Interlayer Quasiparticle Transport in the Vortex State of Josephson Coupled Superconductors

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We calculate the dependence of the interlayer quasiparticle conductivity, \(\sigma_c\), in a Josephson coupled \(d\)-wave superconductor on the magnetic field \(B \parallel c\) and the temperature \(T\). We consider a clean superconductor with resonant impurity scattering and a dominant coherent interlayer tunneling. When pancake vortices in adjacent layers are weakly correlated at low \(T\) the conductivity increases sharply with \(B\) before reaching an extended region of slow linear growth, while at high \(T\) it initially decreases and then reaches the same linear regime. For correlated pancakes \(\sigma_c\) increases much more strongly with the applied field.

Experimental study of quasiparticle properties in high-temperature superconductors so far has focussed almost exclusively on thermodynamic and in-plane transport properties. Two remarkable theoretical results connect these properties with the symmetry of the superconducting gap: a) a massless Dirac spectrum of nodal quasiparticles in a \(d\)-wave superconductor leads to a finite density of states (DOS) at the Fermi level in presence of impurity scattering [1] and to a universal (impurity independent) low-temperature limit of the in-plane thermal conductivity, \(\kappa_{ab} \equiv \lim_{T \to 0} \frac{\kappa_{ab}(0,T)}{T}\), in the absence of a magnetic field. [2,3]; b) in the vortex state the DOS is locally enhanced due to the effect of the supercurrents around the vortex cores on the near-nodal quasiparticles (Volovik effect) [4]. Consequently, the effect of the magnetic field on the electrical and the thermal conductivities is twofold: the net enhancement of the DOS tends to increase the conductivity, while local changes in the phase and amplitude of the order parameter lead to its decrease. [5]

The enhancement of the DOS with the magnetic field, \(B\), has been invoked to explain the increase of the specific heat with \(B\) observed in \(\text{YBa}_2\text{Cu}_3\text{O}_7\text{−}\delta\) (YBCO). [6] The in-plane thermal conductivity, \(\kappa_{ab}\), has been found to have a highly non-trivial dependence on the applied field and the temperature in both YBCO and \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}\) (Bi-2212) compounds. [7] At low temperatures \(\kappa_{ab}\) increases with \(B\), while at \(T \geq 5 - 10\) K it decreases and then often saturates in fields above a few Tesla. In some zero-field cooled Bi-2212 samples a hysteretic field dependence with two plateaus at different values of \(\kappa_{ab}(B)\) has been observed. [8] This indicates that the behavior of \(\kappa_{ab}(B)\) is sensitive to the vortex arrangement in the sample.

Franz [9] has appealed to the effects of disorder in the vortex positions to explain the observed high-field plateau in \(\kappa_{ab}(B)\). He has argued that randomly positioned vortices act similarly to impurities leading to the enhancement of both the DOS and the electron scattering. These two effects compensate each other at high fields (in analogy to the zero-field result being insensitive to impurity concentration) leading to the universal \(\kappa_{ab}(B,T)/T \to \kappa_0\). However, to obtain this result Franz has made questionable approximations: he has replaced the spatial average of the product of two Green’s functions by the product of the averages, and has assumed a Lorentzian shape of the distribution function of the in-plane supervelocity, which differs from a realistic distribution in the vortex state, see below. Clearly, this approach cannot explain the different (non-universal) plateau values. [10]

Recently it has been demonstrated [11] that the interlayer quasiparticle electrical transport can be studied directly in the resistive state of small area samples (mesas) fabricated from the Josephson coupled superconductors such as Bi-2212. In the energy and temperature range below \(\approx 3\) meV, the experimental data are well understood in the framework of a Fermi-liquid model for the near-nodal quasiparticles in a \(d\)-wave superconductor assuming (i) clean limit, (ii) resonant (unitarity) impurity scattering, and (iii) dominant contribution to the \(c\)-axis conductivity from coherent (conserving the in-plane momentum) interlayer tunneling. [12] The authors of Ref. [10] introduced the universal quasiparticle \(c\)-axis conductivity \(\sigma_q(T = 0, B = 0)\). This quantity is insensitive to the impurity vertex corrections which modify the in-plane conductivity as discussed by Durst and Lee. [12]

So far the experimental data on the magnetic field dependence of interlayer quasiparticle conductivity have been quite scarce. Measurements of \(\sigma_q(B)\) in high fields, where \(\sigma_c \approx \sigma_q\), reveal a linear increase of \(\sigma_c\) with \(B\) on the scale of \(40\) T. [12] For mesas Yurgens et al. [10] have reported a weak field dependence for \(B \leq 7\) T without quantitative results for its functional form.

In this Letter we address the effect of vortices on the \(c\)-axis quasiparticle conductivity, \(\sigma_q(B,T)\), as a function
of the magnetic field $\mathbf{B} \parallel c$, in Josephson coupled superconductors in the framework of the approach used in Ref. [11]. We find that at low (high) temperatures for c-axis uncorrelated vortices $\sigma_y(B, T)$ increases (decreases) with $B$, before reaching an extended region of slow linear-in-$B$ increase. In this high field regime the increase in the DOS is largely compensated by the enhancement of the electron scattering at tunneling, due to disorder in the positions of vortices in neighboring layers.

To calculate the effect of vortices on the interlayer transport in the framework of the d-wave model we employ a semiclassical approach. [4] The supercurrents around vortex cores lead to a Doppler shift, $\epsilon_n(\mathbf{k}, \mathbf{r}) = \mathbf{k} \cdot \mathbf{v}_n(\mathbf{r})$, in the quasiparticle spectrum, $\varepsilon(\mathbf{k}, \mathbf{v}_s) = E_k + \epsilon_n$. Here $\mathbf{k}$ is the quasiparticle momentum and $\mathbf{v}_n(\mathbf{r})$ is the supervelocity at point $\mathbf{r}$ inside layer $n$, $E_k \approx (\xi^2_k + \Delta^2(k))/1/2$ is the quasiparticle energy when $B = 0$, $\xi_k = vF \cdot (\mathbf{k} - \mathbf{k}_F)$ and $\Delta(k) = \Delta_0(k^2_x - k^2_y)/\xi^2_k$. As c-axis currents are parallel to the field (in contrast to the in-plane currents), [4] the net interlayer transport is determined by the spatial average of local transport coefficients. In the following we neglect correlations between the positions of impurities and those of vortices, and average over the distribution of each independently, see below. We also neglect the temperature dependence of $\Delta_0$ and small effects due to Zeeman splitting. Then the interlayer quasiparticle conductivity is given by [11]

$$\frac{\sigma_y(B, T)}{\sigma_y(0, 0)} = \frac{\Delta_0}{8TN(0)} \int_{-\infty}^{+\infty} \frac{d\omega}{\cosh^2(\omega/2T)} \int d\mathbf{k} \times (t^2(\mathbf{k})/t^2_0) \langle A(\mathbf{k}, \omega + \epsilon_n)A(\mathbf{k}, \omega + \epsilon_{n+1}) \rangle,$$

where $\sigma_y(0, 0) = 2e^2T_0N(0)\hbar^2/\pi\hbar\Delta_0$ is the universal c-axis conductivity, $N(0)$ is the 2D DOS, $t(\mathbf{k})$ is the interlayer transfer integral, $t_0 = t(k_g)$, $k_g$ are the positions of the gap nodes on the Fermi surface, and $0 < \eta < 1$ is the weight for the coherent tunneling. Further, $\langle \ldots \rangle$ denotes the spatial average, and

$$A(\mathbf{k}, \omega) \approx \frac{1 + \xi_k}{E_k} \left( \frac{L(\omega, E_k)}{2} + \frac{1 - \xi_k}{E_k} \frac{L(\omega, -E_k)}{2} \right) / \omega \cosh^2(\omega/2T),$$

is the spectral density averaged over impurities with $L(\omega, E) = \gamma(\omega)/\pi[(E - \Omega(\omega))^2 + \gamma^2(\omega)]$. The functions $\omega = \Omega(\omega)$ and $-\gamma(\omega)$ are the real and the imaginary part of the self-energy respectively. [4] In the unitarity limit the effective scattering rate of quasiparticles is $\gamma(\omega) \approx \gamma_0 - \omega^2/\gamma_0$ when $\omega \ll \gamma_0$ and $\gamma(\omega) \approx \pi\gamma_0^2/2|\omega|$ when $\omega \gg \gamma_0$, where $\gamma_0 \approx (h\nu_0\Delta_0)/1/2$ and $\nu_0$ is the bare scattering rate. The renormalized frequency is $\Omega(\omega) \approx \omega/2$ when $\omega \ll \gamma_0$ and $\Omega(\omega) \approx \omega$ if $\omega \gg \gamma_0$.

At low temperatures $T \ll \Delta_0$ the quasiparticle current comes mainly from the regions near the gap nodes. In the vicinity of a node we can linearize the quasiparticle spectrum and obtain from Eq. (1)

$$\frac{\sigma_y(B, T)}{\sigma_y(0, 0)} = \int_{-\infty}^{+\infty} d\omega \int_{0}^{3\pi^2E/8T\cosh^2(\omega/2T)} \times \langle L(\omega + \epsilon_{n+1}, E)|L(\omega + \epsilon_n, E) + (1/3)L(\omega + \epsilon_n, -E)\rangle,$$

where $\epsilon_n(\mathbf{r}) = \mathbf{k}_g \cdot \mathbf{v}_n(\mathbf{r})$ and we have set $t(\mathbf{k}) \approx t_0$. The characteristic energy scale for the Doppler shift, $\epsilon_n$, is $\epsilon_B = h\nu_F/\pi a$, where $a = (\Phi_0/B)^{1/2}$ is typical intervortex distance. The semiclassical approach is valid for $\epsilon_B \ll \Delta_0$, i.e. for fields $B \ll B_D$, where $B_D = \Phi_0\Delta_0^2/\hbar^2v_F^2$. This approach also does not account correctly for the quasiparticles in the regions near the vortex cores, which leads to corrections of the order of $\Delta/\Delta_0 \approx \epsilon_B^2/\Delta_0^2$ when $\Delta \approx \eta_0\gamma_0^2/\hbar^2v_F^2$ separates the regimes of impurity dominated and field dominated behavior. Using the parameters $\Delta_0 \approx 25$ meV, $\gamma_0 \approx 2 - 3$ meV [11], and $v_F \approx 1.5 - 2.5 \times 10^7$ cm/s we obtain $\gamma_0 \approx 0.2 - 0.6$ T and $B_D \approx 40 - 80$ T.

We now rewrite the spatial average in Eq. (1) as the average over the probability distribution of the Doppler shift, $\epsilon$, which is fully determined by the vortex arrangement. In Bi-2212 crystals the 3D vortex lattice is destroyed by pinning at the “second peak field” $B \approx 0.02$-0.05 T (see, e.g., [4]), and at higher fields pancakes in neighboring layers are only weakly correlated. Consequently, we consider the limits of c-axis-correlated and uncorrelated pancakes. In both cases the average depends only on the probability distribution function of $\epsilon$ in a single layer, $\mathcal{P}(\epsilon)$, which is related to the distribution $P(p_x)$ of a component of supermomentum $p_x = 2m\nu_s$, where $m$ is the effective mass, by $\mathcal{P}(\epsilon) = (2/v_F)P(p_x = 2\epsilon/v_F)$. The function $P(p_x)$ is determined by the pancake configuration $\mathcal{R}_i = \{X_i, Y_i\}$, and, when $a$ is smaller than the London penetration depth $\lambda_{ab}$, is given by $P(p_x) = \langle \delta(p_x - \hbar \sum_i Y_i/R^2_i) \rangle_{\mathcal{R}_i}$. For an isolated vortex $p \propto \hbar/R$ where $\xi_{ab} \ll R \ll a$ is the distance from the pancake center, and $\xi_{ab}$ is the coherence length. Consequently, $\mathcal{P}(\epsilon)$ has a universal tail $\mathcal{P}(\epsilon) \approx \pi\epsilon^2/(8\epsilon^3)$ at $\epsilon B \ll \epsilon \ll \Delta_0$, while its behavior at $\epsilon \ll \epsilon_B$ depends on the actual positions of vortices. However, in absence of correlation between the positions of vortices and impurities $\mathcal{P}(\epsilon)$ involves a single energy scale $\epsilon_B$ for any $\epsilon$, and depends on $\epsilon$ only via $\epsilon/B$, i.e. $\mathcal{P}(\epsilon)d\epsilon = P_B(\epsilon)/\epsilon\epsilon_B$. In the clean limit we can extend the asymptotic behavior $\mathcal{P}(\epsilon) \propto 1/\epsilon^3$ to infinity when the integral over $\epsilon$ in Eq. (1) converges. Therefore, importantly, up to terms of the order of $\gamma_0/\Delta_0$, the conductivity depends only on the dimensionless parameters $\epsilon_B/\gamma_0 = \sqrt{B/B_D}$ and $T/\gamma_0$.

For c-axis-correlated vortices $\epsilon_n \approx \epsilon_{n+1}$, and the average in Eq. (1) means $\mathcal{F} = \langle \mathcal{F}(\epsilon, \epsilon_{n+1}) \rangle = \int \mathcal{D}\mathcal{P}(\epsilon)\mathcal{F}(\epsilon, \epsilon)$. At low temperatures, $T \ll \gamma_0$, both in weak $\mathcal{B} < B_D$ and in strong $B \gg B_D$ fields the leading field-dependent part of the conductivity varies as $(\epsilon/\gamma_0)^2$. We therefore separate $\sigma_y$ into the contributions from the regions $\epsilon < \gamma_0$ and $\epsilon > \gamma_0$, and use the asymptotic behavior of $\Omega(\omega)$ and $\gamma(\omega)$ in each case to estimate
\[
\sigma_q(B,0) - \sigma_q(0,0) \approx \frac{1}{6} \langle e^2 \rangle_{\epsilon < \gamma_0} + \frac{3\pi^2}{8} \frac{\langle e^2 \rangle_{\epsilon > \gamma_0}}{\gamma_0^2} . \tag{3}
\]

As the average is dominated by the tail of \( P(\epsilon) \) we can evaluate \( \sigma_q \) in weak and strong fields with logarithmic accuracy. Moreover, we interweave between the two limits to obtain also a qualitative description at \( B \sim B_\gamma \),

\[
\frac{\sigma_q(B,T)}{\sigma_q(0,0)} \approx 1 + \pi^2 T^2/18\gamma_0^2 + \frac{3\pi^2 B}{64B_\gamma} \ln \frac{B_\Delta}{(B^2 + B_\Delta^2)^{1/2}}. \tag{4}
\]

Therefore at low \( T \) for the 3D-ordered case \( \sigma_q \) increases quasi-linearly with \( B \) in the entire field range up to \( B_\Delta \).

For the \( c \)-axis uncorrelated vortices the average in Eq. (2) has to be taken independently in each layer, \( \langle F(\epsilon_i, \epsilon_{i+1}) \rangle = \int \text{d} \epsilon_1 P(\epsilon_1) \int \text{d} \epsilon_2 P(\epsilon_2) F(\epsilon_1, \epsilon_2) \). Again, for \( T < \gamma_0 \) and \( B \ll B_\gamma \), we expand in \( \epsilon_\alpha/\gamma_0 \), and obtain

\[
\frac{\sigma_q(B,T)}{\sigma_q(0,0)} \approx 1 + \pi^2 T^2/18\gamma_0^2 + \frac{\pi B}{96B_\gamma} \ln \frac{B_\gamma}{B}. \tag{5}
\]

At high fields the variation of \( \epsilon_\alpha \) is of the order of \( \epsilon_B \gg \gamma_0 \) and consequently \( \langle L(\omega + \epsilon_\alpha, E) \rangle \approx P(E - \omega) \). Then for \( B_\gamma \ll B \ll B_\Delta \) the conductivity is given by

\[
\frac{\sigma_q(B,T)}{\sigma_q(0,0)} = C_1 + \frac{B}{B_0}, \quad C_1 = 2\pi^2 \int_0^\infty d\epsilon |P(\epsilon)|^2. \tag{6}
\]

The most important contribution to the linear, in \( B \), term is due to the increase in the tunneling away from the nodes, \( t_\phi(k) \simeq t_0 + t_1 \phi^2 \), where \( \phi \) is the angle between \( k \) and the nodal direction, and \( t_1 \gg t_0 \), [3] which yields \( B_0 \simeq (t_0/t_1)B_\Delta \). Other contributions, due to deviations of the quasiparticle spectrum from the massless Dirac form of the linearized dispersion and due to corrections to the semiclassical approximation in the vicinity of the vortex cores, only enhance \( \sigma_q \) on the scale of \( B_\Delta \gg B_0 \). Very importantly, due to scaling of \( P(\epsilon) \propto P(\epsilon/\epsilon_B)/\epsilon_B \) (see above), \( C_1 \) in Eq. (6) is a constant which depends solely on the shape of the distribution \( P(\epsilon) \), but not on the magnetic field. If \( P_B(x) \) is monotonous \( C_1 \) depends on its asymptotic decay. For the Lorentzian (when vortices act exactly as the impurity scatterers do at \( B = 0 \)) \( P_B(x) \propto x^{-2} \) at \( x \gg 1 \), and \( C_1 = C_1 = 1 \), while for the Gaussian distribution \( C_1 = C_1 = \pi/2 \). In the vortex state \( P_B(x) \propto x^{-3} \) at large \( x \), and we expect \( 1 < C_1 < \pi/2 \). Therefore to find \( \sigma_q \) we need accurate information on pancake arrangement to determine the supervelocity distribution \( P(\epsilon) \).

Below the irreversibility line \( T_{\text{irr}}(B) \) at high fields and in the vortex liquid state pancakes do not possess long range order inside layers due to pinning and thermal fluctuations. To calculate \( \sigma_q(B,T) \) in these regimes we use the distribution function \( P(p_z) \) obtained by numerical simulation of the 2D pancake liquid at different values of the dimensionless temperature \( t = T_{\text{eff}}/2\pi E_0 \), which characterizes pancake disorder inside the layers. Here \( E_0 = \Phi_0^2/16\pi^2 \lambda_{ab}^2 \) is the characteristic energy of pancake interaction. In the liquid state \( T_{\text{eff}} = T \), while in the glass state it is reasonable to take \( T_{\text{eff}} \approx T_{\text{irr}}(B) \). The calculated function \( P(p_z) \) is shown in Fig. 1 for several values of \( t \); the inset shows that the parameter \( C_1 \) grows with \( t \) from 1.08 to 1.22. The full field dependence of \( \sigma_q(B,T)/\sigma_q(0,0) \) at \( T \ll \gamma_0 \) obtained from these distribution functions is shown in Fig. 2. In computing \( \sigma_q \) we have determined \( \gamma(\omega) \) and \( \Omega(\omega) \) self-consistently [14] in the unitarity limit, choosing the scattering rate so that \( \gamma_0 = 0.1\Delta_0 \), typical of Bi-2212 samples.

Due to short range correlations inherent to strongly interacting vortices at \( B > \Phi_0/\lambda_{ab}^2 \) the calculated function \( P(p_z) \) differs significantly from the Gaussian (obtained in Ref. [17] assuming uncorrelated and random vortex positions). The distribution function depends on a single variable \( \epsilon/\epsilon_B \) only if the positions of vortices and those of impurities are uncorrelated; this is the case for a vortex solid and for the liquid phase in presence of weak pinning. In general, below the irreversibility line impurities act as pinning centers, and the distribution function depends on two variables, \( \epsilon/\epsilon_B \) and \( B/\Phi_0 n_i \), where \( n_i \) is the impurity concentration in a layer. We believe nevertheless that in that regime our approach gives at least the correct qualitative behavior.

At \( T \gg \gamma_0 \), we also consider the intermediate fields, \( B_\gamma \ll B \ll B_\Delta \), and we expect an increase of the interlayer conductivity at low and high temperatures is quite different from the 3D-ordered case.

\[
\frac{\sigma_q(B,T)}{\sigma_q(0,0)} \approx C_2 \frac{\sqrt{B_T}}{B}, \quad C_2 = 3 \pi^2 \ln 2 \int_0^\infty dx P_B^2(x). \tag{7}
\]

At \( B \gg B_T \) Eq. (6) holds. Therefore, for \( c \)-axis uncorrelated pancakes at \( T \ll \gamma_0 \), \( \sigma_q(B,T) \) increases quasi-linearly with \( B \) with the slope \( 1/B_\gamma \), where \( B \ll B_\gamma \), and with a smaller slope \( \sim 1/B_0 \), for \( B_\gamma \ll B \ll B_\Delta \). At higher temperatures, \( T \gg \gamma_0 \), \( \sigma_q(B,T) \) decreases with field for \( B \ll B_T \), before crossing over to the slow linear growth with the slope \( \sim 1/B_0 \), see Fig. 3. Here the conductivity has been computed with a temperature-independent \( \Delta_0 \). In fact \( \Delta_0 \) is reduced with increasing \( T \), and the plateau values increase. Note that linear interpolation of \( \sigma_q(B,T) \) from high fields does not yield \( \sigma_q(0,T) \).

To conclude, we find that: a) the field dependence of the quasiparticle interlayer conductivity is sensitive to the structure of vortex state; b) for the \( c \)-axis uncorrelated vortices the field dependence of the interlayer conductivity at low and high temperatures is quite different at low fields but becomes similar in high fields \( B \gg \max(B_T, B_\gamma) \).

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FIG. 1. The distribution functions $P(p_x)$ of the $x$-component of superstrequency. Here $p_x$ is measured in units $h/a_0$, where $a_0 = (2\Phi_0/\sqrt{3}B)^{1/2}$ is the lattice constant of a triangular vortex lattice. $P(p_x)$ has been calculated using Langevin dynamics simulations of a 2D liquid at different reduced temperatures $t = T/2\pi E_0$. The corresponding absolute temperatures obtained assuming $\lambda_{ab} = 200nm(1 - T^2/T_c^2)^{-1/2}$ are given in brackets. The distribution function of the Doppler shifts [$P(\epsilon)$ is related to $P(p_x)$ by $P(\epsilon) = (2\omega_0/hv_F)P(p_x = 2\omega_0/hv_F)$. Inset: temperature dependence of the parameter $C_1$ in Eq. (6).

FIG. 2. Magnetic field dependence of the normalized quasiparticle conductivity $\sigma(B,0)/\sigma(0,0)$ at low temperatures $T \ll \gamma_0$ for $c$-axis uncorrelated vortex state calculated using functions $P(p_x)$ shown Fig. 1. Here we do not show the weak linear field dependence at $B/B_s \gg 1$, see Eq. (6).

FIG. 3. Field dependence of the quasiparticle conductivity at temperatures $T \geq 0.5\gamma_0$, computed for the pancake liquid state with $P(p_x)$ obtained by simulations (Fig. 1) and $\gamma_0 = 0.16E_0 \approx 30$ K.