Fundamental collapse in cellular automaton process

To cite this article: Yehuda Roth 2019 J. Phys. Commun. 3 045002

View the article online for updates and enhancements.
Fundamental collapse in cellular automaton process

Yehuda Roth
Oranim Academic College, Q. Tivon 36006, Israel
E-mail: yudroth@gmail.com

Keywords: Distinguishable-cells basis of states, rules states, fundamental collapse, artificial inelegance, quantum cellular automaton

Abstract
In this paper, we show that quantum states and operators are well-adapted to describe a cellular automaton (CA) process. Using quantum mechanical terminology, we reveal a fundamental collapse that is naturally embedded in the CA evolution. We further suggest that our formalism can be useful in the field of artificial intelligence.

1. Introduction
A cellular automaton (CA) is a discrete mathematical model that consists of a grid of cells in which each cell is associated with a state (e.g. 0 or 1) and is influenced by the states of its neighbors through a CA rule or set of rules. Typically, the rules for updating the state of a cell is the same for all cells [1, 2].

Integrating quantum mechanics and CA processes into quantum CA (QCA) is very useful to the field of physics. In nanotechnology, CA is one of the top-six emerging technologies having the potential to support building future quantum computers [3]. Here, we briefly review two QCA theories: a cavity with two-level atoms [4] and a quantum-dots construction [3].

In the cavity system, a chain of N two-level atoms is analyzed using CA methods, where each atom is treated as a single cell. This method allows systems to be handled with a large number of particles, such as quantum-optics dissipative systems.

In the quantum dots model, the states are represented by the positions of electrons in a quantum nano-size dot [5]. Usually, a cell comprises four dots, with two mobile electrons that tunnel between neighboring dots. Considering a cell with dots located at the corners of a square, owing to Coulombic repulsion, the electrons tend to occupy the antipodal dots of the cell. However, they never occupy dots that are along the same edge. The two diagonal cites thus generate two stable polarizations. In QCA, these two polarizations provide a binary 1 or 0. The cells exchange information via Coulombic interactions acting among them. This process generates the QCA rule, wherein neighboring cells have the same polarization.

In previously studied examples, states were designed to obey QCA rules. In this paper, we extend the QCA basis of states to those that obey the CA rule and to those that can violate it. We show that, under these conditions, the corresponding Hilbert space is complete. Moreover, we interpret two types of state superpositions. To complete the quantum picture, we reveal a fundamental collapse that mathematically resembles that of quantum mechanics.

2. CA processes expressed in quantum terminology
Consider an automaton that is a one-dimensional grid of cells labeled i, where each can be turned on or off. In our examined model, the evolution of the system is determined by the following rules [6]:

- **Rule 1:** The i-th cell will be on if the neighboring cells are in different states.
- **Rule 2:** The i-th cell will be off when both neighbors are in the same state.
We assume that some concepts of quantum computers are relevant to allow the implementation of a qubit [7]. Thus, the ith cell can assume the states, $|0\rangle_i$ or $|1\rangle_i$, which are identified with the off and on commands, respectively. To implement the CA rule, we define local spaces associated with the states of the ith cell and its neighbors, the $i - 1$th and $i + 1$th cells. For this space, we introduce two types of spanning sets: the distinguishable-cells basis of states and the rules states.

3. Distinguishable-cells basis of states

Following conventional quantum mechanics, the distinguishable basis comprises the following $2^3 = 8$ products of states:

\[
|j\rangle = |\psi_i\rangle_i |\psi_{i+1}\rangle_{i+1} |\psi_{i-1}\rangle_{i-1},
\psi = 0, 1, \quad j = 1, 2, \ldots, 8.
\] (1)

Next, the projection operators are introduced.

\[
P_0 = P_{i+1}(1)P_{i-1}(1) + P_{i+1}(0)P_{i-1}(0)
\]

\[
P_1 = P_{i+1}(1)P_{i-1}(0) + P_{i+1}(0)P_{i-1}(1).
\] (2)

The corresponding observable, $S$, which checks rules validity ($S$ for the separated basis) is

\[
S^{(i)} = \sqrt{\{P_0 P_i(\psi = 1) + P_1 P_i(\psi = 0)\}}
+ \sqrt{\{P_0 P_i(\psi = 0) + P_1 P_i(\psi = 1)\}}.
\] (3)

Note that, instead of defining numerical eigenvalues as the measurement result, we present the concept of ‘eigenstrings,’ denoted by $\sqrt{\cdot}$ and $\sqrt{x}$, meaning obeying or violating the rules in the ith site, respectively. As a demonstration, $S_i$ when implemented on the obeying-the-rules and disobeying-the-rules states, $|1\rangle_i |0\rangle_{i+1} |1\rangle_{i-1}$ and $|1\rangle_i |0\rangle_{i+1} |0\rangle_{i-1}$ are used to obtain

\[
S^{(i)} |1\rangle_i |0\rangle_{i+1} |1\rangle_{i-1} = \sqrt{1} |1\rangle_i |0\rangle_{i+1} |1\rangle_{i-1}
S^{(i)} |1\rangle_i |0\rangle_{i+1} |0\rangle_{i-1} = \sqrt{1} |1\rangle_i |0\rangle_{i+1} |0\rangle_{i-1}.
\] (4)

As in quantum mechanical formalism: given the appropriate states, violating or obeying state rules serve as eigenstates of $S^{(j)}$ with an associated eigenstring. An observer who analyzes the entities of $S^{(j)}$ may run through all 8 states of the distinguishable basis to obtain a table of data that determines which state obeys or violates the rule. This does not mean that the observer will reveal the cellular automaton rules. According to our terminology, all concepts are generated by biological-measuring devices embedded in the observer’s mind. In particular, the terms the same and not the same are the core concepts of CA rules. The measurements, as conducted by $S^{(j)}$, describe the state of each cell and make the relative concepts the same or not the same.

4. Rules states

In contrast to the distinguishable basis of states in which the state of each ith cell is well-defined, rule states define the CA rules rather than the specific state of the ith cell. To demonstrate the difference between these two sets, consider Rule 2 of section 2, which states, ‘an i-cell will be off when both neighbors are in the same state’. Mathematically speaking, Rule 2 is fulfilled if both neighbors are at $|0\rangle_i$ or $|1\rangle_i$-states. Thus, even if the observer knows that the ith cell is in the $|0\rangle_i$ th state, the neighboring states that are either on or off remain undefined.

Under the distinguishable basis, we have 8 states generated by three cells that can be on or off. However, a rough glance at the rules reveals only four possible states:

- The rules-obeying states–
  i. The ith cell is off; both cells are in the same state
  ii. The ith cell is on; the cells are in different states

- The rules violating states–
  i. The ith cell is on; both cells are in the same state
  ii. The ith cell is off; the cells are in different states

Next, we will see that the missing states are correlated to relative phases.
The neighbor’s relative states can be described as teleportation-style entangled states [8, 9].

- **States for Rule 1**: The neighboring cells are in opposite states,
  \[ |x_i, \pm \rangle \equiv \frac{1}{\sqrt{2}} (|0\rangle_{i-1}|1\rangle_{i+1} \pm |1\rangle_{i-1}|0\rangle_{i+1}). \tag{5} \]

- **States for Rule 2**: Both neighboring cells are in the same state,
  \[ |-, \pm \rangle \equiv \frac{1}{\sqrt{2}} (|0\rangle_{i-1}|0\rangle_{i+1} \pm |1\rangle_{i-1}|1\rangle_{i+1}). \tag{6} \]

Observing equations (5) and (6), we find that the number of states is doubled because of a new variable, ± relative phases, such that the dimensionality of the Hilbert space is conserved. However, we enact a rule for handling phases, such as when handling \(|x, +\rangle\) with respect to \(|x, -\rangle\). Doing nothing is also allowed, and this is the approach we adapt. Obeying or violating the rules is determined only using relative statements, like ‘the same’ or ‘different,’ regardless of the relative ± signs. This yields the definition of the *rule basis of states*,

\[ |\sqrt{\pm} \rangle = |0\rangle_i \cdot |x_{\pm} \rangle, \quad |\sqrt{\pm} \rangle = |1\rangle_i \cdot |x_{\pm} \rangle, \]
\[ |x_{\pm} \rangle = |0\rangle_i \cdot |\mp \rangle_i, \quad |x_{\pm} \rangle = |1\rangle_i \cdot |\mp \rangle_i, \]

with the observable (\(\mathbb{E}\) for the entangled states):

\[ \mathbb{E}(\rho) = \sqrt{| \langle x_{\pm} | \rho | x_{\pm} \rangle |} + \langle x_{\pm} | \rho | x_{\pm} \rangle \]
\[ + \langle x_{\pm} | \rho | x_{\pm} \rangle + \langle x_{\pm} | \rho | x_{\pm} \rangle. \]

Mathematically speaking, we can say that \(\mathbb{E}\) is degenerated under the relative ±-phases.

### 5. Fundamental collapse in a local space

#### 5.1. Analogy to the XOR and XNOR gates

Measurement and, in particular, the collapse, is an irreversible process. Before we introduce the fundamental collapse concept and its implementation with CA, we begin with an example from computer science that demonstrates the relationship between the lack of information and irreversible processes.

In computer science, the XOR gate represents the inequality function [10]. It receives two bits of input and processes them into a single-bit output. The true-value, 1, is obtained for different inputs, whereas, if both inputs are the same, the output will be false (0). Logically, XOR simulates a \(\checkmark\)-eigenstring result in the CA process. To complete the analogy, the XNOR gate that logically acts oppositely to the XOR-gate simulates a process that violates the cellular automaton rule and, thus, it represents the \(\times\)-eigenstring.

In computer science, XOR and XNOR are recognized as reversible operators in the sense that there is no loss of information. However, it is irreversible if and only if one knows the value of at least one of the input bytes. If this information is lost, then the XOR and XNOR gates are irreversible. The same happens in quantum computers where the XOR gate is represented by the function [10],

\[ \text{XOR}(X) = a_1 |0\rangle_i |0\rangle_o + a_2 |0\rangle_i |1\rangle_o + a_3 |1\rangle_i |0\rangle_o + a_4 |1\rangle_i |1\rangle_o, \tag{9} \]

where the first \(i\)-bit is reserved for the input, and the \(o\)-th bit is the XOR-gate result. Unitarity is obtained if and only if the \(i\)-bit is conserved in a kind of ‘before-the-gate-was-activated’ memory. This unitarity is lost when we exclude the input knowledge to write the XOR operation:

\[ \text{XOR}(X) = a_1 |+\rangle_i |0\rangle_o + a_2 |\rangle_i |1\rangle_o, \tag{10} \]

where \(|\pm\rangle\) represents the relative phase of the input bits. The same argument holds for the XNOR gate to obtain the quantum operation:

\[ \text{XNOR}(X) = b_1 |\rangle_i |0\rangle_o + b_2 |+\rangle_i |1\rangle_o. \tag{11} \]

Note that the states of equations (10)–(11) are orthogonal, meaning that they can both be included under the same Hilbert space. Consequently, they can be joined to form the \(\mathbb{E}\) operator of equation (8), where the XOR and XNOR gates are associated with the \(\checkmark\) and \(\times\) parts, respectively. Suppose that we have XOR and XNOR operating gates that simulate a CA local space whenever a gate is evoked, or when a red or green light is turned on. If an observer detects, for example, a green light with a numerical output, 1, it can only be known that both inputs were in different states during a process that fulfills the CA rule. This is an irreversible process.
5.2. Fundamental collapse
In physics, there are fundamental concepts that are shared by other fields and disciplines, such as entropy, energy, and matter. We suggest that the collapse also be considered as a transferable fundamental concept. Since it was first introduced by Heisenberg [11], the quantum collapse phenomenon has drawn a great deal of attention from physicists [12, 13] who seek to understand the physical mechanism behind the phenomenon. When extending the definition to other fields and disciplines, we must understand that the term is already recognized as an exclusive feature of quantum mechanics. Therefore, our extended definition refers to the term, fundamental collapse. For the definition, we use the same mathematical tools as those in quantum mechanics theory. However, a fundamental collapse needs no specific explanatory mechanism.

A fundamental collapse is the output of a measurement conducted by a conceptual (or real) observer where the measurement is characterized as follows:

- The measurement is an irreversible process.
- Mathematically speaking, a measuring device entails projecting operators generated by a specific basis of states that span a Hilbert space. Each projecting operator is multiplied by an ‘eigenfeature,’ such as a number, a string, or a symbol that may represent neural activity. When an observer is incapable of measuring a state, the ‘eigenfeature’ will simply be the numerical number, 0. Thus, even if that is some conceptual complete basis of states, the observer may be incapable of measuring some or even most of them. Notice that this process may start with a large amount of data and terminate almost instantaneously with a single value. In the following, we show that our example of a cellular automaton process exhibits fundamental collapse behavior.

5.3. Fundamental collapse in CA process
Suppose that the observer detects the $i$th cell in a state, $|0\rangle$, with a $\check{\mathbf{\bigcirc}}$ as the eigenstring. This means that both neighboring states are the same. Suppose that $E^{(i)}$, is on a system with the $i$th cell and its neighbors. Thus, by using the rules states, the observer comprehends each cell state. Consider an observer who measures rule validity. Applying $E^{(i)}$ to $|1\rangle|0\rangle_{i-1}|1\rangle_{i+1}$, the observer obtains the result,

$$E^{(i)} |1\rangle|0\rangle_{i-1}|1\rangle_{i+1} = \frac{1}{2} |\check{\mathbf{\bigcirc}} \neq \check{\mathbf{\bigcirc}}\rangle.$$  \hspace{1cm} (12)

On the one hand, $|1\rangle|0\rangle_{i-1}|1\rangle_{i+1}$ is not an eigenstate of the operator $E^{(i)}$, disqualifying it from serving as a measurable state to detect the rule fulfilment. On the other hand, the neighboring states, $|1\rangle|0\rangle_{i-1}|1\rangle_{i+1}$, are in different states, meaning that the result should indeed be a $\check{\mathbf{\bigcirc}}$. Whereas $|1\rangle|0\rangle_{i-1}|1\rangle_{i+1}$ is not an eigenstate of $E^{(i)}$, by obeying a fundamental collapse rule, the system terminates at one of the states, $|\check{\mathbf{\bigcirc}}\neq \check{\mathbf{\bigcirc}}\rangle$. Generally, we can say that, in the $E^{(i)}$, measurement, we always obtain the $\check{\mathbf{\bigcirc}}$ or $\check{\mathbf{\bigcirc}}$ states either as eigenstates or via a fundamental collapse.

6. Global CA space
To maintain a cellular automaton process with a series of iterations, we define local operators that maintain rule-obeying states and flip the rule-violating states. Thus, when summing over all sites in the system, the operator ‘fixes’ the cells to ensure that they will all follow the rule. We then observe the states after a large number of iterations. An example of a local operator is

$$U_i = \sum_{k=0}^{1} |\check{\mathbf{\bigcirc}}_{k+1}\rangle \langle \check{\mathbf{\bigcirc}}_{k+1} | + \sum_{k=0}^{1} |\check{\mathbf{\bigcirc}}_{k-1}\rangle \langle \check{\mathbf{\bigcirc}}_{k-1} | + \sum_{k=0}^{1} |\check{\mathbf{\bigcirc}}_{k+1}\rangle \langle \check{\mathbf{\bigcirc}}_{k-1} | + \sum_{k=0}^{1} |\check{\mathbf{\bigcirc}}_{k-1}\rangle \langle \check{\mathbf{\bigcirc}}_{k+1} |.$$  \hspace{1cm} (13)

In this operator, the $\pm$ -relative phase is conserved.

To put into action all cells in the CA’s iterations, we sum over $i$ the $U_i$ in use:

$$E^{(i)} = \sum_{i=2}^{N-1} \alpha_i U_i,$$ \hspace{1cm} (14)
with $N$ being the number of the cell. The superscript, $(1)$, indicates a single iteration, and the set, $\{\alpha_i\}$, contains coefficients indicating the relative contribution of each cell. The index, $i$, goes from $2$ to $N - 1$ and includes only those cells that have neighbors on both sides.

Whereas we cannot distinguish between different cells’ states, those of the $i$th cell and its neighbors can be applied distinguishable ‘true–false’ states.

$$|\Psi_{ij}^{(0)}\rangle = \prod_i |\psi_i\rangle,$$

(15)

where the possible local states, $|\psi\rangle$, are defined by the true–false basis of equations (7). The subscript, $j$, counts the number of product combinations, and the superscript, $(0)$, indicates that the state experiences no iteration. At each iteration of $U^{(1)}$, each local operator, $U_j$, operates on its local $i$th zone, regardless of the other local operators. Therefore, we obtain

$$|\Psi_{ij}^{(1)}\rangle = U^{(1)} |\Psi_{ij}^{(0)}\rangle = \sum_j A_{ij}^{(1)} |\Psi_{ij}^{(1)}\rangle,$$

(16)

where $A_{ij}^{(1)}$ are the expansion coefficients after the first iteration. For $n$ iterations, we apply $U^{(1)}$ $n$ times to obtain the recursion relation.

$$|\Psi_{ij}^{(n)}\rangle = U^{(n)} |\Psi_{ij}^{(n-1)}\rangle = \sum_j A_{ij}^{(n)} |\Psi_{ij}^{(n-1)}\rangle,$$

(17)

with the appropriate coefficients, $A_{ij}^{(n)}$. Observing the composed state superposition, as in equation (17), we can see that each temporary state is a result of all changes that happened to the local states during previous iterations. Thus, we can say that each state, $|\Psi_j\rangle$, of equation (17), defines a concept comprising states generated during the history of the cellular automaton process. In our formalism, the states are not necessarily normalized, and they are not orthogonal.

### 7. Conclusions

In this paper, we presented a CA process consisting of a finite number of cells in a linear array using the terminology of quantum mechanics. We evoked a fundamental collapse scenario that mathematically resembles those of quantum mechanics. It was shown that a defined rule determining the cellular automaton process also corresponds to entangled states that associate each local space with a spanning set (see equations (7)). Consequently, if we observe a cell and its two neighbors with a ‘defined rule–basis of states,’ then, whereas we will find a defined state for that cell, we will also have complete uncertainty about the states of the two neighbors. If we insist on knowing the precise state of the three cells simultaneously, we will lose any knowledge concerning the CA rule. We believe that our formalism can provide a mathematical platform for considering advanced intelligent computers. By designing a measuring device that measures the rule States, we can envision computers that can not only perform logical operations, but also verify if these operations are correct.

Selecting a measuring device introduces a concept, such as the momentum or location, where different measuring devices correspond to state superpositions between the spanning sets. Leveraging equation (17), we observed that, during the history of the QCA process, states of the $n$th iteration came to comprise the superposition of states of the $n-1$th iteration. Therefore, not only can our sophisticated computer verify the validity of the rules, via the history of the operation, it can generate new concepts.

### ORCID iDs

Yehuda Roth https://orcid.org/0000-0002-0968-3245

### References

[1] Wolfram S 1983 Rev. Mod. Phys. 55 601–44
[2] Schiferl J 2011 Cellular Automata: A Discrete View of the World (Hoboken, New Jersey: Wiley)
[3] Zohreh B and Shahidinejad A 2014 Rev. Theor. Sci. 2 1–9
[4] Walczak M and Leonski W 2003 Fortschr. Phys. 51 186–9
[5] Harrison P 2005 Quantum Wells, Wires and Dots: Theoretical and Computational Physics of Semiconductor Nanostructures (Hoboken, New Jersey: WileyInterscience) (https://doi.org/10.1002/0470010827)
[6] De Oliveira P B and Verardo M 2014 The Mathematica Journal 16 1–20
[7] Mermin N D 2007 Quantum Computer Science (Cambridge: Cambridge University Press)
[8] Peres A and Wootters W K 1991 Phys. Rev. Lett. 66 1119
[9] Braunstein S L, Mann A and Revenz M 1992 Phys. Rev. Lett. 68 3259
[10] Nielsen M and Chuang I 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[11] Heisenberg W 1927 *Z. Phys.* 43 172–98
[12] Ghirardi G C, Rimini A and Weber T 1986 *Phys. Rev.* D 34
[13] Bell J S 2004 *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press)