Propagating phase boundaries as sonic horizons

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If certain conditions are met, a propagating phase boundary can be a sonic horizon. Sonic Hawking radiation from such a phase boundary is expected in the quantum theory. The Hawking temperature for typical values of system parameters can be as large as ~ 0.04 K. Since the setup does not require the physical transport of material, it evades the seemingly insurmountable difficulties of the usual proposals to create a sonic horizon in which fluid is required to flow at supersonic speeds. Issues that are likely to present difficulties that are particular to this setup are discussed. Hawking evaporation of the sonic horizon is also expected and is predicted to lead to a deceleration of the phase boundary.

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Condensed matter analogs of gravitational systems have brought purely theoretical ideas on very large-scale gravitational structures to almost within the realm of experimental tests. A key breakthrough was the realization by Unruh [1] that black hole event horizons can be simulated by sonic event horizons around “dumbholes” in fluid flow. Furthermore, theoretical results from the application of quantum field theory in curved spacetimes, such as Hawking radiation [2], should also apply to condensed matter analogs, thus opening the door to novel experimental tests of gravitational phenomena.

The fluid analog of black holes occurs when sub-sonic fluid flow upstream changes to supersonic flow downstream. Now the fluid flow downstream is too rapid to allow sonic perturbations to flow upstream. Sound can travel downstream from upstream but not upstream from downstream, resulting in a “sonic horizon” at the location where the flow velocity equals the sound velocity. To the upstream observer, the downstream region is a hole from which no sound can emanate, that is, it is a dumbhole. This is the classical picture. As in the black hole case, “dumbholes ain’t so dumb” and quantization of the acoustic perturbations leads to sonic Hawking radiation from downstream to upstream. Unruh found that the sonic Hawking temperature is given by:

$$T_{sH} = \left( \frac{\hbar}{2\pi k_B} \right) \frac{d}{dr}(-\nu) \bigg|_{hor}$$

(1)

where $\hbar$ is Planck’s constant divided by $2\pi$, $k_B$ is Boltzmann’s constant, $r$ is the radial coordinate, $v \equiv |\nu|$, and the flow velocity $\nu$ is assumed to be radially inward. The derivative of the velocity is evaluated at the location of the sonic horizon determined by $v = c_s$ where $c_s$ is the sound velocity. A numerical estimate gives:

$$T_{sH} \sim 3 \times 10^{-7} K \left( \frac{c_s}{300 \text{ m/s}} \right) \left( \frac{1 \text{ mm}}{R} \right)$$

(2)

where the velocity gradient is taken to be $c_s/R$. The estimate clearly demonstrates the challenge: even when the fluid undergoes tremendous acceleration – velocity change from 0 to 300 m/s within 1 mm or, equivalently, ~ $10^7 g$ where $g$ is the terrestrial acceleration due to gravity – the sonic Hawking temperature is extremely small.

The immense practical problems in creating a sonic black hole and observing Hawking radiation have been described by Unruh [3], leading to a very grim picture. The crux of the problem is that the flow of fluids and superfluids suffers from instabilities at the tremendous accelerations that are required for creating a sonic horizon with a measurable Hawking temperature. If one chooses to work with a system with low sound velocity, the instabilities can be evaded, but then the Hawking temperature is unmeasurably low. At the moment these difficulties seem to be insurmountable, except possibly in Bose-Einstein condensates [4, 5]. (Proposals to test quantum processes in the presence of cosmological horizons in BEC analogs also appear promising [6].)

For the idea to be described in the present paper, a generalization of Eq. (1) made by Visser [7] will be very important. Visser considered the case when the sound velocity itself is a function of space and time and obtained:

$$T_{sH} = \left( \frac{\hbar}{2\pi k_B} \right) \frac{d}{dr}(c_s - \nu) \bigg|_{hor}$$

(3)

This seemingly small step opens up a conceptually new set of possible dumbhole realizations because now it is not necessary to accelerate the fluid; instead one can manipulate the function $c_s(t, \mathbf{x})$.

Along these lines, the proposal of Jacobson and Volovik [8] (also see [9]) employs a domain wall in $^3$He-A to produce a variation of the ”sound” velocity [10] on microscopic scales (~ 500 Å). Modest velocities of the domain wall lead to an enhanced Hawking temperature. The gain is however somewhat offset by the rather low sound velocity (~ 3 cm/s). The Hawking temperature in their setup is 5µK, whereas reliable thermometry in $^3$He currently only goes down to 100µK. Furthermore, gradients
in the order parameter associated with the presence of the domain wall lead to additional sources of radiation that are not related to the sonic horizon. It remains to be seen if the Hawking radiation can be disentangled from these other effects.

We envisage a setup somewhat related to the Jacobson-Volovik proposal and pictured in Fig. 1. The container is initially filled with a metastable phase (Phase 1) of a material. Suppose that Phase 2 is the stable phase under the conditions of the experiment. Then a perturbation will trigger bubble nucleation of Phase 2 and this bubble will grow. The velocity \( v \) with which the bubble grows will be subsonic in Phase 1: \( v < c_1 \) where \( c_1 \) is the sound velocity in Phase 1. The sound velocity \( c_2 \) in Phase 2 will be different from that in Phase 1. We assume \( c_2 < c_1 \). More crucially we take \( c_2 < v \). For the purposes of this paper, we will simply assume that the conditions \( (c_1 > v > c_2) \) are met in the system we have chosen. Then sound emanating from within Phase 2 cannot catch up with the phase boundary. Thus the phase boundary will be a sonic horizon. By the usual arguments \[, \] sonic Hawking radiation should be seen to emanate from the bubble [11].

The advantage of this setup is that it completely eliminates any instabilities associated with supersonic flow. In certain situations, the bubble wall may push on the material outside. If the system is kept at constant pressure, this would cause some expansion of the occupied volume. The height occupied by the material would increase by a fraction \( (\delta V_f)/V \) where \( \delta V_f \) is the increase in volume occupied by the bubble and \( V \) is the volume occupied initially by Phase 1. This can be made small by choosing a large container. We can also imagine a situation where the transition from Phase 1 to Phase 2, does not involve a significant change in density. This might happen, for example, in the case of a fluid to superfluid transition. The bubble wall then just marks the location where Phase 1 is being converted to Phase 2, without the wall actually exerting any forces on the material outside. In this case, there is no transport of material involved.

In fact, the two phases need not even be fluid, they can be solid, say the normal and superconducting phases of a Type I superconductor. Our only requirement is that we have two different sound velocities and a phase boundary velocity that satisfies \( c_1 > v > c_2 \). If the velocities of ordinary (compressional) sound do not satisfy these conditions, one could consider some other excitations. However, it is advantageous to consider the excitations with the largest velocities that satisfy the conditions since then the Hawking temperature is highest.

In this setup, the Hawking temperature can be relatively large because the gradient of \( c_s - v \) occurs within the phase boundary whose thickness will be of microscopic dimensions, given by the inter-molecular spacing, or the coherence length of Cooper pairs, or some other such length scale. The sound velocity depends on what kind of excitations we are considering. We would like to take a large sound velocity, since this gives a larger Hawking temperature. With ordinary sound (compressional waves) typical sound speeds are \( \sim 300 \) m/s. This leads to the estimate:

\[
T_{sH} \sim 0.04K \left( \frac{\delta c_s}{300 \text{m/s}} \right) \left( \frac{100\text{Å}}{\xi} \right) \tag{4}
\]

where \( \delta c_s \) is the change in the sound velocity across the phase boundary and \( \xi \) denotes the thickness of the phase boundary. This is quite large compared to Hawking temperature estimates in earlier proposals. The thermal frequency is \( \nu = k_B T/h \sim 1 \) GHz and the power emitted is \( \sigma_s T_{sH}^4 \sim 10^4 \) pW/cm\(^2\) where the sonic Stefan-Boltzmann constant \( \sigma_s = \pi^2 k_B^4 / 60 c_s^3 h^3 \) involves the sound speed and two polarizations have been assumed [1].

Now let us consider some specific systems and variations on the general scheme described above. The first point to note is that we must necessarily work at low temperature since the Hawking temperature is of order 0.04 K. All substances except the isotopes of Helium, solidify at such a temperature. So we need either to work with a solid, or with Helium. If we choose \(^4\)He, we will be working with a superfluid. \(^3\)He however need not be in the superfluid phase.

An existence proof that the conditions \( c_1 > v > c_2 \) can be met can be found by considering the Abelian Higgs model in a cosmological setting. (The non-relativistic version of the Abelian Higgs model is the Ginzburg-Landau model used to describe simple superconductors.) For a certain set of parameters, the Abelian Higgs model is known to undergo a first order phase transition, corresponding to Type I superconductors. Imagine that the system is supercooled, so that it is stuck in the metastable state labeled Phase 1 in Fig. 2. Excitations in this metastable state are massless gauge particles (photons) and massive scalars. If these excitations are in equilibrium, they form a relativistic fluid for which the equation of state is \( p = \rho/3 \) (in units where the speed of light is one). So the sound velocity in the supercooled state is
c_1 = 1/\sqrt{3}. If there was no plasma outside the bubble, the bubble wall velocity would be the speed of light. However, the plasma exerts a drag on the bubble wall, and reduces its speed somewhat, and one has v \lesssim c_1. (In the similar case of the cosmological electroweak phase transition, calculations yield v \sim 0.1.\textsuperscript{[12]}) Inside the bubble, in Phase 2, all the excitations are massive and non-relativistic. Now the equation of state is p \sim 0 and the speed of sound c_2 is non-relativistic, say 300 m/s which is much less than v. Therefore the conditions c_1 > v > c_2 are met, and the bubble should emit sonic Hawking radiation. Also note that any heat released by the phase transition within the bubble does not propagate to the outside since the bubble wall is moving at supersonic speeds. So oscillations of the order parameter in the true vacuum are inconsequential. The only danger arises from excitations living outside the bubble (i.e. order parameter oscillations in the metastable state) that might scatter off the bubble wall. However, if the system is sufficiently supercooled, the density of excitations in the metastable state is expected to be very low.

A one dimensional version of our setup consists of a cylindrical container filled with Phase 1 (Fig. 3). A perturbation applied on the right-hand end of the cylinder will trigger bubble nucleation of Phase 2. The bubble of Phase 2 will start growing and the phase boundary (or bubble wall) will propagate to the left. Unlike the bubble of Fig. 1 the area of the sonic horizon does not change with time and hence the setup is easier to analyze. The disadvantage of the one dimensional setup is that there is more interaction of the system with the walls of the container and hence there is greater danger of instabilities. In particular, heat released on the Phase 2 side can be transmitted to the Phase 1 side through the walls of the container.

A one dimensional condensed matter situation where a phase boundary propagates at supersonic speeds occurs in $^3$He. The boundary of a bubble of A phase inside a metastable B phase has been experimentally seen\textsuperscript{[14]} to propagate at speeds of up to 67 cm/s, while fermionic quasiparticles excitations in $^3$He-A propagate at only 3 cm/s. On the $^3$He-B side, these excitations are can propagate at 55 m/s and so the setting seems right for the creation of a sonic horizon. However, note that the bubble wall velocity is in the wrong direction in this case – we would like the wall of an A phase bubble, where the sound velocity is small, to propagate at high velocity into the B phase where the sound velocity is high. It remains to be seen if the A-B phase boundary can provide measurable Hawking radiation\textsuperscript{[8]} (also see Sec. 29.3 of\textsuperscript{[15]}).

A novel feature of our proposal is that it is an example of a “non-driven” sonic horizon. In other examples, one has to arrange external forces to produce a sonic horizon. For example, in fluid flow, the fluid has to be driven under pressure. If the external conditions are designed to maintain a sonic horizon, the Hawking radiation backreaction on the sonic horizon will be lost. In the bubble nucleation proposal, however, the procedure is to nucleate a bubble of Phase 2 and then leave the system to its own devices. If the propagating phase boundary does emit Hawking radiation, it will experience a backreaction that will slow it down. Eventually the bubble wall velocity will become subsonic with respect to the sound velocity in Phase 2. In other words, the prediction is that the condition c_1 > v > c_2 is quantum mechanically unstable and eventually the system will go to a state with c_1 > c_2 > v. This is different from the evaporation of black holes due to Hawking radiation.

To estimate the evaporation time, note that the power in the emitted Hawking radiation is: \(\sim \sigma_s T^4_{BH} \) where \(A = 4\pi R^2\) is the area of the horizon when the bubble radius is \(R = vt\). The energy lost as a function of time is therefore: \(E_{\text{lost}} \sim \sigma_s T^4_{BH} 4\pi v^2 t^3\). The kinetic energy of the phase boundary in the absence of the losses would be: \(E_{\text{kin}} \sim \Sigma A v^2\) where \(\Sigma\) denotes the surface energy density. Therefore the evaporation time scale is given by equating the energy lost to the total kinetic energy:

\[ t_{\text{evap}} \sim 10^{-6} f^2 \left( \frac{\Sigma c^3}{\hbar} \right) \left( \frac{\xi}{100A} \right) \left( \frac{300 \text{m/s}}{c_1} \right) \text{ s} \tag{5} \]

where \(f \equiv v/c_1\). The evaporation time depends very crucially on the surface energy density. We would like to have a large surface density so that the evaporation is relatively slow. The surface density depends on the
system that is being used. However, we can get an idea for its value by considering the phase boundary to be a domain wall. For a domain wall in a $\lambda \phi^4$ model, we have:

$$\frac{\Sigma c^3 \phi}{\hbar} = O\left(\frac{1}{\lambda}\right)$$

Hence if the coupling constant $\lambda$ is small, the evaporation time will be long. This is exactly as for black holes where the Hawking evaporation time is long because gravity is so weakly coupled. One way in which black hole evaporation is different from durbhole evaporation is that the black hole necessarily gets hotter as it gets smaller. The durbhole may get hotter or colder with evaporation, depending on the gradient of $c_s(t, x)$ at the location of the sonic horizon $v = c_s(t, x)$.

If it is difficult to find a system in which the condition $v > c_2$ holds, one could consider driving the phase boundary by some external forces. This might be easy to implement if the substance undergoes a phase transition when an electromagnetic field is applied. Then the electromagnetic fields could be varied externally to drive the phase boundary at high velocities. The only danger is that the application of dynamical external forces could give rise to additional heating that could swamp the signal. We will not consider the “driven” setup any further here.

We now discuss potential difficulties in implementing the current proposal.

The first danger is that the phase boundary may have instabilities during propagation. Such instabilities need to be avoided since they will introduce noise in the Hawking emission. Fingering and other instabilities are system-dependent, and the success of the proposal will depend on finding a system in which such instabilities are absent.

An important issue is that, since the phase boundary is propagating, eventually the whole container will be filled with the second phase. So there is only a limited time $(L/v - L/c_1) \sim L/v$, where $L$ is the size of the container, to make measurements. This time had better be long enough to detect excitations at the thermal frequency $\nu = k_B T / \hbar \sim 1$ GHz and we obtain a lower bound on the required length of the container: $L > \hbar \nu / (k_B T_s) \sim 10^{-5}$ cm. This condition does not seem difficult to meet in the laboratory where samples are typically on the scale of centimeters. However, a sample of 1 cm size will only provide $\sim 10^{-5}$ s to make measurements.

Another potential difficulty is that Phase 1 is metastable and might decay spontaneously. Hence it has to be protected from perturbations. However, the Hawking radiation itself will perturb Phase 1 and the concern is if the system might “self-destruct”. To avoid this possibility, Phase 1 should be immune to sonic perturbations at the Hawking temperature.

The bubble will have perturbations that are produced during the nucleation process. Even quantum nucleation leads to fluctuations on the bubble $^{15}$, $^{16}$. The amplitude of fluctuations is suppressed if the bubble surface density, $\Sigma$, is large. Hence the effects of bubble perturbations can be reduced, as well as the evaporation time made large, by choosing a system with a large $\Sigma$.

In summary, we have shown that propagating phase boundaries can be sonic horizons provided the propagation velocity satisfies the condition $c_1 > v > c_2$. In systems where this condition is met, the Hawking temperature can be relatively large, leading to the exciting possibility that experiments might detect Hawking radiation. In addition, in these systems the sonic Hawking radiation will lead to evaporation of the sonic horizon, providing a yet closer analog of gravitational black holes.

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[1] W.G. Unruh, Phys. Rev. Lett. 46, 1351 (1981).
[2] S.W. Hawking, Nature 248, 30 (1974); Comm. Math. Phys. 43, 199 (1975).
[3] W.G. Unruh, in “Artificial black holes”, eds. M. Novello, M. Visser, and G. Volovik, World Scientific, River Edge, USA (2002).
[4] L.J. Garay, J.R. Anglin, J.I. Cirac and P. Zoller, Phys. Rev. Lett. 85, 4643 (2000).
[5] L.J. Garay, J.R. Anglin, J.I. Cirac and P. Zoller, Phys. Rev. A63, 023611 (2001).
[6] P.O. Fedichev and U.R. Fischer, Phys. Rev. Lett. 91, 240407 (2003).
[7] M. Visser, Class. Quant. Grav. 15, 1767 (1998).
[8] T.A. Jacobson and G.E. Volovik, Phys. Rev. D58, 064021 (1998).
[9] W.G. Unruh and R. Schutzhold Phys. Rev. D68, 024008 (2003).
[10] The sound velocity that varies within the domain wall is not for the usual compressional waves but for certain excitations that are very specific to the $^3$He-A order parameter.
[11] To connect with the Unruh setup, it is more convenient to transform to the rest frame of the phase boundary. In this frame the fluid is flowing with velocity $v$ which is subsonic outside the bubble and supersonic inside. Note that the process of cavitation, in which the bubble wall simply pushes the fluid and leaves behind an empty cavity, does not provide a sonic horizon. In this process, in the rest frame of the bubble wall, the fluid is also at rest.
[12] G.D. Moore, JHEP 0003, 006 (2000).
[13] G.E. Volovik, “The Universe in a Helium droplet”, Oxford University Press (2003).
[14] D.S. Buchanan, G.W. Swift and J.C. Wheatley, Phys. Rev. Lett. 57, 341 (1986).
[15] T. Vachaspati and A. Vilenkin, Phys. Rev. D43, 3846 (1991).
[16] J. Garriga and A. Vilenkin, Phys. Rev. D45, 3469 (1992).