Rule Extracting based on MCG with its Application in Helicopter Power Train Fault Diagnosis

M Wang¹,², N Q Hu¹,² G J Qin¹
¹School of Mechatronic Engineering and Automation, National University of Defense Technology, ChangSha, Hunan, 410073, China
²Corresponding author: E-mail address: hnq@nudt.edu.cn (N Q Hu), wm198063@yahoo.com.cn (M Wang)

Abstract. In order to extract decision rules for fault diagnosis from incomplete historical test records for knowledge-based damage assessment of helicopter power train structure. A method that can directly extract the optimal generalized decision rules from incomplete information based on GrC was proposed. Based on semantic analysis of unknown attribute value, the granule was extended to handle incomplete information. Maximum characteristic granule (MCG) was defined based on characteristic relation, and MCG was used to construct the resolution function matrix. The optimal general decision rule was introduced, with the basic equivalent forms of propositional logic, the rules were extracted and reduction from incomplete information table. Combined with a fault diagnosis example of power train, the application approach of the method was present, and the validity of this method in knowledge acquisition was proved.

1. Introduction

As a key but not redundancy component of helicopter, the structure damage of power train will catastrophically affect helicopter’s safety in flight, so it is important to assess damage level of power train structure on ground before flight. But, in many helicopter building users and repair depots, though the running condition of power train can be monitored, some important physical quantity (vibration, rotational speed, etc) can be tested and recorded by measurement instrument, the fault diagnosis are mostly “man-made”, that is, repairmen and experts diagnose power train fault, and then assess damage level of structure according to their experiment. A key problem hindered development of knowledge-based power train fault diagnosis system is lack of diagnosis knowledge, that is, there is no valid method to extract decision rule for fault diagnosis, which is most common expression form of knowledge, from historical test records. As we know, these records contain test information and diagnosis conclusion of every operation of damage assessment of power train structure on ground, and can be displayed as a diagnosis information table, which is the accumulation of repairmen and experts’ experiment.

Granular Computing (GrC), as a new intelligent computing theory based on partition of problem concepts [1][2], is an important method to study fuzzy and coarse information, and a useful tool to extract decision rule from information table. But in engineering practice, information that historical
records afford is always incomplete, and traditional GrC methods [3][4] is invalid to handle incomplete information table. A method that can directly extract optimal general decision rules from incomplete information table based on GrC was proposed in this paper, and combined with a given information table for fault diagnosis of power train, the application approach of the method was presented.

2. Incomplete information table for fault diagnosis

2.1. Basic concept of information table

Definition 1: An information table \( S \) can be denoted by a quadruple: \( S = (U, A, \{V_a | a \in A\}, \{I_a | a \in A\}) \), in which \( U \) is a finite nonempty set of objects, named universe; \( A \) is a finite nonempty set of attributes; \( V_a \) is a nonempty set of values for \( a \in A \) and \( I_a : U \rightarrow V_a \) is an information function.

If \( A \) can be divided into two finite nonempty sets, \( C \) and \( D \), that is: \( A = C \cup D \) and \( C \cap D = \emptyset \), then \( S \) can be called a decision table, where \( C \) is a set of condition attributes and \( D \) is a set of decision attributes. Corresponding, \( V_a = V_c \cup V_d \) and \( I_a = I_c \cup I_d \). In this paper, if no special explanation, information table is pointed to decision table

Definition 2: An information table is incomplete, if one or more attributes \( a \in A \), \( I_a : U \rightarrow \text{Null} \), where \text{Null} denotes an unknown value.

In this paper, we assumed that “Null” value only appear on condition attribute, in the process of fault diagnosis of power train, if an object \( x \in U \), \( I_a(x) = \text{Null} \), then we deemed that \( x \) is an incorrect object and deleted it.

2.2. Semantic analysis of incomplete information

In the process of damage assessment of power train structure on ground, because of the complex structure of power train, lots of vibration sources, mass test data and variable dynamic environment, there are many uncertain factors that may disturb the acquisition of test information for fault diagnosis. But for an information table, we can divide the unknown attribute value into two semantemes according to the analysis of the reason that why it is unknown:

(1) “do not care” semanteme. Because of the limitation of measurement and record resource, the repairmen and experts cannot measure all vibration test points of power train in an operation of damage assessment. So they select several key physical quantities to test and extract several most important condition features according to their experiment, and then the unselected test points and unrecorded condition features can be considered to be “do not care”. In this paper, the unknown attribute value of “do not care” semanteme is denoted by “*” in the information table.

(2) “lost” semanteme. Because of the disturbance of uncertain factors (such as the abnormal of measurement instrument, data transmission facilities and data recorder, the lowest of signal-to-noise ratio, et al), test information aren’t recorded or have to be deleted because of they are useless. Then the unknown attribute value is considered to be “lost”, and is denoted by “?” in the information table.

2.3. Information table for fault diagnosis of power train

An information table for fault diagnosis of power train structure was given as Table.1, which include 29 test records obtained from a helicopter repair depot’s central database, and is divided into 14
objects of universe. The value denoted by $k$ in the second column is the number of records belongs to the same object. Condition attribute set $C = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\}$, where $c_i$ is the average rotational speed, $c_2$ is the root-mean-square (RMS) of torque, $c_3$, $c_4$, $c_5$, $c_6$, and $c_7$ are the displacement amplitude of six vibration signals on power train (there are many other test information in records, but this paper focus on introduce the method of knowledge acquisition and its application in power train, so we choices the eight attributes as example). the principle of discretization is: after normalized, if $c_i \in [0, 0.5]$, then $c_i = 0$, else $c_i = 1$; if $c_i \in [0, 0.8]$, then $c_i = 0$, else $c_i = 1$; if $c_i \in [0, 0.35]$, then $c_i = 0$, else $c_i = 1$, $i = 3 \sim 8$. Decision attribute set $D = \{d\}$, $V_d = \{H, F_1, F_2, F_3\}$, which represent different damage levels of power train structure (no damage, mildly damage, moderate damage and severely damage), which are determined by repairmen and experts.

Table 1. Incomplete information table for fault diagnosis of power train.

| Object | $k$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ | $c_7$ | $c_8$ | $d$ |
|--------|-----|------|------|------|------|------|------|------|------|----|
| $x_1$  | 2   | *    | 1    | 0    | ?    | 0    | 0    | *    | 1    | $H$|
| $x_2$  | 3   | *    | 1    | *    | 0    | 0    | *    | 0    | 1    | $H$|
| $x_3$  | 1   | 1    | *    | 1    | 1    | 0    | *    | 1    | $H$|
| $x_4$  | 4   | 0    | *    | 1    | 0    | *    | 0    | *    | 1    | $H$|
| $x_5$  | 3   | 1    | 1    | *    | 1    | 1    | 0    | *    | $H$|
| $x_6$  | 2   | *    | 1    | 1    | 1    | 0    | *    | 0    | 1    | $H$|
| $x_7$  | 2   | 0    | *    | *    | 0    | *    | 0    | 1    | 1    | $F_1$|
| $x_8$  | 1   | 1    | 0    | 1    | 1    | 1    | *    | 0    | *    | $F_1$|
| $x_9$  | 3   | *    | 0    | 0    | 0    | *    | 1    | 1    | $F_1$|
| $x_{10}$| 1   | 1    | 0    | *    | 1    | 1    | 0    | 0    | ?    | $F_2$|
| $x_{11}$| 3   | 0    | *    | 0    | 1    | 0    | *    | 1    | *    | $F_2$|
| $x_{12}$| 2   | 1    | 0    | 0    | 1    | *    | 1    | *    | 1    | $F_2$|
| $x_{13}$| 1   | *    | *    | 0    | 0    | 0    | 0    | 1    | 1    | $F_1$|
| $x_{14}$| 1   | *    | 1    | *    | 0    | 1    | 0    | *    | 0    | $F_3$|

3. Granule of incomplete information

An important step in GrC is granulation of universe [5], which can be done based on decision logic language (DL-language) [6] if information table is complete. In the DL-language, an atomic formula is given by $(a, v)$, where $a \in A$ and $v \in V$, an object $x \in U$ is described by atomic formula under the form:

$$x = (a, v) \iff I_a(x) = v$$

(1)

For an attribute $a \in A$, if two objects $x$ and $y$ have the same value on $a$, namely, $I_a(x) = I_a(y)$, they may be assigned to the same granule. So, for a given value $v \in V$, one can obtain a granule with respect to the atomic formula of the DL-language [7], as follows:

$$m(a, v) = \{x \in U \mid I_a(x) = v\}$$

(2)

It consists of all objects whose value on attribute $a$ equals to $v$, and may be interpreted as the granule defined based on an equality constraint. But with an incomplete information table, since there are unknown value, the equality constraint is not tenable, $m(a, v)$ needs to be redefined according to semantic analysis of unknown attribute value.
For unknown attribute value of “do not care” semanteme, since the test points measured and the condition features recorded are selected by repairmen and experts according to their experiment, then from the view of repairmen and experts, information afforded by the unselected test points and condition features can be considered having relatively small contribution for fault diagnosis. So the unknown attribute value of “do not care” semanteme can be considered to be equal to any value in \( V_a \), which means if \( I_a(x) = * \), \( x \) is similar with all other objects in \( U \) on attribute \( a \). But for unknown attribute value of “lost” semanteme, since it is caused by uncertain factors and failures, it cannot be replaced with any value in \( V_a \), so, if \( I_a(x) = ? \), \( x \) is dissimilar with any other objects in \( U \) on attribute \( a \).

According to the analysis above, with an incomplete information table, an object \( x \in U \) should be described by the atomic formula \( (a, v) \) as expression (3):

\[
x = (a, v) \iff I_a(x) = \neg v \lor I_a(x) = *
\]

The granule with respect to the atomic formula \( (a, v) \) is redefined as below:

\[
m(a, v) = \{ x \mid I_a(x) = \neg v \lor I_a(x) = * \}
\]

This granule consists of all objects whose value on attribute \( a \) is equal to \( v \) or in “do not care” semanteme.

4. Maximum characteristic granule

4.1. Extend granule based on characteristic relation

Since equivalence relation is not tenable in incomplete information table, it is difficult for decision rule extraction based on traditional GrC methods, but we extend the granule model by introducing new relation, as people do for Rough Set (RS) model [8][9][10][11]. In this paper, we introduced characteristic relation [10][12].

**Definition 3:** with an incomplete information table, characteristic relation \( K \) is defined as below:

\[
y K x \iff \forall a \in A (I_a(x) = I_a(y) \lor I_a(y) = *, I_a(x) \neq ?)
\]

With the definition 3, one can obtain a granule based on characteristic relation:

\[
K_a(x) = \{ y \mid y K x, y \in \text{Uanda} \in A \}
\]

\( K_a(x) \) is a granule that is composed of all objects which are indistinguishable with \( x \) on attribute set \( A \) based on characteristic relation, and can be calculated through disjunction of atomic formula \( (a, v) \) for all attributes \( a \in A \) for which \( I_a(x) \) is specified and \( I_a(x) = v \):

\[
K_a(x) = \bigcap_{a \in A, I_a(x) = v} \{ y \in U \mid y \in m(a, v) \}
\]

Based on formula (7), for condition attribute set \( C \) of Table.1, all granules based on characteristic relation \( K \) are as below:

\[
K_c(x_1) = \{ x_1, x_2, x_3, x_4, x_5, x_6 \} ; K_c(x_2) = \{ x_2, x_4, x_6 \} ; K_c(x_3) = \{ x_3, x_5, x_6, x_10 \} ; K_c(x_4) = \{ x_2, x_4, x_6, x_7 \} ;
\]
Definition 4: based on characteristic relation, a two-tuples set of $U$, namely $\mathcal{K}(A)$, is defined as below:

$$\mathcal{K}(A) = \{(x, y) \in U \times U \mid y \mathcal{K} x\}$$

The relation of $\mathcal{K}(A)$ and $\mathcal{K}_a(x)$ is given by (11):

$$\mathcal{K}(A) = \{(x, y) \in U \times U \mid y \mathcal{K}_a(x)\}$$

For condition attribute set $C$ of Table.1, $\mathcal{K}(C)$ was expressed as (12):

$$\mathcal{K}(C) = \left\{ (x_1, x_2), (x_1, x_3), (x_1, x_4), (x_1, x_5), (x_1, x_6), (x_1, x_7), (x_1, x_8), (x_1, x_9), (x_1, x_{10}), (x_1, x_{11}), (x_1, x_{12}), (x_1, x_{13}), (x_1, x_{14}), (x_2, x_3), (x_2, x_4), (x_2, x_5), (x_2, x_9), (x_2, x_{10}), (x_2, x_{11}), (x_2, x_{12}), (x_2, x_{13}), (x_2, x_{14}), (x_3, x_4), (x_3, x_5), (x_3, x_6), (x_3, x_7), (x_3, x_8), (x_3, x_9), (x_3, x_{10}), (x_3, x_{11}), (x_3, x_{12}), (x_3, x_{13}), (x_3, x_{14}), (x_4, x_5), (x_4, x_6), (x_4, x_7), (x_4, x_8), (x_4, x_9), (x_4, x_{10}), (x_4, x_{11}), (x_4, x_{12}), (x_4, x_{13}), (x_4, x_{14}), (x_5, x_6), (x_5, x_7), (x_5, x_8), (x_5, x_9), (x_5, x_{10}), (x_5, x_{11}), (x_5, x_{12}), (x_5, x_{13}), (x_5, x_{14}), (x_6, x_7), (x_6, x_8), (x_6, x_9), (x_6, x_{10}), (x_6, x_{11}), (x_6, x_{12}), (x_6, x_{13}), (x_6, x_{14}), (x_7, x_8), (x_7, x_9), (x_7, x_{10}), (x_7, x_{11}), (x_7, x_{12}), (x_7, x_{13}), (x_7, x_{14}), (x_8, x_{10}), (x_8, x_{11}), (x_8, x_{12}), (x_8, x_{13}), (x_8, x_{14}), (x_9, x_{11}), (x_9, x_{12}), (x_9, x_{13}), (x_9, x_{14}), (x_{10}, x_{11}), (x_{10}, x_{12}), (x_{10}, x_{13}), (x_{10}, x_{14}), (x_{11}, x_{12}), (x_{11}, x_{13}), (x_{11}, x_{14}), (x_{12}, x_{13}), (x_{12}, x_{14}), (x_{13}, x_{14}) \right\}$$

4.2. Maximum characteristic granule

Now, we defined a special information granule, named maximum characteristic granule (MCG), which describes a tiny granulation of the universe based on characteristic relation. In the granule, any two objects, $x$ and $y$, satisfy characteristic relation $\mathcal{K}$ and the relation is reflexive, that is, if $y \mathcal{K} x$ is tenable, $x \mathcal{K} y$ is also tenable.

Definition 5: in an incomplete information table, a granule $Y$ is called characteristic granule about attribute set $A$, if $\forall x, y \in Y, (x, y) \in \mathcal{K}(A)$. If there is not a granule $X$, that $Y \subseteq X$ and $X$ is a characteristic granule of $U$, then $Y$ is called a MCG on attribute set $A$.

According to the definition 5, all objects in $Y$ are indistinguishable on attribute set $A$ based on characteristic relation. $Y$ would be used to construct resolution function and to define the generalized decision function in the next section. For condition attribute set $C$ of Table.1, all MCGs were as below:

$$Y_1 = \{x_1\}; Y_2 = \{x_2, x_4, x_6\}; Y_3 = \{x_3, x_5\}; Y_4 = \{x_4, x_6\}; Y_5 = \{x_7, x_9, x_{11}\};$$
$$Y_6 = \{x_3, x_8\}; Y_7 = \{x_5, x_{10}\}; Y_8 = \{x_{11}\}; Y_9 = \{x_{12}\}; Y_{10} = \{x_{14}\}.$$
5.1. Generalized decision function

**Definition 6:** generalized decision function is a set of decision attribute value, with an incomplete information table, for \( \forall x \in U \), generalized decision function based on characteristic relation is defined as below:

\[
\hat{\delta}_c(x) = \{I_y(x) \mid y \in K_c(x) \}
\]

(11)

which includes all decision classes to which \( x \) may be assigned based on usable information. Defining \( \text{card}(\cdot) \) as cardinal number of the set, if \( \text{card}(\hat{\delta}_c(x)) = 1 \), the classification of \( x \) is consistent, else the classification is inconsistent.

With definition 6, all generalized decision functions of Table.1 were given by Table.2:

| \( U \) | \( \hat{\delta}_c(x) \) | \( U \) | \( \hat{\delta}_c(x) \) | \( U \) | \( \hat{\delta}_c(x) \) |
|------|------------------|------|------------------|------|------------------|
| \( x_1 \) | \( \{H, F_1, F_2, F_3\} \) | \( x_2 \) | \( \{H\} \) | \( x_3 \) | \( \{H, F_1, F_2\} \) |
| \( x_4 \) | \( \{H, F_1\} \) | \( x_5 \) | \( \{H\} \) | \( x_6 \) | \( \{H\} \) |
| \( x_7 \) | \( \{H, F_1, F_3\} \) | \( x_8 \) | \( \{H, F_1\} \) | \( x_9 \) | \( \{F_1, F_3\} \) |
| \( x_{10} \) | \( \{H, F_1, F_3\} \) | \( x_{11} \) | \( \{F_2\} \) | \( x_{12} \) | \( \{F_2\} \) |
| \( x_{13} \) | \( \{F_1, F_2, F_3\} \) | \( x_{14} \) | \( \{F_1\} \) |

Generalized decision function can also be defined by MCG.

**Definition 7:** in an incomplete information table, if \( Y \subset U \) is a MCG on condition attribute set \( C \), the generalized decision function of \( Y \) is given by expression (12):

\[
\hat{\delta}_c(Y) = \{I_x(x) \mid x \in Y\}
\]

(12)

With definition 7, all generalized decision functions of MCGs of Table.1 were given by Table.3:

| \( Y \) | \( \hat{\delta}_c(Y) \) | \( Y \) | \( \hat{\delta}_c(Y) \) | \( Y \) | \( \hat{\delta}_c(Y) \) |
|------|------------------|------|------------------|------|------------------|
| \( Y_1 \) | \( \{H\} \) | \( Y_2 \) | \( \{H\} \) | \( Y_3 \) | \( \{H\} \) | \( Y_4 \) | \( \{H, F_1\} \) |
| \( Y_5 \) | \( \{F_1, F_3\} \) | \( Y_6 \) | \( \{H, F_3\} \) | \( Y_7 \) | \( \{H, F_2\} \) | \( Y_8 \) | \( \{F_2\} \) |
| \( Y_9 \) | \( \{F_3\} \) | \( Y_{10} \) | \( \{F_3\} \) |

5.2. Optimal generalized decision rule

In this paper, production rule [15] was used to describe incomplete information table, every object was expressed as the form of \( t \rightarrow s \), in which \( t \) was called condition part of rule and \( s \) was called decision part. The condition part and decision part can be expressed as disjunction / conjunction of atomic formula \( (a, v) \), as follows:

\[
t = \land(a, v) \\
s = \lor(d, w)
\]

(13)

With the atomic formula \( (a, v) \) and using connection words (such as: \( \land, \lor, \) etc) to do finite logic operation, one can obtain two granules, as given by expression (14):
\[ |I| = \bigcap m(a, v) \]
\[ |S| = \bigcup m(d, w) \]  
(14)

\[ |I| \] includes all objects which satisfy the condition part of rule, and \[ |S| \] includes all objects which satisfy the decision part of rule.

Definition 8: with an incomplete information table, an object \( x \in U \) is deemed to uphold a rule for fault diagnosis \( t \rightarrow s \), if and only if \( x \in |I| \) and \( x \in |S| \).

Definition 9: with an incomplete information table, a rule for fault diagnosis \( t \rightarrow s \) is deemed to be a generalized decision rule based on characteristic relation, if and only if expression (15) is tenable.

\[ \{x \in U \mid K_s(x) \cap |S| \neq \emptyset \} \subseteq |I| \]  
(15)

where \( B \subseteq C \) is a condition attribute set that include all attributes in the condition part of rule.

In an incomplete information table, if a decision rule \( t \rightarrow s \) is generalized, \( t \rightarrow s \) is said to be consistent (or compatible) in information table. If the conjunction normal form of all decision rules obtained from information table is generalized, the information table is said to be consistent (or compatible), else to be inconsistent (or incompatible). The consistency of an information table means one can do consistent fault decision, based on the information afforded by the table. An incomplete information table is consistent, if and only if \( \forall x \in U, \, card(\hat{c}_r(x)) = 1 \).

In an incomplete information table, a rule \( t \rightarrow s \) is said to be optimal generalized, if and only if \( t \rightarrow s \) is generalized and any rule constructed by granule \( |I| \) and \( |S| \), which satisfy \( |I'| \subseteq |I| \) and \( |S'| \subseteq |S| \), is not generalized [15].

5.3. Resolution function-matrix of incomplete information table.

When extracting decision rules for fault diagnosis, we always wish the condition part of the rule to be lowest possible, to do which resolution function matrix [17] is an important tool. If the information table is complete, the resolution function matrix is a symmetrical matrix of \( N \) rows and \( N \) columns, \( N = card(U) \). Every item, marked as \( \lambda(x, y) \), is called resolution function of resolving object \( x \) and \( y \), and given by expression (16):

\[ \lambda(x, y) = \begin{cases} \{a \in C \mid f_x(x) \neq f_y(y)\} & f_x(x) \neq f_y(y) \\ \Phi & f_x(x) = f_y(y) \end{cases} \]  
(16)

But in an incomplete information table, since there are unknown attribute values in some objects and equivalence relation has been extended to characteristic relation, resolution function \( \lambda(x, y) \) is also needed to be extended. In this paper, we used maximum characteristic granule as unit to define resolution function of resolving object \( x \) and granule \( Y \).

Definition 10: with an incomplete information table, if a function \( \lambda(x, Y) \) is defined as expression (17), the function is called resolution function of resolving object \( x \) and MCG \( Y \).
The resolution function matrix is a matrix of $N$ rows and $M$ columns, where $M$ is the number of MCG.

With Definition 10, the resolution function matrix of Table.1 is given by Table.4:

|    | $Y_1$ | $Y_2$ | $Y_3$ | $Y_4$ | $Y_5$ | $Y_6$ | $Y_7$ | $Y_8$ | $Y_9$ | $Y_{10}$ |
|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| $x_1$ | $c_7$ | $c_2c_7$ | $c_2c_3c_5$ | $c_2c_4$ | $c_2c_7$ | $c_2c_4$ | $c_2c_6$ |
| $x_2$ | $c_3c_4c_5$ | $c_6$ |
| $x_3$ | $c_3$ | $c_4c_6c_7$ | $c_2c_4c_6c_7$ | $c_2$ | $c_2$ | $c_2c_4$ | $c_2$ | $c_2$ | $c_6$ |
| $x_4$ | $c_3c_4c_5c_7$ | $c_2c_4c_6c_7$ | $c_2$ | $c_2$ | $c_2c_4$ | $c_2$ | $c_2c_4$ | $c_6$ |
| $x_5$ | $c_2c_4$ | $c_7$ | $c_2c_4c_5$ | $c_2$ | $c_2c_4$ | $c_2c_4$ | $c_4$ | $c_4c_6$ |
| $x_6$ | $c_3c_4c_5c_7$ | $c_2c_4c_6c_7$ | $c_2$ | $c_2c_4$ | $c_2$ | $c_2c_4$ | $c_6$ |
| $x_7$ | $c_2c_4c_5c_7$ | $c_2c_4c_6c_7$ | $c_2$ | $c_2c_4$ | $c_2$ | $c_2c_4$ | $c_6$ |
| $x_8$ | $c_2c_4$ | $c_7$ | $c_2c_4c_5$ | $c_2$ | $c_2c_4$ | $c_2c_4$ | $c_4$ | $c_4c_6$ |
| $x_9$ | $c_3c_4c_5c_7$ | $c_2c_4c_6c_7$ | $c_2$ | $c_2c_4$ | $c_2$ | $c_2c_4$ | $c_6$ |
| $x_{10}$ | $c_2c_4c_5c_7$ | $c_2c_4c_6c_7$ | $c_2$ | $c_2c_4$ | $c_2$ | $c_2c_4$ | $c_6$ |
| $x_{11}$ | $c_2c_4$ | $c_7$ | $c_2c_4c_5$ | $c_2$ | $c_2c_4$ | $c_2c_4$ | $c_4$ | $c_4c_6$ |
| $x_{12}$ | $c_2c_4c_5c_7$ | $c_2c_4c_6c_7$ | $c_2$ | $c_2c_4$ | $c_2$ | $c_2c_4$ | $c_6$ |
| $x_{13}$ | $c_2c_4c_5c_7$ | $c_2c_4c_6c_7$ | $c_2$ | $c_2c_4$ | $c_2$ | $c_2c_4$ | $c_6$ |
| $x_{14}$ | $c_2c_4$ | $c_7$ | $c_2c_4c_5$ | $c_2$ | $c_2c_4$ | $c_2c_4$ | $c_4$ | $c_4c_6$ |

6. Extracting the optimal generalized decision rule

The decision rule for fault diagnosis of power train based on incomplete information can be obtained from the conjunctive normal form of resolution function. The conjunctive normal form is given by expression (18):

$$
\Delta(x) = \wedge \vee \lambda(x, Y)
$$

where $\wedge \lambda(x, Y)$ is a disjunction of atomic formula of all attributes in $\lambda(x, Y)$, if $\lambda(x, Y) = \Phi$, then $\wedge \lambda(x, Y) = 1$.

If $\Delta(x)$ is translated equivalently into a disjunction normal form, all reduction of object $x$ can be obtained from the minor formula of the disjunction normal form. According to the definition of minimal reduction, the minor formula with minimum cardinal number is the minimal reduction of object.
As to the incomplete information table of power train (Table.1), from Table.4, one can obtain all of the conjunctive normal forms of resolution function. Translating these conjunctive normal forms equivalently into disjunction normal forms, then calculating generalized decision rules upheld by every object from this disjunction normal forms and using production rule to express these generalized decision rules, one can obtain the set including all optimal decision rules of the whole incomplete information table. The optimal generalized decision rules that can be obtained from Table.1 were showed by Table.5.

| ID | Generalized decision rule | \( k \) | Object |
|----|---------------------------|------|--------|
| 1  | \((c_2, 0) \land (c_3, 0) \land (c_4, 1) \rightarrow (d, H)\) | 5    | \( x_2 \), \( x_3 \) |
| 2  | \((c_2, 0) \land (c_3, 0) \land (c_4, 1) \rightarrow (d, H)\) | 3    | \( x_2 \) |
| 3  | \((c_8, 1) \rightarrow (d, H) \lor (d, F_1) \lor (d, F_3)\) | 1    | \( x_3 \) |
| 4  | \((c_1, 0) \land (c_2, 1) \land (c_3, 1) \rightarrow (d, H) \lor (d, F_1)\) | 4    | \( x_4 \) |
| 5  | \((c_3, 1) \land (c_8, 0) \land (c_9, 1) \rightarrow (d, H) \lor (d, F_1)\) | 4    | \( x_4 \) |
| 6  | \((c_2, 1) \land (c_3, 1) \rightarrow (d, H)\) | 3    | \( x_5 \) |
| 7  | \((c_2, 1) \land (c_3, 1) \rightarrow (d, H)\) | 3    | \( x_5 \) |
| 8  | \((c_2, 0) \rightarrow (d, H) \lor (d, F_1) \lor (d, F_3)\) | 2    | \( x_7 \) |
| 9  | \((c_2, 0) \land (c_3, 1) \rightarrow (d, H) \lor (d, F_1)\) | 1    | \( x_8 \) |
| 10 | \((c_2, 0) \land (c_3, 0) \land (c_4, 0) \rightarrow (d, F_1) \lor (d, F_3)\) | 3    | \( x_9 \) |
| 11 | \((c_2, 0) \land (c_3, 0) \rightarrow (d, H) \lor (d, F_1) \lor (d, F_3)\) | 1    | \( x_{10} \) |
| 12 | \((c_1, 0) \land (c_4, 1) \rightarrow (d, F_2)\) | 3    | \( x_{11} \) |
| 13 | \((c_1, 0) \land (c_4, 1) \rightarrow (d, F_2)\) | 2    | \( x_{12} \) |
| 14 | \((c_3, 0) \land (c_4, 0) \rightarrow (d, F_1) \lor (d, F_2) \lor (d, F_3)\) | 1    | \( x_{13} \) |
| 15 | \((c_3, 0) \land (c_4, 0) \rightarrow (d, F_1)\) | 1    | \( x_{14} \) |

There are fifteen optimal generalized decision rules in Table.5. The value denoted by \( k \) in the third column is the number of cases that uphold the rule in the same row, and the fourth column is the objects that uphold the rule in the same row.

These optimal generalized decision rules, showed in Table.5, is the refined summarization and visualized representation of incomplete information for fault diagnosis in Table.1, and can help to make decision for pattern recognition of damage level of power train structure. These rules also make it convenience to accumulating fault knowledge of power train and improve knowledge-base.

7. Conclusion

A valid method of extracting decision rules for fault diagnosis from historical test records necessary for knowledge-based damage assessment of helicopter power train structure on ground. Aiming at the incompleteness of information afforded by historical records, this paper first did semantic analysis of incomplete information according to the factors cause information acquisition, proposed two semantemes of unknown attribute value, and defined the extended granule which including incomplete information. Then, characteristic relation was introduced, MCG was defined based on characteristic relation, and MCG was used as unit to construct resolution function matrix. Last, the optimal general decision rule was introduced, with the basic equivalent forms of propositional logic, the method of directly extract optimal general decision rules from incomplete table was proposed. An application in
the given incomplete information table of power train fault diagnosis proved the valid of the method proposed in this paper, and presented the approach of the method in engineer practice. The method proposed in this paper is suit to be wildly applied in knowledge acquisition for damage assessment of helicopter power train structure.

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