FPFS Shear Estimator: Systematic Tests on the Hyper Suprime-Cam Survey First-year Data

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Abstract

We apply the Fourier Power Function Shapelets (FPFS) shear estimator to the first-year data of the Hyper Suprime-Cam survey to construct a shape catalog. The FPFS shear estimator has been demonstrated to have a multiplicative bias less than 1% in the absence of blending, regardless of complexities of galaxy shapes, smears of point spread functions (PSFs), and contamination from noise. The blending bias is calibrated with realistic image simulations, which include the impact of neighboring objects, using the COSMOS Hubble Space Telescope images. Here we carefully test the influence of PSF model residual on the FPFS shear estimation and the uncertainties in the shear calibration. Internal null tests are conducted to characterize potential systematics in the FPFS shape catalog, and the results are compared with those measured using a catalog where the shapes were estimated using the re-Gaussianization algorithms. Furthermore, we compare various weak-lensing measurements between the FPFS shape catalog and the re-Gaussianization shape catalog and conclude that the weak-lensing measurements between these two shape catalogs are consistent with each other within the statistical uncertainty.

Unified Astronomy Thesaurus concepts: Weak gravitational lensing (1979); Astronomy data analysis (1858); Observational cosmology (1146)

1. Introduction

Weak lensing provides us with a means of observing the total matter distributions in the universe, including invisible dark matter, by measuring coherent shear distortions on background galaxy images caused by inhomogeneous density distributions along the line of sight (e.g., Kilbinger 2015; Mandelbaum 2018).

Weak lensing has wide applications in cosmology. For instance, the cross-correlation between the shear measured from background galaxies and positions of foreground lens galaxies, which is commonly known as galaxy–galaxy lensing, probes the connection between galaxies and underlying matter fluctuations (e.g., Mandelbaum et al. 2006; Han et al. 2015). Combined with galaxy clustering, galaxy–galaxy lensing can be used to constrain cosmology (e.g., More et al. 2015; Prat et al. 2018). The two-point autocorrelation of shear, which is referred to as cosmic shear, directly measures the amplitude and growth of matter fluctuations and hence can be used to constrain cosmology (e.g., Hildebrandt et al. 2017; Troxel et al. 2018; Hamana et al. 2019; Hikage et al. 2019). One can also directly reconstruct projected mass distribution from an observed shear field (e.g., Jeffrey et al. 2018; Oguri et al. 2018). Such weak-lensing mass maps provide an important means of studying the non-Gaussian features of the matter distributions (e.g., Shan et al. 2018; Martinet et al. 2018).

Given its importance, weak lensing is one of the primary science goals of the following three ongoing stage III surveys, the Kilo-Degree Survey\textsuperscript{8} (KiDS; de Jong et al. 2013), the Dark Energy Survey\textsuperscript{9} (DES; Dark Energy Survey Collaboration et al. 2016), and the Subaru Hyper Suprime-Cam survey\textsuperscript{10} (HSC; Aihara et al. 2018). HSC is a wide-field prime focus camera with a 1.85 diameter field of view mounted on the 8.2 m Subaru telescope (Furusawa et al. 2018; Kawanomoto et al. 2018; Komiyama et al. 2018; Miyazaki et al. 2018). HSC survey has a median i-band seeing of 0.16 and an i-band limiting magnitude of $m_i \sim 26$ for its Wide layer. Such good seeing and deep image enable us to use a large number of galaxies up to $z \sim 2$ for the weak-lensing analysis. The Wide layer of the HSC first-year data covers $\sim 170$ deg$^2$, if we restrict ourselves to the full depth and full color regions without considering the bright star masks (see Mandelbaum et al. 2018b).

Accurate shear estimation is a challenging task in weak-lensing surveys. Several traditional methods (e.g., Kaiser et al. 1995, KSB; Hirata & Seljak 2003, re-Gaussianization) have been proposed to recover the shear from a large ensemble of galaxies. However, these traditional methods suffer from noise contamination (noise bias) and irregular galaxy shapes (galaxy model bias). Therefore, realistic image simulations are used to calibrate the noise bias and model bias for these traditional methods. Several newly developed methods (e.g., Sheldon & Huff 2017, METACALIBRATION; Bernstein et al. 2016, BFD; Zhang et al. 2017, FourierQuard; Li et al. 2018, FPFS) can accurately recover shear with good accuracy for isolated galaxies in the absence of blending. However, for deep ground-based surveys, biases from blending effect should be carefully

\textsuperscript{8} http://kids.strw.leidenuniv.nl/index.php

\textsuperscript{9} http://www.darkenergysurvey.org/

\textsuperscript{10} http://hsc.mtk.nao.ac.jp/ssp/
revised. METADETECTION (Sheldon et al. 2019), which is developed based on METACALIBRATION, offers a solution to biases caused by blending without relying on calibration from external simulations.

In this paper, we apply the FPFS shear estimator, which was recently developed in Li et al. (2018), to the i-band images of the HSC first-year data. Li et al. (2018) proposed the FPFS shear estimator for an accurate shear measurement from large ensembles of galaxy images. The FPFS method conducts the shape measurement on the power function of the galaxy image’s Fourier transform. It projects the Fourier power function of galaxies on shapelet basis vectors after deconvolving the point-spread function (PSF) in Fourier space. Four shapelet modes are used to construct the ellipticity and the corresponding shear response. Li et al. (2018) used HSC-like galaxy image simulations from Mandelbaum et al. (2018a) to show that, in the absence of blending, the systematic biases for the FPFS shear estimator are well below 1%, regardless of the complexities of galaxy morphologies, smears of PSFs, and contaminations from noise. However, in the existence of blending, the first generation of the HSC deblender (Bosch et al. 2018) is used to isolate blended sources before shape measurement and a multiplicative bias of $-5.7\%$ on average is found. Such blending bias is modeled and calibrated with the help of HSC-like image simulations (Li et al. 2018).

Combining the HSC first-year data with HSC-like image simulations, in this paper, we characterize several potential systematics in the FPFS shear estimation, which have not been discussed in Li et al. (2018). One of them is the shear bias caused by the PSF model residual. The PSF model residual refers to the difference between the images of the true PSFs and the images of the PSF models reconstructed by the HSC pipeline (Bosch et al. 2018). Unlike traditional methods such as KSB (Kaiser et al. 1995) and re-Gaussianization (Hirata & Seljak 2003) that use moments of PSF images to correct smears of PSF, the FPFS algorithm deconvolves the PSF in Fourier space (see Zhang 2008; Bernstein & Armstrong 2014; Sheldon & Huff 2017 for similar treatments of the PSF correction) and therefore may have a different dependence on the PSF model residual. Even though Mandelbaum et al. (2018b) have shown that the residuals of shapes and sizes on the coadd images of the HSC first-year data meet the HSC first-year science requirement (Mandelbaum et al. 2018b), the influence of the PSF model residual on the process of PSF deconvolution in Fourier space has not been directly quantified. To quantify the systematic error caused by the PSF residual to the FPFS shear estimator, we use the star images in the HSC survey as input PSFs to deconvolve with the simulated galaxy images but use the corresponding PSF models at the positions of the stars for deconvolution in the procedure of the shape measurement (Lu et al. 2017). Another potential systematic is the calibration residual that refers to the bias caused by the difference between the simulated data used to calibrate the shear estimation and the observed data to which the calibrated shear estimator is applied to. In order to quantify the calibration residual, we apply the shear estimator calibrated by the default image simulation to simulations with different galaxy properties. We show that none of these bias exceeds the HSC first-year science requirement.

Several internal null tests suggested by Mandelbaum et al. (2018b) are conducted on the FPFS shape catalog to demonstrate that the systematics on the FPFS shear estimator are below the level of the HSC first-year science requirement (Mandelbaum et al. 2018b). We also compare the null test results of the FPFS shape catalog with those of the re-Gaussianization shape catalog (Mandelbaum et al. 2018b) to check the consistency between these two catalogs.

We proceed with two major applications of the FPFS shape catalog, namely galaxy–galaxy lensing and mass map reconstruction. First, we measure the galaxy–galaxy lensing signal for different lens catalogs using the FPFS shape catalog. To further check the consistency between the FPFS shape catalog and the re-Gaussianization shape catalog, we compare our measurements with those of the re-Gaussianization catalog. Subsequently, we apply our shape catalog to mass map reconstruction using the Kaiser & Squires (1993) method.

This paper is organized as follows. In Section 2, we describe the FPFS shape catalog based on the images of the HSC first year i-band Wide layer. In Section 3, we conduct several external systematic tests to quantify the bias caused by the PSF residual and the calibration residual. In Section 4, we perform internal null tests and compares the results with those of the re-Gaussianization shape catalog. In Section 5, we apply the FPFS catalog to galaxy–galaxy lensing and compares the results with those of the re-Gaussianization shape catalog. In Section 6, we construct mass maps with the FPFS shape catalog. We give our conclusion in Section 7. Throughout the paper, we adopt $\Omega_M = 0.279$, $\Omega_b = 0.046$, $\Omega_{\Lambda} = 0.721$, $b = 0.7$, $n_s = 0.97$, and $\sigma_8 = 0.82$ (Hinshaw et al. 2013).

2. The FPFS Shape Catalog

2.1. Shear Estimator on Isolated Galaxies

First, we focus on the shape measurement of isolated galaxies. The systematic bias caused by blending is modeled and calibrated with HSC-like image simulations (Li et al. 2018), which will be discussed in Section 2.2.

2.1.1. Fourier Power Function

In order to reduce the uncertainty caused by pixel noise, the FPFS algorithm defines the boundary of each galaxy using a circular top-hat aperture around the centroid of the galaxy (Li & Zhang 2016) and sets the pixels outside the boundary to zero. The ratio between the aperture radius and the galaxy’s half-light radius is set to a constant and denoted as $c$. The half-light radius of each galaxy is calculated from the second-order adaptive moment matrix measured by the re-Gaussianization algorithm (Hirata & Seljak 2003), and the centroid is set to the center of the footprint of deblended galaxy. The Fourier power function of the galaxy is calculated as

$$\tilde{f}_g(k) = \int f_g(x_\theta)e^{-ikx_\theta}d^2x_\theta,$$

$$\tilde{F}_g(k) = \lvert \tilde{f}_g(k) \rvert^2.\quad (1)$$

The Fourier power function defined in Equation (1) is contaminated by the Fourier power function of noise. The Fourier power function of noise depends on the correlation function of noise, which is the weighted inverse Fourier transform of the noise Fourier power function (Li et al. 2018).
Although noise on CCD images (single exposures) does not correlate across pixels (Zhang et al. 2015), noise on coadd exposures correlates across pixels because an ad hoc warping kernel is used to convolve CCD images before repixelization on a common coordinate in the coadding procedure. Noise correlations on coadd exposures are mainly determined by shapes of the warping kernels.

For the HSC first-year data, the HSC pipeline uses a third-order Lanczos kernel to warp the CCD images, projects the warped images onto the common coordinates, and combines the projected images to generate coadd exposures (Bosch et al. 2018). The shapes of the projected warping kernels vary across the common coordinate because the projections of the input CCDs vary. Noise correlation functions depend on the projected warping kernels; therefore, they can be different for galaxies at different positions on coadd images.

In this paper, we use principal component analysis (PCA) to capture the variation of noise correlation functions on coadd exposures. For each coadd exposure, we randomly sample 100 positions and use the projected warping kernels at these positions to derive a group of noise correlation functions. PCA is performed on such group of noise correlation functions. The standard deviation of these noise correlations as a function of the rank of the principal components (PCs) is shown in Figure 1. The average correlation function and the first nine PCs for one of the exposures in Figure 2. The upper-left panel shows the average of the noise correlation function on one coadd exposure. The rest of the panels show the first eight PCs of the noise correlation functions on the coadd exposure.

Using these basis vectors of noise Fourier power function, we fit the Fourier power function of each galaxy at large wavenumbers, where the Fourier power function of the PSF decays to $10^{-4}$ of its maximum. Signals at such scale are mainly dominated by the power function of noise because the galaxy signal is filtered by the PSF and reduced to $10^{-4}$ of its original value. We denote the fitted model of the noise Fourier power function for each galaxy as $F_n(k)$ and subtract it from the galaxy Fourier power function,

$$F_k(k) = F_k(k) - F_n(k).$$

The upper panels of Figure 3 show the stacked Fourier power functions of faint galaxies before subtracting the noise Fourier power function. These images are significantly contaminated by the Fourier power function of noise, especially at large wavenumbers. The lower panels of Figure 3 show the stacked Fourier power functions of faint galaxies after subtracting the noise Fourier power function. From the lower panels of Figure 3, we conclude that our algorithm for subtracting the noise Fourier power function works well on average at least at large wavenumbers. We cannot directly see the performance at small wavenumbers because the signals there are dominated by galaxies.

Because the residual of the noise power function mainly influences the shape measurement on faint galaxies, to mitigate the potential bias caused by imperfect noise power subtraction, conservative magnitude and S/N cuts are applied to our galaxy sample (see Section 2.3). Also, a conservative weighting scheme (see Section 2.1.2) is applied to our shape catalog. The next generation of HSC image simulations will include different noise correlations, and we will quantify the performance of our noise power function subtraction with image simulations.

2.1.2. Shapelets

The PSF Fourier power function, denoted as $G(k)$, is subsequently deconvolved from the observed galaxy’s Fourier
Li et al. (2018) proposed to set \( \alpha = 4, \beta = 0.85 \) and showed that the systematic bias for such a setup is below 1% of the shear signal.

Finally, using these shapelet modes, we define the dimensionless FPFS ellipticity and the corresponding shear response as

\[
\begin{align*}
\epsilon_1 &= \frac{M_{22x}}{M_{00} + C}, \\
\epsilon_2 &= \frac{M_{22s}}{M_{00} + C}, \\
R_{1,2} &= \frac{\sqrt{2}}{2} \frac{M_{00} - M_{40}}{M_{00} + C} + \sqrt{2} \epsilon_{1,2}^2.
\end{align*}
\]

\( M_{nm} \) and \( M_{nms} \) are used to denote the real and imaginary parts of \( M_{nm} \) when \( m > 0 \). The constant \( C \) is the weighting parameter which adjusts the weight between galaxies with different luminosities and reduces noise bias (Li et al. 2018). According to Li et al. (2018), we quantify the spread of \( M_{00} \) with \( \Delta \), which is the value of \( M_{00} \) at which its histogram drops below 1/8 of its maximum on the side of higher \( M_{00} \) and normalize \( C \) with \( \Delta \) as \( \nu = C/\Delta \). We conservatively set \( \nu = 4 \) and the weight for galaxies with different S/Ns can be found from the left panel of Figure 7 in Li et al. (2018).

With the definition of average response \( R = (R_1 + R_2)/2 \), the final shear estimator is

\[
\gamma_{1,2} = -\frac{\langle \epsilon_{1,2} \rangle}{\langle R \rangle}.
\]

There exists a minus sign in the final shear estimator because the ellipticity is defined in Fourier space.

2.2. Calibration for Blending Bias

As observations go deeper, the density of detected sources increases. Due to the limited resolution of ground-based telescopes, there are high possibilities that multiple sources are blended within one detected footprint for deep surveys such as HSC (Bosch et al. 2018). If such blending happens, we deblend the footprint and measure the shear from each isolated source separately. Because these blended sources can be located at different redshifts, sources can be distorted by different shear signals even though they are close to each other on the transverse two-dimensional sky plane.

The HSC pipeline (Bosch et al. 2018) uses the SDSS deblender (Lupton et al. 2001) to isolate the sources within one footprint if multiple sources are detected within the footprint. The HSC pipeline replaces all of the neighboring sources with uncorrelated Gaussian noise. Finally, we apply the FPFS shear estimator to the isolated galaxy images.

Li et al. (2018) uses HSC-like galaxy image simulations (Mandelbaum et al. 2018a) to estimate the bias caused by blending. The image simulation is described as follows. Postage stamps from the COSMOS Hubble Space Telescope (HST) survey ACS field (Koekemoer et al. 2007; Leauthaud et al. 2007) are selected according to the positions of galaxies detected from the HSC Wide-depth coadds overlapping with the COSMOS region (Aihara et al. 2018). Subsequently, these image stamps are deconvolved, distorted by a known shear \( \gamma_{1,2} \), convolved with HSC-like PSFs and contaminated with HSC-like noise to mimic HSC images. The simulated HSC galaxies are separated into 800 subfields. Different subfields have different HSC-like PSFs, distorted by different shears, and contaminated with different HSC-like noise (Mandelbaum et al. 2018a). The input postage

\[
\begin{align*}
\text{Before Noise Subtraction} \\
\text{After Noise Subtraction}
\end{align*}
\]
stamps of the HSC-like simulation (Mandelbaum et al. 2018a) keep the information on the COSMOS–HST images without deblending and masking out the neighboring sources. The inclusion of neighboring sources is critical to reproducing the observed distributions of galaxy sizes and magnitudes. However, we note that there are two limitations of the simulation. The first limitation is that the background information beyond the scale of the postage stamp (64″ × 0′′168) is ignored in our simulation. Such a global background could influence the galaxy detection and the shape measurement of faint galaxies. The influence should differ among different background subtraction algorithms.

The second limitation is that even though the HSC-like simulation keeps all of the local information on neighboring sources recorded on the COSMOS–HST exposures, due to the limited depth of the COSMOS–HST survey, galaxies fainter than 25.2 cannot be distinguished from noise. Therefore, the input galaxy sample is not complete at the faint end of magnitude 25.2. Sheldon et al. (2019) finds a selection bias caused by the shear-dependent galaxy detection. The shear-dependent galaxy detection refers to the fact that the possibility of distinguishing two neighboring sources correlates with the shear distortion. Such shear-dependent detection leads to a few percent multiplicative bias. According to Sheldon et al. (2019), it is important to include the information of neighboring sources in the simulation to calibrate the selection bias caused by shear-dependent galaxy detection. The calibration derived from our simulation contains the shear-dependent selection bias from neighboring sources that are fainter than 25.2 may not be fully included in the calibration.

This work applies a 24.5 CModel magnitude cut as suggested by Mandelbaum et al. (2018a) to mitigate the potential biases caused by the limitations of the simulation. As shown in Figure 4, the simulation reproduces the observed histograms of galaxy sizes, magnitude and also the FPFS shapes, etc. These limitations do not cause significant systematic biases on the measurements of these observables. Therefore, we expect that these limitations do not significantly bias the shear calibration. Further discussions on the influences of the global background and shear-correlated detection bias from galaxies fainter than 25.2 are left to the future works.

The deblender, neighboring source replacer, and shear estimator are successively run on the simulated galaxies to measure the shear, and the measured shear is denoted as $g_{M1,2}$. We assume a linear relation between the measured shears and the input shears based on the fact that the amplitudes of the shear signals are only a few percent for weak lensing:

$$\gamma_{1,2} = \gamma_{1,2}^M + c_{1,2},$$  

where $m_{1,2}$ are multiplicative biases for two shear components and $c_{1,2}$ are additive biases for two shear components. We use $m$ to denote the mean of the multiplicative biases on two shear components, where $m = (m_1 + m_2)/2$. The values of $m$ and $c_{1,2}$ depend on galaxy properties, which is modeled with image simulations.

Note that Mandelbaum et al. (2018a) further model the additive bias as $c_{1,2} = a_{1,2} e_{1,2}^P + c_{1,2}'$, considering the correlation
between additive bias and PSF ellipticity, where \( e_{p,2}^f \) are two components of the PSF ellipticity, \( a_{1,2} \) are two components of the fractional additive bias, and \( c_{1,2} \) are two components of the remaining additive bias. As demonstrated in Appendix A, the amplitude of \( a_{1,2} \) is below 0.5\% for the FPFS shape estimator, which is below the HSC first-year science requirement. Therefore, we ignore \( a_{1,2} \) in this paper.

Li et al. (2018) reported that, for HSC-like galaxy image simulations (Mandelbaum et al. 2018a), a multiplicative bias of \(-5.8\% \pm 0.4\%\) on average is found on the aforementioned pipeline and no additive bias on average beyond the statistical uncertainty was found due to the blending effect. There exists two causes of such multiplicative bias. The first is that the SDSS deblender does not accurately recover the true galaxy shapes from blended footprints, and we refer to it as deblending bias. The other is that, as shown in Sheldon et al. (2019), shear-dependent detection in the presence of blending leads to a few percent multiplicative bias at an HSC-like object density. Here we do not separate the deblending bias from the shear-dependent detection bias and leave it to our future work.

The modeling and calibration of blending bias is conducted as follows. Galaxies are first divided into different bins according to the FPFS flux ratio\(^{12}\) (s):

\[
s = \frac{M_{0g}}{M_{0g} + C}.
\]

Next, shear is estimated independently for galaxies in each bin. By fitting the linear relation between the input shear (\( \gamma \)) and the estimated shear (\( \gamma \text{fit} \)) with Equation (9), the average multiplicative bias is measured for each FPFS flux ratio bin. Finally, multiplicative bias on individual galaxies is modeled as a third-order polynomial function of the FPFS flux ratio:

\[
\hat{m} = a_3 s^3 + a_2 s^2 + a_1 s + a_0,
\]

where \( a_i \) (\( i = 0, 1, 2, 3 \)) are determined by fitting the FPFS flux ratio to the average multiplicative bias in each bin (Li et al. 2018). The calibrated shear estimator is

\[
\gamma_{1,2} = -\frac{\langle e_{1,2} \rangle}{\langle (1 + \hat{m}) R \rangle}.
\]

We model the multiplicative bias as a function of FPFS flux ratio and marginalize its dependence over other properties. It is important to note that the multiplicative bias should also depend on properties other than the FPFS flux ratio. If the simulation has the same galaxy distribution as the observed galaxy sample over the marginalized properties, such calibration provides us with an unbiased shear estimation. However, if this condition is not met, the average multiplicative bias in each FPFS flux ratio bin estimated from the simulation could be biased from that of the observed galaxy sample so that the calibration could be biased. We refer to such bias as calibration residual.

To mitigate the calibration residual, it is necessary to ensure that the galaxy distributions over marginalized properties are the same between the simulation and the observation. Figure 4 shows the galaxy distribution over eight different properties of HSC data and in our HSC-like simulation. We find that the distribution are indeed quite similar between simulations and observation. Moreover, Section 3.2 quantifies the amplitude of the potential calibration residual caused by the differences between simulations and observed galaxy ensemble.

\(^{12}\) We termed \( s \) the FPFS flux in Li et al. (2018) but it is incorrect because \( s \) is dimensionless. Therefore, we rename \( s \) as the FPFS flux ratio.
other hand, the re-Gaussianization shape catalog mask out only galaxies with bad pixels near the centroids of galaxies. We add the same $i$-band magnitude cut and $S/N$ cut as in the re-Gaussianization shape catalog. In addition, we add a cut on the FPFS flux ratio, where $s$ is defined in Equation (10). We confirm that the selection bias caused by these selections is well below the HSC first-year science requirement.

Figure 5. The number density maps of the FPFS galaxy shape catalog for six fields of the HSC first year data. The FPFS shape catalog contains more than $1.45 \times 10^7$ galaxies and the average number density is 28.6 deg$^{-2}$.

Figure 6. The density maps of the FPFS response ($R$) for six fields of the HSC first-year data.
### Table 2

| Column name | Explanation |
|-------------|-------------|
| fpfs_c1_2   | FPFS ellipticity |
| fpfs_RA    | Average response |
| fpfs_flux  | FPFS flux ratio |
| fpfs_m     | FPFS multiplicative bias |

#### 2.4. Shape Catalog

The FPFS catalog contains the FPFS shapes as well as all of the columns shown in Table 3 of Mandelbaum et al. (2018b). The columns for FPFS shapes are listed in Table 2. Note that there are two main differences between the FPFS shapes and re-Gaussianization shapes. The first difference is that the FPFS shear estimator does not apply to the lensing weight. As a result, it does not rely on external simulations for the weight bias calibration. The second difference is that the FPFS shear estimator does not need to calibrate additive biases, at least for the accuracy required by the HSC first-year science, because additive biases are quite small on average and do not correlate with the shape of the PSF (Li et al. 2018).

Section 5.2 describes the procedure for using the FPFS shape catalog to measure the average excess surface densities (ESDs) in galaxy–galaxy lensing. Section 6.1 describes the procedure for using the FPFS shape catalog to reconstruct a mass map with the Kaiser–Squires method (Kaiser & Squires 1993). Although in this paper we do not provide cosmic shear measurement using the FPFS shape catalog, the procedure of using the FPFS shape catalog to measure cosmic shear is described in Appendix B.

### 3. External Tests

#### 3.1. The PSF Residual

The HSC pipeline uses a restructured version of PSFEx (Bertin 2011) to perform a polynomial fit of the PSF as a function of position on every CCD. The stars used to feed PSFEx are selected by a k-means clustering algorithm on each CCD (Bosch et al. 2018). PSF models for galaxies on coadd exposures are constructed using all of the corresponding PSF models from the input CCDs at the same sky coordinates (Bosch et al. 2018). We refer to the difference between the true PSF and the PSF model as the PSF residual.

Bosch et al. (2018) and Mandelbaum et al. (2018b) tested the PSF modeling on the single-visit level and coadd level, respectively, by comparing the size and shape between stars and interpolated PSF model at the positions of stars. They checked the probability distribution functions (PDFs) and the spatial correlations of size residual and ellipticity residual for the PSF model and conclude that these residuals meet the HSC first-year science requirement (Mandelbaum et al. 2018b).

It is important to note that the FPFS algorithm does not use moments of PSF models to remove the influence of PSFs on the shear estimator. Instead, it deconvolves PSFs in Fourier space, as done in several other recently developed shear estimators (e.g., Zhang 2008; Bernstein & Armstrong 2014; Sheldon & Huff 2017). In what follows, we conduct the systematic tests suggested by Lu et al. (2017) to directly quantify the influence of the PSF residual on the FPFS shear estimation using star images in the HSC survey and external galaxy image simulations.

The setups of the PSF tests are described as follows. We first select stars with S/N greater than 500 and i-band magnitude less than 22.5. We also require the stars to have the measurement of the re-Gaussianization moments and to not contain bad pixels near the centroid. For each star, a group of noiseless modeled galaxies fitted to the 25.2 magnitude-limited COSMOS HST galaxy sample (Mandelbaum et al. 2012) are subsequently simulated and distorted with input shears using GalSim, which is an open-source software for galaxy image simulation (Rowe et al. 2015). To remove the shape noise, the galaxies are grouped into orthogonal pairs, whose intrinsic orientations is 90° apart from each other. Then, the galaxies are convolved with each star image. We use the PSF model reconstructed by the HSC pipeline at the corresponding position of the star to deconvolve the simulated galaxies and to measure shear with the FPFS shear estimator. Finally, a linear fitting from the input shear to the measured shear with Equation (9) is performed to determine the multiplicative bias \((m_{1,2})\) and additive bias \((c_{1,2})\) for each star selected by the test. Note that we ignore the difference between two components of the multiplicative bias because it is quite small. The average of two components of the multiplicative bias \(m = (m_1 + m_2)/2\) is used for our analysis.

One reason to use bright stars with S/N > 500 is to reduce the shear measurement bias caused by the noise on star images. Because the simulated galaxies are noiseless and isolated, this PSF test focuses on the shear measurement bias caused by the PSF residual. Another reason to use the bright star is to ensure the pureness of the star sample. The extendedness algorithm (Bosch et al. 2018) used to distinguish stars from galaxies performs well on the bright sources, whereas on the faint end, there could be galaxies mixed into the star sample.

The histograms of multiplicative bias and additive bias caused by the PSF model residual for six different fields in the HSC first-year data are shown in Figure 7. The averages and rms of the multiplicative bias and additive bias are laid out in Table 3. We find that the average of the multiplicative bias is below 1.7%, which meets the HSC first-year science requirement.

In addition to the PDFs of the biases, we also check the spatial correlation functions of the biases caused by the PSF residual. The two components of the measured shear correlation function \(\hat{\xi}_{\pm}\) are influenced by the multiplicative bias and additive bias as

\[
\hat{\xi}_{\pm} = \langle \hat{g}_r \hat{g}_x \rangle \pm \langle \hat{g}_x \hat{g}_r \rangle = (1 \pm 2\langle m \rangle + \xi_m)\xi_{\pm} + \xi_c^{\pm},
\]

where \(\hat{g}_r\) is the component along or perpendicular to the separation (tangential component), and \(\hat{g}_x\) is the component at 45° (cross component). \(\xi_{\pm}\) represent two components of the true shear–shear correlation function. The correlations of the multiplicative bias and additive bias \((\xi_m\) and \(\xi_c^{\pm}\)) are defined as

\[
\xi_m = \langle mm \rangle, \quad \xi_c^{\pm} = \langle c_c c_{c \pm} \rangle \pm \langle c_c c_{c \mp} \rangle.
\]

The correlation functions of the multiplicative bias and additive bias for six fields in the HSC first-year survey are
shown in Figure 8. Mandelbaum et al. (2018b) require $|\xi_\perp| < \xi_{\parallel}/25$ to ensure the systematic bias is below the statistical uncertainty, where $\xi_{\parallel}$ is the expected shear–shear correlation function. Such requirement is shown as gray areas in Figure 8. We conclude that the biases introduced by the PSF model residual are within the HSC first-year science requirement.

3.2. The Calibration Residual

As mentioned in Section 2.2, the differences between the simulation and observed data can lead to systematic bias when applying the calibration derived from the simulated data to the observed data, and we refer to such bias as calibration residual. Figure 4 compares the overall number distributions between the simulation and observation over eight observables, demonstrating the differences between the simulation and observation are very small.

Here we quantify the amplitude of the potential calibration residual caused by the differences between the simulation and the observation. The original galaxy simulation, which is used to derive the calibration factor, is divided into 800 subsamples with different input PSFs, noise variances, and galaxy shapes, and this galaxy sample has a magnitude limit of 25.2 (Li et al. 2018). We bootstrap these 800 subsamples to simulate galaxies with different observational conditions and apply the shear estimator calibrated with the original simulation to the bootstrapped simulations with the additional $i$-band selections summarized in Table 1. The remaining biases, including both the residual of the multiplicative bias and the residual of the additive bias, for each bootstrap realization are estimated, and the distributions of the remaining biases are plotted in Figure 9. As demonstrated in Figure 9, the centers of the distributions of the remaining biases are slightly offset from zero. However, the offsets are below 0.5% for multiplicative bias and below $1 \times 10^{-4}$ for additive bias, which are much smaller than the HSC first-year science requirement. Furthermore, the probability distributions of the remaining biases shown in Figure 9 also demonstrate that the remaining biases are within the HSC first-year science requirement.

We note that such remaining biases not only include the calibration residual caused by the differences between the data used for calibration and the observed data but they include also the contribution of measurement error caused by the photon noise on the galaxy images of each subsamples. They also include the selection bias due to the $i$-band selections. Because we have not found any remaining biases exceeding the HSC first-year science requirement, we leave the separation of calibration residual from the contributions of photon noise and selection bias to our future work.

Another possible cause of systematic bias is that additional cuts are used to select galaxy samples in the real observation. As shown in Tables 1 and 4, several selections based on multiband magnitudes and multiband S/Ns are applied to the shape catalog to ensure the accuracy of the photo-$z$ estimation. Moreover, in the galaxy–galaxy lensing measurement performed in Section 5.2, an additional weight as a function of photo-$z$ is added to optimize the excess surface density measurement.

Table 3

| Field      | $\langle m \rangle (10^{-2})$ | $\sqrt{\langle m^2 \rangle} (10^{-2})$ | $\langle c_1 \rangle (10^{-4})$ | $\sqrt{\langle c_1^2 \rangle} (10^{-4})$ | $\langle c_2 \rangle (10^{-4})$ | $\sqrt{\langle c_2^2 \rangle} (10^{-4})$ |
|------------|------------------------------|--------------------------------------|-------------------------------|--------------------------------------|-------------------------------|--------------------------------------|
| XMM        | 0.56                         | 4.78                                 | 0.02                          | 180.65                               | 0.15                          | 187.55                               |
| GAMA09H    | 0.73                         | 3.67                                 | −0.41                         | 171.58                               | 0.84                          | 169.51                               |
| GAMA15H    | 0.51                         | 2.78                                 | −1.66                         | 137.95                               | 0.35                          | 137.90                               |
| HECTOMAP   | 0.76                         | 2.80                                 | −0.72                         | 124.67                               | 0.96                          | 132.93                               |
| WIDE12H    | 0.64                         | 2.61                                 | 0.42                          | 123.79                               | 0.27                          | 133.53                               |
| VVDS       | 0.81                         | 2.91                                 | −2.02                         | 136.52                               | 0.95                          | 132.97                               |

Note. The averages of the multiplicative biases are within the HSC first-year science requirement, which is 1.7%.
To estimate the potential bias caused by such weighting and selection on photo-$z$, we further test the correlation between the calibration residual and the photo-$z$ by dividing our simulation into several COSMOS photo-$z$ (Ilbert et al. 2009) bins and estimating the calibration residual in each bin. As shown in the middle panel of Figure 10, the calibration residuals weakly correlate with the COSMOS photo-$z$ but they are below the first-year HSC requirement. Therefore, we conclude that the bias caused by the additional cuts and selection on photo-$z$ in the galaxy–galaxy lensing measurement should be within the first-year HSC science requirement.

We also check the correlations between the calibration residual and other marginalized observables such as magnitude and seeing. The left panel in Figure 10 shows the calibration residuals for galaxies in different seeing bins, and the right panel in Figure 10 shows the calibration residuals for galaxies in different COSMOS F814W magnitude bins. We report that no potential bias beyond the first-year HSC science requirement is discovered in these sanity tests.

4. Internal Null Tests

4.1. Mock Catalog

In order to check the significance of any nonzero values, it is necessary to accurately estimate the error bar of the shear estimation, which includes both shape noise and cosmic variance. Error is dominated by shape noise on small scales due to the limited galaxy number whereas it is dominated by...
We present a set of null tests (Mandelbaum et al. 2018b) for the FPFS shape catalog and compare the results with the re-Gaussianization shape catalog. Note that all of the null tests adopt calibrations for multiplicative and additive biases. The results for the re-Gaussianization catalog can be found in Mandelbaum et al. (2018b).

Figure 11 shows the tangential and cross-shear components as functions of angular separations measured with the FPFS shape catalog and the re-Gaussianization shape catalog around random points and bright stars. The tests around random points extend to scales of 100′ with the intent to investigate systematic bias caused by incomplete annuli on large scales. The error bars for the random point tests are estimated with the mock catalogs introduced in Section 4.1 as the error bars are dominated by cosmic variance on large scales. The tests around bright stars focus on small scales within 2′ to investigate the possible systematic bias due to sky background misestimation near bright objects. The error bars for the bright star tests are estimated using the mock catalogs with different realizations. The corresponding reduced χ² and p values, the correlations of error between different radius bins are not taken into account. We do not find any significant detection of a nonzero shear signal from these null tests. Moreover, we do not find a significant difference between the results from two shape catalogs.

Figure 12 shows the average of two shear components (namely \( \langle g_{1,2} \rangle \)) as functions of three i-band properties, which are the CModel S/N, CModel magnitude, and re-Gaussianization resolution. The error bars in Figure 12 are calculated with the mock catalogs introduced in Section 4.1. It is important to note that the errors in different property bins are correlated due to the cosmic variance such that the bin-to-bin correlation coefficients range from 0.3 to 0.6. In conclusion, we do not find strong dependence of the average shear values on these galaxy properties. The averaged signal is persistently below zero in most of the bins. Such a negative signal is likely to be caused by cosmic variance.

Figure 13 shows the results of the systematic test of the star–galaxy shape correlations. As suggested by Mandelbaum et al. (2018b), we define a parameter to quantify the performance of

### Table 4

| photo-z cut | Selection of Source Galaxies According to Photo-z |
|-------------|--------------------------------------------------|
| mlz_std_best < 3 | The uncertainty of the photo-z estimation |
| mlz_conf_best > 0.13 | The confidence of the photo-z estimation |
| mlz_best < 2.5 | The best estimation of photo-z |
| \( \int_{D=2}^{\infty} P(z) > 0.95 \) | \( P(z) \) cut defined in Medezinski et al. (2018) |

where \( \gamma_{1,2} \) is the cosmic shear obtained from all-sky weak-lensing maps presented in Takahashi et al. (2017). The cosmological model used for the all-sky simulation is from the best fitting result of the Wilkinson Microwave Anisotropy Probe (WMAP) nine-year data with \( \Omega_M = 0.279, \Omega_b = 0.046, \Omega_{\Lambda} = 0.721, b = 0.7, n_s = 0.97, \) and \( \sigma_8 = 0.82 \) (Hinshaw et al. 2013). For each galaxy, we randomly assign its redshift following the Machine Learning and photo-Z (MLZ) probability distribution function (Tanaka et al. 2018) of the galaxy. The shear values are obtained from the all-sky weak-lensing maps at two adjacent redshift slices by linear interpolation.

From these procedures, we create mock catalogs with different realizations of the shape noise and cosmic shear. The distributions of the mock FPFS ellipticities are very similar to those of the real FPFS ellipticities.
the PSF correction as
\[
\xi_{\text{sys}} = \frac{\langle \gamma_{\text{sys}} \rangle_{\text{gal}}}{\langle \gamma_{\text{sys}} \rangle_{\star}},
\]
(17)
where \(\gamma_{\star}\) refers to the distortion measured from the star image using the re-Gaussianization second-order moments, and \(\gamma_{\text{gal}}\) is the distortion measured from galaxy images. We use the FPFS shape catalog and the re-Gaussianization shape catalog to measure \(\gamma_{\text{gal}}\) and determine \(\xi_{\text{sys}}\) as functions of the separation distance for two shape catalogs, respectively. As demonstrated in Figure 13, while the overall amplitudes of \(\xi_{\text{sys}}\) measured from two shape catalogs are comparable, the systematic error

Figure 11. The tangential (upper panels) and cross (lower panels) shear components measured by the FPFS shape catalog and the re-Gaussianization shape catalog around random points and bright stars. The red points are the results for FPFS whereas the blue points are the results for the re-Gaussianization shape catalog. The left panels show the results on random points. The number density of random points is 50 deg\(^{-2}\). The right panels show the results on bright stars with i-band magnitude \(<22.5\). The reduced chi squares (\(\chi^2\)) and \(p\) values are shown in each panel. Note that the reduced chi squares (\(\chi^2\)) and \(p\) values are calculated without accounting for the correlation between data point at different radius bins.

Figure 12. The average \(\gamma_1\) (dotted line) and \(\gamma_2\) (dashed line) as functions of CModel magnitude (left panel), CModel S/N (middle panel), and CModel Resolution (right panel). The red lines are for the FPFS shape catalog, and the blue lines are for the re-Gaussianization shape catalog.
Equation for the FPFS shape catalog is slightly smaller than that of the re-Gaussianization shape catalog on large scales (>0.5).

5. Galaxy–Galaxy Lensing

5.1. Lens Catalog

Observationally, a few different approaches have been adopted to identify a sample of galaxies with a high fraction of central galaxies in dark matter halos, including a red sequence cluster finder based on photometric surveys (e.g., Oguri 2014; Rykoff et al. 2014), the construction of galaxy group catalogs based on spectroscopic data sets (e.g., Yang et al. 2007), and the selection of isolated galaxies that are isolated central galaxies (ICGs; e.g., Wang & White 2012; Wang et al. 2019). We perform galaxy–galaxy lensing measurements on different lens catalogs, including the HSC CAMIRA cluster catalog (Oguri et al. 2018), SDSS galaxy group catalog (Yang et al. 2007), SDSS ICG catalog (Wang & White 2012), and GAMA (Driver et al. 2011) ICG catalog. For each lens catalog, the galaxy–galaxy lensing analysis is conducted with two source galaxy catalogs, namely the FPFS shape catalog and the re-Gaussianization shape catalog to check the consistency between the results from two source catalogs. The selections of these four lens catalog are summarized below.

The HSC CAMIRA clusters (Oguri et al. 2018) are constructed with the CAMIRA algorithm (Oguri 2014) from the Wide layer of the HSC first-year data (Aihara et al. 2018). The CAMIRA algorithm is based on the stellar population synthesis (SPS) model of Bruzual & Charlot (2003) to predict colors of red-sequence galaxies at a given redshift for an arbitrary set of bandpass filters. The algorithm applies additional calibration using spectroscopic galaxies to improve the accuracy of the SPS model prediction. Using the calibrated SPS model, the CAMIRA algorithm computes the likelihood of each galaxy being in the red sequence as a function of redshift and creates a richness map based on the likelihood. The CAMIRA clusters are detected as the peaks of the richness map, where the center of the cluster is defined as the center of the brightest cluster galaxy (BCG).

The SDSS group catalog (Yang et al. 2007) is constructed with a modified version of the halo-based group finder developed by Yang et al. (2005) from the NYU Value Added Galaxy Catalog (NYU-VAGC; Blanton et al. 2005), which is based on the spectroscopic main galaxy sample from the fourth data release of the Sloan Digital Sky Survey (SDSS/DR7; Abazajian et al. 2009). This method finds potential group centers and its corresponding group members using the Friend-Of-Friend (FOF) algorithm (Davis et al. 1985) and determines a characteristic luminosity for each tentative group. With an assumed mass-to-light ratio, a tentative mass is assigned to each group and used to estimate the size and velocity dispersion of the underlying halo that hosts the group, which in turn is used to determine group memberships. With the updated group memberships, the group centers are also updated. This procedure is repeated until no further changes occur in group memberships.

ICGs are the brightest galaxies within a radius of 1 Mpc in projected separation and within 1000 km s⁻¹ along the line of sight. The parent sample used to select SDSS ICG is the NYU Value Added Galaxy Catalog (NYU-VAGC; Blanton et al. 2005), which is based on the spectroscopic Main galaxy sample from the seventh data release of the Sloan Digital Sky Survey (SDSS/DR7; Abazajian et al. 2009). To avoid mistakenly selecting galaxies which have brighter physical companions but the companions are missing spectroscopic redshifts due to the fiber collision, the photo-z probability distribution catalog (Cunha et al. 2009) is used to compensate the selection. Candidates are further rejected if they have apparent companions projected within 1 Mpc and the photo-z probability distribution of the companion is compatible with the spectroscopic redshift of the candidate (see Wang & White 2012 for details).

Compared with the SDSS Main galaxy sample, the Galaxy And Mass Assembly (GAMA) Survey (Driver et al. 2011) is about two magnitudes deeper. The same selection criterion is applied to identify isolated central galaxies (ICG) from the public GAMA DR3 spectroscopy catalog (Baldry et al. 2018). The selection steps are the same as for the SDSS Main galaxy sample, except that we do not apply any further selection using photo-z probability distribution catalogs, as the effect of fiber collision in GAMA is much less severe.

Additional redshift cuts are applied to these lens catalogs. The redshift distributions of these four lens catalog after redshift cuts are shown in Figure 14.

5.2. Excess Surface Density

From weak-lensing measurements, the average ESDs of the lens samples described in Section 5.1 can be estimated. The ESD for an isotropic lens system at redshift z_l is defined as

$$\Delta \Sigma(R) = \Sigma(\leq R) - \Sigma(R),$$

(18)

where R is the radius to the center of the lens system, \(\Sigma(\leq R)\) is the mean surface mass density inside a certain radius R, and \(\Sigma(R)\) is the surface mass density at the radius R. The surface mass density refers to the line-of-sight projection of the mass density.
The lens system causes tangential shear to the background galaxies. The tangential shear at radius $R$ and redshift $z_s$ is

$$\gamma_T(R, z_l, z_s) = \Delta \Sigma(R) \Sigma_{cr}^{-1}(z_l, z_s),$$

where the critical surface density is defined for a lens–source system as

$$\Sigma_{cr}^{-1}(z_l, z_s) = \begin{cases} \frac{4\pi G}{c^2}(1 + z_l)\frac{\chi_h}{\chi_s} & (z_l \leq z_s) \\ 0 & (z_l > z_s). \end{cases}$$

(20)

$\chi$ denotes the comoving distance, $G$ is the gravitational constant, and $c$ is the speed of light. For simplicity, we denote $\Sigma_{cr}^{-1}(z_l, z_s)$ as $\Sigma_{cr,ls}^{-1}$. We use the PDF of the photometric redshifts ($P(z_i)$), estimated with the MLZ algorithm (Tanaka et al. 2018), to calculate the expectation of $\Sigma_{cr,ls}^{-1}$,

$$\langle \Sigma_{cr,ls}^{-1} \rangle = \int_{z_l}^{+\infty} dz_s \Sigma_{cr}^{-1}(z_l, z_s) P(z_s).$$

(21)

Because the uncertainties on the photo-z measurements of background galaxies do not correlate with the ESDs of lens systems, the relation between the expectation of tangential shear and the expectation of ESD is

$$\langle \gamma_T(R, z_l, z_s) \rangle = \langle \Delta \Sigma(R) \rangle \langle \Sigma_{cr}^{-1}(z_l, z_s) \rangle.$$

(22)

In the galaxy–galaxy lensing analysis, source galaxies are further selected according to their redshift (Medezinski et al. 2018), which we summarized in Table 4.

We do not correct for the effect of photometric redshift bias on the lensing measurements (S. More et al. 2020, in preparation) using the COSMOS 30 band photo-z data as done in Miyatake et al. (2019) and Murata et al. (2019). Because the photometric bias is well below 1% for our selection criteria for lens and source galaxies (Miyatake et al. 2019).

The FPFS shear estimator divides the expectation of tangential ellipticities by the expectation of responses to estimate the tangential shear. The tangential ellipticity is defined as

$$e_T = -e_1 \cos(2\phi) - e_2 \sin(2\phi),$$

(23)

where $\phi$ is the angular position of the source galaxy with respect to the centroid of the lens system in polar coordinates.

By stacking a large ensemble of lenses in the lens catalog, the stacked lens system is approximately isotropic, and the expectation of the stacked ESDs as a function of radius is

$$\langle \Delta \Sigma(R) \rangle = \sum_i \sum_j w_{ls}(\Sigma_{cr,ls}^{-1})^{-1} e_{T,s} \sum_j (1 + m_j) w_{ls} R_i,$$

(24)

where $w_{ls}$ is the weight for each lens–source pair. Conventionally, the weight is set to

$$w_{ls} = (\Sigma_{cr}^{-1}(z_l, z_s))^2$$

(25)

to optimize the estimation of the ESD (Shirasaki & Takada 2018). Substituting Equation (25) into Equation (24), the estimator of ESDs changes to

$$\langle \Delta \Sigma(R) \rangle = \sum_i \sum_j (\Sigma_{cr,ls}^{-1}) e_{T,s} \sum_j (1 + m_j) (\Sigma_{cr,ls}^{-1})^2 R_i.$$

(26)

Note that Equation (26) is unbiased only if $\langle \Sigma_{cr,ls}^{-1}(z_l, z_s) \rangle$ is not correlated with the ellipticities of galaxies. Here we assume that $\langle \Sigma_{cr,ls}^{-1}(z_l, z_s) \rangle$ does not correlate with galaxy shapes without further validation as $\langle \Sigma_{cr,ls}^{-1}(z_l, z_s) \rangle$ is estimated from the photo-z PDF using multiband images, and we do not have multiband image simulations to validate the assumption.

5.3. Results

We compare the results of the ESD estimations between the re-Gaussianization shape catalog and the FPFS shape catalog on the lens catalogs summarized in Section 5.1. The ESDs as functions of the distance to the center of the lenses for two shape catalogs are shown in the left panel of Figure 15. We focus on the radius larger than 0.1 Mpc $h^{-1}$, because the measurement is noisy due to the limited number of background source galaxies on the smaller scale. The error bars for the ESDs are estimated with 100 realizations of galaxy shape catalogs with randomly rotated shapes.

The right panel of Figure 15 shows the ratio between the measurements from two shape catalogs, where the ratio is defined as

$$\text{ratio} = \frac{\langle \Delta \Sigma_{FP} \rangle}{\langle \Delta \Sigma_{RG} \rangle}.$$

(27)
where \( \langle \Delta \Sigma_{\text{FP}} \rangle \) and \( \langle \Delta \Sigma_{\text{RG}} \rangle \) are the ESDs measured with the FPFS shape catalog and the re-Gaussianization shape catalog, respectively. When calculating the error bars of the ratio, it is necessary to account for the correlation between the ESDs measured with two shape catalogs (Miyatake et al. 2019). The correlation is also estimated from the 100 realizations of the galaxy shape catalogs with randomly rotated shapes for each lens catalog. The ratio of the last radius bin for the SDSS-ICG lens sample is biased high because the error is comparable to the denominator in Equation (27). We report that no difference beyond the statistical uncertainty has been found between these two shape catalogs.

6. Mass Map

6.1. Kaiser–Squires Reconstruction

In addition to measuring the ESDs around lens catalogs, we also construct the projected mass distribution from the observed shear map (Kaiser & Squires 1993).

We smooth the shear field with a Gaussian filter as suggested by Oguri et al. (2018). The Gaussian smoothing filter is defined as

\[
W(\theta) = \frac{1}{\pi \sigma^2} \exp \left( -\frac{\theta^2}{\sigma^2} \right),
\]

where the smoothing scale \( \sigma \) is set to 2' in the following analysis. The smoothed two components of shear field are

\[
\gamma_{l2}(\theta) = \frac{\sum_{i} e_{l2}(\theta_i) W(|\theta - \theta_i|)}{\sum_{i} (1 + \hat{m}(\theta_i)) R(\theta_i) W(|\theta - \theta_i|)},
\]

where \( e_{l2}(\theta_i) \), \( \hat{m}(\theta_i) \), and \( R(\theta_i) \) are the two components of ellipticity, multiplicative bias, and average response for the galaxy at position \( \theta_i \), respectively. The smoothed shear fields are pixelized on a regular grid adopting a flat-sky approximation. The pixel size is 0'5.

Subsequently, the shear field is converted to the convergence field via (Kaiser & Squires 1993)

\[
\kappa(\theta) = \frac{1}{\pi} \int d^2\theta' \frac{\gamma_2(\theta'|\theta)}{|\theta - \theta'|^2},
\]

where \( \gamma_2(\theta'|\theta) \) is a tangential shear at position \( \theta' \) computed with respect to the reference position \( \theta \). The shear-to-convergence relationship is a convolution in the two-dimensional angular plane. Such convolution is computed in Fourier space with the Fast Fourier Transform (FFT) to reduce the computational time. Shear fields are padded with zero beyond their boundary to avoid the periodic boundary condition assumed in FFT. The complex shear field is denoted as \( \gamma(\theta) = \gamma_1(\theta) + i\gamma_2(\theta) \). The Fourier transform of the complex shear field and the complex kappa field is denoted as \( \tilde{\gamma}(\ell) \) and \( \tilde{\kappa}(\ell) \), respectively. Equation (30) can be expressed in Fourier space as

\[
\tilde{\kappa}(\ell) = \pi^{-1} \tilde{\gamma}(\ell) \hat{D}^*(\ell) \text{ for } \ell \neq 0,
\]

where \( \hat{D}(\ell) \) is the Fourier transform of the convolution kernel in Equation (30),

\[
\hat{D}(\ell) = \frac{l_1^2 - l_2^2 + 2il_1l_2}{|l|^2}.
\]

The mass map in configuration space \( \langle \kappa(\theta) \rangle \) is then reconstructed by inverse Fourier-transforming \( \tilde{\kappa}(\ell) \). Note that the real part of the reconstructed mass map is referred to as an E-mode mass map, whereas the imaginary part of the reconstructed mass map is referred to as a B-mode mass map which is used to check for certain types of residual systematics in weak-lensing measurements.

A “sigma map” of the convergence field is constructed as follows. We randomly rotate the ellipticity of every individual galaxy to randomize the shape catalog and construct the mass map with the randomized shape catalog. Such procedure is repeated to create 100 random mass maps with different

\[\begin{array}{c|c|c|c|c}
\text{Ratio} & <\text{ratio}> & 0.98 \pm 0.01 & <\text{ratio}> & 0.99 \pm 0.05 \\
\text{SDSS-ICG} & <\text{ratio}> & 0.96 \pm 0.03 & <\text{ratio}> & 0.98 \pm 0.02 \\
\end{array}\]
realizations of the randomized shape catalogs. Then a standard deviation is calculated for each pixel from the 100 random mass maps to construct a “sigma map” which shows the spatial variation for the statistical noise of the reconstructed mass map.

6.2. Results

Figure 16 shows the S/N maps for six fields of the HSC first-year data. S/N of convergence is defined as a convergence divided by a “sigma” of statistical noise. Therefore, the S/N maps are calculated by dividing the mass maps with the “sigma maps.”

Figure 17 shows the probability density function (PDF) for the pixel values of the E (B)-mode convergence S/N maps. The red points are for the E mode, and the blue points are for the B mode. The dashed line is the PDF of a Gaussian distribution.

for the B-mode PDF, we do not find any tail from both the negative and positive ends. We note that the B-mode PDF is slightly deviated from a Gaussian distribution, which could be caused by the finite survey area and the irregular masks (see Oguri et al. 2018).

7. Conclusion

In this paper, we apply the FPFS shear estimator to the first-year data of the HSC survey. The FPFS shear estimator measures shapelet modes from the Fourier power function of galaxy images after deconvolving the PSF in Fourier space and uses four shapelet modes to construct ellipticity and shear response.

We perform both tests with external image simulation and null tests to validate the accuracy of the FPFS shape catalog. The external galaxy image simulation combined with the star/PSF images of the HSC first-year data is used to demonstrate that the biases caused by the PSF model residual are within the HSC first-year science requirement. Moreover, we use the HSC-like image simulation to demonstrate that the biases, which are caused by the difference between the simulated data used for calibration and the observed data to which the calibration is applied, are also much smaller than that in the HSC first-year science requirement. The internal tests are conducted within the HSC data set. The internal test results of the FPFS shape catalog are compared with those of the re-Gaussianization shape catalog. We find that the results of two catalogs are consistent with each other, and there are no remaining systematic biases beyond the HSC first-year science requirement.

The FPFS shape catalog is used to measure the excess surface density of several lens catalogs and the results are compared with those of the re-Gaussianization shape catalog. We find that the results from the two catalogs are consistent with each other within the statistical errors. Also, we apply the
FPFS shape catalog to mass map reconstructions and we find no $B$-mode signals from the reconstructed mass maps.

Even though the assumptions behind these two shear estimators are different, both of these methods use the same image simulation to calibrate shear bias so the systematic uncertainties of these two shape catalogs are not strictly independent. Nevertheless, we note that the procedures of modeling and calibration of systematic biases are also different among these two methods. The re-Gaussianization shape catalog uses the image simulation to determine optimal weight and response as function of $S/N$ and resolution and calibrates noise bias, model bias, weight bias, selection bias and blending bias using the simulation. On the other hand, FPFS only uses the image simulation to calibrate blending bias. Therefore, we expect that cross-comparisons of any scientific results between these two catalogs of the observed data are still valuable.

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### Appendix A

**Correlation between the Additive Bias and PSF Ellipticity**

In Li et al. (2018, Figure 10), we checked the correlation between the additive bias and the PSF ellipticity for isolated galaxies without considering the PSF model residual. We concluded that the amplitude of fractional additive bias is below 0.5%. Here we further check the correlation between the additive bias and the ellipticity of PSF, taking account of blending and PSF residual.

Figure A1 shows the correlation between the additive bias and the PSF ellipticity for sample 4 of the HSC-like Great3 simulation (Mandelbaum et al. 2018a). Sample 4 in Mandelbaum et al. (2018a) is a realistic galaxy sample containing both isolated galaxies and blended galaxies. Each scatter point is the result for one subfield in the simulation. The fractional additive bias is shown in Figure A1, the amplitude of which is below 0.5%.

However, sample 4 of the HSC-like Great3 simulation does not include the PSF model residual. Figure A2 shows the correlation between the additive bias and the PSF ellipticity for the simulation described in Section 3.1. Each point is the result for one star in the star sample selected in Section 3.1. The fractional additive bias is also shown in Figure A2, the amplitude of which is below 0.5%.
Appendix B

Cosmic Shear Measurement

Here we describe the procedure of using the FPFS shape catalog to measure the shear–shear correlation functions. As shown in Equation (13), the two components of the shear–shear correlation function are

\[ \hat{c}_{\perp}(|\theta|) = \langle \hat{\gamma}_{\parallel}(\theta) \hat{\gamma}_{\parallel}(\theta) \rangle \pm \langle \hat{\gamma}_{\times}(\theta) \hat{\gamma}_{\times}(\theta) \rangle, \]

where \( \hat{\gamma}_\parallel \) and \( \hat{\gamma}_\times \) are the tangential component and cross component with reference to the separation, and the expectations are taken over pairs of galaxies with angular separation \(|\theta| = |\theta_1 - \theta_2|\) within an interval \(\Delta|\theta|\) around \(|\theta|\).

For each source pair, two components of galaxy ellipticity are decomposed into tangential \(e_\|\) and cross \(e_\times\) components. Subsequently, we have

\[ \langle \hat{\gamma}_{\parallel}(\theta) \hat{\gamma}_{\parallel}(\theta) \rangle = \sum_{i,j} \frac{e_{\parallel i}(\theta) e_{\parallel j}(\theta)}{[1 + m(\theta)] R(\theta)[1 + m(\theta)] R(\theta)}, \]

\[ \langle \hat{\gamma}_{\times}(\theta) \hat{\gamma}_{\times}(\theta) \rangle = \sum_{i,j} \frac{e_{\times i}(\theta) e_{\times j}(\theta)}{[1 + m(\theta)] R(\theta)[1 + m(\theta)] R(\theta)}. \]

Figure A1. The correlation between the additive bias and the PSF ellipticity for sample 4 (with blending) of the HSC-like Great3 simulation (Mandelbaum et al. 2018a). Each point is the result for one subfield in the simulation. The solid lines in the two panels show the fitting relations between the additive bias and PSF ellipticity. The x-axes are two components of the PSF ellipticity, and the y-axes are two components of the additive bias.

Figure A2. The correlation between the additive bias and the PSF ellipticity for simulation described in Section 3.1. Each point is the result for one star in the star sample selected in Section 3.1. The solid lines in the two panels show the fitting relations between the additive bias and PSF ellipticity. The x-axes are two components of the PSF ellipticity, and the y-axes are two components of the additive bias.
