Lepton Flavour Violating Decays of Supersymmetric Higgs Bosons

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Abstract

We compute the lepton flavour violating couplings of Higgs bosons in the Minimal Supersymmetric Standard Model, and show that they can induce the decays \((h^0, H^0, A^0) \rightarrow \mu\tau\) at non-negligible rates, for large \(\tan\beta\) and sizeable smuon-stau mixing. We also discuss the prospects for detecting such decays at LHC and other colliders, as well as the correlation with other flavour violating processes, such as \(\tau \rightarrow \mu\gamma\) and \(\tau \rightarrow 3\mu\).

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1 Introduction

The recent important indications of neutrino oscillations [1] reveal that flavour violation also occurs in the lepton sector and further motivate the search for alternative signals of lepton flavour violation (LFV). The Minimal Supersymmetric extension of the Standard Model (MSSM) is a natural framework where several such signals could be significant, provided the mass matrices of the leptons and of the sleptons are not aligned. Well known examples are the LFV radiative decays of charged leptons, $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to e\gamma$. In this Letter we would like to explore another class of such processes, namely the LFV decays of the neutral Higgs bosons ($h^0, H^0, A^0$). An important feature of these decays is that the corresponding amplitudes do not vanish in the limit of very heavy superpartners, since the leading contributions are induced by dimension-four effective operators, at variance with the case of radiative decays.

Related investigations on flavour violating Higgs couplings in the MSSM framework have mainly focused on processes with virtual Higgs exchange (see e.g. [2, 3, 4, 5]) and regard either quark or lepton flavour violation. The decays of physical Higgs bosons into fermion pairs have been explored in the case of quark flavour violation in the MSSM [6, 7], whereas in the case of lepton flavour violation existing studies [8, 9] have mainly used phenomenological parametrizations of the LFV couplings.

This Letter is organized as follows. In Section 2 we present the effective LFV Higgs couplings in the MSSM framework, focusing on the second and third lepton generations. We explicitly compute the one-loop contributions to those couplings and the branching ratios of the decays ($h^0, H^0, A^0 \to \mu\tau$). New results on flavour conserving Higgs couplings are also presented. In Section 3 we give a numerical discussion on the LFV Higgs couplings and branching ratios, and also discuss the prospects at future colliders. Finally in Section 4 we comment on the correlation of the LFV Higgs decays with other LFV processes, such as $\tau \to \mu\gamma$ and $\tau \to 3\mu$, and summarize our results.

2 Higgs-muon-tau effective interactions

The MSSM contains two Higgs doublets $H_1$ and $H_2$, with opposite hypercharges. Down-type fermions, which only couple to $H_1$ at the tree level, also couple to $H_2$ after the inclusion of radiative corrections [11]. In particular, for the charged leptons of second and third generations the tree-level couplings read as

$$\mathcal{L} = -Y_{\mu} H_1^0 \mu^c \mu - Y_{\tau} H_1^0 \tau^c \tau + \text{h.c.},$$

where $H_1^0$ is the neutral component of $H_1$ and $Y_{\mu}, Y_{\tau}$ are the Yukawa coupling constants.

Also the leading effective interactions with $H_2$, which arise once superpartners are integrated

Footnotes:
1 An attempt to study LFV Higgs decays in the MSSM can be found in [10]. However, we believe that in this work the Higgs couplings to the sleptons have not been properly identified.
2 We adopt two-component spinor notation, so $\mu$ and $\tau$ ($\mu^c$ and $\tau^c$) are the left-handed (right-handed) components of the muon and tau fields, respectively. Throughout our discussion we assume CP conservation and therefore all the dimensionless as well as dimensionful parameters are taken to be real.
out, are described by dimension-four operators. These can be either flavour conserving (FC):

\[ \Delta \mathcal{L}_{FC} = -(Y_\mu \Delta_\mu + Y_\tau \Delta'_\mu)H_2^{0*}H^{0*} - Y_\tau \Delta R H_2^{0*}\tau^c \tau + \text{h.c.}, \]  

(2)

or flavour violating (FV):

\[ \Delta \mathcal{L}_{FV} = -Y_\tau \Delta L H_2^{0*}\tau^c - Y_\tau \Delta R H_2^{0*}\mu^c \tau + \text{h.c.}, \]  

(3)

where \( \Delta_\mu, \Delta'_\mu, \Delta_L, \Delta_R \) are dimensionless functions of the MSSM mass parameters, to be described below. In eqs. (2) and (3) we have only retained the dominant terms, proportional to \( Y_\tau \), besides the first term in \( \Delta \mathcal{L}_{FC} \) proportional to \( Y_\mu \). In the following we are mostly interested in the effects induced by the terms in (3). In the mass-eigenstate basis for both leptons and Higgs bosons, the FV couplings read as:

\[ \mathcal{L}_{FV} = -\frac{Y_\tau}{\sqrt{2}\cos \beta} (\Delta_L \tau^c \mu + \Delta_R \mu^c \tau) [h^0 \cos(\beta - \alpha) - H^0 \sin(\beta - \alpha) - iA^0] + \text{h.c.}, \]  

(4)

where \( \tan \beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle \), \( \alpha \) is the mixing angle in the CP-even Higgs sector \( [\sqrt{2}\text{Re}(H_1^0 - \langle H_1^0 \rangle) = H^0 \cos \alpha - h^0 \sin \alpha, \sqrt{2}\text{Re}(H_2^0 - \langle H_2^0 \rangle) = H^0 \sin \alpha + h^0 \cos \alpha] \) and \( A^0 \) is the physical CP-odd Higgs field. The expression in (4) holds up to \( \mathcal{O}(\Delta L, \tan \beta) \) corrections, which arise from eq. (2) and can be \( \mathcal{O}(10\%) \) for large \( \tan \beta \). For our purposes it is not compelling to include and resum such higher-order (\( \tan \beta \)-enhanced) terms.

The effective couplings (4) contribute to LFV low-energy processes, such as the decay \( \tau \to 3\mu \) and other ones, through Higgs boson exchange [3, 4, 5]. We will comment later on \( \tau \to 3\mu \). Here we are interested in a more direct implication of those LFV couplings, i.e. the decays \( \Phi^0 \to \mu^+\tau^- \) where \( \Phi^0 = h^0, H^0, A^0 \). It is straightforward to compute the branching ratios \( BR(\Phi^0 \to \mu^+\tau^-) = BR(\Phi^0 \to \mu^-\tau^+) \), and it is convenient to relate them to those of the flavour conserving decays \( \Phi^0 \to \tau^+\tau^- \):

\[ BR(\Phi^0 \to \mu^+\tau^-) = \tan^2 \beta \left( |\Delta_L|^2 + |\Delta_R|^2 \right) C_\Phi \ BR(\Phi^0 \to \tau^+\tau^-), \]  

(5)

where the \( C_\Phi \) coefficients are:

\[ C_h = \left[ \frac{\cos(\beta - \alpha)}{\sin \alpha} \right]^2, \quad C_H = \left[ \frac{\sin(\beta - \alpha)}{\cos \alpha} \right]^2, \quad C_A = 1. \]  

(6)

Since non-negligible effects can only arise in the large \( \tan \beta \) limit, in eq. (4) we have approximated \( 1/\cos^2 \beta \approx \tan^2 \beta \).

We now present explicit expressions for the quantities \( \Delta_L \) and \( \Delta_R \), i.e. the coefficients of the dimension-four operators in (3). The relevant one-loop diagrams, which involve the exchange of sleptons, gauginos and Higgsinos, are shown in Fig. 1. The diagrammatic computation is consistently performed in the gauge symmetry limit, at zero external momentum [3]. In the superfield basis in which the charged lepton mass matrix is diagonal, the mass matrices

\[ \text{[3]} \text{In particular, the only Higgs insertion we consider is that explicitly depicted in the diagrams. Further Higgs insertions or momentum dependent effects correspond to higher dimension operators and give subleading corrections to } \Phi^0 \to \mu\tau, \text{ in the limit of heavy superpartners and large } \tan \beta. \]
of the left-handed and right-handed sleptons read:

\[
\tilde{M}_L^2 = \begin{pmatrix}
\tilde{m}_{L\mu}^2 & \tilde{m}_{L\tau}^2 \\
\tilde{m}_{L\tau}^2 & \tilde{m}_{L\tau}^2 
\end{pmatrix}, \\
\tilde{M}_R^2 = \begin{pmatrix}
\tilde{m}_{R\mu}^2 & \tilde{m}_{R\mu}^2 \\
\tilde{m}_{R\tau}^2 & \tilde{m}_{R\tau}^2 
\end{pmatrix}.
\] (7)

We are interested in scenarios with large LFV, either in \( \tilde{M}_L^2 \) [((LFV)_L] or in \( \tilde{M}_R^2 \) [((LFV)_R]. Large (LFV)_L means that \( \tilde{m}^2_{L\mu\tau} \) is comparable to \( \tilde{m}^2_{L\mu\mu} \) and \( \tilde{m}^2_{L\tau\tau} \). Similarly, large (LFV)_R means that \( \tilde{m}^2_{R\mu\tau} \) is comparable to \( \tilde{m}^2_{R\mu\mu} \) and \( \tilde{m}^2_{R\tau\tau} \). The flavour states \( \tilde{L}_\mu = (\tilde{\nu}_\mu, \tilde{\mu}_L, \tilde{\tau}_L) \) and \( \tilde{L}_\tau = (\tilde{\nu}_\tau, \tilde{\tau}_L) \) are related to the \( \tilde{M}_L^2 \) eigenstates \( \tilde{L}_2 = (\tilde{\nu}_2, \tilde{\epsilon}_L) \) and \( \tilde{L}_3 = (\tilde{\nu}_3, \tilde{\epsilon}_L) \) by the relations

\[
\tilde{L}_\mu = c_L \tilde{L}_2 - s_L \tilde{L}_3, \\
\tilde{L}_\tau = s_L \tilde{L}_2 + c_L \tilde{L}_3.
\]

Analogous relations hold for the right-handed sleptons:

\[
\tilde{\mu}_R = c_R \tilde{e}_R - s_R \tilde{e}_R, \\
\tilde{\tau}_R = s_R \tilde{e}_R + c_R \tilde{e}_R,
\]

where \( \tilde{e}_{R\alpha} \) and \( \tilde{e}_{R\alpha} \) (\( \alpha = 2, 3 \)) are the eigenvalues of \( \tilde{M}_L^2 \) and \( \tilde{M}_R^2 \), respectively. The other relevant parameters are the Bino (\( \tilde{B} \)) mass \( M_1 \), the Wino (\( \tilde{W}^0, \tilde{W}^\pm \)) mass \( M_2 \) and the \( \mu \) parameter. The latter appears in the Higgsino mass terms \(-\mu (\tilde{H}^0_1 \tilde{H}^0_1 - \tilde{H}^+_1 \tilde{H}^-_1) + \text{h.c.} \) and in the cubic interaction \(-Y_{\tau\mu} H^0_2 \tilde{\tau}_R \tilde{\tau}_L + \text{h.c.} \). The explicit evaluation of the diagrams gives for \( \Delta_L \):

\[
\Delta_L = \Delta_L^{(a)} + \Delta_L^{(b)} + \Delta_L^{(c)} + \Delta_L^{(d)},
\] (9)

Figure 1: Diagrams that contribute to \( \Delta_L \) [(a),(b), (c),(d)] and to \( \Delta_R \) [(e),(f)].
\[ \Delta^{(a)}_L = - \frac{g^2}{16\pi^2} \mu M_1 s_{LCL} \left[ s^2_R \left( I(M^2_1, \tilde{m}^2_{L_2}, \tilde{m}^2_{R_2}) - I(M^2_1, \tilde{m}^2_{R_2}, \tilde{m}^2_{L_2}) \right) \\
+ c^2_R \left( I(M^2_1, \tilde{m}^2_{R_2}, \tilde{m}^2_{L_2}) - I(M^2_1, \tilde{m}^2_{L_2}, \tilde{m}^2_{R_2}) \right) \right], \]

\[ \Delta^{(b)}_L = - \frac{g^2}{32\pi^2} \mu M_1 s_{LCL} \left[ I(M^2_1, \mu^2, \tilde{m}^2_{L_2}) - I(M^2_1, \mu^2, \tilde{m}^2_{R_2}) \right], \]

\[ \Delta^{(c)}_L = \frac{g^2}{32\pi^2} \mu M_2 s_{LCL} \left[ I(M^2_1, \mu^2, \tilde{m}^2_{L_2}) - I(M^2_1, \mu^2, \tilde{m}^2_{L_3}) \right], \]

\[ \Delta^{(d)}_L = \frac{g^2}{16\pi^2} \mu M_2 s_{LCL} \left[ I(M^2_1, \mu^2, \tilde{m}^2_{L_2}) - I(M^2_1, \mu^2, \tilde{m}^2_{L_3}) \right], \]

and for \( \Delta_R \):

\[ \Delta_R = \Delta^{(e)}_R + \Delta^{(f)}_R, \]

\[ \Delta^{(e)}_R = - \frac{g^2}{16\pi^2} \mu M_1 s_{RCL} \left[ s^2_L \left( I(M^2_1, \tilde{m}^2_{L_2}, \tilde{m}^2_{R_2}) - I(M^2_1, \tilde{m}^2_{R_2}, \tilde{m}^2_{L_2}) \right) \\
+ c^2_L \left( I(M^2_1, \tilde{m}^2_{L_2}, \tilde{m}^2_{R_2}) - I(M^2_1, \tilde{m}^2_{R_2}, \tilde{m}^2_{L_2}) \right) \right], \]

\[ \Delta^{(f)}_R = \frac{g^2}{16\pi^2} \mu M_1 s_{RCL} \left[ I(M^2_1, \mu^2, \tilde{m}^2_{R_2}) - I(M^2_1, \mu^2, \tilde{m}^2_{R_3}) \right], \]

The function \( I \), which has mass dimension \(-2\), is the standard three-point one-loop integral:

\[ I(x, y, z) = \frac{xy \log \frac{x}{y} + yz \log \frac{y}{z} + zx \log \frac{z}{x}}{(x-y)(z-y)(z-x)}. \]

Our results for the LFV diagrams in Fig. 11 can be compared with similar ones presented in \[ \text{[3][5]} \]. However, one notices some differences in those works: \textit{i)} there LFV effects were treated at linear order, through the mass insertion approximation; \textit{ii)} only LFV in the left-handed sleptons was considered, since LFV was related to the seesaw generation of neutrino masses; \textit{iii)} the relative signs between the \( \tilde{B} \) diagram and gaugino-Higgsino diagrams differ from ours. This sign is crucial to correctly determine the interference effects, as we will see below. Notice that such a sign discrepancy does not depend on the fact that we use a different sign convention for the \( \mu \) parameter.

For the sake of completeness we also present the expressions of the FC parameters \( \Delta_\mu, \Delta'_\mu, \Delta_\tau \), which are relevant for establishing the relations between the lepton masses (\( m_\mu, m_\tau \)) and the corresponding Yukawa couplings. Such quantities are induced by diagrams analogous to those in Fig. 1 but with the same flavour in the external fermion lines (either muon or tau flavour):

\[ \Delta_\mu = - \frac{g^2}{16\pi^2} \mu M_1 \left[ c^2_L c^2_R I(M^2_1, \tilde{m}^2_{L_2}, \tilde{m}^2_{R_2}) + c^2_L s^2_R I(M^2_1, \tilde{m}^2_{L_2}, \tilde{m}^2_{R_3}) + s^2_L c^2_R I(M^2_1, \tilde{m}^2_{L_3}, \tilde{m}^2_{R_2}) + s^2_L s^2_R I(M^2_1, \tilde{m}^2_{L_3}, \tilde{m}^2_{R_3}) + \frac{1}{2} c^2_L I(M^2_1, \mu^2, \tilde{m}^2_{L_2}) + \frac{1}{2} s^2_L I(M^2_1, \mu^2, \tilde{m}^2_{L_3}) - c^2_R I(M^2_1, \mu^2, \tilde{m}^2_{R_2}) - s^2_R I(M^2_1, \mu^2, \tilde{m}^2_{R_3}) \right] \]
\[ \Delta'_\mu = -\frac{g^2}{16\pi^2} \mu M_1 s_L c_R \left[ I(M_1^2, \tilde{m}_{L_2}, \tilde{m}_{R_2}) - I(M_1^2, \tilde{m}_{L_3}, \tilde{m}_{R_3}) \right] - I(M_2^2, \tilde{m}_{L_2}^2, \tilde{m}_{R_2}^2) + I(M_2^2, \tilde{m}_{L_3}^2, \tilde{m}_{R_3}^2) \]  
\[ \Delta_\tau = -\frac{g^2}{16\pi^2} \mu M_1 \left[ s_L^2 s_R^2 I(M_1^2, \tilde{m}_{L_2}^2, \tilde{m}_{R_2}^2) + s_L^2 c_R^2 I(M_1^2, \tilde{m}_{L_3}^2, \tilde{m}_{R_3}^2) \right] 
+ c_L^2 s_R^2 I(M_2^2, \mu^2, \tilde{m}_{L_2}^2) + c_L^2 c_R^2 I(M_2^2, \mu^2, \tilde{m}_{L_3}^2) + \frac{1}{2} s_L^2 I(M_2^2, \mu^2, \tilde{m}_{L_2}^2) \]  
\[ + \frac{1}{2} c_L^2 I(M_2^2, \mu^2, \tilde{m}_{L_3}^2) - s_L^2 I(M_2^2, \mu^2, \tilde{m}_{R_2}^2) - c_L^2 I(M_2^2, \mu^2, \tilde{m}_{R_3}^2) \]  
\[ + \frac{3g^2}{32\pi^2} \mu M_2 \left[ s_L^2 I(M_2^2, \mu^2, \tilde{m}_{L_2}^2) + c_L^2 I(M_2^2, \mu^2, \tilde{m}_{L_3}^2) \right]. \]  

These formulas are quite general as they include possible LFV in the slepton mass matrices. By setting \( c_L = c_R = 1, s_L = s_R = 0 \) one can easily recover for \( \Delta'_\mu \) and \( \Delta_\tau \) the corresponding cases without LFV, whereas \( \Delta'_\mu \) vanishes as this term requires both (LFV)\(_L\) and (LFV)\(_R\).

Incidentally, notice that \( \Delta'_\mu \) in eq. (2) is multiplied by \( Y_\tau \). Thereby, if (LFV)\(_L\) and (LFV)\(_R\) are both large, the relation between the muon mass and Yukawa coupling could receive large (\( \tan\beta \) and \( Y_\tau / Y_\mu \) enhanced) corrections\(^5\), and the ratios \( BR(\Phi^0 \rightarrow \mu^+ \mu^-) / BR(\Phi^0 \rightarrow \tau^+ \tau^-) \) could differ significantly from the tree level expectation \( (m_\mu/m_\tau)^2 \). However, having simultaneously large (LFV)\(_L\) and (LFV)\(_R\) does not seem very natural if the smallness of \( m_\mu/m_\tau \) is related to an underlying supersymmetric flavour symmetry.

### 3 Numerical results and implications at colliders

Now we give some numerical examples to appreciate the size of the effects we are discussing. For definiteness, we discuss separately the case of large (LFV)\(_L\), with negligible (LFV)\(_R\), and the complementary case of large (LFV)\(_R\), with negligible (LFV)\(_L\). Let us redefine in \( \tilde{m}^2_{L_{\tau\tau}} \equiv \tilde{m}_L^2 \) and \( \tilde{m}^2_{R_{\tau\tau}} \equiv \tilde{m}_R^2 \). As a representative case of large (LFV)\(_L\), we choose \( \tilde{m}^2_{L_{\mu\mu}} = \tilde{m}_L^2 \) and \( \tilde{m}^2_{L_{\mu\tau}} = 0.8 \cdot \tilde{m}_L^2 \), while \( \tilde{m}^2_{R_{\mu\tau}} \sim 0 \). Analogously, for the case of large (LFV)\(_R\) we choose \( \tilde{m}^2_{R_{\mu\mu}} = \tilde{m}_R^2 \) and \( \tilde{m}^2_{R_{\mu\tau}} = 0.8 \cdot \tilde{m}_R^2 \), while \( \tilde{m}^2_{L_{\mu\tau}} \sim 0 \). We show the quantity \( |50 \Delta_L|^2 \) as a function of \( |\mu|/\bar{m}_L \) in Fig. 2 and \( |50 \Delta_R|^2 \) as a function of \( |\mu|/\bar{m}_R \) in Fig. 3 for fixed values of other mass ratios. We have inserted a factor 50 to make it easier the numerical estimate of eq. (3) for the reference case of \( \tan \beta = 50 \). The curves depicted exhibit a

\(^4\)In this limit of vanishing LFV, different expressions for \( \Delta_\tau \) can be found in the literature \(^{12}\) \(^{13}\) \(^{29}\), and some discrepancies exist among them. Our result for \( \Delta_\tau \) is consistent with that in \(^{13}\), taking into account that we use an opposite sign convention for the \( \mu \) parameter and include left-right slepton mixing at linear order. To our knowledge no explicit expression for \( \Delta_\mu \) or \( \Delta'_\mu \) appears in the literature. In principle \( \Delta_\mu \) can be distinct from \( \Delta_\tau \).

\(^5\)In this limit of large (LFV)\(_L\) and (LFV)\(_R\), analogous enhancement effects also appear in the muon magnetic and electric dipole operators, see e.g. \(^{14}\). For similar enhancement effects in the relation between quark masses and Yukawa couplings, see e.g. \(^{7}\).
Figure 2: The quantity $|50\Delta L|^2$ as a function of $|\mu|/\tilde{m}_L$, for $\tilde{m}_{L\mu\tau}^2 = 0.8 \cdot \tilde{m}_L^2$ and four choices of the other relevant mass ratios: 1) $M_1 = M_2 = \tilde{m}_R = \tilde{m}_L$ (solid line); 2) $M_1 = \tilde{m}_L/3$, $M_2 = \tilde{m}_R = \tilde{m}_L$ (dotted line); 3) $M_1 = M_2 = \tilde{m}_L$, $\tilde{m}_R = \tilde{m}_L/3$ (dashed line); 4) $M_1 = M_2 = \tilde{m}_L$, $\tilde{m}_R = 3\tilde{m}_L$ (thin solid line).

Figure 3: The quantity $|50\Delta R|^2$ as a function of $|\mu|/\tilde{m}_R$, for $\tilde{m}_{R\mu\tau}^2 = 0.8 \cdot \tilde{m}_R^2$ and four choices of the other relevant mass ratios: 1) $M_1 = \tilde{m}_L = \tilde{m}_R$ (solid line); 2) $M_1 = \tilde{m}_R/3$, $\tilde{m}_L = \tilde{m}_R$ (dotted line); 3) $M_1 = \tilde{m}_R$, $\tilde{m}_L = \tilde{m}_R/3$ (dashed line); 4) $M_1 = \tilde{m}_R$, $\tilde{m}_L = 3\tilde{m}_R$ (thin solid line).
common behaviour with respect to the ratio $|\mu|/\tilde{m}_L$ or $|\mu|/\tilde{m}_R$: for each curve there is a deep minimum which separates the right-side region, where the pure $B^0$ diagram dominates as that mass ratio increases (diagram (a) for (LFV)$_L$ and diagram (e) for (LFV)$_R$ in Fig. 1), from the left-side one in which the Higgsino-gaugino diagrams dominate. The deep wells for either $|\Delta_L|^2$ or $|\Delta_R|^2$ are due to the destructive interference of the above mentioned diagrams. Notice that the interference would be constructive if the sign of $M_1$ were opposite to that of $M_2$.

In the case of (LFV)$_L$ we can see that values of $|50\Delta_L|^2$ larger than $\sim 5 \times 10^{-4}$ are achieved both in the left and right ranges in Fig. 2. The example with $\tilde{m}_R = \tilde{m}_L/3$ (dashed line) provides larger values in the range $|\mu|/\tilde{m}_L \gtrsim 3$ since the pure $B^0$ diagram is further enhanced by the smaller $\tilde{m}_R$. In the case of (LFV)$_R$, values of $|50\Delta_R|^2$ larger than $\sim 5 \times 10^{-4}$ can be obtained for large values of $|\mu|/\tilde{m}_R$ (see Fig. 3). An enhancement appears for $\tilde{m}_L = \tilde{m}_R/3$ (dashed line), in analogy to the (LFV)$_L$ example mentioned above. On the other hand, in the left-side region the values of $|50\Delta_R|^2$ are smaller with respect to the analogous ones of $|50\Delta_L|^2$. Indeed, in this range $|\Delta_R|^2$ is dominated by the $\tilde{H}-\tilde{B}$ diagram (proportional to $g^2$), while $|\Delta_L|^2$ is dominated by the $\tilde{H}-\tilde{W}$ diagrams (proportional to $g^2$).

We now make contact with the physical observable, i.e. the $BR(\Phi^0 \to \mu^+\tau^-)$ in (3), and discuss the phenomenological implications. We recall that the Higgs boson masses and the angle $\alpha$ in the coefficients $C_\Phi$ are also affected, through radiative corrections, by a set of MSSM parameters not involved in the determination of $\Delta_L, \Delta_R$, such as the mass parameters of the squark-gluino sector (see e.g. [15] and references therein). The latter parameters indirectly affect also the $BR(\Phi^0 \to \tau^+\tau^-)$ through radiative corrections to $BR(\Phi^0 \to b\bar{b})$ (see e.g. [15] [13]). We do not make a definite choice of those parameters and only outline some general features of $BR(\Phi^0 \to \mu^+\tau^-)$ at large tan $\beta$ and the prospects for these decay channels at the Large Hadron Collider (LHC) and other colliders. It is convenient to schematically separate the three Higgs bosons into two groups. The CP-odd and one of the CP-even Higgs bosons have about the same mass, non-standard (enhanced) couplings with down-type fermions and suppressed couplings with up-type fermions and electroweak gauge bosons. These bosons, which are mainly contained in $H^0_1$, correspond to $H^0, A^0 (h^0, A^0)$ for $m_A \gtrsim m_\tau$ ($m_A \lesssim m_\tau$), where $m_\tau \sim 110 - 130$ GeV. The other CP-even Higgs has a mass $\sim m_\tau$ and Standard Model-like couplings with up-type fermions and electroweak gauge bosons. It is mainly contained in $H^0_2$ and corresponds to $h^0$ ($H^0$) for $m_A \gtrsim m_\tau$ ($m_A \lesssim m_\tau$). Let us discuss the different Higgs bosons, assuming for definiteness tan $\beta \sim 50$, $|50\Delta|^2 \sim 10^{-3}$ ($\Delta = \Delta_L$ or $\Delta_R$) and an integrated luminosity of 100 fb$^{-1}$ at LHC.

1. If $\Phi^0$ denotes one of the ‘non-standard’ Higgs bosons, we have $C_\Phi \simeq 1$ and $BR(\Phi^0 \to \tau^+\tau^-) \sim 10^{-1}$, so $BR(\Phi^0 \to \mu^+\tau^-) \sim 10^{-4}$. The main production mechanisms at LHC are bottom-loop mediated gluon fusion and associated production with $b\bar{b}$, which yield cross sections $\sigma \sim (10^3, 10^2, 20)$ pb for $m_A \sim (100, 200, 300)$ GeV, respectively. The corresponding numbers of $\Phi^0 \to \mu^+\tau^-$ events are about $(10^4, 10^3, 2 \cdot 10^2)$. These estimates do not change much if the bottom Yukawa coupling $Y_b$ is enhanced (suppressed)
by radiative corrections, since in this case the enhancement (suppression) of $\sigma$ would be roughly compensated by the suppression (enhancement) of $BR(\Phi^0 \rightarrow \mu^+\tau^-)$.

2. If $\Phi^0$ denotes the other (more ‘Standard Model-like’) Higgs boson, the factor $C_\Phi \cdot BR(\Phi^0 \rightarrow \tau^+\tau^-)$ strongly depends on $m_A$, while the production cross section at LHC, which is dominated by top-loop mediated gluon fusion, is $\sigma \sim 30$ pb. For $m_A \sim 100$ GeV we may have $C_\Phi \cdot BR(\Phi^0 \rightarrow \tau^+\tau^-) \sim 10^{-1}$ and $BR(\Phi^0 \rightarrow \mu^+\tau^-) \sim 10^{-4}$, which would imply $\sim 300 \mu^+\tau^-$ events. The number of events is generically smaller for large $m_A$ since $C_\Phi$ scales as $1/m_A$, consistently with the expected decoupling of LFV effects for such a Higgs boson. However, an enhancement can occur under certain conditions. In particular, for a range of $m_A$ values the (radiatively corrected) off-diagonal element of the Higgs boson mass matrix could be over-suppressed. In this case the $\Phi^0 b\tau^c$, $\Phi^0 \tau\tau^c$ couplings would also be suppressed and as a result the number of $\mu^+\tau^-$ events could be even $O(10^0)$.

The above discussion suggests that LHC may offer good chances to detect the decays $\Phi^0 \rightarrow \mu\tau$, especially in the case of non-standard Higgs bosons. This indication should be supported by a detailed study of the background (which is beyond the scope of this work), for instance by generalizing the analyses in [9]. At Tevatron the sensitivity is lower than at LHC because both the expected luminosity and the Higgs production cross sections are smaller. The number of events would be smaller by a factor $10^2 - 10^3$. A few events may be expected also at $e^+e^-$ or $\mu^+\mu^-$ future colliders, assuming integrated luminosities of 500 fb$^{-1}$ and 1 fb$^{-1}$, respectively. At a $\mu^+\mu^-$ collider an enhancement may occur for the non-standard Higgs bosons if radiative corrections strongly suppress $Y_b$, since in this case both the resonant production cross section $[\sigma \sim (4\pi/m_A^2)BR(\Phi^0 \rightarrow \mu^+\mu^-)]$ and the LFV branching ratios $BR(\Phi^0 \rightarrow \mu^+\tau^-)$ would be enhanced. As a result, for light $m_A$, hundreds of $\mu^+\tau^-$ events could occur.

Finally we notice that, although we have focused on LFV decays of neutral Higgs bosons, also charged Higgs bosons have LFV decays, i.e $H^+ \rightarrow \tau^+\nu_\mu$ and $H^+ \rightarrow \mu^+\nu_\tau$ (and related charge conjugated channels). Also these decays are controlled by the parameters $\Delta_L$ and $\Delta_R$, at lowest order in SU(2)$_W$ breaking effects. The FV couplings with the charged Higgs bosons emerge by taking into account the SU(2)$_W$ completion of eqs. 1 and 3. It is straightforward to find $BR(H^+ \rightarrow \tau^+\nu_\mu) = \tan^2 \beta |\Delta_L|^2 BR(H^+ \rightarrow \tau^+\nu_\tau)$ and $BR(H^+ \rightarrow \mu^+\nu_\tau) = \tan^2 \beta |\Delta_R|^2 BR(H^+ \rightarrow \tau^+\nu_\tau)$. However, it is more natural to compare $H^+ \rightarrow \mu^+\nu_\tau$ with $H^+ \rightarrow \mu^+\nu_\mu$ so that:

$$BR(H^+ \rightarrow \mu^+\nu_\tau) \simeq \left(\frac{m_\tau}{m_\mu}\right)^2 \tan^2 \beta |\Delta_R|^2 BR(H^+ \rightarrow \mu^+\nu_\mu).$$  \hspace{1cm} (17)$$

For $\tan^2 \beta |\Delta_R|^2 \sim 10^{-3}$ this would lead to a 30% enhancement in the channel $H^+ \rightarrow \mu^+ + \text{missing energy}$.
4 Final remarks and conclusions

A few comments are in order about possible correlations between the decays $\Phi^0 \to \mu \tau$ and other LFV processes. We have seen that non-negligible rates for $\Phi^0 \to \mu \tau$ can only be obtained for large $\tan \beta$ and large LFV. In this limit also the decay rate for $\tau \to \mu \gamma$, which is dominated by diagrams analogous to those of Fig. 1 with an extra photon attached [20], is enhanced and could exceed the experimental limit. However, we recall that the rate of $\tau \to \mu \gamma$ decreases as the superparticle masses increase, whereas the rate of $\Phi^0 \to \mu \tau$ does not, since the latter is induced by dimension-four effective operators and only depends on mass ratios. Therefore to obtain an adequate suppression of $\tau \to \mu \gamma$ the superparticle spectrum has to be raised towards the TeV region, although some slepton may be lighter. For instance, in the case 1) of (LFV)$_L$ shown in Fig. 2 (M$_1 = M_2 = \tilde{m}_R = \tilde{m}_L$), for $|\mu|/\tilde{m}_L \sim 1$ we obtain $|50\Delta_L|^2 \sim 6 \times 10^{-4}$. In this particular example the present bound $BR(\tau \to \mu \gamma) < 6 \times 10^{-7}$ [19] constrains $\tilde{m}_L \gtrsim 1.4$ TeV for $\tan \beta = 50$, which implies min($\tilde{m}_L^2, \tilde{m}_R^2, |\mu|) \gtrsim 0.6$ TeV, max($\tilde{m}_L^2, \tilde{m}_R^2, |\mu|) \gtrsim 1.9$ TeV and $M_1, M_2, \tilde{m}_R, |\mu| \gtrsim 1.4$ TeV.

The decays $\Phi^0 \to \mu \tau$ are also correlated to the decay $\tau \to 3\mu$. We recall that the latter receives $\tan \beta$-enhanced contributions of two types: from dipole LFV operators via photon exchange [20] and from the scalar LFV operators [4] via Higgs exchange [3, 5]. The dipole contribution is directly related to the $\tau \to \mu \gamma$ decay rate and is consequently bounded, i.e. $BR(\tau \to 3\mu)^{\tau \to \mu \gamma} \sim 2.3 \times 10^{-3} \times BR(\tau \to \mu \gamma) \lesssim 1.4 \times 10^{-9}$. As for the Higgs-mediated contribution, we obtain the following estimate:

$$BR(\tau \to 3\mu)^{\Phi^0} \sim 10^{-7} \left(\frac{\tan \beta}{50}\right)^6 \left(\frac{100 \text{ GeV}}{m_A}\right)^4 \left(\frac{|50\Delta_L|^2 + |50\Delta_R|^2}{10^{-3}}\right).$$  \tag{18}$$

Therefore, this contribution can exceed the dipole induced one$^8$ and be not far from the present bound, $BR(\tau \to 3\mu) < 3.8 \times 10^{-7}$ [21]. Notice that the parameter region in which this occurs is also the most favorable one for the observation of the $\Phi^0 \to \mu \tau$ decays, so an interesting correlation emerges.

Throughout our work we have focused on the second and third generations, implicitly assuming that large slepton mixing only appears in that sector. In a scenario in which staus are mainly mixed with selectrons rather than with smuons, our discussion and numerical estimates concerning $\Phi^0 \to \mu \tau$ decays can be directly translated to $\Phi^0 \to e \tau$ decays, with obvious substitutions. The case of large smuon-selectron mixing is somewhat different. Although the strong constraints from $\mu \to e \gamma$ can be satisfied by taking sufficiently heavy superparticles, the decays $\Phi^0 \to \mu e$ are generically suppressed by the presence of $Y_{\mu}$. The latter decays could be $Y_{\mu}$-enhanced if both (LFV)$_L$ and (LFV)$_R$ were present, and staus were mixed with both smuons and selectrons.

$^8$Here our conclusion is in qualitative agreement with that drawn by [3]. On the other hand, the authors of [5] conclude that Higgs-mediated contributions to $\tau \to 3\mu$ are subleading compared to the photonic penguin ones. We believe that this different conclusion is partly due to the fact that in [5] the superparticle masses are chosen to lie below the TeV scale, so sizeable values for the LFV Higgs couplings are prevented by the $\tau \to \mu \gamma$ constraint.
In summary, we have studied the LFV couplings of Higgs bosons in a general MSSM framework, allowing for generic LFV entries in the slepton mass matrices, but without invoking any specific mechanism to generate them. We have computed the branching ratios of $\Phi^0 \rightarrow \mu\tau$ decays, which depend on ratios of MSSM mass parameters, and increase for increasing $\tan \beta$ and LFV. Although cancellations can occur in some regions of parameter space, $\mathcal{O}(10^{-4})$ values are achievable, and they are compatible with the bounds on $\tau \rightarrow \mu\gamma$ for a superparticle spectrum in the TeV range. If the Higgs spectrum is relatively light ($m_A \lesssim 300$ GeV), our results indicate that future colliders (in particular LHC) may be able to detect the decays $\Phi^0 \rightarrow \mu\tau$, especially in the case of non-standard Higgs bosons. Moreover, the detection of these decays is closely correlated with that of $\tau \rightarrow 3\mu$, which may be observed in the near future.

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