Is the K-quantum number conserved in the order to chaos transition region?

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Abstract

The study of the order-to-chaos transition in nuclei is here made by investigating the validity of the K-quantum number in the warm rapidly rotating $^{163}$Er nucleus. The variance and covariance of the spectrum fluctuations of the $\gamma$-cascades feeding into low-K and high-K bands are analyzed. Low-K bands are found to be fed by a much larger effective number of cascades than high-K bands. The covariance between pairs of gated spectra shows that the cascades feeding low-K bands are different from those feeding high-K bands. The data are compared to simulated spectra obtained using energy levels and transition probabilities calculated with the band mixing model including the residual interaction and a term representing the effect of the K-quantum number on the rotational energy. K-selection rules are found to be obeyed for decay along excited unresolved rotational bands of heat energy up to around 1.2 MeV and angular momenta $30\hbar \leq I \leq 40\hbar$, forming ridges in two-dimensional $\gamma$-$\gamma$ spectra. At higher heat energy, from about 1.2 to 2.5 MeV, the selection rules are found to be only partially valid.

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The conditions under which $K$, the projection of aligned nucleonic angular momenta on the symmetry axis in deformed nuclei, is a good quantum number remain a topic of much current interest, as testified by the extensive experimental work on high-$K$ isomers [1]. The study of nuclear states with high values of the $K$-quantum number is interesting both from the point of view of the decay out from such states but also in connection with their feeding, which allows to investigate the validity of the associated selection rules at higher excitation energies. In fact, as it was stated by B. Mottelson [2], the question of $K$-quantum number violation in thermally excited states is a key issue in the study of the transition between ordered and chaotic many-nucleon motion caused by the residual interaction and the high level density. This problem has been addressed by studying the $\gamma$-decay from neutron resonances at energy $U \approx 8$ MeV [3, 4] and by studying the $\gamma$ transitions of quasi-continuum nature emitted by nuclei formed in fusion reaction, which are probing the energy region extending up to $\approx 4$ MeV [5, 6]. Since the violation or persistence of the $K$-quantum number depends on the thermal excitation energy, it becomes particularly interesting to focus the attention to where the order-to-chaos transition is predicted to take place, namely at $U \approx 1-2$ MeV [7, 8].

In this paper we present a new study of the warm rotational motion in the $^{163}$Er nucleus based on the measurement of the $\gamma$-transitions forming quasi-continuum patterns in $\gamma-\gamma$ spectra and populating specific configurations with different values of the $K$-quantum number. The data are analyzed with the fluctuation and covariance analysis technique [9, 10, 11]. This particular nucleus has been previously studied [5] and it represents a good case for further and more detailed investigations of the validity of selection rules in the order-to-chaos transition region. Two novelties are presented. First, a more detailed experimental investigation is carried out for the warmest part of the decay. Secondly and most important, a direct and rather realistic comparison between experiment and theory is made for the first time. This comparison is based on simulated spectra constructed using recent calculations on this specific nucleus [12].
The experiment was carried out using the EUROBALL array at the IReS Laboratory (France), employing the reaction $^{18}\text{O} + ^{150}\text{Nd}$, at $E_{\text{beam}} = 87, 93$ MeV. The $^{150}\text{Nd}$ target was made of a stack of two thin foils for a total thickness of 740 $\mu g/cm^2$. The corresponding maximum angular momentum reached in the reaction has been calculated to be 40 and 45 $\hbar$, respectively. Energy-dependent time gates on the Ge time signals were used to suppress background from neutrons. A total of $\approx 3 \times 10^9$ events of triple and higher Ge-folds were finally obtained, with $^{162,163}\text{Er}$ as main evaporation residua. The data have been sorted into a number of $\gamma-\gamma$ matrices in coincidence with specific $\gamma$-transitions of the $^{163}\text{Er}$ nucleus [13]. First, a matrix collecting the entire decay flow of $^{163}\text{Er}$ (named total) has been constructed by gating on the three cleanest low spin transitions. In addition, seven matrices gated by transitions belonging to the low-K ($K=5/2$) signature and parity configurations labeled $A=(1/2,+)$, $B=(-1/2,+)$, $E=(1/2,-)$ and $F=(-1/2,-)$, and by the high-K ($K=19/2$) bands labeled $K1$ (negative parity) and $K2$ and $K4$ (positive parity), as done in ref. [13], have been sorted together with their corresponding two-dimensional (2D) backgrounds. For each 2D spectrum all known peak-peak and peak-background coincidences have been subtracted using the Radware software [14]. The separately gated matrices have also been summed into one low-$K$ ($A+B+E+F$) and one high-$K$ ($K1+K2+K4$) matrix. Figure 1 (left column) shows example of cuts perpendicular to the $E_{\gamma_1} = E_{\gamma_2}$ diagonal, 60 keV wide, in the total, low-$K$ and high-$K$ $\gamma-\gamma$ matrices, at the average transition energy $(E_{\gamma_1} + E_{\gamma_2})/2 = 900$ keV.

The fluctuations of counts in each channel of the selected measured 2D spectra, expressed as variance and covariance, are evaluated by the program STATFIT [10] and stored into 2D spectra. One additional option is applied: all pairs of resolved transitions are removed in the triangular sector $E_{\gamma_1} \geq E_{\gamma_2}$ with the proper intensity from the $\gamma-\gamma$ spectra, before the fluctuations are extracted, since the fluctuations are severely affected by the low lying intense transitions [10]. Because each rotational $E_\gamma$-cascade on the average contributes one count in each $\frac{4\hbar^2}{3}$ interval, the statistical moments are evaluated over sectors of $\frac{4\hbar^2}{3} \times \frac{4\hbar^2}{3}$, corresponding to 60 keV $\times$ 60 keV intervals for rare earth nuclei around $^{163}\text{Er}$. 




















































































































































































































































































From the fluctuation spectra we first extract the effective number of decay paths, which eventually feed into the gate-selected band. The number of decay paths $N_{\text{path}}^{(2)}$ having two $\gamma$ transitions with energies lying in a chosen $60 \text{ keV} \times 60 \text{ keV}$ window in the $\gamma - \gamma$ coincidence spectrum is obtained from the simple expression

$$N_{\text{path}}^{(2)} = \frac{N}{\mu_2 - \mu_1} \times P^{(2)}$$

(1)

where $N$ is the number of events, while $\mu_1$ and $\mu_2$ are the first and second moments of the distribution of counts, all evaluated in a $\frac{4}{3} \hbar^2 \times \frac{4}{3} \hbar^2$ sector of the $\gamma - \gamma$ matrix. The superscript $(2)$ indicates that the extraction of the number of paths is based on first and second moments, while the $P^{(2)}$ factor corrects for the finite resolution of the detector system [9, 10].

The number of paths obtained from the analysis of the first ridge of the 2D matrices gated by individual bands is found, in average, to be $\approx 10$ for both the four low-K and the three high-K configurations. Adding together the number of paths relative to specific configurations in a similar way as described in ref. [11], a total number of $\approx 20$ paths is found for both low-K and high-K states, as shown in the bottom part of figure 2 by open circles and squares, respectively. For the gates at the bottom of the bands, collecting the total decay flow through both low-K and high-K bands, one finds that a total of $\approx 45$ discrete rotational bands exist in the $^{163}\text{Er}$ nucleus, at heat energies below the onset of damping, as shown by triangles in figure 2a).

In contrast to the results of the ridge analysis the number of paths obtained by analyzing the valley region is found to depend significantly on the nuclear configuration. This result is shown in the top part of figure 2 together with the number of paths deduced from the total $E_{\gamma 1} \times E_{\gamma 2}$ spectrum. As the valley is probing the region in which the rotational bands are strongly mixed, this result intuitively suggests that the mixing process is indeed different for high-K and low-K states.

To provide a better understanding of the mixing of states with different $K$ quantum numbers we have studied the correlations in fluctuations between the spectra associ-
ated with low-K and high-K quantum numbers. These correlations are expressed by the covariance of counts, defined as [11]

\[ \mu_{2,\text{cov}}(A, B) = \frac{1}{N_{\text{ch}}} \sum_j (M_j(A) - \bar{M}_j(A))((M_j(B) - \bar{M}_j(B)) \]  

(2)

where \( M(A) \) and \( M(B) \) refer to spectra gated by transitions from two different bands, \( A \) and \( B \). The sum is over a region spanning \( N_{\text{ch}} \) channels (in this case \( 15 \times 15 \)) in a two-dimensional 60 keV×60 keV window, and \( \bar{M} \) denotes an average spectrum, (which in our case is obtained by the routine STATFIT as a numerical smoothed 3rd order approximation to the 2D spectrum). To normalize the covariance and thereby determine the degree of correlation between the two spectra, the correlation coefficient \( r(A, B) \) is calculated:

\[ r(A, B) = \frac{\mu_{2,\text{cov}}(A, B)}{\sqrt{(\mu_2(A) - \bar{\mu}_1(A))(\mu_2(B) - \bar{\mu}_1(B))}} \]  

(3)

Here, \( \mu_2 \) denotes the second moment defined for the same region \( N_{\text{ch}} \), related to the expression for the covariance by \( \mu_2(A) = \mu_{2,\text{cov}}(A, A) \). The first moment \( \bar{\mu}_1 \) is the average of \( M \) over the region \( N_{\text{ch}} \). The subtraction of the first moments in the denominator of eq. 3 corrects for the contribution to \( \mu_2 \) from counting statistics, which is linear in the number of events. The more interesting fluctuations are due to the nature of the finite number of transitions available to each cascade, and their contribution to \( \mu_2 \) is quadratic in the number of events.

Figure 3 shows the average values of the correlation coefficient \( r \) extracted from the covariance analysis of the ridge and valley structures of \( \gamma-\gamma \) coincidence spectra of \(^{163}\text{Er} \). In the case of the ridge analysis, \( r \) is found to be of the order of 0.2 for configurations with similar low-K values (fig. 3a)), while it is approximately zero for combinations of low-K and high-K spectra (fig. 3b)). This shows that there are basically no cross-transitions between the \( \approx 20 \) bands feeding high-K states and the \( \approx 20 \) bands feeding low-K states. For the valley fluctuations, the correlation coefficient is still of the order of 0.2, but now actually somewhat higher for the combination of low-
K with high-K than for low-K versus low-K. Although the error bars are significant, one may notice that for increasing transition energies above 1 MeV, correlation coefficients of the order of 0.5 between high-K and low-K are found, (figure 3 d)), while also the number of paths in the valley gated by low-K and high-K approach each other. This indicates a weakening of selection rules associated with the K-quantum number with increasing rotational frequency and heat energy.

To obtain a more thorough discussion of the validity of the K-quantum number in thermally excited states based on the present experimental data, we have performed simulations using band mixing calculations including both the residual interaction and a term that takes into account the angular momentum carried by the K-quantum number [12]. Inspecting the states resulting from the band mixing calculations, one finds that the onset of K-mixing takes place at around $U = 1.5$ MeV while the onset of damping (mixing of the majority of configurations), occurs at around $U = 1$ MeV. The calculations are made employing 4000 np-nh basis states with the lowest excitation energies to diagonalize the Hamiltonian for each spin and parity $I^\pi$. The truncation corresponds to a cut off of approximately 4 MeV.

Simulated $\gamma - \gamma$ spectra of interest were constructed by a Monte Carlo code, successfully employed to study rotational damping in different region of mass and deformation [15, 16]. The code is based on the levels and E2 transition probabilities microscopically calculated for the specific case of the $^{163}$Er nucleus [12]. In addition, statistical E1 transitions are included in a more schematic way using the calculated level density and a GDR strength function corresponding to a prolate nucleus with quadrupole deformation $\beta = 0.25$ and rotating collectively. In addition, an exponential quenching factor that takes into account the difference in K-quantum number between the initial and final states has been used. Such factor is analogous to the one employed in the analysis of the E1 decay-out from isomeric states [17, 18].

Each $\gamma$-cascade is started from initial values of internal energy and spin randomly chosen from a two-dimensional entry distribution of Gaussian shape, with centroids and widths reproducing the experimental conditions of the $^{163}$Er experiment previ-
ously discussed (i.e. $< U > = 4$ MeV, $FWHM_U = 4$ MeV, $< I > = 44\ h$, $FWHM_I = 20\ h$). In the cascade simulations, E2 transitions tend to keep the heat energy, while E1 transitions cool the nucleus. The choice of these parameters for the entry points and E1 transition probability resulted in a good reproduction of the measured intensities of both ridge structures and low spin yrast transitions [15, 19].

The right column of figure 1 shows examples of 60 keV wide projections perpendicular to the $E_{\gamma_1} = E_{\gamma_2}$ diagonal of simulated $\gamma$-$\gamma$ matrices collecting all cascades (Total, panel d)) and cascades finally feeding into low-K (namely $K \leq 8$, panel e)) and high-K ($K > 8$, panel f)) bands. The projections are taken at the average transition energy $(E_{\gamma_1} + E_{\gamma_2})/2 = 900$ keV, as in the corresponding experimental spectra (left column of figure 1). The asymmetry in the spectra is connected to the discrete lines subtraction which is here performed only in the $E_{\gamma_1} \geq E_{\gamma_2}$ region. It is worth noticing that in the simulated matrices only the yrast and the first excited band (for each parity and signature configuration) have been subtracted from the coincidence spectra, resulting in a more pronounced ridge than in the data, where all discrete transitions known from the level scheme have been removed (see right part of the spectra).

The fluctuations of counts in each channel of the selected 2D spectra, expressed as variance and covariance, are evaluated also for the simulated data and the results obtained for the ridge and valley structures are shown with lines in figures 2 and 3. It is remarkable how well the results of the band mixing model agree with the data, both with respect to the number of paths and to the correlation coefficients from the covariance analysis. This to some extent confirms the general nature of the states and transitions applied in the simulations.

The number of paths on the ridge may be understood by comparing to the total number of discrete bands, as directly extracted from the band mixing calculations (BM) by counting the number of bands branching out to less than 2 states [20]. The good agreement between the number of paths extracted from data, from simulations and calculated from the band mixing calculations tells that the nucleus $^{163}$Er contains about 45 discrete bands at low heat energies before damping sets in [20]. This is a
somewhat higher number than the typically 20 to 25 discrete bands obtained for $^{164,167,168}$Yb [10, 11], and can be attributed to the existence of the additional 20 high-K bands in $^{163}$Er, which do not exist at such low heat energies in the other nuclei.

The smaller number of high-K gated paths in the valley, relatively to the low-K gated is due to a lower level density for high-K states, $\approx 3$ times lower than for the low-K states [12]. Also, the rotational damping width has been measured to be $\approx 30\%$ reduced for high-K states [21]. In schematic evaluations of the number of paths [10], the level density and the rotational damping width enter as quadratic terms, thus explaining roughly the factor of 10 separating the number of high-K and low-K gated paths.

Turning now to the correlation coefficient, the rather low value $r = 0.2$ typically obtained for the low-K versus low-K ridge analysis may be understood from the fact that at most one or two E1 transitions cool down from the excited unresolved bands around $< U > \approx 0.6$ MeV to the low lying resolved bands. A simple quantitative estimate of the correlation coefficient can be deduced from the ratio between path probabilities, as described in ref. [11]. In the case of bands with same parity and different signature, this path probability depends on the relative probability $f$ for emitting an unstretched versus a stretched E1 transition, with $f \approx U^4/(U + \omega)^4$, being $\omega$ the rotational frequency. Altogether, for ridges including also bands with different parities, one obtains the simple expression $r = 2f/(1 + f^2)$. Inserting now $U = 0.6$ MeV and $\omega = 0.4$ MeV, it is found $r \approx 0.25$. One may note that such small correlation coefficients support the specific choice of cooling transitions applied in the simulations. Thus, for example, an even competition between M1 and E1 transitions would lead to much higher correlation coefficients, which is excluded by the experimental results.

The combined effect of selection rules on E1’s together with the straightforward parity-signature rules lead to a smaller value of $r$ relative to the typical value $r \approx 0.2$. This is seen for simulations as well as data for low-K versus high-K ridge correlations. Going up in heat energy, as probed by transitions in the valley, the K-selection rule
should weaken due to the K-mixing of the states, both for the E2’s and E1’s, and this is seen both in simulations and data. In particular, in the case of low-K versus low-K the low value of $r \approx 0.25$ indicates a similar probabilities for E1’s crossing as found in the ridge, although one could expect, on the basis of the previous simple expression, slightly higher values. The somewhat larger values of the correlation coefficients measured for high-K versus low-K as compared to low-K versus low-K could indicate instead a gradual mixing of low and high-K configurations, giving rise to a larger number of common paths. This is consistent with the results obtained by the fluctuation analysis of the valley region (figure 2b)), showing a gradual approach of low-K and high-K data at the highest values of transition energies. The present picture well agrees with the theoretical predictions from the band mixing model on $^{163}$Er [12], which shows that the onset of K-mixing takes place around a heat energy of $U \approx 1.5 - 1.8$ MeV, while the statistical limit of strong K-mixing is approached only above 2-2.5 MeV.

In conclusion, we have discussed the onset of K-mixing in rapidly rotating warm nuclei by means of a comparison of high statistics experimental data on $^{163}$Er and recently developed band mixing calculations for the specific nucleus. The experimental results from the fluctuation and covariance analysis on $\gamma-\gamma$ coincidences address the spin region 30-40 $\hbar$, and heat energies up to around 2.5 MeV. For the lower interval of heat energy up to approximately 1.2 MeV, a rather strict conservation of the K quantum number is found, as deduced by the analysis of the ridge structure which is formed by transitions along discrete unresolved bands. At higher internal energy ($U \sim 1.2$ to 2.5 MeV), probed by the weaker and more numerous transitions forming the valley, a partial conservation of the K quantum number is found, as shown by both the fluctuation and covariance analysis.

Further progress in the interesting topic of the order-to-chaos transition in nuclei will benefit from a better understanding of the thermal energy dependence of the K-mixing problem. For this purpose, future works focusing on high K-bands of larger internal energies should be made.
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References

[1] P. Walker and G. Draculis, Nature 399, (1999) 35.
[2] B.R. Mottelson, Nucl. Phys. A557, (1993) 717c.
[3] J. Rekstad et al., Phys. Rev. C55 (1997) 1805.
[4] V.G. Soloviev, Phys. Lett. B317 (1993) 501.
[5] P. Bosetti et al., Phys. Rev. Lett. 76 (1996) 1204.
[6] A. Bracco and S. Leoni, Rep. Prog. Phys. 65 (2002) 299.
[7] M. Matsuo et al., Nucl. Phys. A620 (1997) 296.
[8] S. Åberg, Phys. Rev. Lett. 64 (1990) 3119.
[9] B. Herskind et al, Phys. Rev. Lett. 68 (1992) 3008.
[10] T. Døssing et al., Phys. Rep. 268 (1996) 1.
[11] S. Leoni et al., Nucl. Phys. A671 (2000) 71.
[12] M. Matsuo et al., Nucl. Phys. A736 (2004) 241.
[13] G.B. Hagemann et al., Nucl. Phys. A618 (1997) 199.
[14] D.C. Radford, Nucl. Inst. Meth. A361, (1995) 297.
[15] A. Bracco et al., Phys. Rev. Lett. 76 (1996) 4484.

[16] A. Bracco et al., Nuc. Phys. A673, (2000) 64.

[17] P. Walker et al., Phys. Lett. B408 (1997) 242.

[18] F.G. Kondev et al., Nuc. Phys. A632 (1998) 473.

[19] A. Bracco et al., to be published.

[20] M. Matsuo et al., Nucl. Phys. A617 (1997) 1.

[21] S. Leoni, in print in Phys. Rev. Lett.
**Figure 1:** 60 keV wide projections perpendicular to the $E_{\gamma_1} = E_{\gamma_2}$ diagonal of experimental and simulated 2D spectra of $^{163}$Er (left and right panels, respectively), at the average transition energy $<E_\gamma>$=900 keV. The spectra collect either the total $\gamma$-decay flow (panel a) and d)) or the $\gamma$-decay in coincidence with low-K (panel b) and e)) or high-K (panel c) and f)) specific configurations. In the simulation, a state is defined as low-K (high-K) if $K \leq 8$ ($K > 8$). The reduced intensity observed in the $E_{\gamma_1} \geq E_{\gamma_2}$ region of the spectra is due to the subtraction of all discrete lines known from the level scheme, in the case of the experimental data, and of the yrast and first excited bands, in the case of the simulation.

**Figure 2:** *panel a):* The number of decay paths extracted from the fluctuation analysis of the ridges structure of $^{163}$Er. The open circles (squares) refer to the number of unresolved rotational bands populating the ridges of $\gamma$-$\gamma$ matrices gated by low-K (high-K) configurations, while the full triangles give the number of discrete paths obtained from the total matrix. The corresponding values for simulated spectra are shown by dotted, dashed and full lines (low-K, high-K and total, respectively). The total number of discrete bands directly extracted from the bands mixing (BM) calculations is also given for comparison (thin solid line). *panel b):* The effective number of transitions among mixed bands obtained from the fluctuation analysis of the valley region is shown by open circles (squares) for low-K (high-K) gated spectra, while the full triangles show the results obtained from the total $\gamma$-$\gamma$ matrix. The dashed and solid lines give the theoretical expectations for total and high-K cascades, as obtained from simulated spectra.

**Figure 3:** The results of the covariance analysis on ridge (bottom panels) and valley (top panels) structures of $^{163}$Er. Panels a) and c) show by open squares the correlation coefficient $r$ obtained experimentally by averaging over pairs of $\gamma$-$\gamma$ spectra gated by low-K configurations, while the correlation coefficient obtained from the experimental analysis of the low-K versus the high-K matrices is shown by full circles in panels b) and d). The theoretical values, as obtained from the covariance analysis of simulated spectra, are represented by dashed and solid lines, respectively.
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