In this paper we analyze the number counts of false peaks resulting from intrinsic ellipticities in lensing convergence maps, including the alignment of source galaxies. The full noise variance in convergence \(\kappa\)-maps can be written as \(\sigma^2_{\kappa} = \sigma^2_{\text{shot}} + \sigma^2_{\text{clean}}\), where \(\sigma^2_{\text{shot}}\) is the noise contributed from randomly oriented source galaxies and \(\sigma^2_{\text{clean}}\) denotes the additional noise from intrinsic alignments. However, it is observationally difficult to measure \(\sigma^2_{\text{clean}}\), and usually only \(\sigma^2_{\text{shot}}\) can be estimated in weak-lensing observations. Thus, the observational signal-to-noise ratio is often defined with respect to \(\sigma_{\text{shot}}\), which we denote as \(\nu_{\text{ran}}\). The true signal-to-noise ratio \(\nu\) in terms of \(\sigma_0\) is then \(\nu = \nu_{\text{ran}}/(1 + \sigma^2_{\text{clean}}/\sigma^2_{\text{shot}})^{1/2}\).

Given a detection threshold on \(\nu_{\text{ran}}\), a larger value of \(\sigma^2_{\text{clean}}/\sigma^2_{\text{shot}}\) leads to a lower threshold on \(\nu\) and therefore a larger expected number of false peaks. With \(\sigma^2_{\text{clean}}/\sigma^2_{\text{shot}} \approx 10\%\), the average number of false peaks with \(\nu_{\text{ran}} \geq 3.5\) nearly doubles compared to that without considering the alignment, and for \(\nu_{\text{ran}} \geq 5\), the number is tripled. As a result, the efficiency of weak-lensing cluster detection declines significantly. The increase of the number of false peaks also affects the likelihood of the existence of dark clumps. On the other hand, if one can observationally distinguish false peaks and true peaks, the number of false peaks can be used to tightly constrain the level of intrinsic alignments. The CFHTLS Deep survey’s 3.61 deg\(^2\) lensing observations and follow-up studies find that 5 out of the 14 peaks with \(\nu_{\text{ran}} > 3.5\) are likely to be false peaks, giving rise to a constraint of \(\sigma^2_{\text{clean}} \leq 1.6 \times 10^{-5}\) (1 \(\sigma\)) at the angular scale of 1’ for galaxies at redshift \(z \sim 1\). The corresponding limit on the correlation of intrinsic ellipticities is 3.2 \(\times 10^{-5}\).

**Subject headings:** cosmology: theory — dark matter — galaxies: clusters: general — gravitational lensing — large-scale structure of universe

### 1. INTRODUCTION

Gravitational lensing effects are the only ways to directly measure the distribution of dark matter in the universe (e.g., Hoekstra et al. 2006). Because of their dependence on the formation of structures, as well as on the geometry of the universe, lensing effects are sensitive to the nature of dark energy and therefore are highly promising in dark energy studies (e.g., Knox et al. 2006; Munshi et al. 2006). Weak gravitational lensing effects are mostly extracted from image distortions of background galaxies (e.g., Bartelmann & Schneider 2001). The intrinsic ellipticities of galaxies therefore present themselves as important errors in lensing observations (e.g., Kaiser & Squires 1993; Schneider 1996). It has been commonly assumed that the intrinsic ellipticities of different background galaxies are statistically uncorrelated and thus that the average shear measured over a large enough number of galaxies gives an unbiased estimate of the lensing effects (e.g., Kaiser & Squires 1993). However, the formation of galaxies is highly affected by their environment, and the shapes of galaxies can well be correlated if they are close enough. With the assumption that the shapes of galaxies are well represented by the shapes of their host dark matter halos, numerical simulations indicate that the shape correlations range from 10\(^{-5}\) to 10\(^{-3}\) on angular scales of a few arcminutes, depending on the halo masses and redshift distributions of source galaxies (e.g., Heavens et al. 2000; Croft & Metzler 2000; Jing 2002; Porciani et al. 2002; Heymans et al. 2006). The existence of such correlations can contaminate lensing signals significantly. Weak-lensing effects are directly related to lensing potentials, and thus only the gradient mode, i.e., the \(E\) mode, is expected (e.g., Crittenden et al. 2001). The presence of the \(B\) mode can therefore be used to reveal the existence of different systematics, including the intrinsic alignments, but the correction to the \(E\)-mode amplitude cannot be done in a straightforward way (e.g., Heymans et al. 2004). Downweighting or removing physically closed pairs of background galaxies in lensing analyses reduces the contamination of intrinsic alignments at the expense of increasing the shot noise if the alignments extend to relatively large scales (e.g., Heymans et al. 2004). Because of the different redshift dependences for lensing signals and for intrinsic alignments, the tomographic method based on template fitting has been proposed to isolate different components, assuming the availability of photometric redshifts for background galaxies (King & Schneider 2003). This method has also been extended to include shear-ellipticity cross-correlations in the analyses (e.g., Hirata & Seljak 2004; King 2005).

The intrinsic alignments of galaxies have been searched observationally. The SuperCOSMOS data on nearby galaxies (with median redshift \(z \sim 0.1\)) reveal a level of 10\(^{-3}\)–10\(^{-4}\) on the correlation of intrinsic ellipticities of galaxies over the angular scales of a few tens of arcminutes (Brown et al. 2002). Analyses on close pairs of galaxies from COMBO-17 data (with \(z \sim 0.6\)) find the intrinsic alignments to be consistent with zero, but with uncertainties on the order of a few times 10\(^{-4}\) on scales of a few arcminutes (Heymans et al. 2004). Investigations of the Sloan Digital Sky Survey (SDSS) main sample with \(z \sim 0.1\) and its subsamples ranging from \(z \sim 0.07\) to \(z \sim 0.21\) conclude that no significant intrinsic alignments are detected (Mandelbaum et al. 2006). The observational results on the intrinsic alignments are close to the lower limits given by different numerical simulations of dark matter halos (e.g., Heymans et al. 2006). It has been pointed out that the existence of misalignment between baryonic matter and dark matter can significantly reduce the intrinsic alignment of background galaxies in comparison with that of dark matter halos and may explain the low observational results found in different surveys (Heymans et al. 2006).
In this paper, we study the influence of the intrinsic alignment on finding mass concentrations through weak-lensing effects. Being the largest virialized objects in the universe, clusters of galaxies are important cosmological probes because their formation and evolution depend sensitively on cosmologies (e.g., Borgani 2006; Fan & Chiuhe 2001). However, large uncertainties exist in linking cluster observables, such as galaxy richness, X-ray brightness, and the Sunyaev-Zel’dovich effect, to their mass, which is the important quantity in cosmological analyses (e.g., Bode et al. 2007). On the other hand, lensing effects are generated through gravitational and depend on the total mass distribution. Thus, it is expected that a cluster sample detected through weak-lensing effects is better suited for cosmological studies, in comparison with those selected by other probes (e.g., White et al. 2002; Hamana et al. 2004; Tang & Fan 2005; Hennawi & Spergel 2005; Fang & Haiman 2007). Without involving complicated gas physics, however, weak-lensing cluster detections have their own shortcomings. Besides observational errors, physical systematics, such as projection effects and complex mass distributions of clusters of galaxies, affect the selection function of weak-lensing clusters considerably. Thus, weak-lensing cluster samples are not truly mass-selected (Tang & Fan 2005). The intrinsic ellipticities of background galaxies result in false peaks in lensing maps and reduce the efficiency of cluster detections significantly (e.g., White et al. 2002). The false peaks could also be misinterpreted as dark clumps, which might lead to a faulty conclusion regarding the validity of a cosmological model. Here we explore how the existence of the intrinsic alignment of background galaxies affects the number of false peaks in lensing convergence maps. We further propose that the number of false peaks can be used to sensitively constrain the level of the intrinsic alignment if one can separate true and false peaks observationally.

The rest of the paper is organized as follows. In § 2 we discuss the correlations of galaxy ellipticities based on the model proposed by Heymans et al. (2004, 2006). In § 3 we study the dependence of the number of false peaks on the intrinsic alignment of background galaxies. In § 4 we analyze the constraints on the intrinsic alignment from the results of the CFHTLS Deep survey on the number of false peaks given by Gavazzi & Soucail (2007). Discussions are presented in § 5.

2. INTRINSIC ALIGNMENTS OF GALAXIES

Galaxies do not form in isolated ways. Environmental effects play important roles in shaping galaxies. Therefore, correlations of the ellipticities of galaxies are expected if they are close enough.

The ellipticity of a galaxy is defined through the second moments of its surface brightness profile $S(x, y)$. Specifically, we adopt the following definitions:

$$2e_1 = \frac{I_{xx} - I_{yy}}{I_{xx} + I_{yy}}, \quad 2e_2 = \frac{2I_{xy}}{I_{xx} + I_{yy}},$$  \hspace{1cm} (1)

where $(I_{xx}$ and $I_{yy}$ have similar forms)

$$I_{xy} = \frac{\int S(x, y)(x - \bar{x})(y - \bar{y}) \, dx \, dy}{\int S(x, y) \, dx \, dy}. \hspace{1cm} (2)$$

Here $(\bar{x}, \bar{y})$ are the coordinates of the center of the galaxy image. Concerning two-point correlations $c_g(r) = \langle e_1(x)e_1(x + r) \rangle$, it is convenient to choose the $x$-axis and the $y$-axis to be, respectively, parallel to and perpendicular to the line joining the two considered galaxies in the projected plane.

Numerical simulations show that $c_{12} = \langle e_1(x)e_2(x + r) \rangle \approx 0$ (e.g., Jing 2002; Heymans et al. 2004, 2006). For $c_{ij} = \langle e_i(x)e_j(x + r) \rangle (i = 1, 2)$, we use the fitting formula provided by Heymans et al. (2004), which is

$$c_i = \frac{0.001A_i}{1 + (r/B_i)^{2}},$$  \hspace{1cm} (3)

Our following analyses primarily concern

$$\eta(r) = \langle e_1(x)e_1(x + r) \rangle + \langle e_2(x)e_2(x + r) \rangle = c_{11} + c_{22},$$

which can also be written as

$$\eta(r) = \frac{0.001A}{1 + (r/B)^{2}}. \hspace{1cm} (4)$$

Heymans et al. (2004, 2006), incorporating different galaxy models in numerical simulations, obtained the fitting values of $A$ and $B$ for each model. Comparing with SDSS observations, Mandelbaum et al. (2006) present their fitting results with $B = 1 h^{-1}$ Mpc and $A = 0.57 \pm 0.72$ (see also Heymans et al. 2006). To investigate their influence on weak-lensing effects, we need to analyze the angular correlation of intrinsic ellipticities, which is related to the three-dimensional correlation $c_g(r)$ through the following equation:

$$C_{ij}(\theta) = \int \frac{r^2\phi(r_1)\phi(r_2)\, dr_1 \, dr_2 \, [1 + \xi(r_{12})]c_{ij}(r_p, \pi)}{r^2\phi(r_1)^2\phi(r_2)^2 \, dr_1 \, dr_2 \, [1 + \xi(r_{12})]}, \hspace{1cm} (5)$$

where $\phi(r)$ and $\xi(r)$ are the selection function and the two-point correlation function for background galaxies, respectively, and $r_p$ and $\pi$ are the comoving separations of two galaxies that are perpendicular to and parallel to the line of sight, respectively. Since the correlations decrease quickly on large scales, in the small-angle limit we have (e.g., Jing 2002)

$$C_{ij}(\theta) = \int \frac{\int r^2\phi(r)^2 \, dr \Sigma_{ij}(\theta)}{\int r^4\phi(r)^2 \, dr \int d\pi \xi(r, \theta, \pi)}, \hspace{1cm} (6)$$

$$\Sigma_{ij}(r_p) = \int d\pi \, [1 + \xi(r_p, \pi)]c_{ij}(r_p, \pi). \hspace{1cm} (7)$$

For weak-lensing effects, both the convergence $\kappa$ and the shear $\gamma$ are determined by the second derivatives of the lensing potential $\phi$, and

$$\kappa = \frac{\nabla^2\phi}{2}, \quad \gamma_1 = \frac{\phi_{,11} - \phi_{,22}}{2}, \quad \gamma_2 = \phi_{,12}, \hspace{1cm} (8)$$

where $\phi_{,ij} = \partial_i \partial_j \phi$. Concerning weak-lensing detections of mass concentrations, we focus on the convergence $\kappa$-field. In the weak-lensing limit, it is related to the shear, $\gamma$, in Fourier space through

$$\tilde{\kappa}(k) = c_{\kappa}(k)\tilde{\gamma}_{\kappa}(k), \hspace{1cm} (9)$$

where summation over $\alpha = (1, 2)$ is implied and $c_{\kappa} = (\cos 2\varphi, \sin 2\varphi)$, with $k = k(\cos \varphi, \sin \varphi)$ (Kaiser & Squires 1993). Observationally, the shear $\gamma$ can be estimated from the ellipticities of galaxy images. In the weak-lensing limit, we have

$$\epsilon^{(0)} = \gamma + \epsilon^{(S)}, \hspace{1cm} (10)$$
where $e$ is defined in equation (1) and the superscripts "O" and "S" denote the observed image and the source, respectively. Then the noisy convergence $\kappa_n$, including the contamination from source ellipticities, follows

$$\tilde{\kappa}_n(k) = c_n(k) e^{(O)}_n(k) = \tilde{\kappa}(k) + c_n(k) e^{(S)}_n(k).$$

(11)

Considering smoothed quantities, we have (e.g., van Waerbeke 2000)

$$\Sigma^{(O)}(\theta) = \Gamma(\theta) + \frac{1}{n_g} \sum_{i=1}^{N_g} W(\theta - \theta_i) e^{(S)}(\theta_i),$$

(12)

$$K_N(\theta) = \int dk e^{-ik\cdot\theta} c_n(k) \tilde{\Sigma}_n^{(O)}(k),$$

(13)

where $\Sigma^{(O)}$, $\Gamma$, and $K_N(\theta)$ are the smoothed versions of $e^{(O)}$, $\gamma$, and $\kappa_n$, respectively, $W(\theta)$ is the smoothing function, and $n_g$ and $N_g$ are, respectively, the surface number density and the number of source galaxies in the field. The noise part of $K_N$ due to the intrinsic ellipticities is then

$$N(\theta) = \frac{1}{n_g} \sum_{i=1}^{N_g} \int dk \tilde{W}(k) e^{-ik\cdot(\theta-\theta_i)} c_n(k) e^{(S)}(\theta_i),$$

(14)

where $\tilde{W}(k)$ is the Fourier transformation of the smoothing function with the form

$$\tilde{W}(k) = \frac{1}{(2\pi)^2} \int d\theta e^{ik\cdot\theta} W(\theta).$$

(15)

Following van Waerbeke (2000), the correlation of $N(\theta)$ is calculated by averaging over both the ellipticities and the positions of source galaxies. Without intrinsic alignments, the correlation of $N(\theta)$ arises only from the smoothing operations, and by ignoring the nonuniform sampling of source galaxies, we have (van Waerbeke 2000)

$$\langle N(\theta)N(\theta') \rangle = \frac{\sigma_e^2}{2n_g} (2\pi)^2 \int dk e^{ik\cdot(\theta'-\theta)} \tilde{W}(k)^2,$$

(16)

where $\sigma_e$ is the intrinsic dispersion of $e^{(S)}$ and the factor $(2\pi)^2$ comes in to be in accord with the definition of $\tilde{W}(k)$ in equation (15).

Including the alignment, the operation by averaging over the ellipticities of source galaxies, denoted by $A$, following van Waerbeke (2000), is

$$A \left[ e^{(O)}_\alpha(\theta) e^{(S)}_\beta(\theta) \right] = \frac{\sigma_e^2}{2} \delta_{\alpha\beta} \delta_{(\theta_1 - \theta_2)} + \delta_{\alpha\beta} C_{\alpha\beta} (|\theta_1 - \theta_2|),$$

(17)

where $C_{\alpha\beta}(|\theta_1 - \theta_2|)$ is given in equation (6). Furthermore, by averaging over positions of galaxies, i.e., by applying the operation $(1/S^2) \int d\theta_1 d\theta_2$, with $S$ being the area of the field (van Waerbeke 2000), we get

$$\langle N(\theta)N(\theta') \rangle = \frac{\sigma_e^2}{2n_g} (2\pi)^2 \int dk e^{ik\cdot(\theta'-\theta)} \tilde{W}(k)^2 + (2\pi)^4 \int \frac{dk}{n_g} e^{ik\cdot(\theta'-\theta)} \tilde{W}(k)^2 \left[ c^2_1(k) \tilde{C}_{11}(k) + c^2_2(k) \tilde{C}_{22}(k) \right],$$

(18)

where $\tilde{C}_{11}(k)$ and $\tilde{C}_{22}(k)$ are the corresponding Fourier transformations of $C_{11}(\theta)$ and $C_{22}(\theta)$ discussed above. Thus, the zero-lag noise variance can be written as $\sigma^2_0 = \sigma^2_{0\text{ran}} + \sigma^2_{0\text{corr}}$, where

$$\sigma^2_{0\text{ran}} = \frac{\sigma^2_e}{2n_g} (2\pi)^2 \int dk |\tilde{W}(k)|^2,$$

(19)

$$\sigma^2_{0\text{corr}} = (2\pi)^4 \int \frac{dk}{n_g} |\tilde{W}(k)|^2 \left[ \frac{1}{2} \tilde{C}_{11}(k) + \tilde{C}_{22}(k) \right],$$

(20)

where the factor of $1/2$ is from the integration of $c^2_i(k)$ [and $c^2_i(k) = \sin^2(2\varphi)$ over $\varphi$].

Considering Gaussian smoothings with

$$W(\theta) = \frac{1}{\pi \theta_G^2} \exp \left(-\frac{\theta^2}{\theta_G^2}\right),$$

(21)

where $\theta_G$ is the angular smoothing scale, we have

$$\sigma^2_{0\text{ran}} = \frac{\sigma^2_e}{2} \frac{1}{2\pi \theta_G^2 n_g},$$

(22)

$$\sigma^2_{0\text{corr}} = \frac{1}{2\pi} \int d\theta_1 \frac{1}{2} |C_{11}(\theta) + C_{22}(\theta)| \frac{1}{\theta_G^2} \exp \left(-\frac{\theta^2}{2\theta_G^2}\right).$$

(23)

In the following analyses, we use equations (4), (6), and (7) to calculate $C_{11}(\theta) + C_{22}(\theta)$, and further, to get $\sigma^2_{0\text{corr}}$ from equation (23).

The angular correlations $C_{\alpha\beta}(\theta)$ depend sensitively on the redshift distribution of the background galaxies. For galaxies distributed in a narrow range around a relatively low redshift, a large fraction of them are physically close to each other, resulting in large values of $C_{\alpha\beta}(\theta)$. We adopt the following functional form to describe the distribution of background galaxies:

$$p(z) = \frac{\beta}{1 + (\alpha / \beta)} \left( \frac{z}{z_t} \right)^{\alpha - 1} \left( 1 - \frac{z}{z_t} \right)^{\beta - 1},$$

(24)

where $\alpha$, $\beta$, and $z_t$ are parameters that can be determined from survey conditions. We take $\alpha = 2$ and $z_t = 0.7$. To see the effect of the width of the distribution, we vary the $\beta$ value with $\beta = 1, 1.5, 3,$ and $6$. The larger the $\beta$ value is, the narrower the distribution is, as can be seen in Figure 1. The corresponding median redshifts for the four distributions are $z_{\text{med}} \approx 1.87, 0.99, 0.62,$ and $0.55$ for $\beta = 1, 1.5, 3,$ and $6$, respectively.

In Figure 2, we show the results of $\sigma^2_{0\text{corr}}$. For the intrinsic alignment, we take $A = 0.57$, the value from the SDSS, in equation (4) (e.g., Heymans et al. 2006). The solid, dotted, dashed, and dash-dotted lines correspond to values of $\beta = 6, 3, 1.5,$ and $1$, respectively. For comparison, we also plot $\sigma^2_{0\text{ran}}$ (dash–double-dotted line), using $\sigma_e = 0.4$ and $n_g = 30$ arcmin$^{-2}$. We see that the result with $\beta = 6$ and $z_{\text{med}} \approx 0.55$ is an order of magnitude larger than that with $\beta = 1$ and $z_{\text{med}} \sim 1.87$, demonstrating clearly the sensitive dependence of $\sigma^2_{0\text{corr}}$ on the redshift distribution of background galaxies. Therefore, for tomographic analyses of weak-lensing effects with source galaxies distributed in narrow redshift bins, the effects of intrinsic alignments can be significant. The angular dependence of $\sigma^2_{0\text{corr}}$ is shallower than that of $\sigma^2_{0\text{ran}}$, and the ratio of $\sigma^2_{0\text{corr}}/\sigma^2_{0\text{ran}}$ increases with the increase of smoothing scales.

In Table 1, we list the values of $\sigma^2_{0\text{corr}}/\sigma^2_{0\text{ran}}$ for various cases. With an upper limit of $A = 1.29$ from the SDSS, the ratio can reach...
as high as about $\sigma_{\text{corr}}^2/\sigma_{\text{ran}}^2 \sim 20\%$ for $\beta = 6$ at $\theta_G = 2'$. Note that $\sigma_{\text{ran}}^2$ and $\sigma_{\text{corr}}^2$ depend differently on the distribution of source galaxies. While $\sigma_{\text{ran}}^2$ depends mainly on the form of the redshift distribution, $\sigma_{\text{ran}}^2 \propto n_g$. Thus, for surveys with a higher surface number density of source galaxies than what we consider here, the ratio $\sigma_{\text{corr}}^2/\sigma_{\text{ran}}^2$ can increase considerably. Results expected for some surveys are presented in Table 2. The survey parameters for COSMOS are taken from Massey et al. (2007). For SNAP, we adopt the parameters used in Semboloni et al. (2007). For deep surveys with large values of $n_g$, tomographic analyses with source galaxies distributed in narrow redshift ranges become possible. For example, with a total of $n_g \sim 100$ arcmin$^{-2}$, as is expected from surveys similar to SNAP, the background galaxies can be divided into three bins, each with $n_g \sim 30$ arcmin$^{-2}$. The effect of intrinsic alignments can be significantly stronger within individual bins than that in total. If we regard the narrow redshift distribution with $\beta = 6$ as one of the bins, it can be seen from Table 1 that the respective values of $\sigma_{\text{corr}}^2/\sigma_{\text{ran}}^2$ for $\theta_G = 1'$ and $2'$ are about 6% and 10% with $A = 0.57$, in comparison with the values of 3.3% and 5% expected for the full sample of galaxies from SNAP, as can be seen in Table 2.

In the next section, we show that the number of false peaks in lensing $\kappa$-maps is very sensitive to the ratio of $\sigma_{\text{corr}}^2/\sigma_{\text{ran}}^2$. Even a relatively low value of $\sigma_{\text{corr}}^2$ can result in a considerable increase of the number of false peaks and therefore reduce the efficiency of cluster detections significantly.

3. STATISTICS OF FALSE PEAKS IN $\kappa$-MAPS RESULTING FROM INTRINSIC ELLIPTICITIES

Weak-lensing cluster detections associate high peaks in $\kappa$-maps reconstructed from shear measurements with clusters of galaxies. Intrinsic ellipticities of background galaxies can produce false peaks and therefore affect the efficiency of cluster detections. It is thus important to understand the statistics of false peaks thoroughly in order to extract reliable cluster samples from weak-lensing surveys. Van Waerbeke (2000) studies the number of false peaks, assuming no intrinsic alignments for source galaxies. In this case, the smoothed quantity $N(\theta)$ defined in equation (14) is approximately a Gaussian random field because of the central limit theorem. When the correlations of the intrinsic ellipticities are included, the statistics of the noise field $N(\theta)$ can be complicated. Relating the intrinsic ellipticities linearly with the tidal field predicts Gaussian statistics. Assuming that they are associated with galaxy spins gives rise to non-Gaussian statistics for the intrinsic ellipticities. On the other hand, $N(\theta)$ is related to the sum of the intrinsic ellipticities of the background galaxies in the smoothing window. Since the intrinsic alignments are relatively weak (on the order of $10^{-6} - 10^{-5}$ at $\theta_G \sim 1'$), according to the central limit theorem, we do not expect a highly non-Gaussian field for $N(\theta)$ if the number of galaxies within the smoothing window is large enough. In our following analyses, we assume Gaussianity for $N(\theta)$. Detailed studies on its statistics will be carried out in our future investigations.

For a two-dimensional Gaussian random field $N$, the differential number density of the peaks can be written explicitly in the following form (Bond & Efstathiou 1987; van Waerbeke 2000):

$$n_{\text{peak}}(\nu) = \frac{1}{2\pi\sigma_\theta^2} \exp\left(-\frac{\nu^2}{2}\right) G(\gamma_\nu, \gamma_\nu, \nu) = \frac{1}{2\pi\sigma_\theta^2} \exp\left(-\frac{\nu^2}{2}\right) G(\gamma_\nu, \gamma_\nu, \nu).$$

TABLE 2

| Survey  | $\langle z \rangle$ | $\langle z \rangle$ | $n_g$ | $\theta_G = 1'$ | $\theta_G = 1'$ |
|---------|-------------------|-------------------|-------|----------------|----------------|
| COSMOS  | 1.2               | 0.8               | 1.13  | 70             | 2.3            |
| SNAP    | 1.2               | 0.8               | 1.13  | 100            | 3.3            |
| SNAP (Deep) | 1.4             | 0.93              | 1.3   | 300            | 8.4            |

Note.—For the redshift distribution, we take $\alpha = 2$ and $\beta = 1.5.$

![Figure 1](image1.png)

**Fig. 1.**—Redshift distribution of background galaxies with the functional form given in eq. (24). Here we take $\alpha = 2$ and $z_\eta = 0.7$. The solid, dotted, dashed, and dash-dotted lines correspond to the results with $\beta = 6$, 3, 1.5, and 1, respectively.

![Figure 2](image2.png)

**Fig. 2.**—Variance contributed by intrinsic alignments. We take $A = 0.57$ in eq. (4) and $\alpha = 2$ and $z_\eta = 0.7$ in eq. (24). The solid, dotted, dashed, and dash-dotted lines correspond to the results with $\beta = 6$, 3, 1.5, and 1, respectively. The dash-double-dotted line indicates $\sigma_{\text{ran}}^2$ with $\sigma_\theta = 0.4$ and $n_g = 30$ arcmin$^{-2}$.
where \( \nu = N/\sigma_0 \) is the significance of a peak with \( N \) being the value of the considered quantity at the peak position, \( b = [2(1 - \gamma_p^2)]^{1/2} \), and

\[
G(\gamma_p, \hat{x}) = \frac{1}{2} \left( \frac{\hat{x}^2 + b^2}{2} - 1 \right) \text{erfc} \left( \frac{\hat{x}}{b} \right) + \frac{1}{2(1 + b^2)^{1/2}} \text{erfc} \left( \frac{\hat{x}}{b\sqrt{1 + b^2}} \right).
\]

(26)

It can be seen that \( n_{\text{peak}}(\nu) \) is fully characterized by \( \gamma_p \) and \( \theta_* \), which are respectively defined as

\[
\gamma_p = \frac{\sigma_1}{\sigma_0 \sigma_2}, \quad \theta_* = \sqrt{2} \frac{\sigma_1}{\sigma_2}, \quad \sigma_n^2 = \int dk k^{2n} \langle |N(k)|^2 \rangle.
\]

(27)

(28)

Considering the noise field \( N(\theta) \) defined in equation (14), with Gaussian smoothings, we have \( \gamma_p = \sqrt{2}/2 \) and \( \theta_* = \theta_G/\sqrt{2} \) in the case without intrinsic alignments (van Waerbeke 2000). Thus, the average cumulative number density of peaks \( N_{\text{peak}}(\nu_{\text{ran}}) = \int_{\nu_{\text{ran}}} n_{\text{peak}}(\nu') d\nu' \) is independent of \( \sigma_{0,\text{ran}} \) and scales with the smoothing angle as \( \theta_*^{-2} \). In this case, given a survey area and a smoothing angle \( \theta_* \), the average number of false peaks in terms of the significance \( \nu_{\text{ran}} \) is fixed regardless of the specific value of \( \sigma_{0,\text{ran}} \). Note that the existence of false peaks is the result of chance alignments of background galaxies. Given \( \nu_{\text{ran}} = N/\sigma_{0,\text{ran}} \) for a false peak, its strength \( N \) is proportional to \( \sigma_{0,\text{ran}} \), which is in turn determined by \( n_g \) and \( \sigma_e \). Thus, the number of false peaks as measured by their strength \( N \) depends on \( n_g \) or larger values of \( \sigma_e \), which give rise to larger values of \( \sigma_{0,\text{ran}} \) and lead to higher probabilities in forming false peaks with large values of \( N \) by chance alignments.

When the intrinsic alignments are included, both \( \gamma_p \) and \( \theta_* \), and thus the number density of false peaks in terms of the true significance \( \nu \), depend on the correlation level. We note from Figure 2 that the level of the intrinsic alignments from current observations is low compared with \( \sigma_{0,\text{ran}} \). Therefore, \( \gamma_p, \theta_* \), and \( n_{\text{peak}}(\nu) \) change only slightly with respect to the case without intrinsic alignments.

The number of false peaks discussed above is given in terms of the true significance \( \nu = N/\sigma_0 \); i.e., the peak height is measured relative to the full noise variance \( \sigma_0 = (\sigma_{0,\text{ran}}^2 + \sigma_{\text{corr}}^2)^{1/2} \), including \( \sigma_{\text{corr}} \). Observationally, however, it is difficult to obtain \( \sigma_{0,\text{corr}} \), and thus usually only \( \sigma_{0,\text{ran}} \) is estimated and used in measuring the significance of a peak. The true significance of the peak corresponding to the observed significance \( \nu_{\text{ran}} = N/\sigma_{0,\text{ran}} \) is then \( \nu = \nu_{\text{ran}}/(1 + \sigma_{0,\text{corr}}^2/\sigma_{0,\text{ran}}^2)^{1/2} \). Given a threshold on \( \nu_{\text{ran}} \), the average number of peaks is \( N_{\text{peak}}(\nu_{\text{ran}}) \propto \int_{\nu_0} n_{\text{peak}}(\nu') d\nu' \). Because \( \nu < \nu_{\text{ran}} \) for nonzero values of \( \sigma_{0,\text{corr}} \), \( N_{\text{peak}} \) increases with the increase of \( \sigma_{0,\text{corr}} \).

In Figure 3, we show \( N_{\text{peak}} - N_{\text{peak}}^{\text{ran}} \), the number of false peaks resulting from intrinsic alignments, with respect to the detection threshold \( \nu_{\text{ran}} \), where \( N_{\text{peak}} \) and \( N_{\text{peak}}^{\text{ran}} \) are the cumulative numbers of false peaks in 1 deg^2 with and without intrinsic alignments. For comparison, we also plot \( N_{\text{peak}}^{\text{ran}} \) (thick solid lines) in each panel. The left and right panels are, respectively, for \( \theta_G = 1' \) and \( \theta_G = 2' \). The top panels are for \( A = 0.57 \), and the bottom panels are for \( A = 1.29 \). The thin solid, dotted, dashed, and dash-dotted lines in each panel are for \( \beta = 6, 3, 1.5, \) and 1, respectively.

It can be seen that \( N_{\text{peak}} - N_{\text{peak}}^{\text{ran}} \) is comparable to \( N_{\text{peak}}^{\text{ran}} \). For \( \theta_G = 1' \), \( N_{\text{peak}} - N_{\text{peak}}^{\text{ran}} \) is comparable to \( N_{\text{peak}}^{\text{ran}} \). For \( \theta_G = 1' \), \( N_{\text{peak}} - N_{\text{peak}}^{\text{ran}} \) is comparable to \( N_{\text{peak}}^{\text{ran}} \). For larger

Fig. 3.—Cumulative number of false peaks resulting from intrinsic alignments in 1 deg^2. The thick solid line shows the cumulative number of false peaks without intrinsic alignment. The thin solid, dotted, dashed, and dash-dotted lines are those for \( \beta = 6, 3, 1.5, \) and 1, respectively.
values of $\beta$, the numbers are larger, and $N_{\text{peak}} - N_{\text{ran}} > N_{\text{ran}}^{\text{peak}}$
when $\nu_{\text{ran}} > 3.7$ and 4.5 for $\beta = 6$ and 3, respectively. For $\theta_G = 2'$,
$N_{\text{peak}} - N_{\text{ran}}^{\text{peak}} > N_{\text{ran}}^{\text{peak}}$ when $\nu_{\text{ran}} > 3, 3.8, \text{and } 5.3$ for $\beta = 6, 3$, and 1.5, respectively. Therefore, the existence of intrinsic
alignments can result in a significant number of extra false peaks in
lensing convergence maps.

In Figure 4, we show the dependence of the ratio $r_{\text{peak}} = N_{\text{peak}} / N_{\text{ran}}$ on the level of intrinsic alignments represented by
the amplitude $A$ for $\theta_G = 1'$. The $\beta$-value in each panel is written
out explicitly. The solid, dotted, dashed, and dash-dotted lines are,
respectively, for $\nu_{\text{ran}} = 5, 4.5, 4,$ and 3.5. For $\beta = 1.5$ and $A = 1.29$, we have $r_{\text{peak}} \sim 1.36, 1.27, 1.2,$ and 1.14 for $\nu_{\text{ran}} = 5, 4.5, 4,$ and 3.5, respectively. With larger smoothing scales, the
relative effect of intrinsic alignments is higher. For $\theta_G = 2'$, the
corresponding ratios change to 1.6, 1.4, 1.3, and 1.2. For $\beta = 6$, the
ratios for $\nu_{\text{ran}} = 5$ reach as high as 3.8 and 7.6 for $\theta_G = 1'$
and $2'$, respectively.

From equations (25) and (26), it can be shown that $N_{\text{peak}}$ depends
largely on the detection threshold, with $N_{\text{peak}} \propto \nu \exp(-\nu^2/2)$
when $\nu > 3$ (e.g., van Waerbeke 2000). Given a detection threshold
on $\nu_{\text{ran}}$, the corresponding threshold for the true significance is
$\nu = \nu_{\text{ran}}/(1 + \sigma_0^{\text{corr}}/\sigma_0^{\text{ran}})^{1/2}$, which decreases with the increase
of $\sigma_0^{\text{corr}}/\sigma_0^{\text{ran}}$. Thus, the value of $r_{\text{peak}}$ is largely determined by the
ratio $\sigma_0^{\text{corr}}/\sigma_0^{\text{ran}}$. In Figure 5, we show $r_{\text{peak}}$ with respect to
$\sigma_0^{\text{corr}}/\sigma_0^{\text{ran}}$ for $\theta_G = 1'$ (top) and $2'$ (bottom). The four sets of lines
from top to bottom in each panel correspond, respectively, to the
threshold values of $\nu_{\text{ran}} = 5, 4.5, 4,$ and 3.5. Note that each set contains
four lines with $\beta = 6, 3, 1.5,$ and 1 and that the lines are largely
overlapped. With $\sigma_0^{\text{corr}}/\sigma_0^{\text{ran}} \sim 5\%$, the values of $r_{\text{peak}}$ are about $1.7, 1.55, 1.4,$ and $1.3$ for $\nu_{\text{ran}} = 5, 4.5, 4,$ and 3.5. For $\sigma_0^{\text{corr}}/\sigma_0^{\text{ran}} \sim 10\%$, the corresponding values of $r_{\text{peak}}$ are $2.9, 2.3, 1.9,$ and 1.6.
A specific value of $\sigma_0^{\text{corr}}/\sigma_0^{\text{ran}}$ depends on the strength of the
intrinsic alignment, the surface number density, the redshift distri-

Fig. 4.—Dependence of the ratio $r_{\text{peak}} = N_{\text{peak}} / N_{\text{ran}}$ on $A$ for $\theta_G = 1'$. Different panels show the results with different $\beta$-values. The solid, dotted, dashed, and dash-dotted lines are for $\nu_{\text{ran}} = 5, 4.5, 4,$ and 3.5, respectively. The solid and dotted vertical lines are, respectively, at the positions of $A = 0.57$ and $A = 1.29$. 

bution of source galaxies, and $\sigma_e$. The dotted vertical lines from
left to right in each panel show the corresponding values of $\sigma_0^{\text{corr}}/\sigma_0^{\text{ran}}$ for $\beta = 1.5, 3, 6$, and 6, where we take $A = 1.29$, $\sigma_e = 0.4$, and $n_g = 30$ arcmin$^{-2}$. It should be noted that $n_g$
usually varies with the redshift distribution of source galaxies.

Surveys that can reach high redshifts typically have large values
of $n_g$. Thus, our estimates on $\sigma_0^{\text{corr}}/\sigma_0^{\text{ran}}$ with a fixed value of $n_g = 30$ arcmin$^{-2}$ may overestimate the ratio for $\beta = 6$. On the
other hand, for deep surveys with large values of $n_g$ (e.g., $n_g \sim 100$ for SNAP and $n_g \sim 300$ for SNAP Deep), we can divide the source galaxies into different bins with $n_g \sim 30$ arcmin$^{-2}$
in each bin. In this case, the narrow distribution with $\beta = 6$ can be
one of these bins, and our above estimate on $\sigma_0^{\text{corr}}/\sigma_0^{\text{ran}}$ with
$n_g = 30$ arcmin$^{-2}$ can be a representative value for galaxies within
the bin.

For weak-lensing cluster surveys, the efficiency measures how
efficiently we can find true clusters from lensing maps. If we assume
the NFW profile (Navarro et al. 1996) for the mass distribution
of clusters of galaxies, we find that the number of peaks resulting from true clusters in lensing $\kappa$-maps is about 6 and 4 deg$^{-2}$
for significances larger than 3.5 and 4, respectively, where the values
of $\sigma_e = 0.4, n_g = 30$ arcmin$^{-2}$, and $\theta_G = 1'$ are used (Hamana
et al. 2004). As we showed previously, the corresponding values of
$N_{\text{peak}}^{\text{ran}}$ are about 2 and 0.3. Then a simple estimate gives an
efficiency of about 75% and 93% for the two detection thresholds
if there are no intrinsic alignments. With $\sigma_0^{\text{corr}}/\sigma_0^{\text{ran}} \sim 10\%$, the
corresponding efficiencies drop to 65% and 87%. Note that we only
consider the contamination from false peaks when estimating
the above efficiencies. The existence of intrinsic ellipticities and
alignments not only results in false peaks, but also affects the
heights of the true peaks, which can further decrease the efficiency
of weak-lensing cluster detections considerably (e.g., Hamana
et al. 2004).
Because weak-lensing effects arise from the gravitational influence of the matter distribution, it is expected that dark clumps without luminous counterparts could be discovered from lensing observations. The existence of massive dark clumps would seriously call into question the current theory of structure formation. There have been such candidates reported in the literature. Erben et al. (2000) present Canada-France-Hawaii Telescope (CFHT) weak-lensing results around the galaxy cluster Abell 1942. They find a high peak (~5 σ in the aperture-mass measurement; Schneider 1996) without associated galaxy overdensities at a location about 7′ south of the main cluster. Faint X-ray emissions from the nearby region of the peak were detected by the Röntgensatellit (ROSAT), but they may not be related to the lensing peak signal. With Hubble Space Telescope (HST) lensing observations in this field, von der Linden et al. (2006) also find a peak at a place consistent with that given by Erben et al. (2000), but with a much lower significance, ~2.9 σ. Further, they divide the source galaxies into three magnitude bins and perform lensing analyses for each of them. For the bright bin, which contains most of the source galaxies used in Erben et al. (2000), they find a 1.9 σ peak with a smoothing scale of 120′. For the faint bin, a 3.3 σ peak is detected. There is no lensing detection from the medium bin, which is unexpected if there is a foreground dark clump. A spatial concentration of galaxies in the medium bin is observed, which could act as the lens for galaxies in the faint bin, but not for those in the bright bin. The lack of lensing detections in the medium bin and the low significance of the peak from HST observations raise questions as to the lensing origin of the peak. It is likely that the peak is a statistical fluke (von der Linden et al. 2006). On the other hand, Erben et al. (2000) estimate the probability that their detected peak is a false one from chance alignments of background galaxies. In order to apply the results given by van Waerbeke (2000), they perform a Gaussian smoothing with θG ≈ 0.5′ to the κ-field and find that the considered peak has a height of νran ≈ 4.5σ. The probability to have such a high peak from chance alignments is very low (Erben et al. 2000).

Our analyses show that the existence of intrinsic alignments can increase the chance for the appearance of false peaks in a given area depending on the ratio of σcorr/σran. In Table 3, we list the probabilities that the detected “dark clumps” are false peaks for different observations. The probability is calculated from Poisson statistics with

$$p_n = e^{-N_{\text{ran}}} \frac{N_{\text{peak}}^n}{n!},$$

where $p_n$ is the probability to have $n$ false peaks in a field and $N_{\text{peak}}$ is the average number of false peaks expected in the field. For the observation of Erben et al. (2000), we estimate that $σ_{\text{corr}}/σ_{\text{ran}} \sim 1.3%$ (with $A = 1.29$) for $θ_G = 0.5′$. With this level of intrinsic alignment, the probability to find one $ν_{\text{ran}} = 4.5$ false peak in a field of $14′ \times 14′$ increases only slightly from 0.9% to 1%. On the other hand, with a noise level comparable to that of Erben et al. (2000), the average surface number density of peaks with $ν_{\text{ran}} \geq 4.5$ resulting from the lensing effects of true mass concentrations is about 3 deg$$^{-2}$$ for $θ_G = 0.5′$ (Hamana et al. 2004). Then the average number of true peaks in a field of $14′ \times 14′$ is about 0.15, and the probability to have one true peak in this field is ~13%. Thus, the $ν_{\text{ran}} = 4.5$ peak found by Erben et al. (2000) is much more likely to be associated with a true mass clump than it is to be a false peak. However, the analyses on HST data by von der Linden et al. (2006) give $ν_{\text{ran}} \sim 3$ for the peak. The average number of false peaks expected in a field of $14′ \times 14′$ with $ν_{\text{ran}} \geq 3$ is about 1.7 with $σ_{\text{corr}} = 0$. For the source galaxies in

Figure 5.—Ratio $r_{\text{peak}}$ with respect to $σ_{\text{corr}}^2/σ_{\text{ran}}^2$. The top and bottom panels correspond to $θ_G = 1′$ and $θ_G = 2′$, respectively. The four sets of solid, dotted, dashed, and dash-dotted lines are, respectively, for $ν_{\text{ran}} = 5, 4, 5, 4$, and 3.5. Each set of lines contains results with $β = 6, 3, 1.5$, and 1, and the lines are largely overlapped. The vertical dotted lines from right to left show the values of $σ_{\text{corr}}^2/σ_{\text{ran}}^2$ with $A = 1.29$ for $β = 6, 3, 1.5$, and 1, respectively.
von der Linden et al. (2006), we estimate that \( \sigma^2_{\text{corr}}/\sigma^2_{\text{ran}} \sim 5\% \) with \( A = 1.29 \). Then the average number of false peaks increases to \( \sim 2 \), and the corresponding probability \( p(\text{corr}) \sim 27\% \). In this case, it is quite possible that the observed peak is a false one. The reason for the difference between the peak heights from CFHT and HST is unclear (von der Linden et al. 2006), and so is the conclusion on the origin of the peak. It is likely that the peak is associated with a small mass clump with its height enhanced by the chance alignment of background galaxies (von der Linden et al. 2006).

Massey et al. (2007) present the COSMOS 2 deg\(^2\) lensing analysis. They note the existence of two high peaks without luminous counterparts near the main cluster. The \( \kappa \)-field in Massey et al. (2007) is reconstructed using the wavelet method, and its noise properties are complicated (e.g., Starck et al. 2006). The significances of the peaks are not clearly given in Massey et al. (2007). Thus, in Table 3, we include the probabilities for different significances. When calculating \( p(\text{corr}) \), we use the value of \( \sigma^2_{\text{corr}}/\sigma^2_{\text{ran}} \sim 5\% \) estimated for the redshift distribution of source galaxies with \( \alpha = 2, \beta = 1.5, \) and \( z_s = 0.8 \). It can be seen that with the intrinsic alignments, the probability that the two peaks are false ones is tripled for values of \( \nu_{\text{ran}} > 5 \). We note that our results are for Gaussian smoothings, and therefore they cannot be used directly to discuss how likely it is for the peaks found by Massey et al. (2007) to be false ones. On the other hand, the COSMOS data can be readily analyzed with the method of Kaiser & Squires (1993) with Gaussian smoothings. Then our studies presented here can be directly applicable.

### 4. CONSTRAINTS ON INTRINSIC ALIGNMENTS FROM THE CFHTLS DEEP SURVEY

Because of its sensitive dependence on intrinsic alignments, the number of false peaks can be used to probe the strength of the intrinsic alignments of source galaxies if one can observationally distinguish false and true peaks. In this section, we analyze the constraints on \( \sigma^2_{\text{corr}} \) from the results of the CFHTLS Deep survey (Gavazzi & Soucail 2007).

The CFHTLS Deep survey shares the same data with the Supernova Legacy Survey (SNLS). It contains four independent fields and includes data from five bands \((g', r', i', z', u')\). The shear measurements are done using the \( \beta \)-band images with magnitudes in the range \( 22 < i' < 26 \). The seeing is \( \sim 0.9'' \). The total working area for weak-lensing analysis is 3.61 deg\(^2\). The photometric redshift is estimated for each source galaxy with the multiband observational data. The redshift distribution for a subsample of source galaxies with reliable photo-\(z\) measurements is presented in Figure 3 of Gavazzi & Soucail (2007), which will be used in our following analysis. The convergence \( \kappa \)-map is constructed from the inferred shear \( \gamma \) with the technique developed by Kaiser & Squires (1993). The shear, and consequently the \( \kappa \)-fields, are smoothed with a Gaussian window function with \( \theta_{\text{w}} = 1' \). The variance of noise in the smoothed \( \kappa \)-field from randomly oriented background galaxies is estimated to be \( \sigma_{\text{ran}} = 0.0196, 0.0225, 0.0202, \) and \( 0.0221 \) for the four fields (Gavazzi & Soucail 2007). The signal-to-noise ratio \( \nu \) is defined as \( \nu_{\text{ran}} = \nu_{\text{ran}}/\sigma_{\text{ran}} \). From the \( \kappa \)-maps of the four fields, Gavazzi & Soucail (2007) detect \( \sim 46 \) peaks with \( \nu_{\text{ran}} > 3 \), 14 peaks with \( \nu_{\text{ran}} > 3.5 \), and 5 peaks with \( \nu_{\text{ran}} > 4 \). Detailed studies are done for the 14 peaks with \( \nu_{\text{ran}} > 3.5 \). With the help of photometric redshift measurements, X-ray observations, and the lensing tomographic analysis, they claim that there are 9 secure cluster detections among the 14 peaks. The other five are likely false peaks. In our study here, we regard these five peaks as false ones resulting from the intrinsic ellipticities of background galaxies and constrain the level of intrinsic alignments on the basis of our analysis presented in \\( \S 3 \). It is worth mentioning that the statistics based on only five peaks is poor, and therefore our analysis mainly aims at demonstrating the feasibility of extracting the information of the intrinsic alignments of background galaxies from the number of false peaks. Also, some of the five peaks may result from dark clumps without luminous counterparts. Larger weak-lensing cluster surveys with more reliable tomographic analyses will provide statistically meaningful results on the intrinsic alignments.

We model the redshift distribution shown in Gavazzi & Soucail (2007) as

\[
p(z) \propto \left( \frac{z}{z_s} \right)^2 \exp \left[ -\left( \frac{z}{z_s} \right)^2 \right] + 0.07 \exp \left[ -\frac{(z - 2.8)^2}{0.6^2} \right],
\]

where \( z_s \) is taken to be 0.8. The second term is added to describe the low bump at \( z = 3 \) that is seen in the redshift distribution of the source galaxies of the CFHTLS Deep survey (Gavazzi & Soucail 2007). This term does not affect our results much because of its low amplitude. We calculate \( \sigma_{\text{corr}}^2 \) from equation (23) with \( \theta_{\text{w}} = 1' \), where \( C_{11} \) and \( C_{22} \) are computed from equations (4)–(7).

In Figure 6, we show the dependence on \( \sigma^2_{\text{corr}}/\sigma^2_{\text{ran}} \) of the expected number of false peaks detected in 3.61 deg\(^2\). The solid line shows the average number of false peaks. The upper and lower dashed lines show the \(-1 \sigma \) and \(-2 \sigma \) Poisson deviations from the mean, i.e., \( N_{\text{peak}} - \sqrt{N_{\text{peak}}} \) and \( N_{\text{peak}} - 2 \sqrt{N_{\text{peak}}} \), respectively. The horizontal dash-dotted line is located at \( N_{\text{peak}} = 5 \). The vertical dotted lines indicate the values of \( \sigma^2_{\text{corr}}/\sigma^2_{\text{ran}} \) for the source galaxies of the CFHTLS Deep survey with \( A = 0.57 \) and \( A = 1.29 \), from left to right, respectively. We see that \( 1 \sigma \) and \( 2 \sigma \) constraints give values of \( \sigma^2_{\text{corr}}/\sigma^2_{\text{ran}} < 4\% \) and \( < 14\% \). With \( \sigma^2_{\text{ran}} = 0.0004 \), we have \( \sigma^2_{\text{corr}} < 1.6 \times 10^{-3} \) and \( \sigma^2_{\text{corr}} < 5.6 \times 10^{-3} \) for \( 1 \sigma \) and \( 2 \sigma \) constraints. From equation (23), it can be seen that \( 2 \sigma_{\text{corr}} \) corresponds to the intrinsic alignment \( C_{11} + C_{22} \) smoothed over

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**TABLE 3**

| Observation                      | Area \((\text{deg}^2)\) | \(n_i\) \((\text{arcmin}^{-2})\) | \(\theta_{\text{w}}\) \((\text{arcmin})\) | \(\nu_{\text{ran}}\) | \(n_{\text{clump}}\) | \(p(\text{ran})\) \((\%)\) | \(p(\text{corr})\) \((\%)\) |
|----------------------------------|--------------------------|---------------------------------|---------------------------------|-----------------|-----------------|------------------|------------------|
| Erben et al. (2000)................| 0.05                     | 20                              | 0.5                             | 4.5             | 1               | 0.9              | 1                |
| Von der Linden et al. (2006).....| 0.05                     | 65                              | 0.5                             | 3               | 1               | 30               | 27               |
| Massey et al. (2007)..............| 2                        | 70                              | 1                               | 4               | 2               | 10               | 15               |
|                                  | 2                        | 70                              | 1                               | 4.5             | 2               | 0.3              | 0.6              |
|                                  | 2                        | 70                              | 1                               | 5               | 2               | 0.003            | 0.01             |

**Note.**—Here \( n_{\text{clump}} \) denotes the number of clumps found in the field, \( p(\text{ran}) \) represents the probability without intrinsic alignments, and \( p(\text{corr}) \) is the probability taking into account intrinsic alignments with \( A = 1.29 \).
the angular scale $\theta_G$. Thus, we find the 1 $\sigma$ and 2 $\sigma$ constraints on $C_{11} + C_{22}$ for $\theta_G = 1'$ to be $<3.2 \times 10^{-5}$ and $<1.1 \times 10^{-4}$. The corresponding limits on the amplitude of the intrinsic alignments $A$ are $A < 2.9$ and $A < 10$. The results are fully consistent with those from SDSS observations with $A < 1.29$.

Schirmer et al. (2007) analyze a total of $\sim 20$ deg$^2$ of data collected from different observations with different observational depths. They present a sample of shear-selected clusters containing a total of 158 candidates identified by two types of statistics. Using only S-statistics, which is similar to the aperture mass statistics but with different filtering functions, they find 91 peaks with significances higher than 4. Among them, there are 48 dark peaks without obvious optical counterparts. It is found that the fraction of dark peaks is relatively high in shallow surveys with a low surface number density of source galaxies. This indicates that a significant number of dark peaks could be false ones resulting from intrinsic ellipticities. Since their filtering functions are complicated with different filtering scales, we cannot do quantitative analyses on the constraints on the intrinsic alignments with these dark peaks. However, we may give some rough estimates. In Schirmer et al. (2007), the filtering scales used in peak identifications range from 1.6' to 19.8', with most of them being larger than 2'. When we compare the functional form of the Gaussian smoothing with the filtering functions used in Schirmer et al. (2007), their filtering should have the effects corresponding to Gaussian smoothings with $\theta_G \geq 1'$. Thus, we use the results of $\theta_G = 1'$ for a conservative discussion. Without intrinsic alignments, our results show that the average number of false peaks in 20 deg$^2$ with $\nu_{\text{ran}} \geq 4$ is about six. The existence of intrinsic alignments enhances the average number of false peaks. If the 48 dark peaks are all false ones and the number is $+1 \sigma$ from the average number of false peaks, we need the average number to be $N_{\text{peak}} \sim 42$. Then we have to have a value of $\sigma^2_{\text{ran}}/\sigma^2_{\text{corr}} \sim 35\%$ to get such a high number of false peaks. For the redshift distribution of source galaxies, Schirmer et al. (2007) give $\alpha = 2$, $\beta = 1.5$, and $z_s = 0.4$ for shallow surveys. The surface number density is $n_g \sim 12$ arcmin$^{-2}$, and $\sigma_r \sim 0.48$ (Schirmer et al. 2007). We then estimate $\sigma^2_{\text{corr}} \sim 0.0015$. Thus, the ratio $\sigma^2_{\text{ran}}/\sigma^2_{\text{corr}} \sim 35\%$ requires the parameter $A$ to be $A \sim 32$, which is much higher than the constraint $A \leq 1.29$ from the SDSS. Therefore, it is very unlikely that the 48 dark peaks are all false ones from intrinsic ellipticities of background galaxies. As we discussed previously, and as is also discussed in Schirmer et al. (2007), the joint effects of small mass clumps and the intrinsic ellipticities could contribute significantly to the number of dark peaks with high significance. It should be pointed out that the functional form and the scale of the filtering function adopted by Schirmer et al. (2007) are optimized to detect clusters with NFW density profiles. With Gaussian smoothings, the number of peaks and their properties may change quantitatively. It is therefore desirable to analyze the observations with Gaussian smoothings so that we can perform detailed analyses on the statistics of false peaks. On the other hand, it is also worthwhile to investigate the noise properties and the associated statistics of false peaks under different smoothing schemes. As the catalog of Schirmer et al. (2007) is the largest one so far, from many aspects careful observational and theoretical studies on these dark peaks are highly valuable.

Future surveys with larger areas will result in many more peaks. If a large number of false peaks from intrinsic ellipticities can be securely identified, we can put tight constraints on the level of intrinsic alignments. Considering Poisson fluctuations, we can estimate, as follows, how well the quantity $x = \sigma_{\text{ran}}^2/\sigma_{\text{corr}}^2$ can be constrained from $N$ false peaks with $\nu_{\text{ran}} \geq 10$. With an average cumulative number of peaks of $N_{\text{peak}} \propto \nu \exp(-\nu^2/2)$ for $\nu \geq 3$, where $\nu$ is the true significance (e.g., van Waerbeke 2000), we have, for the central value of $x$, denoted by $x_c$,

$$\int \frac{N_{\text{ran}}}{\sqrt{1 + x_c}} \exp\left(-\frac{\nu^2}{2}/(1 + x_c)\right) \frac{\nu_0}{\exp(-\nu_0^2/2)} = \frac{N_{\text{ran}}}{\sqrt{1 + x_c}} \exp\left[\frac{\nu^2_0 x_c}{2(1 + x_c)}\right] = N,$$

where $N_{\text{ran}}$ is the average number of false peaks expected in the field without considering intrinsic alignments. The $\pm 1 \sigma$ constraints on $x$ can then be obtained by

$$\frac{N_{\text{ran}}}{\sqrt{1 + x_c + \delta x}} \exp\left[\frac{\nu^2_0 (x_c + \delta x)}{2(1 + x_c + \delta x)}\right] - \sqrt{\frac{N_{\text{ran}}}{\sqrt{1 + x_c + \delta x}}} \exp\left[\frac{\nu^2_0 (x_c + \delta x)}{2(1 + x_c + \delta x)}\right] = N,$$

$$\frac{N_{\text{ran}}}{\sqrt{1 + x_c - \delta x}} \exp\left[\frac{\nu^2_0 (x_c - \delta x)}{2(1 + x_c - \delta x)}\right] + \sqrt{\frac{N_{\text{ran}}}{\sqrt{1 + x_c - \delta x}}} \exp\left[\frac{\nu^2_0 (x_c - \delta x)}{2(1 + x_c - \delta x)}\right] = N.$$

With $\delta x \ll 1$, we have

$$\delta x \approx \frac{1}{\nu_0/2 - 1/2} \sqrt{\frac{N}{1 + 0.5/\sqrt{N}}}.$$

For instance, with $N = 50$ and $\nu_0 = 4$, we have $\delta x \approx 1.8\%$; i.e., the quantity $x$ can be constrained to the level of $x_c$ $\pm 1.8\% (1 \sigma)$. From $\delta x$ to $\delta(\sigma^2_{\text{corr}})$, it depends on $\sigma_{\text{corr}}$, and thus on $n_g$ and $\sigma_r$. Furthermore, from the constraint on $\sigma^2_{\text{corr}}$ to the constraint on $A$, we need the redshift distribution of background galaxies. With $n_g = 30$ arcmin$^{-2}$, $\sigma_r = 0.4$, $\alpha = 2$, $\beta = 1.5$, and $z_s = 0.7$, we...
have \( \delta(\sigma^2_{\text{corr}}) \sim 7 \times 10^{-6} \) and \( \delta(A) \sim 0.9 \). From the relation between \( 2\sigma^2_{\text{corr}} \) and the intrinsic alignment \( C_{11} + C_{22} \) smoothed over the angular scale \( \theta_{\text{eq}} \), the limit \( \delta(\sigma^2_{\text{corr}}) \sim 7 \times 10^{-6} \) leads to \( \delta(C_{11} + C_{22}) \sim 1.4 \times 10^{-5} \) over \( \theta_{\text{eq}} = 1' \).

Intrinsic alignments of galaxies have been estimated from nearby surveys, including SuperCOSMOS (Brown et al. 2002) and SDSS (Mandelbaum et al. 2006), assuming negligible lensing effects. The process of extrapolating their results to redshifts \( z \sim 1 \), which are appropriate for most lensing surveys, suffers many uncertainties (Brown et al. 2002; Mandelbaum et al. 2006). Heymans et al. (2004) estimated the intrinsic alignments in the COMBO-17 survey (\( z \sim 0.6 \)) from close pairs of background galaxies. Their error bar at \( z \sim 1' \) is about a few times \( 10^{-4} \). Our above analysis based on only five false peaks already gives rise to a tighter constraint on the order of \( \delta(C_{11} + C_{22}) \sim 4 \times 10^{-5} \), demonstrating the great potential of our proposed method.

5. DISCUSSION

In this paper, we investigate the effect of the intrinsic alignments of background galaxies on weak-lensing detections of mass concentrations. Focusing on the convergence \( \kappa \)-maps, we analyze the number of false peaks due to the intrinsic ellipticities of background galaxies, taking into account their intrinsic alignments. Under the assumption of Gaussianity for the noise field, the number of false peaks in \( \kappa \)-maps depends on two characteristic parameters, \( \gamma_\nu \) and \( \theta_\kappa \), which are in turn determined by the two-point correlations of the field. Without intrinsic alignments, \( \gamma_\nu = \sqrt{2/2} \) and \( \theta_\kappa = \sqrt{2/2} \theta_{\text{eq}} \) for a Gaussian window. Thus, the number of false peaks in terms of \( \nu_{\text{ran}} = N/\sigma_{\nu_{\text{ran}}} \) does not depend on \( \sigma_{\nu_{\text{ran}}} \) (note that, given \( \nu_{\text{ran}} \), the strength of a peak \( N \) depends on \( \sigma_{\nu_{\text{ran}}} \)). With the intrinsic alignments, however, both \( \gamma_\nu \) and \( \theta_\kappa \) change with \( \sigma_{\nu_{\text{ran}}} \). More importantly, the full noise variance \( \sigma_\nu^2 = \sigma_{\nu_{\text{ran}}}^2 + \sigma_{\nu_{\text{corr}}}^2 \) cannot be measured easily in real observations. Only the quantity \( \sigma_{\nu_{\text{ran}}}^2 \) can be estimated. Therefore, the observationally defined signal-to-noise ratio is often with respect to \( \sigma_{\nu_{\text{ran}}} \) rather than to the true noise variance \( \sigma_\nu \). For a given \( \nu_{\text{ran}} \), the true signal-to-noise ratio \( \nu \) decreases with the increase of \( \sigma_{\nu_{\text{corr}}} \). Because \( N_{\text{peak}}(\nu) \) drops steeply at large \( \nu \), the cumulative number of false peaks given a threshold on \( \nu_{\text{ran}} \) increases sensitively as \( \sigma_{\nu_{\text{corr}}} \) increases. This can result in a large reduction of the efficiency of weak-lensing cluster detections. If a 75% efficiency is expected in the case with \( \sigma_{\nu_{\text{corr}}} = 0 \) for a survey at a detection threshold of \( \nu_{\text{ran}} = 3.5 \), this number goes down to 65% with \( \sigma_{\nu_{\text{corr}}} \approx 0.1 \sigma_{\nu_{\text{ran}}} \). The increase of the number of false peaks with intrinsic alignments can also affect the statistical likelihood in judging whether a dark clump truly corresponds to a dark clump or is a false peak from intrinsic ellipticities of source galaxies.

On the other hand, the number of false peaks can be a sensitive probe to the intrinsic alignments of background galaxies. A value of \( \sigma_{\nu_{\text{corr}}} \approx 0.1 \sigma_{\nu_{\text{ran}}} \) results in

\[
N_{\text{peak}}(\nu_{\text{ran}} \geq 3.5, \sigma_{\nu_{\text{corr}}}) = 1.7N_{\text{peak}}(\nu_{\text{ran}} \geq 3.5, \sigma_{\nu_{\text{corr}}} = 0).
\]

Thus, it is easier to derive information on \( \sigma^2_{\nu_{\text{corr}}} \) from \( N_{\text{peak}}(\nu_{\text{ran}}) \) than it is to directly measure \( \sigma^2_{\nu_{\text{corr}}} \) from two-point correlations. The studies of the CFHTLS Deep cluster survey find that 5 out of 14 peaks with \( \nu_{\text{ran}} > 3.5 \) in an area of 3.61 deg\(^2\) are possibly false ones. We then obtain a constraint of \( \sigma^2_{\nu_{\text{corr}}} \leq 2 \times 10^{-5} \) (1\( \sigma \)), which corresponds to the constraint on \( A \) in equation (4) with \( A < 2.9 \), fully consistent with the limit from SDSS observations. Future large surveys can generate samples containing many lensing-detected candidates. If one can find 50 false peaks, the quantity \( \sigma_{\nu_{\text{corr}}}/\sigma_{\nu_{\text{ran}}} \) can be constrained to \( \delta(\sigma^2_{\nu_{\text{corr}}}/\sigma^2_{\nu_{\text{ran}}}) \sim 1.8\% \). With \( \sigma_{\nu_{\text{ran}}} \sim 4 \times 10^{-4} \), we then have \( \delta(\sigma^2_{\nu_{\text{corr}}}) \sim 7 \times 10^{-6} \) and \( \delta(C_{11} + C_{22}) \sim 1.4 \times 10^{-5} \).

The intrinsic alignments of galaxies carry important information regarding galaxy formation, especially the environmental effects. Previous observational studies show that the intrinsic alignments of galaxies are at the lower end of the theoretical predictions for dark matter halos, indicating the possible existence of misalignment between galaxies and their host halos (e.g., Heymans et al. 2006). The method proposed in this paper allows us to constrain the intrinsic alignments of galaxies to a very high precision and therefore is very promising for detailed studies on the formation of galaxies.

In our analysis, we assume Gaussian statistics for the noise in smoothed \( \kappa \)-fields for cases with and without intrinsic alignments. Although we do not expect a highly non-Gaussian smoothed noise field because of the central limit theorem, its detailed statistical properties deserve thorough investigation. The existence of intrinsic ellipticities, as well as their alignments, not only produces false peaks in \( \kappa \)-maps, but also affects the height of the true peaks that are associated with clusters of galaxies. From \( K_N(\theta) = K(\theta) + N(\theta) \) for smoothed \( \kappa \)-fields, and assuming that \( K \) and \( N \) are independent of each other, we can write the distribution of \( K_N \) in the form \( p(K_N) dK_N = \int [ p_K(K) p_N(K_N - K) ] dK_N \). It can be seen that \( p(K_N) \) depends on the statistics of \( K \) and \( N \). Thus, detailed analyses on the statistical properties of \( K \) and \( N \) are crucial in order to understand how the true peaks are influenced by the noise. Further complications arise due to the shear-ellipticity correlations (e.g., Hirata & Seljak 2004; Mandelbaum et al. 2006). Because the ellipticities of galaxies are associated with the properties of their host halos, correlations between the ellipticities of foreground galaxies and the shear generated by their host halos on background galaxies are expected. Then \( K \) and \( N \) are not independent quantities any more. The effects of intrinsic alignments and shear-ellipticity correlations on lensing analyses depend differentially on the redshift distribution of background galaxies. The narrower the distribution, the stronger the effects of the intrinsic alignments. For shear-ellipticity correlations, the effects are stronger for broader distributions. For tomographic lensing studies, the intrinsic alignments are important for galaxies within the same redshift bins, while the shear-ellipticity correlations are significant in considering the cross-correlations between different bins. Extensive investigations on these problems and their effects on weak-lensing cluster surveys will be pursued in our future research.

Weak-lensing cluster studies, together with other lensing analyses, are sensitive probes of the dark matter distribution, as well as the nature of dark energy (e.g., Fang & Haiman 2007). With fast observational advances and thorough theoretical understanding of different systems, cosmological applications of weak-lensing effects will greatly improve our knowledge of the universe.

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REFERENCES

Bartelmann, M., & Schneider, P. 2001, Phys. Rep., 340, 291
Bode, P., Ostriker, J. P., Weller, J., & Shaw, L. 2007, ApJ, 663, 139
Bond, J. R., & Efstathiou, G. 1987, MNRAS, 226, 655
Borgani, S. 2006, preprint (astro-ph/0605575)
Brown, M. L., Taylor, A. N., Hambly, N. C., & Dye, S. 2002, MNRAS, 333, 501
Crittenden, R. G., Natarajan, P., Pen, U. L., & Theuns, T. 2001, ApJ, 559, 552
Croft, R. A. C., & Metzler, C. A. 2000, ApJ, 545, 561
Erben, Th., van Waerbeke, L., Mellier, Y., Schneider, P., Cuillandeii, J. C., Castander, F. J., & Daniel-Fort, M. 2000, A&A, 355, 23
Fan, Z., & Chiu, T. 2001, ApJ, 550, 547
Fang, W., & Haiman, Z. 2007, Phys. Rev. D, 75, 043010
Gavazzi, R., & Soucail, G. 2007, A&A, 462, 459
Hamana, T., Takada, M., & Yoshida, N. 2004, MNRAS, 350, 893
Heavens, A., Refregier, A., & Heymans, C. 2000, MNRAS, 319, 649
Hennawi, J. F., & Spergel, D. N. 2005, ApJ, 624, 59
Heymans, C., Brown, M., Heavens, A., Meisenheimer, K., Taylor, A., & Wolf, C. 2004, MNRAS, 347, 895
Heymans, C., White, M., Heavens, A., Vale, C., & van Waerbeke, L. 2006, MNRAS, 371, 750
Hirata, C. M., & Seljak, U. 2004, Phys. Rev. D, 70, 063526
Hoekstra, H., et al. 2006, ApJ, 647, 116
Jing, Y. P. 2002, MNRAS, 335, L89
Kaiser, N., & Squires, G. 1993, ApJ, 404, 441
King, L. J. 2005, A&A, 441, 47
King, L. J., & Schneider, P. 2003, A&A, 398, 23
Knox, L., Song, Y., & Tyson, J. A. 2006, Phys. Rev. D, 74, 023512
Mandelbaum, R., Hirata, C. M., Ishak, M., Seljak, U., & Brinkmann, J. 2006, MNRAS, 367, 611
Massey, R., et al. 2007, Nature, 445, 286
Munshi, D., Valageas, P., van Waerbeke, L., & Heavens, A. 2006, preprint (astro-ph/0612667)
Navarro, J., Frenk, C., & White, S. D. M. 1996, ApJ, 462, 563
Porciani, C., Dekel, A., & Hoffman, Y. 2002, MNRAS, 332, 325
Schirmer, M., Erben, T., Hetterscheidt, M., & Schneider, P. 2007, A&A, 462, 875
Schneider, P. 1996, MNRAS, 283, 837
Semboloni, E., van Waerbeke, L., Heymans, C., Hamana, T., Colombi, S., White, M., & Mellier, Y. 2007, MNRAS, 375, L6
Starck, J. L., Pires, S., & Refregier, A. 2006, A&A, 451, 1139
Tang, J. Y., & Fan, Z. H. 2005, ApJ, 635, 60
van Waerbeke, L. 2000, MNRAS, 313, 524
von der Linden, A., Erben, T., Schneider, P., & Castander, F. J. 2006, A&A, 454, 37
White, M., van Waerbeke, L., & Mackey, J. 2002, ApJ, 575, 640