Cosmic Acceleration and Modified Gravity

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Abstract

I briefly discuss some attempts to construct a consistent modification to General Relativity (GR) that might explain the observed late-time acceleration of the universe and provide an alternative to dark energy. I mention the issues facing extensions to GR, illustrate these with two specific examples, and discuss the resulting observational and theoretical obstacles. This article comprises an invited talk at the NASA workshop From Quantum to Cosmos: Fundamental Physics Research in Space.

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I. INTRODUCTION

Approaches to the late-time acceleration of the universe may be divided into three broad classes. First, it is possible that there is some as yet undiscovered property of our existing model of gravity and matter that leads to acceleration at the current epoch. Into this category one might include the existence of a tiny cosmological constant and the possibility that the backreaction of cosmological perturbations might cause self-acceleration.

Second is the idea that there exists a new dynamical component to the cosmic energy budget. This possibility, with the new source of energy density modeled by a scalar field, is usually referred to as dark energy.

Finally, it may be that curvatures and length scales in the observable universe are only now reaching values at which an infrared modification of gravity can make itself apparent by driving self-acceleration. It is this possibility that I will briefly describe in this article, submitted to the proceedings of the NASA workshop *From Quantum to Cosmos: Fundamental Physics Research in Space*.

While I will mention a number of different approaches to modified gravity, I will concentrate on laying out the central challenges to constructing a successful modified gravity model and on illustrating them with a particular simple example. Detailed descriptions of some of the other possible ways to approach this problem can be found in the excellent contributions of Sean Carroll, Cedric Deffayet, Gia Dvali and John Moffat.

II. THE CHALLENGE

Although, within the context of General Relativity (GR), one doesn’t think about it too often, the metric tensor contains, in principle, more degrees of freedom than the usual spin-2 graviton (See Sean Carroll’s talk in these proceedings for a detailed discussion of this).

The reason why one doesn’t hear of these degrees of freedom in GR is that the Einstein-Hilbert action is a very special choice, resulting in second-order equations of motion, which constrain away the scalars and the vectors, so that they are non-propagating. However, this is not the case if one departs from the Einstein-Hilbert form for the action. When using any modified action (and the usual variational principle) one inevitably frees up some of the additional degrees of freedom. In fact, this can be a good thing, in that the dynamics of these new degrees of freedom may be precisely what one needs to drive the accelerated expansion of
the universe. However, there is often a price to pay.

The problems may be of several different kinds. First, there is the possibility that along with the desired deviations from GR on cosmological scales, one may also find similar deviations on solar system scales, at which GR is rather well-tested. Second is the possibility that the newly-activated degrees of freedom may be badly behaved in one way or another; either having the wrong sign kinetic terms (ghosts), and hence being unstable, or leading to superluminal propagation, which may lead to other problems.

These constraints are surprisingly restrictive when one tries to create viable modified gravity models yielding cosmic acceleration. In the next few sections I will describe several ways in which one might modify the action, and in each case provide an explicit, clean, and simple example of how cosmic acceleration emerges. However, I will also point out how the constraints I have mentioned rule out these simple examples, and mention how one must complicate the models to recover viable models.

III. A SIMPLE MODEL: $f(R)$ GRAVITY

The simplest way one could think to modify GR is to replace the Einstein-Hilbert Lagrangian density by a general function $f(R)$ of the Ricci scalar $R$ [6, 7, 20, 21, 22, 23, 24, 25].

$$S = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left[ R + f(R) \right] + \int d^4x \sqrt{-g} \mathcal{L}_m[\chi_i, g_{\mu\nu}] ,$$

(1)

where $M_p \equiv (8\pi G)^{-1/2}$ is the (reduced) Planck mass and $\mathcal{L}_m$ is the Lagrangian density for the matter fields $\chi_i$.

Here, I have written the matter Lagrangian as $\mathcal{L}_m[\chi_i, g_{\mu\nu}]$ to make explicit that in this frame - the Jordan frame - matter falls along geodesics of the metric $g_{\mu\nu}$.

The equation of motion obtained by varying the action (1) is

$$(1 + f_R) R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (R + f) + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f_R = \frac{T_{\mu\nu}}{M_p^2} ,$$

(2)

where I have defined $f_R \equiv \partial f/\partial R$.

Further, if the matter content is described as a perfect fluid, with energy-momentum tensor,

$$T_{\mu\nu}^m = (\rho_m + p_m) U_\mu U_\nu + p_m g_{\mu\nu} ,$$

(3)

where $U^\mu$ is the fluid rest-frame four-velocity, $\rho_m$ is the energy density and $p_m$ is the pressure, then the fluid equation of motion is the usual continuity equation.
When considering the background cosmological evolution of such models, I will take the metric to be of the flat Robertson-Walker form, \( ds^2 = -dt^2 + a^2(t)dx^2 \). In this case, the usual Friedmann equation of GR is modified to become

\[
3H^2 - 3f_R(\dot{H} + H^2) + \frac{1}{2}f + 18f_{RR}H(\dot{H} + 4H\dot{H}) = \frac{\rho_m}{M_p^2}
\]

and the continuity equation is

\[
\dot{\rho}_m + 3H(\rho_m + p_m) = 0 .
\]

When supplied with an equation of state parameter \( w \), the above equations are sufficient to solve for the background cosmological behavior of the space-time and its matter contents. For appropriate choices of the function \( f(R) \) it is possible to obtain late-time cosmic acceleration without the need for dark energy, although evading bounds from precision solar-system tests of gravity turns out to be a much trickier matter, as we shall see.

While one can go ahead and analyze this theory in the Jordan frame, it is more convenient to perform a carefully-chosen conformal transformation on the metric, in order to render the gravitational action in the usual Einstein Hilbert form of GR.

Following the description in [26], consider the conformal transformation

\[
\tilde{g}_{\mu\nu} = \Omega(x^\alpha)g_{\mu\nu} ,
\]

and construct the function \( r(\Omega) \) that satisfies

\[
1 + f_R[r(\Omega)] = \Omega .
\]

Defining a rescaled scalar field by \( \Omega \equiv e^{\beta\phi} \), with \( \beta M_p \equiv \sqrt{2/3} \), the resulting action becomes

\[
\tilde{S} = \frac{M_p}{2} \int d^4x \sqrt{-\tilde{g}} \tilde{R} + \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{1}{2}\tilde{g}^{\mu\nu} (\partial_{\mu}\phi) \partial_{\nu}\phi - V(\phi) \right] + \int d^4x \sqrt{-\tilde{g}} e^{-2\beta\phi} L_m[\chi_i, e^{-\beta\phi}\tilde{g}_{\mu\nu}] ,
\]

where the potential \( V(\phi) \) is determined entirely by the original form (1) of the action and is given by

\[
V(\phi) = \frac{e^{-2\beta\phi}}{2} \left\{ e^{\beta\phi} r[\Omega(\phi)] - f(r[\Omega(\phi)]) \right\} .
\]

The equations of motion in the Einstein frame are much more familiar than those in the Jordan frame, although there are some crucial subtleties. In particular, note that in general, test particles of the matter content \( \chi_i \) do not freely fall along geodesics of the metric \( \tilde{g}_{\mu\nu} \).
The equations of motion in this frame are those obtained by varying the action with respect to the metric $\tilde{g}_{\mu\nu}$

$$\tilde{G}_{\mu\nu} = \frac{1}{M_p^2} \left( \tilde{T}_{\mu\nu} + T^{(\phi)}_{\mu\nu} \right),$$

with respect to the scalar field $\phi$

$$\Box \phi = -\frac{dV}{d\phi}(\phi),$$

and with respect to the matter fields $\chi_i$, described as a perfect fluid.

Once again, I will specialize to consider background cosmological evolution in this frame. The Einstein-frame line element can be written in familiar FRW form as

$$ds^2 = -d\tilde{t}^2 + \tilde{a}^2(t) dx^2,$$

where $d\tilde{t} \equiv \sqrt{\Omega} dt$ and $\tilde{a}(t) \equiv \sqrt{\Omega} a(t)$. The Einstein-frame matter energy-momentum tensor is then given by

$$\tilde{T}^m_{\mu\nu} = (\tilde{\rho}_m + \tilde{p}_m) \tilde{U}_\mu \tilde{U}_\nu + \tilde{p}_m \tilde{g}_{\mu\nu},$$

where $\tilde{U}_\mu \equiv \sqrt{\Omega} U_\mu$, $\tilde{\rho}_m \equiv \rho_m / \Omega^2$ and $\tilde{p}_m \equiv p_m / \Omega^2$.

A. A Simple Example

For definiteness and simplicity focus on the simplest correction to the Einstein-Hilbert action; $f(R) = -\mu^4 / R$, with $\mu$ a new parameter with units of [mass].

The field equation for the metric is then

$$\left(1 + \frac{\mu^4}{R^2}\right) R_{\mu\nu} - \frac{1}{2} \left(1 - \frac{\mu^4}{R^2}\right) R \tilde{g}_{\mu\nu} + \mu^4 \left[g_{\mu\nu} \nabla_\alpha \nabla^\alpha - \nabla_\mu \nabla_\nu\right] R^{-2} = \frac{T^m_{\mu\nu}}{M_p^2}. \quad (14)$$

The constant-curvature vacuum solutions, for which $\nabla_\mu R = 0$, satisfy $R = \pm \sqrt{3}\mu^2$. Thus, there exists a constant-curvature vacuum solution which is de Sitter space. We will see that the de Sitter solution is, in fact, unstable, albeit with a very long decay time $\tau \sim \mu^{-1}$.

The time-time component of the field equations for this metric is

$$3H^2 - \frac{\mu^4}{12(\dot{H} + 2H^2)} \left(2H \ddot{H} + 15H^2 \dot{H} + 2\dot{H}^2 + 6H^4\right) = \frac{\rho M}{M_p^2}.$$

As I have discussed, one may now transform to the Einstein frame, where the gravitational Lagrangian takes the Einstein-Hilbert form and the additional degree of freedom appears as a fictitious scalar field $\phi$, with potential

$$V(\phi) = \mu^2 M_p^2 e^{-2\beta \phi} \sqrt{e^{2\beta \phi} - 1}, \quad (16)$$
FIG. 1: The Einstein-frame potential $V(\phi)$ shown in the figure below.

Denoting with a tilde all quantities (except $\phi$) in the Einstein frame, the relevant Einstein-frame cosmological equations of motion are

$$3\tilde{H}^2 = \frac{1}{M_p^2} \left[ \rho_\phi + \tilde{\rho} \right],$$  \hspace{1cm} (17)

$$\phi'' + 3\tilde{H}\phi' + \frac{dV}{d\phi}(\phi) - \frac{(1 - 3w)}{\sqrt{6}M_p}\tilde{\rho}_M = 0,$$ \hspace{1cm} (18)

where a prime denotes $d/d\tilde{t}$, and where

$$\tilde{\rho}_M = \frac{C}{a^3(1+w)} \exp \left[ -\frac{(1 - 3w)}{\sqrt{6}M_p} \phi \right],$$ \hspace{1cm} (19)

with $C$ a constant, and

$$\rho_\phi = \frac{1}{2}\phi'^2 + V(\phi).$$ \hspace{1cm} (20)

Finally, note that the matter-frame Hubble parameter $H$ is related to that in the Einstein frame $\tilde{H} \equiv \dot{a}/\dot{a}$ by

$$H = \sqrt{\rho} \left( \tilde{H} - \frac{\phi'}{M_p\sqrt{6}} \right).$$ \hspace{1cm} (21)

How about cosmological solutions in the Einstein frame? Ordinarily, Einstein gravity with a scalar field with a minimum at $V = 0$ would yield a Minkowski vacuum state. However, here this is no longer true. Even though $V \to 0$ as $\phi \to 0$, this corresponds to a curvature singularity
and so is not a Minkowski vacuum. The other minimum of the potential, at $\phi \to \infty$, does not represent a solution.

Focusing on vacuum solutions, i.e., $P_M = \rho_M = 0$, the beginning of the Universe corresponds to $R \to \infty$ and $\phi \to 0$. The initial conditions we must specify are the initial values of $\phi$ and $\phi'$, denoted as $\phi_i$ and $\phi'_i$. There are then three qualitatively distinct outcomes, depending on the value of $\phi'_i$.

1. **Eternal de Sitter.** There is a critical value of $\phi'_i \equiv \phi'_C$ for which $\phi$ just reaches the maximum of the potential $V(\phi)$ and comes to rest. In this case the Universe asymptotically evolves to a de Sitter solution (ignoring spatial perturbations). As we have discovered before (and is obvious in the Einstein frame), this solution requires tuning and is unstable.

2. **Power-Law Acceleration.** For $\phi'_i > \phi'_C$, the field overshoots the maximum of $V(\phi)$. Soon thereafter, the potential is well-approximated by $V(\phi) \simeq \mu^2 M_p^2 \exp(-\sqrt{3/2}\phi/M_p)$, and the solution corresponds to $a(t) \propto t^2$ in the matter frame. Thus, the Universe evolves to late-time power-law inflation, with observational consequences similar to dark energy with equation-of-state parameter $w_{DE} = -2/3$.

3. **Future Singularity.** For $\phi'_i < \phi'_C$, $\phi$ does not reach the maximum of its potential and rolls back down to $\phi = 0$. This yields a future curvature singularity.

What about including matter? As can be seen from (18), the major difference here is that the equation-of-motion for $\phi$ in the Einstein frame has a new term. Furthermore, since the matter density is much greater than $V \sim \mu^2 M_p^2$ for $t \ll 14$ Gyr, this term is very large and greatly affects the evolution of $\phi$. The exception is when the matter content is radiation alone ($w = 1/3$), in which case it decouples from the $\phi$ equation due to conformal invariance.

Despite this complication, it is possible to show that the three possible cosmic futures identified in the vacuum case remain in the presence of matter.

Thus far, the dimensionful parameter $\mu$ is unspecified. By choosing $\mu \sim 10^{-33}$ eV, the corrections to the standard cosmology only become important at the present epoch, explaining the observed acceleration of the Universe without recourse to dark energy.

Clearly the choice of correction to the gravitational action can be generalized. Terms of the form $-\mu^2(\rho + p)/R^m$, with $m > 1$, lead to similar late-time self acceleration, which can easily accommodate current observational bounds on the equation of state parameter.

Now, as I mentioned in the introduction, any modification of the Einstein-Hilbert action must, of course, be consistent with the classic solar system tests of gravity theory, as well as numerous other astrophysical dynamical tests. We have chosen the coupling constant $\mu$ to be
very small, but we have also introduced a new light degree of freedom. As shown by Chiba [27],
the simple model above is equivalent to a Brans-Dicke theory with $\omega = 0$ in the approximation
where the potential was neglected, and would therefore be inconsistent with experiment [28]
(although see [29, 30, 31] for suggestions that the conformally transformed theory may not be
the correct way to analyze deviations from GR).

To construct a realistic $f(R)$ model requires a more complicated function, with more than
one adjustable parameter in order to fit the cosmological data and satisfy solar system bounds.
Examples can be found in [13, 32].

IV. EXTENSIONS: HIGHER-ORDER CURVATURE INVARIANTS

It is natural to consider generalizing the action of $\mathcal{L}$ to include other curvature invariants.
There are, of course, any number of terms that one could consider, but for simplicity, focus on
those invariants of lowest mass dimension that are also parity-conserving

$$
P \equiv R_{\mu\nu} R^{\mu\nu},$$

$$
Q \equiv R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}. \quad (22)
$$

We consider actions of the form

$$
S = \int d^4x \sqrt{-g} \left[ R + f(R, P, Q) \right] + \int d^4x \sqrt{-g} \mathcal{L}_M ,
$$

(23)

where $f(R, P, Q)$ is a general function describing deviations from general relativity.

It is convenient to define

$$
f_R \equiv \frac{\partial f}{\partial R} , \quad f_P \equiv \frac{\partial f}{\partial P} , \quad f_Q \equiv \frac{\partial f}{\partial Q} , \quad (24)
$$
in terms of which the equations of motion are

$$
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{1}{2} g_{\mu\nu} f
+ f_R R_{\mu\nu} + 2 f_P R^{\alpha}_\mu R_{\alpha\nu} + 2 f_Q R_{\alpha\beta\gamma\mu} R^{\alpha\beta\gamma\nu}
+ g_{\mu\nu} \Box f_R - \nabla_\mu \nabla_\nu f_R - 2 \nabla_\alpha \nabla_\beta \left[ f_P R^{\alpha}_\mu (\rho^{\beta}_\nu) \right] + \Box (f_P R_{\mu\nu})
+ g_{\mu\nu} \nabla_\alpha \nabla_\beta (f_P R^{\alpha\beta}) - 4 \nabla_\alpha \nabla_\beta [f_Q R^{\alpha}_{(\mu\nu)}] = 8\pi G T_{\mu\nu} . \quad (25)
$$

It is straightforward to show that actions of the form (23) generically admit a maximally-
symmetric solution: $R = \text{a non-zero constant}$. However, an equally generic feature of such
models is that this de Sitter solution is unstable. In the CDTT model the instability is to an accelerating power-law attractor. This is a possibility that we will also see in many of the more general models under consideration here.

Since we are interested in adding terms to the action that explicitly forbid flat space as a solution, I will, in a similar way as in [6], consider inverse powers of the above invariants and, for simplicity, specialize to a class of actions with

$$f(R, P, Q) = -\frac{\mu^{4n+2}}{(aR^2 + bP + cQ)^n},$$

(26)

where $n$ is a positive integer (taken to be unity), $\mu$ has dimensions of mass and $a$, $b$ and $c$ are dimensionless constants. In fact, for general $n$ the qualitative features of the system are as for $n = 1$[14].

A. Another Simple Example

For the purposes of this short talk, I will focus on a specific example - actions containing modifications involving only $P \equiv R_{\mu \nu} R^{\mu \nu}$, with the prototype being $f(P) = -m^6 / P$, with $m$ a parameter with dimensions of mass.

It is easy to see that there is a constant curvature vacuum solution to this action given by $R_{\text{const}}^{(P)} = (16)^{1/3} m^2$. However, we would like to investigate other cosmological solutions and analyze their stability.

From (25), with the flat cosmological ansatz, the analogue of the Friedmann equation becomes

$$3H^2 - \frac{m^6}{8(3H^4 + 3H^2 \dot{H} + \dot{H}^2)^3} \left[ \dot{H}^4 + 11H^2 \dot{H}^3 + 2H \dot{H}^2 \ddot{H} + 33H^4 \dot{H}^2 + 30H^6 \dot{H} + 6H^3 \dot{H}^2 \ddot{H} + 6H^8 + 4H^5 \dddot{H} \right] = 0.$$

(27)

Asymptotic analysis of this equation (substituting in a power-law ansatz and taking the late-time limit) yields two late-time attractors with powers $v_0 = 2 - \sqrt{6}/2 \simeq 0.77$ and $v_0 = 2 + \sqrt{6}/2 \simeq 3.22$.

However, in order to obtain a late-time accelerating solution ($p > 1$), it is necessary to give accelerating initial conditions ($\ddot{a} > 0$), otherwise the system is in the basin of attraction of the non-accelerating attractor at $p \simeq 0.77$ (This type of behavior is generic in some other modified gravity theories [33]). While I’ve given a simple example here, cosmologically viable models are described in [34].
What about the other constraints on these models? It has been shown \cite{35} that solar system constraints, of the type I have described for \( f(R) \) models, can be evaded by these more general models whenever the constant \( c \) is nonzero. Roughly speaking, this is because the Schwarzschild solution, which governs the solar system, has vanishing \( R \) and \( P \), but non-vanishing \( Q \).

More serious is the issue of ghosts and superluminal propagation. It has been shown \cite{36, 37} that a necessary but not sufficient condition that the action be ghost-free is that \( b = -4c \), so that there are no fourth derivatives in the linearised field equations. What remained was the possibility that the second derivatives might have the wrong signs, and also might allow superluminal propagation at some time in a particular cosmological background. It has recently been shown that in a FRW background with matter, the theories are ghost-free, but contain superluminally propagating scalar or tensor modes over a wide range of parameter space \cite{38, 39}. It is certainly necessary to be ghost-free. Whether the presence of superluminally propagating modes is a fatal blow to the theories remains to be seen.

V. CONCLUSIONS

Given the immense challenge posed by the accelerating universe, it is important to explore every option to explain the underlying physics. Modifying gravity may be one of the more radical proposals, but it is not one without precedence as an explanation for unusual physics. However, it is an approach that is tightly constrained both by observation and theoretical obstacles.

In the brief time and space allowed, I have tried to give a flavor of some attempts to modify GR to account for cosmic acceleration without dark energy. I have focused on two of the directions in which I have been involved and have chosen to present simple examples of the models, which clearly demonstrate not only the cosmological effects, but also how constraints from solar system tests and theoretical consistency apply.

There are a number of other proposals for modified gravity and, while I have had neither time nor space to devote to them here, others have discussed some of them in detail at this meeting.

There is much work ahead, with significant current effort, my own included, devoted to how one might distinguish between modified gravity, dark energy and a cosmological constant as competing explanations for cosmic acceleration.
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