Model of Integrated Car Brake-Active Suspension of Quarter Car Model By Using LQR Control Under Periodic Road Excitation

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Abstract. The most important technology in a vehicle is the stability of the direction of the vehicle and the comfort of the passengers. For stability, direct yaw control is used via ABS / TCS / DSC, while for convenience, use the active suspension. These two solutions also contribute to one another. In this research paper is produced and explained about the use of LQR control to control the response of the sprung mass to achieve high damping ratios in isolating the excitation force from nonlinear road surfaces. The simulation results prove that the damping ratio controlled by LQR was better than compared by using PID.

1. Introduction

The final suspension is a research object that is very essential and is being carried out by world researchers because it is one of the important factors of vehicle road handling comfort and stability. The results of previous research on the suspension consisted of 3 important concentrations, namely the first plant model consisting of quarter, half and full suspension. Second, in terms of control, research has been carried out using passive, semi-active and active. and third of many control methods that use PID control, Sliding mode, Observer etc. for that we will review the previous research, before explaining the results of our study. The ideal target control response is minimal rolling and maximum grip-wheel ground surface. S.F. Youness, E.C. Lobusov [1] published a research on suspension control ¼ car model that is no-network and networking, with PID and LQR control approaches, the results of the performance of suspense control concluded that LQR is better than PID both no-network and networking. Prabu Krishnasamy, Jancirani Jayaraj, Dennie John [2] concluded from the results of his research on pneumatic active suspension with PID control compared to passive suspension, all model car models with LabView software. The results of PID control response for pneumatic active suspension are better than passive suspension. Narinder Singh1, Himanshu Chhabra and Karansher Bhangal [3] studied the 2 DOF ¼ car model by comparing 2 control H∞ and μ- synthesis, the results of this study concluded that μ-synthesis is better than H∞ from the robust side. Zhi-Jun Fu, 1 Bin Li, 2 Xiao-Bin Ning, and Wei-Dong Xie [4] show the results of research on ¼ car using optimal control approximate dynamic programming (ADP) instead of LQR. Optimal control applies single adaptive critic NN Hamilton-jacobi-bellman solution. The results of the close loop stability analysis for both controls concluded that the optimal control is better than LQR. Michiel Haemers1, Stijn Derammelaere3, Clara-Mihaela Ionescu1, Kurt Stockman, Jasper De Vlaene, Florian Verbelen1 [5] presents research results from the full-car state space model to meet the dynamic characteristics of the ISO 8608 suspension. State feedback control is used, with parameter optimization using a genetic algorithm (GA). The results of the study explain that this control system is better than those that are not tuned with GA. Rosmazi Rosli, Musa Mailah, Gigih Priyandoko
Thanh-Phong Pham, Olivier Sename, Luc Dugard promoted the results of active suspension control by comparing 2 control methods, namely hybrid active force control (AFC) - iterative learning (IL) with PID. The results of the study concluded that AFC-IL is better than PID. Qinghua Meng Chih-Chiang Chen Pan-Wang Zong-Yao Sun Bingji Li [8] described the results of his investigative research on the differences in the use of 2 active suspension controls, namely active suspension homogenous controller (ASHC) and SMC for passive suspension. The results showed that ASHC was better than SMC. Ammar A. Aldair and William J. Wang [9] was also proposing an Evolutionary Algorithm Based Fractional Order PID + μ Controller Design for a Full Vehicle Nonlinear Active Suspension System with successfully. Abdolvahab Agharkakli, Gohbad Shafiei Sabet, Armin Barouz [10] proposed a Simulation and Analysis of Passive and Active Suspension System Using Quarter Car Model for Different Road Profile. The result active suspension has superior performance than passive. Tongchit Suthisripok, Chanwit Wongrattanapornkul Somchai Poonyaniran, And Adirak Kanchanaharuthai [11] studied a Disturbance Observer Based Control for Active Suspension Systems was better performance compared with PID control.

2. Proposed Turning, Braking/Traction on Random Road Full Car Suspension Model

2.1 Axial excitation

The DoF, it includes unsprung mass, namely \( X_w \), which is the motion of the wheels and sprung mass \( X_s \), which is the motion of the vehicle chassis / body as shown in figure 1.

\[
X_1 = X_b, \quad X_2 = X_w, \quad X_3 = \dot{X}_b, \quad X_4 = \dot{X}_w
\]
The motion equation of the sprung as can be represented by the equation (1)

\[ m_b \ddot{x}_b = k_s (x_w - x_b) + b_s (\dot{x}_w - \dot{x}_b) + f_s \]  

(1)

The equation for the movement of un-sprung mass can be represented by the equation (2)

\[ m_w \ddot{x}_w = k_t (r - x_w) - k_s (x_w - x_b) - b_s (\dot{x}_w - \dot{x}_b) - f_s \]  

(2)

Equations (1) and (2) can be represented in the form of state space 3 as follows

\[ \dot{X} = AX + BU + EW \]  

(3)

Where \( U \) is the controlled force (\( f_s \)) and the highway excitation container (\( r \)) with the state space matrix constants as follows:

\[ A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & k_s & 0 & 1 \\ 0 & 0 & -b_s & 1 \\ k_s & m_b & m_b & m_w \end{bmatrix} \]

\[ B^T = \begin{bmatrix} 0 \\ 0 \\ 1 \\ m_b \end{bmatrix} \]

\[ E^T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_t/m_w \end{bmatrix} \]

2.2 The implication of Vertical and longitudinal brake force excitation as shown in figure 2 make superposition to vertical motion of road excitation. Formula is presented below.

![Figure 2. Model 3 DoF active suspension with brake force vertical superposition](image-url)

The equation for the movement of the sprung mas can be represented by the equation (4)

\[ m_b \ddot{x}_b = k_s (x_w - x_b) + b_s (\dot{x}_w - \dot{x}_b) + f_s \]  

(4)
The motion equation of the un-sprung mass which is excited by the brake force $F_b$, shown in figure 2 implicate to add vertical extra force to the un-sprung mass. It can be represented by the equation

$$m_w \ddot{X}_w = k_t (r - X_w) + F_b \sin \theta - k_s (X_w - X_b) - b_s (\dot{X}_w - \dot{X}_b) - f_s$$  \hspace{1cm} (5)

If the change of road-tire geometry is periodic, the equation becomes

$$m_w \ddot{X}_w = k_t (r - X_w) + F_b \sin \omega \cdot t - k_s (X_w - X_b) - b_s (\dot{X}_w - \dot{X}_b) - f_s$$  \hspace{1cm} (6)

Equations (4) and (5) can be represented in the form of state space as follows

$$\dot{X} = AX + BU + EW + DZ$$  \hspace{1cm} (7)

Where $U$ is the controlled force ($f_s$) and $W$ is the highway excitation ($r$) and $Z$ is the brake force ($F_b \sin \omega \cdot t$). With the state space matrix constants as follows

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_B} & \frac{k_s}{m_B} & -\frac{b_s}{m_B} & \frac{b_s}{m_B} \\ \frac{k_s}{m_w} & -\frac{(k_s + k_t)}{m_w} & \frac{b_s}{m_B} & -\frac{b_s}{m_B} \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_B} \\ \frac{1}{m_w} \end{bmatrix}$$

$$E^T = \begin{bmatrix} 0 & 0 & \frac{k_t}{m_w} \end{bmatrix}$$

$$D^T = \begin{bmatrix} 0 & 0 & \frac{\sin \omega \cdot t}{m_w} \end{bmatrix}$$

2.3 Axial and longitudinal drive force excitation

If the acceleration sensor variable is ($a$) and variable, then the speed of the sprung mass $X_b$ and un-sprung mass $X_w$ and the body level sensor is $X_b (0)$ as the initial condition, it can be formulated by the equations (8), (9), (10):

$$\dot{X}_b = \int a_b \, dt$$ \hspace{1cm} (8)

$$\dot{X}_w = \int a_w \, dt$$ \hspace{1cm} (9)

$$X_{b(0)} = \int \int a_b \, dt \, dt$$ \hspace{1cm} (10)
3. Proposed Suspension LQR Control

To test the LQR control, all the quarter car modeling equations above are compiled into the state space equation \( \dot{X} = AX + BU + EW + DZ \) with \( A, B, C, D \) which have been determined according to the vehicle suspension parameters as depicted in figure 3. The first step is to define the function object as follows:

\[
J = \frac{1}{2} \int_{t_o}^{t_f} \left( \dot{x}(t_i)^T Q x(t_i) + u(t_i)^T R u(t_i) \right) dt
\]

Then second determined the values of \( Q, R \) as control state and excitation state from \( A, B, C, D \) and \( Q, R \) has been obtained \( P \) with Riccati equestion to get the value of \( K \).

\[
P A_{ii} + A_{ii}^T P - P B_{ij} R^{-1} B_{ij}^T P + Q = 0
\]

\[
K = (R^{-1} B^T P)
\]

Then third from here it can be calculated the value of control vector \( U \). Where the control vector \( U \) can be formulated as follows

\[
U_i = (-K x(t_i))
\]

4. Simulation result and discussion

The simulation has been setup with the excitation frequency of the highway geometry as follows 1 rad / sec, 2 rad / sec and 4 rad / sec and constant amplitude of 0.5 meters. In this research, the results of the LQR control are compared to the damper ratio with PID controls with the same plant. Then the most important thing is that the brake excitation force which is projected in a vertical direction is also included while this has not been done by other researchers.

The results of the real time response of the vertical displacement for a street frequency of 1 rad / sec are to produce a damping ratio of 8 as shown in the figure 4 while by using PID reach 7.3 only. Mean it can isolate the sprung mass from road vibration by reducing one eighth compared road amplitude.
Then for the second simulation, the road geometry frequency and vehicle speed have been set so that it becomes 2 rad/sec while maintaining the 0.5 meter amplitude. The result of the damping ratio is 6 as shown in the figure 5 while by using PID reach 5.7 only. Mean it can isolate the sprung mass from road vibration by reducing one sixth compared road amplitude.

In the third experiment, it was tried to increase the frequency geometry and vehicle speed which produced 4 rad/sec with a performance damping ratio of 2 to 3 as shown in the figure 6 while by using PID reach 2 only. Mean it can isolate the sprung mass from road vibration by reducing half to one third compared road amplitude.
5. Conclusion
From the results of the discussion in the chapter above, it can be concluded as follow:

1) In average the performance LQR control suspension in research were better than compare by using PID control

2) Incorporating the longitudinal brake force excitation into a vertical excitation superposition to the vertical force of the road geometry is very significant

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