Voltage dependent conductance and shot noise in quantum microconstriction with single defects.

Ye.S. Avotina(1), A. Namiranian(2), Yu.A. Kolesnichenko(1)

(1) B. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, 47 Lenin Ave., 310164 Kharkov, Ukraine.

(2) Physics Department, Iran University of Science and Technology, Narmak, 16844, Teheran, Iran.

Abstract

The influence of the interference of electron waves, which are scattered by single impurities and by a barrier on nonlinear conductance and shot noise of metallic microconstriction is studied theoretically. It is shown that these characteristics are nonmonotonic functions on the applied bias $V$. 
I. INTRODUCTION

Single defects influence strongly to physical properties of mesoscopic systems. Usually different defects arise during a manufacturing of mesoscopic conductors and an investigation of its effect to the transport characteristics has a practical significance. From other hand the study of the effect of single defects to kinetic coefficients makes it possible to obtain the most detailed information on the electron scattering processes that is important for the basic science. Point contacts and quantum microconstrictions (quantum wires) are one of the classes of mesoscopic systems, which are wide investigated both theoretically and experimentally (for review see [1, 2]). The electrical conductance $G$ of such constrictions is proportional to the number $N$ of propagating electron modes (the number of discrete energy levels $\varepsilon_n < \varepsilon_F$ of transverse quantization, $\varepsilon_F$ is the Fermi energy), each one contributing an amount of $G_0 = 2e^2/h$. The changing of the contact diameter $d$ leads to the changing of the number of occupied levels $\varepsilon_n$ and the $G(d)$ suffers a $G_0$ stepwise change. This effect is a manifestation of the quantum size effect in metals, which first predicted by Lifshits and Kosevich in 1955 [3]. The scattering processes decrease a probability $T_n < 1$ of the $n$-th mode and the conductance at zero temperature $T = 0$ and an applied voltage $V \to 0$ is described by Landauer-Buttiker formula [4, 5].

The shot noise is an important characteristic of the transport properties of mesoscopic conductors [1, 2, 6]. It is originated from the time-dependent current fluctuations. First Kulik and Omelyanchouk [7] noticed that the shot noise in ballistic contacts vanishes in the quasiclassical approximation, if there is no any electron scattering. In quantum microconstriction these fluctuations arise from the quantum mechanical probability of electrons to be transmitted through it. At $T = 0$, a bias at the contact $V \to 0$ and for low frequencies $\omega \to 0$ the shot noise is given by [1]

$$S(0) = 2eV G_0 \sum_{n=1}^{N} T_n (1 - T_n). \tag{1}$$

In perfect ballistic contacts, in which the transmission probability for every mode $T_n = 1$, the shot noise is fully suppressed. However even in adiabatic ballistic constrictions near the values of its diameter, at which the highest energy levels $\varepsilon_N$ is close to $\varepsilon_F$, the probability $T_N < 1$ [8]. According to Eq. (1) at the small bias the shot noise is the linear function of the voltage $V$. 

2
The conductance of quantum microconstriction containing different types of single defects has been investigated theoretically in the papers \cite{9-18}. The most remarkable features of manifestation of electron scattering process in mesoscopic constrictions with only few point-like defects are: (i) the effect of quantum interference between directly transmitted through the contact an electron wave and electron waves scattered by the defects and a barrier in the contact and (ii) the dependence an electron scattering amplitude on a position of the defect inside the constriction. First effect results in the nonmonotonic dependence of the point-contact conductance on the applied bias, which was observed in the experiments \cite{20, 21} and theoretically considered in the papers \cite{9, 21}. Recently new experimental observation of conductance oscillations in quantum contact was reported in Ref. \cite{22}. The second one is the reason for a size dependence of the Kondo anomaly of the quantum conductance \cite{16, 19}. This dependence is due to a non-homogeneity of the local density of the electron states across of the diameter of microconstriction. In the paper \cite{17} based on a numerical simulation the influence of ”dirty” banks on the conductance of quantum point contact was considered and authors had predicted the suppression of conductance fluctuations near the edges of the steps of the function $G(d)$. This effect have been experimentally observed in Ref.\cite{21} and explained by the decreasing of the interference terms in the conductance, if the contact diameter $d$ is closed to the jump of the $G(d)$.

The most important feature of the ballistic microconstriction is the splitting of the Fermi surface by applied voltage \cite{23}. Effectively, there are two electronic beams moving in opposite directions with energies differing at each point of the constriction by exactly the bias energy $eV$. Because of this difference of electron energies $\varepsilon \pm \frac{eV}{2}$, a value of a wave vector $k_z(\varepsilon \pm \frac{eV}{2})$ along the constriction depends on $eV$. The mentioned above the effect of quantum interference between directly transmitted and scattered waves defines by a relative phase shift $\Delta \varphi = 2k_z \Delta z$ of wave functions ($\Delta z$ is a distance between scatterers) and the dependence $k_z(\varepsilon \pm \frac{eV}{2})$ results in oscillations of transmission probabilities $T_n(V)$ as functions on $V$. In this paper we consider an influence of such effect to the conductance and shot noise in long quantum microconstrictions with few defects and the potential barrier.

The paper is organized as follows. In Sec. II the model of microconstriction and the basic equations are discussed. In Sec. III the voltage dependence of conductance and shot noise is studied. The two cases are considered: single impurity in the constriction with a barrier and two impurities in the constriction without the barrier. The expressions for Green’s function
for these cases are given. Also the results of numerical calculations are presented in this
section. We summaries our results in Sec. IV.

II. MODEL OF MICROCONSTRICTION AND FORMULATION OF THE PROBLEM

Let us consider the quantum microconstriction in the form of a long channel with smooth
boundaries and a diameter $2R$ comparable with the Fermi wavelength $\lambda_F$ (Fig.1). A length
of the channel $L$ is much larger than $R$. We assume that the channel is smoothly (over Fermi
length scale) connected with a bulk metal banks, to which the voltage $eV \ll \varepsilon_F$ is applied.
In a center of the constriction a potential barrier ($U(z) = U\delta(z)$) is situated in vicinity of
which there are few point-like defects in positions $r_i$. The Hamiltonian of the system can be
written as

$$\hat{H} = \frac{\hat{p}^2}{2m^*} + U\delta(z) + g \sum_i \delta(r - r_i), \quad (2)$$

where $\hat{p}$ and $m^*$ are a momentum operator and an effective mass of an electron, $g$ is a con-
stant of electron-impurity interaction ($g > 0$, a repulsive impurity). In a ballistic channel
without the barrier and defects ($U = g = 0$) the wave functions and energies of the eigen-
states inside the channel can be separated to the transversal and longitudinal parts with
respect to the constriction axis $z$:

$$\Psi_{\alpha}(r) = \frac{1}{\sqrt{L}} \psi_{\perp \beta}(R) e^{ik_z z}; \quad (3)$$

$$\varepsilon_{\alpha} = \varepsilon_{\beta} + \frac{\hbar^2 k_z^2}{2m^*}; \quad (4)$$

where $\alpha = (\beta, k_z)$ is a full set of quantum numbers consisting of two discrete quantum
numbers $\beta = (m, n)$, which define the discrete energies $\varepsilon_{\beta}$ of conducting modes, and $k_z$ is the
wave vector along the $z$ axis; $r = (R, z)$. The transversal part $\psi_{\perp \beta}(R)$ of the wave function
satisfies to zero boundary conditions at the surface of the constriction. The functions $\Psi_{\alpha}(r)$
are orthogonal and normalized.

According to definition the noise power spectrum is

$$S_{ab}(\omega) = \frac{1}{2} \int dt e^{i\omega t} \left\langle \Delta \hat{I}_a(t) \Delta \hat{I}_b(0) + \Delta \hat{I}_b(0) \Delta \hat{I}_a(t) \right\rangle, \quad (5)$$
where $\Delta \hat{I}_a(t) = \hat{I}_a(t) - I_a$; $\hat{I}_a(t)$ is the current operator in the right $(a, b = R)$ or left $(a, b = L)$ lead; $I_a = \langle \hat{I}_a \rangle$ is the average current in the lead $a$; brackets $\langle ... \rangle$ denotes the quantum statistical average for a system at thermal equilibrium. In this paper we will only be interested in zero frequency noise $S_{ab}(0)$. Note that due to the current conservation $I \equiv I_L = I_R$ we have $S \equiv S_{LL} = S_{RR} = -S_{LR} = -S_{RL}$.

The general formula for a current $I$ through the quantum contact at an arbitrary voltage was obtained by Bagwell and Orlando [24] (see also the book [25]):

$$I = \frac{2e}{h} \int d\varepsilon T(\varepsilon, V) \times (f_L - f_R) ; \quad (6)$$

where is the transmission coefficient of electrons through the constriction

$$T(\varepsilon, V) = Tr \left[ \hat{t}^\dagger (\varepsilon, V) \hat{t}(\varepsilon, V) \right], \quad (7)$$

and $f_{L,R}(\varepsilon) = f_F(\varepsilon \pm \frac{eV}{2})$ is the distribution function of electrons moving in the contact from the left ($f_L$) or right ($f_R$) bank; $f_F(\varepsilon)$ is the Fermi function, $\hat{t}(\varepsilon, V)$ is a scattering matrix. In general case the function $T(\varepsilon, V)$ depends on the applied voltage $V$ because the electron scattering leads to the appearance of nonuniform electrical field inside the constriction [26]. This field must be found self-consistently from the equation of electroneutrality. In an almost ballistic microconstriction containing few scatterers and $\delta$-function potential barrier of the small amplitude $U$ the mentioned electrical field is small and we neglect its effect, assuming that the electrical potential drops at the ends of the constriction.

In the same approximation the noise spectrum $S(0)$ is given by

$$S(0) = \frac{2e^2}{h} \int d\varepsilon \left\{ Tr \left[ \hat{t}^\dagger (\varepsilon) \hat{t}(\varepsilon) \left( \hat{t}^\dagger (\varepsilon) \hat{t}(\varepsilon) \right) \right] \times [f_L (1 - f_L) + f_R (1 - f_R)] \right\}$$

$$+ Tr \left[ \hat{t}^\dagger (\varepsilon) \hat{t}(\varepsilon) \left( \hat{I} - \hat{t}^\dagger (\varepsilon) \hat{t}(\varepsilon) \right) \right] \times [f_L (1 - f_R) + f_R (1 - f_L)] \right\}; \quad (8)$$

where $\hat{I}$ is the unit matrix. The first term in the Eq. (8) corresponds to thermal fluctuations (the equilibrium, or Nyquist-Johnson noise) and vanishes, if the temperature $T \to 0$. The second part of this equation remains finite at $T = 0$, if the bias is applied to the constriction, and it describes the shot noise.

The calculation of the transport properties of the quantum constriction comes to the determination of the scattering matrix $\hat{t}(\varepsilon)$ . Elements of scattering matrix $t_{\beta\beta'}$ can be expressed by means of the advanced Green’s function $G^+ (r, r'; \varepsilon)$ of the system [27]:

$$t_{\beta\beta'}(\varepsilon) = -\frac{i\hbar^2 k_{\beta'}}{m^*} G^+_{\beta\beta'}(z, z'; \varepsilon); \quad z \to -\infty, z' \to +\infty; \quad (9)$$
where
\[ k_\beta (\varepsilon) = \frac{1}{\hbar} \sqrt{2m^* (\varepsilon - \varepsilon_\beta)} \] (10)
is an absolute value of electron wave vector corresponding to the quantum energy level \( \varepsilon_\beta \); \( G_{\beta\beta'} (z, z'; \varepsilon) \) are components of the expansion of Green’s function on the full set of wave functions corresponding to the transverse motion of electrons
\[ G^+(r, r'; \varepsilon) = \sum_{\beta\beta'} \psi_{\perp\beta} (R) \psi_{\perp\beta'}^* (R') G_{\beta\beta'}^+ (z, z'; \varepsilon). \] (11)

The matrix elements \( t_{\beta\beta'} (\varepsilon) \) describes the transmission probabilities for carriers incident in channel \( \beta \) in the left lead \( L \) and transmitted into channel \( \beta' \) in the right lead \( R \). The Green’s function satisfies the Dyson’s equation:
\[ G (r, r'; \varepsilon) = G_b (r, r'; \varepsilon) + g \sum_i G_b (r, r_i; \varepsilon) G (r_i, r'; \varepsilon), \] (12)
where \( G_b (r, r'; \varepsilon) \) is the Green’s function of ballistic microconstriction with the barrier in the absence of defects. It can be found from the equation
\[ G_b (r, r'; \varepsilon) = G_0 (r, r'; \varepsilon) + U \int dR'' G_0 (r; R'', z'' = 0; \varepsilon) G_b (R'', z'' = 0; r'; \varepsilon), \] (13)
where
\[ G_0^+ (r, r'; \varepsilon) = \sum_\beta \frac{m^*}{i\hbar^2 k_\beta} \psi_{\perp\beta} (R) \psi_{\perp\beta}^* (R') e^{ik_\beta |z' - z|} \] (14)
is the Green’s function in the absence of impurities and the barrier. Substituting the expansions (11) and (14) into equation (13) and taking into account the orthogonality of functions \( \psi_{\perp\beta} (R) \) for the coefficients \( G_{b\beta}^+ (z, z'; \varepsilon) \delta_{\beta\beta'} \) of \( G_b^+ (r, r'; \varepsilon) \) in the expansion (11) we obtain the algebraic equation
\[ G_{b\beta}^+ (z, z'; \varepsilon) = \frac{m^*}{i\hbar^2 k_\beta} \left[ e^{ik_\beta |z' - z|} + U e^{ik_\beta |z|} G_{b\beta}^+ (0, z'; \varepsilon) \right]. \] (15)

Taking this equation at \( z = 0 \) we find \( G_{b\beta}^+ (0, z'; \varepsilon) \) and finally \( G_{b\beta}^+ (z, z'; \varepsilon) \) is given by
\[ G_{b\beta\beta'}^+ (z, z'; \varepsilon) = \frac{m^*}{i\hbar^2 k_\beta} \left[ e^{ik_\beta |z' - z|} + r_\beta e^{ik_\beta (|z'| + |z|)} \right], \] (16)
where
\[ r_\beta = -\frac{im^* U}{\hbar^2 k_\beta + im^* U} = \cos \varphi_\beta e^{i\varphi_\beta}; \] (17)
is the amplitude of reflected wave;

\[ \varphi_\beta (\varepsilon) = \arcsin \left[ \frac{1}{\sqrt{1 + (m^* U/\hbar^2 k_\beta)^2}} \right]. \]  

(18)

The amplitude \( t_\beta \) of transmitted wave can be evaluated through the \( r_\beta \) from the condition of continuity of electron wave function at \( z = 0 \)

\[ t_\beta = r_\beta + 1 = \frac{\hbar^2 k_\beta}{\hbar^2 k_\beta + i m^* U} = i \sin \varphi_\beta e^{i \varphi_\beta}. \]  

(19)

The same functions \( r_\beta \) and \( t_\beta \) can be found from the solution of the one dimensional Schrödinger equation of a system with \( \delta \)–function barrier \( U\delta (z) \) [28].

The Eq.(12) can be solved exactly for any finite number of defects. For that the Eq.(12) should be written in all points \( r_i \) of the defect positions and the functions \( G(r_i, r'; \varepsilon) \) are found from the system of \( i \) algebraic equations.

By using the matrix elements \( 11 \) the conductance \( G = \frac{dI}{dV} \) of the microconstriction as well as the shot noise \( S(0) \) can be calculated.

III. VOLTAGE DEPENDENCE OF CONDUCTANCE AND SHOT NOISE.

To illustrate the effect of quantum interference of scattered electron waves to the conductance and the shot noise we present the results for two cases: (i) single impurity in the constriction with the barrier; (ii) two impurities in the constriction without the barrier. For the first case the Green’s function takes the form:

\[ G (r, r'; \varepsilon) = G_0 (r, r'; \varepsilon) + \frac{g G_b (r, r_1; \varepsilon) G_b (r_1, r'; \varepsilon)}{1 - g G_b (r_1, r_1; \varepsilon)} \times \]  

(20)

where \( r_1 \) is the position of the impurity, a Green’s function \( G_b (r, r'; \varepsilon) \) is defined by Eqs.(11), (16). If only two impurities are situated inside the ballistic microconstriction, the solution of the Eq.(12) is

\[ G (r, r'; \varepsilon) = G_0 (r, r'; \varepsilon) + \frac{1}{1 - G_1 (r_1; \varepsilon) G_1 (r_2; \varepsilon) G_0^2 (r_1, r_2; \varepsilon) \times} \]

\[ \sum_{i,k=1,2; i\neq k} \{G_1 (r_i; \varepsilon) G_0 (r, r_i; \varepsilon) [G_0 (r, r'; \varepsilon) + G_1 (r_i; \varepsilon) G_0 (r_i, r_k; \varepsilon) G_0 (r_k, r'; \varepsilon)] \}; \]  

(21)
where
\[ G_1 (\mathbf{r}_i; \varepsilon) = \frac{g}{1 - g G_0 (\mathbf{r}_i; \varepsilon)}; \tag{22} \]
and \( G_0 (\mathbf{r}, \mathbf{r}'; \varepsilon) \) is the Green’s function of the ballistic microconstriction \( (14) \). Using the Eqs. \( (20) \) and \( (21) \) it is easy to find the transmission probabilities \( t_{\beta \beta'} \) \( (9) \).

At zero temperature the nonlinear conductance \( G (V) \) and the noise power \( S (0, eV) \) are given by the following expressions:
\[ G (V) = \sum_{\beta \beta'} \left[ \left| t_{\beta \beta'} (\varepsilon_F + \frac{eV}{2}) \right|^2 + \left| t_{\beta \beta'} (\varepsilon_F - \frac{eV}{2}) \right|^2 \right]; \tag{23} \]
\[ S (0, eV) = \sum_{\beta \beta' \beta'' \beta'''} \int d\varepsilon \left\{ t_{\beta \beta'} (\varepsilon) t_{\beta' \beta''} (\varepsilon) \left[ \delta_{\beta'' \beta'} \delta_{\beta' \beta''} - t^*_{\beta' \beta''} (\varepsilon) t_{\beta'' \beta' \beta''} (\varepsilon) \right] \right\}. \tag{24} \]

To explain the analytical results we present the expansion of the transmission coefficient \( (7) \) on the constant of electron-impurity interaction \( g \) up to linear in \( g \) term for the constriction with one impurity in the point \( \mathbf{r}_1 = (\mathbf{R}_1, z_1) \) and the barrier
\[ T (\varepsilon) = \sum_{\beta} |t_\beta|^2 \left\{ 1 - \frac{2m^* g}{\hbar^2 k_\beta} |r_\beta| \left| \psi_{\perp \beta} (\mathbf{R}_1) \right|^2 \cos \left( 2k_\beta z_1 + \varphi_\beta \right) \right\}; \varepsilon > \varepsilon_\beta, \tag{25} \]
where \( t_\beta, r_\beta, \) and phase \( \varphi_\beta \) are defined by Eqs. \( (19), (17), \) and \( (18) \). This formula is correct for \( \frac{2m^* g}{\hbar^2 k_\beta} \ll 1 \), i.e. far from the end of the step of conductance, where \( k_\beta \to 0 \). The oscillatory term in Eq. \( (25) \) originates from the interference between directly transmitted wave (trajectory 1 in Fig.1) and the wave, which is once reflected by the barrier and after one reflection from the impurity passes through the contact (trajectory 2 in Fig.1). The amplitude of the oscillations depends on the local density of electron states \( \nu_\beta (\mathbf{R}_1, \varepsilon) = m^* \left| \psi_{\perp \beta} (\mathbf{R}_1) \right|^2 / (\hbar^2 k_\beta (\varepsilon)) \) in the point, in which the impurity is situated. In certain points \( \nu_\beta (\mathbf{R}, \varepsilon) \) can be equal to zero and the defect located near such point contribute a little to the oscillatory addition of \( \beta \)th mode to the \( T (\varepsilon) \). In particular, impurities on the surface \( \mathbf{R} = \mathbf{R}_s \) do not influence to the oscillations of \( T (\varepsilon) \), because \( \psi_{\perp \beta} (\mathbf{R}_s) = 0 \). As a result of the reflection from the barrier the oscillations have the additional phase \( \varphi_\beta \). Its dependence on the energy \( \varepsilon \) leads to nonperiodicity of oscillations of function \( T (\varepsilon) \). The
Eq. (25) enables to follow the changing in the amplitude of oscillation with changing of the contact diameter. If the diameter is increased and goes to the end of the conductance step, the energy of the transverse quantum mode $\varepsilon_\beta$ is decreased (see, for example, the Eq. (28) for cylindrical geometry). The wave number $k_\beta$ (10) is increased and according the Eq. (17) the modulus of the reflection probability $|r_\beta|$ is decreased. In opposite situation (the radius is decreased) the decreasing of $k_\beta$ is a reason for the decreasing of the transmission probability $|t_\beta|$ (19). In both cases the amplitude of oscillations of $T(\varepsilon)$ is decreased.

The similar expansion of $T(\varepsilon)$ for the constriction with two defects in points $\mathbf{r}_1 = (R_1, z_1)$ and $\mathbf{r}_2 = (R_2, z_2)$ without barrier is

$$T(\varepsilon) = \sum_{\beta\beta'} \left\{ \delta_{\beta\beta'} - 2 \left( \frac{m^* g}{\hbar^2} \right)^2 \frac{1}{k_\beta k_{\beta'}} \sum_{i=1,2} \left[ |A_{\beta\beta'}^{(ii)}|^2 + \right. \right.$$  

$$\left. \left. \text{Re} \sum_{i \neq j=1,2} A_{\beta\beta'}^{(ii)} A_{\beta'\beta'}^{(jj)} \exp \left( \left( k_\beta + k_{\beta'} \right) (z_j - z_i) + \varphi_\beta + \varphi_{\beta'} \right) \right] \right\}, \quad \varepsilon > \varepsilon_\beta, \varepsilon_{\beta'};$$  

(26)

where

$$A_{\beta\beta'}^{(ii)} = \psi_{\perp \beta}(R_i) \psi_{\perp \beta'}^*(R_i).$$  

(27)

The last term in square brackets describes the interference effect between trajectory 3 in Fig.1 and trajectory 4, which corresponds to two scattering by different impurities, and non-monotonically depends on the energy $\varepsilon$. The discussed above the energy dependence of the transmission coefficient $T(\varepsilon)$ manifests itself in nonmonotonic dependence of conductance and shot noise on the applied bias $eV$.

The general expression for the components $t_{\beta\beta'}(\varepsilon)$ (2) calculated by using the Green’s functions (20) and (21) takes into account a multiple electron scattering by the impurities and barrier. It is valid for any values of parameters. Below we illustrate such situation presenting the plots for the voltage dependencies of conductance and shot noise for some values of parameters, which may be related to experiments.

For numerical calculations we used the model of cylindrical channel, for which in formulas (3) and (4)

$$\psi_{\perp \beta}(\rho, \varphi) = \frac{1}{\sqrt{\pi R} J_{m+1}(\gamma_{mn})} J_m \left( \frac{\rho}{R} \right) e^{in\varphi};$$  

(28)

$$\varepsilon_{mn} = \frac{\hbar^2 \gamma_{mn}^2}{2m^* R^2};$$  

(29)
Here we used the cylindrical coordinates $r = (\rho, \varphi, z)$; $\gamma_{mn}$ is $n$–th zero of Bessel function $J_m$. Also we introduce dimensionless parameters

$$
\tilde{g} = \frac{m^* g}{\pi R^2 \hbar^2 k_F}; \quad \tilde{U} = \frac{m^* U}{\hbar^2 k_F},
$$

(30)

where $k_F$ is the Fermi wave vector. We have performed the calculations for $\tilde{g} = 1$, $\tilde{U} = 0$. For such values of these parameters the amplitude of conductance oscillations is of the order the amplitude, which was observed in Ref. [22]. For the value of the radius $2\pi R = 2.9 \lambda_F$ (one mode channel) the first energy level $\varepsilon_{0,1} < \varepsilon_F$ is comparatively far from the Fermi energy and for $2\pi R = 3.45 \lambda_F$ this level is close to $\varepsilon_F$. For the larger value of radius ($2\pi R = 5 \lambda_F$) there are two opened quantum modes with energies $\varepsilon_{0,1}, \varepsilon_{\pm1,1} < \varepsilon_F$. To illustrate different reasons of appearance of conductance oscillations, in Fig.2 and Fig.3 we show the dependences of the conductance on the applied voltage for the channel without the barrier ($U = 0$) containing two impurities and for the channel with the barrier and single impurity. From comparison of the different curves in Figs. 2, 3 we observe that the amplitude of conductance oscillations is decreased for radius value ($2\pi R = 3.45 \lambda_F$) corresponding the end of first step of conductance. In Fig.4 and Fig.5 the voltage dependences of noise power are plotted. We remark that, as seen from Fig.4, for the one mode channel the shot noise is the strongly nonmonotonic function of $V$. As well as for the conductance, the amplitude of the oscillations of the shot noise is decreased near the end of the first step ($2\pi R = 3.45 \lambda_F$). For the two mode channel the $S(V)$ is almost linear function that can be explained by the effect of a superposition of oscillations with a different periods. In the contact with the barrier the main part of the shot noise $S_0 (V)$ originates from the electron reflection from the barrier potential ($S (V) = S_0 (V)$, if $g = 0$), and it is the monotonic function on $V$. The small nonlinearity of this function arises from the energy dependence transmission probability. The interference of electron wave in the presence of defect leads to nonmonotonic additions, which we show in Fig.5.

IV. CONCLUSION

We have studied theoretically the voltage dependence of the conductance $G$ and the shot noise power $S$ in the quantum microconstriction in the form of long channel (quantum wire). The effect of quantum interference of electron waves, which are scattered by single defects
and the potential barrier inside the constriction, is taken into account. In the framework of the model we have obtained the analytical solution of the problem and found the dependencies of the $G$ and $S$ on such important parameters as the constriction diameter, the constant of electron-impurity interaction, the amplitude of the barrier potential and positions of impurities. In the general case these dependencies are complex and defined by the expression of transmission probability $t_{\beta\beta'}$ \([9]\) by means of Green’s functions \((20), (21)\). For the small constant $g$ of electron-impurity interaction and far from the step of conductance the part of the total transmission coefficient $T(\varepsilon)\,(25)$, which is due to the interference effect, is proportional to $g$ and to the amplitude of the reflected from the barrier wave $r_\beta$ (see, Eq.\,(17)). As a result of that, at small $g$ and $U$ the interference part of conductance and shot noise is proportional to $gU$ or $g^2$ (for $U = 0$) for any number of defects.

We have shown that the conductance and noise are oscillatory functions on the applied bias $V$ and come to the conclusion that the experimentally observed suppression of conductance oscillations \([21]\) could be due to the energy dependence of the transmission probability of electrons through the constriction. In the framework of our model this suppression of conductance oscillations can be explained in the following way: The oscillatory part of conductance is decreased with the decreasing of amplitude $r_\beta$ of reflected from the barrier wave. The reflection probability $r_\beta$ from the barrier has the minimal value, if the energy of quantum mode $\varepsilon_\beta$ is close to $\varepsilon_\beta \lesssim \varepsilon_F$. It is demonstrated that in the one mode constriction containing only impurities the shot noise power is a strongly nonlinear function on $V$. In the contact with the barrier the almost linear dependence $S(V)$ has small oscillatory addition.

We acknowledge fruitful discussion with A.N. Omelyanchouk.

---

[1] Ya. M. Blanter, M. Buttiker, Phys. Rep., 336, 1 (2000).
[2] N. Agrait, A.L. Yeyati, and J.M. van Ruitenbeek, Phys. Rep., 377, 81 (2003).
[3] I.M. Lifshits, and A.M. Kosevich, Izv. AN USSR, ser. phyx., 19, 395 (1955) (in Russian).
[4] R. Landauer, IBM J. Res. Dev., 1, 223 (1957).
[5] M. Buttiker, Phys. Rev. Lett. 57, 1761 (1986).
[6] Sh. Kogan, Electronic Noise and Fluctuations in Solids, Cambridge University Press (1996).
[7] I. O. Kulik and A. N. Omel’yanchuk, Fiz. Nizk. Temp. 10 305 (1984) [Sov. J. Low Temp.
[8] A.G. Scherbakov, E. N. Bogachek, and Uzi Landman, Phys. Rev. B, 57, 6654 (1997).
[9] A. Namiranian, Yu.A. Kolesnichenko, and A.N. Omelyanchouk, Phys. Rev. B, 61, 16796 (2000).
[10] M. E. Flatté, J. M. Byers, Phys. Rev. B, 53, R10536, (1996).
[11] E. Granot, cond-mat/0303347 v1, (2003).
[12] M. I. Molina, H. Bahliouli, Phys. Lett. A, 284, 87 (2002).
[13] I. E. Aronov, M. Jonson, and A. M. Zagoskin, Appl. Phys. Rep., 93-57 (1994).
[14] C. S. Kim, O. N. Roznova, A. M. Satanin, and V. B. Stenberg, ZhETP, 121, 1157 (2002) [JETP, 94, 992 (2002)].
[15] D. Boese, M. Lischka and L.E. Reichl, Phys. Rev. B 62, 16933 (2000).
[16] A. Namiranian, Yu.A. Kolesnichenko, and A.N. Omelyanchouk, Fiz. Nizk. Temp., 26, 694 (2000).
[17] D.L. Maslov, C. Barnes, and G. Kirczenov, Phys. Rev. Lett., 70, 1984 (1993).
[18] Ye. S. Avotina, and Yu.A. Kolesnichenko, Fiz. Nizk. Temp., 30, 209 (2004) [J. Low Temp. Phys., 30, 153 (2004)].
[19] G. Zarand, Jan von Delft, A Zawadowski, Phys. Rev. Lett. 80, 1353 (1998)
[20] C. Untiedt, G. R. Bollinger, S. Vieira, and N. Agraït, Phys. Rev. B, 62, 9962 (2000).
[21] B. Ludoph and J.M. van Ruitenbeek, Phys. Rev. B, 61, 2273 (2000).
[22] A. Halbritter, Sz. Csonka, G. Mihály, O.I. Shklyarevskii, S. Speller, and H. van Kempen, arXiv:cond-mat/031138 v2, (2003).
[23] I.O. Kulik, A.N. Omelyanchuk, R.I. Shekhter Sov. J. Low Temp. Phys. 3, 1543 (1977) [Sov. J. Low Temp. Phys. 3, 740 (1977)].
[24] P.F. Bagwell, and T.P. Orlando, Phys. Rev. B, 40, 1456 (1989).
[25] S. Datta, Electronic transport in mesoscopic systems, Cambridge University Press, Cambridge (1997).
[26] D. Lenstra, and R.T.M. Smokers, Phys. Rev. B, 38, 6452 (1988).
[27] D. S. Fisher and P.A. Lee, Phys. Rev. B, 23, 6851 (1981).
[28] S. Flügge, Practical Quantum Mechanics, V.1, Springer-Verlag (1971).
FIG. 1: The model of quantum constriction in the form of long channel adiabatically connecting with bulk metallic reservoirs. The trajectories (1-4) of electrons, which are scattered by the defects and a barrier are shown schematically.
FIG. 2: The dependences of the conductance on the applied voltage for the channel containing two impurities for different values of radius; impurity positions are $2\pi \rho_1 = 0.3\lambda_F$ and $2\pi \rho_2 = 0.4\lambda_F$, $2\pi (z_1 - z_2) = 35\lambda_F$. 

\[
(2\pi R = 5\lambda_F) \quad (2\pi R = 3.45\lambda_F) \quad (2\pi R = 2.9\lambda_F)
\]
FIG. 3: The dependences of the conductance on the applied voltage for the channel containing the single impurity and the barrier for different values of radius; the impurity position is $2\pi \rho_1 = 0.3\lambda_F$, $2\pi z_1 = 35\lambda_F$. 
FIG. 4: The voltage dependences of noise power on the applied voltage for the channel containing two impurities for different values of radius; impurity positions are $2\pi \rho_1 = 0.3\lambda_F$ and $2\pi \rho_2 = 0.4\lambda_F$, $2\pi (z_1 - z_2) = 35\lambda_F$. 
FIG. 5: The voltage dependences of the nonmonotonic part of noise power on the applied voltage for the channel containing the single impurity and the barrier for different values of radius; the impurity position is $2\pi \rho_1 = 0.3\lambda_F$, $2\pi z_1 = 35\lambda_F$. 