A residual power series technique for solving Boussinesq–Burgers equations

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Abstract: In this paper, a residual power series method (RPSM) is combining Taylor’s formula series with residual error function, and is investigated to find a novel analytical solution of the coupled strong system nonlinear Boussinesq-Burgers equations according to the time. Analytical solution was purposed to find approximate solutions by RPSM and compared with the exact solutions and approximate solutions obtained by the homotopy perturbation method and optimal homotopy asymptotic method at different time and concluded that the present results are more accurate and efficient than analytical methods studied. Then, analytical simulations of the results are studied graphically through representations for action of time and accuracy of method.

Subjects: Science; Mathematics & Statistics; Advanced Mathematics; Analysis - Mathematics; Mathematical Numerical Analysis

Keywords: analytical solution; Boussinesq-Burgers equations; residual power series method

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PUBLIC INTEREST STATEMENT

Many phenomena in the world was described by nonlinear partial differential equations that can be solved numerically. Boussinesq–Burgers equation is one of the famous nonlinear equations. In this work, author solved Boussinesq–Burgers equations numerically by novel and new method called residual power series method (RPSM). Also, the author compared the present work via two another published method and concluded that the present results are more accurate and efficient than analytical methods studied. Action of time and accuracy of the method was studied graphically at last.
1. Introduction

First, we consider the generalized Boussinesq–Burgers equations given by

\[ u_t + auu_x + bw_x = 0 \tag{1.1} \]

\[ w_t + c(uw)_x + du_{xxx} = 0, \tag{1.2} \]

where \( a, b, c, \) and \( d \) are real nonzero constants.

The systems of nonlinear equations are known to describe a wide variety of phenomena in physics, engineering, applied mathematics, chemistry, and biology.

The Boussinesq–Burgers equations arise in the study of fluid flow and describe the propagation of shallow water waves. Here, \( x \) and \( t \) represent the normalized space and time, respectively, \( u(x, t) \) represents the horizontal velocity and at the leading order, it is the depth averaged horizontal field, while \( w(x, t) \) denotes the height of the water surface above the horizontal level at the bottom (Wang, Tian, Liu, Lü, & Jiang, 2011).

Since many systems of nonlinear equations do not have exact analytical solutions, various analytic and numerical methods for finding approximate solutions to Boussinesq–Burgers equations have been proposed. In recent years some works have been done in order to find the numerical solution of this equation (for example Abdel Rady & Khalfallah, 2010; Abdel Rady, Osman, & Khalfallah, 2010; Chen & Li, 2006; Gao, Xu, Tang, & Meng, 2007; Hardik & Meher, 2015; Khalfallah, 2009; Li & Chen, 2005). In this work, the residual power series method (RPSM) is applied to find the analytic solution for the Boussinesq–Burgers equations.

The RPSM first proposed by Arqub (2013) is a powerful method for solving linear and nonlinear problems. This method has been employed by many mathematicians and engineers to solve nonlinear problems such as Arqub (2013), Arqub, Abo-Hammour, Al-Badarneh, and Momani (2013), Arqub, El-Ajou, Bataineh, and Hashim (2013), Arqub, El-Ajou, and Momani (2015), Arqub and Maayah (2016), El-Ajou, Arqub, and Al-Smadi (2015), El-Ajou, Arqub, Momani, Baleanu, and Alsaedi (2015), El-Ajou, Arqub, and Momani (2015), El-Ajou, Arqub, Zhour, and Momani (2013), Komashynska, Al-Smadi, Al-Hababheh, and Ateiwi (2014), Komashynska, Al-Smadi, Ateiwi, and Al-Obaidy (2016) and Wang and Chen (2015).

The RPSM is based on constructing power series expansion solution for different nonlinear equations without linearization, perturbation, or discretization (Arqub, Abo-Hammour, 2013; Arqub, El-Ajou, et al., 2013; El-Ajou et al., 2013, 2015; Wang & Chen, 2015). Recently, Deng et al. discussed global existence and asymptotic behavior of the Boussinesq–Burgers equations (Ding & Wang, 2015). Also, many authors studied existence and uniqueness of Boussinesq–Burgers equations (see Changjiang & Renjun, 2003; Liu, 2016). With the help of residual error concepts, this method computes the coefficient of the power series by a chain of algebraic equations of one or more variables and finally we get a series solution, in practice a truncated series solution. The main advantage of this method over the other method is it can be applied directly to the given problem by choosing an appropriate initial guess approximation and you can discuss the nonlinear problems at large time.

It has been proven that the RPSM is a convenient and effective method in its application. The aim of this paper is solving Boussinesq–Burgers equations analytically using RPSM and compared homotopy perturbation method and optimal homotopy asymptotic method at different time. Also, the accuracy of the present method at different time was discussed.

2. Analysis of RPSM

To illustrate the fundamental scheme of RPSM, we set \( a = c = 2, \) and \( b = d = -1/2 \) in the system of Equations (1.1) and (1.2), the system reduces to the following system:
subject to the initial conditions

\[ u(x, 0) = f(x), \]  
\[ w(x, 0) = g(x), \]

In RPSM, the solution of Equations (2.1) and (2.2) with (2.3) and (2.4) can be expressed as a power series expansion about the initial point \( t = 0 \).

Assume that the solutions takes the expansion

\[ u(x, t) = \sum_{m=0}^{\infty} f_m(x) t^m, \]  
\[ w(x, t) = \sum_{m=0}^{\infty} g_m(x) t^m, \]

where \( m = 0, 1, 2, ... \)

Next, we define \( u_k(x, t) \) and \( w_k(x, t) \) to denote the \( k \)th truncated series of \( u(x, t) \) and \( w(x, t) \), respectively, that is,

\[ u_k(x, t) = \sum_{m=0}^{k} f_m(x) t^m, \]  
\[ w_k(x, t) = \sum_{m=0}^{k} g_m(x) t^m, \]

where \( k = 1, 2, 3, ... \)

Obviously \( u(x, t) \) and \( w(x, t) \) satisfy the initial conditions (2.3) and (2.4), so the 0th RPS approximate solutions of \( u(x, t) \) and \( w(x, t) \) are

\[ u(x, 0) = f_0(x) = f(x). \]  
\[ w(x, 0) = g_0(x) = g(x). \]

On the other hand, from Equations (2.7) and (2.8) the initial guess approximation the first RPS approximate solutions of \( u(x, t) \) and \( w(x, t) \) should be

\[ u_1(x, t) = f(x) + f_1(x) t \]  
\[ w_1(x, t) = g(x) + g_1(x) t \]

Consequently, one can reformulate the expansion of Equations (2.7) and (2.8) as follows:
where \( k = 2, 3, 4, \ldots \). Subsequently in the RPS technique for finding the values of coefficients \( f_m(x) \) and \( g_m(x) \) \( m = 1, 2, 3, 4, \ldots, k \) in the series expansion of Equations (2.13) and (2.14), we define the residual functions as

\[
\text{Res}_u(x, t) = u_t + 2uu_x - \frac{1}{2}w_x
\]

(2.15)

\[
\text{Res}_w(x, t) = w_t + 2(uw)_x - \frac{1}{2}u_{xxx}
\]

(2.16)

and, therefore, the \( k \)th residual functions, \( \text{Res}_{u,k}(x, t) \), and \( \text{Res}_{w,k}(x, t) \), of both the style forms

\[
\text{Res}_{u,k}(x, t) = (u_k)_t + 2u_k(u_k)_x - \frac{1}{2}(w_k)_x
\]

(2.17)

\[
\text{Res}_{w,k}(x, t) = (w_k)_t + 2(u_k w_k)_x - \frac{1}{2}(u_k)_{xxx}
\]

(2.18)

where \( k = 1, 2, 3, 4, \ldots \). As described in (Arqub, Abo-Hammour, et al., 2013; Arqub, El-Ajou, et al., 2013; El-Ajou et al., 2013, 2015; Komashynska et al., 2016), it is clear that \( \text{Res}(x, t) = 0 \) and \( \lim_{k \to \infty} \text{Res}_k(x, t) = \text{Res}(x, t) \) for all \( x \in I \) and \( t \geq 0 \). Then, \( \frac{\partial}{\partial t} \text{Res}(x, t) = 0 \) when \( t = 0 \) for each \( s = 0, k \).

To obtain the coefficients \( f_m(x) \) and \( g_m(x) \) \( m = 1, 2, 3, 4, \ldots, k \), we apply the following subroutine; substitute \( m \)th truncated series of \( u(x, t) \) and \( w(x, t) \) into Equations (2.17) and (2.18), apply the derivative formula \( \frac{\partial}{\partial t} \text{Res}(x, t) = 0 \) on \( \text{Res}_m(x, t) \) \( m = 1, 2, 3, 4, \ldots, k \), substitute \( t = 0 \), in the following formula, equate it to zero, and then lastly solve the obtained algebraic equation to obtain the form of the other coefficients. Any how we need to solve the following algebraic equation:

\[
(\frac{\partial}{\partial t} \text{Res}_s(x, t) / \partial t^s) = 0 \quad (t = 0), \quad s = 1, 2, 3, \ldots, k
\]

(2.19)

In this way, we can find all the required coefficients of the multiple power series of Equations (2.1–2.4) are obtained.

3. Application of RPSM to Boussinesq–Burgers equation

The general Boussinesq–Burgers equation considering as follows:

\[
u_t + 2uu_x - \frac{1}{2}w_x = 0,
\]

(3.1)

\[
w_t + 2(uw)_x - \frac{1}{2}u_{xxx} = 0, \quad 0 \leq x \leq 1,
\]

(3.2)

Subjected to initial conditions

\[
u(x, 0) = \frac{ck}{2} + \frac{ck}{2} \tanh \left( \frac{-kx - \ln(b)}{2} \right),
\]

(3.3)
w(x, 0) = \frac{-k^2}{8}\text{sech}^2\left(\frac{kx + \ln(b)}{2}\right), \quad (3.4)

With exact solution

\[ u(x, t) = \frac{ck}{2} + \frac{ck}{2} \tanh\left(\frac{ck^2 t - kx - \ln(b)}{2}\right), \quad (3.5) \]

\[ w(x, t) = \frac{-k^2}{8}\text{sech}^2\left(\frac{kx - ck^2 t + \ln(b)}{2}\right), \quad (3.6) \]

By iterative of RPSM when \( n = 0, 1, 2, \ldots \), we get:

\[ u_1(x, t) = -\frac{1}{4} + \frac{1}{4} \tanh\left(\frac{x - \ln(2)}{2}\right) + \frac{1}{16} \left(\begin{array}{c}
-\text{sech}^2\left(\frac{x - \ln(2)}{2}\right) \\
-\text{sech}^2\left(\frac{x + \ln(2)}{2}\right)
\end{array}\right) t, \quad (3.7) \]

\[ w_1(x, t) = -\frac{1}{8}\text{sech}^2\left(\frac{-x + \ln(2)}{2}\right) + \frac{1}{32} \left(\begin{array}{c}
\text{sech}^4\left(\frac{x - \ln(2)}{2}\right) \\
-\text{sech}^2\left(\frac{x - \ln(2)}{2}\right)\text{sech}^2\left(\frac{x + \ln(2)}{2}\right) \\
-2\text{sech}^2\left(\frac{x - \ln(2)}{2}\right)\tanh\left(\frac{x + \ln(2)}{2}\right) \\
-2\text{sech}^2\left(\frac{x - \ln(2)}{2}\right)\tanh\left(\frac{-x + \ln(2)}{2}\right) \\
-2\text{sech}^2\left(\frac{-x + \ln(2)}{2}\right)\tanh\left(\frac{-x - \ln(2)}{2}\right)
\end{array}\right) t, \quad (3.8) \]

By same way for \( u_2(x, t), u_3(x, t), \ldots, \) and \( w_2(x, t), w_3(x, t), \ldots \)

Then, the solutions are

\[ u(x, t) = u_1(x, t) + u_2(x, t), \ldots \quad (3.9) \]

\[ w(x, t) = w_1(x, t) + w_2(x, t), \ldots \quad (3.10) \]

4. Numerical results and discussion

This section describes the proposed methods to obtain the approximate analytical solutions by numerical simulations of. From Table 1 compared RPSM with the exact solution and the solutions obtained by the homotopy perturbation method and optimal homotopy asymptotic method at different time, and it is clear that the present method is accurate and provides efficient results. Figures 1–4 showed surface graphic of \( u(x, t) \) and \( w(x, t) \) with exact and RPSM solution at small time, and concluded that the present work is the same exact solution. From Figures 5 and 6 sketched the RPSM solution a wide large time, and it is the advantage of RPSM. Figures 7 and 8 show the effect of space and time described at counter-plot. Figures 9 and 10 show the curve steadiness of problem at \( t = 1 \) between exact and RPSM. Finally, the action and influence of time are examined in substantive Figures 11 and 12.
Table 1. The comparison for the solution of Boussinesq–Burgers equation using three terms approximation for RPSM, HPM and OHAM at various points by absolute errors when \( c = \frac{1}{2} \), \( k = -1 \) and \( b = 2 \)

| (x, t)       | \(|u_{\text{exact}} - u_{\text{RPSM}}|\) | \(|u_{\text{exact}} - u_{\text{HPM}}|\) | \(|u_{\text{exact}} - u_{\text{OHAM}}|\) |
|--------------|----------------------------------------|----------------------------------------|----------------------------------------|
| (0.1, 0.1)   | 4.11577 \times 10^{-12}               | 9.11428 \times 10^{-07}               | 3.15534 \times 10^{-06}               |
| (0.1, 0.3)   | 3.20132 \times 10^{-09}               | 2.53911 \times 10^{-05}               | 1.36454 \times 10^{-06}               |
| (0.1, 0.5)   | 6.81571 \times 10^{-08}               | 1.21007 \times 10^{-06}               | 2.06021 \times 10^{-06}               |
| (0.2, 0.1)   | 9.60588 \times 10^{-13}               | 1.12268 \times 10^{-10}               | 3.34664 \times 10^{-10}               |
| (0.3, 0.3)   | 9.64227 \times 10^{-10}               | 3.08802 \times 10^{-06}               | 1.62153 \times 10^{-06}               |
| (0.3, 0.5)   | 2.63810 \times 10^{-09}               | 1.45333 \times 10^{-06}               | 3.66844 \times 10^{-06}               |
| (0.5, 0.1)   | 3.70697 \times 10^{-12}               | 1.25997 \times 10^{-06}               | 3.72005 \times 10^{-06}               |
| (0.5, 0.3)   | 2.63474 \times 10^{-09}               | 3.62835 \times 10^{-06}               | 5.69591 \times 10^{-06}               |
| (0.5, 0.5)   | 5.48936 \times 10^{-08}               | 1.59629 \times 10^{-06}               | 5.20368 \times 10^{-06}               |

| (x, t)       | \(|w_{\text{exact}} - w_{\text{RPSM}}|\) | \(|w_{\text{exact}} - w_{\text{HPM}}|\) | \(|w_{\text{exact}} - w_{\text{OHAM}}|\) |
|--------------|----------------------------------------|----------------------------------------|----------------------------------------|
| (0.1, 0.1)   | 6.06616 \times 10^{-12}               | 1.19150 \times 10^{-06}               | 5.85344 \times 10^{-07}               |
| (0.1, 0.3)   | 4.67117 \times 10^{-09}               | 3.13655 \times 10^{-05}               | 1.12982 \times 10^{-05}               |
| (0.1, 0.5)   | 1.05252 \times 10^{-07}               | 1.40972 \times 10^{-04}               | 7.71116 \times 10^{-05}               |
| (0.2, 0.1)   | 2.05567 \times 10^{-12}               | 8.94544 \times 10^{-07}               | 2.35740 \times 10^{-06}               |
| (0.3, 0.3)   | 1.39067 \times 10^{-09}               | 2.28283 \times 10^{-06}               | 1.67671 \times 10^{-05}               |
| (0.3, 0.5)   | 2.74299 \times 10^{-08}               | 9.91676 \times 10^{-06}               | 1.65572 \times 10^{-05}               |
| (0.5, 0.1)   | 1.01186 \times 10^{-11}               | 4.59601 \times 10^{-07}               | 5.48493 \times 10^{-06}               |
| (0.5, 0.3)   | 7.51605 \times 10^{-09}               | 1.07397 \times 10^{-06}               | 4.80354 \times 10^{-06}               |
| (0.5, 0.5)   | 1.63669 \times 10^{-07}               | 4.18318 \times 10^{-06}               | 1.24363 \times 10^{-05}               |

Figure 1. The exact solution of the \( u(x, t) \).
Figure 2. The RPSM solution of the $u(x, t)$.

Figure 3. The exact solution of the $w(x, t)$.

Figure 4. The RPSM solution of the $w(x, t)$. 
Figure 5. The RPSM solution of the $u(x, t)$ at large time.

Figure 6. The RPSM solution of the $w(x, t)$ at large time.

Figure 7. The RPSM Contour solution of the $u(x, t)$ at large time.
Figure 8. The RPSM Contour solution of the $w(x, t)$ at large time.

Figure 9. The RPSM and exact solution of the $u(x, t)$, when $t = 1$.

Figure 10. The RPSM and exact solution of the $w(x, t)$, when $t = 1$. 
Figure 11. The effect of time on $u(x, t)$.

Figure 12. The effect of time on $w(x, t)$.

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