Studying a Two-level Supply Chain Including a Manufacturer and Several Retailers with a Wholesale Price Contract between Them

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ABSTRACT

As competition has intensified in today’s market, most of companies have understood that their business performance is highly dependent on the degree of cooperation and coordination in the supply chain. One of the factors causing supply chain coordination is the wholesale price contract. In this study, a two-level supply chain including a producer and several retailers that wholesale price contract is established among them. First, to construct the initial model, the library method is used to make the primary model and then, mathematical methods are used to construct the model. The final goal of this study is to determine the optimum order quantity and optimal production cost and maximizes the supply chain profit in a single-period model with a random demand market. Also, the case where the exchange of goods between retailers is justified under the conditions of shortage and surplus, and retailers are co-operating with each other is also examined.

Keywords: Supply Chain, Pricing Chain And Order Quantity, Coalition of the Retailers, Problem of the Newsvendor

JEL Classifications: L1

1. INTRODUCTION

Co-ordination among the supply chain members is an essential strategic issue. In coordination with the supply chain, for example in the contract system, the top members present, the parameters of a suitable contract (with the negotiations) set down to the downstream members so that they complete it so that they are closer to the target gains (Sana, 2013). Under the executive scope of the contract, the agreements can be singe between members who have a decisive action to execute one or more issues (Govindan et al., 2013). Hoehn et al. (2010) suggest that the contracts signed in the supply chain constitute a coordination in the problem of the newsvendor and contract sharing consist of: (1) wholesale price contract. (2) Return of purchase deal, (3) profit sharing deal, (4) contract of the flexibility quantity, (5) contract of sale discount, (6) contract of quantity discount. The problem of the newsvendor is to find optimal order quantity for single-period sales that maximizes the expected profit. The newsvendor products (such as flash drivers, fashion clothing, PCs. Etc.), are characterized as quarterly or tidal products. The short period of these products will result in the quantity of non-zero or low relinquishing at the end of the season, which makes the contribution and coordination of the supply chain in particular.

The main objective of coordination in the variable issues of coordination in some activities is to minimize costs or maximize profits. The coordination of some activities at different levels of supply chain reduces the supply chain costs. In this study, two-level supply chain in a problem of the newsvendor involve as a producer and a number of retailers with a wholesale price contract between them. First, to contract the initial model, the library method is used and then using mathematical methods to make the model.
In this paper, determining optimal order quantity and optimal production cost and maximizing the supply chain profit in a single-period model with random demand market will studied. It is also consider a case where the exchange of goods between retailers is justify in terms of a dearth of surplus-deficit and retailers unite.

2. LITERATURE REVIEW

Numerous studies have been propose in terms of optimal ordering and optimal price and problem of the newsvendor and the supply chain coordination, where some are refer too.

Arshinder et al. (2011) demonstrate that supply chain members are dependent on reach other for resources and information, in recent years their dependence on information technology has increased because of globalization and rapid innovations. This increase the risk dependence and high uncertainly associated with the benefits that, in order to deal these challenges, the supply chain members should move to a coordination and integrate system.

Malone et al. (1994) believe that addressing the issues of managing relationships between members of a chain in an intricate supply network creates needs that are used to eliminate them from coordination theory. In each system, the mental performance of the members is the result of good coordination of members. However, the exact definition of coordination may be very difficult. Coordination can be viewed from a number of dimensions, for example, based on organizational structure, coordination theory, group coordination theory, and balanced AI theory. The common definition of coordination in the supply chain is the coordination of the art of managing interdependence between members and the combined efforts of members to achieve the designated goals.

The study of Simatupang et al. (2002) shows that Coordination is a prerequisite for achieving bilateral goals in the supply chain.

Cachon (2003) offers the sequence of events in supply chain contracts as follows:
1. The manufacturer of the contract offers the buyer
2. The buyer accepts or denies the contract. Under the assumption that the buyer accepts the contract, the buyer orders Q
3. The manufacturer generates a buyer’s order before the start of the sales season
4. Finally, transfers between institutions are made in accordance with the agreed agreement.

Govindan et al. (2013) point out that the wholesale price contract is the easiest type of coordination contract that is very simple to operate and the information it needs is very low, but since it has only one parameter, it has very little ability to synchronize the supply chain in different conditions.

Cachon (2004) shows that although it has not yet coordinated the wholesale price contract for the supply chain, this contract is worth examining and studying because it is commonly observed in practice that it alone has redeemable features. For example, a wholesale price contract for its use is simple, so if the additional executive responsibility associated with the coordination agreement exceeds the potential profit of the manufacturer. The manufacturer may prefer the wholesale price contract to the coordination agreement.

Chen (2011) examines the wholesale discount price along with the return policy on the issue of the newsvendor, newsvendor problem from a manufacturer and a retailer, in which the maker of the leader and retailer follows. In the newsvendor problem, the goal is to find the optimal order quantity for a single-sale sale that maximizes the expected profit.

\[ D = \text{The retailer expected demand that is categorical.} \]
\[ C_i = \text{Probable demand of the retailer.} \]
\[ X_i = \text{Total retail demand that is probable.} \]
\[ z = \text{The reserve level of the retailer’s confidence level} \]
\[ M_i = z - e_i \text{ (excess scale)} (M \geq 0) \]
\[ K_i = c_i - z \text{ (deficiency)} (k \geq 0) \]
\[ P = \text{Retail prices} \]
\[ C = \text{cost of supplying goods to the producer} \]
\[ g = \text{the cost incurred by the retailer if there is a shortage of goods (cost of shortage).} \]
\[ T_{pi} = \text{the average net profit function of retailer} \]
\[ T_{pm} = \text{the average producer profit} \]
\[ T_{ps} = \text{the average of supply chain profit} \]
\[ W_{(pd)} = \text{the return of goods cost from retailer to producer} \]
\[ V = \text{the return of goods cost from retailer to producer} \]
\[ Q = \text{The quantity of every retailer’s order} \]
\[ Q = D + z \]
\[ W = \text{sales price of producer} \]
\[ Q^* = \text{The amount of optimal retailer order} \]
\[ W^* = \text{optimal cleaning price} \]

3. PROBLEM STATEMENT

In this study, problem of newsvendor has been investigate in the supply chain, which consist of a manufacturer and two retailers, in which the leader and retailers are follower. The producer, as a leader, put the terms of the contract in the form of “take this or leave its” to the retailers. The retailers are supposed to accept the contract if they make their profits positive and the price of selling products to retailers (w) exceeds the retail price (p). Retailers are facing a potential demand (x), which is define in two possible demand (\(\varepsilon\)) categories and expected and absolute (D) demand:

\[ X = D + \varepsilon \]  

(1)

The producer invokes the wholesale price of W per unit purchased from the retailer and sells the product to its customers at retail price P. the retailer consider the risk of holding unsold goods. This simple transaction between the retailer and the supplier is refer to as the wholesale price contract. As a leader, the supplier offers the terms of the contract to the retailer, it is assume that if such an agreement leads to a non-negative profit retailer, the retailer accepts this contract.
At the beginning of the retail sales season, the retailer will make the order at the expense of WQ. If the demand (X) is not above Q in this period, the retailer’s income is PQ, and if the demand is higher than Q, the retailer’s income is PQ, and any commodity deficiency (XQ) that does not satisfy demand is fractional and Z = Q as confidence storage level.

The purpose of this study is to determine the optimal order and optimal quantity of the supplier between the two retailers and one manufacturer with a wholesale price contract. A wholesale price deal in the two states is a non-coalition between retailers and the existence of a coalition between retailers. At the time of the coalition, retailers are committed to exchanging goods if any pf them face a shortage of goods.

4. RESEARCH HYPOTHESES

1. It is a one-time model and is suitable for modeling fashion goods. Therefore, more sales in this period will lead to an increase in the total revenue of the chain.
2. The demand for the cargo is possible and the demand function for each individual retailer is independently defined.
3. Retailer prices and demand distribution for retailers are known.
4. For giving a security about nonnegative of in the (−A, A), is defined that to be A ≤ D.

Determining the quantity of the producer and the optimal order quantity of the two independent retailers and one manufacturer.

In the present of two independent retailers and a producer, the first retailer is facing possible demand (ε₁) that (−A, A) is exposed and (ε₂) there is also a second retailer possibly demand as shown in Figure 1 in the distance (−A, A).

There is no commodity shortages on the line. Assuming that Z₁ = z then ε₁ + ε₂ = z₁ + z₂ = 2z. Above the line, there is a lack of goods (one) and at the bottom of the line (two area) surplus of the product.

**Figure 1:** Surplus area and lack of goods for two retailers

\[ M_i = \int (z - \varepsilon_i) \times \frac{1}{2A} \, d = \frac{A}{4} + \frac{z^2}{2A} \]  

In addition, if z ≥ εₖ is a retailer at the end of the period, the lack of goods for each retailer is calculated using the following equation:

\[ K_i = \int (\varepsilon_i - z) \times \frac{1}{2A} \, d = \frac{A}{4} - \frac{z^2}{2A} \]

f(εₖ) is the probability density function of the probable part of commodity demand for each retailer and F (εₖ) is consider as a cumulative distribution function of ε, and it assumed to be strictly additive F (εₖ), and it is strictly addition F (εₖ).

In light of the fact, the supplier is the leader, so, first, the supplier will maximize the profit of each of the weights separately to estimate the optimal order quantity that given at the price W.

The average profit function of the sales tax is as follow:

\[ TP_i = (p - c)D - cM_i - (p + g - c)K_i = D(-c + p) \]

\[ = -(-c + g + p) \left( \frac{d}{4} + \frac{z}{2A} - \frac{z^2}{4A} \right) + c \left( \frac{d}{4} + \frac{z}{2A} + \frac{z^2}{4A} \right) \]

For a wholesale price (w), there is an optimal level of trust (z*) and a unique \( Q^*(w) \) order quantity for each retailer, which is determine by differentiating the mean of the retailer’s net profit function and placing it equal to zero as follow:

\[ \frac{dTP_i}{dz} = -(-c + g + p) \left( \frac{1}{2} + \frac{z}{2A} \right) + c \left( \frac{1}{2} + \frac{z}{2A} \right) = 0 \]

\[ Z^*_i = A(2c - g - p) \]

\[ Q^*_i = Z^*_i + D = A - \frac{2Ac}{g + p} + D \]

The average utility function is as follow:

\[ = D(-c + w) + \frac{A(g + p - z)(-c + k)}{(z + z)} \]

By writing the average yield function of the producer and substitution of the optimal Z obtained from the pre-and differentiation phase, the optimal, the optimal W quantity is as follows:

\[ \frac{BTP}{B_w} = D + \frac{A(g + p - 2w)}{g + p} - \frac{2A(-c + w)}{g + p} = 0 \]

\[ W^* = 2Ac + Ag + Dg + Az + Dz \]

Given that W is gaining the quantity of the profit function of the retailer’s objective function, the approximate W can be derive from the following:
Based on the Chen, if \( w > w^* \) is, then \( \frac{\partial TPr}{\partial w} > \frac{\partial TPr}{\partial w} \), therefore, it is possible to obtain approximate \( w \) according to the same relationship.

\[
\frac{\partial TP}{\partial w} = -A + D + \frac{2Aw}{g + p} - w - \left( \frac{A - \frac{ZA}{Z - Z^2}}{g + p} \right) - (g + p - w) \left( \frac{A - \frac{ZA}{Z - Z^2}}{Z + P} \right)
\]

(11)

The average supply function of the supply chain is defined as:

\[
TP_i = TP_p + TP_m
\]

(14)

\[
TP_i = (p - w) D - (wM) - (p + g - w) K + (w - c)
\]

D + (w - c) (M_i - K_i)

(15)

Determine the optimal quantity of the producer and the optimal ordering quantity with two coalition partners and a producer

In this case, the two retailers compete with each other without informing each other that is any of them suffers from lack of goods, the retailer will then exchange goods with another retailer in case of surplus.

First, the producer as in the state of an optimal \( W \), determines optimal \( W \) according to its leader in the chain and by declaring \( w > p \) by the producer and they determine the optimal ordering quantity according to the alliances between them. Thus, they specify the quantity of the calculated optimal order quantity to the producer.

The amount of surplus is equal to:

\[
M = \int_{-A}^{z-A} \int_{-A}^{z-A} \frac{2}{A^2} d_d d_z + \int_{-A}^{z-A} \int_{-A}^{z-A} \frac{1}{A^2} d_d d_z = z(A + z) + A^2 - z^2
\]

(16)

The deficiency is equal to:

\[
K = \int_{-A}^{z-A} \int_{-A}^{z-A} (1 + 2 - 2Z) \frac{1}{A^2} d_d d_z = (A - z)^2
\]

(17)

In this case, the profit function of the retailer is determined according to the degree of less and the amount of the joint surplus between them as follow:

\[
TP_r = 2D (p - w) - wM - (p + g - w)K
\]

\[
2D(p - w) - \frac{(g + z - w)(A - z)^2}{2A^2} - w \left( \frac{z(A + z)}{A} + \frac{A^2 - z^2}{2A^2} \right)
\]

(18)

In the above utility function, the amount of surplus and the common deficiency of the two retailers are considered.

Again, it is derive to determine the optimal quantity of \( Z \) and the optimal quantity of the order from the retailer’s profit function and placed equal to zero.

\[
\frac{\partial TP}{\partial z} = \left( \frac{g + p - w}{3A^2} \right) - w \left( \frac{z(A + z)}{A} + \frac{A^2 - z^2}{2A^2} \right)
\]

(19)

The optimal quantity and optimal ordering quantity for the two retailers that they subscribe is as follow:

\[
Z^*_1 = \frac{Z + \frac{Z}{A^2} \cdot \frac{A - z}{A}}{Z + \frac{Z}{A^2}}
\]

(20)

\[
Z^*_2 = \frac{Z + \frac{Z}{A^2} \cdot \frac{A - z}{A}}{Z + \frac{Z}{A^2}}\]

\[
Q^*_1 = z + D \cdot \frac{Z + \frac{Z}{A^2} \cdot \frac{A - z}{A}}{Z + \frac{Z}{A^2}}
\]

(21)
The average supply function of the supply chain function is as follow:

\[ \text{TP}_m + \text{TP}_t = \text{TP}_y \]  \hspace{0.5cm} (23)

\[ (w - c) D + (w - c)(M - K) + (p - w) 2D - wM - (p + g - w) K = \text{TP}_y \]  \hspace{0.5cm} (24)

It is show that the average function of the retailer’s earnings at the coalition is larger than the function of average retail profit when there is no coalition.

\[ \text{TP}_r = (p - w)*D - w* M - (p + g - w)*K = D(p - w) \]

\[ -w \left( z + \frac{A^2}{2(A + z)} - \frac{Z^2}{2(A + z)} \right) - (g + p - w) \]

\[ \left( \frac{A^2}{2(A + z)} - \frac{Az}{A + z} + \frac{z^2}{2(A + z)} \right) \]

\[ (25) \]

\[ \text{TP}_u = (p - w)* D - w* M - (p + g - w)*K \]

\[ = D(p - w) - \left( \frac{g + z - w(A - z)^2}{2A^2} \right) - w \left( \frac{Z(A + z)}{A} + \frac{A^2 - z^2}{2A^2} \right) \]

\[ (26) \]

5. NUMERICAL STUDIES

In order to compute optimal order quantity and optimal producer price, the parameters are consider as follow:

\[ A = 40, \text{D} = 200, g = 10, c = 10, p = 30, z = 10 \]

In this case, to compute by means of the average of function of retailer profit and producer and function of the supply chain utility is plotted using this diagram, an approximate W is obtained. In Figure 2 the W range is consider with respect to quantity of c = 10, p = 30, (10, 30).

As it can be seen in the Figure 2, the slope function of the retailers in the coalition state is less than the function of the function in the coalition state.

**Figure 2:** Change in profits with regard to the producer price at the time of the coalition and the lack of coalition.
non-coalition state, so the given example shows that in the non-return state of goods and price discount if retailers have alliances with each other, the quantity of retailers’ decreases with less speed. Thus, retailers prefer to merge with one another for the exchange of goods. In addition, considering the Figure 2 in the coalition state between retailers is higher than the retail profit function at a time when retailers disagree with each other, indicating that at a certain supplier price, retailers are more profitable at the time of the coalition.

Regarding the W-quantity, in the coalition state it is about 22 and in the non-coalition state is about 21.

According to W and the coalition state, we can decrease W by using \( w < w^* = \frac{-\partial E[T_{Pr}]}{\partial w} \), so to obtain a better quantity for its. Using \( W = 22 \), the amount of \( -\frac{\partial E[T_{Pr}]}{\partial w} \) is equal to 420 and the amount of \( \frac{\partial E[T_{Pm}]}{\partial w} \) is equal to 172, so the bet is established as 172<. In this case, the average quantity function of the retailers and the average quantity of the producer’s profit function is obtain by using the corresponding formulas in the order of 2352, 2535 respectively.

\[
TP_y = 2D(p-w) - wM - (p + g-w) K
\]

\[
2D(p-w) - \frac{(g+p-w)(A-z)^2}{2A} - w \left( \frac{z(A+z)}{A} + \frac{A^2 - Z^2}{2A^2} \right)
\]

\[
= TP_M = (w-c)D + (w-c)(M-K)
\]

Given the different W quantities, the quantity of the retailers and the suppliers and the supply chain are calculate as shown in the Table 1.

As shown in Tables 1 and 2, the proportion of retailers in the coalition is higher than profit when non-integration and the price of retailers in both modes (coalition and non-coalition) increases with the reduction in supplier prices, as well as the supply chain profit margin, while the production rate is reduced by increasing the supplier price. As noted above, by decreasing W by a single unite the quantity of the retailers’ increases to 420 units.

5.1. The Iterative Algorithm for Achieving W and Z
To improve the W quantity, the following iterative algorithm is perform.

The z-quantity is first determined \( (z = z) \). The quantity of W is then obtain by the determined z-quantity. The quantity is calculate w and the constitution in the \( z = \frac{A(g+p-2w)}{g+p} \) formula as the z-quantity. This trend is repeat until distance of two stages is negligible.

Using the equation of \( z = \frac{A(g+p-2w)}{g+p} \) and substitution of \( w = 22 \) is obtain the quantity of \( z = -4 \), it is by substitution of \( z = -4 \) the quantity of \( w = 22/5 \) and by repeating, as shown in the Figure 3, the w quantity is approximately 22.5.

6. CONCLUSION

In this study, the optimal order and optimal quantity of the manufacturer in supply chain, including a producer and multiple retailer and the possibility of return of goods and discount costs investigated, in which the manufacturer and retailers are follower. First, to construct the initial model, the library method was used and then using mathematical methods to make the model. It has also shown the case where the exchange of goods between retailers justified in the dearth of surplus-deficit and retailers unite. The proposed model was examined by the numerical example and the optimal quantity of the order and optimal quantity of the manufacturer was investigated in both the coalition and lack of
coalition periods. It was show if there is a coalition between retailers, the supply chain and the retailers are higher than when the coalition is and in the coalition state, the function is less than the function gradient in the non-coalition state. Therefore, if retailers are alliance with one another, the lower price of retailers decreases with lower rate. It was also show that in the coalition state the profit function between retailers is higher than the retail profit function when retailers disagree with each other, indicating that at a certain supplier price, retailers are more profitable at the time of the coalition.

The problem examined in this study can be examined in future research, which is referred to as some of these issues, this research is a development of a classic model of a newsvendor model that can be considered in a multi-period case. The study also referred to a two-level supply chain between producers and retailers that it would be interesting to examine how the return, discount, and coalition return between retailers in a three-levels supply chain between suppliers, manufacturers and retailers.

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