Winds and radiation in unison: a new semi-analytic feedback model for cloud dissolution

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ABSTRACT

Star clusters interact with the interstellar medium (ISM) in various ways, most importantly in the destruction of molecular star-forming clouds, resulting in inefficient star formation on galactic scales. On cloud scales, ionizing radiation creates H\textsc{ii} regions, while stellar winds and supernovae drive the ISM into thin shells. These shells are accelerated by the combined effect of winds, radiation pressure and supernova explosions, and slowed down by gravity. Since radiative and mechanical feedback is highly interconnected, they must be taken into account in a self-consistent and combined manner, including the coupling of radiation and matter. We present a new semi-analytic one-dimensional feedback model for isolated massive clouds (\(\geq 10^5 M_\odot\)) to calculate shell dynamics and shell structure simultaneously. It allows us to scan a large range of physical parameters (gas density, star formation efficiency, metallicity) and to estimate escape fractions of ionizing radiation \(f_{esc,i}\), the minimum star formation efficiency \(\epsilon_{min}\) required to drive an outflow, and recollapse time scales for clouds that are not destroyed by feedback. Our results show that there is no simple answer to the question of what dominates cloud dynamics, and that each feedback process significantly influences the efficiency of the others. We find that variations in natal cloud density can very easily explain differences between dense-bound and diffuse-open star clusters. We also predict, as a consequence of feedback, a 4 – 6 Myr age difference for massive clusters with multiple generations.

Key words: radiation: dynamics – galaxies: star formation – H\textsc{ii} regions – ISM: clouds – ISM: bubbles – ISM: kinematics and dynamics

1 INTRODUCTION

The formation of stars from the cold, dense interstellar medium (ISM) marks the onset of the conversion of nuclear binding energy into radiative and mechanical energy. Injected back into the immediate surroundings of the stars, this energy drives a rapid chemical and dynamic evolution of the very molecular cloud from which the stars formed. This chain of events, where the creation of stars leads to energy injection by stars which disrupt the clouds, is known as stellar feedback. In the case of massive stellar clusters (\(M_* > 10^3 M_\odot\)), the energetic processes are dominated by three main forms of feedback: ultraviolet radiation, colliding stellar winds, and supernovae (SNe). Each of these processes provides a source of energy and momentum that acts in opposition to gravity (for a review about stellar feedback, see [Krumholz et al. 2014]).

Around young massive clusters, confined interacting winds produce hot (\(T \sim 10^5 – 10^6\) K) bubbles ([Weaver et al. 1977] hereafter W77; [Dunne et al. 2003]). These adiabatically expand, compressing the gas ahead of them into a thin dense shell. The bubbles are characterized by a rarefied, collisionally ionized gas. While this gas remains hot, its high thermal pressure drives the expansion of the surrounding shell (W77). Once the gas cools, however, the winds from the central cluster push the remainder of the gas from the bubble into the shell. Thereafter, the wind momentum is deposited directly into the shell in the form of ram pressure. Supernovae exploding within the bubble add their energy to the existing thermal and mechanical energy of the gas in the bubble.
The optical depth of the gas inside a wind bubble is very low, and so radiation from the central stellar cluster easily reaches the dense shell surrounding the bubble (Townsend et al. 2003; Gupta et al. 2016). Ultraviolet photons with energies $E > 13.6$ eV photoionize hydrogen in this shell, resulting in one of two outcomes: either the entire shell becomes ionized, or only the inner layers become so, with the outer layers of the shell remaining neutral (e.g. Martinez-Gonzalez et al. 2014).

Photons that are absorbed in the shell not only heat it and potentially change its chemical state, but also deposit momentum (Lebedew 1901). Essentially, the radiation exerts a pressure force on the gas and dust that acts radially outwards from the central stellar cluster. If this radiation pressure is sufficiently large, then it can become dynamically significant and can play a major role in driving the evolution of the shell (Mathews 1967; Draine 2011; Kim et al. 2016). One of the key factors that determines whether or not radiation pressure becomes significant is the efficiency with which radiation couples with the shell (Krumholz & Matzner 2009). For ionizing radiation, this is determined by the amount of neutral and molecular material as well as dust absorbing the radiation. When the column density of the gas is high enough to absorb all the ionizing photons (i.e. when the layer is optically thick to ionizing radiation), the system is ”radiation bounded”, coupling is efficient and momentum is transferred effectively. However, the shells surrounding many observed star-forming regions are optically thin to ionizing radiation, suggesting that coupling is not always effective (Pellegrini et al. 2012; Seon 2009). For non-ionizing radiation ($E < 13.6$ eV), the optical depth is again a major factor determining whether or not coupling is efficient, but in this case the dominant source of opacity is provided by dust unless the radiation field is weak (Krumholz et al. 2008).

Previous simplified models of the growth of shells and bubbles around young massive clusters have typically assumed that the dynamics of the shell are dominated by the effect of winds (e.g. W77; Chevalier & Clegg 1985; Mac Low & McCray 1988; Koo & McKee 1992; Canto et al. 2000; Sillich & Tenorio-Tagle 2013, hereafter ST13) or radiation pressure (e.g. Krumholz & Matzner 2009; Murray et al. 2010; Kim et al. 2016). However, as we will see later, in the general case both must be included in order for the model to be self-consistent and hence both processes are important. In addition, in the treatments that do account for radiation pressure, the shell is often assumed to be completely opaque to radiation (Krumholz & Matzner 2009; Murray et al. 2010), whereas in reality the escape fraction can often be significant (see Section 5).

In this paper, we present a new model for the growth of shells around clusters that properly accounts for both winds and radiation, and that carefully treats the structure of the shell and its influence on the fraction of the radiation that is absorbed. In Section 2, we present our model for the structure and dynamics of the shell, and in Section 3, we discuss the evolution of an exemplary cloud and compare to analytic solutions. In Section 4, we examine how well coupled radiation is to the shell and use those results in Section 5 to explore the conditions in which each of the different feedback processes (winds, SNe and radiation) dominates, examining this both as a function of time during the expansion, and in an integrated form over the entire lifetime of the cloud. Our model also allows us to make predictions for the evolution of the escape fraction of ionizing radiation during the growth of the shell, which we present in Section 6. In Section 7, we discuss what we can learn from our model about the star formation efficiency $\epsilon$ of the cloud, and how this varies as a function of the mass, mean density and metallicity of the cloud. We conclude in Section 8 with a summary of the key results of our study.

## 2 MODEL

For our model we consider a spherical cloud with a constant density $\rho_c$. We assume the ISM of the cloud has a standard chemical composition of 1 He atom per 10 H atoms; thus the mean mass per nucleus $\mu = (14/11) m_H$ and the mean mass per particle $\mu_p = (14/23) m_H$, where $m_H$ is the proton mass. The cloud’s radius is given by

$$R_c = 19.7 \, \text{pc} \times \left( \frac{M_c/10^5 M_\odot}{n_{HI}/10^3 \text{cm}^{-3}} \right)^{1/3}, \tag{1}$$

where $M_c$ is the cloud mass, and $n_{HI} = \rho_c/\mu_p$ is the number density of atoms/ions in the cloud. At $t = 0$ a star cluster of mass $M_*$ forms at the cloud’s center. It injects feedback into the surrounding ISM in the form of stellar winds, radiation and eventually supernova explosions. As outlined in the Introduction, the combined effects of radiation and winds from a massive cluster will create an expanding bubble of tenuous and hot ionized gas which is surrounded by a much denser and colder shell of swept-up cloud material. In order to calculate the resulting expansion speed, or – if gravity starts to dominate at some stage – to compute the corresponding contraction velocity, we need to have a detailed understanding of the strength of the different forces acting on the shell. For this we need to take into account the aging population of the star cluster, the morphological and kinematical structure of the bubble and the shell, and their chemical composition. In this Section, we first outline our physical model for the shell dynamics, then discuss the structure of the dense shell, and finally introduce our scheme to couple both together.

### 2.1 Shell Dynamics

We model three phases of expansion of the natal cloud around the cluster. Early expansion is adiabatic and dominated by wind energy which sweeps the cloud interior into a thin shell (Phase I). This phase last so long as the energy is confined and radiative losses are small. After that, shell acceleration is determined by momentum input by winds, radiation and eventually by SN explosions opposing gravity (Phase II & III). In Phase II the expanding shell continues to sweep-up material. Once the whole cloud has been swept up, the shell can freely expand into the ambient ISM (Phase III). These phases are outlined in Figure 1 and are now discussed in more detail. Since we only model isolated clouds we do not take into account any effects of an external galactic potential like shearing, which would introduce differential rotation and tidal torques, or the coupling to the larger-scale turbulent flows in the ISM.
2.1.1 Phase I: Energy-dominated winds

Initially, radiation with $E > 13.6\text{ eV}$ creates a large ionized region around the cluster (the so-called Strömgren sphere). At the same time, winds from the star cluster expand freely into the ISM. Due to its very short duration, however, this initial phase can be neglected (Lamers & Cassinelli 1999). Soon, several distinct zones form around the cluster (W77): An inner free wind zone is surrounded by a hot shocked wind region. Together they make up the wind bubble (red region in Figure 1) which works against a dense shell consisting of swept-up material. Since the density in the shell is higher than in the cloud, the recombination rate increases and the ionization front travels inwards until it lies inside the shell. The shocked wind material reaches temperatures of $10^6 - 10^8\text{ K}$ causing a fast, adiabatic expansion. During this phase we can ignore the effect of gravity and radiation pressure as they are second order effects. If the shell runs into ISM of a constant density, the equation of motion in the thin shell limit according to Bisnovatyi-Kogan & Silich (1995) is

$$\frac{d^2}{dR^2} \left( R^3 \frac{dR}{dt} \right) + (3\gamma - 2) \frac{R}{R} \frac{d}{dt} \left( R^3 \frac{dR}{dt} \right) = \frac{9(\gamma - 1) L_w}{4\pi \rho_c R}. \quad (2)$$

Here, $R$ is the (inner) radius of the shell and $\gamma$ is the adiabatic index, with $\gamma = 5/3$ for an ideal gas. If the mechanical luminosity of the winds $L_w$ is a constant, eq. (2) can be solved analytically, yielding $R \propto t^{3/5}$ (Avedisova 1972, Castor et al. 1975, hereafter C75, and W77). However, stellar evolution models (e.g. Leitherer et al. 2014) show that $L_w$ is time dependent, especially in the Wolf-Rayet and pre-SN phases and we will thus use eq. (2) instead of the analytic solution for constant $L_w$. From Bisnovatyi-Kogan & Silich (1995), during the adiabatic phase of the shell expansion the...
Evaporative flows from the shell gradually increase the density in the shocked wind region, leading to strong radiative cooling. When radiative losses become comparable to the wind energy input, the bubble loses the driving pressure of the hot gas and the adiabatic phase ends. The cooling time $t_{\text{cool}}$ of a hot wind bubble is given by (Mac Low & McCray 1988) as

$$t_{\text{cool}} = 16 \text{Myr} \times (Z/Z_\odot)^{-35/22} n_{\text{cl}}^{-8/11} L_{38}^{3/11},$$

where $Z$ is the metallicity, $n_{\text{cl}}$ is given in cm$^{-3}$ and $L_{38} = L_w/(10^{38} \text{erg s}^{-1})$.

Alternatively, as the shell expands, inhomogeneities or asymmetries in the cloud may provide low density pathways along which the hot gas can escape (Rogers & Pittard 2013). If this occurs, instead of expanding and doing work, the hot gas will escape into the low density/pressure ambient ISM. However, we argue that in a rather high density environment as investigated in this paper, and given the resulting rapid expansion in the adiabatic phase it is reasonable to assume the bubble does not “burst” until the expansion is of the order of the initial cloud radius. At this time, $t_{\text{sweep}}$, the entire cloud has been swept up in the shell. Further expansion begins to stretch the shell without significantly adding to its mass. The shell’s average density begins to decrease, possibly becoming unstable and leading to the formation of channels. Modeling the formation of low density channels goes beyond the scope of a 1D model. For simplicity, we assume that before $t_{\text{sweep}}$, the formation of any leaks gets hampered. After $t_{\text{sweep}}$, we assume the remaining shell structure is coherent, but does not effectively confine the winds. The time, when Phase I transitions to the next phase is thus given by $t_{\text{tran}} = \min(t_{\text{cool}}, t_{\text{sweep}})$.

### 2.1.2 Phase II: Momentum-dominated sweeping

Once the hot X-ray emitting gas in the bubble cools, causing its thermal pressure to drop dramatically, the reverse shock quickly moves towards the shell as the shocked wind region is pushed into the shell (ST13). This evacuates almost all of the remaining gas from the bubble (now represented by the blue region in Figure 1), and therefore during Phase II and Phase III, it is a good approximation to treat the bubble as if it were completely empty. This allows us to assume that the wind thereafter imparts its momentum directly on the shell and that no absorption of radiation occurs before the radiation reaches the shell. In reality, the transition between energy-driving in Phase I and momentum-driving in Phase II will be gradual and even at $t > t_{\text{tran}}$, some thermal pressure from the shocked wind material will be present. However, the remaining hot gas is dynamically weak (Gupta et al. 2016; Rahner et al., in prep.) and we will ignore it here.

Following the evacuation of the bubble, the further expansion of the shell is driven by a combination of radiation pressure and ram pressure from winds and – at later times – SNe, all of which act to oppose gravity. If the hot gas cooled before the cloud was swept up, the shell continues to expand into high density ISM so that the mass of the shell grows as

$$M_{sh} = (4\pi/3)R^3_0 \rho_{sh}$$

(as in Phase I). During this phase, the shell’s equation of motion is

$$\frac{d}{dt} \left( M_{sh} \dot{R} \right) = F_{\text{ram}} + F_{\text{rad}} - F_{\text{grav}},$$

where $F_{\text{ram}}$, $F_{\text{rad}}$, and $F_{\text{grav}}$ are the forces corresponding to ram pressure from stellar winds and type II SNe, radiation pressure, and gravity, respectively. Since we assume that the bubble is efficiently evacuated by feedback from the cluster, its density is too low to exert any significant amount of thermal pressure on the shell. Also, Dale et al. (2012) have shown that massive clouds are largely unaffected by thermal pressure from ionizing radiation. In our model we hence assume that thermal pressure from the bubble is negligible for the dynamics of the shell (thermal pressure does however influence the shell structure, as described in Section 5.2). We note that this argument does not apply for low-mass systems where thermal pressure from H II regions plays a significant role in driving outflows (e.g. Walch et al. 2012; Dale et al. 2012).

The star clusters investigated in this work are large enough that as soon as the first SNe occur, treating them as a continuum process rather than distinct explosions is a good approximation. The ram pressure force term is then

$$F_{\text{ram}} = M_w v_w + M_{\text{SN,SN}} v_{\text{SN}}.$$  

Here $M_w$ and $M_{\text{SN}}$ are the mass loss rates due to stellar winds and SNe and $v_w$ and $v_{\text{SN}}$ are the terminal velocities of the winds and SN ejecta. The ram pressure at the edge of the bubble is then

$$P_b = \frac{F_{\text{ram}}}{4\pi R^2}.$$  

The full amount of the ram pressure is always transmitted to the shell. However, the shell does not absorb all photons emitted by the cluster. Consequently, it will feel only a fraction $f_{\text{abs}}$ of the maximum radiation pressure that the photons from the stellar cluster can potentially exert (c.f. Section 2.2). Additionally, radiation absorbed by dust grains is re-emitted isotropically in the infrared (IR) which leads to an enhancement of radiation pressure. The total force due to radiation pressure is thus given by a direct and an indirect term,

$$F_{\text{rad}} = F_{\text{direct}} + F_{\text{indirect}} 
\approx f_{\text{abs}} \frac{L_{\text{bol}}}{c} (1 + \tau_{\text{IR}}),$$

where $L_{\text{bol}}$ is the bolometric luminosity of the star cluster and $c$ is the speed of light. The quantity $f_{\text{abs}} (1 + \tau_{\text{IR}})$ is sometimes referred to as the trapping factor (e.g. Krumholz & Matzner 2009). The optical depth of the shell in the IR is given by

$$\tau_{\text{IR}} = \int \frac{R_{\text{out}}}{R} \rho_n n_{sh} \, dR,$$

where $\kappa_{\text{IR}}$ is the Rosseland mean dust opacity, $n_{sh}$ is the number density of atoms/ions in the shell, and $R_{\text{out}}$ is the shell’s outer radius. For simplicity we do not relate $\kappa_{\text{IR}}$ to the dust temperature but use a constant $\kappa_{\text{IR}} = 4 \text{ cm}^2 \text{ g}^{-1}$ as
would be appropriate for M17. For more details about the M17 model, see [Pellegrini et al. (2007)](P07).

In our treatment of gravity we consider both gravity between the cluster and the shell and the self-gravity of the shell. Thus,

\[ F_{\text{grav}} = \frac{GM_{\text{sh}}}{R^2} \left( M_\star + \frac{M_{\text{sh}}}{2} \right), \]  

(10)

where \( G \) is the gravitational constant. We do not, however, consider any gravitational collapse by the parts of the cloud that have not yet been incorporated into the shell as we assume the cloud is in virial equilibrium.

### 2.1.3 Phase III: Free expansion into low-density ISM or recollapse

If feedback is strong enough, the shell eventually overtakes the initial cloud radius \( R_\text{cl} \). The shell then expands into the low-density ambient ISM. It is assumed to become leaky at \( t_\text{sweep} \) so that any shocked, hot wind material cools after \( t_\text{sweep} \) at the latest. Thus, if \( t_\text{sweep} < t_\text{cool} \), Phase III follows directly after Phase I (indicated by the dashed white arrow in Figure 1).

Here, we take the ambient ISM to have a mass density \( \rho_{\text{ISM}} = 1.67 \times 10^{-25} \text{ g cm}^{-3} \), corresponding to a number density of \( \sim 0.1 \text{ cm}^{-3} \). The equation of motion is still given by eq. [5] but the mass of the shell is now

\[ M_{\text{sh}} = M_\text{cl} + \frac{\rho_{\text{ISM}}}{3} (R^3 - R_\text{cl}^3) \rho_{\text{ISM}}. \]  

(11)

We also ran tests with \( \rho_{\text{ISM}} = 1.67 \times 10^{-24} \text{ g cm}^{-3} \) and found that this leads to somewhat slower expansions but overall the effect is small.

There are two options now. If feedback is strong enough the shell will expand to very large radii. As the shell expands, it thins out, its density drops and it eventually becomes indistinguishable from the diffuse ambient ISM. Even before this, we can no longer represent the shell using the thin shell limit, and so eq. [5] does not adequately describe its dynamics any longer. To account for this, we stop the integration if the density of the densest part of the shell drops below \( 1 \text{ cm}^{-3} \) for an extended period of time (more than 1 Myr) as we consider the shell dissolved. If we would immediately stop, we might miss the reformation of a shell, e.g. during the Wolf-Rayet phase which drastically increases the wind momentum deposition rate, we need to determine the fraction of absorbed radiation \( f_{\text{abs}} \). Numerical codes like CLOUDY [Ferland et al. (2013)] provide powerful tools for calculating the chemistry, density, and temperature structure of shells. However, here we choose a simpler set of equations which sacrifice a detailed treatment of the chemical and thermal structure of the shell in exchange for a great increase in the speed with which one can calculate the volume of ionized gas. Our simple approach here also makes it easier to assess the relative importance of the different forms of feedback responsible for driving the dynamical evolution of the shell.

During Phase I, dust inside the hot bubble is destroyed by sputtering and hydrogen is collisionally ionized, allowing radiation to pass through unattenuated. During Phases II and III, the density inside the bubble quickly drops below \( 1 \text{ cm}^{-3} \) (see Section 2.1), so that only little attenuation of radiation occurs. Thus, ionizing photons from the cluster can reach and ionize at least the inner edge of the shell (and potentially the whole of the shell, as we explain below).

Beyond the wind bubble, the momentum carried by radiation has a pronounced effect on the density structure of the ISM. Our model assumes the ionized and neutral/molecular phases of the shell are in the quasi-hydrostatic equilibrium described by the equation of state outlined in [P07] (hereafter the P07-EOS). The work by [P07] was the first to validate a hydrostatic equation of state by reproducing an observed \( \text{H}\text{\textsc{i}} / \text{H}/\text{H}_2 \) star-forming ISM interface. The final pressure law defining a hydrostatic shell subject to external radiation states that the total pressure \( P_{\text{tot}} \) at a radius \( r > R \), measured from the star cluster to a point in the shell, equals the sum of the pressure \( P_\text{in} \) at the inner boundary of the shell and a term arising from radiative acceleration \( a_\text{rad} \) from photons deposited in the shell:

\[ P_\text{in}(R) + \int_R^r a_\text{rad} \rho_\text{sh} \, dr' = P_{\text{tot}}(r) \]

\[ = P_{\text{therm}} + P_{\text{turb}} + P_{\text{mag}}. \]

(12)

Here, \( \rho_{\text{sh}} \) is the density of the shell and \( P_{\text{therm}}, P_{\text{turb}}, \) and \( P_{\text{mag}} \) are the thermal pressure, the turbulent pressure, and the magnetic pressure in the shell, respectively.

It is important to understand that a hydrostatic shell is not at constant pressure when exposed to a radiation field. By definition, the condition of hydrostatic equilibrium implies that there is no differential acceleration within the shell. In a hydrostatic shell, at any interior point the net external force (excluding gravity) acting on a layer with thickness \( dr \) is proportional to the amount of stellar radiation absorbed. Since absorption by each previous layer reduces the transmitted flux of ionizing and non-ionizing UV flux, if we want the amount of radiation per unit mass absorbed in each layer (and hence the amount of momentum deposited per unit mass) to remain constant, then the optical depth \( \tau \) of each layer must progressively increase. In ionized gas, this means increasing the gas density of the layer. However,

\[ \text{Note that this assumes that the turbulence is dominated by motions on scales that are small compared to the shell thickness.} \]
if we increase the density of the layer, we also increase its mass, and hence require an even higher momentum deposition rate in order to keep it accelerating at the same rate as the previous layer. This implies that the density of the layer must increase even more, in order to provide the necessary increase in $\tau$. In shells with an outward density gradient due to radiation pressure, a monotonically increasing total pressure is required to produce uniform acceleration.

The terms in the P07-EOS have been validated against the density, chemical and velocity structures of observed multi-phase shells. A very strong magnetic field could provide additional pressure support even in the ionized gas, lowering the gas densities and recombination rate. Following P07, we can estimate the potential importance of the magnetic field by examining the peak magnetic field

$$B = \sqrt{8\pi P_b + \frac{2Q_i h\nu}{R_i^2 c}},$$  \hspace{1cm} (13)

where $Q_i$ is the rate at which ionizing photons are emitted by the central source, $h\nu$ is the average energy of a stellar photon, and $R_i$ is the radius of the ionization front.

We have computed the peak magnetic pressure predicted by this equation for the clusters and gas densities modeled here and find that magnetic pressure is only marginally significant in the ionized gas while $R_i \lesssim R_{cl}$. At larger radii and/or late times when the winds are momentum-driven, magnetic pressure is much smaller than the radiation pressure, and decreases in significance as the shell evolves. The magnetic field may still provide a dominant source of pressure in the atomic gas, but the momentum deposited there is proportional to the dust column only (cf. eq. [20]), and is therefore not affected by the structure of the atomic gas layer. Thus, in our calculations we ignore the effect of magnetic fields.

We also neglect the effects of turbulence, which is unlikely to be important in the ionized gas, unless the turbulence velocity dispersion is large ($\sigma_{\text{rms}} \gg 10 \text{ km s}^{-1}$ in the ionized shell). However, in star forming regions like Orion, the turbulent velocities in the III-region are clearly subsonic (Arthur et al., 2016) and turbulence is thus of limited importance for determining the structure of the ionized shell. In the atomic gas turbulence may play an important role in structuring the material but since, as mentioned above, the absorbed fraction of radiation depends only on the column density, turbulence does not play a significant role in the overall dynamics of the shell.

Detailed studies of observations find that the inner edge of the shell and the wind bubble are in pressure equilibrium (see e.g. P07). In this case, $P_0(R) = P_b$. Neglecting magnetic and turbulent pressure, the number density of the atomic nuclei $n_{\text{sh}}$ at the inner radius of the shell $R$ must then satisfy

$$n_{\text{sh}}(R) = \frac{\mu_p}{\mu_{\text{sh}}} P_b,$$  \hspace{1cm} (14)

where $k$ is the Boltzmann constant and $T_i$ is the temperature of the inner (ionized) region of the shell. The pressure of the bubble $P_b$ is given by eq. ([3]) during Phase I and by eq. ([7]) during Phases II and III. Note that pressure equilibrium implies that the shell is expanding at the same rate as the bubble.

For simplicity, we also assume that the ionized gas is at a constant temperature of $T_i = 10^4$ K. Under these assumptions, the condition of hydrostatic equilibrium, eq. ([12]), dictates that the gradient in the total pressure be offset by the external forces, in this case the force from radiation pressure, leading to

$$\alpha_{\text{sh}} \rho_{\text{sh}} = \frac{d}{dr} \left( \frac{\mu_p}{\mu_{\text{sh}}} n_{\text{sh}} k T_i \right).$$  \hspace{1cm} (15)

The radiative transfer of eq. ([13]), can be reduced to two energy bands: Ionizing radiation (photons with energies above 13.6 eV) which is absorbed by hydrogen and dust, and non-ionizing radiation which is absorbed by dust only. Recombination is assumed to occur only via case B recombination with a recombination coefficient $\alpha_B = 2.59 \times 10^{-13}$ cm$^3$ s$^{-1}$ (Osterbrock & Ferland, 2006). These simplifying assumptions, and a conversion from acceleration times density to force per volume, give rise to the following set of coupled differential equations for the number density of the shell $n_{\text{sh}}(r)$, the attenuation function for ionizing radiation $\phi(r)$ and the optical depth $\tau_i(r)$ of dust in the shell, which have been applied to dusty II regions by Draine (2011) and to shells by Martínez-González et al. (2014):

$$\frac{d}{dr} \left( \frac{\mu_p}{\mu_{\text{sh}}} n_{\text{sh}} k T_i \right) = \frac{1}{4\pi^2 c} \frac{d}{dr} \left( L_i e^{-\tau_i} + L_\phi \right),$$  \hspace{1cm} (16)

$$\frac{d\phi}{dr} = -\frac{4\pi^2}{Q_i} \alpha_B n_{\text{sh}} - n_{\text{sh}} \sigma_d \phi,$$  \hspace{1cm} (17)

$$\frac{d\tau_i}{dr} = n_{\text{sh}} \sigma_d.$$  \hspace{1cm} (18)

Here, $L_i$ and $L_\phi$ are the luminosities of non-ionizing and ionizing radiation. We assume the dust cross section $\sigma_d$ scales linearly with metallicity, $\sigma_d = \sigma_0 Z/Z_\odot$ where $\sigma_0 = 1.5 \times 10^{-21}$ cm$^2$ (Draine, 2011) and neglect any formation or destruction of dust in the shell. During Phase I, with temperatures of the shocked wind material in excess of $10^6$ K, neglecting dust sublimation is certainly not correct. However, we treat this early phase as being dominated by ram-pressure anyway and ignore radiation pressure on dust altogether. At later times, gas temperatures in the shell reach at most $10^4$ K, at which point the dust-to-gas ratio is not so different from the general ISM (Osterbrock & Ferland, 2006). Destruction of dust is only important close to the illuminated face of the shell and even if dust destruction is taken into account, the majority of ionizing photons will continue to be absorbed by dust (Arthur et al. 2004). The formation of dust is never significant at the densities considered in this paper.

Equations ([16], [17], and [18]) hold at all radii $r < R_i$ within the shell. The radius of the ionization front corresponds to the transition between the ionized and non-ionized parts of the shell and hence marks the point at which the ionizing photon flux drops to zero, i.e. $\phi(R_i) = 0$. Beyond the ionization front we assume the gas is purely atomic with $2$ Photons in the energy range 11.2–13.6 eV can also be absorbed in the Lyman-Werner bands of H$_2$, but this is significant in comparison to dust absorption only when the radiation field is relatively soft (Krumholz et al., 2008).
a temperature of $T_a = 100$ K. At radii $r > R_i$, we then have
\[ \frac{d}{dr} (n_{ah} k T_a) = \frac{1}{4 \pi r^2 c} \frac{d}{dr} \left( L_a e^{-\tau_a} \right), \] (19)

Note that the condition of pressure equilibrium between the ionized and the non-ionized gas leads to a discontinuous increase in $n_{ah}$ by a factor $\mu_a T_i/(\mu_B T_a)$ at $R_i$.

Since the density inside the bubble is assumed to be very low, any absorption inside the bubble is negligible and the boundary conditions used for solving eqs. (16), (17), and (18) are given by eq. (14), $\phi(R) = 1$, and $\tau_a(R) = 0$. We stop the integration at a radius $R_{out}$, once we have accounted for all of the shell’s mass, i.e.
\[ \frac{4 \pi \mu_a}{R_{out}} \int_R^{R_{out}} n_{ah}(r) r^2 dr = M_{ah}. \] (20)

Figure 2 shows a sketch of the density, pressure and attenuation of ionizing radiation $\phi$ as a function of radius. The red dashed line shows the pressures of the wind bubble $P_{bol}$, the thermal gas pressure $P_{therm}$ of the shell and lastly of the ambient medium. At very early and late times when the column density of the shell and/or the pressure from winds is low, the shell may be fully ionized (not shown). See also Martínez-González et al. (2014).

where $P_{therm}$ is the thermal gas pressure, $c$ is the speed of sound in the gas, and $\rho$ is the density of the gas.

\[ P_{therm} = \frac{\rho c^2}{\gamma - 1} \] (22)

By ignoring absorption of Lyman-Werner radiation on H$_2$ we underestimate $f_{abs}$. We recalculated some of our shell structure models with CLOUDY to explore the effect chemistry has on opacity and find that a significant amount of H$_2$ only forms when the shell is dense and quite optically thick, i.e. if $f_{abs} \sim 1$. In lower density, expanded shells the interstellar radiation field suppresses the formation of H$_2$, and a more detailed chemical model does not lead to substantially different escape fractions.

A larger caveat is that we fix the dust cross section $\sigma_0$ (for a fixed metallicity). In reality, $\sigma_0$ is a function of the effective stellar temperature and decreases as the massive stars die (Draine 2011). Again using CLOUDY, we find that at later times ($t \gtrsim 3$ Myr) in our simplified treatment we are overestimating $f_{abs}$ by $\sim 25\%$. But since, as we will show, at late times radiation pressure is rarely the dominating source of feedback, this does not strongly affect the dynamics of shells. In a future iteration of our method we plan to self-consistently calculate $\sigma_0$ from the time-variable stellar spectrum.

\[ R \propto t^{3/5} \] (25)

if cooling is neglected and the ISM is assumed to be infinite and homogeneous. ST13 expanded that model and included momentum feedback from winds after the wind energy has been radiated away. Still for an infinite, homogeneous ISM, they show that
\[ R \propto (At^2 + Bt + C)^{1/4} \text{ if } t > t_{cool}. \] (26)

where $A$, $B$ and $C$ depend on wind parameters, the cloud density and the cooling time. Both these models neglect the influence of gravity, radiation pressure and SNe on the dynamics. Kim et al. (2016) study the combined effect of radiation pressure and gravity but neglect winds, similar to Murray et al. (2010) who include energy from hot winds in one of their models but always neglect wind momentum. We also note Krumholz & Matzner (2009) who calculated the dynamics under the influence of radiative momentum deposition, albeit under the assumption of full absorption and while neglecting gravity.

At one point or another all of these models fall short of a full, self-consistent treatment of feedback. The expansion rate of the shell depends on how well-coupled it is to radiation, which in turn depends on the shell structure. However, as we have seen, the shell structure itself depends on the expansion rate of the shell. To complicate things even further, winds, radiation and SNe output depend on cluster mass and age. It is therefore necessary to simultaneously solve for the expansion rate and structure of the shell while accounting for an evolving stellar population.

Expanding shells in the ISM are not truly hydrostatic – in the sense that parts of the shell do not move radially with respect to each other – as they tend to become thicker over time. If the “thickening velocity” $v_t \equiv \frac{d(R_{out} - R)}{dt}$ is lower than the shell’s sound speed $c_s$, the pressure distribution within the shell can readjust itself on a timescale short compared to that on which the shell thickness changes, and
the shell therefore rapidly settles into a quasi-hydrostatic equilibrium. In such a case, the assumption of local hydrostatic equilibrium is a good approximation.

We find that in our models $v_3$ is subsonic except for short times when we switch from the adiabatic phase to Phase II or III and around the occurrence of the first SNe. Over the whole simulated time span, the short periods when the quasi-hydrostatic assumption breaks down are expected to be negligible for the dynamics. Additionally, observations suggest that hydrostatic models as adopted here provide reasonable approximations for expanding gas shells (e.g. Georgy et al. 2012, 2013) for non-rotating stars (fiducial model) and rotating stars (see Appendix A1). The terminal velocity of the SN ejecta $v_{SN}$ is set to a constant 10$^3$ km s$^{-1}$. These feedback parameters as well as $\tau_R$ and $f_{abs}$ (which are 0 at $t = 0$ as no shell yet exists) are used to calculate the shell dynamics via the expansion equations (3) and (6).

3) After a certain time step $\Delta t$ the feedback parameters are updated and the shell structure is modeled via eq. (16) – (20). From the shell structure we get $\tau_R$ and $f_{abs}$. The time step is adaptive: It is small ($\sim 0.02$ Myr) during the early phase and around the time of the first SNe (at $t \sim 3$ Myr), when $f_{abs}$ is strongly time-dependent.

Steps 2 and 3 are repeated until the end of the simulation is reached at a time $t_{end}$. The code WARPFIELD (Winds And Radiation Pressure: Feedback Induced Expansion, colLapse and Dissolution) developed for this work is publicly available for download under https://bitbucket.org/drahner/warpfield.

2.4 Investigated parameter space

We explore the evolution of shells in clouds with masses in the range $10^3 M_\odot \lesssim M_{cl} \lesssim 10^6 M_\odot$, i.e. giant molecular clouds (GMCs) and giant molecular associations. For simplicity, we will refer to them as clouds, independent of their mass. The masses are equally spaced in log-space with $\Delta \log(M_{cl}) = 0.25$. We investigate star formation efficiencies varying from 0.01 to 0.25 with $\Delta \epsilon = 0.01$. The investigated parameter space thus includes a small region where the star clusters are not massive enough to fully sample the IMF ($M_\star \lesssim 10^3 M_\odot$), namely clouds with $M_{cl} < 10^6 M_\odot$ and with very low star formation efficiencies. In the stochastic regime the assumption of continuous SN explosions after $t \sim 3$ Myr and the values for $L_1$ and $L_w$ obtained from scaling down a fully sampled star cluster are not valid any more and hence we do not include this regime in our analysis. Also, shells around low mass GMCs ($M_{cl} \sim 10^3 M_\odot$) with very high star formation efficiencies ($\epsilon \gtrsim 0.2$) are not in quasi-hydrostatic equilibrium as $v_3 > c_6$ after the stellar winds of the most massive stars disappear and the pressure at shell’s inner edge drops significantly, thus leading to a rapid increase in the shell’s thickness. However, these are shells which are close to dissolution and for which radiation pressure is already negligible. Hence, the absolute error we make when calculating the amount of momentum deposited by radiation into such shells is small.

We examine two different natal cloud densities, $n_{cl} = 100$ cm$^{-3}$ and $n_{cl} = 1000$ cm$^{-3}$, corresponding to diffuse and translucent molecular clouds, respectively (Snow & McCall 2006). In later sections we will refer to these as low- and high-density runs. Some GMCs contain clumps and cores in excess of $n = 10^5$ cm$^{-3}$ but on average their density is $\sim 100 - 1000$ cm$^{-3}$. We do not yet include a density profile for our clouds but plan to do so in the future. We also model two different metallicities, $Z = Z_\odot$ and $Z = 0.15 Z_\odot$. Note that $Z$ refers to both the metallicity of the cloud, affecting the amount of dust and the time-scale for radiative cooling, and to the metallicity of the cluster, affecting the energy and momentum output by stellar winds and to a lesser extent by radiation. We call these the solar $Z$ and low $Z$ runs, respectively.

Table 2 lists the parameter space described above. The expansion of the shell is modelled until either it dissolves into the ambient ISM, or it collapses, or 7 free-fall times $\tau_R$ have passed; thus, $t_{end} = \min(t_{dis}, t_{collapse}, 7\tau_R)$. The free-
Table 2. Investigated parameter space

| parameter                      | value          |
|-------------------------------|---------------|
| cloud number density          | $n_{cl}$      |
| metallicity                   | $Z$           |
| cloud mass                    | $M_{cl}$      |
| star formation efficiency     | $\epsilon$    |

| parameter                      | value          |
|-------------------------------|---------------|
| high density                  | $100 - 1000$  |
| solar metallicity             | $Z_{\odot}$   |
| $10^7 - 10^8$                 | $M_{\odot}$   |
| $0.01 - 0.25$                 |               |


Figure 3. Top: Evolution of the inner radius of the shell as a function of time for a model with $M_{cl} = 10^7 M_{\odot}$, $\epsilon = 0.1$, $Z = Z_{\odot}$ and $n_{cl} = 1000 cm^{-3}$. Bottom: Expansion velocity of the shell. The vertical black lines mark the transition between the expansion phases (marked by the Roman numerals I, II and III) at $t_{cool}$ and $t_{sweep}$. The yellow diamond indicates where the shell becomes fully ionized and ionizing radiation starts to leak out. The blue dashed and dotted lines show a continuation of Phase I (assuming no cooling and an infinite mass reservoir) and Phase II (assuming an infinite mass reservoir only), respectively. The actual evolution of the shell is shown by the solid blue line. The red dashed and dotted lines show analytic solutions for comparison, eq. (21) in [W77] and eq. (13) in [ST13].

3 A FEEDBACK-DRIVEN DYNAMIC TIMELINE

We will now attempt to summarize the contribution of each feedback mechanism and its variation with time. Our aim is to highlight the different physical regimes where simple scaling relations fail short. There is no simple answer to the question of which feedback mechanism is dominant. Instead this complex problem must be addressed by quantifying how their relative contributions vary with time in an effort to combat gravity.

We start by showing the expansion of a shell that is driven by feedback from a cluster in a dense molecular cloud with cloud mass $M_{cl} = 10^7 M_{\odot}$ and star formation efficiency $\epsilon = 0.1$ (see Figure 3). An overview of the shell dynamics for a large number of other models can be found in Appendix A2. In this example, both the cloud and the cluster have solar metallicity. Rapid expansion in the adiabatic phase (Phase I) is followed by strong deceleration after $t_{cool} \sim 0.1$ Myr as the thermal pressure from the shocked wind bubble vanishes and the shell accumulates more and more mass (Phase II). At $t_{sweep} \sim 0.8$ Myr the whole cloud has been swept up by the shell. Expanding into low density ISM (Phase III), the shell now accelerates again.

We have also simulated how the cloud would evolve if Phase I (Phase II) would continue indefinitely as one would expect for an infinite ISM reservoir without cooling (after cooling). This allows us to compare our results with analytic solutions for the equation of motion, i.e. eq. (21) in [W77] and eq. (13) in [ST13]. For the particular cloud shown here, [W77] and [ST13] provide good approximations for the shell expansion in Phase I and II (some small deviations towards faster expansion in our model in Phase II are due to radiation pressure). However, for a model with the same cluster mass but a larger cloud size, [ST13] would seriously overestimate the shell’s velocity and radius at late times (due to their neglect of gravity). Even though we follow [W77] in neglecting gravity in Phase I, we do always take into account stellar evolution. This is why at late times our continued Phase I model differs from eq. (25).

An important consequence of including gravity is that for all models investigated here, shells expanding into an infinite ISM reservoir will always recollapse. Sweeping up more and more mass, the shell eventually becomes too massive for gravity to be balanced by the outward forces. If the shell approaches this point asymptotically, it can keep roughly that size until the massive stars have died and feedback decreases. Usually, however, the shell passes the point of force balance with a positive velocity. As soon as this happens, the shell starts to lose momentum and eventually collapses. This is shown by the blue dotted line in Figure 3. If gravity is included and the mass reservoir is infinite, the shell reaches a turning point at $t \sim 2$ Myr as the expansion velocity becomes negative and the radius of the shell starts to significantly deviate from the [ST13] model.

In some models ionizing radiation can completely overpower the shell. This is the moment when ionizing radiation starts to leak out (see yellow diamond in Figure 3). Coupling of radiation and the escape fraction of ionizing radiation will be discussed in the following sections.

4 RADIATION COUPLING

For young star clusters, momentum carried by radiation exceeds momentum carried by winds by a factor of a few for solar metallicity and by a few decades at $0.15 Z_{\odot}$ ([Leitherer et al. 2014]). However, this does not mean that radiation always dominates over winds as a source of feedback. Rather, it is the coupling between radiation and the ISM, quantified by $f_{abs}$ in our model, that ultimately determines which
of these feedback sources dominates. Any attempt to determine how radiation pressure and ram-pressure compare to each other must therefore begin by quantifying \( f_{\text{abs}} \).

Ionizing and non-ionizing radiation behave differently. While \( f_{\text{abs, n}} \) is only influenced by the column density of the shell, \( f_{\text{abs, i}} \) depends also on the volume density (since the recombination rate is proportional to \( n_e^2 \)) and on the rate of ionizing photons \( Q \) emitted by the cluster, cf. eqs. (17) and (18). Thus, \( f_{\text{abs, n}} \) is solely set by how far out the shell has expanded and how much mass it has swept up in the process, whereas \( f_{\text{abs, i}} \) is also dependent on the cluster’s current output in terms of ram pressure from winds and radiation pressure, which set \( n_e(r) \) via eqs. (14) and (15), and its current emission rate of ionizing photons. Since the shell expansion is a result of the history of feedback, we might say that \( f_{\text{abs, n}} \) only cares about the past while \( f_{\text{abs, i}} \) is determined by both the past and the present.

After a dense shell has formed, radiation is initially well-coupled (see Figure 3). However, after the shell enters the free expansion phase (Phase III), the expansion velocity increases while at the same time the mass growth nearly stalls. The gas in the shell thus stretches over an ever-increasing surface area, reducing the shell’s surface density and leading to a decrease of \( f_{\text{abs}} \). At the same time, ram pressure drops as \( R^{-2} \), the volume density decreases and the shell becomes thicker. As a result, \( f_{\text{abs}} \) decreases even further. In the particular example shown in Figure 4, \( f_{\text{abs}} \) starts to differ significantly from unity at \( t \sim 1 \) Myr, just after the cloud has been swept up. The bump at \( t \sim 3 \) Myr is caused by the increase in ram pressure during the Wolf-Rayet and pre-SN phases. At \( t \sim 5 \) Myr, ionizing radiation decouples from the shell. At that point the whole shell is ionized. However, the time period during which ionizing radiation can pass through the shell is short: At late times the output of ionizing radiation is greatly reduced due to the death of the very massive stars. Ionizing radiation is then again fully absorbed by the ISM. At \( t \sim 8 \) Myr, \( f_{\text{abs}} \) drops below 0.1. At this point, less than 10% of the radiation, which has already been diminished due to the aging of the cluster, is transmitted to the shell, greatly reducing the efficiency of radiation pressure as a source of feedback.

As explained above, the gas density of the shell which determines radiation momentum-coupling depends on many quantities in a non-linear way. To reduce the result into a digestible statement, it is useful to examine a fit to the absorption fraction as a function of the most important model parameters. Between 1 and 10 Myr and for fully sampled IMFs (\( M_\ast \gtrsim 10^4 M_\odot \)), \( f_{\text{abs}} \) is well fitted by

\[
f_{\text{abs}} = \begin{cases} 
0 & \text{if } \tilde{f} \leq 0, \\
\tilde{f} & \text{if } 0 < \tilde{f} < 1, \\
1 & \text{if } \tilde{f} \geq 1, 
\end{cases}
\]

with

\[
\tilde{f} = \left( a \log \epsilon + b \log \left( \frac{M_c}{M_\odot} \right) + c \right) \frac{t}{\text{Myr}} + d \log \left( \frac{M_c}{M_\odot} \right) + e.
\]

The fit parameters \( a, b, c, d, \) and \( e \) are provided in Table 3 for the combinations of density and metallicity examined in this study. We also list the reduced chi squared statistic in each case, to indicate the goodness of fit.

From the signs of the fit parameters \( a \) (negative) and \( b \) (positive) we can already draw two conclusions:

- **a)** Keeping the cloud mass constant, an increase in star formation efficiency leads to a faster decoupling with time.
- **b)** Keeping the star formation efficiency constant, an increase in cloud mass leads to a slower decoupling with time.

To understand these trends, imagine a cloud with a given mass and density. If the cloud has a high star formation efficiency, two effects play a role: First, as a more massive cluster outputs more energy and momentum in winds and SNe, the ram pressure at the inner edge of the shell rises. The shell thus becomes denser and ionizing radiation is more coupled. However, there is a second, competing effect. Stronger feedback (both ram and radiation pressure) leads to a faster expansion of the shell. The column density thus drops faster (as soon as the cloud has been swept up), leading to weaker coupling of radiation. The negative sign of \( a \) shows that on average the second effect dominates.

Now consider a fixed cluster mass but a variable cloud mass. The higher the cloud mass, the higher the column density radiation has to pass through. Also, gravity becomes more important as the cloud mass is scaled up, slowing the expansion of the shell down and increasing the coupling of radiation. If instead of a fixed cluster mass, \( \epsilon \) is kept constant, the same arguments applies, albeit in a somewhat weakened form as the cluster mass and its feedback also increase as we increase the cloud mass. In summary, radiation coupling is stronger in massive clouds, explaining the positive sign of \( b \).

### 5 WHICH TYPE OF FEEDBACK DOMINATES?

Now that we have quantified radiation coupling we can start answering the question “Which type of feedback dominates?” When asking this, it is crucial to distinguish between the instantaneous and the cumulative effect of feedback. The current density/chemical structure of the ISM is a bellwether of instantaneous feedback while cumulative feedback is traced by shell dynamics.
Instantaneous feedback, as measured by its exerted force, is highly time-dependent. It is therefore necessary to specify what evolutionary stage one is interested in. To demonstrate this, we show in Figure 5 for two examples the relative contributions from the various forces influencing the shell. These are the forces associated to winds and SNe, \( F_{\text{wind}} \) and \( F_{\text{SN}} \), direct and indirect radiation pressure, \( F_{\text{direct}} \) and \( F_{\text{indirect}} \), as well as gravity \( F_{\text{grav}} \) (cf. Section 2.1.2). To allow easy comparison between the various terms, the forces are normalized to their sum, \( F_{\text{tot}} = F_{\text{wind}} + F_{\text{SN}} + F_{\text{direct}} + F_{\text{indirect}} + F_{\text{grav}} \). The feedback term that dominates at a given time can be read off from the vertical width in Figure 5. Note that here for the sake of comparison gravity receives a positive sign. Therefore, if \( F_{\text{grav}} / F_{\text{tot}} < 0.5 \) the shell gains momentum, otherwise it loses momentum. During the adiabatic phase, the force associated to thermal pressure from shocked winds \( F_{\text{wind}} \) is the only force we consider in our model.

Before we discuss the importance of the different feedback terms, it is also instructive to consider the integrated forces. The momentum \( p \) injected by the various feedback terms (or removed in case of gravity) up to a time \( t \) can be calculated via

\[
p_i(t) = \int_0^t F_i \, dt',
\]

where the index \( i \) stands for the particular feedback term (wind, SN, etc.). The net momentum of the shell is \( p_{\text{tot}} = p_{\text{hot}} + p_{\text{wind}} + p_{\text{SN}} + p_{\text{direct}} + p_{\text{indirect}} - p_{\text{grav}} \). The evolution of \( p \) is shown in Figure 6 for the same models as in Figure 5.

During Phase I, gas pressure from hot winds is the only source driving the shell (cf. Figure 5) but as soon as the shell enters Phase II this force is shut off so that \( p_{\text{hot}} \) remains constant. After the adiabatic phase, direct radiation pressure becomes the main driving force until at \( t \sim 2 - 3 \) Myr first momentum from winds and then from SNe starts to dominate the feedback budget. At the end of the simulation, the cumulative contribution from direct radiation pressure equals that from wind ram pressure in the case of the low-mass cloud (Figure 6 top panel) and exceeds the contribution from wind ram pressure by a factor of 1.5 in the case of the high-mass cloud with higher star formation efficiency (Figure 6 bottom panel). In the low-mass cloud case shown, the absorption fraction drops rapidly after 3 Myr (cf. Figure 3) making radiation pressure a very ineffective feedback process at late times. This coincides with the death of massive stars marking a reduction in wind feedback and an increase in ram pressure from SNe. This additional pressure is not sufficient to raise the shell density, leading to a weak coupling between radiation and the swept-up ISM.

Although SNe become the main driving force at late times, the momentum injected by them over the whole simulation time is lower than that injected by winds or direct radiation pressure, albeit still of the same order of magnitude. In massive clouds, the relative importance of SNe is lower than in less massive clouds, as the exerted force associated with direct radiation pressure remained comparable with the force from SN feedback for a long time span.

Whereas feedback parameters like luminosity scale linearly with a cluster’s mass for a fully sampled IMF, the gravitational force increases quadratically. With increasing cloud mass, \( F_{\text{grav}} \) thus undergoes a super-linear increase, in contrast to the radiation pressure and ram pressure output of a cluster. This is the reason why in the massive cloud case shown, gravity dominates for most of the time after the end of Phase I and the cloud loses momentum. However, the shell still expands with a positive velocity caused by the initial velocity kick from the adiabatic phase (and a smaller kick during the Wolf-Rayet phase). Due to the slow expansion, radiation remains well-coupled. Thus, feedback from

| \( n \) (cm\(^{-3}\)) | \( Z \) (Z\(_{\odot}\)) | \( a \) | \( b \) | \( c \) | \( d \) | \( e \) | \( \chi^2 \) |
|---|---|---|---|---|---|---|---|
| 1000 | 1 | -0.323 | 0.129 | -1.119 | -0.143 | 1.975 | 1.07 |
| 1000 | 0.15 | -0.118 | 0.085 | -0.695 | 0.102 | 0.140 | 2.01 |
| 100 | 1 | -0.109 | 0.063 | -0.579 | 0.084 | 0.363 | 1.18 |
| 100 | 0.15 | -0.020 | 0.037 | -0.312 | 0.097 | -0.034 | 3.18 |
radiation pressure continues to exceed wind ram pressure feedback.

In all but the most massive clouds ($M_\odot \gtrsim 10^7 M_\odot$) which produce very massive and dense shells, the contribution from indirect radiation pressure is small. During the expansion phase, even for a shell that forms in a $10^3 M_\odot$ cloud, $\tau_\odot$ never exceeds $0.8$, supporting findings by Skinner & Ostriker (2015) [Martínez-González et al. 2014]; Reifl et al., in prep. Only at late times during recollapse can $\tau_\odot$ exceed unity, but indirect radiation is still not strong enough to stall the collapse. However, for certain cloud-cluster combinations it can provide just enough momentum to keep the expansion of the shell going until the entire cloud has been swept up and the shell accelerates again. In such a case, indirect radiation pressure can make the difference between continued expansion and collapse.

In order to determine whether the expansion of a shell up to a time $t$ was driven mainly by winds and SNe or by radiation pressure, it is instructive to compare $p_{\text{ram}}$ and $p_{\text{rad}}$ where, as before, $p_{\text{rad}} = p_{\text{direct}} + p_{\text{indirect}}$ and $p_{\text{ram}} = p_{\text{wind}} + p_{\text{SN}}$. We therefore introduce the relative radiation pressure strength parameter

$$
\Omega_{\text{rad}}(t) \equiv \frac{p_{\text{rad}}(t)}{p_{\text{ram}}(t) + p_{\text{hot}}(t)}.
$$

(32)

If $\Omega_{\text{rad}}(t) > 0.5$, radiation pressure dominates over ram pressure from winds and SNe, in the sense that up to time $t$ more momentum has been injected by radiation pressure than by ram pressure. To include the contribution from winds during the adiabatic phase we also introduce the associated relative radiation pressure strength parameter

$$
\Omega'_{\text{rad}}(t) \equiv \frac{p_{\text{rad}}(t)}{p_{\text{rad}}(t) + p_{\text{ram}}(t) + p_{\text{hot}}(t)}.
$$

(33)

Following this definition, if $\Omega'_{\text{rad}}(t) > 0.5$, radiation pressure has injected more momentum than ram pressure and hot gas pressure taken together. In Figure 7 we show the regimes $\Omega'_{\text{rad}}(t_{\text{end}}) > 0.5$ (white area) which corresponds to the regime in which radiation pressure dominates over winds and SNe, $\Omega_{\text{rad}}(t_{\text{end}}) > 0.5$ (light gray area) which corresponds to the regime where radiation pressure only dominates if momentum injected during the adiabatic phase is not taken into account, and $\Omega_{\text{rad}}(t_{\text{end}}) < 0.5$ (dark gray area) which corresponds to the regime where winds and SNe dominate.

Figure 6. Comparison of momentum $p$ deposited by the various feedback terms. The red line labeled “hot” corresponds to feedback from hot shocked wind material during the adiabatic phase, the other terms are as in eq. (3), i.e. ram pressure in blue, radiation pressure in yellow, and gravity, which has a negative contribution, in black. The parameters of the clouds examined in the two panels are the same as in Figure 5.

Figure 7. Regimes in which momentum, integrated over the whole simulation time $t_{\text{end}}$, has mainly been injected by radiation or winds/SNe for the high-density runs with solar metallicity (top) and low metallicity (bottom). In white areas, the total momentum injected by radiation pressure exceeds the total momentum injected by ram pressure from winds/SNe and hot, shocked wind material ($\Omega'_{\text{rad}} > 0.5$). In light gray areas, momentum from radiation pressure exceeds momentum from ram pressure but not momentum from ram pressure and hot gas combined ($\Omega_{\text{rad}} > 0.5$). In dark gray areas, ram pressure dominates over radiation pressure ($\Omega_{\text{rad}} < 0.5$). Black dotted curves indicate lines of constant cluster mass from $10^3 M_\odot$ to $10^7 M_\odot$. © 2017 RAS, MNRAS 470, 11
Figure 7 shows that the dynamics of shells forming in high-mass natal clouds are dominated by radiation pressure while the dynamics of shells in low-mass clouds are dominated by winds (and to a lesser extent SNe). Also, ram pressure tends to dominate for high star formation efficiencies, as was expected from eq. (29).

Interestingly, even in the low metallicity case, where momentum output from winds is roughly one order of magnitude lower than for solar metallicity, there is still a large regime where they dominate over radiation pressure (Figure 4, bottom panel). This has two reasons: First, the low amount of dust in metal-poor cloud leads to radiation being less coupled to the ISM. Second, the low ram pressure on the inner side of the shell causes the shell to be extended and low density; in such shells the recombination rate is small and ionizing radiation can easily escape without depositing its momentum. Thus, even though metallicity of a cluster does not strongly affect its radiative output, the entwinedment between winds and radiation pressure still leads to a weakening of the efficiency with which radiation is deposited in the surrounding gas. A change in ram pressure output is always accompanied by a change in radiation coupling.

Our results show that for dense clouds there is a large parameter range in which radiation pressure dominates. This shed doubts on findings by Martínez-González et al. (2014) who reported that radiation pressure is not the dominant feedback force for dense clouds. Their models, however, were not able to include radiation pressure in their shell expansion model. Instead they relied on an indirect diagnostic.

For our low-density models, ram pressure dominates the whole parameter space. The main reason for this is not that these models were simulated up to later times when SN feedback increases but rather that the shells driven in low density environments have a lower density themselves and are thus less coupled to radiation. However, ram pressure only dominates by a factor of 1 – 4 over radiation pressure, meaning that radiation is still not a negligible driving force.

6 ESCAPE FRACTION OF IONIZING RADIATION

While \( f_{\text{abs}} \) determines how well-coupled the total radiation is to the shell, the escape fraction of ionizing radiation \( f_{\text{esc},i} \) from the whole cloud is of particular interest for larger-scale simulations. For its calculation we have to take into account not only absorptions of ionizing photons by the shell but also – at early times – by the natal cloud. We can estimate the coupling of ionizing radiation at \( t = 0 \) using a Strömgren approximation (Strömgren 1939). For a classic Strömgren sphere, the mass ionized in a constant density cloud \( M_{\text{Strom}} = (4\pi/3)R_{\text{Strom}}^3 \rho_{\text{cl}} \), where \( R_{\text{Strom}} \) is the Strömgren radius, can be formulated as

\[
M_{\text{Strom}} = \frac{Q_i \mu_n}{\alpha_{\text{B}} \rho_{\text{cl}}},
\]

We can calculate the star formation efficiency needed to ionize such a cloud \( (M_{\text{cl}} = M_{\text{Strom}}) \), above which ionizing radiation is no longer fully coupled. Assuming an ionizing photon output that scales linearly with cluster mass \( (Q_i = 4 \times 10^{51} \text{s}^{-1} \times M_*/10^5 M_\odot) \) the star formation efficiency needed to fully ionize a constant density cloud and
decouple radiation dynamically at early times is

\[
e_{\text{load}} = \left( \frac{\mu_n}{\rho_{\text{cl}} \alpha_{\text{B}}} \frac{Q_i}{M_*} + 1 \right)^{-1}.
\]

This corresponds to star formation efficiencies of 0.86 and 0.38 respectively for the 1000 and 100 cm\(^{-3}\) models examined here.

Initial expansion of the wind bubble increases the density of the shell and hence the global cloud recombination rate, which will not decrease until the expansion radius exceeds the initial cloud radius. Therefore, ionizing radiation cannot escape in any of our models as long as the shell is still confined by the cloud. Thus,

\[
f_{\text{esc},i} = \begin{cases} 0 & \text{if } t < t_{\text{wep}}, \\ 1 - f_{\text{abs},i} & \text{otherwise}. \end{cases}
\]

In Figures 8 and 9 we show how the escape fraction varies as a function of time for \( 10^5 M_\odot \) and \( 10^6 M_\odot \) clouds with a range of densities and metallicities. For clouds more massive than \( 10^5 M_\odot \), \( f_{\text{esc},i} \) remains 0 at all times. Note, however, that we do not take into account fragmentation of the shell. Hence, the escape fractions provided here purely consider radiation escaping through the isotropic shell ignoring any holes and clumps. Consequently, in most cases the escape fractions derived here will be lower limits on the true values.

For solar metallicity (Figure 8), \( f_{\text{esc},i} \) reaches its highest values around 5 Myr. We have tested how the escape fraction would evolve if we would continue the expansion of the “shell” even after it has dissolved and found that \( f_{\text{esc},i} \) always drops after \( t \sim 5 \text{Myr} \). At late times the strong reduction in \( L_i \) due to the death of the massive stars causes a decrease.
in $f_{\text{esc,}\, i}$, even though the shell has a low column and volume density by then. Both the time span during which ionizing radiation can escape and the amount of escaping ionizing radiation depend on the cloud mass (more escape for low $M_{\text{cl}}$) and cloud density (more escape for low $n_{\text{cl}}$). Additionally, the fact that the shell dissolves before 10 Myr for some models does not mean that all ionizing radiation can escape. With a decrease in $L_\iota$ at late times, even a diffuse medium of $\lesssim 1$ cm$^{-3}$ can be enough to absorb a large part of the ionizing radiation.

Low metallicity models (Figure 9) have higher integrated ionizing escape fractions than solar metallicity models and $f_{\text{esc,}\, i}$ peaks earlier, at $\sim 2.5$ Myr, as less radiation is absorbed by dust. Also, even at low $\epsilon$, dense clouds become optically thin to ionizing radiation before the first SNe. Thus, the Wolf-Rayet phase and the first SNe lead to a significant reduction in $f_{\text{esc,}\, i}$ between $\sim 3$–4 Myr. Even though we neglect turbulence, which can open and close low density channels in the ISM through which radiation can escape, we show that some strong variability in $f_{\text{esc,}\, i}$ is expected purely due to stellar evolution.

Our results are in good agreement with 3D MHD simulations by Howard et al. (2017) for a cloud with $M_{\text{cl}} = 10^6 M_\odot$, $\epsilon = 0.1$ and $n_{\text{cl}} = 100$ cm$^{-3}$ and solar metallicity even though they include turbulence but neglect stellar winds. Furthermore, our results are consistent with ionization parameter mappings of the Magellanic cloud carried out by Pellegrini et al. (2012), who find average ionizing escape fractions of 0.4. These escape fractions are dominated by H II regions with two types of geometries: blister type H II and classical density-bounded nebulae. Our model is most applicable to the density-bounded regions, which are consistent with fully ionized shells.

7 WHEN FEEDBACK FAILS – RECOLLAPSE AND SEQUENTIAL STAR FORMATION

It is not a given that stellar feedback is always able to overpower gravity and drive an outflow. If $\epsilon$ is lower than some minimum star formation efficiency $\epsilon_{\text{min}}$, the shell eventually collapses back on itself, initiating more star formation. One possible example for this could be the core of 30 Doradus where two distinct stellar clusters of different age coexist (e.g. Sabbi et al. 2012). The collapse time thus sets what we coin the cadence of star formation. Only when $\epsilon > \epsilon_{\text{min}}$ can further star formation be shut off (neglecting triggered star formation in the shell). Since we cannot follow the expansion of each shell ad infinitum we regard shells as non-collapsing if they have either dissolved or have not collapsed by $t = t_{\text{end}}$. We hence might miss a small number of shells that take longer than $7\tau_{\iota}$ to collapse.

Figure 10 shows the collapse time $t_{\text{collapse}}$ for high-density runs. It is remarkable that a vast majority of models that collapse share a similar collapse time: $t_{\text{collapse}} = 2$–$4\tau_{\iota}$ ($\sim 3$–$6$ Myr) for solar metallicity and $t_{\text{collapse}} = 4$–$5\tau_{\iota}$ ($\sim 6$–$7$ Myr) in our low metallicity run. No shell in the investigated range collapsed in less than $2\tau_{\iota}$. Even though in our model all stars formed in an instantaneous star burst we can define a time averaged star formation rate $\langle \dot{M}_* \rangle \equiv \dot{M}_*/t_{\text{collapse}}$. Following Krumholz & McKee (2005) we then define the dimensionless star formation rate per free-fall time

$$\epsilon_{\text{ff}} \equiv \frac{\langle \dot{M}_* \rangle}{M_* + M_{\text{cl}}} \tau_{\iota},$$

which can be rewritten as $\epsilon_{\text{ff}} = \epsilon_{\tau_{\iota}}/t_{\text{collapse}}$. Our re-collapsing models have $\epsilon_{\text{ff}}$ of the order 0.01 and never exceed 0.07, in very good agreement with observations (e.g. Krumholz & Tan 2007).

The dashed contour line between re-collapsing and non-collapsing models shows the minimum star formation efficiency $\epsilon_{\text{min}}$. It increases with increasing cloud mass as gravity prevents outflows in massive clouds. We find that for solar metallicity, $\epsilon_{\text{min}}$ scales linearly with $\log M_{\text{cl}}$ while for the low $Z$, high density model $\log \epsilon_{\text{min}}$ scales linearly with $\log M_{\text{cl}}$. For all but the most massive clouds, $\epsilon_{\text{min}}$ is lower for low metallicity.

The blue area in Figure 10 shows models in which the shells dissolve before $7\tau_{\iota}$ ($\sim 10$ Myr). The earliest dissipations take place after 4 Myr. Using numerical simulations, this is also what Inutsuka et al. (2015) find for the destruction time of $\sim 10^5 M_\odot$ clouds, albeit for lower star formation efficiencies. 4 Myr is clearly shorter than what observational

Figures 5 and 6 see e.g. Wong et al. (2011) but the characteristic metallicity of the gas in these two galaxies is 0.5 Z_⊙ (in between the metallicities we investigated) and 0.2 Z_⊙ (slightly above our low-Z model), respectively.
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Figure 10. Collapse time $t_{\text{collapse}}$ in multiples of $\tau_{\text{ff}}$ (1.4 Myr) as a function of cloud mass and star formation efficiency for high density runs with solar metallicity (top) and low metallicity (bottom). The black dashed line shows the minimum star formation efficiency $\epsilon_{\text{min}}$ (see main text). Shells in the light blue regime have dissolved before $t = 7 \tau_{\text{ff}}$ and are assumed to never recollapse. Black dotted curves indicate lines of constant cluster mass from $10^4 M_\odot$ to $10^7 M_\odot$.

Figure 11. Collapse time $t_{\text{collapse}}$ in multiples of $\tau_{\text{ff}}$ (4.6 Myr) as a function of cloud mass and star formation efficiency for low density runs with solar metallicity (top) and low metallicity (bottom). The black dashed line shows the minimum star formation efficiency $\epsilon_{\text{min}}$ (see main text). Shells in the light blue regime have dissolved before $t = 7 \tau_{\text{ff}}$ and are assumed to never recollapse. Black dotted curves indicate lines of constant cluster mass from $10^4 M_\odot$ to $10^7 M_\odot$. Only star formation efficiencies up to $\epsilon = 0.1$ are shown.

studies usually estimate for the lifetimes of molecular clouds after the onset of star formation, i.e. $\sim 20$ Myr (see Dobbs et al. 2014 for an overview). We note this calls into question the existence of clouds with low masses and high star formation efficiencies.

In Figure 11 we show $t_{\text{collapse}}$ for our low density models. Recollapse is limited to the most massive clouds or small star formation efficiencies in the case of solar metallicity. At low metallicity, only shells that form in clouds with masses close to $10^6 M_\odot$ and $\epsilon \lesssim 0.02$ collapse. Recollapsing low-density models have lower $\epsilon_{\text{ff}}$ values than high density models but are still consistent with observations (e.g. Murray 2011).

The trend of increasing $\epsilon_{\text{min}}$ for increasing cloud mass hints at star formation being more efficient for massive clouds. Observationally, this is hard to test. Some studies that found the opposite trend, i.e. lower $\epsilon$ with increasing cloud mass, were probably limited by sampling and selection effects (Murray 2011).

Kim et al. (2016) present $\epsilon_{\text{min}}$ for various cloud densities. As an example, for a $2 \times 10^6 M_\odot$ cloud with $n_{\text{cl}} = 1000$ cm$^{-3}$ they find $\epsilon_{\text{min}}$ anywhere between 0.2 and 0.7 depending on which of their definitions for $\epsilon_{\text{min}}$ they use. Our results suggest a lower value of $\epsilon_{\text{min}} = 0.12$ for such a cloud. This difference, however, is not surprising since Kim et al. (2016) ignore wind and SN feedback in their model.

Studies of the effect of gas expulsion on star cluster evolution show that a majority of stars remain bound only if $\epsilon \gtrsim 0.1 - 0.2$ (Geyer & Burkert 2001; Baumgardt & Kroupa 2007; Shukirgaliyev et al. 2017). Since clouds with a low gas density or a low mass have a lower minimum star formation efficiency than this value, our model predicts that such clouds will form gravitationally unbound OB associations rather than gravitationally bound star clusters. Similarly, the lower values of $\epsilon_{\text{min}}$ that we find in our lower metallicity models suggest that the formation of unbound associations rather than bound clusters may be more common in these systems.

8 CONCLUSIONS AND SUMMARY

We have developed a new model that simultaneously and self-consistently calculates the structure and the expansion of shells driven by feedback from stellar winds, supernovae and radiation pressure. The model has been put to use to investigate the conditions in which the various different sources of feedback dominate, the amount of radiation that can escape through the shell, and to derive minimum star formation efficiencies for a large parameter space of clouds and clusters. Our main results are summarized below.
8.1 What is the dominant source of feedback?

- Radiation pressure and ram pressure are interconnected. Any attempt to estimate the momentum that radiation injects into the ISM without accounting for ram pressure by winds and SNe will yield incorrect results. Changing the momentum imparted by winds always leads to a change in the efficiency of radiation pressure.
- The evolution of a star-forming molecular cloud is strongly influenced by the effects of stellar evolution. The Wolf-Rayet phase and SN explosions do not only increase the effect of ram pressure but also indirectly increase the effect of radiation pressure (see above). It is thus imperative to include proper stellar evolution when investigating feedback.
- After the shocked wind material has cooled, radiation dominates the driving of the shell as long as the shell remains optically thick. This is usually the case when the star cluster is still young ($t \lesssim 2 - 3$ Myr). In massive clouds, which tend to expand more slowly due to the quadratic dependence of the gravitational force on mass, radiation pressure remains dominant for an even longer time span. Thus, in more massive clouds the time-integrated effect of radiation pressure compared to ram pressure increases. Indirect radiation pressure is negligible for low mass clouds and is only of some importance during the early phases of massive cloud evolution or during recollapse.
- Stellar winds are more important than radiation pressure in dense clouds only if the cloud mass is towards the lower end of the range studied here ($M \sim 10^4 - 10^6 M_\odot$). They always dominate over radiation pressure if the cloud density is low. At low metallicity, the momentum output by winds is decreased but radiation also couples more weakly with the shell, and so winds can still dominate over radiation.
- SNe dominate at late times. However, in most cases, over the whole cloud lifetime SN feedback does not exceed either feedback from winds or from radiation pressure. Also, feedback from SNe is not always sufficient to destroy a molecular cloud.

8.2 How well-coupled is radiation to the shell?

As we have demonstrated, classical Strömgren calculations show a full ionization of a massive molecular cloud by a star cluster is practically impossible. Despite this, we find the escape of ionizing radiation from a spherically-symmetric expanding cloud is significant, and a direct result of the shell structure responding to stellar feedback. This is an unavoidable consequence of the dynamic evolution caused by feedback driving an expansion and stretching the gas over a larger volume, decreasing its density.

- Radiation decouples more rapidly from the ISM for higher star formation efficiency, lower metallicity, lower cloud density or lower cloud mass. This is true for both ionizing and non-ionizing radiation.
- For our calculations of ionizing escape fractions $f_{\text{esc},i}$ we consider the radiation escaping through a shell but neglect any fragmentation of shell. Our escape fractions are thus independent of the solid angle on the sky and, in most cases, are lower limits to real total escape fractions.

8.3 What is the minimum star formation efficiency required to prevent Recollapse?

- We find minimum star formation efficiencies $\epsilon_{\text{min}}$ of a few percent for low mass clouds, increasing to $\sim 25\%$ or more for very massive clouds. Clouds with star formation efficiencies above these values are disrupted by the effects of stellar feedback and do not recollapse.
- The values we recover for $\epsilon_{\text{min}}$ are considerably smaller than those found by Kim et al. (2016), likely because those authors do not account for the effects of stellar winds or SNe.
- The cadence of star formation (i.e. the delay between episodes of star formation in clouds that recollapse) is $3 - 6$ Myr ($2 - 4\tau_f$) for dense clouds with solar metallicity and is somewhat higher for lower metallicity clouds. Low-density clouds are much easier to disrupt by feedback (especially if they are metal-poor), thus suggesting that they earlier shut off further star formation and hence tend to have a lower star formation efficiency.
- Our results suggest that dense, massive and/or metal-rich clouds are more likely to form gravitationally bound star clusters, while less dense, less massive and/or more metal-poor clouds are more likely to form unbound OB associations.

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REFERENCES

Arthur S. J., Kurtz S. E., Franco J., Albarran M. Y., 2004, ApJ, 608, 282
Arthur S. J., Medina S.-N. X., Henney W. J., 2016, MNRAS, 463, 2864
Avedisova V. S., 1972, Soviet Ast., 15, 708
Baumgardt H., Kroupa P., 2007, MNRAS, 380, 1589
Bisnovatyi-Kogan G. S., Silich S. A., 1995, Rev. Mod. Phys., 67, 661
Canto J., Raga a. c., Rodriguez L. F., 2000, ApJ, 536, 896
Castor J., McCray R., Weaver R., 1975, ApJ, 200, L107
Chevalier R. A., Clegg A. W., 1985, Nature, 317, 44
Dale J. E., Ercolano B., Bonnell I. A., 2012, MNRAS, 424, 377
Dobbs C. L., Krumholz M. R., Ballesteros-Paredes J., Bolatto A. D., Fukui Y., Heyer M., Mac Low M.-M., Ostriker E. C., Vázquez-Semadeni E., 2014, Protostars and Planets VI, 3, 25
Draine B. T., 2011, ApJ, 732, 100
Dunne B. C., Choi Y.-H., Chen C.-H. R., Lowry J. D., Townsley L., Gruendl R. A., Guerrero M. A., Rosado M., 2003, ApJ, 590, 306
Ekström S., Georgy C., Meynet G., Massey P., Levesque E. M., Hirschi R., Egggenberger P., Maeder A., 2012, A&A, 542, A29
Ferland G. J., Porter R. L., Van Hoof P. A. M., Williams R. J. R., Abel N. P., Lykins M. L., Shaw G., Henney W. J., Stancil P. C., 2013, Revista Mexicana de Astronomia y Astrofisica, 49, 137
Georgy C., Ekström S., Meynet G., Massey P., Levesque E. M., Hirschi R., Eggenberger P., Maeder A., 2012, A&A, 542, A29
Georgy C., et al., 2013, A&A, 558, A103
Geyer M. P., Burkert A., 2001, MNRAS, 323, 988
Gupta S., Nath B. B., Sharma P., Shekhtinov Y., 2016, Monthly Notices of the Royal Astronomical Society, 462, 4532
Howard C., Podritz R., Klessen R., 2017, ApJ, 834, 40
Inutsuka S.-i., Inoue T., Iwasaki K., Hosokawa T., 2015, A&A, 580, A49
Kim J.-G., Kim W.-T., Ostriker E. C., 2016, ApJ, 819, 137
Koo B.-C., McKee C., 1992, ApJ, 388, 93
Kroupa P., 2001, MNRAS, 322, 231
Krumholz M. R., Bate M. R., Arce H. G., Dale J. E., Gutermuth R., Klein R. I., Li Z.-Y., Nakamura F., Zhang Q., 2014, Protostars and Planets VI, pp 243–266
Krumholz M. R., McKee C. F., 2005, ApJ, 630, 250
Krumholz M. R., Matzner C. D., 2009, ApJ, 703, 1352
Krumholz M. R., McKee C. F., Tumlinson J., 2008, ApJ, 689, 865
Krumholz M. R., Tan J. C., 2007, ApJ, 654, 304
Lamers H. J. G. L. M., Cassinelli J. P., 1999, Introduction to Stellar Winds. Cambridge University Press
Lebedew P., 1901, Annalen der Physik, 311, 433
Leitherer C., Ekström S., Meynet G., Schaerer D., Agienko T. H., 2007, ApJ, 658, 1119
Leitherer C., Schauer D., Goldader J. D., Gonza R. M., Delgado L., Fui Kune D., De Mello L. F., Devost D., Heckman T. M., 1999, ApJS, 123, 3
Levesque E., Leitherer C., Ekstrom S., Meynet G., Schaerer D., 2012, ApJ, 751, 67
Mac Low M.-M., McCray R., 1988, ApJ, 342, 776
Martínez-González S., Silich S., Tenorio-Tagle G., 2014, ApJ, 785, 164
Martins F., Palacios A., 2013, A&A, 560, A16
Mathews W. G., 1967, ApJ, 147, 965
Murray N., 2011, ApJ, 729, 133
Murray N., Quataert E., Thompson T. A., 2010, ApJ, 709, 191
Osterbrock D. E., Ferland G. J., 2006, Astrophysics of gaseous nebulae and active galactic nuclei, 2nd edition. Sausalito, CA: University Science Books
Pellegrini E. W., Baldwin J. A., Brogan C. L., Hanson M. M., Abel N. P., Ferland G. J., Nemala H. B., Shaw G., Troland T. H., 2007, ApJ, 658, 1119
Pellegrini E. W., Oey M. S., Winkler P. F., Points S. D., Smith R. C., Jaskot A. E., Zastrow J., 2012, ApJ, 755, 138
Rogers H., Pittard M. J., 2013, MNRAS, 431, 1337
Sabbi E., Lennon D. J., Gieles M., de Mink S. E., Walborn N. R., Anderson J., Bellini A., Panaglia N., van der Marel R., Maž Apellániz J., 2012, ApJ, 754, L37
Seon K.-I., 2009, ApJ, 703, 1159
Shukirgaliyev B., Parmentier G., Just A., Berczik P., 2017, A&A, (arXiv:1706.03228v2)
Silich S., Tenorio-Tagle G., 2013, ApJ, 765, 43
Skinner M. A., Ostriker E. C., 2015, ApJ, 809, 187
Snow T. P., McCaill B. J., 2006, ARA&A, 44, 367
Strömgren B., 1939, ApJ, 89, 526
Townsley L. K., Feigelson E. D., Montmerle T., Broos P. S., Chu Y.-H., Garmire G. P., 2003, ApJ, 593, 874
Walch S. K., Whitworth A. P., Bissas T., Wünsch R., Hubber D., 2012, MNRAS, 427, 625
Weaver R., McCray R., Castor J., Shapiro P., Moore R., 1977, ApJ, 218, 377
Wong T., et al., 2011, ApJS, 197, 16

Figure A1. Example of the dependence of $f_{\text{esc},1}$ on stellar rotation for $n_{\text{cl}} = 100 \, \text{cm}^{-3}$ and solar metallicity. The dashed lines correspond to the model which includes stellar rotation. The solid lines correspond to the non-rotating model. We show results for clouds with masses $M_{\text{cl}} = 10^{6} \, M_{\odot}$ (black) and $M_{\text{cl}} = 10^{5} \, M_{\odot}$ (red), as in Figure 8. Since the assumed stellar rotation might be too high (see main text), realistic escape fractions are expected to lie in the gray and red shaded areas, respectively.

APPENDIX A:

A1 The Effect of Stellar Rotation
Models that include stellar rotation can better reproduce the observed main sequence width and stellar surface abundances and velocities than models of non-rotating stars and are therefore thought to provide a more realistic view (Ekström et al. 2012). Given that rotating stars produce more ionizing radiation at later times (Levesque et al. 2012), it is interesting to see how stellar rotation effects the escape fractions of ionizing radiation in our models.

We reran all models including stellar rotation and found that the effects on the dynamics of the shell are small. However, since most ionizing radiation gets emitted at late times when the density of the shell has already dropped, $f_{\text{esc},1}$ is larger at late times for rotating stars than for non-rotating stars (see Figure A1). On the other hand, at early times stellar rotation does not considerably decrease $f_{\text{esc},1}$. Taken together, the time-integrated escape fractions of ionizing radiation are higher if stellar rotation is included.

For our simulations we have used the rotating models by Ekström et al. (2012), which assume a stellar rotation velocity of 40% of the break-up velocity on the zero-age main sequence. However, as Martins & Palacios (2013) point out, this value might be too extreme. The results obtained from including such a high rotation velocity should thus be regarded as an upper limit for $f_{\text{esc},1}$ while non-rotating models provide a lower limit.

A2 Overview of Models
On the following pages we provide figures showing the shell radius and velocity, the absorption fraction of ionizing and non-ionizing radiation as well as momentum and force comparisons for models with a cloud mass of $10^{5} \, M_{\odot}$ and star formation efficiencies of 0.1, 0.15, 0.2, and 0.25, (Figure A2) and models with cloud masses $M_{\text{cl}} = 10^{6}, 10^{5}, 10^{4} \, M_{\odot}$ and star formation efficiencies $\epsilon = 0.02, 0.05, 0.1,$ and 0.25 (Figures A3, A4 and A5). Densities of $n_{\text{cl}} = 1000, 100 \, \text{cm}^{-3}$ are shown; the metallicity is solar. Dashed lines in the expansion velocity and momentum plots show negative values.
Figure A2. Models for clouds with $M_{cl} = 10^5 M_\odot$ and $\epsilon = 0.1, 0.15, 0.2, \text{ and } 0.25$. 

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Figure A3. Models for clouds with $M_{cl} = 10^6 M_\odot$ and $\epsilon = 0.02, 0.05, 0.1, \text{and} 0.25$. 

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Figure A4. Models for clouds with $M_{cl} = 10^7 M_{\odot}$ and $\epsilon = 0.02, 0.05, 0.1,$ and 0.25.
Figure A5. Models for clouds with $M_{cl} = 10^8 M_\odot$ and $\epsilon = 0.02, 0.05, 0.1$, and 0.25.