The improved model of incentive-pricing of internet schemes for Cobb-Douglas utility function by using LINGO 13.0

F M Puspita1, E Yuliza1, B J Rezky1, A N Y Simarmata1 and Y Hartono2

1 Mathematics Department, Universitas Sriwijaya, Indonesia
2 Mathematics Study Program, Universitas Sriwijaya, Indonesia

*Corresponding author's e-mail: fitrimayapuspita@unsri.ac.id

Abstract. In this paper, an improved model of the incentive-pricing scheme was designed. Previous research only shows the incentive model with the disadvantages of no considering the utility function to measure the satisfaction of the user. Then, the models are improved by deriving from the incorporation of bundling and reverse charging models obtained from previous studies, as well as taking into account the quality of service to users by using the Cobb-Douglas utility function. Optimal pricing schemes applied to the local data server, which is mail traffic data. The model used is the nonlinear optimization problem and solved by LINGO 13.0 to get the optimal solution. The results show that optimal results in the form of pricing schemes by applying flat fee, usage-based, and two-part tariff for homogeneous consumer's schemes and the improved models show better performance in achieving the profit than the previous incentive pricing models.

1. Introduction

In the era of modern technology at this time, the need to access the internet is indispensable in information is critical both locally and globally. The internet service provider (ISP) is trying to compete to become the best by giving its best services to the satisfaction of its customers. ISPs use several ways or strategies to obtain an optimal result, which results in a minimal cost. The way is to perform the method of bundling and considering the utility function [1]. Bundling strategies are undertaken to increase profits because they utilize the method in the form of a merger or doing a combination of existing services. So the best of services - services that are available can be taken that the maximum benefits [2]. This model also uses a utility function. Utility functions can be interpreted by considering the level of satisfaction of users of services that can provide information to the ISP in improving the quality of services so as to achieve the target for the user who will use the services of the provider [3].

Every network needs a pricing mechanism, which varies depending on the level of service. The price on the internet as an important economic incentive for users can be regarded as an effective mechanism for network management [4]. If each user requires resources to maximize the level of satisfaction, it would require some mechanism to provide an incentive for users to increase the overall benefit. Due to various pricing models that have been widely grown, classification is divided into at
least the pricing model dependency services in the technical network, the corresponding difference of price component as a determinant of price, selection of applicable parameters, and the identification of price [5].

In this paper, the models of Improved Reverse Charging (IRC) based on internet usage on 3G and 4G networks are discussed. IRC models are used by service providers to tailor the use of 3G and 4G networks appropriately in accordance with the conditions and the location of the user. If the user is in a position close enough in a big city or crowded area, then most likely used is a 4G network, but if the distance is far enough, users then automatically use the 3G network. IRC is a scheme that is shaped to fit the environment that meets user demand and maximize profit service providers. Reverse Charging Mechanism enables service providers to charge other users of other network providers to access user data [6][7]. IRC models used in this paper are based on the combined model and incentive mechanism [4][8][9]. The combined model will form the Mixed Integer Non-Linear Programming (MINLP) model with an objective function with nonlinear boundaries and an integer value of the decision variables. This model is solved using Lingo because it has several facilities to solve nonlinear models and can be integrated with Ms. Excel. So, the contribution of the paper will be a new improved model of incentive pricing mechanism by considering utility function to measure the homogenous consumer satisfaction, three pricing scheme, namely flat fee/flat rate, usage-based and two-part tariff schemes, and IRC models that are quite rare to discuss in present literature.

2. Methods
The idea of designing the new model is presented in the diagram, as showed in figure 1.

![Diagram](image_url)

**Figure 1.** Flowchart of designing a new model.
In this paper, the determination of the incentive value on the internet by using a combination of optimization models bundling issue, the utility function Cobb-Douglas, optimization of consumer issues, and models are solved by LINGO 13.0. Calculations on this model are applied to the mail traffic data by using secondary data.

3. Result and discussion

The improved model is based on the combined model and incentive mechanisms stated as follows [4][8][9].

Maximize:

\[ R = \sum_{i=1}^{2} \sum_{k=1}^{2} \left( P_{i} - B_{k} \right) X_{ik} - \sum_{k=1}^{2} MY_{k} - X_{a} Y_{b} + P_{X} X + P_{Y} Y + PZ \sum_{k=1}^{2} \sum_{i=1}^{2} \left( PR_{ik} \pm PQ_{ik} \right) + \left( \alpha + \beta l_{i} \right) P_{ik} X_{ik} \] (1)

Subject to:

\[ S_{i} \geq \left( R_{ik} - P_{k} \right) Y_{k} \] (2)

\[ S_{i} = \sum_{k=1}^{2} \left( R_{ik} - P_{k} \right) \] (3)

\[ \left( R_{ik} - P_{k} \right) X_{ik} \geq 0 \] (4)

\[ \sum_{k=1}^{2} X_{ik} \leq 1 \] (5)

\[ X_{ik} \leq Y_{k} \] (6)

\[ S_{i} \geq 0 \] (7)

\[ P_{i} \geq 0 \] (8)

\[ X_{ik} \in \{0,1\} \] (9)

\[ Y_{ik} \in \{0,1\} \] (10)

\[ X_{ik} \leq X_{i} Z_{i} \] (11)

\[ Y_{ik} \leq Y_{i} Z_{i} \] (12)

\[ U \left( X, Y \right) - P_{X} X + P_{Y} Y + PZ \geq 0 \] (13)

\[ Z_{i} \in \{0,1\} \] (14)

\[ I_{i} d_{ik} x_{ik} \leq a_{i} C \] (15)

\[ \sum_{i=1}^{2} \sum_{k=1}^{1} I_{i} d_{ik} x_{ik} \leq a_{i} C \] (16)

\[ \sum_{i=1}^{2} a_{i} \leq 1 \leq 1; \ a_{i} \in \{0,1\} \] (17)

\[ m_{i} \leq I_{i} \leq 1; \ m_{i} \geq 0 \] (18)

\[ 0 \leq x_{ik} \leq n_{i}; \ x_{ik} \geq 0 \] (19)
This model combines the optimization of bundling issues are as follows. Max

\[ R = \sum_{i=1}^{2} \sum_{k=1}^{2} (P_k - B_k) x_{ik} - \sum_{k=1}^{2} MY_k, \]

utility function based on Quasi-linear is in the form

\[ U(X,Y) = (X^a Y^b), \]

the optimization of consumers issues is max \( \theta = U(X,Y) - P_X X + P_Y Y + P_Z, \) and IRC model of max

\[ R = \sum_{i=1}^{2} \sum_{k=1}^{2} ((PR_{ik} \pm PQ_{ik}) + (\alpha + \beta I_i) P_{ik} X_{ik}). \]

The calculation of this model is divided into four cases. The first case \( PQ_{ij} \) increases, \( x \) increases is with the objective function max \( R = \sum_{i=1}^{2} \sum_{k=1}^{2} ((PR_{ik} \pm PQ_{ik}) + (\alpha + \beta I_i) P_{ik} X_{ik}) \) subject to

\[ PQ_{ik} = \left(1 + \frac{x}{Q_{bik}}\right) PB_{ik} Lx, \]

followed by constraint (2) until constraint (27). Second case \( PQ_{ij} \) increases, \( x \) decreases is with the objective function max \( R = \sum_{i=1}^{2} \sum_{k=1}^{2} ((PR_{ik} \pm PQ_{ik}) + (\alpha + \beta I_i) P_{ik} X_{ik}) \) subject to

\[ PQ_{ik} = \left(1 + \frac{x}{Q_{bik}}\right) PB_{ik} Lx, \]

followed by constraint (2) until constraint (27). Third case \( PQ_{ij} \) decreases, \( x \) increases is with the objective function max \( R = \sum_{i=1}^{2} \sum_{k=1}^{2} ((PR_{ik} \pm PQ_{ik}) + (\alpha + \beta I_i) P_{ik} X_{ik}) \) subject to

\[ PQ_{ik} = \left(1 + \frac{x}{Q_{bik}}\right) PB_{ik} Lx, \]

followed by constraint (2) until constraint (27), and the last case \( PQ_{ik} \) decreases, \( x \) decreases is with

\[ PQ_{ik} = \left(1 + \frac{x}{Q_{bik}}\right) PB_{ik} Lx, \]

followed by constraints (2) until constraint (27). The parameters and decision variables are presented in table 1 and table 2 as follows.
### Table 1. Parameters for each improved model.

| Parameter | Description |
|-----------|-------------|
| $B_j$     | The cost of making a bundle for each service $j$ |
| $M$       | Marginal cost if adding more than one bundle service in the menu |
| $R_{ik}$  | The total order price for each $i$-th customer on each favorite service to $k$ |
| $U_i(X,Y_i)$ | The consumer utility function $i$ for the rush hour and clock rate is not busy |
| $P_S$     | The unit price specified by the service provider on the clock is busy |
| $P_Y$     | The unit price specified by the service provider on the clock is not busy |
| $P$       | Cost to be paid by consumers to follow the service |
| $\alpha$  | The base price for class $k$ |
| $\beta$   | Premium quality for every service |
| $C$       | The total capacity available from the network |
| $k$       | The fee for a connection with QoS provided |
| $m_i$     | Minimum QoS for the service $i$ |
| $n_i$     | Number of users of the service $i$ |
| $d_{ik}$  | The capacity required to service $i$ on the network $k$ |
| $Q_{bik}$ | The nominal value of the QoS attribute in the carrier network (kbps) |
| $F$       | The minimum value that the service provider has set for $a_{ik}$ |
| $H$       | Minimum number of payload allowed for $T_i$ (kbps) |
| $K$       | Maximum number of payloads allowed for $T_i$ (kbps) |
| $G$       | The maximum value that the service provider has set for $a_{ik}$ |

Table 1 presents the parameters used in each improved case with a predetermined value. The values of these parameters will be substituted into the equation used.

### Table 2. Decision variables for each improved model.

| Decision variable | Description |
|-------------------|-------------|
| $X_{ik}$          | \[ \begin{cases} 1, & \text{if consumer } i \text{ selects bundle in service } k \\ 0, & \text{if consumer } i \text{ doesn't select bundle in service } k \end{cases} \] |
| $Y_j$             | \[ \begin{cases} 1, & \text{if the service provider offers a bundle of service } k \\ 0, & \text{if the service provider doesn't offer a bundle of service } k \end{cases} \] |
| $S_i$             | Benefit usage for -$i$ customer |
| $X_i$             | Level of the consumer's maximum consumption $i$ on the peak hour service. |
| $Y_i$             | The rate of the consumer's maximum consumption $i$ on the clock service is not busy. |
| $Z_i$             | \[ \begin{cases} 1, & \text{if customer } i \text{ chose to join program} \\ 0, & \text{if customer } i \text{ didn't choose to join program} \end{cases} \] |
| $PQ_{ik}$         | Cost change as long as QoS changes (rupiah) |
| $PB_{ik}$         | Basic cost for a connection with user $i$ and class $k$ |
| $a_{ik}$          | Linear cost factor that user $i$ has when in class $k$ |
| $I_i$             | The base price of the minimum required for service $i$ |
| $T_i$             | Traffic load |
| $L_x$             | Factor of elasticity |
| $x$               | Number of increase or decrease in QoS value |
| $B$               | The specified linear parameter |
Table 2 presents the decision variables of the model used. This variable will produce the values required as a decision from the model.

We present parameter values for the upgraded model and parameter values for homogeneous users in table 3 and table 4. The model was then completed using LINGO 13.0 for each case and for the three pricing schemes.

### Table 3. Parameter values for improv models.

| Parameter                                      | Value |
|------------------------------------------------|-------|
| Cost of connecting user 1 class 1 ($PR_{11}$) | 0.5   |
| Cost of connecting user 1 class 2 ($PR_{12}$) | 0.6   |
| The cost of connecting users 2 class 1 ($PR_{21}$) | 0.4   |
| The cost of connecting users 2 class 2 ($PR_{22}$) | 0.7   |
| Linear parameters ($a$) | 1     |
| Value limit $a_{11}$ | $0.05 \leq a_{11} \leq 0.15$ |
| Value limit $a_{12}$ | $0.06 \leq a_{12} \leq 0.14$ |
| Value limit $a_{21}$ | $0.07 \leq a_{21} \leq 0.13$ |
| Value limit $a_{22}$ | $0.08 \leq a_{22} \leq 0.12$ |
| Limitation of traffic load for $T_l$ | $50 \leq T_l \leq 1000$ |
| The base price for grade 1 ($\alpha_1$) | 0.1   |
| The base price for grade 2 ($\alpha_2$) | 0.2   |
| Total bandwidth ($Q$) | 102400 |
| Minimum user bandwidth 1 ($V_1$) | 1     |
| Minimum user bandwidth 2 ($V_2$) | 1     |

### Table 4. The parameter value for homogeneous consumers.

| Parameter | Flat fee | Usage-based | Two-Part tariff |
|-----------|----------|-------------|-----------------|
| $V_{11}$  | 500      | 500         | 500             |
| $V_{12}$  | 800      | 800         | 800             |
| $V_{21}$  | 600      | 600         | 600             |
| $V_{22}$  | 900      | 900         | 900             |
| $M$       | 200      | 200         | 200             |
| $B_1$     | 300      | 300         | 300             |
| $B_2$     | 500      | 500         | 500             |
| $a$       | 3        | 3           | 3               |
| $b$       | 4        | 4           | 4               |
| $\bar{X}$ | 53.72    | 53.72       | 53.72           |
| $\bar{Y}$ | 59.57    | 59.57       | 59.57           |

Table 5 presents the solution obtained, for a flat fee scheme, the case of increasing $PQ_k$ with an increase in $x$ gets the highest return, with an optimal solution of 35 iterations of 34.9395, the length of time used to solve and produce the model is expressed in ER of 1 second.
Table 5. The solution of the incentive pricing model of internet-based Cobb-Douglas utility functions for homogeneous consumer pricing flat fee scheme.

| Case | Solver Status | $PQ_{ik}$ increases, $x$ increases | $PQ_{ik}$ increases, $x$ decreases | $PQ_{ik}$ decreases, $x$ increases | $PQ_{ik}$ decreases, $x$ decreases |
|------|---------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
|      | Class of Model | INLP                              | INLP                              | INLP                              | INLP                              |
|      | State         | local Optimal                     | local Optimal                     | local Optimal                     | local Optimal                     |
|      | Objective      | value                             | value                             | value                             | value                             |
|      | Infeasibility  | 34.9395                           | 34.9068                           | 2.2                               | 2.2                               |
|      | iterations     | 35                                | 35                                | 28                                | 28                                |
|      | Extender Solver Status |                                | Best Objective | 34.9395 | 34.9068 | 2.2 | 2.2 |
|      | Steps          | 0                                 | 0                                 | 0                                 | 0                                 |
|      | Interval       | 2                                 | 2                                 | 2                                 | 2                                 |
|      | Updater        | 44                                | 44                                | 44                                | 44                                |
|      | GMU (K)        | 1                                 | 0                                 | 0                                 | 0                                 |
|      | ER (Sec)       | 0                                 | 0                                 | 0                                 | 0                                 |

As table 6 suggests, for an improved model using a usage-based scheme, the highest solution of the incentive pricing model is achieved when $PQ_{ik}$ increases, and $x$ increases. In addition, the model class will be integer nonlinear programming (INLP). The number of iterations to complete is 42 times. Next, in table 7, the improved model continues with a two-part tariff scheme with the same case as the previous pricing scheme showed, which $PQ_{ik}$ increases, and $x$ increases.

Table 6. The solution of incentive pricing Model of Internet-Based Cobb-Douglas Utility Functions for Homogeneous Consumer Usage-based Pricing Scheme.

| Case | Solver Status | $PQ_{ik}$ increases, $x$ increases | $PQ_{ik}$ increases, $x$ decreases | $PQ_{ik}$ decreases, $x$ increases | $PQ_{ik}$ decreases, $x$ decreases |
|------|---------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
|      | Model Class   | INLP                              | INLP                              | INLP                              | INLP                              |
|      | State         | local Optimal                     | local Optimal                     | local Optimal                     | local Optimal                     |
|      | Objective      | value                             | value                             | value                             | value                             |
|      | Infeasibility  | 35.3921                           | 35.3594                           | 2.2                               | 2.2                               |
|      | iterations     | 0                                 | 0                                 | 0                                 | 0                                 |
|      | Extender Solver Status |                                | Best Objective | 35.3921 | 35.3594 | 2.2 | 2.2 |
|      | Steps          | 0                                 | 0                                 | 0                                 | 0                                 |
|      | Interval       | 2                                 | 2                                 | 2                                 | 2                                 |
|      | Updater        | 42                                | 42                                | 42                                | 42                                |
|      | GMU (K)        | 0                                 | 0                                 | 0                                 | 0                                 |
|      | ER (Sec)       | 0                                 | 0                                 | 0                                 | 0                                 |

Table 7 presents the solution obtained, for a Two-Part Tariff scheme, the case of increasing $PQ_{ik}$ with an increase in $x$ gets the highest return, with an optimal solution of 42 iterations of 35.3921.
Table 7. The solution of the Incentive Pricing Model of Internet-Based Cobb-Douglas Utility Functions for Homogeneous Consumer Two-Part Tariff Pricing Scheme.

| Case                  | Solver Status | Model Class | INLP | INLP | INLP | INLP |
|-----------------------|---------------|-------------|------|------|------|------|
|                       | Status        | State       | local Optimal | local Optimal | local Optimal | local Optimal |
| PQ increases, x decreases | PQ decreases, x increases | PQ decreases, x decreases | PQ decreases, x decreases |
| Model Class State     | Objective value | 35.3921     | 35.3594     | 2.2 | 2.2 | 2.2 |
| Infeasibility iterations | 0             | 0           | 0           | 25 | 25 | 25 |
| Best Objective Steps | Extender Solver Status | 35.3921 | 35.3594 | 2.2 | 2.2 | 2.2 |
| Interval Updating     | 2             | 2           | 2           | 2  | 2  | 2  |
| GMU (K)               | 42            | 42          | 42          | 42 | 42 | 42 |
| ER (Sec)              | 0             | 0           | 0           | 0  | 0  | 0  |

Of the three tables of solution for three pricing schemes, it can be seen that for objective results obtained, the usage-based and two-part tariff schemes yield the same values for increasing $PQ_{ik}$ and increasing $x$ case. It means that the user can adopt the two schemes based on their preferences with the incentive obtained from ISP. It also means that for ISP, if the user chooses those pricing schemes, ISP enables to give incentive to users that subscribe to the network.

4. Conclusion
Pricing scheme of incentive model of the internet that has the most optimal solution in the homogeneous consumers by the utility function Cobb-Douglas is the usage-based and two-part tariff pricing scheme of the internet. The model gives the user the best incentive and also gives ISP the most benefit to be achieved.

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