Propagation of Acoustic Waves in Liquid Containing Multilayer Barrier

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Abstract. The dynamics of acoustic waves in a liquid containing multilayer barrier is studied. Using the subroutine of the fast Fourier transform the calculation of the distortion of the acoustic signal at the diagnosis of a multilayer sample comprising a layer of a liquid with polydispersed bubbles is executed. A good agreement between theoretical and experimental data is obtained.

Introduction

It is well known that even an insignificant (on volume or mass) number of bubbles in a liquid strongly affects the acoustic properties of the liquid. The fundamentals of mechanics and thermal physics of bubble liquids and the most essential results on studying the wave processes in such media are presented in [1, 2]. The conditions under which the carrying-liquid compressibility manifests itself for the problems of acoustics of bubble liquids are established in [3]. The results of experimental investigations of bubble-gel samples are given in [4]. In [5, 6], the propagation of weak harmonic perturbations in two-fraction mixes of a liquid with polydisperse bubbles of gas of different composition was investigated. We study pressure-pulse propagation in a polydisperse bubble medium on the basis of [6]. The dependence of the complex wave number on the vibration frequency according to [6] provided that the disperse phase consists only of bubbles of one gas; i.e., the volume content of the second-fraction bubbles is zero and has the form

\[
\left( \frac{K_\alpha}{\omega} \right)^2 = \frac{1}{C_f^2} + \frac{3\alpha_{20}\alpha_{10}^2\rho_{10}}{3\gamma_2\rho_{10} - \langle QS \rangle}
\]

\[C_f = C_1 / \alpha_{10}, \quad Q = 1 - 3(y_2 - 1)(y \cdot \coth(y) - 1) / y^3, \quad y = \sqrt{i\omega \alpha^2 / \kappa^2},\]

\[S = i\omega^2 h_z \rho_{10} / (1 + h_z t_z), \quad h_z = -i\omega + 4\nu_1 / a^2, \quad t_z = a / (C_1(\alpha_{20})^2),\]

\[\langle Q \rangle = \frac{1}{\rho_2} \int N(a) g_0(a) Q da, \quad \langle QS \rangle = \frac{1}{\rho_2} \int N(a) g_0(a) QS da,\]
\[ \rho_2 = \int_{a_{\text{min}}}^{a_{\text{max}}} N(a) g_\sigma(a) \, da, \quad g_\sigma(a) = \frac{4}{3} \pi a^3 \rho_0, \]

where \( \rho_0 \) are the true density and pressure of unperturbed liquid, \( C_1 \) is the sound speed in liquid, \( \nu_1 \) is the kinematic viscosity of liquid, \( \rho_0 \) is the true density of gas in bubbles, \( \gamma_2 \) is the adiabatic exponent of gas in bubbles, \( N(a) \) is the distribution function over bubble sizes \( a \), \( \kappa_2 \) is the thermal diffusivity of gas, and \( c_{p2} \) is the specific heat capacity of gas in bubbles at constant pressure.

**Results**

Further, we calculated the dynamics of the pulse perturbation of the pressure in liquid containing the multilayer barrier and compared the results with the experiment. In [4], a sample consisting of two polycarbonate layers and the layer containing industrial gel with polydisperse bubbles was investigated experimentally.

![Figure 1. Scheme of installation for acoustic diagnostics from experimental data [4].](image)

The scheme of the experiment is shown in figure 1. The sample is placed in a vessel with water between an acoustic-signal source (the piezoelectric converter) and a hydrophone. On the basis of the analysis of the parameters of sound waves transmitted through a sample, the data on its properties are found. The function of the distribution of bubbles over sizes was determined as

\[ N(a) = \frac{n_i}{\sqrt{2\pi\varepsilon}} \exp \left\{ -\frac{\ln(a/a_i)^2}{2\varepsilon^2} \right\}, \quad a_{\text{min}} \leq a \leq a_{\text{max}}. \]

Where \( a_i = 0.08 \text{ mm} \), \( \varepsilon = 0.05 \) is the distribution parameters [4]. For approximating the initial pulse from the experiment, the pressure pulse perturbation is set with the help of the imaginary part of the Morlet wavelet with the splash frequency \( \nu_b = \omega_b / 2\pi = 0.25 \text{ MHz} \):

\[ p'(0,t) = \text{Im} \left\{ \exp[-i2\pi\nu_b(t-r)] \exp \left[ \frac{-(t-r)^2}{2\delta^2} \right] \right\}. \quad (2) \]
The parameters $\tau = 20\,\mu s$ and $\delta = 3\,\mu s$ determine the splash location and width, respectively.

According to [7], the result of transmission of a plane monochromatic wave $\sim \exp[(Kx - \omega t)]$ through a multilayer sample is a plane wave $\sim W\exp[(Kx - \omega t)]$, where $W$ is the wave-transmission coefficient, which is determined through the layer impedances $Z_j$ and the input layer-boundary impedances $Z^{\text{in}}_j$:

$$W = \frac{Z^{\text{in}}_1 + Z_4 e^{i\kappa d_4}}{Z^{\text{in}}_1 + Z_5} \frac{Z^{\text{in}}_3 + Z_4 e^{i\kappa d_3}}{Z^{\text{in}}_3 + Z_5} \frac{Z^{\text{in}}_2 + Z_4 e^{i\kappa d_2}}{Z^{\text{in}}_2 + Z_5} \frac{2Z_1}{Z_1 + Z_2}. \quad (3)$$

$$Z_j = \rho_j \frac{\omega}{K_j}, \quad Z^{\text{in}}_j = \rho_j \frac{Z^{\text{in}}_{j-1} - iZ_j \tan(K_j d_j)}{Z_j - iZ^{\text{in}}_j \tan(K_j d_j)} Z_j, \quad Z^{\text{in}}_1 = Z_1.$$  

Here, $d_j$ and $K_j$ are, respectively, the thickness and wavenumber of the $j$-th layer (for the liquid layer with bubble inclusions $d_j = h, \ K_3 = K_4$).

Using the subroutine of the fast Fourier transform (FFT), we calculate the evolution of pressure pulse (2). With the help of the inverse Fourier transform, we determined the complex amplitudes of the harmonic components of initial pressure pulse (2) at $x = 0$. Then, using equation (3), we determine the amplitudes of harmonic components of the pulse transmitted through a multilayer sample to the hydrophone (at $x = \sum d_j$). The variation of pressure as a function of time in a new position is calculated with the help of the direct Fourier transform.

![Figure 2](image)

**Figure 2.** Pulse perturbation of pressure in purified water ($I$ is the calculation, and $II$ is the experimental data [4]).

The results of calculation of evolution of the pressure pulse perturbation were compared with the experimental results [4]. The data in figure 2 for the initial signal show that function (2) describes well the initial acoustic signal written in the experiment [4]. The number of harmonics in the subroutine FFT was selected from the condition of the pulse motion in purified water (when there is no dispersion or dissipation) without distortion.
Figure 3. Pulse perturbation of pressure in water containing a multilayered sample (I is the calculation, and II is the experimental data [4]).

The experimental results [4] shown in figure 3 are obtained at the following bubble-layer parameters: \( h = 2 \text{ mm} \), \( \alpha_{20} = 0.0015 \), \( a_0 = 0.08 \text{ mm} \), and correspond to the average distance \( l = a_0 / \alpha_2^{1/3} = 0.7 \text{ mm} \) between bubbles. In this case, the wavelength under consideration on a pure liquid is \( L = 2\pi c / \omega = 15 \text{ mm} \). The continuum model under consideration for a bubble liquid is valid for \( L >> l \) and \( h >> l \). Although the second condition is not completely fulfilled, the theoretical curve of the dependence of the pressure at the hydrophone on the reduced time agrees well with the experiment.

Thus, the theory under consideration can be used confidently for calculation of the distortion of the acoustic signal in the diagnostics of multilayer samples with bubble inclusions.

Acknowledgments
This work was funded by the subsidy of the Russian Government to support the Program of Competitive Growth of Kazan Federal University among World’s Leading Academic Centers.

References

[1] Nigmatulin R I 1991 *Dynamics of Multiphase Media* (New York: Hemisphere)
[2] Nakoryakov V E, Pokusaev B G and Shreiber I R 1993 *Wave Propagation in Gas-Liquid Media* (New York: CRC Press)
[3] Nigmatulin R I, Shagapov V Sh and Vakhitova N K 1989 *USSR Academy of Sciences Doklady* 304 1077
[4] Leroy V, Strybulevych A, Page J H and Scanlon M G 2008 *J. Acoust. Soc. Am.* 123 1931
[5] Nigmatulin R I, Gubaidullin D A, Fedorov Yu V 2012 *Doklady Physics* 57 451
[6] Nigmatulin R I, Gubaidullin D A, Fedorov Yu V 2013 *Doklady Physics* 58 261
[7] Brekhovskikh L M 1980 *Waves in layered media* (New York: Academic Press)