Development of methodology for calculation of permissible stresses for evaluation of contact strength of cylindrical gears in case of overload

Yu V Belousov¹², S L Shambina²* and F V Rekach²

¹ Bauman Moscow State Technical University (National Research University), Moscow, Russia
² Peoples' Friendship University of Russia (RUDN University), Moscow, Russia

E-mail: shambina_sl@mail.ru

Abstract. The article focuses on the development of methods for determining the allowable stresses to check the contact strength of the spur and helical gears under overload. Short-term overloads in gears occur under the action of maximum moments during the start-up of the device or due to the other reasons. The calculation of these loads is performed to check the static strength of the transmission teeth. The analysis of the limiting state of the material in the working area of the gear teeth was performed using the Pisarenko-Lebedev strength criterion, which is one of the most common strength conditions. The method of calculation of allowable stresses was developed. The calculation of limit stresses for normalized, improved or hardened wheels, as well as for the wheels after hardening by chemical-thermal treatment. The calculation results were compared with the values recommended by the standard. It was shown that the permissible contact stress for helical gears is slightly higher than for straight gears. Practically for all heat-treated steels used in the manufacture of gears, the norm of permissible limit stresses recommended by the standard is higher (in some cases significantly) compared to the stresses required from the conditions of contact strength.

Keywords: permissible stresses, contact area, strength criterion, stress intensity

1. Introduction

Gears are designed to transmit motion with the necessary change of angular velocity and torque by the magnitude and direction [1]. The force from one element of the coupling pair to another is transmitted by means of teeth, sequentially engaging. Gears are mainly used to transmit rotational motion between parallel shafts, shafts with intersecting and crisscrossing axes, to convert rotational motion into translational motion and vice versa.

If there are parallel shafts they use transmission with cylindrical gears, which may be with straight, slanting or herringbone teeth. These transmissions are usually called cylindrical. Among the mechanical transmission they are the most common. They are used in a fairly large range of loads and operating conditions.

The force interaction of the gear teeth is accompanied by the destruction of their working surfaces. The main types of teeth destruction, that are associated with contact stresses and friction, are fatigue chipping, jamming, wear, as well as residual deformation or brittle destruction of the surface layer under overload.

The method of determining the allowable stress for the evaluation of the last two of these types of damage is presented in the State Standard of Russia (STST) 21354-87. The permissible stresses in this case are designed to eliminate the appearance of unacceptable plastic deformations or brittle destruction of the surface layer of the teeth under the action of the ultimate load.

When the maximum torque $T_{\text{max}}$ is applied, the contact stress $\sigma_{N_{\text{max}}}$ must not exceed the permissible contact stress for the specified service life:
\[
\sigma_{N_{\text{max}}} \leq \sigma_{NP_{\text{max}}}. \tag{1.1}
\]

The stress \(\sigma_{N_{\text{max}}}\) is determined by the formula:

\[
\sigma_N = \sigma_N \sqrt{\frac{T_{\text{max}} K_{N_{\text{max}}}}{T_N K_N}}, \tag{1.2}
\]

where \(T_N\) and \(K_N\) are the nominal torque and the corresponding load factor; \(K_N\) is the load factor, determined at a load of \(T_{\text{max}}\).

Short-term operating maximum loads \(T_{\text{max}}\) are used to check the static strength of the transmission teeth. These loads (moments) include such which number of cycles during the service life of the transmission

\[
N_{G_{\text{N}}} \leq 0.03 N_{G_{\text{H}}} ,
\]

where \(N_{G_{\text{H}}}\) is the number of loading cycles corresponding to the abscissa of the inflection point of the fatigue curve for contact stresses:

\[
N_{G_{\text{N}}} = 30 (HB)^{2.4},
\]

here \(HB\) is the surface hardness of the transmission wheel.

When \(H \geq 560\) HB (HRC 56) take \(N_{G_{\text{N}}} = 12 \cdot 10^7\).

Peak loads are not included in the typical loading conditions, they are indicated separately. If the peak moment \(T_{\text{peak}}\) is not set, its value is found by the starting torque of the electromotor, by the limiting torque of the safety devices, by the inertial moments arising during sudden braking, etc. [2, 3, 13].

According to STST 21354-87, the permissible contact stress at maximum load, which does not cause residual deformation or brittle destruction of the surface layer \(\sigma_{NP_{\text{max}}}\), depends on the method of chemical and thermal treatment of the gear and the nature of the hardness change along the depth of the tooth:

- for teeth subjected to normalization, improvement or quenching with low tempering, they are:

\[
\sigma_{NP_{\text{max}}} = 2.8 \sigma_T ;
\]

- for teeth subjected to cementation or contour hardening they are:

\[
\sigma_{NP_{\text{max}}} = 44 H_{HRCE} ;
\]

- for nitrided teeth they are:

\[
\sigma_{HP_{\text{max}}} = 3 H_{HV}.
\]

According to STST 21354-87 different criteria were used to determine the permissible stresses: Brinell hardness numbers \(HB\), ones by Rockwell \(\sigma_T\). The choice of the criterion depends on the kind of chemical-thermal surface treatment of teeth.

This situation makes it difficult to analyze the load capacity of gears. In addition, all the above dependencies are empirical. Methods of accurate calculation of permissible contact stresses at the moment does not exist [6-8]. The authors of this paper tried to develop a uniform method of calculating the allowable stress.

2. Calculation of allowable stresses for plastic materials.

At first it is necessary to give the definition of allowable stresses for the calculation of the strength of the transmission under short-term overload. In cylindrical gears, the initial contact of the teeth occurs along the line. Then the linear contact passes into contact on a narrow platform bounded by two straight lines. With increasing load elastic deformation is being replaced by elastic-plastic one. The greatest shear stresses initially appear at some depth. With a further increase in the load, the plastic deformation comes to the surface, where it initially occurs at the points of the longitudinal axis of
symmetry of the contact site. It can be assumed that the laws of the theory of elasticity remain valid until the appearance of plastic deformations on the surface [9-12, 16].

For the quantitative description of the material’s limiting state in the contact zone the Pisarenko-Lebedev strength criterion was used. According to this criterion, the limit state of the material is determined by both tangential and normal stresses. The expression for the equivalent stress has the following form:

\[ \sigma_{eq} = x\sigma_i + (1 - x)\sigma_{lar}, \]  

(2.1)

where \( x = \sigma_t/\sigma_c \); \( \sigma_t \) is the ultimate strength of the material under uniaxial tension; \( \sigma_c \) is the absolute value of the compressive strength of the material; \( \sigma_i \) is stress intensity; \( \sigma_{lar} \) is the largest principal stress.

The Pisarenko-Lebedev strength criterion is the most common strength condition for such a case. At \( x = 1 \), which is typical for a plastic material, the Pisarenko-Lebedev strength criterion switches into the stress intensity criterion. At \( x = 0 \) for a very brittle material, this criterion coincides with the criterion of the highest normal stresses.

Let’s express the intensity of stresses through the main stresses:

\[ \sigma_i = \frac{1}{\sqrt{2}} \left( (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right)^{1/2}. \]  

(2.2)

For toothed gears at the points of the longitudinal axis of symmetry of the contact strip:

\[ \sigma_1 = -2\mu p_0; \quad \sigma_2 = \sigma_3 = -p_0, \]  

(2.3)

where \( p_0 \) – is the highest pressure on the contact surface; \( \mu \) is the coefficient of transverse deformation of the material.

Solving together the equations (2) and (3), we obtain:

\[ \sigma_i = p_0(1 - 2\mu). \]  

(2.4)

In [5, 6] it was shown that the contact area of the teeth in a helical gear has the form of a highly elongated ellipse, not a strip. The main stresses will have the greatest value in the center of the contact area [4]. They can be defined by formulas:

\[ \sigma_1 = -\left[ 2\mu + \frac{(1 - 2\mu)b}{a + b} \right] q_0; \quad \sigma_2 = -\left[ 2\mu + \frac{(1 - 2\mu)a}{a + b} \right] q_0; \quad \sigma_3 = -q_0, \]  

(2.5)

where \( q_0 \) – the maximum pressure at the center of the contact area; \( \sigma_1 \) – tension in the direction of the smaller axis of the ellipse; \( \sigma_2 \) – tension in the direction of the larger axis of the ellipse; \( \sigma_3 \) – tension normal to the contact area; \( a \) and \( b \) are the size of the greater and lesser axes of the ellipse, respectively.

Substituting the expression (5) in (2), for the oblique transmission we obtain:

\[ \sigma_i' = (1 - 2\mu)q_0 \frac{\sqrt{a^2 + b^2 - ab}}{a + b}. \]  

(2.6)

With the average ratio of the ellipse half-axes \( a/b \approx 50 \) according to [5] we have:

\[ \sigma_i' = 0.97(1 - 2\mu)q_0. \]  

(2.7)

The gear limit state will occur when the following conditions are true:

- for spur gears expressions (3) and (4) we will have:

\[ \sigma_{eq} = p_0[x(1 - 4\mu) + 2\mu] = \sigma_{ul}, \]  

(2.8)

- for helical gears taking in the account expressions (5) and (7) we will have:

\[ \sigma_{eq} = q_0 \left[ 0.97x(1 - 2\mu) + (1 - x) \left( 2\mu + (1 - 2\mu) \frac{b}{a + b} \right) \right] = \sigma_{ul}, \]  

(2.9)
where \( \sigma_{ul} \) is the value characterizing the limit state of the material (yield strength or ultimate strength). For materials in the plastic state, when \( x = 1 \), the condition of reaching the limit state is:

- for straight gears:
  \[
  \sigma_{eq} = p_0 (1 - 2\mu) = \sigma_T ,
  \]  
  \( 2.10 \)

- for helical gears:
  \[
  \sigma_{eq} = 0.97 q_0 (1 - 2\mu) = \sigma_T .
  \]  
  \( 2.11 \)

For structural materials, which after surface hardening by chemical-thermal treatment were prone to brittle fracture \( \sigma_{eq} = \sigma_T \).

At short-term overloads for plastic materials (wheels subjected to normalization, improvement or throughout quenching with low tempering) basing on formulas (10) and (11) we obtain expressions for the highest pressure \( p_0_{max} (q_0_{max}) \) on the surface of the contact area at the time of the appearance of plastic deformations.

- for straight-toothed gears, they are determined by the following formula:
  \[
  p_{0_{max}} = \frac{\sigma_T}{(1 - 2\mu)} ,
  \]  
  \( 2.12 \)

- for helical gears according to the formula:
  \[
  q_{0_{max}} = \frac{\sigma_T}{0.97(1 - 2\mu)} .
  \]  
  \( 2.13 \)

By the contact stresses \( \sigma_N \) we mean the greatest pressure at the contact site. Therefore, the permissible contact stress \( \sigma_{NP} \) at maximum load:

- for straight gears:
  \[
  \sigma_{NP_{max}} = p_{0_{cr}} = (2.00 ... 2.78)\sigma_T ,
  \]  
- for helical gears:
  \[
  \sigma'_{NP_{max}} = q_{0_{max}} = (2.06 ... 2.87)\sigma_T
  \]  
  when the Poisson's ratio \( \mu \) changes from 0.25 to 0.32.

Standard STST 21354-87 for steels which are usually used for the manufacture of gears and subjected to volumetric heat treatment (normalization, improvement, through hardening with low tempering) recommended the following rule, limiting the value of the permissible limit contact stresses, \( \sigma_{NP_{max}} = 2.8 \sigma_T \). It corresponds only to the extreme upper limit in the obtained dependences. The average values of permissible contact stresses in the obtained dependences are about 15% lower than the recommended ones.

3. Calculation of allowable stresses for materials prone to brittle destruction.

For materials which as a result of hardening heat treatment have a tendency to brittle destruction of the surface layer (for teeth subjected to cementation or contour hardening, for nitrided teeth) permissible contact stress limit:

- for straight gears:
  \[
  \sigma_{NP_{max}} = p_{0_{cr}} = \frac{\sigma_T}{x(1-4\mu)+2\mu} ,
  \]  
  \( 3.1 \)

- for helical gears:
  \[
  \sigma'_{NP_{max}} = q_{0_{cr}} = \frac{\sigma_T}{0.97x(1-2\mu)+(1-x)(2\mu+(1-2\mu)\frac{\mu}{\sigma_{crp}})} .
  \]  
  \( 3.2 \)
For steels subjected to heat treatment, the coefficient characterizing their brittleness \( x = 0.4 \ldots 0.78 \). For the average values of \( x = 0.59 \) and the coefficient of transverse deformation \( \mu = 0.29 \) permissible contact stress limit:

- for straight gears:
  \[ \sigma_{NP_{\text{max}}} = 2.06\sigma_t, \]
- for helical gears:
  \[ \sigma'_{NP_{\text{max}}} = 38.13\ HRC. \]

Tensile strength of steels, which are commonly used for the manufacture of gears and are subjected to improvement and hardening of HDTV along the entire tooth contour and through hardening of HDTV, \( \sigma_t \approx 18.33 \) HRC. For them, the permissible contact limit stress has the following value:

- for straight gears:
  \[ \sigma_{NP_{\text{max}}} = 37.76\ HRC, \]
- for helical gears:
  \[ \sigma'_{NP_{\text{max}}} = 38.13\ HRC. \]

Tensile strength of steels, which are commonly used for the manufacture of gears and are subjected to cementation and contour hardening \( \sigma_t \approx 16.81 \) HRC. For them, the permissible contact limit stress has the following value:

- for straight gears:
  \[ \sigma_{NP_{\text{max}}} = 34.63\ HRC, \]
- for helical gears:
  \[ \sigma'_{NP_{\text{max}}} = 34.96\ HRC. \]

Tensile strength of steels subjected to nitriding \( \sigma_c \approx 18.49 \) HRC. For them, the permissible contact limit stress has the following value:

- for straight gears:
  \[ \sigma_{NP_{\text{max}}} = 38.1\ HRC, \]
- for helical gears:
  \[ \sigma'_{NP_{\text{max}}} = 38.46\ HRC. \]

In the first two cases, the STST 21354-87 recommends the following:

\[ \sigma_{NP_{\text{max}}} = 40\ HRC. \]

In the third case:

\[ \sigma_{NP_{\text{max}}} = (30 \ldots 40)\ HRC. \]

4. Conclusion

Analyzing the obtained dependences, it can be noted that the permissible limit contact stress for helical gears is slightly higher than for straight gears. In the manufacture of gears made of steel, which are subjected to volumetric heat treatment (normalization, improvement, through hardening with low tempering), this difference is 3.3%. In other cases, it does not exceed 1%. The difference here is negligible and it can be assumed that the permissible limit contact stress for helical gears coincides with the same for straight teeth.

For steels that undergo volumetric heat treatment (normalization, improvement, through quenching with low tempering), the norm of permissible limit contact stresses recommended by the standard can be almost 30% higher compared to the values calculated from the dependencies obtained in this work.
For steels, which are usually used for the manufacture of gears and are subject to improvement and hardening of HDTV throughout the tooth contour and through hardening of HDTV, the rate of permissible limit contact stresses is almost 6% higher compared to the values calculated from the dependencies obtained in this work.

For steels subjected to cementation and contour hardening, the permissible limit contact stresses are 16% higher than the values calculated from the dependencies obtained in this work.

For nitrided steels, the values of permissible contact stress limits set by the standard are quite consistent with the calculations according to the above dependencies.

However, it should be noted that for nitrided teeth due to the small thickness of the hardened surface layer, it is possible to press the surface layer into the relatively soft core of the tooth under the action of high contact stresses, as well as the occurrence of deep contact destruction. In this case, the values of permissible contact stresses determined by formulas (11) and (12) should be reduced by approximately 1.35 times. Then for nitrided teeth:

- spur gears:
  \[ \sigma_{NP_{\text{max}}} = 28.49 \text{ HRC}, \]
- helical gears:
  \[ \sigma'_{NP_{\text{max}}} = 28.76 \text{ HRC}. \]

These values are below the standard by 18.6%.

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Acknowledgements
This paper was financially supported by the Ministry of Education and Science of the Russian Federation on the program to improve the competitiveness of Peoples’ Friendship University of Russia (RUDN University) among the world’s leading research and education centers in the 2016-2020. This publication was prepared with the support of the “RUDN University Program 5-100”.