Abstract We examine how the signature of the strange-dibaryon resonances in the $\bar{K}NN - \pi\Sigma N$ system shows up in the scattering amplitude on the physical real energy axis within the framework of Alt-Grassberger-Sandhas (AGS) equations. The so-called point method is applied to handle the three-body unitarity cut in the amplitudes. We also discuss the possibility that the strange-dibaryon production reactions can be used for discriminating between existing models of the two-body $\bar{K}N - \pi\Sigma$ system with $\Lambda(1405)$.

Keywords strange dibaryon · AGS equations · point method

1 Introduction

The structure of $\Lambda(1405)$ with spin-parity $J^P = 1/2^-$ and strangeness $S = -1$ has been studied for a long time. In the constituent quark model, $\Lambda(1405)$ might be considered as p-wave excited state with $uds$ quarks. However, since the mass of $\Lambda(1405)$ is about 30 MeV below the $\bar{K}N$ threshold, it has also been suggested that $\Lambda(1405)$ is the s-wave $\bar{K}N$ quasi bound state due to the strongly attractive s-wave interaction of the $\bar{K}N$ system with $I = 0$ [1]. Akaishi and Yamazaki suggested that this strong attraction will produce a new type of nuclei, the deeply bound kaonic nuclei [2]. The simplest deeply bound kaonic nuclei are the strange dibaryon, which are the resonances in the $\bar{K}NN - \pi\Sigma N$ system. The strange-dibaryons will give a baseline for the systematic investigation of such deeply bound kaonic nuclei, because the many body dynamics can be treated accurately.

The strange dibaryon resonances have been studied with the Alt-Grassberger-Sandhas (AGS) equations [3, 4] and with the variational method [5, 6, 7] using the phenomenological meson-baryon interactions [3, 5, 6] or interactions based on the effective chiral Lagrangian [4, 7]. All the analyses suggest the existence of the strange-dibaryon resonances.

The strange dibaryon resonances can be produced by photon- or kaon-induced reactions on light nuclei such as $d$ and $^3$He, and the signal of the resonances may be observed in the invariant mass and/or missing mass distributions of the decay products. Theoretical studies of the kaon-induced reactions have been done by Koike-Harada and Yamagata et al. within the optical potential approach [8, 9].

In this contribution, we present how the signature of the strange-dibaryon resonances in the $\bar{K}NN - \pi\Sigma N$ system shows up in the three-body scattering amplitude obtained by solving AGS equations on the physical real energy axis, which is the basic ingredient to calculate the cross sections for strange-dibaryon production reactions measured in the experimental facilities such as J-PARC.
The coupled channel equation for the $\bar{Z}$NN = $\pi\Sigma N$ coupled channel system is given by the AGS equation

$$X_{ij}(p_i, p_j, W) = (1 - \delta_{ij})Z_{ij}(p_i, p_j, W) + \sum_{n \neq j} \int d\mathbf{p}_n Z_{i,n}(\mathbf{p}_i, \mathbf{p}_n, W) \tau_n(W - E_n)X_{n,j}(\mathbf{p}_n, \mathbf{p}_j, W),$$

with the separable approximation for the interaction $V$

$$V(q', q) = \lambda g(q')g(q).$$

Here $X_{ij}(p_i, p_j, W)$ is the quasi two-body amplitudes with the particle $i$ ($j$) as the spectator in the final (initial) state; the energy $W$ contains the infinitesimal positive imaginary part, $W = W' + i\epsilon$ with a real $W'$ and a infinitesimal positive $\epsilon$, resulting from the boundary condition of the scattering problem. The driving term $Z_{ij}(p_i, p_j, W)$ for the s-wave depicted in Fig.1 is the particle-exchange interaction given by

$$Z_{ij}(p_i, p_j, W) = 2\pi \int_{-1}^{1} d(cos \theta) \frac{g(q_j)g(q_i)}{W - p_i^2/2m_j - p_j^2/2m_j - p_i^2/2m_j + 2p_ip_j \cos \theta},$$

where $\cos \theta = \hat{p}_i \cdot \hat{p}_j$. Because of the three-body propagator in the integrand of this equation, the interaction $Z_{ij}(p_i, p_j, W)$ has logarithmic singularities in $(p_i, p_j)$ plane known as the moon-shape singularities shown in Fig.2. Methods to handle these singularities are well studied, for example, with the spline method [10] or the point method [11, 12]. In this work, we employ the point method.

2.1 AGS equations

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2.2 Point method

The point method has been proposed by Schlessinger [11] and developed by Kamada et al. [12]. We evaluate the amplitudes $X_{ij}(p_i, p_j, W)$ in Eq (1) at $W = W' + i\epsilon_i$ with a real $W'$ and a finite positive $\epsilon_i$ ($i = 1, 2, \ldots$). With finite $\epsilon_i$, the logarithmic singularities in $Z_{ij}(p_i, p_j, W)$ become mild and numerical calculations can be performed safely. Then, we use the following continued fraction formula to extrapolate the amplitudes to the energy at $W = W' + i\epsilon$ with the infinitesimal positive $\epsilon$:

$$X(W' + i\epsilon) = \frac{X(W' + i\epsilon_1)}{1 + a_1(\epsilon - \epsilon_1)} = \frac{X(W' + i\epsilon_1) a_1(\epsilon - \epsilon_1) a_2(\epsilon - \epsilon_2) \ldots}{1 + a_2(\epsilon - \epsilon_2) \ldots},$$

with

$$a_i = \frac{1}{\epsilon_i - \epsilon_{i+1}} \left(1 + \frac{a_{i-1}(\epsilon_{i-1} - \epsilon_{i-2}) a_{i-2}(\epsilon_{i-2} - \epsilon_{i-3}) \ldots}{1 + \frac{a_{i-2}(\epsilon_{i-2} - \epsilon_{i-3}) \ldots}{1 + \frac{a_{i-3}(\epsilon_{i-3} - \epsilon_{i-4}) \ldots}{1 + \frac{a_{i-4}(\epsilon_{i-4} - \epsilon_{i-5}) \ldots}{1 + a_{i-5}(\epsilon_{i-5} - \epsilon_{i-6}) \ldots}}}ight).$$
Table 1  Cutoff parameters of $\bar{K}N - \pi\Sigma$ interaction.

|            | $A_{I=0}^{L=0\Lambda}(\text{MeV})$ | $A_{I=0}^{L=0\Lambda}(\text{MeV})$ | $A_{I=1}^{L=0\Lambda}(\text{MeV})$ | $A_{I=1}^{L=1\Lambda}(\text{MeV})$ |
|------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| E-indep.   | 1000                                | 700                                 | 920                                 | 960                                 |
| E-dep.     | 1000                                | 700                                 | 725                                 | 725                                 |

Fig. 3  The $S = -1, J^{\pi} = 1/2^-$ $\bar{K}N$ s-wave amplitude on complex energy plane in (a) the E-indep. model and (b) the E-dep. model.

3 Model of Two-body Interaction

We employ the two models for the meson-baryon interaction of the $\bar{K}NN - \pi\Sigma N$ system. One is the model with the energy independent (E-indep.) separable potentials employed in [4]

$$V_{\alpha\beta}(q', q) = -\lambda_{\alpha\beta} \frac{1}{32\pi^2 F_\pi^2} \frac{m_\alpha + m_\beta}{\sqrt{m_\alpha m_\beta}} \left( \frac{\Lambda_\alpha^2}{q'^2 + \Lambda_\alpha^2} \right) \left( \frac{\Lambda_\beta^2}{q^2 + \Lambda_\beta^2} \right),$$

and another is the model with the energy dependent (E-dep.) potentials employed in [13]

$$V_{\alpha\beta}(q', q; E) = -\lambda_{\alpha\beta} \frac{1}{32\pi^2 F_\pi^2} \left( \frac{2E - M_\alpha - M_\beta}{\sqrt{m_\alpha m_\beta}} \right) \left( \frac{\Lambda_\alpha^2}{q'^2 + \Lambda_\alpha^2} \right) \left( \frac{\Lambda_\beta^2}{q^2 + \Lambda_\beta^2} \right).$$

Here, $\alpha$ and $\beta$ specify the meson-baryon channels, $m_\alpha$ ($M_\alpha$) is the meson (baryon) mass of the channel $\alpha$; $q$ and $q'$ are the relative momenta of the channels $\alpha$ and $\beta$ in the center of mass system, respectively; $F_\pi$ is the pion decay constant; $E$ is the total scattering energy of the meson-baryon system, which is determined by $W - p^2/2\eta$ with $\eta$ being the reduced mass between spectator particle and meson-baryon pair in the three-body system; $\lambda_{\alpha\beta}$ is determined by the flavor SU(3) structure of the Weinberg-Tomozawa term, assuming the different off-shell behavior with non-relativistic kinematics. Also, we have introduced the cutoff parameter $\Lambda_\alpha$. These parameters are determined by fitting the $\bar{K}N$ cross sections (The resulting values of the parameters are listed in Table 1). Here we take “non-relativistic kinematics” in this report.

We find that the above two models have a quite different analytic structure of the $\bar{K}N$ s-wave amplitude in the complex energy plane below the $\bar{K}N$ and above the $\pi\Sigma$ threshold energies: the E-indep. model has a pole corresponding to $\Lambda(1405)$ in the $\bar{K}N$ physical and $\pi\Sigma$ unphysical sheet (Fig 3(a)), while the E-dep. model has two poles in the same sheet (Fig 3(b)). The analytic structure of the E-dep. model is similar to that of the chiral unitary model [14]. It will be then interesting to examine how this difference between the models of the two-body interaction emerges in the strange-dibaryon production reactions.

4 Results and Discussion

In this report, we presents the quasi two-body amplitudes by using the most important interactions. For three-body $Z$, we include $\bar{K}$-exchange mechanism but not $\pi$ or baryon exchange mechanism. For two-body interac-

1 In deriving the potentials from the Weinberg-Tomozawa term, we have also assumed $E_\alpha/M_\alpha \sim 1$ for baryons.
tion, we include $\bar{K}N - \pi\Sigma$ interaction. In Fig. 4 we show $|X_{\bar{K}N\Sigma_i\Sigma_j}(p_i, p_j, W)|^2$ on the real energy axis for the E-indep. (thick curves) and E-dep. (thin curves). We observe a peak around $W \sim 2310$ MeV for the E-indep. model and a bump around $W \sim 2350$ MeV for the E-dep. model. These peak and bump appear near the calculated resonance energy of the strange-dibaryons ($W_R = 2329.5 - i23.3$ MeV for the E-indep. model and $W_R = 2352.0 - i22.5$ MeV for the E-dep. model). This result suggests that the signal of the strange-dibaryons can emerge as a clear peak or a bump of the cross sections, which can be calculated from the amplitude-square $|X|^2$ on the real energy axis. The peak structure is pronounced in the E-indep. model, while in the E-dep. model it is rather small and may not be possible to separate from the background contributions. This difference of the three-body amplitudes due to the model dependence of the two-body subsystem suggests that the strange-dibaryon production reactions could provide also the useful information on the $\bar{K}N - \pi\Sigma$ system.

In summary, by making use of the point method, we have calculated the quasi two-body amplitude $X_{i,j}(p_i, p_j, W)$ on the real energy axis. We then have found the bump structure in the amplitude in the energy region where the strange-dibaryons are expected to exist, implying that the signal of the strange-dibaryon resonances is possible to be observed in the physical cross sections. We have also shown that the strange-dibaryon production reactions could also be useful for judging existing dynamical models of $\bar{K}N - \pi\Sigma$ system with $\Lambda(1405)$. In the current work, however, we have not taken account of several reaction mechanisms such as $\pi$-exchanges. The further improvement of the current model and the calculation of the actual cross sections are under investigation.

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