Exploiting electron parity violation: from Standard Model tests to dark matter detection predictions

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Abstract. There has been recent interest in low energy, high luminosity polarized electron beams for studies of parity-violating (PV) electron scattering, such as the MESA accelerator at Mainz or an upgraded FEL facility at Jefferson Lab. Accurate measurements of the PV asymmetry in elastic electron scattering from nuclei can be used to determine Standard Model couplings, such as the weak-mixing angle or higher-order radiative corrections, as well as to extract specific information on the nuclear and nucleon structure. To this end, low uncertainties are required from modelling some confounding nuclear and nuclear structure effects, including isospin mixing, nucleon strangeness content or Coulomb distortion of electron wave functions. We estimate the sizes and theoretical uncertainties of such effects for a carbon 12 target. An experimental precision in the PV asymmetry of a few tenths of a percent may be reachable under certain kinematic conditions, that are also discussed for the same nuclear target.

This high precision PV asymmetry in elastic electron scattering can also be used to relate in a very simple manner the elastic electron-nucleus scattering cross section with the elastic neutrino-nucleus cross section. This novel relationship allows us to exploit experimentally well-determined quantities to predict unknown or recently measured observables, such as coherent neutrino-nucleus cross sections. This idea can be extended to link electron scattering to an even more uncertain magnitude: the direct detection rate of hypothetical weak-interacting dark matter particles through axial and/or vector elastic interactions with nuclei.

1. Introduction to parity violation in elastic electron scattering off nuclei

The Standard Model (SM) weak current contains polar-vector (or simply ‘vector’) and axial-vector (or simply ‘axial’) components, that behave differently under inversion of spatial coordinates, namely, under a parity transformation. As a consequence, parity-violating (PV) observables can be measured in processes involving the weak interaction. One of them is the PV asymmetry in elastic electron scattering off nuclei, which is defined as the relative difference between the differential cross sections of electrons longitudinally polarized parallel \( d\sigma^+ \) and antiparallel \( d\sigma^- \) to their momentum:

\[
A = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}.
\]

The main contribution to the electron-nucleus scattering cross section comes from the electromagnetic (EM) interaction, but the weak neutral (WN) interaction, although much weaker, also plays a role, and is actually responsible for the nonzero value of the PV asymmetry.
When only one gauge boson exchange is considered for each interaction (one photon and one $Z^0$) and when the distortion of the electron wave function due to the nuclear Coulomb field is neglected, the calculations are performed within plane wave Born approximation (PWBA). The PV asymmetry can be then factorized in three parts: a Standard Model factor, containing the coupling constants of the two interactions involved, EM ($\alpha$) and WN ($G_F$); a nuclear structure factor, containing the ratio of the parity-violating (PV) to the parity-conserving (PC) responses of the nuclear target; and a four-momentum transfer squared factor, $|Q^2|$. The asymmetry can then be written as [1]:

$$A = \frac{G_F}{2\pi\alpha\sqrt{2}} \frac{W^{PV}}{W^{PC}} |Q^2|.$$  

The nuclear PC response can be written as $W^{PC} = v_L W_L + v_T W_T$, which contains longitudinal and transverse EM (vector-vector) nuclear tensors, $W_L$ and $W_T$, each multiplied by a kinematical factor related to the projectile tensor, $v_L$ and $v_T$. The nuclear PV response, on the other hand, has its origin in the interference between the EM and the WN amplitudes of the scattering process that gives rise to mixed (EM-WN) nuclear responses $\tilde{W}$, and can be written as $W^{PV} = a_A^c (v_L \tilde{W}_L + v_T \tilde{W}_T) + a_V v_T \tilde{W}_T^\prime$. It consists of two terms, one containing the axial WN coupling of the electron, $a_A^c$, and the longitudinal and transverse vector-vector nuclear tensors, $\tilde{W}_L$ and $\tilde{W}_T$, with their corresponding kinematical factors, $v_L$ and $v_T$; and another one containing the vector WN coupling of the electron, $a_V^c$, and the transverse axial-vector nuclear tensor, $\tilde{W}_T^\prime$, with its corresponding kinematical factor $v_T$.

For a nuclear target with pure isospin zero ($T = 0$) in its ground state, the axial-vector nuclear tensor ($\tilde{W}_T^\prime$) in the PV response vanishes, since the axial isoscalar WN coupling in the Standard Model is zero at tree level. Similarly, for a nuclear target with spin zero ($J = 0$) in its ground state, the transverse tensors in both the PV and the PC responses vanish since only the Coulomb-type monopole operators contribute. Under these conditions, namely a $T = 0, J = 0$ nuclear target, the PV and the PC nuclear responses become proportional [2]:

$$\frac{W^{PV}}{W^{PC}} = \frac{a_A^c \tilde{W}_L}{\tilde{W}_L} = a_A^c \beta_{V(0)} ,$$

where $\beta_{V(0)}$ is the hadronic vector isoscalar WN coupling.

Using the SM values of the electron coupling, $a_A^c = -1$, and of the hadronic coupling, $\beta_{V(0)} = -2 \sin^2 \theta_W$, the PV asymmetry of Eq. 2 becomes:

$$A^{ref} = 8.31 \cdot 10^{-11} |Q^2| \text{MeV}^{-2} ,$$

which can be considered a reference value: the one expected within PWBA for pure $T = 0$ and $J = 0$ nuclear targets.

The condition $T = 0$ implies that the number of neutrons in the target is equal to the number of protons, $N = Z$. When they are different, $N \neq Z$, the PV asymmetry becomes:

$$A \approx 8.99 \cdot 10^{-11} \frac{N}{Z} \frac{F_n}{F_p} |Q^2| \text{MeV}^{-2} ,$$

where $F_n$ and $F_p$ are the neutron and proton nuclear form factors, namely the Fourier transform of the density distribution of neutrons and protons within the target, respectively. This expression is an approximation where the small electric form factor of the neutron and a small additional term ($\sim 0.08 Z/N$) have been neglected [3].
2. Electron parity violation and the structure of nuclei and neutron stars

It is apparent from Eq. 5 that a measurement of the PV asymmetry in a \( N \neq Z \) nucleus can be used to extract the neutron form factor, \( F_n \), provided that the proton form factor, \( F_p \), is known from usual (parity-conserving) electron scattering data [1]. The well known electroweak theory involved in PV electron scattering allows us to extract information on the neutron distribution with much better precision than by using hadronic probes. Another advantage is that weak probes are more sensitive to the neutron distribution in nuclei since the WN charge of the neutrons is more than twelve times larger than that of the protons.

From both form factors one can obtain the difference between the neutron and the proton rms radii, which defines the thickness of the neutron skin, an outer shell of the nucleus where part of the neutrons accumulate. This feature arises in heavy nuclei due to the neutron excess and to the interplay between the Coulomb barrier of the protons, the surface energy and the symmetry energy at different nucleon densities. The energy of the skin increases with the density of neutrons at a rate given by the pressure of neutron-rich nuclear matter; therefore, the larger the pressure, the thicker the neutron skin, and vice versa. The recent experiments PREX and CREX carried out at Jefferson Lab were aimed at measuring the PV asymmetry in \(^{208}\text{Pb}\) and \(^{48}\text{Ca}\) in order to extract their neutron rms radii and skin thicknesses [4].

The pressure is an important ingredient of the nuclear matter equation of state, that relates the volume energy per nucleon with the nucleon density. The equation of state neglects surface effects and the Coulomb repulsion between protons, and therefore plays an important role in the description of neutron stars. These are astrophysical objects with around 1.5 solar masses and radii of the order of 10 km. They are held together by gravity, and their collapse is prevented by the same pressure that gives rise to neutron skins in nuclei. The crust of the neutron star, with a depth between 1 and 2 km below the surface, is a solid (Coulomb crystal) region containing neutron-rich nuclei, with more topologically complex structures showing up as the density increases with depth. The outer core is a region immediately below the crust consisting of a homogeneous fluid of neutrons with some fraction (around 10%) of protons and negatively charged leptons, where the density reaches values close to the saturation nuclear density [5]. Under some reasonable assumptions, one can expect that the thicker the neutron skin in nuclei, the larger the radius of neutron stars, since both are proportional to the pressure of neutron-rich matter. Moreover, one can also expect that the thicker the neutron skin in nuclei, the thinner the crust in neutron stars [6].

3. Electron parity violation and tests of the Standard Model electroweak sector

Eq. 4 can be used to extract the value of parameters or the size of perturbative corrections within the electroweak sector of the Standard Model from experimental data on electron PV asymmetry in \( N = Z \) nuclei. In particular, this kind of measurements have been suggested to evaluate accurately the weak mixing (Weinberg) angle or the size and momentum-dependence of higher-order electroweak radiative corrections. Several high-precision facilities have been proposed recently that implement low-energy, high-luminosity polarized electron beams, such as MESA in Mainz, FEL at Jefferson Laboratory or \( C\beta \) at Cornell University [7].

The simplicity of the reference value of the PV asymmetry in Eq. 4 is due to the fact that the PC and the PV nuclear responses become proportional (Eq. 3) whenever there is no isovector contributions and no additional isoscalar contributions in the weak response. The former condition is not fulfilled, even in \( N = Z \) nuclei, when there are sources of isospin mixing that prevent the ground state from being purely \( T = 0 \): the Coulomb interaction among protons and, possibly, isospin-mixing terms in the strong interaction. On the other hand, when the strangeness content of the nucleons (presence of strange quark-antiquark virtual pairs) is different from zero, an extra isoscalar contribution in the weak nuclear responses shows up.

Thus, the isospin mixing in the nuclear target and the strangeness content of the nucleons
are two important sources of deviations of the PV asymmetry from the reference value in Eq. 4 in $N = Z$ nuclear targets. Other sources are the Coulomb distortion of the incoming and outgoing electron wave functions (since the reference asymmetry is computed within PWBA), the meson exchange currents between nucleons (that affect differently the isoscalar and the isovector nuclear responses when both are present) or the inelastic transitions that may contribute to the measurement of the desired elastic electron PV asymmetry when the energy resolution of the electron detector is poor.

We have computed the relative size of each source of deviation from the reference PV asymmetry for a carbon 12 target ($N = Z = 6$), together with an estimation of the theoretical uncertainty associated to each effect [8]. The latter is the most relevant result of this study, since it determines the precision of the information extracted from experimental PV data. The results are summarized in Table 1, where it can be seen that the largest effect comes from the Coulomb distortion of the electron wave functions (around 3%), although it does not harm the precision of the information extracted from the experimental data because the theory that describes the effect, the distorted wave Born approximation (DWBA), is very accurate (approximately 0.01% relative uncertainty). On the contrary, the nucleon strangeness content effect can be zero or just moderately large (up to around 1%), but its relative uncertainty is large, since it actually covers the interval from 0% to 1%. This range comes from the current experimental uncertainties in the nucleon strangeness content parameters [9], and might prevent us from testing the SM with the desired precision. We therefore suggest to pin down first the strangeness content effect by using a nuclear target with small isospin mixing, such as $^4$He. This isotope was analyzed in the HAPPEX-He experiment [10], but a somewhat better precision on the measured PV asymmetry, that seems certainly achievable, would be required [8].

| Contribution to PV asymmetry | Relative size | Relative uncertainty |
|------------------------------|--------------|----------------------|
| Coulomb distortion of electron wave functions | 3%           | 0.01%                |
| Nuclear isospin mixing (EM origin)         | 0.4%         | 0.05%                |
| Nucleon strangeness content (mainly electric) | 0% - 1%     | 1%                   |
| Meson exchange currents               | $\lesssim 0.1\%$ | $\lesssim 0.1\%$    |
| Inelastic contributions    | $\lesssim 0.1\%$     | –                    |

4. Electron parity violation and coherent neutrino scattering
In coherent lepton scattering off nuclei all the nucleons in the target interact with the incoming lepton. It is a special case of elastic scattering, where the incoming and the outgoing lepton is the same and its energy loss is only due to the recoil of the nuclear target, whose structure and excitation energy does not change in the process. Coherent scattering is important when the wavelength associated to the momentum transfer is of the order of the nuclear size, $q \approx 160 A^{-1/3}$ MeV. It is the only elastic contribution for even-even ($J = 0$), $T = 0$ nuclear targets, and dominant for the rest of targets except for the lightest ones. In the case of neutrinos, coherent scattering is a purely WN process, which is possible for any neutrino flavor without any energy threshold. Given the elusive nature of these particles, the recoil of the nuclear target is the only event that can be detected.

Although coherent neutrino scattering was predicted more than 40 years ago, the first detections have been registered only very recently in the COHERENT experiment [11], the reason being the small nuclear recoil energy; the coherent cross section, however, is proportional to the mass number squared, $A^2$, and is actually larger than in other neutrino-nucleus scatterings.
(charged-current). The study of this process can be used to extract SM electroweak constants at low momentum transfers and the size of higher-order corrections, as well as to test the universality of the weak neutral interaction for different lepton flavors or for charged and neutral leptons in the axial sector. It can also be exploited to obtain information on the structure of the nuclear target, as for instance the axial response in light nuclei [12]. And it may have an impact on our understanding of astrophysical processes where large fluxes of neutrinos are produced that subsequently travel through dense matter, such as stellar core collapses and supernovae or in the URCA cooling mechanism in white dwarfs and neutron stars.

The differential cross section for coherent neutrino-nucleus scattering can be easily predicted within the SM, but following the theme of this paper, we will relate it to the PV asymmetry in elastic electron scattering. It can be expressed as a product of the electron PV asymmetry squared and the coherent electron scattering differential cross section, all of them given at the same momentum transfer [12]:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\nu} = A^2 \left( \frac{d\sigma}{d\Omega} \right)_{e}.
\]

(6)

Both the electron PV asymmetry and the electron cross section can be measured (the latter more easily than the former) and used to predict the neutrino cross section in the SM. This simple expression is valid at tree level when the Coulomb distortion of the electron wave functions is ignored; experimental results can be easily transformed by using distorted-wave theory (DWBA), which, as mentioned in the previous section, is very precise. The expression is valid only in the ultrarelativistic limit, which is a very good approximation for neutrinos and electrons (for heavier charged leptons it depends on their energy). An even simpler relationship can be established using Eq. 4, valid for even-even \((J = 0)\), \(N = Z\) nuclei without isospin mixing and without nucleon strangeness content, since in this case \(A^2 = 6.9 \cdot 10^{-21} |Q|^4 \text{ MeV}^{-1}\). Additionally, the relative uncertainty of the neutrino cross section obtained using Eq. 6 can be related in a very simple manner with the relative uncertainty of the PV asymmetry [12]: \(\varepsilon \left[ \left( \frac{d\sigma}{d\Omega} \right)_{\nu} \right] \approx 2 \varepsilon[A]\).

5. Electron parity violation and direct detection of weakly-interacting dark matter

An expression analogous to Eq. 6 can be given for the coherent scattering of weakly-interacting particles (WIPs), that we define as particles with vector and axial current interactions, as in the SM weak interaction, but with different values of the couplings. It contains, as before, the electron PV asymmetry and the coherent electron scattering differential cross section [13]:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{WIP} = \frac{\kappa^2 \gamma_V^2(0)}{G_F^2 \beta_V^2(0)} (b_V^2 C_V + b_A^2 C_A) A^2 \left( \frac{d\sigma}{d\Omega} \right)_{e},
\]

(7)

where \(\beta_V^{(0)}\) is the SM hadronic vector isoscalar WN coupling, corresponding to the interaction between the nuclear target and the \(Z^0\) exchanged in PV electron scattering, and \(\gamma_V^{(0)}\) is also a hadronic vector isoscalar coupling, but corresponding to the interaction between the nuclear target and a hypothetical non-SM boson exchanged by the WIP. The vector and axial couplings \(b_V\) and \(b_A\) correspond to the interaction between the non-SM boson and the WIP, and each of them is multiplied by a different kinematical factor, \(C_V\) and \(C_A\), that depends on the momentum and scattering angle of the electron in the coherent scattering, as well as on the masses of the WIP and the target nucleus. Finally, \(G_F\) is the overall SM weak interaction coupling of the electron and \(\kappa\) is the overall coupling of the non-SM interaction of the WIP.

WIPs might be part of the dark matter (DM) content of the universe. There are several astrophysical evidences of the existence of DM particles, as for instance the deviation of the galaxy rotation curves from the expected newtonian value obtained if all the galaxy mass is contained in visible matter. A dark matter contribution to the mass of the galaxy is then
conjectured, where dark (non-visible) means that it does not interact electromagnetically. The fact that dark particles have never been detected suggests that their couplings are smaller than the SM weak interaction coupling, implying that they do not take part in strong nor in weak interactions, at least not directly. If the hypothetical, non-SM interaction of DM particles has vector and axial components (not necessarily both together, nor both with the same relative weight), they can be considered WIPs, as defined above, and Eq. 7 is valid for them. WIPs are a type of WIMPs, a generic label for popular DM candidates characterized by large masses and very feeble couplings with themselves and with SM particles.

If WIPs interact via the exchange of a $Z^0$ boson, the hadronic vertex coupling becomes the SM one, and then $\kappa \gamma^2_{V(0)} = G_F \beta^2_{V(0)}$. The WIP-$Z^0$ vertex couplings, however, should be much smaller than the SM ones, in order to fulfil the DM requirements. If the WIP is one of the SM neutrinos, the WIP vertex couplings also become the SM ones, $\kappa b^2 = G_F (a^2_{V}) = G_F$ and $\kappa A^2 = G_F (a^2_{A}) = G_F$, and Eq. 6 is recovered (since $C_{V,A} \to 1/2$ in the ultrarelativistic limit).

An example of non-SM WIP is the lightest neutralino, a hypothetical stable combination of supersymmetric particles, which is of Majorana type (it is the same as its antiparticle) and therefore behaves as if $b_V = 0$ in Eq. 7. Another example is the sterile neutrino, a hypothetical fourth neutrino flavor eigenstate with a large component of a fourth, possibly very massive neutrino mass eigenstate, that has no charged lepton partner and therefore does not take part in the SM weak interaction directly. However, there might be a small probability that the additional mass eigenstate is detected as an active SM neutrino flavor. This process can be described by Eq. 7 considering the interaction of a neutrino exchanging a $Z^0$, but with the overall coupling constant $G_F$ considerably reduced by the sterile-active neutrino mixing parameter.

In summary, parity-violating electron scattering is a rare electroweak process that can be related to other rare events, such as coherent neutrino scattering or dark matter direct detection. It can also be used to test the Standard Model, as well as to explore some elusive properties of particles, including the neutron density distribution in nuclei and neutron stars or the strange quark content of nucleons. All these topics require extremely accurate theoretical calculations and measurements, which are becoming feasible and are therefore drawing increasing attention from both the theoretical and the experimental communities.

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