Additional Strange Hadrons from QCD Thermodynamics and Strangeness Freeze-out in Heavy Ion Collisions

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PACS numbers: 11.10.Wx, 11.15.Ha, 12.38.Gc, 12.38.Mh

We compare lattice QCD results for appropriate combinations of net strangeness fluctuations and their correlations with net baryon number fluctuations with predictions from two hadron resonance gas (HRG) models having different strange hadron content. The conventionally used HRG model based on experimentally established strange hadrons fails to describe the lattice QCD results in the hadronic phase close to the QCD crossover. Supplementing the conventional HRG with additional, experimentally uncharted strange hadrons predicted by quark model calculations and observed in lattice QCD spectrum calculations leads to good descriptions of strange hadron thermodynamics below the QCD crossover. We show that the thermodynamic presence of these additional states gets imprinted in the yields of the ground-state strange hadrons leading to a systematic 5–8 MeV decrease of the chemical freeze-out temperatures of ground-state strange baryons.

Introduction.— With increasing temperature the strong interaction among constituents of ordinary nuclear matter, mesons and baryons, results in the copious production of new hadronic resonances. The newly produced resonances account for the interaction among hadrons to an extent that bulk thermodynamic properties become well described by a gas of uncorrelated hadronic resonances [1]. The hadron resonance gas (HRG) model is extremely successful in describing the hot hadronic matter created in heavy ion experiments [2]. Abundances of various hadron species measured in heavy ion experiments at different beam energies are well described by thermal distributions characterized by a freeze-out temperature and a set of chemical potentials $\mu = (\mu_B, \mu_Q, \mu_S$) for net baryon number ($B$), electric charge ($Q$) and strangeness ($S$) [3]. Nonetheless, details of the freeze-out pattern may provide evidence for a more complex sequential freeze-out pattern [4, 5]. In particular, in the case of strange hadrons arguments have been put forward in favor of a greater freeze-out temperature than that of nonstrange hadrons [6, 8].

At the temperature $T_c = (154 \pm 9)$ MeV [9] strong interaction matter undergoes a chiral crossover to a new phase. In the same crossover region HRG-based descriptions of the fluctuations and correlations of conserved charges for light [10], strange [11, 12], as well as charm [12] degrees of freedom break down. Below $T_c$, bulk thermodynamic properties as well as conserved charge distributions are generally well described by a HRG containing all experimentally observed resonances (PDG-HRG) listed in the particle data tables [13]. However, there are also some notable differences between lattice QCD results and the PDG-HRG predictions. At temperatures below $T_c$, the trace anomaly, i.e., the difference between energy density and 3 times the pressure, is found to be greater in lattice QCD calculations [14, 15] than that of the PDG-HRG. Fluctuations of net strangeness and correlations between net strangeness and baryon number fluctuations are also larger in QCD compared to those of PDG-HRG [17, 18]. It has been argued that the former provides evidence for contributions of additional, experimentally still-unobserved hadron resonances to the thermodynamics of strong interaction matter [19, 20]. Indeed a large number of additional resonances has been identified in lattice QCD [21] and quark model calculations [22, 23]. The presence of such additional flavored hadrons in a thermal medium enhances fluctuations of the associated quantum numbers and modifies their correlations with other quantum numbers. In fact, lattice QCD results on net charm fluctuations and their correlations with baryon number, electric charge, and strangeness fluctuations have also been compared with the expectations from a HRG containing additional, experimentally unobserved charmed hadrons predicted by quark model calculations [12]. Such comparisons have provided evidence for the thermodynamic importance of additional charmed hadrons in the vicinity of the QCD crossover [12].

In this Letter, we show that discrepancies between lattice QCD results and PDG-HRG predictions for strangeness fluctuations and correlations below the QCD crossover can be quantitatively explained through the inclusion of additional, experimentally unobserved strange...
hadrons. The thermodynamic presence of these additional strange hadrons also gets imprinted on the yields of ground-state strange baryons, resulting in observable hadrons. The thermodynamic presence of these additional states is taken over all open strange mesons or baryons listed in the particle data tables (X=PDG) or a larger set including additional open strange mesons [23] and baryons [22].

**Hadron resonance gas models.**—The partial pressure of all open strange hadrons can be separated into mesonic and baryonic components, \( P_{\text{tot}}^{S,X} = P_{M}^{S,X} + P_{B}^{S,X} \),

\[
P_{M/B}^{S,X}(T, \bar{\mu}) = \frac{T^4}{2\pi^2} \sum_{i\in X} g_i \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \times \cosh(B_i \bar{\mu}_B + Q_i \bar{\mu}_Q + S_i \bar{\mu}_S)
\]

Here, \( M (B) \) labels the partial pressure of open strange mesons (baryons), \( g_i \) is the degeneracy factor for hadrons of mass \( m_i \), and \( \bar{\mu}_q \equiv \mu_q/T \), with \( q = B, Q, S \). The sum is taken over all open strange mesons or baryons listed in the particle data tables (X=PDG) or a larger set including additional open strange mesons [23] and baryons [22] predicted by quark models (X=QM). Throughout this work we refer to the HRG model containing these additional, quark model predicted, experimentally undiscovered hadrons as the QM-HRG. In Eq. (1) the classical, Boltzmann approximation has been used which is known to be appropriate for all strange hadrons at temperatures \( T \lesssim T_c \).

The masses and, more importantly, the number of additional states are quite similar in the quark model calculations and the strange hadron spectrum of lattice QCD [21]. HRG models based on either one, thus, give very similar results. As the lattice computations of the strange hadron spectrum have so far been performed with unphysically heavy up and down quark masses, for definiteness we have chosen to compare our finite temperature lattice results with the quark model predictions that generally reproduce the masses of the experimentally known states rather well.

Figure 1 compares partial pressures of open strange mesons and baryons calculated within PDG-HRG and QM-HRG models. The additional strange baryons present in the QM-HRG lead to a large enhancement of the partial baryonic pressure relative to that obtained from the PDG-HRG model. In the mesonic sector, changes are below 5% even at \( T = 170 \text{ MeV} \), i.e., above the applicability range of any HRG [11]. This simply reflects that a large part of the open strange mesons is accounted for in the PDG-HRG model, and the additional strange mesons contributing to the QM-HRG model are too heavy to alter the pressure significantly.

**Strangeness fluctuations and correlations.**—We calculate cumulants of strangeness fluctuations and their correlations with baryon number and electric charge in (2+1)-flavor QCD using the highly improved staggered quark (HISQ) action [24]. In these calculations the strange quark mass \( m_s \) is tuned to its physical value and the masses of degenerate up and down quarks have been fixed to \( m_u = m_s/20 \). In the continuum limit, the latter corresponds to a pion mass of about 160 MeV. In the relevant temperature range, 145 MeV \( \leq T \leq 170 \text{ MeV} \), we have analyzed \((10-16) \times 10^3\) configurations, separated by 10 time units in rational hybrid Monte Carlo updates, on lattices of size \( 6 \times 24^3 \) and \( 8 \times 32^3 \). Some additional calculations on \( 8 \times 32^3 \) lattices have been performed with physical light quarks, \( m_u = m_s/27 \), to confirm that the quark mass dependence of observables of interest is indeed small.

To analyze the composition of the thermal medium in terms of the quantum numbers of the effective degrees of freedom, we consider generalized susceptibilities of the conserved charges,

\[
\chi_{\text{BQS}}^{klm} = \frac{\partial^{(k+l+m)}[P(T, \bar{\mu}_B, \bar{\mu}_Q, \bar{\mu}_S)/T^4]}{\partial \bar{\mu}_B^k \partial \bar{\mu}_Q^l \partial \bar{\mu}_S^m} \bigg|_{\bar{\mu}=0}, \tag{2}
\]

where \( P \) denotes the total pressure of the hot medium. For brevity, we drop the superscript when the corresponding subscript is zero.

The correlation of net strangeness with net baryon number fluctuations normalized to the second cumulant of net strangeness fluctuations, \( \chi_{11}^{BS}/\chi_2^S \), is a sensitive probe of the strangeness carrying degrees of freedom [25]. Consistent continuum extrapolations for this ratio have been obtained with two different staggered fermion discretization schemes [17, 18]. In Fig. 2(top) we show our present, refined results obtained for lattices with temporal extents \( N_t = 6 \) and 8, which are in agreement with the earlier results, together with an improved estimate for the continuum result based on the enlarged statistics for these lattices. In the crossover region and also at lower temperatures in the hadronic regime the lattice QCD results for \( -\chi_{11}^{BS}/\chi_2^S \) are significantly greater than those of the PDG-HRG model predictions.

In the validity range of HRG models, the \( BS \) correlation \( \chi_{11}^{BS} \) is a weighted sum of partial pressures of strange baryons, while the quadratic strangeness fluc-
fig. 2. top: BS correlations normalized to the second cumulant of net strangeness fluctuations. Results are from (2+1)-flavor lattice QCD calculations performed with a strange to light quark mass ratio m_s/m_l = 20 (squares) and m_s/m_l = 27 (diamonds). The band depicts the improved estimate for the continuum result facilitated by the high statistics \( N_r = 6 \) and 8 data. Bottom: Ratios of baryonic \( (\chi_2^2) \) to mesonic \( (M_2^S) \) pressure observables defined in Eq. 3. The dotted and solid lines show results from PDG-HRG and QM-HRG model calculations, respectively. The shaded region denotes the continuum extrapolated chiral crossover temperature at physical values of quark masses \( T_c = (154 \pm 9) \) MeV.

fig. 3. The leading-order result for the ratio of strangeness and baryon chemical potentials versus temperature. Data and HRG model results are for a strangeness neutral thermal system having a ratio of net electric charge to net baryon number density \( N_Q/N_B = 0.4 \). The dashed line shows the QM-HRG result for vanishing electric charge chemical potential. All other curves and labels are as in Fig. 2.

The calculations \( \chi_2^S \) are dominated by the contribution from strange mesons. The larger value of \( -\chi_{11}^{BS}/\chi_2^S \) found in lattice QCD calculations compared to that of PDG-HRG model calculations, thus, reflects the stronger increase of \( P_{BS}^{S,\text{PDG}}/P_{BM}^{S,\text{PDG}} \) compared to \( P_{N_{11}}^{S,\text{QM}}/P_{M_1}^{S,\text{PDG}} \) (see Fig. 1). As a consequence, the QM-HRG model provides a better description of QCD thermodynamics in the hadronic phase. This can be more directly verified by considering the ratio of two observables, which in a HRG model give \( P_{BM}^{S} \) and \( P_{BM}^{S} \), respectively. There is a large set of “pressure-observables” that can be constructed for this purpose by using second- and fourth-order cumulants of strangeness fluctuations and correlations with net baryon number \( \mu_B \). They will all give identical results in a gas of uncorrelated hadrons, but differ otherwise. In particular, they can yield widely different results in a free quark gas.

We use two linearly independent pressure-observables for the open strange meson \( (M_2^S, M_2^S) \) and baryon \( (B_2^S, B_2^S) \) partial pressures, respectively,

\[
M_1^S = \chi_2^S - \chi_{22}^{BS} , \quad M_2^S = \frac{1}{12} \left( \chi_2^S + 11 \chi_{22}^S \right) + \frac{1}{2} \left( \chi_{11}^{BS} + \chi_{13}^{BS} \right) , \quad B_1^S = -\frac{1}{6} \left( 11 \chi_{11}^{BS} + 6 \chi_{22}^{BS} + \chi_{13}^{BS} \right) , \quad B_2^S = \frac{1}{12} \left( \chi_2^S - \chi_{22}^S \right) - \frac{1}{3} \left( 4 \chi_{11}^{BS} - \chi_{13}^{BS} \right) . \tag{3}
\]

Three independent ratios \( B_2^S/M_2^S \) are shown in Fig. 2 (bottom). They start to coincide in the crossover region giving identical results only below \( T \lesssim 155 \) MeV. This reconfirms that a description of QCD thermodynamics in terms of an uncorrelated gas of hadrons is valid till the chiral crossover temperature \( T_c \). Below \( T_c \), the information one extracts from \( B_2^S/M_2^S \) agrees with that of \( -\chi_{22}^{BS}/\chi_2^S \). In the hadronic regime the ratios calculated in lattice QCD are significantly greater than those calculated in the PDG-HRG model. QM-HRG model calculations are in good agreement with lattice QCD. These results provide evidence for the existence of additional strange baryons and their thermodynamic importance below the QCD crossover.

Implications for strangeness freeze-out. — Since the initial nuclei in a heavy ion collision are net strangeness free, the HRG at the chemical freeze-out must also be strangeness neutral. Obviously, for such a strangeness neutral medium, all three thermal parameters \( T, \mu_B \), and \( \mu_S \) are not independent; the strangeness chemical potential can be expressed as a function of \( T \) and \( \mu_B \). While \( \mu_S(T, \mu_B) \) is unique in QCD, for a HRG it clearly depends on the relative abundances of the open strange baryons and mesons. For fixed \( T \) and \( \mu_B \), a strangeness neutral HRG having a larger relative abundance of strange baryons over open strange mesons naturally leads to a larger value of \( \mu_S \).

Calculations of \( \mu_S(T, \mu_B) \) in a strangeness neutral HRG are straightforward. For QCD this can be obtained from lattice QCD computations of \( \mu_S/\mu_B \) using next-to-leading-order Taylor expansion of the net strangeness density \( B_2^S \). The ratio \( \mu_S/\mu_B = s_1(T) + s_3(T)(\mu_B/T)^2 + \mathcal{O}(\mu_B^3) \) is closely related to the ratio \( \chi_{11}^{BS}/\chi_2^S \) shown in Fig. 2. At leading order, it only receives a small correction from nonzero electric charge.
chemical potential $\mu_Q/\mu_B$,

$$\left(\frac{\mu_S}{\mu_B}\right)_{LO} \equiv s_1(T) = \frac{\delta S}{\delta \mu_S} - \frac{\delta Q}{\delta \mu_Q} \frac{\mu_Q}{\mu_B}. \quad (4)$$

The next-to-leading-order correction $s_3(T)(\mu_S/\mu_B)^2$ is small for $\mu_B \lesssim 200$ MeV. We show the leading order result in Fig. 3. At a given temperature, the strangeness neutrality constraint gives rise to a larger value, consistent with lattice QCD results, of $\mu_S/\mu_B$ for the QM-HRG compared to the PDG-HRG. In other words, for a given $\mu_B/T$, the required value of $\mu_S/\mu_B$ necessary to guarantee strangeness neutrality is achieved at a lower temperature in the QCD and QM-HRG model than in the PDG-HRG model.

The relative yields of strange anti-baryons ($\bar{H}_S$) to baryons ($H_S$) at freeze-out are controlled by the freeze-out parameters ($T^f, \mu^f_B, \mu^f_S$),

$$R_H \equiv \frac{\bar{H}_S}{H_S} = e^{-2(\mu^f_B/T^f)\left(1-(\mu^f_S/\mu^f_B)|S|\right)} \quad (5)$$

(Fo simplicity of the argument we will ignore here the influence of a non-vanishing electric charge chemical potential. As shown in Fig. 4 a non-zero $\mu_Q/T$ has only a small influence on $\mu_S/\mu_B$.) While this relation does not explicitly depend on the content and spectra of hadrons in a HRG, the presence of additional strange hadrons implicitly enters through the strangeness neutrality constraint. As discussed before, for different HRG models at fixed $T$ and $\mu_S$ strangeness neutrality leads to different values of $\mu_S/\mu_B$. Once $\mu^f_B/T^f$ and $\mu^f_S/\mu^f_B$ are fixed through experimental yields of strange hadrons, it is obvious from Fig. 3 that a given value of $\mu^f_S/\mu^f_B$ is realized at a larger temperature in the PDG-HRG model than in the QM-HRG model.

Ratios of the freeze-out parameters $\mu^f_B/T^f$ and $\mu^f_S/\mu^f_B$ can be obtained by fitting the experimentally measured values of the strange baryon ratios $R_A = \Lambda/\bar{\Lambda}, R_2 = \Xi^+/\Xi^-$ and $R_Q = \Omega^+/(\Omega/3)$ to $\mu^f/B$ in Eq. 5. Fits of these strange anti-baryon to baryon yields to Eq. 5 result in $(\mu^f_S/\mu^f_B, \mu^f_B/T^f) = (0.213(10), 1.213(30))$ for the NA57 results at $\sqrt{s} = 17.3$ GeV [27] and $(0.254(4), 0.697(20))$ for the STAR preliminary results at $\sqrt{s} = 39$ GeV [28]. In Fig. 3 we show comparisons of these $(\mu^f_S/\mu^f_B, \mu^f_B/T^f)$ values with the lattice QCD, QM-HRG, and PDG-HRG predictions for $\mu_S/\mu_B$ at $\mu_B/T = \mu^f_B/T^f$. By varying the temperature ranges, one can match the values of $\mu_S/\mu_B$ to $\mu^f_S/\mu^f_B$ and, thus, determine the freeze-out temperatures $T^f$. As expected from Fig. 4 the QM-HRG predictions are in good agreement with lattice QCD results and lead to almost identical values for $T^f$. The PDG-HRG-based analysis, however, results in freeze-out temperatures for strange baryons that are larger by about 8 (5) MeV for the smaller (larger) value of $T^f$.

**Conclusions.**— By comparing lattice QCD results for various observables of strangeness fluctuations and correlations with predictions from PDG-HRG and QM-HRG models, we have provided evidence that additional, experimentally unobserved strange hadrons become thermodynamically relevant in the vicinity of the QCD crossover. We have also shown that the thermodynamic relevance of these additional strange hadrons modifies the yields of the ground-state strange hadrons in heavy ion collisions. This leads to significant reductions in the chemical freeze-out temperature of strange hadrons. Compared to the PDG-HRG, the QM-HRG model provides a more complete description of the lattice QCD results on thermodynamics of strange hadrons at moderate values of the baryon chemical potential. This suggests that the QM-HRG model is probably the preferable choice for the determination of the freeze-out parameters also at greater values of the baryon chemical potential beyond the validity of the present lattice QCD calculations.

Finally, we note that the observation regarding the thermodynamic relevance of additional strange hadrons hints that an improved HRG model including further, unobserved light quark hadrons may resolve the current discrepancy between lattice QCD results for the trace anomaly and the results obtained within the PDG-HRG model.

**Acknowledgments.**— This work was supported in part through Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy, through the Scientific Discovery through Advanced Computing (SciDAC) program funded by the U.S. Department of Energy, Office of Science, Advanced Scientific Computing Research and Nuclear Physics, the BMBF under Grants No. 05P12PBCTA and No. 56268409, the DFG under Grant No. GRK 881, EU under Grant No. 283286 and the GSI BILAER Grant. Numerical calculations were performed using GPU clusters at JLab, Bielefeld University, Pader-
born University, and Indiana University. We acknowledge the support of Nvidia through the Cuda Research Center at Bielefeld University.

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