Colour transparency in hadronic basis

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Abstract

The dynamics of nuclear transparency in hard nuclear reactions is studied by an expansion of the correlator of the hard scattering operator on a hadronic basis. Colour transparency appears as an effect of interference between the amplitudes corresponding to the final state interaction of different intermediate baryonic states. We find that colour transparency occurs even at moderate momentum transfers if one chooses the missing momentum of the struck nucleon by appropriate kinematics. In general, the role of the Fermi-motion is found to be crucial; without it, any effect of anomalous nuclear transparency would be impossible.

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The concept of colour transparency [1-5] has attracted great interest in the past few years (see, for example, [6-10]). This concept is based essentially on two ideas:

i) high-momentum transfer nuclear reactions select from the hadron wave function the “compact” component, whose size is smaller than the average size of the hadron;

ii) since pointlike colourless states do not interact in QCD, this compact state will undergo small initial- and final-state interactions with the nuclear medium.

The property of colour transparency is a direct consequence of QCD as a gauge theory and it should manifest itself at high enough energies and momentum transfers. However, very important things are still unclear.

Which energy is “high enough” to enter into the colour transparency domain? How the transition between regimes of normal nuclear attenuation and colour transparency occurs? Simple answers to these questions are still lacking.

Indeed, the only experiment [11] dedicated to the study of colour transparency in \((p, \, 2p)\) reaction, has demonstrated an unexpected oscillatory pattern of the ratio of nuclear to hydrogen cross section. This is in a striking contradiction with theoretical expectations, based on the naive picture of a compact state expanding in the nuclear medium after the hard interaction has occurred. Several attempts to find out a reason of the oscillations inside of the mechanism of hard \(pp\) scattering were made [7,12-14]. However, it is yet unclear whether or not colour transparency should manifest itself at all in the kinematical conditions of experiment [11]. The more detailed, dynamical treatment of the process is thus needed.

To get an insight into the problem, let us note firstly that all the essential information about the space-time evolution of a hadronic state produced by the hard process is contained in the (euclidian: \(\tau^2 = -x^2\)) correlator of the hard scattering operator \(\hat{J}\) inside a proton:

\[
\Pi_J(\tau) = \langle p | T\{\hat{J}(x)\hat{J}(0)\} | p \rangle.
\]  

(1)

This correlator can be represented as a sum of contributions arising from different baryonic states. At small distances the correlator is composed by a great number of baryonic excitations. On the other hand, in the large distance limit the behaviour of the correlator \(\sim \exp(-m\tau)\) is governed by the mass \(m\) of the ground baryonic state.
(proton). The proper formation time $\tau_{form}$ of the proton thus can be defined as a scale at which the correlator approaches this asymptotic behaviour [15] and can be roughly estimated as $\tau_{form} \sim (m_* - m)^{-1}$, where $m_*$ is the mass of the lowest baryonic excitation with proton quantum numbers. The colour transparency can manifest itself if the baryonic state produced by hard scattering interacts with the nuclear medium before it has turned into the proton.

The effect of nuclear medium on the correlator (1) is to attenuate each baryonic state component as well as to cause transitions between them. As we shall demonstrate, this latter effect is the one that ensures the colour transparency property in hadronic basis.

The final state interaction with nuclear medium $\hat{V}$ is soft in the sense that the time scale $\tau_{int}$ at which the correlator

$$\Pi_{V}(\tau) = \langle p | T\{\hat{V}(x)\hat{V}(0)\} | p \rangle$$

(2)

reaches its asymptotics is large: $\tau_{int} > \tau_{form}$. This is true for all the phenomenological mean field nuclear potentials, since their depth $V$ does not exceed the mass spacing between the proton and the first baryonic levels: $V < (m_* - m) \sim 300$ MeV. Therefore, only transitions between the proton and the first baryonic excitation in the intermediate state are effective in the nuclear medium. In this case a simple and economic description can be made in hadronic basis. The dynamics of the process can therefore be described by a system of two coupled equations.

To put this idea on a quantitative basis, we restrict ourselves to the consideration of two baryonic states, namely the proton and its excitation with the same quantum numbers. The reason for this is the following. In vacuum only baryonic states with the same quantum numbers can mix in the correlator (1). The nuclear medium can, in general, cause transitions between baryonic states with different quantum numbers; however it is well known (see, for example, reviews [16,17]) that at high energies diffractive hadronic interactions are dominated by the Pomeron scalar exchange which again favours transitions between states with the same quantum numbers. In practical evaluations we have used the mass of the lowest state with proton quantum numbers, i.e. of the Roper resonance $N(1440)$. 

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Probably the most interesting feature of the approach described above is that it helps to understand the fundamental importance of Fermi-motion for colour transparency.

Let us illustrate this point in detail. Consider a hypothetical nucleus where nucleons have fixed zero three-momentum, i.e. no Fermi-motion exists. The four-momentum of nucleons in the lab is therefore simply \( p_0 = (m_N, 0) \). Suppose also that the kinematics of the process allows one to fix the transferred momentum \( q \). In this case the invariant mass of the state produced by hard scattering is strictly determined: \( s = s_0 = (p_0 + q)^2 \). Substituting this constant value of \( s_0 \) into the correlator (1) one finds that it is composed by only one hadronic state of mass \( m_0 = \sqrt{s_0} \) (if it exists). This state obviously will be attenuated in the nuclear medium in a usual way since its wave function is of the normal hadronic size: colour transparency cannot appear.

The existence of Fermi-motion smears the distribution in the nucleon momentum and therefore causes an intrinsic uncertainty in the invariant mass of the produced state. Let us consider a hard scattering producing a baryonic state \( | k > \) at fixed transferred momentum \( q \) from a nuclear bound proton. The square modulus of the invariant amplitude, summed over the residual nucleus final states, reads

\[
|< k | \hat{J}(q^2) | i >|^2 = Z \int d^3 k \left| g_k(q^2) \right|^2 \frac{m_k \Gamma_k}{[m_k^2 - (k + q)^2]^2 + m_k^2 \Gamma_k^2} n(k),
\]

where \( n(k) \) is the momentum distribution of protons in the nucleus, defined as an integral of the nuclear spectral function \( S(k, E_f) \) over the residual nucleus excitation energies

\[
n(k) = \int dE_f S(k, E_f);
\]

\( Z \) is the number of protons in the nucleus, and \( g_k(q^2) \) is the elementary amplitude of \( | k > \) production on hydrogen. It is clear from eq. (3) that in the presence of Fermi-motion the invariant mass of the state produced by hard scattering is not strictly fixed even at fixed momentum transfer. Owing to the Fermi-motion, at each value of \( q \) several terms in the spectral density contribute to the correlator (1).

However, at low \( q \) the Fermi-motion is still unable to allow for a composition of a sufficiently coherent wave packet. At large momentum transfers the situation changes;
the uncertainty in the invariant mass of the produced state becomes larger than a characteristic mass spacing between physical hadronic states, and a coherent compact wave packet can be produced.

The excited components of the wave packet will then undergo multiple diffractive transitions in the nucleus, accompanied by the longitudinal momentum transfer

$$q_L \simeq \frac{m_k^2 - m^2}{2p}.$$  \hspace{2cm} (5)

The cancellation of the corresponding amplitudes with the amplitude of proton elastic rescattering inside the nucleus will cause colour transparency.

We have thus come to the conclusion that the Fermi-motion plays the fundamental role in exclusive knock-out reactions, allowing for the very existence of colour transparency.

Our treatment, as we shall demonstrate, leads to some new results. In particular, we clarify the fundamental role of Fermi-motion in colour transparency.\footnote{While this paper was in preparation, we became aware of a preprint by Jennings and Kopeliovich [18], which appeared simultaneously with our previous work [19]. In spite of the difference in formalisms we use, the similarity of conclusions is amazing.}

The dynamics of the final state can be described, in the eikonal approximation, by the system of coupled equations:

$$\left( \frac{\partial}{\partial z} - ik_n \right) \Psi_n = \frac{1}{2ik_n} \sum_{l=1,2} V_{nl} \Psi_l, \quad n = 1, 2,$$

$$\vec{r} \equiv z\hat{n} + \vec{b}.$$  \hspace{2cm} (6)

The wave functions $\Psi_1$ and $\Psi_2$ are the proton and the $N^*$ states with the same energy and direction of motion $\hat{n}$, but different mass and absolute values of momentum:

$$E = \sqrt{m^2 + k_2^2} = \sqrt{m^2 + k_1^2}, \quad k_1 \equiv p.$$  \hspace{2cm} (7)

The effective optical potential $\{V_{nm}\}$ describes their interactions with the residual (A–1) nucleus. It induces transitions between the two nucleon states ($1 \to 2$ and $2 \to 1$) as well as elastic interactions (we neglect here spontaneous decay of $N^*$ inside the nucleus,
which is justified at high energies). The momentum transfer in the $2 \to 1$ process is

$$q_2 \equiv k_1 - k_2 = p - \sqrt{p^2 - (m^* - m^2)},$$

(8)

with necessarily $p^2 > m^* - m^2$. We define $\Psi_n(\vec{r}, \vec{r}_o) \ (n = 1, 2)$ as the hadronic state produced by the hard interaction with the momentum transfer $\vec{q}$ at the point $\vec{r}_o$. We require $\Psi_2(\vec{r}_o) = B_n \ (n = 1, 2)$, with $|B_1|^2 + |B_2|^2 = 1$. The outgoing elastic-channel wavefunction $\tilde{\Psi}(\vec{r})$ is the coherent sum of all the contributions from every $\vec{r}_o$ inside the nucleus, with relative phase given by the exponent $e^{i\vec{q}_o \cdot \vec{r}}$, weighted by the bound state function $\phi_0(\vec{r}_o)$. For $z \gg R$ (nuclear radius) eqs. (6) imply

$$\Psi_1(\vec{r}, \vec{r}_o) \equiv \Psi[(z, \vec{b}), (z_o, \vec{b}_o)] \simeq f(\vec{b}, \vec{b}_o, z_o) e^{i\beta z}.$$  

(9)

Then the cross section for the process will be proportional to $|I|^2$, where

$$I(z) = \int d^3 r_o e^{i\vec{q}_o \cdot \vec{r}_o} \phi_0(\vec{r}_o) \Psi_1[(z, \vec{b}), (z_o, \vec{b}_o)]$$

(10)

and $|I|^2$ is independent of $z$ for $z$ larger than the nuclear potential range.

The shape of the functions $V_{ij}$ is a critical point. At high energies for $V_{11}$ we have the Glauber expression

$$\frac{V_{11}(z, \vec{b})}{2i k_1} = \frac{-2\pi i A}{k_1} f_{11}(0) \rho(z, \vec{b}) \simeq -\frac{A}{2} \sigma_{tot} \rho(z, \vec{b}),$$

(11)

where $A$ is the nuclear mass number, $\rho$ is the nuclear density normalized to 1, and $f_{11}(0)$ is the elastic forward transition amplitude; for $\sigma_{tot}$ we have used the value $\sigma_{tot} = 40$ mb. Similarly $V_{22}/V_{11} = \sigma_{2tot}/\sigma_{1tot}$. Since $\sigma_{2tot}$ is unknown, we assume $V_{22} = V_{11}$.

For $V_{12}$ a first guess could be $V_{12}(z, \vec{b}) \equiv V_{11}(z, \vec{b}) f_{12}(0)/f_{11}(0)$. In homogeneous nuclear matter this potential would be $z$-independent, allowing for solutions of the kind const.$\cdot\exp[(ik - b)z]$. This does not agree with the change of momentum $k_2 \to k_1$ during rescattering. Neglecting deflection and assuming axial symmetry we require

$$<1, k_1 | V(b)| 2, k_2> = \frac{f_{12}(0)}{f_{11}(0)} <1, k_1 | V(b)| 1, k_1>.$$  

(12)

Thus we obtain

$$<1, z | V(b)| 2, z'> \equiv V_{12}(z, b) \delta(z - z') = e^{i(k_1 - k_2)z} V_{11}(z, b) \delta(z - z') \frac{f_{12}(0)}{f_{11}(0)}.$$  

(13)
The coefficients $B_n$ depend in a critical way on the detailed kinematics of the hard scattering. As far as their modulus is concerned we can write the general expression as

$$|B_i|^2 = \int d^3k |g_i(t)|^2 R_i(s)n(k). \quad (14)$$

In eq. (14) the variable $s \equiv (q + k)^\mu(q + k)_\mu$ is the squared mass of the intermediate state particle, and $R_i(s)$ is the weight for producing the baryonic state $i$ (not far from the resonance peak),

$$R_i(s) = \frac{m_i \Gamma_i}{(s - m_i^2) + m_i^2 \Gamma_i^2}. \quad (15)$$

In particular for the elastic channel $\Gamma \to 0$ and $R_p(s) = \pi \delta(s - m_p^2)$. The function $g_i(t)$ is a vertex coupling dependent on the momentum transfer $t$. At high $t$ the quark counting rules [20,21] state, on a general ground, that this factor should only depend on the number of valence quarks participating in the interaction. For every baryon ground or excited state we have the same coupling $g(t)$. At lower energies the same result can be obtained within the nonrelativistic quark model [8]. Since only the ratio $B_2/B_1$ is important here we can assume

$$|g_1(t)|^2 = |g_2(t)|^2. \quad (16)$$

The quantities $B_1$ and $B_2$ defined above are functions of the missing momentum

$$\vec{p}_m \equiv \vec{p} - \vec{q}. \quad (17)$$

In the special case of parallel kinematics: $\vec{p} || \vec{q}$. Then $p_m = p - q$ and

$$s - m_1^2 = 2q(p_m - k_z) + p_m^2 - k^2 + m_p^2 - m_i^2 \simeq 2q(p_m - k_z) + m_p^2 - m_i^2. \quad (18)$$

In numerical calculations we use the exact formula, but the last approximate expression inserted in eqs. (15) and (14) makes it clearer the role of events with given missing momentum in exclusive $(e, e'p)$ experiments:

$$|B_1|^2 \propto \int d^3k n(k) \frac{m_i \Gamma_i}{[2q(p_m - k_z) + m_p^2 - m_i^2]^2 + m_i^2 \Gamma_i^2}. \quad (19)$$
Incidentally we note that at the leading order in $p_m/q$, $|B_i|^2 = |B_i(p_m,q)|^2 = |B_i(p-q,q)|^2$ only depends on $\vec{p}_m$ via its component along $\vec{q}$, $(p_m)_z$. The values of $|B_i|^2 \equiv |B_i|^2$ for $q = 4, 8, 40$ GeV are depicted as functions of $p_m$ in fig. The parameters chosen for the Roper resonance are $m_\ast = 1.44$ GeV, $\Gamma_\ast = 350$ MeV.

The integral in eq. (19) attains its maximum when $k_z \equiv 0$, i.e. $p_m = (m_\ast^2 - m_p^2)/2q > 0$. This behaviour is particularly remarkable for low $q$. Of course, for $q \to \infty$ we have $(m_\ast^2 - m_p^2)/2q \to 0$ and the $|B_i|^2$ distribution tends to look like the $|B_p|^2$ distribution, symmetric around $p_m = 0$. This has the strongest consequence on transparency.

We have assumed that the $B$ coefficients have the same phase, which can be strictly justified only in the case of $ep$ interactions and neglecting the finite width of the resonance. This last assumption is not crucial, however.

Numerical calculations have been performed for the $^{27}$Al target nucleus. Then one has to consider bound states up to $l = 2$. To represent them we use harmonic oscillator wavefunctions. The matter density in eq. (11) has been assumed to be an isotropic Wood-Saxon function with radius $1.1 \cdot A^{1/3}$ fm and diffuseness 0.5 fm.

The obtained values of transparency are plotted in fig. 2 versus $p_m$ (in units of the Fermi momentum $p_f$) for $q = 4, 8$ and 40 GeV, and in fig. 3 versus $q$ for $p_m/p_f = 0$, $0.35p_f$ and $0.7p_f$.

Figs. 2 and 3 can be related to fig. 1 where the probability $|B_s|^2$ of the $N_\ast$ excitation is plotted for the same values of $m_\ast$ and $q$ and in the same $p_m$ range of fig. 3.

It is easy to see that the maxima of transparency in fig. 2 occur at practically the same values of $p_m$ as the maxima of the corresponding curves in fig. 1. So an obvious conclusion is that maxima of transparency are attained for those values of $q$ and $p_m$ where $|B_s|^2$ is maximum.

The non trivial conclusion is that only for $p_m \sim 0$ transparency is attained at asymptotic momentum transfer. At non-asymptotic $q$ transparency maxima manifest themselves at other (positive) values of missing momenta. These maxima are strictly related to the maxima of the probability $|B_s|^2$ of exciting baryonic resonance in the hard scattering vertex.
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**Figure captions**

Fig. 1. The values of the squared amplitude $|B_\pi|^2$ of producing a Roper resonance as a function of the missing momentum $p_m$ in units of the Fermi momentum ($p_f = 234$ MeV). Dotted, dashed and solid curves for $q = 4, 8, 40$ GeV, respectively.

Fig. 2. The transparency coefficient as a function of missing momentum $p_m$ in units of the Fermi momentum ($p_f = 234$ MeV). Dotted, dashed and solid curves for $q = 4, 8, 40$ GeV, respectively.

Fig. 3. The transparency coefficient as a function of the momentum transfer $q$. Dotted, dashed and solid curves for $p_m/p_f = 0.0, 0.35, 0.7$.

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