Measurement of train-induced vibration for track parameter identification: Bayesian probabilistic approach

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Abstract. The train-induced vibration response is used to develop a comprehensive finite element model for the railway track, which considered rail-sleeper-ballast interaction. This model can be further use in the development of an operational monitoring system for railway track by following the Bayesian statistical system identification framework. However, it is believed that the uncertainties associated with the measured train-induced test data and model parameters are relatively high, therefore, the corresponding model updating problem will very likely to be in the category of unidentifiable case. Therefore, Markov chain Monte Carlo (MCMC) simulation was used in generating the samples for the approximation of the posterior uncertainties. The measurements also provide not only reference values suitable for model fitting, but also a good insight into the main features of the dynamic behaviour of the railway track system. Finally, the reliability and accuracy of the proposed method are demonstrated by numerically reproducing the system responses. Modelling assumptions are discussed, together with their implications on numerical results of the time-domain analysis, which were found to be in good agreement with experimental results.

1. Introduction

The railway system is one of the most essential component of transportation systems and plays a vital, and irreplaceable role in the development of economies and societies. In 2007, the total length of railway lines in the world was reported to 1,370,782 km. The increase in speed and axle load of the rains has had a negative effect on the life span of the track system. With developments in high-speed train, the rate of railway ballast degradation has increased [1-4]. If remedial work is not carried out, this degradation will result in uneven support of railway tracks and increase the chances of rail buckling and train derailment, which would obviously endanger the safety and comfort of railway users [5].

However, railway ballast is a granular material resting on the subgrade, which provide a support for the concrete sleeper over it and holds the sleepers in position by resisting longitudinal and lateral displacement and preventing vertical settlement. Broken stone such as quartzite and granite of size 50 mm to 65 mm is used [6]. Because of the increased speed, weight of the trains and irregularities of the wheels and rails, subjects the ballast under the sleeper to high dynamic loading condition which accelerate the damage of ballast. From the literature, the stress-strain relationship is nonlinear. Based on the author’s experience, the dynamic behaviour of the rail-sleeper-ballast system can be well approximated by a linear model under small amplitude vibration [7-8]. However, the approximation becomes very bad if the amplitude of vibration is increased (e.g., large force impact hammer test or train
induced vibration). However, one of the main purpose of this study is to consider this nonlinear effect of ballast in the model updating process. As a result, one can improve the accuracy of the existing ballast damage detection method by reducing the level of modelling error [7]. Furthermore, this help in the development of a long-term ballast health monitoring method utilizing train-induced vibration of the ballasted track.

Several efforts have been made to model the dynamic behaviour of railway ballast under concrete sleeper as in the following references [1][7][9-13]. Their modelling methods assumed that the railway ballast is a linearly elastic material and; usually modelled as an elastic foundation or as a series of closely distributed linear springs in a vertical direction. Little or no work has been done to incorporate the nonlinear behaviour of ballast in the modelling of rail-sleeper-ballast system. This study presents a Bayesian finite-element (FE) model updating approach using time-domain MCMC methodology together with the incorporation of nonlinear ballast modelling method based on train-induced vibration test data. Although the use of impact hammer test data could identify the model parameters with reasonable accuracy based on MCMC-based Bayesian approach, however, there is still room for improvement using measured train-induced test data. Field train tests were performed on the railway track system to measure it acceleration histories with different train speeds ranging between 20 km/h and 80 km/h.

The train-induced data is used to develop a comprehensive finite element model for the railway track, which considered rail-sleeper-ballast interaction. This model can be further use in the development of an operational monitoring system for railway track by following the Bayesian statistical system identification framework. It is believed that the uncertainties associated with the measured train-induced test data and model parameters are relatively high, therefore, the corresponding model updating problem will very likely to be in the category of unidentifiable case. Thus, Markov chain Monte Carlo (MCMC) simulation was used in generating the samples for the approximation of the posterior uncertainties. The measurements also provide not only reference values suitable for model fitting, but also a good insight into the main features of the dynamic behaviour of the railway track system. Finally, the reliability and accuracy of the proposed method are demonstrated by numerically reproducing the system responses. Modelling assumptions are discussed, together with their implications on numerical results of the nonlinear time-domain analysis, which were found in good agreement with experimental results.

2. Experimental setup

The measured railway tracks are typical ballasted track (see figure 1) with a continuously welded UIC 60 rails with a mass per unit length of 60 kg/m and moment of inertia $I = 3.217 \times 10^{-5} \text{m}^4$. The rail seat on a resilient pad (TRACKLAST FC105A) with thickness of 10 mm and a static stiffness of about 100 MN/m, for a varying load between 15 kN and 90 kN. The track gauge is 1.5 m with pandrol shoulder and clips used to hold the rails. The UIC 60 rail is supported at every 0.60 m by concrete mono-block sleepers with dimension of length 2.5 m, width 0.285 m, height 0.205 m (under the rail), and 0.177 m (at the middle of the sleeper), a mass of 300 kg and a Young’s modulus $38 \times 10^9 \text{N/m}^2$. Therefore, the sleepers were on the well compacted ballast bed of macadam with thickness ranging between 0.350 and 0.450 m, with the stable density of the of about 1800 kg/m$^3$, the elastic modulus of the ballast is about 110 MPa and modulus of subgrade is about 90 MPa/m. The size of normal ballast is 50 – 65 mm. For the existing railway track, the sub-ballast layer, which is usually comprised of well-graded crushed rock that prevents the penetration of coarse granite ballast into the subgrade is presented. Type 8776A50M3 Kistler accelerometers, as shown in figure 2, with a sensitivity level of around 100 mV/g and a dynamic range of ±50 g was used. Seven accelerometers were installed on the top surface of the concrete sleeper along the central line to capture vertical vibration. Figure 2 shows the locations of the accelerometers on the target sleeper. Based on visual inspection, it was assumed that standard of track geometry was high and identical across the measurement points. However, after all the sensors, has been properly fixed on the sleeper, the moving train at different speed is allowed to pass over the measurement locations. Train-induced vibrations originated from the interaction between the train wheel and track. Each wheel sets act as independent sources of vibration. A total of 10 train passages at speeds varying between 20 km/h and 80 km/h were recorded. Two hours long train-induced measurements were
simultaneously recorded. The Sampling frequency of 6400 Hz was used for train-induced vibration measurements.

Figure 1. Measured ballasted track.

Figure 2. Sensors on small square steel plates on the sleeper.

3. MCMC-based Bayesian methodology

3.1. Modelling of rail-sleeper-ballast system

A single in-situ sleeper is modelled as a Timoshenko beam on a nonlinear elastic foundation with two mass-spring systems representing the dynamic effects of the two rails together with rail-pads as shown in figure 3. In this study, the behaviour of ballast stiffness \( k_B \) is assumed to be dependent on the strain level. When the strain level is low, the ballast stiffness is a constant. But the stiffness value will be reduced when the strain is higher than the yield limit. In summary, the value of ballast stiffness, \( k_B \), can be defined as:

\[
k_B = \begin{cases} 
  k_{B, L} = \frac{\sigma_Y A}{\varepsilon_e h} - \frac{E_{B, L} A}{h} & \text{constant for } \varepsilon \leq \varepsilon_e \\
  k_{B, N} = \frac{A K n}{h} \left( \varepsilon_e - \frac{z}{h} \right)^{n-1} & \text{for } \varepsilon > \varepsilon_e
\end{cases}
\]

where \( k_{B, L}, k_{B, N}, \sigma_Y, A, \varepsilon, h, E_{B, L}, \varepsilon_e, K, n, z, \) and \( \varepsilon_e \) are linear ballast stiffness, nonlinear ballast stiffness, yield stress of the ballast, area of ballast under the element of sleeper, total strain, thickness of ballast, elastic modulus of ballast, yield strain, hardening strength, strain-hardening exponent, vertical displacement (assumed to be positive downward and negative upward) and strain in equilibrium (with the consideration of the self-weight of the sleeper). Readers can also refer to references [8] for more details about the development of equation (1). A self-developed finite element (FE) MATLAB® function was used for the time-domain analysis of the rail-sleeper system. The nominal values for these uncertain parameters are summarised in table 1.
Table 1. Parameters used in modelling nonlinear rail-sleeper-ballast system.

| Model parameters                  | Nominal values       |
|-----------------------------------|----------------------|
| Yield stress                      | $1.70 \times 10^8$ N/m$^2$ |
| Elastic strain                    | $2.273 \times 10^6$ |
| Strain hardening strength         | $5.70 \times 10^8$ N/m$^2$ |
| Exponential factor                | 0.35                 |
| Young’s modulus                   | $34 \times 10^9$ N/m$^2$ |
| Rail mass                         | 60 kg                |
| Rail pad stiffness                | $28 \times 10^7$ N/m$^2$ |

Figure 3. Modelling method of the rail-sleeper-ballast system

3.2. Time-domain MCMC-based Bayesian model updating

Bayesian probabilistic approach aims in calculating the posterior probability density function, PDF, $p(\theta|D,M)$ of the uncertain model parameters $\theta$, conditional on a given set of measured data $D$ and a given class of models $M$ as follows:

$$p(\theta|D,M) = cp(D|\theta,M) p(\theta|M)$$

(2)

where $p(\theta|M)$ is the prior PDF of model parameters; $p(D|\theta,M)$ is the likelihood function; and $c$ is the normalizing constant such that the integration of the PDF over parameter space is equal to unity. Therefore, the uncertainty that gives rise to the likelihood function is basically a result of modelling error and measurement noise. The measured structural response of the $k^{th}$ DOF at the $j^{th}$ time step can be modelled as

$$\hat{x}_j(k) = \hat{y}_j(k,\theta) + \epsilon$$

(3)

where $\hat{y}(k,\theta)$ is the model-predicted response of the $k^{th}$ DOF at time step $j$; and $\epsilon$ is the prediction error, which is modelled as a zero-mean Gaussian random variable with variance $\kappa^2$. It is assumed that the prediction errors at each time step and at each DOF follow independent and identically distributed (i.i.d.) distributions. The train induced vibrations originate from the interaction between the train wheel and track. Individual wheel sets act as an independent source of vibration. Therefore, the measured rail vibration is used as the system input to calculate the sleeper vibration. It is assumed that each impulse in the different data sets is independent, and the PDF of measured data $D$ from $N_f$ impulses can be express in a similar form as in references [1][14].

$$p(D|\theta,M) = m_0 \exp \left\{-\frac{1}{2\kappa_1} \sum_{k=1}^{N_f} \sum_{j=1}^{N_r} \sum_{n=1}^{N_r} \left( \hat{x}_n^n(k) - \hat{y}_j(k,\theta) \right)^2 \right\}$$

(4)

Let $J(\theta)$ be the measure-of-fit function

$$J(\theta) = \frac{1}{N_0} \sum_{k=1}^{N_f} \sum_{j=1}^{N_r} \sum_{n=1}^{N_r} \left( \hat{x}_n^n(k) - \hat{y}_j(k,\theta) \right)^2$$

(5)
where \( N_0 = (N_f N_s N_d) \) is the total number of data points considered in the updating process, and 1-1 is the Euclidean norm. Therefore, the joint posterior PDF of uncertain model parameters \( \theta \) can be calculated by substituting equation (4) into equation (1)

\[
p(\theta|D) = c_1 \exp \left\{ -\frac{1}{2\kappa^2} J(\theta) \right\}
\]

where the normalizing constant \( c_1 \) combines the effects from \( m_0 \) and \( c \).

The posterior PDF of the model parameters as defined in equation 6 cannot usually be obtained in an explicit manner, not only because the posterior PDF may have a complex format, but also because it requires the evaluation of integrals over the space of the model parameters, which is not analytically available and cannot be computed in a straightforward manner by numerical integration due to the higher dimensions of the model parameter vector \( \theta \). Thus, it is proposed in this paper to evaluate the posterior PDF of uncertain model parameters defined in equation 6 by statistical samples drawn from the posterior PDF via Monte Carlo simulation [15-16]. The standard MCMC algorithm may not be efficient when the posterior PDF contains a very sharp peak or multiple peaks. This is because many proposed samples will be rejected if a very sharp peak is present. Furthermore, the samples drawn may become “stuck” on one local peak and cannot move to other peaks in the parameter space of interest. To overcome the inefficiency of direct sampling, an MCMC method was developed in references [17] to generate samples in multiple levels. In this study, this MCMC method was extended from the modal domain to the time domain to identify the uncertain model parameters. The new method proposes a sequence of bridge PDFs that act as bridges to ensure that the samples generated from different levels converge in the important region of the posterior PDF. Readers can also refer to references [1][8][17-18] for more details about formulations.

4. Model updating results

The method presented in section 3.2 is capable of handling a given large number of measured data points and a case where repeated evaluations of the likelihood function \( p(D|\theta,M) \), in the probabilistic optimization search for a nonlinear model is computationally prohibitive. \( \theta \) denote the uncertain model parameters vector that determines the structural model within a selected class of models \( M \). The uncertain model parameters to be identified are the non-dimensional scaling factors

\[
\theta \equiv \{ \theta_e, \theta_e, \theta_n, \theta_E, \theta_{ml}, \theta_{ml}, \theta_{kl}, \theta_{kr} \},
\]

where \( \theta_e, \theta_e, \theta_n \) and \( \theta_E \) are the scaling factors for the strain at equilibrium, elastic strain, exponent factor defining the degree of material nonlinear and Young’s modulus of sleeper, respectively; \( \theta_{ml} \) and \( \theta_{nR} \) are the scaling factors for the left and right rail mass, respectively; and \( \theta_{kl} \) and \( \theta_{kr} \) are the scaling factors for the left and right stiffness values of rail, respectively. The nominal values for these uncertain model parameters are summarized in table 1. The prior distribution is a mean to incorporate initial knowledge about the uncertain model parameters into the identification process, and it is subjective in the sense that people with different experience may use different prior leading to broader ranges of the solution in cases where lesser amount of prior information is available. To update prior knowledge about the uncertain model parameters and hence to obtain the posterior PDFs, the proposed time domain MCMC-based Bayesian model updating method is used, a multilevel Metropolis Hasting algorithm. The prior distributions of model parameters are considered as independent uniform distributions lower bound of 1 and upper bound of 4. In order words, uniform distributions within the bounds \( \theta_i \sim U([1,4]), i = 1,2,\ldots N_0 \) are used as prior PDFs for the \( N_0 \) uncertain model parameters in the FE Bayesian model updating case, Markov Chain samples generated as shown in figure 4. Since it is impossible to present a multi-dimensional plot (there are maximum of 8 uncertain model parameters in this class of models), various 2D projections of the samples were considered. The joint probability density function of the posterior samples is concentrated in the neighbourhood of most probable model, since there is only one
important region in the parameter space of interest. The posterior samples of Young’s modulus of sleeper and the degree of nonlinearity, $\theta_E$ and $\theta_n$, shows clear difference to the prior samples (assumes to be uniform) since they are concentrated around a point. While the posterior samples of other model parameters shown little or no clear difference to the prior samples since they cover the entire area of the prior PDFs.

With the MCMC samples, the marginal posterior PDFs of eight (8) random uncertain model parameters is calculated as shown in figure 5 (solid blue line). It is clear from the figure that the marginal PDFs of degree of nonlinearity and Young’s modulus of sleeper are close to Gaussian distribution but not exactly Gaussian distributions. It is also clear from the figure that the spreading of the distribution $\theta_k$ decay very fast from the optimal value (i.e., the most probable value) showing that the associated posterior uncertainty is relatively low. However, the decay of PDF value from the optimal for other parameters is very slow. It can be concluded that the uncertainties associated with the other model parameters are very high. The uncertainties of the other model parameters are so high that the model updating problem can be considered as unidentifiable which means that there exist an infinite number of most probable models. This means that there is more than one solution point led to a model which can reproduce the subset of measured test data taken from the complete set of measured train-induced test data. The reason for these high uncertainties lie in the fact that the data used for updating are incomplete and therefore not only one single parameters vector constitutes the solution, but there is a set of identified model parameters that forms the solution space for the vector of model parameter. Also, the solution is not unique and the posterior PDFs reveals a certain dispersion. This shows the importance of using MCMC-based method in model updating.

Another important observation, is that the posterior probability distributions of the yield stress, strain hardening strength, left and right rail mass and stiffness values of rails are one-sided because of the lower bound constraint of zero. The posterior PDF of the degree of nonlinearity and Young’s modulus of sleeper are two-sided which suggest that their distribution is Gaussian. The two estimates of the posterior probability distribution of each model parameter, namely the kernel marginal PDF and the approximated Gaussian PDF (dashed red line) are very close for the degree of nonlinearity and Young’s modulus of concrete sleeper, but slightly different for other model parameters. It is clear from the figure that due to the incorporation of the information contained in the measured train-induced test data, the prior distributions are shifted towards the optimal values leading to a considerably better match. As readily observed for the posterior structural parameter values, the distributions of $\theta_E$, $\theta_k$, $\theta_{mL}$, $\theta_{mR}$, $\theta_{cL}$ and $\theta_{cR}$ shows a larger posterior uncertainties due to the presence of the degree of nonlinearity, which is visible through the spreading of the posterior PDFs. Therefore, the closer the kernel PDFs that represent the most probable values (MPV) and coefficient of variation, COV of the samples, the smaller is the correlation between updating parameters. The degree of nonlinearity slightly affect the measured test used for model updating which results in the posterior PDFs of the model parameters in the upper bound of the interval to less probable. In order words, the measured data contain information that model parameter values in the upper bound of the interval of the prior uniform distribution have small probabilities.

To provide more information about the interpretation of the results, an approximated Gaussian probability density functions (dashed red line in figure 5) were employed to fit the marginal posterior PDFs for the estimation of the posterior mean and coefficient of variation (COV) of various uncertain model parameters. The most probable values of all the uncertain parameters are summarized in second row of table 2 together with the estimated posterior COVs. It is clear from the table that the estimated parameters of yield stress, strain hardening strength, rail mass and rail-pad stiffness have larger COVs than that of Young’s modulus of the concrete sleeper and degree of nonlinearity. This is due to the higher sensitivity of the updating parameter, Young’s modulus of the concrete sleeper and degree of nonlinear to the time domain data that is larger observability of these parameters in the view of data. Also, this maybe as a result of modelling error in the reference FE model and the variability in the measured data caused by environmental conditions and measurement error. Also, it can be due to the fact that the rail mass and rail pad stiffness has smaller vibration responses.

With the most probable models in 8-parameter model, the updated time-domain responses of the system were calculated using the response measured on the web of the rail under the moving train.
The matching between the model-predicted (after model updating) and measured time domain responses are plotted in figure 6. The matchings are very good with the acceleration amplitudes very close to each other and the frequency content of the response are also in good agreement.

Figure 4. Markov samples generated at the final level.
Figure 5. The posterior marginal PDFs of 8-parameters model by MCMC (dashed line) and the corresponding Gaussian approximation (red dash lines).

Table 2. Bayesian model updating results of an existing railway track.

| Model parameter | $\theta_s$ | $\theta_c$ | $\theta_n$ | $\theta_E$ | $\theta_{mL}$ | $\theta_{mR}$ | $\theta_{kL}$ | $\theta_{kR}$ |
|------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| MPV              | 0.0167    | 0.9342    | 0.4952    | 1.6028    | 2.3565    | 2.3299    | 1.0452    | 0.7824    |
| COV (%)          | 9.5808    | 7.2683    | 120.07    | 0.0076    | 27.689    | 27.310    | 28.940    | 93.622    |
Figure 6. The ability of the model to replicate the measured train data.
5. Concluding remarks
For the first time, a probabilistic model updating of nonlinear rail-sleeper-ballast system using train-induced vibration field test data, that is essential for the development of ballast damage detection method has been presented. Because of the existence of uncertainties in the estimation of structural model parameter values. These uncertainties are caused by many factors, such as geometry and boundary conditions, the FE model discretization, material variability, inaccurate construction process, material deformation due to typhoon and material deterioration throughout its operational lifetime. However, the use of uncertain model parameter values in assessing the structural dynamic response usually leads to invalid structural reliability. The application of Bayesian model updating provides a way to model these uncertainties of the model parameter values and have the ability to update them into some more reliable ones. Therefore, this study proposes a time domain Markov Chain Monte Carlo (MCMC)-based Bayesian model updating method capable of handling any class of model updating problems and discuss its effectiveness and efficiency in nonlinear dynamic system model updating. The quantification of model uncertainties is carried out by means of the prediction error (difference between the measured field test data and model-predicted responses) which has been analysed in context with updating a finite element model where the data derived from the model are affected by the presence of nonlinearity.

The proposed method was applied to the identification of nonlinear rail-sleeper-ballast system based on the train-induced vibration field test data. Eight (8) model parameters, which included degree of nonlinearity, yield stress, strain hardening strength, Young’s modulus of sleeper, left and right rail masses and stiffness values of the rails, were updated from their prior beliefs into a more accurate one, while nonlinear dynamic analysis of the rail-sleeper-ballast system was carried out by Newmark’s beta method. The proposed Bayesian framework allows for an improvement of the numerical model by incorporating the information contained in the available train-induced vibration field test data. Based on the optimal model, the matching between measured and model-predicted responses were presented. It can be concluded that the verification results based on the field test data from the in-situ sleeper demonstrate the high potential for the practical application of the MCMC-based Bayesian model updating method.

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