Constraints on the R-parity Violating Couplings from $B^\pm \to l^\pm \nu$ Decays

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Abstract

We derive the upper bounds on certain products of R-parity- and lepton-flavor-violating couplings from $B^\pm \to l^\pm \nu$ decays. These modes of $B$-meson decays can constrain the product combinations of the couplings with one or more heavy generation indices which are comparable with or stronger than the present bounds. And we investigate the possible effects of R-parity violating interactions on $B_c \to l\nu$ decays. These decay modes can be largely affected by R-parity violation.

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In supersymmetric extensions of the standard model, there are gauge invariant interactions which violate the baryon number ($B$) and the lepton number ($L$) in general. To prevent occurrences of these $B$- and $L$-violating interactions in supersymmetric extensions of the standard model, the additional global symmetry is required. This requirement leads to the consideration of the so-called R-parity ($R_p$). Even though the requirement of $R_p$ conservation makes a theory consistent with present experimental searches, there is no good theoretical justification for this requirement. Therefore the models with explicit $R_p$-violation have been considered by many authors [1].

In the MSSM, the most general $R_p$-violating superpotential is given by

$$W_{R_p} = \lambda_{ijk} L_i L_j E_k^c + \lambda_{ijk}' Q_i Q_j D_k^c + \lambda_{ijk}'' U_i^c D_j^c D_k^c.$$  \hspace{1cm} (1)

Here $i, j, k$ are generation indices and we assume that possible bilinear terms $\mu_i L_i H_2$ can be rotated away. $L_i$ and $Q_i$ are the $SU(2)$-doublet lepton and quark superfields and $E_i^c, U_i^c, D_i^c$ are the singlet superfields respectively. $\lambda_{ijk}$ and $\lambda_{ijk}'$ are antisymmetric under the interchange of the first two and the last two generation indices respectively; $\lambda_{ijk} = -\lambda_{jk i}$ and $\lambda_{ijk}' = -\lambda_{k i j}'$. So the number of couplings is 45 (9 of the $\lambda$ type, 27 of the $\lambda'$ type and 9 of the $\lambda''$ type). Among these 45 couplings, 36 couplings are related with the lepton flavor violation.

There are upper bounds on a single $R_p$ violating coupling from several different sources [2–5]. Among these, upper bounds from neutrinoless double beta decay [3], $\nu$ mass [4] and $K^+, t$-quark decays [1] are strong. Neutrinoless double beta decay gives $\lambda_{111}' < 3.5 \times 10^{-4}$. The bounds from $\nu$ mass are $\lambda_{133} < 3 \times 10^{-3}$ and $\lambda_{133}' < 7 \times 10^{-4}$. From $K^+$-meson decays one obtain $\lambda_{ijk} < 0.012$ for $j = 1$ and 2. These bounds from $K^+$-meson decays are basis-dependent [3,4]. Here all masses of scalar partners which mediate the processes are assumed to be 100 GeV. Extensive reviews of the updated limits on a single $R_p$-violating coupling can be found in [3].

There are more stringent bounds on some products of the $R_p$-violating couplings from the mixings of the neutral $K$- and $B$-mesons and rare leptonic decays of the $K_L$-meson, the muon and the tau [5], $B^0$ decays into two charged leptons [6], $b\bar{b}$ productions at LEP [11] and muon(ium) conversion, and $\tau$ and $\pi^0$ decays [11], semileptonic decays of $B$ mesons [12], $B \rightarrow X_s l_i^+ l_j^-$ decays [13].

In this paper, we derive the upper bounds on certain products of $R_p$ and lepton flavor violating couplings from $B^\pm \rightarrow l^\pm \nu$ decays in the minimal supersymmetric standard model (MSSM) with explicit $R_p$ violation. These modes of $B$-meson decays can constrain the product combinations of the couplings with one or more heavy generation indices. Here, we assume that the baryon number violating couplings $\lambda''$’s vanish in order to avoid too fast proton decays. Especially in the models with a very light gravitino ($G$) or axino ($\tilde{a}$), $\lambda''$ have to be very small independently of $\lambda'$ from the proton decay $p \rightarrow K^+ G$ (or $K^+ \tilde{a}$); $\lambda_{112}'' < 10^{-15}$ [4]. One can construct a grand unified model which has only lepton number non-conserving trilinear operators in the low energy superpotential when $R_p$ is broken only.
by bilinear terms of the form \( L_i H_2 \) \(^{13}\). And usually it may be very difficult to discern signals of \( B \)-violating interactions above QCD backgrounds \(^{4}\).

In the MSSM with \( R_p \), the terms in the effective lagrangian relevant for the leptonic \( B \)-meson decays are

\[
L^{\text{eff}}(b \bar{q} \rightarrow e_l \bar{\nu}_l) = -V_{qb} \frac{4G_F}{\sqrt{2}} \left[ (\bar{q} \gamma^\mu P_L b)(\bar{e}_l \gamma_\mu P_L \nu_l) - R_l (\bar{q} P_R b)(\bar{e}_l P_L \nu_l) \right],
\]

where \( P_{R,L} = \frac{1}{2}(1 \pm \gamma_5) \), \( R_l = r^2 m_e m_b^Y \), \( r = \tan \beta \), and \( m_{H^\pm} \) is the mass of the charged Higgs fields. An upper index \( Y \) denotes the running quark mass, \( \tan \beta \) is the ratio of the vacuum expectation values of the neutral Higgs fields and \( m_{H^\pm} \) is the mass of the charged Higgs fields. The first term in Eq. (2) gives the standard model (SM) contribution and the second one gives that of the charged Higgs scalars. Neglecting the masses of the electron (\( l = 1 \)) and the muon (\( l = 2 \)), the contribution of the charged Higgs scalars is zero. The contribution of the charged Higgs scalars is not vanishing only when \( l = 3 \); \( b \bar{q} \rightarrow \tau \bar{\nu}_\tau \). We neglect a term proportional to \( m_{Y_c}^2 \) for \( q = c \) since the term is suppressed by the mass ratio \( m_{Y_c}^2 / m_{Y_b}^2 \) and does not have the possibly large \( \tan^2 \beta \) factor.

In the MSSM without \( R_p \), the exchange of the sleptons and the squarks leads to the additional four-fermion interactions which are relevant for the leptonic decays of \( B \)-meson. Considering the fact that the CKM matrix \( V \) is not an identity matrix, the \( \lambda' \) terms of the Eq. (1) are reexpressed in terms of the fermion mass eigenstates as follow

\[
W_{\lambda'} = \lambda'_{ijk} \left( N_i D_j - \sum_p V_{jp}^U E_i U_p \right) D_k^c,
\]

where \( N_i \), \( E_i \), \( U_i \) and \( D_i \) are the superfields with neutrinos, charged leptons, up- and down-type-quarks and \( \lambda' \) have been redefined to absorb some field rotation effects. From Eq. (1) and Eq. (3) we obtain the effective interactions which are relevant for the leptonic decays of \( B \)-meson as follows

\[
L^{\text{eff}}_{R_p}(b \bar{q} \rightarrow e_l \bar{\nu}_n) = -V_{qb} \frac{4G_F}{\sqrt{2}} \left[ A_{ln}^q (\bar{q} \gamma^\mu P_L b)(\bar{e}_l \gamma_\mu P_L \nu_n) - B_{ln}^q (\bar{q} P_R b)(\bar{e}_l P_L \nu_n) \right],
\]

where we assume the matrices of the soft mass terms are diagonal in the fermion mass basis. Note that the operators in Eq. (4) take the same form as those of the MSSM with \( R_p \). Comparing with the SM, the above effective lagrangian includes the interactions even when \( l \) and \( n \) are different from each other. The dimensionless coupling constants \( A \) and \( B \) depend on the species of quark, charged lepton and neutrino and are given by

\[
A_{ln}^q = \frac{\sqrt{2}}{4G_F V_{qb}} \sum_{i,j=1}^{3} \frac{1}{2 m_{d_i}^2} V_{qj} \lambda'_{n3i} \lambda'_{lji},
\]

\[
B_{ln}^q = \frac{\sqrt{2}}{4G_F V_{qb}} \sum_{i,j=1}^{3} \frac{2}{m_{l_i}^2} V_{qj} \lambda_{lmi} \lambda'_{ij3},
\]
where \( l \) and \( n \) are the generation indices running from 1 to 3.

From the numerical values of \( |V_{ij}| \), we find

\[
\mathcal{A}_\text{in} = \sum_{i=1}^{3} \lambda' \left\{ 422\lambda'^*_{1i} \left( V_{ud}/0.9751 - V_{ub}/0.0035 \right) + 96\lambda'^*_{2i} \left( V_{us}/0.2215 - V_{ub}/0.0035 \right) + 1.52\lambda'^*_{3i} \right\} \left( \frac{100 \text{ GeV}}{m_{\tilde{c}_i}} \right)^2,
\]

\[
\mathcal{B}_\text{in} = \sum_{i=1}^{3} \lambda \left\{ 1689\lambda'_{13} \left( V_{ud}/0.9751 - V_{ub}/0.0035 \right) + 384\lambda'_{23} \left( V_{us}/0.2215 - V_{ub}/0.0035 \right) + 6.1\lambda'_{33} \right\} \left( \frac{100 \text{ GeV}}{m_{\tilde{c}_i}} \right)^2,
\]

\[
\mathcal{A}'_\text{in} = \sum_{i=1}^{3} \lambda' \left\{ 8.2\lambda'^*_{1i} \left( V_{cd}/0.221 - V_{cb}/0.041 \right) + 36\lambda'^*_{2i} \left( V_{cs}/0.973 - V_{cb}/0.041 \right) + 1.52\lambda'^*_{3i} \right\} \left( \frac{100 \text{ GeV}}{m_{\tilde{c}_i}} \right)^2,
\]

\[
\mathcal{B}'_\text{in} = \sum_{i=1}^{3} \lambda \left\{ 32.7\lambda'_{13} \left( V_{cd}/0.221 - V_{cb}/0.041 \right) + 144\lambda'_{23} \left( V_{cs}/0.973 - V_{cb}/0.041 \right) + 6.1\lambda'_{33} \right\} \left( \frac{100 \text{ GeV}}{m_{\tilde{c}_i}} \right)^2.
\]

(6)

Note the large numerical factors coming from the big differences between the values of the CKM matrix elements.

First, we consider \( q = u \) case. At present, the measurements of the branching ratios of the \( B^\pm \rightarrow \ell^\pm \nu \) processes give the upper bounds (at 90 % C.L.) \[16\]

\[
BR(B^- \rightarrow e^-\bar{\nu}_e) < 1.5 \times 10^{-5},
\]

\[
BR(B^- \rightarrow \mu^-\bar{\nu}_\mu) < 2.1 \times 10^{-5},
\]

\[
BR(B^- \rightarrow \tau^-\bar{\nu}_\tau) < 5.7 \times 10^{-4}.
\]

(7)

These experimental bounds are much larger than the standard model expectations; \(BR(B^- \rightarrow e^-\bar{\nu}_e)_{SM} \sim 9.2 \times 10^{-12}, BR(B^- \rightarrow \mu^-\bar{\nu}_\mu)_{SM} \sim 3.9 \times 10^{-7}\) and \(BR(B^- \rightarrow \tau^-\bar{\nu}_\tau)_{SM} \sim 8.8 \times 10^{-5}\).

If we assume that the \( R_p \)-violating interactions are dominated, the decay rate of the processes \( B^- \rightarrow e_l\bar{\nu}_n \) reads

\[
\Gamma(B^- \rightarrow e_l\bar{\nu}_n) = \frac{1}{8\pi}|V_{ub}|^2 G_F f_B^2 M_B^2 \left| \mathcal{A}'_\text{in} \frac{m_l}{M_B} - \mathcal{B}'_\text{in} \frac{1 - m_l^2}{M_B^2} \right|^2,
\]

(8)

using the PCAC (partial conservation of axial-vector current) relations

\[
\langle 0 | \bar{b}_\gamma^\mu \gamma_5 q | B_q(p) \rangle = if_{B_q} P^\mu_B,
\]

\[
\langle 0 | \bar{b}_\gamma \gamma_5 q | B_q(p) \rangle = -if_{B_q} \frac{M_{B_q}^2}{m_b + m_q} \approx -if_{B_q} M_{B_q},
\]

(9)

Since the species of the neutrinos cannot be distinguished by experiments and the \( R_p \)-violating interactions allow the different kinds of the charged lepton and the neutrino as
decay products, we should sum the above decay rates over neutrino species to compare with experimental data as follow
\[ \Gamma(B^- \rightarrow e^- \bar{\nu}) \equiv \sum_{n=1}^{3} \Gamma(B^- \rightarrow e^- \bar{\nu}_n). \] (10)

From the Eq. (8), (10) and the upper limit on the branching ratio, Eq. (7), we obtain
\[ \sum_{n=1}^{3} |B_{1n}^u|^2 < 1.5 \times 10^{-2}, \]
\[ \sum_{n=1}^{3} |B_{2n}^u|^2 < 2.1 \times 10^{-2}, \]
\[ \sum_{n=1}^{3} |B_{3n}^u - 0.337A_{3n}^u|^2 < 7.4 \times 10^{-1}, \] (11)

For numerical calculations, we used \( \tau_B = 1.6 \text{ ps} \), \( f_B = 200 \text{ MeV} \), \( m_\tau = 1.78 \text{ GeV} \) and neglected the lepton masses for \( l = e, \mu \) cases.

Under the assumption that only one product combination is not zero, we get the upper bounds on some combinations of the \( \lambda \lambda' \)- and \( \lambda \lambda' \)-type. For the product combinations of \( \lambda \lambda' \) type, we observe that the several bounds are stronger than the previous bounds and list them in the Table I. In the case of the product combinations of \( \lambda \lambda' \) type, there is no stronger bound than the previous ones. The previous bounds are calculated from the bounds on single \( R_p \)-violating coupling, see Table 1 of ref. [7].

Next, we consider \( q = c \) case. Recently charmed \( B \) meson (\( B_c \)) are observed [17]. It is expected that in the near future large data sample of \( B_c \) mesons would be available. The standard model predictions of branching ratios for \( B_c \rightarrow l \nu \) decay modes are \( BR(B_c \rightarrow e\nu)_{SM} \sim 2.5 \times 10^{-9}, \) \( BR(B_c \rightarrow \mu\nu)_{SM} \sim 1.0 \times 10^{-4}, \) \( BR(B_c \rightarrow \tau\nu)_{SM} \sim 2.6 \times 10^{-2} \). These branching ratios can be largely affected by R-parity violation. If we assume that the \( R_p \)-violating interactions are dominated, the decay rate of the processes \( B_c^- \rightarrow e_l \bar{\nu}_n \) reads
\[ \Gamma(B_c^- \rightarrow e_l \bar{\nu}_n) = \frac{1}{8\pi} |V_{cb}|^2 G_F^2 f_{B_c}^2 M_{B_c}^3 |A_{ln} m_l M_{B_c} - B_{ln}^c|^2 \left( 1 - \frac{m_l^2}{M_{B_c}^2} \right)^2. \] (12)

In Table 2, we list the combinations of couplings whose present upper limits allows the branching ratios to have the values of order of \( 10^{-2} \), assuming only one product of \( R_p \)-violating couplings is nonzero. For numerical calculations, we used \( \tau_{B_c} = 0.55 \text{ ps} \), \( f_{B_c} = 450 \text{ MeV} \), \( M_{B_c} = 6.275 \text{ GeV} \) [18].

To conclude, we have derived the more strigent upper bounds on certain products of \( R_p \) and lepton-flavor-violating couplings from the upper limits of \( B^\pm \rightarrow l^\pm \nu \) branching ratios. And we investigate the possible effects of R-parity violating interactions on \( B_c \rightarrow l \nu \) decays. These decay modes can be largely affected by R-parity violation.
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## TABLE I. Upper bounds on the magnitudes of products of couplings derived from $B \to l \nu$.

| Decay Mode | Combinations Constrained | Upper bound | Previous bound |
|------------|--------------------------|-------------|----------------|
| $B^- \to e^- \bar{\nu}$ | $\lambda_{131} \lambda'_{113}$ | $7.3 \times 10^{-5}$ | $4.9 \times 10^{-4a}$ |
| | $\lambda_{131} \lambda'_{123}$ | $3.2 \times 10^{-4}$ | $6.0 \times 10^{-4}$ |
| | $\lambda_{131} \lambda'_{223}$ | $3.2 \times 10^{-4}$ | $6.0 \times 10^{-4}$ |
| | $\lambda_{231} \lambda'_{213}$ | $7.3 \times 10^{-5}$ | $4.9 \times 10^{-4a}$ |
| | $\lambda_{231} \lambda'_{223}$ | $3.2 \times 10^{-4}$ | $5.5 \times 10^{-4}$ |
| | $\lambda_{231} \lambda'_{233}$ | $2.0 \times 10^{-2}$ | $2.0 \times 10^{-2}$ |
| | $\lambda_{231} \lambda'_{323}$ | $3.2 \times 10^{-4}$ | $5.5 \times 10^{-4}$ |
| $B^- \to \mu^- \bar{\nu}$ | $\lambda_{132} \lambda'_{113}$ | $8.7 \times 10^{-5}$ | $6.0 \times 10^{-4a}$ |
| | $\lambda_{132} \lambda'_{123}$ | $3.8 \times 10^{-4}$ | $6.0 \times 10^{-4}$ |
| | $\lambda_{132} \lambda'_{323}$ | $3.8 \times 10^{-4}$ | $6.0 \times 10^{-4}$ |
| | $\lambda_{232} \lambda'_{213}$ | $8.7 \times 10^{-5}$ | $6.0 \times 10^{-4a}$ |
| | $\lambda_{232} \lambda'_{223}$ | $3.8 \times 10^{-4}$ | $5.5 \times 10^{-4}$ |
| $B^- \to \tau^- \bar{\nu}$ | $\lambda_{123} \lambda'_{113}$ | $5.1 \times 10^{-4}$ | $6.0 \times 10^{-4a}$ |
| | $\lambda_{233} \lambda'_{213}$ | $5.1 \times 10^{-4}$ | $5.5 \times 10^{-4}$ |
| | $\lambda_{233} \lambda'_{313}$ | $5.1 \times 10^{-4}$ | $6.0 \times 10^{-4a}$ |

a: Bounds from $B \to l_i^+ l_j^- \bar{\nu}$.

## TABLE II. Maximally allowed branching ratios and the list of combinations whose present upper bounds allow the branching ratios to have the value of order of $10^{-2}$.

| Decay Mode | Combinations | Branching Ratio |
|------------|--------------|-----------------|
| $B_c^- \to e^- \bar{\nu}$ | $\lambda_{131} \lambda'_{123}$ | $1.1 \times 10^{-2}$ |
| | $\lambda_{131} \lambda'_{323}$ | $1.1 \times 10^{-2}$ |
| | $\lambda_{231} \lambda'_{223}$ | $2.2 \times 10^{-2}$ |
| $B_c^- \to \mu^- \bar{\nu}$ | $\lambda_{132} \lambda'_{123}$ | $1.0 \times 10^{-2}$ |
| | $\lambda_{232} \lambda'_{223}$ | $2.1 \times 10^{-2}$ |
| $B_c^- \to \tau^- \bar{\nu}$ | $\lambda_{233} \lambda'_{223}$ | $0.5 \times 10^{-2}$ |