Foundations of the Constituent Quark Model

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Abstract

‘Constituent quarks’ means massive quarks, in contrast to the nearly massless u, d, s quarks of the QCD lagrangian. The dynamical or constituent masses appear owing to the spontaneous chiral symmetry breaking in QCD. A realistic mechanism of the chiral symmetry breaking is provided by instantons. Therefore, I review the present status of the QCD instanton vacuum, emphasizing the mechanism of chiral symmetry breaking and of the generation of the constituent quark mass. I end up with the Nambu–Jona-Lasinio-type effective theory to which QCD is reduced at low momenta.

Keywords
Constituent quarks, chiral symmetry breaking, instantons, effective chiral lagrangian, Nambu–Jona-Lasinio model

1 Introduction

Quantum Chromodynamics (QCD) is the theory of strong interactions. From a theorist’s point of view strong interactions means that all dimensionless quantities of the theory are, generally speaking, of the order of unity. Meanwhile, there are at least two fundamental facts about strong interactions showing that actually it is not exactly so: there are still certain small parameters hidden. One is that nuclei can be, to a good accuracy, viewed as made of weakly bound nucleons — in a true ‘strong interactions’ case they would rather look like a quark soup. The second puzzle is that the nucleon itself can be, to a good accuracy, viewed as built of three constituent quarks — in a true ‘strong interactions’ case it would rather look like a pack of an indefinite number of quarks, antiquarks and gluons.

The second puzzle (and probably the first, too) is qualitatively explained by the spontaneous breaking of the (approximate) chiral symmetry of QCD. The bare ot current masses of the light quarks \(m_u \simeq 4\ MeV, \ m_d \simeq 7\ MeV, \ m_s \simeq 150\ MeV\) are small as compared to the typical scale of strong
interactions, say, the $\rho$-meson mass, 770 MeV, and can be put to zero. In that limit (called the chiral limit) the QCD lagrangian possesses an 18-parameter symmetry, called chiral symmetry. It is the symmetry under independent $U(3) \times U(3)$ rotations of the left- and right-handed components of the $u, d, s$ fields. Equivalently, it is the symmetry under $U(3)_{\text{Vector}} \times U(3)_{\text{Axial}}$ rotations. Were this symmetry exact, we would observe degeneracy between states with opposite parity but otherwise the same quantum numbers. For example, the vector $\rho$-meson would be degenerate with the axial $a_1$-meson; the nucleon ($\frac{1}{2}^+, 940$) would be degenerate with the resonance ($\frac{1}{2}^-, 1535$), etc. Since it is not the case, we conclude that the $U(3)_A$ symmetry is broken spontaneously. The order parameter for symmetry breaking is the quark or chiral condensate,

$$\langle \bar{\psi} \psi \rangle \approx -(250 \text{ MeV})^3, \quad \psi = u, d, s. \quad (1)$$

Its $U(3)_A$ rotations produce nine massless Goldstone bosons – the pseudoscalar nonet.

It should be noted that the above quantity is well defined only for massless quarks, otherwise it is somewhat ambiguous. By definition, this is the quark Green function taken at one point; in momentum space it is a closed quark loop. Had the quark propagator only the "slash" term, the trace over the spinor indices understood in this loop would give an identical zero. Therefore, chiral symmetry breaking implies that a massless (or nearly massless) quark develops a non-zero dynamical mass (i.e. a "non-slash" term in the propagator). There are no reasons for this quantity to be a constant independent of the momentum; moreover, we understand that the dynamical mass should anyhow vanish at large momenta. Its value at zero momentum can be estimated as one half of the $\rho$ meson mass or one third of the nucleon mass, that is about

$$M(0) \approx 350 - 400 \text{ MeV}. \quad (2)$$

This scale is also related to chiral symmetry breaking and should emerge together with the condensate.

The spontaneous breaking of chiral symmetry is the main dynamical happening in QCD: it determines the face of the strong interactions world. Indeed, it explains why the pseudoscalar mesons are light (they are called pseudo-Goldstone bosons since they get a non-zero mass from the small current quark masses which break somewhat the chiral symmetry from the beginning), and why nucleons are heavy and not degenerate in parity. The appearance of the dynamical quark mass (I prefer this word to the constituent mass), related to the chiral symmetry breaking, justifies, qualitatively, the old non-relativistic or semi-relativistic models of hadrons. Indeed, once one gets (2), one can state that mesons are predominantly made of a quark and an antiquark, and baryons are made of three quarks: an addition of a quark-antiquark pair costs $700 - 800 \text{ MeV}$ which is a lot. Moreover, the lightest vector, axial and tensor mesons and baryons should be rather loosely bound: first, because their masses are not far away from the sum of the dynamical quark masses, second, because, after the appearance of a rather large scale in the theory, there is no place for the "infrared catastrophe" — the gluon coupling constant $\alpha_s$ remains finite and in fact, as known from hard hadronic processes,
relatively small. Therefore, one would expect that neither the one-gluon exchange nor the linear confining potential plays a crucial role in determining the world of light hadrons (I shall soon cite direct evidence from lattice experiments substantiating this view). Of course, one would need explicit confining forces to describe highly excited resonances lying on linear Regge trajectories, when hadrons are artificially ”stretched” by centrifugal forces, but they are not so much important for describing low-lying hadrons. The dominant phenomenon is chiral symmetry breaking, and if we want to understand strong interactions we have to understand how to get $1, 2$ from the only dimensional quantity in QCD, that is $\Lambda_{QCD}$ which is the ultraviolet cut-off-independent combination of the cut-off $\mu$ and the gauge coupling constant $g^2(\mu)$ given at that cut-off:

$$\Lambda_{QCD} = \mu \exp \left[ -\frac{8\pi^2}{bg^2(\mu)} \right], \quad b = \frac{11}{3} N_c - \frac{2}{3} N_f,$$

where $N_c$ is the number of colours ($N_c = 3$) and $N_f$ is the number of active quark flavours. The value of $\Lambda_{QCD}$ depends on the regularization scheme used; in the $\overline{MS}$ scheme $\Lambda_{\overline{MS}} \simeq 250 - 300$ MeV.

There is a remarkable puzzle, called the $U(1)$ paradox (Weinberg, 1974): eight out of nine pseudoscalar mesons ($\pi, K, \eta$) look indeed like pseudo-Goldstone bosons and obey the PCAC relations while the singlet $\eta'$-meson does not (the PCAC predicts its mass to be about 400 $MeV$ vs. 958 $MeV$ (exper.)). A related puzzle is the $\eta \to 3\pi$ decay. It has been known from ’t Hooft (1976) that, qualitatively, one needs instantons to cure the $U(1)$ paradox (see also (Diakonov, 1982)). Probably the modest $\eta'$-meson tells us about strong interactions more than any other hadron! It says that one has to take into account instanton configurations in QCD: there is no other way to get a heavy $\eta'$-meson. We shall see that instantons bring in a most realistic scenario of spontaneous chiral symmetry breaking as well.

### 2 Instantons

Instantons are certain configurations of the Yang–Mills potentials for the gluon field $A_{\mu}^a(x)$, satisfying the equations of motion $D_{\mu}^{ab}F_{\mu\nu}^{b} = 0$ in euclidean space, i.e. in imaginary time. The solution has been found by Belavin, Polyakov, Schwartz and Tiupkin (1975); the name ”instanton” has been suggested by ’t Hooft in (1976), who also made a major contribution to the investigation of the instantons properties.

In QCD instantons are the best studied non-perturbative effects, leading to the formation of the gluon condensate (Shifman, Vainshtein and Zakharov, 1979) and of the so-called topological susceptibility needed to cure the $U(1)$ paradox. The QCD instanton vacuum has been studied starting from the pioneering works in the end of the seventies (Callan, Dashen and Gross, 1978; Shuryak, 1982); a quantitative treatment of the instanton ensemble, based on the Feynman variational principle, has been developed by Diakonov and Petrov (1984a). The most striking success of the QCD instanton vacuum is its capacity to provide a beautiful mechanism of the spontaneous chiral symmetry breaking (Diakonov and Petrov, 1984b, 1986a). Moreover, the instanton vacuum leads to a very reasonable
effective chiral lagrangian at low energies, including the Wess–Zumino term, etc., which, in its turn
gives a nice description of nucleons as chiral quark solitons (Diakonov and Petrov, 1986b).

A detailed numerical study of dozens of correlation functions in the instanton medium undertaken
by Shuryak and collaborators (1993a) (earlier certain correlation functions were computed analyt-
ically by Diakonov and Petrov (1984b, 1986b)) demonstrated an impressing agreement with the
phenomenology (Shuryak, 1993b) and, more recently, with direct lattice measurements (Chu et al.,
1994). As to baryons, the instanton-motivated chiral quark soliton model (Diakonov and Petrov,
1986b) also leads to a very reasonable description of dozens of baryon characteristics (see Klaus
Goeke’s contribution to these Proceedings).

More recently the instanton vacuum was studied in direct lattice experiments by the so-called cooling
procedure (Teper, 1985; Ilgenfritz et al., 1986; Polikarpov and Veselov, 1988; Chu et al., 1994). It was
demonstrated that instantons and antiinstantons (I’s and I’s for short) are the only non-perturbative
gluon configurations surviving after a sufficient smearing of the quantum gluon fluctuations. The
measured properties of the Ī medium appeared (Chu et al., 1994; Michael and Spencer, 1995) to
be close to that computed from the variational principle (Diakonov and Petrov, 1984a) and to what
had been suggested by Shuryak (1982) from phenomenological considerations.

Cooling down the quantum fluctuations above instantons kills both the one-gluon exchange and the
linear confining potential (a small residual string tension observed in the cooled vacuum (Polikarpov
and Veselov, 1988; Chu et al., 1994) is probably due to the instanton-induced rising potential at
intermediate distances (Diakonov, Petrov and Pobylitsa, 1989). Nevertheless, in the cooled vacuum
where only I’s and I’s are left, the correlation functions of various mesonic and baryonic currents, as
well as the density-density correlation functions representing the quark wave functions in hadrons,
appear to be quite similar to those of the true or ”hot” vacuum (Chu et al., 1994).

In recent lectures (Diakonov, 1995) I have reviewed the building blocks of the instanton vacuum; an
interested reader is addressed to those lectures or original literature. Here I shall review the main
facts about instantons, in particular the way one gets chiral symmetry breaking and the ‘constituent’
quarks from instantons.

3 Instanton field

Physically, one can think of instantons in two ways: on one hand it is a tunneling process occuring in
imaginary or euclidean time (this interpretation belongs to V.Gribov, 1976), on the other hand it is
a localized pseudoparticle in the euclidean space (A.Polyakov, 1977). The gluon field of the instanton
in the singular gauge is (’t Hooft, 1976):

$$A_{\mu}^{la} = \frac{2\rho^2 O^{ab} \eta_{\mu\nu} (x - z)_\nu}{(x - z)^2 [\rho^2 + (x - z)^2]}, \quad O^{ab} = Tr(U^\dagger t^a U \sigma^i), \quad O^{ai} O^{aj} = \delta^{ij}. \quad (4)$$
Here the 4-dim. vector $z_\mu$ is called the instanton centre, $\rho$ is the instanton size, the rectangular matrix $O^{ai}$, $a = 1, \ldots, (N_c^2 - 1)$, $i = 1, 2, 3$ gives the orientation of the instanton in the colour $SU(N_c)$ space. All in all there are

$$4 \text{ (centre)} + 1 \text{ (size)} + (4N_c - 5) \text{ (orientations)} = 4N_c$$

so called collective coordinates describing the field of the instanton, of which the action is independent. The tensors $\bar{\eta}_{\mu\nu}$ are called 't Hooft symbols ('t Hooft, 1976).

The field strength of an instanton (centered at $z_\mu = 0$) is

$$F^a_{\mu\nu} = -\frac{4\rho^2}{(x^2 + \rho^2)^2}O^{ai}\left(\bar{\eta}_{\mu\nu} - 2\bar{\eta}_{\mu\alpha}x^\alpha x^\nu - 2\bar{\eta}_{\beta\nu}x^\mu x^\beta\right), \quad F^a_{\mu\nu}F^a_{\mu\nu} = \frac{192\rho^4}{(x^2 + \rho^2)^4},$$

and satisfies the self-duality equation $F^a_{\mu\nu} = \tilde{F}^a_{\mu\nu}$. The anti-instanton satisfies the anti-self-dual equation, $F = -\tilde{F}$; it is given by eqs.(4, 6) with the replacement $\bar{\eta} \rightarrow \eta$.

The action of one (anti)instanton is

$$S = \frac{1}{4} \int d^4x F^a_{\mu\nu}F^a_{\mu\nu} = 8\pi^2. \quad (7)$$

### 4 Gluon condensate

The QCD perturbation theory implies that the fields $A^a_i(x)$ are performing quantum zero-point oscillations; in the lowest order these are just plane waves with arbitrary frequencies. The aggregate energy of these zero-point oscillations, $(B^2 + E^2)/2$, is divergent as the fourth power of the cutoff frequency, however for any state one has $\langle F^2_{\mu\nu} \rangle = 2\langle B^2 - E^2 \rangle = 0$, which is just a manifestation of the virial theorem for harmonic oscillators: the average potential energy is equal to that of the kinetic (I am temporarily in the Minkowski space). One can prove that this is also true in any order of the perturbation theory in the coupling constant, provided one does not violate the Lorentz symmetry and the renormalization properties of the theory. Meanwhile, we know from the QCD sum rules phenomenology that the QCD vacuum possesses what is called gluon condensate (Shifman, Vainshtein and Zakharov, 1979):

$$\frac{1}{32\pi^2}\langle F^a_{\mu\nu}F^a_{\mu\nu} \rangle \simeq (200 \text{ MeV})^4 > 0. \quad (8)$$

Instantons suggest an immediate explanation of this basic property of QCD. Indeed, instanton is a tunneling process, it occurs in imaginary time; therefore in Minkowski space one has $E^a_i = \pm iB^a_i$ (this is actually the self-duality equation). In euclidean space the electric field is real as well as the magnetic one, and the gluon condensate is just the average action density. Let us make a quick estimate of its value.
Let the total number of $I$'s and $\bar{I}$'s in the 4-dimensional volume $V$ be $N$. Assuming that the average separations of instantons are larger than their average sizes (to be justified below), we can estimate the total action of the ensemble as the sum of individual actions:

$$\langle F_{\mu\nu}^2 \rangle_V = \int d^4 x F_{\mu\nu}^2 \simeq N \cdot 32\pi^2,$$

(9)
hence the gluon condensate is directly related to the instanton density in the 4-dimensional euclidean space-time:

$$\frac{1}{32\pi^2} \langle F_{\mu\nu}^a F_{\mu\nu}^a \rangle \simeq \frac{N}{V} \equiv \frac{1}{\bar{R}^4}.$$

(10)

In order to get the phenomenological value of the condensate one needs thus to have the average separation between pseudoparticles (Shifman, Vainshtein and Zakharov, 1979; Shuryak, 1982)

$$\bar{R} \simeq \frac{1}{200} \text{MeV} = 1 \text{ fm}.$$

(11)

There is another point of view on the gluon condensate which I describe briefly. In principle, all information about field theory is contained in the partition function being the functional integral over the fields. In the euclidean formulation it is

$$\mathcal{Z} = \int DA_\mu \exp \left( -\frac{1}{4g^2} \int d^4 x F_{\mu\nu}^2 \right) \xrightarrow{T \to \infty} e^{-ET},$$

(12)

where I have used that at large (euclidean) time $T$ the partition function picks up the ground state or vacuum energy $E$. For the sake of brevity I do not write the gauge fixing and Faddeev–Popov ghost terms. If the state is homogeneous, the energy can be written as $E = \theta_{44}V^{(3)}$ where $\theta_{\mu\nu}$ is the stress-energy tensor and $V^{(3)}$ is the 3-volume of the system. Hence, at large 4-volumes $V = V^{(3)}T$ the partition function is $\mathcal{Z} = \exp(-\theta_{44}V)$. This $\theta_{44}$ includes zero-point oscillations and diverges badly. A more reasonable quantity is the partition function, normalized to the partition function understood as a perturbative expansion about the zero-field vacuum. The latter can be distinguished from the former by imposing a condition that it does not contain integration over singular Yang–Mills potentials: instanton potentials are singular at the origins. One has

$$\frac{\mathcal{Z}}{\mathcal{Z}_{P.T.}} = \exp \left[ -(\theta_{44} - \theta_{44}^{P.T.})V \right].$$

(13)

We expect that the non-perturbative vacuum energy density $\theta_{44} - \theta_{44}^{P.T.}$ is a negative quantity since we have allowed for tunneling: as usual in quantum mechanics, it lowers the ground state energy. If the vacuum is isotropical, one has $\theta_{44} = \theta_{\mu\mu}/4$. Using the trace anomaly,

$$\theta_{\mu\mu} \simeq -b \frac{F_{\mu\nu}^2}{32\pi^2}, \quad b = \frac{11}{3} N_c,$$

(14)

one gets (Diakonov and Petrov, 1984a):
\[ \frac{Z}{Z_{P.T.}} = \exp \left( \frac{b}{4} V \langle F_{\mu\nu}^2 / 32\pi^2 \rangle_{NP} \right) \]  

(15)

where \( \langle F_{\mu\nu}^2 \rangle_{NP} \) is the gluon field vacuum expectation value which is due to non-perturbative fluctuations, i.e. the gluon condensate. The aim of any QCD-vacuum builder is to minimize the vacuum energy or, equivalently, to maximize the gluon condensate.

5 One-instanton weight

The words ‘instanton vacuum’ mean that one assumes that the QCD partition function is mainly saturated by an ensemble of interacting \( I \)'s and \( \bar{I} \)'s together with quantum fluctuations about them. Instantons are necessarily present in the QCD vacuum if only because they lower the vacuum energy in respect to the purely perturbative (divergent) one. The question is whether they give the dominant contribution to the gluon condensate, and to other basic quantities. To answer this question one has to compute the partition function (12) assuming that it is mainly saturated by instantons, and to compare the obtained gluon condensate with the phenomenological one. This work has been done a decade ago in ref. (Diakonov and Petrov, 1984a); today direct lattice measurements seem to confirm that the answer to the question is positive: the observed density of \( I \)'s and \( \bar{I} \)'s is in agreement with the estimate (11).

The starting point of this calculation is the contribution of one isolated instanton to the partition function (12), or the one-instanton weight. To get a reasonable result it must be

1. normalized (to the determinant of the free quadratic form, i.e. with no background field),
2. regularized (for example by using the Pauli–Villars method), and
3. accounted for the zero modes. The resulting one-instanton contribution to the partition function (normalized to the free one) is an integral over the \( 4N_c \) collective coordinates of an instanton (‘t Hooft, 1976; Bernard, 1979):

\[ \frac{Z_{1-\text{inst}}}{Z_{P.T.}} = \int d^4z_\mu \int d\rho \int dO \frac{C(N_c)}{\rho^5} \left[ \frac{8\pi^2}{g^2(\mu)} \right]^{2N_c} (\rho \Lambda_{QCD})^{\frac{11}{3}N_c} \exp \left( -\frac{8\pi^2}{g^2(\mu)} \right) \]  

(16)

\[ = \int d^4z_\mu \int d\rho \int dO \frac{C(N_c)}{\rho^5} \left[ \frac{8\pi^2}{g^2(\mu)} \right]^{2N_c} (\rho \Lambda_{QCD})^{\frac{11}{3}N_c}. \]  

(17)

The product of the last two factors in eq. (16) is actually a combination of the cut-off \( \mu \) and the bare coupling constant \( g^2(\mu) \) given at this cut-off, which is cutoff-independent; it can be replaced by \( (\rho \Lambda_{QCD})^{\frac{11N_c}{3}} \), see eq. (3). This is the way \( \Lambda_{QCD} \) enters into the game; henceforth all dimensional quantities will be expressed through \( \Lambda_{QCD} \), which is, of course, a welcome message. The numerical coefficient \( C(N_c) \) depends explicitly on the number of colours; it also implicitly depends on the regularization scheme used.

Note that the \( g^2 \) in the pre-exponent starts to "run" only at the 2-loop level, hence its argument is taken at the ultra-violet cut-off \( \mu \). The 2-loop instanton weight can be found in refs. (Diakonov, 1995; Diakonov, Polyakov and Weiss, 1955).
In both one- and two-loop approximations the integral over the instanton sizes \( \rho \) in eq. (17) diverges as a high power of \( \rho \) at large \( \rho \): this is of course the consequence of asymptotic freedom. It means that individual instantons tend to swell. This circumstance plagued the instanton calculus for many years. If one attempts to cut the \( \rho \) integrals "by hand", one violates the renormalization properties of the YM theory. Actually the size integrals appear to be cut from above due to instanton interactions.

6 Instanton ensemble

To get a volume effect from instantons one needs to consider an \( I\bar{I} \) ensemble, with their total number \( N \) proportional to the 4-dimensional volume \( V \). Immediately a mathematical difficulty arises: any superposition of \( I \)'s and \( \bar{I} \)'s is not, strictly speaking, a solution of the equation of motion, therefore, one cannot directly use the semiclassical approach of the previous section. There are two ways to overcome this difficulty. One is to use a variational principle (Diakonov and Petrov, 1984a), the other is to use the effective YM lagrangian in the instanton field (Diakonov and Polyakov, 1993).

The idea of the variational principle is to use a modified YM action for which a chosen \( I\bar{I} \) ansatz is a saddle point. Exploiting the convexity of the exponent one can prove that the true vacuum energy is less than that obtained from the modified action. One can therefore use variational parameters (or even functions) to get a best upper bound for the vacuum energy. We call it the Feynman variational principle since the method was suggested by Feynman in his famous study of the polaron problem. Todays direct lattice investigation of the \( I\bar{I} \) ensemble seem to indicate that Petrov and I have obtained rather accurate numbers in this terrible problem. Therefore I will cite the numerics from those calculations in what follows.

The main finding is that the \( I\bar{I} \) ensemble stabilizes at a certain density related to the \( \Lambda_{QCD} \) parameter (there is no other dimensional quantity in the theory!):

\[
\langle F_{\mu\nu}^2/32\pi^2 \rangle \simeq \frac{N}{V} \geq (0.75\Lambda_{MS}^2)^4
\]

which would require \( \Lambda_{MS} \simeq 270 \text{ MeV} \) to get the phenomenological value of the condensate. It should be mentioned however that using more sophisticated variational Ansätze one can obtain a larger coefficient in eq. (18) and hence would need smaller values of \( \Lambda \).

The average sizes \( \bar{\rho} \) appears to be much less than the average separation \( \bar{R} \). Numerically we have found for the \( SU(3) \) colour:

\[
\frac{\bar{\rho}}{\bar{R}} \simeq \frac{1}{3}
\]

which coincides with what was suggested previously by Shuryak (1982) from considering the phenomenological applications of the instanton vacuum. This value should be compared with that found from direct lattice measurements (Chu et al., 1994): \( \bar{\rho}/\bar{R} \simeq .37 - .4 \), depending on where one stops the cooling procedure. The packing fraction, i.e. the fraction of the 4-dimensional volume occupied
by instantons appears thus to be rather small, $\pi^2 \bar{\rho}^4 / \bar{R}^4 \simeq 1/8$. This small number can be traced back to the “accidentally” large numbers appearing in the 4-dimensional YM theory: the $11N_c/3$ of the Gell-Mann–Low beta function and the number of zero modes being $4N_c$. Meanwhile, it is exactly this small packing fraction of the instanton vacuum which gives an a posteriori justification for the use of the semi-classical methods. As I shall show in the next sections, it also enables one to identify adequate degrees of freedom to describe the low-energy QCD.

7 Chiral symmetry breaking by instantons: qualitative derivation

The key observation is that the Dirac operator in the background field of one (anti) instanton has an exact zero mode with $\lambda = 0$ (‘t Hooft, 1976). It is a consequence of the general Atiah–Singer index theorem; in our case it is guaranteed by the unit Pontryagin index or the topological charge of the instanton field. These zero modes are 2-component Weyl spinors: right-handed for instantons and left-handed for antiinstantons.

For infinitely separated $I$ and $\bar{I}$ one has thus two degenerate states with exactly zero eigenvalues. As usual in quantum mechanics, this degeneracy is lifted through the diagonalization of the hamiltonian, in this case the hamiltonian is the full Dirac operator. The two ”wave functions” which diagonalize the ”hamiltonian” are the sum and the difference of the would-be zero modes, one of which is a 2-component left-handed spinor $\Phi_1$, and the other is a 2-component right-handed spinor $\Phi_2$. The resulting wave functions are 4-component Dirac spinors; one can be obtained from another by multiplying by the $\gamma_5$ matrix. As the result the two would-be zero eigenstates are split symmetrically into two 4-component Dirac states with non-zero eigenvalues equal to the overlap integral between the original states $\Phi_{1,2}$:

$$\lambda = \pm |T_{12}|, \quad T_{12} = \int d^4x \Phi_1^\dagger (-i \partial) \Phi_2 \to \frac{2\rho_1 \rho_2}{R_{12}} \text{Tr}(U_1^\dagger U_2 R_{12}^+).$$  \hspace{1cm} (20)

We see that the splitting between the would-be zero modes fall off as the third power of the distance between $I$ and $\bar{I}$; it also depends on their relative orientation.

When one adds more $I$’s and $\bar{I}$’s each of them brings in a would-be zero mode. After the diagonalization they get split symmetrically in respect to the $\lambda = 0$ axis. Eventually, for an $II$ ensemble one gets a continuous band spectrum with a spectral density $\nu(\lambda)$ which is even in $\lambda$ and finite at $\lambda = 0$. Meanwhile, there is a general formula relating the chiral condensate with the average spectral density of the Dirac operator at zero $\lambda$ (Banks and Casher, 1980; Diakonov and Petrov, 1984b):

$$\langle \bar{\psi} \psi \rangle = -\frac{1}{V} \text{sign}(m) \pi \nu(0)$$  \hspace{1cm} (21)

Since $I$’s and $\bar{I}$’s lead to a non-zero $\nu(0)$ they automatically give the non-zero chiral condensate,
which signals chiral symmetry breaking.

One can make a quick estimate of $\langle \bar{\psi} \psi \rangle$: Let the total number of $I$’s and $\bar{I}$’s in the 4-dimensional volume $V$ be $N$. The spread $\Delta$ of the band spectrum of the would-be zero modes is given by their average overlap (20):

$$\Delta \sim \sqrt{\int (d^4 R/V) T(R) T^*(R)} \sim \frac{\bar{\rho}}{R^2}$$

(22)

where $\bar{\rho}$ is the average size and $\bar{R} = (N/V)^{-1/4}$ is the average separation of instantons. Note that the spread of the would-be zero modes is parametrically much less than $1/\bar{\rho}$ which is the typical scale for the non-zero modes. Therefore, neglecting the influence of the non-zero modes is justified if the packing fraction of instantons is small enough. From eq. (21) one gets an estimate for the chiral condensate induced by instantons:

$$\langle \bar{\psi} \psi \rangle = -\frac{\pi}{V} \nu(0) \simeq -\frac{\pi}{V} \frac{N}{\Delta} \sim -\frac{1}{R^2 \bar{\rho}}.$$  

(23)

It is amusing that the physics of the spontaneous breaking of chiral symmetry resembles the so-called Anderson conductivity in disordered systems. Imagine random impurities (atoms) spread over a sample with final density, such that each atom has a localized bound state for an electron. Due to the overlap of those localized electron states belonging to individual atoms, the levels are split into a band, and the electrons become delocalized. That means conductivity of the sample. In our case the localized zero quark modes of individual instantons randomly spread over the volume get delocalized due to their overlap, which means chiral symmetry breaking.

I should mention that the idea that instantons can break chiral symmetry has been discussed previously (see refs. (Caldi, 1977; Carlitz and Creamer, 1978; Callan, Dashen and Gross, 1978, Shuryak, 1982), however the present mechanism and a consistent formalism has been suggested and developed in the papers (Diakonov and Petrov, 1984b, 1986a).

8 Chiral symmetry breaking by instantons: quark propagator

Having explained the physical mechanism of chiral symmetry breaking as due to the delocalization of the would-be zero fermion modes in the field of individual instantons, I shall indicate how to compute observables in the instanton vacuum. The main quantity is the quark propagator in the instanton vacuum, averaged over the instanton ensemble. This quantity has been calculated in (Diakonov and Petrov, 1984b; Pobylitsa, 1989). In particular, Pobylitsa has derived a closed equation for the averaged quark propagator, which can be solved as a series expansion in the formal parameter $N \bar{\rho}^4/VN_c$ which numerically is something like $1/250$.
The result of these works is that in the leading order in the above parameter the quark propagator has the form of a massive propagator with a momentum-dependent dynamical mass:

\[ S(p) = \frac{\not{p} + iM(p^2)}{p^2 + M^2(p^2)}, \quad M(p^2) = c\sqrt{\frac{\pi^2 N\bar{\rho}^2}{VM_c}}F(p\bar{\rho}), \] (24)

where \( F(z) \) is a combination of the modified Bessel functions and is related to the Fourier transform of the quark zero mode: it is equal to one at \( z = 0 \) and decreases rapidly with the momentum, measured in units of the inverse average size of instantons; \( c \) is a constant of the order of unity which depends slightly on the approximation used in deriving the propagator. Note that the dynamical quark mass is non-analytical in the instanton density.

Fixing the average density by the empirical gluon condensate (see section 4) so that \( \bar{R} \approx 1 \text{ fm} \) and fixing the ratio \( \bar{\rho}/\bar{R} = 1/3 \) from our variational estimate, we get the value of the dynamical mass at zero momentum,

\[ M(0) \approx (350) \text{ MeV} \] (25)

while the quark condensate is

\[ \langle \bar{\psi}\psi \rangle = i \int \frac{d^4p}{(2\pi)^4} \text{Tr} S(p) = -4N_c \int \frac{d^4p}{(2\pi)^4} \frac{M(p)}{p^2 + M^2(p)} \approx -(255 \text{ MeV})^3. \] (26)

Both numbers, (25) and (26), appear to be close to their phenomenological values.

Using the above small parameter one can also compute more complicated quantities like 2- or 3-point mesonic correlation functions of the type

\[ \langle J_A(x)J_B(y) \rangle, \quad \langle J_A(x)J_B(y)J_c(z) \rangle, \quad J_A = \bar{\psi}\Gamma_A\psi \] (27)

where \( \Gamma_A \) is a unit matrix in colour but an arbitrary matrix in flavour and spin. Instantons influence the correlation function in two ways: i) the quark and antiquark propagators get dressed and obtain the dynamical mass, as in eq. (24), ii) quark and antiquark may scatter simultaneously on the same pseudoparticle; that leads to certain effective quark interactions. These interactions are strongly dependent on the quark-antiquark quantum numbers: they are strong and attractive in the scalar and especially in the pseudoscalar and the axial channels, and rather weak in the vector and tensor channels.

Since we have already obtained chiral symmetry breaking by studying a single quark propagator in the instanton vacuum, we are doomed to have a massless Goldstone pion in the pseudoscalar and axial correlators. Having a concrete dynamical realization of chiral symmetry breaking at hand, we can not only check the general Ward identities of the PCAC (which work of course) but we are in a position to find quantities whose values do not follow from general relations. One of the most important quantities is the \( F_\pi \) constant: it can be calculated as the residue of the pion pole. We get:
\[ F_\pi = \frac{\text{const}}{\bar{\rho}} \left( \frac{\bar{\rho}}{\bar{R}} \right)^2 \sqrt{\ln \frac{\bar{R}}{\rho}} \approx 100 \text{ MeV} \quad \text{vs.} \quad 93 \text{ MeV (exper.)} \quad (28) \]

This is a very instructive formula. The point is, \( F_\pi \) is surprisingly small in the strong interactions scale which, in the instanton vacuum, is given by the average size of pseudoparticles, \( 1/\bar{\rho} \approx 600 \text{ MeV} \).

The above formula says that \( F_\pi \) is down by the packing fraction factor \( (\bar{\rho}/\bar{R})^2 \approx 1/9 \). It can be said that \( F_\pi \) measures the diluteness of the instanton vacuum! However it would be wrong to say that instantons are in a dilute gas phase – the interactions are crucial to stabilize the medium and to support the known renormalization properties of the theory, therefore they are rather in a liquid phase, however dilute it may turn to be.

By calculating three-point correlation functions in the instanton vacuum we are able to determine, e.g. the charge radius of the Goldstone excitation:

\[ \sqrt{r_\pi^2} \approx \frac{\sqrt{N_c}}{2\pi F_\pi} \approx (340 \text{ MeV})^{-1} \quad \text{vs.} \quad (310 \text{ MeV})^{-1} \quad (\text{exper.}) \quad (29) \]

A systematic numerical study of various correlation functions in the instanton vacuum has been performed by Shuryak and collaborators (1993a). In all cases considered the results agree well or very well with experiments and phenomenology. As I already mentioned in the introduction, similar conclusions have been recently obtained from direct lattice measurements (Chu et al., 1994).

9 Chiral symmetry breaking by instantons: Nambu–Jona-Lasinio model

The idea of the first two derivations of chiral symmetry breaking by instantons, presented above, is: "Calculate quark observables in a given background gluon field, then average over the ensemble of fields", in our case the ensemble of \( I \)'s and \( \bar{I} \)'s. The idea of the third derivation is the opposite: "First average over the \( I \bar{I} \) ensemble and obtain an effective theory written in terms of interacting quarks only. Then compute observables from this effective theory". This approach is in a sense more economical; it has been developed in (Diakonov and Petrov, 1986a; Diakonov, 1995; Diakonov, Polyakov and Weiss, 1995).

Quark interaction arises when two or more quarks happen to scatter on the same pseudoparticle; averaging over its positions and orientations results in a four- (or more) fermion interaction term whose range is that of the average size of instantons. The most essential way how instantons influence quarks is, of course, via the zero modes. Since each massless quark flavour has its own zero mode, it means that the effective quark interactions will be actually \( 2N_f \) fermion ones. They are usually referred to as \( \text{'t Hooft interactions} \) as he was the first to point out the quantum numbers of these effective instanton-induced interactions (\( \text{'t Hooft, 1976} \)). In case of two flavours they are four-fermion interactions, and the resulting low-energy theory resembles the old Vaks–Larkin–Nambu–
Jona-Lasinio model (Vaks et al., 1961) which is known to lead to chiral symmetry breaking.

Let me quote the result for the case of two flavours, $N_f = 2$ (Diakonov and Petrov, 1986a). In that case one gets a 4-quark interaction vertex:

$$Y_2^{(+)} = \frac{2N_c^2}{N/V} \int \frac{d^4k_1d^4k_2d^4l_1d^4l_2}{(2\pi)^{12}} \sqrt{M(k_1)M(k_2)M(l_1)M(l_2)}$$

$$\frac{\epsilon^{f_1f_2} \epsilon_{g_1g_2}}{2(N_c^2 - 1)} \left[ \frac{2N_c - 1}{2N_c} (\psi_{L,f_1}^\dagger(k_1)\psi_{L,g_1}^g(l_1))(\psi_{L,f_2}^\dagger(k_2)\psi_{L,g_2}^g(l_2)) \right]$$

where $\psi_{L}^f$ is the left-handed component of the quark of flavour $f$ ($f = u, d$). For the antiinstanton-induced vertex $Y^{(-)}$ one has to replace left handed components by right-handed. $M(k)$ is the same dynamical quark mass as obtained in another approach outlined in the previous section. It is given by ($z = k\rho/2$):

$$M(k) = M(0)F^2(k), \quad M(0) = \lambda(2\pi\rho)^2,$$

$$F(k) = 2z[I_0(z)K_1(z) - I_1(z)K_0(z) - \frac{1}{z}I_1(z)K_1(z) \stackrel{k \to \infty}{\longrightarrow} \frac{6}{k^3\rho^3}, \quad F(0) = 1. \quad (31)$$

The value $M(0)$ (or $\lambda$) is found from the equation (Diakonov and Petrov, 1984b, 1986a) (called sometimes self-consistency or gap equation):

$$\frac{4N_c}{N/V} \int \frac{d^4k}{(2\pi)^4} \frac{M^2(k)}{k^2 + M^2(k)} = 1. \quad (32)$$

It is seen from eq. (32) that the dynamically generated mass is of the order of $M(0) \sim \sqrt{N/(VN_c)}\bar{\rho}$. Knowing the form of $M(k)$ given by eq. (31) and using the "standard" values $N/V = (1 \text{ fm})^{-4}, \bar{\rho} = (1/3) \text{ fm}$ we find numerically $M(0) \simeq 350 \text{ MeV}$. If one neglects $M^2$ in the denominator of eq. (32) (Pobylitsa, 1989) one gets $M(0) \simeq 420 \text{ MeV}$. This deviation indicates the accuracy of the "zero mode approximation" used in this derivation: it is about 15%.

Note that the second (tensor) term in eq. (31) is negligible at large $N_c$. Using the identity

$$2\epsilon^{f_1f_2} \epsilon_{g_1g_2} = \delta^{f_1}_{g_1}\delta^{f_2}_{g_2} - (\tau^A)^{f_1}_{g_1}(\tau^A)^{f_2}_{g_2} \quad (33)$$

one can rewrite the leading (first) term of eq. (30) as

$$(\psi^\dagger\psi)^2 + (\psi^\dagger\gamma_5\psi)^2 - (\psi^\dagger\tau^A\psi)^2 - (\psi^\dagger\tau^A\gamma_5\psi)^2 \quad (34)$$

which resembles closely the Nambu–Jona-Lasinio model. It should be stressed though that in contrast to that at hoc model the interaction (30) i) violates explicitly the $U_A(1)$ symmetry, ii) has a fixed interaction strength and iii) contains an intrinsic ultraviolet cutoff due to the formfactor function.
M(k). This model is known to lead to chiral symmetry breaking, at least at large \(N_c\) when the use of the mean field approximation to the model is theoretically justified.

The bosonization of these interactions has been performed in (Diakonov and Petrov, 1986a); it paves the way to studying analytically various correlation functions in the instanton vacuum.

A separate issue is the application of these ideas to hadrons made of light and heavy quarks (Chernyshov, Nowak and Zahed, 1994).

10 QCD at still lower energies

Using the packing fraction of instantons \(\bar{\rho}/\bar{R} \simeq 1/3\) as a new algebraic parameter one observes that all degrees of freedom in QCD can be divided into two categories: \(i\) those with masses \(\geq 1/\bar{\rho}\) and \(ii\) those with masses \(\ll 1/\bar{\rho}\). If one restricts oneself to low-energy strong interactions such that momenta are \(\ll 1/\bar{\rho} \approx 600\) MeV, one can neglect the former and concentrate on the latter. There are just two kind of degrees of freedom whose mass is much less than the inverse average size of instantons: the (pseudo) Goldstone pseudoscalar mesons and the quarks themselves which obtain a dynamically-generated mass \(M \sim (1/\bar{\rho})(\bar{\rho}^2/\bar{R}^2) \ll 1/\bar{\rho}\). Thus in the domain of momenta \(k \ll 1/\bar{\rho}\) QCD reduces to a remarkably simple though nontrivial theory of massive quarks interacting with massless or nearly massless pions. It is given by the partition function (Diakonov and Petrov, 1984b, 1986a)

\[
Z_{\text{QCD}}^{\text{low mom.}} = \int D\psi D\psi^\dagger \exp \left[ \int d^4x \psi^\dagger \left( i\slashed{\partial} + iMe^{i\pi A} - M \right) \psi \right].
\]

Notice that there is no kinetic energy term for the pions, and that the theory is not a renormalizable one. The last circumstance is due to the fact that it is an effective low-energy theory; the ultraviolet cutoff is actually \(1/\bar{\rho}\).

There is a close analogy with solid state physics here. The microscopic theory of solid states is QED: it manages to break spontaneously the translational symmetry, so that a Goldstone excitation emerges, called the phonon. Electrons obtain a ”dynamical mass” \(m^*\) due to hopping from one atom in a lattice to another. The ”low energy” limit of solid state physics is described by interactions of dressed electrons with Goldstone phonons. These interactions are more or less fixed by symmetry considerations apart from a few constants which can be deduced from experiments or calculated approximately from the underlying QED. Little is left of the complicated dynamics at the atom scale.

What Petrov and I have attempted, is a similar path: one starts with the fundamental QCD, finds that instantons stabilize at a relatively low density and that they break chiral symmetry; what is left at low momenta are just the dynamically massive quarks and massless pions. One needs two scales to describe strong interactions at low momenta: the ultra-violet cutoff, whose role is played by the inverse instanton size, and the dynamical quark mass proportional to the square of the instanton...
density. If one does not believe our variational calculations of these quantities one can take them from experiment.

If one integrates off the quark fields in eq. (35) one gets the effective chiral lagrangian:

$$S_{\text{eff}}[\pi] = -N_c \text{Tr} \ln (i \beta + i M U^\gamma),$$

$$U = \exp(i \pi \tau_A), \quad U^\gamma = \exp(i \pi \tau_A \gamma_5), \quad L_\mu = i U^\dagger \partial_\mu U. \quad (36)$$

One can expand eq. (36) in powers of the derivatives of the pion field and get:

$$S_{\text{eff}}[\pi] = \frac{F_\pi^2}{4} \int d^4 x \text{Tr} L^2_\mu - \frac{N_c}{192 \pi^2} \int d^4 x \left[ 2 \text{Tr}(\partial_\mu L_\mu)^2 + \text{Tr} L_\mu L_\nu L_\mu L_\nu \right]$$

$$+ \frac{N_c}{240 \pi^2} \int d^5 x \epsilon_{\alpha\beta\gamma\delta\epsilon} \text{Tr} L_\alpha L_\beta L_\gamma L_\delta L_\epsilon + \ldots \quad (37)$$

The first term here is the old Weinberg chiral lagrangian with

$$F_\pi^2 = 4 N_c \int \frac{d^4 k}{(2\pi)^4} \frac{M^2(k)}{[k^2 + M^2(k)]^2}; \quad (38)$$

the second term are the four-derivative Gasser–Leutwyler terms (with coefficients which turn out to agree with those following from the analysis of the data); the last term in eq. (37) is the so-called Wess–Zumino term. Note that the $F_\pi$ constant diverges logarithmically at large momenta; the integral is cut by the momentum- dependent mass at $k \sim 1/\bar{\rho}$, so that one gets the same expression as in a different approach described in section 10, see eq. (28).

An ideal field of application of the low-momentum partition function (35) is the quark-soliton model of nucleons (Diakonov and Petrov, 1986b) – actually the model has been derived from this partition function. The size of the nucleon $\sim (250 \text{ MeV})^{-1}$ is much larger than the size of instantons $\sim (600 \text{ MeV})^{-1}$; hence the low-momentum theory (35) seems to be justified. Indeed, the computed static characteristics of baryons like formfactors, magnetic moments, etc., are in a good accordance with the data (see the talk by K.Goeke).

11 Can instantons give confinement?

Our analytical calculations sketched in these lectures, the extensive numerical studies of the instanton vacuum by Shuryak and collaborators and the recent direct lattice measurements – all point out that instantons play a crucial role in determining the world of light hadrons, including the nucleon. Confinement has not much to do with it – contrary to what has been a common wisdom a decade ago and in what many people still believe. Nevertheless, confinement is a property of QCD, and one needs to understand the confinement mechanism. What can be said today is that confinement must be ”soft”: it should destroy neither the successes of the perturbative description of high-energy processes (no ”string effects” there) nor the successes of instantons at low momenta.
Quite recently we have noticed (Diakonov and Petrov, 1995) that one can obtain the area behaviour of large Wilson loops (i.e. confinement) from instantons, provided their size distribution behaves as $\sim d\rho/\rho^3$ for large $\rho$. It is remarkable that only the large-distance behaviour of the interquark potential is sensitive to the tail of the size distribution – all other quantities discussed in this paper are determined rather by the average sizes. Moreover, precisely the ”one over cube” distribution law seems to follow from the statistical mechanics of instantons, when one takes into account the quantum gluon fluctuations in the instanton ensemble.

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