Current distribution and Hall potential landscape towards breakdown of the quantum Hall effect: a scanning force microscopy investigation

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Received 11 June 2014, revised 29 September 2014
Accepted for publication 15 October 2014
Published 26 November 2014
New Journal of Physics 16 (2014) 113071
doi:10.1088/1367-2630/16/11/113071

Abstract
We present line- and area-scans of the Hall potential landscape of a two-dimensional electron system (2DES) in narrow (AlGa)As-based Hall bars under quantum Hall (QH) conditions, obtained by low-temperature scanning force microscopy. For several magnetic field values $B$ in the regime of the QH plateau with Landau level filling factor $\nu = 2$, we measured the evolution of the Hall potential profiles and of the longitudinal voltage drop along the Hall bar as a function of increasing voltage/current bias, leading finally to the electrically induced breakdown of the quantum Hall effect (QHE). Basically two types of evolution were observed: for the low $B$-field side of the QHE plateau, two distinct Hall potential drops appear close to the two edges of a cross section, equally distributed at low bias but continuously developing to an asymmetrical distribution with increasing bias. At high bias, a steady increase of the longitudinal voltage drop is observed, accompanied by a rising slope of the Hall potential drop in the bulk. For the upper $B$-field side of the QH plateau, the Hall voltage drops are broadly distributed across the whole cross section, and the distribution remains almost unchanged until the bias reaches a critical value where the Hall potential profile changes rather abruptly, enhancing locally the Hall field. Beyond this, with further increase of the bias, a steep rise of the longitudinal voltage drop is detected. These findings are naturally explained in the microscopic picture of the QHE, based on the self-consistent evolution of the
compressible and incompressible landscape inside the 2DES with increasing bias.

Keywords: quantum Hall effect, breakdown of the QHE, incompressible stripes, scanning force microscopy, current distribution, Hall potential landscape

1. Introduction

Previous scanning probe microscopy experiments [1–6] have already uncovered interesting details about the Hall potential profiles and, therefore, the current distribution in narrow Hall bars under quantum Hall effect (QHE) conditions. Depending on the magnetic field, the current is distributed either close to the Hall bar edges or in the bulk of the Hall bar. The characteristic changes of the current distribution, which are observed while the magnetic field varies within a quantum Hall QH plateau, can be attributed to the presence and evolution of incompressible stripes starting from the electrostatic depletion regions at the edges of the two-dimensional electron system (2DES), as described by the model of Chklovskii et al [7, 8] and its self-consistent generalization to finite temperatures [9–11]. In contrast to the commonly used model of current carrying edge states for the QHE, incompressible regions extended along the Hall bar carry the dissipationless current, which is driven by the Hall voltage drops over the respective regions. Experimentally this has been confirmed on QH samples based on (Al,Ga)As heterostructures [12], and also on samples based on graphene flakes [13].

Incompressible and compressible regions appear in the 2DES under high magnetic field due to electrostatic screening of superposed electrostatic potential variations, which are naturally present, especially towards the 2DES edges. Transport calculations taking such screening into account and assuming local equilibrium in stationary non-equilibrium states [14–18] show good agreement with the experimental findings. Moreover, Güven et al [14] also analyzed the Hall potential profiles for higher bias and predicted, for the regime with current near the edges, an asymmetric current distribution at both edges accompanied by different widths of the incompressible stripes carrying the current.

The aim of the present work is to investigate how the current distribution in a sample exhibiting the QHE depends on the strength of the current through the sample or on the bias voltage between source and drain. Of special interest here is the regime of strong bias, approaching the electrical breakdown of the QHE.

2. Samples and electrical characterization measurements

The samples used here were made out of two different GaAs/Al$_x$Ga$_{1-x}$As heterostructures ($x = 0.33$), both with the two-dimensional electron system (2DES) at the heterojunction interface 55 nm below the surface. In this paper we show the results of two samples, denoted by A and B. The values for the electron concentration $n$ and electron mobility $\mu$, determined at 1

Layer sequence for sample A: on top of a semi-insulating GaAs substrate, a GaAs/AlGaAs superlattice 50x (10 nm, 10 nm) was grown, followed by 500 nm GaAs, 20 nm AlGaAs spacer, 25 nm AlGaAs doped with Si, 5 nm AlGaAs, and, finally, 5 nm GaAs as cap layer. For sample B, the AlGaAs spacer layer was 25 nm thick, the AlGaAs:Si doping layer 20 nm.
$T = 1.5\ \text{K}$ during the experiments presented here, are $n = 2.8 \times 10^{15}\ \text{m}^{-2}$, $\mu = 38\ \text{T}^{-1}$ for sample A and $n = 3.2 \times 10^{15}\ \text{m}^{-2}$, $\mu = 51\ \text{T}^{-1}$ for sample B.

The Hall bar layouts and the measurement setup for the electrical characterization are shown in figure 1. The cross sections at the contact regions are wider than at the Hall bar center to initiate electrical breakdown of the QHE at first in the Hall bar center area. There the layout dimensions are adjusted to the possible scan range of the scanning probe microscope (max. 20 m by 20 m). Both samples A and B incorporated the two Hall bar layouts, shown in figures 1(a) and (b), but due to technical reasons only one on each sample could be used for scanning the Hall potential landscape: all area-scans shown below were performed on a sample B/ Hall bar layout (b); i.e., electrical breakdown measurements and scanning probe microscopy investigations were performed in parallel on the same six-terminal Hall bar device. All other Hall potential measurements presented here were performed on sample A/ layout (a), while, in addition, electrical breakdown measurements were done on the adjacent six-terminal device (b), present on the same chip.

To characterize the samples in terms of the electrically induced breakdown, the longitudinal voltage drop $V_x$ versus applied source-drain bias voltage $V$ and magnetic field $B$ was measured. Figure 2 shows color-coded the absolute value of $V_x$ versus $V$ and $B$ in the regime of the QHE plateau of Landau level filling factor $\nu = 2$, measured at a temperature of 1.5 K on sample A. In addition, the black line in the figure indicates $|V_x|$ as a function of $B$ for a small value of $V$ and indicates, therefore, the position of the $\nu = 2$ QH plateau in the range $5.6\ \text{T} \lesssim B \lesssim 6.1\ \text{T}$. There is an asymmetry in the behavior around the magnetic field value of $B = 5.9\ \text{T}$. On the lower $B$-field side, the onset of dissipation with increasing source-drain voltage is smooth, while on the upper $B$-field side the onset seems to be rather abrupt. figure 3 shows selected line plots from the data of figure 2 for some magnetic field values. Apparently the onset of a longitudinal voltage drop, i.e., the breakdown of the QHE with increasing source-drain bias voltage, occurs for the red curves, taken at $B \geq 5.97\ \text{T}$, with a much steeper slope than for the green and black curves, taken at $B \leq 5.9\ \text{T}$. This quantifies the mentioned asymmetry in figure 2.
3. Hall potential measurements

The measurement technique used here to determine the Hall potential profiles was introduced previously by P Weitz et al in 1999 [2, 19]. A metallized and electrically connected tip at the end of an oscillating piezoresistive cantilever is scanned at about 50 nm height above the surface across a Hall bar. To avoid local electrostatic depletion of the 2DES, a 2DES/tip voltage is applied to compensate for the work function difference. The response of the cantilever is sensitive to local electrostatic potential changes caused by a modulation voltage applied to the sample. We used a modulation with rectangular voltage pulses with the base line at zero in order to avoid averaging over potential changes present at interim bias voltage values and

Figure 2. Color-coded absolute value of the longitudinal voltage drop $|V_x|$ measured between the potential probes (see figure 1(b)) on sample A as a function of source-drain bias voltage $V$ and magnetic field $B$. The black line shows $V_x$, measured for a bias of $V = 0.65$ mV, versus $B$.

Figure 3. Selected line plots from figure 2. The red curves are taken at the indicated magnetic field values above 5.9 T and show a rather abrupt onset of longitudinal voltage drop. The bias voltage threshold beyond which the onset is observed decreases with increasing magnetic field, i.e., with convergence towards the upper edge of the QH plateau. The green curves belong to $B$ values below 5.9 T, where the longitudinal voltage drop gradually increases with bias.
different current directions, which occurs with a sinusoidal modulation without offset [19]. Our modulation technique means turning on and off the source-drain bias and therefore mapping the local potential difference between the situations with and without bias. Scanning over the Hall bar width twice, two signal traces from the cantilever are taken: the first while modulating source and drain potential simultaneously (2DES without imposed current), and the second while modulating only the drain contact with the same modulation amplitude (2DES sees source-drain bias $V_{SD}$). Dividing both traces results in the normalized Hall potential profile, present in the Hall bar cross section for the applied source-drain bias [2, 19]. As we used only positive bias voltages, the normalized Hall potential ‘0’ corresponds to the ground potential, while a normalized value of ‘1’ corresponds to the applied positive bias voltage $V$. In the following section we show only data for one current direction, since we get consistent results for opposite polarity, obtained by interchanging the electrical connection to source and drain contact. The color code for presenting the Hall potential profiles figures 4(b) and (d) and in the appendix is chosen in such a way that blue means the source potential, which is kept zero, and red means the (positive) drain potential. The spatial resolution of these scans is about 0.1 μm [2], and the resolution of the Hall potential variations is about 2–5 mV.

We observed qualitatively the evolution of two types of Hall potential profiles, most clearly distinguishable at small source-drain bias: for the green and black plots of figure 3, corresponding to $B \leq 5.9$ T, the Hall potential profile is, even for high bias, nearly constant in

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2 The modulation frequency was between 1–3 Hz for the data shown in this paper.
the center of the 2DES, with steep rises near the edges. This can be seen in the representative Hall potential profiles taken at $B = 5.6$ T and shown in figures 4(a) and (b). (Further plots on the evolution of the Hall potential profile in this regime and for other magnetic fields can be found in figure A1 and in [20].) For the red plots of figure 3, corresponding to $B \geq 5.97$ T, on the other hand, the main variation of the Hall potential profile happens over the bulk of the 2DES. Representative plots of the Hall potential profile for this regime can be found in figures 4(c) and (d) (for more magnetic field values see figure A2). Therefore we want to denote the evolution vs. bias voltage shown in the green and black plots with $B \leq 5.9$ T as \emph{edge-dominated} and the evolution shown in the red plots with $B > 5.9$ T as \emph{bulk-dominated}. This classification fits naturally into the microscopic picture for the integer QHE [12, 18].

For the edge-dominated QHE, (see figures 4(a) and (b)), there is with increasing bias an increasing asymmetry in the Hall potential step between left and right edge so that the mean bulk potential continuously rises.\(^3\) In addition, a slope of the Hall potential in the 2DES bulk becomes visible at higher bias voltages. The widths of the Hall potential steps at the edges seem also to change. But since the changes are close to the spatial resolution limit, we do not analyze this in more detail.\(^4\)

Figures 5(c) and (d) show area scans of another sample (B) in the edge-dominated regime, appearing there at $B = 6.1$ T, due to the different electron concentration.\(^5\) (Area scans for additional bias voltage values can be found in figure A3 and in [20].) The corresponding trace of the longitudinal voltage $V_x$ over sample bias is plotted in green in figure 5(a). Apparently for low bias voltage ($V \lesssim 40$ mV), where the longitudinal voltage drop $V_x$ is small and the current is confined to the incompressible stripes near the edges, an equal potential profile (translation invariance) along the sample is a good approximation. But for larger bias, the QHE breaks down and the current spreads out over the bulk and changes its spatial distribution with the distance from the source contact.

In the bulk-dominated QHE, an increase of the source-drain bias has at first little effect on the Hall potential distribution, i.e., the drop happens broadly distributed over the 2DES bulk region (see figure 4(a) for line plots of sample A). But above a certain bias voltage threshold, a rather abrupt change of the Hall potential landscape is observed (best seen for $B = 6.1$ T (sample A) in figure 4(d) at about $V = 48$ mV). For the reverse current we find the same behavior but for slightly different bias (for $B = 6.1$ T at $|V| = 56$ mV).

A similar behavior is observed in sample B: in figures 5(e) and (f), area scans for sample B are shown in the bulk-dominated regime at $B = 6.8$ T \cite{comment3}. The onset of significant changes in the Hall potential landscape only after a bias voltage threshold can also be observed. The changes of the Hall potential landscape are abrupt and disordered, especially when compared to the changes for the edge-dominated regime. (The complete set of area scans can be found in the appendix, figures A3 and A4.)

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\(^3\) The initial asymmetry for low bias can be explained by intrinsic asymmetries of the edges.

\(^4\) The measurement time to acquire one line was 1300 s. The relevant parameters of the SR830 Lock-in amplifier were a time constant of 3 s, a filter slope of 24 dB, and the use of the sync filter option.

\(^5\) 77 neighboring line scans along the sample length were taken, each measured within 68 s and with a Lock-in amplifier time constant of 0.3 s, 24 dB filter slope, and activated sync filter.
4. Discussion

The characteristic structure of the measured Hall potential profiles is explained by the landscape of compressible and incompressible regions in the 2DES \[4, 5\], where relatively small source-drain biases were used to investigate the QHE. The current was found to flow in the incompressible stripes near the 2DES edges for magnetic fields at the low \(B\)-field side of the QH plateau. At the upper \(B\)-field side of the plateau, the incompressible stripes move towards the center of the 2DES, widen, and eventually merge, forcing the current to flow through the 2DES bulk. We find a similar behavior of the current distribution as a function of the magnetic field even for much higher biases, which leads us finally into the breakdown of the QHE. The evolution towards breakdown in the two regimes will be discussed separately in the following section.

**Figure 5.** Normalized Hall potential distribution for the Hall bar region depicted in (b), presented in (c) and (d) for the edge-dominated breakdown regime (sample B at magnetic field \(B = 6.1\ T\)), and in (e) and (f) for the bulk-dominated regime (sample B at magnetic field \(B = 6.8\ T\)). The corresponding longitudinal voltage drop is shown in (a) as the green trace for the edge-dominated and red trace for the bulk dominated breakdown regime. The applied bias voltage is increased from 20 mV to 80 mV. The current flows through the sample along the \(x\)-direction. From (c) to (d) the Hall potential profile along the device remains almost translation invariant. Only the bulk potential is increased. From (e) to (f) the Hall potential profiles show strong local changes, indicating the influence of disorder.
4.1. Edge-dominated breakdown

As already mentioned in the introduction, under QH conditions the current flows, essentially dissipationless, in the incompressible regions. For the low $B$-field side of the QH plateau, there is a well-developed incompressible stripe near both edges of a cross section of the Hall bar (see figure 6(a)). The intrinsic electric fields across the incompressible stripes in the $y$ direction lead to dissipationless, intrinsic diamagnetic Hall currents in the $x$ direction along the stripes, and the currents along stripes near opposite edges in a cross section of the Hall bar have the same magnitude but opposite directions. In the case of same edge properties, the bias current is equally distributed among the two incompressible stripes for low applied bias, as depicted in figure 6(b). In figure 4(a) this situation is observed for low bias and resembles former measurements [4, 5]. With increasing bias, i.e., with increasing current along the Hall bar, the distribution of this current on the two incompressible stripes and, consequently, the increase of the Hall potential drop across these stripes, becomes asymmetric, as predicted in [14]. As seen in the normalized Hall potential profiles in figure 4(a), the Hall potential step across the left stripe becomes larger with increasing bias, whereas the step at the right stripe becomes smaller, so that the bulk values of the normalized Hall potential increase. Figure 6(c) shows schematically the respective Landau level bending. We want to point out that the decrease of the
Hall voltage drop or, equivalently, the decrease of the electrochemical potential drop, always happens at the edge of high Hall potential (i.e., low electrochemical potential, independent of current and magnetic field direction, which was checked experimentally. Self-consistent calculations as described in [18], adapted to the present geometry, yield similar results, as is shown in figure 7 for \( B = 6.0 \) T (we consider only the \( \nu = 2 \) plateau and neglect spin splitting). They also show the total current through a stripe, i.e. the sum of intrinsic and imposed current, has the same direction as the intrinsic diamagnetic current through the stripe. It means that the intrinsic current is enhanced on one edge and reduced on the other edge by the imposed current but is never overcompensated, so that total current is reversed in the incompressible stripe (see figure 6(c)). As a consequence, the fraction of the imposed current, flowing through the stripe with an intrinsic current of opposite direction, is limited, and with increasing \( I \) most of the imposed current will flow through the stripe, where the intrinsic current has the same direction as \( I \). Since this stripe carries the larger amount of the imposed current, it leads to a larger increase of the Hall potential than the other stripe. Electrostatic arguments [7] and self-consistent calculations [18] also show the variation of the confinement potential across this stripe and its width increase with increasing imposed current, and that this stripe becomes the most relevant stripe. This explains the asymmetry observed in figure 7 and in the experimental curves of figure 4(a).

\[ \text{Figure 7. Self-consistent calculations of the Hall potential profile } V_H(y), \text{ normalized by the overall Hall voltage drop, in a cross section of a translation invariant Hall bar (of width } 2d = 3 \mu m, \text{ homogeneous donor density } n_D = 4 \cdot 10^{15} \text{ m}^{-2}, \text{ and mean electron density } n = 2.9 \cdot 10^{15} \text{ m}^{-2} \text{) as described in [18], for different values of the imposed current along the sample, from } I = 0 \text{ (linear response) to } I = 0.5 \mu A, \text{ and for two values of the magnetic field.} \]

6 Here a translation invariant Hall bar is assumed with an electron density that, in equilibrium, i.e., without imposed current (\( I = 0 \)), decreases symmetrically and monotonically from a maximum value in the center towards the 2DES edges. All charges and the confining gates are assumed to be located in the plane \( z = 0 \) [7, 8], and a constant positive background charge density is chosen so that at low temperatures (\( T \lesssim 4 \) K) and for \( B = 6.0 \) T, the \( n = 1 \) Landau level is partly occupied in the center [18].
Qualitatively, this asymmetry can also be understood by the following simple arguments: in thermal equilibrium at sufficiently low temperatures, intrinsic Hall currents of equal magnitude and opposite direction flow through the incompressible stripes near opposite edges of the sample. Their magnitude is determined by the electrostatic potential drop $V_{\text{cyc}} = \frac{\hbar \omega_c}{e}$ (we use the effective mass of electrons in the conduction band of GaAs and neglect spin splitting) across the stripes. An imposed (non-equilibrium) current $I = I_l + I_r$ splits into two components $I_l$ and $I_r$, flowing through the left and the right stripe, respectively, and creating there Hall voltages $V_l$ and $V_r$, which add to or subtract from the intrinsic voltage $V_{\text{cyc}}$, depending on the relative directions of imposed and intrinsic currents. According to the already mentioned electrostatic argument explained in [7], the width $w$ of an incompressible stripe is proportional to the square root of the electrostatic potential drop across this stripe, with a pre-factor depending on the electron density profile at $B = 0$. Assuming this profile to be symmetric, one obtains for the widths $w_l$ and $w_r$ of the left and the right stripe

$$\left[ \frac{w_r}{w_l} \right]^2 = \frac{V_{\text{cyc}} - V_l}{V_{\text{cyc}} + V_l}$$

provided the imposed current has the direction of the intrinsic current in the left stripe. If one now makes the naive assumption that the imposed current contributions $I_l$ and $I_r$, and, as a consequence, the corresponding Hall voltage drops $V_l$ and $V_r$, are proportional to the widths $w_l$ and $w_r$ of the hosting stripes, one obtains the relation $(V_l/V_r)^2 = (V_{\text{cyc}} - V_l)/(V_{\text{cyc}} + V_l)$, which determines the asymmetry. Denoting the total imposed Hall voltage by $V = V_l + V_r$, one obtains for the normalized voltages across the incompressible stripes

$$\frac{V_l}{V} = \frac{1}{2} + \sqrt{\frac{1}{4} + \left( \frac{V_{\text{cyc}}}{V} \right)^2} - \frac{V_{\text{cyc}}}{V}, \quad \frac{V_r}{V} = \frac{1}{2} + \frac{V_{\text{cyc}}}{V} - \sqrt{\frac{1}{4} + \left( \frac{V_{\text{cyc}}}{V} \right)^2}. \quad (2)$$

For large $V$ the drop $V_l$ approaches $V_{\text{cyc}}$ and therefore the right incompressible stripe gets the width zero (see relation (1)).

Drawing equation (2) as a function of $V_{\text{cyc}}/V$ results in two universal curves, independent of magnetic field or filling factor, describing the current asymmetry of the edge dominated regime in general. A comparison of this simple model with our measurements is given in figure 8, and we find an excellent agreement. For the comparison we used the fact that we do not measure any significant voltage drop over the ohmic contacts and thus can set the Hall voltage equal to the bias voltage. The excellent agreement between model and experiment is surprising in view of the numerical results, which show that the edge regions of incompressible stripes carry less current than their interior (see for current density profile in an incompressible stripe [18], figure 3). This would lead to $I_l/I_l < w_l/w_l$ and therefore to a stronger asymmetry in the Hall voltage drops than described by the relations (2).

Although the quoted theory [18] nicely explains the asymmetry observed in the experiment, it apparently has some shortcomings. The experimental Hall potential curves exhibit with increasing bias an increasing slope in the bulk region. This shows that not all of the imposed current can flow dissipationless through the incompressible stripes, which leads to the breakdown of the QHE. Some current is spread out into the compressible bulk, where it suffers dissipation. Several breakdown mechanisms have been discussed in the literature, such as Joule heating [21–23] and inter- [24, 25] and intra- Landau level transitions [26]. It seems necessary
to include these mechanisms in the model calculations. But at present we have no conclusive arguments to decide which of these mechanisms is the most important, although some facts seem to indicate the importance of Joule heating and quasi-elastic inter-Landau-level scattering (QUILLS) [18]. For instance, the calculated broadening of the relevant stripe with increasing bias is not clearly observed in the experiment. This may be due to the restricted resolution in our scanning probe experiment. But it may also be that increasing imposed current stimulates the QUILLS effect, and thus the breakdown of the QHE, since the electric confinement field across the relevant stripe increases faster than its width, so that the states of neighboring Landau levels with equal energy start to overlap [18].

4.2. Bulk-dominated breakdown

The dependence of the current distribution on the bias voltage in the bulk-dominated regime—found at the high $B$-field side of the QH plateau—is obviously different. Figure 4(c) shows that the normalized Hall potential profiles change only a little when the bias is increased up to 40 mV. Up to this bias value the QHE persists, as can be seen in the longitudinal resistance in figure 3, which has started to deviate at this point from zero.

Due to the microscopic picture of the quantum Hall effect derived from scanning probe experiments [12], incompressible strips evolve from the edges towards the bulk, merging to a widely incompressible bulk. Due to inhomogeneities in the electron concentration caused by disorder in the sample, the incompressible bulk region may, interrupted by compressible islands, spread over a large portion of the sample, as sketched in figure 9(a). Then the imposed current is widely spread in the incompressible regions of the bulk—a situation which dominates...
the high $B$-field side of the QH plateau. In figure 4(c), the Hall potential varies for low-bias values essentially over a length of about 80% of the Hall bar width. This is also shown schematically in figure 9(c). For larger bias voltage, where, according to figure 3, the QHE is broken down, a sudden change of the normalized Hall potential profile is observed: it is close to zero (constant) for about 40% of the sample width on the left side and then increases linearly towards the other side. Figure 9(d) sketches the Landau level bending in the cross section for this situation. At even higher bias (60 mV), the linear increase starts closer to the left side of the sample. The importance of inhomogeneities is emphasized by the area scans presented in figures 5(e) and (f) (the complete set can be found in figure A4 of the appendix), which show rather different potential landscapes in the breakdown area for bias voltages larger than 60 mV.

The model calculations presented with figure 7 include the effect of short range disorder in conductivity and Landau level broadening but no long range disorder. They yield that, in a very small range of magnetic fields, the incompressible stripes evolving from the edges merge to a rather narrow incompressible region in the center, which carries all the imposed current, as is shown in figure 7 for $B = 7.33$ T and in [18]. This is not in agreement with our experimental results, presented in figure 4(c). Apparently there is no translation invariance in the scans of figures 5(e) and (f), and therefore the mentioned model will not be able to explain these findings. But some hints for an understanding of the measurements in the bulk-dominated

Figure 9. (a) Sketch of the compressible and incompressible landscape within a bulk-dominated sample. Over a short length, the sample can be viewed as translation invariant. Schemes of Landau levels in the marked sample cross section are shown in (b)–(d). They are drawn to resemble qualitatively the situation in figure 4(d) for (b) no bias, (c) 16 mV, and (d) 50 mV bias. Incompressible regions are located, where the electrochemical potential drawn in red is positioned between the Landau levels. The empty, half filled, and fully filled circles represent the filling of the Landau levels. While (c) resembles the result of linear response theory, (d) deviates strongly. A dominant incompressible stripe evolves in the sample bulk, where the electric field is locally enhanced.
regime can be obtained from a simple generalization of this model and its application on regions with approximate translation invariance, as depicted in figure 9(a). To simulate density fluctuations, we have replaced in the self-consistent calculations the constant donor density $n_D = 4 \times 10^{15} \text{ m}^{-2}$ by the oscillating density $n_{\text{Don}}(y) = n_D[1 + 0.1 \cos(5\pi y/d)]$ and then performed the same calculations as for figure 7. Now we obtain an electron density profile with four relative minima and five relative maxima between $2.9 \times 10^{15}$ and $3.9 \times 10^{15}$ m$^{-2}$ (see upper panel of figure 10), which allows for $B \approx 6.6$ T up to ten incompressible stripes and therefore a current distribution over a wide part (more than half) of the bulk of the Hall bar. The dashed curves in figure 10 refer to $B = 6$ T, broken lines to $B = 7.33$ T. The imposed currents range from $I = 0$ (linear response) to $I = 0.5 \mu$A.

![Figure 10](image_url)

**Figure 10.** Upper panel: modulated donor density $n_{\text{Don}}(y) = n_D[1 + 0.1 \cos(5\pi y/d)]$ and resulting electron density for $B = 0$, $T = 0$ (red lines), and corresponding densities without modulation (black lines). The horizontal blue lines indicate the densities $n_T(B)$, which lead to filling factor $\nu = 2$ for $B = 6$ T and $B = 7.33$ T, respectively. Lower panel: as in figure 7, but for the modulated donor density model. Solid lines refer to $B = 6$ T, broken lines to $B = 7.33$ T. The imposed currents range from $I = 0$ (linear response) to $I = 0.5 \mu$A.

These model calculations demonstrate that density fluctuations in the bulk have little effect on the edge-dominated regime (comparing figure 7 with figure 10) but may become very important in the bulk-dominated situations. The calculations also show that it depends on the details of the confinement potential, which incompressible stripe becomes most relevant for a large imposed current. In real samples, the density fluctuations are of course not translation invariant, and the landscape of compressible and incompressible regions may become very complicated. However, it seems likely that self-consistent feedback effects, which in our model calculations lead to a broadening of just one of several incompressible stripes, lead in real
samples to a current distribution that depends on the strength of the imposed current. Such screening-mediated feedback effects may be the reason for the bias-dependent inhomogeneities of the Hall potential landscape seen in the area scans of figures 5(e) and (f).

The model calculations and the fact that disorder dominates the sample in this regime lead us to the idea that the breakdown can occur in a very narrow region of the sample. An analysis of our area-scan data revealed such a region, which is shown in line-cuts in figure 11. There the increasing the bias does not change the normalized Hall potential profile until above a critical bias voltage—a relevant portion of the total current is driven through the marked narrow region. With increasing bias beyond the critical value, a significant longitudinal voltage drop $V_x$ is measured.

5. Conclusion

With this work we have extended previous scanning force microscope experiments on narrow QH samples to higher source-drain bias, which allows us to investigate the electrically induced breakdown of the QHE. In contrast to our previous experiments at low source-drain bias, where we had used a sinusoidal modulation, here a modulation technique was applied, which switches only between the situations without and with a certain source-drain voltage bias. Thereby we avoid the averaging over the different Hall potential landscapes of the 2DES, which appear in the case of continuously varying the source-drain bias strength and switching the bias polarity.

In accordance with our previous results at low bias, we found different types of Hall potential profiles on the higher and the lower magnetic field side of the QH plateau, and therefore different evolutions of the Hall potential profiles versus bias voltage in these plateau regimes.

In the edge-dominated Hall plateau regime, where the imposed current flows dissipationless in incompressible stripes near the edges of the 2DES, an increase of the imposed bias leads to an increasingly asymmetric distribution of the imposed current between the left and the right side of the Hall bar. This behavior is explained by the self-consistent theory of electrostatic screening present in the QH sample [18]: the nearly dissipationless imposed
current adds to the intrinsic current at the incompressible stripe of the edge with lower Hall potential (i.e., higher electrochemical potential energy) and subtracts from the intrinsic current in the other incompressible stripe. The variation of the total, screened electrostatic confinement potential across such an incompressible stripe is proportional to the total current through this stripe, whereas the variation of the Hall potential, i.e., of the electrochemical potential, across such a stripe is proportional to the imposed current through this stripe. Since the variation of the total potential across an incompressible stripe cannot change its sign, and the width of the stripe decreases with (the square root of) this variation, this limits the total variation of the Hall potential through the narrow stripe to \( \Delta \omega = \frac{\eta}{e} \) and explains the asymmetry of the imposed current density.

As a result of the asymmetry, in addition to Joule heating, which is always present at finite temperatures, different breakdown processes may become possible across the narrow and the broad incompressible stripes. Across the narrow stripe, electrons from the bulk region may relax to states with slightly lower energy in the outer region, e.g., by phonon and/or photon emission, which would be consistent with observations by Ikushima et al [27, 28] on highly biased QH samples. In the wide stripe, the electric field increases with increasing bias stronger than the width of the stripe so that the spatial distance of occupied and unoccupied states of the same

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**Figure A1.** Measurements on sample A for the edge-dominated breakdown regime. (a) Schematic of the sample and the scan line position. (b)–(f) Color-coded normalized Hall potential profiles across the width at the Hall bar center, measured for increasing bias voltage at \( B \) values corresponding to the green and black curves in figure 3. To be comparable, all color-coded Hall potential plots have the same bias voltage scale from 10–60 mV.

energy in adjacent Landau levels decreases. When this distance approaches the typical extent of Landau wave functions, the breakdown mechanism of QUILLS becomes possible [25]. Such peculiar breakdown mechanisms may become important at low temperatures, where the longitudinal conductivity and thus the dissipation in the ‘incompressible’ stripes becomes exponentially small, and, therefore, Joule heating becomes ineffective. Neither of these breakdown mechanisms is included in the model calculations presented here. Nevertheless, from the evolution of the Hall potential profiles with increasing bias it is obvious that the breakdown of the QH effect evolves smoothly, as seen in the experiment by a gradual increase of the longitudinal voltage drop along the Hall bar and also by a gradual increase of a Hall voltage drop in the compressible bulk of the 2DES.

At the upper magnetic field side of the QH plateau, the imposed current flows widely distributed in the bulk of the 2DES, avoiding compressible islands that are embedded due to inhomogeneities in the electron density of the 2DES. In this bulk-dominated situation, the normalized Hall potential profile remains unchanged with increasing bias until beyond a certain bias, a rather abrupt change of the Hall potential is observed, leading to a large Hall voltage drop over a small region. A further increase leads to the breakdown of the QH effect, seen by a steep rise of the longitudinal voltage drop along the Hall bar. Scanning the Hall potential landscape over large areas of the Hall bar shows that enhanced Hall fields appear locally somewhere in the Hall bar, indicating that the inhomogeneities in the electron density are now

\[ \text{Figure A2. Color-coded normalized Hall potential profiles as a function of increasing bias voltage for sample A and for the magnetic field values corresponding to the red curves in figure 3.} \]
important. Model calculations, based on a self-consistent local-equilibrium theory of magneto-transport and screening, indicate that these findings result from a complicated interplay of sample inhomogeneities and a non-linear feedback of the imposed current on the electron density, in particular the spatial distribution of incompressible and compressible areas inside the 2DES. As the incompressible areas are not all identical in width, and in some the intrinsic electrostatic potential drop goes with—in the others, against—the Hall potential drop, they evolve differently with increasing bias, leading finally to the situation that the imposed current is concentrated into one incompressible stripe, accompanied by large Hall fields and triggering the electrically induced breakdown.

Acknowledgments

The authors thank Maik Hauser and Werner Dietsche for providing us with (Al,Ga)As heterostructures and Hans Boschker for suggestions for the manuscript.
Appendix

In the following section we show data for intermediate magnetic field values compared to figure 4, as well as additional bias voltages compared to figure 5. Figure A1 demonstrates the conformity of the breakdown transition for the edge-dominated breakdown regime. For the bulk-dominated regime (figure A2), the breakdown transition happens differently. The area scans presented in figure A3 for the edge-dominated regime show the gradual change of the Hall potential profiles for the entire scanned area. In contrast to the bulk-dominated breakdown regime shown in figure A4, changes in the potential landscape are visible only after a certain bias voltage and are drastic compared to the edge-dominated regime.

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