Spin Filtering in Multiple Scattering off Point Magnets
Areg Ghazaryan, Mikhail Lemeshko, and Artem G. Volosniev
IST Austria (Institute of Science and Technology Austria), Am Campus 1, 3400 Klosterneuburg, Austria

Electron spin filters that produce spin-polarized currents of high intensity have important applications in different branches of physics. In this work, we propose an efficient spin filter based upon scattering off a two-dimensional crystal made of aligned point magnets. We demonstrate that such a crystal greatly increases polarization for specific ‘magic’ values of parameters. While polarization increase is accompanied by higher reflectivity of the crystal, higher output currents can be obtained in scattering off a quantum cavity made of two crystals. Besides being a spin filter, our setup gives some insight into collective scattering of electrons from aligned chiral molecules, and, thus, into the chiral induced spin selectivity effect.

Introduction. The quest for spin filters started directly after the discovery of spin. It turns out that for electrons (in contrast to atoms), this problem is not trivial, since the Lorentz force and the uncertainty principle render it difficult, if not impossible, to achieve spin polarization using magnetic fields alone [1–3]. The quest continues even after a century of research and numerous proposals [4–11]. The applications of polarizers are quite diverse and span atomic, molecular, nuclear, and condensed-matter physics [12–16]. They are used to study magnetization dynamics [17, 18] and in spin and angle resolved photoemission spectroscopy of topological materials [19], to give just a few examples.

At the present time, not only inorganic but also organic systems are being considered as possible spintronic devices [20]. Recent experiments show that electrons become spin-polarized when passing through a molecular monolayer of chiral molecules (such as DNA, oligopeptides, helicine, etc.) [21–28]. This property of chiral molecules is now called chiral induced spin selectivity (CISS), and its existence can lead to novel spin filters [29, 30]. The magnitude of polarization in CISS is quite high, yet the intensity of the outgoing flux is relatively low. Despite the seeming simplicity of the CISS experiments, the observed effect is an outstanding problem in theoretical physics. Several models that rely on scattering off a single molecule have been suggested [31–44]. However, it is still not clear whether the effect can be observed at a single-molecule level or CISS requires electron scattering off multiple molecules. In particular, strong dependence of the asymmetry function on the doping level [45] suggests that multiple scattering might be important.

In this Letter, we address the issues outlined above. First, we propose a spin filter made of point scatterers whose scattering length depends on the spin of incoming electrons. The filter has conceptual similarities with a layer of chiral molecules, thus, it gives insight into CISS as a collective-scattering phenomenon.

We investigate scattering off a two-dimensional (2D) layer of spatially arranged point scatterers (magnets), see Fig. 1(a), which for ‘magic’ parameters acts as a perfect mirror. We show that the layer functions as a spin filter for low-energy electrons in the vicinity of the ‘magic’ point: while one spin component is perfectly reflected, the other one is transmitted. Although the intensity of the outgoing current is quite low for scattering off a single mirror, it can be substantially enhanced through scattering off two identical mirrors, Fig. 1(b). This setup bears some similarity to a 1D spin filter with a spin-dependent energy profile [7]. A possible experimental realization of the suggested spin filter is to dope the outer layer of a GaAs superlattice with two layers of magnetic adatoms. This should be possible without considerable fine tuning, because current state-of-the-art polarizers for microscopy applications are based on negative electron affinity GaAs superlattice photocathodes [46–49]. The observed spin polarization in those set-ups can be larger than 80% and the corresponding quantum efficiency is on the order of several percent.

Our ideas do not employ fundamental properties of electrons, and can be used to implement spin filters in other systems as well. Our proposal could be tested with cold atoms – a tunable testground for studying quantum transport phenomena [50]. Layers of atoms created with optical lattices [51] could simulate point magnets.

Another type of atoms would then be used to simulate electrons, in particular, electron’s spin would be modelled by a hyperfine state of the atom [52]. Realization of our proposal with cold atoms would extend the existing one-dimensional family of cold-atom spin filters [53–55] to the three-dimensional world. Our findings are also connected...
to light scattering from an array of point dipoles [56, 57] (although the latter has an additional complication due to the polarization of light). In particular, cooperative resonances in light scattering allow for a regime where a sheet acts a perfect mirror [58, 59], which is similar to what we find in our model. Our work adds another degree of freedom (spin) to this discussion, and acknowledges spin-filtering capabilities of a layer of point scatterers.

Single Layer. First, we consider electrons impinging perpendicular to an infinite layer of spin-dependent contact potentials see Fig. 1 (a). For the sake of discussion, the scatterers are placed in the nodes of a square lattice, i.e., at $a_{lm} = lb\hat{x} + mb\hat{y} + 0\hat{z}$, where $l$ and $m$ are integers and $b$ is the lattice constant. We have checked that other geometries, e.g., a triangular lattice, lead to similar results. The electron wave function is an eigenstate of a translation operator that shifts the wave function by $a_{lm}$, i.e., $\psi(r + a_{lm}) = e^{i(k\cdot a_{lm})}\psi(r)$, where $k_i$ is the momentum of an incoming electron. We assume that $|k_i| = k$ ($|k_i| = k$), hence $k_i a_{lm} = 0$. The corresponding scattering state reads

$$\psi(r) = e^{ikz} + A \sum_{lm} e^{ik|r - a_{lm}|} \frac{|r - a_{lm}|}{|r - a_{lm}|}$$

(1)

where the incoming flux is given by the plane wave, and the outgoing flux is made of spherical waves propagating away from the point scatterers. The last term in Eq. (1) is defined as the limit: $\lim_{lR \to \infty} A R \sum_{lm} R |r - a_{lm}|$ is a dimensionless cut-off parameter, see the discussion below. The constant $A R$ is determined from the boundary conditions [60]:

$$\psi(r \to a_{lm}) = s_{lm} \left( \frac{1}{|r - a_{lm}|} - \frac{1}{\alpha_s} \right)$$

(2)

where $s_{lm}$ is the normalization constant, $\alpha_s$ is a spin-dependent scattering length that fully determines a zero-range potential. Note that our zero-range approach can model low-energy scattering off potentials that decay faster than $1/r^3$ at large interparticle distances, provided that $b$ is larger than the range associated with the potential. In particular, our model is appropriate for electron-atom interactions ($\sim 1/r^4$ as $r \to \infty$). We assume that $\alpha_+ = a_0 + a_1$ and $\alpha_- = a_0 - a_1$, where $\uparrow$ ($\downarrow$) denotes a spin projection of incoming electrons on the desired quantization axis (for illustration purposes, we have chosen it as the $y$-axis in Fig. 1); $a_0$ ($a_1$) describes the spin-independent (spin-dependent) part of the potential. Imposing the boundary condition, we obtain $A R$:

$$A R = -\alpha_s \left( 1 + \alpha_s \sum_{lm} R \frac{|a_{lm}|}{|a_{lm}|} + i k \alpha_s \right)^{-1}$$

(3)

where $R$ is used to define an upper limit of the sum. Equations (1) and (3) fully determine all properties of scattering.

Zero-Energy Limit. To gain analytical insight, we explore the zero-energy limit ($k \to 0$). The layer of magnets appears to be homogeneous for a distant observer ($|z| \gg b$), allowing us to focus on $r = z\hat{r}$. To write the wave function, we should estimate the sums in Eqs. (1) and (3) for large values of $R$. This can be done using the integral test: $\sum_{lm} \frac{1}{|a_{lm}|} \approx \frac{2\pi}{|r - a_{lm}|} + \frac{2\pi}{|r - a_{lm}|}$ and $\sum_{lm,a_{lm} \neq 0} \frac{1}{|a_{lm}|} \approx \frac{2\pi}{R}$, where $\Delta_0 \geq 0$ is a constant, which depends only on the geometry of the system; it can easily be determined numerically, $\Delta_0 \approx 0.635$. Both sums diverge linearly with $R$ as $R \to \infty$, leading to the wave function:

$$\psi(r) = 1 + \frac{2\pi\alpha_s}{b^2} |z| - \frac{2\pi\alpha_s}{b^2} \Delta_0$$

(4)

which is identical to the 1D wave function that describes zero-energy scattering off the Dirac delta potential, $g_\delta(z) \psi(z)\left( \frac{2\pi}{|z| + 1} \right)$ [61]. This observation allows us to map the 3D problem onto a 1D zero-range model with

$$g_\delta(z) = \frac{4\pi\alpha_s}{b(b - 2\pi\alpha_s\Delta_0)}$$

(5)

Considering finite-energy scattering off the potential $g_\delta(z)$, we determine the transmission and reflection coefficients as $T = 4k^2 / (g^2 + 4k^2)$ and $R = g^2 / (g^2 + 4k^2)$, respectively. The corresponding spin polarization is $P = T / T + R$. From Eq. (5) is accurate only for $k \to 0$, similar relations exist also for finite values of $k$ [62]. It is clear from (5), that the 1D potential amplitude diverges when $b = 2\pi\alpha_s\Delta_0$, and the layer of scatterers makes a perfect mirror. This quantum interference phenomenon is related to the perfect mirror regime observed in light scattering off a layer of point dipoles [56, 58, 59].

Let us analyze the polarization, $P$, in the two limiting cases: $a_0 = 0$ and $|a_1| < |a_0|$. Note that there can be no polarization in scattering off a single zero-range potential with either $a_0 = 0$ or $a_1 = 0$. Therefore, the limits address the importance of multiple scatterings. For $a_0 = 0$, we derive

$$P = -\frac{16\pi^3 a_0^3 b \Delta_0}{k^2 b^2 (b^2 - 4\pi^2 a_0^2 \Delta_0^2) + 4\pi^2 a_0^2 (b^2 + 4\pi^2 a_0^2 \Delta_0^2)}$$

(6)

We can further simplify this expression assuming low-energy scattering, $kb^2 \ll |a_1|$, and $a_1 \ll b$: $P \approx -4\pi a_0 \Delta_0 / b$. In this limit $P \propto \sqrt{n}$, where $n = 1/b^2$ is the density of scatterers. This dependence is a manifestation of coherent scattering, since for incoherent scattering one expects observables to be proportional to $n$. We find that $P \to 0$ for $b \to \infty$, recovering the fact that a single scatterer with $a_0 = 0$ cannot act as a spin polarizer.

In the other limit, $|a_1| \ll |a_0|$, we derive

$$P \approx -\frac{8\pi^2 a_0 a_1 b}{k^2 b^2 (b - 2\pi a_0 \Delta_0)^2 + 4\pi^2 a_0^2 (b - 2\pi a_0 \Delta_0)}$$

(7)
FIG. 2. (a) Dependence of transmission (points connected by solid curves) and the absolute value of polarization (points connected by dotted curves) on the dimensionless scattering length, $a_1/b$, when $a_0 = 0$. Dependence of the polarization coefficient on $a_0/b$ when (b) $a_1/a_0 = 0.1$ and (c) $a_1/a_0 = 0.3$. The transmission coefficient, $T$, is defined as $T = (T_1 + T_2)/2$.

Taking again the limit of $kb^2 \ll |a_0|$ and $|a_0| \ll b$, we obtain $P \approx -2a_1/a_0 - 4\pi a_1 \Delta a/b$, which has the same density dependence as the previous case, although, a single scatterer can act as a spin polarizer if $|a_1| \neq 0$, the corresponding polarization is $P \approx -2a_1/a_0$. For electrons with $k \neq 0$, there is a competition between the two terms in the denominator of Eqs. (6) and (7), which makes the dependence on $n$ more complex. Finally, we note that the polarization (7) diverges for the ‘magic’ lattice spacing, $b_c \approx 2\pi a_0 \Delta a$: the transmission vanishes at the same time. This regime holds promise for constructing a spin filter, as we discuss below.

Full Solution. Having analyzed the zero-energy limit, we now consider finite energies in more detail. For low energies, we establish a 1D mapping similar to Eq. (4) [62]. However, this mapping does not yield any qualitatively new results, and below we simply illustrate the finite-energy solution for certain values of parameters. Figure 2 (a) shows the dependence of transmission and polarization on the dimensionless scattering length $a_1/b$ for different values of electron momenta, assuming that $a_0 = 0$. As was described above, for the ‘magic’ ratio of $a_1/b$ the transmission $T_1$ goes to zero and the layer of magnets acts as a perfect mirror. The polarization is peaked for the same parameters. While our zero-energy results imply that the position of the peak does not depend on $k$, that is no longer the case for the full solution. We do observe a minor change of the peak position in Fig. 2 (a). For small momenta the transmission $T = (T_1 + T_2)/2$ is vanishing everywhere in the region with noticeable polarization, however, the situation changes if $kb$ is increased. At $kb = 1.0$ there is already a range of $a_1/b$ where both transmission and polarization are substantial. Note that a zero-range potential is useful only for small values of $kr_{\text{eff}}$ [60], where $r_{\text{eff}}$ is the effective range, because otherwise electrons start to resolve finer details of the interaction potential, beyond the zero-range treatment. In other words, our results for $kb = 1$ are accurate as long as $b \gg r_{\text{eff}}$. Figure 2 also shows the dependence of polarization on $a_0/b$ for $a_1/a_0 = 0.1$ (b) and $a_1/a_0 = 0.3$ (c). The polarization changes sign in this case at the ‘magic’ point and peaks on both sides of that point. The value of $kb$ does not have any important effect on the position of the peak. Still, working with larger momenta is beneficial, since it modifies transmission considerably (similar to Fig. 2 (a)). Note that the width of the resonance increases with $a_1/a_0$.

Two Layers. Figure 2 clearly shows the transmission-polarization tradeoff present in our set-up: large values of $P$ are possible only for small values of $T$. To overcome this problem, we consider two aligned identical 2D sheets, see Fig. 1 (b). The sheets form a resonating cavity in the vicinity of the ‘magic’ point, which can be used to tune scattering properties. One could introduce some potential dependence in between the layers, either to reflect
some material between layers or as an additional tuning parameter for quantum simulations [55, 63]. We leave this discussion to future studies, as we do not expect a slow-varying potential to qualitatively change the transmission of our setup (see, e.g., Ref. [64]). Similar to the single-layer case, we cast the 3D problem into a 1D one with two delta-function potentials of the strength (5). This approximation is accurate as long as $L \gg b$, where $L$ is the distance between the potentials (layers).

Since for a single layer the results were (almost) energy-independent, we consider below only the zero-energy solution, see [62] for comparison. The transmission coefficient for scattering off two zero-range potentials is

$$T_s = \left| \frac{4k^2}{g_s^2 + (g_a + 2k)^2} \right|^2. \quad \text{we do not present the expression for polarization – it is cumbersome and does not provide us with any further insight. Instead, we analyze scattering for the parameters used to illustrate the single-layer case (see Fig. 2). For the sake of discussion, we assume that $L = 100b$. We have checked that the results do not vary qualitatively with $L$.}

Figure 3 presents the transmission and polarization coefficients for scattering off two layers. Interference inside the cavity leads to additional peaks for both transmission and polarization. These peaks can be used to engineer regions where both polarization and transmission are substantial for any energy of incoming electrons. Our conclusion is that two sheets of quantum scatterers have enough tunability to allow for an efficient spin filter. The fact that the inter-sheet separation can be several orders of magnitude larger than the spacing between quantum scatterers makes it feasible to engineer such a filter with GaAs superlattices as briefly outlined in the introduction.

**CISS.** The considered system is too simplistic to account for the complete physical picture of the CISS effect. Electron energies in the CISS experiments are typically in the range of $0–2$ eV [21, 65], which cannot be fully described with a zero-range treatment. Indeed, $kb \approx 0–10$ is certainly beyond our zero-range model. We have assumed in this estimate that inter-molecular separation and low-energy scattering parameters are all of the order $1$ nm [66, 67]. Still, our model is useful as it addresses the low-energy limit, which is an important reference point for theoretical analysis of chiral molecules [68]. In particular, our model can estimate the importance of many-molecule scattering for CISS.

To model the CISS experiments, we consider a single layer of scatterers with $a_0 \neq 0$ and $a_1 \neq 0$. The parameter $a_0$ describes the spin-independent part of low-energy scattering off molecules. We choose this parameter to be about the molecular diameter, i.e., about $1–2$ nm [66, 67]. The parameter $a_1$ describes the spin asymmetry in low-energy scattering. The spin-orbit coupling is weak for organic molecules, therefore, we consider $|a_1| \ll |a_0|$. Two important corollaries follow from the analysis of this CISS model. First, the polarization depends weakly on the density of scatterers (it scales as $\sqrt{n}$), which shows that collective interference is important for CISS at low energies. This aligns nicely with the fact that the CISS effect is strong for a wide range of inter-molecular separations $b \sim 1–20$ nm [66]. Second, one expects that $a_0$ is comparable to $b$, therefore, it is quite likely that the system operates close to the ‘magic’ parameter regime. The polarization reversal observed in: (i) molecules embedded in the membrane [24], and (ii) experiments with a variable temperature [69] can be a consequence of that. Indeed, both embedding and temperature denaturation of molecules modify scattering, and hence, the $a_0/b$ ratio, which determines the sign of the polarization coefficient (see Fig. 2 (b)).

**Conclusions.** We have considered a 2D sheet of point scatterers, and have shown that quantum interference in this system leads to spin filtering. Even though a single-layer spin filter suffers from a reflection-polarization tradeoff, we have demonstrated that two parallel sheets of scatterers can provide simultaneously high transmission and high polarization. In addition, we have investigated the importance of many-molecule scattering in the CISS effect. In particular, we have argued that the observed reversal of polarization could be explained from scattering close to the mirror regime.

Finally, we note that our work does not address the dependence of the results on the direction of the incoming flux and presence of disorder. These issues are important for experimental realization of the proposed setup, especially in solid-state settings, and will be studied in our upcoming works.

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FINITE ENERGY SOLUTION FOR SCATTERING OFF A SINGLE CRYSTAL

In this section, we derive the transmission and polarization coefficients for a single layer of point scatterers assuming low but finite energies of incoming electrons. Our starting point is the wave function presented in the main text

$$\Psi(r) = e^{ikz} + \lim_{R \to \infty} A_R \sum_{lm} \frac{e^{ik|a_{lm}|}}{|r - a_{lm}|},$$

(S1)

where

$$A_R = -\alpha_s \left(1 + \alpha_s \sum_{lm} \frac{e^{ik|a_{lm}|}}{|a_{lm}|} + ik\alpha_s\right)^{-1},$$

(S2)

and $R$ defines an upper limit of the sum. As in the main text, to estimate the sums in Eqs. (S1) and (S2), we approximate summations by integrals:

$$\sum_{lm} \frac{e^{ik|a_{lm}|}}{|a_{lm}|} = 2\pi ikb^2 \left(e^{ik|z|} - e^{ikbR} - 2\pi \frac{\Delta_k}{b}\right),$$

(S3)

$$\sum_{lm} \frac{e^{ik|a_{lm}|}}{a_{lm} \neq 0} = 2\pi ikb^2 (e^{ikbR} - 1) - 2\pi \frac{\Delta_k}{b},$$

(S4)

where the parameters $\Delta_r$ and $\Delta_k$ denote the difference between the exact values of the sums and the integral approximation. For simplicity, we have assumed that $|z| \gg b$ and $r = z\hat{z}$. The parameters $\Delta_r$ and $\Delta_k$ depend on the momentum of incoming electrons, $kb$. Figure S1 shows this dependence for $R = 400$, although we have checked that the results do not change substantially by changing $R$. For the zero-energy case, $kb = 0$, we have $\Delta_r = 0$ and $\Delta_k = \Delta_0 (\Delta_0 \approx 0.635)$, see the main text. Note that both sums may diverge for $kb \geq 2\pi$ due to constructive interference. We do not discuss this phenomenon here, since the zero-range potential model is applicable only at low energies – our focus is on $kb \leq 1$. In this energy regime $\Delta_r$ and $\Delta_k$ change weakly, allowing one to describe low-energy scattering using the zero-energy solution considered in the main text. Since $\Delta_r \approx 0$, from now on we will ignore $\Delta_r$ and only keep $\Delta_k$.

The $R$-dependent parts of Eqs. (S3) and (S4) do not cancel each other out as in the zero-energy case. Despite that we can ignore them, since an incoming electron beam is a square-integrable wave packet – the $R$-dependent terms are highly oscillatory and for $R \to \infty$ their integral contribution is negligible. In the limit $|z| \gg b$, the wave function reads

$$\Psi(r) \approx e^{ikz} + \frac{2\pi \alpha_s e^{ik|z|}}{ikb^2 - 2\pi \alpha_s - 2\pi \alpha_s \Delta_k kb - k^2 b^2 \alpha_s}.$$

(S5)

Comparing this wave function to the outgoing flux in scattering off a 1D Dirac delta potential [1], $g_s \delta(z)$:

$$\Psi_{1D}(z) = \frac{2ik e^{ikz}}{2ik - g_s},$$

(S6)

we derive a mapping onto a 1D problem, which is similar to the one derived for the zero-energy case. The strength of the corresponding 1D potential is

$$g_s = \frac{4\pi \alpha_s}{b (b - 2\pi \alpha_s \Delta_k + ik\alpha_s b)}.$$

(S7)
the transmission and reflection coefficients have the form

$$T_s = 1 - \frac{4\pi^2\alpha_s^2 + 4\pi k b \alpha_s^2 (k b - 2\pi \Delta R)}{\alpha_s^2 (k b^2 + 2\pi (1-k b \Delta R))^2 + k b^2 (b - 2\pi \alpha_s \Delta R)^2},$$

(S8)

$$R_s = \frac{4\pi^2\alpha_s^2}{\alpha_s^2 (k b^2 + 2\pi (1-k b \Delta R))^2 + k b^2 (b - 2\pi \alpha_s \Delta R)^2},$$

(S9)

where $\Delta R^R$ and $\Delta R^I$ denote the real and imaginary parts of $\Delta R$. The polarization is determined by $P = T_\uparrow - T_\downarrow$. Figure S2 compares the transmission and polarization coefficients from above to the ones obtained using the zero-energy approximation, see the main text. We use parameters as in Fig. 2 of the main text, in fact, Figs. S2 (b,d,f) shows the results of Fig. 2 of the main text. As is evident from Fig. S2, the results obtained for the zero-energy limit in the main text are very similar to those obtained here. The only considerable difference is the dependence of the 'magic' point on $k$, which stems from the dependence of $\Delta R$ on $k$.

FINITE ENERGY SOLUTION FOR SCATTERING OFF TWO PARALLEL CRYSTALS

In this section, we consider scattering off two layers separated by distance $L$. Our ansatz for the wave function is

$$\Psi_L (\mathbf{r}) = e^{i k z} + \lim_{R \to \infty} \sum_i A_R^i \sum_{lm} e^{i k |\mathbf{r} - \mathbf{a}_{lm}^i|},$$

(S10)

where the superscript $i = 0, 1$ implies that the quantity is related either to the first or the second layers, $\mathbf{a}_{lm}^0 = l \mathbf{x} + m \mathbf{y} + 0 \mathbf{z}$ and $\mathbf{a}_{lm}^1 = l \mathbf{x} + m \mathbf{y} + L \mathbf{z}$. The parameters $A_R^i$ are determined from the boundary conditions

$$\Psi_L (\mathbf{r} \to \mathbf{a}_{lm}^i) = s_{lm}^i \left( \frac{1}{|\mathbf{r} - \mathbf{a}_{lm}^i|} - \frac{1}{\alpha_s} \right),$$

(S11)

where, for simplicity, we assume that all scatterers are identical. We obtain from these boundary conditions that

$$A_R^i = \frac{\alpha_s + i \alpha_s^2 k + \alpha_s^2 S_0 - \alpha_s^2 S_1 e^{(-1)^i k L}}{\alpha_s^2 (S_1^2 - S_0^2) - 2\alpha_s^2 k S_0 - 2\alpha_s S_0^2 + \alpha_s^2 k^2 - 2\alpha_s k - 1}.$$  

(S12)

Here $S_0$ and $S_1$ are the sums: $S_i = \sum_{lm} e^{i k |\mathbf{a}_{lm}^i|} / |\mathbf{a}_{lm}^i|$, hence $S_0$ is the same as (S4) and $S_1$ is connected to (S3) if $z = -L$ and $L \gg b$. After approximating the sums by integrals [cf. Eqs (S3) and (S4)] and dropping all highly oscillatory terms, we write the wave function in the limit $z \gg L$ as

$$\Psi_L (\mathbf{r}) \simeq \frac{k b^2 (b - 2\pi \alpha_s \Delta k + i k \alpha_s b)^2 e^{i k z}}{4\pi^2 \alpha_s e^{2 k b L} + (2i \pi \alpha_s + k b^2 - 2\pi \alpha_s b \Delta k + i k^2 \alpha_s b^2)^2}.$$  

(S13)
FIG. S2. (a, b) Dependence of transmission (points connected by solid lines) and absolute value of polarization (points connected by dotted lines) on the dimensionless scattering length $a_1/b$ for $a_0 = 0$. Dependence of polarization on $a_0/b$ when (c, d) $a_1/a_0 = 0.1$ and (e, f) $a_1/a_0 = 0.3$. The left panels (a, c, e) show the results of the zero-range approximation discussed in the main text, while the right panels (b, d, f) present the full solution. The transmission coefficients $T$ is defined as $T = (T_1 + T_2)/2$.

FIG. S3. The same as in Fig S2 but for scattering off two layers of point magnets. The separation between the sheets is $L = 100b$. 
This can again be compared with 1D scattering off two identical Dirac delta potentials located at \(z = 0\) and \(z = L\), for which the outgoing flux is \([1]\)

\[
\Psi_{1D, L}(z) = \frac{4k^2 e^{ikz}}{g_s^2 e^{2ikL} + (ig_s + 2k)^2}, \tag{S14}
\]

If we use in this expression the interaction strength from Eq. (S7), then we derive Eq. (S13). Therefore, one can use the mapping derived for a single crystal to describe scattering off multiple crystals, provided that \(L \gg b\). The transmission coefficients will have the same form as in the zero-energy case (see the main text),

\[
T_s = \left| \frac{4k^2}{g_s^2 e^{2ikL} + (ig_s + 2k)^2} \right|^2, \tag{S15}
\]

with the sole redefinition of \(g_s\) according to Eq. (S7). Similar to the single-layer case, the results derived from the zero-energy approximation agree well with the results derived here, see Fig. S3. The only change is a weak energy dependence of the ‘magic’ point.

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