On the thermal mass shift of nucleons

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Abstract

The mass shift $\Delta m_N$ of a nucleon in the finite temperature pion gas is calculated in the case of an arbitrary nucleon momentum. A general formula is used which relates the in-medium mass shift $\Delta m(E)$ of a particle to the real part of the forward scattering amplitude $\text{Re} f(E)$ of this particle on constituents of the medium and its applicability domain is formulated. The mass shift of the nucleon in thermal equilibrium with pion gas is also calculated.
The problem of how the properties of hadrons change in hadron or nuclear matter in comparison to their free values has attracted a lot of attention recently. Among these properties of immediate interest are the in-medium mass shifts of particles. Different models as well as model independent approaches were used to calculate hadron mass shifts both at finite temperature and finite density (for a review, see e.g. [1]). It is however clear on physical grounds that the in-medium mass shift of a particle is only due to its interaction with the constituents of the medium. Thus one can use phenomenological information on this interaction to calculate the mass shifts. In our recent paper [2] we have argued that the mass shift of a particle in medium at relatively low density of the latter can be related to the forward scattering amplitude $f(E)$ of this particle on the constituents of the medium

$$\Delta m(E) = -2\pi \frac{\rho}{m} \text{Re} f(E)$$

Here $m$ is the vacuum mass of the particle, $E$ is its energy in the rest frame of the constituent particle, and $\rho$ is the density of constituents. The normalization of the amplitude corresponds to the standard form of the optical theorem

$$k\sigma = 4\pi \text{Im} f(E),$$

where $k$ is the particle momentum. All quantities in Eqs.(1) and (2) correspond to the rest frame of the target. In most of the papers on the in-medium hadron mass shifts the hadrons were considered at rest. As seen from Eq.(1) this restriction is not necessary theoretically. Experimentally it is also desirable to have theoretical predictions in broad energy interval, since it extends the possibilities of experimental investigations. As discussed in [2] for the cases of $\rho$ or $\pi$-mesons embedded in nuclear matter the energy dependence of the mass shifts is rather significant at low energies, i.e. in the resonance region. Here we will demonstrate this for the case of the thermal mass shift of a nucleon imbedded in a pion gas.

For the nucleon at rest the mass shift was calculated by Leutwyler and Smilga [3] by using experimental information [4] on the $\pi N$ forward scattering amplitude. We will show how their result follows from Eq.(1) and then generalize it to the case of a moving nucleon. First, let us go from the rest frame of the nucleon to that of a pion. Under this transformation $\rho \rightarrow \rho\sqrt{1 - v^2}$, ($v$ is the pion velocity). Kinematically, we have $m_N E_\pi = m_\pi E_N$, where $E_\pi$ is the pion energy in the nucleon rest frame, $E_N$ is the nucleon energy in the pion rest frame.
Since the cross section $\sigma$ is the same in both the rest frames, we have from the optical theorem $f_{N\pi}(E_N)/E_N = f_{\pi N}(E_\pi)/E_\pi$, where the indices $N\pi$ and $\pi N$ of the amplitudes mean pion and nucleon rest frames correspondingly. Then Eq.(1) can be rewritten as

$$\Delta m_N(E = m_N) = -2\pi \rho \frac{E}{E_\pi} \text{Re}f_{\pi N}(E_\pi),$$

Here $\rho$ is now the pion gas density in the rest frame of the nucleon and $f_{\pi N}(E_\pi)$ is the amplitude of forward scattering of a pion with energy $E_\pi$ on the nucleon in this frame. Since the rest frame of the nucleon is also the rest frame of the pion gas, we recover the formula used in [3]

$$\Delta m_N = -2\pi \sum_{\pi a} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_\pi} \frac{1}{e^{E_\pi/T} - 1} \text{Re}f_{\pi a N}(E_\pi),$$

where $E_\pi = \sqrt{k^2 - m_\pi^2}$. The simplest way to generalize this formula to the case of a nucleon with a finite momentum moving through a pion gas which is at rest is to go to the nucleon rest frame again boosting the pion gas. Since the pion phase space $d^3k/E_\pi$ is Lorentz invariant we only need to boost the pion distribution function which is achieved by substituting $E_\pi$ for $p_\pi u$ in the Bose factor, where $u = (1/\sqrt{1 - v^2}, v/\sqrt{1 - v^2})$ is the 4-velocity of the nucleon. Using $v = p_N/E_N$ and $\sqrt{1 - v^2} = m_N/E_N$ and performing the integral over the angle between $p_\pi$ and $p_N$ we finally get

$$\Delta m_N = -T m_N \sum_{\pi a} \int \frac{dE_\pi}{2\pi} \log \frac{1 - e^{-E_+(E_\pi)/T}}{1 - e^{-E_-(E_\pi)/T}} f_{\pi a N}(E_\pi),$$

where $E_\pm = (E_N E_\pi \pm p_N p_\pi)/m_N$.

We have calculated $\Delta m_N(T)$ for $p_N = 0.5, 1$ and 5 GeV using the same data [4] on the isospin symmetric forward amplitude

$$D^+(E_\pi) = \frac{4\pi}{3} \sum_{\pi a} f_{\pi a N}(E_\pi)$$

as in [3]. The results are shown in Fig.1 and compared to the case of the nucleon at rest [2].
then in its rest frame the scattering of pions is shifted to higher energies. At energies above
the $\Delta$-resonance, $D^+$ is mainly negative. Thus, it could be expected that at sufficiently high
nucleon momentum the effect of $\Delta$ would be washed out, and the dip will disappear.
As is seen from Fig.1 the dip decreases and indeed disappears at $p_N \sim 1$ GeV.

We have also calculated $\Delta m_N(T)$ for the case when the nucleon is in thermal equilibrium
with the heat bath. Strictly speaking in this case we should also average over Maxwell
distribution of nucleon momenta. For simplicity instead we put nucleon energy fixed and
equal to the mean energy in the pion gas, $E_N = m_N + (3/2)T$. The results of calculations
are presented in Fig.2. One can see that the original dip at $T \approx 90$ MeV has practically
disappeared: it has moved to $T \approx 70$ MeV and its depth decreased by a factor of two.

The effective nucleon width (damping rate) in the thermal pion gas is obtained in a
similar way

$$
\gamma = T \frac{m_N}{p_N} \sum_{\pi a} \int_{m_\pi}^{\infty} dE_\pi \frac{(E_\pi^2 - m_\pi^2)^{1/2}}{(2\pi)^2} \log \frac{1 - e^{-E_\pi(E_\pi)/T}}{1 - e^{-E_\pi(E_\pi)/T} \pi \sigma_{\pi a N}(E_\pi)}
$$

The effect of a finite momentum of the nucleon on $\gamma$ is much less than on the mass shift,
since there is no sign change in the integrand of Eq.(7) contrary to the case of Eq.(6). In
the case of nucleon in thermal equilibrium with pion gas we have numerically at $T = 100$
MeV, $\gamma = 33$ MeV. For the nucleon at rest at this temperature $\gamma = 30$ MeV [3].

Let us estimate the applicability domain of our approach. The main restriction arises
from the requirement (see [2]) $|\text{Re} f_{N\pi}(E_N)| < d$, where $d$ is the mean distance between pions
in the gas. We use the relation $f_{N\pi}(E_N) = (E_N/E_\pi)f_{\pi N}(E_\pi)$ and note that in the whole
important energy interval $|\text{Re} f_{\pi N}| < 0.7$ fm (apart from the region of $\Delta$-resonance, where
$-\text{Re} f_{\pi + N}$ reaches 1.5 fm). At $T = 150$ MeV the kinematical factor $E_N/E_\pi \approx 2.6$ for thermal
pions, while $d \approx 2$ fm and rapidly increases at lower $T$. We thus see that this approach
works up to about $T = 150$ MeV, where the approximation of non-interacting pion gas also
becomes invalid. Another restriction, $\lambda_N = 1/p_N \ll d$ which could be violated at low $T$ is
in fact fulfilled for a thermalized nucleon due to the exponential growth of $d$ at $T \to 0$.

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FIGURES

FIG. 1. Temperature dependence of $\Delta m_N$ for $p_N = 0, 0.5, 1, \text{ and } 1.5 \text{ GeV}$ (curves $a, b, c$ and $d$, respectively).

FIG. 2. Temperature dependence of $\Delta m_N$ for a thermalized nucleon, $E_N = m_N + (3/2)T$, (curve $b$) in comparison with the one for the nucleon at rest (curve $a$).
