MECHANICAL ENGINEERING | RESEARCH ARTICLE

Gear fault diagnosis using an improved Reassigned Smoothed Pseudo Wigner-Ville Distribution

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Abstract: The work is aimed: (i) to propose a Joint Time-Frequency Analysis method for gear fault diagnosis by using the combined autoregressive model-based filtering and Reassigned Smoothed Pseudo Wigner-Ville Distribution (RSPWVD) methods; (ii) to investigate the use of both vibration and acoustic measurements for fault diagnosis of a gear system by using the proposed fault diagnosis method. To the best of the authors’ knowledge, such RSPWVD method has not been utilized for gearbox applications due to the problem with the complexity of signals generated by the gearbox. For this purpose, experiments on a single-stage spur gearbox were carried out on a gearbox test-rig using single-defect and double-defect gear tooth faults with vibration and non-contact acoustic sensing. It was experimentally demonstrated that the proposed fault diagnosis method performed better compared to the Continuous Wavelet Transform and the Smoothed Pseudo Wigner Ville-Distribution methods. The proposed method is able to provide a more accurate indication of faults in a gearbox, even for the case of multiple gear defects using both acoustic and vibration measurements.

Subjects: Mechanical Engineering; Nondestructive Testing; Vibration; Acoustical Engineering; Digital Signal Processing

Keywords: vibration and acoustic sensing; gear fault diagnosis; reassignment method; Wigner-Ville distribution; wavelet

ABOUT THE AUTHORS

One of our research interests is in the development of signal processing methods for condition monitoring of rotary machinery and smart health monitoring of bridge structures. The work also includes the investigation into the time-frequency analysis to improve feature extractions of signals acquired from rotary machinery and bridge structures. Other research interests are related to the development of sensing and control systems for smart structures; and in the field of acoustics and vibration, particularly in the development of noise and vibration control methods for a variety of applications.

PUBLIC INTEREST STATEMENT

Gear fault diagnosis has gained a lot of attentions in the last three decades. Up to this day, the main obstacle is the feature extraction of the faulty signals that is not clearly available from the raw measurements. In addition, most of the available techniques have been heavily focused on vibration measurements. Hence, this work attempted to utilize both vibration and acoustic measurements from a gearbox test-rig. A novel gear fault diagnosis algorithm was developed in this work. The results reveal that the proposed signal processing methodology has a superior performance in terms of achieving an improved readability for gear fault interpretation compared to the existing advanced signal processing methods. Moreover, the obtained results also suggested that the non-contact acoustic measurements have a great potential for gear fault diagnosis applications, as an alternative method to the commonly used vibration measurements.
1. Introduction

Gear failures are one of the most common failures normally encountered in industrial machineries. A large number of researchers have investigated different techniques for gear fault diagnosis purposes, typically using vibration measurements obtained from the gear system. In general, when a local fault occurs in a gear system, a non-stationary impulse-type of vibration signal (Dalpiaz, Rivola, & Rubini, 2000; Randall, 1982) is generated. This signal contains abundant information about the state-of-health of the gear system. However, it is still a challenge to be able to extract useful information from such a complex non-stationary vibration signal generated by the gear system, particularly since the signal is generally corrupted by different types of measurement noise.

There are a large number of signal processing methods that can be used for vibration-based gear fault diagnosis. Time Synchronous Averaging (TSA) method is a simple yet powerful method to get rid of asynchronous components that contaminate measurements from rotating machineries. The method can also be used to eliminate the random noise that always exists during the data acquisition process. However, when the gear fault is still in the early stage, the TSA method itself is not capable of providing a satisfactory result. In this case, the residual analysis method can be used as an enhancement of the TSA method, which can be utilized to analyze the transient signals generated by the faulty gear. The demodulation method is also one of the most prominent methods, which is based on TSA or residual analysis. It can be done by directly extracting the envelope of residual signals in time domain or by band-pass filtering in a certain frequency bandwidth of residual signals. Then a statistical analysis (i.e. kurtosis) or a direct visual inspection from the demodulation signals can be performed to diagnose the state-of-health of the gear system (Dalpiaz et al., 2000). However, the main drawback of the demodulation method that is based on the residual analysis, is that the selection of the appropriate frequency bandwidth is not a straightforward task. The sidebands formed in the spectrum of the vibration signals are typically not symmetric with respect to the gearmesh frequency and its harmonics. Therefore, one will find a difficulty in making the right choice of the particular frequency bandwidth for demodulation. In this case, once an incorrect frequency bandwidth is selected, one may infer a wrong conclusion regarding the state-of-health of the gear system (Staszewski & Tomlinson, 1994). Moreover, it has been reported that the kurtosis can be lower as the fault in the gear is getting more severe, which makes it difficult to accurately diagnose a particular fault. The kurtosis will further become lower when multiple faults occur, in contrast to the case for the single faulty gear (Zhan, Makis, & Jardine, 2006). On the other hand, the Fast Fourier Transform (FFT) method is considered to be one of the most common methods for gear fault diagnosis, with its variety of applications can be found in the existing literatures (Hartono, Halim, & Roberts, 2016; Ulus & Erkaya, 2016). However, the impulse signal generated from the faulty gearbox is considered as a non-stationary signal, hence the assumption in Fourier analysis is no longer valid (Staszewski, Worden, & Tomlinson, 1997). In general, all methods described above were able to detect any abnormality in the gears, however they still have limitations in offering informative details regarding the gear faults, such as their severity and precise locations. Thus, other advanced signal processing methodologies will be required to be able to extract fault features from a gearbox system more accurately.

In addition to time domain-based analyses, the time-frequency analysis has been attracted a lot of attentions in the field of gear fault diagnosis due to its capability in providing a simultaneous representation of time and frequency information. The effective identification of characteristic frequencies of the gearbox, such as gearmesh frequency, its harmonics and sidebands, can provide valuable fault diagnosis information. Although the simultaneous time-frequency information provided by JTFA-based methods has been useful in determining the state-of-health of the gearbox (Baydar & Ball, 2000), a number of limitations still need to be addressed. An example of a widely-used JTFA method for gear fault diagnosis is the Continuous Wavelet Transform (CWT) (Staszewski & Tomlinson, 1994). Its capability to provide multi scale features in a single plot offers an advantage compared to the Short Time Fourier Transform (STFT) (Hartono, Halim, Roberts, & Liu, 2016; Yang, Peng, Meng, & Zhang, 2012). However, it is still restricted by the time duration-frequency bandwidth principle which restricts the simultaneous fine time-frequency localization. Other signal processing
methods that can provide an informative time-frequency representation, such as the Chirplet Transform and Warblet Transform, have been recently proposed. However, their current algorithms are primarily restricted for analyzing mono-component signals only (Yang et al., 2012). Moreover, the synchro-squeezing method, which is an improvement of the CWT method, has also been proposed, although it is not designed to analyze transient signals of short temporal duration (Iatsenko, McClintock, & Stefanovska, 2016) that corresponds to the fault characteristics of a faulty gearbox. The other JTFA method, the Wigner-Ville Distribution (WVD) has been proven to have the finest time-frequency localization compared to other existing JTFA method. However, due to its bilinear property, it has the cross term interference (CTI) when analyzing multi-components signal that can mislead the interpretation. Hence, it is not possible to apply such a method to real experimental data, particularly for gearbox signals that do not only contain multi-components signals with its harmonics of gearmesh frequency and sidebands, since the signals are generally corrupted by heavy measurement noise. Therefore, a number of methods have been developed in the last three decades with the attempts to alleviate the CTI of WVD. One of the most versatile methods is the Smoothed Pseudo Wigner-Ville Distribution (SPWVD) method (Baydar & Ball, 2003), although it should be noted that the CTI reduction is done at the expense of the time-frequency localization in WVD (Auger et al., 2013). In this case, a smearing phenomenon in the SPWVD is expected to happen and this is a crucial problem for analyzing multi-components signal like a gearbox signal where several sidebands around the gearmesh frequency can appear at the same time. The SPWVD method has been demonstrated to be effective in removing the CTI of WVD for gear fault diagnosis purposes (Baydar & Ball, 2003). Nevertheless, regardless the benefit of using the existing SPWVD method, a further improvement of WVD method is still highly desirable for achieving more accurate gear fault diagnosis.

To address this problem, the reassignment method has been proposed to improve the readability of the existing JTFA method, by removing the smearing phenomena that always appears in any JTFA, such as SPWVD. The efficient computation of the reassignment method was made possible by Auger et al. (2013) so that it can be used in practical situation. It has been demonstrated that RSPWVD is capable of analyzing transient seismic signals (Wu & Liu, 2009), which has a similar nature to an impulse signal generated by the faulty gear. However, to the best of authors’ knowledge, its application in gear fault diagnosis has yet to be investigated. RSPWVD also has a number of limitations that need to be addressed. Although it can offer very fine time-frequency localization compared to others JTFA method, it is still prone to measurement noise contamination (Iatsenko, McClintock, & Stefanovska, 2015). Signals measured by sensors in a gearbox system can consist of signal contributions associated with other components of gearbox so it is not possible to extract useful fault diagnosis information merely from raw vibration signals without any signal pre-processing. On the other hand, the Auto-Regressive (AR) model-based filtering method has been observed to outperform the residual analysis method in term of denoising and removing other asynchronous component measurements not related to the gear of interest. However, the majority of works using this method are focused on analyzing the gear fault diagnosis performance either in the time domain or frequency domain only (Endo & Randall, 2007; Wang & Wong, 2002). In this case, due to the fact that the generated impulse signal is non-stationary by nature, the use of analysis in the joint time-frequency domain, such as RSPWVD is expected to provide more complete information about the state-of-health of the gearbox.

Therefore, the aims of this present work are two-folds: (i) It proposes a JTFA method for gear fault diagnosis by using the AR model-based filtering combined with RSPWVD; (ii) It investigates the use of both vibration and acoustic measurements for fault diagnosis of a gear system. The cases with normal, broken and double defects gears are investigated. This work integrates the use of AR model-based filtering with the reassignment method of SPWVD (RSPWVD). The problem with the inaccuracy of time-frequency representation, due to high measurement noise contamination and the signal interference associated with other mechanical components of a gearbox system, are addressed by proposing the use of AR model-based filtering. The proposed method is used to pinpoint new features in sidebands generated by the gear faults from the time-frequency plot. The performance of this method is demonstrated by comparing it with the results obtained from the SPWVD and CWT.
methods, showing a better representation of the fault features generated by a faulty gearbox. This work also investigates the use of acoustic signals obtained from the gear system for gear fault diagnosis, particularly with the use of the proposed fault diagnosis method. Although acoustic signals can contain abundant information regarding the state of health of the gear system, very few literatures have been reported so far in the use of acoustic signals for gear fault diagnosis (Amarnath & Praveen Krishna, 2014; Belsak & Prezelj, 2011; Ulus & Erkaya, 2016; Vanraj, Dhami, & Pabla, 2017). It is shown in this work that acoustic signals can also be used to provide accurate fault diagnosis information of a gearbox system.

2. AR model-based filtering and RSPWVD Methods for gear fault diagnosis

This section describes the principles of AR model based filtering and RSPWVD methods that are used in this work.

2.1. AR model-based filtering for gear fault diagnosis

The AR model-based filtering method is used for gear fault diagnosis by first modeling the signal obtained from a healthy gear system as an AR process. This AR model is used to deterministically predict the pattern of the future signal from a healthy gear system, which can be used to differentiate it from the signal obtained from the faulty gear system. In other words, the residual signal can then be extracted, which is the signal difference between the predicted signal from the healthy gear system and the actual signal from the faulty gear system that cannot be predicted by the AR model. In this case, the AR residual signal is associated with the impulse signal generated by the local defect (Endo & Randall, 2007; Wang & Wong, 2002) from the faulty gear system. The expression for the AR model-based filtering or the AR linear predictor is presented in Equations (1) and (2)

\[
\hat{y}(n) = -\sum_{k=1}^{L} a(k)y(n-k)
\]

\[
y(n) = \hat{y}(n) + e(n)
\]

where \(y(n), \hat{y}(n)\)and \(e(n)\) respectively denote the current, predicted and error prediction of the \(n\)-th data point by the AR filter respectively, whereas \(a(k)\) is the \(k\)th coefficient from the AR model and \(L\) is the optimal order chosen for the AR model.

One of the most common methods to obtain the coefficient values of the AR model is by solving the Yule-Walker equations, as shown in Equations (3) and (4). In this work, the Levinson-Durbin Recursion algorithm (Kay, 1988, pp. 156–161) is used to solve the Yule-Walker equations. Lastly, the Akaike Information Criterion (AIC) which is presented in Equation (5) is used to determine the optimal order of the AR model. The lowest value of the AIC indicates the optimal order for the AR model (Akaike, 1974).

\[
r_{yy}[k] = -\sum_{i=1}^{L} a[i] \ast r_{yy}[k-i], \quad \forall k = 1, \ldots, L
\]

\[
r_{yy}[0] = -\sum_{i=1}^{L} a[i] \ast r_{yy}[k-i] + \sigma^2
\]

\[
AIC(L) = N\left(\ln\left(\sigma^2\right)\right) + 2L
\]

\[
\hat{r}_{yy}[j] = \frac{1}{N} \sum_{n=0}^{N-1} y[n]y[n-j], \quad 0 \leq j \leq L - 1
\]
where $r_{yy}[k]$ in Yule-Walker equations are computed by the biased estimate of the autocorrelation function as represented in Equation (6), $\sigma^2_l$ is the error of the $L$th order AR model and $N$ is the total number of data. Equation (5) clearly describes that the smaller the error model, the smaller the AIC value becomes.

2.2.1. Smoothed Pseudo Wigner-Ville Distribution

The use of SPWVD method in this work is motivated by the elimination of the Cross Term Interference (CTI) which is the appearance of a new signal that is located in the midway of any two distinct signals in the time-frequency plane generated by the Wigner-Ville Distribution. In the case of gear fault diagnosis, the appearance of new signals occurs between the fundamental gearmesh frequency, its harmonics and additional sidebands. In particular, when the faults introduced in the gear, the appearance of new sidebands (Dalpiaz et al., 2000; Mark, 2009) needs to be captured accurately for proper fault diagnosis. Several time-frequency distributions have been previously proposed to reduce the CTI, such as the Choi-William distribution (Xiang, Wang, & Liu, 2015) and Zhao-Atlas-Marks distribution (Aharamuthu & Ayyasamy, 2013). The idea is to reduce the cross term interference by introducing a kernel function that will smooth out the interference. However, SPWVD is considered as one of the most versatile methods by providing independent control of smoothing in time and frequency domains, hence this will be incorporated further in this work. The expression of the SPWVD is presented in Equation (7)

$$SPWVD_{g,h}(t, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\mu - t)h(\nu)z(t - \mu + \frac{\nu}{2})z^*(t - \mu - \frac{\nu}{2})e^{-j\omega\nu}d\mu dt$$

where $g$ and $h$ are the time and frequency smoothing windows respectively with $g(0) = h(0) = 1$, whereas $z(t)$ is the Hilbert transform of the signals under investigation. In this present work, the hamming window is used for both the time and frequency smoothing windowing and the processed signals will then be used for further analysis.

2.2.2. Reassignment of Smoothed Pseudo Wigner-Ville Distribution

In this work, the main motivation of using the reassignment method is to improve the readability of the existing joint time-frequency representation (Auger et al., 2013). One of the main issues in gear fault diagnosis is the effective identification of sidebands in time-frequency plot that contain useful fault diagnosis information by using JTFTA methods (Mark, 2009; Randall, 1982). However, this is generally not a straightforward task due to the smearing phenomena that always exists when any JTFTA method is utilized. It does not only exacerbate the time-frequency representation but can also mislead the interpretation. By utilizing RSPWVD, the smearing phenomena between sidebands associated with the faulty gearbox signal, which always occur in the SPWVD, can be smoothened out so a more accurate fault diagnosis can be made. The key point of the reassignment method is by moving each time-frequency value $(t, \omega)$ of the computed joint time-frequency distribution, i.e. SPWVD, to a new time-frequency coordinate system $(\hat{t}, \hat{\omega})$, which is the center gravity of the signal energy distribution as expressed in Equations (8) and (9) (Auger et al., 2013; Wu & Liu, 2009).

$$\hat{t}(t, \omega) = t - \frac{SPWVD_{g,h}(t, \omega)}{SPWVD_{g,h}(t, \omega)}$$

$$\hat{\omega}(t, \omega) = \omega + j\frac{SPWVD_{g,h}(t, \omega)}{SPWVD_{g,h}(t, \omega)}$$

where $D(t)$ and $T(t)$ are the respective differential and product operators with respect to the time, which can be expressed as $Dh(t) = h'(t)$ and $Th(t) = t'h(t)$ respectively. Finally, the value of RSPWVD at any point $(t', \omega')$ in the new coordinate system is the summation of all SPWVD values assigned to this point as expressed in Equation (10).
where $\delta$ is the Dirac delta function with a unit value at its origin and zero everywhere else.

3. Experimental setup of a gearbox system
To implement the proposed fault diagnosis method, experiments on a single-stage spur gearbox were carried out on a test-rig system built at The University of Nottingham Ningbo China as shown both in Figures 1 and 2. In this experiment, an 1 HP DC motor was used as a driver of the system and a magnetic brake was incorporated as a load. Two torque and speed sensors were located at the input and output of the gearbox system to ensure the constant operating speed and torque. The speed of the motor was set to 500 r/min during the experiment. The whole experiment was conducted on the Nexus Optical Table to isolate vibration from external environment. A G.R.A.S free-field microphone was used to pick up the acoustic signals from the gearbox and located 20 cm on the gearbox. On the other hand, a PCB Piezotronics accelerometer was used to obtain vibration measurements from the gearbox. It was located at the bearing attached at the right side of the gearbox input shaft. In addition, a Monarch Instrument tachometer is also incorporated for synchronizing the measured vibration and acoustic signals with the rotation speed of pinion (input gearbox), and also for resampling the vibration and acoustic measurements into the angular domain for the implementation of TSA method. Both the tape used of the tachometer sensing and the accelerometer can be seen in Figure 3.

In this work, a single-stage spur gear was studied with the detail specification of the gear presented in Table 1. The pinion was located at the input shaft to achieve a speed reducer gearbox configuration. The gear defects were artificially created with a milling process at the pinion where two types of defects were considered for further analysis. The first one is the tooth breakage and another one is double defects that consist of both the tooth breakage and a chipped tooth as shown...
in Figure 4(a) and (b) respectively. These defects are used in the experiment because the tooth breakage and chipped tooth are considered as one of the most common type of faults in industrial gearbox applications (Loutridis, 2004). The chipped tooth in the double defect was done by removal from pitch point to the top at 25% removal of its thickness whereas the broken tooth was achieved by 50% depth wise tooth removal along the face width with respect to the total addendum and dedendum of the gear tooth. In addition, the tape of the tachometer was adjusted so that the broken tooth was located at approximately 250–260° of the gear angular location. For the double defects, the broken tooth was located at approximately 320–330° and the chipped tooth was located at around 145–155°. National Instrument CompactRio with NI 9234 module and LabView software were used for data acquisition for both vibration and acoustic signals from the gearbox test-rig. The sampling rate was set to 5.12 kHz to obtain 30 seconds data from vibration and acoustic signals. Furthermore, the data was uploaded to MATLAB to be analyzed further.

Table 1. Gear specifications

|                      | Pinion | Wheel |
|----------------------|--------|-------|
| Module               | 4      | 4     |
| Number of teeth      | 18     | 27    |
| Addendum (mm)        | 4      | 4     |
| Dedendum (mm)        | 5      | 5     |
| Pressure Angle (°)   | 20     | 20    |
| Face width (mm)      | 20     | 20    |

Figure 3. The locations for the accelerometer and the tape used for the tachometer sensing.

Figure 4. Views of: (a) the broken gear tooth (b) double defects gear tooth.
4. Vibration and acoustic analyses for gear fault diagnosis

4.1. Vibration-based fault diagnosis
Ten seconds of raw vibration measurements from the gearbox system are presented in Figure 5. Before proceeding to further analysis, initial validation of the test rig was done by comparing the root mean square (RMS) value of velocity signals to the ISO standards of vibration limits from rotating machines (ISO 10816, 1995). The velocity signals as presented in Figure 6 were obtained by the integration method of raw acceleration vibration measurements from the healthy gearbox. The value of RMS of the velocity signals was 3.5717 mm/s, hence the vibration of the gearbox could be categorized in the allowable region according to the standard for small machines class (<15 kW). Initially, the TSA method was utilized in this work, not only to attenuate the random noise from the raw measurements but also to eliminate the asynchronous component with respect to the defected gear that contributed to the raw vibration measurements. The TSA result from the healthy vibration signals is presented in Figure 7(a).

The next step is to utilize this particular TSA signal from the healthy gear to create the AR model. The model order for the vibration measurements was chosen to be 81. This value was chosen because there was no significant reduction from the AIC value beyond this point and after some trials there was no significant improvement in terms of noise reduction by increasing the model order to beyond 81 order. The AIC plot from vibrations measurements is shown in Figure 8. Furthermore, the model was used to filter out the faulty vibration signals from the broken tooth and double defects. From Figure 7(b), it can be seen clearly that there was an indication of fault located at around 250–260° as expected. In addition, the AR based residual analysis is also capable of extracting the impulse signal to indicate the locations of double defects as shown in Figure 7(c). As mentioned in the
previous section, these two defects are separated around 180° which is approximately captured by the AR residual method. It should be noted that the fault that is located at around 325° is the broken tooth whereas the chipped tooth is located at around 150°, as already mentioned before. Although the AR residual signals already can provide us with a good representation of the fault features from the faulty gear in its angular domain, further investigation was done on the impulse property that was generated by the faulty tooth by analyzing it in the frequency domain. The results from frequency domain analysis using FFT are presented in Figure 9. Although the emerging of amplified sidebands can be easily observed by comparing Figure 9(a) with Figure 9(b) and (c), it is not straightforward task to observe the details of sidebands spacing from the plots. Moreover, the transient nature of signal that makes it unsuitable to be analyzed with the regular frequency domain technique, such as FFT (Ulus & Erkaya, 2016; Yesilurt, 2003). Hence, a signal processing method that can provides a simultaneous representation in both time and frequency domains may be needed so that more accurate fault diagnosis information can be obtained. For this purpose, the SPWVD analysis is conducted further in this work.

In this experiment, the gear mesh frequency is formulated as \( f_m = \left( T \times f_r \right) \) where \( T \) is the total number of the gear tooth and \( f_r \) is the rotational speed of the gear. In this case, with respect to the pinion with 18 teeth and rotating at frequency 8.371 Hz, the gear mesh frequency is determined to be 150 Hz as described in Table 2. When the gear is in normal condition, it should be expected that the fundamental gearmesh frequency can be determined from the observation of dominant energy distribution in the time–frequency plot (Baydar & Ball, 2000) which is the case as represented in Figure 10(a). This result was used as a benchmark to compare the fault diagnosis performance based on SPWVD analysis for the system with defected gears.
Once the faulty gear was assembled in the gearbox, it can be clearly observed that there was a stronger signal intensity at 250–270° compared to other frequencies in the time-frequency plot shown in Figure 10(b). This particular frequency represents the sidebands at 97th order of rotational frequency for the faulty gear. This indicates the presence of an impulse in the signal. In addition, the SPWVD plot of vibration signals generated from double defects is clearly shown in Figure 10(c). However, the smearing effect still makes it hard to quantify clearly the frequency band and also the angular location of the fault. In relation to the broad peak frequency bandwidth generated from the faulty teeth in the SPWVD plot, it can be explained by the fact that the increase in amplitude and number of sidebands in the frequency domain is an indication of faults in the gear (Dalpiaz et al., 2000). The existence of defects in a gear generates the modulation phenomenon that can be observed as an impulse in the time domain, which is characterized by the sidebands in the frequency domain (Dalpiaz et al., 2000; Wang & Wong, 2002). The impulse signal is categorized as a transient.

**Table 2. Gearbox characteristics frequency**

| Transmission ratio | Gearmesh frequency (Hz) | Rotational frequency of pinion (r/min) |
|--------------------|-------------------------|---------------------------------------|
| 27/18              | 150.678                 | 500                                   |

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signal, which is a short duration signal in time domain, so that its spectrum will be spread over a wide frequency bandwidth (Randall, 1982; Wang & Wong, 2002). Therefore, all these factors described above contribute to the broad frequency bandwidth observed in SPWVD plot. On the other hand, the CWT plot of vibration signals from gears with broken tooth and double defects are presented in Figure 11(a) and (b) respectively. It is clear that the CWT method has a better time localization but a worse frequency localization compared to the SPWVD method. The results can be compared to those in the time-frequency plot for double defects gears from the SPWVD analysis in Figure 10(c) and from the CWT analysis in Figure 11(b). It can be observed that there are several dominant peaks in the SPWVD plot generated by the broken tooth for the double defects case in Figure 10(c). These peaks represent the sidebands at 124 and 101 orders. However, the CWT plot was only able to show a broad peak frequency bandwidth of a sideband at 112 order generated by the broken tooth in the double defects. This is caused mainly by the property of the CWT itself that in higher frequency region, it has lower frequency localization but higher time localization instead. In this case, the simultaneous high time and frequency localizations are not possible to achieve due to the restriction of the duration-bandwidth principle in the CWT. From the above analyses, the smearing phenomenon can be observed in both CWT and SPWVD (Baydar & Ball, 2003) methods as expected to occur in gear fault diagnosis applications. In this case, the smearing phenomenon needs to be alleviated to enhance the extraction of gear fault features from the time-frequency plot. Therefore, the implementation of the RSPWVD method for addressing this problem is considered further in this work.

The results of the reassignment method based on the results shown in Figure 10(b) and (c) are shown in Figure 12(a) and (b) respectively. These two plots clearly show that the most concentrated energy in the time-frequency plot from the SPWVD was located at the neighborhood of the fault location. The better fault localization for double defects can be made by using the RSPWVD method. The broken tooth for the double defects was located at angular location between 320–330° whereas the chipped tooth was located at 145–155° as expected. The RSPWVD method is not only able to unmask the smearing effect of SPWVD but it also allows one to distinguish clearly which features are associated with the faulty tooth, differentiating them from the measurement noise, as shown by the SPWVD plots from Figure 10(b) and (c). The proposed algorithm has thus showed that it is sensitive enough to detect the presence of the fault and allows one to differentiate it from the measurement noise by observing the intensity contrast of signal in the time-frequency plot.

4.2. Acoustic-based fault diagnosis

In this work, an alternative method for gear fault diagnosis using non-contact acoustic measurements was also investigated by using acoustic measurements. A similar procedure was used based on acoustic measurements, instead of vibration measurements described in the previous section. The raw measurements from acoustic signals of healthy, broken tooth and double defects gears are shown in Figure 13 after initially pre-processed the signals by using the TSA method. An AR model was then built based on the healthy TSA acoustic signals with the optimal model for the acoustic signal was also chosen to be 81. Based on the same reason used in the previous vibration analysis, there is no more significant reduction from the AIC value at the vicinity of 80 as can be observed from Figure 15. Moreover, after some trials, the increasing the model order did not show any significant improvements in the terms of noise reduction from the AR residual signal.

The AR residual signal for the broken tooth and the double defects from acoustic measurements are presented in Figure 14(b) and (c) respectively. It can be clearly seen that the AR residual method is also capable of extracting impulse-type information from acoustic signals. Figure 14(b) shows an indication of fault located around 250° that is in agreement with the results from the AR residual signals in Figure 7(b). Moreover, the double defect fault indication can still be observed in Figure 14(c). However, the acoustic signals are still corrupted by measurement noise so that the localization of the defects is not as clear as that of vibration signals. Furthermore, the results of the FFT analysis based on the AR residual acoustic signals from normal, broken tooth and double defects gears are presented in Figure 15. Similar to the case with vibration signals, there is no significant
fault diagnosis information that can be obtained rather than the amplification of sidebands when the faults occur in the gearbox.

Furthermore, the SPWVD analysis was conducted for the healthy TSA acoustic signals and the AR residual of acoustic measurements from the faulty gear. From Figure 17(a) there exists the same gear mesh frequency observed from the healthy vibration signals at 150 Hz. However, there also exist several uneven weak frequency components outside the fundamental gear mesh frequency caused by the amplitude-modulation generated by the imperfect meshing although the gear is in the healthy condition. This is mainly caused by the manufacturing error of the gears (Baydar & Ball, 2000). The same phenomenon can also be observed from the SPWVD plot of the AR residual of healthy vibration signals in Figure 10(a).

The SPWVD plot of the acoustic signal from a broken tooth is presented in Figure 17(b). From the plot, there are some slightly stronger indications that the fault might occur at an angle around 250°–270° compared to Figure 17(a). The SPWVD of the double defects is also presented in Figure
17(c). Although it can be concluded that there are fault indications from Figure 17(b) and (c), their values cannot be quantified accurately due to the smearing effect from the SPWVD analysis. However, it can be observed that there is a higher level of noise at 160–190° in the chipped tooth region compared to the exact fault location located at 150°. On the other hand, the CWT plot of
acoustic signals from the case with broken tooth and double defects gears are shown in Figure 18(a) and (b) respectively. Although the CWT analysis can provide a satisfactory angular location of fault in the gear, it cannot still offer sufficiently accurate frequency localization due to the smearing bandwidth. It can be observed clearly that the large smearing bandwidth in CWT analysis prevented it from capturing several fault indications in terms of sidebands for both cases of broken tooth and double defects gears. With the same reason as in the vibration case, the duration-bandwidth principle puts the limitation on a simultaneous time and frequency localization using the CWT method. It is clear that the results need to be improved to allow a more accurate identification of the fault location more accurately using acoustic measurements, which is investigated next.

In order to improve the previous fault diagnosis results, the RSPWVD analysis is undertaken to unmask the smearing in the SPWVD analysis. The reassignment method once again provides an improved readability of the time-frequency plot compared to the SPWVD analysis as shown in Figure 19. The sidebands generated by the impulse can be clearly seen from Figure 19(a) and (b). From Figure 19(a), although there is still a certain measurement noise presents in the plot, the fault features generated by the faulty gear can be clearly distinguished compared to the one generated by the previous SPWVD method. The broad peak frequency bandwidth of frequency generated by the broken tooth, which ranges from 500 Hz -1400 Hz, contains three sidebands components at 71, 97...
and 119 orders. Similar to the case with vibration signal, this frequency band consists of modulation sidebands and also reflects the broad-band impulsive signals that are induced by the local fault in the gear. Moreover, it can be seen a weak signal intensity that corresponds to a sideband at 101 order, generated by the chipped tooth at 150° as shown in Figure 19(b). However, the plot shows stronger noise components at the angular location between 160–200° compared to the true location of faulty gear located at 150°. This can also be observed in the AR-based residual signals shown in Figure 14(c) in terms of an increased signal magnitude between 160–200°. Nevertheless, the AR-based residual signals performed better in providing a clearer identification of angular location of the faulty tooth for the double defects case. It is clear that the use of acoustic measurements for fault diagnosis tends to be influenced with more significant measurement noise compared to that using vibration measurement. This can be expected due to the non-contact nature of acoustic sensing which is more prone to measurement noise associated with various structure-borne and airborne noises. However, it is shown that it is indeed still possible to use acoustic-measurements to provide a useful gear fault diagnosis, having the advantage of non-contact sensing compared to the typical contact-sensing requirement for vibration measurements.

5. Conclusions
A gear fault diagnosis method has been proposed in this work by using an improved RSPWVD analysis using acoustic and vibration measurements. Gear fault cases that were based on a broken gear tooth and double defects gear were considered to experimentally demonstrate the effectiveness of the proposed strategy. The superiority of the proposed signal processing technique in term of better representation of the joint time-frequency plot has been compared with results from the SPWVD and CWT analyses, which are regarded as advanced signal processing techniques commonly used for gear fault diagnosis. The experimental results undertaken in this work indicate that the use of the RSPWVD method alone is generally not sufficient for accurate gear fault diagnosis when there is a significant level of measurement noise, such as for the case of double defects using the acoustic measurements. Thus, a fault diagnosis method has been proposed in this work by using the combination of the RSPWVD and AR residual signal analyses for the fault diagnosis improvement. One of the main advantages of the proposed method is allowing a more effective use of non-contact acoustic sensing for gearbox fault diagnosis since the measurement noise issue can be better addressed. The present work concludes that non-contact acoustic measurements can offer satisfactory fault diagnosis capability that can be used as an alternative sensing solution or to complement vibration-based diagnosis for gear fault diagnosis purposes.

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References
Aharamuthu, K., & Ayyasamy, E. P. (2013). Application of discrete wavelet transform and Zhao-Atlas-Marks transforms in non-stationary gear fault diagnosis. Journal of Mechanical Science and Technology, 27, 641–647. https://doi.org/10.1007/s12206-013-0114-y

Akaike, H. (1974). A new look at the statistical model identification, IEEE Transactions on Automatic Control, AC19, 716–723. https://doi.org/10.1109/TAC.1974.1100705

Annamath, M., & Krishna, J. R. P. (2014). Local fault detection in helical gears via vibration and acoustic signals using EMD based statistical parameter analysis. Measurement, 58, 154–164. https://doi.org/10.1016/j.measurement.2014.08.015

Auger, F., Flandrin, P., Lin, Y. T., McLoughlin, S., Meignen, S., Oberlin, T., & Wu, H. T. (2013). Time-frequency reassignment and synchrosqueezing: An overview. IEEE Signal Processing Magazine, 30, 32–41. https://doi.org/10.1109/MSP.2013.2265316

Baydar, N., & Ball, A. (2000). Detection of gear deterioration under varying load condition by using the instantaneous power spectrum. Mechanical Systems and Signal Processing, 14, 907–921. https://doi.org/10.1006/mssp.1999.1281
Belsak, A., & Prezelj, J. (2011). Identification of complex sound sources produced by gear units. Engineering Failure Analysis, 18, 1831–1841. https://doi.org/10.1016/j.engfailanal.2011.05.011

Dolpić, G., Rivola, A., & Rubini, R. (2000). Effectiveness and sensitivity of vibration processing techniques for local fault detection in gears. Mechanical Systems and Signal Processing, 14, 387–412. https://doi.org/10.1006/mssp.1999.1294

Endo, H., & Randall, R. B. (2007). Enhancement of autoregressive model-based gear tooth fault detection technique by the use of minimum entropy deconvolution filter. Mechanical Systems and Signal Processing, 21, 906–919. https://doi.org/10.1016/j.jmssp.2006.02.005

Hartono, D., Halim, D., Roberts, G. W., & Liu, Q. (2016). Vibration-based fault diagnostic of a spur gearbox. In Proceedings of 3rd International Conference on Manufacturing and Industrial Technologies (ICMIT), MATEC Web of Conferences (pp. 02004). Istanbul, Turkey.

Hartono, D., Halim, D., & Roberts, G. W. R. (2016). Gearbox fault diagnosis via acoustic and vibration measurements. In Proceedings of International Symposium on Green Manufacturing and Applications (ISGMA) (pp. OPOS2). Bali, Indonesia.

Iatsenko, D., McClintock, P. V. E., & Stefanovska, A. (2015). Linear and synchrosqueezed time–frequency representations revisited: Overview, standards of use, resolution, reconstruction, concentration, and algorithms. Digital Signal Processing, 42, 1–26. https://doi.org/10.1016/j.dsp.2015.03.004

Iatsenko, D., McClintock, P. V. E., & Stefanovska, A. (2016). Extraction of instantaneous frequencies from ridges in time–frequency representations of signals. Signal Processing, 125, 290–303. https://doi.org/10.1016/j.sigpro.2016.01.024

ISO 10816, Part 1. (1995). Mechanical vibration—Evaluation of machine vibration by measurements on non-rotating parts—Part 1: General guidelines. Geneva: International Organization for Standardization.

Kay, S. M. (1988). Modern spectral estimation: Theory and application (pp. 156–161). Englewood Cliffs, NJ: Prentice Hall.

Loutridis, S. J. (2004). Damage detection in gear systems using empirical mode decomposition. Engineering Structures, 26, 1833–1841. https://doi.org/10.1016/j.engstruct.2004.07.007

Mark, W. D. (2009). Stationary transducer response to planetary-gear vibration excitation II: Effects of torque modulations. Mechanical Systems and Signal Processing, 23, 2253–2259. https://doi.org/10.1016/j.ymssp.2009.03.005

Randall, R. B. (1982). A new method of modeling gear faults. Journal of Mechanical Design, 104, 259–267. https://doi.org/10.1115/1.3256334

Staszewski, W. J., & Tomlinson, G. R. (1994). Application of the wavelet transform to fault detection in a spur gear. Mechanical Systems and Signal Processing, 8, 289–307. https://doi.org/10.1006/mssp.1994.1022

Staszewski, W. J., Worden, K., & Tomlinson, G. R. (1997). Time-frequency analysis in gearbox fault detection using the Wigner– Ville distribution and pattern recognition. Mechanical Systems and Signal Processing, 11, 673–692. https://doi.org/10.1016/mssp.1997.0102

Ulus, S., & Erkaya, S. (2016). An experimental study on gear diagnosis by using acoustic emission technique. International Journal of Acoustics and Vibration, 21, 103–111.

Vannoy, Dhami, S. S., & Pabla, B. S. (2017). Optimization of sounds sensor placement of condition monitoring of fixed-axis gearbox. Cogent Engineering, 4, 1345673.

Wang, W., & Wong, A. K. (2002). Autoregressive model-based gear fault diagnosis. Journal of Vibration and Acoustics, 124, 172–179. https://doi.org/10.1115/1.1456905

Wu, X., & Liu, T. (2009). Spectral decomposition of seismic data with reassigned smoothed pseudo Wigner–Ville distribution. Journal of Applied Geophysics, 68, 386–393. https://doi.org/10.1016/j.jappgeo.2009.03.004

Xiang, M., Wang, Z., & Liu, J. (2015). Extracting array acoustic logging signal information by combining fractional Fourier transform and Choi-Williams distribution. Applied Acoustics, 90, 111–115. https://doi.org/10.1016/j.apacoust.2014.11.004

Yang, Y., Peng, Z. K., Meng, G., Zhang, W. M. (2012). Characterize highly oscillating frequency modulation using generalized Warlet transform. Mechanical Systems and Signal Processing, 26, 128–140. https://doi.org/10.1016/j.jmssp.2011.06.020

Yesilurt, I. (2003). Fault detection and location in gears by the smoothed instantaneous power spectrum distribution. NDT&E International, 36, 535–542.

Zhan, Y., Makis, V., & Jardine, A. K. S. (2006). Adaptive state detection of gearboxes under varying load conditions based on parametric modelling. Mechanical Systems and Signal Processing, 20, 188–221. https://doi.org/10.1016/j.jmssp.2004.08.004