Active fault tolerant / intrusion tolerant cooperative control of discrete NCS based on sliding mode variable structure

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Abstract. Aiming at the actuator failure in the linear discrete networked control system (NCS) and the denial of service network attack (DoS) on the actuator side, under the discrete event trigger mechanism, an active fault-tolerant/intrusion-tolerant cooperative control method based on sliding mode variable structure is proposed. This method applies the fading adaptive Kalman filter (AKF) to jointly estimate the system state, fault parameters and attack parameters, obtains the estimated value of each parameter vector, and uses the estimated value satisfying the event trigger condition to reconstruct the controller. In this way, system errors caused by actuator faults and attacks are corrected, fault-tolerant and intrusion-tolerant coordinated control is realized, and the purpose of saving network resources can be achieved. Finally, the effectiveness of the control method is verified by simulation.

1. Preface
With the development of modern science and technology, in order to improve the performance and product quality of network control system, the complexity and automation level of technology continue to improve, which makes the probability of system failure and network attack increase. Once the fault or attack occurs, the system performance will decline or even be unstable [1]. As the actuator of the system, the actuator plays an important role in the stability and performance of the system, but the actuator is prone to failure and vulnerable to network attacks, so it is of great significance to perform fault-tolerant/intrusion-tolerant cooperative control of the system's actuators [2].

Since Dr. Beard proposed Fault Detection and Isolation (FDI) [1] in 1971, model-based fault diagnosis technology has received extensive attention from scholars at home and abroad. At present, the most commonly used model-based fault diagnosis technology is the state estimation method [3], in addition, there are the parameter estimation method [4]. For the actual system, the existence of noise is inevitable, and Kalman filter is an algorithm that can estimate the unknown state or parameter well according to the noisy observation signal [5]. Therefore, the state estimation method based on Kalman filter has been widely used in the field of fault diagnosis. Mehra first used the Kalman filter algorithm [6] to detect faults in spacecraft actuators and sensors. In this method, innovation (residual) is used to estimate various faults, and feedback control is used to keep the system running stably. However, the fault-tolerant control of the system is not considered, and the output of the system has errors. Subsequently, Hajiyev and Caliskan proposed an improved Kalman filter FDI method [7], which estimates actuator and sensor faults and corrects the system through fault-tolerant control. This method does not require prior information about faults. Liu proposed a continuous-discrete unscented Kalman filter algorithm for actuator fault detection, and realized active fault-tolerant control by designing and
adjusting feedback control input[8]. For the research on DoS attack detection and intrusion tolerance control, scholars have never stopped. In reference to the problem of distributed denial-of-service (DDoS) attacks on NCS, the literature[9] proposed an algorithm that extends Kalman filtering and control system model parameter identification to detect DDoS attack. Ding et al. used Bernoulli distribution to simulate the randomness of DoS attackers’ attacks, and studied the NCS event-triggered security control under DoS attacks with energy-limited and random characteristics[10]. Reference[11] considers the stability of the tolerance of the network control system when controlling and measuring data packets are transmitted on the communication network under malicious DoS attacks. In the framework of average residence time, the system under attack is modeled as a time-varying delay switching system. For NCS, where actuator faults and network attacks coexist, there are few researches on the comprehensive security control of fault tolerance and intrusion tolerance[12]. However, it is inevitable that faults and attacks coexist in actual NCS. In view of this, the focus of this paper is how to make the system maintain its original performance and realize active fault-tolerant/intrusion-tolerant coordinated control under the condition that unknown faults and unknown attacks occur simultaneously.

This article focuses on the actuator failure and the DoS attack on the actuator side (for the convenience of description, this attack will be recorded as a network attack later), and the design of the linear discrete NCS active fault-tolerant/intrusion-tolerant cooperative controller under the event-triggered communication mechanism is studied. The controller has good suppression performance to the limited external energy disturbance. Therefore, the research ideas of this paper are as follows: first, establish a closed-loop NCS model in which faults and attacks coexist under the discrete event-triggered communication mechanism; then, using the idea of state augmentation, the fault parameters, attack parameters, and system state are augmented into a new state, using adaptive gradual elimination Kalman filter[13] estimates the new state after augmentation, obtains the augmented state estimate, and adjusts the fault-tolerant/intrusion-tolerant cooperative control law based on sliding mode control design online; finally, the MATLAB simulation example is used to verify the improvement of the system performance and the saving of network resources by the introduction of event triggering conditions.

2. Problem description

2.1. System model

Consider a discrete NCS system with actuator failures and cyber attacks:

\[
\begin{align*}
\dot{x}(k+1) &= Ax(k) + Bu(k) + Ef(k) + Na(k) + D_tw(k) \\
y(k) &= Cx(k) + D_yv(k)
\end{align*}
\]

(1)

Where \(x(k) \in \mathbb{R}^n\), \(y(k) \in \mathbb{R}^p\) and \(u(k) \in \mathbb{R}^l\) are the state, output and control input of the system, respectively, \(f(k) \in \mathbb{R}^p\) is the actuator fault vector, \(a(k) \in \mathbb{R}^p\) is the network attack vector on the actuator side, referred to as the actuator attack vector; \(A \in \mathbb{R}^{nxn}, B \in \mathbb{R}^{nxl}, C \in \mathbb{R}^{nxp}, E \in \mathbb{R}^{nxp}\) and \(N \in \mathbb{R}^{nxq}\) are the coefficient matrix of appropriate dimensions, \(w(k) \in \mathbb{R}^p\) and \(v(k) \in \mathbb{R}^p\) are bounded energy Disturbances are all Gaussian white noise sequences with a mean value of 0 and are independent of each other. The covariance matrices are \(Q \in \mathbb{R}^{pp}\) and \(R \in \mathbb{R}^{pp}\), \(D_1 \in \mathbb{R}^{nxp}\) and \(D_2 \in \mathbb{R}^{nxp}\) are disturbance matrices of appropriate dimensions.

2.2. Introduction of DETCS

DETCS refers to the introduction of a communication constraint condition in NCS, which determines whether data is transmitted according to the state of the system or whether the state error meets the event trigger condition during the sampling period: only when the control task meets this condition will it be executed. This will enable the system to save a lot of network resources while maintaining the corresponding expected performance, thereby improving the efficiency of network utilization. The system control structure diagram is shown in Figure 1:
Using the event trigger mechanism created in [17]: Assuming that the initial state $x(0)$ is successfully transmitted, then the next transmission time $t_{k+1}T$ when the event is triggered is

$$t_{k+1}T = t_k T + \min \left\{ |T-k| \left| e^T(kT) \Xi e(kT) \geq \sigma x^T(kT) \Xi x(kT) \right| \right\} \quad (2)$$

Therefore, before the next transmission time $t_{k+1}T$, when $k \in [t_k, t_{k+1})$, data packet $x(t_k)$ will not be transmitted, and satisfies

$$e^T(kT) \Xi e(kT) \leq \sigma x^T(kT) \Xi x(kT) \quad (3)$$

Among them: the state error is

$$e(kT) = x(kT) - x(t_k) \quad (4)$$

$T$ is the sampling period, $\Xi$ is the positive definite symmetric matrix, and $\sigma \in [0,1)$ is the event trigger parameter scalar. If the condition is met, $t_{k+1}T$ is the time of the next data packet to be transmitted. If the event trigger condition is designed reasonably, DETCS will save network resources and increase the utilization rate of network resources on the basis of ensuring system performance.

2.3. Formatting author affiliations

Lemma 1 (Schur's Complementary Lemma) For a given symmetric matrix $Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^T & Z_{22} \end{bmatrix}$, the following three conditions are equivalent:

1) $Z < 0$; 2) $Z_{11} < 0, Z_{22} - Z_{12}^T Z_{11}^{-1} Z_{12} < 0$; 3) $Z_{22} < 0, Z_{11} - Z_{12}^T Z_{12}^{-1} Z_{11} < 0$.

Lemma 2 (Reciprocal Convex Combination[13]) Assuming that $f_1, f_2, \cdots, f_N : R^n \to R$ has a positive value in the subset of open set $D$, and $D \in R^n$, then the reciprocal convex combination of $f_i$ in set $D$ satisfies

$$\min_{\sum a_i \geq 0, \sum a_i = 1} \frac{1}{\sum_{i=1}^n a_i} \sum_{i=1}^n a_i f_i(k) = \sum_{i=1}^n a_i f_i(k) + \max_{\sum_{i,j \geq 0} g_{i,j}(k)} \sum_{i,j} g_{i,j}(k)$$

among them, $\begin{bmatrix} f_i(k) & g_{i,j}(k) \\ g_{i,j}(k) & f_j(k) \end{bmatrix} \geq 0$, \{ $g_{i,j} : R^n \to R, g_{i,j}(k) = g_{i,j}(k) \}$

Lemma 3 (Wirtinger's inequality in discrete form[14]) For a given positive definite matrix $N \in R^{n \times n}$, scalar $0 \leq \rho_1 \leq \rho_2$ and vector function $\eta : [-\rho_2, \rho_2] \to R^{n \times n}$ satisfy the following inequalities:

$$-(\rho_2 - \rho_1) \sum_{s=k+\rho_2}^{k+N-1} \eta^T(s) N \eta(s) \leq - \begin{bmatrix} \Omega_1^T & N & 0 \\ N & 0 & 3N \end{bmatrix} \begin{bmatrix} \Omega_1 \\ \Omega_2 \end{bmatrix}$$

among them,
\( \eta(s) = x(s+1) - x(s), \Omega_1 = x(k - \rho_1) - x(k - \rho_2), \Omega_2 = x(k - \rho_1) + x(k - \rho_2) - \frac{2}{\rho_2 - \rho_1 + 1} \sum_{i=1}^{\rho_2 - \rho_1 + 1} x(s) \)

Lemma 4 (Moon inequality[15]) Suppose \( \alpha \in \mathbb{R}^n, \beta \in \mathbb{R}^m, N \in \mathbb{R}^{m \times n}, \) then the following inequalities hold for any matrix \( X \in \mathbb{R}^{m \times n}, Y \in \mathbb{R}^{m \times n} \) and \( X \in \mathbb{R}^{m \times n} \):

\[
-2\alpha^T N \beta \leq \begin{bmatrix} \alpha \end{bmatrix}^T \begin{bmatrix} X & Y - N \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \begin{bmatrix} X \end{bmatrix}^T \begin{bmatrix} Z \end{bmatrix} \geq 0
\]

where \( \* \) denotes the vector function.

Lemma 5 (B-L inequality in discrete form[16]) Given a positive definite matrix \( R > 0 \), the positive integer \( n > 2 \), and the vector function \( y : [-n, 0] \rightarrow \mathbb{R}^{n \times n} \) satisfy the following inequalities:

\[
\sum_{i=0}^{n-1} y^T(i)Ny(i) \geq \frac{1}{n} \Omega_0^T \Omega_0 + \frac{3}{n} \Omega_1^T \Omega_1 + \frac{5}{n} \Omega_2^T \Omega_2
\]

\[y(i) = x(i+1) - x(i), \Omega_0 = x(n) - x(0), \Omega_1 = x(n) + x(0) - \frac{2}{n+1} \sum_{i=0}^{n} x(i)\]

among them:

\[\Omega_2 = x(n) - x(0) - \frac{6}{n+1} \sum_{i=0}^{n} x(i) - \frac{12}{(n+1)(n+2)} \sum_{i=0}^{n} \sum_{j=0}^{n} x(j)\]

3. Fading adaptive Kalman filter design

In order to estimate the parameters at the same time, the fault parameters and the attack parameters are used as the state augmentation, and the fading adaptive Kalman filter is designed for estimation.

Extend the fault parameters and attack parameters to the state vector, and the augmented state vector is: \( x_a(k) = \begin{bmatrix} x(k) & f(k) & a(k) \end{bmatrix}^T \), and the augmented system is:

\[
\begin{bmatrix} x_a(k+1) \\ y(k) \end{bmatrix} = \begin{bmatrix} A_{11} x_a(k) + B_{12} u(k) + D_{11} w(k) \\ C_{11} x_a(k) + D_{12} v(k) \end{bmatrix}
\]

among them \( A_{11} = \begin{bmatrix} A & E & N \\ 0_{1 \times 2} & 0_{1 \times 2} & 0_{1 \times 2} \end{bmatrix}, B_{11} = \begin{bmatrix} B \\ 0_{1 \times 2} \end{bmatrix}, D_{11} = \begin{bmatrix} D_{1} \\ I_{2 \times 1} \end{bmatrix}, C_{11} = \begin{bmatrix} C \\ 0_{1 \times 2}, D_{12} = D_{2} \end{bmatrix} \)

Since the state functions of the fault parameters and attack parameters are unknown, auxiliary state functions \( f(k+1) = f(k) \) and \( a(k+1) = a(k) \) are introduced, and the other state vectors are the same as the system (1); \( w_r(k) = \begin{bmatrix} w(k) & d(k) & b(k) \end{bmatrix}^T, d(k) \) and \( b(k) \) are respectively the fault parameter noise, and the attack parameter noise are all bounded noises, \( d_i, b_i \) are its upper bounds respectively.

For the augmented system (5), the following fading adaptive Kalman filter is designed:

\[
\hat{x}_a(k+1 | k) = A_{11} \hat{x}_a(k | k) + B_{12} u(k)
\]

\[
P_t(k+1 | k) = \lambda(k+1) A_{11} P(k | k) A_{11}^T + D_{11} Q D_{11}^T
\]

\[
\hat{y}(k+1 | k) = C_{11} \hat{x}(k+1 | k)
\]

\[
e(k+1) = y(k+1) - \hat{y}(k+1)
\]

\[
V(k+1) = \begin{cases} e(1)e^T(1), & k = 0 \\ \rho v(k) + e(k+1)e^T(k+1), & k \geq 1 \end{cases}
\]
\[ N(k+1) = V(k+1) - C(k+1)Q(k)C(k+1)^T - D_2 R(k+1)D_2^T \]  
\[ M(k+1) = C(k+1)A(k+1)PA^T(k+1)C^T(k+1) + D_2 RD_2^T \]  
\[ \lambda_0 = \frac{\text{tr}[N(k+1)]}{\text{tr}[M(k+1)]}, \lambda(k+1) = \begin{cases} \lambda_0, \lambda_0 \geq 1 \\ 1, \lambda_0 < 1 \end{cases} \]  
\[ \Gamma(k+1) = C_{11}P(k+1|k)C_{11}^T + D_2 RD_2^T \]  
\[ K(k+1) = P(k+1|k)C_{11}'\Gamma(k+1) \]  
\[ \hat{x}_n(k+1|k+1) = \hat{x}_n(k+1|k) + K(k+1)e(k+1) \]  
\[ P(k+1|k+1) = [I_n - K(k+1)C_{11}]P(k+1|k) \]

In formula (10), $\rho$ is the forgetting factor, $\rho \in (0,1]$. $\rho$ is generally taken as $\rho=0.95$; $\lambda(k+1) \geq 1$ is the fading factor at time $k+1$. Compared with the standard Kalman basic equation, the only difference is that formula (7) has one more fading factor. Because of $\lambda(k+1) \geq 1$, the filter covariance is magnified by $\lambda(k+1)$ times, and the effect of actual measurement data in state estimation is increased, thereby avoiding the divergence of the filter.

### 4. Design of fault-tolerant/intrusion-tolerant cooperative controller based on sliding mode variable structure

Based on the failure and attack of the adaptive Kalman filter, sliding mode variable structure control is used to design a fault-tolerant/intrusion-tolerant cooperative controller, so that the system maintains its original performance requirements under fault conditions.

#### 4.1. Preliminary Design of Fault-tolerant/intrusion-tolerant Control Law

For discrete domain linear system (1), select the specified system input command as $r(k)$, and its change rate is $dr(k)$, take $R=[r(k); dr(k)]$ and $R_t=[r(k+1); dr(k+1)]$. Use linear extrapolation method to predict $R_t$, namely

\[ r(k+1) = 2r(k) - r(k-1); dr(k+1) = 2dr(k) - dr(k-1) \]

The structural system error $e_s(k)$ is:

\[ e_s(k) = r(k) - x(k) \]

Select sliding surface $s(k)$ as:

\[ s(k) = Ge_s(k) \]

Where $G$ is the matrix to be designed, ignoring $w_t$ and $v_t$, because

\[ \Delta s(k) = G(r(k+1) - Ax(k) - Bu(k) - Ef(k) - Na(k)) - s(k) - s(k) \]

Using sliding mode control based on exponential reaching law, namely

\[ u(k) = (GB)^{-1}(Gr(k+1) - GAx(k) - GEf(k) - GNa(k) - s(k) - s(k)) \]

Among them $ds(k) = -\varepsilon T sgn(s(k)) - qTs(k)$, $q, \varepsilon$ are constants to be designed.

When the system is fault-free and no attack occurs, the control law $u_s(k)$ of the nominal system is:

\[ u_s(k) = (GB)^{-1}(Gr(k+1) - GAx(k) - s(k) - ds(k)) \]

When both faults and attacks occur, the additional active fault-tolerant/intrusion-tolerant control law is:
\[ u_\mu(k) = (GB)^{-1} (-GEf(k) - GNa(k)) \]  

(22)

Theorem 1: When the constants \(q, \varepsilon\) respectively satisfy \(q > 0, \varepsilon > 0\), the system (1) satisfies the existence and reachability conditions of sliding mode.

Proof: Choose the Lyapunov function

\[ V(k) = \frac{1}{2} s(k)^2 \]  

(23)

Substituting equation (19) into equation (23), we can get

\[ \Delta V(k) = s(k) \Delta s(k) = s(k)(Gr(k+1) - Ax(k) - Bu(k) - Ef(k) - Na(k)) - s(k) \]

Substituting formula (20) into, we get \(\Delta V(k) = -qTs(k)^2 - \varepsilon T|s(k)|\)

When the constants \(q, \varepsilon\) satisfy \(q > 0, \varepsilon > 0\) respectively, \(T\) is the sampling period, \(T > 0\) has

\[ \Delta V(k) = -qTs(k)^2 - \varepsilon T|s(k)| \leq 0 \]

It can be seen from the above analysis that the system can meet the requirements of progressive stability, which means that within a limited time, starting from any initial state other than the sliding surface 3, it will reach the sliding surface.

4.2. Real-time adjustment of fault-tolerant/intrusion-tolerant control law

The online fault-tolerant/intrusion-tolerant coordinated control for system (1) can be described as: using the state estimate \(\hat{x}(k)\) of the system state, the actuator fault estimate \(\hat{f}(k)\) and the network attack estimate \(\hat{a}(k)\), and considering the effect of the event triggering condition, it will pass the event. The estimated values \(\hat{x}(t_e), \hat{f}(t_e), \hat{a}(t_e)\) of the trigger are substituted into the control law \(u(k)\), and the adjusted online fault-tolerant/intrusion-tolerant control law \(u(k)\) is:

\[ u(k) = (GB)^{-1}(Gr(k+1) - GA\hat{x}(t_e) - GE\hat{f}(t_e) - GNa(t_e) - s(k) - ds(k)) \]  

(24)

When the system is fault-free and no attack occurs, the control law \(u_c(k)\) of the nominal system is adjusted to:

\[ u_c(k) = (GB)^{-1}(Gr(k+1) - GA\hat{x}(t_e) - s(k) - ds(k)) \]  

(25)

When both faults and attacks occur, the additional active fault-tolerant/intrusion-tolerant control law is adjusted to:

\[ u_{\mu}(k) = (GB)^{-1}(-GE\hat{f}(t_e) - GNa(t_e)) \]  

(26)

The process of sliding mode surface accessibility analysis is the same as theorem 1. In addition, it is known that the nominal system function, failure function, and attack function are all bounded, so the following inequality can be defined:

\[ \|x(k) - \hat{x}(t_e)\| \leq \|e\|, \|f(k) - \hat{f}(t_e)\| \leq \gamma_1, \|a(k) - \hat{a}(t_e)\| \leq \gamma_2 \]  

(27)

then \(\Delta V(k) \leq s(k)\|-\varepsilon T \text{sgn}(s(k)) - qTs(k)\| - \|GAe\| - \|GE\gamma_1\| - \|GN\gamma_2\| \)

When the constant \(q, \varepsilon\) respectively satisfies the following inequalities

\[ \varepsilon \geq T(\|GAe\| + \|GE\gamma_1\| + \|GN\gamma_2\|), q > 0 \]  

Then \(\Delta V(k) \leq -qTs(k)^2 \leq 0 \)

It can be seen from the above analysis that the system can meet the requirements of progressive stability, which means that within a limited time, starting from any initial state other than the sliding surface \(s(k) = 0\), it will reach the sliding surface.

When the system is on the sliding surface, \(s(k) = 0\). Combining formula (24), the equivalent control items of the controller can be obtained as:
\[ u_{eq}(k) = (GB)^{-1} \left( Gr(k+1) - GA\hat{x}(t_k) - GE\hat{f}(t_k) - GN\hat{u}(t_k) \right) \] (28)

Substituting formula (28) into formula (1), the state space model of linear discrete NCS on the sliding surface is

\[
\begin{align*}
\dot{x}(k+1) &= A x(k) + B (GB)^{-1} \left( Gr(k+1) - GA\hat{x}(t_k) - GE\hat{f}(t_k) - GN\hat{u}(t_k) \right) + D_j v(k) \\
y(k) &= C x(k) + D_j v(k) \\
x(k) &= \phi(k), k \in [-\tau_M, 0]
\end{align*}
\] (29)

where \( \phi(k) \) is a real-valued initial function on \([-\tau_M, 0]\).

Based on the design of the aforementioned fading adaptive Kalman filter, equation (29) can be rewritten as:

\[
\begin{align*}
\dot{x}(k+1) &= A x(k) + G^{-1} \left[ Gr(k+1) - GA\hat{x}(t_k) + Ge(t_k) - Ee(t_k) + \tau(k) Ed(k) - Ne(t_k) + N\tau(k) Na(t_k) \right] + D_j v(k) \\
y(k) &= C x(k) + D_j v(k) \\
x(k) &= \phi(k), k \in [-\tau_M, 0]
\end{align*}
\] (30)

Since the system has already met the accessibility condition of the sliding mode surface, in order to ensure stability, it is only necessary to design a suitable matrix \( G \) to make the sliding mode described by equation (30) stable.

Assumption: The fault compensation matrix \( B^* \) satisfies \((I - BB^*) E = 0\).

Theorem 2: Under DETCS, given a positive number \( \tau_M, \sigma, n_1, n_2, n_3, m_1, m_2, m_3, m_4, m_5, m_6 \), for a system(30) with actuator failure \( f(k) \) and network attack \( a(k) \), if there is a symmetric positive definite matrix \( P^* \) and a suitable dimensional matrix \( X', K, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6 \) satisfy

\[
\begin{align*}
[\psi_{i1} \psi_{i2} \psi_{i3} \Xi_{i4} \Xi_{i5} \Xi_{i6} 0 0]
\end{align*}
\] < 0 (31)

\[
\begin{align*}
[\psi'_{i1} \Xi_{i1} \Xi_{i2} \Xi_{i3} \Xi_{i4} \Xi_{i5} \Xi_{i6} 0 0]
\end{align*}
\] < 0 (32)

\[
\begin{align*}
[\psi''_{i1} 0 0 \Xi'_{i10} \Xi'_{i11} 0 0]
\end{align*}
\] < 0 (33)

\[
\begin{align*}
[\psi''_{i1} 0 0 \Xi''_{i10} \Xi''_{i11} 0 0]
\end{align*}
\] < 0 (34)

\[
\begin{align*}
\left[ Q_2 \ E' \ P' \\
\right] > 0, \left[ Q_4 \ n_1 E' \ P' \\
\right] > 0, \left[ Q_6 \ n_2 E' \ P' \\
\right] > 0, \left[ Q'_{10} \ N' \ P' \\
\right] > 0, \left[ Q'_{12} \ n_1 N' \ P' \\
\right] > 0, \left[ Q'_{14} \ n_2 N' \ P' \\
\right] > 0
\end{align*}
\] (35)

Then the designed active fault-tolerant/intrusion-tolerant control law (24) can make the system (30) progressively stable and has \( \gamma \) disturbance suppression performance, that is, satisfy the performance index conditions of the form (36), the controller gain matrix \( G = P^* K^* B^{-1} \) and the event trigger weight matrix \( \Phi^* \) can be obtained collaboratively.
\[
\sum_{k=0}^{\infty} x^T(k)x(k) < \gamma^2 \sum_{k=0}^{\infty} \left[ w^T(k)w(k) + (t_{k+1} - t_k) \left( e_i^T(t_k) e_i(t_k) + e_j^T(t_k) e_j(t_k) + e_o^T(t_k) e_o(t_k) \right) \right]
\] (36)

among them,
\[
\Xi_{i1} = P^T A + A^T P' - n_i P' + n_i P^T + \frac{\tau_i^2}{4} (m_2 + m_o) A^T P'A - 3X' - 3X'^T, \quad \Psi_{26} = \Psi_{26}',
\]
\[
\Xi_{i2} = K_i + n_i P^T + \frac{\tau_i^2}{4} (m_2 + m_o) A^T K_i - \frac{\tau_i^2}{4} (m_2 + m_o) P' + X' - 3X'^T, \quad \Psi_{27} = -\frac{\tau_i^2}{4} (m_2 + m_o) K_i^T N,
\]
\[
\Psi_{28} = \frac{\tau_i^2}{4} (m_2 + m_o) K_i^T D, \quad \Psi_{34} = \Xi_{i1}, \quad \Psi_{35} = -\gamma^2 I, \quad \Psi_{36} = \Psi_{27}, \quad \Psi_{38} = \Psi_{28} \Psi_{36} - \frac{\tau_i^2}{4} (m_2 + m_o) E^T P'D,
\]
\[
\Psi_{66} = -\gamma^2 I + \frac{\tau_i^2}{4} (m_2 + m_o) E^T P'E, \quad \Psi_{67} = \frac{\tau_i^2}{4} (m_2 + m_o) E^T P'N, \quad \Xi_{i11} = \Xi_{i31} = \Xi_{i41} = X', \quad \Xi_{i21} = \Xi_{i25} = \Xi_{i23} = \Xi_{i24} = \Xi_{i25}
\]
\[
\Psi_{77} = -\gamma^2 I + \frac{\tau_i^2}{4} (m_2 + m_o) N^T P'N, \quad \Psi_{78} = -\gamma^2 I + \frac{\tau_i^2}{4} (m_2 + m_o) N^T P'D, \quad \Psi_{88} = -\gamma^2 I + \frac{\tau_i^2}{4} (m_2 + m_o) E^T P'D,
\]

prove:

1) Consider \( e_i(t_k) = 0, e_j(t_k) = 0, e_o(t_k) = 0, w(k) = 0 \) to make the system (30) eventually uniformly bounded.

Construct the Lyapunov-Krasovskii function of the following form:
\[
V(k) = x^T(k)P'x(k) + \sum_{i=k}^{k+1} x^T(i)Q'x(i) + (r_m - \tau(k)) \sum_{i=k}^{k+1} x^T(i+1)R'x(i+1) + (r_m - \tau(k)) \varphi^T(k)S\varphi(k)
\]

Where \( \varphi(k) = x(k) - x(t_k) \) and \( P' = P^T > 0, Q' = Q^T > 0, R' = R^T > 0, S' = S^T > 0 \).

Perform difference calculations on the Lyapunov-Krasovskii functional:
\[
\Delta V(k) = x^T(k+1)P'x(k+1) - x^T(k)P'x(k) + \sum_{i=k+1}^{k+2} x^T(i)Q'x(i) - \sum_{i=k}^{k+1} x^T(i)Q'x(i) - \sum_{i=k}^{k+1} x^T(i)
\]
\[
Q'x(i) + \varphi_i^T(k)S\varphi(k) - (r_m - \tau(k)) \sum_{i=k}^{k+1} x^T(i+1)R'x(i+1) + 2(r_m - \tau(k)) \varphi_i^T(k)S\varphi(k+1)
\]

Use Lemma 1 to deal with \( \sum_{i=k}^{k+1} x^T(i+1)R'x(i+1) \) in \( \Delta V(k) \), namely
\[
- \sum_{i=k}^{k+1} x^T(i+1)R'x(i+1) \leq -\psi_i^T(k)\Theta\psi_i(k)
\]
among them,
\[
\psi_i^T(k) = \left[ x^T(k) \ x^T(t_k) \ \frac{1}{\tau(k)} \Omega_0^T \ x^T(i) \ \frac{1}{\tau(k)} \Omega_0^T \right], \quad \psi_i(k) = \sum_{i=k}^{k+1} \psi_i^T(k), \quad \Theta = XH + H^T X'^T - \tau(k)X\bar{R}X'^T, \quad \bar{R} = \text{diag} \left\{ R^{-1}, \ \frac{1}{3} R^{-1}, \ \frac{1}{5} R^{-1} \right\}, \quad H = \begin{bmatrix} I & -I & 0 & 0 \\ I & I & -2I & 0 \\ I & -I & 0 & -6I \end{bmatrix}
\]

Remember \( M_{31} = [I \ 0 \ 0 \ 0], M_{32} = [I \ -I \ 0 \ 0], M_{33} = [A \ -A \ 0 \ 0], M_{34} = [0 \ I \ 0 \ 0] \).

The
\[
x(k+1) = M_{34} \psi_1(k) + \tau(k) Ed(k), \quad x(k - \tau(k)) = M_{34} \psi_1(k)
\]

Therefore, you can get
\[ \Delta V(k) \leq \psi_1^T(k) \left[ 2M_{31}^T P M_{33} - M_{32}^T S M_{32} + M_{30}^T Q M_{31} - (XH + H^T X)^T - M_{34}^T Q M_{34} - (\tau_m - \tau(k)) \right] \]
\[ + \left( 2M_{33}^T S M_{33} + M_{30}^T R M_{03} \right) + \tau(k) X \overline{R} X^T \right] \psi_1(k) + 2\tau(k) \left( \psi_1^T(k) P' E d(k) + x^T(k) P' N b(k) \right) \]
\[ + \psi_1^T(k) \left( M_{34}^T R^T E d(k) + \phi^T(k) S E d(k) + \phi^T(k) S N b(k) \right) \]
\[ + \psi_1^T(k) \left( \varepsilon^T(k) E^T R^T E d(k) + b^T(k) N^T R' N b(k) + 2d^T(k) E^T R N b(k) \right) \]

denoted by (39)

According to the following inequality

\[ \left[ \begin{array}{c} Q_2 \\ E^T P' \\ \ast \\ * \\ Q_l \end{array} \right] > 0, \left[ \begin{array}{c} Q_4 \\ E^T S' \\ \ast \\ * \\ m_2 P' \end{array} \right] > 0, \left[ \begin{array}{c} Q_6 \\ E^T R' \\ \ast \\ * \\ m_2 P' \end{array} \right] > 0, \left[ \begin{array}{c} Q_3' \\ N^T P' \\ \ast \\ * \\ m_2 P' \end{array} \right] > 0, \left[ \begin{array}{c} Q_3' \\ N^T S' \\ \ast \\ * \\ m_2 P' \end{array} \right] > 0, \left[ \begin{array}{c} Q_4' \\ N^T R' \end{array} \right] > 0 \]

Lemma 4 and equation (39), we can get

\[ \Delta V(k) \leq \psi_1^T(k) \left[ \Sigma_{11} + (\tau_m - \tau(k)) \Sigma_{12} + \tau(k) \Sigma_{22} \right] \psi_1(k) + \kappa \]

among them,

\[ \kappa = \tau_m d_m^2 \lambda_{\max} (Q_2) + \tau_m b_m^2 \lambda_{\max} (Q_4) + \frac{\tau_m^2}{4} b_m^2 \lambda_{\max} (Q_3) + \frac{\tau_m^2}{4} b_m^2 \lambda_{\max} (Q_3) + \frac{\tau_m^2}{4} d_m^2 \lambda_{\max} (Q_6) + \frac{\tau_m^2}{8} d_m^2 \lambda_{\max} (Q_4) + \frac{\tau_m^2}{8} d_m^2 \lambda_{\max} (E^T R') + \frac{\tau_m^2}{8} b_m^2 \lambda_{\max} (E^T R') + \frac{\tau_m^2}{8} d_m^2 \lambda_{\max} (E^T R') \]

If \( \Sigma_{11} + (\tau_m - \tau(k)) \Sigma_{12} + \tau(k) \Sigma_{22} < 0 \), then \( \Delta V(k) \leq -\varepsilon \sum_{k=0}^{\infty} \psi_1^T(k) \psi_1(k) + \kappa \).

Among them, \( \varepsilon = \lambda_{\max} \left[ -\left( \Sigma_{11} + (\tau_m - \tau(k)) \Sigma_{12} + \tau(k) \Sigma_{22} \right) \right] \).

Therefore, when \( -\varepsilon \sum_{k=0}^{\infty} \psi_1^T(k) \psi_1(k) + \kappa < 0 \), that is \( \sum_{k=0}^{\infty} \psi_1^T(k) \psi_1(k) > \frac{\kappa}{\varepsilon} \), holds.

Therefore, when \( \psi_1(k) \) finally converges to the set: \( \Delta = \left\{ \psi_1(k) \right\} \sum_{k=0}^{\infty} \psi_1^T(k) \psi_1(k) > \frac{\kappa}{\varepsilon} \), the system (37) is finally bounded uniformly.

According to the linear convex combination lemma, \( \Sigma_{11} + (\tau_m - \tau(k)) \Sigma_{12} + \tau(k) \Sigma_{22} < 0 \), the necessary and sufficient condition is

\[ \Sigma_{11} + \tau(k) \Sigma_{22} < 0, \Sigma_{11} + \tau(k) \Sigma_{12} < 0 \]

2): Under initial conditions, when \( e_1(t_i) \neq 0, e_j(t_i) \neq 0, e_a(t_i) \neq 0, w(k) \neq 0 \), consider the following \( H_{\infty} \) performance index functions:

\[ J = \Delta V(k) + X^T(k) x(k) - \gamma^2(w^T(k) w(k) + e_1^T(t_i) e_1(t_i) + e_2^T(t_i) e_2(t_i) + e_3^T(t_i) e_3(t_i)) \]

\[ + e_4^T(t_i) e_4(t_i) + e_5^T(t_i) e_5(t_i) \phi_{\alpha}(k) \phi_{\alpha}(k) \]

Triggered by the event, when \( k \in \left[ t_i + \tau_n \cdot t_{k+1} + \tau_n \right] \), there is

\[ e^T(k) \phi_{\alpha}(k) \leq \sigma \tilde{x}^T(t_i) \phi_{\alpha}(t_i) \]

Remember

\[ \psi_2^T(k) = \left[ \begin{array}{c} x^T(k) \\ x^T(t_i) \\ \frac{1}{\tau(k)} Q_6^T \\ \frac{1}{\tau(k)} Q_7^T \\ e_1^T(t_i) \\ e_2^T(t_i) \\ e_3^T(t_i) \\ w^T(k) \\ \tilde{x}^T(t_i) \\ e^T(k) \end{array} \right] \]

The proof process is the same as 1). \( J < 0 \) is equivalent to

\[ \Sigma_{11} + \tau(k) \Sigma_{22} < 0, \Sigma_{11} + \tau(k) \Sigma_{12} < 0 \]

Inequalities (40) and (43) are non-linear. Let \( S' = n_1 P' \), \( R' = n_2 P', Q' = n_3 P' \), and \( Q_1 = m_1 P', Q_2 = m_2 P', Q_3 = m_3 P' \). \( Q_1 = m_4 P', Q_2 = m_5 P', K_i = P' G_B \), then apply Schur's
complement lemma to turn them into linear matrix inequalities to obtain inequalities (31)–(34). Among them, the parameter $G$ to be designed can be obtained by solving $G = P^{-1}K_iB^{-1}$.

Accumulate formula (41) from 0 to $\infty$, we can get

$$\sum_{k=0}^{\infty} \Delta V(k) + \sum_{k=0}^{\infty} x^T(k)x(k) - \gamma^2 \sum_{k=0}^{\infty} x^T(k)x(k) - \gamma^2 (t_{k+1} - t_k)$$

$$\sum_{k=0}^{\infty} (t_{k+1} - t_k) \left[ e^T_i(t_k)e_i(t_k) + e^T_j(t_k)e_j(t_k) + e^T_a(t_k)e_a(t_k) \right] \leq 0$$

Obviously, you can get

$$\sum_{k=0}^{\infty} x^T(k)x(k) - \gamma^2 \sum_{k=0}^{\infty} x^T(k)x(k) - \gamma^2 (t_{k+1} - t_k) \left[ e^T_i(t_k)e_i(t_k) + e^T_j(t_k)e_j(t_k) + e^T_a(t_k)e_a(t_k) \right] \leq 0$$

That is, under zero initial conditions, there are

$$\sum_{k=0}^{\infty} x^T(k)x(k) \leq \gamma^2 \sum_{k=0}^{\infty} x^T(k)x(k) - \gamma^2 (t_{k+1} - t_k) \left( e^T_i(t_k)e_i(t_k) + e^T_j(t_k)e_j(t_k) + e^T_a(t_k)e_a(t_k) \right)$$

established. That is, the fault/attack system meets the h performance index.

5. Simulation verification

This paper takes the control system model given in [17] as Research objects, design parameters are as follows:

$$A = \begin{bmatrix} 0.9879 & 0.0098 \\ -0.0837 & 0.9908 \end{bmatrix}, \quad B = \begin{bmatrix} -0.0029 & -0.0005 \\ -0.1919 & -0.0378 \end{bmatrix}, \quad E = \begin{bmatrix} -0.0029 \\ -0.0019 \end{bmatrix}, \quad N = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

In this simulation, we make the following assumptions about actuator failures and network attacks:

That is, the actuator failure is as follows: $f(k) = 4, 0 \leq k \leq 500$

The cyber attacks are as follows: $a(k) = 2, 0 \leq k \leq 500$

The coefficient matrices after augmentation are:

$$A_{i1} = \begin{bmatrix} 0.9879 & 0.0098 & -0.0029 & 1 \\ -0.0837 & 0.9908 & -0.1919 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B_{i1} = \begin{bmatrix} -0.0029 & -0.0005 \\ -0.1919 & -0.0378 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_{i1} = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}, \quad C_{i1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Suppose the initial conditions of the system are:

$$y(0) = I_4, \lambda = 0.8, \quad Q = 0.0001, R = 0.0025, r(k) = 0, \quad x(0) = \dot{x}(0) = \begin{bmatrix} -30 & 20 & 4 & 2 \end{bmatrix}^T$$

From equation (14), $K = \begin{bmatrix} 11.9169 & -1.1917 \\ 109.1688 & -10.9168 \\ -8.2060 & 0.8206 \\ 0.5501 & -0.0557 \end{bmatrix}$

In Theorem 2, let $n_1 = 1.5, n_2 = 0.9, \quad n_3 = 0.01, \quad m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = 0.1$; then let the maximum delay upper bound $r_m = 1.2$, trigger parameter $\sigma = 0.85$, disturbance suppression rate $\gamma = 2.6562$, and take $q = 5, \xi = 2, \tau = 0.1$. Based on Theorem 2, the sliding mode gain matrix $g$ and the event trigger matrix $f$ can be obtained cooperatively, which are:

$$G = \begin{bmatrix} -5.6995 & 1.0126 \\ -0.656 & 0.8085 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 1.5812 & 0.2811 \\ -0.2657 & 2.6766 \end{bmatrix}$$

The simulation results are shown in Figures 2–8. Figures 2 and 3 show the system attacks and their estimation curves and attack estimation error graphs respectively. Figures 4 and 5 show the actuator failures in the system and their estimation curves and fault estimation errors. Figures 6 and 7 respectively
show the state of the system and its estimation curve and state estimation curve. Figures 8 and 9 are the output response curve of the system when the actuator failure and network attack occur simultaneously, and the non-uniform transmission NCS The data sending time and sending interval.

It can be seen from Figures 2–7 that the adaptive Kalman filter algorithm can estimate the state, faults and attacks of the system well. It can be seen from Figure 8 that when the system encounters an actuator failure and encounters a network attack, the cooperative controller designed in this paper is used to compensate for the failure and attack, and the oscillation is gradually attenuated and tends to a balanced state. The simulation results show that the fault-tolerant/intrusion-tolerant cooperative controller designed in this paper can effectively tolerate faults and attacks actively, and at the same time suppress the influence of disturbance and noise.

From the analysis in Figure 9, when the event trigger parameter $\sigma = 0.85$, within 50s of the simulation time, compared with the traditional PPTCS that needs to transmit 500 data, only 75 data are transmitted under DETCS, the data transmission rate is 15.06%, and the average transmission period is...
\[ T = 0.6647s \] and the maximum sending period is \[ T_{\text{max}} = 2s \]. This shows that the control method proposed in this paper not only ensures the excellent performance of the system, but also effectively saves network communication resources and improves the efficiency of network utilization.

6. Conclusion

Aiming at the linear discrete NCS with time-varying delay, when actuator failures and network attacks occur, under noise interference, a sliding mode variable structure fault-tolerant/intrusion-tolerant coordinated control method based on discrete event trigger mechanism is proposed. This method uses sliding mode control method to design active fault-tolerant/intrusion-tolerant controller on the basis of estimating fault parameters and attack parameters, and considering event trigger mechanism, according to the impact of faults and attacks on the system, so that the system will fail and attack Keep normal operation under the circumstances. The simulation results show that the system can track the changes of fault parameters and attack parameters well, and perform parameter estimation. The cooperative controller can compensate the system well, and at the same time, it can effectively save network communication resources. This control method has good convergence and dual capacity capability for faults and attacks.

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