A Numerical Study of the Three-dimensional Structure of the Taylor-Couette Flow in Eccentric Configuration with Superimposed Cross Flow

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Abstract. The eccentric small gap Taylor-Couette system with rotating inner cylinder and fixed outer cylinder is investigated numerically. The main flow fields were examined and the transition region from the laminar Couette-flow to the Taylor-vortex-flow in different eccentric arrangements of the cylinders. The effect of the eccentricity on flow patterns was studied for different values of the eccentricity between 0 and 0.75 in relation to the mean gap. This flow was further disturbed by the superimposed cross flow entering into the gap through the feed hole with a cross flow rate of 0.1 of the circumferential flow rate. Hence, more complex three dimensional flow structures evolved in the cylinders’ gap, especially in the vicinity of the feed hole.

1. Introduction

The flow between eccentric rotating cylinders is of interest in the fluid dynamics due to the strong effect of both Reynolds number (Re) and eccentricity, both. Also it is of considerable technical importance, for example in the lubrication technology. Here, Re was defined as $Re=U_1H_0/\nu$ where $U_1$ is the rotational speed of the inner cylinder, $H_0$ is the mean value of the gap width and $\nu$ the kinematic viscosity of the fluid.

For the classical Taylor-Couette system with rotating inner cylinder and fixed outer cylinder, the transition of the fluid flow from stable circular Couette flow to steady axisymmetric Taylor vortex flow will occur at a critical value of the Reynolds number $Re_{cr}$. However, when displacing one of the cylinders to an eccentric position, the rotational symmetry is broken and a fully three-dimensional flow will be obtained. The dependency of $Re_{cr}$ on the eccentricity $\varepsilon$ has been studied experimentally and analytically by DiPrima and Stuart [1]. The value of $Re_{cr}$ was found to increase by increasing the eccentricity. Moreover, if the eccentricity is greater than 0.3, recirculation of the flow can be found in the region of the widest gap. The recirculation eddy is located near the wall of the outer cylinder. For $\varepsilon<0.6$ a non-axisymmetric Taylor vortex flow was observed by e.g. Karasudani [4], whereas no steady flow could be observed for $\varepsilon>0.6$. Recent numerical results concerning the effect of eccentricity in the Couette-Taylor system with normalized gap width $\psi=0.5$ were given by Shu et al. [4].
This paper focuses on two issues, which determine the flow inside the annular gap between the cylinders. Firstly, the numerical investigation of the eccentric Taylor-Couette system with rotating inner cylinder and fixed outer cylinder for small gap configurations, i.e. $\psi<0.2$, will be presented. For different eccentric arrangements of the cylinders the main flow field is computed and presented in form of the topology of its velocity vectors, particular interest is given to the transition region from the laminar Couette-flow to the Taylor-vortex-flow. Here the eccentricity $\varepsilon$ is set at 0.25, 0.5 and 0.75 relative to the mean gap width.

Secondly, the numerical study of the flow pattern inside the gap with a superimposed cross flow onto the mean flow of the concentric or eccentric cylinder arrangement will be presented. The cross flow enters into the gap through the feed hole, which is located in the wall of the outer cylinder. In this case more complex and three dimensional flow structures evolve especially in the vicinity of the feed hole.

2. Simulation results of the system with eccentricity

For flow simulation in journal bearings, usually one solves the Reynolds equations, which are an idealized form of the Navier-Stokes equations and do not include the three-dimensional effects occurring especially in the supercritical case of the eccentric system and if the mean flow is disturbed by the superimposed cross flow. Elaborated studies of flow in journal bearings based on the Reynolds equations exists by e.g. Gersten and Herwig [2] and Hutter[3].

An approach which combines the finite-element spatial discretization and the $\theta$-scheme time approximation was used to solve the three-dimensional unsteady incompressible Navier-Stokes equations. Very good agreement of the numerical results obtained by this method and the experimental data for the concentric Couette-Taylor system with asymmetric boundary conditions and small gap, were found in Meincke et al. [5] and Stücke et al. [7, 8, 9], respectively.

A schematic configuration of the three dimensional eccentric system is shown in Fig. 1. The eccentricity of the rotating inner cylinder (radius $R_1$, angular velocity $\omega_1$) and the fixed outer cylinder (radius $R_2$) is represented by the distance $e$. The geometric parameters are the mean gap width $H_0=R_2-R_1$, the normalized clearance $\psi=H_0/R_1$, the eccentricity $\varepsilon=e/H_0$ and the aspect ratio $\Gamma=B/H_0$. The conventional Reynolds number is defined by $Re=R_1\omega_1H_0/\nu$. For the numerical computations a
quadratic system, i.e. $B=2R_i$ is considered. Numerical computations are carried out for three values of the normalized clearance, namely $\Psi=0.077$, 0.1 and 0.2, and for three values of the eccentricity, namely $\varepsilon=0.25$, 0.5 and 0.75.

The velocity field and the recirculation vortex is represented in Fig. 2 in an axial section at $\Psi=0.2$, $\varepsilon=0.75$ and $Re=110$ in the left figure and $Re=200$ in the right one. One can observe that for high Reynolds numbers vortices occur also at the interface between the main flow and the recirculation flow. In Fig. 3 the velocity field is represented for $\Psi=0.077$ and $\varepsilon=0.25$ at $Re=180$ in different azimuthal sections. Taylor vortices occur first in the region downstream of the wide gap and develop by increasing $Re$ also in the small gap zone. This phenomenon can be better observed in Fig. 4, where the isosurface of the velocity norm $|U|=0.5$ is represented in the gap of the system. Downstream from the maximal gap the Taylor vortices were fully developed, but upstream of the maximal gap the radial velocity component is very small. However, all around the cylinder gap the Taylor-vortices number is the same. At $\varepsilon=0.5$ and $Re=280$ the recirculation flow is found between the Taylor vortices, Fig. 5. For $\varepsilon=0.75$ the flow passes from Couette flow directly into a turbulent one, without formation of organized Taylor vortices. Numerical results presented in this paper confirm experimental observations of [1, 4] that the critical Reynolds number increases with increasing the eccentricity.
3. **Simulation results of the cylinders system with superimposed cross flow**

Through a circular feed hole situated on the outer fixed cylinder, a quantity of fluid is introduced permanently into the cylinder gap. This supplied fluid merges with the rotating main flow and leaves the system at the ends of the cylinder, where a constant pressure distribution is assumed. The feed hole diameter is 0.1 in relation to the inner cylinder diameter and the cross flow rate is considered to be 0.1 of the mean circumferential flow.

**Figure 5.** Taylor vortices and recirculation flow at $\Psi=0.077$, $\varepsilon=0.75$ and $Re=280$ in the maximal gap section.

**Figure 6.** Velocity field in an axial section of the concentric system with feed hole at $\Psi=0.1$, $\Gamma=20$ and $Re=150$. 

Also at low Reynolds numbers in the concentric system, when the mean flow is of Couette type, the presence of the feed hole and the cross flow induced three dimensional flow structures at least in the vicinity of the feed hole. The Couette flow in the concentric system of normalized clearance $\Psi=0.1$ is disturbed at $Re=150$ by the cross flow. In Fig. 6 the velocity field in an axial section shows some vortices near the outer cylinder up- and downstream of the feed hole. The circumferential velocity profiles along the sections 1 downstream and 2 upstream of the feed hole, respectively, are mapped in Fig. 7. The reversed flow in the downstream section 1 is more prominent than that in the upstream section 2.

The circumferential pressure distribution is compared in Fig. 8 with analytical results obtained by solving the idealized Reynolds equation, e. g. [3], in the eccentric cylindrical system. At small gap values $\Psi=0.1$ and eccentricity $\varepsilon=0.5$ very good agreement of the pressure distribution is found for low $Re$, whereas, for larger Reynolds numbers, a substantial deviation can be found. By considering a feed hole the external cylinder positioned at $\phi=0^\circ$ (Fig. 8 left) or $\phi=180^\circ$ (Fig. 8 right) in the eccentric system the pressure distribution differs essentially from the idealized distribution given by the Reynolds equation and shows the perturbation produced in the vicinity of the feed hole.

**Figure 7.** Circumferential velocity component in radial direction for (left) section 1; (right) section 2 of Fig. 6.

**Figure 8.** Circumferential pressure distribution in Couette system at $\Psi=0.1$, $\varepsilon=0.5$ and $Re=12$, with feed hole positioned at (left) $\phi=0^\circ$ and (right) $\phi=180^\circ$. 
4. Summary

Numerical simulations of the flow in several different cylindrical arrangements were performed by solving the time dependent three-dimensional Navier-Stokes equations. Firstly, the eccentric Taylor-Couette system with small gap and rotating inner and fixed outer cylinder was considered. The flow patterns were found to be influenced by the Reynolds number, the gap width and the eccentricity. In the supercritical regime an overlapping of the Taylor vortex flow and the recirculation eddy due to the eccentricity was found. Secondly, a cross flow entering in the concentric or eccentric cylinder system through a feed hole was superimposed. In this case the flow patterns were additionally affected by the superimposed flow rate and the feed hole position relative to the eccentricity. In the subcritical regime, when the main flow is of Couette type, the cross flow induced three dimensional flow structures in the vicinity of the feed hole.

Further numerical simulations will be focused on the flow structures in the vicinity of the feed hole covering a wide range of variation of the main parameters: Reynolds number, gap width, eccentricity, cross flow rate and feed hole position relative to the eccentricity.

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