Nonlinear dynamics analysis on intermediate bearing-dual rotor system for a marine gas turbine

JIA Yan\textsuperscript{1,2}, LIU Yongbao\textsuperscript{1}, WANG Qiang\textsuperscript{*1}, LI Mo\textsuperscript{1} and LI Jun\textsuperscript{1}

\textsuperscript{1}College of Power Engineering, Naval University of Engineering. 717 Jiefang Avenue, Wuhan City, Hubei Province, China
\textsuperscript{2}E-mail: 1075786398@qq.com

\begin{abstract}
In this paper, a marine dual-rotor gas turbine is taken as the research object, and a dual-rotor system model with a simplified structure and supported by intermediary bearings is established based on its structural characteristics and nonlinear characteristics of bearings. The Lagrange equation of motion is used to derive the differential equations of motion of the system, and the fourth-order Runge-Kutta method is used to solve the equations. The nonlinear dynamic response and bifurcation chaotic characteristics of the dual-rotor system with speed changes are studied. The results show that when the speed is low, the system is in a single-cycle motion. As the rotational speed increases, the system exhibits multiplicative bifurcation, Hopf bifurcation and chaotic behaviour. This result provides theoretical guidance for the design and safe and stable operation of the rotor system.
\end{abstract}

1. Introduction

Depending on the number of rotors, gas turbine engines can be divided into single-rotor engines, dual-rotor engines, and three-rotor engines. In contrast, the dual-rotor engine has been widely used in aero engines and ship gas turbines. The number of rotors is small, the support structure is simple, assembly and disassembly are convenient, and it is easy to supply, return oil and seal oil and gas \cite{1}. For the dual-rotor engine, the inner and outer rotors are supported by intermediary bearings, and there is a strong coupling relationship between the two rotors, which makes the vibration problem more complicated. At the same time, there are many nonlinear factors in the rotor system supported by rolling bearings, and the vibration phenomenon cannot be explained using linear theory.

At home and abroad, a lot of researches have been done on the nonlinear dynamic characteristics of the intermediary bearing-double rotor system. Reference \cite{2} established a simplified dynamic model for an aero-engine dual-rotor system, considering nonlinear factors such as fractional index nonlinearity and radial clearance of the intermediate bearing and analyzed the influence of the nonlinear characteristics of the intermediary bearing on the nonlinear vibration characteristics of the dual rotor system. Reference \cite{3} discussed the research results of nonlinear dynamic characteristics and quantitative analysis of misalignment of double-rotor system. Reference \cite{4} studied high-precision modeling, model dimensionality reduction and nonlinear system solution methods for the complex dual rotor structure of an aero engine, and based on this, analyzed the impact response characteristics and bifurcation mechanism. Reference \cite{5} built a four-point quasi-intermediate bearing dual-rotor system dynamics model based on bearing clearance, Hertz contact and variable stiffness effects, and obtained the nonlinear dynamic response of the system by numerical integration. Reference \cite{6} considered the coupling effect of the aero-engine dual-rotor intermediary bearing and the influence of the gyroscopic moment, and used the Lagrange equation to establish the dynamic model of the dual-rotor-bearing
bearing support coupling system under maneuvering flight conditions, conducted theoretical and experimental studies on the dynamic characteristics of the system. Reference [7] considered a dual-rotor system with intermediary bearing waviness, used the Lagrange equation and rotor dynamics theory to establish a system differential equation of motion, used a numerical method to solve the nonlinear vibration response of the system, and studied the waviness wavenumber and maximum amplitude of waviness Value, initial amplitude of waviness and speed affect the dynamic response of the system. Reference [8] presented a new high-order Poincaré mapping method, combined with Newton-Raphson iteration to form a new improved shooting method for solving periodic solutions of nonlinear dynamic systems. Reference [9] established a complex rotor system dynamics model with local nonlinear stiffness, combined with FFT and Broyden iterative method to develop a numerical harmonic balance method, which improves the solution efficiency. Reference [10] established a dynamic model of the bending vibration of the dual rotor shaft system considering the stiffness characteristics of the intermediary bearing based on the Timoshenko beam model and the finite element method. The QR decomposition method was used for numerical calculation, and the nonlinear response phenomenon of the single high-pressure rotor system and the nonlinear response phenomenon of the coupled dual rotor system were analyzed. Reference [11] considered the nonlinear factors of the intermediate bearing and the elastic support, used the Lagrange equation to establish the dynamic model of the basic fixed counter-rotating dual-rotor system, used the Runge-Kutta method for numerical calculation and analyzed the influence of rotor unbalance and elastic support nonlinear stiffness coefficient on system dynamic characteristics. Reference [12] analyzed the dynamic characteristics of the asymmetric stiffness matrix of the counter-rotating dual rotors, and obtained the variation rules of critical speed, unbalance response curve and precession direction. Reference [13] used ball bearings instead of elastic supports to establish a dual-rotor-ball bearing coupling dynamics model, deduced the finite element dynamic equations with rubbing forces, and analyzed the impact of rubbing parameters on the nonlinear response of the system.

The above studies are based on the dual-rotor structure of an aero engine, and the system model is established after simplification to perform nonlinear dynamic analysis. Most of the currently studied dual-rotor models are based on a typical aero-engine dual-rotor system, that is, a dual-rotor structure including high and low pressure gas engines and high and low pressure turbines. The effects of various nonlinear factors in the rotor-bearing system on the system dynamic response are studied. Based on the actual structure of a certain marine gas turbine, this paper establishes a dual-rotor-intermediate bearing system with 4 fulcrum supports of 2 sets of deep groove ball bearings and 2 sets of cylindrical roller bearings. The gas generator rotor has only one-stage power turbine, which has a different structural form from the aero engine, and the dynamic response characteristics of the system are obviously different. On this basis, analyzing the nonlinear dynamic response characteristics of the system at different speeds has important guiding significance for the safe and stable operation and design of the gas turbine.

2. Dynamic model of a dual-rotor system for a marine gas turbine

By analyzing the dual-rotor system of a marine gas turbine, the actual dual-rotor system is simplified, as shown in Figure 1. Through the concentrated mass method, the rigid disk 1 is used to simulate the power turbine rotor, which is located at the end of the rotor 1, and the rigid disks 2, 3 are used to simulate the gas generator compressor rotor and the turbine rotor, which are located at both ends of the rotor 2, respectively. The entire dual-rotor system uses four bearings to support, bearings 1, 4 are ball bearings, bearings 2, 3 are cylindrical roller bearings, of which bearing 2 is an intermediary bearing that supports the inner and outer rotors. The rotating shaft of the dual rotor is assumed to be a massless elastic shaft, only the lateral vibration of the rotor is considered, and the torsional vibration and axial vibration are ignored, that is, the degrees of freedom in the x and y directions are considered at each concentrated mass.
Figure 1. Dynamic model of a dual-rotor system for a marine gas turbine

In Figure 1, Ob1, Ob2, Ob3 and Ob4 are the centroids of Bearing 1, Bearing 2, Bearing 3, and Bearing 4, respectively. Op1, Op2 and Op3 are the centroids of Disk 1, Disk 2, and Disk 3. Oc1, Oc2 and Oc3 are the centroids of disk 1, disk 2, disk 3. e1, e2, and e3 are the eccentricity of disk 1, disk 2, and disk 3, respectively. Assume that the structural materials of the rotating shaft 1 and the rotating shaft 2 are the same, and the stiffness relationship between the shaft sections is obtained according to the structural relationship of the plane beam, \( k_1 = \frac{k}{125}, k_2 = 4k, k_5 = \frac{k}{2}, k_3 = k_4 = k_6 = k \).

The dynamic model of the dual-rotor system of the marine gas turbine built in this paper has 7 concentrated masses and 14 degrees of freedom in the model. Based on the Lagrange motion equation, the system nonlinear dynamic differential equations are obtained as follows:

\[
\begin{align*}
M\ddot{X} + C\dot{X} + KX &= F_{hx} + F_{ex} \\
M\ddot{Y} + C\dot{Y} + KY &= F_{hy} + F_{ey} - Mg
\end{align*}
\]

\[
F_{hx} = \begin{pmatrix} F_{h1x}, F_{h2x}, 0, F_{h3x}, F_{h4x}, 0, 0 \end{pmatrix}^T
\]

\[
F_{hy} = \begin{pmatrix} F_{h1y}, F_{h2y}, 0, F_{h3y}, F_{h4y}, 0, 0 \end{pmatrix}^T
\]

\[
X = \begin{pmatrix} x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23} \end{pmatrix}^T
\]

\[
Y = \begin{pmatrix} y_{11}, y_{12}, y_{13}, y_{14}, y_{21}, y_{22}, y_{23} \end{pmatrix}^T
\]

\[
F_{ex} = \begin{pmatrix} 0, 0, m_{p1} e_1 \omega_2^2 \cos \omega_2 t, 0, 0, m_{p2} e_2 \omega_2^2 \cos \omega_2 t, m_{p3} e_3 \omega_2^2 \cos \omega_2 t \end{pmatrix}^T
\]

\[
F_{ey} = \begin{pmatrix} 0, 0, m_{p1} e_1 \omega_2^2 \sin \omega_2 t, 0, 0, m_{p2} e_2 \omega_2^2 \sin \omega_2 t, m_{p3} e_3 \omega_2^2 \sin \omega_2 t \end{pmatrix}^T
\]

\[
M = \text{diag} \left( m_{b1}, m_{b2}, m_{p1}, m_{b3}, m_{b4}, m_{p2}, m_{p3} \right)
\]

\[
C = \text{diag} \left( c_{b1}, c_{b2}, c_{p1}, c_{b3}, c_{b4}, c_{p2}, c_{p3} \right)
\]
\[ K = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 & 0 \\ -k_1 & k_1 + k_2 + k_6 & -k_2 & 0 & 0 & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 & 0 & 0 \\ 0 & 0 & -k_3 & k_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_4 & -k_4 \\ 0 & 0 & 0 & 0 & -k_4 & k_4 + k_5 & -k_5 \\ 0 & 0 & 0 & 0 & -k_5 & k_5 + k_6 \end{bmatrix} \]

In the formula: M is the system mass matrix, C is the system damping matrix, K is the system stiffness matrix, Fbx and Fby are bearing force matrices, and Fex and Fey are rotor unbalanced force matrices.

Dimensionless processing of the system of equations, \( \xi \) is the dimensionless reference value, and dimensionless parameters are introduced:

\[ \tau = \omega_1 \xi, X_j = \frac{x_j}{\xi}, Y_j = \frac{y_j}{\xi}, \dot{X}_j = \frac{\dot{x}_j}{\xi\omega_1}, \dot{Y}_j = \frac{\dot{y}_j}{\xi\omega_1}, \ddot{X}_j = \frac{\ddot{x}_j}{\xi\omega_1^2}, \ddot{Y}_j = \frac{\ddot{y}_j}{\xi\omega_1^2} \]  

(3)

For ball bearings \( h=1/2 \); for cylindrical roller bearings \( h=1/9 \). Among them, \( \omega_1 \) is the rotation speed of the rotor 1, \( \omega_2 \) is the rotation speed of the rotor 2, and the rotation speed ratio \( \alpha=\omega_2/\omega_1 \). The dimensionless formula of the rolling bearing non-linear bearing force is as follows:

\[ F_{bx} = \sum_{j=1}^{2} F_{j,x} \cos(\theta_j) = K_{b} \sum_{j=1}^{2} H(\chi_j) \chi_j^n \cos(\theta_j) \]

\[ F_{by} = \sum_{j=1}^{2} F_{j,y} \sin(\theta_j) = K_{b} \sum_{j=1}^{2} H(\chi_j) \chi_j^n \sin(\theta_j) \]  

(4)

For rolling bearings 1, 4, \( n=3/2 \); for cylindrical roller bearings 2, 3, \( n=10/9 \). The positions of bearing 1, bearing 2, bearing 3, and bearing 4 are shown in FIG1.

According to the dimensionless parameters (3), (4) and the equation (1), the dimensionless equation (5) are obtained, as follows:

\[ \ddot{X} + \frac{CX}{M\omega^2} + \frac{KX}{M\gamma\omega^2} = \frac{\ddot{F}_{bx} + \ddot{F}_{cx}}{M\gamma\omega^2} \]

\[ \ddot{Y} + \frac{CY}{M\omega^2} + \frac{KY}{M\gamma\omega^2} = \frac{\ddot{F}_{by} + \ddot{F}_{cy} - Mg}{M\gamma\omega^2} \]  

(5)

The rotor parameters are: \( m_{b1}=m_{b2}=m_{b3}=m_{b4}=1.2kg, m_{p1}=11kg, m_{p2}=6kg, m_{p3}=8kg, C_{b1}=C_{b2}=C_{b3}=C_{b4}=2200 \text{ N/s/m, } C_{p1}=C_{p2}=C_{p3}=1050 \text{ N/s/m, } k=2.5 \times 10^7 \text{ N/m }, e_1=e_2=e_3=10^{-5} \text{m.} \)

Parameters of bearings 1, 4: inside diameter \( R_i=40.1mm \), outer diameter \( R_o=63.9mm \), number of rolling bodies \( Z=8 \), radial clearance \( G_r=12 \times 10^{-6} \text{m} \), contact stiffness \( k_b=13.34 \times 10^9 \text{N/m}^{3/2} \). Parameters of bearings 2, 3: inside diameter \( R_i=9.37mm \), outer diameter \( R_o=14.13mm \), number of rolling bodies \( Z=9 \), radial clearance \( G_r=12 \times 10^{-6} \text{m} \), contact stiffness \( k_b=7.055 \times 10^9 \text{N/m}^{3/2} \). The differential equations are solved using the fourth-order Runge-Kutta method.

3. Study on nonlinear characteristics of a double rotor system for a marine gas turbine

In this section, the rotational speed of rotor 1 is used as the bifurcation parameter, and the horizontal displacement \( X_p2 \) of disk 2 is used as the observation object. With other parameters of the system
unchanged, the bifurcation chaos characteristics of the system are studied as the rotational speed increases. Figure 2 is a global bifurcation diagram of the horizontal displacement $X_{p2}$ of the disk 2 when the system increases with the rotation speed. As can be seen from the figure, when the system increases with speed, single-cycle, pseudo-cycle, doubling bifurcation, Hopf bifurcation, and chaotic behavior will occur.

![Figure 2. Bifurcation diagram of $X_{p2}$ with speed](image)

When the rotational speed of rotor 1 $\omega_1=1590$ rad/s, as shown in Figure 3, the phase diagram of disk 2 is a closed circle, and there is only one isolated point on the section of Poincaré. The frequency spectrum is mainly the frequency conversion, indicating that disk 2 is in single cycle motion.

![Figure 3. Phase diagram, Poincaré section, and spectrum diagram of disk 2 at 1590 rad/s](image)

When the rotation speed of rotor 1 $\omega_1=1750$ rad/s, as shown in Figure 4, the phase diagram of disk 2 is scattered and interlaced trajectories. The Poincaré cross-section is a number of random scattered points. The frequency conversion and its frequency division are the main factors, indicating that the disk 2 is in a chaotic state.

![Figure 4. Phase diagram, Poincaré section, and spectrum diagram of disk 2 at 1750 rad/s](image)

When the rotation speed of rotor 1 $\omega_1=1900$ rad/s, as shown in Figure 5, the phase diagram of the disk 2 is two closed circles that cross each other. There are two isolated points on the Poincaré section.
It is the frequency conversion and its frequency division, indicating that the disk 2 behaves as a period two motion.

![Figure 5. Phase diagram, Poincaré section, and spectrum diagram of disk 2 at 1900 rad/s](image)

When the rotation speed of rotor 1 $\omega_1=2700$ rad/s, as shown in Figure 6, the phase diagram of the disk 2 is a tire-shaped ring. The Poincaré section is a closed ring, and the spectrum diagram. The frequency conversion and frequency division are the main ones. A Hopf bifurcation occurs at this time.

![Figure 6. Phase diagram, Poincaré section, and spectrum diagram of disk 2 at 2700 rad/s](image)

4. Conclusion
This paper takes the dual-rotor structure of a certain marine gas turbine as the research object, and establishes a simplified intermediary bearing-dual-rotor support system model. The Lagrange equation is used to derive the differential equation of motion, and the fourth-order Runge-Kutta method is used for numerical calculation. Taking disk 2 as an example, the dynamic behaviour of the system with increasing speed is studied. The study found that when the speed of the rotor 1 is around 1125 rad/s, the system behaves as a single-cycle motion. At around 1750 rad/s, the system behaves as a chaotic motion. When the rotation speed is around 1900 rad/s, a multiplying bifurcation occurs, and the system performs two cycles of motion. When the rotation speed is around 2700 rad/s, Hopf bifurcation occurs, and the system performs quasi-periodic motion. This conclusion can provide a theoretical reference for the design of the dual-rotor system of the gas turbine, and is of great significance to the safe and stable operation of the gas turbine.

References
[1] HU Xun 2007 Research on dynamic characteristics of reverse rotating double rotor system D. Nanjing University of Aeronautics and Astronautics
[2] GAO Peng, HOU Lei and CHEN Yunshu 2019 Nonlinear vibration characteristics of double-rotor-intermediate bearing system J. Journal of Sound and Vibration. 38(15):1-10.
[3] ZHANG Hongxian, LI Xuejun, JIANG Lingli, YANG Dalian and CHEN Yumeng 2019 Research progress of aero-engine double rotor system misalignment J. Journal of Aviation. 40(06):42-53.
[4] SUN Chuanzong 2017 Research on high precision dynamic modeling and rubbing response of aero-engine double rotor system D. Harbin Institute of Technology
[5] LI Hongliang 2017 Study on nonlinear vibration characteristics of ball bearing-misalignment
7

rotor system D. Harbin Institute of Technology

[6] LI Jie, CAO Shuqian, GUO Hulun, LI Liqing, WANG Jun and BAI Xuwuchuan 2017 Dynamic modeling and response analysis of double rotor system under maneuvering conditions J. Aerodynamics. 32(04):835-849.

[7] HOU Lei, FU Yiqiang 2017 Nonlinear vibration of double rotor system considering intermediate bearing waviness J. Aerodynamics. 32(03):714-722.

[8] HU Qinghua 2011 Research on nonlinear dynamics analysis and optimization of bearing-rotor system D. Dalian University of Technology

[9] YU Pingchao, MA Yanhong, ZHANG Dayi and HONG Jie 2016 Dynamic model and vibration analysis of complex rotor system with local nonlinear stiffness J. Propulsion Technology. 37(12):2343-2351.

[10] LI Hao 2016 Research on dynamic characteristics of coupling double rotor-combined support system D. Dalian Maritime University

[11] BAI Xuechuan 2014 Research on dynamic characteristics of aero-engine reverse rotating rouble rotor system D. Tianjin University

[12] G Ferraris, V Maisonneuve and M Lalanne 1996 Prediction of the dynamic behavior of non-symmetric alcoaxialco-or counter-rotating rotors J. Journal of Sound and Vibration. 195(4).

[13] SUN Tao 2018 Analysis and optimization of rub-impact dynamics of engine double rotor D. Northwestern Polytechnical University

[14] S Fukata, EH Gad and H Tamura 1985 On the radial vibration of ballbearings: computer simulation J. Bulletin of JSME. 28(239):899-904.