A New Three-Dimensional Noise Modeling Method Based on Singular Value Decomposition and Its Application to CMONOC GPS Network

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Abstract  Construction of noise model is an important task in the analysis of Global Positioning System (GPS) reference station coordinate time series. Ignoring the relationship between the noise on different components within a GPS station network may affect the accuracy of station's velocity and its uncertainty. In view of this problem, we propose to use the singular value decomposition (SVD) method to establish a new three-dimensional (3-D) noise model for GPS station networks. Our simulation tests show that the accuracy of the noise amplitude obtained based on the proposed 3-D noise model is 30–50% higher than that directly obtained from Create and Analyze Time Series (CATS) software, thereby improving the accuracy of the velocity uncertainty by approximately two times. Taking the coordinate time series of 82 GPS stations from the Crustal Motion Observation Network of China (CMONOC) as an example, we confirm that significant correlation exists among noise amplitude estimates in the different components of the CMONOC stations. In general, the variation of white noise (WN) amplitude is 2–6% smaller than that of flicker noise (FN) amplitude, and the FN amplitude in the vertical component is 1% larger than that in the horizontal component. Compared with the velocity uncertainty obtained from CATS software, the variations of the velocity uncertainty obtained from the new 3-D noise model in the horizontal components (7% for North, 8% for East) are slightly less than that in the Up component (9%). However, the velocity estimation is hardly affected by the new 3-D noise model.

1. Introduction

Due to various factors, the random process in the Global Positioning System (GPS) station coordinate time series appears as a combination of white and colored noise (Feng et al., 2016; He et al., 2016; Jiang et al., 2014; Langbein, 2012; Liu et al., 2016; Mao et al., 1999). Until now, the existing noise model establishment of GPS station coordinate time series was mostly based on residual sequences obtained by removing the displacement time series of the station from the coordinate time series in single component of a single station (He et al., 2019; Langbein & Svarc, 2019; Li et al., 2018; Wang et al., 2012; Williams et al., 2004), which ignored the correlation between noise in the different components and of different stations. Recently, using the principal axis regression method, Jiang et al. (2018) confirmed that there were actually correlations between noises in two different components of GPS stations in Southern California. However, this regression method still ignored the noise in the third component of stations; thus, the obtained noise model is incomplete from a theoretical perspective. Moreover, since an accurate noise model would affect the velocity uncertainty of GPS stations (Mao et al., 1999), this kind of ignorance may also result in an inaccurate interpretation of the plate construction signal in the area where the station is located.

The Crustal Motion Observation Network of China (CMONOC) is an important infrastructure based on Global Navigation Satellite System (GNSS) technology, with its main purpose of obtaining the spatial-temporal trends of the large-scale crustal movement throughout Mainland China (Deng et al., 2017; Du et al., 2013; Guo et al., 2017; Huang et al., 2017; Tian, 2011; Wang et al., 2014; Wu et al., 2018; Yang et al., 2017; Zhou et al., 2017; Zhu et al., 2017). Due to the complexity of topography and geological structure, it is thus very important to study the noise characteristics of the CMONOC GPS station coordinate time series, so that more accurate velocity uncertainty can be obtained for crustal motion interpretations. Previous results have already shown that the best noise model for the CMONOC GPS stations are white noise (WN).
plus flicker noise (FN) (Wang et al., 2012). However, all these results were based on a component and a station. Do correlation exist among different components of this large-scale CMONOC GPS network stations? If it is true, would it affect the noise amplitude and the velocity uncertainty? These are also focuses of our research.

In this paper, we define the functional model that describes the relationship of noise amplitude in three components (North, East, Up) of stations in the GPS network as a new three-dimensional (3-D) noise model. Given that the regression equation established by the principal axis regression criterion is actually the best fitting straight line of the noise amplitude estimates in two different components of stations (Wu et al., 2018), a 3-D noise model can also be represented by a spatially best fitting straight line of noise amplitude estimates in the three components. Through maximizing the sum of projected squares from a point set to a line, the traditional singular value decomposition (SVD) method, an important matrix decomposition method used in linear algebra could find the best fitting line of this point set (Kanjilal & Palit, 1995), which satisfies the requirement of a 3-D noise model. Therefore, here we try to develop a new 3-D network noise modeling method by considering the correlation among noise amplitude in the three components of regional GPS stations based on the SVD method.

The remainder of this paper is structured as follows: section 2 describes the simulated and measured GPS data sources, together with the method of building the new 3-D noise model based on SVD. Section 4 uses simulated data to verify the fitting effect of the proposed 3-D noise model. Based on the correlation analysis among noise amplitudes in the different components of the 82 CMONOC GPS stations after removal of the common mode error (CME), a more precise 3-D noise model for CMONOC network is then established in section 3 using our proposed method. Conclusions are drawn in section 5.

2. Data and Methods

2.1. Simulated Station Coordinate Time Series

Since true noise amplitude values for measured GPS coordinate time series cannot be obtained, we try to use simulated time series first to test the effect of our new 3-D noise model on the noise amplitude estimate. We simulate time series of 100 stations in the North (N), East (E), and Up (U) components for 6 years. These time series consist of linear trend, annual and semiannual signals, WN together with FN, among which the WN amplitudes in the horizontal (N and E) components are equal, ranging between 0.325 and 3 mm, while that in the U component are twice of the horizontal components (0.65–6 mm). The FN amplitudes are within 0.55–6.5 and 1.1–13 mm/yr$^{0.25}$ for the horizontal and U components, respectively.

2.2. Measured Station Coordinate Time Series

We use time series of the CMONOC GPS reference stations as an example to construct a more precise 3-D noise model (available at http://www.cgps.ac.cn/). We select stations that are least affected by earthquakes from October 2011 to August 2017 and also remove those with missing data of more than 1 month during this period. Finally, the coordinate time series in the N, E, and U components of 82 stations under the ITRF2014 framework during the period are used. The locations of these stations are shown in Figure 1, and the models used in GPS data processing are shown (ftp://ftp.cgps.ac.cn/doc/processing_manual.pdf). During data preprocessing, obvious “jumps” caused by the replacement of antennas are estimated and removed. At the same time, residuals of greater than three times of standard deviations after removing linear trend of the coordinate time series are considered as gross errors and eliminated (Jiang & Zhou, 2015). Due to limited space, here we only list the coordinate time series of six representative stations in Figure 2, e.g., station BJFS, XJBE, HBXF, YNJP, HRBN, and XIAM in the North, Northwest, Central, Southwest, Northeast, and Southeast region of China, respectively. Readers can access the shared network disk mentioned in the acknowledgment to obtain the complete time series of the 82 stations.

2.3. Singular Value Decomposition

Assume that \( A = [a_1^T a_2^T a_3^T \cdots a_n^T]^T \), among which \( a_i = [a_{i1} a_{i2} a_{i3} \cdots a_{id}] \) is a matrix with \( n \) (number of points) rows and \( d \) (spatial dimension) columns. The SVD of matrix \( A \) can then be obtained by the following formula (Yanai et al., 2011):
where $U$ and $V$ are orthogonal matrices that representing the left and right singular vector of matrix $A$, with $V = [V_1 \ V_2 \ V_3 \cdots V_d]$, among which $V_1 \ V_2 \ V_3 \cdots V_d$ are singular vectors corresponding to singular values, and $V_1$ is the first singular vector of $A$. $D = \text{diag}(\sigma_1, \sigma_2, \sigma_3 \cdots \sigma_d)$, with $\sigma_1, \sigma_2, \sigma_3 \cdots \sigma_d$ denoting singular values arranged from large to small.

To gain insight into SVD, the rows of $n \times d$ matrix $A$ are regarded as $n$ points in a $d$-dimensional space, and the problem of finding the best fitting $k$-dimensional subspace of this point set is considered. The "best" here means to minimize the sum of the squares of the perpendicular distances from these points to the subspace (Hopcroft & Kannan, 2013). In other word, the sum of the squares of the projections of these points to the subspace is the largest. If $k = 1$, the subspace then represents a straight line. Here, since we hope to obtain a unique optimal noise amplitude through SVD, the optimal noise amplitude must be located in a one-dimensional subspace. That is, the subspace is actually a best space fitting straight line of the point set, which represents the noise amplitudes.

Let $v$ be the unit vector parallel to the best-fit straight line of the point set, then the projection length of $a_i(i = 1, 2, 3 \cdots n)$ on $v$ is $|a_i \cdot v|$, and the square sum of the projection length of the point set on the best-fit straight line is $|Av|^2$. To get the best-fit straight line requires that we maximize $|Av|^2$, thus minimizing the sum of the squares of the distance from these points to a straight line. The relationship between the largest singular value $\sigma_1$ and the first singular vector $V_1$ can be expressed by (Hopcroft & Kannan, 2013)}

$$A = UDV^T$$
\[ \sigma_i^2 = |AV_1|^2 = \sum_{i=1}^{n} (a_i \cdot V_1)^2 \quad (2) \]

\[
V_1 = \arg \max |Av| \\
|v| = 1 \quad (3)
\]

where \( V_1 \) is the direction vector of the best-fit straight line (a nonzero vector parallel to the straight line denotes the direction vector of the straight line). After determining the direction of the line, the point that passes through this line must be obtained to establish the equation for the best-fit straight line. Since the best-fit line passes through the average of these points in all directions (Roy, 1994), the best fitting straight line of the point set can then be determined by combining the first singular vector \( V_1 \) and the mean of the point data set. According to our earlier definition in section 1, for a point set consisting of the noise amplitudes in the three components of the GPS station network coordinate time series, its spatial best fitting straight line then represents the 3-D noise model of the regional GPS stations. Therefore, the key to construct a 3-D noise model is to determine the first singular vector using SVD, that is, directing the optimal fitting line.

**Figure 2.** Coordinate time series of the 6 representative stations throughout Mainland China. The blue, red, and yellow curves represent the N, E, and U components, respectively.
2.4. Establishment of a New 3-D Noise Model for Regional GPS Network

The specific steps for establishing a 3-D noise model of regional GPS stations based on SVD are as follows:

1. Based on the best noise combination model, the noise amplitude estimates are firstly obtained. Here, we choose the WN + FN combination model and estimate their noise amplitudes using the Create and Analyze Time Series (CATS) software (Williams, 2008).

2. The same type of noise amplitude estimates in the same component constitutes a noise vector. Take FN as an example, and assume that \( \hat{f}_N, \hat{f}_E, \) and \( \hat{f}_U \) are the FN amplitude estimates for the N, E, and U components of a station. The FN vectors of stations in the three component are \( \hat{f}_N = [\hat{f}_{N1}, \hat{f}_{N2}, \ldots, \hat{f}_{Nn}]^T, \)
\[ \hat{f}_E = [\hat{f}_{E1}, \hat{f}_{E2}, \ldots, \hat{f}_{En}]^T, \] and \( \hat{f}_U = [\hat{f}_{U1}, \hat{f}_{U2}, \ldots, \hat{f}_{Un}]^T, \) respectively, among which \( n \) represents the number of stations. The average FN amplitude estimates in the N, E, and U component are denoted as \( \bar{f}_N, \bar{f}_E, \) and \( \bar{f}_U. \)

3. Compose matrix \( F = [f_N, f_E, f_U] \) using the above FN vectors, with specific form as follows:
\[
F = \begin{bmatrix}
\hat{f}_{N1} & \hat{f}_{E1} & \hat{f}_{U1} \\
\hat{f}_{N2} & \hat{f}_{E2} & \hat{f}_{U2} \\
\vdots & \vdots & \vdots \\
\hat{f}_{Nn} & \hat{f}_{En} & \hat{f}_{Un}
\end{bmatrix} \quad (4)
\]

SVD is then performed on matrix \( F \) according to Equation 1 to obtain the first singular vector \( V_1 = [V_N, V_E, V_U]^T, \) where \( V_N, V_E, \) and \( V_U \) are elements of \( V_1 \) in the N, E, and U directions, respectively.

4. Position of the best space fit straight line could be determined when point \( M_0(\bar{f}_N, \bar{f}_E, \bar{f}_U) \) and the direction vector \( V_1 \) are known. Assume that \( \hat{f}_N, \hat{f}_E, \) and \( \hat{f}_U \) are the obtained new fitted FN noise amplitudes for the N, E, and U components of a station based on the proposed 3-D noise model, the point \( P = P_0(\hat{f}_N, \hat{f}_E, \hat{f}_U) \) is then on the fitted straight line, and the vector \( M_0P_0 = (\hat{f}_N - \bar{f}_N, \hat{f}_E - \bar{f}_E, \hat{f}_U - \bar{f}_U) \) is parallel to the direction vector \( V_1, \) which means that the corresponding coordinates of the two vectors are proportional, as shown in Equation 5:
\[
\frac{\hat{f}_N - \bar{f}_N}{V_N} = \frac{\hat{f}_E - \bar{f}_E}{V_E} = \frac{\hat{f}_U - \bar{f}_U}{V_U} \quad (5)
\]

Equation 5 is the best spatial fit linear equation, thus can be represented as the 3-D FN model of the station coordinate time series within a GPS station network. The mean of estimates \( \bar{f}_N, \bar{f}_E, \) and \( \bar{f}_U \) and the direction elements \( V_N, V_E, \) and \( V_U \) of the first singular vector are then defined as the parameters of the proposed new 3-D noise model. As a symmetry equation for a space line, Equation 5 can also be written as a parametric equation:
\[
\begin{align*}
\hat{f}_N &= \bar{f}_N + V_N t \\
\hat{f}_E &= \bar{f}_E + V_E t \\
\hat{f}_U &= \bar{f}_U + V_U t
\end{align*} \quad (6)
\]
where \( t \) is a parameter whose absolute value represents the distance from point \( P_0 \) to point \( M_0. \) From this point of view, different stations are likely to correspond to different \( t \) values in the 3-D noise model. Figure 3 shows the self-developed scheme of the above work to make readers better understands the process to build a 3-D noise model. In our later experiments, we write a data processing script using
MATLAB software according to the above scheme, among which the SVD function in the script comes from MATLAB. Note that before running the script, special software, for example, here, the CATS software, must be used to estimate the noise amplitude in the GPS station coordinate time series.

2.5. New Fitted Noise Amplitudes Based on the Proposed 3-D Noise Model

According to Formula 6, in order to obtain the new fitted noise amplitudes ($\tilde{f}_N$, $\tilde{f}_E$, and $\tilde{f}_U$) after constructing the proposed 3-D noise model, the premise is to determine the value of parameter $t$, which can be calculated by using the noise amplitude estimates and the 3-D noise model. In an ideal case, if $P(\tilde{f}_N, \tilde{f}_E, \tilde{f}_U)$

Figure 3. Scheme of establishing the proposed new 3-D noise model for a GPS network.
without errors is on the straight line in the space, parameter $t$ can then be obtained by substituting the estimate in any component into Equation 6. However, in reality, point $P$ does not lie in the space fitting line. Therefore, if we substitute the estimates in three components into Equation 6, we will get three different values of $t$:

$$\frac{(\tilde{f}_N - \tilde{f}_N)/V_N = t_N, \ (\tilde{f}_E - \tilde{f}_E)/V_E = t_E, \ and \ (\tilde{f}_U - \tilde{f}_U)/V_U = t_U.}$$

In this way, there will be 3 points distributed on the space fitting line: $P_1(\tilde{f}_N, \tilde{f}_E + V_E t_N, \tilde{f}_U + V_U t_N)$, $P_2(\tilde{f}_N + V_N t_E, \tilde{f}_E, \tilde{f}_U + V_U t_E)$, and $P_3(\tilde{f}_N + V_N t_U, \tilde{f}_E, \tilde{f}_U + V_U t_U)$, as shown in Figure 4. We can see that for coordinates of these three points, the noise amplitudes of only two components are adjusted, and the new noise amplitude in the third component is actually the original CATS estimate. Hence, to obtain the point $P_0(\tilde{f}_N, \tilde{f}_E, \tilde{f}_U)$ that represents the single adjusted noise amplitudes in the three components, the mean value $\overline{t}$ of $t_N, t_E,$ and $t_U$ is substituted into Equation 6, that is, $\tilde{f}_N = \overline{t} \times V_N + \tilde{f}_N, \tilde{f}_E = \overline{t} \times V_E + \tilde{f}_E,$ and $\tilde{f}_U = \overline{t} \times V_U + \tilde{f}_U.$ These resulted values of $\tilde{f}_N, \tilde{f}_E,$ and $\tilde{f}_U$ are then called as the fitted noise amplitude hereafter.

The self-developed scheme to generate the new fitted noise amplitude based on the 3-D noise model is illustrated in Figure 5. The same as section 2.4, the corresponding script is also written in the MATLAB environment during our later experiments.

### 3. Fitting Effect of the 3-D Noise Model Using Simulated Data

As described in section 2.4, CATS software was employed first to estimate the noise amplitudes of the generated simulated time series based on WN + FN model (Williams, 2008). Within the MATLAB script, the noise vectors in the N, E, and U components of the simulated GPS network were formed by stacking different types of estimates in different components of all stations. Then, the new 3-D noise models for WN and FN of the simulated time series were established, respectively. After that, the fitted noise amplitudes were obtained using CATS noise amplitude estimates (CATS estimates for short hereafter) and the 3-D noise model. Figure 6 shows spatial distributions of the true noise amplitudes, CATS estimates, and the fitted noise amplitude values of the simulated data. We observe that for both FN and WN, the CATS estimates are uniformly around the straight line where the true noise amplitude lies, and the straight line where the fitted noise amplitude values are located is almost coincident with the straight line where the truth lies.

Statistical comparison of the CATS estimates and the 3-D fitted noise amplitude values with respect to true noise amplitudes are shown in Table 1. We notice that the differences between CATS estimates and the true values are significant, while those between 3-D fitted noise amplitudes and the true values decrease by about 30–50% compared with the CATS results. Correspondingly, the velocity uncertainty differences obtained from CATS estimates and the true values are also approximately double of those from the new fitted and the true values. Nevertheless, the velocity estimate is almost unaffected by changes in the noise amplitude estimate. Hence, we conclude that using our 3-D noise modeling method to fit the noise amplitude, estimates could obtain more precise noise amplitude of the GPS time series, thereby improving the accuracy of the velocity uncertainty.

From Table 1, we also observe that the difference between CATS estimate and the true value of FN is about 7 times bigger than that of WN, and the difference in the vertical component is about twice of that in the
horizontal component. In addition, although a more accurate FN amplitude can be obtained through 3-D noise modeling, the difference between 3-D fitted FN amplitude and the true value is still much larger than that of WN. This shows that the accuracy of FN estimate is not as good as that of WN estimate, and the accuracy of noise amplitude estimate in the horizontal component is better than that in the vertical component.

For further comparison, statistical results for the two-dimensional (2-D) fitted noise amplitude obtained from regression equations of noise in two different components are also shown in Table 1 (the last column). It can be seen that the RMS values for 2-D result are about 20–50% bigger than that of the 3-D results, indicating that our proposed 3-D noise model is better than the previous regression equation method to obtain more accurate noise amplitude and velocity uncertainty.
4. Application of the Proposed 3-D Noise Modeling Method to CMONOC GPS Network

4.1. Linear Regression Analysis for the Noise Amplitudes of CMONOC Stations

According to Jiang et al. (2018), the correlation between noise amplitudes in any two components is a necessary condition to establish a 2-D noise model (linear regression equation). Similarly, we should first analyze the correlation between noise amplitude estimates in different components of the GPS station coordinate time series before establishing the 3-D noise model, since the 3-D noise model is a spatial straight-line equation that describes the relationship of noise amplitudes among the three components. Figure 7 shows the 3-D distribution of the CATS estimates for WN (left panel) and FN (right panel) models of all the selected 82 CMONOC stations. It can be seen that the CATS noise estimates for each component are approximately linear in the 3-D space, and the straight-line feature of the WN is more pronounced than FN. This finding indicates that there very likely to be a linear relationship between noise amplitudes among different components of real GPS network.

It is already known that CME exists in global and regional GPS station networks, which consists of the main error sources of GPS station coordinate time series (Dong et al., 2006). Since removing CME could help improve signal-to-noise ratio, as well as the accuracy and reliability of the station position (Tian & Shen, 2016), linear regression analysis is performed on the CATS noise amplitude estimates after removing CME in this research, and the statistical results in listed in Table 2. Note that symbols in Table 2 have the same meaning as those in Tab. 2 of Jiang et al. (2018).

From Table 2, we observe that strong correlation exists between the CATS obtained noise amplitude vectors in different components (>0.8), and the correlation coefficients of WN (>0.9) are greater than that of FN (<0.9). Combined with results from Table 1, since the accuracy of WN amplitude estimate is higher than that of FN, we therefore can...
draw the conclusion that the correlation between noise amplitude estimates is related to the accuracy of the noise amplitude estimates. Moreover, the \( p \) values in Table 2 are considerably lower than the significance level of 0.05, indicating that the null hypothesis is rejected, and the regression effect is significant. In another word, the amplitudes of the same type of noise in each pair of components for the CMONOC GPS network have a linear relationship, and they are distributed in the same line in space, which satisfies the prerequisites for establishing a 3-D noise model.

Table 2

| Variable       | Correlation coefficient | \( p \) value |
|----------------|-------------------------|---------------|
| \( w_N, w_E \) | 0.94                    | <<0.01        |
| \( f_N, f_E \) | 0.87                    | <<0.01        |
| \( w_N, w_U \) | 0.96                    | <<0.01        |
| \( f_N, f_U \) | 0.84                    | <<0.01        |
| \( w_E, w_U \) | 0.92                    | <<0.01        |
| \( f_E, f_U \) | 0.83                    | <<0.01        |

Figure 7. Spatial distributions of the CATS noise amplitude estimates (blue dots) of 82 CMONOC stations for WN (left panel) and FN (right panel) models. \( w \) and \( f \) represent the WN and FN amplitudes, while subscripts N, E, and U indicate the North, East, and Up components.

4.2. Construction of the Proposed 3-D Noise Model for CMONOC GPS Stations

Based on the CATS estimates obtained in section 4, we then use the self-developed MATLAB script as described in section 4 to establish the 3-D noise model of the CMONOC GPS stations for WN and FN, respectively, with model parameters as shown in Table 3. It is worth noting that the mean of the estimates in Table 3 has a unit, while the direction element is unitless. Finally, through using the described procedure in section 2.5, the 3-D fitted noise amplitudes are obtained, and the spatial distributions of CATS estimates and the 3-D fitted values in all components of all stations are shown in Figure 8. We can clearly see that the fitted values are distributed on the best-fit straight line, and the CATS
estimates are evenly distributed around the fitted straight line. In addition, the WN amplitudes are much more closer to the fitted straight line than FN, which is mainly due to the higher correlations between WN amplitudes in different components. Hence, we can conclude that the accuracy of the 3-D fitted noise amplitude is higher than the original CATS noise amplitude estimate. The greater the correlation between different components of noise amplitudes, the closer the noise amplitude estimates are to the true values.

### 4.3. Impacts of 3-D Noise Modeling on Noise Amplitude, Velocity, and Velocity Uncertainty

Taking into account linear motion, annual and semiannual motions, the covariance matrices of CMONOC GPS data are constructed using CATS estimates and the 3-D fitted values, respectively, from which we obtain the corresponding velocities and velocity uncertainties of the CMONOC station based on the least squares method. Since true values for the measured data cannot be obtained, here we only compare the changes in noise amplitudes, velocity, and velocity uncertainties before and after constructing 3-D noise model. The statistical results are shown in Table 4. We observe that overall the variation of WN amplitude is 2–6% smaller than that of FN. This is because the accuracy of CATS WN estimate is better than that of CATS FN estimate (Tables 1 and 2), thus leading to greater correlation of CATS WN amplitude than those of FN, and enhanced improvement of the FN amplitude than that of WN obtained from 3-D noise model. In addition, since the accuracy of vertical CATS FN estimate is worse than the horizontal estimate

| Noise type | Mean of estimates | Direction elements |
|------------|------------------|-------------------|
|            | $\bar{a}_N$ | $\bar{a}_E$ | $\bar{a}_U$ | $V_N$ | $V_E$ | $V_U$ |
| WN (mm)    | 0.65            | 0.78             | 2.66       | 0.22 | 0.34 | 0.91 |
| FN (mm/yr$^{0.25}$) | 2.02 | 2.21 | 7.41 | 0.25 | 0.28 | 0.93 |

**Figure 8.** Spatial distributions of CATS estimates (blue dots) and the 3-D fitted estimates (red dots) of 82 CMONOC stations after removing CME. The left panel (a) shows the WN result, and the right panel (b) shows the FN result.
Table 1, the variation of FN amplitude in the vertical component (13%) is slightly larger than that in the horizontal components (both 12% in N and E), and the changes of WN amplitude in the E component (10%) are also bigger than that in the N component (6%).

With respect to velocity and velocity uncertainty, we find that there is almost no difference between the obtained velocity estimates based on CATS estimates and the 3-D fitted values, which is the same as simulated data. However, the changes in velocity uncertainty of the vertical component (9%) are slightly bigger than that in the N (7%) and E (8%) components, respectively. Since the magnitude of velocity uncertainty is related to noise amplitude and FN is the dominant noise in GPS station coordinate time series (Jiang et al., 2018), the above changes can be explained by variations in the FN amplitude.

5. Conclusions

Due to limitations of noise models based on single component, a new 3-D noise modeling method for GPS network is proposed through using SVD. First, simulated data are used to verify the CATS obtained noise amplitude estimates and the 3-D fitted noise amplitudes. We find that the differences between CATS estimates and the true values are significant. The accuracy of the 3-D fitted noise amplitude is 30–50% higher than that of the CATS estimate, thereby approximately doubling the accuracy of stations’ velocity uncertainties. In addition, the differences between 2-D fitted noise amplitudes and the true values are also about 20–50% larger than that between 3-D fitted noise amplitudes and the true values, which further confirms the advantage of our 3-D noise modeling method to obtain a more precise noise amplitude.

We then take the coordinate time series of 82 CMONOC GPS stations as an example and find that the correlation coefficient between CATS obtained WN amplitude estimates in different components of CMONOC station is greater than 0.9, while that between FN is also within the most favorable range (0.8–0.9), thus confirming that strong correlations exist between noise amplitude estimates among different components of the CMONOC stations, thereby satisfying the prerequisite to establish a 3-D noise model.

Finally, the 3-D noise models for WN and FN of the CMONOC stations are established based on the correlation results according to our proposed method, from which the 3-D fitted noise amplitudes, together with their corresponding velocities and velocity uncertainties, are obtained. Compared with the analysis results based on CATS software, we find that our 3-D noise modeling method introduces almost no change in the velocity estimates for the three components. However, the WN amplitude varies at about 6%, 10%, and 8% for the N, E, and U components, respectively. FN amplitude exhibits much larger variations (12% for N, 12% for E, and 13% for U), thus leading to the biggest improvement of velocity uncertainty for the vertical component (9%), and slightly smaller improvement in the N (7%) and E (8%) components, respectively.

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Conflict of Interest

The authors declare no conflict of interest.
Data Availability Statement

The raw time series of these R2 GPS reference stations and their noise amplitude estimates, as well as the procedures for building a three-dimensional noise model, are uploaded to Figshare. Readers can browse the website where the above experimental data are stored according to the (https://doi.org/10.6084/m9.figshare.12812801), and download the experimental data.

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