Need for Two Vectorlike Families in SUSY Composite Models†

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Abstract

Within the context of a viable and economical SUSY preon model, two vector–like families \( Q_{L,R} = (U, D, N, E)_{L,R} \) and \( Q'_{L,R} = (U', D', N', E')_{L,R} \) with masses of order 1 TeV, one of which is a doublet of \( SU(2)_L \) and the other a doublet of \( SU(2)_R \), have been predicted to exist together with the three observed chiral families. The existence of these two vector–like families turns out to be crucial especially for an explanation of the inter–family mass–hierarchy and therefore for the SUSY preon model itself. This paper is devoted to a detailed study of the expected masses, mixings and decay modes of the vector–like fermions. Including QCD renormalization–effects, the masses of the vector–like quarks are expected to lie in the range of 500 GeV to about 2.5 TeV, while those of the vector–like leptons are expected to be in the range of 200 GeV to 1 TeV.
1. Generalities

Recently, it has been shown [1-3] that the idea that quarks, leptons, and Higgs bosons are composites and that their constituents possess local supersymmetry can be realized in the context of a viable and economical preon model which has many attractive features. These include:

- Understanding of the quark and lepton mass matrix, including KM-Matrix and CP-violation [2];
- Understanding family replication [3];
- Cosmological features [4].

In this talk, I would like to concentrate on a specific feature of this preon model, namely the appearance of two families of vector–like quarks and leptons \( Q_{L,R} \) and \( Q'_{L,R} \) in the TeV range [5]. Let me first recall the essential role which these two families play in providing an explanation of the inter–family mass hierarchy. Since \( Q_L \) and \( Q_R \) couple symmetrically to \( SU(2)_L \times U(1)_Y \) gauge bosons, the mass term \((\bar{Q}_L Q_R + h.c)\) and likewise \((\bar{Q}'_L Q'_R + h.c)\) preserve \( SU(2)_L \times U(1)_Y \). This, however, infers that the oblique parameters \( S, T \) and \( U \) [6], do not receive contributions from these vector–like families, in the leading approximation. As a result, the prevailing set of measurements of the electroweak parameters, despite their precision, are not sensitive enough to the existence of vector–like families [7] - unlike the case of a fourth chiral family which is slightly disfavored by the measurement of the \( S, T \) and \( U \– \)parameters. One therefore tends to believe that if new families beyond the three chiral ones are yet to be found, they are more likely to exhibit vectorial rather than chiral couplings to \( W_L \)’s and \( W_R \)’s.

The model under consideration is based on a set of masseless chiral superfields, each belonging to the fundamental representation of the metacolor gauge symmetry \( SU(N) \). The superfields carry also flavor-color quantum numbers according to the gauge group \( SU(4)_c \times SU(2)_L \times SU(2)_R \). It is assumed that the metacolor force becomes strong and confining at a scale \( \Lambda_M \simeq 10^{11} \) GeV, with the following effects:

- Three light chiral families of composite quarks and leptons \( (q^i_{L,R})_{i=1,2,3} \) and two vector-like families \( Q_{L,R} \) and \( Q'_{L,R} \), coupling vectorially to \( W_L \)’s and \( W_R \)’s, are formed;
- Supersymmetry-breaking condensates are formed; they include the metagaugino condensate \( \langle \bar{\lambda} \cdot \lambda \rangle \) and the matter fermion-condensates \( \langle \bar{\psi}^a \psi^a \rangle \). Noting that, within the class of models under consideration, the index theorem prohibits a dynamical breaking of supersymmetry in the absence of gravity [8], so the formation of these condensates must need the collaboration between the metacolor force and gravity. As a result, each of these condensates is expected to be damped by one power of
\[(\Lambda_M/M_{Pl}) \simeq 10^{-8}\] relative to \(\Lambda_M\) \[9\]:

\[
\langle \vec{\lambda} \cdot \vec{\lambda} \rangle = \kappa_\lambda \Lambda_M^3 (\Lambda_M/M_{Pl}) ,
\]

\[
\langle \bar{\psi}^a \psi^a \rangle = \kappa_{\psi a} \Lambda_M^3 (\Lambda_M/M_{Pl})
\]  (1)

Here, the indices \(a\) are running over color and flavor quantum numbers. The condensates \(\langle \bar{\psi}^a \psi^a \rangle\), break not only SUSY but also the electroweak symmetry \(SU(2)_L \times U(1)_Y\) therefore giving mass to the electroweak gauge bosons. The coefficients \(\kappa_\lambda\) and \(\kappa_{\psi a}\), apriori, are expected to be of order unity within a factor of ten (say), although \(\kappa_\lambda\) is expected to be bigger than \(\kappa_{\psi a}\)’s, typically by factors of 3 to 10, because the \(\bar{\psi}\)’s are in the fundamental and the \(\lambda\)’s are in the adjoint representation of the metacolor group.

- Furthermore, supersymmetry-preserving condensates, which however break the gauge group \(SU(4)_c \times SU(2)_L \times SU(2)_R\) to the low-energy gauge group \(SU(3)_c \times SU(2)_L \times U(1)_Y\) are assumed to form as well. They provide a large superheavy Majorana mass to the right-handed neutrinos and may play an interesting role in the discussion of inflationary models \[4\].

Now, the vector-families \(Q_{L,R}\) and \(Q'_{L,R}\) acquire relatively heavy masses through the metagaugino condensate \(\langle \vec{\lambda} \cdot \vec{\lambda} \rangle\) of order \(\kappa_\lambda \Lambda_M (\Lambda_M/M_{Pl}) \sim 1\text{ TeV}\) which are independent of flavor and color. But the chiral families \(q^i_{L,R}\) acquire masses primarily through their mixings with the vector-like families \(Q_{L,R}\) and \(Q'_{L,R}\) which are induced by \(\langle \bar{\psi}^a \psi^a \rangle\). This is because the direct mass-terms cannot be induced through two-body condensates. Thus, ignoring QCD corrections and higher order condensates for a moment, the Dirac-mass matrices of all four types - i.e., up, down, charged lepton, and neutrino - have the form:

\[
M^{(o)} = \begin{pmatrix}
q^i_L & Q_L & Q'_L \\
O & X_{kf} & Y_{kC} \\
Y'^i_{kC} & \kappa_\lambda & O \\
X'^i_{kC} & O & \kappa_\lambda
\end{pmatrix}.
\]  (2)

Here, the index \(i\) runs over three families, \(f,c\) denotes flavor- or color-type condensate, and the quantities \(X, Y, X'\) and \(Y'\) are column matrices in the family-space and have their origin in the detailed vertex structure of the corresponding preonic diagrams. They are expected to be numbers of order \(\simeq 1\). As a result, the Dirac mass-matrices of all four types have a natural see-saw structure.

2. Discussion of the Mass Matrix

A detailed discussion of the mass matrix can be summarized as follows:

- In the absence of electroweak corrections, left-right symmetry and flavor-color independence of the metacolor force guarantee (a) \(X = X'\) and \(Y = Y'\), and (b) the same \(X, Y\) and \(\kappa_\lambda\) enters into up, down, lepton, or neutrino-type matrix. This
results in an enormous reduction of parameters. The whole $5 \times 5$ mass-matrices of the four types is essentially determined by just six effective parameters.

- Since the rank of the matrix is four, one family (the electron-family), is strictly massless to all orders. The essential parameters, however, can be adjusted to give reasonable masses for the second and third chiral family. Inclusion of electroweak corrections (five more parameters of order 10%) do not change the rank, however, they give a possibility to fit the KM-Matrix elements.

- Also, $CP$-violating phases can be rotated away in the above matrix. The inclusion of electroweak corrections does not alter this. In order to gain mass for the electron family, a small contribution from 4-body condensates has to be included into the mass entries for the chiral quarks. These contributions are damped by $(\Lambda_M/M_P)^2$ and yield masses of order $\approx 1MeV$. Interesting enough, they also provide a nonzero phase in the KM Matrix. Therefore the interesting conclusion in this model is, that the mass of the first chiral family and $CP$-Violation has the same origin.

We therefore can summarize that the above fermion mass matrix including radiative corrections on the metacolor scale as well as renormalization group corrections and the appearance of small direct mass entries can give a reasonable understanding of all fermion masses, mixing angles, and $CP$-phases. Naturally, the key ingredient in this prediction is the seesaw mechanism, provided by the existence of the two set of vector–like quarks. The prediction of the masses, coupling constants and decay rates of these quarks and their verification by experiments is therefore a crucial test of the model itself.

### 3. Properties of the heavy vector–like quarks

Heaving fixed our parameters by the known chiral families of quarks and leptons, we now start to make detailed predictions about the behavior of the $Q$-family (the $SU(2)_L$-doublet) and the $Q'$-family (which is an $SU(2)_L$ singlet). Obviously, the off-diagonal entries in the mass matrix yield also a mixing between those two families in second order seesaw, yielding four mass eigenstates $(U_1, D_1, E_1, N_1)_{L,R}$ (mostly weak doublets), and $(U_2, D_2, E_2, N_2)_{L,R}$ (mostly singlets). Furthermore, we have to calculate renormalization group corrections for the running masses from $\Lambda_M$, where the above matrix was defined, down to about 1.5TeV, which is the expected range of the masses of the heavy quarks. In case of $QCD$, this correction is as large as a factor 2.39, whereas the electroweak correction factors vary between 1.001 – 1.23 for the different mass values. Putting things together, we arrive at the following list of mass values for a representative set of parameters:

\[
\begin{align*}
\kappa_\lambda &\approx 575 \text{ GeV}, \quad \kappa_u/\kappa_\lambda = 1/6, \quad \kappa_d/\kappa_u \approx 1/40 \\
\kappa_r/\kappa_\lambda &\approx 1/3, \quad \kappa_l/\kappa_\lambda \approx 1/3 
\end{align*}
\]

(3)

This yields

\[
\begin{align*}
(m_t, m_b, m_\tau)_{1.5 \text{ TeV}} &\simeq (135, 3.4, 1.5) \text{ GeV} \\
(m_t, m_b, m_\tau)_{\text{phys}} &\simeq (157, 4.7, 1.7) \text{ GeV}
\end{align*}
\]

(4)
While the precise values of the masses and the mixing angles depend upon the specific choice of the parameters, a few qualitative features would still remain which are worth noting:

- The masses of the heavy neutrinos are Dirac masses. We rely here in a scenario [10] where the heavy neutrinos do not acquire a superheavy Majorana mass, in contrast to the chiral righthanded neutrinos $\nu^R_i$.

- The smallness of the mixing angles from second order seesaw implies that $D_1$ and even $U_1$ are mostly composed of $Q$–fermions which are $SU(2)_L$–doublets while $U_2$ and $D_2$ are mostly composed of $Q'$–fermions which are $SU(2)_R$–doublets. This is important for their decay modes.

- Note that the pair $U_1$ and $D_1$ are nearly degenerate to within about 10-30 GeV, so also the pair $N_1$ and $E_1$, and to a lesser extent the pair $N_2$ and $E_2$. But the $(U_1, D_1)$ pair is substantially heavier, by about a few hundred GeV, than the pair $(U_2, D_2)$. Similarly the $(N_1, E_1)$ pair is heavier by about 100 GeV than the pair $(N_2, E_2)$. This is because the $(U_1, D_1)$ and also the $(N_1, E_1)$ pair receive enhancement due to a $SU(2)_L$–renormalization factor, which is, however, absent for the $(U_2, D_2)$ and $(N_2, E_2)$–pairs.

- Given this mass–pattern, we see that $U_1 \to D_1 + W$ and likewise $N_1 \to E_1 + W$ are forbidden kinematically, while decays such as $U_1 \to D_2 + W$, $U_1 \to U_2 + Z$, $D_1 \to U_2 + W$, $D_1 \to D_2 + Z$ and possibly $U_2 \to D_2 + W$ are kinematically allowed.

The most dominant decay modes (with rates $\Gamma$ up to 100 GeV) are the decays $U_1 \to U_2 + Z$, $U_2 \to b + W^+$, $D_1 \to U_2 + W^-, t + W^-$, and $D_2 \to t + W^-$. For the leptons, the decays of $E_1, E_2$ into $\tau + Z$ are dominant, but highly suppressed, indicating a small decay rate, whereas the decays of the neutrinos into chiral leptons and $Z$ or $W$ is appreciable.

Production of the heavy quarks in pairs by hadronic colliders at SSC and LHC energies has been studied in a number of papers [11]. These studies typically yield production cross sections of $3 \times 10^{-4}$ nb at $\sqrt{s} = 40$ TeV. Assuming that a future version of the SSC will be built one day in the near future, the production cross section noted above would lead to about $2.5 \times 10^4$ events per year for $m_U = 1$ TeV, with a luminosity of $10^{33} cm^{-2} s^{-1}$.

In summary, two vector–like families, not more not less, with one coupling vectorially to $W_L$’s and the other to $W_R$’s (before mass–mixing), with masses of order $\simeq 1 TeV$, constitute a *hall–mark* and a crucial prediction of our SUSY preon model [1-3]. There does not seem to be any other model including superstring–inspired models of elementary quarks and leptons which have a good reason to predict two such complete vector–like
families with masses in the TeV range.

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