Affleck-Dine baryogenesis in inflating curvaton scenario
with $\mathcal{O}(10 - 10^2 \text{ TeV})$ mass moduli curvaton

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Abstract

We study the Affleck-Dine (AD) baryogenesis in the inflating curvaton scenario, when the curvaton is a moduli field with $\mathcal{O}(10 - 10^2 \text{ TeV})$ mass. A moduli field with such mass is known to be free from the Polonyi problem, and furthermore its decay products can explain the present cold dark matter abundance. In our scenario, it further explains the primordial curvature perturbation and the present baryon density all together. The current observational bound on the baryon isocurvature perturbation, which severely constrains the AD baryogenesis with the original oscillating moduli curvaton scenario, is shown to put practically negligible constraint if we replace the oscillating curvaton with the inflating curvaton.
1 Introduction

The existence of moduli fields, light neutral scalar fields which interact with other fields only with the strength of gravitational interaction, is ubiquitous in candidates of more fundamental theory like supergravity or string theory. Since they interact only with the strength of gravity, their life-time is long and their late decay might destroy the successful scenario of the big-bang nucleosynthesis (BBN). This is the so-called Polonyi problem or cosmological moduli problem [1, 2, 3, 4, 5, 6]. The problem can be circumvented if the mass of the lightest moduli field is larger than some 10 TeV scale so that the reheating temperature is higher than \( \sim 1 \) MeV at which BBN starts [2]. Then, if the origin of the baryon number in the universe is the Affleck-Dine (AD) mechanism [7], the dilution of the baryon number by the decay of the moduli field may correctly explain the baryon asymmetry of the universe [8]. Furthermore, if the non-thermal decay of the moduli field produces the wino-like lightest supersymmetric particles (LSPs), it may account for the cold dark matter abundance in the present universe [9]. Thus, a moduli field with \( \mathcal{O}(10-10^2 \text{ TeV}) \) mass combined with the AD baryogenesis may explain the baryon asymmetry and the dark matter abundance of the universe all together. Recently, it has been argued that string compactifications to four dimensions with stabilized moduli may generically have at least one moduli field with \( \mathcal{O}(10-10^2 \text{ TeV}) \) mass, and the above scenarios have been revisited from this viewpoint [10, 11, 12, 13].

It will be very interesting if \( \mathcal{O}(10-10^2 \text{ TeV}) \) mass moduli field can further explain other features of our universe. In this paper, we examine whether \( \mathcal{O}(10-10^2 \text{ TeV}) \) mass moduli field can explain the primordial curvature perturbation as a curvaton [18, 19, 20, 21]. We assume weak-scale supersymmetry with gravity-mediated supersymmetry breaking, where the gravitino has \( \mathcal{O}(10-10^2 \text{ TeV}) \) mass [2]. In this case the moduli field also naturally has \( \mathcal{O}(10-10^2 \text{ TeV}) \) mass. Without special mechanisms, sfermions also acquire the mass of similar scale. However, if the field whose F-term dominates the supersymmetry breaking in the hidden sector is different from the field whose vacuum expectation value generates the gauge couplings, gaugino masses can be suppressed [23, 10, 11, 12, 24, 13]. We will be interested in the case where the LSP is wino-like, in order to explain the present cold dark matter abundance. The AD baryogenesis in the original oscillating curvaton scenario has been studied in [25] where it was found that the observational bound on the baryon isocurvature perturbation puts an important constraint. The constraint is severe when the curvaton is a

\[1\] The interests to the non-thermally produced wino-like dark matter was also boosted by the PAMELA data [14, 15, 16, 17].

\[2\] The anomaly-mediation also works for the non-thermal production of the cold dark matter, but the AD baryogenesis part does not work in the same way as in the gravity-mediation case studied here. See e.g. [22] and references therein.
moduli field, since there are natural mass scales for the moduli field and the AD scalar field in weak-scale supersymmetry scenarios. Interestingly, this constraint naturally leads us to consider the recently proposed inflating curvaton scenario [26], as we explain in section 5. Our scenario for the baryon asymmetry of the universe has an important difference from the above mentioned scenario, and this difference is crucial for the suppression of the baryon isocurvature perturbation. Our scenario for the baryogenesis can be regarded as AD baryogenesis with a version of low-scale inflation with low reheating temperature [27, 28].

Before entering the main body, let us summarize our scenario for the history of the early universe. The relevant scales are: the Hubble expansion rate of the first inflation $H_1$, the decay rate of the inflaton $\Gamma_I$, the mass of the inflaton $m_I$, the Hubble expansion rate of the second inflation $H_2$, the mass of the inflating curvaton $m_{\sigma}$, the mass of the AD scalar field $m_{AD}$, the decay rate of the curvaton $\Gamma_{\sigma} \sim m_{\sigma}^3/M_P^2$. Here, $M_P$ is the reduced Planck scale: $M_P = (8\pi G)^{-1/2} \sim 2.4 \times 10^{18}$ GeV. Our scenario assumes $H_1 > m_I > \Gamma_I > H_2 > m_{\sigma} > m_{AD} > \Gamma_{\sigma}$:

1. The first inflation with the Hubble expansion rate $H_1$. The curvaton acquires quantum fluctuation, which becomes classical at the horizon exit. The phase part of the AD scalar also acquires quantum fluctuation if the Hubble induced A-term is suppressed, and this leads to baryon isocurvature perturbation.

2. When $H \sim m_I$ the first inflation ends and the inflaton starts coherent oscillations.

3. After the time scale $t \sim \Gamma_I^{-1}$, the inflaton decays and reheats the universe. Then the energy density of the universe is dominated by radiation.

4. As the universe expands, the radiation is diluted and later on the potential energy of the curvaton begins to dominate the energy density of the universe. This leads to the second inflation with the Hubble expansion rate $H_2$. The primordial curvature perturbation is produced around the time when the second inflation starts. The pre-existed radiation is diluted away by the second inflation.

5. When $H \sim m_{\sigma}$, the second inflation ends and the curvaton begins coherent oscillations. The energy density of the universe is dominated by the curvaton matter.

6. When $H \sim m_{AD}$, the AD scalar field begins a spiral motion and produces the baryon number.

7. After $H$ becomes much smaller than $m_{AD}$, the baryon number freezes. The AD scalar eventually decays and the baryon number is transmitted to the Standard
Model (SM) particles. The energy density of the AD scalar is always sub-dominant and its decay does not reheat the universe.

8. At \( t \sim \Gamma^{-1}_\sigma \), the curvaton decays and reheats the universe. The cold dark matter is produced from the decay product of the curvaton.

The organization of this paper is as follows. In section 2 we discuss the inflating curvaton scenario, which corresponds to the epoch 1-5 above. In section 3 we discuss the AD baryogenesis in our scenario, which corresponds to the epoch 5-8. In section 4 we discuss the baryon isocurvature perturbation. In section 5 we review an issue in the AD baryogenesis in original oscillating curvaton scenario for a comparison. We end with summary and discussions in section 6. In appendix A we review the argument of [26] for why and under what conditions the inflating curvaton with a quadratic potential cannot dominate the primordial curvature perturbation. In appendix B we review the non-thermal production of the wino-like cold dark matter from the inflating curvaton decay, which corresponds to the epoch 8 above.

2 Inflating curvaton scenario

An interesting new scenario for the origin of the primordial curvature perturbation, named “inflating curvaton,” was recently proposed in Ref. [26]. This scenario is radically different from the original oscillating curvaton scenario [18, 19, 20, 21]. As briefly reviewed in the introduction, in this scenario the curvaton starts to dominate the energy density while it is still slowly varying, giving a few e-folds of inflation before it starts to oscillate. In section 5 we will argue that the inflating curvaton scenario is quite natural when combined with the AD baryogenesis, considering the observational bound on baryon isocurvature perturbation. In the inflating curvaton scenario, cosmological scales are demanded to be outside the horizon at the time \( t_2 \) when the second inflation starts. Requiring that the shortest cosmological scale, say a region enclosing \( 10^4 \) solar masses, to be outside the horizon at the beginning of the second inflation, the e-folds \( N_2 \) required for the second inflation is given as [26, 29]

\[
N_2 \lesssim 45 - \frac{1}{2} \ln \left( \frac{10^{-5} M_P}{H_2} \right).
\] (2.1)

In this paper we consider the pseudo-Nambu-Goldstone boson (PNGB) type inflating curvaton potential considered in [26]:

\[
V_\sigma(\sigma) = m_\sigma^2 f^2 \left[ 1 - \cos \left( \frac{\sigma}{f} \right) \right].
\] (2.2)
For a moduli field, $m_\sigma \sim m_{3/2}$ and $f \sim M_P$ would be quite natural and we will assume it to be the case in the following. We will discuss a constraint on $f$ from the observations in section 4. With the above parameter region, $H_2 \sim m_\sigma$ and thus the slow-roll condition is not satisfied; we are in the regime of the fast-roll inflation [30]. When $\pi - \sigma/f \ll 1$, the potential (2.2) can be approximated as
\begin{equation}
V_\sigma(\sigma) \sim m_\sigma^2 f^2 - \frac{m_\sigma^2}{2} (\pi f - \sigma)^2.
\end{equation}
When $H$ is approximately constant:
\begin{equation}
H \sim H_2 = \frac{f}{\sqrt{6}M_P} m_\sigma,
\end{equation}
the evolution of the curvaton field is described by the equation
\begin{equation}
\ddot{\sigma} + 3H_2 \dot{\sigma} - m_\sigma^2 \sigma = 0,
\end{equation}
where
\begin{equation}
\tilde{\sigma} \equiv \pi f - \sigma.
\end{equation}
In the above, the dot denotes the derivative with respect to time $t$ in the coordinate system
\begin{equation}
ds^2 = -dt^2 + a^2(t) \sum_{i=1}^{3} (dx^i)^2.
\end{equation}
(2.5) can be solved by an ansatz
\begin{equation}
\tilde{\sigma} = (\pi f - \sigma_2) e^{\omega(t-t_2)},
\end{equation}
where we have set the initial condition when the second inflation starts as $\sigma = \sigma_2 < \pi f$ at $t = t_2$. The inflating curvaton rolls towards the origin $\sigma = 0$. Putting (2.8) into (2.5), we obtain
\begin{equation}
\omega^2 + 3H_2 \omega - m_\sigma^2 = 0,
\end{equation}
thus
\begin{equation}
\omega = \omega_{\pm} \equiv \frac{-3H_2 \pm \sqrt{9H_2^2 + 4m_\sigma^2}}{2}.
\end{equation}
The solution with $\omega = \omega_-$ corresponds to the exponentially decreasing field which rapidly disappears, whereas the solution with $\omega = \omega_+$ corresponds to the exponentially growing field. Below we consider the $\omega = \omega_+$ solution:
\begin{equation}
\tilde{\sigma} = (\pi f - \sigma_2) e^{FH_2(t-t_2)},
\end{equation}
\begin{equation}
H_2 = \frac{f}{\sqrt{6}M_P} m_\sigma.
\end{equation}
where

\[ F = \frac{3}{2} \left( \sqrt{1 + \frac{4m^2_\sigma}{9H^2_2}} - 1 \right). \]  

(2.12)

The field value of the curvaton at the end of the second inflation \( \sigma(t_e) \equiv \sigma_e \) may be well approximated by \( \sigma_e \sim \pi f/2 \). Then the number of e-folds by the second inflation is given by

\[ e^{N_2} = e^{H(t_e - t_2)} \sim \left( \frac{\pi f}{\pi f - \sigma_2} \right)^{1/F}, \]

(2.13)

thus

\[ N_2 \sim \frac{1}{F} \ln \left( \frac{\pi f}{\pi f - \sigma_2} \right). \]  

(2.14)

When \( \sigma \ll \pi f/2 \), the potential (2.2) can be approximated as

\[ V_\sigma(\sigma) \sim \frac{m^2_\sigma}{2} \sigma^2. \]  

(2.15)

The curvaton starts to oscillate when \( \sigma \lesssim \pi f/2 \) and \( H \sim m_\sigma \).

Now let us discuss the primordial curvature perturbation. To the first order in the fluctuation of the energy density \( \delta \rho \), the primordial curvature perturbation \( \zeta(k, t) \) is given as

\[ \zeta(k, t) = -H(t) \frac{\delta \rho(k, t)}{\dot{\rho}(t)} = \frac{1}{3} \frac{\delta \rho(k, t)}{\rho(t) + p(t)}, \]

(2.16)

where \( \rho \) is the energy density and \( p \) is the pressure. The second equality corresponds to the energy continuity condition \( \dot{\rho} = -3H(\rho + p) \). Just from the conservation of the energy-momentum tensor one can show the conservation of the curvature perturbation when the pressure is a unique function of the energy density \[31, 29\]. In any curvaton scenario, \( \zeta \) is generated while \( \rho = \rho_\sigma + \rho_r \) and \( p = p_\sigma + p_r \), where \( \rho_\sigma = \dot{\sigma}^2/2 + V_\sigma(\sigma) \) and \( p_\sigma = \dot{\sigma}^2/2 - V_\sigma(\sigma) \) are the curvaton contributions to the energy density and the pressure respectively, and \( \rho_r \) and \( p_r \) are those from the radiation\[3\]. Let us write

\[ \zeta(k, t) = h(t) \zeta_\sigma(k, t), \]  

(2.17)

where \( h(t) \equiv (\rho_\sigma + p_\sigma)/(\rho + p) \) and \( 3\zeta_\sigma = \delta \rho_\sigma/\rho_\sigma \). There is supposed to be negligible exchange of energy between the two components, so that \( \zeta_\sigma \) is constant if \( p_\sigma \) is a unique function of \( \rho_\sigma \). In inflating curvaton scenario, \( \zeta_\sigma \) becomes constant soon after the second

\[3\]In inflating curvaton scenario those can be from the matter \[26\].
inflation begins \[20\]. The second inflation dilutes the radiation and \( h(t) \) soon becomes close to 1. Thus we have

\[
\zeta(k) \sim \frac{\delta \rho_\sigma(k, t_2)}{3 \dot{\sigma}^2(t_2)}.
\]  

(2.18)

To the first order in \( \delta \sigma \), \( \delta \rho_\sigma \sim V'_\sigma \delta \sigma \). At the horizon exit during the inflation, \( \delta \sigma \sim H_1/2\pi \).

Writing \( \tilde{\sigma}(t_2) \) as a function of the field value of the curvaton at the horizon exit \( \tilde{\sigma}_* \) during the first inflation: \( \tilde{\sigma}(t_2) = g(\tilde{\sigma}_*) \), the primordial curvature perturbation is given as

\[
P_{\zeta}^{1/2} \sim \frac{g'}{3 \dot{\sigma}^2(t_2)} \frac{H_1}{2\pi}.
\]  

(2.19)

This should be compared with the CMB normalization \[32\]

\[
P_{\zeta}^{1/2} \sim 5 \times 10^{-5}.
\]  

(2.21)

We will come back to the comparison with the CMB observation in section 4.

### 3 AD baryogenesis after the second inflation

In this section we study the AD baryogenesis taking place after the second inflation. We assume the AD scalar originates from the flat direction in the Minimal Supersymmetric Standard Model (MSSM) \[33, 34\]. The potential for the AD scalar \( \phi \) is given by

\[
V_{AD}(\phi) = (-cH^2 + m_{AD}^2)|\phi|^2 + \left( \frac{A_H H + A m_3^2}{M_{1/3}} \lambda \phi^p + h.c. \right) + |\lambda|^2 \frac{|\phi|^{2p-2}}{M^{2p-6}}.
\]  

(3.1)

where \( c, A \) and \( \lambda \) are order one constants, \( H \) is the Hubble expansion rate, and \( M \) is the UV cut-off scale. The sign in front of the quadratic part must be negative in order for the AD scalar to have large field value when it starts the spiral motion, which is required in the AD baryogenesis. The coefficient of the Hubble induced A-term \( A_H \) can be order one or can be much suppressed, depending on the symmetry of the inflaton sector. While its magnitude does not affect the produced baryon number density, it is of crucial relevance for the baryon isocurvature perturbation \[35, 36, 37, 38\]. We will discuss it in some detail in section 4. We are interested in the case \( A_H \ll 1 \), and consider this case in the following.
During the second inflation, the AD scalar stays at the minimum of the potential (3.1). We first assume
\[ c H_2^2 \gg m_{AD}^2. \]  
(3.2)
From (2.4), for \( f \sim M_p \), we have \( H_2 \sim m_\sigma \). In section 4 we will see that \( f \) is bounded from above as \( f \lesssim 5M_p \) by the upper bound on the e-folds of the second inflation, see (4.10). We also assume that the effects of A-terms to the minimum of the potential of the AD scalar \( \phi \) is smaller than those from the other terms during the second inflation. This assumption amounts to
\[ 4c(p-1)H^2 \gg p^2 A^2 m_{3/2}^2, \]  
(3.3)
when \( H \sim H_2 \). (3.2) and (3.3) may be satisfied with moderately large \( c \). For example, when \( f = \sqrt{6}M_p \) and \( H_2 = m_\sigma \sim 2m_{3/2} \sim 2m_{AD}, A \sim 1 \) and \( p = 9 \) which are the parameter set we will use later, (3.3) reads
\[ c \gg 0.6, \]  
(3.4)
which may be realized with the order one coefficient \( c \). With the same parameter set, (3.2) can be also satisfied. With the above assumptions, the field value during the second inflation can be estimated as
\[ |\phi| \sim \phi_{\text{min}}(H) \equiv \left( \frac{c}{|\lambda|^2} \right) \frac{1}{p(p-2)} M \left( \frac{H}{\sqrt{p-1}M} \right)^{\frac{1}{p-2}}. \]  
(3.5)
The baryon number density \( n_B \) is given by
\[ n_B = iq \left( \phi \dot{\phi}^* - \dot{\phi} \phi^* \right), \]  
(3.6)
where \( q \) is the baryon number carried by the AD scalar \( \phi \) and the dot denotes the derivative with respect to time \( t \) in the coordinate system (2.7). The evolution of the baryon number density follows from the equation of motion of \( \phi \) and is given by
\[ \dot{n}_B + 3Hn_B = 2q \text{Im} \left[ \phi \frac{\partial V_A(\phi)}{\partial \phi} \right], \]  
(3.7)
where \( V_A(\phi) \) is the A-term part of the AD scalar potential (3.1):
\[ V_A(\phi) = \frac{A_H H + Am_{3/2}}{M_{p-3}} \lambda \phi^p + h.c. \]  
(3.8)
Since we are interested in the case \( A_H \ll 1 \), we will neglect the Hubble induced A-term in the following. Then, during the second inflation the angular part of the complex scalar \( \phi \) will take random value. When \( H \sim m_\sigma \) and \( \sigma \lesssim \pi f/2 \), the inflating curvaton
starts to oscillate and the energy density of the universe is dominated by the curvaton matter. When \( H \sim m_{AD} \), (3.3) is no longer satisfied. Then, the potential for the angular direction from the A-term becomes relevant and the AD scalar starts a spiral motion. By multiplying \( a^3(t) \) (\( a(t) \) is the scale factor in (2.7)) to (3.7) and integrating with respect to the time \( t \), we obtain

\[
a^3(t)n_B(t) \sim qp \int_{t_{sp}}^{t} dt' a^3(t') \frac{m_{3/2}}{M^{p-3}} \text{Im}[A\phi^p] \\
+ qp \int_{t_{sp}}^{t} dt' a^3(t') \frac{m_{3/2}}{M^{p-3}} \text{Im}[A\phi^p].
\]  

(3.9)

The contribution from the second term of (3.9) is small because of the two reasons: (a) \( \text{Im}[A\phi^p] \) changes sign rapidly due to the spiral motion of the AD scalar \( \phi \). (b) The amplitude of \( |\phi|^p \) decreases with time as \( H^p \propto t^{-p} \) in the matter dominant universe and it also makes the contribution from this term small. Therefore, the baryon number density is produced dominantly at the onset of the spiral move \( t_{sp} \). Since \( a \propto t^{2/3} \) in the matter dominant universe, the integrand of the first term of (3.9) is proportional to \( t^{p-4} \). Then the integration gives

\[
n_B \sim qp \frac{m_{3/2}}{M^{p-3}} \left( \frac{|\phi_{sp}|^p}{M^{p-3}} \right) 2 \sin[p\theta_{sp} + \text{arg}(A)] \times \frac{p-2}{2p-6} t_{sp}.
\]  

(3.10)

Here, we set the initial condition for the baryon number density to be zero at the end of the second inflation. From \( a \propto t^{2/3} \) and \( H = \dot{a}/a \sim \frac{2}{3} t^{-1} \) in the matter dominant universe, (3.10) becomes

\[
n_B \sim q p(p-2) \frac{m_{3/2}}{p-3} \left( \frac{|\phi_{sp}|^p}{M^{p-3}} \right) \sin[p\theta_{sp} + \text{arg}(A)] \times \frac{2}{3m_{AD}},
\]  

(3.11)

where we have parametrized the phase part of the AD scalar by \( \theta: \phi = |\phi|e^{i\theta} \), and \( \theta_{sp} \) is the value of \( \theta \) when the AD scalar starts the spiral motion. \( |\phi_{sp}| \) is defined with (3.5) as

\[
|\phi_{sp}| \equiv \phi_{min}(H = m_{AD}) \sim M \left( \frac{m_{AD}}{\sqrt{p-1}M} \right) \frac{1}{p^{1/2}}.
\]  

(3.12)

The ratio of the baryon density to AD scalar at this epoch is given by

\[
\left( \frac{n_B}{n_{\phi}} \right)_{sp} \sim \frac{n_B}{m_{AD}|\phi_{sp}|^2} \sim \frac{2qp(p-2)}{3(p-3)\sqrt{p-1} m_{AD}} \left( \frac{m_{AD}}{m_{AD}} \right)^{p-2} \sin[p\theta_{sp} + \text{arg}(A)].
\]  

(3.13)

When the Hubble expansion rate becomes much less than \( m_{AD} \), the baryon number of the condensate is frozen, and later be converted to the baryon asymmetry. As we check later, the energy density of the AD scalar is always sub-dominant and its decay does not
reheat the universe. The curvaton decays around the time scale $\Gamma_{\sigma}^{-1}$, where $\Gamma_{\sigma} \sim m_{\sigma}^3/M_p^2$. The baryon density at this epoch is given by

$$n_B(t \sim \Gamma_{\sigma}^{-1}) \sim m_{AD} \phi_{sp}^2 \left( \frac{n_B}{n_{\phi}} \right)_{sp} \left( \frac{\Gamma_{\sigma}}{m_{AD}} \right)^2,$$

where the factor $\Gamma_{\sigma}/m_{AD}$ comes from the expansion of the universe in the matter domination: $a \propto t^{2/3}$. After the curvaton decay, the curvaton energy is converted into the radiation with reheating temperature $T_R$. We make an approximation that the curvaton energy density is instantaneously transferred to the energy density of radiation at $t \sim \Gamma_{\sigma}^{-1}$. At this time $H \sim \Gamma_{\sigma}$ and thus

$$\rho_r = \frac{\pi^2 g_*(T_R)}{30} T_R^4 \sim 3 \Gamma_{\sigma}^2 M_p^2,$$

where $g_*(T)$ is the effective number of the contribution of the massless degrees of freedom to the energy density at temperature $T$. From (3.15) we obtain

$$T_R \sim \left( \frac{90}{\pi^2 g_*(T_R)} \right)^{1/4} \sqrt{\Gamma_{\sigma} M_p}.$$

The entropy density at this epoch is estimated as

$$s = \frac{2 \pi^2}{45} g_*(T_R) T_R^3.$$

The baryon number to entropy ratio at the time of the reheating is given by

$$\frac{n_B}{s} \sim \frac{45}{2 \pi^2 g_*(T_R) T_R^3} m_{AD} \phi_{sp}^2 \left( \frac{\Gamma_{\sigma}}{m_{AD}} \right)^2 \left( \frac{n_B}{n_{\phi}} \right)_{sp} \sim \frac{45}{2 \pi^2 g_*(T_R) \sqrt{p - 1}} \frac{1}{M_p} \left( \frac{m_{\sigma}}{M_p} \right)^{3/2} \left( \sqrt{p - 1} M_p \frac{\Gamma_{\sigma}}{m_{AD}} \right)^{p-4} \left( \frac{n_B}{n_{\phi}} \right)_{sp} \sim \frac{45}{2 \pi^2 g_*(T_R) \sqrt{p - 1}} \frac{1}{M_p^2} \left( \sqrt{p - 1} M_p \frac{\Gamma_{\sigma}}{m_{AD}} \right)^{p-4} \left( \frac{n_B}{n_{\phi}} \right)_{sp}.$$

In our scenario there is no further entropy production at later stage and the baryon to entropy ratio (3.18) is fixed until today. To estimate (3.18), let us be a little bit more precise about the decay rate $\Gamma_{\sigma}$. The precise number is model dependent and here we choose

$$\Gamma_{\sigma} = 4 \frac{m_{\sigma}^3}{M_p^2},$$

following [13], as an representative value. Then, for $m_{\sigma} \sim 150$ TeV we have $T_R \sim 70$ MeV, where $g_*(T_R) = 10.75$ is used. For $M \sim M_p$, $p = 9$ and $q \sim 4$, (3.18) gives

$$\frac{n_B}{s} \sim 6 \times 10^{-11} \times \left( \frac{m_{\sigma}}{150 \text{ TeV}} \right)^{3/2} \left( \frac{75 \text{ TeV}}{m_{AD}} \right)^{1/2} \left( \frac{m_{3/2}}{m_{AD}} \right).$$
Here, we have assumed \( \sin[p\theta_{sp} + \arg(A)] \sim 1 \). (3.20) gives the correct order of the baryon to entropy ratio of the present universe \( 9 \times 10^{-11} \) for the representative parameter set. We need relatively large \( p \) to account for the present baryon asymmetry of the universe. This is because the reheating temperature, which depends on the moduli curvaton mass \( \sim \mathcal{O}(10 - 10^2 \text{TeV}) \), is relatively low in our scenario. The choice \( p = 9 \) has a good reason, since larger \( p \) gives larger \( \phi_{sp} \) and thus gives larger contribution to the baryon number, and \( p = 9 \) is the maximal \( p \) in MSSM [39].

In the above we assumed that the energy density of the AD scalar is always sub-dominant. This can be easily checked to be the case. When the AD scalar starts to oscillate, the energy density of the universe is given by

\[
\rho_{tot}(H \sim m_{AD}) \sim 3m_{AD}^2 M_P^2. \tag{3.21}
\]

On the other hand, the energy density of the AD scalar is given by

\[
\rho_{AD} \sim m_{AD}^2 \phi_{sp}^2 \sim m_{AD}^2 M^2 \left( \frac{m_{AD}}{\sqrt{p - 1} M} \right)^2 p^{-2} \ll \rho_{tot}(H \sim m_{AD}), \tag{3.22}
\]

where the last line is satisfied when \( m_{AD} \) is sufficiently smaller than \( M \).

In the analysis so far, we were assuming that the AD condensate evolves homogeneously after it formed. In general, there is a possibility that the AD condensate becomes unstable with respect to spacial perturbations and turns into non-topological solitons called Q-balls [40, 41, 42, 43, 44]. If Q-balls are formed, our scenario for the evolution of the universe may need to be modified. However, while we have not made detailed analysis, it seems likely that Q-balls are not formed in our preferred parameter region \( m_{AD} \gg m_{1/2} \), where \( m_{1/2} \) is the mass scale for the gauginos [45]. In order for the Q-balls to be formed, it is necessary that the potential for the AD scalar is flatter than \( |\phi|^2 \) at large field values. After taking account the one-loop correction, the potential for the AD scalar looks like

\[
V_{AD,1-loop}(\phi) \sim m_{AD}^2 |\phi|^2 \left( 1 + K \ln \frac{|\phi|^2}{M^2} \right) + \ldots, \tag{3.23}
\]

where the coefficient \( K \) is determined from the renormalization group equations, see e.g. [46, 43, 47]. Loops containing gauginos make a negative contribution proportional to \( m_{1/2}^2 \), while loops containing sfermions make a positive contribution proportional to \( m_{AD}^2 \). Thus when the spectrum is such that the gauginos are much lighter than the sfermions i.e. \( m_{AD} \gg m_{1/2} \), which is the case of our interest, \( K \) is likely to be positive and thus Q-balls will not be formed. More complete analysis of the Q-balls is beyond the scope of the current paper and is left to the future investigations.
4 Baryon isocurvature perturbation

As we mentioned in section 3 while the baryon number density is not affected much by the Hubble induced A-term, the magnitude of the coefficient $A_H$ of the Hubble-induced A-term is crucially relevant for the uncorrelated baryon isocurvature perturbation: If there is no sizable Hubble-induced A-term, the phase part of the AD scalar is effectively massless during the first inflation and acquires quantum fluctuations, which leads to uncorrelated baryon isocurvature perturbation. Below we study this case, i.e. $A_H \ll 1$ in eq.(3.1). If the Hubble induced A-term is sizable, there is no uncorrelated baryon isocurvature perturbation and the observational bound on it does not put any constraint on our model.

Since the curvaton dominates the energy density when the baryon number is generated, the baryon number to entropy density ratio does not depend on the curvaton field fluctuations. Therefore, no correlated baryon isocurvature perturbation is produced in our scenario. This is an important difference from the AD baryogenesis in the original oscillating curvaton scenario, as we discuss in section 5.

At the horizon exit during the first inflation, the phase part of the AD scalar acquires fluctuation:

$$\delta \theta = \frac{H_1}{2\pi |\phi_1|}, \quad (4.1)$$

where $\phi_1$ is the field value of the AD scalar during the first inflation. It is given by $\phi_1 \equiv \phi_{\text{min}}(H_1)$, where $\phi_{\text{min}}(H)$ is given in (3.5). The baryon isocurvature perturbation $S_B$ is defined as

$$S_B \equiv \frac{\delta \rho_B}{\rho_B} - \frac{3}{4} \frac{\delta \rho_\gamma}{\rho_\gamma} = \delta \log \left( \frac{n_B}{s} \right). \quad (4.2)$$

Here, $\rho_B$ and $\rho_\gamma$ are the present energy density of baryons and photons, respectively. We have used $\rho_\gamma^{3/4} \propto s$ in the above. Substituting (3.18) (with (3.13)) into (4.2), we obtain

$$S_B \sim p \cot[p \theta_{sp} + \arg(A)] \delta \theta. \quad (4.3)$$

We assume that the curvature perturbation is dominantly from the curvaton. Then, from (4.1) we can estimate the uncorrelated isocurvature perturbation

$$|S_B^{(uncorr)}| \sim \frac{H_1}{2\pi |\phi_1|} \frac{p}{p - \frac{1}{2}} \left( \frac{H_1}{\sqrt{p - 1}M} \right)^{\frac{p-3}{p-2}}. \quad (4.4)$$

In the first line we assumed $\cot[p \theta_{sp} + \arg(A)] \sim O(1)$, though one should keep in mind that the $|\cot|$ function can take any value between $[0, \infty]$. The baryon isocurvature
perturbation is constrained by the current observational bound on the matter isocurvature perturbation \[32\]:

\[ |S^{uncorr}_B| < \frac{\Omega_c}{\Omega_B} \times \sqrt{\alpha_0} \mathcal{P}_\zeta^{1/2} \sim 7 \times 10^{-5}, \tag{4.5} \]

where we have used \( \alpha_0 < 0.08 \). Here, we have used \( \rho_B/\rho_c = \Omega_B/\Omega_c \sim 0.2 \). For \( p = 9 \) and \( M \sim M_P \), by comparing (4.4) and (4.5) we obtain an upper bound on \( H_1 \):

\[ H_1 \lesssim 8 \times 10^{-6} M_P. \tag{4.6} \]

Let us study the implication of this bound to the inflating curvaton model discussed in section 2. Putting (4.6) to (2.21) we obtain

\[ (\pi f - \sigma_2) = \frac{1}{3} \left( \frac{m_\sigma}{F H_2} \right)^2 \frac{H_1}{2\pi \mathcal{P}_\zeta^{1/2}} \lesssim \bar{\sigma}_c(f), \tag{4.7} \]

where

\[ \bar{\sigma}_c(f) = \frac{1}{3} \left( \frac{m_\sigma}{F H_2} \right)^2 \frac{8 \times 10^{-6} M_P}{2\pi \mathcal{P}_\zeta^{1/2}}. \tag{4.8} \]

We made clear that \( \bar{\sigma}_c(f) \) depends on \( f \) through \( H_2 \) and \( F \). Putting (4.7) to (2.14) we obtain

\[ N_2 \gtrsim \frac{1}{F} \ln \left( \frac{\pi f}{\bar{\sigma}_c(f)} \right). \tag{4.9} \]

On the other hand, since \( H_2 \sim m_\sigma \), from (2.1) we have \( N_2 \lesssim 36 \). Thus (4.9) leads to an upper bound on \( f \). The right hand side of (4.9) is slightly complicated function of \( f \), but numerically solving it we obtained

\[ f \lesssim 5M_P. \tag{4.10} \]

On the other hand, for a successful AD baryogenesis we require \( H_2 > m_{AD} \). Then from (2.4) we obtain

\[ f > \frac{m_{AD}}{m_\sigma} \sqrt{6} M_P. \tag{4.11} \]

In order that there is an allowed region for \( f \) we need

\[ \frac{m_{AD}}{m_\sigma} < \frac{5}{\sqrt{6}}. \tag{4.12} \]

This condition is satisfied in our scenario, since we assume \( m_\sigma > m_{AD} \) as summarized in the introduction.
Going in the opposite direction, if we set the ratio $m_{AD}/m_\sigma$, we obtain a theoretical constraint on uncorrelated baryon isocurvature perturbation in our model. However, this bound turns out to be very mild, practically giving no constraint: For the case $m_{AD}/m_\sigma \sim 0.5$, we numerically obtained the bound

$$\alpha_0 \gtrsim 10^{-25}. \quad (4.13)$$

This is an extremely mild constraint since the lower bound would not be detectable in a foreseeable future. Note that this is just a lower bound, meaning the the baryon isocurvature perturbation above the bound might be detected in the future observation.

5 **An issue in the oscillating moduli curvaton scenario with AD baryogenesis**

In this section we review an issue in the AD baryogenesis in the original oscillating curvaton scenario, when the curvaton is a moduli field [25] (see also [48]). Then we argue that this issue naturally leads us to consider the AD baryogenesis in the inflating curvaton scenario. In this section we will use the same notation for the curvaton and related variables as before, but notice that we are discussing a different scenario in this section. It will not cause any confusion if the readers keep in mind that the oscillating curvaton scenario is discussed only in this section.

In the oscillating curvaton scenario, it is assumed that when the curvaton starts to oscillate i.e. $H \sim m_\sigma$, the universe is dominated from the radiation whose energy density is

$$\rho_{r,o} \sim 3m_\sigma^2 M_P^2, \quad (5.1)$$

while the energy density of the curvaton is given by

$$\rho_{\sigma,o} \sim \frac{1}{2} m_\sigma^2 \sigma_o^2, \quad (5.2)$$

where $\sigma_o$ is the field value of the curvaton at this moment. Here, we consider the quadratic potential for the curvaton:

$$V_\sigma(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 \quad (5.3)$$

which is most popular in the oscillating curvaton models, instead of (2.2). We write the scale factor at this moment as $a_o$. From (5.1) and (5.2), by assuming that the radiation dominates the energy density of the universe at this epoch, i.e. $\rho_{r,o} \gtrsim \rho_{\sigma,o}$, we have

$$\sigma_o \lesssim \sqrt{6} M_P. \quad (5.4)$$
After this epoch, the energy density of the curvaton matter decreases as \( \rho_\sigma(a) \propto a^{-3} \), whereas that of the radiation decreases as \( \rho_r(a) \propto a^{-4} \), where \( a \) is the scale factor (see (2.7)). Thus the subsequent evolution is given as

\[
\frac{\rho_\sigma(a)}{\rho_r(a)} = \frac{a \rho_{\sigma,0}}{a_o \rho_{r,0}} = \frac{a}{a_o} \frac{\sigma_\sigma^2}{6M_P^2}. \tag{5.5}
\]

From (5.5), \( a_{eq} \) when the energy density of the curvaton and that of the radiation become equal is given by

\[
a_{eq} = a_o \frac{6M_P^2}{\sigma_\sigma^2}. \tag{5.6}
\]

At this moment the energy density of the curvaton is given by

\[
\rho_\sigma(a_{eq}) = \rho_{\sigma,0} \left( \frac{a_o}{a_{eq}} \right)^3 \]
\[
= \frac{1}{2} m_\sigma^2 \sigma_\sigma^2 \left( \frac{6M_P^2}{\sigma_\sigma^2} \right)^{-3} \]
\[
= \frac{1}{2} m_\sigma^2 \left( \frac{\sigma_\sigma^8}{(\sqrt{6}M_P)^6} \right). \tag{5.7}
\]

At the time of the radiation-curtavon equality, the expansion rate \( H_{eq} \) is obtained from

\[
H_{eq}^2 = \frac{2}{3M_P^2} \rho_\sigma(a_{eq}) = 2m_\sigma^2 \left( \frac{\sigma_\sigma}{\sqrt{6}M_P} \right)^8, \tag{5.8}
\]

thus

\[
H_{eq} = \sqrt{2}m_\sigma \left( \frac{\sigma_\sigma}{\sqrt{6}M_P} \right)^4. \tag{5.9}
\]

The correlated baryon isocurvature perturbation crucially depends on whether which of the following cases is realized \[25\]:

1. The AD field starts to oscillate when the radiation is dominant.
2. The AD field starts to oscillate when the curvaton is dominant.

Whether which case is realized depends on whether the mass of the AD field \( m_{AD} \) is bigger or smaller than \( H_{eq} \). If \( H_{eq} \lesssim m_{AD} \), the case 1 is realized while \( H_{eq} \gtrsim m_{AD} \), the case 2 is realized. Using (5.9), these conditions can be rewritten as

\[
\sigma_\sigma \lesssim \sqrt{6}M_P \left( \frac{m_{AD}}{\sqrt{2}m_\sigma} \right)^{1/4} : \text{case 1} \tag{5.10}
\]
\[
\sigma_\sigma \gtrsim \sqrt{6}M_P \left( \frac{m_{AD}}{\sqrt{2}m_\sigma} \right)^{1/4} : \text{case 2} \tag{5.11}
\]
Let us first look at the case 1. As we have seen in section 3, the baryon number is dominantly generated at the time when the AD scalar starts the spiral motion, \( \ddot{H} \sim m_{AD} \).

The curvaton number density at this epoch is 
\[ n_{\sigma} = m_{\sigma} \sigma_{sp}^2 / 2, \]
where \( \sigma_{sp} \) is the curvaton field value at this epoch. Thus the baryon to curvaton number density ratio is proportional to 
\[ n_{\sigma}^{-1} \propto \sigma_{sp}^{-2}. \]
In the case 1, \( \sigma_{sp} \propto \sigma_o \propto \sigma_*, \) where \( \sigma_* \) is the field value of the curvaton at the horizon exit. The baryon to curvaton ratio is converted into the baryon to entropy ratio when the curvaton decays, and it is fixed until today if there is no entropy production at later time. Thus in this case we have a correlated isocurvature perturbation [25]

\[ S_B^{(corr)} \sim \delta \ln(\sigma_*^{-2}) \sim -2 \frac{\delta \sigma_*}{\sigma_*}. \tag{5.12} \]

In the oscillating curvaton scenario with the quadratic potential (5.3), \( \delta \sigma_* \) is given by

\[ \delta \sigma_* = \frac{H_1}{2\pi}. \tag{5.13} \]

Here, \( H_1 \) is the Hubble parameter at the inflationary stage. Assuming that the primordial curvature perturbation is dominantly produced by the curvaton, \( H_1 \) is related to \( \sigma_* \) through the CMB normalization:

\[ P_\zeta^{1/2} = \frac{H_1}{3\pi \sigma_*} = 5 \times 10^{-5}. \tag{5.14} \]

From (5.12), (5.13) and (5.14) we obtain

\[ |S_B^{(corr)}| \sim 2 \frac{|\delta \sigma_*|}{\sigma_*} \sim 2 \times 10^{-4}. \tag{5.15} \]

This is one order above the current observational bound [32]

\[ |S_B^{(corr)}| \lesssim \frac{\Omega_c}{\Omega_B} \sqrt{\alpha_{-1}} P_\zeta^{1/2} \lesssim 2 \times 10^{-5}, \tag{5.16} \]

where we have used \( \alpha_{-1} < 0.005. \) Thus the case 1 is excluded by the observation.

On the other hand, in the case 2, after the curvaton becomes dominant in the energy density of the universe, the Hubble parameter is determined by the curvaton field value, and vice versa. Put it differently, after the curvaton dominates the energy density, in the gauge where on each time slice the energy density is spatially uniform, the curvaton density is also spatially uniform. In this gauge, the baryon number is also produced uniformly. Thus the baryon to curvaton ratio does not depend on the fluctuation of the curvaton. The baryon to curvaton ratio is later converted to baryon to entropy ratio when the curvaton decays, thus there is no correlated isocurvature perturbation.

From (5.11), the case 2 may be realized by the following two ways:

\[ (i) \quad \sigma_o \gtrsim \sqrt{6} M_P. \tag{5.17} \]
\[ (ii) \quad m_{\sigma} \gg m_{AD}. \tag{5.18} \]
Due to the $1/4$ power dependence on the ratio $m_{AD}/m_\sigma$ in (5.11), in order to realize the case 2 we need a large hierarchy between $m_\sigma$ and $m_{AD}$. In the weak-scale supersymmetry framework, the anomaly mediation can give a hierarchy typically of order $m_{AD}/m_\sigma \sim 10^{-2} - 10^{-3}$ by one-loop factor. If we consider the case of the maximal hierarchy $m_{AD}/m_\sigma \sim 10^{-3}$, $(m_{AD}/m_\sigma)^{1/4} \sim 0.2$ and therefore from (5.11) and (5.4) we obtain

$$0.2 \lesssim \frac{\sigma_0}{\sqrt{6}M_P} \lesssim 1.$$  \hspace{1cm} (5.19)

We do not have an analytical control in the transition region between the case 1 (5.10) and case 2 (5.11), which corresponds to the region around $\sigma_0/(\sqrt{6}M_P) \sim (m_{AD}/m_\sigma)^{1/4} \sim 0.2$. Therefore, one may wonder whether there is an window in the region (5.19) which is not ruled out by the observational bound on the correlated baryon isocurvature perturbation. However, according to the numerical analysis in [25] with the updated WMAP data [32], the region (5.19) seems to be ruled out.

We have seen that even if we assume the maximal hierarchy between $m_\sigma$ and $m_{AD}$ $m_{AD}/m_\sigma \sim 10^{-3}$ which can be naturally realized in the weak-scale supersymmetry scenario to achieve (5.18), the oscillating curvaton scenario is severely constrained, if not ruled out, by the observational bound on the baryon isocurvature perturbation. Actually, as mentioned in the footnote 2 and can be understood from the analysis in section 3, with the mass scales natural in the anomaly mediation the AD baryogenesis does not work as in the gravity mediation case. Thus we considered $m_\sigma \sim 2m_{AD}$ as our reference in the previous sections, which is natural in the gravity mediation. This case is clearly ruled out by the observational bound on the baryon isocurvature perturbation.

While the lower bound in (5.19) is closely tied with the observations and is hard to avoid, the upper bound is just a condition in order to stay in a particular scenario, namely the oscillating curvaton scenario: The upper bound in (5.19) came from the assumption that the radiation dominates the energy density of the universe when the curvaton starts to oscillate, (5.4). Thus this bound can be relaxed if we assume instead that the energy density of the curvaton is comparable or larger than that of the radiation at the time when the curvaton starts to oscillate. However, in this case the curvaton energy density before its oscillation may cause the second stage of inflation and the scenario would need to be modified considerably. This situation may better be studied in the framework of the inflating curvaton scenario, which is the main focus of the current paper.

\footnote{In [26] it was argued that the inflating curvaton with a quadratic potential cannot make a dominant contribution to the primordial curvature perturbation, when both the inflaton and the curvaton have canonical kinetic terms. We review the outline of their arguments in appendix A.}
6 Summary and discussions

The main results of this paper can be summarized as follows:

1. The AD baryogenesis in the inflating curvaton scenario is consistent with the observational bound on baryon isocurvature perturbation. Note that as explained in section 5, the observational bound on correlated baryon isocurvature perturbation severely constrains the AD baryogensis in the original oscillating curvaton scenario when the curvaton is a moduli field.

2. The moduli field with $O(10^{-10} - 10^2 \text{ TeV})$ mass plays multiple key roles in our scenario. It explains the primordial curvature perturbation as well as the baryon density and the cold dark matter density of the present universe (see appendix B for the cold dark matter part).

It will be interesting to realize our scenario in a controlled string compactification with stabilized moduli, which will predict more precise values for the physical input parameters. It will also be interesting to examine whether such $O(10^{-10} - 10^2 \text{ TeV})$ mass moduli field exists in a large class of four-dimensional compactifications of string theory with stabilized moduli [12].

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A Contribution of the inflating curvaton with a quadratic potential to the primordial curvature perturbation

In this appendix we outline the arguments of [26] that the inflating curvaton with a quadratic potential cannot dominate the primordial curvature perturbation when both the inflaton and the curvaton have canonical kinetic terms.

We start from looking at the tilt of the spectrum [19, 49, 29]

$$n(k) - 1 \equiv \frac{d \ln P_\zeta}{d \ln k} = -2\epsilon_{H1} + 2\eta_1 - \frac{2}{M_P^2 P_\zeta(k)} \left( \frac{H_1(k)}{2\pi} \right)^2, \quad (A.1)$$
where
\[ \epsilon_H \equiv \frac{\dot{H}}{H^2}, \quad \eta \equiv \frac{1}{3H^2} \frac{\partial^2 V(I, \sigma, \ldots)}{\partial \sigma^2}. \quad (A.2) \]

Here, \( V(I, \sigma, \ldots) \) is the total potential for the inflaton \( I \) of the first inflation, the curvaton and other scalar fields in the model. The subscripts 1 mean they are the values during the first inflation. The right hand side of (A.1) is evaluated at the horizon exit \( k = aH \). The observation gives \( n - 1 \sim 0.04 \). In many models the last two terms in (A.1) are negligible. Even when they are not negligible, it is unlikely that the terms in the right hand side of (A.1) cancel accurately so that they give the value in the observed tilt. Thus we obtain

\[ \epsilon_{H1} \lesssim 0.02. \quad (A.3) \]

Assuming that the inflaton has a canonical kinetic term, the contribution to the primordial curvature perturbation from the inflaton is given by

\[ \mathcal{P}_{\zeta_i}^{1/2} \sim \frac{1}{\sqrt{2\epsilon_{H1}}} \frac{H_1}{2\pi M_P}. \quad (A.4) \]

On the other hand, when the curvaton also has a canonical kinetic term, in order to realize the second inflation we should require the slow-roll condition \( \epsilon_{H2} \sim \epsilon_2 \equiv M_P^2 (V'/V)^2 \ll 1. \) In this case, the contribution to the primordial curvature perturbation from the curvaton is given by

\[ \mathcal{P}_{\zeta_\sigma}^{1/2} \sim \frac{g' H_1}{\sqrt{2\epsilon_2}} \frac{1}{2\pi M_P}. \quad (A.5) \]

Thus we obtain the ratio

\[ \frac{\mathcal{P}_{\zeta_\sigma}}{\mathcal{P}_{\zeta_i}} \sim \frac{(g')^2 \epsilon_{H1}}{\epsilon_2} \sim 2N_2 \epsilon_{H1} \left( \frac{\sigma_2}{\sigma_*} \right)^2, \quad (A.6) \]

where we have used \( \sigma_2 = g(\sigma_*) \propto \sigma_* \). Since the curvaton rolls down slowly during the first and the second inflation, there should not be much difference between \( \sigma_2 \) and \( \sigma_* \), \( \sigma_2/\sigma_* \lesssim 1 \). Thus from (2.1) and (A.3), we obtain

\[ \frac{\mathcal{P}_{\zeta_\sigma}}{\mathcal{P}_{\zeta_i}} \lesssim 1. \quad (A.7) \]

From (A.7) we conclude that with the naturalness argument above eq. (A.3), the inflating curvaton with a quadratic potential cannot dominate the primordial curvature perturbation when both the inflaton and the curvaton have canonical kinetic terms.
B Non-thermal production of the cold dark matter

For completeness, in this appendix we review the non-thermal production of the cold dark matter density [9] (see also [50, 10, 11, 12, 13, 51]) and confirm that it is realized in our model. The moduli field couples to other fields universally with the strength of the gravitational interaction, thus it decays to the superpartners with a large branching ratio. Each of these superpartners eventually decay to an LSP. When the branching ratio is order one, the LSP to entropy ratio is roughly the same order with the curvaton to entropy ratio before the curvaton decay. It can be calculated in a similar way to the baryon to entropy ratio (3.18) and is given by

\[ n_{\chi} \sim n_{\sigma} \sim \frac{45}{2\pi^2 g_s(T_R)T_R^3} \frac{3m_{AD}^2 M_P^2}{m_\sigma} \left(\frac{\Gamma_\sigma}{m_{AD}}\right)^2. \]  

(B.1)

Here, we have approximated the inflating curvaton potential with the quadratic potential (2.15) when \( H \sim m_{AD} \). The produced LSPs undergo an out-of-equilibrium annihilation if the self-annihilation rate is larger than the expansion rate: \( n_{\chi}\langle v_{rel}\sigma \rangle > H \). This amounts to the following condition:

\[ n_{\chi} \gtrsim n_{\chi}^c \equiv \frac{H}{\langle v_{rel}\sigma \rangle} \bigg|_{T=T_R}. \]  

(B.2)

For the mass of the LSP \( m_\chi \sim 100 \text{ GeV} \) the cross section of the wino of this mass \( \langle v_{rel}\sigma \rangle \sim 3 \times 10^{-7} \text{ GeV}^{-2} \) [9]. Then for \( m_\sigma \sim 150 \text{ TeV} \) the abundance is too large, i.e. \( n_{\chi}/s \sim 10^{-7} \) while \( n_{\chi}^c/s \sim 10^{-12} \). Thus the LSPs further annihilate. The final abundance is determined by the critical number density \( n_{\chi}^c \). The final dark matter to entropy ratio is given by

\[ \frac{n_{\chi}^c}{s} = \frac{45}{2\pi^2 g_s(T)T^3} \frac{H}{\langle v_{rel}\sigma \rangle} \bigg|_{T=T_R}. \]  

(B.3)

We will use \( g_s(T_R) = 10.75 \) as before. (B.3) can be converted into the relic abundance today:

\[ \Omega_\chi = \frac{m_\chi n_{\chi}^c}{\rho_0/s_0} \frac{\rho_0}{s_0} \sim 0.1 h^{-2} \left(\frac{m_\chi}{100 \text{ GeV}}\right) \left(\frac{3 \times 10^{-7} \text{ GeV}^{-2}}{\langle v_{rel}\sigma \rangle}\right) \left(\frac{150 \text{ TeV}}{m_\sigma}\right)^{3/2}. \]  

(B.4)

Here, \( \rho_0/s_0 \) is the ratio between the critical density and the entropy density \( \rho_0/s_0 \sim 3.6 \times 10^{-9}h^2 \text{ GeV} \), where \( h \) is the present Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\). \( \Omega_\chi \) provides a constraint on the branching ratio as small as \( O(10^{-4}) \).

\(^5\)As can be seen from the discussion below, the conclusion would not change up to the branching ratio as small as \( O(10^{-4}) \).
gives the correct order for the present dark matter to critical density ratio $\Omega_c h^{-2} = 0.11$
for the representative set of parameters.\footnote{For a given reheating temperature $T_R$, $m_\chi$ can be adjusted to obtain the observed $\Omega_\chi$ as can be seen from (B.3). But the point is that the wino-mass chosen in this way is quite natural in the current weak-scale supersymmetry scenario.}

References

[1] G. Coughlan, W. Fischler, E. W. Kolb, S. Raby, and G. G. Ross, “Cosmological Problems for the Polonyi Potential,” *Phys.Lett.* B131 (1983) 59.

[2] J. R. Ellis, D. V. Nanopoulos, and M. Quiros, “On the Axion, Dilaton, Polonyi, Gravitino and Shadow Matter Problems in Supergravity and Superstring Models,” *Phys.Lett.* B174 (1986) 176.

[3] T. Banks, D. B. Kaplan, and A. E. Nelson, “Cosmological implications of dynamical supersymmetry breaking,” *Phys.Rev.* D49 (1994) 779–787, arXiv:hep-ph/9308292 [hep-ph].

[4] B. de Carlos, J. Casas, F. Quevedo, and E. Roulet, “Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-d strings,” *Phys.Lett.* B318 (1993) 447–456, arXiv:hep-ph/9308325 [hep-ph].

[5] M. Kawasaki, K. Kohri, and N. Sugiyama, “Cosmological constraints on late time entropy production,” *Phys.Rev.Lett.* 82 (1999) 4168, arXiv:astro-ph/9811437 [astro-ph].

[6] M. Kawasaki, K. Kohri, and N. Sugiyama, “MeV scale reheating temperature and thermalization of neutrino background,” *Phys.Rev.* D62 (2000) 023506, arXiv:astro-ph/0002127 [astro-ph].

[7] I. Affleck and M. Dine, “A New Mechanism for Baryogenesis,” *Nucl.Phys.* B249 (1985) 361.

[8] T. Moroi, M. Yamaguchi, and T. Yanagida, “On the solution to the Polonyi problem with $\mathcal{O}$ (10-TeV) gravitino mass in supergravity,” *Phys.Lett.* B342 (1995) 105–110, arXiv:hep-ph/9409367 [hep-ph].

[9] T. Moroi and L. Randall, “Wino cold dark matter from anomaly mediated SUSY breaking,” *Nucl.Phys.* B570 (2000) 455–472, arXiv:hep-ph/9906527 [hep-ph].
[10] B. S. Acharya, P. Kumar, K. Bobkov, G. Kane, J. Shao, and S. Watson, “Non-thermal Dark Matter and the Moduli Problem in String Frameworks,” JHEP 0806 (2008) 064, arXiv:0804.0863 [hep-ph].

[11] B. S. Acharya, G. Kane, S. Watson, and P. Kumar, “A Non-thermal WIMP Miracle,” Phys.Rev. D80 (2009) 083529, arXiv:0908.2430 [astro-ph.CO].

[12] B. S. Acharya, G. Kane, and E. Kuflik, “String Theories with Moduli Stabilization Imply Non-Thermal Cosmological History, and Particular Dark Matter,” arXiv:1006.3272 [hep-ph].

[13] G. Kane, J. Shao, S. Watson, and H.-B. Yu, “The Baryon-Dark Matter Ratio Via Moduli Decay After Affleck-Dine Baryogenesis,” JCAP 1111 (2011) 012, arXiv:1108.5178 [hep-ph].

[14] P. Grajek, G. Kane, D. Phalen, A. Pierce, and S. Watson, “Is the PAMELA Positron Excess Winos?,” Phys.Rev. D79 (2009) 043506, arXiv:0812.4555 [hep-ph].

[15] G. Kane, R. Lu, and S. Watson, “PAMELA Satellite Data as a Signal of Non-Thermal Wino LSP Dark Matter,” Phys.Lett. B681 (2009) 151–160, arXiv:0906.4765 [astro-ph.HE]. * Brief entry *.

[16] D. Feldman, Z. Liu, P. Nath, and B. D. Nelson, “Explaining PAMELA and WMAP data through Coannihilations in Extended SUGRA with Collider Implications,” Phys.Rev. D80 (2009) 075001, arXiv:0907.5392 [hep-ph].

[17] J. Hisano, M. Kawasaki, K. Kohri, and K. Nakayama, “Positron/Gamma-Ray Signatures of Dark Matter Annihilation and Big-Bang Nucleosynthesis,” Phys.Rev. D79 (2009) 063514, arXiv:0810.1892 [hep-ph].

[18] K. Enqvist and M. S. Sloth, “Adiabatic CMB perturbations in pre - big bang string cosmology,” Nucl.Phys. B626 (2002) 395–409, arXiv:hep-ph/0109214 [hep-ph].

[19] D. H. Lyth and D. Wands, “Generating the curvature perturbation without an inflaton,” Phys.Lett. B524 (2002) 5–14, arXiv:hep-ph/0110002 [hep-ph].

[20] T. Moroi and T. Takahashi, “Effects of cosmological moduli fields on cosmic microwave background,” Phys.Lett. B522 (2001) 215–221, arXiv:hep-ph/0110096 [hep-ph].
[21] D. H. Lyth, C. Ungarelli, and D. Wands, “The Primordial density perturbation in the curvaton scenario,” *Phys.Rev.* **D67** (2003) 023503, arXiv:astro-ph/0208055 [astro-ph].

[22] M. Kawasaki and K. Nakayama, “Affleck-Dine baryogenesis in anomaly-mediated SUSY breaking,” *JCAP* **0702** (2007) 002, arXiv:hep-ph/0611320 [hep-ph].

[23] B. S. Acharya, K. Bobkov, G. L. Kane, P. Kumar, and J. Shao, “Explaining the Electroweak Scale and Stabilizing Moduli in M Theory,” *Phys.Rev.* **D76** (2007) 126010, arXiv:hep-th/0701034 [hep-th].

[24] D. Feldman, G. Kane, E. Kuflik, and R. Lu, “A new (string motivated) approach to the little hierarchy problem,” *Phys.Lett.* **B704** (2011) 56–61, arXiv:1105.3765 [hep-ph].

[25] M. Ikegami and T. Moroi, “Curvaton scenario with Affleck-Dine baryogenesis,” *Phys.Rev.* **D70** (2004) 083515, arXiv:hep-ph/0404253 [hep-ph].

[26] K. Dimopoulos, K. Kohri, D. H. Lyth, and T. Matsuda, “The inflating curvaton,” arXiv:1110.2951 [astro-ph.CO].

[27] L. Randall, M. Soljacic, and A. H. Guth, “Supernatural inflation: Inflation from supersymmetry with no (very) small parameters,” *Nucl.Phys.* **B472** (1996) 377–408, arXiv:hep-ph/9512439 [hep-ph].

[28] T. Asaka, “Affleck-Dine leptogenesis and low scale inflation,” *Phys.Lett.* **B521** (2001) 329–334, arXiv:hep-ph/0110073 [hep-ph].

[29] D. H. Lyth and A. R. Liddle, “The primordial density perturbation: Cosmology, inflation and the origin of structure.”

[30] A. D. Linde, “Fast roll inflation,” *JHEP* **0111** (2001) 052, arXiv:hep-th/0110195 [hep-th].

[31] D. H. Lyth, K. A. Malik, and M. Sasaki, “A General proof of the conservation of the curvature perturbation,” *JCAP* **0505** (2005) 004, arXiv:astro-ph/0411220 [astro-ph].

[32] **WMAP Collaboration** Collaboration, E. Komatsu et al., “Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation,” *Astrophys.J.Suppl.* **192** (2011) 18, arXiv:1001.4538 [astro-ph.CO].
[33] M. Dine, L. Randall, and S. D. Thomas, “Supersymmetry breaking in the early universe,” *Phys.Rev.Lett.* **75** (1995) 398–401, arXiv:hep-ph/9503303 [hep-ph].

[34] M. Dine, L. Randall, and S. D. Thomas, “Baryogenesis from flat directions of the supersymmetric standard model,” *Nucl.Phys.* **B458** (1996) 291–326, arXiv:hep-ph/9507453 [hep-ph].

[35] K. Enqvist and J. McDonald, “Observable isocurvature fluctuations from the Affleck-Dine condensate,” *Phys.Rev.Lett.* **83** (1999) 2510–2513, arXiv:hep-ph/9811412 [hep-ph].

[36] K. Enqvist and J. McDonald, “Inflationary Affleck-Dine scalar dynamics and isocurvature perturbations,” *Phys.Rev.* **D62** (2000) 043502, arXiv:hep-ph/9912478 [hep-ph].

[37] M. Kawasaki and F. Takahashi, “Adiabatic and isocurvature fluctuations of Affleck-Dine field in D term inflation model,” *Phys.Lett.* **B516** (2001) 388–394, arXiv:hep-ph/0105134 [hep-ph].

[38] S. Kasuya, M. Kawasaki, and F. Takahashi, “Isocurvature fluctuations in Affleck-Dine mechanism and constraints on inflation models,” *JCAP* **0810** (2008) 017, arXiv:0805.4245 [hep-ph].

[39] T. Gherghetta, C. F. Kolda, and S. P. Martin, “Flat directions in the scalar potential of the supersymmetric standard model,” *Nucl.Phys.* **B468** (1996) 37–58, arXiv:hep-ph/9510370 [hep-ph].

[40] S. R. Coleman, “Q Balls,” *Nucl.Phys.* **B262** (1985) 263.

[41] A. Kusenko, “Solitons in the supersymmetric extensions of the standard model,” *Phys.Lett.* **B405** (1997) 108, arXiv:hep-ph/9704273 [hep-ph].

[42] A. Kusenko, “Small Q balls,” *Phys.Lett.* **B404** (1997) 285, arXiv:hep-th/9704073 [hep-th].

[43] K. Enqvist and J. McDonald, “Q balls and baryogenesis in the MSSM,” *Phys.Lett.* **B425** (1998) 309–321, arXiv:hep-ph/9711514 [hep-ph].

[44] K. Enqvist and J. McDonald, “B-ball baryogenesis and the baryon to dark matter ratio,” *Nucl.Phys.* **B538** (1999) 321–350, arXiv:hep-ph/9803380 [hep-ph].

[45] R. Allahverdi, S. Hannestad, A. Jokinen, A. Mazumdar, and S. Pascoli, “Supermassive gravitinos, dark matter, leptogenesis and flat direction baryogenesis,” arXiv:hep-ph/0504102 [hep-ph].
[46] H. P. Nilles, “Supersymmetry, Supergravity and Particle Physics,” *Phys.Rept.* **110** (1984) 1–162.

[47] K. Enqvist, A. Jokinen, and J. McDonald, “Flat direction condensate instabilities in the MSSM,” *Phys.Lett.* **B483** (2000) 191–195, arXiv:hep-ph/0004050 [hep-ph].

[48] C. Gordon and A. Lewis, “Observational constraints on the curvaton model of inflation,” *Phys.Rev.* **D67** (2003) 123513, arXiv:astro-ph/0212248 [astro-ph].

[49] D. Wands, N. Bartolo, S. Matarrese, and A. Riotto, “An Observational test of two-field inflation,” *Phys.Rev.* **D66** (2002) 043520, arXiv:astro-ph/0205253 [astro-ph].

[50] M. Kawasaki and K. Nakayama, “Baryon Asymmetry in Heavy Moduli Scenario,” *Phys.Rev.* **D76** (2007) 043502, arXiv:0705.0079 [hep-ph].

[51] D. Feldman and G. Kane, “A Wino-like LSP world: Theoretical and phenomenological motivations,” *In Kane, G.L. (ed.): Perspectives on supersymmetry II* (2010) 288–304.