Internal Transition of Superconducting State by Impurity Doping with a Jump of Isotope-effect Coefficient in Multiband Superconductors

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The impurity effects on the transition temperature $T_c$ and the isotope effect are examined in multiband superconductors with magnetic and nonmagnetic impurities, where the effect of Coulomb repulsion is considered. It is shown that an internal transition of the superconducting state is induced by impurity doping, and that the transition is accompanied by a jump of the isotope-effect coefficient $\alpha$. In particular, the transition is illustrated in a system with two electron bands. In some special cases, extended Abrikosov and Gor’kov (AG) equations for $T_c$ and the expressions of the isotope-effect coefficient $\alpha$ are obtained. Possible relevance of the present mechanism to the experimental results of Sr$_2$RuO$_4$ is discussed.

KEYWORDS: Abrikosov-Gor’kov equation, superconducting transition temperature, isotope effect, multiband superconductor, impurity effect, weakly screened electron-phonon interaction

The isotope-effect coefficient provides important information on the mechanism of superconductivity. For example, the BCS theory based on the electron-phonon interactions is supported in nontransition-metal superconductors, such as Hg and Zn, by their isotope-effect coefficient $\alpha$, which is nearly equal to 0.5. In transition metals and some compounds, large deviations from $\alpha = 0.5$ have been observed, but the deviations can be explained in the context of electron-phonon interactions by taking into account strong Coulomb repulsion, anharmonicity of lattice vibrations, and van Hove singularity.

On the other hand, in exotic superconductors, such as high-$T_c$ cuprates, organic superconductors, Sr$_2$RuO$_4$, and heavy fermion superconductors, nonphonon mechanisms of superconductivity, such as spin-fluctuation exchange interactions, have been examined as possible mechanisms. In such nonphonon mechanisms, the isotope-effect coefficient must deviate from 0.5 markedly, or vanish completely. The presence of the isotope shift ($\alpha \neq 0$) suggests the presence of a contribution to the pairing interaction from electron-phonon interactions.

Therefore, it may be difficult to derive information on the pairing mechanism solely from a single value of $\alpha$. Therefore, it is useful to systematically examine correlations between $\alpha$ and other quantities, such as impurity concentration, transition temperature, hole concentration (for high-$T_c$ cuprates), and pressure.

We examine the impurity effect on the isotope effect in this study. First, we briefly review it in single-band systems. In the presence of impurities, $T_c$ is reduced except for nonmagnetic isotropic impurity scattering and isotropic pairing. The ratio $T_c/T_{c0}$ satisfies the Abrikosov and Gor’kov (AG) equation

$$\ln\left(\frac{T_{c0}}{T_c}\right) = \psi\left(\frac{1}{2} + \frac{\zeta_s}{2\pi T_c}\right) - \psi\left(\frac{1}{2}\right) \equiv \Phi(\zeta_s), \quad (1)$$

where $\zeta_s$ is a scattering rate proportional to $n_{imp}$, and $\psi(x)$ denotes the digamma function. We have introduced a function $\Phi$ for convenience.

If a slight change in the atomic mass does not affect the mechanisms of the pairing interactions and the pair-breaking effect by impurities, it is plausible to assume that the same equation as eq. (1)

$$\ln\left(\frac{T_c(M',0)}{T_c(M',\zeta_s)}\right) = \psi\left(\frac{1}{2} + \frac{\zeta_s}{2\pi T_c(M',\zeta_s)}\right) - \psi\left(\frac{1}{2}\right) \equiv \Phi(\zeta_s) \equiv \frac{1}{\alpha_0} \ln (\frac{T_{c0}}{T_c}), \quad (2)$$

holds in the system with an atomic mass $M'$ slightly different from $M$. From eqs. (1) and (2), we immediately obtain

$$\frac{\alpha}{\alpha_0} = \left(1 - \frac{\zeta_s}{2\pi T_c} \psi'\left(\frac{1}{2}\right) + \frac{\zeta_s}{2\pi T_c}\right)^{-1}, \quad (3)$$

which has been obtained by Carbotte, Greeson and Perez-Gonzalez (CGP) where $\alpha = -\partial \ln T_c/\partial \ln M$ and $\alpha_0 = \alpha(M, 0)$. It should be noted that the relation between $\alpha/\alpha_0$ and $T_c/T_{c0}$, which is obtained by eliminating $\zeta_s$ from eqs. (1) and (3), is universal in the sense that it does not depend on the mechanism of the pairing interactions, properties of impurity scattering, and even the pairing symmetry.

In Sr$_2$RuO$_4$, however, a large deviation from the CGP universal relation has been observed by Mao et al.

When the transition temperature is lowered by defects (as impurities), the coefficient $\alpha$ rapidly increases near $T_c \approx 0.94 T_{c0}$ from the negative value $\alpha \approx -0.1$ near $T_c = T_{c0}$ to the positive value $\alpha \approx 0.2$. Below $T_c \approx 0.94 T_{c0}$, $\alpha$ seems to follow the CGP relation. On the other hand, it has been experimentally suggested that $T_c$ obeys eq. (1). Since the CGP relation is derived directly from eq. (1) as explained above, the large deviation from it appears mysterious.

In this paper, we develop a formulation of the impurity effect on $T_c$ and $\alpha$ in multiband superconductors, and show that an abrupt change in the isotope coefficient $\alpha$ with impurity doping is possible in some conditions, even when $T_c$ practically obeys an AG equation.

The impurity effect in the multiband superconductor has been examined by some authors. Golubov and
Mazin examined this subject and calculated the density of states.\(^6\) Agterberg examined the impurity effect in a two-band case for Sr\(_2\)RuO\(_4\) in the context of the orbital-dependent superconductivity.\(^8,9\) It is known that Sr\(_2\)RuO\(_4\) is a multiband superconductor.\(^10,11\)

The appreciable isotope shifts observed in Sr\(_2\)RuO\(_4\) suggest the presence of the phonon contribution.\(^12\) For simplicity, we ignore any possible additional nonphonon pairing interactions,\(^13\) but the present theory is applicable whether they exist or not.\(^14\) It should be noted that anisotropic pairing can be induced by the phonon interactions.\(^15-17\)

We define the impurity potential which scatters electrons from \(X\)-band to \(X^\prime\)-band by

\[
u_{XX'}(\mathbf{r} - \mathbf{R}_i) + v_{XX'}(\mathbf{r} - \mathbf{R}_i) \mathbf{S}_i \cdot \mathbf{\sigma}(r).
\]

(4)

We introduce symmetry functions \(\gamma_{\alpha}(\mathbf{p}_i)\) with index \(\alpha\) by which the pairing interaction \(V_{XX'}(\mathbf{p}, \mathbf{p'})\) is written in a diagonal form as

\[
V_{XX'}(\mathbf{p}, \mathbf{p'}) = - \sum_{\alpha} \gamma_{\alpha}(\mathbf{p}_i) \gamma_{\alpha}^{(X)}(\mathbf{p}) \gamma_{\alpha}^{(X')}(\mathbf{p}')
\]

(5)

Here, \(p_{\parallel}\) as an argument of \(\gamma_{\alpha}^{(X)}\) denotes the two-dimensional momentum coordinate on the Fermi surface of the \(X\)-band, where band suffixes are omitted from it. For example, in the isotropic case, \(p_{\parallel}\) can be replaced with the polar coordinates (\(\theta, \varphi\)), and we can put \(\alpha = s, p_x, p_y, \cdots\). The symmetry functions \(\gamma_{\alpha}\) are defined so that the orthonormal condition

\[
\int d^2 p_{\parallel} \rho_X(0, p_{\parallel}) \gamma_{\alpha}(\mathbf{p}_i) \gamma_{\alpha}(\mathbf{p}) / N_X(0) = \delta_{\alpha \alpha'}
\]

is satisfied, where \(N_X(0)\) and \(\rho_X(0, p_{\parallel})\) denote the density of states of the \(X\)-band at the Fermi energy and that within the infinitesimal area \(d^2 p_{\parallel}\) near \(p_{\parallel}\), respectively. In particular, we put \(\gamma_{\alpha}(\mathbf{p}_i) = 1\).

The gap function in the \(X\)-band is expanded by the symmetry functions as \(\Delta_X(\mathbf{p}) = \sum_{\alpha} \Delta_{XX} \gamma_{\alpha}(\mathbf{p}_i)\). The impurity potentials are also expanded as

\[
|u_{XX'}(\mathbf{p}, \mathbf{p'})|^2 = \sum_{\alpha} \gamma_{\alpha}(\mathbf{p}_i) \gamma_{\alpha}^{(X)}(\mathbf{p}) \gamma_{\alpha}^{(X')}(\mathbf{p}')
\]

(6)

and the same for \(|v_{XX'}(\mathbf{p}, \mathbf{p'})|^2\), which define \(u_{XX}^{(2\alpha)}\) and \(v_{XX}^{(2\alpha)}\), where the pairing interactions off-diagonal with respect to the suffix \(\alpha\) are ignored for simplicity.

Here, we neglect the momentum dependencies of \(g_{XX}^{(\alpha)}\), \(u\) and \(v\) in the direction perpendicular to the Fermi surface, except that we introduce the cutoff energy \(\omega_c\) (\(\sim \omega_D\)) for the pairing interactions \(g_{XX}^{(\alpha)}\).

For an “\(\alpha\)”-wave superconductor, the gap equation is obtained by the Born approximation\(^6\) as

\[
\Delta_{XX} = 2\pi T \sum_{X'} \sum_{n' = 0}^{n_c} \frac{\lambda_{XX'}^{(\alpha)} \Delta_{XX'}^{n' \alpha}}{\omega_n^{(\alpha)}},
\]

(7)

with

\[
\Delta_{XX} = \Delta_{XX} + \sum_Y M_{XY} \Delta_{Y'\alpha}
\]

(8)

\[
\omega_n^{(\alpha)} = \omega_n + i \text{sgn} (\omega_n) / 2\tau_{1X}.
\]

Here, we have put \(T = T_c\), and defined \(\lambda_{XX'}^{(\alpha)} = g_{XX'}^{(\alpha)} N_X(0), M_{XY} = \Lambda_{XY} / |\omega_n^{(Y)}|, \Lambda_{XY} = 1/2\tau_{2XY}, 1/\tau_{1X} = \sum_Y 1/\tau_{1XY}, \) and

\[
\frac{1}{2\tau_{1X}} = \pi n_{imp} N_y(0) \left[ u_{XY}^{(2\alpha)} + v_{XY}^{(2\alpha)} S(S + 1) \right],
\]

(9)

\[
\frac{1}{2\tau_{2XY}} = \pi n_{imp} N_y(0) \left[ u_{XY}^{(2\alpha)} - c_\alpha v_{XY}^{(2\alpha)} S(S + 1) \right],
\]

where \(c_\alpha = 1\) and \(1/3\) when “\(\alpha\)”-wave pairing is of singlet and triplet pairings, respectively. We introduce matrices \(M, \Lambda, \) and \(\omega\) whose XY elements are \(M_{XY}, \lambda_{XY}^{(\alpha)}, \) and \(\omega_{XY}^{(\alpha)}, \delta_{XY}, \) respectively, and a vector \(\Delta\) whose X element is \(\Delta_X\). Then, we could rewrite eq. (7) as \(\Delta = \Lambda K \Delta\), with \(K = 2\pi T \sum_{n' = 0}^{n_c} \omega^{-1} (1 - M)^{-1}\). Furthermore, if we define

\[
\hat{L} = \hat{\Lambda}^{-1} + \hat{\Phi}
\]

(10)

with \(\hat{\Phi} = L - \hat{K}, \hat{L} = \ln(2\gamma/\omega_c/\tau_{1L})\), and the Euler constant \(\gamma = 0.57721 \cdots\), the gap equation can be rewritten in a compact form

\[
\hat{L} \Delta = L \Delta.
\]

(11)

The matrix \(\hat{\Phi}\) includes all information of the impurity scattering. With the smallest positive eigenvalue \(L\) of the matrix \(L, T_c\) is expressed as

\[
T_c = \frac{2e\gamma}{\pi} \omega_c \exp[-L].
\]

(12)

In particular, when \(\lambda_{XY} \equiv 1/2\tau_{2XY} = 0\) for \(X \neq Y\), the XY element of \(\hat{\Phi}\) becomes \(\Phi_{XX} = \Phi(\zeta_X) \delta_{XY}\) with

\[
\zeta_X' \equiv \frac{1}{2\tau_{1X}} - \frac{1}{2\tau_{2XX}}.
\]

(13)

In the absence of the interband Cooper pair hopping \(g_{XX} = \sigma_{XX} \delta_{XY}\), we obtain \(\ln(T_{cXX} / T_c) = \Phi(\zeta_X'')\) with \(T_{cXX} = (2e^2/\pi) \omega_c \exp[-1/\lambda_{XX}],\) which are the same as the AG equation except that \(\zeta''\) includes \(1/2\tau_{1XX}\) with \(X \neq Y\).

In the general case, i.e., when \(\lambda_{XY} \equiv 1/2\tau_{2XY}\) is not necessarily equal to 0 for \(X \neq Y\), it is difficult to write the equation in a simple form such as that in the above. Therefore, we examine systems with two bands \(A\) and \(B\) to illustrate the mechanism of the transition. The eigenvalues of \(\hat{L}\) are obtained as

\[
L = \frac{1}{2} \left[ \text{tr}(\hat{L}) \pm \sqrt{\left(\text{tr}(\hat{L})\right)^2 - 4\text{det}(\hat{L})} \right].
\]

(14)

The smaller positive one gives \(T_c\) using eq. (12). The explicit forms of the matrix elements of \(\hat{\Phi}\) are obtained by carrying out the summation over \(n'\) as

\[
\Phi_{XX} = \frac{1}{2} \left( 1 + \frac{\zeta''_A B}{\zeta''_A B} \right) \Phi(\zeta_X) + \frac{1}{2} \left( 1 - \frac{\zeta''_A B}{\zeta''_A B} \right) \Phi(\zeta_X),
\]

(15)

\[
\Phi_{XX} = \frac{\Lambda_{XX}}{\zeta''_A B} (\Phi(\zeta_B) - \Phi(\zeta_A)),
\]

with \(\bar{A} \equiv B, \bar{B} \equiv A, \zeta''_A = \zeta_A - \zeta_B, \) and \(\zeta''_B = \zeta_B - \zeta_A\). If \(\zeta_A = \zeta_B\) and \(\zeta''_A = \zeta''_B\), \(\Phi_{XX} = 0\). Therefore, the right-hand side of eq. (14) can be easily evaluated from eq. (10).
Now, we examine the Coulomb effects on the superconductivity. In the single-band system, it is known that the Coulomb interaction $U$ can be taken into account by replacing $\lambda$ with $\lambda = \lambda - \mu^*$, where $\mu^* = U N(0)$ and $U = U/(1 + U N(0)\ln(W/\omega_c))$. We have introduced an effective cutoff energy $W$ of the Coulomb interaction, which is of the order of the bandwidth. Since $\mu^*$ includes $\omega_c$, it affects the isotope effect.

In the multiband system, it is found that the effective Coulomb interaction is obtained by

$$\tilde{U}^{(\alpha)}_{XX'} = U^{(\alpha)}_{XX'} - \sum_{X''} U^{(\alpha)}_{XX''} L^X_{\alpha} \tilde{U}^{(\alpha)}_{XX'} X'^{XX'},$$

with $L^X_{\alpha} = N_X(0)\ln(W/\omega_c)$, when the Coulomb interaction is expanded in a similar manner to that in eqs. (5) and (6). We have assumed that $U^{(\alpha)}_{XX'}$ and $\tilde{U}^{(\alpha)}_{XX'}$ are constants within the ranges $|\xi_p|, |\xi_p'| < W$ and $\omega_c$, respectively, while outside these ranges, $U^{(\alpha)}_{XX'} = 0$ and $\tilde{U}^{(\alpha)}_{XX'} = 0$, where $\xi_p$ denotes the single-particle energy measured from the Fermi energy.

In two-band systems, we obtain

$$\tilde{U}^{(\alpha)}_{XX} = \frac{1}{D} \left[ (1 + L^X_{\alpha} U^{(\alpha)}_{XX}) U^{(\alpha)}_{XX} - L^X_{\alpha} U^{(\alpha)}_{XX} U^{(\alpha)}_{XX} \right]$$

(17)

$$\tilde{U}^{(\alpha)}_{XX} = \frac{1}{D} \left[ (1 + L^X_{\alpha} U^{(\alpha)}_{XX}) U^{(\alpha)}_{XX} - L^X_{\alpha} U^{(\alpha)}_{XX} U^{(\alpha)}_{XX} \right],$$

with $D = (1 + L^X_{\alpha} U^{(\alpha)}_{AB})(1 + L^X_{\alpha} U^{(\alpha)}_{BB}) - L^X_{\alpha} U^{(\alpha)}_{AB} L^X_{\alpha} U^{(\alpha)}_{BB}$. The coupling constants $\lambda^{(\alpha)}_{XX}$ should be replaced with $\tilde{\lambda}^{(\alpha)}_{XX} = \lambda^{(\alpha)}_{XX} - \mu^{(\alpha)}_{XX'}$, where $\mu^{(\alpha)}_{XX'} \equiv U^{(\alpha)}_{XX'} N_{XX'}(0)$. The matrix $\tilde{L}$ is redefined by $\tilde{L} \equiv (\lambda - \mu^*)^{-1} + \tilde{\Phi}$ with a matrix $\tilde{\mu}^*$ whose $XY$ element is $\mu^{(\alpha)}_{XX}$.

The transition temperatures and the isotope-effect coefficients can be explicitly calculated from the set of equations obtained above. However, for simplicity and in order to clarify a physical picture, we concentrate ourselves on a symmetric case in which two bands are equal, namely, $N_A(0) = N_B(0) \equiv N(0)$, $f_{AA} = f_{BB}$ and $f_{AB} = f_{BA}$, where $f = g^{(\alpha)}, u^{(2\alpha)}, v^{(2\alpha)}$ and $U^{(\alpha)}$. In this case, the gap functions $\Delta^{(\pm)}$ with $\Delta_{AA} = \Delta_{BB}$ are the solutions of eq. (11). The eigenvalues are

$$L^{(\pm)} = \frac{1}{\lambda^{(\pm)}} + \Phi^{(\pm)},$$

(18)

where $\tilde{\lambda}^{(\pm)} \equiv \tilde{\lambda}_{AA} \pm \tilde{\lambda}_{AB} = \lambda^{(\pm)} - \mu^{(\pm)}$ with $\lambda^{(\pm)} \equiv \lambda_{AA} \pm \lambda_{AB}$ and $\mu^{(\pm)} \equiv \mu^{(\pm)}_{AA} \pm \mu^{(\pm)}_{AB}$, and $\tilde{\zeta}^{(\pm)} \equiv \tilde{\zeta}_{AA} \pm \tilde{\zeta}_{AB}$ with $\tilde{\zeta}^{(\pm)} \equiv \tilde{\zeta}_{BB}$. From eq. (17), we have

$$\mu^{(\pm)} = \frac{(U^{(\alpha)}_{AA} - U^{(\alpha)}_{AB}) N(0)}{1 + U^{(\alpha)}_{AA} + U^{(\alpha)}_{AB}) N(0) \ln(W/\omega_c)}.$$

(19)

Therefore, we finally obtain

$$\ln \frac{T^{(\pm)}_{c0}}{T_c} = \Phi^{(\pm)} = \psi \left( \frac{1}{2} + \frac{\zeta^{(\pm)}}{2\pi T_c} \right) - \psi \left( \frac{1}{2} \right)$$

(20)

with $T^{(\pm)}_{c0} = 4e^2/\pi \omega_c \exp[-1/\tilde{\lambda}^{(\pm)}]$ and

$$\zeta^{(\pm)} = \left( \frac{1}{2\tau_{1AA}} - \frac{1}{2\tau_{2AA}} \right) + \left( \frac{1}{2\tau_{1AB}} - \frac{1}{2\tau_{2AB}} \right).$$

(21)

From eq. (20), the isotope-effect coefficients for $T^{(\pm)}_{c0}$ are obtained as

$$\frac{\alpha^{(\pm)}}{\alpha_0} = \left[ 1 - \frac{\zeta^{(\pm)}}{2\pi T^{(\pm)}_{c0}} \psi \left( \frac{1}{2} + \frac{\zeta^{(\pm)}}{2\pi T^{(\pm)}_{c0}} \right) \right]^{-1},$$

(22)

with $\alpha_0 = \frac{1}{2} \left[ 1 - (\mu^*/\lambda^{(\pm)})^2 \right]$. In general, $\alpha^{(\pm)}$ are largely different due to the difference in $\mu^{(\pm)}$. The physical origin of these differences is that the Coulomb energy $U^{(\alpha)}_{AA}$ is enhanced or reduced by $U^{(\alpha)}_{AB}$ depending on the signs $\pm$, as explicitly shown in eq. (19).

For the s-wave pairing and the s-wave impurity scattering, $\zeta^{(\pm)} = 0$, while $\zeta^{(\pm)} = 1/2\tau_{1AB}$. Thus, $T^{(\pm)}_{c}$ of the state with $\Delta_{AA} = \Delta_{BB}$ is not suppressed by impurity doping at all, while that of the state with $\Delta_{AA} = -\Delta_{BB}$ is strongly suppressed, which is consistent with the results by Golubov and Mazin and Arseev et al. When $T^{(\pm)}_{c} < T^{(-)}_{c}$, the alternation of the solution and thus the jump of $\alpha$ occurs.

For anisotropic pairing and isotropic impurity scattering, since $1/2\tau_{2XY} = 0$, we have $\zeta^{(\pm)} = \zeta^{(-)}$. Hence, the alternation of the solution does not occur.

Even for anisotropic pairing, it is possible that $\Delta_{AB} = \Delta_{BB}$.
$1/2t_{2AB} \neq 0$, when $u^{(2a)}_{AB} \neq 0$ or $v^{(2a)}_{AB} \neq 0$. It is found in eq. (21) that $\lambda_{AB}$ $\Delta B_{AB} < 0$, one of $T_c(\pm)$ with lower $T_{c0}$ decreases more gradually with impurity doping than the other. In this case, the gap function may alternate from $\Delta_{\alpha} = \pm \Delta_B a_0$ to $\Delta_{\alpha} = \mp \Delta_B a_0$ at some impurity concentration. In the transition from $\Delta_{\alpha} = \pm \Delta_B a_0$ to $\Delta_{\alpha} = \mp \Delta_B a_0$, the isotope-effect coefficient $\alpha$ jumps because of the difference in $\alpha_0(\pm)$ mentioned above.

In Figs. 1 and 2, we show an example. To reproduce the experimental data of $\text{Sr}_2\text{RuO}_4$, we put $\alpha_0^{(+)} = -0.1$, $\alpha_0^{(-)} = 0.2$, $T_{c0}^{(+)} = 1.5K$, $T_{c0}^{(-)} = 0.98T_{c0}^{(0)}$, and $\lambda_{AB}/\zeta = -0.2$. The values of $T_{c0}^{(\pm)}$ and $\alpha_0^{(\pm)}$ are reproduced, for example, by $\omega_c = 410K$, $\lambda_{AA} = 0.33678$, $\lambda_{AB} = 0.028509$, $\mu_{\alpha A} = 0.16276$, and $\mu_{\alpha B} = 0.028203$. In this case, since $\lambda_{AB} > 0$, we have $T_{c0}^{(+)} > T_{c0}^{(-)}$. In Fig. 1, the curves of $T_c(\pm)$ cross at $\zeta^*(T_{c0}) \approx 0.063$ and $T_c/T_{c0} \approx 0.94$. Since the fold of the curve of $T_c$ is slight for the present parameters, it would be hardly detected experimentally. At the crossing point, the superconducting state alternates from the state with $\Delta_{\alpha} = \Delta_B a_0$ to that with $\Delta_{\alpha} = -\Delta_B a_0$. In Fig. 2, it is shown that in this internal transition, $\alpha$ jumps. The present theory could reasonably reproduce the experimental result within the error bar of the data. In experiments, possible inhomogeneity of the samples might smear the transition.

The internal transition and the jump of $\alpha$ could occur in more general situations than in the above example. The eigenstates of the linearized gap equation, eq. (11), are subject to different impurity and Coulomb effects as shown in eqs. (19) and (22) in the example. Since the eigenstate with the highest $T_c$ occurs, the superconducting state alternates from one eigenstate to another with impurity doping for appropriate parameters. In the transition, the isotope-effect coefficient $\alpha$ jumps because the effective Coulomb parameter $\mu^*$ changes.

Lastly, we discuss the application to $\text{Sr}_2\text{RuO}_4$. The $\text{Sr}_2\text{RuO}_4$ compound has three electron bands, called $\alpha$, $\beta$, and $\gamma$, which have separate Fermi surfaces. There are some experimental results to support triplet pairing interactions exist or not. However, for the alternation at $T_c \approx 0.94T_{c0}$ to be quantitatively reproduced, it is necessary that two of the eigenstates have very close $T_{c0}$. It could occur as a result of a combination of three electron bands. The split of the critical field curve observed for parallel fields may suggest an existence of the hidden eigenstate slightly below $T_{c0}$ at the zero field.

We may consider another possibility for $\text{Sr}_2\text{RuO}_4$. The internal degrees of freedom play an essential role in the present mechanism. In the present model, they originate from the multiband nature, but they may originate from the anisotropic gap structure of triplet pairing superconductivity. If we apply the above calculation to a model with a pair of the eigenstates with slightly different $T_{c0}$'s by replacing the band suffixes $A$ and $B$ with the suffix $\alpha$ to express the momentum dependence, the same figures as Figs. 1 and 2 are reproduced for appropriate parameter values. The explicit calculation will be presented in a separate paper.

In conclusion, we have examined the impurity effect in multiband superconductors, and obtained the expressions of $T_c$ and $\alpha$. It has been found that an internal transition of the superconducting state is induced by impurity doping in some condition, and the transition is accompanied by a jump of the isotope-effect coefficient $\alpha$. The $T_c$ dependence of $\alpha$ deviates from the CGP universal relation due to the jump. It is possible under appropriate parameters that $T_c$ appears to obey the standard AG equation simultaneously with a large deviation from the CGP relation.

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