A numerical analysis of metrological properties of Venturi tube in the air-coal particle mixture flow measurement in power industry

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Abstract. The main topic of the study is to evaluate the effect of particle concentration and fractional composition on the differential pressure in the Venturi tube during the measurement of flow rate of the gas-particle mixture. Gas phase flow was described by Euler method, while particles movement by Lagrange method. Calculations were performed using ANSYS Fluent Package. A series of numerical calculations were carried out on the particle loading ratio \(Y \leq 2\) and their diameters of \(10 \mu m \leq d_p \leq 200 \mu m\). It has been found that, except for the \(Y\) concentration loading ratio, the increase in the differential pressure in the venturi tube also affects the Stokes number. It depends on the flow conditions and particles diameter. A correction function has been proposed to determine the effect of particles diameter on the results of the mixture flow measurement. The results of the calculations were compared with the Lee and Crowe experimental data. A good comparison of calculation results to measurement results was found.

1 Introduction

Gas-solid particles mixture flow occurs in many industries where pneumatic conveying is used. As an example power plant industry, food industry, building materials industry and many others. Proper processing and energy conversion processes require a current knowledge of the mass flow or one of its components. This problem is particularly important in the power plant industry, where the uniform supply of burners requires the measurement of the particle coal conveyed by the gas stream. For multiphase flow measurements, a videogrammetry method seems useful, which may help to identify flow regimes [1].

One of the methods of measuring homogeneous mixture flow is Venturi tube flowmeter. It has many advantages and high accuracy when the requirements specified in the relevant normative documents are fulfilled [2]. As a result of the tests, it has been found that selected types of Venturi tubes can be used to measure mixtures flow, but the presence of an additional phase changes the characteristics of the flowmeter. The condition of using a flow meter for the measurement of gas-particle mixture flow is to calibrate it in the conditions similar to those for further use.

The purpose of this paper is to indicate the possibility of applying mathematical modelling and numerical simulation to the initial evaluation of its metrological properties during diluted gas-particle mixture flow measuring. Particular studies have been made for Venturi tube, which is often used in two-phase mixture flow measuring systems, especially in pipe system in the conventional power plant [3, 4].

2 Venturi flow meters in the gas-particle flow measurement systems

Operations on the use of gas-solid mixture flow measurements were undertaken mainly for the needs of the power industry, and more precisely – pneumatic conveying of pulverized materials such a coal dust. In 1948 Carlson and others [5] proposed a dual-reducing system, consisting of a serial ASME nozzle and an orifice. The pressure measurement on the orifice, according to authors practically independent of the solid phase presence, was the basis for calculating the gas mass flow. The pressure drop in the nozzle was used to determine the mass flow of the mixture. As Teisseyre’s later studies [4, 6] show, the Carlson method is not suitable for continuous measurements due to the gradual wear of the orifice and the dependence of the orifice differential pressure on the particle concentration in the mixture.

Fabar [7] implemented the first systematic study on the Venturi tube, created with the recommendations of the ISA standard. The layout of the measuring system, also used in works Antikajn [8] and Szatil [9, 10] is shown in Fig. 1a. As a result of the study, Fabar stated the linear dependence of the pressure drop \(\Delta p_{tp}\) at the two-phase flow from the particle concentration in the mixture:

\[
\Delta p_{tp} = (1 + E_t Y)\Delta p_g
\]

where:
is loading ratio of the particles. The value $\Delta p_g$ corresponds to the differential pressure connected with the pure gas flow with the mass flow $M_g$, and $E_i$ is the Gästerstädt number [11]. Equation (1) is used to calculate the particle concentration in the gas, assuming that the gas mass flow is known at the two-phase flow.

The Farbar method requires an additional measurement of the gas mass flow, which in essence is a two-way method. Szatil [10] has confirmed the suitability of a Venturi tube for the measurement of gas-particle mixtures and found a weak dependence of constant $E_i$ on the solid phase concentration and gas velocity. Modification of the measurement method based on the Venturi tube, Graczyk presented [12, 13] in the form of the so-called three-signal method (Fig. 1a). This method was also used in the works of Teissseyre [4, 6], Leroch [14], Barth [15], Jung [16, 17], Szatil [10] and finally Payne and Crowe [18]. In addition to the pressure drop measurement $\Delta p_{tp}$ (Fig. 1a), Graczyk proposed the pressure drop measurement $\Delta p_{tp}^{II}$ for the whole Venturi tube. As a result of the research, this author stated that one of the important problems is the selection of proper measurement points. $\Delta p_{tp}^{II}$ to provide maximum measurement sensitivity and obtain a linear dependence of the pressure drop from the particle concentration

$$
\Delta p_{tp}^{II} = (1 + E_{II}Y)\Delta p_g
$$

Pressure drop ratio is a basis for calculating the particle concentration $Y$ and the mass flow of both phases according to the formula:

$$
\pi_{tp} = \frac{\Delta p_{tp}^{II}}{\Delta p_{tp}^{I}}
$$

Further, using a single Venturi tube as a mass flow meter simplifies the measurement installation in a significant degree. Graczyk studies have provided the basis for the practical application of the three-signal method. Jung’s research has made a great contribution to the development of Venturi tube [16, 17]. Jung proposed modifying the three-signal method by using a nozzle with extended cylindrical part (Fig. 1b). The pressure drop combined with the flow of the mixture through the tapered cylindrical part was the basis for the calculation of the particle concentration. According to Jung, the differential pressure $\Delta p_{tp}^{I}$ measured in the nozzle depends only on the flow of pure gas and can be the basis for calculating the gas mass flow. With the knowledge $\Delta p_{tp}^{II}$ it is possible to calculate $Y$ and the particle mass flow. The disadvantage of the Jung method is that the influence of the presence of particles on the value $\Delta p_{tp}^{II}$ is neglected, which affects the measurement accuracy. Barth’s work [15, 19] uses a different approach, by placing the measuring points at the end part of the Venturi tube (Fig. 1c).

![Fig. 1. Selected Venturi tube systems (described in the text).](image)
3 Calculation methodology

The gas-particle mixture flow is described by differential equations which can be written in the generalized conservative form, with the isolation of convection, diffusion and source components [26]:

\[
\frac{\partial (\rho \phi)}{\partial t} + \frac{\partial (\rho u_i \phi)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \Gamma \phi \frac{\partial \phi}{\partial x_i} \right) + S_{\phi} + S_{\phi, \gamma} \tag{5}
\]

In Equation (5) \( \phi \) is generalized dependent variable, \( \Gamma \), the diffusion coefficient. The source component will include the other components of differential equations - apart from convective and diffusion. Coefficients \( \Gamma \), \( i \), \( S \), are presented in Table 1.

| Equation | \( \phi \) | \( \Gamma \) | \( S \) | \( S_{\phi} \) |
|----------|-------------|-------------|-------------|-------------|
| Continuity | 1 | 0 | 0 | 0 |
| Momentum equation in the \( \xi \) axis | \( u_i \) | \( \mu_{ef} \) | \( F_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu_{ef} \frac{\partial u_i}{\partial x_j} \right) \) | \( \frac{S_{\mu_{ef}, \phi}}{S_{\mu_{ef}, \gamma}} \) |
| Kinetic turbulence energy | \( k \) | \( \mu_{ef} \) | \( G_k - \rho \varepsilon \) | 0 |
| Turbulent energy dissipation rate | \( \varepsilon \) | \( \frac{\mu_{ef}}{\sigma_t} \) | \( \frac{\varepsilon}{k} \left( C_1 G_k - C_2 \rho \varepsilon \right) \) | 0 |

\( \eta = \frac{\mu_{ef}}{\mu} \cdot \frac{1}{\mu_{ef}} \cdot \frac{\mu_{ef}}{\mu} \cdot \frac{\mu_{ef}}{\mu} = \frac{k^2}{\varepsilon} \),

\( G_k = \frac{\partial u_i}{\partial x_j} \mu_{ef} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \)

\( \sigma_k = 1.00 \), \( \alpha_k = 1.30 \), \( C_1 = 1.44 \), \( C_2 = 1.92 \), \( C_\mu = 0.09 \).

Conversion of momentum between the diluted and continuous phase is described by the source term:

\[
\frac{S_{\mu_{ef}, \phi}}{S_{\mu_{ef}, \gamma}} = \frac{1}{V_E} \eta \int \frac{C_{D1} \beta \rho_{p_{1}}}{8 \pi} \frac{D_{p_{1}}}{D_{p_{1}} - D_{p_{2}}} \left( \bar{u}_j - u_{p_i} \right) dt \tag{6}
\]

where \( V_E \) is the elementary cell volume and \( \eta \) describe the number of particles transported at \( j \)-trajectory in the time unit.

The particle motion was described by the Lagrange method by adding drag coefficient and gravitation:

\[
\frac{d \vec{u}_p}{dt} = \frac{1}{\tau_p} \left( \vec{u} - \vec{u}_p \right) + \vec{g} \tag{7}
\]

where \( \vec{u}_p \) is the particle velocity vector and \( \tau_p \) is the time of dynamic particle relaxation. As a result of the integration of equation (7) in the gas velocity field, trajectories of particle fractions were obtained. Knowledge of the trajectories is the basis for determining interphase interactions according to equation (6). Details of calculations can be found in [26]. ANSYS Fluent program [27] was used for numerical research.

4 Numerical research results

4.1 Boundary conditions of the research

The purpose of the study is to evaluate the degree of non-equilibrium flow phenomena on the metrological properties of the venturi tube in the dispersed flow. In particular, the calculations are carried out to set the connection between empirical constants in one-dimensional mathematical models and the Stokes number.

In this calculation, a Venturi tube with diameter ratio \( \beta = \frac{d}{D} = 0.5 \) was used in a pipe with a diameter \( D = 100 \text{ mm} \). The shape of the venturi tube is shown in Fig. 2, and its dimensions are given in Table 2.

Fig. 2. Scheme of flow system with Venturi tube.

| Table 2. Dimensions of Venturi tube. |
|-------------------|-------------------|
| \( D \) [mm] | 100 | 0.25 | 50 | 135 | 409.8 |

Calculations were made for a range of Reynolds number \( Re = 5 \times 10^4 - 5 \times 10^5 \), with gas density of \( \rho_g = 1.2 \text{ kg/m}^3 \), particles density of \( \rho_p = 1400 \text{ kg/m}^3 \) and their diameter from 10 \( \mu \text{m} \) to 200 \( \mu \text{m} \). The effect of particle concentration on differential pressure \( \Delta p \) at the two-phase flow was also tested. The range of research was proper for the concentration characteristic in pneumatic conveying in a pipe system for coal particle burned in power plants (\( Y \leq 2 \)) [3, 4].

In the calculations, the influence of gravitation was omitted, which simplified the problem to two-dimensional axial-symmetrical flow. In the inlet, a fully developed turbulent flow was assumed. Fig. 3 shows the results of the pure gas test calculations, the symbol \( p_v \) describe pressure value in the inlet cross-section.

The increase a gas velocity, as well as, the Reynolds number \( Re \) cause a significant increase the differential pressure \( \Delta p \). The gas mass flow is calculated from the formula:
\[ M_g = \frac{C}{\sqrt{1 - \beta^4}} \frac{d^2}{4} \sqrt{2 \Delta p_g \rho_g} \]  
where flow coefficient is
\[ C = 4 \frac{M_g}{d^2} \frac{1}{\pi} \sqrt{2 \Delta p_g \rho_g} \]  
(9)

Flow coefficients calculated from the formula (9) were compared to the corresponding values according to the standard [2]. Differences do not exceed ±2%, which is a proof of correctness of accepted calculation methodology.

\[ St = \frac{\rho_s d_p^2 u}{18 \mu_g D} = \frac{1}{18} \frac{\rho_s}{\rho_g} \left( \frac{d_p}{D} \right)^2 Re_g \]  
(10)

In this case, Stokes number is in the range of 0.0324 to 129.630.

4.2 Calculation results for gas-particle mixture flow

A series of calculations were made for gas-particle mixtures. Fig. 4 shows pressure distribution along the Venturi tube for particle diameter \( d_p = 10 \) µm with Reynolds number \( Re = 5 \cdot 10^5 \) for different \( Y \) concentrations.

Similarly, results are shown in Fig. 5. Comparison, the smaller particle diameter creates a much larger pressure drop than for diameter \( d_p = 200 \) µm at the same particle concentration.

The results of further calculations are shown in Fig. 6. The pressure difference measured on the venturi tube is the linear function of the particle flow ratio \( Y \).

The angle of the characteristic inclination \( \frac{\Delta p_{tp}}{\Delta p_g} = f(Y) \) depends to a significant extent on the particle diameter, from Stokes number \( St \) defined as follows:

When the particle diameter \( d_p \) is large, corresponding to \( St \to \infty \), then \( \Delta p_{tp} \approx \Delta p_g \) and the Venturi tube practically does not react to the presence of particles. Otherwise, when \( St \to 0 \), differential pressure \( \Delta p_{tp} \approx \Delta p_g \) corresponds to the flow of a homogeneous mixture with a modified density.
Generally, the differential pressure measured on the Venturi tube is contained in the range

$$\Delta p_h \geq \Delta p_{tp} \geq \Delta p_g$$

(11)

So, if the Stokes number is not taken into account, the differential pressure $\Delta p_{tp}$ does not give more precise information about the mixture mass flow. The results of a wider analysis of this problem show good agreement with numerical calculations results after the introduction of the correction function $f(\Omega)$, which is equivalent to Gästerstädt number $EI$ in equation (1). Then, the differential pressure $\Delta p'_{tp}$ is calculated from the formula:

$$\Delta p'_{tp} = (1 + f(\Omega)Y)\Delta p'_g$$

(12)

where

$$f(\Omega) = (1 + a\Omega^b)^{-1}$$

(13)

and

$$\Omega = \left(\frac{d_p}{D}\right)^2 \frac{\rho_t}{\rho_g} Re_g \beta$$

(14)

Values $a$ and $b$ in formula (13) are empirical constants depending on the fractional composition of the particles and other quantities characterizing the measurement conditions.

The function $f(\Omega)$ fulfills the asymptotic conditions:

$$\lim_{\Omega \to 0} f(\Omega) = 1$$

(15)

$$\lim_{\Omega \to \infty} f(\Omega) = 0$$

(16)

and ensures the fulfillment of the relation (11).

An example of the selection of empirical constants $a$ and $b$ is presented in Fig. 7, for the experimental data Lee and Crowe [23]. From the points distribution, it follows that the function $f(\Omega)$ good describes empirical data in a wide range of Stokes values.

$$M_{tp} = \frac{C}{\sqrt{1 - \beta^4}} \frac{d^2}{4} \frac{2\Delta p'_{tp}\Delta p_g}{1 + f(\Omega)Y}$$

(17)

where function $f(\Omega)$ is defined by equation (13). It follows that determining the mass flow of the gas-particle mixture requires, in addition to the particle concentration, also the knowledge of constant values $a$ and $b$ of a function $f(\Omega)$.

5 Conclusions

On the basis of provided analyses, the following conclusions can be drawn regarding the use of the gas-particle mass flowmeter:

1. Differential pressure $\Delta p_{tp}$ measured on the Venturi tube is caused by the flow of both phases, but in general, it does not correspond to the mass flow of the two-phase mixture.

2. When the Stokes number $St \to 0$, the differential pressure measured on the venturi corresponds to the two-phase mixture flow which can be treated as a modified density homogeneous fluid.

3. For large Stokes number, especially at $St \to \infty$, the presence of particles in the gas does not have a significant effect on the differential pressure increasing, which practically corresponds to the flow of pure gas itself.

4. With a given Stokes number, the pressure increase due to the presence of the dispersion phase is a linear function of particle loading ratio.

5. The results of the measurement of the mixture mass flow or one of its components will always be erroneous if the Stokes number value is not taken into account.
References

1. S. Anweiler, J. Environ. Manage. (2016) (https://doi.org/10.1016/j.jenvman.2017.03.040)
2. PN-EN ISO 5167-1: Measurement of particle flow ratio using measuring nozzles, Book of Polish Normalisation Committee, Warszawa, (2005)
3. D. Giddings, B. J. Azzopardi, A. Aroussi, Pickering SiJ. Powder Technol. 172, 149-156 (2007)
4. M. Teisseyre, M. Mazur, Supply of pulverized coal fired boilers – measurement issues, Book of Wroclaw University of Technology, Wroclaw (2006)
5. H. M. Carlson, P. M. Frazier, R. B. Engdahl, Trans. ASME 70, 65-79 (1948)
6. M. Teisseyre, Books of Wroclaw University of Technology, Energetyka III 143, 111-136 (1966)
7. L. Farbar, Trans. ASME 75, 943-951 (1953)
8. P. A. Antikajn, Teploenergetika 12, 35-37 (1956)
9. A. A. Szatil, Teploenergetika 8, 44-48 (1957)
10. A. A. Szatil, Izmeritelna Technika 9, 46-48 (1961)
11. E. Gasterstädt, Zeitschrift des Vereines Deutcher Ingenieure 24, 617-624 (1924)
12. Cz. Graczyk, Ph.D. Thesis, Silesian University of Technology, Gliwice (1957)
13. Cz. Graczyk, Pomiary Automatyka Kontrola 12, 509-511 (1970)
14. J. Łapa, Measurement of the air-particle mixture flow ratio by three-signals Venturi tube – possibilities and range of application, Books of IMA i BUT, Wroclaw University of Technology, 5, 37-50 (1971)
15. W. Barth, Chem. Ing. Tech. 3, 164-170 (1960)
16. R. Jung, Beiträge angewandter Strömungsfororschung zur Entwicklung Kohlenstaubfeuerung. Book VDI-Forschungsheft, 532 (1969)
17. R. Jung, Brennst.-Warme-Kraft 8, 377-383 (1966)
18. A. L. Payne, C. T. Crowe, Proc. 1981, Symp. Instrum. and Control Fossil Energy Processes, ANL-81-62 (1981)
19. W. Barth, R. Nagel, K. van Wavern, Chem. Ing. Tech. 9, 599-602 (1957)
20. J. Łapa, Pomiary Automatyka Kontrola 2, 68-70 (1974)
21. R. Lerch, Energetyka 7/8, 27-30 (1975)
22. J. Goląbek, Ph.D. Thesis, Silesian University of Technology, Gliwice (1979)
23. J. Lee, C. T. Crowe, J. Fluids Eng. 104, 88-91 (1982)
24. A. Payne, M. Werner, D. Plank, C. T. Crowe, The 1982 Symposium on Controls and Instrumentation of Fossil Energy Systems, Huston, June (1982)
25. L. M. Werner, C. T. Crowe, Proc. Symposium on Instrumentation and Control for Fossil Energy Processes, San Francisco, 8-9 June (1981).
26. B. Dobrowolski, J. Wydrych, JTAM 45, 513-537 (2007)
27. ANSYS Fluent, Ansys Inc. (2016)