Chiral quark models and their applications *

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We give an overview of chiral quark models, both for the pure light sector and the heavy-light sector. We describe how such models can be bosonized to obtain well known chiral Lagrangians which can be inferred from the symmetries of QCD alone. In addition, we can within these models calculate the coefficients of the various pieces of the chiral Lagrangians. We discuss a few applications of the models, in particular, $B - \bar{B}$ mixing and processes of the type $B \to D\bar{D}$, where $D$ might be both pseudoscalar and vector. We suggest how the formalism might be extended to include light vectors ($\rho, \omega, K^*$), and heavy to light transitions like $B \to \pi$.

I. INTRODUCTION

While the short distance (SD) effects in hadronic physics are well understood within perturbative quantum chromodynamics (pQCD), long distance (LD) effects have been hard to pin down. Even if quark models are not QCD itself, various QCD inspired quark models have been useful to make predictions for a limited class of problems. Lattice QCD and QCD sum rules are on more solid ground theoretically, but are in various cases not so easy to apply. In the light quark sector, low energy quantities have been studied in terms of the (extended) Nambu-Jona-Lasinio model (NJL)\cite{NJL}, and also the chiral quark model ($\chi$QM)\cite{chiQM}, which is the mean field approximation of NJL.

Within the $\chi$QM, the light quarks ($u, d, s$) couple to the would be Goldstone octet mesons

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\((K, \pi, \eta)\) in a chiral invariant way, such that all effects are in principle calculable in terms of physical quantities and a few model dependent parameters, namely the quark condensate, the gluon condensate, and the constituent quark mass \([3, 4, 5]\). More specific, one may calculate the coupling constants of chiral Lagrangians by integrating out the quarks by means of the \(\chi\)QM. In this way chiral quark models bridge between pQCD and chiral perturbation theory (\(\chi PT\)) as indicated in Fig. 1.

The ideas from the chiral quark model of the pure light sector \([2, 3, 4, 5]\) has been extended to the sector involving a heavy quark \((c\) or \(b)\) and thereby to heavy-light mesons \([6]\). Such models we name heavy-light chiral quark models (HL\(\chi\)QM). Also in this case, one may integrate out the light and heavy quarks and obtain chiral Lagrangians involving light and heavy mesons \([7]\). That is, we calculate the parameters of chiral Lagrangian terms, where the description of heavy mesons are in accordance with heavy quark effective field theory (HQEFT) \([8]\). In our approach \([9]\) we extended the formalism of \([6]\) to include gluon (vacuum) condensates.
One important motivation for the inclusion of gluon condensates is the possibility to estimate non-factorizable (colour suppressed) contributions in non-leptonic decays. For instance, \( K - \bar{K} \)-mixing and the \( \Delta I = 1/2 \) rule for \( K \to 2\pi \) can be understood in a reasonable way within the \( \chiQM \) \cite{4, 5} including gluon condensates. Especially, the suppression of the \( I = 2 \) amplitude found for \( K \to 2\pi \) is also in agreement with generalized factorization \cite{10}. Furthermore, it allows us for instance to consider decays where the gluonic aspect of \( \eta' \) is relevant \cite{11}, and some aspects of \( D \)-meson decays \cite{12}. The most important application is to calculate non-factorizable contributions to \( B - \bar{B} \)-mixing \cite{13}, where our approach includes \( 1/m_b \) corrections and chiral corrections both from loops and counterterms. Also processes of the type \( B \to D\bar{D} \) are calculable \cite{14}. It should be emphasized that the HL\( \chiQM \) can not, -in its present form, be used for heavy to light transitions like \( B \to \pi K \), where QCD factorization \cite{15} or soft collinear theory(SCET) \cite{16} is often applied. Still, in the last section, we suggest how an extension to this case might be performed. We also suggest how the \( \chiQM \) might be extended to include light vectors (\( \rho, \omega, K^* \)).

II. CHIRAL PERTURBATION THEORY

A. The pure light sector

Quarks are the fundamental hadronic matter. However, the particles we observe are those built out of them: baryons and mesons. In the sector of the lowest mass pseudoscalar mesons (the would-be Goldstone bosons: \( \pi, K \) and \( \eta \)) the interactions can be described in terms of an effective theory, the chiral Lagrangian, that includes only these states. The chiral Lagrangian and chiral perturbation theory (\( \chiPT \)) \cite{17, 18} provide a faithful representation of this sector of the Standard Model after the quark and gluon degrees of freedom have been integrated out. The form of this effective field theory and all its possible terms are determined by \( SU_L(3) \times SU_R(3) \) chiral invariance and Lorentz invariance. Terms which explicitly break chiral invariance are introduced in terms of the quark mass matrix \( \mathcal{M}_q \).

The strong chiral lagrangian is completely fixed to the leading order in momenta by symmetry requirements and the Goldstone boson’s decay amplitudes:

\[
\mathcal{L}^{(2)}_{\text{strong}} = \frac{f^2}{4} Tr \left( D_\mu \Sigma D^\mu \Sigma^\dagger \right) + \frac{f^2}{2} B_0 Tr \left( \mathcal{M}_q \Sigma^\dagger + \Sigma \mathcal{M}_q^\dagger \right),
\]

where the covariant derivative \( D^\mu \) contains the photon field, and \( \mathcal{M}_q = \text{diag}[m_u, m_d, m_s] \).
The quantity $B_0$ is defined by $\langle \bar{q}_i q_j \rangle = -f^2 B_0 \delta_{ij}$, where

\[(m_s + m_d) \langle \bar{q} q \rangle = -f_K^2 m_K^2, \quad (m_u + m_d) \langle \bar{q} q \rangle = -f_\pi^2 m_\pi^2, \tag{2}\]

in the PCAC limit. The quantity $\langle \bar{q} q \rangle$ is the quark condensate, being of order $\langle -240 \text{ MeV} \rangle^3$.

The $SU_L(3) \times SU_R(3)$ field $\Sigma$ contains the pseudoscalar octet $\Pi$:

$$\Sigma \equiv \exp \left( \frac{2i}{f} \Pi \right), \quad \Pi = \frac{\lambda^a}{2} \phi^a(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & K^0 & -\frac{2}{\sqrt{6}} \eta_8 \end{pmatrix}. \tag{3}$$

The quantity $f$ is, to lowest order, identified with the pion decay constant $f_\pi$ (and equal to $f_K$ before chiral loops are introduced).

When the matrix $\Sigma$ is expanded in powers of $f^{-1}$, the zeroth order term obtained from (1) is the free Klein-Gordon Lagrangian for the pseudoscalar particles. From this Lagrangian one might deduce the (left-handed) current

$$J^\mu_n = -i \frac{f^2}{2} Tr \left[ \lambda^a \Sigma D^\mu \Sigma^\dagger \right] \tag{4},$$

where $n$ is a flavour octet index and $\lambda_n$ a $SU(3)$ flavour matrix.

For the next-to-leading order Lagrangian $L^{(4)}_{\text{strong}}$ there are ten terms and thereby ten coefficients $L_i$ to be determined \[18\] either experimentally or by means of some model. Some of these play an important role in the physics of $\epsilon'$ in $K \rightarrow 2\pi$ decays \[19\]. As examples, we display the $L_5$ and $L_8$ terms in governing much of the penguin physics:

$$L_5 B_0 Tr \left[ D_\mu \Sigma^\dagger D^\mu \Sigma \left( M_q^\dagger \Sigma + \Sigma^\dagger M_q \right) \right], \tag{5}$$

and

$$L_8 B_0 Tr \left[ M_q^\dagger \Sigma M_q^\dagger \Sigma + M_q \Sigma^\dagger M_q \Sigma^\dagger \right]. \tag{6}$$

Under the action of the elements $V_R$ and $V_L$ of the chiral group $SU_R(3) \times SU_L(3)$, the field $\Sigma$ transforms as:

$$\Sigma \rightarrow V_L \Sigma V_R^\dagger, \tag{7}$$

and accordingly for the conjugated fields. Formally, $M_q$ is given the same transformation properties as $\Sigma$, and $M_q^\dagger$ as $\Sigma^\dagger$.\[18\]
B. The heavy light sector

The strong chiral Lagrangian for the heavy light sector is [7, 20]:

\[ \mathcal{L}_{\text{str}} = \mp Tr \left[ H^{(\pm)}_{vk} (iv \cdot D_{hk}) H^{(\pm)}_{vh} \right] - g_\Delta Tr \left[ H^{(\pm)}_{vk} A^{\mu}_{hk} H^{(\pm)}_{vh} \right] 
\]

\[ + 2\lambda_1 Tr \left[ H^{(\pm)}_{vk} H^{(\pm)}_{vh} (\bar{M}^V_{qhk}) \right] + \frac{e\beta}{4} Tr \left[ H^{(\pm)}_{vk} H^{(\pm)}_{vh} \gamma_\mu \cdot F (Q^\xi_{qhk}) \right] + \ldots \]  

(8)

where \( k, h \) are \( SU(3) \) triplet indices, and \( v \) is the velocity of the heavy meson. The ellipses indicate other terms (of higher order, say), and \( iD^\mu_{hk} = \delta_{hk} D^\mu + V^\mu_{hk} \). Moreover, \( Q^\xi_q = (\xi^\dagger Q_q \xi + \xi Q_q \xi^\dagger)/2 \), where \( Q_q \) is the \( SU(3) \) charge matrix for light quarks, \( Q_q = \text{diag}(2/3, -1/3, -1/3) \), and \( F \) is the electromagnetic field tensor. \( H^{(\pm)}_{vk} \) is the heavy meson field containing a spin zero and spin one boson:

\[ H^{(\pm)}_{vk} \equiv P_\pm (v)(P^k_\mu \gamma_\mu - iP^k_5 \gamma_5) , \]

\[ H^{(\pm)}_{vk} = \gamma^0 (H^{(\pm)}_{vk})^\dagger \gamma^0 = \left[ (P^k_\mu)\gamma_\mu - i(P^k_5)\gamma_5 \right] P_\pm , \]  

(9)

where

\[ P_\pm (v) = (1 \pm \gamma \cdot v)/2 \]  

(10)

are projection operators. The fields \( P^{(\pm)}(P^{(\pm)}_\mu) \) represent heavy-light mesons, \( 0^- (1^-) \), with velocity \( v \). The signs \( \pm \) refers to particles and anti-particles respectively, and will sometimes be omitted in the following when unnecessary.

The vector and axial vector fields \( \mathcal{V}_\mu \) and \( \mathcal{A}_\mu \) are given by:

\[ \mathcal{V}_\mu \equiv \frac{i}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) \quad ; \quad \mathcal{A}_\mu \equiv -\frac{i}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) . \]

(11)

The fields \( \xi \) and \( H_v \) transform as

\[ \xi \to U \xi V^T_R = V_L U^\dagger \xi , \quad H_v \to H_v U^\dagger , \]

(12)

where \( U \in SU(3)_V \), the unbroken symmetry.

The vector and axial fields transform as

\[ \mathcal{V}_\mu \to U \mathcal{V}_\mu U^\dagger + iU \partial_\mu U^\dagger , \quad \mathcal{A}_\mu \to U \mathcal{A}_\mu U^\dagger . \]

(13)

The vector field \( \mathcal{V}^\mu \) is seen to transform as a gauge field under local \( SU(3)_V \), and can only appear in combination with a derivative as a covariant derivative \( (i\partial^\mu + \mathcal{V}^\mu) \). The quantity \( \widetilde{M}^V_q \) (as well as the orthogonal combination \( \widetilde{M}^A_q \)) is related to the current mass term:

\[ \widetilde{M}^V_q \equiv \frac{1}{2} (\xi^\dagger M_q \xi + \xi M_q \xi^\dagger) \quad ; \quad \widetilde{M}^A_q \equiv -\frac{1}{2} (\xi^\dagger M_q \xi^\dagger - \xi M_q \xi) . \]

(14)
The heavy-light weak current, to zeroth order in $1/m_Q$ and chiral counting, is represented
by:
\[ J^\alpha_k(0) = \frac{\alpha_H}{2} Tr \left[ \xi_{kh}^\dagger \Gamma^\alpha H_{vh} \right], \tag{15} \]
and under $SU(3)_L$ it transforms as
\[ J^\alpha_k \rightarrow J^\alpha_h \left( V_L^\dagger \right)_{hk}. \tag{16} \]
This current has also (counter) terms, of higher order in the chiral counting, needed to make
the chiral loops finite:
\[ J^\mu_k(M) = \frac{\omega_1}{2} Tr \left[ \xi_{h}^\dagger \Gamma^\mu H_{vl} \tilde{M}_{th} \right] + \frac{\omega_1'}{2} Tr \left[ \xi_{kh}^\dagger \Gamma^\mu H_{vh} \right] \tilde{M}_{th}', \tag{17} \]
where the parameters $\omega_1$ and $\omega_1'$ are commented on in section IV-C. To leading order,
$\Gamma^\alpha = \gamma^\alpha L$, where $L$ is the left-handed projector in Dirac space, $L = (1 - \gamma_5)/2$. However,
this is slightly modified by perturbative QCD for $\mu$ below $m_Q$, which gives
\[ \Gamma^\alpha \equiv C_{\gamma}(\mu) \gamma^\alpha L + C_{\nu}(\mu) \nu^\alpha R, \tag{18} \]
where $R$ is the right-handed projector, $R = (1 + \gamma_5)/2$. The coefficients $C_{\gamma,\nu}(\mu)$ are
determined by QCD renormalization for $\mu < m_Q$. They have been calculated to NLO and
the result is the same in $MS$ and $\overline{MS}$ scheme \[21\]. ($C_{\gamma}$ is close to one and $C_{\nu}$ is rather
small). Corrections to the weak current of order $1/m_Q$ will be discussed in section \[\n\]

Before closing this section, we write down the bosonized $b \rightarrow c$ transition current in terms
of the heavy fields
\[ \overline{Q}_v^{(\pm)} \gamma^\mu LQ_{v_c}^{(\pm)} \rightarrow -\zeta(\omega) Tr \left[ H_c^{(\pm)} \gamma^\alpha L H_b^{(\pm)} \right], \tag{19} \]
where $\zeta(\omega)$ is the Isgur-Wise function for the $\bar{B} \rightarrow D$ transition \[22\]. The indices on the
heavy fields here refer to the the $b$- and $c$-quarks with velocities $v_b$ and $v_c$, with $\omega \equiv v_b \cdot v_c$.
The current for $D\bar{D}$ production is:
\[ \overline{Q}_v^{(\pm)} \gamma^\mu LQ_{v}^{(-)} \rightarrow -\zeta(-\lambda) Tr \left[ H_c^{(\pm)} \gamma^\alpha L H_c^{(-)} \right], \tag{20} \]
where the Isgur-Wise function $\zeta(-\lambda)$ is (in general) complex. We have $\lambda = v_c \cdot \bar{v}$, where $\bar{v}$
is the velocity of $\bar{c}$.
III. THE CHIRAL QUARK MODEL ($\chi$QM)

A. The Lagrangians for $\chi$QM

The light quark sector is described by the chiral quark model ($\chi$QM), having a standard QCD term and a term describing interactions between quarks and (Goldstone) mesons [1, 2, 3, 4, 5]:

$$L_{\chi\text{QM}} = \bar{q}_L i\gamma \cdot D q_L + \bar{q}_R i\gamma \cdot D q_R - \bar{q}_L M_q q_R - \bar{q}_R M_q^\dagger q_L - m(\bar{q}_R \Sigma^\dagger q_L + \bar{q}_L \Sigma q_R) \, ,$$  \hspace{1cm} (21)

where $m$ is the ($SU(3)$-invariant) constituent quark mass for light quarks $q^T = (u, d, s)$. The left- and right-handed projections $q_L$ and $q_R$ are transforming after $SU(3)_L$ and $SU(3)_R$ respectively:

$$q_L \to V_L q_L \quad \text{and} \quad q_R \to V_R q_R \, .$$  \hspace{1cm} (22)

From (21) we deduce the Feynman rules. For instance, the $Pq\bar{q}$ coupling is $(m\gamma_5/f)$ times some $SU(3)$ factor ($P$ is a pseudoscalar meson $\pi, K, \eta$). From such Feynman rules, and including the quark propagator $S(p) = (\gamma \cdot p - M_q)^{-1}$, we can calculate amplitudes for, say, $\pi - \pi$ scattering in the strong sector. Here $M_q = m + m_q$ is the total mass. Alternatively one might keep only the constituent mass $m$ in the propagator, and take the current mass $m_q$ as a coupling. Incluing also the Feynman rules for weak vertices, one might calculate amplitudes for non-leptonic decays in terms of quark loops representing $f_\pi$ and the semileptonic form factors $f_{\pm}$, but also for more complicated cases.

Also, as a more exotic example, one may calculate the effect of the electroweak $s \to d$ self-energy transition contribution to $K \to 2\pi$ as shown in Fig. 2. This is an off-shell effect which vanish in the free quark case, but is non-zero for bound quarks and proportional to $m$ within our framework [3].

Chiral Lagrangians, either in the pure strong sector or for non-leptonic decays, are however easier to obtain in a more transparent way within the “rotated version” of the $\chi$QM with flavour rotated quark fields $\chi$ given by:

$$\chi_L = \xi^\dagger q_L \quad ; \quad \chi_R = \xi q_R \quad ; \quad \xi \cdot \xi = \Sigma \, .$$  \hspace{1cm} (23)

The constituent quark fields $\chi_L$ and $\chi_R$ transform in a simple way under $SU(3)_V$:

$$\chi_L \to U \chi_L \quad , \quad \chi_R \to U \chi_R \, .$$  \hspace{1cm} (24)
FIG. 2: Contribution to the process $K \rightarrow 2\pi$ from the non-diagonal $s \rightarrow d$ transition.

In the rotated version, the chiral interactions are rotated into the kinetic term while the interaction term proportional to $m$ in (21) become a pure (constituent) mass term [2, 5]:

$$L_{\chi QM} = \overline{\chi} [\gamma^\mu (iD_\mu + V_\mu + \gamma_5 A_\mu) - m] \chi - \overline{\chi} \tilde{M}_q \chi , \quad (25)$$

which is manifestly invariant under $SU(3)_V$. Moreover,

$$\tilde{M}_q \equiv \tilde{M}_q^V + \tilde{M}_q^A \gamma_5 , \quad (26)$$

where $\tilde{M}_q^{V,A}$ are given in (14).

In the light sector, the various pieces of the strong Lagrangian in section II-A can now be obtained by integrating out the constituent quark fields $\chi$, and these pieces can be written in terms of the fields $A_\mu$, $\tilde{M}_q^V$ and $\tilde{M}_q^A$. This can easily be seen by using the relation

$$A_\mu = - \frac{1}{2i} \xi (D_\mu]^{\dagger} \xi = + \frac{1}{2i} \xi^{\dagger} (D_\mu] \xi^{\dagger} . \quad (27)$$

In our model, the hard gluons are integrated out and we are left with soft gluonic degrees of freedom. These gluons can be described using the external field technique, and their effect will be parameterized by vacuum expectation values, i.e. the gluon condensate $\langle a_\pi G^2 \rangle$. Gluon condensates with higher dimension could also be included, but we truncate the expansion by keeping only the condensate with lowest dimension.

When calculating the soft gluon effects in terms of the gluon condensate, we follow the prescription given in [23]. The calculation is easily carried out in the Fock - Schwinger gauge. In this gauge one can expand the gluon field as :

$$A_\mu^a(k) = - \frac{i(2\pi)^4}{2} G_{\rho\nu}^a(0) \frac{\partial}{\partial k_\rho} \delta^{(4)}(k) + \cdots . \quad (28)$$
FIG. 3: Feynman rule for the light quark -soft gluon vertex.

In some simple cases one may also use the light quark propagator in a gluonic background (to first order in the gluon field):

\[
S_1(p, G) = -\frac{g_s}{4} G_{\alpha\beta}^a t^b \left[ \sigma^{\alpha\beta} (\gamma \cdot p + m) + (\gamma \cdot p + m) \sigma^{\alpha\beta} \right] (p^2 - m^2)^{-2}, \tag{29}
\]

where \(g_s\) is the strong coupling constant, \(a\) and \(b\) are colour octet indices, and \(t^a\) are the colour matrices. In general one should stick to the prescription in \[23\] in order to get correct results. Since each vertex in a Feynman diagram is accomplished with a \(\pi\) integration we get the Feynman rule given in Fig. 3. The gluon condensate contributions are obtained by the replacement

\[
g_s^2 G_{\mu\nu}^a G_{\alpha\beta}^b \rightarrow \frac{4\pi^2}{(N_c^2 - 1)} \delta^{ab} \left( \frac{\alpha_s}{\pi} G^2 \right) \frac{1}{12} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}). \tag{30}
\]

**B. Bosonization of the \(\chi\)QM**

The Lagrangians (21) or (25) from the previous section can now be used for bosonization, i.e. to integrate out the quark fields. This can be done in the path integral formalism, or as we do here, by expanding in terms of Feynman diagrams. Within the \(\chi\)QM, with Feynman rules obtained from (21) one may calculate the simple quark loop amplitude for \(\pi \rightarrow W\) which defines \(f\) (the bare \(f_\pi\)) in terms of a logarithmically divergent integral \(I_2\) times the coupling \(\sim m/f\). Including also the gluon condensate contribution one obtains \[3, 4, 5\]:

\[
f^2 = -i4m^2 N_c I_2 + \frac{1}{24m^2} \left( \frac{\alpha_s}{\pi} G^2 \right), \tag{31}
\]

where \(I_2\) is the following logarithmic divergent integral (\(d\) is the dimension of space within dimensional regularization):

\[
I_2 \equiv \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2)^2}. \tag{32}
\]
Equivalently, one may obtain the (kinetic part of the) strong Lagrangian in (1) by attaching two axial fields $A_\mu$ to a vacuum polarization like quark loop diagram by using (25). Then one obtains:

$$i\mathcal{L}_{\text{str}}^{(2)} = -N_c \int \frac{d^d p}{(2\pi)^d} \text{Tr} \left[ (\gamma_\sigma \gamma_5 A^\sigma) S(p) (\gamma_\rho \gamma_5 A^\rho) S(p) \right] = f^2 \text{Tr} \left[ A_\mu A^\mu \right] ,$$  

where the trace is both in flavor and Dirac spaces (a similar diagram with gluons should also be added). This is easily seen by using the relation (27), provided $f^2$ is given by (31).

The eq. (33) give the Lagrangian (1) in the light sector by applying (27).

The quark condensate is:

$$\langle \bar{q} q \rangle = -i N_c \text{Tr} \int \frac{d^d p}{(2\pi)^d} S(p) = -4i m N_c I_1 - \frac{1}{12m} \left( \frac{\alpha_s}{\pi} G^2 \right) ,$$  

where $I_1$ is the quadratically divergent integral

$$I_1 \equiv \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m^2} .$$  

Here the propagator $S$ has to be understood at the one in the gluon field up to second order and the a priori divergent integrals $I_{1,2}$ have to be interpreted as the regularized ones. The physical values of $I_{1,2}$ are determined by the physical values of $f$ and $\langle \bar{q} q \rangle$. In general, by coupling the fields $A_\mu$ and $\tilde{M}_q^{V,A}$ to quark loops, the chiral Lagrangian terms and their coefficients within the light sector can be obtained.

Similarly, we may bosonize the weak currents. The left handed current can be written

$$\bar{q}_L \gamma^\mu \Lambda^n q_L = \bar{\chi}_L \gamma^\mu \Lambda^n \chi_L ; \quad \Lambda^n \equiv \xi^\dagger \lambda^n \xi .$$  

The lowest order term $O(p)$ is obtained when the vertex $\Lambda^n$ from (36) and axial vertex ($\sim A_\mu$) from (25) are entering a quark loop (see Fig. 4):

$$j^{n}_\mu (A) = -i N_c \int \frac{d^d p}{(2\pi)^d} \text{Tr} \left[ (\gamma_\mu L \Lambda^n) S(p) (\gamma_\sigma \gamma_5 A^\sigma) S(p) \right] \sim \text{Tr} \left[ \Lambda^n A_\mu \right] ,$$  

which coincides with (4) when (27) is used.

As a more non-trivial example, to obtain a non-zero non-factorizable contribution to $D^0 \to K^0 \bar{K}^0$ at tree level, one has to consider the coloured current $j^{n,a}_\mu$ to $O(p^3)$, involving insertions of the “mass fields” $\tilde{M}_q$ in (26) [12]. (This coloured current is obtained by Fierz transformations of the relevant four quark operator). From Fig. 5 one obtains the contribution:
\[ j_{\mu}^{n,a}(G, \xi, A, \widetilde{M}_q) = \frac{g_s}{12m} \frac{\lambda^{n,\alpha}}{16\pi^2} G^{n,\alpha \lambda} \left[ i \varepsilon_{\mu \rho \kappa \lambda} T_{\epsilon}^{n,\rho} + (g_{\mu \kappa} g_{\rho \lambda} - g_{\mu \lambda} g_{\rho \kappa}) T_{\epsilon}^{n,\rho} \right], \]  

where (we have used the analytical computer program FORM [24])

\[ T_{\epsilon}^{n,\rho} = 4 S^{K}_{\rho} - 3(S^{L}_{\rho} + S^{R}_{\rho}), \quad T_{\epsilon}^{n,\rho} = S^{L}_{\rho} - S^{R}_{\rho}. \]

The \( S \)'s are chiral Lagrangian terms:

\[ S^{L}_{\rho} \equiv \frac{1}{2i} \text{Tr} \left[ \Lambda^{n} A_{\rho} \widetilde{M}_{q}^{L} \right] = \frac{1}{2i} \text{Tr} \left[ \lambda^{n}(D_{\rho} \Sigma) \Sigma \Sigma^{\dagger} \right], \]

\[ S^{R}_{\rho} \equiv \frac{1}{2i} \text{Tr} \left[ \Lambda^{n} \widetilde{M}_{q}^{R} A^{\rho} \right] = \frac{1}{2i} \text{Tr} \left[ \lambda^{n} \Sigma \Sigma^{\dagger} (D_{\rho} \Sigma^{\dagger}) \right], \]

\[ S^{K}_{\rho} \equiv \frac{1}{2} \text{Tr} \left[ \Lambda^{n} \left( A^{\rho} \widetilde{M}_{q}^{R} + \widetilde{M}_{q}^{L} A^{\rho} \right) \right] = \frac{1}{4i} \text{Tr} \left[ \lambda^{n} ((D_{\rho} \Sigma)^{\dagger} \Sigma^{\dagger} M_{q}^{L} - \Sigma M_{q}^{L} (D_{\rho} \Sigma^{\dagger})) \right]. \]

The current (39) has to be be combined with the left-handed colour current for \( D \)-meson decay later given in (90) to obtain a contribution to \( D^0 \to K^0 \bar{K}^0 \) [12].
IV. THE HEAVY - LIGHT CHIRAL QUARK MODEL (HL$\chi$QM)

A. The Lagrangian for HL$\chi$QM

Our starting point is the following Lagrangian containing both quark and meson fields:

$$\mathcal{L} = \mathcal{L}_{HQEFT} + \mathcal{L}_{\chiQM} + \mathcal{L}_{Int},$$

where

$$\mathcal{L}_{HQEFT} = \pm Q_v^{(\pm)} i v \cdot D Q_v^{(\pm)} + \frac{1}{2m_Q} Q_v^{(\pm)} (\sigma \cdot G + (iD_{\perp})_{\text{eff}}^2) Q_v^{(\pm)} + \mathcal{O}(m_Q^{-2})$$

is the Lagrangian for heavy quark effective field theory (HQEFT). The heavy quark field $Q_v^{(\pm)}$ annihilates a heavy quark with velocity $v$ and mass $m_Q$. Similarly, $Q_v^{(-)}$ annihilates a heavy anti-quark. Moreover, $D_{\mu}$ is the covariant derivative containing the gluon field (eventually also the photon field), and $\sigma \cdot G = \sigma^{\mu\nu} C_{\mu\nu}^a t^a$, where $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$, and $G_{\mu\nu}^a$ is the gluonic field tensor. This chromo-magnetic term has a factor $C_M$, being one at tree level, but slightly modified by perturbative QCD. (When the covariant derivative also contains the photon field, there is also a corresponding magnetic term $\sim \sigma \cdot F$, where $F_{\mu\nu}$ is the electromagnetic tensor). Furthermore, $(iD_{\perp})_{\text{eff}}^2 = C_D (iD)^2 - C_K (iv \cdot D)^2$. At tree level, $C_D = C_K = 1$. Here, $C_D$ is not modified by perturbative QCD, while $C_K$ is different from one due to perturbative QCD corrections for $\mu < m_Q$. We observe that soft gluons coupling to a heavy quark is suppressed by $1/m_Q$, since to leading order the vertex is proportional to $v_\mu v_\nu G^{\mu\nu} = 0$, $v_\mu$ being the heavy quark velocity.

In the heavy - light case, the generalization of the meson - quark interactions in the pure light sector $\chiQM$ is given by the following $SU(3)$ invariant Lagrangian:

$$\mathcal{L}_{Int} = -G_H \left[ \chi_k \overline{H_v^{(\pm)}} Q_v^{(\pm)} + \overline{Q_v^{(\pm)}} H_v^{(\pm)} \chi_k \right],$$

where $k$ is a triplet $SU(3)$- index and $G_H$ is a coupling constant. Note that in [6], $G_H = 1$ is used. However, in that case one used a renormalization factor for the heavy meson fields $H_v$, which is equivalent. The interaction Lagrangian (44) can, as for the $\chiQM$, be obtained from a NJL model. This has been done in [6] (as for the light sector [1]).
B. Bosonization within the HLχQM

The interaction term $\mathcal{L}_{\text{Int}}$ in (44) can now be used to bosonize the model, i.e. integrate out the quark fields. This can be done in terms of Feynman diagrams as we do here, by attaching the external fields $H^a_v, \overline{H}_v^a, V^\mu, A^\mu$ and $\overline{\tilde{M}}^{V,A}_{q}$ of section II-B to quark loops, and using (25) and (44). In this way one obtains the strong chiral Lagrangian (8) and terms of higher order in the heavy light sector. Some of the loop integrals will be divergent and have to be related to physical parameters, as for the pure light sector [2, 3, 4, 5]. As the pure light sector is a part of our model, we have to keep the relations in (31) and (34) from the pure light sector within the heavy light case studied here. The $1/m_Q$ terms will not be discussed in this section, but will be considered later in section V.

From the diagrams in Fig. 6 we obtain the identification for the kinetic term, which by Ward identities is the same as for the term with the vector field $V^\mu$ attached to the light quark:

$$-iG_H^2 N_c (I_{3/2} + 2m I_2 + i \frac{(8 - 3\pi)}{384 N_c m^3} \frac{\alpha_s}{\pi} G^2 ) = 1,$$

(45)

where $I_2$ is given in (32) and

$$I_{3/2} = \int \frac{d^d k}{(2\pi)^d} \frac{1}{(v \cdot k)(k^2 - m^2)},$$

(46)

which formally depends on $v^2$ which is equal to one. From the same diagram, with the axial field $A_\mu$ attached, we obtain the following identification for the axial vector coupling $g_A$ :

$$g_A = i G_H^2 N_c \left( \frac{1}{3} I_{3/2} - 2m I_2 - i \frac{m}{12\pi} - i \frac{(8 - 3\pi)}{384 N_c m^3} \frac{\alpha_s}{\pi} G^2 \right),$$

(47)

As $I_1$ and $I_2$ are related to the quark condensate and $f_\pi$ respectively, the (formally) linearly divergent integral $I_{3/2}$ is related to $\delta g_A \equiv 1 - g_A$, which is found by eliminating $I_2$ from eqs. (45) and (47) :

$$\delta g_A = -\frac{4}{3} i G_H^2 N_c \left( I_{3/2} - i \frac{m}{16\pi} \right).$$

(48)

Note that the gluon condensate drops out here. Within a primitive cut-off regularization, $I_{3/2}$ is (in the leading approximation) proportional to the cut-off in first power:

$$I_{3/2} = i \frac{\Lambda}{16\pi} \left( 1 + \mathcal{O}\left( \frac{m}{\Lambda} \right) \right),$$

(49)

where the cut-off $\Lambda$, is of the same order as the chiral symmetry breaking scale $\Lambda_\chi$. In contrast, $I_{3/2}$ is finite and proportional to $m$ in dimensional regularization. Note that the
FIG. 6: Bosonization in the strong sector to obtain eq. (8).

cut-off $\Lambda$ is only used in qualitative considerations here and in subsection IV-D. Anyway, $I_{3/2}$ is determined by the physical value of $g_A$.

When attaching $\tilde{M}_q^V$ like in Fig. 6 instead of vector or axial vector fields one finds for the mass correction term in (8):

$$2\lambda_1 \equiv i G_H^2 N_c (I_{3/2} - 2m I_2 - \frac{i}{8\pi} \frac{m}{192 N_c m^3} (\frac{\alpha_s}{\pi} G^2)) .$$  \hspace{1cm} (50)

The electromagnetic $\beta$ term in (51) is obtained by considering diagrams like those in figure 6 but with the vector and axial vector fields $V_\mu$ or $A_\mu$ replaced by a photon field tensor:

$$\beta = \frac{G_H^2}{2} \left\{-4i N_c I_2 + \frac{N_c}{4\pi} - \left(\frac{32 + 3\pi}{576 m^4} \right) \left(\frac{\alpha_s}{\pi} G^2\right) \right\} .$$  \hspace{1cm} (51)

Within the full theory (SM) at quark level, the weak current is:

$$J_k^\alpha = \bar{q}_k L \gamma^\alpha Q$$  \hspace{1cm} (52)

where $Q$ is the heavy quark field in the full theory. Within HQEFT this current will, below the renormalization scale $\mu = m_Q$ (= $m_b, m_c$), be modified in the following way:

$$J_k^\alpha = \bar{x} h_s h_k \Gamma^\alpha Q_v + O(m_Q^{-1}) .$$  \hspace{1cm} (53)
The operator in equation (53) can be bosonized by calculating the Feynman diagrams shown in Fig. 7 which gives the bosonized current in (15) with:

$$\alpha_{H} \equiv -2iG_{H}N_{c}\left(-I_{1} + mI_{3/2} + \frac{i(3\pi - 4)}{384N_{c}m^{2}}\left(\frac{\alpha_{s}}{\pi}G^{2}\right)\right). \quad (54)$$

To first order in the chiral expansion we obtain the bosonized current obtained by attaching one extra field $A_{\nu}$ to the quark loops in Fig. 7:

$$J_{k}^{\alpha}(1) = \frac{1}{2}Tr\left[\xi_{hk}^{+}\Gamma^{\alpha}H_{vh}(\alpha_{H}\gamma_{\nu}\gamma_{5} + \alpha_{H}^{(1)}\nu_{\nu}\gamma_{5})A_{\nu}\right], \quad (55)$$

where the quantities $\alpha_{H}^{(1)}$ are given by expressions similar to (54).

The coupling $\alpha_{H}$ in (15) is related to the physical decay constants $f_{H}$ and $f_{H}^{\ast}$, in the following way (for $H = B, D$):

$$\langle 0|\bar{\psi}\gamma^{\alpha}\gamma_{5}b|H\rangle = -2\langle 0|J_{a}^{\alpha}|H\rangle = iM_{H}f_{H}v^{\alpha}, \quad (56)$$

Taking the trace over the gamma matrices in (15), we obtain a relation for $\alpha_{H}$ and the relations between the heavy meson decay constants $f_{H}$ and $f_{H}^{\ast}$ (for $H = B, D$):

$$\alpha_{H} = \frac{f_{H}\sqrt{M_{H}}}{C_{\gamma}(\mu) + C_{\gamma}(\mu)} = \frac{f_{H}^{\ast}\sqrt{M_{H}^{\ast}}}{C_{\gamma}(\mu)}, \quad (57)$$

where the model dictates us to put $\mu = \Lambda_{\chi}$. Later, in section V-B, we will see how chiral corrections and $1/m_{Q}$ corrections modify this relation.

C. Constraining the parameters of the HL$\chi$QM

The gluon condensate can be related to the chromomagnetic interaction:

$$\mu_{G}^{2}(H) = \frac{1}{2M_{H}C_{M}(\mu)}\langle H|\bar{Q}\frac{1}{2}\sigma \cdot GQ\sigma|H\rangle, \quad (58)$$
where the coefficient $C_M(\mu)$ contains the short distance effects down to the scale $\mu$ and has been calculated to next to leading order (NLO) \[26\]. The chromomagnetic operator is responsible for the splitting between the $1^-$ and $0^-$ state, and is known from spectroscopy,

$$\mu^2_G(H) = \frac{3}{2}m_Q(M_{H^*} - M_H). \quad (59)$$

An explicit calculation of the matrix element in equation (58) gives

$$\mu^2_G = \eta_2 \frac{G_H^2}{m} \langle \frac{\alpha_s}{\pi} G^2 \rangle, \quad \text{where} \quad \eta_2 \equiv \frac{(\pi + 2)}{32}C_M(\Lambda_\chi). \quad (60)$$

Combining the eqs. (31), (34) and (60) we obtain the following relations:

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = \frac{\mu^2_G f^2}{2\eta_2} \frac{1}{\rho}, \quad G^2_H = \frac{2m}{f^2} \rho, \quad (61)$$

where the quantity $\rho$ is of order one and given by

$$\rho \equiv \frac{(1 + 3g_A) + \frac{m\mu_G^2}{\eta_2m^2}}{4(1 + \frac{N_cm^2}{8\pi f^2})}, \quad (62)$$

where $\eta_1 \equiv \frac{\pi}{32}$. In the limit where only the leading logarithmic integral $I_2$ is kept in (45), we obtain:

$$g_A \to 1, \quad \rho \to 1, \quad \beta \to \frac{1}{m}, \quad G_H \to G_H^{(0)} \equiv \frac{\sqrt{2m}}{f}, \quad (63)$$

which for $g_A$ and $\beta$ correspond to the non-relativistic values.

From the eqs. (31), (50), and (60), we find

$$2\lambda_1 = \frac{1}{2}(3g_A - 1) + \frac{(9\pi - 16)\mu_G^2}{384\eta_2m^2}. \quad (64)$$

In the limit (63) we obtain $2\lambda_1 \to 1$, as expected. The parameter $\lambda_1$ is related to the mass difference $M_{H^*} - M_{H_d}$. To leading order, we obtained the following expression for the $\beta$-term:

$$\beta = \frac{\rho}{m} \left\{ 1 + \frac{N_cm^2}{4\pi f^2} - \left( \frac{56 + 3\pi}{576f^2m^2} \right) \langle \frac{\alpha_s}{\pi} G^2 \rangle \right\}, \quad (65)$$

which is rather sensitive to $m$. Choosing $m$ in the range 250-300 MeV we find \[27\] $\beta = (2.5 \pm 0.6)$ GeV$^{-1}$ to be compared with $\beta = (2.7 \pm 0.20)$ GeV$^{-1}$ extracted from experiment.

Using equation (65) and (34) we may write $\alpha_H$ as:

$$\alpha_H = \frac{G_H}{2} \left( -\frac{\langle \bar{q}q \rangle}{m} - 2f^2(1 - \frac{1}{\rho}) + \frac{(\pi - 2)}{16m^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right), \quad (66)$$
Combining (57) with (66), we obtain [12] in the leading limit (taking into account the logarithmic and quadratic divergent integrals only, and let $C_\gamma \to 1$, $C_v \to 0$ and $g_A \to 1$ as in (63) ) we obtain the “Goldberger-Treiman like” relation:

$$f_H \sqrt{M_H} \to \frac{\langle \bar{q}q \rangle}{f_\pi \sqrt{2m}},$$

(67)

which gives the scale for $f_H$ (It is, however, numerically a factor 2 off for the $B$-meson).

Using the relations (45), (47) and (62) we obtain for $\alpha_H^{(1)}$ and $\alpha_H^{(1)}$ in (55):

$$\alpha_H^{(1)} = \frac{2g_A}{G_H},$$

(68)

$$\alpha_H^{(1)} = \frac{4}{3}G_H \left( \frac{f_\pi^2}{2m} \left( \frac{1}{\rho} - 1 \right) + \left[ \frac{mN_c}{8\pi} + \frac{(\pi + 8)}{256m^3} \langle \alpha_s \pi G^2 \rangle \right] \right).$$

(69)

Moreover, for the mass correction to the weak current given in (17) we find that $\omega_1 = -4\lambda_1/G_H$, where $\lambda_1$ is given in equation (50) or (64). The term $\omega'_1$ is subleading in $1/N_c$.

The Isgur-Wise function $\zeta(\omega)$ in (19) relates all the form factors describing the processes $B(B^*) \to D(D^*)$ in the heavy quark limit. This function can be calculated from the diagrams shown in Fig. 8. The result before $1/m_Q$ and chiral corrections is:

$$\zeta(\omega) = \frac{2}{1 + \omega} (1 - \rho) + \rho r(\omega),$$

(70)

where $\rho$ is given in (62) and $r(\omega)$ is the same function appearing in loop calculations of the anomalous dimension in HQEFT:

$$r(\omega) = \frac{1}{\sqrt{\omega^2 - 1}} \ln \left( \omega + \sqrt{\omega^2 - 1} \right).$$

(71)

Note that $\zeta(1) = 1$ as it should.
D. The formal limit \( m \to 0 \)

In this subsection we will discuss the limit of restoration of chiral symmetry, i.e. the limit \( m \to 0 \) [27]. In order to do this, we have to consider the various constraints obtained when constructing the HL\( \chi \)QM [9].

Looking at the equations (31) and (34), one may worry that \( \langle \bar{q}q \rangle \) and \( f \) behaves like \( 1/m \) in the limit \( m \to 0 \) unless one assumes that \( \langle \frac{\alpha_s}{\pi} G^2 \rangle \) also go to zero in this limit. We should stress that the exact limit \( m = 0 \) cannot be taken because our loop integrals will then be meaningless. Still we may let \( m \) approach zero without going to this exact limit. In the pure light sector (at least when vector mesons are not included) there are no restrictions on how \( \langle \alpha_s \pi G^2 \rangle \) might go to zero. In the heavy light sector we have in addition to (31) and (34) also the relations (45) and (47), which put restrictions on the behavior of the gluon condensate \( \langle \alpha_s \pi G^2 \rangle \) for small masses. As \( \langle \alpha_s \pi G^2 \rangle \) has dimension mass to the fourth power, we find that \( \langle \bar{q}q \rangle \) and \( f \) may go to zero if \( \langle \alpha_s \pi G^2 \rangle \) goes to zero as \( m^4 \) or \( m^3 \Lambda \) (eventually combined with \( \ln(m/\Lambda) \)). However, the behavior \( m^3 \Lambda \) is inconsistent with the additional equations (61) and (62). Still, from all equations (45), (61), (62), we find the possible solution

\[
\langle \frac{\alpha_s}{\pi} G^2 \rangle = \hat{c} N_c m^4 K(m) , \quad \text{where} \quad K(m) \equiv (-4iI_2 + \frac{1}{8\pi}) ,
\]

and \( \hat{c} \) is some constant. Then we must have the following behavior for \( G_H^2, g_A \) and \( \mu_G^2 \) when \( m \) approaches zero:

\[
G_H^2 \sim \frac{1}{N_c \Lambda} , \quad (1 + 3g_A) \sim \frac{m}{\Lambda} K(m) , \quad \mu_G^2 \sim \frac{m^3}{\Lambda} K(m) ,
\]

with some restrictions on the proportionality factors. Here, the regularized \( I_2 \) is such that for small \( m \), \( K(m) = (c_1 + c_2 \ln m/\Lambda) \), \( c_1 \) and \( c_2 \) being constants. The behavior of \( G_H^2 \) is in agreement with Nambu-Jona-Lasinio models [1]. Note that in our model, \( \delta g_A \to 4/3 \) (corresponding to \( g_A \to -1/3 \)) for \( m \to 0 \), in contrast to \( \delta g_A \to 2/3 \) for a free Dirac particle with \( m = 0 \). Note that in [9] we gave the variation of the gluon condensate with \( m \) for a fixed value of \( \mu_G^2 \). For the considerations in this subsection, we have to let \( \mu_G^2 \) go to zero with \( m \) in order to be consistent. When \( m \to 0 \), we also find that \( \beta \to 1/\Lambda \), provided that the coefficient \( \hat{c} \) in (72) is fixed to a specific value (which is \( \hat{c} = 576/(3\pi + 32) \approx (1.93)^4 \)).
FIG. 9: Diagrams responsible for $1/m_Q$ terms in the chiral Lagrangian.

V. $1/m_Q$ CORRECTIONS WITHIN THE HLχQM

A. Bosonization of the strong sector

To order $1/m_Q$ one obtains further contributions to chiral Lagrangians (see ref. and references therein):

$$
\mathcal{L}_{str} = -\frac{\varepsilon_1}{m_Q} Tr \left[ \bar{H}_k (i \gamma \cdot D) H_k \right] + \frac{\varepsilon_1}{m_Q} Tr \left[ \bar{H}_k H_k \gamma_\mu \gamma_5 A_\mu \right] \\
+ g_1/m_Q Tr \left[ \bar{H}_k H_k \gamma_\mu \gamma_5 A_\mu \right] + \frac{\varepsilon_2}{m_Q} Tr \left[ \bar{H}_k \sigma^{\alpha\beta} i \gamma \cdot D H_k \sigma_{\alpha\beta} \right] \\
- \frac{\varepsilon_2}{m_Q} Tr \left[ \bar{H}_k \sigma^{\alpha\beta} \gamma_\mu \gamma_5 A_\mu \sigma_{\alpha\beta} H_k \right] + \frac{g_2}{m_Q} Tr \left[ \bar{H}_k \gamma_\mu \gamma_5 A_\mu H_k \right] + \ldots \tag{74}
$$

where the ellipses indicate other terms (of higher order, say), and $D_\mu$ contains the photon field. The new terms of order $1/m_Q$ in (74) are a consequence of the chromomagnetic interaction $O_{mag}$ (the second term in equation (43)), and the kinetic interaction $O_{kin}$ (the
third term in (43)). Calculating the diagrams of Fig. 9 and eliminating the divergent integrals and using equations (32), (34) and (48), gives for example

\[ g_1 = m - G_0^2 \left( \frac{\langle \bar{q} q \rangle}{12m} + \frac{f_\pi^2}{6} + \frac{N_c m^2 (3\pi + 4)}{48\pi} - \frac{C_K}{16} \left( \frac{\langle \bar{q} q \rangle}{m} + 3 f_\pi^2 \right) \right) + \frac{(C_K - 2\pi)}{64m^2} \left( \frac{\alpha_s}{\pi} G_0^2 \right), \]  

\[ g_2 = \left( \frac{\pi + 4}{\pi + 2} \right) \frac{\mu_Q^2}{6m}. \]  

(75)  

(76)

As the $1/m_Q$ terms break heavy quark spin symmetry, the chiral Lagrangian in (74) will split in $H(0^-)$ and $H^*(1^-)$ terms respectively.

**B. The weak current to order $1/m_Q$**

In HQEFT the weak vector current at order $1/m_Q$ is [8]:

\[ J^\alpha = \sum_{i=1,2} C_i(\mu) J_i^\alpha + \frac{1}{2m_Q} \sum_j B_j(\mu) O_j^\alpha + \frac{1}{2m_Q} \sum_k A_k(\mu) T_k^\alpha, \]  

(77)

where the first terms are given in (18) and (53), the $B_j$’s and $A_j$’s are Wilson coefficients, and the $O_j^\alpha$’s are two quark operators

\[ O_1^\alpha = \bar{q}_L \gamma^\alpha i D Q_v, \quad O_4^\alpha = \bar{q}_L \gamma^\alpha (-i v \cdot \overleftrightarrow{D}) Q_v, \]

\[ O_2^\alpha = \bar{q}_L v^\alpha i D Q_v, \quad O_5^\alpha = \bar{q}_L v^\alpha (-i v \cdot \overleftrightarrow{D}) Q_v, \]

\[ O_3^\alpha = \bar{q}_L i D^\alpha Q_v, \quad O_6^\alpha = \bar{q}_L (-i \overleftrightarrow{D}^\alpha) Q_v. \]  

(78)

The operators $T_k$ are nonlocal and is a combination of the leading order currents $J_i$ and a term of order $1/m_Q$ from the effective Lagrangian (43).

Combining (77) with (15), and adding chiral corrections and the $1/m_Q$ corrections indicated in (53), we obtain for $H = B, D$:

\[ f_H = \frac{1}{\sqrt{M_H}} \left[ \left( C_\gamma(\mu) + C_v(\mu) \right) \alpha_H + \frac{\eta_Q}{m_Q} + \frac{\eta_X}{32\pi^2 f_\pi^2} \right], \]  

(79)

where $C_\gamma, v$ are defined in (18). Here the model dictates us to put $\mu = \Lambda_h$. The quantities $\eta_Q$ and $\eta_X$ are given in [8]. One should note that the quantities $\eta_Q$ for $Q = b, c$ depend on the Wilson coefficients $C_i, B_i, A_i$ in (77) and some hadronic parameters, for instance $\varepsilon_{1,2}$ from (74). The Wilson coefficients entering $f_H$ depends on $m_Q$ through $\ln(m_Q/\mu)$, and therefore
Leading order (LO)  
LO $+1/m_Q$  
LO $+\chi +1/m\bar{Q}$

**Fig. 10:** $f_B$ as a function of $m$ for $\langle \bar{q}q \rangle^{1/3} = -240$ MeV.

$f_H$ is a complicated function of $m_Q$, $\langle \bar{q}q \rangle$, $g_A$, $f_{\pi}$, and the constituent light quark mass $m$. Note that $\langle \alpha_s G^2 \rangle^{1/4}$ is fixed to be around 320 MeV. In Fig. 10, $f_B$ is plotted as function of $m$ for standard values of the other parameters. One should note that bigger values of $|\langle \bar{q}q \rangle|$ give higher values of $f_B$. For a discussion of the numerical values of our parameters, see [9] and [27].

## VI. APPLICATIONS

In this section we focus on the chiral quark model aspects, especially contributions proportional to the gluon condensate. There are always additional chiral loop corrections which can be found in [5, 9, 13, 14] and references therein.

### A. $B - \bar{B}$ mixing and heavy quark effective theory

At quark level, the standard effective Lagrangian describing $B - \bar{B}$ mixing is [28]

$$\mathcal{L}_{\text{eff}}^{\Delta B=2} = -\frac{G_F^2}{4\pi^2} M_W^2 (V_{tb}^* V_{tq})^2 S_{1L} (x_t) \eta_B b(\mu) Q_B ,$$

where $G_F$ is Fermi’s coupling constant, the $V$’s are KM factors (for which $q = d$ or $s$ for $B_d$ and $B_s$ respectively) and $S_{1L}$ is the Inami-Lim function due to short distance electroweak
loop effects for the box diagram. The quantity $Q_B \equiv Q(\Delta B = 2)$ is a four quark operator

$$Q_B = \bar{q}_L \gamma^\alpha b_L \bar{q}_L \gamma_\alpha b_L,$$  \hspace{1cm} (81)

where $q_L$ ($b_L$) is the left-handed projection of the $q$ ($b$)-quark field. The quantities $\eta_B = 0.55 \pm 0.01$ and $b(\mu)$ are calculated in perturbative quantum chromodynamics (QCD). At the renormalization scale $\mu = m_b (\simeq 4.8 \text{ GeV})$ one has $b(m_b) \simeq 1.56$ in the naive dimension regularization scheme. The matrix element of the operator $Q_B$ between the meson states is parameterized by the bag parameter $B_{B_q}$:

$$\langle B | Q_B | B \rangle \equiv \frac{2}{3} f_B^2 M_B^2 B_{B_q}(\mu),$$  \hspace{1cm} (82)

where by definition, $B_{B_q} = 1$ within the factorized limit. In general, the matrix element of the operator $Q_B$ is dependent on $\mu$, and thereby $B_{B_q}$ also depends on $\mu$. As for $K - \bar{K}$ mixing, one defines a renormalization scale independent quantity

$$\hat{B}_{B_q} \equiv b(\mu) B_{B_q}(\mu).$$  \hspace{1cm} (83)

The $\Delta B = 2$ operator in equation (81) can for $\Lambda_\chi < \mu < m_b$ be written \cite{29, 30}:

$$Q_B = C_1 Q_1 + C_2 Q_2 + \frac{1}{m_b} \sum_i h_i X_i + \mathcal{O}(1/m_b^2).$$  \hspace{1cm} (84)

The operator $Q_1$ is $Q_B$ for $b$ replaced by $Q_v^{(\pm)}$, while $Q_2$ is generated within perturbative QCD for $\mu < m_b$. The operators $X_i$ are taking care of $1/m_b$ corrections. The quantities $C_1, C_2, h_i$ are Wilson coefficients. The operators are given by

$$Q_1 = 2 \bar{q}_L \gamma^\mu Q_v^{(+)} \bar{q}_L \gamma_\mu Q_v^{(-)},$$  \hspace{1cm} (85)

$$Q_2 = 2 \bar{q}_L \gamma^\mu Q_v^{(+)} \bar{q}_L \gamma_\mu Q_v^{(-)},$$  \hspace{1cm} (86)

$$X_1 = 2 \bar{q}_L iD^\mu Q_v^{(+)} \bar{q}_L \gamma_\mu Q_v^{(-)} + \ldots.$$  \hspace{1cm} (87)

The explicite expressions for the operators $X_i$ are given in \cite{13}. There are also non-local operators constructed as time-ordered products of $Q_{1,2}$ and the first order HQEFT Lagrangian in \cite{13}. The Wilson coefficients $C_1$ and $C_2$ have been calculated to NLO \cite{29} and for $\mu = \Lambda_\chi$ one has $C_1(\Lambda_\chi) = 1.22$ and $C_2(\Lambda_\chi) = -0.15$. The coefficients $h_i$ have been calculated to leading order (LO) in \cite{30}.
In order to find all the matrix element of $Q_{1,2}$, one uses the following relation between the generators of $SU(3)_c$ ($i, j, l, n$ are colour indices running from 1 to 3):

$$\delta_{ij}\delta_{ln} = \frac{1}{N_c}\delta_{in}\delta_{lj} + 2t_a^i t_a^l,$$

(88)

where $a$ is an index running over the eight gluon charges. This means that by means of a Fierz transformation, the operator $Q_1$ in (85) may also be written in the following way (there is a similar expression for $Q_2$):

$$Q_1^F = \frac{1}{N_c}Q_1 + 4q_L t^a \gamma^\mu Q^{(+)} v q_L t^a \gamma^\mu Q^{(-)} v.$$

(89)

The first (naive) step to calculate the matrix element of a four quark operator like $Q_1$ is to insert vacuum states between the two currents. This factorized limit means to bosonize the two currents in $Q_1$ and multiply them (see (53)). The second operator in (89) is genuinely non-factorizable. In the approximation where only the lowest gluon condensate is taken into account, the last term in (89) can be written in a quasi-factorizable way by bosonizing the heavy-light colour current with an extra colour matrix $t^a$ inserted and with an extra gluon emitted as shown in Fig. 11.

We find the bosonized colour current:

$$(\bar{q}L t^a \gamma^\alpha Q^{(\pm)}_v)_1 \rightarrow -\frac{G_H g_s}{8} G_{\mu\nu} Tr \left[ i^\alpha \gamma^\alpha L H^{(\pm)} \left( \pm i I_2 \{\sigma^\mu, \gamma \cdot v\} + \frac{1}{8\pi} \tau^\mu \right) \right],$$

(90)

where $\{ , \}$ symbolizes an anti-commutator. The result for the right part of the diagram with $\bar{B}$ replaced by $B$ is obtained by changing the sign of $v$ and letting $P_5^{ (+)} \rightarrow P_5^{ (-)}$. Multiplying the coloured currents, we obtain the non-factorizable parts of $Q_1$ and $Q_2$ to first order in the gluon condensate by using eq. (30).

Now the bag parameter can be extracted and may be written in the form:

$$\hat{B}_{Bq} = \frac{3}{4} \frac{\tilde{\chi}}{\tilde{b}} \left[ 1 + \frac{1}{N_c} (1 - \delta_B^G) + \frac{\tau_b}{m_b} + \frac{\tau_\chi}{32\pi^2 f^2} \right],$$

(91)
where the parameter $\tilde{b}$ also involves the Wilson coefficients $C_{\gamma,v}$ defined in (18):

$$\tilde{b} = b(m_b) \left[ \frac{C_1 - C_2}{(C_{\gamma} + C_v)^2} \right]_{\mu=\Lambda_\chi}.$$ (92)

The soft gluonic non-factorizable effects are given by

$$\delta^B_G = \frac{N_c \langle \frac{q^2}{\pi^2} G^2 \rangle}{32 \pi^2 f^2 f_B^2} \frac{m}{M_B} \kappa_B \left[ \frac{C_1}{C_1 - C_2} \right]_{\mu=\Lambda_\chi}.$$ (93)

where $\kappa_B$ is a dimensionless hadronic parameter which depends on $m, f, \mu_\gamma^2$ and $g_A$ and is of order 2. Note that we are qualitatively in agreement with [31], where also a negative contribution to the bag factor from soft gluon effects is found. Numerically, $f$ and $f_B$ are of the same order of magnitude, and $\delta^B_G$ is therefore suppressed like $m/M_B$ compared to the corresponding quantity

$$\delta^K_G = \frac{N_c \langle \frac{q^2}{\pi^2} G^2 \rangle}{32 \pi^2 f^4}.$$ (94)

found for $K - \bar{K}$ mixing [5].

However, one should note that $f_B$ scales as $1/\sqrt{M_B}$ within HQEFT, and therefore $\delta^B_G$ is still formally of order $(m_b)^0$. The quantity $\tau_b$ represents the $1/m_b$ corrections due to the operators $X_i$. Furthermore, the quantity $\tau_\chi$ represents the chiral corrections (including counterterms) to the bosonized versions of $Q_{1,2}$. The bag parameter $\hat{B}$ is plotted as function of $m$ in Fig. 12 for the case $B_s$. Our results are numerically in agreement with lattice results [32].
B. The processes $B \to D^{(*)} \overline{D}^{(*)}$

It has been observed\cite{14} that the processes $\bar{B}_{d,s}^0 \to D_{s,d}^0 \overline{D}_{s,d}$, $\bar{B}_{d,s}^0 \to D_{s,d}^{*} \overline{D}_{s,d}$, $\bar{B}_{d,s}^0 \to D_{s,d}^{*} \overline{D}_{s,d}$, and $\bar{B}_{d,s}^0 \to D_{s,d}^{*} \overline{D}_{s,d}$, have no factorized contribution from the spectator mechanism. If one or two of the $D$-mesons in the final state are vectors, there are relatively small contributions from the annihilation mechanism. The effective non-leptonic Lagrangian at quark level has the usual form\cite{28}:

$$\mathcal{L}_W = -4 \frac{G_F}{\sqrt{2}} V_{cb} V_{cq}^* \sum_i a_i \hat{Q}_i(\mu).$$  \hspace{1cm} (95)

In our case there are only two numerically relevant operators (for $q = d, s$):

$$\hat{Q}_1 = (\overline{Q}_L \gamma^\alpha b_L) (\overline{c}_L \gamma_\alpha c_L); \quad \hat{Q}_2 = (\overline{Q}_L \gamma_b b_L) (\overline{c}_L \gamma_\alpha c_L).$$ \hspace{1cm} (96)

At $\mu = m_b$, one has $a_2 \sim 1$ and $a_1 \sim 1/10$.

Using (98), we obtain the Fierzed version of the operators $\hat{Q}_{1,2}$:

$$\hat{Q}_1^F = \frac{1}{N_c} \hat{Q}_2 + 2(\overline{Q}_L \gamma^\alpha t^a b_L) (\overline{c}_L \gamma_\alpha t^a c_L)$$

$$\hat{Q}_2^F = \frac{1}{N_c} \hat{Q}_1 + 2(\overline{Q}_L \gamma^\alpha t^a b_L) (\overline{c}_L \gamma_\alpha t^a c_L)$$ \hspace{1cm} (97)

The genuine non-factorizable $1/N_c$ chiral Lagrangian terms from “coloured quark operators” can be estimated within the $HL\chi QM$. However, in order to do this we have to treat the effective weak non-leptonic Lagrangian in (95) within heavy quark effective theory (HQEFT)\cite{8}. Then $b, c$, and $\overline{c}$ quarks are replaced by their corresponding operators in HQEFT:

$$b \to Q^{(+)\varepsilon}_{b\varepsilon}, \quad c \to Q^{(+)\varepsilon}_{c\varepsilon}, \quad \overline{c} \to Q^{(-)\varepsilon}_{\overline{c}\varepsilon}$$ \hspace{1cm} (98)

up to $1/m_b$ and $1/m_c$ corrections. Then the effective weak non-leptonic Lagrangian (95) can be evolved down to the scale $\mu \sim \Lambda_H \sim 1\text{ GeV}$\cite{33}. At $\mu = 1\text{ GeV}$ we have $a_2 \simeq 1.29 + 0.08i$, and $a_1 \simeq -0.35 - 0.07i$. Note that $a_{1,2}$ are complex for $\Lambda_H < \mu < m_c$\cite{33}.

The bosonized factorized weak Lagrangian corresponding to Fig. 13 and the operator $\hat{Q}_2$ (with the dominating Wilson coefficient $a_2$) is obtained from (15), and (19), and (95):

$$\mathcal{L}_{W-Fact}^{\text{Bos}}(Q_2) = 4 \frac{G_F}{\sqrt{2}} V_{cb} V_{cq}^* a_2 + \frac{a_1}{N_c} \zeta(\omega) H^{(+)} \gamma^\alpha L H^{(+)} \gamma^\alpha \overline{L} H^{(+)} \gamma^\alpha \overline{L} H^{(+)} \gamma^\alpha \overline{L} H^{(+)} \gamma^\alpha \overline{L} H^{(+)} \gamma^\alpha \overline{L} H^{(+)} \gamma^\alpha \overline{L},$$  \hspace{1cm} (99)

where $\omega \equiv v_b \cdot v_c = v_b \cdot \overline{v} = M_B/(2M_D)$. This Lagrangian (corresponding to the spectator mechanism) contributes to the factorized amplitude for the process $\overline{B}^0 \to D^+ D^{-}$, and is the
FIG. 13: Factorized contribution for $\bar{B}^0 \rightarrow D^+ D_s^-$ through the spectator mechanism, which does not exist for the decay mode $\bar{B}^0 \rightarrow D_s^+ D_s^-$. The double dashed lines represent heavy mesons, the double lines represent heavy quarks, and the single lines light quarks.

FIG. 14: Factorized contribution for $\bar{B}^0 \rightarrow D_s^+ D_s^-$ through the annihilation mechanism, which give zero contributions if both $D_s^+$ and $D_s^-$ are pseudo-scalars.

starting point for chiral loop contributions of order $(m_K g_A / 4\pi f)^2$ (which are $1/N_c$ suppressed) to the processes $B \rightarrow D^{(*)} \bar{D}^{(*)}$.

The bosonized factorized weak Lagrangian corresponding to Fig. 14 and the non-dominating Wilson coefficient $a_1$ is

$$\mathcal{L}_{W-\text{Fact}}^{\text{Bos}}(Q_1) = 4 \frac{G_F}{\sqrt{2}} V_{cb} V_{cd}^* (a_1 + \frac{a_2}{N_c}) \frac{\alpha_H}{2} \text{Tr} \left[ \xi^{\mu} \gamma \bar{L} H_b^{(+)} \right] \cdot \text{Tr} \left[ H_c^{(+)} \gamma^\alpha L H_c^{(-)} \right]$$

where $\lambda \equiv \bar{v} \cdot v = (M_B^2/(2M_D^2) - 1)$. Unless one or both of the $D$-mesons in the final state are vector mesons, this matrix element is zero due to current conservation, which is analogous to the decay mode $\bar{D}^0 \rightarrow K^0 \bar{K}^0$.

The genuine non-factorizable part for $\bar{B}^0 \rightarrow D_s^+ D_s^-$ at quark level can, by means of Fierz transformations and the identity $[88]$, be written in terms of colour currents. The left part in Fig. 15 with gluon emission gives us the bosonized colour current which is the same as
FIG. 15: Non-factorizable contribution for $B^0 \rightarrow D_s^+ D_s^-$ through the annihilation mechanism with additional soft gluon emission. The wavy lines represent soft gluons ending in vacuum to make gluon condensates.

for $B - \overline{B}$ mixing in eq. (90). For the creation of a $D \bar{D}$ pair in the right part of Fig. 15 there is an analogue of (90), which can be written:

\[
\left( Q_{\nu_e}^{(+)} t^a \gamma^a L Q_{\nu_e}^{(-)} \right)_{1G} \rightarrow \frac{G_s^2 g_s}{32 \pi m} G_{\mu \nu}^a Tr \left[ H^{(+)}_c \gamma^a L H^{(-)} \gamma^\mu \gamma^\nu \right] X^{\mu \nu}
\]

where

\[
X^{\mu \nu} \equiv \frac{r(-\lambda)}{\pi} \sigma^{\mu \nu} + \frac{1}{4(\lambda - 1)} \{ \sigma^{\mu \nu}, \gamma \cdot t \}
\]

and $t \equiv v_c - \bar{v}$. Multiplying the currents and using (50) we obtain a bosonized effective Lagrangian as the product of two traces. Note that our non-factorizable amplitudes (proportional to the gluon condensate) are proportional to the numerically favourable Wilson coefficient $a_2$.

The gluon condensate contribution obtained from (90) and (101) is a linear combination of terms like:

\[
Tr \left[ \xi^+ \sigma^{\mu \alpha} L H_b^{(+)} \right] Tr \left[ H^{(+)}_c \gamma_{\alpha} L H^{(-)}_c \gamma_{\mu} \right],
\]

\[
Tr \left[ \xi^+ \gamma^\mu L H_b^{(+)} \right] Tr \left[ H^{(+)}_c \gamma^\alpha L H^{(-)} \gamma_{\mu} \sigma_{\alpha} R \right],
\]

\[
Tr \left[ \xi^+ \gamma^\mu L H_b^{(+)} \right] Tr \left[ H^{(+)}_c \gamma_{\mu} L H^{(-)}_c \gamma_{\alpha} \right],
\]

\[
Tr \left[ \xi^+ L H_b^{(+)} \right] Tr \left[ H^{(+)}_c \gamma^\alpha L H^{(-)}_c \gamma_{\alpha} \right],
\]
These terms might have been written down based on the heavy quark symmetry, but the HL$\chi$QM selects a certain linear combination to be realized.

Our amplitudes for $B \to D\bar{D}$, in terms of chiral loop and gluon condensate contributions, are sensitive to $1/m_c$ corrections and counterterms which are not yet calculated [14]. Operators suppressed by $1/m_Q$ are obtained by the replacements

$$Q_v^{(\pm)} \to \frac{1}{m_Q} i\gamma \cdot D_\perp Q_v^{(\pm)}; \quad D_\perp^\nu = D^\nu - v^\nu (v \cdot D),$$

for one of the heavy quarks in (96) and (97). Some new quark operators of order $1/m_Q$ might also be generated by pQCD for $\mu < m_Q$. Counterterms correspond to mass insertion of $\tilde{M}_q$, given by (14) and (26), at light quark lines in the diagrams for $B \to D\bar{D}$ this subsection.

C. Other applications

Within the HL$\chi$QM, the process $B \to D\eta'$ has been estimated [11]. This is done in two steps. First we calculate the subprocess $B \to Dgg^*$. Then the virtual gluon $g^*$ is attached to the $\eta'gg^*$-vertex, and the other end in vacuum and make a gluon condensate together with one of the other soft gluons ($g$) from the $\eta'gg^*$-vertex. Using Fierz transformations for the four quark operators for $b \to c\bar{d}u$, we obtain contributions corresponding to Fig. 16.

We have used existing parameterizations of the $\eta'gg^*$-vertex form factor and assumed that the current for $B \to g^*$ is related to the better known case $B \to \rho$. It turns out that the “factorizable” diagram to the left in figure 16 can be neglected compared to the non-factorizable diagram to the right. For $m$ in the range 230-270 MeV, we obtained the
result \[ Br(B \to D\eta^{'}) = (2.2 \pm 0.4) \times 10^{-4} \]. Here \( 1/m_Q \) and chiral corrections are not included.

Heavy to light non-leptonic processes like \( B, D \to K\pi \) cannot in general be treated within the \( HL\chiQM \) in its present form. (See, however, section VII-B). Still, semileptonic heavy to light processes might be treated at the “no recoil point” \(^2\). The form factors \( f_+(q^2) \) and \( f_-(q^2) \) are defined as:

\[
\langle \pi^+(p_\pi) | \overline{\pi}^{\alpha} b | H \rangle = 2 \langle \pi^+(p_\pi) | J_\gamma^\alpha | H \rangle = f_+(q^2)(p_H + p_\pi)^\alpha + f_-(q^2)(p_H - p_\pi)^\alpha \quad (104)
\]

where \( p_H^\alpha = M_H v^\alpha \) and the index \( \alpha \) corresponds to quark flavour \( u \) and \( q^\mu = p_H^\mu - k_\pi^\mu \). The form factors get contributions from \( J_\gamma^\alpha(0) \) in (15) and \( J_\gamma^\alpha(1) \) in (55) close to the “no recoil point” where \( v \cdot p_\pi \) is small:

\[
f_+(q^2) + f_-(q^2) = -\frac{1}{\sqrt{2} M_H f_\pi} (C_\gamma + C_v - g_A C_\gamma) \alpha_H ,
\]

\[
f_+(q^2) - f_-(q^2) = -C_\gamma \frac{\sqrt{M_H}}{\sqrt{2} f_\pi} \left( \frac{g_A v^\alpha}{v \cdot p_\pi} + \alpha_{H\gamma}^{(1)} \right) ,
\]

where we have neglected terms of first order in \( v \cdot p_\pi \) (where \( \alpha_{H\gamma}^{(1)} \) contributes). The \( 1/v \cdot p_\pi \) term in (106) is due to the \( H^* \) pole. From equation (105) and (106) we see that

\[
(f_+(q^2) + f_-(q^2))/(f_+(q^2) - f_-(q^2)) \sim 1/M_H
\]

which is the well known Isgur-Wise scaling law \(^3\). The equations for the two form factors \( f_+ \) and \( f_- \) should be studied further, and chiral corrections and \( 1/m_Q \) corrections should be added.

**VII. FURTHER POSSIBLE EXTENSIONS OF CHIRAL QUARK MODELS**

In this section we consider two possible extensions of chiral quark models which are not yet worked out in detail. The descriptions are therefore sketchy.

**A. Inclusion of light vectors**

One might include vectors in the chiral perturbation theory \(^35\) and thus it should be possible to use the chiral quark model also in this case. We suggest a Lagrangian

\[
\mathcal{L} = \mathcal{L}_{mass} + \mathcal{L}_{\chiQM} + \mathcal{L}_{IVA} ,
\]

(108)
where the interaction between quarks and the vectors and axial vectors is given by
\[
\mathcal{L}_{IVA} = \chi \left[ h_V \gamma^\mu V_\mu + h_A \gamma_5 A_\mu \right] \chi .
\] (109)
Here \( V \) are given as \( \Pi \) in (3) with \( \pi \) replaced by \( \rho \) etc, and similarly for the axial vector \( A \) where \( \pi \) is replaced by \( a_1 \). The (bare) mass term is
\[
\mathcal{L}_{mass} = \bar{m}_V^2 \text{Tr} [V_\mu V^\mu] + \bar{m}_A^2 \text{Tr} [A_\mu A^\mu] .
\] (110)
After quarks are integrated out, the masses are modified and identified with the physical ones. Then a kinetic term is also generated:
\[
\mathcal{L}_{kin} = -\frac{1}{2} [V_\mu V^\mu] - \frac{1}{2} [A_\mu A^\mu] ;
\] (111)
where for \( X = V, A \):
\[
X_{\mu\nu} = \nabla_\mu X_\nu - \nabla_\nu X_\mu ,
\] (112)
and similar for the axial vector. Here \( \nabla \) is a covariant derivative including the goldstones:
\[
\nabla_\mu X_\nu \equiv \partial_\mu X_\nu + i [V_\mu, X_\nu] .
\] (113)

For the left-handed current for \( vac \rightarrow X = V, A \) we find the \( SU(3) \) octet current
\[
J^n_\mu (vac \rightarrow X) = \frac{1}{2} k_X \text{Tr} [\Lambda^n X_\mu] ,
\] (114)
where the quantity \( \Lambda^n \) is given by (36).

As previously, bosonization gives constraints on the parameters of the vectorial sector. From normalization of the kinetic term(s) we obtain:
\[
\frac{f^2 h_V^2}{3m^2} \left[ 1 - \frac{1}{15m^2 f^2} \left( \frac{\alpha_s}{\pi} G^2 \right) \right] = 1 ,
\] (115)
where \( h_A = h_V \) before chiral corrections. For the currents we obtain
\[
k_V = \frac{1}{2} h_V \left( -\frac{\langle \bar{q}q \rangle}{m} + f^2 - \frac{1}{8m^2} \left( \frac{\alpha_s}{\pi} G^2 \right) \right) ,
\] (116)
and similarly, for the axial case:
\[
k_A = \frac{1}{2} h_A \left( -\frac{\langle \bar{q}q \rangle}{m} - 3f^2 + \frac{1}{8m^2} \left( \frac{\alpha_s}{\pi} G^2 \right) \right) .
\] (117)

The formalism suggested in this subsection might for instance, when combined with HL\( \chi \)QM, give a reasonable description of the weak current for \( D \)-meson decays \( D \rightarrow V \) [36]. This might also be the case for processes like \( D \rightarrow VP \), where \( V \) is a vector meson and \( P \) is a pseudoscalar. In the last case, non-factorizable contributions can be calculated in terms of chiral loops and gluon condensates. However, one should keep in mind that a limitation in this case is that \( V \) is (also formally) light compared to \( D \).
B. Heavy to light transitions

As emphasized in the introduction, the HLχQM is not suited to describe $B \to \pi$ transitions except for semileptonic transitions close to the no recoil point. It might therefore be surprising that we consider a formalism for chiral perturbation theory for $B \to \pi$ transitions (and more general $B \to P$ for $P = \pi, K, \eta$), because the involved pion is hard. However, in general, in a transition $B \to \pi$ and other pions, we might have a configuration where one pion is hard and one (or more) is soft. For such cases we split the pseudoscalar sector in hard and soft pseudoscalars. The soft pseudoscalars are represented as before, while the hard pseudoscalars are represented by an octet $3 \times 3$ matrix $M$ given as $\Pi$ in eq. (3), but transforming as $\Sigma$ under $SU(3)_L \times SU(3)_R$.

Starting with a $\gamma_5$ coupling for quarks coupling to pseudoscalars, we represent the hard light quark with a quark field $q_n \ [16, 37]$ and the soft light quarks by the flavour rotated fields $\chi$ in section III. Then we arrive at an interaction Lagrangian

$$\mathcal{L}_n = G_M \bar{\chi} \left[ \xi^\dagger MR - \xi M^\dagger L \right] q_n ,$$

(118)

for a hard light quark $q_n$ entering a hard pion (kaon) with momentum $p_M = E n$ where $n$ is a lightlike vector and $E$ is the energy of the hard pion(kaon). The hard quark has then momentum $p_q = E n + k$, where $k$ is of order $\Lambda_\chi \sim 1 \text{ GeV}$ or smaller. $G_M$ is a coupling which has to be determined by some physical requirements. For an outgoing hard quark we have

$$\mathcal{L}_n = G_M \bar{q}_n \left[ M \xi^\dagger R - M^\dagger \xi L \right] \chi .$$

(119)

Now one might combine (118) and (119) with HLχQM and use some version of a large energy effective field theory (LEET) [37] to describe the light hard quarks. Using the LEET propagator

$$\frac{\gamma \cdot n}{2n \cdot k}$$

(120)

for the light hard quark, we can write down a quark loop diagram for $B \to P$ with a corresponding amplitude for the heavy-light weak current (to leading order)

$$J_\gamma^\alpha (B \to P) = K Tr \left[ \Gamma^\alpha H_{\text{eh}} \gamma \cdot n \xi M^\dagger \right] .$$

(121)
Given the transformation properties in (7), (12), and (24), the current (121) transforms as in (10).

The behaviour of the quantity (form factor) $K$ is known from theoretical considerations within LEET [37] and soft collinear effective theory (SCET) [16]:

$$K \sim E \zeta^{(v)}(M_B, E),$$

where $\zeta^{(v)}$ is expected to scale as

$$\zeta^{(v)}(M_B, E) \sim \frac{\sqrt{M_B}}{E^2}.\quad (123)$$

Note that a factor $\sqrt{M_B}$ is associated with the heavy ($B$) meson wave function and similarly a factor $\sqrt{E}$ with the wave function of the hard pseudoscalar meson. Within our framework, $K$ will contain the product of the couplings $G_H$ and $G_M$, and some loop integrals involving the heavy quark propagator, the ordinary Dirac propagator for the soft quark, and the LEET propagator in (120). However, it has been pointed out that the LEET propagator is too singular to give meaningful loop integrals [38], and that the LEET is incomplete [16]. Therefore the simple expression in (120) has to be modified in some way, by keeping $n^\mu n_\mu = \delta^2 \neq 0$ with $\delta \sim 1/E$, by adding a small quantity in the LEET propagator denominator, or by modifying the formalism in other ways. And this modification has to be done such that one does not come in conflict with the known scaling properties of $\zeta^{(v)}$. Keeping $\delta \neq 0$ and $\delta \sim 1/E$, some of the involved loop integrals have the same mathematical form as those involved in the Isgur-Wise function in (70), but with $\omega \to 1/\delta$. The most plausible scenario is that $G_M \sim E^{-3/2}$. Anyway, knowledge of $\zeta^{(v)}$ will put restrictions on $G_M$.

The $W \to \pi$ transition is in [15, 16] represented by an integral over a momentum distribution proportional to $x(1 - x)$ dominated at $x \sim 1/2$. However, there are also suppressed contributions (for $E >> \Lambda_\chi$) from momentum configurations where one quark (anti-quark) is hard and the anti-quark (quark) is soft. The left-handed current is in this case given by

$$j^l_\mu = \bar{q}_n \gamma_\mu L (\lambda^l \chi), \quad \text{or} \quad j^l_\mu = \bar{\chi} (\xi^\dagger \lambda^l) \gamma_\mu L q_n,\quad (124)$$

where $l$ is an $SU(3)$ octet index. These quark currents will, when combined with (118) and (119), generate a bosonized current

$$\Delta J^l_\mu = N \bar{n}^\mu Tr \left[ \lambda^l (\Sigma M^I + M\Sigma^I) \right],\quad (125)$$
where $\tilde{n}$ is another (almost) lightlike vector with opposite three momentum compared to $n$ such that $\tilde{n}^2 = \delta^2$ and $\tilde{n} \cdot n = 2 - \delta^2$. In the most plausible scenario mentioned above $N$ scales as a constant ($E^0$), which is suppressed by $1/E$ compared with the leading order current $\sim E f_P^{(0)} \tilde{n}^\mu$. The physical decay constant $f_P$ (for $P = \pi, K, \eta$) is within this scheme given by $f_P^{(0)}$ plus the suppressed contribution $\sim N/E$ from (125).

Now, the product of the currents in (121) and (125) will give a factorized $1/E$ suppressed contribution to $B \to K\pi$ corresponding to the diagram in Fig. 13, with $\bar{c}$ and $c$ replaced by energetic (anti) quarks, $D$ by $\pi$ and $D_s$ by $K$. This contribution can of course not be distinguished from the standard factorized contribution. However, pulling out soft pseudoscalars from $\xi$ and $\Sigma$ in the currents (121) and (125), we obtain $1/E$ suppressed non-factorizable chiral loop contributions to $B \to K\pi$. Similarly there will be $1/E$ suppressed gluon condensate contributions. Such suppressed terms are not in conflict with QCD factorization [15].

VIII. CONCLUSION

We have presented the main features of chiral quark models, both in the pure light and the heavy-light sector. Especially, the HLχQM seem to work well. In that case, it is possible to systematically calculate the $1/m_Q$ corrections as well as chiral corrections. The model may be used to give predictions for many quantities. Especially, it is suitable for calculation of the $B$-parameter for $B - \bar{B}$ mixing [14], and for a study of processes of the type $B \to D\bar{D}$. For heavy to light transitions ($B \to K\pi$, say) the HLχQM cannot be used in its present form. It remains to be seen if the extension indicated in sect VII-B to incorporate light energetic quarks will lead to some understanding of such decays.

In our version of the chiral quark models (pure light and heavy light cases) soft gluon effects are truncated to include only the second order gluon condensate. It has worked reasonably well up to now, but one may wonder if this is enough to accommodate all effects, for instance when light vectors are included. Maybe for instance higher order gluon condensates could be included, but our simple model will of course then be much more complicated.
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