ABSTRACT

We present a numerical study of the response of a thick accretion disc to a localized, external perturbation with the aim of exciting internal modes of oscillation. We find that the perturbations efficiently excite global modes recently identified as acoustic p–modes, and closely related to the epicyclic oscillations of test particles. The two strongest modes occur at eigenfrequencies which are in a 3:2 ratio. We have assumed a constant specific angular momentum distribution within the disc. Our models are in principle scale–free and can be used to simulate accretion tori around stellar or super massive black holes.

Key words: accretion discs — black hole physics — hydrodynamics — stars: neutron — X-rays: binaries

1 INTRODUCTION

Accretion discs around compact objects (black holes and neutron stars) are natural laboratories for the study of strong gravitational fields and their effects (such as Lense–Thirring precession). From the physics that lies behind these accretion processes and the accompanying radiation, it is in principle possible to determine important parameters of the central object, such as mass and spin for the black hole case (Abramowicz & Kluzniak 2001; Wagoner, Silbergleit & Ortega-Rodriguez 2001), and to place constraints on the equation of state of dense matter in the case of neutron stars (Kluźniak, Michelson & Wagoner 1993).

It has also been pointed out that these objects can be potential sources of gravitational waves, arising from different processes, such as nonaxisymmetric instabilities (Zurek & Görski 1983), nonaxisymmetric tori powered by the spin energy of the black hole (van Putten 2001), thick accretion discs (Mineshige, Hosokawa, Mashida & Matsumoto 2002), and global oscillations which induce a time–varying mass quadrupole (Zanotti, Rezzolla & Font 2003). The energy released due to accretion from such systems, if dynamically stable, has also been proposed as a mechanism for the production of cosmological gamma–ray bursts. In this case, nonaxisymmetric tori, fast rotating black hole or neutron star, and a compact binary merger may work in concert to produce the bulk of the burst energy. The accretion disc can be expected to have a strong influence on the dynamics of the gravitational waves generated by the black hole-neutron star binary during the inspiral phase (van Putten 2001).

In the context of low–mass X–ray binaries, observations performed with RXTE (van der Klis 2000) have shown that there are millisecond oscillations in systems containing neutron stars or black holes surrounded by an accretion disc. An important result is that in at least four black hole sources (H1743-322, GRO J1655-40, XTE J1550-564, GRS915+105), two apparently stable peaks in the power spectrum appear at frequencies in the hHz range in a 3:2 ratio. A similar result was found by Abramowicz, Bulik, Bursa & Kluzniak (2003) for the neutron star source Sco X–1. This lends support to the resonance model originally proposed by Abramowicz & Kluzniak (2001), and further developed in terms of parametric resonance in a thin disc by Abramowicz, Kasas, Kluzniak, Lee & Rebusci (2003) and Rebusci (2004). In this interpretation, the frequencies reflect epicyclic motion of perturbed flow lines in the accretion disc, or combinations between these and a fixed, perturbation frequency (Kluźniak, Abramowicz, Kato, Lee & Stergioulas 2003).
likely due to the stellar spin in neutron star sources (Wijnands, van der Klis, Homan, Chakrabarty, Markwardt & Morgan 2003). Pressure coupling allows resonances to occur and manifest themselves in the X-ray lightcurve.

Recently, Zanotti, Rezzolla & Font (2003) have shown that an extended torus can respond to external perturbations in a global fashion, in a series of modes whose frequencies follow the sequence 2:3:4:... . These are attributed to acoustic p-modes within the torus, excited by an impulsive perturbation. Follow-up analytical and numerical work (Rezzolla, Yoshida & Zanotti 2003; Montero, Rezzolla & Yoshida 2004) has extended these results, by calculating the set of corresponding eigenfrequencies (in height-integrated discs) and also by investigating the effects of different background metrics (e.g., Schwarzschild vs. Kerr for a rotating hole). In all cases where a numerical experiment was carried out, the perturbation was impulsive and global, affecting the entire torus. This idea has now been advanced as an explanation for the kHz QPOs in low-mass X-ray binaries containing the black hole candidates where the 3:2 frequency ratios have been reported, as mentioned above (Rezzolla, Yoshida, Maccarone & Zanotti 2003).

In this Letter we show that it is possible to excite these modes in a thick accretion disc for a perturbation that is local, only affecting a small portion of the disc strongly. A frequency ratio of 3:2 for the two strongest modes is apparent. The lower frequency itself is related to the radial epicyclic frequency for test particles in circular motion, shifted to a lower frequency because of the finite extent of the torus. The second frequency maintains a 3:2 relation with the first.

2 INITIAL CONDITIONS AND NUMERICAL METHOD

2.1 Hydrostatic equilibrium for a thick torus

We construct tori which are in hydrostatic equilibrium, have low mass, \( m < M_{BH} \), and a radial extension \( L \sim R \), where \( R \) is the distance separating the torus from the central mass, \( M_{BH} \). They are thick in the sense that their vertical extent, \( H \), is comparable to \( L \). We neglect the self-gravity of the torus, and additionally assume azimuthal symmetry. A polytropic equation of state, \( P = K \rho^\gamma \), has been used for the construction of initial conditions. Integrating the equations of hydrodynamics, it is possible to write:

\[
\frac{\gamma}{\gamma - 1} \frac{P}{\rho} = \Phi_e + \Phi_0 = \text{const},
\]

where \( \Phi_e \) is the effective potential. Here \( \Phi_0 \) can be interpreted as a filling factor of the effective potential well. Through its variation tori of different sizes are constructed (Fig. 1 shows their cross sections over one half the radius plane). The gravitational potential of the central mass is computed with the pseudo-Newtonian expression of Paczynski & Wiita (1980):

\[
\Phi_{PN} = -\frac{GM_{BH}}{R - r_g}
\]

which describes the behaviour of a test particle in a strong gravitational field and, importantly for our purposes, reproduces the existence and positions of the marginally stable and marginally bound orbits in General Relativity (\( r_g = 2GM_{BH}/c^2 \) denotes the gravitational radius throughout). With this potential, and using a constant distribution of specific angular momentum, \( l(r) = \text{cst} \), the effective potential is:

\[
\Phi_e = -\frac{GM_{BH}}{R - r_g} + \int \frac{l(r')^2}{2r'^3} dr.
\]

Finally, we note that with this potential, the frequency of small radial oscillations for a perturbed circular orbit (i.e., the radial epicyclic frequency) is given by

\[
\kappa = \frac{1}{2\pi} \left[ \frac{GM(r - r_g)}{r(r - r_g)^3} \right]^{1/2}.
\]

2.2 Numerical method

For our simulations, we have used Smooth Particle Hydrodynamics (SPH, see Monaghan (1992) for a review), in a two-dimensional version using cylindrical coordinates. We refer the reader to Lee & Ramirez–Ruiz (2002) for details of the implementation. There is no physical viscosity present in the code, only the usual artificial viscosity to model the presence of shocks.

Initial conditions are generated by distributing \( N \) fluid elements, over the torus volume after specifying values for \( M, l(r) \) and \( \Phi_0 \), and relaxing them for several dynamical times in order to obtain a distribution close to equilibrium (see Figure 2). This configuration is then evolved in time with the desired perturbation to study its dynamical behaviour.

Our initial configurations are thus non-accreting tori fully contained within their Roche lobe. Specifically, the inner edge of the disc and the inner Lagrange point, \( L_1 \) are located at \( r_{in} = 2.60r_g \) and \( r_{in} = 2.25r_g \) respectively. During the dynamical evolution described below, no accretion takes place, so the potential produced by the black hole is unaltered.

The analysis of the data is carried out by performing...
the Fourier decomposition of the main hydrodynamic variables as functions of time (e.g., the position of the center of the disc, the maximum and mean densities, and the various total energies).

3 INTRODUCING A PERTURBATION

The perturbation acting on the disc can be considered as arising from the central object, through its magnetic field, a deformation on its surface or a changing radiation field (in the case of neutron stars) or as the emission of gravitational waves from an accreting black hole. Instabilities in the accretion disc itself are another possible source of time variability, which can induce oscillations in the fluid. In either case, these would presumably be more intense at small radii. If the spin of the central object is involved in producing the perturbation, its amplitude will vary and repeat at intervals given by the inverse of the spin period, $\Delta T = 1/\nu_s$.

We have chosen, then, a perturbation which induces an acceleration in the disc given by:

$$a_{\text{pert}} = -\eta a_g \cdot \exp \left( \frac{r_o - r}{\delta r} \right) \cdot \sin (2\pi \nu_s t).$$

Here $a_g$ is the acceleration due to gravity, $r_o$ is the outer edge of the torus and $\eta \ll 1$ is a parameter that modulates the strength of the perturbation. The exponential term decays on a scale $\delta r \simeq R$, the radial extension of the disc, thus reproducing the desired behaviour for this perturbative force, which will be strong near the inner radius and weak in the outer regions. This acceleration induces radial oscillations in the disc, which can be Fourier-analyzed to extract the main frequencies as was done by Lee, Abramowicz & Klu˙zniak (2004) recently for a slender torus.

4 RESULTS AND DISCUSSION

A Fourier transform of the total kinetic energy in the torus in a typical calculation is shown in Figure 3. The black hole mass in this case was 2.5 solar masses, and the perturbation frequency was fixed at $\nu_s = 200$ Hz, which clearly shows up prominently in the spectrum as a narrow peak.

Two additional broad features are clearly seen, centered at $\nu_1 \approx 300$ Hz and $\nu_2 \approx 450$ Hz, and are, to the limit of our resolution, in a 3:2 ratio. The radial epicyclic frequency for a test particle at the locus of maximum density ($4.25 r_g$) is in this case $\kappa = 426$ Hz (see equation 4). Trial runs with tori of different radial extent (Rubio-Herrera, in preparation) and a comparison with the work of Zanotti, Rezzolla & Font (2003) and Rezzolla, Yoshida & Zanotti (2003) show clearly that the lower of the two peaks is simply the epicyclic frequency at the locus of maximum density, shifted to lower values because of the finite extent of the torus, i.e. the fundamental acoustic p-mode. The second peak at higher frequency is the second in the sequence 2:3:4:... of acoustic p-modes.

We draw from these results the following conclusions.

- Global modes of oscillation in thick tori, as studied in the relativistic regime (Zanotti, Rezzolla & Font 2003), can be efficiently excited by a localized perturbation, affecting only the inner edge of the torus strongly. The characteristic
frequencies apparent in the Fourier decomposition of the total internal energy exhibit the sequence 2:3:... .

- The lower of the two frequencies in the series is the first acoustic p–mode, closely tied to the radial epicyclic frequency for test particles, and shifted to lower frequencies because of the finite extent of the torus.

The nature of the excitation mechanism, as stated above, is left as an open question. It could be some disturbance associated with the pulsar spin frequency in the case of neutron star systems, or oscillations in the disc itself which excite these modes. In either case, the modulation in the X–ray lightcurve will presumably occur in the innermost regions of the accretion flow. Since its inner boundary is defined most likely by the effects of strong gravity, one would expect that the frequencies would scale inversely with the mass of the central object, as is indeed the case (Abramowicz, Kluźniak, McClintock & Remillard 2004) for X–ray binaries — and may in principle allow for a mass determination in the intermediate mass black hole candidates (Abramowicz, Kluźniak, McClintock & Remillard 2004).

Two potentially limiting simplifications in this study deserve justification. First, in the context of LMXBs, the ratio of disc mass to black hole mass is very low, $M_d/M_{BH} \ll 1$. Hence it is reasonable to suppose that the gravitational potential of the disc is negligible when compared to that of the black hole, as we have assumed. Second, we have not considered the effects of magnetic fields. This is simply because we wish to carry out a study of global, purely hydrodynamical modes, and does not imply that MHD effects are negligible or irrelevant. Recently, Kato (2004) has obtained interesting results in the context of kHz QPOs through three–dimensional MHD calculations performed in a pseudo–Newtonian potential.

Finally, one may question the choice of a constant distribution of specific angular momentum within the torus, assumed here for simplicity and as a first step. However, we note that in viscous, hydrodynamical flows, even if the injection of matter at large distances occurs at nearly Keplerian values of the angular momentum, the flow near the compact object may follow a flatter distribution (Igumenshchev & Abramowicz 1999). The Papaloizou–Pringle instability (Papaloizou & Pringle 1984) appears when non–axisymmetric perturbations act on non–accreting discs with constant angular momentum. When accretion is taken into account, this instability is suppressed, as was shown by Blaes (1987). This leads to flat distributions of angular momentum near the central object and a power law distribution in the outer region of the torus. We thus believe it is reasonable to assume a constant angular momentum for the discs as a first approximation.

How the oscillations of the fluid in the disc may translate into variations in the X–ray lightcurve and be observed as kHz QPOs is ultimately still unresolved, and requires more complex physical processes than those included here. Assuming that oscillations at such frequencies do in fact occur due to the presence of inhomogeneities in the accretion flow, the luminosity modulation with account of spacetime curvature in the vicinity of a black hole has been investigated by Schnittman & Bertschinger (2004) and Schnittman (2004). Their results indicate that a range of variation would in fact be reflected in the X–rays, which encourages the investigation of simple modes of fluid oscillation. In future work we will discuss the behaviour of discs with non–constant angular momentum subject to various perturbations.

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