WEAK FORM FACTORS OF THE NUCLEON

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We limit ourselves basically to the $SU(2)$ flavor sector of the CKM matrix, as probed in processes like nuclear $\beta$-decay, normal and radiative muon capture and neutrino reactions. Current interests in this primarily stem from our aspirations to interpret the weak form factors of the nucleon in the framework of QCD. Given a standard set of known form factors, we can look also in these processes for possible physics signals beyond the standard model.

1 Introduction

You have all heard from Paul Langecker and others how great is the standard model (SM) in describing strong, electromagnetic and weak interactions. So what I say here will be a modest footnote to that theme. I shall pose the simple question: What do we know about the weak form factors of the nucleon and what interest does this knowledge have in the context of the SM? Interests of the weak form factors are not new: right after the discovery of the parity violation in the weak interaction, these interests started to develop, already quite intensely in the sixties. So what I say will have a familiar ring to the older members of our audience. Current interests in the weak hadron form factors originate in our desire to interpret them in the framework of quantum chromodynamics (QCD). This is a difficult subject in theoretical physics, as the low-$q^2$ QCD is highly non-perturbative and can be only rigorously tackled on the lattice by numerical methods that are highly computer-intensive. The other interest of the weak hadron form factors is to take a set of standard values of them and to see if familiar weak processes lead to any result that is beyond the scope of the SM. I shall focus here on the former, and refer you to reviews, for example, by Herczeg for the latter.

An outline for the remainder of this talk is as follows: In section 2, I shall discuss the traditional versus the QCD motivated ways of looking at the weak nucleon form factors. Informations from nucleonic processes will be reviewed in section 2, while those from the precesses in complex nuclei will be very briefly touched in section 3. I shall close with a brief summary and an outlook.

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My web address: [http://www.rpi.edu/~mukhon](http://www.rpi.edu/~mukhon)
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We restrict ourselves to the nucleon charged current in this talk.

2 Traditional versus QCD-motivated ways of looking at the weak nucleon form factors

2.1 Some definitions

The nucleon charged weak current has the well-known Lorentz structure:

\[ V_\lambda = g_V(q^2)\gamma_\lambda + ig_M(q^2)\sigma_{\lambda\nu}\frac{q^\nu}{2M} + gs(q^2)\frac{q_\lambda}{m_l}, \]

\[ A_\lambda = g_A(q^2)\gamma_\lambda\gamma_5 + gp(q^2)\frac{q_\lambda}{m_l}\gamma_\lambda + ig_T(q^2)\sigma_{\lambda\nu}\gamma_5\frac{q^\nu}{2M}. \]

The notations are standard: \( m_l \) is the charged lepton mass, \( M \), the nucleon mass, \( q_\lambda \), the momentum transfer, \( n_\lambda - p_\lambda \), \( n, p \) being neutrons and protons. Since nucleon has an internal structure,

\[ g_A/g_V \neq 1, \ g_M, \ g_S, \ g_P \ and \ g_T \neq 0, \]

in principle. Hence the name of the last four objects the “induced” nucleon form factors. From T invariance,

\[ Im(g_i) = 0. \]

2.2 T invariance

This can be tested, for example, in the polarized neutron (\( \vec{n} \)) \( \beta \)-decay. The phase \( \phi_{AV} \) of \( g_A \) can be determined by measuring the triple correlation coefficient \( D \) from the component of \( \vec{n} \) spin perpendicular to the decay plane in the \( \beta \)-decay. For T invariance,

\[ D = 0, \]

implying the phase

\[ \phi_{AV} = 0^\circ \ or \ 180^\circ. \]

The Particle Data Group (PDG), in its latest compilation (PDG-98) reports

\[ \phi_{AV} = 180.07 \pm 0.18 \ degrees. \]

We should note that there is a sign difference in \( D \) between the last two measurements which were reported in the seventies. So Freedman and others in the audience should take a note of this and perhaps can mount a modern
No real test for $T$ violation in muon capture so far, though there are theoretical discussions (for $\beta$-decay theory and experiment review, see Herczeg [1] and an experimental proposal at PSI to do this. Deutsch [7] has spoken on this subject at many conferences with a lot of passion. We should note that the discovery of the $T$-violation in the nuclear muon capture (NMC) will be a glimpse at the physics beyond the SM.

2.3 Other constraints on vector form factors

Let us continue our discussion with the vector form factors in Eq. (1). The hypothesis of conserved vector current, which also follows from the SM, implies

$$g_S(q^2) = 0.$$  \hspace{1cm} (8)

This is not well-tested, though some crude tests are available from the NMC.

As regards to the other vector form factors, the $SU(2)$ structure of the electromagnetic and weak currents allows us to relate the weak form factors $g_V(q^2)$ and $g_M(q^2)$ to the electromagnetic form factors of the nucleons. In the usual notation

$$g_V(q^2) = F_1^p(q^2) - F_1^n(q^2),$$  \hspace{1cm} (9)

$$g_M(q^2) = F_2^p(q^2) - F_2^n(q^2).$$  \hspace{1cm} (10)

Thus, their $q^2 \to 0$ limit and $q^2$ dependence are well-known;

$$g_V(0) = 1, \; g_M(0) = \kappa_p - \kappa_n,$$ \hspace{1cm} (11)

where $\kappa_p$ and $\kappa_n$ are the appropriate nucleonic anomalous magnetic moments that are extremely well-known. The dipole dependence of these form factors is theoretically supported, at large $q^2$, by the principles of perturbative QCD; at low $q^2$, this is independently verified by the experiments on neutrino scattering. In this context it is interesting to note that the anomalous magnetic moments $\kappa_i$ enter in the famous spin-dependence sum rule, known as the Drell-Gerasimov-Hearn (DGH) sum rule, in the helicity-separated electromagnetic cross-sections $\sigma_{1/2}$, $\sigma_{3/2}$:

$$- \frac{2\pi^2 \alpha \kappa_i^2}{M^2} = \int \frac{d\nu}{\nu} (\sigma_{1/2} - \sigma_{3/2}),$$ \hspace{1cm} (12)

with usual symbols. Many experimental initiatives are underway to determine the neutron electromagnetic form factors and to test the DGH sum rule. Laboratories doing this include Bonn, Mainz and CEBAF, just to name three.
Continuing with the vector form factors, there are new CVC test proposals presented by van Schagen et al. and Beck et al. at this conference on the mass-12 system, wherein $\beta^+ \text{ decays take place from } ^{12}B \text{ and } ^{12}N \text{ to } ^{12}C_{g.s.}$, and the analogous state in $^{12}C^*$ also decays to the $^{12}C$ ground state. The corresponding reaction in the NMC is

$$^{12}C + \mu^- (1S) \rightarrow ^{12}B_{g.s.} + \nu_\mu,$$

a reaction well-studied theoretically and experimentally. It is a great experimental achievement to have studied the corresponding reactions by neutrino scattering by the LSND and KARMEN collaborations, backed by solid theoretical efforts, which will be summarized by Vogel at this conference. Given the excitement on neutrino oscillation at this conference, we look forward to more excitement from these nuclear reactions induced by neutrinos.

### 2.4 Axial vector form factors

#### 2.4.1 $g_A$

We now come to the nucleon axial-vector form factors. First point to make is the emphasis of the muon-electron universality, a cornerstone of the CKM theory in the SM, modulo radiative corrections, within which

$$\left(\frac{g_A}{g_V}\right)^\mu = \left(\frac{g_A}{g_V}\right)^e, \quad (14)$$

at $q^2 = 0$, where $\mu$ and $e$ respectively refer to the NMC and $\beta$-decay. The former is extracted from the process

$$\mu^- + p \rightarrow n + \nu_\mu. \quad (15)$$

The latter is, of course, given by the classic reaction of neutron $\beta$-decay

$$n \rightarrow p + e^- + \bar{\nu}_e. \quad (16)$$

Of course, this universality test (14) can never rival the one done via the two-body $\beta$-decays of pions, from which one can extract the ratio

$$R_o = \pi_{e2} / \pi_{\mu2}, \quad (17)$$

which is given by the kinematic expression

$$R_o = x(1 - x)^2 y^{-1} (1 - y)^{-2}, \quad (18)$$
on top of which radiative corrections must be included accurately. In Eq. (18),
x and y are given by the mass ratios

\[ x = \left( \frac{m_e}{m_\pi} \right)^2, \quad y = \left( \frac{m_\mu}{m_\pi} \right)^2. \]  

The meson factories have made the measurement of the ratio \( R_o \) an extremely
accurate business, but we do not have time to go into it.

Unfortunately, measurements of \( (g_A/g_V)^e \) from the process (16) have been
accurate, but not without fluctuations in the value of this ratio. Thus, in 1959,
this ratio was known accurately enough, but quite different from its value as
we know now:

\[ (g_A/g_V)^{59}_e = 1.17 \pm 0.02, \quad (g_A/g_V)^{58}_e = 1.2601 \pm 0.0025. \]  

Hence Holstein’s joke that this fluctuation in various epochs seems like fol-
lowing the expectation of Dirac’s law of large numbers!

In order to compare the \( g_A/g_V \) from the \( \mu \)-capture reaction (15) to its
value from the \( \beta \)-decay (16), we have to take out the modest \( q^2 \) dependence in
the former. For this, we use the recent neutrino scattering experiment. This
yields

\[ g_A(q^2)/g_A(0) = \left( 1 - \frac{q^2}{M_A^2} \right)^{-2}, \]  

where the \( M_A \) is extracted from a fit to the neutrino scattering data

\[ M_A = (0.89^{+0.09}_{-0.09}) \text{ GeV}. \]  

The behavior in (21) can be understood in the framework of the extended
Skyrmion model. From the world average of the singlet NMC rate in hydrogen

\[ \Lambda_S = 661 \pm 47 \text{s}^{-1}, \]  

we can extract \( (g_A/g_V)^\mu \) at \( q^2 = 0 \), using (21), (22) and canonical values of
the other form factors, to be

\[ (g_A/g_V)^\mu = 1.24 \pm 0.04. \]  

Comparing (20) and (24), the \( \mu - e \) universality is confirmed, though not at the
accuracy of the ratio (17). Further improvements require a lot more accurate
experimental efforts and theoretical studies on the radiative corrections in
these processes.
2.4.1.1 Axial vector coupling constant as a benchmark:

Though not the principal subject of this talk, it is worthwhile to refer to the important role $g_A$ plays in various areas in physics. We shall not delve here its importance in astrophysics, though that would be a talk in itself.

2.4.1.1.1 Bjorken (Bj) sum rule

The Bj sum rule is a QCD benchmark in which $g_A$ is connected to the physics of deep inelastic scattering (DIS) in nucleon.

To summarize this, we start with polarized structure function $g_1$ and $g_2$, which can be written in terms of the polarization observables

$$d\sigma^{\uparrow\downarrow} - d\sigma^{\uparrow\uparrow} = a g_1(x, Q^2) + b g_2(x, Q^2), \quad (25)$$

where $x$ is the well-known Bj $x$, and $a, b$ are kinematic quantities, $b$ being small relative to $a$. Defining

$$\Gamma_i^1 = \int_0^1 g_i^1(x) dx, \quad i = p, n, \quad (26)$$

we have the famous Bj sum rule, modulo QCD radiative corrections which are well-investigated theoretically. Many experiments at CERN (EMC, SMC), SLAC (E142, E143, E154, E155) and HERA (HERMES) have determined $\Gamma_1^i$, though there are always issues of extrapolation. For example, a typical EMC number for $\Gamma_1^p$ is

$$\Gamma_1^p = 0.126 \pm 0.01 \text{ (stat)} \pm 0.015 \text{ (syst)}. \quad (27)$$

In Fig.1 of Mulders and Sloan, it is demonstrated how the Bj sum rule is tested by taking into QCD corrections to the sum rule to ever higher orders. Thus, this sum rule has become a tool of testing QCD to a high accuracy.

2.4.1.1.2 Connection to resonance physics: Adler-Weissberger sum rule

It is interesting that chiral current algebra allows one to write the famous Adler-Weissberger sum rule in terms of strong interaction observables. In the usual notation, this sum rule relates integral over $\pi^\pm p$ cross-sections for zero mass pions:

$$[g_A(0)]^{-2} = 1 + \text{const} \int_{\nu_z}^{\infty} \frac{d\nu'}{\nu'} \left[ \sigma^0(\pi^- p, \nu') - \sigma^0(\pi^+ p, \nu') \right]. \quad (28)$$
This is an example of the low energy theorem or soft-pion theorem at work. These cross-sections are, of course, resonance dominated, thus bringing a connection between weak and strong interaction dynamics. Adler used his technique of off-mass shell corrections to get the magnitude of \( g_A(0) \)

\[
g_A(0)|_{\text{Adler}} = 1.24 \pm 0.03, \tag{29}\]

which is in fine agreement with the latest experimental result quoted earlier.

### 2.4.1.1.3 Connection to strong interaction dynamics in various QCD-inspired models

In quark model \(^\text{27}\) and Skyrme model \(^\text{19}\), the value of \( g_A \) is not entirely trivial. Indeed, in quark model, one must take into account relativistic effects in some fashion, while, in the Skyrme model, the dynamics goes astray without a lot of finetuning of the so-called anomalous sector \(^\text{19}\). A reasonable description of \( g_A \) has been found in the lattice gauge calculation within its systematic error \(^\text{28}\). In the QCD sum rule approach, \( g_A \) has recently been computed by Ioffe \(^\text{29}\) to be

\[
(g_A)_{\text{QCDSR}} = 1.37 \pm 0.10. \tag{30}\]

### 2.4.2 \( g_P(q^2) \)

Though the discussion of \( g_P(q^2) \) goes back to the old works of Wolfenstein (who is fortunately in the audience), Goldberger and Treiman (GT) \(^\text{30}\), the work of Wolfenstein continues to simulate current theoretical discussions both in the lattice gauge theoretic context \(^\text{28}\) and in the chiral perturbation theoretic (\( \chi \)-PT) framework \(^\text{30}\). It continues to be the primary focus of the NMC in hydrogen \(^\text{4}\) and of the \((e, e'\pi)\) reaction near threshold. Either via once-subtracted dispersion relation of Wolfenstein or via chiral Ward identity \(^\text{4}\), we can write

\[
g_P(q^2) = \frac{2m_\mu g_{\pi NN} F_\pi}{m_\pi^2 - q^2} - \frac{1}{2} g_A(0)m_\mu r_a^2, \tag{31}\]

with \( g_{\pi NN} \) and \( F_\pi \) being the pion-nucleon coupling constant and pion decay constant respectively, \( r_a \) being the nucleon axial vector radius. We recognize the first term on the right-hand side of Eq. (31) as the old Nambu-GT term, the second term being the Wolfenstein correction. For NMC in hydrogen, \( q^2 = q_0^2 = -0.88m_\mu^2 \), and

\[
g_P(q_0^2) = 8.44 \pm 0.23. \tag{32}\]
Recently Bernard et al. have made an important observation that the relation (31) is chirally protected and thus its experimental test is an important physics goal.

2.4.2.1 How good is the GT relation?

Eq(31) can be approximately written as the GT relation (GTR): 

\[ F_\pi g_{\pi NN}(0) = \sqrt{2} M g_A(0), \]  

(33)

where \( \sqrt{2} \) is by convention. One can thus write a discrepancy function for the GTR after Hemmert et al.:

\[ \Delta_\pi = 1 - \frac{\sqrt{2} M g_A(0)}{F_\pi g_{\pi NN}(m_\pi^2)}. \]  

(34)

We can rewrite (34) as

\[ \Delta_\pi = 1 - \frac{g_{\pi NN}(0)}{g_{\pi NN}(m_\pi^2)}. \]  

(35)

Already in 1964, Bjorken and Drell declared that the GTR “agrees with experiment to better than 10%”. Theoretically, the expectation of the agreement of the GTR with experiment is much higher. For example, Holstein reports two numbers. Using the linear \( \sigma \)-model, \( \Delta_\pi \) is estimated to be

\[ \Delta_\pi^{th} \sim 0.02. \]  

(36)

Using the Dashen-Weinstein \( SU(3) \)-theoretic theorem

\[ \Delta_\pi^{th} = 0.028. \]  

(37)

The hypothesis of partial conservation of axial current (PCAC) gives

\[ g_{\pi NN}(0) \sim g_{\pi NN}(m_\pi^2). \]  

(38)

This gives \( \Delta_\pi \) to be identically zero.

We also note here that Hemmert et al. have found a similar relation in the excitation of the \( \Delta(1232) \), originally discussed by Adler, an analogue of the GTR for resonance excitation, with similar expectation of theoretical accuracy. This relation has raised recent theoretical and experimental interests, latter at CEBAF.
2.4.2.2 Status of $g_{\pi NN}$

Ericson has recently raised the question, which must be in the mind of a lot of us, as to how our world would be if $g_{\pi NN}$ changed significantly from what it is known to be! Of course, such questions can be rhetorically raised about any physical constant. In this context, we should note that the VPI and other groups have recently differed quite a bit as to this quantity. To take one example, the value of $g_{\pi NN}$ equal to 13.4 would make the $\Delta_\pi$ to be 4.1%, while its value of 13.05 would yield $\Delta_\pi$ to be 1.5%. This is a point much emphasized by Holstein recently. We may add that the PSI proposal PSI-98.01 by Oades et al. will measure the 1S width of the pionic hydrogen $\Gamma_{1S}(\pi^- p)$ at an accuracy of 1%, which, via the Gell-Mann - Oehme sum rule, would yield a precise estimate of $g_{\pi NN}$ better than what is available. Here is an example of strong interaction quantity influencing mightily our knowledge of the nucleon weak form factors!

2.4.2.3 What do we know about $g_P$ from the NMC?

As you all know, the role of $g_P$ is negligible in the $\beta$-decay, but not in the NMC. Let me recall the argument for the benefit of students, if any, in the audience. The $\beta$-decay matrix element $< ME >_\beta$ is proportional to the following expansion:

$$< ME >_\beta \sim 1 + 6.7 \frac{m_e}{m_\mu} \frac{m_e}{2M} \sim 1 + O(10^{-5}),$$

the second term being the reduction of the $\gamma_5$ operator in the non-relativistic sense. Similarly, for the NMC,

$$< ME >_\mu \sim 1 + 6.7 \frac{m_\mu}{2M},$$

the second term on the right-hand side being considerably larger than the corresponding term in the $\beta$-case and being of the same order as the weak magnetism term $g_M$. Thus, using our present experimental knowledge of $\Lambda_S$ from the NMC in hydrogen, we get, fixing other form factors at their canonical values, the value of $g_P$ to be

$$g_P(q_0^2) = 8.7 \pm 2.9,$$

compared with the PCAC ($\chi$PT), value

$$g_P(q_0^2)|_{PCAC} = 8.44 \pm 0.23,$$
a fine agreement, though there is a lot of room for improvement from better precision of the $\Lambda_S$ measurement to be attempted in future, as reported in a new PSI proposal, presented in a poster session at the WEIN-98 by C. Petitjean and P. Kammel. This proposal wants to reach a precision of $\Lambda_S$ to 1%. It would be great if this goal is reached in future.

Does it mean we are in good shape here? Not quite, as the folks from TRIUMF have come with a disturbing result in the radiative muon capture (RMC) discussed below.

2.4.2.4 What is the latest from the radiative muon capture?

As we know from many discussions in the literature, a very powerful way to determine $g_p$ is from the RMC in hydrogen:

$$\mu^- + p \rightarrow n + \nu_\mu + \gamma$$

wherein the high-energy photon is particularly sensitive to $g_p$. As we all know, the RMC is much weaker than the ordinary NMC and we are talking about a small part of the photon spectrum. Thus, we are talking about a very difficult experiment. The TRIUMF team has really pulled together a great tour de force by using high muon flux on a very pure liquid hydrogen target, avoiding the processes such as:

$$\pi^- p \rightarrow \pi^0 n, \quad (43a)$$
$$\pi^0 \rightarrow \gamma\gamma n (55 - 83 \text{ MeV} \gamma\gamma's), \quad (43b)$$
$$\pi^- p \rightarrow \gamma n (k \sim 129 \text{ MeV}). \quad (43c)$$

The outcome of this great experiment is a puzzle to the theorists. Expressing it in terms of an extracted $g_p$, we get from this experiment:

$$g_p(-0.88 m_p^2) = (9.8 \pm 0.7 \pm 0.3) g_A(0), \quad (44)$$

which is 1.46 times the value expected from the $\chi$PT considerations.

This experiment has generated a cottage industry of higher order $\chi$PT calculations, using the heavy baryon $\chi$PT (HB$\chi$PT). These use the Ecker-Možiš Lagrangian using the development up to order $O(p^3)$. However, these higher order corrections do not alter significantly the leading order result. One point of caution: building $\Delta(1232)$ effects in the $\chi$PT is technically difficult and I cannot judge at this time how good these calculations are, looked from this point of view.

Assuming that the discrepancy between the $\chi$PT theoretical expectation and (44) is not due to any higher order correction in the $\chi$PT (or PCAC), what
are the possible ways out? Let me speculate on two prospects. First is the possibility that the above discrepancy is due to our lack of complete understanding of the complex molecular physics problem in the $p\mu - p$ system involved. This means more precise experiment on the ortho($L = 1$) and para($L = 0$) transition rates. Direct measurement on this is underway at TRIUMF (expt.766-96). Second possibility has been discussed by Opat long ago. He mentioned multiple pion exchange mechanism that distinguishes ordinary NMC and RMC. Is it possible that this mechanism is doing the trick alone? One needs another theoretical study. Of course, there can always be something that we have not considered above!

2.4.2.5 The $\pi NN$ and pseudoscalar form factors from the lattice QCD

Thanks to the heroic efforts by K.-F.Liu and collaborators, we have new insights on the pion-nucleon and the pseudoscalar couplings from a quenched lattice calculation on a $16^3 \times 24$ lattice, implementing thereon the appropriate Chiral Ward identity.

The lattice measurement of the $g_{\pi NN}(q^2)$ cannot distinguish between monopole and dipole behavior, except at very low $q^2$. A quantity $h_A(q^2)$, related to our pseudoscalar form factor, nicely probes the one-pion tail, the classic Nambu behavior, and agrees very well with the experiment of Choi et al. on the electroproduction of pions. Thus, the "lattice" and "real" measurements match very well! An excellent demonstration of QCD in its non-perturbative domain! New technique on the handling of chiral symmetry breaking of fermions on the lattice adds a lot of excitement to the improvement of the lattice result in near future. So please stay tuned!

2.4.3 "Second-class" currents?

Weinberg, in his classic 1958 paper, defined two classes of weak hadron currents. The "first class" ones behave under the G-parity transformation $G(G = Ce^{i\pi \tau_2}$, C, charge conjugation, $\tau_2$, the appropriate isospin generator) as follows:

$$GV_G^{-1} = V_A, GA_A G^{-1} = -A_A.$$  \hspace{1cm} (45)

These which do not behave like (45) are "second-class" and Weinberg surmised that these should be absent or highly suppressed. This important conjecture is also supported by the renormalizability of the gauge theories. The absence
of the second-class currents (SCC) implies

\[ g_S(q^2) = 0, \quad (46) \]

also required by the conserved vector currents (CVC), and

\[ g_T(q^2) = 0. \quad (47) \]

There are related theorems by Cabibbo. \(^{43}\) Search for the SCC's in the \(\beta\)-decay was systematically started by Wilkinson \(^{44}\) (fortunately for us, in the audience) and soon extended to the NMC. \(^{45}\) This search has so far been consistent with its absence, but the subject is still quite healthy. Indeed, the WEIN-98 has seen two contributions, one from van Schagen \(\textit{et al.}\) \(^{12}\) on the possible test of the CVC/SCC in the \(A = 8\) isomultiplet and the other is by Minanisono \(\textit{et al.}\) \(^{46}\) on the correlations in the spin-aligned mirror pairs in the \(A = 12\) system. Let us wait for these studies for better limits than what we have now. The latter work claims that

\[ 2M \frac{g_T}{g_S} = 0.22 \pm 0.05 \pm 0.15(\text{syst}) \]
\[ \pm 0.05(\text{th}), \quad (48) \]

in comparison with the expectation of the QCD sum rule.\(^{47}\)

\[ 2M \frac{g_T}{g_A} = 0.0152 \pm 0.0053, \quad (49) \]

where the G-violation here is proportional to the mass difference of quarks \(m_u - m_d\); these masses also figure in the isospin violation in the pion-nucleon scattering.\(^{48}\)

Muon capture experiments are consistent with the absence of the SCC. Two latest limits are already quite old. First is from Holstein, \(^{49}\)

\[ g_S(0) = -0.4 \pm 2.3, \quad (50) \]

belonging to our survey of the vector form factors earlier, second is from Morita, \(^{50}\)

\[ g_T(q_0^V) = -0.06 \pm 0.49. \quad (51) \]
3 Nuclear muon capture: a brief excursion to a long and old subject which is still very interesting

This is a vast subject that has been reviewed repeatedly in literature(e.g.2). I cannot do justice to the vast subject accumulated over many years here. Let me instead pick up one process in which an enormous progress has been made at PSI. The PSI experiment has measured the statistical capture rate of the muon capture in $^3He$ from the 1S atomic state to an unprecedented accuracy. The reaction of interest is

$$\mu^- + ^3He \rightarrow \nu_\mu + ^3H.$$ (52)

The measured rate is

$$\Lambda_{stat}^{exp} = 1496 \pm 3(stat) \pm 3(sys) \ s^{-1}.$$ (53)

The theory has been treated most recently in a number of works, most thoroughly by Congleton and collaborators. Congleton and Truhlik find a 15% meson exchange current effect, thus a significant contribution and a proof positive on the role of the two-body currents in this clean nuclear reaction(52). They obtain a theoretical rate

$$\Lambda_{stat}^{th} = 1497 \pm 21 \ s^{-1},$$ (54)

in excellent agreement with (54).

The experimental result (54) is so precise that one can explore it for a variety of physics effects, including an explosion of the physics beyond the standard model, as Govaerts has recently attempted. We refer to his papers for further details. Deutsch, Herczeg and Mohapatra have inspired these investigations by emphasizing repeatedly the importance of the NMC as a window on the physics beyond the standard model.

The reaction (52) has been used by Junker and this author in the context of the nuclear PCAC and GTR. Following the method of Wolfenstein, one can write a dispersion relation:

$$D(t) = [2MF_A + \frac{t}{m}F_P]$$

$$= -\sqrt{2}F_p m_e^2 G(t) \left( \frac{1}{t - m_e^2} \right) - \frac{1}{\pi} \int_a^\infty d\tau' \frac{Im(D(t'))}{\tau' - t},$$ (55)

where ”a” represents thresholds for various nuclear anomalous cuts (for $d + n$ and $n + p + n$ channels for the $t$ breakup). Here $F_A, F_P$ are the nuclear analogue
of the nucleon axial vector form factors. From this master equation, one gets an expression for $G^{eff}$, the pion-nuclear coupling constant, yielding:

$$G^{eff}(m_{\pi}^2) = 45.8 \pm 2.4,$$  \hspace{1cm} (56)

compared with the value obtained from the pion-nuclear scattering:

$$G^{eff}(m_{\pi}^2) = (38 \pm 16) \text{ to } (57 \pm 13).$$  \hspace{1cm} (57)

Thus, weak interaction in this context is more powerful than the strong interaction.

There are also recent limits available on the second class current couplings from this reaction.

The asymmetry observable in the polarized muon capture are also very interesting.

For brevity, we omit here the important subject of the effective weak hadron coupling constants in nuclei. Suffice to say that there was a lot of interest in the parallel sessions at this meeting on this topic, as well as many La Fonda hallway discussions on the role of the NMC as a means to explore physics beyond the standard model. Recent works, besides Govaerts, by Barabanov, Ciechanowicz and Popov, Herczeg and Mukhopadhyay, Missimer et al., should be referred for further information.

4 Summary

Let me restate the main points of this talk:

• The $\beta$-decay and the NMC(RMC) are major physics sources for examining old and new issues on weak nucleon form factors, particularly in the context of recent QCD interest.

• Radiative muon capture experiment at TRIUMF poses a challenge to our understanding of the application of the PCAC/\chi PT to this reaction.

• The $^3He \rightarrow ^3H$ reaction is now a new gold standard in the field. It contains lots of valuable information.

• The NMC looks promising in the context of physics beyond the standard model.

High stopping rate of muons, polarized nuclear target and other experimental innovation should make this subject one of continuing interest.
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I hope to see you at a future WEIN conference with better health.

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