A Toy Model of Closed String Tachyon Effective Action

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ABSTRACT: In this paper we propose the toy model of the closed string tachyon effective action that has marginal tachyon profile as its exact solution in case of constant or linear dilaton background. Then we will apply this model for description of two dimensional bosonic string theory. We will find that the background configuration with the spatial dependent linear dilaton, flat spacetime metric and marginal tachyon profile is the exact solution of our model even if we take into account backreaction of tachyon on dilaton and on metric.

KEYWORDS: Closed string tachyon.
1. Introduction

The study of open string tachyon condensation has led to the important insight into
the non-perturbative character of open string theory \(^1\). On the other hand the study
of the closed string tachyon condensation is more difficult \([9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]\). The main problem is that spacetime disapp-
ears altogether when the bulk tachyon condenses and hence perturbation theory breaks down.

One can hope that \(c = 1\) matrix model that provides non-perturbative description of string theory in \(1 + 1\) dimensions (For review, see \([23, 24, 25, 26, 27, 28, 29]\).) could be useful laboratory for the study of the closed string tachyon condensation.
In fact, very interesting results considering time-dependent tachyon condensation in
two dimensions were obtained recently in \([30, 31, 32]\).

Another approach to the problem the time-dependent closed string tachyon con-
densation was given in \([17, 33, 34, 35]\). This approach is based on the fact that there
exists exact, time-dependent solution of the tachyon field theories describing homoge-
nenous tachyon condensation. The worldsheet conformal field theory that describes
this process is governed by the action (suppressing spatial directions and setting
\(\alpha' = 1\))

\[
S = \frac{1}{4\pi} \int d^2 \sigma \left( - (\partial X^0)^2 + 4\pi \mu e^{2\beta X^0} \right).
\] (1.1)

which has negative norm boson and central charge \(c = 1 - 6q^2\), \(q = \beta - 1/\beta\). The potential term in (1.1) can be interpreted as closed string tachyon field that grows exponentially in time.

\(^1\)For reviews of open string tachyon condensation, see \([1, 2, 3, 4, 5, 6, 7, 8]\).
In very interesting paper [36] the possibility of the effective field theory description of such a process was discussed with analogy with the successful effective field theory description of the open string tachyon condensation [37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54]. According to this paper we should search for the analogy of the exact solution in open string theory on unstable D-brane $T = T_+ e^t + T_- e^{-t}$ that describes time-dependent process of the tachyon condensation. It was proposed in [36] that such a closed string solution could be the world sheet interaction

$$\delta L = \lambda \cosh 2x^0.$$  \hspace{1cm} (1.2)

The problem with this term is that it is expected not to be truly marginal and one can expect large backreaction on the dilaton and metric in the late times when we turn on $\lambda$. On the other hand it was argued that the interaction

$$\delta L = \lambda e^{2x^0}$$  \hspace{1cm} (1.3)

is exactly marginal and could be analogue of the rolling tachyon profile in open string theory even if it is not completely clear what suppress the backreaction to the metric and dilaton to the stress tensor of tachyon.

As is well known from the study of non-critical string theory nonzero vacuum expectation value of tachyon is especially important in case of the linear dilaton. For that reason it seems to be natural to study the closed string tachyon dynamics in case of nontrivial dilaton background. Recently we have proposed an effective action for D-brane in the linear dilaton background [52]. This effective action has the rolling tachyon solution on unstable D-brane in the linear dilaton background as its exact solution. Since the tachyon profile on unstable D-brane has similar form as the tachyon exponential profile in the closed string case it is natural to apply the formalism developed in [52] for the construction of the closed string tachyon effective action that will have (1.3) as its exact solution. In fact we should be more modest and say that we are proposing a toy model for the closed string tachyon effective action since our approach is not based on the first principles of the string theory as for example calculation of the tachyon effective action from the partition function on the two sphere. In fact this is the same situation as in the open string case where it was argued [47] that the tachyon effective action that has the marginal tachyon profile as its exact solution should be related by some complicated field redefinition to the tachyon effective action derived from the disk partition function. We mean that the similar tachyon field redefinition that maps our proposed action to the tachyon effective action derived from the sphere partition function could exist as well and for that reason we hope that our toy model could be useful for the study of the closed string tachyon condensation.

The structure of this paper is as follows. In the next section (2) we propose field theory effective action for the tachyon field with the mass square $\mu^2$ that has profile
as its exact solution of the equation of motion. We will calculate the stress energy tensor and dilaton source. Then in section (3) we will apply our proposal to the case of two dimensional string theory where the tachyon effective action considerably simplifies as a consequence of the fact that the potential term in the tachyon effective action vanishes. Since generally nonzero tachyon field forms sources for the dilaton and for the graviton we will study the problem of the backreaction of the tachyon on the metric and dilaton. After inclusion of the additional term to the tachyon effective action that could not be derived from the requirement that the rolling tachyon is the solution of the equation of motion in two dimensional theory we obtain the action for dilaton, graviton and tachyon that has the linear dilaton background in two dimensions with flat Minkowski metric and with exponential tachyon profile as its exact solution even if we take into account the backreaction of the tachyon on dilaton and on metric. We also find another time dependent solution that has the same property. On the other hand we will show that the tachyon effective action poses some exact solutions that generate nonzero dilaton source and stress energy tensor and hence cannot be considered as an exact solution of this toy model of two dimensional effective field theory. In section (4) we will study fluctuations around the tachyon marginal profile. We will find an effective action for fluctuations that has plane waves corresponding to propagating massless modes as its exact solution.

Finally, in conclusion (5) we outline our results and suggest further problems that deserve to be studied.

2. Proposal for closed string tachyon effective action in constant and in the linear dilaton background

In this section we propose the effective action for massive field $T$ that in case of constant dilaton $\Phi_0$ has the leading order condition of marginality

$$\partial_\mu [\eta^{\mu\nu} \partial_\nu T] = -\mu^2 T$$

(2.1)

Using results given in [36, 46, 49, 52] we now presume that the tachyon effective action has the form

$$S = -\int d^D x \mathcal{L} , \quad \mathcal{L} = \frac{1}{(1 + kT^2)} \sqrt{B} , \quad B = 1 + \mu^2 T^2 + \eta^{\mu\nu} \partial_\mu T \partial_\nu T ,$$

(2.2)

where the unknown constant $k$ will be determined from the requirement that following tachyon profile

$$T = T_+ e^{\beta^+_\mu x^\mu} + T_- e^{\beta^-_\mu x^\mu} , \quad \beta^\pm_\mu \eta^{\mu\nu} \beta^\pm_\nu = -\mu^2$$

(2.3)

Our convention is $\eta_{\mu\nu} = (-1,1,\ldots,D-1)$ where $D$ is the number of spacetime dimensions. We also have $i,j = 1,\ldots,D-1$, where $x^i$ label spatial directions.
with $\beta_+^\pm = 0$ is an exact solution of the equation of motion that arises from (2.2)

$$
\begin{align*}
\frac{-2kT\sqrt{B}}{(1 + kT^2)^2} + \frac{\mu^2 T}{(1 + kT^2)\sqrt{B}} - \\
\frac{\partial_\mu \eta^{\mu\nu} \partial_\nu T}{(1 + kT^2)\sqrt{B}} + \frac{2kT \partial_\mu T \eta^{\mu\nu} \partial_\nu T}{(1 + kT^2)^2 \sqrt{B}} &= 0 ,
\end{align*}
$$

where we have presumed that $B$ is constant. This assumption holds for the ansatz (2.3) since in this case we have

$$B = 1 + 2T_+T_-e^{(\beta_+ + \beta_-)^T(\mu^2 + 2(\beta_+ + \beta_-))} = 1 + 2T_+T_-\mu^2 = \text{const} .
$$

Consequently the equation of motion (2.4) reduces for (2.3) to

$$
\begin{align*}
\frac{-2kT\sqrt{B}}{(1 + kT^2)^2} + \frac{2\mu^2 T}{(1 + kT^2)\sqrt{B}} + \frac{2kT(B - 1 - \mu^2 T^2)}{(1 + kT^2)^2 \sqrt{B}} &= 0 .
\end{align*}
$$

The expression given above suggests that we should take $k = \mu^2$ in order to obey equation of motion since then

$$
\begin{align*}
\frac{2\mu^2 T}{(1 + \mu^2 T^2)^2 \sqrt{B}}(-B + B - 1 - \mu^2 T^2) + \frac{2\mu^2 T}{(1 + \mu^2 T^2)\sqrt{B}} &= 0 .
\end{align*}
$$

In summary, the tachyon effective Lagrangian that has tachyon profile

$$T = \lambda e^{\beta \mu e^\Phi} , -\beta_\mu \eta^{\mu\nu} \beta_\nu = \mu^2
$$

as its exact solution takes the form

$$
\mathcal{L} = \frac{A}{1 + \mu^2 T^2 \sqrt{B}} ,
$$

where we have included possible numerical factor $A$ that cannot be determined from this analysis.

Now we apply this strategy to the case of the closed string tachyon in the linear dilaton background. In this case the leading order condition of marginality is

$$
\partial_\mu [e^{-2\Phi} \eta^{\mu\nu} \partial_\nu T] = -\mu^2 T .
$$

Following arguments given in [52] we suggest that the Lagrangian that has the tachyon profile obeying (2.10) as its exact solution has the form

$$
\mathcal{L} = \frac{A}{1 + kT^2 e^{-2\Phi} \sqrt{B}} , B = 1 + e^{-2\Phi}(T^2 \mu^2 + \eta^{\mu\nu} \partial_\mu T \partial_\nu T - 2T \eta^{\mu\nu} \partial_\mu T \partial_\nu \Phi) (2.11)
$$

so that the equation of motion is

$$
\begin{align*}
\frac{-2kT e^{-2\Phi} \sqrt{B}}{(1 + kT^2 e^{-2\Phi})^2} + \frac{\mu^2 T e^{-2\Phi}}{(1 + kT^2 e^{-2\Phi})\sqrt{B}} - \\
\partial_\mu \left[ \frac{e^{-2\Phi} \eta^{\mu\nu} \partial_\nu T}{(1 + kT^2 e^{-2\Phi})\sqrt{B}} \right] - \\
\frac{e^{-2\Phi} \eta^{\mu\nu} \partial_\mu T \partial_\nu \Phi}{(1 + kT^2 e^{-2\Phi})\sqrt{B}} + \partial_\mu \left[ \frac{T e^{-2\Phi} \eta^{\mu\nu} \partial_\nu \Phi}{(1 + kT^2 e^{-2\Phi})\sqrt{B}} \right] &= 0 .
\end{align*}
$$
As in case of constant dilaton reviewed above we determine the constant $k$ from the requirement that the tachyon profile that obeys (2.10) and for which $B$ is constant is an exact solution of the equation of motion. Using the fact that $\sqrt{B} = \text{const}$ the equation of motion (2.12) after some calculation reduces to

$$\frac{2kTe^{-2\phi}}{(1 + kT^2e^{-2\phi})^2\sqrt{B}}(-1 - e^{-2\phi}T^2(\mu^2 - V_\mu V^\mu)) + \frac{2Te^{-2\phi}}{(1 + kT^2e^{-2\phi})\sqrt{B}}(\mu^2 - V_\mu V^\mu) = 0.$$  

(2.13)

We see that the previous equation has solution if we take $k = \mu^2 - V_\mu V^\mu$ since then

$$\frac{2(\mu^2 - V_\mu V^\mu)Te^{-2\phi}}{(1 + kT^2e^{-2\phi})^2\sqrt{B}}(-1 - e^{-2\phi}T^2k) + \frac{2Te^{-2\phi}}{(1 + kT^2e^{-2\phi})\sqrt{B}}(\mu^2 - V_\mu V^\mu) = 0.$$  

(2.14)

Consequently our proposal for the closed string tachyon effective action in bosonic string theory ($\mu^2 = 4$) in the linear dilaton background has the form

$$S = -\int d^Dx \mathcal{L}, \mathcal{L} = \frac{A}{(1 + e^{-2\phi}T^2(4 - V_\mu V^\mu))}\sqrt{B}.$$  

(2.15)

We see that for $D = 2$ the potential term in (2.15) vanishes thanks to the relation between dimension of spacetime and the norm of the dilaton vector $V : 4 - V_\mu V^\mu = 4 - (26 - D)/6 = \frac{D - 2}{6}$ that holds in bosonic string theory. This fact will play significant role in the next section.

In order to calculate the stress energy tensor and dilaton source we will proceed as follows. Firstly we replace $V_\mu V^\mu$ with $\partial_\mu \Phi \eta^{\mu\nu} \partial_\nu \Phi$. Now we presume that the action (2.15) is also valid for any dilaton field. In the same way we replace everywhere the flat spacetime metric $\eta^{\mu\nu}$ with $g^{\mu\nu}$ and also $d^Dx$ with $d^Dx\sqrt{-g}$ and presume that (2.15) is valid for general metric $g$. Then the stress energy tensor is equal to

$$T_{\mu\nu} = -g_{\mu\nu}\mathcal{L} + 2\frac{\delta \mathcal{L}}{\delta g^{\mu\nu}}.$$  

(2.16)

Using the Lagrangian (2.13) we get

$$T_{\mu\nu} = -g_{\mu\nu}\mathcal{L} + 2\frac{\delta \mathcal{L}}{\delta g^{\mu\nu}}.$$  

(2.17)

---

3Generally not all tachyon marginal profiles obeying (2.10) imply $B = \text{const}$. On the other hand we will see that for exponential tachyon profile that obeys (2.10) the expression $B$ is equal to one. For more detailed discussion of this issue, see [52].
and the the dilaton source \( J_\Phi = \frac{\delta S}{\delta \Phi} \)

\[
J_\Phi = -\frac{2A \sqrt{-ge^{-2\Phi} T^2 (4 - \partial_\mu \Phi g^{\mu\nu} \partial_\nu \Phi)} \sqrt{B}}{(1 + e^{-2\Phi} T^2 (4 - \partial_\mu \Phi g^{\mu\nu} \partial_\nu \Phi))^2} + \]

\[+ \partial_\mu \left[ \frac{2A \sqrt{-ge^{-2\Phi} T^2 g^{\mu\nu} \partial_\nu \phi \sqrt{B}}}{(1 + e^{-2\Phi} T^2 (4 - \partial_\mu \Phi g^{\mu\nu} \partial_\nu \Phi))^2} \right] + \]

\[+ A \sqrt{-ge^{-2\Phi} (4T^2 + g^{\mu\nu} \partial_\mu T \partial_\nu T - 2T g^{\mu\nu} \partial_\mu T \partial_\nu \Phi)} \]

\[
\frac{(1 + e^{-2\Phi} T^2 (4 - \partial_\mu \Phi g^{\mu\nu} \partial_\nu \Phi)) \sqrt{B}}{2} + \partial_\mu \left[ \frac{A \sqrt{-ge^{-2\Phi} T g^{\mu\nu} \partial_\nu T}{(1 + e^{-2\Phi} T^2 (4 - \partial_\mu \Phi g^{\mu\nu} \partial_\nu \Phi))^2} \sqrt{B} \right].
\]

(2.18)

These expressions will be useful in the next section when we use the action (2.15) for the effective field theory description of the tachyon dynamics in two dimensions.

3. Toy model of two dimensional string theory

In this section we will formulate simple toy model of two dimensional bosonic string theory. The starting point of this model is the generalisation of the tachyon effective action (2.15) to the case of general metric \( g_{\mu\nu} \) and dilaton \( \Phi \) as was performed in the end of the previous section

\[
S_T = -A \int d^2x \sqrt{-g} \frac{\sqrt{-g}}{(1 + e^{-2\Phi} T^2 (4 - g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi)) \times} \times \sqrt{1 + e^{-2\Phi} (4T^2 + g^{\mu\nu} \partial_\mu T \partial_\nu T - 2T g^{\mu\nu} \partial_\mu T \partial_\nu \Phi)} .
\]

(3.1)

The action for metric and the dilaton field is

\[
S_{g,\Phi} = - \int d^2x \sqrt{-ge^{-2\Phi} (16 + R + 4g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi)} .
\]

(3.2)

The variation of the action \( S = S_{g,\Phi} + S_T \) with respect to \( g^{\mu\nu} \) gives the equation of motion for \( g^{\mu\nu} \)

\[
e^{-2\Phi} \left( G_{\mu\nu} - 2g_{\mu\nu} \nabla^2 \Phi + 2 \nabla_\mu \nabla_\nu \Phi + 2g_{\mu\nu} (\nabla \Phi)^2 - 8g_{\mu\nu} \right) = T^T_{\mu\nu}
\]

(3.3)

and the variation with respect to \( \Phi \) gives

\[
\sqrt{-ge^{-2\Phi}} \left[ 32 + 2R - 8g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \right] + 16e^{-2\Phi} \partial_\mu \left[ \sqrt{-gg^{\mu\nu} \partial_\nu \Phi} \right] = J_\Phi,
\]

(3.4)

where

\[
T^T_{\mu\nu} = \frac{\delta S_T}{\delta g^{\mu\nu}}, J_\Phi = \frac{\delta S_T}{\delta \Phi}.
\]

(3.5)
The explicit form of the components of the stress energy tensor and dilaton source is given in (2.17) and in (2.18).

Let us consider standard linear dilaton background configuration

$$\Phi = V x, g\mu\nu = \eta\mu\nu,$$ \hspace{1cm} (3.6)

where $x^0 = t, x^1 = x$. Without inclusion of the backreaction of $T$ the equation of motion for dilaton and metric implies that $V^2 = 4$. On the other hand we know that the exact solution of the equation of motion that arises from (3.1) is for the ansatz (3.6) equal to

$$T = \mu e^{\beta x}, -\beta^2 + 2\beta V = 4 \Rightarrow \beta = 2.$$ \hspace{1cm} (3.7)

Now we must ask the question whether there is any backreaction of the tachyon on the dilaton and metric. First of all, for the ansatz (3.6) and (3.7) the dilaton current $J_\Phi$ is equal to

$$J_\Phi = \left[6\beta V - 4V^2 - 2\beta^2\right] e^{-2\Phi} T^2 = 0.$$ \hspace{1cm} (3.8)

This result implies that the nonzero tachyon condensate does not modify the dilaton equation of motion and hence ansatz (3.6), (3.7) is solution of the equation of motion for dilaton and tachyon. As a last step we must show that this ansatz also solves the equation of motion for metric. In order to do this we will calculate the stress energy tensor evaluated on the ansatz (3.6), (3.7) and we find

$$T^{T}_{00} = A, T^{T}_{01} = 0, T^{T}_{11} = A(-1 + 4\mu^2).$$ \hspace{1cm} (3.9)

However we would like to have such a toy model of two dimensional string theory that has the background corresponding to the conformal Liouville field theory (3.6), (3.7) as its exact solution. In order find such an action we suggest to introduce additional term into the tachyon effective action that does not have impact on the equation of motion for the dilaton and tachyon as far as two dimensional theory is considered.

More precisely, let us consider following term

$$S_\Lambda = -\int d^2 x \sqrt{-g} \frac{\Lambda}{1 + kT^2 e^{-2\Phi} (4 - \partial_\mu \Phi g^{\mu\nu} \partial_\nu \Phi)},$$ \hspace{1cm} (3.10)

where unknown constants $\Lambda, k$ should be determined from the condition that total stress energy tensor vanishes. As the first step we must confirm that the variation of this term with respect to $T$ and $\Phi$ vanishes for the ansatz (3.6), (3.7). The variation with respect to $T$ gives

$$\frac{\delta S_\Lambda}{\delta T} = 2 \int d^2 x \sqrt{-g} \frac{\Lambda e^{-2\Phi} kT (4 - \partial_\mu \Phi g^{\mu\nu} \partial_\nu \Phi)}{(1 + kT^2 (4 - \partial_\mu \Phi g^{\mu\nu} \partial_\nu \Phi))^2} = 0,$$ \hspace{1cm} (3.11)
where we have used \( V_\mu V^\mu = 4 \). In the same way one can show that
\[
\frac{\delta S_\Lambda}{\delta \Phi} = -2 \int d^2 x \sqrt{-g} \frac{\Lambda k e^{-2\Phi} (4 - \partial_\mu \Phi g^{\mu\nu} \partial_\nu \Phi)}{(1 + k T^2 (4 - \partial_\mu \Phi g^{\mu\nu} \partial_\nu \Phi))^2} + 
\]
\[
+ 2 \int d^2 x \partial_\mu \left[ \frac{\sqrt{-g} k T^2 e^{-2\Phi} g^{\mu\nu} \partial_\nu \Phi}{(1 + k T^2 (4 - \partial_\mu \Phi g^{\mu\nu} \partial_\nu \Phi))^2} \right] = 0 ,
\]
(3.12)
where the first term vanishes for \( V_\mu V^\mu = 4 \) and the second term is equal to zero since the expression in the bracket is constant. Finally let us calculate the variation of (3.10) with respect to \( g^{\mu\nu} \). Since the action has the form \( S = - \int d^2 x \sqrt{-g} \mathcal{L}_\Lambda \) we get the components of the stress energy tensor as
\[
T^{\Lambda}_{\mu\nu} = -g_{\mu\nu} \mathcal{L}_\Lambda + 2 \frac{\delta \mathcal{L}_\Lambda}{\delta g^{\mu\nu}} =
\]
\[
= -g_{\mu\nu} \frac{\Lambda}{1 + k T^2 e^{-2\Phi} (4 - g^{\mu\nu} \partial_\lambda \Phi \partial_\nu \Phi)} + 2 \frac{e^{-2\Phi} k T^2 \Lambda \partial_\mu \Phi \partial_\nu \Phi}{(1 + T^2 e^{-2\Phi} k (4 - g^{\mu\nu} \partial_\lambda \Phi \partial_\nu \Phi))^2}
\]
(3.13)
that for (3.6) and (3.7) are equal to
\[
T_{00} = \Lambda , T_{11} = -\Lambda + 2k \Lambda \mu^2 V^2 = -\Lambda + 8k \Lambda \mu^2 .
\]
(3.14)
We see that for \( \Lambda = -\Lambda , k = \frac{1}{2} \) these terms precisely cancel the contribution from the tachyon stress energy tensor.

To conclude, we have found two dimensional effective action \( S = S_T + S_\Lambda + S_{\Phi,g} \) that has the background (3.6), (3.7) as its exact solution even if we take into account the backreaction of the tachyon on metric and dilaton.

Next question is whether our toy model has another exact solution. For example we could hope to find an analogue of the matrix cosmology solutions [30, 31, 32]. To begin with let us consider following ansatz for the tachyon, dilaton and metric
\[
T = \lambda \epsilon^{\beta_\mu \beta_\nu} , -\beta_\mu \eta^{\mu\nu} \beta_\nu + 2\beta_\mu \eta^{\mu\nu} V_\nu = 4 \\
\Phi = V_\mu x^\mu , V_\mu V^\mu = 4 , g^{\mu\nu} = \eta^{\mu\nu} .
\]
(3.15)
Since \( T \) obeys (2.10) and also for this ansatz \( B = 1 \) it follows that \( T \) solves the equation of motion that arises from (3.1) and (3.10) if we take also account the values of \( g \) and \( \Phi \) given in (3.13). One can also show that the dilaton source is zero for (3.15)
\[
J_\Phi = 2A \partial_\mu [e^{-2\Phi} T^2 \eta^{\mu\nu} V_\nu] - A \partial_\mu [e^{-2\Phi} T^2 \beta_\mu \eta^{\mu\nu}] - A \partial_\mu \left[ e^{-2\Phi} T^2 \beta_\mu \eta^{\mu\nu} V_\nu \right] =
\]
\[
= 2A e^{-2\Phi} T^2 [-\beta_\mu \beta_\mu + 2\beta_\mu V^\mu - V_\mu V^\mu] = 2A e^{-2\Phi} T^2 [4 - V_\mu V^\mu] = 0
\]
(3.16)
and consequently the linear dilaton background is not affected by the nonzero condensate of the tachyon.

On the other hand we can expect that not all values of $V, \beta$ are allowed when we demand the vanishing of the stress energy tensor $T_{\mu\nu} = T^T_{\mu\nu} + T^\Lambda_{\mu\nu}$. To see this note that diagonal components of $T_{\mu\nu}$ are equal to

$$T_{00} = 1 + e^{-2\Phi}T^2 (2V^2_0 + \beta^2_0 - 2\beta_0 V_0) - 1 - V^2_0 e^{-2\Phi}T^2 = e^{-2\Phi}T^2 (\beta^2_0 - 2\beta_0 V_0 + V^2_0) ,
$$

$$T_{11} = -1 + e^{-2\Phi}T^2 (2V^2_1 + \beta^2_1 - 2\beta_1 V_1) + 1 - V^2_1 e^{-2\Phi}T^2 = (V^2_1 + \beta^2_1 - 2\beta_1 V_1)e^{-2\Phi}T^2 .
$$

(3.17)

We see that the diagonal components of the stress energy tensor vanish when

$$\beta_\mu = V_\mu$$

(3.18)

that clearly obeys the condition for tachyon given in (3.13)

$$-\eta^{\mu\nu}\beta_\mu\beta_\nu + 2\eta^{\mu\nu}\beta_\mu V_\nu = -V_\mu V^\mu + 2V_\mu V^\mu = 4 .
$$

(3.19)

As a last check we confirm that the off-diagonal components of the stress energy tensor are equal to zero too

$$T_{01} = T_{10} = e^{-2\Phi}T^2 (2V_0 V_1 + \beta_0 \beta_1 - (\beta_0 V_1 + V_0 \beta_1)) - V_0 V_1 e^{-2\Phi}T^2 = 0 .
$$

(3.20)

In other words the flat metric is solution of the equation of motion. We must however stress that this exact solution is different from the solutions presented in [30] since in this paper the pure spatial dependent dilaton was considered and the tachyon profile for $x \to -\infty$ has the form $T \sim xe^{2x}$. To see whether such a tachyon profile is exact solution of the action (2.15) let us consider an ansatz

$$T = (b + a_\mu x^\mu)e^{\beta_\mu x^\mu}
$$

(3.21)

together with flat metric and linear dilaton background

$$g_{\mu\nu} = \eta_{\mu\nu} , \Phi = V_\mu x^\mu , V_\mu V^\mu = 4 .
$$

(3.22)

Now we demand that the tachyon profile (3.21) obeys (2.10) which implies

$$\partial_\mu [e^{-2\Phi}\eta^{\mu\nu}\partial_\nu T] = -4T \Rightarrow
$$

$$T[\beta_\mu\beta^\mu - 2V_\mu\beta^\mu + 4] + 2e^{\beta_\mu x^\mu} a_\mu (V^\mu - \beta^\mu) = 0 \Rightarrow V_\mu = \beta_\mu
$$

(3.23)

and also

$$B = 1 + e^{-2\Phi}[(4T^2 + \eta^{\mu\nu}\partial_\mu T \partial_\nu T - 2e^{-2\Phi}TV_\mu \eta^{\mu\nu}\partial_\nu T) +
$$

$$+2T a_\mu \eta^{\mu\nu}[\beta_\mu - V_\mu]e^{\beta x} + a_\mu a^\mu e^{2\beta x}] = 1 + a_\mu a^\mu
$$

(3.24)
using the background configuration (3.22). Then it is clear that the tachyon profile (3.21) is solution of the equation of motion that arises from the action (3.1) if we also take into account (3.22). On the other hand one can easily show that \( J_\Phi \) is nonzero for general \( a_\mu \) and hence generates source for the dilaton. For example, for spatial dependent dilaton and tachyon with \( \Phi = 2x, T = (b + ax)e^{2x} \) one finds following dilaton source

\[
J_\Phi = \frac{2A(ax + b)a[4a^2 + 2 - \sqrt{1 + a^2}]}{\sqrt{1 + a^2}} - \frac{a^2A}{\sqrt{1 + a^2}} \tag{3.25}
\]

that diverges at asymptotic regions \( x \to \pm \infty \) and consequently induces large back-reaction on the dilaton. In the same one can show that the tachyon profile (3.21) induces nonzero components of the stress energy tensor that implies that background (3.22) cannot be solution of the equation of motion. In summary, the tachyon profile (3.21) even if it is solution of the tachyon equation of motion in flat spacetime and in the linear dilaton background induces large backreaction on dilaton and metric and hence its meaning in the context of our model is unclear.

4. Analysis of fluctuation

In this section we will study the fluctuations around the classical solution in two dimensional string theory. From the analysis performed in case of the tachyon effective action for unstable D-branes [36, 49] it is known that the effective action of the type (2.13) is suitable for the description of the tachyon dynamics close to the marginal profile. This conclusion implies that it is not quiet correct to perform standard analysis of fluctuations when we split the general field \( T \) into the part obeying the classical equation of motion \( T_c \) and the fluctuation part \( t \) as \( T = T_c + t \) and we also presume that fluctuations are small with respect to the classical part \( T_c \). In order to study the fluctuations around the classical solution we will rather consider field \( T \) in the form

\[
T = t(x^\mu)T_c, T_c = \lambda e^{\beta_\mu x^\mu}, \beta_\mu = V_\mu. \tag{4.1}
\]

For such a field we get

\[
B = 1 + e^{-2\Phi} \left( 4T_c^2 + \eta^{\mu\nu} \partial_\mu T \partial_\nu T - 2T \eta^{\mu\nu} \partial_\mu T V_\nu \right) = 1 + e^{-2\Phi} \left[ T_c^2(4T_c^2 + \eta^{\mu\nu} \partial_\mu T_c \partial_\nu T_c - 2\eta^{\mu\nu} \partial_\mu T_c \partial_\nu T_c V_\nu) + T_c^2(\eta^{\mu\nu} \partial_\mu t \partial_\nu t - 2\eta^{\mu\nu} \partial_\mu t V_\nu) \right. + \left. 2T_c \eta^{\mu\nu} \partial_\mu T_c \partial_\nu t \right] = 1 + e^{-2\Phi} T_c^2[t \eta^{\mu\nu} \partial_\mu t \partial_\nu t + t \eta^{\mu\nu} \partial_\mu t (\beta_\nu - V_\nu)] = 1 + \lambda^2 \eta^{\mu\nu} \partial_\mu t \partial_\nu t \tag{4.2}
\]

After including the contribution from the term (3.10) that is equal for the linear dilaton background to \(-A\) we obtain the action for fluctuation modes around the
classical solution \( T_c \) in the form

\[
S_t = -A \int d^2 x \left( \sqrt{1 + \lambda^2 \eta^{\mu\nu} \partial_{\mu} t \partial_{\nu} t} - 1 \right).
\]  

(4.3)

The equation of motion that arises from (4.3) is

\[
\partial_{\mu} \left[ \frac{\eta^{\mu\nu} \partial_{\nu} t}{\sqrt{1 + \lambda^2 \eta^{\mu\nu} \partial_{\mu} t \partial_{\nu} t}} \right] = 0 \Rightarrow \\
\frac{\partial_{\mu} \eta^{\mu\nu} \partial_{\nu} t}{\sqrt{1 + \lambda^2 \eta^{\mu\nu} \partial_{\mu} t \partial_{\nu} t}} - \lambda^2 \frac{\partial_{\mu} [\eta^{\rho\sigma} \partial_{\rho} t \partial_{\sigma} t] \eta^{\mu\nu} \partial_{\nu} t}{2(1 + \lambda^2 \eta^{\mu\nu} \partial_{\mu} t \partial_{\nu} t)^{3/2}} = 0
\]

(4.4)

For the plane-wave mode \( t_k = e^{ik_{\mu} x^\mu} \) we obtain from (4.4)

\[
\begin{align*}
- \frac{t_k k_{\mu} k_{\nu} \eta^{\mu\nu}}{\sqrt{1 - \lambda^2 \eta^{\mu\nu} k_{\mu} k_{\nu} t_k^2}} + \lambda^2 \frac{\partial_{\mu} [k_{\rho} k_{\kappa} \eta^{\rho\kappa} t_k^2] \eta^{\mu\nu} i k_{\nu} t_k}{2(1 - \lambda^2 \eta^{\mu\nu} k_{\mu} k_{\nu} t_k^2)^{3/2}} &= 0 \\
- \frac{t_k k_{\mu} k_{\nu} \eta^{\mu\nu}}{\sqrt{1 - \lambda^2 \eta^{\mu\nu} k_{\mu} k_{\nu} t_k^2}} - \lambda^2 \frac{k^2 k_{\rho} k_{\kappa} \eta^{\rho\kappa} t_k^3}{(1 - \lambda^2 \eta^{\mu\nu} k_{\mu} k_{\nu} t_k^2)^{3/2}} &= 0 \Rightarrow k_{\mu} k_{\nu} \eta^{\mu\nu} = 0
\end{align*}
\]

(4.5)

so we get the condition that the fluctuation mode is massless which agrees with standard analysis of particle spectrum in two dimensional string theory. However we must stress one important point. As is well known from the open string case \[36, 49\] the tachyon effective action of type (2.9) is presumed to correctly describe the tachyon dynamics in the vicinity of the marginal tachyon profile only. Then one can argue that the fluctuations modes \( t \) should have small momenta \( k_{\mu} \ll 1 \) or equivalently \( \partial_{\mu} t \ll 1 \). In order to describe fluctuations modes with larger derivatives one should consider more general form of the tachyon effective action where terms with derivatives of higher order are included.

5. Conclusion

In this paper we have suggested the field theory effective action for closed string tachyon that has the marginal tachyon profile as its exact solution. Since generally the tachyon marginal perturbation comes with connection with the nontrivial dilaton we have studied this action in the linear dilaton background. We have obtained this action by direct generalisation of the analysis performed in \[52\]. Then we have mainly focused on the two dimensional string theory. We have seen that in two dimensions the analysis simplifies considerably. Moreover, after inclusion of the constant term into the tachyon effective action we have got the action for metric, dilaton and tachyon that has the linear dilaton background with spacelike dilaton, flat spacetime
metric and with marginal tachyon profile as its exact solution. We would like to stress that this is the background that corresponds to the exact Liouville conformal field theory. We mean that this is very interesting property of this model. We have also found another exact solution of this combined system however they meaning is unclear since they contain time-dependent dilaton and tachyon fields. According to our remark given above this background should corresponds to some form of time-like Liouville field theory which is not completely understood at present [33, 35]. We have also found exact solution of the tachyon effective action that has the same form as the tachyon profile that plays significant role in the establishing the correspondence between spacetime two dimensional string theory and its dual matrix model [53]. Unfortunately we have found that this background generates nonzero dilaton source that diverges for $x \to \pm \infty$ and hence cannot be considered as an exact solution of our toy model of two dimensional string theory.

Let us mention some open problems that deserve to be studied. It is clear that our model can be used for description of type 0A and 0B theories [56, 57, 58, 59, 60, 61]. We mean that would be interesting to study whether our model allows more general solution with nontrivial metric and dilaton together with nonzero tachyon and find their possible relations to the solutions of two dimensional string theory that were found in the past 4. It would be also interesting to study this model in $D$-dimensional space-times with $D > 2$ where the tachyon potential does not vanish. We hope to return to these problems in future.

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