Structure Functions Generated by Zero Sound Excitations

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Abstract—The response of symmetric, neutron, and isospin-asymmetric nuclear matter to a weak isovector external field $V_0(\omega, k)$ is considered. It is shown how the response and structure functions are related to earlier constructed solutions to the dispersion zero-sound equation in matter.

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INTRODUCTION

In this work, we study the linear response of nuclear matter to an isovector external field. As in [1, 2], branches of zero sound solutions are considered first. We then study the contribution to the retarded polarization operator (the response function) and the structure functions of zero sound excitations associated with these solutions.

There have been many works describing different types of responses of nuclear matter associated with the excitation of collective states. The isospin response function in asymmetric nuclear matter was studied in [3] in a wide range of variation of the asymmetry parameter. It was shown that the neutrino mean free path depends largely on the presence and asymmetry of collective modes. The effect different contributions of nucleon–nucleon interaction on the response functions of nuclear matter in an isovector external field was studied in [4]. The longitudinal and transverse spin responses in purely neutron matter were studied in a wide range of densities in [5].

The asymmetry parameter and Fermi momenta of protons and neutrons are determined as

$$\begin{align*}
\beta &= (\rho_n - \rho_p)/(\rho_n + \rho_p), \\
\rho_{fn} &= (3\pi^2(1 + \beta)\rho/2)^{1/3}, \\
\rho_{fp} &= (3\pi^2(1 - \beta)\rho/2)^{1/3}.
\end{align*}$$

In this work, we used the approach developed in [6], where the excitation of giant dipole resonances in nuclei was studied within the theory of finite Fermi systems. We considered the response to an external isovector monopole field $V_0(\omega, k) = \lambda \tau_3 e^{i\omega t - i(\omega + \eta)k}$. Response function $\Pi^R(\omega, k)$ was determined by zero sound excitations. The structural function was associated with $\Pi^R(\omega, k)$ by the relation [7, 8]

$$S(\omega, k) = -\frac{1}{\pi} \text{Im} \left( \Pi^R(\omega, k) \right).$$

We used the effective Landau–Migdal interaction between quasi-particles:

$$F_{\text{tot}}(\tilde{\sigma}, \tau; \tilde{\sigma}', \tau') = C_0(F + F' \tilde{\tau}_1 \tilde{\tau}_2 + G(\tilde{\sigma}_1 \tilde{\sigma}_2) + G'(\tilde{\tau}_1 \tilde{\tau}_2)(\tilde{\sigma}_1 \tilde{\sigma}_2),$$

where $\tilde{\sigma}, \tilde{\tau}$ are the Pauli matrices in the spin and isospin space. The normalization factor is $C_0 = N^{-1} = \pi^2/\rho_0 m_0$, where $N$ is the density of states at the Fermi surface. In our calculations, $\rho_0$ is the Fermi momentum, $\rho_0 = 0.268$ GeV, and $m_0 = 0.94$ GeV. The response function to an isovector external field in asymmetric nuclear matter (ANM) is defined as the sum [4]

$$\Pi^R(\omega, k) = \Pi^{pp}(\omega, k) + \Pi^{nn}(\omega, k) - \Pi^{np}(\omega, k).$$

The authors of [6] developed a matrix for effective fields excited in an ANM by an external isovector dipole field. Rewriting this matrix for retarded polarization operators $\Pi^{tr}(\omega, k)$ in an external isovector monopole field [9], we obtain the system of equations

$$\begin{align*}
\Pi^{pp} &= A^p + A^p F^{pp} \Pi^{pp} + A^p F^{pp} \Pi^{np}, \\
\Pi^{np} &= A^n F^{np} \Pi^{pp} + A^n F^{np} \Pi^{np} \\
\Pi^{nn} &= A^n F^{nn} \Pi^{pp} + A^n F^{nn} \Pi^{np}
\end{align*}$$

for $\Pi^{pp}$ and $\Pi^{np}$, where the vertices of interaction between particle-and-hole ($\text{ph}$) pairs are determined by force constants (3)

$$F^{pp} = F^{np} = F + F', \quad F^{nn} = F - F'. $$

0(\omega, k) = \lambda \tau_3 e^{i\omega t - i(\omega + \eta)k}.$$
Functions $A^\tau(\omega, k)$ are Migdal functions:

\begin{align*}
A^p &= A^p(\omega, k) + A^p(-\omega, k), \\
A^n &= A^n(\omega, k) + A^n(-\omega, k), \\
A^\tau(\omega, k) &= -2\frac{m^3}{4\pi^2k}\left(\frac{a^2-b^2}{2}\ln\left(\frac{a+b}{a-b}\right) - ab\right),
\end{align*}

where $\tau = p, n, a = \omega - \frac{k^2}{2m}, b = \frac{kp^2}{m}$. It is important that $A^\tau(\omega, k)$ include logarithmic functions with cuts [1].

Introducing matrix $M$, we rewrite system (5) for $\Pi^{pp}$ and $\Pi^{np}$ in matrix form:

\begin{equation}
M \begin{pmatrix} \Pi^{pp} \\ \Pi^{np} \end{pmatrix} = \begin{pmatrix} A^p \\ 0 \end{pmatrix}, \quad \text{where}
M = \begin{pmatrix} (1 - A^p F^{pp}) & -A^p F^{pn} \\ -A^p F^{np} & (1 - A^p F^{nn}) \end{pmatrix}.
\end{equation}

Solving the system of Eqs. (8), we obtain the analytical expressions for $\Pi^{pp}$ and $\Pi^{np}$. The denominator is the same for all $\Pi^{\tau\tau}(\omega, k)$. We next construct $\Pi^{\tau\tau}(\omega, k)$ as singular functions with poles at the zeros of the denominator. We write the expression for the complete isovector polarization operator (see (4)) in the form

\begin{equation}
\Pi^R = \Pi^{pp} + \Pi^{nn} - \Pi^{pp} - \Pi^{np} = \frac{D^{iv}}{E(\omega, k)},
\end{equation}

where $D^{iv} = D^{pp} + D^{nn} - D^{pp} - D^{np}$ and $E(\omega, k) = \det(M)$. Equating the denominator to zero $E(\omega, k) = 0$, we obtain the dispersion equation that determines the frequencies of zero sound excitations and maxima in the structure functions. Expanding $\det(M)$ in expression (8), we obtain dispersion equation $E(\omega, k) = \det(M) = 0$ in the form

\begin{equation}
E(\omega, k) = (1 - F^{nn} A^p(\omega, k))(1 - F^{pp} A^p(\omega, k)) - (A^p(\omega, k) F^{np})(A^p(\omega, k) F^{np}) = 0.
\end{equation}

This expression can be rewritten in terms of effective quasi-particle interaction constants (4), (6) as

\begin{equation}
1 - C^p(F + F') A^p - C^p(F + F') A^n + 4FF' C^2 A^p A^n = 0.
\end{equation}

For this equation, we obtain three branches of solutions $\omega_\tau(\omega, k)$, $\tau = p, n, np$ in the ANM. In symmetric nuclear matter (SNM), expression (10) is reduced to

\begin{equation}
E(\omega, k) = (1 - C_p FA(\omega, k))(1 - F' A(\omega, k)) = 0,
\end{equation}

where $A = A^p + A^n$. Factorization $E(\omega, k)$ means there are two independent equations in symmetric matter. One equation describes isoscalar excitations caused by $ph$ interaction $F$; the other, isovector excitations caused by interaction $F'$ (3). Factorization (12) suggests that isoscalar and isovector excitations do not interact in symmetric matter. In subsequent calculations, we assume $F = 0$.

There are thus two branches of solutions in the SNM ($\omega_p(\omega, k)$ and $\omega_n(\omega, k)$) [1].

In neutron matter, the Fermi proton momentum is zero ($p_F = 0$), and function $A^p$ vanishes. Equation (10) is then reduced to

\begin{equation}
E(\omega, k) = 1 - C_p F' A^n = 0.
\end{equation}

One branch of solutions $\omega_\tau(\omega, k)$ was obtained in neutron matter, which is considered not as $\beta$-stable nuclear matter but as matter consisting only of neutrons.

Solutions to (10) were presented in [1, 2]. We studied the contribution from these solutions to the response and structure functions. In nuclei, the imaginary parts of the solutions correspond to the widths of the semidirect decay of excited states. In nuclear matter, the imaginary parts of the solutions are due to the escape from collectivization of the excited state of some $ph$ pairs as a result of mixing with noninteracting $ph$ pairs. To obtain solutions, we consider dispersion equations (10)–(13) at the complex frequency plane. Functions $A^\tau(\omega, k)$ (7) contain logarithmic functions. Logarithmic cuts are determined by the energies of noninteracting $ph$ pairs ($ph$ mode). For small $k$, the dispersion equations have real collective zero-sound solutions. As $k$ rises, the collective solution and the cut overlap. The solutions then come under a logarithmic cut on a nonphysical sheet and acquire an imaginary part. The solutions then escape from the cut of the function $A^\tau(\omega, k)$ acquires an imaginary part due to mixing with free proton $ph$ pairs. If the other $A^\tau(\omega, k)$ functions are calculated on the physical sheet, we assume the proton channel is open when $\omega > 0$ and the neutron channel is closed. In nuclei, the decay of these solutions corresponds to the emission of pro-
tons. These solutions are denoted as $\omega_{np}(k)$. When the neutron channel is open (i.e., we construct the solution on the unphysical sheet of the function $A^n(\omega, k)$), we analogously obtain solutions $\omega_{np}(k)$ that decay due to the emission of neutrons. When both proton and neutron channels are open, we obtain a branch of solutions $\omega_{np}(k)$. The imaginary part of this solution corresponds (in the nucleus) to the emission of a nucleon, the isospin of which is not determined in our model.

Note that for $k$ (for which there are complex solutions on unphysical sheets) we cannot find a solution to Eq. (10) if all $A^p(\omega, k)$ are on a physical sheet.

STRUCTURE FUNCTIONS

We present the structure function as a sum over three processes corresponding to the solutions

$$S(\omega, k) = \sum_l S_l(\omega, k)$$  \hspace{1cm} (14)

$l = n, p, np$. We express $\Pi^R(\omega, k)$ (4), (9) as a sum over the poles, which are zeros of dispersion Eq. (10)

$$\frac{1}{E(\omega, k)} = \sum_l \frac{R_l(\omega_l, k)}{\omega - \omega_l(k)} + \text{Reg}_l(\omega, k),$$

where Reg$_l(\omega, k)$ is a smooth function in the region of the poles. Residues $R_l(\omega_l, k)$ at the poles are calculated on the same unphysical sheets on which the poles are located. We write $E'(\omega_l(k)) = dE(\omega, k)/d\omega|_{\omega = \omega_l}$:

$$R_l(\omega_l, k) = \frac{1}{E'(\omega_l(k))} = \frac{\text{Re}(E') - I \text{Im}(E')}{|E'|^2}.$$

The polarization operator (the response function in [7, 8]) is

$$\Pi^R(\omega, k) = \sum_l D^{iv}(\omega, k) \left( \frac{R_l(\omega_l, k)}{\omega - \omega_l(k)} + \text{Reg}_l \right).$$

The structure function can be presented as the sum of pole and regular contributions $S(\omega, k) = S^p(\omega, k) + S^{\text{reg}}(\omega, k)$. Function $S^p(\omega, k)$ denotes the sum of the pole terms:

$$S^p(\omega, k) = -\frac{1}{\pi} \text{Im} D^{iv}(\omega, k) \sum_l \frac{R_l(\omega_l, k)}{\omega - \omega_l(k)}$$ \hspace{1cm} (15)

Function $S^{\text{reg}}(\omega, k)$ does not contain pole terms, but it does include (e.g.) contributions that in nuclei correspond to ones from direct reactions (i.e., the external field knocks out nucleons without the formation of a collective mode). Here we consider only the pole contributions.

RESULTS AND DISCUSSION

Our calculations were performed for an equilibrium density of $\rho_0 = 0.17$ fm$^{-3}$ at a value of $F^* = 1.0$ and a quasi-particle mass of $m = 0.8m_0$.

Figure 1 shows our results for symmetric matter. Branches of solutions and structure functions corresponding to them for $k/p_0 = 0.2$ and 0.6 are presented. Figure 1a shows the branches of solutions $\omega_{s}(k)$ and $\omega_{sp}(k)$. Zero sound branches of solutions $\omega_{s}(k)$ are real for small $k$ and denote ordinary zero sound. As $k$ rises, stable solutions overlap the $ph$ mode for $k = k_s$, $k_s(\beta = 0) = 0.34p_0$. For larger $k$, branch $\omega_{s}(k)$ goes to the nonphysical sheets of both function $A^p(\omega, k)$ and function $A^p(\omega, k)$ and becomes complex [1, 2]. Second solution $\omega_{sp}(k)$ is on the nonphysical sheet of $A^p(\omega, k)$ or $A^p(\omega, k)$. It starts at $k = k_c$, $k_c = 0.52p_0$.  

![Figure 1](image_url)
There is one infinite peak for $\beta = 0.2$ that corresponds to real solution $\omega_s(k)$; there is no solution $\omega_s(k)$ (Fig. 1b). Two solutions have already been presented for $\beta = 0.6$, and they correspond to the two maxima in Fig. 1b. Maximum widths are due to imaginary parts $\omega_s$ and $\omega_p$.

Figure 2a shows the branches of solutions in the ANM with asymmetry parameter $\beta = 0.2$, $\omega_s(k)$, $\omega_p(k)$, and $\omega_{sp}(k)$. We can see that branches start at different values of $k$. There are two solutions for $k/p_0 = 0.2$: $\omega_s(k)$ and $\omega_p(k)$. Figure 2b shows that the line with asterisks has two maxima in $S^e_\text{str}(\omega, k = 0.2p_0)$ corresponding to these solutions. The left one corresponds to solution $\omega_p(k = 0.2p_0)$ (its width in nuclei is determined by the emission of protons); the right one corresponds to solution $\omega_s(k = 0.2p_0)$ (its width in nuclei is determined by the emission of neutrons).

There are three solutions (Fig. 2a) for $k/p_0 = 0.6$, and three maxima corresponding to these solutions (Fig. 2b). Number 1 denotes the maximum corresponding to solution $\omega_s(k = 0.6p_0)$, which decays due to the emission of neutrons (its width is determined by the imaginary part $\omega_s(0.6p_0)$). Number 2 denotes the maximum corresponding to $\omega_p(k = 0.6p_0)$. Number 3 denotes the maximum corresponding to $\omega_{sp}(k = 0.6p_0)$.

There is one solution to $\omega_s(k)$ in neutron matter, and the structure function contains one maximum and $k_s(\beta = 1) = 0.09p_0$. The result is shown in Fig. 3. When $k/p_0 = 0.2$, $\omega_s(k)$ has a small width (Fig. 3a), which corresponds to the high peak in Fig. 3b. When $k/p_0 = 0.6$, imaginary part $\omega_s(k)$ grew and the peak changed its form.

In the future, we plan to apply this technique to specific nuclei, but many questions remain. These include the Coulomb barrier penetration and the influence of the shape of the nucleus.

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