Coupled-channels study of $^{10}\text{Be}+^{4}\text{He}$ structure of highly-excited $^{14}\text{C}$

Y Sakuragi$^{1,2}$, Y Maenaka$^{1}$ and T Furumoto$^{1}$

$^{1}$ Department of Physics, Graduate School of Science, Osaka City University, Sumiyoshi-ku, Osaka 558-8585, Japan
$^{2}$ RIKEN Nishina Center, RIKEN, Wako-shi, Saitama 351-0198, Japan
E-mail: sakuragi@ocunp.hep.osaka-cu.ac.jp

Abstract. Resonance structure observed in the decay spectra of highly-excited $^{14}\text{C}$ ($E_x=14\sim24$ MeV) into $\alpha+^{10}\text{Be}(0^+_1, g.s.)$ and $\alpha+^{10}\text{Be}(2^+_1; 3.37$ MeV) channels has been studied by the coupled-channels (CC) calculation of $\alpha+^{10}\text{Be}$ elastic and inelastic scattering. The CC calculation well reproduces the observed resonance structure both in the elastic and inelastic channels and the spin assignments to the resonance peaks are consistent with the experimental information, suggesting that the observed resonance states have well developed $\alpha+^{10}\text{Be}$ cluster structure. The calculated results are shown to be sensitive to the sign of $Q$-moment of $^{10}\text{Be}(2^+_1)$ state, namely to the shape of deformation either prolate or oblate of the $^{10}\text{Be}$ nucleus.

1. Introduction

Molecular-like cluster structure is now understood as a common feature of many nuclear states, not only in the groud state or low-lying excited states of very light nuclei such as lithium and beryllium isotopes but also in highly-excited region of light and medium-weight stable nuclei, such as $3\alpha$ structure in $^{12}\text{C}$ nucleus, $\alpha+^{12}\text{C}$ one in $^{16}\text{O}$ nucleus, $^{12}\text{C}+^{12}\text{C}$, $^{16}\text{O}+2\alpha$, $^{12}\text{C}+3\alpha$ and $3\alpha+3\alpha$ ones in $^{24}\text{Mg}$ nucleus, and $^{16}\text{O}+^{16}\text{O}$, $^{16}\text{O}+(\alpha+^{12}\text{C})$, $(\alpha+^{12}\text{C})+(\alpha+^{12}\text{C})$ ones in $^{32}\text{S}$ nucleus [1, 2, 3], as predicted by the so-called “Ikeda diagram” [4]. However, cluster structure in unstable neutron-rich or proton-rich nuclei has only been identified in a limited number of isotopes and/or states, such as $^6\text{He}+^6\text{He}$ and $\alpha+^8\text{He}$ structure in $^{12}\text{Be}$.

Existence of a well-developed $^{10}\text{Be}+\alpha$ cluster structure of highly-excited $^{14}\text{C}$ nucleus has been suggested by recent experiments [5, 6] in which highly-excited $^{14}\text{C}$, induced by the $^7\text{Li}(^{10}\text{Be},\alpha)^2\text{H}$ transfer reaction [5] or by the $^{14}\text{C}(^{14}\text{C},\alpha)^{10}\text{Be}$ breakup reaction [6], strongly decays into either $^{10}\text{Be}(\text{g.s.})+\alpha$ channel or $^{10}\text{Be}(2^+; 3.37$ MeV)$+\alpha$ one. The decay spectra show sharp resonances corresponding to highly excited states of $^{14}\text{C}$ at $E_x(^{14}\text{C})=14\sim24$ MeV. The filled dots shown in Figs.1 and 2 are the data observed in the transfer reaction experiment [5]. It is seen that the resonance states populated in the $^{10}\text{Be}(\text{g.s.})+\alpha$ channel are well correlated with those in the $^{10}\text{Be}(2^+; 3.37$ MeV)$+\alpha$ channel. The same is true also in the case of resonance states populated by the breakup reaction experiment [6]. These facts strongly suggest that these resonance states can be generated by the coupling between the $^{10}\text{Be}(\text{g.s.})+\alpha$ and $^{10}\text{Be}(2^+; 3.37$ MeV)$+\alpha$ channels, which are common exit channels of the reactions populating these states [5, 6].

In order to confirm this conjecture and study the origin and nature of these resonance states, we perform a microscopic coupled-channel (MCC) calculation of $^{10}\text{Be}+\alpha$ elastic scattering and inelastic one in which $^{10}\text{Be}$ is excited to the first excited state $^{10}\text{Be}(2^+; 3.37$ MeV).
2. Formalism: microscopic coupled-channels (MCC) method

The microscopic coupled-channels (MCC) method used in the present calculation is essentially the same as those used in Ref. [7] and details of the method was described there, although the isovector contribution of the folding-model interaction is neglected in the present analysis. We call the method "microscopic" because of the use of double-folding-model based on an effective NN interaction in constructing the diagonal and coupling potentials of the colliding system. The coupled-channels equations are written as

\[(T_i + V_i(R) - E_i) \chi_i(R) = - \sum_{j \neq i}^N V_{ij}(R) \chi_j(R) \quad (i = 1 \sim N) . \]

Here, \(V_{ij}(R)\) is the diagonal potential in the \(i\)th channel of the \(^{10}\)Be scattering system and \(V_{ij}(R)\) represents the coupling potential between different channels. In the MCC method, the diagonal and coupling potentials are calculated by the double-folding model using the density-dependent M3Y (DDM3Y) interaction [8]:

\[V_{ij}(R) = \int \rho_{00}^{(\alpha)}(r_1) \rho_{ij}^{(10\text{Be})}(r_2) v_{NN}^{(\text{DDM3Y})}(s, \rho) dr_1 r_2 . \]

Here, \(v_{NN}^{(\text{DDM3Y})}(s, \rho)\) denotes the DDM3Y effective NN interaction, where \(s = |r_1 + R - r_2|\) is the separation distance between the interacting nucleons, one in the projectile nucleus (\(\alpha\)) and the other in the target one (\(^{10}\text{Be}\)), while \(\rho\) is the local density at which the strength of the DDM3Y interaction is evaluated. \(\rho_{00}^{(\alpha)}\) is the nucleon density in the ground state of \(\alpha\) particle, while \(\rho_{ij}^{(10\text{Be})}\) is the diagonal (\(i = j\)) or transition (\(i \neq j\)) density of \(^{10}\text{Be}\). The nucleon densities are the sum of the proton density and the neutron density. We assume the same geometrical form for the proton and neutron densities for which we adopt a simple Gaussian form and the range parameter is determined so as to reproduce the charge radius 1.57 fm of the \(\alpha\) particle.

For the neutron-rich nucleus \(^{10}\text{Be}\), it is important to use different density profile for protons and neutrons. We adopt realistic proton and neutron densities in the ground state of \(^{10}\text{Be}\) obtained by a full microscopic \(\alpha+\alpha+n+n\) four-cluster model calculation by Arai et al. [9].

In the practical MCC calculation, the diagonal or transition proton (neutron) density of \(^{10}\text{Be}\) is decomposed into multipole components depending on the spin states of the \(i\)th and \(j\)th channels:

\[\rho_{ij, p(n)}^{(10\text{Be})}(r) = \rho_{1M, lM', p(n)}^{(10\text{Be})}(r) = \sum_{\lambda \mu} (l'M'|\lambda\mu|IM)Y_{\lambda\mu}^*(\hat{r}) \rho_{lM', p(n)}^{(\lambda)}(r) . \]

What we need to perform the MCC calculation in the present model space are the monopole (\(\lambda = 0\)) component of the proton (neutron) density in the \(^{10}\text{Be}\) ground state, \(\rho_{00, p(n)}^{(0)}(r)\), and that in the \(2^+\) excited state \(\rho_{22, p(n)}^{(0)}(r)\), respectively, and the quadrupole (\(\lambda = 2\)) component (the re-orientation coupling term) in the \(2^+\) excited state, \(\rho_{22, p(n)}^{(2)}(r)\), as well as the transition density between the \(0^+\) ground state and the \(2^+\) excited state, \(\rho_{20, p(n)}^{(2)}(r)\). The \(\lambda = 4\) component, \(\rho_{22, p(n)}^{(4)}(r)\) which exists in the \(2^+\) excited state, is expected to be small and neglected here.

In the present work, we take the monopole component of the proton (neutron) density in the \(2^+\) excited state to be the same as that of the ground state, i.e. \(\rho_{22, p(n)}^{(0)}(r) = \rho_{00, p(n)}^{(0)}(r)\), where the ground state density is given by the microscopic calculation by Arai et al. [9] as mentioned above. On the other hand, we adopt the Bohr-Mottelson type collective model [10] to construct the quadrupole (\(\lambda = 2\)) components of the transition density. This is because we want to
investigate how sensitive the resonance excitation curve predicted by the MCC calculation is to the unknown deformation shape, either prolate or oblate, of the $^{10}$Be nucleus which we can choose as an free parameter within the collective model. Of course, the use of more realistic transition densities obtained by some more elaborated microscopic wave functions would be desirable for precise comparison with the experimental data, which we will leave to a future study.

In the collective model, the radial form of the proton (neutron) transition density is given by the derivative form of the ground state proton (neutron) density $\rho_{00, p(n)}^{(0)}(r)$ as

$$\rho_{20, p(n)}^{(2)}(r) = \delta_{20, p(n)} \frac{d\rho_{00, p(n)}^{(0)}(r)}{dr}, \quad \rho_{22, p(n)}^{(2)}(r) = \delta_{22, p(n)} \frac{d\rho_{00, p(n)}^{(0)}(r)}{dr}. \quad (4)$$

Here, $\delta_{20, p(n)}$ and $\delta_{22, p(n)}$ are the corresponding deformation lengths for protons (neutrons). The proton part of the deformation lengths, $\delta_{20, p}$ and $\delta_{22, p}$, are related to the values of $B(E2; 0^+ \rightarrow 2^+)$ and electric quadrupole moment $Q_2$ of the $2^+$ excited state as

$$B(E2; 0^+ \rightarrow 2^+) = 5 \left| \int_0^\infty \rho_{20, p}^{(2)}(r)r^4dr \right|^2 = |\delta_{20, p}|^2 \times 5 \left| \int_0^\infty \frac{d\rho_{00, p}^{(0)}(r)}{dr}r^4dr \right|^2, \quad (5)$$

$$Q_2 = \left( \frac{16\pi}{5} \right)^{\frac{1}{2}} \left( \frac{2}{7} \right)^{\frac{3}{2}} \int_0^\infty \rho_{22, p}^{(2)}(r)r^4dr = \delta_{22, p} \left( \frac{16\pi}{5} \right)^{\frac{1}{2}} \left( \frac{2}{7} \right)^{\frac{3}{2}} \int_0^\infty \frac{d\rho_{00, p}^{(0)}(r)}{dr}r^4dr. \quad (6)$$

Therefore, if the experimental values of $B(E2; 0^+ \rightarrow 2^+)$ and $Q_2$ are available, one can determine the deformation lengths $\delta_{20, p}$ and $\delta_{22, p}$, for protons. Unfortunately for $^{10}$Be, the $\beta$-unstable neutron-rich nucleus, only the $B(E2; 0^+ \rightarrow 2^+)$ value, 52 e²fm⁴, is available but no measurement was made on the $Q_2$ value. However, in the collective model, $B(E2; 0^+ \rightarrow 2^+)$ and $Q_2$ can be connected by the relation [11],

$$B(E2; 0^+ \rightarrow 2^+) = \frac{5}{16\pi} e^2 \left( \frac{7}{2} \right)^2 |Q_2|^2 \quad (7)$$

if one further assumes that the $0^+$ ground state and the $2^+$ excited state of $^{10}$Be are the members of $K=0$ rotational band. Therefore, we first determine the $|\delta_{20, p}|$ value from the experimental $B(E2; 0^+ \rightarrow 2^+)$ value through Eq.(5) and then calculate the $|\delta_{22, p}|$ value via Eqs.(6) and (7). We further assume that the magnitude of neutron deformation length is the same as the corresponding proton one, namely, $|\delta_{20, n}| = |\delta_{20, p}|$ and $|\delta_{22, n}| = |\delta_{22, p}|$. This assumption would be reasonable because the ratio of proton number to the neutron one is already taken into account in the definition of transition densities by the relationship Eq.(4).

In this above prescription, however, only the magnitude of $\delta_{22, p}$ and $\delta_{22, n}$ are determined and no information about their sign is available. Since the sign of these values corresponds to the sign of quadrupole moment of the excited state nucleus, we have now two possibilities on the intrinsic shape of the proton and neutron distribution in $^{10}$Be, either prolate or oblate. Since the intrinsic quadrupole moment $Q_0$ has an opposite sign to the spectroscopic quadrupole moment $Q_2$ in the case of $K=0$ rotational band via the relation $Q_J = \frac{3K^2 - J(J+1)}{(J+1)(2J+3)} Q_0$ with $J=2$ and $K=0$ in the present case [11]. In the present analysis, we try the following three cases: (A) protons have intrinsic oblate shape ($\delta_{22, p} > 0$) and neutrons have the oblate one ($\delta_{22, n} < 0$), (B) both protons and neutrons have intrinsic oblate shape ($\delta_{22, p} < 0$, $\delta_{22, n} < 0$), and (C) both protons and neutrons have intrinsic prolate shape ($\delta_{22, p} > 0$, $\delta_{22, n} > 0$). (Note that the sign of $\delta_{22}$ is opposite to that of $Q_2$ due to the negative derivative of the ground-state density in Eq.(6), and hence the same as the sign of intrinsic quadrupole moment $Q_0$.)
3. Results and discussion

First, we perform a single-channel (SC) calculation of the elastic scattering and calculate the excitation function of the nuclear part of total cross section as a function of the $^{14}$C excitation energy, $E_x = E_{c.m.} + E_{th}$,

$$\sigma_{tot(N)}^{(el)}(E_x) = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) \left| 1 - S_{\ell}^{(el)}(E_{c.m.}) \right|^2,$$

(8)

where $E_{th} = 12.01$ MeV is the $^{14}$C $\rightarrow$ $^{10}$Be+$\alpha$ threshold energy and $E_{c.m.}$ is the center-of-mass energy of the $^{10}$Be+$\alpha$ scattering system. Figure 1 shows a comparison of the excitation function obtained by the SC calculation with the decay spectrum [5] observed in the elastic channel. The vertical scale is in arbitrary unit because different observables are compared. The three resonance peaks of the calculated excitation curve are of even partial waves, $J = 2$, 4 and 6. They are just the potential resonances produced by the present folding-model potential and do not show any further intermediate-width resonance structure observed in the experimental decay spectrum [5].

Next, we perform the two-channel MCC calculation by including the $2^+$ excited state of $^{10}$Be. The calculated results are shown by the solid curves in Fig. 2 for the elastic and inelastic channels, which are compared with the experimental decay spectra [5] in arbitrary unit. In this calculation, we assume that protons have an intrinsic prolate shape ($\delta_{22}, p > 0$) and neutrons have an oblate one ($\delta_{22}, n < 0$), namely corresponding to the above-mentioned case (A) calculation. The results of the case (B) and case (C) will be discussed later.

It is clearly seen that the MCC calculation well reproduces the characteristic resonance structure observed in the experimental decay spectra in both channels, although the calculated resonance energies are slightly higher (say, by about 0.5 MeV) than those of the experimental data. Such a small amount of resonance-energy shift could be corrected if we change the nuclear-
potential strength by only a few percent from the original one given by the DDM3Y double-folding model, which may be within an uncertainty of the theoretical model adopted, but we dare not modify the strength to keep the theoretical analysis transparent. Therefore, it would be safe to assign the first (lowest) peak of the elastic channel predicted around $E_x(^{14}C)=15$ MeV to either the 14.8 MeV state or the 15.6 MeV in the observed decay spectrum, the second, sharp peak at 16.9 MeV to the state observed at 16.4 MeV, and the third and fourth, rather broad, theoretical peaks around 19 MeV and 20.3 MeV to the 18.5 MeV and 19.8 MeV states in the observed spectrum. A sharp peak at 16.9 MeV also appears in the calculated excitation curve in the $\alpha+^{10}\text{Be}(2^+)$ channel, which indicates that this state is generated by the strong coupling between the elastic and inelastic channels. Other peaks predicted in the inelastic channel also seem to coincide with the observed states in the $E_x(^{14}C)\approx 18\sim 22$ MeV region, although one-to-one assignments are not so clear compared with the elastic-channel case.

In order to identify the spin of the predicted resonance states, we decompose the calculated total cross section into partial-wave components, which are shown in Figs. 3 and 4. As clearly seen, each of the resonance structure predicted both in the elastic and inelastic channels has a single dominant partial-wave component and in some partial waves, say $J=5$ and 6, strong correlations between the two channels are observed.

It is worth noting that the $J^\pi=3^-$ state predicted in the elastic channel around $E_x(^{14}C)=15$ MeV can be a good candidate of the 15.6 MeV state, the spin-parity of which has recently been assigned to be $J^\pi=3^-$ by the angular-correlation measurements [6] of the decay fragments from the state. It should also be mentioned that the $J^\pi=6^+$ sharp state predicted at 16.9 MeV can certainly be a good candidate of the sharp state experimentally observed at 16.4 MeV, as mentioned above, and the spin-parity of the state was temporally assigned to be $J^\pi=6^+$[12] which is consistent with the MCC prediction. To confirm the theoretical spin-parity assignment for these states, we also calculate the angular distribution of the differential cross section on the resonance peak energy for each state. For each state, the angular distribution has a very similar shape as the squared Legendre Polynomial, $|P_J(\cos \theta)|^2$, of the corresponding spin value $J$, particularly at backward angles where the resonance phenomena can be prominent.

Since no clear spin assignment has been made on the other observed states, it would be an experimental challenge to test the validity of the MCC predictions based on the $\alpha+^{10}\text{Be}$ cluster structure for these highly-excited $^{14}C$ states through the experimental spin-parity assignment.

Finally, we perform the same MCC calculation but with different signs on the deformation length parameters, $\delta_{22}, p$ and $\delta_{22}, n$. Figures 5 and 6 show the comparison of the MCC calculations...
Figure 5. Comparison of the MCC calculations for elastic channel with different assumption on the intrinsic shapes of protons and neutrons in $^{10}$Be. See text for details.

Figure 6. Same as Fig.5 but for the inelastic channel.

in the cases (A) to (C) for elastic and inelastic channels. It is clearly seen that the different choice of the deformation shape for protons and neutrons in $^{10}$Be leads to very different results and case (A) seems to give a better agreement with the experimental data than the other two cases. Of course, the present collective-model assumption for the transition densities based on a $K=0$ symmetric rotor picture would be too simplified and might not be so realistic and, hence, it may be too early to conclude that the present MCC analysis verifies the picture suggested by the case (A), a prolate shape for protons and an oblate one for neutrons in $^{10}$Be. However, it should be worthwhile to point out the importance of theoretical analyses of these resonance phenomena based on the MCC type framework, which directly reflects the nuclear structure information on the interaction responsible for the reactions and resonance formation of such exotic nuclear structure.

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