Instanton interactions in dense-matter QCD

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Abstract

A Coulomb gas representation of dense-matter QCD is derived from a dual transformation of the low-energy effective Lagrangian. The point-like charges $Q = \pm 1$ of the gas are identified with the instantons and anti-instantons of such topological charges. An instanton repels another instanton with the same force as it attracts an anti-instanton, in contrast to the semiclassical interaction.

I. INTRODUCTION AND OVERVIEW

Instantons have been argued to play an important role in the nonperturbative dynamics of QCD. Being classical solutions to the field equations, instantons provide the starting point for a phenomenologically successful semiclassical approach to QCD \cite{1}. Yet the instanton calculus, as this approach is known, lacks a controllably small parameter which would put it on a firmer theoretical ground. The most natural parameter — the diluteness of the instanton gas — is numerically small in QCD vacuum, but not to the extent that the interaction between the instantons can be neglected. The instanton “gas” is more similar to a liquid than a dilute gas.

Given its importance for the QCD vacuum, it is interesting to study instanton interaction in regimes where the instanton gas can be made arbitrarily dilute. In this paper we perform such a study for the regime of very high matter density, where the diluteness is controlled by the large ratio of the baryon chemical potential $\mu$ to $\Lambda_{\text{QCD}}$. We find that in this regime the instantons ($I$) and anti-instantons ($\bar{I}$) interact with each other via a four-dimensional Coulomb potential generated by their topological charges.

Our approach is different from the standard semiclassical ones. A standard instanton calculation starts from the classical instanton gas and proceeds to calculating quantum corrections. In this approach the $II$ and $I\bar{I}$ interactions start as fundamentally different: at the classical level an instanton interacts with anti-instantons, but not with another instanton. In contrast, our approach incorporates quantum effects from the very beginning. Our starting point is the low-energy effective Lagrangian for high-density QCD with two massless quark flavors \cite{2}:

$$L = f^2[(\partial_0 \varphi)^2 - u^2(\partial_i \varphi)^2] + a\mu^2 \Delta^2 \cos(\varphi - \theta).$$

(1)
This Lagrangian describes the pseudoscalar isoscalar excitations, \( \varphi \) (similar to the \( \eta \) meson), and the dependence of ground state energy on the QCD vacuum angle \( \theta \). Using a well-known formal trick, we rewrite the partition function of the theory (1) as that of a Coulomb gas. We then notice that \( \theta \) plays the role of an external (electrostatic) potential for this Coulomb gas. Since, in QCD, the \( \theta \) angle is conjugate to the topological charge, we identify the constituents of the Coulomb gas with instantons and anti-instantons. In this manner, we obtain information about instanton interaction at the quantum level, which is not directly available from semiclassical calculations. By comparing the size of the instanton to the average inter-particle distance in the gas, we check that the instanton plasma is indeed dilute for sufficiently large \( \mu \). We also check that the Debye volume contains a large number of instantons, which means that the instanton plasma is weakly nonideal.

II. EFFECTIVE LAGRANGIAN AND THETA DEPENDENCE OF HIGH-DENSITY QCD

Let us briefly recall how Eq. (1) is derived (more details can be found in Ref. [2]). The ground state of high-density QCD with \( N_f = 2 \) light quark flavors (\( u \) and \( d \)) is a color-superconducting phase with two condensates of left- and right-handed quarks [3][4],

\[
\langle q^a_L q^b_L \rangle^* = \epsilon_{\alpha\beta} \epsilon_{ij} \epsilon^{abc} X^c, \\
\langle q^a_R q^b_R \rangle^* = \epsilon_{\alpha\beta} \epsilon_{ij} \epsilon^{abc} Y^c.
\]

(2)

Here the structure of the condensates in spin (\( \alpha, \beta \)), flavor (\( i, j \)) and color (\( a, b \)) indices is written explicitly. In the ground state \( X^c \) and \( Y^c \) have definite length (see below) and are parallel. If quarks are massless, then at the perturbative level there is a degeneracy of the ground state with respect to the relative U(1) phase between \( X^c \) and \( Y^c \), which means that the U(1)\( _A \) symmetry of the classical QCD Lagrangian is spontaneously broken. The effective Lagrangian of the Goldstone boson arising from the breaking of U(1)\( _A \) has the following form

\[
L = f^2 \left[ (\partial_0 \varphi)^2 - u^2 (\partial_i \varphi)^2 \right].
\]

(3)

Since heavier meson excitations have masses of the order of the superconducting gap \( \Delta \), this Lagrangian is applicable only for momenta much smaller than \( \Delta \). For large chemical potentials \( \mu \gg \Lambda_{\text{QCD}} \), the leading perturbative values for \( f \) and \( u \) are [3][4]:

\[
f^2 = \frac{\mu^2}{8\pi^2}, \quad u^2 = \frac{1}{3}.
\]

(4)

The field \( \varphi \), canonically normalized, is the field for the \( \eta \) boson: \( \eta = \sqrt{2} f \varphi \). Strictly speaking, \( \eta \) is not the lightest mode in this theory. There is an unbroken color SU(2) sector which confines at a large scale exponentially larger [8] than the scales we consider in this paper. However, the coupling of \( \varphi \) to this sector is suppressed by a power of \( \Delta/f \) since the coupling has the form \( (\eta/f) \tilde{F} \bar{F} \). Such a sector is absent for the \( N_f = 3 \) case briefly discussed in Sec. [8].

Instantons add a contribution to (3) which is given by [2]:

\[
V_{\text{inst}}(\varphi) = -a \mu^2 \Delta^2 \cos(\varphi - \theta).
\]

(5)
This form is uniquely fixed by the anomaly-mediated relation of chiral rotations to $\theta$ rotations, and by the fact that multi-instanton contributions are suppressed at large $\mu$. The calculation of the dimensionless function $a(\mu)$ is reviewed below. What is important at this moment is that $a$ vanishes in the limit $\mu \to \infty$, which means that the mass of the Goldstone boson becomes much less than $\Delta$ at large $\mu$. This is the necessary condition for the effective theory (1), which contains only one field $\varphi$, to have a region of validity.

If the quark masses, $m_u$ and $m_d$, are non-zero, a new contribution appears in the effective Lagrangian (1). To lowest order in the masses its dependence on $\varphi$ is also fixed by symmetry:

$$V_{\text{mass}} = -b m_u m_d \Delta^2 \cos \varphi,$$

and the coefficient $b \sim 1$ can be calculated using methods of Ref. [6].

The $\theta$ parameter in Eq. (5) is the fundamental parameter of the theory which is equal to, or at least very close to, zero in our real world. However, we would like to keep this parameter explicit because it plays a crucial role in our further arguments. We note here that by minimizing the effective potential $V_{\text{inst}} + V_{\text{mass}}$ with respect to $\varphi$ one can find the theta dependence of the ground state energy. For small $m_u m_d \ll a \mu^2$,

$$E_{\text{vac}}(\theta) \simeq -a \mu^2 \Delta^2 - b m_u m_d \Delta^2 \cos \theta,$$

in agreement with the fact that massless fermions suppress the topological susceptibility (and the theta dependence).

The calculation of $V_{\text{inst}}(\varphi)$ can be found in Ref. [2]. The starting point is the instanton-induced effective four-fermion interaction [3][4][11],

$$L_{\text{inst}} = e^{i \varphi} \int d\rho n_0(\rho) \left( \frac{4}{3} \pi^2 \rho^3 \right)^2 \left\{ (\bar{u}_R u_L)(\bar{d}_R d_L) + \frac{3}{32} \left[ (\bar{u}_R \lambda^a u_L)(\bar{d}_R \lambda^a d_L) - \frac{3}{4} (\bar{u}_R \sigma_{\mu\nu} \lambda^a u_L)(\bar{d}_R \sigma_{\mu\nu} \lambda^a d_L) \right] \right\} + \text{H.c.},$$

By taking the average of Eq. (8) over the superconducting state (2), one finds $V_{\text{inst}}$, and confirms that it is proportional to $\cos(\varphi - \theta)$ as in Eq. (5). Taking the average of (8) in the superconducting ground state, where

$$|X| = |Y| = \frac{3}{2 \sqrt{2\pi}} \frac{\mu^2 \Delta}{g},$$

we find

$$V_{\text{inst}}(\varphi) = -\int d\rho n_0(\rho) \left( \frac{4}{3} \pi^2 \rho^3 \right)^2 12 |X|^2 \cos(\varphi - \theta),$$

where $n_0(\rho)$ is the instanton (size) density at finite chemical potential, which is given by [11][12][11]

$$n_0(\rho) = \frac{0.466 e^{-1.679 N_c} 1.34 N_f}{(N_c - 1)!(N_c - 2)!} \left( \frac{8 \pi^2}{g^2} \right)^{2 N_c} \rho^{-5} \exp \left( -\frac{8 \pi^2}{g^2(\rho)} \right) e^{-N_f \mu^2 \rho^2}.$$  (11)

Taking the $\rho$ integration in Eq. (10), we find

$$a = 5 \times 10^4 \left( \ln \frac{\mu}{\Lambda_{\text{QCD}}} \right)^7 \left( \frac{\Lambda_{\text{QCD}}}{\mu} \right)^{29/3}.$$  (12)

Thus $a \to 0$ when $\mu \to \infty$, so at sufficiently large $\mu$ the instanton calculations are under analytical control.
III. COULOMB GAS REPRESENTATION

The effective low energy dense-QCD Lagrangian (1) is the sine-Gordon (SG) Lagrangian. Many of the special properties of the SG theory apply. One of these properties is the existence of a kink-type solution, corresponding to the domain wall in four dimensions [2]. Another important property is the admittance of a Coulomb gas (CG) representation for the partition function. Such a representation was used previously [13] to argue that, in zero-density QCD, the low energy effective Lagrangian with \( \text{U}(1)_A \) anomaly represents the dual form of instanton contribution to the partition function. However, QCD at zero density is in a nonperturbative regime where no theoretical control is possible [13]. In the present paper all calculations are under theoretical control and, therefore, some reliable and precise statements can be made.

The mapping between the SG theory and its CG representation is well known. All we need to do is reverse the derivation of SG functional representation of the CG in Ref. [14]. The partition function corresponding to the Lagrangian (1) is given by

\[
Z = \int \mathcal{D}\varphi \, e^{-\int d^4x \, d\tau \, L_E} = \int \mathcal{D}\varphi \, e^{-\int d^4x \, d\tau \, f^2u \int d^4x (\partial \varphi)^2} \, e^{\lambda \int d^4x \cos(\varphi(x) - \theta)},
\]

(13)

where we introduced

\[
\lambda \equiv \frac{a\mu^2 \Delta^2}{u}.
\]

\( L_E \) is the Euclidean space Lagrangian. The imaginary time \( \tau \) is rescaled to bring the kinetic term into the Euclidean invariant form in new coordinates with \( x_0 = u\tau \). Leaving alone the integration over \( \varphi(x) \) for the moment, we expand the last exponent in Eq. (13), represent the cosine as a sum of two exponents and perform the binomial expansion:

\[
e^{\lambda \int d^4x \cos(\varphi(x) - \theta)} = \sum_{M=0}^{\infty} \frac{(\lambda/2)^M}{M!} \left( \int d^4x \sum_{Q=\pm 1} e^{iQ(\varphi(x) - \theta)} \right)^M
= \sum_{M_+=0}^{\infty} \frac{(\lambda/2)^M}{M_+!M_-!} \int d^4x_1 \ldots \int d^4x_M \, e^{i\sum_{a=0}^{M} Q_a(\varphi(x_a) - \theta)}.\]

(15)

The last sum is over all possible sets of \( M_+ \) positive and \( M_- \) negative charges \( Q_a = \pm 1 \). The last line in Eq. (13) is a classical partition function of an ideal gas of \( M = M_+ + M_- \) identical (except for charge) particles of charges +1 or −1 placed in an external potential given by \( i(\theta - \varphi(x)) \). It is easy to see that (for a constant or slowly varying potential) the average number of these particles per unit of 4-volume \( \langle M \rangle/V_4 \), i.e., the density, is equal to \( \lambda \). Thus making \( \lambda \) small one can make the gas arbitrarily dilute.

\(^1\)To be precise, the path integral in Eq. (13) should be understood as an integral over low-momentum modes of \( \varphi \) only. The upper limit of the momentum of \( \varphi \) is the ultraviolet cutoff of the effective Lagrangian (1), which should be taken as some scale smaller than \( \Delta \). Only tree graphs contribute to \( Z \) so there is no dependence on the precise value of the cutoff.
While $\theta$ can indeed be viewed as an external potential for the gas (13), $\varphi(x)$ is a dynamical variable, since it fluctuates as signified by the path integration in (13). For each term in (15) the path integral is Gaussian and can be easily taken:

$$\int D\varphi e^{-f^2u \int d^4x (\partial \varphi)^2} e^{i \sum_{a=0}^M Q_a (\varphi(x_a) - \theta)} = e^{-\theta \sum_{a=0}^M Q_a} e^{-\frac{1}{2f^2u} \sum_{a>b=0}^M Q_a Q_b G(x_a - x_b)}.$$  \hspace{1cm} (16)

We see that, for a given configuration of charges $Q_a$, $-i\varphi(x)$ is the Coulomb potential created by such distribution.

The function $G(x)$ is the solution of the four-dimensional Poisson equation with a point source (the inverse of $-\partial^2$):

$$G(x_a - x_b) = \frac{1}{4\pi^2(x_a - x_b)^2}.$$  \hspace{1cm} (17)

Thus we obtain the dual CG representation for the partition function (13):

$$Z = \sum_{M,M' = 0}^{\infty} \frac{(\lambda/2)^M}{M!M'!} \int d^4x_1 \ldots \int d^4x_M e^{-\theta \sum_{a=0}^M Q_a} e^{-\frac{1}{2f^2u} \sum_{a>b=0}^M Q_a Q_b G(x_a - x_b)}.$$  \hspace{1cm} (18)

The two representations of the partition function (13) and (18) are equivalent.

### IV. PHYSICAL INTERPRETATION

The charges $Q_a$ were originally introduced in a rather formal manner so that the QCD effective low energy Lagrangian can be written in the dual CG form (18). However, now the physical interpretation of these charges becomes clear: since $Q_{\text{net}} \equiv \sum_a Q_a$ is the total charge and it appears in the action multiplied by the parameter $\theta$ (see Eq. (13)), one concludes that $Q_{\text{net}}$ is the total topological charge of a given configuration. Indeed, in QCD the $\theta$ parameter appears in the Lagrangian only in the combination with the topological charge density $-i\theta G_{\mu\nu} \tilde{G}_{\mu\nu}/(32\pi^2)$. It is also quite obvious that each charge $Q_a$ in a given configuration should be identified with an integer topological charge well localized at the point $x_a$. This, by definition, corresponds to a small instanton positioned at $x_a$. To corroborate this identification we notice that every particle with charge $Q_a$ brings along a factor of fugacity $\lambda$ which contains the classical one-instanton suppression factor $\exp(-8\pi^2/g^2(\rho))$ in the density of instantons (11).

The following hierarchy of scales exists in such an instanton ensemble for sufficiently large $\mu$. The typical size of the instantons $\tilde{\rho} \sim \mu^{-1}$ is much smaller than the short-distance cutoff of our effective low-energy theory, $\Delta^{-1}$. Therefore, in our low-energy description they are represented by $\delta(x - x_a)$ functions. The average distance between the instantons

\[ \text{One notices that the term } a = b \text{ in the double sum (13) is dropped. This is the self-interaction of each charge. It would renormalize the fugacity } \lambda \text{ by a factor } \exp\left(-G(0)/(f^2u)\right). \]  This factor should be dropped as it represents contribution of very short wavelength fluctuations of $\varphi$. Such fluctuations have to be cutoff at the scale $1/\Delta$. The self-energy of the charges comes from a much smaller scale, of order $1/\mu$, which is already calculated and contained in $a$.  

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2 One notices that the term $a = b$ in the double sum (13) is dropped. This is the self-interaction of each charge. It would renormalize the fugacity $\lambda$ by a factor $\exp(-G(0)/(f^2u))$. This factor should be dropped as it represents contribution of very short wavelength fluctuations of $\varphi$. Such fluctuations have to be cutoff at the scale $1/\Delta$. The self-energy of the charges comes from a much smaller scale, of order $1/\mu$, which is already calculated and contained in $a$. 

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\[ \tilde{r} = \lambda^{1/4} = (\sqrt{a} \mu \Delta)^{-1/2} \] is much larger than both the average size of the instantons and the cutoff \( \Delta^{-1} \). The largest scale is the Debye screening length in the Coulomb gas, \( r_D = (\sqrt{a/2 \mu \Delta} / fu)^{-1} \sim (\sqrt{a} \Delta)^{-1} \). This coincides with the static correlation length of the \( \varphi \) field, which differs from the rest mass of the Goldstone by a factor of \( u \). In short:

\[
\begin{align*}
\text{(size)} & \ll \text{(cutoff)} \ll \text{(distance)} \ll \text{(Debye)} \\
\mu^{-1} & \ll \Delta^{-1} \ll (\sqrt{a} \mu \Delta)^{-1/2} \ll (\sqrt{a} \Delta)^{-1}
\end{align*}
\] (19)

Due to this hierarchy, ensured by large \( \mu / \Lambda_{\text{QCD}} \), we acquire analytical control.

It is also quite interesting that, although the starting low-energy effective Lagrangian contains only a colorless field \( \varphi \), we have ended up with a representation of the partition function in which objects carrying color (the instantons, their interactions and distributions) can be studied. In particular, from the discussions given above, one can immediately deduce that \( II \) and \( I \bar{I} \) interactions are exactly the same up to a sign and are Coulomb-like at large distances.

This looks highly nontrivial since it has long been known that at the semiclassical level an instanton interacts only with anti-instantons but not with another instanton carrying a topological charge of the same sign. As we demonstrated above it is not true any more at the quantum level. Indeed, what we have found is that the interactions between dressed (as opposed to than bare) instantons and anti-instantons after one takes into account their classical and quantum interactions, after integration over their all possible sizes and color orientations, after accounting for the interaction with the background condensates \( X \) and \( Y \) in Eq. (2) (which by itself is nonzero due to the same (anti)instantons as well as due to purely perturbative interactions), must become very simple at large distances as explicitly described by Eq. (18). One could be really amazed how the problem which looks so complicated in terms of the original bare (anti)instantons, becomes so simple in terms of the dressed (anti)instantons when all integrations over all possible sizes, color orientations and interactions with background fields are properly accounted for!

Such a simplification of the interactions is of course due to the presence of an almost massless pseudo-Goldstone boson \( \eta \) which couples to the topological charge. When the instanton gas becomes very dilute all semiclassical interactions (due to zero modes) cannot contribute much, since they fall off with distance faster then the Coulomb interaction mediated by \( \eta \). On the other hand, when the instanton density increases (at lower baryon densities), the Coulomb interaction becomes more screened and, as the Debye length becomes comparable to the inter-instanton distances, we lose analytical control.

Another interesting observation is that the Gauss law manifests itself in the CG partition function by suppressing all configurations with nonzero net charge \( Q_{\text{net}} \). (The weight of such configurations is suppressed by \( \exp(-\#Q_{\text{net}}^2 / L^2) \), where \( L \) is the linear size of the system. This restricts fluctuations of \( Q_{\text{net}} \) to \( O(L) \), negligible compared to normal thermodynamic size \( O(L^2) \).) In QCD vacuum this corresponds to the suppression of the topological charge fluctuations in the chiral limit. Nonzero quark masses give the \( \eta \) particle a mass, turning the long-range Coulomb interaction into a Yukawa one, thus unfreezing fluctuations of \( Q_{\text{net}} \), in agreement with a well known result that topological charge susceptibility is proportional to quark masses (to the product \( m_u m_d \) in our case (4)).
V. CONCLUSIONS AND OUTLOOK

In this paper we study a regime of QCD where the instanton dynamics acquires analytical control — the high baryon density regime. We showed that the partition function \( \langle \mathcal{Z} \rangle \) of the Coulomb gas of instantons is the dual representation of low-energy high-density QCD \( \langle \mathcal{Z} \rangle \) as far as the ground state structure, in particular, its theta dependence, is concerned. We discuss the correspondence between these two representations. The most important physical result is a rather straightforward identification of the charges \( Q_a \) from the statistical ensemble \( \langle \mathcal{Z} \rangle \) with the well-known BPST-instantons \( [15,16] \) which have been under intensive study since 1970’s. We have explicitly demonstrated that at large distances instantons and anti-instantons interact like point-like charged particles. This is in contrast with standard semiclassical approximation when \( II \) and \( I\bar{I} \) interactions are very different from each other.

A similar correspondence between a formal expression for a statistical ensemble of particles and the instanton quarks suspected long ago \( [17] \) was conjectured recently \( [13] \) based on the analysis of the multi-instanton measure. As we have said, no theoretical control was possible in Ref. \( [13] \) to justify that conjecture. Only in the high-density regime considered in the present paper the relation between \( \eta \) physics and dual representation of the ensemble of instantons can be made precise. The properties of the instanton ensemble which we find should be of interest for numerical instanton simulations.

It is possible to generalize our results to the color-flavor-locking (CFL) state of \( N_f = 3 \) QCD \( [18] \). The \( U(1)_A \) symmetry is also spontaneously broken in this case. The role of the \( \eta \) boson is played by the \( \eta' \) meson, which is also light at high densities \( [18,1] \). The instanton-induced \( \eta' \) potential has a form similar to \( [1] \) \( [19,2] \):

\[
V_{\text{inst}}(\phi) = -a' \cdot \mu^2 \Delta^2 \cos(\phi - \theta),
\]

where the evaluation of dimensionless function \( a' \) is very much the same as our calculation of \( a \) in Sec. \( [1] \) or Ref. \( [4] \). We need to insert an extra factor \( m_s \rho \) into \( (10) \) and use \( N_f = 3 \) in \( g(\rho) \) in \( (11) \) to find:

\[
a' = 7 \times 10^3 \left( \frac{m_s}{\mu} \right) \left( \ln \frac{\mu}{\Lambda_{\text{QCD}}} \right)^7 \left( \frac{\Lambda_{\text{QCD}}}{\mu} \right)^9 .
\]

Finally, the same analysis can be also carried in the case of QCD at large isospin density \( \mu_I \) \( [20] \). The instanton interactions can be also shown to be Coulomb-like and the diluteness of the gas is achieved at large \( \mu_I \). In contrast to finite baryon density case, such a system can be simulated on the lattice today.

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