Solution of d-dimension time independent cosmic string using supersymmetry quantum mechanics method for rosen morse, scarf II and scarf I non-central potentials

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Abstract. Solution of d-dimension time independent cosmic string for rosen morse, scarf II and scarf I non-central potentials was solved using supersymmetry quantum mechanics method. Variable separation method was applied to the d-dimension time independent cosmic string equation, and then the d-dimension time independent cosmic string equation was reduced into d one dimensional schrodinger equation. Supersymmetry quantum mechanics method was used to analyze radial wave function, angular wave functions, and energy level. Energy levels were higher by the presence of rosen morse, scarf II and scarf I potentials parameters. The increased value of \( n_r \), \( n_{\theta_1} \) and \( n_{\theta_2} \) caused the increase in energy levels, the decrease of cosmic string \( \alpha \) parameter, caused the decrease of energy levels and the increasing of number \( n_q \) and \( n_{\theta_1} \) caused the decrease of energy levels.

1. Introduction
The cosmic string theory is a theory that combines relativity and quantum mechanics, everything that happen in the universe can be explained by that theory [1]. Cosmic string is a topology defect, the length of cosmic string infinite and has an angular parameter \( \alpha \) [2,3]. The cosmic theory has been reduced to 1+2 dimensions in a vacuum solution of Kerr general relativity then, the cosmic string was expressed in space time. That caused the four dimensional solution was expanded from three dimensional space time [4,5]. A relativistic system or non-relativistic quantum mechanics system was studied on the cosmic string as a topological defect, and cosmic string solution had been solved use cylindrical or spherical coordinates system [6-9]. The solution of cosmic string had been solved by reduce cosmic string equation into Dirac equation, Schrodinger equation and Klein Gordon equation [10-12]. Cosmic string equation for central potential or non-central potential can be solved. Non-central potential is consist of radial part and angular part. Lots of potentials like Poschl Teller Double Ring Shaped Coulomb potential, Double Ring Shape Oscillator [13,14], Woods-Saxon [9], Doubled Ring-Shaped Coulomb Oscillator [15], Coulomb Like Scalar [16] had been solved. The solution can be obtained by using some methods. Supersymmetry quantum mechanics method, Confluent Hypergeometric method, Nikiforov Uvarov method, Ansatz method, can be used to obtain the solution [3,11,16-18].
In this research the solution will be investigated in d-dimension and will be reduced in to d-dimension Schrodinger equation. The d-dimensional system is the physics model that explain unity of two fundamental forces of gravitational and electromagnetic since it is involved in such huge universe. The additional of spatial dimension is from string theory, for a consistent description of gravity is needed more than 3+1 dimension [19]. Study about d-dimension on relativistic and non-relativistic system had been solved. Solution of d-dimension Schrodinger equation, d-dimension Klein Gordon equation and d-dimension Dirac equation had been studied [20-22]. Solution of d-dimensional Schrodinger also had been solved using supersymmetry. Supersymmetry quantum mechanics was explained by Mustafa and Kais [23]. Supersymmetry quantum mechanics method is one of simple methods. Supersymmetry quantum mechanics method was used in solution of three dimension Schrodinger and also had been used to solved in d-dimension Schrodinger [24,25]. In this paper, we investigated solution of d-dimension time independent cosmic string equation using supersymmetry quantum mechanics method for Rosen morse, scarf II and scarf I non-central potential. Potentials can be written:

\[
V(r, \theta_1, \theta_2, \theta_3, \theta_4) = V(r) + \frac{V(\theta_1)}{r^2 \sin^2 \theta_2 \sin^2 \theta_3 \sin^2 \theta_4} + \frac{V(\theta_2)}{r^2 \sin^2 \theta_1 \sin^2 \theta_4} + \frac{V(\theta_3)}{r^2 \sin^2 \theta_1 \sin^2 \theta_2} + \frac{V(\theta_4)}{r^2 \sin^2 \theta_1 \sin^2 \theta_3} \tag{1}
\]

\[
V(r) = \frac{\hbar^2}{2M} \left[ \frac{\nu (\nu + 1)}{\sin^2 (\nu r)} - 2q \cot (\nu r) \right] \tag{2}
\]

\[
V(\theta_1) = \frac{\hbar^2}{2M} \left[ b_1^2 + a_1 (a - 1) \frac{\sin \theta_1}{\cos^2 \theta_1} \right] \tag{3}
\]

\[
V(\theta_2) = \frac{\hbar^2}{2M} \left[ b_2^2 + a_2 (a - 1) \frac{\sin \theta_2}{\cos^2 \theta_2} \right] \tag{4}
\]

Rosen morse potential in radial part, scarf I in \( \theta_1 \) and scarf II in \( \theta_i \), where, \( i = 2, 3, 4 \), \( r \) is displacement, \( 0 \leq r \leq \infty, \ 0 \leq \theta_i \leq \pi, \ q > 1, \ \nu > 1, \ b_i^2 + a_i (a - 1) > 0, \ 2b(a - \nu^2) > 0 \). The Rosen morse potential was proposed by Rosen-Morse and the potential was used to describe the diatomic molecular vibration. Rosen morse potential was also used to describe the quark gluon dynamics. Trigonometric Scarf potential was used to explain strong and electromagnetic interactions and molecule vibrations. Solution of one dimensional Schrodinger and quantum information for Rosen morse was evaluated [26-33]. Rosen morse, scarf II and scarf I non-central potentials is used to explore potential combinations that are focused on effects of parameter on energy and wave functions. Rosen morse, scarf II and scarf I non-central potentials is separable potential based on denominator. Time independent cosmic string for five dimension potentials can be resolve by using variable separation method. In section 2, we begin with basic theory about d-dimension time independent cosmic string and supersymmetry quantum mechanics methods. In section 3, results and discussions and in section 4, we summarize our result.

2. Basic Theory

2.1 d-dimension time independent cosmic string

Time independent cosmic string in d-dimension will be reduced to d-dimension Schrodinger [19]. The metric of cosmic string space time for d-dimension is defined as:

\[
ds^2 = -dt^2 + dr^2 + r^2 \alpha^2 \delta \sin^2 \theta_2 \sin^2 \theta_3 \sin^2 \theta_4 dp_1^2 + r^2 \sin^2 \theta_1 \sin^2 \theta_2 dp_2^2 + r^2 \sin^2 \theta_1 \sin^2 \theta_3 dp_3^2 + r^2 \sin^2 \theta_1 \sin^2 \theta_4 dp_4^2 \tag{5}
\]

Where, range of \( r, \ \theta \) were \( 0 \leq r \leq \infty, \ 0 \leq \theta \leq \pi \). To obtain d-dimension time independent cosmic string equation, variable separation method was applied. The metric of d-dimension time independent cosmic string is written:
By using equation (7), laplacian equation is written by equation (8):

$$\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r^2} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Phi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta \sin^2 \phi} \frac{\partial}{\partial \phi} \left( \sin \phi \frac{\partial \Phi}{\partial \phi} \right)$$

2.2 d-dimension time independent cosmic string for non-central potentials

In this research, the solution of the d-dimension time independent cosmic string will be investigated. To solve d-dimension time independent cosmic string equation, Laplacian equation and non-central potential are substituted to in Schrodinger equation.

Time independent cosmic string for d-dimension can be solved by using variable separation method. This method is used to obtain radial part equation and angular parts equation. Each part from that equation is one dimensional schrodinger like equation. Formulation of each part are written in equation (10) for radial part, equation (11) for \( \theta_1 \) angular part, equation (12) for \( \theta_2 \) angular part, equation (13) for \( \theta_3 \) angular part and equation (14) for \( \theta_4 \) angular part.
2.3 Supersymmetry quantum mechanics method

Supersymmetry quantum mechanics method is applied for one dimensional Schrödinger system. Supersymmetry is consists of a supercharge operator. Supercharge operator is commute with supersymmetry Hamiltonian [23,34]:

\[ [Q_i, H_{ss}] = 0 \text{ with } i = 1,2,...,N \]  

(15)

where \( N \) is the number of super power that meets the anti-commutation relationship. We shall write \( H_{ss} = H_s \). Supersymmetry Hamiltonian are written:

\[ H_s = A^+ A^- = -\frac{\hbar^2}{2M} \frac{d^2}{dx^2} + V_s(x) \quad \text{and} \quad H_s = A^- A^+ = -\frac{\hbar^2}{2M} \frac{d^2}{dx^2} + V_s(x) \]  

(16)

where \( A^+ \) is raising operator, \( A^- \) is lowering operator and \( V_s(x) \) and \( V_r(x) \) are supersymmetry partner for supersymmetry Hamiltonian \( H_s \). Raising and lowering operator are written:

\[ A^+ = \frac{\hbar}{(2M)^{1/2}} \frac{d}{dx} + \phi \quad \text{and} \quad A^- = -\frac{\hbar}{(2M)^{1/2}} \frac{d}{dx} + \phi \]  

(17)

Effective potential is written as:

\[ V_{\psi} = V_r(x;a_j) + \epsilon_0 = \psi^2(x;a_j) - \frac{\hbar}{(2M)^{1/2}} \phi(x;a_j) + \epsilon_0 \]  

(18)

Supersymmetry partner potential following:

\[ V_s(x;a_j) = V_r(x;a_{j+1}) + R(a_{j+1}) \]  

(19)

\[ V_r(x;a_j) = \phi^2(x;a_j) + \frac{\hbar}{(2M)^{1/2}} \phi(x;a_j) \]  

(20)

\[ V_r(x;a_j) = \phi^2(x;a_j) - \frac{\hbar}{(2M)^{1/2}} \phi(x;a_j) \]  

(21)

where \( j = 1,2,3,... \) and parameter a determined sequentially and \( R \) do not depend on \( x \).

\[ E_n = \sum_{i=0}^{n} R(a_i) \]  

(22)

\[ E_n = \epsilon_0 + E_n \]  

(23)

\( \epsilon_0 \) is a ground state energy of the system, and total energy level is obtained.

\[ A^- \cdot \psi_{\psi}(-) = 0 \]  

(24)

\[ \psi_{\psi}^{(-)}(x;a_j) = A^{-1}(x;a_{j-1}) \cdot \psi_{\psi}^{(-)}(x;a_j) \]  

(25)

Ground state wave function is obtained by using lowering operator, which is written as equation (24) and n state wave function was obtained by multiplying raising operator and n-1 state wave function, like equation (25).

3. Results and Discussions

3.1 Angular part solution of d-dimension time independent cosmic string for rosen morse, scarf II and scarf I non-central potentials

To solve \( \theta_1 \) angular part, we assume that \( \theta_1(\theta) = H_s(\theta) \) for equation (14) and then we obtain:

\[ -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial \theta_1^2} H_s(\theta) + \frac{\hbar^2}{2M} \alpha^2 \left[ \frac{\hbar^2}{\cos^2 \theta_1} \right] H_s(\theta) = \frac{\hbar^2}{2M} \lambda_H(\theta) \]  

(26)

\[ V_{\psi} = \frac{\hbar^2}{2M} \alpha^2 \left[ \frac{\hbar^2}{\cos^2 \theta_1} \right] + \frac{2 \hbar}{\cos^2 \theta_1} \cos \theta \]  

(27)

To solve the angular part equation, superpotential is used to obtain raising operator, lowering operator and superpartner potential. Superpotential equation is proposed and written in the formulation:
\[ \phi = A \tan \theta_i + B \sec \theta_i \]  
(28)

Superpotential must be squared and that also must be differentiated, by using equation (18), equation (27) and equation (28) and then we obtain value:

\[ A = \frac{\hbar}{\sqrt{2M}} \left( K_i + \frac{1}{2} \right) \]
where, \( K_i = \sqrt{\left( a_i - \frac{1}{2} \right)^2 - \sqrt{\left( a_i - \frac{1}{2} \right)^2} - \left( \frac{\hbar}{2} \alpha \right)^2 \left( a_i - \frac{1}{2} \right)^2} \]

\[ B = \frac{\hbar}{\sqrt{2M}} L_i \]
where, \( L_i = \frac{\hbar}{K_i} \alpha \frac{a_i - \frac{1}{2}}{\frac{1}{2}} \)

To obtain the angular wave function we need raising and lowering operators, we obtain raising and lowering operators:

\[ A^* = -\frac{h}{\sqrt{2M}} \frac{d}{d\theta_i} + \frac{h}{\sqrt{2M}} \left( K + \frac{1}{2} \right) \tan \theta_i + \frac{h}{\sqrt{2M}} L \sec \theta_i \]

\[ A' = \frac{h}{\sqrt{2M}} \frac{d}{d\theta_i} + \frac{h}{\sqrt{2M}} \left( K + \frac{1}{2} \right) \tan \theta_i + \frac{h}{\sqrt{2M}} L \sec \theta_i \]

To obtain the angular wave function we need raising and lowering operators, we obtain raising and lowering operators:

\[ \phi = \frac{1}{\sin \theta_i} \]

By using \( K_i = K; K_i = K + 1 \) in superpartner potential, energy levels for \( \theta_i \) angular part following formulation:

\[ E = \frac{h^2}{2M} \left( K + \frac{1}{2} + n \right)^2 \]

(37)

Based on equation (27) and equation (37), we obtain value of constant \( \lambda_1 \):

\[ \lambda_1 = \left( K_i + \frac{1}{2} + n_0 \right)^2 \]

(38)

To solve \( \theta_2 \) angular part, we assume \( \Theta_2 = \frac{H_2(\theta_2)}{\sin \beta \theta_2} \) for equation (13) and we obtain:

\[ -\frac{h^2}{2M} \frac{d^2}{d\theta_2^2} H_2(\theta_2) + \frac{h^2}{2M} \left( \frac{b_i^2 + a_i \left( a_i - 1 \right)}{\sin^2 \theta_2} \right) + \frac{2h}{2M} \left( a_i - \frac{1}{2} \right) \cos \theta_2 \]

(39)

To solve this part we propose superpotential. Superpotential equation for this part is written in the formulation:

\[ \phi = A \cot \theta_2 + B \cosec \theta_2 \]

(40)

by using equation (18), equation (39) and equation (40), we obtain:
\[ A = -\frac{\hbar}{2M} \left[ K_s + \frac{1}{2} \right] \] where, \( K_s = \sqrt{\left( a_s - \frac{1}{2} \right)^2 - \left[ \left( a_s - \frac{1}{2} \right)^2 \right] - 4 \left[ b_s \left( a_s - \frac{1}{2} \right) \right]^2} \) \] (41)

\[ B = \frac{\hbar}{2M} L_s \] where, \( L_s = \frac{b_s}{K_s} \left( a_s - \frac{1}{2} \right) \), \( a_s = \sqrt{b_s^2 + a_s (a_s - 1) + \frac{\lambda_s}{\alpha^2} + \frac{1}{2}} \) and \( \epsilon_n = \frac{\hbar^2}{2M} \left[ K + \frac{1}{2} \right]^2 \) \] (42)

This angular part is solved by using the same method like \( \Theta_1 \) angular part. Then, by using lowering operator, ground state wave function for angular part is obtained:

\[ H \left( \Theta_1 \right)_0 = C \sin \theta_1 \left( K_s + \frac{1}{2} \right) \left( 1 - \cos \theta_1 \right)^{-\frac{1}{2}} \] (43)

We obtain first state wave function by using ground state wave function and raising operator. That function is written:

\[ H \left( \Theta_1 \right)_1 = -\frac{\hbar}{2M} C \left[ (2K_s + L_s + 2) \cos \theta_1 - L_s (1 - \cos \theta_1)^{-1} \sin^2 \theta_1 - L_s \] \times \sin \theta_1 \left( K_s + \frac{1}{2} \right) \left( 1 - \cos \theta_1 \right)^{-\frac{1}{2}} \right] \] (44)

Energy for \( \Theta_2 \) angular part is written in equation (45) and value of constant \( \lambda_2 \) is written in equation (46).

\[ E' = \frac{\hbar^2}{2M} \left[ K + \frac{1}{4} + n \right] \] (45)

\[ \lambda_2 = \frac{\left( K_s + \frac{1}{2} + n \right)^2 - \frac{1}{4}}{2} \] (46)

To solve \( \Theta_3 \) angular part, we assume \( \Theta_3 (\theta) = H_3 (\theta_1) \) for equation (12) and we obtain:

\[ -\frac{\hbar^2}{2M} \frac{d^2}{d\theta^2} H_3 (\theta) + \frac{\hbar^2}{2M} \frac{b_s + a_s (a_s - 1) + \lambda_3^2}{\sin^2 \theta_1} - \frac{2b_s \left( a_s - \frac{1}{2} \right) \sin \theta_1}{\sin^2 \theta_1} \] \( H_3 (\theta) = \frac{\hbar^2}{2M} \left( \lambda_3 + 1 \right) H_3 (\theta) \) \] (47)

Superpotential equation for this part is proposed and written in the formulation:

\[ \phi = A \cot \theta_3 + B \cosec \theta_3 \] (48)

by using equation (18), equation (47) and equation (48) we obtain:

\[ A = -\frac{\hbar}{2M} \left[ K_s + \frac{1}{2} \right] \] where, \( K_s = \sqrt{\left( a_s - \frac{1}{2} \right)^2 - \left[ \left( a_s - \frac{1}{2} \right)^2 \right] - 4 \left[ b_s \left( a_s - \frac{1}{2} \right) \right]^2} \) \] (49)

\[ B = \frac{\hbar}{2M} L_3 \] where, \( L_3 = \frac{b_s}{K_s} \left( a_s - \frac{1}{2} \right) \), \( a_s = \sqrt{b_s^2 + a_s (a_s - 1) + \frac{\lambda_3^2}{\alpha^2} + \frac{1}{4}} + \frac{1}{2} \) and \( \epsilon_n = \frac{\hbar^2}{2M} \left[ K + \frac{1}{2} \right]^2 \) \] (50)

\( \Theta_3 \) angular part is also solved by using the same method like the \( \Theta_1 \) angular part. The ground state wave function for \( \Theta_3 \) angular part is:

\[ H \left( \Theta_3 \right)_0 = C \sin \theta_1 \left( K_s + \frac{1}{2} \right) \left( 1 - \cos \theta_1 \right)^{-\frac{1}{2}} \] (51)

The first state wave function for the \( \Theta_3 \) angular part:

\[ H \left( \Theta_3 \right)_1 = -\frac{\hbar}{2M} C \left[ (2K_s + L_3 + 2) \cos \theta_1 - L_3 (1 - \cos \theta_1)^{-1} \sin^2 \theta_1 - L_3 \] \times \sin \theta_1 \left( K_s + \frac{1}{2} \right) \left( 1 - \cos \theta_1 \right)^{-\frac{1}{2}} \right] \] (52)

Energy for \( \Theta_3 \) angular part is written in equation (53) and value of constant \( \lambda_3 \) is written in equation (54).
To solve other angular part, we assume \( \Theta_4(\theta_4) = \frac{H_4(\theta_4)}{\sin^{\frac{3}{2}\theta_4}} \) for equation (11), and then we obtain:

\[
\frac{-\hbar^2}{2M} \frac{d^2}{d\theta_4^2} H_4(\theta_4) + \frac{\hbar^2}{2M} \left[ \frac{b_i^2 + a_i(a_i - 1) + \lambda_i' + \frac{3}{4}}{\sin^2 \theta_4} \right] H_4(\theta_4) = \frac{\hbar^2}{2M} \left( \lambda_i' + \frac{3}{4} \right)^2 H_4(\theta_4) \tag{55}
\]

Superpotential equation is proposed as:

\[
\phi = A \cot \theta_2 + B \csc \theta_2 \tag{56}
\]

by using equation (18), equation (55) and equation (56) and then we obtain:

\[
A = -\frac{\hbar}{\sqrt{2M}} \left[ K_i + \frac{1}{2} \right] \quad \text{where,} \quad K_i = \left[ \left( a_i - \frac{1}{2} \right)^2 - \left( a_i - \frac{1}{2} \right)^2 \right]^{-\frac{1}{2}} - 4 \left[ a_i - \frac{1}{2} \right]^2 \tag{57}
\]

\[
B = \frac{\hbar}{\sqrt{2M}} L_i \quad \text{where,} \quad L_i = \frac{b_i}{K_i} \left( a_i - \frac{1}{2} \right), \quad a_i' = \sqrt{b_i^2 + a_i(a_i - 1) + \lambda_i' + \frac{1}{2}} \quad \text{and} \quad \epsilon_0 = \frac{\hbar^2}{2M} \left[ K_i + \frac{1}{2} \right]^2 \tag{58}
\]

The \( \Theta_4 \) angular part is also solved by using the same method as the \( \Theta_1 \) angular part. The ground state wave function for the \( \Theta_4 \) angular part is:

\[
H(\theta_4)_0 = C \sin \theta_4^{\left( \frac{K_i + \lambda_i + \frac{1}{2}}{2} \right)} \left( 1 - \cos \theta_4 \right)^{-L_i} \tag{59}
\]

The first state wave function for the \( \Theta_4 \) angular part is:

\[
H(\theta_4)_1 = -\frac{\hbar}{\sqrt{2M}} C \left[ (2K_i + L_i + 2) \cos \theta_4 - L_i (1 - \cos \theta_4)^{-L_i} \sin^2 \theta_4 - L_i \right] \times \left[ \sin \theta_4^{\left( \frac{K_i + \lambda_i + \frac{1}{2}}{2} \right)} \left( 1 - \cos \theta_4 \right)^{-L_i} \right] \tag{60}
\]

Energy for the \( \Theta_4 \) angular part is written in equation (61) and value of constant \( \lambda_i' \) is written in equation (62).

\[
E^* = \frac{\hbar^2}{2M} \left( K_i + \frac{1}{2} + n \right)^2 \tag{61}
\]

\[
\lambda_i' = \left( K_i + \frac{1}{2} + n \right)^2 - \frac{9}{4} \tag{62}
\]

### 3.2 Radial part solution of d-dimension time independent cosmic string for rosen morse, scarf II and scarf I non-central potentials

To solved radial part, we assume that \( R(r) = \frac{X(r)}{r} \) to equation (10) and we obtained equation (63).

Where this equation have effective potential in equation (64).

\[
-\frac{\hbar^2}{2M} \frac{d^2}{dr^2} X(r) + \frac{\hbar^2}{2M} \frac{v'(r + 1) + \lambda_i' + 2}{\sin^2 (yr)} - 2q \cot (yr) + (\lambda_i' + 2) d_0 \right] X(r) = EX(r) \tag{63}
\]

\[
V_{\phi} = \frac{\hbar^2}{2M} \frac{v'(r + 1) + \lambda_i' + 2}{\sin^2 (yr)} - 2q \cot (yr) \right] + \frac{\hbar^2}{2M} \gamma^2 \left( \lambda_i' + 2 \right) d_0 \tag{64}
\]

Superpotential equation for the radial part is proposed in the formulation:

\[
\phi = A \cot (yr) - \frac{B}{A} \tag{65}
\]

by using equation (18), equation (64) and equation (65) and then we obtain:
\[
A = -\frac{\hbar}{\sqrt{2}M} \gamma (\nu' + 1) \quad \text{and} \quad B = \frac{\hbar^2}{2M} \gamma^2 q, \quad \text{where:} \quad \nu' = \sqrt{\nu(\nu+1) + \lambda^2 + 2 + \frac{1}{4} - \frac{1}{2}} \tag{66}
\]

\[
\epsilon_0 = \frac{\hbar^2}{2M} \gamma^2 (\nu' + 1)^2 - \frac{\hbar^2}{2M} \gamma^2 \left( \nu + \frac{q^2}{\nu + 1} \right) + \frac{\hbar^2}{2M} \gamma^2 \lambda^2 d_0 \tag{67}
\]

To obtain angular wave functions we need raising and lowering operators. Raising and lowering operators are:

\[
A' = -\frac{\hbar}{\sqrt{2}M} \frac{d}{dr} \frac{\hbar}{\sqrt{2}M} \gamma (\nu' + 1) \cot(\gamma r) + \frac{\hbar}{\sqrt{2}M} \gamma q \tag{68}
\]

\[
A'' = \frac{\hbar}{\sqrt{2}M} \frac{d}{dr} \frac{\hbar}{\sqrt{2}M} \gamma (\nu' + 1) \cot(\gamma r) + \frac{\hbar}{\sqrt{2}M} \gamma q \tag{69}
\]

By using lowering operator, we obtain equation for the radial part ground state wave function:

\[
R_0 = \sin(\gamma r)^{\nu' + 1} \exp \left[ -\gamma \frac{qr}{\nu' + 1} \right] \tag{70}
\]

Then, by using raising operator we obtain first state wave function for radial part. That equation can be written as formulation:

\[
R_1 = \frac{\hbar}{\sqrt{2}M} \gamma \left[ -2(\nu' + 1)^2 \cos(\gamma r) + (2\nu' + 3)q \sin(\gamma r) \right] \sin(\gamma r)^{\nu' + 1} \exp \left[ -\gamma \frac{qr}{\nu' + 1 + 1} \right] \tag{71}
\]

To obtain the solution of energy levels, we use of superpartner potential. Superpartner potential for the radial part are:

\[
V_+(R; v_0') = \frac{h^2}{2M} \gamma^2 \left( v^2 + v' \right) \csc^2(\gamma r) - \frac{h^2}{2M} \gamma^2 \left( v' + 1 \right)^2 + \frac{h^2}{2M} \gamma^2 \left( v + \frac{q^2}{v + 1} \right) - 2 \frac{h^2}{2M} \gamma^2 q \cot(\gamma r) \tag{72}
\]

\[
V_+(R; v_0') = \frac{h^2}{2M} \gamma^2 \left( v^2 + 3v' + 2 \right) \csc^2(\gamma r) - \frac{h^2}{2M} \gamma^2 \left( v' + 1 \right)^2 + \frac{h^2}{2M} \gamma^2 \left( v + \frac{q^2}{v + 1} \right) - 2 \frac{h^2}{2M} \gamma^2 q \cot(\gamma r) \tag{73}
\]

By using \( v_0' = v' ; v_1' = v' + 1 \) in superpartner potential, energy for d-dimension time independent cosmic string for rosen morse, scarf I and scarf II non-central potentials have been obtained. Energy spectrum can be written:

\[
E_{s_i} = \frac{h^2}{2M} \gamma^2 \left( v' + n \right)^2 - \frac{q^2}{(v' + 1 + n)} + \frac{h^2}{2M} \gamma^2 \lambda^2 d_0 \tag{74}
\]
Figure 1 showed that variation of parameter $\alpha$ caused variation in wave amplitude of the ground state wave function for the radial part. In every variation, waveform was damped along the $r$ axis and probability of particle distribution decreased when it was far from the center. If value of parameter $\alpha$ decrease, the amplitude of wave function decrease. Figure 2, showed that variation of parameters also caused variation of wave amplitude in the first state wave function for the radial part. In first state wave function, variation of parameters caused the same increasing amplitude or decreasing amplitude like in ground state wave function. If value of parameter $\alpha$ decrease, the amplitude of wave function also decrease. The greater value of $n$, then more the number of wave was formed and damped along of the $r$ axis.

![Figure 2](image) First state wave function for radial part with $\alpha$ variation

| $n_r$ | $n_{\theta_1}$ | $n_{\theta_2}$ | $n_{\theta_3}$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $\nu$ | Energy for $\alpha = 0.9$ | Energy for $\alpha = 0.6$ | Energy for $\alpha = 0.3$ | Energy for $\alpha = 0.1$ |
|------|----------------|----------------|----------------|------|------|------|------|------|------|------|------|------|----------------|----------------|----------------|----------------|
| 0    | 0              | 0              | 0              | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0.5000         | 0.5000          | 0.5000          | 0.5000          |
| 1    | 0              | 0              | 0              | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 2.0000         | 2.0000          | 2.0000          | 2.0000          |
| 2    | 0              | 0              | 0              | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 4.5000         | 4.5000          | 4.5000          | 4.5000          |
| 3    | 0              | 0              | 0              | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 8.0000         | 8.0000          | 8.0000          | 8.0000          |
| 0    | 1              | 1              | 1              | 1    | 2    | 2    | 2    | 1    | 1    | 1    | 1    | 1    | 6.8827         | 6.8756          | 6.8626          | 6.8474          |
| 1    | 1              | 1              | 1              | 1    | 2    | 2    | 2    | 1    | 1    | 1    | 1    | 1    | 11.1425        | 11.1336         | 11.1175         | 11.1175         |
| 2    | 1              | 1              | 1              | 1    | 2    | 2    | 2    | 1    | 1    | 1    | 1    | 1    | 16.3757        | 16.3651         | 16.3459         | 16.3235         |
| 3    | 1              | 1              | 1              | 1    | 2    | 2    | 2    | 1    | 1    | 1    | 1    | 1    | 22.5971        | 22.5847         | 22.5622         | 22.5362         |

Table 1. Energy levels of d-dimension time independent cosmic string for rosen morse, scarf II and scarf I non-central potentials.

Energy levels of d-dimension time independent cosmic string for rosen morse, scarf II and scarf I non-central potential was shown in Table 1. Value of parameters caused significant effects in energy levels. Energy levels for $\nu \neq 0$, $a_1 \neq 0$, $a_2 \neq 0$, $a_3 \neq 0$, $a_4 \neq 0$, $b_1 \neq 0$, $b_2 \neq 0$, $b_3 \neq 0$ and $b_4 \neq 0$ have higher energy levels than $\nu = 0$, $a_1 = 0$, $a_2 = 0$, $a_3 = 0$, $a_4 = 0$, $b_1 = 0$, $b_2 = 0$, $b_3 = 0$ and $b_4 = 0$. The presence of potentials parameters caused increasing in energy levels.
were higher than energy levels with potentials parameters and increased. But, energy levels were decreasing if value of were increasing. Energy levels also decreased if value of parameter caused the increasing in energy level, but increasing number and scarf I non-central potentials were increase and decrease. Energy levels with potentials radial part equation. Energy levels of d-dimension time independent cosmic string for rosen morse, scarf II and scarf I non-central potentials were obtained from angular part equation. Radial part wave function was obtained from one dimensional Schrodinger equation. To obtained wave function and spectrum we applied variable separation method to d-dimensional Schrodinger like equation to obtained radial part equation and angular parts equation. These equation like one dimensional Schrodinger equation. To solved the solution of d-dimension time independent cosmic string for rosen morse, scarf II and scarf I non-central potentials. We applied variable separation method to d-dimensional Schrodinger like equation to obtained radial part equation and angular parts equation.

Table 2. Energy levels of d-dimension time independent cosmic string for rosen morse, scarf II and scarf I non-central potentials with variation of n.

| n_r | n_{\theta_1} | n_{\theta_2} | n_{\theta_3} | a_1 | a_2 | a_3 | b_1 | b_2 | b_3 | b_4 | Energy |
|-----|--------------|--------------|--------------|-----|-----|-----|-----|-----|-----|-----|--------|
| 1   | 2            | 1            | 1            | 1   | 2   | 2   | 2   | 1   | 1   | 1   | 11.1299 |
| 1   | 3            | 1            | 1            | 1   | 2   | 2   | 2   | 1   | 1   | 1   | 11.1217 |
| 1   | 2            | 1            | 1            | 1   | 2   | 2   | 2   | 1   | 1   | 1   | 11.2292 |
| 1   | 3            | 1            | 1            | 1   | 2   | 2   | 2   | 1   | 1   | 1   | 11.2878 |
| 1   | 2            | 1            | 1            | 1   | 2   | 2   | 2   | 1   | 1   | 1   | 10.7074  |
| 1   | 3            | 1            | 1            | 1   | 2   | 2   | 2   | 1   | 1   | 1   | 10.4450  |
| 1   | 2            | 1            | 1            | 1   | 2   | 2   | 2   | 1   | 1   | 1   | 15.0244  |
| 1   | 3            | 1            | 1            | 1   | 2   | 2   | 2   | 1   | 1   | 1   | 20.1379  |

Based on Table 2, increasing in parameters not only caused increasing in energy levels but also caused decreasing in energy levels. If the value of parameter \( \alpha \) decreased, that caused decreasing in energy levels. Quantum number also associated with energy levels. Increasing of quantum number \( n_r \), \( n_{\theta_1} \) and \( n_{\theta_3} \) caused the increasing in energy level, but increasing number \( n_{\theta_2} \) and \( n_{\theta_2} \) caused decreasing in energy levels. The most significant increase in energy levels was caused by increasing of \( n_r \).

4. Conclusion
In this research, we solved the solution of d-dimension time independent cosmic string for rosen morse, scarf II and scarf I non-central potentials. We applied variable separation method to d-dimensional Schrodinger like equation to obtained radial part equation and angular parts equation. These equation like one dimensional Schrodinger equation. To obtained wave function and spectrum energy, we applied supersymmetry quantum mechanics method to each part equation. Angular part wave function was obtained from angular part equation. Radial part wave function was obtained from radial part equation. Energy levels of d-dimension time independent cosmic string for rosen morse, scarf I and scarf II non-central potentials were increase and decrease. Energy levels with potentials parameters \( \neq 0 \) were higher than energy levels with potentials parameters \( = 0 \). Energy levels were increasing if value of number \( n_r \), \( n_{\theta_2} \) and \( n_{\theta_4} \) increased. But, energy levels were decreasing if value of number \( n_{\theta_1} \) and \( n_{\theta_3} \) were increasing. Energy levels also decreased if value of parameter \( \alpha \) decreased.

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