PHQMD Model for the Formation of Nuclear Clusters and Hypernuclei in Heavy Ion Collisions

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Abstract—A new n-body dynamic transport approach, PHQMD (Parton-Hadron-Quantum-Molecular-Dynamics), is used to describe heavy-ion collisions and the formation of clusters and hypernuclei. The first results are presented from using PHQMD to study the rates of production of strange hadrons, nuclear clusters, and hypernuclei in elementary and heavy-ion collisions at NICA energies. The sensitivity of bulk observables toward the hard and soft equations of state in the PHQMD model is investigated.

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INTRODUCTION

The new Nuclotron-Based Ion Collider–Multipurpose Detector (NICA–MPD) and Baryonic Matter at Nuclotron (BM@N) experiments in Dubna and the Compressed Baryonic Matter (CBM) experiment at the Facility for Antiproton and Ion Research (FAIR) are under construction with the aim of investigating the Quantum Chromodynamics (QCD) phase diagram at low temperatures and high baryon densities by studying heavy-ion collisions in the $\sqrt{s_{NN}} < 11$ GeV range of energies.

Heavy-ion collisions provide a unique opportunity to create and investigate hot and dense matter in the laboratory. A new state of matter, quark–gluon plasma (QGP), forms at the initial stage of the reaction at relativistic incident energies. The final stage is driven by hadronization and the formation of clusters. The capture of the resulting hyperons by clusters of nucleons leads to the formation of hypernuclei, which is a very rare process when the reaction occurs at the threshold energies of strangeness. The dynamic formation of fragments can yield a more accurate description of such observables as flow harmonics or spectra of transverse momentum. It can also help in exploring such new opportunities of physics as the formation of hypernuclei, first-order phase transitions, and the formation of fragments at (ultra-) relativistic energies.
between nucleons has two parts, local Skyrme-type and Coulomb interaction:

\[ V_{i,j} = V(\vec{r}_i, \vec{r}_j, \vec{r}_{i0}, \vec{r}_{j0}, t) = V_{\text{Skyrme}} + V_{\text{Coul}} \]

\[ = \frac{1}{2} t \delta(\vec{r}_i - \vec{r}_j) + \frac{1}{\gamma + 1} t \delta(\vec{r}_i - \vec{r}_j) \]

\[ \times \rho^{-1}(\vec{r}_i, \vec{r}_j, \vec{r}_{i0}, \vec{r}_{j0}, t) + \frac{Z_i Z_j e^2}{2|\vec{r}_i - \vec{r}_j|}, \]

where \( \vec{r}_i, \vec{r}_j, \vec{r}_{i0}, \vec{r}_{j0} \) are coordinates of nucleons.

The analytical form is used for the Skyrme potential:

\[ \langle V_{\text{Skyrme}}(\vec{r}_{i0}, t) \rangle \]

\[ = \alpha \left[ \frac{\rho_{\text{int}}(\vec{r}_{i0}, t)}{\rho_0} \right] + \beta \left[ \frac{\rho_{\text{int}}(\vec{r}_{i0}, t)}{\rho_0} \right]^{\gamma}, \]

where \( \rho_{\text{int}} \) is the density of interaction, obtained by convoluting the density distribution of particles with the distribution functions of all other particles of the surrounding medium.

For a given value of \( \gamma \), parameters \( t_1, t_2 \) in Eq. (1) are uniquely related to coefficients \( \alpha, \beta \) of Eq. (2). Parameter sets for the nuclear equation of state used in the PHQMD model can be found in the Table 1.

An equation of state with a rather low value of compression modulus \( K \) yields weak repulsion against the compression of nuclear matter and thus describes soft matter (denoted by S). A high value of \( K \) causes strong repulsion of nuclear matter under compression (a hard equation of state, H).

CLUSTER FORMATION

The PHQMD approach retains the correlations in the system and does not suppress fluctuations. Since clusters are \( n \)-body correlations, this approach is well suited for studying the creation of clusters and its time evolution.

The simplest way to identify clusters is to use coalescence or minimum spanning tree procedures. The first needs a multitude of free parameters, the second can be used only for identification at the end of the reaction when groups of nucleons are thoroughly separated in the coordinate space, excluding any study of the physical origin of cluster formation [4].

To surmount the limitation that fragments can be identified only at the final stage of the reaction, we can use the coordinate space information in combination with momentum. This idea was first proposed by Dorso et al. [5], and it has developed into the Simulated Annealing Clusterization Algorithm (SACA).

The SACA algorithm consists of several steps. The algorithm first takes the positions and momenta of all nucleons at time \( t \) to identify clusters with a phase space coalescence approach using the Minimum Spanning Tree (MST) technique. In the second step, the MST clusters and individual particles are recombined into fragments in all possible ways, or they are left as single nucleons in order to choose the configuration that has the highest bonding energy. This proce-

| Table 1. Parameter sets for the nuclear equation of state used in the PHQMD model |
|-----------------|-----------------|-----------------|-----------------|
|                | \( \alpha, \text{MeV} \) | \( \beta, \text{MeV} \) | \( \gamma \) | \( K, \text{MeV} \) |
| H              | -130             | 59              | 2.09            | 380             |
| S              | -390             | 320             | 1.14            | 200             |

Fig. 1. Mean multiplicity of (a) \( \pi^+ \) and (b) \( K^+ \) produced in inelastic collisions. PHSD model predictions are indicated by the red \((p + p)\), blue \((p + n)\), and green dashed \((n + n)\) lines; those of the PYTHIA model are drawn with red \((p + p)\), blue \((p + n)\), and green dashed-and-dotted \((n + n)\) lines. Black dots represent a compilation of worldwide experimental data for \( p + p \) collisions [7–14].
dure is repeated many times in the Metropolis procedure, and it automatically leads to the most tightly bound configuration. Note that in the spectator region of the collision clusters chosen in this way early are preliminary fragments of the final state clusters, since fragments are not random collections of nucleons in the end but the initial final state correlations.

**MODEL RESULTS**

**Elementary Reactions**

To be certain about the production of particles in heavy-ion collisions, the production of hadrons in elementary reactions must be controlled. Due to the lack of experimental information, this is not a trivial task,
especially for multi-strange hyperons in the NICA range of energies. In addition, it is important to have the correct isospin decomposition for hadron production in $p + p$, $p + n$, and $n + n$ reactions, since both protons and neutrons participate in collisions in heavy-ion interaction. As an example, Fig. 1 shows the energy dependence of the mean multiplicity of positively charged pions (Fig. 1a) and kaons (Fig. 1b) from PHSD for elementary interactions, of the underlying string model (FRITIOF 7.02 and PYTHIA6.4), the results from the default PYTHIA 8.2 (dashed lines) are also included for reference. As is seen from Fig. 1, PHQMD with the PHSD tunes describes the experimental data on pion and kaon multiplicities in $p + p$ collisions reasonably well.

**Heavy Ion Collisions**

The transverse mass spectra of protons, anti-protons, and produced mesons are compared in Figs. 2, 3 to the STAR experimental data from Au + Au coll-
sions at $\sqrt{s_{NN}} = 11.5$ GeV [15] for the hard and soft equations of state, respectively. The centrality dependence of the spectra of newly produced particles is described well. A hard EoS increases the slope of the spectra at high $p_T$, relative to a soft EoS, while the proton slope is still slightly underestimated at high $p_T$.

**Formation of Hypernuclei**

Figure 4 shows velocity distributions of $Z = 1, Z = 2$ charged particles, heavier clusters ($Z > 2$), $\Lambda$ hyperons, light (\(A \leq 4\)) and heavy (\(A > 4\)) hypernuclei, identified by the MST algorithm for Au + Au collisions at 10 $A$ GeV. We observe an increase in the yields of heavier fragments and hypernuclei in the area of target and beam fragmentation, and an almost constant distribution of $Z = 1$ particles in between. Only a small number of hyperons are bound in small hypernuclei in the area of central velocities, while many of the produced hyperons are bound in larger hypernuclei in the area of target and beam velocities.

**CONCLUSIONS**

We have presented the basic ideas of PHQMD, an $n$-body transport approach now under development. It combines mean-field and QMD propagation of baryons in one code, allowing us to study different descriptions of heavy-ion collisions. The collision integrals have been adopted from the PHSD approach, while the cluster recognition algorithm is executed according to the SACA model or the simpler MST technique. We have shown that PHQMD in the PHSD tune of the string model shows good agreement with experimental data on pion and kaon multiplicities for elementary $p + p$ reactions in the NICA range of energies. We have found that the transverse momenta spectra of hadrons in heavy-ion collisions at NICA energies are sensitive to the equation of state (EoS) of nuclear matter. The PHQMD model recognizes clusters in the central region of velocities and the area of target/projectile fragmentation. Predictions are now being made for clusters and the formation of hypernuclei for future NICA experiments.

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