Modification of generalized vector form factors and transverse charge densities of the nucleon in nuclear matter

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We investigate the medium modification of the generalized vector form factors of the nucleon, which include the electromagnetic and energy-momentum tensor form factors, based on an in-medium modified $\pi\rho\omega$ soliton model. We find that the vector form factors of the nucleon in nuclear matter fall off faster than those in free space, which implies that the charge radii of the nucleon become larger in nuclear medium than in free space. We also compute the corresponding transverse charge densities of the nucleon in nuclear matter, which clearly reveal the increasing of the nucleon size in nuclear medium.

PACS numbers: 12.39.Dc, 21.65.Cd, 21.65.Jk
Keywords: Electromagnetic form factors, energy-momentum tensor form factors, chiral soliton, transverse charge densities, vector mesons in nuclear matter.

I. INTRODUCTION

Understanding the electromagnetic form factors (EMFFs) of the nucleon has been one of the most important issues in hadronic physics, since they reveal the internal quark structure of the nucleon. While the EMFFs of the nucleon have been studied well over several decades, their precise data were obtained only recently by measuring the transverse and longitudinal recoil proton polarisations [11,11]. These new experimental data have drawn a great deal of attention both experimentally and theoretically (see recent reviews and references therein [12–15]). In the meanwhile, form factors of the nucleon can be defined as Mellin moments of the corresponding generalized parton distributions (GPDs) that unveil novel aspect of the internal structure of the nucleon [16,19] (see also the following reviews [20,22]). This new definition of the form factors enables one to get access to the energy-momentum tensor form factors (EMTFFs) and the tensor form factors via the GPDs, which cannot be otherwise directly measured experimentally. In this definition, the energy-momentum tensor form factors can be also understood as the second Mellin moments of the isoscalar vector GPDs of the nucleon.

The Fourier transforms of the generalized vector form factors of the nucleon including the EMFFs and the EMTFFs in the transverse plane, as viewed from a light front frame moving towards a nucleon, makes it possible to see how the charge densities of quarks are distributed transversely [23,24]. These are called transverse charge densities and they provide correctly a probability of finding quarks inside a nucleon in the transverse plane. Transverse charge densities have been already investigated for the unpolarized [25] and transversely polarized [26] nucleons.

Furthermore, it is of equal importance to examine how the EM structure of the nucleon is changed in nuclear matter. Studying the EMFFs of the nucleon in nuclear medium provides a new perspective on EM properties of the nucleon modified in nuclei [27–35]. In fact, the first experimental study of deeply virtual Compton scattering on (gaseous) nuclear targets (H, He, N, Ne, Kr, Xe) was reported in Ref. [36]. While uncertainties of the first measurement are so large that one is not able to observe nuclear modifications of the nucleon structure, future experiments will provide more information on medium modifications of the EM properties of the nucleons.

In the present work, we want to investigate the nucleon EMFFs and the transverse charge densities of quarks inside a nucleon in nuclear matter within the framework of an in-medium modified soliton model with explicit $\pi\rho\omega$ degrees of freedom. The model has certain virtues: it is simple but respects the chiral symmetry and its spontaneous breaking. Moreover, one can easily extend it including the influence of surrounding nuclear environment to the nucleon properties based on modifications of the meson properties in nuclear medium [37–39]. In this context, the EMTFFs of the nucleon, which are yet another fundamental form factors that are related to the generalized EMFFs, have been investigated in free space [39,40] and in nuclear matter within the chiral soliton approaches [41,42]. The results have explained certain interesting features of the modifications of nucleon properties in nuclear matter such as the pressure and angular momentum. Indeed, we will also show in this work how the EM properties of the nucleon are changed in nuclear matter in a simple manner. We will also see that the transverse charge densities expose noticeably how the distribution of quarks undergo changes in the presence of nuclear medium.

The present paper is organized as follows: In Sec. II,
we briefly explain the general formalism of the \( \pi\rho\omega \) soliton model modified in nuclear medium. In Sec. III, we describe how one can compute the generalized vector form factors within this framework. In Sec. IV, we present the results from the present work and discuss them. The final Section is devoted to the summary and the conclusion.

II. GENERAL FORMALISM

We start from the in-medium modified effective chiral Lagrangian with the \( \pi, \rho, \) and \( \omega \) meson degrees of freedom, where the nucleon arises as a topological soliton [43]. The Lagrangian has the form

\[
\mathcal{L}^\ast = \mathcal{L}^\ast_\pi + \mathcal{L}^\ast_\rho + \mathcal{L}^\ast_{\text{kin}} + \mathcal{L}^\ast_{\text{WZ}},
\]

where the corresponding terms are expressed as

\[
\mathcal{L}^\ast_\pi = \frac{f^2_{\pi}}{4} \text{Tr} \left( \partial_\mu U \partial_\mu U^\dagger \right) - \alpha_p \frac{f^2_{\rho} m^2_{\pi}}{4} \text{Tr} \left( \partial_\mu U \partial_\mu U^\dagger \right) + \alpha_s \frac{f^2_{\rho} m^2_{\pi}}{2} \text{Tr} \left( U^\dagger U - 1 \right),
\]

\[
\mathcal{L}^\ast_\rho = \frac{f^2_{\rho}}{2} \text{Tr} \left[ D_\mu \xi^\mu + D_\mu \xi^\mu \xi^\nu \xi^\nu \right],
\]

\[
\mathcal{L}^\ast_{\text{kin}} = -\frac{1}{2g^2_{\pi} \xi_\rho} \text{Tr} \left( F^2_{\mu\nu} \right),
\]

\[
\mathcal{L}^\ast_{\text{WZ}} = \left( \frac{N_c}{2} g_{\omega} \sqrt{\xi_\omega} \right) \omega_\rho \epsilon^{\mu\nu\rho\sigma} \frac{m_{\pi}}{24g^2_{\pi}} \times \text{Tr} \left\{ \left( U^\dagger \partial_\mu U \right) \left( U^\dagger \partial_\mu U \right) \left( U^\dagger \partial_\mu U \right) \right\}.
\]

Here the asterisk designates medium modified quantities. The SU(2) chiral field is written as \( U = \xi_L^\rho \xi_R^\omega \) in unitary gauge, and the field-strength tensor and the covariant derivative are defined, respectively, as

\[ F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\nu, V_\mu], \]

\[ D_\mu \xi_L(R) = \partial_\mu \xi_L(R) - i \nu \delta \xi_L(R), \]

where the vector field \( V_\mu \) includes the \( \rho \)-meson and \( \omega \)-meson fields, \( \nu \mu \) and \( \omega \mu \), respectively, expressed as

\[ V_\mu = \frac{g \sqrt{\xi_\rho}}{2} \left( \tau \cdot \rho_\mu + \omega_\mu \right) \]

with the Pauli matrices \( \tau \) in isospin space.

Note that in Eqs. (3), (4) and (8) the subscript \( V \) generically stands for both the \( \rho \)-meson and the \( \omega \)-meson and for compactness we keep the generic form of those expressions. One can separate Eqs. (3) and (4) into the \( \rho \)- and \( \omega \)-meson parts using the definitions (9), (7) and (8). Then \( g_\rho \) designates \( g_\rho \) for the \( \rho \) meson or \( g_\omega \) for the \( \omega \). \( N_c = 3 \) is the number of colors.

The input parameters of the model in Eqs. (3)-(5) can be classified into two different classes: the parameters \( f_{\pi}, m_{\pi}, g_\rho, g_\omega \) and \( N_c \) are related to the corresponding observables in free space, while \( \alpha_p, \alpha_s \) and \( \zeta_\omega \) are pertinent to properties of pionic atoms and infinite and homogenous nuclear matter 1.

In free space, in-medium parameters are all set equal to one: \( \alpha_p = \alpha_s = \zeta_\rho = \zeta_\omega = 1 \) and the other parameters are fixed by using either experimental or empirical data on the pion and the vector mesons [43]. The pion decay constant and mass are taken to be \( f_{\pi} = 93 \text{ MeV} \) and \( m_\pi = 135 \text{ MeV} \) (the neutral pion mass). The values of the coupling constants for the \( \rho \) and \( \omega \) mesons are given respectively as \( g_\rho = 5.86 \) and \( g_\omega = 5.95 \). The Kawarabayashi-Suzuki–Riazuddin–Fayyazuddin (KSRF) relation connects them to the masses of the vector mesons, i.e. \( m_\rho = 770 \text{ MeV} \) and \( m_\omega = 782 \text{ MeV} \), as follows

\[
2f^2_\pi g^2_\rho = m^2_\rho, \quad 2f^2_\pi g^2_\omega = m^2_\omega.
\]

In general, the parameters \( \alpha_p, \alpha_s \) and \( \zeta_\omega \) stand for the medium functionals which are the essential quantities in the present work. They depend on the nuclear matter density \( \rho \) and are defined as

\[
\begin{align*}
\alpha_p(\rho) &= 1 - \frac{4\pi c_0 \rho / \eta}{1 + (g_\rho 4\pi c_0 \rho / \eta)}, \\
\alpha_s(\rho) &= 1 - 4\pi \eta b_0 m^2_\pi, \\
\zeta_\omega(\rho) &= \exp \left\{ -\frac{\gamma_{\text{num}}}{1 + \gamma_{\text{den}} \rho} \right\}.
\end{align*}
\]

They provide crucial information on how the nuclear-matter environment influences properties of the single soliton [43]. The \( \eta \) is a kinematic factor defined as \( \eta = 1 + m_\pi / m_N \approx 1.14 \). The values of the empirical parameters \( b_0 = -0.024 m^{-1}_\pi \) and \( c_0 = 0.09 m^{-3}_\pi \) are taken from the analysis of pionic atoms and the data on low-energy pion-nucleus scattering. The \( g_\rho = 0.7 \) denotes the Lorentz-Lorenz factor that takes into account the short-range correlations [45].

The additional parameters \( \gamma_{\text{num}} \) and \( \gamma_{\text{den}} \) are introduced phenomenologically to reproduce the saturation point at normal nuclear matter. Two different models have been discussed in the framework of the present approach [43], in order to introduce a nuclear modification in the present soliton approach, which we will briefly explain here. In Model I, one neglects the small mass difference of the \( \rho \) and \( \omega \) mesons in free space \( m_\rho = m_\omega = 770 \text{ MeV}, \ g_\rho = g_\omega = 5.86 \) and assumes that the KSRF relation still holds in nuclear matter

\[
2f^2_\pi g^2_\rho \zeta_\rho = m^2_\rho = m^2_\omega, \quad \zeta_\rho = \zeta_\omega \neq 1.
\]

In Model II, on the other hand, we remove the degeneracy of the vector meson masses in free space \( m_\rho \neq m_\omega = 782 \text{ MeV}, \ g_\rho = g_\omega = 5.95 \), and instead of Eq. (11) assume that the KSRF relation is valid only for the \( \rho \) meson, with the \( \omega \) meson kept as in free space:

\[
2f^2_\pi g^2_\rho \zeta_\rho = m^2_\rho \neq m^2_\omega, \quad \zeta_\rho \neq 1, \quad \zeta_\omega = 1.
\]

1. \( \zeta_\omega \) denotes also a generic form for both \( \zeta_\rho \) and \( \zeta_\omega \) which appear in the corresponding \( \rho \)- and \( \omega \)-meson parts of the Lagrangian.
These two different models are devised to implement possible ways of nuclear modification. We take into account the possibility that the $\rho$ and $\omega$ meson degrees of freedom could respond differently to a nuclear environment [16] [17]. The effects of the $\omega$-mesons are mainly limited to the inner core of the nucleon. Therefore, the two variants of the model describe the situation that the inner core of the nucleon is more (Model I) or less (Model II) affected by medium effects. The latter is a plausible scenario, at least around the normal nuclear matter density.

In practice, these two models yield comparable results in many respects. A notable (and in our context important) difference, however, is the description of the incompressibility of symmetric nuclear matter: Model I yields a smaller value of the incompressibility, while Model II gives a stiffer nuclear binding energy and agrees better with the data (see explanations in Ref. [43]). In both models the values of $\gamma_{\text{num}}$ and $\gamma_{\text{den}}$ are fitted to reproduce the coefficient of the volume term in the empirical mass formula $\alpha_V \approx 26 \text{ MeV}$. Although this is larger than the experimental value $\alpha_V^{\exp} \approx 16 \text{ MeV}$, the relative change of the in-medium nucleon mass is reproduced correctly (See Eq. (12) in Ref. [43] and the corresponding explanation.). In Model I we have $\gamma_{\text{num}} = 2.390 m_\pi^{-3}$ and $\gamma_{\text{den}} = 1.172 m_\pi^{-3}$, whereas in Model II we employ $\gamma_{\text{num}} = 1.970 m_\pi^{-3}$ and $\gamma_{\text{den}} = 0.841 m_\pi^{-3}$. For further details on these two models in relation to nuclear matter properties, and to the classical and quantum solution in free space and in nuclear matter we refer to Refs. [42] [43]. In the next section we concentrate on generalized form factors and the corresponding transverse charge densities.

### III. Generalized Vector Form Factors and Transverse Charge Densities

The generalized vector form factors of the nucleon can be defined as the matrix element of a vector operator as follows:

$$
\langle N(p', s') | \bar{\psi}(0) \gamma_i iD^{\mu_1} \cdots iD^{\mu_n} \psi(0) | N(p, s) \rangle = \bar{u}(p', s') \gamma_i \left( \frac{-i \Delta \sigma^{\alpha} \mu}{2 M_N} \Delta_{\mu_1} \cdots \Delta_{\mu_n} \vec{P}^{\mu_1+1} \cdots \vec{P}^{\mu_n} \right) B_{n+1,i}(\Delta^2) + \frac{\Delta_{\mu_1} \cdots \Delta_{\mu_n}}{M_N} C_{n+1,0}(\Delta^2) \Bigg|_{\text{n, odd}} \right) u(p, s),
\left(13\right)
$$

where $D^\mu_i$ denotes the covariant operator in quantum chromodynamics (QCD) and the braces “{” stand for the symmetrization. Here $\Delta_{\mu_i}$ and the $P^{\mu_i}$ are the momentum transfer and the average of the momenta defined respectively as $\Delta_{\mu_i} = p^{\mu_i} - p^{\mu_i}$ and $P^{\mu_i} = (p^{\mu_i} + p'^{\mu_i})/2$; $u(p, s)$ and $\bar{u}(p', s')$ designate the spinor of the nucleon; $A_{n+1, i}(\Delta^2)$, $B_{n+1, i}(\Delta^2)$, and $C_{n+1,0}(\Delta^2)$ represent the generalized vector form factors (GVFFs) that are related to the Mellin moments of the GPDs, which are given as

$$
\begin{align*}
\int_{-1}^{1} x^n H(x, \xi, t) &= \sum_{i=0, \text{even}}^{n} (-2\xi)^i A_{n+1,i}(\Delta^2) + (-2\xi)^{n+1} C_{n+1,0}(\Delta^2) \Big|_{n, \text{odd}}, \\
\int_{-1}^{1} x^n E(x, \xi, t) &= \sum_{i=0, \text{even}}^{n} (-2\xi)^i B_{n+1,i}(\Delta^2) - (-2\xi)^{n+1} C_{n+1,0}(\Delta^2) \Big|_{n, \text{odd}}.
\end{align*}
\left(14\right)
$$

Here $H(x, \xi, t)$ and $E(x, \xi, t)$ are the twist-2 vector GPDs. The usual Dirac and Pauli form factors are identified as the leading GVFFs:

$$
F_1(\Delta^2) = A_{1,0}(\Delta^2), \quad F_2 = B_{1,0}(\Delta^2),
\left(15\right)
$$

which are defined as the matrix element of the electromagnetic current:

$$
\begin{align*}
\left\langle N(p', s') \big| \bar{\psi}(0) \gamma_\mu \hat{Q} \psi(0) \big| N(p, s) \right\rangle &= \bar{u}(p', s') \left[ \gamma^\mu F_1(\Delta^2) + i \frac{\sigma^{\mu \nu}}{2 M_N} \Delta_{\nu} F_2(\Delta^2) \right] u(p, s).
\end{align*}
\left(16\right)
$$
The nucleon matrix elements of the symmetric EMT operator are parametrized in terms of the EMTFFs as follows \[ \text{Ref. [43].} \]

\begin{align*}
(N(p', s')| \mathcal{T}_{\mu \nu}(0)| N(p, s)) &= \bar{u}(p', s') \left[ M_N^{2}(\Delta^2) \frac{P_\mu P_\nu}{M_N} \right. \\
&+ J(\Delta^2) \frac{i(P_\mu \sigma_\nu + P_\nu \sigma_\mu) \Delta^\rho}{2M_N} \\
&+ d_1(\Delta^2) \frac{\Delta_\mu \Delta_\nu - \Delta_{\mu \nu} \Delta^2}{5M_N} \left] u(p, s). \right. \tag{17}
\end{align*}

Similarly, the EMTFFs are related to the second moments of the vector GFs and in a such way related to the GVFFs in the next-to-leading order (NLO) as follows:

\begin{align*}
A_{2,0}(\Delta^2) &= \int_{-1}^{1} dx \, x \, H(x, 0, \Delta^2) = M_2(\Delta^2), \\
B_{2,0}(\Delta^2) &= \int_{-1}^{1} dx \, x \, E(x, 0, t) = 2J(\Delta^2) - M_2(\Delta^2), \\
C_{2,0}(\Delta^2) &= \frac{1}{5} d_1(\Delta^2). \tag{18}
\end{align*}

In this work we want to examine the modification of the EMFs and the EMTFs of the nucleon in nuclear medium. Let us first consider the EMFs of the nucleon.

In the Breit frame one has \( \Delta = (0, \Delta) \) and \( p' = -p. \) Since it is more convenient to introduce the positive definite square of the momentum transfer \( Q^2 = -\Delta^2 > 0 \) to describe the form factors, we will use it from now on. The Sachs EMFs \( G_E \) and \( G_M \) of the nucleon are expressed in terms of the Dirac and Pauli form factors:

\begin{align*}
G_E(Q^2) &= F_1(Q^2) + \frac{Q^2}{4M_N^2} F_2(Q^2) \tag{19} \\
G_M(Q^2) &= F_1(Q^2) + F_2(Q^2), \tag{20}
\end{align*}

which can be represented respectively as the Fourier transforms of the charge and current densities

\begin{align*}
G_E(Q^2) &= \int d^3 r \, e^{i\Delta \cdot r} J^0(r), \\
G_M(Q^2) &= m_N \int d^3 r \, r \times J(r) \cdot \langle r \rangle, \tag{21}
\end{align*}

Here \( J^0 \) and \( J \) denote respectively the charge and current densities. Note that the EM current \( J^\mu \) is defined as the sum of the baryonic current \( B^\mu \) and the third component of the isovector current \( V^\mu. \) The final expressions for the in-medium modified isoscalar and isovector FFs are derived as

\begin{align*}
G_{E,M}^2(Q^2) &= -\frac{m_{\omega}^2}{3\sqrt{\epsilon}} \int_0^\infty \int \frac{r^2}{2\pi} j_0(Qr) \omega(r) dr, \\
G_{E,M}^2(Q^2) &= -\frac{m_{\omega}^2}{3\sqrt{\epsilon}} \int_0^\infty \int \frac{r^2}{2\pi} j_1(Qr) \phi(r) dr, \\
G_{E}^2(Q^2) &= \frac{4\pi}{\lambda^2} \int_0^\infty j_0(Qr) \left[ \frac{f_\pi^2 \epsilon}{3} \left\{ 4\sin^4 \frac{F}{2} + (1 + 2\cos F) \xi_1 + \xi_2 \right\} + \frac{8\sqrt{\epsilon}}{\pi} \delta F^0 \sin^2 F \right] dr, \\
G_{M}^2(Q^2) &= \frac{8\pi}{3} \frac{m_N}{\lambda} \int_0^\infty \int \frac{r^2}{2\pi} j_1(Qr) \left[ 2f_\pi^2 \left( 2\sin^4 \frac{F}{2} - 2(1 - \alpha_p) \frac{1}{4} \sin^2 F - G \cos F \right) + \frac{3\sqrt{\epsilon}}{4\pi} \delta F^0 \sin^2 F \right], \tag{22-25}
\end{align*}

where the detailed expressions for the profile functions, \( \omega(r), \phi(r), F(r), \xi_1(r), \) and \( \xi_2(r), \) can be found in Ref. [43]. The proton and neutron EMFs are expressed in terms of the isoscalar and isovector FFs \( \tau_3 = \xi_3 \) for a given nucleon isospin state. At the zero momentum transfer \( Q^2 = 0 \) the EMFs are normalized as

\begin{align*}
G_{E,M}^p(0) = 1, \quad G_{E,M}^n(0) = 0, \quad G_{E,M}^{p,n}(0) = \mu_{p,n}. \tag{26}
\end{align*}

Since we already have studied the EMTFs in nuclear matter within the present approach [42], we refer to Ref. [42] for details.

Once we obtain the EMFs and the EMTFs of the nucleon, we can proceed to derive the transverse quark charge densities inside a nucleon, which show how the charges and magnetizations of the quarks are distributed in the transverse plane inside a nucleon [23, 49]. The transverse charge density inside an unpolarized nucleon is defined as the two-dimensional Fourier transform of the Dirac form factor:

\begin{align*}
\rho_{ch} = \frac{1}{(2\pi)^2} \int \frac{dQ}{2\pi} \frac{e^{iQb}}{QJ_0(Qb)} F_1(Q^2) = \int_0^\infty \frac{dQ}{2\pi} \frac{QJ_0(Qb)}{QJ_0(Qb)} G_{E}(Q^2) + \tau G_{M}(Q^2), \tag{27}
\end{align*}

where \( \tau_3 = \xi_3 \) for a given nucleon isospin state. At the zero momentum transfer \( Q^2 = 0 \) the EMFs are normalized as

\begin{align*}
G_{E,M}^p(0) = 1, \quad G_{E,M}^n(0) = 0, \quad G_{E,M}^{p,n}(0) = \mu_{p,n}. \tag{28}
\end{align*}
where $b$ designates the impact parameter, i.e. the distance in the transverse plane to the place where the density is being probed, and $J_0$ denotes the Bessel function of order zero \cite{23, 50}. The anomalous magnetisation density in the transverse plane \cite{50, 51} is defined as

$$\rho_m = -b \frac{d}{db} \rho_2(b) = b \int_0^\infty \frac{dQ}{2\pi} Q^2 J_1(Qb) F_2(Q^2), \quad (29)$$

where $\rho_2(b)$ is directly given by the two-dimensional Fourier transform of the Pauli form factor:

$$\rho_2(b) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(Qb) F_2(Q^2). \quad (30)$$

Assume that the nucleon is transversely polarized along the $x$ axis. Then the polarization of the nucleon can be expressed in terms of the transverse spin operator of the nucleon $\mathbf{S}_\perp = \cos \varphi_S \mathbf{e}_x + \sin \varphi_S \mathbf{e}_y$, so that the transverse charge density inside a transversely polarized nucleon is written as \cite{26}

$$\rho_T(b) = \rho_{ch} - \sin(\varphi_b - \varphi_S) \frac{1}{2M_N} \rho_m(b), \quad (31)$$

where the angle $\varphi_b$ is defined in the position vector $b$ that stands for the impact parameter or the transverse distance from the center of the nucleon in the transverse plane $b = b(\cos \varphi_b \mathbf{e}_x + \sin \varphi_b \mathbf{e}_y)$.

Since the EMFTFs are identified as the generalized vector FFs in the isoscalar channel, one can also define the transverse isoscalar densities in the case of the EMFTFs, which takes the following form:

$$\rho_{20}(b) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(Qb) A_{2,0} (Q^2). \quad (32)$$

When the nucleon is polarized along the $x$ axis in the transverse plane, the transverse isoscalar density inside the polarized nucleon is defined as

$$\rho_{20,T}(b) = \rho_{20}(b) - \sin(\phi_b - \phi_S)$$

\[ \times \int_0^\infty \frac{Q^2 dQ}{4\pi M_N} J_1(Qb) B_{2,0} (Q^2). \quad (33) \]

**IV. RESULTS AND DISCUSSIONS**

In this Section we present the numerical results of the form factors and related observables and discuss their physical implications.

### A. Electromagnetic form factors and Transverse Charge densities

We first show the results for the traditional charge and magnetization radii and the magnetic moments of the proton and the neutron. Table I lists them in free space calculated within two different models. It is already well known that the $\pi$-$\rho$-$\omega$ soliton model overestimates the neutron charge radius in medium turns out to be larger than that in free space. Both the results from Model I and Model II show similar tendencies. It indicates that the nucleon tends to buldge out in nuclear medium. On the other hand, Model I and Model II yield different results. While the neutron charge radius from Model I is almost the same as that in free space, its magnitude from Model II is drastically increased. In fact, the neutron radius is a rather difficult observable to describe theoretically because it comes from the subtraction between the isoscalar and isovector FFs (see Eq. (26)). As will be discussed later, the traditional charge density of the neutron is very different from the transverse charge density. In addition, the medium effects affect strongly the radial dependence

| Table I: The electromagnetic properties of the nucleons in free space. The magnetic moments of the proton and the neutron are given in the unit of the nuclear magneton ($\mu_N$). |
|-----------------|-----------------|-----------------|
| $\langle r_C^2 \rangle_{1/2}$ [fm] | $\langle r_M^2 \rangle_{1/2}$ [fm] | $\langle r_M^2 \rangle_p$ [fm] |
| Model I | Model II | Experiment |
| $0.93$ | $0.93$ | $0.86$ |
| $0.87$ | $0.87$ | $0.78$ |
| $-0.23$ | $-0.23$ | $-0.12$ |
| $0.88$ | $0.88$ | $0.86$ |
| $3.37$ | $3.37$ | $2.79$ |
| $-2.58$ | $-2.61$ | $-1.91$ |
| $1.31$ | $1.30$ | $1.46$ |

| $\rho/N$ | $\mu_p [\mu_N]$ | $\mu_n [\mu_N]$ |
|----------|-----------------|-----------------|
| $1.15$ | $5.23$ | $5.41$ |
| $-4.56$ | $-4.73$ | $-4.73$ |
| $1.15$ | $1.14$ | $1.14$ |

**Table II: The electromagnetic properties of the nucleons in nuclear medium at normal nuclear matter density $\rho_0$.**

| Table II: The electromagnetic properties of the nucleons in nuclear medium at normal nuclear matter density $\rho_0$. |
|-----------------|-----------------|-----------------|
| $\langle r_C^2 \rangle_{1/2}$ [fm] | $\langle r_M^2 \rangle_{1/2}$ [fm] | $\langle r_M^2 \rangle_p$ [fm] |
| Model I | Model II | Experiment |
| $1.17$ | $1.08$ | $1.08$ |
| $1.17$ | $1.14$ | $1.14$ |
| $-0.22$ | $-0.40$ | $-0.40$ |
| $1.18$ | $1.17$ | $1.17$ |
| $5.23$ | $5.41$ | $5.41$ |
| $-4.56$ | $-4.73$ | $-4.73$ |
| $1.15$ | $1.14$ | $1.14$ |
of the neutron charge distribution in comparison with the proton one. Thus, it is difficult to draw any conclusion about the changes of the neutron size in medium, based on the traditional neutron charge density. However, we will soon see that the transverse charge density inside a nucleon will clearly show that both the proton and the neutron swell in nuclear medium. The magnitudes of the magnetic moments of both the proton and the neutron become quite larger in nuclear medium than in free space by approximately 40\%, as shown in Table I. The medium effects turn out to be even larger on the neutron magnetic moment than the proton one as observed in the results of their ratio $|\mu_p^e/\mu_n^e|$. The reason can be found in the fact that the magnetization density becomes broadened in medium, which will be shown soon. Since the operator of the magnetic moment is proportional to the distance from the center of the nucleon, the nucleon magnetic moments in general tend to increase in nuclear medium. This also indicates indirectly that the nucleon swells in nuclear matter.

Figure I depicts the results for the EMFFs of the proton and the neutron as functions of $Q^2$. As was expected from the charge and magnetic radii of the proton, the EMFFs of the proton in medium fall off faster than those in free space as $Q^2$ increases. The general tendency of the form factors remains almost unchanged in the case of both Model I and Model II. When it comes to the electric FF of the neutron, however, the result from Model I is very different from that obtained from Model II. We already have seen that Model I and Model II give rather different results for the neutron charge radii. The difference arises from the fact that the $\omega$ meson is treated in a distinctive way. In Model I, both the $\rho$ and $\omega$ mesons are treated on an equal footing. That is, both the vector mesons undergo changes in the same manner. On the other hand, the $\omega$ meson is kept to be same as in free space in Model II. Since the proton electric FF is given as the sum of the isoscalar and isovector form factors as shown in Eq. (26), the difference between Model I and Model II is marginal (see the results for the electric FFs of the proton in Fig. I). However, the neutron electric FF comes from the subtraction of the isovector FF from the isoscalar one. Considering the fact that the $\rho$ meson contributes only to the isovector FF whereas the $\omega$ meson comes into play only in the isoscalar FF, we can easily see that the changes of both the $\rho$ and $\omega$ mesons are more or less compensated in Model I. However, in Model II, the isoscalar FF remains intact while the isovector FF is modified, which leads to the amplification of the electric FF of the nucleon (see the lower panel of Fig. I). It is interesting to note that the results for the neutron from Model I is very similar to those from Ref. [25]. Considering the fact that the Skyrme term in Ref. [26] is related to the vector mesons by the resonance saturation [27], the characteristics of Model I is closer to the medium-modified Skyrme model in which both the pion kinetic and Skyrme terms are modified, compared to Model II.

The results for the transverse charge and magnetization distributions inside an unpolarized proton are drawn in the upper-left and upper-right panels, respectively, with $b_z = 0$. The medium-modified transverse charge densities near the center of the proton are reduced drastically but get larger as $b$ increases. It indicates that the transverse size of the nucleon becomes larger in nuclear medium. As for the transverse magnetization densities, we find that the densities in medium are shifted and broadened in comparison with that in free space. It also implies that the in-medium nucleon swells relatively to the free space one.

It is already well known that the transverse charge density inside an unpolarized neutron provides a new aspect on the structure of the neutron [25, 50]. Considering the fact that the transverse charge density inside a nucleon has a physical meaning of the probability of finding a quark inside a nucleon, we can see from the results for the transverse charge densities inside a neutron, which are depicted in the lower-left panel of Fig. I, that the negatively charged quarks, i.e., down quarks are more probably found in the vicinity of the center of the neutron whereas the positive charged quarks or up quarks are located in outer regions inside a neutron. This is very much different from the usual and traditional understanding of the neutron charge distribution in which the positive charge is found near the center of the neutron while the negative charge is placed in outer regions. In nuclear matter, the transverse charge density inside a neutron has the same tendency but the magnitude of the densities is reduced and is broadened, as shown in the lower-left panel of Fig. I. It implies that the size of the neutron is also extended in nuclear medium. The transverse magnetization density inside a neutron is similarly modified in nuclear medium as that inside a proton.

|       | Free Space | Model I | Model II |
|-------|------------|---------|----------|
| $\langle b_{ch,m}^2 \rangle_p$ | 0.70 | 0.90 | 0.81 |
| $\langle b_{ch,m}^2 \rangle_n$ | -0.023 | -0.015 | -0.042 |
| $\langle b_{m}^2 \rangle_p$ | -0.85 | -2.89 | -2.89 |

Table III: The transverse charge and magnetization radii of the unpolarized proton and the neutron. The results in free space and in nuclear matter at normal nuclear matter density, $\rho_0$, are presented.

To see the swelling of the nucleon in nuclear matter more clearly, we define the transverse mean square charge and magnetization radii of the nucleon as follows

$$\langle b_{ch,m}^2 \rangle_{p,n} = \int d^2 b b^2 \rho_{ch,m}^{p,n} (b),$$

where the transverse charge density, $\rho_{ch}$, and the transverse magnetization density, $\rho_m$ are defined in Eq. (28) and Eq. (29), respectively. The results for the transverse charge and magnetization radii of the proton and
Figure 1: The electric and magnetic form factors of the proton are drawn respectively in the upper-left and upper-right panels, and those of the neutron are depicted in the lower panels in the same manner as functions of $Q^2$. The solid curve represents the form factors in free space, while the dotted and dotted-dashed ones designate, respectively, those from Model I and Model II in nuclear matter.

Figure 4 illustrates the medium modification of the transverse charge densities inside both the polarized proton and neutron. The general behavior of the transverse charge densities in nuclear medium is very similar to those in free space. However, the extension of the nucleon size is observed in nuclear medium. Examining the results shown in Fig. 4, the effects due to the polariza-
Figure 2: The transverse charge densities inside an unpolarized proton with $b_x = 0$ are plotted in the upper-left and the upper-right panels, respectively, and those inside a neutron are depicted in the lower panels in the same manner as functions of the impact parameter $b$. The solid curve represents the transverse densities in free space, while the dotted and dotted-dashed ones designate, respectively, those from Model I and Model II in nuclear matter.

Figure 3: Transverse charge densities inside the polarized proton (left panel) and neutron (right panel) in free space.
Figure 4: Transverse charge densities inside the polarized proton (upper panels) and neutron (lower panels) in nuclear medium at normal nuclear matter density $\rho_0$ from Model I (left panels) and Model II (right panels), respectively.
tion of the nucleon are lessened in nuclear medium. This can be understood from the medium modification of the transverse charge and magnetization densities inside an unpolarized nucleon as shown in Fig. 2. These densities in medium indicate that the size of the nucleon in medium becomes larger and the effects of the polarizations also get diminished.

B. EMT form factors and Transverse Energy-Momentum densities

Let us now discuss the EMTFFs of the nucleon. Since the EMTFFs correspond to the generalized isoscalar VFFs, we do not need to distinguish the proton from the neutron. The same results hold for the nucleon embedded into isospin-symmetric nuclear matter. The situation will change if one introduces the effects of isospin breaking into the mesonic sector. When one consider more realistic isospin asymmetric nuclear matter, one has to compute both the isoscalar and isovector generalized vector FFs. In this case, the EMTFFs will be regarded only as a part of the GVFFs. In the present work, we concentrate only on isospin-symmetric nuclear medium.

The medium-modified EMTFFs of the nucleon have been already investigated in Ref. [12] in detail. Thus, we will discuss here only the transverse charge and magnetization densities inside a nucleon, which correspond to the EMTFFs. As shown in Eq. (18), the EMTFFs of the nucleon are identified as the GVFFs in the NLO, which arise from the second moments of the vector GPDs. Hence, the transverse charge and magnetization densities from the EMTFFs of the nucleon can be regarded as those inside a nucleon to the NLO.

Figure 5 draws the NLO transverse charge densities inside both the unpolarized nucleon (left panel), \( \rho_{20} \) and the polarized nucleon (right panel), \( \rho_{20,T}^{*} \), with \( b_{z} \) fixed to be zero. Interestingly, the general feature of \( \rho_{20}^{*} \) is almost the same as \( \rho_{\text{ch}}^{*} \) presented in Fig. 2. When the nucleon is polarized, the \( \rho_{20,T}^{*} \) changed drastically, as shown in the right panel of Fig. 5. However, it can be also easily understood as we have discussed previously. The induced electric field will cause the shift of the positive charged quark to the negative \( y \) direction whereas will translate the negative one to the positive \( y \) axis. The strength of the NLO transverse charge densities are much decreased in nuclear medium. In particular, the magnitude of the negative charge is almost suppressed.

V. SUMMARY AND OUTLOOK

In the present work, we investigated the electromagnetic form factors of the nucleon in nuclear medium, based on the \( \pi-p-\omega \) soliton model. We employed two different models: Model I was constructed by changing both the \( \rho \) meson and the \( \omega \) meson in nuclear matter, while in Model II only the \( \rho \) meson undergoes the change but the \( \omega \) meson is intact. This difference yielded the very different results for the neutron electric form factor. We also discussed the transverse charge and magnetization densities inside both the unpolarized nucleon and the polarized nucleon. The densities showed that the nucleon swells in nuclear matter, which was also the case in the medium-modified Skyrme model. The effects of the nucleon polarization turned out to be lessened in nuclear matter. Finally, we presented the results for the next-to-leading order transverse charge densities obtained from the energy-momentum tensor form factors or the generalized vector form factors of the nucleon.

Based on the \( \pi-p-\omega \) soliton model, it is also of great interest to study the spin problem of the nucleon, in particular, the spin densities of the nucleon [54]. While the model does not contain any quark degrees of freedom, it is still possible to study the quark spin distributions inside a nucleon. The present work will shed light on the spin structure of the nucleus from a complementary viewpoint and furthermore on the changes of its spin structure in nuclear medium. The corresponding work is under way.

Acknowledgments

This work is supported by the Basic Science Research Program through the National Research Foundation (NRF) of Korea funded by the Korean government (Ministry of Education, Science and Technology), Grant Numbers 2011-0023478 (JHJ and UY) and NRF-2013S1A2A2035612 (HChK). JHJ also acknowledges a partial support by the "Fonds zur Förderung der wissenschaftlichen Forschung in Österreich via FWF DK W1203-N16."

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Figure 5: The NLO transverse charge densities inside the unpolarized nucleon, $\rho_{20}^*$, in the left panel, and those inside the polarized nucleon, $\rho_{20,T}^*$, in the right panel, with $b_x = 0$. The solid curve depicts those in free space, while the dotted and dotted-dashed ones represent, respectively, those from model I and model II in nuclear matter.

Figure 6: The two-dimensional NLO transverse charge densities inside the polarized nucleon in free space, $\rho_{20,T}$, (upper panel), and those inside the polarized nucleon, $\rho_{20,T}^*$, (lower panel). The lower-left panel depicts $\rho_{20,T}^*$ from Model I and the lower-right panel draws those from Model II.
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