Law of cooling, heat conduction and Stefan-Boltzmann radiation laws fitted to experimental data for bones irradiated by CO2 laser

Luc Lévesque*
Department of Physics, Royal Military College, Kingston, On, K7K 7B4, Canada
*Luc.levesque@rmc.ca

Abstract: The rate of cooling of domesticated pig bones is investigated within the temperature range of 20°C-320°C. Within the afore-mentioned temperature range, it was found that different behaviors in the rate of cooling were taking place. For bones reaching a temperature within the lower temperature range of 20°C-50°C, it was found that the rate of cooling is mostly governed by the empirical Newton’s law of cooling. It is also shown that a transition is taking place somewhere within 50°C-100°C, where both the heat conduction equation and Newton’s law apply. As bones can be raised at a fairly high temperature before burning, it was found that the rate of cooling within the range 125°C-320°C is mostly behaving according to the heat conduction equation and Stefan-Boltzmann radiation law. A pulsed CO2 laser was used to heat the bones up to a given temperature and the change of temperature as a function of time was recorded by non-contact infrared thermometer during the cooling period.

© 2014 Optical Society of America

OCIS codes: (140.0140) Lasers and laser optics; (120.0120) Instrumentation, measurement, and metrology; (140.6810) Thermal effects.

References and links
1. A. L. McKenzie, “Physics of thermal processes in laser-tissue interaction,” Phys. Med. Biol. 35(9), 1175–1210 (1990).
2. K.-W. Guan, Y.-Q. Jiang, C. S. Sun, and H. Yu, “A two-layer model of laser interaction with skin: A photothermal effect analysis,” Opt. Laser Technol. 43(3), 425–429 (2011).
3. J. H. Torres, M. Motamedi, J. A. Pearce, and A. J. Welch, “Experimental evaluation of mathematical models for predicting the thermal response of tissue to laser irradiation,” Appl. Opt. 32(4), 597–606 (1993).
4. M. M. Ivanenko, S. Fahimi-Weber, T. Mitra, W. Wierich, and P. Hering, “Bone Tissue Ablation with sub-μs Pulses of a Q-switched CO2 Laser: Histological Examination of Thermal Side Effects,” Lasers Med. Sci. 17(4), 258–264 (2002).
5. N. M. Fried and D. Fried, “Comparison of Er:YAG and 9.6-μm TE CO2 Lasers for Ablation of Skull Tissue,” Lasers Surg. Med. 28(4), 335–343 (2001).
6. J. Lee, Y. Rabin, and O. B. Ozdoganlar, “A new thermal model for bone drilling with applications to orthopaedic surgery,” Med. Eng. Phys. 33(10), 1234–1244 (2011).
7. Q. Peng, A. Juzeniene, J. Chen, L. O. Svaasand, T. Warloe, K-E Giercksky and J. Moan, “Lasers in Medicine,” Rep. Prog. Phys. 71, 056701 (2008).
8. Y. Yener and S. Kakaç, Heat Conduction, 4th ed. (Taylor & Francis, 2008) pp. 210–212.
9. M. N. Özisik, Heat Conduction (John Wiley & Sons, 1980) p.276.
10. A. Mathur and Y. K. Agrawal, “An overview of methods used for estimation of time since death,” Aust. J. Forensic Sci. 43(4), 275–285 (2011).
11. M. M. Ivanenko and P. Hering, “Wet bone ablation with mechanically Q-switched high-repetition-rate CO2 laser,” Appl. Phys. B 67(3), 395–397 (1998).
12. M. Forrer, M. Frenz, V. Romano, H. J. Altermatt, H. P. Weber, A. Silenok, M. Istomyn, and V. I. Konov, “Bone-Ablation Mechanism Using CO2 lasers of Different Pulse Duration and Wavelength,” Appl. Phys. B 56(2), 104–112 (1993).
13. M. R. Tatara, W. Krupski, B. Tymczyna, and I. Lusczewska-Sierakowsky, “Biochemical bone metabolism markers and morphometric, densitometric and biomechanical properties of femur andibia in female and gonadectomised male Polish Landrace pigs,” J. Pre-Clin. Clin. Res. 6(1), 14–19 (2012).
1. Introduction

1.1 Context of previously published work

Several mathematical models have been developed for predicting thermal response when tissue is irradiated by lasers [1–3]. During the last decade, histological examinations of cartilage and skull tissue using mid-infrared lasers were studied for potential surgical applications [4,5]. From these studies, it is shown that the quality of troughs or cuts created in bones depends upon the laser pulse duration. As more conventional practices involving drilling [6] lead to bone fragmentation, new solutions involving lasers became very attractive in surgical procedures. In material processing and surgery procedures the lasers are usually pulsed to allow sufficient time for heat to diffuse and prevent thermal damage or charring [4,5]. Therefore thermal response of bones must be investigated thoroughly in order to improve the laser performances in a given surgical procedure. In the case of moderate heating in therapeutic applications [7], temperatures involved are much below the vaporization temperature. In this case, mostly the modes of heat transfer by convection and conduction are expected to dominate. In the case of more severe heating that can cause discoloration in bones, heat transfer by radiation will be shown to be more important than conduction and convective heat losses. In this experimental investigation, we will focus our attention on three modes of heat transfer. These modes of heat transfer that will be investigated are those by convection, radiation and conduction.

1.2 Heat transfer by convection

The first mode of heat transfer investigated is described by Newton’s law of cooling, which is often applied in many fields of engineering [8,9] to describe how the temperature $T$ of a material changes as a function of time $t$. It also finds applications in forensic sciences as it can be used to estimate the time since death occurred [10]. Newton’s law of cooling is often used to model heat losses by convection during the process of cooling. This empirical law describes the heat flowing per unit time and per unit area from the solid surface to the surrounded fluid or gases when convection is taking place. Mathematically, it is expressed as:

$$ q^\text{conv} = h(T - T_\infty) $$

where $q^\text{conv}$ is the heat flow per unit time and per unit area in W/m$^2$, $h$ is the heat transfer coefficient which has units of W/(m$^2$ K), $T$ is the temperature and $T_\infty$ is the temperature of the fluid or gas surrounding the material surface. In our investigations $T_\infty$ is the laboratory temperature, which is 21°C. From values found in the literature [6,8,9], $h$ is within the 5-30 W/(m$^2$.K) range for free air convection outdoor and the value of the heat transfer coefficient also depends upon the material of the solid surface during the process of cooling. Some experimental data will be fitted to an expression based on this mode of heat transfer later in the experimental section and the temperature range for which it applies will also be identified.
1.3 Heat transfer by radiation

The mode of heat transfer by radiation is often overlooked as it does not behave linearly with temperature and it is not always possible to solve analytically. In the case of bone’s temperature raised well over 100°C, an important contribution of heat transfer is bound to occur by radiation. The heat lost by radiation is described by the Stefan-Boltzmann radiation law, which is expressed as:

\[ q_r = \varepsilon \sigma (T^4 - T_0^4), \tag{2} \]

where \( q_r \) is the heat flow per unit time and per unit area for the heat loss by radiation, \( \varepsilon \) is the material emissivity and \( \sigma \) is the Stefan-Boltzmann constant, which is equal to 5.67 \times 10^{-8} \, \text{W/(m}^2\text{K}^4\text{)}. Note from Eq. (2), that the radiation loss depends on the fourth power of the temperature, which means that this mode of heat transfer is very important as temperature increases. Some experimental data will be fitted to an expression based on this mode of heat transfer later in the experimental section and it will be shown that this mode of heat transfer is more important as bones start cooling. It will also be shown that it is very much depending upon the bone initial temperature.

1.4 Heat transfer by conduction

The other mode of heat transfer that is investigated is by conduction. The heat lost by conduction mostly depends upon the temperature gradient \( \nabla T \) and is described by Fourier’s law as:

\[ q_{\text{cond.}} = -k \nabla T, \tag{3} \]

where \( k \) is the thermal conductivity of the material and \( q_{\text{cond.}} \) is the heat flow per unit time and per unit area for the heat lost by conduction. Some experimental data will be fitted to an expression based on this mode of heat transfer combined with that of convection later in the experimental section.

2. Methodology

The absorption \( \alpha \) of bone is very large [11,12] at the \( \text{CO}_2 \) laser wavelength \( \lambda \). As the melting point of mineral bones is high [11] (\( T_{\text{melt}} \sim 1280^\circ \text{C} \)), the material can be heated to at least 300°C before getting burned and without affecting much its properties. A \( \text{CO}_2 \) laser Firestar 60t from Synrad was used to investigate all the bone samples. The beam from the \( \text{CO}_2 \) laser aperture was reflected by a system of two mirrors as shown in Fig. 1. The pair of mirrors was adjusted so that the beam propagates in a direction that is parallel to the plane of the optics table. The beam of the \( \text{CO}_2 \) laser was then collimated by a pair of lenses of focal lengths \( f_1 \) and \( f_2 \). Using this pairs of collimation lenses, the beam was also magnified to fill about one third of the lens surface of a third lens of focal length \( f_3 \).
This last lens of focal length $f_3$ is focussing the CO$_2$ beam a distance of 12 cm behind it as shown in Fig. 1 and the bone sample is positioned further away in the diverging field about 15 cm from the focus point. Pig bones were chosen because laser ablation resulting in these animal bones is similar to findings in hard biological tissues such as a human tooth [11]. Also, the cortical bone mineral density of domesticated pig bones is comparable to that of human bones [13]. The domesticated pig bones were cleaned in a 3% H$_2$O$_2$ solution for four hours. Then each bone was soaked for a few minutes in distilled water, wiped with a low abrasive tissue and they were left to dry in a ventilated area for a day. The bone samples used were cut out of flat rib bones about 60 mm long. Their cross sections were oval in shape and their thickness were roughly 5 mm across the center point. The shape of the surface facing the incident CO$_2$ beam (c.f. Fig. 1) was nearly rectangular and the area’s dimensions were roughly 1 cm x 2 cm. The shape of the samples was ideal as they could sit well on a thick glass substrate having a thickness of roughly 2 cm. This thick glass substrate (not shown in Fig. 1) lying directly on the optic table was also absorbing the CO$_2$ radiation during the heating procedure and prevented too much heat transfer from the bone samples to the metallic table underneath. Small or long cylindrical bones were not investigated as the surface in thermal contact with the glass plate was greatly reduced and as a result would be much more difficult to model. At the point where the bone samples were placed, the CO$_2$ beam exposing each sample had a broad Gaussian profile with the beam spot size (at the $1/e^2$) of 20 mm, which was comparable to the longest dimension of the sample. Each bone sample was centered with the CO$_2$ laser beam. Near that working point an IR thermometer is aimed at the surface. The IR thermometer is adjusted to measure the temperature of any object that is lying at the sample position. When taking a water sample from the refrigerator and locating it at this working point, the IR thermometer was reading 4°C. Also, the IR thermometer indicated about 31°C if the operator’s finger was positioned at the working point. As bones are absorbing very much at the CO$_2$ wavelength, only a very small layer confined near the bone surface will be heated. The thickness $\tau$ of this layer confined near the surface is attenuating the laser beam irradiation by direct absorption according to Beer’s law [14,15]. Therefore, in the case of a disk of material the laser irradiation will decay exponentially from the surface as:

$$I(z) = I_o \exp(-\alpha z),$$

where $\alpha$ is the absorption coefficients, $I_o$ is the laser irradiation at the material surface and $z$ is the distance from the surface. At a distance $z = 3\alpha^{-1}$ from the material surface, Eq. (4) predicts that the laser irradiation is roughly 5% of the initial value $I_o$. As a result, in our investigation we will estimate the layer heated by the CO$_2$ laser as $\tau \sim 3\alpha^{-1}$. In our investigations, the CO$_2$ laser is heating bone samples up to a given temperature and then the
laser is turned off. From the time the laser is turned off, the temperature of each bone sample is monitored as a function of time during the cooling period.

3. Temperature as a function of time for all modes of heat transfers

In this section, we will express how temperature should vary as a function of time based on the law describing heat lost in the modes of convection, radiation and conduction.

3.1 Temperature varying as a function of time for heat lost by convection

Heat lost by convection depends upon the first power of temperature \( T \) and is described empirically by Eq. (1). If no vaporization is taking place, the heat \( Q \) released during the process of cooling is given by:

\[
Q = \rho V c T, \tag{5}
\]

where \( \rho \) is the density, \( c \) is the specific heat and \( V \) is the volume of material from which heat is flowing. In our investigation, heat is absorbed in the material within a small layer of thickness \( \tau \), just before it starts cooling. If only convection is taking place during the cooling process, then Eq. (1) and Eq. (5) can be applied to find a differential equation for the temperature as a function of time \( t \). We find that the material temperature is a solution of the first order differential equation:

\[
\frac{dT}{dt} + \left(\frac{h}{\rho \tau c}\right)T = \left(\frac{h}{\rho \tau c}\right)T_o. \tag{6}
\]

All terms in Eq. (6) have been defined earlier. Note in Eq. (6) that the inverse of \( (h/\rho \tau c) \) gives an estimate of the time constant for the cooling process by convection according to Newton law. From values available in the literature for bones [15], \( \rho \sim 2500 \text{ kg/m}^3 \), \( c \sim 1300 \text{ J/kg. ºK} \) and \( \alpha \sim 2500 \text{ cm}^{-1} \). Therefore, \( \tau \sim 3 \text{ (1/2500 cm}^{-1}) \sim 12 \mu \text{m} \) and in the case of free convection indoor [16] \( (h/\rho \tau c)^{-1} \sim 26 \text{ seconds} \). This means that this mode of cooling by convection is fairly slow. The solution to the differential Eq. (6) is given by:

\[
T = T_o + (T_i - T_o) e^{-\beta t}, \tag{7}
\]

where \( T_i \) is the initial temperature and \( \beta \) is equal to \( (h/\rho \tau c) \).

3.2 Temperature varying as a function of time for heat lost by radiation

Heat loss from radiation depends upon the fourth power of temperature and is not expected to be governed by a linear differential equation. If heat is released only by radiation during the process of cooling, Eqs. (2) and (5) can be used. In this case, we find that:

\[
\frac{dT}{dt} = -\varepsilon\sigma \left( T^4 - T_o^4 \right). \tag{8}
\]

For all bone samples investigated in our experiments \( \varepsilon \) was assumed to be equal to 1. By regrouping the variables in Eq. (8) on each side of the equal sign, an analytical solution can be found by integration. The general solution [17] for \( T \) is given by:

\[
\frac{1}{4T_o^3} \ln \left[ \frac{T - T_o}{T + T_o} \right] - \frac{1}{2T_o^3} \arctan \left( \frac{T}{T_o} \right) + C = -k_1 t, \tag{9}
\]

where \( C \) is an integration constant and \( k_1 \) was set to \( (\varepsilon \sigma/\rho \tau c) \). The integration constant is evaluated from the initial condition \( T = T_i \) at \( t = 0 \) and it can be shown that:
In Eq. (10), the initial temperature $T_i$ is the temperature from which the bone sample is starting to cool off just after the laser is turned off. Note in Eq. (10), that the argument of the natural logarithmic function can be approximated for large temperatures $T$ and $T_i$ compared to the laboratory temperature $T_o$. For instance, for $T_i > 10T_o$, the ratio $T_i + T_o \sim 1$ and for $T > 10T_o$, the logarithmic function can be approximated by:

$$
\ln\left[\frac{(T - T_o)(T_i + T_o)}{(T + T_o)(T_i - T_o)}\right] = \ln\left[\frac{1 - \frac{T_o}{T}}{1 + \frac{T_o}{T}}\right] \approx -2x - \frac{2}{3}x^3 + \ldots \quad \text{for} \quad x < 1. \tag{11}
$$

In Eq. (11), $x$ was set to $T_o/T$. As the arc tan functions nearly cancel each other out for $T_i > 10T_o$ and $T > 10T_o$, one can show that to the first order, $t \sim 0.3T_o/T$ for $T_o/T << 1$. In the previous approximation, we used the following values: $\varepsilon = 1$, $\rho = 2500 \text{ kg/m}^3$, $\tau = 12\mu\text{m}$ and $c = 1300 \text{ J/kg} \cdot \text{K}$. As the laboratory temperature $T_o$ is about 21°C and the bone sample temperature $T$ is often larger than 175°C, the approximation $t \sim 0.3T_o/T$ should hold at the start of the cooling process just after the laser is turned off. Note that the cooling rate is very fast just after the laser is turned off and that the cooling by radiation loss should occur at times $t$ that are roughly three orders of magnitude smaller than that predicted from Eq. (6) for convection loss. One important technical aspect should be borne in mind regarding the temperature measurement from the IR thermometer. The IR thermometer is aimed to include the field of view of the material being heated. As the laser beam is directed normal to the sample surface, the IR thermometer is placed along a slightly oblique direction and is most likely looking at a field of view that is larger than the sample’s surface. Therefore, the temperature being measured by the device would be an average over the sample temperature and part of the surrounding medium just around it. As the bone is heated very much within a layer $\tau$ that is confined to the surface, the temperature gradient is expected to be large along the bone surface normal. Therefore, heat flow by conduction is expected to be important and it should also be considered.

### 3.3 Temperature varying as a function of time for heat loss by conduction

As the temperature gradient is very large, the heat flow per unit time will be important. We will consider the heat flow along the direction that is normal to the bone samples. At the heating point, the beam spot size was comparable to the dimensions of the bones. As a result, the radial temperature gradient is assumed to be small and consequently the loss by conduction along the radial direction will be neglected. Therefore, Eq. (3) can be expressed as:

$$
q^{*}_{\text{cond.}} = -k \frac{dT_s}{dt}, \quad \tag{12}
$$

where $T_s$ is the surface temperature as shown in Fig. 2(a). All the power irradiated by the laser should be confined within a small volume $V$ ($\equiv 4\tau$) of thickness $r$, as shown in Fig. 2(a).
Fig. 2(a), the laser beam cross-sectional area is depicted by \( A \). This means that Eq. (12) may also be approximated by:

\[
\frac{\rho V c}{A} \frac{dT_x}{dt} \approx -k \frac{(T_x - T_o)}{\tau}
\]  

(13)

from where we find that:

\[
\frac{dT_x}{dt} + \frac{1}{\beta'} T_x \approx \frac{T_o}{\beta'}
\]  

(14)

where \( \beta' = \frac{\rho c^2}{k} \). Factor \( \beta' \) in the previous differential Eq. (14) gives an estimate for the diffusion time by conduction in bones. Using \( \rho \sim 2500 \text{ kg/m}^3 \), \( c \sim 1300 \text{ J/kg} \cdot ^\circ\text{K} \), \( k = 0.32 \text{ W/m}^2\text{K} \) and \( \tau \sim 12 \mu\text{m} \), we obtain about \( 1.5 \text{ ms} \) for the diffusion time. This estimate is in agreement with diffusion times in the literature for hard bone such as enamel in heat diffusion analysis [12].

An expression can be derived from the heat conduction equation. In 1D, bone sample will be considered as a semi-infinite medium with the z-axis as shown in Fig. 2(b). The problem is formulated as:

\[
\frac{\partial^2 T}{\partial z^2} = \frac{1}{K} \frac{\partial T}{\partial t}
\]  

(15)

\[
-k \frac{\partial T}{\partial z} \bigg|_{z=0} = -h(T - T_o) \bigg|_{z=0}
\]  

(16)

\[
T = T_o \text{ for } z \gg \tau
\]  

(17)
Equation (15) is the heat conduction equation in 1D for the semi-infinite medium shown in Fig. 2(b). Note in Eq. (15), that $K ( = k/\rho c)$ is the thermal diffusivity. Equation (16) is taken into account the convection loss at the interface $z = 0$. In Eq. (17), it is assumed that the bone temperature should be the same as the laboratory temperature for depth that are much larger than $\tau$. As the thermal conductivity $k$ of glass is at least 100 times smaller than that of metal [8], it was deemed necessary to put the bone samples on a glass plate instead of placing them directly on the optic table or a metal plate to prevent too much heat losses. As the bone samples are sitting directly on glass plate that is about 2 cm thick, the heat sink was thus minimized and as a result it was not taken in account in the mathematical model. Lastly, the temperature $T$ is that sensed by the non-contact IR temperature detector just after the laser is turned off and as it starts to cool off. Temperature detected by the non-contact IR sensor is receiving signals from many points distributed within the depth of the sample that could extend to $\tau$ or even more. Therefore, temperature $T(z,t)$ from the solution of the heat equation as formulated by Eqs. (15) to 18 will be calculated at $z = \tau/2 = 6 \mu m$ below the bone’s surface. The value used for $T_i$ is the one measured by the non-contact sensor as the sample starts cooling at $t = 0$. From the problem being formulated by Eqs. (15) to 18, the solution for $T(z,t)$ is given by [18,19]:

$$T(z,t) = T_i - (T_i - T_o)\left[erfc\left(\frac{z}{2\sqrt{Kt}}\right) - \exp\left(H^2Kt + Hz\right)erfc\left(H\sqrt{Kt} + \frac{z}{2\sqrt{Kt}}\right)\right],$$

where $H = h/k$.

4. Experimental

The bones were placed as shown in the experimental set-up illustrated in Fig. 1. A series of CO$_2$ laser pulses at a modulation frequency of 5 kHz were delivered to the sample at a power of about 7 W during 20 seconds. The temperature of each sample was raised very quickly from 21°C to a given temperature within the 65°C-275°C range during 20 seconds. This phase is shown as step A in Fig. 3. Just after the first series of pulses at 5 kHz, a second series of pulses was delivered at the samples for 2 minutes at a higher modulation frequency of 20 kHz and at a lower power of 5.6 W. This second series of pulses at 20 kHz was used to keep the temperature uniform as shown in step B in Fig. 3. This method was also proved to be successful with other materials [20]. Just after this second series of pulses, the laser was stopped and the bone samples were starting to cool off as shown in step C in Fig. 3.
Normal bone contains about 25% of collagen matrix, about 5% of non-collagenous protein and roughly 70% of calcium hydroxyapatite mineral. It is known that denaturation in collagen occurs around 40°C. As a result collagen undergoes irreversible physical changes. Therefore, each curve shown in each step in Fig. 3 is changed as bone samples are raised at high temperatures near 300°C. The temperature reached in step A and the temperature level maintained in step B using laser pulses varies from one trial to the next and a decrease of about 10% was observed when repeating the whole heating procedure (steps A through C) for bone samples raised to a temperature of 275°C. After repeating the heating procedure three times on a given sample the results appeared more stable, but at temperatures greater than 300°C, some discoloration was noticed on bone samples. For this reasons, only data for bone samples raised to temperature below 300°C will be shown in this report. It is also possible that denaturation of collagen occurred near the bone surface during the cleaning process in the solution of H$_2$O$_2$.

Each bone sample underwent the same cycle depicted by steps A, B and C in Fig. 3. Then, the solutions for the temperature $T$ for the heat lost by convection (Eq. (7)), by radiation (Eq. (10)) and by conduction (Eq. (19)) were plotted amongst the data points obtained for step C during the cooling process. Figure 4 shows the results obtained for various samples. Table 1 is showing the data that were used to plot Eqs. (7), (10) and (19) in each prediction shown in Fig. 4. In Fig. 4(a), the bone sample was heated to a constant temperature of 57°C (Step B) and only the data points during the cooling period are shown (Step C). As the bone sample is not raised at a very high temperature, the power loss by radiation is expected to be small. As a result, Eq. (10) is not fitting well the experimental data points. Note that some of the data points appear to fit the theoretical curve from Eq. (19) at the very beginning of the cooling phase. When bone samples are kept at temperatures within 125°C and 175°C before cooling starts as in Figs. 4(c) through 4(f) the theoretical curve for Stefan-Boltzmann law of radiation loss from Eq. (10) and the solution (Eq. (19)) for the heat conduction equation is starting to fit reasonably well the experimental data points. At lower temperatures below 55°C, Newton’s law of cooling from Eq. (7) can be fitted well to the experimental data points. That is why no line or just a small line is shown from the prediction from Eq. (7) in Figs. 4(d) and 4(f), respectively. As the IR thermometer can only measure temperature at each second, note that only the first 5 or 6 points can be fitted to Eq. (10). Nevertheless, the data points at the starts of cooling also fit reasonably well to Eqs. (10) and (19) for bone samples heated to
temperature near 300°C as in Figs. 4(g) and 4(h). Note in Table 1, that values used for the heat transfer coefficient \( h \) are large in the fitting of data to Eq. (19) when the initial temperature \( T_i \) is above 100°C. In some treatments [3], water vapor in the air near the surface of tissue contributes in convective loss and it is modelled using the Lewis number (\( L_e \)). This heat transfer contribution also depends upon the tissue surface temperature. This suggests that relative humidity in the air must play a more prominent role in heat transfer by convection at temperatures greater than 100°C, which would explain why higher values of \( h \) fits the experimental data points when \( T_i \) is above 100°C. Within the range of temperature shown in Fig. 4, Eq. (7) also fits well the experimental data points from 50°C or lower. The values used for heat transfer coefficients \( h \) were within 0.85 and 1.1 W/m²K. The used values of \( h \) to fit Eq. (7) are in agreement with those reported for free convection inside public buildings [16]. After 20 seconds or so, conduction appears to become less significant as a mean of heat transfer and it can be seen from all curves that an exponential curve from Eq. (7) is fitting reasonably well the experimental data points (see Figs. 4(b) and 4(h)). As Newton’s law of cooling predicted by Eq. (7) is not expected to contribute much at high temperatures, Eq. (7) was shown to fit data better within the first 20 seconds at moderate temperatures (see Fig. 4(b)). For bones heated at higher temperatures, note that data fitting using Eq. (7) was either not possible within the first 20 seconds (see Fig. 4(d)) or just starting to be a good fit after 15s of cooling as shown in Figs. 4(f) and 4(h).
Fig. 4. Temperature of bone samples as a function of time measured by the IR thermometer during the cooling off process (Step C). The + symbols are the experimental data points, the dashed line is the prediction from Eq. (10) (Stefan-Boltzmann radiation loss), the black solid line is the prediction from Eq. (7) (Newton’s law of cooling) and the grey line shows the prediction from the heat conduction equation Eq. (19) as formulated by Eqs. (15) to 18. 

a) Bone heated at moderate temperature near 60°C 
b) Graph shown in a) within the first 20s of cooling 
c) Bone heated to temperature near 130°C 
d) Graph shown in c) within the first 15s of cooling 
e) Bone heated to a temperature of 185°C 
f) Graph shown in e) within the first 20s of cooling 
g) Bone heated to a temperature above 250°C 
h) Graph shown in g) within the first 20s of cooling.
Table 1. Data used in predictions from Eqs. (7), (10) and (19) in Fig. 4. In these theoretical predictions we used $\rho = 2500$ kg/m$^3$, $\tau = 12 \mu$m, $c = 1300$ J/°C, $k = 0.32$ W/m°K, $T_o = 21°C$, $\sigma = 5.67 \times 10^{-8}$W/m$^2$°K and $\varepsilon = 1$.

| Eq. (7) | Eq. (10) | Eq. (19) (at z = 6µm) |
|---------|----------|------------------------|
| Figures 4(a) and 4(b) | $h = 1.09$ W/m$^2$°C  
$\beta = 0.028$ s$^{-1}$  
$T_i = 49°C$ | $T_i = 57°C$ | $h = 120$ W/m$^2$°C  
$T_i = 57°C$ |
| Figures 4(c) and 4(d) | $h = 0.86$ W/m$^2$°C  
$\beta = 0.022$ s$^{-1}$  
$T_i = 55°C$ | $T_i = 129°C$ | $h = 405$ W/m$^2$°C  
$T_i = 129°C$ |
| Figure 4(e) and 4(f) | $h = 0.86$ W/m$^2$°C  
$\beta = 0.022$ s$^{-1}$  
$T_i = 80°C$ | $T_i = 185°C$ | $h = 410$ W/m$^2$°C  
$T_i = 185°C$ |
| Figure 4(g) and 4(h) | $h = 0.86$ W/m$^2$°C  
$\beta = 0.022$ s$^{-1}$  
$T_i = 81°C$ | $T_i = 288°C$ | $h = 570$ W/m$^2$°C  
$T_i = 288°C$ |

5. Conclusion

An estimate of the time range for heat transfer by convection and radiation were presented and discussed. Investigations on bones samples show that the three modes of heat transfer discussed are taking place during the start and the end of a cooling process. The equations derived for these three modes of heat loss were also fitted reasonably well to experimental data points in each identified time range as discussed in sections 3 and 4. These results are believed to be useful in determining which heat transfer is dominant within a given temperature range. These data would also be useful to improve laser performances in bone surgical procedures where fragments become problematic in orthopaedics and laser bone ablation.

Acknowledgment

The author would like to thank the Academic Research Program at Royal Military College of Canada for their financial support.