Acoustic black hole in Schwarzschild spacetime: quasi-normal modes, analogous Hawking radiation and shadows

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Abstract

Various properties of acoustic black holes constructed in Minkowski spacetime have been widely studied in the past decades. And the acoustic black holes in general spacetime were proposed recently. In this paper, we first investigate the basic characteristics of ‘curved’ acoustic black hole in Schwarzschild spacetime, including the quasi-normal modes, grey-body factor and analogous Hawking radiation. We find that its signal of quasinormal mode is weaker than that of Schwarzschild black hole. Moreover, as the tuning parameter increases, both the positive real part and negative imaginal part of the quasi-normal mode frequency approach to the horizontal axis, but they will not change sign, meaning that all the perturbations could die off and the system is stable under those perturbations. Since the larger tuning parameter suppresses the effective potential barrier, it enhances the grey-body factor, while the energy emission rate of Hawking radiation is not monotonic as the tuning parameter due to the non-monotonicity of the Hawking temperature. Finally, as a first attempt, we also study the acoustic shadow of the analogous black hole. The radius of shadow is larger as the tuning parameter increases because both the related acoustic horizon and the acoustic sphere become larger. We expect that our studies could help us to further understand the near horizon geometrical features of the black hole and also be detected experimentally in the near future.

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I. INTRODUCTION

The black hole is one of the most intriguing celestial objects in our universe, and it plays a significant role in the study of general relativity, thermodynamics, statistics and quantum mechanics. With the astrophysical detections, it is still difficult to touch the signal of Hawking radiation or anything else with the interaction of quantum field in the gravitational spacetime. The situation turns around as the acoustic black hole proposed by Unruh [1] provide potential connections between astrophysical phenomena and the tabletop experiments.

In the acoustic model of gravity, the equation of motion describes the propagation of sound modes. The acoustic black hole is formed by a moving fluid with speed exceeding the local sound velocity through a spherical surface and the horizon is the boundary where the speed of flow equals the local speed of sound. The features including horizon, ergosphere and Hawking radiation of those analogue black holes were then explored in [2], inspired by which more efforts on the analogue Hawking radiation of acoustic black hole were made in [3, 4]. Moreover, the stability of the static or rotating acoustic black holes has been analyzed by computing the quasi-normal modes [5–8]. A nice review paper about more details of the analogue black holes can be seen in [9].

The first realization of a sonic black hole as Bose-Einstein condensate in experiment has been reported in [10]. More recently, the remarkable experiments reported that the thermal Hawking radiation and the corresponding temperature in an analogue black hole were observed[11] and more research was shown in [12]. Progress on stimulated Hawking radiation has also been made in an optical system[13–15] and some other mechanics[16–18].

Due to those significant realization of astrophysical phenomena in term of acoustic black hole, it nowadays draws more and more focusing. More recent extended study on its analogue Hawking radiation can be seen in [19–22]. The thermodynamic-like description for the 2-
dimensional acoustic black hole has been discussed in [23]. The particle dynamics in the acoustic spacetime was addressed in [24].

Most of the aforementioned studies are based on the acoustic models constructed in the real Minkowski spacetime. However, the authors of [25–28] derived the acoustic black holes from the relativistic fluids with the starting of the Abelian Higgs model. Specially, in [28], the authors studied analogue gravity models by considering the relativistic Gross-Pitaevskii theory and Yang-Mills theory in the fixed curved spacetime geometry, and constructed the acoustic black hole in general curved spacetime. This is significant and interesting because the black holes in our universe could be in the bath of some kind of superfluid or just the cosmological microwave. Moreover, it was addressed in [29] that the acoustic black hole could emerge from black-D3 brane based on holographic approach.

In this paper, we are interested in the acoustic black hole in four dimensional Schwarzschild background, which could be one of the simplest analogue black hole in curved spacetime. We accept that the characteristics appearing in general relativistic black holes, should also appear in their analogues, saying the acoustic black holes. Here we shall concentrate on the basic characteristics near the acoustic horizon, which are related with the possible observable quantities in that kind of ‘curved’ acoustic black hole.

The first characteristic we shall explore is the frequency of quasi-normal modes (QNM) which governs the relaxation of the sound wave perturbation. The real part of the QNM frequencies describes the oscillations of the corresponding perturbation while the imaginary part indicates the damped or undamped of the mode (see [30] and therein for review). In general relativity (GR), the QNMs of various perturbation have been widely studied because it could be one of the fingerprints of a gravity theory or other possible deviations from GR. The study of QNMs could help to test the (in)stability of acoustic black hole and provide the stable regime of the black hole parameters which in this model are the black hole mass and the tuning parameter.

The second characteristic we shall investigate is the grey-body factors, which is equal to the transmission probability for an outgoing wave radiated from the black hole event horizon to reach the asymptotic region [31, 32]. It is noticed that the frequency dependent grey-body factors measures the modification of the pure black body spectrum, and thus gives us significant information about the near-horizon structure of black holes [33]. Moreover, based on the grey-body factors, we shall further evaluate the energy emission rate of analogous Hawking radiation.

The last characteristic we consider is the acoustic black hole shadow. Black hole shadow is another fingerprint of the geometry around the black hole horizon, and it describes the property of the black hole which depends on the gravitational lensing of the nearby radiation [34]. Shadow is important to determine the near horizon geometry and its properties are widely studied, see for example [35–47] and therein. Moreover, in the experimental side, the Event Horizon Telescope group detected the black hole images with the use of the shadow properties [48–50]. The detection of gravitational waves [51] from black holes (or other compact objects) and other observations strongly motivate us to disclose more near horizon geometry of black holes. Thus, as a first attempt we will study the acoustic shadow of the ‘curved’ acoustic black hole. Theoretically, the acoustic shadow is a region of the listener’s sky that is left dumb if there are sonic sources distributed everywhere but not between the listener and the acoustic black hole. We expect this observable property could be detected in the analogue black hole experiment.

The structure of this paper is listed as following. In section II we review the acoustic black
hole in the Schwarzschild spacetime and then present the covariant scalar equation in this background. In Section III we compute the frequencies of quasi-normal modes and analyze the stability of the sector under scalar perturbation. Then in section IV and section V, we study the grey-body factor, Hawking radiation and the shadow properties of the acoustic black hole, respectively. We give the conclusion and discussion in the last section.

II. BACKGROUND AND THE COVARIANT SCALAR EQUATION

A. Acoustic black hole in Schwarzschild spacetime

With the use of the relativistic Gross-Pitaevskii theory, the acoustic black hole in the general curved spacetime has been constructed in [28]. It was addressed that the line element of a static dumb black holes in the background spacetime metric

$$ds^2_{bg} = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2$$

is

$$ds^2 = c_s\sqrt{c_s^2 - v_\mu v^\mu}\left[\frac{c_s^2 - v_r v_r}{c_s^2 - v_\mu v^\mu}g_{tt}dt^2 + \frac{c_s^2}{c_s^2 - v_\mu v^\mu}g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2\right]$$

(1)

where $c_s$ is the velocity of sound wave and $v_\mu$ determines the velocity of the background fluid. Note that the above metric is obtained under the setting $v_t \neq 0, v_r \neq 0, v_i = 0, g_{tt}g_{rr} = -1$ and the coordinate transformation $dt \rightarrow dt - \frac{v_tv_r g_{tt}}{c_s^2 - v_\mu v^\mu} dr$.

We shall focus on the Schwarzschild background with the spacetime metric

$$ds^2_{GR} = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \text{ with } f(r) = 1 - \frac{2M}{r}. \quad (2)$$

Subsequently, one can consider an orbit of a vortex that falls freely along the radial from infinity starting from rest outside a Schwarzschild black hole. Then the radial component $v_r$ is treated as the escape velocity of an observer who maintains a stationary position at Schwarzschild coordinate radius $r$, and it can be set as $v_r \sim \sqrt{2M\xi/r}$. Thus, with simple algebra computation after rescaling $v^\mu v_\mu \rightarrow c_s^2$, we can rewrite the line element (1) as

$$ds^2 = c_s^2\left[-\mathcal{F}(r)dt^2 + \frac{dr^2}{\mathcal{F}(r)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2)\right]$$

(3)

with

$$\mathcal{F}(r) = \left(1 - \frac{2M}{r}\right)\left[1 - \frac{\xi}{r}\left(1 - \frac{2M}{r}\right)\right] \quad (4)$$

for the acoustic black hole metric in Schwarzschild background where $\xi$ is the tuning parameter. We shall set $c_s = 1$ in the following study. It it noticed that (3) recovers the Schwarzschild black hole (2) as $\xi \rightarrow 0$, while as $\xi \rightarrow +\infty$, the whole spacetime should be an acoustic black hole because the escape velocity $v_r$ goes to infinity.

Then by setting $\mathcal{F}(r) = 0$, we shall obtain three solutions. One is the optical event horizon $r_{bh} = 2M$ and other two are $r_{ac \pm} = (\xi \pm \sqrt{\xi^2 - 4\xi})M$ for the acoustic black hole. The real value of $r_{ac \pm}$ requires the tuning parameter to satisfy $\xi \geq 4$. Moreover, as we aforementioned that the whole spacetime is an acoustic black hole as $\xi \rightarrow \infty$, i.e., the
acoustic horizon should satisfy \( r_{ac}(\xi \to \infty) \to \infty \), then the acoustic horizon which satisfies the condition locates at

\[
r_{ac} = r_{ac+} = (\xi + \sqrt{\xi^2 - 4\xi})M.
\]  

We plot the dependence of \( r_{ac} \) on \( \xi \) in Fig. 1. It is obvious that there exists a minimal acoustic black hole with horizon \( r_{ac} = 4M \) at \( \xi = 4 \); As \( \xi \) increases, the horizon for the acoustic black hole increases monotonously, until it fills the whole spacetime as the tuning parameter goes to infinity. Subsequently, in the case with the tuning parameter \( \xi \geq 4 \), an analogue metric would involve in the Schwarzschild spacetime such that the spacetime can be divided into three regions: the inside of black hole is with regime \( r < r_{bh} \), in the regime \( r_{bh} < r < r_{ac} \) the light can escape but the sound cannot, while in the regime \( r > r_{ac} \) both the light and the sound can escape. For more information on how the acoustic black holes emerge from the curved spacetimes, one can refer to [28].

**B. Covariant scalar equation**

We are interested in some basic characteristics of this acoustic black holes, including quasi-normal modes and grey-body factors of scalar field as well as the shadow cast by this analogy black hole. To this end, we consider a probe minimally coupled massless scalar field, and its covariant equation is

\[
\frac{1}{\sqrt{-G}} \partial_\mu (\sqrt{-G} G^{\mu\nu} \partial_\nu \psi) = 0
\]

where \( G_{\mu\nu} \) denotes the metric components of the acoustic black holes (3). Taking the standard ansatz

\[
\psi(t, r, \theta) = \sum_{lm} e^{-i\omega t} \frac{\Psi(r)}{r} Y_{lm}(\theta)
\]

and introducing the tortoise coordinate \( r_* = \int \frac{1}{\mathcal{F}} dr \), we shall obtain the Schrodinger-like formula for the scalar field

\[
\frac{d^2 \Psi}{d r_*^2} + (\omega^2 - V(r)) \Psi = 0
\]
where the effective potential is

\[ V(r) = \mathcal{F} \left[ \frac{l(l+1)}{r^2} + \frac{\mathcal{F}'}{r} \right]. \tag{9} \]

The radial domain of the following study is given by \( (r_{ac}, \infty) \) where \( r_{ac} \) was defined in (5).

FIG. 2. The behavior of effective potential with \( M = 1 \). On the left panel we fix \( \xi = 5 \) and take different angular numbers into consideration, while on the right panel we show the potential at fixed \( l = 1 \) for different \( \xi \).

The effective potential as a function of \( r \) for different cases are present in Fig. 2. As the radial coordinate approaches the near horizon region, the effective potential first shows a barrier and then quickly fall into zero at the acoustic horizon, meanwhile, the tortoise coordinate \( r_* \) reaches the infinity \( r_* \to -\infty \). In the left plot, with fixed \( \xi \) and \( M \), the position of zero potential is not affected by the angular number, which is reasonable because it has no print on the acoustic horizon. However, the potential barrier is promoted by larger \( l \) which is similar to that in Schwarzschild black hole. On the right plot, as we increase the tuning parameter, the position of zero potential is at larger radius because the acoustic horizon increases (see Fig. 1). In addition, the potential barrier is suppressed by larger \( \xi \). This behavior could be reflected by the near horizon characteristics as we shall study in the following.

### III. QUASINORMAL MODES

In this section we shall calculate the quasi-normal modes of the acoustic black hole from the equation (8). Thus, we require the purely outgoing waves at infinity and purely incoming waves at the event horizon for the the scalar field as

\[ \Psi \sim e^{\pm i\omega r_*}, \quad r_* \to \pm \infty. \tag{10} \]

To calculate the frequency of the QNMs, we shall employ the semi-analytical WKB method and the asymptotic iteration method (AIM), both of which are widely applied in the study of QNMs. So here we skip the instruction of the two methods, and the readers
can refer to [52] and [53] (and therein) for the details, respectively. It is noticed that even though our results are quite at good convergence in the 6th order WKB approximation, our calculation is accomplished by WKB method upto the 9th order correction for sufficient precision.

The QNM frequencies for small $\xi$ with samples of angular number, $l$, and overtone number, $n$, are listed in Table I ($l = n = 0$), Table II ($l = 1, n = 0$) and Table III ($l = n = 1$), respectively. The universal properties of the QNM frequency we can extract from the three tables are:

- The real part $Re(\omega)$ are positive and the imaginal part $Im(\omega)$ are negative, which means the acoustic black hole are stable for small tuning parameters under the perturbation. Moreover, their magnitudes for the acoustic black hole are quite smaller than that for the Schwarzschild black hole with $\xi = 0$. This implies the signal of the QNM is weaker than the astrophysical black hole and the perturbation dies off slower.

- With the increase of the tuning parameter $\xi$, the real part of the frequency decreases which denotes that the oscillation of the scalar field damps. While the magnitude of the imaginal part also decreases denotes the loss of the damping rate. The results implies the strength of oscillation is damping as the acoustic horizon $r_{ac}$ grows because $r_{ac}$ is larger as $\xi$ increases (see Fig. 1). This behavior is reasonable because the effective potential barrier is suppressed for larger acoustic black hole (see Fig. 2).

- In each table, the magnitude of $Im(\omega)$ continues decreasing as $\xi$ increases. This indicates that it is possible the $Im(\omega)$ crosses zero and changes sign as we continue increasing $\xi$. Since the system is stable only when $Im(\omega)$ is always negative, therefore, to further check the stability, we must study the QNM frequency for general $\xi$.

| $\xi$   | 9th-order WKB                | AIM                        |
|---------|------------------------------|----------------------------|
| 0       | 0.11031 − 0.10496i          | −                          |
| 4       | 0.02836 − 0.01905i          | −                          |
| 5       | 0.02341 − 0.01656i          | 0.02351 − 0.01660i         |
| 6       | 0.01954 − 0.01471i          | 0.01956 − 0.01471i         |
| 7       | 0.01668 − 0.01310i          | 0.01666 − 0.01307i         |
| 8       | 0.01469 − 0.01168i          | 0.01449 − 0.01171i         |
| 9       | 0.01283 − 0.01066i          | 0.01282 − 0.01058i         |
| 10      | 0.01148 − 0.00971i          | 0.01149 − 0.00964i         |

**TABLE I.** The QNM frequency of acoustic black hole with the mode $l = n = 0$.

The QNM frequencies as a function of general tuning parameter are shown in Fig. 3 and Fig. 4 where we choose samples of modes with fixed $l = 1$ and $n = 0$, respectively. The features of QNM frequency we can obtain from the figures are summarized as:
TABLE II. The QNM frequency of acoustic black hole with the mode \( l = 1 \) and \( n = 0 \).

| \( \xi \) | 9th-order WKB | AIM |
|---|---|---|
| 0  | 0.29294 – 0.09766\(i\) | – |
| 4  | 0.08211 – 0.01744\(i\) | – |
| 5  | 0.06391 – 0.01591\(i\), 0.06390 – 0.01591\(i\) | – |
| 6  | 0.05234 – 0.01402\(i\), 0.05234 – 0.01402\(i\) | – |
| 7  | 0.04434 – 0.01240\(i\), 0.04434 – 0.01240\(i\) | – |
| 8  | 0.03848 – 0.01107\(i\), 0.03848 – 0.01107\(i\) | – |
| 9  | 0.03399 – 0.00998\(i\), 0.03399 – 0.00998\(i\) | – |
| 10 | 0.03045 – 0.00908\(i\), 0.03045 – 0.00908\(i\) | – |

TABLE III. The QNM frequency of acoustic black hole with the mode \( l = n = 1 \).

| \( \xi \) | 9th-order WKB | AIM |
|---|---|---|
| 0  | 0.26431 – 0.30620\(i\) | – |
| 4  | 0.07649 – 0.05359\(i\) | – |
| 5  | 0.06061 – 0.04869\(i\), 0.06061 – 0.04868\(i\) | – |
| 6  | 0.04947 – 0.04314\(i\), 0.04947 – 0.04313\(i\) | – |
| 7  | 0.04171 – 0.03829\(i\), 0.04171 – 0.03828\(i\) | – |
| 8  | 0.03603 – 0.03426\(i\), 0.03604 – 0.03426\(i\) | – |
| 9  | 0.03171 – 0.03095\(i\), 0.03171 – 0.03095\(i\) | – |
| 10 | 0.02832 – 0.02818\(i\), 0.02832 – 0.02818\(i\) | – |

- In both figures for different modes and overtones, as \( \xi \) increases, both the positive real part and negative imaginary part are close to the horizontal axis, however, we find that neither of them changes sign. This indicates that all the perturbation could die off and the acoustic black hole is stable under those perturbations. It is noticed that similar behavior was observed for the counter-rotating waves (negative \( l \)) as the rotation parameter increases in the rotating acoustic flat black hole [5].

- In Fig. 3 with fixed \( l = 1 \), different overtone has different QNM frequency and the difference is more sharp at small \( \xi \). Moreover, for larger \( n \), both \( \text{Re}(\omega) \) and \( \text{Im}(\omega) \) are suppressed, which implies that perturbation with larger \( n \) die off quicker. This property is similar as that for acoustic black hole in flat spacetime as well as that for Schwarzschild black hole.

- In Fig. 4 with fixed overtone, different modes correspond to different QNM frequencies.
As $l$ increases, both $Re(\omega)$ and $Im(\omega)$ are enhanced, indicating that the perturbation with smaller $l$ dies off quicker. This property matches that in acoustic black hole in flat spacetime and Schwarzschild black hole.

FIG. 3. The QNM frequency as a function of $\xi$ for different overtone numbers with fixed $l = 1$.

FIG. 4. The QNM frequency as a function of $\xi$ for different angular numbers with fixed $n = 0$.

Then we further study the effect of the mass $M$ on the QNM frequency in acoustic black hole, and the result with fixed $\xi = 5$ and $l = n = 0$ is shown in Fig. 5. It is obvious that as $M$ increases, the positive $Re(\omega)$ decreases and approaches to zero, while the negative $Im(\omega)$ increases and also approaches to the horizontal axis. We did not find the sign changing as we further increase $M$, such that the sector is stable. This behavior indicates that the existence of heavier black holes or other compact objects would restrain the oscillation amplitude of scalar field, even though it could die off slower. It is noticed that the rule is similar as the effect of $\xi$ because both larger $M$ and $\xi$ corresponds to larger acoustic black hole.
IV. GREY-BODY FACTOR AND HAWKING RADIATION

In this section we investigate the grey-body factor and analogue Hawking radiation for the acoustic black hole. As we have addressed that there exists an acoustic horizon for the acoustic black hole. This could imply that similar to the well-known Hawking radiation emitted by astrophysical black hole, a radiation with the emission of a thermal flux of phonons could produce, which is named as analogue Hawking radiation and the temperature is proportional to the gradient of the velocity field at the acoustic horizon.

There are plenty of approaches proposed to study the Hawking radiation for astrophysical black hole, but it is known that it can not be the black-body radiation since the particles created in the vicinity of horizon without enough energy can not penetrate the potential barrier such that only the part of the particles can be observed at infinity, which makes it just a scattering problem. Thus, here we shall employ the tunneling method [54]. We use the grey-body factor to describe the transmission of particles through the potential and thus working out the energy radiation rate based on the obtained grey-body factor. The effective potential has the form of the potential barrier which monotonically decreases at both infinities thus allowing us to use WKB approach in this scattering problem. We should point out that the WKB method does not work well when \( l = 0 \), so we only consider the case with \( l > 0 \) in our following discussion.

We should consider the wave equation Eq.(8) with the boundary condition allowing incoming waves from infinity which differs the boundary condition for computing QNMs where only outgoing waves are allowed in the infinity. The scattering boundary condition is given by

\[
\Psi = T e^{-i\omega r_*}, \quad r_* \to -\infty, \tag{11}
\]

\[
\Psi = e^{-i\omega r_*} + R e^{i\omega r_*}, r_* \to +\infty \tag{12}
\]

where the \( R \) and \( T \) are the reflection and transmission coefficient which satisfy \( |T|^2 + |R|^2 = 1 \). The grey-body factor is then given by the transmission coefficient for each angular number as[55]

\[
|A_l|^2 = 1 - |R_l|^2 = |T_l|^2 \quad \text{and} \quad |T_l|^2 = (1 + e^{2i\pi K})^{-1} \tag{13}
\]
where the $K$ is determined by the equation

$$K - i \frac{\omega^2 - V_0}{\sqrt{-2V_0''}} - \sum_{i=2}^{i=6} \Lambda_i(K) = 0. \quad (14)$$

Here $V_0$ and $V_0''$ denotes the maximal value of the effective potential and its second derivative with respective to the tortoise coordinate at the maximum, respectively; and $\Lambda_i$ are the higher WKB corrections which are dependent on $K$ and up to $2i$th order derivative of the potential at its maximum\cite{55, 56}.

Once the grey-body factor is at hands, we can then study the Hawking radiation by evaluating the energy emission rate of the boson which is connected with the grey-body factor via \cite{57}

$$\frac{dE}{dt} = \sum_l N_l |A_l|^2 \frac{\omega}{e^{\omega/T_H} - 1} \frac{d\omega}{2\pi}. \quad (15)$$

In the above definition, $T_H$ is the Hawking temperature defined as $T_H = -\mathcal{F}'(r_{ac})/4\pi$ and $N_l$ are the multiplicities satisfying $N_l = 2l + 1$ for the scalar field.

Then, we shall employ the 6th order WKB method to calculate the grey-body factor and energy emission rate of Hawking radiation as the function of frequency. It is noticed that this method was employed to study the properties of Hawking radiation in various models, see for examples \cite{58–61} and therein. Our results are shown in Fig. 6 and Fig. 7.

![FIG. 6. The left panel shows the grey-body factor and the right panel shows the partial energy radiation rate for different angular numbers. For both panels, we fix $\xi = 5$, and the red, green, blue lines correspond to angular numbers $l = 1, 2$ and 3, respectively.](image-url)

In Fig. 6 we fix the tuning parameter $\xi = 5$ and study the effect of the angular number $l$. The left panel shows that the larger frequency corresponds to the higher grey-body as a natural consequence of the fact that the particles with larger energy are more likely to penetrate the potential barrier. On the other hand, it is obvious that larger angular number leads to a lower grey-body factor and this result can be intuitively explained by the effective potential which has higher barrier for larger $l$ as we show in Fig. 2, such that the particles are more likely to be reflected by the potential. For the energy emission rate of the Hawking radiation demonstrated on the right panel, it is observed that the dominant
radiation corresponds to $l = 1$, and the contribution of the higher $l$ is very small and negligible.

In Fig. 7 we fix the angular number $l = 1$ and choose different tuning parameters. The left panel shows that the grey-body factor is enhanced by the larger $\xi$ because of the lower potential barrier as we have shown in Fig. 2. For the energy emission rate of Hawking radiation, we see that at the low frequency region larger $\xi$ corresponds to larger emission rate, but when $\omega$ grows larger, this picture will be changed as a result of its Hawking temperature dependence. It is shown in Fig. 8 that the Hawking temperature grows from zero which corresponds to $\xi = 4$ standing for extremal black holes to a maximum at $\xi = 16/3$, and then start to decrease which suppresses the energy emission rate.

V. ACOUSTIC BLACK HOLE SHADOW

The black hole shadow in general relativity is one of the optical properties. As a first attempt, we shall study the analogous behavior in the acoustic black hole model. We then
treat it as “acoustic shadow” which describes the property of the sound waves. Analogous with the gravitational lensing and the related properties of astrophysical black hole, there exists the inmost unstable sound wave orbit and we call it “acoustic sphere” instead of “photon sphere” in general relativity. Beyond the acoustic sphere, the sound waves are absorbed by the acoustic black hole, so the acoustic sphere describes the “audible” boundary of the sound waves near the acoustic black hole horizon region. This property is related with the acoustic shadow.

Moreover, it is known that the shadow for static and spherical symmetric black holes also has spherical symmetry, and the shape of shadow would be more complex when the rotation of the black hole is involved. Thus, here for the static and spherical symmetric acoustic black hole (3), we simply study the shadow radius and discuss how the tuning parameter and the black hole mass shall affect the shadow radius.

To proceed, we follow the designations of [62] and first find the radius of the “acoustic sphere” \( r_{ah} \) by solving the following equation

\[
\frac{dh^2(r)}{dr} = 0 \quad (16)
\]

where the function \( h(r) \) is defined as \( h(r) = \sqrt{r^2/F(r)} \). Then for a distant static listener locating at \( r_L \), the detected radius of the acoustic shadow is

\[
r_{sh} = \frac{h(r_{ah})}{h(r_L)} r_L. \quad (17)
\]

To study the properties of the acoustic shadow, we assume the static listener is far away from the vicinity of acoustic horizon so that we have \( \frac{r_L}{h(r_L)} \approx 1 \). The radius of the acoustic sphere and the acoustic shadow as functions of the tuning parameter are given in Fig. 9. It is obvious that with fixed \( M \), both \( r_{sh} \) and \( r_{ah} \) increase almost linearly as the parameter \( \xi \), but the slope for the shadow radius is larger than that for the acoustic sphere. Moreover, in the figure, we can also see the influence of the mass parameter \( M \) on the radius, the larger \( M \) corresponds to both larger shadow radius and acoustic radius. This is reasonable because the increase of \( M \) could enlarge the acoustic horizon. It is noticed that the radius of acoustic sphere is much larger than the photon sphere for Schwarzschild black hole which is \( 3M \).

![FIG. 9. The shadow radius \( r_{sh} \) and the acoustic sphere \( r_{ah} \) changing with the tuning parameter \( \xi \).](image)
VI. CONCLUSION AND DISCUSSION

In this paper, we explored the near-horizon characteristic of the ‘curved’ acoustic black hole in the Schwarzschild spacetime. By adding a massless scalar field, we first studied the quasi-normal mode, grey-body factor and analogous Hawking radiation of the sector. Moreover, as a first attempt, we then studied the acoustic shadow which is analogous to the photon shadow caused by the bent light ray in general relativity.

We computed the frequencies of quasinormal mode with the use of both WKB method upto ninth order corrections and the asymptotic iteration method. Our results shew that the signal of quasinormal mode in this acoustic black hole is weaker than that of Schwarzschild black hole. Moreover, the real part is always positive while the imaginal part of the quasi-normal mode frequency is negative, and both of them get closer to the horizontal axis as the tuning parameter increases. We did not see the sign of the QNM frequency changing, which implies that all the perturbations would die off and the black hole is stable under those perturbations. It would be interesting to further test the stability under other types of perturbations around the acoustic black hole, which we will present elsewhere.

We then investigated the analogous Hawking radiation of the acoustic black hole and we employed the WKB approach to solve the scalar equation as a scattering problem. Both the grey-body factor and the energy emission rate of Hawking radiation are affected by the angular number and the tuning parameter in the model which corresponds to different properties of the potential barrier. Especially, the grey-body factor is enhanced by the larger tuning parameter because it corresponds to lower potential barrier, while the energy emission rate of Hawking radiation is not monotonic due to the non-monotonicity of the Hawking temperature on the tuning parameter.

Finally, as a first attempt we studied the acoustic shadow of the acoustic black hole. Since the acoustic black hole we considered is static and spherical symmetric, so the acoustic shadow has spherical symmetry. Thus, we simply analyzed the acoustic shadow radius, and we found that the radius of shadow is larger as the tuning parameter increases. This is acceptable because as the parameter increases, both the related acoustic horizon and the acoustic sphere increases.

It is expected that most of the black holes in the galactic center rotate. Moreover, rotating black holes are closely relative with the gravitational waves and black hole shadows, which are two important directions and open new windows for us to understand our universe. Thus, it would be significant to extend our studies into the acoustic black hole with rotation in the curved spacetime, and then further study the connection between QNM and black hole shadow which was proposed in [63]. We also expect that our theoretical results could be observed in analogous black hole experiment in the near future, and help us to further understand the structure of near horizon geometry of astrophysical black holes.

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