Integrated Design Optimization of a 5-DOF Assistive Light-weight Anthropomorphic Arm

Lelai Zhou, Shaoping Bai  
Department of Mechanical and Manufacturing Engineering, Aalborg University  
Pontoppidanstræde 103, 9220 Aalborg, Denmark  
{1zh, shb}@m-tech.aau.dk

Michael R. Hansen  
Department of Engineering  
University of Agder  
Grooseveien 36, 4876 Grimstad, Norway  
michael.r.hansen@uia.no

Abstract—An integrated dimensional and drive train optimization method was developed for light-weight robotic arm design. The method deals with the determination of optimal link lengths and the optimal selection of motors and gearboxes from commercially available components. Constraints are formulated on the basis of kinematic performance and dynamic requirements, whereas the main objective is to minimize the weight. The design of a human-like arm, which is 10 kg in weight with a load capacity of 5 kg, is described.

I. INTRODUCTION

The design of light and strong robotic arms faces many challenges, varying from power supply, actuators, power transmission and structural parts. An effective approach to achieve a light-weight design will be the integrated design where all major factors can be considered and evaluated.

A majority of research work in robot design optimization is related to the drive train design. Pettersson and Ölvander [1] reported a method of design optimization, in which the drive train of two joints were optimized for an industrial manipulator. DLR’s robotics lab designed a 7-dof (degrees of freedom) torque-controlled light-weight robotic arm with a payload-to-weight ratio of 1, with a payload mass of 14 kg [2]. A drive train optimization method for robot designing was recently reported in [3]. The method is able to optimally select combinations of motors and gearboxes from a catalogue of commercially available components for each joint of a robot arm.

On the other hand, dimensional optimization was also studied for improvement of robotic performance, either kinematic or dynamic one. An optimum robot design method for a specified task was proposed [4], in which dimensions were optimized based on dynamic analysis. It can be noticed that, for most research, dimensional and drive train optimizations were conducted separately. An integrated approach is desired in order to fully utilize the potential of applying optimization techniques to robot design.

This paper reports an integrated dimensional and drive train optimization method for the design of robotic arms. In this method, both link dimensions and drive-train components are design variables of the design optimization to reach minimal weight of the arm. Two sets of constraints are defined, namely, the dynamic constraints on the selection of motors and gearboxes, and the kinematic constraints on the robot’s kinematic performance. The new method is an extension of the authors’ previous work reported in [3], with focus on the integrated design optimization. Through implementing this method, a 5-dof light-weight robotic arm (Fig. 1) is designed and presented.

II. DESIGN OF AN ANTHROPOMORPHIC ARM

The light-weight robot arm considered in this paper has five degrees of freedom (dof), with two dof at the shoulder, one at the elbow, and two at the wrist, as shown in Fig. 2.

A. Arm Mechanism

A modular approach is used in the design. CPU series gearboxes of Harmonic Drive™ are used as transmission elements and, simultaneously, as the mechanical joints, for the different dof’s. To increase the torque capabilities of Joints 1 and 2, a second stage of gearhead is used between Harmonic Drive and the motor. The geared motors and Harmonic Drive gearboxes are mounted inside the joint housings, while the axes of rotation coincide with the joint axes. The physical realization of Joint 2 is illustrated in Fig. 2.
The arm joints are driven by DC motors from Maxon\textsuperscript{TM}. The motors are equipped with encoders having 1000 counts per turn.

**B. Parameterized Dimensions**

Following the Denavit-Hartenberg convention [5], Cartesian coordinate systems are established for each link of the robotic arm, as shown in Figure 3.

The link lengths of the upper arm \(l_1\) and lower arm \(l_2\) (in Fig. 3) are taken as variables in the problem of optimization, while \(h_1\) and \(d_1\) are fixed. To keep the reachable space of the robotic arm constant, the total reaching distance \(L = l_1 + l_2\) is fixed.

One non-dimensional parameter \(r\) is introduced as \(r = l_1/L\). Considering the structural issues, a minimum length is required for both lower and upper arms, which means the link length ratio \(r\) is manipulated in the interval \([r_{\min}, r_{\max}]\).

In practice, a vector \(r\) is defined by discretizing \(r\) from the interval \([r_{\min}, r_{\max}]\) with a step of 0.05.

\[
\mathbf{r} = \{r_{\min} + u_d \cdot 0.05\}_{u_d=1}^{u_d=c}
\]

where \(u_d\) is an index number which acts as design variable of structural dimension in the optimization, and \(c = (r_{\max} - r_{\min})/0.05 + 1\).

**III. INTEGRATED DIMENSIONAL AND DRIVE-TRAIN OPTIMIZATION**

In this method, both robot kinematic and dynamic models are required to evaluate the robotic arm’s performance, such as the GCI and applied torques at joints, etc.

**A. Global Conditioning Index**

The kinematics performance is one of the major concerns in robot design. It is desirable for a robot to have a high kinematic performance, while the drive-drain being optimized. Gosselin and Angeles have developed a global conditioning index (GCI) for the kinematic optimization of manipulators [6]. The GCI is defined with in a workspace \(W\) as

\[
GCI = \frac{\int_W \frac{1}{\kappa} dW}{\int_W dW}
\]

with the condition number \(\kappa\) given by

\[
\kappa = \| J(\theta, \mathbf{u}_d) \| \| J^{-1}(\theta, \mathbf{u}_d) \|
\]

where \(J(\theta, \mathbf{u}_d)\) is the Jacobian matrix, \(\theta\) is the vector of joint angles, and \(\mathbf{u}_d = \{u_{d1}, u_{d2}, \ldots\}\) is an array of structural dimensions. The Euclidean norm \(\| \cdot \|\) of the matrix is defined as

\[
\| J \| = \sqrt{tr(JN^TJ)}
\]

with \(N = \frac{1}{n} I\), where \(n\) is the dimension of the square matrix \(J\), and \(I\) is the \(n \times n\) identity matrix.

In practice, the GCI of a robotic manipulator is calculated through a discrete approach as [7]

\[
GCI = \frac{1}{m} \sum_{i=1}^{m} \frac{1}{\kappa_i}
\]

The GCI is dimension dependent, to keep a high kinematics performance with selected link lengths in the integrated optimization, a constraint is given on the GCI

\[
GCI(\mathbf{u}_d) \geq C_{\min}
\]

where \(C_{\min}\) is the minimum acceptable GCI.

**B. Drive Train Modeling**

The drive train optimization is based on time domain simulation in order to evaluate the design criteria. For that purpose, a dynamic model of the robotic arm has been developed. The governing equation of the arm motion can be written as:

\[
\mathbf{M}(\theta)\ddot{\theta} + \mathbf{v}(\theta, \dot{\theta}) + \mathbf{g}(\theta) = \mathbf{\tau}
\]

where \(\mathbf{M}\) is the mass matrix, \(\mathbf{v}\) is the vector of Coriolis and centrifugal terms of the links, \(\mathbf{g}\) is the vector of gravitational
forces, $\tau$ is the vector of joint torques, and $\theta$ is the vector of joint angles.

A model of a single joint is shown in Fig. 4. The drive train consists of a motor, a linkage and a gearbox for speed reduction. For the harmonic drive gearbox, the gear efficiency varies depending on the output torque. The required motor torque for the $i$th joint can be calculated by

$$\tau_{m,i} = \left( (J_m + J_g) \ddot{\theta}(t) \rho + \frac{\tau(t)}{\rho \eta_g} \right)_i ; \quad i = 1, \ldots, 5 \quad (8)$$

where $\rho_i$ is the gear ratio, $J_m,i$ is mass moment of inertia of the $i$th motor, $J_g,i$ is the equivalent mass moment of inertia of the $i$th gearbox, $\eta_g,i$ is the corresponding gear efficiency, and $\tau(t)$ is the load at the output link which can be solved by (7).

C. Motor Selection Criteria

Motors for robotic arms are usually selected from two motor groups, brushed and brushless DC motors. In selecting motors, the following three criteria are considered:

$$\tau_{rms} \leq T_m \quad (9a)$$

$$\tau_p \leq T_{m,\text{max}} \quad (9b)$$

$$n_p \leq N_{g,\text{max}} \quad (9c)$$

where $\tau_{rms}$ denotes the root mean square (RMS) value of the required motor torque, and $T_m$ is the nominal torque of the motor. $\tau_p = \max\{|\tau_m|\}$ is the required peak torque, and $T_{m,\text{max}}$ is the stall torque of the motor. Moreover, $n_p = \max\{|2\pi \dot{\theta}(t) \cdot \rho|\}$ is the required peak speed corresponding to the motor, and $N_{g,\text{max}}$ denotes the maximum permissible input speed of the motor.

D. Gearbox Selection Criteria

For the selection of gearboxes, the following three criteria apply:

The first criterion is related to the RMC value of the calculated torques ($\tau_{rmc}$). This is recommended by the Harmonic Drive gearbox manufacturer [8]. The RMC value is a measure of the accumulated fatigue on a structural component and reflects typical endurance curves of steel and aluminium [9]. It is therefore relevant to gearbox lifetime, and this criterion has also been used in robotic applications [10]. With this criterion, a constraint is derived as

$$\tau_{rmc} \leq T_g \quad (10a)$$

where $\tau_{rmc} = \frac{\sqrt{1}}{\Delta t} \int_0^{\Delta t} \tau^3(t) dt$, with $\tau(t)$ being the required torque from the gearbox output. $T_g$ is the limit for rated torque of the gearbox.

Other two criteria for gearbox selection include

$$\tau_g \leq T_{g,\text{max}} \quad (10b)$$

$$n_g \leq N_{g,\text{max}} \quad (10c)$$

where $\tau_g = \max\{|\tau(t)|\}$ denotes the required peak torque with respect to the output side, and $T_{g,\text{max}}$ is the allowable peak torque of the gearbox. Moreover, $n_g = \max\{|\dot{\theta}(t) \cdot \rho|\}$ is the required maximum input peak speed, and $N_{g,\text{max}}$ denotes the maximum permissible input speed of a gearbox.

E. Objective Function Formulation

The objective of the optimization is to find the minimum mass of the robotic arm, which requires to find the lightest combination of motors and gearboxes for all five joints and the optimal link lengths that fulfill all constraints associated with the kinematics performance. The objective function, $f(x)$, is defined as

$$\min \quad f(x) = \sum_{i=1}^{5} (m_m(u_m) + m_g(u_g))_i \quad (11)$$

S.T.

$$C_{min} \leq GC1(u_d) \quad (12a)$$

$$T_{m,i} \geq \frac{1}{\Delta t} \int_0^{\Delta t} \left\{ (J_m(x) + J_g(x)) \ddot{\theta}(t) \rho + \frac{\tau(t,x)}{\rho \eta_g} \right\}_i^2 \cdot dt \quad (12b)$$

$$T_{m,\text{max},i} \geq \max \left\{ (J_m(x) + J_g(x)) \ddot{\theta}(t) \rho + \frac{\tau(t,x)}{\rho \eta_g} \right\}_i \quad (12c)$$

$$N_{\text{max},m,i} \geq \max \left\{ 2 \pi \dot{\theta}(t) \cdot \rho \right\}_i \quad (12d)$$

$$T_{g,i} \geq \frac{\sqrt{1}}{\Delta t} \int_0^{\Delta t} \tau_i \cdot dt \quad (12e)$$

$$T_{g,\text{max},i} \geq \max \left\{ |\tau(t,x)| \right\}_i \quad (12f)$$

$$N_{g,\text{max},i} \geq \max \left\{ |\dot{\theta}(t) \cdot \rho| \right\}_i \quad (12g)$$

where design variables of $x$ include the index numbers of motors $u_m = [u_{m1}, \ldots, u_{m5}]$ and gearboxes $u_g = [u_{g1}, \ldots, u_{g5}]$, relative to databases containing commercially available components, and an array of dimensional variables $u_d$. So far, we have formulated the design problem as a discrete optimization problem, which can be solved by commercial available codes. We select a non-gradient method called Complex for this purpose. The implementation is outlined in the next section.

Fig. 4. Schematic view of drive train model for a single joint.
IV. PROCEEDURE OF MINIMIZATION

A. Optimization by Complex

The Complex method is a non-gradient based optimization method, first presented by Box [11]. The set of design variables minimizing the objective function is denoted as the best point \( x_b \), while the one maximizing the objective function is denoted as the worst point \( x_w \). The candidate point is found by the reflection of the worst point through the centroid \( x_c \) with a reflection coefficient \( \alpha \), yielding the following expression for the candidate design point.

\[
x_c = \frac{1}{m} \sum_{i=1}^{m} x^i, \quad x^i \neq x^j
\]

\[
x^{old}_{cand} = x_c + \alpha (x_c - x_w)
\]

\[
x^{new}_{cand} = \frac{1}{2} \left( x^{old}_{cand} + \varepsilon x_c + (1 - \varepsilon) x_b \right)
+ (x_c - x_b)(1 - \varepsilon)(2K - 1)
\]

where the coefficient \( \alpha = 1.3 \), \( K \) is a random number varying in the interval \([0, 1]\), with

\[
\varepsilon = \beta^{-\beta}; \beta = 1 + \frac{k_r - 1}{n_r}
\]

Here \( k_r \) is the number of repeating times the point has repeated itself, and \( n_r \) is a parameter which is recommended as 4 in the program. The convergence criterion of the Complex method is that the difference between the best and worst objective function values is less than a user defined tolerance.

B. Design Variables Programming

The design points in the Complex method are usually continuous. However, the design variables \( u_{m,t} \) and \( u_{g,t} \) have to be integers, since they are the index numbers from the databases of motors and gearboxes. To deal with the integer design variables, a round function is introduced to transfer the design variables into integers. The rounding function is given as

\[
x_{DV} = \text{round}(x)
\]

\[
x_{DV} = \begin{cases} x_{int}; & \text{if } x_{int} \leq x < x_{int} + 0.5 \\ x_{int} + 1; & \text{if } x_{int} + 0.5 \leq x < x_{int} + 1 \end{cases}
\]

where \( x \) is the design variable manipulated by the Complex method, \( x_{int} \) is the integral part of the number \( x \), and \( x_{DV} \) is the rounded design variable. The rounded variable \( x_{DV} \) is used to update the mass of motors and gearboxes in inverse dynamic analysis, as well as the allowable torque and speed values used to examine constraint violations.

C. The Optimization Routine

The implementation of the optimization takes two steps: the optimization routine development and the generation of a parametric simulation model. The optimization program was implemented in Matlab. The flow diagram of the optimization routine is shown in Fig. 5.

V. THE ARM DESIGN OPTIMIZATION

A. Predefined Arm Trajectory

The predefined arm trajectory in the base coordinate system is defined as \( X_{cf}(t) = 50 + 400(1 - \cos(t)) \), \( Y_{cf}(t) = -1000 + 800(1 - \cos(t/2)) \), and \( Z_{cf}(t) = 280 + 250(\cos(t/2) - 1) \), all with unit of \( \text{mm} \). The Euler angles for the end-effector are given as \([0, \cos(\pi/20), 0]\). The payload is defined as a point mass of 5 kg.

The motions of each joint were solved by the inverse kinematics, with the arm link length ratio \( r \in [0.2, 0.8] \). Fig. 6 shows the solved joint velocity and acceleration of Joint 2 for different link length ratios.

B. Candidate Components

Nine candidate motors from the Maxon Motor catalogue are listed in the database. They are listed ascendingly with respect to the mass of motor, as shown in Table I.

The gearboxes used in the robotic arm are selected from Harmonic Drive CPU units, as listed in Table II. For the Harmonic Drive gearboxes, the efficiency is a function of operation speed. In this work, the gear efficiency is set to 0.85 for all gearboxes, which is an average value from product catalog. The candidate motors and gearboxes have to be sorted by mass, as their masses are the main design variables of the objective function.
The gear ratio of each joint is set to $\rho = \{200, 200, 200, 51, 100\}$, orderly from Joint 1 to Joint 5, based on previous investigation of joint torques. Note that Joints 1, 2 and 3 are built with two-stage gearboxes, i.e., combinations of a planetary gearhead and a Harmonic Drive unit. For simplicity, only the mass of the Harmonic Drive gearbox is parameterized, while the mass of the planetary gearhead is set to constant. The Harmonic Drive CPU unit is used in all joints except Joint 4, due to the joint structure consideration. A planetary gearhead is used in Joint 4, so $u_{g, 4} = 0$.

### C. Optimization Results

Optimized designs of motor and gearbox for the robotic arm are listed in Table III. The optimized weight of the robotic arm is 9.92 kg, with a reduction of 41% comparing to the weight of 16.7 kg of the initial combination of motors and gearboxes.

The convergence of the objective function is depicted in Fig. 7, both best value (black dot) and worst value (gray dot) from the Complex algorithm are shown. The solution to the optimal result is achieved at 3500 iterations with 130 population sizes. In this work, the tolerance of convergence is equal to 0.0001. The duration of the optimization increases with respect to increasing of population size.

The convergence of the link length ratio is shown in Fig. 8. The link length ratio is converged to $r = 0.6$. The variance of GCI during the optimization is shown in Fig. 9.

Figure 10(a) illustrates the convergence of motor design variables. Only the convergence curves for Joints 1 and 5 are displayed for clarity. The convergence of gearbox design variables is depicted in Fig. 10(b). Comparing the convergence rate for the motor and gearbox design variables, the gearbox design variables converge faster than the motor design variables. This phenomena is caused by that the mass
difference among Harmonic Drive units is larger than among motors.

Another optimization case with fixed link length ratio \( r = 0.5 \) is also shown in Table III for comparison. In this design case, the weight change is not significant relative to the previous optimized case, but still noticeable. The change is due to the size of the motor at Joints 1 and 3. Referring to Table I, the corresponding nominal torques are 0.0426 Nm (EC 32) and 0.0965 Nm (RE 35) for Joint 1.

VI. PROTOTYPE OF THE ARM

A prototype of the 5-dof robotic arm was built, as shown in Fig. 1. The components of drive-train in the prototype are selected and scaled based on the optimization results shown in Table III. The prototype doesn’t utilize the optimized ratio of \( r = 0.6 \); instead, the length ratio is \( r = 0.5 \).

VII. CONCLUSIONS

An integrated dimensional and drive-train optimization method was developed, which is able to generate optimal link lengths and combinations of drive-train components. The proposed method provides a systemic optimization approach for design of robots. A 5-dof light-weight robotic arm was optimized and prototyped based on the proposed method. The development of the motion controller for assistive tasks with the prototype is under way.

REFERENCES

[1] M. Pettersson and J. "Olvander. Drive train optimization for industrial robots. *IEEE Transactions on Robotics*, 25(6):1419–1423, 2009.
[2] A. Albu-Schäffer, S. Haddadin, C. Ott, A. Stemmer, T. Wimböck, and G. Hirzinger. The DLR lightweight robot: design and control concepts for robots in human environments. *Industrial Robot*, 34(5):376–385, 2007.
[3] L. Zhou, S. Bai, and M. R. Hansen. Design optimization on the drive train of a light-weight robotic arm. *Mechatronics*, 21:560–569, 2011.
[4] P. S. Shiakolas, D. Koladiya, and J. Kebrel. Optimum robot design based on task specifications using evolutionary techniques and kinematic, dynamic, and structural constraints. *Inverse Problems in Science and Engineering*, 10(4):359–375, 2010.
[5] J. Denavit and R. S. Hartenberg. A kinematic notation for lower pair mechanisms based on matrices. *ASME J. Appl. Mech.*, 77:215–221, 1955.
[6] C. Gosselin and J. Angeles. A global performance index for the kinematic optimization of robotic manipulators. *ASME Journal of Mechanical Design*, 113:220–226, 1991.
[7] S. Bai. Optimum design of spherical parallel manipulators for a prescribed workspace. *Mechanism and Machine Theory*, 45:200–211, 2010.
[8] Engineering data for harmonic drive gears. Available from: www.harmonicdrive.de/cms/upload/pdf/en/cpu_h7.pdf.
[9] R. L. Norten. *Machine Design: An Integrated Approach*. Prentice Hall, fourth edition, 2010.
[10] G. G. Antony. Rating and sizing of precision low backlash planetary gearboxes for automation motion control and robotics applications. Available from: www.neugartusa.com/Service/faq/Gear_Rating.pdf.
[11] M. J. Box. A new method of constrained optimization and a comparison with other methods. *Computer Journal*, (8):42–52, 1965.
[12] Maxon motor products catalogue 10/11. Available from: www.maxonmotor.ch/e-paper/blaetterkatalog/pdf/complete.pdf.
[13] Harmonic drive technical data. Available from: www.harmonicdrive.de/cms/upload/German/B Produkte/B_Units/kompl_Produktkapitel_CPU_D-E.pdf.