The influence of thermal properties of anisotropic thermoelement substrate to the temperature distribution and the value of thermal EMF

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Abstract. Temperature and the electric potential distributions in anisotropic thermoelements situated on substrates with different thermal properties are calculated. The influence of the geometric dimensions and thermophysical properties of the thermoelement and the substrate material to the electrical signal of the sensor is investigated. It is shown that at the initial stage of heating the influence of the thermal conductivity anisotropy of the thermoelement is negligible. On further heating the thermal properties of the substrate significantly affect to the temperature and potential distribution. Using a sensors with long thermoelements situated on the substrate with high thermal diffusivity can greatly simplify the processing of the measurement results.

1. Introduction
Traditionally, anisotropic thermoelements are used as thermoelectric power generators and cooling devices [1, 2]. Usually, in such devices, the temperature distribution is known and the subjects of the study are features of thermal EMF generation. Recently, thermoelements were used as sensitive elements of the heat sensors. In this case it is necessary to solve the inverse problem – to determine the heat flux based on electric signal of the sensor obtained in the experiment. In the general case it is difficult to do because of the two-dimensional distribution of temperature due to anisotropy of the thermal conductivity. However, for the same reason, under certain thermal properties of the substrate it can be achieved close to the one-dimensional temperature distribution. It greatly simplifies the processing of the output signal of this type of sensors [5]. In this paper we analyze the influence of thermal properties of substrates on the temperature distribution in the anisotropic thermoelement and the value of thermal EMF in the case of heat sensors.

The heat sensor is a battery of series-connected bismuth anisotropic thermoelements of length $1 \div 10 \text{ mm}$ fixed on mica substrate [3, 4]. They have a fast response, high sensitivity and noise immunity, and are successfully used in various thermophysical experiments [5–7].

2. Mathematical model
Consider the model of the heat sensor that consists of a rectangular anisotropic thermoelement (1) of length $l$ and thickness $h$, placed on a nonconducting substrate (2) of thickness $s$ (Fig. 1). The heat flux $q_h$ passes through the working surface of the thermoelement $y = h$. The side surfaces $x = 0, x = l$ of the thermoelement and substrate are thermal insulated. On the back surface of the substrate $y = s$ occurs the convective heat exchange with the environment $q_s = \alpha_s (T_s - T_e)$, where $\alpha_s$ – the heat...
transfer coefficient, $T_s$ and $T_e$ – the temperature of the substrate and the environment respectively. Electrical insulation boundary condition $j_n = 0$ is set on all the surfaces of thermoelement. All physical properties of the sensor and the substrate are constant.

**Figure 1.** Computational domain: (1) – anisotropic thermoelement, (2) – substrate.

Further we consider two versions of the heat sensor model. In the first version (full model) it takes into account the anisotropy of the thermoelement material properties and the temperature $T(x, y)$ and the potential $\varphi(x, y)$ are found by solving the system of equations [2]:

$$C_p \frac{\partial T}{\partial t} = \text{div} \mathbf{q}$$

where $\mathbf{q} = -\lambda \nabla T$, $\mathbf{j} = -\sigma \nabla \varphi - \sigma \alpha \nabla T$ – heat flux and current density, $T$ – temperature, $\zeta$ – electrochemical potential, $\lambda, \sigma, \alpha$ – tensor of thermal conductivity, electrical conductivity and Seebeck coefficient. The numerical solution of this the system is carried out in the COMSOL Multiphysics. The voltage $\Delta \varphi$ is determined between A and B points located on the working surface of the thermoelement $y = h$. The experimental data obtained in a stationary calibration of gradient heat flux sensor (GHFS) [3] were used to verify the model of the heat sensor. Sensor was a battery of 10 connected in series bismuth thermoelements of length $l = 2.2 \text{ mm}$ and thickness $h = 0.2 \text{ mm}$ mounted on a mica substrate. Heat flux $q_h = 100 \text{ kW/m}^2$ passed through the working surface of the sensor. The output signal of the sensor was $U = 6.8 \text{ mV}$. According to the calculation results corresponding to the conditions of this experiment, the voltage between points A and B was $\Delta \varphi_{AB} = 0.43 \text{ mV}$, and the full voltage of the battery was $\Delta \varphi = 4.3 \text{ mV}$. Difference between the calculation results and the experimental data does not exceed 35%.

In the second version of the model (simplified model) it takes into account only the anisotropy of the Seebeck coefficient while thermal conductivity of the material is isotropic ($\lambda = \lambda_{yy}$). The temperature distribution $T = T(y)$ is obtained from the solution of the one-dimensional heat transfer equation, and the voltage between points A and B is calculated by Thomson formula $\Delta \varphi_T = \alpha_{12} \Delta T \frac{l}{h}$, i.e. considered only transverse component of the thermal EMF [1].

3. **Calculation parameters**

In further calculations we consider the typical bismuth thermoelements of length $l = 2, 5, 10 \text{ mm}$ and thickness $h = 0.2 \text{ mm}$ ($l/h = 10, 25, 50$) which are located on substrates of mica and copper (thermal diffusivity is $a \approx 10^{-7} \text{ m}^2/\text{s}$ and $a \approx 10^{-4} \text{ m}^2/\text{s}$ respectively). Thermal conductivity is equal $\lambda = \left[\begin{array}{cc} 7.6 & -1.2 \\ -1.2 & 8.3 \end{array}\right] \text{ W/m} \cdot \text{K}$, electrical conductivity is equal $\sigma = \left[\begin{array}{cc} 7.9 & -0.9 \\ -0.9 & 8.5 \end{array}\right] \times 10^5 \text{ 1/Ohm} \cdot \text{m}$, Seebeck coefficient is equal $\alpha = \left[\begin{array}{cc} -72.8 & 23.9 \\ 23.9 & -87.2 \end{array}\right] \times 10^{-6} \text{ V/K}$. 


Heat flux \( q_h = 100 \, kW/m^2 \) passes through the working surface of the sensor \( y = h \). At initial state temperature of anisotropic thermoelement and substrate is \( T = T_e \). Termination condition of the calculation is the equality of heat fluxes \( q_s = q_h \). The difference between voltage \( \Delta \phi \) calculated using the full model and \( \Delta \phi_T \) obtained using the simplified model allows us to evaluate the applicability of the one-dimensional model for calculating the required temperature difference \( \Delta T = T_h - T_0 \).

4. Calculation results
Fig. 2 shows the voltages \( \Delta \phi \) and \( \Delta \phi_T \) for two ranges of time. At the initial stage of heating the temperature of thrmoelement back surface \( T_0 \) remains unchanged. For the considered thermoelements that time amounts to \( \approx 0.5 \, ms \) [8]. During which time the results of the calculation for both models are close enough (figures (a) and (b)). That means at this stage the influence of thermal conductivity anisotropy is negligible and temperature distribution is almost one-dimensional. The difference between the curves in the case of a short thermoelement is explained by the comparable contribution of the longitudinal and transverse components. In the case of a long thermoelement, the contribution of only a transverse component is decisive.

On further heating the thermal properties of the substrate material is beginning to affect to the temperature distribution in the thermoelement and to the thermal EMF. In the case of mica substrate the temperature field becomes a two-dimensional, and the longitudinal component of thermal EMF significantly increases. In the case of copper substrate the temperature distribution remains close to a one-dimensional. For short thermoelements \( \Delta \phi \) differs from \( \Delta \phi_T \) for both variants of the substrate material, and in the case of a long thermoelements the voltage curves practically coincide. At that stage the voltage of thermoelements with copper substrate is greater for all aspect ratios \( l/h \).
Figure 2. The voltage at the initial stage of heating (a, b) close to stationary thermal mode (c, d) for thermocouples with aspect ratio \( l/h = 10 \) (a, c) and \( l/h = 50 \) (b, d). Curves (1, 2) correspond to the mica substrate, and copper substrate (3, 4). Curves (1, 3) are calculated using the full model and simplified model (2, 4).

The influence of substrate thermal properties to the voltage is maintained at the steady-state thermal mode (figures (c) and (d)). In the case of mica substrate \( \Delta \varphi \) (curves 1 and 3) and \( \Delta \varphi_T \) (curves 2 and 4) are significantly different, but this difference is less for copper substrates. With the increase of the length of thermoelement, this difference decreases. It should also be noted a shorter time of the sensor signal output from the copper substrate to the quasi-stationary value.

Fig. 3 shows the ratio \( \Delta \varphi_T / \Delta \varphi \) on a sufficiently large time when the thermal mode close to stationary. In the case of mica substrates, using of a simplified model causes to significant errors even for a fairly long thermoelements. For copper substrates, this approach can significantly improve the accuracy in the case of short thermoelements.

![Graph showing the ratio of voltages](image)

Figure 3. The ratio of the voltages, calculated according to the simplified model, to the value obtained for the full model for the different aspect ratios of the thermoelement \( l/h \) at stationary thermal mode. Curve 1 corresponds to the mica substrate and a curve 2 corresponds to the copper substrate.

Fig. 4 shows the temperature and the electric potential distribution in the case of thermoelement with aspect ratio \( l/h = 10 \) at stationary thermal mode. For the same heat flux \( q_h \) passes through the working surface of the sensor the temperature distribution is different. In the case of mica substrate, the isotherms have practically same slope relative to the working surface defined by the thermal conductivity anisotropy. The copper substrate significantly influences the temperature distribution in the thermoelement, and it becomes close to one-dimensional. In this case, the influence of the thermal conductivity anisotropy affects only near ends of thermoelement.

![Temperature distribution](image)
Figure 4. Temperature in anisotropic thermoelement and substrate (a, b); electric potential in thermoelement (c, d) in the case of mica substrate (a, c) and copper substrate (b, d).

Conclusion
The studies show that at the initial stage of heating the simplified model allows to calculate the required temperature difference $\Delta T = T_h - T_0$ based on the electrical signal of heat sensor. When measuring pulsed heat fluxes with duration of $\sim 1 \text{ ms}$ it is better to use sensors with long thermoelements $l/h \approx 50$ as it increases the maximum time of applicability of the simplified model. Using the substrate with high thermal diffusivity (e.g. copper) allows to achieve close to one-dimensional temperature distribution in the thermoelement, and to apply the simplified model during the whole range of times. Also it increases the electrical signal of heat sensor at the same heat flux. The applicability of the simplified model means that there is geometric scalability effect in calculating the value of the sensor signal. It simplifies the preliminary calibration procedure. The calibration procedure should be carried out for the sensor with unit area, and then using a scale factor.

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