Bouncing and cyclic universes in the charged AdS bulk background

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Abstract

We study bouncing and cyclic universes from an \((n + 1)\)-dimensional brane in the \((n + 2)\)-dimensional charged AdS bulk background. In the moving domain wall (MDW) approach this picture is clearly realized with a specified bulk configuration, the 5D charged topological AdS (CTAdS\(_5\)) black hole with mass \(M\) and charge \(Q\). The bulk gravitational dynamics induces the 4D Friedmann equations with CFT-radiation and exotic stiff matters for a dynamic brane. This provides bouncing universes for \(k = 0, -1\) and cyclic universe for \(k = 1\), even though it has an exotic stiff matter from the charge \(Q\). In this work we use the other of the Binetruy-Deffayet-Langlos (BDL) approach with the bulk Maxwell field. In this case we are free to determine the corresponding mass \(\tilde{M}\) and charge \(\tilde{Q}\) because the mass term is usually included as an initial condition and the charge is given by an unspecified solution to the Maxwell equation under the BDL metric. Here we obtain only bouncing universes if one does not choose two CTAdS\(_5\) black holes as the bulk spacetime. We provide a way of avoiding the exotic matter on the brane by introducing an appropriate local matter. Finally we discuss an important relation between the exotic holographic matter and Lorentz invariance violation.

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1 Introduction

Recently there has been much interest in cyclic (oscillating) and bouncing universes of brane cosmology \[1, 2, 3, 4\]. In these studies one started with the moving domain wall (MDW) \[6, 7, 8, 9\]. This is obtained by embedding of an \((n + 1)\)-dimensional brane into the \((n + 2)\)-dimensional bulk background. In this approach one has to choose a specified bulk configuration, for example, the 5D charged topological AdS (CTAdS) black hole with mass \(M\) and charge \(Q\) \[10, 11\]. Then the bulk gravitational dynamics induces the 4D Friedmann equations with CFT-radiation and exotic stiff matters for a dynamic brane \[12, 13, 14, 15, 16\]. This provides bouncing and cyclic universes, even though it has an exotic stiff matter from the black hole charge \(Q\) \[3, 17\].

Without a charge \(Q\), one finds a scenario with a CFT-radiation matter: the universe starts from the black hole singularity (big bang), crosses the horizon of the black hole, and expands up to its maximum size. And then it contracts, crosses the horizon of the black hole again, and finally collapses into the singularity (big crunch) \[18\]. On the other hand we obtain a different evolution of the universe with \(Q \neq 0\). This universe is free from singularities of big bang and big crunch and evolves with a smooth transition between a contracting phase and an eventual expanding one. The presence of charge makes it in such a way. This provides an exotic stiff matter with a negative energy density on the brane. This term dominates for small scale factor and hence induces a singular repulsive potential near the black hole singularity for the brane to be unable to access it.

In addition the presence of both mass \(M\) and charge \(Q\) induces a potential well which makes bouncing and cyclic trajectories of the brane in the bulk spacetime. The key point is that the sign of stiff matter from \(Q\) is opposite with respect to that of radiation matter from \(M\). Although this opposite sign provides bouncing and cyclic universes, we immediately confront with a difficulty that it violates the dominant energy condition (DEC): \(\rho \geq 0\); \(P = \omega \rho\) with \(|\omega| \leq 1\) \[19\]. We remind the reader that the derivation of the DEC is based on the 4D Lorentz invariance. However, it was shown that the Lorentz invariance is apparently violated for an observer on the brane moving in the charged black hole background \[20\]. Nowadays one may accept the negative energy density from the charge of the black hole as well as the violation of the DEC in the brane cosmology \[21\].
On the other hand, the BDL approach is a genuine extension to the higher dimensional (Kaluza-Klein) cosmology to take into account a local matter distribution [22, 23, 24]. An evolution of the brane is possible when the brane is fixed with respect to the extra direction $y$ and the bulk is given by a charged AdS spacetime [15]. For this purpose we have to introduce negative cosmological constants and Maxwell fields for both bulk sides. Then one finds the Friedmann equations with two unknown constants. In this case we are free to determine the corresponding mass and charge because the mass term is usually included as an integration constant with respect to a cosmic time on the brane and the charge is given by an unspecified solution to the Maxwell equation under the BDL metric. One may attempt to identify these constants as mass and charge of the black hole if one chooses a specific bulk spacetime of two CTAdS black holes as did in the MDW case.

In this work, we ask mainly of whether or not bouncing and cyclic solutions for the 4D universe can be derived from the BDL approach.

The organization of this paper is as follows. In Sec.2 we review briefly the MDW procedure in the charged topological AdS black holes. First we study the CFT-thermodynamics for a static brane located near the AdS boundary using the AdS/CFT correspondence. Then we will understand the thermodynamics of a dynamic brane in view of the bulk gravitational dynamics. It is important to note that the CFT-interpretation of radiation and stiff matter is possible only when a MDW was close to the AdS boundary unless its holographic energy density is small. Sec.3 is devoted to the BDL approach in the charged AdS background. For simplicity we use the same bulk stress-energy tensor for both sides. Here we also find the same Friedmann equations with unspecified integration constants. These can be fixed if one chooses a specific bulk spacetime like as two CTAdS black holes. With this choice we find bouncing universes as well as cyclic universe from the BDL approach. If not with this choice, we obtain only the bouncing universes. If the exotic stiff from charge $Q$ is considered still as an unwanted matter, we may answer to a question of how to avoid it. This may be done by introducing an appropriate local matter on the brane in Sec.4. Finally we discuss our results, especially for a relation between the exotic stiff matter and Lorentz invariance violation in the brane world scenarios.
2 MDW cosmology in the charged topological AdS black holes

In general we consider an \((n + 1)\)-dimensional brane with a local matter distribution \(\tau_{\mu\nu}\) including a tension \(\sigma\) which is located at between two \((n + 2)\)-dimensional charged black holes. We introduce the bulk spacetime by \(\mathcal{M}^+\) and \(\mathcal{M}^-\) for the positively and negatively charged black holes and their boundaries by \(\partial\mathcal{M}^+\) and \(\partial\mathcal{M}^-\). These boundaries are fixed to coincide with the brane exactly. We keep the \(Z_2\)-symmetric geometry, although we don’t need to have the \(Z_2\)-symmetry for the electric configuration \[25\]. Actually we have a dipole configuration (\(\leftarrow\)) in flat spacetime as far as the electric flux lines are concerned. Also we assume that there is no source or sink for the charge on the brane. Thus the flux lines are conserved on the brane and thus the Israel junction condition remains unchanged. For this purpose we start with the \((n + 2)\)-dimensional Einstein-Maxwell theory with the cosmological constant \(\Lambda = -\frac{n(n + 1)}{2\ell^2}\):

\[
I = \frac{1}{16\pi G_{n+2}} \int_{\mathcal{M}^+ + \mathcal{M}^-} d^{n+2}x \sqrt{-g} \left( R - F_{MN}F^{MN} - 2\Lambda \right),
\]

(2.1)

where \(R\) is the curvature scalar and \(F_{MN}\) denotes the Maxwell field\[1\]. For a while we consider the right hand side \((\mathcal{M}^+)\) of the bulk spacetime. Varying the action (2.1) yields the equations of motion

\[
G_{MN} \equiv R_{MN} - \frac{1}{2} g_{MN} R = T_{MN}, \quad T_{MN} = 2F_{MP}F_{N}^{\ P} - \frac{1}{2} g_{MN} F^2 - \Lambda g_{MN}, \quad (2.2)
\]

\[
\partial_M(\sqrt{-g}F^{MN}) = 0. \quad (2.3)
\]

For completeness we need the Bianchi identity

\[
\partial_{[M} F_{NP]} = 0. \quad (2.4)
\]

These equations all give us a charged topological AdS(CTAdS) black hole solution \[11, 10\]

\[
ds^2_{CTAdS} \equiv g_{MN}dx^Mdx^N = -h(r)dt^2 + h(r)^{-1}dr^2 + r^2 \gamma_{ij}dx^idx^j, \quad (2.5)
\]

\[
F_{rt} = \frac{n\omega_n Q_+}{4} r^n, \quad (2.6)
\]

\[1\] Hereafter we use indices \(M, N, \cdots\) for \((n + 2)\)-bulk space, \(\mu, \nu, \cdots\) for \((n + 1)\)-brane, and \(i, j, \cdots\) for \(n\)-spatial space in the brane.
where the metric function $h(r)$ is given by

$$
h(r) = k - \frac{m_+}{r^{n-1}} + \frac{q_+^2}{r^{2(n-1)}} + \frac{r^2}{\ell^2}, \quad m_+ = \omega_n M_+, \quad q_+^2 = \frac{n\omega_n^2 Q_+^2}{8(n-1)}, \quad \omega_n = \frac{16\pi G_{n+2}}{nV(M^n)}, \quad (2.7)
$$

Here $\gamma_{ij}$ is the horizon metric for a constant curvature manifold $M^n$ with the volume $V(M^n) = \int d^n x \sqrt{\gamma}$. The horizon geometry is thus extended to include elliptic, flat, and hyperbolic for $k = 1, 0, -1$, respectively. In this sense we call it the topological black hole.

The event horizon $r_H$ is determined by the maximal root of $h(r_H) = 0$. The integration constants $M_+$ and $Q_+$ can be interpreted as the ADM mass and electric charge of the black hole.

In the spirit of the brane world scenario \[26\] and AdS/CFT correspondence \[27\], let us introduce two black hole spacetimes. We glue it ($M^+$) to another ($M^-$). For simplicity we choose the same horizon geometry and cosmological constant for $M_+$ and $M^-$. Then the thermodynamics of two black holes corresponds to that for the boundary CFT with an $R$-charge (or $Q$-potential). We rescale the boundary metric so that it can take a form of $ds_{BCFT}^2 = -d\tau^2 + \ell^2 \gamma_{ij} dx^i dx^j \[28\]$. For simplicity we choose $M_+ = M_- = M$ ($m_+ = m_- = m$), but $Q_+ = -Q_- = Q/2$ ($q_+ = -q_- = q/2$). As a result, we have $T_{+MN} = T_{-MN}$ for the stress-energy tensor $\mathcal{T}$. This implies that a single energy $E = 2E_+ = 2E_-$, temperature $T = T_+ = T_-$, entropy $S = 2S_+ = 2S_-$ and pressure $p = 2p_+ = 2p_-$ can be defined on the brane of two boundaries using the AdS/CFT correspondence \[29\]. Thermodynamic quantities of the corresponding CFT are given by \[30, 31\]

$$
E = 2M_+ \frac{\ell}{r} = \frac{2\ell r_H^{n-1}}{r_\omega_n} \left( 1 + \frac{\omega_n^2 (Q/2)^2}{8(n-1)r_H^{2(n-1)}} \right),
$$

$$
T = T_+ \frac{\ell}{4\pi r r_H} = \frac{\ell}{4\pi r r_H} \left( (n-1) + \frac{(n+1) r_H^2}{\ell^2} - \frac{n\omega_n^2 (Q/2)^2}{8r_H^{2(n-1)}} \right),
$$

$$
\Phi = \phi_H \frac{\ell}{r} = \frac{n\ell \omega_n Q}{8(n-1)r_H^{n-1}},
$$

$$
S = S_{BH} = \frac{r_H^{n-1}}{2G_{n+2}} V(M^n), \quad (2.8)
$$

where $\Phi(= \Phi_+ = -\Phi_-)$ denotes the chemical potential conjugate to the charge of $Q/2$.

\[2\]If not, in brane cosmology we have to solve the Gauss-Godazzi equation for the non-$Z_2$ symmetric evolution by different masses and different charges.
We can rewrite the entropy in Eq.(2.8) as the Cardy-Verlinde formula \[32\]

\[ S = \frac{2\pi r}{n} \sqrt{E_c(2(E - E_q) - E_c)}, \]  

(2.9)

where the Casimir energy \(E_c\) and electromagnetic energy \(E_q\) are given by \[13\]

\[ E_c = \frac{4\ell r^{n-1}}{\omega n r}, \quad E_q = \frac{1}{2} \Phi Q = \frac{n\ell \omega_n Q^2}{16(n-1)rr_H^{n-1}}. \]  

(2.10)

Now we wish to derive a dynamic equation from the moving domain wall approach. Actually the embedding of a moving domain wall into the black hole space is a mapping of \(t \rightarrow t(\tau), r \rightarrow R(\tau)\) with \(\ddot{R}^2/h(R) - h(R)t^2 = -1\). For a good embedding, a small black hole of \(q/\ell < m/\ell < 1\) is usually assumed. And then we obtain an induced metric \(h_{\mu\nu}\) on the brane

\[ ds^2_{n+1} = -d\tau^2 + R^2(\tau)\gamma_{ij}dx^i dx^j \equiv h_{\mu\nu}dx^\mu dx^\nu. \]  

(2.11)

Here the scale factor \(R(\tau)\) will be determined by the Israel junction condition \[33\]. For \((\partial \mathcal{M}_+)_{k=1,n=3}\), the extrinsic curvature components are given by

\[ K_{\tau\tau} = K_{MN} u^M u^N = (h(R)t)^{-1}(\dot{R} + h'(R)/2) = \frac{\dot{R} + h'(R)/2}{\sqrt{\dot{R}^2 + h(R)}}, \]  

(2.12)

\[ K_{\chi\chi} = K_{\theta\theta} = K_{\phi\phi} = -h(R)tR = -\sqrt{\dot{R}^2 + h(R)} R, \]  

(2.13)

where \(u^M\) is a tangent vector along the moving brane with \(u^M u_M = -1\) and prime (’) stands for derivative with respect to \(R\). The presence of any localized matter on the brane including a brane tension implies that the extrinsic curvature jumps across the brane. At this stage let us glue it to another bulk of \(\mathcal{M}_-\) with the same \(R(\tau)\) but an opposite embedding of \(K_{-\mu\nu} = -K_{+\mu\nu} (= -K_{\mu\nu})\). This jump is then described by the Israel junction condition\[3\]

\[ K_{+\mu\nu} - K_{-\mu\nu} = -\kappa^2 \left( \tau_{\mu\nu} - \frac{1}{n} \tau^\lambda h_{\mu\nu} \right), \]  

(2.14)

\[^3\]The Gibbons-Hawking term \(\frac{1}{n} \int d^{n+1}x \sqrt{h} K\) and the Hawking-Ross term \(\frac{2}{\sqrt{n}} \int d^{n+1}x \sqrt{h} F^{MN} n_M A_N\) are necessary for embedding of the brane into the charged black hole. However, the electric flux across the brane is conserved and thus the Israel junction condition remains unchanged even for \(F_{MN} \neq 0\) \[25\]. Here we no longer consider one-sided brane cosmology. Hence we keep the \(Z_2\)-symmetry which was important in two-sided brane cosmology.
with the bulk gravitational constant $\kappa^2 = 8\pi G_{n+2}$. The 4D perfect fluid is introduced for a local stress-energy tensor on the brane

$$\tau_{\mu\nu} = (\rho + p)u_\mu u_\nu + p h_{\mu\nu}. \quad (2.15)$$

Here $\rho = \rho_l + \sigma$ ($p = P_l - \sigma$), where $\rho_l$ ($P_l$) are the energy density (pressure) of a local matter and $\sigma$ is a brane tension. In the case of $\rho_l = P_l = 0$, the right hand side of Eq. (2.14) leads to a form of the Randall-Sundrum case as $-\frac{\kappa^2}{n} h_{\mu\nu}$ [20]. The spatial components of the junction condition (2.14) lead to

$$\sqrt{h(R)} + \dot{R}^2 = \frac{\kappa^2}{2n} \sigma R. \quad (2.16)$$

In two black hole spacetimes, we use the tension $\sigma = 2n/\kappa^2 \ell$ to obtain a critical brane. The above equation leads to the first Friedmann equation

$$H^2 = -k \frac{R^2}{R^2} + \frac{m}{R^{n+1}} - \frac{q^2}{R^{2n}}. \quad (2.17)$$

Its time rate as the second Friedmann equation is given by

$$\dot{H} = k \frac{R^2}{R^2} - \frac{n+1}{2} m \frac{R^{n+1}}{R^{2n}} + \frac{nq^2}{R^{2n}}. \quad (2.18)$$

Introducing a CFT-energy density $\rho_r = E/V$ with $E = 2M\ell/R$, its pressure $P_r = \rho_r/n$, a charge density $\rho_Q = Q/V$, and its electric potential $\Phi = n\ell \omega nQ/8(n-1)R^n$ with the volume of the brane $V = R^n V(M^n)$, two Friedmann equations take the forms

$$H^2 = -k \frac{R^2}{R^2} + \frac{16\pi G_{n+1}}{n(n-1)} \left( \rho_r - \frac{1}{2} \Phi \rho_Q \right), \quad (2.19)$$

$$\dot{H} = k \frac{R^2}{R^2} - \frac{8\pi G_{n+1}}{n-1} \left( \rho_r + P_r - \Phi \rho_Q \right). \quad (2.20)$$

In deriving these, we use an important relation for the two-sided brane scenario

$$G_{n+2} = \frac{2\ell}{n-1} G_{n+1} \quad (2.21)$$

and the conservation law of $\dot{\rho} = -nH(\rho + P)$. This means that the cosmological evolution is derived by the energy density and pressure of the CFT-radiation matter plus those of the electric potential energy. Eqs. (2.19) and (2.20) are identified with those of the one-sided
brane world [16] because two black hole masses and charges on both sides are combined with Eq. (2.21) to give the same ones. These equations can be further rewritten as the cosmological Cardy-Verlinde formula and a defining relation for the Bekenstein-Hawking energy as

\[ S_H = \frac{2\pi R}{n} \sqrt{E_{BH}[2(E - \Phi Q/2) - kE_{BH}]}, \]  
\[ kE_{BH} = n(E + pV - \Phi Q - T_HS), \]

Here the Hubble temperature \( T_H = -\frac{\dot{H}}{2\pi H} \) is expressed in terms of the Hubble parameter and its time rate only.

Another interpretation is also possible for the Friedmann equations on the brane if one defines a charge stiff density \( \rho_{csti} = \Phi \rho Q/2 \) with the equation of state : \( P_{csti} = \rho_{csti} \). Then one finds

\[ H^2 = -\frac{k}{R^2} + \frac{16\pi G_{n+1}}{n(n-1)} \rho_h, \]  
\[ \dot{H} = \frac{k}{R^2} - \frac{8\pi G_{n+1}}{n-1} (\rho_h + P_h), \]

where a holographic energy density \( \rho_h \) and its pressure \( P_h \) are defined by

\[ \rho_h = \rho_r - \rho_{csti}, \quad P_h = P_r - P_{csti} = \frac{\rho_r}{n} - \rho_{csti}. \]

This means that the mass of the black hole gives a CFT-radiation matter holographically, whereas the charge of the black hole induces an exotic CFT-stiff matter on the moving domain wall because the sign in the front of \( \rho_{csti} \) is negative. Thus it is shown that the bulk gravitational dynamics leads to the standard cosmology with a radiation matter of \( \rho_r \sim R^{-(n+1)} \) and a stiff matter of \( \rho_{csti} \sim R^{-2n} \) on the brane. However, this CFT-interpretation on the brane is always not true. This is possible only when the brane is located at large distance from the center of AdS space unless the holographic energy density \( \rho_h \) is small [23]. Applying the Hamiltonian approach to calculation of the exact energy on the brane, the energy density \( \rho_e \) and its pressure \( P_e \) measured by an observer on the brane is given by

\[ \rho_e \equiv \frac{2n}{\kappa^2 \ell^2} \xi(R) = \frac{2n}{\kappa^2 \ell^2} \left[ \sqrt{1 + \frac{\ell^2}{R^2} \left( \frac{m}{R^{n-1}} - \frac{q^2}{R^{2n-2}} \right)} - 1 \right], \]
\[ P_e = -\rho_f + \frac{\ell}{\kappa^2 \xi(R)} \left[ \frac{(n+1)m}{R^{n+1}} - \frac{2nq^2}{R^{2n}} \right]. \]
Here it is not guaranteed that $\rho_f$ is always positive. The Friedmann equations lead to those of the conventional brane cosmology

\begin{align}
H^2 &= -\frac{k}{R^2} + \frac{16\pi G_{n+1}}{n(n-1)} \rho_e + \frac{\kappa^4}{4n^2} \rho_e^2, \quad (2.29) \\
\dot{H} &= \frac{k}{R^2} - \frac{8\pi G_{n+1}}{n-1} (\rho_e + P_e) - \frac{\kappa^4}{4n} \rho_e (\rho_e + P_e). \quad (2.30)
\end{align}

Small $\rho_e$ means either that the brane is close to the AdS boundary:

$$\lim_{R \to \infty} \frac{\ell^2}{R^2} \left( \frac{m}{R^{n-1}} - \frac{q^2}{R^{2n-2}} \right) \to \text{small} \quad (2.31)$$

or that a holographic term is small

$$\left( \frac{m}{R^{n-1}} - \frac{q^2}{R^{2n-2}} \right) \to \text{small} \quad (2.32)$$

which implies a small black hole of $q/\ell < m/\ell < 1$. In this approximation the energy density is given by

$$\rho_e \approx \frac{n\ell}{\kappa^2} \left( \frac{m}{R^{n+1}} - \frac{q^2}{R^{2n}} \right) = \rho_r - \rho_{\text{csti}} = \rho_h \quad (2.33)$$

and for its pressure

$$P_e \approx \frac{\rho_r}{n} - \rho_{\text{csti}} = P_r - P_{\text{csti}} = P_h. \quad (2.34)$$

Neglecting $\rho_e^2$ and $\rho_e P_e$ terms in (2.29) and (2.30), we recover our equations (2.24) and (2.25). This means that our holographic interpretation of the cosmological Cardy-Verlinde formula for Eq.(2.22) has limitation and approximation because it is based on an approximate equation (2.8). Unless we assume small $\rho_e$, a lot of terms are generated in the Friedmann equations. In this case we may establish the unconventional duality, and the holographic duality corresponds to an approximation of the unconventional duality.

### 3 BDL cosmology in the charged background

In the MDW approach the bulk space time is always fixed. This means that we can determine the gravitational dynamics of the brane exactly from the given bulk black hole background. Actually, we don’t know precisely what kind of higher dimensional theory
we should start with, even if string theories provide us some information about it. In this sense the moving domain wall method is considered as a very restrictive approach although it provides us an exact evolution of the brane. The BDL approach is a genuine extension to the Kaluza-Klein cosmology to account for a local matter distribution on the brane. Hence it is very desirable to study the BDL brane cosmology in the charged background. Following \[22\], we assume the Gaussian-normal metric for an \((n+2)\)-dimensional spacetime

\[
ds^2_{\text{BDL}} = -c^2(t, y) dt^2 + a^2(t, y) \gamma_{ij} dx^i dx^j + b^2(t, y) dy^2,
\]

(3.1)

where \(\gamma_{ij}\) is the previous metric of an \(n\)-dimensional space with constant curvature \(n(n-1)k\). In the orthogonal basis, we express the Einstein tensor \(G_{\hat{M}\hat{N}}\) in Eq.(2.2) in terms of the BDL metric as

\[
G_{\hat{t}\hat{t}} = n \left[ \frac{\dot{a}}{a c^2} \left( \frac{n-1}{2} \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) - \frac{1}{b^2} \left( \frac{a''}{a} + \frac{a'}{a} \left( \frac{n-1}{2} \frac{\dot{a}}{a} - \frac{\dot{b}}{b} \right) \right) + \frac{n-1}{2} \frac{k}{a^2} \right],
\]

\[
G_{\hat{y}\hat{y}} = n \left[ \frac{a'}{ab^2} \left( \frac{n-1}{2} \frac{a'}{a} + \frac{c'}{c} \right) - \frac{1}{c^2} \left( \frac{\ddot{a}}{a} + \frac{\dot{a}}{a} \left( \frac{n-1}{2} \frac{\dot{a}}{a} - \frac{\dot{c}}{c} \right) \right) - \frac{n-1}{2} \frac{k}{a^2} \right],
\]

\[
G_{\hat{t}\hat{y}} = n \left( \frac{\dot{a} c'}{abc^2} + \frac{\dot{a} b'}{ab^2 c} - \frac{\dot{a}'}{abc} \right),
\]

\[
G_{\hat{i}\hat{j}} = \frac{\delta_{ij}}{b^2} \left[ (n-1) \frac{a''}{a} + \frac{c''}{c} + \frac{n-1}{2} \frac{a'}{a} \left( (n-2) \frac{a'}{a} + 2 \frac{c'}{c} \right) - \frac{b'}{b} \left( n-1 \frac{a'}{a} + \frac{c'}{c} \right) \right]
\]

\[
+ \frac{\delta_{ij}}{c^2} \left[ -(n-1) \frac{\ddot{a}}{a} + \frac{\dot{b}}{b} \left( \frac{\dot{c}}{c} - (n-1) \frac{\dot{a}}{a} \right) + \frac{n-1}{2} \frac{\ddot{a}}{a} \left( 2 \frac{\ddot{c}}{c} - (n-2) \frac{\dot{a}}{a} \right) \right]
\]

\[-\frac{(n-1)(n-2)}{2} \frac{k}{a^2} \delta_{ij}, \quad (3.2)
\]

where the dot (prime) stand for differentiation with respect to \(t\) (\(y\)). Suppose that an \((n+1)\)-dimensional brane is located at \(y = 0\). In the two-sided brane world, the stress-energy tensors are given by (2.2), but we don’t need to choose the same initially. Furthermore \(T_{\hat{t}\hat{y}} = 0\) implies that there is no local matter flow along the extra direction. A local stress-energy tensor including a brane tension \(\sigma\) is assumed to be

\[
\tilde{\tau}_{\mu\nu} = \frac{\delta(y)}{b} \text{diag}(-\varrho, p, \cdots, p, 0).
\]

(3.3)

Let us denote the gap of a given function \(f\) at \(y = 0\) by \([f] = f(0_+) - f(0_-)\) and its average by \(\{f\} = (f(0_+) + f(0_-))/2\), where \(+(-)\) denote \(y > 0(y < 0)\), respectively. The
functions $a$, $b$, $c$ in the metric (3.1) are continuous at $y = 0$, but their derivatives are not.

So the second derivative takes the form

$$f'' = f''|_{y 
eq 0} + [f']\delta(y).$$

(3.4)

It is then straightforward to write down the gaps in $(\hat{ii})$, $(\hat{yy})$ and $(\hat{ij})$ components of the Einstein equation (2.2), respectively:

$$\frac{n}{b_0^2} \left( -(n - 1) \frac{[a']^2}{a_0 b_0} + \frac{[a']^2 [b']}{a_0 b_0} + \frac{[a']^2}{a_0 b_0} \right) = T_{ii}(0_+) - T_{ii}(0_-),$$

$$\frac{n}{b_0^2} \left( (n - 1) \frac{[a']^2 [a']}{a_0 c_0} + \frac{[a']^2 [c']}{a_0 c_0} + \frac{[a']^2 [c']}{a_0 b_0} - \frac{[a']^2 [b']}{a_0 b_0} \right) = T_{yy}(0_+) - T_{yy}(0_-),$$

$$\frac{(n - 1)}{b_0^2} \delta_{ij} \left( (n - 2) \frac{[a']^2 [c']}{a_0 c_0} + \frac{[a']^2 [c']}{a_0 c_0} + \frac{[a']^2 [c']}{a_0 b_0} - \frac{[a']^2 [b']}{a_0 b_0} \right) - \frac{1}{n - 1} \frac{[b']^2 [c']}{b_0 c_0} - \frac{1}{n - 1} \frac{[b']^2 [c']}{b_0 c_0} = T_{ij}(0_+) - T_{ij}(0_-).$$

(3.5)

where quantities with subscript “0” denote those at $y = 0$. The $\delta(y)$-function parts in $(\hat{ii})$ and $(\hat{ij})$ components of the Einstein equation with Eq.(3.3) lead to the Israel junction condition

$$\frac{[a']}{a_0 b_0} = \frac{-\kappa^2}{n - 2}, \quad \frac{[c']}{b_0 c_0} = \kappa^2 \left( p + \frac{n - 1}{n} \rho \right).$$

(3.6)

On the other hand, the average part of $(\hat{yy})$ component is given by

$$\frac{1}{c_0} \left( \hat{\alpha}_0 - \hat{\alpha}_0 \left( \frac{n - 1}{2} \frac{\hat{\alpha}_0}{a_0} - \frac{\hat{\alpha}_0}{c_0} \right) \right) = -\frac{1}{2n} (T_{yy}(0_+) + T_{yy}(0_-))$$

$$+ \frac{n - 1}{2} \left( -\frac{k}{\hat{c}_0} + \frac{1}{4} \left( \frac{[a']}{a_0 b_0} \right)^2 + \frac{[a']}{a_0 b_0} \right)^2 + \frac{1}{4} \frac{[a']^2 [c']}{a_0 b_0} + \frac{[a']^2 [c']}{a_0 b_0}.$$

(3.7)

The Maxwell equation (2.3) and the Bianchi identity (2.4) under the BDL metric (3.1) have a solution

$$F_{yt} = \frac{Qb_c}{a^n},$$

(3.8)

where $Q$ is an unknown integration constant. Thus the stress-energy tensor including the
Maxwell field $F_{MN}$ and cosmological constant $\Lambda$ is given by
\[
T_{\hat{M}}^{\hat{N}} = \text{diag}\left(\frac{n(n+1)}{2\ell^2} - \frac{Q^2}{a^{2n}}, \frac{n(n+1)}{2\ell^2} + \frac{Q^2}{a^{2n}}, \ldots, \frac{n(n+1)}{2\ell^2} + \frac{Q^2}{a^{2n}}, \frac{n(n+1)}{2\ell^2} - \frac{Q^2}{a^{2n}}\right).
\]
(3.9)

In order to go parallel with the moving domain wall approach, we consider a simple case in which the bulk stress-energy tensors are identical on two sides of the brane and thus the bulk is $Z_2$-symmetric. This is the same situation as was considered in Sec. 2. One then has $\{f'\} = 0$ for $f = a, b, c$. Also $G_{i\bar{g}} = 0$ is satisfied if $c(t, y) = a(t, y)/a_0$ with $b(t, 0) = 1$. This means that the position of the brane is always fixed along the extra direction during whole evolution. This allows us to set $c_0 \equiv c(t, 0) = 1$ so that $d\tau \equiv c(t, 0)dt = dt$. The Hubble parameter $H$ on the brane is defined as $H = \dot{a}_0/a_0 \equiv \dot{R}/R$. The left hand side of Eq. (3.7) can be rewritten as
\[
\frac{1}{c_0^2}\left(\frac{\ddot{a}_0}{a_0} + \frac{\dot{a}_0}{a_0}\left(\frac{n-1}{2} \frac{\dot{a}_0}{a_0} - \frac{\ddot{c}_0}{c_0}\right)\right) = \frac{1}{2R^n} \frac{d}{dR}\left(H^2R^{n+1}\right),
\]
(3.10)

while the right hand side of Eq. (3.7) can be calculated with Eqs. (3.6) and (3.9) to yield
\[
\frac{1}{2R^n} \frac{d}{dR}\left(H^2R^{n+1}\right) = -\frac{\kappa^4}{8n^2} \varrho (2np + (n-1)\varrho) - \frac{n+1}{2\ell^2} - \frac{n-1}{2} \frac{k}{R^2} + \frac{Q^2}{nR^{2n}}.
\]
(3.11)

In general we include a local matter distribution with $\tau_{\mu\nu}$ on the brane. In this case we have $\varrho = \rho_l + \sigma$, $p = P_l - \sigma$ with $P_l = 2\rho_l$, $\sigma = 2n/k^2\ell$. Then the brane becomes a critical one and Eq. (3.11) reduces to
\[
\frac{d}{dR}\left(H^2R^{n+1}\right) = -(n-1)kR^{n-2} + \frac{2Q^2}{nR^n} - \frac{\kappa^2(n\varpi - 1)R^n}{n\ell} \rho_l - \frac{\kappa^4}{4n^2}(2n\varpi + n - 1)R^n \rho_l^2.
\]
(3.12)

Assuming $\rho_l \sim R^{-n(1+\varpi)}$, one integrates this equation to yield
\[
H^2 = -\frac{k}{R^2} + \frac{\mathcal{C}}{R^{n+1}} - \frac{2}{n(n-1)} \frac{Q^2}{R^{2n}} + \frac{16\pi G_{n+1}^{n+1}}{n(n-1)} \rho_l + \frac{\kappa^4}{4n^2} \rho_l^2,
\]
(3.13)

where $\mathcal{C}$ and $Q$ are two integration constants $^{[22, 15]}$. This governs an evolution of the fixed brane sandwiched in the two charged AdS spacetimes.

For $\rho_l = P_l = 0$ case, let us compare our equation (3.13) with (2.17) of the moving domain wall in the CTAdS black holes which can be rewritten further as
\[
H^2 = -\frac{k}{R^2} + \frac{\omega M}{R^{n+1}} - \frac{n\omega Q^2}{32(n-1)R^{2n}}.
\]
(3.14)
By analogy, we rewrite two constants as

\[ C = \omega_n \tilde{M}, \quad Q = \pm \frac{n\omega_n \tilde{Q}}{4}. \quad (3.15) \]

Our equation (3.13) exactly coincides with Eq. (3.14) if \( M = \tilde{M} \) and \( Q = \tilde{Q} \). In case of the moving brane Eq. (3.14), the bulk is just fixed as two CTAdS black holes with the same mass \( M \) and charge \( Q/2 \). These two parameters encode all information of the bulk and are used to describe a strongly coupled CFT on the brane using the finite-temperature AdS/CFT correspondence. On the other hand, in deriving Eq. (3.13), we don’t know what the precise AdS geometry is. But we include the bulk Maxwell fields. Certainly, the two integration constants \( C \) and \( Q \) will carry with information of the bulk, as the initial conditions. This shows that a holographic duality may be realized within the BDL approach [24].

Now we are position to obtain a solution to equation (3.13). As an explicit computation we choose a case of \( n = 3, \rho_l = P_l = 0 \). Introducing a conformal time \( \eta \), defined by \( d\tau = \mathcal{R}(\eta)d\eta \), one finds a solution for \( k = 1 \) closed geometry

\[ \mathcal{R}^{k=1}(\eta) = \sqrt{\frac{\omega_3 \tilde{M}}{2} \left( 1 - \tilde{c}_1 \cos(2\eta) \right)}, \quad \tilde{c}_1 = \sqrt{1 - \frac{3\tilde{Q}^2}{16 \tilde{M}^2}}. \quad (3.16) \]

Here we do not guarantee the reality of \( \tilde{c}_1 \): \( \tilde{Q}/2 < 2\tilde{M}/\sqrt{3} \), approximately, a small black hole condition of \( q/\ell < m/\ell < 1 \). because the values of two parameters \( \tilde{M} \) and \( \tilde{Q} \) are arbitrary. This means that \( \tilde{c}_1 \) may become a pure imaginary. On the other hand, for the MDW case the reality condition of \( \tilde{c}_1 \) comes from the condition for the existence of the event horizon in the 5D AdS Reissner-Nordstrom (AdSRN\(_5\)) black hole [3]. Hence a cyclic solution is allowed. Then the universe evolves periodically between a maximal distance \( \mathcal{R}_{\text{max}} \) and a minimal distance \( \mathcal{R}_{\text{min}} \) with

\[ \mathcal{R}_{\text{max/min}}^{k=1} = \sqrt{\frac{\omega_3 M}{2} (1 \pm c_1)}, \quad c_1 = \sqrt{1 - \frac{3Q^2}{16M^2}} > 0. \quad (3.17) \]

If one chooses \( \tilde{M} = M \) and \( \tilde{Q} = Q \), then we find the above bouncing universe. However, this case rarely occurs in the BDL approach because the bulk spacetime should be chosen specially as two AdSRN\(_5\) black holes.
For $k = -1$ open universe, one has the solution
\[ R^{k=-1}(\eta) = \sqrt{\frac{\omega_3 M}{2} (\tilde{c}_2 \cosh(2\eta) - 1)}, \quad \tilde{c}_2 = \sqrt{1 + \frac{3\tilde{Q}^2}{16M^2}} > 1. \] (3.18)

This is acceptable because $c_2$ is always real. The brane is initially contracting and then bounces to an expansion. At the turning point of $\eta = 0$, the minimal radius takes the form
\[ R^{k=-1}_{\text{min}} = \sqrt{\frac{\omega_3 M}{2} (c_2 - 1)}. \] (3.19)

In the absence of the Maxwell fields ($\tilde{Q} = 0$), one immediately finds $R^{k=-1}_{\text{min}} = 0$.

In the case of $k = 0$ flat universe we have
\[ R^{k=0}(\eta) = \sqrt{\frac{3\tilde{Q}^2\omega_3}{64M} + \omega_3 \tilde{M} \eta^2}. \] (3.20)

Also we have a bouncing universe with the minimal distance $R^{k=0}_{\text{min}} = \sqrt{3\tilde{Q}^2\omega_3/64M}$ at the turning point $\eta = 0$. Obviously this goes to zero in the limit of $\tilde{Q} \to 0$. Hence we find bouncing universes for $k = -1, 0$ cases from the BDL approach. The cyclic closed universe is only allowable if the bulk spacetime are chosen specially as two AdSRN$_5$ black holes. In this case, all solutions remain fixed with respect to the extra dimension $y$. However, there is no restriction on $\tilde{M}$ and $\tilde{Q}$, for example, a small black hole condition of $\tilde{Q}/2 < 2\tilde{M}/\sqrt{3}$ required in the MDW solutions.

4 Brane cosmologies with a local matter

4.1 MDW approach

First of all we wish to remark the sign of $\rho_h = \rho_r - \rho_{\text{csti}}$ in Eq.(2.20). At the very early universe we have a negative holographic energy density of $\rho_h < 0$ because $\rho_{\text{csti}}$ dominates. On the other hand, in the standard cosmology one usually uses the dominant energy condition (DEC) which postulates that the local energy density is non-negative for all observers $\rho \geq 0, P = \omega \rho$ with $|\omega| \leq 1$. But we do not maintain this condition because of $\rho_h < 0$ at the very early universe. In this section we may deal with this problem by
introducing an appropriate local matter distribution on the brane [14]. In the case of \( \rho_l \neq 0 \) with \( P_l = \omega \rho_l \), from Eqs. (2.14) and (2.15) we obtain two Friedmann equations

\[
H^2 = -\frac{k}{R^2} + \frac{16\pi G_{\text{n+1}}}{n(n-1)} (\rho_r - \rho_{\text{csti}} + \rho_l) + \frac{\kappa^4}{4n^2} \rho_l^2, \quad (4.1)
\]

\[
\dot{H} = \frac{k}{R^2} - \frac{8\pi G_{\text{n+1}}}{n-1} (\rho_r - \rho_{\text{csti}} + \rho_l + P_r - P_{\text{csti}} + P_l) - n \frac{\kappa^4}{4n^2} \rho_l (\rho_l + P_l). \quad (4.2)
\]

If a local stiff matter with \( P_{\text{lsti}} = \rho_{\text{lsti}} > \rho_{\text{csti}} \) is chosen, we achieve the positivity of the total energy density \( \rho_T = \rho_h + \rho_{\text{lsti}} = \rho_r + (\rho_{\text{lsti}} - \rho_{\text{csti}}) > 0 \) in Eq. (4.1). Even though we avoid an exotic stiff-matter problem thanks to a local stiff matter, it gives rise to a higher-order term of \( \rho_{\text{lsti}}^2 \sim R^{-4n} \), which have not been found in the standard and brane cosmologies. If this exists, it will contribute to the very early universe.

In order to find another case, we rewrite the above equations using \( \rho_{\text{cdust}}^2 = Q^2 / V^2 \) as

\[
H^2 = -\frac{k}{R^2} + \frac{16\pi G_{\text{n+1}}}{n(n-1)} (\rho_r + \rho_l) + \frac{\kappa^4}{4n^2} (\rho_l^2 - \rho_{\text{cdust}}^2), \quad (4.3)
\]

\[
\dot{H} = \frac{k}{R^2} - \frac{8\pi G_{\text{n+1}}}{n-1} (\rho_r + P_r + P_l) - n \frac{\kappa^4}{4n^2} [\rho_l (\rho_l + P_l) - \rho_{\text{cdust}}^2]. \quad (4.4)
\]

Including a local dust matter with \( P_{\text{dust}} = 0, \rho_{\text{dust}}^2 > \rho_{\text{cdust}}^2 \), then we maintain the positivity of the last term in Eq. (4.3). However, we obtain an effective energy density which is composed of two different matters: a CFT-radiation matter with \( \rho_r \sim R^{-(n+1)} \) plus a local dust matter with \( \rho_{\text{dust}} \sim R^{-n} \).

### 4.2 BDL approach

Considering Eq. (3.13) together with (3.15), we can lead to Eqs. (4.1) and (4.2) but replacing \( \rho_r, \rho_{\text{csti}} \) by \( \tilde{\rho}_r, \tilde{\rho}_{\text{csti}} \) of \( \tilde{M}, \tilde{Q}^2 \). Also another case of Eqs. (4.3) and (4.4) can be recovered from the BDL approach by replacing \( \tilde{\rho}_r, \tilde{\rho}_{\text{cdust}}^2 \) of \( \tilde{M}, \tilde{Q}^2 \). Hence we may avoid an exotic stiff matter due to the bulk Maxwell field by inserting an appropriate local matter on the brane within the BDL scheme. In addition, we have degrees of freedom to make the positivity of the total energy density because there are no restrictions on determining \( \tilde{M} \) and \( \tilde{Q} \) in the BDL approach.
5 Discussions

A moving domain wall in the charged topological AdS black holes provides bouncing and cyclic universes. This solution is possible because an exotic stiff matter term arises from the bulk charge $Q$ of the black hole in a holographic way. On the other hand this term obviously violates the dominant energy condition of keeping the positive energy density for all observers on the brane. In the BDL approach we confirm from Eq.$(3.13)$ that the Maxwell field with a charge $\tilde{Q}$ generates an exotic stiff matter with a negative sign. As was reported in $[20]$, one cannot guarantee the positivity of holographic energy densities. But there is no problem in any local matter on the brane because the 4D Lorentz invariance is always maintained.

The embedding of a moving domain wall into the black hole background is just the 2→1 mapping of $t \rightarrow t(\tau)$, $r \rightarrow R(\tau)$ with a timelike tangent vector $u^M = (\dot{t}, \dot{R}, 0, 0, 0)$, $u^M u_M = -1$ and a spacelike normal vector $n_M = (-\dot{R}, \dot{t}, 0, 0, 0)$, $n^M n_M = 1$, $u^M n_M = 0$. These lead to only one condition of $-h(R)\dot{t}^2 + \dot{R}^2/h(R) = -1$ which makes the timelike brane during whole evolution $[34]$. However, it is well known that for a timelike bulk coordinate $t = t(\tau)$ and spacelike bulk coordinate $r = R(\tau)$, their role is exchanged into each other when the brane crosses the horizon of the black hole at $R = r_H$ with $h(R) = 0$. This means that the induced metric $(2.11)$ will have differently defined Lorentz symmetry at different points along the radial timelike geodesics to maintain a speed of light $c = 1$. This implies that the bulk spacetime of $(2.5)$ globally violates 4D Lorentz invariance, leading to apparent violations of Lorentz invariance in view of an observer on the brane due to the bulk gravitational effects. An important fact of this consequence is that the speed of gravitational waves which propagates through the bulk spacetime would be different from the speed of light waves which propagates along the brane. Explicitly the gravitational wave will arrive at the brane faster than the light wave $[20]$. This could cause apparent violations of the causality from the view of an observer on the 4D brane. On the other hand the dominant energy condition usually requires the causality on the 4D spacetime. Hence it arrives that we don’t need to require the dominant energy condition on the holographic energy density of $\rho_h = \rho_r - \rho_{csi}$ from the bulk configuration. The mass of the black hole happens to give a positive energy density, whereas the charge happens to be a negative
energy density. This is considered as a result of the nature of the bulk configuration.

In the BDL approach we note first that two metrics (2.3) and (3.1) are equivalent even for the charged black hole background [9, 20]. Hence we expect that the dominant energy condition is not necessarily required on the holographic energy density of \( \tilde{\rho}_h = \tilde{\rho}_r - \tilde{\rho}_{\text{csti}} \) from the Maxwell fields. Our result of Eq.(3.13) shows an explicit example for this direction.

In conclusion we obtain the bouncing universes from the BDL approach in the charged bulk spacetime. This method provides a more general approach to study the brane cosmology than the moving domain wall. Especially considering a special bulk of two AdSNR black holes, then we can find a cyclic universe for \( k = 1 \) closed geometry. These solutions arise mainly from an exotic stiff matter term originated from the Maxwell field. If one wishes to avoid this exotic matter on the brane, we may include an appropriate local matter. In this case we may use this model to describe a core of neutron stars consistent with causality [35].

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