Exact Solutions in Bosonic and Heterotic String Theory †

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We review some exact solitonic solutions of string theory with higher-membrane structure. These include an axionic instanton solution of bosonic string theory as well as multi-instanton and multimonopole solutions of heterotic string theory. The heterotic solutions reveal some interesting aspects of string theory as a theory of quantum gravity.

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1. Introduction

In recent work classical solitonic solutions of string theory with higher-membrane structure have been investigated. In this paper we consider some exact bosonic as well as heterotic solutions.

We begin with a review of the results of \[1\], in which the tree-level axionic instanton of \[2\] is extended to an exact solution of bosonic string theory for the special case of a linear dilaton wormhole\[3,4\]. Exactness is shown by combining the metric and anti-symmetric tensor in a generalized curvature, which is written covariantly in terms of the tree-level dilaton field, and rescaling the dilaton order by order in the parameter $\alpha'$. The corresponding conformal field theory is written down.

An exact multi-soliton solution of heterotic string theory\[5\] with YM instanton structure in the four dimensional transverse space can be obtained\[6,7\] by equating the curvature of the Yang-Mills gauge field with the generalized curvature derived in \[1\]. This solution represents an exact extension of the tree-level fivebrane solutions of \[8,9,10\] and combines the gauge and axionic instanton structures.

Finally, we consider the recently constructed exact multimonopole solution of heterotic string theory\[11,12\]. An interesting aspect of this string monopole solution is that the divergences stemming from the YM sector are precisely cancelled by those coming from the gravity sector, thus resulting in a finite action solution.

For both classes of heterotic solutions, the interplay between the gravitational and gauge sectors reveals an interesting aspect of string theory as a finite theory of quantum gravity.

2. Exact Bosonic Solution

In \[2\] a tree-level bosonic axionic instanton solution was written down. Here we review the exact extension of this solution\[1\] for the special case of a wormhole, and write down the corresponding conformal field theory. For this purpose we use the theorem of equivalence of the massless string field equations to the sigma-model Weyl invariance conditions (demonstrated to two-loop order by Metsaev and Tseytlin\[13,14\]), which require the Weyl anomaly coefficients $\beta^G_{\mu\nu}$, $\beta^B_{\mu\nu}$ and $\beta^\Phi$ to vanish identically to the appropriate order in the parameter $\alpha'$. The two-loop solution obtained by this method suggests a representation of the sigma model as the product of a WZW\[15\] model and a one-dimensional CFT (a Feigin-Fuchs Coulomb gas)\[2\]. This representation allows us to obtain an exact solution.
The bosonic sigma model action can be written as

\[ I = \frac{1}{4\pi\alpha'} \int d^2x \left( \sqrt{-\gamma} \gamma^{ab} \partial_a x^\mu \partial_b x^\nu g_{\mu\nu} + i\epsilon^{ab} \partial_a x^\mu \partial_b x^\nu B_{\mu\nu} + \alpha' \sqrt{-\gamma} R^{(2)} \phi \right), \]

where \( g_{\mu\nu} \) is the sigma model metric, \( \phi \) is the dilaton and \( B_{\mu\nu} \) is the antisymmetric tensor and where \( \gamma_{ab} \) is the worldsheet metric and \( R^{(2)} \) the two-dimensional curvature. The Weyl anomaly coefficients are given by

\[
\beta^G_{\mu\nu} = \alpha' (\hat{R}_{(\mu\nu)} + 2\nabla_\mu \nabla_\nu \phi)
\]
\[ + \frac{\alpha'^2}{2} \left( \hat{R}^{\alpha\beta\gamma}_\mu [\hat{R}_{\nu\alpha\beta\gamma} - \frac{1}{2} \hat{R}^\beta\gamma\alpha_\mu \hat{R}_\nu\alpha\beta\gamma + \frac{1}{2} \hat{R}_\alpha(\mu\nu)\beta (H^2)_{\alpha\beta} \right) + \nabla_\mu W_\nu, \]

\[
\beta^B_{\mu\nu} = \alpha' (\hat{R}_{[\mu\nu]} + H_{\mu\nu} \lambda \partial_\lambda \phi)
\]
\[ + \frac{\alpha'^2}{2} \left( \hat{R}^{\alpha\beta\gamma}_\mu [\hat{R}_{\nu\alpha\beta\gamma} - \frac{1}{2} \hat{R}^\beta\gamma\alpha_\mu \hat{R}_\nu\alpha\beta\gamma + \frac{1}{2} \hat{R}_\alpha[\mu\nu]_\beta (H^2)_{\alpha\beta} \right) + \frac{1}{2} H_{\mu\nu} \lambda W_\lambda, \]

\[
\beta^\phi = \frac{D}{6} - \frac{\alpha'}{2} \left( \nabla^2 \phi - 2(\partial \phi)^2 + \frac{1}{12} H^2 \right)
\]
\[ + \frac{\alpha'^2}{16} \left( 2(H^2)^{\mu\nu} \nabla_\mu \nabla_\nu \phi + R^2_{\lambda\mu\nu\rho} - \frac{11}{2} R H H + \frac{5}{24} H^4 + \frac{11}{8} (H^2_{\mu\nu})^2 + \frac{4}{3} \nabla H \cdot \nabla H \right)
\]
\[ + \frac{1}{2} W^\lambda \partial_\lambda \phi, \]

where \( H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}, W_\mu = -(\alpha'^2/24) \nabla_\mu H^2, \nabla H \cdot \nabla H \equiv \nabla_\alpha H_{\beta\gamma\delta} \nabla^\alpha H^{\beta\gamma\delta} \) and \( \hat{R}^i_{jkl} \) is the generalized curvature defined in terms of the standard curvature \( R^i_{jkl} \) and \( H_{\mu\alpha\beta} \) by

\[ \hat{R}^i_{jkl} = R^i_{jkl} + \frac{1}{2} (\nabla_i H^j_{jk} - \nabla_k H^i_{jl}) + \frac{1}{4} (H^m_{jk} H^i_{lm} - H^m_{jl} H^i_{km}). \]

One can also define \( \hat{R}^i_{jkl} \) as the Riemann tensor generated by the generalized Christoffel symbols \( \hat{\Gamma}^\mu_{\alpha\beta} \) where \( \hat{\Gamma}^\mu_{\alpha\beta} = \Gamma^\mu_{\alpha\beta} - (1/2) H^\mu_{\alpha\beta} \). Unless otherwise indicated, all expressions are written to two loop order in the beta-functions, which corresponds to \( O(\alpha') \) in the action.

For any dilaton function satisfying \( e^{-2\phi} \Box e^{2\phi} = 0 \) with

\[
g_{\mu\nu} = e^{2\phi} \delta_{\mu\nu} \quad \mu, \nu = 1, 2, 3, 4,
\]
\[
g_{ab} = \delta_{ab} \quad a, b = 5, ..., 26,
\]
\[
H_{\mu\nu\lambda} = \pm \epsilon_{\mu\nu\lambda\sigma} \partial^\sigma \phi \quad \mu, \nu, \lambda, \sigma = 1, 2, 3, 4
\]

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the \(O(\alpha')\) Weyl anomaly coefficients vanish. This can be seen as follows. For any solution of the form (2.4), we can express the generalized curvature in covariant form in terms of the dilaton field \(\phi\):

\[
\hat{R}^i_{jkl} = \delta_{il} \nabla_k \nabla_j \phi - \delta_{ik} \nabla_l \nabla_j \phi + \delta_{jk} \nabla_l \nabla_i \phi - \delta_{jl} \nabla_k \nabla_i \phi \pm \epsilon_{ijkm} \nabla_l \nabla_m \phi \mp \epsilon_{ijlm} \nabla_k \nabla_m \phi. \tag{2.5}
\]

It follows from (2.5) that

\[
\hat{R}_{(\mu\nu)} = -2\nabla_\mu \nabla_\nu \phi, \tag{2.6}
\]

\[
\hat{R}_{[\mu\nu]} = 0.
\]

It also follows from (2.4) that

\[
\nabla^2 \phi = 0, \tag{2.7}
\]

\[
H^{(\mu\nu)}_\lambda \partial_\lambda \phi = 0,
\]

\[
H^2 = 24(\partial \phi)^2.
\]

From (2.6) and (2.7) it follows that the \(O(\alpha')\) terms in the Weyl anomaly coefficients in (2.2) vanish identically for the ansatz (2.4). A tree-level multi-instanton solution is therefore given by (2.4) with the dilaton given by

\[
e^{2\phi} = C + \sum_{i=1}^{N} \frac{Q_i}{|\vec{x} - \vec{a}_i|^2}, \tag{2.8}
\]

where \(Q_i\) is the charge and \(\vec{a}_i\) the location in the four-space (1234) of the \(i\)th instanton. We call (1234) the transverse space, as the solitons have the structure of 21+1-dimensional objects embedded in a 26-dimensional spacetime.

We now specialize to the spherically symmetric case of \(e^{2\phi} = Q/r^2\) in (2.4) and determine the \(O(\alpha')\) corrections to the massless fields in (2.4) so that the Weyl anomaly coefficients vanish to \(O(\alpha'^2)\). For this solution we notice

\[
\nabla_\mu \nabla_\nu \phi = 0, \tag{2.9}
\]

and therefore from (2.5)

\[
\hat{R}^i_{jkl} = 0, \tag{2.10}
\]
and we have what is called a “parallelizable” space \[13,14\]. To maintain a parallelizable space to \(O(\alpha')\) we keep \(g_{\mu\nu}\) and \(H_{\alpha\beta\gamma}\) in their lowest order form and assume that any corrections to (2.4) appear in the dilaton:

\[
\phi = \phi_0 + \alpha' \phi_1 + \ldots \\
e^{2\phi_0} = \frac{Q}{r^2}, \\
g_{\mu\nu} = e^{2\phi_0} \delta_{\mu\nu}, \\
H_{\mu\nu\lambda} = \pm \epsilon_{\mu\nu\lambda\sigma} \partial^{\sigma} \phi_0.
\]  

(2.11)

It follows from (2.11) that \(H^2 = 24(\partial \phi_0)^2 = 24/Q\) and thus \(W_\mu = 0\). It follows from (2.10) that \(\bar{\beta}_G^{\mu\nu}\) and \(\bar{\beta}_B^{\mu\nu}\) vanish identically to two loop order and that

\[
\bar{\beta}^\phi = \frac{D}{6} + \alpha' \left( (\partial \phi)^2 - \frac{1}{Q} \right) + \frac{\alpha'^2}{16} \left( R^2_{\lambda\mu\nu\rho} - \frac{11}{2} RHH + \frac{5}{24} H^4 + \frac{11}{8} (H^2_{\mu\nu})^2 + \frac{4}{3} \nabla H \cdot \nabla H \right). 
\]  

(2.12)

We use the relations in equation (34) in \[13\] for parallelizable spaces and the observation that \((H^2_{\mu\nu})^2 = 2H^4 = 192/Q^2\) for our solution to get the identities

\[
R^2_{\lambda\mu\nu\rho} = \frac{1}{8} H^4, \\
RHH = \frac{1}{2} H^4, \\
\nabla H \cdot \nabla H = 0. 
\]  

(2.13)

(2.12) then simplifies further to

\[
\bar{\beta}^\phi = \frac{D}{6} + \alpha' \left( (\partial \phi)^2 - \frac{1}{Q} \right) + 2 \frac{\alpha'^2}{Q^2}. 
\]  

(2.14)

The lowest order term in \(\bar{\beta}^\phi\) is proportional to the central charge and the \(O(\alpha')\) terms vanish identically. With the choice \(\nabla \phi_1 = -(1/Q) \nabla \phi_0\), the \(O(\alpha'^2)\) terms also vanish identically. The two-loop solution is then given by

\[
e^{2\phi} = \frac{Q}{r^{2(1 - \frac{\alpha'}{2\pi})}}, \\
g_{\mu\nu} = \frac{Q}{r^2} \delta_{\mu\nu}, \\
H_{\mu\nu\lambda} = \pm \epsilon_{\mu\nu\lambda\sigma} \partial^{\sigma} \phi_0.
\]  

(2.15)
which corresponds to a simple rescaling of the dilaton. A quick check shows that this solution has finite action near the singularity.

We now rewrite $\beta^\phi$ in (2.14) in the following suggestive form:

$$6\beta^\phi = (1 + 6\alpha' (\partial \phi)^2) + \left(3 - 6\frac{\alpha'}{Q} + 12\left(\frac{\alpha'}{Q}\right)^2\right)$$

$$= 4.$$  \hfill (2.16)

The above splitting of the central charge $c = 6\beta^\phi$ suggests the decomposition of the corresponding sigma model into the product of a one-dimensional CFT (a Feigin-Fuchs Coulomb gas) and a three-dimensional WZW model with an $SU(2)$ group manifold \[2,13,14\]. This can be seen as follows. Setting $u = \ln r$, we can rewrite (2.1) for our solution\[2\] in the form

$$I = I_1 + I_3,$$

where

$$I_1 = \frac{1}{4\pi \alpha'} \int d^2x \left( Q(\partial u)^2 + \alpha' R^{(2)} \phi \right)$$  \hfill (2.17)

is the action for a Feigin-Fuchs Coulomb gas, which is a one-dimensional CFT with central charge given by $c_1 = 1 + 6\alpha' (\partial \phi)^2$\[18\]. The imaginary charge of the Feigin-Fuchs Coulomb gas describes the dilaton background growing linearly in imaginary time\[3,4\]. $I_3$ is the Wess–Zumino–Witten\[15\] action on an $SU(2)$ group manifold with central charge

$$c_3 = \frac{3k}{k + 2} \approx 3 - \frac{6}{k} + \frac{12}{k^2} + ...$$  \hfill (2.18)

where $k = Q/\alpha'$, called the “level” of the WZW model, is an integer. This can be seen from the quantization condition on the Wess-Zumino term\[14\]

$$I_{WZ} = \frac{i}{4\pi \alpha'} \int_{\partial S_3} d^2x \epsilon^{ab} \partial_a x^\mu \partial_b x^\nu B_{\mu\nu}$$

$$= \frac{i}{12\pi \alpha'} \int_{S_3} d^3x \epsilon^{abc} \partial_a x^\mu \partial_b x^\nu \partial_c x^\lambda H_{\mu\nu\lambda}$$

$$= 2\pi i \left(\frac{Q}{\alpha'}\right).$$  \hfill (2.19)

Thus $Q$ is not arbitrary, but is quantized in units of $\alpha'$.

We use this splitting to obtain exact expressions for the fields by fixing the metric and antisymmetric tensor field in their lowest order form and rescaling the dilaton order by order in $\alpha'$. The resulting expression for the dilaton is

$$e^{2\phi} = \frac{Q}{r \sqrt{1 + \frac{2\alpha'}{Q}}}.$$  \hfill (2.20)
3. Exact Heterotic Multi-Instanton Solution

We now turn to the heterotic multi-instanton solution of \[6, 7\]. The tree-level supersymmetric vacuum equations for the heterotic string are given by

\[
\delta \psi_M = \left( \nabla_M - \frac{1}{4} H_{MAB} \Gamma^{AB} \right) \epsilon = 0, \\
\delta \lambda = \left( \Gamma^A \partial_A \phi - \frac{1}{6} H_{AMC} \Gamma^{ABC} \right) \epsilon = 0, \\
\delta \chi = F_{AB} \Gamma^{AB} \epsilon = 0,
\]

where \(\psi_M\), \(\lambda\) and \(\chi\) are the gravitino, dilatino and gaugino fields. The Bianchi identity is given by

\[
dH = \alpha' \left( \text{tr} R \wedge R - \frac{1}{30} \text{Tr} F \wedge F \right).
\]

The \((9 + 1)\)-dimensional Majorana-Weyl fermions decompose down to chiral spinors according to \(SO(9,1) \supset SO(5,1) \otimes SO(4)\) for the \(M^{9,1} \to M^{5,1} \times M^4\) decomposition. Let \(\mu, \nu, \lambda, \sigma = 1, 2, 3, 4\) and \(a, b = 0, 5, 6, 7, 8, 9\). Then the ansatz

\[
g_{\mu\nu} = e^{2\phi} \delta_{\mu\nu}, \\
g_{ab} = \eta_{ab}, \\
H_{\mu\nu\lambda} = \pm \epsilon_{\mu\nu\lambda\sigma} \partial^\sigma \phi
\]

with constant chiral spinors \(\epsilon_{\pm}\) solves the supersymmetry equations with zero background fermi fields provided the YM gauge field satisfies the instanton (anti)self-duality condition

\[
F_{\mu\nu} = \pm \frac{1}{2} \epsilon_{\mu\nu}{}^{\lambda\sigma} F_{\lambda\sigma}.
\]

An exact solution is obtained as follows. Define a generalized connection by

\[
\Omega^{AB}_\pm = \omega^A_M \pm H^{AB}_M
\]

embedded in an SU(2) subgroup of the gauge group, and equate it to the gauge connection \(A_\mu\) so that \(dH = 0\) and the corresponding curvature \(R(\Omega_{\pm})\) cancels against the Yang-Mills field strength \(F\). As in the bosonic case, for \(e^{-2\phi} e^{2\phi} = 0\) with the above ansatz, the curvature of the generalized connection can be written in the covariant form

\[
R(\Omega_{\pm})_{\mu\nu}^{mn} = \delta_{\nu\nu} \nabla_m \nabla_{\mu} \phi - \delta_{\mu\mu} \nabla_m \nabla_{\nu} \phi + \delta_{m\mu} \nabla_n \nabla_{\nu} \phi - \delta_{m\nu} \nabla_n \nabla_{\mu} \phi \\
\pm \epsilon_{\mu\nu\alpha} \nabla_\alpha \nabla_{\nu} \phi \mp \epsilon_{\nu\mu\alpha} \nabla_\alpha \nabla_{\mu} \phi.
\]
from which it easily follows that

\[ R(\Omega_{\pm})_{\mu\nu}^{mn} = \pm \frac{1}{2} \epsilon_{\mu\nu}^{\lambda\sigma} R(\Omega_{\pm})_{\lambda\sigma}^{mn}. \] (3.9)

Thus we have a solution with the ansatz (3.5) such that

\[ F_{\mu\nu}^{mn} = R(\Omega_{\pm})_{\mu\nu}^{mn}, \] (3.10)

where both \( F \) and \( R \) are (anti)self-dual. This solution becomes exact since \( A_\mu = \Omega_{\pm\mu} \) implies that all the higher order corrections vanish [19, 20, 21, 3, 7, 22]. The self-dual solution for the gauge connection is then given by the ‘t Hooft ansatz

\[ A_\mu = i \Sigma_{\mu\nu} \partial_\nu \ln f. \] (3.11)

For a multi-instanton solution \( f \) is given by

\[ f = e^{-2\phi_0} e^{2\phi} = 1 + \sum_{i=1}^{N} \frac{\rho_i^2}{|x - \vec{a}_i|^2}, \] (3.12)

where \( \rho_i^2 \) is the instanton scale size and \( \vec{a}_i \) the location in four-space of the \( i \)th instanton. An interesting feature of the heterotic solution is that it combines a YM instanton structure in the gauge sector with an axionic instanton structure in the gravity sector. In addition, the heterotic solution has finite action.

Note that the single instanton solution in the heterotic case carries through to higher order without correction to the dilaton. This seems to contradict the bosonic solution by suggesting that the expansion for the Weyl anomaly coefficient \( \beta^\Phi \) terminates at one loop. This contradiction is resolved by noting that for a supersymmetric ansatz the bosonic contribution to the central charge is given by [23]

\[ c_3 = \frac{3k'}{k' + 2}, \] (3.13)

where \( k' = k - 2 \). This reduces to

\[ c_3 = 3 - \frac{6}{k}, \] (3.14)

\[ = 3 - \frac{6\alpha'}{Q}, \]
which indeed terminates at one loop order. The exactness of the splitting then requires that \( c_1 \) not get any corrections from \( (\partial \Phi)^2 \) so that \( c_1 + c_3 = 4 \) is exact for the tree-level value of the dilaton\(^8\).

The exact heterotic instanton represents a nonperturbative classical solution combining the massless fields in the string gravitational sector with the YM field in the gauge sector. If the extension of this solution to an exact string loop solution is also nonperturbative, the heterotic instanton may provide a potential testing ground for string theory as a finite theory of quantum gravity. In addition, there is the hope that this type of finite action instanton solution may eventually lead to an understanding of the vacuum in string theory, a role analogous to that of the instanton in YM field theory.

4. Exact Heterotic Multimonopole Solution

In this section we consider the exact multimonopole solution of heterotic string theory obtained in \[^{11,12}\] . The derivation of this solution closely parallels that of the multistanton solution reviewed in section 3, but in this case, the solution possesses three-dimensional (rather than four-dimensional) spherical symmetry near each source. The reduction is effected by singling out a direction in the transverse space. An exact solution is now given by

\[
g_{\mu \nu} = e^{2\phi} \delta_{\mu \nu}, \quad g_{ab} = \eta_{ab},
\]

\[
H_{\mu \nu \lambda} = \pm \epsilon_{\mu \nu \lambda \sigma} \partial^{\sigma} \phi,
\]

\[
e^{2\phi} = e^{2\phi_0} f,
\]

\[
A_{\mu} = i \sum_{\mu \nu} \partial_{\nu} \ln f,
\]

where in this case

\[
f = 1 + \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{a}_i|}, \quad (4.2)
\]

where \( m_i \) is the charge and \( \vec{a}_i \) the location in the three-space (123) of the \( i \)th monopole. If we identify the scalar field as \( \Phi \equiv A_4 \), then the \( SU(2) \) gauge and scalar fields may be simply written in terms of the dilaton as \[^{11,12}\]

\[
\Phi^a = -2 \frac{\delta^a_i}{g} \partial_i \phi,
\]

\[
A^a_k = -2 \frac{\epsilon^a_{kj}}{g} \partial_j \phi
\]

(4.3)
for the self-dual solution. For the anti-self-dual solution, the scalar field simply changes sign. Here \( g \) is the YM coupling constant. Note that \( \phi_0 \) drops out in (4.3).

The above solution (with the gravitational fields obtained directly from (4.1) and (4.2)) represents an exact multimonopole solution of heterotic string theory and has the same structure in the four-dimensional transverse space as an analogous multimonopole solution of the YM + scalar field action\[11,12\]. If we identify the (123) subspace of the transverse space as the space part of the four-dimensional spacetime (with some toroidal compactification, similar to that used in \[24\]) and take the timelike direction as the usual \( X^0 \), then the monopole properties of the field theory solution carry over directly into the string solution.

The string action contains a term \(-\alpha' F^2\) which diverges as in the field theory solution (see \[11\]). However, this divergence is precisely cancelled by the term \( \alpha' R^2(\Omega_{\pm}) \) in the \( O(\alpha') \) action. This result follows from the exactness condition \( A_{\mu} = \Omega_{\pm\mu} \) which leads to \( dH = 0 \) and the vanishing of all higher order corrections in \( \alpha' \). Another way of seeing this is to consider the higher order corrections to the bosonic action shown in \[20,21\]. All such terms contain the tensor \( T_{MNPQ} \), a generalized curvature incorporating both \( R(\Omega_{\pm}) \) and \( F \). The ansatz is constructed precisely so that this tensor vanishes identically\[1,22\]. The action thus reduces to its finite lowest order form and can be calculated directly for a multi-source solution from the expressions for the massless fields in the gravity sector.

The divergences in the gravitational sector in heterotic string theory thus serve to cancel the divergences stemming from the field theory solution. This solution thus provides an interesting example of how this type of cancellation can occur in string theory, and supports the promise of string theory as a finite theory of quantum gravity. Another point of interest is that the string solution represents a supersymmetric multimonopole solution coupled to gravity, whose zero-force condition in the gravity sector (cancellation of the attractive gravitational force and repulsive antisymmetric field force) arises as a direct result of the zero-force condition in the gauge sector (cancellation of vector and scalar forces of exchange) once the gauge connection and generalized connection are identified.

5. Conclusion

We outlined in section 2 the bosonic tree-level axionic instanton solution of \[2\] and its exact extension for the case of a single instanton wormhole solution\[1\]. A combination of the YM gauge instanton and the axionic instanton solution led to an exact multi-instanton
solution in heterotic string theory\cite{3,4}, which was discussed in section 3. Finally, in section 4 we turned to the recently constructed exact multimonopole solution of heterotic string theory\cite{11,12}.

For both heterotic instantons and monopoles, if the nonperturbative nature of these classical solutions carries over to their quantum string-loop extensions, we may eventually gain some insight into the nature of string theory as a finite theory of quantum gravity. The heterotic instanton solution may perhaps play a role analogous to that of instantons in field theory by providing an understanding of the structure of the vacuum in string theory.

An interesting aspect of the monopole solution is that the YM divergences of the modified \'{t} Hooft ansatz are precisely cancelled in the string theory solution by similar divergences in the gravity sector, resulting in a finite action solution. This finding is significant in that it represents an example of how string theory incorporates gravity in such a way as to cancel infinities inherent in gauge theories. It will be especially noteworthy if the quantum string loop extension of this solution retains this feature.
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