Towards Interactive Logic Programming

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Abstract: Linear logic programming uses provability as the basis for computation. In the operational semantics based on provability, executing the additive-conjunctive goal $G_1 \& G_2$ from a program $P$ simply terminates with a success if both $G_1$ and $G_2$ are solvable from $P$. This is an unsatisfactory situation, as a central action of $\&$ – the action of choosing either $G_1$ or $G_2$ by the user – is missing in this semantics. We propose to modify the operational semantics above to allow for more active participation from the user. We illustrate our idea via Prolog⊕, an extension of Prolog with additive goals.

keywords: interaction, logic programming, linear logic, computability logic.

1 Introduction

Linear logic programming such as [5] has traditionally lacked mechanisms that model decision steps from the user in the course of execution. This deficiency is an outcome of using provability as the basis for computation. In the operational semantics based on provability such as uniform provability [5, 6, 7], solving the additive-conjunctive goal $G_0 \& G_1$ from a program $P$ simply terminates with a success if both $G_1$ and $G_2$ are solvable from $P$. This semantics, $pr_o$, is shown below:

$$pr_o(P, G_0 \& G_1) \text{ if } pr_o(P, G_1) \text{ and } pr_o(P, G_2)$$

This is an unsatisfactory situation, as an action of choosing either $G_1$ or $G_2$ by the user – the declarative reading of $\&$ – is missing in this operational semantics.

Our approach, inspired by the game semantics of [3], in this paper involves a modification of the operational semantics above to allow for more active participation from the user. Executing the additive-conjunctive goal $G_1 \& G_2$ from a program $P$ now has the new operational semantics, $pr_n$, which is shown below:

$$pr_n(P, G_0 \& G_1) \text{ if } pr_n(P, G_i) \text{ and } pr_o(P, G_{(i+1) \mod 2})$$

where $i (= 0 \text{ or } 1)$ is chosen by the user. In the above definition, the system requests the user to choose $i$ and then proceeds with solving both the chosen goal, $G_i$, and the unchosen goal, $G_{(i+1) \mod 2}$. Both executions must succeed for the current goal to succeed. It is worth noting that solving the unchosen goal, $G_{(i+1) \mod 2}$, must proceed using $pr_o$ rather than $pr_n$ to ensure that there will be no further interactions with the user. It can be easily seen that our new semantics has the advantage over the old semantics: the former respects the declarative reading of $\&$ without losing completeness or efficiency.

As an illustration of this approach, let us consider a fast-food restaurant where you can have the hamburger set or the fishburger set. For a hamburger set, you can have a hamburger, a coke and a side-dish vegetable (onion or cone but they make the choice). For a fishburger set, you can...
have a fishburger, a coke and a side-dish vegetable (onion or cone but they make the choice). This
is provided by the following definition:

\![hburger. \\
!fburger. \\
!coke. \\
!onion. \\
!(hset : − hburger ⊗ coke ⊗ (onion ⊕ cone)) \\
!(fset : − fburger ⊗ coke ⊗ (onion ⊕ cone))

The definition above consists of reusable resources, denoted by \(!\). As a particular example, consider
a goal task \(hset\&fset\). This goal simply terminates with a success in the context of \([5]\) as both
goals are solvable. However, in our context, execution proceeds as follows: the system requests the
user to select a particular burger set. After the set – say, \(hset\) – is selected, the system tries to
solve the first conjunct using the new semantics, whereas it tries to solve the second conjunct using
the old semantics. Now the execution terminates with a success, as both conjuncts are solvable.

As seen from the example above, additive-conjunctive goals can be used to model interactive
decision tasks. We also adopt additive-disjunctive goals which are of the form \(G_1 \oplus G_2\) where \(G_1, G_2\)
are goals. Executing this goal has the following intended semantics: select the true disjunct \(G_i\) and
execute \(G_i\) where \(i(=1 \text{ or } 2)\) is chosen by the system.

To present our idea as simple as possible, this paper focuses on Prolog\(^{\oplus,\&}\), which is a variant of
a subset of Lolli\([5]\). The former can be obtained from the latter by (a) disallowing linear context
and \& in the clauses, and (b) allowing only \(\otimes, \oplus, \&\) operators in goal formulas. Prolog\(^{\oplus,\&}\) can
also be seen as an extension of Prolog with \(\oplus, \&\) operators in goal formulas, as \(\otimes\) in Prolog\(^{\oplus,\&}\)
corresponds to \(\land\) of Prolog.

In this paper we present the syntax and semantics of this extended language, show some exam-
pies of its use. The remainder of this paper is structured as follows. We describe Prolog\(^{\oplus,\&}\) based
on a first-order clauses in the next section and Section 3. In Section 4 we present some examples
of Prolog\(^{\oplus,\&}\). Section 5 concludes the paper.

## 2 The Prolog\(^{\oplus,\&}\) with Old Semantics

The extended language is a version of Horn clauses with additive goals. It is described by \(G\)- and
\(D\)-formulas given by the syntax rules below:

\[
G ::= A | G \otimes G | \exists x \ G | G \& G | G \oplus G \\
D ::= A | G \supset A | \forall x \ D
\]

In the rules above, \(A\) represents an atomic formula. A \(D\)-formula is called a Horn clause with
additive goals, or simply a clause.

In the transition system to be considered, \(G\)-formulas will function as queries and a set of
\(D\)-formulas will constitute a program. We will present the standard operational semantics for
this language as inference rules \([5]\). The rules for executing queries in our language are based on
uniform provability [3][7]. Below the notation $D; P$ denotes $\{D\} \cup P$ but with the $D$ formula being distinguished (marked for backchaining). Note that execution alternates between two phases: the goal reduction phase (one without a distinguished clause) and the backchaining phase (one with a distinguished clause).

**Definition 1.** Let $G$ be a goal and let $P$ be a program. Then the task $pr_o(P; G)$ is defined as follows:

1. $pr_o(A; P, A)$. % This is a success.
2. $pr_o((G_1 \supset A); P, A)$ if $pr_o(P, G_1)$.
3. $pr_o(\forall x D; P, A)$ if $pr_o([t/x]D; P, A)$.
4. $pr_o(P, A)$ if $D \in P$ and $pr_o(D; P, A)$.
5. $pr_o(P, G_1 \otimes G_2)$ if $pr_o(P, G_1)$ and $pr_o(P, G_2)$.
6. $pr_o(P, G_1 \& G_2)$ if $pr_o(P, G_1)$ and $pr_o(P, G_2)$.
7. $pr_o(P, G_1 \oplus G_2)$ if $pr_o(P, G_1)$.
8. $pr_o(P, G_1 \oplus G_2)$ if $pr_o(P, G_2)$.
9. $pr_o(P, \exists x G_1)$ if $pr_o(P, [t/x]G_1)$. Typically, selecting the true term can be achieved via the unification process.

The above rules are based on the focused proof theory of linear logic.

**3 The Prolog$^\oplus,\&$ with New Semantics**

Again, the new rules of Prolog$^\oplus,\&$ are formalized by means of what it means to execute a goal $G$ from the program $P$.

**Definition 2.** Let $G$ be a goal and let $P$ be a program. Then executing $G$ from $P$ — written as $pr_n(P, G)$ — is defined as follows:

1. $pr_n(A; P, A)$. % This is a success.
2. $pr_n((G_1 \supset A); P, A)$ if $pr_n(P, G_1)$.
3. $pr_n(\forall x D; P, A)$ if $pr_n([t/x]D; P, A)$.
4. $pr_n(P, A)$ if $D \in P$ and $pr_n(D; P, A)$.
5. $pr_n(P, G_1 \otimes G_2)$ if $pr_n(P, G_1)$ and $pr_n(P, G_2)$. Thus, the two goal tasks must be done in parallel and both tasks must succeed for the current task to succeed.
6. $pr_n(P, \exists x G_1)$ if $pr_n(P, [t/x]G_1)$. Typically, selecting the true term can be achieved via the unification process.
In the above rules, the symbols $\oplus, \&$ provides choice operations: in particular, the symbol $\oplus$ allows for the mutually exclusive execution of goals \[2\].

The operational notion of execution defined above is intuitive enough and the following theorem – whose proof is rather obvious from the discussion in \[5\] and can be shown using an induction on the length of derivations – shows the connection between the old operational semantics of Lolli \[5\] and the new operational semantics.

**Theorem 1** Let $P$ be a program and $G$ be a goal in Prolog$^{\oplus, \&}$. Executing $\langle P, G \rangle$ terminates with a success if and only if $G$ follows from $!P$ in Lolli (Hence, in intuitionistic linear logic).

### 4 Examples

As an example, let us consider the following database which contains the today’s flight information for major airlines such as Panam and Delta airlines.

\[
\begin{align*}
\% \text{panam}(\text{source, destination, dp\_time, ar\_time}) \\
\% \text{delta}(\text{source, destination, dp\_time, ar\_time}) \\
\text{panam}(\text{paris, nice, 9 : 40, 10 : 50}) \\
\text{panam}(\text{nice, london, 9 : 45, 10 : 10}) \\
\text{delta}(\text{paris, nice, 8 : 40, 09 : 35}) \\
\text{delta}(\text{paris, london, 9 : 24, 09 : 50})
\end{align*}
\]

Consider a goal $\exists dt \exists at \ \text{panam}(\text{paris, nice, dt, at}) \ \& \ \exists dt \exists at \ \text{delta}(\text{paris, nice, dt, at})$. This goal expresses the task of diagnosing whether the user has a choice between Panam and Delta to fly from Paris to London today. Note that this goal is solvable because the user indeed does have a choice in the example above. The system in Section 2 requests the user to select a particular airline. After the airline – say, Panam – is selected, the system produces the departure and arrival time of the flight of the Panam airline, i.e., $dt = 9 : 40, at = 10 : 50$.

### 5 Conclusion

In this paper, we have considered an extension to Prolog with additive goals in linear logic. This extension allows goals of the form $G_1 \oplus G_2$ and $G_1 \& G_2$ where $G_1, G_2$ are goals. In particular, the latter goals make it possible for Prolog to model decision steps from the user.

We plan to connect our execution model to Japaridze’s Computability Logic \[3, 4\] in the near future.

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