Achromatic polarization rotator with tunable rotation angle

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Abstract
We theoretically suggest and experimentally demonstrate a broadband composite optical rotator that is capable of rotating the polarization plane of a linearly-polarized light at any chosen angle. The device is composed of an even number of half-wave plates (WPs) rotated at specific angles with respect to their fast-polarization axes. The frequency bandwidth of the polarization rotator in principal increases with the number of half-WPs. Here we experimentally examine the performance of rotators composed of two, four, six, eight and ten half-WPs.

Keywords: polarization, polarization rotator, tunable rotator, achromatic rotator

(Some figures may appear in colour only in the online journal)

1. Introduction
Broadband polarization manipulation of light has been a topic of great interest in optics for many years [1–9]. First achromatic wave plates (WPs) were developed in combinations of plates having different birefringence dispersions [1], then achromatic retarders composed of two and three WPs of the same material but different thicknesses were demonstrated [2, 3]. Later, achromatic WPs with six [4], ten [5] and arbitrary number WPs [6] were experimentally implemented. In contrast to achromatic WPs, which have long history, the achromatic polarization rotators were proposed [7] and experimentally realized [8, 9] only recently. The demonstrated broadband polarization rotator schemes heretofore use two achromatic half-WPs with the rotation angle being double the angle between the fast optical axis of the two half-WPs [7–9]. This approach uses the universal principle that two crossed half-WPs serve as a polarization rotator, and therefore any combination of two achromatic half-WPs serve as a broadband polarization rotator. In the case of the Messaadi et al [9], the achromatic half-WPs were implemented with two double Fresnel rhombs, while in the case of [7, 8] the achromatic half-WPs were composite half-WPs [6, 10].

In this paper, we further develop the idea of broadband polarization rotator by using a set of an even number of half-WPs rotated at predetermined angles. The even number half-WPs is crucial for the proposed rotator due to the fact that combination of any two rotators is a rotator, therefore we arrange a sequences of rotators, each of which is combination of two half-WPs. The realized rotators are theoretically predicted by the use of the additional free parameters in order to perform of the rotator bandwidth.

2. Theory
The WPs (or retarders) and rotators are two of the basic elements in polarization optics [11, 12]. In the horizontal–vertical (HV) basis, a rotation at an angle \( \theta \) is described by the Jones matrix

\[
R(\theta) = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix}
\]

(1)

The combination of two rotators is also a rotator

\[
R(\theta_2)R(\theta_1) = R(\theta_1 + \theta_2).
\]

(2)

In the HV basis, the Jones matrix for a retarder when the optical axes are aligned with the horizontal and vertical
Table 1. Calculated angles of rotation $\theta_N$ (in degrees) for different numbers $N$ of constituent half-wave plates and different rotator angles.

| $N$ | Rotation angles $\theta_1, \theta_2, \ldots \theta_N$ |
|-----|---------------------------------|
| 2   | $(-3.75; 7.5)$                  |
| 4   | $(40.6; 119.3; 116.4; 22.7)$    |
| 6   | $(64.7; 112.9; 58.6; 43.0; 99.6; 52.1)$ |
| 8   | $(-33.9; 165.7; 175.6; 73.1; 31.3; 53.5; 66.4)$ |
| 10  | $(75.4; 4.9; 57.4; 56.9; 6.0; 124.4; 156.6; 73.1; 31.3; 53.5; 66.4)$ |

30 degree rotator

| $N$ | Rotation angles $\theta_1, \theta_2, \ldots \theta_N$ |
|-----|---------------------------------|
| 2   | $(-7.5; 7.5)$                  |
| 4   | $(183.0; 176.4; 78.8; 70.4)$   |
| 6   | $(119.2; 110.8; 55.4; 103; 83.14; 29.0)$ |
| 8   | $(129.4; 166.3; 51.4; 4.7; 79.5; 128.2; 63.1; 9.2)$ |
| 10  | $(43.4; 125.7; 114.6; 172.2; 112.2; 99.4; 156.0; 55.6; 141.3; 99.4)$ |

45 degree rotator

| $N$ | Rotation angles $\theta_1, \theta_2, \ldots \theta_N$ |
|-----|---------------------------------|
| 2   | $(-11.25; 11.25)$             |
| 4   | $(106.90; 92.68; 170.96; 162.68)$ |
| 6   | $(46.9; 173.2; 47.2; 22.4; 148.8; 24.9)$ |
| 8   | $(-177.6; 16.3; 97.2; 115.5; 173.2; 109.1; 175.4; 64.8)$ |
| 10  | $(123.4; 50.8; 80.6; 175.1; 49.6; 61.7; 172.1; 22.6; 48.7; 161.3)$ |

Table 2. Calculated angles of rotation $\theta_N$ (in degrees) for different numbers $N$ of constituent half-wave plates and different rotator angles.

| $N$ | Rotation angles $\theta_1, \theta_2, \ldots \theta_N$ |
|-----|---------------------------------|
| 2   | $(-15; 15)$                    |
| 4   | $(138.2; 29.3; 9.9; 88.9)$     |
| 6   | $(14.5; 138.2; 14.3; 172.2; 113.9; 162.3)$ |
| 8   | $(-43.4; 23.5; 121.0; 179.7; 122; 12.6; 56.6; 10.4)$ |
| 10  | $(129.0; 88.2; 154.4; 24.9; 105.9; 131.6; 63.0; 110.5; 54.6; 121.7)$ |

60 degree rotator

| $N$ | Rotation angles $\theta_1, \theta_2, \ldots \theta_N$ |
|-----|---------------------------------|
| 2   | $(-18.75; 18.75)$              |
| 4   | $(61.9; 133.8; 117.4; 8.1)$    |
| 6   | $(164.0; 119.1; 179.8; 132.0; 75.5; 130.7)$ |
| 8   | $(111.4; 60.0; 56.1; 133.2; 128.3; 51.7; 126.9; 140.2)$ |
| 10  | $(255.8; 170.3; 54.3; 67.3; 64.3; 98.6; 17.3; 3.9; 37.9; 51.9)$ |

75 degree rotator

| $N$ | Rotation angles $\theta_1, \theta_2, \ldots \theta_N$ |
|-----|---------------------------------|
| 2   | $(-22.5; 22.5)$                |
| 4   | $(188.7; 77.5; 57.9; 124.2)$   |
| 6   | $(126.3; 116.0; 164.8; 90.1; 63.1; 103.0)$ |
| 8   | $(264.9; 84.7; 22.3; 87.5; 125.8; 131.3; 65.4; 129.9)$ |
| 10  | $(31.1; 38.9; 142.9; 3.4; 72.0; 2.7; 6.9; 116.2; 146.4; 193.2; 31.1)$ |

90 degree rotator

| $N$ | Rotation angles $\theta_1, \theta_2, \ldots \theta_N$ |
|-----|---------------------------------|

Now let us consider a sequence of two half-WPs rotated at angles $\theta_1$ and $\theta_2$ with respect to the HV basis. We multiply the Jones matrices of the two half-WPs ($\varphi = \pi$) given in equation (4), to obtain the total propagator

$$J_0(\varphi)J_0(\varphi) = \begin{bmatrix} \cos (2(\theta_2 - \theta_1)) & \sin (2(\theta_2 - \theta_1)) \\ -\sin (2(\theta_2 - \theta_1)) & \cos (2(\theta_2 - \theta_1)) \end{bmatrix}$$

which is a Jones matrix for a rotator up to an unimportant minus sign. For a sequence of $N$ such pairs, the Jones matrix is

$$J_N(\varphi) = [J_0(\pi)J_N(\varphi)]J_0(\pi)J_0(\pi) \cdots J_0(\theta_{N-1})J_0(\varphi).$$

By using the property of equation (2), we obtain

$$J_N(\varphi) = \begin{bmatrix} \cos (\alpha) & \sin (\alpha) \\ -\sin (\alpha) & \cos (\alpha) \end{bmatrix}$$

which is a rotator with a rotation angle $\alpha$ given as

$$\alpha = 2 \sum_{k=1}^{N} (-1)^k \theta_k.$$

It is easy to see that we can have the same rotation matrix $J_N(\varphi)$ if each individual rotator is rotated at an additional angle.
Therefore, we can use the additional angles $\delta_1, \delta_2, \ldots, \delta_N$ as free parameters to optimize the bandwidth performance of our rotator.

We now define the fidelity $\mathcal{F}$

$$\mathcal{F}(\varepsilon) = \frac{1}{2} \text{Tr} \left( \mathbf{R}^{-1}(\varepsilon) \mathbf{J}_a(\pi + \varepsilon) \right).$$

(11)

If the two operators $\mathbf{R}(\alpha)$ and $\mathbf{J}_a(\pi + \varepsilon)$ are identical then $\mathcal{F} = 1$, but if the two matrices differ from one another then fidelity drops. Here $\varepsilon$ represents the systematic deviation from the half-WP. Obviously, for the central wavelength at which the WPs serve as half-WPs, we have $\varepsilon = 0$ and $\mathcal{F}(0) = 1$.

In order to find the optimized angles of rotation of each WP we use the Monte Carlo method and for each number of WPs and each rotator angles and we generate $10^4$ sets of random angles $\theta_1, \theta_2, \ldots, \theta_{2N}$. We pick up solutions, which in the interval of $\varepsilon \in [-\pi, \pi]$ deliver the biggest area of the fidelity $\mathcal{F}(\varepsilon)$ and also ensure a flat top. The angles are presented in tables 1 and 2.

We note that by using the numerical solution of many optimized angles of rotation we managed to derive exact analytic formulas for the angles of rotation for the case of four WPs. A broadband rotator at angle $\alpha$ composed of four half-WPs is given as

$$\mathbf{J}_a(\pi) = J_{\theta_1}(\pi) J_{\theta_2}(\pi) J_{\theta_3}(\pi) J_{\theta_4}(\pi),$$

(12)

with

$$\theta_1 = \alpha / 8,$$
$$\theta_2 = \pi / 2 - \alpha / 8,$$
$$\theta_3 = 3\pi / 2 - 3\alpha / 8,$$
$$\theta_4 = \pi - 5\alpha / 8.$$  

(13a, 13b, 13c, 13d)

Unfortunately, we could not find analytical formulas for longer sequences of WPs, but we suspect such a formula may exist for 8 and 12 WP series.

3. Experiment

3.1. Experimental setup

Experimental investigation of the composite linear polarization rotator described above was performed by analyzing polarization of the passed light beam. The light source was a halogen lamp TUNGSRAM powered by a 6 V d.c. power supply. It covered a broad continuous spectral region from 400 to 1100 nm. A set of two irises and two planoconvex lenses, respectively $L_1, f_1 = 20$ mm and $L_2, f_2 = 150$ mm, were used for producing a collimated white light beam with a diameter of about 3 mm, see figure 1. The light was linearly polarized in the vertical plane by a polarizer $P_1$, borrowed from a Lambda-950 spectrometer (Perkin Elmer, Glan-Taylor type, spectral range of 210–1100 nm). Thereafter, it was directed through the investigated composite rotator. The

Figure 1. Experimental setup. A collimated beam of white light is formed by the light source $S$, irises $I_1$ and $I_2$, and lenses $L_1$ and $L_2$. After the polarizer $P_1$ the light is linearly polarized in the vertical direction and passes through the broadband optical rotator, built by WPs. The second polarizer $P_2$ is used as an analyzer. With the help of lens $L_3$ the beam is focused onto the optical fiber entrance $F$ which is connected to a spectrometer and a computer.
polarization of the output beam was analysed by a second polariser \( P_2 \), the same type as the \( P_1 \). Subsequently, it was directed by a lens \( L_3, f_3 = 15 \text{ mm} \) to the optical fiber \( F \) of the spectrometer AvaSpec—3648 Fiber Optic Spectrometer controlled by the proprietary software AvaSoft—version 7.5.

The linear polarization rotators were built as stacks of an even number of ordinary multi-order quarter-WPs (WPMQ10M-780, Thorlabs Inc.), the fast axes of which were rotated at the respective theoretically calculated angles given in tables 1 and 2. Each WP was 1\( " \) and was assembled onto a RSP1 (Thorlabs Inc.) rotation mount. This mounting allows rotation at 360\( ° \) and accuracy of the rotation angle 1\( ° \). The multi-order WPs are designed to be 11.25 waves and serve as a quarter WPs at 780 nm, while at 763 nm they perform like half-WPs. Here, we focus at the region where the WPs act as half-wave plates.

### 3.2. Measurement procedure

The measurement started with taking the reference signal before applying the rotation procedure. The measurement’s series of steps were similar to those taken in [13]. We saved the dark and reference spectra. The dark spectrum was taken with the light suspended and was further used to automatically correct for hardware offsets. The reference spectrum was taken with the light source on and using a blank specimen instead the specimen under test. For this purpose, the fast axes of the WPs were aligned with the axes of the polarisers \( P_1 \) and \( P_2 \). The integration time of 15 ms and the 1000 data averaging were kept the same during all following measurements. We looked at the transmittance mode of the spectrometer which for the reference signal is 100\%. For each investigated set of \( N \) WPs, where \( N \) is 2, 4, 6, 8, 10 WPs, we took the respective reference signal.

The reference signal presents zero rotation of the linearly polarized light. The next step was to realize the linear polarization rotator by rotating the fast axis of each WP at the respective angle \( \theta_n \). The measurements were accomplished at six rotation angles of the polarization \( \alpha = \{15°, 30°, 45°, 60°, 75°, 90°\} \) for each set. The analysis was done by rotating the polariser \( P_2 \) at the target angle \( \alpha \).

### 3.3. Experimental results

Angle tunable broadband polarization rotators comprising two, four, six, eight and ten WPs were experimentally demonstrated and the results correspond very accurately to the predicted theoretically ones. In figure 2, we present the experimentally measured spectra and each group of curves demonstrates the rotation effect of the chosen sets of WPs at each angle \( \alpha, \alpha \in \{15°, 30°, 45°, 60°, 75°, 90°\} \) for each set. The analysis was done by rotating the polariser \( P_2 \) at the target angle \( \alpha \).

![Figure 2. Measured transmittance versus wavelength for broadband polarization rotator, consisting of different number of half-wave plates \( N \) (2, 4, 6, 8 and 10). The rotation angle \( \alpha \) is denoted in each frame.](image-url)
However the best implementation of the rotator happen when polarization rotators comprise of four and eight WPs, therefore we have reason to believe that a combination of multiples of four WPs give optimal performance. It is also evident from figure 2 that the rotator is more robust against variations in the wavelength in the case of small rotation angles compared to large rotation angles. This is a novel feature of this composite rotator in contrast to previous composite rotators [7, 8], where almost the same spectral shape was observed. In this sense, our composite rotator with four half-WPs outperforms the previous rotator [7, 8] with six half-WPs up to rotation angles \( \pi/4 \).

4. Conclusion

In this paper, we introduced and experimentally demonstrated a novel type of broadband composite polarization rotator that can rotate the polarization plane of a linearly-polarized light at any chosen angle. The experimental results show strong broadening of the bandwidth of the polarization rotator in case of four and eight half-WPs and moderate broadband increase in other cases as the number of half-WPs grow.

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