Directed motion for delta-kicked atoms with broken symmetries: comparison between theory and experiment

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We report an experimental investigation of momentum diffusion in the δ-function kicked rotor where time symmetry is broken by a two-period kicking cycle and spatial symmetry by an alternating linear potential. We exploit this, and a technique involving a moving optical potential, to create an asymmetry in the momentum diffusion that is due to the classical chaotic diffusion. This represents a realization of a type of Hamiltonian quantum ratchet.

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The ratchet effect, in other words the rectification of noise in a system without net bias, was first proposed by Feynmann and has since formed the subject of numerous studies [1]. Recently, there has been further interest and investigations of Hamiltonian chaotic ratchets, where the extrinsic noise is replaced by deterministic chaos. Hamiltonian ratchets offer the additional possibility of a fully quantum ratchet, where some form of directed transport appears in the context of coherent wave dynamics; other types of ratchets in dissipative and noisy quantum systems, corresponding to coherence times which are relatively short, have also been proposed [2]. Previous studies of chaotic Hamiltonian ratchets [3] indicated that directed motion arises if certain symmetries are broken, but persists only in the presence of mixed phase-space dynamics (e.g. a bounded classical phase-space with a mixture of regular tori and chaotic regions). The quantitative analysis of the directed current then relies on the details of the classical phase-space.

In [4, 5] an alternative theoretical proposal for chaotic, but asymmetric momentum diffusion, aimed at a realization with cold atoms in far-detuned pulsed optical lattices, was presented. Experiments with cold atoms in near-detuned, driven optical lattices, had already been shown to provide realizations of classical Brownian and dissipative ratchets [6]. Far-detuned lattices minimize decoherent effects: hence they provided the clearest demonstrations of Hamiltonian quantum chaotic dynamics [7]. In particular, cold atoms in δ-kicked optical lattices can realize the dynamics of the chaotic quantum kicked rotor (QKR) and show the effect of dynamical localization (DL): the momentum diffusion of the cold atoms follows approximately the classical chaotic rate, \( \langle p^2 \rangle \approx \Delta t \), up to a timescale \( t^* \propto \hbar^{-2} \), after which the diffusion stops and the quantum momentum probability distribution \( N(p) \) localizes, with a variance \( \langle p^2 \rangle^{1/2} = L \sim D/h \). DL is a quantum coherent effect due to destructive wave interference [8].

The directed chaotic transport mechanism of [4, 5] is generic in character: a quantum kicked rotor with broken time and space symmetry diffuses asymmetrically for a finite timescale \( t_R \). For the classical system, unfortunately \( L \) grows without limit as \( t \to \infty \). For the quantum dynamics, localization arrests the diffusion, and a momentum asymmetry which is comparable to \( L \) can result. This may be considered a type of quantum ratchet. An early proof-of-principle experimental realization was carried out [9]. Other theoretical proposals for Hamiltonian regular or chaotic directed motion have subsequently also been published by several groups [10].

In this paper we report experimental realizations of the kicked rotor with broken time and space symmetry which permit for the first time quantitative comparisons with the analytical results for the ‘ratchet time’ \( t_R \) and the periodic ‘current reversals’ expected from the theoretical model [4, 5]. We present an accurate formula for the classical ratchet current (a simplified formula was presented in [11]). Good agreement is obtained between theory and experiment. To our knowledge, this remains the only experimental realization of directed transport in a Hamiltonian quantum system.

An optical lattice formed by two counter-propagating laser beams may be used to trap laser-cooled atoms in a one-dimensional periodic potential. Here, an accelerating optical lattice was sometimes also used to apply an additional ‘rocking’ linear potential. In a frame with acceleration \( a \), the Hamiltonian has an additional inertial term [12]:

\[
H = \frac{p^2}{2M} + V_0 \cos(2k_Lx) \pm M a x
\]  

where \( M \) is the mass of the atom, \( k_L = 2\pi/\lambda \) the laser wavevector and \( V_0 \) the potential depth. If the optical lattice is applied as a series of short (δ-function) pulses with period \( T \), then we may as for the usual δ-kicked rotor, write the Hamiltonian including the rocking potential in dimensionless form:

\[
\mathcal{H} = \frac{p^2}{2} + \sum_n [K \cos(\phi) + A(-1)^n \phi] \delta(\tau - n)
\]
where $K$ is the stochasticity parameter which describes the strength of the kick. Here $\rho = 2Tk_{lp}/M$ is a scaled momentum, $\phi = 2k_{l}x$ a scaled position, $\tau = t/T$ a scaled time and $\mathcal{H} = 8\omega_{R}T^{2}H/\hbar$ the scaled Hamiltonian. The commutation relation $[\phi, \rho] = i8\omega_{R}T$ gives the scaled unit of system action or effective Planck constant $\hbar_{eff} = 8\omega_{R}T$ ($\omega_{R}$ the atomic recoil frequency, $2\pi \times 2.1$ kHz for cesium D2) which may be controlled through the period of the pulses.

For a $\delta$-kicked rotor, to lowest order, the momentum diffusion rate is the uncorrelated rate $D \approx K^{2}/2$. Corrections to this arise from correlations between kicks [13]. In [2] it was shown that for the classical $\delta$-kicked rotor where time symmetry is broken by a two-period kicking cycle of periods $T(1+b) : T(1-b)$ (where $b \ll 1$), then, for short times, the corrections produce an asymmetry in the momentum distribution $N(\rho)$.

Here, we consider an ensemble of particles with initial $(t = 0)$ momentum distribution strongly peaked about a value $\rho_{L}$, in the lattice frame, ie $N(\rho) \approx \delta(\rho - \rho_{L})$. Using the method of [13], we calculated a value for the asymmetry $I(t) = \langle (\rho - \rho_{L}) \rangle$ at later times:

$$I(t) = I_{0} \sin((1-b)A - 2b\rho_{L})F(t)$$

where the maximum current is $I_{0} = \frac{KJ(2Kb)}{M\omega(2Kb)}$. Using the method of [13], we calculated a value for the asymmetry $I(t) = \langle (\rho - \rho_{L}) \rangle$ at later times:

$$F(t) = 1 - J_{0}(2Kb)^{2t-2}.$$  

For $t$ small, $F(t) \sim t$ grows linearly with time, but for $t \gg t_{R} \approx 1/(Kb)^{2}$, it saturates, ie $F(t) \to 1$. Hence $t_{R}$ is the classical timescale for the current to develop; if dynamical localization occurs too quickly, ie $t_{s} \ll t_{R}$, no appreciable quantum effect is observed. Conversely if $t_{s} \gg t_{R}$, the asymmetry is negligible compared with $L$, (the asymptotic variance of $N(\rho)$). Optimally, we require $t_{s} \sim t_{R}$.

For a ratchet effect, as usually understood, a distribution initially with zero average momentum $\langle \rho \rangle = 0$, and $\rho_{L} = 0$ is in the rest frame of a potential (giving no net bias), evolves to an asymmetric distribution with $\langle \rho \rangle \geq 0$. In the present experiment, this requires $A \neq 0$; for example, for $A = \pi/2$, and $\rho = 0$, if at $t = 0$, $\langle \rho \rangle = 0$, we obtain $I(t > t_{s}) \approx I_{0}$. However, in order to fully investigate the underlying mechanism, here we also investigated extensively starting conditions with non-zero initial momentum, ie $\rho \neq 0$.

In our experiment we use laser-cooled cesium atoms in a far-off resonant pulsed optical lattice. The lattice is formed by two horizontal counter-propagating laser beams, $1/e$ radius ($0.95 \pm 0.05$ mm), with parallel linear polarizations (see figure 1) which produces a spatial variation of the AC Stark shift proportional to the local intensity, and which hence, is sinusoidal.

The pulses are produced by rapidly switching the drive voltage to the acousto-optic modulators (AOMs) according to a pre-defined sequence. The time between the kicks may be altered in order to produce the two-period $T(1+b) : T(1-b)$ alternating kicking cycle described above. The experiment proceeds as follows. Cesium atoms are trapped and cooled in a standard six-beam magneto-optic trap (MOT) before further cooling in an optical molasses to an rms scaled momentum width of $\sigma_{p} \approx 4$. The molasses light is turned off using an AOM and the periodic “kicking” potential applied. The beams for the kicking potential are derived from a Ti:Sapphire laser with an output power of 1 W at 852 nm, detuned typically 2000 linewidths (natural linewidth $\Gamma = 2\pi \times 5.22$ MHz) to the low frequency side of the D2 cooling transition in cesium, which is sufficient for effects due to spontaneous emission to be neglected. This is split into two equal intensity beams using a half-wave plate and polarizing beam splitter (HWP1 and PBS in figure 1) and each beam sent through an AOM. The two AOMs are driven by separate (phase-locked) radio-frequency synthesizers that are controlled by separate fast radio-frequency switches but triggered by the same arbitrary function generator that produces the kicks. After the kicking the cloud of cold atoms is allowed to expand ballistically for up to 20 ms before a pair of counter-propagating near-resonant laser beams are switched on and the fluorescence from the atoms imaged on a CCD camera. From the spatial distribution of the fluorescence it is then possible to extract the momentum distribution. Using this apparatus we have checked that (for regularly spaced kicks, i.e. $b = 0$ in the above) dynamical localization can be observed as a change to an exponential momentum distribution for $t_{s} \sim K^{2}/\hbar_{eff}$.

To investigate starting conditions with non-zero mean momentum we have used a moving optical lattice formed by laser beams with a controlled frequency difference to make the kicking potential, so that atoms which are stationary in the laboratory frame have a momentum $\rho_{L}$ in the rest frame of the optical potential. This is achieved
with the theoretical prediction of Eq. 3, i.e.,

\[ \rho_0 \approx 4 \] (for larger \( \rho_L \) these effects become important and start to affect the data). An investigation of the effects on the momentum diffusion arising from the finite width of the kicks was presented in [13]. We were able to investigate values of the parameters \( K = 2 \rightarrow 5 \) (with order 10% error arising mainly from the measurement of the beam intensity), \( h_{eff} = 1 \), and values of \( b = 1/8 \rightarrow 1/32 \). Values of \( K \) close to the first maximum of the Bessel function \( J_2(K) \) may be expected to produce the largest maximal currents \( I_0 \) and hence the clearest experimental signature. For values of \( K \approx 2 - 5 \) there are still stable islands in the classical phase-space; nevertheless these are sufficiently small: for \( b \neq 0 \), good quantitative agreement with calculated chaotic diffusive rates is obtained for lower \( K \) than for the Standard Map [5].

Finally, for experiments with \( A \neq 0 \), the linear ‘rocking’ term of alternating sign was included by accelerating the optical lattice. This is done by modulating the frequency of one of the laser beams in a linear manner using a second (phase-locked) arbitrary function generator by an amount \( \pm 6 \Delta f \) in the time of the kick period \( T \). The dimensionless potential gradient \( A \) is related to the magnitude of the frequency modulation (acceleration of the lattice) by \( A = 2\pi t_p \delta f \) for finite square pulses of width \( t_p \). Accelerating the potential thus provides a simple way of controlling the magnitude of \( A \) and hence controlling the phase shift of the momentum-dependent diffusion constant in order to make it locally asymmetric around zero momentum. As the maximum frequency modulation amplitude allowed by the radio-frequency synthesizers was \( \pm 1.25 \) MHz this limits the range of \( A \) achievable to \( \pm 3\pi/4 \). In order to observe one complete oscillation of the momentum diffusion constant, for some experiments an additional constant frequency offset was introduced between the laser beams such that in the rest frame of the lattice the mean atomic momentum was \( \rho_L = 8\pi \).

Figure 2 shows the asymmetry \( \langle \rho \rangle \) (combining \( \rho_L = 0 \) and \( \rho_L = 8\pi \)) plotted as a function of \( \Phi = \pi \left( 2\rho_L b - A \right) / \pi \approx \left( 2\rho_L b - A(1-b) \right) / \pi \) suggested by Eq.3. In particular, the data at \( \Phi = 0.5 \), corresponding to \( \rho_L = 0 \) hence represents a ratchet current \( I_0 \approx 4 \) obtained in the rest frame of the potential. The behavior is in good agreement with quantum simulations carried out here, which yielded \( I(t \geq t^*) \approx I_0 \sin \Phi \approx 3.8 \sin \Phi \). The corresponding classical formula from Eq.3 yields a much larger current \( I_0 \approx 7.5 \). For these values of \( K, b \) the result is close to the simplified classical current formula \( I(t \rightarrow \infty) \approx K^2 \frac{4\Delta f}{\pi} \sin \Phi \) obtained in [11].

In Fig. 3, the dependence on \( \langle \rho \rangle \) on \( b \) is tested more extensively for \( A = 0 \) and is seen to oscillate with a period \( \pi/b \) and is consistent with \( I_0 \propto 1/b \), in agreement with theory. However, in order to test experimentally the time dependence of the classical correlations (i.e. the behavior of \( F(t) \) above), further experiments for smaller \( b \approx 1/4 \), for which there is closer agreement for classical and quan-

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**FIG. 2: Demonstration of a type of quantum ‘ratchet effect’**. The graph shows the final momentum asymmetry \( \langle \rho \rangle \) vs \( \Phi = (2\rho_L b - A) / \pi \) for \( K = 2.6 \), \( b = 1/16 \), \( h_{eff} = 1 \). Filled squares are data for \( \rho_L = 0 \), open squares are \( \rho_L = 8\pi \). For the maximum at \( \Phi = 0.5 \), the cloud is initially at rest relative to the optical lattice (i.e. \( \rho_L = 0 \)) and has no asymmetry; after 120 kicks, there is a constant ‘ratchet current’ \( \langle \rho \rangle \approx 4 \).

**FIG. 3: Behavior as a function of \( b \) and initial momentum \( \rho_L \).** Momentum asymmetry vs starting momentum \( \rho_L \) in the lattice frame for \( K = 3.3 \), \( h_{eff} = 1 \), \( b = 1/32 \) (filled squares) and \( b = 1/16 \) (open triangles). The asymmetry is consistent with the theoretical prediction of Eq.3, i.e., \( \langle \rho \rangle \propto I_0 \sin 2\rho_L b \) and (approximately) \( I_0 \propto 1/b \).

by driving the AOMs at frequencies that differ by \( 2\Delta f \), such that the atomic momentum in the rest frame of lattice is \( \rho_L = m\lambda^2 \Delta f h_{eff} / 4\pi \hbar \). Using this technique, \( \rho_L \) may be varied over a large range in order to sample several periods of the oscillation of the asymmetric diffusion without the beams becoming significantly misaligned from the cloud of cold atoms.

For these experiments the period of the kicks is \( T = 9.47 \) \( \mu s \) and pulses are square with duration typically \( t_p = 296 \) ns \((t_p/T = 1/32 \leq b)\), which is sufficient for there to be no substantial effects on the diffusion constant due to the finite temporal width of the kicks in the region of \( \rho_L \approx 0 \) [14].
tual diffusion (ie $t^* \approx t_R$), were obtained. Hence in Fig. 4 a comparison of the behaviour of the current $\langle \rho \rangle$ is undertaken for quantal, classical (ie Eq. 3) and experimental values and for $K = 2.1, \hbar = 1.4$. The agreement between all three is quite good. Strikingly, there is for this value of $\hbar$, near-perfect agreement between the quantum and classical time dependence. For these parameters, as investigated in Eq. 4, the momentum diffusion has minima (ie $D(p_0)$ is rather small for $p_0 \approx \pi/(4b)$). The quantum cloud localizes as soon as the variance $L \sim \pi/(4b)$, hence $t^* \sim \pi^2/(Kb)^2 \sim t_R$. For the classical cloud, diffusion continues (albeit more slowly) and the momentum variance increases with time but the asymmetry saturates. Hence, the close agreement between the classical and quantal asymmetry is not unexpected. Fig. 5 shows the corresponding calculated quantal and experimental momentum distributions after dynamical localization (120 kicks) for the same parameters as Fig. 4. We note that the method for evaluation of the experimental $N(\rho)$ will tend to somewhat over-estimate $\langle \rho \rangle$; conversely, the theoretical results are for $\sigma_p = 1$: for larger initial momentum widths, there will be a damping of the current $I_0 \to I_0 \exp^{-4\sigma^2}$ although the near exact quantitative agreement at $t \to \infty$ is somewhat fortuitous, theory and experiment are in quite good agreement.

In sum, we have demonstrated experimentally directed transport which relies on the persistence of coherent, unitary quantum evolution over the full timescale of the experiment. This represents a type of Hamiltonian quantum ratchet.

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