Bose-Einstein condensation in dense matter and the third family of compact stars

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Abstract. We investigate antikaon condensation in compact star matter using a relativistic mean field model. Antikaon condensates make the equation of state softer resulting in a smaller maximum mass star compared to the case without condensate. It is found that the equation of state including antikaon condensates gives rise to a stable sequence of compact stars called the third family beyond the neutron star branch.

It was first demonstrated by Kaplan and Nelson \cite{1} within a chiral $SU(3)_L \times SU(3)_R$ model that $K^-$ meson may undergo Bose-Einstein condensation in dense matter formed in relativistic heavy ion collisions. In this model, the effective mass of antikaons decreases with increasing density because of the strongly attractive antikaon-baryon interaction. Consequently, the in-medium energy of $K^-$ mesons in zero momentum state also decreases with density. The $s$-wave $K^-$ condensation sets in when the energy of $K^-$ mesons equals to its chemical potential.

Since the work of Kaplan and Nelson, there is a growing interest to understand (anti)kaon properties in dense matter formed in relativistic heavy ion collisions as well as compact star matter \cite{2,3,4,5,6}. The in-medium properties of (anti)kaons were studied through the analyses of collective flow \cite{2} and particle spectra of (anti)kaons \cite{3} and $K^-$ atomic data \cite{7}. All these experimental results suggest that kaon-nucleon interaction is repulsive whereas it is attractive for antikaon-nucleon interaction \cite{2}. Here, we investigate antikaon condensation in compact star matter and its role on the equation of state (EoS) and the structure of compact stars.

In this calculation, $K^-$ condensation in compact star matter is treated as a first order phase transition. We describe the equilibrium conditions of the pure antikaon condensed and hadronic matter and their mixed phase in a uniform background of electrons and muons. The antikaon condensed phase is consisted of all species of the baryon octet, leptons and antikaons. The baryon-baryon interaction mediated by meson exchange is described by a relativistic mean field model \cite{5,6}. The model also includes hyperon-hyperon interaction through two hidden strangeness mesons $f_0(975)$ (denoted as $\sigma^*$) and $\phi$ (1020). The (anti)kaon-baryon Lagrangian density in the minimal coupling scheme is $L_K = D^\mu K D_\mu K - m_K^* KK$ with the covariant derivative $D_\mu = \partial_\mu + ig_\omega K \omega_\mu + ig_\phi K \phi_\mu + ig_\rho K \rho_\mu$. The effective mass of kaons is $m_\sigma = m_K - g_\sigma K \sigma - g_\sigma K \sigma^*$, where $m_K$ is the bare kaon mass. Various strangeness changing processes such as $n \leftrightarrow p + K^-$ and $e^- \leftrightarrow K^- + \nu_e$ occur in neutron stars. It leads to the chemical equilibrium condition $\mu_K^- = \mu_e$. The onset of $K^-$ condensation is possible when the above condition is satisfied. We also include $\bar{K}^0$ condensation in this calculation and this is treated as a second order phase transition. The threshold condition for $\bar{K}^0$ condensation is $\omega_{\bar{K}^0} = 0$. The constituents of this phase are in
Bose-Einstein condensation in dense matter...

Figure 1. The effective nucleon mass is plotted with baryon number density.

beta-equilibrium and maintain local charge neutrality. The energy density ($\epsilon^K$) of this phase has contributions from baryons, leptons and antikaons [5, 6]. The pressure follows from the relation $P^K = \sum_i \mu_i n_i - \epsilon^K$, where $n_i$ is the number density of i-th species.

To describe the pure hadron phase, we employ the field theoretical model for baryon-baryon interaction as described above. The equation of state of this phase is obtained by solving the meson field equations and effective baryon mass in conjunction with local charge neutrality and beta-equilibrium conditions [5, 6]. The energy density ($\epsilon^h$) and pressure ($P^h$) in this phase are related by $P^h = \sum_i \mu_i n_i - \epsilon^h$.

The mixed phase of antikaon condensed matter and hadronic matter is governed by Gibbs phase rules and global conservation laws [8]. The Gibbs phase rules read, $P^h = P^K$ and $\mu_B^h = \mu_B^K$ where $\mu_B^h$ and $\mu_B^K$ are chemical potentials of baryon B in hadronic and $K^-$ condensed matter respectively. The conditions for global charge neutrality and baryon number conservation are $(1 - \chi)Q^h + \chi Q^K = 0$ and $n_B = (1 - \chi)n_B^h + \chi n_B^K$ where $\chi$ is the volume fraction in the condensed phase. The total energy density in the mixed phase is given by $\epsilon = (1 - \chi)\epsilon^h + \chi\epsilon^K$.

In this calculation, we adopt GM1 parameter set [9] where nucleon-meson coupling constants are determined from the nuclear matter saturation properties. The vector meson coupling constants for (anti)kaons and hyperons are determined from the quark model [10]. The scalar meson coupling constants for hyperons and antikaons are obtained from the potential depths of hyperons and antikaons in normal nuclear matter [6, 10]. The phenomenological fit to the $K^-$ atomic data yielded the real part of antikaon potential as $U_{\bar{K}} = -180 \pm 20$ MeV [7]. On the other hand, the
recent microscopic calculations predict a shallow attractive potential at the saturation density \([11]\). We perform this calculation with an antikaon optical potential of -160 MeV at normal nuclear matter density \((n_0 = 0.153 fm^{-3})\). The coupling constants for strange mesons with (anti)kaons are given as \(g_{\sigma K} = 2.65\) and \(\sqrt{2}g_{\phi K} = 6.04\) \([10]\). The coupling constants for \(\sigma^*\)-hyperons are calculated by fitting them to the potential depth for a hyperon in hyperon matter at the saturation density \([6, 10]\).

The effective nucleon mass is plotted with baryon density in Figure 1. Two vertical dotted lines denote the lower and upper boundary of the mixed phase. The mixed phase begins at \(2.23n_0\) and terminates at \(4.0n_0\). The upper curve shows the behaviour of effective nucleon mass with density in the pure hadronic matter. With the onset of \(K^-\) condensation, a second solution for the effective nucleon mass appears. This result was first obtained by Glendenning and Schaffner-Bielich \([5]\). It shows that the effective mass of nucleons behaves differently in the pure hadronic and antikaon condensed matter. This may be attributed to the behaviour of \(\sigma\) and \(\sigma^*\) fields in those pure phases. The effective nucleon mass in the pure antikaon condensed phase is lower than that of the pure hadronic phase. It is to be noted here that it is energetically favourable for \(K^-\) mesons in the zero momentum state to make the system charge neutral removing electrons and muons. Just after the mixed phase is over, \(K^0\) condensation occurs at \(\sim 4.1n_0\). It is \(\Lambda\) hyperon which appears first in this phase. Neutron and proton fractions become equal after \(\bar{K}^0\) condensation. With the appearance of negatively charged \(\Sigma\) and \(\Xi\) hyperons around \(\sim 7n_0\), the density of \(K^-\) condensate starts falling.

The equation of state for compact star matter with and without antikaon condensates are exhibited in the left panel of Figure 2. The curve indicating the
overall EoS with hyperons and antikaon condensates (solid line) is softer compared with the EoS with hyperons and no condensate (dashed line). The kinks on the lower curve mark the beginning and end of the mixed phase. These kinks may lead to discontinuity in the velocity of sound.

Gerlach first argued that a third family of compact stars beyond white dwarf and neutron star branches could exist in nature [12]. It was attributed to the behaviour of an EoS at high density i.e. a jump in the EoS and consequently a discontinuity in the speed of sound and adiabatic index. Glendenning and Kettner [13] first found the actual physical situation where the EoS corresponding to a first order phase transition from hadronic to quark matter, had the requisite properties. With that EoS, it was noted that the neutron star branch was terminated due to abnormally small adiabatic index and a jump in the adiabatic index thereafter gave rise to another stable branch of compact stars at higher central energy densities. Later, various groups [6, 14] obtained similar solutions in first order phase transitions from hadronic matter to strange matter. It is worth mentioning here that such a solution was only possible for a subspace of the parameter space in each calculations. Here, we study the structure of compact stars using Tolman-Oppenheimer-Volkoff equations and the EoS with and without \( \bar{K} \) condensates. The compact star mass sequences are shown with central energy density in the right panel of Figure 2. For the EoS with \( \bar{K} \) condensates, it is found from the figure that after the positive slope neutron star branch, there is an unstable region followed by another positive slope compact star branch. From the study of fundamental mode of radial vibration, we find that the third family branch is a stable one. However, there is no third family solution for the EoS without \( \bar{K} \) condensate (not shown in the figure). The maximum masses of neutron star and third family branch are 1.571\( M_{\odot} \) and 1.553\( M_{\odot} \) corresponding to radii 12.8 km and 10.7 km. Because of partial overlapping mass regions of the neutron star and third family branch, nonidentical stars having same mass but distinctly different composition and radii could exist. Such a pair of compact stars is called "neutron star twins" [13]. It would be challenging to observe two stars with almost the same mass but different radii as the proof of the existence of a third family solution.

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