IMPLICATION OF THE WEAK PHASE $\beta$ MEASURED IN $B \rightarrow \rho \gamma$ DECAY

C. S. KIM
Department of Physics, Yonsei University, Seoul 120-749, Korea
E-mail: cskim@yonsei.ac.kr

YEONG GYUN KIM
Department of Physics, Korea University, Seoul 136-701, Korea
E-mail: yg-kim@korea.ac.kr

KANG YOUNG LEE *
Department of Physics, KAIST, Daejeon 305-701, Korea
E-mail: kylee@muon.kaist.ac.kr

We explore the exclusive $B^0 \rightarrow \rho^0 \gamma$ decay to obtain the time-dependent CP asymmetry in $b \rightarrow d \gamma$ decay process. We find that the complex RL and RR mass insertion to the squark sector in the MSSM can lead to a large deviation of CP asymmetry from that predicted in the Standard Model.

1 Introduction

In the $B$ meson system, it is strongly required to find a new observables for the CP violation in a way independent of the $B^0$–$ar{B}^0$ mixing since the observed CP violating asymmetry appears only through the mixing so far. Moreover, we may expect that new physics can influence the $\Delta B = 1$ penguin decays in a different way from the $\Delta B = 2$ mixing, e.g. the controversial deviation of the recent measurement of $\sin 2\beta$ in $B \rightarrow \phi K$ decay from that in $B \rightarrow J/\psi K_S$ decays 1, which implies an evidence of a new physics effect beyond the SM 2.

The Cabibbo-suppressed $b \rightarrow d \gamma$ decay provides us a new chance to study the CP violation in a way independent of the mixing. In the present work, we consider the time-dependent CP asymmetry in the neutral $B^0 \rightarrow \rho^0 \gamma$ decay. Although we will be able to determine $V_{td}$ from the inclusive $B \rightarrow X_d \gamma$ decay in a theoretically clean way 3, it suffers from large $B \rightarrow X_s \gamma$ background in the experiment. The charged $B^\pm \rightarrow \rho^\pm \gamma$ decay mode provides clean signal and has a branching ratio twice larger than that of the neutral mode, by the isospin symmetry. However, the long-distance (LD) effect on the charged mode due to dominantly $W^\pm$-annihilation is very large ($\sim 30\%$), which contaminates the CP violating effect 4,5. The exclusive $B \rightarrow \rho \gamma$ decays in the SM and the MSSM have been studied in the literature 6.

The photon has two helicity states $\gamma_L$ and $\gamma_R$ although we cannot discriminate them in the experiment. Since the time-dependent CP violating asymmetry is defined when both $B$ and $\bar{B}$ mesons decay into a same state, there is no interference between final states with the definite helicity. In the SM, the operator which governs $b \rightarrow d \gamma$ decay is chiral and the conjugate operator is suppressed by $m_d/m_b$ and the CP asymmetry also suppressed accordingly. Therefore the new physics beyond the SM is required for a large time-dependent CP asymmetry enough being observed in the experiment 7.

In this work, we consider the supersymmetric models which have non-diagonal elements of the squark mass matrices, parameterized by the mass insertions $(\delta_{ij})_{MN} = (\tilde{m}_{ij}^2)_{MN}/\tilde{m}^2$, where $\tilde{m}$ is the averaged squark mass, $i$ and $j$ are flavor indices and
and so is the right polarized photon emission analysis. We investigate the time-dependent cay, the LD contribution due to cay and the time-dependent CP asymmetry. In section 2, we describe the relevant terms of the effective Hamiltonian for their chiral conjugate operators. The effective Hamiltonian is given by

\[ \mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{q=u,c} \left[ \lambda_q \left( C_1 O_1^q + C_1' O_1'^q \right) - \lambda_t \left( C_7^\text{eff} O_7 + C_7'^\text{eff} O_7' \right) + \cdots \right], \]

where

\[ \lambda_q = V_{qb} V_{q'd}^* \,, \quad O_1^q = (\bar{d}_L^q \gamma_\mu q_L^b)(\bar{q}_L^b \gamma^\mu b_L^q) \,, \quad O_1'^q = (\bar{d}_L^q \gamma_\mu q_L^b)(\bar{q}_L^b \gamma^\mu b_L^q) \,, \quad \text{and} \quad O_7 = (c_{mb}/16\pi^2) \bar{d}_L^q \sigma_{\mu\nu} F^{\mu\nu} b_R. \]

The primed \( O'_i \) are their chiral conjugate operators. The effective Wilson coefficient \( C_7^\text{eff} \) includes the effects of operator mixing.

We write the amplitudes for the final states of polarized photon as

\[ A_L \equiv \langle \gamma_L | \mathcal{H}_{\text{eff}} | B^0 \rangle \sim C_7^\text{eff} \lambda_t^* \langle \gamma_L | O_7^q | B^0 \rangle, \]

\[ A_R \equiv \langle \gamma_R | \mathcal{H}_{\text{eff}} | B^0 \rangle \sim C_7^\text{eff} \lambda_t \langle \gamma_R | O_7^q | B^0 \rangle, \]

\[ \bar{A}_L \equiv \langle \gamma_L | \mathcal{H}_{\text{eff}} | \bar{B}^0 \rangle \sim C_7^\text{eff} \lambda_t \langle \gamma_L | O_7^q | \bar{B}^0 \rangle, \]

\[ \bar{A}_R \equiv \langle \gamma_R | \mathcal{H}_{\text{eff}} | \bar{B}^0 \rangle \sim C_7^\text{eff} \lambda_t \langle \gamma_R | O_7^q | \bar{B}^0 \rangle, \]

up to the factor of \( 4G_F/\sqrt{2} \). We note that

\[ \langle \gamma_L | O_7 | B^0 \rangle = \langle \gamma_L | O_7'^q | B^0 \rangle, \]

\[ \langle \gamma_R | O_7'^q | B^0 \rangle = \langle \gamma_R | O_7 | B^0 \rangle. \]

In the SM, \( C_7^\text{eff} \) is suppressed by the mass ratio \( m_d/m_b \) and so is the right polarized photon emission \( b_L \rightarrow q\gamma_R \). For the neutral \( B \) meson decay, the LD contribution due to \( W \)-exchange is merely a few % from the QCD sum rule calculation 4,5, so it will be ignored in our analysis. We investigate the time-dependent CP asymmetry given by

\[ A_{\text{CP}}(t) = \frac{\bar{\Gamma} - \Gamma}{\bar{\Gamma} + \Gamma} = -C \cos(\Delta m_B t) + S \sin(\Delta m_B t), \]

where \( \bar{\Gamma} = \Gamma(B^0(t) \rightarrow \rho^0 \gamma_L) + \Gamma(\bar{B}^0(t) \rightarrow \rho^0 \gamma_R), \Gamma = \Gamma(B^0(t) \rightarrow \rho^0 \gamma_L) + \Gamma(\bar{B}^0(t) \rightarrow \rho^0 \gamma_R), \) since we cannot distinguish \( \gamma_L \) and \( \gamma_R \) in practice. The coefficients \( C = 0 \) and

\[ S = \frac{|A_L|^2 |\text{Im} \lambda_L| + |A_R|^2 |\text{Im} \lambda_R|}{|A_L|^2 + |A_R|^2}, \]

with the parameter \( \lambda_{L(R)} \) defined by

\[ \lambda_{L(R)} = \sqrt{\frac{M_{12}^2 \bar{A}_{L(R)}}{M_{12}^2 A_{L(R)}}}. \]

The off-diagonal element \( M_{12} \) describes the \( B^0-\bar{B}^0 \) mixing and \( A_{L(R)} \) does the \( b \rightarrow d\gamma \) decays. We define \( 2\beta_\text{mix} = \text{Arg}(M_{12}) \) and \( 2\beta_\text{decay} = \text{Arg}(A_R/A_L) = \text{Arg}(\bar{A}_R/A_L). \) Then the coefficient \( S \) is expressed by

\[ S = -\frac{2 |C_7||C_7'|}{|C_7|^2 + |C_7'|^2} \sin(2\beta_\text{mix} - 2\beta_\text{decay}), \]

where we rewrite

\[ 2\beta_\text{decay} = 2\beta_\text{SM} + \text{Arg}(C_7') - \text{Arg}(C_7). \]

Note that we have an additional factor \( 2 |C_7||C_7'|/(|C_7|^2 + |C_7'|^2) \), which can enhance or suppress \( S \) by the new physics effect \( C_7' \).

### 3 SUSY contributions

By penguin diagrams with gluino-squark loop, the Wilson coefficients \( C_i \) get contribution to produce \( \gamma_R \) at the matching scale \( \mu = m_W \). After the RG evolution, we have

\[ C_7^\text{eff}(m_b) = C_7^\text{SM}(m_b) = -0.31 \]

and

\[ C_7^\text{eff}(m_b) = \frac{\sqrt{2}}{G_F V_{tb} V_{td}^*} \left( 0.67 C_7^\text{SUSY}(m_W) + 0.09 C_8^\text{SUSY}(m_W) \right), \]

where the SUSY contributions at \( \mu = m_W \) are

\[ C_7^\text{SUSY} = \frac{4\alpha_s \pi Q_b}{3m_b^2} \left[ (\delta_{13})_{RR} M_4(x) - (\delta_{13})_{RL} A_4(x) \right], \]
\[ C_8^{\text{SUSY}} = \frac{\alpha_s \pi}{6 \bar{m}^2} \left( (\delta_{13})_{RR}(9M_3(x) - M_4(x)) + (\delta_{13})_{RL} \left( 4B_1(x) - 9B_2(x) \right) \frac{m_\tilde{g}}{m_\tilde{b}} \right), \]

with \( x = (m_\tilde{g}/\bar{m})^2 \). Note that the SUSY contribution is more sensitive to \((\delta_{13})_{RL}\) than \((\delta_{13})_{RR}\) due to the enhancement factor \(m_\tilde{g}/m_\tilde{b}\). The loop functions \(B_i(x)\) are found in the literature\(^9\). Since \(\delta_{RL,RR}\) are complex in general, the Wilson coefficients \(C''_{\tau}^{\text{eff}}(m_\tilde{b})\) has nontrivial phase which affects the phase of \(A/A\).

On the other hand, the \(B\bar{B}\) mixing is affected by the gluino-squark box diagrams in the MSSM. The relevant \(\Delta B = 2\) effective Hamiltonian with the supersymmetric contribution contains new scalar-scalar interaction operators \(\mathcal{O}_{S_2} = (d_\alpha(1+\gamma_5)b_\alpha)(\bar{d}\gamma_5(1+\gamma_5)b_3), \mathcal{O}'_{S_2} = (d_\alpha(1+\gamma_5)b_\alpha)(\bar{d}_\alpha(1+\gamma_5)b_3)\), when we introduce only the RL and RR mass insertions. The Wilson coefficient \(C_1\) corresponding to the SM operator \(O_1 = (\bar{d}\gamma_5(1-\gamma_5)b)(\bar{d}\gamma_5(1-\gamma_5)b)\) consists of the SM part and the supersymmetric contributions, while \(C''_{S_2}\) and \(C''_{S_3}\) corresponding to the above operators are entirely supersymmetric. Their explicit expression at the scale \(\mu = M_\text{SUSY}\) can be found in Refs.\(^{10,11}\). The RG evolved Wilson coefficients from \(m_W\) to \(m_\tilde{b}\) scale ignoring the RG running effects between \(M_\text{SUSY}\) and \(m_W\), are given in Ref.\(^{12}\).

### 4 Numerical results

Figure 1 shows the quantity \(S\) as a function of the phase of \((\delta_{13})_{RL}, \varphi\), assuming \(|\delta_{13})_{RL}| = 0.001\). We vary the weak phase \(\gamma\) from 0 to \(2\pi\). Hereafter we use the input parameters as follows: \(m_B = 5.3\) GeV, \(m_t = 174.3\) GeV, \(m_b = 4.6\) GeV, and \(\alpha_s(m_Z) = 0.118\). The decay constant \(f_\rho = 200 \pm 30\) MeV is the main source of the theoretical uncertainty and the bag parameters are those of Ref.\(^{13}\); \(B_1 = 0.87, B_2 = 0.82, B_3 = 1.02\). The supersymmetric scale is taken to be \(m_\tilde{g} \simeq \bar{m} \simeq M_\text{SUSY} \approx 500\) GeV. We require that the mass difference \(\Delta m_B\) and \(\beta_\text{mix}\) in \(B \to J/\psi K\) decay should be within the experimental limit: \(\Delta m_B = 0.489 \pm 0.008\) ps\(^{-1}\) and \(\sin 2\beta_\text{mix} = 0.734 \pm 0.055\). We do not use \(\text{Br}(B \to \rho/\omega\gamma)\) as a constraint since it involves a large theoretical uncertainty in the form factor. Instead, we assume a moderate upper bound on the branching ratio of the inclusive \(B \to X_d\gamma\) decay \(\text{Br}(B \to X_d\gamma) \leq 1.0 \times 10^{-6}\), following Ref.\(^{11}\). although the inclusive decay is not observed yet. The black region corresponds to the allowed values for the phase of \((\delta_{13})_{RL}\), while the grey (green) region denotes the parameter set which satisfies the \(\Delta m_B\) and \(\sin 2\beta_\text{mix}\) constraints but exceeds the bound on \(\text{Br}(B \to X_d\gamma)\). We find that large CP violating asymmetry is possible.

The plot of \(S\) with respect to \(|\delta_{13})_{RL}|\) is depicted in Fig. 2 when the phase \(\varphi\) is fixed to be zero. The black region and the grey (green) region are defined as in Fig. 1. We see that \(|\delta_{13})_{RL}|\) is strongly constrained by the inclusive branching ratio and a large CP violation is still possible even when \(C''_{\tau}^{\text{eff}}(m_\tilde{b})\) is real. The branching ratio \(\text{Br}(B \to X_d\gamma)\) and CP asymmetry \(S\) provide the complimentary information on \((\delta_{13})_{RL}\).
Figure 2. The time-dependent CP asymmetry $S$ as a function of $|\delta_{13}|_{RL}$. The phase of $(\delta_{13})_{RL}$ is assumed to be 0. The black region and the grey (green) region are defined in Fig. 1.

5 Concluding remarks

If we observe a sizable CP asymmetry in $B^0 \to \rho^0 \gamma$ decay, it will be a clear evidence of the new physics beyond the SM. Although it is hardly expected that the time dependent CP asymmetry of $B^0 \to \rho^0 \gamma$ will be measured in the present $B$-factory, it will be achieved in the next generation of $B$-factory with about 100 times more $B$ mesons produced. Due to the agreement of the SM prediction with the present $\Delta m_B$ data and the CP asymmetry in $B \to J/\psi K$ decay, we favor the new physics which contributes less to the $B-\bar{B}$ mixing but has a strong effect on the $b \to d\gamma$ penguin diagram. In this work, we showed that the RL mass insertion of squark mixing of the MSSM can produce a large CP asymmetry of $B^0 \to \rho^0 \gamma$ decay process.

References

1. K. Abe et al., Belle Collaboration, Phys. Rev. D 66, 071102 (2002); A. Aubert et al., BaBar Collaboration, Phys. Rev. Lett. 89, 201802 (2002).
2. B. Dutta, C. S. Kim and S. Oh, Phys. Rev. Lett. 90, 011801 (2003); B. Dutta, C. S. Kim, S. Oh and G. H. Zhu, arXiv:hep-ph/0312389; S. Khalil and E. Kou, Phys. Rev. D 67, 055009 (2003); J.-P. Lee and K. Y. Lee, Euro. Phys. J. C 29, 373 (2003); G. L. Kane et al., Phys. Rev. Lett. 90, 141803 (2003).
3. C. S. Kim, T. Morozumi and A. I. Sanda, Phys. Rev. D 56, 7240 (1997); D. Atwood, B. Blok and A. Soni, Int. J. Mod. Phys. A 11, 3743 (1996); Nuovo Cimento 109A, 873 (1996).
4. A. Ali and V. M. Braun, Phys. Lett. B 359, 223 (1995).
5. A. Khodjamirian, G. Stoll, and D. Wyler, Phys. Lett. B 358, 129 (1995).
6. A. Ali, L. T. Handoko, and D. London, Phys. Rev. D 63, 014014 (2000); A. Ali and E. Lunghi, Euro. Phys. J. C 26, 195 (2002); K. Kiers, A. Soni and G.-H. Wu, Phys. Rev. D 62, 116004 (2000).
7. D. Atwood, M. Gronau and A. Soni, Phys. Rev. Lett. 79, 185 (1997).
8. A. Ali, V. M. Braun, H. Simma, Z. Phys. C 63, 437 (1994).
9. L. Everett et al., JHEP 0201, 022 (2002); S. Baek et al., Nucl. Phys. B609, 442 (2001); A. J. Buras et al., Nucl. Phys. B566, 3 (2000); T. Besmer, C. Greub, and T. Hurth, Nucl. Phys. B609, 359 (2001).
10. F. Gabbiani et al., Nucl. Phys. B477, 321 (1996).
11. P. Ko, G. Kramer, and J.-h. Park, Euro. Phys. J. C 25, 615 (2002).
12. D. Becirevic et al., Nucl. Phys. B634, 105 (2002).
13. D. Becirevic et al., JHEP 0204, 025 (2002).
14. K. Hagiwara et al., Review of Particle Physics, Phys. Rev. D 66, 010001 (2002).