The Cultural Heritage of Architectural Linear Perspective: The Mural Paintings in Nantang Church

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Abstract: This paper presents a contextual use of the innovative drawing techniques that involved architecture and painting in the Qing court during the first half of the eighteenth century. At this point architectural linear perspective in painting (quadratura) and stage design had become common fields of experimentation for the Chinese and Jesuit artists missionaries. In this conceptual context, Western quadratura was developed in China by Giovanni Gherardini (1655–1729), and especially by Giuseppe Castiglione (1688–1766), who is remembered as an extraordinarily versatile architect–painter. The focus of this paper is on the “illusory mural paintings of architectural perspective in Nantang Church” (Beijing), which has now disappeared, and which spread the influence of the Western Renaissance. The imported Western linear perspective and the fundamentals of architectural drawing facilitated the systematization and dissemination of the quadratura as an unknown technique in China. Based on the text described by the contemporary scholar Yao Yuan Zhi, an original interpretation of the architectural perspective mural paintings in Nantang Church is proposed. These paintings provide an important case study of Sino-European collaboration in the eighteenth century from different points of view: the representation of the light in drawings and the fact that the concept of shadow in some respects was unknown to the Chinese artist.

Keywords: cultural heritage; quadrature; linear perspective; architectural painting

1. Introduction

In order to understand the dynamic of the linear mathematical perspective transmission process to East Asia and its application to architectural painting, there is a need to start with some important antecedents. According to Rossi, “a rich bibliography includes research on its mathematical essence, the role it has played in the representation and history of architecture, its use in theatres and the various projective applications to measurement and representation of surfaces” [1,2]. As a scientific topic in general, perspective traditionally involves various disciplines that are easily distinguished by the specific nature of their approaches. Art history, architecture, mathematics, descriptive geometry, drawing, stage design and other more technical fields of knowledge have applied geometric principles to the solution of practical problems [1] (p. 578). The architectural perspective, culminating in the technique of quadratura, appeared to be exclusive to Italy and was very widespread between the sixteenth and eighteenth centuries [3,4].

The term architectural perspective or quadratura refers to the set of perspective constructions realised on wall and ceiling surfaces to simulate diverse spaces thanks to the possibility of measuring and representing depth of space in an illusory way [3,5] (p. 23).

In the seventeenth century, the mathematician Girard Desargues (1591–1661) developed the basic concept of the principle of projection and section view proposed by the Renaissance artists. In 1636, this scientist derived his famous “perspective theorem,” which states: “When two triangles are in perspective, the points where the corresponding sides meet are collinear.” Desargues’ most important work Brouillon projet d’une atteinte aux événements des rencontres d’une côte avec un plan (proposed draft for an essay on the results of taking plane sections of a cone) was printed in 1639, where Desargues presented innovations...
in projective geometry applied to the theory of conic sections. With this work he founded the use of projective methods in geometry, inspired by the theory of perspective in art. Additionally, the Italian Jesuit Andrea Pozzo (Appendix A, A1) (1642–1709) published *Perspectiva, Pictorum et Arquitectorum* between 1693 and 1698, which argued for perspective drawings based on the two orthogonal projections of an object. The first volume became the most widely published and translated treatise on perspective written to date in 1693. At the end of the first volume, Pozzo included an answer in response to those who advocated the use of multiple viewpoints within a perspective illusion. For the most part, Pozzo employed a single point of view in his *quadratura* works (Figure 1a,b).

![Figure 1. (a) Andrea Pozzo. Perspectiva, Pictorum et Arquitectorum. London, ca 1707. Public domain; (b) Andrea Pozzo. Perspectiva, Pictorum et Arquitectorum. London, ca 1707. Public domain.](image)

On the other hand, “the mathematician Brook Taylor (1685–1731) made breakthroughs in the development of perspective drawing. In 1715 he published *On the Principles of Linear Perspective*. Additionally, Johann Heinrich Lambert (1728–1777) was one of the great mathematicians in eighteenth century. Lambert diagrammatized a few important issues of elementary geometry based on perspective principles and drew perspective with the center of projection at infinity according to the property of affinity correspondence” [6] (p. 108).

### 2. Development of Graphic Drawings in the Late Ming Dynasty and the Early Qing Dynasty

The role of China in the historical development of graphics declined during the Ming (1368–1644) and Qing (1644–1911) dynasties. In these periods, various European missionaries came to China and used the Western perspective. The painters among these missionaries applied the focus perspective in drawing by furthering the landscape and strong stereo feelings, which attracted the attention of the scholars of pictorial works of the Qing dynasty. These scholars absorbed the theories and techniques of western drawings and paintings and disseminated them. Thus, Nian Xiyao (?–1738), high Commissioner of the Ministry of Works during Yongzheng’s reign (1678–1735) became a collaborator of the Jesuit missionary G. Castiglione. Nian Xiyao compiled and published his book *Shi Xue* (Appendix A, A2), *(Study of Vision)*, which illustrated the basics of illusionist paintings and architectural techniques on perspective drawings [7]. The publication of the two versions of *Shi Xue* can be recognized as “excellent work and the first of its kind in the history of graphics in China” [6] (pp. 108–120). Nian specifically acknowledged Castiglione for his continuing instruction in the art of the European linear perspective. Nian adapted Pozzo’s designs for a trompe l’oeil cupola, although in the process of copying, the details were greatly simplified and many elements were combined or eliminated (Figure 2a,b) [8].

When Jesuit missionaries travelled to China, they carried with them some indispensable tools. These tools were constituted on two bases: firstly, the missionary illusion and in second place, the resources available in the Western culture to establish the *inculturation* through the sciences and the arts [9] (pp. 43–57). The linear perspective makes it possible to not only to represent our perception of space on drawings, but also to achieve new spatial
effects on the whole decoration of the painting. These effects lead us to perceive not the actual spatial reality, but a totally fictitious reality created by the illusion of perspective.

Figure 2. (a) Andrea Pozzo. “A Cupola in Horizontal Perspective”. Perspective in Architecture and Painting, London, ca 1707. Public domain; (b) Nian Xiyao. Page from Shixue, 1735 taken from Andrea Pozzo’s Perspectiva Pictorum et Arquitectorum. University of Glasgow, Special Collection. Public domain.

The Italian Giovanni Gherardini (1655–ca. 1729) was among the first painters in China to introduce these perspective techniques [10] (pp. 117–118, 298–299) at the court of Kangxi (r. 1662–1723). Before him, the mathematician Ludovico Buglio (Chinese name Li Leisi) (1606–1682) delighted the emperor Kangxi with paintings completed in perspective and taught the technique to Chinese artists. Gherardini was in charge of an important lineal perspective painting school at the Emperor’s court. This technique is also known as “quadratura” [11] of which the Jesuit artists had a relevant knowledge. In this sense, Gherardini’s references to the “quadratura” style were based not only on the paintings of A. Pozzo but also on those of the two highly experienced masters M. Colonna (1604–1687) and A. Mitelli (1609–1660).

The pictorial quadraturista illusionism of Gherardini enchanted the Chinese courtiers despite the vast differences between the Chinese and European styles of painting [11]. However, Gherardini left Beijing unexpectedly in 1704 due to the decision to return to Europe. After his departure, the arrival of the Italian Jesuit architect–painter Giuseppe Castiglione (1688–1766) (Appendix A, A3) in 1715 allowed Emperor Kangxi to maintain the quadratura works at court. As a consequence, Kangxi finally boasted a European painter of significant importance, and this had a considerable impact on the court’s preference for over half a century. The Jesuit artist became known in Chinese as Lang Shining. His great ability as a painter allowed him a magnificent personal and professional relationship with Emperors Kangxi, Yongzheng, and Qianlong. It was Qianlong in particular who discovered his extraordinary ability for portrait and architectural painting, as well as in architecture.

During the Qianlong era, Giuseppe Castiglione (Lang Shining) and J. Denis Attiret (Chinese name Wang Zhi-Cheng, (1702–1768) [12] (pp. 109–122)) were responsible for teaching oil painting, linear perspective, and quadratura in the court Academy [13]. The enormous attraction aroused by the illusionistic painting explains why it was applied in interior decorations. It is known that this effect is emphasised where walls and ceilings appear as though the space is open from the inside towards infinity. Obviously, the idea of using walls for supporting the rendering scenes is as old as art itself [14] (pp. 585–598). Thus, the Jesuit artists imported the novelty of using mural painting as a new technique from the West. These plaster surfaces were very suitable for painting scenes on the walls in churches and palaces, producing a “trompe l’oeil” effect on the viewer. The purpose of this effect was to create the belief that such wall and ceiling surfaces were not present but opened in a misleading way to infinity.

Thanks to Qianlong’s sponsorship, the Chinese and European artists of the court were able to innovate a new style of painting. To achieve this, the artists combined Renaissance
perspective techniques with Chinese themes and materials. This produced the appearance, for the first time, of the illusionist life-size.

Castiglione employed many baroque stylistic tricks to increase the efficacy of his message including perspective, a discipline of which he was an expert. Through this technique, Castiglione’s mural paintings demonstrate that his motivation and expertise were quite surprising involving the Renaissance perspective, which represented the key and the secret of the geometrical knowledge of Castiglione.

Naturally, quadratura was not limited to the mural technique mentioned above. There was a tradition of paintings also on canvas in large formats. However, all pictorial works, regardless of the technique and format, had a common denominator, which was the study of light and its effects. Nevertheless, these effects were not exclusively artistic, but in most cases had a vigorous symbolic character. It is necessary to highlight that the appearance of two-dimensional paintings with a three-dimensional appearance on ceilings, walls, and vaults pre-supposed two constraints: the importance of the viewer’s position related to the work and secondly the symbolic dimension. Both characteristics occurred in this disappeared heritage. Consequently, the viewer’s position at a certain point and the environment are essential to the location inside or outside the illusion [15] (p. 214).

A second constraint is the symbolic dimension, which was presupposed in Jesuit artists’ paintings. It was an essential tool in the process of inculturation. In this sense, G. Castiglione used the linear perspective, not only as a technique that charmed Emperor Qianlong, but also as a powerful instrument for missionary activity. It must be noted that the main points of contact for potential converts to Western culture were in the four Catholic churches in Beijing. They housed European missionaries and were the centre of the diffusion of science and religion [16] (pp. 18–22). The dual role of Castiglione, the Jesuit lay brother and the Qing imperial painter–architect [17] (pp. 45–59), was definitely a perfect duality for inculturation.

The missionary methodology was taken to an extreme and initiated by the Jesuit Matteo Ricci (Chinese name Li Madou) (1552–1610) [18] who emulated the values of court literati and officials to gain more converts among the Chinese elite. His methods had a powerful influence over his successors at the Jesuit mission in China during the Ming (1368–1644) and Qing (1644–1911) dynasties. Sciences, technology, and arts, particularly architecture and painting, culminated in a period of splendour that would not easily be surpassed.

The apex of Qing Imperial power witnessed the most active and productive period in the architecture and painting fields during the eighteenth century. The interest of the Emperor in Western art and his gratitude for the artistic work carried out by Giuseppe Castiglione made all Jesuits welcome in his court. Marco Musillo reminds us:

> On this basis I shall then show that Castiglione’s artistic process of translation involved a system of dynamic, always shifting points of connection. These points of connection gave Castiglione the opportunity to bring his European professional training to bear on his new Chinese artistic environment and, in response to this; he developed an artistic idiom which was much more than just a Sino-European hybrid, [19] (p. 9).

As an architect, G. Castiglione was a pioneer in the implementation and execution of a monumental architectural project, which materialized in the construction and interior decoration of the European Palaces at the Garden of Perfect Brightness (Yuan Ming Yuan) in Beijing. This cultural heritage was destroyed by Anglo-French troops in 1860. The Jesuit’s projective dimension, as a symbol of the extraordinary influence on Western architecture, had such significance that it would be analysed in future works.

Susan Naquin reminds us that: “Today, he is written about as a man who fused Western and Chinese artistic elements to establish a distinctive high Qing court style” [20] (p. 393).
3. Case Study. Illusory Mural Paintings: “If You Touch It, You Will Suddenly Find It a Wall”

This research focuses on two perspective drawings that Yao Yuanzhi’s description locates in Nantang Church dating from the eighteenth century. This paper presents the author’s interpretation of the cultural heritage of the two paintings in linear perspective and an approximated analysis of these two drawings to determine the accuracy of the geometrical representation.

In Beijing, the organisation of the four Catholic churches depended on the missionaries and the churches were the centre of Western art and religion dissemination. It is worth mentioning the strong cultural shock caused by the illusionistic paintings on viewers who visited the churches for the first time as described by Yao Yuan Zhi (1773–1852). This was mainly because of the different Asian mind-set for the drawing of old painting traditions. Consequently, this was discovered thanks to the European illusionistic techniques that introduced a painting genre that was integrated into real architecture. Therefore, with the transmission of these Western artistic traditions to the East by Jesuit missionaries, new paintings were widely accepted, and above all, this was because they contained an important element: the shadow.

The best-known of the four churches was Nantang Church (Figure 3), which was of historic heritage in imperial China. It was important for the functioning of the mission, but also for its architecture and interior decorative art. This transformed it into a masterpiece of Jesuit art in the East.

According to the written sources, primarily from Ricci’s diary, the beginnings of this church can be traced to a small chapel (1605). The church was not the artistic invention of its initial builder Matteo Ricci (1552–1610) but was mainly the result of gradually completed reconstructions and renovations. In 1652 Nantang Church underwent an important architectural modification and was monumentally enlarged by the German Jesuit, Johann Schall von Bell (1592–1666). From 1690 and between 1703 and 1711 Nantang received significant architectural renewals and transformations [21] (p. 1).

It is noteworthy that the Japanese Mikinosuke Ishida found a Chinese text about the paintings of Castiglione in Nantang [22] (pp. 102–106), which describes its interior decoration in great detail. Based on the existence of this little-known specific source as regards Nantang Church, one may conclude with the performance of Lang Shining in Nantang. Thus, Susan Nanquin writes:

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**Figure 3. Exterior of Nantang Church. Project of Fernando Buenaventura Moggi (1776). Overseas Historical Archive of Lisboa. Public Domain.**
Ishida's thorough research and command of the existing secondary sources in many languages not only surveyed the Jesuit's life but attempted a complete list of the growing number of known works [20] (p. 400).

Furthermore, a mention of these sources is made in [19] (p. 124):

Mikinosuke Ishida has discovered further evidence from certain Chinese sources about the Nan Tang (Southern Church) of the Portuguese residence. For this church Castiglione made two canvases and two mural paintings. The mural works were on the east and west walls. They are described in the Zhuyeting zaji by the contemporary scholar Yao, Yuanzhi.

The murals on the East and West walls were intricately described by Yao Yuanzhi, [22]. In accordance with these sources, the existence of a false dome and four illusionist mural paintings in the church can be confirmed (Appendix A, A4).

It is observed in the text that the illusionistic paintings occupied the entire space of the walls of Nantang and included real objects and shapes. Castiglione’s attention to this aspect of illusionism was equal to the care dedicated to the global construction of the murals.

These drawing constructions, emphasizing the effect of light and shadow with the linear perspective technique, and among others of less importance, merged into one surprising illusion. To obtain this result, it was necessary to imagine an entire three-dimensional structure that set itself beyond the wall surface and that totally involved the real space.

It is worth mentioning that a special feature of these paintings was their ability to activate the sense of touch of the viewer, in order to verify that what was seen was just a painting and had nothing to do with reality. Ingrid Sjöström holds that the main characteristic of the quadratura is the continuity from the false to the real architecture:

\[ \ldots \] continuity implies that the fictive and the real spatial volumes merge, and that the actual border between real and painted architecture is camouflaged or arranged in the composition so as to counteract its disruptive effect [23] (p. 15).

Therefore, the merging of the work of the missionaries and art culminated in the architectural quadratura. The result was a pictorial illusory space of undoubted beauty as it can be read in the sentence of the text: “\[ \ldots \] if you touch it, you will suddenly find it a wall”: there was a clear marriage of art and science.

4. Nantang Murals: Three-Dimensional Model Representation

To achieve a correct representation of the wall paintings, the description of the text and the objects represented were first analysed. Subsequently, and using three-dimensional rendering and ray tracing technologies, the model was built with all the information to finally materialize the representations of the same two-dimensional view in linear perspective in an exact way.

Three-dimensional rendering is a process that uses a computer to generate a two-dimensional image from a digital three-dimensional scene. To generate an image, specific methodologies and special software and hardware are used. Therefore, it has to be understood that three-dimensional rendering is a process.

Single three-dimensional points cannot provide a global illustration of the structure of the object. Thereby, the creation of a three-dimensional object model is required to depict the formation and the real conditions of the object. In this sense, three-dimensional digitization is a complex work that consists mainly of three phases:

(a) Conceptual phase that involves decisions about the technique and method to be adopted;
(b) Digital recording according to the plan from the conceptual phase;
(c) Simulation and data processing which involves the modelling of the digitized object through geometric data processing, texture data processing, colour etc. [24] (pp. 93–98).

The model may be defined as the process through which an idea is developed, simulated, and elaborated into detailed instructions and working tools for the final materialization.
On the other hand, ray tracing technology is a rendering technique that can produce incredibly realistic lighting effects. Essentially, an algorithm can trace the path of light, and then simulate the way that the light interacts with the virtual objects.

Based on these technical considerations, the mural painting on the side walls described in Yao Yuanzhi’s text were interpreted and are represented in Figures 4 and 5.

In the text, Yuanzhi precisely described the existing number of churches in Beijing. The text focused on Nantang Church, giving no doubt that there were at least two mural paintings attributed to Castiglione, painted in the quadratura method.

Nevertheless, despite the amazement of Yuanzhi about the realism of the murals and the new and unknown technique, there were other contemporary artists in the court that did not positively interpret this pictorial methodology and who saw serious flaws in it. This is summarized by one of these artists (Tsou I-kuei) in these words:

*The Westerners are skilled in geometry, and consequently there is not the slightest mistake in their way of rendering light and shade [yang-yin] and distance (near and far). In their...*
This particular interpretation of Western painting by some of the Chinese artists was not understood by the Europeans. It may have been due to the Chinese superiority granted to the spiritual aspect as well as the artist’s vision about the realistic idea of space (Figure 2).

5. Further Consideration and Discussion

The innovative pictorial effect of the linear perspective marked a before and after, and it could create an optical illusion between the two walls and their pictorial representations. However, turning the eyes completely to the right or left, perpendicular to the wall, the two perspectives appeared as a frontal perspective, also called a “one-point perspective”. According to Ishida, Lang Shining managed to create a deep illusionistic architectural design by making physical walls disappear. This was a well-known perspective of the Renaissance, one that positioned the parallelepiped volumes of the architectural painting with one side parallel to the mural [1]. The sidewalls were perpendicular to it and their edges appeared to converge into one single vanishing point (Figure 6). The drawings were a frontal perspective drawn from a station point in front of the east and west walls. The prolongation of supposedly perpendicular lines to the walls indicates the location of the principal point, the orthogonal projection of the station point to the picture plane, and the position of the horizon line.

The integration of elements and objects into the architectural environment create comfortable and distinguished scenery. The illusionistic methods used in these frescoes powerfully suggest a painter with advanced Western pictorial skills. All this occurred due to an extraordinary mastery of perspective through the accurate application of geometry.

In the description in the text of the east wall of the church, he tried to give special importance to the prospect of light (chiaroscuro) using the diffused light from the window described. All this is what generated the concept of shadow. In this way, it may be claimed that not only did the artist manage to explain the projection of the table on the wall and floor, but he also succeeded in representing the impression of shadow of the feather fan, which crowned a Chinese vase (Figure 7a). In [22] Ishida writes:

[... ] Under the sunbeams the shadow of the fan, the shadow of the vase, the shadow of the table all these are perfectly accurate.
As can be observed in these two representations (Figures 4 and 5), the traditional oriental aesthetics attempted to capture the enigmatic world of the shadow. When these effects were suppressed, the beauty lost its magnificence. In the simulated architecture there were lots of descriptions of the lateral decoration. Some objects, typically Chinese, were identified and located on the walls: calligraphic scrolls, and two large bookshelves (Figure 7b). These representations ensured a pleasant visit to a hypothetical observer exploring the width and length of the room. In [22] Ishida writes:

\[ \ldots \] A large quantity of books in scrolls with ivory plates used as indexes and jade axes fill up the book-shelves. There is a magnificent cabinet containing a number of curious which glitter up and down. \[ \ldots \] On the wall are calligraphic scrolls and hanging pairs of rhymed epigraphs in decorative style exhibited for inspection.

Castiglione conducted his illusory scenes taking advantage of the diffused natural light already presented in that real environment, namely the light entering from a window that was half open. The windows, which were a source of filtered light, illuminated each object in the room and at the same time gave the illusion of the distinctive light of a warm climate. This happened in a beautiful harmonious interaction between light and shadow.

The presence of the patio, as well as the set of imagined columns (Figure 8a), suggest a brise soleil and increase the feeling of an imaginary depth. It is represented in an identical sequence, as shown in Perspectiva Pictorum et Arquitectorum of A. Pozzo (Figure 8b).
Again, when describing the West wall (Figure 5), the text insisted on the presence of light and shadow. The decor, which was not forgotten, contained traditional elements such as the folding screen, the bell, and an astronomical instrument. This instrument was possibly similar to the one used by Jesuit astronomers and could have been a quadrant or a sextant. The floral decoration of the ceiling reminds one of the wisterias painted by G. Castiglione in Juanqin Zhai (Appendix A, A5). In [22] Ishida writes:

[... ] By the southern window, the sun shadows three tripod-kettles. Three tables are arranged in a row. The gold glitters (Figure 9a).

[... ] On one of them stands a clock which strikes automatically; on the other, an astronomical instrument is placed. Between the two tables are placed two chairs. On the pillar are placed four lamp-basins on which silver-like candles stand. Looking up to the ceiling, you will see the wood carved into flowers. The middle part is raised to look like stamens and pistils (Figure 9b).

Figure 9. (a) Detail of the west wall. Three tables and windows. Representation of light and shadows (Ray Tracing); (b) Detail of the west wall. An astronomy tool on a table. Representation of light and shadows (Ray Tracing). Source: Author.

The visual impact of the public exhibition of the Nantang murals should have been the result of a strategy designed to approach potential converts. A documented example of this was perhaps the visit by Hong Daeyong (philosopher, mathematician, astronomer, 1731–1783) during the Joseon Dynasty (1392–1910), who was fascinated by the quality and realism of the murals. The paintings, which were deceptively tactile, produced a strong sensorial and emotional response in Hong Daeyong [26] (p. 57). Clearly, in the eighteenth century, through geometry, the observer found the specific place for the specific visual perception.

Castiglione found a favourable environment as a Jesuit lay brother putting into practice his humanistic education and experimenting with his pictorial skills. Through his mural paintings, Castiglione created a new body of knowledge as he matured as a painter and an architect, while at the same time enriching his own missionary vocation.

6. Conclusions

The representation of cultural heritage is a multi-dimensional process. It depends largely on the nature of the object as well as the purpose of its representation.

This research started from the premise that Chinese culture contributed to the development of the linear perspective in China, in addition to receiving concepts and methods from Europe. Thus, it is remarkable that the resurgence of architectural painting during the 17th and 18th centuries, due to the Jesuit missionaries, was a consequence of the political, economic, and social situation during the Ming and Qing dynasties. The main argument...
of this article is that Chinese culture was an important source of inspiration for European artists mainly during the Qing Dynasty. An important consequence of this inspiration was the wall paintings of the Nantang Church (Beijing). Consequently, the cultural heritage of Nantang was a meeting point between the European perspective and Chinese culture.

These paintings are examples of Castiglione’s training in the field of linear perspective and its application in illusory architectural spaces. This training was fundamentally based on the work of A. Pozzo, as is suggested in the comments on Figure 4a,b.

As far as this destroyed heritage is concerned, it is clearly necessary to offer an alternative possibility for access that enables its popularization and cultural appropriation. In this sense, to understand the missionary methodologies more precisely, the mural paintings on the east and west walls of the Church of Nantang were interpreted and represented through the reconstruction of three-dimensional models with render and ray tracing technologies, according to the remarkable text described by Yao Yuanzhi.

Considering that the iconographic heritage in question comprised drawn architectures that merged with real architecture, it is interesting to propose these dynamic interpretative digital models that can associate the virtual tours of the buildings with the reading of the illusory space. Thus, the profound knowledge of the linear perspective of the Jesuit, enhanced by Chinese elements and materials, provided his works with undeniable interest and beauty, thanks to a sensory involvement, which is much more effective than the static vision of the depicted scenes. The digital models allow the observer to grasp the graphic and artistic strength of this destroyed cultural heritage.

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Appendix A

A1. Italian painter and master in the art and science of perspective. For a complete revision of A. Pozzo’s biography, see Vittorio de Feo and Vittorio Martinelli, eds., Andrea Pozzo. (Milano.1996).

A2. The Chinese scholar Cao Yi-qiang has discussed the different ways in which the categories of “art” and “science” were recognized in Europe and China in his research on Nian Xiyao’s, (?–1739) Shixue (Theory of Vision) (1729–1735), which is a translation of selections from Andrea Pozzo’s Perspective, Pictorum et Arquitectorum.

The theories of graphics discussed in Shi Xue cover almost every aspect of graphics. The principles of perspective include the methods of points of distance, double points of distance, intercepts, and upward viewing. These theories cover almost all the fields of engineering graphics in the West. [7] (pp. 110–111).

A3. In relation to the life and work of G. Castiglione, there are magnificent works. The first to highlight is the one by Robert George Loehr, Giuseppe Castiglione, 1688–1766, Pittore di Corte di ch’ienlung, Imperatore della China, (Rome: ISMEO, 1940). A pioneering work on the life and work of Castiglione is by Mikinosuke Ishida entitled, A Biographical Study of Giuseppe Castiglione (Lang-Shinin), a Jesuit Painter in the Court of Peking under the Chi’ng Dynasty, Memoirs of Research in the Department of Tokyo Bunko Tokyo, (1960), where the life of G. Castiglione and his work are exposed. A great monograph of Castiglione can be seen in Cécile and Giuseppe Michel Beurdeley, Castiglione: A Jesuit Painter at the court of the Chinese emperors. (Rutland, Vermont: Charles E. Tuttle Company, 1971). Finally, see the excellent work of Marco Musillo, Reconciling Two Careers: The Jesuit Memoir of Giuseppe Castiglione Lay Brother and Qing Imperial Painter, Eighteenth-Century Studies, 42, 1, (2008), 45–59 and a Review Essay by Susan Naquin entitled Giuseppe Castiglione/ Lang Shining, T’oung Pao 95, (2009), 393–412.
A4. Description of the Nantang’s murals in the Zhuyeting zaji by the contemporary scholar Yao, Yuanzhi:

Within the capital there are four Catholic churches in all. One of them is called His-ts’ang. It was destroyed by fire a long time ago. One sited at Ts’an-ch’ih-k’ou is called Pei-ts’ang. One sited Tung-ts’ang-ts'hu-ts’ang is called Tung-ts’ang. One sited, within the Hsüan-wu-mên at the foot of the Tung-ch’êng is called Nam-ts’ang.

Within the Nan-Tang there are two ‘linear school’ pictures drawn by Lang Shih-ning. They are spread on the two walls, east and west of the parlour, high and large like the walls. If you stand at the foot of the west wall, close one eye, and look toward the east wall, the inner chamber extends to a great depth, with the bead-blinds completely drawn up. The southern window is half open. The sunbeams play on the floor. A large quantity of books in scrolls with ivory plates used as indexes and jade axes fill up the bookshelves. There is a magnificent cabinet containing a number of curios which glitter up and down. On the north end a tall table stands. On the table stands a vase in which peacock feathers are arranged. A brilliant feather-fan is in the sunshine. Under the sunbeams the shadow of the fan, the shadow of the vase, and the shadow of the table all these are perfectly accurate. On the wall are calligraphic scrolls and hanging pairs of rhymed epigraphs in decorative style exhibited for inspection. If you go through the chamber and go east, you will come to the north part of a large court. There a long corridor runs on and on. A set of pillars stand in line. The stone-pavement evenly shines with brightness. If you go east, you will see as if a house exists, and the door seems not yet open. If you lower your head and look out of the window, you will see two dogs playing together on the ground.

If you stand again at the foot of the east wall, and look toward the west wall, you will again see the three chambers of the outer building. By the southern window, the sun shadows three tripod-kettles. Three tables are arranged in a row. The gold glitters. On the top of the pillars in the hall, three large mirrors are hung. On the north end of the hall, screens stand; on the east and west, stand two tables on which red brocade covers are spread. On one of them stands a clock which strikes automatically; on the other, an astronomical instrument is placed. Between the two tables are placed two chairs. On the pillar are placed four lamp-basins on which silver-like candles stand. Looking up to the ceiling, you will see the wood carved into flowers. The middle part is raised to look like stamens and pistils. The lower part hangs down as if left upside down. If you look down upon the floor, you will see it so bright like a mirror that you will be able to count all the square tiles. One white stream along the centre of the tiles shows that it is paved with white stones. If you step further in from the hall, there are two stories of the bedroom. The blinds in the doorway are still and it is profoundly quiet. The table in the room, when seen at a distance, is tidied in perfect order so that you will be tempted to enter. If you touch it, you will suddenly find it a wall. There was no technique of perspective representation in ancient times. Since it is so accurate as this, you will only regret that ancient people had not seen it. Hence this account.

A5. Juanqinzhai (Studio of Exhaustion from Diligent Service, sometimes called Lodge of Retirement), a 320 m² building in the north-eastern section of the Forbidden City, exemplifies and then goes far beyond an architectural style that reached its prime during the middle period of the Qing dynasty.

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