Planck’s constant measurement by Landauer quantization for student laboratories

Desislav S Damyanov, Iliana N Pavlova, Simona I Ilieva, Vassil N Gourev, Vasil G Yordanov and Todor M Mishonov

Faculty of Physics, St. Clement of Ohrid University at Sofia, 5 J. Bourchier Blvd.,
BG-1164 Sofia, Bulgaria

E-mail: desislav.s.damyanov@gmail.com, ilinikpavlova@gmail.com,
simonailieva24@gmail.com, gourev@phys.uni-sofia.bg, vasil.yordanov@gmail.com
and mishonov@gmail.com

Received 5 February 2015, revised 28 May 2015
Accepted for publication 26 June 2015
Published 11 August 2015

Abstract
A simple experimental setup for measuring Planck’s constant, using Landauer quantization of the conductance between touching gold wires, is described. It consists of two gold wires with thickness of 500 µm and 1.5 cm length, and an operational amplifier. The setup costs less than $30 and can be realized in every teaching laboratory in two weeks. The use of an oscilloscope is required.

Keywords: Landauer quantization of conductivity, Planck’s constant, conductance quantum, measurement

(Some figures may appear in colour only in the online journal)

1. Introduction
The purpose of the present work is to describe an experimental setup for observation of Landauer’s conductance quantization and determination of Planck’s constant [1], which can be easily realized in every teaching laboratory.

The quantization of the conductance was predicted by Rolf Landauer in 1957 [2]. See, for example, Landauer’s [3] and Büttiker’s [4] surveys.

In the last few years the observation of the conductance quantum and its use in determining Planck’s constant has become an ordinary experimental work for students [5–8].

1 Author to whom any correspondence should be addressed.
For further information about the history of measuring Planck’s constant see, for instance, Steiner’s survey [9].

The conductance quantization has an elementary explanation, suitable for high school students, which can be considered as an illustration of Bohr’s model of a hydrogen atom [10].

The experimental setup is available in the Atomic Physics Teaching Laboratory. As described in other works [5, 6], it is up to the students’ imagination to find a way to connect and disconnect the wires. However, the experimental setup is robust and repeatable in the time frame of three hours (the duration of the laboratory class). If break junctions are used [8], the measurement becomes more reliable but the flair of the experimental fineness is lost.

The most reproducible experiment on conductance quantization is conducted in technological structures with no atomic movement. The gain voltage is the only regulator of the quantum channels [11–13].

2. Theoretical model

2.1. Conductance of one-dimensional metal excluding scattering

According to the Landauer consideration [2, 3] a conductor can be assumed as a one-dimensional system of free electrons, schematically presented in figure 1, if length \( L \) is much smaller than the mean free path.

The distribution of the electrons over the energy states in a system with a large number of identical particles is described by Fermi–Dirac’s statistics [14]. Fermi–Dirac’s distribution gives the average number of fermions with momentum \( p \) in a single-particle state [15]

\[
\frac{1}{\exp\left(\frac{\epsilon_p - \mu}{k_BT'}\right) + 1},
\]

where \( k_B \) is Boltzmann’s constant, \( T' \) is the absolute temperature, \( \epsilon_p \) is the energy of the single-particle state, supposedly equal to \( \epsilon_p = \frac{p^2}{2m} \), \( \mu \) is the chemical potential and \( p \) is the electron momentum. At zero temperature the chemical potential is a sum of the Fermi energy and the potential energy per electron [16, 17].

There are two possible values for \( n_p \)

\[
\begin{cases} 
1 & \text{in case of } \epsilon_p < \mu \\
0 & \text{in case of } \epsilon_p > \mu 
\end{cases}
\]

which derive from Pauli’s principle.

Now we can present the current as a flux of electrons using summation

\[
I = q_e \sum_{\alpha, \sigma: \{\epsilon_\sigma > 0\}} \frac{\bar{n}_p \nu_p}{L}, \\
\nu_p = \partial_p \epsilon_p = \frac{p}{m},
\]

where \( q_e, \alpha \) and \( \nu \) respectively are: the charge, spin and velocity of the electron. We suppose that \( q_e \) is known. The magnitude \( \bar{n}_p / L \) is averaged space density of electrons, having momentum \( p \) and \( q_e \bar{n}_p / L \) is the electrical density and represents the average number of particles per unit length. Implicitly in the Landauer consideration is supposed ballistic propagation of free particles in periodic boundary conditions with geometrical period \( L \).
We can transform the summation into integration using the phase integral [19]

\[
\sum_p = \frac{L^D}{(2\pi\hbar)^D} \int_0^{p_F} d^Dp, \quad v = \frac{p}{m} \geq 0,
\]

where \(D\) is the space dimension, \(D = 1\) is our case, \(\hbar\) is the reduced Planck constant, \(L^D\) is the considered volume (in this case it is the length of the conductor) and \(p_F\) is the Fermi momentum. Formally summation can be substituted by integration only in \(L \to \infty\) limit.

Therefore, the current can be written as

\[
I = 2q_e L \int_0^{p_F} \frac{p}{2\pi\hbar} \frac{dp}{L},
\]

where the first multiplier 2 takes into account spin summation. The velocity is presented as \(p/m\) and \(\hbar = 2\pi\hbar/2\).

After integration, the equation becomes

\[
I = \frac{2q_e}{2\pi\hbar} \left( \frac{p^2}{2m} \right) \bigg|_0^{p_F} = \frac{2q_e}{2\pi\hbar} E_F = \frac{2q_e}{2\pi\hbar} q_F U = \frac{2q_e^2 U}{h} = \sigma_0 U,
\]

where \(\sigma_0\) is quantum of conductance [8]

\[
\sigma_0 = \frac{q_e^2}{\pi\hbar} = \frac{q_e^2}{R_H} = 77.5 \mu S = \frac{1}{12.9 \text{ k} \Omega}
\]

where \(R_H = 2\pi\hbar / q_e^2 = 25812.807443(84) \Omega\) [18] is the quantized Hall resistance (von Klitzing constant), related to the metrological definition of the resistance unit \(\Omega\). Planck’s constant is defined assuming that the elementary charge is given. It is worth noting that in equation (7) effective mass of quasiparticles \(m\) is cancelled and this result is applicable for all 1D conductors. The length \(L\) is also irrelevant and conductivity quantization can be seen not only in 1D electron waveguides, but even for point contacts for which \(L = 0\), see [3, 12]. For applicability of Landauer quantization for touching wires it is necessary for the size of the contact (of the order of a few Å) to be much smaller than the mean free path in metals which is of the order of 100 Å. The electrons have to fly through the contact area as free particles. That is why a thin oxide layer in copper can smear the conductivity quantization and it is better to use gold wires.
The conductance quantum is also related to Bohr’s velocity

\[ \frac{1}{4\pi\varepsilon_0 R_{\text{H}}} = \frac{1}{2} \frac{\sigma_0}{4\pi\varepsilon_0} = \frac{v_{\text{Bohr}}}{2\pi}, \]

All formulae are written in SI. In the CGS system \(4\pi\varepsilon_0 = 1\), and the conductance has its natural dimensionality velocity \([m^2/s]\). In the Heaviside–Lorentz system \(\epsilon_0 = 1 = c\).

There is a simpler method to calculate the integral for the current in equation (5) using only summation of arithmetic progression \([10]\), which is suitable for high school students.

2.2. Landauer formalism

A more realistic case in observing the current flowing through a conductor is when scattering is taken into account.

Consider the conductor as a two-dimensional (2D) system. The electrons are confined by an infinite 2D potential in the \(x\)-and \(y\)-directions. In this case we should use quantum mechanics, in particular the Schrödinger equation for a particle in a box. Its solution gives a discrete number of eigenstates, which are also called modes (or channels if conduction is considered). The total energy of the conductor can be obtained as a sum of the lateral mode energy and the energy of the one-dimensional solution in the \(z\)-direction.

Using the Landauer formalism for a current flow through small constrictions, the following can be derived

\[ \sigma = \sigma_0 \sum_{i=1}^{N} T_i, \]

where \(\sigma_0\) is the quantum conductance, \(N\) is the number of conduction channels and \(T_i\) is the transmission coefficient for the \(i\)th channel in the wire \([3, 8]\). The factors \(T_i\) are significant since the conductor is not ideal. They represent the probability that an electron will traverse the constriction, travelling through the \(i\)th channel. The value of \(T_i\) differs from 1 when backscattering becomes important in the transport process. If the length of the break junctions is smaller than the mean free path of an electron in a metal (~100 Å), it can be accepted that \(T_i = 1\) for all channels. The Landauer formula becomes

\[ \sigma = \sigma_0 N \text{ or } \frac{\sigma}{\sigma_0} = N, \]

i.e. the conductance divided by quantum conductance is always equal to an integer \([8]\).

In the next section we describe the experimental realization.

3. Experimental part

3.1. Electric circuit

The experiment is conducted using the electric circuit presented in figure 2, where crossed lines represent two gold wires. This circuit is similar to the one used in \([6]\).

For observing the time dependence of the conductance and its quantization, the current through the conducting wires is measured using a current-to-voltage converter, as shown in figure 2.
When the gold wires form one quantum of conductance, they can be presented as a resistor \( r_1 \). In this case, figure 2 shows the scheme of an inverting amplifier, which measures the input voltage from the voltage divider formed by \( R_1 \) and \( R_2 \). For the values of \( R_1 = 10 \, \Omega \) and \( R_2 = 300 \, \Omega \) the voltage drop on \( R_1 \) is approximately 100 mV. The value of \( R_1 \) is chosen in such a way that the voltage divider is not loaded by the input resistance of the inverting amplifier for the first tens of the quantum resistances \( r_N = 1, \ldots, 10 \). The value of \( R_2 \) is chosen so that the output voltage from the divider \( U_{in} \) is less than the maximum output voltage of the inverting amplifier, divided by its gain \( G_n \) for the first tens of the quantum conductance \( U_{in} = 3 \, V \times R_1/(R_1 + R_2) < 9 \, V/G_n \), \( N = 1, \ldots, 10 \).

The voltage gain of an inverting amplifier is given as

\[
G_N = \frac{U_{out}}{U_{in}} = -\frac{R_f}{r_N}
\]  

The used op-amp is TL071. It is very cheap, widespread and available in Dual In-line Package (DIP). It has bandwidth 3 MHz corresponding to gain \( G = 1 \). In the case of one quantum of conductance the gain is \( G_1 = R_1/r_1 = 3.7 \). Since the product of the bandwidth of an op-amp with its gain is constant, the bandwidth of the used inverting amplifier is \( B = 0.8 \, MHz \). This is much smaller than the bandwidth of the used oscilloscopes [21, 22], which makes them appropriate for measurement of the output voltage \( U_{out} \). The bandwidth of the inverting amplifier is also high enough, so that we can measure the first few quantum steps with length greater than \( 1/B = 1.25 \, \mu s \). This is the reason to choose a relatively small value of \( R_f \) of the order of several tens of k\( \Omega \), so that the gain is also kept small and the bandwidth is high. The aim of the experiment is to determine Planck’s constant using the values of quantized conductance between two touching gold wires. This conductance, and corresponding resistance, are related to Planck’s constant.

Figure 2. Electrical scheme for the measurement of Planck’s constant. The current-to-voltage converter formed by the operational amplifier TL071 and the feedback resistor \( R_f \) has output voltage proportional to the input current \( U_{out} = -R_f I \). The input current that flows through the gold wires is given by the conductivity \( I = \sigma U_{in} \), where \( U_{in} \) is the voltage drop on \( R_1 \) from the voltage divider formed by \( R_1 \) and \( R_2 \). The output voltage is proportional to the conductivity of the gold wires \( U_{out} = -\sigma R_1 U_{in} \) and it is recorded with a digital oscilloscope. An alternative analysis of the circuit is that input voltage \( U_{in} = E R_f/(R_1 + R_2) \) is amplified by an inverting amplifier with the feedback resistor \( R_f \) and input resistor \( 1/\sigma \). The amplification coefficient is \( G(t) = -\sigma R_1 \) and the oscilloscope shows the time-dependent output voltage \( U_{out}(t) = G U_{in} < \sigma(t) \). In such a way we can see the quantized time-dependent conductivity between touching gold wires.
The resistance $r_N$ is obtained by measuring the value of $U_{\text{out}}$, corresponding to the $N\text{th}$ quantum level, and taking into account the values of the input voltage $U_{\text{in}}$ and the feedback resistance $R_f$. Therefore, Planck’s constant is

$$r_N = \frac{1}{\sigma} = \frac{1}{N\sigma_0} = \frac{1}{2\eta^2 N}.$$  

(12)
3.2. Construction of the setup

The setup in figure 3 consists of a mechanical and an electrical part, which are connected with coaxial cables. The electrical and the mechanical parts are shown in figures 4 and 5 respectively.

The mechanical part is constructed in such a way to provide slow movement of the gold wires when they form or destroy an electric contact. The gold wires are soldered at the end of big springs (see figure 5). The wires are made of commercial 24-karat gold for jewelry. They are 15 mm long and have a thickness of 0.5 mm. When the springs oscillate, the gold wires

\[ h = 2 q^2 r_N N = -2 q^2 R f N \frac{U_{in}}{U_{out}}. \]  

(13)

It is expressed by physical constants and experimentally determinable parameters and variables of the setup.
touch and detach. Each spring is 1.5 mm copper wire with 9 cm length. It has 20 turns with a diameter of 2 cm. The springs are soldered on a cooper board with split layers in the middle to form two individual conducting surfaces.

The electric circuit described in section 3.1 is implemented in the blue metal box shown on figure 4. The DC voltage source is a 3 V battery cell consisting of two batteries of 1.5 V each. The operational amplifier TL071 is supplied by two batteries of 9 V. The switch breaks the power supply of the op-amp.

The electrical part is connected with the mechanical part and the oscilloscope with 30 cm long coaxial cables. The BNC connectors on the box match the characteristic impedance of the cables, which is 50 Ω.

The blue metal box of STR8 serves as a Faraday cage to suppress electromagnetic interference from the environment.

3.3. Measurement

In the beginning, the switch must be turned on to power the op-amp. Then, we set the oscilloscope’s time scale, voltage scale and trigger level (for further details see section 5).

After a little push, the spring-shaped conductors start to vibrate and the gold wires come in and out of contact. When the wires connect, electrons pass from one of the wires to the other through N opened quantum channels (see section 2.2). Simultaneous to the detachment of the wires, the number of modes decreases and so does the output voltage. The resulting signal is a sequence of steps, due to the conductance quantization of the wires. The height of each step corresponds to an integer quantum of conductance. The value of $U_{\text{out}}$, corresponding to each step, is measured using the oscilloscope. The input voltage of the inverting amplifier $U_{\text{in}}$ is measured directly with a voltmeter as a voltage drop on the resistor $R_1$. The resistance $R_1$ is measured with an ohmmeter. Then, Planck’s constant is calculated using equation (13) and the measured values of $U_{\text{out}}$, for a chosen $N$th step, $U_{\text{in}}$ and $R_1$.

When performing the experiment, two approaches are possible. The first approach is to separate the gold wires. The second approach is to put them in contact.

In the first approach, if the wires are connected in the beginning of the measurement, the output voltage is around −9 V, because $r_N = 0$ and the amplification of the inverting amplifier is close to its open-loop gain and $U_{\text{in}}$ is amplified up to the supplying voltage. In this case the oscilloscope’s trigger level is set at around 700 mV below it. The conductance quantization is observed during the separation of the wires.

In the second approach, if the wires are initially detached, the output voltage is around 0 V, because the gain of the inverting amplifier is zero and also it is disconnected from the input source. In this case the trigger level is set at approximately 700 mV above the zero level, which corresponds to the output voltage $U_{\text{out}}$, when there are several quanta of conductance. The conductance quantization occurs when the wires touch together.

Both methods are appropriate for measurements and seem to be equally effective.

4. Results

The input voltage is measured to be $U_{\text{in}} = 97.3$ mV and it does not change during the experiment. The measured value of $R_1 = (47.0 \pm 0.1)$ kΩ. The accuracy of of this value is much higher than the accuracy of measurements by oscilloscope, that is why the uncertainty of $R_1$ is irrelevant.

Figures 6 and 7 present photographs of the oscilloscope’s display. They illustrate the op-amp output voltage as a function of time, using two different oscilloscopes. Figure 6 shows
Figure 6. Voltage proportional to the conductivity as function of time. One can easily see a step at which the conductivity is equal to the conductivity quantum $\sigma_0$. Regime of disconnection of the gold wires. A 40 MHz bandwidth oscilloscope is used. The X-axis is set to 200 ns and the Y-axis is at 200 mV. The step with height of 440 mV and length of 200 ns corresponds to a single quantum conductance unit. The width of the step with quantized conductance is around 200 ns.

Figure 7. Three steps of quantized conductivity as a function of time can be seen in the regime of connection of the gold wires. A 50 MHz bandwidth oscilloscope is used. The X-axis is set to 100 $\mu$s and the Y-axis is at 500 mV. The step between the markers with height of 360 mV and length of 60 $\mu$s corresponds to a single quantum conductance unit. The horizontal cursors describes the chosen level of the conductivity. The variation of the curve from the horizontal step of quantized conductivity is more informative than an error bar around an experimental point. Note that the width of the steps 50–100 $\mu$s is significantly bigger than the width of the step depicted in figure 6.
the experimental curves, when measurement is performed in a regime of separation of wires, using a Rigol DS5042M digital oscilloscope. The curves in figure 7 are displayed in regime of connection, using a Rigol DS1052E digital oscilloscope. The number of the open quantized channels is a matter of interpretation. We approximate to the nearest integer number \( N \).

Results from the measurements are presented in table 1.

Planck’s constant evaluated as a mean value and standard deviation of our measurements represented in table 1 is

\[
h = (6.54 \pm 0.45) \times 10^{-34} \text{Js.}
\]  

(14)

The deviation from the CODATA 2010 recommended value \((h = 6.62607123(133) \times 10^{-34} \text{Js})\) [18] is 1.3%.

5. Guidelines for students and laboratory teaching assistants

In order to estimate Planck’s constant using the method and experimental setup presented in this work, students should follow the instructions below.

Turn on the switch on the metal box. The switch breaks the voltage supply of the op-amp. Then, measure the voltage immediately before the gold wires, using a voltmeter. The measured value is the input voltage \( U_{in} \) and is approximately 100 mV.

After that, connect the oscilloscope to the electrical part of the setup, using the BNC. We use Rigol DS5042M [21] and Rigol DS1052E [22], but any digital oscilloscope with sampling rate larger than 10 mega-samples per second can be used. This frequency is determined by the time interval of the quantized conductivity. For our setup those times are of the order of \( \mu s \), and in case of good mechanics with very low eigen frequencies time intervals can be even larger, up to 1 ms.

| Output voltage of channels \( U_{out} \) [mV] | Number of channels \( N \) | Planck’s constant \( h \) [10^{-34} \text{Js}] |
|------------------------------------------|-----------------|-----------------|
| 720                                      | 2               | 6.522           |
| 760                                      | 2               | 6.179           |
| 1800                                     | 5               | 6.522           |
| 360                                      | 1               | 6.522           |
| 440                                      | 1               | 5.336           |
| 360                                      | 1               | 6.522           |
| 320                                      | 1               | 7.337           |
| 360                                      | 1               | 6.522           |
| 320                                      | 1               | 7.337           |
| 720                                      | 2               | 6.522           |
| 700                                      | 2               | 6.708           |
| 360                                      | 1               | 6.522           |
| 360                                      | 1               | 6.522           |
| 360                                      | 1               | 6.522           |
| 360                                      | 1               | 6.522           |

Eur. J. Phys. 36 (2015) 055047 D S Damyanov et al.
Set the time scale of the device. Take into account that the stability of the mechanical part of the setup influences the voltage dependence on time. In case of slow connection and disconnection of the gold wires the X scale should be set to 100 μs/div. Otherwise, the X scale should be 0.2–50 μs/div.

Set the voltage scale. The gain of the inverting amplifier is $G_N = -\frac{1}{r_N} R_I = 3.7 N$ for $N$ conductance quanta. As the resulting voltage is $U_{\text{out}} = -G_N, U_{\text{in}}$, the voltage scale should be set to 200-500 mV/div.

Furthermore, a quick check on the reliability of the setup is recommended. When the gold wires are steadily connected, a horizontal line at around $-9$ V appears on the oscilloscope’s screen. The resulting voltage is approximately equal to the supply voltage of the inverting op-amp. When the wires are separated from one another, the resulting voltage is 0 V and a horizontal line at 0 should be on display.

At the start of each measurement the wires can be connected or divided. As the resulting signal is supposed to be a sequence of steps, a trigger level must be set. The appropriate trigger mode is ‘Edge’. An edge trigger occurs when the trigger input passes through a specified voltage level at the specified slope direction. The trigger sweep should be ‘Single’. This way, whenever a trigger event occurs, the oscilloscope acquires one waveform and stops.

If the wires are initially connected, the triggering is on rising slope. In case they are divided, the triggering is on failing slope. The trigger level should be set at around $U_{\text{trig}} = U_{\text{out}}(N = 2) \approx 700$ mV below or above the initial line for the two approaches respectively. Teaching assistants can find more information on triggering in the oscilloscope’s user manual [21, 22].

When performing the experiment, the springs can be shaped in different ways and various actions can be taken to improve the resulting signal. It is possible to resize one or both of the springs, restrict their movement, for example make one of them still, tilt the cooper plate, put a soft fabric under the plate, make bigger springs with lower vibration frequency and much more. Students are given a chance to come up with and test their own ideas and moderate the springs the way they prefer. For example, students from the same course made an alternative realization, using a long lever with an attached micro-metric screw on its long arm. When they turned the screw, the short arms moved a gold wire that touched or detached another gold wire.

When the desired motion manner is achieved, a little push on the side of the copper plate or on the surface of the experimental table would be sufficient to provide a good signal on the oscilloscope’s screen. In most cases, many attempts are made before the goal is reached. It is also very likely that only one step is clearly observable on the oscilloscope. This means that several graphs should be obtained in order to estimate Planck’s constant accurately. Once the graphs are obtained, the height of each step should be calculated. This is the output voltage $U_{\text{out}}$.

After all the measurements are done, the switch must be turned off in order to disconnect the op-amp from the power supply.

Finally, Planck’s constant is calculated using equation (13). The CODATA 2010 value of the elementary charge is $1.602176565(35) \times 10^{-19}$ C [18]. Also, the electron charge can be measured in teaching labs. For this purpose, a setup, made by students, is available at the Atomic Physics Teaching Laboratory. The estimated value of Planck’s constant should be compared to the CODATA 2010 recommended value.
6. Conclusion

Being ubiquitous in our everyday life, it is hard to believe how handy the contact between metal conductors can be for measuring a fundamental physics constant. The Landauer quantization happens to be a surprisingly good basis for a true experiment, leading to miraculously good results for such a simple experimental setup. The opportunity to observe quantum conductance units as steps in the time dependence of voltage, produced by the fine touch between two wires, is remarkable. It took years to measure Planck’s constant with high accuracy, but for a teaching laboratory with an oscilloscope, $30, couple of weeks and a small group of motivated students are all the resources you need to measure within 2%. Undeniably, this price is accessible for every teaching laboratory with the desire to make noticeable achievements by its simplicity and effectiveness. Since different approaches are possible, a dose of enthusiasm and preciseness may become the fuel for a valuable accomplishment, making it worthwhile for the teaching assistant to participate in accuracy.

In order to pass the exam on quantum physics many students gave different realizations of the mechanical part of the setup. The best solution with the longest steps of the conductivity was given by spring-shaped wires with very low frequency of vibration and small velocity of connection and disconnection. Our concrete conclusion is that this setup gives the simplest method for measurement of Planck’s constant by an electronic phenomenon. The simplicity of the setup will have relevance for the teaching and learning of quantum phenomena and at both the university and high school levels.

7. Author contributions and acknowledgments

This work has partially the fulfilled the requirement for the exam on Thermodynamics and Statistical Physics, lecturer T Mishonov, by the students D Damyanov, I Pavlova, S Ilieva. The first prototype of the setup was made by V Yordanov and T Mishonov. The students, supervised by V Gurev, have realized their setup from scratch and now it is in the Atomic Physics Teaching Laboratory. The text in this paper was completely written by the students. They would like to express their gratitude to T Velchev, N Pavlov, S Damyanov and I Iliev for the support. The authors appreciate many creative suggestions made by colleagues after critical readings of the manuscript.

References

[1] Planck M 1901 Ueber das gesetz der energieverteilung im normalspectrum Ann. Phys. 309 553–63
[2] Landauer R 1957 Spatial variation of currents and fields due to localized scatters in metallic conduction IBM J. Res. Dev. 1 223–31
[3] Landauer R 1989 Conductance determined by transmission: Probes and quantized constriction resistance J. Phys.: Condens. Matter 1 8099–110
[4] Büttiker M 1988 Absence of backscattering in the quantum Hall effect in multiprobe conductors Phys. Rev. B 38 9375–89
[5] Tolley R, Silvidi A, Little C and Eid K F 2013 Conductance quantization: a laboratory experiment in a senior-level nanoscale science and technology course Am. J. Phys. 81 14–19
[6] Foley E L, Candela D, Martini K M and Tuominen M T 1999 An undergraduate laboratory experiment on quantized conductance in nanocontacts Am. J. Phys. 67 389–93
[7] Navrocki W 2008 Electrical and thermal conductance quantization in nanostructures J. Phys.: Conf. Ser. 129 012023
[8] Soukiassian L G Measuring the conductance of gold atomic wires: quantized conductance of a break junction PhD Thesis Purdue University
[9] Steiner R 2013 History and progress on accurate measurements of the Planck constant *Rep. Prog. Phys.* **76** 016101

[10] Mishonov T and Stoev M 2005 Quantization of conductivity of nanotechnological point contact
Simple derivation of the Landauer formula *Sci.-Meth. J. Phys.* **30** 16–22 (in Bulgarian)
Mishonov T and Stoev M 2005 Quantization of conductivity of nanotechnological point contact.
Simple derivation of the Landauer formula arXiv:physics/0507131v2 (in English and Slovak)

[11] Wees B J, van Houten H, Beenaker C W J, Williamson J G, Kouwenhoven L P, van der Marel D and Foxon C T 1988 *Phys. Rev. Lett.* **60** 848

[12] Wees B J, Kouwenhoven L P, Willems E M M, Harmans C J P M, Mooij J E, van Houten H, Williamson J G and Foxon C T 1991 *Phys. Rev. B* **43** 12431

[13] Kouwenhoven L P 1992 Transport of electron-waves and single-charges in semiconductor nanostructures PhD Thesis Technische Universiteit Delft, chapter 1

[14] Fermi E 1926 Sulla quantizzazione del gas perfetto monoatomico *Rend. Lincei* **3** 145–9

[15] Reif F 1965 *Fundamentals of Statistical and Thermal Physics* (New York: McGraw-Hill) p 341

[16] Blakemore J S 2002 *Semiconductor Statistics* (New York: Dover)

[17] Kittel C and Kroemer H 1980 *Thermal Physics* 2nd edn (San Francisco, CA: Freeman) p 357

[18] Mohr P J, Taylor B N and Newell D B 2011 The 2010 CODATA Recommended Values of the Fundamental Physical Constants (Gaithersburg, MD: National Institute of Standards and Technology)
Mohr P J, Taylor B N and Newell D B 2012 CODATA recommended values of the fundamental physical constants: 2010 *Rev. Mod. Phys.* **84** 1527

[19] Laurendeau M 2005 *Statistical Thermodynamics: Fundamentals and Applications* (Cambridge: Cambridge University Press)

[20] Purcell M E 1984 *Electricity and Magnetism* Berkeley Physics Course **2** (New York: McGraw-Hill)

[21] User Manual Rigol DS-5000 Series Digital Storage Oscilloscope 2004 www.madelltech.com/DS5000Manual.pdf

[22] User’s Guide Rigol DS1000E, DS1000D Series Digital Oscilloscopes 2014 www.rigol.com/download/Oversea/DS/User_guide/DS1000E(D)_UserGuide_EN.pdf

[23] Texas Instruments, TL071 Low-Noise JFET-Input Operational Amplifiers 2014 www.ti.com/lit/ds/symlink/tl072.pdf