The pseudoscalar and vector excited mesons
in the $U(3) \ast U(3)$ chiral model

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Abstract

A chiral $U(3) \ast U(3)$ Lagrangian containing, besides the usual meson fields, their first radial excitations is constructed. The Lagrangian is derived by bosonization of the Nambu–Jona-Lasinio quark model with separable non-local interactions, with form factors corresponding to 3–dimensional ground and excited state wave functions. The spontaneous breaking of chiral symmetry is governed by the NJL gap equation. The first radial excitations of the pions, kaons and vector meson nonet are described with the help of five different form factors. Each form factor contains only one arbitrary parameter. The masses of these meson states and the weak decay constants $F_{\pi'}$, $F_K$ and $F_K'$ are calculated.

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1 Introduction

Investigation of the radial excitations of the light mesons is of great interest in hadronic physics. So far there are the questions connected with the experimental and theoretical descriptions of the radial excitations of pseudoscalar mesons. For instance, the $\pi'$ meson with the mass $(1300 \pm 100)\text{MeV}$ is usually identified as the first radial excitation of the pion $[1]$. However, indications of a light resonance in diffractive production of $3\pi$–states have lead to speculations that the mass of the $\pi'$ may be considerably lower, at $\sim 750\text{MeV}$ $[2]$. So far there are no experimental data concerning the excited states of the kaons $[1]$. In this paper we try to describe the masses of the excited pions, kaons and vector mesons and the weak decay constants of pseudoscalar mesons in the framework of the $U(3) \ast U(3)$ chiral model.

A theoretical description of radially excited pions poses some interesting challenges. The physics of normal pions is completely governed by the spontaneous breaking of chiral symmetry. A convenient way to derive the properties of soft pions is by way of an effective Lagrangian based on a non-linear realization of chiral symmetry $[3]$. When attempting to introduce higher resonances to extend the effective Lagrangian description to higher energies, one must ensure that the introduction of new degrees of freedom does not spoil the low–energy theorems for pions, which are universal consequences of chiral symmetry.

A useful guideline in the construction of effective meson Lagrangians is the Nambu–Jona-Lasinio (NJL) model, which describes the spontaneous breaking of chiral symmetry at quark level using a four–fermion interaction $[4, 5, 6]$. The bosonization of this model and the derivative expansion of the resulting fermion determinant reproduce the Lagrangian of the linear sigma model, which embodies the physics of soft pions as well as higher–derivative terms. With appropriate couplings the model allows to derive also a Lagrangian for vector and axial–vector mesons. This not only gives the correct structure of the terms of the Lagrangian as required by chiral symmetry, but also quantitative predictions for the coefficients, such as $F_\pi, F_K, g_\pi, g_\rho$, etc., which are in good agreement with phenomenology. One may therefore hope that a suitable generalization of the NJL–model may provide a means for deriving an effective Lagrangian including also the excited mesons.

When extending the NJL model to describe radial excitations of mesons, one has to introduce non-local (finite–range) four–fermion interactions. Many non-local generalizations of the NJL model have been proposed, using either covariant–euclidean $[7]$ or instantaneous (potential–type) $[8, 9]$ effective quark interactions. These models generally require bilocal meson fields for bosonization, which makes it difficult to perform a consistent derivative expansion leading to an effective Lagrangian. A simple alternative is the use of separable quark interactions. There are a number of advantages of working with such a scheme. First, separable interactions can be bosonized by introducing local meson fields, just as the usual NJL–model. One can thus derive an effective meson Lagrangian directly in terms of local fields and their derivatives. Second, separable interactions allow one to introduce a limited number of excited states and only in a given channel. An interesting method for describing excited meson states in this approximation was proposed in $[10]$. Furthermore, the separable interaction can be defined in Minkowski space in a 3–dimensional (yet covariant) way, with form factors depending only on the part of the quark–antiquark relative momentum transverse to the meson momentum $[10, 11, 12]$. This is essential for a correct description of excited states, since it ensures the absence of spurious relative–time excitations $[13]$. Finally, as we have shown $[12]$, the form factors defining the separable interaction can be chosen so that the gap equation of the generalized NJL–model coincides with the one of the usual NJL–model, whose solution is a constant (momentum–independent) dynamical quark mass. Thus, in this approach it is possible to describe radially excited mesons above the usual NJL vacuum. Aside from the technical simplification the latter means that the separable generalization contains all the successful quantitative results of the
In our previous paper [12] the theoretical foundations for the choice of the pion-quark form factors in a simple extension of the NJL model to the non-local quark interaction were discussed. It was shown that we can choose these form factors such that the gap equation conserves the usual form and gives the solution with a constant constituent quark mass. The quark condensate also does not change after including the excited states in the model, because the tadpole connected with the excited scalar field is equal to zero (the quark loop with the one excited scalar vertex - vertex with form factor).

Now we shall use these form factors for describing the first excited states of the pseudoscalar and vector meson nonets in the framework of the more realistic $U(3)\times U(3)$ chiral model [4, 5, 6]. We shall take into account the connections of the scalar and vector coupling constants which have appeared in this model and the additional renormalization of the pseudoscalar fields connected with the pseudoscalar – axial–vector transitions. For simplicity, we shall suppose that the masses of the up and down quarks are equal to each other and shall take into account only the mass difference between (up, down) and strange quarks ($m_u$ and $m_s$). Then we have in this model the five basic parameters: $m_u$, $m_s$, $\Lambda_3$ (3-dimensional cut-off parameter), $G_1$ and $G_2$ (the four–quark coupling constants for the scalar–pseudoscalar coupling ($G_1$) and for the vector – axial–vector coupling ($G_2$)). For the definition of these parameters we shall use the experimental values: the pion decay constant $F_\pi = 93\,MeV$, the $\rho$–meson decay constant $g_\rho \approx 6.14$ ($\frac{g_\rho}{F_\pi} \approx 3$), the pion mass $M_\pi \approx 140\,MeV$, $\rho$–meson mass $M_\rho = 770\,MeV$ and the kaon mass $M_K \approx 495\,MeV$. Using these five parameters we can describe the masses of the four meson nonets (pseudoscalar, vector, scalar and axial–vector) and all the meson coupling constants describing the strong interactions of the meson with each other and with the quarks.

For the investigation of the excited states of the mesons it is necessary to consider the non-local four–quark interactions. We have shown that for description of the excited states of pions, kaons and vector meson nonet we have to use five different form factors in the effective four–quark interactions. Each form factor contains only one arbitrary parameter. We have calculated also the weak decay constants $F_{\pi'}$, $F_K$ and $F_{K'}$. In our next work we are going to calculate the decay widths of the excited mesons and to describe the excited states of the $\eta$ and $\eta'$ mesons.

In section 2, we introduce the effective quark interaction in the separable approximation and describe its bosonization. We discuss the choice of the form factors necessary to describe the excited states of the pseudoscalar and vector meson nonets. In section 3, we derive the effective Lagrangian for the pseudoscalar mesons, and perform the diagonalization leading to the physical meson ground and excited states. In section 4, we perform it for the vector mesons. In section 5, we fix the parameters of the model and evaluate the masses of the ground and excited meson states and weak decay constants $F_{\pi}$, $F_{\pi'}$, $F_{K}$ and $F_{K'}$. In section 6, we discuss the obtained results.

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1. The mass formulae for the axial–vector mesons and, especially, for the scalar mesons give only qualitative results (20 – 30% accuracy).
2. Remind, one more additional parameter, connected with the gluon anomaly, was used in the usual NJL model, when we described the ground states of the $\eta$ and $\eta'$ mesons ($U(1)$ problem) [3].

The problem of the radial excitations of the light mesons, including the $\eta$ and $\eta'$, in the framework of the potential model was discussed in works [14].
2 \( U(3) \times U(3) \) chiral Lagrangian with the excited meson states

In the usual \( U(3) \times U(3) \) NJL model a local (current–current) effective quark interaction is used

\[
L[\bar{q}, q] = \int d^4x \bar{q}(x) \left( i\gamma \cdot \vec{\partial} - m^0 \right) q(x) + L_{\text{int}},
\]

(1)

\[
L_{\text{int}} = \int d^4x \left[ \frac{G_1}{2} (j_S^a(x) j^a_S(x) + j_P^a(x) j_P^a(x)) - \frac{G_2}{2} (j_V^a(x) j^a_V(x) + j_A^a(x) j^a_A(x)) \right],
\]

(2)

where \( m^0 \) is the current quark mass matrix. We suppose that \( m_u \approx m_d \).

\( j_{S,P,V,A}^a(x) \) denote, respectively, the scalar, pseudoscalar, vector and axial–vector currents of the quark field \( (U(3)\text{–flavor}) \),

\[
\begin{align*}
j_S^a(x) & = \bar{q}(x) \lambda^a q(x), \\
j_P^a(x) & = \bar{q}(x) i\gamma_5 \lambda^a q(x), \\
j_V^{a,\mu}(x) & = \bar{q}(x) \gamma^\mu \lambda^a q(x), \\
j_A^{a,\mu}(x) & = \bar{q}(x) \gamma_5 \gamma^\mu \lambda^a q(x).
\end{align*}
\]

(3)

Here \( \lambda^a \) are the Gell-Mann matrices, \( 0 \leq a \leq 8 \). The model can be bosonized in the standard way by representing the 4–fermion interaction as a Gaussian functional integral over scalar, pseudoscalar, vector and axial–vector meson fields \([4,5,6]\). The effective meson Lagrangian, which is obtained by integration over the quark fields, is expressed in terms of local meson fields. By expanding the quark determinant in derivatives of the local meson fields one then derives the chiral meson Lagrangian.

The Lagrangian (2) describes only ground–state mesons. To include excited states, one has to introduce effective quark interactions with a finite range. In general, such interactions require bilocal meson fields for bosonization \([4,6]\). A possibility to avoid this complication is the use of a separable interaction, which is still of current–current form, eq.(4), but allows for non–local vertices (form factors) in the definition of the quark currents, eqs.(3),

\[
\tilde{L}_{\text{int}} = \int d^4x \sum_{i=1}^N \left[ \frac{G_1}{2} \left( j_{S,i}^a(x) j_{S,i}^a(x) + j_{P,i}^a(x) j_{P,i}^a(x) \right) - \frac{G_2}{2} \left( j_{V,i}^a(x) j_{V,i}^a(x) + j_{A,i}^a(x) j_{A,i}^a(x) \right) \right],
\]

(4)

\[
\begin{align*}
j_{S,i}^a(x) & = \int d^4x_1 \int d^4x_2 \bar{q}(x_1) F_{S,i}^a(x_1; x_1, x_2) q(x_2), \\
j_{P,i}^a(x) & = \int d^4x_1 \int d^4x_2 \bar{q}(x_1) F_{P,i}^a(x_1; x_1, x_2) q(x_2), \\
j_{V,i}^{a,\mu}(x) & = \int d^4x_1 \int d^4x_2 \bar{q}(x_1) F_{V,i}^{a,\mu}(x_1; x_1, x_2) q(x_2), \\
j_{A,i}^{a,\mu}(x) & = \int d^4x_1 \int d^4x_2 \bar{q}(x_1) F_{A,i}^{a,\mu}(x_1; x_1, x_2) q(x_2),
\end{align*}
\]

(5–8)

Here, \( F_{i,i}^{a,\mu}(x_1; x_1, x_2) \), \( i = 1, \ldots, N \), denote a set of non–local scalar, pseudoscalar, vector and axial–vector quark vertices (in general momentum– and spin–dependent), which will be specified below. Upon bosonization we obtain

\[
L_{\text{bos}}(\bar{q}, q; \sigma, \phi, P, A) = \int d^4x_1 \int d^4x_2 \bar{q}(x_1) \left( i\gamma \cdot \vec{x}_2 - m^0 \right) \delta(x_1 - x_2)
\]
and redefine the quark masses

\[ m_a = m_a^0 - \langle \sigma_a' > \]  

Then eq. (13) can be rewritten in the form of the usual gap equation

\[ m_i = m_i^0 + 8G_1m_iI_1(m_i), \quad (i = u, d, s) \]  

where

\[ I_n(m_i) = -iN_c \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \frac{1}{m^2 - k^2} \]
and \( m_i \) are the constituent quark masses.

In order to obtain the correct coefficients of kinetic terms of the mesons in the one-quark-loop approximation, we have to make the renormalization of the meson fields in eq. (12)

\[
\sigma_a = g_a^\sigma \sigma^r_a, \quad \phi_a = g_a^\phi \phi^r_a, \quad V^\mu_a = g_V^a V^\mu a^r, \quad A^\mu_a = g_A^a A^\mu a^r, \quad (18)
\]

where

\[
g_a^{\sigma,\phi} = \left[ 4 I_2(m_i, m_j) \right]^{-\frac{1}{2}} \quad I_2(m_i, m_j) = -i N_c \int_{\Lambda_3}^{\Lambda} \frac{1}{(2\pi)^4 (m_i^2 - k^2)(m_j^2 - k^2)}. \quad (19)
\]

\[
g_V^a = \sqrt{6} g_\sigma^a. \quad (20)
\]

After taking into account the pseudoscalar – axial–vector transitions (\( \phi_a \rightarrow A_a \)), the additional renormalization of the pseudoscalar fields appears

\[
g_\phi^a = Z^{-\frac{1}{2}}_a g_\sigma^a. \quad (21)
\]

where \( Z_{\pi} = 1 - \frac{6m_u^2}{M_{a_1}^2} \approx 0.7 \) for pions. (\( M_{a_1} = 1.23 GeV \) is the mass of the axial-vector \( a_1 \) meson, \( m_u = 280 MeV \)). We shall assume that all \( Z_a \approx Z_{\pi} \approx 0.7 \).

After these renormalizations the part of the Lagrangian (12) describing the ground states of mesons takes the form

\[
L(\sigma, \phi, V, A) = -\frac{1}{2 G_1} (g_\sigma^2 \sigma_a^2 + g_\phi^2 \phi_a^2) + \frac{g_V^2}{2 G_2} (V^2_a + A^2_a) - i N_c \text{ Tr } \log \left[ i \partial \bar{m}^0 + \left( g_\sigma^2 \sigma_a + i \gamma_5 g_\phi^2 \phi_a + \frac{g_V^2}{2} (\gamma_\mu V^\mu_a + \gamma_5 \gamma_\mu A^\mu_a) \right) \Lambda^a \right]. \quad (22)
\]

For simplicity we omitted here the index \( r \) of the meson fields.

From the Lagrangian (22) in the one-loop approximation the following expressions for the meson masses are obtained

\[
M_{\pi}^2 = g_\pi^2 \left[ \frac{1}{G_1} - 8 I_1(m_u) \right] = \frac{g_\pi^2 m_u^0}{G_1 m_u} \quad g_\pi^2 = \frac{1}{4 I_2(m_u, m_u)}. \quad (23)
\]

\[
M_K^2 = g_K^2 \left[ \frac{1}{G_1} - 4 (I_1(m_u) + I_1(m_s)) \right] + Z^{-1} (m_s - m_u)^2,
\]

\[
g_K^2 = \frac{1}{4 I_2(m_u, m_s)}. \quad (24)
\]

\[
M_\rho^2 = \frac{g_\rho^2}{4 G_2} = \frac{3}{8 g_2 I_2(m_u, m_u)}, \quad (25)
\]

\[
M_\phi^2 = M_\rho^2 \frac{I_2(m_u, m_u)}{I_2(m_u, m_s)}, \quad (26)
\]

\[
M_{K^*}^2 = M_\rho^2 \frac{I_2(m_u, m_u)}{I_2(m_u, m_s)} + \frac{3}{2} (m_s - m_u)^2. \quad (27)
\]
Now let us fix our basic parameters. For that we shall use the five experimental values:

1) The pion decay constant \( F_\pi = 93 \text{MeV} \).
2) The \( \rho \)-meson decay constant \( g_\rho \approx 6.14 \).

Then from the Goldberger-Treiman identity we obtain

\[
m_u = F_\pi g_\pi \tag{28}
\]

and from eqs. (20) and (21) we get

\[
g_\pi = \frac{g_\rho}{\sqrt{6Z}}, \quad m_u = \frac{F_\pi g_\rho}{\sqrt{6Z}} = 280 \text{MeV}. \tag{29}
\]

3) \( M_\pi \approx 140 \text{MeV} \). The eq. (23) gives \( G_1 = 3.48 \text{GeV}^{-2} \) (see [13]).
4) \( M_\rho = 770 \text{MeV} \). The eq. (24) gives \( G_2 = 16 \text{GeV}^{-2} \).
5) \( M_K \approx 495 \text{MeV} \). The eq. (25) gives \( m_s = 455 \text{MeV} \).

After that the masses of \( \eta, \eta' \) and \( K^*, \phi \) mesons can be calculated with a satisfactory accuracy.

It is possible also to give the qualitative estimations for the masses of the scalar and axial–vector mesons, using the formulae

\[
M_{A_{i,j}}^2 = M_{V_{i,j}}^2 + 6m_im_j, \tag{31}
\]
\[
M_{\sigma_{i,j}}^2 = M_{\phi_{i,j}}^2 + 4m_im_j. \tag{32}
\]

We can calculate the values of all the coupling constants, describing the strong interactions of the scalar, pseudoscalar, vector and axial–vector mesons with each other and with the quarks, and describe all the main decays of these mesons (see [4]).

### 3 The effective Lagrangian for the ground and excited states of the pions and kaons

To describe the first excited states of all the meson nonets, it is enough to use only three different inner parameters \( d_a \) in form factors \( f_a^U(k) \) (see eq. (11))

\[
f_{uu}^{P,V}(k) = c_{uu}^{P,V} (1 + d_{uu} k^2),
\]
\[
f_{us}^{P,V}(k) = c_{us}^{P,V} (1 + d_{us} k^2),
\]
\[
f_{ss}^{P,V}(k) = c_{ss}^{P,V} (1 + d_{ss} k^2). \tag{33}
\]

Following our work [12] we can fix the parameters \( d_{uu}, d_{us} \) and \( d_{ss} \) by using the conditions

\[
I_{1}^{f_{uu}}(m_u) = 0, \quad I_{1}^{f_{us}}(m_u) + I_{1}^{f_{us}}(m_s) = 0, \quad I_{1}^{f_{ss}}(m_s) = 0, \tag{34}
\]

where

\[
I_{1}^{f_{a}f_{a}}(m_i) = -iN_c \int \frac{d^4k}{(2\pi)^4} \frac{f_{a}f_{a}}{\Lambda_3^2 (m_i^2 - k^2)^2}. \tag{35}
\]

\( ^3 \)To calculate the masses of the \( \eta \) and \( \eta' \) mesons, it is necessary to take into account the gluon anomaly \( ^{3} \).
The eqs. (34) allows us to conserve the gap equations in the form usual for the NJL model (see eqs. (16)), because the tadpoles with the excited scalar external fields do not contribute to the quark condensates and to the constituent quark masses.

Using eqs. (34) we obtain for all $d_a$ close values

$$d_{uu} = -1.784 \text{ GeV}^{-2}, \quad d_{us} = -1.7565 \text{ GeV}^{-2}, \quad d_{ss} = -1.727 \text{ GeV}^{-2}. \quad (36)$$

Now let us consider the free part of the Lagrangian (12). For the pseudoscalar meson we obtain

$$L^{(2)}(\phi) = \frac{1}{2} \sum_{i,j=1}^{8} \sum_{a=0}^{2} \phi_i^a(P) K_{ij}^{ab}(P) \phi_j^b(P). \quad (37)$$

Here

$$\sum_{a=1}^{3} (\phi_i^a)^2 = (\pi_i^0)^2 + 2\pi_i^+\pi_i^-, \quad (\phi_i^4)^2 = 2K_i^+K_i^-,$$

$$K_i^0K_i^0, \quad (\phi_i^5)^2 = (\phi_i^u)^2, \quad (\phi_i^8)^2 = (\phi_i^s)^2. \quad (38)$$

$\phi_i^u$ and $\phi_i^s$ are the components of the $\eta$ and $\eta'$ mesons. The quadratic form $K_{ij}^{ab}(P)$, eq.(37), is obtained as

$$K_{ij}^{ab}(P) \equiv \delta^{ab}K_{ij}^a(P),$$

$$K_{ij}^a(P) = -\delta_{ij} \frac{1}{G_1} \sum_{a,b=1}^{8} \frac{1}{k^2 + \frac{1}{2}P - m_q^a - \frac{1}{2}P - m_{q'}^a} i\gamma_5 f_{ia} - \frac{1}{2}P - m_{q'}^a, \quad f_i^a \equiv 1, \quad f_i^a \equiv f_i^a(k). \quad (39)$$

$$m_q^a = m_u (a = 0, \ldots, 7); \quad m_q^8 = m_s;$$

$$m_{q'}^a = m_u (a = 0, \ldots, 3); \quad m_{q'}^8 = m_s (a = 4, \ldots, 8). \quad (40)$$

$m_u$ and $m_s$ are the constituent quark masses ($m_u \approx m_d$). The integral (39) is evaluated by expanding in the meson field momentum, $P$. To order $P^2$, one obtains

$$K_{11}^a(P) = Z_1^a(P^2 - M_1^{a2}), \quad K_{22}^a(P) = Z_2^a(P^2 - M_2^{a2})$$

$$K_{12}^a(P) = K_{21}^a(P) = \gamma^a(P^2 - \Delta^2 \delta_{ab}|_{b=4,\ldots,7}), \quad (\Delta = m_s - m_u) \quad (41)$$

where

$$Z_1^a = 4I_2^a Z, \quad Z_2^a = 4I_2^a Z, \quad \gamma^a = 4I_2^aZ,$$

$$M_1^{a2} = (Z_1^a)^{-1}\left[\frac{1}{G_1} - 4(I_1^a(m_q^a) + I_1^a(m_{q'}^a)) + Z^{-1}\Delta^2 \delta_{ab}|_{b=4,\ldots,7}\right], \quad (42)$$

$$M_2^{a2} = (Z_2^a)^{-1}\left[\frac{1}{G_1} - 4(I_1^a(m_q^a) + I_1^a(m_{q'}^a)) + Z^{-1}\Delta^2 \delta_{ab}|_{b=4,\ldots,7}\right]. \quad (43)$$

Here, $\bar{Z} = 1 - \Gamma^2 \frac{6m^2_{M_2}}{M_{M_2}^2} \approx 1$ (see eq. (49)), $I_n^a, f_{1n}^a$ and $f_{2n}^a$ denote the usual loop integrals arising in the momentum expansion of the NJL quark determinant, but now with zero, one or two factors $f_a(k)$, eqs.(33), in the numerator (see (33) and below)

$$I_2^{f_{1n}(m_q, m_{q'})} = -iN_c \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \frac{f_a(k) \cdots f_a(k)}{(m_q^2 - k^2)(m_{q'}^2 - k^2)}. \quad (45)$$
The evaluation of these integrals with a 3–momentum cutoff is described e.g. in ref. [13]. The integral over \( k_0 \) is taken by contour integration, and the remaining 3–dimensional integral is regularized by the cutoff. Only the divergent parts are kept; all finite parts are dropped. We point out that the momentum expansion of the quark loop integrals, eq.(39), is an essential part of this approach. The NJL–model is understood here as a model only for the lowest coefficients point out that the momentum expansion of the quark loop integrals, eq.(39), is an essential part regularized by the cutoff. Only the divergent parts are kept; all finite parts are dropped. We

\[
\phi^a_i = \sqrt{Z_1^a \phi_i^a}
\]

(46)

the part of the Lagrangian (37), describing the pions and kaons, takes the form

\[
L_\pi^{(2)} = \frac{1}{2} \left[ (P^2 - M_{\pi_1}^2) \pi_1^2 + 2\Gamma_{\pi} P_\pi \pi_2 + (P^2 - M_{\pi_2}^2) \pi_2^2 \right],
\]

(47)

\[
L_K^{(2)} = \frac{1}{2} \left[ (P^2 - M_{K_1}^2 - \Delta^2) K_1^2 + (P^2 - M_{K_2}^2 - \Delta^2) K_2^2 
+ 2\Gamma_K (P^2 - \Delta^2) K_1 K_2 \right].
\]

(48)

Here

\[
\Gamma_a = \frac{\gamma_a}{\sqrt{Z_1^a Z_2^a}} = \frac{I^{(a) \alpha} \sqrt{Z}}{\sqrt{I^{(a) \alpha} I^{(\alpha) \beta} Z}}.
\]

(49)

\[
M_{\pi_1}^2 = (4ZI_2(m_u, m_u))^{-1} \left[ \frac{1}{G_1} - 8I_1(m_u) \right] = \frac{m_u^0}{4Zm_uI_2(m_u, m_u)},
\]

(50)

\[
M_{\pi_2}^2 = (4I_2^f(m_u, m_u))^{-1} \left[ \frac{1}{G_1} - 8I_1^f(m_u) \right],
\]

\[
M_{K_1}^2 = (4ZI_2(m_u, m_s))^{-1} \left[ \frac{1}{G_1} - 4(I_1^f(m_u) + I_1^f(m_s)) \right] + (Z^{-1} - 1)\Delta^2
= \frac{m_u^0 + m_s^0}{4ZI_2(m_u, m_s)} + (Z^{-1} - 1)\Delta^2,
\]

(51)

\[
M_{K_2}^2 = (4I_2^f(m_u, m_s))^{-1} \left[ \frac{1}{G_1} - 4(I_1^f(m_u) + I_1^f(m_s)) \right].
\]

After the transformations of the meson fields (\( \sin\theta_a^0 = \sqrt{\frac{1 + \gamma_a}{2}}, \) see eq.(62))

\[
\phi^a = \cos(\theta_a - \theta_a^0)\phi_1^a - \cos(\theta_a + \theta_a^0)\phi_2^a,
\]

\[
\phi^a = \sin(\theta_a - \theta_a^0)\phi_1^a - \sin(\theta_a + \theta_a^0)\phi_2^a
\]

(52)

the Lagrangians (17) and (48) take the diagonal forms

\[
L_\pi^{(2)} = \frac{1}{2} (P^2 - M_{\pi_1}^2) \pi_1^2 + \frac{1}{2} (P^2 - M_{\pi_2}^2) \pi_2^2,
\]

(53)

\[
L_K^{(2)} = \frac{1}{2} (P^2 - M_{K_1}^2) K_1^2 + \frac{1}{2} (P^2 - M_{K_2}^2) K_2^2.
\]

(54)
Here

\[ M_{(\pi,\pi')}^2 = \frac{1}{2(1 - \Gamma_\pi^2)}[M_{\pi_1}^2 + M_{\pi_2}^2] \]

\[ (-, +) \sqrt{(M_{\pi_1}^2 - M_{\pi_2}^2)^2 + (2M_{\pi_1}M_{\pi_2} \Gamma_\pi)^2} \], \quad (55) \]

\[ M_{(K,K')}^2 = \frac{1}{2(1 - \Gamma_K^2)}[M_{K_1}^2 + M_{K_2}^2 + 2\Delta^2(1 - \Gamma_K^2)] \]

\[ (-, +) \sqrt{(M_{K_1}^2 - M_{K_2}^2)^2 + (2M_{K_1}M_{K_2} \Gamma_K)^2} \]. \quad (56) \]

and

\[ \tan 2\tilde{\theta}_a = \sqrt{\frac{1}{\Gamma_a^2} - 1} \left[ \frac{M_{\phi_a}^2 - M_{\phi_b}^2}{M_{\phi_a}^2 + M_{\phi_b}^2} \right], \quad 2\theta_a = 2\tilde{\theta}_a + \pi. \quad (57) \]

In the chiral limit we obtain: \( M_{\pi_1} = 0, M_{\pi_2} \neq 0 \) (see eqs. (50)) and

\[ M_{\pi}^2 = M_{\pi_1}^2 + O(M_{\pi_1}^4), \quad (58) \]

\[ M_{\pi'}^2 = \frac{M_{\pi_2}^2 + M_{\pi_1}^2 \Gamma_{\pi}}{1 - \Gamma_{\pi}^2} + O(M_{\pi_1}^4). \quad (59) \]

Thus, in the chiral limit the effective Lagrangian eq.(47) describes a massless Goldstone pion, \( \pi \), and a massive particle, \( \pi' \). We obtained similar results for the kaons.

For the weak decay constants of the pions and kaons we obtain (see [12])

\[ F_\pi = 2m_u \sqrt{ZI_2(m_u, m_u)} \cos(\theta_\pi - \theta_\pi^0), \]
\[ F_{\pi'} = 2m_u \sqrt{ZI_2(m_u, m_u)} \sin(\theta_\pi - \theta_\pi^0), \quad (60) \]

\[ F_K = (m_u + m_s) \sqrt{ZI_2(m_u, m_s)} \cos(\theta_K - \theta_K^0), \]
\[ F_{K'} = (m_u + m_s) \sqrt{ZI_2(m_u, m_s)} \sin(\theta_K - \theta_K^0). \quad (61) \]

In the chiral limit we have \( \theta_\pi = \theta_\pi^0 = \theta_0 \)

\[ \sin\theta_\pi^0 = \sqrt{\frac{1 + \Gamma_\pi}{2}}, \quad \cos\theta_\pi^0 = \sqrt{\frac{1 - \Gamma_\pi}{2}} \quad (62) \]

and

\[ F_\pi = \frac{m_u}{g_\pi}, \quad F_K = \frac{(m_u + m_s)}{2g_K}, \quad F_{\pi'} = 0, \quad F_{K'} = 0. \quad (63) \]

Here we used eqs. [19] and (21). Therefore, in the chiral limit we obtain the Goldberger-Treimann identities for the coupling constants \( g_\pi \) and \( g_K \). The matrix elements of the divergences of the axial currents between meson states and the vacuum equal (PCAC relations)

\[ \langle 0|\partial^\mu A_{\mu}^a|\phi^b \rangle = m_\phi^2 F_\phi \delta^{ab}, \quad (64) \]
\[ \langle 0|\partial^\mu A_{\mu}^a|\phi' \rangle = m_\phi^2 F_\phi \delta^{ab}. \quad (65) \]

Then from eqs. (58) and (63) we can see that these axial currents are conserved in the chiral limit, because their divergences equal zero, according to the low-energy theorems.
4 The effective Lagrangian for the ground and excited states of the vector mesons

The free part of the effective Lagrangian (12) describing the ground and excited states of the vector mesons has the form

\[ L^{(2)}(V) = - \frac{1}{2} \sum_{\alpha=0}^{3} \sum_{i,j}^{8} V_{i}^{\mu \alpha}(P) R_{ij}^{\mu \alpha}(P) V_{j}^{\mu \alpha}(P), \]  

(66)

where

\[ \sum_{a=0}^{3} V_{i}^{\mu \alpha} = (\omega_{i}^{\mu})^{2} + (\rho_{i}^{\mu})^{2} + 2 \rho_{i}^{+ \mu} \rho_{i}^{- \mu}, \quad (V_{i}^{4 \mu})^{2} + (V_{i}^{5 \mu})^{2} = 2 k_{i}^{* \mu} k_{i}^{* - \mu}, \]

\[ (V_{i}^{6 \mu})^{2} + (V_{i}^{7 \mu})^{2} = 2 k_{i}^{* 0 \mu} k_{i}^{* 0 \mu}, \quad (V_{i}^{8 \mu})^{2} = (\phi_{i}^{\mu})^{2} \]  

(67)

and

\[ R_{ij}^{\mu \alpha}(P) = - \frac{\delta_{ij}}{G_{2}} g^{\mu \nu} \]

\[ - i N_{c} \operatorname{tr} \int \frac{d^{4}k}{(2\pi)^{4}} \left[ \frac{1}{\not{P} - m_{q}^{2}} \gamma_{\mu} f_{1}^{a,V} \frac{1}{\not{k} - \frac{1}{2} \not{P} - m_{q}^{2}} \gamma_{\nu} f_{2}^{a,V} \right], \]

(68)

To order \( P^{2} \), one obtains

\[ R_{11}^{\mu \alpha} = W_{1}^{\alpha}[P^{2} g^{\mu \nu} - P^{\mu} P^{\nu} - g^{\mu \nu}(\tilde{M}_{1}^{a})^{2}], \]

\[ R_{22}^{\mu \alpha} = W_{2}^{\alpha}[P^{2} g^{\mu \nu} - P^{\mu} P^{\nu} - g^{\mu \nu}(\tilde{M}_{2}^{a})^{2}], \]

\[ R_{12}^{\mu \alpha} = R_{21}^{\mu \alpha} = \gamma^{\alpha}[P^{2} g^{\mu \nu} - P^{\mu} P^{\nu} - \frac{3}{2} \Delta^{2} g^{\mu \nu} \delta^{ab}|_{b=4.7}]. \]  

(69)

Here

\[ W_{1}^{a} = \frac{8}{3} I_{2}^{a}, \quad W_{2}^{a} = \frac{8}{3} I_{2}^{af a}, \quad \gamma^{a} = \frac{8}{3} I_{2}^{f a}, \]

(70)

\[ (\tilde{M}_{1}^{a})^{2} = (W_{1}^{a} G_{2})^{-1} + \frac{3}{2} \Delta^{2} \delta^{ab}|_{b=4.7}, \]

(71)

\[ (\tilde{M}_{2}^{a})^{2} = (W_{2}^{a} G_{2})^{-1} + \frac{3}{2} \Delta^{2} \delta^{ab}|_{b=4.7}. \]  

(72)

After renormalization of the meson fields

\[ V_{i}^{\mu \alpha} = \sqrt{W_{i}^{a}} V_{i}^{\mu \alpha} \]  

(73)

we obtain the Lagrangians

\[ L_{\rho}^{(2)} = - \frac{1}{2} [(g^{\mu \nu} P^{2} - P^{\mu} P^{\nu} - g^{\mu \nu} M_{\rho_{1}}^{2}) \rho_{1}^{\mu} \rho_{1}^{\nu}] + 2 \Gamma_{\rho}(g^{\mu \nu} P^{2} - P^{\mu} P^{\nu}) \rho_{1}^{\mu} \rho_{2}^{\nu} + (g^{\mu \nu} P^{2} - P^{\mu} P^{\nu} - g^{\mu \nu} M_{\rho_{2}}^{2}) \rho_{2}^{\mu} \rho_{2}^{\nu}, \]  

(74)

\[ L_{\phi}^{(2)} = - \frac{1}{2} [(g^{\mu \nu} P^{2} - P^{\mu} P^{\nu} - g^{\mu \nu} M_{\phi_{1}}^{2}) \phi_{1}^{\mu} \phi_{1}^{\nu}] + 2 \Gamma_{\phi}(g^{\mu \nu} P^{2} - P^{\mu} P^{\nu}) \phi_{1}^{\mu} \phi_{2}^{\nu} + (g^{\mu \nu} P^{2} - P^{\mu} P^{\nu} - g^{\mu \nu} M_{\phi_{2}}^{2}) \phi_{2}^{\mu} \phi_{2}^{\nu}, \]  

(75)
\[ L_{K^*}^{(2)} = -\frac{1}{2} [(g^{\mu\nu} P^2 - P^\mu P^\nu - g^{\mu\nu} (\frac{3}{2} \Delta^2 + M_{K^*}^2))] K^{*\mu} K^{*\nu} + 2 \Gamma_{K^*} (g^{\mu\nu} P^2 - P^\mu P^\nu - g^{\mu\nu} (\frac{3}{2} \Delta^2 + M_{K^*}^2))] K^{*\mu} K^{*\nu} + (g^{\mu\nu} P^2 - P^\mu P^\nu - g^{\mu\nu} (\frac{3}{2} \Delta^2 + M_{K^*}^2))] K^{*\mu} K^{*\nu}. \] (76)

Here

\[ M_{K^*}^2 = \frac{3}{8 G_2 I_2 (m_u, m_s)}, \quad M_{K^*_{12}}^2 = \frac{3}{8 G_2 I_2 (m_u, m_s)}, \] (77)

\[ \Gamma_{\alpha, \beta} = \frac{I_2^a (m_i, m_j)}{\sqrt{I_2^a (m_i, m_j) I_2^a (m_i, m_j)}}. \] (78)

After transformations of the vector meson fields, similar to eqs. (52) for the pseudoscalar mesons, the Lagrangians (74, 75, 76) take the diagonal form

\[ L_{V^a, V^a}^{(2)} = -\frac{1}{2} \left[(g^{\mu\nu} P^2 - P^\mu P^\nu - M_{V^a}^2) \bar{V}^{a\mu} V^{a\nu} + (g^{\mu\nu} P^2 - P^\mu P^\nu - M_{V^a}^2) \bar{V}^{a\mu} V^{a\nu}\right], \] (79)

where \( V^a \) and \( \bar{V}^a \) are the physical ground and excited states vector mesons

\[ M_{\rho, \rho}^2 = \frac{1}{2(1 - \Gamma_2^2)} \left[ M_{\rho_1}^2 + M_{\rho_2}^2 \left(1, + \right) \sqrt{(M_{\rho_1}^2 - M_{\rho_2}^2)^2 + (2M_{\rho_1} M_{\rho_2} \Gamma)^2} \right], \] (80)

\[ M_{\phi, \phi}^2 = \frac{1}{2(1 - \Gamma_2^2)} \left[ M_{\phi_1}^2 + M_{\phi_2}^2 \left(1, + \right) \sqrt{(M_{\phi_1}^2 - M_{\phi_2}^2)^2 + (2M_{\phi_1} M_{\phi_2} \Gamma)^2} \right], \] (81)

\[ M_{K^*, K^*}^2 = \frac{1}{2(1 - \Gamma_2^2)} \left[ M_{K^*_1}^2 + M_{K^*_2}^2 + 3 \Delta^2 (1 - \Gamma_2^2) \right] \left(1, + \right) \sqrt{(M_{K^*_1}^2 - M_{K^*_2}^2)^2 + (2M_{K^*_1} M_{K^*_2} \Gamma)^2}. \] (82)

### 5 Numerical estimations

We can now estimate numerically the masses of the pseudoscalar and vector mesons and the weak decay constants \( F_\pi, F_{\pi'}, F_K \) and \( F_{K'} \) in our model.

Because the masses formulae and others equations (for instance, Goldberger – Treiman identity and so on) have new forms in the NJL model with the excited states of mesons as compared with the usual NJL model, where the excited states of mesons were ignored, the values of basic parameters of this model \( (m_u, m_s, \Lambda_3, G_1, G_2) \) could change. However, we see that one can use the former values of the parameters \( m_u, m_s \) and \( \Lambda_3 = 1.03 \text{ GeV} \), because the conditions (34) conserve the gap equation in the old form (16) and one can satisfactory describe
the decay $\rho \to 2\pi$ in the new model using the cut-off parameter $\Lambda_3 = 1.03$ GeV (compare with eq. (30)). In new model $G_1 = 3.469$ GeV$^{-2}$. It is very close to former value $G_1 = 3.48$ GeV$^{-2}$, because $M_\pi \approx M_{\pi_1}$ (see eqs. (50), (55) and (23)). For the coupling constant $G_2$ the new value $G_2 = 12.5$ GeV$^{-2}$ will be used, which more noticeably differs from the former value $G_2 = 16$ GeV$^{-2}$ (see section 2). It is a consequence of the fact that the mass $M_{\rho_1}$ noticeably differs from the physical mass $M_\rho$ of the ground state $\rho$ (see eqs. (27) and (30)).

Using these basic parameters and the internal form factor parameter $d_{uu} = -1.784$ GeV$^{-2}$ (see eq. (36)) and choosing the external form factor parameters $c_{uu}^{\pi} = 1.37$ and $c_{uu}^{\rho} = 1.26$, one finds

$$
M_\rho = 768.3 \text{ MeV}, \quad M_{\rho'} = 1.49 \text{ GeV}, \\
M_\pi = 136 \text{ MeV}, \quad M_{\pi'} = 1.3 \text{ GeV}.
$$

(83)

$$
\Gamma_\pi = 0.474, \quad \Gamma_\rho = 0.545.
$$

(84)

$\Gamma_\pi = \sqrt{\frac{2}{Z}} \Gamma_\rho$ (see eqs. (83) and (84)). The experimental values are equal to

$$
M_{\rho_{\text{exp}}} = 768.5 \pm 0.6 \text{ MeV}, \quad M_{\rho'_{\text{exp}}} = 1465 \pm 25 \text{ MeV}, \\
M_{\pi_{\text{exp}}} = 139.57 \text{ MeV}, \quad M_{\pi'_{\text{exp}}} = 134.98 \text{ MeV}, \\
M_{\pi'} = 1300 \pm 100 \text{ MeV}.
$$

(85)

From eq. (60), one obtains

$$
F_\pi = 93 \text{ MeV}, \quad F_{\pi'} = 0.57 \text{ MeV}, \\
\frac{F_{\pi'}}{F_\pi} \approx \sqrt{\frac{1}{\Gamma_\pi^2 - 1} \left( \frac{M_\pi}{M_{\pi'}} \right)^2}
$$

(86)

Using the internal form factor parameter $d_{us} = -1.757$ GeV$^{-2}$ (see eq. (36)) and choosing the external form factor parameters $c_{us}^K = 1.45$, $c_{us}^{K*} = 1.5$, one finds

$$
M_{K^*} = 887 \text{ MeV}, \quad M_{K^*'} = 1479 \text{ MeV}, \\
M_K = 496 \text{ MeV}, \quad M_{K'} = 1450 \text{ MeV},
$$

(87)

$$
\Gamma_K = 0.412, \quad \Gamma_{K^*} = 0.473.
$$

(88)

The experimental values are equal to

$$
M_{K_{\text{exp}}} = 891.59 \pm 0.24 \text{ MeV}, \quad M_{K^*_{\text{exp}}} = 1412 \pm 12 \text{ MeV}, \\
M_{K^*_{\text{exp}}} = 493.677 \pm 0.016 \text{ MeV}, \quad M_{K^*_{\text{exp}}} = 497.672 \pm 0.031 \text{ MeV}, \\
M_{K^*_{\text{exp}}} = 1460 \text{ MeV}(?)
$$

(89)

From the eq. (61), one gets

$$
F_K = 1.16F_\pi = 108 \text{ MeV}, \quad F_{K^*} = 3.3 \text{ MeV}.
$$

(90)

And for the $\phi$ and $\phi'$ we obtain, using the form factor parameters $d_{ss} = -1.727$ GeV$^{-2}$ (see eq. (36)) and $c_{ss}^\phi = 1.41$

$$
M_\phi = 1019 \text{ MeV}, \quad M_{\phi'} = 1682 \text{ MeV}, \quad \Gamma_\phi = 0.411
$$

(91)

The experimental values are equal to

$$
M_{\phi_{\text{exp}}} = 1019.413 \pm 0.008 \text{ MeV}, \quad M_{\phi'_{\text{exp}}} = 1680 \pm 50 \text{ MeV}.
$$

(92)

We can see that the parameters $c_{us}$ and $c_{ss}$ are close to each other.
6 Summary and conclusions

Let us discuss the obtained results. Conditions (34) allow us to conserve the gap equations in the form usual for the standard NJL model (see eqs.(16)) and to fix all the internal form factor parameters $d_a$. These parameters are equal to each other with accuracy $1.5\%$. As a result, the constituent quark masses $m_u$ and $m_s$ and the cut-off parameter $\Lambda_3$ conserve their former values after introducing excited meson states into the model. (The constant $\bar{g}_\rho$ describing the decay $\rho \to 2\pi$ in the new model approximately equals the former constant $g_\rho$ (eq.(30)) for $\Lambda_3 = 1.03\text{GeV}$.)

We can describe all the excited states of strange mesons (pseudoscalar kaons, vector kaons and the $\phi$-meson) using practically only one value of the external parameter $c_a$, because $c^K_{us} \approx c^K_{us} \approx c^\phi_{ss}$ with accuracy $3\%$.

A more complicated situation takes place in the sector of light mesons consisting of $u$ and $d$ quarks. Here the parameters $c^\pi_{uu}$ and $c^\rho_{uu}$ not only differ noticeably from similar parameters of strange mesons but they are also not equal to each other. This difference equals $8\%$. The coupling constant of the effective four-quark interaction describing the excited $\rho$-meson states, $G^2(c^\rho_{uu})^2$, is 1.4 times less than the coupling constants $G^2c^2_{us}$ and $G^2c^2_{ss}$ describing the excited states of strange mesons.

The mass of the first radial excited state of the pion has an interesting history. Three years ago the new experimental information about excited states in the few-GeV region, e.g on the $\pi'$ meson, was obtained at IHEP (Protvino). Indications of the light resonance in diffractive production of $3\pi$–states have lead to speculations that the mass of the $\pi'$ may be considerably lower at $\approx 750\text{MeV}$ [2]. To describe this pion state in our model, it is necessary to use a much larger value of $c^\pi_{uu} = 1.67$. And for this pion state the decay channel $\pi' \to \rho\pi$ observed in experiment [1] is forbidden. Therefore, we prefer to consider the first radial excited pion state as a state with mass $1.3\text{ GeV}$.

Besides the description of the excited meson masses, the weak decay coupling constants $F_{\pi'}$, $F_K$ and $F_K'$ were calculated. In future we are going to calculate the decay widths of the excited meson nonets. The preliminary result on the decay width $\pi' \to \rho\pi$ satisfies the experimental data.

We have here considered the simplest extension of the NJL–model with polynomial meson–quark form factors and have shown that this model can be useful for describing the excited states of mesons.

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