Probing the shell structure of heaviest nuclei with $\alpha$-decay data

Andreea-Ioana Budaca and Ion Silisteanu
Horia Hulubei National Institute of Physics and Nuclear Engineering, Bucharest-Magurele, RO-077125, Romania, EU
E-mail: abudaca@theory.nipne.ro

Abstract. A standard many-body theory of alpha decay is developed using shell model clustering and resonance scattering amplitudes. Formulas are obtained for the $\alpha$ half-lives and applied extensively to known superheavy nuclei. These formulas are combined with measured decay energy to investigate the structure of the decaying states. The work includes formal considerations, as well as practical computational methods, based on self-consistent models for the nuclear structure and reaction dynamics.

1. Introduction
For the determination of the $\alpha$-formation amplitude we employ here the microscopic shell model wave functions. This method has been initiated for the harmonic oscillator single-particle wave functions [1] and extended to Woods-Saxon wave functions [2]. We use two kinds of formation amplitudes: one-body resonance amplitude that results in Breit procedure [3] and the shell model formation amplitude given on the basis of shell model single-particle wave functions [4]. In case of the $\alpha$-decay of a single resonance state $k$ into a set $n$ of channels, the partial half-live is: $T^k_n = \ln 2 \cdot \hbar / \Gamma^k_n$, the partial decay width being expressed [5] as:

$$\Gamma^k_n = 2\pi \left| \int_{r_{\text{min}}}^{r_{\text{max}}} I^k_n(r) u^0_n(r) dr \right|^2,$$

where $I^k_n(r)$ is the particle (cluster) formation amplitude (FA).

2. Methods
Resonance formation amplitude (RFA) The RFA was defined [5] by means only of the eigenvalues and eigenfunctions. The asymptotic overlap integral or the RFA $I^k_n(r \to \infty) \longrightarrow I^k_n(r) = u^0_n(r) \simeq \tilde{G}_n(r)$.

Shell model $\alpha$-formation amplitude (SM) Following [1] we use in Eq.(2) the w.f. $| \Psi^{SM}_k (r_i) \rangle = \det \psi^{SM}_{nj}(r_i)$, $i=1,A$ and $| \Phi^D_{ni} \rangle = \det \psi^{SM}_{nj}(\eta_{ni}) \rangle$, $i=1,A-4$, and the $\alpha$-particle wave function is chosen as: $\Phi_p(\eta_2) = (2/(1/2)!)^{3/2}(\beta/\pi)^{9/4}(\rho_1^2 + \rho_2^2 + \rho_3^2)(4\pi)^{-3/2}\chi_{00}(s_1s_2)\chi_{00}(s_3s_4)$, where the $\alpha$-particle oscillator parameter is $\beta = 0.484 \, \text{fm}^{-2}$, $\chi_{00}$ is the singlet spin function and the internal spatial
coordinates \( r_1 = (r_1 - r_2) / \sqrt{2}, \rho_2 = (r_3 - r_4) / \sqrt{2}, \rho_3 = (r_1 + r_2 - r_3 - r_4) / 2 \), are connected to the individual coordinates \( r_i \) of four nucleons. Replacing these functions in Eq.(2) we get

\[
I_k^{SM}(r) = r \langle \Psi_k^{SM}(r_1) | \mathcal{A} \{ [\Phi_D^{SM}(\eta_1) \Phi_p(\eta_2) Y_{lm}(\rho_1)]_n \} \rangle. \tag{3}
\]

We used s.p. configurations to \( \alpha \)-half-lives (\( \log T_{\alpha}^{SM} \)) \( Z=102-120 \rightarrow p \) (s.p. proton states): \( 1i_{13/2}, 2f_{7/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}; N=150-178 \rightarrow n \) (s.p. neutron states): \( 2g_{9/2}, 2g_{7/2}, 3d_{5/2}, 3d_{3/2}, 4s_{1/2} \).

**Empirical approximations:**

**Viola-Seaborg formula** [6] (VS): \( \log T_{\alpha}(s) = (aZ_d + b)Q_{\alpha}^{1/2} + (cZ_d + d) + h_{e-o}, \) with the parameters \( a, b, c, d \) [8] and even-odd corrections \( h_{e-o} \) for different \( (Z, N) \) parities.

**Brown formula** [7] (B): \( \log T_{\alpha}(s) = 9.54Z_d^{(0.6)} \cdot Q_{\alpha}^{1/2} - 51.37 + h_{e-o}, \) with the numerical constants being determined from the fit of 119 data points \((T_{\alpha}, Q_{\alpha}), Z_d=74-106,\) only for even-even nuclei. The effective decay energy includes screening corrections \( E_{\alpha} = 6.53 \cdot 10^{-5} \cdot Z^{5/2}_d - 8.0 \cdot 10^{-5} \cdot Z^{5/2}_d \cdot Q_{\alpha}(MeV) = (A/(A-4)) \cdot E_{\alpha} + E_{sc}, \) \( T_{\alpha}^{exp} \) and \( E_{\alpha}^{exp} \) are taken from Refs. [9–12] and therein references.

### 3. Results

The earliest systematics of \( \alpha \)-decay lifetimes of naturally emitters from actinides region was obtained by Geiger and Nutch in 1911 plotting the experimental values of \( \log T_{\alpha} \) vs. \( Q_{\alpha}^{-1/2} \). Fig.1a shows a version of this plot for the g.s.-g.s. \( \alpha \)-half-lives of known SHN calculated for the simplest case with "one body" approximation (Eq.(5)). We see that all values are distributed on parallel lines for different \( (Z, N) \) parities. Inspired by later formulations as Viola-Seaborg or Brown we plot \( \log T_{\alpha}(s) \) vs. \( Z_dQ_{\alpha}^{1/2} \) (Fig.1b). The order of \( Z_d \) lines is now reversed suggesting a possible interpolation between Fig.1a and Fig.1b. After several representations we found that for \( \log T_{\alpha}(s) \) \( (Z_d^{0.6}Q_{\alpha}^{-1/2}) \) the data fall on a single line and have the minimum \( rms \) value (Fig.1c). Although, this dependence does not have a physical interpretation, it came out as well numerically from the WKB calculations performed by Brown in [7].

Further we see that the presently known data points for SHN (Fig.3) also fall on a nearly straight universal line and it is parallel with the Brown fit line. From the fit of the calculated values of \( \log T_{\alpha}^{res} + h_{e-o}(s) \) and experimental ones \( \log T_{\alpha}^{exp}(s) \) as functions \( Z_d^{0.6}Q_{\alpha}^{-1/2} \), we obtain two simple formulas for \( \alpha \) half-life:

\[
\log T_{\alpha}^{res}(s) = 9.93(Z_d^{0.6}Q_{\alpha}^{-1/2}) - 53.83, \quad rms_{res} = 0.37, \tag{4}
\]

\[
\log T_{\alpha}^{exp}(s) = 9.69(Z_d^{0.6}Q_{\alpha}^{-1/2}) - 51.37, \quad rms_{exp} = 0.52. \tag{5}
\]

Eqs.(7,8) can be written in a generalized form as: \( \log T_{\alpha} = AZ_d^{0.6}Q_{\alpha}^{-1/2} + B \) where the parameters \( A \) and \( B \) are determined by a fitting procedure, like in the case of the empirical formulas presented above.

In the case of the SHN the growth of the available data amount has become so considerable as to demand a reexamination of regularities in \( \alpha \)-decay properties [5]. In order to do this, we plotted in Figs.4-9 the \( Q_{\alpha} \) and \( \log T_{\alpha} \) values as function of the neutron number for even-\( Z \) and respectively odd-\( Z \) isotopic chains. Large gaps in \( Q_{\alpha} \) are observed in Fig.4 when passing from \( Z = 110 \) to 108, and from \( Z = 116 \) to 114 regions. This is the direct confirmation of the shell closures at \( Z = 108, N = 162 \) and \( Z = 114, N = 184 \). Similar behavior of \( Q_{\alpha} \) can be seen in Fig.5 when passing from \( Z = 109 \) to 107. The slope of \( Q_{\alpha} \) vs. \( N \) changes significantly for the nuclides with \( Z = 108 \) and 110. Such an effect might be due to the transition from deformed to spherical shapes in successive \( \alpha \)-decays, in agreement with the results of Ref.[11]. The trends of \( T_{\alpha}^{exp} \) and \( T_{\alpha}^{SM} \) vs. \( N \) remain practically the same, excepting the heaviest isotopes of \( Z = 111 - 115 \). These discrepancies can be due to the measurements errors. All the SHN found up to now are believed to be well deformed. The measured \( E_{\alpha}^{exp} \) values and \( \alpha \) half-lives Figs.4-9 confirm the special stability of the neutron deformed shell at \( N = 162 (Z = 108) \), and the the spherical shell in the proximity of \( N = 184 (Z = 114) \). The four magic character of the
Figure 1. Calculated values for $\log T_{\alpha}^{TCS}$ with Eq.(1), $I_k^n$ from Eq.(5). $Q_\alpha$ is the effective decay energy and $Z_d$ is the charge number of the daughter nucleus.
systems $\alpha + (^{270}Hs, ^{298}114)$ manifests through exceptionally large $Q_\alpha$-values (Figs.4,5) and short $\alpha$ half-lives (Figs.6-9). These indicate not only the major influence and importance of magicity for the $\alpha$-decay channels, but also high probabilities in the formation and transmission probabilities.

Since the SHN are joined in long $\alpha$-decay chains, it is possible to interrelate on the energy content basis most different nuclear species and to determine regions of nuclear stability higher than in neighboring regions where the nucleons are less bound. The study of nuclei passing from spherical to deformed shapes in the course of their successive $\alpha$-decays provides valuable information about the influence of significant structure changes on the nuclear decay properties of SHE.

**Figure 2.** Calculated values (Eq.(1), $I_n^k$ from Eq.(5)) of $\log T_{\alpha}^{exp}$ (s), with even-odd corrections $b_{e-o}$ added, are plotted vs. $Z_0^0.6Q_\alpha^{-1/2}$.

**Figure 3.** The values of $\log T_{\alpha}^{exp}$ (s) are plotted vs. $Z_0^0.6Q_\alpha^{-1/2}$. The upper line represent the best fit to the $T_{\alpha}^{exp}$ values, while the lower one is the result of the linear fit from Ref.[7].

**Figure 4.** Experimental values for $Q_\alpha$ (MeV) are plotted vs. neutron number for isotopes of even-Z elements $Z = 102 - 120$.

**Figure 5.** Calculated values for $Q_\alpha$ (MeV) are plotted vs. neutron number for isotopes of odd-Z elements $Z = 103 - 117$. 
Figure 6. Experimental values for $\log T^{exp}_\alpha (s)$ are plotted vs. neutron number for isotopes of even-$Z$ elements $Z = 102 – 120$.

Figure 7. Calculated values for $\log T^{SM}_\alpha (s)$, with even-odd corrections $h_{e-o}$ added, are plotted vs. neutron number for isotopes of even-$Z$ elements $Z = 102 – 120$.

Figure 8. Experimental values for $\log T^{exp}_\alpha (s)$ are plotted vs. neutron number for isotopes of odd-$Z$ elements $Z = 103 – 117$.

Figure 9. Calculated values for $\log T^{SM}_\alpha (s)$, with even-odd corrections $h_{e-o}$ added, are plotted vs. neutron number for isotopes of odd-$Z$ elements $Z = 103 – 117$.

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