Neutrino Mixing in a Democratic-Seesaw-Mass-Matrix Model

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Abstract

On the basis of a seesaw-type mass matrix model for quarks and leptons, $M_f \simeq m_L M_F^{-1} m_R$, where $m_L \propto m_R$ are universal for $f = u, d, \nu$ and $\nu_e$ (up-quark-, down-quark-, neutrino- and charged lepton-sectors), and $M_F$ has a form [(unit matrix)+(democratic-type matrix)], neutrino masses and mixings are investigated. It is tried to understand a large $\nu_{\mu}-\nu_{\tau}$ mixing, i.e., $\sin^2 2\theta_{23} \sim 1$, with $m_{\nu_1} \ll m_{\nu_2} \sim m_{\nu_3}$, which has been suggested by the atmospheric neutrino data.

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1. Introduction

The Kamiokande collaboration [1] has recently suggested a possibility of a large neutrino mixing $\nu_\mu - \nu_x$, $\sin^2 2\theta \simeq 1$, with $\Delta m^2 \simeq 1.8 \times 10^{-2} \text{eV}^2$ for $x = e$ ($x = \tau$) from their atmospheric neutrino data. Although their conclusion is still controversial [2], it seems to be worth while to take it seriously. On the other hand, the solar neutrino data [3] with the Mikheyev-Smirnov-Wolfenstein (MSW) effect [4] have suggested a neutrino mixing $\sin^2 2\theta \simeq 7 \times 10^{-3}$ with $\Delta m^2 \simeq 6 \times 10^{-6} \text{eV}^2$. What is of great interest to us is whether we can give a satisfactory explanation of both the data, [1] and [3], on the basis of an extension of a successful quark mass matrix model to the neutrino sector.

Recently, based on a seesaw-type quark mass matrix model [5], Fusaoka and the author [6] have proposed a quark mass matrix model which can naturally understand the observed facts $m_t \gg m_b$ and $m_u \sim m_d$, without bringing such a parameter as a parameter in $M_u$ takes extremely large value compared with that in $M_d$. They have assumed vector-like heavy fermions $F_i$ in addition to conventional quarks and leptons $f_i$ ($i = 1, 2, 3$) [$f = u$ (up-quarks), $f = d$ (down-quarks), $f = \nu$ (neutrinos) and $f = e$ (charged leptons)]. These fermions belong to $F_L = (1, 1), F_R = (1, 1), f_L = (2, 1)$, and $f_R = (1, 2)$ of SU(2)$_L \times$SU(2)$_R$, respectively. The mass matrix for $(f, F)$ is given by a $6 \times 6$ matrix

$$ M = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} = m_0 \begin{pmatrix} 0 & Z \\ \kappa Z & \lambda O_f \end{pmatrix}, \quad (1.1) $$

where the chiral symmetry breaking terms $m_L$ and $m_R$ are assumed to be $m_L \propto m_R$ and they have a universal structure $Z$ for quarks and leptons $f$ (= $u, d, \nu, e$),

$$ Z = \begin{pmatrix} z_1 & 0 & 0 \\ 0 & z_2 & 0 \\ 0 & 0 & z_3 \end{pmatrix}, \quad (1.2) $$

where $z_i$ are normalized as $z_1^2 + z_2^2 + z_3^2 = 1$. The heavy fermion mass matrix $M_F = m_0 \lambda O_f$ has a structure [7] of [(unit matrix)+(a democratic-type matrix)] and it includes only one complex parameter $b_f e^{i\beta_f}$ which depends on $f = u, d, \nu, e$:

$$ O_f = 1 + 3b_f e^{i\beta_f} X, \quad (1.3) $$
where $1$ are a $3 \times 3$ unit matrix and $X$ is a democratic-type matrix [8]

$$X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$ (1.4)

The mass matrix (1.1) leads to the well-known seesaw form $M_f \simeq m_L M_F^{-1} m_R$ for $\text{Tr} M_F \gg \text{Tr} m_R, \text{Tr} m_L$. Note that the inverse matrix of $O_f$ again takes the form $[(\text{unit matrix})+(\text{democratic-type matrix})]$,

$$O_f^{-1} = 1 + 3 a_f e^{i \alpha_f} X ,$$ (1.5)

with

$$a_f e^{i \alpha_f} = -\frac{b_f e^{i \beta_f}}{1 + 3 b_f e^{i \beta_f}} .$$ (1.6)

The limit $b_f e^{i \beta_f} \rightarrow -1/3$ leads to $|a_f| \rightarrow \infty$. Therefore, a slight difference between $b_u$ and $b_d$ around $b_f \simeq -1/3$ can induce an extremely large difference between $m_t$ and $m_b$. On the other hand, we can keep $m_u \sim m_d$ because the democratic mass matrix makes only the third family heavy. Thus, they [6] have given a natural explanation of the observed facts $m_t \gg m_b$ and $m_u \sim m_d$.

In order to fix the parameters $z_i$, they have assumed that $b_e = 0$, i.e.,

$$M_e \simeq m_0 \frac{\kappa}{\lambda} Z^2 ,$$ (1.7)

so that $z_i$ are given by

$$\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}} .$$ (1.8)

By taking $\kappa/\lambda = 0.02$, $b_u = -1/3$ ($\beta_u = 0$) and $b_d = -1$ ($\beta_d = -18^\circ$), they have obtained reasonable quark mass ratios and Kobayashi-Maskawa (KM) [9] matrix parameters.

In their model, the variety of the quark and lepton mass matrices come form the variety of the corresponding heavy fermion mass matrices which are characterized by the parameter $b_f e^{i \beta_f}$. They have concluded that the parameter values $b_u = -1/3$, $b_d = -1$ and $b_e = 0$ are favorable to the observed mass spectra and mixings. However, why the nature chooses such values of $b_f$ is an open question. In order to obtain a clue to such a question, in the present paper, we investigate
what value of $b_\nu$ is required from the phenomenological study of neutrino masses and mixings.

2. Neutrino mass matrix

In the model in Ref. [6], the mass matrices of the charged leptons and quarks have been given by (1.1). In order to understand why neutrino masses are so negligibly small, we must consider that a value of the parameter $\lambda$ in (1.1) in neutrino sector takes extremely large value compared with those in charged lepton and quark sectors, or that a extremely large Majorana mass term causes the so-called seesaw mechanism [10] doubly. The former case is not natural from the standpoint of the unified description of quark and lepton mass matrices. For the latter case, two possibilities are considered: one is that the heavy neutrinos $N_{Li}$ and $N_{Ri}$ have large Majorana masses $M_{Mi}$, and another is that the right-handed neutrinos $\nu_{Ri}$ have large Majorana masses $M_{Mi}$. Roughly speaking, for $\text{Tr}M_M \gg \text{Tr}M_D$ (for convenience, we denote the Dirac masses $M_F$ in (1.1) as $M_D$), the former and latter cases lead to mass matrices for the left-handed neutrinos $\nu_{Li}$,

$$M_{\nu_i} \simeq -(1/2)^2 m_L M_M^{-1} m_T L ,$$

and

$$M_{\nu_i} \simeq -(1/2)^4 m_L M_D^{-1} m_R M_M^{-1} m_T R (M_D^{-1})^{-1} m_T L ,$$

respectively. In the former case, in order to give neutrino mixings, we must consider some structure of $M_M$, which may be independent of that of $M_D$, so that the mass matrix $M_{\nu_i}$ cannot be related to the mass matrices of charged leptons and quarks. In the present paper, we investigate the latter possibility.

The 6×6 mass matrix which is sandwiched by $(\nu_L, \nu_R, N_L, N_R)$ and $(\nu_L^c, \nu_R^c, N_L^c, N_R^c)^T$ is given by

$$M = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} m_L \\ 0 & M_M & \frac{1}{2} m_T R & 0 \\ 0 & \frac{1}{2} m_R & 0 & M_D \\ \frac{1}{2} m_T L & 0 & M_D^T & 0 \end{pmatrix} ,$$

so that the 3×3 light-neutrino mass matrix is given by (2.2). We assume that $M_M$ is simply given by $M_M = m_0 \xi 1$, while $M_D$ is given by a universal structure $M_D = m_0 \lambda O_f = m_0 \lambda (1 + 3b_\nu e^{i\beta} X)$ as well as those in quark sectors. Then, we obtain

$$M_{\nu_i} \simeq \frac{1}{16} \frac{\kappa^2 m_0}{\lambda^2 \xi} Z O^{-1}_\nu Z \cdot Z O^{-1}_\nu Z .$$
In Fig. 1, we illustrate the behavior of the neutrino masses versus the parameter \( b_\nu \), which is similar to that of the quark masses (see Fig. 1 in Ref. [6]). For the case of \( \beta_\nu = 0 \), at \( b_\nu = -1/2 \) (\( b_\nu = -1 \)), the mass levels \( m_{\nu_3} \) and \( m_{\nu_2} \) (\( m_{\nu_3} \) and \( m_{\nu_1} \)) degenerate each other. Therefore, we can expect that large neutrino mixings occur at \( b_\nu = -1/2 \) and \( b_\nu = -1 \). For the case of \( \beta_\nu \neq 0 \), the degeneracies between \( m_{\nu_i} \) and \( m_{\nu_j} \) disappear, so that the large mixings \( \sin^2 2\theta_{ij} \approx 1 \) become mild.

### 3. Masses and mixings for typical three cases of \( b_\nu \)

Let us show the neutrino masses \( m_i \) and mixing matrix \( U_{\nu L} \) for typical three cases of \( b_\nu \): \( b_\nu \approx -1/3 \), \( b_\nu \approx -1/2 \) and \( b_\nu \approx -1 \). Here, the mixing matrix \( U_{\nu L} \) is defined by

\[
\nu_\alpha = \sum_{i=1}^{3} (U_{\nu L})_{\alpha i} \nu_i ,
\]

where \( \nu_\alpha (\alpha = e, \mu, \tau) \) are flavor eigenstates and \( \nu_i (i = 1, 2, 3) \) are mass eigenstates. For simplicity, we consider the case of \( \beta_\nu = 0 \). Then, we obtain the following approximate expressions:

\[
m_{\nu_1} \approx \left( \frac{3}{4} \frac{m_e}{m_\tau} \right)^2 m_0^\nu , \quad m_{\nu_2} \approx \left( \frac{m_\mu}{m_\tau} \right)^2 m_0^\nu , \quad m_{\nu_3} \approx \left( \frac{\sqrt{2}}{27|\varepsilon|} \right)^2 m_0^\nu ,
\]

(3.2)

\[
U_{\nu L} \approx \begin{pmatrix}
1 & \frac{1}{2} \sqrt{m_e/m_\mu} & \frac{1}{2} \sqrt{m_e/m_\tau} \\
-\frac{1}{2} \sqrt{m_e/m_\mu} & 1 & -\frac{1}{2} \sqrt{m_\mu/m_\tau} \\
-\frac{1}{2} \sqrt{m_e/m_\tau} & \frac{1}{2} \sqrt{m_\mu/m_\tau} & 1
\end{pmatrix} ,
\]

(3.3)

for \( b_\nu \approx -1/3 \) (\( \varepsilon \equiv b_\nu + 1/3 \)),

\[
m_{\nu_1} \approx \left( \frac{m_e}{m_\tau} \right)^2 m_0^\nu , \quad m_{\nu_2} \approx m_{\nu_3} \approx \left( \frac{1}{2} \sqrt{m_\mu/m_\tau} \right)^2 m_0^\nu ,
\]

(3.4)

\[
U_{\nu L} \approx \begin{pmatrix}
1 & \frac{1}{\sqrt{2}} \left( \sqrt{m_\mu/m_\mu} + \eta \sqrt{m_\mu/m_\tau} \right) & \frac{1}{\sqrt{2}} \left( \sqrt{m_\mu/m_\mu} - \eta \sqrt{m_\mu/m_\tau} \right) \\
-\sqrt{m_e/m_\mu} & \frac{1}{\sqrt{2}} & -\frac{\eta}{\sqrt{2}} \\
-\sqrt{m_e/m_\tau} & \frac{\eta}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix} ,
\]

(3.5)

for \( b_\nu \approx -1/2 \), and

\[
m_{\nu_1} \approx m_{\nu_2} \approx \left( \frac{1}{2} \sqrt{m_e m_\mu/m_\tau^2} \right)^2 m_0^\nu , \quad m_{\nu_3} = \left( \frac{1}{4} \right)^2 m_0^\nu ,
\]

(3.6)
\[ U_{\nu L} \approx \left( \begin{array}{ccc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\eta \frac{1}{\sqrt{2}} \\ \eta \frac{1}{\sqrt{2}} \left( m_\mu / m_\tau \right) & \frac{1}{\sqrt{2}} \left( m_e / m_\tau \right) & -\sqrt{m_\mu / m_\tau} \\ \eta \frac{1}{\sqrt{2}} \left( m_\mu / m_\tau \right) & -\sqrt{m_e / m_\tau} & -\sqrt{m_\mu / m_\tau} \end{array} \right), \] 

for \( b_\nu \approx -1 \), where \( m_0^\nu \) is defined by

\[ m_0^\nu = \left( \frac{\kappa}{2\lambda} \right)^2 \frac{m_0}{\xi}. \]  

Here, in (3.5) [(3.7)], the factor \( \eta \) is defined as \( \eta = \pm 1 \) for \( b_\nu = b_{23}^{0 \pm} \) and \( \sim -1/2 \) \( (1 \gg \varepsilon > 0) \) \( b_\nu = b_{23}^{0 \pm} \) \( \varepsilon \sim -1 \) \( (\varepsilon > 0) \), where \( b_{23}^{0 \pm} \) is the value of \( b_\nu \) at which the values of \( m_2 \) and \( m_3 \) exactly degenerate. As shown in (3.5) and (3.7), the mixing elements \( (U_{\nu L})_{a2} \) and \( (U_{\nu L})_{a3} \) \( (|U_{\nu L}|_{a1} \text{ and } (U_{\nu L})_{a2} \text{ } \right) \) are exchanged each other at \( b_\nu = b_{23}^{0 \pm} \) \( b_\nu = b_{12}^{0 \pm} \), because the mass levels of \( m_2 \) and \( m_3 \) \( \nu_1 \text{ and } \nu_2 \) cross each other at \( b_\nu = b_{23}^{0 \pm} \) \( b_\nu = b_{12}^{0 \pm} \) as seen in Fig. 1.

The result (3.3) for the case \( b_\nu \sim -1/3 \) has been reported in Ref. [11]. The mixing matrix element \( U_{\nu e} \equiv \sin \theta_{e2} \) leads to \( \sin^2 2\theta_{e2} \approx m_e / m_\mu = 4.8 \times 10^{-3} \), which is in good agreement with the MSW solution of solar neutrino data [3] \( \sin^2 2\theta \approx 7 \times 10^{-3} \). However, in this paper, we will direct our attention to the atmospheric neutrino data [1] as well as the solar neutrino data [3].

4. Numerical study

We consider that the atmospheric neutrino data [1] show \( \nu_\mu - \nu_\tau \) mixing, while the solar neutrino data [3] show \( \nu_e - \nu_\mu \) mixing.

For reference, in Fig. 2, we illustrate \( \Delta m_{21}^2 \equiv m_{\nu_2}^2 - m_{\nu_1}^2 \) versus \( \sin^2 2\theta_{e2} \equiv 4|U_{e2}|^2(1 - |U_{e2}|^2) \) and \( \Delta m_{32}^2 \equiv m_{\nu_3}^2 - m_{\nu_2}^2 \) versus \( \sin^2 2\theta_{\mu 3} \equiv 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) \) in the case of \( \beta_\nu = 0 \). Note that the value of \( \sin^2 2\theta_{e2} \) is discontinuous at \( b_\nu \sim -1/2 \) because the value of \( b_\nu \) crosses the value \( b_{0 \pm} \sim -1/2 \).

We interests in the case of \( b_\nu \sim -1/2 \), because the case yields \( \sin^2 2\theta_{\mu 3} \approx 1 \) with \( m_{\nu_1} \ll m_{\nu_2} \approx m_{\nu_3} \). In Fig. 3, we illustrate the behaviors of \( \sin^2 2\theta_{e2} \) and \( \sin^2 2\theta_{\mu 3} \) versus \( b_\nu \). For reference, we also illustrate the ratio \( \Delta m_{32}^2 / \Delta m_{21}^2 \) in the figure. The observed values \( \Delta m_{32}^2 \approx 1.6 \times 10^{-2} \text{ eV}^2 [1] \) and \( \Delta m_{21}^2 \approx 6 \times 10^{-6} \text{ eV}^2 [3] \) give the ratio \( \Delta m_{32}^2 / \Delta m_{21}^2 \approx 2.7 \times 10^3 \). As seen in Fig. 3, there is no solution which gives \( \sin^2 2\theta_{\mu 3} \approx 1 \), \( \sin^2 2\theta_{e2} \approx 0.007 \) and \( \Delta m_{32}^2 / \Delta m_{21}^2 \approx 3 \times 10^3 \) simultaneously. If we reduce the requirement of the maximal mixing \( \sin^2 2\theta_{\mu 3} \approx 1 \), for example, to \( \sin^2 2\theta_{\mu 3} \approx 0.4 \), we can find satisfactory solutions of \( b_\nu \). For the case of \( \beta_\nu = 0 \), the choice \( b_\nu \sim -0.41 \) can give the plausible values of \( \sin^2 2\theta_{\mu 3}, \sin^2 2\theta_{e2} \)
and $\Delta m_{32}^2/\Delta m_{21}^2$ as seen in Fig. 3. For the case of $\beta_\nu \neq 0$, we take $b_\nu = -1/2$ by way of trial, because the value is a simple fractional number which gives $b_\nu \sim -0.5$. Then, the choice $\beta_\nu \simeq 22^\circ$ can give favorable predictions. We list numerical results for some special cases of $(b_\nu, \beta_\nu)$ in Table 1.

In Table 1, the values $\xi m_0$ have been estimated as follows: from (1.7), we obtain

$$m_0 \kappa/\lambda = m_\tau + m_\mu + m_e = 1.883 \text{ GeV},$$

so that from the definition (3.8), we obtain

$$\xi m_0 = (m_\tau + m_\mu + m_e)^2/4m_\nu^\nu.$$  \hspace{1cm} (4.1)

Here, the values of $m_\nu^\nu$ have been obtained from $(\Delta m_{21}^2)_{\text{theory}}/(\Delta m_{21}^2)_{\text{input}}$ with $(\Delta m_{21}^2)_{\text{input}} = 6 \times 10^{-6} \text{ eV}^2$. We find that the Majorana masses of $\nu_R$ are of the order of $10^9 \text{ GeV}$.  

5. Discussions

As seen in Fig. 3 and Table 1, if we want a solution which gives the largest possible $\nu_\mu$-$\nu_\tau$ mixing with $\Delta m_{32}^2 \geq 10^{-2} \text{ eV}^2$ (for the input $\Delta m_{21}^2 = 6 \times 10^{-6} \text{ eV}^2$), the solution $b_\nu = -0.41$ with $\beta_\nu = 0$ is favorable rather than the case of $\beta \neq 0$: the mixing matrix $U_{\nu_L}$ and neutrino masses $m_{\nu_i}$ are given by

$$U_{\nu_L} = \begin{pmatrix} 0.9988 & 0.0387 & 0.0310 \\ -0.0484 & 0.9061 & 0.4203 \\ -0.0117 & -0.4212 & 0.9069 \end{pmatrix},$$  \hspace{1cm} (5.1)

$m_{\nu_1} = 2.4 \times 10^{-8} \text{ eV}$, $m_{\nu_2} = 0.0024 \text{ eV}$ and $m_{\nu_3} = 0.099 \text{ eV}$, respectively.

However, from the phenomenological study [6] of quark masses and KM mixings, we have known that the values $b_u = -1/3$ and $b_d = -1$ for the input $b_e = 0$ are favorable. If we take notice of an empirical rule that $(b_e, Q_e) = (0, -1)$, $(b_d, Q_d) = (-1, -1/3)$ and $(b_u, Q_u) = (-1/3, +2/3)$, where $Q_{f_i}$ is the charge of the fermions $f_{i}$, we can speculate [12] $(b_\nu, Q_\nu) = (+2/3, 0)$ for the neutrino sector. The value $b_\nu = 2/3$ with $\beta_\nu = 0$ $(\beta_\nu = \pi)$ predicts $\sin^2 2\theta_{e2} = 3.2 \times 10^{-3}$ (0.074), $\sin^2 2\theta_{\mu3} = 0.021$ (0.52) and $\Delta m_{32}^2/\Delta m_{21}^2 = 1.2 \times 10^5$ (4.1 $\times 10^3$). The predicted values of $\sin^2 2\theta_{\mu3}$ and $\Delta m_{32}^2/\Delta m_{21}^2$ in the case of $(b_\nu, \beta_\nu) = (+2/3, \pi)$ [i.e., $(b_\nu, \beta_\nu) = (-2/3, 0)$] are favorable to the observed data, but the predicted value $\sin^2 2\theta_{e2} = 0.074$ is larger by one order than the the MSW-suggested value $\sin^2 2\theta_{e2} \simeq 7 \times 10^{-3}$. If we suppose $b_\nu = 2/3$ with $\beta_\nu \simeq \pi$, we must discard the
neutrino mixing $\sin^2 2\theta_{e2} \simeq 7 \times 10^{-3}$ with $\Delta m_{21}^2 \simeq 6 \times 10^{-6}$ eV$^2$, which is suggested from the solar neutrino data. On the other hand, if we suppose $b_\nu = 2/3$ with $\beta_\nu \simeq 0$, we must discard the neutrino mixing $\sin^2 2\theta_{\mu3} \sim 1$ with $\Delta m_{32}^2 \simeq 1.6 \times 10^{-2}$ eV$^2$, which is suggested from the atmospheric neutrino data. If we want an explanation both for the atmospheric and solar neutrino data, we must accept the choice $(b_\nu, \beta_\nu) \simeq (-0.41, 0)$, but it is an open question how we understand the parameter value $b_\nu \simeq -0.41$ with $\beta_\nu \simeq 0$ from the point of view of a unified description of $b_f$ ($f = \nu, e, u, d$).

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Table 1. Numerical results for special cases of \((b_\nu, \beta_\nu)\). The input value \(\Delta m_{21}^2 \equiv 6 \times 10^{-6} \text{ eV}^2\) is used in order to fix the value of \(m_0^\nu\).

\[
\begin{array}{|c|c|c|c|c|}
\hline
(b_\nu, \beta_\nu) & (-0.41, 0^\circ) & (-0.40, 0^\circ) & (-1/2, 20^\circ) & (-1/2, 22^\circ) \\
\hline
\Delta m_{21}^2 & 6 \times 10^{-6} \text{ eV}^2 & 6 \times 10^{-6} \text{ eV}^2 & 6 \times 10^{-6} \text{ eV}^2 & 6 \times 10^{-6} \text{ eV}^2 \\
\sin^2 2\theta_{e2} & 6.1 \times 10^{-3} & 5.9 \times 10^{-3} & 1.4 \times 10^{-2} & 1.4 \times 10^{-2} \\
\hline
\Delta m_{32}^2 & 0.97 \times 10^{-2} \text{ eV}^2 & 2.7 \times 10^{-2} \text{ eV}^2 & 0.65 \times 10^{-2} \text{ eV}^2 & 1.1 \times 10^{-2} \text{ eV}^2 \\
\sin^2 2\theta_{\mu3} & 0.58 & 0.52 & 0.49 & 0.41 \\
\hline
m(\nu_1) & 2.4 \times 10^{-8} \text{ eV} & 2.6 \times 10^{-8} \text{ eV} & 7.4 \times 10^{-8} \text{ eV} & 8.2 \times 10^{-8} \text{ eV} \\
m(\nu_2) & 2.4 \times 10^{-3} \text{ eV} & 2.4 \times 10^{-3} \text{ eV} & 2.4 \times 10^{-3} \text{ eV} & 2.4 \times 10^{-3} \text{ eV} \\
m(\nu_3) & 0.099 \text{ eV} & 0.16 \text{ eV} & 0.081 \text{ eV} & 0.103 \text{ eV} \\
\hline
m_0^\nu & 0.46 \text{ eV} & 0.50 \text{ eV} & 1.25 \text{ eV} & 1.44 \text{ eV} \\
\hline
\xi m_0 & 1.9 \times 10^9 \text{ GeV} & 1.8 \times 10^9 \text{ GeV} & 0.71 \times 10^9 \text{ GeV} & 0.62 \times 10^9 \text{ GeV} \\
\hline
\end{array}
\]

Figure Captions

Fig. 1. Neutrino masses (in unit of \(m_0^\nu\)) versus the parameter \(b_\nu\). The solid and broken lines correspond to the cases \(\beta_\nu = 0\) and \(\beta_\nu = 20^\circ\), respectively.

Fig. 2. \(\Delta m_{ij}^2\) [in unit of \((m_0^\nu)^2\)] versus \(\sin^2 2\theta_{\alpha i}\): (a) \(\Delta m_{21}^2\) versus \(\sin^2 2\theta_{e2}\) and (b) \(\Delta m_{32}^2\) versus \(\sin^2 2\theta_{\mu3}\). The dots denote points \(b_\nu = +10, +1, +0.1, -0.1, -0.2, -0.3, -0.4, -0.5, -0.6, -0.7, -0.8, -0.9, -1.0, -10\).

Fig. 3. \(\sin^2 2\theta_{e2}, \sin^2 2\theta_{\mu3}\) and \(\Delta m_{32}^2/\Delta m_{21}^2\) versus the parameter \(b_\nu\). The solid and broken lines correspond to the cases \(\beta_\nu = 0\) and \(\beta_\nu = 20^\circ\), respectively.
Fig. 1

Neutrino Mass

$m_{\nu 3}$

$m_{\nu 2}$

$m_{\nu 1}$
Fig. 2
Fig. 3