Parallax in the Park

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Abstract

This article describes a parallax experiment performed by undergraduate physics students at Queensland University of Technology. The experiment is analogous to the parallax method used in astronomy to measure distances to the local stars. The result of one of these experiments is presented in this paper. A target was photographed using a digital camera at five distances between 3 and 8 metres from two vantage points spaced 0.6 m apart. The parallax distances were compared with the actual distance measured using a tape measure and the average error was 0.5 ± 0.9 %.

Keywords: parallax, stars, Hipparchus, distance

Introduction

Parallax is the primary method of measuring distances in the local universe. As far as we know the first person to make parallax measurements was the German mathematician and astronomer Johannes Kepler (1571-1630). It is quite remarkable that he made parallax observations accurate enough to derive his three planetary laws before the invention of the telescope.

For example, six years before Galileo pointed the newly invented telescope towards the heavens, in 1604 Kepler used parallax to determine that a supernova was too far away to exhibit parallax and therefore was considered to be in the sphere of the stars. At this time the stars were thought to be equidistant from the central Earth and stuck onto a crystalline sphere. SN1604 is actually 20 000 light years from the Earth.
In Kepler’s day, only the five naked eye planets were known – Mercury, Venus, Mars, Jupiter and Saturn. Saturn exhibits the smallest parallax since it is the farthest planet from the Earth. The parallax of the nearest star to the Earth, Alpha Centauri is 0.74 pc, in other words over a six month period the star moves 0.74 seconds of arc, one second of arc being just 1/3600 of a degree. (The angular shift is actually twice 0.74 seconds of arc since the parallax angle is defined as half the total angular shift). The resolution of the eye is of the order of one minute of arc – 60 times greater than an arc second and therefore it would have been impossible for Kepler to measure stellar parallax. Kepler was able to plot the orbits of the planets scaled in terms of the Astronomical Unit (AU) the distance between the Earth and Sun.

Kepler discovered that the orbits of the planets around the Sun were elliptical, and not perfect circles, as supposed by the ancients. The main purpose of Cook’s voyage to the antipodes was to observe the 1769 transit of Venus to measure the distance to Venus in kilometres or miles using the parallax method. Up until that time the size of the Solar System was known in terms of the Astronomical Unit (AU), the average distance between the Earth and Sun, but not in kilometres or miles. The parallax technique is relatively simple and suitable as an exercise for high school and undergraduate students (Cenadelli et al, 2009, Stewart 2011).

The achievement of men such as Kepler seem even more remarkable when placed against the ubiquitous reliance on electronic gadgets in the modern world.

After the invention of photographic film, the parallax method was used to measure distances to the local stars. Ground based telescopes can measure distances within 100 pc of the earth using parallax (Webb 1999, de Grijs 2011). Space borne telescopes extend the range by a factor of 10 (Kovalevsky 1998). In 1989 the European Space Agency (ESA) launched a satellite called Hipparcos, which measured stellar distances out to 1 000 pc (sci.esa.int/hipparcos) until ceasing operations in 1993. The Hipparcos Catalogue contains accurate distances to 188 218 stars.

In 2013, ESA launched the GAIA (Global Astrometric Interferometer for Astrophysics) spacecraft with the aim of measuring the parallax of one billion stars in the Milky Way to produce a 3D map (http://sci.esa.int/gaia/).

This paper describes a parallax experiment performed by undergraduate astrophysics students in the Brisbane city Botanical Gardens, adjacent to the Gardens Point campus of Queensland University of Technology. The technique is directly analogous to the technique used in astronomy, except photographs are taken more or less at the same time rather than six months apart. Figure 1 shows the method of measuring distances using the parallax. In the Botanic Gardens exercise, the Earth is replaced by a digital camera, the star by a bright orange target, and the fixed stars by the buildings of the Central Business District (CBD) of Brisbane.
Figure 1. Measuring distances using the parallax method. The local star is photographed against the distant stars when the earth is on opposite sides of the Sun, i.e. 6 months apart. The diameter of the Earths’ orbit is the baseline of a triangle with an apex angle of $\theta$. The parallax angle is $\theta/2$.

A photograph of a star is taken six month apart so that the diameter of the Earth’s orbit around the Sun forms the baseline of a surveying triangle. The distance to a star is calculated using

$$d = \frac{b/2}{\sin(\theta/2)}$$

Where $d$ is the distance to the star and $b$ the length of the base-line which is the diameter of the Earth’s orbit around the Sun (300 million km). The total angular shift of the star against the distant backdrop of the stars is $\theta$. The parallax angle is designated as half of the total shift, i.e. $\theta/2$.

Normally in astrophysics, angle is specified in seconds of arc (arcseconds), which is defined as 1/3600 of a degree. The inverse of the parallax angle specified in arcseconds is the distance to a star in parsecs (pc). A star must be at a distance of 3.26 light years from the Earth to exhibit a parallax of one second of arc. The closest visible star to the Earth is Alpha Centauri at a distance of 4.4 light years (1.3 pc).

**Method**

The experiment described in this paper was performed in the Brisbane City Botanic Gardens adjacent to the Gardens Point campus of Queensland University of Technology. The photographs were taken by a group of second year physics students, although author SH analysed the images independently and obtained similar results.
Two tripods were placed on the lawn. The distance between the tripod bolts that screw into the base of the camera was measured as $60 \pm 1$ cm. The central axis of the optical system of the camera will, in most cases, be offset from the tripod bolt by a few centimetres. If the camera axes are parallel the error will be close to zero, but when the cameras axes are non-parallel because they are pointing at the target for example, there will be a small error in the baseline distance. However, this will be encompassed by the $\pm 1$ cm in the baseline distance.

Initially, a meter rule was placed on a tripod 3.38 m from the camera. This distance was chosen so that the rule filled most the field of view. The photo is shown in figure 2. This image was used to calculate the physical size of the pixels in the CCD (Charge Coupled Device) chip in the camera. The physical size of the CCDs is required to convert pixel distances to physical distance in the camera. The physical distance is combined with the focal length to calculate the parallax angle ($\theta$) as shown in figure 1. The technique described in this paper is based on the principle of similar triangles, i.e. the triangle inside the camera is a scale model of the external parallax triangle.

Figure 2. Photo of a metre rule on a tripod for obtaining the physical size of the CCD pixels in the camera.
The target shown in figure 3 was placed at a distance 3.38, 4.78, 5.69, 6.82 and 7.82 ± 0.01 m from the midpoint of the line between the tripod camera attachment bolts. There was no particular reason for these distances other than the target was already at 3.38 when the calibration image was taken and then the target was moved back about one metre at a time for each subsequent measurement.

A photo of the target was taken with a Canon Power Shot S45 4 Mega Pixel camera placed first on one tripod and then on the other. The focus was set to infinity so that both the target and buildings of the CBD were in focus. The camera does not have to point exactly at the target.

![Figure 3. A QUT student performing the parallax experiment. The display of nine fingers indicates that the photo is the 9th image in the sequence. In this case the target was 7.8 m away from the camera baseline. The diagonal cross in the circle in the top right of the picture is the reference used for calculating the parallax angle.](image)

After the experiment the photos were placed on the university Blackboard teaching website to enable students to download the images for further analysis using a freely available image analysis program such as ImageJ (http://imagej.nih.gov/ij/). The first step is to calculate the physical size of the CCD pixels. This is shown in figure 4 and following equations.
Figure 4. Schematic diagram of the experimental geometry for finding the physical size of the CCD pixels in the camera. The symbols in the figure are as defined in the text. The triangle with sides $x$ and $f$ inside the camera is similar to the triangle with sides $r$ and $d$ outside the camera.

We begin by noting that the triangle with sides $x$ and $f$ inside the camera is similar to the triangle with sides $r$ and $d$ outside the camera.

\[
\frac{x}{f} = \frac{r}{d} \Rightarrow x = \frac{rf}{d} \quad \text{pixel size} = \frac{x}{n} \quad \theta = \tan^{-1}\left(\frac{x}{f}\right)
\]

Where $f =$ focal length of the lens, $x =$ physical distance of a point on the image from the centre-line, $d =$ distance along centre-line to rule, $r =$ distance from centre-line to a point on the rule, $n =$ no of pixels from centre-line.

The one metre rule was photographed at a distance of $3.38 \pm 0.01$ m with a camera focal length of $21.3225$ mm (given in the image file). The horizontal distance of a $50$ cm section of the rule was measured as $980$ pixels using the line function of ImageJ. Therefore the physical pixel size on the CCD chip was

\[
x = \frac{rf}{d} = \frac{500 \times 21.3225}{338} = 3.154 \pm 0.009 \text{ mm} = 3.154 \pm 0.009 \mu\text{m}
\]

\[
\text{pixel size} = \frac{x}{n} = \frac{3.154}{980} = 0.003218 \pm 0.000016 \text{ mm} = 3.218 \pm 0.016 \mu\text{m}
\]

The error in $r$ was taken to be $\pm 1$ cm and the error in $n \pm 2$ pixels. The error in the pixel size was calculated by reducing the distance to the rule from $3.38$ to $3.37$ m and the horizontal pixel length from $980$ to $978$. The length of the rule used in the
calibration was assumed to be fixed at 0.5 m and the camera focal length 21.3225 mm as specified in the image header.

For each pair of images (2-3, 4-5, 6-7, 8-9, 10-11) the following procedure was used. A reference point, or origin, was chosen in the image — as marked on the image in figure 3. The horizontal distance between the reference and the target was measured on the two parallax images (figure 5). This was done by finding the difference between the x coordinate of the reference point and the x coordinate of the target on the first image ($\Delta x_1$). This was repeated for the second image ($\Delta x_2$). (N.B. the ordering of each pair of images is not important). The two $\Delta x$ values were then subtracted from each other and the modulus taken, i.e. $\Delta(\Delta x) = |\Delta x_1 - \Delta x_2|$ . The results are shown in table 1.

![Image 1](image1.png)

![Image 2](image2.png)

Figure 5. The horizontal pixel distance between a reference point and centre of the target is measured for images take from two different vantage points.

For each $\Delta(\Delta x)$, the physical CCD distance was found by multiplying the value by the pixel size. This was used in conjunction with the lens focal length to calculate the
parallax angle using equation (1). When the errors in the measurements were propagated the overall error was found to be ±17 cm. This is the size of the error bars on the plot in figure 7. Note that the linear regression line passes through each error bar indicating that this is a reasonable estimate of the error.

The group of students who acquired the images shown in this paper used a slightly different method to calculate the parallax distance. They matched the distant background in each pair of images and measured the horizontal pixel distance between the target centres directly as shown in figure 6, so this is another option for analysing the data.

The pixel distance was converted to a physical distance by multiplying the pixel distance by the physical size of each CCD element. The parallax angle was then calculated using the focal length of the lens in the equation \( \theta = \tan^{-1}(\Delta x/f) \). The distance to the target \((d)\) based on the parallax angle was then calculated using equation (1).

**Results**

Table 1 shows the data obtained from five pairs of images (provided in supplementary materials). Table 2 shows the measurements in a more convenient form with the
associated percentage error for each pair of values. Figure 7 shows a plot of the
distance as measured using the parallax angle versus the tape measure distance ($L$).

Table 1. The basic data obtained from the images. $L$ is the distance to the target measured using the
tape measure and $d$ the distance calculated using the parallax equation (1).

| $L$/m | x coordinate | Image 1 | Image 2 | $\Delta$(d) | $x$/mm | $\theta$/rads | d/m  |
|-------|--------------|---------|---------|-------------|--------|---------------|------|
| 7.82  | Target       | 1272    | 1766    | -506       | 6      | 1.65          | 0.077|7.76 |
|       | Reference    | 1778    | 1760    | -          |        |               |      |
| 6.82  | Target       | 1668    | 1218    | 22         | -560   | 582           | 1.87 | 0.088|6.83 |
|       | Reference    | 1646    | 1778    |            |        |               |      |
| 5.69  | Target       | 916     | 1732    | -608       | 86     | 694           | 2.23 | 0.105|5.73 |
|       | Reference    | 1524    | 1646    |            |        |               |      |
| 4.78  | Target       | 1950    | 826     | 132        | -710   | 842           | 2.71 | 0.127|4.72 |
|       | Reference    | 1818    | 1536    |            |        |               |      |
| 3.38  | Target       | 894     | 1939    | -1082      | 111    | 1193          | 3.84 | 0.180|3.33 |
|       | Reference    | 1976    | 1828    |            |        |               |      |

Table 2. Percentage error of parallax distance measurements.

| Tape (m) | Parallax (m) | % error |
|----------|--------------|---------|
| 3.38     | 3.33         | -1.34   |
| 4.78     | 4.72         | -1.22   |
| 5.69     | 5.73         | 0.66    |
| 6.82     | 6.83         | 0.13    |
| 7.82     | 7.76         | -0.75   |
| mean     |              | 0.50    |
| sd       |              | 0.87    |
Discussion
This parallax experiment can achieve accurate results. The largest source of error is undoubtedly using a flexible tape to measure distance as the error due to the sag is very difficult to quantify. Figure 7 suggests that the error bars actually overestimate the error. The experiment could perhaps be improved by using a laser range finder to measure the distance between the centre of the camera tripods and target.

Students use a single camera supplied for the experiment. At the present time, students are able to acquire data inside an hour and so two groups of students can collect the required images in a 2-hour session. Images could be acquired even faster if two cameras were used to avoid the need for moving a single camera from one tripod to the other. This would reduce the error in baseline distance since the chance of bumping the tripods when removing and attaching the camera is diminished.

Nowadays, two students out of a group of four will often have the same camera, for example an iPhone, that could be used for the experiment. If this is the case then only three tripods are required. As it stands this experiment gives students hands-on experience of measuring distance using parallax.
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