Single-Spin Asymmetries in Inclusive and Exclusive Hadronic Processes

Dae Sung Hwang*, Jong Hyun Kim, and Seyong Kim†

Department of Physics, Sejong University, Seoul 143–747, Korea
 e-mail: dshwang@sejong.ac.kr*, skim@sejong.ac.kr†

Abstract

We investigate the single-spin asymmetries in inclusive and exclusive processes of electron-proton scattering and electron-positron annihilation. In the decomposition of hadronic tensor, a Lorentz symmetric spin-dependent term is generally present. The existence of such a term implies single-spin asymmetries in these processes and these single-spin asymmetries for baryons can be understood in a unified manner. We argue that it is important to measure the single-spin asymmetries in both inclusive and exclusive processes for the Λ production at the present B-factories. This will lead to the first measurement of the structure function of the symmetric spin-dependent hadronic tensor in these processes.

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1 Introduction

There have been important progresses in understanding the structure of the proton recently. The Jefferson Lab Hall A Collaboration [1] obtained the ratio of the proton’s elastic electromagnetic form factors $G_E/G_M$ by the measuring the polarization transfers, whose result showed that the ratio decreases systematically as $Q^2$ increases from 0.5 to 3.5 GeV$^2$. The BABAR Collaboration [2] measured, using the initial state radiation, the $p\overline{p}$ mass dependence of the ratio of the time-like proton electric and magnetic form factors and found that the ratio is significantly larger than 1 for masses up to 2.2 GeV. These new experimental results are more accurate than previous ones and show the $q^2$ dependences of the form factors which are different from old experimental results. These progresses are valuable for improving our understanding of the nucleon structure and motivate theoretical and experimental exploration on the hadronic structures.

We are interested in the time-like nucleon form factors. They have imaginary parts, differently from the space-like form factors, and their complex phases can be measured by finding out the single-spin asymmetry, $P_y$, where $P_y$ is defined as the polarization of the produced nucleon in the direction normal to the production plane. The dependence of $P_y$ on the models of the nucleon form factors were presented in [3]. This single-spin asymmetry may be measured at the future GSI $p\overline{p}$ experiment and the up-graded DAFNE experiment. Also, one can measure such $P_y$ of produced $\Lambda$ particles at the present B-factories by measuring the angular distribution of the daughter particles $p$ and $\pi$ [4]. In this paper we interpret the single-spin asymmetry, $P_y$, in terms of the structure function of the symmetric spin-dependent hadronic tensor, which has its origin in the final-state strong-interaction phases. We call this structure function $G_3$. Existence of $G_3$ permits single-spin asymmetries not only in the exclusive processes but also in the inclusive processes without violating the $T$-invariance in the time-like reactions since $G_3$ can be made from the final-state interactions in the time-like reactions. If the single-spin asymmetry effect in the
inclusive Λ production process in addition to the exclusive Λ production processs is sizeable, its effect in both processes can be measured at the present B-factories, and then the structure function $G_3$ can be extracted. In particular, the results for Λ will be useful for the studies of the nucleon structure. We can expect that the sizes of these single-spin asymmetries are around 15 to 20 % since that of the process $pp \rightarrow \Lambda ^{\uparrow} X$ is of that size [5].

Symmetric spin-dependent structure function has been discussed in various contexts previously. Christ and Lee [6] proposed to test the $T$-invariance by measuring the correlation function $S_m \cdot (k \times k')$ in the inelastic scattering $l^\pm + N \rightarrow l^\pm + \Gamma$, where $S_m$ is the polarization vector of the initial nucleon $N$, $k$ and $k'$ the initial and final momenta of the lepton $l^\pm$, $\Gamma$ all possible final states. They noted that this $T$-non-invariant correlation by the single-photon exchange is independent of the sign of the lepton charge, whereas such a $T$-invariant correlation by the interference between the single-photon and the two-photon exchange processes is proportional to the sign of the lepton charge. Christ and Lee also showed that the unpolarized nucleon target can produce spin-asymmetry. Gourdin [7] introduced the symmetric spin-dependent hadronic tensor in his study of the space-like deep inelastic scattering of the longitudinally polarized leptons from polarized nucleons. He noted that this symmetric spin-dependent hadronic tensor measures a violation of the time reversal invariance. These argument can be applied to time-like processes by crossing and annihilation of lepton–anti-lepton pair into baryons can produce spin-polarized baryon.

Unified understanding of single-spin asymmetries in electron-proton scattering process and electron-positron annihilation process in terms of symmetric spin-dependent structure function extends naturally to inclusive processes as well because the structure function, $G_3$ is allowed by the Lorentz symmetry. In this paper we investigate the single-spin asymmetries in the inclusive and exclusive processes of electron-proton scattering and electron-positron annihilation. These single-spin asymmetries are presented in a unified manner through a symmetric spin-dependent hadronic tensor. It should be important to measure the single-spin asymmetries in both inclusive and
exclusive processes for the $\Lambda$ production at the present B-factories, which will lead to the first measurement of the structure function of the symmetric spin-dependent hadronic tensor.

2 Space-like Processes

2.1 $e^{-}p \rightarrow e^{-}X$

The structure of the proton is probed by the inelastic scattering $e^{-}p \rightarrow e^{-}X$ in which the scattered electron is detected at a fixed energy and angle, and $X$ indicates all possible states. The structure functions are defined by

$$W_{\mu\nu} = (-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2}) W_1(q^2, \nu) + \frac{1}{M^2} \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left( P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) W_2(q^2, \nu)$$

$$+ \frac{1}{M^3} \left[ \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \epsilon_{\nuabc} s^a P^b q^c + \left( P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) \epsilon_{\muabc} s^a P^b q^c \right] G_3(q^2, \nu), \quad (1)$$

in which $M$ is the proton mass and $q = k - k'$ where $k$ and $k'$ are the momenta of the initial and final electrons, respectively. $M \nu = P \cdot q$ and we use $\epsilon_{0123} = 1$. $-q^2 > 0$ and $1 < \omega \equiv \frac{2M^2}{|q^2|} < \infty$ for the space-like processes.

In (1) $W_1$ and $W_2$ are usual spin-independent structure functions, and $G_3$ is a symmetric spin-dependent structure function which measures a violation of the time reversal invariance in electromagnetic interactions [7]. In this section we study the single-spin asymmetry which would be induced by $G_3$ in the space-like deep inelastic scattering.

Using the hadronic tensor $W_{\mu\nu}$ in (1) and the symmetric part of the leptonic tensor given by

$$L^{\mu\nu(S)} = 2 \left( k^\mu k'^\nu + k^\nu k'^\mu - k \cdot k' g^{\mu\nu} \right), \quad (2)$$

we get the differential cross section in the rest frame of the target proton

$$\frac{d\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{|q^2|^2} \sin^2 \frac{\theta}{2} \left[ 2W_1(q^2, \nu) + \cot^2 \frac{\theta}{2} W_2(q^2, \nu) \right.

$$+ \left. (\vec{s} \cdot \vec{y}) \frac{4(E + E')EE'}{M |q^2|} |\sin \theta| G_3(q^2, \nu) \right] , \quad (3)$$
where \( E (\hat{k}) \) and \( E' (\hat{k}') \) are the initial and final energies (momentum directions) and \( \theta \) the scattering angle of the electron, and \( \hat{y} = (\hat{k} \times \hat{k}') / |\hat{k} \times \hat{k}'| \). From (3) the single-spin asymmetry is given by

\[
\mathcal{P}_y = \frac{d\sigma (\vec{s} = \hat{y}) - d\sigma (\vec{s} = -\hat{y})}{d\sigma (\vec{s} = \hat{y}) + d\sigma (\vec{s} = -\hat{y})} = \frac{4(E + E')EE'}{M|q^2|} \sin \theta \frac{G_3(q^2, \nu)}{2W_1(q^2, \nu) + \cot^2 \frac{q^2}{2M} W_2(q^2, \nu)}.
\]

Checking experimentally whether the above \( \mathcal{P}_y \) is really zero corresponds to the study by Christ and Lee on the \( T \)-violation [6].

### 2.2 \( e^- p \rightarrow e^- p \)

The Dirac and Pauli form factors \( F_1(q^2) \) and \( F_2(q^2) \) for a spin-\( \frac{1}{2} \) composite system are defined by

\[
\langle P' | J^\mu(0) | P \rangle = u(P') \left[ F_1(q^2) \gamma^\mu + F_2(q^2) \frac{i}{2M} \sigma^{\mu \alpha} q_\alpha \right] u(P),
\]

where \( u(P) \) is the bound state proton spinor and \( q = P' - P \). The electric and magnetic form factors are defined by

\[
G_E(q^2) = F_1(q^2) + \frac{q^2}{4M^2} F_2(q^2), \quad G_M(q^2) = F_1(q^2) + F_2(q^2),
\]

where \( G_E(0) = 1 \) and the proton magnetic moment is defined by \( \mu = \frac{e}{2M} G_M(0) \).

From Eqs. (5) and (6) we have the relations

\[
W_1^{ex}(q^2, \nu) = -\frac{q^2}{4m^2} |G_M(q^2)|^2 \delta(2M \nu + q^2),
\]

\[
W_2^{ex}(q^2, \nu) = \frac{|G_E(q^2)|^2 - \frac{q^2}{4M^2} |G_M(q^2)|^2}{1 - \frac{q^2}{4M^2}} \delta(2M \nu + q^2),
\]

\[
G_3^{ex}(q^2, \nu) = -\frac{\text{Im}[G_M^* G_E]}{2 \left( 1 - \frac{q^2}{4M^2} \right)} \delta(2M \nu + q^2),
\]

and then, we get the differential cross section given by

\[
\frac{d\sigma}{d\cos \theta} = \frac{\pi \alpha^2 E' \cos^2 \frac{\theta}{2}}{2E^3 \sin^2 \frac{\theta}{2} \left( \frac{1}{1 + \tau} \right)} \left[ |G_E(q^2)|^2 + \frac{\tau}{\epsilon} |G_M(q^2)|^2 \right]
\]

\[
- (\vec{s} \cdot \hat{y}) \frac{E + E'}{M} \tan \frac{\theta}{2} |\text{Im}[G_M^* G_E]|,
\]

\[ \]

5
where \( \hat{y} = (\hat{k} \times \hat{k}')/|\hat{k} \times \hat{k}'| \), \( \tau = \frac{|q^2|}{3M^2} \), and \( 1/\epsilon = 1 + 2(1 + \tau)\tan^2 \theta_0/2 \). The scattering angle dependences of \( \tau \) and \( E' \) are given by \( \tau = \left( \frac{(E/M)^2 \sin^2 \theta_2}{1 + 2E/M \sin^2 \theta_2} \right) \) and \( E' = E \left( 1 + 2E/M \sin^2 \theta_2 \right) \). From (8) the single-spin asymmetry is given by

\[
P_y = -E + E' M |\tan \theta_2| \text{Im}[G_M^* G_E] / |G_E(q^2)|^2 + \frac{1}{2} |G_M(q^2)|^2.
\] (9)

In reality, the above \( P_y \) is expected to be zero since the Hermiticity of the electromagnetic current renders the space-like form factors real. However, it is useful to check experimentally whether \( P_y \) in Eq.(9) really vanishes.

### 3 Time-like Processes

#### 3.1 \( e^+ e^- \rightarrow pX \)

The structure of the proton is probed by the inelastic scattering \( e^+ e^- \rightarrow pX \) in which the produced proton is detected at a fixed energy and angle, and \( X \) indicates all possible states. The structure functions are defined by

\[
\begin{align*}
\overline{W}_{\mu \nu} &= \left( -g_{\mu \nu} + \frac{q_{\mu} q_{\nu}}{q^2} \right) \overline{W}_1(q^2, \nu) + \frac{1}{M^2} \left( P_{\mu} - \frac{P \cdot q}{q^2} q_{\mu} \right) \left( P_{\nu} - \frac{P \cdot q}{q^2} q_{\nu} \right) \overline{W}_2(q^2, \nu) \\
&+ \frac{1}{M^3} \left[ \left( P_{\mu} - \frac{P \cdot q}{q^2} q_{\mu} \right) \epsilon_{abc} s^a P^b q^c + \left( P_{\nu} - \frac{P \cdot q}{q^2} q_{\nu} \right) \epsilon_{abc} s^a P^b q^c \right] G_3(q^2, \nu),
\end{align*}
\] (10)

in which \( M \) is the proton mass and \( q = k + k' \) where \( k \) and \( k' \) are the momenta of the initial electron and positron, respectively. \( M \nu = P \cdot q \) and we use \( \epsilon_{0123} = 1 \). \( q^2 > 0 \) and \( 0 < \omega \equiv \frac{2M \nu}{q^2} < 1 \) for the time-like processes.

In (10) \( \overline{W}_1 \) and \( \overline{W}_2 \) are useal spin-independent structure functions, and \( G_3 \) is a symmetric spin-dependent structure function which has its origin in the strong-interaction phases. In this section we study the single-spin asymmetry which would be induced by \( G_3 \) in the time-like deep inelastic scattering.

Using the hadronic tensor \( W_{\mu \nu} \) in (10) and the symmetric part of the leptonic tensor given by

\[
L^{\mu \nu(S)} = 2 \left( k^{\mu} k'^{\nu} + k'^{\mu} k^{\nu} - k \cdot k' g^{\mu \nu} \right),
\] (11)
we get the differential cross section in the center of mass frame of the initial electron
and positron

\[
\frac{d^2\sigma}{dE\cos\theta} = 2\pi\alpha^2 M \left( \frac{2M\nu}{q^2} \right) \left( 1 - \frac{q^2}{\nu^2} \right)^{\frac{1}{2}} \left[ 2W_1(q^2, \nu) + \left( \frac{2M\nu}{q^2} \right) \left( 1 - \frac{q^2}{\nu^2} \right) \sin^2\theta \frac{\nu W_2(q^2, \nu)}{2M} \right]
\]

+ \left( \mathbf{s} \cdot \mathbf{\hat{y}} \right) \left( \frac{q^2}{4M^2} \right) \left( \frac{2M\nu}{q^2} \right)^2 \left( 1 - \frac{q^2}{\nu^2} \right) \sin 2\theta \left( \frac{G_3(q^2, \nu)}{W_1(q^2, \nu)} \right),
\]

(12)

where \(E\) is the energy of the produced proton, \(\mathbf{k}\) and \(\mathbf{p}\) the initial electron and produced proton momentum directions, and \(\theta\) the angle between \(\mathbf{k}\) and \(\mathbf{p}\), and \(\mathbf{\hat{y}} = (\mathbf{k} \times \mathbf{p}) / |\mathbf{k} \times \mathbf{p}|\). From (12) the single-spin asymmetry is given by

\[
P_y = \frac{\left( \frac{q^2}{4M^2} \right) \left( \frac{2M\nu}{q^2} \right)^2 \left( 1 - \frac{q^2}{\nu^2} \right) \sin 2\theta \left( \frac{G_3(q^2, \nu)}{W_1(q^2, \nu)} \right)}{\frac{2W_1(q^2, \nu)}{W_1(q^2, \nu)} + \left( \frac{2M\nu}{q^2} \right) \left( 1 - \frac{q^2}{\nu^2} \right) \sin^2\theta \left( \frac{\nu W_2(q^2, \nu)}{2M} \right)}. \quad (13)
\]

We also have

\[
\frac{1}{2} \left( \frac{d^2\sigma(\mathbf{s} = \mathbf{\hat{y}})}{dE\cos\theta} - \frac{d^2\sigma(\mathbf{s} = -\mathbf{\hat{y}})}{dE\cos\theta} \right) = \frac{\pi\alpha^2 M^3}{2M^2} \left( \frac{2M\nu}{q^2} \right)^3 \left( 1 - \frac{q^2}{\nu^2} \right)^{\frac{3}{2}} \sin 2\theta \left( \frac{G_3(q^2, \nu)}{W_1(q^2, \nu)} \right). \quad (14)
\]

In the Bjorken limit of the deep inelastic process [8, 10], we have

\[
M W_1 = -F_1(\omega), \quad \nu W_2 = F_2(\omega), \quad \text{and} \quad F_1(\omega) = \frac{\omega}{2} F_2(\omega). \quad (15)
\]

Then, the differential cross section is given by

\[
\frac{d^2\sigma}{dE\cos\theta} = \frac{2\pi\alpha^2 M}{(q^2)^{\frac{1}{2}}} \omega \left( 1 - \frac{4M^2}{q^2} \frac{1}{\omega^2} \right)^{\frac{1}{2}} \times \left( \frac{2}{M} \left( - F_1(\omega) \right) \left[ 1 - \frac{1}{2} \left( 1 - \frac{4M^2}{q^2} \frac{1}{\omega^2} \right) \sin^2\theta \right] \right)
\]

+ \left( \mathbf{s} \cdot \mathbf{\hat{y}} \right) \frac{(q^2)^{\frac{3}{2}}}{4M^3} \omega^2 \left( 1 - \frac{4M^2}{q^2} \frac{1}{\omega^2} \right) \sin 2\theta \left( \frac{G_3(\omega)}{W_1(q^2, \nu)} \right),
\]

(16)

and the single-spin asymmetry is given by

\[
P_y = \frac{G_3(\omega) \left( \frac{q^2}{4M^2} \right) \omega^2 \left( 1 - \frac{4M^2}{q^2} \frac{1}{\omega^2} \right)}{\left( - F_1(\omega) \right)^{\frac{2}{M^2}}} \left[ 1 - \frac{1}{2} \left( 1 - \frac{4M^2}{q^2} \frac{1}{\omega^2} \right) \sin^2\theta \right]. \quad (17)
\]

We present the graph of Eq. (17) in Fig. 1.
Figure 1: single-spin asymmetry, \( P_y \left( -\frac{2}{M} \right) \frac{F_1(\omega)}{G_3(\omega)} \) from Eq. (17)

3.2 \( e^- e^+ \rightarrow p\bar{p} \)

The Dirac and Pauli form factors \( F_1(q^2) \) and \( F_2(q^2) \) for a spin-\( \frac{1}{2} \) composite system are defined by

\[
\langle pp'|J^\mu(0)|0\rangle = \bar{u}(P) \left[ F_1(q^2) \gamma^\mu + F_2(q^2) \frac{i}{2M} \sigma^{\mu\alpha} q_\alpha \right] v(P') ,
\]

where \( u(P) \) and \( v(P') \) are bound state proton and anti-proton spinors, where \( q = P' + P \). The electric and magnetic form factors, \( G_E(q^2) \) and \( G_M(q^2) \), are defined by

\[
G_E(q^2) = F_1(q^2) + \frac{q^2}{4m^2} F_2(q^2) , \quad G_M(q^2) = F_1(q^2) + F_2(q^2) .
\]
From Eqs. (5) and (6) we have the relations

\[ W_1^{ex}(q^2, \nu) = \frac{q^2}{4M^2} |G_M(q^2)|^2 \delta(2M\nu - q^2), \]

\[ W_2^{ex}(q^2, \nu) = - \frac{|G_E(q^2)|^2 - \frac{q^2}{4M^2} |G_M(q^2)|^2}{1 - \frac{q^2}{4M^2}} \delta(2M\nu - q^2), \]

\[ G_3^{ex}(q^2, \nu) = \frac{\text{Im}[G_M^* G_E]}{2\left(1 - \frac{q^2}{4M^2}\right)} \delta(2M\nu - q^2). \tag{20} \]

and then, we get the differential cross section given by

\[
\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2 \sqrt{1 - \frac{4M^2}{q^2}}}{2q^2} \left[ |G_M(q^2)|^2(1 + \cos^2\theta) + \frac{4M^2}{q^2} |G_E(q^2)|^2 \sin^2\theta 
- (\hat{s} \cdot \hat{y}) \frac{2M}{\sqrt{q^2}} \sin2\theta \text{Im}[G_M^* G_E] \right], \tag{21} \]

where \( \hat{y} = (\hat{k} \times \hat{p})/|\hat{k} \times \hat{p}|. \) From (8) the single-spin asymmetry is given by

\[ P_y = - \frac{2M}{\sqrt{q^2}} \sin2\theta \text{Im}[G_M^* G_E]. \tag{22} \]

4 Conclusion

A symmetric spin-dependent structure function, \( G_3(q^2, \nu), \) is allowed in the decomposition of hadronic tensor and this structure function can exist in the time-like processes of electron-positron annihilation into hadrons from the strong phases which are made by the final-state interactions.

By use of this symmetric spin-dependent structure function, we present single-spin asymmetries in inclusive and exclusive processes involving baryons in the final state in a unified way and strongly suggests that single-spin asymmetries are generic phenomena. The existence of \( G_3(q^2, \nu) \) can be measured by \( \sin 2\theta \) distribution in the differential cross-section given in Eq. (12).

We argue that it is important to measure the single-spin asymmetries in both inclusive and exclusive processes for the \( \Lambda \) production at the present B-factories. This will lead to the first measurement of the structure function of the symmetric spin-dependent hadronic tensor in these processes.
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