Bloch oscillations: atom optical interpretation, realizations, and applications

Karl-Peter Marzlin and Jürgen Audretsch

Fakultät für Physik der Universität Konstanz
Postfach 5560 M 674
D-78434 Konstanz, Germany

Abstract

The cyclic motion of particles in a periodic potential under the influence of a constant external force is analyzed in an atom optical approach based on Landau-Zener transitions between two resonant states. The resulting complex picture of population transfers can be interpreted in an intuitive diagrammatic way. The model is also applied to genuine atom optical systems and its applicability is discussed.

PACS: 42.50.Vk

Introduction

Bloch oscillations have been predicted by Bloch and Zener in connection with the electronic transport in crystal lattices: If a constant static electric field is acting on electrons moving in a periodic potential with period \( L \), the electron momentum turns out to be periodic in time with frequency \( \omega_B = eEL/\hbar \). Accordingly the driving electric field \( E \) leads to oscillations of the electrons. In normal crystals these oscillations cannot be seen because of dephasing collisions of the electrons. But in semiconductor superlattices the accompanying radiation has been observed. The recent experiments of Ben Dahan et al. represent a new step in the detection of Bloch oscillations. They are for the first time detected in the domain of atom optics. The authors study atoms (instead of electrons) moving in a periodic potential whereby the constant external force is simulated in tuning linearly in time the two counterpropagating laser waves which create the potential. Other atom optical systems too show Bloch oscillations. Evidently the related oscillations in space may be used to trap and store atoms. An example based on a three-level atom has been discussed by the authors in Ref. On the background of growing interest in Bloch oscillations of atoms it seems to be desirable to give a genuine atom optical interpretation of Bloch oscillations. We will show below that in this approach Landau-Zener transitions, which are known in connection with level crossing and the related population transfer between the levels of 2-level atoms, represent the dominating physical effect. Based on this we study atom optical realizations of Bloch oscillations and show how the respective systems can be used to drive trapped atomic clouds with almost constant velocity thus forming an "atomic elevator". The range of application where the model may be used is discussed.

Bloch oscillations from an atom optical point of view

A typical problem in theoretical solid state physics is the motion of electrons in a periodic potential with constant driving force being applied, in addition. This force \( F \) may go back to a static static electric field or a gravitational field. In the one-dimensional case the Hamiltonian is given by \( H_0 = p^2/(2M) - Fx + V(x) \) where the potential \( V \) is periodic with lattice distance \( L \), \( V(x + L) = V(x) \). If \( V \) fulfills the Dirichlet conditions (roughly, it can have only a finite number of finite jumps) then \( V \) can have written as

\[
V(x) = \frac{α_0}{2} + \sum_{l=1}^{∞} \left\{ α_l \cos(2πlx/L) + β_l \sin(2πlx/L) \right\}
\]

where \( α_0 \) is a constant term.

1e-mail: peter.marzlin@uni-konstanz.de
2e-mail: juergen.audretsch@uni-konstanz.de
with real coefficients $\alpha_l$ and $\beta_l$. In momentum space and in the interaction picture with respect to the acceleration potential (i.e., taking $H_0 = -Fx$) the Schrödinger equation is given by

$$i\hbar \partial_t \psi(\tilde{p}, t) = \left[ \frac{\tilde{p}^2 + Ft}{2M} + \frac{\alpha_0}{2} \right] \psi(\tilde{p}, t) + \frac{1}{2} \sum_{l=1}^{\infty} \left\{ \Omega_l^* \psi(\tilde{p} + i\hbar k_0) + \Omega_l \psi(\tilde{p} - i\hbar k_0) \right\}$$

(2)

where $k_0 := 2\pi/L$ is the border of the first Brillouin zone and $\Omega_l := \alpha_l + i\beta_l$. $\tilde{p}$ is a time independent momentum parameter. $|\psi(\tilde{p}, t)|^2$ is the probability to measure at the time $t$ the momentum $\tilde{p} + Ft$ so that $\tilde{p}$ may be interpreted as initial ($t = 0$) momentum. Obviously only a discrete ladder of states is coupled so that it is favorable to introduce the notation

$$\psi_n(t) := \psi(\tilde{p}_0 + n\hbar k_0, t) \quad n = 0, \pm 1, \pm 2, \ldots$$

(3)

where $\tilde{p}_0$ is a momentum parameter. Setting

$$\varepsilon_n := \frac{1}{\hbar} \left\{ \frac{(\tilde{p}_0 + n\hbar k_0 + Ft)^2}{2M} + \frac{\alpha_0}{2} \right\}$$

(4)

we obtain the dynamical equation

$$i\hbar \partial_t \psi_n = \varepsilon_n \psi_n + \frac{1}{2} \sum_{l=1}^{\infty} \left\{ \Omega_l^* \psi_{n+l} + \Omega_l \psi_{n-l} \right\}.$$  

(5)

which can be written in matrix form.

$i\hbar \begin{pmatrix} \psi_1 \\ \psi_0 \\ \psi_{-1} \\ \psi_{-2} \end{pmatrix} = \begin{pmatrix} \cdots & \cdots & \cdots & \cdots \\ \varepsilon_1 & \Omega_1 & \Omega_2 & \Omega_3 & \cdots \\ \Omega_1^* & \varepsilon_0 & \Omega_1 & \Omega_2 & \cdots \\ \Omega_2^* & \Omega_1^* & \varepsilon_{-1} & \Omega_1 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} \cdots \\ \psi_1 \\ \psi_{-1} \\ \psi_{-2} \\ \cdots \end{pmatrix}$

(6)

What types of transitions are caused by the matrix in Eq. (6) between the states $\psi_n$ and $\psi_{n+l}$, for example? To see this it is important to concentrate on resonances. The states $\psi_n$ and $\psi_{n+l}$ are coupled by the $2 \times 2$ matrix containing $\varepsilon_n$, $\Omega_l$, $\Omega_l^*$, and $\varepsilon_{n+l}$. Because of Eq. (4) the difference between the related diagonal elements is given by

$$\varepsilon_{n+l} - \varepsilon_n = \frac{k_0}{M} \left\{ \frac{1}{2} (2n + l)\hbar k_0 + \tilde{p}_0 + Ft \right\}.$$

(7)

Accordingly there will be a transition when this difference becomes small (resonance case). It happens at a time $t_{n,l} > 0$ when an initial momentum $\tilde{p}_0$ has been changed by the influence of the force to a multiple of $\hbar k_0/2$:

$$\tilde{p}_0 + Ft_{n,l} = -\hbar k_0 \left( n + l/2 \right).$$

(8)

The r.h.s. agrees with the momentum of the particle at the beginning of the transition. To work out the time $t_{n,l}$, i.e., to fix $n$ and $l$ for the transition between two states $\psi_a$ and $\psi_b$ with $a > b$, one has to take into account that $n + l > n$. Accordingly one can read off directly that $n$ is equal to the smaller index $(b = n, l = a - b)$.

We now assume that the recoil shift $\hbar k_0^2/(2M)$, which determines the magnitude of the most important term (quadratic in $l$) in Eq. (7), is large compared to $|\Omega_l|$. In this case all transitions between $\psi_n$ or $\psi_{n+l}$ and any other state will be far off resonance at the time $t_{n,l}$. Each individual transition (beside $\Omega_l$) is then (in general) highly suppressed and we will assume that around $t = t_{n,l}$ the transition with $\Omega_l$ is the only one which has to be taken into account for the pair of states $\psi_n$ and $\psi_{n+l}$. Eq. (6) shows that more than two states cannot be in resonance with each other and reveals the physical structure of the fundamental transition involved. The energy difference (7) approaches zero linearly in time. Accordingly the important observation is that we have in resonance a Landau-Zener transition (LZT) (8).
The efficiency of the LZT is determined by the probability that the particles stay in the initial state
\[ P_{\text{stay}}(l) = \exp \left\{ -\pi \frac{|\Omega_l|^2}{4|\hbar k_0a|} \right\}. \] (9)

This is an exact expression for the asymptotical transition probability which is a good approximation if the time between two subsequent transitions being in resonance is much larger than the LZT time
\[ t_{\text{L,Z}} := \frac{|\Omega_l|^2}{|\hbar k_0a|^{3/2}}. \] (10)

Characteristic for the LZT between \( \psi_n \) and \( \psi_{n+l} \) is that the states in resonance are at equal kinetic energy and that according to Eq. 3 there is a momentum transfer of \( \pm \hbar k_0 \) depending on the direction of the transition.

Let us now describe the resulting time development in detail starting with a particular initial state. Parallel to the discussion of the equations above we will sketch a more intuitive diagrammatic picture to make the underlying physics transparent. For this we show in the figure the kinetic energy of the particles as a function of their momentum \( p(t) \). The horizontal arrows denoted by \( |\Omega_l| \) represent LZTs. The respective transition probabilities \( P_{\text{trans}} = 1 - P_{\text{stay}} \) are given by Eq. 4. They fall off rapidly with increasing \( l \) if the \( \Omega_l \) do not grow with \( l \).

With regard to later applications we assume that \( F \) represents the gravitational acceleration, \( F = -Mg \) with \( q > 0 \). The x-Axis points vertically upwards. Let us for simplicity assume that the LZTs have vanishing duration. In this case they happen at the times \( t_{n,l} \) of Eq. 3. Between the LZTs the particles fall almost freely (i.e., move along the parabola to the left). We start at \( t = 0 \) in the state \( \psi_0 \) with momentum \( \tilde{p}_0 \). Transitions from \( \psi_0 \) are possible with \( |\Omega_l| \) to \( \psi_{0,l} \) at the time \( t_{0,l} \) (compare the remark after Eq. 3) and to \( \psi_{-l} \) at the time \( t_{-l,1} \) with momentum transfer \( -\hbar k_0 \). Let us assume for example \( +1/2 < \tilde{p}_0/\hbar k_0 < 1 \). Then it can be read off from Eq. 3 that the smallest \( t_{0,l} \) or \( t_{-l,1} \) is obtained for \( l = +1, n = -1 \). The corresponding transition is \( \psi_0 \rightarrow \psi_{-1} \) with a momentum change \(-\hbar k_0 \), compare Eqs. 3 and 4. Turning to the figure this means that we start with \( p(t = 0) = \tilde{p}_0 \), move in free fall along the parabola until at the time \( t_{-1,1} \) the momentum \( p = \hbar k_0/2 \) is reached. Then the first LZT (in this case with \( |\Omega_1| \)) may happen. The transition ends in the state \( \psi_{-1,1} \) with momentum \(-\hbar k_0/2 \). The transition probability depends on \( P_{\text{stay}} \) and is therefore determined by \( |\Omega_1| \). To follow the time development the most transparent procedure is to start our clock anew. We therefore take now as initial state \( \psi_0 \) with \( p = \tilde{p}_0 = -\hbar k_0/2 \). Then the smallest \( t_{n,l} \) is because of Eq. 3 obtained for \( n + l/2 = 1 \) and accordingly for \( l = 2 \) and \( n = 0 \). This is the transition for which \( \epsilon_2 = \epsilon_0 \) in Eq. 3 vanishes. Therefore we have \( \psi_0 \rightarrow \psi_2 \). The related momentum transfer is \(+2\hbar k_0 \) so that we end with momentum \( p = \hbar k_0 \). In the figure this corresponds to the free fall from \( p = -\hbar k_0/2 \) to \( p = \hbar k_0 \) and the \( |\Omega_2| \) transition to the right with a probability obtained from Eq. 3 with \( l = 2 \) and \( |\Omega_2| \).

After this, those particles which have made the transition fall again and so on.

The larger \( l \) of \( \Omega_l \) is the larger is according to Eq. 4 the possibility that the particles do not make the \( |\Omega_l| \) transition but follow the parabola to the left (free fall). If \( \Omega_l \) does not increase with \( l \), the \( |\Omega_1| \) transition is the most effective one. Accordingly, if particles start with \(-1/2 < \tilde{p}_0/\hbar k_0 < +1/2 \) they will fall until they reach \( p(t)/\hbar k_0 = -1/2 \) when, for small \( P_{\text{stay}} \), they will practically all be kicked to \( p(t)/\hbar k_0 = +1/2 \) from where they fall again until \( p(t)/\hbar k_0 = -1/2 \), are kicked again and so on. These are the simple Bloch oscillations with Bloch frequency \( \omega_B = |F|L/h \) . But as we have seen above there is the non vanishing probability \( P_{\text{stay}}(l = 1) \) that the particles carry on falling (Zener tunneling) and may then be subject to higher order Bloch oscillations with LZTs \( |\Omega_2|, |\Omega_3|, \ldots \) in both directions if terms with \( l \neq 1 \) in the potential \( V(x) \) of Eq. 3 don’t vanish so that a rather complex situation with many different oscillations arises. For \( \Omega_l \neq 0 \) the potential is of trigonometric shape and the only oscillations which may happen for \( \tilde{p}_0 > 0 \) are simple Bloch oscillations. This completes our atom optical approach in which the involved dynamics related to the general periodic potential \( V(x) \) of Eq. 3 is based on LZTs in 2-level systems.

**Atom optical realizations**

Atoms with only a few energy levels are well studied objects of atom optics. Many effective realizations can be found. It is therefore not surprising that several experimental set ups can be proposed where Bloch oscillations are the dominating effect. We have made one proposal in Ref. 2 in connection with the gravito-optical trapping of 3-level atoms. A 3-level A-system is exposed to two counterpropagating laser
fields (inducing Raman transitions). It is closed by a magnetic hyperfine field tuned to be in resonance with the transitions between the two ground states. The influence of a homogeneous gravitational driving field is taken into account. A discussion in terms of dressed states leads to Eq. (34) of Ref. [8] which agrees with Eq. (3) with \( \Omega_l = 0 \) for \( l \neq 1 \). Again for appropriate initial conditions a sequence of up and down motions is obtained which are simple Bloch oscillations.

Making again use of the Earth’s gravitational acceleration \( g \) one may alternatively consider a 2-level atom in a resonant standing laser wave with the Hamiltonian

\[
H_{2 \times 2} = 1\left\{ \frac{\vec{p}^2}{2M} - M\vec{g} \cdot \vec{x} \right\} + \frac{\hbar \omega_0}{2} \sigma_3 - \Omega \cos(\vec{k} \cdot \vec{x}) \cos(\omega_0 t) \sigma_1
\]  

(11)

where \( \sigma_i \) are the Pauli matrices and \( \omega_0 \) is the resonance frequency. Turning to dressed states by means of the unitary transformation

\[
U' = \begin{pmatrix}
\exp[-i\omega_0 t/2] & \exp[i\omega_0 t/2] \\
\exp[-i\pi \sigma_2/4] & \exp[-i\pi \sigma_2/4]
\end{pmatrix}
\]  

(12)

\( (|\psi'\rangle = U|\psi\rangle) \) we find after the rotating wave approximation

\[
H' = \left\{ \frac{\vec{p}^2}{2M} - M\vec{g} \cdot \vec{x} \right\} 1 - \frac{1}{2} \hbar \Omega \cos(\vec{k} \cdot \vec{x}) \sigma_3.
\]  

(13)

For each of the two states this is equivalent to the Hamiltonian \( H_s \) with \( \Omega_l = 0 \) for \( l \neq 1 \). This means that if the Hamiltonian for the \( \psi_t \) is written as a matrix that it is tridiagonal. The Landau-Zener structure of the transitions is particularly clear in this example.

An experimental realization of pure Bloch oscillations of atoms has been discussed by Ben Dahan et al. [9]. Here the atoms are not subject to a potential \( -M\vec{g} \cdot \vec{x} \) but are exposed to two linearly chirped running laser waves. That this is equivalent to the situation above can be seen by applying the unitary transformation \( \exp[iM\vec{g} \cdot \vec{x} t/\hbar] \exp[-i\vec{p} \cdot \vec{g} t^2/(2\hbar)] \exp[-iM\vec{g}^2 t^3/(6\hbar)] \) to Eq. (11). Physically this represents a transition to an accelerated reference frame. The result of the transformation is a Hamiltonian with two chirped running waves

\[
H = 1\left\{ \frac{\vec{p}^2}{2M} + \frac{\hbar \omega_0}{2} \sigma_3 + \frac{\hbar \Omega}{2} \left\{ \cos(\omega_0 t + \vec{k} \cdot \vec{g} t^2/2 + \vec{k} \cdot \vec{x}) + \cos(\omega_0 t - \vec{k} \cdot \vec{g} t^2/2 - \vec{k} \cdot \vec{x}) \right\} \right\} \sigma_1
\]  

(14)

Handling atoms with an atomic elevator

The atom optical system presented in Eq. (11) allows a generalization which is of interest for the manipulation of cold atomic clouds. Imagine that the electromagnetic field is not a standing laser wave but is composed out of two counterpropagating running laser waves with constant frequencies \( \omega_0 \pm \Delta \omega \). By making a Galilei transformation to a moving reference frame with velocity \( v = k\Delta \omega/k^2 \), which amounts to the unitary transformation \( U = \exp[i\vec{p} \cdot \vec{v} t/\hbar] \), we essentially are back to the original Hamiltonian (11). Therefore the two-level atom performs Bloch oscillations from the point of view of this moving frame. But this means that in the original frame which is at rest the atomic velocity oscillates around the velocity \( v \) which was determined by the laser detuning \( \Delta \omega \). A limited extension in space for a moving atomic cloud can be obtained if simple Bloch oscillations (\( |\Omega_1| \) transitions in the figure) are induced by appropriate initial conditions. This effect can be employed to move atoms in a cloud up or down in the Earth’s gravitational field without a recognizable acceleration.

Range of application

We now argue that the above model can only be applied to atoms moving in electromagnetic fields with frequencies at least in the optical regime if gravitation is the driving force. To do so, we first note that the time \( \Delta t \) between two subsequent LZTs must be much longer than the time \( t_{LZ} \) required to complete one individual LZT. On the other hand, \( P_{\text{stay}} \) must be much less than 1 in order to have an effective LZT. These conditions lead to the inequality

\[
ka \ll \Omega^2 \ll \frac{\hbar k^{5/2} \alpha^{1/2}}{2M}
\]  

(15)

which implies \( k \gg (2Ma^{1/2}/\hbar)^{2/3} \). Inserting for \( a \) the Earth’s acceleration setting \( M = 10^{-26} \) kg (light atoms) one sees that \( k \) must lie well above \( 10^6 \) m\(^{-1}\). Due to the small mass of electrons the condition on \( k \) is much less restrictive in a crystal: \( k_0 \) needs only to be well above \( 10^3 \) m\(^{-1}\). Since typical lattice spacings \( L \) in crystals are less than 1 nm this is easily fulfilled by the electrons.
References

[1] F. Bloch, Z. Phys. 52, 555 (1929).

[2] C. Zener, Proc. R. Soc. Lond. A 145, 523 (1934).

[3] C. Waschke et al., Phys. Rev. Lett. 70, 3318 (1993).

[4] M. Ben Dahan et al., Bloch oscillations of atoms in an optical potential, to appear in Phys. Rev. Lett.

[5] K.-P. Marzlin and J. Audretsch, Actio implies reactio: Gravito-optical trapping of three-level atoms, to appear in Phys. Rev. A.

[6] L.D. Landau, Phys. Zeitschrift 2, 46 (1932).

[7] C. Zener, Proc. R. Soc. Lond. Ser. A 137, 696 (1932).
Figure caption:
A quantum particle moves in a periodic potential with Fourier coefficients $\Omega_l$ and periodicity length $L$ under the influence of a constant force $F$. The kinetic energy $E$ as function of its momentum $p$ is shown ($k_0 = 2\pi/L$). The arrows represent Landau-Zener transitions between resonant momentum states. The particle moves along the parabola with $p(t) = \hat{p}_0 + Ft$ until one of these states is reached. Then there is a certain probability determined by $l$ and $\Omega_l$ to stay on the parabola or to make a transition to the resonant state and to follow the parabola afterwards to a new resonance point where the next transition may happen. For a particle with initial momentum $|p| < \hbar k_0/2$ the $\Omega_1$ transition leads to simple Bloch oscillations of the momentum with Bloch frequency $\omega_B = |F|L/\hbar$. 
\[ p = \frac{\hbar k_0}{2} \]