KamLAND and the determination of neutrino mixing parameters in the post SNO-NC era

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Abstract. We study in detail the ability of the reactor experiment KamLAND for discriminating existing solutions to the solar neutrino problem and giving accurate information on neutrino masses and mixing angles. We include in this analysis the information obtained from the latest SNO-NC (neutral current measurement) results and other solar data. Assuming that the expected signal corresponding to various ‘benchmark’ points in the two-dimensional ($\Delta m^2$, $\tan^2 \theta$) mixing plane, we develop a fully fledged $\chi^2$ analysis which includes the KamLAND spectrum and other existing solar evidence. A complete model of statistical and known systematical errors for 1 and 3 years of observations is included and exclusion plots are presented.

We find a much higher sensitivity, in particular, for values of $\Delta m^2$ lying in the central part of the large mixing angle (LMA) region. The situation would be more complicated for values that are closer to the edge of the LMA region (the HLMA region, i.e. $\Delta m^2 \lesssim 2 \times 10^{-5}$ eV$^2$ and $\Delta m^2 \gtrsim 8-9 \times 10^{-5}$ eV$^2$ or $\tan^2 \theta$ far from $\sim0.5$). In this case KamLAND, with or without any solar data, will only be able to select multiple regions in the parameter space, in the sense that different possible values of the parameters would produce the same signal. Finally, in conclusion, we point out that there is not only a problem in the determination of $\Delta m^2$ in the HLMA region, but also in the discrimination of the values of the mixing angle if one considers values that are not close to the present best-fit points.
1. Introduction

The publication of the recent SNO results [1]–[3] has made an important breakthrough in solving the long-standing solar neutrino [4]–[12] problem (SNP) possible. These results provide the strongest evidence to date for flavour oscillation in the neutral lepton sector.

Soon the reactor experiment, KamLAND [13, 14], is expected to further improve our knowledge of neutrino mixing. In fact it should be able to further improve our understanding of the region of the mixing parameter space corresponding to the so-called large mixing angle (LMA) solution of the SNP ($\Delta m^2 \sim 10^{-5} - 10^{-4}$ eV$^2$ and $\tan^2 \theta \sim 10^{-1} - 1$). This is of prime interest owing to the fact that the LMA region is the one preferred by the solar neutrino data at present.

The previous generation of reactor experiments [15, 16] were performed with a baseline of about 1 km. They attained a sensitivity of $\Delta m^2 \geq 10^{-3}$ eV$^2$ [15, 17] and, by finding that the initial flux did not disappear, demonstrated that the atmospheric neutrino anomaly [18] is not due to muon–electron neutrino oscillations. The KamLAND experiment is the successor of such experiments on a much larger scale in terms of baseline distance and total incident flux.

This experiment relies upon a 1 kton liquid scintillator detector located at the old, enlarged, Kamiokande site. It searches for the oscillation of antineutrinos emitted by several nuclear power plants in Japan. The nearby 16 (of a total of 51) nuclear power stations deliver a $\bar{\nu}_e$ flux of $1.3 \times 10^6$ cm$^{-2}$ s$^{-1}$ for neutrino energies $E_\nu > 1.8$ MeV at the detector position. About 78% of this flux comes from six reactors forming a well defined baseline of 139–214 km. Thus, the flight range is limited in spite of using several reactors; owing to this fact the sensitivity of KamLAND will increase by nearly two orders of magnitude compared to previous reactor experiments.

It has been estimated that even with the most conservative scenario [13], in which the background has to be determined from the reactor power fluctuations, the LMA solution is completely within the estimated sensitivity after three years of data recording. Moreover, KamLAND should be able to determine the mixing angle and mass difference with a 20% accuracy at a 99% confidence level (CL) [13]–[21]. As has been previously highlighted [22] problems might arise if the value of the squared mass difference ($\Delta m^2$) lies in the upper region of the LMA solution; the so-called HLMA region.
Let us briefly recall some independent conclusions obtained from the SNO results [4]. Different quantities can be defined in order to make the evidence for the disappearance and appearance of the neutrino flavours explicit. In addition to the SNO data, it is also possible to define two useful ratios, $\phi_{CC}/\phi_{ES}$, $\phi_{CC}/\phi_{NC}$, from the three fluxes measured by SNO and deviations of these ratios with respect to their standard value are powerful tests for the occurrence of new physics. For the first ratio, one obtains [4]

$$\frac{\phi_{CC}}{\phi_{ES}} = 0.73^{+0.10}_{-0.07},$$

a value which is $\sim 2.7\sigma$ away from the expected no-oscillation value of one. The ratio of the CC and NC fluxes gives the fraction of the electron neutrinos remaining in the solar neutrino beam, with the value obtained in [4] being

$$\frac{\phi_{CC}}{\phi_{NC}} = 0.34^{+0.05}_{-0.04}.$$

This value differs greatly from the standard model case.

Finally, if, in addition to the SNO data, one considers the flux predicted by the solar standard mode one can define, following [23] the quantity $\sin^2 \alpha$, the fraction of ‘neutrinos which oscillated into active ones’, one finds the following result:

$$\sin^2 \alpha = \frac{\Phi_{NC} - \Phi_{CC}}{\Phi_{SSM} - \Phi_{CC}} = 0.92^{+0.39}_{-0.20},$$

where the SSM flux is taken as the $^8\text{B}$ flux predicted in [24]. The central value is clearly below one (only-active oscillations). Although electron neutrinos are still allowed to oscillate into sterile neutrinos the hypothesis of transitions to only sterile neutrinos is rejected at nearly $5\sigma$.

The aim of this work is to study the KamLAND discriminating power and to understand in which regions of the parameter space that are still allowed by the solar neutrino experiments KamLAND might give a satisfactory level of accuracy. We fully include in this analysis all the available solar data including the latest SNO-NC results which are very important in solving the SNP. In conclusion of our work, we will see how there is not only a problem in the determination of $\Delta m^2$ in the HLMA region, but also in the discrimination of the values of the mixing angle if one considers values that are not too close to the present best-fit points.

The structure of this work is as follows. In section 2 we discuss the main features of the KamLAND experiment that are relevant for our analysis: we derive updated numerical expressions for the reactor fuel cycle-averaged antineutrino flux and the absorption antineutrino cross section. The next section is devoted to the salient aspects of the procedure we are adopting. The results of our analysis are presented and discussed in section 4 and, finally, in section 5 we draw our conclusions and discuss possible future scenarios.

2. A KamLAND overview

Electron antineutrinos from nuclear reactors with energies above 1.8 MeV are measured in KamLAND by detecting the inverse $\beta$-decay reaction $\bar{\nu}_e + p \rightarrow n + e^+$. The time coincidence, the space correlation and the energy balance between the positron signal and the 2.2 MeV $\gamma$-ray produced by the capture of an already-thermalized neutron on a free proton make it possible to identify this reaction unambiguously, even with a rather large background.

The two principal ingredients in the calculation of the expected signal in KamLAND are the reactor flux and the antineutrino cross section protons. These ingredients are considered next.
2.1. The reactor antineutrino flux

We first describe the flux of antineutrinos coming from the power reactors. A number of short baseline experiments (see [25] and references therein) have measured the energy spectrum of reactors at distances where oscillatory effects have been shown to be inexistent. They have shown that the theoretical neutrino flux predictions are reliable within 2% [13].

The effective flux of antineutrinos released by the nuclear plants is a rather well understood function of the thermal power of the reactor and the amount of thermal power emitted during the fission of a given nucleus, which gives the total amount, and the isotopic composition of the reactor fuel which gives the spectral shape. Detailed tables for these magnitudes can be found in [25].

For a given isotope \((j)\) the energy spectrum can be parametrized by the following expression [26]:

\[
\frac{dN_j}{dE_\nu} = \exp(a_0 + a_1 E_\nu + a_2 E_\nu^2),
\]

where the coefficients \(a_i\) depend on the nature of the fissionable isotope (see [25] for explicit values). Along the year, between periods of refueling, the total effective flux changes with time as the fuel is expended and the isotope relative composition varies. The overall spectrum is at a given time

\[
\frac{dN_\nu}{dE_\nu} = \sum_{j=\text{isotopes}} c_j(t) \frac{dN_j}{dE_\nu}.
\]

To compute a fuel-cycle averaged spectrum we have made use of the typical evolution of the relative abundances \(c_j\), which can be seen in figure 2 of [25]. This averaged spectrum can be again fitted very well by the same functional expression (1). The isotopic energy yield is then properly taken into account. As the result of this fit, we obtain the following values (which will be used in the rest of this work):

\[
a_0 = 0.916, \quad a_1 = -0.202, \quad a_2 = -0.088.
\]

Although individual variations of the relative abundances \(c_j\) along the fuel cycle can be very high, the variation in the two most important ones is highly correlated: the coefficient \(c(235\text{U})\) increases in the range \(~0.5–0.7\) while \(c(239\text{Pu})\) decreases \(~0.4–0.2\). This correlation makes the effective description of the total spectrum by a single expression \((1)\) useful. With the fitted coefficients \(a_i\) above, the difference between this effective spectrum and the real one is typically 2–4% along the yearly fuel cycle.

2.2. Antineutrino cross sections

We now consider the cross sections for antineutrinos on protons. We will sketch the form of the well known differential expression and more importantly we will give updated numerical values for the transition matrix elements which appear as coefficients.

In the limit of infinite nucleon mass, the cross section for the reaction \(\bar{\nu}_e + p \rightarrow n + e^+\) is given by [27, 28] \(\sigma(E_\bar{\nu}) = k E_{e^+} p_{e^+}\), where \(E, p\) are the positron energy and momentum and \(k\) is a transition matrix element which will be considered below. The positron spectrum is monoenergetic and \(E_{e^+}\) and \(E_\nu\) are related by \(E_\nu^{(0)} = E_{e^+}^{(0)} + \Delta M\), where \(M_n, M_p\) are the neutron and proton masses, respectively, and \(\Delta M = M_n - M_p \simeq 1.293\) MeV.

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Nucleon recoil corrections are potentially important in relating the positron and antineutrino energies in order to evaluate the antineutrino flux. As the antineutrino flux $\Phi(E_{\nu})$ would typically decrease quite rapidly with energy, the lack of adequate corrections will systematically overestimate the positron yield.

At the highest orders, the positron spectrum is not monoenergetic and one has to integrate over the positron angular distribution to obtain the positron yield. The differential cross section at the first-order $1/M_p$ is of the form

$$\left( \frac{d\sigma}{d\cos \theta} \right)^{(1)} = \frac{\sigma_0}{2} [(f^2 + 3g^2) + (f^2 - g^2)v_e^{(1)} \cos \theta]E_e^{(1)} p_e^{(1)}.$$

The complete expressions and notation can be found in [26]. Here we only want to pay attention to the overall coefficient $\sigma_0$ which is related to the transition matrix element $k$ above.

The matrix transition element can be written in terms of measurable quantities as

$$k = 2\pi^2 \log 2 / (m_e^5 f t_{1/2})$$

with the free neutron decay $t_{1/2}$, the phase-space factor $f$ and $m_e$. The value of $f = 1.71465 \pm 0.00015$ follows from calculation [29], while $t_{1/2} = 613.9 \pm 0.55$ s is the latest published value for the neutron half-life [30]. This value has a significantly smaller error than previously quoted measurements.

Using these values we obtain the extremely precise value

$$k = (9.5305 \pm 0.0085) \times 10^{-44} \text{ cm}^2 \text{ MeV}^{-2}.$$

From here the coefficient which appears in the differential cross section is obtained as (with vector and axial vector couplings $f = 1, g = 1.26$): $k = \sigma_0(f^2 + 3g^2)$. In summary, the differential cross section which appears in KamLAND is very well known and its theoretical errors are negligible if the updated values are employed.

### 3. The computation of the expected signals

In order to obtain the expected number of events at KamLAND, we sum the expectations for all the relevant reactor sources weighting each source by its power and distance to the detector (table 2 in [25]), assuming the same spectrum originates from each reactor. The average number of positrons $N_i$ which are detected per visible energy bin $\Delta E_i$ is given by the convolution of various quantities as follows: $\bar{P}$, the oscillation probability averaged over the distance and power of the different reactors, the antineutrino capture cross section given as before, the antineutrino flux spectrum given by expression (1), the relative reactor–reactor power normalization which is included in the definition of $\bar{P}$, the energy resolution of KamLAND which is rather good [14] and the expression $\sigma(E)/E \sim 10\% / \sqrt{E}$.

To run a standard nuclear plant power and fuel schedule for one year with a 600 ton fiducial mass, all the reactors operating at $\sim 80\%$ of their maximum capacity and an averaged, time-independent, fuel composition equal for each detector, the experiment expects about 550 antineutrino events (this number agrees with other estimations. i.e. [14]). We will consider this number as our KamLAND year in what follows. We will not add any background events as we will suppose that they can be distinguished from the signal with sufficiently high efficiency. This, however, should be read with caution and we will dedicate a few words to the experimental background at the end of section.
We compute the expected signal in a set of 0.5 MeV width total positron energy bins in the range 2.0–8 MeV. Quantitively, the main information about observable spectrum shape is summarized by the first moment of the distribution, the average spectrum energy. This first moment is defined as

$$\langle E(\theta, \Delta m^2) \rangle = \frac{\sum_i \tilde{E}_i R_i(\theta, \Delta m^2)}{\sum_i R_i(\theta, \Delta m^2)}$$

where $\tilde{E}_i$ is the centre positron energy in the $i$th energy bin and the normalized signals relative to the no-oscillation case are $R_i = N_i(\theta, \Delta m^2) / N_0^i$.

The expected value of this first moment as a function of $\Delta m^2$ for some selected values of the mixing angle is represented in figure 1(b). We want to illustrate the potentiality of this magnitude as an indicator of neutrino oscillations at KamLAND. In this plot we can graphically see the fact that KamLAND is sensitive to neutrino oscillations in the LMA region and, in particular, to the $\Delta m^2$ parameter. The first moment can vary up to $\sim 12\%$ in the sizeable regions of the parameter space (this can be seen beautifully in a three-dimensional plot). A variation of a fraction of this size should be clearly identifiable from the KamLAND data after one year of effective running. In order to see whether this result depends on the choice of the bin size, we reproduced it with larger, 1 MeV bins, and found that there was no significant difference.

In figure 1(a) we show the visible positron energy spectrum at KamLAND for some chosen oscillation parameters (see table 1). We present the integrated signal at every 0.5 MeV bin normalized to the no-oscillation expectation. We can see from the plot how the shape of the signal is very sensitive to the oscillation parameters. It can greatly change through the LMA region. From the point of view of background reduction, in some favourable cases the spectrum peaks above 5 MeV; this suggests the extension of the fiducial energy thresholds beyond the 8 MeV level.

In addition to the antineutrino signal, two classes of background can be distinguished \cite{13, 25, 31}. The random coincidence background is due to the contamination of the detector scintillator by U, Th and Rn. From MC studies and assuming that an adequate level of purification can be obtained, the background coming from this source is expected to be $\sim 0.15$ events/day/kt which...
Table 1. Benchmark points in the two-neutrino parameter plane used in the analysis.

| Label | $\tan^2 \theta$ | $\Delta m^2$ (eV$^2$) |
|-------|-----------------|---------------------|
| A     | 0.38            | $5.70 \times 10^{-4}$ |
| B     | 0.60            | $2.04 \times 10^{-4}$ |
| C     | 0.50            | $1.01 \times 10^{-4}$ |
| D     | 0.50            | $3.60 \times 10^{-5}$ |
| E     | 0.56            | $2.37 \times 10^{-5}$ |
| F     | 0.99            | $5.07 \times 10^{-5}$ |

is equivalent to a signal to background ratio of $\sim 1\%$. Other works [32] conservatively estimate a 5\% level for this ratio. More importantly for what it follows, one expects that the random coincidence backgrounds will be a relatively steeply falling function of energy. The assumption of no background should be relatively safe only at high energies (above $\sim 5$ MeV).

The second source of background, the so-called correlated background is mostly caused by cosmic ray muons and neutrons. KamLAND’s depth is the main tool for suppressing those backgrounds. MC methods estimate a correlated background of around 0.05 events/day/kt distributed over all the energy range up to $\sim 20$ MeV.

We also need to know the expected signals in different solar neutrino experiments. These are obtained by convoluting solar neutrino fluxes, Sun and Earth oscillation probabilities, neutrino cross sections and detector energy response functions. We closely followed methods that have already been fully explained in previous works [4, 33]–[35] and will mention here only a few aspects of this computation. We determine the neutrino oscillation probabilities using the standard methods found in the literature [36], as explained in detail in [33] and in [4]. We use a thoroughly numerical method for calculating the neutrino evolution equations in the presence of matter for all the parameter space. For the solar neutrino case the calculation is split in three steps, corresponding to the neutrino propagation inside the Sun, in the vacuum (where the propagation is computed analytically) and in the Earth. We find the average neutrino production point inside the Sun and we estimate the electron number density, $n_e$, in the Sun using the BPB2001 model [24]. The averaging over the annual variation of the orbit is also exactly performed. To take the Earth matter effects into account, we adopt a spherical model of the Earth density and chemical composition [37]. The gluing of the neutrino propagation in the three different regions is performed exactly using an evolution operator formalism [36]. The final survival probabilities are obtained from the corresponding (non-pure) density matrices that are built from the evolution operators in each of these three regions.

In this analysis, in addition to the night probabilities we also need the partial night probabilities corresponding to the six zenith angle bin data presented by SK [38]. These are obtained using the appropriate weights depending on the neutrino impact parameter and the sagitta distance from the neutrino trajectory to the Earth’s centre for each detector’s geographical location.
4. Analysis and results

In order to study the potentiality of KamLAND for resolving the neutrino oscillation parameter space, we have developed two kinds of analysis. In the first case (analysis A) we will deal with the expected KamLAND global signal. We will assume that the experiment measures a certain global signal with a given statistical and systematic error over a certain period of data recording (1 or 3 years) and will perform a complete $\chi^2$ statistical analysis also including current solar evidence. In the second case, analysis B, we will include the full KamLAND spectrum information. Instead of giving arbitrary values to the different bins, we will assume a number of oscillation models that are characterized by their mixing parameters ($\Delta m^2, \theta$). After including the solar evidence we will perform the same $\chi^2$ analysis as before.

4.1. Analysis A

The total $\chi^2$ value is given by the sum of two distinct contributions; one from all the solar neutrino data and the other from the KamLAND experiment, as follows:

$$
\chi^2 = \chi^2_\odot + \chi^2_{\text{glob,KL}},
$$

with

$$
\chi^2_{\text{glob,KL}} = \left( \frac{R_{\text{exp}} - R_{\text{teo}}(\Delta m^2, \theta)}{\sigma_{\text{stat+sys}}} \right)^2.
$$

(3)

For the ‘experimental’ signal ratio $R_{\text{exp}}$ we assume different values varying from a very strong suppression $R \sim 0.3$ to no observation of the neutrino oscillations $R \sim 1$. The total error $\sigma$ is computed as the sum of assumed systematic deviations, $\sigma_{\text{sys}}/S \sim 5\%$, mainly coming from flux uncertainty ($3\%$), energy baseline calibration and others (see [14, 25]) and statistical errors $\sigma_{\text{stat}} \sim \sqrt{S}$. The nominal periods of data recording that we consider, 1 and 3 years, are generic representative cases where systematical or statistical errors are taken as being predominant. The solar neutrino contribution can be written in the following way:

$$
\chi^2_\odot = \chi^2_{\text{glob}} + \chi^2_{\text{SK}} + \chi^2_{\text{SNO}}.
$$

(4)

The function $\chi^2_{\text{glob}}$ corresponds to the total event rates measured at the Homestake experiment [39] and at the gallium experiments: SAGE [40, 41], GNO [42] and GALLEX [43]. We follow closely the definitions used in previous works (see [4] for definitions and table 1 in [4] for an explicit list of results and other references).

The contribution to the $\chi^2$ from the SuperKamiokande data ($\chi^2_{\text{SK}}$) has been obtained by using double-binned energy and zenith angle data (see table 2 in [38] and also [44]): i.e., eight energy bins of variable width and seven zenith angle bins (one day bin and six night ones). The definition is given by

$$
\chi^2_{\text{SK}} = (\alpha \mathbf{R}^h - \mathbf{R}^{\text{exp}})^{(\sigma^2_{\text{unc}} + \sigma^2_{\text{cor}})^{-1}}(\alpha \mathbf{R}^h - \mathbf{R}^{\text{exp}}).
$$

(5)

The theoretical and experimental $\mathbf{R}$ quantities are matrices of dimension $8 \times 7$. The factor $\alpha$ is a flux normalization with respect to the value measured by SNO-NC. The covariance quantity $\sigma$ is a four-rank tensor constructed in terms of statistic errors, energy and zenith angle bin-correlated and uncorrelated uncertainties. The data and errors for the individual energy bins for the SK spectrum were obtained from [38].
The contribution of SNO to \( \chi^2 \) is given by
\[
\chi^2_{\text{SNO}} = \left( \frac{\alpha - \alpha^\text{th}}{\sigma_\alpha} \right)^2 + \chi^2_{\text{spec-SNO}}.
\] (6)
The presence of the first term is due to the introduction in our analysis of the flux normalization factor \( \alpha \) with respect to the SNO-NC flux, whose central and error values are given in table 1 of [4]. The second term in formula (6) corresponds to
\[
\chi^2_{\text{spec-SNO}} = \sum_{d,n} (\alpha R^{th} - R^{exp})^T (\sigma^2_{\text{stat}} + \sigma^2_{\text{syst}})^{-1} (\alpha R^{th} - R^{exp}),
\] (7)
where the day and night \( R \) vectors of dimension 17 are made up by the values of the total (NC + CC + ES) SNO signal for the different bins of the spectrum. The statistical contribution to the covariance matrix, \( \sigma_{\text{stat}} \), is obtained directly from the SNO data. The part of the matrix relating to the systematical errors was computed by us studying the influence on the response function of the different sources of the correlated and uncorrelated errors reported by SNO collaboration (see table 2 of [2]) and we assume full correlation or full anticorrelation according to each source.

To test a particular oscillation hypothesis against the parameters of the best fit and find the permitted regions in parameter space we perform a minimization of the three-dimensional function \( \chi^2(\Delta m^2, \tan^2 \theta, \alpha) \). For \( \alpha = \alpha_{\text{min}} \), a given point in the oscillation parameter space is allowed if the globally subtracted quantity fulfils the condition \( \Delta \chi^2 = \chi^2(\Delta m^2, \theta) - \chi^2_{\text{min}} < \chi^2_n(\text{CL}) \) where \( \chi^2_n(90\%, 95\%, \ldots) \) are the quantiles for three degrees of freedom.

In figure 2 we graphically show the results of this analysis. This shows exclusion plots including KamLAND global rates, given a hypothetical experimental global signal ratio under strong and medium suppression, \( S/S_0 = 0.3, 0.6 \), respectively, and no oscillation, \( S/S_0 = 1.0 \), for one and three years of KamLAND observations. As can be seen in the figures, as the KamLAND experimental signal decreases, the LMA region is singled out. The periodic shape in \( \Delta m^2 \) of the 90% CL (red regions) which becomes apparent in the 3-year plot is due to the periodicity of the response function: in order to distinguish between these different equally-likely solutions, one must analyse the energy spectrum, as will done in the main analysis to be presented in the following sections. Obviously, only if KamLAND sees some oscillation signal (i.e. \( S/S_0 \ll 1.0 \)) does the LMA region become the only solution. If we consider a hypothetical signal closer to 1.0 than 0.3, we see that the LOW region survives, although it is less favoured.

### 4.2. Analysis B

Here we use the expected binned KamLAND signal for some benchmark, arbitrarily chosen, points in parameter space that we show in table 1. For any of these points we obtain the expected spectrum after 1 or 3 years of observations made under the ‘standard’ conditions described earlier. Next we perform a standard \( \chi^2 \) analysis introducing statistical and assumed systematics errors including current evidence from various solar experiments (CL, GA, SK and SNO).

In the present study, the total \( \chi^2 \) value is given by the sum of two distinct contributions from the solar neutrino data and the KamLAND experimental data, as follows:
\[
\chi^2 = \chi^2_\odot + \chi^2_{\text{spec,KL}}.
\] (8)
The contribution from the solar neutrino experiments \( \chi^2_\odot \) is described in detail in the previous section and the contribution from the KamLAND experiment is
\[
\chi^2_{\text{spec,KL}} = (R^{th,h} - R^{th})^T (\sigma^2_{\text{unc}} + \sigma^2_{\text{corr}})^{-1} (R^{th,h} - R^{th}).
\] (9)
Figure 2. Exclusion plots including KamLAND global rates (analysis A), given a hypothetical experimental global signal ratio: respectively $S/S_0 = 0.3$, 0.6, and no-oscillation evidence $S/S_0 = 1.0$. Statistical and assumed systematic ($\sim 5\%$) errors are included. (a) 1 year of and (b) 3 year of KamLAND observations. The coloured areas are allowed at 90, 95, 99 and 99.7\% CL relative to the absolute minimum. The region above the upper thick line is excluded by the reactor experiments [17].

Note that the addition to $\chi^2$ of a constant term $N_{dof}$ has no practical importance as regards the main purpose of this kind of analysis since the determination of the exclusion regions proceeds from the minimum subtracted quantity $\chi^2 - \chi^2_{min}$. The $R$ variables are length 12 vectors containing the binned spectrum (0.5 MeV bins ranging from 2 to 8 MeV) normalized to the non-oscillation expectations. The theoretical vectors are a function of the oscillation parameters:
Figure 3. Permitted areas in the two-neutrino parameter space after 3 years of KamLAND observations (analysis B). Permitted regions belonging to the same point are labelled with the corresponding letter and the position of the point itself is labelled with a solid star (see table 1). The coloured lines separate allowed regions at 90, 95, 99 and 99.7% CL relative to the absolute minimum. (a) Results with just the KamLAND spectrum alone. (b) KamLAND spectrum plus solar (CL, GA, SK, SNO) evidence. Crosses are situated in the position of the $\chi^2$ minima.

$R^{th} = R^{th}(\Delta m^2, \theta)$. The ‘experimental’ vectors are defined in similar way, for any of the benchmark points $(\Delta m^2_0, \theta_0)$ we have $R^{th,0} = R^{th,0}(\Delta m^2_0, \theta_0)$.

We generate the acceptance contours (at 90, 95 and 99% CL) in the $(\Delta m^2, \tan^2 \theta)$ plane in a similar manner as explained in the previous section. For the sake of comparison we have also obtained exclusion regions that are derived from the consideration of $\chi^2_{spec,KL}$ alone.

In figure (3) we graphically show the results of this analysis for a selection of points and for 3 years of observations restricting ourselves to the LMA region of the parameter space where, as we have noted before, the KamLAND spectrum information is especially significant. In each plot the permitted regions corresponding to various starting points are superimposed and every region is distinguished with a label. The positions of the initial points are shown with solid stars.

The first case, the study of the KL spectrum alone ($\chi^2_{spec,KL}$) is represented by figure 3(a). The permitted parameter space corresponding to each particular point is formed by a number of, highly degenerated, disconnected regions that are symmetrical with respect to the line $\tan \theta = 1$. These regions can extend far from the initial point especially in terms of $\Delta m^2$ but also on some occasions in terms of $\tan^2 \theta$. For example, the point ‘A’ located at $(\Delta m^2 = 5.7 \times 10^{-4}, \tan^2 \theta = 0.38)$ gives rise to two sets of thin regions that are situated at $\Delta m^2 \sim 10^{-3}, 10^{-4}$, respectively, and a third region situated at $\Delta m^2 \sim 10^{-5}$ which practically covers the full range $\tan^2 \theta \sim 0.1–10$. Similar behaviour is observed for point ‘B’. Of course this situation is not very favourable for the future phenomenologist trying to extract conclusions from the KamLAND data. A much more comfortable situation is found for points that are nearer the centre of the
LMA region. Note how the regions corresponding to points ‘D, E’ and especially ‘F’ only extend very gently around the initial location.

The results of the full analysis are summarized in figure 3(b). The position of the minima of $\chi^2$, marked in the plot with crosses, is practically identical to the position of the initial points except in the case where the difference is not significant anyway. The general effect of the inclusion of the solar evidence in $\chi^2$ is the breaking of the symmetry in $\tan^2 \theta$ as expected and the general reduction of the number of disconnected regions corresponding to each point. Note, however, that point ‘A’ still gives rise to a small permitted region situated nearly one order of magnitude smaller in $\Delta m^2$. The ‘B’ region shrinks in size near its initial location (as happens to the rest of points). The conclusion to be drawn from these plots is that KamLAND combined with other solar experiments will be able to resolve the neutrino mixing parameters with a precision of $\delta \log \Delta m^2 \sim \pm 0.1$ practically everywhere. However, for values of $\Delta m^2 > 10^{-4}$ the problem of the coexistence of multiple regions with similar statistical significance levels will still be present.

5. Summary and conclusions

We have analysed our current experimental knowledge of the neutrino mixing parameters in the region of the parameter space that is relevant for solar neutrinos and we have studied in detail how this knowledge should improve with the forthcoming reactor experiment KamLAND.

In this work we have presented in some detail the characteristics of the KamLAND experiment including antineutrino reactor fluxes and absorption cross sections for which we have given updated values. We find that the present theoretical errors in this cross section are negligible.

We have studied the expected KamLAND spectrum for different possible ('benchmark points') values of the mixing parameters selected inside the LMA region. The shape of the spectrum shows a significant dependence on the values of the mixing parameters. The spectrum distortions caused by oscillation have been characterized by the first moment of the positron energy distribution. The results confirm that KamLAND is very sensitive to neutrino oscillation in the LMA region. In particular, the value of the first moment changes greatly for values of $\Delta m^2$ varying inside the region $\Delta m^2 \simeq 10^{-5}$–$10^{-4}$ eV$^2$. The dependence on the value of the mixing angle is also evident, but somewhat milder. In the LOW and SMA regions, however, the moment value is essentially constant. We have also verified that the result does not depend in a significant way on the choice of the bin size.

In order to investigate the discrimination power of KamLAND, we have selected different points ('benchmark points') in the LMA region and studied which information KamLAND will be able to give after 1–3 years. We have included a full model of the statistical and systematic uncertainties involved. The regions selected by KamLAND alone (all symmetrical with respect to $\tan^2 \theta = 1$) have a large spread in the mixing angle. The experiment should have, however, a much higher sensitivity to the mass difference parameter, especially for values of $\Delta m^2$ lying in the central LMA region. The situation would be more complicated for values that are closer to the border of the LMA region or beyond the HLMA region (i.e. $\Delta m^2 \leq 2 \times 10^{-5}$ eV$^2$ and $\Delta m^2 \geq 8–9 \times 10^{-5}$ eV$^2$ or $\tan^2 \theta$ far from 0.5). In this case KamLAND, with or without solar evidence, will only be able to select multiple regions in the parameter space, in the sense that different possible values of the parameters would produce the same signal.

We have performed a similar analysis by adding to the information from the KamLAND would-be signal all the evidence already available on solar neutrinos. By using $\chi^2$ analysis, we have produced exclusion plots. KamLAND will help to select the values of the mixing parameters especially in the case where the solutions lies in the LMA region. If, instead, one
moves towards values of the KamLAND signal that are closer to the no oscillation value (which at present seems to be strongly disfavoured by the other experiments) the absolute $\chi^2$ minimum moves from the LMA to the LOW solution and one is left with small permitted regions not only in the LOW, but also in the SMA region. In summary, KamLAND together with the other solar experiments will be able to find the neutrino mixing parameters with a high precision practically everywhere. However, for values of $\Delta m^2 > 10^{-4}$ (graphically emphasized by the two benchmark points labelled ‘A’ and ‘B’ in the figures) the problem of the coexistence of multiple regions with similar statistical significances will still be present. In conclusion, we see how there is not only a problem in the determination of $\Delta m^2$ in the HLMA region, but also in the discrimination of the values of the mixing angle if one considers values that are not too close to the present best-fit points.

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