Performance of steel fiber reinforced concrete beams under sustained service loads

Y Li¹,², Y Liu¹ and K H Tan²

¹College of Resources and Civil Engineering Northeastern University, Shenyang, China;
²National University of Singapore, Singapore.

Abstract. In order to evaluate the performance of steel fiber reinforced concrete beams under sustained service loads, the influences of two key parameters, sustained load level and steel fiber content, are investigated by series of bending tests. Specimens Group I are subjected to a sustained load equal to 50% of the ultimate load (i.e. 0.5ₚᵤ) and mix with steel fibers varying as 0%, 0.5%, 1.0%, 1.5% and 2.0%. Specimens Group II mix with steel fibers with the content of 1.0% and differ in the sustained load level varying as 0.35ₚᵤ, 0.5ₚᵤ, 0.59ₚᵤ, 0.65ₚᵤ and 0.80ₚᵤ. Considering the influences of degradation of material strength, reduction in reinforcing steel bar area and bond properties with service time, an evaluation model for the serviceability limit state of cracking and deflection is established using time-dependent uncertainty coefficients. The model is used to analyze the time-dependent reliability indices of beams bearing sustained service load. Research results show that reliability index increases with the increasing of steel fiber content in specimens Group I and decreases with the increasing of sustained load level in specimens Group II. In early stage, the reliability index decreases rapidly, whereas with longer period of sustained loading, it decreases gradually. Under long-term sustained loads, the incorporation of steel fiber improves the cracking characteristics ductility of the material, causing effective crack and deflection control in steel fiber reinforced concrete beam.

1. Introduction

Steel fiber-reinforced concrete (SFRC) has wide applications in the field of civil engineering as it can be used to effectively improve the cracking and ductility of concrete. Compared with studies on the ultimate limit state, there has been little study on deflection and crack width of reinforced concrete structure due to insufficient test data and uncertain failure criteria, even though structural cracks and deflections present important problems in engineering practice [1]. In 2011, the time-dependent reliability of prestressed concrete railway sleeper is analyzed by using Monte Carlo Method [2]. Later, the reliability index of in-service pre-stressed concrete bridge is calculated by using neural network and sampling methods [3]. In another study, Creep was found to influence the reliability of reinforced concrete by Jung. [4]. The influence of fire on the time-dependent resistance of reinforced concrete columns prepared with different reinforcement materials was analyzed by Christopher. [5]. Wei Chen et al. (2013) studied the long-term deformation of prestressed steel-concrete composite beams under sustained service loads for two years [6]. However, most previous studies focused on ordinary concrete and elements subjected to sustained loads over a relatively short period.
The addition of steel fibers makes the material more complex, and it is difficult to ensure the stability of sustaining loads over a long period. Thus, influences on the reliability index as a result of long sustained load period have not been reported for SFRC beams. With increased service time, concrete material will deteriorate and the cross-section area of reinforcement will be reduced due to environmental factors such as corrosion. Additionally, the bond between the steel and concrete will be reduced. In this work, these influencing factors were analyzed, together time-dependency of uncertainty coefficients. Crack width and deflection served as the basis for the model of time-dependent limit state equation. This model was then used to analyze the characteristics of SFRC beams under sustained service load.

2. Statistical parameters

2.1. Tensile strength of concrete
The average and standard deviation of time-dependent tensile strength for concrete grade C10–C110 are, respectively, as follows [7].

\[ \mu_{f_t(t)} = 0.24 \mu_{f_{cu}(t)}^{2/3} \]  
\[ \sigma_{f_t(t)} = 0.24 \sigma_{f_{cu}(t)}^{2/3} \]

where: \( \mu_{f_t(t)} \) = average of time-dependent tensile strength; \( \sigma_{f_t(t)} \) = standard deviation of time-dependent tensile strength; \( \mu_{f_{cu}(t)} \) = average of time-dependent tensile strength; \( \sigma_{f_c(t)} \) = the standard deviation of time-dependent tensile strength.

2.2. Effective cross-section area of reinforcement
Due to corrosion caused by carbonation, the reduced steel reinforcement area at time \( t \) can be described as [8]:

\[ A_t(t) = \begin{cases} A_{0}, & (t \leq t_c) \\ A_{0}[1 - \frac{\lambda}{r_0}(t - t_c)], & (t > t_c) \end{cases} \]

where: \( A_{0} \) = the initial area of rebar; \( \lambda \) = the rebar corrosion rate; \( r_0 \) = the initial radius of rebar; \( t_c \) = the initial corrosion time.

2.3. Bond strength between rebar and concrete
With extended service time, the term \( \varphi(t) \) is used to account for the loss in bond strength between steel and concrete; \( \varphi(t) = 0 \) when there is no corrosion of the steel bars, and 0.95 when the steel has started to rust [9].

2.4. Uncertainty coefficients of evaluation model
Based on crack width, the uncertainty coefficient of the evaluation model \( P \) can be generally expressed as:

\[ p = \frac{\omega_{t,max}}{\omega_{c,max}} \]

where \( \omega_{t,max} \) = measured maximum crack width; and \( \omega_{c,max} \) = calculated maximum crack width.

Based on deflection, the uncertainty coefficient of the model \( P' \) can be generally expressed as:

\[ p' = \frac{f_{t,max}}{f_{c,max}} \]

where \( f_{t,max} \) = maximum measured deflection; and \( f_{c,max} \) = maximum calculated deflection.
3. Time-dependent reliability analysis

3.1. Time-dependent reliability model based on crack control

3.1.1. Maximum crack width of SFRC beams. According to the contribution of load effect and long-term effects of load, the maximum calculated crack width can be generally expressed as (CECS 38-2004) [10]:

$$\omega_{c,\text{max}} = \omega_{c,\text{max}}(1 - k_{cw}\lambda_f) = \frac{\alpha_{cr}}{E_s} \left(1.1 \frac{M_g + \varphi_0 M_Q}{0.87 h_0 A_s} - 0.65 \frac{f_{tk}}{\rho_{te}} \right) \times (1.9 c + 0.08 \frac{d_{eq}}{\rho_{te}})(1 - k_{cw}\lambda_f) \quad (6)$$

where $k_{cw}$ = influence coefficient of steel fiber on the crack width; $\lambda_f$ = characteristic value of steel fiber content ($\lambda_f = \rho_f \frac{l_f}{d_f}, \rho_f$ = volume fraction of steel fibers, $l_f$ = fiber length, $d_f$ = fiber diameter); $\alpha_{cr}$ = coefficient of the mechanical characteristic($\alpha_{cr}=1.9$); $E_s$ = elastic modulus of reinforcement; $M_g$ = moment under dead load; $\varphi_0$ = effect coefficient of quasi-permanent value; $h_0$ = effective section height; $A_s$ = area of tensile bars; $f_{tk}$ = standard value of concrete tensile strength, based on SFRC strength; $\rho_{te}$ = longitudinal reinforcement ratio (when $\rho_{te} < 0.01$, take $\rho_{te} = 0.01$); $c$ = distance from the edge of the longitudinal reinforcement to the bottom edge of the tensile zone (when $c < 20$ mm, take $c = 20$ mm; when $c > 65$ mm, take $c = 65$ mm); $d_{eq}$ = equivalent diameter of longitudinal reinforcement.

3.1.2. Limit state equation. For serviceability limit states, the crack width limit specified in current codes is regarded as the resistance $R$ of the SFRC, and the maximum crack width under actual load is regarded as the load effect $S$. With these values, the limit state equation can be proposed as:

$$Z = R - S = \omega_{lim} - p \omega_{c,\text{max}} \quad (7)$$

where $\omega_{lim}$ = crack width limit in current codes; $\omega_{c,\text{max}}$ = maximum crack width; $P$ = uncertainty coefficient of the calculation model.

Substituting Eq. (6) into Eq. (7), the limit state equation based on crack width for SFRC beam can be obtained as:

$$Z = \omega_{lim} - p \frac{\alpha_{cr}}{E_s} \left(1.1 \frac{M_g + \varphi_0 M_Q}{0.87 h_0 A_s} - 0.65 \frac{f_{tk}}{\rho_{te}} \right) \times (1.9 c + 0.08 \frac{d_{eq}}{\rho_{te}})(1 - k_{cw}\lambda_f) \quad (8)$$

Considering ($\nu$=1 (relative bond characteristic coefficient between steel bar and concrete)) and $d_1$ (diameter of reinforcement), the equivalent diameter can be expressed as $d_{eq} = d_1$. According to CECS 38-2004, the equivalent transformation can be expressed as [10]: where $c = h - h_0 - \frac{d_1}{2}$;

$$A_s = \frac{n \pi d_1^2}{4}, \rho_{te} = \frac{A_s}{0.5bh} = \frac{n \pi d_1^2}{4 \times 0.5bh}$$

where $n$ = number of tension reinforcing bars; $b$ = section width of beam; $h$ = the section height of the beam.

The limit state equation of an SFRC beam based on crack width can be re-written as:

$$Z = \omega_{lim} - p \frac{\alpha_{cr}}{E_s} \left(4.4 \frac{M_g(t) + \varphi_0 M_Q(t)}{0.87 h_0 \pi n d_1^2(t)} - 0.65 \frac{2bh f_{tk(t)}}{n \pi d_1^2(t)} \right) \times \left[1.9 \left(h - h_0 - \frac{d_1}{2}\right) + 0.16 \frac{bh}{n \pi d_1^2(t)} \right](1 - k_{cw}\lambda_f) \quad (9)$$

3.1.3. Time-dependent limit state equation Considering the deterioration of beam with service time, the time-dependent limit state equation of an SFRC beam based on crack width can be written as:

$$Z(t) = \omega_{lim} - p(t) \frac{\alpha_{cr}}{E_s} \left[4.4 \frac{M_g(t) + \varphi_0 M_Q(t)}{0.87 h_0 \pi n d_1^2(t)} - 0.65 \frac{2bh f_{tk(t)}}{n \pi d_1^2(t)} \right] \left[1.9 \left(h - h_0 - \frac{d_1(t)}{2}\right) + 0.16 \frac{bh}{n \pi d_1^2(t)} \right](1 - k_{cw}\lambda_f) \quad (10)$$

where $p(t)$ = time-dependent calculation model uncertainty coefficient based on crack width control, because the maximum measured crack width and calculated values are considered time-dependent; $f_{tk(t)}$ = time-dependent tensile strength of concrete; $d_1(t)$ = change in longitudinal tensile bar diameter over time.
3.2. Time-dependent reliability model based on deflection control

3.2.1. Maximum deflection of reinforced concrete beams

The short-term stiffness of reinforced SFRC flexural members can be calculated as:

\[ B_{fs} = B_s (1 + k_B \lambda_f) \]  

(11)

where \( B_{fs} \) = short-term stiffness of reinforced SFRC members under standard load combinations; \( B_s \) = short-term stiffness of reinforced concrete flexural members under standard load combination; \( k_B \) = influence coefficient of steel fiber on short-term stiffness of reinforced steel fiber-reinforced concrete members, and is equal to 0.35 for steel fiber concrete of strength grades CF20 ~ CF80.

The time-dependent stiffness of reinforced SFRC concrete flexural members can be calculated according to equation (12) [11],

\[ B_{ft} = \frac{M_k}{M_{fs}(1-\epsilon)+M_k} B_{fs} \]  

(12)

where \( \theta = \) influence coefficient; when \( \rho' = 0 \) where \( \rho = A_s/(bh_0) \)

The maximum deflection \( f \) of a SFRC beam is [11]:

\[ f = \frac{5}{48} \left[ M_G \theta + M_Q (\theta \psi_q - \psi_q + 1) \right] \left[ \frac{1.465}{E_c \lambda_s h_0^2} - \frac{0.650325 f_{tk}}{M_{q \rho te} E_s h_0} + \frac{6}{E_c b h_0^2} \right] l_0^2 \cdot \frac{1}{1 + k_B \lambda_f} \]  

(13)

where \( l_0 = \) span of the beam; \( \psi_q = \) strain unevenness coefficient of the longitudinal tensile reinforcement between the cracks; \( E_c = \) elastic modulus of concrete.

3.2.2. Limit state equation

For the normal use limit state, the deflection limit specified in GB 50010-2010 "Concrete Structure Design Code" is used as the resistance \( R \) of the SFRC beam and the maximum deflection generated under the actual load is used as the load effect \( S \). These two values are used to establish the limit state equation:

\[ Z = R - S = f_{lim} - p' f_{c, max} \]  

(14)

where \( f_{lim} = \) specified beam deflection limit; \( f_{c, max} = \) maximum deflection of the beam; \( p' = \) uncertainty coefficient based on deflection control. Substituting Eq.(13) into Eq.(14) gives the limit state equation of SFRC beam based on deflection control:

\[ Z = f_{lim} - p' \cdot \frac{5}{48} \left[ M_G \theta + M_Q (\theta \psi_q - \psi_q + 1) \right] \left[ \frac{1.47}{E_c \lambda_s h_0^2} - \frac{0.65 f_{tk}}{M_{q \rho te} E_s h_0} + \frac{6}{E_c b h_0^2} \right] l_0^2 \cdot \frac{1}{1 + k_B \lambda_f} \]  

(15)

Similarly, \( S \) is expressed as a function of completely independent random variables. From Section 3.1.2, \( A_s \) and \( \rho_{te} \) are transformed as:

\[ Z = f_{lim} - p' \cdot \frac{5}{48} \left[ M_G \theta + M_Q (\theta \psi_q - \psi_q + 1) \right] \left[ \frac{5.86}{n \pi d^4 E_s h_0} - \frac{1.30 b h f_{tk}}{n \pi d^2 M_{q \rho te} E_s h_0} + \frac{6}{E_c b h_0^2} \right] l_0^2 \]  

\[ \frac{1}{1 + k_B \lambda_f} \]  

(16)

3.2.3. Time-dependent limit state equation

The time-dependent limit state equation of steel fiber reinforced concrete beam based on deflection control can be written as:

\[ Z = f_{lim} - p(t)' \cdot \frac{5}{48} \left[ M_Q (t) (\theta - 1) + M_k (t) \right] \left[ \frac{5.86}{n \pi d^4 (t) E_s h_0} - \frac{1.30 b h f_{tk}}{n \pi d^2 (t) M_Q (t) E_s h_0} + \frac{6}{E_c b h_0^2} \right] l_0^2 \]  

\[ \frac{1}{1 + k_B \lambda_f} \]  

(17)

Where \( p(t)' = \) a time-dependent calculation mode uncertainty coefficient based on deflection control.
4. Test overview

4.1. Test specimen design

In the test, ten simply supported beam specimens were designed to withstand ten years of continuous loading (to ensure the stability of long-term loads, concrete blocks were used in the basket). Two different groups that varied in the sustained load level and steel fiber content were tested. Group I specimens had 0.5\% load and steel fiber of 0\%, 0.5\%, 1.0\% 1.5\% and 2.0\%. Group II differed in the sustained load level (0.35\%, 0.5\%, 0.59\%, 0.65\% and 0.80\%), and had a steel fiber content of 1.0\%. The steel fiber content and sustained load level of the test beams are listed in Table 1. The beams had a cross-section dimension of 100 mm × 125 mm, with a total length of 2 000 mm and a span of 1 800 mm. The concrete material consisted of ordinary Portland cement, natural sand, and granite gravel with a maximum particle size of 10 mm.

| Group | Test specimen | Steel fiber content \(\rho_f\) /\% | Sustained load level |
|-------|---------------|--------------------------------|---------------------|
| I     | A-50          | 0                             | 0.50                |
|       | B-50          | 0.5                           | 0.50                |
|       | C-50          | 1.0                           | 0.50                |
|       | D-50          | 1.5                           | 0.50                |
|       | E-50          | 2.0                           | 0.50                |
| II    | C-35          | 1.0                           | 0.35                |
|       | C-50          | 1.0                           | 0.50                |
|       | C-59          | 1.0                           | 0.59                |
|       | C-65          | 1.0                           | 0.65                |
|       | C-80          | 1.0                           | 0.80                |

The test beams were constructed with hooked end steel fibers, with a length of 30 mm and a diameter of 0.5 mm, longitudinal tensile steel bars of 10 mm and HRB335 steel, and compressive longitudinal steel bars with a diameter of 6 mm and HRB335 steel. The ratio of cement: sand: stone: water was 1:1.5:2.5:0.5, and the concrete compressive strength was 40 MPa.

4.2. Loading scheme and measurement

To assess long-term loading, steel plates and concrete blocks were placed at the quarter points of the beam span. For Group I specimens (A-50, B-50, C-50, D-50, and E-50), the load level was 50\% \(P_{ul}\), where \(P_{ul}\) is the design ultimate load carrying capacity of Beam A-50, and \(P_{ul} = 23.3KN\). For Group II specimens (C-35, C-50, C-59, C-65, and C-80), the load level was set as 35\%, 50\%, 59\%, 65\%, and 80\% of the ultimate load \(P_{ul}\). Two dial gauges were placed at the mid-span of the beam to measure the deflection. The gauges were positioned 10 mm from the front and rear edges to reduce the measurement error result from beam torsion, because the imbalance of the support and the loading device can make beam torsion. The average of two measurements was used as the mid-span deflection. A hand-held microscope was used to measure the crack width with an accuracy of ± 0.02 mm. The monitoring time for deflection and crack from the initial applied load were: 1d, 50d, 138d, 230d, 370d, 2 284d (6.25 a), 3 678 d (10 a).

5. Reliability index

According to the resistance decay model, the statistical parameters of resistance were obtained. The time-dependent reliability index results of steel fiber reinforced concrete beams based on crack width control were calculated by combining the second moment method and the equivalent resistance method. Figure 1 shows the time-dependent reliability index of steel fiber reinforced concrete beams.
based on deflection control. The calculation results are shown in Figure 2. The reliability index of the beam lacking steel fiber, A-50, was significantly lower than that of beams containing steel fiber. The higher the steel fiber content as in Beam E-50, the greater the reliability index, and the smaller the crack width and deflection deformation. It can be seen from Figure 1 and Figure 2 that the results were different for specimens at the same sustained load level (0.50$P_{u}$) but with different fiber volumes (0, 0.5%, 1.0%, 1.5%, 2.0%). The order of the reliability index of the Group I of specimens is: $\beta(E - 50) > \beta(D - 50) > \beta(C - 50) > \beta(B - 50) > \beta(A - 50)$. It can be seen from Figures 1b and 2b that the smaller the load level, the smaller were crack width and deflection of the beam. For Group II specimens with the same steel fiber content (1.0%) but different load level (0.35$P_{u}$, 0.50$P_{u}$, 0.59$P_{u}$, 0.65$P_{u}$, and 0.80$P_{u}$), the reliability index values were in the following order: $\beta(C - 35) > \beta(C - 50) > \beta(C - 59) > \beta(C - 65) > \beta(C - 80)$. The lower the load level, the greater the reliability index based on crack width and deflection control. The reliability levels of specimens C - 50 and C - 59 were the closest, indicating similar reliability index values for similar applied loads, which is consistent with the test results.

![Figure 1](image1.png)  
**Figure 1.** Crack width time-dependent reliability index.

![Figure 2](image2.png)  
**Figure 2.** Deflection time-dependent reliability index.

6. **Conclusions**

(1) Based on crack width and deflection, the model of time-dependent reliability was established. In the model, influence mechanisms of time varying on materials were analyzed, as well as the time-dependency of uncertainty coefficients.
(2) The incorporation of steel fiber improved the crack-resistance and toughening efficiency of the material, and effectively inhibited crack width development and deflection deformation of the concrete beam. The reliability index of the beam without steel fibers (A-50) was significantly lower than that of the beam with steel fibers. Moreover, steel fiber content was positively correlated with reliability index. The smaller the crack width, the smaller the deflection deformation.

Acknowledgements
This work presented is jointly supported by Liaoning province natural science fund (20180510019), Fundamental Research Funds for the Central Universities (N17010829) Program for Liaoning Excellent Talents in University (LR2015024).

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