Ferromagnetism in tilted fermionic Mott insulators

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We investigate the magnetism in tilted fermionic Mott insulators. With a small tilt, the fermions are still localized and form a Mott-insulating state, where the localized spins interact via antiferromagnetic exchange coupling. While the localized state is naively expected to be broken with a large tilt, in fact, the fermions are still localized under a large tilt due to the Wannier-Stark localization and it can be regarded as a localized spin system. We find that the sign of the exchange coupling is changed and the ferromagnetic interaction is realized under the large tilt. To show this, we employ the perturbation theory and the real-time numerical simulation with the fermionic Hubbard chain. Our simulation exhibits that it is possible to effectively control the speed and time direction of the dynamics by changing the size of tilt, which may be useful for experimentally measuring the out-of-time ordered correlators. Finally, we address the experimental platforms, such as ultracold atoms in an optical lattice, to observe these phenomena.

I. INTRODUCTION

The effect of a tilted potential in periodic systems has been studied for a long time. This is because the linear potential corresponds to a static electric field and the electric-field effects in solids are an important issue in condensed matter physics from both fundamental and application viewpoints. In this direction, various interesting and significant phenomena, such as the Bloch oscillation \textsuperscript{[1]}, the Zener tunneling \textsuperscript{[2]}, and the Wannier-Stark ladder \textsuperscript{[3]–\textsuperscript{4]}, have been found. While the understanding of these phenomena has been largely advanced, these phenomena in strongly interacting systems have not been fully understood yet and have been an intriguing topic \textsuperscript{[5]–\textsuperscript{12}}. In recent years, a tilted potential is realized in atomic-molecular-optical (AMO) systems such as ultracold atoms and it is used as a convenient tool to induce or control various quantum many-body phenomena. For instance, the bosonic version of the Bloch oscillation \textsuperscript{[13]–\textsuperscript{16}} and the Zener tunneling \textsuperscript{[17]–\textsuperscript{19}} have been studied. Quantum phase transitions induced by a tilt have also been widely investigated \textsuperscript{[20]–\textsuperscript{25}}.

Very recently, a tilted potential has been gathering renewed attention in the context of thermalization problems in quantum systems \textsuperscript{[26]–\textsuperscript{35}}. Remarkably, it has been found that interacting fermions in a tilted lattice show similar behavior to the many-body localization in disordered systems \textsuperscript{[26]–\textsuperscript{27}}. It is called the Stark many-body localization and has been studied extensively. In similar setups, new mechanisms preventing the thermalization, called the Hilbert space fragmentation \textsuperscript{[28]} and Hilbert space shuttering \textsuperscript{[29]}, have been proposed. Quantum many-body scars in a tilted Hubbard model have also been studied \textsuperscript{[30]}. Furthermore, related experiments in ultracold atoms \textsuperscript{[31]–\textsuperscript{33}}, trapped ions \textsuperscript{[34]}, and superconducting qubits \textsuperscript{[35]} have been conducted. The tilted potential system have become an important platform for investigating quantum many-body phenomena.

In this paper, we investigate the magnetism, which is one of the most important quantum many-body phenomena, in tilted fermionic Mott insulators. This is experimentally relevant to both Mott insulator materials under a static electric field and AMO systems with a linear potential. One of the authors has studied the magnetism of fermionic Mott insulators with a small tilt, schematically shown in Fig. 1(a) \textsuperscript{[36]}. In Ref. \textsuperscript{[36]}, it
was demonstrated that the antiferromagnetic coupling is enhanced with a tilt, which was used for controlling various magnetic phases with electric fields. Recently, this idea has been shown to be applicable to more generic setups and useful for controlling other types of magnetic interactions [37, 38]. In this paper, we address a broader parameter range of tilt including a much larger one. With a large tilt, it is naively expected that the Mott-insulating state is broken through the many-body Zener breakdown [5–12]. This is true for the size of the tilt per site at the same order as the on-site interaction. However, with a much larger tilt, the fermions can be localized even under the tilt. This is induced by the Wannier-Stark localization [3 4 20 27], which freezes the charge degree of freedom. Thus, the system is still described as localized spins under a large tilt. Our question is what kind of magnetism emerges in this localized spin system and how the large tilt regime is connected to the small tilt one.

To tackle this issue, we study the one-dimensional Hubbard model with a tilt. One approach is the perturbation theory. We derive the effective spin model for the generic size of tilt and find that the ferromagnetic interaction appears in the large tilt regime. The other approach is to solve the many-body Schrödinger equation numerically for tracking the spin dynamics. The numerical result is consistent with the perturbation theory. The dynamics under a tilt itself is also interesting because we can control the speed and time direction by changing the size of the tilt. We mention the application of this dynamics to the experimental measurement of out-of-time ordered correlators [39, 40]. Finally, we discuss platforms for experimentally observing the signature of the ferromagnetism.

II. MODEL

We study the one-dimensional fermionic Hubbard model with a linear potential [41]. The Hamiltonian is given by

$$H = -t_h \sum_{j=1}^{L-1} \sum_{\sigma=\uparrow, \downarrow} (c_{j+1\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + U \sum_{j=1}^{L} n_{j\uparrow} n_{j\downarrow} + \sum_{j=1}^{L} \sum_{\sigma=\uparrow, \downarrow} E n_{j\sigma},$$

where $c_{j\sigma}$ ($c_{j\sigma}^\dagger$) is the annihilation (creation) operator of a fermion at the $j$-th site with the spin $\sigma(=\uparrow, \downarrow)$ and $n_{j\sigma} = c_{j\sigma}^\dagger c_{j\sigma}$. Here, we choose the open boundary condition, which corresponds to the realistic setup in the AMO systems such as ultracold atoms in an optical lattice [31 33]. $t_h$ and $U$ represent the hopping amplitude and the on-site interaction energy respectively. Throughout this paper, we use $t_h$ ($t_h^{-1}$) as the unit of energy (time). The size of the tilt is denoted by $E$, which is related to the physical electric field $E$ in electronic systems as $E = |e|aE/h$ where $e$ and $a$ are the elementary charge and the lattice constant. To study the properties as localized spin systems, we focus on the half-filled and repulsive ($U > 0$) case through this paper. For the later convenience, we introduce the other gauge choice. With a time-dependent gauge transformation $U(t) = \exp[-iEt\sum_j \sigma j n_{j\sigma}]$, the Hamiltonian is transformed into $\tilde{H}(t) = U^\dagger H_U - iU^\dagger \partial_t U$, which is calculated as

$$\tilde{H}(t) = -t_h \sum_{j=1}^{L-1} \sum_{\sigma=\uparrow, \downarrow} (e^{iEt} c_{j+1\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + U \sum_{j=1}^{L} n_{j\uparrow} n_{j\downarrow}. $$

This gauge choice is called velocity gauge, whereas the one in Eq. (1) is called length gauge [12].

III. LOCALIZATION OF THE CHARGE DEGREE OF FREEDOM

Let us start by seeing the charge dynamics. We numerically solve the many-body Schrödinger equation of the model (2) directly with the fourth-order Runge–Kutta method, which can treat only small sizes but provide the information independent from approximations as long as we adequately choose the time step $\Delta t$ so that the effect of time discretization is negligible. Note that we use the velocity-gauge Hamiltonian (2) instead of the length-gauge Hamiltonian (1) because of the numerical efficiency [43]. We choose the Neél state $|\uparrow\downarrow\uparrow\downarrow\cdots\rangle$ as the initial state and see the time evolution of doublon
number per site,
\[ N_{\text{double}}(t) = \frac{1}{L} \sum_{j=1}^{L} \langle \Psi(t) | n_j \uparrow n_j \downarrow | \Psi(t) \rangle, \]  
(3)

where \(|\Psi(t)\rangle\) is the many-body wavefunction at time \(t\). The field-strength dependence of the doublon dynamics is shown in Fig. 2. For small fields of \(E < U\), the doublon number is small under a tilt because the Mott insulating state is still preserved. At the resonant points \(nE = U\) \((n = 1, 2, \ldots)\), the number becomes very large just after applying the electric field, which breaks the Mott insulators. In contrast, for larger values of \(E > U\), the double occupancy takes smaller values, of the same order of magnitude as in the Mott insulating regime \((E < U)\). This supports the realization of a localized spin system. To study the magnetism in the Mott and Wannier-Stark regimes, we start with the effective spin model based on the perturbation theory. We consider the strong coupling regime \(U > t_h\) and treat the hopping term [the first term in Eq. (1)] as the perturbation. Let us begin with the small tilt case \(|E| < U\). Here, we assume that the value of \(E\) is away from the resonant condition \(U = n|E|\) \((n = 1, 2, 3, \ldots)\), where the double occupancy becomes very large as shown in Fig. 3 (c) and thus the localized states are broken. In contrast, except for the resonant points, the Mott-insulating state survives even under a small tilt. Thus, the system can be described as localized spins [Fig. 3 (a)]. The effective model for the spins is the Heisenberg chain with a field-dependent coupling,

\[ H_{\text{eff}} = \sum_{j=1}^{L} J_{\text{eff}}(E) S_j \cdot S_{j+1}, \]  
(5)

\[ J_{\text{eff}}(E) = \frac{J_0}{1 - (E/U)^2}, \]  
(6)

with \(J_0 = 4t_h^2/U\). Here, \(S_j = (S_j^x, S_j^y, S_j^z)\) denotes a spin operator at the \(j\)-th site. Eq. (6) shows that

\[ \langle \Psi(t) | n_j \uparrow n_j \downarrow | \Psi(t) \rangle = \frac{1}{L} \sum_{j=1}^{L} \langle \Psi(t) | n_j \uparrow n_j \downarrow | \Psi(t) \rangle, \]  
(4)

with \(t_0 = 10\) and \(t_1 = 100\) when we turn on the tilt at \(t = 0\). The averaged doublon numbers for different points of \((E, U)\) are summarized in Fig. 1 (c). This figure shows that we have a well-defined Wannier-Stark localized regime and it has a broad range of parameters. Below, we study the magnetism in the Mott-insulating regime and the Wannier-Stark localized regime where the fermions behave as localized spins.

**IV. EFFECTIVE SPIN HAMILTONIAN**

To study the magnetism in the Mott and Wannier-Stark regimes, we start with the effective spin model based on the perturbation theory. We consider the strong coupling regime \(U > t_h\) and treat the hopping term [the first term in Eq. (1)] as the perturbation. Let us begin with the small tilt case \(|E| < U\). Here, we assume that the value of \(E\) is away from the resonant condition \(U = n|E|\) \((n = 1, 2, 3, \ldots)\), where the double occupancy becomes very large as shown in Fig. 3 (c) and thus the localized states are broken. In contrast, except for the resonant points, the Mott-insulating state survives even under a small tilt. Thus, the system can be described as localized spins [Fig. 3 (a)]. The effective model for the spins is the Heisenberg chain with a field-dependent coupling,

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\[ J_{\text{eff}}(E) = J_+(E) + J_-(E), \]  
(7)

\[ J_{\pm}(E) = \frac{1}{2} \frac{4t_h^2}{U \pm E}. \]  
(8)

The contributions \(J_+\) and \(J_-\) in Eq. (7) correspond to the perturbation process (i) and (ii) shown in Fig. 3 (a) respectively. These two contributions become inequivalent under the field. For simplicity, we focus on \(E > 0\). In this regime, as shown in Fig. 3 (b), the dominant contribution is \(J_-\) and its denominator decreases by \(E\). This means that the energy cost gets smaller due to the tilted potential and the antiparallel spin configuration becomes more favored.

Let us move to the large tilt case \(|E| > U\). The most important point is that the derivation for the small tilt is directly applicable to this case. It is because the derivation is formally just using the second-order perturbation starting from the singly occupied state and its physical origin is irrelevant to the derivation. As shown in Sec. II, the doublon number in the Wannier-Stark regime has the same order of magnitude as in the Mott regime and thus we can apply the perturbation theory. Therefore, the effective spin interaction in the large tilt case is also given by Eq. (6). In the large tilt case, the denominator of Eq. (6) changes sign and the interaction becomes ferromagnetic. For \(E > U\), the dominant contribution is \(J_-\) and the corresponding energy cost \(E - U\) becomes negative. Thus, the antiparallel spin configuration is energetically unfavorable. This is the origin of the ferromagnetism.

We also obtain a consistent effective model using the Floquet theory. Applying the Floquet theory to the
th data for \( E \leq U \) and \( E \geq U \) are shown in the panel (a) and (b) respectively. In the panel (c), we show all the data with a rescaled time defined in Eq. (11). \( t_h (t_h^{-1}) \) is used as the unit of energy (time).

V. REAL-TIME SPIN DYNAMICS

In the previous section, we have derived the effective model under a tilt and pointed out the emergence of ferromagnetism. However, the discussion has been based on the perturbation theory and it is still unclear whether the result is robust beyond the approximation. Also, it is not clear yet how to find the signature of ferromagnetism in the observable quantities. Naively, the magnetization induced by a tilt might be regarded as clear evidence, but it is difficult to observe when we start from the untitled Mott insulator. This is because the time-evolution with Eq. (1) (equivalent Eq. (2)) conserves the total magnetization and the original Mott insulator has zero magnetization in the low-temperature state [52].

To clarify these points, we study the real-time spin dynamics and show how to extract the information of the effective interaction. As in Sec. [11], we solve the many-body Schrödinger equation with the Hamiltonian Eq. (2) using the fourth-order Runge–Kutta method and obtain the time evolution starting from the Neél state \( |\uparrow\downarrow\uparrow\downarrow\cdots\rangle \) [13]. To see the spin dynamics, we study the local spin imbalance

\[
\mathcal{I}_j = \langle \Psi(t)|n_{j\uparrow} - n_{j\downarrow}|\Psi(t)\rangle. \tag{10}
\]

To suppress the boundary effect, we focus on \( j = |L/2| \) in our numerical calculation [53]. In order to see the sharp contrast between \( E = 0 \) and \( E \neq 0 \), we first consider the time evolution without a tilt from \( t = 0 \) to \( t = t_{\text{on}} = 50 \) and then apply the electric field after \( t = t_{\text{on}} \).

To clarify the effect of the exchange interaction on the real-time dynamics, we consider the time-evolution operator with the effective Hamiltonian \( \hat{H}_{\text{eff}}(t) = e^{-i\hat{H}_\text{eff}t} = \exp[-if(E)t\sum_{j=1}^L J_0 \hat{S}_j \cdot \hat{S}_{j+1}] \) where \( J_{\text{eff}}(E) = f(E)J_0 \) and \( f(E) = 1 - (E/U)^2 \). Under the strong coupling condition \( U \gg t_h \), the dynamics is
expected to be governed by this operator \[53\]. The remarkable feature is that the \(E\)-dependence only appears as \(f(E)t\) and \(f(E)\) works as the scale factor in the time direction. This means that the spin dynamics is accelerated in the Mott-insulating regime where \(J_{\text{eff}}\) is enhanced. In contrast, the time evolution is reversed in the Wannier-Stark regime where \(J_{\text{eff}}\) becomes negative. Indeed, these features are seen in the time evolution of the local spin imbalance shown in Fig. 4 (a) and (b). To clearly see whether the dynamics follows the effective Hamiltonian, we show these data with a rescaled time \(t_{\text{rescaled}}\), defined as

\[
t_{\text{rescaled}} = \begin{cases} 
  t & (t < t_{\text{on}}), \\
  t_{\text{on}} + f(E)(t - t_{\text{on}}) & (t > t_{\text{on}}),
\end{cases}
\]

in Fig. 4 (c). As seen in this figure, the data collapse into two curves except for the resonant regime \(U \simeq E\). This means that the spin dynamics is well-described with the effective Hamiltonian \[4\] in both the Mott-insulating and the Wannier-Stark regime. It shows that the exchange coupling under a tilt is given by Eq. (6) and thus ferromagnetism is realized with a large tilt. We emphasize that these results are obtained only from the Hubbard model and do not depend on any specific approximation such as the perturbation theory. This is the most important result in this paper. Note that a similar time-evolution of spins in the Hubbard model under oscillating electric fields was already studied in Ref. \[55\].

An interesting application of this dynamics is the realization of time reversal. Since \(f(E)\) takes \(-1\) at \(E = \sqrt{2}U\), the time evolution is exactly reversed. In Fig. 5 we plot the time evolution when the field \(E = 70.71 \sim \sqrt{2}U\) is turned on at various \(t_{\text{on}} = (50, 75, 100)\) and switched off at \(t_{\text{off}} = t_{\text{on}} + \delta t\) with \(\delta t = 20\). The spin dynamics is obviously reversed for the duration of \(\delta t\) in Fig. 5. Also, all the data almost coincide after \(t = 120\) in Fig. 5. This demonstrates the accuracy of the time-reversal. The time-reversal dynamics is known to be useful for experimentally measuring out-of-time ordered correlators (OTOC), which are used as diagnostics for chaos in quantum systems \[39, 40\]. While there have been many efforts for realizing the time reversal \[40, 57\], it is still not an easy task. Our finding suggests that the time reversal in the strongly correlated Hubbard model is achieved just by adding a tilt. Since the controls of tilt have been already achieved in various AMO systems \[22, 31, 33, 51\], our protocol can be useful for experimentally measuring the OTOC.

VI. DISCUSSION AND SUMMARY

Finally, we discuss the experimental platforms to observe the ferromagnetism in tilted Mott insulators. For this purpose, AMO systems are promising because many-body quantum phenomena induced by a large tilted potential have been already studied experimentally in various AMO systems, such as ultracold atoms \[15, 16, 18, 22, 31, 33, 50, 51\], trapped ions \[34\], and superconducting qubits \[35\]. In particular, ultracold fermionic atoms in an optical lattice \[31, 33\] provide an ideal platform for the fermionic Hubbard model \[1\] and thus they are the most promising platform for our study. The spin-resolved dynamics can be obtained in this setup and thus the imbalance dynamics shown in Fig. 4 will be directly observable. In solid-state electronic systems, it is difficult to realize a large electric field that can achieve the Wannier-Stark regime \[58\]. To avoid this difficulty, the synthetic structured systems, such as semiconductor superlattices \[59, 60\], have been used. The ferromagnetism in the Wannier-Stark regime can be observed in such systems. For this purpose, an array of quantum dots is a good candidate because the fermionic Hubbard model \[51\] and the effective Heisenberg spin chain \[62\] become possible to be simulated in this setup thanks to the recent developments in experimental techniques. A more challenging direction is the observation in bulk solids. Recently, the transient signature of the Wannier-Stark ladder is observed in a bulk semiconductor \[58\], and thus similar pump-probe type measurement in strongly correlated materials may enable us to observe the ferromagnetic signature in Mott insulator materials.

In this paper, we have studied the spin interaction in fermionic Mott insulators with a tilt. Using the perturbation theory and the direct calculation of the real-time evolution, we have revealed the effective interaction in the small and large tilt regime and found the appearance of the ferromagnetism. This appears as the change of the speed and time direction in the real-time dynamics, which can be observed in various experimental platforms.
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[41] We study a one-dimensional system because it is easier to study the real-time dynamics via solving the Schrödinger equation directly. However, the perturbation theory in Sec. [IV] is applicable to any dimensions and thus the ferromagnetic interaction can appear in higher-dimensional systems.
[42] These two gauges are equivalent in the open boundary condition as far as we study the gauge-invariant quantities. A subtle point arises in the periodic boundary condition. The models [1] and [2] with a periodic boundary condition are not connected with the gauge transformation because of the boundary term. However, as discussed in Supplemental Material [S3] our results do not depend too much on the gauge choice even with the periodic boundary condition.
[43] For the further detail of the numerical method, see Supplemental Material [S3].
[44] Note that this protocol is different from the one in Fig. 2 where the tilt is turned on at t = 50.
In principle, the magnetization can be generated if we introduce the dissipation which breaks the symmetry. However, the nature of the Stark MBL is known to be qualitatively changed with a bath [63] and thus we need further study to clarify this point.

While the boundary effect becomes larger near the system edges, the qualitative behaviors do not depend much on the site. For this point, see Supplemental Material [S3].

The dynamics for smaller $U$ are presented in Supplemental Material [S4]. While the dynamics deviates from the one for the effective Hamiltonian with approaching the resonant point $|E| \sim U$, the signature in the spin imbalance survives until around $U \simeq 10$.

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[45] The derivation is presented in Supplemental Material [S1].

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[54] The dynamics for smaller $U$ are presented in Supplemental Material [S4]. While the dynamics deviates from the one for the effective Hamiltonian, approaching the resonant point $|E| \sim U$, the signature in the spin imbalance survives until around $U \simeq 10$.

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Supplemental Material: Ferromagnetism in tilted fermionic Mott insulators

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S1. STATIC PERTURBATION THEORY

We derive the effective model (3) in the main text. This derivation is formally the same as in Ref. [39], where only the small tilt case was discussed. First, we start with a generic Hubbard model where we do not specify the form of the on-site potential and the lattice structure. After deriving the effective model, we will apply the result to our model (1) in the main text. Let us consider the fermionic Hubbard model with an arbitrary on-site potential term,\(^{1}\)

\begin{equation}
H = \sum_{ij} \sum_{\sigma=\uparrow,\downarrow} t_{ij} c_i^{\dagger} c_j + U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_i \sum_{\sigma=\uparrow,\downarrow} V_i n_{i\sigma}, \quad (S1)
\end{equation}

\begin{equation}
H = H_s + H_U + H_V \quad (S2)
\end{equation}

where the indices \(i\) and \(j\) run over all the sites. Here, we consider repulsive interaction \(U > 0\) and the half-filled case.

To derive the effective spin Hamiltonian, we introduce a projection operator \(P_g\) which projects states into the singly-occupied subspace. We also define the complementary projection operator \(P_e \equiv 1 - P_g\) where 1 is the identity operator on the total Hilbert space. Using these operators and the second order perturbation theory, we can write down the effective Hamiltonian up to the second order correction as

\begin{equation}
H_{\text{eff}} = H_{gg} + H_{ge} \frac{1}{E_g - H_{ee}} H_{eg}, \quad (S3)
\end{equation}

where \(H_{\alpha\beta} = P_{\alpha} H P_{\beta}\) \((\alpha, \beta = g, e)\) and \(E_g\) is defined by \(H_{gg} |\Psi_g\rangle = E_g |\Psi_g\rangle\). We apply this formula (S3) to the Hubbard model (S2). This perturbation theory is usually applied to the Mott-insulating state because the singly-occupied states form the ground state subspace and the effective Hamiltonian gives the important information about the low-temperature physics. However, we do not have to specify the physical origin of the singly-occupied state when we apply the perturbation theory. If a state close to the singly-occupied state is realized, this effective Hamiltonian gives a reasonable description, regardless of its origin. This is the reason why we can apply the same effective model to the Wannier-Stark localized regime.

Let us consider \(H_{eg}\). Among the three terms \(H_s, H_U\) and \(H_V\), only the hopping \(H_s\) has a matrix element between the singly occupied states and the other states. Therefore, \(H_{eg}\) is written as

\begin{equation}
H_{eg} = P_e \left( \sum_{ij} \sum_{\sigma=\uparrow,\downarrow} t_{ij} c_i^{\dagger} c_j \right) P_g. \quad (S4)
\end{equation}

Considering the exclusion principle, \(H_{eg}\) survives only when the spin indices \(\sigma\) on the \(i\)-th and \(j\)-th sites are different. Thus, we can rewrite \(H_{eg}\) as

\begin{equation}
H_{eg} = \sum_{ij} \sum_{\sigma=\uparrow,\downarrow} t_{ij} c_i^{\dagger} c_j (S_i^z - S_j^z)^2. \quad (S5)
\end{equation}

Next we compute the energy difference between the singly-occupied state and the intermediate states which are shown in Fig. 3(a) in the main text. Let us focus on the two sites (\(i\)-th site and \(j\)-th site) and a hopping process from the \(j\)-th site to the \(i\)-th site. In the singly occupied states, both sites have a single particle respectively and thus the energy contribution from these sites is \(V_i + V_j\). In contrast, the \(i\)-th site is doubly occupied and the \(j\)-th site is vacant in the intermediate states. Thus, the energy is \(U + 2V_i\). Summing up all the contribution, we obtain

\begin{equation}
\frac{1}{E_g - H_{ee}} H_{eg} = \sum_{ij} \sum_{\sigma=\uparrow,\downarrow} \frac{1}{V_i + V_j} t_{ij} c_i^{\dagger} c_j (S_i^z - S_j^z)^2 = - \sum_{ij} \sum_{\sigma=\uparrow,\downarrow} \frac{1}{U - \Delta V_{ij}} t_{ij} c_i^{\dagger} c_j (S_i^z - S_j^z)^2, \quad (S6)
\end{equation}
with $\Delta V_{ij} = V_i - V_j$.

Finally, we operate the $\mathcal{H}_e$ to Eq. (S6) and then the second-order perturbation term is calculated as follows:

$$\mathcal{H}_e \frac{1}{E_g - \mathcal{H}_{ee}} \mathcal{H}_{eg} = -p_g \sum_{i'j',\sigma'=\uparrow,\downarrow} t_{i'j'} c_{j'\sigma'}^\dagger c_{i'\sigma'} \sum_{ij} \frac{1}{U - \Delta V_{ij}} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} (S_i^z - S_j^z)^2$$

$$= -\sum_{ij} \frac{|t_{ij}|^2}{U - \Delta V_{ij}} c_{j\sigma}^\dagger c_{i\sigma} c_{j\sigma} c_{i\sigma} (S_i^z - S_j^z)^2 - \sum_{ij} \frac{|t_{ij}|^2}{U - \Delta V_{ij}} c_{j\sigma}^\dagger c_{i\sigma} c_{j\sigma} c_{i\sigma} (S_i^z - S_j^z)^2$$

$$= -\sum_{ij} \frac{|t_{ij}|^2}{U - \Delta V_{ij}} (S_i^z - S_j^z)^2 + \sum_{ij} \frac{|t_{ij}|^2}{U - \Delta V_{ij}} (S_i^z S_j^z + S_j^z S_i^z)$$

$$= \sum_{ij} \frac{2|t_{ij}|^2}{U - \Delta V_{ij}} S_i \cdot S_j + \text{const.},$$

(S7)

where we have defined $\tilde{\sigma} = -\sigma$. Since $\mathcal{H}_{gg}$ only gives a constant term, the effective Hamiltonian up to the second-order is

$$H_{\text{eff}} = \sum_{ij} \frac{2|t_{ij}|^2}{U - \Delta V_{ij}} S_i \cdot S_j + \text{const.}$$

$$= \sum_{\langle ij \rangle} \frac{4|t_{ij}|^2}{U - \Delta V_{ij}^2}(\frac{1}{1 - \left(\frac{\Delta V_{ij}}{U}\right)^2}) S_i \cdot S_j + \text{const.},$$

(S8)

where the summation is taken over the every pair $\langle i, j \rangle$ in the last line. When we apply this result to our model in the main text, we reach the effective model in the main text.

### S2. FLOQUET PERTURBATION THEORY

In this section, we derive the effective model in the main text using the Floquet theory.

#### A. Effective Hamiltonian and high-frequency expansion in Floquet theory

We start with the velocity-gauge Hamiltonian (2). Since this model is time-periodic with the period $2\pi/E$, we can apply the Floquet theory, which is a theoretical framework for time-periodic systems [10]. In the Floquet theory, the effective static Hamiltonian in the high-frequency limit plays an important role and the methods to calculate it have been established [46]. Since the frequency is $E$ in the model (2), the high-frequency expansion is expected to work in a large tilt (i.e., Wannier-Stark) regime. Thus, we apply them to the model (2) and derive the effective model for the Wannier-Stark regime.

To introduce the effective Hamiltonian in the Floquet theory, we define the time-evolution operator $U(t, t_0) = \mathcal{T} \exp[-i \int_{t_0}^t dt H(t)]$ and consider its decomposition as $U(t, t_0) = e^{-i K(t)} e^{-i H_{\text{eff}}(t-t_0)} e^{i K(t_0)}$. Here, $H_{\text{eff}}$ is a static operator called an effective Hamiltonian and $K(t)$ is a time-periodic operator called a kick operator. The operators $H_{\text{eff}}$ and $K(t)$ are known to be perturbatively expanded in the power of $1/\omega$ [49]. Using this expansion, the effective Hamiltonian is given as

$$H_{\text{eff}}^{(n_0)} = \sum_{n=0}^{n_0} H_{\text{eff}, n}$$

$$H_{\text{eff}, 0} = H_0$$

$$H_{\text{eff}, 1} = \sum_{m=1}^{\infty} \frac{[H_m, H_{-m}]}{m\omega}$$

$$H_{\text{eff}, 2} = \sum_{m \neq 0} \frac{[H_m, H_0, H_{-m}]}{2m^2\omega^2} + \sum_{m \neq 0, m' \neq 0} \frac{[H_m, H_{m'-m}, H_{-m'}]}{3mm'\omega^2},$$

(S9)(S10)(S11)(S12)
where \( H_n = \frac{1}{T} \int_{-T/2}^{T/2} dt H(t) e^{-i\omega t} \). \( n_0 \) is the truncation order and we consider \( n_0 = 2 \) in this study. For latter convenience, we summarize the Fourier modes of the model (2) as

\[
H = \sum_{n_{\uparrow \downarrow}} n_{\uparrow \downarrow}\| \uparrow \downarrow \rangle.
\]

Using these terms, we calculate the effective Hamiltonian order by order.

**B. Zero-th order**

The zero-th order contribution is given as

\[
H_{\text{eff,0}} = H_0 = U \sum_{i=1}^{L} n_{i\uparrow} n_{i\downarrow}.
\]

**C. First order**

The first order term is \( H_{\text{eff,1}} = \sum_n [H_{+n}, H_{-n}] / (nE) = [H_{+1}, H_{-1}] / E \) and this commutator is computed as

\[
[H_{+1}, H_{-1}] = \frac{t_h^2}{E} \sum_{\sigma=\uparrow, \downarrow} \sum_{i,j=1}^{L-1} [c_{i+1 \sigma}^{\dagger} c_{i\sigma}, c_{j+1 \sigma}^{\dagger} c_{j\sigma}]
\]

\[
= t_h^2 \frac{L-1}{2} \sum_{\sigma=\uparrow, \downarrow} \sum_{i=1}^{L-1} c_{i+1 \sigma}^{\dagger} c_{i\sigma} - t_h^2 \frac{L-1}{2} \sum_{\sigma=\uparrow, \downarrow} \sum_{i=1}^{L-1} c_{i\sigma}^{\dagger} c_{i+1 \sigma}
\]

\[
= t_h^2 \sum_{\sigma=\uparrow, \downarrow} \sum_{i=1}^{L-1} (n_{i\sigma} - n_{i+1 \sigma}).
\]

Therefore,

\[
H_{\text{eff,1}} = \frac{t_h^2}{E} \sum_{\sigma=\uparrow, \downarrow} \sum_{i=1}^{L-1} (n_{i\sigma} - n_{i+1 \sigma})
\]

Note that this term vanishes if we adopt the periodic boundary condition.

**D. Second order**

Since the higher order Fourier components \( n > 1 \) are zero, the second order term is simplified as

\[
H_{\text{eff}}^{(2)} = \frac{[H_{+1}, H_0], H_{-1}}{2E^2} + \frac{[H_{-1}, H_0], H_{+1}}{2E^2}.
\]

We calculate the nested commutators \([H_{+1}, H_0], H_{-1}\) and \([H_{-1}, H_0], H_{+1}\). First, we obtain

\[
[H_{+1}, H_0] = -t_h U \sum_{i=1}^{L-1} \left( c_{i+1 \uparrow}^{\dagger} c_{i \downarrow} - c_{i+1 \downarrow}^{\dagger} c_{i+1 \uparrow} + c_{i \uparrow}^{\dagger} c_{i+1 \downarrow} c_{i+1 \uparrow} c_{i+1 \downarrow} - c_{i+1 \uparrow}^{\dagger} c_{i+1 \downarrow} c_{i \uparrow}^{\dagger} c_{i+1 \downarrow} c_{i+1 \uparrow} c_{i+1 \downarrow} + c_{i+1 \downarrow}^{\dagger} c_{i+1 \uparrow} c_{i+1 \downarrow} c_{i+1 \uparrow} c_{i+1 \downarrow} - c_{i \uparrow}^{\dagger} c_{i+1 \downarrow} c_{i+1 \uparrow} c_{i+1 \downarrow} c_{i+1 \uparrow} c_{i+1 \downarrow} \right),
\]

\[
[H_{-1}, H_0] = -t_h U \sum_{i=1}^{L-1} \left( c_{i \uparrow}^{\dagger} c_{i+1 \downarrow} c_{i+1 \downarrow} c_{i \downarrow} - c_{i \downarrow}^{\dagger} c_{i+1 \uparrow} c_{i+1 \downarrow} c_{i \downarrow} + c_{i \downarrow}^{\dagger} c_{i+1 \uparrow} c_{i+1 \downarrow} c_{i \uparrow} c_{i+1 \uparrow} c_{i+1 \downarrow} - c_{i \uparrow}^{\dagger} c_{i+1 \downarrow} c_{i+1 \uparrow} c_{i+1 \downarrow} c_{i \downarrow} \right).
\]
Then, we calculate the nested ones as

\[
[H_{+1}, H_0], [H_{-1}] = i^2 h \sum_{i=1}^{L-1} \left( c_i \dagger c_i+1 \uparrow + c_{i+1} \dagger c_i \uparrow - c_i \dagger c_{i+1} \uparrow + c_{i+1} \dagger c_i \uparrow + c_i \dagger c_{i+1} \downarrow - c_{i+1} \dagger c_i \downarrow - c_i \dagger c_{i+1} \downarrow + c_{i+1} \dagger c_i \downarrow \right)
\]

(S20)

\#

\[
\sum_{i=1}^{L-1} \left( c_i \dagger c_i+1 \uparrow + c_{i+1} \dagger c_i \uparrow - c_i \dagger c_{i+1} \uparrow + c_{i+1} \dagger c_i \uparrow + c_i \dagger c_{i+1} \downarrow - c_{i+1} \dagger c_i \downarrow - c_i \dagger c_{i+1} \downarrow + c_{i+1} \dagger c_i \downarrow \right)
\]

(S21)

Therefore,

\[
H_{\text{eff,2}} = -\frac{4t^2 U}{E^2} \sum_{i=1}^{L} n_i \downarrow n_i \uparrow + 2t^2 h (n_{i+1} \uparrow n_i \downarrow + n_i \uparrow n_{i+1} \downarrow) + \frac{2t^2 U}{E^2} \sum_{i=1}^{L-1} \left( c_i \dagger c_i+1 \uparrow + c_{i+1} \dagger c_i \uparrow + c_i \dagger c_{i+1} \downarrow + c_{i+1} \dagger c_i \downarrow \right)
\]

(S22)

E. Effective Hamiltonian \(H_{\text{eff}}^{(2)}\)

Using the above results (S14), (S16), and (S22), we obtain the effective Hamiltonian up to the second order as

\[
H_{\text{eff}}^{(2)} = H_{\text{int1}} + H_{\text{int2}} + H_{\text{Hub}} + H_{\text{pair}} + H_b,
\]

(S23)

with

\[
H_{\text{int1}} = \frac{2t^2 U}{E^2} \sum_{i=1}^{L} (n_i \uparrow n_i \downarrow + n_i \uparrow n_{i+1} \downarrow),
\]

(S24)

\[
H_{\text{int2}} = -\frac{2t^2 U}{E^2} \sum_{i=1}^{L-1} \left( c_i \dagger c_i+1 \uparrow c_i \downarrow + c_i \dagger c_{i+1} \uparrow \right),
\]

(S25)

\[
H_{\text{Hub}} = U \left( 1 - \frac{4t^2 h}{E^2} \right) \sum_{i=1}^{L} n_i \uparrow n_i \downarrow,
\]

(S26)

\[
H_{\text{pair}} = -\frac{t^2 h}{E^2} \sum_{i=2}^{L-1} \left( c_i \dagger c_i+1 \uparrow c_i \downarrow + c_i \dagger c_{i+1} \uparrow n_{i \uparrow} n_{i \downarrow} \right),
\]

(S27)

\[
H_b = \frac{t^2 h}{E} \sum_{\sigma = \uparrow, \downarrow} (n_{N \sigma} - n_{1 \sigma})
\]

(S28)

The sum of \(H_{\text{int1}}\) and \(H_{\text{int2}}\) is rewritten with the spin operator \(S_i\). Using the following relation

\[
S_i \cdot S_j - \frac{n_i n_j}{4} = \frac{1}{2} \left( S_i^+ S_j^- + S_i^- S_j^+ \right) + S_i^0 S_j^0 - \frac{n_i n_j}{4}
\]

\[
= \frac{1}{2} \left( c_i \dagger c_i+1 \uparrow c_j \downarrow + c_i \dagger c_{i+1} \uparrow c_j \downarrow - n_i \uparrow n_j \downarrow - n_i \downarrow n_j \uparrow \right),
\]

(S29)
the sum is written as

\[ H_{\text{int1}} + H_{\text{int2}} = \frac{2t^2U}{E^2} \sum_{i=1}^{L-1} \left( n_{i+1\uparrow}n_{i\downarrow} + n_{i+1\downarrow}n_{i\uparrow} - c_{i\uparrow}^\dagger c_{i\downarrow} c_{i+1\downarrow}^\dagger c_{i+1\uparrow}^\dagger - c_{i\downarrow}^\dagger c_{i\uparrow} c_{i+1\uparrow}^\dagger c_{i+1\downarrow}^\dagger \right) \]

\[ = -\frac{4t^2U}{E^2} \sum_{i=1}^{L-1} \left( \mathbf{S}_i \cdot \mathbf{S}_{i+1} - \frac{n_in_j}{4} \right) \equiv H_{\text{FM}}. \]  

(S30)

We arrive at the form of the effective Hamiltonian shown in Eq. [9] in the main text.

S3. DETAIL OF THE NUMERICAL COMPUTATION

We employ the fourth-order Runge-Kutta method for computing the real-time evolution of the wavefunction of the system according to the time-dependent Schrödinger equation

\[ i\frac{\partial}{\partial t}\psi(t) = H(t)\psi(t). \]  

(S31)

For time step \( \Delta t \), the wavefunction at time \( t + \Delta t \) is approximated by defining \( |k_0\rangle = 0 \) and computing

\[ |k_j\rangle = -i\Delta t H(t + \alpha_j \Delta t) (|\psi(t)\rangle - \beta_j \Delta t |k_{j-1}\rangle) \quad (j = 1, 2, 3, 4), \]

\[ |\psi(t + \Delta t)\rangle = |\psi(t)\rangle + \sum_{j=1}^{4} a_j |k_j\rangle; \]  

(S32)  

(S33)

\( (\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (0, 1/2, 1/2, 1), \quad (\beta_1, \beta_2, \beta_3, \beta_4) = (0, 1/2, 1/2, 1), \quad (a_1, a_2, a_3, a_4) = (1/6, 1/3, 1/3, 1/6). \)

We adopt the set of basis diagonal in \( n_{j,\sigma} \) for all \((j, \sigma)\). For the half-filled system with zero net spin polarization, the Hilbert space dimension is \( \left( \frac{L}{L/2} \right)^2 \), while the matrix representation of the tight-binding Hamiltonian has at most \( 4L - 3 \) \((4L + 1)\) non-zero matrix elements for the open (periodic) boundary condition.

A. Gauge choice

For the length gauge Hamiltonian \( \text{(1)} \), the contribution from the site energy term to the diagonal matrix elements is a multiple of \( E \), which can be \( \mathcal{O}(10^4 t_h) \) in our simulation. This is significantly larger compared to the case of the velocity gauge \( \text{(2)} \), with only the on-site interaction, which is at most \( (L/2)U \), contributing to the diagonal matrix elements, while the absolute value of the non-zero off-diagonal matrix elements is always \( t_h \).

For the time evolution by the Runge-Kutta method to be accurate, the time step \( \Delta t \) has to satisfy \(|H\psi|\Delta t \ll 1\) for each normalized input state \( \psi \), where \(|\phi\rangle \equiv \sqrt{\langle \phi | \phi \rangle}\). We have confirmed that with sufficiently small values of \( \Delta t \), the results for the two gauge choices agree completely for the open boundary condition, and do not significantly differ in the periodic boundary condition for \( E, U \gg 1 \). The latter is because when \( E \) is large so that the hopping between any two neighboring sites is suppressed, the site energy difference of \( (L - 1)E \) in the length gauge between sites 1 and \( L \) is even larger, therefore the hopping between these two sites is more strongly suppressed.

B. Boundary effect

Here, we examine the effect of the boundary. In Fig. [S1] we have plotted the local spin imbalance in the case of the periodic boundary condition, while we have studied the open boundary condition in the main text. The settings studied in Fig. [S1] are the same as in Fig. [4] in the main text except for the boundary condition. While the details of the spin dynamics is different, the local spin imbalance overlaps with each other when plotted against the rescaled time both for \( E \ll U \) (unless \( U/E \) is an integer) and \( E \gg U \). In Fig. [S2] we have plotted the site-dependence of local spin imbalance. For the open boundary condition, the site dependence of the dynamics is significant, however they are reversed at the same time when the electric field is switched on at \( t_{\text{on}} = 50 \).
FIG. S1. Time evolution of the local spin imbalance $I_j = (-1)^{j-1}I_j$ [See Eq. (10) in the main text] for $L = 10$ with the periodic boundary condition, before and after the field $E$ is switched on at $t_{on} = 50$. We set $U = 50$ and $\Delta t = 1/3200$, and choose the singly occupied state $|\uparrow\downarrow\uparrow\cdots\uparrow\downarrow\rangle$ as the initial state. We vary $E$ for $0 \leq E \leq 2U$ and the data for $E \leq U$ and $E \geq U$ are shown in the panel (a) and (b) respectively. In the panel (c), we show all the data with a rescaled time defined in Eq. (11). $t_h$ ($t_h^{-1}$) is used as the unit of energy (time).

FIG. S2. Local spin imbalance for $U = 50$ before and after $E = 100$ is switched on at $t_{on} = 50$. We set $\Delta t = 1/3200$ and use the velocity gauge. For the periodic boundary condition (PBC), $I_j = (-1)^{j-1}I_j$, which is $j$-independent with PBC, is plotted. For the open boundary condition (OBC), the local value $\tilde{I}_j = (-1)^{j-1}I_j$ ($j = 1, 2, \ldots, 5$) and the averaged value $\sum_{i=1}^{5} \tilde{I}_j / 5$ are plotted. $t_h$ ($t_h^{-1}$) is used as the unit of energy (time).

S4. SMALLER $U$

Here we discuss the robustness of the effective spin picture presented in the main text down to smaller values of $U$, taking $U = 10$ as an example. In Fig. S3, we plot the local spin imbalance at one of the center sites for $L = 10$, with the singly occupied state as the initial state as in the main text (see Fig. 4) and the electric field $E$ is switched on at $t = 10$. Here we have excluded the cases with $E = 5(= U/2)$ and $E \sim 10(= U)$, for which the doublon occupancy after $t = 10$ becomes significant. The change of the local spin imbalance is more rapid compared to the $U = 50$ case, reflecting the larger value of $J_0(\propto 1/U)$. The plots against $t_{rescaled}$ again overlap with each other, exhibiting dynamics similar to the $E = 0$ case for $E < U$ and its reverse for $E > U$, though with larger discrepancies compared to the $U = 50$ case.

In Fig. S4 we plot the doublon number per site against time, for values of $E$ including $E = 5$ and $E \sim 10$. While the quantity rapidly increases from zero to $\sim 0.03$ at the start of the dynamics reflecting the smaller cost of double occupancy compared to the $U = 50$ case in the main text, the ranges of the values for $E = U/2$ and $E = U$ are comparable to those shown in Fig. 2. For $E \gtrsim 15$, the double occupancy stays below around 0.1.
FIG. S3. Time evolution of the local spin imbalance \( I_{L/2} \) [Eq. (10) in the main text] for \( L = 10 \) before and after the field \( E \) is switched on at \( t_{\text{on}} = 10 \). We set \( U = 10 \) and \( \Delta t = 1/1600 \), and choose the singly occupied state \( |\uparrow\downarrow\uparrow\downarrow\cdots\uparrow\downarrow\rangle \) as the initial state. We vary \( E \) for \( 0 \leq E \leq 2U \) and the data for \( E \leq U \) and \( E \geq U \) are shown in the panel (a) and (b) respectively. In the panel (c), we show all the data with a rescaled time defined in Eq. (11) in the main text. \( t_{\text{h}} \left( t_{\text{h}}^{-1} \right) \) is used as the unit of energy (time).

FIG. S4. Time evolution of the doublon number per site \( \mathcal{N}_{\text{double}}(t) \) [Eq. (3) in the main text], where electric field \( E \) is switched on at \( t = 50 \). The data for \( E \leq (\geq) 10 \) are shown in the left (right) panel. We set \( U = 10, L = 10, \) and \( \Delta t = 1/1600 \), and choose the singly occupied state \( |\uparrow\downarrow\uparrow\downarrow\cdots\uparrow\downarrow\rangle \) as the initial state. \( t_{\text{h}} \left( t_{\text{h}}^{-1} \right) \) is used as the unit of energy (time).