Radiative heat transfer in honeycomb structures-New simple analytical and numerical approaches

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Abstract Porous Honeycomb Structures present the interest of combining, at the same time, high thermal insulating properties, low density and sufficient mechanical resistance. However, their thermal properties remain relatively unexplored. The aim of this study is the modelling of the combined heat transfer and especially radiative heat transfer through this type of anisotropic porous material.

The equivalent radiative properties of the material are determined using ray-tracing procedures inside the honeycomb porous structure. From computational ray-tracing results, simple new analytical relations have been deduced. These useful analytical relations permit to determine radiative properties such as extinction, absorption and scattering coefficients and phase function functions of cell dimensions and optical properties of cell walls. The radiative properties of honeycomb material strongly depend on the direction of propagation.

From the radiative properties computed, we have estimated the radiative heat flux passing through slabs of honeycomb core materials submitted to a 1-D temperature difference between a hot and a cold plate. We have compared numerical results obtained from Discrete Ordinate Method with analytical results obtained from Rosseland-Deissler approximation. This approximation is usually used in the case of isotropic materials. We have extended it to anisotropic honeycomb materials. Indeed a mean over incident directions of Rosseland extinction coefficient is proposed. Results tend to show that Rosseland-Deissler extended approximation can be used as a first approximation. Deviation on radiative conductivity obtained from Rosseland-Deissler approximation and from the Discrete Ordinated Method are lower than 6.7% for all the cases studied.

Keywords: Honeycomb structures, conduction-radiation coupled heat transfer, radiative properties, Ray-Tracing, equivalent thermal conductivity, analytical model

NOMENCLATURE

| Symbol | Description |
|--------|-------------|
| a      | Cellular dimension in m |
| b      | Cellular dimension in m |
| Dcell  | Cell diameter in m |
| gₐ     | Asymmetry parameter |
| Iₗₑ(θ) | Spectral radiation intensity in W/(m²Sr) |
| Iₑₗ(θ) | Spectral radiation intensity emitted by black body in W/(m²Sr) |
| kair   | thermal conductivity of air in W/(mK) |
1. Introduction

The geometry of porous honeycomb structures (Fig. 1.a) presents the advantage to minimize the amount of used material to reach minimal weight and minimal material cost. Porous honeycomb structures present the interest of combining, at the same time, high thermal insulating properties, low density and sufficient mechanical resistance. Honeycomb materials in aluminum, fiberglass, cellulosic and advanced composite materials are widely used in the aerospace industry, they have been featured in aircraft and rockets since the 1950s. They can also be found in many other fields, from packaging materials in the form of paper-based honeycomb cardboard, to sporting goods like skis and snowboards. They are also used for the thermal insulation of solar collectors, storage tanks or pipelines of hot fluids.

Four different groups of researchers have conducted experimental or theoretical investigations on heat transfer through insulating materials with honeycomb structure: Morris et al. [1], Hollands et al.[2,3], Suehrcke et al. [4] and Arulanantham and Kaushika [5,6]. All of these studies have shown that heat transfer, in this kind of materials, is due to heat conduction but also to the propagation of thermal radiation while the convective heat transfer is generally negligible. They used ray-tracing model or zone method to evaluate theoretically the radiative transfer through the honeycomb structure. The local radiation balance in the medium requires integrating the incoming radiation from the cell faces and the surrounding faces. These studies are not well-adapted to a parametric study. These numerical tools are not based on Homogeneous Phase Approach. They do not require to determine radiative properties of homogeneous media. As a result radiative properties remain relatively unexplored in the literature. In order to predict the thermal performances of honeycomb structures, we have adopted another modelling approach, the so-called Homogeneous Phase Approach (HPA). This approach implies that the thermal behaviour of the composite material can be matched faithfully by an equivalent homogeneous semi-transparent conductive medium. It has been successfully used for the modelling of other type of highly porous materials such as open or closed cell foams [7,8] or fibrous insulations [9]. The principle is to assimilate the porous structure to a homogeneous semi-transparent material with equivalent conductive and radiative properties. The heat transfer is then governed by the Energy and Radiative Transfer Equations expressing the coupling between conductive and radiative heat transfer. This system of equations can be solved numerically using a combination of the volume method and the Discrete Ordinates Method (DOM).
The major difficulty is to determine the properties of the equivalent homogeneous medium that permits to match satisfactorily the thermal behaviour of this anisotropic porous material. A similar homogenization problem for anisotropic system of parallel ducts has been studied by Tancrez et al. [10]. In the present study, the equivalent radiative properties of the honeycomb structures have been evaluated using ray-tracing (RT) procedures inspired from the methodologies proposed by prior researchers. A special emphasis is put to deduce new simple analytical relations permitting to determine equivalent radiative properties functions of cell dimensions and cell walls optical properties. About the conductive properties, we used the classical parallel thermal resistance model to compute the effective thermal conductivity $k_c$. It is particularly well-suited for this type of materials. One of the advantages of the present approach using analytical relations to determine radiative and conductive properties is that it is well-adapted to a parametric study and to the optimization of these architectured materials.

2. Radiative transfer in semi-transparent media/ Homogeneous Radiative Transfer Equation

In the present study, we propose to model the heat transfer in the porous core materials by assimilating them to homogeneous semi-transparent conductive media. Indeed, this approximation has already proved its suitability to depict the radiation propagation in other types of porous materials. We can cite as example fibrous media [11,12] or cellular materials such as polymer [6,7], carbon [13] or metal foams [8]. The prediction of the equivalent radiative properties of the core material is the key issue of the present model. These properties must reproduce the radiation-matter interaction inside the honeycomb structure.

Porous media are often treated as continuous semi-transparent media that refract, absorb, emit and scatter radiation [14]. This radiative behavior may also vary with the radiation wavelength $\lambda$ considered. The propagation of radiation in these materials is entirely characterized by the spectral radiative properties : the monochromatic extinction coefficient, $\beta_\lambda$ (m$^{-1}$), the monochromatic scattering coefficient, $\sigma_\lambda$ (m$^{-1}$), the monochromatic absorption coefficient, $\kappa_\lambda$ (m$^{-1}$), the scattering phase function, $\Phi_\lambda(\Delta'\rightarrow\Delta)$, describing the angular distribution of the energy scattered by the

Figure 1: Illustration of the structure of the honeycomb and of the RT procedures developed to compute the radiative properties.
medium characterizes the probability, for a radiation incident in direction $\tilde{\Delta}'$, to be scattered in an elementary solid angle around the direction $\tilde{\Delta}$.

It is necessary to solve the Homogeneous Radiative Transfer Equation (HRTE) governing the spatial and angular distribution of monochromatic radiation intensity $I_\lambda(z, \mu)$ in the medium. Up to now, most of the composite materials studied using this approach (Expanded Polystyrene foams, fibrous materials), were generally considered as isotropic materials from a radiative point of view so that the radiative properties were independent of the propagation direction. In the case of honeycomb materials, the porous structure is anisotropic and the radiative properties are strongly dependent on the angle $\theta$ between the z-axis and the radiation direction. Moreover, the honeycomb structure presents an azimuthal symmetry. Therefore, for a 1-D heat transfer in the z-direction, the HRTE can be expressed by:

$$\mu \frac{\partial I_\lambda(z, \theta)}{\partial z} = -\beta_\lambda(\theta) I_\lambda(z, \theta) + \kappa_\lambda I_\lambda^0(T) + \frac{1}{2} \int_{\Omega \rightarrow 4\pi} \sigma_\lambda(\theta) \Phi_\lambda(\mu \rightarrow \mu) I_\lambda(z, \theta') d\Omega$$

(1)

with the boundary conditions

$$I_\lambda(0, \theta_{\mu>0}) = \varepsilon_{h,\lambda} I_\lambda^0(T_h) + 2(1 - \varepsilon_{h,\lambda}) \int_{\mu<0} I(0, \theta') \mu' d\mu'$$

$$I_\lambda(L, \theta_{\mu<0}) = \varepsilon_{c,\lambda} I_\lambda^0(T_c) + 2(1 - \varepsilon_{c,\lambda}) \int_{\mu>0} I(L, \theta') \mu' d\mu'$$

(2)

Several other characteristics derived from $\beta_\lambda$, $\omega_\lambda$, and $\Phi_\lambda(\tilde{\Delta}' \rightarrow \tilde{\Delta})$ are usually considered:

- the asymmetry parameter $g_\lambda(\tilde{\Delta}) = \frac{1}{4\pi} \int_{\Omega \rightarrow 4\pi} \Phi_\lambda(\tilde{\Delta}' \rightarrow \tilde{\Delta}) \tilde{\Delta} \times \tilde{\Delta} d\Omega$

(3)

which describes the way the energy is scattered: $0 < g < 1$ means that the radiation is predominantly scattered in forward directions whereas $-1 < g < 0$ means that the radiation is predominantly scattered in backward directions

- the weighted scattering and extinction coefficients

$$\sigma^+_{\lambda}(\tilde{\Delta}) = \sigma_{\lambda}(\tilde{\Delta})(1 - g_{\lambda}(\tilde{\Delta})) ; \beta^+_{\lambda}(\tilde{\Delta}) = \kappa_{\lambda}(\tilde{\Delta}) + \sigma_{\lambda}(\tilde{\Delta})(1 - g_{\lambda}(\tilde{\Delta})) = \kappa_{\lambda}(\tilde{\Delta}) + \sigma^+_{\lambda}(\tilde{\Delta})$$

(4)

Contrary to $\sigma_{\lambda}$, $\sigma^+_{\lambda}$ not only quantifies the probability of an intercepted ray to be scattered but encompasses the whole scattering phenomenon by taking into account the way the radiative energy is redistributed via the asymmetry factor $g_{\lambda}$. 

3. Modeling of the radiative of Porous Insulating Materials with honeycomb structures

The common feature of natural or man-made honeycomb structures is an array of hollow cells formed between thin vertical walls. The cells are often columnar and hexagonal in shape. The insulating materials that are investigated in this work are composed of a man-made honeycomb core sandwiched between two composite skins. The cells forming the hexagonal structure are partially transparent. The honeycomb cross section is considered to be hexagonal with the dimensions $a$, $b$ and $D_{cell}$ indicated on Fig. 1.b.

3.1 General Hypotheses.

Several hypotheses concerning the structure and characteristics of the core material are made to simplify the solution:

- The Geometric Optics Approximation (GOA) is used for the honeycomb structure, i.e., the radiation can be treated as pencils of rays propagating according to straight lines. The GOA is valid for the structures considered since the size parameter $x = \pi D_{cell} / \lambda >> 1$. In the present study, the radiation wavelengths are in the I.R ($\lambda = 10 \mu m$) whereas the size of the cells is several
mm. Thus, the GOA is fulfilled: \( x = \frac{\pi D_{\text{cell}}}{\lambda} \gg 1 \). Therefore, the propagation of the radiation in the materials can be modelled using the geometric optics laws of reflection and refraction. Moreover, the scattering of radiation due to diffraction is neglected since the directions of the rays diffracted by large scatterers \(( x \gg 1)\) are very close to the incident directions. Finally, the interparticle distance is much greater than the wavelength and the particles are assumed randomly dispersed so that the interference effects are negligible.

- The sheets forming the honeycomb structures are assumed partially transparent and reflecting with a transmittivity \( T_{r,1} \) and a reflectivity \( R_{1} \) which are assumed independent of the angle of incidence. The absorptivity \( \alpha_{0} \) is given by \( \alpha_{0} = 1-T_{r,1}-R_{1} \). Moreover, we take into account an intermediate type of reflection between perfectly specular or totally diffuse using a single parameter: the specularity parameter \( p_{s,\lambda} \). For \( p_{s,\lambda} = 1 \), the reflection is assumed perfectly specular whereas it is totally diffuse for \( p_{s,\lambda} = 0 \). For \( 0 < p_{s,\lambda} < 1 \), the incident rays have a probability \( p_{s,\lambda} \) to be reflected specularly and \( 1-p_{s,\lambda} \) to be reflected diffusely.

### 3.2 Theoretical basis.

Most of the previous studies about the radiative properties of porous materials have treated the problem by dividing the porous structure into particles whose shape and size permit to reproduce the internal structure of the porous material. Then, under the assumption of independent scattering, the equivalent radiative properties of the porous medium could be calculated by simply adding the radiative characteristics of each particle present in an elementary volume \([14,15]\).

However, although they are commonly used, these methods suffer from inaccuracies since the dependent scattering between the particles is neglected. Moreover, they are poorly adapted to materials with anisotropic radiative behaviour. That is the reason why we propose to use another approach for predicting the radiation-matter interaction in the core material. The method is based on RT procedures inside the exact structure and has been firstly proposed in recent publications \([16,17]\). It has been recently modified by Randrianalisoa and Baillis \([18]\) in order to reduce the computing time and to take into account the possible directional dependence of the properties\([19]\). This method is only applicable when the interaction of radiation with matter can be treated by the GOA.

The proposed method consists in computing the absorption, scattering and extinction mean free paths \( l_{\text{abs}}(\theta) \), \( l_{\text{scat}}(\theta) \) and \( l_{\text{ext}}(\theta) \) and the distribution of scattering directions of the radiation bundles \( W(\theta,\theta') \). The length of honeycomb cell is assumed very large compared to the cell diameter. These mean free paths correspond to the mean distance travelled by the rays between two consecutive extinctions, absorptions or scatterings and are related to the absorption, scattering and extinction coefficients by:

\[
\kappa(\theta) = \frac{1}{l_{\text{abs}}(\theta)} ; \quad \sigma(\theta) = \frac{1}{l_{\text{scat}}(\theta)} ; \quad \beta(\theta) = \sigma(\theta) + \kappa(\theta) = \frac{1}{l_{\text{ext}}(\theta)} = \frac{1}{l_{\text{abs}}(\theta)} + \frac{1}{l_{\text{scat}}(\theta)}
\]  

(5)

While the scattering phase function can be computed from \( W(\theta) \):

\[
\Phi(\theta,\theta') = \frac{W(\theta,\theta')}{\int_{\Omega=4\pi} W(\theta,\theta') d\Omega'}
\]  

(6)

\( l_{\text{abs}}(\theta), \ l_{\text{scat}}(\theta) \) and \( l_{\text{ext}}(\theta) \) and \( W(\theta,\theta') \) are computed through RT procedures inside the 3-D structure.

The principle of these procedures is to trace the histories of a great number of radiation bundles, denoted by \( N_{\text{rays}} \), propagating through the real medium under investigation until they are absorbed or scattered by the porous structure. During the travel of bundles, their histories are stored. In the case of honeycomb structures, the mean free paths and \( W(\theta,\theta') \) depend on the direction of propagation considered and therefore, they had to be evaluated for every direction in the range \( \theta = 0, \pi/2 \) (where \( \theta \) is the angle between the starting direction and the vertical z-axis; see Fig. 1.c).
In the RT procedure (illustrated on Fig. 1.c), the initial location of the ray can be chosen as if it corresponds to the location of a ray after an extinction event (i.e. scattering or absorption). For honeycomb Transparent Insulating Materials (TIM), given that the fluid phase is considered transparent (air), the rays are generated from the solid-fluid interface only. Practically, we randomly choose a point of the solid surface from which the ray will be emitted. The starting direction of the ray corresponds to the direction in which the radiative properties have to be computed. As explained before, all the starting directions in the range \( \theta = 0, \pi/2 \) have to be investigated. In practice, the radiative properties are evaluated for every degree of angle: \( \theta_0 = 0^\circ, \theta_1 = 1^\circ, \ldots, \theta_{90} = 90^\circ \).

Alternative RT procedure consists in choosing the initial location of the ray as if it corresponds to the location of an emission ray of the two isothermal hot and cold boundary plates (perpendicular to the lateral walls of honeycomb).

The path of the ray is tracked until it is intercepted by the solid phase (absorbed or reflected). The distance \( l \) travelled by the ray before interception is stored by incrementing the counter \( L_{\text{ext}} (L_{\text{ext}} = L_{\text{ext}} + l) \) and the type of extinction (absorption or reflection) is memorized by incrementing the counter \( N_{\text{abs}} \) \((N_{\text{abs}} = N_{\text{abs}} + 1, \text{if absorption}) \) or \( N_{\text{SCA}} \) \((N_{\text{SCA}} = N_{\text{SCA}} + 1, \text{if scattering}) \). If the ray is reflected, the reflecting angle \( \theta' \) between the incident and scattering directions is computed and the parameter \( W(\theta, \theta') \) is incremented by unity. At the end of the RT procedure, we have:

\[
L_{\text{ext}} = \frac{L_{\text{ext}}}{N_{\text{ray}}}; \quad l_{\text{sc}} = \frac{l_{\text{sc}}}{N_{\text{sc}}}; \quad L_{\text{ext}} = \frac{L_{\text{ext}}}{N_{\text{ray}}} \quad N_{\text{ray}} = \frac{N_{\text{ray}}}{N_{\text{ray}}}
\]  

The destiny of a ray impacting the solid phase is determined using randomly generated numbers. More details of the method can be found in the previous papers [18,19].

It is worth noting that in our computation, all the rays hitting the cell walls are not necessarily considered as scattering. Some rays intercepted by the cell walls are transmitted through the cell faces without any deviation and continue their path in the medium as if they had not impacted the solid phase. This is due to the fact that the cell walls are considered to be partially transparent. As a consequence, the extinction mean free path and thus the extinction coefficient may vary with the wavelength considered due to the variations of the transmittance of the sheets.

3.3 **Evolution of the radiative properties of Honeycomb structures.**

The radiative properties of honeycomb material strongly depend on the angle \( \theta \) between the direction of propagation and the vertical \( z \)-axis (see Fig. 1.c). It can be noted that radiative properties have been calculated functions of azimuthal angle, \( \phi \). Results have shown that \( \beta \) and \( \kappa \) are nearly independent of \( \phi \) whereas the asymmetry factor \( g \) is totally independent of \( \phi \). As a result azimuthal symmetry is assumed. We have computed the extinction, scattering and absorption coefficients as well as the scattering phase function for every angle \( \theta_i \) with \( i = 0, 90 \). The computations were conducted for different cell sizes (geometrical parameters \( a, b \) and \( D_{\text{cell}} \)) and different optical properties of the walls (\( \Tr, \Rs, \alpha_i \) and \( \p_{\alpha} \)). Moreover, for these investigations, we have also considered that the optical properties are grey in order to simplify the problem. The results are obtained in terms of extinction coefficient \( \beta \), scattering albedo \( \omega = \sigma/\beta \) and scattering phase function \( \Phi(\theta, \theta') \).

3.3.1 **Extinction coefficient**

The numerical results, for a given direction, only depend on the size of the cells and on the transmittivity of the walls. The two possible choices of initial location are investigated. From computational results, simple analytical relations have been deduced. Computational results calculated for transmittivity values comprised between 0 and 1 reveal that the following relation is verified:

\[
\beta_R = \frac{(1 - \Tr)}{a} \sqrt{(1 - \mu^2)} C
\]

The constant \( C \) depends on the choice of the initial location. When initial location of the rays is chosen on one of the six lateral walls of the hexagonal cell of honeycomb (corresponding to the location of a ray after an extinction event, case 1 on Fig. 1.c), the constant \( C = 0.87 \). When initial location of the ray is chosen as if it corresponds to the location of an emission ray of the two isothermal hot and cold boundary plates (perpendicular to the lateral walls of honeycomb, case 2 of Fig. 1.c): \( C = 1.2207 \)

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In the next part we only consider the case 1 in order to shorten the paper. But similar study could be performed for the case 2.

### 3.3.2 Scattering albedo

The scattering albedo $\omega$ provides information concerning the relative importance of absorption and scattering in the extinction phenomena. We have already shown in a previous study (Coquard and Baillis, 2004) that when the reflectivity $R$ is assumed independent of the incident angle, $\omega$ is independent of the shape of the structure or the type of reflection.

$$\omega = \frac{R}{R + \alpha}$$  \hspace{1cm} (9)

### 3.3.3 Scattering Phase Function

We illustrate the shapes of the scattering phase functions computed for walls with totally diffuse ($p_s=0$) on Fig. 2 and with perfectly specular reflection ($p_s=1$) on Fig. 3.

One can notice that the scattering phase function obtained assuming diffusely reflecting walls are isotropic for all the propagation directions, $\theta$ (Fig.2). The fact that the scattering phase functions are identical for all propagation angles $\theta$ is not surprising since, for each reflection event, the direction of scattering is chosen independently of the incident direction on the wall.

When specular reflection is assumed, the scattered energy is redirected in a scattering direction with an angle $\theta' = \theta$ since the walls encountered are parallel to the z-axis (Fig. 3). Thus, from an energetic point of view, all is going on as if the rays were not reflected by the walls and the asymmetry parameter $g$ is equal to 1 for all propagation directions $\theta$.

As a result the phase function can be expressed with the simple relation by using the Dirac function $\delta$:

$$\Phi(\theta, \theta') = p_s \cdot \Phi(\theta - \theta') + (1 - p_s)$$  \hspace{1cm} (10)

### 4. Evolution of the equivalent thermal conductivity of honeycomb Porous Insulating Materials

From the radiative properties computed using the method described in the previous section, we have estimated the radiative heat flux passing through slabs of honeycomb core materials submitted to a 1-D temperature difference between a hot and a cold plate. We compare numerical results obtained from DOM with results obtained from Rosseland-Deissler analytical approximation.

#### 4.1 Numerical solution of the coupled heat transfer

The HRTE can be solved numerically by using a combination of the control volume method and the Discrete Ordinates Method (DOM) [14,20,21]. It is one of the most frequently used method among others (spherical harmonics method, zone method of Hottel, RT methods…) and gives accurate results. The principle is to divide the slab thickness of core material in elementary control volumes. The temperature of the medium at the center of each volume, i.e. temperature distribution in the medium, has to be calculated. The equivalent and “radiative” conductivities are computed by solving the Energy Equation and HRTE. For the computation of $k_r$, radiative equilibrium has to be considered.

In both cases, we have to solve the following system of equations:

$$\left\{ \begin{array}{l}
\frac{\partial q^z_r}{\partial z} = - \frac{\partial q^z_c}{\partial z} = k_c \cdot \frac{\partial^2 q^z}{\partial z^2} \quad \text{with} \quad k_c = k_{\text{air}} \cdot \varepsilon + k_s \cdot (1 - \varepsilon) \quad \text{for} \quad k_{\text{eqv}} ; \\
\frac{\partial q^z_c}{\partial z} = 0 \quad \text{for} \quad k_r
\end{array} \right.$$  \hspace{1cm} (11)

with $q_c(z, \overline{\Delta}) = \int_{\Omega=4\pi}^\infty I_1(z, \overline{\Delta}) \overline{\Delta} \, d\Omega \, d\lambda$  \hspace{1cm} (12)

and $k_r$ is simply given by:

$$k_r = \frac{q^z_r}{\Delta T / L}$$  \hspace{1cm} (13)

The DOM has already been explained in numerous previous publications as (Siegel and Howell, 1992, Dombrovsky and Baillis, 2010). The accuracy of the results is entirely dependent on the angular
Therefore, in the present study, we choose a very fine discretization dividing the $2\pi$ radians in 180 directions of $1^\circ$ and whose weighting factors are proportional to the solid angle they encompass.

Figure 2. Evolution of the scattering phase functions $\Phi(\theta, \theta')$ computed for different direction of propagation $\theta$ and assuming a totally diffuse reflection by the walls ($p_s=0$)

4.2 Rosseland-Deissler analytical Approximation

The Rosseland-Deissler approximation is the following (Deissler 1964):

$$k_r = \frac{4\sigma_{SB} T^3 L}{\left(\frac{1}{\varepsilon_c} + \frac{1}{\varepsilon_h} - 1\right) + \frac{3}{4} \beta_R L}$$

where $T_m = (T_c + T_h)/2$ and $\beta_R = \langle \kappa \rangle + \langle \sigma(1 - g) \rangle = \langle \beta \rangle \times (1 - \omega g)$.

$\langle \beta \rangle$ is the mean calculation over $\theta$ angle of $\beta(\theta)$. Extinction coefficients have been proved to verify analytical relation Eq. (8). As a result the calculation of $\langle \beta \rangle$ depends on the mean calculation $\sqrt{1 - \mu^2}$. But the question arises how to calculate this mean value. In order to answer this question, the mean $\sqrt{1 - \mu^2}$ is obtained by identification by minimizing the relative deviation between results obtained from reference DOM and Rosseland-Deissler calculation for different specular parameter $p_s=0.5$ or 1, different albedo values comprised between 0 and 1 (0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9) and different thickness of honeycomb slab comprised between 0.001 and 1 m (0.001, 0.008, 0.027, 0.064, 0.125, 0.216, 0.343, 0.512, 0.729 and 1 m).

The identified value verifies $\sqrt{1 - \mu^2} = 0.4855$ and thus $\beta_R = \frac{(1 - T_m)}{a} 0.4855 C$.

The mean relative deviation between DOM and Rosseland-Deissler approximation using this identified value is 6.7%. Figure 4 shows as example some cases of the evolution of the radiative equivalent conductivities obtained with the thickness of the slab for honeycomb structures obtained for DOM and Rosseland with this identified value. Note that it tends to show that Rosseland mean extinction can be approximatively obtained by the mean of the extinction mean free path weighted by $\mu d\mu$

$$\frac{1}{\beta_R} \approx \frac{1}{\beta} \mu d\mu = 0.5$$

5. Conclusions

The radiative heat transfer in Porous Insulating Materials with honeycomb structures has been modelled by assuming the core material to an equivalent semi-transparent medium with spectral radiative properties depending on its porous structure and composition. This approach is rather new in
the field of the modelling of heat transfer in honeycomb structures. The equivalent radiative properties of the core material have been modelled using a methodology based on the GOA which was recently proposed by Coquard and Baillis (Coquard and Baillis, 2004; Coquard and Baillis, 2005) and Randrianalisoa and Baillis (Randrianalisoa and Baillis, 2010). It is particularly well-adapted to the honeycomb structures studied given that the geometrical conditions ($x = \frac{\pi D_{cell}}{\lambda} >> 1$) are largely fulfilled and that the RT procedures used in the modelling permit to take into account the particular porous structure very accurately. The particularity of honeycomb cellular material concerns the high anisotropy of the radiative properties.

From computational RT results, new simple useful analytical relations have been deduced to determine radiative properties such as extinction, absorption and scattering coefficients and phase function.

From the radiative properties computed, we have estimated the total heat flux passing through slabs of honeycomb core materials submitted to a 1-D temperature difference between a hot and a cold plate. We have compared numerical results obtained from DOM with analytical results obtained from Rosseland-Deissler approximation. Rosseland mean extinction has been proved to verify a simple analytical relation obtained from the inverse of the mean of the extinction mean free path. The mean is calculated over the polar angle.

Results tend to show that Rosseland-Deissler can be used as a first approximation. Deviations on radiative conductivity obtained between the Rosseland-Deissler approximation and the DOM results are lower than 6.7%.

![Figure 3](image_url) Evolution of the scattering phase function $\Phi(\theta', \theta)$ computed for different direction of propagation $\theta'$ and assuming a specular reflection by the walls ($p_s=1$)

![Figure 4](image_url) Evolution of the radiative equivalent conductivities with the thickness of the slab for honeycomb structures with $D_{cell}=10.0$ mm, $T_r=0$, $p_s=1$, $\varepsilon_r=\varepsilon_h=1$ $T_m=293K$ and $\Delta T=10K$

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