Frequency-dependent squeezing generation with EPR entanglement

B Garaventa¹, M Bawaj², M De Laurentis³, S Di Pace⁴, B D’Angelo¹, I Khan⁵, L Naticchioni⁴, C Nguyen⁶, V Sequino¹, F Sorrentino¹, J P Zendri⁷

¹ Department of Physics, University of Genova, Italy and INFN section of Genova
² INFN section of Perugia
³ Department of Physics, University of Napoli “Federico II”, Italy and INFN Napoli
⁴ Department of Physics, University of Roma “La Sapienza”, Italy and INFN Roma1
⁵ Gran Sasso Science Institute, Italy and INFN Roma2
⁶ Department of Physics, University of Paris-Diderot, APC laboratory, France
⁷ INFN section of Padova

E-mail: barbara.garaventa@ge.infn.it

Abstract. The sensitivity of gravitational wave interferometric detectors is ultimately limited by the quantum noise, which arises from the quantum nature of light and it is driven by vacuum fluctuations of the optical field entering from the dark port of the interferometer. One way to improve the sensitivity of gravitational wave interferometers is to inject squeezed vacuum into the dark port. This has been already demonstrated for the main gravitational wave detectors (GEO, Advanced LIGO and Advanced VIRGO). We are studying tricks to produce a “frequency-dependent squeezing”: a standard method is to filter the squeezed optical field with one or more optical cavities (300 m long cavities). An alternative method using a pair of squeezed EPR (Einstein-Podolsky-Rosen) entangled beams to produce frequency-dependent squeezing by a non-degenerate OPO (Optical Parametric Oscillator) will be discussed in this paper. This method promises to achieve a frequency-dependent optimization of the injected squeezed light fields without the need for an external filter cavity.

1. Introduction
Gravitational waves (GW) are space-time deformations predicted by Einstein Theory of General Relativity generated by violent astrophysics events, such as supernova explosions or black holes mergers. GW detectors operating with a sensitivity enough to detect them are the two LIGO (in the United States, 4 km arms long) and VIRGO (in Italy, 3 km arms long). They use laser interferometry to detect the influence of GWs on “the optical path” of the light that is moving back and forth between test masses [3]. In order to detect the distance changes of less than $10^{-18}$ m among test masses suspended in vacuum, GW interferometers use Fabry-Perot cavity in each arm to artificially increase the length of the arms [1].

Many sources of noise limit the sensitivity of GW detectors, in particular quantum noise. This has two complementary features: the shot noise (SN), which depends on phase fluctuations of the optical field disturbing the detector at high frequencies, and radiation pressure noise, which depends on amplitude fluctuations of the optical field perturbing the position of suspended mirrors at low frequencies. Radiation pressure noise (RPN) does not limit the current sensitivity...
Figure 1. Sensitivity curve of Advanced Virgo at design input power value assuming 125 W entering the interferometer. In the next generation detectors radiation pressure noise will be relevant for the detector sensitivity. The purple curve is the quantum noise. The black curve is the envelope of all noises.

of interferometric GW detectors, being covered by technical noises. However in the next future, when low-frequency technical noises will be reduced or when squeezed light injection level will be higher, RPN will be relevant for the detector sensitivity (Fig. 1).

A KTP (Potassium Titanyl Phosphate) Optical Parametric Oscillator (OPO) squeezer converts classical vacuum into squeezed vacuum [2], with reduced phase fluctuations, i.e. reduced shot noise. At the same time, due to the Heisenberg uncertainty principle, the amplitude fluctuations (RPN) are larger. For this reason we want to produce a “frequency-dependent squeezing (FDS)”. In this paper, we propose to inject a pair of squeezed EPR (Einstein-Podolsky-Rosen) entangled beams into the dark port of the interferometer to achieve broadband squeezing of the total quantum noise.

1.1. Squeezed states of light
Laser light can be described by coherent states |α⟩ with a Poissonian distribution photon number, where α is a complex number. Coherent states are defined as eigenstates of the annihilation and creation operators ˆa, ˆa† [1]. Mathematically, they can be described applying the displacement operator to the vacuum state:

|α⟩ = e^{(αˆa†−α^∗ˆa)} |0⟩ (1)

Every state has to obey to the Heisenberg uncertainty principle:

Δ^2X · Δ^2Y ≥ 1, (2)

where X and Y are the amplitude and phase quadrature operators defined by the annihilation and creation operators. We define the generalized quadrature operator with an arbitrary angle θ between the amplitude and the phase quadrature applying a rotation: Xθ = X cos θ + Y sen θ.

Squeezed states are states with a reduced variance for at least one angle θ, called squeeze angle,
i.e. with $\Delta^2 \hat{X}_\theta$ the lowest of all quadratures. We define the squeeze operator $\hat{S}(\xi)$:

$$\hat{S}(\xi) = e^{\frac{1}{2}\left((\xi^* a)^2 - (\xi^* a^\dagger)^2\right)},$$  

(3)

where $\xi = re^{i\theta}$ and $r$ and $\theta$ are respectively squeeze parameter and angle.

![Figure 2](image)

**Figure 2.** Phase space representations. Representation of a vacuum state (on the left) and a squeezed vacuum state (on the right) of light.

For coherent states, their quantum uncertainty is depicted with a circle around their amplitudes. Squeezed states of light have a reduced variance below the vacuum variance for at least one angle and their quantum uncertainty is depicted with an ellipse around their amplitude (Fig. 2). So, when we inject phase squeezing, we reduce the uncertainty of the phase quadrature with an increase of the uncertainty of the amplitude quadrature and vice versa. For this reason we want to produce frequency dependent squeezing because depending on the frequency we need a squeezing in phase or in amplitude: in FDS the squeezing ellipse changes its orientation depending on the frequency. This will be obtained by rotating the squeezing angle [2].

1.2. Nonlinear optics

To generate nonclassical states like squeezed states we need higher order polarization effects in media. Consider the polarization $P(E(t))$ due to an electro-magnetic wave traveling through a nonlinear medium:

$$P(E(t)) = \epsilon_0(\chi^{(1)}E(t) + \chi^{(2)}E(t)^{(2)} + \chi^{(3)}E(t)^{(3)} + ..),$$  

(4)

where $\chi$ is the susceptibility and $E(t) = E_0cos\omega_p$. We are interested in those materials that manifest a second order nonlinearity. In the Second-Harmonic Generation (SHG), that is an example of second order nonlinear process, two photons at the frequency $\omega_p$ create one photon at the frequency $2\omega_p$. This has been considered a parametric up conversion process. Parametric down conversion is the reverse process, in which the energy of one photon at the frequency $2\omega_p$ is taken to create two photons at the frequency $\omega_p$ each. This process has been used to produce squeezed states of light. An OPO converts an input laser wave at frequency $2\omega_p$ into two output beams at lower frequency, preserving the energy conservation principle. If the two output beams has the same frequency $\omega_p$, we have a degenerate OPO.

![Figure 3](image)

**Figure 3.** Not-degenerate optical parametric oscillator
If the down conversion process produces two output waves at frequencies $\omega_p \pm \Delta \omega_p$, we have a non-degenerate OPO (Fig. 3), where $\Delta \omega_p$ is the frequency of the sidebands with respect to the carrier at frequency $\omega_p$. In the EPR entanglement experiment we will use a non-degenerate OPO consisting of KTP crystal inside an optical cavity.

2. EPR entanglement

Quantum entanglement is a physical phenomenon of a system described in a Hilbert space $H$ consisting of subsystems [1]. If a system is separable, it can be described by the tensor product of the subsystems; if a system is not separable it is called entangled state. According to EPR, in the case of perfect entanglement in the quadrature operators of two subsystems, there are states such that:

$$\Delta^2(\hat{X}_1 - \hat{X}_2) = 0 \quad \Delta^2(\hat{Y}_1 + \hat{Y}_2) = 0$$

(5)

Consider squeezed states of light at the upper and the lower frequency $\omega_p \pm \Delta \omega$, the variances of the sum and of the difference of the operators depend on squeeze parameter $r$ and they are below the vacuum variance due to their vacuum correlations:

$$\Delta^2(\hat{X}(\omega_p + \Delta \omega) + \hat{X}(\omega_p - \Delta \omega)) = e^{-2r} \leq 1$$

(6)

$$\Delta^2(\hat{Y}(\omega_p + \Delta \omega) - \hat{Y}(\omega_p - \Delta \omega)) = e^{-2r} \leq 1$$

(7)

where, in this case, we have done an amplitude squeezed states, since the amplitude quadrature operators are anti-correlated and the phase quadrature operators are correlated.

2.1. Experimental setup

EPR experiment produces two beams detuned with respect to each other (Fig. 4): one has the frequency of the interferometer (signal), the other (idler) perceives the interferometer like a detuned cavity, i.e. like a filter cavity. Detecting idler we receive information also about signal (the other way around) because these beams are entangled, i.e. they are correlated between them. A fundamental role is played by the noise sidebands because they are precisely correlated (Fig. 4C). The interferometer has a dual use as both the GW detector and the filter, eliminating the need for external filter cavities.

We consider a pump beam at a certain frequency $\omega_p = 2\omega_0 + \Delta$, which is detuned of a $\Delta$ quantity with respect to the interferometer frequency $\omega_0$. By a down conversion process two other beams, with frequencies such that their sum is equal to that of pump, are produced:

$$\omega_s = \omega_0 \quad \omega_i = \omega_0 + \Delta,$$

(8)

where $\omega_s$ is the frequency of signal, $\omega_i$ is the frequency of idler. So, for the idler beam, the interferometer acts as a detuned filter cavity. When traveling out of the interferometer, the collinear signal and idler beams are separated and filtered by the output mode cleaners and measured by beating with local oscillators at frequencies $\omega_0$ and $\omega_0 + \Delta$, respectively. The homodyne measurement of a fixed quadrature of the out-going idler beam conditionally squeezes the input signal beam in a frequency dependent way, thereby achieving the broadband reduction of quantum noise.

A table-top experiment (Fig. 4A) that will make use of a test cavity to test the EPR technique, is under development at the European Gravitational Observatory (EGO) in Cascina, Italy. We will use three lasers with different functions: the first is for the SHG that goes into the OPO and then produces the signal and the idler that will go into the cavity test; the second makes the
Figure 4. On the left (scheme A), conceptual scheme proposal for frequency dependence squeezing generation with EPR entanglement. In the scheme B, as the beams see the interferometer. The idler sees the interferometer detuned. We carry out measurements of an idler fixed quadrature (idler frequency-dependent-squeezed). The signal has the frequency of the interferometer. In the scheme C, correlated sidebands idler and signal: the higher sideband of idler is correlated to the lower sideband of signal and vice versa [3].

local oscillator for the idler; the third creates the local oscillator for the signal. An important role will be played by the Acousto Optic Modulator (AOM), that shift the frequency of light using sound waves (usually at radio-frequency). With these devices we will obtain the squeezer control beams to lock the OPO and the test cavity. Finally, we will also get coherent control beams, one for the idler and one for the signal.

3. Conclusion
EPR entangled beams injected will allow FDS generation necessary in GW interferometers where, depending on the frequency, we need phase or amplitude squeezing. With this method we will obtain the rotation of squeeze angle, i.e. the squeezing ellipse changes its orientation. A table-top EPR experiment is under construction at the EGO in Cascina. The goals of this experiment are to verify the rotation of squeezing angle using a pair of entangled beams and also to evaluate the reduction of RPN noise on a system sensitive to it, that will be provided by another experiment, SIPS (Suspended Interferometric Ponderomotive Squeezer), under development in the University of Rome "La Sapienza". The improvement of the GW interferometric detectors sensitivity will enhance the quality and quantity of GW observations, allowing a deeper study of the sky and unexpected discoveries.

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