SPONTANEOUS LORENTZ SYMMETRY BREAKING IN NONLINEAR ELECTRODYNAMICS

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We review some of the basic features and predictions of a gauge invariant spontaneous Lorentz symmetry breaking model arising from the nonzero vacuum expectation value of the electromagnetic tensor and leading to a nonlinear electrodynamics. The model is stable in the small Lorentz invariance violation approximation. The speed of light is independent of the frequency and one of the propagating modes is highly anisotropic. The bound $\Delta c/c < 10^{-32}$ is obtained for such anisotropy measured in perpendicular directions.

1. Introduction

Many candidate theories for describing the structure of spacetime at the microscopic level, like string theory, models of quantum gravity and non-commutative theories, for example, lead to the picture that spacetime has a discrete nature for very small scales, instead of the continuum description in which most modern physics is based. This poses the natural question of whether or not such granular structure will leave measurable imprints upon the dynamics of particles at Standard Model energies. The analogy of particle propagation in crystals suggests that modifications will indeed arise. Thus, one of the open problems of these spacetime theories is to determine the nature of these modifications, in case they are produced. The possibility that such corrections may incorporate Lorentz invariance violation (LIV) was suggested in Ref. 1 and it has recently been the subject of intense study through astrophysical observations. Moreover, some heuristic calculations, inspired in loop quantum gravity, provide also support to this conjecture. This possibility adds additional interest to the search for LIV, specially given that many observations and experiments have already attained Planck scale sensitivities. In such a way, these results will serve as physical constraints to select the right quantum theory of space-
time among the competing proposals, once they are able to fill the gap between the quantum and the semiclassical scales. Since this goal has not yet been achieved and even though the contact between these two regimes may require the introduction of a completely different conceptual structure in modern physics, standard effective quantum field theories, in the form of the so called Standard-Model Extension (SME),\(^4\) provide an adequate tool to study such modifications at Standard Model energies.

In this contribution, the work done in collaboration with J. Alfaro\(^5\) regarding a model of nonlinear gauge invariant electrodynamics arising from spontaneous Lorentz symmetry breaking (SLSB) is reviewed. This model is complementary to related studies of SLSB in the literature. On one hand there are theories where the photon emerges as the Goldstone boson of such breaking and which allow to recover electrodynamics, in a nonlinear gauge, at the tree and one loop level, thus providing a dynamical setting for \(U(1)\) gauge invariance.\(^6\) Also, models with SLSB arising from the vacuum expectation value (VEV) of antisymmetric tensors \(B_{\mu\nu}\) coupled to gravity have been studied.\(^7\) Here the two-form field \(B = B_{\mu\nu}dx^\mu \wedge dx^\nu\) is considered as the potential producing the field strength \(H = dB\) that enters in the kinetic term of the action. The model described in this contribution is intermediate among those two: gauge invariance is always preserved, SLSB is induced by a VEV of \(F_{\mu\nu}\) whose excitations around the minimum turn out to be the usual electromagnetic field, which is ultimately described by a vector potential \(A_\mu\) with the standard kinetic term for electrodynamics. The interpretation of the Goldstone mechanism in our case differs from the standard one related to massless excitations and its description is postponed for future work.

2. The model

We start from the Lagrangian

\[
L(F_{\alpha\beta},X_\mu) = -V(F_{\alpha\beta}) - \bar{F}^{\mu\nu}\partial_\nu X_\mu, \quad F_{\alpha\beta} = -F_{\beta\alpha},
\]

where \(\bar{F}_{\mu\nu}\) is the dual of \(F_{\mu\nu}\). The fields \(X_\mu\) are Lagrange multipliers which ultimately will impose the condition that the excitations of \(F_{\mu\nu}\) are derived from a vector potential, thus recovering a nonlinear electrodynamics. The potential \(V(F)\) provides a minimum for the VEV \(C_{\mu\nu}\) of \(F_{\mu\nu}\). In Ref. 5 we have made plausible the appearance of such a potential, starting from a conventional gauge theory including fermions, gauge fields and Higgs fields which provide masses to the gauge bosons, except for the photon potential
\[ \tilde{A}_\mu. \] For our purposes here it is enough to start with the standard Ginzburg-Landau parametrization of such potential
\[ V(F_{\mu\nu}) = \frac{1}{2} \alpha F^2 + \frac{\beta}{4} (F^2)^2, \quad \beta > 0. \] (2)

The vacuum configuration \( C_{\mu\nu}, C_\mu \) is obtained by minimizing the energy of the system, obtained from the Lagrangian (1) via Noether’s theorem, and requiring constant field configurations in order to preserve translational invariance. The action for the excitations \( a_{\alpha\beta} \) and \( \bar{X}_\mu \) around such minima is subsequently obtained and the elimination of the Lagrange multiplier \( \bar{X}_\mu \) introduces the potential \( A_\mu \) such that
\[ a_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + l C_{\mu\nu}, \] where \( l \) is a constant. After making some rescaling we arrive at the action
\[ S(A_\alpha) = \int d^4x \left( - \frac{1}{4} D^2 B D^2 - \frac{f_{\mu\nu} f^{\mu\nu}}{4} - B \left( D_{\mu\nu} f^{\mu\nu} + f_{\mu\nu} f^{\mu\nu} \right)^2 \right), \] (3)
which defines the model. Here \( f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu; D_{\mu\nu} \), which replaces the VEV \( C_{\mu\nu} \), is the arbitrary constant antisymmetric tensor characterizing the vacuum, \( D^2 = D_{\mu\nu} D^{\mu\nu} \) and \( B \) is a positive constant.

3. The symmetry algebras of the broken theory

As proposed in Ref. 7, the simplest parametrization of the vacuum \( D_{\mu\nu} \), written in terms of the usual electric and magnetic components, is given by two independent quantities for each of the following cases: (i) \( e = \{0, 0, e\}, b = \{0, 0, b\} \), when at least one of the electromagnetic invariants is not zero (the choice \( \psi = 0 \) in Ref. 5) and (ii) \( e = \{0, e, 0\}, b = \{0, 0, b\} \), when both electromagnetic invariants are zero (the choice \( \psi = \pi/2 \) in Ref. 5). The remaining symmetries of the broken action are obtained by requiring that the vacuum be invariant under the transformations \( G^{\mu}_\alpha D^{\alpha\nu} + G^{\nu}_\alpha D^{\mu\alpha} \), generated by \( G^\nu_\alpha \) which denote the standard infinitesimal Lorentz algebra generators, plus dilatation transformations \( (x^\mu \partial_\mu) \) which are represented by a multiple of the identity in this restricted algebra. The case (i) leads to \( T(2) \) as the remaining symmetry algebra, while the case (ii) leads to \( \text{HOM}(2) \).

4. Dispersion relations and polarizations

The propagation properties of the model arise from the quadratic terms in the effective Lagrangian
\[ L_0 = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - B (f_{\mu\nu} D^{\mu\nu})^2. \] (4)
The equations of motion are
\[
(\partial^2 A_\beta - \partial_\beta \partial^\alpha A_\alpha) = -8BD_{\alpha\beta} \partial^\alpha (D^{\mu\nu} \partial_\mu A_\nu)
\].
(5)

Introducing the definitions
\[
D = E + 8Be(B \cdot b - E \cdot e), \quad H = B + 8Bb(B \cdot b - E \cdot e),
\]
(6)
Eqs. (5) adopt the standard form of Maxwell's equation in a medium. The dispersion relations and polarization properties of a plane wave propagating with momentum \(k_\alpha\) are: (1) when \(D_{\alpha\beta}A^\beta = 0\), the triad \(E, B, k\) together with the dispersion relation are the standard ones; (2) when \(D_{\alpha\beta}A^\beta \neq 0\) we have
\[
E \cdot B = 0, \quad k \cdot B = 0, \quad k \cdot E \neq 0 \quad \text{and} \quad \omega = |k| \times \text{F(angles)},
\]
(7)
where angles refer to those between \(k\) and the vectors characterizing the vacuum. Here \(A^\alpha\) is in the Coulomb gauge. In the approximation \(Be^2, Bb^2, B|e||b| \ll 1\), the speed \(c_{1w}(\hat{k}) = |\nabla_k \omega|\) in case (2) is
\[
c_{1w}(\hat{k}) = 1 + 8B(e^2 + b^2) - 4B \left( (b \cdot \hat{k})^2 + (e \cdot \hat{k})^2 - 2\hat{k} \cdot (e \times b) \right).
\]
(8)

5. Embedding in the SME

The propagating sector can be embedded in the SME via the identification
\[
-\mathcal{B}(f_{\mu\nu}D^{\mu\nu})^2 = -\frac{1}{4} (k_F)^{\kappa\lambda\mu\nu} f_{\kappa\lambda} f_{\mu\nu},
\]
(9)
which produces
\[
(k_F)^{\kappa\lambda\mu\nu} = 4\mathcal{B}D^{\kappa\lambda}D^{\mu\nu} + \left[ 2\mathcal{B}D^{\kappa\mu}D^{\lambda\nu} - \frac{1}{2} \mathcal{B}D^2 \eta^{\kappa\mu} \eta^{\lambda\nu} - (\kappa \leftrightarrow \lambda) \right].
\]
(10)
We have explicitly verified that the above realization of \((k_F)^{\kappa\lambda\mu\nu}\) satisfies all the required identities. The relation (10) allows to express the components of \((k_F)^{\kappa\lambda\mu\nu}\) in terms of two independent parameters \(Be^2, Bb^2\) according to the cases described at the beginning of Sec. 3. In this way, the stringent astrophysical bounds\(^8\) \(\tilde{\kappa}_{e+}^{ij}, \tilde{\kappa}_{o-}^{ij} < 10^{-32}\) are summarized in the condition
\[
\mathcal{B}(e^2 + b^2) < 2.5 \times 10^{-33},
\]
(11)
which satisfies all the less stringent remaining bounds. Defining the two-way speed of light \(c_{2w}(\hat{k}) = [c_{1w}(\hat{k}) + c_{1w}(\hat{-k})]/2\) leads to the following bound upon the anisotropy of such velocity, measured along perpendicular trajectories
\[
\Delta c/c \equiv \left| c_{2w}(\hat{k}) - c_{2w}(\hat{q}) \right| / c < 10^{-32}, \quad \hat{q} = \hat{k} \times (\hat{e} \times \hat{b}).
\]
(12)
The recent bound \((\Delta c/c)_{LAB} \sim 10^{-17}\) is recovered provided we use the corresponding restrictions for the accessible parameters in the laboratory \(\tilde{\kappa}_{c-}^{ij}, \tilde{\kappa}_{c+}^{ij}\). In Ref. 10 we find the latest bound for \(\tilde{\kappa}_{c+}^{ij}\) which is \(1.6 \times 10^{-14}\). Assuming that the vacuum parameters \(\mathbf{e}, \mathbf{b}\) might represent some relic fields in the actual era, and that the constant \(\rho \simeq 1/2 (\mathbf{b}^2 - \mathbf{e}^2)\) in (3) can be associated with the cosmological constant \(|\rho_\Lambda| < 10^{-48} \text{ (GeV)}^4\), we obtain the bound \(|\mathbf{b}| < 5 \times 10^{-5} \text{ Gauss}\), by performing a passive Lorentz transformation to a reference frame where \(\mathbf{e} = 0\), which we assume to be concordant with the standard inertial reference frame. This result is consistent with observations of intergalactic magnetic fields.

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