Complexity growth and shock wave geometry in AdS-Maxwell-power-Yang-Mills theory

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Abstract

We study the effects of a matter described by a non-abelian gauge field in the holographic characteristics like the evolution of computational complexity. For this reason we choose the Yang-Mills field in a Maxwell-power-Yang-Mills theory in AdS spacetime. We seek the impact of this new charge of YM field on the complexity growth rate by using “CA” conjecture. We also investigate the spreading of perturbations near the horizon and the complexity growth rate in local shock wave geometry and try to find the effects of the new charge on it.

1 Introduction

In the context of AdS/CFT duality a thermal system on field theory could be expressed by a gravity model in AdS spacetime. By considering various model of black hole solution in the bulk we can explore field theory behaviors which may be complicated when they are studied in a quantum field theory. One of the important aspects of field theory is the computational complexity as the number of qubit gates in the smallest quantum circuit [1,2] or in the another definition is the minimal depth of a quantum circuit [3]. Actually computational complexity regarding AdS/CFT duality could explain something about the inside of black holes and its geometry. There are two conjectures that relates complexity on the boundary to the geometry of bulk: At the first and older one complexity is supposed to be equal to the maximal volume in the spacelike slice into the bulk [4], or complexity = volume (CV) as

\[ C(t_L, t_R) \sim \frac{V}{G\ell}, \]  

(1.1)
where \( G \) is the Newtonian constant and \( V \) is the volume of spacelike slice or the Einstein-Rosen bridge (ER) with connected points \( t_L \) and \( t_R \) corresponding to the left and right boundaries, respectively.

At the newer conjecture quantum complexity is proportional to classical action in the bulk which is defined in "Wheeler De-Witt" patch, or \( \text{complexity} = \text{action} \) (CA) [5,6]. The privilege of this conjecture rather than the older one because it is needless to any length scale chosen by hand, such as the AdS radius "\( \ell \)" or the event horizon radius,

\[
C(\Sigma) = \frac{A_{\text{WDW}}}{\pi \hbar},
\]

(1.2)

in which \( \Sigma \) is a time slice equals to the intersection of asymptotical boundary and Cauchy surface in the bulk [5,6]. Action for \( WDW \) patch is given by the summation of the action and boundary of this patch which is defined between the times \( t_L \) and \( t_R \) on the boundaries and restricted by Lloyd bound [7] as follows:

\[
A_{\text{WDW}} = A_{\text{bulk}} + A_{\text{boundary}} \leq 2E(t_L + t_R),
\]

(1.3)

in which \( E \) is the excited energy of the boundary quantum state. To obtain the growth rate of complexity on the boundary we must calculate the derivative of this action with respect to the corresponding time on the boundary while keep the another time fixed. If we take \( t \) for the time with respect to which action is taken derivative, therefore complexity growth by attention to "CA" conjecture is given by

\[
\frac{dC}{dt} \leq \frac{2E}{\pi \hbar}.
\]

(1.4)

In the neutral black hole the above inequality saturates when energy \( E \) equals to the mass of the black hole. By adding any charge to the black hole this growth rate decreases, so the neutral black holes are the fastest in the nature.

Another important aspects of thermal systems is chaos which could be described with its dual in the bulk as the shock waves near the horizon of AdS black holes [8,9,10]. Actually a perturbation disturbs the geometry of the black hole and then grows by time due to the back reaction effects. During the chaos behaviors the similar initial perpendicular quantum states changes to the totally different states. An interesting point to be valued the studying shock wave geometry is the reflection of the complexity on the boundary as the existence of a firewall. When the perturbation on the boundary depends on transverse coordinates then corresponding complexity is closely connected
to the speed of the perturbation spreading or "butterfly velocity" in spatial directions. This butterfly velocity is studied by out-of-time order four-point function between pairs of local operators \( V(t = 0) \) and \( W(t) \) which are separated in space such that \([11]\)

\[
\langle V_x(0)W_y(t)V_x(0)W_y(t) \rangle_\beta,
\]

in which \( \beta \) or the inverse of the temperature stands for thermal expectation value. After the scrambling time \( t_* \) the butterfly effects could be seen by a sudden decay as follows \([12]\).

\[
\frac{\langle V_x(0)W_y(t)V_x(0)W_y(t) \rangle_\beta}{\langle V_x(0)W_x(0) \rangle_\beta \langle W_y(t)W_y(t) \rangle_\beta} \sim 1 - e^{\lambda_L (t-t_*+\frac{|x-y|}{v_B})},
\]

where \( \lambda_L = 2\pi/\beta \) stands for the Lyapunov exponent and \( v_B \) is the butterfly velocity. There are a wide variety of works has been devoted to the calculation of butterfly velocity for gravity theories in the bulk such as \([13]\) for planar black hole in the Einsteins general relativity framework, \([12]\) for the topologically massive gravity (TMG) and the new massive gravity (NMG), \([14]\) for the Einstein-Gauss-Bonnet gravity and other works \([8,9,15,16]\).

The action growth could be affected in the shock wave geometry by attention to the characteristics of gravity models. The fields sources can change the butterfly velocity and changing the butterfly velocity is proportional to the changing of the action growth \([17,18]\). In this paper we are interested in the Yang-Mills (YM) field which dominates inside nuclei in spite of electromagnetic Maxwell field related to long range effects and outside the nuclei of natural matter. We devotes our work to the black hole solution including both Maxwell and YM fields in the Einstein-Maxwell-power-Yang-Mills gravity. We seek the constraints and circumstances on the parameter of YM theory when the Lloyd bound applied on the complexity growth rate.

The outline of this work is as follows: in section 2 we obtain the evolution of complexity growth by time and check the Lloyd bound by the presence of the YM field and find a constraint on the parameters of the gravity theory. In section 3 we focus on the effects of a disturbance on the boundary and the spreading of shock wave in this theory. We also discuss the effects of the parameters of this theory on the butterfly velocity. In the last section we summarize the results and write the conclusion.
2 The complexity growth

As we know from [5,6] the growth rate of the action of a Wheeler-DeWitt (WDW) patch of the two-sided black hole at the late time, i.e. \( t_L + t_R \gg \beta \), corresponds to the rate of increasing of complexity of the boundary state. At the late time and without any shock wave the contribution of the region behind the past horizon goes to zero exponentially. By adding any kind of conserved charges WDW patch terminates slower than neutral case and Lloyd bound must be generalized due to changing of average energy of the quantum state related to the ground state.

In this section we are about to consider a Maxwell-Yang-Mills theory for the black hole inside the bulk and study action growth rate and Lloyd bound in the presence of conserved charges related to this model. The action for Einstein-Maxwell-power-Yang-Mills gravity with a negative cosmological constant in 4-dimension is given by [19,20]:

\[
\mathcal{A} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left( R + \frac{6}{\ell^2} - F_{\mu\nu}F^{\mu\nu} - [\text{Tr}(F_{\mu\nu}F^{(a)\mu\nu})] \gamma \right) \\
+ \frac{1}{8\pi G} \int_{\partial M} d^3x \sqrt{-h} K,
\]

(2.1)

in which the first integral represents the action in the bulk with \( \ell \) as its radius in AdS spacetime and \( R \) for Ricci scalar. \( \gamma \) is a real positive parameter and the fields are defined as follows:

\[
F_{\mu\nu}^{(a)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} + \frac{1}{2\sigma} C_{(b)(c)}^{(a)} A^b_\mu A^c_\nu,
\]

(2.2)

for Yang-Mills field in which \( \sigma \) is a coupling constant and \( C_{(b)(c)}^{(a)} \) is the structure constants of \((d-1)(d-2)/2\) parameter Lie group \( G \) in general \( d \)-dimensional theory and \( A_\mu^{(a)} \) is the \( SO(d-1) \) gauge group Yang-Mills potentials. According to Wu-Yang ansatz the YM invariant \( \mathcal{F} \) reduces to [20]

\[
\mathcal{F}_{YM} = \text{Tr}(F_{\mu\nu}^{(a)}F^{(a)\mu\nu}) = -\frac{2q^2_{YM}}{r^4}.
\]

(2.3)

The electromagnetic field tensor also defined by the usual Maxwell potential \( A_\mu \) as follows:

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,
\]

(2.4)
so leads to invariant $F$ for Maxwell theory as

$$F = F_{\mu\nu}F^{\mu\nu} = -\frac{2q_E^2}{r^4}. \quad (2.5)$$

Metric ansatz as a solution for such a spherical symmetric line element in 4-dimension could be:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2d\Omega^2. \quad (2.6)$$

The black hole solution in the Einstein-Maxwell-power-Yang-Mills theory for $\gamma \neq \frac{3}{4}$ is\[21]:

$$f(r) = 1 - \frac{2M}{r} + \frac{r^2}{\ell^2} + \frac{q_E^2}{r^2} + \frac{Q}{r^{4\gamma-2}}, \quad Q = \frac{2^{\gamma-1}}{4\gamma-3}q_{YM}. \quad (2.7)$$

with $q_E$ and $q_{YM}$ for electric and Yang-Mills charge, respectively. Writing $f(r)$ like an equipotential surface $f(r) = constant$ the first law could be derived. To do that we variate the gravitational potential as a function of all thermodynamic quantities:

$$df(S, M, P, q_E, q_{YM}) = \frac{\partial f}{\partial S}dS + \frac{\partial f}{\partial M}dM + \frac{\partial f}{\partial P}dP + \frac{\partial f}{\partial q_E}dq_E + \frac{\partial f}{\partial q_{YM}}dq_{YM} = 0, \quad (2.8)$$

into which $S = \pi r^2$ is the entropy of black hole and $P = \frac{3}{8\pi\ell^2}$ is the pressure of AdS spacetime. We obtain a relation for the mass or the first law of thermodynamic as follows:

$$dM = TdS + VdP + \phi_Edq_E + \phi_{YM}dq_{YM}, \quad (2.9)$$

where $T = \partial_S f$ is the temperature, $V = 4\pi r^3/3$ is the thermodynamic volume, $\phi_E = q_E/r$ stands for electric potential and

$$\phi_{YM} = \left(\frac{\gamma}{4\gamma-3}\right)\frac{2^{\gamma-1}}{r^{4\gamma-3}}q_{YM}^{2\gamma-1}. \quad (2.10)$$

\footnote{it will be shown in this section that the case for $\gamma = \frac{3}{4}$ is not useful to study}
for the Yang-Mills potential.

Regarding the above solution, Ricci scalar could be achieved as:

\[
R = -\frac{12}{\ell^2} - \frac{4Q}{r^{4\gamma}} (4\gamma^2 - 7\gamma + 3).
\]  

(2.10)

Therefore, the growth rate of the bulk action is calculated as follows:

\[
\frac{dA_{bk}}{dt} = \frac{1}{16\pi G} \int \int_{r_-}^{r_+} r^2 \left[ -\frac{6}{\ell^2} - \frac{4Q}{r^{4\gamma}} (4\gamma^2 - 7\gamma + 3) + \frac{2q_E^2}{r^4} + \frac{2\gamma q_{YM}^2}{r^{4\gamma}} \right] dr d\Omega_2
\]

\[
= -\frac{1}{2\ell^2} (r_+^3 - r_-^3) - \frac{q_E^2}{2} \left( \frac{1}{r_+} - \frac{1}{r_-} \right) + \left( \frac{2\gamma - 3}{4\gamma - 3} \right)^2 q_{YM}^2 \left( \frac{1}{r_+^{4\gamma-3}} - \frac{1}{r_-^{4\gamma-3}} \right).
\]

(2.11)

In the above integral equations we put \( \Omega_2/4\pi G = 1 \).

In the other side, the boundary part of the action growth rate is given by:

\[
\frac{dA_{bd}}{dt} = \frac{1}{8\pi G} \int_{\partial M} d\Omega_2 (\sqrt{-h} K) = \frac{1}{2} \left[ r^2 \sqrt{f(r)} \left( \frac{2}{r} \sqrt{f(r)} + \frac{f'(r)}{2\sqrt{f(r)}} \right) \right]_{\partial M},
\]

where the extrinsic curvature is defined by

\[
K = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \sqrt{f(r)}) = \frac{2}{r} \sqrt{f(r)} + \frac{f'(r)}{2\sqrt{f(r)}}.
\]

(2.13)

So by attention to our metric we can obtain:

\[
\frac{dA_{bd}}{dt} = (r_+ - r_-) + \frac{3}{2\ell^2} (r_+^3 - r_-^3) + \frac{q_E^2}{2} \left( \frac{1}{r_+} - \frac{1}{r_-} \right)
\]

\[
+ \frac{(3 - 2\gamma)Q}{2} \left( \frac{1}{r_+^{4\gamma-3}} - \frac{1}{r_-^{4\gamma-3}} \right).
\]

(2.14)

(2.15)

Hence the total growth rate of the action is achieved:

\[
\frac{dA}{dt} = (r_+ - r_-) + \frac{r_+^3 - r_-^3}{\ell^2}.
\]

(2.16)
It is useful to rewrite the total growth action equation with respect to black hole quantities like charges and mass. By using the horizon equations \( f(r_+) = f(r_-) = 0 \) one can obtain following relations for the electric charge and mass of the black hole.

\[
q_E^2 = r_+r_- \left[ 1 + \frac{1}{\ell^2} \left( \frac{r_+^3 - r_-^3}{r_+ - r_-} \right) + \frac{2^{\gamma-1}}{4 \gamma - 3} q_{YM}^2 \left( \frac{r_+^{-(4\gamma-3)} - r_-^{-(4\gamma-3)}}{r_+ - r_-} \right) \right], \quad (2.17)
\]

and,

\[
M = \frac{1}{2} \left[ (r_+ + r_-) + \frac{1}{\ell^2} \left( \frac{r_+^4 - r_-^4}{r_+ - r_-} \right) + \frac{2^{\gamma-1}}{4 \gamma - 3} q_{YM}^2 \left( \frac{r_+^{-4(\gamma-1)} - r_-^{-4(\gamma-1)}}{r_+ - r_-} \right) \right]. \quad (2.18)
\]

By attention to these definitions one could rewrite the total action growth rate as follows:

\[
\frac{dA}{dt} = \frac{q_E^2}{r_+ - r_-} - \left( \frac{2^{\gamma-1}}{4 \gamma - 3} q_{YM}^2 \left( \frac{1}{r_+^{4\gamma-3}} - \frac{1}{r_-^{4\gamma-3}} \right) \right). \quad (2.19)
\]

So the action growth at the late time approximation by attention to the conjugated potentials, which are derived earlier in this section, would be summarized as:

\[
\frac{dA}{dt} = (M - \phi_E + q_E - \frac{1}{\gamma} \phi_{YM} + q_{YM}) - (M - \phi_E - q_E - \frac{1}{\gamma} \phi_{YM} - q_{YM}), \quad (2.20)
\]

for \( \gamma \neq \frac{3}{4} \).

It is simple to conclude that the Lloyd bound [7] would be satisfied if only \( \gamma \geq 1 \), so the case for \( \gamma = \frac{3}{4} \) violates this bound and must be thrown away. So the Lloyd bound for \( \gamma \geq 1 \) for the complexity growth rate reads as usual form:

\[
\frac{dC}{dt} \leq \frac{2E}{\pi \hbar}. \quad (2.21)
\]
in which maximum rate devotes to \( \gamma = 1 \) and "\( E \)" is the average energy of the quantum state relating to its ground state.

### 3 The complexity growth in a shock wave geometry

In this section we are about to study our model when a shock wave perturbed the geometry. This shock wave perturbs the black hole solution by injection
of a small amount of energy from the boundary of AdS spacetime towards the horizon. This perturbation grows in time due to the back-reaction effects and propagate on the horizon. By attention to the work of Dary andt Hooft [22], we rewrite the solution in Kruskal coordinates \((u, v)\).

\[
uv = -\exp\left(\frac{4\pi}{\beta} r_\ast\right), \quad u/v = -\exp\left(-\frac{4\pi}{\beta} t\right),
\]

(3.1)
in which \(\beta\) is proportional to the inverse of Hawking temperature and \(r_\ast\) is a function of \(r\) which is defined as \(dr_\ast = \frac{dr}{f(r)}\). The effect of shock wave geometry is considered as the effect of a massless particle at \(u = 0\) which moves in the direction of \(v\) with the speed of light. So geometry for \(u < 0\) stays unchanged like (2.6) and in Kruskal form as:

\[
ds^2 = -2A(u, v)du dv + B(u, v)d\vec{x}^2,
\]

(3.2)
this is a general form into which:

\[
A(u, v) = -\frac{4}{uv} \frac{f(r)}{[f'(r_\ast)]^2}, \quad B(u, v) = r^2.
\]

(3.3)

In other hand, when \(u > 0\) another coordinate which the particle moves in its direction shifted as \(v \rightarrow v + \alpha(x)\), where \(\alpha(x)\) is the shift function. So in general case and for all values of \(u\) we can determine old coordinates with new coordinates by using step function \(\theta(u)\) like,

\[
\hat{u} \equiv u \\
\hat{v} \equiv v + \theta(u)\alpha(x) \\
\hat{x} \equiv x.
\]

(3.4)
By these transformations the metric and the energy momentum tensor are affected. If there is some sources of matter fields then its non-zero components has changed to new form. This new geometry and new energy momentum tensor still satisfies the Einstein equation, \(\hat{\mathcal{G}} = \hat{T}_{\text{matter}}\), in which \(\hat{\mathcal{G}}\) and \(\hat{T}_{\text{matter}}\) is the Einstein tensor and the energy momentum tensor for all sources of matter fields in spacetime in the new coordinate, respectively. After acting a scalar operator at \(tw < 0\) and producing the shock wave, this perturbation propagates along \(\hat{u} = 0\) and its stress-energy tensor will have only \(\hat{u}\hat{u}\) component [6],

\[
\hat{T}_{\{\text{shock}\}\hat{u}\hat{u}} \sim \delta(\hat{u}) \exp\left[\frac{2\pi|tw|}{\beta}\right].
\]

(3.5)
By adding this part of perturbation to the stress-energy tensor and solving
the Einstein equation we find a relationship for shift function $\alpha(x)$. If we
consider this function independent of the transverse coordinates, $\theta$ and $\phi$, so
it will obtained as:

$$\alpha \sim \exp\left[\frac{2\pi}{\beta}(|t_w| - t_s)\right], \quad (3.6)$$

in which $t_s = \frac{\beta}{2\pi} ln(S)$, with $S$ stands for the entropy, is the scrambling time
which is a delay on the action growth due to the "switchback effect". In
other side when the shock wave is local, the shift function depends on the
transverse coordinates. By solving the equations in this case we have an
extra term in the above exponential part. If there is just one transverse
coordinates, named $x$, so the shift function yields:

$$\alpha(x) \sim \exp\left[\frac{2\pi}{\beta}(|t_w| - t_s - \frac{|x|}{v_B})\right], \quad (3.7)$$

where $v_B$ is called the butterfly velocity and present the speed of the local
shock wave on the boundary. $v_B$ is defined as follows:

$$v_B = \sqrt{\frac{f'(r_h)}{4r_h}}, \quad (3.8)$$

with $r_h$ for the outer horizon achieved by $f(r_h) = 0$ and could be computed
for any model of gravity. In our model with $f(r)$ in (2.7) the butterfly velocity
reads:

$$v_B = \frac{1}{2} \sqrt{\frac{1}{r_h^2} + \frac{3}{\ell^2} - \frac{q_E^2}{r_h^4} - \frac{1}{2}\left(\frac{2q_{YM}^2}{r_h^4}\right)^\gamma}. \quad (3.9)$$

As one can see the butterfly velocity in the presence of Yang-Mills fields
takes an extra term which depends on the Yang-Mills charge and $\gamma$, which
was showed in the previous section must be $\gamma \geq 1$. It would be useful the
comparison of $v_B$ in the presence and absence of this Yang-Mills field. To do
this and for simplicity we ignore the effect of Maxwell fields. In the spacetime
without the electric and yang-Mills charge we have simply the Schwarzschild
spacetime with its own single event horizon ($r_h$) and the butterfly velocity
reduces to

$$v_B = \frac{1}{2} \sqrt{\frac{1}{r_h^2} + \frac{3}{\ell^2}}. \quad (3.10)$$
By attention to (2.7) for non-electrical charge case and for fixed mass we have $f(r) > f(r)$, in which $f(r)$ is defined for the Schwarzschild spacetime and $\gamma > 1$ for the Yang-Mills solution. This inequality is true for all radius such as the event horizon $r_h$, so:

$$f(r_h) = 0 > f(r_h),$$

(3.11)

hence $f(r_h) < 0$ and since $f(r_h) = 0$ as well, then we lead to the following conclusion:

$$f(r_h) < f(r_h) \Rightarrow r_h < r_h.$$

(3.12)

It would be simple to see that the butterfly velocity has an inverse relationship with the event horizon and therefore by adding Yang-Mills fields to the spacetime, the spreading of the shock wave on the boundary would be increased, namely $v_B > v_B$. Of course it must be interesting to know that the value of butterfly velocity is decreased by increasing $\gamma$, actually it has the maximum value when $\gamma = 1$, when the Yang-Mills field acts similar to the electric field.

4 Conclusion and summary

In this work we choose the black hole solution with the Maxwell and Yang-mills sources. The complexity growth rate has been investigated in this model by using ”complexity=action” conjecture [5,6]. The Lloyd bound [7] leads to a restriction on the power parameter $\gamma$ for which all values of this parameter smaller than 1 has been discarded. The maximum value for the action growth (or the complexity growth) happens for $\gamma = 1$ and by increasing its value the growth rate would be decreased.

In the other hand, when the boundary is disturbed by a small amount of energy and the spacetime takes a shock wave geometry [4,8], the spreading of perturbation near the horizon with the butterfly velocity affects the complexity growth rate. We show that the existence of the Yang-Mills field causes the increasing of butterfly velocity and also by increasing of $\gamma$ this velocity decreased, so $\gamma = 1$ is the maximum butterfly velocity for fixed all thermodynamics quantities.

References

1. L. Susskind, ” Computational Complexity and Black Hole Horizons, hep-th/1402.5674.
2. L. Susskind, "Addendum to Computational Complexity and Black Hole Horizons, hep-th/1403.5695.

3. P. Hayden and J. Preskill, Black holes as mirrors: Quantum information in random subsystems, JHEP 0709, 120 (2007) hep-th/0708.4025.

4. D. Stanford and L. Susskind, Complexity and Shock Wave Geometries, Phys. Rev. D 90, no.12, 126007 (2014) hep-th/1406.2678.

5. A. R. Brown, D. A. Roberts, L. Susskind, B. Swingle and Y. Zhao, Phys. Rev. Lett. 116, 191301, (2016).

6. A. R. Brown, D. A. Roberts, L. Susskind, B. Swingle and Y. Zhao, Phys. Rev. D 93, 086006, (2016).

7. S. Lloyd, Nature 406, 1047 (2000).

8. S.H. Shenker and D. Stanford, Black holes and the butterfly effect, JHEP 1403, 067 (2014); hep-th/1306.0622.

9. S.H. Shenker and D. Stanford, Multiple shocks, JHEP 1412, 046 (2014); hep-th/1312.3296.

10. D.A. Roberts, D. Stanford and L. Susskind, Localized shocks, JHEP 1503, 051 (2015); hep-th/1409.8180.

11. E. Perlmutter, Bounding the space of holographic CFTs with chaos, JHEP 10, 069 (2016).

12. M. Alishahiha, A. Davody, A. Naseh, and S. F. Taghavi, On butterfly effect in higher derivative gravities, JHEP 11, 032, (2016); hep-th/1610.02890.

13. X. H. Feng and H. Lu, Butterfly Velocity Bound and Reverse Isoperimetric Inequality, Phys. Rev. D 95, 066001, (2017); hep-th/1701.05204.

14. W. H. Huang, Holographic Butterfly Velocities in Brane Geometry and Einstein-Gauss-Bonnet Gravity with Matters,, Phys. Rev. D 97, 066020 (2018); hep-th/1710.05765.

15. Y. Ling, P. Liu and J. P. Wu, Note on the butterfly effect in holographic superconductor models, Phys. Lett.B 768, 288 (2017) hep-th/1610.07146.
16. R. G. Cai, X. X. Zeng and H. Q. Zhang, Influence of inhomogeneities on holographic mutual information and butterfly effect,, JHEP 1707, 082, (2017); hep-th/1704.03989.

17. Y. G. Miao and L. Zhao, Complexity/Action duality of the shock wave geometry in a massive gravity theory, Phys. Rev. D 97, 024035 (2018); hep-th/1708.01779.

18. S. A. Hosseini Mansoori and M. M. Qaemmaqami, hep-th/1711.09749.

19. H. El Moumni. Revisiting the phase transition of AdS-Maxwell-power-Yang-Mills black holes via AdS/CFT tools. Phys. Lett., B776, 124, (2018).

20. S. H. Mazharimousavi, M. Halilsoy and Z. Amirabi: Classical and Quantum Gravity 28, 025004 (2011).

21. M. Zhang, Z. Ying Yang, D. C. Zou, W. Xu and R. H. Yue , P-V criticality of AdS black hole in the Einstein-Maxwell-power-Yang-Mills gravity, Gen.Rel.Grav. 47, 14, (2015).

22. T. Dray and G.t Hooft, The gravitational shock wave of a massless particle,Nucl. Phys.B253, 173 (1985).