Phase field modeling of multi-cluster hydraulic fracturing in horizontal wellbore with inconsistent direction to minimum principal stress

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Abstract. Zhongjiang gas field is a typical narrow channel tight sandstone gas reservoir. In order to obtain enough horizontal section length in high-quality reservoir, horizontal wellbore azimuth is basically consistent with direction of narrow channel sand body, resulting in a certain included angle $\theta$ between wellbore azimuth and in-situ minimum principal stress orientation. In this study, a seepage-stress-damage coupled dynamic fracture propagation model is established through phase field method to investigate the effect of angle $\theta$ on multi-cluster fracture simultaneous propagation based on geological engineering data of JS-X well. Propagation of hydraulic fracture is automatically tracked through phase field evolution without additional criteria or grid direction limitation. Simulation results show that: (1) Fracture tip of each cluster would approach to stress diversion zone created by adjacent clusters. Simultaneous propagating fractures of each cluster are prone to deflect, interconnect with each other and form one single fracture eventually. (2) Increasing cluster number or decreasing cluster spacing in the same fracturing stage would further promote diverting of each
cluster fracture and accelerates fractures merging into one single fracture. We suggest that azimuth of horizontal wellbore in tight gas reservoir should be consistent with in-situ minimum principal stress orientation to reduce the effect of angle $\theta$.

1. Introduction

Zhongjiang gas field is a typical narrow channel tight sandstone gas reservoir. High-quality reservoirs are generally distributed in the center of channels. Multi-cluster hydraulic fracturing in horizontal wellbore is an important technical means to improve development efficiency of Zhongjiang gas field. In order to obtain enough horizontal section length in high-quality reservoirs, horizontal wellbore azimuth is basically consistent with direction of narrow channel sand body, resulting in a certain included angle $\theta$ between wellbore azimuth and in-situ minimum principal stress orientation (Figure 1). The effect of included angle $\theta$ on hydraulic fracture propagation of each perforation cluster is not clear.

Phase field method is an elegant continuous fracture simulation method, which is derived from Griffith fracture theory based on variational principle. Phase field method does not require preliminary assumptions on shape of fractures and propagation of fracture is automatically tracked through energy minimization principle without additional criteria or grid direction limitation. Aiming at simulating hydraulic fracturing process, current framework of most phase field method research is how to better couple flow of reservoir domain and fracture domain. Zhou et al. [1] deal with flow transition domain between two domains. Biot theory and porous media theory (TPM) are main theories for constructing basic equations of reservoir domain. Darcy flow in reservoir domain and Reynolds flow in fracture domain [2, 3], and Darcy flow in reservoir domain and Stokes flow in fracture domain [4, 5] are two combinations for constructing fluid flow in different domains. Lee et al. [6] derive power-law fluid flow in fracture domain. Phase field parameter is often used as a bridge between reservoir domain and fracture domain. Chukwudozie et al. [7] establish a unified fracture-porous media flow model, and it is no longer necessary to use phase field parameters as indicator functions or weight functions to distinguish different calculation domains. Subsequent scholars have applied phase field method to study specific problems of hydraulic fracturing recently. Guo et al. [8] quantitatively study branching mechanism of hydraulic fractures in heterogeneous formation. Mollaali et al. [9] simulate CO2 fracturing under isothermal condition based on phase field method. Li and Zhou [10] examine multizone hydraulic fracture propagation in porous elastic medium. Shovkun and Espinoza [11] investigate propagation of hydraulic fractures in reactive porous media. Yi et al. [12] simulate hydraulic fracture propagation in porous media with natural fracture. Liu et al. [13] investigate hydraulic fracturing behavior in bedded shale with pre-existing fractures. Zeng et al. [14] propose a phase field based discrete fracture model to simulate fluid flow in fractured porous media. Zhuang et al. [15] examine hydraulic fracture propagation in naturally-layered porous media. Liu et al. [16] investigate influence of natural cavities on hydraulic fracture propagation.

In this study, a seepage-stress-damage coupled dynamic fracture propagation model is established based on phase field method. This model is proposed to investigate the effect of included angle $\theta$ on
multi-cluster fracture simultaneous propagation based on engineering geological data of JS-X well in Zhongjiang gas field.

**Figure. 1.** Plane graph of seismic amplitude energy of channel sand bodies (favourable tight gas reservoir) and JS-X well trajectory

### 2. Phase field modeling

#### 2.1 Phase field

Before establish phase field evolution equation, consider a solid $\Omega \subset R^n$ with a fracture set $\Gamma \subset R^{n-1}$ ($n$ is number of space dimension) as shown in Figure. 2, which is characterized by displacement field $\mathbf{u}$ and strain field $\mathbf{e}$.

Phase field models in the mechanics community originate from variational formulation of Griffith brittle fracture. From Griffith fracture theory, fracture propagation is a dynamic energy conservation process, which is described as followed:

$$\Psi_s + \Psi_e - \mathcal{P} = 0 \quad \text{\textbullet MERGEFORMAT (1)}$$

where stored strain energy $\Psi_s$, fracture surface dissipation energy $\Psi_e$ and external potential energy $\mathcal{P}$ are defined as:

$$\Psi_s = \int_\Omega \psi_0 (\epsilon (\mathbf{u}), \Gamma) dV \quad \Psi_e = \int_\Gamma G_e dA \quad \mathcal{P} = \int_\Omega \mathbf{b} \cdot \mathbf{u} dV + \int_{\partial \Omega} \mathbf{t} \cdot \mathbf{u} dA \quad \text{\textbullet MERGEFORMAT (2)}$$

In order to prevent compression failure, elastic energy density function $\psi_0$ is split into tension part and compression part,

$$\psi_0 (\epsilon (\mathbf{u}), \Gamma) = g(s) \psi_0^+ (\epsilon) + \psi_0^- (\epsilon) \quad \text{\textbullet MERGEFORMAT (3)}$$

where energy degradation function $g(s)$ is simplified as $g(s) = (1 - s)^2$, and $s$ represents diffusive definition of fracture topology ($s = 0$ stands for intact state, while $s = 1$ stands for complete damaged state).

Based on viscous regularization of rate-independent formulation proposed by Miehe et al. [17], rate-dependent dissipation function is:

$$G_e = \beta \dot{s} - \frac{1}{\eta} \langle \beta - g_e \cdot \delta_s \gamma \rangle_+ \quad \text{\textbullet MERGEFORMAT (4)}$$

where $\beta$ is local driving force, $g_e$ is threshold value of Griffith fracture energy, and $\delta_s \gamma$ is variational derivative of fracture surface density functional $\gamma$. 

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Fracture surface density functional \( \gamma \) is defined as:

\[
\gamma(s, \nabla s) = \frac{1}{2l} s^2 + \frac{1}{2} |\nabla s|^2 \quad \text{\(* MERGEFORMAT (5)*}
\]

where length parameter \( l \) scales width of diffuse fracture as shown in Figure 2.

Insert three parts of energy functional expressions into energy conservation equation, and apply Gauss theorem to get:

\[
\int_{\Omega} \left\{ -\text{Div}(\partial_s \psi) + b \right\} \cdot \mathbf{u} + \left[ \partial_s \psi + \partial_s G_c - \text{Div}(\partial_s G_c) \right] \mathbf{s} \, dV + \int_{\partial\Omega} (\partial_s \psi \cdot \mathbf{n} - t) \cdot \mathbf{u} \, dA + \int_{\partial\Omega} (\partial_s G_c \cdot \mathbf{n}) \mathbf{s} \, dA = 0 \quad \text{\(* MERGEFORMAT (6)*}
\]

For viscous model, three-field formulation of energy balance equation proposed by Miehe et al. [9] is:

\[
\text{Div} \left[ (1 - s)^2 \partial_s \right] = 0
\]

\[
g_c l \Delta s + 2(1 - s) \psi_0 (\nabla_s \mathbf{u}) - \beta = 0 \quad \text{\(* MERGEFORMAT (7)*}
\]

\[
\dot{s} - \frac{1}{\eta} \left( \beta - \frac{g_c}{l} \right)_s + 0
\]

Phase field evolution equation is acquired from (7), which is expressed as:

\[
\dot{s} = \frac{1}{\eta} \left[ 2(1 - s) \psi_0^2 - g_c \left( \frac{s}{l} - 1 \right) \right] \quad \text{\(* MERGEFORMAT (8)*}
\]

2.2 Displacement field

Motion equation of tight sandstone micro element is written as:

\[
\begin{align*}
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \alpha \frac{\partial p}{\partial x} + B_x &= \rho a_x \\
\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} + \alpha \frac{\partial p}{\partial y} + B_y &= \rho a_y \\
\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \alpha \frac{\partial p}{\partial z} + B_z &= \rho a_z
\end{align*} \quad \text{\(* MERGEFORMAT (9)*}
\]

where \( p_i \) \((i = x, y, z)\) is fluid pressure, \( B_i \) \((i = x, y, z)\) is body force acting on the element, \( a_i \) \((i = x, y, z)\) is acceleration, and \( \rho \) is density of tight sandstone.

2.3 Fluid pressure field

Continuity equation of fracture flow is:

\[
- \left[ \frac{\partial (\rho v_x \phi_f)}{\partial x} + \frac{\partial (\rho v_y \phi_f)}{\partial y} + \frac{\partial (\rho v_z \phi_f)}{\partial z} \right] dx dy dz dt + m_* = \frac{\partial}{\partial t} (\rho \phi_f) dx dy dz dt \quad \text{\(* MERGEFORMAT (10)*}
\]

where \( \phi_f \) is fracture volume ratio, \( v_i \) \((i = x, y, z)\) is fluid velocity in \( i \) direction, and \( m_* \) is source sink term of mass. 

Fracture volume ratio \( \phi_f \) depends on fracture width \( w \) and micro element length \( dL \), which is expressed as \( \phi_f = \frac{w}{dL} \).
Motion equation of fracture plate flow is:

\[
\begin{align*}
    v_x &= - \frac{w^2}{12\mu} \left( \frac{\partial p}{\partial x} - \rho g \frac{\partial H}{\partial x} \right) \cdot n_x \\
    v_y &= - \frac{w^2}{12\mu} \left( \frac{\partial p}{\partial y} - \rho g \frac{\partial H}{\partial y} \right) \cdot n_y \\
    v_z &= - \frac{w^2}{12\mu} \left( \frac{\partial p}{\partial z} - \rho g \frac{\partial H}{\partial z} \right) \cdot n_z
\end{align*}
\]

where \( \mu \) is fluid viscosity, \( H \) is reservoir depth and \( \rho \) is fluid density. When fluid flow direction is vertical to fracture normal direction, then \( n_i = 1 \) \((i = x, y, z)\). Otherwise, \( n_i = 0 \).

For two-dimensional fracture propagation, governing equation by combining continuity equation and motion equation can be obtained:

\[
\begin{align*}
    \frac{n_z}{12\mu dL} \left[ w^3 \frac{\partial^2 p}{\partial x^2} + \frac{\partial w}{\partial x} \frac{\partial^3 p}{\partial x^3} \right] + \frac{n_y}{12\mu dL} \left[ w^3 \frac{\partial^2 p}{\partial y^2} + \frac{\partial w}{\partial y} \frac{\partial^3 p}{\partial y^3} \right] + \frac{Q_s}{L} = \frac{1}{dL} \left( w_c + \frac{1}{K_n} \right) \frac{\partial p}{\partial t}
\end{align*}
\]

where \( K_n \) is fracture stiffness in normal direction, \( c \) is fluid compression coefficient, and \( Q_s \) is injection rate or leak off rate.

### 2.4 Numerical implementation and model validation

Numerical solution of above three field equations is realized by finite difference method. Fluid pressure will be applied to micro element when calculated order parameter exceed critical value, which is set as 0.9 in this paper.

Flow chart of numerical implementation is presented in Figure 3. Reliability and accuracy of present model have been verified by previous study [8, 16].

![Flow chart of numerical implementation](image)

**Figure 2.** a solid \( \Omega \) with a fracture set \( \Gamma \): (a) sharp description of a fracture and (b) diffuse description of a fracture based on phase field method
3. Model construction

Based on engineering geological data of JS-X well, this section establishes basic physical model to study the influence of included angle $\theta$ on multi-cluster fracture simultaneous propagation. Multi-cluster horizontal well fracturing physical model is implemented in 2-D on a 300 m $\times$ 200 m rectangle plane as shown in Figure. 4, which includes a stage of horizontal wellbore section. Direction of *in-situ* maximum principal stress $\sigma_H$ and minimum principal stress $\sigma_h$ are along long and short side of rectangle plane respectively. Angle between horizontal wellbore section and *in-situ* minimum principal stress $\sigma_h$ is so-called included angle $\theta$. Multiple perforation clusters are evenly distributed on horizontal wellbore section with the same cluster spacing and direction of perforation clusters is along *in-situ* maximum principal stress $\sigma_H$.

Input parameters of multi-cluster fracture simultaneous propagation simulation according to engineering geological data of JS-X well in Zhongjiang gas field are represented in Table. 1.

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**Figure. 3.** Flow chart of numerical implementation

**Figure. 4.** Multi-cluster horizontal well fracturing physical model considering included angle $\theta
Table 1. Parameters for hydraulic fracturing simulation

| Properties                        | Value     |
|-----------------------------------|-----------|
| Maximum horizontal stress $\sigma_H$ | 72 MPa    |
| Minimum horizontal stress $\sigma_h$ | 62 MPa    |
| Porosity of rock matrix           | 9 %       |
| Permeability of rock matrix       | 0.2 mD    |
| Young’s modulus of rock matrix    | 18000 MPa |
| Poisson’s ratio of rock matrix    | 0.22      |

4. Results and discussion

4.1 Effect of included angle $\theta$

Figure. 5 demonstrates multi-cluster fracture simultaneous propagation simulation results considering included angle $\theta$ based on phase field method. Although every fracture initially propagates along direction of in-situ maximum principal stress orientation, fracture tip of each cluster would approach to stress diversion zone (Blue Zone) created by adjacent clusters as shown in stress field distribution pattern (Figure. 5 (a)). Fractures of each cluster are prone to deflect, interconnect with each other and form one single fracture eventually. The effectiveness of fracture propagation in each cluster is limited, which is shown in fluid pressure distribution pattern in fracture of each cluster (Figure. 5 (b)).

Figure. 6 demonstrates multi-cluster fracture simultaneous propagation simulation results under different included angle $\theta$. As included angle $\theta$ increases gradually, number of clusters and cluster spacing remains unchanged, but the distance between clusters in the direction of in-situ minimum principal stress decreases. Fracture tip of each cluster become closer to stress diversion zone and easier to deflect and interconnect. Under the same number of clusters and cluster spacing, presence of included angle $\theta$ would greatly inhibit fracture propagation effectiveness.

![Stress Diversion Zone and Stress Zone of Perforation Cluster](image1)

**Figure. 5.** Simulation results of multi-cluster fracture simultaneous propagation under included angle 20°: (a) stress field distribution around perforation cluster and (b) fluid pressure distribution in fracture of each cluster
Figure 6. Simulation results of multi-cluster fracture simultaneous propagation under different included angle

4.2 Effect of cluster number and cluster spacing

Effect of cluster number and cluster spacing on multi-cluster fracture simultaneous propagation considering included angle $\theta$ are investigated. As can be seen from Figure 7, increasing number of clusters or decreasing cluster spacing in the same fracturing stage of horizontal wellbore section would further reduce the distance between clusters in the direction of in-situ minimum principal stress, which further promotes diverting of each cluster fracture and accelerates fractures merging into one single fracture. Increasing cluster number would also enhance inter-cluster stress interference as more fractures are created. Increasing cluster spacing cannot prevent fracture deflecting and coalescing under existing included angle $\theta$, but it would extend duration of this process. By analyzing simulation results, we suggest that azimuth of horizontal wellbore in tight sandstone gas reservoir should be consistent with in-situ minimum principal stress orientation as much as possible to reduce effect of included angle $\theta$. Besides, increasing cluster spacing and selecting appropriate number of clusters are beneficial to weaken inter-cluster stress interference effect and slow down fracture deflecting and interconnecting under existing included angle $\theta$. 
Figure. 7. Simulation results of multi-cluster fracture simultaneous propagation under combinations of included angle, clusters number and cluster spacing

5. Conclusion
This paper investigates the effect of included angle $\theta$ between wellbore azimuth and in-situ minimum principal stress orientation on multi-cluster hydraulic fracture simultaneous propagation based on phase field computational framework. Due to the existence of included angle, fracture tip of each cluster would be closer to stress diversion zone created by adjacent clusters. Fractures are prone to deflect, interconnect with each other and form one single fracture eventually. Increasing number of clusters or decreasing cluster spacing in the same fracturing stage would promote diverting and coalescing process. Increasing cluster spacing would extend duration of this process. Simulation results suggest that azimuth of horizontal wellbore in tight sandstone gas reservoir should be consistent with in-situ minimum principal stress orientation as much as possible. Increasing cluster spacing and selecting appropriate number of clusters help to slow down fracture deflecting and interconnecting process under existing included angle.

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