The effect of turbulence on the dynamics of two bubbles in hydrodynamic cavitation

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Abstract. Hydrodynamic cavitation is commonly found in hydraulic machineries, causing damages to equipment, reducing efficiency and making noise, which has been researched intensely in past years. In these processes, cavitation bubbles play essential roles, and their behaviors and interactions influence the effects of cavitation significantly. In this paper, the effect of turbulence on the dynamics of two bubbles generated by an orifice plate is studied. The oscillation of the two bubbles and the internal maximum temperature of the two bubbles during collapse under the influence of turbulence are investigated numerically. The numerical results show that the existence of the larger bubble exert restrained impact on the oscillation behavior of the smaller bubble. When the distance between the two bubbles decreases, the inhibition effect grows stronger. The maximum internal temperature of the smaller bubble rises violently, while the maximum internal temperature of the larger bubble is affected little. The influences of two parameters (i.e., cavitation number, aperture ratio) on the fluctuation frequency and intensity of turbulence affecting bubble behaviors are also investigated.

1. Introduction
Cavitation bubbles can be generated in hydraulic machineries, such as valves, pipes, pumps, marine propellers, and turbines. The collapse of these bubbles will produce high speed jets, shock waves, extreme high pressure and high temperature. These effects will cause damages to overflow components, reducing efficiency and even causing safety accidents. Owing to these negative outcomes, cavitation researches have received great attentions. In recent years, the advantages of the effects created by cavitation have been noticed in many fields. Hydrodynamic cavitation can be used to treat water and wastewater with its effective removal of pharmaceuticals, bacteria, microalgae and viruses [1]. For instance, Jyoti and Pandit [2] found that cavitation could improve the production of potable water as a potential physical water disinfection technique. Tesch et al. [3] proposed that hydrodynamic cavitation is an effective method to produce emulsions.

Hydrodynamic cavitation can be generated by changing liquid environment pressure by means of applying flow system with special geometric structure. For instance, when the fluid flows through the orifice plate, the local pressure is lower than critical pressure of cavitation, resulting in generating a number of cavitation bubbles, which will collapse when the pressure recovery in downstream pipe. Due to their simple installation and energy conservation, orifice plate is a commonly used equipment for studying hydrodynamic cavitation and the subsequent effects [4-6]. Pawar et.al [4] reported orifice plate has low density of cavitation bubbles in flow by using the method of CFD simulation and high speed photographs. Ebrahimi et al. [5] proposed a characterization of high-pressure cavitation flow by using a
thick orifice plate in a pipe of constant cross section. Pradhan and Gogate [6] studied the treatment efficiency of orifice plate on wastewater under different operating conditions. The variation of the structural parameters of the orifice plate may cause the change of the frequency and the intensity of turbulence, which influence dynamics of single bubble and the pressure in the bubble [7]. The dynamics of a two-bubble system under a linear pressure recovery in downstream of orifice plate have also been investigated [8]. However, above researches did not combine turbulence pulsation with two-bubble interactions.

Therefore, in this study, we investigate the influence of the structural parameters of the orifice plate, e.g., cavitation number and aperture ratio, on the oscillation behaviors of a two-bubble system in water numerically. And the maximum internal temperature in two interacting cavitation bubbles are discussed.

2. Calculation model

2.1 Bubble dynamic equations

For calculation simplicity, several important assumptions are adopted as follows. 1) The bubble is always spherical. 2) Only radial motion of the bubble is considered. 3) The liquid is homogeneous and isotropic. 4) The gas inside the bubble is ideally adiabatic (i.e., $\kappa = 1.4$ ). 5) There is no gas or vapor exchange between the bubble and the surrounding medium.

The bubble dynamics are modeled by the Keller-Miksis equation including the viscosity, surface tension and compressibility of the liquid [9]. For two interacting bubbles, the equations of bubbles should be (Ref. [10], equation (7))

$$\left(1 - \frac{\dot{R}_1}{c_i}\right) \ddot{R}_1 \dot{R}_1 + \frac{3}{2} \left(1 - \frac{\dot{R}_2}{c_i}\right) \ddot{R}_2 = \left(1 + \frac{\dot{R}_1}{c_i}\right) \frac{p_1(R_1, t) - p_i(t)}{\rho_i}$$

$$+ \frac{R_1}{\rho_i c_i} \frac{d}{dt} \left[p_1(R_1, t) - p_i(t)\right] - \frac{1}{L} \frac{d}{dt} \left(\dot{R}_1 \dot{R}_2^2\right)$$

$$\left(1 - \frac{\dot{R}_2}{c_i}\right) \ddot{R}_2 \dot{R}_2 + \frac{3}{2} \left(1 - \frac{\dot{R}_1}{c_i}\right) \ddot{R}_1 = \left(1 + \frac{\dot{R}_2}{c_i}\right) \frac{p_2(R_2, t) - p_i(t)}{\rho_i}$$

$$+ \frac{R_2}{\rho_i c_i} \frac{d}{dt} \left[p_2(R_2, t) - p_i(t)\right] - \frac{1}{L} \frac{d}{dt} \left(\dot{R}_1 \dot{R}_2^2\right)$$

where

$$p_1(R_1, t) = \left(\frac{P_0 + \frac{2\sigma}{R_{01}}}{R_{01}/R_1}\right)^{3\kappa} - \frac{2\sigma}{R_1} - \frac{4\mu_i}{R_1} \dot{R}_1$$

$$p_2(R_2, t) = \left(\frac{P_0 + \frac{2\sigma}{R_{02}}}{R_{02}/R_2}\right)^{3\kappa} - \frac{2\sigma}{R_2} - \frac{4\mu_i}{R_2} \dot{R}_2$$

Here, $R_1$ and $R_2$ are the instantaneous bubble radii of bubbles 1 and 2 respectively; $L$ is the distance between the centers of bubbles 1 and 2; $c_i$ is the speed of sound in the liquid; $P_0$ is the atmospheric pressure; $p_i$ is the instantaneous local static pressure; $\rho_i$ is the density of the liquid; $\sigma$ is the surface tension coefficient; $R_{10}$ and $R_{20}$ are the equilibrium bubble radii of bubbles 1 and 2 respectively; $\kappa$ is the polytropic exponent; $\mu_i$ is the viscosity of the liquid; overdot denotes the time derivative. The last item of equation (1) is the radiation pressure generated by bubble 2 giving influences on the bubble 1. Vice versa can be found in equation (2).

The maximum internal temperature at the end of the bubble collapse is calculated by (Ref. [11], equation (11)) following equation
\[ T_{\text{max}} = T_\infty \left( \frac{R_{\text{max}}}{R_{\text{min}}} \right)^{\gamma - 1} \]  

(5)

Here, \( T_\infty \) is the liquid temperature; \( R_{\text{max}} \) is the maximum bubble radius at the expansion; \( R_{\text{min}} \) is the minimum bubble radius at the collapse.

### 2.2 Turbulence model

The geometry of orifice plate is shown in Figure 1. The instantaneous pressure of any point in downstream of orifice plate is predicted by the turbulence model, which is analogous to Moholkar and Pandit [7] and Cai et al. [12]. The fluctuating pressure is incorporated into two bubbles equations, and the bubble dynamics can be obtained as functions of structural parameters of orifice plate.

#### Figure 1. Geometry for orifice plate.

In the study of hydrodynamic cavitation, cavitation number is an important parameter. It can be defined as

\[ C = \frac{p_2 - p_v}{1/2 \rho v_0^2} \]  

(6)

Here, \( p_2 \) is the recovered pressure in pipe; \( p_v \) is the vapor pressure under operating temperature; \( v_0 \) is the fluid velocity at the orifice.

Because the volume flow in throat equals to that in pipe, the velocity in pipe can be obtained as below

\[ v_p = \left( \frac{d_0}{d_p} \right)^2 v_0 = \beta^2 v_0 \]  

(7)

Here, \( d_0 \) and \( d_p \) are the diameters of the hole and the pipe respectively; \( \beta \) is aperture ratio, which is defined as the ratio of diameter of the hole to that of the pipe.

Length scale of eddy is defined as

\[ l = \left( 0.07 d_0 + 0.007 d_p \right) \left( \frac{1}{2} \right) \]  

(8)

In this turbulent flow, instantaneous velocity in the flow direction is given as

\[ v_m = v_i + \overline{v} \sin \left( 2 \pi f_T t \right) \]  

(9)

Here, \( v_i \) is the local mean velocity; \( \overline{v} \) and \( f_T \) are the fluctuating velocity and the dominant frequency of turbulence respectively, which can be given as

\[ P_m = \left( \overline{v} \right)^3 / l \]  

\[ f_T = \overline{v}' / l \]  

(10)

Here, \( P_m \) is the energy dissipation per unit mass in liquid

\[ P_m = (\Delta P \cdot Q) / M_1 \]  

(12)

\[ Q = v_p \pi d_p^2 / 4 \]  

(13)

\[ M_1 = \rho_l \pi d_p^2 / 4 \]  

(14)
Here, $Q$ is the volume flow rate; $M_1$ is the total mass of the liquid in the area of pressure recovery; $\Delta P$ is the pressure loss determined by

$$\Delta P = (p_x - p_z) + \rho_1 v_0^2/2 - \rho_1 v_p^2/2$$

(15)

Here, $v'$ and $f_T$ can be obtained by solving the equations (5)-(15) and then the pressure at any point in flow can be estimated as below

$$p_t = p_x + \rho_1 v_0^2/2 - \rho_1 v_m^2/2 - \Delta P$$

(16)

3. Numerical results and Discussion

Equations (1)-(4) are solved by 4-5 Runge-Kutta method with the pressure at any point in flow, which is obtained by solving equation (16). The initial conditions are $t = 0$, $R_1 = R_{10}$, $R_2 = R_{20}$, $dR_1/dt = 0$, $dR_2/dt = 0$. If not specified, the following physical properties are employed in present paper: $R_{10} = 10 \mu m; R_{20} = 100 \mu m; L = 1000 \mu m; T_w = 293.15 K; \rho_1 = 1000 kg/m^3; \sigma = 0.072 N/m; \mu = 1.005 \times 10^{-3}$ Pa·s; $p_x = 2338$ Pa; $P_0 = 1 \times 10^5$ Pa; $p_2 = 1 \times 10^5$ Pa; $c = 1480$ m/s; $d_p = 0.03$ m; $C = 0.7$; $\beta = 0.5$.

According to these parameters, the turbulence frequency is 614.84 Hz. The normalized pressure at any point in flow is shown in Figure 2. Turbulence pulsation causes violent fluctuation of the fluid pressure, which makes bubbles oscillating around their equilibrium radii. The pressure is negative at the initial stage of the pulsation resulting in the bubble growth violently and then the pressure gradually increases to the recovery pressure resulting in the bubble collapse. The dynamics of bubbles at the initial stage ($t = 2P$) are investigated in the following discussion.

Figure 2. Normalized pressure at any point in flow versus normalized time $t/P$. $P$ is defined as $P = 1/f_T$.

Figure 3 shows the dynamics of two coupled bubbles with different distances between two bubbles centers. Under the condition of $L = \infty$, there is no coupling effects on two bubbles, and the dynamics of each bubble are similar to single bubble oscillation. The expansion ratios (i.e., max($R$)/$R_0$) of the bubble with small initial radius are much larger than that of the bubble with large initial radius. When distance between the centers of bubbles 1 and 2 decreasing, the mutual interactions play important roles in two bubbles dynamics. The oscillation behavior of the smaller bubble is suppressed by the larger bubble. The smaller the distance is, the stronger the inhibition effect is. In contrast, the smaller bubble has little effect on the oscillation of the larger bubble.
Figure 3. Radial motion of two bubbles with different distances between the centers of bubbles 1 and 2. $L = \infty$, 3000 μm, 2000 μm and 1000 μm.

The expansion ratios and the maximum internal temperatures at the end of two bubbles collapse with distances ranging from 1000 μm to 3000 μm are shown in Figure 4. The variation tendency of the maximum internal temperatures are similar to that of the expansion ratios. According to Figure 4, we know the maximum internal temperature in smaller bubble increases with increasing distance, and rises violently with distance ranging from 1500 μm to 2500 μm. But the maximum temperature in larger bubble has no obvious change with distance varying. At small L, the maximum internal temperature in smaller bubble is less than that of in larger bubble. At large L, the reverse is true.
**Figure 4.** The expansion ratios and the maximum internal temperatures in two bubbles with different distances between the centers of bubbles. The solid line and dashed line are the curve of the maximum internal temperatures inside two bubbles and the expansion ratios of two bubbles respectively. The circle symbols and triangle symbols represent the dynamics of smaller bubble and larger bubble respectively.

The variations of orifice plate structure parameters can cause the change of fluctuation frequency (shown in Figure 5) and intensity of turbulence (shown in Figure 6), resulting in affecting the behaviors of bubbles, which are shown in Figure 7 and Figure 8.

Figure 5 shows that turbulence frequency decreases monotonously with cavitation number \((C)\) increasing. The turbulence frequency also change with different aperture ratio \((\beta)\), reaching its maximum value when \(b = 0.5\). Figure 6 shows the influence of \(C\) and \(\beta\) on pressure at any point in flow. As shown in Figure 6(a), the value of pressure at any point in flow changes sharply. Figure 6(b) illustrates that the absolute value of the minimum pressure and the maximum pressure during a period increase with \(C\) decreasing.

**Figure 5.** The influence of the hydrodynamic cavitation number \((C)\) and aperture ratio \((\beta)\) on the turbulence frequency.

**Figure 6.** Normalized pressure at any point in flow versus normalized time under different structural parameters. Figure (a) shows normalized pressure change with different cavitation number \((C)\). Figure (b) shows normalized pressure change with different aperture ratio \((\beta)\).

The influence of \(C\) on the expansion ratios and the maximum internal temperatures in two bubbles are summarized in Figure 7. The variation tendency of the maximum internal temperatures are similar
to that of the expansion ratios. We can see the $T_{\text{max}}$ in smaller bubble decreases with $C$ increasing. $T_{\text{max}}$ in larger bubble changes little when $C$ in the range of 0.3–0.6 and decreases significantly within 0.6–0.8.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure7}
\caption{The expansion ratios and the maximum internal temperatures in two bubbles with different cavitation number ($C$). The solid line and dashed line are the curve of the maximum internal temperatures inside two bubbles and the expansion ratios of two bubbles respectively. The circle symbols and triangle symbols represent the dynamics of smaller bubble and larger bubble respectively.}
\end{figure}

Figure 8 shows the influence of $\beta$ on the expansion ratios and the maximum internal temperatures in two bubbles. The variation tendency of the maximum internal temperatures in larger bubble is similar to that of the expansion ratios. The change of $\beta$ has little on effect on the expansion ratio of smaller bubble. $T_{\text{max}}$ reaches its maximum value when $\beta = 0.6$ and decreases when $\beta$ is larger than 0.6 in two bubbles. $T_{\text{max}}$ in larger bubble is always greater than that in smaller bubble.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure8}
\caption{The expansion ratios and the maximum internal temperatures in two bubbles under different aperture ratio ($\beta$). The solid line and dashed line are the curve of the maximum internal temperatures inside two bubbles and the expansion ratios of two bubbles respectively. The circle symbols and triangle symbols represent the dynamics of smaller bubble and larger bubble respectively.}
\end{figure}

Through the above results, we find turbulence fluctuating pressure and frequency affect greatly on the bubble dynamics. Especially, the turbulence fluctuating pressure over the first cycle is closely related to the maximum initial temperature.

4. Conclusion
In this paper, the dynamics of two bubbles in orifice plate are investigated numerically with taking the interaction between two bubbles into account. The oscillation of smaller bubble is suppressed by larger bubble. And the suppression effect and the maximum internal temperature in smaller bubble increases with the distance between the two bubbles decreasing. But the existence of smaller bubble has little influence on the dynamics of larger bubble. The turbulence fluctuating pressure and frequency varies with changing the parameters of orifice plate. The maximum internal temperatures in two bubbles decreases with increasing cavitation number, and the smaller bubble is greatly affected by the cavitation number. When changing aperture ratio, the maximum internal temperatures in two bubbles have a maximum value near $\beta = 0.6$. The results of this paper provide theoretical supports for prevention of cavitation damages and design of cavitation reactor of higher efficiency.

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References

[1] Dular M, Griessler-Bulcbe T, Gutierrez-Aguirrec I, Heathd E, Kosenčičbe A K, Oderb M, Petkovška M, Račkic N, Ravnikarc M, Šarca A, Široka B, Zupanec M, Žitnikb M and Komparee B 2016 Use of hydrodynamic cavitation in (waste)water treatment. Ultrason. Sonochem. 29 577-588.

[2] Jyoti K K and Pandit A B 2001 Water disinfection by acoustic and hydrodynamic avitation. Biochem. Eng. J. 7 201-212.

[3] Tesch S, Freudig B and Schubert H 2003 Production of Emulsions in High – Pressure Homogenizers – Part I: Disruption and Stabilization of Droplets. Chem. Eng. Technol. 26 569-573.

[4] Pawar S K, Mahulkar A V, Pandit A B, Roy K and Moholkar A S 2017 Sonochemical Effect Induced by Hydrodynamic Cavitation:Comparison of Venturi / Orifice Flow Geometries. Aiche. J. 63 4705-16.

[5] Ebrahimi B, He G L and Tang Y J 2017 Characterization of high-pressure cavitating flow through a thick orifice plate in a pipe of constant cross section. Int. J. Therm. Sci. 114 229-240.

[6] Pradhan A A and Gogate P R 2010 Removal of p-nitrophenol using hydrodynamic cavitation and Fenton chemistry at pilot scale operation. Chem. Eng. J. 156 77-82.

[7] Moholkar V S and Pandit A B 1997 Bubble Behavior in Hydrodynamic Cavitation: Effect of Turbulence. Aiche. J. 43 1641-48.

[8] Li F C, Cai J, Huai X L and Liu B 2013 Interaction Mechanism of Double Bubbles in Hydrodynamic Cavitation. J. Therm. Sci. 22 242-249.

[9] Keller J B and Miksis M 1980 Bubble oscillations of large amplitude. J. Acoust. Soc. Am. 68 628–633.

[10] Mettin R, Akhatov I, Parlitz U, Ohl C D and Lauterborn W 1997 Bjerknes forces between small cavitation bubbles in a strong acoustic field. Phys. Rev. E. 56 2924–31.

[11] Merouani S, Hamdaoui O, Rezzeg Y and Guemini M 2015 Computer simulation of chemical reactions occurring in collapsing acoustical bubble: dependence of free radicals production on operational conditions. Res. Chem. Intermediat. 41 881-897.

[12] Cai J, Huai X L and Li X F 2010 Investigation on cavitation bubble dynamics in compressible liquid under turbulence (in Chinese). Chin. Sci. Bull. 55 857-866.