GENERAL RELATIVISTIC EFFECTS OF GRAVITOMAGNETIC CHARGE ON PULSAR MAGNETOSPHERES AND PARTICLE ACCELERATION IN THE POLAR CAP

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ABSTRACT

We study the magnetospheric structure surrounding a rotating, magnetized neutron star with nonvanishing NUT (Newman-Unti-Tamburino) parameter. For simplicity, the Goldreich-Julian charge density is analyzed for an aligned neutron star, with zero potential differences between different parts of its surface. The cascade generation of an electron-positron plasma in the polar cap region (Sturrock 1971; Ruderman & Sutherland 1975; Zhu & Ruderman 1997) means that the magnetosphere of a neutron star is filled with plasma, screening the longitudinal electric field. This screening results in the corotation of the plasma with the neutron star. Such rotation is not possible outside the light cylinder, and thus it creates essentially different groups of field lines: closed, that is, those returning to the stellar surface, and open, that is, those crossing the light cylinder and going to infinity. As a result, plasma may leave the neutron star along the open field lines, and it is generally thought that pulsar radio emission is produced in the open field line region well inside the light cylinder (the radius at which the corotation speed equals \(c\)).

The study of plasma modes along the field lines was boosted by the pioneering work of Goldreich & Julian (1969), Sturrock (1971), Mestel (1971), Ruderman & Sutherland (1975), and Arons & Scharlemann (1979). Subsequent achievements and some new ideas have been reviewed by numerous authors (e.g., Michel 1991; Arons 1992; Mestel 1992; Muslimov & Harding 1997). Although a self-consistent pulsar magnetosphere theory has yet to be developed, the analysis of plasma modes in the pulsar magnetosphere based on the aforementioned papers provides firm ground for the construction of such a model.

The existence of strong electromagnetic fields is one of the most important features of rotating neutron stars observed as pulsars. It has been shown, starting with the pioneering paper of Deutsch (1955), that an electric field is induced by the rotation of a highly magnetized star. The general relativistic effect of dragging of inertial frames is very important in pulsar magnetospheres (Beskin 1990; Muslimov & Tsygan 1990) and a source of additional electric field.

The field-aligned parallel electric field is driven by deviations of the space charge from the Goldreich-Julian charge density, which is determined by the magnetic field geometry. Therefore, as has been noted by several authors (e.g., Beskin 1990; Muslimov & Tsygan 1990; Muslimov & Harding 1997; Mofiz & Ahmedov 2000; Dyks et al. 2001; Beskin 2005), the effect of general relativistic frame dragging on the field geometry in the plasma magnetosphere of a rotating neutron star is a first-order effect, which has to be carefully included in a self-consistent model of pulsar magnetospheric structure and the associated electromagnetic radiation. It was first shown by Muslimov & Tsygan (1990) and independently by Beskin (1990) that general relativistic effects due to frame dragging are crucial for the formation of the field-aligned electric field and particle acceleration in the magnetosphere. General relativistic effects are also important for the vacuum electromagnetic fields produced by slowly rotating magnetized neutron stars (Rezzolla et al. 2001a, 2001b).

At present, there is no observational evidence for the existence of a gravitomagnetic monopole—that is, of the exotic spacetime called NUT space (Newman-Uni-Tamburino; see Newman et al. 1963). Therefore, it is interesting to study the electromagnetic fields and processes in NUT space with the aim of finding new tools with which to study new and important general relativistic

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effects associated with nondiagonal components of the metric tensor that have no Newtonian analogs.

Muslimov & Tsygan (1992b) initiated the detailed general relativistic derivation of the magnetospheric electromagnetic fields around rotating, magnetized neutron stars. In this paper, we attempt to extend the work of Muslimov & Harding (1997) by including the NUT parameter in a study of particle acceleration along the open field lines of a rotating neutron star. In § 2, the general relativistic equations describing the electrodynamics of a rotating NUT star are formulated. The equations are rewritten in a frame of reference corotating with the NUT star. A detailed analysis of the Goldreich-Julian charge density is also carried out.

A general equation governing the electrostatic potential in the magnetosphere is derived. As has been shown by several authors, in particular Sakai & Shibata (2003), the effects of general relativity on particle motion in the pulsar magnetosphere cannot be neglected. For this reason, in § 3 we solve the equations of motion for charged particles in the region near the magnetic pole just above the stellar surface. By studying the particle motion along the field lines, it is shown that the effect of the NUT parameter, coupled with the frame-dragging effect, plays a noticeable role in the particle dynamics. In § 4, we conclude our findings and discuss their utility for further investigations.

Throughout, we use a spacelike signature (−, +, +, +) and a system of units in which $G = 1 = c$ (however, for those expressions with an astrophysical application we write the speed of light explicitly). Latin indices run 1, 2, 3 and Greek ones from 0 to 3.

2. PLASMA MAGNETOSPHERE OF A SLOWLY ROTATING, MAGNETIZED NUT STAR

As shown by Muslimov & Tsygan (1986, 1992b) from the system of Maxwell’s equations, assuming the magnetic field of a neutron star to be stationary in the corotating frame, the following Poisson equation for the scalar potential $\Phi$ can be derived:

$$\nabla \cdot \left( \frac{1}{N} \nabla \Phi \right) = -4\pi (\rho - \rho_{\text{GJ}}),$$

where $N \equiv (1 - 2Mr/R)^{1/2}$ is the gravitational lapse function, $M$ is the total mass of the star, $I$ is its angular momentum, $\rho - \rho_{\text{GJ}}$ is the effective space charge density responsible for production of the unscreened parallel electric field, and $\rho_{\text{GJ}}$ is the Goldreich-Julian charge density, which we discuss here.

In a pioneering work, Goldreich & Julian (1969) showed that a strongly magnetized, highly conducting neutron star rotating about its magnetic axis will spontaneously build up a charged magnetosphere. The essence of the argument is that the rotation induces a charge in the magnetosphere that is subject to enormous unbalanced electric forces parallel to the magnetic field $\mathbf{B}$. Goldreich & Julian hypothesized that a far better approximation for the magnetosphere would be obtained by shorting out the component of the electric field $\mathbf{E}$ along $\mathbf{B}$ with charges extracted from the stellar surface. The magnetospheric charges that maintain $\mathbf{E} \cdot \mathbf{B}$ are themselves subject to the $\mathbf{E} \times \mathbf{B}$ drift, which sets them into corotation with the star. Here we analyze the Goldreich-Julian charge density within general relativity in the spacetime of a slowly rotating NUT star.

An expression for the Goldreich-Julian charge density $\rho_{\text{GJ}}$ can be calculated in terms of the vector $q_i = -g_{0i}/g_{00}$ through the formula

$$\rho_{\text{GJ}} = -\frac{1}{4\pi} \nabla \cdot (Ng \times \mathbf{B}).$$

In the metric of a slowly rotating star with nonvanishing NUT parameter,

$$ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) - 2(4N^2 l \sin^2 \frac{\theta}{2} + \omega^2 r^2 \sin^2 \theta) d\phi dt,$$

the vector $g$ looks like the following:

$$g = \frac{1}{N^2} \left[ (\Omega - \omega) \times r - \frac{4N^2 l \sin^2 \frac{\theta}{2}}{r^2 \sin^2 \theta} \hat{z} \times r \right].$$

This is a stationary, axially symmetric solution of the vacuum Einstein field equations with three parameters. Equation (3) is the linear approximation to the Kerr-Taub-NUT metric (see, e.g., Dadhich & Turakulov 2002; Bini et al. 2003) with respect to the specific angular momentum $a = J/M$ and the gravitomagnetic monopole $l$. The first term on the right-hand side of equation (4) takes into account the well-known effect of the dragging of inertial frames of reference (the Lense-Thirring effect), with angular velocity $\omega = 2aMr^2$. Here $\Omega$ is the angular velocity of the star’s rotation. Obviously, when $l = 0$ equation (3) reduces to the external Hartle-Thorne metric (Hartle & Thorne 1968). On the other hand, when $\omega$ vanishes, the spacetime reduces to the NUT one (Newman et al. 1963).

To obtain this equation, we recall that in Cartesian coordinates $r = (x, y, z)$, $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, and $z = r \cos \theta$, and with $\hat{z} = (0, 0, 1)$, the following relation holds:

$$(\hat{z} \times \mathbf{r}) \cdot dr = r^2 \sin^2 \theta d\phi$$

(see Bini et al. 2003). Inserting equation (4) into the expression for the Goldreich-Julian charge density $\rho_{\text{GJ}}$ (eq. [2]), we obtain

$$\rho_{\text{GJ}} = -\frac{1}{4\pi} \nabla \left\{ \frac{1}{N} \left[ 1 - \frac{\kappa}{\eta^3} - L \left( \frac{1 - \epsilon}{\eta} \right)^2 \frac{4 \sin^2 \frac{\theta}{2}}{\sin^2 \theta} \right] \mathbf{u} \times \mathbf{B} \right\},$$

where we have written $L \equiv cl/\Omega R^2$ with $R$ the stellar radius and $\mathbf{u} = \Omega \times \mathbf{r}$, $\eta = r \sin \theta$ is the dimensionless radial coordinate, the parameter $\kappa \equiv \epsilon \beta$, $\epsilon = 2MR/R$ is the compactness parameter, and $\beta = l/I_0$ is the stellar moment of inertia in units of $I_0 = MR^2$.

Under the assumption of a dipole-like configuration for the stellar magnetic field, the nonvanishing components of the magnetic field $\mathbf{B}$ measured by a zero angular momentum observer with 4-velocity

$$u_\alpha = \{-N^2, 0, 0, 0\}$$

take the form

$$B^\theta = B_0 \frac{f(\eta)}{f(1)} \eta^{-3} \cos \theta,$$

$$B^\phi = \frac{B_0 N}{2} \left[ -2f(\eta) \left( \frac{3}{1 - \epsilon/\eta} + \frac{\epsilon}{\eta} \right) \right] \eta^{-3} \sin \theta$$

(see Muslimov & Tsygan 1992b), where

$$f(\eta) = -3 \left( \frac{\eta}{\epsilon} \right)^3 \left[ \ln \left( 1 - \frac{\epsilon}{\eta} \right) + \frac{\epsilon}{\eta} \left( 1 + \frac{\epsilon}{2\eta} \right) \right]$$

and $B_0 \equiv 2\mu/R^3$ is the Newtonian value of the magnetic field at the pole of star. Here hats label the orthonormal components and $\mu$
is the magnetic moment. A solution of this type was first obtained by Ginzburg & Ozernoy (1964) and then reproduced by a number of authors.

The polar angle $\Theta$ of the last open magnetic line as a function of $\eta$ looks like

$$
\Theta \sim \sin^{-1} \left\{ \left[ \frac{f(1)}{\eta f(\eta)} \right]^{1/2} \sin \Theta_0 \right\},
$$

$$
\Theta_0 = \sin^{-1} \left( \frac{R}{R_{L,C} f(1)} \right)^{1/2}
$$

(see Muslimov & Tsygan 1991, 1992a). The parameter $\Theta_0$ has the meaning of the magnetic colatitude of the last open magnetic line at the stellar surface, and $R_{L,C} = c/\Omega$ is the radius of the light cylinder.

Further algebraically transforming equation (6), and taking into account equation (8), we come to the following expression for the Goldreich-Julian charge density:

$$
\rho_{\text{GJ}} = -\frac{\Omega B_0}{2\pi c} \frac{f(\eta)}{N\eta^3 f(1)} \left[ 1 - \frac{\kappa}{\eta^3} - L \left( 1 - \frac{\epsilon}{\eta} \right) \frac{1}{\eta^2} \frac{4 \sin^2 \frac{1}{2} \theta}{\sin^2 \theta} \right].
$$

(11)

Hereafter, for simplicity of calculation we assume that the angle between the magnetic axis and the axis of rotation of the star is equal to zero. Figure 1 presents the radial dependence of the obtained general relativistic expression for the Goldreich-Julian charge density $\rho_{\text{GJ}}$, normalized to its Newtonian expression, for several different values of the NUT parameter $l$. One can see that even for comparatively small values of $l$, its influence on $\rho_{\text{GJ}}$ (eq. [11]) is important. The value of $\rho_{\text{GJ}}$ is more sensitive to the NUT parameter at the surface of the star. The tendency seen from this figure is that $\rho_{\text{GJ}}$ is inversely proportional to the NUT parameter at the surface of the star and asymptotically reaches the Newtonian expression. Here we have taken as typical neutron star parameters $R = 10$ km, $M = 2$ km, and $P = 0.1$ s.

For a relativistic plasma, the charge density $\rho$ is proportional to the magnetic field, with the proportionality coefficient being constant along a given magnetic field line such that

$$
\rho = \frac{\Omega B_0}{2\pi c} \frac{f(\eta)}{N\eta^3 f(1)} A(\xi)
$$

(12)

(see, e.g., Muslimov & Tsygan 1992a), where $\xi = \theta/\Theta$ is a dimensionless angular variable and $A(\xi)$ is an unknown function to be determined from the boundary conditions. One can insert equations (11) and (12) into the Poisson equation (eq. [1]) and, under the approximation of small angles $\theta$, obtain the following differential equation:

$$
R^{-2} \left\{ N \frac{1}{\eta^2} \frac{\partial}{\partial \eta} \left( \eta^2 \frac{\partial \Phi}{\partial \eta} \right) + \frac{1}{N\eta^3 \theta} \left[ \frac{\partial}{\partial \theta} \left( \theta \frac{\partial \Phi}{\partial \theta} \right) + \frac{1}{\theta} \frac{\partial^2 \Phi}{\partial \phi^2} \right] \right\} \Phi
$$

$$
= -4\pi \frac{\Omega B_0}{2\pi c} \frac{f(\eta)}{N\eta^3 f(1)} \left[ 1 - \frac{\kappa}{\eta^3} - L \left( 1 - \frac{\epsilon}{\eta} \right) \frac{1}{\eta^2} + A(\xi) \right].
$$

(13)

Here we have used that fact that in the small-angle limit

$$
4 \sin^2 \frac{1}{2} \theta \sim 1.
$$

(14)

Our further exposition is based on extending the work of Muslimov & Tsygan (1992b) to NUT spacetime. Using the dimensionless function $F \equiv \eta \Phi/\Phi_0$, where $\Phi_0 = \Omega B_0 R^2$, and the variables $\eta$ and $\xi$, we can rewrite equation (1) for the dimensionless electrostatic potential as

$$
\left[ \frac{d^2}{d\eta^2} + \Lambda^2(\eta) \frac{1}{\xi} \frac{\partial}{\partial \xi} \left( \xi \frac{\partial}{\partial \xi} \right) \right] F
$$

$$
= -\frac{2}{\eta^2(1 - \epsilon/\eta)} \frac{f(\eta)}{f(1)} \left[ 1 - \frac{\kappa}{\eta^3} - L \left( 1 - \frac{\epsilon}{\eta} \right) \frac{1}{\eta^2} + A(\xi) \right]
$$

(15)

with $\Lambda(\eta) = [\eta \Theta(\eta)(1 - \epsilon/\eta)^{1/2}]^{-1}$. After performing a Fourier-Bessel transform

$$
F(\eta, \xi) = \sum_{i=1}^{\infty} F_i(\eta) J_0(k_i \xi),
$$

$$
F_i(\eta) = \frac{2}{|J_1(k_i)|^2} \int_0^1 \xi F(\eta, \xi) J_0(k_i \xi) \, d\xi,
$$

(16)

and using the relation

$$
\sum_{i=1}^{\infty} \frac{2}{k_i J_1(k_i)} J_0(k_i \xi) = 1,
$$

(17)

one can obtain equation (15) in the form

$$
\left[ \frac{d^2}{d\eta^2} - \gamma_i^2(\eta) \right] F_i
$$

$$
= -\frac{2}{\eta^2(1 - \epsilon/\eta)} \frac{f(\eta)}{f(1)} \times \left\{ \frac{2}{k_i J_1(k_i)} \left[ 1 - \frac{\kappa}{\eta^3} - L \left( 1 - \frac{\epsilon}{\eta} \right) \frac{1}{\eta^2} + A(\xi) \right] \right\},
$$

(18)

where $\gamma_i^2 = k_i^2 \Lambda^2$ and the $k_i$ are positive zeros of the function $J_0$.

Consider a region near the surface of the star, where $z = \eta - 1 \ll 1$. Using the boundary conditions (equipotentiality of the stellar surface and zero steady state electric field at $r = R$)

$$
F_i|_{z=0} = 0, \quad \partial F_i/\partial z|_{z=0} = 0,
$$

(19)
one can find an expression for the scalar potential $\Phi$ near the surface of the star and, corresponding to this potential, the component of the electric field parallel to the magnetic field:

$$
\Phi = \frac{12\Phi_0}{\eta} \sqrt{1 - \epsilon(\kappa - L)c_0^2} \\
\times \sum_{i=1}^{\infty} \left\{ \exp \left[ \frac{k_i(1 - \eta)}{\Theta_0 \sqrt{1 - \epsilon}} \right] - 1 + \frac{k_i(\eta - 1)}{\Theta_0 \sqrt{1 - \epsilon}} \right\} J_0(k_i\xi) k_i^2J_i(k_i),
$$

(20)

$$
E_{1} = -\frac{12\Phi_0}{R} (\kappa - Lc_0^2) \sum_{i=1}^{\infty} \left\{ 1 - \exp \left[ \frac{k_i(1 - \eta)}{\Theta_0 \sqrt{1 - \epsilon}} \right] \right\} J_0(k_i\xi) k_i^2J_i(k_i)
$$

(21)

(see Muslimov & Tsygan 1992b). It should be noted that these formulae differ from those obtained by Muslimov & Tsygan in that the parameter $\kappa$ has been replaced with $\kappa - Lc_0^2$. Considering now the region $\Theta_0 \ll \eta - 1 \ll R_1/c_0$, where $|d^2F_i/d\eta^2| \approx \gamma_1^2(\eta)|F_i|$, one can see that equation (18) becomes

$$
-\gamma_1^2(\eta)F_i = -\frac{2}{\eta^2(1 - \epsilon/\eta)} f(\eta) \times \left\{ \frac{2}{k_iJ_i(k_i)} \left[ \left( 1 - \frac{\kappa}{\eta^2} \right) - L \left( 1 - \frac{\epsilon}{\eta^2} \right) + A_i \right] \right\}.
$$

(22)

from which it immediately follows that

$$
F_i = \frac{2}{k_i^2} \theta^2(\eta^2) f(\eta) \frac{d}{d\eta} \left[ \frac{2}{k_iJ_i(k_i)} \left( \left( 1 - \frac{\kappa}{\eta^2} \right) - L \left( 1 - \frac{\epsilon}{\eta^2} \right) + A_i \right) \right].
$$

(23)

Using this expression for $F_i$, one can obtain the scalar potential at distances greater than the polar cap size as

$$
\Phi = \frac{\Phi_0}{\eta} F = 2\Phi_0 \Theta_0^2 (\kappa - Lc_0^2) \left( 1 - \frac{1}{\eta^2} \right) \sum_{i=1}^{\infty} \frac{2J_0(k_i\xi)}{k_i^2J_i(k_i)}
$$

$$
= \frac{\Phi_0 \Theta_0^2}{2} (\kappa - Lc_0^2) \left( 1 - \frac{1}{\eta^2} \right) (1 - \xi^2)
$$

$$
= \frac{\Omega R^2 B_0 \Theta_0^2}{2} (\kappa - Lc_0^2) \left( 1 - \frac{1}{\eta^2} \right) (1 - \xi^2).
$$

(24)

Corresponding to this potential, the parallel component of the electric field becomes

$$
E_{||} = -\frac{1}{R} \frac{\partial \Phi}{\partial \eta} \bigg|_{\xi=\text{const}} = -E_{\text{vac}} \frac{3(\kappa - Lc_0^2)}{2\eta^4} \left( 1 - \xi^2 \right).
$$

(25)

where $E_{\text{vac}} \equiv (\Omega R/c_0) B_0$ is the characteristic Newtonian value of the electric field generated near the surface of a neutron star rotating in vacuum (Deutsch 1955). In Figure 2, we show the radial dependence of $E_{||}$ in terms of $E_{\text{vac}}$ for different values of the NUT parameter.

The energy losses from the polar cap of a rotating star with nonvanishing NUT parameter can now be calculated. According to Muslimov & Harding (1997), the expression for the total power carried away by relativistically moving particles is

$$
L_p = 2 \left( -c \int \rho \Phi \, dS \right).
$$

(26)

In the spacetime of a slowly rotating NUT star,

$$
-\rho \Phi \approx \frac{1}{4\pi} \left( \frac{\Omega B_0}{c} \right)^2 \frac{R^2 \Theta_0^2 f(\eta)}{N \eta^2 f(1)} \times \left\{ (\kappa - Lc_0^2) \left( 1 - \kappa - L(1 - \epsilon) \right) \right\} (1 - \xi^2).
$$

(27)

Inserting equation (27) into equation (26) and evaluating the integral, one finds that

$$
(L_p)_{\text{max}} = \frac{1}{2} \left\{ (\kappa - Lc_0^2) \left( 1 - \kappa - L(1 - \epsilon) \right) \right\} E_{\text{rot}}.
$$

(28)

where

$$
E_{\text{rot}} \equiv \frac{1}{6} \frac{\Omega^2 R^2 \Theta_0^2}{c^2 f(1)} = \frac{1}{f(1)} (E_{\text{rot}})_{\text{Newt}}
$$

(29)

with $(E_{\text{rot}})_{\text{Newt}}$ is the standard Newtonian expression for magnetic dipole losses in the flat-spacetime approximation.

In the limiting case $l \rightarrow 0$,

$$
(L_p)_{\text{max}} (l=0) = \frac{1}{2} \frac{\kappa (1 - \kappa)}{\kappa (1 - \kappa)} E_{\text{rot}}
$$

(30)

(Muslimov & Harding 1997). The ratio

$$
\frac{(L_p)_{\text{max}} (l=0)}{(L_p)_{\text{max}} (l=0)} = 1 - \frac{L(\kappa + \epsilon - 2\kappa\epsilon)}{\kappa (1 - \kappa)} + \frac{L^2 \epsilon(1 - \epsilon)}{\kappa (1 - \kappa)}
$$

(31)

as a function of the NUT parameter is presented in Figure 3. The dependence has a parabolic form. Namely, for small values of the NUT parameter the energy losses are decreasing. Then the graph has a minimum, and with further growth of the NUT parameter, more energy is lost. Physically, this is due to the fact that contributions to the accelerating electric field from the different parameters have opposite signs.

Taking into account that $\kappa = \epsilon \beta \sim \epsilon$, one can see that the overall magnitude of the additional terms arising from the NUT
parameter is determined by the magnitude of $L$. For a millisecond pulsar with $l \sim 10^5$ cm, $\Omega \sim 10^{-6}$ s$^{-1}$, and $R \sim 10^6$ cm, we have

$$L = \frac{c l}{\Omega R} \sim 1.$$  

(32)

This shows that corrections to the energy losses related to a non-zero NUT parameter cannot be neglected and, in principle, can provide important information that can help in attempts to detect the gravitomagnetic charge.

Equation (28) makes physical sense only if $(L_p)_{\text{max}} < \dot{E}_{\text{rot}}$, which means that the power of a polar-cap accelerator in principle cannot exceed the total spin-down power of the pulsar if one supposes that the electromagnetic radiation is powered by rotation. Using equations (28), (30), and (31), one can see that physical sense is preserved for the magnetosphere model under discussion (in which the electric field created by rotation of the star is dominant) if

$$L^2 \lesssim \frac{2}{3\kappa(1 - \kappa)} - 1.$$  

(33)

Using a value $\kappa = 0.15$ (Muslimov & Tsygan 1992b), we obtain an upper limit for the value of the NUT parameter of $I \lesssim 1000$ cm.

For practical use, equation (28) can be rewritten in terms of the pulsar’s observable characteristics, the period $P$ and its time derivative $\dot{P} \equiv dP/dt$:

$$(PP)_{\text{max}} = \frac{3}{4} \{(\kappa - Lc)[1 - \kappa - L(1 - c)]\} \frac{1}{f^2(1)} (PP)_{\text{Newt}},$$  

(34)

where the expressions

$$(L_p)_{\text{max}} = -\bar{I}(\Omega \bar{\Omega})_{\text{max}},$$  

(35)

$$(PP)_{\text{Newt}} \equiv \frac{2\pi^2}{3c^3} \frac{R^6 B_0^2}{I}$$  

(36)

have been taken into account. In equation (35), $\bar{I}$ is the general relativistic moment of inertia of the star,

$$\bar{I} \equiv \int d^3x \sqrt{\gamma} e^{-\Phi(r)} \rho^2 \sin^2 \theta$$  

(37)

(see, e.g., Rezzolla & Ahmedov 2004), where $e^{-\Phi(r)} \equiv 1/(-g_{00})^{1/2}$, $\rho(r)$ is the total energy density, $\gamma$ is the determinant of the 3-metric, and $d^3x$ is the coordinate volume element.

The period and its time derivative have been very precisely measured for a large number of pulsars (e.g., Kaspi et al. 2006 present a $P-\dot{P}$ diagram for 1403 cataloged rotation-powered pulsars; see also Arons 2008). Thus, equation (34) for $PP$ may reveal the possible existence and magnitude of the NUT parameter. The main difficulty encountered on this path nowadays is the uncertainty in estimating the neutron star moment of inertia. But in the future, when the moment of inertia can be determined more precisely, one in principle will be able to obtain the value of the NUT parameter from observational data.

3. CHARGED-PARTICLE ACCELERATION IN THE POLAR CAP OF A SLOWLY ROTATING NUT STAR

The origin of radio emission from the polar cap is one of the most mysterious of the remaining unsolved questions in the physics of pulsars. One likely scenario is that particles are accelerated along open magnetic field lines and emit $\gamma$-rays that subsequently convert into electron-positron pairs in the strong magnetic field. The combination of the primary beam and the pair plasma then provides the mechanism of radio emission. For this reason, it is interesting to study particle acceleration conditions and the equations of motion in the pulsar magnetosphere in the presence of the NUT parameter.

In an earlier paper (Motiz & Ahmedov 2000), the effects of general relativity on plasma modes along the open field lines of a rotating, magnetized neutron star were studied. Here we investigate the equations of motion for a charged particle in the region just above the polar cap surface of a neutron star with non-zero NUT parameter by extending the results of Sakai & Shibata (2003) to a spacetime with gravitomagnetic charge.

The equations of motion for a particle with mass $m$, charge $e$, proper time $\tau$, and 4-velocity $u \equiv dx/d\tau$ look like

$$m \left( \frac{d^4u}{d\tau^4} + \Gamma_{\mu\nu}^\rho u^\mu u^\nu \right) = eF^{\nu\mu}u_\nu,$$  

(38)

where $\Gamma_{\mu\rho}^\nu$ is the affine connection, $F^{\mu\nu} = u^\mu E^\nu - u^\nu E^\mu + \eta^{\mu\nu}\gamma^\rho u_\rho B_\nu$ is the electromagnetic field tensor, and $\eta^{\mu\nu}\gamma^\rho$ is the Levi-Civita tensor. In the approximation of small angles $\theta$ in the polar cap region, the equations of motion can be rewritten in the form

$$\frac{N d}{s^2} \left( \frac{s^2 d\phi}{ds} \right)^2 - \frac{l(l + 1)}{N s^2} \phi = B_0 N \frac{j}{V - \bar{j}},$$  

(39)

$$\frac{d}{ds} (N \gamma) = \frac{1}{N^2} \frac{d\phi}{ds}$$  

(40)

in the spacetime outside the star described by the Kerr-Taub-NUT metric, in the slow-rotation limit (eq. [3]). These formally coincide with the equations of motion from Sakai & Shibata (2003). Here the Lorentz factor $\gamma \equiv -u^\nu u_\nu = N^\nu_\nu = 1/(1 - V^2)^{1/2}$, the relative 3-velocity $V^i \equiv u^\mu h_{\mu i}/\gamma$, the projection tensor $h^{\alpha \beta} \equiv g^{\alpha \beta} + u^\alpha u^\beta$, and the normalized variables

$$j \equiv -\frac{2\pi N(s)s^2}{\Omega B(s)} = \text{const},$$  

$$\phi(s) \equiv \frac{e}{m} \Phi(s),$$  

$$s \equiv \sqrt{\frac{2\Omega B_0 e}{mc^2}}, \quad j \equiv -\frac{2\pi N(s)s\Omega(s)}{\Omega B(s)}$$  

(41)
have been introduced. To obtain equations (39) and (40), the scalar potential $\Phi$ was expanded in spherical harmonics $Y_{lm}(\theta, \phi)$ as $\Phi = \sum_{lm} \Phi(r) Y_{lm}(\theta, \phi)$, and only the mode of the polar cap scale $l \approx \pi / \Theta_0$ was taken. In equation (41), $J$ is the electric charge current density and $B \equiv |B| = B^2 + O(\theta^2)$. The approximate value of the Goldreich-Julian charge density (eq. [6]) is then

$$\rho_{GJ} = \frac{1}{2\pi} \frac{B \Omega}{N} \left(1 - \frac{\omega}{\Omega} \right) \frac{N^2l}{\Omega},$$

and consequently the NUT parameter will provide an additional radial dependence to $J$.

According to a recent paper by Beloborodov (2008), the motion of charged particles in the vicinity of the polar cap of a pulsar is governed by the equation

$$\frac{dp}{dt} = \frac{eE}{mc}$$

(where the momentum of the particle is given in units of $mc$).

And, for the parallel electric field $E_\parallel$ one can obtain the equation

$$\nabla \cdot E = \frac{dE_\parallel}{dz} = 4\pi (\rho - \rho_{GJ}), \quad z \ll r_{pc},$$

where $r_{pc}$ is the radius of the polar cap of the star and $z$ is the distance from the stellar surface. Rewriting equation (44) in terms of $d/dt \equiv v d/dz$, $a \equiv j/c\rho_{GJ}$, $\rho = j/v$, and $v = c(1 + p^2)^{-1/2}$, one can obtain

$$\frac{dE_\parallel}{dt} = 4\pi j \left(1 - \frac{ap}{\sqrt{1 + p^2}}\right)$$

(see Beloborodov 2008), where

$$a(z) = a_0 \frac{1 - \kappa + Lc}{1 - (\kappa + Lc)(1 + z/R)^{-3}},$$

Finally, the system of equations for $p(z)$ is solved graphically. Figure 4 presents $p(z)$ for several values of $Lc$, with $\kappa = 0.15$ (Muslimov & Tsygan 1992b) and $a_0 = 0.999$, and shows that the influence of the NUT parameter significantly changes the period of oscillations. From Figures 5 and 6, one can further see that when $Lc \sim 1$, oscillations take place even for large $a_0$. For values of $Lc$ very close to 1, the character of the graphs has almost no dependence on the value of $a_0$. For old, recycled neutron stars with almost zero angular momentum, the effects connected with the NUT parameter may be the sole remaining mechanism for pulsar radiation.

Following Arons (1998) and using equations (11), (12), and (14), one can obtain for the accelerating potential drop

$$\Delta \Phi_\parallel \approx (\kappa - Lc)\Phi_{pole}(1 - \eta^{-3}),$$

while in the Newtonian case this appears as

$$\Delta \Phi_\parallel \approx \Phi_{pole}(R/\rho_B),$$

where $\rho_B$ is the radius of curvature of the magnetic field lines, $\Phi_{pole} = \Omega^2 \mu/c^2$, and $\mu$ is again the magnetic moment. Taking into account the effect of frame dragging noticeably improves comparisons between theory and observations. As $R/\rho_B \sim 10^{-2}P^{-1/2}$ and $\kappa \sim 10P^{-1/2}$, the energy of curvature gamma rays in the relativistic frame-dragging case may rise to 1000 times more than in the pair creation theory of Arons & Scharlemann (1979; see Arons 1998). As the effect of the NUT parameter seems to be of the same order as the effect of frame dragging, it makes an additional contribution to the energy of curvature gamma photons and thus may play a noticeable role in the formation of the plasma magnetosphere of neutron stars with gravitomagnetic charge.
4. CONCLUSION

We have considered astrophysical processes in the polar caps of pulsar magnetospheres, assuming the spacetime of a slowly rotating NUT star. In particular, general relativistic corrections to the Goldreich-Julian charge density, electrostatic scalar potential, and accelerating component of the electric field (parallel to the magnetic field lines) in the polar cap region due to the presence of a gravitomagnetic charge are found. The presence of the NUT parameter slightly modulates the Goldreich-Julian charge density near the surface of the star. However, as already known, the effective electric charge density—that is, the difference between the Goldreich-Julian charge density $\rho_{GJ}$ (which is proportional in the case of a flat spacetime to $\Omega \cdot B$) and the electric charge density (proportional to $B$) in the magnetosphere—is responsible for the generation of the electric field parallel to the magnetic field lines. This difference is zero at the surface of the star and changes with distance from it because of the fact that $\rho$ cannot compensate for $\rho_{GJ}$. General relativistic terms arising from the dragging of inertial frames and the presence of the gravitomagnetic charge make very important additional contributions to this difference. Both these terms depend on the radial distance from the star as $1/r^3$ and have equally important influences on the value of the accelerating electric field component generated in the magnetosphere near the surface of the neutron star.

These results were applied to find an expression for the gravitomagnetic energy losses along the open magnetic field lines of a slowly rotating NUT star. It is found that in the case of a non-vanishing NUT parameter, an additional and important term appears in the coefficient of the standard expression for the magnetic dipole energy losses. A comparison of the effect of the NUT parameter with the already known effects shows that it cannot be neglected. The newly obtained dependence may be combined with astrophysical data on pulsar period slowdowns and be useful in further investigations of the possible detection of the gravitomagnetic monopole.

We have also shown that the presence of a gravitomagnetic charge has an influence on the conditions governing particle motion in the polar cap region. From the derived results, it can be seen that the NUT parameter modulates the period of oscillations of the particles’ momentum. As this effect connected to the NUT parameter has the angular velocity of the star’s rotation in the denominator, it will increase for old stars, for which the frame-dragging effect will be decreased. Thus, possible candidates to verify the existence of the NUT parameter are old, compact objects with extremely low rotation period.

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