Decoherence processes in a current biased dc SQUID

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A current bias dc SQUID behaves as an anharmonic quantum oscillator controlled by a bias current and an applied magnetic flux. We consider here its two level limit consisting of the two lower energy states |0⟩ and |1⟩. We have measured energy relaxation times and microwave absorption for different bias currents and fluxes in the low microwave power limit. Decoherence times are extracted. The low frequency flux and current noise have been measured independently by analyzing the probability of current switching from the superconducting to the finite voltage state, as a function of applied flux. The high frequency part of the current noise is derived from the electromagnetic environment of the circuit. The decoherence of this quantum circuit can be fully accounted by these current and flux noise sources.

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In the past years, coherent manipulation of two and multi-level quantum systems, efficient quantum readouts, entanglement between quantum states have been achieved[1, 2] demonstrating the full potential of quantum logic in solid state physics. At present, future developments require longer coherence times[8, 9, 10]. However a complete and consistent understanding of decoherence processes acting on various systems remains a priority for a full-control of quantum experiments. Different models for the noise sources have been proposed to describe the decoherence processes acting on various qubits[7, 8, 9, 10]. However a complete and consistent understanding of decoherence remains a current and open problem. In this paper, we study decoherence processes of a phase qubit: the current biased dc SQUID.

This superconducting circuit consists of two Josephson junctions (JJ), each with a critical current $I_0$ and a capacitance $C_0$. The junctions are embedded in a superconducting loop of inductance $L$, threaded by a flux $\Phi_b$. In the limit where $L_sI_0 \approx \Phi_0/2\pi$, the phase dynamics of the two junctions can be mapped onto a fictitious particle following a one dimensional path in a 2D-potential $U$. If the biasing current $I_b$ is smaller than the SQUID critical current $I_c$, the particle is trapped in a cubic potential well characterized by its bottom frequency $\omega_p(I_b, \Phi_b)$ and a barrier height $\Delta U(I_b, \Phi_b)$ (Fig. 1.a). The quantum states in this anharmonic potential are denoted $|n⟩$, with corresponding energies $E_n, n = 0, 1, ...$. In the following, only the lowest states $|0⟩$ and $|1⟩$ will be involved. For $I_b$ well below $I_c$, these two levels are stable and constitute a phase qubit.

When the bias current $I_b$ is close to $I_c$, $\Delta U$ decreases and becomes of the order of a few $\hbar \omega_p$. The ground state can tunnel through the potential barrier and the SQUID switches to a voltage state[11]. The tunnelling rate $\Gamma_0$ of the ground state $|0⟩$ is given by the well-known MQT formula for underdamped JJ[12]:

$$\Gamma_0(I_b, \Phi_b) = a \omega_p \exp(-36\Delta U/5\hbar \omega_p),$$

where $a$ is of order unity.

The environment of the dc SQUID induces fluctuations of the bias current and the bias flux. In this work, we show how the current and flux noise sources can be separately quantified. This is achieved by escape measurements of the SQUID at specific working points where it is mostly sensitive to current or flux noise. Using these identified noise sources, the measured decoherence times are fit precisely as a function of bias current.

Experimental results are analyzed assuming a linear coupling between the SQUID and the environment degrees of freedom. We suppose that current $\delta I$ and flux $\delta \Phi$ noises are generated by independent gaussian sources. Here, $\delta x$ ($x = I$ or $\Phi$) is an operator acting on the environment. Their fluctuations are specified by the quantum spectral densities $S_x(\nu)$. In presence of flux microwave (MW) excitation, the total Hamiltonian $\hat H$ in the SQUID eigenstates basis $\{|0⟩, |1⟩\}$ reads:

$$\hat H = -\frac{\hbar}{2}h\nu_0|\hat \sigma_x - h\nu \cos(2\pi\nu t)\hat \sigma_z| + \hat N$$

where $\hat \sigma_x$ and $\hat \sigma_z$ are Pauli matrices and $\nu_0 = (E_1 - E_0)/\hbar$. The first term is the qubit Hamiltonian and the second term describes the MW excitation of reduced amplitude $h\nu_0$ at frequency $\nu$. In this notation, $\nu_R$ is also the Rabi precession frequency for a tuned excitation $(\nu = \nu_0)$.

The last term is the coupling to the noise sources. For our circuit it is, within linear response,

$$\hat N = -\frac{\hbar}{2}r_I(\theta)\hat \sigma_z\left[\frac{r_I(\theta)}{2\pi\sqrt{C_0|\nu_0|}}\hat \delta I + \frac{r_\Phi(\theta)}{\pi L_s\sqrt{C_0\hbar|\nu_0|}}\hat \delta \Phi\right]$$

$$-\frac{\hbar}{2}r_\Phi(\theta)\hat \delta \Phi + \left(\frac{\partial \nu_0}{\partial \Phi_b}\right)\hat \sigma_z - \frac{i}{\pi}$$

where $\eta$ is the asymmetry inductance parameter (see below), $r_I(\theta) = \cos \theta + \eta \sin \theta$, $r_\Phi(\theta) = \sin \theta$ and $\theta$ is the angle between the escape and the mean slope directions in the 2D potential[11, 13]. To first order, the transverse noise proportional to $\sigma_z$ only induces depolarization. The longitudinal term proportional to $\sigma_z$ induces "pure" dephasing. The qubit sensitivity to
longitudinal noise is given by the partial derivatives $(\partial n_1 / \partial I_b)$ and $(\partial n_1 / \partial \Phi_b)$. They depend strongly on the experimental working point and increase near the critical current.

The measured SQUID consists of two large aluminum JJs of 15 $\mu m^2$ area ($I_0 = 1.242 \mu A$ and $C_0 = 0.56$ pF) enclosing a 350 $\mu m^2$-area superconducting loop. The two SQUID branches of inductances $L_1$ and $L_2$ contribute to the total loop inductance $L_s = 280$ pH with the asymmetry parameter $\eta = (L_1 - L_2)/L_s = 0.414$. The immediate electromagnetic environment of the SQUID is designed to decouple the circuit from the external world. It consists of two cascaded LC filters (see Fig. 1b). A large on-chip inductance $L_{oc} = 9$ nH is made of two long and thin superconducting wires which value, derived from the normal state resistance, is dominated by the kinetic inductance. The gold thin film parallel capacitor, $C_p \approx 150$ pF, introduces a finite resistor. Its dc value at 30 mK is $R_s = 0.1$ $\Omega$ giving the gold resistivity $\rho_g = 1.21 \times 10^{-8}$ $\Omega$m.

The second filter consists of the bounding wires, with an estimated inductance $L_f = 3$ nH, and a surface mounted (SMC) capacitor $C_{SMC} = 2$ nF and four 500 $\Omega$ SMC resistors. The nominal room temperature microwave signal is guided by 50 $\Omega$ coaxial lines, attenuated at low temperature before reaching the SQUID through a mutual inductance $M_s = 1.3$ pH. Special care was taken in magnetic shielding and bias lines filtering. All these electrical parameters were determined independently [11, 12]. Our environment model predicts two resonances (resistance dips in Fig. 1c). They were observed in a similar set-up and the associated resonance frequencies were in precise agreement with the model.

The current noise through the SQUID comes mostly from its immediate environment thermalized at $T = 30$ mK ($\nu_T \equiv k_B T / h = 600$ MHz $\ll \nu_0$). The quantum spectral density of the current noise, $S_I(\nu)$ in this environment is set by the fluctuation-dissipation theorem: $S_I(\nu) = \hbar \nu [\coth (\hbar \nu / 2 k_B T) + 1] R_{eff}(\nu)^{-1}$ where $R_{eff}(\nu)^{-1}$ is the real part of the environment circuit admittance. $R_{eff}(\nu)$ is calculated using the electrical circuit shown in Fig. 1b and is plotted in Fig. 1c. To a good approximation, the root mean square (RMS) current fluctuations are of order $\sqrt{k_B T / L_{oc}} = 6$ nA. Most of the noise is peaked around 30 MHz, a frequency much smaller than $\nu_T$. A simple estimate of the flux noise produced by the inductive coupling to the 50 $\Omega$ coaxial line shows it can be neglected in the following.

The escape probability $P_{esc}(I_b)$ out of the superconducting states is measured at fixed flux using dc current pulses with $\Delta t = 50$ $\mu s$ duration and $I_b$ amplitude. Each measurement involves 5000 identical current pulses and the total acquisition time is $T_m = 10$ s. The escape current $I_{esc}$ is defined as the current $I_b$ where the escape probability $P_{esc}(I_b) = 0.5$ and the width of the switching curve $\Delta I = |I_b - I_{esc}|$ is the distance between the currents where $P_{esc}(I_b) = 0.9$ and $P_{esc}(I_b) = 0.1$. In Fig. 2, the dependence of $I_{esc}$ and $\Delta I$ on $\Phi_b$ are plotted. By fitting the escape current curve $I_{esc}(\Phi_b)$, the experimental parameters of the SQUID ($I_0, C_0, L_s, \eta$) are determined.

Moreover, escape measurements are a sensitive tool to characterize noise (frequency range and amplitude). If noise frequencies exceed the inverse of a current pulse duration $\Delta t^{-1}$, the tunnel rate fluctuates during each current pulse. The escape probability is controlled by the average $\langle \Gamma_0 \rangle$ escape rate in the frequency window $[\Delta t^{-1}, \nu_T]$: $P_{esc} = 1 - \exp \left[ - \langle \Gamma_0 (I_b + \delta I, \Phi_b + \delta \Phi) \rangle \Delta t \right]$. The current noise produced by the electrical
environment lies in this frequency interval. Its effect is to decrease $I_{esc}(\Phi_b)$ by about 6 nA, the RMS current fluctuations (unobservable in Fig. 2a). Similarly, the width of the switching curve is not affected.

On the other hand, if noise frequencies are slower than $\Delta t^{-1}$, the tunnel rate is constant during a pulse, but fluctuates from pulse to pulse. In this limit, the escape probability becomes $P_{esc} = 1 - \exp\left[ -t_1(I_b + \delta I, \Phi_0 + \delta \Phi)\Delta t \right]$, where the statistical average $\langle \rangle$ is now in frequency range from $T_m^{-1}$ to $\Delta t^{-1}$. To first order, low frequency noise does not affect $I_{esc}$, but increases the width $\Delta I$. Thus $\Delta I$ is the best quantity to probe the origin and the magnitude of the low frequency fluctuations: if the flux $\Phi_0$ is set at the value $\Phi_{0\beta}$ which maximizes $I_c$, the SQUID is only sensitive to current fluctuations since $\frac{dI}{d\Phi_b} = 0$. In the vicinity of this flux, the measured width is explained by the usual MQT theory. The measured RMS current fluctuations in the $[T_m^{-1}, \Delta t^{-1}]$ interval (low frequency current noise) is below 0.5 nA, the error bar in $\Delta I$ measurements. This is consistent with the 0.1 nA RMS value derived from the spectral density of noise at frequencies below $\Delta t^{-1}$. For other applied fluxes, the width is slightly larger than MQT prediction, indicating a residual low frequency flux noise. The dependence of $\Delta I$ on $\Phi_0$ shown in Fig. 2b is perfectly explained by a gaussian low frequency flux noise. Its RMS amplitude, $\langle \delta \Phi^2_{LF} \rangle^{1/2} = 5.5 \times 10^{-4} \Phi_0$, is extracted from the fit shown in Fig. 2b and is attributed to the flux noise in the [100 mHz, 20 kHz] frequency interval. The origin of flux noise may be due to vortices trapped in the four aluminium contact pads located at a 0.5mm distance from the SQUID.

Hereafter we discuss dephasing and relaxation induced by the noise sources previously identified. These incoherent processes are experimentally studied with low power spectroscopy and energy relaxation measurement. As described in Ref. 1, a MW flux pulse is applied followed by a 2 ns duration dc flux pulse to perform a fast but adiabatic measurement of the quantum state of the SQUID (Fig. 3c inset). The duration $T_{MW} = 300$ ns of MW pulses is sufficient to reach the stationary state where the population $p_1$ of the level $|1\rangle$ only depends on $\nu$ and the amplitude $\nu_R$. The microwave amplitude $\nu_R$ is calibrated using Rabi like oscillations. In the two level experiments discussed in this paper, the measured escape probability $P_{esc}$ induced by the dc flux pulse can be interpreted as $P_{esc} = P_{esc}^{(0)} + (P_{esc}^{(1)} - P_{esc}^{(0)}) \times p_1(\nu, \nu_R)$. $P_{esc}^{(0)}$ denotes the escape probability out of the pure state $|n\rangle$. In Fig. 3a and 3b, the escape probability versus microwave frequency $\nu$ are plotted at two different biasing points. The experimental curves present a resonant peak which position and full width at half maximum define the resonant frequency $\nu_{01}$ and $\Delta \nu$. Spectroscopy experiments are performed in the linear regime and $\Delta \nu$ is experimentally checked to be independent of the MW amplitude. Relaxation measurements were performed by populating the $|1\rangle$ state with low power MW tuned at $\nu_{01}$ during a time $T_{MW} = 300$ ns, and measuring its population with increasing time delay $T_{delay}$ after the end of the MW pulse. As shown in Fig. 3c, the escape probability follows an exponential relaxation with a characteristic time $T_1$. In Fig. 4 measured resonant frequency $\nu_{01}$, relaxation time $T_1$ and the inverse of microwave width $\Delta \nu^{-1}$ are plotted versus current bias for the two different applied fluxes $\Phi_{b1}$(close to $\Phi_{0\beta}$) and $\Phi_{b2}$ shown in Fig. 2. $\nu_{01}$, $T_1$ and $\Delta \nu^{-1}$ decreases as $I_b$ gets closer to $I_c$. For these two applied fluxes, the $\nu_{01}$ dependence fits perfectly the semiclassical formulas for a cubic potential using the same SQUID electrical parameters as those extracted from escape measurements.

The depolarization rate $T_1^{-1}$ is given by the sum $T_1^{-1} = T_{R}^{+} + T_{E}$ of the relaxation $\Gamma_R$ and the excitation $\Gamma_E$ rates. These two rates are calculated using Fermi golden rule. At low temperature, excitation can be neglected and $\Gamma_R$ reads:

$$\Gamma_R = \frac{r_2^2(\theta)}{4 C_0 \hbar \nu_{01}} S(\nu_{01}) + \frac{r_3^2(\theta)}{2 C_0 \hbar \nu_{01}} S(\Phi(\nu_{01})).$$

Neglecting the high frequency part of the flux noise, one obtains $T_1 = 2 R_{esc}(\nu_{01}) C_0 / r_2^2(\theta)$ where $R_{esc}(\nu_{01}) = (2\pi L_{esc} \nu_{01})^2 / R_{S}(\nu_{01})$. $R_S(\nu) = \alpha R_s$ is the high frequency resistance of the gold capacitor where $R_s = \sqrt{\mu_0 \rho_{ys}}$ is the surface resistance and $\alpha$ is a dimensionless geometrical parameter. The $T_1$ versus $I_b$ dependence is well fitted with $\alpha = 200$ as the only adjustable parameter (Fig. 4b). This value is the right order of magnitude for the geometrical parameter.
First, we consider the time evolution of the reduced density matrix in the basis 

\[ \rho(\tau) = \exp(-iH\tau)\rho(0)\exp(iH\tau) \]

where \( H \) is the Hamiltonian of the system. The linear coupling to noise sources induces a time decay of the amplitude \( f(t) \) of the coherence terms. Since current and flux noises are independent, \( f(t) \) is factorized as \( f(t) = f_1(t)f_0(t) \exp(-2t/T_1) \) where \( f_1(t) \) and \( f_0(t) \) are respectively the "pure" dephasing contributions due to current and flux noises.

The current contribution is given by the well-known gaussian noise formula:

\[ f_1(t) = \exp \left[ -\frac{1}{2}t^2 \right] \]

with the width dominated by current noise. At low microwave power where the quantum circuit can be reduced to a two level system, analyzing the experimental curves. In Fig. 4c, the theoretical width \( \Delta \nu \) extracted from the curve \( f_{coh}(\nu) \) is in very good agreement with experimental data without free parameter. When \( I_b \) gets close to \( I_c \), the partial derivatives \( (\partial \nu_01/\partial I_b) \) and \( (\partial \nu_01/\partial \Phi_b) \) increase: the noise sensitivity increases and \( \Delta \nu \) broadens. For bias points corresponding to \( \Phi_{b2} \), the width is due to current and flux noise.

Within linear response, the shape of the resonance curve is proportional to the Fourier transform (FT) of \( f_{coh}(t) \). Resonance curves in Fig. 3a and 3b are fit using \( P_{esc} - P_{esc}^{(0)} \propto FT\{f_{coh}\}(\nu-\nu_0) \) (continuous line). Our model explains perfectly the shape of the experimental curves. In Fig. 4c, the theoretical width \( \Delta \nu \) extracted from the curve \( f_{coh}(\nu) \) is in very good agreement with experimental data without free parameter.

In conclusion, we have shown how the flux and current noise present in this controlled quantum circuit can be separately identified. We measured the decoherence times at low microwave power where the quantum circuit can be reduced to a two level system. Analyzing the coupling of the SQUID to the known noise sources, the measured relaxation times and the resonance width can be fully understood.

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\[ S_\alpha(\nu) = -\frac{i}{2\pi} \left< \frac{\partial^2}{\partial \nu^2} \right\rangle \left< \frac{\partial^2}{\partial \tau^2} \right\rangle_{\nu,\tau} \]

where \( \nu \) and \( \tau \) are not observed. Other noise mechanism may blur the predicted satellite peaks.

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In fact, We kept \( \Phi_b \) as constant as possible during experiments, because of a static flux hysteresis about \( 10^{-3} \Phi_0 \).