Spectral curves and Nahm transform for doubly-periodic instantons

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Abstract

We explore the role played by the spectral curves associated with Higgs pairs in the context of the Nahm transform of doubly-periodic instantons previously defined by the author. More precisely, we show how to construct a triple consisting of an algebraic curve plus a line bundle with connection over it from a doubly-periodic instanton, and that these coincide with the Hitchin’s spectral data associated with the Nahm transformed Higgs bundle.
1 Introduction

In [6, 7], we have established a correspondence between instantons on $\mathbb{R}^4$ which are periodic in two directions (so-called doubly-periodic instantons) and certain singular Higgs pairs over a 2-dimensional torus. On the other hand, Hitchin has shown that Higgs pairs are equivalent to a pair consisting of an algebraic curve (the spectral curve) in the total space of the cotangent bundle plus a “line bundle” over it.

In this third installment of the series initiated by [6, 7], we shall explore the relation between Hitchin’s spectral data and the Nahm transform for doubly-periodic instantons defined in the previous papers.

The motivation comes from the Hitchin’s work on monopoles [3]. He has shown that monopoles on $\mathbb{R}^3$ (that is, instantons on $\mathbb{R}^4$ which are translation invariant in one direction) are equivalent to certain singular solutions of Nahm’s equations and to pair consisting of an algebraic curve plus a line bundle over it.

The paper is organized as follows. In section 2 we briefly review the Nahm transform of doubly-periodic instantons discussed in the previous papers. We then show how to construct the spectral data from the instanton (section 3) and from the Higgs bundle (section 4). The main purpose of this paper is to prove that these two sets of spectral data coincide when the Higgs bundle is the Nahm transform of a doubly-periodic instanton; this is done in section 5.

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2 Nahm transform

Let $T$ be a complex torus, and consider the product $T \times \mathbb{C}$ equipped with the product flat metric. Let $E \to T \times \mathbb{C}$ be a rank 2 complex vector bundle with an irreducible $SU(2)$ instanton (i.e. anti-self-dual) connection $A$ such that $|F_A| = O(|w|^{-2})$, where $w$ is a complex coordinate on $\mathbb{C}$. As usual, its total energy is given by:

$$\int_{T \times \mathbb{C}} |F_A|^2 = 8\pi^2 k$$

where $k$ is a positive integer, the instanton number.

As it was shown in [1], the toroidal components of the connection $A$ have a well-defined limit as $r \to \infty$ given by a constant flat connection $\Gamma$ over $T$. General theory tells us that a constant flat connection on a bundle $S \to T$ determines uniquely a holomorphic structure on $S$. Moreover, $S$ must split, holomorphically, as the sum of two flat line bundles, i.e. $S = \xi_0 \oplus -\xi_0$, uniquely up to $\pm 1$. Here, $\pm \xi_0$ are seen as points in $\hat{T}$, the torus dual to $T$.

The following result has been proved in [6, 7]:

**Theorem 1.** The Nahm transform is a bijective correspondence between the following objects:

- irreducible $SU(2)$ instanton connections on $E \to T \times \mathbb{C}$ with fixed instanton number $k$ and asymptotic state $\xi_0$; and

- admissible $U(k)$ solutions of the Hitchin’s equations over $\hat{T}$, such that the Higgs field has at most simple poles at $\pm \xi_0 \in \hat{T}$, with semisimple residues of rank $\leq 2$ if $\xi_0$ is an element of order 2 in the Jacobian of $T$, and rank $\leq 1$ otherwise.

For the purpose of this paper, it is enough to recall one way of the above correspondence. For simplicity, let us assume that $\xi_0 \neq -\xi_0$.

Recall that $\hat{T}$ parametrises the set of line bundles with flat connection on $T$. Indeed, given a point $\xi \in \hat{T}$, we denote by $L_\xi$ the trivial line bundle
with the constant connection $\omega_\xi = i\xi$. Let $\pi_1 : T \times \mathbb{C} \to T$ be the obvious projection, and consider the twisted bundles $E(\xi) = E \otimes \pi_1^* L_\xi$ with the corresponding instanton connection:

$$A_\xi = A \otimes \text{Id} + \text{Id} \otimes \omega_\xi$$

For each $\xi \neq \pm \xi_0$ we have the elliptic complex

$$0 \to L^2_2(\Omega^0 E(\xi)) \xrightarrow{\bar{\nabla}_{A_\xi}} L^2_1(\Omega^{0,1} E(\xi)) \xrightarrow{-\bar{\nabla}_{A_\xi}} L^2(\Omega^{0,2} E(\xi)) \to 0 \quad (1)$$

whose 0th and 2nd cohomologies vanish, while $H^1(\xi)$ (which coincides with the cokernel of the coupled Dirac operator $D_{A_\xi}$) has complex dimension $k$. Thus, the cohomology of the above monad defines a rank $k$ vector bundle $V \to (\hat{T} \setminus \{\pm \xi_0\})$, with fibres $V_\xi = H^1(\xi) = \ker D^*_{A_\xi}$, plus an unitary connection $B$ obtained as follows. Let $\mathcal{H}$ be the trivial Hilbert bundle over $\hat{T} \setminus \{\pm \xi_0\}$ with fibres given by $L^2_1(\Omega^{0,1} E(\xi))$. Then $V$ is a naturally a subbundle of $\mathcal{H}$, and we denote by $P$ the projection $\mathcal{H} \to V$ and by $\iota_V \to \mathcal{H}$ the inclusion $V \hookrightarrow \mathcal{H}$. We define:

$$\nabla_B = P \circ d \circ \iota_V \to \mathcal{H}$$

where $d$ denotes the trivial connection on $\mathcal{H}$.

A Higgs field $\Phi \in \text{End}(V) \otimes K_{\hat{T}}$ is defined as follows. Let $\psi$ be a section of $V$, i.e. for each $\xi \in \hat{T} \setminus \{\pm \xi_0\}, \psi[\xi] \in \ker D^*_{A_\xi}$. For a fixed $\xi$, the Higgs field will act on $\psi[\xi]$ by multiplication by the plane coordinate $w$ composed with projection to $\ker D^*_{A_\xi}$:

$$(\Phi(\psi))[\xi] = \frac{1}{\sqrt{2}} P_\xi(w\psi[\xi]) d\xi \quad (2)$$

where $P_\xi : L^2_2(\Omega^{0,1} E(\xi)) \to \ker D^*_{A_\xi}$ is the natural projection operator.

As it was shown in 

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1Recall that on a Kähler manifold the Dirac operator $D$ can be written as $D = \bar{\partial} - \partial^\ast$. 

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4
**Kobayashi-Hitchin correspondence for doubly-periodic instantons.**

It is also useful to recall that given an doubly-periodic instanton connection $A$ on the bundle $E \to T \times \mathbb{C}$ there is a holomorphic bundle $\mathcal{E} \to T \times \mathbb{P}^1$, so called *instanton bundle* such that:

- $\mathcal{E}|_{T \times \{\mathbb{P}^1 \setminus \infty\}} = (E, \nabla_A)$;
- $c_1(\mathcal{E}) = 0$ and $c_2(\mathcal{E}) = k$;
- $\mathcal{E}|_{T \times \infty} = L_{\xi_0} \oplus L_{-\xi_0}$.

Instanton bundles also satisfy a certain stability condition, but that is not relevant here; see [1] for the complete statement.

It is then easy to show that $h^0(\mathcal{E}(\xi)) = h^2(\mathcal{E}(\xi)) = 0$, while $H^1(\mathcal{E}(\xi))$ can be identified with $\ker D_{A_{\xi}}^*$ [7].

Furthermore, every instanton bundle is *generically fibrewise semistable*, that is $\mathcal{E}|_{T_p}$ is semistable for generic $p \in \mathbb{P}^1$. The instanton bundle is said to be *fibrewise semistable* if $\mathcal{E}|_{T_p}$ is semistable for every $p \in \mathbb{P}^1$. It is *regular* if it is fibrewise semistable and $h^0(\text{End}(\mathcal{E}|_{T_p})) = 2$ for all $p \in \mathbb{P}^1$ [4].

The instanton $A$ is said to be regular if the corresponding instanton bundle is regular. As we will see below, regular instantons form a Zariski open subset of the moduli space of doubly-periodic instanton.

Finally, it is important to remind how the Higgs field $\Phi$ can be constructed out of this holomorphic data.

Start by fixing two sections $s_0$ and $s_{\infty}$ generating $H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(1))$, such that $s_0$ vanishes at $0 \in \mathbb{C}$ and $s_{\infty}$ vanishes at the point added at infinity. For each $\xi \in \hat{T} \setminus \{\pm \xi_0\}$, we define the map:

$$H^1(T \times \mathbb{P}^1, \mathcal{E}(\xi)) \times H^1(T \times \mathbb{P}^1, \mathcal{E}(\xi)) \xrightarrow{\Psi_{\xi}} H^1(T \times \mathbb{P}^1, \tilde{\mathcal{E}}(\xi))$$

$$\alpha, \beta \mapsto \alpha \otimes s_0 - \beta \otimes s_{\infty}$$

\(2\)Recall that every semistable, rank 2 vector bundle over an eliptic with trivial determinant either splits as a sum of flat line bundles or it is the unique nontrivial extension of a flat line bundle of order 2 by itself. Such bundle is regular if it is not the sum of flat lines bundle of order 2.
If \((\alpha, \beta) \in \ker \Psi_\xi\), we define \(\Phi \in \text{End}(H^1(T \times \mathbb{P}^1, \mathcal{E}(\xi))) = \text{End}(V_\xi)\) as follows:

\[\varphi_\xi(\alpha) = \beta\] (4)

To check that the two definitions coincide at \(C = \mathbb{P}^1 \setminus \infty\), just note that \(s_\infty(w) = w\) for any trivialisation of \(O_{\mathbb{P}^1}(1)\) with local coordinate \(w\).

With this formulation, it is not difficult to show that the eigenvalues of the Higgs field \(\Phi\) have at most simple poles at \(\pm \xi_0\). Moreover, the residues of \(\Phi\) are semisimple and have rank \(\leq 2\) if \(\xi_0\) is an element of order 2 in the Jacobian of \(T\), and rank \(\leq 1\) otherwise; see [7] for the details.

3 The instanton spectral data

Our first step is to construct a complex curve \(S \hookrightarrow \hat{T} \times \mathbb{C}\) associated to a doubly-periodic instanton \(A\).

Let \(D^*_A\xi(w)\) denote the restriction of the coupled Dirac operator \(D_A\xi\) to the torus \(T_w\). We define:

\[S = \{((\xi, w) \in \hat{T} \times \mathbb{C} | \ker \{D^*_A\xi(w)\} \neq 0}\} \] (5)

Since \(D_A\xi(w) = \overline{\partial}_A\xi|_{T_w} - \overline{\partial}^*_A\xi|_{T_w}\), it is easy to see that:

\[\ker \{D^*_A\xi(w)\} = H^1(T_w, \mathcal{E}(\xi)|_{T_w}) = H^1(T_w, \mathcal{E}(\xi)|_{T_w})\]

Note also that \(S\) can be compactified to a curve \(\overline{S} \hookrightarrow \hat{T} \times \mathbb{P}^1\) by adding the two points \((\pm \xi_0, \infty)\) corresponding to the asymptotic states.

Assuming that the instanton bundle is fibrewise semistable, we conclude that \(\overline{S}\) is a branched double cover of \(\mathbb{P}^1\); the branch points correspond to those \(w \in \mathbb{C}\) such that \(\mathcal{E}(\xi)|_{T_w}\) is an extension of the trivial line bundle by itself.

On the other hand, index theorem tells us that \(\overline{S}\) is a \(k\)-fold cover of \(\hat{T}\). Hence there are \(4k\) branch points, and the genus of \(\overline{S}\) is \(2k - 1\). Moreover, all spectral curves belong to the linear system \(k \cdot [\hat{T}] + 2 \cdot [\mathbb{P}^1] \subset \hat{T} \times \mathbb{P}^1\). The curve \(\overline{S}\) is smooth provided \(A\) is regular.
Line bundle with connection. Let $\pi_1 : \hat{T} \times \mathbb{P}^1 \to \hat{T}$ and $\pi_2 : \hat{T} \times \mathbb{P}^1 \to \mathbb{P}^1$ be the natural projection maps; we will also use $\pi_1$ and $\pi_2$ to denote the projections $\bar{S} \to \hat{T}$ and $\bar{S} \to \mathbb{P}^1$.

To each $s \in \bar{S}$, we attach the vector space:

$$L_s = \ker \left\{ D_{A_{\pi_1(s)}(\pi_2(s))} \right\} = H^1(T_{\pi_2(s)}, \mathcal{E}(\pi_1(s))|_{T_{\pi_2(s)}})$$

(6)

If $\mathcal{E}$ is only fibrewise semistable, then $L$ is only a coherent sheaf on the (singular) spectral curve. However, when the instanton bundle is regular $L$ becomes a line bundle.

So now let us assume that $A$ is a regular doubly-periodic instanton, and consider the bundle $\pi_1^*\mathcal{H} \to S$. There is a bundle map $T : \pi_1^*\mathcal{H} \to L$, which is given by the following composition on each fibre:

$$L_s^*(\Omega^{0,1}E(\pi_1(s))) \xrightarrow{\mathcal{P}} \ker \left\{ D_{A_{\pi_1(s)}} \right\} \xrightarrow{\mathcal{r}} \ker \left\{ D_{A_{\pi_1(s)}(\pi_2(s))} \right\}$$

(7)

where $r$ denotes the restriction map. Let $\iota_{\mathcal{L} \to \mathcal{H}}$ denotes the inclusion $\mathcal{L} \hookrightarrow \pi_1^*\mathcal{H}$, which makes sense in terms of distributions. A connection $\Gamma$ on the line bundle $\mathcal{L} \to S$ is defined by:

$$\nabla_\Gamma = T \circ \pi_1^*d \circ \iota_{\mathcal{L} \to \mathcal{H}}$$

(8)

4 Hitchin’s spectral data

We now look at the other side of the correspondence in theorem \[ and review Hitchin’s construction of spectral curves associated to Higgs bundles \[.

Recall that $V \to \hat{T} \setminus \{\pm \xi_0\}$ is a rank $k$ vector bundle, and $\Phi$ is an endomorphism valued $(1, 0)$-form with simple poles at $\pm \xi_0$. So, for any fixed $\xi \in \hat{T} \setminus \{\pm \xi_0\}$, $\Phi[\xi]$ is a $k \times k$ matrix and one can compute its $k$ eigenvalues. As we vary $\xi$, we get a $k$-fold covering, possibly branched, of $\hat{T} \setminus \{\pm \xi_0\}$ inside $\hat{T} \times \mathbb{C}$. This curve of eigenvalues is what we want to define as our Higgs spectral curve; more precisely:

$$C = \left\{ (\xi, w) \in \hat{T} \times \mathbb{C} \mid \det(\Phi[\xi] - w \cdot I_k) = 0 \right\}$$

(9)
In other words, $C$ is the set of points $(\xi, w) \in \hat{T} \times \mathbb{C}$ such that $w$ is an eigenvalue of the endomorphism $\Phi[\xi] : V_{\xi} \to V_{\xi}$.

Since we are assuming that $\Phi$ has simple poles at $\pm \xi_0$, the curve $C \hookrightarrow \hat{T} \times \mathbb{C}$ can be compactified to a curve $\overline{C} \hookrightarrow \hat{T} \times \mathbb{P}^1$ by adding the points $(\pm \xi_0, \infty)$.

The following proposition is a familiar fact from the theory of Higgs bundles.

**Proposition 2.** If $\xi_0 \neq -\xi_0$, the spectral curve associated to a generic Higgs bundle $(V, B, \Phi)$ is smooth.

Note that if $\xi_0 = -\xi_0$, then all spectral curves have a double-point at $(\pm \xi_0, \infty)$, but are generically smooth elsewhere.

**Defining the spectral bundle.** As before, we will denote the projections $\overline{C} \to \hat{T}$ and $\overline{C} \to \mathbb{P}^1$ by $\pi_1$ and $\pi_2$. We define a coherent sheaf $\mathcal{N}$ on $\overline{C}$ with stalks given by:

$$\mathcal{N}_c = \text{coker} \left\{ \Phi[\pi_1(c)] - \pi_2(c) \cdot \text{Id}_k \right\}$$

(10)

i.e. the dual of the $\pi_2(c)$-eigenspace of $\Phi[\pi_1(c)]$. Generically, one expects the eigenvalues to be distinct, so that $\mathcal{N}$ becomes a line bundle over the smooth curve $\overline{C}$.

Assuming that Higgs bundle $(V, B, \Phi)$ is generic, we define a connection $\Lambda$ on the line bundle $\mathcal{N} \to C$. First note that $\mathcal{N}$ is naturally a subbundle of $\pi_1^*V$; let $\iota_{\mathcal{N} \to V}$ be the inclusion and $E : \pi_1^*V \to \mathcal{N}$ the fibrewise projection. We define:

$$\nabla_\Lambda = E \circ \pi_1^* \nabla_B \circ \iota_{\mathcal{N} \to V}$$

(11)

5 Matching the spectral data

We are finally in a position to state and prove the main result of this paper:
Theorem 3. If \((V, B, \Phi)\) is the Nahm transform of a regular instanton \((E, A)\), then the instanton spectral data \((\mathcal{S}, \mathcal{L}, \Gamma)\) is equivalent to the Higgs spectral data \((\mathcal{C}, \mathcal{N}, \Lambda)\), in the sense that the curves \(S\) and \(C\) coincide pointwise and there is a natural isomorphism \(\mathcal{L} \to \mathcal{N}\) preserving the connections.

Proof. Clearly, both spectral curves already have the points \((\pm \xi_0, \infty)\) in common. So let \(\xi \neq \pm \xi_0\) and suppose that \(\alpha\) is an eigenvector of \(\Phi[\xi]\) with eigenvalue \(\epsilon < \infty\). In particular, the point \((\xi, \epsilon) \in \hat{T} \times \mathbb{C}\) belongs to the Higgs spectral curve \(C\). By definition, we have:

\[
\Phi[\xi](\alpha) = \epsilon \cdot \alpha \Rightarrow \alpha \otimes (s_0 - \epsilon \cdot s_\infty) = 0
\]

Clearly, \(s_\epsilon = s_0 - \epsilon \cdot s_\infty\) is a holomorphic section in \(H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(1))\) vanishing at \(\epsilon \in \mathbb{P}^1 \setminus \{\infty\}\). Therefore it induces the following exact sequence:

\[
0 \to \mathcal{E}(\xi) \otimes s_\epsilon \to \tilde{\mathcal{E}}(\xi) \to \tilde{\mathcal{E}}(\xi)|_{T_\epsilon} \to 0
\]

which in turn induces the cohomology sequence:

\[
0 \to H^0(T_\epsilon, \tilde{\mathcal{E}}(\xi)|_{T_\epsilon}) \to H^1(T \times \mathbb{P}^1, \mathcal{E}(\xi)) \otimes s_\epsilon \to H^1(T \times \mathbb{P}^1, \tilde{\mathcal{E}}(\xi)) \to 0
\]

Thus \(\alpha \in \ker(\otimes s_\epsilon) = H^0(T_\epsilon, \tilde{\mathcal{E}}(\xi)|_{T_\epsilon}) = H^0(T_\epsilon, \mathcal{E}(\xi)|_{T_\epsilon}) = H^1(T_\epsilon, \mathcal{E}(\xi)|_{T_\epsilon})^*\).

In particular, \(H^1(T_\epsilon, \mathcal{E}(\xi)|_{T_\epsilon}) = \ker\{D_{\mathcal{A}(w)}\}\) is non-empty, hence \((\xi, \epsilon) \in \hat{T} \times \mathbb{C}\) also belongs to the instanton spectral curve \(S\). The same argument clearly provides the converse statement. Thus the curves \(C\) and \(S\) must coincide pointwise.

It also follows from the cohomology sequence \((12)\) that the dual of the \(\epsilon\)-eigenspace of \(\Phi[\xi]\) is exactly \(H^1(T_\epsilon, \tilde{\mathcal{E}}(\xi)|_{T_\epsilon}) = H^1(T_\epsilon, \mathcal{E}(\xi)|_{T_\epsilon})\). In other words, there are canonical identifications between the fibres \(\mathcal{N}_{(\xi, \epsilon)}\) and \(\mathcal{L}_{(\xi, \epsilon)}\), and the line bundles are isomorphic.

Finally, let us check that the connection \(\nabla_{\Gamma}\) and \(\nabla_{\Gamma}\) also coincide. Noting that the projection \(E : \pi_1^*V \to \mathcal{N} = \mathcal{L}\) is just the restriction map

\[
\pi_1^*V = \mathcal{N} = \mathcal{L}
\]

which completes the proof.
\[ r : \ker \left\{ D_{A_{x_1(s)}}^* \right\} \to \ker \left\{ D_{A_{x_1(s)}(\pi_2(s))}^* \right\} \text{ on each } s \in S = C, \text{ it is easy to see that } T = E \circ \pi_1^* P. \] Therefore, we have:

\[
\nabla_{\Gamma} = T \circ \pi_1^* d \circ \iota_{N \to H} = \\
= E \circ (\pi_1^* P \circ \pi_1^* d \circ \iota_{V \to H}) \circ \iota_{N \to V} = E \circ \pi_1^* \nabla_B \circ \iota_{N \to V} = \nabla_A
\]

Remark 1: More generally, the above argument shows that the pairs \((\overline{S}, L)\) and \((\overline{C}, N)\) also coincide when \(A\) is fibrewise semistable.

Remark 2: Cherkis and Kapustin used a similar argument to establish the analogous result for periodic monopoles \[2\]. More precisely, they considered monopoles on \(S^1 \times \mathbb{R}^2\), so that the Nahm transformed object is a Higgs pair on \(S^1 \times \mathbb{R}\). Each of these objects can be associated to a spectral pair consisting of an algebraic curve on \(\mathbb{R}^2 \times (\mathbb{R}^2 \setminus \{0\})\) plus a line bundle over it. If the Higgs pair is the Nahm transform of a periodic monopole, Cherkis and Kapustin have shown that both spectral data coincide.

6 Relation with Fourier-Mukai transform

The instanton spectral pair \((\overline{S}, L)\) could also be constructed via Fourier-Mukai transform in the following way.

Let \(F\) be a sheaf on \(T \times \mathbb{P}^1\) and consider the diagram:

\[
\begin{array}{ccc}
T \times \hat{T} \times \mathbb{P}^1 & \xrightarrow{\Pi} & \hat{T} \times \mathbb{P}^1 \\
\downarrow \Pi & & \downarrow \hat{\Pi} \\
T \times \mathbb{P}^1 & \xleftarrow{\leftarrow} & \hat{T} \times \mathbb{P}^1
\end{array}
\]

The Fourier-Mukai transform of \(F\) is given by

\[
\Psi(F) = R\hat{\Pi}_* (\Pi^* F \otimes \mathcal{P})
\]
where $\mathcal{P}$ denotes the pullback of the Poincaré bundle from $T \times \hat{T}$ to $T \times \hat{T} \times \mathbb{P}^1$. If $F$ is torsion-free and generically fibrewise semistable, then $\Psi(F)$ is a torsion sheaf on $\hat{T} \times \mathbb{P}^1$.

It is simple to show that if $F$ is locally-free and fibrewise semistable (as we have assumed throughout the paper), then $\Psi(F)$ is supported exactly over the spectral curve $\mathfrak{S}$, and the restriction to its support coincides with $\mathcal{L}$. Furthermore, it is also easy to see that $V = \pi_{14}(R^1\hat{\Pi}_4(\Pi^*F \otimes \mathcal{P}))$.

Therefore, the holomorphic version of the Nahm transform [3, 4] can be seen as a Fourier-Mukai transform composed with Hitchin’s correspondence. However, the Nahm transform (and the spectral construction of section 3) also contains some differential-geometric information (i.e. the instanton $A$, the transformed connection $B$, and the spectral connection $\Gamma$) in addition to the holomorphic information encoded into the Fourier-Mukai transform.

Of course, such differential-geometric information is usually encoded into the holomorphic data in the form of a stability condition. Such condition is well-known for Higgs bundles [4]. For doubly-periodic instantons, the appropriate concept of stability for the corresponding instanton bundles is discussed in [1]. It is less clear, though, what is the stability condition to be imposed on the spectral pairs $(\mathfrak{S}, \mathcal{L})$; such question is addressed in [8].

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