Improving Automated Symbolic Analysis for E-voting Protocols: A Method Based on Sufficient Conditions for Ballot Secrecy

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ABSTRACT
We advance the state-of-the-art in automated symbolic analysis for e-voting protocols by introducing three conditions that together are sufficient to guarantee ballot secrecy. There are two main advantages to using our conditions, compared to existing automated approaches. The first is a substantial expansion of the class of protocols and threat models that can be automatically analysed: we can systematically deal with (a) honest authorities present in different phases, (b) threat models in which no dishonest voters occur, and (c) protocols whose ballot secrecy depends on fresh data coming from other phases. The second advantage is that it can significantly improve verification efficiency, as the individual conditions are often simpler to verify. E.g., for the LEE protocol, we obtain a speedup of over two orders of magnitude.

We show the scope and effectiveness of our approach using ProVerif in several case studies, including FOO, LEE, JCJ, and Belenios. In these case studies, our approach does not yield any false attacks, suggesting that our conditions are tight.

1 INTRODUCTION
There have been substantial advances during the last years in the field of e-voting protocols. Many new approaches have been developed, the relevant security properties have become better understood and agreed upon [7, 15, 16]. The most important security property is that voters’ votes remain private, which is known as ballot secrecy. However, designing protocols that achieve this has proven subtle: many vulnerabilities have been found in previously proposed protocols [16, 25], motivating the need for improved analysis techniques. Because of its high level of automation, symbolic formal verification has proven to be very effective.

For classical security protocols, there is mature tool support [4, 8, 17, 27], which enables detecting many flaws during the protocol design phase, or later, as new threat models are considered. However, they traditionally did not handle e-voting protocols [18]. Recently, automated methods have been proposed [5, 10, 21, 28] to analyse e-voting protocols. However, their applicability is still extremely limited both in the type of protocols that they can deal with and the type of security properties that they analyse.

The reasons for these limitations interact in a complex way with existing approaches. The initial restrictions arise from the fact that (a) ballot secrecy is an behavioural equivalence-based property and not a reachability property, and (b) most proposed e-voting protocols use less standard primitives that are not covered by all tools. This naturally restricts the set of tools that can be applied, as acknowledged by [13, 18, 28].

Tools that deal with equivalence properties and an unbounded number of sessions are ProVerif [8] and Tamarin [27]. Since recent developments [21], both Tamarin and ProVerif can deal with typical primitives that are used in e-voting protocols, such as zero knowledge proofs, blind signatures, trapdoor commitments, designated verifier proofs [5, 10, 21]. ProVerif and Tamarin can check for so-called diff-equivalence, which is a property that implies behavioural equivalence. However, in practice, and in the context of the typical encoding of protocols and ballot secrecy, many protocols fail to satisfy diff-equivalence, even if they may satisfy behavioural equivalence. This effectively limits the class of e-voting protocols to which existing tools can be successfully applied. Recent efforts (e.g. swapping approach [10], small-attack property [28], swapping approach using multisets [21]) have aimed to increase the class of e-voting protocols that ProVerif and Tamarin can deal with, but many relevant protocols and threat models still cannot be usefully analysed automatically. Moreover, the proofs of ballot secrecy in Tamarin from [21] target very simple and limited threat models and require additional manual effort.

Contributions. In this work, we advance the state-of-the-art in automated symbolic verification of e-voting protocols. We focus on the most important privacy property: ballot secrecy. We revisit the state-of-the-art definition of ballot secrecy [19, 26] and propose a more precise variant whose automated verification does not rely on synchronisation barriers [10, 19–21, 26]. We believe that modularity is key when facing such complex systems & security properties and thus adopt a security via sub-conditions approach.

We develop three tight conditions on e-voting protocols and a main theorem that states that together, they imply ballot secrecy. The three conditions in our theorem are directly inspired from our analysis of the different types of attacks on ballot secrecy. Since each condition focuses on one specific aspect of ballot secrecy, it is typically simpler to check the combination of the three conditions than to verify ballot secrecy directly, as was done in prior works. We also provide an algorithm for automatically verifying all conditions. This gives rise to a new method to verify ballot secrecy, improving the state of the art in several aspects.

First, our approach significantly expands the class of protocols and threat models that can be analysed. Notably, unlike prior work, we can deal with the following features:

- Honest authorities that are present in different phases. For instance, one encounters such cases to model a registrar distributing credentials in a registration phase and then committing credentials of eligible voters to the bulletin box in a later phase, as in JCJ and Belenios.
- Threat models in which no dishonest voter is assumed.

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• Protocols whose ballot secrecy also relies on the freshness of some data coming from previous phases. For example, such data can be credentials created during a registration phase, as in JCJ and Belenios.

Second, our approach can significantly improve verification efficiency. The increased efficiency can occur for two main reasons. First, because each of our conditions focuses on one aspect of the problem and simplifies parts not related to that aspect, it involves smaller processes that are typically easier to verify. Second, because previous techniques involve guessing swaps of processes at the start of each phase, they suffer from an exponential blow up related to the number of processes in each phase. In practice, we typically observe a speedup of over two orders of magnitude and even cause the analysis to terminate in cases where it did not before.

Third, we use our approach to analyse several new case studies. Thanks to the flexibility and the large class of protocols we can deal with, we are able to analyse a multitude of different threat models allowing comprehensive comparisons. Moreover, thanks to the aforementioned advantages, our approach is able to systematically take the registration phase into account, whereas prior works often consider registrars as honest and not model them. We successfully automatically analysed FOO, Lee, JCJ, and Belenios. We show that our theorem applies to Okamoto as well.

We conduct a comprehensive comparison with related work in Section 6 and provide support for the above claims. While we present our work in the ProVerif framework, our results are applicable beyond this specific tool. Indeed, our conditions and our Main Theorem are stated in a standard applied \( \pi \)-calculus framework. Finally, we believe that our conditions shed light on three crucial aspects that e-voting protocols should enforce; thus improving our understanding of the complex notion ballot secrecy.

**Links & Ballot Secrecy.** Ballot secrecy boils down to ensuring that an attacker cannot establish a meaningful link between a specific voter and a specific vote (that this voter is willing to cast). For instance, a naive example of such a link occurs when a voter outputs a signed vote in the clear, explicitly linking his vote to his identity. However, in more realistic e-voting protocols, such links can be very complex, possibly relying on long transitive chains of different pieces of data from different outputs. For example, if an attacker is able to link a credential with the identity of the recipient voter during a registration phase, and then the voter anonymously sends his vote along with the credential during a casting phase, then the attacker can link the vote to the voter.

As noted before, diff-equivalence (as an under-approximation of behavioural equivalence) is rarely appropriate to directly verify ballot secrecy [10, 18]. An underlying reason for this is that considering diff-equivalence gives the attacker more additional structural links than when considering (the intended) behavioural equivalence. This often leads to negative results even for secure e-voting protocols, which manifest themselves as false attacks.

**Informal Presentation of the Conditions.** We analysed typical attacks and the underlying links. We classified them and identified three classes of links leading to privacy breaches. The purpose of each condition is to prevent links from the corresponding class. Our Main Theorem states that those conditions are sufficient to enforce ballot secrecy.

**Dishonest Condition** By adopting a malicious behaviour, the attacker may be able to link messages that would not be linkable in the intended, honest execution. For instance, if the attacker sends tampered data to a voter, the attacker may be able to observe the tampered part in later actions knowing it comes from the same voter allowing this attacker to establish possibly harmful links. Our first condition essentially requires that a voting system is indistinguishable for the attacker from a voting system where at each beginning of phase, all agents forget everything about past phases and pretend that everything happened as expected, i.e., as in an honest execution. The previous example would violate the condition, because in the second system, the attacker would not be able to observe the tainted data. Interestingly, this condition is mostly a reachability property that does not suffer from the lack of precision of diff-equivalence.

**Honest Relations Condition** Even in the expected honest execution, the attacker may be able to exploit useful links. Thanks to the previous condition, we can focus on a system where each role is split into sub-roles for each phase. This allows us to verify the absence of the former relations using diff-equivalence, without giving the attacker spurious structural links, as mentioned above.

**Tally Condition** We take into account the tally outcome, which enables establishing more links. Typically, the attacker may link an identity to a vote if it can forge valid ballots related to (i.e., containing the same vote) data that can be linked to an identity. This introduces a bias in the tally outcome that can reveal the vote in the forged ballot. This attack class strictly extends ballot independence attacks [16]. The Tally Condition requires that when a valid ballot was forged by the attacker then it must have been forged without meaningfully using voter’s data already linked to an identity.

**Outline.** In Section 2, we briefly present the symbolic model we use to represent protocols and security properties. We then describe our framework in Section 3 notably defining the class of e-voting protocols we deal with as well as ballot secrecy. Next, we formally define our conditions and state our Main Theorem in Section 4. We show the practicality of our approach in Section 5 by explaining how to verify our conditions and presenting case studies. Finally, we discuss related work in Section 6 and conclude in Section 7.

Note that this is a significantly improved version of a previously submitted paper. We provide a summary of previous reviews and explanations on how we addressed reviewer concerns in Appendix A.

## 2 MODEL

We model security protocols using the standard process algebra in the style of the dialect of Blanchet et al. [9] (used in the ProVerif tool), that is inspired by the applied \( \pi \)-calculus [1]. Participants are modelled as processes, and the exchanged messages are modelled in a term algebra.

Since most of the e-voting protocols are structured in a sequence of phases (e.g., registration phase, voting phase, tallying phase), we consider a model featuring phases. We briefly present this model in this section; a detailed presentation can be found in Appendix B.
Term algebra. We now present the term algebra used to model messages built and manipulated using various cryptographic primitives. We assume an infinite set $N$ of names, used to represent keys and nonces; and two infinite and disjoint sets of variables $X$ (to refer to unknown parts of messages expected by participants) and $W$ (called handles, used to store messages learned by the attacker).

We consider a signature $\Sigma$ (i.e. a set of function symbols with their arity). $\Sigma$ is the union of two disjoint sets: the constructor $\Sigma_c$ and destructor $\Sigma_d$ symbols. Given a signature $\Sigma$, and a set of initial data $A$, we denote by $T(\Sigma, A)$ the set of terms built using atoms in $A$ and function symbols in $T$. The terms in $T(\Sigma_c, N)$ are called messages. Sequences of elements are shown bold (e.g. $x, n$). The application of a substitution $\sigma$ to a term $u$ is written $u\sigma$, and $\text{dom}(\sigma)$ denotes its domain.

As in the process calculus presented in [9], messages are subject to an equational theory used for modelling algebraic properties of cryptographic primitives. Formally, we consider a congruence $\equiv$ on $T(\Sigma_c, N \cup X)$, generated from a set of equations $E$ over $T(\Sigma_c, X)$. We say that a function symbol is free when it does not occur in $E$. We can also give a meaning to destructor symbols for this. For this, we need a notion of computation relation $\Rightarrow$ on $T(\Sigma, N) \times T(\Sigma, N)$. While its precise definition is unimportant for this paper, we describe how it can be obtained from rewriting systems and give a full example in Appendix B.1.2. For modelling purposes, we also split the signature $\Sigma$ into two parts, namely $\Sigma_{\text{pub}}$ (public function symbols, known by the attacker) and $\Sigma_{\text{priv}}$ (private function symbols).

An attacker builds his own messages by applying public function symbols to terms he already knows and that are available through variables in $W$. Formally, a computation done by the attacker is a recipe (noted $R$), i.e. a term in $T(\Sigma_{\text{pub}}, W)$.

Example 2.1. Consider the signature
\[
\Sigma_c = \{\text{eq.}(\cdot), \text{sign}, \text{pk}, \text{blind}, \text{unblind}, \text{commit}, \text{ok}\},
\]
\[
\Sigma_d = \{\text{verSign}, \text{open}, n_1, n_2, \text{eq}\}.
\]

The symbols $\text{eq.}(\cdot)$, $\text{sign}$, $\text{pk}$, $\text{blind}$, $\text{unblind}$, $\text{commit}$ and $\text{ok}$ represent equality test, pairing, signature, signature verification, blind signature, unblind, commitment and commitment opening. The symbols $n_1, n_2$ and $\text{pk}$ of arity 2 are projections and verification key. Finally, $\text{ok}$ is a constant symbol (i.e. arity 0). To reflect the algebraic properties of the blind signature, we may consider $\equiv$ generated by the following equations:

\[
\begin{align*}
\text{unblind}(\text{sign}(\text{blind}(x_m, y), z_2), y) &= \text{sign}(x_m, z_2) \\
\text{unblind}(\text{blind}(x_m, y)) &= x_m.
\end{align*}
\]

Symbols in $\Sigma_c$ can be given a semantics through the following rewriting rules:

1. $\text{open}(\text{commit}(x_m, y)) \rightarrow x_m, \pi_1, \pi_2 = (x_1, x_2)$; $x \rightarrow x$, $y \rightarrow y$.
2. $\text{verSign}(\text{sign}(x_m, z_2), \text{pk}(z_2)) \rightarrow x_m$ and $\text{eq}(x, y) \rightarrow \text{ok}$.

Process algebra. We assume $C_{\text{pub}}$ and $C_{\text{priv}}$ are disjoint sets of public and private channel names and note $C = C_{\text{pub}} \cup C_{\text{priv}}$. Protocols are specified using the syntax in Figure 1. Most of the constructions are standard. The constructor let $x = v \in P$ else $Q$ tries to evaluate the term $v$ and in case of success, i.e. when $v \in \Lambda$ for some message $u$, the process $P$ in which $x$ is substituted by $u$ is executed; otherwise the process $Q$ is executed. Note also that the let instruction together with the eq theory (see Example 2.1) can encode the usual conditional construction. The replication $i.P$ behaves like an infinite parallel composition $P|P|\ldots$. The construct $P,Q := 0$ indicates that the process $P$ may be executed only when the current phase is $i$. The construct $vn.P$ allows to create a new, fresh name $n$. For a sequence of names $n$, we may note $vn.P$ to denote the sequence of creation of names in $n$ followed by $P$.

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The operational semantics of processes is given by a labelled transition system over configurations (denoted by $K$) ($\mathcal{P}$, $\mathcal{E}$) i.e. a multiset $\mathcal{P}$ of guarded ground processes, $i$ in $\mathcal{N}$ the current phase, and a $\mathcal{E}$ if it has no free variable (i.e. variable not in the scope of an input or a let construct). A process is guarded if of the form $i.P$.

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In \((i : \text{in}(c, x). P \cup P; \phi; i) \xrightarrow{t r_{\text{in}}} (i : P[x \mapsto u] \cup P; \phi; i)\) with \(c \in C_{\text{pub}}\) where \(R\) is a recipe such that \(R \phi \downarrow u\) for some \(u\) and \(w\) a fresh variable in \(W\) with \(c \in C_{\text{pub}}\) and \(R\) is a recipe such that \(R \phi \downarrow u\) for some \(u\).

\(\text{Out} (i : \text{out}(c, u) \cup P; \phi; i) \xrightarrow{t r_{\text{out}}} (i : P \cup P; \phi; w \mapsto u; i)\) with \(c \in C_{\text{pub}}\) and \(w\) a fresh variable in \(W\) with \(c \in C_{\text{pub}}\) and \(w\) a fresh variable in \(W\).

\(\text{Con} (i : \text{in}(c, x). P \cup P; \phi; i) \xrightarrow{t r_{\text{con}}} (i : P \cup P; \phi; \{w \mapsto u\}; i)\) when \(v \mapsto u\) for some \(v\).

\(\text{Let} (i : \text{let}(c, x) \cup P; \phi; i) \xrightarrow{t r_{\text{let}}} (i : P \cup P; \phi; i)\) with \(c \in C_{\text{pub}}\) and \(\phi\) is a symbolic recipe with \(c \in C_{\text{pub}}\) and \(\phi\) is a symbolic recipe.

\(\text{New} (i : v. P \cup P; \phi; i) \xrightarrow{t r_{\text{new}}} (i : P \cup P; \phi; i)\) when \(v\) is a fresh name from \(N\).

\(\text{Next} (\langle \phi; \Sigma \rangle, (\phi'; \Sigma') for some \(j \in N\) such that \(j > i\) \xrightarrow{t r_{\text{next}}} (\langle \phi; \Sigma \rangle, (\phi'; \Sigma')\) for some \(j \in N\) such that \(j > i\).

\(\text{Phase} (i : j). P \cup P; \phi; i) \xrightarrow{t r_{\text{phase}}} (i : P \cup P; \phi; i)\) with \(c \in C_{\text{pub}}\) and \(\phi\) is a symbolic recipe with \(c \in C_{\text{pub}}\) and \(\phi\) is a symbolic recipe.

**Figure 2:** Semantics for processes

\(k\{j\}; 1\). It notably has an execution \(K_{1} \xrightarrow{t r_{\text{tr}}}(\emptyset; \phi; 2)\), where:

\(tr_{b} = \tau. r. \text{out}(c, w_{1}). \text{in}(c, R). r_{\text{then}}. \text{r.phase}(2)\),

\(\text{out}(c, w_{2}). \text{in}(c, (C, w_{2})). r_{\text{then}}. \text{out}(c, w_{3})\)

and where \(C\) is any constant in \(\Sigma_{c} \cap \Sigma_{\text{pub}}\): \(\phi = \{w_{g} \mapsto k_{g}, w_{1} \mapsto \langle\text{pk}(k_{g}), s\rangle, w_{2} \mapsto \text{sign}(M, k_{g}), w_{3} \mapsto (n; M; k_{g})\}, s, M\) are as specified above and \(R = \text{sign}(\text{verSign}(\pi_{2}(w_{1}), \pi_{1}(w_{1})), w_{g})\). This corresponds to a normal, expected execution of one protocol session.

**Discussion on Phases.** Our notion of phases, also known as stages or weak phase [9, 20], faithfully model the notion of phases with deadlines in the context of e-voting protocols. Once the deadline of a phase \(i\) has passed (i.e., the action phase\((i + 1)\) has been triggered for \(j > i\)) then, no remaining actions from phase \(i\) can be executed. It also can be modelled in ProVerif (see [9, 11]). Note that in the literature, phases are often modelled with synchronisation barriers [10, 20] (also called strong phases). The latter are a much stronger notion of phases that require all initial processes to reach the next phase before the system can progress to the next phase (i.e., no processes can be dropped). In our view, synchronisation barriers model phases in e-voting protocols less faithfully than our (weak) phases, and come with limitations that we discuss in Section 3.2.

**Trace equivalence.** Trace equivalence is commonly used [18] to express many privacy-type properties such as ballot secrecy. Intuitively, two configurations are trace equivalent if an attacker cannot tell whether he is interacting with one or the other. The latter is based on a notion of indistinguishability between frames, called static equivalence. Intuitively, two frames are statically equivalent, if there is no computation (or equality test) that succeeds in one frame and fails in the other one. Then, trace equivalence is the active counterpart taking into account the fact that the attacker may interfere during the execution of the process in order to distinguish between the two situations. We define obs\((tr)\) to be the subtrace of \(tr\) obtained by erasing all the \(r\), \(r_{\text{then}}\), \(r_{\text{else}}\) actions. Intuitively, trace equivalence holds when any execution of one configuration can be mimicked by an execution of the other configuration having some observable actions and leading to statically equivalent frames. We give a formal definition in Appendix B.3.

**Example 2.3.** Consider the frame \(\phi\) from Example 2.2. The fact that the attacker cannot statically distinguish the resulting frame from a frame obtained after the same execution but starting with \(V(A, v_{2})\) instead of \(V(A, v_{1})\) is modelled by the following static equivalence: \(\phi \xrightarrow{} \phi'\) where \(\phi' = \phi(v_{1} \mapsto v_{2})\) which does actually not hold (see witness given in Appendix B.1.2). Consider \(K_{1} = ((V(A, v_{i})): \{w_{g} \mapsto k_{g}\}; 1)\) for \(i \in \{1, 2\}\). We may be interested whether \(K_{1} \simeq K_{2}\). This trace equivalence does actually not hold because there is only one execution starting with \(K_{1}\) (resp. \(K_{2}\)) following the trace \(\text{obs}(tr_{b})\) (see Example 2.2) and the resulting frame is \(\phi\) (resp. \(\phi'\)). But, as shown above, \(\phi \neq \phi'\). Therefore, \(K_{1} \neq K_{2}\). However, ballot secrecy is defined differently (in Section 3.2) and we will see that the FOO protocol actually satisfies it.

**Diff-equivalence.** Trace equivalence is hard to verify, in particular because of its forall-exists structure: for any execution on one side, one has to find a matching execution on the other side. One approach is to consider under-approximations of trace equivalence by over-approximating the attacker’s capabilities. Diff-equivalence is such an under-approximation. It was originally introduced to enable ProVerif to analyse some form of behavioural equivalence, and was later also implemented in Tamarin and Maude-NPA.

Such a notion is defined on bi-processes, which are pairs of processes with the same structure that only differ in the terms they use. The syntax is similar to above, but each term \(u\) has to be replaced by a bi-term written \(\text{choice}([u_{1}, u_{2}])\) (using ProVerif syntax). Given a bi-process \(P\), the process \(\text{fst}(P)\) is obtained by replacing all occurrences of \(\text{choice}([u_{1}, u_{2}])\) with \(u_{1}\) (similarly with \(\text{snd}(P)\)). The semantics of bi-processes is defined as expected via a relation that expresses when and how a bi-configuration may evolve. A bi-process reduces if, and only if, both sides of the bi-process reduce in the same way (e.g., a conditional) but still produce the same observable actions. Phrased differently: diff-equivalence gives the attacker the ability to see not only the observable actions, but also the processes’ structures. This strong notion of diff-equivalence is sufficient to establish some properties but is too strong to be useful for establishing ballot secrecy off-the-shelf (we discuss this at greater length in Section 6).

**3 FRAMEWORK**

In this Section we present our framework that we need to establish our results notably e-voting protocols and ballot secrecy.

**Preliminaries.** We first define symbolic traces which are traces whose recipes are symbolic; i.e., they are from \(T(\Sigma_{\text{pub}} \cup W \cup \xi)\), where \(\xi\) is a new set of second-order variables. Intuitively, a symbolic recipe \(R\) is a partial computation containing unknown parts
symbolised by second-order variables. Symbolic traces represent attacker’s behaviours with non-fully specified recipes. A symbolic trace can be instantiated to a concrete trace by replacing the second-order variables by recipes (i.e., in \( T(\Sigma_{\text{pub}}, \mathcal{W}) \)). To an honest trace \( \theta \), we associate a distinguished instantiation called the \textit{idealised trace of} \( \theta \) that can be obtained from \( \theta \) by replacing each variable \( Y \in \mathcal{Y} \) by a fixed, public constant \( C_Y \) that we add to \( \Sigma_{\text{c}} \cap \Sigma_{\text{pub}} \).

Example 3.1 (Resuming Example 2.2). The recipe of the last input of \( \theta_b \) could be replaced by the symbolic recipe \( (X,w_2) \) with \( X \in \xi \) (i.e., reflecting that the choice of \( C \) is unimportant) resulting in a symbolic trace \( \theta \). The idealised trace is \( \theta(X \mapsto C_X) \), where \( C_X \in \Sigma_{\text{c}} \cap \Sigma_{\text{pub}} \).

3.1 Class of e-voting protocols

We explain in this section how we model e-voting protocols and the considered scenarios. Essentially, we may consider an arbitrary number of honest voters plus all necessary authorities (e.g., bulletin box, registrar, tally), which can perform an unbounded number of sessions. Depending on the threat model, we also consider an arbitrary number of dishonest voters. We use role to refer to a specific role of the protocol, such as voter, authority, etc. Together, the agents performing the roles are able to produce a public bulletin board of ballots from which the tally computes the final result (i.e., multisets of accepted votes).

First, the protocol should specify a fixed finite set of possible votes as a set of free, public constants \( V \) (e.g., \( V \approx \{\text{yes, no}\} \) for a referendum). We also distinguish a specific free, public constant \( \bot \) modelling the result of an invalid ballot.

Roles. E-voting protocols specify a process for each honest role (in particular, the voter role). Dishonest roles can be left unspecified because they will be played by the environment. Those processes may use e.g. phases, private data, private channels but no replication nor parallel composition, as a role specifies how a single agent behaves during one session.

Definition 3.2. An honest role is specified by a process of the form \( i : \text{vn.A} \), where \( A \) is a process without parallel composition, replication nor creation of names. There should be at least a process for the voter role and one for the bulletin box role (noted \( A_b \)). Moreover, for the specific case of voter role, the corresponding process noted \( V(id, v) \) should be parameterized by id (modelling an identity) and \( v \) (modelling the vote this voter is willing to cast). Finally, initial attacker’s knowledge is specified through a frame \( \phi_0 \).

The process \( A_b \) shall contain (at least) one output on the distinguished public channel \( c_p \in C_{\text{pub}} \). Intuitively, each session of the bulletin box processes inputs data and may output a ballot on channel \( c_p \) (this may depend on private checks). We eventually define the bulletin box itself as the set of messages output on channel \( c_p \). W.l.o.g., we assume that role processes do not feature creation of names. There should be at least a process for the bulletin board itself as the set of messages output on channel \( c_p \).

\[ \psi_b \in \{\psi_b \mid \exists b \in B(b, \phi), \text{Extract}(ba) \downarrow v \in V' \}^* \]

The bulletin board is the multisets of messages that pass the \( \psi_b \) condition and channel \( c_p \). Then, the tally’s outcome is the multi-set of votes obtained by applying \( \text{Extract(·)} \) on the bulletin board. While our notion of Tally seems very restrictive, note that many operations can be performed by roles (e.g., \( A_b \)) such as mixnets as done e.g. in [10] where the shuffling is done between two phases.

Example 3.3 (Continuing Example 3.3). The public test \( \psi_b \) is defined as the following term with hole: \( \psi_b[\cdot] = \text{and}(\text{verSign}(\pi_1(\pi_2([\ ]]), \text{pk}(\text{sk}_{\mathcal{B}})), \text{open}(\text{getMess}(\pi_1(\pi_2([\ ])), \pi_2([\ ])))) \).

Example 3.4. The bulletin board and the tally are specified through a public term \( \psi_b \) \in \{\psi_b \mid \exists b \in B(b, \phi), \text{Extract}(ba) \downarrow v \in V' \}^* \).

Honest Trace. As said before, no process is given for dishonest roles. However, we require a notion of honest trace that itself specifies what behaviour should be expected from dishonest roles.

Definition 3.6. The protocol shall specify a symbolic trace \( th = th^0 \circ \text{out}(c_p, w_b) \) (i.e., the last action corresponds to the casting of a ballot) and a distinguished execution, called the honest execution, of the form: \( (V(id, v)) \uparrow \exists b \in B(b, \phi), \text{Extract}(ba) \downarrow v \in V \) and a free constant id, with \( th \) the idealised trace associated to \( th \). Additionally, we assume that \( th \) contains the action phase(k) for all \( 2 \leq k \leq k_f \) (no phase is skipped).
The honest trace describes the honest expected execution of one voter completing the voting process until casting a ballot possibly through an interaction with different roles. Here, the notion captures the fact that some corrupted roles are played by the attacker. Hence the fact that the honest trace is a symbolic trace with sub-messages that are unknown and not specified because chosen by the attacker. Note that the honest trace specifies how conditionals are expected to evaluate thanks to the $\tau_{\text{hen}}/\tau_{\text{the}}$ dichotomy.

Example 3.7 (Resuming Example 3.3). We consider the following extension of the symbolic trace described in Example 3.1 (where $X \in \xi$ and $R_1 = \sigma(\text{verSign}(\tau_1(w_1)), \pi_1(w_1)), w_1)):
\[\text{th} = r.r.\text{out}(c, w_1).\text{in}(c, R_1).\tau_{\text{hen}}.r.phase(2).\text{out}(c, w_2).\in(c, X, w_2).\tau_{\text{hen}}.\text{out}(w_3).r.\text{in}(w, w_3).\text{out}(c_0, w_3)\]

Definition 3.8 (E-voting Protocols). An e-voting protocol is given by a tuple $(V; \phi_0; \text{Id}(i,i); R; (\Psi_i[]), \text{Extract}[]); \text{th})$ where $V$ are the allowed votes (i.e. free, public constants), $\text{Id}(i,i)$ and $R$ are the processes modelling honest roles and $\phi_0$ is the attacker’s initial knowledge as in Definition 3.5. $\Psi_i[]$ and $\text{Extract}[]$ model the bulletin board and the tally as in Definition 3.4, and $\text{th}$ describes the intended, honest execution as in Definition 3.6.

Flexible threat models. Our generic definition of e-voting protocols allows to model many different threat models. First, the processes that model roles may use different kinds of channels. For instance, by using private channels for some inputs and outputs, we model communication channels that prevent the attacker to eavesdrop on or tamper with those exchanged messages. By using public channels and adding the identity of voter in exchanged data, we model an insecure, non-anonymous communication channel. In contrast, by using only a single public channel, we model an anonymous communication channel, since all voters will use the same channel. Moreover, some roles can be considered dishonest or honest yielding different threat models. Finally, different frames $\phi_0$ allow to model different initial attacker knowledge (e.g. secret keys of some roles).

Annotated Processes. Finally, we equip the semantics with annotations that will help subsequent developments. We assume a notion of annotations over processes so that we can keep track of role sessions and specific voters throughout executions. Each action can then be labelled by this information. For a voter process $V(\text{Id}(i), v)$, we note $[\text{Id}(i), v]$ the annotation given to actions produced by this process. Formally we may define such annotations by giving explicit annotations to processes in the initial multiset and modify the semantics so that it keeps annotations on processes as one could expect. Those notations notably allow to define when a specific voter casts a ballot as shown next.

Definition 3.9. Consider an e-voting protocol $(V; \phi_0; \text{Id}(i), v); R; (\Psi_i[]), \text{Extract}[]); \text{th})$. We say that a voter $V(\text{Id}(i), v)$ casts a ballot $w$ in an execution $(\mathcal{P}, \Psi) \models (\text{Id}(i), v); \text{Id}(i), v); \phi_0; 1) \models \mathcal{K}$ when there exists an output $\text{out}(c, w) \in tr \text{ annotated } [\text{Id}(i), v]$ and a bulletin box (i.e. $A_k$) session $s_b$ such that actions from $tr$ annotated $s_b$ are $\in(c, w'_b)$. If $\text{out}(c, w) \in tr$ then $w_b(\phi(K)) = w'_b(\phi(K))$. We say that $V(\text{Id}(i), v)$ casts a valid ballot $w$ when, in addition, $\Psi_i[w \phi(K)]$.

3.2 Ballot Secrecy

Next, we define the notion of ballot secrecy that we aim to analyse. Intuitively, ballot secrecy holds when the attacker is not able to observe any difference between two situations where voters are willing to cast different votes. However, we cannot achieve such a property by modifying just one vote, since the attacker will always be able to observe the difference on the final tally outcome. Example 2.3 illustrates this problem: one has that $K_1 \neq K_2$ while the FOO protocol actually ensures ballot secrecy. Instead, we shall consider a swap of votes that preserves the tally’s outcome as usually done [19, 26]. More formally, we are interested in comparing $S$ and $S_r$ as defined next (where, for a multiset of processes $Q$, $\mathcal{Q}$ refers to $\{P | P \in Q\}$):
\[
S = (\{R\} \cup \{V(A, c_0), V(B, c_1)\})
\]
\[
S_r = (\{R\} \cup \{V(A, c_1), V(B, c_0)\})
\]

where $c_0, c_1$ are two distinct votes in $V$ and $A, B$ are two distinct free, public constants. Because the attacker should neither be able to distinguish $S$ and $S_r$ when having access to the tally’s outcome, we are actually interested in the trace equivalence between $S \cup \{\text{Tally}\}$ and $S_r \cup \{\text{Tally}\}$ where the Tally is a process computing the e-voting protocol’s outcome: e.g. Tally = !\text{in}(c_0,x).let $z = \Psi_i[x] \in \text{out}(c, \text{Extract}(x))$. This is the most well-established definition of ballot secrecy in symbolic model introduced in [26].

However, many e-voting protocols in our class would not satisfy such a property because the attacker may force a particular voter (e.g. $A$) not to cast any ballot in order to infer, from the tally’s outcome, the vote that the other voter (e.g. $B$) has cast. This is well-known and usually addressed by modelling phases as synchronisation barriers as already acknowledged in [26]: “when we omit the synchronisation […] privacy is violated.” With such synchronisation barriers, all participants shall reach the same barrier in order to move to the next phase preventing the previous scenario from happening. However, the use of barriers (as done e.g. in [10, 19–21, 26]) also limits the range of e-voting protocols one can model and analyse. For instance, no synchronisation barrier can be put under a replication, which forbids modelling authorities that act during several phases or threat models with no dishonest voter.

In contrast, we choose to model e-voting protocols as weak phases to avoid those limitations and thus need an extra assumption as a counterpart to synchronisation barriers. We shall restrict our analysis to fair executions where, at each beginning of phase, the voter $A$ and $B$ are still present and $A$ casts a ballot, if and only if, $B$ does so. Note that all executions of protocols modelled with synchronisation barriers are necessarily fair. We are thus conservative over prior definitions. Our fairness assumption can also be seen as a variation of the Tally’s assumption in [28] that process the bulletin boards only if they contain both Alice and Bob’s ballots.

Definition 3.10. Consider an e-voting protocol $(V; \phi_0; \text{Id}(i), v); R; (\Psi_i[]), \text{Extract}[]); \text{th})$. An execution $(\mathcal{P}, \phi_0; 1) \models \mathcal{K}$ for $\mathcal{P} \in \{S, S_r\}$ is said to be fair for voter [i, v] when at each beginning of phase i, there is a process annotated [i, v] at phase i. Such an execution

---

\(^3\)This attack is captured by the model but is unrealistic in practice. Indeed, in practical scenarios, to break the ballot secrecy of a particular voter, it would require the attacker to prevent all other voters from casting a vote.

\(^4\)This should not be mixed up with the fairness property [16, 28] that is one of the security properties often required from e-voting protocols.
is said to be fair when, for some \(v, v' \in V\), (i) it is fair for \([A, v]\) and \([B, v']\) and (ii) \([A, v]\) casts a ballot if, and only if, so does \([B, v']\).

Finally, we give below the definition of ballot secrecy. We could have defined it as the trace equivalence (by symmetry) between \(S \cup \{\text{Tally}\}\) and \(S_r \cup \{\text{Tally}\}\) with a restriction over the explored traces (i.e. the ones that are fair) but we prefer our formulation in the interest of clarity. Note that the fairness assumptions get rid of strictly less behaviours than the use of synchronisation barriers, and are therefore more precise from that point of view.

**Definition 3.11 (Ballot Secrecy).** An e-voting protocol \((V; \phi_0; V(id, v); R; \Psi_k[i], \text{Extract}[]); th) ensures ballot secrecy when for any fair execution \((S; \phi_0; 1) \xrightarrow{\text{tr}} K_r\), there exists a fair execution \((S_r; \phi_1; 1) \xrightarrow{\text{tr}} K_r\) such that:

- the attacker observes same actions: \(\text{obs(tr)} = \text{obs(tr')}\);
- the attacker observes same data: \(\phi(K) \sim \phi(K_r)\);
- the attacker observes same tally’s outcome: \(\text{Res(tr, } \phi(K)) = \text{Res(tr', } \phi(K_r))\).

4 CONDTIONS

We introduce three conditions and prove that together, they imply ballot secrecy. In Section 4.1 we provide intuition for our approach and formally define the support notions. We then define the conditions (i.e., Dishonest, Honest Relations, Tally Condition) in sections 4.2 to 4.4. We state in Section 4.5 that our conditions are sufficient.

4.1 Protocol phases and their links

**Identity-leaking vs. Vote-leaking Phases.** In a nutshell, ballot secrecy boils down to the absence of link between an identity and the vote this identity is willing to cast. However, as illustrated by the next example, the attacker is able to link different actions performed by the same voter as long as they take part in the same phase. Thus, each phase of the e-voting protocol must hide and protect either the identity of voters or the votes voters are willing to cast. It is thus natural to associate to each phase, a leaking label: either the phase (possibly) leaks identity (we call such phases id-leaking) or it (possibly) leaks vote (we call such phases vote-leaking).

In order to ensure ballot secrecy, the Honest Relations Condition, which we define later, will enforce that the attacker cannot establish meaningful links (i.e. links that would hold for \(S\) but not for \(S_r\)) between id-leaking phase outputs and vote-leaking phase outputs.

**Example 4.1.** Consider a voter’s role process \(V(id, v) = 1 : \text{out}(a, id), \text{out}(a, v)\) (other components are unimportant here). This trivial protocol is an abstraction of a registration phase (voter sends its identity) followed by a voting phase (voter sends its vote). We show this does not ensure ballot secrecy (see also the full witness in Appendix C). Consider the (fair) execution starting with \((S; \emptyset; 1)\) and producing the trace \(tr = \text{out}(a, w_1), \text{out}(a, w), \text{out}(a, w_2)\), whose first two actions are performed by the voter \(A\) (resp. \(B\)). This execution has no indistinguishable counterpart in \(S_r\). Indeed, because the first message reveals the identity of the voter, the attacker can test that the first output is performed by \(A\). After the first output \(A\), the \(S_r\) side can only output either \(B\) or \(v_1\) but not \(v_0\). However, because the second message reveals the vote, the attacker can make sure the output vote is \(v_0\) and not \(v_1\). Thus, this protocol does not ensure ballot secrecy because in a single phase (i.e., phase 1), there is one output revealing the identity of the voter and one output revealing the voter’s vote. However, the process \(V(id, v) = 1 : \text{out}(a, id), 2 : \text{out}(a, v)\) ensures ballot secrecy and does not suffer from the above problem. The attacker cannot force \(A\) to execute its first message leaking identity and then immediately its second message leaking its vote, because doing so would kill the process \(V(B, v_1)\) (which is still in phase 1) preventing the whole execution from being fair. Thus, the attacker has to trigger all possible first-phase actions of \(A\) and \(B\) before moving to the second phase. After the first phase, we end up with the processes \(\{\text{out}(a, v_0), \text{out}(a, v_1)\}\) on the \(S\) side and \(\{\text{out}(a, v_1), \text{out}(a, v_0)\}\) on the \(S_r\) side, which are indistinguishable.

Thus, in this first iteration, we split outputs revealing identity and outputs revealing votes in distinct phases. This enables breaking links between identity and vote.

As we will later see, our approach requires that we associate a leaking label to each phase which is a binary label indicating whether we consider the phase to be vote-leaking or id-leaking. Our only goal is not find such a labelling for which our conditions hold, implying ballot secrecy. In practice and on a case-by-case basis, we can immediately associate the appropriate leaking label to a phase. However, we explain in Section 5.1 how those labels can be automatically guessed.

**Example 4.2 (Continuing Example 3.7).** We consider phase 1 (resp. phase 2) as id-leaking (resp. vote-leaking).

**Id-leaking vs. Vote-leaking Names.** As illustrated by the next example, a name presents in different outputs can also be exploited to link those outputs. This is problematic when phases of those outputs have different leaking labels since it would enable linking those phases and thus maybe an identity with a vote. That is why, similarly to phases, we associate a leaking label to each name created by role processes. Note that the phase in which the name is created is irrelevant. What really matters is where the name is used and to what kind of data it can be linked. Again this classification is easily done on a case-by-case basis in practice but we present simple heuristics to automatically infer it in Section 5.1.

**Example 4.3.** We continue Example 4.1 and consider \(V(id, v) = 1 : \text{vr.out}(a, id @ r), 2 : \text{out}(a, v @ r)\) where @ denotes the exclusive or operator. This new protocol seems similar to the previous iteration. However, it does not satisfy ballot secrecy: Now, the attacker can use the name \(r\) to link the action of the id-leaking phase with the action of the vote-leaking phase (see witness in Example 4.10), defeating the role of the phase which previously broke this link. Note that only names can lead to the this issue: all other kinds of data (e.g., public constants) are uniform and do not depend on a specific voter session (e.g., replace \(r\) by a constant \(k \in \Sigma_e\) and the resulting protocol ensures ballot secrecy).

**Example 4.4 (Continuing Example 4.2).** We consider the names \(k, k'\) to be vote-leaking.

"Divide & Conquer". One reason that ballot secrecy is hard to verify using existing techniques, is the fact that diff-equivalence is too rigid w.r.t. phases: it does not allow any flexibility at the beginning of phases. We should be able to stop there, and start again
with a new pairing left-right, a new biprocess. A core ingredient of our technique is to split each role into independent, standalone sub-roles (each sub-role playing one phase of the initial role), which allows us to consider much more pairings including ones (left-right) that are not consistent over phases. One of our conditions will require that the attacker cannot distinguish the voter and other roles processes from standalone sub-role processes that do not need to know the execution of past phases. This is also important to ensure ballot secrecy, because otherwise the attacker might link two actions coming from two different phases and then learn that they came from the same voter.

We now formally define the sub-roles. Let \( n_i^k \) be the vector made of the constant \( v_i \) and all vote-lease-mailing annotations (with indices \( i \)). Let \( n_i^k \) be the vector made of the constant id; and all id-leasing names (with indices \( i \)). The pair \( (n_i^k, n_i^j) \) (deterministically) describes the initial data needed to start one full honest interaction of one voter with all necessary role sessions.

**Definition 4.5.** Recall that the voter process is of the form \( V(\text{id}, c) = k : v . V(\text{id}, v) \) where \( V \) is without creation of names. We define \( V(n_i^k, n_i^0) \) as the process \( k : v . V(\text{id}, v) \sigma \) where \( \sigma \) maps names in \( n \) to corresponding names in \( n_i^k \cup n_i^0 \). We similarly define \( A(n_i^k, n_i^0) \) for \( A \in \mathcal{A} \). Finally, we define \( R_0(n_i^k, n_i^0) = \{ A(n_i^k, n_i^0) | A \in \mathcal{R} \} \).

**Example 4.6 (Resuming Example 4.3).** Assuming \( r \) is said to be vote-lease-mailing, one has \( n_i^0 = \text{id} \) and \( n_i^0 = v_i \) \( r_j \), and, \( V(n_i^k, n_i^0) = 1 : \text{out}(a, \text{id} \oplus r_j) \).\( 2 : \text{out}(a, v_j \oplus r_j) \).

Intuitively, the process \( V(n_i^k, n_i^0) \) corresponds to the voter role process of identity id; and vote \( v_i \) that will use all given names instead of creating fresh ones. Similarly for authorities. Note that in the vectors \( n_i^k \), there may be names that are never used in some roles; we still give the full vectors as arguments though. We remark that given names \( n_i^k, n_i^0 \), there is a unique (modulo \( \equiv \)) execution of \( (V(n_i^k, n_i^0) \uplus \mathcal{R}_0(n_i^k, n_i^0); \varnothing) \) following the idealised trace that is (up to some \( \tau \)-actions) the bijective renaming of the honest execution (see Definition 3.6) from names used in the honest execution to names in \( n_i^k, n_i^0 \). We call that execution the idealised execution for \( n_i^k, n_i^0 \).

**Definition 4.7 (Phase Roles).** Given \( n_i^k, n_i^0 \), consider the unique idealised execution for \( n_i^k, n_i^0 \) (\( (V(n_i^k, n_i^0) \uplus \mathcal{R}_0(n_i^k, n_i^0); \varnothing) \)) \( (\mathcal{R} \equiv \) ) to \( k \) for some \( P \) if it exists; and \( 0 \) otherwise.

Finally, the phase role of \( A \) for \( k \), noted \( A^k(n_i^k, n_i^0) \), is the process one obtains from \( A^k(n_i^k, n_i^0) \) by replacing by \( 0 \) each sub-process of the form \( l : P \) for some \( P \) and \( l > k \). Further, the process \( V^k(n_i^k, n_i^0) \) is the sequential composition of all \( A^k(n_i^k, n_i^0) \). Finally, \( V^k(n_i^k, n_i^0) \) and \( V^k(n_i^k, n_i^0) \) are defined similarly.

In a nutshell, phase roles describe how roles behave in each phase, assuming that previous phases followed the idealised executions for the given names. A crucial property we eventually deduce from our conditions is that it is sufficient (i.e., we do not lose behaviours and hence neither attacks) to analyse the phase roles in parallel instead of the whole e-voting system. Note that, by doing so, we do not only put processes in parallel that were in sequence, we also make them forget the execution of past phases cutting out some potential links that rely on that aspect. Indeed, the voter process in a phase \( i \) may use data received in a phase \( j < i \) creating links between those two (e.g. via malicious tainted data). When put in parallel, all parts are standalone processes that are no longer linked by past execution. Note also that we put standalone processes in parallel that behave as if previous phases followed one specific instantiation of the honest trace (i.e., the idealised trace) thus reducing a lot possible behaviours. This will be crucial for defining Honest Relations Condition via biprocesses that could not be defined otherwise.

**Definition 4.8.** The id-leaking sub-roles (respectively vote-leaking sub-roles) are as follow:

- \( R^\text{id}(n_i^k, n_i^0) = \{ A^k(n_i^k, n_i^0) \} | A \in \mathcal{R}_o \cup \{ V \} \) id-leak.
- \( R^\text{vote}(n_i^k, n_i^0) = \{ A^k(n_i^k, n_i^0) \} | A \in \mathcal{R}_o \cup \{ V \} \) vote-leak.

**Example 4.9 (Continuing Example 4.4).** The phase roles are:

\[
V^k(n_i^k, n_i^0) = 1 : M = \text{commit}(v_i, k_i) \\
= \text{blind}(M, k_i, s = \text{sign}(c, \text{key}(id)), \\
\text{out}(c, pk(\text{key}(id)); s)), \text{in}(c, x); \\
\text{if} \text{verify}(x, pk(k_y)) = e \text{ then 0}
\]

\[
V^2(n_i^k, n_i^0) = 2 : M = \text{commit}(v_i, k_i), \\
\text{out}(c, \text{sign}(M, k_g)), \text{in}(c, y); \\
\text{if} y = (y_1; M) \text{ then out}(c, (y_1, M, k_i))
\]

Finally, the sub-roles are \( R^\text{id} = \{ V^1 \}, R^\text{vote} = \{ V^2, A_b \} \).

**Honest Interactions.** We will show that under our conditions, \( S \) is indistinguishable from an e-voting system based on the reunion of id-leaking and vote-leaking sub-roles. To achieve this property we eventually require that when a voter reaches a phase \( k \) then it must be the case that it had an honest interaction so far. This notion of honest interaction is captured by the honest trace \( th \) as formally defined next.

For two traces \( tr_1, tr_2 \) and a frame \( \phi \), we note \( tr_1 \equiv \phi tr_2 \) if \( tr_1 \) and \( tr_2 \) are equal up to recipes and for all recipes \( M_1 \) of \( tr_2 \) we have that \( M_1 \phi \downarrow = M_2 \phi \downarrow M_2 \) being the corresponding recipe in \( tr_2 \). For some \( i < j \leq k_f \), we say that a trace \( tr_1 \) and a frame \( \phi \) follow a trace \( tr_2 \) up to phase \( j \) (resp. follow \( tr_2 \)) if \( tr_2 \equiv \phi (tr_{j+1}^\phi) \) where \( tr_{j+1}^\phi \) is such that \( tr_2 \equiv tr_{j+1}^\phi (\phi(\cdot)) \) for some \( tr_{j+1}^\phi \) (resp. \( tr_{j+1}^\phi = tr_2 \)) and \( \phi \) is some bijection of handles (so that the notion is insensitive to choices of handles). The above ensures that \( tr_1 \) and \( tr_2 \) compute messages having the same relations w.r.t. outputs (handles). For instance, if \( tr = \text{out}(c, w), \text{in}(c, w) \) is some trace, we would like to capture the fact that for a frame \( \phi \), any trace \( \text{out}(c, w), \text{in}(c, \phi) \) follows as long as \( \phi \) computes the same message as \( w \) (i.e. \( w \phi = w \)). Finally, given an execution, we say that a voter had an honest interaction up to phase \( j \) (resp. had an honest interaction) when there exist role session annotations \( s_1, \ldots, s_n \) such that the sub-trace made of all actions labelled by this voter or \( s_i \) with the resulting
frame follow an instance of the honest trace up to phase \(j\) (resp. follow an instance of the honest trace).

Remind that the trace \(tr^B\) from the honest execution is an instance of the honest trace \(th\) and both are equal when \(th\) has no unknown part (i.e. second-order variable). The purpose of the idealised trace in subsequent developments is to consider an arbitrary, fixed, instantiation that is uniform for all voters and sessions.

### 4.2 Honest Relations Condition

This condition aims at ensuring the absence of id-vote relations in honest executions. Let \(n^id_A\) (resp. \(n^id_B\)) be the public identity \(id_A\) (resp. \(id_B\)) and as many as necessary (depending on the protocol) fresh names. We may use \(A\) (resp. \(B\)) to refer to \(id_A\) (resp. \(id_B\)). Let \(n^v_B\) (resp. \(n^v_A\)) be the public vote \(v_0\) (resp. \(v_1\)) and fresh names. We require these names to be pairwise distinct. We define the biprocess \(B\) at the core of the Honest Relations Condition:

\[
B = (\langle R^1(a, id_A \oplus choice[n^id_A, n^v_A]), R^2(choice[n^id_A, n^id_B], n^v_B), R^{eq}(n^id_A, choice[n^id_A, n^id_B], n^v_B), R^{eq}(choice[n^id_B, n^id_A], n^v_A)\rangle, \菹)\]

The biprocess \(B\) represents a system where votes (and vote-leaking names) are swapped in id-leaking phase and identities (and id-leaking names) are swapped in vote-leaking phase. The attacker should not be able to observe any difference in the absence of relation between identity plus id-leaking names and vote plus vote-leaking names.

Note that the swaps are inconsistent across phases (i.e. we do not swap same things in all phases). We could not have defined such non-uniform swaps by relying on the roles’ processes. Instead, this has been made possible thanks to our divide & conquer approach.

**Example 4.10 (Resuming Example 4.6).** One has \(B = ((1 : out(a, id_A \oplus choice[r_0, r_1]), 2 : out(a, v_0 \oplus r_0), A_B), 1 : out(a, id_B \oplus choice[r_1, r_0]), 2 : out(a, v_1 \oplus r_1), A_B, A_B); \菹, 1)\). We argue that this biprocess is not diff-equivalent. Indeed, the attacker can xor \(id_A, v_0\), an output of phase 1, and an output of phase 2. For one choice of the outputs, the attacker may obtain 0 on the left. This cannot happen on the right. The same interaction is also an attack trace for ballot secrecy.

One requirement of the Honest Relations Condition is the diff-equivalence of \(B\). However, this alone does not prevent the honest trace to make explicit links between outputs of id-leaking phases and inputs of vote-leaking phase (or the converse). This happens when the honest trace is not phase-oblivious as defined next.

**Definition 4.11.** The honest trace is phase-oblivious when:

- in all input \(in(c, M)\) of \(th\) in a phase \(i\), handles in \(M\) must not come from phases with a different leaking labels (i.e. vote or id-leaking) than that of phase \(i\) and
- a variable \(X \in \xi\) of \(th\) must not occur in two phases having different leaking labels.

**Condition 1. (Honest Relations)** The Honest Relations Condition is satisfied if \(B\) is diff-equivalent and the honest trace is phase-oblivious.

### 4.3 Tally Condition

The Tally Condition’s goal is to prevent ballot secrecy attacks that exploit the tally’s outcome. Intuitively, the Tally Condition requires that for any valid ballot produced by \(S\), either (i) the ballot stems from an honest execution of \(A\) or \(B\), or (ii) it is a dishonest ballot and in that case, it must be that the vote the Tally would extract from that ballot is the same before or after the swap \(A \leftrightarrow B\). Formally, we deal with the case (ii) by considering executions of \(B\) so that we always can compare ballots before or after the swap \(A \leftrightarrow B\) (i.e. intuitively in \(S\) or in \(S_0\)).

**Condition 2. (Tally)** We assume that \(B\) is diff-equivalent. The Tally Condition holds if for any execution \(B^{\text{ trie}}, B^T\) leading to two frames \(\phi_1, \phi_2\) such that the corresponding execution on the left is fair, it holds that for any ballot ba \(\in \text{BB}(tr, \phi_2)\) (with \(w\) as the handle) then either:

1. there exists a voter \(V(id, w)\) which had an honest interaction and cast a valid ballot \(w\) (it stems from an honest voter);
2. or there exists some \(v \in \\text{V}'(\\{\lambda\})\) such that \(\text{Extract}(w, \phi_1) \downarrow v\) and \(\text{Extract}(w, \phi_2) \downarrow v\) (it may correspond to a dishonest ballot that should not depend on \(A's\) or \(B's\) vote).

The Tally Condition does not forbid making copies of a ballot completely “blindly” (i.e. without being able to link this ballot to a specific voter/identity). Indeed, votes in vote-leaking phases are identical on both sides of \(B\) and the second case (2) will thus trivially hold. This actually improves the precision of the condition since such copies are not harmful w.r.t. ballot secrecy. In fact, the attacker may observe a bias that he might exploit to learn the vote contained in a specific ballot, but the attacker would be unable to link this ballot (and its vote) to a specific voter. Therefore, our condition captures a refined notion of ballot independence attacks [16].

### 4.4 Dishonest Condition

This condition prevents attacks based on actively dishonest interactions where the attacker deliberately deviates from the honest trace in order to exploit possibly more links (e.g. see tainted data example from Introduction). The idea of the condition is to be able to reduce the behaviours of the voter system to the parallel composition of all phase roles that are based on the idealised execution for some names chosen non-deterministically. The condition requires that if a voter process moves to the next phase in an execution of the e-voting system then it must be the case that it had an honest interaction up to that phase and all agents involved in that honest interaction are not involved in others. When \(th = tr^B\) (no unknown part in the honest execution), this is sufficient to show that roles are indistinguishable from the parallel composition of phase roles. However, when \(th \neq tr^B\), some attacker choices for second second-order variables of \(th\) may break the latter. For that case, the condition thus requires an additional diff-equivalence between the system based on roles and the system based on the sequential composition of the phase roles (i.e. processes \(A^T\)). To make sure that the tally’s outcome could not break this equivalence, we test the former in presence of an oracle opening all ballots.

Formally, we let \(V^{\text{trie}}(n^id_A, n^id_B)\) be the biprocess obtained by the (straightforward) merge of the two following processes: (1) \(V(n^id_A, n^id_B)\)
and (2) \( V^V(n^\text{id}_id, n^\text{\_id}_i) \) (i.e. see Definition 4.7). Recall that the process \( V^V \) forgets the past execution at the beginning of each phase and behaves as if the past execution followed the idealised trace. In particular, it forgets previous (possibly malicious) input messages. We similarly define biprocesses \( A^D \) for \( A \in \mathcal{R}_n \). Given an identity id and a vote \( v \), we define a process:

\[
S_f(id, v) = n^\text{id}_id \cdot n^\text{\_id}_i(id, n^\text{\_id}_i(v, n^\text{\_id}_i))
\]

where \( \Pi \) denotes the parallel composition and \( n^\text{id}_id \) (resp. \( n^\text{\_id}_i(v, n^\text{\_id}_i) \)) is made of all id-leaking names except the identity (resp. vote). Intuitively, \( S_f \) starts by creating all necessary names and is then ready to complete one voter session (according to processes \( V \) and \( A \in \mathcal{R}_n \) on the left and \( V^V, A^D \) on the right) using those names. Next, the oracle opening all valid ballots is as follows: OpenBal = \( k_f : 1n(c_v, x) \), let \( z = \Psi[x] \), let \( v = \text{Extract}(x) \cdot 1n(c_v, x) \) where \( e_c \) is some public channel and \( k_f \) is the last phase that occurs in the honest trace \( \text{th} \). Finally, we define: \( B^D = (S_f(A_0, \emptyset), S_f(B, !\text{OpenBal}), !\text{\_OpenBal}) \).

Example 4.12 (Continuing Example 4.9). The process \( V^D \) associated to the FOO protocol is shown below:

1. \( M = \text{commit}(c_v, k_1) \)
2. \( e = \text{blind}(M, k_1) \), \( s = \text{sign}(e, \text{key}(id)) \)
3. \( \text{out}(c_v((pk(\text{key}(id))) \cdot 1n(c_v, x), x)) \).
   - if \( \text{verSign}(x, pk(k_1)) = e \)
   - then \( 2 : \text{out}(a, \text{choice}[\text{unblind}(x, k_1), \text{sign}(M, k_1)]) \).
4. \( 1n(c_v, y) \).
   - if \( y = (y_1; 1) \)
   - then \( \text{out}(c_v((y_1, (\text{\_M}, k_1))) \).

**Condition 3. (Dishonest)** The Dishonest Condition holds when:

1. For any fair execution (\( \mathcal{S}; \phi_0, 1\cdot \text{tr\_phase}(j) = (\mathcal{P}; \phi'j) \), if a process at phase \( j \) annotated \( \text{id} = v \) for \( \text{id} \in \{A, B\} \) and \( v \in \mathcal{V} \) is present in \( \mathcal{P} \) then it has an honest interaction in \( \text{tr} \), \( \phi \) up to phase \( j \). Moreover, authority sessions \( \psi_i \) involved in this interaction are not involved in other honest interactions.
2. If \( \text{th} \) has some unknown part \( (i.e. \text{th} \neq \text{tr}^k) \), then the biprocess \( B^D \) is diff-equivalent.

### 4.5 Main Theorem

Our main theorem states that our three conditions together enforce ballot secrecy. It is based on the following Lemma that states the essential property we deduce from the Dishonest Condition. Note that the definition of having honest interactions is straightforwardly extended to executions performed by phase sub-roles. For instance, \( V^V(n^\text{id}_id, n^\text{\_id}_i) \) would be annotated \( [\text{id}, \text{id}_i] \). We give full proofs of the lemma and our Main Theorem in Appendix D.

**Lemma 4.13.** Let \( v_1, v_2 \) be some distinct votes in \( \mathcal{V} \) and \( t \) a trace of the form \( t_0, \text{phase}(k), t_k \), for some \( 1 \leq k \leq k \), where no phase(\( ) \) action occurs in \( t_k \). If the dishonest condition then there exists a fair execution \((V(id_1, v_1), V(id_2, v_2)) \cup \mathcal{R}; \phi_0, 1\cdot \text{tr\_phase}(\mathcal{P}; \phi') \) if, and only if, there exist pairwise distinct names \( n^\text{id}_id, n^\text{\_id}_i, n^\text{id}_id, n^\text{\_id}_i \) (not including vote or identity), a trace \( \text{tr}' = t_0', \text{phase}(k), t_k' \) and a fair execution

\[
(\mathcal{R}^V(id_1, n^\text{id}_id, (v_1, n^\text{\_id}_i)), \mathcal{R}^V(id_2, n^\text{id}_id, (v_2, n^\text{\_id}_i)), \mathcal{R}^D(id_1, n^\text{id}_id, (v_1, n^\text{\_id}_i)), \mathcal{R}^D(id_2, n^\text{id}_id, (v_2, n^\text{\_id}_i)))
\]

where \([id_1, v_1] \) and \([id_2, v_2] \) had an honest interaction in \( \text{tr}' \), phase \( k \), up to phase \( k \), \( \psi \). In both directions, we additionally have that \( \text{obs}(\text{tr}') = \text{obs}(\text{tr}) \), \( \psi \sim \psi \) and \( \text{Res}(\text{tr}, \psi) = \text{Res}(\text{tr}', \psi) \).

**Theorem 4.14.** If a protocol ensures the Dishonest Condition, the Tally Condition, and the Relation Condition then it ensures ballot secrecy.

### 5 CASE STUDIES

We now apply our technique to several case studies, illustrating its scope and effectiveness. We show in Section 5.1 how ProVerif can be used to verify the three conditions. In Section 5.2, we present and benchmark several e-voting protocols within our class, and explore several threat models for one of those protocols in Section 5.3.

#### 5.1 Verifying the conditions

We explain in this section how to leverage ProVerif to verify the three conditions via systematic encodings producing ProVerif models. We present at the end of the section, an algorithm showing that writing those encodings can be automated. However, we leave the task of implementing such an algorithm as future work. We also show that the time spent by the algorithm computing those encodings is negligible compared to the time ProVerif spends to verify the produced models.

**Guessing leaking labels** While it would be reasonable to require from users leaking labels for given e-voting protocols, very simple heuristics to guess them allow to conclude on all our case studies. First, the registration phase is often the first phase. Hence, guessing that the first phase is the only id-leaking phase always allow to conclude on our examples. Similarly, the following heuristic for guessing leaking labels of names proved to be precise enough: if the name is output then it takes the leaking label of the corresponding phase, if the name is used as signature key then it is id-leaking and otherwise it takes the leaking label of the phase of its first use.

**Sound Verification of The Tally Condition.** It is possible to verify the Tally Condition by analysing the diff-equivalence of the biprocess \( B \) in presence of an oracle opening all ballots (i.e. OpenBal defined in Section 4.4). The diff-equivalence of \( B^D = B \cup \{\text{OpenBal}\} \) implies the diff-equivalence of \( B \) and for all executions and valid ballots, item 2 of the Tally Condition. We formally state and prove the former in Appendix E. We also describe in Appendix E an independent way to establish the condition based on trivial syntactical checks that always imply the Tally Condition but that is less tight.

**The Dishonest Condition.** Let us explain how to verify item (1) of the Dishonest Condition using correspondence properties of events that ProVerif can verify. We can equip the e-voting system \( S \) with events that are fired with each input and output containing exchanged messages and session annotations. Then, the fact that a specific voter passes a phase or casts a valid ballot can be expressed by such an event. Further, the fact that a specific voter had an honest interaction (up to a certain phase or not) can be expressed as implications between events. For instance, if \( \text{th} = \text{out}(c, w), \text{in}(c, X; w) \) then one would write \( \text{EventIn}(a, (X; y_{\text{\_v}})) \Rightarrow \text{EventOut}(id, v, y_{\text{\_v}}) \) where \( \text{id} \) and \( v \) are voter annotations and \( a \) is a role session annotation, \( x \) and \( y_{\text{\_v}} \) are variables and open messages in events must pattern-match with the exchanged messages. Note that this has already be
Algorithm for verifying all conditions. The input format of our algorithm is a valid ProVerif file containing at least: public constants modelling $\phi_0$ and $\varphi$, function and reduction rules modelling $\Psi_k$ and Extract and a biprocess for each role describing $V, R, A$ and the idealised execution. Formally, the left part of a biprocess associated to a role $A$ should model $A(n^A, n^A)$ while the right part should model $A'(n^A, n^A)$ where input messages are replaced by messages received in the honest execution. Moreover, constants in such messages corresponding to second-order variables in the honest execution shall be given distinguished names. Therefore, the right part of those biprocesses both specify the idealised execution and the honest trace. Hence, the user is just required to specify an e-voting protocol according to Definition 3.8.

As explained, from such a file, the honest trace $th$ can be retrieved (by syntactical equality between inputs and parts of outputs) and the fact that $th$ is phase-oblivious can be checked via a linear-time syntactical check. Exploiting the right part of the given biprocesses, the algorithm can compute $A'(n^A, n^A)$ and thus $A(n, n^A)$ for all $1 \leq k \leq k_f$. Using the aforementioned heuristic, the algorithm guesses leaking labels for names and phases and deduce $R^V(n, n^V)$ and $R^D(n, n^V)$. The algorithm then deduces $B$ and, using the two functions modelling $\Psi_k$ and Extract, it also deduces $B^T$. Finally, when $th$ does contain second-order variables, the algorithm computes $B^D$ from the left part of the given biprocesses and roles $A'(n^A, n^A)$.

Finally, the algorithm produces two or three files: one containing $B^T$, one containing correspondence properties using encoding described above for modelling the Dishonest Condition, item (1), and, if $th \neq tr^h$, one containing $B^D$. Then, ProVerif is used to verify the diff-equivalence of $B^D$, all the correspondence properties and, when necessary, diff-equivalence of $B^D$. If all checks hold then the algorithm deduces that the given e-voting protocol ensures ballot secrecy.

All the described tasks the algorithm should perform are linear-time syntactical manipulations of the given input data. Therefore, the cost of computing the three ProVerif files is negligible compared to the time spent by ProVerif for verifying the files. In our benchmarks, we thus only measured the latter.

5.2 E-voting protocols

We now describe the different e-voting systems we verified with our approach and compare (when possible) with prior approach (see Figure 3). We benchmark the verification times of our approach vs. the only prior comparable approach, i.e., the direct encoding of ballot secrecy in ProVerif with synchronisation barriers [10], which tries to swap processes at the beginning of phases. Other approaches are not automated (require non systematic manual efforts), or do not deal with the protocols and threat models we consider (see discussions in Section 6). Notably, while Tamarin is now expressive enough to describe our case studies [21], it does not allow to automatically prove them\(^5\).

We summarise our results in Figure 3 and provide all our ProVerif models at [3]. We ran ProVerif 1.94 on a single 2.67GHz Xeon core with 48GB of RAM. $\dagger$ means non-termination in 45 hours or consumption of more than 30GB of RAM. In non-termination cases, we also tried to approximate the model by collapsing some phases. We use $\star$ when the approach yielded false attacks, and hence is not precise enough to conclude.

\begin{figure}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Protocol & Ballot Secrecy & Our verif. time & Swapping technique verif. time \\
\hline
FOO & ✓ & 0.04 & 0.26 \\
Lee 1 & ✓ & 0.04 & 46 \\
Lee 2 & ✓ & 0.05 & $\dagger$ (collapsed-phases: 45.33) \\
Lee 3 & ✓ & 0.01 & $\dagger$ (collapsed-phases: 269.06) \\
Lee 4 & $\Box$ & 6.64 & 169.94 \\
JCJ & ✓ & 18.79 & $\star$ \\
Belenios & ✓ & 0.02 & $\star$ \\
\hline
\end{tabular}
\caption{Analysed protocols and threat models.}
\end{figure}

FOO. We verified the three conditions for FOO [22] as described in our running example (with a dishonest registrar and considering dishonest voters). We use the same modelling and threat model as in [10].

JCJ. We analysed the JCJ protocol [28] used in the Civitas system [12]. We adapted models from [6]. We fully modelled the registration phase in which voters request and then obtain credentials from the registrar. We considered a threat model where the registrar and the tally are honest but their secret keys are compromised. A voter requests a credential on an unsecure channel and receives it on an unattapable channel. Voters send ballots on an anonymous channel. Unlike the model of [28], we took the registration phase into account. Note that [6] analyses JCJ for coercion-resistance, a stronger property than ballot secrecy. However, they considered a simpler threat model where the registration phase is completely hidden from the attacker. Moreover, their approach required non-negligible human efforts since one has to "explicitly encode in the biprocess the proof strategy" according to [6].

Belenios. We analysed the Belenios protocol [14] (in its mixnet version) which builds on the Helios protocol [2] and considered the same threat model as for JCJ. Again, contrary to [28], we took the registration phase into account.

Okamoto. We finally discuss the protocol due to Okamoto [29] as modelled in [19]. This protocol features trap-door commitments that ProVerif is currently unable to deal with. However, this protocol lies in our class and our theorem thus applies. This could both ease manual verification and future automated verification (e.g. recent analysis [21]).

5.3 Threat models

We now support the claim that our class of e-voting protocols is expressive enough to capture a large class of threat models by analysis several threat models for Lee et al. (variant proposed in [19]). This protocol contains two phases. In the registration phase, each

\(^5\)One might tackle this by devising some non-trivial and non-systematic helping lemmas.
voter encrypts her vote with the Tally’s public key, signs the ciphertext and (output i) sends both messages to the registrar. The registrar verifies the signature, re-encrypts the ciphertext using a fresh nonce and (output ii) sends to the voter this signed ciphertext along with a Designated Verifier Proof (DVP) of re-encryption. The voter can then verify the signature and the DVP.

Lee 1. The first threat model we consider is the only one analysed in [10]. It considers the registrar’s signature key and the tally’s private key corrupted, and considers infinitely dishonest voters. The channel of outputs (i) and (ii) is assumed untappable (i.e. everything is completely invisible to the attacker) while the channel of output (iii) is anonymous. Since the registrar’s signing key is corrupted, the dishonest voters do not need to have access to registrar sessions (they can be played by the environment). Similarly, there is no need to explicitly model the tally. This simplifies the models one needs to consider, partly explaining the effectiveness of the direct encoding approach (46s).

Lee 2. Now, we no longer consider the tally’s key corrupted. When verifying ballot secrecy without our conditions, it is thus mandatory to explicitly model the tally. This change to the model causes ProVerif to not terminate on the direct encoding of ballot secrecy. We thus tried to approximate the model. We collapsed the two phases into one, which enables ProVerif to terminate in 45.33 seconds on the direct-encoding. Unfortunately, this approximation does not always solve the problem: if the security relies on the phases, one would obtain false attacks. For instance, removing all phases causes ProVerif to return a false attack. More importantly, this approximation is not sound in general (we may miss some attacks). In contrast, the verification of our conditions only takes a fraction of a second without the above approximation.

Lee 3. Continuing Lee 2, we additionally consider a secure registrar signing key. We now need to explicitly model a registrar for dishonest voters. As for the previous model, ProVerif is unable to directly conclude. After collapsing phases, it terminates in 269.06 seconds. In contrast, our approach concludes in under 0.1 seconds.

Lee 4. We modify the previous threat model and weaken the output channel’s security (i) to be unsecure instead of untappable. In this case, ballot secrecy no longer holds. ProVerif returns an attack on the tally condition (verified using the ballot-opening oracle; see Section 5.1). Relying on the latter, we can immediately infer the attack on ballot secrecy.

6 RELATED WORK
As mentioned before, diff-equivalence is known to be too imprecise to analyse vote-privacy via a direct encoding (acknowledged e.g. in [5, 10, 19, 20]).

Swapping Approach. The swapping approach originates from [20], and 8 years later, was formally proven and implemented in ProVerif [10]. It aims to improve the precision of diff-equivalence for protocols with synchronisation barriers. The main idea is to guess some process permutations at the beginning of each barrier and then verify a modified protocol based on these permutations. We give an example showing this mechanism in Appendix F.1. Theoretically, the permutations do not break trace equivalence since they transform configurations into structurally equivalent configurations. This approach is only compatible with replication in a very constrained way: all barriers above and below a replication must be removed, which reduces precision. Given a model with barriers, the front-end first generates several biprocesses without barriers, each corresponding to a possible swap strategy (i.e. the permutation done at each barrier). The equivalence holds if one of the produced biprocesses is diff-equivalent (proven in [10]). Similar techniques [21] have been used in the tool Tamarin relying on multisets. Essentially, all agents are put in a multiset at synchronisation barriers and a rule allows to shuffle this multiset before moving to the next phase. Therefore, the same limitations w.r.t. replications hold. Moreover, Tamarin will also have to explore all possible swaps. The fact that no replication can be put under a barrier notably forbids to model authorities crossing phases (because one needs to consider unbounded number of sessions of them) as well as threat models where no dishonest voter is considered for the same reason. We give more details on this in Appendix F.2.

Small-attack property. A different line of work is to devise small attack properties, as for example in [28] for ballot secrecy. They show that proving ballot secrecy for some specific finite-scenarios implies ballot secrecy for the general, unbounded case. The focus in [28] is on complex ballot weeding mechanisms, as used for example in Helios [2]. In contrast to our work, they require that the pre-tally part contains only one voting phase that must be action-determinate (same actions yield statically equivalent frames). This approach is therefore unable to deal with e-voting protocol models that involve more than one phase, like the ones we consider in Section 5. Moreover, considering only one phase greatly simplifies the verification since it hides the diff-equivalence problems mentioned previously. Moreover, the finite-scenarios still lead to state space explosion problems. Because of this, they were unable to automatically verify the JCJ protocol, even without modelling the registration phase.

The analysis method we develop in this paper borrows the methodological approach of [23]: devise sufficient conditions implying a complex privacy property hard to verify, via a careful analysis and categorisation of known attacks. However, we target a different class of protocols and property and devise different conditions.

7 CONCLUSION
We presented three conditions that together imply ballot secrecy. They proved to be tight enough to be conclusive on several case studies. Verifying ballot secrecy in a modular way via our conditions constitutes a new approach which outperforms prior works: we cover a greater class of e-voting protocols and threat models and their analysis is more efficient.

Based on the main limitations of our current work, we identify several avenues for future work. First, the class of tally we can deal with is rather small. Hence, our method is currently unable to deal with revotes. While adding revotes might be directly achievable, we conjecture that considering revoting policies (e.g. ballot weeding in Helios as analysed in [28]) is a more intricate challenge. Further, our notion of tally cannot deal with homomorphic tallying and only produces a set of votes while e-voting protocols satisfying verifiability should also produce verification data (e.g. ZK proofs of correct decryption). We would like to extend our class of e-voting protocols accordingly. Second, our notion of fairness and
our Dishonest Condition lacks precision in presence of some mixnet roles. For instance, a degenerated mixnet such as \( M = 1 : 1 (c, x), 2 : out(c, dec(x, k)) \) (where \( c \) is public) introduces spurious attacks that are not prevented by our fairness condition and that are detected by our Dishonest Condition (the problem disappear when \( c \) is private). Third, we believe that our conditions can be adapted to enforce more complex privacy-type properties in e-voting protocols as receipt-freeness and coercion-resistance [6, 19]. Finally, we would like to implement the algorithm we presented for verifying all conditions as a ProVerif front-end.

We think that the modular privacy via sufficient conditions methodology we presented advances the state-of-the-art for the verification of privacy-related properties, and paves the way for further improvements.

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A REVIEWs AND IMPROVEMENTs

We have submitted a preliminary version of this work and received several reviews. Some of them raised concerns about aspects of this preliminary work. We now summarize those concerns and explain how we took advantages of those reviews to improve our method and the paper in many ways.

A.1 Notion of Ballot Secrecy

In the previous form of this work, we defined ballot secrecy differently. Our definition was based on a very strong form of fairness that essentially requires that both \( A \) and \( B \) should cast a vote. Some reviewers were rightly concerned about this definition and its relation with the state-of-the-art definition [26] in the symbolic model. We have deeply reworked our fairness condition and our ballot secrecy notions. Essentially, our fairness condition requires that the attacker does not prevent a specific voter to progress to a next phase. As explained in Section 3.2, this is less restrictive than using synchronisation barriers as done in most of the literature. Our notion of ballot secrecy is now closely related to other existing definitions and we consider our notion of fairness itself as a (small) contribution since it plays the role of usual synchronisation barriers without its limitations discussed in Section 3.2 and in Section 6. Obviously, those modifications to ballot secrecy have required many improvements on the conditions side as well in order to keep our Main Theorem sound whose the proof has been deeply retooled.
A.2 Automate the Verification of our Conditions

In the previously submitted paper, definitions of phase roles, idealised execution for n, m, etc. were more complex. Furthermore, no reviewer complained about our claim that our method is automated. Now, phase roles are defined as bijective renaming of the honest execution that is part of the given e-voting protocol. We explain in detail how all necessary objects can be constructed in linear-time when describing our algorithm, which allow for verifying all our conditions in Section 5.1. We notably provide a simple heuristic for Guessing leaking labels that is precise enough to conclude on all our case studies. Overall, we think that we have addressed concerns about automation of our method. We also think that is now clear why it is fair to compare the time needed for prior techniques to conclude with the time for our technique to verify the sufficient conditions, since all encodings of the pre-processing can be done in linear-time.

While we agree that providing an actual implementation would have strengthened our contributions, we believe that it is important to ground the approach on strong foundations first while making sure everything could be implemented easily as witnessed by the algorithm we described in Section 5.1. We also would like to point out the fact that 8 years passed between the theoretical design of the "switching approach" [20] (2008) and its actual implementation in ProVerif [10] (2016).

A.3 On our Class of E-voting Protocols and its Limitation

A reviewer was surprised to find no discussions about potential limitations of the class of e-voting protocols we are dealing with.

We think that is no longer the case as we now identify in Section 7 numerous such limitations and future work for addressing them.

B MODEL AND DEFINITIONS

B.1 Term algebra

We now present the term algebra used to model messages built and learned by the attacker. We consider a participants, while variables in in of arity , and the following rewriting rule:

\[ \text{blind}(\text{sign}(\text{blind}(x_m, y), z_k), y) = \text{sign}(x_m, z_k) \]

\[ \text{blind}(\text{sign}(x_m, y), y) = x_m. \]

B.1.1 Equational Theories. As in the process calculus presented in [9], constructor terms are subject to an equational theory used for modelling algebraic properties of cryptographic primitives. Formally, we consider a congruence \( \equiv \) on \( T(\Sigma_c, N \cup X) \), generated from a set of equations \( E \) over \( T(\Sigma_c, X) \). We say that a function symbol is free when it does not occur in \( E \).

Example B.2. To reflect the algebraic properties of the blind signature, we may consider the equational theory generated by the following equations:

\[ \text{unblind}(\text{sign}(\text{blind}(x_m, y), z_k), y) = \text{sign}(x_m, z_k) \]

B.1.2 Computation relation and rewriting systems. We can also give a meaning to destructor symbols. For this, we need a notion of computation relation \( \downarrow \) on \( T(\Sigma, N) \times T(\Sigma, N) \) such that for any term \( t \), \( t \downarrow u \iff t \downarrow u' \) if, and only if, \( u \equiv u' \). While the precise definition of the computation relation is unimportant for this paper, we describe how it can be obtained from rewriting systems.

A rewriting system is a set of rewriting rules \( g(u_1, \ldots, u_n) \rightarrow u \) where \( g \) is a destructor, and \( u_1, \ldots, u_n \in T(\Sigma_c, X) \). A ground term \( t \) can be rewritten into \( t' \) if there is a position \( p \) in \( t \) and a rewriting rule \( g(u_1, \ldots, u_n) \rightarrow u \) such that \( t_p = g(v_1, \ldots, v_n) \) and \( v_1 \equiv u_1 \theta, \ldots, v_n \equiv u_n \theta \) for some substitution \( \theta \), and \( t' = t[\theta]_p \) (i.e. \( t \) in which the sub-term at position \( p \) has been replaced by \( u \)).

Moreover, we assume that \( u_1 \theta, \ldots, u_n \theta \) and \( u \theta \) are messages. Finally, for some term \( t \), we have \( t \downarrow u \) when \( u \) is a message such that \( t \rightarrow u \). We write \( t \downarrow \) to denote that there exists some \( u \) such that \( t \downarrow u \), and write \( t \downarrow \) otherwise.

Example B.3. The properties of symbols in \( \Sigma_d \) (see Example B.1) are reflected through the following rewriting rules:

open(commit(x_m, y), y) \rightarrow x_m eq(x, x) \rightarrow ok

verSign(sign(x_m, z_k), pkv(z_k)) \rightarrow x_m

\[ \pi_i((x_1, x_2)) \rightarrow x_i \quad \text{for} \quad i \in \{1, 2\}. \]

This rewriting system is convergent modulo the equational theory \( E \) given in Example B.2, and induces a computation relation as described above. For instance, we have that \( \text{open}(\text{verSign}(t, \text{pkv}(sk_A)), k_c) \downarrow u \) where \( t = \text{unblind}(\text{sign}(\text{blind}(\text{commit}(v, k_c), k_b), sk_A), k_b) \) because \( t \equiv \text{sign}(\text{commit}(v, k_c), sk_A) \).

Example B.4. We are able to model the boolean operators using a destructor and in \( \Sigma_c \cap \Sigma_{pub} \) and the following rewriting rule:

and(x, y) \rightarrow ok. The induced computational relation satisfies for any terms \( t_1, t_2 \) that \( t_1 \downarrow t_2 \downarrow \) ok, and only if, \( t_1 \downarrow \) and \( t_2 \downarrow \).

For the purposes, we also split the signature \( \Sigma \) into two parts, namely \( \Sigma_{pub} \) (public function symbols, known by the attacker).
\(F, Q := 0 \text { null}\)
| \(\text{in}(c, x).P\) input
| \(\text{out}(c, u).P\) output
| let \(x = \text{vin} P\) else \(Q\) evaluation
| \(P \parallel Q\) parallel
| \(\nu n.P\) replication
| \(i : P\) phase

where \(c \in C, x \in X, n \in N, u \in T(\Sigma_c, N \cup X), i \in \mathbb{N}\), and \(v \in T(\Sigma, N \cup X)\).

**Figure 4:** Syntax of processes

\[
V(id, v) = 1 : v. k. k'. out(c, (pk(\text{id})); s)). \text{in}(c, x),
\]
if \(\text{verSign}(x, pk(k')) = e\)

\[
\text{if} \; z = \text{out}(c, \text{unblind}(x, k')) \text{. in}(c, y),
\]
if \(y = (y_1; M)\)

\[
\text{then} \; \text{out}(c, (y_1, (M, k)))
\]

**Figure 5:** Voter role of FOO (for some channel \(c \in C_{\text{pub}}\) where \(M = \text{commit}(c, k), e = \text{blind}(M, k')\) and \(s = \text{sign}(e, \text{key(id)})\))

and \(\Sigma_{\text{priv}}\) (private function symbols). An attacker builds his own messages by applying public function symbols to terms he already knows and that are available through variables in \(\mathcal{W}\). Formally, a computation done by the attacker is a recipe (noted \(R\), i.e., a term in \(T(\Sigma_{\text{pub}} \cdot \mathcal{W})\)).

### B.2 Process algebra

We assume \(C_{\text{pub}}\) and \(C_{\text{priv}}\) are disjoint sets of public and private channel names and note \(C = C_{\text{pub}} \cup C_{\text{priv}}\). Protocols are specified using the syntax in Figure 1.

Most of the constructions are standard. The construct let \(x = v\) \(in\ P\) else \(Q\) tries to evaluate the term \(v\) and in case of success, \(i.e.,\) when \(v \downarrow u\) for some message \(u\), the process \(P\) in which \(x\) is substituted by \(u\) is executed; otherwise the process \(Q\) is executed. Note also that the let instruction together with the eq theory (see Example B.3) can encode the usual conditional construction.

The replication \(\nu n.P\) is an infinite parallel composition \(P \parallel P \parallel \ldots \). We shall always deal with guarded processes of the form \(i : P\). Such a construct \(i : P\) indicates that the process \(P\) may be executed only when the current phase is \(i\). The construct \(v. n.P\) allows to create a new, fresh name \(n\). For a sequence of names \(n\), we may note \(v. n.P\) to denote the sequence of creation of names in \(n\) followed by \(P\). For brevity, we sometimes omit “else 0” and null processes at the end of processes. We write \(fv(P)\) for the set of free variables of \(P\), i.e. the set of variables that are not in the scope of an input or a let construct. A process \(P\) is ground if \(fv(P) = \emptyset\).

The operational semantics of processes is given by a labelled transition system over configurations (denoted by \(K\) \((\mathcal{P}; \phi; i)\) made of a multiset \(\mathcal{P}\) of ground processes, \(i \in \mathbb{N}\) the current phase, and a frame \(\phi = \{w_1 \mapsto u_1, \ldots, w_n \mapsto u_n\}\) (i.e. a substitution where \(\forall i, w_i \in \mathcal{W}, u_i \in T(\Sigma_c, N)\)). The frame \(\phi\) represents the messages known to the attacker. Given a configuration \(K\), \(\phi(K)\) denotes its frame. We often write \(P \cup \mathcal{P}\) instead of \(\{P\} \cup \mathcal{P}\) and implicitly remove null processes from configurations.

**Example B.5.** We use the FOO protocol [22] (modelled as in [10]) as a running example. FOO involves voters and a registrar role. In the first phase, a voter commits and then blindly its vote and sends this blinded commit signed with his own signing key \(\text{key(id)}\) to the registrar. The function symbol \(\text{key(.)}\) is a free private function symbol associating a secret key to each identity. The registrar is then supposed to blindly sign the committed vote with his own signing key \(k_r \in \Sigma_c \cap \Sigma_{\text{priv}}\) and sends this to the voter. In the voting phase, voters anonymously output their committed vote signed by the registrar and, on request, anonymously send the opening for their committed vote.

The process corresponding to a voter session (depending on some constant id and a constant \(v\)) is depicted in Figure 4. A configuration corresponding to a voter \(A\) ready to vote \(v_1\) with an environment knowing the registrar’s key is \(K_1 = \{(V(A, v_1); \{w \mapsto k_r\}); 1\}\).

The operational semantics of a process is given by the relation \(\Rightarrow\) defined as the least relation over configurations satisfying the rules in Figure 2. For all constructs, phases are just handover to continuation processes. Except for the phases, the rules are quite standard and correspond to the intuitive meaning of the syntax given in the previous section.

The rules \(\text{In}, \text{Out}, \text{Next}\) are the only rules that produce observable actions (i.e., non-\(\tau\)-actions). The relation \(\mathcal{A}_1 \ldots \mathcal{A}_n\) between configurations (where \(\mathcal{A}_1 \ldots \mathcal{A}_n\) is a sequence of actions) is defined as the transitive closure of \(\Rightarrow\).

### B.3 Trace Equivalence

Continuing Section 2, we give the formal definition of static equivalence.

**Definition B.6.** A frame \(\phi\) is statically included in \(\phi'\) when \(\text{dom}(\phi) = \text{dom}(\phi')\), and

- for any recipe \(R\) such that \(R \phi' \downarrow u\) for some \(u\), we have that \(R \phi \downarrow \text{null} u'\) for some \(u'\);
- for any recipes \(R_1, R_2\) such that \(R_1 \phi \downarrow u\) and \(R_2 \phi' \downarrow u\) for some message \(u\), there exists a message \(v\) such that \(R_1 \phi' \downarrow v\) and \(R_2 \phi' \downarrow v\).

Two frames \(\phi\) and \(\phi'\) are in static equivalence, written \(\phi \sim \phi'\), if the two static inclusions hold.

**Example B.7.** Continuing Example 2.3, we give the formal witness of statically inequivalence. Remind that \(\phi\) is as in Example 2.2 and \(\phi' = \phi(v_1 \mapsto v_2)\) i.e. the frame one obtains from \(\phi\) by replacing the constant \(v_1\) by the constant \(v_2\). If we let \(R_o\) be the recipe \(\text{open}(\pi_2(\pi_2(w_1)), \pi_2(\pi_2(w_1)))\) and \(R_1\) be the recipe \(v_1\), one would obtain \(R_0 \phi\downarrow v_1\) and \(R_1 \downarrow v_2 \neq v_1\) while \(R_0 \phi' \downarrow v_2\) but \(R_1 \downarrow v_1 \neq v_2\).

Remind that \(\text{obs(tr)}\) is defined as the subsequence of \(\text{tr}\) obtained by erasing all the \(\tau\) actions (i.e. \(\tau_{\text{then}}, \tau_{\text{else}}\)).

**Definition B.8.** Let \(K_1\) and \(K_2\) be two configurations. We say that \(K_1\) is trace included in \(K_2\), written \(K_1 \subseteq K_2\), when, for any \(K_1 \uparrow S\) there exists \(K_2 \uparrow S\) such that \(\text{obs(tr)} = \text{obs(tr')}\) and \(\phi(K_1') \sim \phi(K_2')\). They are in trace equivalence, written \(K_1 \approx K_2\), when \(K_1 \subseteq K_2\) and \(K_2 \subseteq K_1\).
B.4 Diff-Equivalence

The semantics of bi-processes is defined as expected via a relation that expresses when and how a bi-process may evolve. A bi-process reduces if, and only if, both sides of the bi-process reduce in the same way triggering the same rule: e.g., a conditional has to be evaluated in the same way on both sides. For instance, the Tzen and Elsif rules for the let construct are depicted in Figure 6.

When the two sides of the bi-process reduce in different ways, the bi-process blocks. The relation \( \text{sb} \) on bi-processes is therefore defined as for processes. This leads us to the following notion of diff-equivalence.

**Definition B.9.** A bi-configuration \( K_0 \) satisfies diff-equivalence if for every bi-configuration \( K = (\mathcal{P}, \phi) \) such that \( K_0 \xrightarrow{\text{tr}} K \) for some trace \( \text{tr} \), we have that:

- both sides generate statically equivalent frames: \( \text{fst}(\phi) \sim \text{snd}(\phi) \);
- both sides are able to execute same actions: if \( \text{fst}(K) \xrightarrow{id} K' \) then there exists a bi-configuration \( K' \) such that \( K_0 \xrightarrow{\text{tr}} K' \) and \( \text{fst}(K') = A_2 \) (and similarly for \( \text{snd} \)).

C CONDITIONS

**Example C.1.** Consider a voter’s role process \( V(id, v) = 1 \) : out(a, id), out(a, v) (other components are unimportant here). This trivial protocol is an abstraction of a registration phase (voter sends its identity) followed by a voting phase (voter sends its vote).

We show this does not ensure ballot secrecy. Consider the following fair execution: \((S; \emptyset; 1) \xrightarrow{w_0}(\emptyset; \emptyset; 1) \) with \( \text{tr} = \text{out}(a, w_{id}), \text{out}(a, w_{v}) \).

\( \text{out}(a, w_{id}', \text{out}(a, w_{v}') \) and \( \phi = \{w_{id} \rightarrow A, w_{v} \rightarrow v_0, w_{id}' \rightarrow B, w_{v}' \ightarrow v_1\} \). This execution has no indistinguishable counterpart in \( S_1 \). Indeed, because the first message reveals the identity of the voter, the attacker can make sure that the voter \( A \) executes the first output \( (i.e. w_{id}) \) on the \( S_1 \) side as well. After the first output \( w_{id} \rightarrow A \), the \( S_1 \) side can only output either \( B \) or \( v_1 \) (because A votes \( v_1 \) in \( S_1 \) ) but not \( v_0 \). However, because the second message reveals the vote, the attacker expects a message \( v_0 \) and can test whether \( w_{v} \) equals \( v_0 \) or not. To summarise, the only executions of \((S_1; \emptyset; 0) \) following the same trace \( \text{tr} \) produce frames that are never statically equivalent to \( \phi \). Thus, this protocol does not ensure ballot secrecy because in a single phase (i.e., phase 1), there is one output revealing the identity of the voter and one output revealing the voter’s vote.

However, the process \( V(id, v) = 1 \) : out(a, id), 2 : out(a, v) ensures ballot secrecy and does not suffer from the above problem. The attacker cannot force \( A \) to execute its first message leaking identity and then immediately its second message leaking its vote, because doing so would kill the process \( V(B, v_1) \) (which is still in phase 1) preventing the whole execution from being fair. Thus, the attacker has to trigger all possible first-phase actions of \( A \) and \( B \) before moving to the second phase. After the first phase, we end up with the processes \( \{\text{out}(a, v_0), \text{out}(a, v_1)\} \) on the \( S_1 \) side and \( \{\text{out}(a, v_2), \text{out}(a, v_3)\} \) on the \( S_2 \) side, which are indistinguishable.

Thus, in this first iteration, we split outputs revealing identity and outputs revealing votes in distinct phases. This enables breaking links between identity and vote.

D PROOFS

**Lemma D.1.** Let \( v_i, v_j \) be some distinct votes in \( V \) and \( tr \) a trace of the form \( tr_0, \phi \cdot phase(k) \cdot tr_{1} \) for some \( 1 \leq k \leq k' \) where no phase- \( \cdot \) action occurs in \( tr_1 \). If the dishonest condition holds then there exists a fair execution \( \{\{V(id_A, v_i), V(id_B, v_j)\} \cup \{R_i, v_0\} \} \xrightarrow{\text{tr}} (\mathcal{P}; \phi; k) \). If, and only if, there exist pairwise distinct names \( n_{id_A}^{id}, n_{id_B}^{id}, n_{v}^{id}, n_{v}^{id} \) (not including vote or identity), a trace \( tr'' = tr'_0, phase(k) \cdot tr''_{1} \) and a fair execution

\[
(\{\{V(id_A, n_{id_A}^{id}), (v_i, n_{v}^{id})\}, \{V(id_B, n_{id_B}^{id}), (v_j, n_{v}^{id})\}\} \cup \{R_i, v_0\}) \xrightarrow{\text{tr}} (\mathcal{P}; \phi; k)
\]

where \( [\{id_A, v_i\}] \) and \( [id_B, v_j] \) had an honest interaction in \( tr'_0, phase(k) \) up to phase \( k \). In both directions, we additionally have that \( \text{obs}(tr') = \text{obs}(tr) \) and \( \text{Res}(tr, \phi) = \text{Res}(tr, \phi) \).

**Proof.** (\( \Rightarrow \)) By fairness, we deduce that, in the given execution after the action phase \( k \) of the processes, there are processes annotated \( [id_A, v_i] \) and \( [id_B, v_j] \) both at phase \( k \). First item of the Dishonest Condition implies that \( A \) and \( B \) (disjoint) honest interactions in \( tr_0, phase(k) \) up to phase \( k \). This allows us to define properly \( n_{id_A}^{id}, n_{id_B}^{id}, n_{v}^{id}, n_{v}^{id} \) and all (disjoint) authorities sessions involved in the two disjoint honest interactions (note that names used in authorities starting in phases \( k' > k \) can be chosen arbitrarily). As a slight abuse of notation, we may omit the vote or the identity from those vectors and write for instance \( n_{id_A}^{id} \) to refer to \( (id_A, n_{id_A}^{id}) \).

Moving backward some \( r \) actions, one obtains a fair execution

\[
(\{\{V(n_{id_A}^{id}, v_i), \{v_i, n_{v}^{id}\}\}, \{V(n_{id_B}^{id}, v_j), \{v_j, n_{v}^{id}\}\}\}) \sqcup \{R; v_0\} \xrightarrow{\text{tr}} (\mathcal{P}; \phi; k)
\]

with \( \text{obs}(tr) = \text{obs}(tr') \) where \( tr'' = tr'_0, phase(k) \cdot tr''_{1} \). We now distinguish two cases whether \( tr = tr^h \) or not to prove that the latter execution can also be executed starting with roles \( A^h \) instead of \( A \).

If \( tr = tr^h \) then there is only one instantiation of the honest trace. Therefore, voters \( A \) and \( B \) followed the idealised trace \( tr_0, phase(k) \) up to phase \( k \). Therefore, their executions up to the action phase \( k \) can be exactly mimicked by executions of process \( A^h \) for \( A \in \mathcal{R}_0 \cup \{V\} \) with appropriate names. Indeed, once names \( n_{id_A}^{id}, n_{id_B}^{id} \) are fixed, there is only one possible execution (module \( = \) relation) following the trace \( tr^h \) and this is, by definition of those processes, an execution \( A^h \) can play. Moreover, the resulting processes for \( A \) and \( B \) right after the action phase \( k \) are the same starting with \( A(\cdot, \cdot) \) for \( A \in \mathcal{R}_0 \cup \{V\} \) or with \( A^h(\cdot, \cdot) \); they are indeed of the form \( V^h \). Hence, the final sub-trace \( tr''_1 \) can equally be executed by roles \( A^h \). Note also that the resulting frame is \( \psi = \phi \) and the resulting trace has same observable actions as \( tr' \).

Otherwise (i.e. \( tr = tr^b \)) we remark that the multiset of processes at the beginning of Execution 1 can be reached from the multiset of processes \( \{S_A(A, v_i), S_B(A, v_j)\} \sqcup \{R\} \). The Execution 1 can thus be played by the right side of the biprocess \( B^D \) (by a biprocess one can obtain from it by applying a bijection of free public constants). Applying diff-equivalence of \( B^D \) (i.e. second item of the Dishonest Condition) and the fact that diff-equivalence is stable by bijection of free, public constants, allows us to replace \( V \) (resp. role process
Let 
\[(i : \text{let } x = \text{choice}[u_l, u_r] \text{ in } P \text{ else } Q) \!
\stackrel{\text{when } u_l \downarrow u_l \text{ and } u_r \downarrow u_r}{\longrightarrow} (i : P)\]

Let-Fail 
\[(i : \text{let } x = \text{choice}[u_l, u_r] \text{ in } P \text{ else } Q) \!
\stackrel{\text{when } u_l \downarrow \bot \text{ and } u_r \downarrow \bot}{\longrightarrow} (i : \bot)\]

Figure 6: Two rules of the semantics for bi-processes

A) by \(V^Y\) (resp. \(A^P\)) in the latter execution whilst preserving the excutability of the same observable actions (i.e. \(\text{obs}(tr')\)) and leading to a frame \(\phi' \sim \phi\).

Next, we remark that thanks to the appropriate multi-set of processes at the beginning.

In order to conclude, we shall prove \(\text{Res}(tr, \phi) = \text{Res}(tr', \phi')\). In the case \(th = tr_h\), it follows from \(\phi' = \phi\) and that the frame \(tr_r'\) has some observable actions as tr. Otherwise, we assume \(\text{Res}(tr, \phi) \neq \text{Res}(tr', \phi')\) for the sake of the contradiction. We remark that \(\text{BB}(tr, \phi) = \text{BB}(tr', \phi')\) follows from \(\phi \sim \psi\) and \(\text{obs}(tr) = \text{obs}(tr')\). Further, there exists an handle \(w\) such that \(\text{out}(cw, w) \in tr\) (and thus \(\text{out}(cw, w) \in tr')\).

By symmetry, we assume \(\text{Extract}(w) \downarrow v_l \in V' \cup \{\bot\}\). Extract\((w)\) \(\downarrow v_l \in V' \cup \{\bot\}\) with \(v_l \neq v_r\). At least on one side, the extraction does not lead to \(\bot\) (otherwise we have \(w = \bot\)).

We now consider a straightforward extension of the previously considered execution of the bi-process \(B_{O}^{\Omega}\) by adding the trace \(tr_{O} = r_{\mathcal{T}}(n_{l}, w)_{\mathcal{T}}(n_{l}, w_{c})\). This trace corresponds to the replication of the OpenBal process and a usage of one instance of the oracle which tries to extract a vote from the ballot \(w\). Note that the given execution can be extended with this trace on the left (the conditional holds) and because \(B_{O}^{\Omega}\) is diff-equivalent, on the right as well. We call \(\phi_{O}^{\prime}\) (resp. \(\phi_{O}^{\prime'}\)) the resulting frame on the left (resp. on the right).

By diff-equivalence, it holds that \(\phi_{O}^{\prime} \sim \phi_{O}^{\prime'}\). We remark that \(w_{c} \psi_{O}^{\prime} = v_l\) and \(w_{c} \phi_{O}^{\prime} = v_r\). Remind that \(v_l\) and \(v_r\) are two different public terms not involved in equations in \(E\). Therefore, \(w_{c} \phi_{O}^{\prime} = \mathcal{E} v_{l}\). Hence the recipe \(e(w_{c}, v_{l})\) (remind that \(v_l\) is a public constant) does not fail when applied on the left (i.e. on \(\phi_{O}^{\prime}\)) but does fail when applied on the right (i.e. on \(\phi_{O}^{\prime'}\)) contradicting \(\phi_{O}^{\prime} \sim \phi_{O}^{\prime'}\).

\(\Rightarrow\) Using the fact that \([id_{A}, v_{l}]\) and \([id_{B}, v_{l}]\) had an honest interaction in \(tr_{O} \phi_{O}^{\prime}\) up to phase \(k\), one can show by gluing together phase reads the existence of a similar execution of the form

\[
\begin{align*}
&((V(n_{A}^{id}, n_{A}^{id}), V(n_{B}^{id}, n_{B}^{id}))) \psi ! \mathcal{R} \\
&\cup_{A \in R_{A}} \{A(n_{A}^{id}, n_{A}^{id}), A(n_{B}^{id}, n_{B}^{id})\}; \phi_{O}^{\prime} \!
\stackrel{[\mathcal{R}(n_{A}^{id}, n_{B}^{id}) R(n_{A}^{id}, n_{B}^{id})]; \phi_{O}^{\prime'}}{\longrightarrow}(P^{\prime}, \phi_{O}^{\prime' \prime}); k).
\end{align*}
\]

(2)

Via a similar proof to the former one for \(\Rightarrow\), we make use of the diff-equivalence of \(B_{O}^{\Omega}\) to get an execution with real role processes when \(A\) and \(B\) did not follow the idealised trace; otherwise, we get the execution with real role processes from the uniqueness of executions following \(tr_{O}\). The execution from \(\{(V(n_{A}^{id}, v_{l}), V(id_{B}, v_{l})) \cup \mathcal{R}; \phi_{O}^{\prime} \} \!
\stackrel{[\mathcal{R}(n_{A}^{id}, n_{B}^{id}) R(n_{A}^{id}, n_{B}^{id})]; \phi_{O}^{\prime'}}{\longrightarrow}(P^{\prime}, \phi_{O}^{\prime' \prime}); k)\) can then be obtained by creating appropriate names since they are pairwise distinct.
occurs in $tr'$ then $w_l$ induces a valid ballot on the left if, and only if, it induces a valid ballot on the right. This is because $\phi_l \sim \phi_r$ and $\phi_{l_r}[]$ is a public term (and thus induces a recipe). Let us first assume that either $id_A$ or $id_d$ casts a ballot. In such a case, by fairness, both cast a ballot. Therefore, there are two handles $w_0, w_1$ corresponding to the honest casting of $[id_A, v_0]$ (resp. $[id_A, v_1]$) and $[id_B, v_1]$ (resp. $[id_B, v_0]$) on the left (resp. on the right). We have that $\text{Extract}(w_i \phi_l \phi_r) \vdash v_j$ for all $j = 0, 1$. We can now split the set of valid ballots into ballots $w_0, w_1$ and the others: for $d \in \{l, r\}$, one has $b_d = (w_0 \phi_l \phi_r, w_1 \phi_l \phi_r) \cup \{b_d^1, \ldots, b_d^l\}$ where $b_d^i = w_0 \phi_d \phi_r (w_0 \phi_l \phi_r) = \text{Extract}(w_0 \phi_l \phi_r, w_1 \phi_l \phi_r) = (v_0, v_1)^d$. It remains to show that $\text{Extract}(b_d^1, \ldots, b_d^l) = \text{Extract}(\{b_1^d, \ldots, b_l^d\})$. We actually have that $\text{Extract}(b_d^i) = \text{Extract}(b_d^i) = \text{Extract}(b_d^i)$ for all $1 \leq i \leq l$ as a direct consequence of the Tally Condition. If neither $A$ nor $B$ cast a ballot then $\text{Res}(tr', \phi_l) = \text{Res}(tr', \psi)$ stems from $\text{Extract}(b_d^i) = \text{Extract}(b_d^i)$ for all $1 \leq i \leq l$.

If $k = 1$ then no phase($\phi$) action occurs in $tr$. In particular, the phase roles for phase 1 can also perform this execution. Formally, replacing $S$ by the following process allows for performing the same execution: $\cup_{A \in R \cup L} \{A(A(n_0^A, n_1^A), A(n_0^B, n_1^B)) \cup R\}$. As a subset of the latter, $\mathcal{R}^{(\mathcal{A}(n_0^A, n_1^B), n_0^A, n_1^B), \mathcal{R}^{(n_0^A, n_1^B), \mathcal{R}^{(n_0^B, n_1^B)}} \cup R\}$ is also able to perform the given execution. We conclude by the Honest relation condition. The proof that Res($\phi$) is preserved is a particular case of the previously discussed proof for the $k > 1$ case.

\qed

E CASE STUDIES

(Resuming Section 5.1) (Syntactical check) If (i) the vote $v_d$ does not syntactically occur at all in outputs of id-leaking phases (i.e. in $\mathcal{R}^{(n_0^A, n_1^B), n_0^A, n_1^B}$) and (ii) there is no vote-leaking phase before an id-leaking phase in the honest trace then the condition is satisfied since item 2 trivially holds then.

We now state that $B^T$ can be used to verify the Tally Condition.

Lemma E.1. If $B^T$ is diff-equivalent then $B$ is diff-equivalent and the protocol ensures the Tally Condition.

Proof. Diff-equivalence is stable by removal of processes in the initial multisets. Formally, for a biprocess $B$, if $B = (\{P\} \cup Q; \phi)$ is diff-equivalent then $(Q; \phi)$ is diff-equivalent. This implies that $B$ is diff-equivalent as well. Now, let us show that the Tally condition holds.

We can actually prove that the diff-equivalence of $B^T$ implies that for any execution of $B$, the two executions on both sides yield exactly the same tally’s outcome (w.r.t. Res($\phi$)). The proof of this is the same as the proof of the Lemma 4.15 in Appendix D.

\qed

F ADDITIONAL EXPLANATIONS ON THE SWAPPING APPROACH

F.1 Example of modified protocol using swapping

We consider the voter process from Example 4.1; i.e. $V(id, v) = 1 : \text{out}(a, id), 2 : \text{out}(a, v)$. We have that $\{(V(A, \text{choice}[v_1, v_2]), V(B, \text{choice}[v_2, v_2]); \emptyset)\}$ is not diff-equivalent since after the two outputs $\text{out}(a, A), \text{out}(a, B)$, the resulting multisets of processes is $\{(\text{out}(a, \text{choice}[v_1, v_2]), \text{out}(a, \text{choice}[v_2, v_2]))\}$ which is obviously not diff-equivalent. However, if one is allowed to swap the order of the two processes on the right side of that biprocess, he would obtain $\{(\text{out}(a, \text{choice}[v_1, v_1]), \text{out}(a, \text{choice}[v_2, v_2]))\}$ which is diff-equivalent.

F.2 Restrictions of the swapping approach

We here provide support for the claims we made in the introduction about the different problems the swapping approach cannot tackle (while our approach can). First, it cannot deal with honest roles present in different phases (except for the voter role). Indeed, such roles would require synchronisation barriers. Moreover, because a potentially unbounded number of dishonest voters communicate with those authorities, this requires modelling an unbounded number of sessions for them. However, in the swapping approach, there cannot be a replication underneath a synchronisation barrier, which mean we cannot model such roles. We encountered this problem for a simplified variant of JCJ and Belenios: it was impossible to model an unbounded number of sessions of the registrar role that creates and sends credentials to voters in the registration phase and then send encrypted credentials to the bulletin box (so that the latter can verify eligibility).

Second, and similarly, it cannot tackle threat models without a dishonest voter, because this would require explicitly modelling an arbitrary number of honest voters. Since they are present in multiple phases, this would require replication.

Third, when it comes to leveraging the swapping approach in ProVerif, false attacks arise when the protocol’s security also relies on the freshness of some data from previous phases. The problem is that for the generated processes, ProVerif considers two different sessions of a certain phase using the same data resulting from one single session of a previous phase, as a valid execution. The reason is that, for a fixed swap strategy, ProVerif replaces synchronisation barriers of a process $P$ by private communications exchanging all data the process currently knows with another process $Q$ with which the swap occurs. Those new private communications are abstracted by the Horn-Clause approximations used by ProVerif: an input on a private channel is not consumed upon use, and can be replayed later on. Therefore, ProVerif also explores the possibility of swapping data with an old session of $Q$ whose data has already been swapped before. This caused the false attacks for JCJ and Belenios (see Figure 3); the credential being the fresh data coming from the registration phase and used during the voting phase.

Finally, the swapping approach suffers from exponential blow up. Indeed, the compiler produces $\prod_{i=0}^{n_i} (n_i)$ processes where $n_i$ is the number of processes active at phase $i$ (e.g. 3 phases, $n_1 = 3$ lead to 216 processes to verify). This is unsurprising since the compiler has to generate as many processes as possible swaps, to guess one that yields security. The same problem arises in Tamarin implementing the swapping approach through multisets [21] since the tool may have to explore all possible shufflings of multisets for each phase.