STRING DEFECTS IN CONDENSED MATTER SYSTEMS
AS OPTICAL FIBERS

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ABSTRACT

We analyze the core structure of string defects in various condensed matter systems, such as nematic liquid crystals and superfluid helium, and argue that in certain cases the variation of the refractive index near the core is such that it can lead to total internal reflection of light travelling along the string core. These strings thus behave as optical fibers providing a qualitatively new approach to optical fibers. We present a candidate for such a fiber by looking at string segments in a thin nematic liquid crystal film on water. We discuss various possibilities for constructing such fibers as well as possible technological applications.

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The study of strings has been a subject of great interest in the context of the early Universe as well as various condensed matter systems [1, 2, 3]. Recently, the formation and dynamical evolution of string defects has been experimentally studied in nematic liquid crystals with results which are in good agreement with theoretical expectations [4]. In this letter we investigate an interesting implication of the existence of string defects. We consider strings in certain transparent condensed matter systems and study the variation of the refractive index near the core of the string. We then argue that for certain systems the spatial variation of the refractive index is such that it leads to total internal reflection of light travelling along the string core. These strings therefore behave like optical fibers and can be used for light transmission.

First let us briefly recall the essential properties of an optical fiber [5]. An optical fiber basically consists of an inner glass (or transparent plastic) wire with an outer coating of another transparent material such that the refractive index of the inner material is larger than that of the outer coating. This leads to the total internal reflection of any light ray travelling along the inner wire for not too large incidence angles. There are also fibers which have many coatings with successively decreasing refractive indices. The thinnest fibers are typically 10-100 microns thick.

Now we consider the structure of string cores in condensed matter systems. Consider a condensed matter system which has two phases separated by a phase transition such that the low temperature phase supports strings. Examples are liquid crystals, superfluid helium etc. [1]. We consider only transparent systems as we are considering optical fibers. We are investigating extensions of these ideas to strings in other systems, such as flux tubes in type II superconductors and cosmic strings, and hope to present it in a future work. Consider now the core of the string. The core of the string will typically consist of the high temperature phase and as one goes away from the center, the phase will gradually become the low temperature phase. In other words, if there is an order parameter \( \phi \) such that \( \phi = 0 \) for large temperatures and \( \phi = \) some constant, say \( \eta \), for low temperatures, then \( \phi = 0 \) on the axis of the string and \( \phi \) gradually rises to the value \( \phi = \eta \) as we go away from the center of the string. The distance over which \( \phi \) becomes almost \( \eta \) is the radius of the string.

The existence of such strings is typically associated with a winding number describing the rotation of the order parameter as one encircles the string. However, The only aspects of such a string, which are relevant for our discussion, are the nature and stability of the core of the string and not the winding of the order parameter field around the core. We will therefore generally not refer to the winding number and will loosely refer to the
magnitude of the order parameter as the order parameter itself.

Now suppose that the refractive index (RI) corresponding to a given wavelength of light has a value $n_1$ in the low temperature phase (where $\phi = \eta$) and $n_2$ in the high temperature phase (where $\phi = 0$). Consider the case when $n_1 < n_2$. Let us now look at the structure of the core of the string. As we discussed above, the core of the string consists of the high temperature phase with $\phi = 0$ and as we go away from the core, $\phi$ gradually becomes equal to $\eta$ as the low temperature phase is achieved. This then implies that the refractive index has the larger value $n_2$ inside the core of the string and as we go away from the core, the refractive index decreases to the smaller value $n_1$ as $\phi$ rises to the value $\eta$. We note that this is the same as the configuration of an optical fiber as described above. Any light (with wavelength for which the refractive index has the assumed values) travelling along the core will undergo total internal reflection for not too large incidence angles. In fact, from any physical considerations one will expect that RI will monotonically decrease from $n_2$ to $n_1$ as $\phi$ increases from 0 to $\eta$. This string therefore is like an optical fiber which consists of very large number of coatings all with successively decreasing RIs.

This provides a qualitatively new approach to optical fibers. Typically optical fibers are manufactured by creating wires and then coating them; this process limiting the thinness of the fiber due to technological limitations. Strings, on the other hand, are typically of microscopic thickness and for suitable systems may provide the thinnest possible optical fibers for a given wavelength of light. Another interesting property of these string fibers is due to the fact that typically the core thickness of a string is temperature dependent and can grow by large factors as one approaches the phase transition temperature. These fibers will then be such that one could control their thicknesses by simply changing the temperature so that they are suitable for different wavelengths of light or for different image resolution requirements. In contrast the thickness of conventional fibers is pretty much fixed. The main problem with these string fibers may be that it will be difficult to create long lengths of these strings, thereby limiting their possible use only to small scale devices. Let us now consider some examples of condensed matter systems where these string fibers may actually be realized.

First consider vortices in superfluid $^4$He. Generally these vortices have cores of atomic dimensions but at temperatures very close to the $\lambda$ point the cores can be very large, of the size of few thousand Å [6]. At these temperatures the core of the vortex will typically consist of normal fluid with a circulation of superfluid in the outer region. The refractive index data for superfluid $^4$He shows [7] that the value of $(n - 1)$, where $n$ is the refractive
index, at temperature $T = 1.6K$ is equal to 0.028488 and $(n - 1)$ monotonically rises to a value of 0.028668 at the $\lambda$ point, $T_\lambda = 2.17K$. The wavelength of the light used in\[\text{is}\] was 5462.27 Å. As the temperature is increased above 2.17$K$ $(n - 1)$ starts decreasing, though this has no consequence for us. The important thing for us is that RI is smaller in the superfluid phase than in the normal phase for temperatures below $T_\lambda$. Therefore, in the temperature regime where the vortex core is large and consists of normal fluid the RI should decrease as one goes away from the core of the vortex. From our earlier discussion we then see that these vortices should behave like optical fibers. There are certain points one has to be careful about. For example the refractive index data refers to the change in RI at different temperatures which does not necessarily mean that the RI for normal fluid will be larger than that of the superfluid when both are at the same temperature (as in a vortex configuration). These issues will be discussed further in a paper under preparation \cite{8} where we will also discuss the case of vortices in superfluid $^3$He which typically have very large cores with richer structure.

Now let us consider the example of strings in nematic liquid crystals (NLCs). NLCs consist of rod like molecules which tend to locally align in the low temperature nematic phase. In the high temperature isotropic phase the directions of these molecules are completely random. The order parameter for isotropic-nematic phase transition can be taken to be the director $D$ giving the average orientation (direction and magnitude) of these molecules in a small region. In the nematic phase $D$ will have some fixed magnitude and in the isotropic phase random directions of the molecules will average out to give $D = 0$. In the nematic phase there are various kinds of topological defects, strings being one of them. There are strings of strength $S$ where the director $D$ rotates by $2\pi S$ around the core. For half integral $S$ the core has to be in the isotropic phase but for integral values of $S$ the core may be in the nematic phase by escape into the third dimension \cite{2,3}. The thickness of the isotropic core of the string is of the order 100 Å, though the thickness of the string itself (characterized by the distance from the core where $D$ becomes significantly non-zero) can be much larger.

We first describe an experiment where we have observed segments of strings which seem to behave like optical fibers. We used the nematic liquid crystal K15 (4-cyano-4′-n-pentylbiphenyl) manufactured by BDH Chemicals, Ltd.. The transition temperature for K15 is 35.3°C. We prepared thin films of nematic liquid crystal on water and studied them through a phase contrast microscope (Olympus, Inc. model BH) with a 10X objective. The film was illuminated from underneath by a standard light source. The microscope was connected to a monochrome television camera and the image was recorded by a standard
videocassette recorder. The temperature of water was chosen so that the NLC film is in the nematic phase.

As the NLC film formed on water surface, we saw bright dots of light which were the ends of string segments. As the strings shrank, these bright dots came together and annihilated. When most of the strings disappeared, a slight shaking of the water surface again created many string segments. Fig. 1 shows pictures where one sees strings ending in bright dots. The strings are always much fainter than the ends which are typically very bright.

These pictures are very suggestive that these strings may be trapping light which is entering the string from underneath and then (due to the curvature of the string) exiting from the ends of the string. Though we would like to emphasize that this should only be taken as a suggestion in the sense that there may be other explanations for the bright ends. For example, it is possible that the ends of the string segments scatter light in a different way, especially due to the deformation of the NLC surface near the string ends, (see [2]), and that makes it bright. Therefore, for nematic liquid crystals, the following discussion is to be taken as suggesting strong possibility and our pictures as possible candidates for optical fibers.

It is known that in the nematic phase there are two RIs depending on the angle between the director $D$ (the order parameter) and the electric vector $E$ of the light [9]. If $D$ and $E$ are parallel (extra-ordinary ray) then $RI = r_e$ and when $D$ is perpendicular to $E$ (ordinary ray) then $RI = r_o$ such that $r_e > r_o$. In the isotropic phase when $D = 0$, $RI = r_i$ such that $r_o < r_i < r_e$. For example, for NLC ethyl p-(p-ethoxybenzyl-ideneamino)-$\alpha$-methylcinnamate, the values of RI near the transition temperature $T_c \approx 115°C$ are, $r_e \approx 1.75$, $r_o \approx 1.55$ and $r_i \approx 1.62$, see [9].

Consider now a cylindrical configuration of a string of strength $S = 1$ along the $z$ axis where $D$ lies in the x-y plane and rotates by full $2\pi$ as we go around the string. This string is topologically unstable in the sense that an isolated string of $S = 1$ can be continuously deformed so that the director $D$ points in the same direction everywhere. However, this deformation may not be possible if the NLC is in some container where $D$ typically assumes fixed orientation at the boundaries. For example, for liquid crystals in thin glass tubes with certain types of coatings in the inner wall of the tube, $D$ becomes normal to the wall and forms strings of strength $S = 1$ (where $D$ rotates by $2\pi$) [10]. There is a barrier for breaking this string inside the glass tube. Further assume that, as we approach the center of the cylinder, a core of isotropic phase is achieved with $D = 0$. Note that in this case $D$ does not have to become zero as it can start pointing along
the length of the string. However, it is known that there are many situations where it is energetically more favorable to have $D = 0$ in the middle instead of having nematic phase throughout (see [10]). This is the most suitable candidate for an optical fiber for a NLC.

Let us assume that unpolarized light is travelling along the core of such a string. Now suppose that along any direction away from the string axis where the director $D$ becomes non-zero, $D$ points in a certain direction. Then 50% of light which has $E$ parallel to $D$ in that region, will get out of the string rather quickly as for that portion of light the string acts as an anti optical fiber in the sense that the RI in the core (where the liquid crystal is in the isotropic phase) is smaller than the RI away from the core (where the liquid crystal is in the nematic phase).

However, for the remaining 50% of light $E$ will be normal to $D$ and this light will undergo total internal reflection at that boundary. In fact when it bounces off the opposite wall, it will again undergo total internal reflection as $D$ at opposite points are parallel. Thus, as long as the reflections remain in a plane, 50% of light will undergo total internal reflection at all walls of the string and for that much of the light the string is like an optical fiber. Note that it is important for this that as we go along the string axis, $D$ does not twist. Twists in $D$ will be generally suppressed, especially at low temperatures, since they cost energy. As for the elementary string of strength $1/2$, the problem in repeating the above argument is that as the light bounces on opposite walls it will not find $D$ with the same relative angle so there will be some light loss. (The string segments we see in NLC films on water are probably of this type, though $S = 1$ strings are also possible due to special boundary conditions at the NLC-air interface.) Fixed boundary conditions or applied external electric field may also give a suitable distribution of $D$ for this case. We will discuss these points in more detail in [8].

The above examples show that there are numerous examples where the fundamental string defects in condensed matter systems have the property of trapping light and behave like optical fibers. One needs to investigate other condensed matter systems to look for such strings. In the following we now consider possible ways of creating such fibers. We also comment on the possibilities of technological applications, though at present these ideas are at very elementary level.

As we mentioned above, for NLCs the construction of strength 1 string seems rather simple. One simply takes a glass tube with appropriate coating on the inner wall and fills it with NLC. This creates a string of strength 1 in the tube which under suitable conditions will have isotropic core [10] and from the above discussion should behave like an optical fiber. From the above discussion it should be clear that for this case it may be
necessary to constrain any bending of the tube (and the trajectory of the light) to remain
in a plane to avoid the loss of light being larger than 50%.

For $^4$He, the creation of vortices is even simpler. Imagine that a cylindrical tube is
filled with superfluid helium and this tube is rotated about its cylindrical axis (initially
we may assume this to be a straight cylinder, though this tube can be bent later and
actually even the rotation can be achieved while the tube is already bent). This process
is well known to create a bundle of vortices spreading from one end to another end of
the cylinder [6]. This bundle of vortices is completely topologically stable and it may be
possible to use it for image transmission (or for vortex imaging itself). One has to be
careful though as there may be interchanges in the relative positions of the ends of the
vortices which may scramble the image. However, these strings repel each other and in
steady ground state their ends form a well defined pattern (such as a triangular lattice),
see [6]. One will expect that each vortex will remain parallel to the local axis of the
cylinder, this being the shortest length for the vortex and energetically preferred. This
suggests that there will not be any scrambling of image if one does not keep moving the
cylinder and lets it settle to its lowest energy state.

The important point about such an image transmitting device being that all these
fibers are so thin that they are typically of the order of the wavelength of the light
(or smaller, so one has the worry about diffraction effects and one may use very short
wavelengths) which is anyway absolute limit to the resolution of an image. Note that
typical size here can be about 1/10 - 1/100 of the size of conventional fibers, implying
about 100- 10,000 times more information in the image. Note that in this case even the
number of these vortex fibers can be easily manipulated by changing the angular speed
of the cylinder, higher speeds producing larger number of vortices.

We conclude by emphasizing again the novel aspects of our approach to optical fibers.
Here one just looks for appropriate transparent condensed matter systems which support
string like topological excitations and finds whether the refractive index has correct prop-
erties. These strings then can behave like optical fibers. As these strings are of microscopic
thickness and typically may not be very long, presumably they can find applications only
in small devices.

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Figure Caption

Figure 1: These pictures show bright dots which are the ends of string segments. Strings are much fainter than their ends which appear bright. (a) shows one end of a string. (b) shows a string segment with both ends visible. (c) shows two string segments with their ends almost touching each other. The scale of the pictures is shown in (d).