The CP/T Experiment

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Abstract

In this talk I describe a proposed Fermilab Main Injector experiment to carry out a program of measurements on the physics of $K^0$ mesons. The experiment is designed to maximize the interference between $K_L$ and $K_S$ mesons near their production target, and hence have excellent sensitivity to CP violation in many decay modes. The extremely accurate CP violation measurements we will be able to make will allow us to test CPT symmetry violation with sensitivity at the Planck scale.

The experiment will use an RF-separated $K^+$ beam striking a target at the entrance to a hyperon magnet to make the $K^0$ beam by charge exchange. The decay region, magnetic spectrometer, electromagnetic calorimeter, and muon detector follow immediately to observe interference between $K_L$ and $K_S$ near the target.

I. INTRODUCTION

This talk is a description of an experiment that has been proposed to run at the Fermilab Main Injector to study CP violation, test CPT symmetry conservation, and search for rare decays of the $K_S$ meson. Here I will only discuss the CPT symmetry nonconservation search.

CPT symmetry conservation is a subject under theoretical attack. Studies of Hawking radiation [1] and of string theory (the leading contender for a unified theory of all four forces of nature) [2] have shown the CPT theorem [3] to be invalid in real life (rather than in the three-force approximation we call the standard model). Many physicists are reluctant to
accept the possibility that CPT symmetry violation may occur at the Planck scale. One reason for this reluctance is that we have, so far, only theoretical hints that this is the case. Another reason is the great success of the standard model. It may be quite a few years before a theory that unifies all four of the forces of nature becomes mature enough so that convincing theoretical statements can be made about the CPT structure of the world.

Fortunately we don’t have to wait. The $K_L - K_S$ system provides a way of testing the validity of CPT symmetry conservation where it is possible to perform extremely accurate experiments \cite{4}. In this document we propose to do an experiment that will reach the Planck scale. Finding CPT symmetry nonconservation would be a major discovery that would change in a fundamental way how physicists view the world. If we don’t find it we will strongly constrain several quantum theories of gravity \cite{5}, \cite{6} and provide a powerful benchmark against which future theories must be measured.

II. CPT THEORY AND PHENOMENOLOGY

The CPT theorem \cite{3} is based on the assumptions of locality, Lorentz invariance, the spin-statistics theorem, and asymptotically free wave functions. All quantum field theories (including the standard model of the elementary particles) assume CPT symmetry invariance.

There is a theoretical hint of the level at which CPT symmetry might be violated. This comes from the fact that gravity can’t be consistently included in a quantum field theory, and the proof of the CPT theorem assumes Minkowski space \cite{3}. To include gravity in a unified theory of all four forces of nature, many physicists think that a more general theory is needed, which would have quantum field theory embedded in it. In this more general theory the CPT theorem will be invalid.

One expects to see quantum effects of gravity at what is called the Planck scale: at energies of $M_{\text{Planck}}c^2 = \sqrt{\hbar c^5/G} = 1.2 \times 10^{19}$ GeV, or at distances of the order of $10^{-33}$ cm. The quantum effects of gravity are expected to be very small in ordinary processes.
However, in a place where the standard model predicts a null result, like CPT violation, quantum effects of gravity would stand out. Therefore, it would be very interesting to test CPT symmetry conservation at the Planck scale.

One might think that string theory, as a candidate for the more general theory that has quantum field theory embedded in it, would give us guidance. CPT conservation is artificially built into string theories, first by G. Veneziano [10].

Kostelecky and Potting [6] suggested that spontaneous CPT violation might occur in string theory; i.e., they put the CPT violation in the solutions rather than in the equations of motion. One of the problems with string theory in general is that it’s not known how to relate string effects at the Planck scale to effects seen at current accelerator energies, and Kostelecky and Potting have the same difficulty. They have tried to remedy this by writing the most general additions to the Standard Model Lagrangian that maintain the SU(3) x SU(2) x U(1) effective structure of the theory but violate CPT symmetry. This allows them to classify the various types of CPT violation that might be seen (in the lepton sector, quark sector, etc.) and have a parameterization that includes all these effects. They find that the largest CPT violating effect is a change in quark propagators that has the opposite sign for antiquarks. This leads to a nonzero value of $|M_{K^0} - M_{\bar{K}^0}|$ coming from indirect CPT violation. This is much larger than any direct CPT violation effect. This is precisely the signature that this experiment would search for.

The $K^0 - \bar{K}^0$ system provides us with an incredibly finely balanced interferometer that magnifies small perturbations such as CPT violating effects. It is a natural place to search for CPT symmetry violation since it exhibits C, P, and CP symmetry violation (and is the only place to date where CP violation has been seen). In the final analysis, the conservation or violation of CPT symmetry is an experimental question, and the search for this effect is of the utmost interest.

In $K^0$ physics, one can observe CPT violating effects through mixing or decays (called indirect or direct CPT violation respectively). In mixing, one introduces a parameter $\Delta$ which is both CP and CPT violating. One can also have direct CPT violation. Eqn. (2.1)
shows the mixing of $K_L$ and $K_S$ in terms of the CP eigenstates $K_1$ and $K_2$.

\[
\begin{align*}
K_S &= K_1 + (\epsilon + \Delta)K_2 \\
K_L &= K_2 + (\epsilon - \Delta)K_1
\end{align*}
\] (2.1)

There are several measurements that would signify CPT violation: a difference between the phase of $\epsilon$ and the phase of $\eta_{+-}$, evidence for a non-zero $\Delta$ in the Bell-Steinberger relation, a difference between the phases of $\eta_{+-}$ and $\eta_{00}$, or certain interference terms between $K_L$ and $K_S$ in semileptonic decays. In this report we will concentrate on the first two methods, measuring the phase of $\eta_{+-}$ and comparing it to the calculated value of the phase of $\epsilon$, and evaluating the Bell-Steinberger relation, since from them we can make the most accurate measurements.

We now consider the CPT test based on measuring the phase of $\eta_{+-}$ and calculating the phase of $\epsilon$. For what follows we adopt the Wu-Yang phase convention. Figure 1 shows the relationships between $\epsilon, \epsilon', \Delta$, and $\eta_{+-}$. $\epsilon'$ and $\Delta$ are shown greatly enlarged for clarity.

![The Wu-Yang Diagram](image)

FIG. 1. The Wu-Yang Diagram

The size of $|\epsilon'/\epsilon|$ is of order $10^{-3}$, and the phase of $\epsilon'$ is very close to that of $\epsilon$, so the phase of the vector $\epsilon + \epsilon'$ is the same, to good accuracy, to the phase of $\epsilon$ ($\epsilon'$ is too small to
have an affect on the calculation of the phase of $\epsilon$ at the level in which we are interested).

We can see from the figure that the component of $\Delta$ perpendicular to $\epsilon$, $\Delta_\perp$, is

$$\Delta_\perp = |\eta_{+\mp}|(\phi_{+\mp} - \phi_\epsilon) \tag{2.2}$$

where $\phi_{+\mp} (\phi_\epsilon)$ is the phase of $\eta_{+\mp} (\epsilon)$. In general, in terms of the elements of the kaon decay matrix $\Gamma$ and mass matrix $M$, $\Delta$ is given by [11]:

$$\Delta = (\Gamma_{11} - \Gamma_{22}) + i(M_{11} - M_{22}) \tag{2.3}$$

The mass term has a phase perpendicular to $\phi_{SW}$, the superweak phase, which is defined as $\tan \phi_{SW} = 2(M_L - M_S)/(\Gamma_S - \Gamma_L)$. $\phi_{SW}$ is approximately equal to $\phi_\epsilon$. The decay term is parallel to $\phi_{SW}$. We can solve Eqns. (2.2) and (2.3) for $M_{11} - M_{22}$, which is the mass difference between the $K^0$ and $\bar{K}^0$ mesons, and get an equation which we can use to search for indirect CPT violation:

$$\frac{|M_{K^0} - M_{\bar{K}^0}|}{M_{K^0}} = \frac{2(M_L - M_S)}{M_{K^0}} \frac{|\eta_{+\mp}|}{\sin \phi_{SW}} |\phi_{+\mp} - \phi_\epsilon| \tag{2.4}$$

In Eqn. (2.4), Nature has been kind: the constant factors multiplying $|\phi_{+\mp} - \phi_\epsilon|$ are exceedingly small. $(M_L - M_S)$ is $10^{-6}$ eV, and when one divides by $M_{K^0}$ the ratio is of order $10^{-15}$. $|\eta_{+\mp}|$ is of order $10^{-3}$. The product of all the factors multiplying $|\phi_{+\mp} - \phi_\epsilon|$ is $4 \times 10^{-17}$. By the Planck scale we mean

$$\frac{|M_{K^0} - M_{\bar{K}^0}|}{M_{K^0}} = \frac{M_{K^0}}{M_{Planck}} = 4.1 \times 10^{-20} \tag{2.5}$$

so a measurement of $|\phi_{+\mp} - \phi_\epsilon|$ accurate to 1 milliradian would test a CPT violating effect at the accuracy of the Planck scale.

Some CP/T experiment collaborators were part of Fermilab experiment E773. In this experiment we placed the limit (at 90% confidence level) [4],

$$\frac{|M_{K^0} - M_{\bar{K}^0}|}{M_{K^0}} < 1.3 \times 10^{-18} \tag{2.6}$$

so the result of Ref. [4] stands at 31 times the Planck scale.
That publication actually compared the phase of \( \eta^{+\pm} \) to the superweak phase (and stated clearly that in doing so the assumption was being made that CP violation would not be unexpectedly large in modes other than \( \pi \pi \)). In the calculation of the phase of \( \epsilon \), there are three corrections that should be made to the superweak phase: from \( Im(x) \), the \( \Delta S = \Delta Q \) rule violation parameter, from \( Im(\eta^{+\mp 0}) \), and from \( Im(\eta_{000}) \). Together they have an uncertainty of 2.7 degrees which should be added in quadrature with the approximately 1 degree accuracy of Ref. [4].

Several CP/T experiment collaborators are part of the KTeV experiment as well. There we expect to make an improvement of a factor of 3 to 5. In KTeV interference is seen very clearly. But the interference term from which \( \phi^{+\mp} \) is measured, \( 2|\eta^{+\mp}||\rho|\cos(\Delta mt + \phi_\rho - \phi_{+\mp})\exp(-t/2\tau_s) \), is reduced by the regeneration amplitude \( |\rho| \simeq 0.03 \), and \( \phi^{+\mp} \) and \( \phi_\rho \) are hard to disentangle. Using the regeneration method will be difficult beyond the KTeV level [12].

It should be understood clearly that measuring the phase of \( \eta^{+\mp} \) and comparing it to the superweak phase does not constitute a complete test of CPT symmetry conservation: the corrections to the superweak phase have larger uncertainties than existing experimental measurements of \( \phi^{+\mp} \). For example, if a significant difference between \( \phi^{+\mp} \) and \( \phi_{SW} \) were found in an experiment it would NOT prove that CPT symmetry was violated. More accurate measurements of \( Im(x), Im(\eta^{+\mp 0}), \) and \( Im(\eta_{000}) \) must be made before this could be proved. An interference experiment located just downstream of the production target is needed for these measurements. In a regeneration experiment the interference in \( 3\pi \) decays is reduced in size by a factor of \( \rho \), the regeneration amplitude, which is about 0.1 at most (at Main Injector energies), compared to an experiment near the production target, and it’s extremely difficult for a regeneration experiment to measure \( Im(x), Im(\eta^{+\mp 0}), \) and \( Im(\eta_{000}) \) to the required accuracy.

III. TWO TESTS OF CPT SYMMETRY CONSERVATION
A. The Phase Difference between $\eta_{+-}$ and $\epsilon$

After the KTeV experiment we expect to stand an order of magnitude above the Planck scale. To close that gap we will want to do an interference experiment near the kaon production target. The interference term is then $2D|\eta_{+-}|\cos(\Delta m t - \phi_{+-})\exp(-t/2\tau_s)$. Here $\phi_{+-}$ appears alone, and $|\rho|$ is replaced with the dilution factor, $D = (K^0 - \overline{K}^0)/(K^0 + \overline{K}^0)$ at the target. To maximize D and hence the interference, we choose to make our $K^0$ beam from a $K^+$ beam by charge exchange. Then at medium to high Feynman x, $D \simeq 1$. The charge exchange cross section is large, about 20% of the total cross section. To maximize the flux of $K^+$ made from the 120 GeV/c protons from the Fermilab Main Injector we choose a $K^+$ momentum of 25 GeV/c. We would use a hyperon magnet to define the $K^0$ beam, similar to the one in the Fermilab Proton Center beam line. In the calculations described below we assume the use of a vee spectrometer, a lead glass electromagnetic calorimeter, and a muon detector.

In Ref. [4] $\phi_{+-}$ was measured to 1° accuracy. A CPT-violating mass difference exactly at the Planck scale would result in $|\phi_{+-} - \phi_{\epsilon}| = 0.06°$. We set ourselves the goal of measuring $\phi_{+-}$ and $\phi_{\epsilon}$ to sufficient accuracy to see such a CPT-violating effect.

We have calculated the statistical sensitivity of the CPT measurements assuming that we have a 1 year long run with $3 \times 10^{12}$ protons per pulse at 52% efficiency.
FIG. 2. Proper Time Distributions for $\pi^+\pi^-$ Decays a) Distributions are shown both with interference (dark squares) and without (light squares). b) The ratio of the two distributions in part a).

Fig. 2 shows the proper time distribution of accepted events. The figure shows the actual proper time distribution and also what the distribution would look like if there were no interference. The second part of the figure shows the ratio of those two curves. Between 5 and 20 $K_S$ lifetimes the interference is first a 40% destructive effect then is a 65% constructive effect.

We calculated the distribution of events in momentum and proper time for the resulting 20 billion events and fit this distribution using MINUIT, with fitting parameters $|\eta_{+-}|, \phi_{+-}, D$, three parameters describing the normalization and shape of the kaon momentum spectrum, $\tau_S$ and $\Delta m$ (the $K_S$ lifetime and the $K_L - K_S$ mass difference). The uncertainty that results from this fit is 0.040 degrees. This will meet our goal of testing CPT symmetry conservation at the Planck scale. This number ±0.040 degrees has another meaning: it is the statistical (including fitting) uncertainty of this measurement, and sets
the scale against which all other aspects of the $|\phi_{+-} - \phi_\epsilon|$ measurement should be compared.

In this experiment we measure $\phi_{+-}$, but we must also determine $\phi_\epsilon$. The leading contribution to $\phi_\epsilon$ is the superweak phase, $\phi_{SW}$, given by $\tan(\phi_{SW}) = 2\Delta m/\Delta \Gamma$. The superweak phase will be measured by KTeV to accuracy sufficient for our purposes here. We next describe some corrections to this contribution.

For this experiment, $\epsilon'$ will have no meaningful effect. Assuming CPT invariance, the phase of $\epsilon'$ is known to be $(48 \pm 4)$ degrees [13]. Its magnitude is unknown, but if we assume it to be the central value from E832 we find that the maximum possible difference it can provide between $\phi_{+-}$ and $\phi_\epsilon$ is 0.012 degrees, a factor of 5 smaller than the contribution of CPT violation at the Planck scale.

The full formula for $\phi_\epsilon$ is [14]

$$\tan \phi_\epsilon = \frac{2\Delta m}{\Gamma_S - \Gamma_L} \cos \xi + \frac{\sin \xi}{\delta}$$

(3.1)

where $\xi = \arg(\Gamma_{12}A_0A_0^*)$ and $\delta = 2Re(\epsilon)$. Here $A_0$ is the isospin 0 part of the $\pi^+\pi^-$ decay amplitude. In the Wu-Yang phase convention, $A_0$ is real, and $\Gamma_{12}$ gives contributions from two sources: semileptonic decays through $Im(x)$, the $\Delta S = \Delta Q$ violation parameter, and $3\pi$ decays through $Im(\eta_{+0})$ and $Im(\eta_{000})$.

In the standard model we expect $x \simeq 10^{-7}$, which is too small to affect this experiment, but $Im(x)$ is known experimentally only to an accuracy of $\pm 0.026$. This results in an uncertainty in $\phi_\epsilon$ of 1.7 degrees. To prove that an observed difference between $\phi_{+-}$ and $\phi_\epsilon$ were due to CPT violation one would have to measure $Im(x)$ about 40 times more accurately than today’s level. The way we will do this is described below.

The contribution to $\phi_\epsilon$ from the $3\pi$ modes in the standard model is 0.017 degrees, which is smaller than the accuracy we are trying to obtain. But if one takes into account the current world’s knowledge, the uncertainty these decay modes contribute is 2.2 degrees. So they also have to be measured better.

The best experimental approach to measuring these three quantities, $x$, $\eta_{+0}$, and $\eta_{000}$, is the same: choose an experiment with high dilution factor and observe interference between
$K_L$ and $K_S$ close to the target; i.e. the experiment described here. These measurements should be thought of as being an integral part of this experiment. We have performed a calculation of the sensitivity of this experiment for these quantities, and we estimate that we can reach at least the required sensitivity. We conclude that we can determine $\phi_\epsilon$ to the required accuracy.

We used the same Monte Carlo and fitting programs to estimate the sensitivity of our experiment to the measurements necessary for the calculation of $\phi_\epsilon$, $Im(x)$, $Im(\eta_{+0})$, and $Im(\eta_{000})$, and conclude that we will have the required sensitivity. We find that the uncertainty in $Im(x)$ contributes much more than $Im(\eta_{+0})$ and $Im(\eta_{000})$ to the uncertainty in $\phi_\epsilon$.

### B. CPT Test via the Bell-Steinberger Relation

The next test of CPT symmetry conservation comes through an evaluation of the Bell-Steinberger relation. Our ability to measure CP violation parameters (and also $Im(x)$) very accurately will make it possible to reduce the uncertainties in the Bell-Steinberger relation by two orders of magnitude, which will make this CPT test be sensitive at the Planck scale also.

The Bell-Steinberger relation is a statement of the conservation of probability in $K^0 - \bar{K}^0$ decays, in which, through Eq. (2.1), $\Delta$ appears. It is usually written as:

$$(1 + i \tan \phi_{SW})[Re(\epsilon) - i Im(\Delta)] = \sum_f \alpha_f$$

where the sum runs over all decay channels $f$, and $\alpha_f = \frac{1}{\Gamma_S} A^*(K_S \to f) A(K_L \to f)$. The most recent published evaluation of the Bell-Steinberger relation is ref. [7].

The biggest uncertainties in the Bell-Steinberger relation at this time come from $\eta_{000}$, $Im(x)$, and $\delta_l$ (the charge asymmetry in $K_L$ semileptonic decays). Although $\delta_l$ doesn’t explicitly appear in the Bell-Steinberger relation, it is the best way of evaluating $Re(\epsilon)$. The proposed experiment will be able to make excellent measurements of the first two of these
quantities, and KTeV will measure $\delta_l$ quite accurately. For the next level of accuracy in the Bell-Steinberger relation the uncertainties of the $\alpha_{+-}$ and $\alpha_{00}$ terms must be reduced. These uncertainties depend on those of $|\eta_{+-}|$, $Re(\epsilon'/\epsilon)$, and $\Delta\phi = \phi_{00} - \phi_{+-}$. The latter two quantities will be measured by the KTeV experiment to sufficient accuracy for our purposes here.

We will have good sensitivity for the $|\eta_{+-}|$ measurement. In our fits to the proper time dependence of $\pi^+\pi^-$ events we have excellent statistical sensitivity for measuring $|\eta_{+-}|$. In the interference term, however, it is highly correlated with $D$, the dilution factor. We will measure $D$ using semileptonic decays. The semileptonic charge asymmetry at zero proper time equals $D$. We calculate that we will be able to measure $D$ to better than 0.1% for momenta above 13 GeV/c. We should be able to measure $|\eta_{+-}|$ to 0.1% accuracy, about 10 times better than it is currently known.

The most accurate way to determine $|\eta_{00}|$ will be by using the KTeV value of $\epsilon'/\epsilon$ and our measurement of $|\eta_{+-}|$. The most accurate way of determining $\phi_{00}$ will use the KTeV value of $\Delta\phi$ and our measurement of $\phi_{+-}$.

We should be able to reduce the uncertainties in the Bell-Steinberger relation by about two orders of magnitude from their present values. The limit on $Re(\Delta)$ will be about $5 \times 10^{-6}$, about twice the contribution of CPT violation at the Planck scale, and will be set by the uncertainty in $\delta_l$. For $Im(\Delta)$ the limit will be about $1 \times 10^{-6}$, dominated by the uncertainty in $Im(\epsilon)$, which would allow us to place a $2\sigma$ limit at the Planck scale. Since the Bell-Steinberger measurement is sensitive to $Re(\Delta)$ and $Im(\Delta)$ independently, these limits would be valid even if $\Delta$ is parallel to $\epsilon$, in contrast to the CPT violation limits from $|\phi_{+-} - \phi_{\epsilon}|$, which are sensitive only to the component of $\Delta$ perpendicular to $\epsilon$.

**IV. CONCLUSION**

We have described an experiment to carry out a systematic program of measurements in $K_S - K_L$ interference physics. We will search for CPT symmetry violation in the decays of
$K^0$ mesons with the sensitivity to reach the Planck scale, measure CP violation parameters to test the detailed predictions of the Standard Model, and study rare kaon decays.

Our design uses protons from the Fermilab Main Injector to make an RF separated $K^+$ beam. With this we make a tertiary neutral kaon beam created in just the way to maximize the interference between $K_S$ and $K_L$ while maintaining high flux. We use a “closed geometry” hyperon magnet for beam definition. A standard Vee spectrometer, with drift chambers, an electromagnetic calorimeter, and a muon detector, is used to make the measurement.
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