On physical insignificance of null naked singularities

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Abstract

In this work we study collapse of a general matter in a most general spacetime i.e., a spacetime with any matter and without (assuming) any symmetry. We show that the energy is completely trapped inside the null singularity and therefore they cannot be experimentally observed. This most general result implies, there is no physical significance of the null naked singularities irrespective of their existence. This conclusion strongly supports the essence of cosmic censorship hypothesis.

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I. INTRODUCTION

Black holes are one of the most well studied objects in general relativity. When combined with theory, several astronomical observations exhibit strong evidence for existence of stellar mass (few times solar mass $M_\odot$) \cite{1} as well as supermassive (mass $10^5 - 10^9 M_\odot$) \cite{2} black holes. The evidence of stellar mass black holes typically comes from observations of binary system, while by recent observations the supermassive black holes are expected to exist at the center of almost every galaxy. In recent times there are some observational evidences \cite{3} for intermediate mass black holes. Existence of primordial black holes \cite{4} in the early universe is also speculated. Most of the primordial black holes are expected to have very small mass. Since the black holes have a trapped surface, singularity theorems \cite{5} guarantee a singularity inside their horizon, although, they are silent about their structural details. Singularities also form without or before formation of a trapped surface or an event horizon. If such a singularity forms, then non-spacelike geodesics come out of it and in principle the singularity can be visible to an outside i.e., nonsingular observer; therefore, it is called a naked singularity. There are speculations \cite{6} that gamma ray bursts originate from such naked singularities.

We cannot foretell laws of physics at a singularity, hence existence of a naked singularity can lead to breakdown of predictability. Normally, we do not want such a situation to arise in nature. With this in mind Penrose \cite{7} proposed the Cosmic Censorship Hypothesis (CCH). The CCH asserts that, naked singularities should not/do not form from collapse of a reasonable matter field when we start with a generic nonsingular initial data. There are many investigations supporting the CCH as well as many examples of existence of naked singularities \cite{8–36}. Normally, these investigations put several restrictions on the model. Almost all these studies assume a very specific and very restrictive form of matter. They also assume only a certain kind of metric, typically, either with spherical or axial symmetry. Therefore, they may not satisfy the genericity of initial data or reasonable matter field criteria. Hence, they do not prove or disprove the CCH. The main difficulty arises as in general we cannot establish the relationship of initial data to the formation and structure of the singularity even after assuming lot of symmetry and taking a very simplistic equation for matter. Therefore, proving or disproving the CCH remains one of the most important open problems in classical general relativity.
In such a situation we need to investigate, whether some other effects safeguard the physics. It may happen that, even if naked singularities exist geometrically, they have no physical or observational consequences. i.e. they cannot affect the world outside them and other physical effects make them benign. Naked singularities and the CCH get lot of attention and is important as general relativity is a physical theory which describes nature. Unpredictability is a serious issue for a physical theory and needs to be fixed. Therefore, it is important to find out whether a naked singularity can affect the physics outside it, i.e., whether a nonsingular observer can distinguish it observationally. With this in mind we studied the spherically symmetric dust collapse model [37] and calculated the redshift and luminosity of light rays coming out of the singularity. Subsequently, we also studied extremely wide class of (any type II matter) spherically symmetric collapse models for the same purpose. In both the cases we showed that at the most along one singular null geodesic coming out of a null (naked) singularity the redshift is finite while redshift diverges along all other outgoing singular null geodesics. Hence, we concluded that the null naked singularities will not be physically troublesome, as no energy can come out of them.

Here, we generalize the proof for any null singularity. In this work, we consider a completely general spacetime metric without any symmetries. Apart from weak energy condition [5], no restrictions on form of matter are assumed. We show that if a null singularity forms in the collapse, then at the most for one singular null geodesic the redshift is finite while it is infinite for all other (infinite family) of singular geodesics. Hence no energy or information can come out of a null naked singularity. We need a wavepacket to carry energy, thereupon, redshift should be finite for a finite (though it can be very small) duration to get out energy from the singularity. As no energy can come out of the null singularity, it cannot affect the physics outside and there should not be any danger of breakdown of predictability. This means, physically the null naked singularities are not important or they are not dangerous.

We also show that the redshift is always finite for null geodesics coming out of a timelike naked singularity. This means that, in principle such a singularity can be observed and they can be more problematic. However, as such we expect them to be very rare [37].
II. METHODOLOGY

We use the characteristic or null cone formulation. The spacetime is foliated using a family of outgoing null hypersurfaces. We use the Bondi-Sachs \[38, 39\] coordinates. These coordinates are commonly used in characteristic approach to numerical relativity \[40–43\]. The null hypersurfaces are labelled by coordinate \(u\); \(r\) is the area radius coordinate and the angular coordinates \(x^A (A = 2, 3)\) label the outgoing null rays. The metric in Bondi-Sachs coordinates \[38, 39\] is written as

\[
ds^2 = -\left(\frac{V}{r}e^{2\beta} - r^2h_{AB}U^A U^B\right)du^2 -2e^{2\beta}dudr -2r^2h_{AB}U^B dx^A + r^2h_{AB}dx^A dx^B, \tag{1}\]

where \(h^{AB}h_{BC} = \delta_A^C\) and \(det(h_{AB}) = det(q_{AB})\), with \(q_{AB}\) a unit sphere metric. All the metric variables, i.e., \(\beta, V, U^A\) and \(h_{AB}\) are functions of \((u, r, x^A)\). This metric in general does not have any symmetries and any spacetime can be written in this form. The field \(\beta\) gives expansion of light rays, \(V\) is akin to the Newtonian potential, \(h_{AB}\) gives the conformal geometry of \(r = \text{constant}\), \(u = \text{constant}\) surface (or the deviation from spherical symmetry), \(U^A\) represents the shift vector. \(e^{2\beta}V/r\) is analogous to square of the lapse function.

The Einstein equations are decomposed as constraint equations and evolution equations. The constraint equations are basically hypersurface equations. For our purpose here, we need only a couple of hypersurface equations, which are written as \[40–42\]

\[
rR_{rr} = 4\beta_{,rr} - \frac{1}{4}r h^{AC} h^{BD} h_{AB,rr} h_{CD,rr}, \tag{2}\]

\[
r^2g^{AB}R_{AB} = -2e^{-2\beta}V_{,r} + R - 2D^A D_A \beta - 2D^A \beta D_A \beta + r^{-2}e^{-2\beta} D_A (r^4 U^A)_{,r} - \frac{1}{2}r^4 e^{-4\beta} h_{AB} U^A_{,r} U^B_{,r}, \tag{3}\]

where \(D_A\) is the covariant derivative and \(R\) the curvature scalar of the 2-metric \(h_{AB}\). We do not need the evolution equations for our purpose.

At the regular center various metric quantities have to obey some regularity conditions. They are given as, \(U^A = O(r)\), \(h_{AB} = q_{AB} + O(r^2)\), \(V = r + O(r^3)\) and \(\beta = O(r^4)\). The form of \(h_{AB}\) near the center also tell us that, at the center we have \(R = 2\). As we are using the outgoing null coordinates, \(du = dx^A = 0\) represent the outgoing null geodesics. We will call these Outgoing Radial Null Geodesics (ORNGs). The geodesic tangent vector \(K^r\) is given
as

\[ K^r = \frac{dr}{d\lambda} = C_1 e^{-2\beta}, \]  

(4)

where \( \lambda \) is the affine parameter and \( C_1 \) is a constant of integration.

Let \( u^{a}_{(s)} \) and \( u^{a}_{(o)} \) be the four-velocities of the source and the observer and let \( E_1 \) and \( E_2 \) be two events connecting the source and the observer through the ORNG. The redshift \( z \) is given by \[ 1 + z = \frac{[K_a u^a_{(s)}]_{E_1}}{[K_a u^a_{(o)}]_{E_2}}, \]  

(5)

where the numerator and denominator are evaluated at events \( E_1 \) and \( E_2 \), at the source and observer respectively, with 

\[ u^{a}_{(s)} = -(e^{2\beta} V/r - r^2 h_{AB} U^A U^B)_{(s)}^{-1/2} \delta^a_u \]  

and 

\[ u^{a}_{(o)} = -(e^{2\beta} V/r - r^2 h_{AB} U^A U^B)_{(o)}^{-1/2} \delta^a_u. \]

Taking the source as the naked singularity at \( r = 0 \) or \( r = \text{constant} \) we get

\[ 1 + z = \left( \frac{V e^{2\beta} - r^2 h_{AB} U^A U^B}{} \right)_{(s)}^{1/2} \left( \frac{V e^{2\beta} - r^2 h_{AB} U^A U^B}{} \right)_{(o)}^{1/2}. \]  

(6)

We assume that a singularity forms in the gravitational collapse. We can choose our coordinates such that a point where the singularity forms is at \( r = \text{constant} \). Now if the singularity is null singularity, then after it forms \( r = \text{constant} \) should be null, i.e. we should have \( dr/du = 0 \) for the ingoing null geodesic, while outgoing null geodesics will be represented by \( du = 0 \) by our coordinate choice. This gives

\[ \frac{dr}{du} = \left( \frac{V}{r} - r^2 h_{AB} U^A U^B e^{-2\beta} \right) = 0. \]  

(7)

This with Eq. (6) tell us that the redshift diverges for geodesics coming out of a null singularity. Our argument may not apply to the first point of singularity formation as it is a boundary point and there the redshift could be finite or infinite. But that is just an instant and finite (non-zero) amount of energy cannot come out of it.

We can choose coordinates suitable for our purpose, e.g., we can choose them such that the singularity forms at \( r = 0 \). In spherically symmetric case we can choose \( h_{AB} = q_{AB} \) and \( U^A = 0 \) and equations (2,3) simplify a lot. Different choices for \( U^A \) will lead to different sets of null geodesics along \( u = \text{constant}, r = \text{constant} \). In spherically symmetry a shell focusing naked singularity can form only at \( r = 0 \) when weak energy condition holds.

If the \( dr = dx^A = 0 \) curve is timelike, then from the metric (Eq. (11)), \( (e^{2\beta} V/r - r^2 h_{AB} U^A U^B) \) is finite and from Eq. (6) we get that the redshift is always finite. Please
note that if $e^{2\beta}$ diverges, then weak energy condition does not hold from Eq. (2). If the singularity is spacelike, then it is always covered and the question of redshift of radiation coming out of the singularity does not arise.

We have checked doing the coordinate transformations from the commonly used coordinates for various spherically symmetric models to the retarded Bondi-Sachs coordinates near the central singularity, namely, for the cases of dust [17, 18], general matter [37] and also Vaidya radiation collapse [31]. Those calculations also confirm that our conclusions are correct.

As such we have not used $r = \text{constant}$ is a singularity to get the redshift and in general the curvature or the Einstein equations will tell us if it is a singularity. In most cases some components of curvature diverge at singularity. We basically derive the redshift using only the geometry of the spacetime and do not explicitly need the Einstein equations. Hence the results are extremely general. Just the proper time of a (singular) source and an observer tells us that the redshift for any null (singularity) surface will diverge. Basically if $\Phi=0$ is ekonal of the wave, then the frequency in geometrical optics approximation is [44, 45]

$$\nu = \frac{d\Phi}{dT},$$

where $T$ is the proper time. Redshift is ratio of the frequency at source to the frequency at observer. For the null source proper time is zero i.e. $dT = 0$ and so redshift diverges for any observer which is not at the source. In figure II we have shown a Penrose diagram for a globally naked null singularity. It is clear from the Penrose diagram that no ingoing timelike geodesic can reach the null singularity. Any source which is at the singularity for finite time has to be null. That means the redshift along any rays coming out of the null singularity will be infinite. For a timelike source, the proper time is finite i.e., nonzero; therefore, redshift is always finite, though it can be very large.

The observed intensity $I_p$ of a point source [46] is

$$I_p = \frac{P_0}{A_0(1 + z)^2},$$

where $P_0$ is the power radiated by the source into the solid angle $\delta\Omega$, and $A_0$ is the area sustained by the rays at the observer. The redshift factor $(1 + z)^2$ appears because the power radiated is not the same as power received by the observer; there is a effect of time dilation as well as frequency change. As the redshift diverges along geodesics coming out of
a null (naked) singularity and $A_0$ is finite, classically the observed luminosity of a null naked singularity is zero. It means that no energy can reach an outside (nonsingular) observer from any null singularity. In essence, we need the redshift to be finite for a finite period to carry any energy outside from a source. For null singularities it is finite at the most for an instant i.e., along the first point/ray coming out of the null naked singularity. Hence no energy can reach outside from a null naked singularity; it cannot affect the physics outside and therefore, there will be no breakdown of predictability. A timelike singularity can in principle be visible to an outside observer as the redshift is always finite for it.

Our result about redshift and luminosity is not very surprising and is also expected from special relativity. In the special theory of relativity, as the source velocity approaches velocity of light the redshift of the emitted radiation starts diverging and the emission cone becomes narrower and narrower. When the source is moving at the speed of light all the radiation is directed only in that source’s direction and for other rays going out from the source redshift is infinite. In our case the source is null (singularity) and so for any observer not sitting on the source (singularity) the redshift of the rays coming to it from the null source is infinite and energy reaching it from the null source is zero. That means our general relativistic result is expected from special relativity.

III. SUMMARY

In summary, in this work, writing the metric in the Bondi-Sachs form we have shown that no energy can come out of any null naked singularity. The advantage of using null formulation is that it is designed to study the null geodesics which we want to explore. Hence
no extra work is needed to get out the quantities of our interest. We have not imposed any symmetries on the spacetime nor we have assumed any specific form of matter. Essentially the assumption that the singularity which forms is null and just the geometry of spacetime is enough for us to reach this conclusion. We also showed that a timelike singularity is in principle likely to be visible to an outside/nonsingular observer as redshift is always finite for rays coming out from it. However, we expect formation of a timelike singularity to be rare when energy conditions hold, as it seems [37] creation of such a singularity needs fine balance of factors; we require the collapse to stop as soon as the singularity is formed. That indicates they will be non-generic. The known examples of naked singularities are also mostly null, or else, they are formed in a very unusual matter fields and need lot of fine tuning.

We have also given the fundamental principle/logic which drive the result. As we have explained above, the result is essentially very similar to the special relativistic result for a null (or timelike) source. For the null singularity (surface) the redshift basically diverges as the proper time goes to zero on null surface. So if a ray has finite frequency outside the singularity (surface) and it originates at the singularity, then it has to have infinite frequency at the singularity. Similarly for timelike singularity as the proper time is always finite (nonzero) at the singularity so the redshift is finite, though it can be very large.

Our result is valid for any form of matter as it is purely based on the geometry at the source i.e., geometry at the singularity. That means our result(s) are most general and should be valid in any theory of gravity as well as, in higher dimensions. One can generalize these results easily along the timelike geodesics. The results basically means that, though the null singularity is geometrically naked (i.e., null geodesics can come out of it) essentially physically it is not visible (naked), as no energy can come out of it due to infinite redshift. That implies we cannot get any information from the null naked singularity and it will not have any undesirable physical effect outside. Therefore, null naked singularity cannot cause breakdown of predictability and they have no special physical significance; i.e. the fact that non-spacelike geodesics come out of null naked singularity does not have any significance. Our conclusion is extremely general and is valid for all possible spacetimes. This strongly supports/preserves the essence of cosmic censorship for null singularities. However, we need to study the timelike naked singularities in more detail for this purpose.

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