On the applicability of the electrodynamic approximation in the simulation of the electrovortex flow in the presence of an external magnetic field

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Abstract. Experimental and numerical results on the effect of an external axial magnetic field on the structure of the flow which occurs when an electric current passes through a conducting medium between two hemispheres are presented. Results show that for high external axial magnetic field it is absolutely necessary to take into account the induced currents generated by the flow. These results can be used in electrometallurgy for a priori estimates of the flow structure in the direct-current-arc furnaces.

1. Introduction
Electrovortex flow (EVF) is formed as a result of interaction between the non-uniform electric current passing through the liquid metal and the own magnetic field of this current [1]. Such flows significantly affect many processes in mechanical engineering (electro-welding) and electrometallurgy (electroslag remelting, various electric melting furnaces). In particular, electrovortex flow determines the hydrodynamic structure in the baths of DC-arc furnaces, which are increasingly used in the industry [2].

Here is the brief review of works about electrovortex flows. The first articles associated with investigation of electrovortex flow were about flow calculation at the small currents using Stokes approximation [3, 4]. Article [5] was devoted to possible reasons for rotation of the EVF. Issues related to the surface deformation were presented in [6]. Problems connected with the influence of the external axial magnetic field started to be considered in [7, 8, 9]. In [7] we showed that even the weak external magnetic field of Earth can lead to essential hydrodynamical effect – the rotation of conductive liquid with velocities up to ~10 cm/s and formation of the second upward vortex. In [8], authors gave an estimate of value for the external magnetic field when the secondary vortex fully suppresses the main electrovortex flow, but in our opinion it was not quite correct. In [9] we investigated self-oscillations in the flow appearing at the collision of two vortices in opposite directions.

In this paper we continue to investigate the influence of the external axial magnetic field (MF) on the EVF of liquid metal in the hemispherical container. This geometry is very close to the configuration of the industrial DC-arc furnaces. The flows inside the furnace determine the processes
of heat and mass transfer which affect the important parameters such as the melting time of the metal, temperature distribution in the bath, the shape of the melting and crystallization front, and time and quality of mixing the metal. Flow in the bath of the furnace is very sensitive to external MF, so managing the flow with the help of MF is the natural way which should be used with care.

Let us consider the system where electric current propagates from the small hemispherical electrode through the conductive media (liquid metal) to the big hemispherical electrode. Electromagnetic force \( \mathbf{F} = \mathbf{J} \times \mathbf{B} \) (where \( \mathbf{J} \) is the current density and \( \mathbf{B} \) is the own magnetic field) causes the liquid to move.

![Elecrovortex structure](image)

**Figure 1.** Elecrovortex structure, l - small electrode, 2 - liquid metal, 3 - big electrode. a) Toroidal EVF; b) occurrence of the azimuthal swirl; c) occurrence of the secondary vortex; d) double-vortices rotating EVF.

In such an axisymmetric system, without the influence of external magnetic fields, EVF develops which has the form of a toroidal vortex (figure 1a). The influence of the axial external magnetic field causes the azimuthal swirl of the flow (figure 1b). At that, due to the azimuthal rotation, a secondary vortex appears, it is rotating in the opposite direction to the main EVF (figure 1d). This secondary vortex pushes the EVF towards the periphery of the bath. The azimuthal velocity leads to significant redistribution of axial and radial velocity components in liquid metal.

Usually problems associated with the calculation of the evrotors flow are carried out using the so-called electrodynamic approximation, according to which the effect of the motion of the liquid under the action of electric current in the liquid can be neglected. The electromagnetic force is calculated only based on the current density distribution and the own magnetic field calculated earlier. According to [10], it was shown that electrodynamic approximation is acceptable for applied electric currents \( I < 30 \text{kA} \). When the value of the current is greater than \( \sim 30 \text{kA} \) one should apply non-inductive approximation. In this situation electric currents induced by the liquid flow must be taken into account in the expression of the Lorentz force, but the magnetic fields induced by these currents can be neglected. In the present system, external axial magnetic field leads to intense of rotation due to the swirl effect, so that the electrodynamic approximation is probably not applicable. The present paper is devoted to the investigation of applicability of the electrodynamic approximation in evrotors flow of the liquid metal in external magnetic field.

2. The experimental technique

Experimental studies were carried out at the setup shown in the figure 2. An eutectic indium-gallium-tin alloy (weight content: Ga-67%, In-20.55%, Sn-12.5%, physical properties: melting point +10.5°C, \( \rho = 6482 \text{kg/m}^3 \), \( \nu = 4.3e-7 \text{m}^2/\text{s} \), \( \sigma = 3.3e6 \text{Sm} \)) was used as the working liquid in the experiments. Alloy filled a copper hemispherical container with the radius \( R_2 = 188 \text{mm} \) which also served as a large electrode. Small electrode - copper or steel cylinder with the radius \( R_1 = 2.5 \text{mm} \) with hemispherical tip was immersed into the alloy in the middle of the working bath. Power source developed on the basis
of three-phase AC rectifier ($I\leq1500$ A) was used to supply an experimental setup. To create an external longitudinal magnetic field, the coil consisting of 228 turns of special lacquered copper wire with diameter 2.77 mm was used. Coil power was supplied from a stabilized power source, providing smooth control of DC within the range from 0 to 50 A with voltage up to 30 V. This system allowed getting the magnetic field with induction $B_z=0.05$ T in the middle of the working area at a current of ~50 A.

For measuring the azimuthal velocity, hydrochloric acid was added on surface of liquid metal (figure 3). Chemical reaction between hydrochloric acid and In-Ga-Sn leads to appearance of the hydrogen bubbles, which were used as markers for observation and video fixation of the azimuthal velocity. The motion of the liquid metal was filmed by the camera with resolution of 1920×1080 and frame rate 50 fps. To measure coordinates of the hydrogen bubble on the free surface, special rulers were placed above the liquid metal surface.

3. Numerical technique

To solve Navier–Stokes equation, we used the finite volume method on an unstructured 2D-axisymmetric grid (figure 4) in cylindrical coordinates ($\partial{}/\partial{\phi}=0$, $U_r\neq0$). Calculation area was a quarter of circle with internal radius $R_1=2.5$ mm and external radius $R_2=94$ mm and consisted of 5000 quadrangular cells. The symmetry axis was coincided with z-axis and free surface was coincided with r-axis direction.

Figure 2. Experimental setup. 1 - solenoid, 2 - small electrode, 3 - eutectic alloy In-Ga-Sn, 4 - hemispherical container, 5 - heat exchanger, 6 - programmable power supply.

Figure 3. Hydrogen bubbles on the free surface of the liquid metal.

Figure 4. Typical 2D axisymmetric grid.
The Navier-Stokes equation has the form:
\[
\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{U} + \frac{1}{\rho} \mathbf{F}
\]
where \( \mathbf{U} \) is the velocity of the liquid, \( \rho \) is the density of the liquid, \( \nu \) is the kinematic viscosity coefficient, \( p \) is the pressure, and \( \mathbf{F} \) is the electromagnetic force.

Calculation was carried out in electrodynamic and non-induction approximations. For electrodynamic approximation (ED) electromagnetic force has the form:
\[
\mathbf{F}_{\text{ED}} = \mathbf{J} \times (\mathbf{B}_{\text{EVF}} + \mathbf{B}_{\text{ext}})
\]
where \( \mathbf{J} \) is the current density, \( \mathbf{B}_{\text{EVF}} \) is the own magnetic field, \( \mathbf{B}_{\text{ext}} \) is the external magnetic field with the only z-component \( B_z \).

In axisymmetric cylindrical geometry equations for current density and own magnetic field can be found analytically:
\[
J_r = \frac{I_r}{2\pi \sqrt{(r^2 + z^2)^3}}
\]
\[
J_z = \frac{I_z}{2\pi \sqrt{(r^2 + z^2)^3}}
\]
The magnetic field can be found from the Maxwell’s equation:
\[
\nabla \times \mathbf{B} = \mu_0 \mathbf{J}.
\]
So the magnetic field will be:
\[
B_\phi = \frac{\mu_0 I \left( \sqrt{r^2 + z^2} - z \right)}{r \sqrt{r^2 + z^2}}.
\]
For non-inductive approximation (NI) electromagnetic force has the form:
\[
\mathbf{F}_{\text{NI}} = (\mathbf{J} + \sigma \mathbf{U} \times (\mathbf{B}_{\text{EVF}} + \mathbf{B}_{\text{ext}})) \times (\mathbf{B}_{\text{EVF}} + \mathbf{B}_{\text{ext}})
\]
where \( \sigma \) is electrical conductivity.

Let us analyze the electromagnetic force in nonductive approximation which consists of the following three components:
\[
F_r = -J_z B_\phi - \sigma U_r B_\phi^2 - \sigma U_r B_z^2,
\]
\[
F_z = J_z B_\phi + \sigma U_\phi B_\phi B_z - \sigma U_z B_\phi^2,
\]
\[
F_\phi = -(J_z B_\phi + \sigma U_\phi B_\phi^2 + \sigma U_z B_\phi B_z).
\]

Azimuthal component \( F_\phi \) leads to appearance of the azimuthal velocity, and increasing of the external magnetic filed \( B_z \) leads to increasing of the term \( \sigma U_\phi B_\phi^2 \), which includes the square of the magnetic field and negative \( U_\phi \) (azimuthal velocity is directed along the main component of the force - \( J_z \)), so this term decreases azimuthal electromagnetic force. Thus, using electrodynamic approximation (not taking into account this mechanism) should entail overstating of the azimuthal velocity.

4. Results

Two cases with an electric current of \( I = 10 \) A, and two different external magnetic field of \( 10^{-3} \) T and \( 2 \times 10^{-2} \) T passing through the liquid metal were investigated experimentally and numerically. Calculation of the velocity field of the electrovortex flow in electrodynamic and non-induction approximations were carried out for each case. In consequence of the calculations, the dependence of the azimuthal velocity (on the free surface) on the radius was obtained. Then the values of the azimuthal velocity at the radius \( r = 10 \) mm were used to plot the dependence of this velocity on the external magnetic field. Radius \( r = 10 \) mm was chosen because the maximal azimuthal velocity is reached approximately at this radius.
The dependences of the azimuthal velocity on radius for the external magnetic field \( B_z = 10^{-3} \) T and \( 2\times10^{-2} \) T are shown in the figures 5 and 6. Experimental and calculation results are in satisfactory agreement. The experimental and calculation results in the case of magnetic field \( B_z = 10^{-3} \) T have more discrepancy than those in the case of \( B_z = 2\times10^{-2} \) T. The magnetic field of the coil is included in the term \( \sigma U_\phi B_z^2 \) quadratically, so the inhomogeneity of the field must have a noticeable effect.

![Figure 5](image1)

**Figure 5.** Dependence of the azimuthal velocity on radius of hemisphere container \( B_{\text{ext}} = 10^{-3} \) T. 1 - electrodynamic approximation, 2 - non-induction approximation, 3 - experiment.

![Figure 6](image2)

**Figure 6.** Dependence of the azimuthal velocity on radius of hemisphere container \( B_{\text{ext}} = 2\times10^{-2} \) T. 1 - electrodynamic approximation, 2 - non-induction approximation, 3 - experiment.

Dependences of the azimuthal velocity on external magnetic field are presented on the figure 7, the difference between electrodynamic and non-induction approximation reaches about 10% at the external magnetic field \( B_z = 1.25\times10^{-3} \) T.

![Figure 7](image3)

**Figure 7.** Dependence of the azimuthal velocity on the external magnetic field. 1 - electrodynamic approximation, 2 - non-induction approximation, 3 - experiment.

5. **Conclusions**

Experiments and calculations of the azimuthal velocity in electrovortex flow with the presence of the external axial magnetic field in electrodynamic and non-induction approximations were carried out. The dependences of the azimuthal velocity on the radius and the azimuthal velocity on the external axial magnetic field were obtained. It was established that with influence of external axial magnetic field \( B_z > 1.25\times10^{-3} \) it is necessary to consider influence of induced electric currents on hydrodynamic structure of electrovortex flow.
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