Three-Nucleon Forces for the New Millennium
J. L. Friar\textsuperscript{a}

\textsuperscript{a}Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico, 87545

Most nuclear physics ranges from insensitive to relatively insensitive to many-nucleon forces. The dominant ingredient in calculations of nuclear properties is the nucleon-nucleon potential. Three-nucleon forces nevertheless play an important role in nuclear physics because of the great precision of modern calculational methods for systems of relatively few nucleons. We explore the reasons why many-body forces are weak in nuclei by using a classification scheme for such forces that is based on dimensional power counting, which is used to organize chiral perturbation theory. An assessment will be made of how close we are to a “standard” three-nucleon force. Recent advances in determining the significance of three-nucleon forces will also be discussed.

1. INTRODUCTION

The turn of the century is a good time to assess the importance and impact of three-nucleon forces (3NFs) on the development of the field of few-nucleon physics. It has been 67 years since Wigner\textsuperscript{[1]} first raised the possibility that three-nucleon forces might be significant in the triton: “... one must assume a certain potential energy ... or a three-body force.” It is significant that the triton had not yet been discovered, although he predicted it would be bound by nucleon-nucleon (NN) forces alone. Since that time we have relied on field-theoretic techniques, phenomenology, and sophisticated symmetry arguments to construct 3NFs, and the most modern and advanced experimental facilities have recently been used to validate these forces.

In a very real sense we are fortunate that three-nucleon potentials are not too strong or too weak. Indeed, I wouldn’t be giving this talk if they were. Imagine these forces to be 1 – 2 orders of magnitude stronger than they actually are. In that case 3NFs would be comparable to NN forces and (without stretching the imagination) 4NFs, ... could also be comparable. In this scenario nuclear physics would be intractable, and in all likelihood this conference would not be held. On the other hand we could imagine such forces to be 1 – 2 orders of magnitude weaker than they actually are. In this case they would play almost no role in nuclear physics, and although few-nucleon physics would be healthy and this conference would be held, the topic would be vacuous and there would be no such talk.
2. THREE-NUCLEON FORCE SCALES

What sets the scales such that our universe lies between these limits, where 3NFs are weak but significant? The answer lies in the scales associated with QCD, which I will introduce later. For better or worse, these scales allow me to stand before you today and discuss these most interesting of forces. Indeed, these scales allow a qualitative discussion of many aspects of few-nucleon physics, and I will rely on this approach to find common ground.

My first task is to estimate the size of the effect of three-nucleon forces using scales. This can be achieved by a handwaving argument that is nevertheless correct in its essence.

Figure 1. Generic nuclear force (dashed line) is shown in (a), while the additional effect of a third nucleon is indicated in (b) by a wiggly line, and the emission by a nucleon of a pion with four-momentum $q^\mu$ is illustrated in (c).

Figure (1a) shows two nucleons interacting via an NN potential, $V$ (dashed line). Adding another nucleon makes this a three-body system (Fig. (1b)) and, in addition to the normal NN interaction between the original two nucleons, that force $V$ will somehow feel the effect of the additional nucleon (wavy line), and the size of this additional effect on the energy should scale as $V \cdot V = V^2$, since all of the nucleons are the same. This quantity unfortunately no longer has the dimensions of energy and we need to divide by an additional energy scale in order to obtain a final estimate. We motivate this scale in Fig. (1c) by showing a virtual pion (with four-momentum $q^\mu$) emitted by a nucleon. Normally we ignore the time component of $q^\mu (q^0)$, which scales as the difference of kinetic energies of the initial and final nucleon; that is, it scales as $1/M$, where $M$ is the nucleon mass. Thus we might suspect that the additional effect of the third nucleon on the potential energy of the original pair of nucleons scales as

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$$\Delta V \sim \frac{V^2}{M c^2} ,$$

where we have placed a factor of $c^2$ to make the dimensions correct. This simple result, which in chiral perturbation theory has $M c^2$ replaced by $\Lambda \sim 1 \text{ GeV}$ (a generic large-mass QCD scale) gives us a quick estimate of the energy shift. Using $\langle V \rangle \sim 20-30 \text{ MeV/pair}$ we find $\langle \Delta V \rangle \sim \frac{1}{2} - 1 \text{ MeV}$, for what is either a three-nucleon force effect, a relativistic effect.
(because of the $1/c^2$), or an off-shell effect (this latter is not obvious, but is intimately related to the $q^0$ in our “derivation”; it is an essential part of the “quasipotential” \[2\] problem).

One important and obvious caveat for theorists is that it will be difficult to interpret calculations that have numerical errors greater than 1 MeV, if our goal is to understand three-nucleon forces. Indeed, we should do much better than that and restrict triton errors to $\sim 0.1$ MeV $\sim 1\%$ of the triton binding energy. In addition, 1% absolute experiments are extremely difficult and rare. Calculations with numerical errors $\lesssim 1\%$ have consequently become the standard and are called “exact,” “complete,” or “rigorous”. They are one of the biggest success stories in our field in the past 50 years.

### 3. THREE-NUCLEON SYSTEMS AND CALCULATIONS

A bit of history is always a good way to start a discussion about the future. As scientists we naturally tend to concentrate on our unsolved problems, and successes are often overlooked. In the process of giving my views on where the field is going, I will also enumerate a few of the many successes in our business, which highlight the progress that we have made\[3,4\].

One can conveniently categorize few-nucleon calculations as follows: (a) bound states (i.e., $^3$H and $^3$He); (b) Nd scattering below deuteron-breakup threshold; (c) Nd scattering above deuteron-breakup threshold; (d) transitions between bound and continuum states. All of these types of calculations have been performed, and benchmarked comparisons between different methods exist for all categories except pd scattering (i.e., including a Coulomb force between protons\[5\]) at finite energies. The ability to perform these extremely difficult calculations, especially the scattering calculations, has been one of the major successes in few-nucleon physics. When one considers this together with the incredible accomplishments of Vijay Pandharipande\[6\] and his collaborators (including my colleague, Joe Carlson) for $A > 3$, this area is one of the most successful in all of nuclear physics, and goes far beyond even the dreams that theorists had 25 years ago.

I summarize this part of the talk by noting that 1% calculations are needed in order to disentangle systematically the relatively small effects of three-nucleon forces (or relativistic effects, off-shell effects, ...). Such calculations are now possible using many different techniques. Most observables agree very well with experiment; indeed, most are insensitive to 3NFs. The trick is to find the proper observables to investigate.

### 4. HISTORY OF THREE-NUCLEON FORCES

Wigner’s mention of three-nucleon forces in his calculation of $^3$H was subsequently ignored. There was no hope then (and little now) of being able to calculate and interpret results for a strongly interacting system dominated by such forces. While we have significant and extensive experimental information on the NN force, we have very little knowledge with which to constrain three-nucleon forces. We are forced to rely almost entirely on theory, particularly on field theory. Early efforts involved primitive models that have not left their mark on the field. One calculation that has left an indelible mark is the august Fujita-Miyazawa model[7], based entirely on isobar intermediate states and pion propagation within nuclei. This was motivated by the dominant role of nuclear resonances in some processes.
Examples both familiar and unfamiliar are shown in Fig. (2). In Fig. (2a) a pion emitted by one nucleon interacts in a complicated way with a second nucleon and then is absorbed by a third nucleon. The Fujita-Miyazawa (FM) approximation to the entire process is shown in Fig. (2b). One can also replace one (or both) pions by a heavy meson (as shown in Fig. (2d)). A variety of such short-range mechanisms are depicted in chiral perturbation theory by Fig. (2c), where all of the short-range processes (induced by heavy-mass particles) are shrunk to a point. We will first discuss the $2\pi$-exchange forces.

The second noteworthy calculation was performed by one of our conference organizers, Shin Nan Yang[8]. The Yang model was the first three-nucleon potential model based on chiral-symmetry considerations, although there were previous calculations of effects based on PCAC. This model was used in a variational calculation to estimate $\langle \Delta V_{3NF} \rangle \sim 2$ MeV. This set the stage for the development of the most widely used 3NF, the Tucson-Melbourne (TM) model[9], which exploited current algebra and PCAC (and treated off-shell effects in a serious manner) in deriving that force. The various parameters of this model incorporate phenomenology (including isobars) in a more meaningful way than the FM model. In addition to the models already mentioned, there are the Urbana-Argonne (UA)[10] [an offshoot of FM], the RuhrPot[11], the Brazil[12], and the [χPT-based] Texas models[13].

It is not a good thing to have so many different models for what should be a single correct physical force. Indeed, one of our tasks for the new millennium is to force a “convergent evolution” on these models by incorporating proper amounts of the correct physics. Recent progress has been made in that direction. The UA model now incorporates a chiral-symmetry-breaking term (the “a” term of TM) while a direct appeal to $\chi$PT[14] leads to the elimination of a rather unimportant term of short-range + pion-range character (the “c” term) in the TM model, leaving the $2\pi$-exchange parts of the two models essentially equivalent except for minor differences in parameter values. Thus $\chi$PT has produced a recent unification of $2\pi$-exchange forces.

We emphasize that in order to accomplish this we must incorporate two key pieces of physics:
• adequate phenomenology (such as isobars)
• chiral constraints.

5. QCD AND $\chi$PT

Before discussing the road to the future for 3NFs that have a short-range component, it is necessary to implement an organizational scheme. There are simply too many possible operators that can contribute. Chiral perturbation theory fortunately allows us to sort 3NFs into classes and identify the terms that should be dominant. Indeed, this sorting process is the organizational scheme of $\chi$PT. How does it work?

The “natural” degrees of freedom of QCD are quarks and gluons, whose interactions manifestly reflect the symmetries of QCD. We are not required to use these degrees of freedom, however, and the traditional degrees of freedom of nuclear physics, namely nucleons and pions, are the most effective ones. If we imagine somehow mapping QCD expressed in terms of quarks and gluons onto the Hilbert space of all particles, and then freezing out the effect of the heavy particles (e.g., all nucleon resonances and all mesons with mass $\Lambda \gtrsim 1$ GeV) we arrive at chiral perturbation theory[15]. The freezing-out process is familiar in nuclear physics as Feshbach $[P,Q]$ reaction theory[16], and is known in that context to lead to complicated operators. It is nevertheless possible to implement the important chiral-symmetry constraints of QCD in this “QCD in disguise” theory. Even more important is the power counting that makes this scheme work as a field theory[17].

Power counting is a kind of dimensional analysis based on (only) two energy scales associated with QCD. One scale is $f_\pi$, the pion (beta-)decay constant ($\sim 93$ MeV) that controls the Goldstone bosons (such as the pion), while the second is $\Lambda \sim 1$ GeV, the scale of QCD bound states (the $\rho$ and $\omega$ mesons, nucleon resonances, etc.), above which we agree to freeze out all excitations. That these scales are all that is needed is not only not obvious, but it’s a little bit miraculous! Using these scales it can be shown that a given (Lagrangian) building block should scale as

$$L^{(\Delta)} \sim \frac{c}{f_\pi^3 \Lambda^\Delta} \text{ (times various fields)} .$$

Two vital properties of this simple construction are: (1) $\Delta \geq 0$ because of chiral symmetry; (2) $c$ is a dimensionless constant that satisfies $|c| \sim 1$, which is the condition of “naturalness”. The latter is also not obvious, but is extremely important. If $|c|$ could vary over many orders of magnitude in a problem, this organizational scheme would be useless. Moreover, unless positive powers of $\Lambda$ exist in the denominator (even in the presence of vacuum fluctuations) this would not lead to an expansion in powers of (small/large).

This formal scheme can be implemented in nuclei to estimate the sizes of various types of operators in the nuclear medium[18]. An additional nuclear scale is needed in order to characterize the medium, and this is given by the effective nuclear momentum (or inverse correlation length): $Q \sim m_\pi c$, where $m_\pi$ is the pion mass. Then it is possible to show that the one-pion-exchange NN potential satisfies[18]

$$\langle V_\pi \rangle \sim \frac{Q^3}{f_\pi \Lambda} \sim 30 \text{ MeV/pair} ,$$
and

\[ \langle V_{3NF} \rangle \sim \frac{Q^6}{f^2 \Lambda^4} \sim \frac{\langle V_{\pi} \rangle^2}{\Lambda} \sim 1 \text{MeV/triplet}, \]  

(4)

and we have reproduced our previous result with \( \Lambda \sim Mc^2 \). Note that this size estimate applies only to the leading-order terms; smaller terms exist that might be significant in special situations.

6. STATUS OF 3NF CALCULATIONS

There are 7 basic types of 3NFs in leading order. Four are of two-pion range, two are of mixed pion-range + short-range, and there is a class of short-range + short-range forces. Figure 2 shows several examples. The generic two-pion-exchange force is given by Fig. (2a) and can be broken down into the “a”, “b”, and “d” terms of the TM force, plus the so-called Born terms. The latter have been derived but have never been used in their entirety (there are many terms) in any \(^3\text{H}\) calculation. The two mixed terms are those represented generically in Fig. (2c), one specific mechanism in this category being that of Fig. (2d) (the so-called \(d_1\)-term). These terms have been investigated only once or twice. Finally, there are purely short-range terms of the type incorporated in the UA 3NF.

In the near future it will be necessary to investigate thoroughly the importance of this set. Most urgent are the Born terms. The local terms are almost certainly unimportant compared to the isobar mechanism, but none of the nonlocal terms have been incorporated into existing calculations. A preliminary and not wholly satisfactory set of calculations exists for the mixed short-range + pion-range forces. These need to be extended and refined. Finally, it is by no means certain that the effects of these different forces are entirely linear when added together (as indicated in Ref. [20]). Thus a lot of different calculations need to be performed in different combinations and for as many different observables as possible. Completion of this exercise will indeed provide us with a solid base in this area from which we can extend few-nucleon physics into the new millennium.

7. EVIDENCE FOR THREE-NUCLEON FORCES

We have postponed until the end a discussion of evidence for these forces, both direct and indirect. The indirect evidence is strong but not compelling. With all modern “second-generation” NN forces the triton is underbound by roughly \( \frac{1}{2} - 1 \) MeV, in agreement with our earlier estimate, and \(^4\text{He}, \ldots\) are also underbound. Unless our understanding of these NN forces is badly deficient, such NN forces require an additional three-nucleon force.

Better evidence is provided by a recent analysis of the tail of the \(pp\) potential, which generated very strong support for calculations of two-pion-exchange forces obtained from \(\chi\)PT. Several effective coupling constants (for pion-nucleon scattering operators) were fit in that analysis, and they agree with the same couplings that are used in two-pion-exchange 3NFs, validating the latter mechanisms. In other words, once the building blocks have been established it makes little difference what those blocks are used to construct.

Direct evidence is available in the Sagara discrepancy, which is shown in Fig. (3) in the differential cross section for \(pd\) scattering at 65 MeV. If we ignore the effects of
the Coulomb interaction in the forward direction, agreement between the experimental data and the calculation with NN forces alone (dashed line) is very good, except in the diffraction minimum where the data lie above the calculation. If one adds a 3NF the solid curve results, which nicely fills in the minimum, and agreement with the data is quite good. This is rather strong evidence for 3NFs and it exists at other energies. An estimate of the effect of the 3NF alone that is based on DWBA is shown in the long-dashed curve. This general behavior is very familiar in Glauber scattering at high energy, where a dominant single-scattering term falls rapidly with increasing angle, until the double-scattering amplitude (which decreases more slowly with angle) becomes dominant, and so on.

![Figure 3](image.png)

Figure 3. Differential cross section for 65 MeV proton-deuteron scattering, showing calculations with $N-N$ forces only (dashed lines), a full calculation that includes the TM 3NF (solid line), and an estimate of the effect of the 3NF alone (long-dashed line).

Finally, we discuss the effects of the short-range + pion-range terms, and in particular the one depicted in Fig. (2d). This is the so-called $d_1$-term that has recently been shown[20] to have a nonnegligible effect on the nucleon asymmetry ($A_y$) scattering observable. That observable has been a serious problem for theorists for a long time. Calculations using NN forces alone are significantly smaller than the data, particularly at low energies (e.g., 3 MeV), both for the pd and nd scattering. A variety of explanations have been proposed (this observable is very sensitive to spin-orbit interactions), but the most
plausible is the 3NF mechanism \[23\]. An example of this is Nd scattering at 3 MeV. Calculations that incorporate only an NN force are about 30% smaller than the experimental data at the maximum. Incorporating the TM 3NF removes about $\frac{1}{4}$ of the discrepancy. Adding the $d_1$-term (that we discussed before) with a dimensionless strength coefficient $c_1 = -1$ removes another $\frac{1}{4}$ of the discrepancy. Technical problems prevented calculations with stronger versions of this 3NF. It is nevertheless clear that 3NFs of various types make significant contributions to this observable. Much more work is needed on this problem. We note that discrepancies also exist for electromagnetic reactions and in the four-nucleon problem.

Special circumstances may dictate that classes of 3NF operators smaller than leading order will play a role. An example of this is neutron matter (or neutron-rich nuclei). The isospin dependence of three-nucleon forces takes 3 forms: $\tau_1 \cdot \tau_2 \times \tau_3$, 1, and $\tau_i \cdot \tau_j$. The first vanishes for three neutrons. Because 3 neutrons exist in a $T = \frac{3}{2}$ state, only the projection $(3 + \tau_i \cdot \tau_j)/4$ contributes to that state, while the projection $(1 - \tau_i \cdot \tau_j)/4$ vanishes. Some mechanisms (such as isobar configurations) that prefer large isospins may be enhanced, as shown by Vijay Pandharipande and his collaborators. In these special circumstances the dimensionless isospin factors (which typically average to about 1) can conspire to give enhanced coupling strengths. This has not yet been investigated in detail, but it should be.

8. SUMMARY AND PROGNOSIS

We have made great advances in the past 25 years in our understanding of both three-nucleon systems and three-nucleon forces. We stand poised to make further advances, based on recent technical developments. Hopefully we will soon be able to develop a consensus “standard model” of 3NFs with all significant features incorporated, which will then allow us to pursue three-nucleon physics into the new millennium.

We summarize this section as follows.

- Most three-nucleon observables are insensitive to 3NFs.
- 3NFs are small in size but appear necessary for the $^3$H binding energy, the Sagara discrepancy, and the $A_y$ puzzle.
- Chiral symmetry provides a unified approach to 3NFs; power counting identifies dominant mechanisms.
- The leading-order (dominant) $2\pi$-exchange 3NFs have been calculated; they have large isobar contributions.
- New short-range plus pion-range mechanisms may help resolve the $A_y$ puzzle.
- Although much remains to be investigated, a consensus appears to be developing for the bulk of 3NF terms.

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