A stable vacuum of the tachyonic $E_8$ string

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Abstract

We consider tachyon condensation in unstable ten-dimensional heterotic string theory with gauge group $E_8$. In the background of a lightlike linear dilaton rolling to weak coupling, we find an exact solution in which the theory decays to a stable ground state. The final state represents a new, modular-invariant perturbative string theory, tachyon-free in nine spacetime dimensions with a spacelike dilaton gradient, $E_8$ gauge group and no spacetime supersymmetry.

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1 Introduction

Weakly coupled heterotic string backgrounds with ten-dimensional Poincaré symmetry were classified some time ago [1–8]. Among them is the unstable heterotic string with a single $E_8$ gauge group realized on the worldsheet as a level-two current algebra (hereafter, the UHE string). This background has been the subject of much interest. In contrast to the supersymmetric 10D heterotic string with $E_8 \times \bar{E}_8$ gauge group, the UHE string has a single real tachyon $T$ and broken supersymmetry.

One prominent role for the UHE string emerges from the study of spacetime-destroying decay modes of nonsupersymmetric M-theory backgrounds [9]. The supersymmetric heterotic background has been interpreted as the $R_{11} \to 0$ limit of 11-dimensional M-theory compactified on an interval of length $R_{11}$, on which the 11-dimensional gravitino has supersymmetric boundary conditions. Changing the boundary condition so that the gravitino has half-integral rather than integral moding on the interval produces a nonsupersymmetric theory in 10 dimensions, with a nonzero Casimir energy causing the two endpoints of the interval to attract one another.

In [9], Fabinger and Hořava proposed that the UHE background could be a limit of such a nonsupersymmetric configuration, following a phase transition in which the $E_8 \times \bar{E}_8$ gauge group is broken spontaneously to a diagonal subgroup. It was also conjectured that the remaining singlet tachyon can condense, generating a “decay to nothing,” in the sense of [10–17]. Utilizing and extending the methods of [18], an exact solution was studied recently by Hořava and Keeler [19,20] describing such a decay in the background of a lightlike linear dilaton rolling to weak coupling in the future.

In this paper we follow the logic of [18,21–24] to analyze the condensation of the singlet tachyon of the UHE theory. We show that, in addition to the solution studied in [19,20], there exists another exact solution with the same initial state, leading to a notably different outcome. Our solution is dimension-changing in the sense described in [21], with a final state described by a stable string theory in nine dimensions with no spacetime supersymmetry.

We analyze the UHE theory in the background of a lightlike linear dilaton rolling to weak coupling in the direction $X^- \equiv \frac{1}{\sqrt{2}}(X^0 - X^1)$. This theory admits deformations with a tachyon $T$ growing exponentially in the complementary lightcone direction $X^+ \equiv \frac{1}{\sqrt{2}}(X^0 + X^1)$. These deformations are exact solutions to the beta-function equations, and the motion

\footnote{The first exact heterotic bubble of nothing solution was written in [21].}
of a string in this theory is integrable. It has been suggested [18] that such null propagation of the tachyon arises automatically from the evolution of localized field inhomogeneities in the background of a linear dilaton.

The organization of the paper is as follows. After reviewing the UHE theory and the general properties of tachyon condensation in Section 2, we consider two cases: first, we review a solution in which the tachyon $T$ depends only on the lightcone direction $X^+$; second, we go on to consider a more generic case, in which the tachyon profile is not assumed to preserve any special symmetry. We review the more symmetric case (studied in [19, 20]) in Section 3, where 8D Poincaré invariance is imposed on the directions $X_i$ transverse to the lightcone. Under this restriction, the most generic solution to the equation of motion is of the form $\Phi = -\frac{q}{\sqrt{2}} X^-, T = \mu \exp (\beta X^+)$, with $q\beta = \sqrt{2}/\alpha'$, possibly including fields that decay to zero at late times.

In Section 4 we break Poincaré invariance in the eight directions $X_i$ to obtain late-time endpoints that differ qualitatively from the bubble of nothing. If the real tachyon $T$ is allowed to depend in an arbitrary way on $X_i$, $T$ generically develops zeroes along loci of real codimension one in the eight directions spanned by $X_i$. To simplify the analysis, we take the limit in which the field $T$ varies on long distance scales in the $X_i$ directions, as compared to the string scale. In this limit, each component of the zero locus is well approximated by a flat, isolated component of codimension one, with the tachyon $T$ varying linearly in the direction transverse to the component. We show that the physics of our solution is that of dimension quenching [21], where the number of spacetime dimensions reduces dynamically from ten to nine. In the limit $X^+ \to \infty$, strings propagate in only 8+1 dimensions, confined to the locus $T = 0$ by a potential barrier that becomes infinitely steep and high.

In Section 5 we show that the stable, late-time limit is described by a novel perturbative heterotic string theory with $E_8$ gauge symmetry, a linear dilaton gradient in a spacelike direction, and no spacetime supersymmetry in nine dimensions. To simplify the exposition, we refer to this nine-dimensional heterotic theory as the HE9 theory. We calculate the one-loop partition function in the HE9 theory and verify that it is indeed modular invariant. Unlike its 10D parent, the 9D heterotic theory supported inside the bubble is stable. It follows that the dynamical dimension-reducing solution is indeed generic in the space of solutions: any normalizable on-shell perturbation decays or disperses at late times. Section 6 contains conclusions and general comments.
2 Review of the unstable $E_8$ string in 10 dimensions

In this section we review several properties of the UHE string theory that are salient to the present discussion. We establish conventions, present the worldsheet supersymmetry algebra and outline the free-fermion construction of the level-two $E_8$ current algebra.

2.1 Worldsheet dynamics of the UHE string

The UHE string \cite{1,9}, like all perturbative heterotic string theories, is described in (super)conformal gauge as a (0,1) superconformal field theory (SCFT). There are ten embedding coordinates $X^\mu$, transforming in the standard way under 10D Poincaré invariance. Each embedding coordinate is paired with a right-moving worldsheet superpartner $\psi^\mu$, and the total central charge of the right-moving SCFT is equal to $c_R = 15$.

The left-moving side has an $E_8$ current algebra at level two, and a single Majorana-Weyl fermion $\tilde{\lambda}$. The current algebra has central charge $c_{\text{alg}} = 31/2$, and the fermion $\tilde{\lambda}$ contributes central charge $1/2$, so the total central charge of the left-moving side, including the ten bosonic coordinates $X^\mu$, is $c_L = 26$. The level-two $E_8$ current algebra has a free-fermion representation based on 31 left-moving Majorana-Weyl fermions $\tilde{\lambda}^A$.

The worldsheet degrees of freedom transform as follows under the (0,1) worldsheet supersymmetry:

$$[Q, X^\mu] = i \sqrt{\frac{\alpha'}{2}} \psi^\mu , \quad \{ Q, \psi^\mu \} = \sqrt{2 \alpha'} \partial_+ X^\mu ,$$

$$\{ Q, \tilde{\lambda} \} = F , \quad [Q, F] = i \partial_+ \tilde{\lambda} ,$$

$$\{ Q, \tilde{\lambda}^A \} = F^A , \quad [Q, F^A] = i \partial_+ \tilde{\lambda}^A .$$

(2.1)

2.2 Properties of the current algebra

To better understand the discrete gauge symmetry of the UHE string, we review the free-fermion construction of the $E_8$ current algebra at level two. It was observed in \cite{1} that there exists a discrete symmetry $(\mathbb{Z}_2^5)_L$ (where the subscript $L$ indicates the group acting only on left-moving excitations) acting on a set of 31 free fermions $\tilde{\lambda}^A$, such that gauging $(\mathbb{Z}_2^5)_L$
leads to a level-two current algebra with group $E_8$. With $(\mathbb{Z}_2^5)_L$ taken to be generated by $g_1, \cdots, g_5$, the action of $(\mathbb{Z}_2^5)_L$ on left-moving fermions is specified by:

$$
\begin{align*}
    g_1 &= \sigma^3 \otimes 1 \otimes 1 \otimes 1 \otimes 1 , \\
    g_2 &= 1 \otimes \sigma^3 \otimes 1 \otimes 1 \otimes 1 , \\
    g_3 &= 1 \otimes 1 \otimes \sigma^3 \otimes 1 \otimes 1 , \\
    g_4 &= 1 \otimes 1 \otimes 1 \otimes \sigma^3 \otimes 1 , \\
    g_5 &= 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma^3 , \\
\end{align*}
$$

(2.2)

where these matrices act in the basis $(\tilde{\lambda}, \tilde{\lambda}^1, \cdots, \tilde{\lambda}^{31})$. The single fermion $\tilde{\lambda}$ does not participate in the current algebra, and is neutral under all elements of $(\mathbb{Z}_2^5)_L$.

The generators $g_i$ act with a minus sign on subsets of $(\tilde{\lambda}, \tilde{\lambda}^A)$ in blocks of 16 at a time. As a result, every sector twisted by an element of $(\mathbb{Z}_2^5)_L$ is level-matched. As usual, we also gauge the operation $(-1)^{F_w}$, which acts on all left- and right-moving fermions simultaneously with a $-1$. The action of $(-1)^{F_w}$ defines an R-parity, meaning that it acts with a $-1$ on the right-moving supercurrent $G(\sigma^+)$. There are no left-moving currents in the untwisted sectors, since any fermion bilinear (without derivatives) is odd under at least some element in $(\mathbb{Z}_2^5)_L$. All $E_8$ currents come from the twisted sectors. Each of the 31 non-identity elements of $(\mathbb{Z}_2^5)_L$ defines a twisted NS sector in which 16 of the 32 current algebra fermions are periodic. Prior to imposing projections, the number of fermion ground states is $2^{16} = 256$. Imposing five independent $g_i$ projections cuts the number of ground states by a factor of $2^5$, leaving eight ground states in each twisted sector, for a total of $8 \cdot 31 = 248$. The ground-state weight in each of the twisted sectors is $\frac{16}{16}$ from 16 periodic fermions, so there are exactly 248 weight-one currents, generating the 248-dimensional Lie algebra $E_8$. The remaining fermion $\tilde{\lambda}$ is invariant under $(\mathbb{Z}_2^5)_L$ and transforms as an $E_8$ singlet, but it plays a role as the left-moving matter part of the tachyon vertex operator.

### 2.3 One-loop partition function of the UHE string

We can summarize the spectrum of the UHE string by computing its one-loop partition function. We will take the standard notation for the path integral on a torus of complex structure $\tau$, with two real fermions transforming with signs of $(-1)^{a+1}$ and $(-1)^{b+1}$ around the spacelike and (Euclidean) timelike cycles, respectively. The partition functions are:

$$
\begin{align*}
    Z^a_b &\equiv \frac{1}{\eta(\tau)} \theta_{ab}(0, \tau) , \\
    \tilde{Z}^a_b &\equiv \frac{1}{\eta(\tilde{\tau})} \theta_{ab}(0, \tilde{\tau}) , \\
\end{align*}
$$

(2.3)
for right- or left-movers, respectively.

In the untwisted sector – that is, with no action of \((Z_2^5)_L\) on fermions when transported around the spacelike cycle – the partition functions for the left-moving fermions \((\tilde{\lambda}, \tilde{\lambda}^4)\) are

\[
L_{0}^{0\text{(untw.)}}(\bar{\tau}) = \frac{1}{32} (\bar{Z}_0^0)^8 \left[ (\bar{Z}_0^0)^8 + 31 (\bar{Z}_1^0)^8 \right],
\]

\[
L_{1}^{0\text{(untw.)}}(\bar{\tau}) = \frac{1}{32} (\bar{Z}_1^0)^8 \left[ (\bar{Z}_1^0)^8 + 31 (\bar{Z}_0^0)^8 \right],
\]

\[
L_{0}^{1\text{(untw.)}}(\bar{\tau}) = \frac{1}{32} (\bar{Z}_0^1)^{16},
\]

\[
L_{1}^{1\text{(untw.)}}(\bar{\tau}) = 0. \tag{2.4}
\]

Here we have averaged over insertions of elements of \((Z_2^5)_L\) to implement the projection onto invariant states. In the sectors twisted by a nontrivial element \(g \in (Z_2^5)_L\), we have

\[
L_{0}^{0\text{(twist=g)}}(\bar{\tau}) = \frac{1}{32} (\bar{Z}_0^0)^8 \left( (\bar{Z}_1^0)^8 \right),
\]

\[
L_{1}^{0\text{(twist=g)}}(\bar{\tau}) = \frac{1}{32} (\bar{Z}_1^0)^8 \left( (\bar{Z}_0^0)^8 \right),
\]

\[
L_{0}^{1\text{(twist=g)}}(\bar{\tau}) = \frac{1}{32} (\bar{Z}_0^1)^8 \left[ (\bar{Z}_0^0)^8 + (\bar{Z}_1^0)^8 \right],
\]

\[
L_{1}^{1\text{(twist=g)}}(\bar{\tau}) = 0. \tag{2.5}
\]

Summing over twisted sectors, we obtain

\[
L_{0}^{0}(\bar{\tau}) = \frac{1}{32} (\bar{Z}_0^0)^8 \left[ (\bar{Z}_0^0)^8 + 31 (\bar{Z}_1^0)^8 + 31 (\bar{Z}_0^1)^8 \right],
\]

\[
L_{1}^{0}(\bar{\tau}) = \frac{1}{32} (\bar{Z}_1^0)^8 \left[ (\bar{Z}_1^0)^8 + 31 (\bar{Z}_0^0)^8 + 31 (\bar{Z}_0^1)^8 \right],
\]

\[
L_{0}^{1}(\bar{\tau}) = \frac{1}{32} (\bar{Z}_0^1)^8 \left[ (\bar{Z}_1^0)^8 + 31 (\bar{Z}_0^1)^8 + 31 (\bar{Z}_0^0)^8 \right],
\]

\[
L_{1}^{1}(\bar{\tau}) = 0. \tag{2.6}
\]
As usual, the partition function for the right-moving fermions and superghosts is \((Z^a_b)^4\).
The full path integrals for the fermions and superghosts are then
\[
F^a_b(\tau, \bar{\tau}) \equiv (Z^a_b)^4 L^a_b ,
\]
where we have summed over twisted sectors and projected onto \((\mathbb{Z}_2^5)^L\)-invariant states. Under
\(\tau \to \tau + 1\), the nonvanishing \(F^a_b\) transform as
\[
F^0_0 \to -F^0_1 , \quad F^0_1 \to -F^0_0 , \quad F^1_0 \to +F^1_0 ,
\]
while under \(\tau \to -1/\tau\), the \(F^a_b\) transform classically:
\[
F^a_b \to F^b_a .
\]
The path integrals in the NS\(\pm\) and R\(\pm\) sectors are therefore
\[
I_{NS\pm} \equiv \frac{1}{2} (F^0_0 \mp F^0_1) ,
\]
where the sign flip comes from the odd fermion number of the superghost ground state in
the NS sector. The partition function in the Ramond sector is
\[
I_{R+} = I_{R-} = \frac{1}{2} F^1_0 .
\]
The path integral over bosons and reparametrization ghosts is independent of the sector,
and equal to
\[
iV_{10}(4\pi^2\alpha'\tau)^{-5}|\eta(\tau)|^{-16} ,
\]
for ten free embedding coordinates, where \(V_{10}\) is the (infinite) volume of the flat ten-dimensional spacetime. Multiplying the two partition functions against the factor of \(d\tau d\bar{\tau}/4\tau_2\)
from the gauge-fixed path integral measure \([25]\) and integrating over the modular fundamental region \(\mathcal{F}\), we find
\[
\int_{\mathcal{F}} \frac{d\tau d\bar{\tau}}{16\pi^2\alpha'\tau_2^2} (4\pi^2\alpha'\tau_2)^{-4}|\eta(\tau)|^{-16} (I_{NS+} - I_{R\pm}) ,
\]
with the minus sign in front of \(I_{R\pm}\) implementing the fermionic spacetime statistics of the
Ramond states.
The measure $d\tau d\bar{\tau}/\tau_2^2$ is itself modular invariant, as is the combination $\tau_2|\eta(\tau)|^4$. The fermionic partition functions transform under $\tau \to \tau + 1$ as

$$I_{NS\pm} \to \pm I_{NS\pm}, \quad I_{R\pm} \to + I_{R\pm}. \quad (2.14)$$

Since the $F^a_b$ transform classically under $\tau \to -1/\tau$, (2.9) the combination

$$I_{NS+} - I_{NS-} = \frac{1}{2} \left( F^0_0 - F^0_1 - F^1_0 \right) \quad (2.15)$$

remains unchanged. It follows that the integral (2.13) is modular invariant, for either choice of GSO projection in the Ramond sectors.

The factor $iV_{10}(4\pi^2\alpha'^2\tau_2)^{-5}$ in the modular integrand comes from the integral over bosonic zero modes: i.e., the momentum integral for physical states. Removing this and the measure factor, and performing the $\tau_1$ integral that implements level matching, we obtain the partition function for the masses of physical states:

$$Z_{mass}(\tau, \bar{\tau}) \equiv \sum_{physical \ states} (-1)^{F_S} \exp\left(-\pi\alpha' m^2 \tau_2\right) = \int_0^1 d\tau_1 |\eta(\tau)|^{-16} (I_{NS+} - I_{R\pm})$$

$$\equiv Z^{NS}_{mass}(\tau, \bar{\tau}) - Z^{R}_{mass}(\tau, \bar{\tau}). \quad (2.16)$$

The expansion of these functions yields

$$Z^{NS}_{mass}(\tau) = (q\bar{q})^{-\frac{1}{2}} + 2,048 + 148,752 \ (q\bar{q})^{\frac{1}{2}} + O\left((q\bar{q})^1\right),$$

$$Z^{R}_{mass}(\tau) = 3,968 + 9,404,416 \ (q\bar{q})^1 + O\left((q\bar{q})^2\right), \quad (2.17)$$

where we have defined $q \equiv \exp(2\pi i \tau)$.

The first term in the NS partition function corresponds to the tachyon $T$ in the spectrum, with a single state at mass $m^2 = -\frac{2}{\alpha'}$. The second term corresponds to several different massless fields, including the graviton $G_{\mu\nu}$, with 35 polarizations, the B-field $B_{\mu\nu}$ with 28 polarizations, the dilaton with 1 state, and the $E_8$ gauge field $A_\mu$ with $8 \cdot 248 = 1,984$ massless states, for a total of 2,048 massless states. The next term represents the first massive level, with 148,752 states at $m^2 = \frac{2}{\alpha'}$.

The leading term in the Ramond partition function represents a pair of Majorana-Weyl fermions (one of each chirality), each in the adjoint of $E_8$, for a total of $2 \cdot 8 \cdot 248 = 3,968$ physical states at $m^2 = 0$. The next term appears at $m^2 = \frac{4}{\alpha'}$. In particular, there is no room for a gravitino, which, if present, would represent another 56 physical states at $m^2 = 0$.  

8
3 The Hořava-Keeler bubble of nothing

In this section we review the Hořava-Keeler exact bubble of nothing, with the intent of contrasting it with the dimension-reducing solution described in the next section.

3.1 Linear dilaton background

We consider the heterotic string in a lightlike linear dilaton background, with dilaton gradient given by $V_+ = V_i = 0$ and $V_- = -q/\sqrt{2}$. We take $q$ to be positive, but its magnitude is frame dependent: assuming it is positive, we can boost it to any other positive value under a Lorentz transformation.

The worldsheet theory is conformally invariant, with a free, massless Lagrangian given by

$$L_{\text{kin}} = \frac{1}{2\pi} G_{\mu\nu} \left[ \frac{2}{\alpha'} (\partial_+ X^\mu) (\partial_- X^\nu) - i \psi^\mu (\partial_- \psi^\nu) \right] - \frac{i}{2\pi} \bar{\lambda}^A (\partial_+ \bar{\lambda}^A) - \frac{i}{2\pi} \bar{\lambda} (\partial_+ \bar{\lambda}).$$

(3.1)

The dilaton coupling takes the form

$$L_{\text{dilaton}} = \frac{1}{4\pi} \left( \Phi_0 - \frac{q}{\sqrt{2}} X^- \right) \mathcal{R}^{(2)},$$

(3.2)

where $\mathcal{R}^{(2)}$ is the worldsheet Ricci scalar, and $\Phi_0$ is the value of the dilaton at $X^- = 0$. To allow the closure of worldsheet supersymmetry off shell, we supplement the action with kinetic terms for the nondynamical auxiliary fields $F$ and $F^A$:

$$L_{\text{aux}} = \frac{1}{2\pi} \left( F^2 + (F^A)^2 \right).$$

(3.3)

3.2 Tachyon deformation

To obtain the Hořava-Keeler solution, we must deform the lightlike linear dilaton background by letting the tachyon acquire a nonzero value obeying the equations of motion. The single real tachyon $T$ is a singlet of $E_8$, and couples to the worldsheet as a superpotential

$$W \equiv \bar{\lambda} : T(X) :.$$

(3.4)
where the component action comes from integrating the superpotential over a single Grassmann direction $\theta_+$ (to form a $(0,1)$ superspace integral):

$$
\Delta L \equiv -\frac{1}{2\pi} \int d\theta_+ W
= -\frac{1}{2\pi} \left( F : T(X) : -i \sqrt{\frac{\alpha'}{2}} : \partial_\mu T(X) : \bar{\lambda} \psi^\mu \right).
$$

(3.5)

Integrating out $F$ yields a potential of the form

$$
\Delta L = -\frac{1}{8\pi} : T(X)^2 :,
$$

(3.6)

and a modified supersymmetry transformation $\{Q, \bar{\lambda}\} = F = \frac{1}{2} : T :$.

The linearized equation of motion for the tachyon is

$$
\partial^\mu \partial_\mu T - 2V^\mu \partial_\mu T + \frac{2}{\alpha'} T = 0.
$$

(3.7)

The Hořava-Keeler solution takes the tachyon gradient to lie in a lightlike direction:

$$
T = \mu \exp \left( \beta X^+ \right),
$$

(3.8)

with $q\beta = \sqrt{2}/\alpha'$. For diagrammatic reasons (discussed in [18, 21, 22]), the worldsheet theory defined by this superpotential has exactly vanishing beta function, so the tachyon profile represents a solution beyond linearized order.

The bosonic potential on the string worldsheet is positive-definite and increasing exponentially toward the future:

$$
\Delta L = -\frac{1}{8\pi} \mu^2 \exp \left( 2\beta X^+ \right) + \frac{i}{2\pi} \sqrt{\frac{\alpha'}{2}} \beta \mu \exp \left( \beta X^+ \right) \bar{\lambda} \psi^+.
$$

(3.9)

The classical worldsheet potential of this model is identical to that described in [18, 26, 27]. Consequently, classical solutions for strings moving in the UHE bubble of nothing are the same as those in the bubble of nothing in Ref. [18]. That is, the solution describes a Liouville wall moving at the speed of light in the negative $X_1$ direction. Every string state eventually meets the wall and is accelerated outward to the left.

The worldsheet theory of the UHE bubble of nothing differs from that of the bosonic bubble of nothing due to the presence of worldsheet fermions. However, it is not straightforward to understand the physical meaning of quantum effects contributed by the fermionic degrees
of freedom. Fermionic interactions are only supported in the region of positive $X^+$, where string states are energetically forbidden from penetrating. In [19][20], worldsheet techniques were developed for studying the UHE bubble of nothing deep inside the tachyon condensate. In particular, the authors of [19][20] made a non-standard gauge choice for the local worldsheet supersymmetry in which the region of nonzero tachyon condensate could be studied with greater ease. The aim of [20] was to understand the “nothing phase” of the UHE string, possibly in terms of a conjectured topological phase of M-theory [28] describing the region behind the Hořava-Witten wall [29].

Our focus is more pedestrian. The easiest observables to understand are associated with regions whose future infinity has vanishing potential on the string worldsheet. In the next section, we will focus on solutions to the UHE string that contain such regions. We study a different class of exact bubble solutions to the UHE string theory, where the interior of the bubble supports a stable, nine-dimensional string theory rather than a topological or “nothing” phase.

4 Dimension-changing solutions of the type UHE string

The bubble of nothing solution described above imposes 8D Poincaré symmetry in the directions transverse to the lightcone defined by the dilaton gradient $V_\mu$. In this section we relax this assumption and examine solutions that break the 8D Poincaré symmetry. Within this more general ansatz there are exact classical solutions of the UHE model where the total number of spacetime dimensions decreases from 9+1 to 8+1. These solutions are qualitatively similar to the decays of the type HO$^+$ background [30] (i.e., the unstable supercritical heterotic model with nondiagonal GSO projection and gauge group $SO(32) \times SO(D-10)$). The similarity is that the tachyon is not constrained to vanish at any particular locus defined in advance by the initial state. Our discussion of the transition from 9+1 to 8+1 dimensions in the type UHE string applies equally well to the transition from 10+1 to 9+1 dimensions in the type HO$^{+(1)}$ string described in [30], with the exception that in the present case the tachyon and dilaton gradients are lightlike, rather than timelike.

Our initial conditions are such that the (9+1)-dimensional theory at $X^0 = -\infty$ has FRW spatial slices in the form of the maximally Poincaré-invariant nine-dimensional $\mathbb{R}^9$. The final theory will be a stable heterotic theory with gauge group $E_8$ (at level two) in 8+1
dimensions, with spacelike dilaton gradient and flat string-frame metric. The solutions we study are exact to all orders in $\alpha'$. Furthermore, there is a readjustment of the dilaton and the string-frame metric that comes from a one-loop effect on the worldsheet, just as in the dimension-changing transitions discussed in [21].

4.1 Inhomogeneous profiles for the tachyon $T$

At this point we allow the tachyon $T$ to vary in the eight dimensions $X_2, \ldots, X_9$ transverse to the lightcone directions $X^\pm$. We assume that the tachyon has a smooth vanishing locus $T = 0$, and we take the limit in which the typical scale of variation $|k_{2-9}|^{-1}$ of the tachyon is large compared to the string length $\sqrt{\alpha'}$. We can then approximate the tachyon as a linear function of the direction normal to the locus $T = 0$. Having done so, we can always perform an 8D Poincaré transformation to set the zero locus precisely at $X_9 = 0$. We lose no generality, then, by taking

$$T(X) = \sqrt{\frac{2}{\alpha'}} \exp (\beta X^+) \left[ \mu X_9 + O(kX^2) \right], \quad (4.1)$$

with

$$q\beta = \frac{\sqrt{2}}{\alpha'}. \quad (4.2)$$

Taking the long-wavelength limit and assuming a smooth vanishing locus for the tachyon amounts to dropping the $O(kX^2)$ terms. The superpotential is then of the form

$$W = \mu \sqrt{\frac{2}{\alpha'}} \exp (\beta X^+) \tilde{\lambda} X_9, \quad (4.3)$$

which means that the interaction Lagrangian equals

$$\mathcal{L}_{\text{int}} = -\frac{\mu^2}{4\pi\alpha'} \exp (2\beta X^+) : X_9^2 : + \frac{i\mu}{2\pi} \exp (\beta X^+) \tilde{\lambda} \left( \psi_9 + \beta X_9 \psi^+ \right). \quad (4.4)$$

This superpotential is the same as that discussed in the heterotic section (Section 3) of [21], restricted to the case in which a single real tachyon acquires a vev.
The equations of motion take the form (with \( i = 2, \cdots, 8 \)):

\[
\begin{align*}
\partial_+ \partial_- X^+ &= \partial_- \psi^+ = \partial_+ \partial_- X_i = \partial_- \psi_i = \partial_+ \tilde{\lambda}^A = 0 , \\
\partial_+ \partial_- X_9 &= -\frac{\mu^2}{4} \exp(2\beta X^+) \ X_9 + \frac{i \mu \beta \alpha'}{4} \exp(\beta X^+) \tilde{\lambda} \psi^+ , \\
\partial_+ \partial_- X^- &= \frac{\mu^2 \beta}{4} \exp(2\beta X^+) \ X_9^2 - \frac{i \mu \beta \alpha'}{4} \exp(\beta X^+) \tilde{\lambda} \left( \psi_9 + \beta X_9 \psi^+ \right) , \\
\partial_+ \tilde{\lambda} &= \frac{\mu}{2} \exp(\beta X^+) \left( \psi_9 + \beta X_9 \psi^+ \right) , \\
\partial_- \psi^- &= \frac{\beta \mu}{2} \exp(\beta X^+) \ X_9 \tilde{\lambda} .
\end{align*}
\]

This 2D worldsheet theory is integrable at both the classical and quantum levels. As a result, the properties of string trajectories in this theory are particularly simple, and can be summarized as follows:

- In infinite worldsheet volume, the general classical solution can be written in closed form.
- In finite worldsheet volume, trajectories with \( X^\pm , \psi^\pm \) independent of the spatial worldsheet coordinate \( \sigma^1 \) can be written in a simple closed form involving Bessel functions (see, e.g., Eqn. (2.17) of Ref. [21]). The Virasoro constraints and null-state gauge equivalences are sufficient to put a general physical solution into such a form.
- The classical and quantum behavior of these solutions can be understood from general principles, using the adiabatic and virial theorems. The excitation numbers in the massive modes \( X_9 , \psi_9 , \tilde{\lambda} \) are frozen into constant values at late times.
- String behavior relative to the bubble wall at \( X^+ \sim 0 \) depends on asymptotic occupation numbers. Strings can enter the bubble wall and exist in the \( X^+ \rightarrow \infty \) phase if the occupation numbers in the massive modes are all equal to zero. Otherwise, the string is pushed outward along the bubble wall at \( X^0 \sim -X^1 \), accelerated to the speed of light.
4.2 Effective worldsheet theory at large $X^+$

As with the examples studied in [18,21,24], the properties that render the theory exactly solvable at the classical level also make the quantum theory particularly simple. Namely, all connected correlators of free fields have quantum perturbation expansions that terminate at one-loop order. The structure of the quantum corrections can be summarized as follows:

- The two-point function for the light-cone fields $X^\pm$ and their superpartners $\psi^\pm$ are proportional to $G^{\mu\nu}$, so the propagators only connect “+” fields to “−” fields. Propagators for the massless $X^\pm$ multiplets are therefore oriented, and we represent them as dashed lines with arrows pointing from $+$ to $−$. The massive multiplet $X_9$, $\psi_9$, $\tilde{\lambda}$ is correlated with itself, so its propagators are represented by solid, unoriented lines.

- Fundamental vertices representing the classical potential and Yukawa couplings have arbitrary numbers of outgoing dashed lines emanating from them, and exactly two solid lines.

- Every connected tree diagram with multiple vertices therefore has the structure of an ordered sequence of vertices with a single solid line passing through, and arbitrary numbers of dashed lines emanating from each vertex.

- Interaction vertices have only outgoing (dashed) lines, and no two vertices can be connected with a dashed line. A connected Feynman diagram can have either zero or one loop, but not two loops or more.

- Connected loop diagrams consist strictly of a closed solid line with dashed lines emanating from an arbitrary number of points. This exhausts the set of connected Feynman diagrams in the theory, and every connected correlator is exact at one-loop order.

4.3 Dynamical readjustment of the metric and dilaton gradient

Now we would like to study the dynamics of states that penetrate into the far interior of the bubble, at $X^+ \to \infty$. These strings necessarily have all massive modes in their ground states, frozen out with exponentially increasing mass. It is possible to integrate out $X_9$, $\psi_9$ and $\tilde{\lambda}$ exactly to obtain an effective action for the remaining worldsheet degrees of freedom. All but a finite number of terms have canonical dimension greater than 2, coming from derivatives
and fermions. By scale invariance, such terms always appear dressed with real exponentials of $X^+$, with negative exponent. We are ultimately interested in the limit $X^+ \to \infty$, so all terms in the action of canonical dimension greater than two can be ignored. Furthermore, no terms of canonical dimension less than two are generated, since there are no such operators that are invariant under the $(\mathbb{Z}_2^5)_L$ symmetry and $(0,1)$ supersymmetry.

The only terms generated that survive the $X^+ \to \infty$ limit are thus a renormalization of the Ricci term $\sqrt{g}R^{(2)}$, and a renormalization of the kinetic term for $X^+$, $\psi^+$. In the language of spacetime physics, these represent dynamical readjustments of the string-frame metric and the dilaton due to the backreaction of the condensing tachyon. As described in [21], these readjustments arise entirely from one-loop renormalizations on the string worldsheet, so they are exactly calculable. The relevant diagrams are depicted in Fig. 1 where solid lines are drawn to indicate massive fields, and dashed oriented lines indicate the massless $X^\pm$ fields. We will refer to quantities in the final-state theory with a hatted notation. The readjusted metric and dilaton are thus denoted by $\hat{G}$ and $\hat{\Phi}$, respectively, with the gradient of the readjusted dilaton denoted by $\hat{\nabla}_\mu$.

$$\Delta(\partial_+ \Phi) = \begin{array}{c} \text{\ldots} \\
\end{array}$$

$$\Delta G_{++} = \begin{array}{c} \text{\ldots} \\
\end{array}$$

Figure 1: Diagrams contributing to the nonvanishing renormalizations of the dilaton and metric, $\Delta(\partial_+ \Phi)$ and $\Delta G_{++}$. Solid lines indicate massive fields, while dashed oriented lines represent propagators of the massless lightcone fields $X^\pm$.

The calculation of the renormalized worldsheet couplings is straightforward (see Refs. [21, 31] for details). Defining $M \equiv \mu \exp(\beta X^+)$, the dilaton readjustment is

$$\Delta \Phi = \frac{1}{4} \ln \left( \frac{M}{\tilde{\mu}} \right),$$

where $\tilde{\mu}$ is an arbitrary mass scale entering the definition of the path integral measure. The
renormalization can be expressed as
\[ \Delta \Phi = (\text{const.}) + \frac{\beta}{4} X^+ , \quad \Delta V_+ = \frac{\beta}{4} . \]  
(4.7)

As noted, there is also a nonzero renormalization of the string-frame metric. For a generalized mass term \( \frac{1}{4\pi\alpha'} M(X)^2 X_9^2 \), where \( M(X) \) depends arbitrarily on all coordinates other than \( X_9 \), the metric \( G_{\mu\nu} \) is renormalized by an amount
\[ \Delta G_{\mu\nu} = \frac{\alpha'}{4} \partial_\mu M \partial_\nu M . \]  
(4.8)

For \( M = \mu \exp(\beta X^+) \), this gives the renormalized metric
\[ \hat{G}_{++} = -\hat{G}_{--} = \frac{\alpha' \beta^2}{4} , \]  
with all other components unrenormalized.

The linear dilaton central charge at \( X^+ \to \infty \) is therefore given by (including the renormalized dilaton gradient \( \hat{V}_\mu \)):
\[ c_{\text{dilaton}} = 6 \alpha' \hat{G}^{\mu\nu} \hat{V}_\mu \hat{V}_\nu = \frac{3 q \beta \alpha'}{\sqrt{2}} - \frac{3 \beta^2 q^2 \alpha'^2}{4} . \]  
(4.10)

With \( q \beta = \sqrt{2}/\alpha' \), the final dilaton contribution to the central charge is
\[ c_{\text{dilaton}} = \frac{3}{2} . \]  
(4.11)

We therefore find that exactly 3/2 units of central charge have been transferred from the fields \( X_9, \psi_9, \tilde{\lambda} \) into the strength of the dilaton gradient at \( X^+ = \infty \). The total central charge, including free-field and dilaton contributions, is again the same at \( X^+ = \infty \) as at \( X^+ = -\infty \), and in particular equal to (26,15).

4.4 Stability properties of the tachyon solution

We will demonstrate explicitly in Section 5 that the nine-dimensional string theory describing the final state of the transition is stable, meaning it has no spatially normalizable perturbations that grow with time. For the purposes of the discussion in this subsection, however, we will assume the stability of the final state. No normalizable perturbation of the transition as a whole should be able to affect the qualitative nature of the endpoint: a perturbation of
the full, time-dependent background will always evolve into a perturbation of the late-time background. The latter is stable, so the late-time limit of such a perturbation cannot lead to a further instability changing the phase of the final state. In Ref. [23], transitions having this property were referred to as stable transitions, and many of the transitions studied in [21] are in fact stable. In particular, the transitions whose final states are spacetime-supersymmetric or two-dimensional are such that a generic perturbation (satisfying the appropriate normalizability condition) of the full, time-dependent solution will not alter the solution qualitatively at late times. In this subsection we wish to refine this classification by drawing a further important distinction between different types of stable transitions. For purposes of our discussion, we will consider bubble of nothing solutions to be stable transitions, in that no normalizable perturbation changes their qualitative behavior.

**Absolute vs. local stability**

We first define **absolutely stable** transitions as those that exhibit a stable final endpoint whose qualitative nature is completely determined in advance by the nature of the initial state. For instance, the transitions from supercritical type HO\(^{+/-}\) (which has diagonal GSO projection) on certain orbifolds to supersymmetric type HO string theory in ten dimensions have the property of absolute stability. Even a large, non-infinitesimal change in the initial configuration of the tachyon will not result in any final state other than a single supersymmetric type HO theory in ten dimensions. This supercritical orbifold does have other possible end-states. For instance, it can fragment into disconnected baby universes, of which precisely one supports supersymmetric type HO theory, with the others containing unstable type HO in various dimensions. However, such alternate endpoints are always fine-tuned. Untuned transitions lead to a unique and universal final state: a single component, supporting supersymmetric type HO in ten dimensions. A phase space of absolute-stable transitions is depicted in Fig. 2.

We define a second type of stable transition as possessing the weaker property of being **locally stable** in the space of solutions. That is, a linearized perturbation around a locally stable solution will always preserve the qualitative nature of the final state. However, a sufficiently large deformation of a locally stable solution can lead to a qualitatively different but stable outcome. A model phase space of locally stable transitions is presented in Fig. 3. The string theories of type HO\(^{+}\) discussed in [30] were shown to have the property of local
stability, but not absolute stability. Starting with type HO$^+$ in supercritical dimensions leads generically to a number of disconnected universes supporting stable, supersymmetric type HO string theory. However, the number of such universes, and the chirality of the gravitini and gaugini in the baby universes, depends on the number of zeroes of the tachyon configuration, as well as on tachyon derivatives at the vanishing points. In some open sets of tachyon configuration space there are no zeroes at all, and the universe is destroyed from within by a bubble of nothing.

**Local stability of dimension-changing type UHE transitions**

We now wish to demonstrate that our dimension-reducing solutions of UHE string theory are locally stable, rather than absolutely stable transitions. Consider a tachyon profile of the form $T(X) \equiv \mu \exp(\beta X^+) f(X_i, X^+)$, with $i = 2, \cdots, 9$, and $f(X_i, 0)$ taken to be a completely arbitrary real function. We assume that $T(X)$ is an operator of definite weight $(\frac{1}{2}, \frac{1}{2})$, which means that $f(X_i, X^+)$ satisfies the diffusion equation

$$\partial_+ f = \frac{\beta \alpha'}{2} \partial_i^2 f .$$

(4.12)

For $\mu \exp(\beta X^+) \gg 1$, the background is described by a string theory in nine dimensions,
Figure 3: Phase diagram of endpoints in locally stable transitions. Contours depict loci of non-generic, fine-tuned unstable endpoints of the transition. The regions bounded by these loci indicate stable but inequivalent endpoints. For these purposes, the “nothing” state will be considered a stable endpoint, in that nearby solutions converge to it at large $X^+$. 

Supported on the vanishing locus of $f$. For $\sqrt{\alpha'} \gg |\partial_i f|$, higher orders of conformal perturbation theory can be neglected, and the leading order, given by equation (4.12), is a good approximation to the dynamics. Using equation (4.12), we demonstrate that two initial tachyon profiles can lead to different outcomes for the final behavior of the system.

Consider one initial profile, given by $f(X,0) = X_8^2 + X_9^2 - R_{\text{init}}^2$, where we assume $R_{\text{init}}^2 \gg \alpha'$. Given this initial condition, the subsequent evolution of the tachyon profile is

$$f(X, X^+) = \frac{1}{\alpha'}(X_8^2 + X_9^2 - R_{\text{init}}^2 + 2\beta \alpha' X^+)$$.

At a particular lightcone time $X_{\text{crit}}^+ = R_{\text{init}}^2/(2\beta \alpha')$, the circle (defined by $T = 0$) shrinks to zero size, and the minimum of the classical worldsheet potential rises above zero:

$$V_{\text{ws}} = T^2 = \mu^2 \exp\left(2\beta X^+\right) f^2$$

$$= \frac{\mu^2}{\alpha'^2} \exp\left(2\beta X^+\right) \left[X_8^2 + X_9^2 + 2\beta \alpha' (X^+ - X_{\text{crit}}^+)\right]^2,$$

which is greater than $4\beta^2 \mu^2(X^+ - X_{\text{crit}}^+)^2$ when $X^+ > X_{\text{crit}}^+$. A positive-definite worldsheet potential increasing exponentially with lightcone time has qualitative behavior akin to the
bubble of nothing described previously. It does not describe the static $(8 + 1)$-dimensional heterotic string theory at large $X^+$, or anything in the universality class thereof.

For a second initial profile, one may change the sign of the $X_9^2$ term from $+1$ to $-a$, with $a > 1$. The tachyon profile then evolves as:

$$f(X_i, X^+) = f(X_i, 0) = \frac{1}{\alpha'} \left[ X_8^2 - aX_9^2 - R_{\text{init}}^2 + (1 - a)\beta\alpha' X^+ \right]. \quad (4.15)$$

The effective value of $R_{\text{init}}^2$ increases as $X^+ \to \infty$, so the qualitative behavior of the background at late times is that of the stable nine-dimensional heterotic theory propagating in two disconnected baby universes, described by the two branches of the hyperbola in the $X_{8,9}$ plane where $f(X_i, X^+)$ vanishes.

This establishes that two simple choices of initial conditions in the same linear dilaton background of the same theory can lead to qualitatively different behaviors at late times, each of which is stable under small perturbations. In one case, we have a universe-destroying bubble of nothing. In the second case we obtain a bubble of new vacuum in which the universe bifurcates into two stable nine-dimensional components. This indeterminate behavior, with multiple possible stable endpoints following from the same initial starting point, is suggestive of a randomly populated landscape of vacua.

5 The stable HE9 string theory in nine dimensions

We have established that a final state of our dimension-reducing solution describes a nine-dimensional theory, which we refer to as the HE9 theory. In the transition, the fields $X_9$, $\psi_9$ and the neutral fermion $\tilde{\lambda}$ have decoupled from the worldsheet, and the dilaton gradient and string-frame metric have acquired renormalizations associated with one-loop quantum corrections to the free worldsheet theory. As in the cases studied in \cite{21, 24}, the effect is to transfer the central charge contributions of the decoupled worldsheet degrees of freedom into the strength of the dilaton gradient, keeping the total central charge constant.

In this section we analyze directly the HE9 final state of our solution. The final vacuum has a flat string-frame metric and linear dilaton with spacelike gradient. Furthermore, the final vacuum has no tachyon degree of freedom. The lightest excitations are the metric $\hat{G}$, an NS two-form $\hat{B}$, the dilaton $\hat{\Phi}$, an $E_8$ gauge field $\hat{A}$ and a fermion $\hat{\Lambda}$ transforming as a $16$-real-dimensional Majorana spinor of $SO(8, 1)$, in an adjoint representation $248$ of $E_8$. 20
All of these fields descend in obvious ways from the degrees of freedom present in the initial 10D UHE theory.

We have also seen that the final \((8 + 1)\)-dimensional state has a dilaton gradient that is spacelike, with norm-squared \(\hat{V}^2 = \frac{1}{4\alpha'}\). We can choose a new set of coordinates \(Y^m, m = 0, \cdots, 8\) that put the final dilaton and metric into a simple form; the appropriate coordinate transformations can be expressed in terms of the original \(X^\pm\) variables as

\[
Y_8 = -\sqrt{2\alpha'}qX^- + \frac{\beta\sqrt{\alpha'}}{2}X^+ ,
\]

\[
Y^0 = \frac{2}{\beta\sqrt{\alpha'}}X^- ,
\]

\[
Y_m = X_m , \quad m = 2, \cdots, 8 .
\]  

(5.1)

In the \(Y^m\) system, the \(m, n, \cdots\) indices are raised and lowered with the hatted metric \(\hat{G}_{mn}\), and the dilaton is given by

\[
\hat{\Phi} = \hat{\Phi}_0 + \hat{V}_m Y^m , \quad \hat{V}_{0,2,3,\cdots,8} = 0 , \quad \hat{V}_1 = \hat{q} ,
\]  

(5.2)

with

\[
\hat{q} = \frac{1}{2\sqrt{\alpha'}} , \quad \hat{G}_{mn} = \eta_{mn} .
\]  

(5.3)

### 5.1 Partition function

The computation of the partition function for the HE9 theory proceeds as for the UHE theory. The major difference is that the contributions from the boson \(X_9\) and the fermions \(\psi_9, \tilde{\lambda}\) are absent. The linear dilaton does not couple to the torus, and has no effect on the path integral. The fermions \(\psi_9, \tilde{\lambda}\) are neutral under \((\mathbb{Z}_2)_L\), and have the same transformations under \((-1)^F\). The path integral on the torus with spin structure \(a, b\) is therefore the same as that in the UHE theory, with one less factor of \((4\pi^2\alpha'\tau_2)^{-\frac{1}{2}}|\eta(\tau)|^{-2}\) in the boson path integral, one less factor of \((\tilde{Z}_{a_b})^{\frac{1}{2}}\) in the left-moving fermion path integral, and one less factor of \((Z_{a_b})^{\frac{1}{2}}\) in the right-moving fermion path integral.
The left-moving fermion path integrals in the HE9 theory are thus

\[ L_{900}(\bar{\tau}) = \frac{1}{32} \left( \hat{Z}_{00} \right)^{\frac{33}{2}} \left[ \left( \hat{Z}_{00} \right)^8 + 31 \left( \hat{Z}_{01} \right)^8 + 31 \left( \hat{Z}_{10} \right)^8 \right], \]

\[ L_{901}(\bar{\tau}) = \frac{1}{32} \left( \hat{Z}_{01} \right)^{\frac{15}{2}} \left[ \left( \hat{Z}_{01} \right)^8 + 31 \left( \hat{Z}_{00} \right)^8 + 31 \left( \hat{Z}_{10} \right)^8 \right], \]

\[ L_{910}(\bar{\tau}) = \frac{1}{32} \left( \hat{Z}_{10} \right)^{\frac{15}{2}} \left[ \left( \hat{Z}_{10} \right)^8 + 31 \left( \hat{Z}_{00} \right)^8 + 31 \left( \hat{Z}_{01} \right)^8 \right], \]

\[ L_{911}(\bar{\tau}) = 0. \quad (5.4) \]

The full path integral for fermions and superghosts, with spin structure \( a, b \) is

\[ F_{9a,b}(\tau, \bar{\tau}) \equiv \left( Z_{a,b} \right)^{\frac{7}{2}} L_{9a,b}. \quad (5.5) \]

These are then combined into projected partition functions in NS and R sectors:

\[ I_{9_{\text{NS}}\pm} \equiv \frac{1}{2} \left( F_{900} \mp F_{901} \right), \]

\[ I_{9_{\text{R}}} \equiv I_{9_{\text{R}}^-} = \frac{1}{2} F_{910}. \quad (5.6) \]

The path integral for bosons and reparametrization ghosts is

\[ iV_9(4\pi^2\alpha'\tau_2)^{-\frac{9}{2}}|\eta(\tau)|^{-14}, \quad (5.7) \]

and the measure for moduli takes the usual form. The total partition function is then:

\[ \int_{\mathcal{F}} \frac{d\tau d\bar{\tau}}{16\pi^2\alpha'\tau_2^2} (4\pi^2\alpha'\tau_2)^{-\frac{7}{2}}|\eta(\tau)|^{-14} \left( I_{9_{\text{NS}}^+} - I_{9_{\text{R}}^\pm} \right). \quad (5.8) \]

The functions \( F_{9a,b}, I_{9_{\text{NS}}\pm} \) and \( I_{9_{\text{R}}} \) exhibit the same modular transformation properties as their ten-dimensional counterparts (see, e.g., Eqns. (2.14, 2.15)), so the modular invariance of Eqn. (5.8) follows. Defining mass partition functions for the HE9 theory in parallel with those of the UHE theory, we find

\[ Z_{9_{\text{NS}}}^{\text{mass}}(\tau) = (q\bar{q})^{\frac{1}{2}} \left[ 1,785 + 108,500 \left( q\bar{q} \right)^{\frac{1}{2}} + O (q\bar{q}) \right] , \]

\[ Z_{9_{\text{R}}}^{\text{mass}}(\tau) = 1,984 + 4,058,880 \left( q\bar{q} \right)^{1} + O \left( (q\bar{q})^2 \right). \quad (5.9) \]
We now pause to emphasize several points regarding the spectrum. As expected, the NS sector is tachyon-free. The lowest NS states gain an effective mass-squared of $V^2 = \frac{1}{4\alpha'}$ from their coupling to the background dilaton gradient $[^32]$, as is usual in subcritical string theory $[^33][^34]$. The first NS mass level consists of $\frac{7\cdot 8}{2} - 1 = 27$ graviton polarizations, $\frac{7\cdot 6}{2}$ B-field polarizations, and one dilaton, together with $7 \cdot 248 = 1,736$ polarizations of the $E_8$ gauge field, for a total of 1,785 physical states. These states would be massless in a background with constant dilaton.

The lowest Ramond mass level is a massless Majorana spinor in the adjoint of $E_8$. A Majorana spinor in 9D has 16 degrees of freedom off shell, which reduces to eight upon imposing the on-shell conditions. Multiplying by the dimension of the adjoint representation, we obtain $8 \cdot 248 = 1,984$ physical states. Interestingly, the spin-$\frac{1}{2}$ fermion does not obtain a nonzero mass from its coupling to the dilaton gradient.

The masslessness of the lowest state is an inevitable consequence of the effective field theory. There is only a single Majorana adjoint fermion at the lowest level. A single Majorana fermion can have no mass term with itself in $8k$ spatial and one time dimension: if $C$ is the charge-conjugation matrix acting on spinors, then $C$ and $C\Gamma^m$ are both symmetric. A Majorana spinor $\hat{\Lambda}$ obeys $C_{\alpha\beta}\bar{\hat{\Lambda}}_\beta = \hat{\Lambda}_\alpha$, so terms such as $M\bar{\hat{\Lambda}}\hat{\Lambda}$ and $\bar{\hat{\Lambda}}(\hat{\Phi}\hat{\Phi})\hat{\Lambda}$ vanish identically by Fermi statistics. Therefore, the rescaling from string frame to a canonically normalized fermion cannot introduce couplings that give $\hat{\Lambda}$ a nonzero physical mass. We summarize the field content and spectral properties in the low-lying energy levels of both the ten-dimensional parent UHE theory and the HE9 final state in Table 5.1.

5.2 Absence of supersymmetry

The HE9 theory has no unbroken spacetime supersymmetry. One way to see this is to demonstrate the complete absence of Bose-Fermi mass degeneracy. The $m^2 = 0$ adjoint fermions $\hat{\Lambda}$ are split from the gauge field $\hat{A}$ in the spectrum by an amount $\Delta m^2 = \frac{1}{4\alpha'}$; the canonically normalized modes of $\hat{A}$ have an effective mass equal to $\frac{1}{2\sqrt{\alpha'}}$. Furthermore, the multiplicities of the gauge field and adjoint fermions do not agree: for each of the 248 gauge generators, there are seven physical polarizations of the gauge field, but eight physical polarizations of $\hat{\Lambda}$, which transforms as a Majorana spinor of SO(8,1). There are also no spin-1/2 or spin-3/2 particles that could serve as degenerate superpartners of the dilaton $\hat{\Phi}$, NS two-form $\hat{B}$ or graviton $\hat{G}$. Indeed, the lightest spin-3/2 fields and gauge-singlet spin-1/2
Table 1: Summary of field content and multiplicities in the lowest-lying mass levels of the ten-dimensional UHE string theory and its nine-dimensional endpoint, following the transition. The tachyon is absent in the HE9 final state. Fields in the massless NS sector acquire an effective mass $m^2 = 1/(4\alpha')$ after the transition, due to the coupling to the dilaton background. Furthermore, the massless R sector loses half of its particle content in the transition. Interestingly, the lightest state in the HE9 theory is a fermion rather than a boson. (Hatted quantities are reserved for the nine-dimensional theory.)

| theory | sector | mass | field content | mult. |
|--------|--------|------|---------------|-------|
| UHE    | NS     | $m^2 = -2/\alpha'$ | $T$ | 1 |
| NS     | $m^2 = 0$ | $\Phi(1) + G(35) + B(28) + A(1984)$ | 2048 |
| R      | $m^2 = 0$ | $\Lambda_+(1984) + \Lambda_-(1984)$ | 3968 |
| HE9    | NS     | $m^2 = +1/(4\alpha')$ | $\hat{T}(1) + \hat{G}(27) + \hat{B}(21) + \hat{A}(1736)$ | 1785 |
| R      | $m^2 = 0$ | $\hat{\Lambda}(1984)$ | 1984 |

Table 2: The lowest-lying normalizable string modes in the HE9 theory. The canonically normalized modes of the graviton, dilaton and B-field acquire an effective mass $\Delta m = \frac{1}{2\sqrt{\alpha}}$ from their coupling to the background dilaton gradient, as does the $E_8$ gauge field. The $E_8$ adjoint fermion, on the other hand, does not acquire a positive mass: the canonically normalized field $\hat{\Lambda}$ obeys the massless Dirac equation.

| string state | $m^2 \equiv -k_m k^m$ | field name | transversality/Dirac equation | gauge invariance |
|--------------|----------------------|------------|-------------------------------|-----------------|
| $e^{mn}\hat{a}_{-1/2}^{m} \psi_{-1/2}^{n} | k ; 0 \rangle_1$ | $\frac{1}{4\alpha'}$ | $\hat{G}_{mn}, \hat{B}_{mn}, \hat{\Phi}$ | $(k + i\hat{V})^m e_{mn} = 0$ | $\Delta e_{mn} = \xi_m (k - i\hat{V})_n$ |
| $e^{m}\psi_{-1/2}^{m} | k ; 0 \rangle_{gl}$ | $\frac{1}{4\alpha'}$ | $\hat{A}_m$ | $(k + i\hat{V})^m e_m = 0$ | $\Delta e_m = (k - i\hat{V})_m \xi$ |
| $|k ; \hat{\alpha}\rangle_{gl,(-1)Fw}$ | $0$ | $\hat{\Lambda}_{\alpha}$ | $\phi \hat{\Lambda} = 0$ | - |
fields enter at $m^2 = \frac{4}{\alpha'}$, which is four times as heavy as the normalizable excitations of $\hat{\Phi}$, $\hat{B}$ and $\hat{C}$.

Despite the lack of spacetime supersymmetry, our theory is tachyon-free. The tachyon vertex operator would have to be built from a matter primary of weights $(\tilde{h}, \tilde{h} - 1/2)$, with $\tilde{h} < 1$ prior to momentum dressing. The only left-moving fields with $\tilde{h} < 1$ are the 31 current algebra fermions $\tilde{\lambda}^A$, which have $\tilde{h} = 1/2$. (All twisted operators in the current algebra have weight at least one.) Each of the 31 fermions $\tilde{\lambda}^A$ transforms nontrivially under some element of the left-moving gauge group $(\mathbb{Z}_2^5)_L$, and none can therefore enter a physical vertex operator unaccompanied by other current algebra fermions. Hence, there are no tachyons in the HE9 final state.

Finally, the background has no moduli. The lightest field is the dilaton, whose normalizable excitations have $m = \frac{1}{2\sqrt{\alpha'}}$. Even the constant mode of the dilaton $\delta \hat{\Phi} = \text{const.}$ does not represent a modulus in the spacelike linear dilaton background. Shifting $\hat{\Phi}$ can be compensated by a redefinition of the spatial coordinate $Y_8$, so even this degree of freedom is not truly a modulus; rather, it is pure gauge.

6 Discussion and conclusions

In addition to the bubble of nothing solution studied by Hořava and Keeler, the UHE theory in ten initial dimensions admits exact solutions that transit to a stable, nine-dimensional theory with no moduli and no spacetime supersymmetry. This transition is depicted schematically in Fig. 4, where our solution focuses on the upper left-hand region of the spacetime diagram.

Generally, the absence of supersymmetry with no tachyons and no moduli is interesting. Few completely stable nonsupersymmetric string theories are known in dimensions above $D = 2$. The $O(16) \times O(16)$ theory in ten dimensions is nonsupersymmetric and tachyon-free at tree level, but its massless fields (such as the dilaton and scale factor of the metric) acquire potentials from higher-genus string diagrams. The HE9 theory, by contrast, has no moduli at tree level, and the effective mass shift due to the dilaton gradient renders the background stable against quantum corrections. Background shifts due to higher-genus string diagrams are finite away from the strong coupling region, and can be incorporated via the Fischler-Susskind mechanism [35,36].
Figure 4: The dynamical spacetime transition from ten dimensions outside a bubble wall to nine dimensions in the interior. Our solution focuses on the upper left-hand corner of the diagram, where the bubble is a domain wall moving to the left at the speed of light. The final phase is a stable theory with a single $E_8$ gauge group and no spacetime supersymmetry.

The qualitative nature of the final state of the UHE string depends on details of the initial tachyon profile.\footnote{As noted above, similar non-deterministic behavior also occurs in the supercritical type HO$^+$ string\cite{30}.} This situation is reminiscent of the behavior of an eternally inflating universe with a complicated scalar potential, in which different, inequivalent vacua are populated by the random evolution of scalar fields across a jagged landscape of local minima\cite{37,38}. Tachyonic starting points that can make locally stable (but not absolutely stable) transitions suggest a possible arena for studying the string landscape concept in a weakly-coupled limit.

Cosmological evolution in quantum gravity can produce qualitatively different outcomes from the same initial conditions. The stable HE9 theory is separated in phase space from the theory studied in\cite{19,20}, though both theories descend from the same unstable parent theory (the UHE background). Two such basins of attraction must be separated by a transition with a characteristic critical behavior. It would be interesting to find the set of unstable, fine-tuned endpoint theories that separate these two stable attractors in phase space. These
theories lie along critical loci in phase space, joining a patchwork of locally stable endpoints as depicted in Fig. 3 above.

A broader question is, what are the possible stable endpoints of tachyon condensation descending from unstable parent theories other than type UHE? A census of unstable, ten-dimensional heterotic string theories was taken by Kawai, Lewellen and Tye in Ref. [1]. They classified a total of six tachyonic backgrounds, with gauge groups $SO(32)$, $O(16) \times E_8$, $O(8) \times O(24)$, $(E_7 \times SU(2))^2$, $U(16)$ and $E_8$, the last belonging to the UHE theory studied in this paper. It would be interesting to analyze the remaining unstable backgrounds in this classification, with the hope of finding new string vacua of the type studied here.

Of the six, the UHE theory is distinguished by having a sensible 11-dimensional interpretation [9]. In this interpretation, the string-theoretic bubble of nothing is lifted to 11 dimensions as a cosmological spacetime with a particular geometry and topology. Given that the same UHE state can also transition to the HE9 background, the 11-dimensional description of type UHE and its instabilities could provide insight into noncritical and non-supersymmetric string vacua through the lens of M-theory.

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