On Emergent Gauge and Gravity Theories

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Abstract

We present some general approach to emergent gauge theories and consider in significant detail the emergent tensor field gravity case. In essence, an arbitrary local theory of a symmetric two-tensor field $H_{\mu\nu}$ in Minkowski spacetime is considered, in which the equations of motion are required to be compatible with a nonlinear $\sigma$ model type length-fixing constraint $H_{\mu\nu}^2 = \pm M^2$ leading to spontaneous Lorentz invariance violation, SLIV ($M$ is the proposed scale for SLIV). Allowing the parameters in the Lagrangian to be adjusted so as to be consistent with this constraint, the theory turns out to correspond to linearized general relativity in the weak field approximation, while some of the massless tensor Goldstone modes appearing through SLIV are naturally collected in the physical graviton. The underlying diffeomorphism invariance emerges as a necessary condition for the tensor field $H_{\mu\nu}$ not to be superfluously restricted in degrees of freedom, apart from the constraint due to which the true vacuum in the theory is chosen by SLIV. The emergent theory appears essentially nonlinear, when expressed in terms of the pure Goldstone tensor modes and contains a plethora of new Lorentz and $CPT$ violating couplings. However, these couplings do not lead to physical Lorentz violation once this tensor field gravity is properly extended to conventional general relativity.

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1 Introduction

It is conceivable that spontaneous Lorentz invariance violation (SLIV) could provide a dynamical approach to quantum electrodynamics, gravity and Yang-Mills theories with photon, graviton and gluons appearing as massless Nambu-Goldstone bosons \[1\] (for some later developments see \[2, 3, 4\]). However, in contrast to spontaneous violation of internal symmetries, SLIV seems not to necessarily implies a physical breakdown of Lorentz invariance. Rather, when appearing in a gauge theory framework, this may eventually result in noncovariant gauge choice in an otherwise gauge invariant and Lorentz invariant theory.

Remarkably, a possible source for such a kind of the unobserved SLIV could provide the nonlinearly realized Lorentz symmetry for underlying vector field \( A_\mu \) through its length-fixing constraint

\[
A_\mu A^\mu = n^2 M^2, \quad n^2 \equiv n_\mu n^\mu = \pm 1
\]  

(1)

(where \( n_\mu \) is a properly oriented unit Lorentz vector, while \( M \) is the proposed SLIV scale) rather than some vector field potential. This constraint in the gauge invariant QED framework was first studied by Nambu \[5\] a long ago, and in more detail (including the higher order corrections, extensions to spontaneously broken massive QED and non-Abelian theories etc.) in the last years \[6\]. The constraint (1), which in fact is very similar to the constraint appearing in the nonlinear \( \sigma \)-model for pions \[7\], means in essence that the vector field \( A_\mu \) develops some constant background value and the Lorentz symmetry \( SO(1,3) \) formally breaks down to \( SO(3) \) or \( SO(1,2) \) depending on the time-like (\( n^2 > 0 \)) or space-like (\( n^2 < 0 \)) nature of SLIV. The point is, however, that, in sharp contrast to the nonlinear \( \sigma \) model for pions, the nonlinear QED theory, due to the starting gauge invariance involved, ensures that all physical Lorentz violating effects are proved to be strictly cancelled.

Extending the above argumentation, we consider here spontaneous Lorentz violation realized through a nonlinear length-fixing tensor field constraint of the type

\[
H_{\mu\nu} H^{\mu\nu} = n^2 M^2, \quad n^2 \equiv n_\mu n^{\mu} = \pm 1
\]  

(2)

where \( n_{\mu\nu} \) is a properly oriented ‘unit’ Lorentz tensor, while \( M \) is the scale for Lorentz violation. Such a type of SLIV implemented into the tensor field gravity theory, which mimics linearized general relativity in Minkowski space-time, induces massless tensor Goldstone modes some of which can naturally be collected in the physical graviton \[8\]. Again, the theory appears essentially nonlinear and contains a plethora of new Lorentz and CPT violating couplings. However, these couplings do not lead to physical Lorentz violation once this tensor field gravity is properly extended to conventional general relativity.

2 Emergent gauge symmetries

Speaking still about vector field theories, the most important side of the nonlinear vector field constraint \(1\) was shown \[9\] to be that one does not need to specially postulate the starting gauge invariance in the framework of an arbitrary relativistically invariant Lagrangian which is proposed only to possess some global internal symmetry. Indeed, the SLIV conjecture \(1\) happens to be powerful enough by itself to require gauge invariance,
provided that we allow the parameters in the corresponding Lagrangian density to be adjusted so as to ensure self-consistency without losing too many degrees of freedom. Namely, due to the spontaneous Lorentz violation determined by the constraint (1), the true vacuum in such a theory is chosen so that this theory acquires on its own a gauge-type invariance, which gauges the starting global symmetry of the interacting vector and matter fields involved. In essence, the gauge invariance (with a proper gauge-fixing term) appears as a necessary condition for these vector fields not to be superfluously restricted in degrees of freedom.

Let us dwell upon this point in more detail. Generally, while a conventional variation principle requires the equations of motion to be satisfied, it is possible to eliminate one component of a general 4-vector field $A_{\mu}$, in order to describe a pure spin-1 particle by imposing a supplementary condition. In the massive vector field case there are three physical spin-1 states to be described by the $A_{\mu}$ field. Similarly in the massless vector field case, although there are only two physical (transverse) photon spin states, one cannot construct a massless 4-vector field $A_{\mu}$ as a linear combination of creation and annihilation operators for helicity $\pm 1$ states in a relativistically covariant way, unless one fictitious state is added [10]. So, in both the massive and massless vector field cases, only one component of the $A_{\mu}$ field may be eliminated and still preserve Lorentz invariance. Once the SLIV constraint (1) is imposed, it is therefore not possible to satisfy another supplementary condition, since this would superfluously restrict the number of degrees of freedom for the vector field. In fact a further reduction in the number of independent $A_{\mu}$ components would make it impossible to set the required initial conditions in the appropriate Cauchy problem and, in quantum theory, to choose self-consistent equal-time commutation relations [11].

We now turn to the question of the consistency of a constraint with the equations of motion for a general 4-vector field $A_{\mu}$ Actually, there are only two possible covariant constraints for such a vector field in a relativistically invariant theory - the holonomic SLIV constraint, $C(A) = A_{\mu}A^{\mu} - n^2 M^2 = 0$ (1), and the non-holonomic one, known as the Lorentz condition, $C(A) = \partial_{\mu}A^{\mu} = 0$. In the presence of the SLIV constraint $C(A) = A_{\mu}A^{\mu} - n^2 M^2 = 0$, it follows that the equations of motion can no longer be independent. The important point is that, in general, the time development would not preserve the constraint. So the parameters in the Lagrangian have to be chosen in such a way that effectively we have one less equation of motion for the vector field. This means that there should be some relationship between all the (vector and matter) field Eulerians ($E_A, E_{\psi}, ...$) involved$^1$. Such a relationship can quite generally be formulated as a functional but by locality just a function - of the Eulerians, $F(E_A, E_{\psi}, ...)$, being put equal to zero at each spacetime point with the configuration space restricted by the constraint $C(A) = 0$:

$F(C = 0; E_A, E_{\psi}, ...) = 0$ . (3)

This relationship must satisfy the same symmetry requirements of Lorentz and translational invariance, as well as all the global internal symmetry requirements, as the general starting Lagrangian $L(A, \psi, ...)$ does. We shall use this relationship in subsequent sections

$^1E_A$ stands for the vector-field Eulerian $(E_A)^{\mu} \equiv \partial L/\partial A_{\mu} - \partial_{\nu}[\partial L/\partial(\partial_{\nu}A_{\mu})]$. We use similar notations for other field Eulerians as well.
as the basis for gauge symmetry generation in the SLIV constrained vector and tensor field theories.

Let us now consider a “Taylor expansion” of the function $F$ expressed as a linear combination of terms involving various field combinations multiplying or derivatives acting on the Eulerians. The constant term in this expansion is of course zero since the relation must be trivially satisfied when all the Eulerians vanish, i.e. when the equations of motion are satisfied. We now consider just the terms containing field combinations (and derivatives) with mass dimension 4, corresponding to the Lorentz invariant expressions

$$\partial_\mu (E_A)^\mu, \ A_\mu (E_A)^\mu, \ E_\psi \bar{\psi}, \ \bar{\psi} E_\psi.$$  

(4)

All the other terms in the expansion contain field combinations and derivatives with higher mass dimension and must therefore have coefficients with an inverse mass dimension. We expect the mass scale associated with these coefficients should correspond to a large fundamental mass (e.g. the Planck mass $M_P$). Hence we conclude that such higher dimensional terms must be highly suppressed and can be neglected. A priori these neglected terms could lead to the breaking of the spontaneously generated gauge symmetry at high energy. However it could well be that a more detailed analysis would reveal that the imposed SLIV constraint requires an exact gauge symmetry. Indeed, if one uses classical equations of motion, a gauge breaking term will typically predict the development of the “gauge” in a way that is inconsistent with our gauge fixing constraint $C(A) = 0$. Thus the theory will generically only be consistent if it has exact gauge symmetry.

3 Deriving diffeomorphism invariance

We now illustrate these ideas by the example of the emergent tensor field gravity case. Let us consider an arbitrary relativistically invariant Lagrangian $L(H_{\mu\nu}, \phi)$ of one symmetric two-tensor field $H_{\mu\nu}$ and one real scalar field $\phi$ as the simplest possible matter in the theory taken in Minkowski spacetime. We restrict ourselves to the minimal interactions. In contrast to vector fields, whose basic interactions contain dimensionless coupling constants, for tensor fields the interactions with coupling constants of dimensionality $sm^1$ are essential.

We first turn to the possible supplementary conditions which can be imposed on the tensor fields $H_{\mu\nu}$ in the Lagrangian $L$, possessing still only a global Lorentz (and translational) invariance, in order to finally establish its form. The SLIV constraint, as it usually is when considering a system with holonomic constraints, leads to massive QED and massive Yang-Mills theories [11].

$$L'(H_{\mu\nu}, \phi, \lambda) = L(H_{\mu\nu}, \phi) - \frac{1}{4} \lambda \left( H_{\mu\nu} H^{\mu\nu} - n^2 M^2 \right)^2$$  

(5)

The Eulerians are of course just particular field combinations themselves and so this “expansion” at first includes higher powers and higher derivatives of the Eulerians.

The other possible Lorentz covariant constraint $\partial_\mu A^\mu = 0$, while also being sensitive to the form of the constraint-compatible Lagrangian, leads to massive QED and massive Yang-Mills theories [11].
and varying with respect to the auxiliary field \( \lambda(x) \) one has just the SLIV condition \( \text{(2)} \). Our choice of the quadratic form of the Lagrange-multiplier term \( \text{[12]} \) is only related to the fact that the equations of motion for \( H_{\mu\nu} \) in this case are independent of the \( \lambda(x) \) which entirely decouples from them rather than acts as some extra source of energy-momentum density, as it would be for the linear Lagrange multiplier term that could make the subsequent consideration to be more complicated. So, as soon as the constraint \( \text{(2)} \) holds

\[
C(H_{\mu\nu}) = H_{\mu\nu}H^{\mu\nu} - n^2 M^2 = 0 \quad \text{(6)}
\]

one has the equations of motion for \( H_{\mu\nu} \) expressed through its Eulerian \( (\mathcal{E}_H)^{\mu\nu} \)

\[
(\mathcal{E}_H)^{\mu\nu} \equiv \partial \mathcal{L}/\partial H_{\mu\nu} - \partial \rho [\partial \mathcal{L}/\partial (\partial \rho H_{\mu\nu})] = 0 \quad \text{(7)}
\]

which is determined solely by the starting Lagrangian \( \mathcal{L}(H_{\mu\nu}, \phi) \).

Despite the SLIV constraint \( \text{(2)} \) the tensor field \( H_{\mu\nu} \), both massive and massless, still contains many superfluous components which are usually eliminated by imposing some supplementary conditions. In the massive tensor field case there are five physical spin-2 states to be described by \( H_{\mu\nu} \). Similarly, in the massless tensor field case, though there are only two physical (transverse) spin states associated with graviton, one cannot construct a symmetric two-tensor field \( H_{\mu\nu} \) as a linear combination of creation and annihilation operators for helicity \( \pm 2 \) states unless three (and \( 2j - 1 \), in general, for the spin \( j \) massless field) fictitious states with other helicities are added \( \text{[7, 10]} \). So, in both massive and massless tensor field cases only five components in the 10-component tensor field \( H_{\mu\nu} \) may be at most eliminated so as to preserve the Lorentz invariance. However, once the SLIV constraint \( \text{(3)} \) is already imposed, four extra supplementary conditions are only possible. Normally they should exclude the spin 1 states which are still left in the theory and are described by some of the components of the tensor \( H_{\mu\nu} \). Usually, they (and one of the spin 0 states) are excluded by the conventional harmonic gauge condition

\[
\partial^\mu H_{\mu\nu} - \partial_\nu H_{tr}/2 = 0. \quad \text{(8)}
\]

or some of its analogs (see section 4). In fact, there should not be more supplementary conditions - otherwise, this would superfluously restrict the number of degrees of freedom for the spin 2 tensor field which is inadmissible.

Under this assumption of not getting too many constraints, we shall now derive gauge invariance of the Lagrangian \( \mathcal{L}(H_{\mu\nu}, \phi) \). Actually, we turn to the question of the consistency of the SLIV constraint with the equations of motion for a general symmetric tensor field \( H_{\mu\nu} \). For an arbitrary Lagrangian \( \mathcal{L}(H_{\mu\nu}, \phi) \), the time development of the fields would not preserve the constraint \( \text{(4)} \). So the parameters in the Lagrangian must be chosen so as to give a relationship between the Eulerians for the tensor and matter fields of the type

\[
\mathcal{F}^\mu(C = 0; \mathcal{E}_H, \mathcal{E}_\phi, ... = 0 \quad (\mu = 0, 1, 2, 3)). \quad \text{(9)}
\]

\(^4\)These spin 1 states must necessarily be excluded as the sign of the energy for spin 1 is always opposite to that for spin 2 and 0
which, in contrast to the relationship \(3\), transforms in general as a Lorentz vector in the tensor field case. As a result, four additional equations for the tensor field \(H_{\mu\nu}\) which appear by taking 4-divergence of the tensor field equations of motion \(7\)

\[
\partial_\mu (\mathcal{E}_H)^{\mu\nu} = 0
\]

will not produce supplementary conditions at all once the SL IV condition \(6\) occurs. In fact, due to the relationship \(9\) these equations \(10\) are satisfied identically or as a result of the equations of motion of all the fields involved. This implies that in the absence of the equations of motion there must hold a general off-shell identity of the type

\[
\partial_\mu (\mathcal{E}_H)^{\mu\nu} = P^{\nu}_{\alpha\beta} (\mathcal{E}_H)^{\alpha\beta} + Q^\nu \mathcal{E}_\phi
\]

where \(P^{\nu}_{\alpha\beta}\) and \(Q^\nu\) are some operators acting on corresponding Eulerians of tensor and scalar fields (for this form of \(\partial_\mu (\mathcal{E}_H)^{\mu\nu}\) the second equation in \(10\) is trivially satisfied). The simplest conceivable forms of these operators are

\[
P^{\nu}_{\alpha\beta} = p_1 \eta^{\nu\rho} (H_{\alpha\rho} \partial_\beta + H_{\rho\beta} \partial_\alpha + \partial_\rho H_{\alpha\beta}) ,
\]

\[
Q^\nu = q_1 \eta^{\nu\rho} \partial_\rho \phi
\]

in which only terms with constants \(p_1\) and \(q_1\) of dimensionality \(cm^1\) appear essential. This identity \(11\) implies then the invariance of \(\mathcal{L}(H_{\mu\nu}, \phi)\) under the local transformations of tensor and scalar fields whose infinitesimal form is given by

\[
\delta H_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + p_1 (\partial_\mu \xi^{\rho} H_{\rho\nu} + \partial_\nu \xi^\rho H_{\mu\rho} + \xi^\rho \partial_\rho H_{\mu\nu}) ,
\]

\[
\delta \phi = q_1 \xi^\rho \partial_\rho \phi
\]

where \(\xi^\mu(x)\) is an arbitrary 4-vector function, only being required to conform with the nonlinear constraint \(2\). Conversely, the identity \(11\) in its turn follows from the invariance of the Lagrangian \(\mathcal{L}(H_{\mu\nu}, \phi)\) under the transformations \(13\). Both direct and converse assertions are in fact particular cases of Noether’s second theorem \(13\).

An important point is that the operators \(P^{\nu}_{\alpha\beta}\) and \(Q^\nu\) \(12\) were chosen in a way that the corresponding transformations \(13\) could generally constitute a group (that is the Lie structure relation holds). This is why all three terms in the symmetric operator \(P^{\nu}_{\alpha\beta}\) \(12\) are taken with the same constant, though the third term in it might enter with some different constant. Remarkably, though the transformations \(13\) were only restricted to form a group, this emergent symmetry group is proved, as one can readily confirm, to be nothing but the diff invariance. Indeed, for the quantity

\[
g_{\mu\nu} = \eta_{\mu\nu} + p_1 H_{\mu\nu}
\]

the tensor field transformation \(13\) may be written in a form

\[
\delta g_{\mu\nu} = p_1 (\partial_\mu \xi^\rho g_{\rho\nu} + \partial_\nu \xi^\rho g_{\mu\rho} + \xi^\rho \partial_\rho g_{\mu\nu})
\]
which shows that $g_{\mu\nu}$ transform as the metric tensors in the Riemannian geometry (the constant $p_1$ may be included into the transformation 4-vector parameter $\xi^\mu(x)$) with general coordinate transformations, $\delta x^\mu = \xi^\mu(x)$. So, we have shown that the imposition of the SLIV constraint (2) supplements the starting global Poincare symmetry with the local diff invariance. Otherwise, the theory would superfluously restrict the number of degrees of freedom for the tensor field $H_{\mu\nu}$, which would certainly not be allowed.

This SLIV induced gauge symmetry (13) completely determines now the Lagrangian $\mathcal{L}(H_{\mu\nu},\phi)$. Indeed, in the weak field approximation (when $\delta H_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$) this symmetry gives the well-known linearized gravity Lagrangian

$$\mathcal{L}(H_{\mu\nu},\phi) = \mathcal{L}(H) + \mathcal{L}(\phi) + \mathcal{L}_{\text{int}}$$

consisted of the $H$ field kinetic term of the form

$$\mathcal{L}(H) = \frac{1}{2} \partial_\lambda H^{\mu\nu} \partial^\lambda H_{\mu\nu} - \frac{1}{2} \partial_\lambda H_{\mu\nu} \partial^\lambda H_{\mu\nu}$$

($H_{tr}$ stands for a trace of the $H_{\mu\nu}$, $H_{tr} = \eta^{\mu\nu} h_{\mu\nu}$), and the free scalar field Lagrangian $\mathcal{L}(\phi)$ and interaction term $\mathcal{L}_{\text{int}} = (1/M_P) H_{\mu\nu} T^{\mu\nu}(\phi)$, where $T^{\mu\nu}(\phi)$ is a conventional energy-momentum tensor for scalar field. Besides, the proportionality coefficient $p_1$ in the metric (14) was chosen to be inverse just to the Planck mass $M_P$. It is clear that, in contrast to the free field terms given above by $\mathcal{L}(H)$ and $\mathcal{L}(\phi)$, the interaction term $\mathcal{L}_{\text{int}}$ is only approximately invariant under the diff transformations in the weak field limit.

To determine a complete theory, one should consider the full variation of the Lagrangian $\mathcal{L}$ as function of metric $g_{\mu\nu}$ and its derivatives (including the second order ones), and solve a general identity of the type

$$\delta \mathcal{L}(g_{\mu\nu}, g_{\mu\nu,\lambda}, g_{\mu\nu,\lambda\rho}; \phi, \phi_\lambda) = \partial_\mu X^\mu$$

(subscripts after commas denote derivatives) which contains an unknown vector function $X^\mu$. The latter must be constructed from the fields and local transformation parameters $\xi^\mu(x)$ taking into account the requirement of compatibility with the invariance of the $\mathcal{L}$ under transformations of the Lorentz group and translations. Following this procedure [14] for the metric and scalar field variations (15, 13) conditioned by SLIV constraint (2), one can eventually find the total Lagrangian $\mathcal{L}$ which is turned out to be properly expressed in terms of quantities similar to the basic ones in the Riemannian geometry (like as metric, connection, curvature etc.). Actually, this theory successfully mimics general relativity that allows us to conclude that the Einstein equations could be really derived in the flat Minkowski spacetime provided that Lorentz symmetry in it is spontaneously broken.

### 4 Graviton as a Goldstone boson

Let us turn now to spontaneous Lorentz violation in itself which is caused by the nonlinear tensor field constraint (2). This constraint means in essence that the tensor field $H_{\mu\nu}$
develops the vev configuration

\[ \langle H_{\mu\nu}(x) \rangle = n_{\mu\nu} M \]  \hspace{1cm} (19)

determined by the matrix \( n_{\mu\nu} \), and starting Lorentz symmetry \( SO(1,3) \) of the Lagrangian \( \mathcal{L}(H_{\mu\nu}, \phi) \) given in (16) formally breaks down at a scale \( M \) to one of its subgroup thus producing a corresponding number of the Goldstone modes. In this connection the question about other components of a symmetric two-index tensor \( H_{\mu\nu} \), aside from the pure Goldstone ones, naturally arises. Remarkably, they are turned out to be the pseudo-Goldstone modes (PGMs) in the theory. Indeed, although we only propose the Lorentz invariance of the Lagrangian \( \mathcal{L}(H_{\mu\nu}, \phi) \), the SLIV constraint (2) possesses formally a much higher accidental symmetry. This is in fact \( SO(7,3) \) symmetry of the length-fixing bilinear form (2). This symmetry is spontaneously broken side by side with Lorentz symmetry at scale \( M \). Assuming a minimal vacuum configuration in the \( SO(7,3) \) space with the vevs (19) developed on only one \( H_{\mu\nu} \) component, we have the time-like \( (SO(7,3) \rightarrow SO(6,3)) \) or space-like \( (SO(7,3) \rightarrow SO(7,2)) \) violations of the accidental symmetry depending on the sign of \( n_{\alpha\beta} H_{\alpha\beta} = \frac{M^2 - n^2 h^2}{2M} + O(1/M^2) \) \hspace{1cm} (21)

taking, for definiteness, the positive sign for the square root and expanding it in powers of \( h^2/M^2 \). Putting then the parameterization (20) into Lagrangian \( \mathcal{L}(H_{\mu\nu}, \phi) \) given in (16), one readily comes to the truly Goldstonic tensor field gravity Lagrangian \( \mathcal{L}(h_{\mu\nu}, \phi) \) containing infinite series in powers of the \( h_{\mu\nu} \) modes, which we will not display here due to its excessive length (see [8]).

Now, one can rewrite the Lagrangian \( \mathcal{L}(H_{\mu\nu}, \phi) \) in terms of the Goldstone modes explicitly using the SLIV constraint (2). For this purpose let us take the following handy parameterization for the tensor field \( H_{\mu\nu} \) in the Lagrangian \( \mathcal{L}(H_{\mu\nu}, \phi) \):

\[ H_{\mu\nu} = h_{\mu\nu} + \frac{n_{\mu\nu}}{n^2}(n_{\alpha\beta} H^{\alpha\beta}) \hspace{1cm} \text{where} \hspace{1cm} n_{\mu\nu} h_{\mu\nu} = 0 \]  \hspace{1cm} (20)

where \( h_{\mu\nu} \) corresponds to the pure Goldstonic modes, while the effective “Higgs” mode (or the \( H_{\mu\nu} \) component in the vacuum direction) is

\[ n_{\alpha\beta} H^{\alpha\beta} = (M^2 - n^2 h^2) \frac{1}{2} = M - \frac{n^2 h^2}{2M} + O(1/M^2) \]  \hspace{1cm} (21)

taking, for definiteness, the positive sign for the square root and expanding it in powers of \( h^2/M^2 \). Putting then the parameterization (20) with the SLIV constraint (21) into Lagrangian \( \mathcal{L}(H_{\mu\nu}, \phi) \) given in (16), one readily comes to the truly Goldstonic tensor field gravity Lagrangian \( \mathcal{L}(h_{\mu\nu}, \phi) \) containing infinite series in powers of the \( h_{\mu\nu} \) modes, which we will not display here due to its excessive length (see [8]).

Together with the Lagrangian \( \mathcal{L}(h_{\mu\nu}, \phi) \) one must be also certain about the gauge fixing terms, apart from a general Goldstonic “gauge” \( n_{\mu\nu} h_{\mu\nu} = 0 \) given above (20). Remarkably, the simplest set of conditions being compatible with the latter is turned out to be

\[ \partial^\rho(\partial_\rho h_{\mu\nu} - \partial_\nu h_{\mu\rho}) = 0 \]  \hspace{1cm} (22)
(rather than harmonic gauge conditions (8)) which also automatically eliminates the (negative-energy) spin 1 states in the theory. So, with the Lagrangian $L(h_{\mu\nu}, \phi)$ and the supplementary conditions (20) and (22) lumped together, one eventually comes to the working model for the Goldstonic tensor field gravity. Generally, from ten components in the symmetric-two $h_{\mu\nu}$ tensor, four components are excluded by the supplementary conditions (20) and (22). For a plane gravitational wave propagating, say, in the $z$ direction another four components can also be eliminated. This is due to the fact that the above supplementary conditions still leave freedom in the choice of a coordinate system, $x^\mu \rightarrow x^\mu - \xi^\mu (t - z/c)$, much as takes place in standard GR. Depending on the form of the vev tensor $n_{\mu\nu}$, the two remaining transverse modes of the physical graviton may consist solely of Lorentz Goldstone modes or of Pseudo Goldstone modes or include both of them.

5 Summary and outlook

We presented here some approach to emergent gauge theories and considered in significant detail the emergent tensor field gravity case. Our main result can be summarized in a form of a general Emergent Gauge Symmetry (EGS) conjecture:

Let there be given an interacting field system $\{A_\mu, \ldots, \phi, \psi, H_{\mu\nu}\}$ containing some vector field (or vector field multiplet) $A_\mu$ or/and tensor field $H_{\mu\nu}$ in an arbitrary Lorentz invariant Lagrangian of scalar ($\phi$), fermion ($\psi$) and other matter field multiplets, which possesses only global Abelian or non-Abelian symmetry $G$ (and only conventional global Lorentz invariance in a pure tensor field case). Suppose that one of fields in a given field system is subject to the nonlinear $\sigma$ model type "length-fixing" constraint, say, $A_\mu^2 = M^2$ (for vector fields) or $\phi^2 = M^2$ (for scalar fields) or $H_{\mu\nu}^2 = M^2$ (for tensor field). Then, since the time development would not in general preserve this constraint, the parameters in their common Lagrangian $L(A_\mu, \ldots, \phi, \psi; H_{\mu\nu})$ will adjust themselves in such a way that effectively we have less independent equations of motion for the field system taken. This means that there should be some relationship between Eulers of all the fields involved to which Noether’s second theorem can be applied. As a result, one comes to the conversion of the global symmetry $G$ into the local symmetry $G_{loc}$, being exact or spontaneously broken depending on whether vector or scalar fields are constrained, and to the conversion of global Lorentz invariance into diffeomorphism invariance for the constrained tensor field.

Applying the EGS conjecture to tensor field theory case we found that the only possible local theory of a symmetric two-tensor field $H_{\mu\nu}$ in Minkowski spacetime which is compatible with SLIV constraint $H_{\mu\nu}^2 = \pm M^2$ is turned out to be linearized general relativity in the weak field approximation. When expressed in terms of the pure tensor Goldstone modes this theory is essentially nonlinear and contains a variety of Lorentz and CPT violating couplings. Nonetheless, as was shown in the recent calculations [8], all the SLIV effects turn out to be strictly cancelled in the lowest order gravity processes as soon as the tensor field gravity theory is properly extended to general relativity. So, the nonlinear SLIV condition being applied both in vector and tensor field theories, due to which true vacuum is chosen and Goldstonic gauge fields are generated, may provide a dynamical setting for all underlying internal and spacetime local symmetries involved. However, this gauge theory
framework, uniquely emerging for the length-fixed vector and tensor fields, makes in turn this SLIV to be physically unobservable.

From this standpoint, the only way for physical Lorentz violation to appear would be if the above local invariance were slightly broken at very small distances controlled by quantum gravity [15]. The latter could in general hinder the setting of the required initial conditions in the appropriate Cauchy problem thus admitting a superfluous restriction of vector and tensor fields in degrees of freedom through some high-order operators stemming from the quantum gravity influenced area. This may be a place where the emergent vector and tensor field theories may drastically differ from conventional gauge theories that could have some observational evidence at low energies.

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