Leveraging Conflicting Constraints in Solving Vehicle Routing Problems

Sabino Francesco Roselli and Remco Vader and Martin Fabian and Knut Åkesson

Abstract—The Conflict-Free Electric Vehicle Routing Problem (CF-EVRP) is a combinatorial optimization problem of designing routes for vehicles to visit customers such that a cost function, typically the number of vehicles or the total travelled distance, is minimized. The CF-EVRP involves constraints such as time windows on the delivery to the customers, limited operating range of the vehicles, and limited capacity on the number of vehicles that a road segment can simultaneously accommodate. In previous work, the compositional algorithm ComSat was introduced and that solves the CF-EVRP by breaking it down into sub-problems and iteratively solve them to build an overall solution. Though ComSat showed good performance in general, some problems took significant time to solve due to the high number of iterations required to find solutions that satisfy the road segments’ capacity constraints. The bottleneck is the Path Changing Problem, i.e., the sub-problem of finding a new set of shortest paths to connect a subset of the customers, disregarding previously found shortest paths. This paper presents an improved version of the PathsChanger function to solve the Path Changing Problem that exploits the unsatisfiable core, i.e., information on which constraints conflict, to guide the search for feasible solutions. Experiments show faster convergence to feasible solutions compared to the previous version of PathsChanger.

I. INTRODUCTION

We consider scheduling a fleet of mobile robots, in the sequel referred to as Automated Guided Vehicles (AGVs), that pick-up and deliver components to workstations within specified time-windows. The AGVs move on a predefined road network, where each road segment has a maximum number of AGVs it can accommodate at a specific time. The problem is motivated by an industrial need to develop more flexible logistic systems to deliver components just-in-time to an assembly line.

In this scenario, in addition to time-windows in which the components should be delivered, a scheduler needs to consider additional constraints. First, AGVs have a limited operating range and need to recharge their battery when the state-of-charge becomes low. Second, jobs have specific requirements on the AGV eligible to execute them. Finally, the number of AGVs on road segments and workstations are limited to allow low-level trajectory planning problems to be feasible. Thus, we define the capacity of the road segments, intersections, and workstations and include capacity constraints. A schedule is said to be conflict-free if it fulfills the capacity constraints at all times.

The problem of computing conflict-free routes was first introduced in [1] and tackled by means of column generation. In [2], conflict-free routing in combination with scheduling of jobs for flexible manufacturing systems is discussed. An ant colony algorithm is applied to the problem of job shop scheduling and conflict free routing of AGVs by [3]. In [4], a collision-free path planning for multi AGV systems based on the A* algorithm is presented. Another heuristic approach to solve the conflict-free routing problem with storage allocation is presented by [5]. In [6], a MILP formulation to design conflict-free routes for capacitated vehicles is presented. In [7], is presented a hybrid evolutionary algorithm to deal with conflict-free AGV scheduling in automated container terminals, and [8] handles the problem of conflict-free routing of AGVs by a meta-heuristic improvement strategy based on large neighbourhood search. Hence, conflict-free routing and scheduling has been addressed previously, but to the best of our knowledge, there is no work in the literature that tackles all above mentioned constraints at once. Therefore, [9] introduced the Conflict-Free Electric Vehicle Routing Problem (CF-EVRP). The CF-EVRP is an extension of the vehicle routing problem (VRP) [10], involving the additional constraints. In [11], a compositional algorithm, ComSat, for solving the CF-EVRP is proposed. ComSat breaks down CF-EVRP into sub-problems and iteratively solves these to find a feasible solution to the overall problem. Experimental and analytical evaluation shows that ComSat generates high-quality but not necessarily optimal solutions. Briefly, ComSat computes routes to serve the customers, and assigns vehicles to the routes attempting to make the execution of the system conflict-free. In a plant there can be several ways to travel from one customer’s location to another. Initially, ComSat uses the shortest paths among the customers’ locations when designing the routes. However, if a feasible schedule cannot be achieved using the shortest paths, alternative paths have to be found, which is handled by the Conflict-free Paths Search (CFPS). CFPS is composed of two main functions; the PathsChanger function, that finds alternative sets of paths if the current schedule violates the capacity constraints, and the CapacityVerifier function, that checks whether the schedule is conflict-free or not.

Experiments show that when a solution computed using the shortest paths violates the capacity constraints, finding alternative paths using the PathsChanger function may require multiple iterations. This does not come unexpected, since the number of possible paths in a graph can be high, and minimizing the cumulative length while looking for alternative paths does not guarantee that the schedule will be conflict-free. In this paper we focus on the CFPS and present improved versions of the PathsChanger and CapacityVerifier that, in many cases, find feasible solutions faster.

We gratefully acknowledge financial support from Chalmers AI Research Centre (CHAIR), ITEA3-projekt AIToC (Artificial Intelligence supported Tool Chain in Manufacturing Engineering), and the Wallenberg AI, Autonomous Systems and Software program (WASP) funded by the Knut and Alice Wallenberg Foundation. 1Department of Electrical Engineering, Chalmers University of Technology, Göteborg, Sweden {rasabino, fabian, knut}@chalmers.se. 2Department of Mechanical Engineering, Eindhoven University of Technology, Netherlands r.m.vader@student.tue.nl
The sub-problems in ComSat are modelled as Satisfiability Modulo Theory (SMT) problems [12], [13], as SMT solvers have shown to be efficient in solving combinatorial problems [14].

Moreover, some SMT solvers come with algorithms that allow them to deal with optimization problems [15]. Two sub-problems in ComSat, marked by the round boxes in Fig. 1 (see below) are optimization problems.

For the CFPS polynomial time algorithms exist to find paths in graphs, [13]. However, modelling the Path Changing Problem as an SMT problem is beneficial as it allows to define problem-specific requirements, such as not returning solutions that are already proven infeasible because they violate the capacity constraints. Moreover, when a problem is infeasible, SMT solvers have the ability to return a Minimal Unsatisfiable Core (MUC) [17], i.e., one of the (possibly many) smallest subsets of constraints that make the problem infeasible. The MUC can provide useful information about why a problem is infeasible and can therefore be used to guide the search towards a feasible solution [13].

When dealing with the CF-EVRP, the MUC can be extracted when the Capacity Verification Problem is infeasible and used to define additional constraints for the Path Changing Problem, to increase the chances of finding a feasible schedule.

The contributions in this paper are: (i) exploitation of SMT solvers’ MUC to extract information about the infeasibility of an SMT formula representing a conflicting schedule for a VRP; (ii) use of such information to find conflict-free schedules; (iii) performance comparison between the unguided and MUC guided paths search over a set of CF-EVRP problem instances.

The remainder of the paper is organized as follows. Preliminaries are presented in Section II. Section III presents the mathematical models of the sub-problems that form the CFPS and how it is improved using the MUC from the Capacity Verification Problem. Proof of soundness and completeness of the procedure is provided in Section IV. In Section V the results of the analysis over a set of problem instances are presented. Finally, conclusions are drawn in Section VI.

II. PRELIMINARIES

In the CF-EVRP the plant layout is represented by a finite, strongly connected, weighted, directed graph, where edges represent road segments and nodes represent either intersections between road segments or customers’ locations. A customer is defined by a unique (numerical) identifier, a location, and a time window, i.e., a lower and upper bound that represent the earliest and latest arrival time allowed to serve the customer. Edges have two attributes, the first representing the road segment’s length, and the second its capacity. The capacity is 2 if two vehicles can simultaneously travel in opposite directions, 1 otherwise.

The following definitions are provided:

- **Node**: a location in the plant. A node can only accommodate one vehicle at a time unless it is a hub node that can accommodate an arbitrary number of vehicles.
  - \( N \): a finite set of nodes.
  - \( N_H \subseteq N \): the set of hub nodes.

- **Edge**: a road segment that connects two nodes.
  - \( E \subseteq N \times N \): the finite set of direct edges.
  - \( \bar{e} \): the reverse edge of edge \( e \in E \).
  - \( d_e \in \mathbb{R}_+ \): the length of edge \( e \in E \).
  - \( g_e \in \{1, 2\} \): the capacity of edge \( e \in E \).

- **Time horizon**: a fixed, continuous point of time when all jobs have ended, assuming they start at time 0.
  - \( T \): the time horizon.

- **Customer**: entity representing a task to be executed by a vehicle, e.g., a pickup or delivery of material, that needs to be visited exactly once by the vehicle. A customer is always associated with a node where the pickup/delivery operation is executed, and has a time window indicating the earliest and latest time at which it can be visited. Unless explicitly given, the time window is the entire time span [0, T].
  - \( K \): the finite set of all customers, and let \( l_k, u_k \in \mathbb{R} \), \( k \in K \) be the time window’s lower (\( l_k \)) and upper (\( u_k \)) bound for customer \( k \) such that \( u_k > l_k \). Also let \( s_k \in \mathbb{R}_+ \) and \( l_k \in N \), for \( k \in K \), be the service time and location of customer \( k \), respectively.
  - **Route**: an ordered set of unique customers.
    - \( r \subseteq K \), \( m \leq |K| \)
  - **Route set**: a set of routes such that each customer belongs to exactly one route, thus guaranteeing that all customers are served.
    - \( R = \{r_1, \ldots, r_m\}, m \leq |K| \)
  - A route contains at least one customer, hence \( m \leq |K| \).
  - **Route start**: the starting time \( \tau_r \) of route \( r \), computed by the function \( \text{Assign} \). \( \Gamma \) is the set that contains the route start of each route.
    - \( \Gamma = \{\tau_r \in \mathbb{R} | r \in R\} \)
  - **Pair Set of route** \( r \): set containing the sequence of customers of a route \( r = \{k_1, \ldots, k_m\} \), grouped as pairs in sequence.
    - \( \mathcal{P}_r = \{(k_1, k_2), (k_3, k_4), \ldots, (k_{m-1}, k_m)\} \)
  - **Path**: ordered set of unique nodes. It is used to keep track of how vehicles are travelling among customers of routes, since each pair of customers in a route is connected by a path.
    - \( \theta_p = \{n_1, \ldots, n_m\}, p \in \mathcal{P}_r, m \leq |N| \)
    - \( n_i \in N, i = 1, \ldots, m \)
  - **Edge sequence**: ordered set of unique edges for a given path \( \theta_p \).
    - \( \delta_p = \{e_1, \ldots, e_m\}, p \in \mathcal{P}_r, m = |\theta_p| - 1 \)
    - \( e_i \in E, i = 1, \ldots, m \)

In order to clarify which part of ComSat is analyzed and improved in this work, let us recap briefly how the algorithm works. Fig. 1 shows a simplified flowchart of ComSat that illustrate the concepts of this paper. The first step of ComSat is to design a set of routes \( R \) to serve all the customers; at this point, the shortest path between any two customers is computed using Dijkstra’s algorithm [19].
This optimization problem is handled by the function Router and must guarantee that the routes meet specific requirements such as maximum length, specific ordering among the customers and time windows. If this step is infeasible the CF-EVRP instance has no solution and the algorithm terminates. If this step is feasible, the function Assign will try to allocate available vehicles to the routes and compute a start time \( \tau_r, \forall r \in R \), to the routes. If this step is infeasible then Router will try to find different routes, but if it is feasible, the CapacityVerifier checks if the current set of routes is conflict-free. More details on the functions Router and Assign can be found in [11].

### A. The minimal Unsat Core

For infeasible problems, there can be identified a subset of the constraints that conflict, meaning they cannot all simultaneously be satisfied. Such a subset is called an Unsat Core. An Unsat Core with the property that removing any one of the constraints makes the Unsat Core feasible, is said to be minimal.

Formally, given an SMT formula \( \varphi \) and set of conflicting constraints \( C \subseteq \varphi \), \( C \) is a MUC of \( \varphi \) if removing any constraint \( C_i \in C \) makes \( C \setminus C_i \) no longer infeasible; removing \( C \) removes the particular conflict represented by the MUC. Consequently, for an infeasible problem with a MUC \( C \), adding to the problem a constraint that prevents all the constraints in \( C \) to be simultaneously active will resolve this particular conflict.

The naive approach to MUC extraction, [20], successively removes constraints and solves the problem again; if the problem is still infeasible after a constraint has been removed that constraint does not belong to a MUC. There exist more efficient approaches though; the MUC [21] algorithm based on efficient manipulation of Binary Decision Trees guarantees the extraction of a minimal Unsat Core. [22] presents an algorithm based on the resolution graph [23] for MUC extraction. [24] improves the resolution based algorithm using model rotation and path strengthening.

### III. THE CONFLICT-FREE PATHS SEARCH

In this section the two sub-problems that form the CFPS are presented. The Capacity Verification Problem is modelled as a job shop problem (JSP), in order to exploit the good performance of the SMT solver Z3 [25] in dealing with JSPs, as demonstrated in [26]. The model formulation for the Path Changing Problem is inspired by [27].

The following logical operators are used as a shorthand to express cardinality constraints [28] in the sub-problems:

1. \( \text{EN}(A, n) : \) exactly \( n \) variables in the set \( A \) are true;
2. \( \text{If}(c, o_1, o_2) : \) if \( c \) is true returns \( o_1 \), else returns \( o_2 \).

We will write \( \text{EN}_{m \in M}(m, n) \) to denote \( \text{EN}( \bigcup_{m \in M} \{ m \}, n) \) in order to shorten the notation.

#### A. The Capacity Verification Problem

The Capacity Verification Problem aims to find a feasible schedule for the vehicles, where the routes that the vehicles are assigned to satisfy the capacity constraints of the edges.

In this work the Capacity Verification Problem, as defined in [11], has been extended to account for pairs as well, since the information about conflicts must be related to a specific pair to define additional constraints in the PathsChanger.

Let \( vp_n \) be the node visited before edge \( e \) of pair \( p \) of route \( r \), and let \( e_{pn} \) be the node visited before node \( n \) on pair \( p \) of route \( r \). Similarly, let \( e_{pn}^r \) be the node visited after edge \( e \) of pair \( p \) of route \( r \), and let \( e_{pn}^r \) be the edge visited after node \( n \) on pair \( p \) route \( r \). Let \( p_{1}^r \) be the first pair of route \( r \) and \( n_{r}^* \) be its starting node.

#### Example of Routes, Pairs, Nodes, and Edges:
Let \( K = \{ k_1, \ldots, k_7 \} \) and \( N = \{ n_1, \ldots, n_{20} \} \). Let \( L_{k_3} = n_{11} \) and \( L_{k_2} = n_{17} \), and assume two routes designed to serve all customers: \( r_1 = \langle k_1, k_2, k_5, k_7 \rangle \), \( r_2 = \langle k_3, k_4, k_6 \rangle \).

In order to clarify the notation introduced above, let us analyze \( r_1 \). First, the set of pairs for \( r_1 \) is defined as \( P_{r_1} = \{ \langle k_1, k_2 \rangle, \langle k_2, k_3 \rangle, \langle k_3, k_7 \rangle \} \).

Then, let us assume that the path and edge sequence for pair \( \langle k_1, k_2 \rangle \) are the following:

\[
\begin{align*}
\theta_{k_1,k_2} &= \langle n_1, n_2, n_4, n_5, n_7 \rangle, \\
\delta_{k_1,k_2} &= \langle n_1, n_2, n_4, n_5, n_7, n_5 \rangle.
\end{align*}
\]

Then \( p_{r_1}^0 = \langle k_1, k_2 \rangle \) and \( n_{r_1}^0 = n_{11} \). Also, let \( p = \langle k_1, k_2 \rangle \); then for \( e = \langle n_1, n_2 \rangle \), \( n_{r_1} = n_{11} \) and \( n_{r_1}^e = n_2 \); for \( n = n_{11} \), \( e_{r_1} = \langle n_1, n_2 \rangle \), and for \( n = n_{12} \), \( e_{r_1} = \langle n_1, n_2 \rangle \).

For each node it must also be specified whether there exists a time window, since some of the nodes are only intersections of road segments in the real plant, while others are actual customers. Let \( l_{pn} \) and \( u_{pn} \) be the earliest and latest arrival time, respectively, at node \( n \) of pair \( p \) of route \( r \); let \( s_{rpn} \) be the service time at node \( n \) of pair \( p \) of route \( r \). Finally, let \( \gamma > 0 \) be a small real constant used to prevent swapping of vehicles’ positions between a node and the previous or following edge.

The Capacity Verification Problem decision variables are:
x_{pni}: non-negative real variable that models when a vehicle executing route r starts using node n in pair p;
y_{rpe}: non-negative real variable that models when a vehicle executing route r starts using edge e in pair p.

The model for the **Capacity Verification Problem** is:

\[ x_{rpn} \geq \tau_r, \forall r \in \mathcal{R} \]  
(1)

\[ y_{rpe} \geq x_{pni} + x_{pni}, \forall r \in \mathcal{R}, p \in \mathcal{P}_r, e \in \delta_p \]  
(2)

\[ x_{rpn} = y_{rpe} + d_{c_{rpn}}, \forall r \in \mathcal{R}, p \in \mathcal{P}_r, n \in \delta_p \]  
(3)

\[ x_{rpn} \geq \tau_{rpn} \land x_{rpn} \leq \tau_{rpn}, \forall r \in \mathcal{R}, p \in \mathcal{P}_r, n \in \delta_p \]  
(4)

\[ x_{rpn} \geq y_{rpn} + \gamma \lor x_{rpn} \geq y_{rpn} + d_{c_{rpn}} + \gamma, \forall r \in \mathcal{R}, r \neq r2, p \in \mathcal{P}_r1, p2 \in \mathcal{P}_r2 \]  
(5)

\[ y_{rpe} \geq y_{rpe} + \gamma \lor y_{rpe} \geq y_{rpe} + \gamma, \forall r \in \mathcal{R}, r \neq r2, n \in \mathcal{N}_r \]  
(6)

\[ y_{rpe} \geq y_{rpe} + d_{e_{rpe}} \lor y_{rpe} \geq y_{rpe} + d_{e_{rpe}} + \gamma, \forall r \in \mathcal{R}, r \neq r2, p \in \mathcal{P}_r, p2 \in \mathcal{P}_r2, e \in \delta_p1 \land \delta_p2 \]  
(7)

1. constrains the start time of a route; 2. and 3. define the precedence among nodes and edges to visit in a route; 4. enforces time windows on the nodes that correspond to the customers; 5. prevents vehicles from using the same node at the same time; 6. and 7. transit the constraint of vehicles over the same edge. If two vehicles are using the same edge from the same node, one has to start at least \( \gamma \) after the other and if two vehicles are using the same edge from opposite nodes, one has to fully transit before the other one can start.

Based on the model described above, the algorithm **CapacityVerifier (CV)** is defined, that takes a set of routes \( \mathcal{R} \), the start times in \( \Gamma \), and the current set of paths \( CP \) as input and returns:

- **CFS**, a list that expresses where each vehicle is at each time; this is empty if the problem is infeasible.
- \( \mathcal{C} \), the **Unsat Core** relative to constraints 5-7 (see Section III.C); this is empty if the problem is feasible.

**B. Paths Changing Problem**

In the **Paths Changing Problem**, alternative paths are computed to connect the consecutive customers of each route. Finding alternative paths may be necessary when, for a given set of routes \( \mathcal{R} \) and starting times \( \Gamma \), no feasible schedule exists. The **Capacity Verification Problem** may be infeasible due to the current set of paths that connect the customers’ locations, therefore a different set may lead to a feasible solution. A route is defined as a sequence of customers, and for any two consecutive customers there is a path (a sequence of edges) connecting them. Therefore, for a route containing \( i + 1 \) customers we will have \( i \) paths and for each path we can define a start and an end node, \( \xi_i \) and \( \pi_i \), respectively. The sets of outgoing and incoming edges for a certain node \( n \) are denoted \( O_n \) and \( I_n \), respectively.

Decision variables used to build the model are:

- \( w_{rpn} \): Boolean variable that represents whether the pair \( p \) of route \( r \) is using node \( n \);
- \( z_{rpe} \): Boolean variable that represents whether the pair \( p \) of route \( r \) is using edge \( e \).

This problem can be split into \( r \cdot i \) sub-problems (assuming all routes have \( i + 1 \) customers) that find paths for each route separately; simpler and smaller models are faster to solve. Unfortunately it may be necessary to explore different combinations of paths, so to retain the information we have only one model. Therefore, let the optimal solution to the **Path Changing Problem** found at iteration \( h \) be

\[ CP = \bigcup_{r \in \mathcal{R}, p \in \mathcal{P}_r, e \in \mathcal{E}} \{ z_{rpe}^* \}, \]

where \( z_{rpe}^* \) is the value of \( z_{rpe} \) in the current solution; also, let \( PP \) be the set containing the optimal solutions found until the \((h-1)\)-th iteration. The model is then:

\[ \min_{r \in \mathcal{R}, p \in \mathcal{P}_r, n \in \mathcal{N}_r} \sum_{r \in \mathcal{R}, p \in \mathcal{P}_r, r \in \mathcal{R}} \sum_{r \in \mathcal{R}, p \in \mathcal{P}_r, e \in \mathcal{E}} \text{If}(z_{rpe}, d_{e}, 0) \]

(8)

\[ \text{EN}_{e \in \mathcal{O}_n}(z_{rpe}, 1), \forall p \in \mathcal{P}_r, r \in \mathcal{R} \]

(9)

\[ \text{EN}_{e \in \mathcal{I}_n}(z_{rpe}, 1), \forall p \in \mathcal{P}_r, r \in \mathcal{R} \]

(10)

\[ \text{EN}_{e \in \mathcal{I}_n}(z_{rpe}, 0) \land \text{EN}_{e \in \mathcal{O}_n}(z_{rpe}, 0), \forall p \in \mathcal{P}_r, r \in \mathcal{R} \]

(11)

The cost function 8 to minimize is the cumulative length of the used edges; 9 guarantees that, for each path of each route, the start and end nodes are used; 10 and 11 make sure that exactly one outgoing (incoming) edge is incident with the start (end) node of a route; 12 makes sure that a path is not allowed to use both an edge and its reverse; 13 guarantees that if a node (different from the start or end) is selected, exactly one of its outgoing and one of its incoming edges will be used. On the other hand, if a node is not used, none of its incident edges will be used; finally, 14 rules out all the previously found solutions.

Based on the model described above the function **Paths-Changer (PC)** is defined, that takes the previous paths \( PP \) as input and returns a new set of paths \( NP \). If the **Paths Changing Problem** is infeasible then \( NP = \emptyset \).

Up to this point, unless specified otherwise, the models presented are taken from [11].

**C. Exploiting the MUC**

Experiments reported in [11], show that ComSat performs well for many problem instances, however, for some specific instances ComSat failed to find feasible solutions in reasonable time. Investigations revealed the **PC** to be the culprit. The reason is that it searches blindly through the possible paths that connect any two customers, while minimizing the paths’
cumulative length. A conflict-free solution may involve paths that are quite longer than the current ones though, and the PC will have to explore many shorter solutions before finding the right one. Improving the performance of the PathsChanger would be beneficial for the overall performance of ComSat, and letting the MUC guide the paths changing is such an improvement.

When extracting the MUC, it is possible to only track specific constraints. This feature can be exploited to focus only on the capacity constraints violations. In fact, since time windows and service time are not flexible, it is of little use to track constraints represented by $\mathcal{C}_1-\mathcal{C}_4$. Also, an infeasible formula $\varphi$ may have multiple MUCs; in the CF-EVRP this means that conflicts may arise at different locations in the plant. In order to catch all of them, it is possible to iteratively relax the conflicting constraints from the initial formula and solve it again, until it becomes feasible. The formula will indeed become feasible eventually, since it is based on a feasible solution $\mathcal{R}$ and only the capacity constraints can make it infeasible; in the worst case all such constraints will be removed during the iterations. Note that, since not all constraints are tracked, the set of constraints $\bar{\mathcal{C}}$ returned is not an actual Unsat Core, since $\bar{\mathcal{C}}$ would only make the problem infeasible in conjunction with the untracked constraints. Nonetheless, it provides the information about the conflicts needed to guide the search of paths.

Let $\varphi_0$ be the conjunction of constraints $\mathcal{C}_1-\mathcal{C}_7$. Assume that $\varphi_0$ is infeasible, and let $\mathcal{C}_0$ be the subset of a MUC retrieved by tracking constraints $\mathcal{C}_5-\mathcal{C}_7$. Then let $\varphi_1 = \varphi_0 \setminus \mathcal{C}_0$, also infeasible, and let $\mathcal{C}_1$ be the subset of a MUC retrieved by tracking constraints defined by $\mathcal{C}_5-\mathcal{C}_7$, not including the ones in $\mathcal{C}_0$. In general, the constraints in $\mathcal{C}_{i-1}$ can be iteratively relaxed to obtain a new formula $\varphi_i$, until a feasible $\varphi_n = \varphi_0 \setminus (\mathcal{C}_0 \cup \ldots \cup \mathcal{C}_{n-1})$ is found. Then $\bar{\mathcal{C}} = \mathcal{C}_0 \cup \ldots \cup \mathcal{C}_{n-1}$ contains all the conflicts due to the capacity constraints.

Each constraint represented by $\mathcal{C}_5-\mathcal{C}_7$ is defined over two routes $r_1$ and $r_2$ and their pairs $p_1$ and $p_2$ for a specific node $n$ or edge $e$; therefore, if the constraint is part of $\bar{\mathcal{C}}$, the routes and pairs that caused the conflict over $n$ or $e$ can be identified. If the conflict was generated by a constraint from $\mathcal{C}_5$, then the following constraint is added to $\mathcal{C}_5-\mathcal{C}_7$:

$$\neg(w_{r_1p_1n}) \vee \neg(w_{r_2p_2n}).$$ (15)

On the other hand, if the conflict was caused by constraint from $\mathcal{C}_6$ or $\mathcal{C}_7$, the following constraint is added to $\mathcal{C}_5-\mathcal{C}_7$:

$$\neg(z_{r_1p_1e}) \vee \neg(z_{r_2p_2e}).$$ (16)

Constraints (15) and (16) force at least one of the routes involved in the conflict to avoid the specific node (edge, respectively) when computing a path for the pairs involved in the conflict. The constraint is formulated so that the choice of the route to change is left to the solver, including the possibility of changing both routes; since the problem is an optimization, the solver will choose the change that leads to the shortest cumulative paths length.

Based on the model described by (5)-(16), the function $\text{MUC-Guided-Paths-Changer (GPC)}$ is defined, that takes the previous paths $PP$ and $\bar{\mathcal{C}}$ as input and returns a new set of paths $NP$. If the Path Changing Problem is infeasible $NP = \emptyset$.

Since for each constraint in $\bar{\mathcal{C}}$ a new constraint is added to the GPC, it is imperative that the Unsat Core returned when the CV is infeasible is minimal. This is so because if the Unsat Core is not minimal, it could contain constraints that are not actually causing capacity conflicts. These constraints would in turn lead to defining constraints (15) and (16) in the GPC that may remove feasible solutions.

Fig. 2 summarizes the steps required to find a conflict-free schedule $CFS$, if such exists, using the improved paths searching algorithm GPC. As mentioned, it is assumed that routes $\mathcal{R}$ and their start times $\Gamma$ have already been computed. The shortest paths between any two customers are computed using Dijkstra’s algorithm and then set as the current paths $CP$ to travel among customers. Also, $CP$ are added to the list of previous paths $PP$.

Then the CV will check such routes against the capacity constraints; if this sub-problem has a feasible solution the algorithm terminates and a conflict-free schedule is returned. Otherwise $\bar{\mathcal{C}}$ is extracted as described in the previous paragraph and the the GPC algorithm is invoked. GPC will use the information about previously computed paths $PP$ and the information about conflicts from $\bar{\mathcal{C}}$ to compute new paths $NP$, which will be set as the current paths and stored in $PP$. At this point the CV is run again using the new paths. The iterations between the two algorithms continue until either the CV is feasible, or the GPC is infeasible, i.e., there are no feasible, conflict-free paths to execute the routes $\mathcal{R}$ with the start times $\Gamma$.

IV. PROOF OF SOUNDNESS AND COMPLETENESS

In this section, proof of soundness and completeness of the Unsat Core Guided CFPS is provided. The underlying idea for the proof is the following. There exists a finite number of solutions to the Path Changing Problem; the GPC can enumerate at least all feasible solutions to the Path Changing Problem; if a solution that satisfies the Capacity Constraints does exists, the GPC will eventually find it, otherwise it will declare the problem infeasible.
Let \( S \) be the set of possible solutions to a Path Changing Problem; let us divide \( S \) into the set of conflict-free solutions \( \mathcal{F} \) and the set of conflicting solutions \( \mathcal{U} \). In other words a solution to the Path Changing Problem from \( \mathcal{F} \) will make the Capacity Verification Problem feasible, while a solution from \( \mathcal{U} \) will not. If the CFPS is infeasible, then \( S = \mathcal{U} \) and \( \mathcal{F} = \emptyset \). In this case, even if the GPC is not able to find all feasible solutions \( \mathcal{F} \), there is none to find.

In case the CFPS is feasible though, in order to prove completeness it is necessary to guarantee that at least all feasible solutions \( \mathcal{F} \) can be found by GPC. This is proven for the PC, since each call of the PC function will find the next optimal solution to the Path Changing Problem, whether it belongs to \( \mathcal{F} \) or not, until all solutions are enumerated. However in the GPC there are additional constraints that may remove feasible solutions. In the proof it is shown that such additional constraints only remove infeasible solutions.

Observation 1: The Path Changing Problem is a satisfiability problem in propositional logic. The Capacity Verification Problem falls into the category of difference logic (a fragment of linear arithmetic). Thus, both problems are decidable.

Observation 2: The Path Changing Problem is bounded. In fact, the Path Changing Problem involves only a finite number of Boolean variables, so its domain is finite.

Lemma 1: Given a finite, directed, weighted graph, the number of paths that connect two arbitrary nodes is finite.

Proof: By definition, a path is an ordered set of nodes such that no node appears more than once. If the number of nodes in the graph is finite, there cannot be an infinite number of paths.

Lemma 2: For a given set of routes \( \mathcal{R} \) and start times in \( \Gamma \), repeated calls to the PC function will enumerate all feasible solutions to the Path Changing Problem, either belonging to \( \mathcal{F} \) or \( \mathcal{U} \), before returning infeasible.

Proof: Let \( \varphi_0 \) be the conjunction of constraints (6)-(13), a relaxation of the Paths Changing Problem, and let \( CP_0 \) be a solution to \( \varphi_0 \). Then, if another solution \( CP_1 \) for \( \varphi_0 \) exists, it can be found by solving \( \varphi_0 \land \neg CP_0 = \varphi_1 \). In general, the \( n \)-th solution can be found by solving \( \varphi_0 \land \neg CP_0 \land \ldots \land \neg CP_{n-1} = \varphi_n \). Because of Lemma 1 we know that the number of solutions to the Paths Changing Problem, \( |S| \), is finite and we can enumerate them all by solving \( \varphi_0, \ldots, \varphi_{|S|-1} \).

Lemma 3: Using the PC and CV is a sound and complete procedure to solve the CFPS

Proof: Because of Observation 1 we know there is a finite number of solutions to the Path Changing Problem, and because of Lemma 2 we know that the PC function can enumerate them all. If a solution that belongs to \( \mathcal{F} \) exists the PC will find it, otherwise it will return all solutions belonging to \( \mathcal{U} \); the CV will then check whether they are conflict-free. Therefore, using the PC and CV in combination will correctly solve the CFPS.

Lemma 4: For a given set of routes \( \mathcal{R} \), the GPC is able to find at least all solutions in \( \mathcal{F} \).

Proof: For each set of current paths \( CP \), \( \tilde{C} \) only contains constraints defined by (5), (6), and (7). The constraints in \( \tilde{C} \) are iteratively retrieved from minimal Unsat Core and therefore represent combinations of nodes and edges in the graph where the conflicts happen. Since each constraint defined by (15) and (16) addresses one constraint from \( \tilde{C} \), (15) and (16) only define constraints over nodes or edges that cause conflicts. Hence these constraints only remove solutions of the Path Changing Problem that belong to \( \mathcal{U} \).

Theorem 1: Using the GPC and CV is a sound and complete procedure to solve the CFPS.

Proof: The PC and the GPC are identical, except for constraints (15)-(16), and because of Lemma 4 we know that the addition of these constraints only removes solutions from \( \mathcal{U} \). Thus, since the CFPS using the PC is sound and complete (Lemma 4), so is the CFPS using the GPC.

V. EXPERIMENTS

In order to evaluate the goodness of the proposed method and its performance against the previous version of the CFPS algorithm, a set of problem instances is designed and used for testing. Both the PC and GPC are embedded in the ComSat algorithm. However, since the goal is to compare the search for alternative paths, problems are designed in such a way that there is only one feasible set of routes \( \mathcal{R} \) to serve the customers; also, only the running time for search of conflict-free paths is measured. The algorithms called by ComSat used the SMT solver Z3 4.8.9 to solve the models. All the experiments were performed on an Intel Core i7 6700K, 4.0 GHz, 32GB RAM running Ubuntu-18.04 LTS.

Table I shows the results of the evaluation of five problem instances of the CF-EVRP solved using ComSat. Each instance was solved twice, once using the PC and once using the GPC; in each case the number of iterations and the time (in seconds) required to find a feasible solution is reported. The problem instances presented are increasingly hard to solve, in terms of plant size (represented by the number of nodes), number of routes and number of customers in each route. The customers’ locations and time windows so that conflicts will arise due to the capacity constraint when the shortest paths are used and a search for alternative paths will be necessary in order to find a conflict-free schedule.

For instances 1 through 4 it took only one iteration to the GPC to find a feasible solution, while the PC required an increasing number of iterations to find a feasible solution, as the instances grew more complicated. The gap in the running time between the GPC and the PC follows the same trend; for instance 1 it only takes 2 iterations to the PC to find a feasible solution, while it takes 24 and 54 iterations to find a solution to instances 2 and 3. This number drops to 15 iterations for instance 4. On average, a single iteration of the PC takes less time than an iteration of the GPC, but due to the larger number of iterations required, the overall running time for the PC is always larger.

Instance 5 is the odd one out, as it only takes one iteration of the PC to find a feasible solution, and, as for the other instances, the running time for the single iteration is shorter.

Results and Discussion

The experiments show that for most of the instances the GPC performed better than PC in terms of running time and

---

1The implementation of the GPC presented in Section III-C and the problem instances are available in the UNSATCore folder at https://github.com/sabinoroselli/VRP.git.
number of iterations. To be more specific, one iteration of the GPC is slower than one iteration of the PC, but the number of iterations required by the PC is always higher, and therefore the overall execution time is longer. As the instances become larger, the gap between the running time for one iteration of each method increases too. However, since the number of iterations required for more complex instances grows as well, the GPC shows increasing good performance for harder-to-solve instances. On the other hand, Instance 5 shows a different result, since both the PC and the GPC take only one iteration. As for the other instances, a single iteration of the PC is faster, hence the PC beats the GPC on Instance 5. We can conclude that for some instances, the PC may be able to quickly find feasible solutions and outperform the GPC. However this is behaviour is highly dependent on the instance and as instances grow larger the chances could grow smaller, as the number of possible paths available increases. Moreover, a detailed analysis of the solutions to the Path Changing Problem for each instance confirms that, for the PC, there is no convergence to a feasible solution as the number of iterations increases, since the number of conflicts does not always decrease at the following iteration. On the other hand, the GPC shows a consistent behaviour as it always takes only one iteration to find feasible solutions.

TABLE I: Comparison of the PC and GPC over a set of instances of the CF-EVRP. For each instance the number of iterations and the total running time (in seconds) required to find a feasible solution is reported.

| Inst. | |N| |R| |K| |Iterations| |Time| |
|-------|-------------------------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|       | |      |      |      |      | |PC| |GPC| |PC| |GPC| |
| 1     | 3 | 2 | 4 | 2 | 1 |          | 0.25 |          | 0.16 | |
| 2     | 8 | 3 | 6 | 24 | 1 |          | 8.81 |          | 0.40 | |
| 3     | 5 | 5 | 4 | 8 | 54 | 1 |          | 35.92 |          | 1.08 | |
| 4     | 64 | 4 | 28 | 15 | 1 |          | 643.40 |          | 184.60 | |
| 5     | 64 | 4 | 28 | 1 | 1 |          | 21.20 |          | 128.40 | |

VI. CONCLUSIONS

This paper presents an algorithm to search for conflict-free paths for a set of routes to serve customers in a conflict-free electric vehicle routing problem (CF-EVRP). The algorithm exploits the SMT solvers’ ability to return a MUC when a formula is infeasible, to guide the search for paths. Soundness and completeness of the algorithm are proved, and preliminary experimental data based on a set of generated CF-EVRP problem instances are provided. The experiments show that the new MUC based algorithm consistently finds feasible paths taking only one iteration and significantly shorter time than the previous naive method. Future work includes to run extensive computational analyses to strengthen the claims made in this paper, and further development of the MUC guided paths search by improving the information extraction from the MUC.

REFERENCES

[1] N. N. Krishnamurthy, R. Batta, and M. H. Karwan, “Developing conflict-free routes for automated guided vehicles,” Operations Research, vol. 41, no. 6, pp. 1077–1090, 1993.

[2] A. I. Correa, A. Langevin, and L.-M. Rousseau, “Scheduling and routing of automated guided vehicles: A hybrid approach,” Computers & Operations Research, vol. 34, no. 6, pp. 1688–1707, 2007.

[3] M. Saidi-Mehrabad, S. Dehnavi-Arani, F. Evazabadian, and V. Mahmoudian, “An ant colony algorithm (ACA) for solving the new integrated model of job shop scheduling and conflict-free routing of AGVs,” Computers & Industrial Engineering, vol. 86, pp. 2–13, 2015.

[4] R. Yuan, T. Dong, and J. Li, “Research on the collision-free path planning of multi-AGVs system based on improved A* algorithm,” American Journal of Operations Research, vol. 6, no. 6, pp. 442–449, 2016.

[5] E. Thanos, T. Wauters, and G. Vandendriessche, “Dispatch and conflict-free routing of capacitated vehicles with storage stack allocation,” Journal of the Operational Research Society, pp. 1–14, 2019.

[6] K. Murakami, “Time-space network model and MILP formulation of the conflict-free routing problem of a capacitated AGV system,” Computers & Industrial Engineering, vol. 141, p. 106270, 2020.

[7] M. Zhong, Y. Yang, Y. Dessouky, and O. Postolache, “Multi-AGV scheduling for conflict-free path planning in automated container terminals,” Computers & Industrial Engineering, vol. 142, p. 106371, 2020.

[8] Z. Chen, J. Alonso-Mora, X. Bai, D. D. Harabor, and P. J. Stuckey, “Integrated task assignment and path planning for capacitated multi-agent pickup and delivery,” IEEE Robotics and Automation Letters, vol. 6, no. 3, pp. 5816–5823, 2021.

[9] S. F. Roselli, M. Fabian, and K. Åkesson, “Solving the conflict-free electric vehicle routing problem using SMT solvers,” in 2021 29th Mediterranean Conference on Control and Automation (MED), IEEE, 2021, pp. 542–547.

[10] G. B. Dantzig and J. H. Ramser, “The truck dispatching problem,” Management science, vol. 6, no. 1, pp. 80–91, 1959.

[11] S. Roselli, M. Fabian, and K. Åkesson, “A compositional Algorithm to solve the Conflict-Free Electric Vehicle Routing Problem,” 2022 IEEE Transactions on Automation Science and Engineering. Submitted for publication, 2022. Available on arXiv.org.

[12] C. W. Barrett, R. Sebastiani, S. A. Seshia, C. Tinelli, et al., “Satisfiability modulo theories,” Handbook of satisfiability, vol. 185, pp. 825–885, 2009.

[13] L. De Moura and N. Björner, “Satisfiability modulo theories: Introduction and applications,” Commun. ACM, vol. 54, no. 9, pp. 69–77, Sep. 2011.

[14] T. Weber, S. Conchon, D. Déharbe, M. Heizmann, A. Niemetz, and G. Reger, “The SMT competition 2015–2018,” Journal on Satisfiability, Boolean Modeling and Computation, vol. 11, no. 1, pp. 221–259, 2019.

[15] R. Sebastiani and P. Trentin, “OptiMathSAT: A tool for optimization modulo theories,” Journal of Automated Reasoning, vol. 64, no. 3, pp. 423–460, 2020.

[16] J. L. Gross and J. Yellen, Handbook of graph theory. CRC press, 2003.

Details of the problem instances are discussed in the file Instances_Results.pdf in the UNSAT_Core folder of the Github repository.
[17] A. Cimatti, A. Griggio, and R. Sebastiani, “Computing small unsatisfiable cores in satisfiability modulo theories,” *Journal of Artificial Intelligence Research*, vol. 40, pp. 701–728, 2011.

[18] D. Selsam and N. Bjørner, “Guiding high-performance SAT solvers with unsat-core predictions,” in *International Conference on Theory and Applications of Satisfiability Testing*, Springer, 2019, pp. 336–353.

[19] E. W. Dijkstra, “A note on two problems in connexion with graphs,” *Numerische mathematik*, vol. 1, no. 1, pp. 269–271, 1959.

[20] N. Dershowitz, Z. Hanna, and A. Nadel, “A scalable algorithm for minimal unsatisfiable core extraction,” in *International Conference on Theory and Applications of Satisfiability Testing*, Springer, 2006, pp. 36–41.

[21] J. Huang, “MUP: A minimal unsatisfiability prover,” in *Proceedings of the ASP-DAC 2005. Asia and South Pacific Design Automation Conference*, 2005, IEEE, vol. 1, 2005, pp. 432–437.

[22] A. Nadel, “Boosting minimal unsatisfiable core extraction,” in *Formal Methods in Computer Aided Design*, IEEE, 2010, pp. 221–229.

[23] D. Kroening and O. Strichman, *Decision procedures*. Springer, 2016.

[24] A. Nadel, V. Ryzchin, and O. Strichman, “Efficient MUS extraction with resolution,” in *2013 Formal Methods in Computer-Aided Design*, IEEE, 2013, pp. 197–200.

[25] N. Bjørner, A.-D. Phan, and L. Fleckenstein, “νZ-an optimizing SMT solver,” in *International Conference on Tools and Algorithms for the Construction and Analysis of Systems*, Springer, 2015, pp. 194–199.

[26] S. F. Roselli, K. Bengtsson, and K. Åkesson, “SMT solvers for job-shop scheduling problems: Models comparison and performance evaluation,” in *2018 IEEE 14th International Conference on Automation Science and Engineering (CASE)*, IEEE, 2018, pp. 547–552.

[27] F. A. Aloul, B. Al Rawi, and M. Aboelaze, “Identifying the shortest path in large networks using boolean satisfiability,” in *2006 3rd International Conference on Electrical and Electronics Engineering*, IEEE, 2006, pp. 1–4.

[28] C. Sinz, “Towards an optimal CNF encoding of boolean cardinality constraints,” in *International conference on principles and practice of constraint programming*, Springer, 2005, pp. 827–831.