THE IMPACTS OF RETAILERS’ REGRET AVERSION ON A RANDOM MULTI-PERIOD SUPPLY CHAIN NETWORK

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Abstract. Most current studies on the equilibrium decision-makings of a supply chain network (SCN) consider only completely rational retailers who always try to maximize their expected profits under the situations that demands are random. However, many evidences show that a retailer wants to choose the decision-making involving random demand which provides him with minimum regret. In this paper, we consider the impacts of retailers’ anticipated regret aversion and experiential regret aversion on the equilibrium decision-makings of a random SCN with multiple production periods. Due to the random demand, the decision-makings of retailers are influenced not only by their anticipated regret, but also by their experiential regret after they have encountered bad or good experiences during past periods. The equilibrium conditions of the model are established. A numerical example is solved to illustrate the benefit of retailers obtained by considering their regret-averse behaviors. Moreover, it is found that retailers should consider their anticipated regret aversion or experiential regret aversion according to different situations of the demand markets.

1. Introduction. In a supply chain network (SCN), there are multiple manufacturers and multiple retailers competing for products in a non-cooperative manner. The random demand plays an important role for manufacturers and retailers to consider their best strategies. Thus, equilibrium models have been developed to help manufacturers and retailers to find the best production quantities and transaction quantities in random situations. Equilibrium models of a random SCN with single production period were investigated by Dong et al. (2004) [14], Wan et al.

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(2018) [36], Nagurney et al. (2019) [29], and Wang et al. (2019) [38], whereas equilibrium models with multiple production periods were investigated by Chan et al. (2018) [6], Cruz and Liu (2011) [12] and Saberi et al. (2018) [34]. All the above models assumed that the decision makers of the random SCN attempt to maximize their expected profits with a completely rational behavior. However, many experimental studies ([35]) and decision-making researches ([16], [19]) showed that the actual decision-makings obviously deviate from the predictable values of the expected utility which assumes that the decision makers are completely rational.

Regret theory is a non-expected utility theory ([4], [26]). Neuroscientists have obtained the supportive evidences for the regret theory in experiments [41]. Camille et al. (2004) found that people always compare the chosen gamble with the unchosen gamble, and they feel regret when the outcome of the chosen gamble is worse than that of the unchosen gamble [5]. Furthermore, learning from their emotional experiences, such people try to choose their future gamble with a minimum anticipated regret. There are a lot of real-life applications in regret theory ([1], [2], [8], [10], [15], [17], [23], [24], [32], [37], [39]), in which the decision-makings under uncertainty are chosen with a mini-max anticipated regret criterion. All these papers show that regret-averse behavior exists in a lot of human activities and can affect the decision makers’ optimal decisions in various applications when they face random demands. Therefore, it is important to incorporate regret-averse behavior into supply chain management.

From the description of the previous paragraphs, decision makers are not completely rational in their decision-making processes. In business environments, the decision-making processes of regret-averse retailers are impacted by both their actual net profits and the maximum net profits obtainable from their businesses. When the decision makers’ actual net profit is less than the maximum net profit, they exhibit the regret behavior. Thus, by capturing the incomplete rational behaviors of the retailers mentioned above, we describe the utility of a regret-averse retailer as a function of both the actual net profit and the regret. Since the demands of retailers are unknown but follow a certain probability distribution function, we use the maximum net profit as a reference point for defining the amount of regret. Then, we use the expected value of ‘actual net profit minus regret’ to define the expected utility of the retailers (See [20] for more details.).

So far we are focusing on the role of the regret as an ‘anticipatory’ emotion, whereby the retailers anticipate their regrets in advance and hence adjust their decisions (i.e., the decisions on the quantity they should order from the manufacturers and the price they should fix in the demand markets during each production period) accordingly in order to eliminate or reduce their regrets. However, in the SCN with multiple production periods, due to the random demand, apart from this ‘anticipatory emotion’, the retailers also have ‘unanticipated emotion’ or ‘experiential emotion’ in which the magnitude of the regret does not stay constant at each period \( t \), where \( t \) is an integer greater than one; but it increases or decreases in accordance with the bad or good regret experienced by the retailer in the period just preceding period \( t \), i.e., in period \( t + 1 \) (See [11] for details.). Thus, due to the uncertain demand in each production period, it is vital to incorporate this experiential regret-averse behavior in the SCN with multiple production periods.

In this paper, in contrast with the existing models of the anticipated regret aversion, we incorporate not only retailers’ anticipated regret aversion into a multi-period SCN model, but also their experiential regret aversion. In the experiential
regret consideration, we increase or decrease the degree of the regret-averse behavior in each period in accordance with the bad or good experiences encountered by the retailers in the previous period. In this way, we can reflect the retailers’ regret-averse behavior (consisting of both anticipated regret aversion and experimental regret aversion) more closely in their decision-making processes. Numerical results show that the expected utilities and profits of experiential regret-averse retailers are better than those of anticipated regret-averse retailers, while anticipated regret-averse retailers can achieve higher expected utilities and profits than those of regret-neutral retailers. Numerical results also show that the utilities and profits of the retailers increase with the degree of the regret aversion, for both anticipated regret and experiential regret. Hence, our numerical results clearly illustrate that retailers obtain more profits by considering their regret-averse behaviors and their performances.

The contributions of this paper are as follows:

(1) This paper investigates retailers’ regret-averse behaviors in a random SCN with multiple production periods. Unlike the conventional methods that maximize retailers’ expected profits, this paper maximizes the regret-averse retailers’ utilities which depend not only on their profits but also on their worst-case regret. The worst-case regret is measured by comparing the actual outcome from the chosen decision with that from the best decision, if we know the best decision in advance. An equilibrium model of the SCN with multiple production periods considering retailers’ regret-averse aversion is constructed.

(2) Moreover, this paper investigates with numerical examples not only the impacts of retailers’ anticipated regret aversion on the performance of the retailers, but also the impacts of their experiential regret aversion after they have encountered bad or good experiences during the past periods.

In the remainder of this paper, literature review is presented in Section 2, and the equilibrium of the SCN with multiple production periods is presented in Section 3, in which retailers’ regret aversion is considered. In Section 4, a numerical example is solved to investigate the impacts of the regret aversion and the experiential regret aversion on the performance of the retailers. Managerial insights and conclusions are presented in the last two sections.

2. Literature review. SCN equilibrium models considering single production period have been investigated by Nagurney et al. (2002) in deterministic demand situation [28], Dong et al. (2004) in random demand situation [14]. Chan et al. (2019) established the SCN equilibrium strategies considering multi-attribute behaviors [7]. Li et al. (2018) considered the SCN competition in quality [25]. Nagurney et al. (2019) studied the financial and logistical equilibrium decisions for humanitarian organizations [29]. Wang et al. (2019) studied the SCN problem with retailer’s collection under legislation [38]. Deng et al. (2020) studied competitions among retailers in the VIM system [13].

In the multi-period SCN, any manufactured products can be produced and sold in different production periods. Thus, SCN equilibrium models with multiple production periods have been investigated by Cruz and Liu (2011) for multi-period effects of social relationship [12], Hamdouch (2011) for production capacity constraints [18], Chan et al. (2015) for uncertain new product and recycled product [6], Saberi et al. (2015) for green technology investment [34] and Zhou et al. (2015) for loss-averse retailers [42]. By using these multi-period equilibrium strategies, the
manufacturers and the retailers in the SCN can get their best schedule plans to optimize their profits or utilities for the whole production periods.

Regret-averse theory ([4], [26]) is another non-expected utility theory (like risk aversion [3], loss aversion [43] and fairness preference [40]), which has a lot of real-life applications. Ahmed and Kwon (2014) studied the optimal contract problem with a regret-averse decision maker [1]. Chan et al. (2019) considered a multiple decision making problem including regret aversion [7]. Chorus (2012) [9] and Ramos et al. (2018) [33] showed that the anticipated regret can influence the investment choices and route choices, respectively. Engelbrecht-Wiggans and Katok (2008) studied how the decision maker’s regret behavior can affect the outcomes of auctions [15]. Perakis and Roels (2008) incorporated regret theory into the newsvendor model to find the order quantities which minimize the maximum regret [32]. Ayvaz-Cavdaroglu et al. (2016) studied the revenue management [2], and Conde et al. (2018) studied the shortest path with minimax regret [10]. Chassein and Goerigk (2017) [8], Gilbert and Spanjaard (2017) [17], Li et al. (2018) [24] and Xidonas et al. (2017) [39] extended minimax regret to the optimization problem. Kuang and Ng (2018) [23] and Wang and Xiao (2017) [37] considered perished products and sustainable products with regret. All these applications show that the decision makers’ negative emotion of anticipated regret affect their optimal decision-makings. Hence, it is vital to investigate the impact of regret aversion on the optimal decision-makings.

As mentioned earlier, in this paper, we incorporate both retailers’ anticipated regret aversion and their experiential regret aversion into a multi-period supply chain. The decisions of the retailers are impacted not only by their anticipated regret aversion for a single period, but also by their experiential regret aversion after having encountered a bad or a good experience during the past period. Thus, this paper develops the first theoretical model that illustrates the impact of the decision makers’ regret aversion on the decision-makings of the SCN with multiple production periods.

3. The equilibrium model of the SCN with regret-averse retailers. In Figure 1, we illustrate a SCN which consists of $m$ manufacturers, $n$ retailers. We assume that $m$ manufacturers ($n$ retailers) are competing with a single product in a non-cooperative manner for $T$ production periods. Each individual manufacturer and each individual retailer are cooperating with each other for product transaction. In addition, the demands faced by retailers are random. In Figure 1, the top tier nodes represent $m$ manufacturers, and the bottom tier nodes represent $n$ retailers at each $t$ period. The link between manufacturer $i$ (a typical manufacturer) and retailer $j$ (a typical retailer) represents the transaction link. Manufacturers produce products to sell them to retailers. Retailers sell the products to the consumers assuming that the retailers are regret-averse due to the random demand. We construct an equilibrium model of this SCN with regret-averse retailers. We need to investigate not only the impacts of retailers’ anticipated regret aversion on the performance of the retailers, but also the impacts of their experiential regret aversion after having encountered bad or good experiences during the past periods.

3.1. Notations and assumptions. (1) The notations for the decision variables at period $t$ are as follows:

- $\hat q_i(t)$: the nonnegative quantity produced by manufacturer $i$.
- $I_i(t)$: the nonnegative inventory quantity of manufacturer $i$. (Since, there is no
inventory kept at the last period, we have $I_i(T) = 0$.

$q_{ij}(t)$: the nonnegative transaction quantity associated with manufacturer $i$ and retailer $j$.

$\rho_j(t)$: the nonnegative price at demand market $j$.

Let $\hat{q}(t) \in \mathbb{R}^m_+$ be the vector consisting of all manufacturers’ production quantities at period $t$, and $\hat{q} \in \mathbb{R}^m_+ T$ be the vector consisting of all manufacturers’ production quantities at all periods. Let $I \in \mathbb{R}^{m(T-1)}_+$ be the vector consisting of all manufacturers’ inventory quantities at all periods. Let $Q(t) \in \mathbb{R}^{mn}_+$ be the vector consisting of all transaction quantities at period $t$, and $Q \in \mathbb{R}^{mnT}_+$ be the vector consisting of all transaction quantities at all periods. Let $\rho \in \mathbb{R}^{nT}_+$ be the vector consisting of all demand markets’ prices at all periods.

(2) The notations for the costs at period $t$ are as follows:

$f_i(\hat{q}(t))$: manufacturer $i$’s production cost.

$c_{ij}(q_{ij}(t))$: manufacturer $i$ and retailer $j$’s transaction cost.

$H_i(I_i(t))$: manufacturer $i$’s inventory cost.

$c_j(Q(t))$: retailer $j$’s handling cost.

(3) The notations for the parameters are as follows:

$\delta$: the coefficient of the regret aversion.

$k_j(t)$: the weight of the regret aversion of retailer $j$ relative to his profit at period $t$.

$\tau$: the adjustment parameter, which is defined as the percentage increase in the degree of regret aversion from one period to the next period considered by the experiential regret-averse retailer. In other words, $\tau = \frac{k_j(t+1)-k_j(t)}{k_j(t)} \times 100\%$.

**Assumption A.** (A1) All vectors are column vectors.
(A2) To reflect that competition among manufacturers exists, $f_i(\hat{q}(t))$, the production cost, is the function of the entire production vector.
(A3) To reflect that competition among retailers exists, $c_j(Q(t))$, the handling cost,
is the function of the entire transaction vector.

(A4) All the production costs, inventory costs, transaction costs and handling costs are convex and differentiable.

3.2. The equilibrium decisions of the manufacturers. Manufacturers compete for their profit in a noncooperative manner. They produce a single product to sell to retailers. The decisions of each manufacturer are his production quantity, his transaction quantity with each of the retailers, and his inventory level at each period $t$. Manufacturers simply sell the goods to retailers at a certain price, without considering the randomness of the demand from the demand markets. In period $t$, manufacturer $i$ produces $\hat{q}_i(t)$ and sells $q_{ij}(t)$ unit to retailer $j$ with a price $\hat{\rho}_{ij}(t)$, where $\hat{\rho}_{ij}(t)$ is an endogenous price each item of sale products. The inventory quantity kept by manufacturer $i$ at the end of period $t$ is $I_i(t)$. Thus, the production quantity, the transaction quantity and the inventory quantity in each production period satisfy the following equation:

$$I_i(t) + \sum_{j=1}^{n} q_{ij}(t) = I_i(t-1) + \hat{q}_i(t), \forall t. \quad (1)$$

Manufacturer $i$’s objective, considering all production periods, is to maximize his total profit. His profit at period $t$, denoted by $M_i(t)$, includes the revenue and the costs (the production cost, the inventory cost, and the transaction cost). The objective of manufacturer $i$ considering all production periods becomes

$$\max \sum_{t=1}^{T} M_i(t) \quad \text{s.t.} \quad I_i(t) + \sum_{j=1}^{n} q_{ij}(t) = I_i(t-1) + \hat{q}_i(t), \forall t, \quad (3)$$

$$\hat{q}_i(t) \geq 0, q_{ij}(t) \geq 0, I_i(t) \geq 0, \forall j, \forall t. \quad (4)$$

The equality constraints (3) can be changed into the following two inequalities:

$$I_i(t) + \sum_{j=1}^{n} q_{ij}(t) - I_i(t-1) - \hat{q}_i(t) \leq 0, \forall t, \quad (5)$$

$$-I_i(t) - \sum_{j=1}^{n} q_{ij}(t) + I_i(t-1) + \hat{q}_i(t) \leq 0, \forall t. \quad (6)$$

Constraints (3) are satisfied if and only if constraints (5) and (6) are both satisfied. The purpose of changing the equality constraints (3) into two inequality constraints (5) and (6) is to allow these inequality constraints to be incorporated into the original objective function (2) to form a new augmented objective function by using the Lagrangian multiplier method. Thus, the equilibrium decisions of all manufacturers can be obtained by a variational inequality (VI) problem. More precisely, equality constraints (3) are ‘conservation of products’ constraints which are equality constraints. It is known that whenever the VI has an equality constraint, the Lagrange multiplier corresponding to this equality constraint can be positive, zero, or negative. Under this situation, it is difficult to use the modified projection algorithm given in Section 3.5 to solve the VI to obtain the equilibrium decisions. Thus, for the sake of using this algorithm to find the equilibrium decisions of all the manufacturers, we need to convert equality constraints (3) into two inequality
constraints (5) and (6). The method of finding these equilibrium decisions are given in Theorem 3.1.

**Theorem 3.1.** Since manufacturers are competing in Nash ([30, 31]) noncooperative games, the equilibrium decisions of all manufacturers, i.e., \( \hat{q}_i(t), q_i^*(t), I_i^*(t), \) \( \forall i, \forall j, \forall t \) (the equilibrium solution is denoted by \( * \)), are obtained by solving VI (7): Determine \( (\hat{q}^*, Q^*, I^*, \gamma_1^*, \gamma_2^*) \in R_{mT+mnT+m(T-1)+2mT}^{n} \) such that

\[
\begin{align*}
&\sum_{t=1}^{T} \sum_{i=1}^{m} \left[ \frac{\partial f_i(\hat{q}_i^*(t))}{\partial \hat{q}_i(t)} - \gamma_1^*(t) + \gamma_2^*(t) \right] \times [\hat{q}_i(t) - \hat{q}_i^*(t)] + \\
&\sum_{t=1}^{T} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ -\hat{p}_{ij}(t) + \frac{\partial c_{ij}(q_{ij}^*(t))}{\partial q_{ij}(t)} + \gamma_1^*(t) - \gamma_2^*(t) \right] \times [q_{ij}(t) - q_{ij}^*(t)] + \\
&\sum_{t=1}^{T-1} \sum_{i=1}^{m} \left[ \frac{\partial H_i(I_i^*(t))}{\partial I_i(t)} + \gamma_1^*(t) - \gamma_2^*(t) - \gamma_1^*(t+1) + \gamma_2^*(t+1) \right] \times [I_i(t) - I_i^*(t)] + \\
&\sum_{t=1}^{T} \sum_{i=1}^{m} [I_i(t-1) + \hat{q}_i^*(t) - I_i^*(t) - \sum_{j=1}^{n} q_{ij}^*(t)] \times [\gamma_1^*(t) - \gamma_1^*(t)] + \\
&\sum_{t=1}^{T} \sum_{i=1}^{m} [-I_i^*(t-1) - \hat{q}_i^*(t) + I_i^*(t) + \sum_{j=1}^{n} q_{ij}^*(t)] \times [\gamma_2^*(t) - \gamma_2^*(t)] \geq 0,
\end{align*}
\]

\[\forall (\hat{q}, Q, I, \gamma_1, \gamma_2) \in R_{mT+mnT+m(T-1)+2mT}, \quad (7)\]

where \( \gamma_1^*(t) \in R_{mT}^n \) and \( \gamma_2^*(t) \in R_{mT}^n \) are the Lagrangian multipliers of the inequality constraints (5) and (6), respectively.

**Proof.** From the first-order optimality conditions of (2), (4), (5) and (6), we obtain VI (7). (See [27] for explanation.) \( \square \)

The endogenous price per item of sale products (\( \hat{p}_{ij}(t) \)) will be determined once the optimal decisions of the SCN have been obtained. Thus, from the second term of VI (7), we have \( \hat{p}_{ij}(t) = \frac{\partial c_{ij}(q_{ij}^*(t))}{\partial q_{ij}(t)} + \gamma_1^*(t) - \gamma_2^*(t) \).

### 3.3. The equilibrium decisions of the retailers.

Retailers, who compete in a noncooperative manner, sell the products to the consumers in demand markets, where the demand is random. The retailers are regret averse due to the random demand. In other words, the retailers compare the chosen decisions with the unchosen decisions, and they feel regret when the outcome of the chosen decisions is worse. Hence, retailers under uncertainty chose the decisions with a minimum regret.

The demand of retailer \( j \), denoted by \( \hat{d}_j(\rho_j(t)) \), is affected by the price of demand market, \( \rho_j(t) \). The relationship between the cumulative density \( P_j(x, \rho_j(t)) \) and the probability density \( f_j(x, \rho_j(t)) \) of \( \hat{d}_j(\rho_j(t)) \) is

\[
P_j(x, \rho_j(t)) = P(\hat{d}_j(\rho_j(t)) \leq x) = \int_{0}^{x} f_j(\xi, \rho_j(t)) d\xi.
\]

(8)

Retailer \( j \) ’s order quantity, denoted by \( s_j(t) \), is the total transaction quantities obtained from all manufacturers, i.e., \( s_j(t) = \sum_{i=1}^{m} q_{ij}(t) \). Moreover, we assume that retailer \( j \) ’s actual sale is \( \min\{s_j(t), \hat{d}_j(\rho_j(t))\} \), his over-stocking quantity is

\[
\max\{s_j(t) - \hat{d}_j(\rho_j(t)), 0\},
\]

(9)
and his under-stocking quantity is
\[
\max\{\hat{d}_j(\rho_j(t)) - s_j(t), 0\}. \tag{10}
\]
We also assumed that the costs per over-stocking item and per under-stocking item are \(\varepsilon^+_j(t) (\varepsilon^+_{j}(t) \geq 0)\) and \(\varepsilon^-_j(t) (\varepsilon^-_{j}(t) \geq 0)\), respectively.

At period \(t\), retailer \(j\)'s net profit, denoted by \(\pi_j(\hat{d}_j(\rho_j(t)), q_{ij}(t))\), which includes the revenue and the costs, is
\[
\pi_j(\hat{d}_j(\rho_j(t)), q_{ij}(t)) = \rho_j(t) \min\{s_j(t), \hat{d}_j(\rho_j(t))\} - \varepsilon^+_j(t) \max\{0, s_j(t) - \hat{d}_j(\rho_j(t))\} - \\
\varepsilon^-_j(t) \max\{0, \hat{d}_j(\rho_j(t)) - s_j(t)\} - c_j(Q(t)) - \sum_{i=1}^{m} \hat{\rho}_{ij}(t) q_{ij}(t), \tag{11}
\]
where \(\rho_j(t)\) is the price of the product. Then, the expected profit at period \(t\) is
\[
\int_{0}^{\pi_j(\hat{d}_j(\rho_j(t)), q_{ij}(t))} [\rho_j(t)x - \varepsilon^+_j(t)(s_j(t) - x)]f_j(x, \rho_j(t))dx + \\
\int_{\pi_j(\hat{d}_j(\rho_j(t)), q_{ij}(t))}^{+\infty} [\rho_j(t)s_j(t) - \varepsilon^-_j(t)(x - s_j(t))]f_j(x, \rho_j(t))dx \\
- c_j(Q(t)) - \sum_{i=1}^{m} \hat{\rho}_{ij}(t) q_{ij}(t). \tag{12}
\]

Let \(\pi^\text{max}_j(q_{ij}(t))\) be the ex-post optimal profit of retailer \(j\), which is the optimal profit if retailer \(j\) knows in advance which scenario will surely occur, i.e., if there is no uncertainty in the demand. From (11), it is clear that retailer \(j\) can obtain the following ex-post optimal profit:
\[
\pi^\text{max}_j(q_{ij}(t)) = \rho_j(t)s_j(t) - c_j(Q(t)) - \sum_{i=1}^{m} \hat{\rho}_{ij}(t) q_{ij}(t). \tag{13}
\]

Let \(\Delta v_j(t)\) be the difference between retailer \(j\)'s ex-post optimal profit, i.e., \(\pi^\text{max}_j(q_{ij}(t))\) as defined in (13), and his actual profit, i.e., \(\pi_j(\hat{d}_j(\rho_j(t)), q_{ij}(t))\) as defined in (11). Then
\[
\Delta v_j(t) = \pi^\text{max}_j(q_{ij}(t)) - \pi_j(\hat{d}_j(\rho_j(t)), q_{ij}(t)). \tag{14}
\]

Since retailers compare the actual profits with the ex-post optimal profits, they feel regret when the actual profits are worse than the ex-post optimal profits. Thus, regret is a painful feeling. In general, a regret function of the regret-averse decision maker depends negatively on the actual profit and positively on the ex-post optimal profit, given the uncertainty resolution. We use the following regret function of retailer \(j\) in this model, denoted by \(R_j(\Delta v_j(t))\), to describe the regret values of retailer ([9]):
\[
R_j(\Delta v_j(t)) = (\Delta v_j(t))^\delta, \tag{15}
\]
where \(\delta (0 < \delta \leq 1)\) is the coefficient of regret aversion, and \(R(0) = 0\). Then \(R_j(\Delta v_j(t))\) denotes the amount of retailer \(j\)'s regret due to the random demand. Figure 2 shows the regret function \(R_j(\Delta v_j(t))\) with different \(\delta\).
In this paper, we consider the worse-case regret, that is \( \delta = 1 \). From (11) and (13)-(15), retailer \( j \)'s regret value at period \( t \) is

\[
R_j(\Delta v_j(t)) = \begin{cases} 
(\rho_j(t) + \varepsilon_j^+(t))(s_j(t) - \hat{d}_j(\rho_j(t))), & \text{if } \hat{d}_j(\rho_j(t)) \leq s_j(t), \\
\varepsilon_j^-(t)(\hat{d}_j(\rho_j(t)) - s_j(t)), & \text{if } s_j(t) < \hat{d}_j(\rho_j(t)).
\end{cases}
\]

(16)

Then, his expected regret value is

\[
\int_0^{s_j(t)} (\rho_j(t) + \varepsilon_j^+(t))(s_j(t) - x)f_j(x, \rho_j(t))dx + \int_{s_j(t)}^{+\infty} \varepsilon_j^-(t)(x - s_j(t)))f_j(x, \rho_j(t))dx.
\]

(17)

The main concept of the regret theory ([4]) concerns with incorporating the anticipated regret feeling into the utility function of the regret-averse decision maker. Thus, in this model, retailer \( j \)'s utility has two parts, namely, the current profit and the regret obtained by comparing the profit of his actual decision with the ex-post optimal profit. At period \( t \), retailer \( j \)'s utility function considering the anticipated regret aversion, denoted by \( U_j(t) = U_j(\hat{d}_j(\rho_j(t)), q_{ij}(t)) \), includes the profit \( \pi_j(\hat{d}_j(\rho_j(t)), q_{ij}(t)) \) and the worse-case regret \( R_j(\Delta v_j(t)) \), such that

\[
U_j(t) = \pi_j(\hat{d}_j(\rho_j(t)), q_{ij}(t)) - k_j(t)R_j(\Delta v_j(t)),
\]

(18)

where the parameter \( k_j(t) (k_j(t) > 0) \) is the weight of retailer \( j \)'s regret relative to his profit. Thus, retailer \( j \)'s regret-averse utility function of all periods, denoted by \( U_j \), can be expressed as

\[
U_j = \sum_{t=1}^{T} (\pi_j(\hat{d}_j(\rho_j(t)), q_{ij}(t)) - k_j(t)R_j(\Delta v_j(t))).
\]

(19)
From (19), (12) and (17), retailer $j$ obtains the optimal decision $q^*_j(t)$ by considering the maximum utility $E(U_j)$, where

$$E(U_j) = \sum_{t=1}^{T} \int_0^{s_j(t)} \left[ \rho_j(t) x - \varepsilon^+_j(t) (s_j(t) - x) \right] f_j(x, \rho_j(t)) dx + \int_{s_j(t)}^{+\infty} \left[ \rho_j(t) s_j(t) - \varepsilon^-_j(t) (x - s_j(t)) \right] f_j(x, \rho_j(t)) dx - \varepsilon^+_j(Q(t)) - \sum_{i=1}^{m} \hat{\rho}_{ij}(t) q_{ij}(t) - k_j(t) \left[ \int_{s_j(t)}^{s_j(t)} \left( \rho_j(t) + \varepsilon^+_j(t) \right) (s_j(t) - x) f_j(x, \rho_j(t)) dx + \int_{s_j(t)}^{+\infty} \varepsilon^-_j(t) (x - s_j(t)) f_j(x, \rho_j(t)) dx \right].$$

(20)

Coricelli et al. (2005) found that healthy people not only minimize their anticipated regret in decisions, but also exhibit increasing their regret aversion after having repeatedly experienced regret [11]. Thus, if retailer $j$ experiences more regret at period $t$, he will increase the regret-averse weight at period $t + 1$. In other words, he imposes the condition $k_j(t + 1) > k_j(t)$ into the utility function. On the other hand, if retailer $j$ experiences less regret at period $t$, he will decrease the regret-averse weight at period $t + 1$. In other words, he imposes the condition $k_j(t + 1) < k_j(t)$ into the utility function. Finally, if retailer $j$ experiences the same amount of regret at period $t$, he will use the same regret-averse weight at period $t + 1$. In other words, he imposes the condition $k_j(t + 1) = k_j(t)$ into the utility function. Let $k_j(t + 1)$ be adjusted in the following manner:

$$k_j(t + 1) = \begin{cases} 
(1 + \tau)k_j(t), & \text{if retailer } j \text{ experiences more regret at period } t + 1, \\
k_j(t), & \text{if retailer } j \text{ experiences the same regret at period } t + 1, \\
(1 - \tau)k_j(t), & \text{if retailer } j \text{ experiences less regret at period } t + 1,
\end{cases}$$

(21)

where $\tau$ ($0 < \tau < 1$) is the parameter controlling the magnitude of the adjustment.

**Proposition 1.** If $\rho_j(t) > \varepsilon^-_j(t) - \varepsilon^+_j(t)$, retailer $j$’s expected utility ($E(U_j)$) is a concave function of his decision $q_{ij}(t)$, $\forall i, \forall j$ and $\forall t$.

The proof is given in Appendix.

Proposition 1 states that the concavity of the utility of retailer $j$ can be guaranteed only when the purchasing price of his product ($\rho_j(t)$) is larger than the difference between his under-stocking price ($\varepsilon^-_j(t)$) and the over-stocking price ($\varepsilon^+_j(t)$). This implies that his optimal transaction quantity with manufacturer $i$ ($q_{ij}(t)$) for maximizing his utility only exists when the above condition for concavity is satisfied. In other words, retailer $j$ is unable to obtain any profit by doing transaction with manufacturer $i$ if the above condition is not satisfied. Since the above condition is quite easily satisfied by retailer $j$, the concavity of the utility of retailer $j$ can be guaranteed.

**Corollary 1.** Suppose that the conditions of Proposition 1 hold. The optimal decision of retailer $j$ ($q^*_{ij}(t)$) is monotone decreasing with the regret-averse weight $k_j(t)$, $\forall i, \forall j, \forall t$.

The proof is given in Appendix.

From Corollary 1, we know that when retailer $j$ pays more attention to his regret aversion, he decreases his optimal transaction quantities with the manufacturers to obtain an outcome that provides him with less regret. Furthermore, if retailer $j$
experiences more regret at period $t$, it is clear from (21) that he will increase the weight of regret aversion in the next period. Thus, from Corollary 1, when the retailer has a bad regret experience, he will decrease the transaction quantities with the manufacturers in the next period. Similarly, when the retailer has a good regret experience, he will decrease the regret-averse weight and increase the transaction quantities with the manufacturers in the next period.

**Theorem 3.2.** Since retailers compete in Nash ([30, 31]) noncooperative games, the equilibrium decisions of all regret-averse retailers, i.e., $q_{ij}^*(t)$, $\forall i, \forall j, \forall t$ (the equilibrium solution is denoted by $*$), are obtained by solving VI (22): Determine $Q^* \in R_{+}^{mnT}$ such that

$$
\sum_{t=1}^{T} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[-(\rho_j^*(t) + \varepsilon_j^-(t))(1 - P_j(\sum_{i=1}^{m} q_{ij}^*(t), \rho_j^*(t))) + \varepsilon_j^+(t) P_j(\sum_{i=1}^{m} q_{ij}^*(t), \rho_j^*(t)) + \frac{\partial C_j(Q^*(t))}{\partial q_{ij}(t)} + \delta q_{ij}(t) + k_j(t)((\rho_j(t) + \varepsilon_j^+(t)) P(\sum_{i=1}^{m} q_{ij}(t), \rho_j^*(t)) - \varepsilon_j^-(t)(1 - P(\sum_{i=1}^{m} q_{ij}^*(t), \rho_j^*(t)))) \times [q_{ij}(t) - q_{ij}^*(t)] \geq 0, \forall Q \in R_{+}^{mnT}. \tag{22}
$$

**Proof.** In view of (20) and the fact that $s_j(t) = \sum_{i=1}^{m} q_{ij}(t)$, by setting the optimality condition $\frac{\partial E(U_j)}{\partial q_{ij}(t)} \geq 0$, we obtain (22). (See [27] for explanation.)

3.4. **The equilibrium decisions of the demand markets.** At period $t$, the equilibrium decisions for demand market $j$ satisfy the complementary conditions (23):

$$
E(\hat{d}_j(\rho_j(t))) \begin{cases} 
\leq \sum_{i=1}^{m} q_{ij}^*(t), & \text{if } \rho_j^*(t) = 0, \\
= \sum_{i=1}^{m} q_{ij}(t), & \text{if } \rho_j^*(t) > 0,
\end{cases} \tag{23}
$$

where $E(\hat{d}_j(\rho_j(t)))$ is the expected value of the demand. In fact, these complementary conditions are exactly the same as the economic equilibrium conditions given in [27].

**Theorem 3.3.** The equilibrium decisions for all the consumers at demand markets, i.e., $\rho_j^*(t)$, $\forall j, \forall t$ (the equilibrium solution is denoted by $*$), can be changed to VI (24) as follows: Determine $\rho^* \in R_{+}^{nT}$ such that

$$
\sum_{t=1}^{T} \sum_{j=1}^{n} \sum_{i=1}^{m} \left| q_{ij}^*(t) - E(\hat{d}_j(\rho_j^*(t))) \right| \times [\rho_j(t) - \rho_j^*(t)] \geq 0, \forall \rho \in R_{+}^{nT}. \tag{24}
$$

**Proof.** From the equivalence of complementary conditions and VI ([21]), we obtain VI (24) from (23).

From VI (24), we know that if $\rho_j^*(t) > 0$, then retailer $j$’s order quantity, i.e., $\sum_{i=1}^{m} q_{ij}^*(t)$, is exactly the same as the expected value of the demand.

3.5. **The equilibrium decisions of the SCN.** At Nash ([30, 31]) equilibrium, no decision maker of the SCN can improve his profit or his utility by changing his decisions, which implies that the equilibrium decisions of all manufacturers (7), all retailers (22), as well as all consumers at demand markets (24) must hold simultaneously.
Theorem 3.4. The equilibrium decisions of the SCN are obtained by solving VI (25): Determine \((q^*, Q^*, \rho^*, I^*, \gamma^2_1, \gamma^2_2) \in R^+=mnT+mnT+nT+m(T-1)+2mT\) (the equilibrium solution is denoted by \(\ast\)) such that

\[
\begin{align*}
\sum_{t=1}^{T} \sum_{i=1}^{m} \left[ \frac{\partial f_i(q^*_i(t))}{\partial q_i(t)} - \gamma^*_i(t) + \gamma^*_2(t) \right] & \times \left[ \hat{q}_i(t) - q^*_i(t) \right] + \\
\sum_{t=1}^{T} \sum_{i=1}^{m} \left[ \frac{\partial c_{ij}(q^*_j(t))}{\partial q^*_j(t)} \right] & + \gamma^*_1(t) - \gamma^*_2(t) - (\rho^*_j(t) + \varepsilon^{-}_j(t))(1 - \\
P_j \sum_{i=1}^{m} q^*_j(t), \rho^*_j(t))) & + \varepsilon^+_j(t)P_j \left( \sum_{i=1}^{m} q^*_j(t), \rho^*_j(t) \right) \right) - \\
\sum_{t=1}^{T} \sum_{j=1}^{n} \sum_{i=1}^{m} q^*_j(t) & - E(\hat{d}_j(\rho^*_j(t))) \times [\rho_j(t) - \rho^*_j(t)] + \\
\sum_{t=1}^{T-1} \sum_{i=1}^{m} \left[ \frac{\partial H_i(I^*_i(t))}{\partial I_i(t)} \right] & + \gamma^*_1(t) - \gamma^*_2(t) - \gamma^*_1(t+1) + \gamma^*_2(t+1) \times [I_i(t) - I^*_i(t)] + \\
\sum_{t=1}^{T} \sum_{i=1}^{m} \left[ I^*_i(t-1) + \hat{q}^*_i(t) - I^*_i(t) - \sum_{j=1}^{n} q^*_j(t) \right] \times [\gamma^*_1(t) - \gamma^*_1(t)] + \\
\sum_{t=1}^{T} \sum_{j=1}^{n} \left[ -I^*_i(t-1) - \hat{q}^*_i(t) + I^*_i(t) + \sum_{j=1}^{n} q^*_j(t) \right] \times [\gamma^*_2(t) - \gamma^*_2(t)] \geq 0, \\
\forall (q, Q, \rho, I, \gamma_1, \gamma_2) \in R^+=mnT+mnT+nT+m(T-1)+2mT. 
\end{align*}
\]

Proof. Adding the equilibrium decisions of all manufacturers (7), all retailers (22), and all consumers at demand markets (24), and performing some algebraic simplification, we obtain VI (25).

Theorem 3.5. Suppose that the conditions of Proposition 1 hold and \(E(\hat{d}_j(\rho_j(t)))\) is a strictly monotone decreasing and has bounded first-order derivative, \(\forall j, \forall t\). Suppose that the costs are strictly convex functions and have bounded second-order derivatives. Then, the monotone and Lipschitz continuity of VI (25) hold, which ensures that the equilibrium decisions of the SCN exist and are unique.

Proof. The proof is similar to those given in Theorem 3.1 and Theorem 3.4 of [42].

If the monotone and Lipschitz continuity of VI (25) hold, the modified projection algorithm [22] can be used to solve VI (25) to find the equilibrium decisions of the SCN considering retailers’ regret aversion. Dong et al. (2004) has used this algorithm to find the equilibrium decisions for an uncertain SCN [14], and Cruz and Liu (2011) has used this algorithm for an uncertain SCN with multiple production periods [12]. Thus, we choose this algorithm to obtain the equilibrium decisions of
the SCN considering retailers’ regret aversion. The full details for solving VI (25) is briefly described in the next paragraph.

**Modified projection algorithm for solving VI (25)**

Let $X = (\tilde{q}, Q, \rho, I, \gamma_1, \gamma_2)$ be the decision vector and $X^* = (\tilde{q}^*, Q^*, \rho^*, I^*, \gamma_1^*, \gamma_2^*)$ be the decision vector at equilibrium.

For the sake of ease of description of this algorithm, we need to first convert the problem of solving VI (25) into the following problem $(Q)$

**Problem (Q).** Determine $X^* \in R_+^{mT+mnT+nT+m(T-1)+2mT}$ such that

$$
\langle F(X^*), X - X^* \rangle \geq 0, \forall X \in R_+^{mT+mnT+nT+m(T-1)+2mT},
$$

where $F(X)$ is the vector-valued function preceding the multiplication signs of VI (25), and $\langle a, b \rangle$ is the inner product of vector $a$ and $b$.

**Step 1.** Initialization. Choose an initial vector $X^0 \in R_+^{mT+mnT+nT+m(T-1)+2mT}$ such that all the components of $X^0$ are set as 1. The step length $\alpha$ is set as 0.01. Set $\kappa = 1$.

**Step 2.** Computation. Compute $\bar{X}^\kappa \in R_+^{mT+mnT+nT+m(T-1)+2mT}$ obtained from the following Problem $(Q)$:

$$
\langle \bar{X}^\kappa + \alpha F(X^\kappa - 1), X - \bar{X}^\kappa \rangle \geq 0, \forall X \in R_+^{mT+mnT+nT+m(T-1)+2mT}.
$$

That is, compute $\bar{X}^\kappa = (\tilde{q}^\kappa, Q^\kappa, \rho^\kappa, I^\kappa, \gamma_1^\kappa, \gamma_2^\kappa) \in R_+^{mT+mnT+nT+m(T-1)+2mT}$ by solving VI (27):

$$
\sum_{i=1}^{T} \sum_{l=1}^{m} \frac{\partial f_i(q_i^\kappa(t))}{\partial q_i(t)} - \gamma_i^{\kappa - 1}(t) - \gamma_i^{\kappa - 1}(t) = 0,
$$

$$
\sum_{t=1}^{T} \sum_{i=1}^{m} \gamma_i^{\kappa - 1}(t) + \alpha \left( \sum_{j=1}^{n} \gamma_j^{\kappa - 1}(t) \right) - \gamma_1^{\kappa - 1}(t) - \gamma_2^{\kappa - 1}(t) = 0,
$$

$$
\left( \rho_j^{\kappa - 1}(t) + \epsilon_j^{-}(t) \right) \left( 1 - P_j \left( \sum_{i=1}^{m} q_{ij}^{\kappa - 1}(t), \rho_j^{\kappa - 1}(t) \right) \right) + \rho_j^{\kappa - 1}(t) P_j \left( \sum_{i=1}^{m} q_{ij}^{\kappa - 1}(t), \rho_j^{\kappa - 1}(t) \right) = 0,
$$

$$
\frac{\partial c_{ij}(q_{ij}^{\kappa - 1}(t))}{\partial q_{ij}(t)} + k_j(t) \left( \rho_j^{\kappa - 1}(t) + \epsilon_j^{+}(t) \right) P_j \left( \sum_{i=1}^{m} q_{ij}^{\kappa - 1}(t), \rho_j^{\kappa - 1}(t) \right) = 0,
$$

$$
\sum_{i=1}^{m} \left( \rho_j^{\kappa - 1}(t) E(q_{ij}^{\kappa - 1}(t)) - q_{ij}^{\kappa - 1}(t) \right) - \gamma_j^{\kappa - 1}(t) \left( \gamma_j^{\kappa - 1}(t) \right) = 0,
$$

$$
\sum_{i=1}^{T} \left( \sum_{j=1}^{n} \gamma_j^{\kappa - 1}(t) \right) + \alpha \left( \sum_{i=1}^{m} \gamma_j^{\kappa - 1}(t) \right) - \gamma_1^{\kappa - 1}(t) - \gamma_2^{\kappa - 1}(t) = 0,
$$

$$
\sum_{i=1}^{T} \sum_{j=1}^{n} \left( I_i(t) - I_i^*(t) \right) + \alpha \left( \sum_{i=1}^{m} \gamma_j^{\kappa - 1}(t) \right) = 0,
$$

$$
\gamma_1^{\kappa - 1}(t+1) + \gamma_2^{\kappa - 1}(t+1) - I_i^{\kappa - 1}(t) \times \left( \gamma_j^{\kappa - 1}(t) \right) = 0,
$$

where $F(X)$ is the vector-valued function preceding the multiplication signs of VI (25), and $\langle a, b \rangle$ is the inner product of vector $a$ and $b$. 

**Step 3.** Termination. If $\|X^\kappa - X^{\kappa - 1}\|_2 < \epsilon$, terminate; otherwise, set $\kappa = \kappa + 1$ and go to Step 2.

**Step 4.** Output. Output $X^{\kappa}$ as the approximate solution of Problem (Q).
Step 3. Adaption. Compute $X^\kappa \in \mathcal{R}_+^{mT+mnT+nT+m(T-1)+2mT}$ obtained from the following Problem (Q):

$$\langle X^\kappa + AF(\bar{X}^\kappa) - X^{k-1}, X - \bar{X}^\kappa \rangle \geq 0, \forall X \in \mathcal{R}_+^{mT+mnT+nT+m(T-1)+2mT}. $$

That is, compute $X^\kappa = (\hat{q}^\kappa, Q^\kappa, \rho^\kappa, I^\kappa, \gamma_1^\kappa, \gamma_2^\kappa) \in \mathcal{R}_+^{mT+mnT+nT+m(T-1)+2mT}$ by solving VI (28):

$$\sum_{t=1}^T \sum_{i=1}^m [\bar{\gamma}^\kappa_{1i}(t)] + \alpha(\bar{I}^\kappa_{i}(t-1) - \bar{I}^\kappa_{i}(t) - \sum_{j=1}^n \bar{q}^\kappa_{ij}(t)) - \gamma^\kappa_{1i}(t)] \times [\gamma_{1i}(t) - \bar{\gamma}^\kappa_{1i}(t)] +
$$

$$\sum_{t=1}^T \sum_{i=1}^m [\bar{\gamma}^\kappa_{2i}(t)] + \alpha(-I^\kappa_{i}(t-1) * (t-1) - \bar{q}^\kappa_{i}(t) + I^\kappa_{i}(t) + \sum_{j=1}^n \bar{q}^\kappa_{ij}(t)) - \gamma^\kappa_{2i}(t)] \times [\gamma_{2i}(t) - \bar{\gamma}^\kappa_{2i}(t)] \geq 0,
$$

$$\forall(\hat{q}, Q, \rho, I, \gamma_1, \gamma_2) \in \mathcal{R}_+^{mT+mnT+nT+m(T-1)+2mT}. $$

(28)
Step 4. Convergence. If \(|X^\kappa_l - X^{\kappa -1}_l| \leq \varepsilon\) for all \(l = 1, \ldots, mT + mnT + nT + m(T - 1) + 2mT\), where \(\varepsilon\) is the parameter for convergence, then stop; else let \(\kappa = \kappa + 1\) and return to Step 2.

4. Numerical results. To show the benefit of retailers considering regret aversion and the impacts of bad and good regret experience on the performance of the retailers, a numerical example with 2 manufacturers, 2 retailers, and 3 production periods in the SCN is solved in this section. Firstly, we compare the results obtained by considering retailers’ anticipated regret-averse behaviors with those obtained by considering retailers’ regret-neutral behaviors. Secondly, we compare the results obtained by considering retailers’ anticipated regret-averse behaviors with those obtained by considering retailers’ experimental regret-averse behaviors.

The cost functions, chosen from Example 1 of [42], are as follows:

- the production cost: \(f_i(q(t)) = (2.5 + (t - 1))(q_i(t))^2 + q_i(t)q_{i-1}(t) + 2q_i(t)\),
- the inventory cost: \(H_i(I_i(t)) = (1.5 + (t - 1))I_i(t)\),
- the transaction cost: \(c_{ij}(q_{ij}(t)) = 0.5q_{ij}^2(t) + 3.5q_{ij}(t)\),
- the handling cost: \(c_j(Q(t)) = 0.5(\sum_{i=1}^{\rho} q_{ij}(t))^2\),
- the over-stocking cost: \(\varepsilon^+_j(t) = 1\),
- the under-stocking cost: \(\varepsilon^-_j(t) = 1\),

where \(i = 1, 2\), \(j = 1, 2\), and \(t = 1, 2, 3\).

Some research papers, such as such as [14], [36] and [42], used the uniform distribution for the random demand in the SCN. Chan et al. [6] considered two different random demands involving the uniform distribution functions and the exponential distribution functions. It was found that irrespective of the distribution function of the demand, the variables of the decisions at equilibrium change according to the variation in the actual demand rate as well as the seasonality of the demand, in the same direction. Thus, we only use the uniform distribution demands to solve the illustrative example in this section, that is, at period \(t = 1, 2, 3\), the probability density function of retailer \(j\)’s \((j = 1, 2)\) random demand which is assumed to be uniformly distributed in \([0, b_j(t)\rho_j(t)]\).

In order to analyze the impacts of retailers’ experiential regret aversion on the performance of the retailers after having encountered a bad or a good regret experience, we will focus on two situations of demand markets: the rise situation and the depletion situation. The scale of demand at period \(t\) is set as \(b_j(t) = 100 + (t - 1)40\) for the rise situation, whereas it is set as \(b_j(t) = 180 - (t - 1)40\) for the depletion situation.
situation \((\forall j, \forall t)\). Hence, the uncertainty of demand increases with period \(t\) for the rise situation, whereas it decreases with period \(t\) for the depletion situation.

In our analyses, we compare the results obtained by considering retailers’ anticipated regret-averse behaviors with those obtained by considering their regret-neutral behaviors, i.e., the results obtained by maximizing retailers’ expected profits only. Thus, we use the weight of the regret attribute \(k_j(t) = 0 \ (\forall j, \forall t)\) to obtain the results corresponding to the regret-neutral behaviors. Then, we use the weight of the regret attribute \(k_j(t) = 0.2, 0.4, 0.6, \text{and } 0.8 \ (\forall j, \forall t)\) to solve VI (25) to obtain the equilibrium results corresponding to different degrees of regret aversion considered by the retailers. The equilibrium decisions of all decision makers in the two situations are shown in Table 1 and Table 2, respectively. The performances of all decision makers in the two situations are shown in Figure 3 and Figure 4, respectively.

Table 1 and Table 2 show that, in both the rise situation and the depletion situation, the transaction quantities \(\hat{q}_{ij}(t)\) obtained by using \(k_j(t) = 0 \ (\forall j, \forall t)\) (i.e., when the retailers are regret-neutral) in all periods are less than those obtained by using \(k_j(t) > 0 \ (\forall j, \forall t)\) (i.e., when the retailers are regret-averse). Moreover, the transaction quantities decrease when the weight of the regret attribute increases. Therefore, the findings in Corollary 1 is verified. In other words, regret-averse retailers have less transaction quantities with all the manufacturers. As a result, the production quantities and inventories of the manufacturers in all the periods decrease. On the other hand, the prices of all demand markets in all the periods increase.

Table 1. The equilibrium in the rise situation of Example 1

| \(i = 1, 2, j = 1, 2\) | regret-neutral | anticipated regret-averse |
|-------------------------|----------------|---------------------------|
| \(k_j(t)\)               |                |                           |
| \(k_j(t)\)               | 0              | 0.2                       | 0.4 | 0.6 | 0.8 |
| \(\hat{q}_{i1}(t), t = 1\) | 1.8633         | 1.6578                    | 1.4318 | 1.1823 | 0.9097 |
| \(\hat{q}_{i2}(t), t = 2\) | 1.5945         | 1.4185                    | 1.2248 | 1.0108 | 0.7769 |
| \(\hat{q}_{i3}(t), t = 3\) | 1.4934         | 1.3291                    | 1.148  | 0.9476 | 0.728  |
| \(I_{i1}^*(t), t = 1\)   | 0.5254         | 0.4672                    | 0.4026 | 0.3309 | 0.2523 |
| \(I_{i1}^*(t), t = 2\)   | 0.4356         | 0.3887                    | 0.3364 | 0.2779 | 0.213  |
| \(q_{i1}(t), t = 1\)     | 0.6692         | 0.5938                    | 0.5125 | 0.4237 | 0.3272 |
| \(q_{i1}(t), t = 2\)     | 0.8424         | 0.7469                    | 0.6431 | 0.5297 | 0.4064 |
| \(q_{i1}(t), t = 3\)     | 0.9648         | 0.8571                    | 0.7396 | 0.6102 | 0.4686 |
| \(\rho_{i1}^*(t), t = 1\) | 35.8181        | 40.1667                   | 46.2534| 55.4406| 70.7135 |
| \(\rho_{i1}^*(t), t = 2\) | 40.228         | 45.183                    | 52.1475| 62.7221| 80.4573 |
| \(\rho_{i1}^*(t), t = 3\) | 45.2266        | 50.7262                   | 58.4607| 70.2238| 90.025  |

From Figure 3 and Figure 4, it can be seen that, in both the rise situation and the depletion situation, retailers have less expected regret, more expected profit and expected utility in all the periods when the retailers are regret-averse. Moreover, when the degree of the regret attribute \(k_j(t)\) increases, that is, when the retailers are more regret averse, their expected regrets decrease and their expected profits and utilities both increase. On the other hand, manufacturers’ profits decrease with the degree of retailers’ regret-averse attributes. In the rise situation, the total profit
Figure 3. The profit, regret and utility of all the decision makers in the rise situation of Example 1

Table 2. The equilibrium in the depleted situation of Example 1

| $i = 1, 2, j = 1, 2$ | regret-neutral | anticipated regret |
|---------------------|----------------|---------------------|
|                     | $k_j(t)$       |                     |
| $q_i^*(t), t = 1$   | 2.1408         | 1.899               |
| $q_i^*(t), t = 2$   | 1.6228         | 1.4407              |
| $q_i^*(t), t = 3$   | 1.253          | 1.1216              |
| $I_i^*(t), t = 1$   | 0.0457         | 0.044               |
| $I_i^*(t), t = 2$   | 0              | 0                   |
| $q_j^*(t), t = 1$   | 1.0476         | 0.9261              |
| $q_j^*(t), t = 2$   | 0.8344         | 0.7408              |
| $q_j^*(t), t = 3$   | 0.6275         | 0.559               |
| $\rho_j^*(t), t = 1$| 42.1631        | 47.4737             |
| $\rho_j^*(t), t = 2$| 40.6374        | 45.5225             |
| $\rho_j^*(t), t = 3$| 37.6092        | 42.0773             |

of the SCN is 167.3594 when the retailers are regret-neutral, whereas it increases from 190.537 to 253.6376 when the weight of the regret attribute increases from
The profit of manufacturer $i$ ($i = 1, 2$) (b) The expected profit of retailer $j$ ($j = 1, 2$)

(c) The expected regret of retailer $j$ ($j = 1, 2$) (d) The expected utility of retailer $j$ ($j = 1, 2$)

**Figure 4.** The profit, regret and utility of all the decision makers in the depleted situation of Example 1

0.2 to 0.8. In the depletion situation, the total profit of the SCN also has positive correlation with the weight of the regret aversion.

The above results show that regret aversion provides retailers with more expected profits, more expected utilities and less expected regrets. Moreover, for anticipated regret, the expected utilities and the profits of the retailers increase with the degree of the regret aversion considered by the retailers.

We now compare the results obtained by considering retailers’ experiential regret aversion and anticipated regret-aversion in the two situations of the demand markets.

In order to conduct the comparison, we first need to describe how a retailer develops his experiential regret-averse behavior from his anticipated regret-averse behavior. For illustrative purpose, we simply use the weight of the regret attribute $k_j(t) = 0.6$ ($\forall j, \forall t$). From Figure 3 (c) and Figure 4 (c), in the rise situation, the regret values of the retailers obtained by using this weight in the three periods are 11.4879, 16.3266 and 21.0433, respectively, and those obtained in the three periods of the depletion situation are 21.4181, 16.2894 and 11.2515, respectively. (The occurrence of these phenomenon can be explained as follows:

In the rise situation, the uncertainty of the demand increases with period $t$, whereas in the depletion situation, the uncertainty of the demand decreases with period $t.$)
Table 3. The equilibrium decisions, profit, regret and utility in experiential regret-averse model of Example 1

| i = 1, 2 | in the rise situation | in the depleted situation |
|---------|------------------------|---------------------------|
| j = 1, 2 | anticipated regret | experiential regret | anticipated regret | experiential regret |
| τ = 0% | τ = 10% | τ = 20% | τ = 0% | τ = 10% | τ = 20% |
| \( \hat{q}_{ij}^*(t), t = 1 \) | 1.1823 | 1.111 | 1.0317 | 1.3421 | 1.3922 | 1.4385 |
| \( \hat{q}_{ij}^*(t), t = 2 \) | 1.0108 | 0.9324 | 0.8457 | 1.0214 | 1.0765 | 1.1285 |
| \( \hat{q}_{ij}^*(t), t = 3 \) | 0.9476 | 0.8502 | 0.744 | 0.813 | 0.8763 | 0.9326 |
| \( \hat{q}_{ij}^*(t), t = 1 \) | 0.2779 | 0.2087 | 0.135 | 0 | 0.0302 | 0.0597 |
| \( \hat{q}_{ij}^*(t), t = 2 \) | 0.3209 | 0.2645 | 0.1918 | 0.0395 | 0.0863 | 0.1325 |
| \( \hat{q}_{ij}^*(t), t = 3 \) | 0.4237 | 0.4215 | 0.4187 | 0.6492 | 0.6506 | 0.6505 |
| \( \rho_j^*(t), t = 1 \) | 55.4406 | 56.6548 | 58.0501 | 66.5564 | 65.7988 | 65.2914 |
| \( \rho_j^*(t), t = 2 \) | 62.7221 | 67.4657 | 73.9462 | 62.818 | 59.1709 | 56.155 |
| \( \rho_j^*(t), t = 3 \) | 70.2238 | 79.7648 | 93.8077 | 57.565 | 52.5551 | 48.7155 |
| \( \Sigma M_i(t) \) | 12.1586 | 10.1711 | 8.2357 | 11.8539 | 13.1645 | 14.4529 |
| \( \Sigma E(\pi_j(t)) \) | 105.2413 | 110.8787 | 116.4356 | 105.4868 | 101.6604 | 97.9989 |
| \( \Sigma E(R_j) \) | 48.8578 | 48.3116 | 47.7065 | 48.959 | 49.2566 | 49.5897 |
| \( E(U_j) \) | 75.9265 | 78.371 | 80.7928 | 76.1115 | 74.4315 | 72.8693 |
| Total profit | 234.7998 | 242.0996 | 249.3426 | 234.6814 | 229.6498 | 224.9036 |

Due to the random demand, retailers should adjust the weights of their regret attitude in accordance with the amount of their emotional regret. Thus, retailers’ anticipated regret-averse behaviors become their experiential regret behaviors.

In the rise situation, retailers experience more regret than that of the last period. Hence, from (21), retailers increase the regret-averse weight using the adjustment parameter \( \tau \), where \( \tau \) is defined as the percentage increase in the degree of regret aversion from one period to the next period considered by the experiential regret-averse retailers. In the depletion situation, retailers experience less regret than that of the last period. Thus, from (21), retailers should decrease the regret-averse weight using the adjustment parameter \( \tau \). We then solve VI (25), using the adjustment parameter \( \tau = 10\% \) and \( 20\% \), respectively, to find the equilibrium of the experiential regret aversion model. These equilibrium results are shown in Table 3. In Table 3, we also show the equilibrium of the anticipated regret aversion model i.e., the model with the adjustment parameter \( \tau = 0\% \).

Table 3 shows that, in the rise situation, the transaction quantities (\( q_{ij}(t) \)) obtained by using the adjustment parameter \( \tau > 0 \) (i.e., when the retailers consider their experiential regret aversion) is less than those obtained by using \( \tau = 0\% \) (i.e., when the retailers consider their anticipated regret aversion only and do not consider their experiential regret aversion). Moreover, in all periods, these transaction quantities decrease with the adjustment parameter \( \tau \), whereas retailers’ expected profits and utilities in all periods increase with the adjustment parameter \( \tau \). Thus, the retailers increase the degree of the regret attribute whenever they have bad regret experience in the rise situation. This adjustment has positive impacts on the performances of retailers’ profits and utilities as well as the total profits of the SCN, but it has negative impacts on the performances of the manufacturers’ profits.
Table 3 also shows that, in the depletion situation, the transaction quantities between manufacturers and retailers obtained by using the adjustment parameter $\tau > 0$ (i.e., when the retailers consider their experiential regret aversion) is larger than those obtained by using $\tau = 0\%$ (i.e., when the retailers consider their anticipated regret aversion only and do not consider their experiential regret aversion). Moreover, in all periods, these transaction quantities increase with the adjustment parameter $\tau$, whereas in all periods, retailers’ expected profits and utilities decrease with the adjustment parameter $\tau$. Thus, if the retailers still increase the weight of the regret attribute whenever they have good regret experience in the depletion situation, this adjustment have negative impacts on the performances of retailers’ profits, utilities and the total profits of the SCN. However, it has positive impacts on the performances of the manufacturers’ profits.

Our equilibrium results concerning retailers’ experiential regret aversion are different from those concerning retailers’ anticipated regret aversion. If the retailers encounter a bad regret experience, they should increase their regret-averse weights so that their expected profits and utilities can be improved. However, if the retailers encounter a good regret experience, they should not adjust their regret-aversion weights. (In other words, they should consider their anticipated regret-aversion behaviors only.) These results are consistent with those given in [11], i.e., people exhibit increasing regret aversion after having experienced regret repeatedly.

5. Managerial insights. From the discussion of our numerical results in Section 4, the following managerial insights can be drawn:

(1) The results in Table 1 and Table 2 show that when the SCN has multiple production periods, the equilibrium results of our model considering retailers’ regret-averse behaviors are different from those of the conventional models that do not consider retailers’ regret aversion behaviors. From Figure 3 and Figure 4, it can be seen that regret-averse retailers have less regret, more expected profits and utilities than the regret-neutral retailers. Thus, they should consider including this natural behavior (i.e. regret-aversion) in their decision-makings in the SCN.

(2) The results in Table 3 show that the profits and utilities of the experiential regret-averse retailers are better than those of the anticipated regret-averse retailers, which are much better than those of the regret-neutral retailers; moreover, retailers’ expected profits and utilities increase with the degree of the regret aversion, for both anticipated regret and experiential regret.

(3) From Table 3, it can be seen that the experiential regret-averse retailers’ profits and utilities increase with the parameter $\tau$ in the rise situation, where $\tau$ is defined as the percentage increase in the degree of regret aversion from one period to the next period considered by the experiential regret-averse retailers; in the depletion situation, the experiential regret-averse retailers’ expected profits and utilities decrease with the adjustment parameter $\tau$. Thus, in the rise situation, the experiential regret-averse retailers should increase the adjustment parameter $\tau$ as much as possible, whereas in the depletion situation, the experiential regret-averse retailers should use $\tau = 0\%$. In other words, the experiential regret-averse retailers in the depletion situation should consider the anticipated regret aversion only, not the experiential regret aversion.

(4) From Table 1 and Table 2, it can be seen that manufacturers’ productions, inventories and profits have negative correlations with the degree of regret aversion considered by retailers. Hence, manufacturers must decrease their raw materials and
production capacities. In addition, they need to consider new policy in response to the regret-averse retailers, e.g. the buy-back policy, in order to increase the transaction quantities with retailers.

(5) From Table 3, it can be seen that the profit of the SCN have increased under the retailers’ regret-averse behavior, when compared with the retailers regret-neutral consideration. The profit of the SCN, as well as retailers’ expected profits and utilities, can achieve the highest values under the experiential regret-averse behavior in the rise situation. On the other hand, the total profit of the supply chain, as well as retailers’ expected profits and utilities can achieve the highest values under the anticipated regret-averse behavior in the depletion situation.

6. Conclusions. This paper incorporates retailers’ anticipated regret-averse behaviors and experiential regret-averse behaviors into the SCN with multiple production periods. Due to the random demand, retailers considering their anticipated regret-averse behaviors have less regret, more profits and utilities. Due to the multiple production periods in the SCN, experiential regret-averse retailers should choose either the anticipated regret-averse decisions or the experiential regret-averse decisions according to the different situations of the demand markets.

The optimal productions, inventories and profits of manufacturers decrease with the degree of regret aversion of retailers, while the total profit of the supply chain have positive correlations. Hence, facing retailers’ regret-averse behaviors, manufacturers must adjust their production policy, e.g. the policy concerning buying raw materials and increasing the production capacities, so that their production costs can be reduced. They may also need to consider new policies in order to increase their transaction quantities with retailers.

There are still some limitations in our study. We only consider retailers’ regret aversions, without considering manufacturers’ regret aversion. In other words, the random demands in the demand markets only affect retailers’ regret-averse behaviors. Future research should include considering both retailers’ and manufacturers’ bounded rationality behaviors and analyzing the randomness of all the manufacturers’ profits caused by the random demand in the SCN. For the existence and uniqueness for the new SCN involving both retailers’ and manufacturers’ bounded rationality behaviors, we need to impose some additional conditions on the cost functions. Future research should also involve finding the equilibrium decisions of a practical SCN with both manufacturers’ and retailers’ bounded rationality behaviors and using another efficient algorithm, such as the one used in Deng et al. (2020) [13], to find the equilibrium decisions for this practical SCN.

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Appendix A.

Proof of Proposition 1. From (20), we have

\[
\frac{\partial E(U_j)}{\partial q_{ij}(t)} = \frac{\partial}{\partial q_{ij}(t)} \int_0^{s_j(t)} \left[ \rho_j(t)x - \varepsilon_j^+(t)(s_j(t) - x) \right]f_j(x, \rho_j(t))dx + \frac{\partial}{\partial q_{ij}(t)} \int_{s_j(t)}^{\infty} \left[ \rho_j(t) + \varepsilon_j^-(t)(s_j(t) - x) \right]f_j(x, \rho_j(t))dx - \frac{\partial c_j(Q(t))}{\partial q_{ij}(t)} - \hat{\rho}_{ij}(t) - k_j(t) \frac{\partial}{\partial q_{ij}(t)} \int_0^{s_j(t)} \left[ \rho_j(t)x - \varepsilon_j^+(t)(s_j(t) - x) \right]f_j(x, \rho_j(t))dx + \frac{\partial}{\partial q_{ij}(t)} \int_{s_j(t)}^{\infty} \varepsilon_j^-(t)(x - s_j(t))f_j(x, \rho_j(t))dx \right].
\]
From (13) and (16) and the fact that \( s_j(t) = \sum_{i=1}^{m} q_{ij}(t) \), we have

\[
\frac{\partial}{\partial q_{ij}(t)} \left[ \int_{0}^{s_j(t)} \left( \rho_j(t)x - \varepsilon_j^+(t)(s_j(t) - x) \right) f_j(x, \rho_j(t)) \, dx \right] + \\
\frac{\partial}{\partial q_{ij}(t)} \left[ \int_{s_j(t)}^{\infty} \rho_j(t)s_j(t) - \varepsilon_j^-(t)(x - s_j(t)) f_j(x, \rho_j(t)) \, dx \right]
\]

\[
= \left( \rho_j(t) + \varepsilon_j^-(t) \right) \left[ 1 - P_j(s_j(t), \rho_j(t)) \right] - \varepsilon_j^+(t) P_j(s_j(t), \rho_j(t)),
\]

and

\[
- k_j(t) \left[ \int_{0}^{s_j(t)} (\rho_j(t) + \varepsilon_j^+(t))(s_j(t) - x) f_j(x, \rho_j(t)) \, dx \right] + \\
\frac{\partial}{\partial q_{ij}(t)} \left[ \int_{s_j(t)}^{\infty} \varepsilon_j^-(t)(x - s_j(t)) f_j(x, \rho_j(t)) \, dx \right]
\]

\[
= -k_j(t) \left[ (\rho_j(t) + \varepsilon_j^+(t)) P(s_j(t), \rho_j(t)) - \varepsilon_j^-(t) \left[ 1 - P(s_j(t), \rho_j(t)) \right] \right].
\]

Thus, the first-order derivative of \( E(U_j) \) becomes

\[
\frac{\partial E(U_j)}{\partial q_{ij}(t)} = \left( \rho_j(t) + \varepsilon_j^-(t) \right) \left[ 1 - P_j(s_j(t), \rho_j(t)) \right] - \varepsilon_j^+(t) P_j(s_j(t), \rho_j(t)) - \\
\frac{\partial c_j(Q(t))}{\partial q_{ij}(t)} - \dot{\rho}_j(t) - k_j(t) \left[ (\rho_j(t) + \varepsilon_j^+(t)) P_j(s_j(t), \rho_j(t)) \right] \, dx - \\
\varepsilon_j^-(t) \left[ 1 - P_j(s_j(t), \rho_j(t)) \right].
\]

Moreover, the second order derivative of \( E(U_j) \) is

\[
\frac{\partial^2 E(U_j)}{\partial q_{ij}(t)^2} = \partial \left( \frac{\partial E(U_j)}{\partial q_{ij}(t)} \right) / \partial q_{ij}(t),
\]

\[
= - (\rho_j(t) + \varepsilon_j^-(t) + \varepsilon_j^+(t)) f_j(s_j(t), \rho_j(t)) - \frac{\partial^2 c_j(Q(t))}{\partial q_{ij}(t)^2} - \\
k_j(t) f(s_j(t), \rho_j(t))(\rho_j(t) + \varepsilon_j^+(t) - \varepsilon_j^-(t)).
\]

If \( \rho_j(t) + \varepsilon_j^+(t) > \varepsilon_j^-(t) \), then we obtain from the convexity property of \( c_j(Q(t)) \) that

\[
\frac{\partial^2 E(U_j)}{\partial q_{ij}(t)^2} \leq 0.
\]

Hence, \( E(U_j(t)) \) is a concave function of \( q_{ij}(t) \).

\[ \Box \]

**Proof of Corollary 1.** From the first-order derivative of \( E(U_j) \), we obtain the optimal transaction quantity between manufacturer \( i \) and retailer \( j \) at period \( t \) \( (q_{ij}^*(t)) \) from the following equation

\[
\frac{\partial E(U_j)}{\partial q_{ij}^*(t)} = 0.
\]

By calculating the total differential of the above equation with respect to \( k_j(t) \), we obtain

\[
\frac{\partial^2 E(U_j)}{\partial q_{ij}^*(t)} \frac{dq_{ij}^*(t)}{dk_j(t)} + \frac{\partial^2 E(U_j)}{\partial q_{ij}^*(t) \partial k_j(t)} = 0.
\]
Then,
\[
\frac{dq^*_{ij}(k_j(t))}{dk_j(t)} = -\frac{\partial^2 E(U_j)/\partial q^*_{ij}(t)\partial k_j(t)}{\partial^2 E(U_j)/\partial q^2_{ij}(t)}.
\]

From Theorem 1, we have
\[
\frac{\partial^2 E(U_j)}{\partial q_{ij}(t)^2} \leq 0.
\]

From the first-order derivative of \(E(U_j)\) and the condition of Proposition 3.1, we have
\[
\frac{\partial^2 E(U_j)}{\partial q^*_{ij}(t)\partial k_j(t)} = -[(\rho_j(t) + \varepsilon^+(t) - \varepsilon^-(t))P(s_j(t), \rho_j(t)) + \varepsilon^-(t)] \leq 0.
\]

Thus, we have
\[
\frac{dq^*_{ij}(k_j(t))}{dk_j(t)} \leq 0,
\]
and \(q^*_{ij}(t)\) is a monotone decreasing function of the regret-averse weight \(k_j(t)\). \(\square\)