**Abstract**

Simple SUSY GUT models based on the gauge group $SO(10)$ require $t - b - \tau$ Yukawa coupling unification, in addition to gauge coupling and matter unification. The Yukawa coupling unification places strong constraints on the expected superparticle mass spectrum, with scalar masses $\sim 10$ TeV while gaugino masses are quite light. A problem generic to all supergravity models comes from overproduction of gravitinos in the early universe: if gravitinos are unstable, then their late decays may destroy the predictions of Big Bang nucleosynthesis. We present a Yukawa-unified $SO(10)$ SUSY GUT scenario which avoids the gravitino problem, gives rise to the correct matter-antimatter asymmetry via non-thermal leptogenesis, and is consistent with the WMAP-measured abundance of cold dark matter due to the presence of an axino LSP. To maintain a consistent cosmology for Yukawa-unified SUSY models, we require a re-heat temperature $T_R \sim 10^6 - 10^7$ GeV, an axino mass around $\sim 0.1 - 10$ MeV, and a PQ breaking scale $f_a \sim 10^{12}$ GeV.

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1 SO(10) SUSY GUTs and Yukawa unification

Grand unified theories (GUTs) are amongst the most compelling ideas in theoretical physics. Their beauty is only enhanced via a marriage to supersymmetry (SUSY). The SU(5) theory\[^{1}\] unifies the Standard Model (SM) gauge symmetries into single Lie group, while explaining the ad-hoc hypercharge assignments of the SM fermions, and successfully predicting the $m_b/m_\tau$ ratio. Adding SUSY to the SU(5) theory stabilizes the hierarchy of interactions, but also receives experimental support from the celebrated unification of gauge couplings at scale $M_{GUT} \approx 2 \times 10^{16}$ GeV.

The SO(10) SUSY GUT theory has even further successes\[^{2}\]. For one, it explains the ad-hoc anomaly cancellation within the SM and SU(5) theories. Further, it unifies all matter of a single generation into the 16-dimensional spinor representation $\hat{\psi}_{(16)}$, provided one adds to the set of supermultiplets a SM gauge singlet superfield $\hat{N}_c$ ($i = 1 - 3$ is a generation index) containing a right-handed neutrino.\[^{3}\] Upon breaking of SO(10), a superpotential term $\hat{f} \ni \frac{1}{2} M_N \hat{N}_{c_i} \hat{N}_{c_i}^c$ is induced which allows for a Majorana neutrino mass $M_N$ which is necessary for implementing the see-saw mechanism for neutrino masses.\[^{4}\] In addition, the SO(10) theory allows for unification of Yukawa couplings of each generation. This applies calculationally especially to the third generation, where in simple SO(10) SUSY GUTs, we may expect $t - b - \tau$ Yukawa coupling unification in addition to gauge coupling unification at scale $Q = M_{GUT}$\[^{5, 6}\].

In spite of these impressive successes, GUTs and also SUSY GUTs have been beset with a variety of problems, most of them arising from implementing GUT gauge symmetry breaking via large, unwieldy Higgs representations. Happily, in recent years physicists have learned that GUT theories– as formulated in spacetime dimensions greater than four– can use extra-dimension compactification to break the GUT symmetry instead\[^{7}\]. This is much in the spirit of string theory, where anyway one must pass from a 10 or 11 dimensional theory to a 4-d theory via some sort of compactification.

Regarding Yukawa coupling unification in SO(10), the calculation begins with stipulating the $b$ and $\tau$ running masses at scale $Q = M_Z$ (for two-loop running, we adopt the $\overline{DR}$ regularization scheme) and the $t$-quark running mass at scale $Q = m_t$. The Yukawa couplings are evolved to scale $Q = M_{SUSY}$, where threshold corrections must be implemented\[^{8}\], as one passes from the SM effective theory to the Minimal Supersymmetric Standard Model (MSSM) effective theory. From $M_{SUSY}$ on to $M_{GUT}$, Yukawa coupling evolution is performed using two-loop MSSM RGEs. Thus, Yukawa coupling unification ends up depending on the complete SUSY mass spectrum via the $t$, $b$ and $\tau$ self-energy corrections.

In this letter, we adopt the Isajet 7.75 program for calculation of the SUSY mass spectrum and mixings\[^{9}\] and IsaReD\[^{10}\] for the neutralino relic density. Isajet uses full two-loop RG running for all gauge and Yukawa couplings and soft SUSY breaking (SSB) terms. In running from $M_{GUT}$ down to $M_{weak}$, the RG-improved 1-loop effective potential is minimized at an optimized scale choice $Q = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$, which accounts for leading two-loop terms. Once a tree-level SUSY/Higgs spectrum is calculated, the complete 1-loop corrections are calculated for all SUSY/Higgs particle masses. Since the SUSY spectrum isn’t known at the beginning of the calculation, an iterative approach must be implemented, which stops when an appropriate convergence criterion is satisfied.

\[^{1}\]Here, we adopt the superfield “hat” notation as presented in Ref. \[^{3}\].
Yukawa coupling unification has been examined in a number of previous papers\textsuperscript{[5] [6] [11] [12] [13] [14]}. The parameter space to be considered is given by

\[ m_{16}, m_{10}, M_D^2, m_{1/2}, A_0, \tan \beta, \text{sign}(\mu) \]  

along with the top quark mass, which we take to be \( m_t = 171 \) GeV. Here, \( m_{16} \) is the common mass of all matter scalars at \( M_{\text{GUT}} \), \( m_{10} \) is the common Higgs soft mass at \( M_{\text{GUT}} \) and \( M_D^2 \) parameterizes either \( D \)-term splitting (DT) or Higgs-only soft mass splitting (HS). The latter is given by \( m_{H_u,d}^2 = m_{10}^2 \pm 2M_D^2 \). As in the minimal supergravity (mSUGRA) model, \( m_{1/2} \) is a common GUT scale gaugino mass, \( A_0 \) is a common GUT scale trilinear soft term, and the bilinear SSB term \( B \) has been traded for the weak scale value of \( \tan \beta \) via the EWSB minimization conditions. The latter also determine the magnitude (but not the sign) of the superpotential Higgs mass term \( \mu \).

What has been learned is that \( t-b-\tau \) Yukawa coupling unification does occur in the MSSM for \( \mu > 0 \) (as preferred by the \((g-2)_\mu \) anomaly), but only if certain conditions are satisfied.

- The scalar mass parameter \( m_{16} \) should be very heavy: in the range 5-20 TeV.
- The gaugino mass parameter \( m_{1/2} \) should be as small as possible.
- The SSB terms should be related as \( A_0^2 = 2m_{10}^2 = 4m_{16}^2 \), with \( A_0 = -2m_{16} \) (in our sign convention). This combination was found to yield a radiatively induced inverted scalar mass hierarchy (IMH) by Bagger \textit{et al.}\textsuperscript{[15]} for MSSM+right hand neutrino (RHN) models with Yukawa coupling unification.
- \( \tan \beta \sim 50 \).
- EWSB can be reconciled with Yukawa unification only if the Higgs SSB masses are split at \( M_{\text{GUT}} \) such that \( m_{H_u}^2 < m_{H_d}^2 \). The HS prescription ends up working better than DT splitting\textsuperscript{[13] [12]}.

In the case where the above conditions are satisfied, then Yukawa coupling unification to within a few percent can be achieved. The resulting sparticle mass spectrum has some notable features.

- First and second generation matter scalars have masses of order \( m_{16} \sim 5-20 \) TeV.
- Third generation scalars, \( m_A \) and \( \mu \) are suppressed relative to \( m_{16} \) by the IMH mechanism: they have masses on the \( 1-2 \) TeV scale. This reduces the amount of fine-tuning one might otherwise expect in such models.
- Gaugino masses are quite light, with \( m_{\tilde{g}} \sim 350-500 \) GeV, \( m_{\tilde{Z}_1} \sim 50-80 \) GeV and \( m_{\tilde{W}_1} \sim 100-150 \) GeV.

The sparticle mass spectra from \( SO(10) \) SUSY GUTs shares some features with spectra generated in “large cutoff supergravity” or LCSUGRA, investigated in Ref.\textsuperscript{[16]}. LCSUGRA also has high mass scalars– typically with mass around 5 TeV– and low mass gauginos. The \( SO(10) \) SUSY GUT models are different from LCSUGRA in that they have a large \( A_0 \), with
\( A_0 \sim -2m_{16} \), and a \( \mu \) term of around 1-2 TeV. This means \( SO(10) \) SUSY GUTs have a dominantly bino-like \( \tilde{Z}_1 \) state, whereas the LCSUGRA authors adopt the mSUGRA model focus point region, which has a mixed higgsino-bino \( \tilde{Z}_1 \) state. The latter can easily give the measured abundance of cold dark matter (CDM) in the form of lightest neutralinos.

Since the lightest neutralino of \( SO(10) \) SUSY GUTs is nearly a pure bino state, it turns out the neutralino relic density \( \Omega_{\tilde{Z}_1} h^2 \) is calculated to be extremely high, of order \( 10^2 - 10^4 \). This conflicts with the WMAP-measured value\([17]\), which gives

\[
\Omega_{\text{CDM}} h^2 = \frac{\rho_{\text{CDM}}}{\rho_c} = 0.111^{+0.011}_{-0.015} \ (2\sigma).
\]  

(2)

where \( h = 0.74 \pm 0.03 \) is the scaled Hubble constant.

Several solutions to the \( SO(10) \) SUSY GUT dark matter problem have been proposed in Refs. \([18, 14]\). Here, we will concentrate on the most attractive one: that the dark matter particle is in fact not the neutralino, but the axino \( \tilde{a} \). Axino dark matter occurs in models where the MSSM is extended via the Peccei-Quinn (PQ) solution to the strong \( CP \) problem\([19]\). The PQ solution introduces a spin-0 axion field into the model; if the model is supersymmetric, then a spin-\( \frac{1}{2} \) axino is also required. It has been shown that the \( \tilde{a} \) state can be an excellent candidate for cold dark matter in the universe\([20]\). In this paper, we will find that \( SO(10) \) SUSY GUT models with an axino DM candidate can 1. yield the correct abundance of CDM in the universe, 2. avoid the gravitino/BBN problem and 3. have an compelling mechanism for generating the matter-antimatter asymmetry of the universe via non-thermal leptogenesis.

## 2 The gravitino problem

An affliction common to all models with gravity mediated SUSY breaking (supergravity or SUGRA) models is known as the gravitino problem. In realistic SUGRA models (those that include the SM as their sub-weak-scale effective theory), SUGRA is broken in a hidden sector by the superHiggs mechanism, which induces a mass for the gravitino \( \tilde{G} \), which is commonly taken to be of order the weak scale. The gravitino mass \( m_{\tilde{G}} \) ends up setting the mass scale for all the soft breaking terms, so then all SSB terms end up also being of order the weak scale.

The coupling of the gravitino to matter is strongly suppressed by the Planck mass, so the \( \tilde{G} \) in the mass range considered here \( (m_{\tilde{G}} \sim 5 - 20 \) TeV) is never in thermal equilibrium with the thermal bath in the early universe. Nonetheless, it does get produced by scatterings of particles that do partake of thermal equilibrium. Thermal production of gravitinos in the early universe has been calculated in Refs. \([21]\), where the abundance is found to depend naturally on \( m_{\tilde{G}} \) and on the re-heat temperature \( T_R \) at the end of inflation. Once produced, the \( \tilde{G} \)s decay into all varieties of particle-sparticle pairs, but with a lifetime that can exceed \( \sim 1 \) sec, the time scale where Big Bang nucleosynthesis (BBN) begins. The energy injection from \( \tilde{G} \) decays is a threat to dis-associate the light element nuclei which are created in BBN. Thus, the long-lived \( \tilde{G} \)s can destroy the successful predictions of the light element abundances as calculated by nuclear thermodynamics.

The BBN constraints on gravitino production in the early universe have been calculated by several groups\([22]\). The recent results from Ref. \([23]\) give an upper limit on the re-heat temperature as a function of \( m_{\tilde{G}} \). The results depend on how long-lived the \( \tilde{G} \) is (at what stage
of BBN the energy is injected), and what its dominant decay modes are. Qualitatively, for $m_{\tilde{G}} \gtrsim 5$ TeV, $T_R \gtrsim 10^9$ GeV is required; if this is violated, then too many $\tilde{G}$ are produced in the early universe, which destroy the $^3He$, $^6Li$ and $D$ abundance calculations. For $m_{\tilde{G}} \sim 5 - 50$ TeV, the re-heat upper bound is much less: $T_R \lesssim 5 \times 10^7 - 10^9$ GeV (depending on the $^4He$ abundance) due to overproduction of $^4He$ arising from $n \leftrightarrow p$ conversions. For $m_{\tilde{G}} \gtrsim 50$ TeV, there is an upper bound of $T_R \lesssim 5 \times 10^9$ GeV due to overproduction of $\tilde{Z}_1$ LSPs due to $\tilde{G}$ decays.

Solutions to the gravitino BBN problem then include: 1. having $m_{\tilde{G}} \gtrsim 50$ TeV but with an unstable $\tilde{Z}_1$ (no $T_R$ bound), 2. having a gravitino LSP so that $\tilde{G}$ is stable or 3. keep the re-heat temperature below the BBN bounds. We will here adopt solution number 3. In the case of $SO(10)$ SUSY GUT models, with $m_{\tilde{G}} \sim m_{16} \sim 5 - 20$ TeV, this means we need a re-heat temperature $T_R \lesssim 10^8 - 10^9$ GeV.

### 3 Non-thermal leptogenesis

The data gleaned on neutrino masses during the past decade has lead credence to a particular mechanism of generating the baryon asymmetry of the universe known as leptogenesis[24]. Leptogenesis requires the presence of heavy gauge singlet Majorana right handed neutrino states $\psi_{N_i}(\equiv N_i)$ with mass $M_{N_i}$ ($i = 1 - 3$ is a generation index). The $N_i$ states may be produced thermally in the early universe, or perhaps non-thermally, as suggested in Ref. [25] via inflaton $\phi \rightarrow N_iN_i$ decay. The $N_i$ may then decay asymmetrically to elements of the doublets– for instance $\Gamma(N_1 \rightarrow h^+e^-) \neq \Gamma(N_1 \rightarrow h^-e^+)$– owing to the contribution of CP-violating phases in the tree/loop decay interference terms. Focussing on just one species of heavy neutrino $N_1$, the asymmetry is calculated to be[26]

$$\epsilon \equiv \frac{\Gamma(N_1 \rightarrow \ell^+) - \Gamma(N_1 \rightarrow \ell^-)}{\Gamma_{N_1}} \simeq -\frac{3}{8\pi} \frac{M_{N_1}}{v_u^2} m_{\nu_3} \delta_{eff},$$

where $m_{\nu_3}$ is the heaviest active neutrino, $v_u$ is the up-Higgs vev and $\delta_{eff}$ is an effective CP-violating phase factor which may be of order 1. The ultimate baryon asymmetry of the universe is proportional to $\epsilon$, so larger values of $M_{N_1}$ lead to a higher baryon asymmetry.

To find the baryon asymmetry, one may first assume that the $N_1$ is thermally produced in the early universe, and then solve the Boltzmann equations for the $B - L$ asymmetry. The ultimate baryon asymmetry of the universe arises from the lepton asymmetry via sphaleron effects. The final answer[27], compared against the WMAP-measured result $\frac{n_B}{s} \simeq 0.9 \times 10^{-10}$ for the baryon-to-entropy ratio, requires $M_{N_1} \gtrsim 10^{10}$ GeV, and thus a re-heat temperature $T_R \gtrsim 10^{10}$ GeV. This high a value of reheat temperature is in conflict with the upper bound on $T_R$ discussed in Sec. 2. In this way, it is found that generic SUGRA models are apparently in conflict with leptogenesis as a means to generate the baryon asymmetry of the universe.

If one instead looks to non-thermal leptogenesis, then it is possible to have lower reheat temperatures, since the $N_1$ may be generated via inflaton decay. The Boltzmann equations for the $B - L$ asymmetry have been solved numerically in Ref. [28]. The $B - L$ asymmetry is then converted to a baryon asymmetry via sphaleron effects as usual. The baryon-to-entropy ratio
is calculated in [28], where it is found
\[ \frac{n_B}{s} \simeq 8.2 \times 10^{-11} \times \left( \frac{T_R}{10^6 \text{ GeV}} \right) \left( \frac{2M_{N_1}}{m_{\phi}} \right) \left( \frac{m_{\nu_3}}{0.05 \text{ eV}} \right) \delta_{\text{eff}}, \] (4)

where \( m_{\phi} \) is the inflaton mass. Comparing calculation with data, a lower bound \( T_R \gtrsim 10^6 \text{ GeV} \) may be inferred for viable non-thermal leptogenesis via inflaton decay.

4 Axino dark matter

The sparticle mass spectrum described in Sec. [1] is characterized by 5 – 20 GeV scalars, but very light gauginos, with a \( \mu \) parameter of order 1-2 TeV. As a consequence, the neutralino \( \tilde{Z}_1 \) ends up being nearly pure bino. Since all the scalars are quite heavy, the predicted neutralino relic abundance ends up being very high: the calculation of Refs. [18, 14] find values in the range \( \Omega_{\tilde{Z}_1} h^2 \sim 10^2 - 10^4 \), which is 3 – 4 orders of magnitude beyond the WMAP-measured abundance.

A solution was advocated in Ref. [14] that in fact the \( \tilde{Z}_1 \) state is not the LSP, but instead the axino \( \tilde{a} \) makes up the CDM of the universe. The axino is the spin-1/2 element of the axion supermultiplet which is needed to solve the strong \( CP \) problem in supersymmetric models. The axino is characterized by a mass in the range of keV-GeV. Its couplings are of sub-weak interaction strength, since they are suppressed by the Peccei-Quinn symmetry breaking scale \( f_a \), which itself has a viable mass range \( 10^{10} - 10^{12} \text{ GeV} \). While the axino interacts very feebly, it does interact more strongly than the gravitino.

If the \( \tilde{a} \) is the lightest SUSY particle, then the \( \tilde{Z}_1 \) will no longer be stable, and can decay via \( \tilde{Z}_1 \rightarrow \tilde{a} \gamma \). The relic abundance of axinos from neutralino decay (non-thermal production, or \( NTP \)) is given simply by
\[ \Omega_{\tilde{a}}^{NTP} h^2 = \frac{m_{\tilde{a}}}{m_{\tilde{Z}_1}} \Omega_{\tilde{Z}_1} h^2, \] (5)
since in this case the axinos inherit the thermally produced neutralino number density. Notice that neutralino-to-axino decay offers a mechanism to shed large factors of relic density. For a case where \( m_{\tilde{Z}_1} \sim 50 \text{ GeV} \) and \( \Omega_{\tilde{Z}_1} h^2 \sim 1000 \), as can occur in \( SO(10) \) SUSY GUTs, an axino mass of less than 5 MeV reduces the DM abundance to below WMAP-measured levels.

The lifetime for these decays has been calculated, and it is typically in the range of \( \tau(\tilde{Z}_1 \rightarrow \tilde{a} \gamma) \sim 0.03 \text{ sec} \)[20]. The photon energy injection from \( \tilde{Z}_1 \rightarrow \tilde{a} \gamma \) decay into the cosmic soup occurs well before BBN, thus avoiding the constraints that plague the case of a gravitino LSP[29]. The axino DM arising from neutralino decay is generally considered warm or even hot dark matter for cases with \( m_{\tilde{a}} \gtrsim 1 - 10 \text{ GeV} \)[31]. Thus, in our Yukawa-unified scenario, where \( m_{\tilde{a}} \gtrsim 80 \text{ MeV} \), we always get warm DM from neutralino decay.

Even though they are not in thermal equilibrium, axinos can still be produced thermally in the early universe via scattering processes. The axino thermally produced (TP) relic abundance has been calculated in Ref. [20, 30], and is given by
\[ \Omega_{\tilde{a}}^{TP} h^2 \simeq 5.5g_s^6 \ln \left( \frac{1.108}{g_s} \right) \left( \frac{10^{11} \text{ GeV}}{f_a/N} \right)^2 \left( \frac{m_{\tilde{a}}}{0.1 \text{ GeV}} \right) \left( \frac{T_R}{10^4 \text{ GeV}} \right) \] (6)
where \( g_s \) is the strong coupling evaluated at \( Q = T_R \) and \( N \) is the model dependent color anomaly of the PQ symmetry, of order 1. The thermally produced axinos qualify as cold dark matter as long as \( m_{\tilde{a}} \gtrsim 0.1 \text{ MeV} \).

5 A consistent cosmology for axino DM from \( SO(10) \) SUSY GUTs

At this point, we are able to check if we can implement a consistent cosmology for \( SO(10) \) SUSY GUTs with axino dark matter. Our first step is to select points from the \( SO(10) \) parameter space Eq. 1 that are very nearly Yukawa-unified. In Ref. [14], Yukawa unified solutions were searched for by looking for \( R \) values as close to 1 as possible, where

\[
R = \frac{\max(f_t, f_b, f_\tau)}{\min(f_t, f_b, f_\tau)}
\]

where the \( f_t \), \( f_b \) and \( f_\tau \) Yukawa couplings were evaluated at \( M_{\text{GUT}} \). Thus, a solution with \( R = 1.05 \) gives Yukawa unification to 5%.

We would like solutions where the axino DM is dominantly CDM. For definiteness, we will insist on \( \Omega_{\tilde{a}}^{NTP} h^2 \sim 0.01 \), while \( \Omega_{\tilde{a}}^{TP} h^2 = 0.1 \). Thus, in step 1., we select models from the random scan of Ref. [14] that have \( R < 1.05 \), and \( m_{16} : 5 - 20 \text{ TeV} \). In step 2., from the known value of \( m_{\tilde{Z}_1} \) and \( \Omega_{\tilde{Z}_1} h^2 \), we next calculate the axino mass needed to generate \( \Omega_{\tilde{a}}^{NTP} h^2 = 0.01 \) according to Eq. 5. In step 3, we plug \( m_{\tilde{a}} \) into Eq. 6, where we also take \( g_s = 0.915 \) (the running \( g_s \) value at \( \sim 10^6 \text{ GeV} \)), and PQ scale \( f_a/N = 10^{12} \text{ GeV} \). By insisting that \( \Omega_{\tilde{a}}^{TP} h^2 = 0.1 \), we may calculate the value of \( T_R \) that is needed.

Our results are plotted in the \( m_{\tilde{a}} \) vs. \( T_R \) plane in Fig. 11 and occupy the upper band of solutions. In this plane, solutions with \( T_R \gtrsim 3 \times 10^7 - 5 \times 10^8 \text{ GeV} \) are allowed by the gravitino constraint (with \( m_{\tilde{G}} \sim 5 - 20 \text{ TeV} \)) and BBN. Solutions with \( T_R \gtrsim 10^6 \text{ GeV} \) can generate the matter-antimatter asymmetry correctly via non-thermal leptogenesis. Solutions with \( m_{\tilde{a}} \gtrsim 10^{-4} \text{ GeV} \) give dominantly cold DM from TP of axinos. Solutions with \( m_{16} > 15 \text{ TeV} \) are denoted by filled (turquoise) symbols, while solutions with \( m_{16} < 15 \text{ TeV} \) have open (dark blue) symbols.

We see that a variety of points fall in the allowed region. These points give rise to a consistent cosmology for \( SO(10) \) SUSY GUT models! Of course, there is some uncertainty in these results. We can take higher or lower values of the PQ breaking scale, higher or lower fractions of \( \Omega_{\tilde{a}}^{NTP} h^2 \), and the \( T_R \) upper (and lower) bounds have some variability built into them. As an example, the lower band of solutions is obtained with \( \Omega_{\tilde{a}}^{NTP} h^2 = 0.03 \), \( \Omega_{\tilde{a}}^{TP} h^2 = 0.08 \) and \( f_a/N = 5 \times 10^{11} \text{ GeV} \). In this case, some of the previously excluded solutions migrate into the allowed region to give a consistent cosmology with somewhat different parameters.

6 Conclusion

Our main conclusion can be summarized briefly. For Yukawa unified supersymmetric models, as expected in \( SO(10) \) SUSY GUT models, we find one can implement a consistent cosmology
Figure 1: Plot of Yukawa unified solutions with $R < 1.05$ and $5 \text{ TeV} < m_{16} < 20 \text{ TeV}$ in the $m_\tilde{a}$ vs. $T_R$ plane. The upper band of solutions has $\Omega_{\tilde{a}}^{NTP} h^2 = 0.01$, $\Omega_{\tilde{a}}^{TP} h^2 = 0.10$ and $f_a/N = 10^{12}$ GeV, while the lower band of solutions has $\Omega_{\tilde{a}}^{NTP} h^2 = 0.03$, $\Omega_{\tilde{a}}^{TP} h^2 = 0.08$ and $f_a/N = 5 \times 10^{11}$ GeV.

including the following: 1. BBN safe mass spectra owing to the multi-TeV value of $m_{16}$, which arises in SUGRA models from a multi-TeV $m_G$. 2. a WMAP-allowed relic density of CDM that consists dominantly of thermally produced axinos, and 3. the re-heat temperature needed to fulfill the relic density falls above the lower bound required by non-thermal leptogenesis, and below the upper bound coming from gravitino/BBN constraints.

We feel that the fact that Yukawa unified $SO(10)$ SUSY GUT models pass these several cosmological tests makes them even more compelling than they were based on pure particle physics reasons. In any case, with a spectrum of light gluinos, charginos and neutralinos, they should easily be tested by experiments at the CERN LHC even with low integrated luminosities of just $\sim 0.1 \text{ fb}^{-1}$.

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