The Monitoring of Milling Tool Tipping by Estimating Holder Exponents of Vibration

CHANG’AN ZHOU1, ZHENXI JIANG2, CHAO SUN1, AND ZHAOJU ZHU1

1School of Mechanical Engineering and Automation, Fuzhou University, Fuzhou 350108, China
2AVIC Chengdu Aircraft Industrial (Group) Company Ltd., Chengdu 610091, China

Corresponding author: Zhaoju Zhu (zhuzhaoju0216@163.com)

ABSTRACT The gradual tool wear is unavoidable in the machining process, it directly influences the surface integrity and dimensional tolerances of the components. The tool condition monitoring (TCM) systems have the capacity to make full use of the cutting potential of the cutting tools, which is of great significance for improving production efficiency and ensuring product quality. In milling titanium alloys, tool tipping is one of the main failure modes. Therefore, this study focuses on establishing a tool tipping monitoring approach for the milling process. The singularity analysis method based on wavelet transform was employed to characterize the variation of vibration waveforms quantitatively with the Holder Exponent (HE) index. The probability density distribution and statistical analysis were adopted to extract effective HE features from the HE indexes to correlate with the different tool conditions. The mutual information method was adopted to rank the discriminability of the HE statistical parameters. Then, several machine learning (ML) models were established with the screened HE features. Finally, the results of the experiments indicate that the Support Vector Machine (SVM) model has the highest classification accuracy and can provide practical guides on the tool changes.

INDEX TERMS Tool tipping, titanium alloys, vibrations, Holder Exponents, support vector machine.

I. INTRODUCTION
With the increasingly fierce global manufacturing competition, many manufacturing companies have taken the improvement of the automation and intelligence of manufacturing systems as the main transformation direction [1]. And establishing robust process monitoring systems has become one of the research hotspots, especially for the aviation and aerospace industries. Due to the high specific strength and excellent corrosion resistance, Titanium alloys have become one of the main raw materials of the main monolithic components in these fields. However, titanium alloy is a typical difficult-to-machine material. Its low thermal conductivity, poor plasticity, and lively chemical properties lead to high cutting resistance and high cutting temperature, which cause cutting tools to be easily worn or damaged. And tipping is one of the main tool failure modes. If the tool is not replaced in time after the tipping occurs, it is easy to cause adverse effects such as poor surface integrity, burned parts, and even machine tool damages. Therefore, the effective monitoring of tipping is essential to improve production efficiency, ensure manufacturing quality, and reduce the rejection rate [2].

For the monitoring purposes in machining, the majority of the current researches on TCM are employing the indirect sensor-driven monitoring means. The commonly-used sensing signals include cutting force [3], [7], vibration [8], [9], acoustic emission [10], [11], sound [12], temperature [13], spindle current or power [14], [15], etc. The correlation model between the sensor signals and tool conditions would be established to implement the TCM tasks. How to extract the most relevant information from the raw sensor signals, which directly affects the final monitoring performances. Milling is a typical discontinuous cutting process and generates a large number of nonstationary signals. The traditional time-domain and frequency-domain analysis methods can no longer provide more detailed information for the intelligent process monitoring systems. The time-frequency analysis methods have become a hot topic in signal processing technology in recent years, which can provide joint distribution information in the time and frequency domain [16], such as
Wavelet analysis, Wavelet packet analysis, Empirical Mode Decomposition, etc. Among them, due to the non-sensitivity to the varied cutting conditions, wavelet analysis has achieved excellent application results in TCM.. [16], [17].

From the signal processing perspective, tool tipping would lead to the abrupt changes in the signals’ waveforms. This phenomenon can be characterized by the singularity analysis method, which employs the characteristics of wavelet decomposition coefficients to characterize signal waveform changes quantitatively. The singularity analysis was originally applied to image edge identification and image compression [18]. Then it was employed to the machine fault diagnosis [19], [20]. The TCM approaches established on singularity analysis was firstly introduced in turning with acoustic emission signals [21]. Bukkapatnam et al. [22] used the fractal features to characterize the singularities in cutting force and vibration signals for proposing a turning TCM approach. Zhu et al. [23] employed the probability density distributions of singularity index, i.e. Holder Exponent (HE), estimated from the cutting force to classify different tool statuses in micro-milling. Zhou et al. [24], [25] studied the difference in adopting singularity analysis to estimate the cutting force and vibration of the milling process, and established a TCM approach with good performances. However, there are few researches on employing singularity features to establish tool tipping monitoring models published.

In this study, the singularity analysis based on wavelet transform was employed to extract the correlated features from the vibration signals, then the HE features were adopted to establish a tool tipping monitoring (TTM) approach for the milling process. The paper is organized as follows. Section II builds the singularity analysis procedure, and section III presents the experiments and results, then the singularity features are extracted and the tool tipping monitoring approach are established in section IV. Section V concludes the results.

II. SINGULARITY ANALYSIS

A. SIGNAL WAVEFORM SHAPES AND TOOL TIPPING

Vibration signals are discovered extremely correlated to different tool conditions and very appropriate for achieving the TCM tasks [26]. The conceptual idea of this study is that the tipping of the cutting edges will lead to abrupt variations in signals’ waveforms, and this characteristic could be made use of indicating the tool conditions. Moreover, the degree of this variation can be characterized by singularity. This basic scheme is displayed in Figure 1. The Holder Exponent (HE) index can characterize the singularity quantitatively [17].

B. HOLDER EXPONENT

If a function \( f(t) \) is not continuity at point \( t = v \); in other words, this point does not have 1st order or \( n \)th order derivatives, we define this point as a singular point. These discontinuity, disorder or un-smoothness can be qualitatively characterized by Holder Exponent (HE, \( \alpha \)) index [17].

Generally, the smaller the amplitude of the HE indexes, the more singular and disorder the points are.

If \( f(t) \) is said to be HE \( \alpha \geq 0 \) at point \( v \), and if a constant \( A \) \((A > 0)\) and the \( m^{th} \) \((m \text{ is an integer and } m \leq n)\) order Taylor polynomial \( p_n(t) \) exist, then

\[
f(t) = p_v(t) + \varepsilon_v(t)
\]

where \( \varepsilon_v(t) \) is the fitting error of the polynomial \( p_n(t) \), then the HE \( \alpha \) determines the upper bound of this error as shown in (2). Then the supremum of \( \alpha \) is defined as the HE of \( f(t) \) at \( v \). Generally, smaller \( \alpha \) indicates that the \( f(t) \) is more singular. If \( f(t) \) is \( n \)-time differentiable, and the discontinuity happens to its \( n \)th-order derivative at \( v \), the \( \alpha \) also describes this singularity with \( n < \alpha < n + 1 \). Nevertheless, the computation of HE is extremely complex. It was found that the variation of the wavelet transform modulus maxima (WTMM) along the scale \( s \) can be employed to calculate the HE index [17].

Firstly, after obtained the wavelet transform \( WTf(u,s) \), the partial differentiation of \( WTf(u,s) \) was calculated and set to zero at \( u \) to search for the local extrema along with \( s \).

\[
\frac{\partial WTf}{\partial u} = 0.
\]

where \( s \) is scale factor, and \( u \) is space factor of wavelet transform \( WTf(u,s) \). Generally, the WTMMs generated by the signal’s singularity can be connected to form a curve along with \( s \), which is said to be the modulus maxima line. Along this line, the wavelet transform possesses the scaling ability around point \( t \), expressed as [17]:

\[
|WTf_s(t)|A \leq s^{\alpha +1/2}.
\]

To reduce the amount of calculation as much as possible, the discrete wavelet transform \((s = 2)\) was adopted to obtain the wavelet coefficients, and this discrete scale would not lead

![FIGURE 1. Signal waveform changes before and after tool tipping.](image-url)
to lines’ discontinuities [17]. Then function (4) can be written as

\[ \log_2 |WT_2 f(t)| \leq \log_2 A + j (\alpha + 1/2). \quad (5) \]

By setting equal in (5), the constant \( A \) and variable \( \alpha \) can be estimated. The range of HE depends directly on which derivative of \( f(t) \) the singularity appears. For example, the ramp signal is 1-time differentiable with HE = 1; the step function, i.e. the 1\(^{st}\)-order derivative of a ramp signal has HE of 0; and the white noise is singular almost everywhere, so its HE = \(-0.5 - \varepsilon\) (\(\varepsilon > 0\)). It can be perceived that the noise would produce negative HE values, and these noises are almost singular everywhere. While calculating WTMMs, the samples dominated by noise would result in a large number of pseudo-WTMMs. And these pseudo-WTMMs would increase remarkably as the \( s \) reduces, this characteristic can be employed to remove these contaminations as a de-noising method [25].

III. EXPERIMENTS

A. EXPERIMENTAL SETUP

In order to obtain the original signal before and after tipping during the cutting process of titanium alloys, an experiment of milling titanium alloy (TC4) was designed and performed. Figure 2 shows the experimental setup. A TC4 work-piece with a size of 400 mm \( \times \) 100 mm \( \times \) 150 mm was mounted on the machine table by a vise. A brand-new solid carbide mill cutter was employed; its detailed parameters are displayed in Table 1. The milling paths are illustrated in Figure 3, each milling path is 400 mm. The work-piece was machined with a down milling mode; the cutting parameters are shown in Table 2. The experiments continued to cut until the cutter reached its wear limit [2], a total of 85 cuts were performed. During each cut, a triaxial accelerometer (Type-O3PZ0110009 produced by MARPOSS) was mounted on the spindle head to acquire the vibration signals with a sampling frequency of 5120 Hz, and an optical microscope (Dino-lite AM7013MZT4) was employed to measure the tool wear of each edge after every cut.

B. ANALYSIS OF EXPERIMENTAL RESULTS

When machining titanium alloys, a difficult-to-machine material, multiple wear mechanisms may occur on the cutting edges at the same time, such as abrasion, adhesion, chemical wear [2], [26], etc. In order to quantify the tool wear, the maximum width of the flank wear land (\( V_B \)) is usually adopted and most recommended. Figure 4 shows the maximum \( V_B \) of all five cutting edges after each cut throughout the whole cutting life of the cutter. It can be observed that the tipping happened right after the 55\(^{th}\) mill path. By then, the cutter had been utilized for 30.25 min.

Figure 5 shows the flank wear lands of all five cutting edges, it can be observed that the tipping occurred on the...
Edge #1 with \( VB \) of 0.155mm, which was much more serious than the other edges. From the perspective of the raw signals' waveform, Figure 1 shows the vibration signal in the feed direction \( (a_x) \) right before and after the tipping occurred. It can be found that the tipping made the amplitudes of the signal slightly increase, and the main difference is that there are more abrupt increasing points in the signal after tipping. Therefore, we can use these characteristics of these abrupt increasing points as indexes of tool tipping.

IV. RESULTS AND DISCUSSIONS

A. HE ESTIMATION PROCEDURE

The selection of wavelet basis functions is critical for the singularity analysis of sensory signals. Zhou et al. [24], [25] found the wavelet basis with two vanishing moments were the most appropriate wavelet to detect the singularities in the vibration signals of the milling process. And Gaussian wavelet can ensure the continuity of the modulus maxima line [18]. Thence, this paper uses a Mexican Hat Wavelet (also named as the 2\(^{nd}\) order derivative of Gaussian function) as the wavelet basis function to analyze the singularity of the vibration signal. Figure 7 illustrates the HE index estimation procedure. The vibration signals of 100 spindle rotations (the duration is 0.052s, 16000 data points in total) were extracted from the milling segment of each cut to form the sample sets. Since the amount of WTMMs’ calculation will increase rapidly with the increase of the maximum of \( j \) (scale \( s = 2^j \), \( j = 1, 2 \ldots J \)) [19], so the \( J = 4 \) was selected to reduce the calculation amount as much as possible. Firstly, the wavelet decomposition of sample sets was obtained, and all the WTMMs were detected on the \((u, s)\) plane. Then, based on the change trends of the WTMMs with the scale \( s \), the sample points controlled by noise were eliminated, and the HE values of the remaining WTMMs were estimated.

B. HOLDER EXPONENT ANALYSIS

Based on the HE estimation procedure, the singular points of the vibration signals of all three channels were detected, and their HE values and corresponding positions are shown in Figure 7. It can be observed that the variation of HE estimation results of vibration in the feed direction \( (a_x) \) is more significant before and after tipping, as shown in Figure 7 (a). This is because that the feed direction bears the largest cutting resistance force, and the tipping of the cutting edges causes the vibration in this direction to deteriorate rapidly. It can be observed that there are only several points with stronger singularity, i.e. with lower HE values, before tool tipping. This is caused by the initial abnormal wear of the cutting edges. And after the tool tipping, it can be seen that the similar discrete singular points have increased significantly, and the overall distribution range of HE values becomes wider. This is mainly because the tipping only occurred on edge #1, which result in the discrete distribution. And the singularity caused by tipping is quite strong, so the amplitude of HE is much smaller, which widens the HE range.

In order to estimate the change of HE values more intuitively, the probability density distribution of the HE value estimated from the signal before and after tipping was calculated, as shown in Figure 8. Here, the HE density probability distributions of the four sets (NO. 53~ NO.56) of cutting vibration signals before and after tipping were calculated continuously. It can be perceived that they all meet the Gaussian distribution (also named standard normal distribution). Before tipping, the distributions of HE values are quite narrow, after the tipping happens, the HE range becomes wider and shorter, which will inevitably lead to the changes in the main parameters (Mean \( \mu \) and Standard deviation \( \sigma \)) of the probability density function. In order to provide quantitative indicators for online monitoring of tipping, we calculated the statistical features commonly used to describe the probability
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FIGURE 7. HE variation before and after tipping happened. (a) HE estimation of vibration \(a_x\), (b) HE estimation of vibration \(a_y\), (c) HE estimation of vibration \(a_z\).

FIGURE 8. HE probability densities of \(a_x\) before and after tipping happen.

density distributions, i.e. Mean, Standard deviation (STD), Skewness, Kurtosis, Maximum value and Minimum value of HE values. Besides, when detecting singular points in raw vibration signals, the quantities of singular points were also found related to different tool statuses. Therefore, the number of singular points was extracted as another parameter.

Figure 9 displays the variation of the extracted HE statistical features with the gradual tool wear. It can be found that the Mean of HE values and Number of singular points have changed most significantly after the appearance of tipping, as shown in Figure 9 (a) and (b). Due to the stronger singularity of the vibration caused by tipping, the Mean of HE values decreased dramatically. And the tipping produced more singular points resulting in the increase of the Number. At the same time, it can also be observed that the three other statistical features, i.e. Standard deviation, Skewness, and Kurtosis, also changed after tipping slightly. Therefore, it can be perceived that these statistical features could be employed as an indicator to detect the tipping during the cutting process.

Besides the vibration signals in the feed direction, the statistical features of the HE values of the vibration signals in the other two directions were also extracted, so we obtained a total of 21 features.

C. HE FEATURES SELECTION

After the HE statistical features were extracted, a total of 21 indicators were obtained. Based on the analysis of the previous section, it can be found that there are some poorly correlated features. Through the direct observation, it can be perceived that the two features, i.e. Mean of HE values and Number of singular points of \(a_x\), are highly correlative with tipping. However, in order to quantify these correlations, we employed the mutual information (MI) method [27] to score the discriminability of the 21 features. Since the MI method can only analyze discrete data, then the Minimum Description Length Principle (MDLP) was employed to discretize these continuous HE features initially [28].

The MI method can describe the correlation between any two variables \(X\) and \(Y\), or between variables and a class of variables quantitatively. In order to calculate the MI index \(I(X; Y)\) [27], the entropy measure \((H(X))\) of variable \(X\) should be calculated first:

\[
H (X) = - \sum_{X \in \Omega_X} p (x) \log_2 (p (x)) .
\]

Then, the conditional entropy \((H(X|Y))\) of \(Y\) under the occurrence of \(X\) need to be calculated by:

\[
H (X | Y) = - \sum_{X \in \Omega_X} \sum_{Y \in \Omega_Y} p (x, y) \log_2 (p (x | y)) .
\]
FIGURE 9. Relationship between HEs’ statistical features and gradual tool wear (VB). (a) Mean of HE index, (b) Number of singular points, (c) Standard deviation, (d) Skewness, (e) Kurtosis, (f) Maximum of HE index.

where \( \Omega_x \) and \( \Omega_y \) are the variable spaces of \( X \) and \( Y \). And \( p(x), p(y), p(x,y) \) are the probability density functions of \( X \), \( Y \) and \( (X,Y) \). \( p(x|y) \) can be estimated as:

\[
p(x|y) = \frac{p(x,y)}{p(y)}.
\] (8)

Then, the MI index \( I(X;Y) \) can be calculated by

\[
I(X;Y) = H(X) - H(X|Y).
\] (9)

After \( I(X;Y) \) is obtained, the symmetric uncertainty \( SU(X,Y) \) analysis is adopted to score all the features, which is computed as:

\[
SU(X;Y) = 2 \frac{I(X,Y)}{H(X) + H(Y)}.
\] (10)

The top 10 \( SU \) results of the HE statistical features are displayed in Figure 10. It can be seen that the Mean of HE values and Number of singular points of vibration in the feed direction \( (a_x) \) score the highest, which are consistent with the analysis of the previous section. And the score of these two features are much higher than the rest, then they are selected as indicators to establish a tool tipping monitoring approach in the following section.

D. TOOL TIPPING ESTIMATION WITH HE INDICATORS

The monitoring of tool tipping is a typical pattern classification; an effective classifier will be able to accomplish the monitoring of the tool tipping. The pattern classification belongs to the category of the weak Artificial Intelligence (AI), and the Machine Learning (ML) models are the core technology to implement these weak AI applications. Various classifiers based on ML models have achieved good results in TCM [29]–[31]. Therefore, multiple commonly used ML algorithms were adopted to establish the classifier models for tool tipping, including Decision Tree (DT), Support vector machine (SVM), k-Nearest Neighbor (KNN), Ensemble Learning (EL), etc., with the selected HE indicators.

Figure 11 illustrates the establishing procedure of the Tool Tipping Monitoring (TTM) model. The tool condition can be divided into two categories according to whether the tool tipping occurs as: State 1 is before tipping occurs; State 2 is after tipping occurs. A total of 300 groups of raw vibration signal of 30 spindle rotations, i.e. 4800 sample points, were extracted from the stable milling segments of each cut (150 groups of State 1, and 150 groups of State 2). Then, the selected indicators, i.e. the Mean of HE values and Number of singular points of vibration in the feed direction \( (a_x) \), were calculated to compose of the Training set. Moreover, another 200 groups of HE indicators were estimated in the same way to form the Test set (100 groups of State 1, and
100 groups of State 2), and their raw vibration signals are different from the Training set. The Training set was normalized firstly, and then was employed to train several ML models and optimize the kernel parameter of each ML models. Moreover, 10-fold cross-validation was adopted to obtain the training accuracy. Afterwards, to compare the classification accuracy of all ML models, the optimized ML models were used to classify the Test set.

Table 3 shows the training accuracy, kernel parameters and classification accuracy of each ML models. It can be seen that the SVM model with Gaussian kernel function scores the highest training accuracy of 99.4%. And it also achieves the highest classification accuracy of 98.7% on the Test set. This is mainly because that the SVM algorithm has strong small sample analysis ability and classification accuracy with a solid mathematical basis. Table 4 displays the classification performance of the state of the art researches on tool tipping monitoring for milling. It can be observed that the classification result of the proposed TCM approach is comparable to the above studies. The classification performance indicates that this SVM model can provide accurate estimation results for the occurrence of tool tipping and provide an effective guide for tool change in the factories.

TABLE 3. Performances of ML models.

| ML models                | Kernel parameters | Training accuracy | Classification accuracy |
|-------------------------|-------------------|-------------------|------------------------|
| Support Vector Machine  | Kernel function   | 99.4%             | 99.3%, 98%, 98%        |
|                         | Scale             |                   |                        |
| k-Nearest Neighbor      | Number of neighbors | 95.3%, 98%, 93.7% |                        |
| Decision Tree           | Max number of splots | 97.6%, 94%, 96.3% |                        |
| Ensemble Learning       | Ensemble method   | 98.2%, 100%, 96.3%|                        |

V. CONCLUSIONS

A tool tipping monitoring approach has been established for the milling process with the singularity features of the vibration. The singularity analysis was adopted to de-noise and extract the Holder Exponents (HE) from the raw vibration signals. The HE indexes of the vibration in the feed direction $a_x$ were found correlated to the occurrence of tipping. The probability density distributions of HE indexes of $a_x$ before and after tipping have significant differences. The statistical features of HE indexes of $a_x$, i.e. the Mean of HE values and Number of singular points, have the strongest discriminability for tool tipping based on the ranking scores of the Mutual Information method. The SVM model established on the selected HE features has the highest training and classification accuracy performance, 99.4% and 98.7%, which can provide a practical guide for tool change in the factories.

Future work will be focused on applying this method to tool wear condition monitoring, chatter detection and surface roughness prediction, etc.

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