Kinematic Analysis of Underactuated Robotic Finger Design

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Abstract. This paper presents a kinematic analysis of a novel underactuated robotic finger design. The design finger is a development of an index finger of the Ottobock hand. Namely, it consists of three phalanges with 3-degrees of freedom. A four-bar mechanism was used to make the finger self-adaptive with the grasped object. The Solidworks software was used to create the design, and the ANSYS software was used to analyze the design. The kinematic equations of the novel design are derived to get the optimum values of the links dimension that achieved the optimum grasping force by using the genetic algorithm. The normal force was measured by using the grasping force measuring mechanism. The models were manufactured using a 3D printer with hard Polylactic acid (PLA) printing material. The contact points' normal force between phalaxxes and grasping force measuring mechanism was measured using a load cell. The experimental results of the normal force of the finger were closed to the theoretical results.

Keywords. Robotic finger, Underactuated, Four-bar linkages, Design optimization, Genetic algorithm.

1. Introduction

The hand is one of the essential parts of the human body, and it is useful for performing various functions in everyday life. Having a disabled limb means loss of hand function. Therefore, restoring these functions represents the primary goal of research by designing advanced machines that help amputees reduce effort and time [1]. In this field, the typical products of the prosthetic hand depend on the two basic types of the driving mechanism, which are tendon drive and connecting rod drive [2]. The tendon drive was developed by Stanford University [3], while the connecting rod motor was developed by Iowa State University, USA [4]. Low cost, high performance, simple design [5], aesthetics, and low weight [6] definitely make the prosthesis get acceptable reliability for users and very promising for future development.

M. C. Carrozza et al. [7] introduced Cyber-Hand that offers 16 degrees of freedom and 6 operations (i.e., 6-degrees of mobility): Each Cyber-Hand finger has three degrees of freedom (flexion/extension). The size of the Cyber-Hand can be compared to human hands and can generate many different versions and control is limited to a subset of the functional handles: side grip, cylindrical and ball grip, and tripod. The five-finger bending triggers are placed in a groove and occupy a total volume of 250 cc, while the stroke actuation is in the palm of the hand. The palm consists of an outer shell made of carbon fiber, divided into dorsal and lateral parts, and an inner frame, which holds the fingers and contains the thumb mechanism.
F. Lotti et al. [8] developed a prototype hand updated with a standard structure with four identical fingers that are fused, and thumb is attached to the wrist frame, which will be attached to a wrist. The dimensions of the suggested hand are very similar to the human hands. A compatible layer, which reproduces the role of the human soft tissue, covers the internal structure. The internal structure is designed according to the concept of a "compatible mechanism" so that the phalanx movement is obtained using flexible connectors. Corresponding elements are made of closely wound coil springs that bend under the influence of tendon pull. A limited number of coils suffice to obtain a large displacement while avoiding permanent deformations and flexion phenomena.

C. Meijneke et al. [9] described the main requirements for designing the Prototype (DH-2) and the design options designed to meet these requirements. The model is used to improve the performance. In DH-2, the operating torque is distributed between the fingers and the phalanges by differential mechanisms. The transmission and gears were differential in the (DH-2) to distribute torque to the three fingers to ensure durability. Each finger consists of phalanges that are driven by a four-bar linkage mechanism. To prevent objects from sticking between joints, the finger parts are designed to form a closed shell. The power or position sensor was not used to reduce the required mechanical and electronic parts and achieved a cheap and straightforward design. The fingers can also be open and close using the MAXON RE13 3.5W with GP13 67: 1 gearbox and motor 24 volts and give a constant torque applied to the base of the finger and 0.5 Nm to each of the corresponding fingers.

Nikolai Dechev [10] presents a three-part finger design because the human finger has three parts, known as phalanges. Designing the third clip. Each finger consists of a prototype of six links; All four fingers are precisely the same, allowing for ease of manufacture and ease of service due to interchangeable parts. The links (1, 2, and 3) correspond to the three parts within the typical finger. Links (4, 5, and 6) are the links that drive the first three links' movement.

The finger is driven by a driving link (6) left for the extension and right for bending through a pin that passes through the hole on its right end. In this research, the prosthetic hand (Otto bock hand) was developed by making the finger consist of three phalanges. Namely, each finger will have 3 degrees of freedom to give it the ability to embrace things using the four-column mechanism in each phalanx. The equations governing the proposed design were derived to achieve optimal bond dimensions using the genetic algorithm. Then, the pattern is printed according to the optimal dimensions obtained. The model is assembled with the special mechanism provided by the sensors to measure the contact forces in each battalion and compare them with the theoretical results.

2. Mechanism of robotic finger
In this research, Figure (1) illustrates the design of a robotic finger mechanism similar in shape and function to the human hand fingers. The robotic finger mechanism has three degrees of freedom and consists of three phalanges connected by three joints. Each hinge has a torsional spring that is used for the return. The 1st phalanx and the 2nd phalanx are attached to the second joint, and the 2nd phalanx is directed to wrap around link 2 when the object touches the link. To wrap around the third joint when the object touches the link. These turns give the finger the ability to adopt different shapes and materials such as eggs, fruits, mugs, etc. Figure (1) illustrates that the robotic fingers' design similar in shape and function to the human hand fingers.
3. Robotic finger analysis
To find the grasping force in each phalanx, as shown in Figure (2), the mathematical equations for the proposed design are derived during the grasping process as follows:

Figure (3) shows a schematic of the finger with the four-bar mechanism used to make the finger self-adaptive around the grasped body. Table (1) shows the list of symbol definitions.
**Figure 3.** The mechanism analysis of the index finger.

**Table 1.** Symbols definitions.

| Symbol          | Definition                                                                 | Unit      |
|-----------------|---------------------------------------------------------------------------|-----------|
| $\theta_1$, $\theta_{12}$ | The angles of rotating for the proximal, middle, and the distal phalanges, respectively | Deg.      |
| $\theta_2$      | The angle of the second phalanx with link.2                               | Deg.      |
| $\phi_1$        | The angle between link 2 and the vertical axis                            | Deg.      |
| $\phi_2$        | The angle between link 4 and the vertical axis                            | Deg.      |
| $\mu$           | The coefficient of the friction                                            |           |
| $F_1^p, F_1^t$  | The normal and the tangential forces, respectively                         | N         |
| $F_a$           | Force of actuator                                                         | N         |
| $T_{\text{index}}, T_2, T_3$ | The torque of first, second, and third phalanges, respectively | N.mm      |
| $K_2, K_3$      | The torsional spring stiffness in second and third joint angles, respectively | N.mm/rad  |
| $K_1$           | The tension spring stiffness in the first joint angle                      | N/mm      |
| $l_1, l_2, l_3$ | The lengths of the proximal, middle, and distal phalanges, respectively   | mm        |
| $x_1$           | The location of normal force on link 2                                    | mm        |
| $x_2$           | The location of the normal force on link.4                                 | mm        |
| $x_3$           | The location of the normal force on the third phalanx                      | mm        |
| $t$             | The radius of the finger                                                   | mm        |
| $a$             | The distance from the second joint to the fixing point of link 1           | mm        |
| $b$             | The distance from the second joint to the fixing point of link 2           | mm        |
| $c$             | The distance from the third joint to the fixing point of link 4            | mm        |
| $d$             | The distance from the third joint to the fixing point of link 3            | mm        |
| $l_a, l_b, l_c, l_d$ | The length of link 2, link 1, link 4, and link 3, respectively         | mm        |
The applied force by a phalanx on the rigid body surface. According to the contact of the rigid body properties, the external forces affected by the center of the grasped object can be determined. After ignoring the relative velocity of the fingertips, the grasping force components (as shown in Figure (3)) have been calculated as follows:

\[
\begin{align*}
\phi_1 &= \theta_1 - \theta_{12} \\
\phi_2 &= \theta_{12} + \theta_{23} + \theta_2 \\
F_1^n &= \mu. F_1^n \\
F_2^n &= \mu. F_2^n \\
F_3^n &= \mu. F_3^n
\end{align*}
\]

The equilibrium about the first joint for the first, second, and third phalanges with simplification produces the following equation:

\[
F_1^n = \frac{T_{\text{index}} - \alpha. F_2^n - \beta. F_3^n}{U}
\]

Where:

\[
U = l_1(\cos(\theta_1 + \mu. \sin(\phi_1)) + b(\cos(\theta_1) + \mu. \sin(\theta_1))) - x_1
\]

\[
\alpha = l_1(\cos(\phi_2 - \mu. \sin(\phi_2)) + l_2(\cos(\theta_{12} - \phi_2) + \mu. \sin(\theta_{12} - \phi_2)) + c(\cos(\theta_2) + \mu. \sin(\theta_2))) - x_2
\]

\[
\beta = l_1(\cos(\theta_{12} + \theta_{23}) + \mu. \sin(\theta_{12} + \theta_{23})) + l_2(\cos(\theta_{23}) + \mu. \sin(\theta_{23})) + l_3
\]

From the equilibrium relations, the normal force component at the second and third phalanges (see Figure (4)) can be determined as follows:

\[
F_2^n = \frac{T_2 + K_2. \theta_{12} - R. F_3^n}{S}
\]
Where:
\[ R = l_2 (\cos(\theta_{23}) + \mu \sin(\theta_{23})) + l_3 \]
\[ S = l_2 (\cos(\theta_2 - \theta_{23}) + \mu \sin(\theta_2 - \theta_{23})) + c(\cos(\theta_2) + \mu \sin(\theta_2)) - x_2 \]

In the same way, the normal force component for the third phalanx (see Figure (5)) is equal to:

\[ F^n_3 = \frac{T_3 - K_3 \cdot \theta_{23}}{x_3 + \mu \cdot t} \quad (5) \]

The transmission torque from the 1st phalanx to the 2nd phalanx can be calculated using Equation (6), as shown in Figure (6).

\[ T_2 = F^n_1 [(x_1 - b \cdot \cos\theta_1) - \mu \cdot b \cdot \sin\theta_1] \quad (6) \]

The transmission torque from the 2nd phalanx to the 3rd phalanx can be calculated using Equation (7), as shown in Figure (7).

\[ T_3 = F^n_2 [(x_2 - c \cdot \cos\theta_2) - \mu \cdot c \cdot \sin\theta_2] \quad (7) \]
Figure 7. Schematic of the torque transmissions for the second stage.

By substituting equation (5, 6, and 7) into the equation (4):

\[ F_2^n = \frac{W \cdot F_1^n + K_2 \cdot \theta_{12} + \frac{R \cdot K_3 \cdot \theta_{23}}{x_3 + \mu \cdot t}}{S + \frac{N \cdot R}{x_3 + \mu \cdot t}} \]  

(8)

Where:

\[ W = [x_1 - b \cdot \cos(\theta_1) - \mu \cdot b \cdot \sin(\theta_1)] \]

\[ N = [x_2 - c \cdot \cos(\theta_2) - \mu \cdot c \cdot \sin(\theta_2)] \]

By substituting Equations (6 and 8) into Equation (5) will get:

\[ F_3^n = \frac{W \cdot N \cdot F_1^n + N \cdot K_2 \cdot \theta_{12} + \frac{N \cdot R \cdot K_3 \cdot \theta_{23}}{x_3 + \mu \cdot t}}{S + \frac{N \cdot R}{x_3 + \mu \cdot t}} - K_3 \cdot \theta_{23} \]  

(9)

By substituting equation (8 and 9) into equation (3) will get:

\[ T_{index} + \frac{K_2 \cdot \theta_{12} \cdot \pi}{180 \cdot S + \frac{R \cdot N}{x_3 + \mu \cdot t}} \left( \alpha + \frac{\beta \cdot N}{x_3 + \mu \cdot t} \right) - \frac{R \cdot K_3 \cdot \theta_{23} \cdot \pi}{180 \cdot S + \frac{R \cdot N}{x_3 + \mu \cdot t}} \left( \alpha + \frac{\beta \cdot N}{x_3 + \mu \cdot t} \right) \]  

\[ F_1^n = \frac{U + \frac{W \cdot R \cdot N}{S + \frac{R \cdot N}{x_3 + \mu \cdot t}} \left( \alpha + \frac{\beta \cdot N}{x_3 + \mu \cdot t} \right)}{U + \frac{W \cdot R \cdot N}{S + \frac{R \cdot N}{x_3 + \mu \cdot t}} \left( \alpha + \frac{\beta \cdot N}{x_3 + \mu \cdot t} \right)} \]  

(10)
From the geometry of the linkage mechanism shown in Figure (8) for the first stage, the following geometrical relations can be written as follows:

\[
\begin{align*}
\theta_1 &= 2 \cdot \tan^{-1} \left( \frac{-B_{11} \pm \sqrt{B_{11}^2 - 4 \cdot A_{11} \cdot C_{11}}}{2 \cdot A_{11}} \right) \\
\end{align*}
\]

(11)

Where:

\[
\begin{align*}
A_{11} &= (1 - v_{12}) \cdot \cos(\theta_{12}) + v_{13} + v_{11} \\
B_{11} &= -2 \cdot \sin(\theta_{12}) \\
C_{11} &= v_{11} + v_{13} - (1 - v_{12}) \cdot \cos(\theta_{12}) \\
\end{align*}
\]

\[
\begin{align*}
v_{11} &= \frac{-b}{a}, v_{12} = \frac{-b}{l_a} \text{ and } v_{13} = \frac{l_a^2 - l_b^2 + a^2 + b^2}{2 \cdot a \cdot l_a}
\end{align*}
\]

\[l_A: \text{Length of link } A, \quad l_B: \text{Length of link } B\]

Also, for the second stage, the geometry relations of the linkage mechanism shown in Figure (9) can be written as follows:

\[
\begin{align*}
\end{align*}
\]

Figure 8. Schematic of the four-bar linkage mechanism for the first stage.

Figure 9. Schematic of the linkage mechanism for the second stage.
\[
\theta_2 = 2 \times \tan^{-1}\left(\frac{-B_{22} \pm \sqrt{B_{22}^2 - 4 \cdot A_{22} \cdot C_{22}}}{2 \cdot A_{22}}\right)
\]  

(12)

Where:

\[A_{22} = (1 - v_{22}) \cdot \cos(\theta_{23}) + v_{23} + v_{21}\]

\[B_{22} = -2 \cdot \sin(\theta_{23})\]

\[C_{22} = v_{21} + v_{23} - (1 - v_{22}) \cdot \cos(\theta_{23})\]

\[v_{21} = -\frac{d}{c}, v_{22} = -\frac{d}{l_c} \text{ and } v_{23} = \frac{l_c^2 - l_d^2 + c^2 + d^2}{2c \cdot l_c}\]

\[l_c: \text{Length of link } C, l_d: \text{Length of link } D\]

4. Geometric optimization:

It is important to have an acceptable configuration of the robot finger to employ it for the gripping function; therefore, it is imperative to improve the job parameters. Because of the complexity of the system, it is very complicated to separate each parameter. To fix the problem, a genetic algorithm (GA) was chosen. These criteria are specified to find the parameters:

1. First criterion: The grasp forces
2. Second criterion: Squeezing force

Some parameters are defined as input parameters shown in Table (2), which shows the improvement input parameters. Parameters 1, 2, and 3 were taken as the original length of a human finger (44, 25, 18) mm [11], and we found the optimal parameters using the GA. It is a machine learning model that generates behavior patterns from the representation of evolution mechanisms. The model is achieved through a generation within-population machine of entities defined by genetics. The population of people will go through the boom phase. The growth will note not to be an auxiliary mechanism. So, there is no evidence to support the theory that the purpose of evolution is to generate humans. Therefore, the mechanisms of existence come to different people competing for services in the world. The following steps are used to obtain alignment with the GA (10) [12], Figure 10.

| Parameters | Unite | Min. Values | Max. Values | Optimum Values |
|------------|-------|-------------|-------------|----------------|
| \(\theta_{12}\) | Deg. | 30 | 90 | - |
| \(\theta_{23}\) | Deg. | 30 | 90 | - |
| a | mm | 4 | 8 | 7 |
| b | mm | 4 | 8 | 6 |
| c | mm | 4 | 8 | 5 |
| d | mm | 4 | 8 | 7 |
| l_a | mm | 15 | 28 | 24 |
| l_b | mm | 12 | 22 | 19 |
| l_c | mm | 15 | 20 | 18 |
| l_d | mm | 12 | 16 | 14 |
5. Experimental work

5.1. Manufacturing of robotic finger

The developed robotic finger in this work was built of hard plastic using rapid prototyping. This technique allowed the manufacture of prototypes and their presentation within a short time using the 3D printer machine. 3D-printers give the ability to test the product designs easily using models made of PLA (Polylactic acid). Polylactic acid plastic is tough sufficient to use it as working parts, as shown in Figure (11).

5.2. Manufacturing of Grasping force mechanism

In this work, we needed to measure the force exerted by each phalanx in the finger. Therefore, we designed a mechanism used to measure the force. It comprises several links connected to several joints; these parts were made by a 3D printer with PLA material, as shown in Figure (12).
Figure (13) shows the final form of the measuring device of the grasping force after assembling the parts obtained by the 3D printer and the finger and load cells installed. The most important feature of the device that can be adjusted the position of the exerted force by changing the height of the load cell or change the angle to adapt with all objects.

![Figure 13. Assembly of the grasping force mechanism.](image)

5.3. Measuring system component
The measurement system consists of the following:
1) PC (1), 2) Arduino MEGA 2560 (1), 3) HX711 amplifier (4), 4) Load cell (4), 5) Potentiometer (2), and 6) Wires (several). The schematic of the Measuring System can be shown in figure (14).

![Figure 14. Schematic diagram of the measuring system.](image)

5.4. Calibration process
Instrument calibration is one of the fundamental processes used to maintain instrument accuracy. Calibration is the process of creating a tool to present a sample result within an acceptable range. Eliminate or minimize factors as inaccurate measurements are an essential aspect of instrument design. The calibration was done by using measured weights and comparing them to the device readings, and the horizontal axis represents the signal obtained from the device, while the vertical axis represents the true reading of the force. Figure (15a, 15b, 15c, and 15d) shows calibrated results for load cell, Figure (16a and 16b) for potentiometer, Figure (17) for tension spring, and Figure (18) for torsional spring.
Figure 15. Calibrated results for loads.
Figure 16. Calibration of the potentiometer at the joints.

(a) The second joint.

(b) The third joint.

Figure 17. Calibration of tension springs.
6. Results and discussion

Static structure analysis by ANSYS was used to find the safety factor and stress of the grasping force mechanism to satisfy the experimental work conditions. A PLA material with a magnitude of (20 N) force, which is greater than the total grasping force (16.7 N) [13], is used as input to the program. Figure (19) shows the safety factor of the grasping force mechanism where the minimum value of safety factor was (1.66) so that it is enough to withstand the force exerted by each phalanx in the finger. Also, the resulting stress from this force on the structure of the grasping force mechanism is less than (9.8 Mpa), as shown in Figure (20), while the yield stress of PLA is (35.9 Mpa) [14], so it is safe.

Figure 18. Calibration of the torsional spring.

Figure 19. Safety factor of grasping force mechanism.

Figure 20. Equivalent stress on the grasping force mechanism.
Figures (21a, 21b, and 21c) show the normal force distribution with the joint angles ($\theta_{12}$, $\theta_{23}$), where the results show that the normal force is always positive and this is an excellent feature of the proposed design, and it is almost constant within $(30^\circ:60^\circ)$ for both joint angles; for example, when holding large and medium objects such as the water cup and increase for more than $(60^\circ)$ for both joint angles for example, when holding small objects such as a pen.

(a) The first phalanx with respect to joint angles.

(b) The second phalanx with respect to joint angles.

(c) The third phalanx with respect to joint angles.

**Figure 21.** The distribution of Contact forces on the phalanxes with respect to joint angles.
Figures (22a, 22b, and 22c) show the theoretical and experimental values of contact force (normal force) with different joint angles values. Where the red color represents the theoretical values, and the blue color is the experimental values. Figure (22a) shows the theoretical and experimental values of the contact force for the first phalanx.

Figure (22b) shows the theoretical and experimental values of the contact force for the second phalanx, and Figure (22c) shows the theoretical and experimental values of the contact force for the third phalanx. Through the previous figures, we found that the theoretical and experimental values converged with a small difference depending on the measurement devices’ accuracy. This convergence indicates the validity of the mathematical model of the proposed design that is used in optimization to find the optimum values of finger parameters.

(a) The first phalanx.

(b) The second phalanx.

(c) The third phalanx.

Figure 22. Theoretical and experimental values of the contact force at the phalanxes.
7. Conclusion
1- Developed a new finger design with three degrees of freedom and derive a mathematical model for it.
2- Multi objectives function for robotic finger is optimized depending on two design criteria (the grasp forces and squeezing force) to get an optimal solution, increasing the fingers' performance at grasping tasks. The genetic algorithm is used for optimization.
3- The design of a robotic finger with an underactuated mechanism produces to minimize the cost and make the weight of the design more agreeable to use for industrial and prosthesis robotic hands that demand force for grasping operations.
4- The results exhibit that the adaptation of the finger is accepted.

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