Tactical preference based objectives for solving an evasion problem of fighter in air combat

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Abstract. Evasive maneuvers of fighter only consider the objective of miss distance in traditional air combat decision problems. The amalgamative tactical demands of achieving self-conflicting evasive objectives in actual air combat are taken into account in this paper. A method to generate a strategy of evasive maneuvers based on tactical preference is proposed. Tactical preference is given to higher miss distance, less energy consumption, and higher terminal superiority. The evasion problem in air combat is defined and reformulated into a multi-objective optimization problem, which is solved by a multi-objective evolutionary algorithm. Simulation results show the feasibility and effectiveness of the proposed method, and a set of approximate Pareto-optimal solutions are obtained to satisfy the tactical preference.

1. Introduction

Fighter aircraft are faced with increasingly threatening high-precision air-to-air missiles (AAM) in the modern air combat mode. How to minimize the lethality of enemy AAM through evasive maneuvers is a necessary skill for the fighter, which is of great significance to improve the survival probability of the fighter. Therefore, the decision of evasive maneuvers of the fighter has received widespread attention.

Imado [1] and Akdag [2] made a comparative study of some features of different evasive maneuvers against a tactical missile through mathematical simulation and analysis. The analytic solution of optimal maneuver has been solved by a zero-sum two-person differential game method [3-4]. Ong [5] treated the problem as an approximated parameter optimization problem, and sequential quadratic programming was used to solve it. A receding horizon control scheme for obtaining near-optimal controls in a feedback form was addressed in [6]. A similar problem was solved in [7] using nonlinear model predictive control, and the Gauss Pseudospectral method was used to solve the model. The search for an optimal evading solution was performed by using parallel evolutionary programming in [8]. Besides, an evasive maneuver decision was presented based on the artificial neural network in [9] for evading incoming missile.

Evasive maneuvers of fighter only consider the objective of miss distance in traditional air combat decision problems. However, the fighter usually has multiple tactical objectives when confronting incoming missiles in actual air combat [10], such as higher miss distance, less energy consumption [11] and higher terminal superiority. Therefore, this problem that involves several objectives has no unique optimal solution but rather a set of approximate Pareto-optimal solutions, which can exhibit different tactical preferences of the pilot [12]. The evasion problem in air combat is defined and reformulated into a multi-objective optimization problem. Then a multi-objective evolutionary algorithm based on
2. Problem analysis and formulation

2.1 Description of the problem
Ensuring tactical superiority and survivability in air combat is crucial for the fighter. In the face of incoming missile, the essential prerequisite of the fighter threatened by AAM is to avoid being hit. On this basis, consider that air warfare is a continuous and multi-round process with missile attacks. Therefore, the fighter needs to consider the whole battle efficiency and tactical superiority, not just successful evasion.

In this research, we will consider multiple tactical objectives of evasion in realistic air combat, including higher miss distance, less energy consumption, and higher terminal superiority. Maximizing the miss distance means increasing the survival probability, decreasing energy consumption means more energy for subsequent multi-round combat after a successful evasion, and maximizing the terminal superiority means a superior situation in the next missile duel.

This research focuses on defining and reformulating the evasion problem in air combat into a multi-objective optimization problem, then finding a non-dominant and feasible solution set with tactical preference for the problem. Firstly, establish dynamical models and constraints of the fighter and missile. Then simulation end conditions of the evasion problem in three-dimensional space are defined. The details of the optimization model of evasive maneuvers strategy are presented, and the objective space is solved by digital simulation. Finally, the MOEA/D is designed to find the approximate Pareto-optimal strategy set of evasive maneuvers.

It should be noted that several assumptions that simplify the problem to a certain extent without losing practicability are listed:
(a) The fighter can obtain real-time status information of the missile.
(b) Not consider the case of adopting active or passive jamming measures.
(c) The fighter can not evade the missile without maneuver.
(d) Ignore the change in fighter weight and the earth surface effect during evasion.

2.2 Fighter model
The influence of the fighter’s trajectory on evasive effects is focused in this study, so simplified dynamic equations represented by a three-degree-of-freedom and point-mass model are used to solve the evasive trajectory of the fighter. The equations of motion are

\[
\begin{align*}
\dot{x}_f &= v_f \cos \theta_f \cos \varphi_f \\
\dot{y}_f &= v_f \sin \theta_f \\
\dot{z}_f &= v_f \cos \theta_f \sin \varphi_f \\
\dot{v}_f &= \frac{1}{m_f} (\eta T_{f,\text{max}} \cos \alpha_f - D_f) - g \sin \theta_f \\
\dot{\theta}_f &= \frac{\cos \mu_f (\eta T_{f,\text{max}} \sin \alpha_f + L_f)}{m_f v_f} - \frac{g}{v_f} \cos \theta_f \\
\dot{\varphi}_f &= \frac{\eta T_{f,\text{max}} \sin \alpha_f + L_f}{m_f v_f \cos \theta_f} \sin \mu_f
\end{align*}
\]

Six variables used to describe the motion in (1) are down range \(x_f\), altitude \(y_f\), cross range \(z_f\), velocity \(v_f\), flight-path angle \(\theta_f\), and heading angle \(\varphi_f\) of the fighter, respectively. \(L_f\), \(D_f\), and...
$T_{f_{\text{max}}}$ are lift force, drag force and the maximum thrust, respectively, which are functions of $y_f$, $\alpha_f$, and $v_f$. The $T_{f_{\text{max}}}$ is assumed to be constant in this paper. Angle of attack $\alpha_f$, bank angle $\mu_f$, and thrust coefficient $\eta$ are used for the control of the fighter.

The dynamic equations are constrained in this research for the fighter, that is,

$$\begin{align*}
\alpha_{f_{\text{min}}} & \leq \alpha_f \leq \alpha_{f_{\text{max}}} \\
\mu_{f_{\text{min}}} & \leq \mu_f \leq \mu_{f_{\text{max}}} \\
0 & \leq \eta \leq 1 \\
y_{f_{\text{min}}} & \leq y_f \leq y_{f_{\text{max}}} \\
n_f(\alpha_f, y_f, v_f) - n_{f_{\text{max}}} & \leq 0
\end{align*}$$

where $\alpha_{f_{\text{min}}}$, $\alpha_{f_{\text{max}}}$, $\mu_{f_{\text{min}}}$, $\mu_{f_{\text{max}}}$, $y_{f_{\text{min}}}$, $y_{f_{\text{max}}}$, $n_f$, $n_{f_{\text{max}}}$ refer to minimum angle of attack, maximum angle of attack, minimum bank angle, maximum bank angle, minimum altitude, maximum altitude, overload, and maximum overload of the fighter, respectively.

2.3 Missile model

The dynamic equations of the missile are described by

$$\begin{align*}
\dot{x}_m &= v_m \cos \theta_m \cos \varphi_m \\
\dot{y}_m &= v_m \sin \theta_m \\
\dot{z}_m &= \dot{v}_m - \frac{1}{m_m(t)}(T_m(t) - D_m) - g \sin \theta_m \\
\dot{\theta}_m &= \frac{g}{v_m}(n_{my} - \cos \theta_m) \\
\dot{\varphi}_m &= \frac{n_{m\varphi}}{v_m \cos \theta_m}
\end{align*}$$

The definitions of the motion equations of the missile are similar to that of the fighter, that are, down range $x_m$, altitude $y_m$, cross range $z_m$, velocity $v_m$, flight-path angle $\theta_m$, and heading angle $\varphi_m$, respectively. The mass of the missile $m_m$ and the thrust $T_m$ are functions of the missile’s flight time $t$. $g$ is the acceleration of gravity. The drag force $D_m$ is given as tabular data. The guidance law adopts the proportional navigation guidance scheme, and be given through first-order lag from command signals [10]. $n_{my}$ and $n_{m\varphi}$ are the overload control commands of the missile in pitch and yaw channels, respectively. The dynamic equations are constrained in this research for the missile, that is,

$$\begin{align*}
|n_{my}| & \leq n_{m_{\text{max}}} \\
|n_{m\varphi}| & \leq n_{m_{\text{max}}} \\
t & \leq t_{\text{max}}
\end{align*}$$

where $n_{m_{\text{max}}}$ and $t_{\text{max}}$ refer to the maximum available overload and maximum flight time of the missile, respectively.
2.4 End condition of simulation

The results of the evasion can be determined through simulation if the initial conditions are certain, which include the following possible scenarios:

(a) Failed evasion.

If the distance between the missile and fighter $r \leq r_d$ ($r_d$ is the damage radius of the missile) or the trajectory of the fighter exceed the height constraint in (2), the missile is believed to have hit the fighter, that is, a failed evasion.

(b) Successful evasion

In the simulation, if $t > t_{max}$ or $v_m < v_{min}$ ($v_{min}$ is the minimum controlled velocity of the missile), that will lead to the self-destruction of the missile as out of control. Besides, if $r > r_d$ in the terminal phase, the fighter is also believed to have successfully evaded the missile.

3. Solution of the evasion problem

3.1 Evasive objectives based on tactical preference

As previously mentioned, the multi-objective of evasive maneuver include: maximizing the miss distance as much as possible to increase the survival probability, decreasing the energy consumption as much as possible for subsequent multi-round combat after a successful evasion, and maximizing the terminal superiority as much as possible for a superior situation in the next missile duel.

Based on this tactical preference in evasion, the optimization model of evasive maneuver strategy for the fighter is defined as

$$\min J(u) = (J_m(u), J_e(u), J_s(u))$$

s.t. $\dot{x} = f(x, u, t)$ $x(t_0) = x_0$ (6)

$$g(x, u) \leq 0$$ (7)

Where system model (6) is constituted by (1) and (3), and $x_0$ is the initial state. The state vector refers to $x = (x_f, y_f, z_f, v_f, \theta_f, \phi_f, x_m, y_m, z_m, v_m, \theta_m, \phi_m)$, and the control vector is $u = (\eta, \alpha_f, \mu_f)$.

Equation (7) represents the constraints of the system model, which is constituted by (2) and (4).

$J_m(u)$, $J_e(u)$, and $J_s(u)$ are objective functions of miss distance, energy consumption, and terminal superiority, respectively, which are designed as follows.

(a) Miss distance

The miss distance is the distance when the closing velocity between the fighter and the missile is zero. The most basic objective of the fighter is to make the miss distance greater than the missile's damage radius, that is, $r(t_f) > r_d$, where the terminal time $t_f$ satisfies $\dot{r}(t_f) = 0$.

$$J_m(u) = \frac{1}{r(t_f)}$$ (8)

However, if the fighter is hit, $J_m(u)$ is set as a large constant for punishment.

(b) Energy consumption

As can be seen from dynamic equations of the fighter in (1), thrust coefficient $\eta$ directly reflects the energy consumption of the fighter with a positive correlation relation. Therefore, in this paper, it is designed as

$$J_e(u) = \frac{1}{M \cdot \Delta t_d} \sum_{k=0}^{M} \Delta t_d \cdot \eta(k)$$ (9)

where $\Delta t_d$ is the decision-making period in simulation, and $M$ refers to the terminal period.
(c) Terminal superiority
This research designs the terminal superiority as the total energy divided by the weight of the fighter, which contains the altitude and the velocity states, that is
\[ J_s(u) = \frac{E}{E_{\text{max}}} = \frac{y_f(t_f) + v_f^2(t_f) / 2g}{E_{\text{max}}} \]  
(10)
Where \( E_{\text{max}} = E(y_{f_{\text{max}}}, v_{f_{\text{max}}}) \), and \( v_{f_{\text{max}}} \) is the maximum velocity of the fighter.

3.2 Solution algorithm
(a) Normalized MOEA/D
According to the MOEA/D [13], the number of the subproblems is denoted by \( N \), then let \( \lambda = (\lambda_1, \lambda_2, ..., \lambda_N) \) be a weight vector \( \lambda^i = 1, \lambda^i \geq 0, i = 1, ..., N \), where \( \lambda^i = (\lambda_1^i, \lambda_2^i, \lambda_3^i) \). The multi-objective optimization problem is decomposed into a series of subproblems \( \min g_j(X)(i = 1, 2, ..., N) \) through the above weight vector. The objective function of each subproblem is the aggregate function of all objective components. The aggregation function is constructed by the tchebycheff approach in this paper. Besides, to reduce the error caused by the value range difference of objective functions, the value of objective function needs to be performed normalization. Therefore, the \( i \) th subproblem could be defined as
\[
\min g_j(X | \lambda^i, z^*) = \max_{j=m,e,s} \left\{ \lambda^i_j \left[ \frac{J_j(X) - z_j^i}{\bar{z}^i_j - z_j^i} \right] \right\} \]  
(11)
where \( z^* = (z^*_m, z^*_e, z^*_s) \) is the reference point, that is \( z_j^i = \min J_j(X)(j = m, e, s) \), \( z_j^i = \min \{J_j(X) | X \in \text{pop}\} \), and \( \bar{z}^i_j = \max \{J_j(X) | X \in \text{pop}\} \). \( \text{pop} \) denotes the population of current generation.

(b) Evolutionary strategy
An individual of the evolutionary strategy in this research reflects a set of fighter control history over span of time \( t_f \), that is \( u = (\eta, \alpha_f, \mu_f) \) at every decision-making period \( \Delta t_f \). The mode of real coding can enhance the search capability of evolutionary algorithms, so the following transformations are used for the coding of initial angle of attack \( \alpha_{f_0} \), initial bank angle \( \mu_{f_0} \), and thrust coefficient \( \eta_0 \).

\[
\alpha_{f_0} = \alpha_{f_{\text{min}}} + C_\alpha \cdot (\alpha_{f_{\text{max}}} - \alpha_{f_{\text{min}}}) \]  
(12)
\[
\mu_{f_0} = \mu_{f_{\text{min}}} + C_\mu \cdot (\mu_{f_{\text{max}}} - \mu_{f_{\text{min}}}) \]  
(13)
\[
\eta_0 = C_\eta \]  
(14)
Where \( C_\alpha, C_\mu, \) and \( C_\eta \) are random number evenly distributed within \([0,1]\). The simulated binary crossover and the polynomial mutation are used for crossover and mutation operation in the process of genetic manipulation, respectively. When the code in a new individual exceeds their corresponding boundary, it will be replaced by the boundary.

The individual is evaluated by running a simulation of evasion in three-dimensional space, then returning the fitness results of the individual, i.e., \( J_m(u), J_e(u), \) and \( J_s(u) \). In the simulation, the fighter uses the control variables extracted from the individual during the whole period of the evasion. An individual constitutes an evasive trajectory of the fighter. The missile uses the proportional navigation guidance scheme to intercept the fighter. Finally, the remaining excellent individuals constitute the required non-dominant solution set.
4. Simulation and analysis
Simulations are used to demonstrate the feasibility and effectiveness of the approach in this paper. Algorithm parameters and model parameters of the fighter and the missile are given as follows. The max generation, the number of the subproblem, the neighbourhood list size, and the external population size are set as 1500, 400, 30, and 200, respectively. Parameters of the fighter are set as \( \alpha_{f_{\text{min}}} = 0\), \( \alpha_{f_{\text{max}}} = 60^\circ\), \( \mu_{f_{\text{min}}} = -180^\circ\), \( \mu_{f_{\text{max}}} = 180^\circ\), \( y_{f_{\text{min}}} = 0.5 \text{km}\), \( y_{f_{\text{max}}} = 16 \text{km}\), \( n_{f_{\text{max}}} = 8.5\), \( T_{f_{\text{max}}} = 54597 N\), and \( m_f = 9298 kg\). Parameters of the missile are set as \( m_m(0) = 108 kg\), \( T_m(0) = 12.6 kN\), \( n_{m_{\text{max}}} = 50\), \( t_{\text{max}} = 30 s\), \( v_{m_{\text{min}}} = 400 m / s\), \( r_d = 12 m\), and \( \Delta t_d = 2 s\).

4.1 Simulation experiment 1
Initial states of the fighter are set as \([x_f, y_f, z_f, v_f, \psi_f, \phi_f] = [10 \text{km}, 9 \text{km}, 20 \text{km}, 250 m / s, 0^\circ, 60^\circ]\), and initial states of the missile set to \([x_m, y_m, z_m, v_m, \psi_m, \phi_m] = [0 \text{km}, 9 \text{km}, 20 \text{km}, 250 m / s, 0^\circ, 30^\circ]\). An approximate Pareto front of the evasion problem is obtained based on the proposed method in this paper through digital simulation (see Figure 1). Besides, three objective functions are performed normalization in Figure 1.

4.2 Simulation experiment 2
To visualize the impact of tactical preferences on flight paths, Figure 2 shows an evasive trajectory that satisfies a specific tactical preference. The tactical preference is expressed by weighting the objective functions, that is, \( a_i J_m(u) + a_j J_c(u) + a_k J_s(u) \), \( \sum_{i=1}^{3} a_i = 1\), \( 0 \leq a_i \leq 1\), \( i = 1, 2, 3\). Weights reflect the importance of different objectives. \( a_1 = 0.5, a_2 = 0.2, a_3 = 0.3\) in the Figure 2.
Figure 2. Evasive trajectory of the fighter against the missile for a specific tactical preference.

The tactical preference in the Figure 2 is characterized by security objective first, followed by energy consumption and terminal superiority. Therefore, the obtained trajectory of the fighter has lower energy consumption and higher terminal superiority than the trajectory with miss distance as the only objective.

5. Conclusion and future work
This paper studies evasive maneuver strategy of the fighter with multiple tactical objectives when confronting incoming missile. The amalgamative tactical demands of achieving self-conflicting evasive objectives in air combat are taken into account, such as higher miss distance, less energy consumption and higher terminal superiority. The evasion problem is defined and reformulated into a multi-objective optimization problem. Using scenario-based simulations, a set of approximate Pareto-optimal solutions (i.e., non-dominated evasive strategies) are obtained, which satisfy different tactical preferences of the fighter while ensuring security. In general, the proposed method is feasible and effective for solving the problem.

Future research directions mainly include considering the uncertainty of information, building more accurate models, enhancing algorithm efficiency, and improving objective functions.

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