Determination of shear forces inside the proton

V.D. Burkert,¹ L. Elouadrhiri,¹, ² and F.X. Girod², ³

¹Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA
²Center for Nuclear Femtography, SURA, 1201 New York Ave. NW, Washington, DC 20005, USA
³Department of Physics - The George Washington University 725 21st Street, NW, Washington, DC 20052

(Dated: April 12, 2021)

We report on the first determination of the shear forces on quarks inside the proton from experimental data on deeply virtual Compton scattering. The maximum shear force of $40 \pm 20$ MeV fm$^{-1}$ occurs near 0.6 fm from the proton center, indicating where confinement forces may be strongest. On the macroscopic scale of the earth surface, this force corresponds to the weight of a mass of $\approx 650$ kg. The shear forces in the proton reverse direction at $r \approx 0.45$ fm from the center.

PACS numbers:

Protons and neutrons, generally referred to as nucleons, are the fundamental building blocks of nuclei and make up nearly 100% of the mass of ordinary matter in the universe. They are composed of elementary objects, quarks and gluons. The latter are the carrier of the strong force. The distribution of quarks inside the proton is governed by the strong interaction as described by the theory of strong interaction, Quantum-Chromodynamics (QCD).

The internal mass and energy distributions, the forces on the quarks, and the angular momentum distribution inside the proton are largely unknown. These properties are encoded in the proton’s matrix element of the energy-momentum tensor [1, 2] and are expressed in gravitational form factors (GFFs) [3]. For decades the GFFs were considered unmeasurable due to the extreme weakness of the gravitational interaction. More recent theoretical development however showed that the GFFs may be indirectly probed in deeply virtual Compton scattering (DVCS) [4, 5]. DVCS allows for the extraction of the internal proton structure expressed in the generalized parton distributions (GPDs) [3, 6], and are the basis for the exploration of its gravitational properties [7]. This new direction of nucleon structure research has recently resulted in the first estimate of the pressure distribution inside the proton based on experimental data [8]. In this paper we report the first results of the tangential (shear) forces and their spatial distribution inside the proton.

The critical part is the $D$-term, which represents the last unknown global property of the proton which, until recently, has remained unconstrained. In previous work the corresponding form factor $D(t)$ was determined in a range of 4-momentum-squared $-t$, and used to study the pressure distribution inside the proton. In this letter we employ $D(t)$ to determine the forces on the quarks in the proton, using the DVCS process as an effective proxy of the gravitational interaction. We first summarize the steps involved in this process.

The basic process is the handbag diagram shown in Fig. 1 in leading twist approximation. All particles involved, the scattered electron, the emitted photon and the recoil proton are measured in coincidence to establish the exclusive DVCS process. The two high-energy photons each with spin $J_\gamma = 1$ that couple to the same quark in the proton, have the characteristics as a single graviton of spin $J_G = 2$ coupling to the quark, when integrated over the quark propagator.

![FIG. 1: Left graph: The graviton-proton interaction as a probe of the gravitational form factors. Gravity is a spin $J_G = 2$ tensor coupling. Right graph: The graviton-proton coupling is mimicked with the two $J_\gamma = 1$ photon vertices in the leading handbag diagram in DVCS when integrating over the quark propagator.](image-url)
into the systematic uncertainties of the final results.

The GPD \( H \) is directly mapped to the gravitational form factors \( d_1(t) \) and \( M_2(t) \) in a sum rule \( [4] \) involving the second Mellin moment of the GPD \( H \):

\[
\int dx x H(x, \xi, t) = M_2(t) + \frac{4}{5} \xi^2 d_1(t),
\]

where the GFF \( d_1(t) \) encodes the distribution of shear forces on the quarks and the pressure distribution in the proton. Ideally, one would determine the integral by measuring GPD \( H \) in the entire \( x \) and \( \xi \) space in a large range of \( t \). Given the current state of the DVCS experiments, such an approach is impractical as the GPDs are not directly accessible in experiment in the full \( x \)-space, but only at \( x = \pm \xi \). We therefore proceed with a more phenomenological approach and express the GPD \( H(x, \xi, t) \) in terms of the Compton Form Factor \( \mathcal{H}(\xi, t) \) through the convolution integral defined as

\[
\text{Re}\mathcal{H}(\xi, t) + i\text{Im}\mathcal{H}(\xi, t) = \int_0^1 dx \left[ \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right] H(x, \xi, t),
\]

where we have traded the real function of 3 parameters \( H(x, \xi, t) \) with the complex function of 2 parameters \( \text{Re}\mathcal{H}(\xi, t) \) and \( \text{Im}\mathcal{H}(\xi, t) \), which can be related more directly to experimentally accessible observables.

\[\text{FIG. 2: The form factor } D(\xi) \text{ as determined in the fit to the DVCS data. The hatched area represents the magnitude of the estimated systematic uncertainties.}\]

This analysis uses experimental data of DVCS with polarized electron beams that have been measured using the CLAS detector at Jefferson Laboratory in limited kinematics of \( \xi \) and \( t \). The imaginary part and the real part of \( \mathcal{H}(\xi, t) \) are extracted in local fits to the CLAS beam-spin asymmetry data \( [9] \) and to the cross section data \( [10] \), respectively. In order to cover the full \( \xi \) range we use parameterizations \( [16, 17] \) that were obtained from global fits to world data. The global and local fits show good agreement in \( \xi \) and \( t \) kinematics where they overlap.

\[\text{Im}\mathcal{H}(\xi, t) \text{ and } \text{Re}\mathcal{H}(\xi, t) \text{ are related through a dispersion relation at fixed values of } t.\]

\[\text{Re}\mathcal{H}(\xi, t) \overset{\text{lo}}{=} D(t) + \mathcal{P} \int_0^1 dx \left[ \frac{1}{\xi - x} - \frac{1}{\xi + x} \right] \text{Im}\mathcal{H}(\xi, t),\]

In this approach, we parameterized both \( \text{Im}\mathcal{H}(\xi, t) \) and \( D(t) \) and fitted these parameters directly to the data, with the dispersion relation included in the fit. Therefore we only have a few parameters adjusted to the entire available kinematic space. The parameterization provides strong constraints, which is currently the only way to obtain physical results given the status of the experiments. The systematic uncertainties inherent in this approach are carefully evaluated and are included in the results.

From the global fit we determine \( D(t) \) for each value of \( \xi \). \( D(t) \) is directly related to \( d_1(t) \) in eq. \( [1] \) and to the mechanical properties of the proton \( [15, 18, 19] \). The GFF \( d_1(t) \) appears as the first coefficient in the expansion of the \( D(t) \)-term in terms of Gegenbauer polynomials in \( \xi \). Details of the global fit and the extraction of \( D(t) \) from the experimental data are discussed in a forthcoming article \( [25] \).

In Fig. \( [2] \) the results of the \( D(t) \) form factor extraction are displayed, and the fit to the multipole form:

\[D(t) = D \left[ 1 + \frac{t}{M^2} \right]^{-\alpha},\]

\[\text{where } D, \alpha \text{ and } M^2 \text{ are the fit parameters. Our fits result in the following parameters:}\]

\[D = -1.47 \pm 0.06 \pm 0.14 \quad (3)\]

\[M^2 = +1.02 \pm 0.13 \pm 0.21 \text{ GeV}^2 \quad (4)\]

\[\alpha = +2.76 \pm 0.23 \pm 0.48 \quad (5)\]

\[\text{where the first error is the fit uncertainty, and the second error is due to the systematic uncertainties.}\]

\[\text{Approaches that do not rely on parameterizations or other constraints, such as techniques used in } [23, 24], \text{ have so far not been successful in determining mechanical properties of the proton from data.}\]

Next we relate \( D(t) \) to the static energy-momentum tensor (EMT) components. The shear forces \( s(r) \) and the pressure distribution \( p(r) \) are related to the \( ij \) components of the static energy momentum tensor as:

\[T^{ij}(r) = \left( r^i r^j - \frac{1}{3} \delta^{ij} \right) s(r) + \delta^{ij} p(r) \quad (6)\]
FIG. 3: The distribution of the shear forces $s(r)$ in the proton. The middle solid line represents the fit result. The outer blue-shaded area marks the range of uncertainties when only data prior to the CLAS data are included. The middle (light-green) areas are based on the CLAS data, and the inner (red) area represent projections when expected results from the ongoing and planned experiments are included in the fits. The widths of the bands are dominated by systematic uncertainties in parameterizations used in the integration. The dashed black curve is a model prediction [11].

The shear forces are defined separately for quarks and gluons. Therefore by using $D(t)$ from the DVCS process the shear forces $s(r)$ and normal pressure $p(r)$ are separately determined for the quark content of the proton. In the Breit-frame they are expressed through the integral [11]:

\[
\begin{align*}
    s(r) &= -\frac{1}{4m^2} \frac{d}{dr} \left( \frac{d}{dr} r \frac{d}{dr} \tilde{D}(r) \right) \\
    p(r) &= \frac{1}{6m^2} \frac{d}{dr} \left( \frac{d^2}{dr^2} r \frac{d}{dr} \tilde{D}(r) \right) \\
    \tilde{D}(r) &= \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\Delta \cdot r} D(-\Delta^2).
\end{align*}
\]

Where $\Delta$ is the 3-momentum transfer to the proton. We neglect contributions from the GFF $\tilde{c}$ [11] which would cancel with the gluons in the total Energy Momentum Tensor. Further extensions and discussions of other reference frames are included in [12].

When carrying out the integral over the entire kinematical range, we extract $s(r)$ and $p(r)$. Fig. 3 shows the results for shear stress. This result represents the first effort in determining the shear forces in the proton using the experimental DVCS process and its relation to the GFFs. While our results still have significant systematic uncertainties due to the limited kinematic range covered in the DVCS data, they lead to interesting conclusions. The shear forces in the proton are tangential to the surface, and they change their direction near 0.45 fm radial distance from the center. The maximum shear force of $40 \pm 20$ MeVfm$^{-1}$ occurs near 0.55 fm from the proton center, indicating the location where confinement forces may be dominant. On the macroscopic scale of the earth
surface, this force corresponds to the weight of a mass of 650 kg. A 2-dimensional display of both the normal and shear forces is presented in Fig. 4. We want to stress that the arrows represent the forces acting along the orientation of the surface. So if one changes the direction of the normal to the surface one also changes the arrow direction, so that pressure acts equally on both sides of a surface immersed in the system.

Shear forces in the proton have been computed in models 11, 12, 13, 14, 20, 26, 27. New experiments are currently underway 29 to measure the DVCS process with much extended kinematical coverage and higher statistical precision. This should lead to significantly reduced systematic uncertainties in the extraction of the gravitational properties of protons and neutrons, including their shear forces, as indicated in Fig. 3 as the innermost (red) shaded area.

I. ACKNOWLEDGMENTS

We are grateful for useful comments by Cedric Lorce and Maxim Polyakov. The material discussed in the paper is based on research supported by the US Department of Energy, Office of Science, Office of Nuclear Physics under contract DE-AC05-06OR23177.

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