Modeling of nonlinear elastoplastic behavior after stress reversal for high strength steel

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Abstract. Material characteristics have significant impact on simulation of sheet metal forming. The accuracy of springback prediction depends on the estimation of strain recovery after die release. It is well known that the experimentally obtained unloading behavior for steel sheets is nonlinear stress-strain relationship, and the response during unloading and reloading shows a hysteresis loop. This behavior should be modeled by a material model and considered in FE-simulations for accurate predictions. In this study, the in-plane stress reversal tests for high strength steel were carried out to observe the elastoplastic behaviors after stress reversal. A material model that considers the nonlinear behavior was newly developed and implemented into the FEM software. The accuracy of springback prediction with the developed material model was validated by the draw bending tests and its springback simulations. The simulations with the developed material model show better agreement with the experimentally measured springback profile as compared to the other material models.

1. Introduction

Springback has been a long-discussed problem especially since high strength steel began to be used in automotive parts for weight saving. Development of the automotive parts can take a longer period if significant springback incurs several die modifications. For this reason, considerable efforts have been made to predict springback accurately by numerical simulations in the tool design stage. For a better springback prediction, a material model used in finite element method (FEM) software should properly consider material behaviors. Unloading behavior is particularly important for springback prediction since springback is caused by the relief of a portion of the elastic stresses during unloading to attain equilibrium. Stress-strain behavior after stress reversal including unloading and reloading shows nonlinearity at the small strain region that has been classically considered linear elasticity [1-3]. Yoshida et al. [4] investigated average Young’s modulus and proposed an exponential equation representing the decrease in the average Young’s modulus with plastic deformation. Application of the average Young’s modulus leads to a better springback prediction [5-7]. Sun and Wagoner [8] evaluated recoverable nonlinear elastic strain as Quasi-Plastic-Elastic (QPE) strain, and developed a two-surface kinematic material model.

In most of the studies, the unloading behaviors were evaluated in tension-unloading deformation. However, re-yielding behavior at transition from elasticity to plasticity after stress reversal should be critical since the material is subject to complex stress-strain history during forming other than tension-unloading. The stress in the material tends to go to the opposite side (tension to compression,
compression to tension) even in springback deformation. The inner forces in the material are hardly balanced in the condition that stress is exactly zero after springback especially at a bending part.

In this study, the stress strain behaviors after stress reversal including unloading and reloading for high strength steel sheet were observed by in-plane stress reversal tests. Then a material model considering the nonlinear stress-strain behavior was newly developed. The material model was applied to FE-simulation of draw bending, and the accuracy of springback prediction was evaluated.

2. Observation of stress-strain relationship by in-plane stress reversal test

2.1. Material

The material used for in-plane stress reversal test was 590DP with the thickness of 1.2mm. The mechanical properties of the steel are shown in Table 1.

| Property | Value |
|----------|-------|
| YS /MPa | 398   |
| TS /MPa | 632   |
| El /%   | 29    |
| E /GPa  | 210   |

2.2. Experimental procedure

The geometry of the specimen for the in-plane stress reversal test is shown in Figure 1. The specimen was prepared by laser cutting in R.D.. The strain in the tensile direction was measured with a strain gauge installed at the center of the specimen. Kuwabara et al. [9] developed a test apparatus for the in-plane stress reversal test by using comb-type dies installed on and under the specimen to prevent buckling. This study followed the test apparatus. 5 kN of force was applied to the dies in the thickness direction from above. The reversal points from tension to compression were set by pre-strain, 0.025 as the first reversal point and 0.05 as the second reversal point. The reversal points from compression to tension were set by three levels of compressive stress, -200 MPa, -400 MPa and -500 MPa.

![Figure 1. Geometry of test specimen for uniaxial stress reversal test.](image)

2.3. Experimental results

Figure 2 shows the cyclic stress-strain curves obtained by the in-plane stress reversal tests. Closed hysteresis loops are observed in all curves. In order to evaluate stress-strain behavior in detail, the instantaneous stress-strain slope $d\sigma/d\varepsilon$ was obtained in each unloading (U1–U6) and reloading (R1–R6). The slopes during unloading after 0.025 of pre-strain and reloading after -200 MPa of compressive stress are shown in Figure 3(a). The slopes decrease continuously with stress in both unloading and reloading immediately after stress reversal due to nonlinearity of stress-strain relationship. In order to compare the stress-strain slopes in unloading and reloading, the amount of stress change from stress reversal $\sigma_{\text{chng}}$ was newly defined in this study, as shown in equation (1).

$$\sigma_{\text{chng}} = |\sigma_{\text{rev}} - \sigma|$$

where $\sigma$ denotes the current stress and $\sigma_{\text{rev}}$ denotes the stress at stress reversal. $\sigma_{\text{chng}}$ is a stress value that becomes zero when stress reversal occurs and increases with monotonic deformation in which stress reversal does not occur. Figure 3(b) shows stress-strain slopes organized by $\sigma_{\text{chng}}$ on the
horizontal axis based on the results in Figure 3(a). The slopes during unloading and reloading coincide with each other. Furthermore, the slopes during unloading after 0.025 and 0.05 of pre-strain organized in the same way are shown in Figure 4(a), and the slopes during reloading at each compression level after 0.025 of pre-strain are shown in Figure 4(b). It can be observed that the magnitude of the slopes and its change are nearly the same. Hence, these results show that the nonlinear stress-strain relationship cannot be influenced by pre-strain, stress before reversal and deformation mode (unloading or reloading) and be determined by the amount of stress change from stress reversal $\sigma_{\text{chng}}$.

![Figure 2](image)

**Figure 2.** Cyclic stress-strain curves with three levels of compressive stress: (a) -200 MPa, (b) -400 MPa, (c) -500 MPa.

![Figure 3](image)

**Figure 3.** Stress-strain slope: (a) relation with stress, (b) relation with amount of stress change from stress reversal.

![Figure 4](image)

**Figure 4.** Stress-strain slope with amount of stress change: (a) two pre-strain levels, (b) three compression levels.

3. Material model

3.1. Framework of material model

A kinematic hardening model that describes the nonlinear behavior after stress reversal was newly developed. Figure 5 shows a concept of the proposed material model. This material model consists of two surfaces, yield surface and bounding surface. The inner yield surface adopts kinematic hardening and moves in the outer bounding surface. The criterion for the subsequent yield surface $f$ is given by

$$ f = \phi(\sigma - \alpha) - Y = 0 $$

where $\phi$ expresses the equivalent stress calculated by yield functions, and $\sigma$ and $\alpha$ express the Cauchy stress and the back stress of the yield surface, respectively. $Y$ is a material parameter that expresses the size of the yield surface, i.e., yield stress. In general cases, for parameter identification of material models, the parameter $Y$ should be nearly the 0.2% offset yield strength for better description of the stress-strain curve. However, the stress-strain relationship during unloading is represented linearly, because it is assumed to be elastic deformation. In the proposed model, the size of the yield surface $Y$ is much smaller than in the conventional material models. The large part of the stress-strain behavior after stress reversal becomes nonlinear relationship resulting from an algorithm of plastic deformation.

The bounding surface adopts isotropic hardening. The bounding surface $F$ is given by

$$ F = \phi(\sigma) - (B + R) = 0 $$

(3)
where \( B \) is a material parameter that expresses the initial size of the bounding surface, and \( R \) expresses the amount of isotropic hardening of the bounding surface. The evolution of \( R \) is shown by the following equation:

\[
dR = m(R_{sat} - R)d\varepsilon
\]  

(4)

where \( m \) is a material parameter for the rate of isotropic hardening, \( R_{sat} \) is a material parameter for the saturated value of \( R \) and \( d\varepsilon \) expresses the effective plastic strain rate.

### 3.2. Evolution equation of back stress

Movement of the yield surface directly denotes the hardening behavior in kinematic hardening. Therefore, a modification of the evolution equation of back stress is effective to describe the stress-strain behavior at a “small strain region” just after stress reversal. As discussed above, the experimental results revealed that the stress-strain behavior during unloading and reloading can be expressed by the amount of stress change after reversal. Therefore, this stress value (tensor) was introduced as \( X \) to the decreasing term in the evolution equation of back stress \( d\alpha \), as follows:

\[
d\alpha = C \left( \frac{1}{Y} \right) (\sigma - a) - \rho X d\varepsilon
\]  

(5)

where, the coefficient \( C \) denotes the rate of kinematic hardening and \( a \) denotes the difference of the size between the yield surface and the boundary surface, as given by

\[
a = B + R - Y = a_0 + R
\]  

(6)

where \( a_0 \) is a material parameter of initial value of \( a \).

\( X \) denotes the distance of the yield surface from stress reversal. \( X \) and \( \rho \) are defined depending on the two cases, as shown in Figure 6. Case 1 is the case in which the current equivalent stress \( \bar{\sigma} \) is equal to the maximum value of equivalent stress, and case 2 is the case in which the current equivalent stress \( \bar{\sigma} \) is less than the maximum value. \( X \) and \( \rho \) in each case are given by

\[
X = a, \quad \rho = 1 \quad \text{when } \bar{\sigma} = \bar{\sigma}_{max} \text{ (Case 1)}
\]

\[
X = a' - a, \quad \rho = 0.5 \delta \quad \text{when } \bar{\sigma} < \bar{\sigma}_{max} \text{ (Case 2)}
\]  

(7)

In case 1, \( X \) is simply equal to the current back stress \( \alpha \) because current deformation can be considered monotonic deformation without stress reversal. Once a stress reversal occurs (case 2), \( X \) changes into the difference between \( a' \), back stress at the moment of stress reversal, and \( \alpha \). \( a' \) does not change until the next stress reversal. In order to distinguish whether the current deformation is stress reversal or not, \( \delta \) is defined by an inner product value of the two vectors, as shown in below.
\[ \delta = \sigma_{\text{max}} \cdot (\sigma - \alpha) \]  

(8)

where \( \sigma_{\text{max}} \) expresses the maximum stress vector. When the value of \( \delta \) becomes negative value, the current deformation is considered stress reversal.

**Figure 6.** Illustrations of deformation modes: (a) just after stress reversal, (b) adequate deformation after stress reversal.

### 3.3. Modification of coefficient \( C \) in evolution of kinematic hardening

As mentioned above, the size of the yield surface is set to be small enough to describe the nonlinear behavior after stress reversal. Figure 7 shows the comparison of the stress-strain curves between the experimental results and the calculated results obtained by the proposed model applying the equation (5) with three different values of the coefficient \( C \). The size of the yield surface \( Y \) is set at 50 MPa. Most of the calculated stress-strain curves are successfully nonlinear. However, these curves don’t well describe the experimental curve, regardless of the coefficient \( C \). In this study, the coefficient \( C \) is treated as a variable and formulated using the distance of the yield surface from stress reversal \( X \).

\[ C = C_0 + C_C \exp \left( \frac{\overline{X}^n}{A} \right) \]  

(9)

where \( \overline{X} \) expresses the equivalent value of \( X \), and \( C_0 \) and \( C_C \) are material parameters for the maximum value of \( C \) and the maximum amount of increase in \( C \), respectively. \( A \) and \( n \) are material parameters that denote the convergence rate of \( C \), and change into two values \((A_1 \text{ or } A_2, n_1 \text{ or } n_2)\) depending on the stress state as well as equation (7).

**Figure 7.** Comparison of stress-strain curves between experimental results and calculated results of model proposed applying equation (5) with different values of parameter \( C \).

### 3.4. Description of permanent softening

The permanent softening is a stress offset between monotonic and reversed flow stresses [4,10]. The proposed model describes the permanent softening by the parameter \( \lambda \) used in equation (7). The degree of the permanent softening becomes more significant when the value of \( \lambda \) is more than 1 as shown in Figure 8.
3.5. Accuracy evaluation of stress-strain curves

In the validation of the calculated stress-strain curves, the proposed model and the Yoshida-Uemori model [11] (hereafter Y-U model) considering the average Young’s modulus obtained by tension-unloading test was evaluated. The material parameters for the proposed model are shown in Table 2. Figure 9(a) shows the calculated stress-strain curves with the experimental data. The proposed material model represents the experimental hysteresis loops accurately. Figure 9(b) and (c) shows the stress-strain slopes during unloading and reloading after 0.025 and 0.05 of pre-strain. In case of the Y-U model, the slopes are constant until the following re-yielding. On the other hand, the proposed model describes the gradual decreases in the slope both in unloading and reloading.

![Figure 8](image)

**Figure 8.** Description of permanent softening by parameter \( \lambda \).

![Figure 9](image)

**Figure 9.** Comparison between experimental and calculated results: (a) stress-strain curves, (b) stress-strain slopes after 0.025 of pre-strain, (c) stress-strain slopes after 0.05 of pre-strain.

| \( Y \) /MPa | \( a_0 \) /MPa | \( C_0 \) | \( m \) | \( R_{sat} \) /MPa | \( C \) /MPa | \( A_1 \) /MPa | \( A_2 \) /MPa | \( n_1 \) | \( n_2 \) | \( \lambda \) |
|------------|-------------|--------|------|-----------------|---------|----------|----------|------|------|-------|
| 50         | 470         | 120    | 11   | 270             | 5000    | 200      | 460      | 4.00 | 1.45 | 1.01  |

4. Application to FE-simulation

4.1. Draw bending test and definition of springback

Figure 10 shows a schematic view of the draw bending tools. The blanks of DP590 with a dimension of 360 mm (R.D.) × 80 mm were used. The dies with different die radii \( R_d \) (3 mm, 5 mm, 10 mm) were used in the condition that blank holder force was set to 100 kN. Three levels of blank holder force (50 kN, 100 kN, 200 kN) were applied in the condition that the dies with 5 mm radius was used. A pad was installed against the punch, and pad force was set to 50 kN. For springback evaluations, the radius of curvature \( \rho \) was measured at the designated portion of the sidewall as shown in Figure 11.
4.2. FE-simulation

The proposed material model was implemented into LS-DYNA ver. 971 by a user subroutine, and simulations for the above-mentioned forming were performed. In addition to the proposed material model, the Swift-type isotropic hardening model (hereafter IH model) and the Y-U model, which were implemented into LS-DYNA by user subroutines, were also utilized in this accuracy evaluation. The IH model and the Y-U model considered the average Young’s modulus obtained by uniaxial tension-unloading tests.

4.3. Results and discussion

Figure 12 shows the curvature $1/\rho$ simulated by three material models with experimental results. The IH model and the Y-U model overestimate the amount of springback regardless of radius of die and blank holder force. On the other hand, the results of the proposed material model totally agree well with the experimental results.

Figure 13 shows an illustration of the local stress-strain history on the upper surface at the sidewall portion. The blank is subjected to tension-compression (O-A-B) during forming, after that it is unloaded (B-C) during springback. The dominant factors in springback are the local stress at the bottom dead point $\sigma_B$ and the recovered strain during springback $\varepsilon_{SB}$. Figure 14 shows the absolute values of $\sigma_B$ and the recovered strains $\varepsilon_{SB}$ calculated by three material models. The stress calculated by the IH model is larger than those calculated by the other material models since the Bauschinger effect is not considered in the IH model. This is the reason why the IH model estimates larger springback than the others.

The recovered strains vary with material model. The difference between the IH model and the Y-U model can be simply attributed to the difference of stress at the bottom dead point. The recovered strain of the proposed model is less than that of Y-U model. Y-U model represents unloading behavior as a linear relationship with the average Young’s modulus, which is obtained from unloading curves in the condition that the specimen is completely unloaded (zero stress). However, in the actual forming,
this is not the case. Figure 15 shows the local stress calculated by the proposed model at each integration point through thickness before and after springback. The amount of the released stress differs by integration point and certain level of stress still remains in the material even after springback. This result indicates that modeling of the whole nonlinear behavior is necessary for a better estimation of recovered strain. The proposed material model, which properly estimates the stress at the bottom dead point and the recovered strain during springback, enables accurate springback prediction.

![Figure 13](image1.png)  
**Figure 13.** Stress-strain history during forming and springback at sidewall.

![Figure 14](image2.png)  
**Figure 14.** Stress at bottom dead point and recovered strain during springback at sidewall.

![Figure 15](image3.png)  
**Figure 15.** Local stress at the each integration point through thickness before and after springback.

5. **Conclusion**
A new material model which considers nonlinear behavior after stress reversal was proposed and implemented into LS-DYNA. The results of springback prediction in draw bending utilizing the proposed model showed good agreement with experimentally-obtained results. It is concluded that the nonlinear behavior after stress reversal has an impact on the springback prediction and should be considered by material model for accurate prediction.

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