Owing to the rapid development of sensor technology, hyperspectral (HS) remote sensing (RS) imaging has provided a significant amount of spatial and spectral information for the observation and analysis of Earth’s surface at a distance of data acquisition devices. The recent advancement and even revolution of HS RS techniques offer opportunities to realize the potential of various applications while confronting new challenges for efficiently processing and analyzing the enormous HS acquisition data. Due to the maintenance of the 3D HS inherent structure, tensor decomposition has aroused widespread concern and spurred research in HS data processing tasks over the past decades. In this article, we aim to present a comprehensive overview of tensor decomposition, specifically contextualizing the five broad topics in HS...
data processing: HS restoration, compressive sensing (CS), anomaly detection (AD), HS–multispectral (MS) fusion, and spectral unmixing (SU). For each topic, we elaborate on the remarkable achievements of tensor decomposition models for HS RS, with a pivotal description of the existing methodologies and a representative exhibition of experimental results. As a result, the remaining challenges of the follow-up research directions are outlined and discussed from the perspective of actual HS RS practices and tensor decomposition merged with advanced priors and even deep neural networks. This article summarizes different tensor decomposition-based HS data processing methods and categorizes them into different classes, from simple adoptions to complex combinations with other priors for algorithm beginners. We expect that this survey provides new investigations and development trends for experienced researchers to some extent.

INTRODUCTION
HS RS imaging has gradually become one of the most vital achievements in the field of RS since the 1980s [1], [2]. Varied from an initial single-band pansharmonic image, a three-band red–green–blue (RGB) image, and a several-band MS image, an HS image contains hundreds of narrow and contiguous spectral bands, promoted by the development of spectral imaging equipment and improvement of spectral resolutions. The broader portion of the HS spectrum can scan from the ultraviolet, extend into the visible spectrum, and eventually reach the near infrared, or shortwave infrared [3]. Each pixel of HS images corresponds to a spectral signature and reflects the electromagnetic properties of the observed object. This enables the identification and discrimination of underlying objects, especially some that have similar properties in single- and several-band RS images (such as panchromatic, RGB, and MS) in a more accurate manner. As a result, the wealth of spatial and spectral information in HS images has extremely improved the perceptual ability of Earth observation, which makes the HS RS technique play a crucial role in fields such as precision agriculture (e.g., monitoring the growth and health of crops), space exploration (e.g., searching for signs of life on other planets), pollution monitoring (e.g., the detection of ocean oil spills), and military applications (e.g., the identification of military targets) [4], [5], [6].

Over the past decade, massive efforts have been made to process and analyze HS RS data after data collection. Initial HS data processing considers either a gray-level image for each band or the spectral signature of each pixel. From one side, each HS spectral band is regarded as a gray-level image, and traditional 2D image processing algorithms are directly introduced band by band [7], [8]. From another side, spectral signatures that have similar visible properties (e.g., color and texture) can be used to identify materials [9]. Furthermore, extensive low-rank (LR) matrix-based methods are employed to explore the high correlation of spectral channels, with the assumption that the unfolding HS matrix has an LR property [10], [11], [12]. Given an HS image of size \( h \times v \times z \), the recovery of an unfolding HS matrix \( (hv \times z) \) usually requires singular value decomposition (SVD), which leads to a computational cost of \( O(hv^2z+z^2) \) [13], [14], [15]. In some typical tensor decomposition-based methods, the complexity of the tensor SVD (t-SVD) is about \( O(hvz\log z + hv^2z) \) [16], [17], [18]. Compared to matrix forms, tensor decompositions achieve excellent performance with a tolerable increment of computational complexity. However, these traditional LR models reshape each spectral band as a vector, leading to the destruction of the inherent spatial neighborhood similarity and correlation between the spatial and spectral information of HS images. Correct interpretations of HS images and the appropriate choice of intelligent models should be determined to reduce the gap between HS tasks and the advanced data processing technique. Both 2D spatial information and 1D spectral information are considered when an HS image is modeled as a three-order tensor.

Tensor decomposition, which originates from Hitchcock’s work in 1927 [19], touches upon numerous disciplines, but it has become significant in the fields of signal processing, machine learning, data mining, and fusion over the past 10 years [20], [21], [22]. Early overviews focused on two common decomposition ways: Tucker decomposition and canonical decomposition/parallel factor analysis (CP) decomposition. In 2008, these two decompositions were first introduced into HS restoration tasks to remove Gaussian noise [23], [24]. Tensor decomposition-based mathematical models avoid converting the original dimensions and, to some degree, enhance the interpretability and completeness of problem modeling. Different types of prior knowledge (e.g., nonlocal similarity in the spatial domain and spatial and spectral smoothness) in HS RS are considered and incorporated into tensor decomposition frameworks. However, on the one hand, additional tensor decomposition methods have been proposed recently, such as block term (BT) decomposition, t-SVD [25], tensor train (TT) decomposition [26], and tensor ring (TR) decomposition [27]. On the other hand, as a versatile tool, tensor decomposition related to HS image processing has not been reviewed. In this article, we mainly present a systematic overview from the perspective of state-of-the-art tensor decomposition techniques for HS data processing in terms of the five burgeoning topics previously mentioned, as presented in Figure 1.
FIGURE 1. The main tensor decomposition-based methods for HS data processing. PCA: principal component analysis.
Figure 2 displays the dynamics of tensor decompositions used for HS data processing in the HS community. The listed numbers contain both scientific journal and conference papers published in IEEE Xplore, which regards “hyperspectral” and “tensor decomposition” as the main keywords in abstracts. To highlight the increasing trend of the number of publications, the time period has been divided into four equal time slots [i.e., 2007–2010, 2011–2014, 2015–2018, and 2019–2022 (29 September)]. In this article, we mainly present a systematic overview from the perspective of state-of-the-art tensor decomposition techniques for HS data processing in terms of the five burgeoning topics previously mentioned:

1) To the best of our knowledge, this is the first comprehensive survey of the state-of-the-art tensor decomposition techniques for processing and analyzing HS RS images. More than 100 publications in this field are reviewed and discussed, most of which were published during the past five years.

2) For each HS topic, major representative works are scrupulously presented in terms of the specific categories of tensor decomposition. We introduce and discuss the pure tensor decomposition-based methods and their variants with other HS priors in sequence. Experimental examples are given for validating and evaluating theoretical methods, followed by a discussion of remaining challenges and further research directions.

3) This article makes a connection between tensor decomposition modeling and HS prior information. Table 1 summarizes the publication years, with a brief description and prior information. Either beginners or experienced practitioners are expected to obtain insight into the tensor decomposition-based frameworks for HS RS. Code displayed in Table 1 can be used with real data from Table 2 for the sake of repeatability and further studies.

**TENSOR THEORY PRELIMINARIES**

We present some materials and references for practitioners to establish readers’ awareness of tensor-related theory before learning tensor decomposition-based HS applications, including Kolda and Bader’s tensor decomposition and applications [20], Sidiropoulos’ research on signal processing and machine learning [22], and Kilmer’s theoretical framework with applications in imaging [25]. These contribute to a quick understanding of the characteristics and principles of tensor decomposition. Beginners can benefit from relevant resources, such as this section’s basic tensor theory, available HS RS data in Table 2, and open source code links, with a specific classification provided in Table 1.

In this section, we first introduce some vital preliminaries that are widely used in HS RS applications, containing needful tensor operations, definitions, and notations. Definitions are provided for various types of fundamental and frequently used tensor decompositions. These preliminaries are part of the theoretical foundation of tensors and run through this review.

**MATHEMATICAL NOTATIONS**

The basic notations listed in Tables 3 and 4 give the main abbreviations used in this article. Common tensor operations and definitions are reported as follows, and their corresponding toolbox can be found at http://www.tensortoolbox.org/.

**DEFINITION 1 (T-PRODUCT [120])**

The T-product of two three-order tensors $A \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ and $B \in \mathbb{R}^{n'_{1} \times n_2 \times n_3}$ is denoted by $C \in \mathbb{R}^{n_1 \times n'_1 \times n_2 \times n_3}$:

$$C(i, k, :) = \sum_{j=1}^{n_3} A(i, j, :) \cdot B(j, k, :)$$

where $\cdot$ represents the circular convolution between two tubes.

**DEFINITION 2 (TENSOR $n$-MODE PRODUCT [20])**

The $n$-mode product of a tensor $A \in \mathbb{R}^{n_1 \times n_2 \times \cdots \times n_N}$ and matrix $B \in \mathbb{R}^{n_{1} \times n}$ is the tensor $X \in \mathbb{R}^{n_1 \times n_2 \times \cdots \times n_N}$, defined by

$$X = A \times_n B.$$  

The unfolding matrix form of (2) is

$$X_{(n)} = B \times A_{(n)}.$$  

**DEFINITION 3 (FOUR SPECIAL TENSORS [121])**

The conjugate transpose of a three-order tensor $X \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ is the tensor $\text{con}(X) = X' \in \mathbb{R}^{n_3 \times n_2 \times n_1}$, which can be obtained by conjugately transposing each front slice and reversing the order of transposed frontal two through $z$.

The identity tensor denoted by $I \in \mathbb{R}^{h \times v \times z}$ is the tensor whose first frontal slice is an identity matrix, and all other frontal slices are zero. A three-order tensor $Q$ is orthogonal...
| CATEGORY    | YEAR | METHOD            | BRIEF DESCRIPTION                           | PRIOR INFORMATION                       | CODE LINKS                                      |
|-------------|------|-------------------|---------------------------------------------|------------------------------------------|-------------------------------------------------|
| Restoration | 2008 | LRTA [23]         | Tucker decomposition                        | Spectral correlation                     |                                                 |
|             | 2008 | PARAFAC [24]      | CP decomposition                            | Spectral correlation                     |                                                 |
|             | 2013 | RITD [28]         | Rank 1 tensor decomposition                 | Spectral correlation                     |                                                 |
|             | 2017 | LRTR [16]         | Tensor nuclear norm (TNN)                   | Spectral correlation                     |                                                 |
|             | 2019 | NTRM [29]         | Logarithmic TNN                             | Spectral correlation                     |                                                 |
|             | 2020 | 3DTNN/3DLogTNN [18]| Three-directional TNN/log-based TNN         | Spectral correlation                     | https://yubangzheng.github.io/homepage/         |
| Denoising   | 2014 | TDL               | Tucker decomposition with dictionary learning| Spectral correlation + nonlocal similarity | http://www.cs.cmu.edu/~deyum/                   |
|             | 2018 | NSNTD [30]        | Nonlocal similarity-based non-negative Tucker decomposition | Spectral correlation + nonlocal similarity |                                                 |
|             | 2019 | GNWTTN [31]       | Global and nonlocal weighted TTN            | Spectral correlation + nonlocal similarity |                                                 |
|             | 2015 | NLTA-LSM [32]     | Tensor decomposition with Laplacian scale mixture | Spectral correlation + nonlocal similarity |                                                 |
|             | 2016 | ITS [33]          | CP + Tucker decomposition                    | Spectral correlation + nonlocal similarity | https://gr.xjtu.edu.cn/web/dymeng/3              |
|             | 2019 | NLR-CPTD [34]     | CP + Tucker decomposition                    | Spectral correlation + nonlocal similarity |                                                 |
|             | 2017 | LLRT [35]         | Hyper-Laplacian prior + unidirectional LRTR  | Nonlocal similarity + spectral smoothness | https://owuchangyuo.github.io/                 |
|             | 2019 | NGmeet [36]       | Spectral subspace-based unidirectional LRTR  | Spectral correlation + Nonlocal similarity | https://prowdy.github.io/weihe.                  |
|             | 2020 | WLRTR [37]        | Weighted Tucker decomposition                | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/                 |
|             | 2020 | NLTR [38]         | NLTR decomposition                           | Spectral correlation + nonlocal similarity | https://chenyong1993.github.io/yongchen.github.io/|
|             | 2018 | TLRTV [39]        | TNN + 2DTV/3DTV                             | Spectral correlation + spatial and spectral smoothness |                                                 |
|             | 2018 | SSTV-LRTF [17]    | TNN + SSTV                                  | Spectral correlation + spatial–spectral smoothness |                                                 |
|             | 2021 | MLR-SSTV [40]     | Multidirectional weighted TNN + SSTV         | Spectral correlation + spatial–spectral smoothness |                                                 |
|             | 2018 | LRTDTV [41]       | 3DwTV + Tucker decomposition                | Spectral correlation + spatial–spectral smoothness | https://github.com/zhaoxile                     |
|             | 2021 | TLR-L1-2SSTV [42]| L1−2SSTV + local patch TNN                  | Spectral correlation + spatial–spectral smoothness |                                                 |
| Deblurring  | 2019 | LRTDGS [43]       | Weighted group sparsity-regularized TV + Tucker decomposition | Spectral correlation + spatial–spectral smoothness | https://chenyong1993.github.io/yongchen.github.io/|
|             | 2019 | LRTFL0 [44]       | l0 gradient constraint + LR BT decomposition | Spectral correlation + spatial–spectral smoothness | http://www.xiongfuli.com/cv/                   |
|             | 2021 | TLR-L0TV [45]     | l0TV + TNN                                  | Spectral correlation + spatial–spectral smoothness | https://github.com/minghuawang123/TLR-L0TV     |
|             | 2019 | SNLRSF [46]       | Subspace-based NLR and sparse factorization  | Spectral correlation + nonlocal tensor subspace | https://github.com/AlgnersYJW/                  |
|             | 2020 | LRTF-DFR [47]     | Double-factor-regularized LR tensor factorization | Subspace spectral correlation + spatial and spectral constraints | https://yubangzheng.github.io/homepage/          |
|             | 2021 | DNTSLR [48]       | Difference continuity + nonlocal tensor subspace | Spectral correlation + Non-local tensor subspace |                                                 |
| Inpainting  | 2020 | WLTR [37]         | Weighted Tucker decomposition                | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/                 |
|             | 2021 | OLRT [49]         | Joint spectral and NLR tensor               | Nonlocal similarity + spectral smoothness   | https://owuchangyuo.github.io/                 |
|             | 2015 | TMac [50]         | LRTC by parallel matrix factorization       | Spectral correlation                      |                                                 |
|             | 2015 | TNCP [51]         | TNN + CP decomposition                      | Spectral correlation                      |                                                 |
|             | 2017 | AWTC [52]         | High-accuracy LRTC (HaLRTC) with well-designed weights | Spectral correlation                      |                                                 |
| CATEGORY | YEAR | METHOD | BRIEF DESCRIPTION | PRIOR INFORMATION | CODE LINKS |
|----------|------|--------|------------------|------------------|------------|
| Destriping | 2019 | LRRTC [53] | Logarithm of the determinant + TTN | Spectral correlation |  |
| | 2020 | LRTC [54], [55] | t-SVD | Spectral correlation |  |
| | 2019 | TRTV [56] | TR decomposition + spatial TV | Spectral correlation + spatial smoothness |  |
| | 2020 | WLTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity |  |
| | 2021 | TVWTR [57] | WTR decomposition + 3DTV | Spectral correlation + spatial–spectral smoothness |  |
| | 2018 | LRTD [58] | Tucker decomposition + SSTV | Spectral correlation + spatial–spectral smoothness |  |
| Destriping | 2018 | LRNLTV [59] | Matrix NN + nonlocal TV | Spectral correlation + nonlocal similarity |  |
| | 2020 | GLTSA [60] | Global and local tensor sparse approximation | Sparsity + spatial and spectral smoothness |  |
| | 2020 | WLRTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity |  |
| | 2017 | JTenRe3DTV [61] | Tucker decomposition + weighted 3DTV | Spectral correlation + spatial and spectral smoothness |  |
| | 2017 | PLTD [62] | Nonlocal Tucker decomposition | Spectral correlation + nonlocal similarity |  |
| | 2019 | NTSRLR [63] | TRN + Tucker decomposition | Spectral correlation + nonlocal similarity |  |
| | 2020 | SMLTR [64] | Combined Tucker decomposition + subspace representation | Nonlocal similarity + spectral smoothness |  |
| | 2015 | 3D-KCHSI [65] | Kronecker CS (KCS) with independent sampling dimensions | Spectral correlation |  |
| | 2015 | T-NCS [66] | Tucker decomposition | Spectral correlation |  |
| | 2013 | NBOMP [67] | KCS with a tensor-based greedy algorithm | Spectral correlation |  |
| | 2016 | BOSE [68] | KCS with beamformed mode-based sparse estimator | Spectral correlation |  |
| | 2020 | TBR [69] | KCS with multidimensional block sparsity | Spectral correlation |  |
| AD | 2015 | LTDD [70] | Tucker decomposition + unmixing | Spectral correlation |  |
| | 2016 | TenB [71] | Tucker decomposition + principal component analysis (PCA) | Spectral correlation |  |
| | 2019 | TDCW [72] | Tucker decomposition + clustering | Spectral correlation |  |
| | 2020 | TEELRD [73] | Tucker decomposition + endmember extraction | Spectral correlation + subspace representation |  |
| | 2019 | LRASHT [74] | Tucker decomposition + TNN | Spectral correlation + subspace representation |  |
| | 2018 | TPCA [75] | TPCA + Fourier transform | Spectral correlation |  |
| | 2020 | PTA [76] | Truncated NN + spatial TV | Spectral correlation + spatial smoothness |  |
| | 2022 | PCA-TLRR [77] | Weighted TNN + multisubspace | Spectral correlation + subspace |  |
| SR | 2018 | STEREO [78] | CP decomposition | Spectral correlation |  |
| | 2020 | NCTCP [79] | Nonlocal coupled CP decomposition | Spectral correlation + nonlocal similarity |  |
| | 2018 | SCUBA [80] | CP decomposition with matrix factorization | Spectral correlation |  |
| | 2018 | CSTF [81] | Tucker decomposition | Spectral correlation |  |
| | 2021 | CT/CB-STAR [82] | Tucker decomposition with interimage variability | Spectral correlation + spatial–spectral variability |  |

(Continued)
| CATEGORY | METHOD | BRIEF DESCRIPTION | PRIOR INFORMATION | CODE LINKS |
|----------|--------|-------------------|-------------------|------------|
| 2021     | Coupled nonnegative Tucker decomposition [83] | Nonnegative Tucker decomposition | Spectral correlation | |
| 2018     | CSTF-I [84] | Tucker decomposition | Spectral correlation | |
| 2020     | SCOTT [85] | Tucker decomposition + higher-order SVD (HOSVD) | Spectral correlation | |
| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
| 2020     | WLRTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/publications/WLRTR |
| 2018     | CSTF-II [84] | Tucker decomposition | Spectral correlation | |
| 2020     | SCOTT [85] | Tucker decomposition + higher-order SVD (HOSVD) | Spectral correlation + nonlocal similarity | |
| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
| 2020     | WLRTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/publications/WLRTR |
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| 2020     | SCOTT [85] | Tucker decomposition + higher-order SVD (HOSVD) | Spectral correlation + nonlocal similarity | |
| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
| 2020     | WLRTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/publications/WLRTR |
| 2018     | CSTF-II [84] | Tucker decomposition | Spectral correlation | |
| 2020     | SCOTT [85] | Tucker decomposition + higher-order SVD (HOSVD) | Spectral correlation + nonlocal similarity | |
| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
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| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
| 2020     | WLRTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/publications/WLRTR |
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| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
| 2020     | WLRTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/publications/WLRTR |
| 2018     | CSTF-II [84] | Tucker decomposition | Spectral correlation | |
| 2020     | SCOTT [85] | Tucker decomposition + higher-order SVD (HOSVD) | Spectral correlation + nonlocal similarity | |
| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
| 2020     | WLRTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/publications/WLRTR |
| 2018     | CSTF-II [84] | Tucker decomposition | Spectral correlation | |
| 2020     | SCOTT [85] | Tucker decomposition + higher-order SVD (HOSVD) | Spectral correlation + nonlocal similarity | |
| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
| 2020     | WLRTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/publications/WLRTR |
| 2018     | CSTF-II [84] | Tucker decomposition | Spectral correlation | |
| 2020     | SCOTT [85] | Tucker decomposition + higher-order SVD (HOSVD) | Spectral correlation + nonlocal similarity | |
| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
| 2020     | WLRTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/publications/WLRTR |
| 2018     | CSTF-II [84] | Tucker decomposition | Spectral correlation | |
| 2020     | SCOTT [85] | Tucker decomposition + higher-order SVD (HOSVD) | Spectral correlation + nonlocal similarity | |
| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
| 2020     | WLRTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/publications/WLRTR |
| 2018     | CSTF-II [84] | Tucker decomposition | Spectral correlation | |
| 2020     | SCOTT [85] | Tucker decomposition + higher-order SVD (HOSVD) | Spectral correlation + nonlocal similarity | |
| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
| 2020     | WLRTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/publications/WLRTR |
| 2018     | CSTF-II [84] | Tucker decomposition | Spectral correlation | |
| 2020     | SCOTT [85] | Tucker decomposition + higher-order SVD (HOSVD) | Spectral correlation + nonlocal similarity | |
| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
| 2020     | WLRTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/publications/WLRTR |
| 2018     | CSTF-II [84] | Tucker decomposition | Spectral correlation | |
| 2020     | SCOTT [85] | Tucker decomposition + higher-order SVD (HOSVD) | Spectral correlation + nonlocal similarity | |
| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
| 2020     | WLRTR [37] | Weighted Tucker decomposition | Spectral correlation + nonlocal similarity | https://owuchangyuo.github.io/publications/WLRTR |
| 2018     | CSTF-II [84] | Tucker decomposition | Spectral correlation | |
| 2020     | SCOTT [85] | Tucker decomposition + higher-order SVD (HOSVD) | Spectral correlation + nonlocal similarity | |
| 2020     | NNSTF [86] | Tucker decomposition + HOSVD | Spectral correlation + nonlocal similarity | |
| 2020     | WLRTR [37] | Weighted Tucker decomposition | Spectrical...
The TR decomposition can be rewritten as

\[ \text{METHOD} \]

Weighted NLR + TV

\[ \text{CODE LINKS} \]

Spectral correlation + sparsity

https://gitSUb.com/

WNLTDSU [118]

SCNMTF [114]

NLR-T V [115]

where

**MATRICIZATION**

DEFINITION 5 (SECOND MODE-

A tensor. For a tensor

DEFINITION 5 (SECOND MODE-

MATRICIZATION [27])

For a tensor \( X \in \mathbb{R}^{l_1 \times l_2 \times \cdots \times l_k} \), its second mode-

MATRICIZATION

DEFINITION 6 (MODE-k PERMUTATION [122])

For a tensor \( X \in \mathbb{R}^{l_1 \times l_2 \times \cdots \times l_k} \), this operator, noted by \( X^k \) = permutation(\( X, k \)), changes its permutation order \( k \) times and

\[
G^{(n+1)}(I_1, I_2, \ldots, I_n) = G^{(n)}(I_1, I_2, \ldots, I_n, I_{n+1})
\]

for \( i_1 = 1, 2, \ldots, I_n; j_{i+1} = 1, 2, \ldots, I_{i+1} + 1 \).

From this, the multinomial product of all the TR factors can be induced as

\[
G = g^{(1)}g^{(2)}g^{(3)} \cdots g^{(n)} \in \mathbb{R}^{l_1 \times l_2 \times \cdots \times l_n}
\]

The TR decomposition can be rewritten as

\[ X = \Phi(G) \]

where \( \Phi \) is a dimensional shifting operator.
## TABLE 2. THE LISTS OF REAL HS DATASETS.

| H S A P P L I C A T I O N | D A T A S E T | S O U R C E | D E S C R I P T I O N | O R I G I N A T E S F R O M |
|-------------------------|-------------|-----------|--------------------|--------------------------|
| HS restoration          | Indian Pine | AVIRIS    | 145 × 145 pixels with 224 spectral bands in the wavelength range of 400–2,500 nm | https://www.ehu.eus/ccwintco/index.php/Hyperspectral_Remote_Sensing_Scenes |
|                         | Salinas     | AVIRIS    | 512 × 217 pixels and 224 spectral bands with a spatial resolution of 3.7 m | https://www.ehu.eus/ccwintco/index.php/Hyperspectral_Remote_Sensing_Scenes |
|                         | Pavia Center | ROSIS    | 1,096 × 1,096 pixels and 102 spectral bands with a spatial resolution of 1.3 m | https://www.ehu.eus/ccwintco/index.php/Hyperspectral_Remote_Sensing_Scenes |
|                         | Pavia University | ROSIS | 610 × 610 pixels and 103 spectral bands with a spatial resolution of 1.3 m | https://www.ehu.eus/ccwintco/index.php/Hyperspectral_Remote_Sensing_Scenes |
|                         | Kennedy Space Center | AVIRIS | 512 × 614 and 176 spectral bands with center wavelengths from 400 to 2,500 nm | https://www.ehu.eus/ccwintco/index.php/Hyperspectral_Remote_Sensing_Scenes |
|                         | Botswana    | HyMap     | 1,476 × 256 pixels and 145 spectral bands covering the 400–2,500-nm range | https://engineering.purdue.edu/~biehl/MultiSpec/hyperspectral.html |
|                         | Washington, D.C. | HYDICE | 1,280 × 307 pixels and 191 spectral bands covering the 400–2,500-nm range | https://www1.cs.columbia.edu/~cscll/remote-sensing-datasets/database/HS-MS fusion |
| HS CS                   | CAVE        | Time-multiplexed 31-channel camera | 32 scenes containing 512 × 512 pixels and 31 spectral bands | http://xudongkang.weebly.com/data-sets.html |
|                         | Harvard     | Time-multiplexed 31-channel camera | 1,040 × 1,392 pixels and 31 spectral bands covering the 420–720-nm range | https://aviris.jpl.nasa.gov/data/free_data.html |
|                         | Moffett Field | AVIRIS | 224 of the spectral channels covering the 400–2,500-nm range | https://aviris.jpl.nasa.gov/data/free_data.html |
|                         | Low Altitude | AVIRIS | 224 of the spectral channels covering the 400–2,500-nm range | https://aviris.jpl.nasa.gov/data/free_data.html |
| HS AD                   | San Diego   | AVIRIS    | 307 × 307 pixels and 210 spectral bands with a spatial resolution of 3.5 m | https://www.doi.org/10.1109/TGRS.2015.2493201 |
|                         | HyMap image | HyMap     | 280 × 800 pixels and 126 spectral bands with a spatial resolution of 3 m | http://dirsapps.cis.rit.edu/blindtest/... |
|                         | Cambridge Research and Instrumentation image scene | Nuance | 400 × 400 pixels and 46 spectral bands | https://www.doi.org/10.1007/s10618-010-0182-x |
|                         | MUUFL Gulfport | Gemini lidar and ITRES CASI-1500 | 325 × 220 and 64 spectral bands | https://www.ehu.eus/ccwintco/index.php/Hyperspectral_Remote_Sensing_Scenes |
|                         | Airpor Beach Urban | AVIRIS/ROSIS | Four airports, four beaches, and five urban scenes, with 100 × 100 and 150 × 150 pixels | http://xudongkang.weebly.com/data-sets.html |
| HS--MS fusion           | Xuchang City | Gaofen-2 and Gaofen-5 | 2,083 × 2,008 pixels, with 330 channels in the HS image regime, and 7,304 × 7,304 pixels, with four channels in the MS image regime | https://www.doi.org/10.1109/TGRS.2021.3135501 |
|                         | Lafayette   | Sentinel-2A and Hyperion | 341 × 365 pixels, with 83 channels in the HS image regime, and 1,023 × 1,095 pixels, with four channels in the MS image regime | https://www.doi.org/10.3390/rs10050800 |
|                         | University of Houston | Optech D-8900 and ITRES CASI 1500 | 4,172 × 1,202 pixels, with 48 channels in the HS image regime, and 83,440 × 24,040 pixels, with three channels in the MS image regime | https://hyperspectral.ee.uh.edu/ |
|                         | Paris       | Advanced Land Imager and Hyperion | HS images with a spatial resolution of 30 m and MS images with a spatial resolution of 30 m | https://eospso.gsfc.nasa.gov/ http://eol.usgs.gov/sensors/ali http://eol.usgs.gov/sensors/hyperioncoverage |
|                         | Hong Kong   | Terra/Aqua MODIS and Landsat 7 Enhanced Thematic Mapper Plus | HS images with 36 channels and MS images with six channels | https://www.doi.org/10.1109/TGRS.2013.2253612 |
| HS SU                   | Cuprite     | AVIRIS    | 250 × 190 pixels with 224 spectral bands ranging from 370 to 2,480 nm | https://www.ehu.eus/ccwintco/index.php/Hyperspectral_Remote_Sensing_Scenes... |
|                         | Samson      | Samson    | 952 × 952 pixels and 156 spectral bands covering the wavelengths from 401 to 889 nm | https://rslab.ut.ac.ir/~remote-sensing-datasets |
|                         | Japser Ridge | AVIRIS | 512 × 614 pixels and 224 spectral bands ranging from 380 to 2,500 nm | https://rslab.ut.ac.ir/~remote-sensing-datasets |
|                         | Urban       | HYDICE    | 307 × 307 pixels and 210 spectral bands ranging from 400 to 2,500 nm | https://rslab.ut.ac.ir/~remote-sensing-datasets |

AVIRIS: Airborne Visible/Infrared Imaging Spectrometer; ROSIS: Reflective Optics System Imaging Spectrometer; HYDICE: Hyperspectral Digital Imagery Collection Experiment; MODIS: Moderate-Resolution Imaging Spectroradiometer.
**LEMMA 1 (CIRCULAR DIMENSIONAL PERMUTATION INVARiance [27])**

If the TR decomposition of $X$ is $X = \Phi(G^{(1)}, G^{(2)}, \ldots, G^{(N)})$, $\bar{X} \in \mathbb{R}^{l_1 \times l_2 \times \cdots \times l_N}$ is defined as circular shifting the dimensions of $X$ by $n$, we obtain the following relation:

$$\bar{X} = \Phi(G^{(1)}, G^{(n+1)}, \ldots, G^{(N)}, G^{(1)}, G^{(2)}, \ldots, G^{(n)}).$$  \hspace{1cm} (5)

**DEFINITION 8 (MIXED $l_{1,0}$ PSEUDONORM [124])**

Given a vector $y \in \mathbb{R}^m$ and index sets $\theta_1, \ldots, \theta_n (1 \leq n \leq m)$, where each $\theta_i$ is a subset of $1, \ldots, m$, $\theta_i \cap \theta_j = \emptyset$ for any $i \neq j$, and $\bigcup_{i=1}^{n} \theta_i = 1, \ldots, m$, the mixed $l_{1,0}$ pseudonorm of $y$ is defined as

$$\|y\|_{1,0}^{\theta} = \|y_{\theta_1}, \ldots, y_{\theta_n}, y_{\bar{\theta}}\|_0$$  \hspace{1cm} (6)

where $y_{\theta_i}$ denotes a subvector of $y$, with its entries specified by $\theta_i$, and $\|\|_0$ calculates the number of the nonzero entries in $()$.

**DEFINITIONS OF COMMON TENSOR DECOMPOSITIONS**

Before providing a comprehensive survey of five practical HS applications, we summarize eight types of tensor decompositions widely used in HS data processing. Practitioners can obtain a basic theoretical knowledge of the relevant background in preparation for the following research.

**TENSOR TRACE NORM**

This is the sum of the nuclear norm (SNN) of the mode-$k$ unfolding matrix for a three-way HS tensor [125]:

$$\|X\|_{\text{SNN}} := \sum_{k=1}^{n} a_k \|X_{(k)}\|$$  \hspace{1cm} (7)

where weights $a_k$ satisfy $a_k \geq 0 (k = 1, 2, 3)$ and $\sum_{k=1}^{n} a_k = 1$.

**TUCKER DECOMPOSITION**

The Tucker decomposition of an $N$-order tensor $X \in \mathbb{R}^{l_1 \times l_2 \times \cdots \times l_N}$ is defined as [20], [126], [127]

$$X = A \times_1 B_1 \times_2 B_2 \cdots \times_N B_N$$  \hspace{1cm} (8)

where $A \in \mathbb{R}^{l_1 \times r_1 \times \cdots \times r_N}$ stands for a core tensor and $B_k \in \mathbb{R}^{l_k \times l_k}$, $n = 1, 2, \ldots, N$ represent factor matrices. The Tucker ranks are represented by rank$_{\text{Tucker}}(X) = [r_1, r_2, \ldots, r_N]$.

Here, one variation of Tucker decomposition is worth noting. The “Tucker 1” decomposition of a third-order array sets two of the factor matrices to be the identity matrix:

$$X = A \times_1 B_1$$  \hspace{1cm} (9)

which is equivalent to standard 2D principal component analysis (PCA) since

$$X_{(1)} = B_1 \times_1 A_{(1)}. \hspace{1cm} (10)$$

“Tucker 1” can find those components that obtain the variation in mode $n$, independent of the other modes, later known as higher-order SVD (HOSVD). Lathauwer et al. [126] were convinced that HOSVD is a generalization of the matrix SVD and discussed how to compute the leading left singular vectors of $X_{(n)}$. Lathauwer et al. [128] proposed and named the higher-order orthogonal iteration. This more efficient technique iteratively calculates factor matrices, specifically computing only the dominant singular vectors of $X(n)$ and using an SVD rather than an eigenvalue decomposition or even just an orthonormal basis of the dominant subspace.

**CANONICAL DECOMPOSITION/PARALLEL FACTOR ANALYSIS DECOMPOSITION**

The CP decomposition of an $N$-order tensor $X \in \mathbb{R}^{l_1 \times l_2 \times \cdots \times l_N}$ is defined as [20], [129], [130]

| TABLE 3. THE NOTATIONS USED IN THIS ARTICLE. |
|---------------------------------------------|
| **NOTATION** | **DESCRIPTION** |
| $X$ | Scalars |
| $x$ | Vectors |
| $\mathbf{X}$ | Matrices |
| $\text{vec}(X)$ | $\text{vec}(X)$ stacks the columns of $X$ |
| $\theta_i$ | Index sets |
| $x_{\theta_i}$ | Subvector of $x$, with its entries specified by $\theta_i$ |
| $X_{(i)}$ | Tensors with three modes |
| $1$ | Identity tensor |
| $X_{(h,v,z)}$ | $(h,v,z)$ element of $X$ |
| $X(:,:,i)$ | $i$th horizontal, lateral, and frontal slices |
| $\|X\|_{I_1}$ | $I_1$ norm |
| $\|X\|_{F}$ | Frobenius norm |
| $\|X\|_{\text{TNN}}$ | Tensor nuclear norm |
| $\sigma_i(X)$ | Singular values of matrix $X$ |
| $\|X\|_{\text{SN}}$ | Nuclear norm |
| $\|x\|_{I_1}$ | $I_1$ norm |
| $\|x\|_{I_2}$ | $I_2$ norm |
| $\hat{X}$ | Fourier transformation of $X$ along mode 3 |
| $X_{(3)}$ | Unfold($X_3$) |
| $X_{(k)}$ | Second mode-$k$ unfolding |
| $X^{(k)}$ | Mode-$k$ permutation |
| $\hat{X}^{(k)}$ | Circular shifting the dimensions of $X$ by $n$ |
| $[G]$ | Multiproduct of $G^{(n)}$ |
\[ X = \sum_{r=1}^{R} \tau_r \cdot b^{(1)}_r \cdot b^{(2)}_r \cdots b^{(N)}_r \]  

(11)

where \( \tau_r \) are nonzero weight parameters and \( b^{(1)}_r \cdot b^{(2)}_r \cdots b^{(N)}_r \) denotes a rank 1 tensor, with \( b^{(i)}_r \in \mathbb{R}^{p_i} \). The CP rank denoted by \( \text{rank}_{\text{CP}}(X) = R \) is the sum of the rank 1 tensors. The core consistency diagnostic (CORCONDIA) has gained popularity as a useful tool for calculating the number of components in parallel factor analysis and Tucker decomposition models. Several modifications for CORCONDIA have been proposed to improve efficiency [131], [132]. To address this problem of rank estimation, Shi et al. [133] regarded the weight vector of the orthogonal CP decomposition as the vector of singular values. Das et al. [134] proposed a noise-invariant tensor-based rank estimation approach and verified the efficiency of our proposed estimation in noise cases.

**BLOCK TERM DECOMPOSITION**

The BT decomposition of a three-order tensor \( X \in \mathbb{R}^{k \times s \times x} \) is defined as [135]

\[ X = \sum_{i=1}^{R} G_i \times_1 A_i \times_2 B_i \times_3 C_i \]  

(12)

where \( G_i \) is an \( f \)-diagonal tensor. The details of t-SVD are described in Algorithm 1.

**TABLE 4. THE MAIN ABBREVIATIONS USED IN THIS ARTICLE.**

| ABBREVIATION | FULL NAME |
|--------------|-----------|
| AO           | Anomaly detection |
| ADMM         | Alternating direction method of multipliers |
| AVIRIS       | Airborne Visible/Infrared Imaging Spectrometer |
| CP           | Canonical decomposition/parallel factor analysis |
| CS           | Compressive sensing |
| BT           | Block term |
| HOSVD        | Higher-order SVD |
| HYDICE       | Hyperspectral Digital Imagery Collection Experiment |
| HS           | Hyperspectral |
| HR           | High resolution |
| LMM          | Linear mixing model |
| LR           | Low rank |
| MS           | Multispectral |
| MODIS        | Moderate-Resolution Imaging Spectroradiometer |
| NMF          | Nonnegative matrix factorization |
| PCA          | Principal component analysis |
| ROSIS        | Reflective Optics System Imaging Spectrometer |
| RS           | Remote sensing |
| SNN          | Sum of the nuclear norm |
| SU           | Spectral unmixing |
| SVD          | Singular value decomposition |
| TNN          | Tensor nuclear norm |
| t-SVD        | Tensor SVD |
| TV           | Total variation |
| TT           | Tensor train |
| TTN          | Tensor trace norm |
| TR           | Tensor ring |

**TENSOR NUCLEAR NORM**

Let \( X = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V} \) be the t-SVD of \( X \in \mathbb{R}^{k \times s \times x} \). The tensor nuclear norm (TNN) is the sum of singular values of \( X \), that is,

\[ \|X\| := \sum_{k=1}^{s} S(k, k, 1) \]  

(15)

and it can also be expressed as the SNN of all the frontal slices of \( \hat{X} \):

\[ \|X\| := \sum_{k=1}^{s} \|\hat{X}(:, :, k)\| \]  

(16)

For a more intuitive understanding of the preceding tensor decompositions, examples of third-order tensors are provided in Figure 3, which benefits the consequent tensor decomposition-based research of third-order HS data.

**TENSOR SINGULAR VALUE DECOMPOSITION**

Here, \( X \in \mathbb{R}^{k \times s \times x} \) can be factorized by t-SVD as [54]

\[ X = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V} \]  

(17)

where \( \mathbf{U} \in \mathbb{R}^{k \times x}, \mathbf{V} \in \mathbb{R}^{x \times x} \) are orthogonal tensors and \( \mathbf{S} \in \mathbb{R}^{s \times x} \) is an f-diagonal tensor. The details of t-SVD are described in Algorithm 1.

**TENSOR TRAIN DECOMPOSITION**

The TT decomposition [26] of an N-order \( X \in \mathbb{R}^{l_1 \times l_2 \times \cdots \times l_N} \) is represented by cores \( \mathcal{G} = \{G^{(1)}, \ldots, G^{(N)}\} \), where \( G^{(n)} \in \mathbb{R}^{l_1 \times l_2 \times \cdots \times l_N} \), \( n = 1, 2, \ldots, N \), and \( r_0 = r_N = 1 \). The rank of TT decomposition is defined as \( \text{rank}_{\text{TT}}(X) = \{r_0, r_1, \ldots, r_N\} \). Each entry of the tensor \( X \) is formulated as

\[ X(i_1, \ldots, i_N) = G^{(1)}(:, i_1, :)G^{(2)}(:, i_2, :) \ldots G^{(N)}(:, i_N, :) \]  

(18)
TENSOR RING DECOMPOSITION

The purpose of TR decomposition [27] is to represent a high-order \( X \) by multilinear products of a sequence of three-order tensors in circular form. Three-order tensors are named TR factors \( \{G^{(n)}\}_{n=1}^{N} = \{G^{(1)}, G^{(2)}, \ldots, G^{(N)}\} \), where \( G^{(n)} \in \mathbb{R}^{h \times v \times w} \), \( n = 1, 2, \ldots, \) and \( N, r_0 = r_N \). In this case, the element-wise relationship of TR decomposition with factors \( G \) can be written as

\[
X(i_1, i_2, \ldots, i_N) = Tr(G^{(1)}(:, i_1) G^{(2)}(:, i_2) \cdots G^{(N)}(:, i_N))
\]

\[
= Tr(\prod_{i=1}^{N} G^{(n)}(:, i_n))
\]

(19)

where \( Tr \) denotes the matrix trace operation.

HYPERSONICAL RESTORATION

In the actual process of HS data acquisition and transformation, the observed HS images are often degraded by different sources related to external environmental change and internal equipment conditions. The atmosphere, through absorption, scattering, clouds, and the reflection of solar radiation, can have a significant impact on HS RS data. Artifacts and noise are produced by imaging systems, such as striping in HS push broom imaging systems. These inevitably lead to noises, blurs, and missing data (including clouds and stripes) [136], [137], which degrades the visual quality of HS images and efficiency of subsequent HS data applications, such as a fine HS RS classification for crops and wetlands [138], [139] and the refinement of spectral information for target detection [140], [141]. Figure 4 depicts HS RS degradation and restoration. Therefore, HS image restoration appears as a crucial preprocessing step for further applications.

Mathematically, an observed degraded HS image can be formulated as follows:

\[
T = M(X) + S + N
\]

(20)

where \( T \in \mathbb{R}^{h \times v \times z}, X \in \mathbb{R}^{h \times v \times z}, S \in \mathbb{R}^{h \times v \times z} \), and \( N \in \mathbb{R}^{h \times v \times z} \) represent an observed HS image, the restored HS image, the sparse error, and the additive noise, respectively. The additive noise is modeled as an independent signal, usually

---

**FIGURE 3.** Six tensor decompositions of a third-order tensor: (a) Tucker decomposition, (b) CP decomposition, (c) BT decomposition, (d) t-SVD, (e) TT decomposition, and (f) TR decomposition.
known as Gaussian noise, when massive photons are collected and with a signal-dependent variance in a low-flux regime [136].

Imaging results are degraded to varying degrees by various sources in different imaging conditions, resulting in a variety of HS restoration issues, such as HS image denoising, HS image blurring, HS image inpainting, and HS image destriping. Here, $M(\cdot)$ denotes different linear degradation operators for different HS restoration problems:

1) When $M(X)$ keeps $X$ constant, i.e., $M(X) = X$, (20) is reformulated as a HS destriping problem ($T = X + S$) or HS denoising problem (consider only Gaussian noise $T = X + N$ or mixed noise $T = X + S + N$).

2) When $M(\cdot)$ is a binary operation, i.e., one for original pixels and zero for missing data, (20) turns into a HS inpainting problem.

3) When $M(\cdot)$ is a blur kernel, also called a point spread function (PSF), (20) becomes an HS deblurring problem.

The HS restoration task is to estimate recovered HS images $\hat{X}$ from the given HS images $T$. This ill-posed problem suggests that extra constraints on $X$ need to be enforced for the optimal solution of $X$. These additional constraints reveal the HS desired property and various types of HS prior information, such as nonlocal similarity, spatial and spectral smoothness, and subspace representation. The HS restoration problem can be summarized as

$$\min \frac{1}{2} \| T - M(X) - S \|_F^2 + \tau f(X) + \lambda g(S)$$

(21)

where $\tau$ and $\lambda$ are regularization parameters and $f(X)$ and $g(S)$ stand for the regularizations to explore the desired properties on the recovered $X$ and sparse part $S$, respectively. Various regularization aspects are summarized and divided into different types for systematic learning in the “Rank Minimization Approaches” section. For example, sparsity measurement is critical in HS reconstruction tasks. Willett et al. [142] reviewed the state of the art of HS image models, with an emphasis on sparse regularizations. Das et al. [143] designed an efficient criterion to explore multiway sparsity for HS noise removal. Das et al. [144] developed a novel semisupervised parameter-free algorithm for library pruning that made use of sparsity measures, which verified the computational efficiency and proficiency of the proposed method in the presence of noise.

**Hyperspectral Denoising**

Observed HS images are often corrupted by mixed noise, including Gaussian noise, salt-and-pepper noise, and dead line noise. Several noise types of HS images are available in Figure 5. Figure 5(a) shows an urban dataset containing mixed noise, especially heavy stripes and dead lines. These two types of noise have a directional and linear structure, destroying the inherent structure of the original HS image. As shown in Figure 5(b), the Indian Pines dataset is corrupted by salt-and-pepper noise and Gaussian noise. The salt-and-pepper noise damages the spectral characteristics of the original signal and causes
the spectral curve to be severely distorted. In Figure 5(c), the Gaussian noise is densely spread over the whole image of the Salinas dataset, resulting in severe blurring of the image edge information.

The wealth of spatial and spectral information in HS images can be extracted by different prior constraints, such as the LR property, sparse representation (SR), nonlocal similarity, and total variation (TV). Different LR tensor decomposition (LRTD) models are introduced for HS denoising. Consequently, one or two kinds of other prior constraints are combined with these tensor decomposition models.

LOW-RANK TENSOR DECOMPOSITION

In this section, LRTD methods are divided into two categories: 1) factorization-based approaches and 2) rank minimization-based approaches. The former need to predefined rank values and update decomposition factors. The latter directly minimize tensor ranks and update LR tensors.

FACTORIZATION-BASED APPROACHES

Two typical representatives are used in the HS image denoising literature, namely, Tucker decomposition and CP decomposition. Renard et al. [23] considered Gaussian noise and suggested an LR tensor approximation (LRTA) model to complete an HS image denoising task:

\[
\min_{\mathcal{T}, \mathcal{X}} \| \mathcal{T} - \mathcal{X} \|^2 \\
\text{s.t. } \mathcal{X} = \mathcal{A} \times_1 \mathbf{B}_1 \times_2 \mathbf{B}_2 \times_3 \mathbf{B}_3
\] (22)

where \( \mathcal{A} \) represents a core tensor and \( \mathbf{B}_1, \mathbf{B}_2, \) and \( \mathbf{B}_3 \) denote factor matrices. Nevertheless, users should manually predefine the multiple ranks along all modes before running the Tucker decomposition-related algorithm, which is intractable in reality. In (22), the Tucker decomposition constraint is easily replaced by another tensor decomposition, such as CP decomposition. Liu et al. [24] used a parallel factor analysis decomposition algorithm and still assumed that HS images were corrupted by white Gaussian noise. Guo et al. [28] presented an HS image noise reduction model via rank 1 tensor decomposition, which was capable of extracting the signal-dominant features. However, the smallest number of rank 1 factors was served as the CP rank, which has a high computation cost to be calculated.

RANK MINIMIZATION APPROACHES

Tensor rank bounds are rarely available in many HS noisy scenes. To avoid the occurrence of rank estimation, another kind of method focuses on minimizing the tensor rank directly, which can be formulated as follows:

\[
\min_{\mathcal{X}} \text{rank}(\mathcal{X}) \\
\text{s.t. } \mathcal{T} = \mathcal{X} + \mathcal{S} + \mathcal{N}
\] (23)

where \( \text{rank}(\mathcal{X}) \) denotes the rank of HS tensor \( \mathcal{X} \) and includes different rank definitions, such as Tucker rank, CP rank, TT rank, and tubal rank. Due to the preceding rank minimizations belonging to nonconvex problems, these problems are NP-hard to compute. Nuclear norms are generally used as the convex surrogate of nonconvex rank function. Zhang et al. [121] proposed a tubal rank-related TNN to characterize the 3D structural complexity of multilinear data. Based on the TNN, Fan et al. [16] presented an LR tensor recovery (LRTR) model to remove Gaussian noise and sparse noise:

\[
\min_{\mathcal{X}, \mathcal{S}, \mathcal{N}} \| \mathcal{X} \|_1 + \lambda_1 \| \mathcal{S} \|_1 + \lambda_2 \| \mathcal{N} \|_2
\]
\[
\text{s.t. } \mathcal{T} = \mathcal{X} + \mathcal{S} + \mathcal{N}
\] (24)

where parameter \( \lambda_1 \) controls the strength of sparse noise \( \mathcal{S} \) and parameter \( \lambda_2 \) is used to adjust the additive Gaussian noise strength.

The alternating direction method of multipliers (ADMM) framework has become popular to solve constrained optimization problems, such as (24), of tensor decomposition for HS data processing. Auxiliary variables are introduced in the ADMM. An equivalent problem is derived with a separable unconstrained function, which is subject to a linear compatibility constraint between the original and auxiliary variables [145]. The original variables, auxiliary variables, and dual variables are alternately updated to solve the converted problem. Assisted by proper auxiliary variables, each update step reduces to a simple subproblem whose solution is often found with closed-form terms. Meanwhile, the ADMM hardly depends on the smoothness of the optimization problem and quickly converges to one optimal solution with moderate accuracy [146]. Therefore, the ADMM has become an attractive choice for solving large-scale optimization problems, such as tensor decomposition and HS data processing.

Xue et al. [29] applied a nonconvex logarithmic surrogate function to a tensor trace norm (TTN) for tensor completion (TC) and tensor robust PCA tasks. Zheng et al. [18] explored the LR properties of tensors along three directions and proposed two tensor models: a three-directional TNN and three-directional log-based TNN as its convex

**FIGURE 5.** HS datasets with different noise types: (a) heavy stripes and dead lines on the urban dataset, (b) salt-and-pepper noise and Gaussian noise on the Indian Pines dataset, and (c) heavy Gaussian noise on the Salinas dataset.
and nonconvex relaxation. Although these pure LRTD approaches utilize the LR prior knowledge of HS images, they are hardly effective to suppress mixed noise, due to the lack of other useful information.

OTHER PRIORS—REGULARIZED LOW-RANK TENSOR DECOMPOSITION

Various types of priors are combined with an LRTD model to optimize the model solution, including nonlocal similarity, spatial sparsity, spatial and spectral smoothness, and subspace representation.

NONLOCAL SIMILARITY

An HS image often possesses many repetitive local spatial patterns, and thus, a local patch always has many similar patches across an HS image [147]. Peng et al. [148] designed a tensor dictionary learning (TDL) framework. In Figure 6, an HS image is segmented into 3D full-band patches (FBPs). Similar FBPs are clustered together as a 4D tensor group to simultaneously leverage the nonlocal similarity of spatial patches and spectral correlation. TDL is the first model to exploit the nonlocal similarity and the LR tensor property of 4D tensor groups, as demonstrated in Figure 6(b). Instead of a traditional alternative least-squares-based Tucker decomposition, Bai et al. [30] improved a hierarchical least-squares-based nonnegative Tucker decomposition method. Kong et al. [31] incorporated weighted tensor norm minimization into the Tucker decompositions of 4D patches.

Differing from [30], [31], and [148], other works [32], [33], [34], [35], [36], [38] obtained a 3D tensor by stacking all nonlocal similar FBPs converted as matrices with a spatial mode and spectral mode [Figure 6(d)]. Based on a nonlocal similar framework, Dong et al. [32] proposed a Laplacian scale mixture-regularized LRTA method for denoising. Xie et al. [33] conducted a tensor sparsity regularization, intrinsic tensor sparsity (ITS), to encode the spatial and spectral correlation of nonlocal similar FBP groups. With the nonlocal similarity of FBPs, X is estimated from its corruption T by solving the following problem:

$$\min_{X} \Lambda(X) + \frac{1}{2}\|T - X\|_F^2$$

(25)

where the sparsity of a tensor X is $$\Lambda(X) = \|A\|_0 + (1 - t)\Pi_{n=1}^N \text{rank}(X_{[n]})$$ and A is the core tensor of X via the Tucker decomposition $$X = AX_1B_1 \times_2 B_2 \times_3 B_3$$. Xue et al. [34] presented a nonlocal LR (NLR)-regularized CP tensor decomposition algorithm. However, the Tucker and CP decomposition-related methods are subject to heavy computational burden issues.

Chang et al. [35] discovered the LR property of nonlocal patches and used a hyper-Laplacian prior to model additional spectral information. He et al. [36] developed a new paradigm, called the nonlocal meets global (NGmeet) method, to fuse the spatial nonlocal similarity and global spectral LR property. Chen et al. [38] analyzed the advantages of a novel TR decomposition over Tucker and CP decompositions. The proposed nonlocal TR (NLTR) decomposition method for HS image denoising is formulated as

$$\min_{X, g} \frac{1}{2}\|T - X\|_F^2 \quad \text{s.t.} \quad X = \Phi([g])$$

(26)

The nonlocal similarity-based tensor decomposition methods focus on removing Gaussian noise from corrupted HS images and unavoidably cause a computational burden in practice.

SPATIAL AND SPECTRAL SMOOTHNESS

HS images are usually captured by airborne and spaceborne platforms far from Earth’s surface. The low measurement accuracy of imaging spectrometers leads to low spatial resolutions of HS images. In general, the distribution of ground objects varies. Moreover, high correlations exist among different spectral bands. HS images always have relatively smoothing characteristics in the spatial and spectral domains.

An original TV method was proposed by Rudin et al. [8] to remove the noise of gray-level images, due to the ability to preserve edge information and promote piecewise smoothness. HS image smoothness can be constrained by either an isotropic TV norm or an anisotropic TV norm [149]. Obvious blurring artifacts are hardly eliminated in the denoised results of the isotropic model [150]. Thus, anisotropic TV norms for HS image denoising are investigated in this article. We take the Washington, D.C. (WDC)
dataset as a typical example to depict the gradient images along three directions in Figure 7. The smoothing areas and edge information of gradient images are much clearer than the origin.

Inspired by TV applications to gray-level images, the 2D spatial TV norm of $\mathcal{X}$ is easily introduced to an HS image in a band-by-band manner [13]. This simple band-by-band TV norm is defined as follows:

$$\|\mathcal{X}\|_V = \|D_h\mathcal{X}\| + \|D_v\mathcal{X}\|,$$  \hspace{1cm} (27)

where $D_h$ and $D_v$ stand for first-order linear difference operators corresponding to the horizontal and vertical directions, respectively. These two operators are usually defined as

$$\|D_h\mathcal{X}\| = \left\{ \begin{array}{ll} \mathcal{X}(i, j, k+1) - \mathcal{X}(i, j, k), & 1 \leq j < v \\ 0, & j = v \end{array} \right. \hspace{1cm} (28)$$

$$\|D_v\mathcal{X}\| = \left\{ \begin{array}{ll} \mathcal{X}(i+1, j, k) - \mathcal{X}(i, j, k), & 1 \leq i < h \\ 0, & i = h \end{array} \right. \hspace{1cm} (29)$$

To enforce the spatial piecewise smoothness and spectral consistency of HS images, a 3DTV norm [149] and spatial-spectral TV (SSTV) norm [151] are formulated, respectively:

$$\|\mathcal{X}\|_{\text{3DTV}} = \|D_h\mathcal{X}\| + \|D_v\mathcal{X}\| + \|D_z\mathcal{X}\|,$$  \hspace{1cm} (30)

$$\|\mathcal{X}\|_{\text{SSTV}} = \|D_z\mathcal{X}\|,$$  \hspace{1cm} (31)

where $\|D_z\mathcal{X}\|$ is a 1D finite-difference operator along the spectral direction and defined as

$$\|D_z\mathcal{X}\| = \left\{ \begin{array}{ll} \mathcal{X}(i, j, k+1) - \mathcal{X}(i, j, k), & 1 \leq i < h \\ 0, & k = z \end{array} \right. \hspace{1cm} (32)$$

Considering the degraded model with mixed noise, Chen et al. [39] integrated both 2DTV and the 3DTV regularizations into the TNN. Fan et al. [17] injected the preceding SSTV norm into LR tensor factorization. Wang et al. [152] used an SSTV term in a multidirectional weighted LR tensor framework. Based on the different contributions of the three gradient terms to 3DTV regularization, Wang et al. [41] proposed the TV-regularized LRTD method:

$$\min_{\mathcal{X}, S, N} \gamma \|\mathcal{X}\|_{\text{3DTV}} + \lambda \|S\| + \beta \|N\|$$  \hspace{1cm} (33)

subject to

$$\mathcal{X} = \mathcal{X} + S + N,$$

where

$$\|\mathcal{X}\|_{\text{3DTV}} = w_1 \|D_h\mathcal{X}\| + w_2 \|D_v\mathcal{X}\| + w_3 \|D_z\mathcal{X}\|.$$  \hspace{1cm} (34)

Zeng et al. [42] integrated the advantages of both a global $L_{1,\gamma}$ SSTV and the local patch TNN. Chen et al. [43] exploited the row-sparse structure of gradient images and proposed a weighted group sparsity-regularized TV combined with LR Tucker decomposition (LRTDGS) for HS mixed-noise removal.

Due to the mentioned TV norms penalizing only large gradient magnitudes and easily blurring real image edges, a new $l_0$ gradient minimization was proposed to sharpen image edges [152]. Actually, the $l_1$ TV norm is a relaxation form of the $l_0$ gradient. Xiong et al. [44] and Wang et al. [40] applied the $l_0$ gradient constraint in an LR BT decomposition and Tucker decomposition, respectively. However, the degrees of smoothness of this $l_0$ gradient form are controlled by a parameter without any physical meaning. To alleviate this limitation, Ono [124] proposed a novel $l_0$ gradient projection that directly adopts a parameter to represent the smoothing degree of the output image. Wang et al. [45] extended the $l_0$TV model into an LR tensor framework to preserve more information for classification tasks after HS image denoising [154], [155]. The optimization model of TLR-$l_0$TV is formulated as

$$\min_{\mathcal{X}, S, T} \sum_{k=1}^{K} \alpha_k E_k(\mathcal{X})_a + \lambda \|S\| + \beta \|T\|,$$  \hspace{1cm} (35)

subject to

$$\|BD\mathcal{X}\|_a \leq \gamma$$

where the functions $E_k(\mathcal{X})_a$ are set to be $\|\mathcal{X}_{(k)}\|_{\text{ss}}$ in the weighted sum of weighted nuclear norm (WSWN)-$l_0$-TV-based method and $\|\mathcal{X}_{(k)}\|_{\text{ss}}$ in the weighted sum of weighted tensor nuclear norm (WSWNN)-$l_0$-TV-based method. Operator $B$ forces boundary values of gradients to be zero when $i = h$ and $j = v$. Operator $D$ is an operator to calculate both horizontal and vertical differences. Compared with many other TV-based LRTD, TLR-$l_0$TV achieves better denoising performance for mixed-noise removal in

---

**FIGURE 7.** The spatial smooth properties of the WDC dataset: (a) the original band, (b) the gradient image along the spatial horizontal direction, (c) the gradient image along the spatial vertical direction, and (d) the gradient image along the spectral direction.
HS images. In particular, HS classification accuracy is improved more effectively after denoising by TLR-l0TV.

**SUBSPACE REPRESENTATION**

As Figure 8 illustrates, an unfolding matrix $X$ of a denoised HS image can be projected into an orthogonal subspace; i.e., $X = EZ$. Here, $E \in \mathbb{R}^{2 \times l}$ represents the basis of the subspace $S_l$, and $Z \in \mathbb{R}^{l \times h}$ denotes the representation coefficient of $X$ with respect to $E$. Also, $E$ is reasonably assumed to be orthogonal; i.e., $E^T E = I$ [156].

Cao et al. [46] combined an LR and sparse factorization with the nonlocal tensor constraint of subspace coefficients (SNLRTSF). Each spectral band of an observed HS image $T \in \mathbb{R}^{h \times w \times l}$ is reshaped as each row of an HS unfolding matrix $T \in \mathbb{R}^{2 \times h \times l}$. The spectral vectors are assumed to lie in a $l$-dimensional subspace $S_l \ (l \ll z)$, and the optimization model can be written as

$$
\arg\min_{T, E, Z} \frac{1}{2} \| T - EZ - S \|_F^2 + \lambda_1 \| S \|_1 + \lambda_2 \| \epsilon \|_1 + \| L_c \|_{TV} \\
\text{s.t. } E^T E = I
$$

![Figure 8. The subspace representation.](image)

**EXPERIMENTAL RESULTS AND ANALYSIS**

**GAUSSIAN NOISE CASE**

An HS subimage is selected from the Pavia University dataset, which can be found on and downloaded from the website in Table 2. Zero-mean Gaussian noise of noise variance 0.12 is added to each band and shown in Figure 9(b). Five effective denoising methods are selected, including LRTA (Tucker decomposition), TDL (Tucker decomposition with dictionary learning), ITS (CP and Tucker decomposition), Hyper-laplacian regularized unidirectional low-rank tensor recovery (LLRT) (with nonlocal similarity), and NGmeet (with subspace representation). These denoising algorithms are based on the Gaussian noise degradation model.

**MIXED-NOISE CASE**

The same Gaussian noise is adopted. Each band is corrupted by salt-and-pepper noise, with a proportion of 0%–20%. Dead lines are randomly added from band 61 to band 80, with the width of the generated stripes ranging from one to three, and the number of stripes is randomly selected from three to 10. In addition, bands 61–70 are corrupted by stripes, with the number randomly selected from 20 to

| INDEX | LRTA | TDL | ITS | LLRT | NGmeet |
|-------|------|-----|-----|------|--------|
| PSNR  | 32.14| 34.54| 34.38| 35.96| 37.06  |
| SSIM  | 0.9097| 0.9484| 0.9466| 0.9637| 0.9707 |
| ERGAS | 5.7044| 4.3392| 4.3981| 4.0462| 3.2344 |
| MSAD  | 6.672 | 5.0701| 5.0912| 4.2402| 3.7804 |
| Time (s) | 1.48 | 13.77 | 650.49 | 506.84 | 29.58 |

![Figure 9. The different methods of Gaussian noise removal: (a) the original HS image, (b) Gaussian noise, (c) LRTA, (d) TDL, (e) ITS, (f) LLRT, and (g) NGmeet. LLRT: Hyper-laplacian regularized unidirectional low-rank tensor recovery.](image)
We employ five other representative approaches based on the mixed-noise degradation model to evaluate the denoising performance, quantitatively, containing LRTR (with TNN), LRTDTV (using Tucker decomposition with TV), LRTDGS (using Tucker decomposition with sparsity-regularized TV), three-directional TNN (using TNN), and TLR-L₀TV (with L₀TV).

In the preceding, we showed specific noise configurations for different noise cases to compare the performance of denoised approaches through several quantitative quality assessments. Real-world data experiments can be carried out when one denoising algorithm performs well in different simulated data experiments. The origin of the simulation and real datasets are listed in the second row of Table 2. Ten denoising methods adapt their own decompositions without/with different priors, with their code links in Table 1.

Four different quantitative quality indices are chosen: the mean of the peak signal-to-noise ratio (MPSNR), mean of structural similarity (MSSIM), relative dimensional global error in synthesis (ERGAS), and mean spectral angle distance (MSAD). Larger MPSNR and MSSIM values indicate better denoised image quality. These two indices pay attention to the restoration precision of spatial pixels. In contrast, smaller ERGAS and MSAD values illustrate better performance of denoised results.

For Gaussian noise removal, all the competing approaches achieve good results to some degree in Figure 9, in which enlarged subregions are delineated by red boxes. But residual noise remains in the result denoised by LRTA. Compared with TDL, ITS fails to preserve detailed spatial information. LLRT provides a rather similar result to NGmeet. Consistent with visual observation, NGmeet outperforms the other methods and obtains the highest metric values among the denoising models in Table 5. The NLR tensor methods, including ITSReg, TDL, and LLRT, gain better performance than LRTA, due to exploiting two types of HS prior knowledge. The LRTA method is the fastest among all the competing algorithms since LRTA considers only the spectral correlation.

Figure 10 gives the restoration results of five different methods in a heavy-noise case. Dead lines remaining in the images denoised by LRTR and three-directional TNN are more obvious than in those restored by LRTDTV and LRTDGS. The LR tensor-based model is employed in LRTR and three-directional TNN yet LRTDTV, LRTDGS, and TLR-L₀TV consider two kinds of prior knowledge: spectral correlation and spatial–spectral smoothness. LRTDTV and LRTDGS are more sensitive to dead lines than TLR-L₀TV, leading to more or fewer artifacts in the denoised results. TLR-L₀TV removes most of the mixed noise and preserves image details, such as texture information and edges. To further evaluate

**TABLE 6. A QUANTITATIVE COMPARISON OF DIFFERENT SELECTED ALGORITHMS FOR MIXED NOISE REMOVAL.**

| INDEX | MPSNR | MSSIM | ERGAS | MSAD | Time (s) |
|-------|-------|-------|-------|------|----------|
| LRTR  | 26.89 | 0.8157| 10.8842| 9.9624| 19.67    |
| LRTDTV| 30.76 | 0.8821| 7.8154 | 7.3689| 35.29    |
| LRTDGS| 30.76 | 0.7852| 9.5527 | 10.4568| 25.11    |
| THREE-DIRECTIONAL TNN | 30.2 | 0.8945 | 7.4915 | 7.4712 | 44.25    |
| TLR-L₀TV | 31.59 | 0.8973 | 7.1748 | 7.2055 | 325.67   |

**FIGURE 10.** The different methods for mixed-noise removal: (a) the original HS image, (b) mixed noise, (c) LRTR, (d) LRTDTV, (e) LRTDGS, (f) three-directional TNN, and (g) TLR-L₀TV.

**FIGURE 11.** The OLRT for different blur cases: (a) the original WDC image, (b) the light Gaussian blur on the WDC image (8 × 8; σ = 3) and (c) corresponding deblurred image, (d) the heavy Gaussian blur on the WDC image (17 × 17; σ = 7) and (e) corresponding deblurred image, and (f) the Uniform blur and (g) corresponding deblurred image.
the differences among competing denoising methods, we calculate four quality indices and include them in Table 6, with the best results in bold. TLR-LT-TV obtains the highest denoising performance among all the approaches. For MPSNR, LRTDTV, and LRTDGS, the values are slightly larger than those of three-directional TNN, whereas the SSIM and ERGAS values of LRTDTV and LRTDGS are better than those of three-directional TNN. LRTL and LRTDGS are the first- and second-fastest, but they hardly handle the complex mixed-noise case, with some dead lines remaining.

**HYPERSPECTRAL DEBLURRING**

Atmospheric turbulence and the fundamental deviation of some imaging systems often blur HS images during the data acquisition process, which unfortunately damages the high-frequency components and edge features of HS images. HS deblurring aims to recover sharp latent images from blurred ones. Chang et al. [49] discussed LR correlations along HS spatial, spectral, and nonlocal similarity modes and proposed a unified optimal LR tensor (OLRT) framework for multiple HS restoration tasks. But a matrix nuclear norm is used to constrain the LR property of unfolding nonlocal patch groups. Consequently, Chang et al. [37] proposed a weighted LRTR (WLRTR) algorithm with a reweighted strategy. Considering spectral correlation and nonlocal similarity, the HS deblurring optimization problem can be formulated as follows:

\[
\min_{X,A,B,-} \frac{1}{2} \|T - M(X)\|^2_F + \gamma \sum_i (\|R_i X - A_i \times_1 B_1 \times_2 B_2 \times_3 B_3\|^2_F + \sigma^2 \|w_i A_i\|_i) 
\]

(37)

where \(w_i\) is a reweighting factor inversely proportional to singular values of \(L_i\) with \(L_i = A_i \times_1 B_1 \times_2 B_2 \times_3 B_3\) and HOSVD is applied to see the different sparsities of higher-order singular values, i.e., the LR property. The last term, \(\|Y - M(X)\|^2_F\), is a data fidelity item, which can be replaced by \(\|T - M(X) - S - N\|^2_F\) for HS inpainting, destriping, and denoising problems.

An experimental example is given to display the deblurred performance of the OLRT for the Gaussian blur with different levels and uniform blur in the WDC dataset. In the first case, the Gaussian blur kernel has a size of \(8 \times 8\), with a standard deviation of \(\sigma = 3\). The second case tests the heavy Gaussian blur kernel with a size of \(17 \times 17\) and \(\sigma = 7\). Figure 11 provides visual results under different blur cases. The specific texture information is hardly distinguished in the three blurred images in Figure 11(b), (d), and (f). The prior knowledge of the OLRT reliably reflects the intrinsic structural correlation of HS images, which benefits the recovery of structural information and image edges. Quantitative results in different blur cases are reported in Table 7.

**HYPERSPECTRAL INPAINTING**

In this section, we introduce and discuss LR tensor-based methods for HS inpainting. These methods are also suitable for missing data recovery of high-dimensional RS (HDRS) images. RS images, such as HS, MS, and multitemporal images, often have missing data problems, such as dead pixels, thick clouds, and cloud shadows. Figure 12 presents several real examples of missing data problems. The goal of inpainting is to estimate missing data from observed images, which can be regarded as a TC problem.

LRTC theory has been successfully applied for HS inpainting [37], [49], [50], [51], [53], [125], [157]. Liu et al. [51] suggested a trace norm-regularized CP decomposition for missing data recovery. Ng et al. [52] learned from high-accuracy LRTC (HaLRTC) [125] for recovering the missing data of HDRS and proposed an adaptive weighted TC (AWTC) method. The proposed AWTC model is expressed as

\[
\min_{X,A} \frac{1}{2} \|T - M(X)\|^2_F + \sum_{i=1}^5 w_i \|X_{i0}\|_F 
\]

(38)

where \(w_i\) is a well-designed parameter related to the singular values of \(X_{i0}\). Xie et al. [53] proposed an LR regularization-based TC, fusing the logarithm of the determinant with a TTN. With the definitions of a new TNN and its t-SVD [54], Wang et al. [55] and Srindhuna et al. [158] proposed new low-tubal-rank TC methods to estimate missing values in HDRS images. Consequently, a novel TR decomposition is formulated to represent a high-dimensional tensor by circular multilinear products on a sequence of third-order tensors [27]. Based on TR theory, He et al. [56] fused spatial TV into the TR framework and developed two solving algorithms: the augmented Lagrangian method and alternating least squares (ALS). Similarly, Wang et al. [57] incorporated

**TABLE 7. A QUANTITATIVE EVALUATION OF THE OLRT FOR DIFFERENT BLUR CASES.**

| BLUR CASE         | MPSNR | MSSIM | ERGAS | MSAD  | TIME (S) |
|-------------------|-------|-------|-------|-------|----------|
| Gaussian blur     |       |       |       |       |          |
| (8 \times 8; \sigma = 3) | 43.5  | 0.9912| 1.591 | 1.9486| 314.5    |
| Gaussian blur     |       |       |       |       |          |
| (17 \times 17; \sigma = 7) | 39.63 | 0.9807| 2.5407| 3.0819| 305.7    |
| Uniform blur      |       |       |       |       |          |
|                   | 39.39 | 0.9784| 2.9332| 3.8355| 314.28   |

**FIGURE 12.** The missing information problems of RS data: (a) dead lines in Aqua Moderate Resolution Imaging Spectroradiometer band 6, (b) the scan-line-corrector-off problem in the **Landsat 7** Enhanced Thematic Mapper Plus, and (c) thick cloud obscuration in a **Landsat image**.
a 3DTV regularization into a novel weighted TR (WTR) decomposition framework. The proposed TV-WTR model is formulated as

$$
\min_{X,\alpha} \sum_{n=1}^{N} \sum_{i=1}^{I} \left[ \phi \|G^{(n)}_{i}\|_1 + \frac{\lambda}{2} \|X - \Phi(G)\|_F^2 + \tau \|X\|_{DTV}\right]
$$

s.t. $X\alpha = \mathcal{I}_{\alpha}$.

(39)

For HS image inpainting tasks, we test three methods: HaLRRTC, LRTC, and WTR decomposition + 3DTV (TVWTR) on a random missing data problem and text removal problem. A subimage is chosen from the Houston 2013 dataset for our experimental study. Figure 13 conveys the results of the Houston 2013 dataset before and after recovery, under a ratio of 80%. Although missing pixels disappear in the results of HaLRRTC and LRTC, these methods produce more or fewer artifacts in the top-right corner of the zoomed area. The TVWTR method performs best among all the compared algorithms and recovers details, such as the red square center of the zoom area. In Figure 14, original HS bands are corrupted by different texts that do not appear randomly, as in previous cases. The text corruption is eliminated by three tensor decomposition-based algorithms. Few text artifacts exist in the enlarged area of LRTC. Due to the consideration of the spectral correlation and the spatial–spectral smoothness, TVWTR provides the best result by reconstructing most information of the original image.

**FIGURE 13.** The inpainting results of different methods under an 80% missing ratio: (a) an original Houston 2013 image, (b) the missing information, (c) HaLRRTC, (d) LRTC, and (e) TVWTR.

**FIGURE 14.** The inpainting results of different methods for the text removal case: (a) an original Houston 2013 image, (b) the missing information, (c) HaLRRTC, (d) LRTC, and (e) TVWTR.
The corresponding quantitative results of two inpainting tasks are given in Table 8. Taking account of two types of prior knowledge, TVWTR gives a significantly fortified performance in two cases as compared with the other competing methods. HalLRTC and LRTC are the fastest and second-fastest among all the comparing methods.

**HYPERSONTICAL DESTRIPING**

In the past three decades, plenty of airborne and spaceborne imaging spectrometers have adopted a whisk broom sensor or a push broom sensor. The former is built with linear charge-coupled device (CCD) detector arrays. The corresponding HS imaging systems scan the target pixel by pixel and then acquire a spatial image by tracking scanning with a scan mirror forward motion [159]. The latter contains area CCD arrays. A push broom sensor scans the target line by line, one direction of which is utilized for spatial imaging and the other for spectral imaging. The incoherence of the system’s mechanical motion and failure of CCD arrays lead to the nonuniform response of neighboring detectors, mainly generating stripe noise. The periodic and nonperiodic stripes generally distributed along the scanning direction have a certain width and length. The values of stripes are brighter or darker than their surrounding pixels. The inherent property of stripes, i.e., $g(S)$, should be considered in the HS destriping model.

Chen et al. [58] took the first to develop an LRTD for an MS image destriping task. The high correlation of the stripe component along the spatial domain is depicted by an LR Tucker decomposition. The final minimization model for solving the destriping problem is expressed as follows:

$$\min_{X,S,\hat{S}\in\mathbb{R}^{n	imes k}} \left\| Y - X - S \right\|_F^2 + \eta_1 \| D_1 X \|_1 + \eta_2 \| D_2 X \|_1 + \rho \ \text{rank}(S)$$

subject to $\hat{S} = A \times_1 B_1 \times_2 B_2 \times_3 B_3$, $B_i = I (i = 1, 2, 3)$ (40)

where $\| S \|_1 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sqrt{\sum_{k=1}^{k} S_{ijk}}$.

Cao et al. [59] implemented the destriping task by the matrix nuclear norm of stripes and nonlocal similarity of image patches in the spatiotemporal volumes. WLRTR and the OLRT [37], [49] are also effective for an HS destriping task. Chang et al. [49] simultaneously considered the LR properties of the stripe cubics and nonlocal patches. The OLRT algorithm is reformulated for modeling both the recovered and stripe components, as follows:

$$\min_{X,L,S} \frac{1}{2} \left\| T - X - S \right\|_F^2 + \rho \ \text{rank}(S)$$

$$+ \omega_1 \sum_{i} \left( \frac{1}{2} \left\| R_i X - L_i \right\|_F^2 + \text{rank}(L_i) \right)$$

(41)

where the $k$ rank of $X$ denotes the rank of the matrix unfolding $X_0$. In the optimization process, $\text{rank}(S)$ is replaced by $\| S_{(i)} \|_1; \| L_{(i)} \|_1$ is used to denote $\text{rank}(L_i)$, and (41) is transformed into a typical optimization that has a closed-form solution and is easily solved by a singular value thresholding algorithm. In [60], an HS destriping model is transformed into a tensor framework in which the tensor-based nonconvex sparse model uses both $l_0$ and $l_1$ sparse priors to estimate stripes from noisy images.

We take an example with nonperiodic stripes of intensity 50 and stripe ratio 0.2, which is presented in Figure 15(b). Figure 15 (c) and (d) display the destriping results of WLRTR and LRTD. The stripes are estimated and removed correctly by WLRTR and LRTD since both models consider nonlocal similarity and spectral correlation. Considering the third type of prior knowledge—spatial and spectral smoothness, LRTD moderately preserves more details, such as clear edges, than WLRTR. A quantitative comparison is in accordance with the previously mentioned visual results. Table 9 provides destriping results with four quantitative indices. LRTD achieves higher evaluation values than WLRTR.

**FUTURE CHALLENGES**

Various tensor optimization models have been developed to solve the HS restoration problem and show impressive performance. Nevertheless, these models can be further improved for future work.

Since prior information is effective for finding the optimal solution, novel tensor-based approaches should utilize as many types of priors as possible. Therein, how best to simultaneously design a unified framework for nonlocal similarity, spatial and spectral smoothness, and subspace representation is a crucial challenge.

The addition of different regularizations leads to the manual adjustment of corresponding parameters. For example, a noise-adjusted parameter predefinition strategy needs to be studied to enhance the robustness of tensor optimization models.

It is worth noting that we are usually blind to the location of stripes and clouds. The locations of stripes and clouds. The locations of stripes and clouds.

---

**TABLE 8. A QUANTITATIVE EVALUATION OF DIFFERENT METHODS FOR INPAINTING.**

| INDEX | MPSNR | MSSIM | ERGAS | MSAD | Time (s) |
|-------|-------|-------|-------|------|----------|
| MISS | 36.54 | 0.9555 | 0.9947 | 0.9685 | 0.9975 |
| TVWTR | 49.05 | 0.2921 | 1.0108 | 1.1345 | 0.4012 |
| TEXT | HalLRTC | 50.29 | 1.2424 | 1.1345 | 0.5497 |
| REMOVAL | LRTC | 53.39 | 1.1134 | 1.1345 | 0.5497 |
| | TVWTR | 57.31 | 3.8388 | 1.1276 | 345.66 |

**TABLE 9. A QUANTITATIVE COMPARISON OF DIFFERENT SELECTED ALGORITHMS FOR DESTRIPING.**

| MISS | MPSNR | MSSIM | ERGAS | MSAD |
|------|-------|-------|-------|------|
| MISS | 39.77 | 0.9844 | 7.7883 | 5.1765 |
| TVWTR | 47.87 | 0.9912 | 3.8388 | 1.1276 |

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mixed noise between neighbor bands are often different and need to be estimated. How best to predict the degradation positions and design blind estimation algorithms deserves further study in future research.

Due to some HS images containing hundreds of spectral bands, the high dimensions of an HS tensor cause a time-consuming problem. The model complexity of tensorial models should be reduced, with a guarantee of efficiency and accuracy of HS restoration.

**HYPERSPECTRAL COMPRESSIVE SENSING**

Traditional HS imaging techniques are based on the Nyquist sampling theory for data acquisition. A signal must be sampled at a rate greater than twice its maximum frequency component to ensure unambiguous data [160], [161]. This signal processing needs huge computing and storage space. Meanwhile, the ever-increasing spectral resolution of HS images also leads to the high expense and low efficiency of transmission from airborne and spaceborne platforms to ground stations. The goal of compressive sensing (CS) is to compressively sample and reconstruct signals based on SR to reduce the cost of signal storage and transmission. In Figure 16, based on the image forming principle of a single-pixel camera that uses a digital micromirror device to accomplish CS sampling, an HS sensor can span the necessary wavelength range and record the intensity of the light reflected by the modulator in each wavelength [162]. Since the CS rate can be far

![Figure 15](image15.png)

**FIGURE 15.** The destriping results of different methods: (a) an original Houston 2018 image, (b) nonperiodic stripes, (c) WLRTR, and (d) LRTD.

![Figure 16](image16.png)

**FIGURE 16.** The HS CS process. DMD: digital micromirror device.
lower than the Nyquist rate, the limitation of high cost caused by the sheer volume of HS data will be alleviated. A contradiction usually exists between the massive HS data and limited bandwidth of satellite transmission channels. HS images can be compressed first to reduce the pressure on channel transmission. Therefore, the HS CS technique is conducive to onboard burst transmission and real-time processing in RS [163].

The CS of HS images aims to precisely reconstruct HS data \( X \in \mathbb{R}^{k \times v \times z} \) from a few compressive measurements \( y \in \mathbb{R}^m \) through effective HS CS algorithms. The compressive measurements \( y \) can be formulated by

\[
y = \Psi (X)
\]

where \( \Psi \) is a measurement operator instantiated as \( \Psi = D \cdot H \cdot P \), where \( D \) is a random downsampling operator, \( H \) is a random permutation matrix, \( P \) is a Walsh–Hadamard transform, and the mapping of \( \Psi \) is \( \mathbb{R}^{k \times v \times z} \rightarrow \mathbb{R}^m \) (the sampling ratio is \( m = hvz \)). The strict reconstruction of \( X \) from \( y \) will be guaranteed by CS theory when \( \Psi \) satisfies the restricted isometry property. This compressive operator has been successfully adopted for various HS CS tasks [164], [165], [166], [167]. However, operator \( \Psi \) can be replaced with real demands. Apparently, it is an ill-posed inverse problem to directly recover \( X \) from (42). Extra prior information needs to be investigated to optimize the HS CS problem. The HS CS task can be generalized for the following optimization problem:

\[
\min_\lambda \| y - \Psi (X) \|_F^2 + \lambda F (X)
\]

where \( F (X) \) denotes the additional regularization term to use different types of HS prior information, such as spectral correlation, spatial and spectral smoothness, and nonlocal similarity.

**TENSOR DECOMPOSITION-BASED HYPERSONTAL COMPRESSIVE SENSING RECONSTRUCTION METHODS**

Recently, CP decomposition, Tucker decomposition, and tensor-based multilinear SVD have been used for HS compression. For example, Fang et al. [168] decomposed an HS cube into rank 1 tensors and utilized the sparsity of those components to compress the original one. Das et al. [169] developed Tucker decomposition for HS image and video compression, which gained better efficiency in terms of the compression ratio and SNR. Renu et al. [170] extended a 3D multilinear SVD for compressing HS images spatially and spectrally.

Subsequently, Tucker decomposition-based methods have drawn wide attention for HS CS. Tucker decomposition was first introduced into the compression of HS images to constrain the discrete wavelet transform coefficients of spectral bands [171]. Most of the following works try to study the Tucker decomposition-based variants for HS CS [61], [63], [66], [164], [172].

**TUCKER DECOMPOSITION WITH TOTAL VARIATION**

In one earlier work [164], a 2D TV norm was penalized in an LR matrix framework, which robustly recovers a large HS image when the sampling ratio is only 3%. A spectral LR model is rarely enough to depict the inherent property of HS images. Joint tensor Tucker decomposition with a weighted 3DTV (JTenRe3DTV) [61] injected a weighted 3DTV into the LR Tucker decomposition framework to model the global spectral correlation and local spatial–spectral smoothness of an HS image. Considering the disturbance \( E \), the JTenRe3DTV optimization problem for HS CS can be expressed as

\[
\min_\lambda \frac{1}{2} \| E - \lambda I \|_F + \lambda \| X \|_{\text{DN2TV}}
\]

s.t. \( y = \Psi (X), X = A_1 \times B_1 \times B_2 \times B_3 + E \).

In [173], the LR tensor constraint of (44) was replaced by the TNN.

**TUCKER DECOMPOSITION WITH NONLOCAL SIMILARITY**

Tucker decomposition methods with nonlocal similarity either cluster similar patches into a 4D group or unfold 2D patches into a 3D group. Du et al. [62] represented each local patch of HS images as a 3D tensor and grouped similar tensor patches to form one 4D tensor per cluster. Each tensor group can be approximately decomposed by a sparse coefficient tensor and a few matrix dictionaries. Xue et al. [63] unfolded a series of 3D cubes into 2D matrices along the spectral modes and stacked these matrices as a new 3D tensor. The spatial sparsity, nonlocal similarity, and spectral correlation were simultaneously employed to obtain the proposed model:

\[
\begin{align*}
\min_{\lambda, A_p, B_{p0}, B_{p1}, B_{p2}, B_{p3}} & \sum_{p=1}^P \frac{\lambda}{2} \| X_p - A_p \times B_{p0} \times B_{p1} \times B_{p2} \times B_{p3} \|_F^2 \\
& + \lambda I \| A_p \|_F + \lambda L (X_p) \\
\text{s.t. } & y = \Phi x, X_p = A_p \times B_{p0} \times B_{p1} \times B_{p2} \times B_{p3} \\
& B_{p0} B_{p1} B_{p2} - I (t = 1, 2, 3)
\end{align*}
\]

where \( p = 1, \ldots P \) and \( P \) denotes the group number, \( x \in \mathbb{R}^{hvz} \) denotes the vector form of \( x \), and \( L (X) \) is the TTN of \( X \).

**TENSOR RING-BASED METHODS**

Unlike Tucker decomposition methods [62], [63] that directly capture the LR prior in the original image space at the cost of high computation, a novel subspace-based NLTR (SNLTR) decomposition approach projects an HS image into a low-dimensional subspace [64]. The nonlocal similarity of the subspace coefficient tensor is constrained by a TR decomposition model. The SNLTR model is presented as

\[
\begin{align*}
\min_{E, X, \mathcal{L}, \mathcal{G}} & \frac{1}{2} \| y - \Psi (E Z) \|_F^2 + \lambda \sum_i \frac{1}{2} \| \mathcal{R} (X - \mathcal{L}) \|_F \\
\text{s.t. } & E = I, \mathcal{L} = \Phi (G) \|
\end{align*}
\]
HYPERSONTRAL KRONNECKER COMPRESSIVE SENSING METHODS

Unlike the current 1D or 2D sampling strategy, Kronecker CS (KCS) consists of Kronecker-structured sensing matrices and sparsifying bases for each HS dimension [66], [174]. Based on multidimensional multiplexing, Yang et al. [65] used a tensor measurement and nonlinear sparse tensor coding to develop a self-learning tensor nonlinear CS (SLTNCS) algorithm. The sampling process and SR can be represented as the model based on Tucker decomposition. Generally, an HS image $X \in \mathbb{R}^{m_1 \times m_2 \times m_3}$ can be expressed as the following Tucker model:

$$X = S \times_1 \Phi_1 \times_2 \Phi_2 \times_3 \Phi_3$$  \hspace{1cm} (47)

where $S \in \mathbb{R}^{m_1 \times m_2 \times m_3}$ stands for an approximate block-sparse tensor in terms of a set of three basis matrices $\Phi_j \in \mathbb{R}^{m_j \times k_j}$, with $m_j < k_j, j = 1, 2, 3$.

In the context of KCS, three measurement and sensing matrices denoted by $\Psi_{j}, j = 1, 2, 3$ of size $n_j \times k_j$, with $n_j < k_j$, are used to reduce the dimensionality of the measurement tensor. The compressive sampling model is given as

$$Y = X \times_1 \Psi_1 \times_2 \Psi_2 \times_3 \Psi_3 = S \times_1 Q_1 \times_2 Q_2 \times_3 Q_3$$  \hspace{1cm} (48)

where $Q_j = \Phi_j \Psi_j, j = 1, 2, 3$.

Zhao et al. [65] designed a 3D HS KCS mechanism to achieve independent samplings in three dimensions. Suitable sparsifying bases were selected, and the corresponding optimized measurement matrices were generated, which adjusted the distribution of the sampling ratio for each dimension of HS images. Yang et al. [66] constrained the nonzero number of the Tucker core tensor to explore the spatial–spectral correlation. To address the issue of the computational burden on the data reconstruction of early HS KCS techniques, researchers have proposed several tensor-based methods, such as the tensor form greedy algorithm, N-way block orthogonal matching pursuit [67], beamformed mode-based sparse estimator [68], and tensor-based Bayesian reconstruction (TBR) [69]. The TBR model exploited the multidimensional block-sparsity of tensors, which was more consistent with the sparse model in HS KCS than conventional CS methods. A Bayesian reconstruction algorithm was developed to achieve the decoupling of hyperparameters by a low-complexity technique.

EXPERIMENTAL RESULTS AND ANALYSIS

An HS data experiment is employed to validate the effectiveness of tensor-based models on HS CS with four different sample ratios, i.e., 1%, 5%, 10%, and 20%. The Reno dataset selected for the experiment has a size of $150 \times 150 \times 100$. The randomly permuted Hadamard transform is adopted as the compressive operator. Table 10 compares the reconstruction results of SLTNLS and JTenRe3DTV. They have quality decays, with sample ratios decreasing, but SLNTCS obtains poorer results than JTenRe3DTV in lower sampling ratios.

In the light of visual comparison, one representative band in sampling ratio 10% is presented in Figure 17. The basic texture information can be found in the results of two HS CS algorithms. As shown in the enlarged area, SLTNLS causes some artifacts, but JTenRe3DTV produces a more acceptable result with the smoothing white area than SLTNLS.

FUTURE CHALLENGES

The low acquisition rate of CS inspires a novel development potentiality for HS RS. Many tensor-based methods have been proposed to achieve remarkable HS CS reconstruction results at a lower sampling ratio. However, here, we briefly point out some potential challenges.

Some novel tensor decomposition approaches need to be explored. In past research works, Tucker decomposition has been successfully applied for HS CS. But with the development of tensorial mathematical theory, many tensor decomposition models have been proposed and introduced in other HS applications. Therefore, how best to

| METHOD | INDEX | 1% | 5% | 10% | 20% |
|--------|-------|----|----|-----|-----|
| SLTNLS | MPSNR | 18.7 | 24.44 | 27.72 | 32.14 |
|         | MSSIM | 0.3273 | 0.6593 | 0.8047 | 0.9159 |
|         | ERGAS | 23.3411 | 12.1203 | 8.3119 | 5.0263 |
|         | MSAD  | 22.0035 | 11.2031 | 7.6354 | 4.6003 |
| JTenRe3DTV | MPSNR | 27.91 | 34.54 | 36.28 | 37.41 |
|         | MSSIM | 0.8116 | 0.9443 | 0.9638 | 0.9709 |
|         | ERGAS | 8.2422 | 4.0139 | 3.299 | 2.9124 |
|         | MSAD  | 7.5545 | 3.5703 | 2.9233 | 2.5723 |

FIGURE 17. The inpainting results of different methods under a 10% sampling ratio: (a) an original Reno image, (b) SLTNLS, and (c) JTenRe3DTV.
find a more appropriate tensor decomposition for HS CS is a vital challenge.

Noise degradation usually has a negative influence on HS CS sampling and reconstruction, which is hardly ignored in the real HS CS real imaging process. As a result, considering the noise interference and enhancing the robustness of noise in the CS process remain challenging.

**HYPERSPECTRAL ANOMALY DETECTION**

HS AD aims to discover and separate potential man-made objects from observed image scenes, which is typically constructive for defense and surveillance developments in RS fields, such as mine exploration and military reconnaissance. Anomalies can be a pixel, a set of pixels, a feature, and a series of features. They rarely occur in HS images, and their spectral signatures differ from the surrounding background environment. For instance, unknown aircraft and vehicles suddenly appear in a suburb or bridge scene, usually marked as anomalies. In Figure 18, AD can be regarded as an unsupervised two-class classification problem where anomalies occupy small areas compared with their surrounding background. The key to coping with this problem is to exploit the discrepancy between anomalies and their background. Anomalies commonly occur with low probabilities, and their spectral signatures are quite different from their neighbors.

HS images containing two spatial dimensions and one spectral dimension are intrinsically considered a three-order tensor. Tensor-based approaches have been gradually gaining attention in HS AD in recent years. Tucker decomposition is the first and essential type of tensor decomposition method used for HS AD. Therefore, in the following sections, we mainly focus on Tucker decomposition-based methods and a few other types of tensor-based methods.

**TENSOR DECOMPOSITION-BASED HYPERSPECTRAL ANOMALY DETECTION METHODS**

**TUCKER DECOMPOSITION-BASED METHODS**

An observed HS image $T$ can be decomposed into two parts by Tucker decomposition; i.e.,

$$T = X + S$$

where $X$ is an LR background tensor and $S$ is a sparse tensor consisting of anomalies. The Tucker decomposition for AD is formulated as the following optimization:

$$\begin{align*}
X &= A \times_1 B_1 \times_2 B_2 \times_3 B_3 \\
S &= T - X
\end{align*}$$

Many Tucker decomposition-based variants have been studied to improve AD accuracy. Li et al. [70] proposed an
LRTD-based AD model, which employed Tucker decomposition to obtain the core tensor of the LR part. The final spectral signatures of anomalies were extracted by an unmixing approach. After Tucker decomposition processing, Zhang et al. [71] utilized a reconstruction error-based method to eliminate background pixels and retain anomaly information. Zhu et al. [72] advocated a weighting strategy based on tensor decomposition and cluster weighting (TDCW). In TDCW, Tucker decomposition was adopted to obtain the anomaly part; k-means clustering and segmenting were assigned as post-processing steps to achieve a performance boost. Song et al. [73] proposed a tensor-based endmember extraction algorithm, where Tucker decomposition and k-means are employed to construct a high-quality dictionary.

Based on Tucker decomposition, Qin et al. [74] proposed an LR and sparse tensor decomposition (LRASTD). The LRASTD can be formulated as

$$\min_{X, S} \| A \| + \beta \| A \|_1 + \lambda \| S \|_{2,1}$$

s.t. $X = A \times_1 B_1 \times_2 B_2 \times_3 B_3 + S$  \hspace{1cm} (51)

where $\| S \|_{2,1} = \sum_{i=1}^{I} \| S(:,::,k) \|$.

OTHER TENSOR-BASED METHODS

Chen et al. [75] presented a tensor principal component analysis-based preprocessing method to separate a principal component part and a residual part. Li et al. [76] proposed a prior-based tensor approximation (PTA) approach, where the background was constrained by a truncated nuclear norm regularization and spatial TV. The proposed PTA can be expressed as

$$\arg\min_{X,S} \frac{1}{2} \| D_H X_{(1)} \| + \| D_S X_{(2)} \| + \alpha \| X_1 \| + \beta \| S_1 \|_{2,1}$$

$$\begin{align*}
Y &= X + S \\
X_1 &= \text{unfold}_1(X) \\
X_2 &= \text{unfold}_2(X) \\
X_3 &= \text{unfold}_3(X) \\
S_1 &= \text{unfold}_1(S)
\end{align*}$$

s.t. (52)

where $D_H \in \mathbb{R}^{(h-1) \times h}$ and $D_S \in \mathbb{R}^{(r-1) \times r}$ are defined as

$$D_H = \begin{bmatrix}
1 & -1 \\
-1 & 1 \\
& \ddots \\
& & 1 & -1 \\
& & -1 & 1
\end{bmatrix}$$

$$D_S = \begin{bmatrix}
1 & -1 \\
-1 & 1 \\
& \ddots \\
& & 1 & -1 \\
& & -1 & 1
\end{bmatrix}$$

Wang et al. [77] proposed a novel tensor LR representation method with a PCA preprocessing step, namely, PCA-TLRR, which was the first to expand the concept of Tensor LR representation in HS AD and exploited the 3D inherent structure of HS images. Assisted by the multisubspace learning of the tensor domain and sparsity constraint along the joint spectral–spatial dimensions, the LR background and anomalies are separated in a more accurate manner.

EXPERIMENTAL RESULTS AND ANALYSIS

Here, we apply PTA and PCA-TLRR on three HS datasets for AD. The San Diego dataset [175] was captured by the Airborne Visible/Infrared Imaging Spectrometer (AVIRIS) sensor over San Diego International Airport, in California, USA. Three flights are observed in the selected region with a size of $100 \times 100 \times 189$. Airport-1 and Airport-2 [176] were also acquired by the AVIRIS sensor. As shown in the second column of Figure 19, flights are regarded as anomalies in different airport scenes.

As the detection maps of Figure 19 indicate, most flights are detected by PTA and PCA-TLRR. But PCA-TLRR preserves sharp image edges and texture information. Except for visual observation of the resulting anomaly maps, we employ the receiver operating characteristic (ROC) curve [177] and area under the ROC curve (AUC) [178] to quantitatively assess the detection accuracy of the tensor-based method. The ROC curve plots the varying relationship between the probability of detection (PD) and false alarm rate (FAR) for extensive possible thresholds. The area under this curve is calculated as the AUC, whose ideal result is one. The AUC values derived from PTA and PCA-TLRR are higher than 0.9, but PCA-TLRR achieves higher AUC values than PTA on three datasets. The last column of Figure 19 presents the ROC curves of PTA and PCA-TLRR. In the upper right, the PD of PCA-TLRR reaches one, even when the FAR is 0.01, whereas the other is lower than about 0.8. PCA-TLRR is capable of obtaining a high detection rate and low FAR.

FUTURE CHALLENGES

Tucker decomposition-based models have been well developed by researchers, yet other types of tensor decompositions are rarely investigated in the HS AD community. In other words, how best to introduce other novel tensor decomposition frameworks into AD is a key challenge.

Although most anomalies are successfully detected, some background pixels, such as roads and roofs, usually remain. More complex backgrounds and fewer targets make the difficulty of AD increase. To solve this problem, researchers need to explore multiple features and suitable regularizations.

The background and anomalies are often modeled as the LR part and sparse part of HS images. The 3D inherent structure of HS images is exploited by tensor decomposition-based methods. The spatial sparsity and 3D inherent structure of anomalies should be considered through a consolidated optimization strategy.

HYPSPECTRAL–MULTISPECTRAL FUSION

HS images provide abundant and varied spectral information yet hardly contain high spatial resolution, owing to the limitations of sun irradiance [179] and imaging systems [180], [181]. On the contrary, MS images are captured with low spectral resolution and high spatial resolution. HS–MS fusion aims to improve the spatial resolution of HS images.
with the assistance of MS images and generate final HS images with high spatial resolution and original spectral resolution. High-quality fused HS images benefit the in-depth recognition of and insight to materials, which contributes to many RS real applications, such as object classification and change detection of wetlands and farms [182], [183], [184], [185], [186], [187], [188].

Figure 20 depicts an HS–MS fusion process to generate a high-spatial–spectral-resolution HS (HR-HS) image. Suppose that a desired HR-HS image, low-resolution HS (LR-HS) image, and high-resolution MS (HR-MS) image is denoted by $X$, $Y$, and $Z$, respectively. An LR-HS image is seen as a spatially downsampled and blurred version of $X$, and an HR-MS image is the spectrally downsampled version of $X$. The two degradation models are expressed as follows:

$$
Y(l, c, m) = X(l, c, m)K + N_h
$$

$$
Z(l, c, k, l, c, m) = GX(l, c, m) + N_m
$$

where $R = BK$, $B$ denotes a convolution blurring operation, $K$ is a spatial downsampling matrix, and $G$ represents a spectral response function of an MS image sensor, which can be regarded as a spectral downsampling matrix; $N_h$ and $N_m$ stand for noise.

According to [189], [190], and [191], $R$ and $G$ are assumed to be given in advance of solving the HS SR problem:

$$
\min_X \|Y(l, c, m) - X(l, c, m)R\|^2 + \|Z(l, c, k, l, c, m) - GX(l, c, m)\|^2 + \tau f(X)
$$

where the first and second F-norms are data fidelity terms with respect to models (53) and (54) and $f(X)$ represents the prior regularization pertinent to the desired property on the HR-HS $X$. In the following section, we review currently advanced HS SR methods from two categories: tensor decomposition and prior-based tensor decomposition models.

---

**FIGURE 19.** Original HS images, ground truth maps, detection maps, and area under the receiver operating characteristic curve (AUC) curves of PTA on different datasets: (a) San Diego, (b) Airport-1, and (c) Airport-2.
**Tensor Factorizations for Sparse Representation**

**Canonical Decomposition/Parallel Factor Analysis Decomposition Model**

Initially, Kanatsoulis et al. [78] employed a coupled CP decomposition framework for HS SR. The CP decomposition of an HR-HS tensor $X$ can be expressed as

$$X = \sum_{r=1}^{R} a_r \otimes b_r \otimes c_r \equiv [A, B, C]$$

(56)

where the latent LR factors are $A = [a_1, ..., a_l], B = [b_1, ..., b_l]$, and $C = [c_1, ..., c_l]$. In [78], the coupled CP decomposition gave the following assumption:

$$Y = [P, A, P, B, C]; \quad Z = [A, B, P, C]$$

(57)

where $P_1 \in \mathbb{R}^{l \times l}$, $P_2 \in \mathbb{R}^{l \times l}$, and $P_3 \in \mathbb{R}^{l \times l}$ are three linear degradation matrices. The identifiability of HS SR based on the algebraic properties of CP decomposition is guaranteed under relaxed conditions. However, LR properties of different dimensions are treated equally, which is rarely suitable for real HS SR. Subsequently, Kanatsoulis et al. [80] introduced an SR cube algorithm that combined the advantages of CP decomposition and matrix factorization. Xu et al. [79] improved the CP decomposition-based method by adding a nonlocal tensor extraction module.

**Tucker Decomposition Model**

Li et al. [81] extended a coupled sparse tensor factorization (CSTF) approach in which the fusion problem was transformed into the estimation of dictionaries along three modes and corresponding sparse core tensors. When a tensor $X$ is decomposed by Tucker decomposition,

$$X = W \times_1 A \times_2 B \times_3 C.$$  

(58)

The LR-HS and HR-HS degradation models are rewritten as

$$Y = W \times_1 (P_1 A) \times_2 (P_2 B) \times_3 C = W \times_1 A' \times_2 B' \times_3 C$$

(59)

$$Z = W \times_1 A \times_2 B \times_3 (P_3 C) = W \times_1 A \times_2 B \times_3 C'$$

(60)

where $A' = P_1 A, B' = P_2 B$, and $C' = P_3 C$ are the downsampled dictionaries along three modes. Taking the sparsity of core tensor $W$, Li et al. formulated the fusion problem as follows:

$$\min_{A', B, C} \left\| Y - W \times_1 A' \times_2 B' \times_3 C \right\|_F^2 + \left\| Z - W \times_1 A \times_2 B \times_3 C' \right\|_F^2 + \lambda \| W \|_2.$$  

(61)

The $l_1$ norm in (61) was replaced by an $l_2$ norm in [84]. Prévost et al. [85] assumed that HR-HS images assessed approximately low multilinear rank and developed an SR algorithm based on coupled Tucker tensor approximation with HOSVD. In the Tucker decomposition framework and

**Figure 20.** HS–MS fusion: (a) a low-spatial-resolution HS image, (b) a low-spectral-resolution MS image, and (c) a high-spatial-spectral-resolution HS image.
the BT of decomposition framework [named coupled Tucker/coupled block term decompositions for hyperspectral Super-resolution with vAriability (CT/CB-STAR)] [82], an additive variability term was admitted for the study of general identifiability with theoretical guarantees. Zare et al. [83] offered a coupled nonnegative Tucker decomposition method to constrain the nonnegativity of two Tucker spectral factors.

NONLOCAL TUCKER DECOMPOSITION
Wan et al. [86] grouped 4D tensor patches by using the spectral correlation and similarity under Tucker decomposition. Dian et al. [87] offered a nonlocal sparse tensor factorization (NLSTF) method, which induced core tensors and corresponding dictionaries from HR-MS images and spectral dictionaries from LR-HS images. A modified NLSTF_Semi-Blind Fusion (SMBF) version was developed for the semi-blind fusion of HS and MS [88]. However, the dictionary and core tensor for each cluster are estimated separately by NLSTF and NLSTF_SMBF.

TUCKER DECOMPOSITION PLUS MANIFOLD
Zhang et al. [91] suggested a spatial–spectral-graph-regularized LRTD (SSGLRTD). In SSGLRTD, the spatial and spectral manifolds between HR-MS and LR-HS images are assumed to be similar to those embedded in HR-HS images. Bu et al. [92] presented a graph Laplacian-guided coupled tensor decomposition model that incorporated global spectral correlation and complementary submanifold structures into a unified framework.

TUCKER DECOMPOSITION PLUS TOTAL VARIATION
Xu et al. [89] presented a Tucker decomposition model with unidirectional TV. Wang et al. [90] advocated an NLR Tucker decomposition and SU-based approach to leverage spectral correlations, nonlocal similarity, and spatial–spectral smoothness. Consequently, several fusion approaches are proposed to provide useful hints, including simultaneously making use of SU and transfer learning [192], the visualization-oriented fusion of HS image bands [193], and the motion cue for superresolving a scene [194].

BLOCK TERM DECOMPOSITION MODEL
Zhang et al. [93] discovered identifiability guarantees in [78] and [81], at the cost of a lack of physical meaning for the latent factors under CP and Tucker decomposition. Therefore, they employed an alternative coupled nonnegative BT tensor decomposition (NN-CBTBD) approach for HS SR. The NN-CBTBD model with rank \((L_r, L_r, 1)\) for HS SR is given as

\[
\min_{\mathbf{A}, \mathbf{B}} \left\| \mathbf{Y} - \sum_{r=1}^{G} (\mathbf{P}_r \mathbf{A}_r \cdot (\mathbf{P}_r \mathbf{B}_r)^\top) \cdot \mathbf{C} \right\|^2 + \left\| \mathbf{Z} - \sum_{r=1}^{G} (\mathbf{A}_r \cdot (\mathbf{B}_r)^\top) \cdot \mathbf{P}_r \cdot \mathbf{C} \right\|^2
\]

s.t. \( \mathbf{A} \succeq 0, \mathbf{B} \succeq 0, \mathbf{C} \succeq 0. \) \hspace{1cm} (62)

Compared with a conference version [93], the journal version [94] additionally gave more recovery ability analysis and a more flexible decomposition framework by using an advocated rank-\((L_r, L_r, 1)\) model and a block coordinate descent algorithm. Jiang et al. [95] introduced a graph manifold, the graph Laplacian, into the CBTD framework.

TENSOR TRAIN DECOMPOSITION MODEL
Dian et al. [96] proposed a low TT rank (LTTR)-based HS SR method. An LTTR prior was designed for learning correlations among the spatial, spectral, and nonlocal modes of 4D FBP patches. The HS SR optimization can be obtained as

\[
\min_{\mathbf{X}(i)} \left\| \mathbf{Y}(i) - \mathbf{X}(i) \right\|_F + \left\| \mathbf{Z}(i) - \mathbf{G} \mathbf{X}(i) \right\|_F + \tau \sum_{k=1}^{K} \left\| \mathbf{X}_k \right\|_{\text{tr}} \hspace{1cm} (63)
\]

where \( K \) denotes the number of clusters. The TT rank of tensor \( \mathbf{X}_k \) is defined as

\[
\left\| \mathbf{X}_k \right\|_{\text{tr}} = \sum_{i=1}^{3} \alpha_i \mathbf{L}_i(S_k(i)) \hspace{1cm} (64)
\]

and \( \mathbf{L}(\mathbf{X}) = \Sigma_i \log(\sigma_i(\mathbf{X}) + \varepsilon) \), with a small positive value \( \varepsilon \). Li et al. [97] presented NLR tensor approximation and NLR SR that formed the nonlocal similarity and spatial–spectral correlation by the TT rank constraint of 4D nonlocal patches.

TENSOR RING DECOMPOSITION MODEL
The TR decomposition of an HR-HS tensor \( \mathbf{X} \in \mathbb{R}^{I \times J \times K} \) is represented as

\[
\mathbf{X} = \Phi [\mathbf{G}^{(1)}, \mathbf{G}^{(2)}, \mathbf{G}^{(3)}] \hspace{1cm} (65)
\]

where three TR factors are denoted by \( \mathbf{G}^{(1)} \in \mathbb{R}^{I \times H \times r_1}, \mathbf{G}^{(2)} \in \mathbb{R}^{J \times V \times r_2}, \) and \( \mathbf{G}^{(3)} \in \mathbb{R}^{K \times \mathbb{B} \times r_3} \), with TR ranks \( r = [r_1, r_2, r_3] \). Based on TR theory, an LR-HS image is rewritten as

\[
\mathbf{Y} = \Phi [\mathbf{G}^{(1)} \times_1 \mathbf{P}_1, \mathbf{G}^{(2)} \times_2 \mathbf{P}_2, \mathbf{G}^{(3)} \times_3 \mathbf{P}_3] \hspace{1cm} (66)
\]

and an HR-MS image can be expressed as

\[
\mathbf{Z} = \Phi [\mathbf{G}^{(1)}, \mathbf{G}^{(2)}, \mathbf{G}^{(3)} \times_3 \mathbf{P}_3]. \hspace{1cm} (67)
\]

He et al. [98] presented a coupled TR factorization (CTRF) model and modified CTRF version [nuclear norm regularized CTRF (NCTRF)] with the nuclear norm regularization of third/spectral TR factor. The NCTRF model is formulated as

\[
\min_{\mathbf{G}^{(1)}, \mathbf{G}^{(2)}, \mathbf{G}^{(3)}} \left\| \mathbf{Y} - \Phi [\mathbf{G}^{(1)} \times_1 \mathbf{P}_1, \mathbf{G}^{(2)} \times_2 \mathbf{P}_2, \mathbf{G}^{(3)}] \right\|_F^2
\]

\[
+ \left\| \mathbf{Z} - \Phi [\mathbf{G}^{(1)}, \mathbf{G}^{(2)}, \mathbf{G}^{(3)} \times_3 \mathbf{P}_3] \right\|_F^2 + \lambda \left\| \mathbf{G}^{(3)} \right\|_F^2 \hspace{1cm} (68)
\]

Equation (68) becomes the CTRF model when removing the final term. In [98], the benefit of TR decomposition for SR is elaborated via theoretical and experimental proof related to a low-dimensional TR subspace. The relationship between the TR spectral factors of LR-HS images and HR-MS images was explored in [99] with a high-order representation of the original HS image. The spectral
structures of HR-HS images were kept consistent with LR-HS images by graph Laplacian regularization. Chen et al. [100] presented a factor-smoothed TR decomposition to capture the spatial–spectral continuity of HR-HS images. Based on the basic CTRF model, Xu et al. [101] advocated LR TR decomposition based on the TNN, which exploited the LR properties of nonlocal similar patches and their TR factors.

TENSOR RANK MINIMIZATION FOR SPARSE REPRESENTATION

Based on t-SVD, Dian et al. [102] developed a subspace-based low tensor multirank (LTMR) that induced an HR-HS image by the spectral subspace and corresponding coefficients of grouped FBPs. The specific LTMR model is expressed as

$$\min_r \|Y_{(3)} - X_{(3)}R\|^2 + \|Z_{(3)} - GX_{(3)}\|^2 + \tau \sum_k \|X_k\|_{2,1} \quad (69)$$

where the multirank of tensor $X$ is defined as $|X|_{TMR} = (1/B^3) \sum_{a=1}^{B^3} I(S(X(:, :, b)))$ and $B$ is the dimension number of the third mode of $X$. To speed up the estimation of the LTMR, Long et al. [103] introduced the concept of the truncation value and obtained a fast LTMR algorithm. Xu et al. [104] presented a nonlocal patch tensor SR model that characterized the spectral and spatial similarities among nonlocal HS patches by the T-product-based tensor SR.

Considering HS image degradation from the perspective of noise, some researchers study the noise-robust HS SR problem. Li et al. [105] proposed a TV-regularized tensor low-multilinear-rank model to improve the performance of the mixed-noise-robust HS SR task. Liu et al. [106] transformed the HS SR problem to a convex TNN optimization, which permitted an SR process robust to an HS image striping case.

EXPERIMENTAL RESULTS AND ANALYSIS

In this section, we select five representative tensor decomposition-based HS–MS fusion approaches: a Tucker decomposition-based method, i.e., CSTF [81]; a CP decomposition-based method, i.e., Blind-Super-resolution Tensor REConstruction (STERO) [78]; a TT decomposition-based method, i.e., LTTR [96]; a BT decomposition-based method, i.e., SC-LL1 [94]; and a t-SVD-based method, i.e., LTMR [102].

The quality assessment is conducted within a simulation study following Wald’s protocol [195]. One RS-HS dataset is selected for data fusion, i.e., the University of Houston campus used for the 2018 IEEE Geoscience and Remote Sensing Society Data Fusion Contest. The original data were acquired by an ITRES CASI 1500 HS camera, covering a 380–1050-nm spectral range with 48 bands, at a 1-m ground sample distance. A subimage of $400 \times 400 \times 46$ is chosen as the ground truth after discarding some noisy bands. The input HR-MS image is generated by the reference image by using the spectral response of WorldView 2, and the input LR-HS image is obtained via a Gaussian blurring kernel whose size equals five. Five quantitative metrics are used to assess the performance of the reconstructed HR-HS image, including the MPSNR, ERGAS, root-mean-square error (RMSE), spectral angle mapper (SAM), and cross correlation (CC). The SAM measures the angles between the HR-HS image and the reference image, and smaller SAM scores correspond to better performance. CC is a score between zero and one, where one represents the best estimation result.

Figure 21 presents reconstructed false-color images, enlarged local images, SAM error maps, and mean relative absolute error (MRAE) heat maps of five HS–MS fusion methods. From Figure 21, all five methods provide good spatial reconstruction results. However, the LTMR and LTTR produce severe spectral distortions at the edge of the objects. In Table 11, the conclusion of a quantitative evaluation is consistent with that of the visual one. In other words, the LTTR and LTMR perform poorly in spectral reconstruction quality. The other three methods show a competitive ability in HS–MS fusion. In particular, CSTF gains the best MPSNR and ERGAS scores, and SC-LL1 achieves the best SAM, RMSE, and CC values among the competing approaches.

FUTURE CHALLENGES

Although tensor decomposition-based HS–MS fusion technology has been promoted rapidly in recent years and shows a promising reconstruction ability due to its strong exploitation of spatial–spectral structure information, a number of challenges remain:

1) Nonregistered HS–MS fusion: Tensor decomposition-based HS–MS fusion methods focus on pixel-level image fusion, which implies that image registration between two input modalities is a necessary prerequisite and that the fusion quality heavily depends on the registration accuracy. However, most of the current methods pay more attention to the follow-up fusion step, ignoring the importance of registration. As a challenging task, image registration handles the inputs of two modalities acquired from different platforms and times. In the future, efforts should be made to accomplish nonregistered HS–MS fusion tasks.

2) Blind HS–MS fusion: Existing tensor decomposition-based HS–MS fusion methods contribute to the appropriate design of handcrafted priors to derive desired reconstruction results. However, degradation models are often given without the estimation of the real PSF and spectral response function in most tensor-based methods. It is intractable to obtain precisely the degradation functions of real cases, due to the uncertainty of sensor degradation. How to devise blind HS–MS fusion methods with unknown degradation functions is an important challenge.

3) Interimage variability: The different times and platforms of two HS and MS modalities lead to discrepancies in terms of interimage variability. However, tensor decomposition-based approaches usually assume that two modalities are acquired under the same condition and hence ignore the spectral and spatial variability that usually
happens in practice. Taking the interimage variability phenomenon into consideration when modeling the degradation process is a key challenge for future research.

**HYPERSONTICAL UNMIXING**

Owing to its acquired continuous abundance maps, SU has widely solved the inversion problems of typical ground object parameters, such as the vegetation index, surface temperature, and water turbidity, in the past several decades [196], [197], [198] and has been successfully applied in some RS applications, such as forest monitoring and land cover change detection [199]. In addition, due to the mixing phenomenon caused by heterogeneity and the stratified distribution of ground objects, SU can effectively realize crop identification and monitoring [200], [201], [202].

When the mixing scale is macroscopic and each incident light-reaching sensor has interacted with just one material, the measured spectrum is usually regarded as a linear mixing, as described in Figure 22(a). However, due to the existence of nonlinear interactions in real scenarios, several physics-based approximations of the nonlinear linear mixing model (NLMM) have been proposed, mainly covering two types of mixing assumptions: intimate mixture [Figure 22(b)] and multilayered mixture [Figure 22(c)]. The former describes the interactions suffered by a surface composed of particles at a microscopic scale. Intimate mixture usually occurs in scenes containing sand and mineral mixtures and requires a certain kind of prior knowledge of the geometric positioning of the sensor to establish the mixture model. The latter characterizes the light reflectance of various surface materials at a macroscopic scale. Multilayered mixture usually occurs in scenes composed of materials with height differences, such as forests, grassland, and rocks, containing many nonlinear interactions between the ground and canopy. In general, a multilayered mixture consisting of more than two orders is ignored owing to its negligible interactions. For the second-order multilayered mixture model, the family of bilinear mixing models is usually adopted to solve the NLMM. Due to the low spatial resolution of sensors, many pixels mixed with different pure materials exist in HS imagery, which inevitably

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**TABLE 11. A QUANTITATIVE COMPARISON OF DIFFERENT METHODS FOR HS–MS FUSION.**

| INDEX | BLIND-STEREO | CSTF | LTR | LTR | SC-LL1 |
|-------|--------------|------|-----|-----|--------|
| MPSNR | 53.49        | 54.28| 38.11| 39.58| 54.13  |
| ERGAS | 0.3584       | 0.3317| 1.9761| 1.5851| 0.3076 |
| SAM   | 1.0132       | 0.8841| 3.7971| 3.0495| 0.8213 |
| RMSE  | 0.0027       | 0.0025| 0.0165| 0.0132| 0.0023 |
| CC    | 0.9993       | 0.9994| 0.9819| 0.987 | 0.9995 |

**FIGURE 21.** The fusion results of five different HS–MS fusion methods: (a) the reference image, (b) Blind-STEREO, (c) CSTF, (d) LTR, (e) LTTR, and (f) SC-LL1. MRAE: mean relative absolute error.
conceals useful information and hinders high-level image processing. SU aims to separate the observed spectrum into a suite of basic components, also called endmembers, and their corresponding fractional abundances.

**LINEAR MIXING MODEL**

With the assumption of a single interaction between the incident light and material, representative SU methods are based on the following LMM [203], [204]:

\[
X = EA + N \tag{70}
\]

where \( X \in \mathbb{R}^{r \times h} \), \( E \in \mathbb{R}^{r \times e} \), \( A \in \mathbb{R}^{r \times b} \), and \( N \in \mathbb{R}^{r \times h} \) denote the observed unfolding HS matrix, endmember matrix, abundance matrix, and additional noise, respectively. LMM-based methods have drawn much attention due to their model simplicity and desirable performance [205], [206], [207]. Among them, a structured matrix factorization model, such as nonnegative matrix factorization (NMF), has physically meaningful interpretation in solving the inverse ill-posed problem for SU [208]. Over the past decades, numerous NMF-based methods have been developed to pursue better unmixing performance by introducing prior information based on (70), such as spatial smoothness [209], [210], [211], sparseness [212], [213], and volume regularization [214], [215], [216]. However, current LMM-based matrix factorization methods usually convert the 3D HS cube into a 2D matrix, leading to the loss of spatial information in the relative positions of pixels. Tensor factorization-based approaches have been dedicated to SU to overcome the limitation of the LMM.

**CANONICAL DECOMPOSITION/PARALLEL FACTOR ANALYSIS AND TUCKER DECOMPOSITION MODEL**

Zhang et al. [107], [108] first introduced nonnegative tensor factorization (NTF) into SU via CP decomposition. However, this NTF-SU method hardly considers the relationship between the LMM and NTF, giving rise to a lack of physical interpretation. Imbiriba et al. [109] considered the underlying variability of spectral signatures and developed a flexible approach, the unmixing with LR tensor regularization algorithm, accounting for spectral variability. The ranks of the abundance tensor and endmember tensor were estimated with only two easily adjusted parameters. Sun et al. [116] first introduced Tucker decomposition for blind unmixing and increased the sparse characteristic of the abundance tensor.

**BLOCK TERM DECOMPOSITION MODEL**

In terms of tensor notation, an HS data tensor can be represented by the sum of the outer products of an endmember (vector) and its abundance fraction (matrix). This enables a matrix–vector third-order tensor factorization that consists of \( R \) component tensors:

\[
X = \sum_{r=1}^{R} \mathbf{E}_r \cdot \mathbf{B}_r^T \cdot \mathbf{C}_r + \mathbf{N} \tag{71}
\]

where \( \mathbf{E}_r \), calculated by the product of \( \mathbf{A}_r \) and \( \mathbf{B}_r^T \), denotes the abundance matrix; \( \mathbf{c}_r \) is the endmember vector; and \( \mathbf{N} \) represents the additional noise. Apparently, this matrix–vector tensor decomposition has the same form as BT decomposition, setting up a straightforward link with the previously mentioned LMM model. Qian et al. [110] proposed a matrix–vector NTF (MVNTF) unmixing method by combining the characteristics of CP decomposition and Tucker decomposition to extract the complete spectral–spatial structure of HS images. The MVNTF method for SU is formulated as

\[
\min_{k \leq R} \| X - \sum_{r=1}^{R} \mathbf{E}_r \cdot \mathbf{C}_r \|_F^2 \\
\text{s.t. } \mathbf{A}_r, \mathbf{B}_r^T, \mathbf{C}_r \geq 0. \tag{72}
\]

MVNTF essentially derived BT decomposition and established a physical connection with the LMM. Compared with NMF-based unmixing approaches, MVNTF can achieve better unmixing performance in most cases. Nevertheless, the abundance results extracted by MVNTF may be oversmooth and lose detailed information, due to...
the strict LR constraint of NTF. Various spatial and spectral structures, such as spatial–spectral smoothness and nonlocal similarity, are proved to tackle the problem of pure MVNTF.

Xiong et al. [111] presented a TV-regularized NTF (NTF-TV) method to make locally smooth regions share similar abundances among neighboring pixels and suppress the effect of noises. Zheng et al. [112] offered a sparse and LR tensor factorization method to flexibly achieve the LR and sparsity characteristics of the abundance tensor. Feng et al. [113] installed three additional constraints, namely, sparseness, volume, and nonlinearity, in the MVNTF framework to improve accuracy on impervious surface area fraction/classification maps. Li et al. [114] integrated NMF into MVNTF by making full use of the approaches’ individual merits to characterize intrinsic structure information. Besides, a sparsity-enhanced convolutional operation (SeCoDe) method [117] incorporated a 3D convolutional operation into MVNTF for the blind SU task.

MODE 3 TENSOR REPRESENTATION MODEL

Under the definition of the tensor mode-$p$ multiplication, the LMM (70) is equivalent to

$$\mathbf{X} = \mathbf{A} \times_3 \mathbf{E} + \mathbf{N} \tag{73}$$

where $\mathbf{A} \in \mathbb{R}^{h \times k \times r}$ denotes the abundance tensor containing $R$ endmembers. In [115], the NLR tensor and 2DTV regularization of the abundance tensor were introduced to extract the spatial contextual information of HS data. With an abundance nonnegative constraint (ANC) and abundance sum-to-one constraint (ASC) [217], the objective function of NLR-TV for SU is expressed as

$$\min_{\mathbf{A}} \frac{1}{\lambda} \| \mathbf{X} - \mathbf{A} \times_3 \mathbf{E} \|_F^2 + \lambda \| \mathbf{A} \|_{2,DTV} + \lambda \| \mathbf{A} \|_{2,NL} + \sum_{k=1}^{K} \| \mathbf{A}^k \|_{NL} \tag{74}$$

s.t. $\mathbf{A} \geq 0$, $\mathbf{A} \times_3 \mathbf{1}_p = \mathbf{1}_{h \times k \times r}$

where $\mathbf{1}_p$ is a $P$-dimensional vector of all one, $\mathbf{1}_{h \times k \times r}$ denotes a matrix of element one, and the NLR regularization is defined as

$$\| \mathbf{A}^k \|_{NL} = \sum_{i=1}^{p} \text{LS}(\mathbf{A}^{(i)}) \tag{75}$$

The ANC, ASC, and sparseness of abundance are often introduced into sparse unmixing models [218], [219], which produces endmembers and corresponding abundance coefficients by a known spectral library instead of extracting endmembers from HS data [220], [221]. Sun et al. [118] developed a weighted NLR Tucker decomposition method for HS sparse unmixing by adding collaborative sparsity and the 2D TV of the endmember tensor into a weighted NLR tensor framework. The LR constraint and joint sparsity in the nonlocal abundance tensor were imposed in a nonlocal tensor-based sparse unmixing algorithm [222].

NONLINEAR MIXING MODEL

Due to physical interactions among multiple materials in a scene, NLMMs have been studied in SU by modeling different-order scattering effects and producing more accurate unmixing results [223], [224], [225]. As a typical representative of NLMMs, bilinear mixture models (BMMs) have a clear physical interpretation and can almost characterize the radiative transfer process in a scene. To further improve the description of nonlinear interactions, several BMM-based models are proposed to characterize macroscopic and microscopic-scale effects, such as the Nascimento model [226], fan model [227], polynomial postnonlinear mixing model [228], and generalized bilinear model [229]. However, traditional BMMs usually transform an HS cube into a 2D matrix and have the same fault as LMMs [229], [231], [232].

To effectively address the nonlinear unmixing problem, Gao et al. [119] expressed an HS cube $\mathbf{X} \in \mathbb{R}^{h \times k \times r}$ based on tensor notation in the following format:

$$\mathbf{X} = \mathbf{A} \times_3 \mathbf{C} + \mathbf{B} \times_3 \mathbf{E} + \mathbf{N} \tag{76}$$

where $\mathbf{C} \in \mathbb{R}^{h \times k \times r}$, $\mathbf{B} \in \mathbb{R}^{h \times k \times r(R-1)/2}$, and $\mathbf{E} \in \mathbb{R}^{h \times k \times r(R-1)/2}$ represent the mixing matrix, nonlinear interaction abundance tensor, and bilinear interaction endmember matrix, respectively. A nonlinear unmixing method [119] was first based on NTF by taking advantage of the LR property of the abundance maps and nonlinear interaction maps, which validated the potential of tensor decomposition in nonlinear unmixing. Note that the optimization of tensor-based unmixing methods can be effectively solved by the ADMM.

EXPERIMENTAL RESULTS AND ANALYSIS

Urban HS datasets obtained by the Hyperspectral Digital Imagery Collection Experiment sensor over Texas, USA, are selected for evaluating the performance of different unmixing methods qualitatively, including MVNTF [110], MVNTF-TV [111], a SeCoDe [117], and LR-NTF [119]. For a fair comparison, HS signal subspace identification by minimum error [233] and vertex component analysis [234] algorithms are adopted to determine the number of endmembers (NOE) and endmember initialization. The urban data contain 307 × 307 pixels and 210 bands ranging from 0.4 to 2.5 $\mu$m. Due to water vapor and atmospheric effects, 162 bands remained after removing affected channels. Four main materials in this scene are investigated, that is, asphalt, grass, trees, and roofs. Two quantitative metrics are utilized to evaluate the extracted abundance and endmember results, namely, the RMSE and SAD.

For illustrative purposes, Figures 23 and 24 display the extracted abundances and corresponding endmember results of different tensor decomposition-based SU approaches. The quantitative results of the urban data are reported in Table 12, where the best results are marked in bold. MVNTF yields poor unmixing performance for both endmember extraction and abundance estimation compared with other...
tensor-based unmixing methods since it considers only the tensor structure to represent the spectral–spatial information of HS images and ignores other useful prior regularizations. Compared with MVNTF, MVNTF-TV integrates the advantage of TV and tensor decomposition, bringing certain performance improvements in terms of the SAD, MSAD, and RMSE. The SeCoDe effectively addresses the problem of spectral variabilities in a convolutional decomposition fashion, thereby yielding a further performance improvement of the endmember and abundance results. Differing from the SeCoDe, LR-NTF considers the nonlinear unmixing model of tensor decomposition and

![Abundance maps of different methods on the urban dataset](image1)
**FIGURE 23.** The abundance maps of different methods on the urban dataset: (a) the ground truth, (b) MVNTF, (c) MVNTF-TV, (d) SeCoDe, and (e) LR-NTF.

![Reflectance plots for different methods](image2)
**FIGURE 24.** The endmember results of different methods on the urban dataset: (a) asphalt, (b) grass, (c) a tree, and (d) a roof.
Several advanced tensor decomposition-based methods have recently achieved effectiveness in HS SU. Nonetheless, there is still a long way to go toward the definition of statistical models and the design of algorithms. In the following, we briefly summarize some aspects that deserve further consideration.

The SU workflow usually contains the preprocessing step of estimating the NOE, and it plays a crucial role in the field of SU [235], [236]. However, almost all NOE approaches convert HS data into 2D and perform matrix-based estimation. Recent research in [134] demonstrates the potential of tensor-based NOE methods in preventing data loss. In this way, these methods based on tensor decomposition can further be exploited to yield good performance for NOE estimation, especially in scenes with high mixing and outliers.

The most commonly utilized evaluation indices for HS SU include the RMSE (which measures the error between the estimated abundance map and reference abundance map) and SAD (which assesses the similarity of the extracted endmember signatures and true endmember signatures). However, the RMSE and SAD contribute to a quantitative comparison of SU results only when ground truth for abundances and endmembers exists. If there are no references in a real scenario, meaningful and suitable evaluation metrics should be developed in future work.

Traditional NLMMs are readily interpreted as matrix factorization problems. The tensor decomposition-based NLMM has been springing up in recent years. We should consider complex interactions, such as intimate and multilayered mixture, for establishing general and robust tensor models. In addition, the rise of deep learning techniques layered mixture, for establishing general and robust tensor models. In addition, the rise of deep learning techniques has led to significant unmixing performance improvement.

Another important challenge is the time consumption of high-performance SU architectures, which hinders their applicability in real scenarios. In particular, as the NOE and size of an image increase, current NTF-based unmixing methods have difficulty dealing with this situation, owing to high computational consumption. Therefore, the exploration of more computationally efficient tensor-based approaches will be an urgent research direction in the future.

CONCLUSION

The HS technique accomplishes the acquisition, utilization, and analysis of nearly continuous spectral bands and permeates a broad range of practical applications, having attracted increasing attention from researchers worldwide. In HS data processing, large-scale and high-order properties are often involved in collected data. The ever-growing volume of 3D HS data puts higher demands on processing algorithms to replace 2D matrix-based methods. Tensor decomposition plays a crucial role in both problem modeling and methodological approaches, making it possible to leverage the spectral information of each complete 1D spatial signature and spatial structure of each complete 2D spatial image. In this article, we presented a comprehensive and technical review of five representative HS topics, including HS restoration, CS, AD, HS–MS fusion, and SU. Among these, we reviewed current tensor decomposition-based methods, with main formulations, experimental illustrations, and remaining challenges. The most important and compatible challenges related to consolidating tensor decomposition techniques for HS data processing should be emphasized and summarized in five aspects: model applicability, parameter adjustment, computational efficiency, methodological feasibility, and multimission applications:

1) Model applicability: Tensor decomposition theory and practice offer us versatile and potent weapons to solve various HS image processing problems. A high-dimensional tensor is often decomposed by different categories of tensor decomposition into several decomposition factors/cores. One sign reveals that the mathematical meaning of different factors/cores should be made in connection with the physical properties of the HS structure. Another sign is that each HS task contains multiple modeling problems, such as various types of HS noise (i.e., Gaussian noise, stripes, and mixed noise) caused by different kinds of sensors and external conditions. Tensor decomposition-based models should be capable of characterizing specific HS properties and use in different scenarios. Beginners and experienced practitioners are expected to learn basic knowledge of tensor decompositions from tutorials and references [20], [22], [25], which contribute to building readers’ awareness of tensorial theory. We also recommend several materials and references related to HS RS applications to master the imaging properties and image features, including the spectral and spatial information of HS data, further yielding significant unmixing performance improvement.

FUTURE CHALLENGES

| METHOD | SAD | Asphalt | Grass | Tree | Roof | MSAD | RMSE |
|--------|-----|---------|-------|------|------|------|------|
|        |     | 0.3738  | 0.2572| 0.1474| 0.2825| 0.2652| 0.2638|
|        |     | 0.2606  | 0.1722| 0.145  | 0.2273| 0.2013| 0.2588|
|        |     | 0.219   | 0.0876| 0.0854 | 0.3681| 0.1839| 0.1453|
|        |     | 0.1127  | 0.1349| 0.0632 | 0.0395| 0.0876| 0.1451|
|        |     |         |       |       |      |      |      |
Parameter adjustment: In the algorithmic solution, parameter adjustment is an indispensable portion to achieve significant HS data processing performance. Parameters can be gradually tuned via extensive simulated experiments, while sometimes, they should be reset for various datasets, due to the uncertainty of data size. In practice, users are most likely to be nonprofessional, with little knowledge of a special algorithm, leading to improper parameter setting and unsatisfactory processing results. Therefore, in the future, efforts should be made to design a fast proper-parameter search scheme and reduce the number of parameters to increase algorithmic practicability. Based on this overview, beginners are encouraged to make good use of open source codes listed in Table 1 and HS RS data given in Table 2. After long-term learning and actual operations, beginners will be proficient in properly adjusting parameters and increasing algorithmic feasibility.

Computational consumption: Tensor decomposition-based methods have achieved satisfactory results in HS data processing, yet they sometimes cause high computational consumption. For instance, for an NLR tensor denoising model, TDL spends more than 10 min for a dataset of 200 × 200 × 80. As the image size increases, the increasing number of nonlocal FBP will cause greater time consumption. Thus, there exists vast room for promotion and innovation in improving the optimization efficiency of HS data processing. Parallel and distributed implementations of big tensor decompositions are beneficial to accelerate the optimization process. A classic tutorial [244] is presented for practitioners to understand the parallel and distributed computation of big tensors from the perspective of mathematical analytics. Several state-of-the-art techniques [245], [246], [247], [248] related to this architecture have emerged in recent years. For example, Rolingera et al. [245] extended three parallel tools to implement CP decomposition using ALS fitting: SPLATT, DFacTo, and ENSIGN. Tensor products can be reduced to multiple sparse matrix operations, and how such operations are used to execute CP-ALS has a significant practical influence on total memory usage and parallelizability.

Methodological feasibility: Unlike deep learning-based methods, designing handcrafted priors is the key to tensor decomposition-based methods. Existing methods exploit the structure information of the underlying target image by implementing various handcrafted priors, such as LR, TV, and nonlocal similarity. However, different prior assumptions apply to specific scenarios, making it challenging to choose suitable priors according to the characteristics of HS images to be processed. Deep learning-based methods automatically learn prior information implicitly from datasets themselves, without the trouble of manually designing a manual regularizer. As an advisable approach, deep learning can be incorporated into tensor-based methods to mine essential multifeatures and enhance methodological feasibility. For instance, recently, transform-based TNN minimization approaches have been developed to capture underlying LR tensor structures in HS RS applications. A non-linear transform is learned by a nonlinear multilayer neural network under self-supervision [249], [250].

Multimission applications: The extremely broad field of HS imagery makes it impossible to provide an exhaustive survey of all the promising HS RS applications. It is certainly of significant interest to develop tensor decomposition-based models for other noteworthy processing and analysis chains in future work, including classification, change detection, large-scale land cover mapping, and image quality assessment. Some HS tasks serve as a preprocessing step for high-level vision. For example, the accuracy of HS classification can be improved after an HS denoising step. How to apply tensor decomposition for high-level vision and even multimission frameworks may be a key challenge. To try tensor decomposition-based models for multimission applications, we should confirm the significance of specific multimission research, which needs to master the whole HS RS imaging process and consequent data analysis. Simultaneously, we recommend several existing multimission techniques, such as coupled denoising and unmixing [251], denoising and classification [252], unmixing using AD [253], and so on.

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Author Information Minghua Wang (wangmh@aircas.ac.cn) received her B.S. degree in 2016 from the School of Automation, Harbin Institute of Technology, Harbin, China, where she received her Ph.D. degree in control science and engineering in 2021. She was also a visiting Ph.D. student at the University of Grenoble Alpes, Grenoble, France, in 2019–2020. She is currently a postdoc with the Aerospace Information Research Institute, Chinese Academy of Sciences, Beijing 100094 China. Her research interests include hyperspectral image denoising, anomaly detection, tensor learning, and low-rank representation. She is a Member of IEEE.

Danfeng Hong (hongdf@aircas.ac.cn) received his M.Sc. degree (summa cum laude) in computer vision from the College of Information Engineering, Qingdao University, Qingdao, China, in 2015 and his Dr.-Ing degree (summa
cum laude) through the Signal Processing in Earth Observation project, Technical University of Munich, Munich, Germany, in 2019. He is currently a professor with the Aerospace Information Research Institute, Chinese Academy of Sciences (CAS), Beijing 100094 China. Before joining the CAS, he was a research scientist and led the Spectral Vision Working Group, Remote Sensing Technology Institute, German Aerospace Center, Oberpfaffenhofen, Germany. He was also an adjunct scientist at the Grenoble Images Speech Signal and Control lab, University of Grenoble Alpes, Grenoble, France. His research interests include signal/image processing, hyperspectral remote sensing, machine/deep learning, artificial intelligence, and their applications in Earth vision. He is an associate editor of IEEE Transactions on Geoscience and Remote Sensing (TGRS) and editorial board member and editorial advisory board member of ISPRS Journal of Photogrammetry and Remote Sensing. He was a recipient of the 2021 and 2022 IEEE TGRS Best Reviewer Award, 2021 and 2022 IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing Best Reviewer Award, 2021 Jose Bioucas Dias Award for an outstanding paper at the IEEE Workshop on Hyperspectral Image and Signal Processing: Evolution in Remote Sensing, 2022 Remote Sensing Young Investigator Award, and 2022 IEEE Geoscience and Remote Sensing Society Early Career Award, and he was a highly cited researcher (Clarivate Analytics/Thomson Reuters) in 2022. He is a Senior Member of IEEE.

Zhu Han (hanzhu19@mails.ucas.ac.cn) received his B.S. degree in electrical engineering from North China University of Technology, Beijing, China, in 2019. She is pursuing her Ph.D. degree in cartography and geographic information systems at the Key Laboratory of Computational Optical Imaging Technology, Aerospace Information Research Institute, Chinese Academy of Sciences, Beijing 100094 China. Her research interests include hyperspectral image processing, deep learning, and artificial intelligence. She is a Student Member of IEEE.

Jiaxin Li (lijiaxin203@mails.ucas.ac.cn) received his B.E. degree from Chongqing University, Chongqing, China, in 2020. He is currently pursuing his Ph.D. degree in cartography and geographic information systems with the Key Laboratory of Computational Optical Imaging Technology, Aerospace Information Research Institute, Chinese Academy of Sciences, Beijing 100094 China. His research interests include multimodal remote sensing data fusion, hyperspectral image processing, and deep learning.

Jing Yao (yaojing@aircas.ac.cn) received his Ph.D. degree in mathematics from Xi’an Jiaotong University, Xi’an, China, in 2021. He is currently an assistant professor with the Key Laboratory of Computational Optical Imaging Technology, Aerospace Information Research Institute, Chinese Academy of Sciences, Beijing 100094 China. From 2019 to 2020, he was a visiting student at the Signal Processing in Earth Observation project, Technical University of Munich, Munich, Germany, and the Remote Sensing Technology Institute, German Aerospace Center, Oberpfaffenhofen, Germany. His research interests include hyperspectral and multimodal remote sensing image analysis, mainly including optimization and deep learning-based methods for image processing and interpretation tasks. He was the recipient of the 2021 Jose Bioucas Dias Award for an outstanding paper at the IEEE Workshop on Hyperspectral Imaging and Signal Processing: Evolution in Remote Sensing. He serves as a guest editor of Remote Sensing and is a Member of IEEE.

Lianru Gao (gaolr@aircas.ac.cn) received his B.S. degree in civil engineering from Tsinghua University, Beijing, China, in 2002, and Ph.D. degree in cartography and geographic information systems from the Institute of Remote Sensing Applications, Chinese Academy of Sciences (CAS), Beijing, China, in 2007. He is currently a professor with the Key Laboratory of Computational Optical Imaging Technology, Aerospace Information Research Institute, CAS, Beijing 100094 China. He was a visiting scholar at the University of Extremadura, Cáceres, Spain, in 2014 and Mississippi State University, Starkville, MS, USA, in 2016. His research interests include hyperspectral image processing and information extraction. In the past 10 years, he has been the principle investigator of 10 scientific research projects at national and ministerial levels, including projects by the National Natural Science Foundation of China (2016–2019, 2018–2020, and 2022–2025) and Key Research Program of the CAS (2013–2015). He has published more than 180 peer-reviewed papers and has more than 100 journal papers in the Science Citation Index. He was a coauthor of three academic books, including Hyperspectral Image Information Extraction, and has obtained 29 national invention patents in China. He was awarded the 2016 CAS Outstanding Science and Technology Achievement Prize, was supported by the China Natural Science Fund for Excellent Young Scholars in 2017, and won a second-place 2018 State Scientific and Technological Progress Award. He was a better reviewer of IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing in 2015 and IEEE Transactions on Geoscience and Remote Sensing in 2017. He is a Senior Member of IEEE.

Bing Zhang (zb@radi.ac.cn) received his B.S. degree in geography from Peking University, Beijing, China, in 1991 and M.S. and Ph.D. degrees in remote sensing from the Institute of Remote Sensing Applications, Chinese Academy of Sciences (CAS), Beijing, China, in 1994 and 2003, respectively. Currently, he is a full professor at and the deputy director of the Aerospace Information Research Institute, CAS, Beijing 100094 China, where he has led numerous scientific projects in the area of hyperspectral remote sensing for more than 25 years. His research interests include the development of mathematical and physical models and image processing software for the analysis of hyperspectral remote sensing data in many areas. He has developed five software systems for image processing and applications. His achievements were rewarded with 10 prizes from the Chinese government and special allowances from the Chinese State Council. He was recognized with the 2013 National
Science Foundation for Distinguished Young Scholars of China award and 2016 CAS Outstanding Science and Technology Achievement Prize, the highest-level award for CAS scholars. He has authored more than 300 publications, including more than 170 journal papers. He has edited six books and contributed book chapters on hyperspectral image processing and subsequent applications. He is a Fellow of IEEE, currently serving as an associate editor of IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing. He has served as a Technical Committee member of the IEEE Workshop on Hyperspectral Image and Signal Processing since 2011, the president of the Hyperspectral Remote Sensing Committee of the China National Committee of International Society for Digital Earth since 2012, and a standing director of the Chinese Society of Space Research since 2016. He was a Student Paper Competition Committee member of the International Geoscience and Remote Sensing Symposium from 2015 to 2019.

Jocelyn Chanussot (jocelyn@hi.is) received his M.Sc. degree in electrical engineering from the Grenoble Institute of Technology (Grenoble INP), Grenoble, France, in 1995, and Ph.D. degree from the Université de Savoie, Annecy, France, in 1998. Since 1999, he has been with Grenoble INP, Grenoble 38000 France, where he is currently a professor of signal and image processing. His research interests include image analysis, hyperspectral remote sensing, data fusion, machine learning, and artificial intelligence. He has been a visiting scholar at Stanford University (Stanford, CA, USA), KTH Royal Institute of Technology (Stockholm, Sweden), and the National University of Singapore (Singapore). Since 2013, he has been an adjunct professor of the University of Iceland, Reykjavik, Iceland. In 2015–2017, he was a visiting professor at the University of California, Los Angeles, Los Angeles, CA, USA. He holds the AXA chair in remote sensing and is an adjunct professor at the Aerospace Information Research Institute, Chinese Academy of Sciences, Beijing, China. He was the founding president of the IEEE Geoscience and Remote Sensing Society (GRSS) French Chapter (2007–2010), which received the 2010 GRSS Chapter Excellence Award. He has received multiple outstanding paper awards. He was a vice president of the GRSS, in charge of meetings and symposia (2017–2019). He was the general chair of the first IEEE GRSS Workshop on Hyperspectral Image and Signal Processing: Evolution in Remote Sensing. He was the chair (2009–2011) and cochair of the GRSS Data Fusion Technical Committee (2005–2008). He was a member of the IEEE Signal Processing Society Machine Learning for Signal Processing Technical Committee (2006–2008) and the program chair of the IEEE International Workshop on Machine Learning for Signal Processing (2009). He is an associate editor of IEEE Transactions on Geoscience and Remote Sensing and Proceedings of the IEEE. He was the editor-in-chief of IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing (2011–2015) and an associate editor of IEEE Transactions on Image Processing. In 2014, he served as a guest editor of IEEE Signal Processing Magazine. He is a Fellow of IEEE, a former member of the Institut Universitaire de France (2012–2017), and a highly cited researcher (Clarivate Analytics/Thomson Reuters).

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