Reference calculations for subthreshold Ξ production

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Abstract. We present a minimal statistical model designed for the description of rare-hadron multiplicities in nucleus-nucleus collisions at energies below the threshold of the particle production in binary elementary collisions. Differences to more conventional canonical statistical model are explained. The minimal statistical model is applied to the description of multiplicity ratios involving Ξ hyperons, which are measured by the HADES collaboration at GSI-SIS. It is argued that the HADES data cannot be reproduced by the model based on the statistical equilibrium and the strangeness conservation. The data remain underpredicted even when in-medium potentials acting on hadrons are taken into account. This hints to non-equilibrium production of the Ξ hyperons and their continuous freeze-out.

1. Introduction
Data obtained by HADES collaboration [1] show that doubly strange Ξ hyperon can be produced in nuclear collisions at energies well below the production threshold for nucleon-nucleon collisions. In Ar+KCl collisions at $E_{\text{beam}} = 1.76$ AGeV the observed ratio of Ξ to $(\Lambda + \Sigma^0)$ multiplicities is $M_\Xi/M_{\Lambda + \Sigma^0} = (5.6 \pm 1.2^{+1.5}_{-1.7}) \times 10^{-3}$, and some Ξ's are also seen in Au+Au collisions at 1.23 AGeV. Statistical model with canonical suppression of strangeness production predicts a value of the order $10^{-4}$ for the ratio.

2. Isospin symmetry and total strangeness content
The colliding nuclei with the atomic number $A$ have usually different number of protons ($Z$) and neutrons ($A-Z$) and thereby the matter created in their collisions has a non-zero total isospin projections quantified by the asymmetry coefficient $\eta = (A-Z)/Z$ being different from one. In our calculations we assume that this asymmetry is reflected in the observed multiplicities of hadrons belonging to one isospin multiplet, e.g,

$$\frac{M_{K^0}}{M_{K^+}} = \frac{M_{K^-}}{M_{K^0}} = \frac{M_{\Xi^-}}{M_{\Xi^0}} = \frac{M_{\Xi^-}}{M_{\Xi^0}} = \frac{M_n}{M_p} = \eta. \quad (1)$$

For the Ar+KCl collisions we have $\eta \approx 1.14$.

We can use the relations (1) to determine the total amount of produced $s\bar{s}$ quark pairs (distributed among hadrons). In a baryon-rich environment, created in collisions at SIS energies, the $\bar{s}$ antiquarks can only be accommodated in kaons. Furthermore, under such conditions kaons do not find partners for inelastic scattering and once produced they are guaranteed to leave the system. Thus the number of $s\bar{s}$ pairs can be reconstructed as

$$M_{s\bar{s}} = M_{K^+} + M_{K^0} = (1 + \eta)M_{K^+}. \quad (2)$$
Once the numbers of produced $s$ and $\bar{s}$ quarks are known we assume in our minimal statistical model [2] that the $s$ quarks are distributed among strange hadrons (anti-kaons and hyperons) with relative abundances determined by thermal and chemical equilibria and by the strangeness conservation. For our analysis we use the ratios which formally do not depend on the produced strangeness [2]

$$\frac{M_{K^-}}{M_{K^+}} = 2.54^{+1.21}_{-0.91} \times 10^{-2}, \quad \frac{M_{\Lambda^0 + \Sigma^0}}{M_{K^+}} = 1.46^{+0.49}_{-0.37}$$

$$\frac{M_{\Sigma^+ + \Sigma^-}}{2M_{K^+}} = 0.30^{+0.23}_{-0.17}, \quad \frac{M_{\Xi^-}}{M_{K^+} + M_{\Lambda^0 + \Sigma^0}} = 0.20^{+0.16}_{-0.12}. \quad (3)$$

These ratios have the same total number of strange and anti-strange quarks in both numerator and denominator.

3. The minimal statistical model

At subthreshold energies no strange particles are created in incident nucleon-nucleon collisions and all strangeness must be produced during the evolution of the hot fireball created in the collision. Creation of strangeness is a very rare event, with $K^+$ multiplicity

$$M_{K^+} = (2.8 \pm 0.4) \times 10^{-2}. \quad (4)$$

The amount of $s\bar{s}$ pairs is thus far below the saturation and can be treated perturbatively. It will not only scale with the volume, but also we can assume that it will steadily grow with time. If — for simplicity — we assume ideal hydrodynamics, then the only length scale is given by $V^{1/3}$, where $V$ is the fireball volume, and it must determine also the lifetime [3]. At fixed impact parameter, the probability of creating one $s\bar{s}$ pair is given as

$$W = \lambda V^{4/3}. \quad (5)$$

For a fixed volume $V$, the probability to have $n$ of the $s\bar{s}$ pairs is Poissonian

$$P_{s\bar{s}}^{(n)} = e^{-\lambda V^{4/3}} \frac{W^n}{n!}. \quad (6)$$

The HADES data have been obtained after averaging over collisions with different centralities. The dependence on $V^{4/3}$ in (5) makes the averaging non-trivial. If we denote the centrality averaging by angle brackets, $\langle \cdots \rangle$, we can relate the parameter $\lambda$ to the measured kaon multiplicity as follows

$$\lambda = \frac{\langle W \rangle}{\langle V^{4/3} \rangle} = \frac{M_{s\bar{s}}}{\langle V^{4/3} \rangle} = \frac{M_{K^+}(1 + \eta)}{\langle V^{4/3} \rangle}. \quad (7)$$

In the minimal statistical model we assume that in an event with volume $V$ and $n$ pairs of $s\bar{s}$ quarks, the $s$ quarks are distributed into hadrons according to the probability

$$P_{a}^{(n)} = (z_s^{(n)})^{s_a} V p_a = (z_s^{(n)})^{s_a} V e^{B_a \mu_B / T} \frac{m_a^2 T}{2\pi} K_2 \left( \frac{m_a}{T} \right). \quad (8)$$

This is the probability to find hadron $a$ with the mass $m_a$, baryon number $B_a$ and strangeness content $s_a$. The chemical potential is determined by the fireball freeze-out density taken 0.6$\rho_0$, where $\rho_0 = 0.16$fm$^{-3}$ is the nuclear saturation density. The probability (8) is normalised — for each $n$ separately — by the factor $z_s^{(n)}$. 

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Our result for the Ξ multiplicity ratio is

\[ \frac{\mathcal{M}_\Xi}{\mathcal{M}_{\Lambda+\Sigma^0}\mathcal{M}_{K^+}} = \frac{(V^{5/3})(V)}{2(V^{4/3})^2} \frac{p_\Xi/(p_K + p_\Lambda + p_\Sigma)}{(p_\Lambda + \frac{mp_{\Sigma}}{\eta^2+\eta+1})}. \]  

In the pre-factor we collected the modifications with respect to the “usual” statistical model with the canonical suppression. This and all other calculated ratios are displayed in Fig. 1 by dashed lines as functions of temperature. We see that the calculated \( \mathcal{M}_\Xi/\mathcal{M}_{\Lambda+\Sigma^0}\mathcal{M}_{K^+} \) is clearly below the measured error band. In Fig. 1 we also revised the data on Σ/\( K^- \) ratio based on the actual isospin ratio from (1), see [2].

By trying to improve the agreement of our calculations with the data we have introduced density-dependent scalar and vector potentials \( S_a \) and \( V_a \) [2]. They modify the statistical distribution function by replacing the dispersion relation to \( E(p) = \sqrt{(m_a + S_a)^2 + p^2 + V_a} \). The chosen values were \( V_\Sigma = V_\Delta = \frac{3}{2}V_\Lambda = 3V_\Sigma = 130 \text{ MeV} \rho_B/\rho_0 \), and \( S_N = -190 \text{ MeV} \rho_B/\rho_0 \). The scalar potentials for strange particles follow from the relation \( S_a = (U_a - V_a(\rho_0))\rho_B/\rho_0 \), where we use \( U_\Lambda = -27 \text{ MeV}, U_\Sigma = 24 \text{ MeV}, U_\Xi = -14 \text{ MeV}, U_K = -75 \) or \(-150 \text{ MeV} \) and \( V_K = 0 \). This leads to changes in our theoretical results which allow to explain \( K^-/K^+, \Lambda/K^+ \) and \( \Sigma/K^+ \) ratios by one freeze-out temperature \( \sim 70 \text{ MeV} \). However, the changes are not large enough to bring the \( \Xi/\Lambda/K^+ \) ratio into agreement with the measured data.

It is interesting that accounting for the trigger on central collisions actually makes the disagreement even bigger. The culprit is in the volume factors which reduce the overall result when selection on larger volumes is made.

Figure 1. Multiplicity ratios as functions of the temperature. Red dotted lines with purple bands show the measured data values (3). Dashed lines are minimal statistical model predictions according to (10). Solid lines are predictions with in-medium potentials included. For the \( \mathcal{M}_{K^-}/\mathcal{M}_{K^+} \) ratio we show results with two values of \( K^- \) optical potential acting on \( \bar{U} = -75 \) and \(-150 \text{ MeV} \) (other curves are calculated with \(-75 \text{ MeV} \) ). Dash-doubly-dotted curves show ratios with only nucleon potentials included.
5. A prediction for Au+Au collisions at 1.23 GeV

Although the model disagrees with the Ar+KCl data, for curiosity we can make a prediction also for the Au+Au collisions recently recorded by HADES. As seen from Fig. 1, the temperature dependence of $\frac{M_{\Xi}}{M_{A+^{39}K^+}}$ is very weak. The crucial difference to the smaller system is in the volume which is almost five times bigger for Au+Au than for Ar+KCl. As the ratio (10) is proportional to $V^{-1}$, we expect that it will be five-times smaller in gold-induced reactions than in Ar+KCl. Below the strangeness production threshold this statement is independent of the collision energy.

6. Conclusions

In comparison to the canonical statistical model, the minimal statistical model assumes that strangeness must be produced during the fireball evolution and takes into account the proper scaling of rare species multiplicity with the volume. This leads to non-trivial differences between the two models and the result of the minimal statistical model on centrality-averaged multiplicities depends on a set of averaged volume factors.

In any case, the conclusion we could draw from our study is that cascade production is non-statistical. We speculate that it might be caused by immediate decoupling of the produced $\Xi$’s from the bulk matter. At an early hot stage some $\Xi$’s are produced which are later not annihilated at lower temperature. Those particles chemically decouple from the system.

We speculate that the most viable for such a production of $\Xi$ would be the strangeness recombination reactions, either involving $\bar{K}$ and a hyperon, or two hyperons. Note that we have provided parametrised cross-sections for such reactions in [4]. An interesting option appears in the $\bar{K}\Lambda$ channel, which in vacuum has a high reaction threshold of 1609 MeV. Due to effective lowering of kaon mass in the baryon-rich environment it can decrease so that eventually it hits the $\Xi^*$ resonance around 1530 MeV. This could increase the cross section for $\Xi$ production considerably.

Kinetic calculations aiming the validation of these mechanisms are planned for the near future.

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