ABSTRACT
In order to design and implement preventive and remedial actions, a continuous performance of fast security analysis is imperative amid outages of system components. Following the contingency of a system component, State estimation and Load flow techniques are the two popular techniques used to determine system state variables leading to estimation of flows, losses and violations in nodal voltages and transmission line flows. But the dynamic state and complexity of the system requires faster means of estimations which can be achieved by linear distribution factors. The use of Distribution factors in form of Power Transfer Distribution Factors (PTDF) and Line Outage Distribution Factors (LODF) which are transmission line sensitivities with respect to active power exchanges between buses and transmission line outages offer an alternative to these two techniques being linear, quicker, and non-iterative. Following the estimation of the linear distribution factors from a reference operating point (base case) and contingency cases involving line outage, generator output variation and outage of a Six bus network using Matlab programs, the results show that by means of Linear Distribution factors quick estimates of post outage line flows can be made which match flow results obtained from DC load flow analysis.

Keywords— Contingency, Distribution Factors, LODF, Outages, Post Contingency Flows, PTDF

I. INTRODUCTION
Beyond the planning stage of any power system, optimal operation is vital for reliable performance of the system. On account of the limitations imposed by the system design and economy of operation, it is the responsibility of the system operators to ensure the reliable operation of system components. Since power system components are connected for ease of disconnection, system contingency is highly probable and inevitable. Contingencies define those events that tend to disrupt or disturb the normal operation of the system such as a generator, transformer or transmission line. Scheduled outage on account of maintenance or forced component outage on account of fault makes it impossible to operate a power system with all its components online at all times. Therefore, the ability to predict the impact of an outage on the system is instructive to guarantee secure and stable system operation. Power system security, an aspect of reliability is comprised of all practices designed to ensure that the system maintains operation despite a component failure. The aim of power system security is to ensure that the power system operations under stable conditions or is restored to it, such that single component outage or failure (N-1 contingency) does not lead to cascaded tripping and blackout [4]. Power system security analysis is achieved by three major approaches namely; System Monitoring (SM), Contingency Analysis (CA) and Security constrained Optimal Power Flow (SCOPF). CA is the study of the impacts of conditional changes or deviations with respect to the scheduled or unscheduled outage of component(s) during steady state operation of a power system. The effect of a component contingency on the remaining system may cause undesired operating conditions that may be disastrous if response or prevention are not in place to control their impact. The instructive question is whether it is possible to predict a post-
The idea of sensitivity analysis in power systems has been widely used to avoid recalculation of the power flow solution. In transmission systems, the parameters used in these analyses are the power transfer distribution factors (PTDF) and the line outage distribution factors (LODF). PTDFs are defined as the changes in the line power flows due to a change in power injection at a particular bus. LODFs are defined as the changes in the line power flows due to the disconnection of a particular line [1]. The calculation of these distribution factors has gained more interest recently due to the need for fast online readjustments in modern power systems. There are generally two approaches to calculate distribution factors for power systems. Considering a more realistic model, power flow equations form a nonlinear system of equations. In order to find sensitivities, one need to find the Jacobian matrix at a particular solution of the network, which yields distribution factors that are only valid for small variations around the operating point [2] and [3]. The second approach is to find an approximate linear model that describes the system for relatively large variations in operating point and find the distribution factors for the approximate linear system. The decoupled power flow equations, for instance, are used in [4] to find the sensitivity of reactive power flows to transmission line and power transformer outages. The Fast Decoupled power flow method was adopted in [5] [5] to derive AC distribution factors for transmission systems. These factors were then used to formulate a re-dispatch optimization problem for congestion management. The assumptions of Fast Decoupled power flow are not as strongly hold for distribution systems as they are for transmission systems (X/R ratio is smaller in distribution systems). Also, those factors depend on the operating condition of the system and, therefore, are valid for small changes in power flow patterns. A linear approximation of the power flow equations is the so-called DC power flow, in which all voltage magnitudes are assumed to be one per-unit, line resistances are ignored, and voltage angles are assumed to be small enough so that their Sine are approximately equal to the angles in radians. Based on the linear DC power flow equations, the PTDFs and LODFs can be derived for a transmission system [1]. The assumptions of DC power flow are not valid for distribution systems since the voltage magnitude plays an important role and cannot be dismissed. Besides analytical studies, there have been some efforts recently to derive the distribution factors based on real time data provided by phasor measurement units. [6] observed that distribution factors may be derived without the need for a power flow model. Distribution factors have many applications in transmission systems, such as optimal transmission switching [7], (N –1) security assessment [8], congestion management [9], generation rescheduling, etc. Using distribution factors, it is possible to calculate the changes in other system quantities such as losses and generation cost. Despite broad discussions on derivations and applications of distribution factors at the transmission level, there is rather limited work on the distribution system counterpart. Distribution systems have high R/X ratios, radial configurations, a mixture of cables and overhead lines, and unbalanced loads. Due to these unique features, the power flow algorithms and sensitivity analysis derived for transmission systems are not always valid for distribution systems. Power flow algorithms specifically designed for distribution systems have been proposed in the literature, for the linear power flow (LPF) [10]. Using the ad joint network method, which is based on the application of Tellegen’s theorem to power systems, the authors of [11] derived the sensitivities of power losses and voltage magnitudes with respect to power injection at any node in the system. Since this method is only valid for radial distribution systems and cannot be used to calculate the branch outage distribution factors (BODF) due to voltage dependence on load. Similar to the case of transmission systems, there are many applications for distribution factors in distribution systems. [12] used a set of Linear Power Flow (LPF) equations to describe two kinds of distribution factors namely Current Transfer Distribution factors (CTDF) and Branch Outage Distribution Factors (BODF) whose application was illustrated using the optimal placement of distributed generation/capacitor banks as well as minimum-loss network re-configuration of a distribution network.

II. LINEAR DISTRIBUTION FACTORS

The problem of studying thousands of possible outages becomes very difficult to solve if the desire is to present quick results. One of the easiest ways to provide a quick calculation of possible overloads is to use linear distribution factors [13]. Essentially, the linear distribution factors approximately estimate the changes in different line flows for any particular outage condition. When their values have been established, subsequent transmission lines flow can be estimated with high DC defined accuracy while avoiding the tedious AC load flow analysis. So basically, linear distribution factors are of two types: 

**Power Transfer Distribution Factors (PTDF)**

PTDF shows linearized impact of power transfer [10]. It is the relative change in power flow on a particular line due to an injection and withdrawal of power on a pair of buses [12]. They represent the sensitivity of the flow on line to a shift of power from bus to bus. Power Transfer Distribution Factors are also known by other names such as Generation Shift Factors (GSFs), Power Distribution Coefficients (PDCs), Effectiveness Factors and Impedance
Factors. The PTDF has four (4) attributes; a particular Line (with reference direction), particular Bus, Value of the transfer factor and a Reference bus.

The value of the PTDF of line with respect to bus is defined to be the change (or sensitivity) of active (MW) power flow in a reference direction on line with respect to a change in injection at bus and a corresponding change in withdrawal at the reference bus [8].

The PTDF factors are designated and have the following definition

\[ \text{LODF} = \text{Pre-contingency real power flow on line (post outage flow)} - \text{Post-contingency real power flow on line (pre outage flow)} \]

Where

- Bus where power is injected (contingency bus)
- Bus where power is taken out (post-contingency bus)
- Power transferred from bus to bus
- Change in megawatt power flow on line when is made between and
- Then the post outage flow can be derived with
- When bus \( k \) is the reference bus, then

\[ = \text{Post-contingency real power flow on line (post outage flow)} \]
\[ = \text{Pre-contingency real power flow on line (pre outage flow)} \]

If the pre-contingency and post-contingency generations of a generator are and then the change in generation due to total outage or output-reduction is

Once the “post contingency flow” on each line has been gotten, then they may be compared to its pre-specified secure limit and those exceeding their limits flagged. This informs system operator of an overloading implication on the monitored transmission line. The numerical range of PTDF is: \( \frac{-1}{1} \). implies that all of the transferred power from \( t \) must flow through line \( t \). means that all of the transferred power from \( t \) will flow through line \( t \), but in a reverse direction while \( t \) means that none of the power transferred from \( t \) will pass through line \( t \).

2.2 Line Outage Distribution Factor (LODF)

Line outage distribution factors are linear estimates of the change in flow on adjacent lines when transmission lines are lost and are normally applied for checking overloads on the lines following the line loss [2]. They represent the sensitivity of the flow on line to a line failure in the network [13]. LODF shows linearized impact of power transfer and aid in estimating the impact opening (outage) a transmission line will have on other online transmission lines. The value of the LODF of line with respect to loss of line \( i \) is defined to be the change (or sensitivity) of active (MW) power flow on line with line out.

From definition the LODF is expressed as:

\[ \text{LODF} = \frac{\text{Outaged transmission line index}}{\text{Pre-contingency/pre-outage flow on line before line was opened}} - \frac{\text{Outaged transmission line index}}{\text{Pre-contingency/pre-outage flow on line before it was opened}} \]

Change in MW flow on line

Since the real power flows on line and on line are known, and then the post-outage flow on line with lineout can be determined using “LODF” factors expressed as:

The LODF matrix is stored such that each row and column corresponds to one line in the network, with rows corresponding to monitored line and columns corresponding to the outaged lines. The LODF for a particular outage and monitored transmission line is obtained by finding the monitored line down the rows and then finding the outaged line along the row in the appropriate column. By pre-calculating the LODFs, a very fast procedure can be set up to test all lines in the network for overload for the outage of a particular line. Furthermore, this procedure can be repeated for the outage of each line in turn, with overloads reported to the operations personnel in form of alarm messages.

III. METHODOLOGY

Direct Current (DC) Load Flow of Sample Network

Unlike the AC load flow, the DC power flow is a non-iterative as it simplifies the Fast decoupled AC derivations under certain assumptions. Following these assumptions, the predominant relationship from the Fast decoupled method relates and , expressed as

Where the diagonal and off diagonal of the reactance matrix B is

and

But where, the resistance of the transmission line is significant, then

The DC load flow is adequate in estimating approximately accurate MW flows on transformers and transmission lines while ignoring the MVAR and MVA flows. Consequently, the real or MW power flow on a lossless transmission line connected between bus and bus using DC power flow is

\[ = \text{reactance of line between bus and bus} \]
\[ = \text{Calculated Real power schedule at bus and respectively} \]
\[ = \text{Real power flowing through transmission line connected between bus and bus} \]
\[ = \text{Voltage bus angle at bus and respectively} \]

3.1 Description of Sample Network

One-line diagram of network is shown in figure 1 with generators at buses 1, 2 and 3 while loads are...
distributed at buses 4, 5 and 6. The Bus/Generator and Branch Data are given in Table 1 and 2 respectively. Using 100MVA as base power,

\[
\begin{align*}
\text{Bus 1:} & \\
\text{Bus 2:} & \\
\text{Bus 3:} & \\
\text{Bus 4:} & \\
\text{Bus 5:} & \\
\text{Bus 6:} & 
\end{align*}
\]

Neglecting bus 1 which is the slack bus, the corresponding and diagonal and off-diagonal elements of the matrix is formulated from

Since

The inverse of the B matrix yields

Solving the matrix multiplication for the bus voltage phase angles (in Radians), yields

With line losses ignored, the power-flow on any line placed between bus and bus can be obtained using,

\[
\begin{align*}
\text{Bus 3} & \\
\text{Bus 2} & \\
\text{Bus 6} & \\
\text{Bus 4} & \\
\text{Bus 5} & 
\end{align*}
\]

IV. RESULT AND DISCUSSION

Consider the load flow results shown in table 3, the base case power flow on line L2 is 76.66 MW. Following the outage of the generator at bus 2, the network transfers its generator output demand of 50MW from generator at bus 2 to the slack bus, bus 1.

| Line Code | Bus From | Bus To | Power (MW) | Reactive Power (MVAr) | Capacity (MVA) | Voltage (V) |
|-----------|---------|--------|------------|-----------------------|---------------|-------------|
| L1        | Bus 1   | Bus 2  | 10.0       | 5.0                   | 150           | 100         |
| L2        | Bus 1   | Bus 4  | 10.0       | 5.0                   | 150           | 100         |
| L3        | Bus 1   | Bus 5  | 10.0       | 5.0                   | 150           | 100         |
| L4        | Bus 2   | Bus 3  | 6.0        | 3.0                   | 150           | 100         |
| L5        | Bus 2   | Bus 4  | 6.0        | 3.0                   | 150           | 100         |
| L6        | Bus 2   | Bus 6  | 6.0        | 3.0                   | 150           | 100         |
| L7        | Bus 2   | Bus 6  | 6.0        | 3.0                   | 150           | 100         |

Note that

Power scheduled at is derived using

\[
\begin{align*}
\text{Bus 3} & \\
\text{Bus 2} & \\
\text{Bus 6} & \\
\text{Bus 4} & \\
\text{Bus 5} & 
\end{align*}
\]
The change in power at bus 2 is the post generator outage flow on line L2 is 92.41MW. To estimate the PTDF we employ **Evaluating PTDF and LODF**

The AC and DC MW line flows/losses for base and contingency cases are captured on composite table 6. The base case DC network composition had been the basis for the computed PTDF (table 4) and LODF (table 5) with which post contingency line flows are estimated.

| Code | Bus 1 | Bus 2 | Bus 3 | Bus 4 | Bus 5 | Bus 6 |
|------|-------|-------|-------|-------|-------|-------|
| L 1  | 0     | -0.4706 | -0.4026 | -0.3149 | -0.3217 | -0.4064 |
| L 2  | 0     | -0.3149 | -0.2949 | -0.5044 | -0.2711 | -0.296 |
| L 3  | 0     | -0.2145 | -0.3026 | -0.1807 | -0.4072 | -0.2976 |
| L 4  | 0     | 0.0544 | -0.3416 | 0.0116 | -0.1057 | -0.1907 |
| L 5  | 0     | 0.3115 | 0.2154 | -0.379 | 0.1013 | 0.2208 |
| L 6  | 0     | 0.0993 | -0.0342 | 0.0292 | -0.1927 | -0.0266 |
| L 7  | 0     | 0.0642 | -0.2422 | 0.0189 | -0.1246 | -0.41 |
| L 8  | 0     | 0.0622 | 0.289 | 0.0183 | -0.1207 | 0.1526 |
| L 9  | 0     | -0.0077 | 0.3695 | -0.0023 | 0.015 | -0.3433 |
| L10  | 0     | -0.0034 | -0.0795 | 0.1166 | -0.1698 | -0.0752 |
| L11  | 0     | -0.0565 | -0.1273 | -0.0166 | 0.1096 | -0.2467 |

Where appears as the intersection of column 2 (the contingent bus) with row 2 (the monitored line). The PTDF values for the other 10 monitored lines considering the outage of generator at bus 2, is reflected in column 2 of the table 3. Following several other contingencies involving nodal power variation/outage, all bus PTDF can and have been estimated from the base case. This is shown in table 4 below.

The LODF of any monitored line following the outage of another line has been expressed as

From DC load flow, the pre-outage flow on L8, upon the outage of L6 whose pre-outage flow, the post outage flow on L8,

| L 8 | 3 | 5 | 6 | 0 | 124 | 0.2 | 12.54 | 0 |
| L 9 | 3 | 6 | 0 | 52.2 | 0.55 | 51.14 | 0 |
| L10 | 4 | 5 | 6 | 0 | 82.1 | 0.14 | 8.69 | 0 |
| L11 | 5 | 6 | 0 | 663 | 0.05 | 6.18 | 0 |

This LODF value can be found as the intersection of the eighth row with the sixth column of table 5.

This table contains all LODF for the eleven transmission lines when monitored with respect to the outage of any line including itself. The values in this table can then be used to predict the flow on any transmission line due to an outage of another line with respect with the base case operating point.

**Estimating Line Flows**

If from a reference operating point (base case) of a power system, it is desired to predict the real power flow on a monitored transmission line following the outage of a particular line, then the sensitivity factor table/matrix for that network similar to tables 3 and 4 be prepared. What follows is how to calculate the post contingency line flows using pre-calculated distribution factors.

**Predicting Flow Using PTDF**

Following power outage or injected power variation at a bus, the post contingency real power flow on any line can be estimated using

Assume that the output of the generator at bus 3 is increased from 50MW to 100MW and that the slack generator bus decreased its output in response to generator 3 and the unchanged network load of 300MW. Without resorting to AC or DC load flows, using PTDF, the real power flow on any monitored line can be predicted. Let’s monitor the flow on L4.

Where Monitored transmission line index; Bus where power is taken out (contingency bus) ; Bus where power is injected (post-contingency bus) ; Change in power at bus i;

**Predicting Flow Using LODF**

The LODF for a monitored transmission line on account of a particular outage is obtained by locating the monitored line along the rows of the LODF matrix and then tracing along the row to the appropriate outaged line column. The post-outage flow on the monitored line can be gotten using equation

Pre-contingency/pre-outage flow on line before it was outaged (opened); = Post - Outage flow on line with line out/open; = Pre-contingency/pre-outage flow on line, before line was outaged (opened)

From figure 2, following the outage of line index 6, and from table 4.

If we desire to know the real power that flows on a monitored line say line indexed 10.

Table 5 contains the AC and DC load flow results for the following operating conditions; Base case and Four Contingency cases (Generator at bus 3 Output variation: reduction and increment, Outages of generator at bus 2 and Line L6).

V. CONCLUSION
The justification for the use of DC load flow for quick estimation of transmission line flows is that its estimates compare proportionally to those from AC load flow so that they are effective when applied for contingency and security analyses. However, the use of base distribution factors to estimate transmission line flows from a known operation point yields faster results that matches in exactness the estimations from DC load flow analysis. These distribution factors in the form of PTDF and LODF are calculated and stored for a network and remains valid for use as long as the network is unmodified with the addition of a bus, loads, generators or transmission lines. Consequently, from any known operating point, the transmission line flow or loading/overload of any other operation point following the outage or variation of generator power output or the contingency of a transmission line may be estimated using the stored network values of PTDF and LODF. Unlike transmission flow results derived from AC techniques, PTDF and LODF flow estimates are not only non iterative but linear and has the exact value with flows from DC load flow analysis. Therefore, a quicker, linear and non-iterative method has been postulated to estimate transmission line flows from a known operating point with the slack bus responsible for active power exchanges.

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Table 5: LODF of monitored lines with respect to line Outages

| M | Contingent | Outaged | Line |
|---|------------|---------|------|
|   | L 1 | L 2 | L 3 | L 4 | L 5 | L 6 | L 7 | L 8 | L 9 | L 10 | L 11 |
| L 1 | -1 | 0.6353 | 0.5427 | -0.113 | -0.503 | -0.21 | -0.122 | -0.137 | 0.0135 | 0.0096 | 0.1316 |
| L 2 | 0.5948 | -1 | 0.4573 | -0.033 | 0.6121 | -0.062 | -0.036 | -0.04 | 0.004 | -0.327 | 0.0387 |
| L 3 | 0.4052 | 0.3647 | -1 | 0.1458 | -0.109 | 0.2721 | 0.158 | 0.1772 | -0.017 | 0.3174 | 0.1703 |
| L 4 | -0.103 | -0.0323 | 0.1783 | -1 | 0.1242 | 0.2262 | 0.4662 | -0.4 | -0.525 | 0.1706 | -0.132 |
| L 5 | -0.588 | 0.7647 | -0.1708 | 0.1591 | -1 | 0.2969 | 0.1724 | 0.1933 | -0.019 | -0.673 | 0.1858 |
| L 6 | -0.188 | -0.0589 | 0.325 | 0.2209 | 0.2264 | -1 | 0.2394 | 0.2685 | -0.026 | 0.311 | -0.258 |
| L 7 | -0.121 | -0.0381 | 0.2102 | 0.5073 | 0.1464 | 0.2667 | -1 | -0.199 | 0.5842 | 0.2011 | 0.4433 |
| L 8 | -0.118 | -0.0369 | 0.2036 | -0.376 | 0.1418 | 0.2583 | -0.172 | -1 | 0.4747 | 0.1948 | -0.425 |
Table 6: Pre and Post Contingency Line Loading for AC and DC load flow

| Line Code | Bus From-To | Line Type | Base Case Line Flow (MW) | Generator 3 Decrease | Generator 3 Increase | Generator Outage 2 | Line 6 Outage |
|-----------|-------------|-----------|--------------------------|----------------------|---------------------|-------------------|---------------|
| L 1       | 1 - 2       | 100       | 62.2 60.65 64.35 62.66 41.25 40.52 | 87.67 84.18 57.95 55.96 |
| L 2       | 1 - 4       | 100       | 82.8 76.66 84.49 78.13 66.36 61.92 | 100.41 92.4 81.54 75.28 |
| L 3       | 1 - 5       | 100       | 68.62.69 69.69 64.21 51.3 47.57 | 80.55 73.42 74.16 68.75 |
| L 4       | 2 - 3       | 60        | 148 13.68 16.58 15.39 -3.18 -3.4 | 12.94 10.96 20.34 18.72 |
| L 5       | 2 - 4       | 60        | 28.9 32.03 27.63 30.95 40.97 42.8 | 12.33 16.46 35.23 38.64 |
| L 6       | 2 - 5       | 60        | 21.9 22.26 22.05 22.43 20.93 20.55 | 16.32 17.3 0 0 |
| L 7       | 2 - 6       | 60        | 4.3 42.67 44.23 43.88 30.98 30.56 | 39.25 39.46 49.27 48.61 |
| L 8       | 3 - 5       | 60        | 124 12.54 11.09 11.09 25.98 26.98 | 9.94 9.43 18.52 18.29 |
| L 9       | 3 - 6       | 60        | 522 51.14 50.37 49.3 70.84 69.62 | 52.85 51.53 51.63 50.43 |
| L 10      | 4 - 5       | 60        | 821 8.69 8.59 9.09 4.58 4.72 | 8.14 8.86 13.23 13.92 |
| L 11      | 5 - 6       | 60        | 663 6.18 7.22 6.82 -0.26 -0.18 | 9.72 9.01 1.22 0.96 |

Total Load: 300 3 0 0 3 0 0 3 0 0 3 0 0 3 0 0 3 0 0 3 0 0 3 0 0 3 0 0
Gen 1 output: 213 2 0 0 218.54 2 0 5 158.91 1 5 0 268.63 2 5 0 213.65 2 0 0
Gen 2 output: 5 0 5 0 5 0 5 0 5 0 5 0 - - 5 0 5 0
Gen 3 output: 5 0 5 0 4 4 5 4 5 0 0 1 0 0 5 0 5 0 5 0 5 0