Bound monopoles in Brans-Dicke theory

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Abstract

We consider axially symmetric SU(2) Yang-Mills-Higgs (YMH) multi-monopoles in Brans-Dicke theory for winding number \( n > 1 \). In analogy to the spherically symmetric \( n = 1 \) solutions, we find that the axially symmetric solutions exist for higher values of the gravitational coupling than in the pure Einstein gravity case. For large values of the gravitational coupling, the solutions collapse to form a black hole which outside the horizon can be described by an extremal Reissner-Nordström solution. Similarly as in the pure Einstein gravity case, like-charged monopoles reside in an attractive phase in a limited domain of parameter space. However, we find that the strength of attraction is decreasing for decreasing Brans-Dicke parameter \( \omega \).

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I. INTRODUCTION

Topological defects \[1\] are believed to have formed during phase transitions in the early universe and are related to spontaneously broken symmetries. Depending on the topology of the vacuum manifold, \(d\)-dimensional objects form. The \(d = 0, 1, 2\) defect is the magnetic monopole, the string and the domain wall, respectively. Two basic types of topological defects have been considered in the literature: a) defects which are described by theories with a broken global symmetry and b) defects which are described by theories with a broken local, i.e. gauge symmetry such as the Nielsen-Olesen vortex \([2]\) and the ’tHooft-Polyakov monopole \([3]\). While the latter exhibit particle-like properties such as finite energy and a well defined core, the former have divergent energies. In turn, due to their long-range behaviour, global defects have much stronger gravitational effects \([4–7]\) than their local counterparts \([8,14]\).

The local magnetic monopole results from a spontaneous symmetry breaking of an SU(2) gauge symmetry down to a U(1) gauge symmetry. The mapping of spatial infinity to the vacuum manifold is characterized by the winding number \(n\). The spherically symmetric \(n = 1\) solution is the famous ’tHooft-Polyakov monopole \([3]\). Since this was proved to be the unique spherically symmetric solution \([9]\), the construction of multimonopoles longed for an axially symmetric Ansatz which was introduced in \([10]\).

In flat space like-charged monopoles are non-interacting \([11]\) in the limit of vanishing Higgs boson mass. In this so-called Bogomol’nyi-Prasad-Sommerfield (BPS) \([12,9]\) limit, the attraction of the long-range Higgs field exactly compensates the repulsion caused by the long-range U(1) field. These configurations saturate a lower energy bound, the so-called Bogomol’nyi bound such that their energy is proportional to the winding number \(n\). For finite Higgs mass, the monopoles are repelling \([13]\) since now the Higgs field is exponentially decaying.

Gravitating monopoles have been studied in the framework of General Relativity \([14,15]\). Interestingly, it was found that gravitating monopoles can - for specific choices of the gravitational constant and the Higgs self-coupling constant - reside in an attractive phase in the sense that their energy per winding number is decreasing with increasing \(n\) \([15]\).

Among alternative gravity theories, the scalar-tensor theory introduced by Brans andDicke \([16]\) is one of the most popular. Since Mach’s principle \([17]\) leads to problems within the framework of General Relativity, Brans and Dicke introduced a scalar field which plays an analog role than the inverse of Newton’s constant and is coupled to the system by the so-called Brans-Dicke (BD) parameter \(\omega\). However, the interest in BD theory in recent years was mainly motivated by the fact that the scalar-tensor part of the low energy effective action of superstring theory resembles BD theory \([15]\).

Global topological defects in Brans-Dicke like theories have been studied extensively \([19]\). In \([20]\) spherically symmetric solutions of SU(2) Brans-Dicke-Yang-Mills-Higgs (BDYMH) theory in Schwarzschild-like coordinates have been constructed. This includes both the globally regular monopole as well as the corresponding black hole solutions.

In this paper, we construct axially symmetric monopole solutions of SU(2) BDYMH theory. We give the Lagrangian, the Ansatz and the boundary conditions in Section II and present our numerical results in Section III. The conclusions are summarized in Section IV.
II. SU(2) BRANS-DICKE-YANG-MILLS-HIGGS (BDYMH) THEORY

In [20] it was argued that for conformally invariant fields such as the Yang-Mills fields, the Lagrangian in the Einstein frame equals that in the BD frame. In the following, all solutions - unless otherwise stated - are those obtained in the Einstein frame.

A. The Lagrangian

The Lagrangian $\tilde{\mathcal{L}}$ of Brans-Dicke theory reads [16] :

$$\tilde{\mathcal{L}} = \frac{1}{16\pi G} \left( \tilde{\Psi} \tilde{\tilde{R}} - \frac{\omega}{\tilde{\Psi}} \tilde{\partial}_\mu \tilde{\Psi} \tilde{\partial}^\mu \tilde{\Psi} \right) + \tilde{\mathcal{L}}_m$$  (1)

$G$ denotes Newton’s constant and $\omega > -3/2$ is the Brans-Dicke parameter. For $|\omega| \rightarrow \infty$ standard Einstein gravity is recovered. A conformal transformation to the Einstein frame yields the Lagrangian $\mathcal{L}$ [20] :

$$\mathcal{L} = \frac{1}{16\pi G} R - \frac{1}{2} \partial_\mu \Psi \partial^\mu \Psi + \frac{2\omega + 3}{2\omega + 4} \mathcal{L}_m$$  (2)

with $\Psi$ given by the BD scalar field $\tilde{\Psi}$ :

$$\Psi = \frac{1}{\gamma \sqrt{8\pi G}} \left( \ln(\tilde{\Psi}) - \ln\left(\frac{2\omega + 4}{2\omega + 3}\right) \right), \quad \gamma = (\omega + \frac{3}{2})^{-1/2}.$$  (3)

The matter Lagrangian $\mathcal{L}_m$ is given by :

$$\mathcal{L}_m = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu,a} - \frac{1}{2} e^{-\gamma \sqrt{8\pi G} \Psi} D_\mu \Phi^a D^\mu \Phi^a - e^{-2\gamma \sqrt{8\pi G} \Psi} V(\Phi^a),$$  (4)

with Higgs potential

$$V(\Phi^a) = \frac{\lambda}{4} (\Phi^a \Phi^a - \eta^2)^2,$$  (5)

the non-abelian field strength tensor

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + e \varepsilon_{abc} A_\mu^b A_\nu^c,$$  (6)

and the covariant derivative of the Higgs field in the adjoint representation ($a = 1, 2, 3$)

$$D_\mu \Phi^a = \partial_\mu \Phi^a + e \varepsilon_{abc} A_\mu^b \Phi^c.$$  (7)

$\lambda$ and $\eta$ are the Higgs field’s self-coupling constant and vacuum expectation value, respectively and $e$ denotes the gauge coupling constant. The prefactors of the covariant derivative and the Higgs potential in (4) result from the conformal transformation from the BD to the Einstein frame :

$$g_{\mu\nu} = \frac{2\omega + 3}{2\omega + 4} \tilde{\Psi} \tilde{g}_{\mu\nu}$$  (8)

Note that the Lagrangian (2) resembles that of Einstein-Yang-Mills-Higgs-dilaton theory [22,23] with a specific coupling of the scalar field.
B. Axially symmetric Ansatz

The axially symmetric Ansatz for the metric in isotropic coordinates reads \[24\] :
\[
 ds^2 = -f dt^2 + \frac{m}{f} \left( dr^2 + r^2 d\theta^2 \right) + \frac{l}{f} r^2 \sin^2 \theta d\varphi^2 .
\] (9)

For the gauge fields we choose the purely magnetic Ansatz \[10\] :
\[
 A_t^a = 0 , \quad A_r^a = \frac{H_1}{e r} v_r^a ,
\] \[
 A_{\theta}^a = \frac{1-H_2}{e} v_{\theta}^a , \quad A_{\varphi}^a = -\frac{n}{e} \sin \theta (H_3 v_r^a + (1-H_4) v_{\theta}^a) .
\] (10)

while for the Higgs field, the Ansatz reads \[10\] :
\[
 \Phi^a = \eta (\Phi_1 v_r^a + \Phi_2 v_{\theta}^a) .
\] (11)

The vectors $\vec{v}_r, \vec{v}_\theta$ and $\vec{v}_{\varphi}$ are given by:
\[
 \vec{v}_r = (\sin \theta \cos n\varphi, \sin \theta \sin n\varphi, \cos \theta) ,
\]
\[
 \vec{v}_\theta = (\cos \theta \cos n\varphi, \cos \theta \sin n\varphi, -\sin \theta) ,
\]
\[
 \vec{v}_{\varphi} = (-\sin n\varphi, \cos n\varphi, 0) .
\] (13)

Here the winding number $n$ enters the Ansatz for the fields. Since we are considering static axially symmetric solutions, the function $f, l, m, H_1, H_2, H_3, H_4, \Phi_1, \Phi_2$ and $\Psi$ depend only on $r$ and $\theta$. The spherically symmetric Ansatz for the construction of $n = 1$ solutions in isotropic coordinates is recovered if the dependence on $\theta$ is dropped and additionally $m = l$, $H_1 = H_3 = \Phi_2 = 0$, $H_2 = H_4$.

We introduce the following dimensionless variable $x$ and the dimensionless field $\psi$ :
\[
 x = e\eta r , \quad \psi = \sqrt{8\pi G} \Psi
\] (14)

With this rescaling, the set of differential equations arising from the variation
\[
 \delta S = \delta \left( \int \mathcal{L} \sqrt{-g} d^4 x \right) = 0
\] (15)

depends only on $\omega$ and the following dimensionless coupling constants :
\[
 \alpha = \sqrt{4\pi} \frac{M_W}{e M_{Pl}} = \sqrt{4\pi G \eta} , \quad \beta = \frac{M_H}{\sqrt{2} M_W} = \frac{\sqrt{\lambda}}{e}
\] (16)

where $M_W = e\eta$ is the gauge boson mass, $M_H = \sqrt{2\lambda \eta}$ is the Higgs boson mass and $M_{Pl} = G^{-1}$ is the Planck mass.

The energy $E(n)$ of the solutions for this choice of Ansatz is given in terms of the derivative of the metric function $f$ at infinity :
\[
 E(n) = \frac{1}{2\alpha^2} \lim_{x \to \infty} x^2 \partial_x f .
\] (17)
C. Boundary conditions

We have to impose 10 conditions on each of the boundaries $x = 0$, $x = \infty$, $\theta = 0$ and $\theta = \frac{\pi}{2}$ to solve the set of 10 second order partial differential equations. Regularity at the origin requires:

$$\partial_x f(0, \theta) = \partial_x l(0, \theta) = \partial_x m(0, \theta) = 0, \quad \partial_x \psi(0, \theta) = 0$$  \hspace{1cm} (18)

$$H_i(0, \theta) = 0, \quad i = 1, 3, \quad H_i(0, \theta) = 1, \quad i = 2, 4, \quad \Phi_i(0, \theta) = 0, \quad i = 1, 2$$  \hspace{1cm} (19)

At infinity, the requirement for finite energy and asymptotically flat solutions leads to the boundary conditions:

$$f(\infty, \theta) = l(\infty, \theta) = m(\infty, \theta) = 1, \quad \psi(\infty, \theta) = 0$$  \hspace{1cm} (20)

$$H_i(\infty, \theta) = 0, \quad i = 1, 2, 3, 4, \quad \Phi_1(\infty, \theta) = 1, \quad \Phi_2(\infty, \theta) = 0$$  \hspace{1cm} (21)

In order to obtain the right symmetry for the solutions, we set on the $z$-axis as well as on the $\rho$-axis ($\theta = 0$ and $\theta = \pi/2$, respectively):

$$H_1 = H_3 = \Phi_2 = 0$$  \hspace{1cm} (22)

and

$$\partial_\theta f = \partial_\theta m = \partial_\theta l = \partial_\theta H_2 = \partial_\theta H_4 = \partial_\theta \Phi_1 = \partial_\theta \psi = 0$$  \hspace{1cm} (23)

III. NUMERICAL RESULTS

We have solved the set of partial differential equations numerically. First, we studied the behaviour of the solutions for fixed $\omega$, $\beta$ and increasing gravitational coupling $\alpha$. In FIG. 1, we show the energy per winding number $E/n$ for $\beta = 0$, $n = 2$ and three different values of $\omega$. $\omega = \infty$ corresponds to the pure Einstein gravity limit, studied in [15] and for $\alpha = 0$, the BPS multimonopoles are recovered with energy per winding number (in our rescaled variables) equal to unity. For $\omega < \infty$ and $\alpha = 0$, the energy per winding number is that of the BPS multimonopoles but rescaled by the prefactor $(2\omega + 3)/(2\omega + 4)$ of the matter Lagrangian $L_m$ (see (2)). The reason for this is that for $\alpha = 0$, the Brans-Dicke function and the metric functions are trivial $\psi \equiv 0$ and $f = m = l \equiv 1$, respectively. Thus the matter Lagrangian $L_m$, which is proportional to the energy density, is that of the flat space Yang-Mills-Higgs multimonopoles. This reasoning is, of course, also true for $\beta \neq 0$. Thus:

$$E(\alpha = 0, n, \omega, \beta) = \frac{2\omega + 3}{2\omega + 4} E(\alpha = 0, n, \omega = \infty, \beta).$$  \hspace{1cm} (24)

For $\alpha \rightarrow \alpha_{\text{max}}$ the branch of Brans-Dicke multimonopoles bifurcates with the branch of extremal Reissner-Nordström (RN) solutions. The extremal RN solution in the model studied here is given by:
\[ f(x) = \left( \frac{x}{x + \alpha n} \right)^2, \quad \hat{\alpha} = \alpha \sqrt{\frac{2\omega + 3}{2\omega + 4}} \]  
\[ (25) \]

and

\[ H_i(x) = \Phi_2(x) = 0, \quad i = 1, 2, 3, 4, \quad \Phi_1(x) = 1, \quad \psi(x) = 0 \]  
\[ (26) \]

Note that the horizon of the extremal RN solution in isotropic coordinates is located at \( x = x_h = 0 \). The functions of the limiting solution thus correspond to those of the RN solution on the full interval \( x \in [0 : \infty] \). The solution (25) and (26) has magnetic charge \( P \) and energy \( E(n) \) depending on the Brans-Dicke parameter \( \omega \):

\[ P = n\alpha \sqrt{\frac{2\omega + 3}{2\omega + 4}}, \quad E(n) = P/\alpha^2 \]  
\[ (27) \]

In FIG. 1, we demonstrate that for all chosen values of \( \omega \), the branch of globally regular monopole solutions bifurcates with the corresponding branch of extremal RN solutions. Moreover, we observe that \( \alpha_{\text{max}}(\omega) \), the maximal value of \( \alpha \) for which globally regular monopole solutions exist, is increasing drastically for decreasing \( \omega \) (see also FIG. 3). We find, e.g. for \( \beta = 0, n = 2 \):

\[ \alpha_{\text{max}}(\infty) \approx 1.49, \quad \alpha_{\text{max}}(0) \approx 1.89, \quad \alpha_{\text{max}}(-1) \approx 2.80 \]  
\[ (28) \]

This agrees with the results in [20], where it was found that for the \( n = 1 \) solutions the ratio \( \alpha_{\text{max}}(0)/\alpha_{\text{max}}(\infty) \approx 1.3 \).

In FIG. 2, we demonstrate the behaviour of the metric function \( f \) in the limit \( \alpha \to \alpha_{\text{max}} \) for \( \omega = 0, n = 2 \) and \( \beta = 0 \). \( f \) is shown as function of the compactified coordinate \( z = x/(1 + x) \) [20]. For increasing \( \alpha \), the value of the metric function at the origin, \( f(0) \), decreases to the RN value \( f_{\text{RN}}(0) = 0 \). Moreover, the angle dependence diminishes indicating that the limiting solution is the spherically symmetric RN solution. For \( \alpha \to \alpha_{\text{max}} \approx 1.89 \), \( f \) approaches the function given by (25).

For \( \omega = \infty \) and \( \alpha = 0 \), like-charged monopoles are either non-interacting (in the limit of vanishing Higgs boson mass) or repelling. When gravity comes into play, it is possible for the multimonopoles to reside in an attractive phase [15]. In the BPS limit (\( \beta = 0 \)), this is not surprising, for \( \beta \neq 0 \) however it is apparent that for small values of \( \beta < \tilde{\beta}(n) \) gravity is able to overcome the repulsion of the long-range magnetic field. In [14] it was found that \( \tilde{\beta}(n = 2) \approx 0.21 \). Nevertheless, the extension of the attractive phase is limited by the fact that the solutions exist only up to an \( n \)- and \( \beta \)- dependent maximal value of the gravitational coupling \( \alpha \). We find that like Einstein gravity Brans-Dicke gravity is able to overcome the repulsion between like-charged monopoles for sufficiently high values of the gravitational coupling \( \alpha \). In FIG. 3, we show \( \alpha_{\text{eq}}(\beta, \omega) \), the value of \( \alpha \) for which the mass of the \( n = 1 \) monopole and the mass per winding number of the \( n = 2 \) multimonopole equal one another, as function of \( \omega \) for two different values of \( \beta \). For both values of \( \beta = 0.1 \) and \( \beta = 0.2 \), \( \alpha_{\text{eq}} \) is increasing for decreasing \( \omega \) from its value at \( \omega = \infty \). Equally, the value \( \alpha_{\text{max}} \) is increasing with decreasing \( \omega \). The multimonopoles reside in an attractive phase in the parameter space above the \( \alpha_{\text{eq}} \)-curve and below the corresponding \( \alpha_{\text{max}} \)-curve. Thus in analogy to the \( \omega = \infty \) limit Brans-Dicke gravity is able to overcome the repulsion of the long-range U(1) field for
\[ \beta = 0.1 \text{ and } \beta = 0.2 \text{ and sufficiently high values of } \alpha. \text{ Since } \alpha_{\text{max}} \text{ is increasing drastically for small } \omega, \text{ the extent of the attractive phase for small } \omega \text{ is much bigger than in the } \omega = \infty \text{ limit. However, we find that the order of magnitude of the value } \Delta = E(n = 1) - E(n)/n, \text{ which is an indicator for the strength of attraction between like-charged monopoles doesn’t change significantly within the whole domain of attraction in the } \alpha-\omega \text{-plane. This can be seen from the table below where we show } \Delta(\omega) = E(n = 1, \omega) - E(n = 2, \omega)/2 \text{ for } \alpha_{\text{max}}(n = 2, \omega)/k, \ k = 2, 3, 4:\]

| \(k\) | 2    | 3    | 4    |
|------|------|------|------|
| \(\Delta(\omega = \infty)\) | 0.0051 | 0.0024 | 0.0022 |
| \(\Delta(\omega = -1)\)     | 0.0052 | 0.0033 | 0.0018 |

In FIG. 4, we show the difference between the energy of the \(n = 1\) monopole and the \(n\) multimonopoles, \(\Delta = E(n = 1) - E(n)/n\), as function of \(\omega\) for \(\alpha = 1, \beta = 0\) and \(n = 2, 3, 4\), respectively. \(\Delta\) stays nearly constant for a large range of \(\omega\) and decreases for decreasing \(\omega\). For \(\omega \to \omega_0 = -3/2\), \(\Delta\) decreases to zero which is due to the fact that the prefactor \((2\omega + 3)/(2\omega + 4)\) is equal to zero for \(\omega = -3/2\) and thus the mass of the solutions itself vanishes. It can be clearly deduced from FIG. 4 that the strength of attraction between like-charged monopoles is smaller in Brans-Dicke theory than in pure tensor gravity theory. This can be compared with the results in [27] and [28] for two different Einstein-Yang-Mills-Higgs-dilaton (EYMHD) models. EYMHD theory - like Brans-Dicke theory - constitutes a theory of gravity in which the metric tensor has a scalar companion, in the case of EYMHD, the dilaton. While in [27] the standard coupling of the dilaton in 4 dimensions was studied, the model studied in [28] arose from dimensional reduction of an Einstein-Yang-Mills system in (4 + 1) dimensions [29]. In both models the dependence of the strength of attraction on the dilaton coupling was studied. It was found that the value \(\Delta\) was increasing for increasing dilaton coupling with \(\alpha\) and the dilaton coupling not too close to their maximal values. Since in the model studied here, \(\sqrt{8\pi G\gamma(\omega)}\) (see (3)) can be interpreted as the analog of a dilaton coupling and is increasing for decreasing \(\omega\) and fixed \(G\) (i.e. fixed \(\alpha\)), the strength of attraction is decreasing for increasing ”dilaton” coupling.

Apparently, the strength of attraction is increasing for increasing \(n\) which suggests that large clumps of monopoles might be possible. However, FIG. 4 also gives a hint that monopoles in such clumps will be more bound in pure Einstein gravity \((\omega = \infty)\) than in Brans-Dicke gravity for equal gravitational coupling \(\alpha\).

**IV. CONCLUSIONS AND SUMMARY**

We have studied axially symmetric multimonopoles with \(n > 1\) in Brans-Dicke gravity. We find that for a maximal value of the gravitational coupling \(\alpha\) these solutions collapse to form an abelian black hole which outside the horizon is described by an extremal Reissner-Nordström (RN) solution with trivial Brans-Dicke scalar field. The RN solution has mass and magnetic charge depending on the Brans-Dicke parameter \(\omega\). This behaviour can be compared to that in Einstein-Yang-Mills-Higgs-dilaton (EYMHD) theory [23,27]. When
either the gravitational coupling or the dilaton coupling reaches its maximal value, the branch of monopole solutions bifurcates with the branch of extremal Einstein-Maxwell-dilaton (EMD) solutions. These have a naked singularity and a non-trivial scalar dilaton field.

In analogy to the \( n = 1 \) solutions, the maximal value of the gravitational coupling up to where the globally regular monopole solutions exist, is increasing for decreasing Brans-Dicke parameter. Equally, \( \alpha_{eq} \), the value of \( \alpha \) where the mass of the \( n = 1 \) monopole and the energy per winding number of the \( n = 2 \) multimonopole equal one another, is increasing for decreasing \( \omega \). At the same time, \( \alpha_{max} \), the maximal value of the gravitational coupling \( \alpha \) up to where globally regular multimonopole solutions exist, is increasing. Remarkable is that the rate of increase for decreasing \( \omega \) is much bigger for \( \alpha_{max} \) than for \( \alpha_{eq} \). Thus, the extension of the attractive phase is much bigger for small \( \omega \) than for \( \omega = \infty \). We conclude that at comparable values of the gravitational coupling \( \alpha \), the Brans-Dicke monopoles are less bound than the pure Einstein gravity monopoles, and though the former can exist to much bigger values of the gravitational coupling than the latter, the strength of attraction is of the same order of magnitude for all values of \( \alpha \) and \( \omega \) for which bound multimonopoles exist. These phenomena can be explained by noticing that in the gravitational field equations always the combination \( \alpha_{eff} := \alpha \sqrt{\frac{2\omega + 3}{2\omega + 4}} \) appears. This can be interpreted as an "effective" gravitational constant. Clearly, for \( \alpha \) fixed and \( \omega \) decreasing from infinity, this expression is decreasing. This explains why the value \( \alpha_{max} \) up to where the Brans-Dicke multimonopoles exist is increasing for \( \omega \) decreasing and also why at fixed \( \alpha \), the strength of attraction is decreasing for decreasing \( \omega \). Only at comparable values of \( \alpha \) for different \( \omega \) (e.g. at the \( k \)-th part of \( \alpha_{max}(\omega) \)) is the strength of attraction of the same order of magnitude in Einstein theory and Brans-Dicke theory, respectively.

In [20] the corresponding spherically symmetric black hole solutions were studied with emphasis on their thermodynamic properties. Axially symmetric black hole solutions in Einstein-Yang-Mills-Higgs theory have been studied recently [30] in the context of the so-called "Isolated horizon framework" [31]. It would be interesting to analyse in which sense the corresponding black hole solutions of the model studied in this paper fulfill the predictions of this framework.

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FIG. 1. The energy per winding number $E/n$ of the axially symmetric $n = 2$ Brans-Dicke monopole (solid) is shown for $\beta = 0$, $\omega = \infty$, 0 and $-1$ as function of $\alpha$. Also shown is the energy per winding number of the corresponding Reissner-Nordström (RN) solution (dotted).
FIG. 2. The metric function $f$ is shown as function of the compactified coordinate $z = x/(1+x)$ for $n = 2$, $\omega = 0$, $\beta = 0$ and several values of $\alpha$ including $\alpha = \alpha_{\text{max}} \approx 1.89$. 
FIG. 3. The value $\alpha_{eq}$, where the mass per winding number of the $n = 2$ multimonopole and the $n = 1$ monopole equal one another is shown for $\beta = 0.1$ and $\beta = 0.2$ as function of $\omega - \omega_0$, $\omega_0 = -\frac{3}{2}$. Also shown is $\alpha_{max}$ for $n = 2$ and the same values of $\beta$. Note that the values of $\alpha_{max}$ increase drastically with decreasing $\omega$, e.g. $\alpha_{max}(\beta = 0.1, \omega = -1) \approx 2.7$ and $\alpha_{max}(\beta = 0.2, \omega = -1) \approx 2.6$. 
FIG. 4. The difference $\Delta = E(n = 1) - E(n)/n$ between the energy per winding number of the $n = 1$ monopole and the multimonopoles with winding number $n$ is shown as function of $\omega - \omega_0$, $\omega_0 = -\frac{3}{2}$, for $\alpha = 1.0$, $\beta = 0$ and $n = 2, 3, 4$, respectively.