Hysteretic transition from laminar to vortex shedding flow in soap films

V.K. Horváth*, J.R. Cressman, W.I. Goldburg, and X.L. Wu

Department of Physics and Astr., Univ. of Pittsburgh, Pittsburgh, PA 15260, USA

*Contact e-mail: vhorvath@pitt.edu

1 Introduction

Studies of flow behind a single cylinder have greatly contributed to our understanding of the development of complex flows like the Kármán vortex street or turbulent flows. At the first instability in such a system the laminar flow (LF) becomes time dependent and vortices appear. It has been shown that this transition to the vortex shedding phase (VS) is well described by the supercritical Hopf bifurcation.

2 Hysteretic Primary Instability in Soap Films

Soap films have been shown to be a particularly useful for the study of two dimensional flows. Here we present measurements demonstrating that the LF→VS transition can be hysteretic in a (quasi-two dimensional) soap film penetrated by a glass cylinder. Our experimental setup consists of a rapidly flowing soap film formed between two vertically positioned thin nylon lines (see Fig. 1). The film is fed continuously from the top through a valve. To investigate the flow at different Reynolds numbers \( Re = \frac{d}{\nu} \) we changed the mean velocity \( V \) by either opening and closing the valve at different rates or by changing the separation of the nylon lines, using computer controlled stepping motors (here \( \nu \) is the kinematic viscosity). At the center of the parallel segment of the set-up, a glass rod penetrates the film in the z direction. The velocity measurements were made using a dual head laser Doppler velocimeter (LDV). The fluctuating horizontal component \( V_x(t) \) and \( V_y(t) \) were measured simultaneously below and above the rod respectively.

On increasing the flow rate a transition appears from the LF state to the VS state. Below the transition, the flow is characterized by two counter-rotating vortices underneath the rod (Fig. 2a). In the VS state these recirculating vortices peel off the rod and flow downstream. The continuously generated counter-rotating vortices form the Kármán vortex street (Fig. 2b). All experiments start

\[
Re = \frac{\nu d}{\nu}
\]
Figure 1: Schematic of the experimental setup. The diameter $d$ of the rod is 1mm. The 0.2mm thick nylon lines are tied to a weight at the bottom.

Figure 2: Interference pictures of the flowing soap film below the rod in (a) the laminar state and (b) the vortex shedding state.

with slowly increasing $V$ in the LF regime. The critical velocity, where VS commences, is called $V_{up}$. After some waiting time in the VS state, $V$ is slowly decreased. At $V_{down}$ the system undergoes a reverse transition, i.e. from VS into LF. This cycle was repeated several times in each run with $V_x(t)$ and $\mathbf{V}$ being recorded simultaneously.

In order to determine the transition velocities, we have calculated the velocity probability distribution function $P(V_x)$ from $V_x(t)$ by a standard binning procedure. In Fig. 3 the magnitude of $P(V_x, t)$ is mapped into gray scale values and is shown as a function of $V_x$ and $t$. It can be seen that sharp changes in $P(V_x, t)$ precisely indicate the transition times $t^{*}_d$ and $t^{*}_u$ and therefore $V_{down} = \mathbf{V}(t^{*}_d)$ and $V_{up} = \mathbf{V}(t^{*}_u)$ too. It is apparent in Fig. 3 that $V_{up}$ is not equal to $V_{down}$, that is, there is no unique critical velocity for the transition. The Reynolds number is roughly 50 in the hysteretic gap. A large uncertainty in this value comes from the poorly determined two-dimensional soap film viscosity $\nu$, which depends on the film thickness.

It is worth pointing out that if we keep the mean flow rate constant in the interval $[V_{down}, V_{up}]$, then the laminar state can persist for an indefinite length of time. However applying sufficiently large acoustic or pulse-like mechanical perturbations, the system could be driven into the VS state. The system remains in this state even if the perturbation is turned off. Applying similar perturbations, VS→LF transition was never observed at constant $\mathbf{V}$. As a result we
can conclude that the observed static hysteresis is not a result of some delay in the response of the system to the bifurcation. Auxiliary experiments were also performed to exclude wetting properties and the effect of the air boundary near the film as possible sources of hysteresis.

In the absence of any available theory, we consider a fifth order Landau equation\[ d\hat{v}(t)/dt = \hat{a} \hat{v} + 2b\hat{v}^2 \hat{v} - c\hat{v}^4 \hat{v}, \]
for the complex velocity $\hat{v} = v_x + iv_y$. Here all variables denoted by tilde are complex and $v_i = V_i - \langle V_i \rangle_t (i=x \text{ or } y)$. The complex velocity can also be written as $\hat{v} = ve^{i\phi}$, where $v$ is the amplitude. The real part of this equation describes the evolution of $v$. Its non-trivial positive stationary solution describes the VS state:

\[ v_{VS}^2 = \frac{b_r}{c_r}(1 + \sqrt{1 + a_r c_r / b_r^2}), \]

where the subscript $r$ is used to designate the real part of complex numbers.

Although the application of this model for data fitting purposes is not obvious (the relationship between the parameters $a_r$, $b_r$, $c_r$, and the control parameter of the experiment is unknown), typically $a_r$ is considered to be proportional to $(\overline{V} - V_c^{up})$. As one can see in Fig. 4, this generic model of the hysteresis is in qualitative agreement with our observations. However none of our 3 best fits are very satisfactory. It is interesting to note that a simple three-parameter phenomenological equation $P_0(\overline{V} - V_c) + 2P_1 v - P_2 v^2 = 0$ provides a surprisingly good fit to our data in the VS state (see inset of Fig. 4.). Using least square fitting to the solution of this equation provides the following result: $v_x = 0.04 \pm \sqrt{V / 39.06 - 0.0113}$, where the velocities are in units of $m/s$.  

**Figure 3:** Here the function $\overline{V}(t)$ is shown together with the $P(v_x, t)$ map at $|d\overline{V}/dt|=0.004 \text{ m/s}^2$. The gray scale corresponds to the probability density of $v_x$. The solid and dashed lines are intended to guide the eye for the transitions VS→LF and LF→VS.

**Figure 4:** This figure shows our experimental data together with different fits (see text). The circles and squares represent two different experimental run. Closed and opened symbols are used for data taken at increasing and decreasing $\overline{V}$ respectively.
Auxiliary experiments suggest that the hysteresis may be connected with the fact that the film may become very thin in the recirculating region. Recognizing that the viscosity of a soap film depends on its thickness, the observed hysteresis effect can be qualitatively understood; the Reynolds number can now take different values at the same value of $\overline{V}$, when the parameter is lowered than when it is raised.

3 Conclusions

- We have observed unexpected hysteresis at the onset of the Kármán vortex street in a quasi two-dimensional soap film.
- The fifth order amplitude equation is in qualitative agreement with our observations, but a simple phenomenological equation provides a better numerical fit to the experimental data in the vortex shedding regime.
- A phenomenological picture is suggested to explain the origin of the hysteresis.

This material is based upon work supported by the NATO under a Grant DGE-9804461 awarded to W.I. Goldburg and V.K. Horváth. The work was also supported by NSF grant DMR-9622699, NASA grant 96-HEDS-01-098 and by additional support from the Hungarian Science Foundation grant OTKA F17310.

References

[1] T. von Karman, Gott. Nachr., 24 (1911).

[2] M. Provansal, C. Mathis, and L. Boyer, J. Fluid Mech. 182, 1 (1987); K. R. Sreenivasan, P. J. Strykowski, and D. Olinger, in ASME Forum on Unsteady Separation, edited by K. N. Ghia, volume 52, page 1, 1986.

[3] Y. Couder and C. Basdevant, J. Fluid Mech. 173, 225 (1986); M. Gharib and P. Derango, Physica D 3, 406 (1989).

[4] V.K. Horváth, J.R. Cressman, W.I. Goldburg, and X.L. Wu, to appear in Phys. Rev. E, Rapid Comm., (2000).

[5] S. H. Strogatz, Nonlinear Dynamics and Chaos, Addison Wesley, New York, 1994.
This figure "fig1.jpg" is available in "jpg" format from:

http://arxiv.org/ps/cond-mat/0007344v1
This figure "fig2.jpg" is available in "jpg" format from:

http://arxiv.org/ps/cond-mat/0007344v1
This figure "fig3.jpg" is available in "jpg" format from:

http://arxiv.org/ps/cond-mat/0007344v1
This figure "fig4.jpg" is available in "jpg" format from:

http://arxiv.org/ps/cond-mat/0007344v1