WHAT MIGHT WE LEARN FROM A FUTURE SUPERNOVA NEUTRINO SIGNAL?

PETR VOGEL
Physics Department, Caltech
Pasadena, CA 91125, USA
E-mail: vogel@lamppost.caltech.edu

ABSTRACT
Neutrinos from a future Galactic supernova will be detected by several large underground detectors, in particular by SuperKamiokande (SK) and the Sudbury Neutrino Observatory (SNO). If, as expected, the $\nu_\mu$ and $\nu_\tau$ neutrinos have somewhat higher energy on average that the electron neutrinos, they will dominate the neutral current response. The ways to separate the neutral and charged current signals will be discussed, and the best strategy to measure the possible time delay of the neutral current events will be outlined. Given the expected count rates, one will be able to measure in this way the $\nu_\tau$ mass down to about 30 eV in SNO and to 50 eV in SuperKamiokande. Another application to be discussed is the supernova localization by the neutrino signal, prior to or independently of the electromagnetic signal. The accuracy with which this can be accomplished using the angular distributions of the reactions will be estimated. With two or more detectors one can, in principle, attempt triangulation based on the arrival time of the neutrinos. It will be argued that for realistic parameters this method will be very difficult and likely leads only to crude localization.

1. Introduction
When the core of a large star ($M \geq 8M_\odot$) runs out of nuclear fuel, it collapses and forms a proto-neutron star. The total energy released in the collapse, i.e., the gravitational binding energy of the core ($E_B \sim G_N M^2_\odot/R$ with $R \sim 10$ km), is about $3 \times 10^{53}$ ergs; $\sim 99\%$ of that is carried away by neutrinos and antineutrinos, the particles with the longest mean free path. It is believed that neutrinos of all three flavors are emitted with approximately equal luminosities over a timescale of several seconds.

Those flavors which interact the most with the matter will decouple at the largest radius and thus the lowest temperature. The $\nu_\mu$ and $\nu_\tau$ neutrinos and their antiparticles have only neutral-current interactions with the matter, and therefore leave with the highest temperature, about 8 MeV (or $\langle E \rangle \simeq 25$ MeV). The $\bar{\nu}_e$ and $\nu_e$ neutrinos have also charged-current interactions, and so leave with lower temperatures, about 5 MeV ($\langle E \rangle \simeq 16$ MeV) and 3.5 MeV ($\langle E \rangle \simeq 11$ MeV), respectively. The $\nu_e$ temperature is lower because the material is neutron-rich and thus the $\nu_e$ interact more

*Invited talk at the 8th Int. Workshop on “Neutrino Telescopes”, Venice, February 23-26, 1999.
than the $\bar{\nu}_e$. The observation of supernova $\nu_\mu$ and $\nu_\tau$ neutrinos and their antiparticles would allow the details of the picture above to be tested.

In this talk I concentrate on two aspects of the neutrino signal:

- The possibility of measuring or constraining the mass of the $\nu_\tau$ and/or $\nu_\mu$.
- The possibility of locating the supernova by its neutrino signal, independently of or prior to the optical observation.

The physics of these task is straightforward, but there are complications due to:

- The finite statistics of the neutrino signal.
- The finite time duration of the signal.

The details of the work reported here can be found in the joint work with John Beacom of Caltech. One can find a much more complete list of the relevant earlier references there.

Numerical supernova models suggest that the neutrino luminosity rises quickly over a time of order 0.1 s, and then falls over a time of order several seconds. The rise is so fast that the details of its shape are largely irrelevant for our task. We model the luminosity fall by an exponential with time constant $\tau = 3$ s. The luminosity then has a width of about 10 s, consistent with the SN 1987A observations. Later, I will show how the conclusions depend on the luminosity decay time constant $\tau$.

2. Neutrino mass determination

The requirement that neutrinos do not overclose the universe gives a bound for the sum of masses of stable neutrinos (see e.g., [3]):

$$\sum_{i=1}^{3} m_{\nu_i} \leq 100 \text{ eV}. \quad (1)$$

However, laboratory kinematic tests of neutrino mass currently give limits for the masses compatible with the above cosmological bound only for the electron neutrino, $m_{\bar{\nu}_e} \leq 5$ eV [4]. For the $\nu_\mu$ and $\nu_\tau$ they far exceed the cosmological bound: $m_{\nu_\mu} < 170$ keV [5], and $m_{\nu_\tau} < 18$ MeV [6]. It is very unlikely that these mass limits can improve by the necessary orders of magnitude any time soon.

When neutrinos are emitted by a supernova, even a tiny mass will make the velocity less than for a massless particle, and will cause a measurable delay in the arrival time. A neutrino with a mass $m$ (in eV) and energy $E$ (in MeV) will experience an energy-dependent delay (in s) relative to a massless neutrino in traveling over a distance $D$ (in 10 kpc) of

$$\Delta t(E) = 0.515 \left( \frac{m}{E} \right)^2 D. \quad (2)$$
For a supernova at 10 kpc distance (approximately at the center of the galaxy), the delay for $\nu_e$ and $\bar{\nu}_e$ will be negligible, and their signal can be used as a reference clock. The $\nu_\mu$ and $\nu_\tau$ neutrinos and their antiparticles will interact only by the neutral current. Thus, in order to determine the $\nu_\tau$ and/or $\nu_\mu$ mass, we should find ways of separating the neutral and charged current signals, and of determining the possible time delay of the former with respect to the latter.

There are three neutral current reactions that give rise to potentially measurable signals: a) neutrino-electron scattering (we will show below that it is difficult to separate the charged and neutral current events in that case), b) neutral current excitation of $^{16}$O nuclei in water, followed by the $\gamma$ emission as suggested in $^7$, and c) the neutral current deuteron disintegration (relevant for SNO). In Table 1 I show the corresponding numbers of events (see $^1$, $^2$ for details how the table was made and refs. $^8$, $^9$ for description of the detectors) for the individual reactions, calculated for the “standard” supernova defined above.

Table 1: Calculated numbers of events expected in SK and SNO. In SNO events in 1 kton of D$_2$O and in 1.4 kton of H$_2$O are added. By $\nu_x$ we denote the combined effect of $\nu_\mu$ and $\nu_\tau$, each accounts for half of the events. In all except the top row, the events caused by $\nu$ and $\bar{\nu}$ are added.

| reaction | events in SK | events in SNO |
|----------|--------------|---------------|
| $\nu_e + p \rightarrow e^+ + n$ | 8300 | 365 |
| $\nu_e + d \rightarrow e^- + p + p$ | - | 160 |
| $\nu_e + d \rightarrow e^+ + n + n$ | - | 160 |
| $\nu_x + d \rightarrow \nu_x + n + p$ | - | 400 |
| $\nu_x + ^{16}$O $\rightarrow \nu_x + \gamma + X$ | 710 | 50 |
| $\nu_x + ^{16}$O $\rightarrow \nu_x + n + ^{15}$O | - | 15 |
| $\nu_e + e^- \rightarrow \nu_e + e^-$ | 200 | 15 |
| $\nu_x + e^- \rightarrow \nu_x + e^-$ | 120 | 10 |

Given the assumed known time dependence of the supernova luminosity $L(t)$, and assuming that all flavors develop in time the same way and keep their temperatures constant, the arrival time of massive neutrinos is then described by $L(t - \Delta t(E_\nu))$. Since in the neutral current scattering one cannot determine the incoming neutrino energy $E_\nu$, we have at our disposal only the time distribution of the events

$$\frac{dN}{dt} = C \int dE_\nu f(E_\nu) \sigma(E_\nu) L(t - \Delta t(E_\nu)),$$

where $C$ is a constant proportional to $1/(D^2 \times \langle E_\nu \rangle)$ and $f(E_\nu)$ is the thermal neutrino spectrum.

The only way one can decide whether there is a time delay or not is to compare the neutral current time distribution (called “Signal”) with the time distribution of
the charged current events (called “Reference”). Since $\nu_e$ and $\nu_\mu$ neutrinos and their antiparticles have higher energies than $\nu_e$ and $\bar{\nu}_e$, the neutral current events will contain a substantial fraction of possibly delayed events, while the charged current events will have no delay.

It turns out, see (1), that the most efficient way to accomplish this is also the simplest one, i.e., to use the difference in the mean arrival time:

$$\langle t \rangle_S = \sum_k t_k/N_S, \quad \langle t \rangle_R = \sum_k t_k/N_R,$$

where $N_S(N_R)$ is the total number of the Signal (Reference) events, and $t_k$ are the arrival times of the individual events. The signature of neutrino mass is then the inequality

$$\langle t \rangle_S > \langle t \rangle_R,$$

valid with significance beyond statistical fluctuations.

---

**Fig. 1.** The results of the $\langle t \rangle$ analysis for a massive $\nu_\tau$ in SK using the $\gamma$ following $^{16}$O excitation. In the upper panel, the relative frequencies of various $\langle t \rangle_S - \langle t \rangle_R$ values are shown for a few example masses. In the lower panel, the range of masses corresponding to a given $\langle t \rangle_S - \langle t \rangle_R$ is shown. The solid line is the 50% confidence level, and the upper and lower dashed lines are the 10% and 90% confidence levels, respectively.

The analysis below is based on the assumption that only one of the neutrino flavors is massive, say $\nu_\tau$, and the other one, $\nu_\mu$ in this case, is either massless or has so much
smaller mass that the corresponding time delay is negligible. The “Signal” then consist of part that is delayed and another part that is not because it is either caused by the massless $\nu_\mu$ or belongs to background that cannot be separated from the signal since it has the same energy and angle, etc. With these assumptions, the neutrino-electron scattering signal in SK will contain 60 delayed events and 700 background events, since a rather large number of the charged current $\nu_e + p \rightarrow e^+ + n$ events will be present in the forward cone. For the $\gamma$ signal from the $^{16}$O excitation, the ratio delayed/background is a more favorable 355/885. And in SNO the neutral current deuteron disintegration, with a single neutron and no charged lepton, is characterized by the ratio 219/316.

The last two ratios above also show that events which look like the neutral current (i.e., the true neutral current plus background with similar characteristics) are dominated by the response to $\nu_\tau$ and $\nu_\mu$ neutrinos. Indeed, since the cross sections are the same for these two flavors, one can simply multiply the numerators by a factor of two and make the corresponding adjustment in the denominator, to obtain the fraction of events caused by the $\nu_\tau$ and $\nu_\mu$ neutrinos. So, for the $\gamma$ signal from the $^{16}$O excitation, that contribution is about 57%, and for the deuteron disintegration it is about 82%. On the other hand, for the neutrino-electron scattering it is only about 16%. By measuring the total number of the Signal events one can determine the temperatures of the $\nu_\tau$ and $\nu_\mu$ neutrinos (see (11)) with reasonable accuracy.

To judge the statistical significance of the delay, we used the Monte Carlo simulation of a large number of supernovae for each mass value. We then histogram the differences $\langle t \rangle_S - \langle t \rangle_R$, and find the 10%, 50%, and 90% confidence levels. Representative cases are plotted in Fig. 1 for the $\gamma$ from $^{16}$O excitation in SK. In the upper panel one can see that if, e.g., $m_\nu = 75$ eV the most probable difference in average arrival times is about 0.2 s, and the 10 - 90 % CL band is 0.1 - 0.3 s, clearly separated from the massless case. In fact, the smallest recognizable mass is about 50 eV. In SNO, using the deuteron disintegration, the smallest recognizable mass is even smaller, about 30 eV.

How does the mass sensitivity depend on the assumptions we made above? Much of the analysis can be made analytically. We shall concentrate on the dependence on the assumed temperatures $T$, distance $D$, and the constant which characterizes the time duration of the neutrino signal, $\tau$. The time delay and its error depend on

$$\langle t \rangle_S - \langle t \rangle_R \sim (m/T)^2 D ; \delta(\langle t \rangle_S - \langle t \rangle_R) \sim \tau D/\sqrt{T}.$$ (6)

Since the significance of the result, and thus the smallest recognizable mass $m_{\text{lim}}$, is the ratio of these two quantities, we conclude that this neutrino mass limit is, remarkably, independent on the distance $D$, and

$$m_{\text{lim}} \sim \sqrt{T}^{3/4}.$$ (7)

Clearly, the shorter the duration of the neutrino pulse (i.e., smaller $\tau$), the better the ability to determine the neutrino mass. (We verified that detailed numerical
simulation closely follows the $\sqrt{\tau}$ scaling above) Also, naturally, if e.g. the $\nu_\mu$ and $\nu_\tau$ masses are close to each other, the mass limit is improved, roughly by $\sqrt{2}$.

3. Supernova localization with neutrinos

A future core-collapse supernova in our Galaxy will be detected by several neutrino detectors around the world. The neutrinos escape from the supernova core over several seconds from the time of collapse, unlike the electromagnetic radiation, emitted from the envelope, which is delayed by a time of order hours. In addition, the electromagnetic radiation can be obscured by dust in the intervening interstellar space. The question therefore arises whether a supernova can be located by its neutrinos alone. The early warning of a supernova and its location might allow greatly improved astronomical observations.

There are two types of techniques to locate a supernova by its neutrinos. The first one is based on angular distributions of the neutrino reaction products, which can be correlated with the neutrino direction. In this case, a single experiment can independently announce a direction and its error. However, to suppress false alarms one can demand coincidence with other experiments. The second method of supernova location is based on triangulation using two or more widely-separated detectors. This technique would require significant and immediate data sharing among the different experiments. The theme of this section (for more details and more complete reference list, see [3]) is a careful and realistic assessment of this question, taking into account the statistical significance of the various neutrino signals.

3.1. Reactions with angular dependence

Neutrino-electron scattering occurs for all flavors of neutrinos and antineutrinos, and is detected by observing the recoil electrons with kinetic energy $T$. The scattering angle is dictated by the kinematics and is given by

$$\cos \alpha = \frac{E_\nu + m_e}{E_\nu} \left( \frac{T}{T + 2m_e} \right)^{1/2}. \quad (8)$$

With threshold of about $T_{\min} = 5$ MeV, the recoil electrons will be sharply forward scattered, i.e., pointing away from the supernova, with the combined average $\langle \cos \alpha \rangle = 0.98$, corresponding to about 11°. However, multiple scattering will smear the Čerenkov cone, resulting in a one-sigma width of $\sim 25^\circ$. In order to evaluate the pointing ability of this signal we have to take into account the finite statistics, the two-dimensional form of the resulting distribution, and the presence of the unavoidable background. The background worsens the pointing ability from the simple expectation by a factor

$$\delta x = \frac{\sigma}{\sqrt{N_S}} \times C(R); \text{ with } C(R) \approx \sqrt{1 + 4R}, \quad (9)$$
where \( R \) is the ratio (at the peak) of the flat background and the signal with \( N_S \) events. For SK and SNO the background reduction factor is \( C(R) \approx 2 - 3 \), and with our standard supernova parameters we find that the one-sigma error based on the neutrino-electron scattering will be about 5° in SK and about 20° in SNO. This is by far the most accurate pointing ability at our disposal.

The reaction with the most events is \( \bar{\nu}_e + p \rightarrow e^+ + n \), with \( \approx 10^4 \) events expected in SK, and \( \approx 400 \) events expected in the light water of SNO. In Čerenkov detectors one can determine the direction of the positrons, whose angular distributions with respect to the direction of the neutrino beam is of the form

\[
\frac{dN}{d\cos\alpha} = \frac{N}{2} (1 + a \cos\alpha).
\]

It is relatively easy to show that the error in the pointing ability for \( N \) observed events in this case is given by

\[
\delta(\cos\alpha) = \frac{2}{|a|} \frac{1}{\sqrt{N}}.
\]

Since, in general \( a = a(E_{\nu}) \), we have to investigate further the neutrino energy dependence of this coefficient, and perform the necessary energy averaging.

![Fig. 2. Upper panel: total cross section for \( \bar{\nu}_e + p \rightarrow e^+ + n \); bottom panel: \( \langle \cos\theta \rangle \) for the same reaction; both as a function of the antineutrino energy. The solid line is the \( \mathcal{O}(1/M) \) result and the short-dashed line is the \( \mathcal{O}(1) \) result. The long-dashed line is the result of Eq.(3.18) of Ref. 11, and the dot-dashed line contains our threshold modifications to the same.](image)

In the limit where the nucleon mass \( M \) is taken to be infinite, i.e., zeroth order in \( 1/M \) (\( \mathcal{O}(1) \)), the asymmetry coefficient \( a \) is independent of \( E_{\nu} \) and is given
simply by the competition of the non-spin-flip (Fermi) and spin-flip (Gamow-Teller) contributions, and is

\[ a^{(0)} = \frac{f^2 - g^2}{f^2 + 3g^2} \simeq -0.10 \quad ; \quad f = 1, g = 1.26 , \]  

(12)

and thus the angular distribution of the positrons is weakly backward.

However, \( a(E_\nu) \) is substantially modified when weak magnetism and recoil corrections of \( \mathcal{O}(1/M) \) are included. It turns out \( \mathcal{O}(1/M) \) that the inclusion to this order gives a very accurate formula for \( \langle \cos \theta \rangle \). This quantity and the total cross section are shown in Fig. 2, evaluated in various approximations (see \( \mathcal{O}(1/M) \)). At high energies, the formula (3.18) of \( \mathcal{O}(1/M) \), valid to all orders in \( 1/M \), but neglecting the threshold effects, is applicable. One can see in Fig. 2 that the dot-dashed line smoothly interpolates between the correct low energy and high energy behaviour.

As far as the pointing ability of the \( \bar{\nu}_e + p \to e^+ + n \) reaction is concerned, due to the rather small angular asymmetry (and its energy dependence) we estimate that the uncertainty \( \delta(\cos \alpha) \approx 0.2 \) even for the high statistics detector like SK. Nevertheless, it would be important and useful to use this additional information constraining the supernova direction.

### 3.2. Triangulation

For two detectors separated by a distance \( d \), there will be a delay between the arrival times of the neutrino pulse. The magnitude of the delay \( \Delta t \) depends upon the angle \( \theta \) between the supernova direction and the axis connecting the two detectors. Given a measured time delay \( \Delta t \), the unknown angle \( \theta \) and its error are then:

\[ \cos \theta = \frac{\Delta t}{d} ; \quad \delta(\cos \theta) = \frac{\delta(\Delta t)}{d} . \]

(13)

Thus two detectors define a cone along their axis with opening \( \cos \theta \) and thickness \( 2 \times \delta(\cos \theta) \) in which the supernova can lie. Obviously, in order to have a reasonable pointing accuracy from triangulation, one will need \( \delta(\Delta t) \ll d \). (The Earth diameter is \( d \approx 40 \text{ ms} \).) Following \( \mathcal{O}(1/M) \) I discuss whether an appropriate time delay can be defined, and what its error would likely be. Basically, the question can be reduced to the following problem in statistics: given \( N \) events of duration \( \tau \), is the uncertainty \( \delta(\Delta t) \) equal to \( \tau/N \) (i.e., the interval between events) or the much larger \( \tau/\sqrt{N} \)?

The answer, of obvious practical significance, requires a degree of subtlety. Let us model, as before, the time dependence of the neutrino pulse (i.e. the supernova luminosity \( L(t) \)) by two exponentials, the increasing sharp rise with time constant \( \tau_1 \) and the slow decay with the time constant \( \tau_2 \) (\( \tau_1 \ll \tau_2 \)). Now take the limit (unrealistic) of zero risetime (\( \tau_1 \to 0 \)). Then, in fact, the first answer is applicable, i.e., \( \delta(\Delta t) \to \tau_2/N \). One would then simply determine the arrival time of the first
event in each detector, and the triangulation would be feasible, though still not very accurate.

But any finite leading edge, or background, would invalidate this picture. Moreover, we know that the leading edge has a finite duration related to the shock propagation time in the supernova. The best strategy then is to try to determine the rather sharp point of maximum rate $t_0$. The error in its determination depends on the duration of the leading edge $\tau_1$, which can be measured in the largest detector, and on the number of events $N_1$ in the leading edge for the given detector,

$$ (\delta t_0)_{min} \approx \frac{\tau_1}{\sqrt{N_1}}. $$

At the same time, the number of events $N_1$ in the leading edge depends somewhat indirectly on the total duration of the pulse, since $N_1 \approx N \tau_1 / \tau_2$. For the existing detectors, this leads to rather large uncertainty, $\delta (\cos \theta) \approx 0.5$. Nevertheless, if there will be several large detectors available in not too distant future, triangulation would offer another handle to the supernova localization, besides the obvious benefit of the false alarm elimination by the coincidence requirement.

4. Conclusions

In this talk, which is based on the results of Refs. 1, 2, 3, 10, I have shown that:

- The supernova signal caused by $\nu_\tau + \nu_\mu$ and their antiparticles can be isolated.
- By measuring the average arrival time difference of the neutral and charged current events, one will be able to (conservatively) determine the upper limit for $m_{\nu_e}$ of 30-50 eV, representing an improvement by $10^6$ when compared to the existing limits.
- Neutrino electron scattering can be used for pointing with accuracy of about 5°.
- $\bar{\nu}_e + p \to e^+ + n$ can be also used for crude pointing, provided the correct differential cross section is used. (Remembering that the naive formula suggests that the positron are slightly backward while in reality they should be slightly forward.)
- Triangulation appears to be difficult if the supernova signal is going to last more than one second. But it would be useful if more than two detectors (and even better if they are going to be large) will participate in the warning network.

5. Acknowledgements

The collaboration with John Beacom is gratefully acknowledged. This work was
supported in part by the US Department of Energy under Grant No. DE-FG03-88ER-40397.

6. References

1) J. F. Beacom and P. Vogel, Phys. Rev. D\textbf{58} (1998) 053010.
2) J. F. Beacom and P. Vogel, Phys. Rev. D\textbf{58} (1998) 093102.
3) J. F. Beacom and P. Vogel, astro-ph/9811350 and Phys. Rev. D, to be published.
4) G.G. Raffelt, \textit{Stars as Laboratories for Fundamental Physics} (University of Chicago Press, 1996).
5) A.I. Belesev, et al., Phys. Lett. B\textbf{350}, 263 (1995).
6) Particle Data Group., Europ. Phys. J. \textbf{3}, 1 (1998).
7) K. Langanke, P. Vogel, E. Kolbe, Phys. Rev. Lett. \textbf{76} (1996) 15.
8) Y. Fukuda et al., Phys. Rev. Lett. \textbf{81} (1998) 1158; M. Nakahata et al., hep-ex/9807027.
9) H. H. Chen, Phys. Rev. Lett. \textbf{55} (1985) 1534; G. T. Ewan, Nucl. Inst. Meth. A\textbf{314} (1992) 373.
10) P. Vogel and J. F. Beacom, \texttt{hep-ph/9903554} and Phys. Rev. D, submitted.
11) C. H. Llewellyn Smith, Phys. Rep. \textbf{3}, 261 (1972).