Reduction of the value of the bend deflection arising due to external forces and gravity by applying the tensile stress to the axial alignment

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Abstract. The paper describes the methodology of designing geometric dimensions of elements of various structures \( h/L < 0.01 \), where \( h \) means height and \( L \) length. It is useful for designing the optimum span of elements for which reaching the minimum value of \( \beta \) bend deflection is required. Creating an additional „artificial” stress acting axially in the direction perpendicular to the beam deflection reduces the value of the \( \beta \) bend deflection in a macro scale. The experiment has shown that the deflection of an element is minimised if the additionally applied force operating along the inert \( x \) axis is approximately 1 884 times greater than the forces causing the arrow to deflect in the \( y \) axis. The results confirmed that the introduction of an additional tensile force affecting an element in the system along the axis and the creation of an additional „artificial” tensile stress allows for reducing the bend deflection to 0. The developed methodology of designing geometrical elements \( h/L < 0.01 \) consists in introducing additional forces causing stretching of the element along the \( x \) axis and checking the state of stresses in extreme fibres. If the fibres on both sides of the inert \( x \) axis are stretched, the deflection value on the macro scale is negligible.

1. Introduction
In order to reduce the arrow of deflection of bridges, embankments and many other components in their middle part, one may implement a change in shape (larger dimensions in the middle part of the bent component). The deflection of components in case of technological processes of cutting of a band of „polymer film” material is an unfavourable phenomenon. The arrow of deflection of the system may cause non-linearity of the cut, insufficient cutting, and may hinder the positioning of the material being cut. The undertaking of research aimed at the development of methods of design of components with the possibility of reduction of the value of the arrow of deflection towards zero is substantiated industrially.

2. Variants of technical solutions and experiment description
Technological processes of mass processing of elastic and highly workable components (e.g. polymer films reinforced with cord along one direction), cutting takes place directly above the transporter [1, 2]. The band „film” is moved over the knife impact surface, and the cut is made. Hence, the component making up the knife impact surface cannot be too high. For such components, the ratio of height \( h \) to
length $L$ should be less than 0.01. This ratio is referred to in the present paper as the height to length ratio, and is marked $c$:

$$c = \frac{h}{L} < 0.01$$

(1)

where: $c$ – height-length ratio, $h$ – component height (mm), $L$ – component length (mm).

In an industrial design, the height and length of a component was defined as $h = 22$ mm and $L = 2757$ mm, respectively, and it was assumed that the maximum deflection of a component towards the floor should not exceed 0.5 mm. The height of the profile cannot be increased, as a synthetic material band, which cannot be deformed, will precisely be transported onto the component. Calculations show that the coefficient $c = 0.007979$ mm:

$$c = \frac{22}{2757} = 0.007979 < 0.01$$

(2)

In order to verify the methodology of design of the above defined structural components, an experiment was prepared that will enable an analysis of the phenomenon of the reduction of the arrow of deflection for a component having a coefficient of $c < 0.01$. For this purpose, a threaded rod was used with a diameter of M3 and a length of 1000 mm. For the calculations it was assumed that the rod height $h = d_{\text{M3}} = 2.387$ mm:

$$c_{\text{M3}} = \frac{2.387}{1000} = 0.002387 < 0.01$$

(3)

The experiment reflects the assumptions made in the engineering design. Figure 1 shows the variants of the concepts of execution of supports for the experiment. Initial calculations and the experiment were carried out for:

1. a classic solution, i.e. a movable support and a fixed support, in two options:
   a. the first is a test of deflection due to the influence of gravity,
   b. a test of deflection being the sum of deflection caused by gravity and an additional focused force;
2. a variant with two fixed supports, in two options:
   a. the first is a test of deflection due to the influence of gravity,
   b. a test of deflection being the sum of deflection caused by gravity and an additional focused force;
3. for a variant with tensioning of the component, with two options:
   a. the first is a test of deflection due to the influence of gravity,
   b. a test of deflection being the sum of deflection caused by gravity and an additional focused force.

Table 1 shows the results of initial calculations performed with the use of calculational procedures of CAD software [3] in order to find the most favourable solutions, meaning, as small a deflection of the component for the different support variants. A set of photographs made during the experiment, along with their descriptions, is included in table 2. The results of the experiment may be shown in a mathematical model described in publications [4, 5] in the field of material strength. Tensions present in the simultaneously stretched and bent components take the following form:

$$\sigma = \sigma_r + \sigma_g$$

(4)

$$\sigma_{\text{max}} = \frac{P}{A} + \frac{M_g}{W} \leq k_r$$

(5)

where: $\sigma_r$ – maximum elongation tension constituting additional „artificial” tensioning (MPa), $\sigma_g$ – maximum bending tension (MPa), $\sigma_{\text{max}}$ – maximum tension present in the tested component (MPa), $P$ – force of elongation applied along the neutral axis $x$ (N), $M_g$ – bending moment causing the bending of the bar along the $y$ axis (Nm), $W$ – strength indicator ($m^3$), $k_r$ – material tensile strength (MPa).

A graphical interpretation of the above interdependencies is shown in figure 2.

The conducted experiment shows (a description of the results of the experiment is given in table 2) that following the removal of the test bed, the rod did not suffer permanent deformation, hence, all the introduced tensions did not cause a permanent deformation of the tested component, which is uncommonly important from the point of view of industrial practice.
The sum of tension values for the tested component may be summarised as follows:

$$\sigma = \sigma_r + (\sigma_{g1} + \sigma_{g2})$$  \hspace{1cm} (6)

Writing out the above equation, one arrives at two terms (the first $P/A$ being the „artificial tension” and the second term constituting the sum of bending tension forces of the rod):

$$\sigma = \frac{P}{A} + \left(\frac{M_{g1}}{W} + \frac{M_{g2}}{W}\right)$$  \hspace{1cm} (7)

Figure 1. Figure showing the assumptions of the experiment. An M3 threaded rod with a total mass of 0.040 kg and a length of 1 m supported on three various types of supports. The rod will be charged in the middle with a component weighing 0.065 kg, which will reflect the influence of a focused force.

Figure 2. Graphical representation of the complex condition of tensions for simultaneous elongation and bending.
For the executed experiment, the strength coefficient was calculated using the following relationship:

\[
W = \frac{t_x}{y_{\text{max}}} = \frac{t_x}{d} = \frac{2 \cdot I_x}{d} = \frac{2 \cdot \pi \cdot d^4}{64 \cdot d} = \frac{\pi \cdot d^3}{32}
\]  

(8)

\[
I = \frac{\pi \cdot d^4}{64} = \frac{\pi \cdot 2.387^4}{64} = 1.594 \text{ (mm}^4)\n\]  

(9)

\[
W = \frac{\pi \cdot d^3}{32} = \frac{\pi \cdot 2.387^3}{32} = 1.334 \text{ (mm}^3) = 1.335 \cdot 10^{-9} \text{ (m}^3)\n\]  

(10)

Then, the size of the surface of the cross-section of the component (the M3 screw) was calculated in order to determine elongating tensions acting along the \(x\) axis:

\[
A = \pi \cdot r^2 = \pi \cdot \left(\frac{d_1}{2}\right)^2 = \pi \cdot \left(\frac{2.387}{2}\right)^2 = 4.475 \text{ (mm}^2)\n\]  

(11)

A further stage was the determination of the maximum values of bending moments \(M_{b1}\) and \(M_{b2}\) caused by gravity and the additional load (the focused force):

\[
M_{b1} = \frac{q \cdot l^2}{8} = \frac{0.0004 \cdot (N/mm) \cdot 1000 \cdot (mm^2)}{8} = 50 \text{ (Nmm)} = 0.05 \text{ (Nm)}
\]  

(12)

\[
M_{b1} = \frac{P \cdot l}{4} = \frac{0.637 \cdot (N) \cdot 1000 \cdot (mm)}{4} = 159 \text{ (Nmm)} = 0.159 \text{ (Nm)}
\]  

(13)

The maximum tension present in the tested component takes the following form:

\[
\sigma = \frac{1940 \text{ (N)}}{4.475 \text{ (mm}^2)} + \left(\frac{50 \text{ (Nmm)}}{1334 \text{ (mm}^3)} + \frac{159 \text{ (Nmm)}}{1334 \text{ (mm}^3)}\right)
\]  

(14)

\[
\sigma = 434 \left(\frac{N}{\text{mm}^2}\right) + \left(37.5 \left(\frac{N}{\text{mm}^2}\right) + 119.2 \left(\frac{N}{\text{mm}^2}\right)\right) = 590 \left(\frac{N}{\text{mm}^2}\right)
\]  

(15)

The sum total of tension values that are present in extreme fibres of the tested component below the neutral axis (\(x\) axis) was calculated according to the following formula:

\[
\sigma_d = 434 \left(\frac{N}{\text{mm}^2}\right) + 156.7 \left(\frac{N}{\text{mm}^2}\right) = 590 \left(\frac{N}{\text{mm}^2}\right)
\]  

(16)

The sum total of tension values present in the extreme fibres of the tested component above the neutral axis (\(x\) axis) is the difference described by the following formula:

\[
\sigma_{r+d} = 434 \left(\frac{N}{\text{mm}^2}\right) - 156.7 \left(\frac{N}{\text{mm}^2}\right) = 277.3 \left(\frac{N}{\text{mm}^2}\right)
\]  

(17)

It can be shown on the basis of the conducted tests that if the extending force acting symmetrically on the component in the \(x\) axis is higher by a factor of approx. 1 884 (1 940 N) than the forces causing the bending of the rod (1.0294 N), and if the extending tension is higher by a factor of approx. 2.8 than the bending tension, then the component behaves just like a rod upon which expanding forces are acting, and no deflection occurs in the macroscale. For components with a height-length ratio of \(c < 0.01\), the bending load acting along the \(y\) axis constituting approx. 0.07 % of the value of additionally introduced elongating tension acting along the \(x\) axis reduces the value of the arrow of deflection to 0. This is possible if the system would utilise supports transferring longitudinal forces, crosswise forces and bending moments and if the complex tensions present in fibres farthest from the neutral axis are elongating tensions (figure 2).
Table 1. List of preliminary rod deflection calculations.

| Diagram of forces, moments and the deflection | Description of calculation variant |
|---------------------------------------------|-----------------------------------|
| ![Diagram 1.a](image1.png)                  | **Variant 1.a.**
Maximum deflection for the movable support and the fixed support amounts to approx. 15 mm and the maximum tension is approx. 37 (N mm\(^{-2}\)). |
| ![Diagram 1.b](image2.png)                  | **Variant 1.b.**
Maximum deflection for the movable support and the fixed support amounts to approx. 53 mm, and the maximum tension is approx. 155 (N mm\(^{-2}\)). |
| ![Diagram 2.a](image3.png)                  | **Variant 2.a.**
Maximum deflection for two fixed supports amounts to approx. 15 mm, and the maximum tension is approx. 37 (N mm\(^{-2}\)). |
| ![Diagram 2.b](image4.png)                  | **Variant 2.b.**
Maximum deflection for two fixed supports amounts to approx. 53 mm, and the maximum tension is approx. 155 (N mm\(^{-2}\)). |
| ![Diagram 3.a](image5.png)                  | **Variant 3.a.**
Maximum deflection for two immobilising supports amounts to approx. 3 mm, and the maximum tension is approx. 434 (N mm\(^{-2}\)). |
| ![Diagram 3.b](image6.png)                  | **Variant 3.b.**
Maximum deflection for two immobilising supports amounts to approx. 13 mm, and the maximum tension is approx. 434 (N mm\(^{-2}\)). |
Table 2. List of photographs taken during the execution of the experiment, along with descriptions.

| Diagram of forces, moments and the deflection | Description of calculation variant |
|---------------------------------------------|-----------------------------------|
| ![Diagram](image1) | **Variant 1.a.**  
Measured maximum deflection for the movable support and the fixed support amounts to approx. 23 mm (according to the calculations, approx. 15 mm). Difference in results is 8 mm. |
| ![Diagram](image2) | **Variant 1.b.**  
Measured maximum deflection for the movable support and the fixed support amounts to approx. 63 mm (according to the calculations, 53 mm). Difference in results is 10 mm. |
| ![Diagram](image3) | **Variant 2.a.**  
Measured maximum deflection for two fixed supports amounts to approx. 21 mm (according to the calculations, 15 mm). Difference in results is 6 mm. |
| ![Diagram](image4) | **Variant 2.b.**  
Deflection was not measured, as it was so high that the rod slid off the support. Such a solution is prohibited in industrial practice (according to the calculations, 53 mm). |
| ![Diagram](image5) | **Variant 3.a.**  
Measured maximum deflection for two immobilising supports amounts to approx. 0 mm (according to the calculations, 3 mm). Difference in results is 3 mm. |
| ![Diagram](image6) | **Variant 3.b.**  
Measured maximum deflection for two immobilising supports amounts to approx. 0 mm (according to the calculations, 13 mm). Difference in results is 13 mm. |

The described experiment can be used in industrial practice, e.g. during the design of a cutting knife system. An example model of a component generated in a CAD application and calculations performed using MES software Nastran in CAD, are shown in figure 3. The acquired simulation results for the designed component show that the deflection in the y axis amounts to just 0.08 mm. Figure 4 shows a photograph of the knife system in a machine, which was built using the reduction of the arrow of deflection by way of introduction of an additional „artificial tension“.
3. Conclusion
An important result of the conducted research work was the execution of the experiment, the analysis of theoretical questions and the confirmation of test results in industrial practice, that the value of the crosswise deflection (arrow of deflection) is reduced through introduction of an additional „artificial” tension. As the value of the force causing the tensioning of the component in the $x$ axis is increased, its deflection in the $y$ axis is reduced (axis, along which the forces act causing the deflection of the component). When the tensioning forces of the structural component (e. g. bar) are significantly higher than the forces and moments causing the deflection (deflection perpendicular to the tensioning), then the deflection of this component is very limited, and may be omitted in the macro scale (deflection within the tolerance ranges assumed for the structural solution). Further tests in this regard are being continued in order to expand and perfect the mathematical model describing the above phenomenon. Calculation procedures constitute a certain simplification of reality, and hence, for important and responsible structural solutions, the execution of an experiment is suggested in order to verify the calculations and the assumed methodology.

4. References
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