservative in this regard.

4.4 Comparisons with previous work

Previous 1D models of CR-driven outflows have focused on diffuse winds (Breitschwerdt et al. 1991, 1993; Dorfi & Breitschwerdt 2012). They take the disc–halo interface as the inner boundary conditions of the flow and assume straight, open magnetic field lines rising above the disc. Their runs include CR diffusion and streaming and an Alfvén wave pressure. The fast, diffuse flows of the standard wind picture presumably groomed the halo’s magnetic field into this coherent structure, and the model thus appears self-consistent. Our results are broadly comparable with this work, particularly the time-dependent simulations of Dorfi & Breitschwerdt (2012). In particular, they find that local CR enhancements close to the disc drive mass loaded winds powered by CR pressure.

As described in the Introduction, Enßlin et al. (2007) and Jubelgas et al. (2008) carried out simulations with the SPH code GADGET that included CRs using a somewhat more sophisticated model than this present work. They found the CRs impacted the structure and SFR of their galaxies, particularly those with circular velocities below $80\,\text{km}\,\text{s}^{-1}$. Most runs did not include CR diffusion, but they did include it for two runs of low-mass haloes, where they found it significantly impacted the SFR. It is unclear whether or not these runs featured significant winds, as found here.

More recently, Uhlig et al. (2012) simulated idealized galaxies in three-dimensions, including CR feedback using a modified version of the SPH code GADGET. Although they did not include diffusion, they implemented CR streaming, where rays flow down gradients in the CR energy density at speeds proportional to the local sound speed. Within this similar setting, they likewise found CR-driven outflows, albeit with some key differences. They found the inclusion of CR streaming crucial to this result for a similar reason as CR diffusion proves crucial to our present study: both mechanisms allow CRs to leave the densest star-forming regions, where they are subdominant in the dynamics, into regions of lower gas density, where they can transfer energy via plasma waves and accelerate mass loaded flows of gas. Their implementation of streaming involves terms in the CR energy density equation beyond adiabatic expansion/compression of the second fluid, one of which behaves in an identical fashion to our CR diffusion, with an effective $k_{\text{CR}}$ squarely within our own explored parameter space. An additional loss term serves to capture the excitement of hydromagnetic waves that quickly get damped in the plasma, which they set proportional to $c_s/|\mathbf{V}_{\text{CR}}|$. Uhlig et al. (2012) found their outflow mechanism does feature appreciable mass loading for haloes below $10^{11}\,\mathcal{M}_{\odot}$, though they did not choose to simulate a halo as massive as in the present work. They motivate this cut-off on analytic grounds, considering a 1D flux tube out of the galaxy plane, but choose to restrict the analysis to cases where $P_{\text{CR}}$ at the base of this tube, in the disc, is roughly that observed in the solar neighbourhood. Our runs show that in a multiphase disc of a star-forming system, $P_{\text{CR}}$ may rise significantly higher in more evacuated regions. Their runs featured lower SFRs, using a subgrid model better suited to quiescent galaxies with smooth, regulated SFRs (Springel & Hernquist 2003). In addition their runs begin with gas in a spherical hydrostatic equilibrium. When they turn on radiative cooling, early outflows need to battle the ram pressure of inflowing gas raining down on to the disc. Our model instead begins with a more rotationally supported structure and a clear halo, which may better reflect the realities of a cosmological setting where gas streams in aligned filaments. Though they included non-adiabatic CR cooling and loss mechanisms, their winds were not strongly regulated by these processes, suggesting our decision to ignore CR cooling is justified.

5 SUMMARY

We performed the first 3D, high-resolution, AMR simulations of an isolated starbursting galaxy that includes a basic model for the production, dynamics and feedback of galactic CRs. This is one of the first 3D galactic disc simulations to include isotropic CR diffusion. We find CRs produced via SNe-driven shock acceleration in star-forming regions represent an important form of feedback, capable of suppressing star formation and driving mass loaded, multiphase winds from a starburst galaxy within a $10^{12}\,\mathcal{M}_{\odot}$ halo.

We implemented and tested a basic two-fluid model for the evolution of the thermal gas and the relativistic (CR) plasma, which captures the non-linear interaction and evolution of these two components. We model additional relevant physics in our runs, including radiative cooling, shocks, self-gravity, star formation, SNe feedback into both the thermal and CR gas and isotropic CR diffusion, while we ignore other key components of realistic galaxies, including an explicit treatment of magnetic fields, CR streaming and loss processes, radiation pressure, stellar winds and chemistry. Our galactic disc lies in a $10^{12}\,\mathcal{M}_{\odot}$ halo within a 500-kpc box, with adaptive resolution of up to 60 pc.

We ran a total of 21 simulations, exploring the consequences of various parameter choices related to the composition of our CR fluid, the details of our star formation algorithm and the key numerical parameters in our software, such as resolution. Below we summarize the key results of this work.

(i) The CR fluid is live and continually replenished during star formation, providing additional pressure support to the ISM and suppressing the global SFR.

(ii) A diffusive CR fluid can drive strong, massive, bi-polar outflows from a MW-sized ($10^{12}\,\mathcal{M}_{\odot}$) starbursting galaxy, with a mass loading factor of 0.3 ($\mathcal{M}/\text{SFR} \approx 30$ per cent) for our fiducial case. For other reasonable parameter choices, the mass loading can exceed unity.
(iii) We find that a mechanism such as CR diffusion (or possibly streaming) is crucial to this process. Without diffusion, no wind is launched; however, as the diffusion coefficient decreases, the mass loading factor of the wind increases, pointing to a picture in which diffusion moves CRs from high-density star-forming regions to more diffuse areas of the disc where their pressure gradient can drive outflows. Lower diffusion rates allow the CR pressure gradient to persist for longer, launching more massive winds.

(iv) These CR-driven outflows stand in contrast to thermal- and momentum-driven wind models, where hot gas ram pressure must rapidly entrain and accelerate the rest of the ISM. Instead, we see a massive, multiphase wind with slowly rising radial velocities over 10s of kpc. The relatively gentle acceleration results in a multiphase wind, which includes a cool, dense component that is generally not present in high-resolution thermally driven winds.

(v) The outflows strengthen when the SFR rises, the CR diffusion mean free path shrinks or when a larger percentage of star formation feedback is apportioned to CR production. Although the relative strength of these outflows varies, their presence persists across wide swaths of parameter space, insensitive to the precise choices in our star formation model, the tuning of our CR fluid physics, the CR-diffusion mean free path and numerical parameters such as resolution.

Our work suggests a new physical model for the generation of outflows from star-forming galaxies. Although traditionally it has been argued that diffusion will lead to a homogeneous distribution of CRs, we find that rapid star formation can maintain an enhanced CR presence at the disc midplane capable of driving mass loaded outflows by gently accelerating material and liberating appreciable gas from the halo. Thus CRs may play dynamically important roles in galaxy formation and evolution.

There are a number of enhancements to this work which should be investigated. One is to augment the CR physics captured by (i) modelling the CR spectrum explicitly and thus allowing a basic treatment of radiative losses, and (ii) including MHD and anisotropic CR diffusion. Another is to include the current CR model in cosmological simulations of galaxy formation, although it may prove challenging to match the resolution of the runs in this paper. Finally, it would be interesting to investigate the observational implications of the simulations described here, to see how well the outflows match observed absorption-line studies of the circumgalactic medium. Our work suggests that studying the detailed morphology of starburst super-winds can provide insight into the relative importance of various baryonic fluid components and the underlying structure of galactic magnetic fields, particularly at the disc–halo interface.

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REFERENCES

Ackermann M. et al., 2012, ApJ, 750, 3
Beck R., Wielebinski R., 2013, in Oswalt T. D., Gilmore G., eds, Magnetic Fields in Galaxies. Springer Science+Business Media, Dordrecht, p. 641
Behroozi P. S., Conroy C., Wechsler R. H., 2010, ApJ, 717, 379
Blandford R., Eichler D., 1987, Phys. Rep., 154, 1
Boulareas A., Cox D. P. 1990, ApJ, 368, 544
Breitschwerdt D., McKenzie J. F., Voelk H. J., 1991, A&A, 245, 79
Breitschwerdt D., McKenzie J. F., Voelk H. J., 1993, A&A, 269, 54
Brook C. B., Stinson G., Gibson B. K., Wadsley J., Quinn T., 2012, MNRAS, 424, 1275
Bryan G., 1999, Comput. Sci. Eng., 1, 46
Bryan G. L., Norman M. L., 1997, in Clarke D. A., West M. J., eds, ASP Conf. Ser. Vol. 123, Computational Astrophysics. Astron. Soc. Pac., San Francisco, p. 363
Bryan G. L. et al., 2013, ApJS, preprint (arXiv:1307.2265)
Cappellari M. et al., 2012, Nature, 484, 485
Cen R., Ostriker J. P., 1992, ApJ, 399, L113
Cesarsky C. J., 1980, ARA&A, 18, 289
Chen H.-W., Helsby J. E., Gauthier J.-R., Shectman S. A., Thompson I. B., Tinker J. L., 2010, ApJ, 714, 1521
Chevalier R. A., Clegg A. W., 1985, Nature, 317, 44
Conroy C., Wechsler R. H., Kravtsov A. V., 2006, ApJ, 647, 201
Cowie L. L., Songaila A., Kim T.-S., Hu E. M., 1995, AJ, 109, 1522
Creasey P., Theuns T., Bower R. G., 2013, MNRAS, 429, 1922
Dekel A., Silk J., 1986, ApJ, 303, 39
Dorfi E. A., Breitschwerdt D., 2012, A&A, 540, A77
Drury L. O., 1985, in Kahn F. D., ed., Cosmical Gas Dynamics. VNU Science Press, Utrecht, p. 131
Drury L. O., Falle S. A. E. G., 1986, MNRAS, 223, 353
Ellison D. C., Patnaude D. J., Slane P., Raymond J., 2010, ApJ, 712, 287
Enßlin T. A., Pfrommner C., Springel V., Jubelgas M., 2007, A&A, 473, 41
Everett J. E., Zweibel E. G., Benjamin R. A., McCammon D., Rocks L., Gallagher J. S., III, 2008, ApJ, 674, 258
Guedes J., Callegari S., Madau P., Mayer L., 2011, ApJ, 742, 76
Guo Q., White S., Li C., Boylan-Kolchin M., 2010, MNRAS, 404, 1111
Heckman T. M., Armus L., Miller G. K., 1990, ApJS, 74, 833
Joung M. R., Mac Low M.-M., Bryan G. L., 2009, ApJ, 704, 137
Jubelgas M., Springel V., Enßlin T., Pfrommner C., 2008, A&A, 481, 33
Jun B.-L., Jones T. W., 1997, ApJ, 481, 253
Jun B., Clarke D. A., Norman M. L., 1994, ApJ, 429, 748
Kang H., Jones T. W. 2006, Astropart. Phys., 25, 246
Klypin A., Zhao H., Somerville R. S., 2002, ApJ, 573, 597
Kulsrud R. M., 2005, Plasma Physics for Astrophysics. Princeton Univ. Press, Princeton, NJ
Kulsrud R. M., Cesarsky C. J., 1971, Astrophys. Lett., 8, 189
Kulsrud R., Pearce W., 1969, in Wentzel D. G., Tidman D. A., eds, Plasma Instabilities in Astrophysics. Gordon & Breach, New York, p. 358
Larson R. B., 1974, MNRAS, 169, 229
McGaugh S. S., Schombert J. M., Bothun G. D., de Blok W. J. G., 2000, ApJ, 533, L99
McGaugh S. S., Schombert J. M., de Blok W. J. G., Zagursky M. J., 2010, ApJ, 708, L14
McKenzie J. F., Breitschwerdt D., Volk H. J., 1987, in Kozyvarinsky V. A., Lidvansky A. S., Tulopova T. I., Tsyabuk A. L., Voedovsky A. V., Volgenut N. S., eds, Proc. 20th International Cosmic Ray Conference, Vol. 2, p. 119
Mac Low M.-M., Ferrara A., 1999, ApJ, 513, 142
Mandelbaum R., Seljak U., Kauffmann G., Hirata C. M., Brinkmann J., 2006, MNRAS, 368, 715
Martin C. L., 1999, ApJ, 513, 156
Melioli C., de Gouveia Dal Pino E. M., Geraisette F. G., 2013, MNRAS, 430, 3235
Minniti F., Ryu D., Kang H., Jones T. W., Cen R., Ostriker J. P., 2000, ApJ, 542, 608
Murray N., Quataert E., Thompson T. A., 2005, ApJ, 618, 569
Murray N., Ménard B., Thompson T. A., 2011, ApJ, 735, 66
Nath B. B., Silk J., 2009, MNRAS, 396, L90
Navarro J. F., Frenk C. S., White S. D. M., 1997, ApJ, 490, 493
Norman M. L., Bryan G. L., 1999, in Miyama S. M., Tomisaka K., Hanawa T., eds, Astrophysics and Space Science Library, Vol. 240, Numerical Astrophysics. Kluwer, Boston, MA, p. 19
O’Shea B. W., Bryan G., Bordner J., Norman M. L., Abel T., Harkness R., Kritsuk A., 2004, in Plawa T., Linde T., Weirs V. G., eds, Springer Lecture Notes in Computational Science and Engineering, Adaptive Mesh Refinement – Theory and Applications. Springer-Verlag, Berlin, p. 341
