Correlations between the Dark Matter and Baryonic Properties of CLASH Galaxy Clusters

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Abstract. We study the total and dark matter (DM) density profiles as well as their correlations for a sample of 15 high-mass galaxy clusters by extending our previous work on several clusters from Newman et al. Our analysis focuses on 15 CLASH X-ray-selected clusters that have high-quality weak- and strong-lensing measurements from combined Subaru and Hubble Space Telescope observations. The total density profiles derived from lensing are interpreted based on the two-phase scenario of cluster formation. In this context, the brightest cluster galaxy (BCG) forms in the first dissipative phase, followed by a dissipationless phase where baryonic physics flattens the inner DM distribution. This results in the formation of clusters with modified DM distribution and several correlations between characteristic quantities of the clusters. We find that the central DM density profiles of the clusters are strongly influenced by baryonic physics as found in our earlier work. The inner slope of the DM density for the CLASH clusters is found to be flatter than the Navarro–Frenk–White profile, ranging from $\alpha = 0.30$ to 0.79. We examine correlations of the DM density slope $\alpha$ with the effective radius $R_e$ and stellar mass $M_e$ of the BCG, finding that these quantities are anti-correlated with a Spearman correlation coefficient of $\sim -0.6$. We also study the correlation between $R_e$ and the cluster halo mass $M_{500}$, and the correlation between the total masses inside 5 kpc and 100 kpc. We find that these quantities are correlated with Spearman coefficients of 0.68 and 0.64, respectively. These observed correlations are in support of the physical picture proposed by Newman et al.

Keywords: Galaxy Clusters, Galaxy Formation, Weak Gravitational Lensing
1 Introduction

The Λ cold dark matter (ΛCDM) paradigm gives a plethora of correct predictions [1–5]. However, some of its predictions are at odds with observations. N-body simulations in ΛCDM predict that the spherically averaged density profiles of self-gravitating structures, ranging from dwarf galaxies to galaxy clusters, are cuspy and well approximated by the Navarro–Frenk–White (NFW) profile [6, 7]. However, observations [8–11] and theoretical studies [12–17] have shown that the inner slopes of the density profile in dwarf galaxies and low-surface-brightness galaxies (LSBs) are usually flatter than simulations, and there is a strong diversity of the dark-matter (DM) distribution in these low-mass systems [the so-called “diversity problem”, 17–19].

On the observational side, the small dynamic range of observations can cause a degeneracy in the mass profile determination [see 27], and this degeneracy cannot be fully broken due to the lack of HI observations in dwarf spheroidals (dSPhs) and elliptical galaxies. Determinations of their DM structure are thus much more complicated. In the case of dSPhs, there are discrepant results on the cusp-core nature of the density profile [28–31], sometimes even in the case of the same object studied with different techniques. Similar uncertainties are present in cluster of galaxies, but X-ray observations, lensing and galaxies dynamics overcome them in an easier manner than for the cases of dSPhs or ellipticals.

While the NFW profile [6, 12] describes well the observed total density profiles in galaxy clusters as found in several studies [32–41], it was also found that the inner DM structure is characterized by a flatter slope within typical scales of the brightest cluster galaxy (BCG; from some kpcs to some tens of kpcs). Hence, the cusp-core problem [17, 19] appears to be present in galaxy clusters as well. This discrepancy can be alleviated when the effects of baryonic physics are properly accounted for in N-body simulations [see 13–16, 42–50].

In order to study how baryons modify the formation and evolution of clusters, we consider in [51] baryonic clumps interacting with the DM model introduced in [13]. In addition to finding that the central baryonic concentration within 10 kpc plays an important role in shaping the cluster density profile, we reproduced the observed cluster profiles for several massive systems [33, 34, 36], namely A611, A383, MACSJ1423.8+2404, and RXJ1133.
Table 1 Parameters derived for the CLASH sample. First column: cluster name; second: \(M_{500}\) as given in [41]; third and fourth: innermost 2D density slopes inferred directly from the observed [41] profiles and obtained from our semi-analytical model; fifth: inner 3D density slope from our model; sixth and seventh: stellar and baryonic fractions from our model.

| Name     | \(M_{500}\) [\(10^{14}M_{\odot}\)] | \(\alpha_{2D}\) | \(\alpha_{2D,T}\) | \(\alpha_{3D}\) | \(f_{\text{star}}\) | \(f_{b}\) |
|----------|----------------------------------|------------------|------------------|----------------|----------------|---------|
| A383     | 5.88 ±1.73                       | 0.71 ±0.26       | 0.70 ±0.09       | 0.37 ±0.09     | 0.0201 ±0.002 | 0.1355 ±0.008 |
| A209     | 9.64 ±1.97                       | 0.67 ±0.29       | 0.68 ±0.09       | 0.60 ±0.1      | 0.0167 ±0.002 | 0.1417 ±0.01 |
| A2261    | 15.65 ±3.05                      | 0.77 ±0.26       | 0.79 ±0.09       | 0.63 ±0.09     | 0.0140 ±0.002 | 0.1480 ±0.012 |
| RXJ2129  | 4.48 ±1.16                       | 0.49 ±0.26       | 0.49 ±0.09       | 0.55 ±0.09     | 0.0222 ±0.002 | 0.1323 ±0.006 |
| A611     | 10.73 ±2.65                      | 0.59 ±0.27       | 0.58 ±0.09       | 0.79 ±0.09     | 0.0161 ±0.002 | 0.1431 ±0.01 |
| MS2137   | 8.28 ±2.57                       | 0.86 ±0.25       | 0.85 ±0.09       | 0.65 ±0.08     | 0.0177 ±0.002 | 0.1398 ±0.009 |
| RXJ2248  | 12.45 ±3.62                      | 0.45 ±0.28       | 0.44 ±0.09       | 0.55 ±0.09     | 0.0152 ±0.002 | 0.1450 ±0.011 |
| MACSJ1115| 10.67 ±2.22                      | 0.33 ±0.30       | 0.34 ±0.09       | 0.39 ±0.1      | 0.0161 ±0.002 | 0.1430 ±0.01 |
| MACSJ1931| 10.51 ±4.05                      | 0.69 ±0.28       | 0.70 ±0.09       | 0.65 ±0.09     | 0.0162 ±0.002 | 0.1428 ±0.01 |
| MACSJ1720| 9.96 ±2.53                       | 0.59 ±0.26       | 0.60 ±0.09       | 0.56 ±0.09     | 0.0165 ±0.002 | 0.1421 ±0.01 |
| MACSJ0429| 6.85 ±2.1                        | 0.45 ±0.28       | 0.44 ±0.09       | 0.48 ±0.09     | 0.0190 ±0.002 | 0.1374 ±0.008 |
| MACSJ1206| 12.24 ±2.49                      | 0.56 ±0.26       | 0.57 ±0.09       | 0.50 ±0.09     | 0.0153 ±0.002 | 0.1448 ±0.011 |
| MACSJ0329| 6.51 ±1.37                       | 0.65 ±0.27       | 0.64 ±0.09       | 0.70 ±0.09     | 0.0193 ±0.002 | 0.1368 ±0.008 |
| RXJ1347  | 22.33 ±4.89                      | 0.39 ±0.30       | 0.40 ±0.09       | 0.30 ±0.1      | 0.0123 ±0.002 | 0.1528 ±0.014 |
| MACSJ0744| 11.94 ±2.81                      | 0.54 ±0.27       | 0.53 ±0.09       | 0.55 ±0.09     | 0.0155 ±0.002 | 0.1445 ±0.011 |

Table 2 Physical parameters derived for the CLASH sample. First column: cluster name; second: BCG mass derived from our model; third: BCG effective radius; fourth and fifth: spherical total masses inside 5 kpc and 100 kpc.

| Name     | \(M_{c}\) [\(10^{11}M_{\odot}\)] | \(R_{c}\) [kpc] | \(M_{5kpc}\) [\(10^{11}M_{\odot}\)] | \(M_{100kpc}\) [\(10^{13}M_{\odot}\)] |
|----------|---------------------------------|----------------|-------------------------------|-------------------------------|
| A383     | 9.16 ±0.29                      | 28.7 ±1.5     | 0.98 ±0.15                    | 1.96 ±0.3                    |
| A209     | 7.84 ±0.29                      | 25 ±1.5       | 1.31 ±0.15                    | 3.21 ±0.3                    |
| A2261    | 10.5 ±0.29                      | 40 ±1.5       | 1.75 ±0.15                    | 4.32 ±0.3                    |
| RXJ2129  | 13.73 ±0.29                     | 33 ±1.5       | 2.43 ±0.15                    | 3.49 ±0.3                    |
| A611     | 12.25 ±0.29                     | 34.6 ±1.5     | 1.63 ±0.15                    | 3.0 ±0.3                     |
| MS2137   | 6.55 ±0.29                      | 14 ±1.5       | 1.95 ±0.15                    | 3.5 ±0.3                     |
| RXJ2248  | 12.45 ±0.29                     | 38.5 ±1.5     | 2.17 ±0.15                    | 4.3 ±0.3                     |
| MACSJ1115| 11.44 ±0.29                     | 44.5 ±1.5     | 2 ±0.15                       | 3.56 ±0.3                    |
| MACSJ1931| 8.19 ±0.29                      | 31 ±1.5       | 1.37 ±0.15                    | 3.5 ±0.3                     |
| MACSJ1720| 8.99 ±0.29                      | 35.8 ±1.5     | 1.5 ±0.15                     | 3.32 ±0.3                    |
| MACSJ0429| 13.27 ±0.29                     | 41 ±1.5       | 2.32 ±0.15                    | 3.28 ±0.3                    |
| MACSJ1206| 11.96 ±0.29                     | 43 ±1.5       | 2.08 ±0.15                    | 4.08 ±0.3                    |
| MACSJ0329| 7.41 ±0.29                      | 20 ±1.5       | 1.24 ±0.15                    | 2.17 ±0.3                    |
| RXJ1347  | 13.8 ±0.29                      | 46.9 ±1.5     | 2.45 ±0.15                    | 4.5 ±0.3                     |
| MACSJ0744| 10 ±0.29                        | 37.1 ±1.5     | 1.67 ±0.15                    | 3.98 ±0.3                    |

In [5], we reproduced the correlations found by [38, 39],\(^2\) for MS2137, A963, A383, A611, A2537, A2667, and A2390. For these clusters, the total mass density profiles are well fitted by an NFW profile, while the central DM distribution is shallower than the total mass.

\(^2\)The quoted authors found correlations of the inner slope of the DM profile with the size of the BCG, the core radius, namely the constant density core of the cored NFW density profile [see Eq. 2 of 39], and the BCG mass, and finally the correlation between the masses contained inside 5 kpc and 100 kpc.
distribution.

The formation picture proposed by [38, 39] is characterized by a dissipational formation of BCGs, followed by a dissipationless phase. In this phase, as described by [13, 17, 42, 43, 47, 50–55], baryon clumps interact with DM through dynamical friction, “heating” DM and reducing the central cusp.

Our aims here are to use high-quality gravitational lensing observations from the CLASH survey [56] and investigate if CLASH clusters exhibit correlations that are similar to those observed in the [38, 39] clusters, to characterize the mass distributions of CLASH clusters, and to test the physical picture that was proposed by [38, 39] and confirmed by [5]. To this end, we perform an improved analysis on a sample of 15 X-ray-selected CLASH clusters compared to our previous work [5, 51].

In the present work, we will characterize the total mass density profiles of 15 CLASH clusters by means of a modified version of the semi-analytical model developed by [5, 51]. Here we take into account the following effects:

1. adiabatic contraction (AC) responsible for the steepening of the inner density profiles in the early stage of cluster formation,
2. interaction between baryonic clumps and DM through dynamical friction, which is responsible for “heating” the DM component and flattening the density profile,
3. supernovae (SN) feedback,
4. AGN feedback and other baryonic effects described in detail in Appendix.

The paper is organized as follows. In section 2, we describe the data used and provide a brief summary of our model. In section 3, we discuss the results, and section 4 is devoted to conclusions.

Throughout this paper, we adopt a concordance ΛCDM cosmology with \( \Omega_m = 0.27 \), \( \Omega_\Lambda = 0.73 \), and \( h = 0.7 \) with \( H_0 = 100h \text{ km s}^{-1}\text{Mpc}^{-1} \).

2 Data used and summary of the model

In this study, we use lensing data obtained from the CLASH survey [56], which studied the mass distributions of 25 high-mass clusters using high-quality gravitational lensing observations. Here we focus on a subsample of 15 X-ray-regular CLASH clusters for which strong and weak lensing data are available from both 16-band *Hubble Space Telescope (HST)* and wide-field weak-lensing observations [41]. The wide-field weak-lensing data were taken primarily with Suprime-Cam on the Subaru Telescope [40, 57, 58].

We exclude high-magnification CLASH clusters from our analysis because they are found to be highly disturbed merging systems [56]. We also exclude one X-ray-regular CLASH cluster (RXJ1532) for which no secure identification of multiple images has been made [59] and hence no central strong-lensing information is available. These selection criteria result in our sample of 15 CLASH clusters (Table 1).

The data we use are taken from [41] and given in the form of binned surface mass density profiles, spanning the radial range from 10 arcsec to 16 arcmin, and their bin-to–bin covariance matrices. It was found in [41] that the ensemble-averaged surface mass density profile of these X-ray-regular clusters can be well described by cuspy, sharply steepening density profiles, such as the NFW and Einasto profiles. Assuming the spherical NFW profile
for each cluster, [41] also found that the concentration–mass relation for the CLASH X-ray-selected subsample is in agreement with ΛCDM predictions, when the CLASH selection function is taken into account.

In [41], the binned surface mass density profiles were derived for a sample of 16 X-ray-regular and 4 high-magnification CLASH clusters using the weak- and strong-lensing data of [40, 59]. The mass profile solution for each cluster, \[ \Sigma = \{\Sigma_i\}_{i=1}^N \] with \( N = 15 \) bins, was obtained from a joint likelihood analysis of strong-lensing, weak-lensing shear and magnification data [41, see their Fig. 11]. The total covariance matrix \( C_{ij} \) accounts for the observational errors, the cosmic-noise contribution due to projected uncorrelated large scale structure, the systematic errors due to the residual mass-sheet degeneracy, and the intrinsic variations of the projected cluster mass profile due to halo triaxiality and correlated substructures.

In this paper, we generate 3D cluster density profiles whose surface density profiles match those obtained by [41], \( \Sigma \). For a given 3D density profile \( \rho(r) \), we compute the surface mass density \( \Sigma(R) \) by integrating \( \rho(r) \) along the line of sight,

\[
\Sigma(R) = 2 \int_0^\infty \rho(R, l) dl
\]

with \( R \) the projected cluster-centric radius [see also Secs. 5.2 and 5.2.2 of 41].

A set of the parameters \( p \) that specify the model can be inferred by minimizing the \( \chi^2 \) function [see 41],

\[
\chi^2(p) = \sum_{i,j=1}^N [\Sigma_i - \hat{\Sigma}_i(p)] C^{-1}_{ij} [\Sigma_j - \hat{\Sigma}_j(p)],
\]

where \( \hat{\Sigma}_i \) is the surface mass density predicted by the model.

Estimates of cluster mass and its radial distribution can be obtained in different ways. The standard approach, as adopted by [41], is to use the NFW profile, which gives a good approximation to the projected total density profile for cluster-size halos out to their virial radius [60]. The NFW density profile is given by

\[
\rho(r) = \frac{\rho_c \delta_c}{(r/r_s)(1 + r/r_s)^2},
\]

where \( r_s = r_\Delta/c_\Delta \) is the scale radius, \( c_\Delta \) the concentration parameter, \( \rho_c \) the critical density of the universe, \( r_\Delta \) the radius inside which the mean density is \( \Delta \times \rho_c \), and

\[
\delta_c = \frac{c_\Delta^3}{3 \ln(1 + c_\Delta) - c_\Delta/(1 + c_\Delta)}.
\]

The total mass enclosed within a sphere of radius \( r_\Delta \) is denoted as \( M_\Delta = (4\pi/3)\Delta \rho_c r_\Delta^3 \). The NFW mass and concentration parameters for the CLASH sample are reported in Tables 2 and 3 of [41]. The typical mass and concentration for the X-ray-selected CLASH sample are \( M_{200} \approx 1.0 \times 10^{15} M_\odot h^{-1} \) and \( c_{200} \approx 3.8 \) at \( z \sim 0.35 \) [41, 61].

In order to characterize the 3D density profile of the clusters, we use a modified version of the physical cluster model described in [5]. Here we provide a qualitative summary of this model (see also Appendix).

In our model, the protostructure contains baryons and DM. After growing in a linear way, the density contrast becomes large enough to stop the protostructure expansion with
the Hubble flow, making it recollapse. DM collapses first and baryons follow. Clumps are formed because of radiative processes, and collapse to the protostructure center to form stars. At high redshifts ($z \simeq 5$ in the case of a protostructure of $10^9 M_\odot$), the collapsing DM compresses baryons (adiabatic contraction). The formed clumps transfer energy and angular momentum to DM through dynamical friction. Then the amplitude of DM random motions increases, and DM moves toward the outskirts of the protostructure, resulting in a reduction of the central DM density of the forming structure and erasing or flattening of the initial cuspy profile. Protostructures giving rise to rotation supported galaxies suffer from a further flattening due to the acquisition of angular momentum in the collapse phase. SN explosions at a later epoch ($z \simeq 2$) produce expulsion of gas, and the smallest clumps remaining after star formation are disrupted [50]. AGN feedback has a similar effect on larger scales to that of SN feedback [48].

We constrain our model for each of the CLASH clusters as follows. In [5, 51], the DM density profile was expressed as $\rho_{DM} = F(M_{500}, F_b, j)$, with $M_{500}$ being the cluster halo mass, $f_b = M_b/M_{500}$ the cluster baryon fraction, and $j$ the random angular momentum In [5, 51], the density profile of the clusters studied were reproduced by a) assuming that the cluster final mass in the model is the same as the observed clusters, b) assuming that the cluster baryon fraction is equal to that calculated with the [62] data, and c) adjusting the value of the random angular momentum to reproduce the observed clusters profiles.

In the present paper, the CLASH density profiles have been reproduced in a slightly different manner. As in [5, 51], we assume that the final mass of the protostructure generating each of the CLASH clusters is equal to the observed mass of that cluster, namely $M_{500}$ from [41]. The cluster baryon fraction $F_b$ is not fixed as in [5, 51]. Instead, we assume that the system initially has a baryon fraction that is equal to the “universal” baryon fraction $f_b = 0.17 \pm 0.01$ [66], while the final baryon fraction is determined from the star-formation processes (see Appendix). The final baryon fraction, obtained in this way, has been compared with the baryon and gas fractions of the clusters in [62] and is found to be in agreement with their results.

The “random” angular momentum has been fixed following [67, 68]. In our study, we do not have the DM and baryon density profiles but just the total density profile from lensing. Hence, unlike [5, 51], the DM content for the CLASH clusters cannot be determined from fitting the model to the total and baryon density profiles. In order to derive the DM density profiles for the CLASH clusters, we subtract from the total density (given by the data) the baryon density given by the model.

3 Results and discussion

In [5, 13, 17, 51], we showed how the environment, angular momentum, and baryon content influence the characteristics of cluster structure. The inner density profiles of clusters are flatter in clusters with larger angular momentum [67–74] and larger baryon fraction (especially in the central region). In [5], we reproduced the total and DM density profiles of [38, 39] and those correlations found by these authors.

The results obtained by [38, 39] are based on strong-lensing, weak-lensing, and improved stellar kinematics with respect to their previous work [33]. The [39] data reduced the

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3 We recall that clusters of galaxies are not supported by rotation, and that their “ordered” angular momentum, coming from tidal torques, has very similar and low values (some km s$^{-1}$) [63, 64], for all clusters, and in terms of the spin parameter $\lambda$ can be fixed to the typical value $\lambda = 0.03$ [65].
Figure 1 Surface mass density profiles for A383 (top-left) and A611 (top-right). The diamond symbols with error bars show the data from [41], while the central and external lines represent the best-fit NFW profile and the 68% CL. The bottom-left and bottom-right panels show the 3D density profiles of A383 and A611, respectively. The blue, magenta, and brown bands represent the stellar, DM, and total matter density profiles, respectively. The width of the band indicates the 1σ uncertainty [see Sec. 4.3 of 39].

The degeneracy between the stellar and DM masses, thanks to the determination of the stellar mass scale and by accounting for the BCGs homogeneity [39, see their Sec. 4]. This resulted in a more physically consistent analysis. They showed that the total density profile is well approximated by a cuspy NFW profile, while the DM profile is found to be flatter.

In Fig. 1 (top panels), we compare the total surface mass density profiles Σ obtained by [41] (diamond symbols with error bars) with our model predictions (the three lines, rep-
resenting the best-fit NFW profile and the 68% CL), for the case of A383 (left panel) and A611 (right panel). As shown here, our model reproduces well the surface mass density profiles and shows that, similarly to what was found in [5, 38, 39], the total mass profile is well approximated by a cuspy NFW-like profile for both clusters. In the bottom panels, we show the 3D density profiles of A383 and A611 obtained with our semi-analytical model. The blue band represents the stellar content, the magenta one is the DM content, the brown one is the total mass, the dashed lines represent the total mass from the [41] data, and the line segment indicates the slope of the NFW profile. In each case, the width of the band indicates the 1σ uncertainty [see Sec. 4.3 of 39]).

Tables 1 and 2 summarize all the CLASH parameters found in our analysis. The 2D density profile slope, $\alpha_{2D}$, was obtained using 3 adjacent radial bins fitted with a 2-parameter power-law profile, accounting for the averaging effect [41]. The 3D DM density slope $\alpha$ is obtained from fitting the spherical DM profile with a generalized NFW profile (gNFW).[5]

In the inner $\approx 5 - 10$ kpc region [see 39, Fig. 3] of A383 and A611, where the BCG mass becomes comparable or larger than the DM mass, Fig. 1 (bottom panels) shows that the density profiles flatten (similarly for the other clusters, whose plots were not shown). The increased role of the baryon mass at these radii (mainly the BCG mass) steepens the total density profile compared with the DM density profile, whereas for radii $\geq 5 - 10$ kpc, in all clusters, DM dominates over the baryon component. As a result, the outer total and DM density profiles are very similar, and their slopes outside the inner region are comparable in the different clusters, in agreement with the NFW profile. Since the total density profile is consistent with the NFW profile at $r \approx 5 - 30$ kpc, this implies a “tight coordination” between the stellar distribution and the inner DM profile, as found by [39]. Since the total mass (DM and baryonic matter) in the inner cluster region follows the NFW form, while the baryonic component is dominant in the 5 − 10 kpc central region, the DM central density profile is flatter than the NFW profile.

These trends are more clearly visible in Fig. 2 and better in Figs. 3 and 4.

In Fig. 2 we show the correlation between the 3D DM density slope $\alpha$ and the baryon fraction $F_b$ (left panel) and the correlation between $\alpha$ and the stellar mass fraction $f_{\text{star}}$ (right panel). The Spearman correlation coefficients are $\approx 0.43$ and $\approx -0.44$, respectively. We note that these correlations are stronger than those of the 2D slope of the total density ($\alpha_{2D}$) with $F_b$ and $f_{\text{star}}$ (not shown). This is because, unlike the 2D total mass profile, the DM density profile varies significantly from cluster to cluster.

Figure 3 compares the inner 3D DM slope ($\alpha$) and the BCG stellar mass ($M_e$). The DM slope parameter $\alpha$ was obtained by parameterizing each cluster with a gNFW profile. From the figure, it is evident that the larger the cluster’s BCG mass, the flatter its inner density profile, in agreement with [5, 39, 51]. This is expected for the following reason: Since the total (DM + baryons) density profile is well represented by a cuspy NFW profile and is dominated by stellar baryons inside 5 − 10 kpc, this implies that the steeper the baryon inner profile, the flatter the DM profile. The best-fit line is derived from orthogonal distance

$\rho_{\text{DM}}(r) = \frac{\rho_s}{(r/r_s)^\alpha(1 + r/r_s)^{3-\alpha}},$ (3.1)

with a central cusp slope given by $d \log \rho_{\text{DM}}/d \log r \to -\alpha$ for $r \to 0$.
Figure 2  Inner 3D slope of the DM density $\alpha$ versus cluster baryon fraction $F_b$ (left), and $\alpha$ versus stellar mass fraction $f_{\text{star}}$ (right), obtained with our model (square symbols with error bars). In each panel, the solid line represents the orthogonal distance regression (ODR) fit to the data. The Spearman correlation coefficients for the respective relations are 0.43 and -0.44.

regression (ODR), which takes into account the uncertainties in the variables different from the least-square method. We calculated again the Spearman correlation coefficient, which in this case is $-0.6$, with a $p$-value of 0.02,\(^6\) testifying for the correlation between $\alpha$ and $M_e$.

Here, we cannot make a statistical comparison between the parameters derived from our modeling and those measured directly from observations, because there is just one systematic study of CLASH BCGs \cite{75} in the literature, and it does not investigate the $\alpha-M_e$ correlation. The $\alpha-M_e$ relation was studied just for three of the CLASH clusters, namely MS2137, A383, and A611 by \cite{5, 39}. Our results for the $\alpha-M_e$ relation are in agreement with \cite{39}. Here, we would like to point out that a comparison of the BCG masses common to \cite{75} and \cite{5, 39} (MS2137, A383, A611) shows that while those in \cite{5} are in agreement with \cite{39}, they do not agree with \cite{75} (except for A383).

In Fig. 4, we show the $\alpha-R_e$ relation, with $R_e$ the BCG effective radius (radius containing half of the total light). The figure shows that a cluster with larger $R_e$ have a shallower inner slope, which is in line with what was discussed above. That is, clusters with larger BCGs contain more stellar baryons and thus less DM ($M_{\text{DM}} = M_{\text{total}} - M_b$) in their central region, resulting in a flatter DM slope. The solid line in the figure represents the ODR fit, with Spearman correlation coefficient of $-0.63$ and a $p$-value of 0.0012.

In Fig. ??, we show the correlation between $R_e$ and the halo mass $M_{500}$ (at $r_{500} \sim 1.4\, \text{Mpc}$ for our sample). The solid line represents again the ODR fit. This comparison exhibits a positive correlation with a Spearman correlation coefficient of $-0.68$ and a $p$-value of 0.005. This is in agreement with previous studies \cite[e.g.,][]{76}, but in contradiction again with \cite{75}. The latter found no correlation between the BCG mass and the cluster.

\(^6\)The probability that the “null” hypothesis (the true correlation is zero) is true.
Figure 3 Correlation between the inner DM slope $\alpha$ and the BCG mass $M_e$. The BCG stellar mass $M_e$ was obtained using our model. For each cluster, the uncertainty of the BCG mass is assumed to be 0.07 dex [5, 39]. The solid line is the ODR fit to the data. The Spearman correlation coefficient is -0.6.

halo mass. They ascribed this lack of correlation to a selection bias in the CLASH sample (whereby clusters with BCGs in a narrow mass range have been selected), but without a clear justification. However, as previously noticed, the BCG mass estimates between [75] and [39] in their overlapping clusters are in striking conflict with each other. Moreover, the inclusion of the high-magnification CLASH clusters in [75] may imply a bias in their results, because they are highly disturbed merging clusters.

Our model [13, 17] can explain the resulting density profiles (Fig. 1) and correlations (Figs. 3 and 4) as follows: The DM protostructure starts to collapse at high $z$ (linear phase), forming potential wells for baryons to fall in. In their collapse, baryons radiate away part of their energy and form clumps, condensing into stars [see 77, Secs. 2.2.2 and 2.2.3], while compressing DM [“adiabatic contraction”, 78, 79]. At around $z \geq 5$ [13, see Figs. 3 and 5 therein], this process dissipationally produces a steep density profile, the main structure of the BCG [see also 80, 81] with scale radius $R_e \approx 30$ kpc, similar to size-scales of high-redshift massive galaxies [38, 39, 82]. Extra stars are added in the outer regions by satellite mergers onto the proto-BCG [e.g., 83, 84].

Moreover, dynamical friction from DM particles induces orbital decay of the baryon
Figure 4 Correlation between the inner DM slope $\alpha$ and the BCG effective radius $R_e$. The line represents the ODR fit to the data, with a Spearman correlation coefficient of $-0.63$ and a $p$-value of 0.0012.

clumps. As a result, DM particles move outward as the baryon clumps move toward the cluster center, thus reducing the central DM density [13, 42, 43, 47, 50, 55, 85].

AGN feedback has been proposed as an alternate mechanism to flatten the DM profile [e.g., 48]. However, it appears to be “too effective” because the 10 kpc core produced is much larger than what is observed [56].

Our physical model thus predicts, at the same time, a flattening of the inner DM density profile and an anti-correlation between the inner DM slope $\alpha$ and the cluster’s central baryon content [13, 17, 43]. This is because the density profile shape is primarily the result of interaction between DM and baryons clumps through dynamical friction and subdominantly of the action of SN/AGN feedback.

Dissipative baryonic formation of the proto-BCG ($z \geq 2$) follows in our model, where stars merge onto the BCG [e.g., 83, 84] and satellites infall toward the cluster center, thus kinematically “heating” DM and flattening the inner DM slope [13, 42, 47, 85].

A further correlation is expected between the star-dominated, innermost cluster mass and the cluster mass within a region that is already established before the BCG formation and thus almost unchanged subsequently [86]. This has been examined in Fig. 6 by comparing the the cluster mass within 5 kpc (composed mainly of stars) and that within 100 kpc (mainly of
Figure 5 Correlation between the BCG effective radius $R_e$ and the cluster halo mass $M_{500}$. The solid line shows the ODR fit, with a Spearman correlation coefficient of $-0.68$ and a $p$-value of 0.005.

DM) for our CLASH sample. We find a Spearman correlation coefficient of 0.64 and a $p$-value of 0.011. Such a strong correlation implies (see also Sec. 4) that the cluster’s progenitor halo and the innermost BCG region were formed at higher redshifts, and they have been subject to little evolution subsequently, whereas the cluster outskirts have grown via the secondary mass accretion. This “inside-out” growth scenario of cluster formation has been proposed in the framework of $\Lambda$CDM [e.g., $87, 88$] and recently confirmed from a joint lensing and X-ray analysis of the cluster-halo fundamental plane for the CLASH sample $[89, 90]$.

These correlations confirm the central role of baryons in shaping the DM density profile in galaxy clusters, in particular for the central $\lesssim 10$ kpc region. The BCG characteristics correlate with baryonic and cluster masses. The total density profile agrees well with the NFW form, while the DM density profile exhibits a shallower inner slope with $\alpha < 1$.

Our findings are consistent with $[33, 36]$ on the clusters MS2137 and A383. Our results are also in line with $[91]$, who found that the innermost region of the (non-CLASH) cluster A2589 has a shallow DM profile assuming reasonable values of mass-to-light ratios. Our cored DM profiles have a mean slope of $\alpha = 0.54 \pm 0.05$, in agreement with $[48, 49]$ and $[39]$.

DM-only simulations produce halos with cuspy, steeper density profiles. In the Phoenix project, $[92]$ analyzed zoomed-in re-simulations of cluster-size halos drawn from a cosmologically representative volume in $\Lambda$CDM, finding that the central density cusp (at their
Figure 6 Total cluster mass contained within 100 kpc (dominated by DM) versus that contained within 5 kpc (dominated by stellar baryons). The solid line is the ODR fit with a Spearman correlation coefficient of 0.64 and a p-value of 0.011.

The innermost resolved radius of $r \sim 2 \times 10^{-3} r_{200}$ has an average logarithmic slope of $\alpha \simeq 1.05$ with a halo-to-halo scatter of $\sim 20\%$. This level of “diversity” is apparently at odds with the observed shallow slopes of the inner DM density. Including the dominant central baryonic physics on top of such DM simulations can reconcile the discrepancy and reproduce the cored DM distribution, by accounting for dynamical friction from baryonic clumps [13, 42, 47, 53, 85] and density flattening driven by SN/AGN feedback [48, 93]. These mechanisms are described in Appendix.

On the other hand, more radical solutions have also been proposed: alternative DM models [e.g., 94–97], modifications of the matter power spectrum at small scales [e.g. 98], and even modified gravity.

We find that the total density profile $\rho_{\text{tot}}(r)$ of the X-ray-selected CLASH sample is NFW-like, as found in previous studies [40, 41, 58]. The average slope of the total density for our sample is $\langle \gamma_{\text{tot}} \rangle = 1.05 \pm 0.02$, where $\gamma_{\text{tot}} = -d \log \rho_{\text{tot}} / d \log r$, and the average slope is obtained by linear fitting in the plane of $\log r$–$\log \rho_{\text{tot}}$ in the radial range $r = (0.003 - 0.03) \times r_{200}$.\footnote{The BCG and the DM halo are distinct components of the cluster model, while $\gamma_{\text{tot}}$ is a derived, composite parameter.} This is in agreement with collisionless DM simulations ($\gamma_{\text{tot}} \simeq 1$), and in
line with the finding of [38, Sec. 9], \( \langle \gamma_{\text{tot}} \rangle = 1.16 \pm 0.05^{+0.05}_{-0.07} \) defined in the same range \( r = (0.003 - 0.03) \times r_{200} \).

Observational results of the asymptotic total density profiles also agree with our results (i.e., NFW-like at \( r \gtrsim 5 - 10 \text{kpc} \)): \( \alpha_{\text{tot}} = 0.96^{+0.31}_{-0.49} \) for MACSJ1206 [99] and \( \alpha_{\text{tot}} = 1.08 \pm 0.07 \) for A383 [100]. \(^8\) Excluding the innermost \( 40 \text{kpc} h^{-1} \) region, the stacked strong+weak lensing analysis of four “superlens” clusters (A1689, A1703, A370, Cl0024+17) gives \( \alpha_{\text{tot}} = 0.89^{+0.27}_{-0.39} \) [35].

4 Conclusions

In this paper, we have studied and characterized the total density profiles for a sample of 15 X-ray-selected CLASH clusters by improving our earlier analysis [5] based on several clusters from [38, 39]. The primary goal of this study was to test the physical picture of cluster formation proposed by [38, 39] in the framework of a modified version of the physical model developed by [5, 51]. To this end, we analyzed binned surface mass density profiles of [41] derived from their strong-lensing, weak-lensing shear and magnification analysis of high-quality HST and Subaru data. For each cluster, we extracted the radial profile of the total 3D density assuming spherical symmetry. We have used our semi-analytical model to interpret the total 3D density profile, which allows us to compute the baryon density profile and thus the DM density profile for the cluster.

The total 3D mass density profile for our sample is characterized by a logarithmic slope \( \langle \gamma_{\text{tot}} \rangle = 1.05 \pm 0.02 \) in the radial range \( r = (0.003 - 0.03) \times r_{200} \), in agreement with several previous studies (e.g. [38, 39]).

Stellar mass dominates the total mass at \( r \lesssim 5 - 10 \text{kpc} \), while the cluster outskirts are dominated by DM. Such segregation reveals a “tight coordination” between the inner DM and stellar distributions, as also implied by interplay of DM and baryons that generates the NFW-like total density profile. The correlation between the mass inside 5 kpc and that inside 100 kpc (Fig. 6) further supports such a tight coordination and points to similar formation time-scales of the BCG and the inner cluster region. Thus, the cluster’s final configuration depends on the baryonic content and their formation process [17]. Therefore, in the context of hierarchical structure formation models, we should expect tight correlations between the final inner baryonic content and the BCG mass, as well as between the total baryonic and cluster masses [see 101].

Since the DM and baryon contents sum to the total mass and the baryons dominate the inner 5 – 10 kpc region, the inner slope of the DM density must be shallower than that of the total density, that is, \( \alpha < 1 \), as shown in Fig. 1. The observed inner DM slopes \( \alpha \) span the range \( [0.30, 0.79] \).

Correlations were also examined between several of the characteristic quantities of clusters (e.g., \( R_e, M_e, M_{500} \)). Our findings are summarized as follows:

a. The inner 3D slope of the DM density, \( \alpha \), is anti-correlated with the BCG effective radius \( R_e \). The anti-correlation reflects the balance between DM and the BCG in the cluster center. For an NFW-like total mass profile, clusters with more massive BCGs contain less central DM, implying a flatter DM slope.

\(^8\)It is important to stress that, similarly to [38], \( \langle \gamma_{\text{tot}} \rangle \) is the average slope of the total density measured in the range \( r = (0.003 - 0.03) \times r_{200} \) and is different from \( \alpha_{\text{tot}} \), which is the asymptotic inner slope of the total density assuming the gNFW profile.
b. Similarly, the inner DM slope $\alpha$ and the BCG mass $M_e$ are anti-correlated with each other. This indicates again that a larger content of the central baryons gives rise to flatter DM profiles.

c. The cluster halo mass $M_{500}$ and the BCG effective radius $R_e$ are correlated with each other, as found in previous studies [76].

d. The cluster mass inside 5 kpc, dominated by the stellar baryons, and the cluster mass inside 100 kpc, dominated by DM, are correlated with each other. This hints at early formation of the BCG and the inner cluster region, while subsequent, continuous mass accretion played a fundamental role in the growth of cluster outskirts [e.g., 89, 90].

These observed correlations are in support of the physical picture proposed by [38, 39], that clusters form from a dissipative phase that leads to steepening the central stellar density, followed by a second dissipationless phase in which interactions between baryonic clumps and DM through dynamical friction kinematically heat the latter, leading to flat DM density profiles [17, 42, 47, 50, 51, 53, 85].

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A Appendix: the model

In this paper, we used, as in several previous studies [5, 15–17, 51], a semi-analytical model (SAM) introduced in [13] and extended in [102, 103]. This model incorporates a secondary infall model (SIM) [68, 73, 104, 105] that takes into account the effect of DM adiabatic contraction [78, 79, 106] of ordered and random angular momentum [73, 107], as well as of baryon-DM angular momentum transfer through dynamical friction [13, 15, 42–44, 47, 50], in contrast to previous SIMs. It also accounts for cooling, reionization, star formation, and SN/AGN feedback (see the following).

It starts from the Hubble flow expansion of a perturbation starting in the linear phase, following its evolution until the maximum expansion (turn-around) and recollapse to a final density, given by [108, 109]

$$\rho(x) = \frac{\rho_{a}(x_{m})}{f(x_{i})^{2}} \left[ 1 + \left. \frac{d \ln f(x_{i})}{d \ln x_{m}(x_{i})} \right]^{-1} \right.$$

with $f(x_{i}) = x/x_{i}(x_{i})$, the so-called collapse factor [see Eq. A18, 13], and

$$x_{m} = g(x_{i}) = x_{i}\frac{1 + \delta_{i}}{\delta_{i} - (\Omega_{i}^{-1} - 1)} .$$

Here $x_{m}$ gives the turn-around radius $x_{m}(x_{i})$, where the initial density parameter is $\Omega_{i}$, and a given shell’s average overdensity reads $\delta_{i}$. 


The perturbation contains DM and baryons, the latter initially in the gas phase, with the amount set by the cosmic baryon fraction $f_b = 0.17 \pm 0.01$ [66] [while it is 0.167 in 1], itself obtained from the star-formation processes described in the following.

The tidal torques of larger scales on smaller-scale structures produce the “ordered” angular momentum $h$ through the tidal torque theory [107, 110–113], while random velocities generate “random” angular momentum [114], $j$, expressed in terms of the eccentricity $e = r_{\text{min}}/r_{\text{max}}$ [67]. Here $r_{\text{max}}$ denotes the apocentric radius, and $r_{\text{min}}$ the pericentric radius.

The dynamical state of the system induces eccentricity to be corrected as in simulations of [68],

$$e(r_{\text{max}}) \simeq 0.8 \left( \frac{r_{\text{max}}}{r_{\text{ta}}} \right)^{0.1}$$ (A.3)

for $r_{\text{max}} < 0.1 r_{\text{ta}}$.

Dynamical friction was introduced in the equation of motion with a deceleration term [Eq. A14, 13], whose coefficient proceeds as in [115] and [13, Appendix D].

Baryon collapse induces adiabatic contraction (AC) of DM, according to the following mechanism. From a protostructure of $f_b = M_b/M_{500} \ll 1$ baryons and $1 - f_b$ DM, the baryons cool and collapse toward the halo center, forming the distribution $M_b(r)$. This compresses DM, relocating particles from $r_i$ to

$$r \left[ M_b(r) + M_{\text{DM}}(r) \right] = r_i M_i(r_i)$$ (A.4)

[78], with $M_i(r_i)$ the initial total mass and $M_{\text{DM}}$ the final DM distribution. Assuming equal initial baryon and DM distributions [116–119] and the final Hernquist baryon distribution [118–120], $M_i(r_i)$ and $M_b(r_i)$ are given, and the DM mass distribution in the absence of shell crossing is obtained by solving Eqs. (A.4) and (A.5),

$$M_{\text{DM}}(r) = (1 - F_b) M_i(r_i)$$ (A.5)

to find the final halo distribution. Conservation of angular momentum, represented by the product of the orbit-averaged radius $\bar{r}$ with its inner mass [79],

$$M(\bar{r}) \bar{r} = \text{const.},$$ (A.6)

using

$$\bar{r} = \frac{2}{T_r} \int_{r_{\text{min}}}^{r_{\text{max}}} r \frac{dr}{v_r},$$ (A.7)

and $T_r$ as the radial period, improves the model.

Our treatment of star formation, gas cooling, reionization, and supernovae feedback (SNF) follows [77, 121, Secs. 2.2.2 and 2.2.3].

With reionization as in [77], the baryon fraction is reduced in the redshift range $z = [11.5, 15]$ and modified as

$$f_{b, \text{halo}}(z, M_{\text{Vir}}) = \frac{f_b}{[1 + 0.26 M_F(z)/M_{\text{Vir}}]^3},$$ (A.8)

calculated with $M_F$ the “filtering” mass [see 122] and the virial mass, $M_{\text{Vir}}$ ($\Delta = 200$) was converted to the cluster halo mass $M_{500}$ following [123]. Although the [107] treatment yields

$^9$The gas mass, $M_{\text{gas}}$, and mass in stars, $M_*$, combine into the total baryonic mass, $M_b$. 

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similar results, a classical cooling flow [e.g., 124] [see Sec. 2.2.2 of 77] is used here for gas cooling.

We follow [121] to account for star-formation processes. SNF follows the treatment of [125]. The blast-wave SNF [126] was used in [127]. Although our choice of formalism is similar, it is not very fundamental, and our model differs from their SNF model [e.g., 127] in the occurrence of the flattening process before star formation and in our gravitational source of energy, whereas the SNF flattening process occurs after star formation with stellar feedback, which is in act as the energy source after core formation disrupting the gas clouds.

The stellar formation occurs once a disc is formed from gas with the star formation rate

$$\psi = 0.03 M_{\text{sf}} / t_{\text{dyn}},$$

(A.9)

which is computed with the disc dynamical time, $t_{\text{dyn}}$, and the gas mass $M_{\text{sf}}$ contained in regions where its density is above a given density threshold, $n > 9.3 \text{ cm}^{-3}$, in the same way as in [127]. We use the Chabrier [128] initial mass function (IMF), forming stars with

$$\Delta M_* = \psi \Delta t$$

(A.10)

per units of time $\Delta t$.

SNF injects energy in the interstellar medium (ISM) at a rate of

$$\Delta E_{\text{SN}} = 0.5 c_{\text{halo}} M_* V_{\text{SN}}^2,$$

(A.11)

obtained with the energy injected per supernova and per unit solar mass, $V_{\text{SN}}^2 = \eta_{\text{SN}} E_{\text{SN}}$. The fixed efficiency $c_{\text{halo}} = 0.35$ [77] of the disc gas reheating by this energy injection is computed with the supernova number per solar mass, $\eta_{\text{SN}} = 8 \times 10^{-3} / M_\odot$, assuming a Chabrier IMF [128], and with the typical energy released in a SN explosion, $E_{\text{SN}} = 10^{51}$ erg.

This energy injection reheats the gas in proportion to the star-formation number,

$$\Delta M_{\text{reheat}} = 3.5 \Delta M_*,$$

(A.12)

inducing a thermal energy change

$$\Delta E_{\text{hot}} = 0.5 \Delta M_{\text{reheat}} V_{\text{Vir}}^2,$$

(A.13)

where $V_{\text{Vir}} = (10GH(z)M_{\text{Vir}})^{1/3}$ with $H(z)$ being the Hubble constant at redshift $z$.

The last will eject a quantity of the hot gas equal to

$$\Delta M_{\text{eject}} = \frac{\Delta E_{\text{SN}} - \Delta E_{\text{hot}}}{0.5 V_{\text{Vir}}^2}$$

(A.14)

from the halo with the condition $\Delta E_{\text{SN}} > \Delta E_{\text{hot}}$.

All the quantities evaluated at virial radius (e.g., $M_{\text{Vir}}, V_{\text{Vir}}$) were converted to corresponding quantities at $\Delta = 500$, by following [123].

Subsequently, the hot gas can be accreted by the halo into its hot component in link with the central galaxy [125, 129].

Accounting for AGN quenching is required for masses $M \simeq 6 \times 10^{11} M_\odot$ [130]. Here, we use the prescription of [48, 131] to implement AGN feedback: A conjunction of the star density above $2.4 \times 10^6 M_\odot \text{kpc}^{-3}$, the gas density at 10 times the stellar density, and the 3D velocity dispersion exceeding 100 km s$^{-1}$ creates an initial (seed) super-massive black hole (SMBH) with $10^5 M_\odot$, which grows with accretion and produces AGN feedback following a variant of the [132] model modified according to [48].

Finally, the robustness of the model has been demonstrated in various manners as summarized below.
a. DM heating by collapsing baryonic clumps that induce cusp flattening was predicted in 2009 for galaxies, and 2012 for clusters. The predictions are in agreement with the studies of [42, 47, 50, 53–55, 85]. Our model is compared with the smoothed particle hydrodynamics (SPH) simulations of [14] in Fig. 4 of [16].

b. Correct predictions for the galaxy density profiles [13, 44] and the cluster density profiles [51] were made before SPH simulations of [14, 133] and [49], respectively.

c. The dependence of the inner DM density slope on the halo mass [15, Fig. 2a, solid line] was predicted before the quasi similar results of [127, Fig. 6, solid lines] presented in terms of the rotation velocity, $V_c$ ($V_c = 2.8 \times 10^{-2} M_{\text{vir}}^{0.316}$ [134]).

d. The dependence of the cluster baryon fraction on the inner DM slope was also predicted in [51], which was later studied and confirmed in SPH simulations [127].

e. Figure 1 of [102, 103] compares the dependence of the inner DM slope on the halo mass with the [127] simulations. Predictions for the Tully–Fisher relation, the Faber–Jackson relation, and the $M_* – M_{\text{halo}}$, relation are compared with simulations in Figs. 4 and 5 of [102, 103].

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