Spin 3/2 particle as a dark matter candidate: an effective field theory approach

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Abstract

There is no indication so far on the spin of dark matter particles. We consider the possibility in this work that a spin-3/2 particle acts as dark matter. Employing the approach of effective field theory, we list all possible 4-fermion effective interactions between a pair of such fields and a pair of ordinary fermion fields. We investigate the implications of the proposal on the relic density, the antiproton to proton flux ratio in cosmic rays, and the elastic scattering off nuclei in direct detection. While the relic density and flux ratio are sensitive to all interactions albeit at different levels, the direct detection is only sensitive to a few of them. Using the observed data and experimental bounds, we set constraints on the relation of couplings and dark particle mass. In particular, we find that some mass ranges can already be excluded by jointly applying the observed relic density on the one side and the measured antiproton to proton flux ratio or the upper bounds from direct detection on the other.

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1 Introduction

There is compelling evidence from astronomical observations that the dominant component of matter in our universe is invisible, dubbed dark matter (DM). After years of efforts the evidence is still mainly restricted to the scope of gravitational effects. There are now many on-going or approved astronomical and laboratory projects that will hopefully reveal in the near future whether the dark matter is composed of particles or astrophysical objects like massive compact halo objects or it is not required at all because of modified Newton dynamics. For brief overviews on the current experimental status, see for instance the recent talks in Ref. [1].

From the theoretical point of view we are not short of candidates if the dark matter turns out to be composed of particles. The most popular one is the lightest supersymmetric particle, perhaps the lightest neutralino, in supersymmetric models. There are also extensive discussions suggesting that the lightest Kaluza-Klein particle in extra dimension models [2] or the lightest T-odd particle in little Higgs models with T parity [3] could act as DM, and so on. For detailed reviews, see for instance, Ref. [4]. Since the models on which the proposals mentioned are based are yet to be verified, it is important not to forget about other alternatives. In this context, effective field theory serves as a useful approach since one can focus on the interactions relevant to DM searches and parameterize unknown underlying dynamics in terms of effective couplings [5], while leaving the dynamics to be identified in dedicated particle experiments like high energy colliders.

The effective field theory approach has been widely employed to study the detection of a scalar, spin-1/2 fermion and vector DM particle [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. In this work we consider the possibility of a spin-3/2 particle as a DM candidate, and investigate its features in various detection experiments. Although we do not know yet a quantum field theory for such a particle of a higher spin that would be renormalizable in the conventional sense, this is by no means a reason to exclude its physical relevance: there are hadronic resonances of higher spin, e.g., Δ(1232) of spin 3/2, which play an important role in nuclear physics. Another example of spin-3/2 particles is the well-motivated hypothetical gravitino, which is a gauge particle associated with spontaneously broken localized supersymmetry. We are aware that even the effective interactions involving the Δ(1232) resonance are still controversial, see for instance [21] for a summary of the relevant issues. Our results in this work are nevertheless immune from such uncertainties since what we will need is not more than the Lorentz covariance from which the polarization sum of a spin-3/2 particle is constructed. We also know that the gravitino itself was previously suggested as a DM particle. It interacts with ordinary particles with an essentially gravitational strength, and the leading interactions at low energies are those involving a single gravitino field (and another superpartner); see Ref. [22] for a practical introduction to the relevant issues. In contrast, what we will consider in the following are the effective interactions that contain a pair of spin-3/2 fields. Such interactions naturally preserve certain parity if the latter is required to stabilize the DM particle.

The paper is organized as follows. In the next section we exhaust all possible 4-fermion effective interactions that involve a pair of spin-3/2 fields and a pair of ordinary spin-1/2 fields. We then compute the cross sections for the annihilation and elastic scattering processes. These
results will be utilized in section 3 to compute the secondary particle fluxes in the cosmic rays, the relic density, and the effective cross sections between the DM particles and nucleons. Using the observational and laboratory data we set constraints on the couplings of the effective interactions as a function of the DM mass. It turns out some of the mass ranges can be excluded using the currently available data. We recapitulate our results in the last section.

2 Effective interactions and cross sections

Suppose the stability of the spin-3/2 particle is protected by certain parity, its leading effective interactions would involve a pair of it. Amongst the possible interactions that are relevant to their detection are those that couple to a pair of ordinary fermions. We therefore restrict ourselves in this work to the 4-fermion interactions.

A free particle of spin $3/2$ and mass $M$ can be described by a field $\Psi_\mu$ that has the mixed transformation properties of a Dirac four-component field and of a four-component vector field \cite{23}. Its equation of motion is

\[ (i\not\partial - M)\Psi_\mu = 0, \]  

augmented with the constraint, $\gamma^\mu \Psi_\mu = 0$. Multiplying eq (1) from the left by $\gamma^\mu$ and applying the constraint one gets $\partial^\mu \Psi_\mu = 0$ as a consequence. The wavefunction of such a particle satisfies the same equations and thus has four independent components as desired, that correspond to the four spin states in its rest frame. As a matter of fact, the wavefunction for a particle with four-momentum $p$ and helicity $\lambda$, $U_\mu(p, \lambda)$, can be constructed using the Clebsch-Gordan coefficients in terms of the ones for a spin-1/2 Dirac spinor $u(p, s)$ of helicity $s$ and a spin-1 polarization $\epsilon(p, \kappa)$ of helicity $\kappa$ \cite{24}. Our following calculation will not depend on the explicit form of $U_\mu$ but its polarization sum $P_{\mu\nu}(p) = \sum_\lambda U_\nu(p, \lambda)\bar{U}_\mu(p, \lambda)$ which is evaluated to be (see for instance, Ref. \cite{22} upon correcting the sign of the $M$ term)

\[ P_{\mu\nu}(p) = -(\not\partial + M)\left( T_{\mu\nu}(p) - \frac{1}{3} \gamma^\rho T_{\rho\mu}(p)T_{\nu\sigma}(p)\gamma^\sigma \right), \]  

with $T_{\mu\nu}(p) = g_{\mu\nu} - p_\mu p_\nu/p^2$ and $p^2 = M^2$. Note that the factor $(\not\partial + M)$ can be equally well put on the rightmost. The equation of motion and the constraint imply that

\[ \gamma^\mu P_{\mu\nu}(p) = P_{\mu\nu}(p)\gamma^\nu = 0, \]
\[ p^\mu P_{\mu\nu}(p) = P_{\mu\nu}(p)p^\nu = 0, \]
\[ (\not\partial - M)P_{\mu\nu}(p) = P_{\mu\nu}(p)(\not\partial - M) = 0, \]  

which may be employed to verify eq (2). The polarization sum $Q_{\mu\nu}(p)$ for the antiparticle and its relations can simply be obtained from the above by $M \rightarrow -M$. We will assume in this work that the spin-3/2 particle is of Dirac nature. For a Majorana-type particle, the amplitudes to be computed later are either multiplied by a factor of two or just vanish.
The leading 4-fermion interactions between a pair of $\Psi_\mu$ fields and a pair of ordinary fermion fields $f$ are of dimension six. For simplicity we will not consider flavor-changing interactions. The independent Hermitian bilinears of $f$ are

$$(a) : \bar{f} f, \bar{f} i\gamma_5 f, \bar{f} \gamma_i f, \bar{f} \gamma_\mu \gamma_5 f, \bar{f} \sigma_{\mu\nu} f.$$  

The bilinears of $\Psi_\mu$ are similar with the only difference in that they have two additional vector indices:

$$(i) : \bar{\Psi}_\alpha \Psi_\beta, \bar{\Psi}_\alpha i\gamma_5 \Psi_\beta, \bar{\Psi}_\alpha \gamma_\rho \Psi_\beta, \bar{\Psi}_\alpha \gamma_5 \Psi_\beta, \bar{\Psi}_\alpha \sigma_{\rho\sigma} \Psi_\beta.$$  

Consider the self-contraction of a pair of indices in the list $(i)$. Since the interactions will be exploited in the cases where the spin-3/2 particles are on-shell, the constraint for the free field still applies. This means that it is not necessary to consider the contraction between the fields and the $\gamma$ matrices. For instance, after a little algebra, we find $g^{\alpha\rho} \Psi_\alpha \sigma_{\rho\sigma} \Psi_\beta = -i\bar{\Psi}_\alpha \Psi_\beta$, which however was already covered in the list $(i)$. The contraction is thus restricted to be between the two factors of the $\Psi_\mu$ field:

$$(ii) : \bar{\Psi}_\alpha \Psi_\alpha, \bar{\Psi}_\alpha i\gamma_5 \Psi_\alpha, \bar{\Psi}_\alpha \gamma_\rho \Psi_\alpha, \bar{\Psi}_\alpha \gamma_5 \Psi_\alpha, \bar{\Psi}_\alpha \sigma_{\rho\sigma} \Psi_\alpha,$$

while further contraction produces nothing new.

All possible interactions are exhausted by multiplying the terms in the list $(a)$ with those in the list $(i)$ and $(ii)$ respectively and contracting remaining indices with the signature tensor $g^{\mu\nu}$ or the totally antisymmetric tensor $e^{\mu\nu\rho\sigma}$. Some of the terms so obtained can be removed as redundant. For instance, using $e^{\mu\nu\rho\sigma} \sigma_{\rho\sigma} = -i2 \sigma^{\mu\nu} \gamma_5$ (in our convention $\gamma_5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3$ and $e^{0123} = +1$), we have $e^{\alpha\beta\rho\sigma} \bar{f} f \Psi_\alpha \sigma_{\rho\sigma} \Psi_\beta = -2\bar{f} f \Psi_\alpha \gamma_5 \Psi_\alpha, e^{\alpha\beta\rho\sigma} \bar{f} i\gamma_5 f \Psi_\alpha \sigma_{\rho\sigma} \Psi_\beta = -2\bar{f} i\gamma_5 f \bar{\Psi}_\alpha \gamma_5 \Psi_\alpha$, and so on. The final list contains the following 14 operators:

\begin{align*}
D^{f}_{1,4} &= \bar{f} \sigma_{\mu\nu} f \bar{\Psi}_\mu^i \Psi^i_\nu, \quad \bar{f} \sigma^{\mu\nu} i\gamma_5 f \bar{\Psi}_\mu^i \Psi^i_\nu, \\
D^{f}_{5,8} &= \bar{f} f \bar{\Psi}_\alpha \Psi^\alpha, \quad \bar{f} i\gamma_5 f \bar{\Psi}_\alpha \Psi^\alpha, \\
D^{f}_{9,12} &= \bar{f} \gamma_\mu f \bar{\Psi}_\alpha \gamma^\mu \Psi^\alpha, \quad \bar{f} \gamma_\mu \gamma_5 f \bar{\Psi}_\alpha \gamma^\mu \gamma_5 \Psi^\alpha, \\
D^{f}_{13,14} &= (\bar{f} \sigma_{\mu\nu} f, \bar{f} \sigma_{\mu\nu} i\gamma_5 f) \bar{\Psi}_\alpha \sigma^{\mu\nu} \Psi^\alpha.
\end{align*}

The effective interactions are summarized as

$$L_{\text{int}} = \sum_{f}^{14} G^f_i D^f_i,$$

where all couplings $G^f_i$ are real and have the same dimensions as the Fermi constant $G_F$.

To prepare for the numerical analysis in the next section, we display the cross sections for DM annihilation and elastic scattering off a nucleus. Since $L_{\text{int}}$ contains many possible interactions, it looks sensible to treat one interaction at a time. When $\Psi$ in a specific model happens to interact in several ways with an ordinary fermion of a given flavor, one should sum coherently their contributions to a scattering amplitude. The spin-summed and -averaged
cross section in the center-of-mass frame for the annihilation process \( \Psi \bar{\Psi} \rightarrow f \bar{f} \) through the interaction \( \mathcal{O}_f \), is

\[
\sigma_f = N_f \frac{(G_f^i)^2 s}{16\pi} \sqrt{\frac{s-4m_f^2}{s-4M^2}} A_i(m_f^2/s, M^2/s),
\]

where \( s \) is the center-of-mass energy squared, \( m_f \) and \( M \) are respectively the masses of the final \((f)\) and initial \((\Psi)\) particles, and \( N_f = 1 \) (3) when \( f \) is a lepton (quark). The dimensionless functions \( A_i \) for various operators are listed in the Appendix.

In direct detection of dark matter one measures the recoil energy of nuclei that have been struck by a DM particle in the local halo. The event rate and energy deposited are determined by the cross section between the two. The calculation of the latter is a hard task, connecting microscopic interactions of DM particles with quarks to effective interactions with nuclei through the intermediate chiral dynamics of nucleons, incurring uncertainties at each step; see the first article in Ref. [4] for a review. The difficulty is alleviated to some extent by the fact that the collision is nonrelativistic. In this case the DM particles only feel the spin or mass of a nucleus [5]. In the remainder of this section we outline the procedure of this calculation relevant to our case and present the results for cross sections between \( \Psi \) and a nucleus.

To begin with, one builds effective interactions between \( \Psi \) and nucleons from those for quarks shown in eq (5). One assumes that the Lorentz structures are not changed but the interaction strengths get corrected by chiral dynamics. Since the nucleons in a nucleus can be treated nonrelativistic for the purpose here, the relevant nucleon bilinears (and thus quark bilinears) are restricted to the spin-independent (SI) and spin-dependent (SD) ones. In the approximation of zero momentum transfer, one then takes the diagonal matrix element of the nucleon bilinears for a static nucleus. The SI part essentially counts the numbers of the protons and neutrons in the nucleus while the SD part gets its main contribution from the unpaired nucleon spin. For a reasonable estimate of the nuclear matrix elements one has to appeal to nuclear models; in particular, when the momentum transfer must be taken into account, a form factor is necessary that will reduce the cross section with nuclei.

Following the above procedure, the effective interactions in eq (5) are first classified into the SI and SD parts for nonrelativistic quarks:

\[
\mathcal{L}^{\text{SI}}_{x,s} = B_x(\Psi) \sum_q G^q S_q, \quad x = 5, 6, \quad (7)
\]

\[
\mathcal{L}^{\text{SI}}_{y,v} = B_y(\Psi) \sum_q G^q V_q, \quad y = 9, 10, \quad (8)
\]

\[
\mathcal{L}^{\text{SD}}_z = B^z(\Psi) \sum_q G^q A^k_q, \quad z = 1, \ldots, 4, 11, \ldots, 14, \quad (9)
\]

where \( S_q = \bar{q} q, \quad V_q = \bar{q} \gamma_0 q, \quad A^k_q = \bar{q} \gamma^k \gamma_5 q, \) and \( B_{x,y}(\Psi) \) and \( B^z(\Psi) \) are the \( \Psi \) bilinears without or with a free spatial index respectively. Although the nucleon matrix elements of the pseudoscalar quark bilinears do not vanish (see the second paper in Ref. [25]), the induced pseudoscalar nucleon bilinear has a nuclear matrix element that is suppressed by the velocity of nucleons.
Thus the operators $\mathcal{O}_{7,8}$ do not contribute at leading order to the scattering of $\Psi$ off a nucleus $N$ as in the usual practice. The scattering amplitudes for the $\Psi$-$N$ scattering are

$$\mathcal{A}_{x,s}^{SI} \approx 2m_N f_N^x B_s(U),$$

(10)

$$\mathcal{A}_{x,y}^{SI} \approx 2m_N b_N^x B_y(U),$$

(11)

$$\mathcal{A}_{z}^{SD} \approx 4m_N g_N^z (J_N) f_i B_i^z(U),$$

(12)

where $m_N$ is the mass of the nucleus with atomic mass number $A$ and charge $Z$, and $(J_N) f_i$ is the matrix element of the $k$-th component of the nuclear spin operator. The scalar SI effective coupling gets contributions from the protons and neutrons contained in $N$,

$$f_N^x = Z f_p^x + (A - Z) f_n^x,$$

(13)

$$f_p^x = \sum_{q = u,d,s} G_q^x m_{p(n)} m_q f_{Tq}^{(p(n))} + \frac{2}{27} f_{Tq}^{(p(n))} \sum_{q = c,b,t} G_q^x m_{p(n)} m_q,$$

(14)

where $m_{p(n),q}$ is the proton (neutron, quark) mass. For light quarks the constants $f_{Tq}^{(p(n))}$ are related to the pion-nucleon sigma term $[25]$, while for heavy quarks $f_{Tq}^{(p(n))} = 1 - \sum_{q = u,d,s} f_{Tq}^{(p(n))}$ enter via the trace anomaly $[26]$. The vector SI effective coupling is easiest to get since $V_q$ just counts the number of valence quarks when sandwiched between the nucleon states:

$$b_N^y = Z b_p^y + (A - Z) b_n^y, b_p^y = 2G_y^u + G_y^d, b_n^y = G_y^u + 2G_y^d.$$  

Finally, the SD effective coupling is,

$$g_N^z = \sum_q G_q^z \lambda_q^N,$$  

(16)

$$\lambda_q^N = \frac{1}{J_N} \langle S_p \Delta q^p + S_N \Delta q^n \rangle,$$  

(17)

where $\Delta q^{p(n)}$ measures the fraction of the proton (neutron) spin carried by the quark $q$ $[27]$, and $\langle S_{p(n)} \rangle$ is the expectation value of the $z$-th component proton (neutron) spin operator in the nuclear state with the highest $J_N$ $[28]$. The spin-summed and -averaged cross sections for $\Psi$-$N$ scattering at zero momentum transfer are,

$$\sigma_0 = \frac{1}{16\pi (M + m_N)^2} \sum_{\text{spins}} |\mathcal{A}|^2,$$  

(18)

where the amplitudes squared are evaluated in the standard manner

$$\sum_{\text{spins}} |\mathcal{A}_{x,s}^{SI}|^2 = (2m_N f_N^x)^2 \frac{1}{4} \sum_{\Psi \text{ spins}} B_s(U) B_s^\dagger(U),$$

(19)

$$\sum_{\text{spins}} |\mathcal{A}_{x,y}^{SI}|^2 = (2m_N b_N^x)^2 \frac{1}{4} \sum_{\Psi \text{ spins}} B_y(U) B_y^\dagger(U),$$

(20)

$$\sum_{\text{spins}} |\mathcal{A}_{z}^{SD}|^2 = (4m_N g_N^z)^2 J_N (J_N + 1) \frac{1}{4} \sum_{\Psi \text{ spins}} B_z^k(U) B_z^{k\dagger}(U).$$  

(21)
The $\Psi$ spin sums are computed using its polarization sum. In the nonrelativistic limit, this is facilitated by noting that only the spatial components are nonvanishing

$$P_{ij}(p) = M(\gamma_0 + 1) \left( \delta_{ij} + \frac{1}{3} \gamma_i \gamma_j \right).$$

The end results for the SI and SD cross sections due to various interactions are respectively

$$\sigma_0^5 = \frac{\mu^2}{\pi} \left( f_N^5 \right)^2,$$

$$\sigma_0^9 = \frac{\mu^2}{\pi} \left( b_N^9 \right)^2,$$

$$\sigma_0^{1,12,13} = \frac{\mu^2}{\pi} J_N(J_N + 1) \left( g_N^{1,12,13} \right)^2 \times \left[ \frac{20}{3}, \frac{20}{3}, \frac{80}{3} \right],$$

where $\mu = m_N M / (m_N + M)$ is the reduced mass for the $\Psi$-$N$ system. That other operators do not contribute in the nonrelativistic limit can also be understood explicitly. Since $U_\mu(p, \lambda)$ is built from $u(p, s)$ and $e_\mu(p, \kappa)$, its static limit can be readily obtained. We have $U_0(p, \lambda) = 0$ either from $p^\mu U_\mu(p, \lambda) = 0$ or by choosing physical polarizations with $e_0 = 0$. Independently of the Lorentz index in $U_\mu$, the limits for a Dirac spinor bilinear also apply to $U_\mu$, with the nonvanishing bilinears being restricted to $\bar{U}_\alpha \gamma^0 U_\beta \approx \bar{U}_\alpha U_\beta$, $\bar{U}_\alpha \gamma^i U_\beta \approx \bar{U}_\alpha U_\beta$, $\bar{U}_\alpha \sigma^{ij} \gamma^5 U_\beta$.

3 Constraints from observations and experiments

3.1 Relic density

The dark matter produced in the early universe would either be depleted too much or over dense in the current epoch, depending on the interaction strengths in eq (5). The observed value for its relic density can therefore set constraints on the relevant parameters. To obtain the relic number density $n_\Psi$, one solves the Boltzmann equation

$$\frac{dn_\Psi}{dt} + 3H n_\Psi = - \langle \sigma | v \rangle [ (n_\Psi)^2 - (n_\Psi^\text{eq})^2 ],$$

where $H = \sqrt{8\pi \rho / 3M_{\text{Pl}}^2}$ is the Hubble expansion rate and $n_\Psi^\text{eq}$ is the value at thermal equilibrium. Assuming the DM particles have negligible chemical potential we have $n_\Psi = n_\Psi^\text{eq}$ so that $n_{\text{DM}} = 2n_\Psi$. $\langle \sigma | v \rangle$ is the thermally averaged annihilation cross section for a relative velocity $v$, and can be calculated in the reference frame where one of the $\Psi$ particles is at rest [29]. Using $s = 2M^2 (1 + (1 - v^2)^{-1/2}) \approx 4M^2 (1 + v^2 / 4)$ for nonrelativistic DM particles, one expands eq (6) as, $\sigma | v | = a + bv^2 + O(v^4)$. Eq (26) is then solved numerically to yield [30],

$$\Omega_{\text{DM}} h^2 \approx 2 \times 1.04 \times 10^9 x_F \text{ GeV}^{-1}$$

$$\frac{x_F}{M_{\text{Pl}} \sqrt{g_* (x_F)} (a + 3b / x_F)},$$

$$\Omega_{\text{DM}} h^2 \approx 2 \times 1.04 \times 10^9 x_F \text{ GeV}^{-1}$$

$$\frac{x_F}{M_{\text{Pl}} \sqrt{g_* (x_F)} (a + 3b / x_F)},$$

7
where $g_*(x_F)$ is the number of relativistic degrees of freedom at the freeze-out temperature $T_F$, and $x_F = M/T_F$. The latter is solved self-consistently by

$$x_F = \ln \left[ c(c + 2) \sqrt{\frac{45}{8}} gM_{\text{Pl}}(a + 6b/x_F) \frac{2\pi^3}{g_*} \right],$$

where $c$ is an order one parameter (we take $c = 1/2$), and $g = 4$ is the spin degrees of freedom of the $\Psi$ particle. We employ the values of $g_*$ as a function of temperature $T$ obtained in Ref. [31].

Figure 1: The current relic density (dashed curves) of the spin-3/2 DM, $\Omega_{\text{DM}}h^2$, is predicted as a function of its mass for various interactions with couplings in scenarios I (left panels) and II (right panels) respectively. The horizontal solid band shows the observed range with the best-fit value, $\Omega_{\text{DM}}h^2 = 0.1123 \pm 0.0035$ [32]. The number next to the legend indicates the operator $\mathcal{O}_i$.

We show in Fig. 1 the predicted current relic density for various interactions shown in eq (5)
as a function of the mass $M$. For simplicity, we consider two scenarios for the couplings $G_i^f$. In scenario I, we assume a universal value for all interactions, $G_i^f = 10^{-5}, 10^{-6}, 10^{-7}$ GeV$^{-2}$. For the operators involving a chirality-flip bilinear of ordinary fermions, i.e., those excluding $O_{9,...,12}$, it is easy to imagine that they might be proportional to the mass of the involved fermion. We therefore study the scenario II in which $G_i^f(1 \text{ GeV}/m_f) = 10^{-5}, 10^{-6}, 10^{-7}$ GeV$^{-2}$. For the purpose of comparison we include the results for $O_{9,...,12}$ in scenario II. The predicted density decreases as the coupling $G_i$ (mass $M$) increases for a fixed mass (coupling). Also shown (horizontal band) is the range of the observed DM relic density, corresponding to the best-fit value, $\Omega_{DM} h^2 = 0.1123 \pm 0.0035$ [32]. Assuming that one of the interactions in eq (5) be responsible for the observed relic density we plot in Fig. 2 the required couplings as a function of $M$. Since $|\sigma_v|$ increases with $G_i$ and $M$, $G_i$ has to decrease as $M$ increases in order to match the observed relic density.

The predicted relic density drops abruptly when a new annihilation channel is opened with increasing $M$. Similarly, for the relic density fixed to the observed value the required coupling $G_i$ drops suddenly at each new threshold as $M$ increases. This is especially obvious in scenario II at the $\tilde{t}\tilde{t}$ threshold where the effect is significantly enhanced. Most curves fall into one of the two groups while the one corresponding to the operator $O_{12}$ stands alone. This arises from different behavior in their thermally averaged cross sections, $\langle |\sigma_v| \rangle \approx a + b\langle v^2 \rangle$, where the coefficients $a$ and $b$ correspond to the $s$- and $p$-wave annihilation respectively. While the operators $O_{1,3,5,7,10}$ only give a $b$ term, all others have both $a$ and $b$ terms. In addition, amongst the latter operators only $O_{12}$ has an $a$ term that is proportional to $m_f^2$, which explains its unique behavior in the figures.

Figure 2: Assuming one of the interactions in eq. (5) produces the observed relic density, its coupling is shown as a function of the DM mass $M$ for both scenarios I (left panel) and II (right).

### 3.2 Direct detection

The experimental results in direct detection of dark matter are conventionally presented in terms of cross sections on nucleons. We follow this practice to show our results on the cross sections $\sigma_{\Psi p(n)}^i$ for nonrelativistic scattering of $\Psi$ off a proton (neutron) due to various operators $O_i^q$. 

![Graph showing the predicted density as a function of mass M for both scenarios I and II.](image-url)
Figure 3: The spin-independent $\Psi$-proton cross sections (dashed curves) are plotted as a function of $M$ at different couplings for the scalar ($g_5^f$, upper panels) and vector ($g_9^f$, lower panels) interactions. The left panels are for scenario I with $g_{5,9}^f = 10^{-5}, \ldots, 10^{-8}$ GeV$^{-2}$, while the right panels are for scenario II with $g_{5,9}^f \times (1 \text{ GeV}/m_f) = 10^{-4}, \ldots, 10^{-8}$ GeV$^{-2}$. The solid curves are the upper bounds from the experiments CDMS II (2010) [34], EDELWEISS-II (2011) [35], XENON100 (2011) [36], XENON10 (2008) [37], and ZEPLIN-III (2011) [38].
As we explained in the last section, out of many possible interactions there are only two types of them that contribute to the SI cross section and three types to the SD one. \( \sigma_{\Psi p(n)}^{I} \) are still given by eqs \( (23, 24, 25) \) for the SI and SD cases respectively, with the following substitutions:

\[
\begin{align*}
& f_{N}^5 \rightarrow f_{p(n)}^5, \quad b_{N}^5 \rightarrow b_{p(n)}^5, \quad g_{N}^5 \rightarrow \sum_q G_{q}^5 \Delta_{q}^{p(n)}, \quad J_{N} \rightarrow 1/2, \quad \mu \rightarrow Mm_{p(n)}/(M + m_{p(n)}). \\
& f_{T_u}^{(n)} = 0.020 \pm 0.004, \quad f_{T_d}^{(n)} = 0.026 \pm 0.005, \quad f_{T_s}^{(n)} = 0.118 \pm 0.062 \quad \text{for the proton}, \\
& f_{T_u}^{(n)} = 0.014 \pm 0.003, \quad f_{T_d}^{(n)} = 0.036 \pm 0.008, \quad f_{T_s}^{(n)} = 0.118 \pm 0.062 \quad \text{for the neutron}. \\
& \text{For the nucleon spin fractions carried by quarks that are required in the SD matrix element, we assume the values in Ref. [10]: } \Delta_{u}^{p} = \Delta_{d}^{n} = 0.78 \pm 0.02, \quad \Delta_{d}^{p} = \Delta_{u}^{n} = -0.48 \pm 0.02, \quad \text{and } \Delta_{s}^{p} = \Delta_{s}^{n} = -0.15 \pm 0.02.
\]

In Fig. 3 we display the SI \( \Psi \)-proton cross sections \( \sigma_{\Psi p}^{s, 9} \) as a function of the mass \( M \) in both scenarios of couplings, while in Fig. 4 we plot the SD \( \Psi \)-neutron cross sections \( \sigma_{\Psi n}^{I, 12} = \sigma_{\Psi n}^{I, 13} \). In scenario II, both light and heavy quarks contribute equally to the scalar SI \( \sigma_{\Psi p(n)}^{s} \) while for the vector SI \( \sigma_{\Psi p(n)}^{v} \) both contributions are suppressed by either a light quark mass or a vanishingly small content of heavy quarks in the nucleon. In the same scenario, the contributions to the SD \( \sigma_{\Psi p(n)}^{1, 12, 13} \) from both light and heavy quarks are significantly suppressed by either light quark masses or tiny spin fractions of heavy quarks in the nucleon. In scenario I with universal couplings both SI and SD cross sections are dominated by light quarks. Since the current experimental bounds on the SD cross sections are several orders of magnitude weaker...
than the SI ones, the upper bounds that one can get on the SD effective couplings are also much weaker.

3.3 Indirect detection

The DM particles in our Galaxy can annihilate through the interactions in eq (5) to produce leptons and quarks that fragment and interact further with the interstellar gas to create more secondaries, including gamma rays, neutrinos, positrons and antiprotons. By comparing the observed cosmic ray fluxes with known astrophysical sources, it is possible to infer the properties of DM particles in our galactic halo and constrain their annihilation rate. We have employed the public computer code GALPROP [41] to simulate the antiproton to proton flux ratio. The code solves numerically with appropriate boundary conditions the transport equation for the number density of cosmic particles that takes into account diffusion and convection effects amongst others. For our purpose here, the source term in the equation will contain a piece due to the $\Psi\bar{\Psi}$ annihilation

$$Q_{q}^\bar{p}(r,E) = \left[\frac{\rho(r)}{2M}\right]^2 \sum_q \langle \sigma^q | v | \rangle \left(\frac{dN}{dE}\right)_q^\bar{p},$$

where the sum is over all channels of quark production, $(dN/dE)_q^\bar{p}$ is the antiproton number per unit energy produced in the $q\bar{q}$ channel, and $\rho(r)$ is the mass density distribution of the DM particles. We use the Monte-Carlo program PYTHIA [42] to simulate the $(dN/dE)_q^\bar{p}$ spectrum.

In our numerical analysis, we take the NFW profile [43]:

$$\rho(r) = \rho_\odot \left[\frac{1 + r_\odot/R}{1 + r/R}\right]^2,$$

where $\rho_\odot$ is the DM density at the solar location, $r_\odot$ the distance of the sun to the Galactic center, and $R$ the scale radius. We adopt the following values for these parameters from Table 3 in Ref.[44]: $\rho_\odot = 0.389$ GeV cm$^{-3}$, $r_\odot = 8.28$ kpc, and $R = 20$ kpc. The Galactic DM particles should follow the Maxwell-Boltzmann velocity distribution. We choose the velocity dispersion $\bar{v} \equiv \sqrt{\langle v^2(r_\odot) \rangle} = \sqrt{3/2}v_c(r_\odot)$ with $v_c(r_\odot) = 243.75\text{km s}^{-1}$ being the local circular velocity [44], so that $\langle v^2 \rangle = 2\langle v^2(r_\odot) \rangle$.

In the calculation of the $\bar{p}/p$ flux ratio with GALPROP, the diffusion region of cosmic rays is described by a thick disk of thickness $2L \approx 8$ kpc and radius $R \approx 20$ kpc, with the thin galactic disk of thickness $2h \approx 200$ pc and radius $R$ lying in the middle. The charged particles traversing the solar system are affected by the solar wind, which results in a shift in the spectrum observed at the Earth compared to the interstellar one [45, 46]. We have scanned the solar modulation potential $\Phi$ from 300 to 1000 MV, and found that $\Phi = 330$ MV yields the minimal $\chi^2$ for the background flux.

Although the PAMELA data on the $\bar{p}/p$ flux ratio can be accounted for by GALPROP based on the conventional propagation model of cosmic rays, it cannot exclude a small portion of contribution from DM annihilations. This will set a stringent bound on the annihilation cross section $\langle \sigma | v | \rangle$. By varying it within the acceptable deviation ranges of the PAMELA data and
evaluating the $\chi^2$ value, we obtain the $3\sigma$ upper bounds on the couplings $G^f_i$ for a given value of the mass $M$. The results are shown in Fig. 5 for both scenarios I and II. The behavior of the curves is quite similar to that shown in Fig. 2, but the drop is less steep as $M$ increases.

We do not consider here the PAMELA positron fraction excess and related effective interactions for a few reasons. It has been proposed that the excess could originate from some astrophysical sources such as supernova remnants or nearby pulsars that were not accounted for earlier, see for instance Ref. [48] for a status review. If the excess is due mainly to the dark matter annihilation, a strong tension arises between the excess and the relic density that is tentatively parameterized by a ‘boost factor’ as large as a few hundreds or even a thousand, whose origin however is unclear [49]. Thus the excess itself cannot yet result in useful constraints on interactions with leptons. And finally we want to work out combined constraints in the next section where only the interactions with quarks are relevant in direct detections.

![Figure 5: The $3\sigma$ upper bounds on various couplings $G^f_i$ as a function of $M$ as imposed by the PAMELA $\bar{p}/p$ spectrum [47], in scenarios I (left panel) and II (right).](image)

### 3.4 Combined constraints

In this section, we present the combined constraints from the observed relic density, direct and indirect detection data discussed in previous subsections. Since the direct detection is only sensitive to the operators $O_{1,5,9,12,13}$, we show our results for these types of interactions. In Fig 6 we show the combined constraints on their couplings $G^f_{1,5,9,12,13}$ as a function of mass $M$ in both scenarios I (left panels) and II (right ones). For the SI types of interactions $O_{5,9}$ the direct detection generally imposes a stronger constraint than the indirect detection except for $O_9$ in scenario II. For the SD types of interactions the direct detection is advantageous in some cases while the indirect detection is better in others.

When the upper bound curves from direct detection (SI and SD) and PAMELA $\bar{p}/p$ data are located below the relic-density allowed curves, they can provide more stringent constraints and be used to exclude some regions of parameters for various types of interactions. In the scenario I of universal couplings, the PAMELA data excludes respectively the operators $O_{12,13}$ in the mass ranges of $(10, 17)$ GeV and $(10, 47)$ GeV for any couplings. While the SI direct detection
Figure 6: Combined constraints on the couplings $G_{1,5,9,12,13}$ are obtained as a function of $M$ from the observed relic density, direct detection experiments of XENON10 (SD $\Psi$-neutron scattering) and XENON100 (SI $\Psi$-proton scattering), and the observed PAMELA $\bar{p}/p$ flux ratio. Left panels for scenario I and right ones for scenario II.
completely excludes the operators $\mathcal{O}_{5,9}$, the SD one dominates for the operator $\mathcal{O}_1$ and excludes it in the mass range $(12, 37)$ GeV. The situation for scenario II is similar but less stronger. For instance, $\mathcal{O}_1$ is not sensitive to either the SD direct detection or the PAMELA data so that any mass would be allowed, while only a smaller mass range $(10, 49)$ GeV for the operator $\mathcal{O}_9$ is excluded by PAMELA.

## 4 Conclusion

We have considered the option that a spin 3/2 particle acts as dark matter, and investigated the constraints on it imposed by the current observations and experiments. We worked in the approach of effective field theory and wrote down all possible 4-fermion effective interactions that involve a pair of spin-3/2 DM fields and a pair of ordinary fermion fields. Assuming one interaction at a time is responsible for the dark matter, we studied its implications on the relic density, the antiproton to proton flux ratio $\bar{p}/p$ in cosmic rays, and the elastic dark matter scattering off nuclei in direct detection. While the relic density and the flux ratio $\bar{p}/p$ are virtually sensitive to all interactions at different levels and for different scenarios of couplings, only a few are relevant to the direct detection experiments. These observational and experimental results can be employed in a complementary manner. Using the observed relic density one can predict the relation between the effective coupling and the DM mass for a given interaction. When this relation curve lies above the upper bounds set by the direct or indirect detection, we can exclude some of the parameter regions for the DM particle. For example, the SI XENON100 data excludes the whole mass range that we studied for the interactions $\mathcal{O}_{5,9}^f$ when the couplings are flavor universal. Depending on the types of interactions and scenarios of couplings, the SD direct detection data and measured flux ratio $\bar{p}/p$ can also exclude portions of mass ranges. Further precise measurements will help narrow down the survival windows thus far.

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*Notes added.* While this manuscript was being finished, a new preprint appeared [50] in which the spin-3/2 particle was also studied as a DM candidate. After this work was submitted to the arXiv, we were informed of a recent paper [51] in which effective operators involving a pair of spin-3/2 fields were studied together with other operators involving the standard model Higgs and gauge fields. A spin-3/2 particle was also proposed earlier [52] in an attempt to reconcile puzzling results in direct detections, as a charged effective degree of freedom that is bound with primordial helium to form the so-called dark atoms. We thank the authors of those papers for their electronic communications.
Appendix Functions $A_i(r, R)$

The functions $A_i(r, R)$ with $r = m_f^2/s$ and $R = M_f^2/s$ appearing in the annihilation cross section are obtained upon finishing the phase space integration:

\[
\begin{align*}
A_1 &= -\frac{13}{54} \frac{46r}{27} + \frac{1}{108R^2} + \frac{2r}{27R^2} - \frac{r}{3R} + \frac{10R}{27} + \frac{200rR}{27}, \\
A_2 &= -\frac{54}{27} - \frac{34r}{27} + \frac{1}{108R^2} + \frac{2r}{27R^2} + \frac{r}{3R} + \frac{7r}{10R}, \\
A_3 &= -\frac{54}{27} + \frac{20r}{27} - \frac{1}{108R^2} - \frac{2r}{27R^2} + \frac{5}{27R} - \frac{160rR}{27}, \\
A_4 &= -\frac{54}{27} + \frac{32r}{27} + \frac{1}{108R^2} - \frac{2r}{27R^2} + \frac{11r}{27R} - \frac{11r}{10R}, \\
A_5 &= -\frac{7}{6} + \frac{14r}{9} + \frac{1}{36R^2} - \frac{5}{9R^2} - \frac{18R}{9} + \frac{10r}{9} - 2R + 8rR, \\
A_6 &= \frac{5}{18} \frac{18R}{9} - \frac{1}{18R}, \\
A_7 &= \frac{5}{6} + \frac{36R^2}{18R} - 2R, \\
A_8 &= \frac{18}{5} + \frac{36R^2}{18R} - \frac{1}{18R}, \\
A_9 &= -\frac{2}{27} + \frac{2}{27} - \frac{1}{27R^2} + \frac{2r}{27R^2} + \frac{2}{27R^2} + \frac{27R}{27R} - \frac{4r}{27R} + \frac{4R}{3} + \frac{8rR}{3}, \\
A_{10} &= \frac{26}{27} + \frac{52r}{27} + \frac{1}{27R^2} + \frac{2r}{27R^2} + \frac{16r}{27R} - \frac{40R}{3} + \frac{80rR}{27}, \\
A_{11} &= -\frac{2}{27} + \frac{8r}{27} + \frac{1}{27R^2} - \frac{1}{27R^2} - \frac{27R}{27R} + \frac{27R}{27R} - \frac{3}{3} + \frac{3}{16rR}, \\
A_{12} &= \frac{26}{27} + \frac{128r}{27} + \frac{1}{27R^2} - \frac{4r}{27R^2} + \frac{8}{27R} + \frac{40R}{27} + \frac{280rR}{27}, \\
A_{13} &= \frac{4}{27} + \frac{184r}{27} + \frac{2}{27R^2} + \frac{4r}{27R^2} - \frac{4}{27R} + \frac{40R}{27} + \frac{800rR}{27}, \\
A_{14} &= \frac{4}{27} + \frac{200r}{27} + \frac{2}{27R^2} + \frac{4r}{27R^2} - \frac{4}{27R} + \frac{56r}{27} + \frac{40R}{27} - \frac{640rR}{27}.
\end{align*}
\]

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