Low-energy Variety of Asymmetric SUSY Flavor Structure

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Abstract

In this letter we study the low-energy phenomenology and cosmological implications of supersymmetric grand unified theory with asymmetric flavor structure, which is suggested by the recent observation of fermion masses and mixing angles. The predictions of the scenario are rather dependent on a Yukawa parameter fixed by group-theoretical argument. A reduced value of the parameter gives a resolution to the sign problem of supersymmetric Higgs mass $\mu$, with which the theory becomes simultaneously consistent with the experimental data of bottom/tau mass ratio, flavor-changing rare decay of bottom quark, and the muon anomalous magnetic moment. The relic abundance of the lightest superparticle as cold dark matter of the universe is also investigated in light of the three-years WMAP result. A new source of flavor violation is found in the D-term induced scalar masses, that is a distinctive signature of the generation asymmetry.
The minimal supersymmetric standard model is conceived to be one of the promising theoretical candidates beyond the standard model for its brevity and attractive features. A remarkable property of supersymmetric standard models is the unification of gauge coupling constants at some high-energy scale \[1\]. This motivates us to promote it into grand unified theory (GUT) \[2\] and various attempts have been devoted to constructing realistic models and understanding their phenomenological implications.

The quarks and leptons are generally unified into multiplets of GUT gauge group. That seems to conflict with disparate flavor structures, namely the observed fermion masses and mixing angles. For example, the well-known GUT relation between down-quark and charge-lepton Yukawa couplings leads to the same mass eigenvalues of quarks and leptons, which is successful only for the third generation. Further if each one-generation matter is combined into a single multiplet, the quark and lepton mixing matrices have similar forms, inconsistent with the experimental observation. Therefore a naive unification hypothesis should be modified in the flavor (Yukawa coupling) sector.

Lopsided flavor structure in the generation space is one of promising ways to remedy the flavor difficulties in grand unification. It has recently been shown, for example, that the flavor asymmetry is dynamically realized by introducing matter multiplets in a generation-dependent fashion. With such a experimentally suggested form of flavor couplings, novel behaviors emerge in the electroweak gauge symmetry breaking and the mass spectrum of superparticles. Extending the previous study \[3\], in this letter we further discuss phenomenological and cosmological issues such as flavor-violating rare processes and the relic dark matter abundance in the universe.

In the analysis of this letter, we assume the following general form of Yukawa couplings at the GUT scale, where the electroweak gauge couplings meet:

\[ Y_u = Y_\nu = \begin{pmatrix} \vdots \\ y_t \end{pmatrix}, \quad Y_d = \begin{pmatrix} \vdots \\ x_d y' y \end{pmatrix}, \quad Y_e = \begin{pmatrix} \vdots \\ y' y \end{pmatrix}, \quad (1) \]

at the leading order. The blank entries take negligibly small values. The top quark and tau neutrino Yukawa coupling \( y_t \) come from the minimal Higgs coupling. On the other hand, the down-quark and charged-lepton Yukawa matrices, \( Y_d \) and \( Y_e \), have flavor asymmetrical forms. The latter structure of Yukawa couplings is incorporated in grand unified framework \[4\] and more reasonably is achieved dynamically by extra matter (and also Higgs) multiplets \[5, 6, 7\]. For example, in the SO(10) language, the third-generation fermions and the up-type Higgs
field come from 16 and 10-plets, respectively. The second generation fermions are generally given by some mixtures of 16 and 10-plets. Consequently the down-type Higgs is a mixture of the above $10_H$-plets and other pieces: $H_d \subset 10_H \cos \theta + (\text{other}) \sin \theta$. This leads to a GUT relation $y = y_t \cos \theta$. The Yukawa couplings solve the aforementioned flavor mixing problem in unified theory; the observed large lepton mixing is explained if $y \simeq y'$, while this does not disturb the small (left-handed) quark mixing.

Another important quantity is $x_d$: the ratio of $(Y_d)_{23}$ and $(Y_e)_{32}$. A deviation from the naive unified relation $x_d = 1$ solves the mass eigenvalue problem of down quarks and charge leptons in unified theory. The parameter $x_d$ is controlled by group-theoretical factors. Two simple limits are $x_d = 1$ and $x_d = -1/3$. The former coupling is seen as the limit of Yukawa unification. The latter one, as a renormalizable coupling, originates from a higher-rank Higgs field and is suitable for reproducing the second (and first) generation fermion masses. These two limits respectively well-capture the distinct phenomenology in the cases that $|x_d|$ is large ($\gtrsim 0.5$) and small ($\lesssim 0.5$). While $x_d$ could generally take a discrete and predictive value, we consider in this letter these limits as reasonable examples, and call $x_d = 1$ as the aligned bottom/tau case and $x_d = -1/3$ as the reduced bottom/tau case.

Without specifying the details of GUT model, we consider the twist parameter $\theta$ as free and the ratio $x_d$ as its typical values $x_d = 1$ and $-1/3$. The other couplings $y$ and $y'$ are responsible for the top quark mass and the large mixing of atmospheric neutrinos, and therefore experimentally determined. In the following analysis, the two parameters $\theta$ and $x_d$ play important roles to clarify the phenomenological and cosmological implications of flavor asymmetry.

Let us start by discussing about the third-generation fermion masses from the Yukawa matrices. The mass prediction at low-energy region depends on the couplings $\theta$ and $x_d$, and are also affected by supersymmetry-breaking parameters through low-energy threshold corrections at the decoupling of superparticles. The latter effect is known to provide an important suggestion for phenomenologically viable classes of superparticle mass spectrum.

We now evaluate the bottom quark mass and $\tan \beta$, the ratio of the two electroweak Higgs doublets, from the observed values of top quark mass $m_t^\text{pole} = 172.7$ GeV \[9\] and tau lepton mass $m_\tau^\text{pole} = 1777$ MeV \[10\]. The right-handed neutrinos are assumed to decouple at $10^{14}$ GeV and induce light neutrino Majorana masses. (For the other details of the evaluation procedure, see Ref. \[3\].) In Fig. the predictions of bottom quark mass and $\tan \beta$ are displayed as the functions of $\theta$ and $x_d$. The analysis does not include the parameter region...
Figure 1: The predictions of bottom quark mass and $\tan \beta$. The two-loop renormalization group evolution down to the electroweak scale is solved. The low-energy threshold corrections are fixed as typical values for the top quark and tau lepton masses ($\Delta_t = 0.05$ and $\Delta_\tau = -0.02$). In the yellow (green) shaded region, the correction $\Delta_b$ needs to be larger than 0.05 (smaller than $-0.05$) to have the correct mass eigenvalue.

$\theta \lesssim 40^\circ$ and $|x_d| \gtrsim 1$. This is because in the former region the electroweak breaking vacuum is unstable and the latter region is less preferable for the strange quark and muon masses. It is found from the left figure that the twist parameter $\theta$ controls a relative strength between top and bottom Yukawa couplings, thereby the value of $\tan \beta$, whereas another parameter $x_d$ is almost irrelevant to $\tan \beta$.

The bottom quark mass prediction shown in the right figure does not include the low-energy threshold correction $\Delta_b$ to the bottom quark mass. (The threshold corrections $\Delta_i$ is defined by $y'_i = y_i(1 + \Delta_i)$ and evaluated at the decoupling scale of superparticles.) It can be therefore extracted from the figure what amount of $\Delta_b$ is required, depending mainly on the parameter $x_d$. Generally speaking, a smaller value of $|x_d|$ leads to a smaller bottom/tau mass ratio and in turn needs a constructive contribution $\Delta_b > 0$. This is why we call $|x_d| \ll 1$ as the reduced bottom/tau case. In the yellow (green) shaded region in Fig. 1, the threshold correction should satisfy $\Delta_b > 0.05$ ($\Delta_b < -0.05$) in order to obtain the experimental range $m_b(m_b) = 4.2 \pm 0.07$ GeV in the MS scheme [10]. On the other hand, the $x_d = 1$ case leads to equal tree-level masses of bottom quark and tau lepton at the GUT scale, and is referred to as the aligned bottom/tau case throughout this letter. For a not so small $\tan \beta$, the correction $\Delta_b$ is dominated by finite pieces [11] and not decoupled. It has the following property; (i)
the sign of correction is equivalent to that of the supersymmetric Higgs mass parameter $\mu$, and (ii) the size of correction is suppressed if the superparticle mass spectrum is hierarchical, i.e. the scalar quarks are much heavier than the gauginos and higgsinos. These features lead to a general, important implication for superparticle mass parameters in the present model. For the reduced bottom/tau case, a non-vanishing $\Delta b > 0$ is needed, from which fact one expects that the $\mu$ parameter is positive and superparticles tend to be degenerate in mass. On the other hand, the aligned bottom/tau case makes the situation opposite, resulting in a negative $\mu$ and hierarchical superparticle spectrum, i.e. relatively heavy scalar quarks. The mass prediction of superparticles is thus found to have a correlation with the parameter $x_d$ via threshold corrections to the bottom quark mass. It is mentioned that there may be a possibility that small corrections are supplied by a cancellation among various diagrams, but we do not consider such a model-dependent case in this letter.

The next issue is to analyze the superparticle mass spectrum in order to realize the radiative electroweak symmetry breaking (EWSB) \cite{12} in addition to the third-generation fermion masses discussed above. The property of the EWSB vacuum depends on the parameters $\theta$ and $x_d$ via radiative corrections. At the GUT scale, the standard gaugino masses are $M_{1/2}$ and all scalar trilinear couplings are set to $A_0$. The Higgs mixing mass parameters are determined by solving the EWSB conditions at the electroweak scale. We assume there are no flavor off-diagonal elements in supersymmetry-breaking parameters at the boundary scale. They are therefore given by the following form (irrespectively of $x_d$):

$$m_{Q_{ij}}^2 = m_{\tilde{u}_{ij}}^2 = m_{\tilde{e}_{ij}}^2 = (m_0^2 - D) \delta_{ij},$$

$$m_{d_{1i}}^2 = m_{\tilde{L}_{1i}}^2 = m_{d_{33}}^2 = m_{\tilde{L}_{33}}^2 = m_0^2 + 3D,$$

$$m_{d_{22}}^2 = m_{\tilde{L}_{22}}^2 = m_0^2 - 2D,$$

$$m_{\tilde{\nu}_{ij}}^2 = (m_0^2 - 5D) \delta_{ij},$$

$$m_{H_u}^2 = m_0^2 + 2D,$$

$$m_{H_d}^2 = m_0^2 + (-2 \cos^2 \theta + 3 \sin^2 \theta)D,$$

where $m_{Q}^2, m_{\tilde{u}}^2, m_{\tilde{e}}^2, m_{\tilde{q}}^2, m_{L}^2, m_{\tilde{\nu}}^2, m_{H_u}^2$ and $m_{H_d}^2$ are the scalar masses for quark doublets, up-type quarks, charged leptons, down-type quarks, lepton doublets, right-handed neutrinos, up-type Higgs and down-type Higgs, respectively. We generally include the D-term contribution associated with the breakdown of unified gauge symmetry [e.g. SO(10) in the above example]. It is noticed that, while the D-term contribution comes from the gauge interaction, its effects
appear in a flavor-dependent way in $m_d^2$ and $m_e^2$. The down-type Higgs mass also deviates from the universality and depends on $\theta$. These facts are important consequences of the generation twisting, and would induce sizable flavor-violating effects, as will be discussed.

Given the boundary conditions of coupling constants, we solve the renormalization group evolution from the GUT scale to the electroweak one. The requirements for the EWSB vacuum are then expressed approximately as the predictions of physical particle masses:

\begin{align}
|\mu|^2 &\simeq -m_{H_u}^2 - \frac{M_Z^2}{2}, \\
M_A^2 &\simeq m_{H_d}^2 - m_{H_u}^2 - M_Z^2,
\end{align}

where $M_Z$ is the $Z$ boson mass. The quantity $\mu$ is related to the higgsino masses, and $M_A^2 = m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2$ is the CP-odd neutral Higgs mass. The former is given by the up-type Higgs soft mass and the latter one by the difference between up and down-type Higgs soft masses. A given set of the Yukawa parameters $\theta$ and $x_d$, the EWSB predictions are derived from the mass parameters at the GUT scale. For example,

\begin{align}
|\mu|^2 &= +0.06 m_0^2 + 1.42 M_{1/2}^2 + 0.08 A_0^2 - 0.24 A_0 M_{1/2} - 1.96 D - \frac{M_Z^2}{2}, \\
M_A^2 &= -0.11 m_0^2 + 0.12 M_{1/2}^2 - 0.00 A_0^2 - 0.02 A_0 M_{1/2} - 1.43 D - M_Z^2,
\end{align}

for $\theta = 45^\circ$ and $x_d = 1$, corresponding to $\tan \beta \simeq 51$. The coupling dependences of the numerical coefficients are found in [3].

In fact, the formula of the higgsino mass $|\mu|$ is almost insensitive to $\theta$ and $x_d$ because the EWSB prediction of $|\mu|$ is correlated only to the up-type Higgs mass and not directly connected to the down-quark and charged-lepton sectors at one-loop level. It is found from eq. (10) that a negative contribution, which make $|\mu|$ small, is obtained only through a positive $D$ term. Such a suppressed value of $\mu$ parameter is obtained in the large $|x_d|$ case for the bottom quark mass being within the experimentally allowed range. Otherwise, the higgsino mass scale becomes larger than the gaugino mass scale, and the threshold correction $\Delta_b$ is not generally unsuppressed.

Another EWSB prediction of $M_A$ has a particular response to the parameters $\theta$ and $x_d$. In the reduced bottom/tau case (with small values of $|x_d|$ and $\cos \theta$) decreases the effects of down-quark and charged-lepton Yukawa couplings in the renormalization-group evolution of down-type Higgs mass $m_{H_d}^2$. That makes $m_{H_d}^2$ larger in the infrared regime and the CP-odd neutral Higgs mass is raised. In the above formula of $M_A^2$, the coefficient of $m_0^2$ therefore increases as $\theta$ and changes its sign from negative to positive around $\theta \simeq 50^\circ$ depending on $x_d$. 

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Figure 2: The parameter space analysis for scalar and gaugino masses $m_0$ and $M_{1/2}$. The left (right) panel exhibits the result for $x_d = 1$, $\theta = 60^\circ$ and $D/m_0^2 = 0.05$ ($x_d = -1/3$, $\theta = 55^\circ$ and $D/m_0^2 = 0.2$). All the experimental mass bounds of superparticles are included. The constraints from the lightest Higgs mass, $b \rightarrow s \gamma$ and $\tau \rightarrow \mu \gamma$ rare decays are shown as the gray, green and blue shaded regions, respectively.

As will be seen, the correlation between the mixing angle $\theta$ and the CP-odd neutral Higgs mass $M_A$ is important for cosmological study of the model.

The parameter space analysis is given in Fig. 2 for the well-motivated limits of $x_d$: the aligned ($x_d = 1$) and reduced ($x_d = -1/3$) bottom/tau cases. We simply set $A_0 = 0$ and show the bottom quark mass predictions in the figures. The detail of trilinear couplings is irrelevant to our qualitative discussion. The parameter analysis includes the experimental lower bounds on superparticle masses [10], the flavor-changing decays of third-generation fermions ($b \rightarrow s \gamma$ [13] and $\tau \rightarrow \mu \gamma$ [14]), and the lightest Higgs mass bound. The general aspect of parameter space is that, (i) if scalar superparticles are much heavier than gauginos, the EWSB vacuum becomes unstable or the $\mu$ parameter is too small, and (ii) the opposite side in the figure with light scalars is disfavored in that the lightest superparticle is not charge neutral. Excepting these trivially-excluded regions, in the green-shaded area in Fig. 2 the prediction of $b \rightarrow s \gamma$ decay rate is outside the experimental result, which we conservatively take as $2.0 \times 10^{-4} < B(b \rightarrow s \gamma) < 4.5 \times 10^{-4}$ [15]. In the blue-shaded region, the $\tau \rightarrow \mu \gamma$ branching ratio does not satisfy the upper bound $B(\tau \rightarrow \mu \gamma) < 6.8 \times 10^{-8}$ (at 90% C.L.) [16]. Finally the gray-shaded area leads to the lightest Higgs mass being lower than the experimental lower bound 114.4 GeV [17], and hence excluded.
It is interesting to note that the constraints from flavor-violating rare decays are rather different depending on the Yukawa parameters $\theta$ and $x_d$. In the aligned bottom/tau case with a large $|x_d|$ (the left panel in Fig. [2]), the $b \to s\gamma$ constraint is important in a wide region of parameter space with light superparticle spectrum. This is mainly due to the negative sign of $\mu$ parameter imposed by the analysis of bottom quark mass (see the right panel of Fig. [1]). Since the standard model prediction of $\mathcal{B}(b \to s\gamma)$ is consistent with the observed value, supersymmetric contribution should be suppressed. For $\mu < 0$ as in the usual Yukawa unification models with universal supersymmetry-breaking parameters, relatively heavy superparticles are required to suppress the amplitude, as seen in Fig. [2] (the green-colored area in the left panel). On the other hand, the reduced bottom/tau case with a small $|x_d|$ predicts a positive $\mu$, which sign makes a cancellation among various diagrams operative, and the $b \to s\gamma$ constraint is not effective. In addition, a positive $\mu$ is suitable for a supersymmetric resolution to the observed anomalous magnetic moment of the muon [18]. The lepton flavor violating decay $\tau \to \mu\gamma$ is however found to exclude a large portion of parameter space in the reduced bottom/tau case (the blue-colored area in the right panel). A source of flavor violation comes from the D term which induces generation-dependent scalar lepton masses (3) and (4). As we mentioned, a reduced bottom/tau ratio allows a small $m_0$, leading to a large $|D|/m_0^2$, and the $\tau \to \mu\gamma$ branching ratio is amplified.

The distinctive types of superparticle spectra for the aligned and reduced bottom/tau cases lead to different properties of the lightest super particle (LSP). In the present model, LSP is the lightest neutralino $\chi$ which is a linear combination of neutral gauginos $\tilde{B}$, $\tilde{W}$ and higgsinos $\tilde{H}_u$, $\tilde{H}_d$:

$$\chi = a_B \tilde{B} + a_W \tilde{W} + a_u \tilde{H}_u + a_d \tilde{H}_d.$$  \hspace{1cm} (12)

We find that the LSP is a mixture of gauginos and higgsinos in the allowed parameter region for the aligned bottom/tau case, while it becomes almost a bino for the reduced bottom/tau case.

The LSP nature provides important information of the cold dark matter in our universe. Assuming the R parity conservation as in the usual supersymmetric standard models (to avoid too rapid nucleon decays), the lightest neutralino $\chi$ is a suitable candidate for the cold dark matter. The recent data brought out from the WMAP satellite gives a strong limit on the relic density of the lightest neutralino as dark matter [19]. The central value of the dark matter density extrapolated from the WMAP three years data is $\Omega_{\text{CDM}}h^2 = 0.1045$ [20]. It has been
known that a theoretical prediction of neutralino relic density $\Omega_\chi$ tends to much exceed $\Omega_{\text{CDM}}$, in particular for a bino-like LSP in the minimal supersymmetric standard model. In order to suppress the predicted value of $\Omega_\chi$, the annihilation cross section of the lightest neutralino must be enhanced. In our scenario, there are two ways to have the enhancement: The first option is to increase the amplitude $\chi\chi \rightarrow W^+W^-$ through the t-channel chargino exchange, which requires a non-negligible component of higgsinos in the LSP. The LSP inclusion of higgsino components needs a small size of the $\mu$ parameter. The second one is to enhance the annihilation into pairs of standard-model fermions through the resonance near the CP-odd neutral Higgs boson. This can be achieved with a tuned parameter region $2m_\chi \simeq M_A$.

Figs. 3 displays the predictions of neutralino relic density $\Omega_\chi$ on the allowed parameter spaces described in Fig. 2. For the aligned bottom/tau case with a large $|x_d|$ (= 1 in the left panel), both of the options are available: the relic density is suppressed around the upper boundary of the correct EWSB region where the $\mu$ parameter becomes small for $\Delta_b$ and the dark matter somehow contains light higgsino components (we will see later what amount of higgsinos is predicted). Another region is near $m_0 \simeq M_{1/2}$, in which the LSP suppression is supplied by the second option, that is, the CP-odd Higgs resonance. Around these two regions, the cosmologically viable dark matter density $\Omega_{\text{CDM}}h^2 \simeq 0.1$ is explained by the LSP neutralino. It is however noticed that the CP-odd Higgs boson resonance is incompatible with other experimental data of the bottom quark mass and/or the flavor-changing rare decay $b \rightarrow s\gamma$. 
For the reduced bottom/tau case with a small $|x_d|$ ($= 1/3$ in the right panel of Fig. 3), the neutralino relic density has rather different behavior from the aligned bottom/tau case. We find that only the second option, the CP-odd Higgs resonance, is available to avoid the overproduction of the LSP neutralino. This is because, as mentioned before, a small value of $|x_d|$ reduces the bottom/tau mass ratio and requires a non-negligible (positive) threshold correction $\Delta_b$, which in turn imposes a not-so-small $\mu$ parameter producing non-negligible higgsino components in the lightest neutralino. Therefore the resonant condition $2m_\chi \sim M_A$ should be realized. It is remind of the EWSB fitting formula shown above that, for some suitable value of the mixing parameter $\theta$, the CP-odd neutral Higgs mass $M_A$ has a slight dependence of scalar soft mass $m_0$. In this case, the gaugino mass $M_{1/2}$ determines $M_A$ and also the (almost bino) LSP mass. Therefore they are naturally correlated to each other and the resonant condition is realized. This fact can be seen in Fig. 3 (the right panel) that the relic abundance of the LSP neutralino does not largely depend on the universal scalar mass $m_0$. We checked that the prediction of CP-odd neutral Higgs mass is also insensitive to $m_0$, and the relation $2m_\chi \sim M_A$ is approximately achieved in most of the parameter space.

Finally, to clarify the distinctive classes of mass spectrum and LSP nature discussed above, we explore a wider parameter space by varying the mixing parameter $\theta$ and the D term. The analysis is done for fixed values of $x_d$ which is expected to be given by somewhat discrete group-theoretical factor.

For the aligned bottom/tau case with a large $|x_d|$, heavier scalar superparticles than gauginos are consistent with the observed bottom quark mass and $b \to s\gamma$ branching ratio. The lightest neutralino contains a sizable amount of higgsino components and the relic abundance is suppressed. To see this, we introduce the gaugino fraction of the lightest neutralino which is defined as $R_\chi \equiv |a_B|^2 + |a_W|^2$. For a large value of $|x_d| \gtrsim 0.5$, a reduced $R_\chi$ is consistent with the observed bottom quark mass. Fig. 4 (the left panel) shows the prediction of LSP relic density for the aligned bottom/tau case with $x_d = 1$. In the figures, the experimental results for superparticles, the $b \to s\gamma$ ratio, the masses of third-generation fermions and Higgs bosons are taken into account. As examples, the mass parameters are set to be $M_{1/2} = 500$ GeV and $m_0 = 4$ TeV. The mixing parameter $\theta$ and D term are scanned. The $\mu$ parameter is negative and heavy scalar superparticles are compatible with the $b \to s\gamma$ constraint. It is seen from the figure that there is a relation between the gaugino fraction and the dark matter density. To explain the WMAP data, the gaugino fraction is restricted as $R_\chi \lesssim 0.8$, that is, the lightest neutralino is given by a mixture of bino and higgsinos. This fact confirms the previous result.
that the abundance of LSP neutralino is suppressed by the χ fusion.

The scanned parameters θ and D are limited by various experimental results. For the aligned bottom/tau case with a large value of |x_d|, θ is bounded from below by the b → sγ constraint as 55° ≲ θ. This is converted to a moderate value of tan β from the argument of Δ_b, which results in 38 ≲ tan β ≲ 48. The mixing parameter θ also has an upper bound, θ ≲ 65°, from a cosmological reason by demanding Ω_χ ≲ Ω_{CDM}. The allowed range of D term is given by 0.02 ≲ D/m_0^2 ≲ 0.06. A positive D term decreases |μ| [see eq. (10)] and the upper bound of D is given by the EWSB conditions or the mass bounds for the lightest chargino and neutralino. As we have shown, heavy scalar superparticles are required from b → sγ. Therefore the τ → μγ decay rate is sufficiently small and difficult to be experimentally observed.

Another limit is the reduced bottom/tau case with x_d = −1/3. For such a small value of |x_d|, superparticle masses tend to be degenerate in low-energy regime. The CP-odd neutral Higgs mass is naturally correlated to that of the lightest neutralino within an appropriate range of θ. The LSP is almost bino-like and the gaugino fraction R_χ becomes almost unity. Fig. 4 (the right panel) shows the prediction of LSP relic density as a function of the τ → μγ branching ratio in the reduced bottom/tau case with a typical example x_d = −1/3. All experimental constraints are included while the parameters θ and D are scanned. We present
the results for two parameter examples: \( m_0 = 800 \text{ GeV} \) and \( M_{1/2} = 500 \text{ GeV} \) (crosses) \( m_0 = 1 \text{ TeV} \) and \( M_{1/2} = 500 \text{ GeV} \) (dots). It is seen from the figure that the neutralino relic density is suppressed around the dips where the annihilation amplitude is enhanced through the CP-odd neutral Higgs resonance; \( 2m_\chi \simeq M_A \). The resonance condition is rather independent on scalar masses, and therefore the flavor violation is easily made within the current experimental bound.

The scanned regions of \( \theta \) and D are limited in the cosmologically allowed range. To keep the relation \( 2m_\chi \sim M_A \), we find \( \theta \) in the range \( 45^\circ \lesssim \theta \lesssim 62^\circ \). This corresponds to \( 43 \lesssim \tan \beta \lesssim 53 \), which is larger than the aligned bottom/tau case. The D term is constrained to a range \( 0.03 \lesssim D/m_0^2 \lesssim 0.4 \) by the observed bottom quark mass and the requirement for dark matter being charge neutral. Thus in the reduced bottom/tau case, \( \tan \beta \) and D become larger than the aligned bottom/tau case. That is a general property for \( |x_d| \lesssim 0.5 \).

It is interesting to note that flavor-changing transitions are important in the reduced bottom/tau case. This is due to the following two reasons. The first is the light mass spectrum of scalar superparticles than that in the aligned bottom/tau case. Another is the generation-dependent D-term contribution to scalar masses, which leads to the non-degeneracy in the down/charged lepton sectors \( [3] \) and \( [4] \). As seen before, a smaller \( |x_d| \) (the reduced bottom/tau case) predicts a larger value of D term (and a larger \( \tan \beta \)), and enhances the flavor-violating decay amplitudes. The \( \tau \to \mu \gamma \) branching ratio is shown in the right panel of Fig. 4. The current experimental upper limit \( B(\tau \to \mu \gamma) < 6.8 \times 10^{-8} \) gives a lower bound of superparticle masses. For \( m_0 \simeq 1 \text{ TeV} \), the decay rate becomes smaller than the current upper limit and is within a range of near future experiments.

As a final remark, we consider the dileptonic rare decay of B meson \( B^0 \to \mu^+ \mu^- \) recently studied. For a large value of \( \tan \beta \), the branching ratio approximately scales as \( \tan^6 \beta/M_A^4 \) \([21]\). The experimental upper bound of the ratio gives a constraint for model parameters unless rather heavy CP-odd neutral Higgs boson is assumed. In our scenario, \( \tan \beta \) is relatively large and the predicted branching ratio is expected to be. In particular, for the reduced bottom/tau case, the CP-odd neutral Higgs boson is allowed to have a similar mass to the LSP for suppressing the neutralino relic density. As a result, the gaugino mass parameter \( M_{1/2} \) would be rather restricted. For the aligned bottom/tau case, the \( b \to s\gamma \) constraint requires TeV-scale CP-odd neutral Higgs boson and the constraint from dileptonic decay is not severe.

In summary, we have studied the supersymmetric grand unified theory with the Yukawa...
coupling form (1). This flavor structure is suggested by the recent experimental data of fermion masses and mixing angles. The two parameters \( \theta \) and \( x_d \) in the flavor sector are important and control the low-energy property of the model. These parameters are related to supersymmetry-breaking threshold corrections to the bottom quark mass and the successful electroweak symmetry breaking. In this letter we have investigated the flavor-changing rare decays of heavy fermions and also the LSP neutralino relic as dark matter of the universe. For the aligned bottom/tau case with \( x_d \sim \mathcal{O}(1) \), a hierarchical superparticle spectrum is obtained; heavier scalars than gauginos and higgsinos. The lightest neutralino contains a significant amount of higgsino components. We have found that the reduced bottom/tau case with a small \( |x_d| \) is particularly interesting to low-energy phenomenology: (i) the superparticles are light and degenerate in masses, like a recent analysis in [22]. (ii) the \( \mu \) parameter becomes positive, which provides a simultaneous supersymmetric solution to the experimental data of bottom/tau mass ratio, the \( b \to s\gamma \) rare decay, and the muon anomalous magnetic moment. (iii) a gaugino-like LSP is expected and the CP-odd neutral Higgs boson is as light as the LSP for explaining the dark matter density of the universe, and (iv) finally the D term is significant for the dark matter and the lepton flavor violation. Its contribution to scalar masses is generation dependent due to the lopsided flavor structure. These features induces distinct and detectable signatures in low-energy phenomenology and future searches for supersymmetry will probe the validity of the scenario.

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