Relaxation dynamics in quantum electron-glasses

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Abstract

It is experimentally shown that, depending on the carrier-concentration of the system n, the dynamics of electron-glasses either slows down with increasing temperature or it is independent of it. This also correlates with the dependence of a typical relaxation time (or ‘viscosity’) on n. These linked features are argued to be consistent with a model for dissipative tunneling. The slow relaxation of the electron glass may emerge then as a manifestation of friction in a many-body quantum system. Our considerations may also explain why strongly-localized granular metals are likely to show electron-glass effects while semiconductors are not.

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Coupling to environment is known to have non-trivial effects on the dynamics of quantum systems. This has been discussed in the context of a two-state-system coupled to an oscillator-bath [1]. Much less is understood in real systems, most notably in systems that contain many, and possibly strongly interacting two-state-systems [2].

Here we use the ratio \( \frac{\Delta G(0)}{G(t)} \equiv \Delta G - \Delta G_{eq} = G(0) - 1 - \Delta G_{eq}; \Delta G_{eq} \equiv \frac{G(V_g(2))}{G(V_g(1))} \) (c.f., inset to Fig. 1), and \( \Delta G(t_w) \equiv G(t) - 1 \). Note that \( \Delta G(t_w) \) depends on how far the sample conductance has drifted in phase-space during \( t_w \) towards its equilibrium state (set by \( V_g(2) \) and the external conditions, e.g., \( T \)). If, for example, a full equilibrium is reached during \( t_w \), \( \Delta G(t_w) \) will obviously equal \( \Delta G(0) \) yielding \( \eta = 1 \). If, on the other hand, relaxation is infinitely slow, \( \Delta G(t_w) \) will be zero (\( \eta = \infty \)). The origin of \( \Delta G_{eq} \) is the thermodynamic field effect. This physical quantity is associated with the anti-symmetric contribution to the field effect as illustrated in the inset to Fig. 1, which also depicts the conductance-excitation (DCE) technique more fully described in [3], which also includes a comprehensive discussion of other techniques for measuring dynamics and their associated caveats. The procedure is illustrated in Fig. 1 along with the data components that are involved in the analysis. Starting with a voltage \( V_g(1) \) held at the gate, and the sample equilibrated under the fixed external conditions (temperature \( T \), or electric field \( F \)), one monitors the conductance as function of time \( G(t) \) to obtain the equilibrium conductance. Next, the gate voltage is switched to \( V_g(2) \) and is maintained there for a “waiting-time” \( t_w \) that for all the experiments reported here was 180 s. Finally, the gate voltage is switched back to \( V_g(1) \), and \( G(t) \) is measured for an additional period of time.

FIG. 1: Typical runs of the DCE protocol at two temperatures. The dependence of the conductance \( G \) on time is plotted as squares and the gate voltage as dashed line. Sample shown is \( \text{In}_2\text{O}_3 \) deposited on a 0.5 \( \mu \text{m} \) SiO\(_2\) spacer thermally grown on Boron-doped Si wafer. Inset shows the memory-dip of this sample measured in an independent set of experiments after allowing the sample to relax for 28 hours under \( V_g = 0 \) V at \( T = 4.1 \) K. The difference between \( G(2) \) and \( G(1) \) is the basis for estimating \( \Delta G_{eq} \) (see [3] for fuller details).
the characteristic width $\Gamma^*$ of the memory-dip $[5]$. Note that $\Delta G(0)$ is a proper normalization; when the external conditions are changed, the degree of sample excitation (when $V_g(2)$ is switched to $V_g(1)$ or vice versa) will in general change too, and it will be reflected in $\Delta G(0)$. Likewise, the value of $\Delta G_{eq}$ is in general $\Delta G_{eq}(T)$ and should be measured for each temperature separately. The values of $\Delta G(0)$ and $\Delta G(t_w)$ are extracted from the $G(t)$ data as in Fig. 1 as follows. First, the times $t_1$ and $t_2$ where $V_g(2)$ reaches its final value, and $V_g(1)$ is reinstated respectively are noted. These are used as the origin ($t = 0$) for the two relaxations of $G(t)$; the first after the $V_g(1)$ to $V_g(2)$ switch, the second after the switch back. The respective values of $G(t_0)$ are found by extrapolation to $t_0$ using the logarithmic law $[4]$ ($t_0$ is the resolution time of the measurement, typically $t_0 = 1$ s), and are used to calculate $\Delta G(0)$ and $\Delta G(t_w)$ by subtracting the appropriate baseline (i.e., the equilibrium $G$ which for $\Delta G(0)$ is different than for $\Delta G(t_w)$ due to the anti-symmetric component shown in the inset to Fig. 1).

Results for $\eta$ versus temperature based on DCE measurements are shown in Fig. 2 for six of the studied samples. Three of these are for $In_2O_{3-x}$ films, two of which are doped with gold (Fig. 3a), and the other three are the amorphous version (Fig. 3b). For each sample, data were taken starting at the lowest temperature, where the sample was allowed to equilibrate for at least 24 hours. A two hours equilibration period was used for each higher temperature.

As a further check, the dependence of the dynamics on temperature was also assessed by measuring the excess conductance versus time $\Delta G(t)$. Such data are available anyhow as part of the more sensitive DCE procedure $[4]$, and few examples for the properly normalized $\Delta G(t)$ are given in Fig. 3. The results of DCE measurements on the 17 samples studied in this work are shown in Fig. 4. These suggest that, at the studied range of temperatures, the relaxation of the electron glass either slows down upon increase of temperature, or it is temperature independent. Note that data are shown for films of $In_2O_{3-x}$, $In_xO$ (with a broad range of $x$, i.e., stoichiometry), Au-doped $In_2O_{3-x}$, and the granular aluminum studied by Grenet et al. $[6]$. These are quite different systems in terms of microstructure, electron-phonon coupling, and carrier concentrations, and they usually obey different $G(T)$ laws (though all are activated). It is thus remarkable that their $\partial \eta/\partial T$ exhibits a unified correlation with a single parameter -- the width $\Gamma^*$ of the memory-dip (c.f., inset to Fig. 1).

Note that the temperature dependent dynamics is observed only for samples that exhibit $\Gamma^* \lesssim 2V$. For small $\Gamma^*$, $\partial \eta/\partial T$ decreases fast with $\Gamma^*$, and saturates for $\Gamma^* \gtrsim 2V$ (Fig. 4). This functional dependence of $\partial \eta/\partial T [\Gamma^*]$ is strikingly similar to the dependence of the typical relaxation time $\tau [\Gamma^*]$ measured by the two-dip experiment in a series of $In_xO$ films (all with similar values of $R_C$) $[6]$. It has been further shown by Vaknin et al. $[5, 7]$ that $\Gamma^*$ increases monotonically with the carrier concentration $n$ of the system that, for the range of $\Gamma^*$ in Fig. 4, spans $n$ values from $\approx 3-10^{19}$ cm$^{-3}$ for undoped $In_2O_{3-x}$ to $\approx 10^{23}$ cm$^{-3}$ for granular metals. The pattern that emerges from our results is that, all other things being equal, the wider is the memory-dip $\Gamma^*$ of a sample (or equivalently, the larger is $n$), the slower is its relaxation
FIG. 4: Temperature dependence of the relaxation dynamics vs. $\Gamma^*$ (evaluated on basis of a 0.5 \(\mu\)m thick SiO$_2$ spacer between the sample and the gate to allow comparison with the data in the inset). Empty diamonds are undoped $In_2O_{1-x}$, filled diamonds are Au-doped $In_2O_{1-x}$, and circles are $In_xO$ samples. Dashed line is a guide to the eye.

(higher ‘viscosity’). In addition to the systematic correlation noted with the $In_xO$ data, this can be seen e.g., in Fig. 3c comparing the dynamics in two $In_2O_{4-x}$ samples with identical $R_C$ and the sample with the larger $\Gamma^*$ exhibits slower dynamics. These results may be then summarized as follows: Dynamics in the electron-glass is more sluggish and less temperature dependent the higher is the carrier concentration $n$ ( or $\Gamma^*$) of the system. These correlations are the basis for the main conclusions of the paper.

The linkage between temperature dependence and rate of relaxation lead us to consider the relevance of a quantum dissipation scenario where relaxation rate and $T$ dependence are inherently connected. To make contact with the model, the following plausible assumptions are made: (i) The relaxation process in the electron-glass involves hopping transitions of electrons (or group of electrons) between localized states. Each of these may be modeled as two-state-system, with a ‘bare’ Rabi frequency $\Delta$. (ii) These tunneling events interact strongly with other (localized) band electrons, and thus coupling to the environment is Ohmic [8]. (iii) The two-state-systems that are relevant for the sluggish relaxation involve configurations with $\varepsilon \ll k_BT$ where $\varepsilon$ is the energy difference between the two states [9, 10]. To have a concrete expression for the sake of the discussion, we use a specific form of the Leggett et al model, which gives a renormalized tunneling rate $\Delta^*$ as [1]:

$$\Delta^* \propto \left( \frac{\Delta^2}{\omega_c} \right) \frac{\Gamma(a)}{\Gamma(a + \frac{1}{2})} \left[ \frac{\pi k_BT}{\hbar\omega_c} \right]^{2a-1}$$

(1)

where $\Gamma$ is the gamma function, $\omega_c$ is the cut-off frequency of the oscillators-bath that represents the environment, and $\alpha$ is the associated coupling constant.

Eq. 1 should not be taken too literally in the context used here. The relaxation process of the electron-glass involves a wide spectrum of tunneling rates [11], and these events are presumably inter-dependent (hierarchical). To characterize this convoluted dynamics by a single ‘effective’ $\Delta^*$ (and a single ‘effective’ $\alpha$) as we proceed to do next, cannot be expected to be more than a crude approximation. Other reservations will be mentioned below. Nevertheless it seems plausible that, if the underlying physics is relevant, the qualitative dependence on either $T$ or $\alpha$ predicted by Eq. 1 would be reflected in the experimental results.

Note first and foremost that Eq. 1 allows for the uncommon $\partial \eta/\partial T > 0$ dependence observed in the limit of small $\Gamma^*$ (i.e., by assigning $\alpha \leq 1/2$), a non-trivial feature that is hard to explain in a classical scenario. Also, the consistency of (ii) with the data can be readily seen by interpreting the temperature dependence of the dynamics (Fig. 4) through Eq. 1. This obviously yields $\alpha$ increasing with $\Gamma^*$. As mentioned above, $\Gamma^*$ increases monotonically with the carrier concentration $n$ of the system [2]. Therefore it is natural to accept that the dissipative environment is the electronic sea. Secondly, over a considerable range of $\Gamma^*$ (and thus $n$), $\partial \eta/\partial T \approx 0$. This means that $\alpha$, while increasing with $n$, saturates at $\approx 1/2$ for large $n$. Intriguingly, this behavior is theoretically anticipated [8] for a metallic bath where it was shown that 1/2 is the maximum value of $\alpha$ attainable in the high $n$ limit [12]. That a similar situation may occur in an Anderson insulator, while not impossible (at least in the sense of $\alpha$ saturating for large $n$), remains an open question.

Another non-trivial aspect of the results is the dependence of dynamics on $\Gamma^*$ at a given temperature. A pronounced decrease of $\Delta^*$ with $\alpha$ is in fact what one gets from Eq. 1 for the range of parameters relevant for these experiments. Associating $\omega_c$ with the Fermi energy [12] $E_F$ gives $\omega_c \approx 10^{13} - 5 \cdot 10^{14}$ sec for the $In_xO$ series. Then using Eq. 1 for $T = 4$ K results in a sharp increase of $\Delta^*$ (by orders of magnitude) when $\alpha$ changes from 0.1 to 1/2 (which occurs over the very narrow range of $\Gamma^*$, c.f., Fig 4). The correlation between the data in Fig. 4 and the data in the inset in terms of the sharp change at the same value of $\Gamma^*$ is then quite consistent with this picture [12].

The dramatic slowdown of the tunneling rate with $n$ naturally explains why sluggish relaxation is peculiar to Anderson insulators with relatively high carrier concentrations. This feature is implicit in Eq. 1 as both $\omega_c$ and $\alpha$ increase with the carrier concentration. The underlying mechanism beyond these quantum ‘friction’ effects is the Anderson orthogonality catastrophe (AOC) [3, 8], which may naturally account for the relevance of the memory-dip $\Gamma^*$. As conjectured by several authors, the memory-dip is a reflection of an underlying density-
of-states modulation brought about by disorder and interactions [13], and the AOC is the generic mechanism responsible for these effects [16].

The AOC is usually discussed only in the metallic regime. However, as pointed out by Ng, the long range Coulomb interaction makes it a viable mechanism for the strongly localized regime as well, although a real divergence may occur only in 3D [14]. At finite temperatures no divergence is expected anywhere as the frequencies that contribute to the AOC are cut-off at \( \omega \approx k_B T / h \). A state localized over a length \( \xi \) introduces \( \omega \xi \approx h / (m \xi^2) \) as a cut-off (\( m \) is the electron mass). Therefore, at \( T \approx 4 \) K, regions of the system where the localization length \( \xi \) is larger than \( \approx 10^2 \) Å are not more restrictive to the AOC than a metal would be at this temperature. Note that these \( \xi \)'s contain \( \gtrsim 30 \) electrons even for samples with the smallest \( n \) used in our studies. This presumably is the reason for the similar electron-glass properties shared by granular metals and the \( \text{In}_2\text{O}_3-x \) and \( \text{In}_x\text{O} \) samples and the lack of these effects in semiconductors. Semiconductors in the localized regime typically have \( n \ll 10^{20} \) \( \text{cm}^{-3} \) we then expect that they exhibit very fast relaxation rates, and their (correspondingly narrow) memory-dip will be anyhow masked by the huge sensitivity of \( G \) to changes in \( V_g \) as already remarked elsewhere [18]. Sluggish relaxations observed in systems with small \( n \) are probably due to coupling to a slowly varying extrinsic potential, e.g., structural defects.

In summary, we have shown that at liquid helium temperatures, the dynamics of several electron-glasses exhibit some surprising similarities with the behavior expected of a single two-state-system coupled to an electronic bath. These include an unusual temperature dependence of dynamics, and a strong suppression of relaxation rates caused by coupling to a dissipative bath. It would be interesting to further test some of the conjectures raised in this paper by carrying out similar studies at lower temperatures (where \( \eta \) should become \( T \) independent). On the theoretical side, the intriguing question is how the orthogonality catastrophe and in particular the behavior of the coupling constant \( a \) as function of \( n \) are affected when the metallic system crosses over to the localized regime. These issues clearly deserve a serious theoretical elucidation.

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[1] A. J. Leggett et al, Rev. Mod. Phys. 59, 1 (1987) and references therein.