**D^0 D^0 \pi^+** mass distribution in the production of the T_{cc} exotic state

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We perform a unitary coupled channel study of the interaction of the D^{*+}D^0, D^{*0}D^+ channels and find a state barely bound, very close to isospin I = 0. The width obtained is small, of the order of 80 keV, tied to the width of the D^* states, short of the experimental one, but which would certainly be bigger upon consideration of the experimental resolution. We perform a detailed study of the D^0 D^0\pi^+ spectrum and compare with experiment, suggesting that the investigation of this state in other decay channels would bring additional new information concerning the nature of this state.

The recent discovery of the T_{cc} state by the LHCb Collaboration has added a new exotic hadron state to an already long list of states discovered in the latest years that challenge the q\bar{q} nature of the standard mesons or qqq of the standard baryons. The novelty with respect to many states containing hidden charm is that now there are two charm quarks open. This finding follows the discovery of the X_{(2866)} and X_{(2904)} which have an open charm quark and a strange quark with manifestly tetraquark structure.[1]

On the theory side there have been quite a few works devoted to the study of tetraquarks with two heavy quarks,[5-21], with quite a wide range of predictions going from about 250 MeV below the energy reported for the T_{cc} to 250 MeV above in the case of two open charmed quarks.

The mass of the T_{cc} state is remarkably close to the D^{*+}D^0 and D^{*0}D^+ thresholds, its value is given by[1-3]

\[ m_{\text{exp}} = 3875.09 \text{ MeV} + \delta m_{\text{exp}}, \]  
(1)

where 3875.09 MeV is the threshold of the D^{*+}D^0 state and

\[ \delta m_{\text{exp}} = -273 \pm 61 \pm 5^{+4}_{-14} \text{ keV}. \]  
(2)

The width reported for the T_{cc} state is[1-3]

\[ \Gamma = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}. \]  
(3)

As we can see, the mass is very close to the D^{*+}D^0 threshold and the width is very small.

The closeness to the D^* D threshold makes one think immediately about the possibility that this state could be a molecular state of D^* D, and in fact such structure was anticipated in Refs.[20,22,23]. Independent of the structure of the T_{cc} state, the proximity of the D^* D threshold makes unavoidable the explicit consideration of the D^* D channels in its study, as shown in the detailed study of threshold structures in Ref.[24]. The possible D^* D bound state would have an analogous structure to the D^* D^* molecular state already studied in Ref.[25], where such open charm molecular structures were reported for the first time. It is interesting to mention that also in Ref.[26] predictions were made for another exotic state of D^* K^* nature that matches correctly the X_{0}(2866) state reported in Ref.[4] (see update in Ref.[26]).

The reaction of the theory community to the experimental finding has been fast. In Ref.[27] a reminder was given that in Ref.[22] a prediction for a molecular D^* D state had been done matching perfectly the mass found in the experiment. At the same time it was stressed that the analogous exotic state, the D^* D^* bound state, had been earlier predicted in Ref.[25]. In Ref.[28] the width of the T_{cc} state is studied with the D^{*+}D^0 and D^{*0}D^+ coupled channels and found small compared with the experimental one. The same conclusion is obtained in Ref.[30] where a single channel D^* D molecule is assumed. The QCD sum rules method also brings its contribution to

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1 The couplings of T_{cc} to the D^{*+}D^0 and D^{*0}D^+ channels obtained in Ref.[28] (version v1 of the ArXiv) are under revision[29].
the subject showing that such a state appears at a central value of 3868 MeV for the mass, with the typical large uncertainties of the sum rules method, about 124 MeV in this case \[34\].

In the present work we report on how a molecular state of $D^*D^0, D^{*0}D^+$ nature naturally emerges from the interaction of these two coupled channels, and we make a study of the invariant mass distribution of $D^0D^0\pi^0$ in the production of this state, which is the mode where it has been observed in Refs. \[1–3\]. We use as a source of interaction the exchange of vector mesons provided by the local hidden gauge Lagrangians \[32–35\]. In the case of $VP$ (vector-pseudoscalar) interaction one can also exchange the pseudoscalar mesons, but comparatively to the vector exchange their contribution is small \[36–38\]. In any case the coupled channels unitary approach requires the use of the $G$ functions, the loop functions of the intermediate $D^*D$ states, which have to be regularized, and missing pieces of the interaction can be accommodated by means of an appropriate choice of the cut off or the subtraction constant, in the cut off or dimensional regularization methods, which are fine tuned to the precise value of the mass of the state.

We use a unitary method with the coupled channels $D^*D^0$ and $D^{*0}D^+$, paying attention to the exact masses and widths. The interaction is obtained from the extended local hidden gauge Lagrangians \[32,35\] and they correspond to the vector exchange of mesons in the diagrams of Fig. 1.

In diagrams Fig. 1(a) and (b) one can exchange $\rho^0$ and $\omega$, but assuming equal masses for the $\rho$ and $\omega$, there is an exact cancellation.

The Lagrangians used are

$$L_{VP} = -ig \langle [P, \partial_\mu P] V^\mu \rangle, \quad L_{VV} = ig \langle [V^\nu, \partial_\mu V_\nu - \partial_\mu V^\nu V_\nu] V^\mu \rangle,$$

$$g = \frac{M_V}{2f}, \quad (M_V = 800 \text{ MeV}, \quad f = 93 \text{ MeV}).$$

with $\langle \rangle$ meaning the trace of the matrices in the SU(4) space, where $P$ and $V$ stand for the pseudoscalars and vectors respectively, and they correspond to the $q_i\tilde{q}_j$ matrices written in terms of the corresponding mesons, which can be found in Ref. \[39\]. Since we are close to the $D^*D$ threshold we neglect the $\rho^0$ components of the vectors and work with the vector polarizations $\hat{e}, \hat{e}'$. Calling $D^{*+}D^0, D^{*0}D^+$ the 1,2 channels, the interaction that we obtain is

$$V_{ij} = C_{ij} g^2 (p_1 + p_3) \cdot (p_2 + p_4) \hat{e} \cdot \hat{e}'$$

$$\rightarrow C_{ij} g^2 \frac{1}{2} \left[ 3s - (M^2 + m^2 + M'^2 + m'^2) \right] \hat{e} \cdot \hat{e}'$$

$$- \frac{1}{s} (M^2 - m^2)(M'^2 - m'^2) \hat{e} \cdot \hat{e}' ,$$

where $M, m$ are the initial vector, pseudoscalar masses and $M', m'$ the corresponding final ones. The second expression in Eq. (5) follows after projection in $s$-wave, which is what we study. The matrix $C_{ij}$ is given by

$$C_{ij} = \left( \begin{array}{cc} \frac{m_D}{M_{J/\psi}} & \frac{1}{m_D} \\ \frac{1}{m_D} & \frac{m_D}{M_{J/\psi}} \end{array} \right).$$

One finds that individually the $D^{*+}D^0, D^{*0}D^+$ states have a weak and repulsive interaction due to $J/\psi$ exchange and hence they do not bind by themselves, but the coupled channels have the virtue of making a bound state possible. Indeed, if we take the isospin combinations (our isospin doublets are $(D^+, -D^0)$ and $(D^{*+}, -D^{*0})$)

$$|D^*D, I = 0 \rangle = -\frac{1}{\sqrt{2}} (D^{*+}D^0 - D^{*0}D^+),$$

$$|D^*D, I = 1, I_3 = 0 \rangle = -\frac{1}{\sqrt{2}} (D^{*+}D^0 + D^{*0}D^+),$$

we find (the indices indicating the isospin)

$$C_{00} = \frac{1}{M_{J/\psi}} - \frac{1}{m_D'}; \quad C_{11} = \frac{1}{M_{J/\psi}} + \frac{1}{m_D'}; \quad C_{01} = 0; \quad (8)$$

which means that we find an attraction, and not weak, in $I = 0$ and repulsion in $I = 1$. An approximate solution can be obtained using the single channel with $I = 0$, which implies using average masses for $D^*$'s and $D$'s, but we wish to be accurate and will use the coupled channels method with the exact masses.

We solve then the Bethe-Salpeter equation in coupled channels and have in matrix form

$$T = [1 - VG]^{-1} V,$$

with $G = \text{diag}[G_1, G_2]$, where $G_i$ are the $D^*D$ loop functions, which we regularize using dimensional regularization as in Ref. \[40\], with the value of $\mu = 1500 \text{ MeV}$ and the subtraction constant $\alpha_H$ having value close to $\alpha = -1.15$. The value of $\alpha_H$ is fine tuned to get the experimental binding of $T_{cc}$ state. Yet, to get a finite width for the state below the $D^*D$ thresholds we need to consider the width of the $D^*$ states. This is accomplished performing a convolution of the $G$ functions with the spectral function (mass distribution) of the $D^*$ states, as done in Ref. \[41\] (see Eqs.(4),(5) of that reference), with the width of the $D^*$ states showing the energy dependence:

$$\Gamma_{D^{*+}}(M_{inv}) = \Gamma(D^{*+}) \left( \frac{m_{D^{*+}}}{M_{inv}} \right)^2 \left[ \frac{2}{3} \left( \frac{p_\pi}{p_{\pi, on}} \right)^3 + \frac{1}{3} \left( \frac{p_\rho}{p_{\rho, on}} \right)^3 \right],$$

where $p_\pi$ is the $\pi^+$ momentum in $D^{*+} \rightarrow D^0 \pi^+$ decay with $D^{*+}$ mass $M_{inv}$, and $p_{\pi, on}$ the same one with the physical mass of $D^{*+}$ taken from the PDG \[22\]. Analogously, $p_\rho', p_{\rho, on}'$ are the same magnitudes for $D^{*+} \rightarrow D^{*0}$. The width $\Gamma(D^{*+})$ is taken from the PDG, $\Gamma(D^{*+}) = 83.4 \text{ keV}$. For the $D^{*0}$, we take

$$\Gamma_{D^{*0}}(M_{inv}) = \Gamma(D^{*0}) \left( \frac{m_{D^{*0}}}{M_{inv}} \right)^2 \left[ 0.647 \left( \frac{p_\pi}{p_{\pi, on}} \right)^3 + 0.353 \right],$$

(11)
where the second term corresponds to the $D^{*0} \to D^0\gamma$ decay, which does not change appreciably with the small changes in $M_{inv}$ of our problem, and we have taken the branching fractions from the PDG and the value of $\Gamma(D^{*0}) = 55.3$ keV from Ref. 43. The values of $p_{\pi}, p_{\gamma}$, on correspond now to the $D^{*0} \to D^0\pi^0$ decay. The results are practically indistinguishable if we use $\Gamma(D^{*0}) = 55.9$ keV from 44 or 77.7 keV from 45, indicating that the $D^{*+}D^0$ channel is the one playing a major role in the state given its proximity to the $D^{*+}D^0$ threshold.

In Fig. 2 we show the results for $|T_{D^{*+}D^0, D^{*+}D^0}|^2$ as a function of $\sqrt{s}$ for the case of two channels. We have taken two subtraction constants $\alpha_H = -0.87$ for $D^{*+}D^0$ and $\alpha_{H} = -1.03$ for $D^{*0}D^+$, and we find a neat peak around the experimental mass. We see that the explicit consideration of the $D^*$ widths provides a width at the peak. The width of the peak for the two channels case is

$$\Gamma \simeq 80 \text{ keV}. \quad (12)$$

This value is small compared with the experimental width, Eq. (3), even considering the large errors, but not too far, and is about 60% larger than those obtained in Refs. 28 and 30 of the order of 50 keV, using the couplings of $T_{cc}$ to the $D^*D$ components. The explicit consideration of the coupled channels with the convolutions done using the energy dependent widths is responsible for this increased width.

It is interesting to see which are the couplings of the resonance in the case of two channels for the state corresponding to the peak of Fig. 2. They are obtained from $T_{11}$ and $T_{12}$ as $g_2^* = \lim_{s \to s_R}(s - s_R)|T_{11}|, g_2^* = g_1T_{21}/T_{11}$. In the easy case of neglecting the width of the $D^*$ states where the state appears as bound and there is no problem in defining the Riemann sheet we get

$$g_{T_{cc}, D^{*+}D^0} = 3658.7 \text{ MeV}; \quad g_{T_{cc}, D^{*+}D^0} = -3921.0 \text{ MeV};$$

and, as we can see, they are basically opposite to each other indicating that we have indeed a quite good $I = 0$ state, in spite of using different masses for the components and being close to thresholds.

Next we make a study of the $D^0D^0\pi^+$ mass distribution in the decay of the resonance, the channel observed in the experiment. This corresponds to a diagram like the one depicted in Fig. 3.

The decay of whichever object producing the $T_{cc}$ and decaying to $D^0D^0\pi^+$ can be obtained with the standard formula

$$\frac{d\Gamma}{dM_{12}^2 dM_{23}^2} = \frac{1}{2} \left(\frac{1}{2\pi}\right)^3 \frac{1}{s^{3/2}} |t|^2, \quad (14)$$

where $t$ is obtained from the diagram of Fig. 3, symmetrizing over the two $D^0$ momenta and the factor $\frac{1}{2}$ is added in the formula. The amplitude $t$ for this process

FIG. 1. Diagrams considered for the interaction $VP$.  

FIG. 2. $|T_{D^{*+}D^0, D^{*+}D^0}|^2$ as a function of $\sqrt{s}$. Dashed vertical line, $D^{*+}D^0$ threshold. Continuous vertical line, $D^{*0}D^+$ threshold. 

FIG. 3. Mechanism for $D^0\pi^+D^0$ decay of the $T_{cc}$ state. The diagram with $D^{*0}D^+$ decay does not lead to final $D^0D^0\pi^+$. 
is given by
\[
t = CT_{D^+D^0,D^0D^+D^0}(\sqrt{s}) \left[ \frac{\epsilon \cdot (\bar{p}_1 - \bar{p}_2)}{M_{12}^2 - m_{D^+}^2 + iM_{12}\Gamma_{D^+}(M_{12})} 
+ \frac{\epsilon \cdot (\bar{p}_3 - \bar{p}_2)}{M_{23}^2 - m_{D^+}^2 + iM_{23}\Gamma_{D^+}(M_{23})} \right],
\]
where \( C \) is an arbitrary constant and \( \epsilon \) stands for the polarization vector of the \( T_{cc}(1^+) \). Upon summing over the polarizations \( \epsilon \) in \( |t|^2 \) for \( T_{cc} \) at rest, we have terms
\[
\sum_{\text{pol.}} (\epsilon \cdot q_1)(\epsilon \cdot q_2) = \epsilon_\mu q_\mu^1 q_\nu^2 \left( -g^{\mu\nu} + \frac{P_\mu P_\nu}{M_{T_{cc}}^2} \right) q_\mu^1 q_\nu^2,
\]
where \( q_1 \) and \( q_2 \) can be \( (\bar{p}_1 - \bar{p}_2) \) and \( (\bar{p}_3 - \bar{p}_2) \). We convert the terms \( q_1 \cdot q_2 \) into invariants which can be written in terms of \( M_{12} \) and \( M_{23} \) using that \( M_{12}^2 + M_{12}^2 + M_{23}^2 = M_{T_{cc}}^2 + m_{D^+}^2 + m_{D^+}^2 + m_{\pi^+}^2 \). Integrating Eq. (14) over \( M_{12} \) and \( M_{23} \), using the limits of the PDG for the Dalitz boundary, we obtain the mass distribution \( \Gamma(\sqrt{s}) \) shown in Fig. 4.

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