Dissipative motion in galaxies with non-axisymmetric potentials

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Abstract

In contrast to the case with most other topics treated in these proceedings, it is not clear whether galactic gas dynamics can be discussed in terms of standard hydrodynamics. Nevertheless, it is clear that certain generic properties related to orbital structure in a given potential and the effect of dissipation can be used to qualitatively understand gas motion in galaxies. The effect of dissipation is examined in triaxial galaxy potentials with and without rotating time dependent components. In the former case, dissipative trajectories settle around closed loop orbits when these exist. When they do not, e.g., inside a constant density core, then the only attractor is the centre and this leads to mass inflow. This provides a self regulating mechanism for accession of material towards the centre — since the formation of a central masses destroys the central density core and eventually stops the accession. In the case when a rotating bar is present, there are usually several types of attractors, including those on which long lived chaotic motion can occur (strange attractors). Motion on these is erratic with large radial and vertical oscillations.

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1 Assumptions of standard galactic dynamics

It is usually assumed that test particles moving in a galaxy are influenced by the mean field produced by all the material in the galaxy, discreteness effects being negligible. This is no trivial simplification: it reduces a problem with $3N$ coupled degrees of freedom to $N$, 3 degrees of freedom problems (with $N \sim 10^{10} - 10^{11}$). It is also customary to assume that present day galaxies are in a steady state. Each object moving in the time independent galactic potential will thus conserve its total energy. In addition, until recently, most systems studied had some “special symmetries”, such that additional quantities were conserved along the motion of test particles. For example, in spherical systems, the three components of angular momenta are also integrals of motion, in axisymmetric systems, one of these is conserved.

The structure of the Hamiltonian equations, them being “skew symmetric” — symmetric with respect to the conjugate variables except for the minus sign in one of them — requires that conserved quantities come in pairs. Thus, by finding three constants of the motion, one secures all six constants required for solving the motion of a particle in three dimensions. Only two constants are required in two dimensions. If these
were found by symmetry arguments, the system is said to be integrable. Such models of galaxies include those with “trivial” symmetries, such as spherically symmetric systems, or thin axi-symmetric ones (where motion in the vertical direction can be assumed to be de-coupled), as well as systems with more subtle symmetries such as the non-axisymmetric Stackel potentials (for an extensive discussion of large classes of non-axisymmetric galaxy models see de-Zeeuw and Pfenniger 1988).

Integrable systems are characterised by the interesting property that the motion can be decoupled into independent one dimensional oscillations, once a suitable coordinate system has been found. Thus, a three degrees of freedom problem is further reduced to three one degree of freedom ones. This is again not a trivial simplification. One has now reduced the a gravitational $N$-body problem to that of studying a collection of independent oscillators. In practice, strictly speaking, no $N$-body ($N > 2$) gravitational system can be exactly integrable, since it was proved by Poincaré that one cannot find enough constants of motion which exist for all initial conditions. Nevertheless, it is assumed that at least for times comparable to a Hubble time, where discreteness effects are small, some gravitational systems can be thus described. Another limitation is that completely integrable systems are rather rare (they actually form a set of measure zero in the space of possible systems). Nevertheless, the celebrated KAM theorem (e.g., Arnold 1987) guarantees that as long as the considered systems are not too different from integrable ones, most initial conditions will lie on trajectories that are qualitatively similar to those of neighbouring completely integrable systems. In the following section we discuss an example of a potential where the above conditions hold for a large range of parameters and initial conditions.

2 Of potentials and pendulums

Observations indicate that, after an initial rise which is usually more or less linear, the circular velocity of test particles in galactic potentials is constant over a large range of radii. Such a situation can be modelled by a particularly simple potential of the form

$$\Phi = \frac{1}{2} \frac{v_0^2}{R_0^2} \log( R_0^2 + R^2 ) ,$$

(1)

where $R_0$ is the “core radius”, beyond which the rotation curve flattens. This potential can be generalised to simulate a triaxial figure and has been studied in some detail in the book by Binney and Tremaine (1987) (BT). In general, it can be written in terms of Cartesian coordinates as

$$\Phi_H = \frac{1}{2} \frac{v_0^2}{R_0^2} \log \left( R_0^2 + x^2 + py^2 + qz^2 \right) .$$

(2)

The equipotential are ellipsoids with ratios $1/p^2$ and $1/q^2$ between the middle and long and short and long axis respectively.

Within the core radius, the density is nearly constant and solutions of Poisson’s equation predict a nearly quadratic potential. In such harmonic potentials the motion is separable in Cartesian coordinates. In the $x − y$ plane for example, orbits can be represented as independent superpositions of oscillations in the $x$ and $y$ directions. They are said to be parented by the $x$ and $y$ axial orbits which move up and down these axes. Such “box orbits” have no definite sense of rotation: their net, time averaged, angular momentum is zero. In the region outside the core radius other types of orbits may exist. In particular, loop orbits, which do have definite sense of rotation, appear. These are parented by the closed loops, which are oval, non-self-intersecting closed periodic orbits.

Since the motion of most orbits in nearly integrable potentials can be represented as a superposition of one dimensional oscillations, it is instructive to invoke the analogy
with this most familiar of 1-d oscillations: the simple pendulum. For small oscillations, the motion is dominated by the linear term in the force field and resembles that of a linear oscillator — where the period of oscillation is independent of the amplitude. For larger amplitudes, the period of oscillation increases with amplitude and becomes infinite at the separatrix. Beyond this, a different type of qualitative motion appears. Now, instead of oscillating, orbits of the pendulum have a definite sense of rotation about the centre. 

In polar coordinates then, box orbits can be characterised as two independent librations, while loop orbits are similar to one librating motion and rotating one. Since the frequencies will, in general, be incommensurable, most orbits of both types will not close. The exception being the periodic orbits. In the box family these resemble the familiar Lissagleou figures, in the loop family these are closed loops. 

Deep inside the harmonic core, the motion then resembles a superposition of two uncoupled harmonic oscillators. Beyond that, it is possible to have rotation in one of the coordinates, and the oscillations change frequency with amplitude. It turns out however that the ratio of the rotation frequency to that of oscillation around the closed loop orbits is usually more or less constant (BT), thus these continue to behave as if they are uncoupled and loop orbits, when they exist, are usually stable. The situation with the box orbits is different however. At large radii the oscillations become coupled and therefore the motion can no longer be represented as independent oscillations and no longer mimics the motion in integrable systems. This gives rise to “chaotic” trajectories which eventually populate most of the region once occupied by the box orbits (Schwarschild 1993).

The advent of chaos can also be understood with the help of the pendulum analogy (see, e.g., Zaslavsky et al. 1991 for a fuller account of the following discussion). Near the separatrix the quantity $\frac{1}{\omega} \frac{d\omega}{dA}$ goes to infinity. That means that any small change in the amplitude $A$ will lead to large changes in the frequency $\omega$ — which implies large changes in the phases of trajectories with nearby initial conditions differing slightly in amplitude. In addition, very small perturbations can transform oscillating trajectories into rotating ones (and vice versa), and also cause rotating trajectories to change their sense of rotation. Thus, introducing periodic perturbations, for example, can lead to extremely complicated trajectories around the separatrix. These “homoclitic tangles” are the types of trajectories Poincaré thought were so complicated that he did not even attempt to draw them. Schematic representation however was achieved by Arnold and these could be found in many standard texts (e.g., Lichtenberg and Lieberman 1992 (LL)). The larger the perturbation, the larger this “separatrix layer” where the chaotic motion described above takes place. 

In multidimensional systems, the role of the unstable equilibrium point around the separatrix is replaced by unstable periodic orbits. When these form a dense set, the chaotic regions merge. In the process of this transition to chaotic behaviour two points are of importance: nonlinearity and symmetry. Without nonlinearity, the frequencies are constant with amplitudes and so the ratios of frequencies of different degrees of freedom are likely to be irrational — i.e., no periodic orbits and no separatrix layers. As with the pendulum, equilibrium solutions are produced by a break in symmetry: in the pendulum, when gravity is turned on, we get the stable and unstable equilibria at the bottom and top respectively. The stable equilibrium parents the oscillations and the unstable one repels nearby trajectories. Integrable systems have the exceptional properties of having only a finite number of periodic orbits that are stable or unstable at a given energy — the rest being marginally stable (see BT). The stable periodic orbits parent the regular general orbit families (e.g., the box and loop orbits) which occupy almost all of the phase space. Lack of symmetry produces periodic orbits (originally in pairs of stable and unstable ones, but further increase in the perturbation leads to
increase in the number of unstable orbits). In this case there is a non-zero measure of chaotic orbits. Finally, in a multidimensional system, the “perturbation” that may give rise to chaotic orbits can either be external (as in a time dependent term in the potential) or coupling between different degrees of freedom. This coupling requires non-linearity and asymmetry.

3 The effect of dissipation: attractors

The skew symmetric form of the Hamiltonian equations ensures that any expansion of the phase flow in any direction will be counteracted by contraction in a conjugate one (which thus ensures conservation of phase space volume: for a discussion of the geometry of this symplectic structure see e.g., Arnold 1989; Marsden & Ratiu 1994; von Westenholtz 1978). Dissipative systems, which have velocity dependent force terms, on the other hand have a “time arrow”. The irreversible behaviour of dissipative systems is manifested in the contraction of the phase space volume (corresponding to a set of initial conditions and evolved dynamically). Trajectories thus end in “attractors”, which in general will have dimension less than that of the embedding phase space (for an excellent review of concepts related to the behaviour of dissipative dynamical systems see Eckmann and Ruelle 1985; collections of original articles can be found in Hao 1989).

Any given system can have more than one attractor. On which attractor a given initial condition will end up will depend on which basin of attraction it started from. Asymptotic dissipative motion in generic galactic mass distributions is expected to exhibit more than one attractor. Here again the analogy with pendulae is helpful. Intuitively, one can see that friction would force a system of rotating pendulae to move at the same rotation speed — or if this is not possible, then at least with the least relative velocity (so as to be compatible with the least possible dissipation). Thus, the symmetry of the problem is such that long lived states where there is common rotation can persist. Translated into the language of galactic dynamics, this implies that a collection of loop orbits interacting in a dissipative manner can keep their definite sense of rotation (assuming that the dissipation mechanism does not lead to net loss in the total orbit averaged angular momentum). The oscillatory motion however would have to die out — again in a system of pendulae oscillating around the stable equilibrium, dissipation, by symmetry, would have to lead to all pendulae ending up at rest at the stable equilibrium.

The above argument implies that in a non-axisymmetric galactic potential, the fate of dissipative trajectories will depend on their type. Loop orbits would end up following up the closed periodic loops — since the radial oscillations would die out while net rotation can persist — while box orbits would dissipate towards the centre (because oscillations in both coordinates would die out). The former being long lived limit cycles while the later are attracting fixed points. The situation in the chaotic region of the original Hamiltonian system will depend on whether there is any energy input. If the Hamiltonian decreases constantly, then the dissipative orbit passes by successive chaotic and regular regions before finally dissipating towards the centre — such behaviour is sometimes labelled “transient chaos” (see, e.g., LL). If, on the other hand, there is some forcing in such a way that the Hamiltonian does not not decrease monotonically, then some trajectories may end up on strange attractors on which long lived chaotic motion can occur (for more details and references on this intricate subject see Pfenniger and Norman 1990 (PN)).
4 Modelling dissipation

The motion of stars in galaxies is essentially collisionless. Effects due to dissipation are therefore expected to be important only for the gaseous component, which plays an important role in the evolution of (especially disk) galaxies. One important characteristic that has to be taken into account when attempting to model the interstellar medium is its highly clumpy, non-uniform nature. This property renders standard hydrodynamical treatments based on the continuum approximation and an equation of state not including gravitational effects inadequate, since the assumption of local thermodynamic equilibrium stemming from a certain separation of “fast” and “slow” processes is no longer satisfied in a straightforward manner. Standard thermal physics and the accompanying hydrodynamic equations therefore no longer apply (Pfenniger 1998) — since things like the Chapman-Enskog approximation (Andersen 1966) are no longer valid. Thus, in this paper, as opposed to most presentations in these proceeding, “fluid dynamics” is not synonymous with standard hydrodynamics. This is true of systems whose character is determined by gravitational instability, which guarantees that they behave like systems near a phase transition, which in turn lack characteristic scale. The lack of such a characteristic separation of space and time scales renders standard hydrodynamics inaccurate (e.g., Lebowitz et al. 1988). The clumpy and apparently scale invariant nature of the gaseous interstellar medium has inspired attempts to treat it as a fractal object (Pfenniger & Combes 1994). This method still awaits detailed application to realistic situations.

Nevertheless, there are indeed fast and slow processes which can be separated. These are the collision time between gas clouds, which is of order of one to ten Myr, and the dynamical time which is much longer — being of the order of 60 to 600 Myr. This leads to a formulation where one considers the hydrodynamics of collections of gas clouds which interact with each other over time-scales which are short compared to the dynamical time (details of the grounds on which such an approach may be justified are given by Scalo & Struck-Marcel 1984). A new set of hydrodynamic equations, more appropriate to this situation, is thus obtained. In these equations a “fluid element” is therefore a region small enough so that the macroscopic gravitational field can be considered roughly constant while large enough to contain a fair sample of gas clouds. According to Combes (1991) the total mass of molecular Hydrogen in the Milky Way for example is about \(2 - 3 \times 10^9\) solar masses concentrated in clouds of mass greater than \(10^5\) solar masses. At about 10 kpc these are concentrated in a region of a hundred pc from the plane of the disk. Therefore the volume mentioned above would be of the order, say, 500 by 500 by 100 pc, but of course will be smaller in the central areas where the concentration of molecular hydrogen increases significantly (Combes 1991).

Assuming that the hydrodynamic effects (e.g., pressure, viscosity etc.) are small (i.e., gas clouds move primarily under gravitational forces) and that any evolutionary effects are slow one can imagine the full hydrodynamic equations of these macroscopic elements to be perturbations to the collisionless Boltzmann equation. Here we will follow Pfenniger & Norman (1990) (PN) and assume that in the context discussed above, the main hydrodynamic effect can be approximated to first order as a velocity dependent viscous force, the value of which is small in comparison to the value of the mean gravitational field. In PN, dissipative perturbations of the form

\[
F_{\text{fric}} = -\gamma v v,
\]

where \(\gamma\) is a constant which determines the strength of the dissipation, were used. This form is similar to the one obtained if one assumes that the individual clouds are composed of infinitely compressible isothermal gas. Then in each collision (Quinn 1991)
the acceleration during an encounter of two clouds is given by

\[ a_{en} \sim \frac{u^2}{r}, \]  

(4)

where \( u \) is the absolute value of the relative velocity of the clouds and \( r \) is their separation.

One can to first order estimate \( \gamma \) using the usual “mean free path” approximation of (linear) viscosity:

\[ \gamma_l = n\lambda v_{rms}, \]  

(5)

where \( n \) is the volume number density of gas clouds, \( \lambda \) the mean free path and \( v_{rms} \) is the average random velocity. Supposing that the volume density is of the order of a few hundred or so per cubic kpc and the random velocity of the order of a few parts per hundred kpc/Myr, so that the mean free path is also of that order if the mean free time is about 1-10 Myr. This gives a value \( \gamma_l = \gamma \times v_{rms} \sim 0.01 - 0.1 \).

PN used even more conservative values for \( \gamma \) and still found highly non-trivial effects.

It was shown in the aforementioned paper that even extremely small dissipation rates can be greatly amplified in the presence of resonances. The main resonance existing in the disk-halo systems is the 1:1 resonance. This is the “separatrix” between the area where loop orbits occur and the nearly harmonic core. This resonance is not accompanied by widespread instability because the closed orbits that bifurcate from it are the stable closed 1:1 loops. Nevertheless, the effect is still palpable, for even in completely integrable one-dimensional systems, separatrix crossing can lead to faster dissipation (Parson 1986). Dissipation is basically a product of disorganised motion, in that context either chaotic resonance layers or the effect of the abrupt changing of qualitative motion at a separatrix can cause acceleration of dissipation even for systems as simple as pendulae (PN). In particular bifurcations leading to the destruction of stable limit cycles where dissipative orbits can settle are particularly effective as we shall see below.

5 Dissipation in non-rotating systems

Using a value of \( \gamma = 0.005 \) we have integrated the equations of motion for particles moving in the logarithmic potential and being influenced by dissipative forces of the form given by Eq. (3). The parameters for the logarithmic potential are taken as \( R_0 = 6 \) kpc and \( v_0 = 0.19 \) kpc/Myr, with axis ratios \( b/a = 0.9 \) and \( c/a = 0.8 \).

The quantity \( v \) in Eq. (3) is taken to be the vector difference between the velocity of the particle and that of a particle moving with the local circular velocity. Such motion cannot exist, because in a non-axisymmetric potential the orbits with smaller radial departure from the mean are ovaly distorted closed loop orbits. Because the dissipation rate is small however, far from the core, where these orbits are fairly close to circular, the difference in velocities is fairly small and an orbit oscillates around the closed loops for very long times (Fig. 3). This means that effectively our dissipative point particles representing fluid elements of gaseous disks settle into regular quasiperiodic limit cycles. In the case where we would calculated \( v \) in terms of the difference between a particle’s trajectory and the local periodic loop orbit we would obtain a true limit cycle as the asymptotic attractor.

The situation is rather different however as one moves nearer the halo core. In this case, the closed loop orbits become more and more eccentric and cannot parent any orbits that spend all their time inside the core. One can then say, for trajectories inside the core, the only attractor is the centre. That is, these trajectories are outside the basin of attraction of limit cycles represented by the closed loops.
In Fig. 2 we plot the radial coordinate of a trajectory of a particle which starts on the $x$ axis at a distance of 6 kpc. As can be seen, as one moves closer and closer towards the centre, the dispersion in the radial coordinate of the particle increases as the local loop orbits become more and more eccentric. At the separatrix beyond which no loop general loop orbits can exist (which occurs at about 4 kpc) there is a sharp transition in the dissipation rate and the particle quickly moves towards the centre.

Fig. 3 displays the spatial evolution of this orbit, which shows it to follow a sequence of loop orbits followed by what essentially are sections of box orbits. This latter behaviour, which starts roughly at about 16000 Myr and increases the dissipation rate dramatically, is shown in the plot on the right hand side of this figure. This process of course will lead to the growth of central mass concentrations. However it appears that this process slows down significantly as the central mass increases. The central mass ruins the harmonic nature of the potential; the 1 : 1 resonance is brought inwards nearer to the centre of the potential. In addition the potential of course becomes much more symmetric in the inner regions so that the closed loop orbits, which now exist in the central areas, are close to circular.

The top diagram in Fig. 4 shows the behaviour starting from the same initial conditions as in Fig. 2 and exactly the same potential except that a central mass of $GM_c = 0.01$ is present (in the units we are using $G = 1$, distance is measured in kpc and time in Myr, so that $GM = 1 \sim 2 \times 10^{11}$ solar masses). Clearly the behaviour previously observed — that is the accelerated dissipation rate — is no longer present. Smaller central masses lead to more eccentric loop orbits. However, even for very small central masses (anything greater than $GM_C \sim 0.00001$) loop orbits exist deep inside the core. Subsequent plots of Fig. 4 clearly show that there is a certain region between the halo core radius and the centre where the trajectories are extremely eccentric and the dissipation is very large. The scope of this region increases with decreasing value of the central mass. If this central mass is large enough, then the whole core region would support stable loop orbits. Otherwise there will be an annulus where no stable non self-intersecting periodic orbits exist. In the Hamiltonian limit, this region will contain chaotic orbits and higher order box orbits. Dissipative trajectories will therefore rapidly spiral towards the centre, until the region where loop orbits exist is reached. As they move through the chaotic region, it is also possible for dissipative trajectories to be $z$ unstable given the right kind of circumstances. This behaviour has been observed for small $\gamma$ (about 0.0005) and when the central mass was also small. In this case, the a dissipative trajectory visits the $z$ instability strips in the “chaotic sea” and does so slowly enough so as to acquire a large $z$ excursion, before reaching the stable limit cycle around the closed loops in the centre. Although this final motion is regular, any stars born in the intermediate process will have chaotic trajectories with large vertical excursions (which will be enhanced once dissipation ceases).

6 Dissipation in systems with rotating bars

Our dissipative force tends to circularise the motion. In the case when no bar was present this meant that general orbits would oscillate around the stable loop orbits, unless none were to be found in which case they dissipated rapidly towards the centre. In the cases discussed here however such orbits can be unstable, especially around the various axisymmetric resonances. For while for a nonrotating system these are not important (since ratio of the rotation frequency to that of small perturbation around nearly circular orbits is nearly constant), this is generally not the case in a rotating system (where for a general pattern speed $\Omega_P$ and rotation frequency $\Omega$, the ratio of the small perturbation to $\Omega - \Omega_P$ can vary significantly with radius). The addition of a
central mass increases this effect.

A general dissipative trajectory starting from the basin of attraction of the centre will pass through the various vertical and horizontal instability strips as it moves inwards. The erratic flow across the resonances will lead to large oscillations in both the $R$ and $z$ cylindrical coordinates. This will in turn lead to two effects (discussed in detail in PN). The first will be again an increased dissipation rate and the consequent decay in the $R$ coordinate. The second effect will be the scattering of particle trajectories out of the disk plane, as one passes through the vertical resonances. For certain dissipation laws, and especially if the potential is time dependent, it is also possible to find strange attractors, in addition to fixed points (like the centre) and limit cycles, as the invariant limit sets of dissipative trajectories. In this case, the motion is chaotic for infinite times and not just when passing through resonances (this latter situation, which is much less well defined than the case when a strange attractor exists and which is similar to the situation described near the end of the previous section, is sometimes labelled `transient chaos' or `intermittent chaos': see, e.g., LL for a review). We have not attempted here an extensive search of the phase space for all possible limit sets. Nevertheless, a fairly large number of initial conditions and parameters (chosen more or less by random trials) were tested. In the following we describe a few (we hope representative) examples. The trajectories described here move in galaxy models with the same logarithmic potential parameters used in the previous section. A Miyamoto Nagai disk with potential

$$\Phi_D = -\frac{GM_D}{\sqrt{x^2 + y^2 + \left(a_d + \sqrt{b_d^2 + z^2}\right)^2}},$$  

(6)

with $a = 3$ kpc, $b = 1$ kpc and $GM_D = 0.3$, represents the bulge-disk contribution (in the units we are using the gravitational constant is $G = 1$ and time is measures in Myr which makes $GM = 1$ equivalent to about $2 \times 10^{11}$ solar masses). A second order Ferrers bar (BT; Pfenniger 1984) with axes of 6 kpc, 1.5 kpc and 0.6 kpc and mass $GM_B = 0.2$ is also added. The systems studied below will therefore represent disk galaxies with stellar bars and triaxial dark matter haloes. The pattern speed $\Omega_P$ of the bar is chosen so that corotation in the disk halo potential is at 6 kpc (i.e., the bar ends at corotation, as is believed to be the case in most systems: Sellwood and Wilkinson 1993).

In the axisymmetric halo case, the triaxial perturbation is small. Near the end of the bar, closed loop orbits, which are termed $x_1$ in rotating systems, exist (see, e.g., Sellwood & Wilkinson 1993 for a review of the orbital structure in barred potentials). A trajectory starting with the circular velocity in the azimuthally averaged potential ends up closely following one of the periodic $x_1$ orbits elongated along the bar. This situation is analogous to the loop orbits outside the core radius of a non-rotating potential (except that in the latter case, loop orbits are elongated normal to the elongation of the mass distribution). Some trajectories may alternatively end up librating about one of the Lagrangian points near the end of the bar. As one moves towards the centre of the potential however, the gaps in the $x_1$ family are affected by lower order resonances. The strongest of these is fourth order “ultra-harmonic resonance”. As trajectories pass this resonance they experience rapid decay in their radial coordinate — an effect that is by now familiar. There is also some scattering in the $z$ direction for some initial conditions. However this is not very dramatic (a few hundred pc) for the parameters chosen here. Since the potential does not contain any other lower order resonances near the centre, the dissipation rate therefore slows down considerably and the trajectory settles down into a quasi-steady limit cycle around the $x_1$ orbits. Here, because the halo spherical, these may exist near the centre.

The addition of even a small central mass broadens the existing resonances and moves them outwards, and creates new lower order ones near the centre (PN). Thus, while in
the case of a non-rotating potential the loop orbits were stabilised by the addition of a central mass, here we have the opposite. The central mass creates and broadens the axi-symmetric resonances and destroys the loop orbits. This is due to the effect described in the opening paragraph of this section. The effect of dissipation therefore becomes much more dramatic, with an attracting point at the centre quickly reached. The scattering in the vertical direction is also enhanced. It is however still comparatively small, and becomes significant (about a kpc or so) only when the dissipation rate is small ($\gamma \sim 0.0002$). These values are comparable to what was obtained by PN but are far smaller than in the Hamiltonian case or when a triaxial halo is present.

We now turn our attention to the case where the halo is non-axisymmetric. We fix the halo potential axis ratio in the disk plane to be $b/a = 0.8$ while normal to the plane of the disk we take the axis ratio to be $c/a = 0.7$. Fig. 5 shows the time evolution of the radial coordinate of a particle started from the edge of the bar with the velocity of a circular orbit in the azimuthally averaged potential. As can be seen from that figure, even trajectories starting this far out from the centre of the potential can now be transported there. It can be noted however that after the initial rapid decay in the radial coordinate, the trajectory settles down to a state where this coordinate oscillates about a value of roughly 0.5 kpc, with little further decay in what appears to be a regular limit cycle. Trajectories started near the ultra-harmonic resonance are not attracted further in but are caught in what appears to be either a limit cycle a strange attractor. Distinguishing between the two cases would require testing with a suitable indicator of chaos (e.g., calculation of Liapunov exponents), in this case however the distinction is not really crucial practical importance, since the behaviour is similar. However, when the dissipation was decreased (by lowering the value of $\gamma$ to $\gamma = 0.0005$), it was found that some trajectories end up in states that can be seen to be highly erratic even by simple inspection. Fig. 6 displays the radial coordinate time series of such an orbit. This behaviour is also usually accompanied by non-negligible excursions in the vertical direction. These are suppressed by high dissipation rates. It is also interesting to note that orbits starting from as small an initial radial coordinate as 2.5 kpc where particles are transported towards the outside and all the way to the end of the bar appear to end in this attractor at least as far as the radial coordinate behaviour is concerned (Fig. 7).

In the presence of a stronger bar and in the absence of dissipation (e.g., $GM_B = 0.04$), many of the trajectories starting near the end of bar escape in the Hamiltonian limit. In the presence of significant dissipation however most of these trajectories end up in stable attractors outside the bar. In general the motion may be extremely complicated with trajectories violently oscillating in and out through the disk plane as well as normal to it. This is especially true if the dissipation is weak.

7 Concluding remarks

The object of this paper was to present simplified models of the effects of dissipation in galaxies with non-axisymmetric potentials. In a way, the study of this type of motion is much simpler than that of the Hamiltonian limit, since the phase space collapses into a few asymptotic attractors along with their attracting basins, as opposed to the complicated ‘mixed’ phase space of low dimensional Hamiltonian systems.

The approach used here boils down to adding weak (compared to the gravitational field) dissipative perturbations to trajectories in non-axisymmetric galaxy models. This approach avoids any detailed assumptions on the details of the dissipation process and reveals generic features resulting from the structure of the phase space independent of the dissipation law used (this has been checked). This avoids such questions as to what type of detailed thermal physics should be used in far from equilibrium systems such a
the clumpy structures of the inter-stellar medium where conventional thermodynamics fails (further discussion of this point can be found in Section 4; see also the interesting discussion on this subject given by Pfenniger 1998).

In the case when no rotating bar is present, the main conclusions are as follows. Long term attractors are nearly circular closed periodic orbits when these exist. When they don’t, which is the case in a nearly homogeneous harmonic core, the only available attractor is the centre. In that case, one expects significant mass inflow towards the centre. This process is however **self regulating**. When enough mass has reached the centre, the resulting central concentration destroys nearly constant density of the core and leads to the existence of stable closed loop orbits (around which dissipative trajectories may settle). The central mass that is required to stop further gas inflow is about 0.05% of the total mass of the galaxy at 20 kpc.

The above conclusions can also be reached by considering the dynamics of a collection of locally interacting “sticky” particles. In this scenario, particles move influenced only by gravity until they come close enough together, when they collide inelastically. Although such a procedure might seem at first sight artificial and somewhat trivial, it can actually be fairly rigorously justified under certain conditions, if some refinements are introduced (El-Zant 1998) — and is certainly no less justified than a continuum approximation with a perfect gas equation. The results indicate that the time-scale for gas inflow is about a Gyr. The aforementioned processes leading to gas accesssion towards the centre provides a self regulating mechanism that does not require the existence of bars. This is important since it is now thought (Mulchaey and Regan 1997) that the correlation between activity in the centres of galaxies and the existence of stellar bars is weak.

When the non-axisymmetric perturbation is rotating (e.g., as in bar of a disk galaxy), dissipative orbits will follow closed loop orbits when these are stable. Otherwise, they will dissipate towards the centre, crossing the variety of resonance regions populated by the chaotic replacements of the $x_2$ orbits. During this interval the motion would exhibit “transient” chaotic behaviour (e.g., LL).

In the case when both a rotating bar and and a nonrotating non-axisymmetric perturbation are present, the potential is inherently time dependent. Dissipative trajectories can therefore end in strange attractors on which long lived chaotic motion can occur. Depending on the parameters of the system (dissipation and bar strengths etc.), the radial motion could be confined to “rings” around the bar or large erratic motions in the cylindrical $R$ coordinate can occur. There can also be large excursions in the vertical direction for orbits starting near the disk plane.

The above effects can have several observable consequences on the evolution of galaxies, including the building of bulges and its relation with the shapes of dark matter haloes, the survival and evolution of bars, star formation activity etc. These are discussed in some detail in El-Zant (1998). However, as long as detailed treatment of the processes that dominate the evolution of the inter-stellar medium are still not very well understood, such description will only be schematic. Nevertheless, as we have seen here, it is still possible to understand the **generic** effects of dissipation in galaxy models in fairly simple manner, which can be understood from elementary principles of dissipative dynamics. It is then perhaps appropriate, when modelling effects in the inter-stellar medium, to isolate the essential phenomena and model them in as simple a manner as possible — the most sophisticated hydrodynamic scheme is useless, if fundamental assumptions like local thermal equilibrium (essential to that type of description) are not valid — by using methods that are easier to implement and interpret.
References

[1] Andersen H.C., 1966, Derivation of hydrodynamic equations from the Boltzmann equation. In: Wu T.Y. (ed) Kinetic equations of gases and plasmas. Addison-Wesley, New York

[2] Arnold V.I., 1987, In Mackay R.S., Weiss J.D. (eds) Hamiltonian dynamical systems. J.W. Arrowsmith ltd., Bristol

[3] Arnold V.I., 1989, Mathematical methods of classical mechanics. Springer, New York

[4] Binney J.J. & Tremaine S., 1987, Galactic dynamics. Princeton Univ. Press, Princeton (BT)

[5] Combes F., 1991, ARAA 29,195

[6] de-Zeeuw T., Pfenniger D., 1988, MNRAS 235, 949

[7] Eckmann J.P., Ruelle D., 1985, Rev. Mod. Phys. 57, 617

[8] El-Zant A.A., 1998, New Astronomy (in press)

[9] Hao B.L., 1989, Chaos. World scientific, Singapore

[10] Lebowitz J.L, Presutti E., Spohn H., 1988, Jour. Stat. Phys. 51, 841

[11] Liechtenberg A.J., Lieberman M.A., 1992, Regular and Chaotic Dynamics. Springer, New York (LL)

[12] Marsden J.E., Ratiu T., 1994, Introduction to mechanics and symmetry: A basic exposition of classical mechanical systems. Springer, New York

[13] Mulchaey J.S., Regan M.W., 1997, ApJ 482,L135

[14] Parson R.P., 1986, Chem Phys. Lett., 129,87

[15] Pfenniger D., 1984, A&A 134,384

[16] Pfenniger D., 1998, Which thermal physics for gravitationally unstable media? (astro-ph/9806150)

[17] Pfenniger D., Combes F., 1994, A&A 285,94

[18] Pfenniger D., Norman C.A., 1990, ApJ 363,391

[19] Quinn T., 1991, Particle simulations of polar rings. In: Warped disks and inclined rings around galaxies. Cambridge University Press, Cambridge

[20] Schwarzschild M., 1993, ApJ 409,563

[21] Scalo J.M., Struck-Marcell C., 1984, ApJ 276,60

[22] Sellwood J.A., Wilkinson A., 1993, Rep. on Prog. in Phys. 56,173

[23] von Westenholtz C., 1978, Differential forms in mathematical physics. Elsevier science publishing company, Amsterdam

[24] Zaslavsky G.M., 1991, Sagdeev R.Z., Chernikov A.A., Usikov D.A., Weak chaos and quasi-regular patterns. Cambridge University Press, Cambridge
Figure 1: Evolution of radial coordinate of trajectory starting from an initial value of 14 kpc

Figure 2: Evolution of radial coordinate of trajectory starting from an initial value of 6 kpc

Figure 3: Trajectory in the $x - y$ plane of orbit with radial coordinate represented in Fig. 2 and a more detailed view of the segment of the trajectory from 1750 Myr to end of the run at 50 000 Myr (bottom)

Figure 4: Same as in Fig. 2 but when a central mass of (from top to bottom) $GM_C = 0.01$, $GM_C = 0.0005$, $GM_C = 0.0001$, $GM_C = 0.00001$

Figure 5: Trajectory started from edge of a rotating bar in logarithmic potential with axis ratios $b/a = 0.8$ and $c/a = 0.7$

Figure 6: Left: Same as in Fig. 5 but with $\gamma = 0.0005$. Right: Corresponding evolution of the absolute value of the $z$ coordinate

Figure 7: Evolution of radial coordinates for trajectories starting at a only 2.5 kpc (left) and 4.5 kpc from centre of the mass distribution but appearing to end up in the same attractor as the trajectory in Fig. 6. Here also $\gamma = 0.0005$
Figure 1:

Figure 2:
Figure 3:
Figure 4:
Figure 5:

Figure 6:

Figure 7: