Composite Signalling for DFRC: Dedicated Probing Signal or Not?

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Abstract

Dual-functional radar-communication (DFRC) is a promising new solution to simultaneously probe the radar target and transmit information in wireless networks. In this paper, we study the joint optimization of transmit and receive beamforming for the DFRC system. Specifically, the signal to interference plus noise ratio (SINR) of the radar is maximized under the SINR constraints of the communication user (CU), which characterizes the optimal tradeoff between radar and communication. In addition to simply using the communication signal for target probing, we further consider to exploit dedicated probing signals to enhance the radar sensing performance. We commence by studying the single-CU scenario, where a closed-form solution to the beamforming design problem is provided. It is then proved that a dedicated radar probing signal is not needed. As a further step, we consider a more complicated multi-CU scenario, where the beamforming design is formulated as a non-convex quadratically constrained quadratic programming. The optimal solutions are obtained by applying semidefinite relaxation with guaranteed rank-1 property. It is shown that under the multi-CU scenario, the dedicated probing signal should be employed to improve the radar performance at the cost of implementing an additional interference cancellation at the CU. Finally, the numerical simulations are provided to verify the effectiveness of the proposed algorithm.

Index Terms

Spectrum sharing, radar-communication, signal to interference and noise ratio, probing signal, communication signal, joint beamforming.

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I. INTRODUCTION

Communication and radar spectrum sharing (CRSS) has recently drawn significant attention due to the scarcity of the commercial wireless spectrum. For instance, the millimeter wave (mmWave) band is occupied by variety of radars [1], and has also been assigned as a new licensed band to the 5G network [2]. It is well-recognized that communication and radar signals have some common features in their waveforms. Although their purposes are dramatically different, it is feasible to use one type of signal for the other types purpose. Nevertheless, the use of radar (communication) signals for communication (radar) functionalities, introduces a number of challenges [3]–[5]. To address these challenges, the research of dual-functional radar-communication (DFRC) is well-underway [6]–[8].

In general, the aim of the DFRC is to implement both communication and radar functionalities on the same hardware platform. Based on information theory, the work in [9] unified the radar and communication performance metric and discussed the performance bounds of the DFRC system. Furthermore, the weighted sum of the estimation and communication rates was analyzed as the performance metrics in the DFRC system [10]. By leveraging the simple time-division scheme, the radar and the communication signals can be transmitted within different time slots, which avoids the mutual interference [11]. To exploit the favorable time-frequency decoupling property of the orthogonal frequency division multiplexing (OFDM) waveform, the OFDM communication signal was adopted for target detection, where the range and Doppler processing are independent with each other [12]. From a signal processing perspective, the implementation of mmWave DFRC systems was fully studied in [13]. The unimodular signal design was discussed in [14] for DFRC architecture, where the information of downlink communication was modulated via the ambiguity function (AF) sidelobe nulling in the prescribed range-Doppler cells.

Beamforming design is essential to improve the performance of the DFRC signal processing in the spatial domain, which has been widely studied in the literature. Aiming to realize the dual functionalities, the work of [15] designed a transmit beampattern for multiple input multiple output (MIMO) radar with the communication information being embedded into the sidelobes of the radar beampattern. Considering both the separated and the shared antenna deployments, a series of optimization-based transmit beamforming approaches for the DFRC system were studied in [16], where the communication signal was exploited for target detection. By imposing the constraints of the radar waveform similarity and the constant modulus, the interference of the multiple communication users (CUs) was suppressed to improve the communication performance in [17]. Based on IEEE 802.11ad wireless local area network (WLAN) protocol, a joint waveform for automotive radar and a potential mmWave vehicular communication system were proposed
in [18]. The work of [19] further studied the feasibility of an opportunistic radar, which exploited the probing signals transmitted during the sector level sweep of the IEEE 802.11ad beamforming training protocol. In order to reduce the hardware complexity and the associated costs incurred in the mmWave massive MIMO system, a hybrid analog-digital beamforming structure was proposed for the DFRC transmission in [20].

It is worth pointing out that all the aforementioned works on the DFRC beamforming design focused on formulating a desired transmit beampattern without considering the receive beamforming. To the best of our knowledge, the joint optimization of the transmit and receive beamforming under the communication constraints has never been studied for the DFRC system before, despite the fact that the joint optimization of transmit and receive beamforming for the MIMO radar system has been extensively investigated in the recent literature [21]–[27]. To detect an extended target, the joint optimization of waveforms and receiving filters in the MIMO radar was considered in [21]. In order to guarantee the constant modulus and similarity properties of the radar waveforms, numerous approaches were provided to maximize the signal to interference plus noise (SINR) of the radar, e.g., the sequential optimization algorithms (SOAs) in [22], the successive quadratically constrained quadratic programming (QCQP) refinement method in [23], the block coordinate descent (BCD) framework in [24], and the general majorization-minimization (MM) framework in [25]. To detect multiple targets, the joint optimization of waveforms and receiving filters in the MIMO radar was further studied in [26]. The problem of beampattern synthesis with sidelobe control was studied in [27] using constant modulus weights. The jointly design of the transmit and receive beamforming was provided in [28] based on a priori information on the locations of target and interferences.

All these works improved the receive SINR of the echo signal based on the prior information of the target and clutter. In contrast, for the DFRC system, there exist both probing signal and communication signal, which are coupled together with each other for drastically different purposes. As a consequence, the known MIMO radar-only designs are inapplicable for the latter. In this paper, we study the joint optimization of transmit and receive beamforming for the DFRC system. Specifically, the SINR of the radar is maximized under the SINR constraints of the CUs. Depending on the component of the DFRC transmit signal, we consider both the non-dedicated probing signal case and the dedicated probing signal case. For the non-dedicated probing signal case, the DFRC transmit signal is only composed of the communication signal of the CUs. For the dedicated probing signal case, besides the communication signal, the dedicated probing signal is added to the DFRC transmit signal to improve the radar performance. Under the single CU scenario, closed-form solutions of the optimized beamforming are provided for both cases.
And it can be proved that there is no need to employ dedicated probing signal for the single CU scenario. On top of that, we consider a more complicated scenario with multiple CUs. For the non-dedicated probing signal case, the beamforming design is formulated as a non-convex quadratically constrained quadratic programming (QCQP), and we show that the globally optimal solution can be obtained by applying semidefinite relaxation (SDR) with rank-1 property. For the dedicated probing signal case, the rank-1 property after applying SDR can be also proved, and the corresponding optimal solution shows that the dedicated probing signal should be employed to improve the SINR of the radar receiver. The main contributions of this paper can be summarized as follows.

- **Transmit-receive DFRC beamforming**: We provide the solutions of the joint transmit-receive beamforming optimization to maximize the SINR of the radar under the SINR constraints of the CUs. For the single CU scenario, the closed-form solutions of the optimized beamforming are derived. For the multiple CUs scenario, the optimal solutions are obtained by applying SDR with rank-1 property.

- **Dual-functional performance tradeoff**: The optimal performance tradeoff between the radar and the communication is characterized in terms of SINR. Compared to the time-sharing scheme between the radar and the communication, the joint optimization of transmit and receive beamforming yields a more favorable tradeoff performance.

- **Dedicated radar probing signal or not**: Compared to the existing works only exploiting communication signals for radar functionality, we consider the use of a dedicated radar probing signal to improve the radar performance. For the single CU scenario, our analysis shows that there is no need of dedicated probing signal. For the multi-CU scenario, on the other hand, it is beneficial to employ the dedicated probing signal.

The remainder of the paper is organized as follows. Section II presents the system model. Section III studies a simplified single CU scenario. The study is further extended to a more complicated multiple CUs scenario in Section IV. Simulation results are provided in Section V, followed by concluding remarks in Section VI.

**Notation**: We use boldface lowercase letter to denote column vectors, and boldface uppercase letters to denote matrices. Superscripts \((\cdot)^H\) and \((\cdot)^T\) stand for Hermitian transpose and transpose, respectively. \(\text{tr}(\cdot)\) and \(\text{rank}(\cdot)\) represent the trace operation and the rank operator, respectively. \(\mathcal{C}^{m \times n}\) is the set of complex-valued \(m \times n\) matrices. \(x \sim \mathcal{CN}(a, b)\) means that \(x\) obeys a complex Gaussian distribution with mean \(a\) and covariance \(b\). \(E(\cdot)\) denotes the statistical expectation. \(\|x\|\) denotes the Euclidean norm of a complex vector \(x\).
II. SYSTEM MODEL

As illustrated in Fig. 1, we consider a DFRC MIMO system, which simultaneously probes the radar target and transmits information to the CUs. To be specific, it is composed of a DFRC base station (BS) with $N_t$ transmit antennas and $N_r$ receive antennas, $K$ single-antenna CUs indexed by $k \in \{1, \cdots, K\}$.

Suppose there is a target and $I$ signal-dependent interference sources indexed by $i \in \{1, \cdots, I\}$. The target is located at angle $\theta_0$ and the interference sources are located at angle $\theta_i$, $i \in \{1, \cdots, I\}$. Given the transmit signal $x \in \mathbb{C}^{N_t \times 1}$, the received signal of the radar receiver is

$$y_0 = \alpha_0 a_r (\theta_0) a_t^T (\theta_0) x + \sum_{i=1}^{I} \alpha_i a_r (\theta_i) a_t^T (\theta_i) x + z_0$$

$$= \alpha_0 A (\theta_0) x + \sum_{i=1}^{I} \alpha_i A (\theta_i) x + z_0,$$  \hspace{1cm} (1)

where $\alpha_0$ and $\alpha_i$ are the complex amplitudes of the target and the $i$-th interference source, respectively, $a_t (\theta) = [1, e^{-j2\pi \Delta_t \sin \theta}, \cdots, e^{-j2\pi (N_t-1) \Delta_t \sin \theta}]^T$ and $a_r (\theta) = [1, e^{-j2\pi \Delta_r \sin \theta}, \cdots, e^{-j2\pi (N_r-1) \Delta_r \sin \theta}]^T$ with $\Delta_t$ and $\Delta_r$ being the spacing between adjacent antennas normalized by the wavelength, respectively, and $z_0 \in \mathbb{C}^{N_r \times 1}$ is the additive white Gaussian noise (AWGN) with each element subjects to $\mathcal{CN}(0,1)$. The symbol index is omitted for simplicity. Then, the output of the radar receiver is

$$r = w^H y_0$$

$$= \alpha_0 w^H A (\theta_0) x + w^H \sum_{i \in I} \alpha_i A (\theta_i) x + w^H z_0,$$  \hspace{1cm} (2)
where \( w \in \mathbb{C}^{N_r \times 1} \) is the receive beamforming vector for SINR maximization.

Further, given the transmit signal \( x \in \mathbb{C}^{N_t \times 1} \), the received signal of the CU \( k \) is

\[
y_k = h_k^H x + z_k,
\]

where \( h_k \in \mathbb{C}^{N_t \times 1} \) is the multiple input single output (MISO) channel vector between the DFRC BS and the CU \( k \), and \( z_k \sim \mathcal{CN}(0, 1) \) is the AWGN of the CU \( k \).

In this paper, we consider two cases according to the component of the DFRC BS’s transmit signal \( x \).

- **Case 1** (Non-dedicated probing signal): In this case, the transmit signal of the DFRC BS is only composed of the communication signals of the CUs. That is

\[
x = \sum_{k=1}^{K} x_k,
\]

where \( x_k \) is the communication signals of the CU \( k \) and the radar functionality is realized by the sum of the CUs’ communication signal.

- **Case 2** (Dedicated probing signal): In this case, the transmit signal of the DFRC BS is composed of both the communication signal of the CUs and the dedicated probing signal. That is

\[
x = \sum_{k=1}^{K} x_k + x_0,
\]

where \( x_0 \) is the dedicated probing signal to enhance the radar performance.

In addition, we impose the following two assumptions in this paper. 1) For the radar function, the angles of the target \( \theta_0 \) and the interference \( \{\theta_i\} \) are assumed to be known to the DFRC BS. 2) For the communication function, the channel is assumed to be known to the DFRC BS, and the dedicated probing signal \( x_0 \) is pseudo-random and assumed to be known in prior to the CUs.

### III. Single CU Scenario

In this section, we consider a simplified scenario with single CU in the network. The beamforming design of the DFRC BS is discussed for the non-dedicated probing signal case, and the closed-form solution is provided. Then, the dedicated probing signal case is studied, and it can be proved that there is no need of dedicated probing signal with single CU.
A. Non-dedicated Probing Signal Case

For the non-dedicated probing signal case, the transmit signal of the DFRC BS in (4) with single CU can be rewritten as

\[ x = u s, \quad (6) \]

where \( u \in \mathbb{C}^{N_t \times 1} \) and \( s \in \mathbb{C}^{N_r \times 1} \) are the beamforming vector and the information symbol of the CU, respectively.

According to the output in (2), the SINR of the radar receiver can be expressed as

\[
\gamma^{(1)}_R = \frac{|\alpha_0 w^H A(\theta_0) x|^2}{\mathbb{E} \left[ w^H \sum_{i=1}^I \alpha_i A(\theta_i) x \right]^2 + w^H w} + w^H \left[ \sum_{i=1}^I |\alpha_i|^2 A(\theta_i) u u^H A^H(\theta_i) + I \right] w . \quad (7)
\]

And the output SINR of the radar receiver depends on the choice of the receive beamforming vector \( w \). The design of \( w \) can be expressed as

\[
\max_w \frac{|w^H A(\theta_0) x|^2}{w^H \left[ \sum_{i=1}^I |\alpha_i|^2 A(\theta_i) uu^H A^H(\theta_i) + I \right] w}, \quad (8)
\]

which is equivalent to the well-known minimum variance distortionless response (MVDR) problem, and its solution can be given by [21]

\[
w^* = \frac{\Sigma_1(u)^{-1} A(\theta_0) x}{x^H A^H(\theta_0) \Sigma_1(u)^{-1} A(\theta_0) x}, \quad (9)
\]

where

\[
\Sigma_1(u) = \left[ \sum_{i=1}^I |\alpha_i|^2 A(\theta_i) uu^H A^H(\theta_i) + I \right]. \quad (10)
\]

Substituting (9) into (7), the SINR of the radar receiver can be calculated as

\[
\gamma^{(1)}_R = x^H \Phi_1(u) x, \quad (11)
\]

where
\[ \Phi_1 (u) = |\alpha_0|^2 A^H (\theta_0) \Sigma_1 (u)^{-1} A (\theta_0). \] (12)

And the average SINR of the radar receiver can be given by

\[ \tilde{\gamma}_R^{(l)} = E [x^H \Phi_1 (u) x] = u^H \Phi_1 (u) u. \] (13)

For the CU \( k \), the received signal in (3) can be rewritten as

\[ y_k = h^H u_s + z_k, \] (14)

where \( h \in \mathbb{C}^{N_t \times 1} \) is the MISO channel vector between the DFRC BS and the CU. And the average SINR of the CU can be calculated as

\[ \tilde{\gamma}_C = |h^H u|^2, \] (15)

Then, we consider the beamforming optimization problem that maximizes the SINR of the radar receiver and satisfies the SINR of the CU, i.e.,

\[ \begin{array}{ll}
\max_u & \tilde{\gamma}_R^{(l)} = u^H \Phi_1 (u) u \\
\text{s.t.} & \tilde{\gamma}_C = |h^H u|^2 \Gamma, \\
& u^H u \leq P_0
\end{array} \] (P1.1)

where \( \Gamma \) is the threshold of the CUs SINR, and \( P_0 \) is the transmit power constraint of the DFRC BS.

Because \( \tilde{\gamma}_R^{(l)} \) is a nonlinear function of the transmit beamforming vector \( u \), Problem (P1.1) is generally non-convex. Thus, we adopt the sequential optimization to find the transmit beamforming vector \( u \) in an iterative fashion. Specifically, at the \( m \)-th iteration, we first compute \( \Phi_0 = \Phi_1 [u^{(m-1)}] \), where \( u^{(m-1)} \) is obtained in the \( (m - 1) \)-th iteration. Thus, Problem (P1.1) can be rewritten as

\[ \begin{array}{ll}
\max_u & \tilde{\gamma}_R^{(l)} = u^H \Phi_0 u \\
\text{s.t.} & \tilde{\gamma}_C = |h^H u|^2 \geq \Gamma, \\
& u^H u \leq P_0
\end{array} \] (P1.2)

And the closed-form solution of Problem (P1.2) can be given by the following proposition.
**Proposition 1.** (Optimal beamforming with single CU) For the non-dedicated probing signal case, the optimal beamforming vector of the DFRC BS with single CU i.e., the optimal solution to Problem (P1.2), can be given by

\[
\mathbf{u}^* = \begin{cases} 
\sqrt{P_0} \hat{\mathbf{g}}, & \Gamma \leq P_0 \| \mathbf{h}^H \hat{\mathbf{g}} \|^2 \\
(\alpha \hat{\mathbf{h}} + \beta \hat{\mathbf{g}}_\perp), & P_0 \| \mathbf{h} \|^2 \geq \Gamma > P_0 \| \mathbf{h}^H \hat{\mathbf{g}} \|^2
\end{cases},
\]

(18)

\[
\alpha = \sqrt{\frac{\Gamma}{\| \mathbf{h} \|^2}} \frac{\alpha_g}{\| \alpha_g \|};
\]

(19)

\[
\beta = \sqrt{P_0 - \frac{\Gamma}{\| \mathbf{h} \|^2}} \frac{\beta_g}{\| \beta_g \|};
\]

(20)

where \( \mathbf{g} \) is the dominant eigenvector of \( \mathbf{\Phi}_0 \), \( \hat{\mathbf{g}} = \mathbf{g} / \| \mathbf{g} \| \), \( \hat{\mathbf{h}} = \mathbf{h} / \| \mathbf{h} \| \), \( \mathbf{g}_\perp = \mathbf{g} - (\hat{\mathbf{h}}^H \hat{\mathbf{g}}) \hat{\mathbf{h}} \) denoting the projection of \( \mathbf{g} \) into the null space of \( \hat{\mathbf{h}} \), \( \hat{\mathbf{g}}_\perp = \mathbf{g}_\perp / \| \mathbf{g}_\perp \| \) and \( \mathbf{g} \) can be expressed as \( \mathbf{g} = \alpha_g \hat{\mathbf{h}} + \beta_g \hat{\mathbf{g}}_\perp \).

**Proof.** The proof is given in Appendix A.

\( \square \)

**B. Dedicated Probing Signal Case**

For the dedicated probing signal case, the transmit signal of the DFRC BS in (5) with single CU can be rewritten as

\[
\mathbf{x} = \mathbf{us} + \mathbf{vs}_0,
\]

(21)

where \( \mathbf{v} \in \mathbb{C}^{N_t \times 1} \) and \( \mathbf{s}_0 \sim \mathcal{C}\mathcal{N}(0,1) \) are the beamforming vector and the symbol of the dedicated probing signal, respectively. In addition, \( \mathbf{s} \) and \( \mathbf{s}_0 \) are independent and identically distributed (i.i.d.).

According to the output in (2), the SINR of the radar receiver can be expressed as

\[
\gamma_R^{(II)} = \frac{|\alpha_0 \mathbf{w}^H \mathbf{A}(\theta_0) \mathbf{x}|^2}{\mathbf{w}^H \mathbf{w} + \mathbf{w}^H \mathbf{w}}
\]

(22)

By solving an equivalent MVDR problem, the corresponding receive beamforming vector to maximize the output SINR can be given by
\[ w^* = \frac{\Sigma_2(u, v)^{-1} A(\theta_0)x}{x^H A^H(\theta_0) \Sigma_2(u, v)^{-1} A(\theta_0)x}, \tag{23} \]

where

\[ \Sigma_2(u, v) = \left[ \sum_{i=1}^{I} |\alpha_i|^2 A(\theta_i) (uu^H + vv^H) A^H(\theta_i) + I \right]. \tag{24} \]

Substituting (23) into (22), the SINR of the radar receiver can be calculated as

\[ \gamma_{R}^{(II)} = x^H \Phi_2(u, v)x, \tag{25} \]

where

\[ \Phi_2(u, v) = |\alpha_0|^2 A^H(\theta_0) \Sigma_2(u, v)^{-1} A(\theta_0). \tag{26} \]

And the average SINR of the radar receiver can be given by

\[ \bar{\gamma}_{R}^{(II)}(u, v) = \mathbb{E}[x^H \Phi_2(u, v)x] \]

\[ = u^H \Phi_2(u, v)u + v^H \Phi_2(u, v)v. \tag{27} \]

For the CU, it has a priori information of probing signal. After probing signal interference cancelling, its received SINR \( \bar{\gamma}_C(u) \) can also be expressed as (15). The beamforming optimization problem that maximizes the SINR of the radar receiver and ensures the SINR requirement of the CU can be expressed as

\[
\begin{align*}
(P2.1) & \quad \max_u \quad \bar{\gamma}_{R}^{(II)} = u^H \Phi_2(u, v)u + v^H \Phi_2(u, v)v \\
& \quad \text{s.t.} \quad \bar{\gamma}_C = |h^Hu|^2 \geq \Gamma, \\
& \quad \quad \quad \quad u^Hu + v^Hv \leq P_0.
\end{align*}
\]

Similarly, the sequential optimization can be adopted to find the transmit beamforming vector \( u \) and \( v \) in an iterative fashion, where we first compute \( \Phi_0 = \Phi_2[u^{(m-1)}v^{(m-1)}] \) at the \( m \)-th iteration with \( u^{(m-1)} \) and \( v^{(m-1)} \) being obtained from the \( (m - 1) \)-th iteration. Thus, the beamforming optimization problem can be given by

\[
\begin{align*}
(P2.2) & \quad \max_u \quad \bar{\gamma}_{R}^{(II)} = u^H \Phi_0u + v^H \Phi_0v \\
& \quad \text{s.t.} \quad \bar{\gamma}_C = |h^Hu|^2 \geq \Gamma, \\
& \quad \quad \quad \quad u^Hu + v^Hv \leq P_0.
\end{align*}
\]
Algorithm 1 Beamforming design of DFRC BS for the single CU scenario.

Initialize \( \{\theta_0, \theta_1, \cdots, \theta_I\} \) and \( \{\alpha_0, \alpha_1, \cdots, \alpha_I\} \).

Initialize \( u^{(0)} = [1, \cdots, 1]^T/\sqrt{P_0 N_t} \).

Initialize the convergence threshold \( \Delta \), and \( m = 0 \).

\begin{algorithmic}
	\Repeat
		Set \( m = m + 1 \).
		Calculate \( \Phi_0 = \Phi_1 [u^{(m-1)}] \) and \( \tilde{\gamma}_R^{(l)} [u^{(m-1)}] \) according to (12) and (13), respectively.
		Calculate \( u^{(m)} \) according to (18) in Proposition 1.
		Calculate \( \tilde{\gamma}_R^{(l)} [u^{(m)}] \) according to (13).
	\Until \( \left| \tilde{\gamma}_R^{(l)} [u^{(m)}] - \tilde{\gamma}_R^{(l)} [u^{(m-1)}] \right| \leq \Delta \).
\end{algorithmic}

And the closed-form solution of Problem (P2.2) can be given by the following proposition.

**Proposition 2.** (No need of dedicated probing signal with single CU) For the dedicated probing signal case, the optimal beamforming vector of the dedicated probing signal with single CU, i.e., the optimal solution to Problem (P2.2), can be given by \( v^* = 0 \). Thus, there is no need to design the dedicated probing signal with single CU, and the optimal beamforming vector of the communication signal \( u^* \) is also given by (18).

**Proof.** The proof is given in Appendix B. \( \square \)

Finally, the beamforming optimization algorithm for the single CU scenario can be summarized as Algorithm 1, which repeatedly updates \( u^{(m-1)} \) based on \( u^{(m)} \) until the improvement of the radar receivers SINR becomes insignificant.

IV. MULTIPLE CU SCENARIO

In this section, we proceed to consider a more complicated scenario with multiple CUs in the network. For the non-dedicated probing signal case, the beamforming design is formulated as a non-convex QCQP, and the globally optimal solutions can be obtained by applying SDR with rank-1 property. For the dedicated probing signal case, the rank-1 property after applying SDR can also be proved, and the corresponding optimal solution shows that the dedicated probing signal should be employed to improve the SINR of the radar receiver.

A. Non-dedicated Probing Signal Case

For the non-dedicated probing signal case, the transmit signal of the DFRC BS in (4) with multiple CUs can be given by
where $u_k \in C^{N_t \times 1}$ and $s_k \in C \mathcal{N} (0, 1)$ are the beamforming vector and the i.i.d. information symbol of the CU $k$, respectively.

According to the output in (2), the SINR of the radar receiver can be written as

$$\gamma_R^{(l)} = \frac{\left| \alpha_0 w^H A (\theta_0) x \right|^2}{E \left[ w^H \sum_{i=1}^{I} \alpha_i A (\theta_i) x \right]^2 + w^H w}$$

$$= \frac{\left| \alpha_0 \right|^2 \left| w^H A (\theta_0) x \right|^2}{w^H \left[ \sum_{i=1}^{I} |\alpha_i|^2 A (\theta_i) \left( \sum_{k=1}^{K} u_k u_k^H \right) A^H (\theta_i) + I \right] w}. \tag{31}$$

By solving an equivalent MVDR problem, the corresponding receive beamforming vector to maximize the output SINR can be given by

$$w^* = \frac{\Sigma_3 (\{u_k\})^{-1} A (\theta_0) x}{x^H A^H (\theta_0) \Sigma_3 (\{u_k\})^{-1} A (\theta_0) x}, \tag{32}$$

where

$$\Sigma_3 (\{u_k\}) = \left[ \sum_{i=1}^{I} |\alpha_i|^2 A (\theta_i) \left( \sum_{k=1}^{K} u_k u_k^H \right) A^H (\theta_i) + I \right]. \tag{33}$$

Substituting (32) into (31), the SINR of the radar receiver can be calculated as

$$\gamma_R^{(l)} = x^H \Phi_3 (\{u_k\}) x, \tag{34}$$

where

$$\Phi_3 (\{u_k\}) = |\alpha_0|^2 A^H (\theta_0) \Sigma_3 (\{u_k\})^{-1} A (\theta_0). \tag{35}$$

And the average SINR of the radar receiver can be given by

$$\overline{\gamma}_R^{(l)} = E \left[ x^H \Phi_3 (\{u_k\}) x \right]$$

$$= \sum_{k=1}^{K} u_k^H \Phi_3 (\{u_k\}) u_k \tag{36}$$

The received signal of the CU $k$ in (3) can be rewritten as
\[ y_k = h_k^H u_k s_k + \sum_{j \neq k} h_k^H u_j s_j + z_k, \]  
(37)

and the average SINR of the CU \( k \) can be calculated as

\[ \bar{\gamma}_{C,k} \left( \{u_k\} \right) = \frac{|h_k^H u_k|^2}{1 + \sum_{j \neq k} |h_k^H u_j|^2}. \]  
(38)

Again, we consider the beamforming optimization problem that maximizes the SINR of the radar receiver by imposing the individual SINR constraints of the CUs, i.e.,

\[
\begin{align*}
\max_{\{u_k\}} \quad & \bar{\gamma}_R^{(l)} = \sum_{k=1}^{K} u_k^H \Phi_3 \left( \{u_k\} \right) u_k \\
\text{s.t.} \quad & \bar{\gamma}_{C,k} = \frac{|h_k^H u_k|^2}{1 + \sum_{j \neq k} |h_k^H u_j|^2} \geq \Gamma_k, \forall k \\
& \sum_{k=1}^{K} u_k^H u_k \leq P_0
\end{align*}
\]  
(P3.1)

Note that Problem (P3.1) is also non-convex. Thus, we adopt the sequential optimization to find the transmit beamforming vector \( u \) in an iterative way. Specifically, at the \( m \)-th iteration, we first compute \( \Phi_0 = \Phi_3 \left( \{u_{k(m-1)}\} \right) \), where \( \{u_{k(m-1)}\} \) is obtained in the \( (m-1) \)-th iteration. As a result, Problem (P3.1) can be rewritten as

\[
\begin{align*}
\max_{\{u_k\}} \quad & \bar{\gamma}_R^{(l)} = \sum_{k=1}^{K} u_k^H \Phi_0 u_k \\
\text{s.t.} \quad & \bar{\gamma}_{C,k} = \frac{|h_k^H u_k|^2}{1 + \sum_{j \neq k} |h_k^H u_j|^2} \geq \Gamma_k, \forall k \\
& \sum_{k=1}^{K} u_k^H u_k \leq P_0
\end{align*}
\]  
(P3.2)

While Problem (P3.2) is still a non-convex QCQP, and we can solve it via SDR with \( U_k = u_k u_k^H \), i.e,

\[
\begin{align*}
\max_{\{U_k\}} \quad & \bar{\gamma}_R^{(l)} = \sum_{k=1}^{K} \text{tr} (\Phi_0 U_k) \\
\text{s.t.} \quad & \bar{\gamma}_{C,k} = \frac{\text{tr} \left( H_k U_k \right)}{\Gamma_k} - \sum_{j \neq k} \text{tr} \left( H_k U_j \right) \geq 1, \forall k \\
& \sum_{k=1}^{K} \text{tr} (U_k) \leq P_0, U_k \succeq 0, \forall k
\end{align*}
\]  
(P3.3)

where \( H_k = h_k h_k^H \).
Algorithm 2 Beamforming design of DFRC BS for multiple CUs scenario without dedicated probing signal.

Initialize \( \{\theta_0, \theta_1, \ldots, \theta_I\} \) and \( \{\alpha_0, \alpha_1, \ldots, \alpha_I\} \).

Initialize \( \{u_k^{(0)}\} = [1, \ldots, 1]^T/\sqrt{KP_0N_t} \).

Initialize the convergence threshold \( \Delta \), and \( m = 0 \).

repeat

Set \( m = m + 1 \).

Calculate \( \Phi_0 = \Phi_3 \left[ \{u_k^{(m-1)}\} \right] \) and \( \bar{\gamma}_R^{(l)} \left[ \{u_k^{(m-1)}\} \right] \) according to (35) and (36), respectively.

Optimize \( \{U_k^*\} \) according to Problem (P3.3).

Calculate \( \{u_k^*\} \) satisfying \( u_k^*(u_k^*)^H = U_k^* \) based on the Proposition 3.

Calculate \( \bar{\gamma}_R^{(l)} \left[ \{u_k^{(m)}\} \right] \) according to (36).

until \( |\bar{\gamma}_R^{(l)} \left[ \{u_k^{(m)}\} \right] - \bar{\gamma}_R^{(l)} \left[ \{u_k^{(m-1)}\} \right]| \leq \Delta \).

Although the rank-1 constraints have been removed for the convexity of the problem, the optimal solution can be proved to have the rank-1 property by the following proposition.

**Proposition 3.** (Rank-1 property of Problem (P3.3)) For the non-dedicated probing signal case with multiple CUs, there is always a solution to Problem (P3.3) satisfying that \( \text{rank} (U_k^*) = 1 \), \( \forall k \). Thus, the optimal beamforming vector for the CU \( k \), i.e., the optimal solution to Problem (P3.2) can be given by \( u_k^* \) with \( u_k^*u_k^{*H} = U_k^* \).

**Proof.** The proof is given in Appendix C.

Above all, the beamforming optimization algorithm for the multi-CU scenario and the non-dedicated probing signal case can be provided as Algorithm 2. It repeatedly updates \( \{u_k^{(m)}\} \) given \( \{u_k^{(m-1)}\} \) until convergence.

**B. Dedicated Probing Signal Case**

For the dedicated probing signal case, the transmit signal of the DFRC BS in (5) with multiple CUs can be given by

\[
x = \sum_{k=1}^{K} u_k s_k + \nu s_0,
\]

where \( \nu \in \mathbb{C}^{N_t \times 1} \) and \( s_0 \sim \mathcal{CN} (0, 1) \) are the beamforming vector and the symbol of the dedicated probing signal, respectively. And \( s_k, \forall k \) and \( s_0 \) are i.i.d..
According to the output in (2), the SINR of the radar receiver can be expressed as

\[
\gamma_{R}^{(II)} = \frac{\left| \alpha_0 w^H A(\theta_0) x \right|^2}{E\left[ w^H \sum_{i=1}^{I} \alpha_i A(\theta_i) x \right]^2 + w^H w} \]

(43)

By solving an equivalent MVDR problem, the corresponding receive beamforming vector to maximize the output SINR can be given by

\[
w^* = \frac{\Sigma_4(\{u_k\}, v)^{-1} A(\theta_0) x}{x^H A^H(\theta_0) \Sigma_4(\{u_k\}, v)^{-1} A(\theta_0) x},
\]

(44)

where

\[
\Sigma_4(\{u_k\}, v) = \sum_{i=1}^{I} |\alpha_i|^2 A(\theta_i) \left( \sum_{k=1}^{K} u_k u_k^H + vv^H \right) A^H(\theta_i) + I.
\]

(45)

Substituting (44) into (43), the SINR of the radar receiver can be calculated as

\[
\gamma_{R}^{(II)} = x^H \Phi_4(\{u_k\}, v) x,
\]

(46)

where

\[
\Phi_4(\{u_k\}, v) = |\alpha_0|^2 A^H(\theta_0) \Sigma_4(\{u_k\}, v)^{-1} A(\theta_0).
\]

(47)

And the average SINR of the radar receiver can be given by

\[
\bar{\gamma}_{R}^{(II)} = E\left[ x^H \Phi_4(\{u_k\}, v) x \right]
\]

\[
= \sum_{k=1}^{K} u_k^H \Phi_4(\{u_k\}, v) u_k + v^H \Phi_4(\{u_k\}, v) v
\]

(48)

For the CU \( k \), it has a priori information on the probing signal. After the probing signal interference cancelling, its received SINR \( \bar{\gamma}_{C,k} \) can also be expressed as (38). Thus, the beamforming optimization problem that maximizes the SINR of the radar receiver and satisfies the SINR constraints of the CUs can be formulated as
\[
\begin{align*}
\max_{\{u_k\}, v} & \quad \bar{\gamma}_R^{(II)} = \sum_{k=1}^K u_k^H \Phi_4 u_k + v^H \Phi_4 v \\
\text{(P4.1) s.t.} & \quad \bar{\gamma}_{C,k} = \frac{|h_k^H u_k|^2}{1 + \sum_{j \neq k} |h_j^H u_j|^2} \geq \Gamma_k, \forall k \tag{49}
\end{align*}
\]

Let us then employ the sequential optimization to find the transmit beamforming vector \( \{u_k\} \) and \( v \) in an iterative fashion, where we first compute \( \Phi_0 = \Phi_4\{u_k^{(m-1)}\}, v^{(m-1)}\) at the \( m \)-th iteration with \( \{u_k^{(m-1)}\} \) and \( v^{(m-1)} \) being obtained from the \((m-1)\)-th iteration. Thus, the beamforming optimization problem can be again formulated as

\[
\begin{align*}
\max_{\{u_k\}, v} & \quad \bar{\gamma}_R^{(II)} = \sum_{k=1}^K u_k^H \Phi_0 u_k + v^H \Phi_0 v \\
\text{(P4.2) s.t.} & \quad \bar{\gamma}_{C,k} = \frac{|h_k^H u_k|^2}{1 + \sum_{j \neq k} |h_j^H u_j|^2} \geq \Gamma_k, \forall k \tag{50}
\end{align*}
\]

which can be solved using the SDR by letting \( U_k = u_k u_k^H \) and \( V_k = v_k v_k^H \), i.e.,

\[
\begin{align*}
\max_{\{u_k\}, v} & \quad \bar{\gamma}_R^{(II)} = \sum_{k=1}^K \text{tr} (\Phi_0 U_k) + \text{tr} (\Phi_0 V) \\
\text{(P4.3) s.t} & \quad \bar{\gamma}_{C,k} = \frac{\text{tr} (H_k U_k)}{\Gamma_k} - \sum_{j \neq k} \text{tr} (H_k U_j) \geq 1, \forall k \tag{51}
\end{align*}
\]

Although the rank-1 constraints have been removed for the convexity of the problem, the optimal solution can be guaranteed to have rank-1 property by the following proposition. Furthermore, the dedicated probing signal should be employed according to the following proposition.

**Proposition 4.** (Rank-1 property of Problem (P4.3)) For the non-dedicated probing signal case with multiple CUs, there is always a solution to Problem (P4.3) satisfying that \( \text{rank} (U_k^*) = 1, \forall k \) and \( \text{rank} (V^*) \leq 1 \). Thus, the optimal beamforming vector for the CU \( k \) and the dedicated probing signal, i.e., the optimal solution to Problem (P4.2) can be given by \( u_k^* \) and \( v^* \) with \( u_k^* u_k^{*H} = U_k^* \) and \( v^* v^{*H} = V^* \). Specifically, \( v^* = \sqrt{\tau P_0} \hat{g} \), where \( \hat{g} = g / \|g\| \), \( g \) is the dominant eigenvector of \( \Phi_0 \), and \( 0 \leq \tau \leq 1 \).
Algorithm 3 Beamforming design of DFRC BS for multiple CUs scenario with dedicated probing signal.

Initialize $\{\theta_0, \theta_1, \cdots, \theta_I\}$ and $\{\alpha_0, \alpha_1, \cdots, \alpha_I\}$.

Initialize $\{u_k^{(0)}\} = v^{(0)} = [1, \cdots, 1]^T / \sqrt{(K+1) P_0 N_t}$.

Initialize the convergence threshold $\Delta$, and $m = 0$.

repeat

Set $m = m + 1$.

Calculate $\Phi_0 = \Phi_4 \left[ \left\{ u_k^{(m-1)} \right\}, v^{(m-1)} \right]$ according to (47) and $\bar{\gamma}^{(II)}_R \left[ \left\{ u_k^{(m-1)} \right\}, v^{(m-1)} \right]$ according to (48).

Optimize $\{U_k^*\}$ and $V^*$ according to Problem (P4.3);

Calculate $\{u_k^*\}$ and $v^*$ satisfying $u_k^*(u_k^*)^H = U_k^*$ and $v^*v^*H = V^*$ based on the Proposition 4.

Calculate $\bar{\gamma}^{(II)}_R \left[ \left\{ u_k^{(m)} \right\}, v^{(m)} \right]$ according to (48).

until $|\bar{\gamma}^{(II)}_R \left[ \left\{ u_k^{(m)} \right\}, v^{(m)} \right] - \bar{\gamma}^{(II)}_R \left[ \left\{ u_k^{(m-1)} \right\}, v^{(m-1)} \right]| \leq \Delta$.

Proof. The proof is given in Appendix D.

Remark 1. (Dedicated probing signal is employed or not) It can be observed that any feasible solution to Problem (4.2) is also feasible for Problem (3.2) with $v^* = 0$, and vice versa. If $v^* \neq 0$, a higher SINR of radar receiver can be achievable, which will also be verified by the simulation results in the next section. Therefore, it is beneficial to employ the dedicated probing signal for multiple CUs scenario. The benefit is achieved at the cost of implementing an additional interference cancellation with a priori known probing signals by all CUs.

Finally, we can also use the solution $\{u_k^{(m-1)}\}$ and $v^{(m-1)}$ to update $\{u_k^{(m)}\}$ and $v^{(m)}$, and it is repeated until the improvement of the radar receivers SINR becomes insignificant as illustrated in Algorithm 3.

V. Simulation Results

In this section, we evaluate the performance of our proposed design via numerical simulations. We assume that both the DFRC BS and the radar receiver are equipped with uniform linear arrays (ULAs) with the same number of elements. The interval between adjacent antennas of the DFRC BS and the radar receiver is half-wavelength. The transmit power constraint of DFRC BS is set as $P_0 = 20$ dBm. A target is located at the spatial angle $\theta_0 = 0^\circ$ with power $|\alpha_0|^2 = 10$ dB,
and four fixed interferences are located at the spatial angles $\theta_1 = -60^\circ$, $\theta_2 = -30^\circ$, $\theta_3 = 30^\circ$, $\theta_4 = 60^\circ$, respectively. The power of each interference is $|\alpha_i|^2 = 30$ dB, $\forall i$. The channel vector of each CU is randomly generated from i.i.d. Rayleigh fading.

A. Single CU Scenario

In Fig. 2, the tradeoff between the SINR constraint of CU and the SINR of radar is shown for the single CU scenario by varying the threshold of the CU’s SINR $\Gamma$. It is easy to identify two boundary points of the tradeoff, i.e., “Radar benchmark” and “Communication benchmark”. When the constraint of the CU’s SINR is inactive, the radar’s SINR is around 38 dB and the CU’s SINR is around 20 dB. When the constraint of the CU’s SINR is active, the maximum feasible constraint of the CU’s SINR is around 28 dB, and the corresponding radar’s SINR is around 25 dB. The optimal tradeoff between the SINR of CU and the SINR of radar is characterized by the solid line. It can be observed that the SINR of the radar decreases with the increase of the CU’s SINR constraint, when the CU’s SINR constraint is above 20 dB. Also, the tradeoff between the SINR of CU and the SINR of radar can be achieved by time sharing, which is shown by the dotted line. However, the optimal beamforming design for the DFRC BS yields better tradeoff performance.

In Fig. 3, the optimized beampatterns with different constraints of the CU’s SINR are illustrated. The nulls are clearly placed at the locations of interferences, i.e., $\theta_1 = -60^\circ$, $\theta_2 = -30^\circ$, $\theta_3 = 30^\circ$, $\theta_4 = 60^\circ$, and the target is located at $\theta_0 = 0^\circ$. When the SINR constraint of the CU is 15 dB, it is inactive, and the beampattern is actually the optimal beampattern to maximize
Figure 3. Optimized beampatterns with different constraints of the CU’s SINR, $N_t = N_r = 8$, $h = [0.21 - 0.02i, -0.56 + 0.65i, 0.57 - 0.23i, -0.93 + 0.47i, -0.19 + 1.35i, -0.19 + 0.11i, 1.05 - 0.21i, 1.02 - 0.35i]$.

the radar’s SINR. When the SINR constraint of the CU increases to 25 dB, the performance of beampattern becomes worse from the radar’s viewpoint. Particularly, both the peak to sidelobe ratio (PSLR) and the main beam power both decrease with the increase of the SINR constraint of the CU. Furthermore, the optimized beampatterns with different numbers of the transmit and receive antennas are shown in Fig. 4. The SINR constraint of the CU is fixed as 25 dB. When the number of the DFRC BS antennas increases, the performance of beampattern becomes better from the radar’s viewpoint. In particular, the PSLR increases with the number of the DFRC BS antennas, and the main beam width decreases with the number of the DFRC BS antennas.

B. Multiple CUs Scenario

In Fig. 5, we evaluate the convergence performance of the proposed algorithm for different numbers of CUs. The SINR of the radar versus the number of iterations is provided. The SINR of the radar converges to a fixed value with only 2 iterations. And the converged SINR performance decreases with the increase of the number of the CUs. Furthermore, the same converged SINR of the radar for dedicated probing signal case and non-dedicated probing signal case is observed, when the number of CU is 1. And the converged SINR of the radar improves with employing dedicated probing signal for the multiple CUs scenario.

The average performance of the tradeoff between the SINR constraints of CUs and the average SINR of radar is evaluated in Fig. 6 through $10^4$ Monte Carlo simulations. The SINR of radar exponentially decreases with the increase of the CUs’ SINR constraint. And the average SINR of radar becomes worse when the number of the CU increases. That is because the feasible region
Figure 4. Optimized beampatterns with different numbers of the DFRC BS antennas, $N_t = N_r$, $h_1 = [1.43 - 0.47i, -0.83 - 0.56i, 0.05 - 0.73i, -0.32 + 0.31i, 0.21 + 0.68i, -0.56 - 0.95i]$, $h_2 = [h_1, 0.23 - 0.62i, 0.90 + 0.60i]$, $h_3 = [h_2, -0.22 - 0.83i, 0.56 - 1.22i]$, $\Gamma = 20$ dB.

Figure 5. Convergence performance of the proposed algorithm for different numbers of CU, $N_t = N_r = 6$, $\Gamma = 20$ dB, $h_1 = [-0.33 - 1.17i, -0.47 - 0.61i, 0.42 - 0.82i, 0.22 - 0.71i, 0.21 + 0.58i, -0.19 + 0.13i]$, $h_2 = [-0.69 + 0.67i, -0.38 + 0.79i, -0.32 + 1.00i, 1.04 - 0.02i, -0.17 + 0.05i, -2.01 - 0.60i]$, $h_3 = [0.80 - 0.48i, -0.68 - 0.23i, 0.10 - 0.17i, -0.01 - 0.19i, 0.06 + 0.62i, -0.34 - 0.03i]$ of the radar beamforming optimization becomes smaller when either the CUs’ SINR constraint or the number of the CUs increase. Furthermore, it can be observed that the average SINR of radar is the same for the dedicated probing signal case and non-dedicated probing signal case, when the number of CU is 1. That verifies Proposition 2. And, it can also observed that the average SINR of the radar improves with employing dedicated probing signal for multiple CUs scenario. In the case that the CUs’ SINR constraints are loose, the improvement is not obvious...
because the CUs’ SINR constraints are inactive. When the CUs’ SINRs are constrained to be large, the improvement becomes obvious. For example, when the number of CU is 2 and CUs’ SINR constraint is 10 dB, it improves about 1 dB.

In Fig. 7, the average SINR of radar versus the numbers of CUs is illustrated with different number of transmit and receive antennas through $10^4$ Monte Carlo simulations. The average SINR of radar decreases with the increase of the number of CUs, and it increases with the increase of the number of transmit and receive antennas. When the number of transmit and receive antennas is large, the decrease in the average SINR of radar with the number of CUs is not obvious. It can also be observed that the average SINR of radar improves by employing the dedicated probing signal. And the improvement increases with the number of CUs.

VI. CONCLUSION

In this paper, we have proposed the joint optimization of transmit and receive beamforming for the DFRC system. The optimal tradeoff of SINR between radar and communication has been characterized through maximizing the SINR of the radar under the SINR constraints of the CUs. For the single CU scenario, we have given the closed-form solution of the optimized beamforming, and it has been proved that there is no need of dedicated probing signals. For the multiple CUs scenario, the beamforming design has been formulated as a non-convex QCQP. We have obtained the optimal solutions by applying SDR with rand-1 property, and it has been proved that the dedicated probing signal should be employed to improve the SINR of the radar. Numerical results have been provided to show that our algorithm is effective.
Future research may focus on beamforming designs with further practical constraints, e.g. constant modulus constraints. Also, our work is based on the prior information of both radar and communication. Thus, the robust design will be considered with the imperfect prior information of the radar and the imperfect CSI of the communication.

**APPENDIX A**

**PROOF OF PROPOSITION 1**

If $\Gamma \leq |h^H\hat{g}|^2$, the SINR constraint of the CU is inactive. Thus, Problem (P1.2) reduces to the MIMO radar beamforming optimization problem without communication constraints, i.e.,

$$
\max_{u} \quad \tilde{\eta}_{R}^{(l)} = u^H\Phi_0 u \\
\text{s.t.} \quad u^Hu \leq P_0
$$

and the corresponding optimal solution is $u^* = \sqrt{P_0}\hat{g}$.

If $P_0|h|^2 \geq \Gamma > P_0|h^H\hat{g}|^2$, the SINR constraint is active. The optimal beamforming vector should lie in the pace jointly spanned by $\hat{h}$ and the projection of $g$ into the null space of $\hat{h}$, i.e., $\hat{g}_\perp$, where $g^H\hat{h} = 0$. The transmit power of $u^*$ allocated in the direction of $\hat{h}$ should satisfy the SINR constraint with the coefficient $\alpha$ in (19), and the left power is allocated in the direction of $\hat{g}_\perp$ with the coefficient $\beta$ in (20), which improves the SINR of the radar receiver without influencing the SINR of the CU.

If $\Gamma > P_0|h|^2$, the SINR constraint cannot be satisfied even with maximum ratio transmission to the CU.
Assuming the power of the dedicated probing signal is $\tau P_0$ with $0 \leq \tau \leq 1$, Problem (P2.2) can be decomposed into the following two problems, i.e.,

$$\max_v v^H \Phi_0 v \quad \text{s.t.} \quad v^H v \leq \tau P_0$$  \hspace{1cm} (P2.3) 

and

$$\max_u u^H \Phi_0 u \quad \text{s.t.} \quad \bar{\gamma} C = |h^H u|^2 \geq \Gamma \cdot (1 - \tau) P_0$$  \hspace{1cm} (P2.4) 

For Problem (P2.3), the optimal beamforming vector of the probing signal can be calculated as

$$v^* = \sqrt{\tau P_0} \hat{g},$$  \hspace{1cm} (55) 

where $\hat{g} = g / \|g\|$ and $g$ is the dominant eigenvector of $\Phi_0$. For Problem (P2.4), assuming $P_1 = \tau P_0$, the optimal beamforming vector of the probing signal can be calculated according to Proposition 1, i.e.,

$$u^* = \begin{cases} \sqrt{(1 - \tau) P_0} \hat{g}, & \Gamma \leq (1 - \tau) P_0 |h^H \hat{g}|^2 \\ (\alpha' \hat{h} + \beta' \hat{g}_\perp), & (1 - \tau) P_0 |h|^2 \geq \Gamma \\ (1 - \tau) P_0 |h^H \hat{g}|^2 & \Gamma > (1 - \tau) P_0 |h^H \hat{g}|^2 \end{cases}$$  \hspace{1cm} (56) 

where $\alpha' = \sqrt{\dfrac{\Gamma}{\|h\|^2} \dfrac{\alpha_g}{\|\alpha_g\|}}$, $\beta' = \sqrt{(1 - \tau) P_0 - \dfrac{\Gamma}{\|h\|^2} \dfrac{\beta_g}{\|\beta_g\|}}$, $\hat{h} = h / \|h\|$, $\hat{g}_\perp = g - (\hat{h}^H g) \hat{h}$ denoting the projection of $g$ into the null space of $\hat{h}$, and $g$ can be expressed as $g = \alpha_g \hat{h} + \beta_g \hat{g}_\perp$. According to (55), one has
Thus, $u^* + v^*$ can achieve the best performance when $\tau = 0$. That is the optimal beamforming vector of the probing signal $v^* = 0$ for Problem (P2.2), which completes the proof.

APPENDIX C
PROOF OF PROPOSITION 3

Consider a separable semidefinite program (SDP) as follows:

$$\min_{\{X_l\}} \sum_{l=1}^{L} \text{tr} \left( B_l X_l \right)$$

s.t. $\sum_{l=1}^{L} \text{tr} \left( C_{ml} X_l \right) \triangleright_m b_m, m = 1, \ldots, M,$

$$X_l \succ 0, l = 1, \ldots, L.$$ \hfill (60)

where $\{B_l\}, \{C_{ml}\}$ are all Hermitian matrices (not necessarily positive semidefinite), $b_m \in \mathcal{R}, \forall m, \triangleright_m \in \{\geq, \leq, =\}, \forall m$, and $\{X_l\}$ are all Hermitian matrices.

Suppose the above SDP is feasible and bounded, where the optimal value is attained. Then, according to [29], it always has an optimal solution $\{X_l^*\}$ such that

$$\sum_{l=1}^{L} \left[ \text{rank} \left( X_l^* \right) \right]^2 \leq M.$$ \hfill (61)

Based on the above result, it can be proved that there is always a solution to Problem (P3.3) satisfying that

$$\sum_{k=1}^{K} \left[ \text{rank} \left( U_k^* \right) \right]^2 \leq K + 1.$$ \hfill (62)

Meanwhile, due to the SINR constraints of each CU, one has $U_k^* \neq 0, \forall k$ and then $\text{rank} \left( U_k^* \right) \geq 1, \forall k$. Thus, $\text{rank} \left( U_k^* \right) = 1, \forall k$ according to (62), which completes the proof.

APPENDIX D
PROOF OF PROPOSITION 4

According to the rank constraints of the SDP in Appendix C of [29], it can be proved that there is always a solution to Problem (P3.3) satisfying that
\[ \sum_{k=1}^{K} [\text{rank} \left( U_k^* \right)]^2 + [\text{rank} \left( V^* \right)]^2 \leq K + 1. \tag{63} \]

Meanwhile, due to the SINR constraints of each CU, \( U_k^* \neq 0, \forall k \), and then \( \text{rank} \left( U_k^* \right) \geq 1, \forall k \). Thus, one has \( \text{rank} \left( U_k^* \right) = 1, \forall k \) and \( \text{rank} \left( V^* \right) \leq 1 \) according to (63).

Next, we prove the necessity of employing the dedicated probing signal. The Lagrangian function of Problem (P4.3) is

\[ \mathcal{L}_3 = \mu P_0 - \sum_{k=1}^{K} \lambda_k + \sum_{k=1}^{K} \text{tr} \left( D_k U_k \right) + \text{tr} \left( E V \right), \tag{64} \]

where

\[ D_k = \Phi_0 + H_k \Gamma_k - \sum_{j \neq k} \lambda_j \text{tr} \left( H_j \right) - \mu I, \tag{65} \]

and

\[ E = \Phi_0 - \mu I, \tag{66} \]

with \( \lambda_k \geq 0, \forall k \) and \( \mu \geq 0 \) being the dual variables associated with the SINR constraint of the CU \( k \) and the transmit power constraint, respectively. And the dual problem is

\[ \min_{\{\lambda_k, \mu\}} \mu P_0 - \sum_{k=1}^{K} \lambda_k \quad \text{s.t.} \quad D_k \preceq 0, \forall k, E \preceq 0 \tag{67} \]

Note that Problem (P4.3) is convex. Thus, strong duality holds and the KKT conditions are necessary and sufficient for any optimal solution to Problem (P4.3). Due to the fact that \( E = \Phi_0 - \mu I \preceq 0 \). We have that \( \mu^* \geq \kappa \), where \( \kappa \) is the dominant eigenvalue of \( \Phi_0 \). If \( \mu^* = \kappa \), we have that \( \nu^* = \sqrt{\tau P_0} \hat{g} \), with \( 0 < \tau \leq 1 \). If \( \mu^* > \kappa \), we have that \( \nu^* = 0 \). It completes the proof.

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