A three-degree-of-freedom bistable piezoelectric energy trapping method

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Abstract: The use of piezoelectric energy harvester to convert environmental low-frequency vibration energy into electrical energy has become a research hotspot in the field of new energy. However, the piezoelectric energy harvester developed at present has high natural frequency and narrow energy trapping band, which has become one of the main reasons restricting the application of this technology. To solve this problem, based on the principle of nonlinear three degrees of freedom, this paper proposes a three-degree-of-freedom bistable piezoelectric energy trapping method. Firstly, the three-degree-of-freedom bistable energy capture method is modeled, and the energy capture frequency and nonlinear force of the model are derived; then use COMSOL software to simulate the model. The simulation results prove that this model not only reduces the natural frequency of the piezoelectric energy harvester, but also the introduced nonlinear magnetic force significantly broadens the energy trapping band. Finally, the actual measurement and comparative analysis of the proposed method prove the effectiveness of the proposed method.

1. Introduction

In recent years, the use of piezoelectric energy harvester to convert environmental low-frequency vibration energy into electrical energy has become a research hotspot in the field of new energy. At present, the conversion of force to electricity realized by piezoelectric energy harvesters is a linear conversion with low conversion efficiency. The main reason is that the natural frequency of the cantilever beam in the energy harvester is higher than the vibration frequency of the environment. In response to this problem, Xiaoqing Ma et al\textsuperscript{[1]} studied the influence of the position of the mass block on the natural frequency of the cantilever beam, and proposed an optimization method for the best position of the mass block; Zhao Wei et al\textsuperscript{[2]} analyzed the influence of the length, thickness and substrate thickness of the piezoelectric ceramics on the natural frequency of the piezoelectric cantilever beam, and proposed an optimization method for the piezoelectric cantilever beam structure; A. Mukhanov et al\textsuperscript{[3]} proposed a method for optimization design of bimorph piezoelectric cantilever beam.

The above method reduces the natural frequency of the piezoelectric cantilever beam to a certain extent, but the energy trapping ability of the energy trap is not greatly improved. The problem is that the vibration frequency of the environment is constantly changing, and the power generation of the improved piezoelectric cantilever beam is only increased at the first-order resonance frequency, and the working frequency band is narrow. In response to this problem, literature\textsuperscript{[4-6]} proposed a nonlinear
bistable energy trapping method, which has made great progress in broadening the energy trapping frequency of piezoelectric cantilever beams. However, the premise of the method is that the bistable system can vibrate on the high-energy orbit between the wells under the environmental low-frequency and low-amplitude vibration conditions. This problem has become one of the main reasons restricting the application of bistable systems in practice, and has received widespread attention. For example, Wanga HY et al. [7] proposed a two-degree-of-freedom bistable piezoelectric power generation system based on magnetic coupling. Under low-frequency and low-amplitude vibration conditions, this system can achieve bistable inter-well vibration to a certain extent. Compared with traditional bistable state, it broadens the output frequency band of electric energy; Thomas H et al. [8] constructed an electromagnetic bistable generator model and analyzed the vibration conditions of the model on the high-energy orbit between the traps; Zhang Yu et al. [9] theoretically analyzed the vibration conditions of the bistable state on the high-energy orbit between the wells, and established a bistable piezoelectric energy trapping dynamic model with an elastic amplifier.

The above-mentioned research has provided research foundation and experience for the practical application of bistable system, but the ability to broaden the energy capture bandwidth of bistable system is still limited. Based on the existing research, this paper proposes a three-degree-of-freedom bistable piezoelectric energy trapping method, which aims to further improve the energy trapping efficiency of the bistable system. Firstly, the three-degree-of-freedom bistable piezoelectric energy harvester is modeled and theoretically analyzed; then simulated the theoretical research results to verify the correctness of the theoretical research results; finally, it is compared with the method in the literature to verify the effectiveness of the method proposed in this article.

2. Bistable system and problem analysis

![Bistable energy capture system](image)

Figure 1(a) is a bistable piezoelectric energy trapping device [10], which introduces magnetic force on the basis of a traditional piezoelectric cantilever beam. Figure (b) is the bistable potential energy function curve of Figure (a). It can be seen from the figure that there is a potential barrier between the double wells of the bistable potential energy function. Under external excitation, the bistable system in Figure (a) can only improve the energy trapping ability of the system when it vibrates on the high-energy orbit between the wells (as shown in Figure (b) in c). That is, only high-amplitude excitation can realize the high-energy orbital vibration between the wells.

Environmental vibrations are mostly low-frequency and low-amplitude vibrations, and they can only perform single-well small-period vibrations (as shown in a in Figure (b)). The output characteristics are similar to linear systems, and the energy capture efficiency of power generation devices is relatively low. The current research can make the bistable state in the alternating vibration state of a and b (as shown in b in Figure (b)), but cannot stabilize the vibration of the bistable system on the high-energy orbit between the wells. This is mainly because the current research is still limited in broadening the energy capture bandwidth of the bistable system. In order for the bistable system to be applied in practice, it is necessary to further broaden the energy capture frequency band of the bistable system.
3. Three-degree-of-freedom bistable piezoelectric energy harvester modeling and theoretical analysis

Figure 2 Three-degree-of-freedom bistable piezoelectric energy harvester model

Based on the principle of nonlinear three-degree-of-freedom, this paper proposes a three-degree-of-freedom bistable piezoelectric energy harvester model. Figure 2 shows the physical structure of the model. The physical model in this paper consists of a pair of permanent magnets, piezoelectric cantilever beams, two-stage transmission beams and support frames. Among them, the transmission beam is made of elastic material, and the piezoelectric cantilever beam is composed of a metal substrate, a piezoelectric ceramic sheet and a magnet block. The basic principle of the model can be explained by formula (1)[11]:

\[ U(x) = \frac{1}{2} K_0 l^2 x^2 + U_{mb} \]

In the formula, \( U(x) \) is the potential energy of the model, \( U_{mb} \) is the magnetic potential energy of the system, \( x \) is the displacement of the cantilever beam oscillator, and \( K_0 \) and \( l \) are the parameters related to the structure. It can be seen from equation (1) that the potential energy of the model in this paper is formed by the superposition of elastic potential energy and magnetic potential energy. The nonlinear restoring force of the model is realized by the magneto-elastic force between two magnets.

3.1. Natural frequency analysis of the model

For the system with \( n \) degrees of freedom, the centralized parameter modeling method is adopted, and the dynamic model[12] is as follows:

\[ M \ddot{x} + K x = 0 \]  

(2)

In the formula, \( x = (x_1) \) is the coordinate array; \( M = (m_{ij}) \) is the function of generalized coordinates, which represents the mass matrix; \( K = (k_{ij}) \) is the generalized coordinate function, which represents the stiffness matrix. And \( K \) and \( M \) are both symmetric positive definite matrices of order \( n \). From the related knowledge of advanced mathematics, the special solution that satisfies the initial conditions is:

\[ x_j = A_j \sin(\omega t + \theta) \quad (j = 1,2,\ldots,n) \]  

(3)

In the formula, \( A \) and \( \theta \) represent the amplitude and initial phase angle of the free vibration of the system, which depend on the initial conditions. The above formula indicates that when the position coordinates of each point in the \( n \)-order system deviate from the equilibrium position, they all perform resonant motion with different amplitudes, the same frequency and the same initial phase angle. The matrix form is:

\[ x = A \sin(\omega t + \theta) \]  

(4)

Among them, \( A = (A_j) \) is the \( n \)-order array matrix composed of the amplitude of each coordinate. The generalized equations of the matrices \( K \) and \( M \) can be obtained:

\[ (K - \omega^2 M)A = 0 \]  

(5)

Let \( \lambda = \omega^2 \), then:

\[ (K - \lambda M)A = 0 \]  

(6)

In order to make matrix \( A \) have a non-zero solution, the following conditions are met:

\[ |K - \lambda M| = 0 \]  

(7)
For a two-degree-of-freedom cantilever beam system, the stiffness matrix and mass matrix of the system are:

\[
K = \begin{bmatrix}
k_1 + k_2 & -k_0 \\ -k_0 & k_0
\end{bmatrix}
\]  
(8)

\[
M = \begin{bmatrix}
m_1 & 0 \\ 0 & m_0
\end{bmatrix}
\]  
(9)

Combine the above formulas to get the eigen equation of the system:

\[
\begin{bmatrix}
k_1 + k_0 - \lambda m_1 & -k_0 \\ -k_0 & k_0 - \lambda m_0
\end{bmatrix} = 0
\]  
(10)

Expand the above eigen equation into:

\[
\lambda^2 - \left(\frac{k_0}{m_0} + \frac{k_1 + k_0}{m_1}\right)\lambda + \frac{k_0 k_1}{m_0 m_1} = 0
\]  
(11)

Solving the above equation, we can get two roots, \(\omega_1^2\) and \(\omega_2^2\).

For a three-degree-of-freedom system, the stiffness and mass matrices are:

\[
K = \begin{bmatrix}
k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_0 + k_0 & k_0 \\ 0 & -k_0 & k_0
\end{bmatrix}
\]  
(12)

\[
M = \begin{bmatrix}
m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3
\end{bmatrix}
\]  
(13)

Combining the above formulas, the eigen equation of the three-free system is:

\[
\lambda^3 - \left[\frac{k_0}{m_0} + \frac{k_1 + k_2}{m_1} + \frac{k_0 + k_2}{m_2}\right]\lambda^2 + \left[\frac{k_0 k_2}{m_0 m_2} + \frac{k_0 (k_1 + k_2)}{m_1 m_2} + \frac{k_1 k_2 + k_0 (k_2 + k_1)}{m_1 m_2}\right] \lambda - \frac{k_0 k_1 k_2}{m_0 m_1 m_2} = 0
\]  
(14)

In the same way, solve the above equations and get three roots, which are the first three resonance frequencies: \(\omega_{11}^2\), \(\omega_{22}^2\), \(\omega_{33}^2\).

Assuming \(m_1 = m_2 = m, m_0 = 2m, k_1 = 2k, k_2 = k_0 = k\). Derive the natural frequency of a single degree of freedom system \(\omega_{11}^2 = 0.5k/m\), natural frequency of two-degree-of-freedom system is \(\omega_{21}^2 = 0.31k/m\), \(\omega_{22}^2 = 3.18k/m\). The natural frequency of the three-degree-of-freedom system is \(\omega_{31}^2 = 0.16k/m\), \(\omega_{32}^2 = 1.67k/m\), \(\omega_{33}^2 = 3.66k/m\). Comparing these three degrees of freedom cantilever beam structure, it is found that \(\omega_{31}^2 < \omega_{21}^2 < \omega_{32}^2 < \omega_{22}^2\).

In summary, the natural frequency that can be reduced by increasing the degree of freedom of the system, and compared to the two-degree-of-freedom structure, the first two-order natural frequencies of the three-degree-of-freedom structure are also significantly reduced.

3.2. Non-linear forces of the model
Assuming that the magnetic fields in magnets A and B are evenly distributed, the geometric relationship between the two magnets is shown in Figure 3. The force model between the magnetic dipoles is used to analyze the magnetic potential energy between the magnets.

\[B_{BA} = \frac{\mu_0}{4\pi} \left[3(m_B \times r_{BA})r_{BA} - \frac{m_B}{|r_{BA}|^3}\right]
\]  
(15)

Figure 3 Magnetic force model

The magnetic induction intensity of magnet B at A is:
In the formula, \( \mu_0 \) is the permeability of the medium, \( m_A \) and \( m_B \) are the magnetic dipole pitch of magnets A and B respectively, and \( r_{BA} \) is the distance between magnets A and B, which can be expressed as:

\[
m_A = [M_AV_A \cos \alpha, M_AV_A \sin \alpha, 0] \quad (16)
\]
\[
m_B = [-M_TV_B, 0, 0] \quad (17)
\]
\[
r_{BA} = [-d, Z, 0] \quad (18)
\]

In the above formula, \( M_A \) and \( M_B \) are the magnetization intensity of the permanent magnet, \( V_A \) and \( V_B \) are the volume of the permanent magnet, \( Z \) is the amplitude of the vibration displacement of the permanent magnet, and the potential energy between the two magnets can be expressed as:

\[
U_m = -B_{BA} \cdot m_A = \frac{\mu_0 M_AV_AV_B \left(2d^2 - 3dZ^2 - Z^2\right)}{4\pi \sqrt{Z^2 + 1(Z^2 + d^2)^5}} \quad (19)
\]

The nonlinear magnetic force received by magnet A is:

\[
F_m = \frac{\partial U_m}{\partial Z} \quad (20)
\]

When the piezoelectric cantilever is excited by vibration, if there is no magnetic force, the system only has elastic potential energy. Through the above analysis, it can be seen that after introducing magnetic force to the system, when the piezoelectric cantilever is excited to vibrate up and down, the magnet provides magnetic potential energy for the system. At this time, the energy of the energy capture system is formed by the superposition of elastic potential energy and magnetic potential energy, and the system has enough power to cross the potential barrier and move on the high-energy orbit.

4. Model performance simulation analysis

The purpose of designing this simulation is to verify that the three-degree-of-freedom bistable piezoelectric trap model proposed in this paper can effectively expand the energy trapping bandwidth. Since piezoelectric energy capture involves the effects of multiple physical fields, this article uses COMSOL Multiphysics software to simulate, mainly to complete the following three analysis tasks: the first is to verify the theoretical analysis results; the second is to analyze the influence of magnetic force on the model's energy capture; the third is to analyze the output characteristics of the model.

According to existing research, the smaller the Young’s modulus of the cantilever substrate material and piezoelectric material, the lower the natural frequency of the cantilever beam. Therefore, this paper uses copper as the substrate material of the cantilever beam, and the piezoelectric material is PZT-5H. The transmission beam of the model uses PMMA with excellent elasticity and insulation, and the permanent magnet is neodymium iron boron, a rare earth material. The structural parameters and material parameters of the model are shown in Table 1.

| Component name      | Material     | Length/width/height (mm) | Density (g/m³) | Young’s modulus (GPa) | Poisson’s ratio |
|---------------------|--------------|--------------------------|----------------|-----------------------|----------------|
| Metal substrate     | Copper       | 50x8x0.5                 | 8920           | 120E9                 | 0.34           |
| Piezoelectric material | PZT-5H     | 40x8x0.3                 | 7500           | 7649                  | 0.3            |
| Magnet              | NdFeB        | 8x8x0.5                  | 7500           | 160E9                 | 0.24           |
| First-order beam    | PMMA         | 40x10x0.2                | 1190           | 3E9                   | 0.4            |
| Second-order beam   | PMMA         | 40x10x0.4                | 1190           | 3E9                   | 0.4            |

4.1. Natural frequency simulation of the model

The traditional cantilever beam and the three-degree-of-freedom bistable energy harvester were analyzed in the frequency domain. The output characteristics of the frequency range from 0 to 200 Hz were analyzed under the same load force and other factors, as shown in Figure 4.
It can be seen from Figure 4 that the first-order natural frequency of the single-degree-of-freedom system is between 80 and 90 Hz; the three-degree-of-freedom bistable energy trap has three peaks between 0 and 200 Hz. The first-order and second-order natural frequencies are significantly lower than the single-degree-of-freedom system, and the third-order natural frequencies are also close to the first-order natural frequencies of the single degree of freedom. The natural frequency of the three-degree-of-freedom bistable energy harvester is significantly reduced, which is consistent with the theoretical value. Therefore, compared with the traditional energy harvester (single degree of freedom), the three-degree-of-freedom bistable energy harvester significantly improves the energy harvesting capacity. The output characteristics of the energy harvester are also related to its mode shape. When vibrating in the z direction, the output voltage amplitude at the resonance frequency is higher, and when vibrating in the x-y direction, the output voltage amplitude at the resonance frequency is lower. Figure 5 shows the first three modes of the two energy harvesters.

4.2. The influence of magnetic force on model energy capture

Different from the linear system, the bistable system adds a pair of magnets. From the perspective of finite element analysis, this is equivalent to adding another physical field. There are two ways to add magnetic field force to the finite element model:
One is to create an air domain in the finite element model, and directly add magnetic fields and magnets. Since the nonlinear magnetic force changes dynamically with the vibration of the cantilever beam, the calculation amount of this method will be very large; the second is to first establish a finite element model of a pair of magnets, perform a fitting analysis of its nonlinear magnetic force, and express it as a function of the vibration displacement of the piezoelectric cantilever beam, and then add it as a vibration excitation to the linear finite element model. This method reduces the amount of calculation in the analysis process. This article adopts method two, and the finite element magnet model established is shown in Figure 6.

![Finite element magnet model](image)

Figure 6 Finite element magnet model

Calculate the magnetic distance \( d \) and the magnetic force \( F \) according to the model, and then fit the curve in MATLAB according to the calculated value, and obtain the relationship between the magnetic force \( F \) and the magnetic distance \( d \) as shown in Equation 20; figure 7 is based on the equation 20 draw the fitted curve.

\[
F_1 = -0.0002 \times d^3 + 0.008 \times d^2 - 0.169 \times d + 1.497
\]

Fig. 7 Variation curve of magnet F with magnetic spacing d

It can be seen from Fig. 7 that there is a nonlinear magnetic force between the magnet spacing \( d \) between 0 and 16mm. When \( d > 16 \)mm, the nonlinear magnetic force disappears and the system degenerates into a linear system.

![Output voltage vs. frequency](image)

(a) No magnetic force (b) Magnetic force

Figure 8 The influence of magnetic force on energy capture

In order to verify the influence of magnetic force on energy capture, the comparative simulation shown in Figure 8 was done. Figure 8(a) is the relationship between the output of a linear three-degree-of-freedom energy trap without magnetic force and the frequency; Figure 8(b) is the relationship between the output of a three-degree-of-freedom bistable energy trap with magnetic force and the
frequency. It can be seen from Fig. 8 that, compared with no magnetic force, the voltage amplitude of
the model output is significantly higher when there is magnetic force, the resonance frequency band is
obviously widened, and the resonance frequency band shifts to low frequency.

4.3. Analysis of model output characteristics
In order to verify the three response characteristics of the bistable system at different vibration
frequencies, and to verify whether the model in this paper can vibrate on a high-energy track, the
following simulation analysis is performed: apply a harmonic force $F = 1N \times \sin(2\pi \omega t)$ to the free end
of the model. According to the actual situation of environmental vibration, three vibration frequencies
are selected as the excitation of the model, namely $\omega = 10Hz$, $\omega = 20Hz$, $\omega = 30Hz$, set the magnetic spacing
to 3mm, and calculate the phase plane, output voltage and vibration displacement of the model under
three frequency excitations. The results are shown in Figure 9 to Figure 11.

It can be seen from the above figure that when the excitation frequency $\omega = 10Hz$, the model vibrates
in a small period around a balance point, and the output displacement vibration amplitude is small,
resulting in a small output voltage; when the vibration frequency increases to 20Hz, the model large
periodic vibrations can be made around two equilibrium points, the displacement amplitude of the
vibration is greatly improved, and the output voltage is also greatly increased; when the vibration frequency reaches 30Hz, the vibration of the model is stable on the high-energy orbit, and the vibration displacement is the largest at this time, and the output The voltage is also the largest.

5. Model performance measurement analysis

5.1. Experimental system and principle
The purpose of designing this experiment is to verify the effectiveness of the model proposed in this article. The experimental system in this article is composed of signal generator, power amplifier, oscillator, traditional cantilever beam, three-degree-of-freedom bistable energy trap, data acquisition card and computer; among them, the three-degree-of-freedom bistable energy harvester is a real object made according to the model in this article. The principle of the experimental system is: the signal generator outputs an excitation signal of a certain frequency. After the signal is amplified by the power amplifier, the oscillator is excited to vibrate. The energy trap is placed on the oscillator and vibrates with the oscillator. The output voltage signal passes through the data. The acquisition card collects, input into the computer and use LabVIEW software for processing, calculation and display. Figure 12 is a schematic diagram of the experimental system in this article.

![Figure 12 schematic diagram of the experimental system](image)

5.2. Experimental methods and measured results
The voltage amplitude output by the signal generator is 5Vpp and remains unchanged; the output frequency starts from 0 and gradually increases in steps of 2 Hz, and ends at 200 Hz. Observe the relationship between the output of traditional cantilever beam, linear three-degree-of-freedom energy harvester and three-degree-of-freedom bistable energy harvester with excitation frequency. Figure 13 is a graph drawn based on experimental data.

![Figure 13 Experimental frequency response diagram of piezoelectric cantilever beam](image)

It can be seen from Figure 13 that the natural frequency of the three-degree-of-freedom system is significantly lower than that of the single-degree-of-freedom system, indicating that the increase in degrees of freedom is beneficial to reducing the natural frequency of the energy capture system. It can also be seen from Figure 13 that the bandwidth of the bistable system is significantly greater than that of the linear system, indicating that the increased magnetic force of the bistable system can broaden the
energy capture band of the system. The experimental data strongly shows that the three-degree-of-freedom bistable energy harvester designed in this paper can effectively reduce the natural frequency of energy capture, and can also effectively broaden the energy capture frequency band of the system, which is beneficial to energy harvesting in low-frequency environments.

6. Conclusion
This paper proposes a three-degree-of-freedom bistable energy capture method. Compared with the single-degree-of-freedom and two-degree-of-freedom energy capture methods, the energy capture frequency is significantly reduced; compared with the linear three-degree-of-freedom energy capture method, it is clearly broaden the energy capture band. The research in this paper can show that increasing the degree of freedom of the system within a certain range can reduce the energy trapping frequency; introducing nonlinear magnetic force into the system can broaden the energy trapping frequency band.

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