Using Small Displacement Torsor to Simulate the Machining Processes for 3D Tolerance Transfer

Heping Peng 1* and Zhuoqun Peng 2

1 School of Electromechanical & Architectural Engineering, Jianghan University, Wuhan 430056, China
2 School of Machinery and Automation, Wuhan University of Science and Technology, Wuhan 430081, China

hbjhun_penn@outlook.com

Abstract. This paper proposes a method for 3D tolerance transfer by using small displacement torsor (SDT) to simulate the machining processes. In this method, the machining process of a mechanical part is regarded as a mechanism mainly composed of machine tool, part-holders, machined part, and cutting tools. And the small displacement torsors are employed to describe all defects in the machining process including the variation deviations of the machined surfaces of a part with regards to their nominal positions, the gap deviations associated to each joint between two contact surfaces, etc. Then, the 3D manufacturing tolerancing chains based on the functional tolerances can be established to realize 3D tolerance transfer. The proposed method is applied to a machining part to demonstrate its effectiveness.

1. Introduction
Tolerance is the important product information at each stage of product design, manufacturing, verification, assembly, testing, etc. And it is the concrete reflection of product precision, the result of trade-off between product requirements and development costs [1]. Tolerance transfer, also considered as tolerance analysis and synthesis in manufacturing process planning, is the approach to convert design tolerances into manufacturing tolerances [2]. Jaballi et al. [3] developed 3D tolerance transfer method based on the Technologically and Topologically Related Surface (TTRS). Jiang et al. [4] studied transfer of cylindrical datum in manufacturing process with T-Map model. Villeneuve et al. [5] put forward 3D model to perform manufacturing tolerancing for mechanical parts in which the small displacement torsor (SDT) concept is employed to model the machined parts, part-holders, and machining operations. And Vignat and Villeneuve [6] performed 3D manufacturing tolerancing for turning operation by using the SDT approach. And since then, the SDT approach was further extended to manufacturing tolerance synthesis by Villeneuve and Vignat [7]. In addition, Villeneuve and Vignat [8] proposed the model of manufactured part (MMP) for 3D transfer of tolerances by simulating successive machining processes.

This paper presents a general 3D modeling method for tolerance transfer in the context of manufacturing tolerancing. The proposed method uses small displacement torsors to model all variations in the successive set-ups of the machining process regardless of the causes of these variations. Then, the influence of the process planning on the respect of functional tolerances is
modelled as 3D tolerances chain to explore the manufacturing tolerancing transfer. Finally, application to a machining part is introduced to verify feasibility of the proposed method.

2. Small displacement torsor

Generally, any variation of a geometrical feature from the nominal position can be represented by using the SDT with three rotation components \((\alpha, \beta, \gamma)\) and three translation components \((u, v, w)\) [9]. SDT \(\{\tau_{PP}\}_{(O,R)}\) synthesizes the position and orientation of an associated feature \(P_i\) relative to its nominal feature at a given point \(O\) in a local reference frame \(\{R_i\}\) and consists of original vector \(\varphi\) and dual vector \(\varepsilon_{O}\) [10].

\[
[\tau_{PP}]_{(O,R)} = \begin{bmatrix} \varphi \\ \varepsilon_{O} \end{bmatrix} = \begin{bmatrix} \alpha & u \\ \beta & v \\ \gamma & w \end{bmatrix}_{(O,R)}
\]

(1)

For example, the torsor represented the relative positions of the associated plane \(P_i\) with regard to its nominal plane at a given point \(O\), in the local reference frame \(\{R_i\}\) with axis \(z_i\) is the normal of the plane can be expressed as:

\[
[\tau_{PP}]_{(O,R)} = \begin{bmatrix} \varphi \\ \varepsilon_{O} \end{bmatrix} = \begin{bmatrix} \alpha & U \\ \beta & U \\ U & w \end{bmatrix}_{(O,R)}
\]

(2)

where capital \(U\) represents the undetermined component in the expression of a torsor.

The two following properties will be defined to calculate operations on these torsors [5]:

\[
\forall a \in R, \quad a + U = U
\]

(3)

\[
\forall a,b \in R^2, \quad a \cdot U + b \cdot U = U
\]

(4)

Suppose \(O\) is a given point of the global reference frame \(\{R_0\}\), the expression of relative translation vector \(\varepsilon_{O}\) at point \(O\) can be derived by using a linearization of the relative rotations, from which, one obtains the transfer expression of the torsor:

\[
[\tau_{PP}]_{(O,R)} = \begin{bmatrix} R_{0,i} \cdot \varphi \\ R_{0,i} \cdot (\varepsilon_{O} + (R_{0,i}^T \cdot \overrightarrow{O_0i}) \times \varphi) \end{bmatrix}
\]

(5)

where \(R_{0,i}\) is the rotation matrix from \(\{R_0\}\) to \(\{R_i\}\), \(R_{0,i}^T\) is the transposed matrix of \(R_{0,i}\), and \(\overrightarrow{O_0i}\) is the translation vector from \(\{R_0\}\) to \(\{R_i\}\) expressed in \(\{R_0\}\).

3. Modelling of variations in a machining process

In production practice, the process engineers make process plans by selecting the appropriate machining processes and production equipment to ensure the functional requirement of machined parts. Considering that a process plan consists of several different set-ups, for each set-up there are several SDTs need to be defined as follows:

- **For machined part**

  \(\tau_{PP}^{S_i}\) : global SDT characterizes the positioning variations of the machined part with respect to its nominal position in set-up \(S_i\).

  \(\tau_{PP}^{S_k}\) : deviation torsor of the machined surface \(P_i\) with regards to its nominal position in set-up \(S_k\) (variations of the machining operation).

  \(\tau_{PP}^{P}\) : deviation torsor expresses the orientation and position variations of surface \(P_i\) relatively to its nominal position on part \(P\). Suppose surface \(P_i\) is manufactured in set-up \(S_k\), this deviation torsor can be calculated as:

  \[
  \tau_{PP}^{P} = \tau_{PP}^{S_k} - \tau_{PP}^{S_i}
  \]

  (6)

- **For part-holder**

  \(\tau_{HR}^{S_i}\) : global SDT of part-holder \(H\) relatively to its nominal position in set-up \(S_k\).
\( \tau_{H/H}^S \): deviation torsor expresses the geometric variations of surface \( H_i \) relatively to its nominal position on part-holder in set-up \( k \).

- For machining operation
  \( \tau_{M/R}^S \): torsor characterizes variations of a machine tool relatively to its nominal position in set-up \( S_k \).
  \( \tau_{M/M}^S \): deviation torsor expresses the geometric variations of surface \( M_i \) relatively to its nominal position on the machining operation \( M \) in set-up \( S_k \).

- For interface of part and part-holder
  \( \tau_{P/H}^S \): gap torsor that expresses the variations of the interface between surface \( P_i \) of the machined part and the corresponding surface \( H_i \) of the part-holder in set-up \( k \). Given that the parts do not interpenetrate at the contacts, each fixed component of the torsors is regarded as nil.

The main link types of the interfaces between the machined surfaces and the positioning surfaces in manufacturing operations and corresponding torsors are list in table 1. The undetermined components in the link torsors are denoted by “\( U \)”, which show the components that leave the characteristic invariance of corresponding direction.

| Link types | Coordinate system | Link torsors | Link types | Coordinate system | Link torsors |
|------------|------------------|--------------|------------|------------------|--------------|
| Planar     |                  | \( \alpha \ U \) \n\( \beta \ U \) \n\( \gamma \ w \) | Edge Slider |                 | \( U \ U \) \n\( \beta \ U \) \n\( U \ w \) |
| Prismatic  |                  | \( \alpha \ U \) \n\( \beta \ v \) \n\( \gamma \ w \) | Point Slider |                 | \( U \ U \) \n\( \beta \ U \) \n\( U \ w \) |
| Cylindrical|                  | \( U \ U \) \n\( \beta \ v \) \n\( \gamma \ w \) | Cylindrical Slider |                 | \( U \ U \) \n\( \beta \ U \) \n\( U \ w \) |
| Spherical  |                  | \( U \ u \) \n\( U \ v \) \n\( U \ w \) | Spherical Slider |                 | \( U \ U \) \n\( U \ U \) \n\( U \ w \) |

The developed manufacturing model regards each manufacturing set-up as a mechanism consisting of a machine tool, several manufacturing operations, a machined part and its surface, and a part-holder and its surface. As shown in figure 1, for any set of two interacting surfaces \((P_i, H_i)\), we have:

\[
\tau_{P/H_i} = \tau_{P/P}^S + \tau_{P/R}^S \tau_{R/H}^S \tau_{H/H_i}
\]  

(7)

![Figure 1. Various torsors of contact surfaces.](image)
4. Three-dimensional tolerance transfer
Tolerance transfer refers to as tolerance analysis and synthesis in process planning. The purpose of tolerance analysis is to accumulate the effects of various variations in machining operation to determine if each functional tolerance of the part will be respected. Tolerance synthesis is to obtain more loose machining tolerances in each set-up while respecting the functional tolerances of the part. Torsor \( \mathbf{\tau}_{P_{i}/P_a} \) can represent the functional tolerance between two machined surfaces \( P_a \) and \( P_b \) of part \( P \). It can be expressed by the equation:

\[
\mathbf{\tau}_{P_{i}/P_a} = \mathbf{\tau}_{P_{i}/P} + \mathbf{\tau}_{P/P_a} = \mathbf{\tau}_{P_{i}/P} - \mathbf{\tau}_{P_{i}/P_a}
\]

Suppose surfaces \( P_a \) and \( P_b \) are machined in set-ups \( S_1 \) and \( S_2 \) respectively, according to equations (6) and (7), equation (8) becomes:

\[
\mathbf{\tau}_{P_{i}/P_a} = (\mathbf{\tau}^{S_1}_{P_{i}/R} + \mathbf{\tau}^{S_2}_{P/P} - \mathbf{\tau}^{S_2}_{P_{i}/R} - \mathbf{\tau}^{S_1}_{P_{i}/R} - (\mathbf{\tau}^{S_1}_{P_{i}/R} + \mathbf{\tau}^{S_2}_{P/P} - \mathbf{\tau}^{S_2}_{P_{i}/R} - \mathbf{\tau}^{S_1}_{P_{i}/R})
\]

Thus, the relationship between functional tolerances and various variations in each process planning can be established. This relationship can guide process engineers to perform manufacturing tolerance analysis and synthesis.

5. Illustrative example
A mechanical part shown in figure 2 is used to demonstrate the proposed method. The functional requirements being studied are a location and a parallelism of plane \( P_7 \) with respect to datum \( A \) on plane \( P_1 \). Figures 3-5 show the machining process of this part, which consists of three set-ups performed on a numerical control (NC) machine-tool.

5.1 Set-up 10
As shown in figure 3, the machined surfaces are marked as \( P_a \), the local reference frame \( R_i \) \( (O_i, x_i, y_i, z_i) \) for each machined surface is defined as: Axis \( z_i \) is normal to surface \( P_i \) pointing towards the outside of the entity; Origin \( O_i \), axes \( x_i \) and \( y_i \) of the reference frame belong to surface \( P_i \).

In this set-up, all surfaces are primarily machined, that is, there are no positioning surfaces. In the global reference frame \( R_0(O, x_0, y_0, z_0) \) of figure 3, we consider: \( \{\mathbf{\tau}_{P_{i}/R_i}\}_{(O,R_0)} = \{0 \quad 0\} \).

And:

\[
\mathbf{\tau}_{P_{i}/P} = \mathbf{\tau}_{P_{i}/R} + \mathbf{\tau}_{P/R} = \mathbf{\tau}_{P_{i}/P}.
\]

So we have:

\[
\{\mathbf{\tau}_{P_{i}/P}_{(O,R)}\} = \begin{bmatrix}
\alpha_p & U_p \\
\beta_p & U_p \\
U_p & w_i
\end{bmatrix}, \quad i \in \{1, 2, \ldots, 6\}
\]

4
5.2 Set-up 20
In this set-up, surface 1 of the part is machined. The machined part is positioned on a plane (main positioning surface \(H_P/H_B\)), a cylinder (2nd positioning surface \(H_B/P_2\)) and a sphere (3rd positioning surface \(H_B/P_6\)) in an isostatic set-up as shown in figure 4. Here, the part-holder support points are also marked as \(O_i\) in a local frame \(R_i\) \((O_i, x_i, y_i, z_i)\), \(i \in \{2, 4, 6\}\).

The global SDT \(\tau_{P/R}^{20}\) of the machined part can be obtained by combining the torsors associated with joints between the part and the part-holder. In a local frame \(R_i\) \((O_i, x_i, y_i, z_i)\), \(i \in \{2, 4, 6\}\), suppose the global variations of part-holder \(H\) are integrated within the deviation torsor of surface \(H_i\) relative to its nominal position in this set up for the main positioning surface \(H_s\), and combine with the expression of the torsor matrices in table 1, we have:

\[
\tau_{H_i/H_s}^{20}(O_i, x_i) = \begin{bmatrix}
0_{H_i} \\
B_{H_i}^{20} \\
V_{H_i}^{20} \\
U_{H}^{20}
\end{bmatrix}
\quad \text{and} \quad
\tau_{H_i/H_s}^{20}(O_i, x_i) = \begin{bmatrix}
U_{H}^{20} \\
V_{H_i}^{20} + r U_H \\
W_{H_i}^{20}
\end{bmatrix}
\]

where the 2nd positioning surface \(H_2\) is a cylindrical surface with radius \(r_2\), the 3rd positioning surface \(H_6\) is a spherical surface whose radius is \(r_6\).

\[\text{Figure 4. Set-up 20: Milling of surface } P_1.\]

\[\text{Figure 5. Set-up 30: Milling of surface } P_7.\]

Then we can calculate the torsors \(\tau_{P,P_i}^{20}(O_{i,R_i})\) and \(\tau_{H_i/H_s}^{20}(O_{i,R_i})\) \(i \in \{2, 4, 6\}\) at point O expressed in the global reference frame \(R_0\) \((O, x_0, y_0, z_0)\) by using equation (5), suppose the origin coordinate of the local reference frame \(R_i\) is \((a_i, b_i, c_i)\), \(i \in \{2, 4, 6\}\) (see figure 3), the rotation matrices \(R_{0,i}\) from \(\{R_i\}\) to \(\{R_0\}\) respectively are:

\[
R_{0,i} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \quad \text{and} \quad R_{0,i} = \begin{bmatrix}
0 & 0 & -1 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{bmatrix}
\]

Then we can get the expressions of \(\tau_{P,P_i}^{20}(O_{i,R_i})\) and \(\tau_{H_i/H_s}^{20}(O_{i,R_i})\) \(i \in \{2, 4, 6\}\). And according to equation (7), we can get the gap SDT \(\tau_{P/H_s}^{20}(O_{i,R_i})\) at point O expressed in global reference frame \(R_0\) \((O, x_0, y_0, z_0)\).
The torsor describing the defects due to the machining operation with regards to the machine is:

\[
\begin{align*}
\{\tau_{P,H}\}_{O,R,I} &= \begin{bmatrix}
{a}_p + a^p - a^H - a^P \\
{b}_p + b^P + b^H - b^P \\
{w}_p - w^P - w^H - b^P a^H + b^P a^P + a^P b^H - a^P b^P
\end{bmatrix}
\end{align*}
\]

Using equation (17) to calculate the gap vector \( \{\tau_{P,H}\}_{O,R,I} \) at point \( O \), expressed in local reference frame \( R_i(0, x_i, y_i, z_i) \) and considering the properties (3) and (4), one gets:

\[
\{\tau_{P,H}\}_{O,R,I} = \begin{bmatrix}
{a}_p + a^p - a^H - a^P \\
{b}_p + b^P + b^H - b^P \\
{w}_p - w^P - w^H - b^P a^H + b^P a^P + a^P b^H - a^P b^P
\end{bmatrix}
\]

Considering the hierarchy of the part/part-holder positioning in an isostatic set-up, the components of \( \{\tau_{P,H}\}_{O,R,I} \) are nil because the contact between the two main positioning surfaces (\( H_2/P_2 \)) has no interpenetrating parts. One gets the following equations:

\[
\begin{align*}
{a}_p + a^p - a^H - a^P &= 0 \\
{b}_p + b^P + b^H - b^P &= 0 \\
w^P - w^P - w^H - b^P a^H + b^P a^P + a^P b^H - a^P b^P &= 0
\end{align*}
\]

Similarly, for the second and third positioning surfaces, we have:

\[
\begin{align*}
{b}_p - b^P + b^H - b^P &= 0 \\
w^P + w^P + w^H + c^P a^P + c^P a^H + a^P 2H - a^H 2H &= 0 \\
w^P - w^P - w^H - c^P a^P + c^P a^H + b^P 2P - b^P 2H &= 0
\end{align*}
\]

Then, \( \{\tau_{P,H}\}_{O,R,I} \) can be derived as:

\[
\begin{align*}
\{\tau_{P,H}\}_{O,R,I} &= \begin{bmatrix}
{a}_p + a^p - a^H + a^P \\
{b}_p - b^P + b^H - b^P \\
w^P - w^P - w^H + c^P a^P + c^P a^H + a^P b^H - a^P b^P
\end{bmatrix}
\end{align*}
\]

Using equation (17), we get:

\[
\begin{align*}
\{\tau_{P,H}\}_{O,R,I} &= \begin{bmatrix}
{b}_p - b^P + b^H - b^P \\
{w}_p + w^P + w^H + c^P a^P + c^P a^H + a^P 2H - a^H 2H \\
w^P - w^P - w^H + c^P a^P + c^P a^H + a^P b^H - a^P b^P
\end{bmatrix}
\end{align*}
\]

The torsor describing the defects due to the machining operation with regards to the machine is:
\{\tau_{\text{pr}}^{30}\}_{(O,R_i)} = \left\{ \begin{array}{c}
a_{n_i} \\ \beta_{n_i} \\ U_p \\ w_n \\
\end{array} \right\} \quad (26)

And
\{\tau_{n_i}^{30}\}_{(O,R_i)} = \left\{ \begin{array}{c}
a_{n_i} + a_{n_i} - \alpha_{n_i}^{20} - a_{n_i}^{30} \\ \beta_{n_i} - \gamma_{n_i}^{20} + \beta_{n_i}^{20} \\ U \\ w_n - v_n^{20} + c a_{n_i} - c a_{n_i}^{20} + a \beta_{n_i} + a \beta_{n_i}^{20} - c c_{n_i}^{20} + a \alpha_{n_i} + a \alpha_{n_i}^{20} + a \beta_{n_i} + a \beta_{n_i}^{20} \\
\end{array} \right\} \quad (27)

Then:
\{\tau_{p_{\text{PR}}}^{30}\}_{(O,R_i)} = \{\tau_{n_i}^{30}\}_{(O,R_i)} - \{\tau_{\text{pr}}^{30}\}_{(O,R_i)} \quad (28)

According to equation (5), for the machined surface \(P_i\) in this set-up we have:
\{\tau_{p_{\text{PR}}}^{30}\}_{(O,R_i)} = \left\{ \begin{array}{c}
a_{n_i} + a_{n_i} - \alpha_{n_i}^{20} - a_{n_i}^{30} \\ \beta_{n_i} - \gamma_{n_i}^{20} + \beta_{n_i}^{20} \\ U \\ - w_n + w_n - v_n^{20} - w_n^{20} + c a_{n_i} - c a_{n_i}^{20} + a \beta_{n_i} - a \beta_{n_i}^{20} - a \beta_{n_i} + c a_{n_i} \\
\end{array} \right\} \quad (29)

5.3 Set-up 30

In this set-up, surfaces 7 and 8 are machined. As shown in figure 5, the part is positioned on a plane (main positioning surface \(H_2/P_2\)), a cylinder (2nd positioning surface \(H_1/P_1\)) and a sphere (3rd positioning surface \(R_0/P_0\)) in an isostatic set-up. For each positioning device \(H_i\) of the part-holder in a local frame \(R_i\) (O, x, y, z), \(i \in \{1, 4, 6\}\).

Consider the main positioning surface \(H_4\), and suppose the global variations of part-holder \(H\) are integrated within the deviation torsor of surface \(H_i\) relative to its nominal position in set up 30, we have:
\{\tau_{n_i}^{30}\}_{(O,R_i)} = \{\tau_{n_i}^{20}\}_{(O,R_i)} + \{\tau_{n_i}^{30}\}_{(O,R_i)} \quad (30)

Suppose the origin coordinate of the local reference frame \(R_i\) is \(O_i\) (a, b, c), \(i \in \{1, 4, 6\}\) (see figure 3), we can calculate the torsors \{\tau_{p_{\text{PR}}}^{30}\}_{(O,R_i)}\) and \{\tau_{n_i}^{30}\}_{(O,R_i)}\) at point \(O\) expressed in the global reference frame \(R_0\) (O, x, y, z) by using equation (5). According to equation (7), we can get the gap SDT \{\tau_{p_{\text{PR}}}^{30}\}_{(O,R_i)}\) at point \(O\) expressed in global reference frame \(R_0\) (O, x, y, z). Using equation (17) to calculate the gap SDT \{\tau_{p_{\text{PR}}}^{30}\}_{(O,R_i)}\) at point \(O\) expressed in reference frame \(R_i\) (O, x, y, z). So, \{\tau_{p_{\text{PR}}}^{30}\}_{(O,R_i)}\) can be derived as:
\{\tau_{p_{\text{PR}}}^{30}\}_{(O,R_i)} = \left\{ \begin{array}{c}
a_{n_i} + a_{n_i} - \alpha_{n_i}^{20} - a_{n_i}^{30} \\ \beta_{n_i} - \gamma_{n_i}^{20} + \beta_{n_i}^{20} \\ w_n - v_n^{20} - \alpha_{n_i}^{20} + c a_{n_i} - c a_{n_i}^{20} + a \beta_{n_i} - a \beta_{n_i}^{20} + a \beta_{n_i} + a \beta_{n_i}^{20} + c a_{n_i}^{20} + a \alpha_{n_i} + a \alpha_{n_i}^{20} + a \beta_{n_i} + a \beta_{n_i}^{20} \\
\end{array} \right\} \quad (31)

And \{\tau_{p_{\text{PR}}}^{30}\}_{(O,R_i)} = \{\tau_{p_{\text{PR}}}^{30}\}_{(O,R_i)} - \{\tau_{p_{\text{PR}}}^{30}\}_{(O,R_i)} \quad (32)

For the machined surface \(P_i\) in set-up 30 one gets:
\{\tau_{p_{\text{PR}}}^{30}\}_{(O,R_i)} = \left\{ \begin{array}{c}
a_{n_i} + a_{n_i} - \alpha_{n_i}^{20} - a_{n_i}^{30} \\ \beta_{n_i} - \gamma_{n_i}^{20} + \beta_{n_i}^{20} \\ U_p - a \beta_{n_i}^{20} + c a_{n_i}^{20} - w_n^{20} - w_n^{20} - \alpha_{n_i}^{20} - c a_{n_i}^{20} + a \beta_{n_i} - a \beta_{n_i}^{20} - a \beta_{n_i}^{20} + a \beta_{n_i}^{20} + c a_{n_i}^{20} + a \alpha_{n_i} + a \alpha_{n_i}^{20} + a \beta_{n_i} + a \beta_{n_i}^{20} \\
\end{array} \right\} \quad (33)

5.4 Respect of the functional tolerances

In this section, we establish the relations between the functional tolerances and the variations in a machining process to verify the validity of the manufacturing process.
Consider the parallelism surface requirement between surface $P_1$ and $P_7$. The variation torsor of surface $P_7$ relative to surface $P_1$ (datum surface) is:

$$
\{ \tau_{P_7;P_1}^{U \alpha \beta \gamma} \}_{(o,n)} = \left\{ \begin{array}{c}
 \alpha_x a_1 + \alpha_y a_2 + \alpha_z a_3 \\
 \beta_x a_4 + \beta_y a_5 + \beta_z a_6 \\
 \gamma_x a_7 + \gamma_y a_8 + \gamma_z a_9 \\
 \end{array} \right\} (34)
$$

The variation between surface $P_7$ and its nominal position related to the datum plane $P_1$ is defined by the displacement of any point of $P_7$ compared to the corresponding point of $P_1$. And this variation only depends on rotation variations, which can be calculated as:

$$
30(-\alpha_6^{30} + \alpha_2^{30} + \alpha_7^{30} - \alpha_1^{30}) - 80(\beta_6^{30} + \gamma_6^{30} + \beta_1^{30}) \leq 0.02
$$

Consider the location tolerance requirement between surface $P_1$ and $P_7$. The tolerance requirement being studied is a location of plane $P_7$ with respect to datum $A$ on plane $P_1$. Unlike the perpendicularity tolerance requirement, the geometrical constraint of location tolerance depends on both rotation and translation variations. In order to respect the location tolerance requirement, the following equations should be satisfied:

$$
|w_x^{30} + a_1^{30} - c_1 a_4^{30} - \gamma_x^{30} + w_y^{30} + c_1 a_5^{30} + a_7^{30} - c_1 a_6^{30} + 30(-\alpha_6^{30} + \alpha_2^{30} + \alpha_7^{30} - \alpha_1^{30}) - 80(\beta_6^{30} + \gamma_6^{30} + \beta_1^{30})| \leq 0.015
$$

6. Conclusions

In the context of tolerance transfer in process planning, our interest focuses onto 3D modeling of manufacturing deviations. The proposed model uses small displacement torsors to represent the geometrical deviations induced by the machining operations and positioning dispersions during the successive machining set-ups, and considers each manufacturing set-up as a single mechanism. Thus, the influence of the process planning on the respect of functional tolerances can be modeled as a chain of small displacement torsors. After having obtained all SDT chains according to set-ups and functional tolerances, we can identify which surfaces and which set-up affect the compliance with functional tolerances. Based on the SDT chains, one can evaluate the effect of these variations on the parts and check whether functional tolerances are respected, or determine the manufacturing tolerances according to the functional tolerances. An example proves the validity of the proposed method.

Under the environment of integrated design and manufacturing, our future research works will focus on: The mathematical modeling of 3D manufacturing variations for integrating product and manufacturing process design; the development of the methods and tools for manufacturing tolerance analysis and synthesis in multi-stage manufacturing processes.

7. References

[1] Qin Y, Lu W, Qi Q, Liu X, Huang M, Scott P and Jiang X 2018 Towards an ontology-supported case-based reasoning approach for computer-aided tolerance specification Knowl-Based Syst. 141 pp 129-147

[2] Thimm G, Wang R and Ma Y 2007 Tolerance transfer in sheet metal forming Int. J. Prod. Res. 45 pp 3289-3309

[3] Jaballi K, Bellacchio A, Louati J, Riviere A and Haddar M 2011 Rational method for 3D manufacturing tolerancing synthesis based on the TTRS approach R3DMTSyn Comput. Ind. 62 pp 541-554

[4] Jiang K, Davidson J K, Liu J and Shah J J 2014 Using tolerance maps to validate machining tolerances for transfer of cylindrical datum in manufacturing process Int. J. Adv. Manuf. Technol. 73 pp 465-478

[5] Villeneuve F, Lego O and Landon Y 2001 Tolerancing for manufacturing: a three-dimensional model Int. J. Prod. Res. 39 pp 1625-1648

[6] Vignat F and Villeneuve F 2003 3D transfer of tolerances using a SDT approach: application to turning process J. Comput. Inf. Sci. Eng. 3 pp 45-53
[7] Villeneuve F and Vignat F 2003 3D synthesis of manufacturing tolerances using a SDT approach
*The 8th CIRP International Seminar on Computer Aided Tolerancing*, Charlotte, North Carolina, *41* pp 279-290

[8] Villeneuve F and Vignat F 2005 Manufacturing process simulation for tolerance analysis and synthesis In: Bramley A, Brissaud D et al (eds) *Advances in integrated design and manufacturing in mechanical engineering*. Presented at the 5th International Conference on Integrated Design and Manufacturing in Mechanical Engineering, Bath, England, New York: Springer, pp 189-200

[9] Peng H P and Lu W L 2018 Modeling of Geometric Variations within Three-dimensional Tolerance Zones *Journal of Harbin Institute of Technology (New Series)* *25* pp 41-49

[10] Teissandier D, Couëtard Y and Gérard A 1999 A computer aided tolerancing model: proportioned assembly clearance volume *Comput.-Aided. Des.* *31* pp 805-817

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