Hadronic production of $B_s^{(*)}$ at TEVATRON and LHC

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Abstract

We study the hadronic production of $B_s$ and $B_s^*$ mesons within the fixed-flavor-number scheme, in which the dominant gluon-gluon fusion mechanism is dealt with by using the complete $\alpha_s^4$ approach. Main theoretical uncertainties for $B_s$ and $B_s^*$ production at TEVATRON and LHC are presented. It is found that when $m_s$ increases by steps of 0.1 GeV, the integrated cross section of $B_s^{(*)}$ decreases by 80% − 100%, when $m_b$ increases by steps of 0.1 GeV, it changes to be $\sim 10\%$. While the uncertainties caused by the parton distribution function and the factorization scale varies within the region of $\frac{1}{5}$ to $\frac{1}{3}$. Considering possible kinematic cut on the transverse momentum and the rapidity cut for the detectors at TEVATRON and LHC, we also make estimations on the $B_s$ and $B_s^*$ production with various kinematic cuts.

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Since Run II at the TEVATRON Collider started in 2001, the CDF and D0 experiments have successfully collected $B_s$ data \[1, 2, 3, 4\]. One can use $B_s$ meson to study those interesting topics as QCD model building, physics beyond the Standard Model, the electroweak symmetry breaking mechanism, charge-parity (CP) violation and etc. \[5, 6, 7, 8\].

Taking into account the prospects of $B_s$ production at Fermilab TEVATRON and at the newly running CERN LHC, the future numerous data require more accurate theoretical predictions, especially on its hadronic production.

According to the QCD factorization formula, the hadronic production of $B_s$ and $B_s^*$ can be written as

$$d\sigma(S, p_T, \cdots) = \sum_{ij} \int \int dx_1 dx_2 F_{H_1, H_2}^i(x_1, \mu_F^2) \cdot F_{H_2, P_2}^j(x_2, \mu_F^2) \cdot d\hat{\sigma}_{ij \rightarrow B_s^{(*)} X}(P_1, P_2, x_1, x_2, \mu_F^2, Q^2, \hat{s}, p_T, \cdots),$$

where $\sqrt{S}$ stands for the total collision energy of the incoming hadrons, $F_{H_1, P_1}^i(x_1, \mu_F^2)$ and $F_{H_2, P_2}^j(x_2, \mu_F^2)$ are the parton distribution functions (PDFs) of incoming hadrons $H_1$ (momentum $P_1$) and $H_2$ (momentum $P_2$) for parton $i$ (with momentum fraction $x_1$) and parton $j$ (with momentum fraction $x_2$) respectively. $Q^2$ is the “characteristic energy scale of the subprocess squared” and $\mu_F$ stands for the factorization scale for the PDF and the hard subprocess. A detailed discussion on the choice of $Q^2$ and $\mu_F$ can be found in Ref.\[9\], here for simplicity, we shall take $Q^2 = \mu_F^2$ for the present perturbative QCD calculation. $d\hat{\sigma}_{ij \rightarrow B_s^{(*)} X}$ stands for the differential cross-section of the relevant hard subprocess, in which $\hat{s} = x_1 x_2 S$ is the c.m.s. energy of the subprocess and $P_T$ is the transverse momentum of $B_s^{(*)}$.

Within the fixed-flavor-number (FFN) scheme \[10\], where only light quark/antiquark and gluon should be considered in the initial state of the hard scattering subprocess, it can be found that $B_s^{(*)}$ hadronic production are dominated by the gluon-gluon fusion mechanism, which is through the sub-process $g + g \rightarrow B_s^{(*)} + b + \bar{s}$ and is of order $\alpha_s^4$. In addition to the gluon-gluon fusion mechanism, there are several different mechanisms for the production, such as that via the quark-antiquark annihilation sub-process $q \bar{q} \rightarrow B_s^{(*)} + b + \bar{s}$ and etc.. However, it can be found that the contributions to the production from quark-antiquark annihilation are much smaller (only about 1%) than those from gluon-gluon fusion, which is due to the fact that the ‘luminosity’ of gluons is much higher than that of quarks in $pp$ collisions (LHC) and in $p\bar{p}$ collisions (TEVATRON), and there is a suppression factor due to
the virtual gluon propagator in the annihilation, which is similar to the case of $B_c$ production \[11, 12\]. Hence in the present letter, we shall concentrate our attention on the gluon-gluon fusion mechanism. And to be useful experimentally, we shall discuss the main uncertainties in estimating the hadronic production of $B^{(s)}_s$, which includes the choices of the factorization energy scale $\mu_F$, the various versions of parton distribution functions (PDFs), the values of the bound state parameters and etc..

As for the dominant gluon-gluon fusion mechanism, its hard subprocess $g + g \rightarrow B^{(s)}_s + \bar{b} + s$ includes 36 Feynman diagrams, whose typical ones are plotted in FIG. 1. To derive the analytical squared amplitude for this subprocess is a tedious task, since it contains non-Abelian gluons and massive fermions. Fortunately however, very recently, a generator BCVEGPY \[13, 14, 15\] for hadronic production of $B_c$ meson has been available, where to deal with the subprocess $g + g \rightarrow B_c + \bar{b} + c$, the so called ‘helicity amplitude approach’ \[16, 17\] has been adopted to derive analytic expressions at the amplitude level and then do the numerical calculation just at the amplitude level. Here we adopt the same method of Refs. \[13, 14, 15\] to deal with the present $B^{(s)}_s$ production, which can be obtained by suitable changing the $c$-quark lines defined in Ref. \[13\] to the present $s$-quark lines. To short the paper, we shall only present the main idea on how to deal with those 36 Feynman diagrams based on the ‘helicity amplitude approach’, and the interesting reader may consult Ref. \[13\] for detailed calculation technology. The main idea is to convert the problem into an equivalent

\footnote{It should be noted that Ref. \[16\] presented only the formulae for massless spinor lines, so proper changes as have been done in Ref. \[13\] should be made so as to deal with the massive spinor lines.}
'massless' one that is well solved in literature, i.e. by transforming the massive quark lines to be massless ones, and then to apply the symmetries as much as possible. To extend the symmetries for the amplitude corresponding to 36 Feynman diagrams, we first focus on the numerator of the amplitude related to typical fermion lines, and neither consider the color factors nor distinguish the flavor of the fermion lines at the moment. Then, because of Feynman diagram symmetries, these diagrams can be grouped into a few typical ones according to the different type of fermion lines. And then we implement proper factors for the fermion lines: color factors, suitable denominator and spinors and etc., so as to obtain an exact and full typical fermion line that appears in Feynman diagrams. When all kinds of typical fermion line factors, factors for external lines of gluons and gluon propagators are ‘assembled’, then the full term, corresponding to the Feynman diagram of the amplitude, is achieved. Next, to do the phase space integration, we first use RAMBOS [18] routine to generate the requested phase space points and then use VEGAS [19] program to perform the integrations.

Based on the above calculation technology, we present the numerical results. As for the present LO estimation, \( f_{B_s} \) appears in the amplitude as a linear factor, so the production cross sections are proportional to it squared. Therefore, the uncertainties in the production from \( f_{B_s} \) can be figured out straightforwardly, so throughout the paper, we take \( f_{B_s} = 0.209 \text{ GeV} \) [20]. And because the spin splitting effects are ignored here, so there is no difference for the decay constant between the spin stats \([ ^1S_0 \] and \([ ^3S_1 \]. Further more, we shall study the uncertainties in ‘a factorization way’ throughout the paper, i.e., all of the parameters vary independently in their reasonable regions. For instance, when focussing on the uncertainties from the constitute s-quark mass \( m_s \), we let it be a basic ‘input’ parameter varying in a possible range

\[
0.4\text{GeV} \leq m_s \leq 0.7\text{GeV}, \quad (2)
\]

with all the other factors, including the \( B_s \)-meson mass, the decay constant \( f_{B_s} \) and etc. being fixed.

In TAB[II we show the uncertainties from \( m_s \), where the other factors are fixed precisely as: \( m_b = 4.9 \text{ GeV} \), the PDFs are taking as CTEQ6L [23]; the strong coupling \( \alpha_s \) is in LO and the factorization energy scale is taken to be \( \mu^2_F = p^2_{T_{B_s}} + m^2_{B_s} \). Note that for the mass of \( B_s \), the experimental result is \( m_{B_s} = 5.3663 \pm 0.0006 \text{ GeV} \) [21], while the prediction by
TABLE I: Total cross section for the hadronic production of $B_s[1^1S_0]$ and $B_s^*[1^3S_1]$ with varying $m_s$, where $m_b = 4.9$ GeV, $m_{B_s} = m_s + m_b$, the gluon distribution function is taken from CTEQ6L, $\mu_F^2 = p_{T_{B_s}}^2 + m_{B_s}^2$ and $\alpha_s$ is of leading order.

| $m_s$ (GeV) | TEVATRON ($\sqrt{S} = 1.96$ TeV) | LHC ($\sqrt{S} = 14$. TeV) |
|------------|---------------------------------|-----------------------------|
| 0.4        | 48.25                           | 512.6                       |
| 0.5        | 24.43                           | 262.2                       |
| 0.6        | 14.33                           | 155.1                       |
| 0.7        | 9.326                           | 101.5                       |
| $\sigma_{B_s}$ (nb) | 165.5                           | 1739.                       |
| $\sigma_{B_s^*}$ (nb)  | 82.18                           | 871.8                       |

FIG. 2: $B_s$ differential distributions versus its $P_T$ and rapidity $y$ with varying $m_s$. The gluon distribution function is taken from CTEQ6L, the characteristic energy scale is taken as $Q^2 = p_{T_{B_s}}^2 + m_{B_s}^2$, and $\alpha_s$ is of leading order. The solid line, dashed line, dotted line and dash-dot line stands for $m_s = 0.4, 0.5, 0.6$ and 0.7 GeV respectively. The upper (lower) four lines corresponding to the distributions at LHC (TEVATRON) respectively.

lattice QCD is about 5.37 GeV [22]. Thus with $m_b = 4.9$ GeV and $m_{B_s} = m_b + m_s$, the obtained $m_{B_s}$ is in the region of theoretical prediction as well as experimental measurement. In Table II the total cross-section for the hadronic production of $B_s[1^1S_0]$ and $B_s^*[1^3S_1]$ at TEVATRON and LHC are computed. From TABH one may observe that the total cross

This relation should be satisfied according to the gauge invariance of the hard subprocess. Further more, at the present, we treat $B_s$ as non-relativistic and weak binding state, then at LO the relative momentum between the constitute quarks can be ignored.
TABLE II: Total cross section for the hadronic production of $B_s[1^1S_0]$ and $B_s^*[1^3S_1]$ with various $m_b$, where $m_s$ is fixed to be 0.5 GeV, the gluon distribution function is taken from CTEQ6L, the factorization energy scale is chosen $\mu_F^2 = p_{T_Bs}^2 + m_{Bs}^2$ and $\alpha_s$ is of leading order.

| $m_b$ (GeV) | TEVATRON ($\sqrt{S} = 1.96$ TeV) | LHC ($\sqrt{S} = 14.$ TeV) |
|------------|----------------------------------|-----------------------------|
|            | $\sigma_{B_s}(nb)$               | $\sigma_{B_s^*}(nb)$       |
| 4.8        | 22.60                            | 280.9                      |
| 4.9        | 24.43                            | 262.2                      |
| 5.0        | 26.49                            | 245.7                      |
|            |                                  |                             |
| $\sigma_{B_s}(nb)$ | 88.86                            | 818.1                      |
|            |                                  |                             |
| $\sigma_{B_s^*}(nb)$      | 82.18                            | 871.8                      |

section of $B_s^*[1^3S_1]$ is about 3 times bigger that of $B_s[1^1S_0]$, which roughly agree with the naive counting of spin states. $m_s$ affects the total cross section greatly, e.g. when $m_s$ increases by steps of 0.1 GeV, then the cross section of $B_s$ or $B_s^*$ decreases by about 80% − 100%. To show this point more clearly, we draw the $B_s$-$P_T$ and rapidity $Y$ distributions with $m_s = 0.4, 0.5, 0.6$ and 0.7 GeV respectively in FIG.2. This implies that the present treatment of $s$-quark as heavy quark is reasonable but may be not too accurate, and a more accurate one, e.g. by including proper relativistic effects into the bound state, maybe improve the estimation, which is out of the range of the present letter. A similar calculation by varying $m_b$ within its reasonable region but with fixed $m_s = 0.5$ GeV, as shown by TAB II which shows that when $m_s$ increases by steps of 0.1 GeV, the cross section of $B_s$ or $B_s^*$ changes slightly, which is around 10%. More precise values of $m_b$ and $m_s$ from potential model or lattice QCD can make our estimations more reliable. In the following parts of the paper when examining the uncertainties from other factors, we shall always take the center values of $m_s = 0.5$ GeV and $m_b = 4.9$ GeV, for the quark masses.

PDF is of non-perturbative nature, which can be obtained through global fitting of the experimental data. Here we take CTEQ6L [23] and MRST2001L[24] as typical examples to study the uncertainty caused by PDF. As shown in Eq.(1) PDF can be factorized out at the energy scale $\mu_F^2$ with the help of pQCD factorization theorem. The factorization scale $\mu_F^2$ can be usually taken as the characteristic energy scale for the hard subprocess ($Q^2$), i.e. $\mu_F^2 = Q^2$. For the present case, the gluon-gluon fusion subprocess is of three-body final state and contain heavy quarks, so there are ambiguities in choosing $Q^2$ and various choices of $Q^2$ would generate quite different results. Since such kind of ambiguity cannot be justified
TABLE III: Total cross-section for the hadronic production of $B_s[1^1S_0]$ and $B_s^*[1^3S_1]$ at TEVATRON and at LHC with LO running $\alpha_s$ and the characteristic energy scale Type A: $Q^2 = \hat{s}/4$; Type B: $Q^2 = p_T^2 + m_{B_s}^2$ and Type C: $Q^2 = p_T^2 + m_b^2$.

| $Q^2$ | $\sigma_{B_s}(nb)$ | $\sigma_{B_s^*}(nb)$ |
|-------|-------------------|-------------------|
| Type A | Type B | Type C | Type A | Type B | Type C |
| TEVATRON ($\sqrt{S} = 1.96$ TeV) | | |
| CTEQ6L | CTEQ6L | MRST2001 | CTEQ6L | CTEQ6L | MRST2001L | CTEQ6L |
| 17.55 | 24.43 | 20.82 | 25.01 | 203.5 | 262.3 | 221.7 | 264.1 |
| LHC ($\sqrt{S} = 14.$ TeV) | | |
| CTEQ6L | CTEQ6L | MRST2001L | CTEQ6L |
| 58.37 | 82.19 | 69.83 | 83.89 | 675.8 | 871.8 | 734.2 | 876.5 |

FIG. 3: $B_s$ differential distributions versus its transverse momentum $P_T$ and rapidity $y$ for different LO PDFs, where the solid line and the dashed line are for CTEQ6L and MRST2001L respectively. The characteristic energy scale is taken as $Q^2 = p_T^2 + m_{B_s}^2$. The upper and lower two lines corresponding to the distributions in LHC and TEVATRON accordingly.

by the LO calculation itself, so we take it as the uncertainty of the LO calculation. In the following we choose three typical examples to study this kind of uncertainties: Type A: $Q^2 = \hat{s}/4$, the C.M. energy squared of the subprocess that is divided by 4; Type B: $Q^2 = p_T^2 + m_{B_s}^2$, the transverse mass squared of the $B_s$ meson; and Type C: $Q^2 = p_T^2 + m_b^2$, the transverse mass squared of the $b$ quark. For comparison between TEVATRON and LHC and to pinpoint the uncertainties from PDFs, $\alpha_s$ running and the choices of the characteristic energy scale $Q^2$, we calculate the production cross sections according to two types of PDFs,
FIG. 4: $B_s$ differential distributions versus its transverse momentum $p_T$ and rapidity $y$ for typical choices of $Q^2$, where the solid and the dashed lines are for Type A and Type B respectively. The gluon distribution is chosen as CTEQ6L and the running $\alpha_s$ is in leading order. The upper and lower two lines corresponding to the distributions in LHC and TEVATRON accordingly.

the strong coupling $\alpha_s$ fixed by the corresponding PDFs and the characteristic $Q^2$ chosen as Type A, Type B and Type C. The obtained results are shown in TABLE III. From TABLE III it is found that the difference caused by the two LO PDFs is small, which is $\sim 15\%$. The choice of $Q^2$ as Type A and Type B cause changes is somewhat larger, i.e. $20\% - 30\%$, while the choose of Type B and Type C leads to negligible changes to the cross section (less than 1%). The total cross section of the $B_s(\ast)$ production at LHC are at least one order larger in magnitude than that at TEVATRON. This is mainly due to the fact that LHC ($\sqrt{S} = 14$ TeV) has much higher collide energy than TEVATRON ($\sqrt{S} = 1.96$ TeV), so the lowest boundary of the gluon momentum fractions $x_i$ ($i = 1, 2$) at LHC are much smaller than that at TEVATRON and then there are more interacting gluons that have a C.M. energy above the threshold for the subprocess, in the collision hadrons at LHC than at TEVATRON. This can be shown more clearly by the $P_T$ and $y$ differential cross sections. More explicitly, we draw the curves for pseudo-scalar meson $B_s$ in FIGs.(3,4). FIG.3 shows that the differential distributions for the two PDFs CTEQ6L and MRST2001L and FIG.4 shows that the differential distributions for the two $Q^2$ Type A and Type B. In regions of comparatively small $p_T$ and $|y|$, the distributions of MSRT2001L are smaller than that of CTEQ6L. From the figure, we also see that the $p_T$ distributions in TEVATRON are steeper than those in LHC. From TABLE III we know that changes in the cross section caused by
TABLE IV: Dependence of $R = \left( \frac{\sigma_{\text{TEVATRON}}}{\sigma_{\text{LHC}}} \right)$ on $P_{\text{cut}}$ for $B_s[1^1S_0]$ and $B_s^*[1^3S_1]$.

| $P_{\text{cut}}$(GeV) | $B_s$     |       | $B_s^*$    |       |
|-----------------------|-----------|-------|------------|-------|
|                       | 0 5 20 35 50 | 0 5 20 35 50 |
| $R(\times10^{-2})$    | 9.32 7.71 3.00 1.67 0.70 | 9.43 7.80 3.12 1.85 0.87 |

FIG. 5: $B_s$ differential distributions versus its $y$ with various $p_{\text{T cut}}$ in TEVATRON (left diagram) and in LHC (right diagram). Solid line corresponds to the full production without $p_{\text{T cut}}$; dashed line to $p_{\text{T cut}} = 5.0$ GeV; dot line to $p_{\text{T cut}} = 20.0$ GeV; the dash-dot line to $p_{\text{T cut}} = 35.0$ GeV; the short dash line to $P_{\text{T cut}} = 50.0$ GeV.

Type B and Type C is quite small (less than 1%), and the curves of the production obtained by Type B and Type C are almost overlap, so we do not draw the curves for Type C.

Experimentally, when the produced $B_s$ and $B_s^*$ mesons with a small $P_T$ or a large rapidity $y$ are too close to the collision beam, they cannot be measured, so only ‘detectable’ events should be taken into account, i.e. events with proper kinematic cuts on $P_T$ and $y$ should be properly set in the estimates. As a comparison, we define a ratio $R = \left( \frac{\sigma_{\text{TEVATRON}}}{\sigma_{\text{LHC}}} \right)_{P_{\text{T cut}}}$ to show how $P_{\text{T cut}}$ affects the integrated cross sections at TEVATRON and LHC, and the results is shown in TABLE IV. It can be found that without $P_{\text{T cut}}$, the integrated cross section at LHC is about one order higher than that at TEVATRON, and the value of $R$ decreases greatly with the increment of $P_{\text{T cut}}$, at about $P_{\text{T cut}} \approx 45$ GeV, $R \sim 1$.

Next, as an explicit example to show how the different cuts affect the production, we study
TABLE V: Values of $R_{PTcut}$ for the hadronic production of pseudo-scalar $B_s$ meson in TEVATRON and LHC.

| $pT_{cut}$ | $y_{cut}$ | 0.0 GeV | 5 GeV | 20 GeV | 35 GeV | 50 GeV |
|-----------|-----------|---------|-------|--------|--------|--------|
| 0.0 GeV   | 1.0       | 1.5     | 2.0   | 1.0    | 1.5    | 2.0    |
| 5 GeV     | 0.47      | 0.66    | 0.80  | 0.48   | 0.67   | 0.81   |
| 20 GeV    | 0.59      | 0.79    | 0.92  | 0.68   | 0.86   | 0.95   |
| 35 GeV    | 0.69      | 0.92    | 0.99  | 0.69   | 0.92   | 0.99   |
| 50 GeV    | 0.74      | 0.97    | 1.0   | 0.73   | 0.93   | 1.0    |

The distributions of $P_T$ and $y$ for $B_s$. For the present purpose, we take CTEQ6L for PDF, LO running $\alpha_s$ and Type B energy scale to carry out the study. The correlations between $P_T$ and $y$ are interesting, so we plot the $y$-distributions with various $P_T$-cuts over a wide range $P_{Tcut} : 5.0 \sim 35$ GeV in FIG.5. FIG.5 shows that the dependence of the differential distributions on rapidity $y$ with different $P_{Tcut}$ at LHC exhibits a broader profile than that at TEVATRON. The $p_T$-distributions of the production vary with $y_{cut}$ mainly due to the fact that as $p_T$ increases, the dependence of the distribution on $y$ becomes smaller as the value of $y_{cut}$ becomes less important. To analyze the quantitative difference of the differential distributions with regard to $P_{Tcut}$ and $y_{cut}$, we introduce a ratio for the integrated hadronic cross sections, $R_{PTcut} = \left( \frac{\sigma_{y_{cut}}}{\sigma_0} \right)_{P_{Tcut}}$, where $\sigma_{y_{cut}}$ and $\sigma_0$ are the hadronic cross section with and without $y_{cut}$ respectively. The ratio $R_{PTcut}$ varies with $P_{Tcut}$ and $y_{cut}$, and its values are given in TABLE V. TABLE V shows that for a fixed $y_{cut}$, the value of $R_{PTcut}$ becomes larger with increasing $P_{Tcut}$. It is understandable that the differential distributions versus the rapidity $y$ decrease with the increment of $P_T$, so the contributions to the hadronic cross section surviving after the cut, i.e. ($|y| \leq y_{cut}$), increase with the increment of $P_{Tcut}$.

To summarize: We have presented quantitative studies on the uncertainties in estimates of the $B_{s}^{(*)}$ meson hadronic production within the FFN scheme. The investigated quantitatively uncertainties involve the PDF, the values of $m_b$ and $m_s$, and the characteristic energy scale $Q^2$ of the process and etc.. It is found that when $m_s$ increases by steps of 0.1 GeV, the integrated cross section of $B_{s}^{(*)}$ decreases by about 80%–100%, when $m_b$ increases by steps of 0.1 GeV, it is about 10%. While the uncertainties caused by the parton distribution function and the factorization scale varies within the region of $1/5$ to $1/3$. We have also shown the
differences between LHC and TEVATRON for various observable with reasonable kinematic
cuts, such as the cuts on the $B_{s}^{(*)}$ meson transverse momentum $P_{Tcut}$ and rapidity $y_{cut}$.
Our results show that the experimental studies of the $B_{s}^{(*)}$ meson at the two colliders are
complimentary and stimulative. Concerning the prospects of $B_{s}$ production at Fermilab
TEVATRON and at CERN LHC, the obtained results may be as useful references for these
experiments. Since LHC has much higher luminosity and higher collision energy than that of
TEVATRON, it seems that the particularly interesting topics on $B_{s}$ may be more accessible
and fruitful at LHC than that at TEVATRON. Further more, it is reasonable to assume
that, similar to the hadronic production of $J/\Psi$, $B_{c}$ and $\Xi_{cc}$, the ‘heavy quark
mechanisms’ via the sub-processes $g + s \rightarrow B_{s}^{(*)} + ...$ and $g + \bar{b} \rightarrow B_{s}^{(*)} + ...$ may be as
important as the gluon-gluon fusion mechanism, which should be treated on the equal footing
in comparison to that of the gluon-gluon fusion mechanism. However to be consistant
theoretically and to deal with the possible double counting from all these mechanisms,
one should work in the general-mass variable-flavor-number (GM-VFN) scheme [28, 29, 30] in stead of the FFN scheme. A detailed discussion on the GM-VFN scheme, and a
comparison of $B_{s}^{(*)}$ production within these two schemes is in preparation and shall be
presented elsewhere.

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