Probing Lorentz violation effects via a laser beam interacting with a high-energy charged lepton beam

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In this work, the conversion of linear polarization of a laser beam to circular one through its forward scattering by a TeV order charged lepton beam in the presence of Lorentz violation correction is explored. We calculate the ratio of circular polarization to linear one (Faraday Conversion phase ∆φ) of laser photons interacting with either electron or muon beam in the framework of the quantum Boltzmann equation. We discuss how the interaction of laser beam and charged lepton beams can provide a suitable situation to constrain Lorentz violation coefficients, especially for cµν coefficients. It is shown that ∆φ depends on the direction of both interacting beams and the location of the lab which can help to determine cµν components individually. It should be mentioned that the laser and charged lepton beams considered here to reach an experimentally measurable ∆φ, are currently available. This study provides a valuable supplementary to other theoretical and experimental frameworks for measuring and constraining Lorentz violation coefficients.

I. INTRODUCTION

Usually, radiation can be both linearly and circularly polarized. It is properly known that when initially unpolarized photon scatter off a free electron through Compton scattering results in linear polarization but not the circular polarization of scattered radiation. However, it is shown that Compton scattering in the presence of external background fields similar to strong magnetic field [1–4] or theoretical (non-trivial) backgrounds such as non-commutativity in space-time [2, 5, 6] and Lorentz symmetry violation [2], can produce circular polarization. Moreover, nonlinear effects such as nonlinear Euler-Heisenberg can cause converting photons linear polarization to circular polarizations [3, 7, 8].

In this Letter, we consider Compton scattering through the collision of laser photon and high energy charged lepton beams in presence of Lorentz Violation (LV) effects as a background to study the generation of circular polarization in the earth-based laboratory. Lorentz symmetry is a fundamental symmetry of the standard model in flat space-time and quantum field theory. However, it can be violated by an underlying theory at PLANCK scale [9, 10] which would cause a background field and a preferred direction. There are many theories in which the Lorentz symmetry is violated spontaneously such as string theory [11–13], quantum gravity [14–17] and Non-Commutative space-time [18–20]. Meanwhile, it is also possible to study LV in a general model-independent way in the context of effective field theory known as the Standard Model Extension (SME). In the SME Lagrangian, the observer Lorentz symmetry (i.e. change of coordinate) is protected while the particle Lorentz symmetry (i.e. boosts on particles and not on background fields) is violated [21, 22]. The SME contains all feasible Lorentz breaking operators created by known fields of the standard model of dimension 3 or more [21, 23, 24] that can describe small Lorentz symmetry violation at available energies. Generally, the number of coefficients is infinite. By the way, it is possible to choose a minimal subset of the SME with finite coefficients. The minimal SME contains renormalizable operators which are invariant under gauge group of the standard model, SU(3) × SU(2) × U(1). In recent years, new studies have provided new types of constraints on the Lorentz violation parameters [25-29]. Among them astrophysical [30-32] and earthly [33-35] systems have brought the stronger bounds on the LV parameters [36]. Observation of circularly polarized photon in lepton and photon
scattering can be a proof of LV and resulting in new physics beyond the standard model. In contrast, constraints on circular polarization might improve the available bounds on parameters of the SME.

The paper is organized as follows: In section II we briefly introduce the Stokes parameters formalism and the generalized Boltzmann equation. In section III we study the effect of LV on the collision of the relativistic lepton beam (electron/muon) and the laser. In section IV we give the value of the Faraday conversion phase shift of laser beam through this interaction. Finally, we discuss the results in the last section.

II. STOKES PARAMETERS AND QUANTUM BOLTZMANN EQUATION

Normally the polarization of a laser beam can be described by well-known Stokes parameters with 4 dimension, I, Q, U and V. I denotes the Intensity of laser beam, V shows the difference between left- and right- circular polarization Q and U indicate the linear polarization. Q is defined by the intensity difference between polarized components of electromagnetic wave in direction of x and y axes. U quantifies discrepancy between 45° and 135° counted from the positive x axis, to the reference plane [37]. The linear polarization can also be shown by vector \( P = \sqrt{Q^2 + U^2} \) [38].

The Stokes parameters can be specified by superposition of two opposite right-, \( \hat{R} \), and left-, \( \hat{L} \), hand circular polarization contributions:

\[
E_L = E_{0L} \cos[\omega_0 t - \phi_L], \quad E_R = E_{0R} \cos[\omega_0 t - \phi_R].
\]

The Stokes parameters can be defined as:

\[
\begin{align*}
I & = \langle E_L^2 \rangle + \langle E_R^2 \rangle, \\
Q & = 2 \langle E_L E_R \cos \eta \rangle, \\
U & = 2 \langle E_L E_R \sin \eta \rangle, \\
V & = \langle E_R^2 \rangle - \langle E_L^2 \rangle,
\end{align*}
\]

where \( \eta = \phi_R - \phi_L \) is phase difference between left- and right- hand circularly polarized waves. Therefore the Stokes parameters Q and U, mix as follows:

\[
\dot{U} = -2Q \frac{d\Delta \phi_{FR}}{dt} \quad \text{and} \quad \dot{Q} = -2U \frac{d\Delta \phi_{FR}}{dt},
\]

which is known as Faraday Rotation (FR). If the linearly polarized light propagates via the cold magnetized plasma, its polarization rotates and Faraday rotation will occur. Simultaneously, charged particles passing through the cold plasma emit cyclotron radiation which is circularly polarized [37]. Now the phase difference results mixing between the U and V Stokes parameters known as Faraday Conversion (FC). The evolution of Stokes parameter V given by this mechanism is obtained as [1, 39]

\[
\dot{V} = 2U \frac{d\Delta \phi_{FC}}{dt},
\]

where \( \Delta \phi_{FC} \) is Faraday Conversion phase shift. Generally, light traversing relativistic plasma undergoes both Faraday Conversion and rotation.

Let us consider an ensemble of photons which is described by density matrix \( \rho_{ij} \equiv \langle |\epsilon_i \rangle < |\epsilon_j \rangle |tr\rho \rangle \). The density matrix can be written based on the Stokes parameters as

\[
\rho = \frac{1}{2} \begin{pmatrix}
I + Q & U - iV \\
U + iV & I - Q
\end{pmatrix}.
\]

The density matrix \( \rho_{ij} \) is related to the number operator \( \hat{D}_{ij}(p) = \hat{a}_i^\dagger(p)\hat{a}_j(p) \) as

\[
\langle \hat{D}_{ij}(p) \rangle = (2\pi)^3 2\rho^0 \delta^{(3)}(0) \rho_{ij}(p).
\]

In order to figure out the time evolution of the density matrix (Stokes parameters), it is convenient to use the Heisenberg equation

\[
\frac{d}{dt} D^{(0)}_{ij}(p) = i[H_I, D^{(0)}_{ij}(p)],
\]
where \( H_I \) is the interacting Hamiltonian of photons with the standard model particles. Obtaining the expectation value of Eq (4) and replacing with eq (6), the Boltzmann equation for the number operator of photons in terms of density matrix elements \( \rho_{ij} \) and Stokes parameters can be written as [37]:

\[
(2\pi)^3 \delta^3(0) 2 p^0 \frac{d}{dt} \rho_{ij}(0, \mathbf{p}) = i \{ \langle H_I^0(t), \hat{D}_{ij}(\mathbf{p}) \rangle \} - \int dt \{ \langle [H_I(t), \hat{D}_{ij}(\mathbf{p})] \rangle \}. \tag{8}
\]

Here we consider only photons and charged leptons contribution in the interacting Hamiltonian. The first term on the right-hand side is a forward scattering term and the second one is a higher order collision term. Forward scattering means most of the photons travel straightforwardly without changing the momentum. Considering the Compton scattering as a dominant process in the standard model, the time evolution of circular polarization is zero, \( \dot{V} = 0 \). However, we will show in the following that the SME as a background for Compton scattering can generate circular polarization. In general, the interacting Hamiltonian for photon-charged lepton beam interaction up to leading order is given by [23]:

\[
H_I^0 = \int d\mathbf{q} d\mathbf{p} (2\pi)^3 \delta^3(\mathbf{q}' + \mathbf{p}' - \mathbf{p} - \mathbf{q}) \exp[i(q'^0 + p'^0 - q^0 - p^0)t] \left[ b^\dagger_{\nu} a^\dagger_{\nu} (\mathcal{M}) a_{\nu} b_{\nu} \right], \tag{9}
\]

where \( b_{\nu}(\mathbf{p}) \) and \( b^\dagger_{\nu}(\mathbf{p}) \) are charged fermion annihilation and creation operators, \( \mathcal{M} \) is the amplitude of scattering matrix, \( d\mathbf{p} = \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2p^0} \) and \( d\mathbf{q} = \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{m_f}{2q^0} \) are the phase space of photon and charged lepton, respectively, with similar definition for \( d\mathbf{p}' \) and \( d\mathbf{q}' \).

### III. THE GENERATION OF CIRCULAR POLARIZATION IN THE SME

We express the scattering amplitude \( M \) of the laser beam photons and charged lepton beam in the presence of LV background by using the minimal SME Lagrangian in QED sector [24]:

\[
\mathcal{L}^V_{\text{QED}} = \bar{\psi}(i\Gamma^\mu \hat{D}_\mu - M)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (k_F)_{\alpha\beta\mu\nu} F^{\alpha\beta} F^{\mu\nu} + \frac{1}{2} (k^A_F)^{\alpha} \epsilon_{\alpha\beta\mu\nu} A^\beta F^{\mu\nu}, \tag{10}
\]

where \( \hat{D}_\mu \) is the usual covariant derivative, \( \psi \) is the charged lepton filed, \( \gamma^\mu \) is the usual Dirac matrices with

\[
M = m + a^\mu \gamma_\mu - b^\mu \gamma^\mu \gamma^5 + \frac{1}{2} H^{\mu\nu} \sigma_{\mu\nu} + im_5 \gamma^5,
\]

\[
\Gamma^\mu = \gamma^\mu + e^{\mu\nu} \gamma_\nu - d^{\mu\nu} \gamma_\nu \gamma^5 + e^\mu + if^{\mu\nu} \gamma^5 + \frac{1}{2} g^{\alpha\mu\nu} \sigma_{\alpha\nu}, \tag{11}
\]

and where parameters \( \{e^{\mu\nu}, d^{\mu\nu}, a^\mu, b^\mu, H^{\mu\nu}\} \) denote LV lepton sector coefficients and \( \{(k_F)_{\alpha\beta\mu\nu}, (k^A_F)^{\alpha}\} \) are the photon sector LV coefficients. Setting coefficients equal to zero cause usual Dirac Lagrangian. LV coefficients emerged in M are mass-like terms and important at low energies and can easily be neglected at high energies whereas parameters in \( \Gamma^\mu \) which are momentum-like are considerable at high energies. In the laser and the fermion beam interaction at high energy we only require to consider the parameters \( e^{\mu\nu} \) and \( d^{\mu\nu} \) in electron LV sector which are dominant. These parameters are Hermitian coefficients, dimensionless with both symmetric and asymmetric space-time components.

Considering Eq (9) and Eq (10), the time evolution of density matrix components are given as follows [21]:

\[
2k^0 \frac{d}{dt} \rho_{ij} = \frac{e^2}{m_f} \int d\mathbf{q} n_f(x, \mathbf{q}) \left[ \delta_{si} \rho_{rj}(k) - \delta_{ri} \rho_{sj}(k) \right] \left[ \frac{1}{2k^0 q} \left( e^{\mu\nu} e^s_{\mu} \left( q \cdot k e^r_{\nu} - q \cdot e^r_{\nu} k \right) ight. \\
+ \left. e^{\mu\nu} e^s_{\nu} \left( q \cdot k e^r_{\nu} - q \cdot e^s_{\nu} k \right) \right) + e^{\mu\nu} q_{\mu} \left( q \cdot e^r_{\mu} + q \cdot e^r_{\nu} \right) \right] \times \left( q_{s} q_{r} + k_{s} k_{r} \right) \left( q \cdot e^s \cdot q \cdot e^r \right), \tag{12}
\]

where \( n_f(x, \mathbf{q}) \) indicates the distribution function of charged lepton, \( e^s_{\mu}(k) \) is the photon polarization states with \( s, r = 1, 2 \). The energy density \( \epsilon_f(x) \), number density \( n_f(x) \) and averaged momentum \( \bar{q} \) of leptons are defined as:

\[
\frac{n_f(x)}{\bar{q}} = \int \frac{d^3q}{(2\pi)^3} \frac{n_f(x, \mathbf{q})}{q}, \quad n_f(x) = \int \frac{d^3q}{(2\pi)^3} n_f(x, \mathbf{q}), \quad \epsilon_f(x) = g f \int \frac{d^3q}{(2\pi)^3} q^0 n_f(x, \mathbf{q}). \tag{13}
\]
where \( g_f \) is the number of spin states. In the calculation, we assume \( n_f(\mathbf{x}, \mathbf{q}) \sim \exp[-|\mathbf{q} - \mathbf{q}|/|\mathbf{q}|] \) for charged lepton beam which means that the most of leptons are moving with the same momentum \( \mathbf{q} \simeq \mathbf{q} \) and in the same direction. We should note that at the forward scattering of photon-lepton, the term including \( d^{\mu\nu} \) correction does not generate any circular polarization, i.e. \( \dot{V} = 0 \). However, \( d^{\mu\nu} \) may contribute to circular polarization in higher order correction which is proportional to \( (d^{\mu\nu})^2 \) and negligible. Hence, \( c^{\mu\nu} \) is the only source of LV which we will consider for the rest.

In many phenomenological points of view, it is more convenient to consider \( c^{\mu\nu} \) as a symmetric and traceless tensor which caused having nine independent components for \( c \) coefficients. In fact, the anti-symmetric part at leading order is equivalent to the redefinition in the representation of the Dirac matrices \([40]\). Therefore the physical quantities are independent of anti-symmetric part of \( c \) coefficient at leading order. Then time evolution of the Stokes parameters given in Eq.\((\ref{eq:stokes})\), are obtained as follows \([2]\):

\[
\begin{align*}
\dot{I}(k) &= 0 \quad , \quad \dot{V}(k) = \rho_Q Q(k) + \rho_V U(k) \\
\dot{Q}(k) &= -\rho_Q V(k) \quad , \quad \dot{U}(k) = -\rho_V V(k). \quad (14)
\end{align*}
\]

Solving above coupled equations leads to:

\[
\dot{V} = \dot{\rho}_Q Q + \dot{\rho}_U U - \rho^2 V, \quad (15)
\]

with

\[
\begin{align*}
\rho_Q &= -\frac{ie^2}{2k_0 m} \int d\mathbf{q} \ n_f(\mathbf{x}, \mathbf{q}) \frac{c^{\mu\nu}}{4(q \cdot k)^2} \bigg[ 2(q \cdot k)^2((\epsilon_\mu^2 \epsilon_\nu^1 + \epsilon_\mu^1 \epsilon_\nu^2) + q \cdot k \left( q \cdot e^2 ((\mu_k - k_\mu) \epsilon_\nu^1) \\
+ (q_\nu - k_\nu) \epsilon_\mu^1) + q \cdot e^1 ((\mu_k - k_\mu) \epsilon_\nu^2 + (q_\nu - k_\nu) \epsilon_\mu^2) \right) + 8(q \cdot e^1 q \cdot c^2(q_\mu q_\nu + k_\nu k_\mu) \bigg], \quad (16a)
\end{align*}
\]

\[
\begin{align*}
\rho_U &= -\frac{ie^2}{2k_0 m} \int d\mathbf{q} \ n_f(\mathbf{x}, \mathbf{q}) \frac{c^{\mu\nu}}{4(q \cdot k)^2} \bigg[ 2(q \cdot k)^2((\epsilon_\mu^1 \epsilon_\nu^1 - \epsilon_\mu^2 \epsilon_\nu^2) + q \cdot k \left( q \cdot e^2 ((\mu_k - k_\mu) \epsilon_\nu^2) \\
+ (k_\nu - q_\nu) \epsilon_\mu^2) + q \cdot e^1 ((\mu_k - k_\mu) \epsilon_\nu^1 + (q_\nu - k_\nu) \epsilon_\mu^1) \right) + 4(q \cdot e^1 q \cdot e^1 - q \cdot e^2 q \cdot e^2)(q_\nu q_\mu + k_\nu k_\mu) \bigg]
\end{align*}
\]

and \( \rho = \sqrt{\rho_Q^2 + \rho_U^2} \), \( c^{\mu\nu} \) means symmetric part of \( c^{\mu\nu} \) tensor, \( q \) and \( k \) are the momenta of leptons and photons, respectively. In particular, available bounds on coefficients of SME are given in the Standard Sun-centered non-rotating inertial reference frame. However, experimental set up is usually managed in the earth. Therefore, for parameterizing our result, it is necessary to introduce Cartesian coordinates on the earth frame and suitable basis of vectors for a non-rotating frame. Let us define a coordinate system in three-dimensional space. \((X, Y, Z)\) shows the non-rotating basis in which \( \hat{Z} \) points along the earth’s axis on the north direction. Note that we just take usual definition for laboratory frame. Then the non-relativistic transformation to a lab basis \((\hat{x}, \hat{y}, \hat{z})\) at any time \( t \) is given by \([41]\):

\[
\begin{pmatrix}
\hat{x} \\
\hat{y} \\
\hat{z}
\end{pmatrix} = \begin{pmatrix}
\cos \chi \cos \Omega t & \cos \chi \sin \Omega t & -\sin \chi \\
-\sin \Omega t & \cos \Omega t & 0 \\
\sin \chi \cos \Omega t & \sin \chi \sin \Omega t & \cos \chi
\end{pmatrix}
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix},
\]

where \( \Omega \simeq 2\pi/(23h \ 56 \ \text{min}) \) is the sidereal rotation frequency of earth and \( \chi = \hat{Z} \cdot \hat{z} \) varies as \( 0 \leq \chi \leq \pi \). Since lab frame rotates with rotation of the earth, the LV components on the earth will depend on location and time i.e. the experimental observable would vary with time and location. By transforming the time-like and space-like vectors of
TABLE I: Location dependence of the independent components of $c$ parameter.

| Parameters | Location Dependence |
|------------|---------------------|
| $c_{00}$   | $c_{TT}$            |
| $c_{22}$   | $\frac{c_{XX} + c_{YY}}{2}$ |
| $c_{33}$   | $\frac{1}{2}((c_{XX} + c_{YY})\sin^2(\chi) + 2c_{ZZ}\cos^2(\chi))$ |
| $c(12)$   | 0                   |
| $c(13)$   | $\sin \chi \cos \chi (c_{XX} + c_{YY} - 2c_{ZZ})$ |
| $c(23)$   | 0                   |
| $c(01)$   | $-c_{(TZ)} \sin \chi$ |
| $c(02)$   | 0                   |
| $c(03)$   | $c_{(TZ)} \cos \chi$ |

the tensor $\epsilon^{\mu\nu}$ we obtain:

$$
c_{00} = c_{TT},
$$

$$
c_{22} = \cos^2 \kappa (c_{XX} \cos^2 \Omega t + (c_{XY} + c_{YX}) \sin \Omega t \cos \Omega t + c_{YY} \sin^2 \Omega t) - \sin \kappa \cos \kappa (c_{XZ} + c_{ZX}) \cos \Omega t,
$$

$$
+ (c_{YZ} + c_{ZY}) \sin \Omega t + c_{ZZ} \sin^2 \kappa,
$$

$$
c_{33} = c_{XX} \sin^2(\Omega t) - (c_{XY} + c_{YX}) \sin \Omega t \cos \Omega t + c_{YY} \cos^2 \Omega t,
$$

$$
c_{01} = \cos \kappa (c_{TX} + c_{XT}) \cos \Omega t + (c_{TY} + c_{YT}) \sin \Omega t) - (c_{TZ} + c_{ZT}) \sin \kappa,
$$

$$
c_{02} = (c_{TY} + c_{YT}) \cos \Omega t - (c_{TX} + c_{XT}) \sin \Omega t,
$$

$$
c_{03} = \sin \kappa (c_{TX} + c_{XT}) \cos \Omega t + (c_{TY} + c_{YT}) \sin \Omega t) + (c_{TZ} + c_{ZT}) \cos \kappa,
$$

$$
c_{12} = \cos \kappa ((c_{YY} - c_{XX}) \sin 2\Omega t + (c_{XY} + c_{YX}) \cos 2\Omega t) + \sin \kappa ((c_{XZ} + c_{ZX}) \sin \Omega t - (c_{YZ} + c_{ZY}) \cos \Omega t),
$$

$$
c_{13} = \sin 2\kappa (c_{XX} \cos^2 \Omega t + (c_{XY} + c_{YX}) \sin \Omega t \cos \Omega t + c_{YY} \sin^2 \Omega t - c_{ZZ}) + \cos^2 \kappa (c_{XZ} \cos \Omega t + c_{YZ} \sin \Omega t)
$$

$$
- \sin^2 \kappa (c_{XX} \cos \Omega t + c_{YY} \sin \Omega t) + \cos^2 \kappa (c_{XZ} \cos \Omega t + c_{YZ} \sin \Omega t) - \sin^2 \kappa (c_{XZ} \cos \Omega t + c_{YZ} \sin \Omega t),
$$

$$
c_{23} = \sin \kappa ((c_{YY} - c_{XX}) \sin 2\Omega t + (c_{XY} + c_{YX}) \cos 2\Omega t) + \cos \kappa ((c_{YZ} + c_{ZY}) \cos \Omega t - (c_{XZ} + c_{ZX}) \sin \Omega t),
$$

where $c_{ij} = c_{ij} + c_{ji}$ with $i, j = 0, \ldots, 3$. The time dependence of LV parameter causes to a day-night asymmetry in Faraday Conversion. However, the interaction of a photon with lepton beam occurs at a very short time of order $\approx 10^{-15} - 10^{-14}$ seconds which we will show it in the following, so the asymmetry is not needed to be considered. Then we can safely average over the time. Time averaging of $c_{(12)}, c_{(23)}, c_{(01)}$ are vanished as $<\cos \Omega t>$ and $<\sin \Omega t>$ are equal to zero. Components of $\epsilon^{\mu\nu}$, after time averaging, in earth coordinate and on the non-rotating frame are given in Table. I. Data-table for upper bounds on components of $c_{(ij)}$ in Sun-centered reference frame are given in Table. II.

As explained above the $\rho_Q$ and $\rho_U$ given in Eq. (16) are almost time independent. Therefore, the above differential equations (Eq. (15)) reduce to

$$
\ddot{V} + \rho^2 V = 0.
$$

The general solution for above equation is:

$$
V(t) = A \sin(\rho t) + B \cos(\rho t),
$$

with applying initial condition at $t = 0$, $V(0) = 0$, for a totally linear polarized laser beam, Eq. (14) can be rewritten as follows:

$$
\Delta \phi_{FC} = \frac{\rho_Q Q_0 + \rho_U U_0}{2\rho} \sin(\rho \Delta t),
$$

where $\Delta t$ is the time that beams are in the interaction. Considering Eq. (16), we obtain

$$
\Delta \phi_{FC} \sim \mathcal{N} \sin \left( \frac{3}{16} \frac{\sigma_T}{\alpha k_0 q_0} \sqrt{\frac{w_Q^2 + w_U^2}{q_0}} \frac{c_{(x,q)}}{q_0} \Delta t \right),
$$

with

$$
w_U = -2 \frac{(q \cdot e^1 q \cdot e^1 - q \cdot e^2 q \cdot e^2)(q \cdot e^1 \cdot q + k \cdot e^1 \cdot k)}{(q \cdot k)^2},
$$

$$
w_Q = 4 \frac{q \cdot e^1 q \cdot e^2 q \cdot e^1 \cdot q + k \cdot e^1 \cdot k}{(q \cdot k)^2}.
$$

(23)
TABLE II: Experimental bounds on components of $c$. 

| $c_{\mu\nu}$ | Experimental bounds | System                  |
|---------------|---------------------|-------------------------|
| $c_{\tau\tau}$ | $2 \times 10^{-15}$ | Collider physics [42]  |
| $c_{\gamma\gamma}$ | $3 \times 10^{-15}$ | Astrophysics [40]       |
| $c_{Z\bar{Z}}$ | $5 \times 10^{-15}$ | Astrophysics [40]       |
| $c_{(X,Y)}$    | $3 \times 10^{-15}$ | Astrophysics [40]       |
| $c_{(Y,Z)}$    | $1.8 \times 10^{-15}$ | Astrophysics [40]       |
| $c_{(X,Z)}$    | $3 \times 10^{-15}$ | Astrophysics [40]       |
| $c_{(T,X)}$    | $-30 \times 10^{-14}$ | Collider physics [42]  |
| $c_{(T,Y)}$    | $-80 \times 10^{-19}$ | Collider physics [12]  |
| $c_{(T,Z)}$    | $-11 \times 10^{-15}$ | Collider physics [12]  |

Here we should note that in deriving above equations, we only consider dominant terms in Eq(16), and the normalization factor is $N = \frac{w_Q Q_0 + w_Q U_0}{\sqrt{w_Q^2 + w_Q^2}}$.

IV. SET UP OF LASER AND CHARGED BEAMS COLLISION

In the lab frame, we set the direction $\hat{k}$ and polarization vectors $\bar{\epsilon}_2(k)$ of incident laser beam photons in observer’s Minkowskian frame coordinate system as follows:

$$\hat{k} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta),$$
$$\bar{\epsilon}_1(k) = (\cos \theta \cos \varphi, \cos \theta \sin \varphi, -\sin \theta),$$
$$\bar{\epsilon}_2(k) = (-\sin \varphi, \cos \varphi, 0),$$

(24)

which $\hat{k} = \frac{k}{|k|}$ makes an angle of $\theta$ with $z$-axis and the 4-vector momentum of charged lepton $q$ has been assumed on $\hat{z}$-direction.

There are many charged lepton beams with different energies. The most noticeable charged lepton beams are the future muon and electron beams available at colliders such as multi-TeV muon collider, Muon accelerator program (MAP) [43, 44], electron-positron International Linear Collider (ILC) [45] and Compact Linear Collider (CLIC) which is planned to obtain the center of mass energy up to $\sqrt{s} = 3 \text{ TeV}$ [46, 47].

The basic picture of our experiment is fairly simple. We measure the generation of circular polarization for the laser beam via its forward scattering with the charged lepton beam in the presence of LV effects. Eq(22) shows that in order to generate a wide range of circularly polarized photon, the intensity of lepton and laser beam should be large enough. However, in high-intensity laser, the effects of magnetic part of Lorentz force on charged lepton become important. In this case, magnetic field as a trivial background will produce circular polarization [1] [2]. Moreover at strong electromagnetic fields, the nonlinear Compton scattering effective become significant [48]. In addition to the collision of the laser beam with electron high energy beam, photons would possibly obtain high energy by back-scattering off high energy electron beam, i.e. inverse Compton scattering. Therefore, pair production is possible as a result of Breti-Wheeler reaction. However, the mentioned effects at low intensity laser beam are negligible and can safely be ignored. Another point is that low-intensity lasers are available sources of monochromatic radiation. They are lightweight and low-cost devices.

While electromagnetic fields and the coherent time duration of the laser pulse increase, the number of laser photons colliding with an electron beam will increase as well. Moreover, forward scattering of linearly polarized photons on polarized lepton such as the electron can cause the circular polarization of photons [49]. To reduce unwanted backgrounds effect as explained above which are particularly important at high frequencies and intensities, we suppose the lepton beam is nearly unpolarized and the laser beam has a low intensity and energy. As shown in Eq(14), in addition to Faraday Conversion, Faraday rotation is also resulted by forward scattering in the presence of LV effects.

For a typical relativistic electron beam with energy order of $E_e \sim \mathcal{O}(\text{TeV})$, the number of electron per bunch $n_e \sim \mathcal{O}(10^{10} \text{ cm}^{-3})$ and the size of beam bunch $\sim \mathcal{O}(\mu \text{m})$, the average energy of flux per bunch can be estimated as

$$\bar{\epsilon}_e(x, \bar{q}) \approx |\bar{q}| n_e(x, \bar{q})c \sim 10^{10} \text{ TeV/(cm}^2\text{s)},$$

(25)
which is normalized to the number density of electron $n_e$. The interacting time $\Delta t$ can be obtained by taking into account the size of two beams at the interacting point as $\Delta d \simeq c \Delta t$, where $\Delta d$ is the spatial interval of the interacting spot. It means that the beam with a larger size would be more effective for our aim. Then $\Delta t_e \sim 10^{-15}$ sec.

Therefore, the Faraday Conversion phase $\Delta \phi^{[LV]}_e$ for different components of $c_{\mu\nu}$ through forward scattering of the electron beam and the laser beam with the energy of photons $k_0 \sim 0.1eV$ as a function of scattering angle $\theta$ is given in Fig. 1. For simplicity we set $\varphi = \chi = \pi/4$ and polarization of incoming laser beam as $U_0 = Q_0 = I_0/2$ where $I_0$ is the intensity of laser beam.

Furthermore, for a typical relativistic muon beam with energy order of $E_\mu \sim \mathcal{O}(\text{TeV})$, the number of muon per bunch $n_\mu \sim \mathcal{O}(10^{12}\text{cm}^{-3})$, the size of beam bunch $\sim \mathcal{O}(10\mu m)$, the average energy of flux per bunch is given as following

$$\bar{\epsilon}_\mu(x, \bar{q}) \approx |\bar{q}| n_\mu(x, \bar{q}) c \sim 10^{12}\text{TeV}/(\text{cm}^2\text{s}).$$

(26)

Then $\Delta t_\mu \simeq \Delta d/c \sim 10^{-14}$ sec. The Faraday Conversion phase $\Delta \phi^{[LV]}_\mu$ for different components of $c_{\mu\nu}$ through forward scattering of the muon beam and the laser beam is shown in Fig. 2 which are plotted as a function of scattering angle $\theta$ with the energy of photons $k_0 \sim 0.1eV$, $\varphi = \chi = \pi/4$ and $U_0 = Q_0 = I_0/2$. 

FIG. 1: FC, $\Delta \phi^{[LV]}_e$ as a function of scattering angle of the laser and the charged lepton beam interaction for $\varphi = \chi = \pi/4$. 

FIG. 2: FC, $\Delta \phi^{[LV]}_\mu$ as a function of scattering angle of the laser and the charged lepton beam interaction for $\varphi = \chi = \pi/4$. 

...
FIG. 2: FC, $\Delta \phi|_{LV}^{c\mu}$ as a function of scattering angle of the laser and the charged lepton beam interaction for $\varphi = \chi = \frac{\pi}{4}$.

V. DISCUSSION AND CONCLUSION

We finally review how the laser beam interacting with charged lepton beams can provide a better situation to constrain $c^{\mu\nu}$ coefficients. The $d^{\mu\nu}$ coefficients, as noted previously, has no contribution for the generation of circular polarization via forward scattering of laser and charged lepton beams.

As our results depend on both the direction of beams and the location of the lab, there might be an optimal beam direction and position for the lab to observe $c^{\mu\nu}$ coefficients. According to Tab. I, the coefficient $c_{00}$ and $c_{22}$ after time averaging no longer depend on earth’s latitude and laboratory location $\chi$. As an example, we choose $\chi = \frac{\pi}{4}$ and obtain Faraday Conversion for typical electron and muon beams with energy $q_0 = 1$TeV, $n_e = 10^{10}$cm$^{-3}$ and $n_\mu = 10^{12}$cm$^{-3}$, note these suggested beams are experimentally available [44–46]. The results are shown in Fig. 1 and Fig. 2. Plots show that $\Delta \phi|_{e,\mu}$ are very sensitive to the scattering angle for different components of $c$. Roughly speaking, all $\Delta \phi|_{e,\mu}$ get their maximum values at specific scattering angles which are repeated periodically. This could be one of the most important characters of our results to distinguish the contribution of $c^{\mu\nu}$ from other sources of circular polarization. As an example, for all coefficients given in Tab. I, $\Delta \phi|_{e,\mu}$ are maximum at $\theta \simeq 1$ rad. Based
on the current constraint on the $\epsilon'^{\mu\nu}$ components and available sensitivity level to detect circular polarization or Faraday Conversion phase $[50,52]$. The most of our results are in the range of current experimental precision. We have considered the energy and number density of available charged lepton beams and we do not need to use very high-intensity laser beams which helps us to avoid other background effects.

We should also mention that any backgrounds can be a possible source to generate circular polarization. For example, in addition to LV correction to Compton scattering, interaction of photon with a LV background can also create circular polarization which is linearly proportional to LV coefficient for photon sector $k_F$ and $k_{AP}$ $[52]$. This effect does not modify the Compton scattering but the dynamic of the laser beam can be influenced $[2]$. Briefly we sure that our study provides a valuable supplement to other theoretical and experimental frameworks for improving the available constraints on LV coefficients.

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