CP violation in $B$ decays and supersymmetry

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Abstract

CP violation in hadronic $B$-decays is studied in a definite and well motivated framework for flavour physics and supersymmetry. Possible deviations from the standard model both in mixing and in decay amplitudes are discussed. An attempt is made to describe an experimental strategy for looking at these deviations and for measuring the relevant parameters.

1 Introduction

Supersymmetry may lead to deviations from the expectations of the Standard Model in a few crucial observables in flavour physics. Other than the usual Cabibbo-Kobayashi-Maskawa matrix in charged current weak interactions, in a generic supersymmetric model there are at least two new possible sources of significant flavour violation. One is the flavour violating interaction driven by the top Yukawa coupling to the charged Higgs, with its supersymmetric counterpart. The other is a possible flavour violation, originated by misaligned fermion and sfermion mass matrices, that appears in gaugino (and higgsino) interactions of matter supermultiplets.

The nature of these two new sources of flavour violation is very different. The first is there in any realistic supersymmetric extension of the SM, it is essentially unrelated to supersymmetry breaking and its flavour structure is controlled by the same CKM matrix. Its effects may be significant in some loop-induced $B$ decays, like $b \to s\gamma$ or $b \to s\ell\bar{\ell}$, but are less important in $\Delta B = 2$ or $\Delta S = 2$ transitions. In any event it will not introduce any new CP-violating phase nor it will affect leptons.

The second potential source of flavour violation, on the contrary, arises from new CKM-like mixing matrices $W$ occurring in the gaugino ($\lambda$) interactions

$$\tilde{f}_L^i W_{i_L l_L} f_L \lambda + \tilde{f}_R^i W_{i_R l_R} f_R \lambda + \text{h.c.}$$

(1)

of matter fermions of given charge, $f = u, d$ or $e$, and chirality, $L$ or $R$, with their superpartners $\tilde{f}$. The presence of a non trivial $W$-matrix in (1) has everything to do with supersymmetry breaking, since, in the supersymmetric limit, $W = 1$. In particular, it is essential that the supersymmetry breaking masses of the scalars $\tilde{f}$ be non degenerate in generation space, since, otherwise, any non-trivial mixing matrix can be rotated away.

There are at least two independent physical motivations that lead to a particular realization of (1), which is of interest in this paper. One case is supersymmetric unification with a hardness scale, defined as the highest scale at which supersymmetry breaking masses appear as local interactions, higher than the unification scale $M_G$. The other is the presence of a flavour symmetry, e.g. a $U(2)$ symmetry, that might relate the flavour structure of the fermion mass matrices to the one of the scalar mass matrices. As pointed out in previous work, from this physical picture we expect possibly significant deviations from the SM in lepton flavour violating processes (mostly $\mu \to e\gamma$ and $\mu \to e$ conversion in atoms), in Electric Dipole Moments (EDMs) of the electron and of the neutron, in CP-violation in the $K$-system and, finally, in mixing and CP-violation in the $B$-system. This last issue is the subject of this paper.

\footnote{For alternatives, see ref. and references therein.}
2 The framework defined

For the present purposes, the physical situation can be characterized as follows:

i) The d-type quarks of given chirality, \( d = \{ d, s, b \} \), and their superpartners \( \tilde{d} \) have gluino and photino interactions of the form \([5]\) with \( W \) matrix elements of the order of the corresponding CKM matrix elements. Of special interest are four parameters, independent from phase conventions in the physical d-quark basis,

\[
\begin{align*}
\omega_{d_L} &= \frac{W_{b_L}^* W_{b_L}^T}{V_{ta} V_{tb}}, & \omega_{d_R} &= \frac{W_{b_R}^* W_{b_R}^T}{V_{ta} V_{tb}} \\
\omega_{s_L} &= \frac{W_{b_L}^* W_{b_L}^T}{V_{ta} V_{tb}}, & \omega_{s_R} &= \frac{W_{b_R}^* W_{b_R}^T}{V_{ta} V_{tb}}
\end{align*}
\]

(2)

We expect that the various \( \omega \) be complex numbers with modulus and phase of order unity.

ii) The \( \tilde{d} \) and \( \tilde{s} \) squarks, Left and Right, are degenerate to a high degree, with squared mass \( m_{12}^2 \) and \( m_{22}^2 \), whereas \( \tilde{b}_L \) and \( \tilde{b}_R \) have squared masses \( m_{12}^2 \) and \( m_{22}^2 \) which can differ from \( m_{12}^2 \) by relative order of unity. Furthermore, it is more likely that \( m_{b_L,R} < m_{12} \) \([4]\).

iii) In the basis where the \( \omega \) are defined, there is still a small admixture between the \( \tilde{d}_L \) and \( \tilde{d}_R \) squarks. We assume, for simplicity, that the \( A \)-terms do not contain new flavour violations. More precisely, for the scalar mixing terms we take the form

\[
m_b(A_b + \mu \tan \beta)(\tilde{d}_L W_{d_L} \bar{b}_L W_{b_R} \tilde{d}_R + \text{h.c.}).
\]

Being proportional to the relatively small \( b \)-quark mass, we can treat these terms as perturbations.

Given this framework, loops of supersymmetric particles contribute with extra terms to the CP-violating parameter in \( K \)-physics, \( \varepsilon_K \), to the neutron EDM, \( d_N \), and to the \( B \bar{B} \) matrix elements of the effective Hamiltonian, \( M_{12}(B) \equiv \langle B^0 | H_{\text{eff}} | B^0 \rangle \). Useful approximate formulæ for these contributions are

\[
\begin{align}
|\varepsilon_K|^\text{SUSY} &\approx \alpha_3 \frac{f_K^2 m_K^3}{9 \sqrt{2} m_t^2 \Delta m_K} |\omega_{d_L} \omega_{d_R} \omega_{s_L}^* \omega_{s_R}^* (V_{ta} V_{tb})^2| \frac{\eta_{L,R} \eta_{L,R}}{\max(m_{b_L}^2, m_{b_R}^2, M_3^2)} \\
d_N|^\text{SUSY} &\approx \frac{2 \alpha_3}{81 \pi} m_b \text{Im}(\omega_{d_L} \omega_{d_R}) |V_{ub}^2| (A_b + \mu \tan \beta) M_3 \frac{\eta_{L,R} \eta_{L,R}}{\max(m_{b_L}^2, m_{b_R}^2, M_3^2)} \\
M_{12}(B_d)^|\text{SUSY} &\approx \frac{2 \alpha_3}{9} f^2_{B_d} m_{B_d} (V_{ta} V_{tb})^2 \left[ \frac{\omega_{d_L}^2 \eta_{L,R}^2}{\max(m_{b_L}^2, M_3^2)} + \frac{\omega_{d_R}^2 \eta_{L,R}^2}{\max(m_{b_R}^2, M_3^2)} + 4 \frac{\omega_{d_L}^* \omega_{d_R} \eta_{L,R} \eta_{L,R}}{\max(m_{b_L}^2, m_{b_R}^2, M_3^2)} \right] \\
M_{12}(B_s)^|\text{SUSY} &\approx M_{12}(B_d)^|\text{SUSY} \text{ with } d \rightarrow s
\end{align}
\]

(3a) (3b) (3c) (3d)

where we have used standard notations for the various quantities, \( M_3 \) is the gluino mass, and

\[
\eta_{L,R} \equiv 1 - \frac{m_{b_L,R}^2}{m_{12}^2} = 1 - \frac{m_{b_L,R}^2}{m_{12}^2},
\]

(4)

is a super-GIM suppression factor. At the unification scale we expect \( \eta_{L,R}^G \) of order unity. A large gluino mass, however, can significantly reduce \( \eta_{L,R} \) at the Fermi scale, since\([4]\)

\[
\eta_{L,R} \approx \frac{\eta_{L,R}^G}{1 + 5.3(M_3^2/m_{12}^2) |G_0|}/M_3
\]

(5)

(“gluino focusing”), a fact to be taken into account in realistic estimates of various effects \([4]\).
Figure 1: Scatter plots of the supersymmetric corrections to the phases of the $B_d$ mixing, $2\varphi_{B_d}$, and of the $B_d \to \phi K_S$ decay amplitude. The empty points (‘◦’ are excluded by too large supersymmetric contributions to $\varepsilon_K$ or $d_N$. The darker area is our estimate of where these effects could be detected in view of the theoretical uncertainties.

3 Supersymmetric effects in the decay amplitudes

To study CP-violation in $B$-decays, one has to investigate the possible contributions from supersymmetric loops not only to the $\Delta B = 2$ mixing amplitudes but also to direct decay amplitudes. In general, this is not an easy task. There are many possible different channels. The SM amplitudes themselves are largely uncertain. Finally, unlike the neutron EDM and the $\Delta S = 2$ and $\Delta B = 2$ mixings, the $\Delta B = 1$ non-leptonic amplitudes receive a priori comparable contributions from three different kinds of supersymmetric loops: “electric” penguins, “magnetic” penguins and box diagrams. Furthermore, to assess the possible size of these contributions, it is essential to take into account the constraints coming from the other observables mentioned above, which have either already been measured ($\varepsilon_K$, $\Delta m_{B_d}$) or are subject to stringent bounds (EDMs, $\mu \to e\gamma$, $\mu \to e$ conversion).

The interesting things to compare are supersymmetric loops with SM penguins [8]. Anticipating the following discussion, we expect the second to dominate over the first. As such, channels with negligible SM penguins, like $B_d \to \psi K_S$, will not be affected by supersymmetric loops as well. On the contrary, one has to concentrate on modes arising, in the SM, from pure loops, like those induced by $b \to sdd$ and $b \to sss$. Furthermore, $b \to sss$ is particularly interesting since in the SM the corresponding amplitude only depends on one overall weak phase to a very good approximation [4]. Confining to the appendices a detailed description of the calculation of the $b \to s\bar{ss}$ effective Hamiltonian and of the (uncertain) estimates of the matrix elements for the $B_d \to \phi K_S$ decay amplitude, we discuss the naive expectation for this amplitude.

In the SM, the amplitude for a $b \to s\bar{ss}$ decay has a parametric dependence on the gauge couplings and on particle masses of the form

$$A_{SM} \sim \frac{\alpha_2 \alpha_3}{M_W^2} \ln \frac{m_t^2}{m_c^2}. \quad (6)$$

The appearance of the charm mass in the argument of the logarithm in (6) reflects an infrared divergence in the SM penguin, making its absolute estimate particularly uncertain. At the same time, the supersymmetric

\[A_{SM} \sim \frac{\alpha_2 \alpha_3}{M_W^2} \ln \frac{m_t^2}{m_c^2}.\]
contribution has the form

\[ A_{\text{SUSY}} \sim \frac{\alpha_3^2}{\max(m_b^2, M_W^2)} \eta \]  

(7)

where we do not distinguish between \( L \) and \( R \) in \( m_\text{ SUSY} \) and \( \eta \). Eq. (7) is appropriate for “electric” penguins, but box diagrams and “magnetic” penguins may give comparable contributions.

In an analogous way, the SM box diagrams for the \( \Delta B = 2 \) mixings has the parametric dependence

\[ M_{12}^{\text{SM}} \sim \frac{\alpha_2^2}{M_W^2} \]  

(8)

whereas, from eq. (3c),

\[ M_{12}^{\text{SUSY}} \sim \frac{\alpha_2^2}{\max(m_b^2, M_W^2)} \eta^2 \]  

(9)

Taking all \( \omega \)'s equal to unity, eq.s (8-9) lead to the naive estimate of the relative importance of supersymmetric corrections in decay amplitudes compared to mixing as

\[ \frac{A_{\text{SUSY}}/A_{\text{SM}}}{M_{12}^{\text{SUSY}}/M_{12}^{\text{SM}}} \sim \frac{\alpha_2}{\alpha_3} \frac{1}{\eta \ln m_t^2/m_\text{ SUSY}^2} \sim \frac{3\%}{\eta} \]  

(10)

Sizable supersymmetric contributions in the decay amplitudes appear less likely than in the mixing. Although the double ratio in (10) increases when \( \eta \) decreases, both \( A_{\text{SUSY}} \) and \( M_{12}^{\text{SUSY}} \) get suppressed in this case.

This naive expectation is confirmed by the detailed calculation described in the appendices, whose results are shown in figure [in the form of scatter plots]. We compare the expected correction to the phase of the \( B_d \to \phi K_S \) decay amplitude with the correction to the phase of the \( B_d-B_s \) mixing. In each graph the physical gluino mass is taken fix as indicated, as are the factors \( \eta \) at the unification scale, \( \eta_{\text{G}} = \eta_{\text{G}} = 0.75 \) (\( m_\text{ SUSY}^2 = m_\text{ SUSY}^2/2 \)). On the contrary we allow variations of the sbottom masses \( m_b^2 \) in the range \( m_b^2 = m_b^2 = (1/3 \pm 3)M_4^2 \). The Higgs soft masses are varied in the same range, so that, taking a moderate \( \tan \beta = (1.5 \pm 5) \) at the electroweak scale, the \( \mu \) term is determined (up to its sign) from the minimization equations. The \( A \)-term \( A_\text{G} \) is varied in the range \( (-3 \pm 3)M_4^2 \). Finally, all the \( \omega \) factors at the unification scale are varied between \( \frac{1}{3} \) and 3 in modulus with random phases. The empty dots correspond to points of parameter space that give rise to too large corrections to \( \epsilon_K \) and/or to the neutron electric dipole. Possibly stronger bounds are obtained from effects in the leptonic sector (like \( \mu \to e\gamma \)) but a reliable comparison is possible only under specific assumptions on the sfermion spectrum and the mixing angles. Most of the points with small relative corrections both to mixing and decay have a strong super-GIM suppression caused by “gluino focusing” (small \( \eta \) at low energy). The only possible enhancement of \( A_{\text{SUSY}} \) with respect to the naive estimate might be due to “magnetic” penguins that give a contribution proportional to \( (A_\text{G} + \mu \tan \beta)/M_3 \). This factor cannot be too large, however, because it also enhances the neutron EDM. As discussed below, the darker areas in fig. [correspond to our estimate of where these effects could be detected in view of the theoretical uncertainties. We have done an analogous study for the \( B_s \)-system, which shows similar effects to the \( B_d \) case, although in general not strictly correlated to it.

### 4 Measurements

Suppose that supersymmetric loops indeed contribute to CP-violating observables in \( B \)-decays. How can they be disentangled from pure SM effects and how the relevant parameters can be finally measured? The answers to these questions will crucially depend on the precise evolution of the experimental programme, which is difficult to anticipate. Furthermore, these same questions have already attracted interest in the literature [8, 10]. Still, we find useful to summarize our view on this issue, having in mind the special physical situation outlined above.

As customary [11, 12], we define two new phases, \( \varphi_{B_d} \) and \( \varphi_{B_s} \) as

\[ M_{12}^{\text{SM}}(B_{d,s}) + M_{12}^{\text{SUSY}}(B_{d,s}) = M_{12}^{\text{SM}}(B_{d,s})r_{d,s} \exp[2i\varphi_{B_{d,s}}] \]  

(11)

\(^4\text{The resulting effects are not different in minimal SO(10) unification, even if this model does not give new CKM-like phases in }\omega_d \text{ and } \omega_s \)
with \( r_{d,s} \) real positive numbers. These phases have direct physical meaning, as the usual angles \( \alpha, \beta, \gamma \) of the CKM unitarity triangle (\( \alpha + \beta + \gamma = \pi \)), being independent from any phase convention for the various fields.

A first point, already mentioned, is that the \( B \)-decay amplitudes largely dominated in the SM by tree level transitions will not be affected by supersymmetric corrections. In particular the CP-asymmetries

\[
\frac{\Gamma(B^0(t) \to f) - \Gamma(B^0(t) \to \bar{f})}{\Gamma(B^0(t) \to f) + \Gamma(B^0(t) \to \bar{f})} = -a(B \to f) \sin \Delta m_B t, \quad \Delta m_B > 0,
\]

for the modes \( B_d \to \psi K_S \) or \( B_s \to \psi \phi \) will measure well defined combinations of CP-phases

\[
a(B_d \to \psi K_S) = \sin 2(\beta + \varphi_{B_d}),
\]

\[
a(B_s \to \psi \phi) = \sin 2\varphi_{B_s}.
\]

While \( 13b \) could allow a direct measurement of \( \varphi_{B_d} \), the study of such ‘clean’ \( B \)-decays alone, however, will not allow the extraction of the CKM angles \( \alpha, \beta, \gamma \), unless supplemented with independent informations on the lengths of the sides of the unitarity triangle, i.e. on \( |V_{ub}| \) and \( |V_{td}| \). Some \( B \)-decays may provide this information, but only if hadronic uncertainties are kept under control. Notice on the contrary that the ratio of the mixing parameters

\[
\left| \frac{M_{12}(B_d)}{M_{12}(B_s)} \right| = \frac{r_d}{r_s} \left| \frac{V_{td}}{V_{ts}} \right|^2,
\]

up to perhaps controllable SU(3)-breaking effects, will involve an extra unknown, \( r_d/r_s \).

To go further, one would like to check if the supersymmetric corrections to the decay amplitudes are indeed relatively suppressed, as indicated by the calculation described above. A sign that they are not, would be a detection of a large difference in appropriate asymmetries, sufficiently free from theoretical uncertainties, like

\[
a(B_d \to \phi K_S) \neq a(B_d \to \psi K_S).
\]

On the basis of the previous calculation, we expect a phase difference \( \delta \varphi \) between the phases entering the left and right hand sides of \( 13b \) not exceeding 0.2. The major uncertainty in determining the maximal effect in \( \delta \varphi \) is represented by the estimate of the matrix element of the magnetic penguin operator. In the case of \( 13b \), however, a phase difference \( \delta \varphi \geq 0.1 \) would be sufficient to indicate an effect beyond the SM.

Let us suppose that we can neglect supersymmetric corrections in decay amplitudes. How could one then proceed in order to measure the angles \( \alpha, \beta, \gamma \) and \( \varphi_{B_d}, \varphi_{B_s} \)? Let us also take the point of view that the various hadronic uncertainties will deteriorate the experimental informations obtainable on the sides of the unitarity triangle in a significant way. Although this may be pessimistic, the problem of the theoretical uncertainties is an obvious matter of concern if one wants to extract new effects from the experimental data.

Although not at the first step of a \( B \)-factory, the measurements of \( B_d \to \pi \pi \), with an appropriate isospin analysis \( 13b \), and of \( B \to D K \) \( 16 \) will give access to \( \alpha - \varphi_{B_d} \) and \( \gamma \) respectively. Notice however that even these informations, together with \( 13b \), are not enough to separate \( \alpha, \beta \) and \( \varphi_{B_s} \).

The possibility that has been considered in the context of the SM is a cumulative study \( 17, 18, 19 \) of \( B \to \pi \pi, \pi K \) and \( KK \). In the present situation we would summarize this possibility as follows. Of special interest are the decays \( B^0_d \to \pi^+\pi^-, B^+ \to \pi^+K^0, B^0 \to \pi^-K^+ \) and their charge conjugate. It is useful to parametrize the corresponding amplitudes as

\[
A(B^0_d \to \pi^+\pi^-) = T^\pi + P_d
\]

\[
A(B^+ \to \pi^+K^0) = P_s
\]

\[
A(B^0_d \to \pi^-K^+) = T^K + P_s
\]

The \( B_s \) system is different from \( B_d \) because of the large mixing and the non negligible \( \Delta \Gamma/T \), which is ignored here. In view of this, untagged rates might be more useful to extract \( 2\varphi_{B_s} \) from \( B_s \to \psi \phi \) \( 13b \).

Independent clean informations \( 14 \) could come from \( K^+ \to \pi^+\nu\bar{\nu} \) and \( K_L \to \pi^0\nu\bar{\nu} \) which are not affected by the supersymmetric corrections considered in this paper \( 14 \).
where, to a good accuracy, $T$ represents tree level contributions whereas $P$ arises from gluonic penguins. Defining

$$T^\pi = V_{ud}V_{ub}^{*}t^\pi, \quad T^K = V_{us}V_{ub}^{*}t^K,$$

\[ (17a) \]

$$P_d = V_{cd}V_{tb}p_d + V_{cd}V_{td}^* \Delta p_d, \quad P_s = V_{ts}V_{tb}^* p_s + V_{ts}V_{td}^* \Delta p_s,$$

\[ (17b) \]

\[ (17c) \]

one gets the amplitudes for the charge conjugate modes by complex conjugation of the CKM matrix elements $V_{ij}$, whereas the complex amplitudes $t, p, \Delta p$ remain unchanged.

The measurements of $\Gamma(B^0 \rightarrow \pi^+ K^0), \Gamma(B_d^0 \rightarrow \pi^- K^+), \Gamma(B_s^0 \rightarrow \pi^- K^+), \Gamma(B_d^0(t) \rightarrow \pi^+ \pi^-)$ and $\Gamma(B_s^0(t) \rightarrow \pi^+ \pi^-)$ give six relations (five widths and $a(B_d \rightarrow \pi^+ \pi^-)$) among the amplitudes $T, P$ and the mixing phase $\beta + \varphi_{B_d}$. In general, however, even taking into account that $t^\pi$ and $t^K$ can be taken real by phase redefinitions, this is not enough to constrain the many more unknowns in (17). To get such constraints it is possible to explore the SU(3) relations among the different amplitudes and/or, but this may be more risky, the fact that $|\Delta p/p|$ could be sufficiently small.

In the exact SU(3) limit one has

$$t^\pi = t^K \equiv |t|,$$

\[ (18a) \]

$$p_d = p_s \equiv |p|e^{i\delta_p},$$

\[ (18b) \]

$$\Delta p_d = \Delta p_s \equiv |\Delta p|e^{i\delta_p},$$

\[ (18c) \]

i.e. five real parameters. To the extent that SU(3) violations maybe accounted for without introducing new unknowns, the programme of measuring all the CP phases could be successfully accomplished. Measurements of the suppressed modes $B \rightarrow KK$ will also provide additional information.

Altogether, it seems possible to extract $\varphi_{B_d}$ from the data in different ways, all involving some amount of hadronic uncertainty.

5 Conclusions

In conclusion, we have studied the possible deviations from the expectation of the standard model in signals of CP-violation in hadronic $B$ decays, as they occur in a definite and well motivated framework for flavour physics and supersymmetry. Such deviations can arise either though extra phases in $B_d, B_s$ and $B_s, B_s$ mixing, or in the decay amplitudes. To assess both the absolute and the relative importance of these new effects, we have considered the constraints coming from other flavour observables either already measured or subject to strong bounds.

We find that visible effects are possible in a portion of the parameter space. To make them manifest, however, it will be essential to study appropriate observables with minimal contamination from various theoretical uncertainties, as already suggested by several authors.

A Supersymmetric corrections to mixing-induced CP violation

The SM contribution to the $B_q^0 \bar{B}_q^0$ mixing ($q = \{d, s\}$) is

$$M_{12}(B_q)|_{SM} = \frac{\alpha_s^2}{8M_W^2} \left( \frac{3}{2} f_B^2 m_B \right) (V_{ub}V_{tb})^2 \eta^2 - 2.5 \quad (A.1a)$$

Here $\eta = (\alpha_3(M_Z)/\alpha_3(m_b))^{3/46} \approx 0.954$ gives the leading order QCD renormalization effect and we have computed the matrix elements in the vacuum saturation approximation. In the same approximation the supersymmetric gluino correction is given by

7 "Annihilation" diagrams and electro-weak penguins are not included in (17). They are unlikely, however, to play any significant role.
\[ M_{12}(B)_{\text{SUSY}} = \frac{\alpha_i^3}{9 M_3^2} \left( f_B m_B (V_{tb} V_{ts})^2 \left[ \eta^2 \omega_{LL}^2 \{11B_{\gamma} + B_{\infty}\} \{\ell_3 - \ell_{12}\} \{\ell_3 - \ell_{12}\} \right] + \right. \\
+ \eta_3 \omega_{LL} \omega_{BB} \left[ \frac{20 \eta^2 - 14 \eta_{16}}{3} B_{\gamma} + \frac{\eta^2 + 56 \eta_{16}}{3} B_{\infty}\right] \{\ell_3 - \ell_{12}\} \{r_3 - r_{12}\} + \\
\left. + \eta_3 \omega_{RR} \{11B_{\gamma} + B_{\infty}\} \{r_3 - r_{12}\} \{r_3 - r_{12}\} \right] \] 

where, in order to avoid long expressions, we have introduced the following compact notations:

\[ \ell_3 \equiv \frac{m_{bl}^2}{M_3^2}, \quad \ell_{12} \equiv \frac{m_{12}^2}{M_3^2}, \quad r_3 \equiv \frac{m_{br}^2}{M_3^2}, \quad r_{12} \equiv \frac{m_{12r}^2}{M_3^2} \]

and

\[ f(\{a_1 \pm a_2\}) \equiv f(a_1) \pm f(a_2), \quad \{f_1 \pm f_2\}(a) \equiv f_1(a) \pm f_2(a) \]

The various loop functions are defined below in eqs \([3,4]\). The QCD corrections have been taken from \([21]\).

The other supersymmetric contribution are not considered here. The ones mediated by charged Higgs and by the higgsino component of the charginos have the same phase and angles of the SM contribution and smaller size. The ones mediated by the weak gauginos are proportional to the new mixing parameters \(\omega\) but have smaller vertex factors.

## B Supersymmetric corrections to direct CP violation

The dominant SM contribution to the effective Hamiltonian for \(b \to s \bar{s}s\), recalled here to establish the notation and to define our approximations, is well known \([4]\) and is given by “electric” penguins, that are enhanced by an infrared logarithmic factor, \(\ln M_W^2/m_b^2\). For this reason there are two significant contributions from top and charm loops. Since \(V_{tb} V_{ts}^* \approx -V_{tb} V_{ts}^*\) these two contributions have the same CKM phase so that the phase of the SM decay amplitude is well predicted. We also include “magnetic penguins”, not enhanced by infrared divergences in the limit \(m_c \to 0\). We neglect electroweak penguins and box diagrams that are suppressed by powers of \(\alpha_2/\alpha_3\).

The inclusion of QCD effects from the \(M_W\) scale to the \(m_b\) scale is done in the usual effective field theory approach. At the \(M_W\) scale the heavy particles are integrated out and the eight current-current operators,

\[ O_{AB}^\mu \equiv (\bar{s}\gamma_\mu P_A b)/(q\gamma_\mu P_B q), \quad (B.1a) \]

\[ O_{AB}^\nu \equiv (\bar{s}\gamma_\mu P_A b)/(q\gamma_\mu P_B q), \quad (B.1b) \]

the two cromo-magnetic penguin operators,

\[ O_A^\nu \equiv m_b g_3(\bar{s}\gamma_\mu T_A^\nu P_A b)G_{\nu\nu}^3, \quad (B.1c) \]

and the ‘charm operators’;

\[ O_c^\nu \equiv (\bar{s}\gamma_\mu P_L c)/(q\gamma_\mu P_L b)', \quad (B.1d) \]

\[ O_c^\nu \equiv (\bar{s}\gamma_\mu P_L c)/(q\gamma_\mu P_L b)', \quad (B.1e) \]

have to be included in the effective Hamiltonian

\[ H_{\text{eff}}^{SM}(b \to s \bar{s}s) = -\frac{g_2^2}{2 M_W^2} V_{tb} V_{ts}^*(C_c^\nu O_c^\nu + C_c^\nu O_c^\nu + C_{LL}^\nu O_{LL}^\nu + C_{LR}^\nu O_{LR}^\nu + C_{LR}^\nu O_{LR}^\nu + C_{LR}^\nu O_{LR}^\nu). \]

In these equations \(i,j\) are color indices and \(A,B = \{L,R\}\). At the electroweak scale \(C_c^\nu = 1, C_c^\nu = 0\) and \(C_L = O(\alpha_3/4\pi)\) while operators with a right-handed s-quark have negligible coefficients. At the \(m_b\) scale the Wilson coefficients \(C_{LA}\) of the current-current operators \(O_{LA}\) get an infrared enhancement due to the mixing with the charm operators. These coefficients are thus of order \(\ell \equiv (\alpha_3/4\pi)\ln M_W^2/m_b^2 \approx 0.1\) or, more precisely,

\[ C_c^m = +1.1, \quad C_{LL}^m = +0.014, \quad C_{LR}^m = +0.009, \quad C_R^m = -0.15 \frac{1}{(4\pi)^2}. \]

In the RGE evolution we have kept the leading-order terms of order \(\ell^0\) and the terms of order \((\alpha_3/4\pi)\). The smaller next-to-leading-order terms of order \((\alpha_3/4\pi)\ell^1\) (with \(n \geq 1\)) (that we have systematically neglected) contain all the technical problems typical of a full NLO computation. At the \(m_b\) scale we can finally compute the charm loop in the effective theory, getting a further contribution to the \(C_{LA}\) coefficients of order \(\alpha_3(m_b)/(4\pi)\). Computing this charm
The coefficients for the current-current operators.

For simplicity, we have given their expressions in the limit where the supersymmetric contributions to the Wilson coefficients compute the charm loop at quark level, so that we have $B_{\text{loop}} - \text{hadronic level}$ could give a larger, and maybe dominant, contribution to the decay amplitude $[22]$. Here we consider

\[
\mathcal{O}^{i,j}_{\text{LL}} \leftrightarrow \mathcal{O}^{i,j}_{\text{RR}}
\]

where $\omega$ is an arbitrary mass scale.

In order to include the perturbative QCD corrections at scales between $m_t$ and $m_b$, we write the effective Hamiltonian for the $b \rightarrow s\bar{s}s$ decay as

\[
\mathcal{H}_{\text{eff}}^{\text{SUSY}}(b \rightarrow s\bar{s}s) = -V_{tb}^{*} V_{cb} \left[ \sum_{i,j} \frac{\alpha^2_{i,j}}{M_i^2} (c_{AB}^{ij} \mathcal{O}^{i,j}_{AB} + c_{AB}^{ij} \mathcal{O}^{i,j}_{AB}^\infty) + \frac{\alpha_3}{4\pi M_3^2} (c_R^{ij} \mathcal{O}_R^{ij} + c_L^{ij} \mathcal{O}_L^{ij}) \right]
\]

where the sum now extends over all the eight current-current operators, defined in eq.s $[B.1]$. The gluino-mediated supersymmetric contributions to the Wilson coefficients $c$ at the weak scale are

\[
c^{\text{LL}}_L = \frac{1}{2} \omega_{sL} P_\omega (\{\ell_3 - \ell_12\}) + \omega_{sL} \frac{1}{7} (5B_{\infty} + 21B_{\infty}) (\{\ell_3 - \ell_12\}, \ell_12), \quad (B.4a)
\]

\[
c^{\text{LL}}_R = -\frac{3}{2} \omega_{sL} P_\omega (\{\ell_3 - \ell_12\}) + \omega_{sL} \frac{1}{7} (-3B_{\infty} + B_{\infty}) (\{\ell_3 - \ell_12\}, \ell_12), \quad (B.4b)
\]

\[
c^{\text{LR}}_L = -\frac{1}{2} \omega_{sL} P_\omega (\{\ell_3 - \ell_12\}) + \omega_{sL} \frac{1}{18} (20B_{\infty} + 21B_{\infty}) (\{\ell_3 - \ell_12\}, \ell_12), \quad (B.4c)
\]

\[
c^{\text{LR}}_R = -\frac{3}{2} \omega_{sL} P_\omega (\{\ell_3 - \ell_12\}) + \omega_{sL} \frac{1}{18} (B_{\infty} - 12B_{\infty}) (\{\ell_3 - \ell_12\}, \ell_12). \quad (B.4d)
\]

The coefficients for the current-current operators $\mathcal{O}_{RA}$ with a right-handed bottom quark are obtained interchanging $L \leftrightarrow R$, $\omega_{sL} \leftrightarrow \omega_{sR}$ and $\ell \leftrightarrow r$ in the previous expressions. The coefficients of the magnetic penguin operators are

\[
c^{R}_{L} = \frac{3}{2} \omega_{sL} \left[ V_{bLbL}^2 \frac{A_{b} + \mu \tan \beta}{M_3} P_{\omega} (\{\ell_3 - \ell_12, r_3\} + P_{\omega} (\{\ell_3 - \ell_12\}) \right], \quad (B.4e)
\]

\[
c^{L}_{L} = \frac{3}{2} \omega_{sL} \left[ V_{bLbL}^2 \frac{A_{b} + \mu \tan \beta}{M_3} P_{\omega} (\{r_3 - \ell_12, \ell_3\} \right]. \quad (B.4f)
\]

For simplicity, we have given their expressions in the limit where the $A$-terms are generation universal at the Fermi scale. The expressions are easily modified if $A_d = A_s \neq A_u$.

The inclusion of QCD effects is well known $[11]$. The coefficients of the two cromomagnetic penguins renormalize multiplicatively as $c'^{\alpha}(m_B) = \eta^{-1/233}c^{\alpha}(M_3)$, where $\alpha = \{L, R\}$ and $\eta$ has been defined as $\alpha_3(M_3)/\alpha_3(m_b)$. The four current-current operators with a left-handed b-quark mix among themselves. The other four current-current operators with a right-handed b-quark also mix among themselves, with the mixing described by the same mixing matrix.

We finally list all the loop functions used above. The penguin loop functions that appear in the supersymmetric contributions are

\[
P_{\pm} = P_{F} - \frac{1}{9} P_{B}, \quad P_{\infty} = P_{FE} - \frac{1}{9} P_{BE}, \quad P_{\infty} = P_{F1} - \frac{1}{9} P_{B1} \quad (B.5)
\]

where

\[
P_{F}(r) = \frac{1}{36} \frac{1}{(r-1)^4} [7 - 36r + 45r^2 - 16r^3 - (18 - 12r)r^2 \ln r] \quad (B.6a)
\]

\[
P_{B}(r) = \frac{1}{36} \frac{1}{(r-1)^4} [11 - 18r + 9r^2 - 2r^3 + 6 \ln r] \quad (B.6b)
\]

\[
P_{FE}(r) = \frac{1}{12} \frac{1}{(r-1)^4} [1 - 6r + 3r^2 + 2r^3 - 6r^2 \ln r] \quad (B.6c)
\]

\[
P_{BE}(r) = \frac{1}{12} \frac{1}{(r-1)^4} [2 + 3r - 6r^2 + r^3 + 6r \ln r] \quad (B.6d)
\]

\[
P_{F1}(r) = \frac{1}{2} \frac{1}{(r-1)^3} [4r - 1 - 3r^2 + 2r^2 \ln r] \quad (B.6e)
\]

\[
P_{B1}(r) = \frac{1}{2} \frac{1}{(r-1)^3} [r^2 - 1 - 2r \ln r] \quad (B.6f)
\]

The box loop functions are defined as

\[
B_{\pm} (r_1, r_2) = i(4\pi)^2 \int \frac{d^4k}{(4\pi)^4} \frac{1}{k^2 - M^2} \frac{M^2}{(k^2 - r_1^2 M^2)(k^2 - r_2^2 M^2)} \quad (B.6g)
\]

\[
B_{\infty} (r_1, r_2) = i(4\pi)^2 \int \frac{d^4k}{(4\pi)^4} \frac{M^4}{(k^2 - M^2)^2(k^2 - r_1^2 M^2)(k^2 - r_2^2 M^2)} \quad (B.6h)
\]

where $M$ is an arbitrary mass scale.
C Hadronic matrix elements for \( b \to s\bar{s}s \) decays

We give here all the relevant hadronic matrix elements computed in the vacuum-insertion approximation.

Neglecting the ‘annihilation diagrams’, that have a strongly suppressed form factor \( [23] \), all the necessary matrix elements can be expressed in terms of

\[
\langle \phi(p_\phi, \epsilon_\phi)|\bar{s}\gamma_\mu s|0\rangle = f_\phi m^2_{\phi} \epsilon^\mu_{\phi},
\]

\[
\langle K^0(p_K)|\bar{s}\gamma_\mu b|\bar{B}_d(p_B)\rangle = F^{BK}_+ t(p_B + p_K)_\mu + F^{BK}_- (p_B - p_K)_\mu,
\]

where \( t \equiv (p_B - p_K)^2 \). Matrix elements of scalar operators are obtained using the equations of motion for the quark fields. Matrix elements of tensor operators, like \( \langle K|\bar{s}\gamma_{\mu\nu} b|B\rangle \), are obtained using the heavy-quark effective theory \( [24] \), while \( \langle \phi|\bar{s}\gamma_{\mu\nu} s|0\rangle \) can be estimated using a quark model as in \( [25] \). In terms of a single combination of form-factors \( [23, 26] \), \( H \equiv (p_B \cdot \epsilon_\phi) 2 f_\phi m^2_s F^{BK}_+ (m^2_c) \), that cancels in the CP-asymmetries, we obtain

\[
\langle \phi K_S|O^c_{LL}|\bar{B}_d^0\rangle = \langle \phi K_S|O^c_{RR}|\bar{B}_d^0\rangle = \frac{1}{4} H (1 + \frac{1}{N_c}) \quad (C.1a)
\]

\[
\langle \phi K_S|O^c_{LL}'|\bar{B}_d^0\rangle = \langle \phi K_S|O^c_{RR}'|\bar{B}_d^0\rangle = \frac{1}{4} H (1 + \frac{1}{N_c}) \quad (C.1b)
\]

\[
\langle \phi K_S|O^c_{LR}'|\bar{B}_d^0\rangle = \langle \phi K_S|O^c_{RL}'|\bar{B}_d^0\rangle = \frac{1}{4} H \quad (C.1c)
\]

\[
\langle \phi K_S|O^c_{RR}'|\bar{B}_d^0\rangle = \langle \phi K_S|O^c_{LL}'|\bar{B}_d^0\rangle = \frac{1}{4} H \frac{1}{N_c} \quad (C.1d)
\]

\[
\langle \phi K_S|O^c_L'|\bar{B}_d^0\rangle = \langle \phi K_S|O^c_R'|\bar{B}_d^0\rangle \approx (1.2 \pm 0.4) g^2_3 H \frac{N^2_c - 1}{2N^2_c} \quad (C.1e)
\]

where \( N_c \) is the number of colors. The computation of the matrix elements of the current-current operators is straightforward. More lengthy is the computation of the matrix element of the cromo-magnetic penguin operator. We have estimated it in the following way. To begin with, the cromo-magnetic penguin of eq. (\( [B.1c] \)) has been converted into a four-quark operator attaching a \( s\bar{s} \) pair to the gluon. Since the \( s\bar{s} \) pair is in an octet state, in the factorization approximation only the Fierz transformed operator contributes. Simplifying the operator using the Dirac equation we get

\[
\langle \phi K_S|O^c_{LL,R}'|\bar{B}_d^0\rangle = \frac{g^2_3 N^2_c - 11}{k^2} \left\{ 2 m_b \langle \phi|\bar{s}\gamma_\mu s|0\rangle \langle K_S|\bar{s}\gamma_\mu b|\bar{B}_d^0\rangle +
\right.
\]

\[
+ m_b (p_B + p_{s\bar{s}})_\mu \times \left[ \langle \phi|\bar{s}\gamma_{\mu\nu} s|0\rangle \langle K_S|\bar{s}b|\bar{B}_d^0\rangle +
\right.
\]

\[
+ i \langle \phi|\bar{s}\gamma_{\mu\nu} s|0\rangle \langle K_S|\bar{s}\gamma^{\nu} b|\bar{B}_d^0\rangle - i \langle \phi|\bar{s}\gamma^{\nu} s|0\rangle \langle K_S|\bar{s}\gamma_{\mu\nu} b|\bar{B}_d^0\rangle \right\}.
\]

We notice that the first ordinary current-current term gives the largest contribution to the full cromo-magnetic hadronic matrix element, so that the numerical coefficient in eq. (\( C.1e \)) is not very uncertain. Adding the contributions from the other terms, and estimating the gluon momentum \( k^2 \approx \frac{1}{2}(m^2_{\bar{B}_d} - \frac{1}{2} m^2_{s\bar{s}} + m^2_4) \) assuming that each one of the two \( s \) quarks inside the \( \phi \) carry similar momentum \( p_{s\bar{s}}/2 \) \( [25] \), we obtain the estimate of eq. (\( C.1e \)). Working in the factorization approximation, we have neglected \( \phi \) hadronization from \( s\bar{s} \) quarks in octet color state, that is kinematically enhanced by a smaller transferred momentum, \( k^2 \approx m^2_{s\bar{s}} \). This same kinematical factor enhances also the contribution from photonic magnetic penguins, that remain however unimportant due to the small d-quark electric charge.
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