Study of the stress state in coated bodies on the layer separation boundary under contact interaction

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Abstract. The issue of stress state in coated bodies is a persistent problem in mechanical engineering and materials science that has congregated numerous efforts, both from experimental and fundamental points of view. The given paper concentrates on the contact problem considering the interaction between a punch and a dual-layered strip base that serves as a model of the interaction between loaded coating and the substrate. It also suggests strategies for applying analytical approaches and using the finite element method to solve the set of goals. The calculations and conducted study of the stress state in the base at the layer interface demonstrate that tensile normal stresses occur at the layer interface, which can lead to the top-layer delamination from the bottom one.

1. Introduction
The development of modern vehicles places large demands on machine operating characteristics. As a rule, machine lifetime is determined by the surfaces condition of the parts directly located in the friction unit. In mechanical engineering the problem of increasing the durability of such friction units is solved with antifriction or hardening coatings applied to the parts in the contact interaction areas. In this regard, one of the key issues is to study the stress state at the coating-substrate boundary, since the process of operation may lead to coating delamination that is connected with the methods of application, their mechanical and geometric characteristics.

A significant number of publications are devoted to contact problems for multilayered bases (see [1–4] and others). Studies of the stress state in the internal regions were carried out in [5] and others.

A model flat contact problem was considered to study the stress state at the coating-substrate (parts) interface, taking into account the friction forces in the interaction between a punch and a dual-layered base in the form of a strip fixed on the foundation. The experiment conditions assume that the layers are rigidly interconnected. The punch foundation is flat; the normal and tangential forces act on the punch. It should be noted that the punch – dual-layered base system is in equilibrium and the punch is restricted to move forward.

The study of the problem was carried out by reducing the corresponding elastic boundary value problem to the solution of an integral equation (IE), whose kernel transform was obtained explicitly, as well as on the basis of the finite element method.

The direct collocation method allowed gaining IE solutions [6]. The precise IC solution [7] was obtained after a special approximation of its kernel transform. Besides, we calculated stress
distribution in the contact area of the punch and the strip and at the layer interface depending on the geometric and mechanical parameters of the layers. The calculation results obtained by analytical methods were compared with the findings of the finite element method.

2. Problem statement
We considered an area $-h_2 \leq y \leq h_1$ consisting of two layers: $0 \leq y \leq h_1$ (layer 1) and $-h_2 \leq y \leq 0$ (layer 2) in Cartesian coordinate system $x, y$. The layers having different elastic constants are interconnected, and the face $y = h_1$ of layer 1 interacts with a punch influenced by a normal force $P$ applied with an eccentricity $e$ and a horizontal force $T = \mu P$. We assume that in the contact area, normal and tangential stresses are related by the Coulomb’s law $\tau_{xy} = \mu \sigma_y$ ($\mu$ is the coefficient of friction). The punch has a flat foundation.

In the case of plane strain, we seek the solution in a boundary value problem for the Lamé equations with boundary conditions:

$$
\sigma_y^1 \tau_{xy}^1 = 0 \quad (y = h_1, x < -a, x > a), \quad \tau_{xy}^1 = \mu \sigma_y^1, \quad v^1 = \delta \quad (y = h_1, -a \leq x \leq a),
$$

$$
v^1 = v^2, \quad u^1 = u^2, \quad \sigma_y^1 = \sigma_y^2, \quad \tau_{xy}^1 = \tau_{xy}^2 \quad (y = 0), \quad v^2 = 0, \quad u^2 = 0 \quad (y = -h_2),
$$

where $u^i, v^i$ – displacements in elastic layers along axes $x, y$ respectively; $\sigma_y^i, \tau_{xy}^i$ – normal and tangential stresses (indices 1 and 2 refer to layer 1 and 2 respectively), $\delta$ – punch displacement in vertical direction.

3. Analytical solution
Using the Fourier transform, the abovementioned boundary value problem is reduced to the following integral equation (IE) [2] with respect to unknown normal contact stresses under the punch:

$$
\int_{-a}^{a} g(\xi) k \left( \frac{\xi - x}{h_1} \right) d\xi = \pi \theta \delta, \quad (-a \leq x \leq b), \quad \theta = \frac{G_1}{1-v_1},
$$

$$
k(t) = k_1(t) - \mu k_2(t), \quad \theta = \frac{\mu (1-2v_1)}{2(1-v_1)},
$$

$$
k_1(t) = \int_{0}^{\infty} \frac{L_4(u)}{u} \cos ut du, \quad k_2(t) = \int_{0}^{\infty} \frac{L_2(u)}{u} \sin ut du, \quad L_4(u) = L_{41}(u)/L_{42}(u).
$$

Functions $L_{ij}(u)$ ($i, j = 1, 2$) have an explicit form. The expressions for them are given in [2].

It also shows that

$$
L_1(u)/u = A_0 + O(u) \quad (u \to 0), \quad L_4(u) = 1 \quad (u \to \infty),
$$

$$
L_2(u)/u = O(u) \quad (u \to 0), \quad L_2(u) = 1 \quad (u \to \infty),
$$

$$
A_0 = \frac{1 - 2v_1}{2(1-v_1)^2} + \frac{H_2(1-2v_1)}{2G_2(1-v_1)(1-v_2)}, \quad H_2 = h_2/h_1
$$

4. Integral equation solving
On the basis of estimates (2), it is manifested that IE kernel (4) has a logarithmic singularity and can be represented as [4]

$$
k(t) = -\ln|t| + F(t), \quad F(t) = -F_1(t) - \theta \left[ \frac{\pi}{2} \text{sgn}(t) - F_2(t) \right],
$$
where the integrals \( F_i(t) \) converge at any \( t \) values from \(-2a/h \leq t \leq 2b/h\) interval.

The solution of IE (2) with the kernel (5), (6) is obtained by the collocation method, using the results of [1 and others], in [6] the corresponding justification is given. As a result, we obtained a system of linear algebraic equations to find the values of contact stresses \( q(x) \) at the collocation points \( x = x_i = -a + \varepsilon/2 + \varepsilon(i - 1) \):

\[
\begin{align*}
\varepsilon \sum_{j=1}^{N} a_{ij}q_j &= b_i \quad (i = 1,...,N), \quad a_{ij} = a_{j1} - \delta a_{ij}^2, \\
a_{m}^{m} &= k_m \left( \frac{\xi_j - x_i}{h_1} \right) \quad (i \neq j, m = 1,2), \quad b_i = \pi \theta \delta, \quad a_{ii} = \varepsilon \left( \ln \frac{\varepsilon}{2h_1} - 1 \right),
\end{align*}
\]  

(7)

where \( q_j = q(\xi_j) \) are the values of contact stresses at collocation points, \( \xi_j = -a + \varepsilon/2 + \varepsilon(j - 1), \quad x_i = -a + \varepsilon/2 + \varepsilon(i - 1), \quad \varepsilon = (a + b)/N \) is a collocation interval, \( N \) is a number of collocation points.

Systems (7)–(8) have a diagonal structure, and between the coefficients of the system there is the following relationship

\[
a_{m+1,j+1} = a_{m}^{m} \quad (j \geq i, m = 1,2), \quad a_{m}^{m} = a_{11}^1, \quad a_{1}^{2} = -a_{11}^2,
\]  

(9)

which allows calculating all the coefficients of the system via the coefficients of the first row.

The force \( P \) and moment \( M \) applied to the punch through contact stresses can be found by the formulas

\[
P = \varepsilon \sum_{k=1}^{N} q_k, \quad M = \varepsilon \sum_{k=1}^{N} x_k q_k.
\]  

(10)

If there is no friction under the punch, we construct the solution of IE in closed form using the scheme outlined in [6]. It requires the approximation of the transform \( L_4(u)/u \) and IE kernel with the function \( L_4(u) = th(Aq/u) \). It should be noted that function behavior is the same both at zero and infinity. The error of such an approximation does not exceed 15%.

Such an approximation allows presenting the IE solution at \( \mu = 0 \) in closed form [6]

\[
q(x) = \theta \frac{\Delta}{a} \frac{\mu_0}{\mu_0} \left[ K \left( \sqrt{1 - h^2} \frac{\mu_0}{\mu} \right) \right] \left[ \frac{\varepsilon}{a} \sqrt{h^2 \mu_0^2 - \varepsilon h^2 \mu_0^2} \right]^{-1}, \quad \mu_0 = \frac{\pi a}{2 A Q h_1}
\]  

(11)

where \( K(\varepsilon) \) is a complete elliptic integral.

Values of \( q(x) = q(x/a)A/A(\theta \delta) \) were compared with the results given in [3] in case the layers have the same elastic constants for \( h_1 + h_2 = 4 \) and \( a = 1 \). The values were calculated on the basis of IE solving by the method of collocations and on the basis of the kernel transform approximation. While applying the collocation method, \( N=1001 \) was assumed. Along with analytical approaches, the modeling and calculation of the problem by the finite element method was performed in the Ansys software package. The comparison demonstrates quite acceptable correlation of results obtained.

The corresponding ratios were constructed to calculate the displacements and stresses in the internal area of the layered base via the found contact stresses. Here we present only expressions for normal stresses \( \sigma_y(x, y) \).
\[
\sigma_y(x, y) = \left[ \int_{-a}^{a} q(\xi) d\xi \int_{0}^{\infty} Q_1^{1}(\alpha, y) \cos \frac{\alpha(\xi - x)}{h_1} d\alpha \right] + \\
\left[ \int_{-a}^{a} \tau(\xi) d\xi \int_{0}^{\infty} Q_2^{2}(\alpha, y) \sin \frac{\alpha(\xi - x)}{h_1} d\alpha \right] , \quad \tau(x) = \mu \sigma(x). \quad (12)
\]

Since the functions \( Q^1_y(\alpha, y) \) obtained using the analytical calculation software have a rather cumbersome structure, they are not given here. Numerical approaches were exploited to calculate the integrals, when figuring out the stresses \( \sigma_y(x, y) \) according to formulas (12) inside the multilayer base, including at the layer interface. Tables 1 shows the results of comparing the values of normal stresses \( \sigma^*(x) = \sigma_y(x/a, 0) a l(\theta \delta) \) at the layer interface, calculated by the proposed methods for \( a=1 \) and \( N=1001 \) respectively.

**Table 1.** Normal stresses values at the interface between the coating and the substrate, calculated by different methods.

| \( h_1=0.5 \) | \( h_2=0.5 \) | \( G_2=0.5G_1 \) at interface | \( x/a \) | \( \sigma^*(x) \) |
|--------------|--------------|-------------------------------|--------|---------|
| \( \text{kol} \) | 0.22 | 0.750 | 0.377 | -0.0229 | -0.0176 | -0.0115 |
| \( \text{app} \) | 1.28 | 0.772 | 0.384 | -0.0244 | -0.0184 | -0.0119 |
| \( \text{Ansys} \) | 1.00 | 0.55 | 0.25 | -0.025 | -0.0195 | -0.0113 |

The above calculations showed a fairly good agreement of the results obtained by the proposed methods, which allows us to draw the right conclusions. The key conclusion is that tensile normal stresses occur at the coating-substrate interface at some distance from the punch, as evidenced by the change in the stress sign.

The following graphs (figures 1–3) depict normal stresses \( \sigma^*(x) \) at the interface of the layers for different thickness values, which were calculated analytically using the collocation method \( \text{(kol)} \) and the finite element method in \( \text{Ansys} \). The results are given for \( x>0 \) due to the symmetry of the problem.

**Figure 1.** \( \sigma^*(x) \) for \( h_1=0.5 \) \( h_2=1 \) \( G_2=0.5G_1 \).  **Figure 2.** \( \sigma^*(x) \) for \( h_1=0.25 \) \( h_2=1 \) \( G_2=0.5G_1 \).

The presented calculations prove that the coating thickness affects the position of the transition point from normal compressive stresses to normal tensile stresses \( (\sigma^*(x)=0) \). The decrease in the coating thickness leads to the transition point shifting towards the punch projection boundary \( (x=1) \).

The horizontal force \( T = \mu P \) was applied to the punch to assess the shear effect on normal stresses at the interface. In the contact area, normal and tangential stresses were related by the Coulomb law \( \tau_{xy} = \mu \sigma_y \) (\( \mu \) is the coefficient of friction). The calculations indicated that the symmetry in the
results relative to the punch center is broken in the presence of a horizontal force. The change in normal stresses at the layer interface is observed only in the direction of the force application. Figure 4 shows a comparison of normal stresses values at the layer interface in the presence and absence of horizontal force ($\mu = 0.75$ and $\mu = 0$, respectively) for the same geometric and mechanical parameters of the model.

![Figure 3. $\sigma^*(x)$ for $h_1=0.1$ $h_2=1$ $G_2=0.5G_1$](image1)

![Figure 4. $\sigma^*(x)$ for $h_1=0.25$ $h_2=0.25$ $G_2=0.5G_1$](image2)

From the presented graph it is seen that in the presence of horizontal force, the transition point at which normal stresses change sign is closer to the border of the stamp projection ($x = 1$), while the maximum value of the normal tensile stress is significantly higher than in the case of $\mu = 0$.

5. Conclusions
The calculations showed that normal tensile stresses may occur at the bottom coating boundary at certain values of the geometric and mechanical parameters of the coating and the substrate. Which indicates possible coating delamination from the base. The transition point at which stresses change the sign and the value of normal tensile stresses substantially depend on the coating thickness. Thus, the transition point shifts towards the punch projection boundary ($x = 1$) with a decrease in the coating thickness, and the values of normal tensile stresses decrease.

During frictional interaction, the appearance of a horizontal load leads to an even greater increase in the values of normal tensile stresses, and their maximum approaches the border of the punch projection ($x = 1$). Consequently, a significant stress decrease occurs on a relatively small area of the coating-substrate interface, which can adversely affect the layer adhesion strength.

Hence, when choosing a coating in order to prevent delamination, it is necessary to take into account both the mechanical and geometric parameters of the coating, as well as loading and operation specifics of a certain unit.

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References
[1] Ivanochkin P G, Kolesnikov V I, Fleck B N and Chebakov M I 2007 *Mechanics of Solids* 1 183–92
[2] Chebakov M I 2006 *Ecological Bulletin of Scientific Centers of Black Sea Economic Cooperation* 1 60–6
[3] Vorovich I I , Alexandrov V M and Babeshko V A 1974 *Non-Classical Mixed Problems of the Theory of Elasticity* (Moscow: Science) 456
[4] Alexandrov V M and Kovalenko E V 1986 *The Problems of Continuum Mechanics with Mixed Boundary Constitutions* (Moscow: Science) 336
[5] Jamison R D and Shen Y L 2016 Surface and Coatings Technology 303 Part A 3–11
[6] Voronin V V and Tsetsekho V A 1981 Journal of Computational Mathematics and Mathematical Physics 21 40–53
[7] Alexandrov V M and Chebakov M I 2007 Introduction to Mechanics of contact interactions (Rostov-on-Don: Valeological Centers of Russian Universities Publishing House) 114