On $V_{ud}$ determination from kaon decays

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The pion $\beta$ decay $\pi^+ \to \pi^0 e^+ \nu$ proceeds through pure weak vector hadronic currents and, therefore, the theoretical prediction for it is more reliable than for the processes with axial-vector current contribution. For example, recently the pion $\beta$ decay has been used for $V_{ud}$ determination. The main aim of this letter is to point that kaon $\beta$ decay $K^0 \to K^+ (\pi^+ \pi^0) e^- \bar{\nu}$ analogously can be used for this purpose.

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I. INTRODUCTION

Flavor transitions within and between different quark generations due to the weak charged interactions are described by the Cabibbo–Kobayashi–Maskawa (CKM) unitary mixing-matrix $V_{CKM}$.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$  \hspace{1cm} (1)

This approach is founded on a pure phenomenological basis and the determination of the matrix elements is completely based on experimental data. If different types of experiments provide a consistent values for a particular matrix element, this should point to the correctness of our results. Therefore, new ideas about systematically independent measurements are welcome.

Often all new things are well forgotten old ones, however, from time to time they appear in the scientific literature without citing the sources. The subject that we will present concerns the determination of the well defined matrix element $V_{ud}$. Therefore, it could be sometimes and somewhere discussed. We apologize for eventual plagiarism, nevertheless, we would like to point to a new possibility to measure this quantity in kaon $\beta$ decays.

Up to now the most precise determination of this value follows from the superallowed $0^+ \to 0^+$ nuclear $\beta$-decay experiments $\cite{2}$. Based on the data from over 100 different experiments and using a new method for controlling hadronic uncertainties in the radiative correction to superallowed nuclear beta decays along with refinements from $\cite{2}$, Marciano and Sirlin $\cite{2}$ have derived the adopted at present PDG value $\cite{2}$

$$|V_{ud}| = 0.97377 \pm 0.00027.$$  \hspace{1cm} (2)

However, a recent modern determination of the $Q$-value $\cite{2}$ of the superallowed decay of the radioactive nuclei $^{46}$V, obtained from the mass difference of $^{46}$V and its decay daughter $^{46}$Ti, gives a new $Q$-value and invalidates the set of its seven previous measurements. This value affects the evaluation of $V_{ud}$ from superallowed nuclear decays and leads to a somewhat lower value for $V_{ud}$. It may indicate a problem with $Q$-values of the other superallowed emitters used for $V_{ud}$ determination.

Therefore, independent determination of $V_{ud}$ from other experiments is needed. The second precise evaluation of $V_{ud}$ value, with bigger than superallowed transition uncertainties, is obtained from the measurements of the neutron lifetime and the $\beta$ asymmetry coefficient $A_0$ $\cite{2}$. The later measurements are necessary in order to fix the unknown contribution of the axial-vector nuclear matrix element into the neutron decay rate, which is the main source of uncertainty for $V_{ud}$ extraction.

Using the most precise updated value for the ratio $\lambda_{\nu}^{\text{exp}} = g_A / g_V = -1.2733(13)$ $\cite{2}$ of the axial coupling constant to the vector coupling constant and the PDG value for the neutron lifetime $\tau_n = 885.7(8)$ s, one can evaluate $V_{ud}$ as $\cite{2}$

$$|V_{ud}| = 0.97218 \pm 0.00101.$$  \hspace{1cm} (3)

This value is 1.5$\sigma$ lower than the extracted one $\cite{2}$ from superallowed nuclear decays and may indicate the presence of new interactions. Their effect, predicted in the $\cite{2}$, leads to the corrected value of $\lambda = -1.2714(13)$ and to more consistent $|V_{ud}| = 0.97339(101)$ value.

However, the extracted from the neutron decays $V_{ud}$ value depends on the experimentally measured neutron lifetime, for which present situation is unclear due to the most recent result $\cite{2}$ $\tau_n = 878.5(8)$. Nevertheless, precision and consistent $\lambda$ determinations from several correlation coefficient measurements, which are ongoing and planned, would indicate reliable experimental results and would be able to put more stringent constraints on new physics.

And the last but not least important source of information about $V_{ud}$ is the very clean theoretically $0^- \to 0^-$ pion $\beta$ decay $\pi^+ \to \pi^0 e^+ \nu$. It is a pure vector transition and is free from nuclear structure uncertainties. However, due to the small pion mass difference it has a very weak branch, of the order of $10^{-8}$, which leads to severe experimental difficulties. Nevertheless, the PIBETA Collaboration $\cite{2}$, using the Paul Scherrer Institute facilities, has improved the experimental uncertainty for this mode up to 0.6% and quotes

$$|V_{ud}| = 0.9728 \pm 0.0030.$$  \hspace{1cm} (4)

Therefore, to reach the precision of $V_{ud}$ determination from superallowed nuclear decay $\cite{2}$ tenfold improvements both in statistics and systematics are necessary.
One hopes that with the development of high-intensity proton drivers, this aim can be reached.

It is worth nothing here, that besides excellent possibility for pion and muon physics, these facilities give a unique possibility for kaon physics as well. For example, a CP violation beyond the Standard Model can be searched in rare kaon decays with branching ratios \(10^{-10} - 10^{-12}\). One of the background processes is \(0^{-} \rightarrow 0^{-}\) kaon \(\beta\) decay \(K^0 \rightarrow K^+ e^-\), which can give an additional information about \(V_{ud}\) value. We are going to discuss this in the next section.

## II. KAON BETA DECAYS

The kaon \(\beta\) decay \(K^0 \rightarrow K^+ e^-\) is completely analogous to the pion beta decay \(\pi^- \rightarrow \pi^0 e^-\). It can serve as a possibility to extract \(V_{ud}\) matrix element, because the strange quark \(s\) does not participate in the weak interactions and play a spectator role. As far as the final kaon is not a stable particle, it can be registered through its escape from registration. Therefore, pure hadronic modes, mainly \(K^+ \rightarrow \pi^+ \pi^0\) decays, are very suitable for this.

It is interesting to note, that the experimental signature of these decays \(K^0 \rightarrow \pi^+ \pi^0 e^-\) does not fulfill \(\Delta S = 2\Delta Q\) selection rule, in contrast to the allowed \(K_{e4}\) decays \(K^0 \rightarrow \pi^- \pi^0 e^+\), but indicates the presence of \(\Delta S = -\Delta Q\) weak transitions. This situation is completely analogous to the experimental puzzle of the beginning of sixties with the observation of \(\Sigma^+ \rightarrow n \mu^+\nu\) decays \([13],[14]\), which later have been realized as background \([15]\).

So, let us consider the background events

\[K^0 \rightarrow K^+ e^-\rightarrow \pi^+ \pi^0 e^-\] (5)

to get a valuable information about the first CKM matrix element. In order to obtain competitive with \([16]\) result, we need to keep all uncertainties of the order of \(10^{-3}\).

The amplitude of the first semileptonic decay in (5)

\[M = -\frac{G_F}{\sqrt{2}} V_{ud} \left[ f_+ (q^2) (p + p')_\mu + f_- (q^2) q_\mu \right] \ell^\mu \] (6)

is expressed through the form factors \(f_+\) and \(f_-\) of hadronic matrix element \(\langle K^+ (p') | \bar{u} \gamma^\mu d | K^0 (p) \rangle\) multiplied by the leptonic current

\[\ell^\mu = \bar{e} \gamma^\mu (1 - \gamma^5) \nu.\] (7)

In general, the form factors depend on the square of the momentum transfer to the lepton pair \(q_\mu = (p - p')_\mu\). However, even for the pion \(\beta\) decay the Dalitz plot integral is practically insensitive to this dependence \([16]\), and the form factors can be considered as constants. We can also neglect the form factor \(f_-\), which is proportional to the small isospin mass difference \(m_{u}^2 - m_{d}^2\), and in \(SU(2)\) symmetry limit is equal to zero. Moreover, it is multiplied on the momentum transfer \(q_\mu\), which effectively leads to the small contribution in the Dalitz plot distribution, proportional to \((m_{e}/m_{K^0})^2 \approx 10^{-6}\).

Furthermore, for kaon \(\beta\) decay, in which the initial and final hadrons belong to an \(I = 1/2\) multiplet, \(f_+ = 1\) with good precision. Isospin corrections in first non-zero approximation are given by the formula \([16]\)

\[\delta f_+ = H_{K+K-} \approx -6.5 \times 10^{-6}.\] (8)

They are negligibly small in accordance to the Ademollo-Gatto theorem \([17]\).

Therefore, the rate of kaon \(\beta\) decay is given by the well-known formula \([18]\)

\[\frac{1}{\tau_{K^0}} = \frac{G_F^2}{60 \pi^3 |V_{ud}|^2} \left(1 - \frac{\Delta}{2m_{K^0}}\right)^3 \Delta^2 f(\epsilon, \Delta)(1 + \delta),\] (9)

where

\[\Delta = m_{K^0} - m_{K^+} = 3.972 \pm 0.027 \text{ MeV}\] (10)

is kaon mass difference \([9]\), \(\epsilon = (m_e/\Delta)^2\), and the Fermi function \(f\) is given by

\[f(\epsilon, \Delta) = \sqrt{1 - \epsilon} \left[ 1 - \frac{9}{2} \epsilon - 4\epsilon^2 + \frac{15}{2} \epsilon^2 \ln \left(\frac{1 + \sqrt{1 - \epsilon}}{\sqrt{\epsilon}}\right) - \frac{3}{7} \frac{\Delta^2}{(m_{K^0}^2 + m_{K^+}^2)} \right],\] (11)

while \(\delta\) represents the effect of the radiative corrections.

The second decay in \([9]\) is pure hadronic decay with an experimental branching ratio \([9]\)

\[B(K^+ \rightarrow \pi^+ \pi^0) = (20.92 \pm 0.12)\%\]. (12)

Therefore, the rate of \(K^0 \rightarrow \pi^+ \pi^0 e^-\) decay \([9]\) can be estimated as

\[\frac{1}{\tau_{K^0 \rightarrow \pi^+ \pi^0 e^-\nu}} = \frac{B(K^+ \rightarrow \pi^+ \pi^0)}{\tau_{K^0 \beta}} \approx 0.02 \frac{1}{8}.\] (13)

This decay channel has been proposed to measure the kaon mass difference \(\Delta\), due to its clear signature and very high sensitivity of the kaon \(\beta\) decay to the latter \([19]\).

However, \(K^0\) and \(K^0\) states are not invariant under CP symmetry transformation and do not represent physical states. Instead \(|KS\rangle = p|K^0\rangle + q|K^0\rangle\) and \(|KL\rangle = p|K^0\rangle - q|K^0\rangle\) combinations are assigned to the physical mesons \(KS\) and \(KL\), respectively. In the case of CP invariance, \(q = p = 1/\sqrt{2}\), \(KS\) represents CP even and \(KL\) CP odd states. Using PDG fit value for mean life of \(KL\) meson \([7]\)

\[\tau_{KL} = (5.114 \pm 0.021) \times 10^{-8} \text{ s}\] (14)

one can estimate the branching ratio of the sum of the two-step decay \([9]\) and its CP transformed

\[K_L \rightarrow \pi^+ \pi^0 e^-\nu + \pi^- \pi^0 e^+\nu\] (15)

as

\[B(K_L \rightarrow \pi^+ \pi^0 e^-\nu) = \frac{\tau_{KL}}{\tau_{K^0 \rightarrow \pi^+ \pi^0 e^-\nu}} \approx 10^{-9}.\] (16)
III. DISCUSSION AND CONCLUSIONS

In this section we would like to discuss the experimental possibility of $V_{ud}$ extraction from decays \[^{14}\] with an accuracy not worse than the one from pion $\beta$ decays. As far as this decays have very small branching ratio, one needs high-intensity beam provided in average $10^7 K_L$-decays per second. Probably, the best place for such measurements is the 50 GeV Proton Synchrotron at JHF. In order to derive the rate of the two-step process \[^{10}\] one needs to know with a good precision the lifetime of $K_L$ meson or some of its branching ratios. The present accuracy of the lifetime \[^{13}\] $\delta\tau_{K_L}/\tau_{K_L} \approx 4 \times 10^{-3}$ is already good enough and contributes to $V_{ud}$ error at the level of $2 \times 10^{-3}$.

The uncertainty in the determination of the rate of kaon $\beta$ decays, according to \[^{14}\], comes from the experimental accuracy of the branching ratio \[^{12}\] of hadronic mode of the charged kaon decays $\delta B/B \approx 6 \times 10^{-3}$. Its contribution into $V_{ud}$ error, $3 \times 10^{-3}$, is also competitive with pion $\beta$ decay uncertainty. It may be improved in future, because the last direct measurement of this ratio has been done more than thirty years ago \[^{21}\].

Speaking about the experimental selection of the rare process \[^{4}\] we should note that the main background to it comes from $K_{s4}$ decay with branching ratio $(5.21 \pm 0.11) \times 10^{-5}$ \[^{21}\]. The difference in $\Delta S = \Delta Q$ selection rule can be used for discrimination of these decays in the case of tagged $K^0$, $\bar{K}^0$ beams. However, it cannot help in the case of $K_L$ beam, containing both $K^0$ and $\bar{K}^0$ meson states. Nevertheless, these decays can be well separated kinematically. First of all, the two pions in the final state of \[^{14}\] come with definite invariant mass $M_{\pi\pi} = m_{K^+}$ from the two-particle $K^+$ decay and apart from $K_L$ decay point. In the same time $K_{s4}$ process is three body decay and its distribution in $M_{\pi\pi}$ variable has a continuous spectrum with the maximum at $340$ MeV and follows decreasing to the end. Good knowledge of the form factors \[^{22}\] of $K_{s4}$ process allows us to subtract background under the peak from process \[^{11}\] around $m_{K^+}$.

The last and the main source of the uncertainty in $V_{ud}$ determination comes from the $K^0 - K^+$ mass difference \[^{11}\], which enters into the rate of kaon $\beta$ decay \[^{14}\] in the fifth power. It leads to inappropriate contribution to $\delta V_{ud}/V_{ud} \approx 1.7 \times 10^{-2}$. Experimental situation at present resembles the one in the 1986 for $m_{\pi^+} - m_{\pi^0}$ pion mass difference, when its uncertainty was almost completely determined by the uncertainty in the neutral-pion mass measurements \[^{23}\]. Taking into account the recent considerable progress in kaon study, one hopes that new measurements of $K^0$ mass \[^{24}\] and directly $K^0 - K^+$ mass difference will be made.

We did not discuss the radiative corrections $\delta$ and their uncertainties for kaon $\beta$ decay. Most probably they can be calculated in the same lines as for pion $\beta$ decay \[^{14}\].

In this letter we have proposed the theoretical possibility to extract $V_{ud}$ matrix element from kaon $\beta$ decay. We have given only primeval experimental insights for the registration of this process and the estimation of the main uncertainties. Of course, in order to provide an experimental realization of this project, a systematical study of such a project is necessary. It will be interesting to analyze the possibility of such measurements within the proposed project \[^{25}\] searching $K_L \rightarrow \pi^0\nu\bar{\nu}$ decay.

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[1] N. Cabibbo, Phys. Rev. Lett. \textbf{10}, 531 (1963).
[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. \textbf{49}, 652 (1973).
[3] J. C. Hardy and I. S. Towner, Phys. Rev. Lett. \textbf{94}, 092502 (2005); Phys. Rev. C \textbf{71}, 055501 (2005).
[4] G. Savard et al., Phys. Rev. Lett. \textbf{95}, 102501 (2005).
[5] A. Czarnecki, W. J. Marciano and A. Sirlin, Phys. Rev. D \textbf{70}, 093006 (2004).
[6] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. \textbf{96}, 032002 (2006).
[7] W.-M. Yao et al., Journal of Physics G \textbf{1}, 1 (2006).
[8] H. Abele et al., Phys. Rev. Lett. \textbf{88}, 211801 (2002).
[9] D. Mund et al., in Proc. \textit{Ultra Cold and Cold Neutrons, Physics and Sources}, St. Petersburg, 13-18 July, 2005.
[10] M. V. Chizhov, \texttt{hep-ph/0411098}.
[11] A. Serebrov et al., Phys. Lett. B \textbf{605}, 72 (2004).
[12] D. Počanić et al., Phys. Rev. Lett. \textbf{93}, 181803 (2004).
[13] A. Barbaro-Galtieri et al., Phys. Rev. Lett. \textbf{9}, 26 (1962).
[14] J. Cronin, in Proc. \textit{14th ICHEP}, Vienna, Aug. 28 -Sept. 5, 1968, eds. J. Prentki , J. Steinberger, p.281, Geneva: CERN (1968).
[15] V. Cirigliano et al., Eur. Phys. J. C \textbf{27}, 255 (2003).
[16] J. Gasser and H. Leutwyler, Nucl. Phys. B \textbf{250}, 517 (1985).
[17] R. E. Behrends and A. Sirlin, Phys. Rev. Lett. \textbf{4}, 186 (1960); M. Ademollo and R. Gatto, Phys. Rev. Lett. \textbf{13}, 264 (1964).
[18] G. Källén, \textit{Elementary Particle Physics} (Addison-Wesley, Reading, Massachusetts, 1964);
[19] A. Sirlin, Rev. Mod. Phys. \textbf{50}, 573 (1978).
[20] D. E. Jaffe, internal note KOPIO TN086 (2004).
[21] I. H. Chiang et al., Phys. Rev. D \textbf{6}, 1254 (1972).
[22] J. R. Batley et al., Phys. Lett. B \textbf{595}, 75 (2004).
[23] A. Pais and S. B. Treiman, Phys. Rev. \textbf{168}, 1858 (1968).
[24] J. F. Crawford et al., Phys. Rev. D \textbf{43}, 46 (1991).
[25] A. Lai et al., Phys. Lett. B \textbf{533}, 196 (2002).
[26] Y. B. Hsiung et al., "Measurement of the $K_L \rightarrow \pi^0\nu\bar{\nu}$ Branching Ratio", submitted to the J-PARC Committee for Nuclear and Particle Physics Experimental Facility, Dec. 2002.