A physical model for parton densities in hadrons, based on Gaussian momentum fluctuations of partons and hadronic baryon-meson fluctuations, is presented. The model has previously been shown to describe proton structure function data, and is now applied to sea quark asymmetries and shown to describe the $\bar{d} - \bar{u}$ asymmetry of the proton. By considering fluctuations involving strange quarks, the model gives an asymmetry between the momentum distributions of $s$ and $\bar{s}$, which would reduce the significance of the NuTeV anomaly.

1 Introduction

Asymmetries of sea quark distributions of the nucleon have for quite some time been an intriguing problem. For the part of the nucleon sea arising from gluon splittings $g \rightarrow q\bar{q}$ in perturbative QCD, symmetry is expected in the distributions of quarks and antiquarks, i.e. $q(x) = \bar{q}(x)$, and also $\bar{u}(x) = \bar{d}(x)$. Conventional parameterizations of quark momentum distributions assume these symmetries also for the $x$-distributions at the start of the perturbative QCD evolution. However, for these sea distributions arising from the non-perturbative dynamics of the bound state nucleon experiment has shown that there are asymmetries between $\bar{u}$ and $\bar{d}$ [1]. There are also no symmetry arguments to prevent asymmetries between quark and anti-quark. This is of great interest, especially in connection to the NuTeV anomaly, where the value of $\sin^2 \theta_W$ was found to differ from the Standard Model fitted value by almost three standard deviations [2]. The anomaly can however, at least in part, be due to an asymmetry between the momentum distribution of $s$ and $\bar{s}$ in the nucleon sea [3, 4].

Here, we report on recent progress (to be more comprehensively presented in [5]) to understand these asymmetries based on a simple and phenomenologically successful

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model for the parton distributions in hadrons. In particular, we have shown that nucleon fluctuations into $|\Lambda K\rangle$, where the $s$ quark is in the heavier $\Lambda$ baryon and the $\bar{s}$ is in the lighter $K$ meson, gives a harder momentum distribution for the $s$ than the $\bar{s}$. This would reduce the NuTeV anomaly to about two standard deviations.

The model

Previously, we have presented a physical model giving the momentum distributions of partons in the nucleon \cite{6}, as illustrated in Fig. 1. The model gives the four-momentum $k$ of a single probed valence parton by assuming that, in the nucleon rest frame where there is no preferred direction, the shape of the momentum distribution for a parton of type $i$ and mass $m_i$ is then taken as a Gaussian

$$f_i(k) = N(\sigma_i, m_i) \exp \left\{ -\frac{(k_0 - m_i)^2 + k_x^2 + k_y^2 + k_z^2}{2\sigma_i^2} \right\}$$ (1)

which may be motivated as a result of the many interactions binding the parton in the nucleon. The width of the distribution should be of order hundred MeV from the Heisenberg uncertainty relation applied to the nucleon size, i.e. $\sigma_i = 1/d_N$. The momentum fraction $x$ of the parton is then defined as the light-cone fraction $x = k_+/p_+$. In order to obtain a kinematically allowed final state, we impose the following constraints. The scat-
tered parton must be on-shell or having a time-like virtuality, \( i.e. \) have a mass-squared in the range \( m_i^2 \leq j^2 < W^2 \) (\( W \) is the invariant mass of the hadronic system). Furthermore, the hadron remnant \( r \) is obtained from energy-momentum conservation and must have a time-like virtuality. These constraints also ensure that \( 0 < x < 1 \).

Using a Monte Carlo method these parton distributions are integrated numerically without approximations. The normalization of the valence distributions is provided by the sum rules \( \int_0^1 dx u_v(x) = 2 \) and \( \int_0^1 dx d_v(x) = 1 \), to get the correct quantum numbers of the proton (and similarly for other hadrons). The gluon normalization is given by the momentum sum rule \( \sum_i \int_0^1 dx x f_i(x) = 1 \), where the sum also includes sea partons.

To describe the dynamics of the sea partons, we note that the appropriate basis for the non-perturbative dynamics of the bound state nucleon is a hadronic quantum mechanical basis. Therefore we consider hadronic fluctuations, \( e.g. \) for the proton

\[
|p\rangle = \alpha_0|p_0\rangle + \alpha_{p\pi}|p\pi^0\rangle + \alpha_{n\pi}|n\pi^+\rangle + \ldots + \alpha_{\Lambda K}|\Lambda K^+\rangle + \ldots
\]  

(2)

Probing a parton \( i \) in a hadron \( H \) of such a fluctuation (Fig. 1b) gives a sea parton with light-cone fraction \( x = x_H x_i \) of the target proton, \( i.e. \) the sea distributions are obtained from a convolution of the momentum \( K \) of the hadron and the momentum \( k \) of the parton in that hadron. The momentum of the probed hadron is given by a similar Gaussian as Eq. (1) but with a separate width parameter \( \sigma_H \). The kinematical constraints to be applied in this case are \( m_i^2 \leq j^2 < x_H W^2 \) and that the remnants (see Fig. 1b) have time-like virtualities. Here \( x_H \sim M_H/(M_{\text{baryon}} + M_{\text{meson}}) \), giving a harder spectrum for the heavier baryon than the lighter meson, for details see [4]. The normalization of the sea distributions is given by the amplitude coefficients \( \alpha \). These are partly given by Clebsch-Gordan coefficients, but depend primarily on non-perturbative dynamics that cannot be calculated from first principles in QCD and are, therefore, taken as free parameters.

This model results in valence and sea parton \( x \)-distributions as shown in Fig. 1(c). These apply at a low scale \( Q_0^2 \), and the distributions at higher \( Q^2 \) are obtained using perturbative QCD evolution. The distributions shown in Fig. 1(c) is the result of the fits described below.

**Comparison to data and parameter fitting**

After QCD evolution, the proton structure function \( F_2(x, Q^2) \) can be calculated and the model parameters fitted to data from deep inelastic scattering. This results in the
Figure 2: The proton structure function $F_2(x, Q^2)$ from our model compared to HERA H1 data [7].

parameter set

\[
\begin{align*}
\sigma_u &= 180 \text{ MeV}, \quad \sigma_d = 150 \text{ MeV}, \quad \sigma_g = 135 \text{ MeV} \\
\sigma_H &= 100 \text{ MeV}, \quad \alpha_{\text{sea}}^2 = 0.06, \quad Q_0^2 = 0.6 \text{ GeV}^2
\end{align*}
\]

where $\alpha_{\text{sea}}^2$ is the fraction of the proton momentum carried by sea quarks at the scale $Q_0^2$. Such inclusive data can only be used to determine the overall normalization $\alpha_N^2$ for the dominating light quark sea from fluctuations with pions in Eq. (2). The model reproduces $F_2$ data well in view of the its simplicity, see Fig. 2 and [6].

The model also reproduces the observed asymmetry between the $\bar{u}$ and $\bar{d}$ distributions as a result of the suppression of fluctuations with a $\pi^-$ relative to those with a $\pi^+$, since the former require a heavier baryon (e.g. $\Delta^{++}$). With only one additional parameter the model gives a nice fit to data on this asymmetry (see Fig. 3), the parameter in question being the normalization of the $|n\pi^+\rangle$-fluctuation relative to the $|p\pi^0\rangle$-fluctuation. The value of this parameter turns out to be $\alpha_N^2/\alpha_p^2 = 1/2$, which might seem surprising in view of the fact that from isospin the relationship should be 2 : 1. However, these fluctuations implicitly include the effects of heavier fluctuations like $|\Delta\pi\rangle$. The SU(6) Clebsch-Gordan coefficient for $|\Delta^{++}\pi^-\rangle$ is much larger than the coefficient for $|\Delta^0\pi^+\rangle$, giving a larger fraction of $\bar{u}$ relative to $\bar{d}$, thus mimicking a larger $|p\pi^0\rangle$ fraction (see e.g. [9]).

To fix the normalization of the strange sea in the model, we assume that all fluctuations including strange quarks (such as $|\Lambda K^+\rangle$, $|\Sigma K\rangle$) can be implicitly included in
The NuTeV anomaly and an asymmetric strange sea

In the NuTeV experiment [2], the value of $\sin^2 \theta_W$ was extracted from neutral and charged current cross-sections of neutrinos and anti-neutrinos. The value they find differs by about 3σ from the value obtained in Standard Model fits to data from other experiments:

The strange sea asymmetry $s^{-}(x) = x s(x) - x \bar{s}(x)$ (at $Q^2 = 20$ GeV$^2$) from the model and combined with the function $F(x)$ accounting for NuTeV’s analysis [11].
\[ \sin^2 \theta^\text{NuTeV}_W = 0.2277 \pm 0.0016 \] while \[ \sin^2 \theta^\text{SM}_W = 0.2227 \pm 0.0004. \] A number of possible explanations for this discrepancy have been suggested, both in terms of extensions to the Standard Model and in terms of effects within the Standard Model. One explanation of the latter kind would be if the momentum distributions for \( s \)-quarks in the nucleon differs from that of \( \bar{s} \)-quarks. In our model, such an asymmetry arise since the \( s \) quark is in the heavier \( \Lambda \) baryon and the \( \bar{s} \) in the lighter \( K \) meson, giving a harder momentum distribution for the \( s \) than the \( \bar{s} \).

In Fig. 4 (right plot) we show the resulting asymmetry \( s^-(x) = xs(x) - x\bar{s}(x) \) and its combination with a folding function \( F(x) \) provided by NuTeV to account for their analysis and give the shift in the extracted value of \( \sin^2 \theta_W \). We obtain the integrated asymmetry \( S^- = \int_0^1 dx s^-(x) = 0.00165 \), and the shift \( \Delta \sin^2 \theta_W = \int_0^1 dx s^-(x)F(x) = -0.0017 \). Thus, the NuTeV value would be shifted to 0.2260 which is only 2.0\( \sigma \) above the Standard Model value, leaving no strong hint of physics beyond the Standard Model.

References

[1] For a review see e.g. R. Vogt, Prog. Part. Nucl. Phys. 45 (2000) S105.
[2] G. P. Zeller et al. [NuTeV Collaboration], Phys. Rev. Lett. 88, 091802 (2002).
[3] S. Davidson et al., JHEP 0202, 037 (2002).
[4] J. Alwall and G. Ingelman, Phys. Rev. D 70 (2004) 111505.
[5] J. Alwall and G. Ingelman, in preparation.
[6] A. Edin and G. Ingelman, Phys. Lett. B 432, 402 (1998).
[7] C. Adloff et al. [H1 Collaboration], Eur. Phys. J. C 21 (2001) 33.
[8] R. S. Towell et al. [FNAL E866/NuSea], Phys. Rev. D 64 (2001) 052002.
[9] S. Kumano, Phys. Rept. 303 (1998) 183 [arXiv:hep-ph/9702367].
[10] A. O. Bazarko et al. [CCFR Collaboration], Z. Phys. C 65, 189 (1995).
[11] G. P. Zeller et al. [NuTeV Collaboration], Phys. Rev. D 65, 111103 (2002).