Detection limits for super-Hubble suppression of causal fluctuations

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We investigate to what extent future microwave background experiments might be able to detect a suppression of fluctuation power on large scales in flat and open universe models. Such suppression would arise if fluctuations are generated by causal processes, and a measurement of a small suppression scale would be problematic for inflation models, but consistent with many defect models. More speculatively, a measurement of a suppression scale of the order of the present Hubble radius could provide independent evidence for a fine-tuned inflation model leading to a low-density universe. We find that, depending on the primordial power spectrum, a suppression scale modestly larger than the visible horizon can be detected, but that the detectability drops very rapidly with increasing scale. For models with two periods of inflation, there is essentially no possibility of detecting a causal suppression scale.

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I. INTRODUCTION

If we assume that fluctuations in the Universe were created by causal processes, then an unambiguous feature of the power spectrum is the maximum scale over which such fluctuations are correlated. In principle this scale is a characteristic of all density fields in the Universe, matter, cosmic microwave background radiation (CMB) scalar and tensor fluctuations, primordial magnetic fields etc. However, the effects of this scale are generally difficult to detect, as they inevitably occur on a very large scale. Indeed, in inflationary models, the scale may exceed the Hubble length $1/H$, and we will refer to this scale as the super-Hubble suppression scale.

Efforts to understand the super-Hubble scale primarily have focused on the CMB scalar perturbation power spectrum. For scalar perturbations, as well as any matter perturbations, if they are created by local causal processes, it is well known that causality requires the power spectrum to diminish at least as fast as $k^4$ at small $k$. The argument for this was first put forth by [1] and over the years it has been refined [2–5].

The advent of inflationary cosmology has placed specific focus on the size of the super-Hubble scale, in particular relative to the Hubble distance $1/H$ (we set $c = 1$ throughout). Whereas the Hubble distance characterizes the maximum distance over which causal interactions can occur during one expansion time, in distinction, the super-Hubble length scale $\lambda_H$ characterizes the absolute largest scale over the lifetime of the Universe, on which causal interactions can seed density perturbations. In particular the super-Hubble scale accounts for the growth of all local perturbations since their conception up to the present due to the expansion of the universe. For example, in a flat universe, a perturbation of the maximum causal physical length created at time $t_i$ expands to a physical scale at the present time $t_0$

$$\lambda(t_0; t_i) = \frac{R(t_0)}{R(t_i)} \int_{t_i}^{t_0} \frac{dt'}{R(t')} ,$$

(1)

where $R(t)$ is the cosmic scale factor. Depending on the behaviour of the cosmological scale factor $R(t)$, this can exceed the present Hubble radius $1/H_0$. In the regime of Standard Hot Big Bang cosmology $R(t) \sim t^{1/2}$ and $t^{2/3}$ in the radiation- and matter-dominated regimes respectively. On the other hand, inflation is a regime in which the Hubble parameter $H \equiv \dot{R}(t)/R(t)$ is approximately constant and the scale factor is accelerating, $\ddot{R}(t) > 0$, with generically a quasi-exponential growth. For example, if at $t_i$ during inflation a local perturbation is created at scale $1/H$, by the end of inflation at $t_f$ it will imply a super-Hubble scale of $\lambda(t_f) \approx (1/H) \exp[H(t_f - t_i)]$. Thus immediately after even a few $e-$foldings of inflation, $\lambda \gg 1/H$.

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Outside the inflation regime, the causal horizon in the standard Big-Bang regime grows in physical units as $\lambda \sim t$. In principle this horizon eventually will grow bigger than the one set by inflation, which in physical units grows as $\lambda R(t)/R(t_f) \sim t^a\lambda$ where $a \sim 1/2$ during radiation-domination and $a \sim 2/3$ during matter-domination. However the generic prediction of most inflation models is that $\lambda(t_0) \gg \eta_0 / 2H_0$, where $\eta_0$ is the physical distance from last scattering or equivalently the causal horizon for events generated since last scattering.

Thus, the generic expectation for density perturbations produced solely within non-inflationary regimes is $\lambda \sim 1/H_0$, whereas isentropic inflation models $[6–10]$ generally predict $\lambda > 1/H_0$ and non-isentropic inflation models $[11,12]$ are fairly impartial to any particular scale between these two limits. As such, the super-Hubble scale, in principle, is an ideal parameter for discriminating between inflationary and non-inflationary models of density perturbations. However the practical limitation is that the super-Hubble scale can be measured accurately only if it no greater than about $1/H_0$. Thus at best one hopes to distinguish from the data whether $\lambda \approx 1/H_0$ or $\lambda \ll 1/H_0$. If such a measurement could be reliably made, it would be far-reaching: evidence for a maximum causal scale which was smaller or of order the Hubble radius would not be expected in inflation, but could be produced by, say, topological defect models generating fluctuations causally after the Big Bang. A more speculative possibility is that inflation produced only just enough e-foldings to solve the horizon problem, but not enough to drive the present Universe to flatness. The fine-tuning required for this is problematical, but measurement of a super-Hubble scale modestly larger than $1/H_0$ could provide evidence, independent of a measurement of $\Omega_0$, for such non-flat inflation.

For the Cosmic Background Explorer experiment (COBE), a likelihood fit to the four-year data with respect to the index, amplitude and super-Hubble suppression scale was done in $[13]$. Their results interestingly preferred a super-Hubble suppression scale $\lambda \approx 4/H_0$ (or $k_{\min} \eta_0 \sim 3.5$). Since the preference was within one-sigma of an infinite suppression scale, the conclusion of their analysis was that no super-Hubble suppression scale $\lambda > 1/H_0$ was excluded. However, the fact that this analysis did not give a clear-cut measurement of an infinite suppression scale leaves some margin of doubt. Due to the importance that detection of a small suppression scale would have in discriminating cosmological models, it is advantageous at this stage to explore the possibilities for measuring this scale.

What are the possibilities of improving on this measurement with new experiments? The first point is that since the signal for super-Hubble suppression is confined to low-order multipoles in the CMB signal, instrumental noise is not the dominant error, and the higher signal-to-noise of MAP and Planck will give little advantage over COBE. However, there are still two advantages. One is that the CMB power spectrum at low multipoles is determined not just by the suppression scale, but other parameters as well: the newer experiments will determine the other parameters rather precisely, thus reducing the marginal error on the suppression scale. Second, the better frequency coverage of the later experiments (especially Planck) should allow better control of systematic errors, especially through effective removal of foreground sources. For the quadrupole, the successful removal of as much foreground emission from the Milky Way galaxy itself will be essential; the systematic uncertainty in the COBE quadrupole is about 70% of its measured value $[14]$. This very large uncertainty is serious for measurement of the super-Hubble suppression scale. On the one hand, this systematic uncertainty could fully account for the suppressed quadrupole found by COBE and thus cast doubt on the claimed measurement in $[3]$. On the other hand, this large uncertainty implies the size of the super-Hubble suppression scale remains an open question and conceivably it may be small enough to be detected.

In this paper, we take an optimistic view that foreground subtraction and parameter estimation (other than of the suppression scale) can be performed precisely, and calculate the error with which the suppression scale could be determined from CMB measurements. We do this via a likelihood analysis (Section II). Section III explores how an open inflation model, leading to $\Omega_0 < 1$, can lead to a measurably small super-Hubble scale, which might provide independent support for such a model. In Section IV we explore the theoretical consequences of a super-Hubble scale suppression, and draw our conclusions in section V.

II. $k_{\min}$ SUPER-HUBBLE DETECTION: FLAT UNIVERSE

In this section we compute the errors expected on a measurement of the suppression scale in a flat universe. For a large suppression scale, the signal will come from the low-order multipoles in the CMB, which are dominated by the Sachs-Wolfe effect. The power spectrum of scalar temperature fluctuations is $C_\ell \equiv \langle |a_\ell^m|^2 \rangle$, where $a_\ell^m \equiv \int d\Omega \Delta T(\Omega)/T$ and $T(\Omega)$ is the temperature on the sky. For low $\ell$ the power spectrum may be written in terms of the mass-density power spectrum $P(k)$ (e.g. $[15]$):

$$C_\ell = \frac{4\pi}{25} \int_0^\infty \frac{dk}{k^2} j_\ell^2(\eta_0 k) P(k)$$

where $j_\ell$ is a spherical Bessel function, and $\eta_0$ is the coordinate distance of the last scattering surface, approximately $2/H_0$ (aside from factors of $c$, we use dimensional units).

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For the power spectrum, we assume minimal suppression as suggested by \[1\] and a primordial power-law spectrum:

\[
P(k) \propto \frac{k^n}{1 + \left( \frac{k}{k_0} \right)^{n-4}}.
\] (3)

We assume \(n = 1\) in what follows. \[3\] find that \(k_0 \simeq 2.24/D\), where \(D\) is the scale beyond which there are no mass-density correlations.

Assuming all-sky coverage, estimates of \(C_\ell\) are uncorrelated, and we can compute the probability of \(k_0\) given the data as a product of individual likelihoods (assuming a uniform prior for \(k_0\)). To estimate the error, and in this section only, we assume the likelihood is gaussian (valid for high \(\ell\), but not particularly accurate for low multipoles), and compute the Fisher information matrix (scalar for a single parameter) \(F = -\langle \partial^2 \ln L / \partial k_0^2 \rangle\). The error on \(k_0\) is \(\sigma_{k_0} = F^{-1/2}\). Up to an additive constant, \[
\ln L(k_0) \simeq -\sum \ln \sigma_\ell - \frac{1}{2} \sum (\hat{C}_\ell - C_\ell(k_0))^2 / \sigma^2_\ell\] (4)

where \(\hat{C}_\ell\) are the observed values, \(C_\ell(k_0)\) is the theoretical value for given \(k_0\), and \(\sigma^2_\ell = 2C^2_\ell / (2\ell + 1)\) is the variance assuming the temperature map is a gaussian random field. The error on \(k_0\) from each multipole is then simply obtained from the Fisher matrix:

\[
\sigma_{k_0} = \frac{C_\ell}{\sqrt{\ell + 1/2}|C|}\] (5)

where \(C'_\ell \equiv \partial C_\ell / \partial k_0\), and, in the case of all-sky coverage, the estimators of \(k_0\) may be close to independent and can optimally be combined with inverse-variance weighting.

FIG. 1. Fisher estimate of the error on \(k_0\), for individual multipoles from \(\ell = 2 \ldots 5\), from left to right. The model assumes a sharp cutoff in the power spectrum at \(k_0\). \(\eta_0\) is the coordinate of the visible horizon. \(\eta_0 = 2\) for an Einstein-de Sitter universe.

The contribution to the error from the low multipoles is illustrated in Fig. 1 for a model where we assume the limit of causal processes introduces an abrupt cutoff in \(P(k)\). We see, as expected, that the ability to detect the suppression scale is a very sensitive function of the suppression scale itself. There is a rather sharp threshold of detectability at \(k\eta_0 = 4\). Note that for flat models with positive \(\Lambda\), the detection of a given physical suppression scale is easier, since \(\eta_0\) is larger than for an Einstein-de Sitter model. No analytic expression exists for \(\eta_0\) in flat models with non-zero \(\Omega\Lambda\) and \(\Omega_0\): \(\eta_0\) rises from \(2/H_0\) when \(\Omega_\Lambda = 0\) to \(3.4/H_0\) when \(\Omega_\Lambda = 0.7\).

This illustrative calculation is approximate because it assumes a gaussian likelihood, and secondly, the likelihood surface is not well-approximated by a gaussian near the peak. Consequently, we compute from now the full likelihood curve, assuming the proper probability distribution for each multipole:

\[
\ln L(k_0) = \sum \ln P(\hat{C}_\ell|C_\ell(k_0))\] (6)

where \(P(\hat{C}|C)\) is a \(\chi^2\) distribution with \(2\ell + 1\) degrees of freedom. Note, however, that the Fisher method, which approximates \(\ln(L)\) as parabolic around the peak, may not be very accurate, since the likelihood is badly approximated by a gaussian if the signal-to-noise is moderate, with a significant probability tail extending to \(k_0 = 0\). This is explored further in section \[\text{III}\].
This case is more interesting theoretically. As the Universe inflates, it is driven towards flatness. In addition, the particle horizon is inflated to larger physical scales. It is possible that inflation can produce a low-density universe, but only with a degree of fine-tuning in current models. Thus observations implying a low density are considered problematic for inflation. However, the modest inflation of the particle horizon offers the possibility of an independent check, as the relatively small super-Hubble scale in this case may be detectable. In this section, we explore this possibility.

We follow in part the notation of [16], and make the following assumptions: perturbations are generated by causal processes up to the particle horizon size; the Universe is radiation-dominated prior to inflation; the scale factor increases by a factor $10^2$ during inflation, and by $10^{27-\Delta}$ after inflation. $\Delta$ is set by the physics of inflation and for most models lies near zero. As argued in [16], such a model can account for any present-day density parameter, depending on the pre-inflation density parameter $\Omega_i$ and the amount of inflation:

$$1 - \Omega_0 = E^2(1 - \Omega_i)$$  \hspace{1cm} (7)

where $E = 10^{27-\Delta-2}\Omega_i^{1/2}$ is the fractional change in $HR$ between the onset of inflation and the present day (assuming zero cosmological constant after inflation). $\Omega_i = 4.22 \times 10^{-5}h^{-2}$ is the present-day radiation density parameter, written in terms of the Hubble parameter $h \equiv H_0/(100\text{km s}^{-1}\text{Mpc}^{-1})$. Thus, for a given $\Omega_i$ and particle physics model (via $\Delta$), [6] gives us the present-day (open) density parameter in terms of the number of inflation 10-foldings.

The suppression scale is also related to the number of 10-foldings: the largest physical scale which is causally-connected is the immediate pre-inflation particle horizon, expanded to the present-day. This scale is

$$D = \frac{R(t_0)}{R(t_i)} \int_{t_i}^{t_0} \frac{dt}{R(t)}$$  \hspace{1cm} (8)

which for radiation-domination prior to inflation, is

$$D = \frac{E}{H_0} \cosh^{-1} \left( \Omega_i^{-1/2} \right).$$  \hspace{1cm} (9)

From [6], this gives a minimal suppression wavenumber $k = 2.24/D$, which we write in terms of the distance to the last-scattering surface, $\eta_0$. We approximate this by the particle horizon assuming matter-domination throughout, $\eta_0 = \text{arccosh}(2/\Omega_0 - 1)$. This is plotted in Fig. 2 for a range of pre-inflation density parameters.

![FIG. 2](image)

The suppression wavenumber $k_0$ in length units of the distance to the last-scattering surface $\eta_0$, as a function of the present density parameter, for a range of pre-inflation density parameters, $\Omega_i$. From the top, $\Omega_i = 0.8, 0.3, 0.1, 0.01$ and 0.001. Also shown is the line showing the minimum value of $k_0\eta_0$ which can be distinguished from $k_0 = 0$ for $n = 1$ (top) and the Ratra-Peebles power spectrum (bottom). See text for details.

The CMB power spectrum in open models is related to the matter spatial power spectrum $P_R$ (defined in a non-standard way; $P_R$ = constant is a scale-invariant spectrum) by [17] (see also [18], [19]):

$$C_\ell(k_0) = 2\pi^2 \int_0^\infty \frac{dk}{k} P_R(k) I_{kl}^2$$  \hspace{1cm} (10)
The kernel $I_{kl}$ depends on a number of functions defined in [17] and [18], and includes Sachs-Wolfe and Integrated Sachs-Wolfe effects. We mention one practical issue: one of these sets of functions ($\tilde{\Pi}_{kl}$) was computed using the recurrence relation (15) in [17]. For small $r$ the recurrence relation is unstable, and we instead used the asymptotic form $\tilde{\Pi}_{kl} \to A_l(k)r^l$, computing the coefficients using the recurrence relation in the limit $r \to 0$. The kernel functions $I_{kl}$ are shown in Fig. 3 for $l = 2$ to 9. Note the presence of some supercurvature modes ($k < 1$) in these models.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{kernel_functions.png}
\caption{Kernel functions giving the contribution to multipoles $C_\ell$; $\ell = 2, \ldots, 9$ (from left to right) from power at wavenumber $k$, for an Open Universe with $\Omega_0 = 0.2$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{likelihood.png}
\caption{Likelihood for the suppression wavenumber $k_0$, from individual multipoles $C_\ell$; $\ell = 2, \ldots, 9$ (from bottom to top), and the combined likelihood (dashed), from a suppressed scale-invariant power spectrum, for an Open Universe with $\Omega_0 = 0.2$. The true $k_0$ is marked by the dotted line. Clearly in this case there is still some information from higher multipoles than $\ell = 9$.}
\end{figure}

In Fig. 4 we also show the detectability limits for the suppression wavenumber, for two choices of the primordial power spectrum, a scale-invariant spectrum (top) and the power spectrum proposed for open universes (bottom) by [20]: $P(q) \propto (4 + q^2)^2/|q|(1 + q^2)$ - see [19]. The lines show the boundary above which the value of $k_0\eta_0$ for which the relative likelihood for $k_0 = 0$ (compared with the true value) is less than $e^{-4} = 0.018$ (see Fig. 4 for an example of the full likelihood curve). This illustrates that, to a first approximation, the detectability limit is determined by the present conformal time: $\lambda \approx \eta_0$. We see that it is, in principle, possible to detect the suppression scale in such an inflationary model, provided $\Omega_i$ is large enough for a given $\Omega_0$. For example, we require $\Omega_i > 0.3$ for $\Omega_0 = 0.1$ and a scale-invariant spectrum. Note also that, roughly, the detectability scale for the scale-invariant power spectrum is the visible horizon size, i.e. $2\pi/k_0 \simeq \eta_0$; in particular, it is possible to detect a cutoff on scales larger than the curvature radius in low-density models.

\section*{IV. THEORETICAL CONSEQUENCES}

Any theoretical model has an implied super-Hubble scale. In most models, it also is assumed that density perturbations are produced from an initially smooth background, so that beyond the super-Hubble scale the power is
suppressed. The analysis in the earlier sections is applicable for such types of models. It is worth mentioning that alternative possibilities are conceivable beyond the super-Hubble scale, such as the opposite case of a highly enhanced power spectrum and, in principle, any possibility between these two limits. Similar analyses as in the previous sections could be done for other behaviour besides suppression beyond the super-Hubble scale. Nevertheless, the analysis in the previous sections is more sensitive to detecting a disparate change in the power spectrum at large scale rather than to the precise nature of this disparity. Indeed, for large enhancements of power on super-Hubble scales, detection is much easier [21]. Thus, we believe detection limits found in the earlier sections for the super-Hubble scale are generic limits, fairly insensitive to the nature of the power beyond this scale. A conclusive test of this belief would be to fully invert the power spectrum and determine the intrinsic measurability limits imposed by cosmic variance, as suggested by [22]. However, this will be left for future work.

Models of primordial density perturbations can be generally classified as either inflationary or non-inflationary. Inflationary models are further differentiated between isentropic expansion which results in a supercooled inflationary regime and non-isentropic expansion which results in a warm inflationary regime. For inflationary models, primordial density perturbations are produced from quantum fluctuations in supercooled inflation [23, 24] and thermal fluctuations in warm inflation [20, 21]. In non-inflationary models, primordial density fluctuations emerge during the radiation- and/or matter-dominated regimes from various possible particle physics mechanisms such as cosmic strings [25, 26, 27] and late-time phase transitions [30, 31].

In comparing supercooled inflation models to non-inflation models, the prediction for the super-Hubble scale is distinctively different. For supercooled inflation models, the super-Hubble scale generically is predicted to be many orders of magnitude larger than the Hubble radius so that effectively \( k_{\text{min}} \approx 0 \) (there are some supercooled inflation models that are exceptions, in which a small number of e-folds can occur [30, 32]). In contrast, for non-inflationary models, the causal horizon is about the same size as the present-day Hubble radius, thus \( k_{\text{min}} \sim H_0 \) in these models. Specifically, in an analysis based on general arguments found that the largest length scale on which sub-Hubble scale perturbations can generate significant power is \( \lambda \lesssim 3 H_0 \) which corresponds to \( k_{\text{min}} H_0 \lesssim 2.2 \). Particular particle physics non-inflation models of density perturbations are consistent with this general estimate. For example cosmic strings [25, 26, 27] plus cold or hot dark matter find \( k_{\text{min}} H_0 \sim 2.1 - 7.9 \) and models of late-time phase transitions [30, 31] find \( k_{\text{min}} H_0 \gtrsim 2.2 \).

The predictions for the super-Hubble scale in warm inflationary models are intermediate to supercooled inflation and non-inflation models. Unlike supercooled inflation models which generally favor a very large number of e-foldings, phenomenological warm inflation [12] and first-principles warm inflation [20, 21] models are more democratic towards any number of e-foldings. One generic feature of warm inflation, that is of potential interest for observation, is the number of e-folds of inflation are correlated with the decrease in temperature of the universe during the inflationary epoch [12]. In distinct contrast, such a correlation between the e-folds of inflation and the temperature of the universe is not present in supercooled inflation. Thus, if future data supports a small super-Hubble suppression scale, within the measurability bounds, consistency of this finding with warm inflation models could be checked through examining correlations with the post-inflation temperature. Such consistency checks would be much less robust for supercooled inflation models. Conversely, if particular post-inflation temperatures are found by independent theoretical and/or observational arguments, for warm inflation models, such arguments also would imply predictions for the super-Hubble scale. In particular, arguments favoring higher post-inflation temperatures, when applied to any given warm inflation model, also would favor smaller number of e-folds and thus a smaller super-Hubble scale. Finally, in the warm inflationary era, isocurvature perturbations also are generated due to thermal fluctuations in the radiation field [37, 38]. Since isocurvature modes do not contribute to curvature perturbations (in the comoving gauge), the contribution from isocurvature modes should be absent from super-horizon perturbations, leading to a drop in amplitude of the scalar perturbations at the horizon scale, thus imitating a super-Hubble suppression scale. Estimates of isocurvature perturbations and their relative magnitudes compared to adiabatic perturbations have been computed for a large class of warm inflation models in [38].

In the open universe case, the commonly accepted particle physics model for open-inflation is based on the Gott bubble nucleation mechanism [39, 40]. In recent times, the observational predictions from such models has been calculated to a high degree of precision [41, 42]. In this scenario there are two stages of inflation. The first is a stage of old inflation in which the universe is trapped in a metastable state. Inside this sea of inflating false vacuum, a bubble is nucleated. In the scalar field description of this scenario, the bubble nucleation process is described by an O(4) symmetric bounce solution [43, 44]. The key point is that the interior of the bubble is a homogeneous and isotropic open universe, which solves the horizon problem. Inside the bubble, initially \( \Omega \) is very close to zero. Then a second stage of inflation is pictured, this time of the slow-roll new inflation type. This inflation is fine tuned to a small number of e-folds, just enough to achieve the desired flatness of the universe with \( \Omega \sim 0.1 \).

In this picture, the purpose of the first stage of old inflation arises because bubbles typically nucleate at finite radius and not at a point. This requires that the universe should be smooth on the length scale of size the initial
bubble when it nucleates, and the first epoch of old inflation achieves this smoothness. The observable implications of this first stage of inflation are primarily through the initial fluctuations inside the bubble, and these have been demonstrated to have minimal measurable consequences in [41,42].

In these open inflation models, the behavior of the background cosmology is the same as the treatment in [11] and applied in Sect. III. to the extent of relating $\Omega_0$, $\Omega_1$ and the number of $e$-folds of inflation. We have not attempted a detailed analysis of these models, but some general remarks are appropriate. In these open inflation models, the relation of these quantities to the super-Hubble scale is not the same as in Sect. III since here two stages of inflation occur. In these models, the entire visible universe sits inside a nucleated bubble and this bubble sits inside a larger universe which was created by the first stage of old inflation. The super-Hubble scale contains this entire system and so is very large, $k_{min} \approx 0$, thus unmeasurable. However these models have an intermediate scale smaller than this super-Hubble scale and larger than the Hubble radius inside the nucleated bubble, and this is the distance to the bubble wall. There clearly are fluctuations on this scale [46], and this might be measurable by the methods of this paper. From our analysis in Sect. III, it can be inferred that any effect just slightly larger than the Hubble radius could be detected. In this respect, these open universe models do not rule out the possibility that the distance to the bubble wall may be just a little larger than the curvature scale. In this case, in the low density regime, where the curvature scale and the Hubble radius are about the same size, the bubble wall will create some sort of super-Hubble effect that is not too far outside the Hubble radius. Although we have not considered any detailed model for this situation, from the analysis in Sect. III it can be inferred that such effects may be measurable.

V. CONCLUSION

We have investigated the extent to which a large-scale suppression scale in primordial fluctuation power might be detectable in flat and open universe models. We have concentrated on observations of the microwave background radiation, as direct measurement of the suppression scale from the galaxy power spectrum is likely to be impossible, since the scale is likely to be at least a substantial fraction of the Hubble radius. The suppression itself is expected if the fluctuations arise from causal processes. The limit of detectability is dependent a little on cosmology, and on the primordial power spectrum, but it is no surprise that it is set roughly by the size of the visible horizon today.

In flat models, a positive cosmological constant makes the suppression scale easier to detect, as the visible horizon is the primordial power spectrum, but it is no surprise that it is set roughly by the size of the visible horizon today. In flat models, a positive cosmological constant makes the suppression scale easier to detect, as the visible horizon is larger. Defect models will be distinguished from inflationary models most easily by their high-$\ell$ power spectrum or by non-gaussian tests (e.g. [47]). Should inflation models be ruled out, then the suppression scale may become a useful discriminant between defect models, as it should appear on scales smaller than the visible horizon, and be readily detectable. Clearly, it would also act as a useful cross-check.

For single-inflation models, the suppression scale depends on both the final value of the matter density parameter, and also on the radiation density parameter immediately prior to inflation. For low final densities, there is the possibility of detecting a moderately small suppression scale, which would provide some sort of independent evidence on what would be a fine-tuned model. For open inflation models involving two stages of inflation, there appears to be no possibility of detection of the causal suppression scale set by the first stage of old inflation. However, as mentioned in Sect. V, we can infer from the analysis in Sect. III that any significant effect that is slightly super-Hubble scale could be detectable. In particular, if the bubble wall created in the second stage of inflation is just beyond the Hubble radius, the type of measurements we have been considering could be applied to detect it. However, we have not made any detailed analysis of this possibility.

We have considered only scalar modes here; in principle it is possible to include tensor modes to increase the detectability, but in fact the transition between a scale which is easily detectable to one which is not is so sharp that such additional information would change the results very little. We expect the suppression scale signal to be contained in the low-order multipoles, which were measured by COBE. Future experiments such as MAP and Planck will improve on COBE not principally because of better signal-to-noise, but via improved foreground subtraction through their better frequency coverage. In addition, their better resolution will allow better determination of cosmological parameters. The reduced uncertainty in other parameters will decrease the marginal error on the suppression scale. This is quantifiable in principle, but the number of cosmological parameters is large (up to $\sim 20$ [18]). It is in the better foreground subtraction that MAP and Planck should gain: the systematic uncertainty in the COBE quadrupole is comparable to the measured signal [11], large enough that the apparent suppression of the quadrupole in the COBE data [13] could be entirely due to this rather than super-Hubble suppression. Provided that foregrounds can be adequately removed and other cosmological parameters are determined accurately from the high-order power spectrum, a factor of 3 improvement in the quadrupole noise is expected, and this should allow the super-Hubble scale suppression to be estimated to the accuracy presented in this paper. If such a scale is indeed measured, then, as with other measurements, polarization measurements can be used as a consistency check.
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