Experimental Study of Resonance Frequency at Prime Mover Thermoacoustic Standing Wave

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Abstract: Thermoacoustic prime movers work by using thermal energy to produce acoustic energy in the form of sound wave through thermoacoustic effect which occurs in a porous medium called stack. This paper describes an experimental study on the relation between the order of resonance frequencies generated by a thermoacoustic prime mover and the length of the resonator and the viscous penetration depth. Extending the resonator length will decreasing the resonance frequency which result in the increasing in the viscous penetration depth. Generally, the generated sound consists of only one frequency, that is the first-order one. However, under certain conditions, the sound has only the second-order frequency or comprises two frequencies of the first-order and second-order resonance frequencies. This phenomenon can be explained by considering the comparison between the effective hydraulic radius of stack ($r_{ef}$) and the viscous penetration depth ($\delta_v$). It is found that the first-order frequency appears when $r_{ef} > \delta_v$, while when $r_h < \delta_v$ (with $\delta_v$ calculated by using the first-order frequency) then the second order frequency is produced so that $\delta_v$ is back to a smaller value and therefore the condition of $r_{ef} > \delta_v$ is recovered. In addition, when of $r_{ef} \approx \delta_v$ the thermoacoustic prime mover will generate the first and second order frequencies together.

Keyword: prime mover thermoacoustic, standing wave, resonance frequency, effective hydraulic radius, viscous depth penetration

1. Introduction

Thermoacoustic is a field of study of interaction between heat and sound, which is about conversion of thermal energy into acoustic energy and vice versa. The device that used to convert thermal energy into acoustic energy is prime mover. Device that pump heat from low temperature to high temperature is called thermoacoustic heat pump and device that pump heat from high temperature to low temperature is called thermoacoustic refrigerator.

Thermoacoustic devices have attracted the interest and attention of scientists in recent years, because they are environmentally friendly, simple structure and high endurance that can be made and maintained easily. Specially, the operation of prime mover thermoacoustic can use solar energy (Chen & Garret, 1998; Adeff & Hoffler, 2000) or waste heat (Gardner & Howard, 2009) as the source of thermal energy, and does not
produce exhaust gases such as carbon dioxide. On the other hand, thermoacoustic refrigerator may use inert gases such as air and noble gases as working gases (Tijani, 2010).

Prime mover thermoacoustic have deficiency, that is generally low efficiency. So far, thermal efficiency of prime mover thermoacoustic type traveling wave is 30% made by Backhauss and Swift (Backhauss & Swift, 2000). Thermoacoustic prime mover traveling wave device has higher thermal efficiency than prime mover type standing wave device, that is generally 20% (Swift, 1992; Backhauss & Swift; 2000). That because prime mover type standing wave works with thermodynamic cycle that is intrinsically non-reversible different from prime mover type traveling that operates with stirling cycle.

Although prime mover thermoacoustic has low thermal efficiency, the ability of these devices work using waste heat. This makes the application of prime mover thermoacoustic to be attractive and profitable to improve the overall thermal system. Prime mover thermoacoustic is usually applied to generate electrical energy by combining it with a linear alternator (Backhauss et al, 2004; Kitadani et al, 2010). In addition, when a thermoacoustic prime mover is combined with a thermoelectric refrigerator, it can obtain a cooling system without moving parts (Yu et al, 2011; Saechan et al., 2013).

In its application, the sound produced by the prime mover thermoacoustic have an important role as a source of electricity and energy sources for thermoacoustic refrigerator. Therefore one of the important parameters in the thermoacoustic prime mover is the resonant frequency. This paper describes the resonant frequency generated by the thermoacoustic prime mover standing wave device by varying the length of the resonator and the effect of the viscous penetration depth with the result of resonance frequency.

2. Theory

2.1. The Thermoacoustic principle

The thermoacoustic prime mover device consists of a resonator tube, a stack that is a porous medium, a working gas and two heat exchangers. Resonator tubes are filled with working gases such as air, noble gases or other inert gases. The stack is enclosed by a hot heat exchanger (HHX) and an ambient heat exchanger (AHX). Combined stack and two heat exchangers are placed inside the resonator tube. Both heat exchangers will provide a large temperature gradient along the stack in the direction of the resonator axis. The temperature gradient is required for thermoacoustic energy conversion processes to occur. The minimum temperature difference between the two ends of the stack required to initiate spontaneous oscillation of working gas (a sound wave arises) is called the temperature difference of onset ($\Delta T_{\text{onset}}$).

The process of appear of sound waves can be explained by following the things that happen to a gas packet inside the stack as shown in Figure 1. The working gas resonator pipe is seen as a gas package undergoing a thermodynamic cycle consisting of four stages (Setiawan, 2015). In step (1) the hot gas pack is heated by absorbing a certain
amount of heat $dQ_1$ and undergoing thermal expansion. In this case, work $dW_1$ is done by gas package. Phase (2) due to the expansion process, this gas package spreads to the cold side and does work of $dW$. This causes the hot side area run into pressure drop. Stage (3) Upon reaching the cold side, as the gas pack is hotter than the pore wall, the gas package releases the $dQ_2$ heat in the pore wall of the stack and undergoes cooling and thermal contraction. In this case, the $dW_2$ is work a gas package. Stage (4), the gas package returns to the hot side to fill the low pressure area and undergoes compression. In this step, $dW$ effort is done on the gas package. The gas package will reheat and the thermal expansion processes begin again.

Figure 1. Four stages of thermodynamic process.¹

Figure 2 shows the schematic of pressure and volume diagrams for the four thermodynamic processes undertaken by the gas package as described above, the process known as the Brayton cycle (Jin, 2015). The resulting closed-loop area $\oint p\,dV$ indicates the total effort used by the prime mover thermoacoustic type standing wave device to produce sound power.

Figure 2. Schematic of the pressure-volume diagram undertaken by the gas package.²

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¹ Panhuis, 2009

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The resonator used in the study is a straight tube made of stainless steel with one end open. The resonance frequency for the $n$ order is

$$f_m = (2m - 1) \frac{v}{4L}. \quad (1)$$

where $L$ is the pipe length of the organa, $v$ is the rapid sound voiding in the air in the range of 348 m/s and $m$ is the resonance order.

The viscous penetration depth is the distance of the viscosity frictional force (frictional fluid / gas force) that spreads over the time interval of one period of voice oscillation $T = 1/f$. The viscosity penetration depth is also a measured distance from the pore wall of the stack, and it is assumed by the formula (Saechan, 2014)

$$\delta_v = \sqrt{\frac{2\mu}{\omega \rho_m}} = \sqrt{\frac{2v}{\omega}} \quad (2)$$

with $\mu$ and $v$ are respectively dynamic viscosities of $1.8 \times 10^{-5}$ Ns/m$^2$ and kinetic viscosities of $1.6 \times 10^{-5}$ m$^2$/s, which are viscosity-style components resulting in reduced sound power. Where $\omega$ is the angular frequency and $\rho_m$ is the average of the working gas type of 1.17 kg/m$^3$. In the conversion of thermal energy into sound the required condition is that the effective hydraulic radius is greater than the viscous penetration depth ($r_{ef} < \delta_v$), so the heat transfer from the gas to the stack wall becomes effective. In order that $\delta_v$ is always smaller than $r_{ef}$ the angular frequency ($\omega$) shall to be great value or the speed of sound in the working gas ($v$) shall to be little value. To make $\omega$ of great value then the resonance frequency ($f_m$) produced by a straight tube must also be large.

3. Research methods

The schematic diagram of prime mover thermoacoustic type standing wave with a straight resonator made in this study is shown in Figure 3. In the resonator there is a stack with effective radius ($r_{ef} = 0.275$ mm thick 3 cm made of stainless steel wire arranged tightly, hot exchanger heat (HHX), ambient heat exchanger (AHX) and air as a working gas with atmospheric pressure (1 atm) and room temperature (27°C) rapidly air voiding ($v$) in the air about 348 m/s.

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2 (Jin et al, 2015)
The sound produced by the prime mover thermoacoustic arises because there is a temperature gradient on both sides of the stack. The heat stack temperature comes from HHX which is wired by a heating cable with 35 watt input power. While the cold temperature of the stack comes from the AHX that flows through the water, so the gas contained in the stack will oscillate due to the thermoacoustic effect and produce sound. At each point on the resonator mounted pressure transducer. The sound pressure data captured by the pressure transducer will be changed using FFT (Fast Fourier Transform) and generate the resonance frequency spectrum. The length of the resonator will be

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3 D D Cahyadi, Y N Adhitama, I Setiawan, A B S Utomo

Figure 3. prime mover thermoacoustic type standing wave scheme.3
varied starting from 30 cm, 105 cm, 130 cm, 180 cm, and 205 cm to obtain the resonant frequency data.

4. Results and Discussion

Frequency data of each resonator pipe length 105 cm 130 cm 155 cm 180 cm and 205 cm with 3 cm thick stack is shown in figure 4. From the results obtained, the main parameters used to adjust the sound wave frequency of the prime mover thermoacoustic device the length resonator. The longer the resonator is given the resulting frequency will be smaller and vice versa. From figure 4 it is found that at the length of 105 cm and 130 cm has a frequency corresponding to equation (1) with the first resonance order \( m = 1 \), while the resonator length of 130 cm 155 cm 180 cm and 205 cm has a second resonance order \( m = 2 \). So the 130 cm long resonator has two frequencies. The difference in the order obtained by the effect of the viscous penetration depth \( \delta_v \) on the efficient radius \( r_{ef} \) as illustrated in figure 5.

![Figure 4. Relationship between resonator length and resonance frequency](image)

Figure 4 gives the viscous viscous depth distances \( \delta_v \) generated at each of the sound frequencies obtained. According to equation (2), the depth of viscous penetration depends on the frequency of sound obtained. For the length of 105 cm to produce the frequency of sound at the first order of 87 Hz. The sound frequency at that order has a smaller viscous penetration depth than the effective radius \( \delta_v \) \( (\delta_v (m=1) < r_{ef} ) \) so that the prime mover thermoacoustic device is capable to producing sound.

While the length of the resonator 155 cm, 180 cm and 205 cm has a frequency on the second order. This occurs because, when the working gas oscillates at first order, the viscous penetration depth obtained is higher than the effective radius \( \delta_v (m=1) > r_{ef} \) so
it does not produce sound. To overcome this, the working gas will adjust to produce sound, by decreasing the viscous penetration depth. To decrease the viscous penetration depth, the frequency is higher than the first order. Therefore, the working gas adjusts to the viscous penetration depth in order to produce a higher sound of the second order. The viscous penetration depth is less than the effective radius \( \delta_v (m=2) < r_{ef} \). The process is similar to that of Sakamoto and Watanabe (2006) that the second order frequency is generated by prime mover thermoacoustic device because the working gases adjust from the viscous penetration depth by increasing their frequency to produce sound.

Different than other resonator lengths, for 130 cm long resonators have multiple frequencies, it occurs because the first order frequency (68 Hz) has a viscous penetration depth approaching the effective radius \( \delta_v \approx r_{ef} \). Thus, the gas oscillation becomes double, there is a fixed working gas on the oscillation and generates a first order frequency. However, there are some working gases that adapt to the viscosity depth by increasing the oscillation of the gas to produce a higher frequency of second order. Therefore, there are two frequencies of 68 Hz (order 1) and 199 Hz (order 2).

![Figure 5. Effect of resonator length on viscous penetration depth parameter \((\delta_v)\)^5](image)

The sound quality level produced by the prime mover thermoacoustic is expressed in acoustic power. In the study conducted Achmadin (2015) concluded that the higher the amplitude of the resulting pressure the acoustic power obtained will be greater. The resulting resonance frequency also affects the amplitude of the pressure obtained. Figure 6 describes the amplitude relationship of the sound pressure generated by the variation of the resonator length.

At 105 cm length of resonator that produces frequency 87 Hz (1\(^{st}\) order) only has a low enough amplitude of pressure that is 1.97 kPa. However, when the resonator length is 130 cm with a double frequency, both frequencies have a very low amplitude of pressure that is 1.21 kPa for frequency of 199 Hz (2\(^{nd}\) order) and 0.99 kPa for frequency 0.99 kPa for frequency.
68 Hz (1st order). This happens because the conversion of thermal energy into a lot of sound energy is wasted to produce multiple frequencies, so that the amplitude of the pressure obtained is very low. As for the length of the resonator 155 cm, 180 cm, and 205 cm has a high amplitude because at that length only produce second-order frequency, where the frequency at the second order has a pressure amplitude much higher than the first order.

![Graph showing the relation of resonator length with pressure amplitude.](image)

**Figure 6.** Relation of resonator length with pressure amplitude.6

### 5. Conclusion

The conclusions obtained from this study are first-order frequencies appear when the efficient hydraulic radius of stack is greater than the viscous penetration depth (r_{ef} > \delta_v). While the double frequency of first order and second order occurs when the efficient hydraulic radius of stack is almost equal to the viscous penetration depth r_{ef} \approx \delta_v. The second order frequency occurs when the efficient hydraulic radius of stack is smaller than the viscous penetration depth (r_{ef} < \delta_v). Since the prime mover thermoacoustic will not produce sound when r_{ef} < \delta_v, the working gas will adjust to the viscous penetration penetration and will oscillate more to produce a higher frequency (2nd order). That way, the viscous penetration depth that is there will be smaller and the main requirement of sound will be fulfilled that is r_{ef} > \delta_v.

From this research is able to choose the best frequency so that its application can work optimally. The best frequency is the frequency that has high order with high frequency value, because at the highest frequency of the pressure amplitude is also high. So that in the future capable of producing high acoustic power as well.
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