THE SOLAR NEUTRINO PROBLEM:
NEITHER ASTROPHYSICS NOR OSCILLATIONS? 

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Abstract

There is no consistent solar model which can describe all experimental data on the solar neutrinos. The problem can be formulated essentially in a model independent way. The key points are the comparison of the Homestake and the Kamiokande data as well as the comparison of the GALLEX and SAGE results with minimal signal estimated from the solar luminosity. It is argued than in such a comparison one should use the Homestake-II data (only after 1986) with caution. The results of the model independent analysis show strong suppression of the beryllium neutrino flux. The data can be well described by the resonant flavor conversion. For the “low flux model” which can accommodate the Kamiokande signal, a consistent solution can be found for the neutrino mass squared difference $\Delta m^2 = (0.3 - 1.0) \cdot 10^{-5} \text{ eV}^2$ and values of mixing angle $\sin^2 2\theta > 5 \cdot 10^{-4}$ (“very small mixing solution”).

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1 Introduction

The solar neutrino problem is usually formulated as disagreement of the experimental signals [1 - 4] with the predictions of the “reference” standard solar models [5 - 9]. The first GALLEX results (1992) have boosted new attempts to find non-neutrino physics solution of the problem, and main points are the following.

1. GALLEX and later SAGE signals exceed the signal from the standard pp-neutrino flux as well as the minimal signal estimated from the luminosity of the Sun. The pp-neutrinos compose the bulk of the solar neutrino flux and its prediction is the most accurate and reliable. Some suppression (0.6 - 0.7) of the observed Ga - signal can be related to smaller fluxes of high energy neutrinos for which the predictions are strongly model dependent and not as reliable as for pp-neutrinos.

2. Kamiokande II+III gives

\[(signal) = (0.5 - 0.7) \times (SSM),\]

i.e. the signal is in agreement with prediction within theoretical uncertainties estimated as \(\sim 40\%\).

3. The energy distribution of the Kamiokande events agrees with undistorted energy spectrum of the boron neutrinos. However, the experimental errors are rather large and the distortions implied by a number of neutrino physics solutions of the problem can not be excluded.

4. The Homestake signal after 1986 is rather high: about 3 SNU. So that there is no direct contradiction between the Homestake and the Kamiokande results. The boron neutrino flux extracted from the Homestake data and the flux measured by Kamiokande agree within error bars.

5. The difference between the Homestake results before 1986 and after 1986 may be just a statistical fluctuation. The results before 1986 show time dependence which can be due to some unknown systematics. Comparing the Homestake and the Kamiokande results one should use the Homestake data during the time of operation of the Kamiokande experiment. In this period the signals from both experiments have no appreciable time dependence.

6. There are essential uncertainties in the extrapolated (to solar energies) cross-sections \(\sigma_{17}, \sigma_{34}\). Some latest experimental and theoretical studies indicate that the cross sections can be 30 - 40 % below those used in the reference models [10,11].

\(\dagger\) Some authors however refer to the problem as to the impossibility to explain the data without introducing new neutrino properties.
7. The collective plasma effects are not taken into account appropriately. A number of corrections may result in diminishing of the opacity, and consequently, of the central temperature of the Sun up to 2 - 3% [12].

8. There are another unresolved problems (e.g. \(^{7}\text{Li}\)-surface concentration ) which may testify for incomplete understanding of properties of the Sun (inner convection ?). This in turn, puts the question mark on the reliability of the solar flux predictions, although it is unclear what could be the impact of the solution of the \(^{7}\text{Li}\) problem on the inner structure of the Sun and on the solar fluxes.

In this paper we will consider present status of the solar neutrino problem.

## 2 Data versus predictions

### 2.1 The data

1). The average Ar-production rate during all observation time (runs 18 - 126) equals after background subtraction

\[
Q_{Ar} = 2.32 \pm 0.16(stat) \pm 0.21(syst) \text{ SNU.} \quad (1)
\]

For the present discussion it is instructive to divide the data into two parts: the data before pump breaking in 1985 and after resuming the experiment in 1986. For the sake of brevity we will call the data before 1986 as data from Homestake-I and after 1986 as data from Homestake-II. The averaged signals are:

\[
\begin{align*}
Q^I_{Ar} &= 2.07 \pm 0.25 \text{ SNU} \quad (\text{Homestake – I, runs 18 – 89}) \\
Q^{II}_{Ar} &= 2.76 \pm 0.31 \text{ SNU} \quad (\text{Homestake – II, runs 90 – 126}).
\end{align*} \quad (2)
\]

The latest data do not confirm the anticorrelation with solar activity: large number of the sunspots in 1990 - 1991 was accompanied by high counting rate; relatively small signal was observed during quiet 1992. On the other hand, the data confirm 2 - 3 years period variations of signal. An impressive increase of the Ar-production rate has been observed in the time of solar flare in June 1991. This may testify for incomplete understanding of the physics of the Sun or the Homestake experiment itself.

2). The boron neutrino flux measured by Kamiokande II+III in the units \(\Phi_0^{SSM}\), where \(\Phi_0^{SSM} \equiv 5.8 \cdot 10^6 \text{ cm}^{-1} \text{ s}^{-1}\) is the central value of flux predicted by the standard solar model[3], is

\[
R^{II+III}_{\nu e} = \frac{\Phi^{exp}}{\Phi_0^{SSM}} = 0.50 \pm 0.04(stat.) \pm 0.06(syst.), \quad (1\sigma). \quad (3)
\]
The data agree with constant neutrino flux. No anticorrelations (or correlations) with solar activity were found. Possible time variations should not exceed 30%. The energy distribution of events can be fitted with practically the same probabilities by constant and MSW-nonadiabatic suppression factors.

3). The average Ge-production rate measured by GALLEX-I+II is

\[ Q_{Ge}^{I+II} = 79 \pm 10(stat) \pm 7(syst) \ (1\sigma \ prelim.). \] (4)

The combined error is \( \sim 12 \) SNU. The data are \( \sim 3\sigma \) below the expected value 125 - 130 SNU.

4). Statistical analysis of the Ge-production rate in 15 runs of the experiment SAGE gives

\[ Q_{Ge} = 74 \pm 19(stat) \pm 10(syst) \ SNU \ (1\sigma \ prelim.). \] (5)

2.2 Comparison with model predictions

A comparison of experimental data with model predictions is given in Fig.1. There are several immediate observations.

All the experiments have detected signals which are lower or much lower than the predictions of the “reference” standard solar models [15 - 19].

New measurements of nuclear cross-sections as well as plasma effects revision probably will result in appreciable reduction of the boron neutrino flux. However, even the “low flux models” [13, 14] which could accommodate the Kamiokande result predict the argon production rate \( Q_{Ar} \sim (4.2 - 4.5) \) SNU, whereas the experimental value of \( Q_{Ar} \) averaged over all observation time is \( \approx 8\sigma \) smaller. For Ge-production rate these models give about 110 SNU, new GALLEX result is 2.5\( \sigma \) lower (fig.1).

Suppression of signals in different experiments is different. The Homestake signal is suppressed stronger than the Kamiokande one: the double ratio, \( (data/model)_{Hom}/(data/model)_{Kam} \), is practically the same for all models:

\[ \frac{R_{Ar}}{R_{\nu e}} = \begin{cases} 0.58 \pm 0.12, \ [6] \\ 0.57 \pm 0.12, \ [7] \\ 0.58 \pm 0.11, \ [14] \end{cases} \] (6)

This statement can be relaxed if one takes the Cl- Ar data only for a period of the operation of Kamiokande: \( \frac{R_{Ar}}{R_{\nu e}} = 0.69 \pm 0.22 \).

The difference in \( R_i \) testifies for the energy dependence of the suppression effects.
3 Homestake versus Kamiokande

3.1 Model independent comparison. Contradiction?

One can perform a direct test of consistency of the Cl-Ar and Kamiokande results in the model independent way \[13\]. Suppose that

1). there is no distortion of the energy spectrum of boron neutrinos and
2). the Kamiokande signal is due to the electron neutrino scattering only.

In this case Kamiokande measures immediately the flux of the electron neutrinos from boron decay:

\[ \Phi_B = (2.9 \pm 0.42) \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1}. \]  

(7)

This flux gives the contribution to Ar production rate:

\[ Q_{Ar}^B = 3.1 \pm 0.45 \text{ SNU} \]  

(8)

which exceeds the total measured rate by \(~ 2\sigma\): \( Q_{Ar}^B > Q_{Ar}^{total} \). The inclusion of the contributions from Be- and other neutrinos strengthens the disagreement.

Main objection is that one should use the data only from Homestake-II when compare with Kamiokande. The result (8) agrees with Homestake-II signal (2) within 1\(\sigma\) if the contributions from all other neutrinos are strongly suppressed.

3.2 Boron neutrino flux from Homestake and Kamiokande

One can confront the Homestake and Kamiokande data comparing the boron neutrino fluxes measured by Kamiokande and extracted from the Homestake experiment. It was claimed in \([1, 14]\) that the Kamiokande flux and the flux extracted from Homestake-II are in agreement. However this statement is the result of selection of the data. Indeed, in \([1]\)

1). the data from Kamiokande-II where used only,
2). the threshold \( E^{th} = 9.3 \text{ MeV} \) was chosen,
3). it was suggested that the contribution to \( Q_{Ar} \) from the boron neutrinos is \( r_B \equiv Q_{Ar}^B / Q_{Ar} = 0.77 \) of all signal. This number corresponds to the Bahcall-Ulrich model. However for the models with low flux of boron neutrinos the contribution of boron neutrinos is typically smaller: \( r_B = 0.69 - 0.70 \).

Using the data from both the Kamiokande-II and the Kamiokande-III, the threshold \( E^{th} = 7.5 \text{ MeV} \) and the boron contribution \( r_B = 0.7 \), one gets the picture shown in fig.2. The flux extracted from the Homestake-II is systematically lower than that measured by Kamiokande. The average value of flux is

\[ \Phi_B(H-II) = (1.8 \pm 0.2) \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1} \]  

(9)
i.e. about $2.5\sigma$ lower than the Kamiokande flux (7). This situation can be described also in “boron - beryllium” neutrino plot (see sect.5.1).

### 3.3 Statistical fluctuation?

In principle, it is correct to use only the Homestake-II data when comparing with Kamiokande. However, let us consider the Homestake-II results more carefully. In fig.3a is shown the distribution of number of runs $n$ with a given Ar-production rate $N_{Ar}$: $n = n(N_{Ar})$ for Homestake-II. Fig. 3b and 3c. show the distribution of the Homestake-I runs as well as the result of Monte Carlo simulation for average production rate corresponding to $Q_{Ar} = 2.3$ SNU. As follows from these figures the shape of the Homestake-II distribution disagrees with both Monte Carlo and with the Homestake-I results. Note that the latter is well described by simulations. The distribution of Homestake-II runs has two components: one component is in a good agreement with simulation for $Q_{Ar} = 2.3$ SNU. Another component is a thin peak in the interval $N_{Ar} = 0.7 - 0.9$ at/day, i.e. there is an excess of runs with high counting rate. 14 runs (among 34) where found in the indicated interval, whereas the simulation gives 4 runs only. Also the shape of the Homestake-II distribution can not be reproduced by Monte Carlo simulation with average $Q_{Ar} = 3$ SNU.

For comparison we show the corresponding distribution for the GALLEX experiment (fig. 3d), where the number of runs (30) is about that in Homestake-II.

What could be the interpretation of the excess observed by Homestake-II? The peak in the distribution can be a statistical fluctuation. In this case one expects in future that average value after 1986 will approach present average value for all runs. The peak could be a result of some systematics (background ?). After subtraction of the peak one gets the result which agrees with average value 2.3 SNU.

Using the Homestake-II result one should explain the shape of the distribution as well as the difference of the Homestake-I and Homestake-II signals, i.e. the change of the signal with time.
4 Splitting of cycle. Cross sections. Luminosity normalization

The important relations and dependencies concerning the solar neutrino fluxes can be obtained immediately from some general astrophysical notions without modeling.

4.1 Splitting of the cycle

The pp-cycle is split: there are several branches of the reactions (fig.4). It is this branching that results in the uncertainties of the predictions of the neutrino fluxes. No branching - no (or almost no) problem. Indeed, without splitting the fluxes of the neutrinos produced in the first, say pp-, and the subsequent, say B-decay, reactions are equal and coincide with number of chains per second: \( \Phi_{pp} = \Phi_{B} = N \). (Strictly speaking this implies also the nuclear reactions equilibrium, i.e. that there is enough time for the termination of chains). Moreover, if the Sun is in thermal equilibrium, the number of chains is fixed by the total solar luminosity. It is the branching that depends on the solar conditions as well as on the nuclear cross-sections.

\( ^3\text{He} \)-branching. There are two main possibilities for \(^3\text{He}\): to interact with another nuclei \(^3\text{He}\) which means that in this chain the second pp-neutrino is produced or to interact with \(^4\text{He}\), producing \(^7\text{Be}\), and consequently beryllium or boron neutrinos. The “branching ratio” \( r \) is determined by corresponding cross-sections and concentrations \((n_3, n_4)\):

\[
\frac{r}{1-r} = \frac{<\sigma_{34}\nu>}{<\sigma_{33}\nu>} \cdot \frac{n_4}{n_3}.
\]

(10)

Since in both reactions the electric charges of nuclei are the same, the dependence of branching on the temperature of the Sun is rather weak. The most important dependence of the branching is that on the cross-sections (astrophysical factors). The concentration of the \(^3\text{He}\) itself depends on the cross-section \(\sigma_{33}\). Main channel of the \(^3\text{He}\) disappearance is the \(^3\text{He} + ^3\text{He} \)-reaction; its probability is proportional to \(W \propto n_3^2\sigma_{33}\), therefore the equilibrium concentration of \(^3\text{He}\) equals \(n_3 \propto 1/\sqrt{\sigma_{33}}\). Adding the effect of another channel, \(^3\text{He} + ^4\text{He}\), gives \(n_3 \propto (\sigma_{33})^{-0.56}\). Substituting this relation in (10) one finds the dependence of branching ratio on the cross-sections (astrophysical factors): \(r \propto \sigma_{34}/\sqrt{\sigma_{33}}\)

(11)

\( ^7\text{Be} \)-branching. \(^7\text{Be}\) can capture the electron, emitting the beryllium neutrino, or can interact with proton, producing the boron - 8 which in turn decays with emission of the
boron neutrino. The branching ratio, $r'$,

$$\frac{r'}{1 - r'} = \frac{\langle \sigma_{17}\nu \rangle}{W_e} \cdot \frac{n_1}{n_e}$$  \hspace{1cm} (12)$$

strongly depends on the temperature due to the Coulomb barrier for the $p - ^7\text{Be}$-reaction. Evidently it is proportional to the cross section $\sigma_{17}$. In (12) $n_1$ and $n_e$ are the concentrations of the protons and the electrons.

Using the parameters $r$, $r'$ one can find the following relations between the number of chains and the fluxes of different neutrinos:

$$\Phi_{pp} = \frac{N}{2}(2 - r) , \quad \Phi_{Be} = \frac{N}{2}r(2 - r') , \quad \Phi_{B} = \frac{N}{2}rr' ,$$  \hspace{1cm} (13)$$

(compare with the toy case).

4.2 Solar luminosity and the normalization of the neutrino flux

The chain of the nuclear reactions results in hydrogen burning, production of the $^4\text{He}$

$$4p + 2e^- \rightarrow ^4\text{He} + 2\nu_e + Q ,$$  \hspace{1cm} (14)$$

and energy release $Q = M(^4\text{He}) - 4M_p + 2M_e \approx 26.7$ MeV. It is supposed that the Sun is in thermal equilibrium, i.e. total luminosity of the Sun equals the nuclear energy release $Q_N$.

If the energy release is approximately constant then one can write the equality

$$L_{\odot} = Q_N - L_{\nu} ,$$  \hspace{1cm} (15)$$

here $L_{\nu}$ is the neutrino luminosity. The total energy release and the neutrino luminosity can be expressed in terms of the neutrino fluxes $\Phi_i$ , i = pp, Be, B, ... as

$$Q_N = \frac{Q}{2} \sum_i \Phi_i , \quad L_{\nu} = \sum_i E_i \Phi_i ,$$  \hspace{1cm} (16)$$

where in the first equality we have taken into account that in each chain of reactions two neutrinos are emitted. In the second equality $E_i$ is the average energy of the neutrino from the i-reaction. Substituting (16) in (15) one gets the desired luminosity normalization condition for the neutrino fluxes:

$$\sum_i \left( \frac{Q}{2} - E_i \right) \Phi_i = L_{\odot} .$$  \hspace{1cm} (17)$$

Note that according to the SSM the pp-neutrinos compose practically 93% of the sum in (17), Be-neutrinos give only 7%, the contributions from other neutrinos are negligible. Therefore
the luminosity condition is sensitive mainly to the pp-, and the beryllium neutrinos. Neglecting also the contribution from the $\nu_{Be}$ one gets the estimation of the pp-neutrino flux:

$$\Phi_{pp} \approx \frac{2L_\odot}{Q - 2E_{pp}}.$$  \hfill (18)

### 4.3 Time variations of the energy release?

One remark is in order. Using the equality (15) one should keep in mind the difference in time. Present (electromagnetic) luminosity of the Sun $L_\odot$ is determined by energy release about $t_d \sim 10^6 - 10^7$ years ago, and more precisely, the condition of thermal equilibrium should be written as

$$L_\odot(t) = Q_N(t - t_d) - L_\nu(t - t_d).$$  \hfill (19)

The variations of the energy release on the time scales $t \ll t_d$ are averaged out and luminosity gives an information about the averaged energy release and the average neutrino fluxes. In the Standard Solar Models there is no appreciable changes of $Q_N$ during $10^7$ years. However, short term variations of $Q_N$ ($10^5$ years, 22 years, 2 years, months, hours?) related to some instabilities in the core of the Sun are not excluded.

### 5 Analysis of all data

#### 5.1 “$\nu_B - \nu_{Be}$”-plot

It is convenient to analyze the data using the $\nu_B - \nu_{Be}$-plot – the plot of the boron and beryllium neutrino fluxes \[3\], \[10\] (fig.5). Let us measure the neutrino fluxes, $\Phi_i$ in the units of fluxes $\Phi_i^0$, predicted by a certain reference SSM:

$$\phi_i = \frac{\Phi_i}{\Phi_i^0}.$$  \hfill (20)

For definiteness we take for $\Phi_i^0$ the central values of Bahcall-Ulrich model. In terms of $\phi_i$ the signals in different experiments can be written as:

$$R_{\nu e} = \phi_B,$$  \hfill (20)

$$Q_{Ar} = 6.1\phi_B + 1.1\phi_{Be} + Q_{Ar}^{other},$$  \hfill (21)

$$Q_{Ge} = 14\phi_B + 34\phi_{Be} + 71\phi_{pp} + Q_{Ge}^{other}.$$  \hfill (22)

In (20) it was suggested that there is no distortion of the boron neutrino spectrum and the signal is stipulated by the electron neutrino only. In (21,22) the numerical coefficients
(in SNU’s) correspond to chosen reference model. For fixed reference model the coefficients are determined by the cross-sections of the neutrino interactions in detectors. \(Q_{Ar}^{\text{other}}\) and \(Q_{Ge}^{\text{other}}\) are the contributions from other fluxes which are typically smaller than the explicitly indicated contributions.

The flux of the pp-neutrinos can be extracted from the \(L_{\odot}\) normalization condition. Neglecting all the contributions in (17) apart from pp- and Be- contributions one gets

\[
\phi_{pp} + k\phi_{Be} = 1 + k, 
\]

where

\[
k \equiv \frac{Q - 2E_{Be}}{Q - 2E_{pp}} \cdot \frac{\Phi_{Be}^0}{\Phi_{pp}^0},
\]

and \(k \approx 0.075\) in SSM [5]. Substituting (23) in (22) one finds:

\[
Q_{Ge} - Q_{Ge}^{\text{other}} - 71(1 + k) = (34 - 71k)\phi_{Be} + 14\phi_B. 
\]

The experimental data on \(Q_{Ar}, Q_{Ge}\) and \(R_{\nu e}\) and the estimations of \(Q^{\text{other}}\) give according to (20, 21, 24) the allowed regions (strips) for each experiment shown on the “boron - beryllium” plot.

5.2 Confronting all data

In fig.5 are shown the regions allowed at 1\(\sigma\) level by the Homestake, Kamiokande and GALLEX experiments. We have suggested that \(Q^{\text{other}} = Q^0/2\). Zero values of \(Q^{\text{other}}\) slightly relax the bounds (see below). As follows from fig.5 the Homestake and Kamiokande results imply strong suppression of the \(\nu_{Be}\)- flux. There is no intersection of corresponding 1\(\sigma\)-allowed regions in the \(\nu_B - \nu_{Be}\)-plot. The intersection of 2\(\sigma\)-allowed regions appears for values of \(\Phi(\nu_{Be})\) being 2.5 times smaller than the predictions of the reference models.

The disagreement between the Kamiokande and Homestake relaxes (but does not disappear) if one takes the Homestake-II results. In this case the intersection of 1\(\sigma\)-allowed regions appears for \(\Phi(\nu_{Be}) < 10^9 \text{ cm}^{-2}\text{s}^{-1}\), i.e. for 5 times smaller flux than reference models predict. Moreover, as we have mentioned the data after 1986 show some statistical inconsistency.

New GALLEX-I+II results have small errors which allow to make some important conclusions. GALLEX signal is just slightly higher than the signal expected from the pp-neutrinos. Therefore if there is no conversion of pp-neutrinos for which one has rather accurate predictions, the GALLEX data testify for strong suppression of the contributions from all other components of the neutrino flux and first of all from the beryllium neutrinos.

The intersection of 1\(\sigma\) regions allowed by GALLEX and Kamiokande on “\(\nu_B - \nu_{Be}\)”-plot gives the same (factor of 5) suppression of \(\Phi(\nu_{Be})\) as Homestake and Kamiokande give.
6 Neither astrophysics nor oscillations?

6.1 Astrophysics

The astrophysical solutions fit the conditions formulated in sect. 3.1 and consequently, meet the problems discussed above. A number of modifications of solar models were suggested which result in decrease of the central temperature of the Sun, $T_c$. However, $T_c$ decrease suppresses the boron neutrino flux stronger than the beryllium neutrino flux, and consequently, the double ratio in (6) becomes even smaller. Essentially for this reason a combined fit of all the data for arbitrary astrophysical parameters is rather bad - any astrophysical solution is excluded at 98% C.L. \[16\].

6.2 Nuclear physics solution

Using (10) one can estimate the dependence of the fluxes on the cross-sections:

$$\frac{\Delta \Phi_{Be}}{\Phi_{Be}} \sim (1 - r) \frac{\Delta \sigma_{34}}{\sigma_{34}}. \tag{25}$$

Precise study gives the coefficient 0.81 \[5\]. Boron neutrino flux has similar dependence on $\sigma_{34}$. Evidently $\Delta \Phi_B/\Phi_B \sim \Delta \sigma_{17}/\sigma_{17}$. A decrease of the astrophysical factor $S_{17}$ allow to suppress the boron neutrino flux without changes of the Be-neutrino flux as well as the solar model. However, present data testify for strong suppression of $\phi_{Be}$. Certainly, 30 - 40 % reduction of the $S_{34}$ is not enough to solve the problem. According to (fig.5) the desired suppression of Be-branch of the pp-cycle implies strong (factor 3 - 5) suppression of the astrophysical factor $S_{34}$ or even more strong (10 times) increase (see (11)) of $S_{33} \[9\]$. An appreciable increase of $S_{33}$ could be related to the existence of the hypothetical $^3$He - $^3$He resonance. Obviously, strong suppression of the Be-branch gives also strong reduction of $\nu_B$-flux. The “astrophysical” suppression of the Be-flux may imply an essential modification of solar models, since Be-neutrinos are related (according to (17)) to $\approx 7\%$ of the Sun luminosity fixed with 0.2% accuracy. The plasma effects are basically reduced to diminishing of the central solar temperature which in turn results in more strong suppression of $\nu_B$-flux than $\nu_{Be}$-flux, i.e. does not solve the problem.

The increase of the cross-section of the pp-reaction works essentially as the decrease of the central temperature \[10\].

Formally one could suppress the beryllium line by strong diminishing $\sigma_{34}$ (or increasing $\sigma_{33}$), the corresponding decrease of the boron flux could be compensated by increase of $\sigma_{17}$ or/and the temperature.

6.3 Large, small, very small?
All the data obtained so far can be easily reconciled with predictions of the reference standard solar models by the resonant flavor conversion (MSW-effect) \[18\] \( \nu_e \rightarrow \nu_\mu (\nu_\tau) \). The best description of the data can be obtained for small mixing angles when the suppression pit is rather thin, so that the pp-neutrinos are outside the pit, the Be-neutrinos are at the bottom and the boron neutrinos are on the non-adiabatic edge (see fig. 6). Note that in this case the conditions of sect. 3.1 are broken: the Homestake and Kamiokande data can be reconciled due to more strong suppression of low energy part of the boron neutrino spectrum which does not contribute to the Kamiokande signal and due to an additional contribution to Kamiokande from the scattering of the converted \( \nu_\mu (\nu_\tau) \) via neutral currents. For the reference models [6] the data pick up the region of parameters (see fig. 7):

\[
\Delta m^2 = (0.5 - 1.2) \cdot 10^{-5} \text{ eV}^2, \quad \sin^2 2\theta = (0.3 - 1.0) \cdot 10^{-2}.
\] (26)

Also the region of large mixing solution is not excluded:

\[
\Delta m^2 = (1 - 3) \cdot 10^{-5} \text{ eV}^2, \quad \sin^2 2\theta = (0.65 - 0.85).
\] (27)

Although the solar neutrino problem can be formulated in a model independent way the implications of the results, and in particular, the appropriate regions of the neutrino parameters depend on the predicted fluxes. The boron neutrino flux being strongly involved in this determination has rather large uncertainties. In the model [6] they estimated to be at the level 40%. The changes of the cross sections and plasma effect revision may reduce the predicted boron neutrino flux to that measured by Kamiokande. From this one can conclude that

1). the uncertainties of the \( \nu_B \)-flux can not be considered as just the statistical ones,

2). probably, these uncertainties will not be essentially diminished in near future (at least to the moment when new solar neutrino experiments will start to work),

3). one should try to solve the problem without referring to the original (theoretical) value of the boron neutrino flux. This flux can be considered as free parameter which should be determined from the experiments.

Let us discuss in this connection the solution of the problem in the context of the “low flux models” which predict the boron neutrino flux at the level of that measured by Kamiokande. In other words let us suppose that \( R_{\nu e} \rightarrow 1 \). Strong suppression of the beryllium flux and weak suppression of the pp- as well as the boron neutrino fluxes can be obtained for very small mixing angles when the high energy part of the boron neutrino spectrum is at the top, whereas the beryllium neutrinos are at the bottom of the nonadiabatic edge (see fig. 6).
For small mixing angles (down to $\sin^2 2\theta \sim 3 \cdot 10^{-4}$) the nonadiabatic edge can be well described by the Landau-Zener formula [19]:

$$P_B = P_{LZ} \equiv \exp \left( - \frac{E_{na}}{E} \right),$$

(28)

where

$$E_{na} = \frac{\Delta m^2 l_n \sin^2 2\theta}{\cos 2\theta}$$

(29)

and $l_n \equiv \left| \frac{d}{dx} \ln n_e \right|$. Taking into account the effect of the neutral currents one has

$$R_{\nu e} \approx P_B + \frac{1}{7}(1 - P_B).$$

(30)

Using (28, 29) one finds the desired value of mixing angle:

$$\sin^2 2\theta \approx \frac{E_B}{\Delta m^2 l_n \ln P_B},$$

(31)

where $E_B \sim 10$ MeV is the average energy of the detected part of the boron neutrino spectrum. The suppression of the beryllium neutrinos can be found from

$$P_{Be} = (P_B)^{E_B/E_{Be}}.$$  

(32)

According to (31), with increase of $P_B$ the allowed region of the neutrino parameters is shifted to small mixings by factor $\ln P_B/\ln P_B^0$. For $P_B = 0.8$ or 0.9 one gets the factors 4 and 9 correspondingly and

$$\sin^2 2\theta = (0.5 - 1.5) \cdot 10^{-3}.$$  

(33)

The values of the mass difference, $\Delta m^2 \sim (0.3 - 1) \cdot 10^{-5}$ eV$^2$, are fixed essentially by the condition $E_{pp}^{max} < E_a < E_{Be}$, i.e. that the adiabatic edge ($E_a$) of the pit is between the highest energy of the pp-spectrum and the energy of the beryllium line.

The “very small mixing” solution is characterized by weak distortion of high energy part of the boron neutrino spectrum and by small effect of the neutral currents in this energy region. Indeed, according to (28) the change of the suppression factor, $\Delta P_B \equiv P_B(E_2) - P_B(E_1)$, in the energy interval $E_1 - E_2$ is

$$\Delta P_B = P_B(E_2) - (P_B(E_2))^{E_2/E_1}.$$  

(34)

Suppose $E_2/E_1 = 2$, then using (34) and (30) one gets $\Delta R_{\nu e} = 0.07$ for $P_B = 0.9$, whereas $\Delta R_{\nu e} = 0.21$ for reference value $P_B = 0.5$. The contribution of the neutral currents to the $\nu e$ scattering is $\approx 6$ times smaller than in case of “reference” models. Therefore, measuring the
distortion of the energy spectrum and the effects of neutral currents in future experiments one can find the values of neutrino parameters and the original flux of boron neutrinos (for details see [20]).

Note that the region of very small angles (33) may be natural for mixing of the first and third generations so that the solar neutrino deficit could be explained by the $\nu_e \rightarrow \nu_\tau$ conversion. Such a scenario can be realized in the supersymmetric $SO(10)$ with unique scale of symmetry violation. Although mixing (33) can be described without fine tuning by formula

$$\theta_{e\mu} = \sqrt{\frac{m_e}{m_\mu} - e^{i\phi} \theta_\nu},$$

which corresponds to the $\nu_e - \nu_\mu$ mixing, here $m_e$ and $m_\mu$ are the masses of the electron and muon, $\phi$ is a phase and $\theta_\nu$ is the angle related to diagonalization of the neutrino mass matrix. The relation (35) between the angles and the masses is similar to the relation in quark sector which follows naturally from the Fritzsch ansatz for mass matrices. Such a possibility can be realized in terms of the see-saw mechanism of the neutrino mass generation.

6.4 “Detection solution”

It is not excluded that the problem has the “detection” solution, i.e. that some experimental results are interpreted incorrectly.

The GALLEX results are rather stable and convincing. It is difficult to expect appreciable changes of numbers. The calibration experiment may result probably in increase of the measured neutrino flux. The SAGE experiment confirms the GALLEX results.

Kamiokande results are stable and the experiment had been (at least partly) calibrated. Homestake experiment shows the strongest suppression of signal. There is no calibration. Some features of the data have small statistical probability, e.g., very low signal during 1978 (five runs with near to zero counting rate), high signal during 1986 - 1992, the peak in the distribution “number of runs with a given counting rate” in the region $N_{Ar} \sim 0.7 - 0.8$ at/day. There is no explanation of the increase of signal during some solar flares.

What kind of changes in the existing experimental data could make the non-neutrino physics solution preferable?

(i). The signal from Cl - experiment at the level $Q_{Ar} \geq 4$ SNU certainly changes the status of the problem. Such a situation could be explained by lower central solar temperature (e.g. as the consequence of the plasma effects revision) or/and by the astrophysical factors $S_{17}$ and $S_{34}$ being (30 - 40)% smaller. In this case for the Gallium experiments one expects $Q_{Ge} \geq 100$ SNU, i.e. $2\sigma$ higher than GALLEX result.
(ii). Suppose that the $\nu e$-scattering experiment gives the boron neutrino flux suppressed by factor of 5 (instead of 2) in comparison with reference model flux. If then the Be-neutrino flux is diminished by 40%, one gets the Ar-production rate $Q_{Ar} \sim 2.3$ SNU in agreement with Homestake result. For the Ga-experiment again a large effect is predicted: $Q_{Ge} \gtrsim 100$ SNU. The indicated situation can be reproduced by both the decrease of the central temperature and the cross-sections (especially, $S_{17}$).

Let us underline in this connection the importance of present GALLEX result as well as further diminishing of the experimental errors.

### 7 Conclusion

1. There is no consistent solar model which can explain all existing experimental data on the solar neutrinos.

2. Boron neutrino flux measured by Kamiokande II+III gives the argon production rate which exceeds total signal measured by the Homestake experiment. (It is supposed that the Kamiokande signal is due to $\nu e$-scattering). Including the effects of neutrinos from other reactions strengthens the disagreement.

3. The disagreement between the Kamiokande and Homestake results relaxes (but does not disappear) if one takes the Homestake results after 1986, i.e. during the time of operation of Kamiokande II+III. The boron neutrino flux extracted from the Homestake-II data is about $2.5\sigma$ below the Kamiokande flux. However one should use the Homestake data alone with caution. The Homestake-II signal is appreciably higher than the Homestake-I signal. Moreover, the shape of the distribution: number of runs with a given Ar-production rate, $n = n(N_{Ar})$, after 1986 does not agree with Monte-Carlo simulation which describes rather well the distribution of Homestake-I runs. After 1986 the distribution has a thin peak in the interval $N_{Ar} = 0.7 - 0.9$ at/day. If the excess is the statistical fluctuation, one expects in future the convergence of the Homestake-II results to the average value for all runs. The peak could be a result of some systematics (background ?); its removing gives the average after 1986 in agreement with the average during all observation time.

The Homestake and Kamiokande results imply strong suppression of the $\nu_{Be}$-flux. New GALLEX-I+II results with smaller errors give the important bounds. They (as well as SAGE results) are at the level of minimal signal which follows from the luminosity normalization condition. The signal practically coincides with sum of signals induced by unsuppressed pp-neutrino flux (according to luminosity normalization) and by boron neutrino flux as measured by Kamiokande.
6. Present data testify for strong suppression of the beryllium neutrino flux. Certainly, 30 - 40 % reduction of the astrophysical factor $S_{34}$ is not enough to solve the problem. The desired suppression of Be-branch of the pp-cycle implies strong increase of $S_{33}$ ($^3$He - $^3$He resonance ?). Plasma effects are basically reduced to the decrease of the central solar temperature which in turn results in more strong suppression of $\nu_B$-flux than $\nu_{Be}$-flux, i.e. does not solve the problem.

7. New measurements of nuclear cross-sections as well as plasma effects revision will result probably in essential reduction of the boron neutrino flux. However, even “Minimal flux models” which could accommodate the Kamiokande flux predict the argon production rate $Q_{Ar} \sim (4.2 - 4.5) \text{ SNU}$, whereas experimental value of $Q_{Ar}$ averaged over all observation time is $\approx 8\sigma$ smaller. For germanium production rate these models give about 110 SNU, new GALLEX result is $2.5\sigma$ lower.

8. The situation when the fluxes of the pp-neutrinos and the boron neutrinos are unsuppressed (or weakly suppressed), whereas the beryllium neutrino flux is strongly suppressed can be easily reproduced by resonant flavor conversion (MSW). The solution corresponds to thin suppression pit (as function of energy) which is realized at $\Delta m^2 \sim (0.3 - 1) \cdot 10^{-5} \text{ eV}^2$ and small values of mixing angle $\sin^2 2\theta = (0.8 - 1.5) \cdot 10^{-3}$. In this case for high energy part of $\nu_B$ spectrum one predicts a weak distortion and small effect of neutral currents.

9. It is not excluded that the problem may have a “detection solution”, i.e. that interpretation of the results of some (one ?) experiments is incorrect.

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References

[1] R. Davis Jr., Proc of the 23th ICRC, Calgary, Canada (1993), Prog. in Nucl. and Part. Phys. 32 (1994).

[2] K. S. Hirata et al Phys. Rev. Lett. 66 (1991) 9; Phys. Rev. D44 (1991) 2241; Y. Suzuki, in TAUP-93.
[3] P. Anselmann et al., Phys. Lett. B285 (1992) 376, 390; P. Anselmann et al., Phys. Lett. B314 (1993) 445; Preprint LNGS 94/89

[4] A. I. Abasov et al., Phys. Rev. Lett. 67 (1991) 3332; V. N. Gavrin, Talk given at Int. Symposium "Neutrino Telescopes", Venice, February (1994).

[5] J. N. Bahcall and R. K. Ulrich, Rev. Mod. Phys. 60 (1989) 297.

[6] J. N. Bahcall and M. M. Pinsonneault, Rev. Mod. Phys. 64 (1992) 885.

[7] S. Turck-Chieze and I. Lopez, Astroph. J., 408 (1993) 347; S. Turck-Chieze et al, Phys. Rep. 230 (1993) 57.

[8] G. Bertomieu, J. Provost and J. Morel, Astron. Astroph. 268 (1993) 775.

[9] V. Castellani, S. Degl’Innocenti, G. Fiorentini, Phys. Lett. 303B (1993) 68; Astron. and Astroph. 271 (1993) 601.

[10] T. Motobayashi et al., Yale-40609-1141, Rikkyo RUP 94-2.

[11] K. Langanke and T.D. Shopa, Calt Map-168, 1994.

[12] V. Tsytovich, Talk given at the Int. workshop "Solar neutrino problem: astrophysics or oscillations", Gran Sasso, Italy, February 28 - March 1, 1994.

[13] D. N. Schramm and X. Shi, FERMILAB-Pub -9#/-400-A.

[14] A. Dar and G. Shaviv, Technion-Ph-94-5 (1994).

[15] I. Barabanov, unpublished, J. N. Bahcall and H. A. Bethe, Phys. Rev. D47 (1993) 1298.

[16] S. Bludman et al., Preprint UPR-0572T.

[17] V.S. Berezinsky, to be published in Comm. Nucl. Part. Phys. (1994).

[18] S. P. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl. Phys. 42 (1985) 913, Sov. Phys. JETP 64 (1986) 4; L. Wolfenstein, Phys. Rev. D17 (1978) 2369.

[19] S. J. Park, Phys. Rev. Lett., 57 (1986) 1275; W. Haxton, Phys. Rev. Lett., 57 (1986) 1271.

[20] P. I. Krastev and A. Yu. Smirnov, in preparation.
Figure Captions

Fig. 1. Comparison of the observed signals (hatched regions) with predictions of different solar models: 1, 2 - Bahcall-Pinsonneault (with and without diffusion) [6], 3 - Castellani et al. [4], 4 - Turck-Chieze - Lopez [7], 5 - Bertomieu et al [8], 6 - Schramm and Shi [13], 7 - Dar and Shaviv [14].

Fig. 2. A comparison of the boron neutrino flux measured by Kamiokande-II+III (solid) with that extracted from Homestake results (dashed).

Fig. 3. The number of runs of the Homestake experiment with a given Ar-production rate. a). Homestake-II (runs 90 - 126); b). the Homestake-I runs; c). Monte-Carlo simulation (from [1]). In fig.3d we show for comparison the corresponding distribution of runs for GALLEX experiment (30 runs).

Fig. 4. Branching of the pp-nuclear reaction chain.

Fig. 5. $\nu_B - \nu_{Be}$-plot. The lines restrict 1σ-regions allowed by the Homestake, GALLEX and Kamiokande experiments. Figures at the curves are the Ge-production rate in SNU’s. Dotted lines show the 1σ region allowed by Homestake measurements after 1986 (Homestake-II).

Fig. 6. The suppression factor due to the MSW-effect as function of the neutrino energy for different values of $\sin^2 2\theta$ (figures at the curves). Also shown is the energy spectrum of solar neutrinos (hatched).

Fig. 7. The $\Delta m^2 - \sin^2 2\theta$ regions of the solutions of the solar neutrino problem for different values of the original boron neutrino flux: $\Phi_B = r\Phi_{SSM}^{B}$, where $\Phi_{SSM}^{B}$ is from [6].
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