

Born-Infeld-de Sitter gravity: Cold, ultracold and Nariai black holes

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Abstract

In this paper, we have presented interesting properties of the static charged Born-Infeld-de Sitter black hole. They can have time-like as well as space-like singularities depending on the parameters of the theory. The degenerate black holes lead to cold, ultra cold and Nariai black holes. The geometry of such black holes are discussed. A comparison is done with the Reissner-Nordstrom-de Sitter black holes.

Key words: static, charged, Nariai, cold, ultra-cold, lukewarm

1 Introduction

Cosmological constant ($\Lambda$) was introduced by Einstein as a method to obtain a static universe from General Theory of Relativity. Later, when experiments revealed that the universe is expanding, Einstein decided to remove $\Lambda$ from the equations. In several observations in the recent history, it has been found that not only the universe is expanding, but the expansion is accelerating [1][2]. The acceleration is driven by some mysterious dark energy which is not well understood yet. One of the proposals for the dark energy is the cosmological constant with a state parameter $\omega = -1$ and negative pressure $p = -\rho$. However, in order to agree with the observable data, the cosmological constant has to be fine tuned up to 120 orders of magnitude [3].

Born-Infeld theory of non-linear electrodynamics was proposed by Max Born and Leopold Infeld [4] to cure the divergences of the self energy of a charged point like particle in Maxwell’s electrodynamics. Born-Infeld theory has received interest in the recent past due to its relation to string theory [5][6][7]. In Born-Infeld electrodynamics, the electric field of a point-like charge is given by $E = Q/\sqrt{r^4 + \frac{Q^2}{\beta^2}}$ which is

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clearly finite at \( r = 0 \). Hence its total energy is also finite e.g. [8]. The maximum value of \( E \) is \( \beta \). From the string theory point of view, the maximum field strength \( \beta = \frac{1}{2\alpha'} \), where \( \alpha' \) is the inverse of the string tension. Born-Infeld theory has exact \( SO(2) \) electromagnetic duality inspite of it's non-linear nature [9].

There are many interesting black hole solutions related to Born-Infeld electrodynamics in the literature. Asymptotically flat charged black hole solutions in Born-Infeld gravity was presented by Breton in [10]. Stability properties of the asymptotically flat charged black hole in Born-Infeld gravity were studied by the current author in [11][12][13]. Melvin universe type solutions coupled to Born-Infeld electrodynamics were studied by Gibbons and Herderio [14]. Exact solutions to Lovelock-Born-Infeld black holes were studied by Aiello et.al [15]. Non-abelian black hole solutions with Born-Infeld electrodynamics in higher dimensions were studied in [16]. Attractor mechanism for extreme black hole solutions in Einstein-Born-Infeld-dilaton gravity was studied by Gao [17]. Black hole solutions in Born-Infeld action coupled to Einstein gravity with a cosmological constant has been found by several authors in [18][19][20][21]. In this paper, we focus on a static Born-Infeld black hole with a positive cosmological constant.

The paper is organized as follows: In section 2, an introduction to the Born-Infeld-de Sitter black hole is given followed by a comparison with the Reissner-Nordstrom-de Sitter black hole in section 3. In section 4, the extreme black holes are discussed. In section 5, the topology is described for the degenerate horizons. Finally, the conclusion is given in section 6.

## 2 Born-Infeld-de Sitter black holes

The Born-Infeld black hole is derived from the action,

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R - 2\Lambda}{16\pi G} + L(F) \right]
\]

(1)

where the function \( L(F) \) is given by,

\[
L(F) = 4\beta^2 \left( 1 - \sqrt{1 + \frac{F_{\mu\nu}F^{\mu\nu}}{2\beta^2}} \right)
\]

(2)

Here, \( \beta \) has dimensions \( \text{length}^{-2} \) and \( G \text{ length}^2 \). In the paper, we will assume, \( 16\pi G = 1 \). Note that when \( \beta \rightarrow \infty \), the Lagrangian \( L(F) \rightarrow -F^2 \) which corresponds to Maxwell’s electrodynamics. The static charged, spherical symmetric black hole derived from the action in eq.(1) is given by the metric,

\[
ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)
\]

(3)

with,

\[
f(r) = 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3} + \frac{2\beta^2 r^2}{3} \left( 1 - \sqrt{1 + \frac{Q^2}{r^4\beta^2}} \right) + \frac{4Q^2}{3r^2} \ {}_2F_1 \left( \frac{1}{4}, \frac{1}{2}; \frac{5}{4}, -\frac{Q^2}{\beta^4 r^4} \right)
\]

(4)
Here \( {}_2F_1 \) is the hypergeometric function. The parameters in the metric are as follows; \( M \) is the mass, \( Q \) is the charge, \( \beta \) is the non-linear parameter and \( \Lambda \) is the cosmological constant. The electric field strength for the black hole is given by,

\[
E = \frac{Q}{\sqrt{r^4 + \frac{Q^2}{\beta^2}}} \tag{5}
\]

Notice that \( E \) is finite for \( r \to 0 \) as expected. In the limit \( \beta \to \infty \), the elliptic integral can be expanded to give,

\[
f(r)_{RN} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3} \tag{6}
\]

which is the function \( f(r) \) for the Reissner-Nordstrom-de Sitter black hole for Maxwell’s electrodynamics. Near the origin, the function \( f(r) \) has the behavior,

\[
f(r) \approx 1 - \frac{(2M - A)}{r} - 2\beta Q + r^2 \left( \frac{2\beta^2 - \Lambda}{3} \right) + \frac{\beta^3}{5} r^4 \tag{7}
\]

Here,

\[
A = \frac{1}{3} \sqrt{\frac{\beta}{\pi}} Q^{3/2} \Gamma \left( \frac{1}{4} \right)^2 \tag{8}
\]

Depending on \( M \) and \( A \), it is possible to have three types of black hole solutions:

**Case 1: \((M > \frac{A}{2})\)**
In this case, for \( r \to 0 \), \( f(r) \to -\infty \). Hence the black hole will be similar to the Schwarzschild-de Sitter black hole. The function \( f(r) \) is plotted in the Fig.1.

**Case 2: \((M < \frac{A}{2})\)**
In this case, for \( r \to 0 \), \( f(r) \to \infty \). Hence the black hole will be similar to the Reissner-Nordstrom-de Sitter black hole. The function \( f(r) \) is plotted in the Fig.2.

**Case 3: \((M = \frac{A}{2})\)**
In this case, for \( r \to 0 \), \( f(r) \to (1 - 2Q\beta) \). Hence \( f(r) \) is finite at \( r = 0 \) and single valued as given in Fig.3. The Kretschmann scalar still diverge at \( r = 0 \) for these black holes. So, the singularity exits. One could call them marginal black holes.

The above three cases can be described by a simple graph of \( M \) vs \( Q \) given in Fig.4. Depending on the values of the parameters in the theory, the function \( f(r) \) can have three roots, two roots, one root or none. When \( f(r) \) has three roots, the behavior of \( f(r) \) is similar to the Reissner-Nordstrom-de Sitter black hole. When there are three roots, the smallest is the black hole inner horizon, the second largest is the black hole event horizon and the largest becomes the cosmological horizon. The graph in Fig.5 shows how for a given \( M \), different number of horizons possible. When it has two roots, the black hole behave similar to the Schwarzschild-de Sitter
black hole. In this case, one will be the black hole horizon and the other will be the cosmological horizon. There are also black holes with degenerate horizons which will be discussed in detail later. Hence the Born-Infeld-de Sitter black hole is interesting since it possess the characteristics of the most well known black holes in the literature.

Figure 1. The figure shows the graphs for $f(r)$ vs $r$ for various masses $M$. In this case, $M > A/2$. Here $Q = 0.2307$, $\beta = 1.4$ and $\Lambda = 0.5$.

Figure 2. The figure shows the graphs for $f(r)$ vs $r$ for various values of $M$ and $Q$. Here $M < A/2$ and $\beta = 1.4$ and $\Lambda = 0.5$. 
Figure 3. The figure shows the graphs for $f(r)$ vs $r$. In all the cases, $M = A/2$. Here $\beta = 1.4$. $M, Q$ and $\Lambda$ are varied.

Figure 4. The figure shows the graphs for $M$ vs $Q$. Here, $Q = 0.5, M = 0.2$ and $\Lambda = 0.5$.

Figure 5. The figure shows the graphs for $M$ vs $r_h$ for $\Lambda = 0.5, \beta = 1.4$. For the first graph $Q = 0.431$ and for the second graph $Q = 0.357$.

In Fig. 6, the black horizon radius is plotted against the non-linear parameter $\beta$. One can observe that for large $\beta$ the black hole is smaller.
The Hawking temperature for the Born-Infeld black hole, $T$ is obtained by the usual relation, $T = \frac{f'(r)}{4\pi}$ as,

$$T = \frac{1}{4\pi r_h} \left[ 1 + (2\beta^2 - \Lambda)r_h^2 - 2\beta \sqrt{(Q^2 + r_h^4\beta^2)} \right] \quad (9)$$

In the Fig.7, the temperature for the cosmological horizon and the black hole event horizon is plotted vs $\beta$. The black hole considered in this case is Schwarzschild-de Sitter type. From the graphs, the temperature for the both horizons increase with $\beta$.

Figure 7. The figure shows the graphs for Temp vs $\beta$. The first graph shows the temperature for the black hole horizon while the second graph shows the temperature for the cosmological horizon. Here, $Q = 0.5$, $M = 0.2$ and $\Lambda = 0.5$.

### 3 Comparison of the Born-Infeld-de Sitter black hole with the Reisnner-Nordstrom-de Sitter black hole

In order to fully appreciate the properties of the Born-Infeld-de Sitter black hole, it is important to compare it with the charged black hole in the Maxwell’s electrodynamics, which is Reissner-Nordstrom-de Sitter black hole whose geometry is defined by the metric in eq.(3) with $f(r)$ given by eq.(6). In the Fig.8, the functions $f(r)$ for both black holes are plotted. It is observed that while the cosmological horizon are close (not the same), the inner black hole horizon is smaller for the Born-Infeld-de Sitter black hole.
Figure 8. The figure shows the graphs for $f(r)$ vs $r$ for the Born-Infeld-de Sitter and the Reissner-Nordstrom-de Sitter black hole for the same mass, charge and the cosmological constant. Here, $\beta = 1.4$, $Q = 0.6$, $\Lambda = 0.5$ and $M = 0.593$.

We would like to review some interesting properties of the Reissner-Nordstrom-de Sitter black holes here. Such black holes could have three horizons, $r_+, r_{++}$ and $r_c$. The first two are black hole inner and outer horizons while the third is the cosmological horizon. In general, the black hole outer horizon and the cosmological horizon are not in thermal equilibrium. The only time this happens is when there is a Nariai type degeneracy (as given in Fig. 2(d)) or in the lukewarm black holes [22]. A description of thermodynamics and instantons of Reissner-Nordstrom-de Sitter black holes are given in [23] [24] [25] [26] [27]. Nariai black holes in other theories are discussed in [28] [29]. Nariai black holes in higher dimensions are discussed by Cardoso et.al. in [30].

The Hawking temperature is plotted for both black holes in Fig.9. From the Figure, it is clear that the Born-Infeld-de Sitter black hole is hotter than its counterpart in Maxwell’s electrodynamics.

Figure 9. The figure shows the graphs for Temp vs $M$ for the Born-Infeld-de Sitter and the Reissner-Nordstrom-de Sitter black hole for the same charge and the cosmological constant. Here, $\beta = 1.4$, $Q = 0.6$ and $\Lambda = 0.5$. 

7
4 Extreme black holes

The main focus in this section is to study the degenerate horizons of the Born-Infeld-de Sitter black holes. For degenerate horizons, \( f(r) = f'(r) = 0 \). By solving these two equations one arrives at,

\[
r_{ex}^2 = \frac{(2\beta^2 - \Lambda) \pm \sqrt{\beta^4 - 4Q^2\beta^4\Lambda + Q^2\beta^2\Lambda}}{\Lambda(4\beta^2 - \Lambda)}
\]

We can also write \( r_{ex}^2 \) as,

\[
r_{ex}^2 = \frac{(2\beta^2 - \Lambda) \pm \sqrt{\delta}}{\Lambda(4\beta^2 - \Lambda)}
\]

where

\[
\delta = (2\beta^2 - \Lambda)^2 + \Lambda(1 - 4\beta^2Q^2)(4\beta^2 - \Lambda)
\]

It may look \( r_{ex} \) to be independent of the mass. However, the mass of the corresponding black hole will be obtained by using \( f(r_{ex}) = 0 \).

Now, we will study the type of horizons for various values of the parameters in the theory.

4.1 \( (2\beta^2 - \Lambda) > 0 \)

In this case, \( (4\beta^2 - \Lambda) > 0 \), Hence \( \delta > 0 \) and \( \sqrt{\delta} \) is real.

Case1: \( (1 - 4\beta^2Q^2) > 0 \) or \( Q\beta < \frac{1}{2} \)

In this case \( \sqrt{\delta} > (2\beta^2 - \Lambda) \). Hence \( r_{ex}^2 \) will have only one root with the “+” sign in front of the \( \sqrt{\delta} \). Hence, in this case, the black hole will be degenerate Schwarzschild-de Sitter type as given in the Fig.10.
Figure 10. The figure shows the graphs for $f(r)$ vs $r$ for a degenerate black hole of the Schwarzschild-de Sitter type. Here, $\beta = 1.4, Q = 0.2307, \Lambda = 0.5$ and $M = 0.49036$.

**Case 2:** $(1 - 4\beta^2 Q^2) < 0$, or $Q\beta > \frac{1}{2}$

In this case, $\sqrt{\delta} < (2\beta^2 - \Lambda)$. Hence there will be two values for $r_{ex}^2$ with the + and the − in front of $\sqrt{\delta}$. Hence, in this case, the black hole will be degenerate Reissner Nordstrom-de Sitter type as given in the Fig.11.

Figure 11. The figure shows the graphs for $f(r)$ vs $r$ for a degenerate black hole of the Reissner-Nordstrom-de Sitter type. Here, $\beta = 1.4, Q = 0.2307$ and $\Lambda = 0.5$.

**Case 3:** $(1 - 4\beta^2 Q^2) = 0$, or $Q\beta = \frac{1}{2}$

In this case, $\sqrt{\delta} = (2\beta^2 - \Lambda)$. Hence,

$$r_{ex}^2 = 0; \quad r_{ex}^2 = \frac{2(2\beta^2 - \Lambda)}{\Lambda(4\beta^2 - \Lambda)} \quad (13)$$

The black hole in this case is degenerate Schwarzschild-de Sitter type as given in the Fig.10.
4.2 \[ Q = \frac{\beta}{\sqrt{\Lambda(4\beta^2 - \Lambda)}} \]

In this case \( \delta = 0 \). Hence,

\[ r_{ex}^2 = \frac{(2\beta^2 - \Lambda)}{\Lambda(4\beta^2 - \Lambda)} \]  \hspace{1cm} (14)

This is indeed a triple root of \( f(r) \) since \( f''(r) = 0 \) for this case as shown in Fig.12.

![Figure 12](image)

Figure 12. The figure shows the graphs for \( f(r) \) vs \( r \) for a degenerate black hole of the Reissner-Nordstrom-de Sitter type. Here, \( \beta = 1.4, Q = 0.7307, \Lambda = 0.5 \) and \( M = 0.6803 \).

4.3 \( (4\beta^2 > \Lambda > 2\beta^2) \)

Here, \( (2\beta^2 - \Lambda) < 0 \) and \( (4\beta^2 - \Lambda) > 0 \). There are two cases to consider:

Case 1: \( Q\beta < \frac{1}{2} \)

In this case, by observing the expression for \( \delta \) in eq. (12), it is clear that \( \sqrt{\delta} > |(2\beta^2 - \Lambda)| \). Hence only + will give a real value for \( r_{ex} \) as,

\[ r_{ex}^2 = \frac{2\beta^2 - \Lambda + \sqrt{\delta}}{\Lambda(4\beta^2 - \Lambda)} \]  \hspace{1cm} (15)

The extreme black hole will be Schwarzschild-de Sitter type as given in Fig. 10.

Case 2: \( Q\beta > \frac{1}{2} \)

In this case for \( \delta \) to be positive, \( Q < \frac{\beta^2}{\sqrt{4\beta^2 \Lambda - \beta^2 \Lambda^2}} \). If both conditions are satisfied, \( \sqrt{\delta} < |(2\beta^2 - \Lambda)| \). Since \( (2\beta^2 - \Lambda) < 0 \), there wont be a real solution for \( r_{ex} \). Hence no extreme solution for such parameters.
4.4 \((\Lambda > 4\beta^2)\)

Here, \((2\beta^2 - \Lambda) < 0\) and \((4\beta^2 - \Lambda) < 0\). By looking at the eq.(12), one can see that \(\delta\) is always positive. Again there are two cases to consider:

**Case 1**: \(Q\beta > \frac{1}{2}\)

In this case, \(\sqrt{\delta} > |(2\beta^2 - \Lambda)|\). For \(r_{ex}\) to be real, the – sign has to be chosen in front of \(\sqrt{\delta}\). Hence it seems that there is one root for \(r_{ex}\) leading to Schwarzschild-de Sitter type extreme black hole with,

\[
 r_{ex}^2 = \frac{(2\beta^2 - \Lambda) - \sqrt{\delta}}{\Lambda(4\beta^2 - \Lambda)} \tag{16}
\]

However, when the mass is calculated for the parameters, it was noticed that \(M < A/2\). Hence the solution has to be Reissner-Nordstrom-de Sitter type according to the discussion in section(2). In fact when we plotted the graph there were no extreme black holes; it was a Reissner-Nordstrom-de Sitter type black hole with a single horizon which was not a degenerate one. Hence the conclusion is that for the above conditions, there are no extreme black holes.

**Case 2**: \(Q\beta < \frac{1}{2}\)

In this case, \(\sqrt{\delta} < |(2\beta^2 - \Lambda)|\). For \(r_{ex}\) to be real, both signs can be chosen in front of \(\sqrt{\delta}\). Hence it seems that there are two roots for \(r_{ex}\) leading to Reissner-Nordstrom-de Sitter type extreme black hole. However, when the mass is calculated, \(M > A/2\) for one of the roots which signals that the solutions should be Schwarzschild-de Sitter type. For the other root, \(M < A/2\) which signals the solutions to be Reissner-Nordstrom-de Sitter type. The solution which works is the first one given by,

\[
 r_{ex}^2 = \frac{(2\beta^2 - \Lambda) + \sqrt{\delta}}{\Lambda(4\beta^2 - \Lambda)} \tag{17}
\]

leading to Schwarzschild-de Sitter type extreme black hole.

**Case 3**: \(Q\beta = \frac{1}{2}\)

In this case, \(\sqrt{\delta} = |(2\beta^2 - \Lambda)|\). Hence there is one non-zero real value for \(r_{ex}\) as,

\[
 r_{ex}^2 = \frac{2(2\beta^2 - \Lambda)}{\Lambda(4\beta^2 - \Lambda)} \tag{18}
\]

This is a Schwarzschild-de Sitter type extreme black hole.
5 Topology of the Extreme black holes

5.1 Nariai type black holes

In Nariai black holes, the cosmological horizon and the black hole event horizon coincide. For a nearly extreme black hole of this type, one can approximate the function \( f(r) \) as \([31]\),

\[
f(r) = \frac{f''(r_{ex})}{2} (r - r_1)(r - r_2)
\]

Here, \( r_1 \) and \( r_2 \) represents the two close horizons, \( r_{++}, r_c \). One can introduce new coordinate \( \chi \) as,

\[
r = r_{ex} + \epsilon \cos \chi
\]

around the close horizons. Here \( \epsilon \) is small. Hence, \( \chi = 0 \) corresponds to \( r_1 \) and \( \chi = \pi \) corresponds to \( r_2 \). A new time-like coordinate \( \psi \) is also introduced such as,

\[
t = \frac{2\psi}{\epsilon f''(r_{ex})}
\]

Note that \( f''(r_{ex}) < 0 \) for this type of black holes due to the nature of the function \( f(r) \) at \( r = r_{ex} \). Now, by substituting the new coordinates, the metric can be written as,

\[
ds^2 = \frac{-2}{f''(r_{ex})} \left(-\sin^2 \chi d\psi^2 + d\chi^2\right) + r_{ex}^2 d\Omega^2
\]

The above geometry corresponds to \( dS^2 \times S^2 \). The \( dS^2 \) has a scalar curvature,

\[
R_{dS^2} = |f''(r_{ex})|
\]

For the Born-Infeld black hole,

\[
f''(r_{ex}) = \frac{Q^2(4\beta^2 - \Lambda) - 2r_{ex}^4 \beta^2 \left(\Lambda - 2\beta^2 + 2\beta^2 \sqrt{1 + \frac{Q^2}{r_{ex}^2\beta^2}}\right)}{Q^2 + r_{ex}^4\beta^2}
\]

Reissner-Nordstrom-de Sitter black hole also has the same topology near the degenerate horizon.

Even though the horizons coincide in the Schwarzschild-like coordinates, the proper distance between them is not-zero. It can be calculated in the new coordinate system as,

\[
\int_0^\pi \frac{\sqrt{2}d\chi}{\sqrt{|f''(r_{ex})|}} = \frac{\sqrt{2}\pi}{\sqrt{|f''(r_{ex})|}}
\]

Also, the Hawking temperature, which is defined by \( T = \frac{\kappa}{2\pi} \) with \( \kappa \) being the surface gravity, seems like zero with the usual definition \( \kappa = \frac{f'(r_{ex})}{2} \). However, Cho and Nam \([32]\) redefined the surface gravity for the Nariai-type black holes as,

\[
\tilde{\kappa} = \frac{f''(r_{ex})}{2}
\]
Hence the temperature at the degenerate horizon is,

\[ \hat{T} = \sqrt{\frac{f''(r_{ex})}{2\sqrt{2\pi}}} \]  \hspace{1cm} (27)

with \( f''(r_{ex}) \) given by eq.(24).

### 5.2 Cold black holes

In cold black holes, the black hole inner horizon and the black hole outer horizon coincides. This is clearly given in the figure (a) in Fig.2. In cold black holes \( f(r) = f'(r) = 0 \) at the degenerate horizon. To understand the topology of the cold black holes, we can choose a new coordinate \( y \) to describe the near extreme geometry as \[ r = r_{ex} - \epsilon y \]  \hspace{1cm} (28)

The function \( f(r) \) can be expanded around \( r = r_{ex} \) as,

\[ f(r) \approx \frac{f''(r_{ex})}{2}(\epsilon y)^2 \] \hspace{1cm} (29)

A new time coordinate is defined as \( \psi = \epsilon t \). With these new coordinates, the metric is approximated to be,

\[ ds^2 = -\frac{f''(r_{ex})}{2}y^2 d\psi^2 + \frac{2}{f''(r_{ex})} \frac{dy^2}{y^2} + r_{ex}^2 d\Omega^2 \] \hspace{1cm} (30)

Since \( f''(r_{ex}) > 0 \) for the cold black hole, the above geometry represents \( AdS_2 \times S^2 \) topology. The \( AdS_2 \) has the curvature \(-f''(r_{ex})/2\). The topology is the same for the cold Reissner-Nordstrom-de Sitter black hole but with curvature \( \Lambda \).

### 5.3 Ultra-cold black holes

From section(4), for the ultra cold black hole, all three horizons coincide as,

\[ r_+ = r_{++} = r_c = \sqrt{\frac{(2\beta^2 - \Lambda)}{\Lambda(4\beta^2 - \Lambda)}} \] \hspace{1cm} (31)

Here, \( f(r_{ex}) = f'(r_{ex}) = f''(r_{ex}) = 0 \).

To understand the geometry near the horizon, a new coordinate is defined for the near extreme case as, \( y = \eta \sqrt{f''(r_{ex})}/2 \). One can substitute the new coordinate and take the limit \( f''(r_{ex}) \to 0 \) which leads to,

\[ ds^2 = -\eta^2 d\psi^2 + d\eta^2 + r_{ex}^2 d\Omega^2 \] \hspace{1cm} (32)
The above geometry has the topology, $R^2 \times S^2$. This is similar to the topology of the ultra-cold Reissner-Nordstrom-de Sitter black hole near the degenerate horizon. The temperature is zero.

6 Conclusions

In this paper, we have studied the properties of the Born-Infeld-de Sitter black holes. Due to the presence of the cosmological constant, the black holes have a cosmological horizon. The black holes in Born-Infeld-de Sitter gravity are interesting in the sense that there are variety of possibilities when it comes to the number of horizons and the nature of the singularity. Depending on the mass, charge, and the non-linear parameter, the black holes could have up-to three horizons. These could be similar to Reissner-Nordstrom-de Sitter, Schwarzschild-de Sitter or a marginal black hole with finite $f(r)$. It is also possible to have naked singularities. The non-linear nature of the electrodynamics change the singularity at the origin drastically. The behavior of the function $f(r)$ is dominated by $M/r$ term rather than a $Q^2/r^2$ term as in Maxwell’s electrodynamics couple to gravity. In the Reissner-Nordstrom-de Sitter black hole, the singularity is time-like, while in the Born-Infeld counterpart, it could be time-like or space-like depending on the parameters of the theory. Also the Born-Infeld black holes are colder.

In this paper, we have focused on the extreme black hole with degenerate horizons. A thorough analysis is done as to what type of degenerate horizon would emerge for the parameters of the theory. There are cold black holes where the black hole inner horizon and the outer horizon merge with zero temperature. These black holes have topology $AdS_2 \times S^2$. There are Nariai type black holes where the black hole outer horizon and the cosmological horizon merge with the topology, $dS_2 \times S^2$. The temperature and the distance between the horizons are calculated for such black holes. The ultra cold black holes has the topology, $R^2 \times S^2$. The geometries of these extreme black holes are similar to the corresponding ones in the Reissner-Nordstrom-de Sitter black holes. These topologies are discussed in more detail in the book by Griffiths & J. Podolský [34].

As for future work, it would be interesting to study the thermodynamics of these interesting class of black holes. In a recent paper, Dolan et.al. [35] did an analysis of de Sitter black holes considering the cosmological constant as a thermodynamical variable. It would be interesting to use such an approach to study thermodynamical properties of Born-Infeld-de Sitter black holes. It is also interesting to study the possibility of pair creation similar to what happens in Reissner-Nordstrom-de Sitter black holes [27]. We have not discussed lukewarm black holes where the black hole horizon and the cosmological horizon has the same temperature [22]. That would be an interesting aspect to study.
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