Spontaneous emergence of free-space optical and atomic patterns

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Abstract

The spontaneous formation of patterns in dynamical systems is a rich phenomenon that transcends scientific boundaries. Here, we report our observation of coupled optical–atomic pattern formation, which results in the creation of self-organized, multimode structures in free-space laser-driven cold atoms. We show that this process gives rise to spontaneous three-dimensional Sisyphus cooling even at very low light intensities and the emergence of self-organized structures on both sub- and super-wavelength scales.

1. Introduction

Optical lattices are a useful experimental tool for studying the interaction between light and ultracold atoms. The lattice creates a dipole force that results in spatial organization of the atoms. Spatially organized atoms can be used to simulate quantum systems and to study novel states of matter. These applications often require only a small number of atoms per lattice site, but new physics is expected when there are many atoms per site.

In the high-atom-number regime, the light scattered by the atoms can be substantial and affect the atoms’ center-of-mass degrees-of-freedom. Early experiments exploring this regime used single-mode optical cavities to enhance the light–atom coupling strength. Alternatively, we have shown theoretically that strong light–atom coupling can be achieved in free space by allowing many sub-Doppler-cooled atoms to spatially organize in the intensity maxima of an optical lattice. Free-space systems are advantageous for many reasons, including experimental simplicity and access to an intrinsically multimode system. Strong light–atom interactions in multimode geometries allow access to different physics, such as continuous symmetry-breaking and the realization of spin glasses.

In this article, we realize enhanced light–atom interactions in a multimode, free-space cloud of ultracold atoms driven by counterpropagating optical fields. Above a threshold value of the nonlinear refractive index, denoted $n_{NL}$, we observe an instability that simultaneously generates new optical fields and new real-space gratings of atoms, which we refer to as optical/atomic pattern formation. The optical and atomic patterns enhance each other because the generated fields give rise to atomic cooling and real-space self-organization, which in turn give rise to increased optical scattering. This results in a runaway process, known as an absolute instability, where an initial atom/field fluctuation triggers macroscopic optical and atomic pattern formation.

While other cold-atom systems have observed the spontaneous emergence of multimode optical fields, they have not directly measured corresponding real-space emergent structures in the atoms. Previous experiments that directly measure emergent structures in atoms are restricted to single-mode cavities or ring cavities in the case of collective atomic recoil lasing, which do not provide access to high-order spatial modes.

Here, we demonstrate the spontaneous emergence of real-space atomic structures in a naturally multimode free-space system using parametric resonance techniques, which allows us to both verify coupled optical/atomic pattern formation and quantitatively measure the characteristics of the self-organized gratings. Due to the mutual interaction among the applied fields, the generated fields, and the atoms, we also find that the atoms spontaneously cool in 3D despite only applying fields in one dimension. Three-dimensional Sisyphus cooling has previously been studied using 3D applied optical lattices to realize longer coherence times, but it has not yet been observed to occur spontaneously, as it does in our system.
Our work provides a means to study non-equilibrium phenomena in a multimode geometry. We present our observation of continuous symmetry-breaking, which is only accessible in the multimode regime. We show that with the observation of spontaneous 3D cooling, we observe long-range multimode self-organization on both sub- and super-wavelength spatial scales, which allows access to studies of more complex self-organization phenomena and nonlinear light–atom interactions. In addition, we observe pattern formation at ultra-low intensities by enhancing the refractive index of the atoms, which allows for studies of low-light-level nonlinear optics and provides a novel material for studying both the spontaneous formation of patterns and the nature of absolute instabilities.

2. Methods

Our free-space experiment consists of an elongated sample of sub-Doppler-cooled $^{87}\text{Rb}$ atoms created in a magneto-optical trap (MOT) of length $L = 3$ cm and diameter $w \sim 400$ $\mu$m that are initially cooled to a temperature $T \approx 30$ $\mu$K [11], well below the Doppler temperature $T_D = 146$ $\mu$K. The typical on-resonance optical depths are $50 \sim 100$ along the long trap axis with an average density of $n_a \sim 10^{15}$ atoms $\text{cm}^{-3}$. After loading the MOT for 97 ms, we apply counterpropagating optical fields (wavelength $\lambda \sim 780$ nm, $1/e^2$ beam waist $\sim 410$ $\mu$m) in a linear, linear polarization configuration along the long axis of the atomic cloud for 3 ms, as depicted in figure 1(a). These fields form an imposed 1D optical lattice with $\sim 200$ atoms per site. The total applied electric field is given by $\vec{E}(z, t) = \vec{E}_0(z)e^{-i\omega t} + \text{c.c.}$, where $\vec{E}_0(z) = F(z)e^{i\zeta z} + iB(z)e^{-i\zeta z}$, $\zeta$ is the vacuum wavenumber, $\omega$ is the frequency, and $\zeta$ is an arbitrary phase chosen to set the polarization to $\sigma^-$ at $z = 0$. This electric field gives rise to a spatially-modulated AC Stark shift in the atomic energy states [12]. When the pump fields have a detuning $\Delta = \omega - \omega_{eg}$ below the atomic resonance frequency $\omega_{eg}$ ($\Delta < 0$), the atoms undergo initial Sisyphus cooling along the long $\hat{z}$-axis [13]. After $\sim 30$ $\mu$s, the mean temperature of the gas along $\hat{z}$ is cooled to $T_z \sim 2 - 3$ $\mu$K. Below threshold for pattern formation, the mean temperature in the radial directions is still $T_{rad} \approx 30$ $\mu$K [14], but as we show below, pattern formation gives rise to additional radial cooling. We typically use $\Delta = -4\Gamma$ to $-10\Gamma$, where $\Gamma = 2\pi \times (6$ MHz$)$ is the natural linewidth of the

![Figure 1](image-url)
5^2S_1/2 (F = 2) \to 5^2P_{3/2} (F = 3)\) atomic transition, which is close enough to the resonance to observe pattern formation, but far enough to avoid substantial absorption [15]. This regime is atypical for lattice experiments, where typically \(|\Delta| \gg 100 \ \Gamma\) [16], and atom-field interaction strengths are comparatively weak.

The spatially-varying AC Stark shift of the applied optical lattice forms two superimposed dipole potentials, depicted in the inset of figure 1(b). The total dipole potential goes as \(U(z) = U^+ (z) + U^- (z)\) [13], where

\[
U^\pm (z) = U_0 \left[ 1 \pm \frac{1}{2} \cos(2k' z) \right]
\]

(1)

correspond to the dipole potentials for the \(m_f = \pm 1/2\) ground states in a \(J = 1/2 \to J' = 3/2\) atomic transition. This simplified fine-structure model provides a good qualitative understanding of our experiment because most (~90%) of the atoms are tightly bunched at regions of pure \(\sigma^+\) polarization and pumped into the \(F = 2, m_f = \pm 2\) ground states [14]. The wavenumber of the pump fields propagating through the atomic cloud is \(k' = kn\) for atoms with an effective index of refraction \(n\), defined below. The maximum well depth in the low-intensity limit is given by

\[
U_0 = \frac{\hbar \Delta I}{I_{sat}},
\]

(2)

where \(I = 2 \varepsilon_0 c |E_0 (z)|^2\) is the total intensity, \(I_{sat} = h^2 \varepsilon_0^2 \varepsilon_c / [2 |\mu|^2 C^2 (1 + 4 \Delta^2 / \Gamma^2)]\) is the off-resonant saturation intensity [18], \(\varepsilon_c\) is the permittivity of free space, \(c\) is the speed of light, \(|\mu|\) is the magnitude of the dipole moment for \(m_f = \pm 1/2 \to m_f = \pm 3/2\) transitions, and \(C^2 = 2/3\) is the difference between the square of the Clebsch–Gordon coefficients for the \(m_f = \pm 1/2 \to m_f = \pm 3/2\) and \(m_f = \pm 1/2 \to m_f = \mp 1/2\) transitions [15].

The dipole potential in equation (1) gives rise to an imposed density distribution

\[
\eta (z) = \frac{n_a}{2} \left[ \sum_{j=\infty}^{\infty} \eta^+ e^{2ik' (z - \pi/2) \eta j} + \sum_{j=\infty}^{\infty} \eta^- e^{2ik' (z + \pi/2) \eta j} \right],
\]

(3)

where the grating wavevector for bunched atoms in a given spin state is \(k' = \pm 2k' n\). The two sums in equation (3) represent atomic bunching at locations of pure \(\sigma^\pm\) field polarizations, which gives rise to bunched ‘pancakes’ of atoms with a lattice constant of \(X/4\), where neighboring pancakes have opposite spins, and \(X = 2\pi/k'\). For a gas in thermal equilibrium [15], the Fourier coefficients appearing in equation (3) are given by

\[
\eta_j^\pm = \frac{I_j [\pm \zeta]}{I_{0j} [\pm \zeta]},
\]

(4)

where \(I_j\) refers to a modified Bessel function of the first kind of order \(j\), and \(\zeta = (|\Delta| / \Gamma)(I / I_{sat})(T_e / T_0)\) is the ratio of \(U_0\) to the atomic thermal energy.

The first-order Fourier coefficients \(\eta_j^\pm\) provide a measure of atomic bunching, where \(\eta_j^\pm = 0\ (\zeta \to 0)\) indicate a homogeneous gas, and \(|\eta_j^\pm| = 1\ (|\zeta| \to \infty)\) indicate maximum bunching, i.e., infinitely thin sheets of atoms [19]. By increasing \(\eta_j^\pm\) for \(|\Delta| < 0\), the light–atom coupling strength is enhanced because atoms bunch tightly at the intensity maxima of the lattice. As can be seen from equation (4), atomic bunching provides a new mechanism to achieve enhanced nonlinear light–atom interactions even for \(I / I_{sat} \ll 1\) by using small \(|\Delta|\), which increases the dipole potential well depth, and by using small \(T_e\) (e.g., via Sisyphus cooling) [16]. This bunching-induced nonlinearity is the primary mechanism that gives rise to pattern formation in our system, in contrast to others where the saturable nonlinearity dominates [20]. In this tight-bunching regime with \(I / I_{sat} \ll 1\), the effective refractive index is

\[
\eta = 1 + \frac{X_{lin}}{2} [1 + \eta_j^\pm] = n_{lin} + n_{NL},
\]

(5)

where \(X_{lin} = -6\pi c^3 (2|\Delta| / \Gamma) n_a C^2 / [\varepsilon_c^2 (1 + 4 \Delta^2 / \Gamma^2)]\) is the linear susceptibility [15]. Here, the intensity-independent terms are the linear refractive index \(n_{lin}\). For a sufficiently strong \(n\) imposed by the applied pump fields, we reach the threshold for generating coupled optical–atomic pattern formation in free space.

2.1. Pattern formation

The threshold for the pattern-forming instability occurs when the nonlinear optical phase shift \(k n_{NL} L \gtrsim \pi/2\) [21], which is achievable at low intensities using our long atomic sample and sub-Doppler temperatures [6]. The minimum observed threshold for pattern formation in our system is \(I / I_{sat} \approx 10^{-2}\) for \(\Delta = -4 \Gamma\ (\eta_j^\pm \approx 0.7\) and \(|\zeta| \approx 2\) and an optical depth of \(\sim 50\), where \(k n_{NL} L \approx 1.6\). This threshold intensity is two orders of magnitude smaller than for optical pattern formation in a warm vapor [22]. Our typical experimental parameters are \(10^{-2} \leq I / I_{sat} \leq 0.4\), where \(\eta_j^\pm \approx 0.7 - 0.99\) and \(|\zeta| \approx 2 - 120\). From equation (3), the peak density at each 1D lattice site at threshold is \(\sim 5n_{sat}\), and the width of each pancake is \(\sim \lambda/13\) [23]. This indicates that the atoms are cooled substantially and tightly bunched in the applied 1D optical lattice, as depicted in
The instability triggers a wave-mixing process that generates new, frequency-degenerate optical fields, which propagate at a small angle $\theta \sim 3–10$ mrad relative to the applied fields [14], as dictated by a phase-matching condition [21]. In theory, the generated fields arise anywhere along a cone centered on the $\hat{z}$-axis, as depicted in figure 1(b). In the far-field transverse plane, we observe multi-spot optical patterns shown in figure 1(c) rather than a continuous ring, which indicates a spontaneous breaking of a continuous symmetry [24]. In addition, we observe different patterns under essentially the same experimental conditions, as shown in figure 1(c). Such shot-to-shot fluctuations are a hallmark of non-equilibrium phenomena, where symmetry-breaking results in self-organization into different modes [5, 25]. In our instability-driven system, we observe that these fluctuations occur in as quickly as 50 $\mu$s, which is on the order of the time it takes for an atom to move to a neighboring pancake and thus contribute to exciting a different pattern [15].

The synergistic coupling between the optical patterns and the atoms implies that there are also corresponding real-space patterns of bunched atoms that form spontaneously with the optical patterns and are enhanced as the power in the optical patterns increases. This synergy is simulated in figure 2, which depicts the dipole potentials that are generated by two- and six-spot far-field optical patterns. The atoms are attracted to the potential minima and self-organize into corresponding ‘atomic patterns’. We emphasize that the self-organized patterns occur within each pancake of atoms, which are generated by the applied 1D optical lattice, and the dipole potentials depicted in figure 2 are weak when compared to the applied dipole potential that forms the atomic pancake. However, we distinguish the self-organized structures using parametric resonance and Bragg scattering techniques, described below.

There are two types of atomic patterns: one with a short-period grating (spacing $d_\rho \approx \pi / [2k \cos(\theta/2)] \approx 195$ nm) overlap strongly with the imposed pump–pump grating (spacing $d_0 = \pi / 2k$) and arise due to the interaction of the atoms with a generated optical field and a nearly counterpropagating pump field, as depicted in figure 3(a). The long-period gratings (spacing $d_\rho \approx \pi / [2k \sin(\theta/2)] \approx 80$ $\mu$m) arise due to the interaction of the atoms with a generated optical field and a nearly co-propagating pump field, as depicted in figure 3(b). These interactions generate new dipole potentials that overlap with the pancakes of atoms in the imposed 1D lattice, simulated in figure 2, which gives rise to self-organization.

The total (applied and generated) electric field for a two-spot optical pattern is denoted by

$$\tilde{E}(z, r, t) = [\tilde{E}_0(z) + \tilde{E}_\rho(z, r)]e^{-i\omega t} + \text{c.c., where}$$

$$\tilde{E}_\rho(z, r) = i f_\rho(z, r)e^{i[k(z,z)\sin\theta\hat{y} + b(z, t)e^{i[-2\cos\theta \pm \sin\theta]x}].$$

Here, we ignore higher-order spatial-mode
patterns for simplicity. With these additional field terms, equations (1) and (3) are also modified so that the density distribution incorporates the self-organized patterns, with corresponding grating wavevectors. The short-period grating wavevectors, \( \hat{g}_s \approx k' [\pm 1 + \cos(\theta)] \hat{z} \pm \sin(\theta) \hat{\ell} \), are nearly along \( \hat{z} \), and the long-period grating wavevectors, \( \hat{g}_s' \approx k' [\pm 1 - \cos(\theta)] \hat{z} \pm \sin(\theta) \hat{\ell} \), are nearly along \( \hat{\ell} \). The Fourier coefficients for the self-organized density gratings are analogous to equation (4), but with \( \zeta_{\ell,\ell'} = U_{\text{dip},(\ell,\ell')} / k_B T_{\text{sat},\ell} \), where \( U_{\text{dip},(\ell,\ell')} \) refers to the effective dipole potential of the short-period (s) and long-period (\( \ell' \)) gratings, and \( T_{\text{sat},\ell} \) refers to the atomic temperature along \( \hat{g}_{\text{sat},\ell} \). Both \( U_{\text{dip},(\ell,\ell')} \) and \( T_{\text{sat},\ell} \) are measured experimentally, as described below.

3. Multimode self-organization

To directly observe and characterize the real-space atomic self-structuring that is coupled to the formation of optical patterns, we perform a parametric resonance experiment to measure the motional states of the atoms. The motional frequencies of atoms in the lowest-energy bound states oscillate according to

\[
\omega_{\text{vib},(p,s,\ell')} \approx \frac{\pi^2 U_{\text{dip},(p,s,\ell')}}{2 m d_{(p,s,\ell')}} ,
\]

where \( m \) is the atomic mass, and we assume the potentials are nearly harmonic. By parametrically driving at the resonance \( 2\omega_{\text{vib}} \) along the direction of the grating wavevectors, atoms can be excited out of their dipole potential wells [26], which reduces the efficiency of the wave-mixing process and the power in the generated fields. Because \( \hat{g}_s' \) is nearly orthogonal to \( \hat{g}_p \) and \( \hat{g}_s \), we use different experimental methods to parametrically drive atoms in each type of grating.

To excite atoms in the imposed 1D lattice and the short-period self-organized gratings, we use an electro-optic phase modulator placed in the path of one of the pump fields and driven at frequency \( \omega_{\text{mod}} \). This perturbs the dipole potentials along \( \pm \hat{z} \) and can parametrically drive atoms in both the imposed gratings and the short-period gratings, whose grating wavevectors are along \( \pm \hat{z} \) and \( \hat{g}_s \approx \pm \hat{\ell} \), respectively. During this experiment, we slightly misalign the pump beams to observe a stationary two-spot optical pattern and avoid detection errors due to pattern rotation [15]. We observe three distinct resonances, as seen in figure 3(c). The high-frequency resonance at \( \omega_{\text{mod}}/2\pi \approx 632 \pm 30 \text{ kHz} \) (width \( 258 \pm 26 \text{ kHz} \)) corresponds to atoms trapped in the pump–pump (imposed) lattice. We expect this resonance at \( 2\omega_{\text{vib}}/2\pi \approx 686 (+123/-109) \text{ kHz} \) according to equation (6) for \( U_{\text{dip},p} \equiv U_0 \) with experimental parameters \( \Delta = -2\pi (28 \pm 2) \text{ MHz} \) and \( I/I_{\text{sat}} = 0.28 \pm 0.06 \). The predicted resonances, the error, and the intermediate resonance at \( \omega_{\text{mod}}/2\pi = 177 \pm 15 \text{ kHz} \) are
discussed further below. Using equation (6), we find that the pump--pump dipole potential well depth is \( U_{\text{dip},p} \sim 635 \mu K \), where the energy is normalized by \( k_B \). Thus, the atoms at \( T_f = 2 - 3 \) \( \mu K \) are tightly confined in the imposed dipole potential wells.

The low-frequency resonance at \( \omega_{\text{mod}}/2\pi = 92 \pm 1 \) kHz (width \( 33 \pm 2 \) kHz) corresponds to atoms in the short-period self-organized gratings, which we expect to occur at \( \omega_{\text{mod}}/2\pi = 95 \) (+12/−9) kHz for a measured generated field intensity of \( (17 \pm 6) \mu W \text{ cm}^{-2} \). The effective dipole potential well depth for atoms in the short-period gratings is therefore \( U_{\text{dip},p} \sim 13 \mu K \). This measurement together with the temperature measurement described below indicate that the atoms self-organize into the short-period structures, which are not imposed on the atoms by the applied fields. For the 6-spot pattern shown in figure 1(c), we estimate there are \( \sim 7 \) short-period-grating lattice sites per pancake with \( \sim 20 \) atoms per site, as portrayed in figure 2.

To investigate the motional properties of atoms in the long-period self-organized gratings, we apply a weak elliptically-shaped optical field to the side of the atomic cloud and periodically modulate its amplitude. This perturbs the dipole potentials along \( \pm \hat{r} \) and can parametrically drive atoms in the long-period gratings, whose grating wavevectors are along \( \hat{g}_s \approx \pm \hat{r} \). To detect the long-period, low-frequency resonances, we operate the experiment in the steady-state regime, where we load the MOT for 97 ms and then leave the MOT beams on at 15% of their initial intensity while we run experiments. In this regime, the patterns persist for \( \sim 2 \) s, compared to 2.4 ms in the transient regime, where the MOT beams are shut off completely [15]. Here, we observe a parametric resonance at \( \omega_{\text{mod}}/2\pi = 134 \pm 2 \) Hz (figure 3(d)) (width \( 50 \pm 7 \) Hz), which we expect to occur at \( \omega_{\text{mod}}/2\pi = 191 \) (+78/−62) Hz. From this measurement, we find that \( U_{\text{dip},e} = 8 \pm 4 \mu K \). Because \( g_{s} \approx \hat{r} \), one might expect \( T_f \approx T_{\text{rad}} \) which would imply \( \zeta_r < 1 \) and negligible bunching into the long-period gratings. However, as we show below, the atoms undergo additional, spontaneous Sisyphus cooling along \( \hat{g}_s \), which facilitates self-structuring of atoms into the long-period gratings. We therefore observe spontaneous emergence of atomic patterns not only on the sub-wavelength, short-period scale, but also on a super-wavelength, long-period scale.

3.1. Analysis of the parametric resonances

To predict the parametric resonances expected in this experiment, we calculate the effective intensity that generates the imposed and self-organized dipole potentials. The intensity of a single pump field is \( I_p \sim 32 \pm 7 \) mW cm\(^{-2} \), where the error is due to the measured beam size (100 \( \pm 15 \) \( \mu m \)) after the pump-beam-reshaping effect shown in figure 1(c). This value of \( I_p \) also accounts for the 10% reduction in pump power from 11.2 to 10 \( \mu W \) that results from the pump-beam reshaping, discussed further below. We measure the intensity of the generated fields by measuring their output power and predicting their near-field size based on a calibration of our imaging system. We find the output intensity of a single generated field is \( \sim 17 \pm 6 \) \( \mu W \) cm\(^{-2} \), where the error is due to a slight asymmetry in beam size between the two spots in the optical pattern. However, because the wave-mixing process gives rise to an exponential increase in the generated field intensity across the length of the atomic cloud [27], we take the effective intensity inside the cloud to be the approximate intensity at the center of the cloud—180% of the output intensity, or \( I_p \sim (3 \pm 1) \times 10^{-3} \) \( \mu W \) cm\(^{-2} \).

We apply these effective intensities to \( U_{\text{dip},p,l,s,e} = h\Delta I/I_{\text{sat}}(1 + 4\Delta^2/I^2) \) in equation (6) with the experimental parameter \( \Delta = \omega - 2\pi (28 \pm 2) \) MHz and the resonant saturation intensity \( I_p^0 = 1.3 \) \( \mu W \) cm\(^{-2} \). For the pump--pump (imposed) gratings, \( I = I_p \). For the self-generated gratings, \( I = 2\sqrt{I_p/I_{\text{sat}}} \), where the factor of 2 is included because there exist two sets of self-generated gratings everywhere, e.g., the interference between \( F(z) e^{ikz} \) and \( F_i(z, r) e^{ik\left(z-\cos\theta-r\sin\theta\right)} \) and that between \( B(z) e^{-ikz} \) and \( F_i(z, r) e^{ik\left(z+\cos\theta+r\sin\theta\right)} \) give rise to spatially overlapping dipole potentials. For this experiment, we also take \( C^2 \to 1 \) because we operate well above the threshold where the fields only act on atoms that are tightly bunched in regions of pure \( \hat{r} \pm \) polarizations and thus only give rise to stretched-state transitions.

Based on this analysis, we predict the parametric resonances are 2\( \omega_{\text{vib},s} = 686 \) (+123/−109) kHz, 2\( \omega_{\text{vib},e} = 95 \) (+12/−9) kHz, and 2\( \omega_{\text{vib},e} = 191 \) (+78/−62) Hz, where the larger error for the long-period resonance is due to the error in \( \theta = 4 \pm 1 \) mrad. These predicted values agree with the measured values of 2\( \omega_{\text{vib},p} = 632 \pm 30 \) kHz, 2\( \omega_{\text{vib},s} = 92 \pm 1 \) kHz, and 2\( \omega_{\text{vib},e} = 134 \pm 2 \) Hz to within the experimental error.

For atoms in the short- and long-period dipole potentials, the location of the parametric resonance is a function of both \( U_{\text{dip},p} \) and \( n \); e.g., it decreases with lower pump beam intensities. Overall, we observe that the short-period-grating parametric resonance occurs between 90 and 133 kHz and the long-period-grating parametric resonance occurs between 11 and 134 Hz for \( I/I_{\text{sat}} = 0.05 \) to 0.3.

We also attribute the parametric resonance at \( \omega_{\text{mod}}/2\pi = 177 \pm 15 \) kHz (width 213 ± 34 kHz) to the dipole potential that arises due to the small ring around the pump beams, appearing in figure 1(c). This ring is pump-beam reshaping effect that arises because the pump size is comparable to the width of the cloud of atoms. We model this ring as an LG\(_{10}\) mode and find it contains \( \sim 10\% \) of the power contained in the central pump spot. The peak intensity of the ring is therefore 3.6% of the pump intensity. We predict that the dipole potentials generated by this ring and a nearby counterpropagating pump should have a parametric resonance at...
Because $e = \text{t}_{A}$ is characterized by a modulation of the dipole potentials due to a ring and a nearly copropagating pump field, we also perform a Bragg scattering experiment to both determine the temperature of the atoms in the self-ordered gratings and provide an additional characterization of their real-space structure. We apply the counterpropagating pump fields and inject a weak probe beam along either the $\hat{z}$-direction. We only collect light from a portion of one of the emission cones, as shown in figure 1(a). In this case, a probe beam traveling along $-\hat{z}$ ($+\hat{z}$) is Bragg-matched to scatter into multiple directions, but it will only reach the detector if it back-scatters (forward-scatters) off the short-period (long-period) gratings depicted in figures 3(a) and (b). This technique provides further evidence of real-space self-organization and allows us to distinguish between the two scales of long-range ordering.

Bragg scattering allows us to extract the temperature of the atoms in the short- and long-period gratings because the mean free path in our system ($\approx 23$ mm) is much longer than any of the lattice constants. Ballistic expansion of atoms out of their gratings results in the decay of the probe signal [29], as shown in figure 4. We note that the atoms move ballistically, rather than diffusively, once released from the optical lattice because the mean free path in our system ($\approx 23$ mm) is much longer than any of the lattice constants. Ballistic expansion is modeled using a Gaussian decay function $f_d(t) = a \exp(-t^2/\tau_e^2)$ [29], where $\tau_e$ corresponds to the time it takes for $n_{NL}$ to reduce to $1/e$ of its initial value. Because $d_e \gg d_i$, the peak density in the long-period gratings decays more slowly and consequently $\tau_e \gg \tau_s$.

**4. Bragg scattering**

We also perform a Bragg scattering experiment to both determine the temperature of the atoms in the self-organized gratings [29] and provide an additional characterization of their real-space structure. We apply the counterpropagating pump fields and allow the patterns to form and persist for $200 \mu$s. We then shut off the pump fields and inject a weak probe beam along either the $\pm \hat{z}$-direction. Time evolution of the scattered probe power. Pattern formation occurs during $-200 \mu$s $\leq t \leq -65$ ns with pump beam intensities $I/\text{I}_{\text{sat}} = 0.1$. A probe beam is turned on at $t = 0$ in the (a) $-\hat{z}$-direction and (b) $+\hat{z}$-direction. The red (dashed) curve is a best fit to a Gaussian, with (a) $\tau_e = 9.9 \pm 0.02 \mu$s and (b) $\tau_e = 120 \pm 2 \mu$s. The rectangular error bars at (a) $t = 0.5 \mu$s and (b) $t = 100 \mu$s indicate the typical statistical confidence interval of the fit. The temperature of atoms in the (c) short-period and (d) long-period self-organized gratings are extracted from data similar to (a) and (b) for various $I/\text{I}_{\text{sat}}$. The error bars define the statistical standard deviation.
4.1. Analysis of the atomic temperatures

To extract an atomic temperature from the Bragg scattering data shown in figures 4(a) and (b), we use a heuristic model of the atomic density distribution to relate $\tau_{s,f}$ to $T_{s,f}$. The density distribution for the short-period gratings before ballistic expansion is approximately

$$\eta_s(z) \approx \frac{n_a}{2\hbar \langle -\zeta \rangle} \left[ e^{-\zeta \cos(2k_z z)} + e^{-\zeta \cos(2k_z z - \ell)} \right],$$

where we use the experimentally measured value of $U_{\text{edge}}$ in $\zeta$. We note that $k'_s$ is slightly larger than $k'$ because the pattern-forming optical fields experience a different index of refraction [15, 21]. However, for the distance scales of relevance in this problem, it is a good approximation to take $k'_s \approx k'$.

The density distribution after ballistic expansion for the short-period gratings can be approximated as

$$\eta'_s(z) \approx n_a \left[ 1 + f(t) \cos(4k'_s z) \right]$$

in the case where $f < 1$. The constraint $f < 1$ maintains normalization, and it is valid for our experimental regime. We calculate the magnitude of $|f(t - \tau_s)|$ for a given $\eta_s(z)$ such that the peak density of $\eta'_s(z)$ is $1/e$ that of $\eta_s(z)$. Example density distributions at $t = 0$ and $t = \tau_s$ are shown together in figure 5(a) in blue (solid) and green (dashed), respectively.

We fit the ‘before’ and ‘after’ density distributions between $\pm \lambda/8$ to a Gaussian envelope in order to calculate the ‘before’ and ‘after’ grating widths. We apply this as the characteristic distance $d$ in a simple kinematics model of $v_0 = d/\tau$, where $v_0 = \sqrt{2\kappa a T/m}$. Leaving $T_s$ as an unknown parameter in $\zeta$, we find a numerical solution for the temperature as a function of $\tau_s$. An example of this predicted relationship is shown in figure 5(b) for $\Delta = -5$ and $1/L_{\text{lat}} = 0.01$. Using the same procedure, we relate $\tau_f$ and $T_f$ for the long-period gratings, whose results are shown in figure 5(c).

From these relationships, we extract an atomic temperature from our experimentally measured decay constants of the Bragg-scattered probe signal. We perform multiple Bragg scattering experiments to extract $\tau_s$ and $\tau_f$ for various pump intensities, which generate the data shown in figures 4(c) and (d). Figures 4(c) and (d) show the atomic temperature is $T_s \approx 1.8 - 2.8 \mu K$ along $\hat{g}_s$ and $T_f \approx 1.9 - 2.7 \mu K$ along $\hat{g}_f$ for all pump intensities. We interpret the slight upward trend as a result of the fact that Sisyphus cooling operates optimally over a limited range of intensities [13].

The low temperatures along $\hat{g}_s$ are expected because atoms in these gratings substantially overlap with the imposed gratings and therefore have the advantage of undergoing initial Sisyphus cooling due to the applied fields. However, we also observe that atoms in the long-period gratings cool to comparable temperatures despite having a momentum nearly orthogonal to $\hat{z}$.

Such transverse cooling can arise due to a weak transverse dipole force in a 1D optical lattice [30], Raman cooling [31], or due to 3D Sisyphus cooling that arises above threshold for pattern formation due to the interaction of the generated fields and the pump fields. However, the weak transverse dipole force imposed by the 1D lattice does not give rise to efficient cooling, i.e., it would take approximately 600 $\mu$s for the atoms to move a distance $d_r$ under the influence of this weak force alone. In contrast, we have observed the signature of these long-period gratings as soon as 20 $\mu$s after turning on the pump beams, and thus, this force cannot be responsible for the cooling timescales that we observe. In addition, we have never observed Raman transitions to
other ground states, and thus we can only attribute the observed cooling to frequency-degenerate schemes such as Sisyphus cooling.

We thus conclude that the only mechanism that can cool and trap atoms in the long-period gratings so quickly and effectively is Sisyphus cooling, where the expected damping time is \( \tau = \left( \frac{\hbar^2}{\Delta^2 / (2m \Gamma)} \right)^{-1} \approx 10 \mu s \) [12]. The observed 3D Sisyphus cooling process occurs spontaneously as a result of the optical/atomic pattern-forming instability. Our observation of 3D cooling by only applying fields along one dimension allows us to achieve longer coherence times with a simplified geometry in comparison to lattice experiments where laser beams are applied in all three dimensions [10]. This is also supported by our observation that the patterns persist in the transient regime for up to 2.4 ms [15] compared to \( \sim 300 \mu s \) in similar wave-mixing experiments where 3D cooling is absent [8], thus giving rise to more rapid atom loss.

We also perform the same Bragg scattering experiment for various pump times and find that when applying the pump beams for longer than \( \sim 30 \mu s \), the temperatures are consistent with 2-3 \( \mu K \). For applied pump times of less than \( \sim 30 \mu s \), the Bragg scattered signal to noise ratio is too small to obtain reliable data because the optical/atomic pattern formation has not yet reached peak power/density during this time period, over which the atoms are still undergoing Sisyphus cooling [15].

5. Conclusion

In conclusion, we directly measure the spontaneous emergence of real-space atomic structures in a multimode geometry using parametric driving and Bragg scattering microscopy, and we observe spontaneous three-dimensional Sisyphus cooling. Our system exhibits continuous symmetry-breaking and long-range atomic self-structuring on multiple spatial scales. Our work represents an important step towards studying non-equilibrium phenomena in multimode geometries and provides a simplified system in which one can observe low-light-level, multidimensional nonlinear optical effects.

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References

[1] Bloch I 2012 Nat. Phys. 1 23
[2] Greiner M, Mandel O, Esslinger T, Hänsch T W and Bloch I 2002 Nature 415 6867
[3] Gopalakrishnan N, Lev B L and Goldbart P M 2009 Nat. Phys. 5 845
[4] Ritsch H, Domokos P, Brennecke F and Esslinger T 2013 Rev. Mod. Phys. 85 553
[5] Baumann K, Gerulic C, Brennecke F and Esslinger T 2010 Nature 464 1301
[6] Schmittberger B L and Gauthier D J 2014 Phys. Rev. A 90 013813
[7] Labeyrie G, Tesio E, Gomes P M, Oppo G L, Firth W J, Robb G R M, Arnold A S, Kaiser R and Ackemann T 2014 Nat. Photon. 8 321
[8] Greenberg J A and Gauthier D J 2012 Phys. Rev. A 86 013823
[9] von Cube C, Slama S, Kruse D, Zimmermann C, Courteille P W, Robb G R M, Piovella N and Bonifacio R 2004 Phys. Rev. Lett. 93 083601
[10] Raithel G, Birkl G, Kastberg A, Phillips W D and Rolston S L 1997 Phys. Rev. Lett. 78 630
[11] Greenberg J A, Oria M, Dawes A M C and Gauthier D J 2007 Opt. Express 15 17699
[12] Metcalf H J and van der Straten P 1999 Laser Cooling and Trapping (Berlin: Springer)
[13] Castin Y, Dalibard J and Cohen-Tannoudji C 1991 Light Induced Kinetic Effects in Atoms, Ions and Molecules ed L Moi et al (Pisa, Italy: ETS Editrice)
[14] Greenberg J A, Schmittberger B L and Gauthier D J 2011 Opt. Express 19 22535
[15] Schmittberger B L and Gauthier D J 2016 J. Opt. Soc. Am. B 33 1543
[16] Schilke A, Zimmermann C, Courteille P W and Guerin W 2011 Phys. Rev. Lett. 106 223903
[17] Lezama A, Cardoso G C and Tabosa J W R 2000 Phys. Rev. A 63 013805
[18] Boyd R W 2008 Nonlinear Optics 3rd edn (New York: Academic)
[19] Deutsch I H, Sreenw R J C, Rolston S L and Phillips W D 1993 Phys. Rev. A 52 1394
[20] Camara A, Kaiser R, Labeyrie G, Firth W J, Oppo G L, Robb G R M, Arnold A S and Ackemann T 2015 Phys. Rev. A 92 013820
[21] Chiao R Y, Kelley P L and Garmire E 1966 Phys. Rev. Lett. 17 1158
[22] Dawes A M C, Billing L, Clark S M and Gauthier D J 2005 Science 308 672
[23] Jessen P S, Gerz C, Lett P D, Phillips W D, Rolston S L, Sreenue R J C and Westbrook C J 1992 Phys. Rev. Lett. 69 69
[24] Geddes J B, Andrik R A, Moloney J V and Firth W J 1994 Phys. Rev. A 50 3471
[25] Black A T, Chan H W and Vuletić V 2003 Phys. Rev. Lett. 91 203003
[26] Raithel G, Birkl G, Phillips W D and Rolston S L 1997 Phys. Rev. Lett. 78 2928
[27] Silberberg Y and Bar-Joseph I 1984 J. Opt. Soc. Am. B 1 662
[28] Fletcher N H 2002 Am. J. Phys. 70 1205
[29] Mitsunaga M, Yamashita M, Koashi M and Imoto N 1998 Opt. Lett. 23 640
[30] Blatt S, Thomsen J W, Campbell G K, Ludow A D, Swallows M D, Martin M J, Boyd M M and Ye J 2009 Phys. Rev. A 80 052703
[31] Kasevich M and Chu S 1992 Phys. Rev. Lett. 69 1741