Restricted Supergauge invariance,
N=2 Coadjoint Orbits
and N=2 Quantum Supergravity

W.A. Sabra
Physics Department
Birkbeck College
University of London
Malet Street
London WC1E 7HX

ABSTRACT

It is shown that the N=2 superconformal transformations are restricted N=1 supergauge transformations of a supergauge theory with Osp(2,2) as a gauge group. Based on this result, a canonical derivation of the Osp(2,2) current algebra in the superchiral gauge formulation of N=2 supergravity is presented.
1. Introduction

The importance of two dimensional (2d) gravity and supergravity is crucial in providing an understanding of non-critical string theories and lattice models formulated on random surfaces. In the continuum formulation of these models, it became evident from the work of Polyakov et al. [1], that a further insight into the quantization of induced 2d quantum gravity is obtained if one employs a particular gauge, known as the chiral gauge. This particular choice of gauge led Polyakov to discover that the theory possesses a gauge symmetry based upon the non-compact group SL(2,R), a result which was obtained by an explicit calculation of the correlation functions for the gravitational field surviving the chiral gauge. The extension of these results to the cases of both N=1 and N=2 2d supergravity has also been performed [1,2,3,4,5].

It has now been established that the appearance of the SL(2,R) symmetry in induced 2d quantum gravity is connected to the fact that the structure of conformal symmetry exhibits a hidden SL(2,R) current algebra symmetry [6,7,8].

In [6], it was demonstrated that diffeomorphisms can be obtained from restricted SL(2,R) gauge transformations. One starts with a two dimensional gauge theory described by the gauge fields $A_z \lambda a$ and $A_{\bar{z}} \lambda a^*$ where $a$ takes values in the set \{+,-,0\} and is the SL(2,R) group index. Now partially fix a gauge by imposing the three conditions,

$$A_z \lambda^+ = 1, \quad A_z \lambda 0 = 0, \quad A_z \lambda^- = T. \quad (1)$$

In this gauge the residual gauge transformation of $T$ becomes the action of Virasoro algebra on the spin-2 stress energy tensor and therefore the dynamics of the restricted gauge theory describes the geometric quantization of the Virasoro algebra [17]. The reason why the gauge field $A_z \lambda^-$ becomes a spin two field is because in

* we parametrize the two dimensional space time with coordinates $(x,t)$ by \( z = t + x \) and \( \bar{z} = t - x \).
the “background field” $A_{\lambda+} = 1$, the internal isotopic space becomes equivalent to a two dimensional space-time [6]. This geometrical observation can then be employed to explain the relationship between the Wess-Zumino-Novikov-Witten (WZNW) action [9,10,11,12] with the group SL(2,R) and that of Polyakov’s two dimensional gravitation action. An equivalent analysis has been performed in [7,8] using the method of Hamiltonian reduction.

In [14] the results of [6] were generalized to the case of N=1 superconformal symmetry. It was shown that N=1 superdiffeomorphisms can be obtained from restricted N=1 Osp(1,2) supergauge transformations. By exploiting this, the relationship existing between induced N=1 2d supergravity and N=1 Osp(1,2) WZNW is analysed.

In this paper, the case of N=2 superconformal symmetry will be considered. It will be shown that the N=2 superconformal symmetry [19] can be obtained from restricting the supergauge transformations of a (1,0) supergauge theory with Osp(2,2) as a gauge group. Based on this result, the method of [6,14] is generalized to the case of induced (2,0) 2d supergravity. We also comment on the case of induced (2,2) 2d supergravity. The case of N=2 2d supergravity is of particular interest since unlike the case of N=0 and N=1 2d supergravity, its chiral gauge formulation is valid for any space-time dimension. Thus it is relevant to the study of supersymmetric four dimensional noncritical strings.

This work is organised as follows. In section 2, the formulation of induced (2,0) 2d supergravity in the superchiral gauge [3,4] is reviewed. The relationship between the action constructed on the coadjoint orbit (of purely central extension) of the N=2 superVirasoro group [15] and that of induced (2,0) 2d supergravity in the superchiral gauge is also presented. In section 3, a (1,0) supergauge theory with the gauge group Osp(2,2) is considered. By fixing a partial gauge, a reduced theory is obtained which has the N=2 superconformal symmetry. The relationship between the induced (2,0) 2d supergravity action in the superchiral gauge formulation and the geometric action of N=2 superVirasoro group is then derived, providing a
canonical derivation of the Osp(2,2) current algebra symmetry in induced (2,0) 2d supergravity. In addition, a composition formula for the geometric action of the N=2 superconformal group is also derived. This is the (2,0) supergravitational analogue of the Polyakov-Wiegmann identity of the WZNW model [12]. In the last section, we discuss our results and suggest a relationship linking, respectively, the super current algebras Osp(1,2) and Osp(2,2) with the N=1, 2 superconformal algebras.

2. (2,0) SUPERGRAVITY IN THE SUPERCHIRAL GAUGE

In this section, the formulation of induced (2,0) 2d supergravity in the superchiral gauge is reviewed. The (2,0) superspace is described by two Grassmann coordinates $(\theta^{\lambda+1}, \theta^{2\lambda+})$, which can be combined into a single complex Grassmann coordinate, $\theta$. The (2,0) superspace coordinates are thus given by the set $(z, \bar{z}, \theta, \bar{\theta})$. The rigid (2,0) supersymmetry algebra can then be described as,

$$\{D_\theta, D_{\bar{\theta}}\} = 2\partial_z, \quad \{D_\theta, D_\theta\} = \{D_{\bar{\theta}}, D_{\bar{\theta}}\} = [D_\theta, \partial_z] = [D_{\bar{\theta}}, \partial_{\bar{z}}] = 0, \quad (2)$$

where $D_\theta = \partial_\theta + \bar{\theta}\partial_z$, $D_{\bar{\theta}} = \partial_{\bar{\theta}} + \theta\partial_z$. In curved superspace, the supercovariant derivatives are given by,

$$\nabla_A = E_\lambda M_A D_M + w_A M, \quad (3)$$

where $E_A \lambda M$ are the vielbeins, $w_A$ are the spin connections and $M$ is the Lorentz generator. The constraints in (2,0) 2d supergravity [18] are

$$\{\nabla_\theta, \nabla_{\bar{\theta}}\} = 2\nabla_z, \quad \{\nabla_\theta, \nabla_\theta\} = \{\nabla_{\bar{\theta}}, \nabla_{\bar{\theta}}\} = 0,$$

$$\nabla_\theta, \nabla_{\bar{\theta}}\} = iG_{\bar{\theta}} D_\theta = 2\Sigma \lambda \bar{\theta} M, \quad (4)$$

$$\nabla_{\bar{\theta}} = \Sigma \lambda \theta \nabla_\theta + \Sigma \lambda \bar{\theta} \nabla_{\bar{\theta}} + RM,$$

where $\Sigma \lambda \bar{\theta}$ and $R$ are the superfields whose first components are the supercovariant field strengths for the component gravitino and graviton respectively. In order for
the constraints to satisfy the Bianchi identities, the following relations must hold,

\[ \nabla_\theta G_{\bar{z}} = \Sigma \lambda \bar{\theta}, \quad \nabla_\theta \Sigma \lambda \bar{\theta} = 0, \]
\[ \nabla_\theta \Sigma \lambda \theta + \nabla_{\bar{\theta}} \Sigma \lambda \bar{\theta} = R, \quad \nabla_\theta R = 2 \nabla_{\bar{z}} \Sigma \lambda \bar{\theta}. \]

(5)

In solving the constraints and choosing the superchiral gauge, it was found [3,4] that the theory can only be described in terms of the unconstrained superfield \( H_{\bar{z}z} \) and that the solution of the constraints is given by

\[ \nabla_\theta = D_\theta, \quad \nabla_{\bar{\theta}} = D_{\bar{\theta}}, \quad \nabla_{\bar{z}} = \partial_{\bar{z}}, \]
\[ \nabla_{\bar{z}} = \partial_{\bar{z}} + \frac{1}{2}(D_{\theta} H_{\bar{z}z}) D_{\bar{\theta}} + \frac{1}{2}(D_{\bar{\theta}} H_{\bar{z}z}) D_{\theta} + H_{\bar{z}z} \partial_{\bar{z}} + \partial_{\bar{z}} H_{\bar{z}z} M, \]
\[ R = \partial \lambda 2 \bar{z} H_{\bar{z}z}, \quad \Sigma \lambda \bar{\theta} = \frac{1}{2} \partial_{\bar{z}} D_{\theta} H_{\bar{z}z}, \quad G_{\bar{z}} = \frac{1}{4i} [D_{\theta}, D_{\bar{\theta}}] H_{\bar{z}z}. \]

(6)

The equation of motion of the superfield \( H_{\bar{z}z} \) is derived by using the anomaly equation [3,4],

\[ \partial_{\bar{z}} [D_{\theta}, D_{\bar{\theta}}] H_{\bar{z}z} = 0. \]

(7)

Expanding \( H_{\bar{z}z} \) in terms of a set of fields \( J_{\lambda} a \) as,

\[ H_{\bar{z}z} = z \lambda 2 J_{\lambda} - 2z J_{\lambda 0} + J_{\lambda 1} - \theta z J_{\lambda} - \frac{1}{2} - \bar{\theta} z J_{\lambda} - \frac{1}{2} + \theta J_{\lambda} \frac{1}{2} + \bar{\theta} J_{\lambda} \frac{1}{2} + i \theta \bar{\theta} J_{\lambda} U(1), \]

(8)

and then calculating the Ward identities of the theory in terms of the fields \( J_{\lambda a} \), it was demonstrated [3,4] that the theory possesses an associated Osp(2,2) current algebra.

Before discussing the relation of \((2, 0) 2d\) supergravity in the superchiral gauge to the coadjoint orbits of the N=2 superVirasoro group, we briefly review the construction of dynamical systems having the coadjoint orbit [16,17] of a Lie group as a phase space. On the coadjoint orbits of a Lie group \( G \), a \( G \)-invariant symplectic structure can be defined [16], that is, there exists a natural antisymmetric bilinear form which is both closed and nondegenerate. This symplectic structure can be
constructed as follows. An element \( u \) of the Lie algebra \( \mathcal{G} \) of \( G \) maps a coadjoint vector \( a \) of the smooth dual space \( \mathcal{G}\lambda^* \) to the coadjoint vector \( u(a) \) defined by

\[
(u(a))(v) = -a([u, v]); \quad \forall v \in \mathcal{G}, \quad a \in \mathcal{G}\lambda^*. \tag{9}
\]

Fix a covector \( b \) and represent by \( W_b \), the orbit of \( b \) obtained by the action of \( G \) on \( b \). Let \( a \) and \( a' \) be coadjoint vectors in \( W_b \), tangent to the orbit at \( b \), being reached by applying an infinitesimal group transformation at \( b \). Thus, there exist two vectors \( u \) and \( u' \) satisfying,

\[
u(b) = a; \quad u'(b) = a'. \tag{10}\]

The symplectic 2-form \( \omega \), is given by [16],

\[
\omega(a, a') = b([u, u']). \tag{11}
\]

The action \( S \) of a dynamical system defined on \( W_b \) can now be constructed by integrating the two form \( \omega \) over a two dimensional submanifold \( \Sigma \) of the coadjoint orbit,

\[
S = \int_{\Sigma} b([u, u']). \tag{12}
\]

The above algorithm has been used to construct actions on the coadjoint orbits of the Kac-Moody and Virasoro groups and their supersymmetric extensions. (For a review see [15] and references therein.)

In the case of the \( N=2 \) superVirasoro group, the elements of the group are given by the superdiffeomorphisms \( X \) and \( \Theta \), satisfying the chirality and the superconformal conditions,

\[
D_\theta \Theta = 0, \quad D_{\bar{\theta}} \bar{\Theta} = 0, \quad D_\theta X = \Theta D_\theta \bar{\Theta}, \quad D_{\bar{\theta}} X = \bar{\Theta} D_{\bar{\theta}} \Theta. \tag{13}
\]

The action constructed on the coadjoint orbit of purely central extension is given
by

\[ S\lambda(2,0)_{s,\text{vir}} = \int d\lambda z d\lambda 2\theta \left( \frac{\partial z \Theta \partial \bar{z} \bar{\Theta} - \partial z \bar{\Theta} \partial \bar{z} \Theta}{(D_\theta \Theta)(D_\bar{\theta} \bar{\Theta})} \right). \]  

(14)

The relation of the induced \((2,0)\) 2d supergravity action, when formulated in the superchiral gauge to (14) becomes transparent if one parametrizes the superfield \(H_{\bar{z}z}\) as

\[ H_{\bar{z}z} = \frac{\partial z f + \bar{\psi} \partial \bar{z} \psi + \psi \partial z \bar{\psi}}{\partial z f + \psi \partial \bar{z} \psi + \bar{\psi} \partial z \psi}, \]  

(15)

where \(f\) and \(\psi\) are respectively Bose and Fermi \((2,0)\) superfields satisfying the same conditions as \(X\) and \(\Theta\), i.e.,

\[ D_\theta \psi = 0, \quad D_\bar{\theta} \bar{\psi} = 0, \quad D_\theta f = \psi D_\theta \bar{\psi}, \quad D_\bar{\theta} f = \bar{\psi} D_\bar{\theta} \psi. \]  

(16)

Under the infinitesimal \((2,0)\) superdiffeomorphisms,

\[ z \rightarrow z + \delta z, \quad \theta \rightarrow \theta + \delta \theta, \quad \bar{\theta} \rightarrow \bar{\theta} + \delta \bar{\theta}, \]  

(17)

the transformation of the new superfields are given by

\[ \delta \psi = \mathcal{E} \lambda z \partial z \psi + \frac{1}{2} D_\theta \mathcal{E} \lambda z D_\bar{\theta} \psi, \]

\[ \delta \bar{\psi} = \mathcal{E} \lambda z \partial \bar{z} \bar{\psi} + \frac{1}{2} D_\bar{\theta} \mathcal{E} \lambda z D_\theta \psi, \]  

\[ \delta f = \mathcal{E} \lambda z \partial z f + \frac{1}{2} D_\theta \mathcal{E} \lambda z D_\bar{\theta} f + \frac{1}{2} D_\bar{\theta} \mathcal{E} \lambda z D_\theta f. \]  

(18)

where \(\mathcal{E} \lambda z = \delta z + \theta \delta \bar{\theta} + \bar{\theta} \delta \theta\). Using the relations (15), (16) and (18), one can recover the transformation of the superfield \(H_{\bar{z}z}\), which is given by,

\[ \delta H_{\bar{z}z} = \partial z \mathcal{E} \lambda z + \mathcal{E} \lambda z \partial z H_{\bar{z}z} + \frac{1}{2} D_\theta \mathcal{E} \lambda z D_\bar{\theta} H_{\bar{z}z} + \frac{1}{2} D_\bar{\theta} \mathcal{E} \lambda z D_\theta H_{\bar{z}z} - \partial z \mathcal{E} \lambda z H_{\bar{z}z}. \]  

(19)

The induced \((2,0)\) 2d supergravity, can then be obtained from (14) via the following set of transformations,

\[ X(f, \bar{z}, \psi, \bar{\psi}) = z, \quad \Theta(f, \bar{z}, \psi, \bar{\psi}) = \theta, \quad \bar{\Theta}(f, \bar{z}, \psi, \bar{\psi}) = \bar{\theta}. \]  

(20)
3. SUPER GAUGE TRANSFORMATIONS AND SUPERDIFFEOMORPHISMS

In this section, we show that the action describing the dynamics of the N=2 superconformal algebra, i.e., the action (14), can be obtained by partially fixing a certain gauge in the (1, 0) supersymmetric Osp(2,2) gauge theory. The relationship between the (2, 0) 2d supergravity action to the coadjoint action is a consequence of the properties of the super WZNW model.

The orthosymplectic group Osp(2,2) is generated by four bosonic generators \{l_0, l_{-1}, l_1, l_u\} and four fermionic generators \{(l_{\frac{1}{2}})_1, (l_{\frac{1}{2}})_2, (l_{-\frac{1}{2}})_1, (l_{-\frac{1}{2}})_2\}, which are represented as follows,

\[
\begin{align*}
l_0 & = \begin{pmatrix} 1/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, & \quad l_1 & = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, & \quad l_{-1} & = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\
l_u & = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, & \quad (l_{\frac{1}{2}})_1 & = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, & \quad (l_{\frac{1}{2}})_2 & = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \\
(l_{-\frac{1}{2}})_1 & = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, & \quad (l_{-\frac{1}{2}})_2 & = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.
\end{align*}
\]

(21)

We consider a (1, 0) supersymmetric two dimensional gauge theory with Osp(2,2) as a gauge group. The (1, 0) superspace is described by the coordinates \((z, \bar{z}, \theta\lambda+)\) where \(\theta\lambda+\) is a real Grassmann coordinates. The rigid (1, 0) supersymmetry algebra can then be described as,

\[
\{D_+, D_+\} = 2\partial_z, \quad [D_+, \partial_z] = [D_+, \partial_{\bar{z}}] = 0,
\]

(22)

where \(D_+ = \partial_{\theta\lambda+} + \theta\lambda + \partial_z\). The (1, 0) supergauge theory has two sectors, a
supersymmetric left-moving sector described by the chiral spinor $A_{\theta\lambda+}$ and a right bosonic sector described by the chiral vector $A_z$. The gauge transformations of the gauge fields are given by,

$$
\delta A_{\theta\lambda+}\lambda a = D_{\theta\lambda+}\epsilon\lambda a = D_{\theta\lambda+}\epsilon\lambda a - f_{\lambda}\lambda a_{bc} A_{\theta\lambda+}\lambda b\epsilon\lambda c, \\
\delta A_z\lambda a = D_z\epsilon\lambda a = \partial_z\epsilon\lambda a - f_{\lambda}\lambda a_{bc} A_z\lambda b\epsilon\lambda c,
$$

(23)

where $f_{\lambda}a_{bc}$ are the structure constants of the Osp(2,2) algebra and $\epsilon\lambda a$ are the gauge parameters. The effective action of the gauge field $A_{\theta\lambda+}$ is given by,

$$
S(A_{\theta\lambda+}) \sim \log \text{sdet}(D_{\theta\lambda+} - A_{\theta\lambda+}).
$$

(24)

Its variation under gauge transformations is,

$$
\delta S(A_{\theta\lambda+}) = \int d\lambda 2 z d\theta \lambda + \text{str}(J_z \delta A_{\theta\lambda+}),
$$

(25)

where $J_z$ is the gauge current satisfying the anomaly equation,

$$
D_{\theta\lambda+} J_z = - k \partial_z A_{\theta\lambda+},
$$

(26)

and $k$ is a constant. Using this anomaly equation together with the gauge transformations of the gauge fields, we then obtain,

$$
\delta S(A_{\theta\lambda+}) = \int d\lambda 2 z d\theta \lambda + \text{str}(J_z D_{\theta\lambda+} \epsilon) \\
= - \int d\lambda 2 z d\theta \lambda + \text{str}(\hat{\epsilon} D_{\theta\lambda+} J_z) \\
= k \int d\lambda 2 z d\theta \lambda + \text{str}(\hat{\epsilon} \partial_z A_{\theta\lambda+}),
$$

(27)

where $\epsilon$ is a matrix gauge parameter taking values in the algebra of Osp(2,2) (and $\hat{\epsilon}$ is obtained from $\epsilon$ by multiplying its fermionic elements by a factor of minus
one). If we parametrize $A_{θλ+}$ by

$$A_{θλ+} = D_{θλ+}ggλ−1, \quad g(z, \bar{z}, θλ+) ∈ Osp(2,2), \quad (28)$$

then the action $S(A_{θλ+})$ is given by a $(1, 0)$ WZNW model $S_1(g)[20]$, with Osp(2,2) as a gauge group. Similarly one can parametrize $A_{\bar{z}} = \partial_z hhλ−1$, where $h$ is a group element of Osp(2,2) and find that the effective action of the gauge field $A_{\bar{z}}$ is also given by a $(1, 0)$ Osp(2,2) WZNW model, $S_2(h)$. The final form of the total effective action is then,

$$S_{eff}(g,h) = S_1(g) + S_2(h) - k \int dλ2zdθλ+str(D_{θλ+}ggλ−1\partial_z hhλ−1), \quad (29)$$

where the last term is added to insure gauge invariance. In terms of the new parameters, a finite gauge transformation on $A_{θλ+}$ and $A_{\bar{z}}$ is given by,

$$g → Ug, \quad h → Uh, \quad U ∈ Osp(2,2). \quad (30)$$

As the effective action is invariant under this transformation, this implies the following symmetry,

$$S_{eff}(g,h) = S_{eff}(Ug,Uh). \quad (31)$$

If we set $U = hλ−1$ or $U = gλ−1$, we can then deduce that

$$S_{eff}(g,h) = S_1(hλ−1g) = S_2(gλ−1h), \quad (32)$$

and in particular,

$$S_1(hλ−1) = S_2(h). \quad (33)$$

Finally, using (29), (32) and (33), we arrive at the $(1, 0)$ supersymmetric extension...
of the Polyakov-Weigmann composition formula [12],

\[ S_1(h\lambda-1g) = S_1(g) + S_1(h\lambda-1) - k \int d\lambda 2z d\theta \lambda + \text{str}(D_{\theta\lambda} + gg\lambda - 1 \partial_z hh\lambda - 1). \] (34)

We will now partially fix a gauge by imposing the following conditions,

\[ A_{\theta\lambda + \lambda 0} = A_{\theta\lambda + \lambda 1} = (A_{\theta\lambda + \lambda - \frac{1}{2}})_1 = 0, \quad (A_{\theta\lambda + \lambda \frac{1}{2}})_1 = 1, \]

\[ (A_{\theta\lambda + \lambda - \frac{1}{2}})_2 = \text{unfixed}, \quad A_{\theta\lambda + \lambda - 1} = \text{unfixed}. \] (35)

In explicit components, the gauge transformations of the supersymmetric left-moving part of the theory are given by,

\[ \delta A_{\theta\lambda + \lambda - 1} = D_{\theta\lambda} + \epsilon \lambda - 1 + 2(A_{\theta\lambda + \lambda - \frac{1}{2}})_1 \epsilon_2 \lambda - \frac{1}{2} + 2(A_{\theta\lambda + \lambda - \frac{1}{2}})_2 \epsilon_1 \lambda - \frac{1}{2} + A_{\theta\lambda + \lambda - 1} \epsilon_0 \lambda - 0 - A_{\theta\lambda + \lambda 0} \epsilon \lambda - 1, \]

\[ \delta A_{\theta\lambda + \lambda 1} = D_{\theta\lambda} + \epsilon \lambda - 1 - 2(A_{\theta\lambda + \lambda - \frac{1}{2}})_1 \epsilon_1 \lambda - \frac{1}{2} - 2(A_{\theta\lambda + \lambda - \frac{1}{2}})_2 \epsilon_2 \lambda - \frac{1}{2} + A_{\theta\lambda + \lambda 0} \epsilon \lambda - 1 - A_{\theta\lambda + \lambda 1} \epsilon \lambda 0, \]

\[ \delta A_{\theta\lambda + \lambda u} = D_{\theta\lambda} + \epsilon \lambda - 1 + (A_{\theta\lambda + \lambda - \frac{1}{2}})_1 \epsilon_2 \lambda - \frac{1}{2} + (A_{\theta\lambda + \lambda - \frac{1}{2}})_2 \epsilon_1 \lambda - \frac{1}{2} - (A_{\theta\lambda + \lambda + \frac{1}{2}})_1 \epsilon_2 \lambda - \frac{1}{2} - (A_{\theta\lambda + \lambda + \frac{1}{2}})_2 \epsilon_1 \lambda - \frac{1}{2}, \]

\[ \delta A_{\theta\lambda + \lambda 0} = D_{\theta\lambda} + \epsilon \lambda - 1 + 2(A_{\theta\lambda + \lambda - \frac{1}{2}})_2 \epsilon_2 \lambda - \frac{1}{2} + 2(A_{\theta\lambda + \lambda - \frac{1}{2}})_2 \epsilon_1 \lambda - \frac{1}{2} = 2(A_{\theta\lambda + \lambda - \frac{1}{2}})_2 \epsilon \lambda - 1 + 2(A_{\theta\lambda + \lambda - \frac{1}{2}})_2 \epsilon \lambda 0, \]

\[ \delta (A_{\theta\lambda + \lambda - \frac{1}{2}})_1 = D_{\theta\lambda} + \epsilon \lambda - 1 + \frac{1}{2} A_{\theta\lambda + \lambda 0} \epsilon_1 \lambda - \frac{1}{2} - \frac{1}{2} (A_{\theta\lambda + \lambda - \frac{1}{2}})_2 \epsilon \lambda 0, \]

\[ \delta (A_{\theta\lambda + \lambda - \frac{1}{2}})_2 = D_{\theta\lambda} + \epsilon \lambda - 1 - \frac{1}{2} A_{\theta\lambda + \lambda 0} \epsilon_2 \lambda - \frac{1}{2} - \frac{1}{2} (A_{\theta\lambda + \lambda - \frac{1}{2}})_2 \epsilon \lambda 0, \]

\[ \delta (A_{\theta\lambda + \lambda - \frac{1}{2}})_1 = D_{\theta\lambda} + \epsilon \lambda - 1 + \frac{1}{2} A_{\theta\lambda + \lambda 0} \epsilon \lambda - \frac{1}{2} - \frac{1}{2} (A_{\theta\lambda + \lambda - \frac{1}{2}})_2 \epsilon \lambda 0, \]

\[ \delta (A_{\theta\lambda + \lambda - \frac{1}{2}})_2 = D_{\theta\lambda} + \epsilon \lambda - 1 - \frac{1}{2} A_{\theta\lambda + \lambda 0} \epsilon \lambda - \frac{1}{2} - \frac{1}{2} (A_{\theta\lambda + \lambda - \frac{1}{2}})_2 \epsilon \lambda 0. \] (36)
In order that (35) be consistent with the gauge transformations (36), the gauge parameters must satisfy,

\[
\begin{align*}
\epsilon_1 \lambda - \frac{1}{2} & = - \frac{1}{2} D_{\theta \lambda} \partial_z \epsilon \lambda 1 - (A_{\theta \lambda+} + \lambda - \frac{1}{2}) \epsilon_2 \lambda - \frac{1}{2} + A_{\theta \lambda+} \lambda - 1 \epsilon \lambda 1, \\
\epsilon_1 \lambda - \frac{1}{2} & = \frac{1}{2} D_{\theta \lambda+} \epsilon \lambda 1, \\
\epsilon \lambda 0 & = \partial_z \epsilon \lambda 1, \\
\epsilon \lambda u & = - D_{\theta \lambda+} \epsilon_2 \lambda - \frac{1}{2} + (A_{\theta \lambda+} + \lambda - \frac{1}{2}) \epsilon \lambda 1, \\
\epsilon \lambda - 1 & = D_{\theta \lambda+} \epsilon_1 \lambda - \frac{1}{2} + A_{\theta \lambda+} \lambda - 1 \epsilon_1 \lambda - \frac{1}{2} - (A_{\theta \lambda+} + \lambda - \frac{1}{2}) \epsilon \lambda u, \\
\epsilon_2 \lambda - \frac{1}{2} & = D_{\theta \lambda+} \epsilon \lambda u - (A_{\theta \lambda+} + \lambda - \frac{1}{2}) \epsilon_1 \lambda - \frac{1}{2}. 
\end{align*}
\]  

(37)

These equations, when substituted back into the transformations of \(A_{\theta \lambda+} + \lambda - 1\) and \((A_{\theta \lambda+} + \lambda - \frac{1}{2})^2\) give,

\[
\begin{align*}
\delta A_{\theta \lambda+} + \lambda & = - \frac{1}{2} \partial_z \lambda 2 D_{\theta \lambda+} \epsilon \lambda 1 + \partial_z A_{\theta \lambda+} + \lambda - 1 \epsilon \lambda 1 + \frac{3}{2} A_{\theta \lambda+} + \lambda - 1 \partial_z \epsilon \lambda 1 + \frac{1}{2} D_{\theta \lambda+} A_{\theta \lambda+} + \lambda - 1 D_{\theta \lambda+} \epsilon \lambda 1 \\
& \quad - 2(A_{\theta \lambda+} + \lambda - \frac{1}{2}) \partial_z \epsilon_2 \lambda - \frac{1}{2} - \partial_z(A_{\theta \lambda+} + \lambda - \frac{1}{2}) \epsilon_2 \lambda - \frac{1}{2} + D_{\theta \lambda+}(A_{\theta \lambda+} + \lambda - \frac{1}{2})^2 D_{\theta \lambda+} \epsilon_2 \lambda - \frac{1}{2}, \\
\delta (A_{\theta \lambda+} + \lambda - \frac{1}{2})^2 & = - \partial_z D_{\theta \lambda+} \epsilon_2 \lambda - \frac{1}{2} + \partial_z(A_{\theta \lambda+} + \lambda - \frac{1}{2}) \epsilon \lambda 1 + (A_{\theta \lambda+} + \lambda - \frac{1}{2}) \partial z \epsilon \lambda 1 + A_{\theta \lambda+} \lambda - 1 \epsilon_2 \lambda - \frac{1}{2} \\
& \quad - \frac{1}{2} D_{\theta \lambda+}(A_{\theta \lambda+} + \lambda - \frac{1}{2})^2 D_{\theta \lambda+} \epsilon \lambda 1. 
\end{align*}
\]  

(38)

If we write

\[
\begin{align*}
G_1 + \theta \lambda + T & = - k A_{\theta \lambda+} + \lambda - 1, \\
U - \theta \lambda + G_2 & = - k (A_{\theta \lambda+} + \lambda - \frac{1}{2})^2, \\
\epsilon \lambda z + \theta \lambda + \epsilon_1 \lambda - \frac{1}{2} & = \epsilon \lambda 1, \\
\epsilon \lambda - \frac{1}{2} + \theta \lambda + \epsilon & = - 2 \epsilon \lambda - \frac{1}{2}, 
\end{align*}
\]  

(39)
then the transformations (38) in components, give us the following equations,

\[ \delta_{\varepsilon_z} T = \frac{k}{2} \partial_z \lambda \varepsilon \lambda z + \partial_z T \varepsilon \lambda z + 2T \partial_z \varepsilon \lambda z, \]
\[ \delta_{\varepsilon_1 \lambda_2} T = -\frac{1}{2} \partial_z G_1 \varepsilon_1 \lambda \frac{1}{2} - \frac{3}{2} G_1 \partial_z \varepsilon_1 \lambda \frac{1}{2}, \]
\[ \delta_{\varepsilon_2 \lambda_2} T = -\frac{1}{2} \partial_z G_2 \varepsilon_2 \lambda \frac{1}{2} - \frac{3}{2} G_2 \partial_z \varepsilon_2 \lambda \frac{1}{2}, \]
\[ \delta_{\varepsilon} T = U \partial_z \varepsilon. \]

(40)

\[ \delta_{\varepsilon_{\lambda z}} G_1 = \varepsilon \lambda z \partial_z G_1 + \frac{3}{2} G_1 \partial_z \varepsilon \lambda z, \]
\[ \delta_{\varepsilon_{1 \lambda_2}} G_1 = \frac{k}{2} \partial \lambda_2 \varepsilon_1 \lambda \frac{1}{2} + \frac{1}{2} \varepsilon_1 \lambda \frac{1}{2} T, \]
\[ \delta_{\varepsilon_{2 \lambda_2}} G_1 = + \frac{1}{2} \varepsilon_2 \lambda \lambda_2 \partial z U + U \partial_z \varepsilon_2 \lambda \frac{1}{2}, \]
\[ \delta_{\varepsilon} G_1 = + \frac{1}{2} \varepsilon G_2, \]

(41)

\[ \delta_{\varepsilon_{\lambda z}} G_2 = \varepsilon \lambda z \partial_z G_2 + \frac{3}{2} G_2 \partial_z \varepsilon \lambda z, \]
\[ \delta_{\varepsilon_{1 \lambda_2}} G_2 = -\frac{1}{2} \varepsilon_1 \lambda \lambda_2 \partial z U - U \partial_z \varepsilon_1 \lambda \frac{1}{2}, \]
\[ \delta_{\varepsilon_{2 \lambda_2}} G_2 = \frac{k}{2} \partial \lambda_2 \varepsilon_2 \lambda \frac{1}{2} + \frac{1}{2} \varepsilon_2 \lambda \lambda_2 T, \]
\[ \delta_{\varepsilon} G_2 = -\frac{1}{2} \varepsilon G_1, \]

(42)

\[ \delta_{\varepsilon_{\lambda z}} U = \varepsilon \lambda z \partial_z U + U \partial_z \varepsilon \lambda z, \]
\[ \delta_{\varepsilon_{1 \lambda_2}} U = \frac{1}{2} G_2 \varepsilon_1 \lambda \frac{1}{2}, \]
\[ \delta_{\varepsilon_{2 \lambda_2}} U = - \frac{1}{2} G_1 \varepsilon_2 \lambda \frac{1}{2}, \]
\[ \delta_{\varepsilon} U = - \frac{k}{2} \partial_z \varepsilon. \]

(43)

The equations (40), (41), (42) and (43) represent the N=2 infinitesimal superconformal transformation [19], where T is the spin-2 current generating diffeomorphisms, G_1 and G_2 are the two spin-3/2 supercurrents generating the two supersymmetries and U is a spin-1 current generating the \( \hat{U}(1) \) Kac-Moody algebra. The
extra spin acquired by \( T, G_1, G_2 \) and \( U \) is due to the fact that in the background \( (A_{\theta \lambda}, \lambda)_{1}^{1} = 1 \), the isospin \(-1/2\) is equivalent to the spin \( 1/2 \).

In the partially fixed theory, \( \delta S(A_{\theta \lambda}) \) reduces to

\[
\delta S(A_{\theta \lambda}) = k \int d\lambda 2z d\theta \lambda + \left( \epsilon \lambda 1 \partial_z A \lambda - 1 - 2 \epsilon \lambda^2 \frac{1}{2} \partial_z (A_{\theta \lambda} + \lambda - \frac{1}{2}) z \right).
\]  

(44)

With the identification (39), Eq. (44) describes the dynamics of the N=2 stress-energy tensor, \( U_z \),

\[
\delta S(U_z) = - \int d\lambda 2z \lambda 2 \epsilon \delta z U_z,
\]

(45)

where \( \delta S(U_z) \) represents the variation of \( S(U_z) \) under the N=2 superconformal transformation, parametrized by the \((2,0)\) superfield \( \epsilon \lambda z \). In addition, we can also write \( \delta S(U_z) \) as,

\[
\delta S(U_z) = \int d\lambda 2z \lambda 2 \epsilon L \bar{z} \bar{z} \delta U_z,
\]

(46)

where \( L \bar{z} \bar{z} \) is some function of \( U_z \). Comparing the above two equations and using

\[
\delta U_z = - \frac{k}{4} [D, D] \delta z \lambda z + \frac{1}{2} D \delta \epsilon \lambda z D \delta U_z + \frac{1}{2} D \delta \epsilon \lambda z D \delta U_z + \partial_z U_z \epsilon \lambda z + U_z \partial_z \epsilon \lambda z
\]

(47)

we deduce that \( L \bar{z} \bar{z} \) must satisfy the following equation,

\[
\left( \partial_z - L \bar{z} \bar{z} \partial_z - \frac{1}{2} (D \delta L \bar{z} \bar{z}) D \delta - \frac{1}{2} (D \delta L \bar{z} \bar{z}) D \delta - (\partial_z L \bar{z} \bar{z}) \right) U_z = - \frac{k}{4} [D, D] \partial_z L \bar{z} \bar{z}.
\]

(48)

Defining the action \( S(H \bar{z} \bar{z}) \) as the Legendre transform of \( S(U_z) \) \([6]\), its transformation under superdiffeomorphisms is then given by,

\[
\delta S(H \bar{z} \bar{z}) = \int d\lambda 2z \lambda 2 \theta Z_z \delta H \bar{z} \bar{z},
\]

(49)

where \( Z_z \) satisfies the following equation,

\[
\left( \partial_z - H \bar{z} \bar{z} \partial_z - \frac{1}{2} (D \delta H \bar{z} \bar{z}) D \delta - \frac{1}{2} (D \delta H \bar{z} \bar{z}) D \delta - (\partial_z H \bar{z} \bar{z}) \right) Z_z = - \frac{k}{4} [D, D] \partial_z H \bar{z} \bar{z},
\]

(50)

and the transformation of the superfield \( H \bar{z} \bar{z} \) is given by (19). Finally, we define
the combined action

\[ W(H_{\bar{z}z}, U_z) = S(H_{\bar{z}z}) + S(U_z) - \int d\lambda 2z d\lambda 2\theta H_{\bar{z}z} U_z. \]  

(51)

It can be easily checked that this combined action is invariant under the transformations given by equations (47) and (19).

We turn now to find a solution for the action \( S(H_{\bar{z}z}) \). Parametrizing \( H_{\bar{z}z} \) as

\[ H_{\bar{z}z} = \frac{\partial_{\bar{z}} f + \bar{\psi}\partial_{\bar{z}} \psi + \psi\partial_{\bar{z}} \bar{\psi}}{\partial_{\bar{z}} f + \psi\partial_{\bar{z}} \psi + \bar{\psi}\partial_{\bar{z}} \bar{\psi}}, \]

then the anomaly equation (50) is solved by

\[ Z_z = -\frac{k}{2} (S(\psi)) = -\frac{k}{2} \left( \frac{\partial_{\bar{z}} D_\theta \bar{\psi}}{D_\theta \psi} - \frac{\partial_{\bar{z}} D_{\bar{\theta}} \psi}{D_{\bar{\theta}} \bar{\psi}} - 2\partial_{\bar{z}} \psi \partial_{\bar{z}} \bar{\psi} \right), \]

(52)

where \( S(\psi) \) is the N=2 super-Schwartzian derivative [21], and the action \( S(H_{\bar{z}z}) \) is given by,

\[ S(H_{\bar{z}z}) = \frac{k}{2} S_{s.grav}(2,0)(f, \psi, \bar{\psi}). \]  

(53)

Obviously, the action of \( S(U_z) \) describes the geometric quantization of the N=2 superVirasoro algebra and is given by the action constructed on the coadjoint orbit of purely central extension of N=2 superVirasoro group,

\[ S(U_z) = -\frac{k}{2} S_{s.vir}(2,0)(\lambda)(f, \psi, \bar{\psi}). \]

(54)

where

\[ L_{z\bar{z}} = \partial_{z\bar{z}} X + \Theta \partial_{z\bar{z}} \Theta + \Theta \partial_{z\bar{z}} \Theta, \]

\[ U_z = -\frac{k}{2} \left( \frac{\partial_{\bar{z}} D_\theta \Theta}{D_\theta \Theta} - \frac{\partial_{\bar{z}} D_{\bar{\theta}} \Theta}{D_{\bar{\theta}} \Theta} - 2\partial_{\bar{z}} \Theta \partial_{\bar{z}} \Theta \right) = -\frac{k}{2} S(\Theta). \]

(55)

The action (54) is the reduced (1, 0) Osp(2,2) supergauge action. The original supergauge theory is described by a (1, 0) WZNW model which has an N=1 Osp(2,2)
current algebra in its left-moving sector, while its right-moving sector is described by a bosonic Osp(2,2) current algebra. In the reduced supergauge theory, the symmetry of the left-moving part is reduced to the N=2 superconformal symmetry, while the right-moving sector still has the current algebra symmetry.

The finite form of (47) and (19) can be represented, respectively, by

\[
\begin{align*}
X(z, \bar{z}, \theta, \bar{\theta}) &\rightarrow X(X_1, \bar{z}, \Theta_1, \bar{\Theta}_1), \\
\Theta(z, \bar{z}, \theta, \bar{\theta}) &\rightarrow \Theta(X_1, \bar{z}, \Theta_1, \bar{\Theta}_1), \\
\bar{\Theta}(z, \bar{z}, \theta, \bar{\theta}) &\rightarrow \bar{\Theta}(X_1, \bar{z}, \Theta_1, \bar{\Theta}_1), \\
f(z, \bar{z}, \theta, \bar{\theta}) &\rightarrow f(X_1, \bar{z}, \Theta_1, \bar{\Theta}_1), \\
\psi(z, \bar{z}, \theta, \bar{\theta}) &\rightarrow \psi(X_1, \bar{z}, \Theta_1, \bar{\Theta}_1), \\
\bar{\psi}(z, \bar{z}, \theta, \bar{\theta}) &\rightarrow \bar{\psi}(X_1, \bar{z}, \Theta_1, \bar{\Theta}_1),
\end{align*}
\]

which for convenience will be written as,

\[
\begin{align*}
(X, \Theta, \bar{\Theta}) &\rightarrow (X, \Theta, \bar{\Theta}) \bullet (X_1, \Theta_1, \bar{\Theta}_1), \\
(f, \psi, \bar{\psi}) &\rightarrow (f, \psi, \bar{\psi}) \bullet (X_1, \Theta_1, \bar{\Theta}_1).
\end{align*}
\]

Using this notation, \((X\lambda-1, \Theta\lambda-1, \bar{\Theta}\lambda-1)\) is defined as,

\[
(X, \Theta, \bar{\Theta}) \bullet (X\lambda-1, \Theta\lambda-1, \bar{\Theta}\lambda-1) = (z, \theta, \bar{\theta}).
\]

The invariance of the combined action \(W(H_{z,\bar{z}}, U_z)\) under superdiffeomorphisms implies the relationship,

\[
W\left((X, \Theta, \bar{\Theta}), (f, \psi, \bar{\psi})\right) = W\left((X, \Theta, \bar{\Theta}) \bullet (X_1, \Theta_1, \bar{\Theta}_1), (f, \psi, \bar{\psi}) \bullet (X_1, \Theta_1, \bar{\Theta}_1)\right).
\]

If we set \((X_1, \Theta_1, \bar{\Theta}_1) = (f\lambda-1, \psi\lambda-1, \bar{\psi}\lambda-1)\) or \((X_1, \Theta_1, \bar{\Theta}_1) = (X\lambda-1, \Theta\lambda-1, \bar{\Theta}\lambda-1)\).
, then the above equation gives

\[
W\left( (X, \Theta, \bar{\Theta}), (f, \psi, \bar{\psi}) \right) = -\frac{k}{2} S_{s.vir}(2, 0) \left( (X, \Theta, \bar{\Theta}) \cdot (f \lambda - 1, \psi \lambda - 1, \bar{\psi} \lambda - 1) \right)
\]

\[
= \frac{k}{2} S_{s.grav}(2, 0) \left( (f, \psi, \bar{\psi}) \cdot (X \lambda - 1, \Theta \lambda - 1, \bar{\Theta} \lambda - 1) \right),
\]

and in particular,

\[
-\frac{k}{2} S_{s.vir}(2, 0) \left( (X, \Theta, \bar{\Theta}) \right) = \frac{k}{2} S_{s.grav}(2, 0) \left( (X \lambda - 1, \Theta \lambda - 1, \bar{\Theta} \lambda - 1) \right).
\]

(60)

This relation is the supergravitational analogue of Eq.(33),

\[
S_1(h \lambda - 1) = S_2(h)
\]

and explains why one obtains the induced (2, 0) 2d supergravity action from the geometric action when

\[
(X, \Theta, \bar{\Theta}) \cdot (f, \psi, \bar{\psi}) = (z, \theta, \bar{\theta}).
\]

(62)

Finally, we obtain the supergravitational composition formula

\[
S_{s.vir}(2, 0) \left( (X, \Theta, \bar{\Theta}) \cdot (f \lambda - 1, \psi \lambda - 1, \bar{\psi} \lambda - 1) \right) = S_{s.vir}(2, 0) \left( (X, \Theta, \bar{\Theta}) \right)
\]

\[
+ S_{s.vir}(2, 0) \left( (f \lambda - 1, \psi \lambda - 1, \bar{\psi} \lambda - 1) \right) - \int d\lambda 2\pi d\lambda 2\theta \frac{\partial z f}{\partial \theta} + \bar{\psi} \partial z \psi + \psi \partial z \bar{\psi}) S(\Theta).
\]

(63)

This is the (2, 0) supergravitational analogue of the composition formula (34) of the (1, 0) super WZNW model.

Let us summarize what we have done. The relationship existing between the induced (2, 0) 2d supergravity in the superchiral gauge formulation to the geometric action describing N=2 superconformal group is verified. Also, the action \( S_{s.vir}(2, 0) \) is derived as a constrained supergauge theory. The constrained theory has a left-moving section with the N=2 superconformal symmetry, while the right-moving part has an Osp(2,2) current algebra. Thus our analysis provides a canonical derivation of the Osp(2,2) current algebra of the induced (2, 0) 2d supergravity theory in the superchiral gauge.
4. Discussion

The analysis of this paper can be easily generalized to the case of $(1, 1)$ Osp$(2,2)$ supergauge theory. In this case, the original theory has both its left and right-moving sectors described by an N=1 Osp$(2,2)$ current algebras. Imposing the conditions (35) on the left-moving sector gives a reduced theory with the left-moving sector having the N=2 superconformal symmetry and the right-moving sector having the N=1 Osp$(2,2)$ current algebra. The reduced action can be simply deduced by replacing $\partial \bar{z}$ by $D_{\theta \lambda -}$ in the action (54)

$$S = \frac{k}{2} \int d\lambda 2zd\lambda 2\theta d\theta \lambda - \left( \frac{\partial \bar{z} \Theta D_{\theta \lambda -} \bar{\Theta} - \partial \bar{z} \bar{\Theta} D_{\theta \lambda -} \Theta}{(D_{\theta \lambda})(D_{\theta \lambda})} \right),$$

where $D_{\theta \lambda -} = \partial \lambda - + \theta \lambda - \partial \bar{z}$, $\theta \lambda -$ is a right-moving Grassmann coordinate and $\Theta = \Theta(z, \bar{z}, \theta, \bar{\theta}, \theta \lambda -)$ satisfies the conditions (13). We suggest that the action (64) could be relevant to the case of induced $(2,2)$ 2d supergravity. However the superchiral formulation of this case and the derivation of the associated current algebra remains to be analysed.

In the introduction it was pointed out that a similar analysis to the one in [6] has been done in [8,7] using the Hamiltonian reduction method. In this method, one imposes a constraint on the phase space of the SL$(2,\mathbb{R})$ WZNW model, then the constrained model has a residual symmetry which is then used to gauge away a further degree of freedom giving a model describing the geometric action of Virasoro algebra. The same method was also used in [13] where it was shown that the N=1 and N=2 superconformal algebra has a hidden Osp$(1,2)$ and Osp$(2,2)$ current algebra respectively. However, our calculations together with the analysis of [14] suggests that the symplectic structure of N=1, 2 superconformal algebras can also be obtained via the Hamiltonian reduction from that of N=1 Osp$(1,2)$, Osp$(2,2)$ current algebra respectively. A complete analysis of this suggestion based on the free field realization [22,23] of the super current algebras will be reported on in a separate publication.
ACKNOWLEDGEMENT

I would like to thank C. Hull for useful conversations.

REFERENCES

1. A.M. Polyakov, Mod. Phys. Lett. A2 (1987) 893;
   V.G. Knizhnik, A.M. Polyakov and A.B. Zamolodchikov, Mod. Phys. Lett. A3 (1988) 819;
   A.M. Polyakov and A.B. Zamolodchikov, Mod. Phys. Lett. A3 (1988)1213;
   Lectures given by Polyakov at Les Houches summer school on Fields, Strings and Critical Phenomena (1988).

2. M.T. Grisaru and R.M. Xu, Phys. Lett. 205B (1988) 486.

3. W. A. Sabra, Int. J. of Mod. Phys. A 6, 755 (1991)

4. R. Xu, Two − Dimensional Quantum (0, 2) Supergravity, preprint UTTG-09-90

5. T. Kuramoto, Nucl. Phys B346 (1990) 527.

6. A. M. Polyakov, Int. J. Mod. Phys. A5(1990) 833.

7. A. Alekseev and Shatashvilli, Nucl. Phys B323 (1989) 719.

8. M. Bershadsky and H. Ooguri Comm. Math. Phys. 2(1989)49.

9. J. Wess and B. Zumino, Phys. Lett. 37B (1971) 95.

10. S. P. Novikov, Sov. Math. Dokl. 24 (1981) 222.

11. E. Witten, Comm. Math. Phys. 92 (1984) 455.

12. A.M. Polyakov and P.B. Wiegmann, Phys. Lett. 141B (1984) 233.

13. M. Bershadsky and H. Ooguri, Mod. Phys. Lett., 229B (1989) 374.

14. W. A. Sabra, Hidden Kac − Moody symmetry and 2D Quantum Supergravity, to appear in Nucl. phys B

19
15. G. Delius, P. van Nieuwenhuizen and V.G.J. Rodgers, Int. J. Mod. Phys. A5 (1990) 3943.

16. Elements of the Theory of Representations, A.A. Kirillov, Springer Verlag (1976).

17. E. Witten, Comm. Math. Phys. 114 (1988) 1.

18. R. Brooks, F. Muhammad and S.J. Gates, Nucl. Phys. B261 (1986) 599.

19. W. Boucher, D. Friedan and A. Kent, Phys. Lett. 172B (1986) 316; S. Nam, Phys. Lett. 172B (1986) 323; P. Di Vecchia, J.L. Petersen, M. Yu and H. B. Zhug, Phys. Lett. 174B (1986) 280; A.B. Zamolodchikov and V.A. Fateev, Zh. Eksp. Theor. Fiz. 90 (1986) 1553.

20. V.G. Kac and T. Todorov, Comm. Math. Phys. 102 (1985) 337; Y. Kazama and H. Suzuki, Nucl. Phys. B321 (1989) 232.

21. J. D. Cohen, Nucl. Phys. B284 (1987) 349.

22. M. Wakimoto, Comm. Math. Phys. 104 (1986) 605.

23. A.B. Zamolodchikov, unpublished