Dibaryons as canonically quantized biskyrmions

T. Krupovnickas\textsuperscript{1} E. Norvaisas\textsuperscript{1} and D.O. Riska\textsuperscript{2,3}

\textsuperscript{1}Institute of Theoretical Physics and Astronomy, Vilnius, 2600 Lithuania
\textsuperscript{2}Helsinki Institute of Physics, 00014 University of Helsinki, Finland
\textsuperscript{3}Department of Physics, 00014 University of Helsinki, Finland

Abstract

The characteristic feature of the ground state configuration of the Skyrme model description of nuclei is the absence of recognizable individual nucleons. The ground state of the skyrmion with baryon number 2 is axially symmetric, and is well approximated by a simple rational map, which represents a direct generalization of Skyrme’s hedgehog ansatz for the nucleon. If the Lagrangian density is canonically quantized this configuration may support excitations that lie close and possibly below the threshold for pion decay, and therefore describe dibaryons. The quantum corrections stabilize these solutions, the mass density of which have the correct exponential fall off at large distances.
1. Introduction

The ground state solutions to Skyrme’s topological soliton model for baryons \[1\] with baryon numbers that are larger than 1 have intriguing geometric structures with striking polyhedral symmetry \[2\]. The simplest example is the system with baryon number 2, which has axial symmetry \[3\]. The deuteron may be viewed as a quantized version of this ground state configuration \[4\]. It remains an open issue whether direct semiclassical quantization of these ground state configurations represent physical systems \[3, 5\].

The numerical construction of these multiskyrmion configurations is a demanding task \[2, 6\]. Fortunately is is possible in many cases to find simple rational maps, which provide remarkably accurate approximations for the multiskyrmion ground state configurations \[7\]. Such rational maps may be viewed to represent direct formal generalizations of Skyrme’s original hedgehog ansatz for the the system with baryon number 1. Employment of the rational map approximation greatly simplifies the study of the quantized modes of the multiskyrmion systems \[8\]. In the case of \(B = 2\) the ground state solution is 6-dimensional, whereas the physical solution in the deuteron channel corresponds to a mode in a 12-dimensional space. We here consider the possibility for dibaryon solutions other than the deuteron, which may be represented as spin-isospin excitations of the ground state solution.

The approach adopted here is to canonically quantize the rational map ansatz for the ground state solution of the baryon number 2 system in representations of arbitrary dimension for \(SU(2)\), using the method of refs.\[9, 10, 11\]. The corresponding states have \(I = J\), and therefore represent dibaryon states other than the deuteron. By the conventional semiclassical quantization method the lowest one of these states is very deeply bound, with vibrational excitations that for \(J = 0\) may lie below the threshold for decay to two nucleons and a pion \[8\]. When canonical \emph{ab initio} quantization is employed the ground state moves to or above the threshold for two-nucleon decay for all representations. In this case the representations of lowest order admit several states with \(I = J = 0, 1\), which fall below the threshold for pion decay. From the phenomenological perspective the Skyrme model thus can accommodate narrow dibaryon states, although it does not demand their existence. The overwhelming experimental evidence is against the existence of narrow dibaryon states, although some intriguing signals have been recently been seen in the missing mass spectrum of the reaction \(pd \to pX\) reaction \[12, 13\].
This paper is organized into 5 sections. In section 2 the rational map ansatz for the baryon number 2 skyrmion is generalized to representations of arbitrary dimension. This “biskyrmion” is canonically quantized in section 3. In section 4 the resulting equations of motion are solved numerically, and the spectrum is obtained. Section 5 contains a concluding discussion.

2. The axially symmetric soliton with $B = 2$

The Skyrme model is a Lagrangian density for a unitary field $U(r, t)$ that is described by any representation of $SU(2)$. In an irreducible representation this unitary field may be expressed in terms of three unconstrained Euler angles $\vec{\alpha} = (\alpha^1, \alpha^2, \alpha^3)$ as

$$U(x, t) = D^j(\vec{\alpha}(x, t)).$$  

Here the elements of the matrices $D^j$ are Wigner D-functions. In an arbitrary reducible representation unitary field can be decomposed into direct sum of $D^j$ functions. The Euler angles $\vec{\alpha}$ then are the dynamical variables of the theory.

The Skyrme model is defined by the chirally symmetric Lagrangian density:

$$\mathcal{L}[U(x, t)] = -\frac{f_\pi^2}{4}\text{Tr}\{R_\mu R^\mu\} + \frac{1}{32e^2}\text{Tr}\{[R_\mu, R_\nu]^2\},$$

where the "right" current is defined as

$$R_\mu = (\partial_\mu U)U^\dagger,$$

and $f_\pi$ (the pion decay constant) and $e$ are parameters.

The rational map ansatz for the Skyrme field with $B = 2$ (biskyrmion) in the fundamental representation may be generalized to an arbitrary irreducible representation of $SU(2)$ in the following way:

$$e^{i(\hat{n} \cdot \vec{\tau})F_R(r)} \Rightarrow U_R(r) = \exp\{2i\hat{n}^a \cdot \vec{J}_a F_R(r)\},$$

Here $F_R(r)$ is a scalar function (“the chiral angle”) and $\hat{n}$ is a unit vector, which may be defined by its circular components:
\[ \hat{n}_{n+1} = -\hat{n}_{n-1} = -\frac{\sin^2 \vartheta}{\sqrt{2} (1 + \cos^2 \vartheta)} e^{2i\varphi}, \]
\[ \hat{n}_0 = \hat{n}_{n} = \frac{2 \cos \vartheta}{1 + \cos^2 \vartheta}, \]
\[ \hat{n}_{n-1} = -\hat{n}_{n+1} = \frac{\sin^2 \vartheta}{\sqrt{2} (1 + \cos^2 \vartheta)} e^{-2i\varphi}. \] (5)

The boundary condition of the chiral angle \( F_R(r) \) at the origin is \( F_R(0) = \pi \). In terms of Euler angles the generalized rational map ansatz may be expressed as

\[ \alpha_1 = 2\varphi - \arctan \left( \frac{2 \cos \vartheta \tan F_R(r)}{1 + \cos^2 \vartheta} \right) - \frac{\pi}{2}, \]
\[ \alpha_2 = -2 \arcsin \left( \frac{\sin^2 \vartheta \sin F_R(r)}{1 + \cos^2 \vartheta} \right), \]
\[ \alpha_3 = -2\varphi - \arctan \left( \frac{2 \cos \vartheta \tan F_R(r)}{1 + \cos^2 \vartheta} \right) + \frac{\pi}{2}. \] (6)

Given the rational map ansatz (5), the Lagrangian density (2) reduces to the following rather simple form

\[ \mathcal{L} = -\frac{1}{3} j(j+1)(2j+1) \left\{ f_\pi^2 \left[ F_R'^2(r) + 8 \frac{\sin^2 \vartheta \sin^2 F_R(r)}{r^2 (1 + \cos^2 \vartheta)} \right] \right. \]
\[ \left. + \frac{8 \sin^2 \vartheta \sin^2 F_R(r)}{r^2 (1 + \cos^2 \vartheta)} \left[ F_R'^2(r) + 2 \frac{\sin^2 \vartheta \sin^2 F_R(r)}{r^2 (1 + \cos^2 \vartheta)} \right] \right\}. \] (7)

The classical Lagrangian density depends on the dimension of the representation \( j \) only through the overall scalar factor \( N = \frac{2}{3} j(j+1)(2j+1) \), which can be absorbed by a renormalization of the parameters [14].

In contrast to hedgehog form for the \( B = 1 \) skyrmion the rational map Lagrangian for the skyrmion with \( B = 2 \) depends on both the polar angle \( \vartheta \) and the radius \( r \). Integration and normalization yields the following expression for the mass of the biskyrmion:

\[ M_0(F_R) = 2\pi f_\pi \int_0^\infty d\bar{r} \left[ \bar{r}^2 F_R'^2(\bar{r}) \left( 1 + \frac{4 \sin^2 F_R(\bar{r})}{\bar{r}^2} \right) \right. \]
\[ \left. + \sin^2 F_R(\bar{r}) \left( 4 + \left( \frac{8}{3} + \pi \right) \frac{\sin^2 F_R(\bar{r})}{\bar{r}^2} \right) \right\}. \] (8)
Here the dimensionless variable \( \tilde{r} \) is defined as \( \tilde{r} = e f_\pi r \). Variation of this expression for the mass leads to the differential equation for the “chiral angle”:

\[
F''_R(\tilde{r}) \left( 1 + 4 \frac{\sin^2 F_R(\tilde{r})}{\tilde{r}^2} \right) + 2 F''_R(\tilde{r}) \frac{\sin 2 F_R(\tilde{r})}{\tilde{r}^2} + \frac{2}{\tilde{r}} F'_R(\tilde{r}) \\
- \frac{\sin 2 F_R(\tilde{r})}{\tilde{r}^2} \left( 2 + \left( \frac{8}{3} + \pi \right) \frac{\sin^2 F_R(\tilde{r})}{\tilde{r}^2} \right) = 0.
\]  
(9)

At large distances \( \tilde{r} \to \infty \) this equation has the asymptotic form:

\[
F''_R(\tilde{r}) + \frac{2}{\tilde{r}} F'_R(\tilde{r}) - \frac{4}{\tilde{r}^2} F_R(\tilde{r}) = 0.
\]  
(10)

This equation differs from the corresponding asymptotic equation for the chiral angle of the B=1 hedhehog solution only in the value of the coefficient in the numerator of the last term on the l.h.s. (4 instead of 2). The solution of this asymptotic equation (10) is

\[
F_R(\tilde{r}) = C \tilde{r}^{-\frac{1+\sqrt{17}}{2}}.
\]  
(11)

The fall off rate is somewhat larger here than in the case of B=1, as the power of \( \tilde{r} \) in (12) is -2.56 whereas it in the case B=1 is -2.

### 3. Quantization of the biskyrmion

We shall employ collective rotational coordinates to separate the variables which depend on the time and spatial coordinates:

\[
U(r, q(t)) = A(q(t)) U_0(r) A^\dagger(q(t)) .
\]  
(12)

We consider the Skyrme model quantum mechanically \emph{ab initio} and thus treat the generalized (collective) coordinates \( q(t) \) and the corresponding velocities \( \dot{q} \) as dynamical variables, which satisfy the commutation relations

\[
[\dot{q}^i, q^j] = -i R f^{i j k}(q).
\]  
(13)

The functions \( R f^{i j k} \) depend on the generalized coordinates \( q \), the explicit form of which is determined by the canonical commutation relations below
After substitution of (12) into the Lagrangian density (2) the dependence of Lagrangian on the generalized velocities may be expressed as

$$
\dot{L}(\dot{q}, q, F_R) = \frac{1}{\pi} \int \dot{L}(n, \dot{q}(t), q(t), F_R(r)) r^2 \sin \vartheta \, dr \, d\vartheta \, d\varphi = \frac{1}{2} q^\alpha Rg_{\alpha\beta}(q) \dot{q}^\beta + \mathcal{O}(\dot{q}^0).
$$

(14)

Here we have used the notation

$$
Rg_{\alpha\beta}(q) = \sum_m (-)^m C^{(m)}_\alpha(q) R a_m(F_R) C^{(m)}_{\beta}(q)
$$

= \begin{pmatrix}
2(a_0 \cos^2 q^2 + a_1 \sin^2 q^2) & 0 & 2a_0 \cos q^2 \\
0 & 2a_1 & 0 \\
2a_0 \cos q^2 & 0 & 2a_0
\end{pmatrix}. 
$$

(15)

The functions of dynamical variables $C^{(m)}_\alpha(q)$ are defined in [11].

Because of the axial symmetry of the rational map configuration there are only two different moments of inertia, which may be defined as:

$$
a_0 = Ra_0(F_R) = \frac{\pi}{3e^3 f_\pi} \int_0^\infty d\tilde{r} \tilde{r}^2 \sin^2 F_R \left( (12 - 3\pi)(1 + F_R^2) + 8 \frac{\sin^2 F_R}{\tilde{r}^2} \right),
$$

(16)

$$
a_1 = Ra_1(F_R) = Ra_{-1}(F_R) = \frac{\pi}{3\sqrt{2}e^3 f_\pi} \int_0^\infty d\tilde{r} \tilde{r}^2 \sin^2 F_R \left( 3\pi(1 + F_R^2) + 16 \frac{\sin^2 F_R}{\tilde{r}^2} \right).
$$

(17)

The generalized momentum operators, which are canonically conjugate to $q$, are defined as

$$
p_\alpha = \frac{\partial L}{\partial \dot{q}^\alpha} = \frac{1}{2} \{ \dot{q}^\beta, Rg_{\alpha\beta}(q) \}.
$$

(18)

These operators satisfy the canonical commutation relations

$$
[p_\alpha, q^\beta] = -i \delta_{\alpha\beta},
$$

(19)

from which it follows that the explicit form for the matrix $Rf^{\alpha\beta}(q, F_R)$ in eq. (13) is
\[ R F^{\alpha\beta}(q, F_R) = C^{\alpha}_{(a)}(q) \left( a^{-1}(F_R) \right)^{ab} C^{\beta}_{(b)}(q). \] (20)

The group parameter manifold of \( SU(3) \) is the hypersphere \( S^3 \). It is convenient to introduce the following angular momentum operators \([11]\) on the sphere:

\[ \hat{J}^b = \frac{i}{\sqrt{2}} \left\{ p^\beta, C^{\beta}_{(b)}(q) \right\} = (-)^b \frac{i}{\sqrt{2}} a_b(F_R) \left\{ \dot{q}^\beta, C^{(-b)}_{\beta}(q) \right\}, \] (21)

The components of this operator satisfy the standard commutation relations. Note that in eq. (21) there is no summation over the index \( b \).

By some lengthy manipulation the Lagrangian (2) brought into the following explicit form

\[ \hat{L}(\dot{q}, q, F_R) = -M_0(F_R) - \Delta M_j(F_R) + \frac{1}{4} \left[ \frac{1}{a_0} \hat{J}^2 + \left( \frac{1}{a_0} - \frac{1}{a_1} \right) \hat{J}_0^2 \right]. \] (22)

Here \( \Delta M_j(F_R) \) represents the quantum mass correction, which may be written as

\[ \Delta M_j(F_R) = \frac{1}{a_0} \Delta M_0 + \frac{1}{a_0a_1} \Delta M_{01} + \frac{1}{a_1} \Delta M_1. \] (23)

Here the three terms on the r.h.s are quantum corrections that are due to the different moments of inertia, which may be written in the form

\[ \Delta M_0 = \frac{1}{e^{f_2}} \Delta \tilde{M}_0, \quad \Delta M_{01} = \frac{1}{e^{f_2}} \Delta \tilde{M}_{01}, \quad \Delta M_1 = \frac{1}{e^{f_2}} \Delta \tilde{M}_1. \] (24)

The dimensionless moments of inertia here are defined as

\[ \Delta \tilde{M}_0 = -\frac{\pi}{\sqrt{2}} \int_0^\infty d\tilde{r} \tilde{r}^2 \left\{ \frac{4 - \pi}{8} \sin^2 F_R + \frac{32 - 9\pi}{80} (2j - 1)(2j + 3) \sin^4 F_R \right. \\
+ \frac{F_R^2 \sin^2 F_R}{80} \left[ 8 (16j (j + 1) - 7) - \pi (36j (j + 1) - 17) \\
- \frac{2}{3} (32 - 9\pi) (2j - 1)(2j + 3) \sin^2 F_R \right] + \frac{32(j + 1)(a_0 + 1)}{75} \sin^4 F_R \right\} \] (25)
\[
\Delta \tilde{M}_{01} = -\sqrt{2\pi} \int_0^\infty d\tilde{r}\tilde{r}^2 \left\{ \frac{4\pi}{8} \sin^2 F_R + \frac{\pi}{80} (2j - 1) (2j + 3) \sin^4 F_R \\
+ \frac{F_R^2 \sin^2 F_R}{80} \left[ 40 + \pi (4j (j + 1) - 13) - \frac{2}{3} (16 - 3\pi) (2j - 1) (2j + 3) \sin^2 F_R \right] - \frac{(2j - 1)(2j + 3) \sin^4 F_R}{75} \right\} 
\] 
(26)

\[
\Delta \tilde{M}_1 = -\sqrt{2\pi} \int_0^\infty d\tilde{r}\tilde{r}^2 \left\{ \frac{3\pi - 4}{8} \sin^2 F_R + \frac{7\pi}{80} (2j - 1) (2j + 3) \sin^4 F_R \\
+ \frac{F_R^2 \sin^2 F_R}{80} \left[ -40 + \pi (28j (j + 1) + 9) - \frac{2}{3} (15\pi - 16) (2j - 1) (2j + 3) \sin^2 F_R \right] + \frac{52j(j + 1) + 11 \sin^4 F_R}{75} \right\}. 
\] 
(27)

The normalized eigenstates with fixed spin and isospin \( \ell \) of the corresponding Hamilton operator are

\[
\left| \ell \atop m, m' \right> = \sqrt{\frac{2\ell + 1}{4\pi}} D_{m,m'}^{\ell}(\mathbf{q}) |0\rangle.
\] 
(28)

The eigenvalues that correspond to these states give the masses of the quantum biskyrmion states as:

\[
M_R = M_0(F_R) + \Delta M_0(F_R) + \frac{1}{4} \left[ \frac{1}{a_1} \ell(\ell + 1) + \left( \frac{1}{a_0} - \frac{1}{a_1} \right) m_t^2 \right].
\] 
(29)

Here \( m_t \) is the third component of the isospin. The chiral angle \( F_R \) is determined by the solution of the integrodifferential equation that is obtained by minimization of this expression (29) for the biskyrmion mass:
Here the mass parameters $\Delta M_j$ and the moments of inertia are integrals of the chiral angle that is determined by solution.
At large distances the equation (30) reduces to the asymptotic form

\[ \ddot{r}^2 F''_R + 2\dot{r} F'_R - (4 + \tilde{m}^2 \ddot{r}^2) F_R = 0. \]  

(31)

Here the quantity \( \tilde{m}^2 \) is defined as

\[ \tilde{m}^2 = \epsilon^4 \left[ -\frac{12 - 3\pi}{6a_0a_1} \left( 2\tilde{a}_1 \Delta \tilde{M}_0 + \tilde{a}_0 \Delta \tilde{M}_{01} \right) - \frac{3\pi}{12a_0a_1} \left( \tilde{a}_1 \Delta \tilde{M}_{01} + 2\tilde{a}_0 \Delta \tilde{M}_1 \right) \right. \]

\[ - \frac{1}{32} \left( \frac{4-\pi}{a_0^2} + \frac{2(4-\pi)}{a_{01}^2} + \frac{3\pi - 4}{a_1^2} + 2\pi \frac{\ell(\ell+1)-m^2}{a_1^2} + 16 \left( 1 - \frac{\pi}{4} \right) \frac{m^2}{a_0^2} \right]. \]  

(32)

This mass parameter describes the behavior of the chiral angle at infinity:

\[ F_R(\tilde{r}) = C \left( \tilde{m} + \frac{2}{\tilde{m}^2} \right) e^{-\tilde{m} \tilde{r}}. \]  

(33)

The quantity \( m = e f_\pi \tilde{m} \) represents an effective pion mass, which governs the asymptotic fall off \( \exp(-2mr) \) of biskyrmion mass density. In the case of the \( B = 1 \) skyrmion the corresponding asymptotic fall off is \( \exp(-mr) \) and represents the Yukawa form of the pion cloud around the nucleon.

4. Numerical results

We have solved numerically the integrodifferential equation (30) for the chiral angle of the rational map ansatz for the biskyrmion (4), which provides a good approximation to the ground state solution for the \( B = 2 \) skyrmion. The resulting biskyrmion mass values are given in Table 1. In the numerical calculation we employed the same values for the two parameters of the the model, \( f_\pi \) and \( e \), as were obtained in ref.\[15\] by fitting the empirical values for mass and the the isoscalar radius of the nucleon. We also solved the eq. (9) for the classical case using the same parameter values. The calculated values for the nucleon mass in the classical case are 978 MeV, 1028 MeV and 1090 MeV respectively for the three sets of parameter values used below to reproduce the empirical value 939 MeV with canonical quantization.

In the classical treatment the rational map ansatz leads to a deeply bound biskyrmion solution with \( I = J = 0 \) that is stable against decay to two
nucleons. This is indicated the negativity of the parameter \( \Delta M = M - 2M_N \), where \( M \) is the biskyrmion mass and \( M_N \) is the calculated mass of the nucleon. In the canonically quantized case this state moves up to or above the threshold for nucleon decay. In the fundamental representation the \( I = J = 0 \) state is marginally stable against decay to two nucleons with the present choice of parameter values. This is not the deuteron state, which has spin 1 and isospin 0. In view of the very small energy by which this solution falls below the sum of two nucleon masses (- 5 MeV) and the absence of any such state in all other representations of \( SU(2) \) we believe interpret this result to be an accidental consequence of the approximate character of the rational map ansatz, and thus that, within the margin of error, there is no bound state.

The \( \ell = I = J = 0 \) state is found to lie below the threshold for decay into two nucleons and a pion in the canonically quantized case in both the fundamental representation and the three dimensional representation \((j = 1)\). There are three states with \( \ell = I = J = 1 \) in the \( j = 1 \) representation, which lie below the threshold of pion decay of the dibaryon. With the rational map ansatz the energies of the state with \( I = J = 0 \) is found to be roughly 1950 MeV and the energies of the states with \( I = J = 1 \) and isospin projection \( m_t = 0 \) at 2000 MeV and with \( m_t = \pm 1 \) at 2010 MeV respectively. In the 4 dimensional representation \( j = 3/2 \) all these states are found to lie well above the threshold for pion decay, when the same parameter values are used. Should the recent empirical indications \[13\] for three "supernarrow" dibaryons at 1904 MeV, 1926 MeV and 1942 MeV be confirmed, they could thus be accommodated within the Skyrme model framework, as long as their spin and isospin equal 0, 1, 1 respectively. The \( \gamma \) decay pattern of those states, do however suggest that they all have isospin 1 and \( J^P = 1^\pm \) \[12, 16\].

Typical contours of constant classical and quantum mass density of the dibaryon are plotted in Fig.1 for the case \( j = 1/2 \) and \( \ell = 0 \). The corresponding typical contours of constant baryon number density are shown in Fig.2. Comparison of the classical and quantum solutions in Figs.1 and 2 show that the quantum soliton is concentrated is more compact as the densities fall off exponentially at infinity in for the latter.

5. Discussion
Table 1: Dibaryon parameter dependence on $SU(2)$ group representation

| $j$ | $\ell$ | $m_\ell$ | $e$ | $f_\pi$ (MeV) | $M$ (MeV) | $\Delta M$ (MeV) | $m$ (MeV) |
|-----|-------|---------|----|--------------|--------|--------------|--------|
| $\frac{1}{2}$ | Classical | 4.46 | 59.8 | 1918.5 | -36.9 |
| $\frac{1}{2}$ | 0 | 0 | 4.46 | 59.8 | 1873.0 | -5.0 | 62.9 |
| 1 | Classical | 4.15 | 58.5 | 2017.0 | -38.8 |
| 1 | 0 | 0 | 4.15 | 58.5 | 1949.9 | 71.9 | 88.1 |
| 1 | 1 | 0 | 4.15 | 58.5 | 1998.5 | 120.5 | 53.7 |
| 1 | 1 | ±1 | 4.15 | 58.5 | 2012.0 | 134.0 | 45.9 |
| $\frac{3}{2}$ | Classical | 3.86 | 57.7 | 2138.9 | -41.1 |
| $\frac{3}{2}$ | 0 | 0 | 3.86 | 57.7 | 2049.7 | 171.7 | 100.6 |
| $\frac{3}{2}$ | 1 | 0 | 3.86 | 57.7 | 2090.5 | 212.5 | 78.8 |
| $\frac{3}{2}$ | 1 | ±1 | 3.86 | 57.7 | 2101.8 | 223.8 | 74.6 |

The present work represents an exploratory calculation of the possible quantum excitations that the the axisymmetric ground state configuration of the $B = 2$ skyrmion may support, once self consistent canonical quantization is imposed. We have here considered the possible states with $I = J$, which represent dibaryons other than the deuteron, which has been considered in ref. [4].

The calculation is based on the rational map ansatz of ref.[7], which provides a good approximation to the actual ground state solution. The calculation, within the margin of error that is associated with the rational map approximation does not yield states that would be stable against decay to two nucleons. One dibaryon state with $I = J = 0$ that is stable against decay into two nucleons and a pion appears in both the fundamental two dimensional as well as in the the three dimensional representation. The three dimensional representation also may accomodate two such dibaryon states with $I = J = 1$. It is of course intriguing that there are some recent empirical indications for such states [13], and that for those $I = 1$ would be favored by the photon decay pattern [16].

The possibility for a dibaryon state with $I = 0$ has drawn considerable experimental and theoretical interest over the past 10 years, but no confirmed evidence for such has yet been found [7]. Should in the end no such dibaryon state be found, it would put constraints on the choice of representation to be used with the rational map ansatz in the canonically quantized case, and require employment of dimension greater than or equal to 4, as in
such the dibaryon states lie above the threshold of pion decay.

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Figure Captions

Fig.1 Typical contours of constant classical (dashed line) and quantum (solid line) mass densities of the biskyrmion $j = \frac{1}{2}$, $\ell = 0$ case.

Fig.2 Typical contours of constant classical (dashed line) and quantum (solid line) baryon number densities of the biskyrmion $j = \frac{1}{2}$, $\ell = 0$ case.
