Exploring the hydrostatic mass bias in MUSIC clusters: application to the NIKA2 mock sample

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ABSTRACT
Clusters of galaxies are useful tools to constrain cosmological parameters, only if their masses can be correctly inferred from observations. In particular, X-ray and Sunyaev-Zeldovich (SZ) effect observations can be used to derive masses within the framework of the hydrostatic equilibrium. Therefore, it is crucial to have a good control of the possible mass biases that can be introduced when this hypothesis is not valid. In this work, we analyzed a set of 260 synthetic clusters from the MUSIC simulation project, at redshifts $0 \leq z \leq 0.82$. We estimate the hydrostatic mass of the MUSIC clusters from X-ray only (temperature and density) and from X-ray and SZ (density and pressure). Then, we compare them with the true 3D dynamical mass. The biases are of the order of 20%. We find that using the temperature instead of the pressure leads to a smaller bias, although the two values are compatible within 1σ. Non-thermal contributions to the total pressure support, arising from bulk motion and turbulence of the gas, are also computed and show that they are sufficient to account for this bias. We also present a study of the correlation between the mass bias and the dynamical state of the clusters. A clear correlation is shown between the relaxation state of the clusters and the bias factor. We applied the same analysis on a subsample of 32 objects, already selected for supporting the NIKA2 SZ Large Program.

1 INTRODUCTION
Galaxy clusters are the most massive gravitationally bound objects in the Universe and they are mainly composed by Dark Matter, that amounts to 80% of the total mass (for a full review see Kravtsov & Borgani 2012). About 8% is composed by galaxies and the remaining 12% is represented by the so called Intra Cluster Medium (ICM), i.e. the hot gas located between galaxies. This gas component provides significant physical information, as it can be observed in the X-ray band and in millimeter wavelengths through the Sunyaev-Zeldovich effect (Sunyaev & Zeldovich 1972).

The emission in the X-ray band is mainly due to the thermal bremsstrahlung. From this emission we can directly measure the temperature, which determines the bremsstrahlung cut-off, and the electron density, to which the spectrum normalization is proportional (for a review see Boehringer & Werner 2009). X-ray observations occurred to be particularly successful because the emission is proportional to the square of the gas density (for a review on the methods adopted to reconstruct the mass profiles in X-ray luminous galaxy clusters see Ettori et al. 2013). In the last two decades, X-ray

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observatories with improved sensitivity and angular resolution, like, for instance, XMM-Newton and Chandra, has conducted cluster X-ray emission studies over large areas of the sky (Pacaud et al. 2007), and deeper studies of previously known objects (e.g. Vikhlinin et al. 2009; Mantz et al. 2010).

X-ray observations are mainly exploring the central regions of the clusters, where the electron number density is high, although there are also X-ray projects exploring the cluster outskirts, such as X-COP (Eckert et al. 2017). A more efficient way to map the cluster outskirts is through the thermal Sunyaev-Zeldovich effect (SZ, Sunyaev & Zeldovich 1972). This effect depends linearly on the pressure, so it is less sensitive to the density decrease at high radial distances from the cluster centre. The SZ effect is a redshift independent probe that consists of the spectral distortion of the CMB radiation due to the inverse Compton scattering of the CMB photons with the hot electrons of the ICM (for a review see Carlstrom et al. 2002; Kitayama 2014; Mroczkowski et al. 2019). It can be used to directly measure the CMB pressure distribution. The latter can be combined with the electron density from an X-ray observation to infer the cluster mass profile. This method avoids the time-consuming evaluation of the temperature with X-ray observations. To date, the SZ effect induced by thermal electrons (tSZ) has been detected for more than a thousand galaxy clusters, including more than 200 new clusters previously unknown by any other observational means (Kitayama 2014) thanks to new observing facilities such as the South Pole Telescope (SPT) (Williamson et al. 2011; Reichardt et al. 2013; Bleem et al. 2020), the Atacama Cosmology Telescope (ACT) (Marriage et al. 2010; Hasselfield et al. 2013; Hilton et al. 2018, 2020), and the Planck satellite (Planck Collaboration 2011, 2013c, 2016b).

Both X-ray and SZ observations can be used to infer the mass of a cluster. In particular, from the former we can exploit the temperature and the electron density, from the latter the pressure profile. In order to use this information, we need to make two fundamental assumptions: the gas must trace the cluster potential well and it must be in hydrostatic equilibrium (HE, Kravtsov & Borgani 2012 for a review). The mass inferred through this method is called hydrostatic mass (see Section 2). An easy way to track the error made when using this equation is called hydrostatic mass bias. It is defined as the difference of the cluster total mass to the one estimated by hydrostatic equilibrium, divided by the total mass $b = (M_{\text{tot}} - M_{\text{HE}})/M_{\text{tot}}$.

Galaxy clusters, and, in particular, their number counts are a fundamental cosmological tool. The abundance of clusters and its evolution with redshift are particularly sensitive to the cosmic matter density, $\Omega_m$, and the present amplitude of density fluctuations, characterized by $\sigma_8$, i.e. the rms linear overdensity in spheres of radius $8h^{-1}$ Mpc. The CMB primary anisotropies, on the other hand, are related to the density perturbation power spectrum at the time of recombination. A comparison of the amplitude of density perturbations from recombination until today, allows us to look for possible extensions to the concordance ΛCDM model, such as non-minimal neutrino masses or non-zero curvature contributions (Planck Collaboration 2016a; Salvati et al. 2018).

In Planck Collaboration (2013b), a tension between the number of clusters detected by SZ signal, and the number of clusters predicted from the cosmological parameters inferred from the primary CMB power spectrum is reported. This issue was later confirmed in Planck Collaboration (2016a) and argues that before the hydrostatic mass bias is fixed to a constant value of 0.2 in cluster cosmological analyses. The hydrostatic mass bias $b$ plays an important role in the number counts, because it leads to a modification of the cluster population at a given mass. In particular, it significantly affects the value of $\sigma_8$: the lower $(1-b)$ is, the higher is $\sigma_8$ (Salvati et al. 2018; Ruppin et al. 2019a). Salvati et al. (2018) published an update of the constraints on cosmological parameters from the clusters observed by Planck. They find that the bias needed to reconcile CMB constraints with those from the tSZ number counts is $(1-b) = 0.62 \pm 0.07$, which is compatible with the value $(1-b) = 0.58 \pm 0.04$ found in Planck Collaboration (2016a). This value is confirmed by Koulouridou et al. (2020), who cross-correlate Planck maps of the tSZ Compton-y parameter with the galaxy distribution.

These values derived from observations are nevertheless in disagreement with the value of $(1-b)$ estimated from simulations, which is about 0.8. This topic will be deeply discussed later in this work. Moreover, as it was noticed in the 2016 results of Planck (Planck Collaboration 2016a), and confirmed by Salvati et al. (2018), there was a factor of 2.5 more clusters predicted than observed when taking into account the CMB cosmology and a value $(1-b)$ of 0.8. This value of the bias, derived from simulations, has been recently confirmed by Makiya et al. (2020). They perform a joint analysis of power spectra of the tSZ and the cosmic weak lensing shear, in order to obtain a $(1-b)$ constraint which is independent from the primordial CMB spectrum. They find $(1-b) = 0.73^{+0.08}_{-0.07}$ and conclude that the late-time probes (tSZ and cosmic weak lensing shear) cosmologies are consistent with each other, but they could not be totally consistent with the CMB cosmology, which is leading to a different value of the mass bias.

However, the cluster number counts are limited by systematic effects, in particular those affecting the mass estimates. The tSZ power spectrum, in turn, is not measured with sufficient accuracy, especially at small angular scales, to reduce the tension with the CMB. The tSZ cosmological analysis can be improved by considering more realistic and complex hypotheses on the mass bias (e.g. redshift and/or mass dependence), the pressure profile and mass function (Salvati et al. 2018; Ruppin et al. 2019a).

The tension between the cluster number counts from the tSZ and the CMB power spectrum arising from fixing the hydrostatic bias to 0.2 in cosmological analysis has led the scientific community to investigate this discrepancy in more detail. Several studies have been made on the HE mass bias. In this work, a simulated dataset of almost 260 clusters from the MUSIC simulations is analysed. We focus on the determination of cluster masses assuming the hydrostatic equilibrium hypothesis, at different redshifts, and compare them with the true cluster mass derived from the simulation data. The HE masses can be estimated using two different equations, depending on which ICM thermodynamic quantities are used, i.e. the pressure and electron density (hereafter referred to as SZ mass) or the temperature and electron density (hereafter X-ray mass). There are also different ways of computing the radial gradients of these quantities. In this work, we present a complete study of the HE mass derivations from the different assumptions. Moreover, we also correlate the results for the mass bias with the dynamical state of the MUSIC clusters. A correction to the HE mass, which takes into account the non thermal pressure contribution arising from gas motions in the ICM, is applied.

We repeated the same analysis for a sub sample of 32 objects from MUSIC, in the redshift range $0.5 < z < 0.9$. This sample, named the NIKA2 twin sample, has been selected to be representative of the clusters observed in the NIKA2 SZ Large Program (Mayet, F. et al. 2020, LPSZ). The LPSZ uniquely exploits the excellent match in sensitivity and spatial resolution of XMM-Newton and the NIKA2 camera, which is a millimetre camera installed at the 30-m radio telescope of the Institut de Radioastronomie Millimétrique (IRAM) in Pico Veleta, Spain (Adam et al. 2018; Perotto et al. 2020). A previous analysis of the gas pressure profiles re-
construction on this sample by applying the NIKA2 data reduction pipeline has been already presented in Ruppin et al. (2019b).

The paper is organized as follows. In Section 2, we describe the hydrostatic equilibrium, and the estimate of the cluster mass from different approaches. In Section 3, we review the estimates of the HE mass bias from different numerical hydrodynamical simulations that have been published in literature, showing the relatively large variations in the bias results from the different simulation suites. In Section 4, we briefly introduce the MUSIC simulations and the data used in this analysis, classifying clusters by their dynamical state. The ICM profiles are presented in Section 5. The distribution of the results for HE masses and their biases are described in Section 6, along with the modelling of the non-thermal correction. In Section 7 we focus on the HE masses and biases for the NIKA2 twin sample. Finally, in Section 8, the main conclusions from these analyses are given.

2 HYDROSTATIC EQUILIBRIUM

We use the hydrostatic equilibrium (HE) hypothesis to estimate the mass of each cluster, see Kravtsov & Borgani (2012); Pratt et al. (2019) for a review. It assumes that the gas thermal pressure is balanced by the gravitational force, so that the cluster is in equilibrium. Further assuming that the system is in spherical symmetry and the gas pressure is purely thermal, the total mass inside a sphere of radius , can be written as

\[
M_{\text{HE}}(r) = - \frac{r^2}{G \mu m_p n_e(r)} \frac{dP_{\text{th}}(r)}{dr}
\]

(1)

where is the gravitational constant, is the mean molecular weight of the ICM, here 0.59, is the proton mass, and are the numerical electron density and the thermal pressure of the gas. Assuming the equation of state of an ideal gas, it follows that the cluster mass can also be derived from the electron density and temperature profiles, as

\[
M_{\text{HE}}(r) = \frac{r k_B T(r)}{G \mu m_p} \left[ \frac{d \ln n_e(r)}{dr} + \frac{d \ln T(r)}{dr} \right]
\]

(2)

We refer to Eq. (1) as \(M_{\text{HE, SZ}}\), given that the pressure is estimated by SZ observations, while we refer to Eq. (2) as \(M_{\text{HE, X}}\) because the temperature and the electron density are usually estimated from X-rays observations (Ansarifard et al. 2020).

Deviations from equilibrium could have an impact on observable properties of clusters and may cause systematic errors when Eq. (1) and (2) are used to estimate cluster masses. A direct comparison with the real mass is usually quantified in the form of a hydrostatic mass bias (see references in Section 3). It should be stressed that the mass bias can be estimated only if the exact cluster total mass is available, which is the case in simulations. The real mass \(M_{\text{true}}\) of a simulated cluster can be easily computed by summing all the dark matter, stars and gas particle/cells masses inside an aperture radius. From the gas density, temperature and pressure profiles, the \(M_{\text{HE}}\) is derived using Eq. (1) or (2) and the mass bias \(b_{\text{SZ}}\) or \(b_{\text{X}}\) at a specific radius, is defined as

\[
b = \frac{M_{\text{true}} - M_{\text{HE}}}{M_{\text{true}}}
\]

(3)

The bias defined in this way is usually a positive quantity, since the HE mass often underestimates the true mass. It can happen also the contrary, leading to a negative bias. Sometimes in literature the opposite difference between the masses is chosen.

3 HYDROSTATIC MASS BIAS, STATE OF THE ART

The comparison between tSZ cluster number counts and CMB Planck results has led the community to carefully account for the impact of the hydrostatic mass bias on cosmological constraints. Here we focus on the hydrostatic mass bias \(b\), Eq. (3), computed at \(R_{500}\). This parameter has been extensively studied with a variety of numerical hydrodynamical simulations.

3.1 Previous work in literature

In Fig. 1 we present a compilation of results published in the literature, including the error estimates of the mean values for \(1-b\). The mean values are represented as vertical white rectangles, the errors corresponding to each value are the blue coloured regions. The different shades of blue represent the physical processes included in each simulation: non-radiative (light blue), cooling+star formation and supernovae feedback (medium blue) and those including also Super Massive Black Hole Feedback (SMBH) (dark blue). In the vertical axis, after the authors reference, we indicate, in parenthesis, the type of bias estimated in each work: SZ for HE masses derived from pressure and density profiles and X-ray for HE masses computed from temperature and density profiles, see Section 2.

Some authors define a positive bias, see the definition in Eq. (3), others negative, but in Fig. 1, in order to compare them, all biases are taken as positive. Here we represent the biases from the mass-weighted temperature profiles, in order to focus mainly on the degree of hydrostatic bias, since the spectroscopic-like temperature profiles (Mazzotta et al. 2004) are sensitive, in addition, to observational biases in the derived gas density and temperature profiles (see discussion in Ansarifard et al. 2020).

As it can be seen in the Figure, most of the published results for \(1-b\) are in the range 0.75 to 0.9, or \(b \sim 0.25 - 0.10\). The majority of these results are in disagreement with respect to the bias needed to reconcile CMB constraints with those from the tSZ number counts of \((1-b) = 0.58 \pm 0.04\), shown as the vertical red dashed line and shaded region (Planck Collaboration 2013b, 2016a). Only 3 bias values over 22 are compatible with it within 1\(\sigma\).

The mean and standard deviation are reported for all the works except for Biffi et al. (2016), where median and MAD (Median Absolute Deviation, computed as \(\text{median}(|b - \text{median}(b)|)/n_{\text{clusters}}\)) are presented. We find a X-ray bias of \(b_{\text{X}} = 0.16^{+0.10}_{-0.12}\), using the median and the percentiles (16th and 84th). Also Gupta et al. (2017) report median and percentiles, while Barnes et al. (2020) report the bootstrap errors. There are also some authors, represented without the vertical white rectangle, who give a range of values for \(b\) but not a central value with an error (Ameglio et al. 2009; Kay et al. 2012; Martizzi & Agrusa 2016; Le Brun et al. 2017; Henson et al. 2017; Pearce et al. 2020).

3.2 Bias dependence on simulation and sample properties

This compilation shows a wide spread in the determination of the bias parameter and their errors. It is obvious that these differences might be attributed to the particularities of the simulations used in each work. To shed some light on this problem, we also account for features like mass resolutions (Fig. 2, central panel); the range

\[1\] The radius where the cluster density is 500 times the Universe critical density \(\rho_c\) at that time, \(\rho_c = 3H(a)^2/(8\pi G)\) where \(H(a)\) is the Hubble function. \(M_{500}\) is the mass inside a sphere with radius \(R_{500}\).
of analyzed halo masses (Fig. 2, left panel) and the statistics of the total number of clusters (Fig. 2, right panel). Moreover, the physical processes included in each simulation should also be considered. They are represented in Fig. 1 and Fig. 2 by the different shades of blue of the bars, as already explained above.

In Fig. 2, we indicate the type of code used in each simulation in parenthesis next to the reference of the study. The majority of them are based on different flavours of the Smoothed-Particle Hydrodynamics (SPH). Lau et al. (2009); Nelson et al. (2014a); Shi et al. (2016); Martizzi & Agrusa (2016) use finite volume eulerian hydrodynamics with Adaptive Mesh Refinement algorithms. The Illustris simulations (Barnes et al. 2020), using the Moving Mesh code (MM) AREPO (Springel 2010) have the lowest DM particle mass, along with the RAMSES code used by Martizzi & Agrusa (2016), Gupta et al. (2017) with Magnetincum simulations, and Le Brun et al. (2017) with cosmo-OWLS, employ the largest number of clusters and have one of the lowest errors on the bias estimate.

We compared the HE biases in Fig. 1 with the main features of each simulation shown in Fig. 2. We further plot the biases as a function of each quantity. For the sake of brevity, we do not show these figures, but only conclude here that we cannot draw any clear dependence between the hydrostatic bias and the cluster mass range (left panel of Fig. 2), the particle mass resolution (central panel) or the number of clusters included in the analysis (right panel).

3.3 Bias dependence on other quantities or measurements

The more compelling questions about the HE mass bias are its dependence on redshift, on cluster dynamical state or on whether the bias is calculated from spectroscopic-like temperature. The redshift dependence has been studied by several authors (Piffaretti & Valdarnini 2008; Lau et al. 2009; Le Brun et al. 2017; Henson et al. 2017). The expectation is that clusters, going at higher redshifts and being less relaxed, tend to have more mass in substructures (Neto et al. 2013b, 2016a). The three different shades of blue represent three classes: those which includes the gravitational and non relativistic physics (NR, light blue in Fig. 1); the ‘middle’ set, which includes also radiative processes, like cooling, star formation and Supernovae feedback (CSF, medium blue); and the ‘complete’ set, which, in addition, includes also the feedbacks from super massive black holes (CSF+SMBH, dark blue). As it can be clearly seen, all the simulations which have the SMBH feedback, have lower errors on the bias, such as Kay et al. (2012); Battaglia et al. (2013); Biffi et al. (2016); McCarthy et al. (2016); Martizzi & Agrusa (2016); Gupta et al. (2017); Pearce et al. (2020); Ansarifard et al. (2020). Even in Meneghetti et al. (2010) the bias error is low mostly for two main reasons: their simulations considered 1/3 of Spitzer thermal conductivity which homogenize the medium and the statistics is limited due to the small sample size (see Fig. 2).
MUSIC and NIKA2 twin sample HE mass bias

4 THE SIMULATED DATASET

4.1 MUSIC simulations

The simulated clusters analysed in this work are taken from the MUSIC\(^2\) project (Sembolini et al. 2013) which consists of two sets of resimulated clusters extracted from two large volume simulations: the MUSIC-1 sample, extracted from the 500\,h\(^{-1}\) Mpc MareNostrum Universe simulation box (Gottlöber & Yepes 2007), and the MUSIC-2 sample, coming from the 1h\(^{-1}\) Gpc MultiDark (MD) simulation box (Prada et al. 2012).

In this work, the 258 zoomed regions around the most massive clusters in the MUSIC-2 database were analysed. From a low-resolution version (256\(^3\) particles) of the two simulations, the particles inside a sphere of 6h\(^{-1}\) Mpc radius at \(z = 0\) are mapped back to the initial redshift, using the Klypin et al. (2001) zooming technique, to identify their corresponding Lagrangian regions. These regions are then resimulated with high resolution and populated with SPH gas particles. The original MD dark-matter-only simulation was performed with L-Gadget2 code (Klypin et al. 2016) and adopting WMAP7+BAO+SN1 cosmology: \(\Omega_M = 0.27\), \(\Omega_b = 0.0469\), \(\Omega_{\Lambda} = 0.73\), \(\sigma_8 = 0.82\), \(n = 0.95\) and \(h = 0.7\) (Komatsu et al. 2011).

All the resimulations are done with the TreePM+SPH GADGET code (Springel 2005), and include three different classes of physical processes, labelled as fl\(\overset{\text{avours}}{\text{ors}}\): Non Radiative (NR), Cooling and Star Formation (CSF) and Active Galactic Nuclei (AGN). The NR flavour includes only the gravitational and gasdynamical effects, while the CSF flavour includes radiative processes, like star formation, feedback from supernovae, UV photoionization, and radiative cooling (see Sembolini et al. 2013 for more details), and finally AGN, where the AGNs and their feedback are added, using the models for super massive black hole feedbacks as in Planelles et al. (2013).

The clusters were identified using a Bound Density Maxima halo finder (see also AHF halo finder, Knollmann & Knebe 2009). Since we take all the objects above a given mass, the MUSIC-2 catalogue constitutes a complete volume limited sample. Our cluster masses \(M_{900}\) range from 1.9 \(\times\) 10\(^{14}\)\,\(M_\odot\) to 1.7 \(\times\) 10\(^{15}\)\,\(M_\odot\) at \(z = 0\). All of them were resimulated with NR, CSF and AGN fl\(\overset{\text{avours}}{\text{ors}}\) with a DM and gas mass resolution of \(m_{\text{DM}} = 1.29 \times 10^{8}\,M_\odot\) and \(m_{\text{gas}} = 2.7 \times 10^{8}\,M_\odot\). MUSIC-2 cluster regions have been saved at specific redshifts, so in this analysis we also studied the cosmic evolution of these objects, using only the main progenitors of the \(z = 0\) clusters at redshifts 0.11, 0.33, 0.43, 0.54, 0.67 and 0.82.

4.2 The NIKA2 LPSZ twin sample

In Section 7, we will focus on a subsample of objects selected in Ruppin et al. (2019b). The clusters of this sub sample were extracted from the MUSIC-2 dataset, which has been selected to reproduce the observed clusters in the NIKA2 Large Program SZ (LPSZ) catalogue (Mayet, F. et al. 2020). NIKA2 (Adam et al. 2018; Calvo et al. 2016; Perotto et al. 2020), is the new multipixels camera at 150 and 260 GHz installed at the 30-m telescope of the Institut de Radioastronomie Millimétrique (IRAM). The NIKA2 SZ large program consists of mapping the tSZ signal of a representative sample of 50 galaxy clusters at high angular resolution (\(18''\) and \(11''\) in two bands) and in the 0.5 < \(z\) < 0.9 redshift range. The cluster sample was extracted from the tSZ catalogues established by the Planck and ACT collaborations (Planck Collaboration 2016b; Hasselfield et al. 2013), and the selected clusters homogeneously populate the mass range with \(M_{500} > 3 \times 10^{14}\,M_\odot\) (Mayet, F. et al. 2020) and redshift range. The MUSIC NIKA2 twin sample closely matches the same mass-redshift space as the NIKA2 tSZ large program. For the redshift bin 0.5 < \(z\) < 0.7 eighteen clusters were chosen from the MUSIC-2 catalogue at redshift 0.54. For the 0.7 < \(z\) < 0.9 bin, 14 clusters from MUSIC redshift 0.82 were also selected. The same mass cut applied to the Planck and ACT catalogue of \(M_{500} > 3 \times 10^{14}\,M_\odot\) was applied to the MUSIC sample, so only the clusters with a hydrostatic mass estimate above this threshold are used, in order to make the two sample comparable. A previous analysis on the gas pressure profiles recovered from the NIKA2 twin sample has been already performed (Ruppin et al. 2019b; De Petris et al. 2020).

4.3 Characterization of the cluster dynamical state and morphology

Throughout this analysis the dynamical state of the clusters has been inferred by two 3D indicators (Neto et al. 2007; Sembolini et al. 2007; Angelinelli et al. 2019) risking to violate the hydrostatic equilibrium hypothesis. However, this behaviour has not yet been confirmed in any work.

According to their dynamical state, clusters are usually classified in two main classes: the relaxed ones, well described by spherical symmetry and HE, and disturbed clusters, the opposite. Non-thermal gas motions in the ICM and the non-spherical symmetry of a cluster (usually disturbed) will most likely lead to a larger HE bias and a larger scatter, with respect to the more regular (relaxed) clusters. Several authors analysed the mass bias dependence with the dynamical state, like Piffaretti & Valdarnini (2008); Rasia et al. (2012); Nelson et al. (2014a); Henson et al. (2017); Ansarifard et al. (2020). They all find no significant distinction between the mass bias of regular and disturbed clusters, given the large dispersion (Cialone et al. 2018). However, Biffi et al. (2016) observe that Cool-Core (CC) and Non Cool-Core (NCC) clusters behave differently, with a larger bias for NCC, especially in the innermost cluster regions.

The spectroscopic-like temperature, see Section 5.3, is estimated by weighting the temperature by the X-ray emission of each gas particle. Usually the HE mass bias estimated from this temperature by combining it with the electron density is larger than the one from the mass-weighted temperature and shows a dependence with the true mass of the halos. This is mainly due to temperature inhomogeneities (Rasia et al. 2006, 2012, 2014; LeBrun et al. 2017; Henson et al. 2017; Pearce et al. 2020; Barnes et al. 2020). Moreover, Piffaretti & Valdarnini (2008) find that the spectroscopic bias also depends on the dynamical state of clusters, with large biases found in the most disturbed clusters (see also Biffi et al. 2014).

In the last years, the HE mass biases (SZ and X-ray) were often studied together (Pearce et al. 2020; Ansarifard et al. 2020). Ansarifard et al. (2020), for example, analyse more than 300 simulated massive clusters, from 'The Three Hundred Project' (Cui et al. 2020), for example, analyse more than 300 simulated massive clusters, from 'The Three Hundred Project' (Cui et al. 2018). They find that a robust correction to the hydrostatic mass bias can be inferred when the gas inhomogeneity from X-ray maps are combined with the asymptotic external slope of the gas density or pressure profiles, which can be derived from X-ray and SZ effect observations. Both SZ and X-ray biases are estimated, with values of 10\%, by using models to fit ICM radial profiles.
et al. 2013; Cialone et al. 2018, for a novel alternative approach to infer the cluster morphology and dynamical state through the Zernike polynomials see Capalbo et al. 2020). In our case, we consider an aperture radius smaller than the virial one used in the previous works. Here we focus on the dynamical state of the clusters inside \( R_{500} \), which is more in agreement with what is measured in observations. The considered 3D dynamical state estimators are

- \( M_{\text{sub}}/M_{500} \), the ratio between the mass of the most massive cluster substructure and the total cluster mass inside \( R_{500} \). This indicator is mainly sensible to strong mergers. Therefore it is useful to find the really disturbed clusters. An alternative definition of \( M_{\text{sub}} \) would be to account for the mass of all the substructures within the aperture radius. This approach is more sensible to a cluster relaxation state (Cui et al. 2017 and De Luca et al., in prep);

- \( \Delta_{\delta} \), the offset between the central density peak, \( r_{\delta} \), and the centre of mass of the cluster, \( r_{\text{cm}} \), normalized to the aperture radius \( R_{500} \):

\[
\Delta_{\delta} = \frac{r_{\delta} - r_{\text{cm}}}{R_{500}}.
\]

In order to have a relaxed cluster, both indicators should be smaller than a given threshold, which varies depending on the authors (Macciò et al. 2007; D’Onghia & Navarro 2007). Here, following Cialone et al. (2018), both indicators should be lower than 0.1 to define a cluster as relaxed, and greater than 0.1 to have a disturbed one. In the cases in which the two indicators provide contradictory answers, the cluster is classified as intermediate or hybrid. According to this selection for the dynamical state estimators, as seen in Table 1, the MUSIC dataset contains roughly 50% of relaxed clusters for all the redshift intervals considered. The same happens for the NIKA2 sub sample.

We can combine the information provided by the two dynamical indicators in a single ‘relaxation’ parameter \( \chi_{DS} \), defined as in Haggar et al. (2020), but dropping the virial ratio \( \eta \). In order to keep the same number of relaxed clusters from the definition of the two separate dynamical indicators, \( M_{\text{sub}}/M_{500} \) and \( \Delta_{\delta} \) (see De Luca et al., in prep). \( \chi_{DS} \) is a continuous, non binary, estimate of the dynamical state

\[
\chi_{DS} = \sqrt{\frac{\Delta_{\delta}^2}{(0.1)^2} + \left( \frac{M_{\text{sub}}/M_{500}}{0.1} \right)^2}.
\]

All clusters that have this parameter higher than 1 are dynamically relaxed. We will study how this parameter is correlated with \( b \) (in Section 6.3.1).

### 5 ICM PROFILES OF THE MUSIC CLUSTERS

For each MUSIC cluster we compute the 3D radial profiles of the ICM thermodynamic properties. The cluster is divided into spherical shells, from 0 (the core) to 3\( R_{500} \) (the outskirts), each with a thickness of 10 kpc. The gas pressure and the electron density, are evaluated as the median of all the SPH gas particles inside each spherical shell, while the temperature used is the mass-weighted one. The associated uncertainty of the median value is estimated by MAD

\[
MAD = \text{median}(|X_i - \text{median}(X)|)
\]

The uncertainties associated to the median profiles usually increase in the cluster outskirts due to the deviations from the spherical symmetry and from a homogenous distribution of the ICM density and temperature (e.g. presence of clumps or disturbances generated by accreting material).

#### 5.1 Pressure profile

The cluster pressure profile is well modelled by the generalized Navarro-Frenk-White (gNFW) model, introduced by Nagai et al. (2007)

\[
P(r) = \frac{P_0}{x^{\alpha} (1 + x^2)^{\beta/2}}
\]

with \( x = r/r_s \) a dimensionless radial distance normalised to the scale radius \( r_s = R_{500}/c_{500} \), where \( c_{500} \) is the concentration parameter. The parameters \( b \) and \( c \) are the slopes for outer and inner region radii respectively and \( \alpha \) is the steepness of the transition between the two regimes. The normalization \( P_{500} \) is inferred by the scaling relation between the pressure content and the cluster total mass in the self-similar model (Arnaud et al. 2010, hereafter A10) purely based on gravitation:

\[
P_{500} = 1.65 \times 10^{-3} E_z^{8/3} \left( \frac{M_{500}}{3 \times 10^{14} h_{70}^{-1} M_{\odot}} \right)^{2/3} h_{70}^{-2} \text{keV/cm}^3.
\]

\( E_z \) is the ratio of the Hubble constant at redshift \( z \) to its present value \( H_0 \), and \( h_{70} = H_0/[70 \text{ km/s/Mpc}] \).

A worthwhile step is to compare our simulated cluster sample with observations. A10 computed the mean pressure profile of galaxy clusters using observed clusters from REXCESS, a representative sample of 33 local clusters (\( z < 0.2 \)) drawn from the REFLEX catalogue and observed with XMM-Newton satellite, and three large samples of simulated clusters at redshift zero extracted from hydrodynamical simulations (Borgani et al. 2004; Nagai et al. 2007; Piffaretti & Valdarnini 2008). The fit on the profile was performed in the radial range [0.03-4]\( R_{500} \). In this radial range, the observed profile is limited to \( R_{500} \) and the region outside this radius was extrapolated according to the predictions from numerical simulations. They describe the resulting pressure profiles as universal, since it fits well both data from simulations and observations, with parameters listed in Table 2, fifth row.

The Planck Collaboration (Planck Collaboration 2013a, P13) compared the median gNFW pressure profile of 62 nearby (\( z < 0.5 \)) massive observed clusters with the A10 profile, finding that there is a very good agreement in the cluster intermediate radii between...
the two. However, within the core, i.e. $R < 0.15 R_{500}$, the observed profile lies significantly below the A10 profile. The fit was done in the radial range $[0.02-3] \times R_{500}$ and the parameters are reported in the fourth row of Table 2.

To compare our simulated sample with observations, we compare the median pressure profiles (dots in Fig. 3) for all the MUSIC cluster sets at low redshifts ($z < 0.5$, a sample of almost 1050 clusters), with the A10 and P13 gNFW profiles, represented in green and violet respectively. The three panels show the MUSIC median profiles for the simulation flavours, AGN, CSF and NR from left to right, with the MAD, see Eq.(6), as associated uncertainty. The yellow line and the shaded regions represent the MUSIC pressure profile fit, with the gNFW model parameters for AGN, CSF and NR listed in the first three rows of Table 2. In the bottom plot of each panel we present the relative difference between the MUSIC pressure profile fit, $f_M$, and the profile from A10 or P13

$$\Delta = \frac{f_M - f_{A10/P13}}{f_M}. \tag{9}$$

The AGN flavour is the dataset showing better agreement with both A10 and P13 profiles, especially for the A10’s universal profile, a very good approximation until $R_{200}$. In the case of the CSF flavour, the MUSIC profiles are steeper starting from $0.1 R_{500}$, while in the NR case the MUSIC profiles are higher than the observed profiles within even a larger region (approximately $0.3 R_{500}$). Since only the AGN set provides a reliable description of the observed profiles, we used this set to check the redshift dependence of the universal profile, extended to the high redshift regime ($0.54 < z < 0.82$).

As we can see from Fig. 4, our data match well enough both the P13 profile and the A10 one with a relative difference of the order of 0.25. This difference is basically at the cluster core, inside $0.1 R_{500}$, and in the outskirts, beyond $2 R_{500}$. Contrary to the low $z$ case, now the P13 parameters are in better agreement with our simulations. The best fit parameters for the high-$z$ case are listed in Table 3.

We have also studied whether the dynamical state of the clusters could have an impact in the comparisons with observations. While we segregated extremely relaxed and disturbed clusters in MUSIC from the dynamical state parameters (see 4.3), it is not that straightforward with P13 observed data. In that case, the closer classification between relaxed and disturbed clusters is to distinguish between cool-core (CC) and non cool-core (NCC) clusters. In fact, the cool core clusters are associated to relaxed objects, see e.g. Hudson et al. (2010), although not always true (see Biffi et al. 2016). In A10 the segregation between the cluster dynamical state is different: the clusters are divided into cool-core and disturbed clusters. In Fig. 5 the median pressure profiles for relaxed and disturbed clusters are plotted both at low redshifts for the AGN flavour. The parameters of the fit are listed in Table 2, only for the AGN flavour because is the one that better matches real observed clusters, as shown in Fig. 3. We can see that the disturbed profile from A10 and the NCC P13 profile match very well the MUSIC disturbed profile inside $R_{500}$. This means that the disturbed clusters from A10 and from the MUSIC simulation have similar features as the NCC clusters in P13. This does not happen for the relaxed MUSIC clusters, showing a shallower slope of the profile in the cluster core with respect to the CC clusters from A10 and P13. The reason for this behaviour could be that selecting dynamically relaxed clusters based on the indicators described in Section 4.3 does not allow us to discriminate cool-core clusters and clusters with a disturbed core. Therefore, we expect the inner slope of the MUSIC pressure profile estimated on relaxed clusters to be shallower than the one obtained by studying only CC clusters. Another possible explanation is that our AGN feedback model is too effective and expels more gas from the cluster core.

### 5.2 Electron density profile

The 3D electron density profiles, $n_e(r)$, of our simulated clusters are modeled using the analytical function proposed by Vikhlinin et al. (2006)

$$n_p n_e(r) = \frac{3}{2} \frac{1}{(1 + (r/r_s)^3)^{3b+1}}$$

which is a modification of the $\beta$–model (Cavaliere & Fusco-Femiano 1978) to represent the observed features of X-ray observations. It is based on two terms. The first term represents a cuspy profile near the cluster centre plus another power law to describe the steepening of the profile for $r > r_s$, the extra parameter $c$ controls the width of the transition region. The second term is another $\beta$-model with a small central core to make the function flexible to fit the data in the central region of the clusters. All clusters profiles were fitted using the same parameter constraints as in Vikhlinin et al. (2006), that is fixing $c = 3$ and $e < 5$.

### 5.3 Temperature profile

The temperature profile is estimated as the mass-weighted average over an ensemble of gas particles within each spherical shell:

$$T = \frac{\sum_i (m_i \times T_i)}{\sum_i m_i}. \tag{11}$$

where $i$ run over each particle inside a spherical bin. We considered only particles with temperature $kT > 0.5$ keV to account only for the X-ray emitting gas. The analytical model describing the mass weighted temperature was introduced also by Vikhlinin et al. (2006)

$$T(r) = T_0 \frac{x + \frac{r}{r_s}}{x + 1} \left(1 + (r/r_s)^{3b+1}\right)^{1/(3b+1)}$$

where $x = (r/r_{cool})^3$. The radial temperature profile has a broad peak at $r < 0.1 R_{500}$ and decreases at larger radii, there is also a temperature decline towards the cluster center, probably because of the presence of radiative cooling, represented by the central expression. Outside the central cooling region, the temperature profile is represented as a broken power law with a transition region, the last term, where $r_s$ is a scale radius. This model has 8 free parameters, none of them were fixed.

The mass weighted temperature is the value that better relates to the mass of the cluster, in fact it reflects the gravitational potential of the system (Biffi et al. 2014). Nevertheless, there are various other ways of estimating the cluster temperature, for instance weighting the temperature by the X-ray emission of each gas particle, such as the spectroscopic-like temperature $T_d$ (Mazzotta et al. 2004), in order to better explore the X-ray observable properties of simulated galaxy clusters and to compare against real observations. The importance of temperature structures was studied deeply in Rasia et al. (2014) through the comparison between the spectroscopic-like and the mass-weighted temperatures. Biffi et al. 2014 have computed the spectroscopic-like temperature for the MUSIC clusters by fitting
6 RESULTS FOR THE FULL MUSIC SAMPLE

In this section we present the results for the hydrostatic masses defined by Eq.(1) and (2), for the complete MUSIC sample. We will devote Section 7 to present the specific results for the NIKA2 twin sample. As we mentioned above, the HE mass estimates, and consequently, the mass bias, depend on different factors: the observable quantities used, the simulation flavour, the numerical method used to estimate the spatial gradients, and finally the redshift and the dynamical state of the clusters. In this section we explore how the HE masses and biases depend on these factors.

6.1 Methods to estimate the HE masses

In this paper, we use two methods to estimate the HE masses accordingly to the ways of computing the derivatives in Eq.(1) and Eq.(2).
In the first approach, the ICM radial profiles are estimated from the SPH gas particles within each spherical bin. Then the derivatives can be directly computed numerically from the binned profiles. This is a more direct estimation of the gradients but can also suffer from noise associated to the bin size and particle sampling. An alternative estimation is to first fit the numerical profiles by the analytical functions (described in Section 5) and take the derivative from the fitted function.

The first approach was applied in several studies (Lau et al. 2009; Ameglio et al. 2009; Sembolini et al. 2013; Battaglia et al. 2013; Nelson et al. 2014a; Biffi et al. 2016; Shi et al. 2016; Martizzi & Agrusa 2016; Le Brun et al. 2017; Cialone et al. 2018). In this work, each cluster is divided in spherical shells with 100 kpc width, which has been found to be the optimal binning for our purpose. This procedure corresponds to smoothing the profile, still keeping its most important features when comparing with the original binning.

The second method is based on the analytical derivative of the fitting functions of the ICM radial profiles. Also in this case, several other works have made use of this method (Piffaretti & Valdarnini 2008; Meneghetti et al. 2010; Kay et al. 2012; Rasia et al. 2012; McCarthy et al. 2016; Gupta et al. 2017; Ruppin et al. 2019b; Pearce et al. 2020; Ansari-Fard et al. 2020; Barnes et al. 2020). The radial profiles of each cluster thermodynamic quantity are fitted with parametric models (see Sections 5.1, 5.2, 5.3) using the Python function `curve_fit`. The profile bootstrap errors have been estimated instead of the MADs, which mainly in the outskirts show too large values, often of the order of the median quantity. The bootstrap is performed using several realizations, in each one of them one particle, chosen randomly, is extracted and the mean over this new sub sample is performed. The bootstrap error is estimated as the standard deviation over all the realizations means. The fits are done in the radial range $[0.1 – 1] \times R_{500}$. So, we neglect the core and the outskirts of the clusters, since in observed cores there is a large variation from cluster to cluster and it is still challenging to reach the external regions in X-ray. We fix a maximum value of 10 for the reduced $\chi^2$, hereafter $\tilde{\chi}^2$, to include those fits which are still, visually, a good approximation to the profiles due to the slight variations and the small errors. In this way, we have almost 50% of the total number of clusters which have reliable fits ($\tilde{\chi}^2 \leq 10$). The group of reliable fit clusters changes depending on the HE mass chosen (X-ray or SZ). In fact, having a cluster a reliable fit for one of the thermodynamic quantities does not necessary means that it has a good fit also for the other quantities.

### 6.2 Hydrostatic mass estimates

In Fig. 6 we show the HE masses computed at $R_{500}$, from SZ and X-ray observables estimated using the analytical fitting method, only for clusters that present reliable fits ($\tilde{\chi}^2 < 10$) and for the AGN simulation flavour. They are plotted as a function of the cluster true mass $M_{500}$ at $z = 0$. The HE mass is proportional to the true mass. Clusters at other redshifts show a similar behaviour, as Le Brun et al. (2017) also find. We represent the relaxed clusters with magenta diamonds, the disturbed with blue squares and the intermediate with black circles. We fit the linear relation between the HE mass and the true mass. We have also done the same analyses for all the simulation flavours and redshift bins. The values of the slope $a$, which is equivalent to the bias, $1 - b$, for all the simulation flavours and redshifts can be seen in Table 4. We do not find any clear dependence on redshift, with values of $1 - b$ that are around 0.8 – 0.9 for all the flavours, roughly in agreement also with the results using the numerical derivative method, not shown here. Taking into account only clusters with reliable fits, leads to smaller slope values with respect to the numerical derivative method and to the situation in which we do not neglect any clusters with bad fits. In particular, in the last two methods usually the slope errors are larger than the only reliable fit method, and ultimately the results are compatible within 1$\sigma$. However, fitting the profiles implies that we are not sensitive to extreme local ICM fluctuations, meaning that we are neglecting information about the clusters and that we could underestimate the hydrostatic mass. On the other hand, gas substructures and fluctuations do influence the bias, possibly leading to an overestimation of it.
Table 4. Slopes and 1σ errors from the fit $M_{\text{HE}} = a M_{\text{500}}$ in Fig. 6 at all the redshifts and for all the simulation flavour with the analytical fitting method using only the clusters with reliable fits.

| $z$   | AGN Slope $\alpha_{\text{SZ}}$ | NR          | AGN Slope $\alpha_{X}$ | NR          |
|-------|-----------------|-------------|-----------------|-------------|
| 0.0   | 0.808 ± 0.014   | 0.790 ± 0.013 | 0.763 ± 0.016 | 0.916 ± 0.016 | 0.909 ± 0.015 | 0.890 ± 0.018 |
| 0.11  | 0.819 ± 0.014   | 0.800 ± 0.014 | 0.767 ± 0.016 | 0.914 ± 0.012 | 0.922 ± 0.015 | 0.897 ± 0.014 |
| 0.33  | 0.788 ± 0.015   | 0.787 ± 0.013 | 0.731 ± 0.019 | 0.876 ± 0.015 | 0.860 ± 0.013 | 0.871 ± 0.018 |
| 0.43  | 0.783 ± 0.016   | 0.766 ± 0.010 | 0.716 ± 0.011 | 0.907 ± 0.017 | 0.886 ± 0.013 | 0.889 ± 0.018 |
| 0.54  | 0.758 ± 0.014   | 0.739 ± 0.012 | 0.758 ± 0.015 | 0.880 ± 0.015 | 0.886 ± 0.011 | 0.855 ± 0.014 |
| 0.67  | 0.778 ± 0.013   | 0.752 ± 0.009 | 0.764 ± 0.016 | 0.898 ± 0.017 | 0.923 ± 0.013 | 0.983 ± 0.026 |
| 0.82  | 0.763 ± 0.014   | 0.789 ± 0.014 | 0.712 ± 0.010 | 0.902 ± 0.018 | 0.895 ± 0.015 | 0.902 ± 0.021 |


6.3 Hydrostatic mass biases

In this section the dependence of the bias on the redshift and on the 3D dynamical state indicators are studied, as well as the bias radial profile.

From the hydrostatic mass, the bias is estimated using Eq. (3). According to this definition, a mass bias value of 0 suggests a HE mass equal to the true mass, therefore the HE equilibrium represents a good cluster mass approximation. This approximation does not account for contributions like, for instance, the non-thermal pressure (see Section 6.4).

At redshift 0, using the analytical fitting and the only reliable fits, we find, at $R_{\text{500}}, b_{\text{SZ}} = 0.23_{-0.09}^{+0.14}$ and $b_{X} = 0.16_{-0.10}^{+0.12}$ (represented in Fig. 1) for the AGN flavour. These results are given as median and 16th and 84th percentiles, since the biases distributions are not Gaussian, see Appendix A for more details. The biases at different redshifts and for different simulation flavour are listed in Tab. B1. We can see that the X-ray hydrostatic mass tends to give a better estimation of the real mass, with respect to the SZ one.

6.3.1 Mass bias dependence on dynamical state

In order to study the link between the hydrostatic mass bias and the cluster dynamical state, we test the dependence on the relaxation parameter, $\chi_{\text{DS}}$ (see definition in 4.3). All of the clusters that have a $\chi_{\text{DS}} > 1$ are dynamically relaxed. The biases as a function of $\chi_{\text{DS}}$, at redshift 0, are shown in Fig. 7 for the analytical fitting method, only for reliable fits. The AGN flavour is shown here, but the situation is similar for CSF and NR. The relaxed clusters present a lower scatter, compared to the other cases and also a smaller overall HE bias, as it is shown in this plot and in Table B1. For the SZ and X-ray bias the Pearson correlation coefficient is respectively -0.4 and -0.3, resulting in a moderate correlation and the slope of the linear relation between the bias and $\chi_{\text{DS}}$ is smaller and different from zero at 3-sigma level. The latter observation is true also for the numerical derivative method (which employs all the clusters of the sample), yet the correlation is weaker, around -0.2 for both biases.

6.3.2 Mass bias dependence on radial profile

It is interesting to study the variation of the mass bias across the considered radial range. To this purpose, the numerical derivative method appears powerful to explore the hydrostatic mass bias even in the large radial range. The median radial profiles and the MADs of $b_{\text{SZ}}$ and $b_{X}$ at $z = 0$ are shown in Fig. 8, from 0.2 to 2 $R_{\text{500}}$. In the left panel of each row there is the median profile over all the clusters, in the middle only relaxed and in the right only disturbed clusters. The NR, CSF and AGN profiles are represented in red, green and blue respectively. The bias profiles from all and only relaxed classes are very regular and, as expected, increase at large radii, as found also in other works (e.g., Meneghetti et al. 2010; Cialone et al. 2018; Ansarifard et al. 2020). This could be most likely the presence of higher non-thermal processes in the cluster outskirts, see Section 6.4. On the contrary, the trend of the disturbed clusters has a lot of variations and a large scatter, with a dip between $R_{\text{500}}$ and 1.5$R_{\text{500}}$, which seems not due to the non-thermal pressure, as a matter of fact it is still present after applying the non-thermal correction, see Section 6.4. We will study in deep this behaviour in a future work, but we attempt here a possible explanation. It could be that among the disturbed clusters there are several objects that are merging clusters and therefore present a merger shock which could boost either the ICM temperature (and therefore $M_{\text{HE}}$) and consequently the gas pressure.

6.3.3 Bias dependence on baryon models

Taking advantage of exploring the same objects with different flavours, we are able to compare HE mass bias values, without worrying about the simulation features, like resolution, integrating box size, cluster mass range and number of objects, focusing only on the differences due to the physics included in the simulation. At redshift 0 and at $R_{\text{500}}$, using the analytical fitting and the only reliable fits, we find $b_{\text{SZ}} = 0.23_{-0.09}^{+0.14}$ and $b_{X} = 0.14_{-0.11}^{+0.13}$ for the AGN flavour, $b_{\text{SZ}} = 0.26_{-0.10}^{+0.12}$ and $b_{X} = 0.14_{-0.11}^{+0.12}$ for the CSF flavour and $b_{\text{SZ}} = 0.27_{-0.13}^{+0.12}$ and $b_{X} = 0.15_{-0.13}^{+0.14}$ for NR. As we can see, the simulation flavour does not influence much the bias value.
The bias dependence on redshift

The bias dependence on the redshift is presented in the first row of Fig. 9. The results are from the analytical fitting method with only the reliable fits and the AGN flavour (in the case of the other flavours the situation is similar). With the current errors, we do not detect any dependence with the redshift, for either the bias (as in Le Brun et al. 2017; Henson et al. 2017; Salvati et al. 2018; Koukoufilippas et al. 2020) or its uncertainties. In all cases, the disturbed clusters have the largest errors, as already pointed out in Section 6.3.1. We see the same behaviour for the numerical derivative method, with the only difference that the disturbed clusters percentiles are almost twice larger than the percentiles shown in the analytical fitting method. In this figure the bootstrap error is represented too, for the three cases.

6.3.5 The bias mass dependence

Regarding a possible dependence of the mass bias with the cluster mass, we do not find any dependence (in agreement with Ansarifard et al. 2020), either using the mass-weighted or the spectroscopic-like temperature. We know that several authors detect a dependence when using the spectroscopic-like temperature, see e.g. Rasia et al. (2006); Piffaretti & Valdarnini (2008); Le Brun et al. (2017); Henson...
6.3.6 Correlation between $b_X$ and $b_{SZ}$

We find that the two bias factors are strongly correlated, with a Pearson coefficient of 0.8. To estimate the scatter between the two biases we can use the mean absolute difference

$$MD = \frac{\sum_i (b_{X,i} - b_{SZ,i})^2}{N(N-1)}$$

with $N$ being the total number of clusters. At all redshifts $MD$ is of the order of $10^{-1}$.

In the simulations the pressure of each fluid particle is computed assuming the ideal gas equation of state (used to derive $M_{\text{HE,SZ}}$ from Eq. (1), and Eq. (2) for $M_{\text{HE,X}}$)

$$P_i = n_{e,i} T_i,$$

where $n_{e,i}$ and $T_i$ are the electron density and the temperature of each particle of fluid. Then the median over the spherical shell is performed. While, in this work, the median pressure profile is replaced by the product of the median electron density profile and the mass-weighted temperature profile. For this reason we expect the two HE masses to be different.

To verify that the scatter between the two biases is really due to the previous consideration, we need to quantify it, for instance using

$$\frac{M_{\text{HE,SZ}}}{M_{\text{HE,X}}} = 1 - b_{SZ}/b_X$$

(15)

which, replacing the masses with the Equations (1) and (2), becomes

$$\frac{1 - b_{SZ}}{1 - b_X} = \frac{P}{n_e T} \frac{d \ln P/d \ln r}{d \ln (n_e T)/d \ln r} \sim \frac{P}{n_e T}$$

(16)

where the derivatives fraction is always of the order of 1, no matter the flavour or the redshift.

We find that the profile of $P/n_e T$ decreases very slowly from a value of almost 1 in the core. In the case of AGN, $z = 0$, the deviation from 1 of this ratio is $1 - P/n_e T = 0.12$, but in general it is always of the order of $10^{-1}$. This is of the same order of the scatter $MD$, so we can conclude that the difference between the X-ray and SZ bias is mainly due to the use of the median and mass-weighted profiles.

6.4 The non-thermal pressure contribution

The hydrostatic equilibrium approximation does not take into account non-thermal motions of the ICM, which could have a significant contribution to the pressure support of the gas within the cluster gravitational potential. The non thermal pressure contribution comes mostly from turbulence or bulk motions of the ICM, neglecting other sources such as magnetic fields or cosmic rays pressure. Recent studies have shown that it can contribute as much as 30 per cent or more to the overall gas pressure at $R_{500}$ (Lau et al. 2009; Nelson et al. 2014a; Shi et al. 2016; Biffi et al. 2016; Martizzi & Agrusa 2016; Angelinelli et al. 2019; Pearce et al. 2020). This is the main origin of the cluster mass bias.

The non-thermal pressure component is modelled as

$$P_{\text{nth}} = \alpha(r) P_{\text{tot}}$$

assuming $P_{\text{tot}} = P_{\text{th}} + P_{\text{nth}}$. From simulations, the non-thermal pressure (Nelson et al. 2014b) can be estimated as

$$P_{\text{nth}} = \rho \sigma^2$$

(18)

where $\sigma^2 = \sum_j \sigma_j^2$, with $j = (x,y,z)$, is the square of the three-dimensional velocity dispersion of gas particles and

$$\sigma_j^2 = \frac{\sum_i (m_i v_{ij} - \bar{v}_j)^2}{\sum_i m_i}$$

(19)

is the velocity dispersion in each spatial direction computed from all the gas particles within each spherical shell around the cluster centre.

Then, introducing $P_{\text{tot}}$ in the formula for the HE, we can derive the corrected hydrostatic masses (Pearce et al. 2020)

$$M_{\text{HE,SZ,corr}} = \frac{1}{1 - \alpha} \left[ M_{\text{HE,SZ}} - \frac{\alpha}{1 - \alpha} \frac{P_{\text{th}}}{G \mu m_p n_e} d \ln \alpha \right]$$

(20)

$$M_{\text{HE,X,corr}} = \frac{1}{1 - \alpha} \left[ M_{\text{X,SZ}} - \frac{\alpha}{1 - \alpha} k_B T_r d \ln \alpha \right].$$

(21)

We note that the temperature profile used in the formula above corresponds to the mass weighted temperature definition. For the
where in the AGN simulation at Figure 10.

Similarly, as shown in Fig. 10, where the ratio at larger radii (e.g., Pearce et al. 2020; Biffi et al. 2016). We confirm in the B1) is larger than the non-corrected case due to the large scatter in the biases. In the bottom panels of Fig. 9, indeed, the relaxed and the disturbed clusters are consistent within their errors. Applying \( R \) to see Rasia et al. (2006).

The mean radial profile of \( P_{\text{th}} / P_{\text{tot}} \) for MUSIC clusters in the non-thermal correction applied on HE mass using the spectroscopic-like temperature see Rasia et al. (2006).

Therefore, in order to correct the HE masses we need to know the value of the \( \alpha(r) \) function. We use the fitting formula originally proposed by Nelson et al. (2014b) for \( R_{500} \), adapted to the smaller aperture radius \( R_{\text{tot}} \).

\[
\alpha(r) = 1 - A \left[ 1 + \exp \left( -\frac{r/R_{500}}{B} \right) \right] ^ C
\]

where \( A, B \) and \( C \) are three free parameters (Pearce et al. 2020).

The contribution of non thermal pressure becomes significant at large radii (e.g., Pearce et al. 2020; Biffi et al. 2016). We confirm a similar behaviour as shown in Fig. 10, where the ratio \( \alpha \) is plotted. We can see that, at \( R_{500} \), it goes from \( \sim 15\% \) to \( \sim 40\% \), depending on the morphological state of the clusters. The most disturbed clusters have a higher contribution to \( \alpha \), even if the two classes of objects (relaxed and disturbed) are consistent within their errors. Applying the non-thermal correction to the mass should result in minimising the biases. In the bottom panels of Fig. 9, indeed, the \( b_X \) and \( b_{SZ} \) are close to 0, but the scatter (represented by the percentiles in Table B1) is larger than the non-corrected case due to the large scatter in the \( \alpha \) profiles, in agreement with the results from Pearce et al. (2020). In addition, we also evince that there is not a significant dependence on the redshift range analyzed in this work. Moreover, the correction on \( b_X \), which already gave an estimation of the mass nearer to 0 before the correction, is more effective than on the \( b_{SZ} \). This leads to zero X-ray bias for the relaxed and all clusters cases, and to an over correction for the disturbed cases. Therefore, the dynamical state has an impact on the strength of the correction, but not on its radial profile, as stated before and shown in Fig. 10.

The median radial profiles of the two corrected biases \( b_{SZ, \text{corr}} \) and \( b_{X, \text{corr}} \) are presented in Fig. 11 for redshift 0 and all flavours. The increasing radial dependence of the X-ray and SZ bias was canceled when including the non thermal correction, as expected. However, there is still a large variation in the bias profiles around \( R_{500} \) for the disturbed clusters, which was already present before the correction. This could be explained by a more intriguing effect, which does not depend on the non-thermal pressure or the simple correction formula does not account for the extreme non thermal pressure from the disturbed clusters. We also note that deviations from spherical symmetry as well as the presence of substructures impacting the multi-phase structure of the gas can play a significant role in the outskirts. As we pointed out before, the scatter (represented by the MADs in this plot) is larger than the non corrected bias. Moreover, the SZ bias has larger scatter than the X-ray bias, especially for the CSF and NR flavours. The NR biases have less regular profiles with respect to the other flavours, but there is not a substantial difference between them. All the different flavours profiles are in agreement.

### 7 RESULTS FOR THE NIKA2 TWIN SAMPLE

The NIKA2 twin sample is composed of 32 MUSIC clusters, chosen at two redshifts: 0.54 (14 clusters) and 0.82 (18 clusters) in order to homogeneously populate the \( Y - z \) plane as the LPSZ sample to be observed with NIKA2 camera. To build the sample we chose clusters at \( z = 0.82 \) which are not the progenitors of the clusters at \( z = 0.54 \), as it would be also in real observations. Due to the small number of clusters, the reliable fit subsample with \( \chi^2 < 10 \) having seven clusters in total, was not applied. Therefore, in the analysis, all the 32 clusters are taken into account.

The median pressure profile is plotted in Fig. 12 and the parameters are listed in Table 5, first row. In this case the profiles from A10 and P13 do not approximate well the data, even though the sub-sample is taken from MUSIC, where those two profiles are acceptable at high redshifts (Fig. 4, Table 3). To check whether this behaviour is expected, 10 sub samples were randomly extracted from MUSIC, keeping the same features in mass, redshift and dynamical state than the NIKA2 twin sample, see Section 4. In all these cases, the gNFW profile parameters are consistent (1σ), see Table 5, second row, but different to the MUSIC ones in Table 3. Repeating the same procedure, without constraining the mass range, leads to random samples which have gNFW profile parameters compatible with the MUSIC one, an example of parameters is listed in the third row of Table 5. The difference may be due to the number of high mass clusters in the NIKA2 twin sample, larger in proportion than the MUSIC one.

In Fig. 13 the hydrostatic masses, estimated for the NIKA2 twin sample, are shown as a function of the true cluster mass. The relaxed and the disturbed clusters are shown in magenta and blue colors, respectively, while the intermediate ones are plotted in black. These HE masses were estimated using the analytical fitting, for both redshifts and all simulation flavours. In this figure, we just show the results for the AGN simulations, both at \( z = 0.54 \), (open symbols), and \( z = 0.82 \), (solid symbols). Also in this case a fit of the type \( M_{\text{HE}} = a M_{500} \) was performed, the slopes \( a \) are listed in Table 6. The slopes and their errors on the NIKA2 twin sample fits are higher with respect to the full MUSIC sample, (see Table 4), due to the difference in the mass distributions.

The hydrostatic mass biases of the NIKA2 twin sample are represented in the first row of Fig. 14 along the redshifts, using the analytical fitting method. They are slightly higher than the corresponding MUSIC cases in Fig. 9, at redshifts 0.54 and 0.82, but still in agreement with them. Contrary to MUSIC full sample, here the disturbed clusters show a bias close to 0 at \( z = 0.54 \). Large errors make these biases still compatible with the whole sample. We have to report that this unexpected behaviour could be also due to having not excluded the clusters with non reliable fits. However, the disturbed clusters show the same behaviour using the numerical derivative method.
The study of the bias in recovering the mass of galaxy clusters under the hydrostatic equilibrium assumption is crucial for the understanding of structures formation and cosmology. In this work the 3D ICM radial profiles of a synthetic set of almost 260 clusters from the MUSIC hydrodynamical simulations were used. The analysis has been applied on three different simulation flavours: NR, CSF and AGN, each of them includes different physical processes. The main difference among them is the absence of radiative processes in NR, while they are included in both CSF and AGN, where also stellar and AGN feedbacks are considered, respectively. We considered seven different redshifts in the range $0.0 \leq z \leq 0.82$. Moreover a MUSIC sub sample, the NIKA2 twin sample, was analysed to have some hints of the median pressure profile of the full sample at low redshifts in the range $z \approx 0.0$.

| $z$ | AGN | CSF | NR | AGN | CSF | NR |
|-----|-----|-----|----|-----|-----|----|
| 0.54+0.82 | 0.783±0.028 | 0.948±0.071 | 0.775±0.034 | 0.970±0.037 | 0.952±0.030 | 0.972±0.050 |

The same problem reflects on the corrected bias. The correction on the biases has been applied to this sample as well, their median values are shown in the second row of Fig. 14. We see that, at redshift 0.54, the non-thermal correction works well, giving bias values close to 0, compatible with the MUSIC sample. Instead, at redshift 0.82 the corrected bias is slightly lower than 0, although still compatible with it within 2 $\sigma$. At $z = 0.54$, the correction does not work for the case of disturbed clusters, but it is still in agreement with the MUSIC value.

Based on these analyses, we can say that the NIKA2 twin sample has compatible biases within 1 $\sigma$ with the MUSIC sample, even if it has a larger percentage number of high mass clusters.

8 CONCLUSION

The study of the bias in recovering the mass of galaxy clusters under the hydrostatic equilibrium assumption is crucial for the understanding of structures formation and cosmology. In this work the 3D ICM radial profiles of a synthetic set of almost 260 clusters from the MUSIC hydrodynamical simulations were used. The analysis has been applied on three different simulation flavours: NR, CSF and AGN, each of them includes different physical processes. The main difference among them is the absence of radiative processes in NR, while they are included in both CSF and AGN, where also stellar and AGN feedbacks are considered, respectively. We considered seven different redshifts in the range $0.0 \leq z \leq 0.82$. Moreover a MUSIC sub sample, the NIKA2 twin sample, was analysed to have some hints of the median pressure profile of the full sample at low redshifts ($z < 0.5$) was compared with two models describing observed local clusters from Planck (P13, Planck Collaboration 2013a) and XMM-Newton (A10, Arnaud et al. 2010) in order to check whether there is consistency between simulations and the analytic models used in literature.

The SZ and X-ray estimations of the hydrostatic mass were
computed from the ICM thermodynamic radial profiles using two strategies to compute the gradients: the numerical and analytical fitting. The correlation between the hydrostatic masses and the true mass $M_{500}$ was studied.

Therefore, the HE mass biases were estimated. Possible dependencies with redshift and dynamical state, as defined by the relaxation parameter $\chi_{DS}$, were analysed, together with the bias radial profile. We also studied the impact of non-thermal pressure support arising from the bulk motion and turbulence of the gas.

All these analyses were done for both the MUSIC full dataset and the NIKA2 sub sample.

The main results of this work can be summarized as follows:

- In MUSIC, the AGN flavour better approximates the real clusters properties. In fact the pressure profile from the AGN simulation better matches the model from the observed clusters in P13 and A10. On the contrary, the median pressure profile of the NIKA2 twin sample is not well approximated by the two models. A possible explanation is the different cluster masses distribution in the two samples.
  - The SZ and X-ray hydrostatic masses have a linear dependence with the true mass $M_{500}$. The slope of the fit, corresponding to the value $(1 - b)$, gives a bias $b$ of the order of 0.2, independent on the considered redshift.
  - Also the hydrostatic biases at $R_{500}$ are of the order of 0.2, independently from X-ray or SZ formulations and derivative estimations and they do not depend on the redshift. These results are in agreement with other simulations. The full sample and the relaxed-cluster-only biases are always in agreement with each others, the disturbed population shows very high biases, with large errors. This can be simply explained because the hydrostatic equilibrium is not fully satisfied.
  - The different approaches to fit the ICM radial profiles have an impact on the scatter of the biases. The numerical derivative method gives on average a larger scatter with respect to the analytical fitting one, with the condition of excluding from the analysis the clusters with a non reliable fit.
  - The dynamical state has an impact on the scatter of the biases. Disturbed clusters show large dispersion. We find that biases depend on the continuous relaxation parameter $\chi_{DS}$, which we use to describe the dynamical state of the clusters.
  - While the median radial profile of the bias for the disturbed clusters shows large fluctuations with a large scatter, the relaxed and the full sample radial biases are flatter. Interestingly, in the case of these two populations the bias profile increases radially stressing the impact of non thermal pressure contribution in the most external regions.
  - It is evident that non thermal pressure contribution has an impact especially in the cluster outskirts. Correctly adding this contribution to the hydrostatic one, the estimated mass of the cluster really gets closer to the true one. Unfortunately, the scatter is larger, mainly due to the scatter in the fraction $P_{\text{nth}}/P_{\text{net}}$, used to estimate the correction.
• The mass biases from the NIKA2 sub sample are compatible with the ones from the full sample, even though they are slightly higher, especially for the relaxed clusters.

The NIKA2 twin sample will help improve the analysis methodology of the ICM properties of the NIKA2 tSZ large program. Thanks to the results of this work, the sample will also help to probe different fundamental hypothesis, like spherical symmetry and hydrostatic equilibrium.

Nowadays, the main systematic uncertainty associated with cluster cosmological constraints is the uncertainty on the hydrostatic mass bias. Using the non thermal correction to obtain mass values closer to the real ones, but with a larger scatter, is not an efficient approach. On the other hand, several other effects still need to be carefully considered, as temperature dishomogeneities, deviation from spherical symmetry and the presence of substructures, in order to give a more accurate measure of the HE mass bias.

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DATA AVAILABILITY
The profiles analyzed in this work were produced with the MUSIC simulations (Sembolini et al. 2013). These data can be accessed following the instructions on the website https://music.ft.uam.es/. The data specifically shown in this paper will be shared upon request to the authors.

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APPENDIX A: MASS BIAS DISTRIBUTIONS

Examples of bias distribution are in Fig.A1 for the analytical fitting method, without discriminating for the goodness of the fits $\chi^2$. The bias values for all the clusters are represented with the dark cyan ($b_{SZ}$) and light green histograms ($b_X$) for the AGN simulation at redshift $z = 0$. In the Table A1, we report the means and the standard deviations from the data and from a Gaussian fit for both methods. Almost half of the sample are relaxed, 52%, see Table 1, Planck Collaboration 2011, A&A, 536, A8

| $b_X$ | $b_{SZ}$ |
|-------|----------|
| Data  | Fit      |
| $\mu$ | $\sigma$ |
| 0.09  | 0.23     |
| 0.12  | 0.20     |
| 0.08  | 0.13     |
| 0.12  | 0.34     |

There should not be any difference between the results obtained with $\rho_g$ and those obtained with $\mu m_p n_e$ , because they should be interchangeable quantities. However, We find that this relation sometime is not satisfied, and probably for this reason we have different biases for the electron and for the gas density. The gas density $\rho_g$ in the MUSIC simulations is computed as the total gas mass in the spherical shell divided by the shell volume. The numerical electron density (in cm$^{-3}$) is evaluated from the gas density using

$$n_e(r) = N_e \rho_e(r) \left( \frac{1 - Z - Y_{He}}{m_p} \right),$$

(A1)

where $\rho_e$ is the fraction of free electrons per hydrogen particle, $Z$ is the metallicity, $Y_{He}$ is the nuclear helium concentration and $m_p$ is the proton mass (Sembolini et al. 2013). The numerical electron density takes into account the local distribution of electrons and gas particles, if the gas is completely ionized, we can relate the electron and gas density using

$$\rho_e(r) = 1.8 \mu m_p n_e(r).$$

(A2)

Table A1. The results in this Table refer to the $b_X$ and $b_{SZ}$ distributions at redshift $z = 0$ and for the AGN runs. The value of the mean $\mu$ and standard deviation $\sigma$ estimated both from the Data and from a Gaussian fit of the bias distributions. Moreover, the bootstrap error is written. All these informations are reported for both methods, the Analytical fitting and the Numerical derivative one.

|          | Analytical | Numerical |
|----------|------------|-----------|
| $\mu$    | $\sigma$   | $\sigma$  |
| $b_X$    | Data       | Fit       |
| $\mu$    | $\sigma$   | $\sigma$  |
| $b_{SZ}$ | Data       | Fit       |
| $\mu$    | $\sigma$   | $\sigma$  |

We can say that the two methods to compute the derivative lead to very similar and compatible results, nevertheless it is worth noticing that the biases from the numerical derivative method show a broader distribution. Computing the mass bias through the X-ray and SZ equations based on different ICM properties also leads to similar results on the hydrostatic bias. However, in the case of the analytical fitting method, $b_{SZ}$ and $b_X$ have both negative skewness for all the redshifts, meaning that they have a more pronounced tail on the left side of the distribution, i.e. toward the 0 bias, as found also in Ansarifard et al. (2020). They study the distributions of the $b_{SZ}$ and $b_X$ estimated using the gas density $\rho_e$ instead of $\mu m_p n_e$ in

Eq. (1) and (2), finding that $b_{SZ}$ shows a larger scatter and that $b_X$ is more symmetric than $b_{SZ}$. These results disagree with what we found here. We conclude that the differences are due to the radial gas density profiles considered to estimate $M_{He}$. In fact, if we use $\rho_g$, we find the same distribution features in Ansarifard et al. (2020).

Moreover, for the same reason, we find that some disturbed and bulkmotions or turbulence, start to be important (see Section 6.4). We find that this approximation does not hold in the outskirts of the cluster, where the contribution of non-thermal motions, like sound waves, where $\mu$ is not satisfied, and probably for this reason we have different biases for the electron and for the gas density. The gas density $\rho_g$ in the MUSIC simulations is computed as the total gas mass in the spherical shell divided by the shell volume. The numerical electron density (in cm$^{-3}$) is evaluated from the gas density using

$$n_e(r) = N_e \rho_e(r) \left( \frac{1 - Z - Y_{He}}{m_p} \right),$$

(A1)

where $\rho_e$ is the fraction of free electrons per hydrogen particle, $Z$ is the metallicity, $Y_{He}$ is the nuclear helium concentration and $m_p$ is the proton mass (Sembolini et al. 2013). The numerical electron density takes into account the local distribution of electrons and gas particles, if the gas is completely ionized, then we can relate the electron and gas density using

$$\rho_e(r) = 1.8 \mu m_p n_e(r).$$

(A2)
Figure A1. Histograms of the $b_{SZ}$ and $b_X$ values corresponding to AGN simulations of the complete MUSIC sample, using the analytical fitting method, for the whole MUSIC sample, without discriminating for the goodness of the fit $\chi^2$. In dark cyan (left figure) and light green (right figure) there are the biases of all the clusters, in magenta, for only relaxed and in blue for only disturbed clusters. In the legend, the values of the skewness and the kurtosis are reported.

Figure A2. The fraction of the gas density and numerical electron density at $R_{500}$ is represented for each cluster in the AGN flavour at $z = 0$. The relaxed, disturbed and intermediate clusters are represented in magenta diamonds, blue squares and black circles respectively. The grey line is the expected value of 1.8.

APPENDIX B: MASS BIAS RESULTS

We present in Table B1, the HE mass biases for all $z$ and for all flavours, using only the reliable fits from the analytical fitting method.
Table B1. A compilation of the hydrostatic mass bias estimates $b_{SZ}$ and $b_{X}$ and their corresponding corrected ones (at the bottom part of the Table) computed at $R_{200}$ for clusters with reliable fits to the analytical profiles in the MUSIC sample, for different redshifts (first column). The biases are listed taking into account All the clusters and only the Relaxed or Disturbed ones. The results for the three AGN, CSF and NR simulation flavours are listed as median values with 16th and 84th percentile errors.

| $z$ | Clusters | AGN | $b_{SZ}$ | $b_{X}$ | $b_{SZ,cort}$ | $b_{X,cort}$ | CSF | $b_{SZ}$ | $b_{X}$ | $b_{SZ,cort}$ | $b_{X,cort}$ | NR | $b_{SZ}$ | $b_{X}$ | $b_{SZ,cort}$ | $b_{X,cort}$ |
|-----|-----------|-----|---------|---------|---------------|---------------|-----|---------|---------|---------------|---------------|-----|---------|---------|---------------|---------------|
| 0.0 | All       | 0.230.14 | 0.140.11 | 0.260.12 | 0.140.16 | 0.270.15 | 0.150.14 | 0.120.18 | 0.280.13 | 0.140.16 | 0.120.18 | 0.280.13 | 0.140.16 | 0.120.18 | 0.280.13 | 0.140.16 |
|     | Rel       | 0.190.05 | 0.080.06 | 0.230.09 | 0.110.01 | 0.280.03 | 0.140.02 | 0.120.03 | 0.260.02 | 0.120.03 | 0.260.02 | 0.120.03 | 0.260.02 | 0.120.03 | 0.260.02 | 0.120.03 |
|     | Dis       | 0.250.01 | 0.140.18 | 0.320.28 | 0.120.23 | 0.260.39 | 0.210.32 | 0.140.28 | 0.400.35 | 0.240.26 | 0.250.32 | 0.160.09 | 0.120.09 | 0.400.35 | 0.240.26 | 0.250.32 |
| 0.11| All       | 0.280.11 | 0.140.13 | 0.250.14 | 0.120.18 | 0.280.13 | 0.140.16 | 0.120.18 | 0.280.13 | 0.140.16 | 0.120.18 | 0.280.13 | 0.140.16 | 0.120.18 | 0.280.13 | 0.140.16 |
|     | Rel       | 0.220.12 | 0.110.08 | 0.270.09 | 0.110.03 | 0.220.07 | 0.110.09 | 0.140.23 | 0.220.07 | 0.110.09 | 0.140.23 | 0.220.07 | 0.110.09 | 0.140.23 | 0.220.07 | 0.110.09 |
|     | Dis       | 0.240.17 | 0.150.02 | 0.350.21 | 0.140.28 | 0.360.42 | 0.220.16 | 0.140.28 | 0.400.35 | 0.240.26 | 0.250.32 | 0.160.09 | 0.120.09 | 0.400.35 | 0.240.26 | 0.250.32 |
| 0.33| All       | 0.260.13 | 0.130.14 | 0.280.1 | 0.150.12 | 0.310.09 | 0.190.14 | 0.150.12 | 0.310.09 | 0.190.14 | 0.150.12 | 0.310.09 | 0.190.14 | 0.150.12 | 0.310.09 | 0.190.14 |
|     | Rel       | 0.220.12 | 0.140.09 | 0.270.09 | 0.140.22 | 0.270.12 | 0.140.12 | 0.150.12 | 0.310.09 | 0.190.14 | 0.150.12 | 0.310.09 | 0.190.14 | 0.150.12 | 0.310.09 | 0.190.14 |
|     | Dis       | 0.270.25 | 0.130.23 | 0.350.14 | 0.150.4 | 0.360.32 | 0.230.19 | 0.150.4 | 0.360.32 | 0.230.19 | 0.150.4 | 0.360.32 | 0.230.19 | 0.150.4 | 0.360.32 | 0.230.19 |
| 0.43| All       | 0.250.11 | 0.140.12 | 0.280.11 | 0.120.15 | 0.310.15 | 0.280.13 | 0.120.15 | 0.310.15 | 0.280.13 | 0.120.15 | 0.310.15 | 0.280.13 | 0.120.15 | 0.310.15 | 0.280.13 |
|     | Rel       | 0.250.11 | 0.140.12 | 0.280.11 | 0.120.15 | 0.310.15 | 0.280.13 | 0.120.15 | 0.310.15 | 0.280.13 | 0.120.15 | 0.310.15 | 0.280.13 | 0.120.15 | 0.310.15 | 0.280.13 |
|     | Dis       | 0.270.13 | 0.170.23 | 0.300.23 | 0.130.29 | 0.160.24 | 0.070.29 | 0.130.29 | 0.160.24 | 0.070.29 | 0.130.29 | 0.160.24 | 0.070.29 | 0.130.29 | 0.160.24 | 0.070.29 |

MNRAS 000, 000–000 (2020)