R&D Strategy Document
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Research: the process of discovering fundamental new knowledge and understanding.

Development: the process by which new knowledge and understanding is applied.

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1 Introduction

We provide an overview of the R&D focus at the Zurich cell of Olsen Ltd. By detailing the general conceptual framework and by identifying key themes embedded in it, we outline what we believe are the prerequisites and building-blocks for successfully devising trading models (TMs) and other financial applications.

Chapters 2, 3, and 4 are based on Appendix A, Chapter 5 on Appendix C of Glatthöfer [2010].

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2 In a Nutshell: Science and Laws of Nature

Laws of nature can be seen as regularities and structures in a highly complex universe. They depend critically on only a small set of conditions, and are independent of many other conditions which could also possibly have an effect.

Science can be understood as a quest to capture laws of nature within the framework of a formal representation, or model. Naively one would expect science to adhere to basic notions of common sense, like logic, empiricism, causality, and rationality.

The philosophy of science deals with the assumptions, foundations, methods and implications of science. It tries to describe what constitutes a law of nature and tries to answer the question of what knowledge is and how it is acquired.

In the philosophy of science, the programs of logical empiricism and critical rationalism have been unsuccessful in conclusively answering the above mentioned questions and in providing an ultimate justification for science based on common sense. Indeed, the streams of postmodernism, constructivism and relativism explicitly question the notions of objectivity, rationality, absolute truths and empiricism.

But then, why has science been so successful at describing reality? And why is science producing the most amazing technology at breakneck speed? It is a great feature of reality, that the formal models which the human mind discovers/devises find their match in the workings of nature. We will return to this enigma later on.

In being pragmatic and disregarding the conceptual problems, one can identify two domains of nature and two modes of describing reality. In the following section, we give a short overview of this weltanschauung. In Secs. 3 and 4, the details are described.

2.1 Overview: The Outline at a Glance

The functioning of nature can broadly speaking be separated into two categories, either belonging to the domain of fundamental processes or complex systems. As examples, elementary particles in a force-field represent the former, while a swarm of birds constitutes the latter.

It has been possible to describe nature with two methods: the analytical and algorithmic approach. The analytical approach is what most people are familiar with.
Physical problems are translated into mathematical equations which, when solved, give new insights. The algorithmic approach simulates the physical system in a computer according to algorithms, where the dynamics of the real system are described by the evolution of the simulation.

In Fig. 1 a simple illustration of this categorization is seen. Four combinations emerge: both fundamental processes and complex systems can be tackled analytically or algorithmically.

An obvious challenge is to identify the most successful method to investigate a certain problem. This means not only choosing the formal representation but also identifying the reality domain it belongs to. Most of science can be seen to be related to strategy $A$, as is discussed in Sec. 3. Strategy $B$ has been successful in addressing real-world complex systems, which is detailed in Sec. 4.

The possibilities $C$ and $D$ have only been sparsely explored. Regarding the former, some authors have recently argued that complex systems can and should be tackled with mathematical analysis. See for instance Sornette [2008]. Strategy $D$ is mostly uncharted, and some tentative efforts include describing space-time as a network in some fundamental theories of quantum gravity (e.g., spin networks in loop quantum gravity) or deriving fundamental laws from cellular automaton networks Wolfram [2002].

We make two choices we believe to be instrumental to the success of understanding real-world financial markets and devising profitable TMs:

1. understand the problem as originating from the domain of complex systems;
2. tackle the problem with an algorithmic approach.

Hence, from our point of view, we see $B$ as the most promising strategy, see also Sec.
3 Fundamental Processes and Mathematical Models (A)

As mentioned, science can be understood as the quest to capture the processes of nature within formal mathematical representations. In other words, “mathematics is the blueprint of reality” in the sense that formal systems are the foundation of science.

3.1 The Success of Mathematical Models

This notion is illustrated in Fig. 2. The left-hand side of the diagram represents the real world, i.e., the observable universe. Scientists focus on a well-defined problem or question, identifying a relevant subset of reality, also called a natural system. To understand more about the nature of the natural system under investigation, experiments are performed yielding new information. Robert Boyle was instrumental in establishing experiments as the cornerstone of physical sciences around 1660. Approximately at the same time, the philosopher Francis Bacon introduced modifications to Aristotle’s (nearly two thousand year old) ideas, introducing the so-called scientific method where inductive reasoning plays an important role. This paved the way for a modern understanding of scientific inquiry.

In essence, guided by thought, observation and measurement, natural systems can be “encoded” into formal systems, depicted on the right-hand side of Fig. 2. Representing nature as mathematical abstractions is understood as a mapping from the real world to the mathematical world. Using logic (e.g., rules of inference) in the formal system, predictions about the natural system can be made. These predictions can be understood as a mapping back to the physical world, “decoding” the knowledge gained from from the abstract model. Checking the predictions with the experimental outcome shows the validity of the formal system as a model for the natural system.

The following two examples should underline the great power of this approach, where new features of reality where discovered solely on the requirements of the mathematical model. Firstly, in order to unify electromagnetism with the weak force (two of the three non-gravitational forces), the theory postulated two new elementary particles: the W and Z bosons. Needless to say, these particles where hitherto unknown and it took 10 years for technology to advance sufficiently to prove their existence. Secondly, the fusion of quantum mechanics and special relativity lead to the Dirac equation which demands the existence of an, up to then, unknown flavor of matter: antimatter. Four years after the formulation of the theory, antimatter was experimentally discovered.

3.2 The Paradigm of Fundamental Processes

Is it possible to isolate a single idea that has been instrumental to the success of describing fundamental processes?

It can be argued that the most fruitful paradigm in the study of fundamental processes in nature has been:
P1. Mathematical models of reality are independent of their formal representation.

This idea leads to the notions of symmetry and invariance. To illustrate, imagine an arrow located in space. It has a length and an orientation. In the mathematical world, this can be represented by a vector, labeled $a$. By choosing a coordinate system, the abstract entity $a$ can be given physical meaning, for instance $a = (3, 5, 1)$. The problem is however, that depending on the choice of the coordinate system, which is arbitrary, the same vector is described very differently: $a = (3, 5, 1) = (0, 23.34, -17)$. The paradigm above states that the physical content of the mathematical model should be independent from the choice of how one represents the mathematical model.

Although this sounds rather trivial, for physics it has very deep consequences. For instance, the requirements that physical experiments should be unaffected by the time of day and geographic location they are performed at, are formalized as time and translational invariance, respectively. These requirements alone give rise to the conservation of energy and momentum (Noether’s theorem).

Generally, the requirement of invariance and symmetry results in a great part of physics, from quantum field theories to general relativity, see Fig. 3.

In general relativity the vectors are somewhat like multidimensional equivalents called tensors and the common sense requirement, that the calculations involving tensor do not depend on how they are represented in space-time, is covariance. See the right-hand side of Fig. 3. It is quite striking, but there is only one more ingredient needed in order to construct one of the most aesthetic and accurate theories in physics. It is called the equivalence principle and states that the gravitational force is equivalent to the forces experienced during acceleration. Again, this may sound trivial, has however very deep implications.

The Standard Model of elementary particle physics unites the quantum field the-
ories describing the fundamental interactions of particles in terms of their (gauge) symmetries. See the left-hand side of Fig. 3.

3.3 The Challenges

We now come back to the questions raised in Sec. 2. Why has science been so successful in describing reality and why is science producing amazing technology at breakneck speed?

The simple answer is that there is no reason for this to be the case, other than the fact that it is the way things are. The Nobel laureate Eugene Wigner captures this salient fact in his essay “The Unreasonable Effectiveness of Mathematics in the Natural Sciences”: [Wigner 1960]:

“[…] the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and […] there is no rational explanation for it.”

“[…] it is not at all natural that ‘laws of nature’ exist, much less that man is able to discover them.”

Especially when recalling that the attempts to found science on common sense notions have been unsuccessful.
“[…] the two miracles of the existence of laws of nature and of the human mind’s capacity to divine them.”

“[…] fundamentally, we do not know why our theories work so well.”

Over the past 300 years, physics has been very successful with this approach, describing most of the observable universe. In essence, this formalism works well for the fundamental workings of nature (strategy A).

However, to explain real-life complex phenomena, one needs to adopt a more systems oriented focus. This also means that the interactions of entities becomes an integral part of the formalism.

Some ideas should illustrate the change in perspective:

- most calculations in physics are idealizations and neglect dissipative effects like friction;
- most calculations in physics deal with linear effects, as non-linearity is hard to tackle and is associated with chaos; however, most physical systems in nature are inherently non-linear [Strogatz 1994];
- the analytical solution of three gravitating bodies in classical mechanics, given their initial positions, masses, and velocities, cannot be found; it turns out to be a chaotic system which can only be simulated in a computer; there are an estimated hundred billion galaxies in the universe.

4 Complex Systems and the Algorithmic Approach (B)

A complex system is usually understood as being comprised of many interacting or interconnected parts. A characteristic feature of such systems is that the whole often exhibits properties not obvious from the properties of the individual parts. This is called emergence. In other words, a key issue is how the macro behavior emerges from the interactions of the system’s elements at the micro level. Moreover, complex systems also exhibit a high level of adaptability and self-organization. The domains complex systems originate from are mostly socio-economical, biological or physio-chemical.

The study of complex systems appears complicated, as it is different to the reductionistic approach of established science. A quote from Anderson [1972] illustrates this fact:

“At each stage [of complexity] entirely new laws, concepts, and generalizations are necessary […] Psychology is not applied biology, nor is biology applied chemistry”.

This means that the knowledge about the constituents of a system doesn’t reveal any insights into how the system will behave as a whole; so it is not at all clear how you get from quarks and leptons via DNA to a human brain and consciousness.

Moreover, complex systems are usually very reluctant to be cast into closed-form analytical expressions. This means that it is generally hard to derive mathematical quantities describing the properties and dynamics of the system under study.
4.1 The Paradigms of Complex Systems

The paradigms of complex systems are straightforward:

**PI.** Every complex system is reduced to a set of objects and a set of functions between the objects.

**PII.** Macroscopic complexity is the result of simple rules of interaction at the micro level.

Paradigm I is reminiscent of the natural problem solving philosophy of object-oriented programming, where the objects are implementations of classes (collections of properties and functions) interacting via functions (public methods). A programming problem is analyzed in terms of objects and the nature of communication between them. When a program is executed, objects interact with each other by sending messages. The whole system obeys certain rules (encapsulation, inheritance, polymorphism, etc.).

Indeed, in mathematics the field of category theory defines a category as the most basic structure: a set of objects and a set of morphisms (maps between the sets) [Hillman 2001]. Special types of mappings, called functors, map categories into each other. Category theory was understood as the “unification of mathematics” in the 1940s.

A natural incarnation of a category is given by a complex network where the nodes represent the objects and the links describe their relationship or interaction. Now the structure of the network (i.e., the topology) determines the function of the network. There are many excellent introductory texts, surveys, overviews and books covering the many topics related to complex networks: [Strogatz 2001], [Albert and Barabási 2002], [Dorogovtsev and Mendes 2002, 2003], [Newman 2003], [Newman et al. 2006], [Caldarelli 2007], [Costa et al. 2007], [Vega-Redondo 2007].

This also highlights the paradigm shift from mathematical (analytical) models to algorithmic models (computations and simulations performed in computers). In other words, the analytical description of complex systems is abandoned in favor of algorithms describing the interaction of the objects, also called agents, in a system according to rules of local interactions. This approach has given rise to the prominent field of *agent-based modeling*. In addition, a key realization is that also the structure and complexity of each agent can be ignored when one focuses on their interactional structure. Hence the animals in swarms, the ants foraging, the chemicals interacting in metabolic systems, the humans in a market, etc., can all be understood as being comprised of featureless agents and modeled within this paradigm.

Paradigm II is perhaps as puzzling as the “unreasonable effectiveness of mathematics in the natural sciences”. To quote Stephen Wolfram’s reaction, to the realization that simplicity encodes complexity, from [Wolfram 2002, p. 9]:

“And I realized, that I had seen a sign of a quite remarkable and unexpected phenomenon: that even from very simple programs behavior of great complexity could emerge.”
Figure 4: The properties of complex systems and the paradigms describing them.

“Indeed, even some of the very simplest programs that I looked at had behavior that was as complex as anything I had ever seen. It took me more than a decade to come to terms with this result, and to realize just how fundamental and far-reaching its consequences are.”

In summary, Paradigms I and II can be seen to belong to strategy B, as seen in Fig. 4.4.2 The Success of Simulating Complex Systems

It is remarkable that simple interactions result in complex behavior: emergence, adaptivity, structure-formation and self-organization. In essence, complexity does not stem from the number of agents but from the number of interactions. For instance, there are roughly 30000 genes in a human vs. about 55000 genes in a grain of rice.

In Fig. 4 an illustrated overview of an agent-based simulation is given: in a computer simulation agents are interacting according to simple rules and give rise to patterns and behavior seen in real-world complex systems.

This also highlights the departure from a top-down to a bottom-up approach to complexity. As an example, swarming behavior in nature can be easily modeled by agents obeying three simple and local rules, reproducing its emergent and adaptive characteristics:

1. move in the same direction as your neighbors;
2. remain close to your neighbors;
3. avoid collisions with your neighbors.

In addition, biological (temporal-spatial) pattern formation, population dynamics, pedestrian/traffic dynamics, market dynamics etc., which where hitherto impossible to tackle with a top-down approach, are well understood by the bottom-up approach given by the paradigms of complex systems.

However, to be precise, there is still some mathematical formalism used in the study of complex systems. For instance, at the macro level, the so-called Fokker-Planck differential equation gives the collective evolution of the probability density function of a system of agents as a function of time. While at the micro level, a single agent’s
behavior can be described by the so-called Langevin differential equation. The two formalism can be mapped into each other [Gardiner 1985]. However, as an example, 10000 agents following Langevin equations in a computer simulation approximate the macro dynamics of the system more efficiently than an analytical investigation attempting to solve the equivalent Fokker-Planck differential equation.

4.3 An Event-Based Methodology: Intrinsic Time

By understanding complexity as arising from the interaction of dynamical systems, it is natural to adopt another paradigm in their study:

PIII. The passage of time is defined by events, i.e., system interactions.

In this time ontology\textsuperscript{2} time rests in the absence of events. In contrast to physical time, only interactions, or events, let a system’s clock tick. Hence this new methodology is called intrinsic time [Olsen 1983, Müller et al. 1993]. Implicit in this approach is that a system is made up exclusively of interactions, becoming a dynamic object with a past, present and future. Every interaction with other systems is a new system. This event-based approach opens the door to a model that is self-referential, does not rely on static building blocks and has a dynamic frame of reference.

This approach was a crucial ingredient in the formulation of the new empirical scaling laws [Glattfelder et al. 2011]. This is described in Secs. 5.3 and 6.2.

5 Scaling Laws

The empirical analysis of real-world complex systems has revealed unsuspected regularities, such as scaling laws, which are robust across many domains [Müller et al. 1990, Mantegna and Stanley 1995, West et al. 1997, Amaral et al. 1998, Albert et al. 1999, Pastor-Satorras et al. 2001, Newman et al. 2002, Garlaschelli et al. 2003, Newman 2005, Glattfelder et al. 2011]. This has suggested that universal or at least generic mechanisms are at work in the structure-formation and evolution of many such systems. Tools and concepts from statistical physics have been crucial for the achievement of these findings [Dorogovtsev and Mendes 2003, Caldarelli 2007]. In essence,

scaling laws can be seen as laws of nature found for complex systems.

A scaling law, or power law, is a simple polynomial functional relationship

\[ f(x) \propto x^{-\alpha}. \]  

(1)

Two properties of such laws can easily be shown:

\textsuperscript{2}See also Van Benthem 1991.
• a logarithmic mapping yields a linear relationship;
• scaling the function’s argument \( x \) preserves the shape of the function \( f(x) \), called scale invariance.

See for instance Newman [2005], Sornette [2000].

Scaling-law relations characterize an immense number of natural processes, prominently in the form of

1. scaling-law distributions;
2. scale-free networks;
3. cumulative relations of stochastic processes.

5.1 Scaling-Law Distributions

Scaling-law distributions have been observed in an extraordinary wide range of natural phenomena: from physics, biology, earth and planetary sciences, economics and finance, computer science and demography to the social sciences West et al., [1997], Amaral et al., [1999], Albert et al., [1999], Sornette [2000], Pastor-Satorras et al., [2001], Bouchaud [2001], Newman et al., [2002], Caldarelli et al., [2002], Garlaschelli et al., [2003], Gabaix et al., [2003], Newman [2005], Lux [2005], Di Matteo [2007].

It is truly amazing, that such diverse topics as

• the size of earthquakes, moon craters, solar flares, computer files, sand particle, wars and price moves in financial markets;
• the number of scientific papers written, citations received by publications, hits on web-pages and species in biological taxa;
• the sales of music, books and other commodities;
• the population of cities;
• the income of people;
• the frequency of words used in human languages and of occurrences of personal names;
• the areas burnt in forest fires;

are all characterized by scaling-law distributions. First used to describe the observed income distribution of households by the economist Vilfredo Pareto in 1897 [Pareto 1897], the recent advancements in the study of complex systems have helped uncover some of the possible mechanisms behind this universal law. However, there is still no conclusive understanding of the origins of scaling law distributions. Some insights have been gained from the study of critical phenomena and phase transitions, stochastic processes, rich-get-richer mechanisms and so-called self-organized criticality Bouchaud [2001], Barndorff-Nielsen and Prause [2001], Farmer and Lillo [2004], Newman [2005].
Processes following normal distributions have a characteristic scale given by the mean of the distribution. In contrast, scaling-law distributions lack such a preferred scale. Measurements of scaling-law processes yield values distributed across an enormous dynamic range (sometimes many orders of magnitude), and for any section one looks at, the proportion of small to large events is the same. Historically, the observation of scale-free or self-similar behavior in the changes of cotton prices was the starting point for Mandelbrot’s research leading to the discovery of fractal geometry \cite{Mandelbrot1963}.

It should be noted, that although scaling laws imply that small occurrences are extremely common, whereas large instances are quite rare, these large events occur nevertheless much more frequently compared to a normal (or Gaussian) probability distribution. For such distributions, events that deviate from the mean by, e.g., 10 standard deviations (called “10-sigma events”) are practically impossible to observe. For scaling law distributions, extreme events have a small but very real probability of occurring. This fact is summed up by saying that the distribution has a “fat tail” (in the terminology of probability theory and statistics, distributions with fat tails are said to be leptokurtic or to display positive kurtosis) which greatly impacts the risk assessment. So although most earthquakes, price moves in financial markets, intensities of solar flares, etc., will be very small, the possibility that a catastrophic event will happen cannot be neglected.

5.2 Scale-Free Networks

The degree distribution of most complex networks follows a scaling-law probability distribution $P(k) \propto k^{-\alpha}$, see also \cite{Barabasi1999,Albert2002,Caldarelli2007}. Scale-free networks are characterized by high robustness against random failure of nodes, but susceptible to coordinated attacks on the hubs. Theoretically, they are thought to arise from a dynamical growth process, called preferential attachment, in which new nodes favor linking to existing nodes with high degree \cite{Barabasi1999}. Although alternative mechanisms have been proposed \cite{Caldarelli2002}.

5.3 Cumulative Scaling-Law Relations

Next to distributions of random variables, scaling laws also appear in collections of random variables, called stochastic processes. Prominent empirical examples are financial time-series, where one finds empirical scaling laws governing the relationship between various observed quantities. As an example, \cite{Glattfelder2011} uncovered 18 novel empirical scaling-law relations, 12 of them being independent of each other.

In finance, where frames of reference and fixed points are hard to come by and often illusory, these new scaling laws provide a reliable framework. We believe they can enhance our study of the dynamic behavior of markets and improve the quality of the inferences and predictions we make about the behavior of prices. The new laws represent the foundation of a completely new generation of tools for studying volatility,
measuring risk, and creating better forecasting and trading models. The new laws also substantially extend the catalogue of stylized facts and sharply constrain the space of possible theoretical explanations of the market mechanisms.

See also Müller et al. [1990], Mantegna and Stanley [1995], Guillaume et al. [1997a], Galluccio et al. [1997], Dacorogna et al. [2001], Glattfelder et al. [2011] and Sec. 6.2.

6 FX Market as a Complex System

The foreign exchange (FX) market can be characterized as a complex network consisting of interacting agents: corporations, institutional and retail traders, and brokers trading through market makers, who themselves form an intricate web of interdependence. With an average daily turnover of three to four trillion USD [Bank for International Settlements 2007, 2010], and with price changes nearly every second, the FX market offers a unique opportunity to analyze the functioning of a highly liquid, over-the-counter market that is not constrained by specific exchange-based rules.

What relevance do the insights presented in Sec. 2 have for real-world markets, in particular the FX market?

6.1 Algorithmic vs. Analytical Approach

There has been a long history of attempting to understand finance from an analytical point of view (i.e., within strategy A). Indeed, the field of mathematical finance has produced a vast body of mathematical tools characterized by a very high level of abstraction Hull [1993], Voit [2005].

We believe this is a misleading approach to fundamentally understand markets and to devise TMs. On the one hand, to make the equations tractable, often stringent constraints and unrealistic assumptions have to be imposed. On the other hand, the reality of markets being an epitome of a complex system is ignored. Hence our shift to strategy B.

We choose to tackle the problem of financial markets in accordance with Paradigm I, viewing it in terms of interacting agents. Applying the algorithmic approach given by Paradigm II, entails understanding the observed market complexity at the macro level arising from the interaction of heterogeneous agents at the micro level according to simple rules. The heterogeneity is given for instance by the traders geographical locations, trading time horizons, risk profiles, dealing frequencies, trade sizes etc.

In essence, our TMs are agent-based models. An agent is defined by a position $p_i$, comprised of the set $\{\bar{x}_i, g_i\}$, where $\bar{x}$ is the current average (or entry price) and $g$ is the gearing (position size). Each position also has a set of event-based and simple rules. The agent’s interaction with each other are constrained by the price curve.

6.2 Event Time: Directional Change Algorithm and Overshoots

There is a very concrete application of Paradigm III, the event-based methodology, to FX time series. As mentioned in Sec. 4.3 it is tightly connected with the existence of
cumulative scaling-law relations.

In [Guillaume et al., 1997b] the direction change algorithm was introduced. Fig. 5, taken from [Glattfelder et al., 2011], depicts how the price curve is dissected into so-called directional-change and overshoot sections. The dissection algorithm measures occurrences of a price change $\Delta x_{dc}$ from the last high or low (i.e., extrema), if it is in an up or down mode, respectively. At each occurrence of a directional change, the overshoot segment associated with the previous directional change is determined as the difference between the price level at which the last directional change occurred and the extrema, i.e., the high when in up mode or low when in down mode. The high and low price levels are then reset to the current price and the mode alternates. [Guillaume et al., 1997b] presented the directional-change count scaling law

$$N(\Delta x_{dc}) = \left( \frac{\Delta x_{dc}}{C} \right)^E,$$

where $N(\Delta x_{dc})$ is the number of directional changes measured for the threshold $\Delta x_{dc}$.

Extending this event-driven paradigm further enables us to observe new, stable
patterns of scaling. In [Glattfelder et al. 2011] this event-based approach was crucial for discovering eight of the 12 primary scaling law relations, and four of the six secondary ones. This establishes the fact, that moving from the empirical time series to their event-based abstractions provides a unique point of view, from which patterns can be observed which would otherwise remain hidden. This is illustrated in Fig. 6.

Moreover, the discovery of the overshoot scaling law\(^3\) was instrumental in extending the event-based methodology to accommodate a second type of event. This scaling law relates the length of the average overshoot segment to the directional change threshold

\[
\langle |\Delta x^{os}| \rangle = \left( \frac{\Delta x_{dc}}{C} \right)^E.
\]

It turns out that the average length is about the same size as the threshold: \(\langle |\Delta x^{os}| \rangle \approx \Delta x_{dc}\).

This finding motivates the dissection of the price curve into, not only directional-change events, but also overshoot events, occurring during the overshoot segments. Crucially, both are defined by the same threshold \(\Delta x_{dc}\). As a result, by applying the direction change algorithm to empirical price moves, we can reduce the level of complexity of the real-world time series. In detail, the various fixed event thresholds of different sizes define focal points, blurring out irrelevant details of the price evolution. In Fig. 7 an example of an empirical price curve and its associated array of events for a chosen directional-change threshold is shown. This effectively unveils the key structures of the market. An example of this is highlighted in Fig. 8: the coastline, comprised solely of directional changes and overshoots, with no constraints coming from physical time. This implies, that the coastline faithfully maps the activity of the market: during low volatility, the coastline is shrunk, whereas active market environments get stretched and all their details are exposed. Hence, by construction, this procedure is adaptive.

In a nutshell,

the coastlines associated with different directional-change thresholds \(\Delta x_{dc}\) are taken as the basis for our R&D effort, coupled with the \textit{weltanschauung} coming from Strategy B and Paradigms I to III.

\(^3\)Or more precisely: the average overshoot length scaling law, see [Glattfelder et al. 2011].
Figure 7: (Top) the original EUR\_USD price curve for about two days (2008-12-14 22:10:56 to 2008-12-16 21:58:20); (bottom) the price curve is overlaid with the directional-change and overshoot events defined by $\Delta x_{dc} = 0.25\%$; the red triangles represent directional-change and the blue bullets overshoot events.
Figure 8: The coastline, defined by the threshold of 0.25%, is a pure event-based price curve and lacks any reference to physical time; its derivation is seen in Fig. 7 by measuring the various coastlines for an array of thresholds, multiple levels of event activity are considered.
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