Adptive Multi-scale Finite Element Considering Local Buckling of Members for Latticed Frames

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Abstract: To consider the local buckling of the members in the reticulated frames, based on the multi-scale simulation, the part of member that having local buckling is divided by the shell elements as a micro-model, and the other part of the member was simulated by beam elements as a macro-model. The incremental displacement constraint equations for the nodes on the section between the two models are established based on the plane section premise of classical beam theory. A method to introduce the constraint equations is derived based on the Updated Largrangian method. The location of the micro-model is predicted by the deflection function of the beam, and the length of the collapsed part is estimated. Its accuracy proves to be satisfied by the pure shell element model.

1. Introduction

The grid structure is a kind of mesh space rod structure, which is arranged by many rod pieces based on certain rules and connected by nodes, such as grid frame and grid shell. The grid structure is usually discreted by using beam elements. M.Elchalakni \cite{3-5} and other scholars have found that such thin-walled rod pieces tend to occur local buckling when subjected to force. Engineering accidents and model experiments show that the local buckling of the rod pieces is an important cause of the thin-walled rods structural damage. Since the displacement of each particle point on the beam element section needs to comply with the plane cross-section assumption, an ideal plastic hinge will be formed after bending occurred. The plastic hinge can stably transmit the bending moment, and if the structure is not degenerated into a mechanism, the axial force still can be stably transmitted. It can be seen that the beam element model cannot consider the influence of the local buckling of the rod piece on the structure, which will cause a large error. Zhang Ailin also found this phenomenon in the static experiment of the suspendome structure \cite{6}. Therefore, it is necessary to propose a finite element analysis method for grid structures considering local buckling of rod pieces.

At present, the research on local buckling is mainly for a single rod piece, and there are few studies on the structure. In order to solve this problem, Liew \cite{6-7} and other scholars proposed the advanced plastic hinge method, which has a small amount of calculation, but assumes too many conditions, and can not directly calculate the stress distribution at the local buckling of the rod. However, if using a full-shell unit to disperse the rod structure, it will result in a large amount of calculation.

According to the idea of multi-scale finite element \cite{10-11}, in this paper, the part of the rod piece that may undergo local buckling is taken as the micro-scale model and the shell element dispersion is used. The rest of the rod is taken as the macro-scale model and the beam element dispersion is adopted; The displacement increment constraint equations on the interface of the two scale models are used to realize the connection of the two scale models; the position of the microscale model is determined by the deflection equation of the beam; and the length of the micro-scale model is determined according
to the Zienkiewicz-Zhu method. Reasonable. Taking the analysis of local buckling as an example, the effectiveness and feasibility of this method are illustrated.

2. Selection of microscale models

2.1 Preliminary determination of the length of the microscale model

After the thin-walled circular tube is compressed and bent, the whole round tube can be divided into three parts [3-5]: (1) The elastic part. This part has no plastic deformation, and its stress-strain relationship conforms to the plane cross-section assumption; 2) The flattened part. This part has a certain plastic deformation, the cross section of the rod piece changes from a circle to an ellipse, but the ratio of the short axis to the long axis is larger than 0.9, and using the plane cross-section assumption will not cause much error. (3) The local buckling part. This part is located at the maximum bending moment of the rod, and is a bulge or depression, which stress-strain relationship is very complicated, and does not conform to the plane cross-section assumption of the beam, as shown in Figure 1. Therefore, this portion of the rod in which the local buckling occurs in the rod can be regarded as a micro-scale model, and the shell element dispersion is adopted, and the rest of the rod is used as a macro-scale model, and the beam element dispersion is adopted, as shown in Figure 2. It should be pointed out that the length of the micro-scale model determined here is only an estimate value and does not need to be very accurate. There is an adaptive finite element algorithm in the third section to judge whether its length is reasonable.

Figure 1 Local buckling of a member

Figure 2 Multi-scale model

2.2 Determination of the position of the microscale model

The rods in the grid structure are generally elongated rods, which ratio of length to radius is generally larger than 30. And the estimated length of the microscale model is only 4R. Therefore, it is important to accurately determine the position of the microscale model.

Commonly, when the local buckling occurs, the pressure will have a relatively large influence on the bending moment distribution. Therefore, in this paper, the position of the micro-scale model is determined based on the differential equation of the deflection curve of the beam.

Establish a local coordinate system oxy, which is a right-handed coordinate system. Its x-axis direction is parallel to the deformed line of the beam element nodes 1, 2, and the coordinate oxy-plane is parallel to the global coordinate system Z-axis of the grid structure finite element model. The length of the beam element is l. The nodal force, the axial force concentration and the shear force concentration of the beam element in the oxy-plane are shown in Figure 2.
When $N$ is the pressure, any one micro-section of the unit 1-2 is selected, as shown in Figure 3, and based on the bending moment balance equation, it can be obtained:

\[
\begin{align*}
M + N dy - Q dx + \frac{1}{2} q_2 dx dx - M - dM &= 0 \\
N - (N + dN) + q_1 dx &= 0 \\
Q - Q - dQ - q_1 dx &= 0 \\
M_x - M_x - dM_x &= 0
\end{align*}
\]  \tag{1}

Since the external load of the grid structure is directly applied to the nodes of the grid structure, the rod pieces are not subjected to other loads except gravity, so $q_1$ and $q_2$ are constant. If the $Z$ coordinate of the global coordinate system of the grid structure is parallel to the direction of the gravity field and the direction is the same, then:

\[q_2 = \cos \alpha \rho Ag \quad q_1 = \sin \alpha \rho Ag\]

Where: $\rho$ is the density of the rod; $A$ is the cross-sectional area of the beam element after deformation; $g$ is the acceleration of gravity; $\alpha$ is the angle between the global coordinate system and the $Z$-axis and the $x$-axis of the local coordinate system, and the clockwise direction is positive.

It can be seen that the change of the axial force on the thin-walled beam is only related to the gravity of the rod piece, and the variation is small. Therefore, the position where the rod is local buckling should be at the maximum bending moment. When $y''' = 0$, $y''$ is the maximum value. Thus, the position where the bending moment is the largest in the rod piece, which also is the coordinate of the centroid of the microscopic model, is: $x=0$, $x=l$ or $x=\arctan(A/B)*l/\nu$.

3. Adaptive adjustment of microscale model length

3.1 The basic concept of Zienkiewicz-Zhu error

Set:
\[ e_\sigma = \sigma - \sigma^* \]
\[ e_u = u - u^* \]

\[ Z^2 \text{ use the energy norm to represent the unit error:} \]
Define the error of the system as:
\[ \|e\| = \int_\Omega (\sigma - \sigma^*)^T D^{-1} (\sigma - \sigma^*) \|^{0.5} \]

The relative error of the defined unit is:
\[ e = \sigma - \sigma_* \]

That is the energy norm representing the relative error:
\[ \eta = \frac{\|e\|}{\left(\|e\|^2 + \|\sigma_*\|^2\right)^{0.5}}. \]

3.2 Solving the exact solution of relative stress

The displacement solution obtained by the finite element method is continuous, and the stress solution is discontinuous. Therefore, the \( Z_2 \) method calculates the relative error \( \eta \) by using the stress field obtained after smoothing as a relatively accurate solution.

The shell element and beam element essence can be regarded as a simplified physical unit, as shown in the figure. Therefore, the virtual nodes 1 to 8 are set on the 8 vertices of the shell unit, and the improved nodal force \( \sigma \) on the 8 virtual nodes is obtained by the node averaging method. The formula is
\[ \sigma_i = \frac{1}{m} \sum_{j=1}^{m} \sigma_i^j \]

\( m \) is the number of all cells around node node \( i \), for virtual node 1, \( i = 3 \); \( \sigma_i^j \) is the node force of the first unit;

![Figure 5 Balance status of a slight beam segment](image-url)

Then the stress improvement solution in the cell is still an interpolation function, which can be set to:

Where \( \sigma_i \) is the improved node stress value to be sought; \( n_e \) is the number of nodes, and \( \hat{N}_i \) is the interpolation function matrix.
\[ \sigma = \sum_{i=1}^{n_e} \hat{N}_i \sigma_i \]

In the error estimation region \( \Omega \), the improved stress solution and the stress solution \( \sigma^* \) obtained by the finite element method should satisfy the principle of weighted least squares, and the following functional is established:

Where \( M \) is the total number of cells.

Substituting it and then using variational method, considering that \( \delta \sigma \) is arbitrarily, it can be
obtained:
\[
A(\sigma^*, \sigma) = \sum_{e=1}^{M} \int_{V_e} \frac{1}{2} (\sigma^* - \sigma)^T D^{-1} (\sigma^* - \sigma) dV
\]  

(7)

\(M\) is the number of cells, and \(S\) is the number of stress components.

Since this paper relates to the magnitude of the error at the interface between the microscale model and the macroscale model, the integral region \(\Omega\) is determined as the set of shell elements adjacent to the beam element. The relative error can be obtained by substituting the relative exact solution shown in the equation. When the relative error \(\eta\) of the adjacent unit is greater than the certain error control standard is \(\varepsilon\), it is necessary to increase the length of \(S\) and re-distribute the micro-scale model.

4. Numerical test

A steel pipe rod has a diameter of 114 mm, a thickness of 0.003 m and a length of 2.9 m. The bottom end of this rod is fixed and the top load \(P\) is perpendicular to the rod axis.

Three kinds of finite element models are used to calculate the force of the rod piece: (1) Beam element 1) model. This model uses 8 beam elements to be discrete, requiring a total of 9 nodes; (2) Multi-scale model. The part that the bottom end length is 4R is used as the micro-scale model, and the shell element dispersion is used. The rest of the rod piece is taken as the macro-scale model, and the beam element dispersion is used. This model requires 300 units and 400 nodes. (3) The full-shell element model. This model uses the shell element to be discrete. The model requires 5000 units and 5000 nodes, which is computationally intensive, but with high precision. It is used to test the accuracy of multi-scale models and beam element models. Three forms of finite element models are shown in Figure 10.

![Figure 6](image1.png)

Figure 6 3 kinds of finite element model of a beam

![Figure 7](image2.png)

Figure 7 Balance status of a slight beam segment

Under the action of the load \(P\), the load-displacement curve displacement of the rod tip point obtained by the three models is shown in Figure 11, and it can be seen that:
(1) During the entire loading process, the displacement-load curves of the multi-scale model and the full-shell model are substantially coincident. It is indicated that the multi-scale model can accurately simulate the local buckling behavior of the rod piece. Meanwhile, in the nonlinear iterative process, the maximum value of $\epsilon$ is only 6.34%, indicating that the stress-strain relationship is consistent with the plane cross-section assumption of the beam at the interface between the microscopic model and the macroscopic model, and the length of the microscopic model is reasonable; It can be seen that the multi-scale model is basically the same as the full-shell model, but the calculation load is greatly reduced;

(2) In the elastic range, the displacement-load and strain-displacement curves of the three models are basically coincident. It shows that the beam unit can accurately simulate the behavior of the rod in the elastic range.

(3) After the plastic yielding of the rod occurred, the displacement load curve of the beam element model and the multi-scale model are greatly different. This is because after the plastic yielding of the beam element model, the displacement of each particle point on the beam section still follows the plane cross-section assumption, and each section is always a plane and the area is constant. Thus, an ideal plastic hinge is formed, which can stably withstand bending moments. Therefore, after the full-section yielding of the beam element model, the displacement of the rod end $T$ increases, and the load $P$ hardly changes. Thus, the displacement load curve of the beam element model in Figure 11 after yielding is almost straight.

In fact, after the rod occurs the plastic yielding, it will soon undergo local buckling at the bottom end and cannot stably withstand the bending moment. Thus, as the displacement increases, the load $P$ decreases rapidly. Therefore, if the local buckling behavior of the rod piece has not been considered, the bearing capacity of the structure will be estimated too high.

5. Conclusion
(1) In order to consider the influence of local buckling of the rod on the performance of the reticulated shell, based on the idea of multi-scale finite element, the part of the rod that may undergo local buckling is taken as the micro-scale model and the shell element dispersion is used, and the rest of the rod is taken as the macro-scale model and the beam element dispersion is used. And based on the plane cross-section assumption of classical beam theory, the displacement increment constraint equation on the interface of two scale models is derived.

(2) According to the local buckling theory of thin-walled circular tube, the initial estimated length of the micro-scale model is determined, which is 4 times of the radius of the rod piece; the position of the micro-scale model is determined according to the deflection differential equation of the beam; and a method to check whether the length and position of the microscale model are reasonable is given.

(3) Dispersed each micro-scale model only requires less than 400 nodes. At the same time, when the grid structure is unstable or collapsed, only a few rod pieces will usually occur local buckling. Therefore, with the rapid development of computer technology, the calculation load increased by the multi-scale finite element method is affordable.

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