Perturbation Theory Remixed: Improved Nonlinearity Modeling beyond Standard Perturbation Theory

Zhenyuan Wang (王震远), 1 Donghui Jeong, 1, 2 Atsushi Taruya, 3, 4 Takahiro Nishimichi, 3, 4 and Ken Osato5, 3

1 Department of Astronomy and Astrophysics and Institute for Gravitational and the Cosmos, The Pennsylvania State University, University Park, PA 16802, USA
2 School of Physics, Korea Institute for Advanced Study, Seoul, South Korea
3 Center for Gravitational Physics and Quantum Information, Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
4 Kavli Institute for the Physics and Mathematics of the Universe, Todai Institutes for Advanced Study, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan
5 Center for Frontier Science, Chiba University, Chiba 263-8522, Japan

(Dated: September 2, 2022)

We present a novel nEPT (n-th-order Eulerian Perturbation Theory) scheme to model the non-linear density field by the summation up to n-th-order density fields in perturbation theory. The obtained analytical power spectrum shows excellent agreement with the results from all 20 DarkQuest suites of N-body simulations spreading over a broad range of cosmologies. The agreement is much better than the conventional two-loop Standard Perturbation Theory and would reach out to \( k_{\text{max}} \approx 0.4 \, h/\text{Mpc} \) at \( z = 3 \) for the best-fitting Planck cosmology, without any free parameters. The method can accelerate the forward modeling of the non-linear cosmological density field, an indispensable probe of cosmic mysteries such as inflation, dark energy, and dark matter.

Introduction — The observations of the Universe’s large-scale structure (LSS) traced by Cosmic Microwave Background (CMB) radiation [1, 2], the distribution of galaxies [3–7], and shape distortion of galaxies [8, 9] have led to the concordance ΛCDM cosmology [10] with most parameters measured to a sub-percent accuracy.

Parallel to the data collection programs has been the theoretical development based upon which we interpret the observation. Starting from the 40s [11], relativistic theory for the evolution of density perturbations in the Friedmann–Lemaître–Robertson–Walker Universe has been developed and used to interpret the LSS data. In particular, its linearized version [12–14] has been so successful in explaining the power spectrum of CMB temperature anisotropies and polarizations to all scales ob-

served by WMAP [15] and Planck [10] satellites. The concordance ΛCDM cosmology model would not be possible without such an accurate linear-theory model.

The remaining big questions in cosmology are to uncover the nature of building blocks of the ΛCDM cosmology, such as inflation, dark energy, and dark matter. To address these questions, modern galaxy surveys are mapping the distribution and shape distortion of galaxies with unprecedented depth and volume [16–22].

These observational developments call for a novel theoretical model beyond the linear theory that is only applicable on large scales where the accuracy of the usual LSS observation is limited by cosmic variance. Using a feature such as Baryon Acoustic Oscillation that is insensitive to the nonlinearities has proven successful for measuring the geometry of the Universe [23–25]. Upon modeling nonlinearities, however, using the full power-spectrum shape can improve the measurement accuracy by a factor of few [26, 27]. In addition, the full-shape analysis enables the measurement of the growth rate of the LSS [28] and features in the galaxy clustering carved by massive neutrinos [29] and primordial physics [30].

A diversity of modeling methods have been developed ranging from simulation-based methods such as emulator [31–34], fast simulations [35–38], and machine learning [39, 40] to analytical methods such as Standard Perturbation Theory (SPT) [41, 42], Lagrangian Perturbation Theory (LPT) [41, 43, 44], Effective-Field Theory of Large Scale Structure (EFTofLSS) [45–47], and various Renormalized Perturbation Theory (RPT) [48–59], including RegPT [60, 61].

Traditionally, the analytical methods focus on obtaining the expressions for the ensemble mean of the summary statistics such as the power spectrum and bispectrum, or n-point correlation functions. Such expressions usually involve high-dimensional integrals whose complexity increases quickly for the higher-order loop calculations. While Refs. [62–66] have developed fast methods for computing nonlinear power spectrum and bispectrum by using FFTlog algorithm [67] or response function expansion [60, 68], the analytical computation beyond the two-loop proves challenging [69].

The PT-based analytical methods can also be used to model the cosmic density field at the field level [70–76]. Instead of computing the ensemble mean of the summary statistics, the field-level computation provides nonlinear density fields from a given realization of the stochastic linear field. In this method, the computation of higher-order summary statistics is much easier than the analytical methods because we can simply take the average over the multiple realizations. For example, Ref. [73] shows that the field-level modeling provides a fast way to compute the summary statistics and their covariance ma-
traces incorporating survey window function due to non-trivial geometry and varying depth. In addition, Ref. [76] presents the two-loop power spectrum and one-loop bispectrum of matter in redshift-space with this grid-based method. The possibility of field-level inference bypassing the summary statistics [77] further strengthens the motivation for the field-based method.

In this letter, we present a novel nEPT (nth-order Eulerian Perturbation Theory) scheme for modeling the nonlinear density field. For the field-level SPT calculation, we use the GridSPT [71] that, unlike LPT, directly generates the density and velocity fields on grids without using particles. While using the recursion relations of the SPT to compute nonlinear fields at each order, nEPT differs from the other PT methods in computing the summary statistics: namely, nEPT first adds all nonlinear contributions to the density field up to the nth order, then compute the summary statistics. By contrast, the SPT computes the summary statistics by collecting the contributions at the fixed order in the linear density contrast $\delta_L$.

In what follows, we show that nEPT models the nonlinear LSS with stunning accuracy, much better than the current state-of-the-art two-loop PT predictions.

GridSPT and nEPT — For a given realization of the linear density field on regular grid points, the GridSPT [71] provides a way to compute the matter density field $\delta$ and the velocity field $v$ of LSS perturbatively by solving the fluid equations:

\begin{equation}
\dot{\delta} + \nabla \cdot [(1 + \delta)v] = 0, \tag{1}
\end{equation}

\begin{equation}
\dot{v} + (v \cdot \nabla)v + \frac{\dot{a}}{a} v = - \nabla \phi, \tag{2}
\end{equation}

along with the Poisson equation:

\begin{equation}
\nabla^2 \phi = 4\pi G \bar{\rho}_m a^2 \delta. \tag{3}
\end{equation}

Here, $\dot{}$ represents the conformal-time derivative, $d\tau = dt/a$ with $a(t)$ being the scale factor and $t$ being the cosmic time, $\nabla$ is comoving-coordinate derivative, $\bar{\rho}_m$ is the mean matter density, and $\phi$ is the peculiar gravitational potential. The set of equations describes the non-relativistic-matter (cold-dark matter and baryon) fluid on scales larger than the baryonic Jeans scale. Following the standard practice of SPT, we assume irrotational velocity and expand the density field and the normalized velocity-divergence field $\theta \equiv - (\nabla \cdot v) / (n H f)$ as

\begin{equation}
\delta(\tau, \mathbf{x}) = \sum_n \left[ D(\tau) \right]^n \delta^{(n)}(\mathbf{x}), \tag{4}
\end{equation}

\begin{equation}
\theta(\tau, \mathbf{x}) = \sum_n \left[ D(\tau) \right]^n \theta^{(n)}(\mathbf{x}). \tag{5}
\end{equation}

Making use of the fast Fourier transform, the GridSPT enables us to quickly generate the nth order quantities $\delta^{(n)}$ and $\theta^{(n)}$ at each grid point following the configuration-space SPT recursion relation [71]. Here, $D$ denotes the linear growth factor and $f \equiv d \ln D/d \ln a$.

The crucial difference between nEPT and the usual PT is that in nEPT, we first compute the nonlinear density field in Eq. (4) up to a fixed order $n$, then estimate the summary statistics, such as power spectrum and bispectrum, directly from $\delta$. For example, for $n = 5$, the power spectrum from 5EPT reads

\begin{equation}
P_{5EPT} = D^2 P_{11} + 2D^3 P_{12} + D^4 \left( 2P_{13} + 2P_{22} \right)
+ D^5 \left( 2P_{14} + 2P_{23} \right) + D^6 \left( 2P_{15} + 2P_{24} + P_{33} \right)
+ D^7 \left( 2P_{25} + 2P_{34} + 2D^5 P_{44} \right)
+ 2D^9 P_{45} + D^{10} P_{55}, \tag{6}
\end{equation}

which clearly differs from the nonlinear power spectrum in the usual PT:

\begin{equation}
P_{PT}^{(2-loop)} = D^2 P_{11} + D^4 \left( 2P_{13} + P_{22} \right)
+ D^6 \left( 2P_{15} + 2P_{24} + P_{33} \right). \tag{7}
\end{equation}

Here, we use the shorthand notation of

\begin{equation}
\langle \delta^{(n)}(k) \delta^{(m)}(k') \rangle \equiv (2\pi)^3 P_{nm}(k) \delta^D(k + k'), \tag{8}
\end{equation}

and suppress the $\tau$ and $k$ dependencies to avoid the clutter. The first (second) bracket in Eq. (7) is called one-loop (two-loop) contribution in SPT.

N-body simulations — We test the performance of the nEPT modeling of the nonlinear power spectrum by comparing the nEPT prediction in Eq. (6) against a series of N-body simulations. First, we use the baseline N-body simulation in Ref. [71]: $1024^3$ particles in $L_{box} = 1 \text{ Gpc}/h$ box with flat-$\Lambda$CDM cosmology ($\Omega_m = 0.279, h = 0.701, n_s = 0.96, \sigma_8 = 0.8159$) consistent with WMAP 5-year results [78]. Then, we use the N-body simulation results from the Dark Quest project [79] aiming to model the cosmological dependence of halo and matter statistics in the six-parameter $\omega$CDM cosmologies. The 20 simulations are for the 20 test cosmologies arranged uniformly over the six-dimensional hyperrectangle based on a maximin distance Latin hypercube design. In particular, we use the high-resolution suite with $2048^3$ mass elements in $(1 \text{ Gpc}/h)^3$ periodic comoving boxes. For both cases, we measure the matter power spectrum employing $1024^3$ grid points for FFT, with the aliasing artifact and the cloud-in-cells mass assignment kernel corrected in Fourier space [80, 81]. The measurement error is much less than 1% up to the Nyquist frequency of $k = 3.2 h/\text{Mpc}$.

$P(k)$ comparison: nEPT vs. N-body — To make a face-to-face comparison with the N-body results, we calculate the GridSPT nonlinear density field using the same initial linear density field that generates the initial condition for corresponding N-body simulations.

When computing the Fourier-space quantities using real-space recursion relations, one must apply the cut-off to reduce the spurious impact from the small-scale
FIG. 1. The ratios of the model power spectra to the \( N \)-body results for the baseline WMAP-5yr cosmology at redshifts \( z = 0, 0.5, 1, 2, 3, \) and 5. The thin dashed lines are the one-loop (green), and two-loop (blue) power spectra from SPT calculations. The thick solid lines are the result from \( n \)EPT calculations: 2EPT (green), 3EPT (blue), 4EPT (magenta) and 5EPT (cyan). Both SPT and \( n \)EPT results are measured from the density field in GridSPT using the same initial linear density field generating the initial condition of the \( N \)-body simulation. The two thin solid lines are the two-loop results of RegPT+ (brown) and IR-resummed EFT (olive) using the smooth (theory) linear power spectrum. The yellow and lavender bands indicate the \( \pm 1\% \) and \( \pm 2\% \) regions. We truncated RegPT+ and IR-resummed EFT beyond the \( k_{\text{max}} \) that gives rise to the minimum reduced \( \chi^2 \).

(UV) modes. For the baseline calculation, we use the cut-off wavenumber \( k_{\text{cut}}^{\text{UV}} = 256k_F = 1.61 \ h/\text{Mpc} \), but we shall also present the results with different \( k_{\text{cut}} \) later. Here, \( k_F = 2\pi/L_{\text{box}} \) is the fundamental wavenumber. To avoid the aliasing effect, we adopt the generalized Orszag rule \([74, 76]\) to zero-pad \( k > 2/(n + 1)k_{\text{Nyquist}} \) in linear density field for computing the \( n \)-th order field, where \( k_{\text{Nyquist}} = \pi N_{\text{grid}}/L_{\text{box}} \) is the Nyquist wavenumber with the one-dimensional grid size \( N_{\text{grid}} \). Requiring that \( k_{\text{cut}}^{\text{UV}} < 2/(n + 1)k_{\text{Nyquist}} \) sets the minimum \( N_{\text{grid}} \) that we use for the GridSPT calculation. For the baseline computation, we use \( N_{\text{grid}} = 1536 \).

We have computed up to fifth-order GridSPT density fields that are sufficient for calculating SPT power spectrum to two-loop level and 5EPT by using, respectively, Eq. (6) and Eq. (7). Fig. 1 shows the ratios of various model nonlinear power spectra to the baseline \( N \)-body power spectrum at following six redshifts: \( z = 0, 0.5, 1, 2, 3, \) and 5. Models plotted here are SPT (dashed lines), \( n \)EPT (thick solid lines), RegPT+ (thin brown line [61]), and IR-resummed EFT (thin olive line [61]). To facilitate the comparison, we highlight the one- and two-percentage ranges by yellow and lavender bands at the center and extend the three high-redshift (right) panels to \( k = 0.8 \ h/\text{Mpc} \).

First, we note that the agreement between \( n \)EPT and \( N \)-body improves significantly as \( n \) increases for \( z \gtrsim 0.5 \), and the 5EPT (the cyan lines) agrees with \( N \)-body results better than one percent to larger wavenumber than two-loop SPT \( P(k, z) \) for \( z \gtrsim 1 \). Such accuracy of 5EPT can only be matched with two-loop results of the RegPT+ and IR-resummed EFT that employ, respectively, one and three free parameters. Here, we show the RegPT+ and IR-resummed EFT to the maximum wavenumber minimizing the reduced \( \chi^2 \) assuming the diagonal covariance matrix with \( \sigma[P(k)] = P(k)/\sqrt{N_k} \) \([82]\). Note that for RegPT+ with \( z \leq 3 \), we find \( k_{\text{max}} = 0.25 \ h/\text{Mpc} \). It is worth reminding the readers that \( n \)EPT requires no free parameters.

The \( n \)EPT and SPT results are also much smoother
than the RegPT+ and IR-resummed EFT results. This is because the latter two models are the ensemble averages while nEPT and SPT are computed with the input linear density field of the N-body. In addition, we have added the odd-order terms (with odd power of $D$ in Eq. (6)) to one-loop and two-loop power spectra. Although much smaller than the even-power terms, these odd-power terms indeed make the power spectrum from the same realization closer to the N-body result [71, 83], especially for the large-scale Fourier modes [76].

We note the characteristic up-turn feature in the nEPT results that is absent in SPT [42]; namely, nEPT does not suffer from the poor convergence in SPT whose residual shows alternating-series-like behavior, which motivates the development of Renormalized PT [48]. Instead, nEPT enjoys well-regulated high-$k$ behavior from the fact that contributions coming from higher order are stiffer by a factor of $k^2$, as we show in Fig. 2.

Finally, one noteworthy feature in Fig. 1 is that no wiggling feature appears in the ratio between nEPT and N-body around the Baryon Acoustic Oscillation (BAO) scales, which means that the damping of BAO has been accurately captured by nEPT, and the IR-resummation [84, 85] might not be necessary for nEPT.

We confirm that the same conclusion also holds for cosmological models different from the WMAP-5yr cosmology by comparing the nEPT results to the outcome from 20 Dark Quest simulations, at 21 redshifts from $z = 0$ to 1.48. Furthermore, we find that the $k_{\text{max}}$ maximum wavenumber below which nEPT models the N-body result to one-percent accuracy, depends primarily on the $\sigma_8(z) = \sigma_8 D(z)$ value at the redshift. The top panel of Fig. 3 shows four representative results with different $\sigma_8(z)$, and the left panel of Fig. 4 shows the $k_{\text{max}}$ measured from nEPT as a function of $\sigma_8(z)$.

Here, we test the effect of the UV cutoff by calculating the nEPT power spectra with two other UV cut-offs, $(k_{\text{cut},1}^{UV}, k_{\text{cut},2}^{UV}) = (200, 340) k_F = (1.26, 2.14) h/$Mpc, and show the result as shaded regions in Fig. 3, and as ranges in Fig. 4. As expected, the higher-order nEPT is much more sensitive to the UV cutoff than the lower-order nEPT. Although going to 5EPT can significantly improve the accuracy of modeling the nonlinearities in matter clustering, for example, 5EPT is accurate up to $k_{\text{max}} = 0.35 (0.40) h$/Mpc at redshift $z = 2 (3)$ in Planck cosmology (dashed vertical lines in Fig. 4), one must be cautious on the UV sensitivity.

The UV-cutoff-dependence of nEPT, however, can be absorbed into the EFT-like counter terms. Motivated by Fig. 2, we have included the EFT correction as

$$\tilde{P}_{n\text{EPT}}(k) = P_{n\text{EPT}}(k) - \sum_{i=1}^{n-1} \alpha_i k^{2i} P_{i1}(k), \quad (9)$$

where $P_{i1}$ is the linear power spectrum of the N-body simulation, and $\{\alpha_i\}$ are free parameters that we fit from the measured power spectrum. As we have done for the RegPT+ and IR-resummed EFT, we find the $k_{\text{max}}$ at which $\tilde{P}_{n\text{EPT}}(k)$ provides the best fit to the N-body results. The shades in the lower panel of Fig. 3 and the ranges in the right panel of Fig. 4 are too narrow to be identified, which indicates that the EFT counterterms in Eq. (9) absorb the UV sensitivity in nEPT. Furthermore, as shown in the right panel of Fig. 4, the EFT correction improves the $k_{\text{max}}$ of all nEPT power spectra significantly, especially at low $\sigma_8(z)$. For instance, with EFT correction, 5EPT can work accurately up to $k_{\text{max}} = 0.6 h$/Mpc at $z = 2$.

Conclusion — In this letter, we present a novel nEPT resummation scheme and show that the nEPT outperforms one-loop and two-loop SPT as well as two-loop results of the RegPT+ and the IR-resummed EFT without employing any free parameters. The resummation scheme also offers well-regulated convergence behavior at each successive $n$, bypassing the pathological behavior shown in SPT.

To be a successful theory for modeling observed galaxy clustering, the nEPT still needs to incorporate the galaxy bias and the redshift-space distortion, but we anticipate that nEPT must still thrive, at the very least, by following the prescriptions in SPT and EFTofLSS. However, taking advantage of having both density and velocity at each grid point, one can directly implement the non-linear redshift-space distortion mapping to improve the modeling accuracy further [76]. Upon the addition of galaxy bias and redshift-space distortion, the field-level modeling with nEPT will be a powerful data-analysis tool for future high-redshift galaxy surveys.

Finally, while we have only demonstrated the accuracy of the nEPT scheme, a more in-depth theoretical study of the underlying reason for such behavior is desired.
FIG. 3. (Top) The ratios of the real-space power spectra from nEPT (solid lines) and SPT (dashed lines) to the N-body results in four representative Dark Quest cosmologies. The colors are the same as that in Fig. 1. The shades show the range of nEPT power spectrum with different $k_{\text{UV}}$ between 1.26$h$/Mpc and 2.14$h$/Mpc. (Lower) The same as the Top panel but for the nEPT power spectrum with EFT correction in Eq. (9).

FIG. 4. (Left) The anti-correlation between the $k_{\text{max}}$, the maximum wavenumber where nEPT matches N-body result to 1% accuracy, of nEPT and $\sigma_8(z)$ for Dark-Quest simulation’s all 20 cosmologies at 21 redshifts from $z = 0$ to $z = 1.48$. The error bars show the range of $k_{\text{max}}$ with varying UV cutoff between (1.26, 2.14)$h$/Mpc. The dashed lines indicate the value of $\sigma_8(z)$ in Planck cosmology at redshifts $z = 0, 0.5, 1, 1.5, 2, 3$. (Right) The same as the left panel but for the nEPT power spectrum with EFT correction in Eq. (9).

DJ is supported by KIAS Individual Grant PG088301 at Korea Institute for Advanced Study. This work was supported in part by MEXT/JSPS KAKENHI Grant Number JP19H00677 (TN), JP20H05861, JP21H01081 (AT and TN), JP21J00011, JP22K14036 (KO), and JP22K03634 (TN). We also acknowledge financial support from Japan Science and Technology Agency (JST) AIP Acceleration Research Grant Number JP20317829 (AT and TN). Numerical computations were carried out at the ROAR supercomputer at Penn State University, Yukawa Institute Computer Facility, and Cray XC50 at Center for Computational Astrophysics, National Astronomical Observatory of Japan.

[1] C. L. Bennett, D. Larson, J. L. Weiland, N. Jarosik, G. Hinshaw, N. Odegard, K. M. Smith, R. S. Hill, B. Gold, M. Halpern, E. Komatsu, M. R. Nolta, L. Page, D. N. Spergel, E. Wollack, J. Dunkley, A. Kogut, M. Limon, S. S. Meyer, G. S. Tucker, and E. L. Wright, Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results, Astrophys. J. Supp. 208, 20 (2013), arXiv:1212.5225 [astro-ph.CO].

[2] N. Aghanim, Y. Akrami, F. Arroja, M. Ashdown, J. Aumont, C. Baccigalupi, M. Ballardini, A. J. Banday, R. Barreiro, N. Bartolo, et al., Astron. Astrophys. 641,
F. Simpson, A. Taylor, S. Thomas, R. Trotta, L. Verde, F. Vernizzii, A. Vollmer, Y. Wang, J. Weller, and T. Zlosnik, Cosmology and fundamental physics with the Euclid satellite, Living Reviews in Relativity 21, 2 (2018), arXiv:1606.00180 [astro-ph.CO].

[21] R. Maartens, F. B. Abdalla, M. Jarvis, and M. G. Santos, Cosmology with the SKA — overview, arXiv e-prints, arXiv:1501.04076 (2015), arXiv:1501.04076 [astro-ph.CO].

[22] O. Doré, J. Bock, M. Ashby, P. Capak, A. Cooray, R. de Putter, T. Eifler, N. Flagey, Y. Gong, S. Habib, K. Heitmann, C. Hirata, W.-S. Jeong, R. Katti, P. Kornogut, E. Krause, D.-H. Lee, D. Masters, P. Manskef, G. Melnick, B. Mennesson, H. Nguyen, K. Öberg, A. Pullen, A. Raccanelli, R. Smith, Y.-S. Song, V. Tolls, S. Unwin, T. Venumadhav, M. Viero, M. Werner, and M. Zemcov, Cosmology with the SPHEREx All-Sky Spectral Survey, arXiv e-prints, arXiv:1412.4872 (2014), arXiv:1412.4872 [astro-ph.CO].

[23] F. Beutler, C. Blake, M. Colless, D. H. Jones, L. Staveley-Smith, L. Campbell, Q. Parker, W. Saunders, and F. Watson, The 6dF Galaxy Survey: baryon acoustic oscillations and the local Hubble constant, Mon. Not. R. Astron. Soc. 416, 3017 (2011), arXiv:1106.3366 [astro-ph.CO].

[24] S. Alam, M. Ata, S. Bailey, F. Beutler, D. Bizyaev, J. A. Blazek, A. S. Bolton, J. R. Brownstein, A. Burden, C.-H. Chuang, J. Compast, A. J. Cuesta, K. S. Dawson, D. J. Eisenstein, S. Escoffier, H. Gil-Marín, J. N. Grieb, N. Hand, S. Ho, K. Kinemuchi, D. Kirkby, F. Kitaura, E. Malanushenko, V. Malanushenko, C. Maraston, C. K. McBride, R. C. Nichol, M. D. Olmstead, A. Roga, D. V. J. Pasquali, A. Raich, F. Prada, A. M. Price-Whelan, B. A. Reid, S. Rodríguez-Torres, N. A. Roe, A. J. Ross, N. P. Ross, G. Rossi, J. A. Rubiño-Martín, S. Saito, S. Salazar-Albornoz, L. Samushia, A. G. Sánchez, S. Satpathy, D. J. Schlegel, D. P. Schneider, C. G. Scoccola, H.-J. Sea, E. S. Sheldon, A. Simmons, A. Slosar, M. A. Strauss, M. E. C. Swanson, D. Thomas, J. L. Tinker, R. Tojeiro, M. V. Magaña, J. A. Vazquez, L. Verde, D. A. Wake, Y. Wang, D. H. Weinberg, M. White, W. M. Wood-Vasey, C. Y. Y. Zhao, and G.-B. Zhao, The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR12 galaxy sample, Mon. Not. R. Astron. Soc. 470, 2617 (2017), arXiv:1607.03155 [astro-ph.CO].

[25] A. J. Ross, L. Samushia, C. Howlett, W. J. Percival, A. Burden, and M. Manera, The clustering of the SDSS DR7 main galaxy sample - I. A 4 per cent distance measure at z = 0.15, Mon. Not. R. Astron. Soc. 449, 835 (2015), arXiv:1409.3242 [astro-ph.CO].

[26] M. Shoja, D. Jeong, and E. Komatsu, Extracting Angular Diameter Distance and Expansion Rate of the Universe From Two-Dimensional Galaxy Power Spectrum at High Redshifts: Baryon Acoustic Oscillation Fitting Versus Full Modeling, Astrophys. J. 693, 1404 (2009), arXiv:0805.4238 [astro-ph].

[27] O. H. E. Philcox, M. M. Ivanov, M. Simonović, and M. Zaldarriaga, Combining full-shape and BAO analyses of galaxy power spectra: a 1.6% CMB-independent constraint on $H_0$, JCAP 2020, 032 (2020), arXiv:2002.04035 [astro-ph.CO].

[28] Y. Kobayashi, T. Nishimichi, M. Takada, and H. Miyatake, Full-shape cosmology analysis of the SDSS-III BOSS galaxy power spectrum using an emulator-based halo model: A 5% determination of $\sigma_8$, Phys. Rev. D 105, 083517 (2022), arXiv:2110.06969 [astro-ph.CO].

[29] J. Lesgourgues and S. Pastor, Massive neutrinos and cosmology, Phys. Rep. 429, 307 (2006), arXiv:astro-ph/0603494 [astro-ph].

[30] O. H. E. Philcox and M. M. Ivanov, BOSS DR12 full-shape cosmology: A CDM constraints from the large-scale galaxy power spectrum and bispectrum monopole, Phys. Rev. D 105, 043517 (2022), arXiv:2112.04515 [astro-ph.CO].

[31] K. Heitmann, D. Bingham, E. Lawrence, S. Bergner, S. Habib, D. Higdon, A. Pope, R. Biswas, H. Finkel, N. Frontiere, and S. Bhattacharya, The Mira-Titan Universe: Precision Predictions for Dark Energy Surveys, Astrophys. J. 820, 108 (2016), arXiv:1508.02654 [astro-ph.CO].

[32] J. DeRose, R. H. Wechsler, J. L. Tinker, M. R. Becker, Y.-Y. Mao, T. McClintock, S. McLaughlin, E. Rozo, and Z. Zhai, The AEMULUS Project. I. Numerical Simulations for Precision Cosmology, Astrophys. J. 875, 69 (2019), arXiv:1804.05865 [astro-ph.CO].

[33] T. Nishimichi, M. Takada, R. Tabashashi, K. Osato, M. Shirasaki, T. Oogi, H. Miyatake, M. Oguri, R. Murata, Y. Kobayashi, and N. Yoshida, Dark Quest. I. Fast and Accurate Emulation of Halo Clustering Statistics and Its Application to Galaxy Clustering, Astrophys. J. 884, 29 (2019), arXiv:1811.09504 [astro-ph.CO].

[34] R. E. Angulo, M. Zennaro, S. Contreras, G. Aricò, M. Pellejero-Ibañez, and T. Stüber, The BACCO simulation project: exploiting the full power of large-scale structure for cosmology, Mon. Not. R. Astron. Soc. 507, 5869 (2021), arXiv:2004.06245 [astro-ph.CO].

[35] S. Tassev, M. Zaldarriaga, and D. J. Eisenstein, Solving large scale structure in ten easy steps with COLA, JCAP 2013, 036 (2013), arXiv:1301.0322 [astro-ph.CO].

[36] P. Monaco, E. Sefusatti, S. Borgani, M. Crocce, P. Fosalba, R. K. Sheth, and T. Theuns, An accurate tool for the fast generation of dark matter halo catalogues, Mon. Not. R. Astron. Soc. 433, 2389 (2013), arXiv:1305.1505 [astro-ph.CO].

[37] F. S. Kitaura and S. Hess, Cosmological structure formation with augmented lagrangian perturbation theory., Mon. Not. R. Astron. Soc. 435, L78 (2019), arXiv:1212.3514 [astro-ph.CO].

[38] C.-H. Chuang, F.-S. Kitaura, F. Prada, C. Zhao, and G. Yepes, EZmocks: extending the Zel'dovich approximation to generate mock galaxy catalogues with accurate clustering statistics, Mon. Not. R. Astron. Soc. 446, 2621 (2015), arXiv:1409.1124 [astro-ph.CO].

[39] S. He, Y. Li, Y. Feng, S. Ho, S. Ravanbakhsh, W. Chen, and G.Yepes, EZmocks: extending the Zel’dovich approximation to generate mock galaxy catalogues with accurate clustering statistics, Mon. Not. R. Astron. Soc. 446, 2621 (2015), arXiv:1409.1124 [astro-ph.CO].

[40] F. Villaescusa-Navarro, D. Angéls-Alcázar, S. Genel, D. N. Spergel, R. S. Somerville, R. Dave, A. Pillepich, L. Hernquist, D. Nelson, P. Torrey, D. Narayanan, Y. Li, O. Philcox, V. La Torre, A. Maria Delgado, S. Ho, S. Hassan, B. Burkhardt, D. Wadekar, N. Battaglia, G. Contrado, and G. L. Bryan, The CAMELS Project: Cosmology and Astrophysics with Machine-learning Simu-
A. Taruya, T. Nishimichi, S. Saito, and T. Hiramatsu, M. Pietroni, Flowing with Time: A New Approach to Baryonic Oscillations in the Real-Space Matter Power Spectrum, Astrophys. J. 651, 619 (2006), arXiv:astro-ph/0604075 [astro-ph].

M. White, The Zel’dovich approximation, Mon. Not. R. Astron. Soc. 439, 3630 (2014), arXiv:1401.5466 [astro-ph.CO].

S.-F. Chen, Z. Vlah, E. Castorina, and M. White, Redshift-space distortions in Lagrangian perturbation theory, JCAP 2021, 100 (2021), arXiv:2012.04636 [astro-ph.CO].

D. Baumann, A. Nicolis, L. Senatore, and M. Zaldarriaga, Cosmological non-linearities as an effective fluid, JCAP 2012, 051 (2012), arXiv:1004.2488 [astro-ph.CO].

J. J. M. Carrasco, M. P. Hertzberg, and L. Senatore, The effective field theory of cosmological large scale structures, Journal of High Energy Physics 2012, 82 (2012), arXiv:1206.2926 [astro-ph.CO].

M. P. Hertzberg, Effective field theory of dark matter and structure formation: Semianalytical results, Phys. Rev. D 89, 043521 (2014), arXiv:1208.0839 [astro-ph.CO].

M. Croce and R. Scoccimarro, Renormalized cosmological perturbation theory, Phys. Rev. D 73, 063519 (2006), arXiv:astro-ph/0509418 [astro-ph].

M. Croce and R. Scoccimarro, Memory of initial conditions in gravitational clustering, Phys. Rev. D 73, 063520 (2006), arXiv:astro-ph/0509419 [astro-ph].

M. Croce and R. Scoccimarro, Nonlinear evolution of baryon acoustic oscillations, Phys. Rev. D 77, 023533 (2008), arXiv:0704.2783 [astro-ph].

P. Valageas, Large-N expansions applied to gravitational clustering, Astron. Astrophys. 465, 725 (2007), astro-ph/0611849.

A. Taruya and T. Hiramatsu, A Closure Theory for Nonlinear Evolution of Cosmological Power Spectra, Astrophys. J. 674, 617-635 (2008), arXiv:0708.1367.

A. Taruya, T. Nishimichi, S. Saito, and T. Hiramatsu, Non-linear Evolution of Baryon Acoustic Oscillations from Improved Perturbation Theory in Real and Redshift Spaces, Phys. Rev. D 80, 123503 (2009), arXiv:0906.0507 [astro-ph.CO].

T. Matsubara, Resumming Cosmological Perturbations via the Lagrangian Picture: One-loop Results in Real Space and in Redshift Space, Phys. Rev. D 77, 063530 (2008), arXiv:0711.2521 [astro-ph].

M. Pietroni, Flowing with Time: A New Approach to Nonlinear Cosmological Perturbations, JCAP 0810, 036 (2008), arXiv:0806.0971 [astro-ph].

F. Bernardeau, M. Croce, and R. Scoccimarro, Multipoint propagators in cosmological gravitational instability, Phys. Rev. D 78, 103521 (2008), arXiv:0806.2334 [astro-ph].

F. Bernardeau, M. Croce, and R. Scoccimarro, Constructing regularized cosine propagators, Phys. Rev. D 85, 123519 (2012), arXiv:1112.3895 [astro-ph.CO].

D. Blas, M. Garny, M. M. Ivanov, and S. Sibiryakov, Time-sliced perturbation theory for large scale structure I: general formalism, JCAP 2016, 052 (2016), arXiv:1512.05807 [astro-ph.CO].

D. Blas, M. Garny, M. M. Ivanov, and S. Sibiryakov, Time-sliced perturbation theory II: baryon acoustic oscillations and infrared resummation, JCAP 2016, 028 (2016), arXiv:1605.02149 [astro-ph.CO].

A. Taruya, F. Bernardeau, T. Nishimichi, and S. Codis, Direct and fast calculation of regularized cosmological power spectrum at two-loop order, Phys. Rev. D 86, 103528 (2012), arXiv:1208.1191 [astro-ph.CO].

K. Osato, T. Nishimichi, F. Bernardeau, and A. Taruya, Perturbation theory challenge for cosmological parameters estimation: Matter power spectrum in real space, Phys. Rev. D 99, 063530 (2019), arXiv:1810.10104 [astro-ph.CO].

M. Schmittfull, Z. Vlah, and P. McDonald, Fast large scale structure perturbation theory using one-dimensional fast Fourier transforms, Phys. Rev. D 93, 103528 (2016), arXiv:1603.04405 [astro-ph.CO].

J. E. McEwen, X. Fang, C. M. Hirata, and J. A. Blazek, FAST-PT: a novel algorithm to calculate convolution integrals in cosmological perturbation theory, JCAP 2016, 015 (2016), arXiv:1603.04826 [astro-ph.CO].

X. Fang, J. A. Blazek, J. E. McEwen, and C. M. Hirata, FAST-PT II: An algorithm to calculate convolution integrals of general tensor quantities in cosmological perturbation theory, JCAP 2017, 030 (2017), arXiv:1609.05978 [astro-ph.CO].

M. Simonović, T. Baldauf, M. Zaldarriaga, J. J. Carrasco, and J. A. Kollmeier, Cosmological perturbation theory using the FFTLog: formalism and connection to QFT loop integrals, JCAP 2018, 030 (2018), arXiv:1708.08130 [astro-ph.CO].

K. Osato, T. Nishimichi, A. Taruya, and F. Bernardeau, Implementing spectra response function approaches for fast calculation of power spectra and bispectra, Phys. Rev. D 104, 103501 (2021), arXiv:2107.04275 [astro-ph.CO].

A. J. S. Hamilton, Uncorrelated modes of the non-linear power spectrum, Mon. Not. R. Astron. Soc. 312, 257 (2000), arXiv:astro-ph/9905191 [astro-ph].

T. Nishimichi, F. Bernardeau, and A. Taruya, Moving around the cosmological parameter space: A non-linear power spectrum reconstruction based on high-resolution cosmic responses, Phys. Rev. D 96, 123515 (2017), arXiv:1708.08946 [astro-ph.CO].

M. Schmittfull and Z. Vlah, Reducing the two-loop large-scale structure power spectrum to low-dimensional, radial integrals, Phys. Rev. D 94, 103530 (2016), arXiv:1609.00349 [astro-ph.CO].

T. Baldauf, E. Schaan, and M. Zaldarriaga, On the reach of perturbative methods for dark matter density fields, JCAP 2016, 007 (2016), arXiv:1507.02255 [astro-ph.CO].

A. Taruya, T. Nishimichi, and D. Jeong, Covariance of the matter power spectrum including the survey window
function effect: N-body simulations versus fifth-order perturbation theory on grids, Phys. Rev. D **103**, 023501 (2021), arXiv:2007.05504 [astro-ph.CO].

[74] F. Schmidt, An n-th order Lagrangian forward model for large-scale structure, JCAP **2021**, 033 (2021), arXiv:2012.09837 [astro-ph.CO].

[75] M. Schmittfull, M. Simonović, M. M. Ivanov, O. H. E. Philcox, and M. Zaldarriaga, Modeling galaxies in redshift space at the field level, JCAP **2021**, 059 (2021), arXiv:2012.03334 [astro-ph.CO].

[76] A. Taruya, T. Nishimichi, and D. Jeong, Grid-based calculations of redshift-space matter fluctuations from perturbation theory: UV sensitivity and convergence at the field level, Phys. Rev. D **105**, 103507 (2022), arXiv:2109.06734 [astro-ph.CO].

[77] A. Andrews, J. Jasche, G. Lavaux, and F. Schmidt, Bayesian field-level inference of primordial non-Gaussianity using next-generation galaxy surveys, arXiv e-prints , arXiv:2203.08838 (2022), arXiv:2203.08838 [astro-ph.CO].

[78] E. Komatsu, J. Dunkley, M. R. Nolta, C. L. Bennett, B. Gold, G. Hinshaw, N. Jarosik, D. Larson, M. Limon, L. Page, D. N. Spergel, M. Halpern, R. S. Hill, A. Kogut, S. S. Meyer, G. S. Tucker, J. L. Weiland, E. Wollack, and E. L. Wright, Five-Year Wilkinson Microwave Anisotropy Probe Observations: Cosmological Interpretation, Astrophys. J. Supp. **180**, 339 (2009), arXiv:0803.0547 [astro-ph].

[79] T. Nishimichi, M. Takada, R. Takahashi, K. Osato, M. Shirasaki, T. Oogi, H. Miyatake, M. Oguri, R. Murata, Y. Kobayashi, and N. Yoshida, Dark Quest. I. Fast and Accurate Emulation of Halo Clustering Statistics and Its Application to Galaxy Clustering, Astrophys. J. **884**, 29 (2019), arXiv:1811.09504 [astro-ph.CO].

[80] Y. P. Jing, Correcting for the Alias Effect When Measuring the Power Spectrum Using a Fast Fourier Transform, Astrophys. J. **620**, 559 (2005), arXiv:astro-ph/0409240 [astro-ph].

[81] E. Sefusatti, M. Crocce, R. Scoccimarro, and H. M. P. Couchman, Accurate estimators of correlation functions in Fourier space, Mon. Not. R. Astron. Soc. **460**, 3624 (2016), arXiv:1512.07295 [astro-ph.CO].

[82] D. Jeong, Cosmology with high ($z > 1$) redshift galaxy surveys, Ph.D. thesis, University of Texas, Austin (2010).

[83] R. Takahashi, N. Yoshida, T. Matsubara, N. Sugiyama, I. Kayo, T. Nishimichi, A. Shirata, A. Taruya, S. Saito, K. Yahata, and Y. Suto, Simulations of baryon acoustic oscillations - I. Growth of large-scale density fluctuations, Monthly Notices of the Royal Astronomical Society **389**, 1675 (2008), arXiv:0802.1808 [astro-ph].

[84] L. Senatore and M. Zaldarriaga, The IR-resummed Effective Field Theory of Large Scale Structures, JCAP **2015**, 013 (2015), arXiv:1404.5954 [astro-ph.CO].

[85] Z. Vlah, U. Seljak, M. Yat Chu, and Y. Feng, Perturbation theory, effective field theory, and oscillations in the power spectrum, JCAP **2016**, 057 (2016), arXiv:1509.02120 [astro-ph.CO].