Investigating and improving introductory physics students’ understanding of the electric field and superposition principle

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Abstract

We discuss an investigation of the difficulties that students in a university introductory physics course have with the electric field and superposition principle and how that research was used as a guide in the development and evaluation of a research-validated tutorial on these topics to help students learn these concepts better. The tutorial uses a guided enquiry-based approach to learning and involved an iterative process of development and evaluation. During its development, we obtained feedback both from physics instructors who regularly teach introductory physics in which these concepts are taught and from students for whom the tutorial is intended. The iterative process continued and the feedback was incorporated in the later versions of the tutorial until the researchers were satisfied with the performance of a diverse group of introductory physics students on the post-test after they worked on the tutorial in an individual one-on-one interview situation. Then the final version of the tutorial was administered in several sections of the university physics course after traditional instruction in relevant concepts. We discuss the performance of students in individual interviews and on the pre-test administered before the tutorial (but after traditional lecture-based instruction) and on the post-test administered after the tutorial. We also compare student performance in sections of the class in which students worked on the tutorial with other similar sections of the class in which students only learned via traditional instruction.

We find that students performed significantly better in the sections of the class in which the tutorial was used compared to when students learned the material via only lecture-based instruction.

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Introduction

Electrostatics is an important topic in most high school and college introductory physics courses. Although Coulomb’s law and the superposition principle are taught extensively in a majority of these courses for science and engineering majors, these concepts are challenging for many students to grasp after traditional lecture-based instruction only. Despite the fact that students may have learned the superposition principle in the context of forces in introductory mechanics, this learning does not automatically transfer from mechanics to the abstract context of electrostatics and students often struggle in applying the superposition principle due to the different ‘surface’ features [1, 2] of the electrostatics problems compared to mechanics problems. However, finding the net electric field due to a charge distribution requires understanding the principle of superposition for the electric field and students must learn to add the field vectors at a point due to various charges in the region. Since these concepts are challenging, developing research-validated learning tools to help students learn these concepts can help them develop a more coherent knowledge structure and can also improve students’ problem solving and reasoning skills [1–7].

Investigation of student difficulties related to electricity and magnetism (E&M) concepts is important for designing instructional strategies to reduce the difficulties and help students develop a good grasp of physics concepts. The origin of student difficulties in learning physics concepts can broadly be classified into two categories: gaps in students’ knowledge and misconceptions [2]. Cognitive theory suggests that learning is incremental and new knowledge builds on prior knowledge [8]. Knowledge gaps can arise from many sources, e.g., a mismatch between the pedagogical approaches and levels at which the material is presented in a course and student’s prior knowledge [1–6]. Deep-rooted misconceptions can also seriously impede the learning process at all levels in physics instruction [1–6].

Here we discuss an investigation of student difficulties with the electric field and superposition principle and how that research on student difficulties was used as a guide in the development and evaluation of a tutorial to help introductory physics students develop a functional understanding of these concepts. We find that students in the sections of the course in which the tutorial was used performed significantly better on the post-test than those who did not use them.

Prior investigations in introductory E&M

Several prior studies have focused on the difficulties of introductory physics students with E&M and strategies that may help students learn these concepts better [9–27]. For example, the investigation by Eylon and Ganiel suggests that macro–micro relationships may be the missing link between electrostatics and electrodynamics in students’ reasoning [9]. Zuza et al have investigated student difficulties with Faraday’s law and strategies to help students learn related concepts better [10–12]. Guruswamy et al [13] studied student understanding of transfer of charge between conductors and Guisasola et al [14, 15] have carried out investigations related to student understanding of capacitors. Dębowska et al reported on robust multimedia resources to help students learn E&M [16]. Baird et al [17] have used spreadsheets to improve student learning of Gauss’s law and Savelsbergh et al have investigated the role of situational knowledge in learning E&M [18]. Sujarittham et al [19] have developed
guided active learning worksheets while Efthimiou et al [20] and Lenaerts et al [21] have 
used pedagogical approaches to help students learn, e.g., by using peer instruction. Itza-Ortiz 
et al studied students’ models of Newton’s second law in mechanics and electromagnetism 
and found that three different models were prevalent in student reasoning [22]. Alejandro and 
Zavala have contrasted students’ understanding of electric field and electric force [23]. 
McDermott and Shaffer performed an investigation of the difficulties students have with 
electrical circuits and developed an enquiry-based curriculum that significantly reduces these 
difficulties [24]. Thacker et al investigated student understanding of transients in direct 
current electric circuits [25]. Singh and collaborators have investigated student reasoning 
difficulties with electrical circuit elements, non-identical light bulbs in series and parallel, 
Gauss’s law, conductors and insulators and the roles of peer instruction and representation in 
learning electrostatics [26, 27]. Several research-based assessment tools that focus on intro-
ductive E&M have been developed. For example, Maloney et al [28] and Ding et al [29] 
developed tests that broadly survey many important concepts covered in the introductory 
E&M. Engelhard et al [30] have developed a conceptual assessment related to circuits, Singh 
a conceptual assessment on symmetry and Gauss’s law and Li et al a conceptual assessment 
on magnetism [31]. These surveys show that introductory students have conceptual difficul-
ties with concepts related to E&M.

Methodology

The students who participated in this investigation were enrolled in different ‘equivalent’ 
sections of a second semester college introductory physics course, mainly taken by engi-
neering, chemistry, mathematics and physics majors. Approximately, one fifth of the students 
in these courses are females. This course covers E&M and some wave optics and calculus is 
used in the course since students are supposed to have taken calculus before they take this 
course. It is taken after the first introductory physics course, which covers mechanics and 
waves. Most of the several hundred students in different sections of this course were college 
freshmen who had completed high school (we do not know how many of these students had 
taken high school physics course but the pre-test was administered to students in all sections 
after traditional lecture-based instruction in the college course). The students in this course 
had 4 h of lecture time and 1 h of recitation time. The different sections of the course were 
generally taught by different instructors and the recitations were taught by graduate teaching 
assistants. All of the sections of the course discussed in this investigation primarily had 
traditional lecture-based instruction in the 4 h of lecture time, and in the recitations, the 
graduate teaching assistants fielded questions about the homework from the students and 
solved example problems on the board (except in the sections of the course that we designate 
the experimental group in which students worked on the tutorial in class immediately after 
lecture-based instruction in those concepts).

The reason there are several sections of this same course offered in the same semester at 
the university where the investigation was carried out (University of Pittsburgh) is that this 
course is mandatory for several hundred engineering freshmen and also for other majors, e.g., 
from chemistry, mathematics etc. The content covered by all the different sections of the 
course (both experimental and comparison groups) is the same. Each week students were 
asked to do homework which was from the textbook (introductory physics textbook by 
Halliday, Resnick and Walker) except that in the experimental group, the part of the tutorial 
that students did not complete in class, they were asked to complete as homework in addition 
to the textbook homework. In all sections of the course, each week after students submitted
the homework on a particular topic, they were given a recitation quiz in the last 15–20 min of the recitation class.

We note that this investigation employs a quasi-experimental design [32] in that we did not have control over whether a particular student will be in the section of the course in which the tutorial was used and we did not have control over who their instructor and teaching assistant (for the recitations) would be. Also, although students in all sections of the course used the same textbook, we did not have control over the textbook homework assignments or the midterm and final exam given by the instructors of different sections (since the instructor of each section in the same semester had full control of their section of the course). However, the conceptual survey of E&M [28] given to some of the sections in the previous years as a pre-test and post-test suggests that student performance on average in various sections of the course is comparable at the beginning of the course (pre-test) and on the post-test after traditional lecture-based instruction.

The development of the research-validated guided enquiry-based tutorial was carried out with the following core issues in mind: (1) the tutorial must build on students’ prior knowledge so it is important to investigate the difficulties students have related to relevant concepts before the development of the tutorial, (2) the tutorial must create an active learning environment where students get an opportunity to build a good knowledge structure in which there is less room for misconceptions, (3) the tutorial must provide scaffolding support, guidance and feedback to students and opportunity to organise, reconstruct, and extend their knowledge.

The process of the development and validation of the questionnaire and tutorial spanned three years and started with a cognitive task analysis from the perspective of an expert and an investigation of the common difficulties that introductory physics students in this course have with the electric field and superposition principle. Thus, the preliminary version of the questionnaire (which was refined into pre-/post-test questions) and the tutorial not only used research on student difficulties as a guide but also a cognitive task analysis of the underlying concepts from an expert perspective. The cognitive task analysis from the perspective of an expert involves making a fine-grained flow chart of the concepts involved in solving a specific class of problems. Such analysis can help identify some stumbling blocks where students may have difficulty. However, investigation of students’ difficulties using written tests and interviews was critical for developing the tutorial because theoretical analysis from the perspective of an expert often does not capture all of the difficulties students have with relevant concepts.

Table 1 summarises this process for data collection before, during and after the development and validation of the questionnaire and different versions of the tutorial. The student difficulties were investigated by administering validated open-ended and multiple-choice questions in written form to introductory students in various sections of the course after traditional instruction in relevant concepts and via individual interviews with a subset of students. These open-ended and multiple-choice questions were validated with the help of physics instructors who had taught this course several times (the questions were iterated with them to ensure that they were robust and interpreted unambiguously by experts) and introductory physics students to ensure, e.g., that they interpreted the questions as intended. Since the validated versions of the open-ended questions became the pre-test and post-test questions, we will focus on student difficulties vis a vis their performance on the pre-test and post-test questions. The validation of the multiple-choice questions is discussed elsewhere [31]. Individual interviews were conducted using a semi-structured, think-aloud protocol to better understand the rationale for student responses before, during, and after the development of different versions of the tutorial and the corresponding pretest and posttest. During the semi-structured interviews, introductory students were asked to verbalise their thought processes.
while they answered the questions. Students read the questions related to the electric field and superposition principle and answered them to the best of their ability without being disturbed. We prompted them to think aloud if they were quiet for a long time. After students had finished answering a particular question to the best of their ability, we asked them to further clarify and elaborate issues that they had not clearly addressed earlier.

The guided enquiry-based tutorial on the electric field and the superposition principle is conceptual in nature and similar in spirit to the introductory physics tutorials developed by the University of Washington group [24]. The tutorial was also implemented in a somewhat similar manner to that in which the Washington tutorials are implemented [24] and students worked on them in small groups. However, in our implementation, whatever part of the tutorial students could not finish, they were asked to finish as part of their homework. All students who were individually interviewed signed a consent form (however, it was not needed for the in-class administration of the tutorial since it was implemented as part of the course material and was part of the evaluation of the effective teaching approaches for the introductory physics course).

During the development of the tutorial, in the individual interviews, we administered the pre-test, tutorial and post-test to some introductory physics students, who were asked to talk aloud while working on them. After each administration, we modified the tutorial based upon the feedback obtained from student interviews. We note that although a few of the interviewed students were females, we will use male pronouns to refer to all interviewed students here in order to hide the gender identity of students. The tutorial was also iterated several times with four physics faculty members who had taught the introductory E&M course for their feedback, and modified after each feedback. These individual administrations helped fine-tune the tutorial and improve its organisation and flow. In summary, the development of the tutorial went through a cyclic, iterative process which included the following stages before the in-class implementation in several sections of the introductory physics course.

1. Development of the preliminary version based on a cognitive task analysis from an expert perspective of the underlying knowledge and research on student difficulties.
2. Implementation and evaluation of the tutorial by administering it individually to students and obtaining feedback from faculty members who teach the relevant introductory course.

### Table 1. Procedure for data collection before, during and after the development and validation of the questionnaire and different versions of the tutorial.

| Before the development of the questionnaire and tutorial | During and after the development of the preliminary version of tutorial: individual interviews with students and individual discussions with faculty members |
|----------------------------------------------------------|---------------------------------------------------------------------------------------------------------------|
| • Cognitive task analysis from an expert perspective of the relevant concepts | • One-on-one interviews with students (N = 10) examining responses to |
| • Development of questions and discussions with faculty teaching this course about them to validate them | ○ tutorial pretest questions |
| • Several years of examining student responses to | ○ enquiry-based learning sequences in tutorial |
|   ○ open-ended questions | ○ tutorial posttest questions |
| • Individual interviews with students (N = 5) | • Discussions with faculty members |
|                                                | • Refinement based upon feedback from students and faculty |

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(3) Determining its impact on student learning and assessing what difficulties were not adequately addressed by the tutorial.

(4) Refinements and modifications based on the feedback from the implementation and evaluation.

Table 2 is a summary of activities in the experimental group (tutorial group) and comparison group (non-tutorial group) in different sections of the college introductory physics course.

| Tutorial group or Experimental group (worked through the tutorial after traditional instruction) | Pretest (quiz graded for completeness) | Learning activities after lecture-based instruction and pretest | Posttest (quiz graded for correctness) | End-of-Semester standardized test on relevant concepts |
|---------------------------------|-------------------------------------|-------------------------------------------------------------|----------------------------------|--------------------------------------------------|
| Traditionally lecture-based instruction in relevant concepts for both groups | $N_1 = 83$ | Worked on tutorial in class in small groups and whatever students could not finish in class was finished individually as homework in addition to textbook homework. Students had opportunity to ask questions. | $N_1 = 83$ | $N = 278$ (four sections of the course including three sections for which the pretest and posttest data are available). |
| (but less time spent by instructor solving example problems during lecture in the tutorial group since students in that group worked on tutorial during part of class) | $N_2 = 61$ | $N_1 = 59$ | $N_2 = 61$ | $N_1 = 59$ |

| Non-tutorial group (did not work through the tutorial) | Pretest (quiz graded for completeness) | Learning activities after lecture-based instruction and pretest | Posttest (quiz graded for correctness) | End-of-Semester standardized test on relevant concepts |
|---------------------------------|-------------------------------------|-------------------------------------------------------------|----------------------------------|--------------------------------------------------|
| Traditional textbook homework problems on relevant concepts | $N_1 = 66$ | Worked on textbook homework problems on relevant concepts. Students had opportunity to ask questions about concepts (including the textbook homework). | $N_1 = 66$ | $N = 135$ (two sections of the course including one section for which the pretest and posttest data are available). |

In Table 2, the timeline or chronological order is from left to right as depicted by the blue arrow. As can be seen from Table 2, both groups had traditional lecture-based instruction in relevant concepts but there was less time spent by the instructor solving example problems during lecture in the tutorial group since students in that group worked on the tutorial during part of class. After the traditional lecture-based instruction, all students (regardless of whether they were in the experimental or comparison group) took the pre-test in class as a quiz which was graded for completeness (all students obtained full quiz grade for trying their best). Then the students in the experimental group worked on the tutorial in small groups in class and finished whatever part of the tutorial they could not finish individually as part of the homework. The comparison group (non-tutorial group) only had traditional textbook homework. After all students submitted their homework on these concepts, they had the opportunity to ask questions before they were given the post-test as a recitation quiz. The post-test quiz was graded for correctness but the weight assigned to the post-test was the same as the pre-test and each counted for less than 1% of the course grade.

To summarise, in the tutorial group, as shown in Table 2, the number of matched students (who were present when both the pre-test and post-test were administered) in the three sections of the course in which the tutorial and the pre-/post-tests were administered ranged from 59 to 83. The pre-test was administered individually after traditional instruction right
before students worked on the tutorial in groups. Although the pre-test and post-test accompanying the tutorial assess the same concepts, the same test was not used as both the pre-test and post-test to minimise the effect of pre-test on post-test. Also, the pre-test was neither returned to the students nor was it discussed by the instructors with the students. Since not all students completed the tutorial during the class, they were asked to complete it as part of their homework assignment. At the beginning of the next class, students were given an opportunity to ask for clarification on any issue related to the part of the tutorial they completed at home and then after they submitted the tutorial, they were administered the corresponding post-test individually. While the pre-test was not returned, the post-test was returned after grading.

Table 2 also shows that at the end of the semester, students in both the experimental and comparison groups were given an end-of-the semester standardised test that included these concepts. We note that the tutorial was administered in four sections of the course but in one of these sections, the pre-test and post-test were not photocopied before returning them to the students (however, the data from the end-of-the semester standardised test was available for all the four tutorial sections). In the comparison group, the pre-test and post-test data were available from one section of the course but the data from the end-of-the semester standardised test were available for two sections.

We note that in all of the sections of the course in which the tutorial was administered, the total time devoted to the electric field and superposition principle is not significantly different from what the instructors in other sections of the course in which the tutorial was not used allocated to this material. As noted, the main difference between the tutorial group (experimental group) and the non-tutorial group (comparison group) of the course is that fewer solved examples were presented by the instructor during lecture in the tutorial sections of the course (students worked on many problems themselves in the tutorial). Also, since the tutorial was administered during the lecture portion of the class, the instructor and the teaching assistant were present during these tutorial sessions to ensure smooth facilitation. Typically, students worked in groups of three and they were asked to raise their hands for questions and clarifications. Once the instructor and teaching assistant knew that a group of students was making good progress, that group was invited to help other groups in the vicinity with similar questions. Thus, students not only worked in small groups discussing issues, some of them also had an opportunity to help students in the other groups.

**Student difficulties**

Before we discuss student difficulties found, we note that students in the sections of the class in which the tutorial was used as a pedagogical tool were less likely to have the difficulties after working on the tutorial (on the post-test) than on the pre-test (after traditional lecture-based instruction only). For reference, the entire pre-test and post-test for the tutorial are included in the appendix. Also, the difficulties on the pre-test and post-test were similar in nature for both the tutorial sections of the class and an equivalent comparison group consisting of students who did not work on the tutorial. The main difference between these groups was that the tutorial group students were significantly less likely to have the difficulties after working on the tutorial (the results are discussed later in the tutorial evaluation section).

All of the questions on the pre-/post-tests require the use of the superposition principle to find the electric field at various points for a given charge distribution. Many students had great difficulty with the principle of superposition and in calculating the vector sum of the fields
due to the individual point charges to obtain the net field at a point if more than one point charge is present in the region. In other words, the performance of many students on the pre-/post-tests was closely tied with their understanding of the principle of superposition pertaining to the electric field.

Below, we first discuss data on the common student difficulties found without separating the performance of the experimental group (tutorial group) and comparison group (non-tutorial group) on the pre-/post-tests. In order to come up with these categories, two researchers separately came up with the categories of the difficulties and most of their categories were overlapping which were kept. A few categories that were not on both researchers’ lists were discussed and some of those categories, e.g., confusion between electric field, force and charge, invoking the dynamics of charges in an electrostatics problem, solving problems from rote memory etc were included as separate categories after discussion. We note that later, in the tutorial evaluation section, we will present quantitative data separately related to how students performed in the tutorial and non-tutorial groups. But we do not report on the number of students who had specific difficulties in interviews because the interview data is used as a means to provide qualitative descriptions of how students reason about the concepts. For the in-class administration of the pre-test and post-test discussed later, the performance of students who took the pre-test and post-test in the different sections of the course was analysed and is represented in terms of the performance data on the tutorial pre/post test questions (see the appendix).

Incorrectly assuming that the electric field is the same at points which are not symmetrically positioned with respect to the charge distribution

Students were asked to compare the electric field at points due to more than one charge in situations in which there was not sufficient symmetry to claim that the magnitudes of the electric field are the same. However, many students incorrectly claimed that the magnitudes of the net electric field are the same at those points because the electric fields produced by different charges will somehow compensate for each other. These kinds of arguments from students typically made no mention of the vector nature of the electric field and were solely based upon arguments involving gut feeling.

For example, Question 1 on the pre-test (see the appendix) involves a situation with an isolated system of three identical point charge \(+q\) arranged in a straight line such that adjacent ones are equidistant and there are four different points where they are asked to find the electric field. Three points A, B, and C are on a parallel line and point D is equidistant from two of the charges on the straight line joining them. Students are asked a series of questions about this situation. For example, one question asks for the points (out of the four points A, B, C and D) at which students can exactly predict the angle the net electric field makes with the horizontal (to the right) without knowing the numerical value of the charge or the distances \(L\) and \(d\). To answer these types of questions correctly, students should understand how individual charges impact the field at a point and be familiar with the addition of vectors using the superposition principle to find net field at that point. For point C, the vertical components of the electric field due to the two symmetrically situated charges cancel out. The net field is the sum of the fields due to the middle charge and the horizontal components of the fields due to the other two charges. Thus, the net field at point C points horizontally. For point D, because D is equidistant from the top two charges and they are both positive, the effect of these two charges will cancel out. The net field at point D is due to the third charge and points in the vertical direction making a 90° angle with horizontal. For points A and B, it is impossible to exactly predict the direction of field without knowing the charge \(Q\) and the distances from
charges to points (length $L$ in terms of length $d$). In different sections of the introductory physics course combined, 70\% of students provided correct response for at least one point C or D. However, only 30\% of them provided a correct response for both points C and D.

Moreover, we find that 13\% of the introductory students incorrectly thought that the magnitude of the field is the same at points A, B and C but the directions are different. Interviews suggest that some of these students thought that the magnitude of the field at these points should be the same because these points were at the same perpendicular distance from the straight line joining the three charges. Some interviewed students provided more detailed reasoning. Instead of viewing this problem as a problem involving the addition of three electric field vectors, they often made guesses by looking at the distances of points A, B, and C from the three charges and hoping that the field will somehow work out to be the same at the three points. They often claimed, e.g., that point A is closer to one charge and farther away from the other two charges than point B, which is equidistant from the two charges and not as far away from the third charge as point A. Therefore, the field at points A and B will have the same magnitude if we take into account all the three charges. In one on one discussion, some students with such difficulties were explicitly asked about the case with only two point charges. When they were asked to compare the field at two different points such that the distances from the two point charges added up to the same value at each point but the symmetries of those points were different with respect to the two charges, they often continued to claim that the magnitude of the electric field produced by the two charges must work out to be the same at both points since the distances of the charges from the point added up to the same value in both cases.

Similarly, the most common difficulty with question (2) on the post-test (see the appendix) was that students agreed with the statement. Some students claimed that since one charge is closer and the other two charges are (for most points) farther from a point on the dashed triangle, the magnitude of the net electric field will work out to be the same at all points on the dashed triangle. The following are typical student responses that illustrate this type of incorrect claim.

- I agree because on the dashed triangle, the closer you get to one charge, you must move that much farther from the other charges. This keeps everything in equilibrium.
- I agree because everywhere on the dashed line the sum of the distances is the same from all 3 charges.
- Agree. Since it is a symmetrical situation, net electric field is equal everywhere.
- Agree because when the magnitude from one charge weakens because of increase in distance, the next charge evens it out. So the electric field magnitude is equal.

Interviews suggest that students with these types of responses, in general, had great difficulty in adding the electric field vectors due to the three charges at various points of the dashed triangle to find the field and realising that the magnitude of the field will not work out to be the same at all points of the dashed triangle. After the individual interviews, some students wanted the interviewer to discuss the correct answers for each question with them. The interviewer focused on the fact that vector addition was involved. She tried to explain to students that the magnitude of the field cannot work out to be the same at all points by choosing two points on the dashed triangle, one point at the vertex of the dashed triangle and another equidistant from two vertices on the dashed triangle, drawing individual contributions to the field due to the three charges and arguing that the vector sum need not have the same magnitude at the two points. However, some students argued that the interviewer had not been able to convince them. For example, one student said: ‘I do not see how you can convince anybody that those two points on the dashed triangle will not have the same
magnitude electric field without using numerical values for charges and distances’. The interviewer further tried to explain to the student by taking a limiting case such that the ring with three charges was very small compared to the dashed triangle so that the three point charges can approximately be lumped into a single point charge. She tried to explain that in this case, the field magnitudes cannot be the same at different points on the dashed triangle because the distances of different points on the dashed triangle from the single point charge ‘lump’ are different without even accounting for the fact that a vector addition is required to find the net field. The student was still not convinced and claimed that he did not see how the three point charges can be lumped into a single point charge if they are actually supposed to be arranged in a triangular shape no matter how large the dashed triangle is compared to the ring on which the charges are. When the interviewer had further discussion with the student about what a point charge is made of, the student initially said that it was probably a single proton or electron. When the interviewer pointed out that the point charges can have magnitudes much larger than the magnitude of the charge on a single electron or proton, which is $1.6 \times 10^{-19}$ C, the student looked somewhat worried. The discussion with this introductory physics student is typical of the common difficulty in conceptual reasoning and in making approximations that many other introductory students had. If students are given a quantitative problem with numerical values of charges and distances of the charges from the point where they have to find the electric field, students may succeed by using an algorithmic approach to solve for the electric field without even thinking about the symmetry of the charge distribution and conceptual implications of those results. Thus, the conceptual questions can sometimes be more challenging because students cannot use an algorithmic approach and must use their conceptual understanding and reasoning to answer these question correctly [33].

Some students claimed that the field will have the same magnitude at every point on the triangle except at the three vertices. For example, one student claimed: ‘disagree. The electric field is radial so for this effect to equal at every point on the dashed triangle, the corners would need to be rounded slightly’. The use of the word ‘slightly’ also suggests that the student may not have understood that the laws of physics are precise and the word ‘slightly’ that may be useful in everyday life may not convey the precision in physics.

Some students who correctly disagreed with the statement in question (2) on the post-test (see the appendix), did not explicitly mention anything about the vector nature of the field and claimed that the magnitude of the field cannot be the same at all points on the dashed triangle because different points of the triangle are closer from some charges and farther from the other charges. In the written responses, students obtained full credit if they noted that the magnitude of the field will not be equal at all points on the triangle and explained this by mentioning that the vector sum of the field due to the three charges at different points cannot be the same or invoked a reasoning involving the distances of different points on the dashed triangle not being the same from the three charges. For example, the following response was given full credit although the explanation is far from perfect: ‘Disagree because some parts of the dashed lines are closer to the charges than other parts. Thus, some parts will be less affected by some charges and the magnitude of the electric field at different points on the dashed triangle cannot be the same’.

In response to question (2) on the post-test, one of the interviewed students noted that the field magnitude is not the same at every point on the dashed triangle because the electric field is proportional to $1/r^2$. However, he incorrectly claimed that the sum of the distances from the three point charges of any point on the dashed triangle is the same and the field magnitude would have been the same at every point on the dashed triangle if it were proportional to $1/r$ instead of $1/r^2$. Further prodding showed that the student was not able to distinguish between a quantity proportional to $r$ versus $1/r$ and thought that the functional dependence $1/r$ is a
linear dependence on the distance $r$. Moreover, the student was ignoring the vector nature of the field and assuming that the net field due to the three charges will work out to be the same at all points on the triangle if the field were proportional to $1/r$. Similar difficulties were observed in other student responses as well.

**Only the nearest charge contributes to the electric field at a point**

Some students claimed that only the nearest charge will contribute to the field at a point. When asked to find the net field at a point, they only took into account the field at that point due to the nearest charge. For example, on question (1) on the pre-test, 28% of the students claimed that the net electric field at point D should be zero. Interviews suggest that many of them only considered the effect of the closest charges and ignored the effect of the third charge, which is further away from the point. The following response is typical among students who ignored the effect of the farther charge: ‘Yes, it is zero at point D. Because the two charges of $Q_1$ and $Q_2$ cancel each other out (the student had labelled two of the charges as $Q_1$ and $Q_2$) since they are the same magnitude but opposite direction’. Interviews also suggest that this type of reasoning is used by some students when they are asked to draw the directions of the field at points A, B and C. In particular, some students drew the directions of the electric field at each of those points (A, B and C) to the right (horizontal) because the field due to the charge that is closest to point A or point C (or charges that are closest to point B) is to the right (horizontal). Moreover, some of these students invoked this idea selectively and claimed that only the nearest charge will contribute to the field if they thought that the other charges are sufficiently farther away and could not contribute to the field at the point. For example, in question (2) on the post-test, the following are two typical sample responses provided by students with such notion: ‘The triangle will have less electric field at its corner because only one charge will act on it’. ‘The centre of dashed side of the triangle will have more electric field because more than one charge will act on it’. Similarly, on question (3) on the post-test (see the appendix), some students claimed that only the nearest charge will produce the field at points A, C or E.

**Charges in a straight line that are blocked by other charges do not contribute to the electric field at a point**

Some students claimed that if several charges are in a straight line, the charges that were ‘blocked’ by other charges do not contribute to the electric field at a point. For example, in response to question (3) on the pre-test (see the appendix), some students claimed that the electric field at point D is zero because the fields due to the two charges symmetrically positioned on opposite sides cancel out. Interviews confirm that some of these students were ignoring the effect of the third charge assuming it was blocked by the other charge and could not influence the electric field at point D.

**Confusion between the electric field due to an individual charge and the net electric field due to all charges in a region**

When the students were asked to draw the net electric field at a point, many students drew several arrows instead of one arrow. The responses to the pre-/post-test questions both in written administration and interviews suggest that some students could not distinguish between the field at a point due to the individual charges and the net field. These students often drew the fields at a point from the individual point charges present and did not add them vectorially even though they were asked about the net field. In other words, they often drew
several arrows to show the contributions of various point charges to the field at a point but did not realise they had to add them vectorially to find the net field. Interviews suggest that this difficulty is coupled with the difficulty students have in applying the superposition principle to determine the direction of the net field.

Confusion between electric field, electric force and electric charge

Some students had difficulty differentiating between the electric field, force and charge. They used the words ‘electric force’, ‘electric field’ and ‘electric charge’ interchangeably. For example, in response to various questions about the electric field at a point, some students claimed that the ‘charge’ or the ‘force’ at that point is in a particular direction. As noted, before the development of tutorial, we gave several open-ended and multiple-choice questions to students. In a multiple-choice question given to 541 introductory students, 10% of the students identified electric charge as a vector. In one-on-one interview situations, to justify why the electric charge is a vector, some students claimed that the positive charges point outward and the negative charges point inward. It appears from the responses that students were often referring to the field but calling it ‘charge’.

Electric field can only be found at points where there is a charge present

Some students thought that there must be a charge present at the point where they are asked to find the electric field. This type of confusion is often coupled with the difficulty in differentiating between the electric force and field. While discussing the pretest question about the field due to three identical point charges in a straight line, one interviewed student stated that the charge at point A will be repelled from other charges. He was explicitly asked by the interviewer why there was a charge at point A. The student first appeared a little concerned but then argued that there could not be any attraction or repulsion if there was no charge present at point A and it would not make sense to talk about electric field at that point. Further discussion suggests that the student was confusing ‘electric field’ at a point produced by charges in that region with the ‘electric force’ on a charge placed at that point. Since $F = qE$, students are often taught that in order to find the direction of the electric field at a point, they should place an imaginary positive test charge at that point. Then, the direction of the net force on that charge and the field at that point will be in the same directions. Interviews with individual students suggest that over-generalisation of this explanation may be partially responsible for some students thinking that there must be a point charge present at the point where the field is to be calculated. Students had difficulty rationalising that while the electric field is defined as the force per unit charge, the field and force do not even have the same units.

On question (1) on the pre-test (see the appendix), some interviewed students concluded that the electric field has the same magnitude at points, B and D, based on two incorrect assumptions: (1) only the closest charges would produce a field and (2) considering the closest points to B, which are A and C, as point charges. For example, one student stated: ‘It is point B. It is also right in the middle of two positive charges’. Interviews suggest that students with these types of responses often realised that point B is equidistant from points A and C and assumed that it is like point D which is on the midpoint between two identical point charges. Then, they claimed that points B and D have the same magnitude of electric field by ignoring the fact that there are no charges present at points A and C. What is interesting is that some students were specifically told in interviews that there are no charges at points A, B, C
and D when they initially reasoned as though there were point charges at those points, but they often continued to treat those points as point charges in their reasoning.

Assuming that the electric field due to a positive charge points towards it and a positive charge attracts all ‘points’ around it

Some students incorrectly thought that the electric field due to a positive point charge points towards the charge. For example, when asked to draw the direction of the electric field at points A, B and C in the pre-test question (1), one common difficulty was drawing the field arrows towards the three point charges as though the three charges shown in the figure are negative. One interviewed student explicitly explained his drawing by incorrectly arguing that the positive charges will attract points A, B and C because positive charges attract all ‘points’ around them. When asked to elaborate, the student could not explain the reasoning but stated that this is what he remembers from his class. Further discussions with him suggest that this confusion may partly be due to the fact that the student had solved some problems in which negative charges were present and they were attracted towards a positive charge and the student interpreted that the arrows for the electric field were always towards a positive charge. The student was not making a distinction between the electric field and force and claimed that the positive charge attracts all ‘points’ (even when no charge is present at those points). Similar difficulties were observed in other student responses as well.

Confusion due to electric field line representation and interpretation of electric field using it

The electric field line representation is used as a tool to obtain the direction of the field and to obtain a qualitative feel for the magnitude of the field at various points for a given charge distribution. In this representation, the tangent to the field line at a point gives information about the direction of the field at that point. If the field lines are closer together in a region, the field is stronger in that region. Unfortunately, this representation can often be misleading for introductory students. Some students incorrectly claimed that the direction of the field at a point is given by the curved electric field lines rather than the tangent to those lines at various points. A common difficulty was discerning the connection between the electric field line representation, the electric field at a point due to individual charges and the net electric field at that point. For example, in response to question (1) on the pre-test, two sample responses are shown in figure 1.

Figure 1. Sample responses on the pre-test question (1).
Similarly, in response to question (1) on the post-test (see the appendix), some students drew electric field lines of a dipole. If students had interpreted the field line representation correctly, they would have predicted the direction of the field correctly at both points A and B. Some students who correctly drew the electric field lines for the dipole had an incorrect notion of what the ‘direction’ of the field at a point means and did not know how to interpret the direction of the field at points A and B using the field line representation. For example, in response to question (1) on the post-test, a typical sample response shown in figure 2 points to this type of difficulty.

The student whose response is shown in figure 2 and who drew the electric field lines for an electric dipole claimed that the ‘negative’ ion attracts positive charge and electric field flows in that direction’. When a student with this type of response was interviewed and was specifically asked to draw the direction of the field at point B, the student drew a field line passing through point B. He kept pointing to the field line he had drawn passing through point B and stated that the curve gives the direction of the field at point B. When the interviewer insisted that he state something specifically about the direction of the field at point B, the student incorrectly claimed that point B will get pulled towards the negative charge.

Many other students who drew field lines often used this representation inappropriately in a similar manner and focused on the curve connecting, e.g., point B to the two charges as representing the direction of the field at point B rather than drawing the tangent to the field line at point B where they were asked to find the field. Even those students who correctly noted that the field at point A points towards the negative charge were often confused about the direction of the field at point B. They often claimed (similar to this interviewed student) that the field at point B is pointing towards the negative charge because the field line curves towards the negative charge or because point B is attracted to the negative charge.

As can be seen from the sample student’s response to question (1b) on the post-test in figure 2, the student thought that the field at point A is zero. Incidentally, some students who drew the field lines correctly and had a field line pointing to the right at point A also incorrectly claimed that the net field at point A is zero in response to question 1(b) on the
post-test. The most common reason cited was that the effect of the positive and negative charges will cancel at midpoint. It was clear from student responses that there was a disconnect between the electric field line they had drawn connecting the positive and negative charges passing through point A and what it implied for the electric field at point A. Some interviewed students drew arrows showing the electric field lines emanating in all directions from a single point charge and claimed that the electric field due to a point charge cancels out since it points in all directions.

**Electric field cannot be zero at any point in a region if only positive charges are present**

Some students claimed that both the positive and negative charges must be present in a region for the electric field to be zero at a point in that region. In response to question (3) on the pre-test (see the appendix), many students correctly noted that the field cannot be zero at any of the points shown because all the three charges present are positive. While the field is not zero at any point shown in the figure, many students provided reasoning that was incorrect for why the field cannot be zero. They incorrectly argued that, in order for the field to be zero at a point, there must be both positive and negative charges present in the region and the field cannot be zero at any location in situations where only positive charges were present as in the pre-test question (3). This statement is incorrect, e.g., the field is zero on a straight line joining two identical positive charges equidistant from each charge. An interviewed student who stated that the field cannot be zero at any point in pre-test question (3) explained: ‘can not cancel the effect because the effect of positive needs to be cancelled by negative’. The student further argued that this must be true because opposite charges repel and cancel each other out. As noted below in the discussion related to the field due to an electric dipole, students often invoked the idea that the positive and negative charges negate each other so both must be present in a region to make the net electric field zero at a point.

**Difficulty with the electric field due to an electric dipole at points on the perpendicular bisector**

On post-test question 1 (see the appendix), students are given two charges which have the same magnitude but opposite signs. Question 1(a) on the post-test is about the direction of the net electric field at point A midway between the two charges of an electric dipole and the direction of the electric field at point B on the perpendicular bisector (but not on the straight line joining the two charges). Question 1(b) in the post-test asks students whether the net electric field at point A is zero. Using Coulomb’s law and the superposition principle, the field due to both the positive and negative charges at point A is to the right. Thus, the net field at point A is to the right (and not zero). For point B, the vertical components of the field due to the two charges cancel out so that the net field at point B is also to the right.

Without trying to draw the directions of the electric field due to each charge and finding the net field by using vector addition, some students intuitively concluded that the net field at point A is zero. For example, one student stated: ‘I agree (the electric field at point A should be zero). Because the point is right in between a positive point charge and negative point charge of equal magnitude. The opposite magnitudes cancel each other and cause a net charge of zero’. As noted in an earlier subsection, it was also difficult for many students to determine the direction of the field at point B shown in figure 3. For example, some students claimed that the direction of the net field at point B is vertically down towards point A. Some students who knew how to draw the field line for a dipole claimed that the direction of the field at point B is along the curved field line (see figure 3). These students were confused between the field lines and the direction of the field at a point. Some students claimed that the direction of the
field at point B is towards the negative charge (along the straight line joining point B with the negative charge). The individual interview with a student who had this difficulty suggests that his reasoning was that the field lines end at the negative charge and the field lines are not the same as the field at each point so he concluded that the field at point B must point towards the negative charge and drew an arrow that points to the negative charge to display the direction of the field

(see figure 3).

Students who incorrectly claimed that the field is zero at both points A and B ignored the vector nature of the electric field. The most common difficulty with the field at point A was assuming that the field is zero at that point. The three most common difficulties with the direction of the field at point B were the following in order of their prevalence.

(I) It points towards the negative charge (e.g., because point B is positively charged and is pulled towards the negative charge or the electric field line pulls point B towards the negative charge).

(II) It is zero because the effects of the negative and positive charges (which are equal in magnitude and opposite in sign) cancel out at the perpendicular bisector of the line joining the two charges.

(III) It is vertically downward (e.g., because both charges will pull point B towards them with an equal magnitude force so that the net field is downward).

The following are typical sample incorrect responses for post-test Question 1(a):

• B will be drawn into the middle of the field by opposing forces and point down. A will remain stationary due to the cancelling forces.

• They cancel at both points because charges are opposite and they attract. Being equal they are equally attracted to one another.

• Net field for both points is zero because they are equidistant from opposite forces of the same magnitude.

• A has no net electric field and B is downward.

• The charges emitted from +Q will be drawn towards the −Q charge so at point B they will be at an angle towards −Q.

As noted, Question 1(b) on the post-test asked students to consider the following statement about point A between the two charges of an electric dipole: ‘The net electric field at point A is zero’ and students were asked to explain why they agree or disagree with the statement. The following are typical sample responses from students who incorrectly agreed with the statement:

• Agree because the forces cancel each other out.

• Agree, the two charges cancel each other out.
• Agree, charges of equal magnitude but opposite sign will pull against each other so point A is pulled in each direction equally.
• Agree assuming point B is not charged. The charges have equal magnitude so they cannot dump a net charge on A.
• Agree since opposites attract. They are putting the same force on A that cancels out.
• Agree as long as A remains equal distance from the two charges.

Invoking the dynamics of charges in an electrostatics problem

In electrostatics problems, students sometimes invoked the dynamics of charges and discussed how charges would accelerate. For example, when students were asked to calculate the net electric field for the charge distributions in the pre-test and post-test, some students described how charges will move around due to the attraction and repulsion between them in their responses. None of these students who described the dynamics of charges in this manner (and how it will affect the electric field) mentioned that the positive and negative charges will essentially collide and the same type of charges will move infinitely away from each other if the electrostatic force was the only force acting on them. For example, a student who noted that the opposite charges will move towards each other in question (1) on the post-test, incorrectly agreed with the statement in question (2) about three point charges. He agreed with the statement claiming that the magnitude of the net field is the same everywhere on the dashed imaginary triangle and provided the following reasoning: ‘I agree because like charges repel and so these charges will maintain the same distance between them’.

Confusion about symmetry of the charge distribution versus symmetry of the object on which charges are distributed

Students often had difficulty in evaluating the symmetry of the charge distribution in a given situation and confused the symmetry of the charge distribution with the symmetry of the object on which charges are embedded. The most common difficulty with question 3(b) on the post-test (see the appendix) was assuming that all the points shown have the same magnitude of the net field as point A. Students often justified this by incorrectly citing that the problem had ‘circular’ symmetry, confusing the symmetry of the object on which charges are embedded with the symmetry of the charge distribution. Question 3(c) on the post-test was very difficult for students and many students agreed with the statement. They often claimed that the electric field is radially outward everywhere on the dashed circle. The following are typical examples of student responses for those who incorrectly agreed with the statement:

• Agree. At any point on the circle the three point charges will cause there to be a radially outward electric field.
• Agree. This has to do with the fact that the three charges are in a circle making the final outcome radial.
• Agree, the tangent to any point on the imaginary circle would point to the centre.
• I agree because there are only positive charges which push out.
• Agree. This region is infinitely concentric so the electric field vector cannot point anywhere else but radially outward.
• Agree because charges will always balance out and produce the same electric field going radially outward.

We note that students were explicitly told on the pre-test and post-test that the meaning of ‘radially’ outward is straight out from the centre. Interviews suggest that those who thought
that all points on the circle have an electric field that is radial often also claimed that the magnitude of the electric field is the same at all points on the dashed circle.

Solving problems from rote memory

Interviews suggest that what some students did for the case of the electric dipole was rote; they knew the procedure for calculating the electric field in that situation. But they had difficulty in extending this procedure to situations involving more than two discrete charges. Interviews suggest that some of these students had a lack of conceptual understanding and had never carefully thought about what it means to calculate the net electric field due to a charge distribution using Coulomb’s law and the superposition principle.

The difficulties described in the previous section indicate that many students struggle with the concepts related to the electric field. These difficulties include, e.g., the difficulty with the principle of superposition and in recognising whether sufficient symmetry exists to predict whether the magnitude of the electric field should be the same at various points for a particular charge distribution, assuming that only the nearest charge contributes to the electric field at a point, assuming that charges in a straight line that are blocked by other charges do not contribute to the field, confusion between the electric field due to an individual charge and the net field due to all charges in a region, difficulty in distinguishing between the electric charge, field and force, assuming that the field can only be found at points where there is a charge present, assuming that the electric field due to a positive charge points towards it and a positive charge attracts all points around it, confusion due to electric field line representation and interpretation of electric field using this representation, difficulty with the electric field due to an electric dipole at points on the perpendicular bisector, incorrectly invoking the dynamics of charges in an electrostatics problem, confusion about the symmetry of the charge distribution versus symmetry of the object on which charges are distributed and solving problems from rote memory.

An overview of the tutorial

The tutorial was developed using the research on students’ conceptual difficulties as a guide. The entire tutorial can be found at http://per-central.org/items/detail.cfm?ID=12620. The guided enquiry-based tutorial strives to help students learn about the electric field and superposition principle in the context of a discrete charge distribution and consists of three parts. In the first part, students focus on the idea that the magnitude of the field at different points depends on the symmetry of the charge distribution and not on the symmetry of the object on which charges are embedded. In the second part, students are guided to use the superposition principle to find the net electric field due to multiple discrete charges. In the third part, the tutorial strives to help students solidify the idea of using the superposition principle to find the net field and helps them focus on the role of symmetry in determining the field at various points due to a given charge distribution.

At the beginning of the tutorial, students are asked questions about a single point charge to help them review Coulomb’s law. They are asked to draw the direction of the field and calculate the magnitude of the field at various points. They are also asked to describe all of the points in the three-dimensional space that will have the same magnitude of field as some of the points explicitly given in a diagram provided. In order to help students reason about the fact that the magnitude of the field is not the same at each point on a symmetric surface and depends on the symmetry of the charge distribution, students are given an imaginary sphere with a point charge $+Q$ inside, which is off centre (see figure 4). They are asked to draw
arrows to indicate the direction and relative magnitude of the field at the four points shown in the figure and told that the larger the relative magnitude is, the longer their arrows depicting the field should be. Then, they are asked to draw arrows straight out from the centre of the imaginary sphere passing through the four points on the spherical surface shown in figure 4. By comparing the relative lengths of the arrows and the angle between the radial direction and the direction of the net field, students are guided to learn that the direction of the field at various points on the sphere due to the off-centre point charge is not radial. Moreover, an analogy between the magnitude of the electric field at different points on an imaginary sphere due to an off-centre point charge within the sphere and the brightness of an off-centre light bulb within a sphere is introduced to help students learn relevant concepts. Students are asked to consider the magnitude of the electric field at different points on the sphere by making analogy with the brightness of a light bulb treating it to be a point source of light which radiates uniformly in all directions. It is easier for students to internalise that only when the light bulb is at the centre of a sphere, all points on the surface of the sphere can be uniformly lit.

In order to help students learn the superposition principle to find the net electric field due to multiple point charges, students are first provided a situation involving only two point charges (see figure 5) and have to first evaluate the validity of statements by two people about whether the field at a point is only influenced by the nearest charges (a misconception prevalent on the pre-test). Then students are provided guidance and scaffolding support as they learn to systematically find the net electric field at different points in figure 5 using the superposition principle. Students are initially asked to draw two arrows, each showing the...
contribution to the net field due to the individual charges at each labelled point in figure 5. They are also asked to predict and draw an approximate direction for the net field at each of the labelled points. Based on their drawings, they are asked whether they can exactly predict the direction of the net electric field at any of the labelled points and what is special about that point or points (e.g., point C in figure 5). They are also asked to reason about the labelled points in figure 5 that have the same field magnitude and whether they can determine the exact direction of the field at point B without elaborate calculation if \( q_1 = q_2 \). They are also asked why the magnitude and direction of the electric field will be different when \( q_1 = q_2 \) versus \( q_1 > q_2 \) (which was the given situation) and are guided to learn that the net field magnitude will be different in these two situations. They are guided to learn that in these two situations even though the direction of the electric field at point B due to each individual charge will not change, the direction of the ‘net’ electric field (which is the vector sum of the individual fields) will change (due to the differences in the magnitude of the field produced by individual charges at point B in the two situations).

The students are also asked to consider the net field at the point which is in the middle of the straight line that connects two equal magnitude positive point charges. Students have to consider a conversation between two people about this situation in which one person says that the net field in the middle is zero and another says that it cannot be zero since both point charges have the same sign. Students are asked to explain with whom they agree and why.

In order to help students learn to become facile at the symmetry considerations in determining the electric field, the tutorial provides students an opportunity to work on several examples, e.g., one that contains four point charges distributed on a ring (see figure 6). Students are asked to find the direction of the net field at the four points shown in figure 6 by applying the superposition principle. They are also asked about the points that have the same magnitude of the field as at point A based on symmetry. Student learning is scaffolded to realise that not all of the points A, B, C and D in figure 6 have the same field magnitude and the direction of the field at all of these points is not necessarily radially outward. Students are presented with conversations between two people to help them and asked why they agree or disagree with each to solidify these ideas about Coulomb’s law and the superposition principle. Moreover, justifying the validity of each statement in the conversation can also help students understand that it is not true that whenever a few charges are embedded on a circle, the electric field magnitude is the same at all points at a fixed distance away from the centre in the plane of the circle. The tutorial helps them reflect upon the fact that the symmetry of the charge distribution and the symmetry of the non-conducting ring are two different things and it is the symmetry of the charge distribution that determines whether different points have the same electric field. After the non-conducting ring shown in figure 6, students consider a
uniformly charged ring and are asked similar questions to help them learn the difference between a uniformly charged ring and the ring shown in figure 6.

In the last part of the tutorial, students consider a square non-conducting object with four equi-spaced point charges or uniformly distributed charge and asked questions similar to the ring. Working through these examples via a guided enquiry-based approach not only allows students to practice using Coulomb’s law and the superposition principle, but also helps them internalise that even with charges uniformly distributed on a square, the points on a concentric square will have different magnitude of electric field because a square is not sufficiently symmetric.

Assessment: performance of the experimental and comparison groups

The pre-test and post-test (see the appendix) were graded by two individuals based upon an agreed rubric, and the inter-rater reliability was better than 85%. The grading rubric scores each answer as correct or incorrect (based upon whether the student responses were conceptually correct or not) and if there was an explanation required, student responses for that part of the question were graded on a three point scale (full point for correct explanation, zero point for incorrect or no explanation and half point for partially correct). Table 3 shows the average pre-/post-test scores on each question and also overall for three section of the course in which the tutorial was administered. In the fourth section of the course, the post-test was returned without photocopying them so we only have data on student performance for that tutorial class on the standardised conceptual test [32] which was administered at the end of the semester (performance on this test is discussed at the end of this section). As shown in table 3, an additional question was included in the pre-test for sections 1 and 3 after analysis of data for section 2. Table 3 shows that the average performance was significantly better on the post-test compared to the pre-test for the tutorial group. The differences in the performance of different sections of the course on the pre-/post-test in table 3 may partly be due to the differences in student samples, instructor/TA differences or the manner in which the tutorial was administered.

Table 4 shows the pre-/post-test data from a comparison group which consists of a section of the class in which the students did not work on the tutorial. The pre-test was given to the comparison group immediately after relevant instruction similar to the tutorial group. The post-test was given the following week as part of the weekly recitation quizzes after students had the opportunity to complete all the homework problems related to those topics.
The results of a $t$-test that compares the performance of the tutorial and non-tutorial sections of the class on pre-/post-tests in tables 3 and 4 show that regardless of which section of the class the students belong to (i.e., whether they belonged to the tutorial or comparison groups), their performance on the pre-test was poor after traditional lecture-based instruction (the averages for tutorial and non-tutorial sections of the class are not statistically significantly different with a $p$ value of 0.23). On the other hand, students in the comparison group performed significantly worse on the post-test than the sections in which students worked on the tutorial ($p$ value of $<0.0001$). As a comparison of the difficulties discussed earlier, on the post-test question (1), 87% of the students in the tutorial sections of the class provided the direction of the electric field at points A and B correctly. In the non-tutorial section of the class, however, 46% provided the direction of the field at point A correctly and only 30% of them provided the direction of the field at point B correctly.

Table 5 shows the performance of students in the tutorial group on the pre-/post-tests partitioned into three separate subgroups based upon the performance on the pre-test (see the pre-test range column). As can be seen from table 5, the tutorial generally helped all students including those who performed poorest on the pre-test. Table 6 shows the performance of students in the comparison group (non-tutorial group) on the pre-/post-tests partitioned into three separate subgroups based upon the pre-test performance. As can be seen from comparing tables 5 and 6, students in the comparison group did not perform on-par with the tutorial group on the post-test for any of the three pre-test ranges.

In order to evaluate retention of learning by different student populations at the end of the semester on these concepts, we administered a 25 question multiple-choice standardised conceptual test [31] in which five questions focus on the tutorial content and analysed performance of

| Pre-test range % | $N_1$ (class 1) | $N_2$ (class 2) | $N_3$ (class 3) |
|------------------|-----------------|-----------------|-----------------|
| All              | 83              | 61              | 85              |
| 0%–34%           | 24              | 28              | 75              |
| 34%–67%          | 29              | 21              | 92              |
| 67%–100%         | 30              | 12              | 94              |
different groups. Table 7 shows the average scores on the five questions on the content relevant for the tutorial on the standardised survey administered to different student populations. In table 7, N refers to the total number of students in each group.

**Table 6.** Percentage average total pre-/post-test scores (matched pairs) divided into three groups according to the pre-test performance for students who did not work on the tutorial and only had traditional lecture-based instruction on the relevant content. The symbol N denotes the total number of students in each range who took both the pre-/post-tests.

| Pre-test range % | N  | Pre | Post |
|------------------|----|-----|------|
| All              | 66 | 47% | 49%  |
| 0%–34%           | 20 | 17% | 40%  |
| 34%–67%          | 29 | 49% | 50%  |
| 67%–100%         | 17 | 78% | 59%  |

**Table 7.** The average score on five questions (and average percentage score including all five questions) on the content focusing on tutorial topics on a standardised test [31] administered to different student groups (including PhD students). N refers to the total number of students in each group.

| Question # | Intro without tutorial N = 135 | Intro honours students N = 182 | Before instruction N = 33 | After instruction N = 28 | Intro with tutorial N = 278 | First year PhD students N = 33 |
|------------|---------------------------------|--------------------------------|--------------------------|--------------------------|-----------------------------|--------------------------------|
| 2          | 45%                             | 64%                            | 52%                      | 57%                      | 84%                         | 64%                            |
| 4          | 22%                             | 31%                            | 33%                      | 39%                      | 67%                         | 73%                            |
| 14         | 54%                             | 49%                            | 55%                      | 64%                      | 72%                         | 85%                            |
| 15         | 23%                             | 26%                            | 36%                      | 25%                      | 52%                         | 61%                            |
| 16         | 13%                             | 14%                            | 9%                       | 36%                      | 45%                         | 52%                            |
| Avg        | 31%                             | 37%                            | 37%                      | 44%                      | 64%                         | 67%                            |
Summary

We investigated the difficulties of students in college introductory physics course with the electric field and superposition principle and used that research as a guide in the development and evaluation of a research-validated tutorial on these concepts to help students learn these concepts better. The tutorial uses a guided enquiry-based approach to learning similar to the approach used in the Washington tutorial [24] and involved an iterative process of development and evaluation. The tutorial begins with the electric field due to a single point charge and then provides guidance and support to help students learn to find the electric field due to a charge distribution using the superposition principle. The tutorial guides students to learn about the vector nature of the electric field, how to apply the superposition principle to find the net field and recognise the symmetry of the charge distribution (and distinguish it from the symmetry of the object on which charges may be embedded). Students work on examples in which the symmetry of the charge distribution (and hence the electric field) is the same but the charges are embedded on objects of different shapes (e.g., four equidistant charges on a plastic ring versus a plastic square). Common misconceptions are explicitly brought out, sometimes by having two people discuss an issue in a specific context. Students are asked to identify the person with whom they agreed and justify their reasoning. Then the students are provided guidance and support to build on their prior knowledge and develop a good knowledge structure related to these topics so that there is less room for misconceptions.

The final version of the tutorial was administered in several sections of the course after traditional lecture-based instruction in relevant concepts. We compared the performance of students in individual interviews and on the pre-test administered before the tutorial (but after traditional lecture-based instruction) and on the post-test administered after the tutorial in three sections of the class. We also compared student performance in the sections of the class in which students worked on the tutorial with other sections of the class in which students only learned via traditional lecture-based instruction. The data from the pre-/post-tests suggest that the tutorial is effective in improving student understanding of these concepts. We find that students performed significantly better in the sections of the class in which the tutorial was used compared to when students learned the same concepts via traditional lecture-based instruction only. Moreover, the tutorial appears to be helpful for students who obtained low scores (0%–33%) on the pre-test after traditional instruction, which is encouraging.

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Appendix. Pre-/Post-tests

Pre

Assume all insulators (non-conductors) are non-polarisable.

Consider an isolated system of three identical point charges \( +Q \) arranged in a line such that adjacent ones are equidistant. We measure the net electric field at four points (three on a parallel line and point D equidistant from two of the charges on the straight line joining them) as shown:
(1) Draw arrows to show the approximate directions of the net electric field at each labelled point.

(2) At which point (or points) can you exactly predict the angle (in degrees) the net electric field makes with the horizontal without knowing the numerical value of Q and the distances L and d? Explain your reasoning.

(3) Is the net electric field zero at any of the points shown? If so, where? Explain.

(4) Is the magnitude of the net electric field at any of the points (A, B, C) shown definitely the same as it is at point D? Which ones? Explain.

Post

Assume all insulators (non-conductors) are non-polarisable.

(1) Shown below are two charges which have the same magnitude but opposite signs.

Points A and B are on the perpendicular bisector of the straight line joining the charges with point A being located on the straight line joining the two charges.

(a) Draw the directions of the net electric field at points A and B. Explain your reasoning.

(b) Consider the following statement from Emily: ‘The net electric field at point A is zero’. Explain why you agree or disagree with her.

(2) A thin non-conducting ring has three identical positive point charges which make an equilateral triangle with each other (see figure). The dashed triangle shows an imaginary equilateral triangle concentric with the ring and with the same orientation as the equilateral triangle whose corners are the three point charges:
Consider the following statement from Sam: ‘The magnitude of the net electric field is the same everywhere on the dashed imaginary triangle because it has the same symmetry as that of the charges’. Explain why you agree or disagree with him.

(3) Now consider an imaginary dashed circle concentric with the same ring as in question (2) and six equally spaced points on it as shown below. The three charges are on the line segments connecting the centre of the ring to points A, C, and E.

(a) At which of the points shown above is the direction of the net electric field radially outward (straight out from the centre)? Explain.

(b) At which of the points shown above is the magnitude of the net electric field the same as it is at point A? Explain.

(c) Consider the following statement from Susan: ‘The net electric field due to the three charges at every point on the imaginary ring is radially outward. This includes those points not labelled in the figure above’. Explain why you agree or disagree with her. If you disagree with her, label one point where the net electric field is not radial.

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