Investigating students' concept image in understanding variables

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Abstract. Concept image is the overall structure associated with a concept, including all mental images and their related properties and processes. The concept image develops continuously through experience and changes as the individual faces stimulation and maturity. This study aims to analyze and describe the concept image of students in understanding the variable concept. This study is qualitative research with a phenomenological method. Data were collected by means of interviews and tests, then analyzed using the content analysis method. Participants involved in this study were ten eighth-grade students, while the object in this study was students’ concept image in understanding the variable concept. The findings revealed that most of the students understood a variable as something whose value is unknown, whilst some other students comprehend a variable as a substitute for something whose value is unknown. Understanding students' thinking or concept image provides educators with an insight of the learning process aligned with students’ needs as a way to increase their understanding of mathematical concepts.

1. Introduction

Mathematics is inseparable from solving problems to train students' thinking and knowledge skills. Mathematics provides a systematic and planned way of thinking so that by learning mathematics, students are expected to be critical, systematic, logical, creative and careful. One of the topics that can explore students' mathematical thinking is algebra. Algebra is a gateway to understanding mathematics further [1]. In algebra, children are introduced to variables and various mathematical symbols used to translate sentences into mathematical models in solving problems related to everyday life. Many such problems can be solved using algebra. Thus, it is important to learn algebra as a provision for facing future lives, as the National Council of Teachers of Mathematics (NCTM) suggested that algebra be essential for adult life as preparation both for jobs and higher education [2].

Algebra is a prominent part of mathematics in solving other mathematical problems since in algebra, we learn mathematical symbols and how to manipulate them [3]. Manipulating mathematical symbols requires a comprehensive understanding of the basic algebraic concepts, that is, how to interpret and understand the concept of variables. In fact, several studies have reported that students have difficulty in understanding algebraic concepts, such as variables and algebraic expressions [4]–[6].

Vinner mentioned that a person’s concept image can be a visible representation, in addition to a set of impressions or stories related to the concept [7]. Vinner also stated that a concept image is cumulated and modified over time; it is not a static object saved in memory. Its particular factors can be activated...
in reaction to a few questions or hassle situations, at the same time as its different factors cannot [7]. Hence, Tall and Vinner outlined an evoked idea of image as the “part of a person’s concept image activated at a selected time” [8]. Understanding variables is a fundamental concept that is highly important to note. By a correct understanding of variables, students will be able to write the algebraic expressions accurately. Tanisli and Kose conducted a study on pre-service teachers, aiming to evaluate the ability of basic mathematics teachers and investigate students’ thinking processes, as well as to predict students’ difficulties and misunderstandings on the variable and equation concepts. The study revealed that students made several mistakes. Suppose the first equation is $4x - 1 = 0$. When students were asked to determine the value of $x$ in the equation, they made a mistake. They wrote $4x = 1$, then $x = 1 - 4$, so $x = -3$. Tanisli and Kose stated that when solving the equation, the students generally did an error $4x = 1, x = 1 - 4$, which is called the other inverse error, and they focused on the inverse of the addition operation, instead of the inverse of the multiplication operation [9]. In addition, another common error is found when students represent mathematical problems into a mathematical model. In this case, the error is related to balancing expressions. For instance, given $2 + 3x$ equals to $5x$. Such habits are associated with students’ desire to use algebraic expressions as a “closed result” in the same way that they obtain a final result for $2 + 3 = 5$ [10]. For this reason, the present study aims to investigate students’ understanding of the variable concept based on their knowledge of the definition of the variable itself. In this article, the author will analyze students’ concept image in understanding variables which are important parts of algebra.

2. Method

This study is qualitative research. The term qualitative research is a general term that refers to several research traditions and strategies that have certain similarities. There is an emphasis on processes, or how things happen, and a focus on attitudes, beliefs, and thoughts–how people make sense of their experiences as they interpret their world [11]. This study also used a phenomenological research method, the most suitable method for research that aims to understand the experiences and general thoughts of participants about a phenomenon [12]. Participants in this study involved ten eighth-grade students labelled S1, S2, …, and S10. The participants were selected based on the ability levels (low, medium and high) taken from the students’ daily test scores. The limited number of research participants is based on the consideration that the qualitative research emphasizes more on comprehensive information than a large number of research subjects [13]. The object in this study was students’ concept image in understanding the variable concept. Data were collected by carrying out interviews and tests to explore the students’ concept image in understanding the variable concept in algebra. Since we employed a qualitative approach, the research instrument in this study is the researchers themselves. The unit of analysis is related to the focus or component under study which includes certain individuals who are in accordance with the focus of the problem. The test used to explore students' concept images was adapted from the intermediate algebra book [14]. Meanwhile, the unstructured interviews were designed to freely trace in various aspects and directions in order to get as complete information as possible [15]. Data analysis included preparing and organizing data for further analysis (e.g., text data in transcripts, or image data in photos), selecting and adjusting data to the research topic through a coding process, and presenting data in the form of images, tables or discussion [12]. This is a common process in qualitative research. The final objective of phenomenological data analysis is to present an in-depth analytical picture of the phenomenon under study, which describes the participants’ experience in understanding the meaning of variables in algebra.

3. Result and discussion

The current study aims to explore students’ concept image in understanding the variable concept in algebra. To get an insight of their concept image, an in-depth interview was conducted. In the interview, the respondents were given a test related to the variable concept. The interview results found that most respondents understood the variable as something unknown, as described by Respondent S2.

Interviewer: What do you think about variable?
S2: A variable is something denoted by $x$ and $y$.

Interviewer: Why are $x$ and $y$?

S2: Because when studying systems of linear equations, I remember that there are always $x$ and $y$, and both are always called variables.

Interviewer: Then, is $a$, $u$ or $z$ not a variable?

S2: Hmm...

Interviewer: If you understand that the variable is something denoted by $x$ and $y$, then what do you think about the meaning of $x$ and $y$?

S2: The meaning of $x$ and $y$ is something that has unknown values.

Interviewer: You may want to explain this in a mathematical model.

S2: $2x + 10$

Interviewer: Try to solve the following problem.

Jaka has the following problem to solve: “Find the value of $x$ in the expression: $x + x + x = 12$.” Then, Jaka answers in the following ways: (a) 2, 5, 5; (b) 10, 1, 1; and (c) 4, 4, 4. Which one is correct? (Please circle the correct letter: a, b, or c) and give your rationale.

S2: (S2 provides the following solutions)

|   |   |
|---|---|
| a | 2 + 5 + 5 = 12 |
| b | 10 + 1 + 1 = 12 |
| c | 4 + 4 + 4 = 12 |

Rationale: Because the results are the same.

**Figure 1.** Respondent S2's strategy.

Based on the solutions given by the student, it seems that the student had not been able to understand and solve the problems correctly. The student's understanding of the variable concept had not been comprehensive yet. The student understood the variable as something unknown. The notion of “unknown” could become an epistemological obstacle when trying to conceptualize the meaning of the variable [16]. There are many variable terms which frequently lead to students' difficulties in understanding variables. The study of Schoenfeld and Arcavi has shown that some students grapple with the distinction between the name of an object (e.g., Michael as a person), the attribute name (e.g., Michael's height), or the measurement or quantity (e.g., $h$ units) [17]. MacGregor explained that students can manipulate variables without fully understanding the power and flexibility of symbols [18]. Students also can interpret letters or algebraic expressions based on intuition, guessing, or comparing them with
other symbol systems they know [19]. In addition, there is evidence that students' understanding of variables is fixed or unchanged over the years in school, even up to the college level [18]. Yet, variables are at the heart of algebra; therefore, understanding their nature and behavior is critical to success in algebra.

A comprehensive understanding of the variable concept in algebra will affect students' ability to write algebraic expressions correctly. It can be seen from Respondent S7's understanding described below.

Interviewer: What do you think about variable?
S7: Variable is a substitute for something whose value is unknown.

Interviewer: What do you mean by substitute?
S7: If there is an unknown value that we need to find, we can write it as $x$, for example.

Interviewer: Is it written as $x$ only or something else?
S7: No, we can replace it with anything, like $y$, $z$, $a$ and so on.

Interviewer: Okay, I have a question like this: Mother's money is increased by IDR 5,000.00. Can you give me a mathematical expression for the statement?
S7: Suppose mother's money is $x$, then $x + 5000$.

Interviewer: Okay, try to solve the following problem.
Jaka has the following problem to solve: “Find the value of $x$ in the expression: $x + x + x = 12$.” Then, Jaka answers in the following ways: (a) 2, 5, 5; (b) 10, 1, 1; and (c) 4, 4, 4. Which one is correct? (Please circle the correct letter: a, b, or c) and give your rationale.
S7: (S7 gives the following solutions)

C. 4, 4, 4
$x$ is the sum of the multiples of the same number
$x + x + x = 12$ is $x^3$
$x + x + x = 12$
If $x$ is 4, then $4 + 4 + 4 = 12$

Figure 2. Respondent S7’s strategy.

Based on the answer given by Respondent S7, it shows that the student have a comprehensive understanding of the variable concept. In the answer, the student explained that $x$ is the sum of the multiples of the same number so that the correct choice, according to S7, is the option C. The findings revealed that the student understood the variable as an unknown value and as a substitute for something whose value is unknown. Several past studies in students’ understanding of the variable concept in algebra also showed similar findings [17], [20]–[22]. With regard to variables, some middle school students view symbols as objects representing one particular value [17]. The overall concept image of students in understanding the variable concept in algebra is presented in table 1.

A comprehensive understanding of a concept will have a positive influence on students' ability to understand and solve math problems [23]. Understanding the definition of a concept does not mean that students have a comprehensive understanding when they are able to formally state the definition. Sometimes students have their own interpretation and language to comprehend a concept, and we as teachers have to provide appropriate corrections to their understanding. Tall and Vinner describe a student’s personal concept definition as the phrases used to explain a mathematical concept. This
concept definition may not be aligned with the formal definition of the concept. Although teachers prefer students to seek advice from a proper definition of a concept, it is common for them to only ask for an explanation of their concept image, irrespective of how intently it aligns with the formal definition [24].

Table 1. Summary of respondents' answers about the concept of variables.

| Theme                                      | Respondents |
|--------------------------------------------|-------------|
| Variable as something denoted by \(x\) and \(y\) | S1, S2, S5, S8 |
| Variable as something whose value is unknown | S2, S4, S5, S6, S8 |
| Variable is a substitute for something whose value is unknown | S3, S7, S10 |
| Do not know (did not give any answer)      | S9          |

A student’s evoked concept image does no longer always correspond to the mathematical definition of a term, and we assume that these inconsistencies impact students’ analysis of mathematical texts. Sometimes students’ responses are formed by means of their choice to delight their instructors or some other authority figures. When this happens, the students will exhibit pseudo-conceptual behavior giving solutions that replicate a cognitive concept, but not really derived from the cognitive concept [25]. While defining a concept, a person identifies the essential properties of the concepts, constructs arguments, and uses a verbal exchange of mathematics.

Vinner introduced that a person’s concept image may be a visible representation as well as a set of impressions or reports related to that concept. Instead of being a static object in memory, a concept image is gradually increased and modified over a period of time. Its particular factors can be activated in reaction to a few questions or hassle situations, at the same time as its different factors cannot [26]. The study findings suggested that the students had a good concept image in understanding the variable concept. Students and teacher having informal definitions may also mean that they no longer preserve the formal definitions. However, they enhance their concept image from their experience of creating informal definitions.

4. Conclusion
Students’ responses related to the variable concept discussed in this article revealed that the students understood variables as unknown values and as a substitute for something whose value is unknown. This study highlights the importance of a comprehensive understanding of the variable concept. In addition to understanding students’ concept image of variables, this study will be valuable for educators in understanding students’ thinking in the algebraic domain. Finally, understanding students’ concept image, especially related to the variable concept, will provide opportunities for educators to learn students’ needs in order to improve students’ understanding of mathematical concepts.

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