A Modeling of Lead-Acid Battery for Electric Vehicles and Simulation Utilizing C MEX S-Function in Matlab/Simulink

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Abstract. Nowadays, the need for better performance of batteries as an energy source is increasing following the growth of electrical vehicles (EVs). Therefore, modeling the battery system is important to analyze how the battery is able to perform better so that further increases the EV's performance. This study is addressed to analyze and simulate circuit modeling of lead-acid battery into a pretty complex RLC circuit, voltage source of battery itself, and a connected load with provided conditions for obtaining varying parameters which are used by the battery with a permanent initial state of charge (SoC). This circuit is simulated using C/C++ programming where it is implemented to some connected C MEX S-Function blocks in Simulink. The result of the study is proved that series circuits increase their voltage and parallel circuits increase their ampere. For satisfying the EV’s specs such as \( V_{out} = 400 \text{ V} \) and the capacity \( = 100 \text{ Ah} \), arranging batteries in 400 series since \( n = 15 \) produce around 13 V.

1. Introduction

Battery is a very essential component to many fields such as electrical tools, mechanical tools, and so on. And, one example of battery usage in electrical tools is the battery as the main power source to operate Electric Vehicles (EVs). The battery's usage as the main power source in EVs directly states the importance and vital influence of the battery's performance [1]. One battery type used in EVs is the lead-acid battery. The lead-Acid battery is accessible and has the highest maximum voltage value in its category [2]. Lead-Acid batteries are typically used in the automotive industry, where they undergo periodic deep discharge and charge, and in stationary applications, where they remain in charged state most of their life [3]. Since EVs require decent battery performance, it is important to model a battery that is suitable for EVs characteristics. The value chain of EVs batteries consists of seven steps: component production (including raw materials); cell production; module production; assembly of modules into the battery (including an electronic control tuning and a cooling system); integration of the battery pack into the vehicles; used during the life of the vehicle; and reuse and recycling [4]. Before running the processes mentioned above, it is important to simulate the battery model before applied in physical EVs. The simulation can be run using various methods and software. One of them is Matlab Simulink using C-Mex S-Function, which is applied using the programming language. The program adopted the derivative equation of the circuit from [5].

A recent study has underlined the importance of a standard battery module to achieve massive electrification of off-road vehicles [6]. According to the study, it is possible to use a standard module of 4 series-connected cells to build up the battery of many types of off-road vehicles [7]. The standard module has a 12.8 V nominal voltage. Its implementation with three capacity values: 30, 60, and 100 Ah
is reported [8]. The large numbers of cells are connected to construct a battery pack to satisfy the voltage and capacity requirement of an electric vehicle [9]. Cell failure and imbalance are critical problems in battery storage systems, especially in series-connected battery strings [10]. So, we need a simulation for knowing and observing batteries’ behaviour, particularly in series-parallel connections. In this study, the simulation will be observed in Matlab Simulink using C-Mex S-Function. It is a novel process since the simulation mostly will take part in coding using C language, so we need more knowledge about C and information about the battery, series-parallel connections, and others.

The general battery used is MSE-100Ah with a nominal voltage of 6 V and a capacity value of 100Ah. Once the required voltage, current and capacity are defined for the target application, it is easy to calculate the number of necessary cells. The battery voltage is set by the number of series-connected cells, whereas the battery capacity and the maximum battery current are given by the number of parallel-connected cells. In [5], the equivalent circuit below is the so-called Randles equivalent circuit with an additional inductance.

![Figure 1. Equivalent Circuit of Lead-Acid Battery](image)

Usually, the equivalent circuit for batteries uses only the Randles equivalent circuit. In this case, the inductance of the storage battery can be said as the form of the energizing part of the storage battery. The inductance occurs due to the wiring straps between the cell element and battery terminal/pole and between one cell element and the other cell element inside the battery. This paper also used an electric circuit parameter at each state of charge in [5].

| State of Charge | R_1 (mΩ) | R_2 (mΩ) | C (F) | L(µH) |
|-----------------|----------|----------|-------|-------|
| 100%            | 1.92     | 0.9      | 12    | 0.75  |
| 80%             | 2.06     | 0.81     | 13.1  | 0.81  |
| 60%             | 2.2      | 0.69     | 13.6  | 0.78  |
| 40%             | 2.28     | 0.61     | 12.5  | 0.68  |
| 20%             | 2.72     | 0.67     | 11.1  | 0.79  |
| 0%              | 3.06     | 1.06     | 4.25  | 0.85  |
The processes that will be explained are to derive a circuit to the state-space equation that can be transformed to the discrete domain and then implemented to C-Mex S-Function.

2. Method
2.1 Mathematical Model of Lead-Acid Battery

According to the equivalent circuit in Figure 1, the equation to determine voltage value of the battery ($V_{out}$) is as below:

$$V_{out} = V_C + V_L + V_B + V_{R1}$$  \hspace{1cm} (1)

Where $V_C$, $V_L$, $V_B$, and $V_{R1}$ is voltage of capacitance component C, inductance component L, battery internal voltage, and resistance $R_1$ component respectively.

The value of the $V_{out}$ can be determined by using state-space equation below:

$$\begin{bmatrix} \dot{V}_C \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} -1/(C \times R_1) & -1/C \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_C \\ I_L \end{bmatrix}$$ \hspace{1cm} (2)

From the state-space equation above in mind, we can have equation to calculate $V_C$ as below:

$$(V_C)_n = \sum_{n=0}^{\infty} \left[ (V_C)_{(n-1)} + \frac{-1}{C} (I_{b Batt})_{(n-1)} + \frac{V_C}{R_2} \right]$$ \hspace{1cm} (3)

Where,

$I_{out} = I_L = I_{b Batt}$

$C =$ Capacitance value of component C

$R_2 =$ Capacitance value of component $R_2$

$R_1 =$ Capacitance value of component $R_1$

$I_{out}$ is the output current of the battery, $I_L$ is the current that pass-through inductance component L, and $I_{b Batt}$ is current value inside the battery. The equation above will be written in C programming language in discrete structure using sampling period $\Delta t$ which is equal to $10^{-3}$ seconds. The $(V_C)_n$ have to be multiplied to $\Delta t$.

To determine $V_L$, we use equation as below:

$$(V_L)_n = \sum_{n=0}^{\infty} \left[ -L \left( I_{b Batt} - (I_{b Batt})_{(n-1)} \right) \right]$$ \hspace{1cm} (4)
Where,
$L = $ Capacitance value of component L. The $(V_L)_n$ have to be divided by sampling period time $\Delta t$.

To determine the value of $V_{R1}$, the equation below is used:

$$V_{R1} = -I_{out} \times R_1$$  \hspace{1cm} (5)

We also need to determine the State-of-Charge SoC value of the Lead Acid battery. To determine $SoC_k$ we are using equation below:

$$SoC_k = \sum_{k=0}^{n} \left[ SoC_{(k-1)} - \left( \frac{a}{b} \right) (I_{batt})_k \right]$$  \hspace{1cm} (6)

Where,
$n = $ battery cells  
$Q = $ battery capacity

The $SoC_k$ has to multiply to $\Delta t$. From Figure 2, we did interpolation process to generate $3^{rd}$ order polynomial equation for each parameter of R1, R2, C, and L. Below are the equation of each parameter:

$$R_1 = -0.8796x^3 + 2.3016x^2 - 2.5728x + 3.0797$$  \hspace{1cm} (7)

$$R_2 = -2.4306x^3 + 5.0387x^2 - 2.7677x + 1.0555$$  \hspace{1cm} (8)

$$C = 23.553x^3 - 56.468x^2 + 40.596x + 4.4329$$  \hspace{1cm} (9)

$$L = -1.2037x^3 + 2.0556x^2 - 0.9582x + 0.8632$$  \hspace{1cm} (10)

2.2 Cmex S Function

The mathematical model that was previously discovered is able to be run in simulink using Cmex S-Function block. Cmex S-Function is a feature provided by simulink so it is possible to build a block in simulink using code created in MATLAB, C, C++, or Fortran. Other than MATLAB, it can be compiled using another compiler which one of them is MinGW Compiler. Compiling C, C++, or Fortran can be done using the mex command on the available command window. Cmex S-Function also uses callback methods on its structures that consist of mdlInitializeSizes, mdlIntilizaeSampleTimes, mdlOutputs, and mdlTerminate. MdlInitializeSizes specifies the sizes of various parameters in the SimStruct, such as the number of output ports for the block. MdlInitializeSampleTimes specifies the sample time(s) of the block. MdlOutputs calculates the output of the block. MdlTerminate performs any actions required at the termination of the simulation. If no actions are required, this function can be implemented as a stub. The Cmex routines will be shown in Figure 3 clearer.

To code the Cmex S-Function in C programming language, there are three provided structures, which are structure A, B, and C. Each of them has its characteristic: structure A accommodates calculation in discrete or continuous routines. Therefore, it does not mdlUpdate nor mdlDerivatives function since it does not have any state as well. The structure B accommodate calculation in discrete, and the most notable characteristic of structure B is it has any discrete states, and it has mdlUpdates function, which contains integral equation that will be used, which named solver algorithm since MATLAB is not able to process integral function system. In this structure, we must describe the time sampling of the process. This structure commonly figured for the controller program of a system. On the other hand, structure C accommodates calculation in continuous routines, and its main attribute is the structure have any continuous states, and it has mdlDerivatives function, which contains the differential equation, so that we don’t need to write the solver or the integral equation, since, MATLAB will act as the solver. There are many kinds of the solver such as ODE45, ODE23, ODE Bogacki and so on. This structure is mainly used for the plant program of a system. These structures (A, B, and C) can
be chosen depending on the equation form and the required output of the system. These explanations could be simplified in Figure 3 and Figure 4.

![Figure 3. Block Diagram for The S-Function](image1)

![Figure 4. Cmex Routines](image2)

2.3. Implementation
The explanation from the previous section can be implemented to a simulation described in Figure 5.

![Figure 5. Parallel configuration s-function block](image3)

![Figure 6. Series configuration s-function block](image4)
The simulation process starts from ‘BATTERY’ s-function block. In this s-function block, there exists a programmed process that runs using Cmex S-Function. In more detail, this Cmex S-function block has one input which is current feedback from a 'CURRENT' S-Function block and it has five output which are \( i_{\text{act}} \); SoC; \( V_{\text{out}} \); and \( R_1 \), \( R_2 \), \( C \), and \( L \) to validate parameter value using 3rd order polynomial equation that had obtained. Then, \( V_{\text{out}} \) result is set as first input of the ‘CURRENT’ block, and a preset value of resistance is set as second input of the block. The output of the block is \( V_{\text{out}} \), \( I_{\text{battery}} \) value to monitor and validate calculation results of the block. The calculation and equation inside the ‘CURRENT’ block using Kirchhoff Law no. 1 which is:

\[
V = iR \tag{11}
\]

From Figure 5 to 7, we can go further to determine batteries arrangement whether parallel, series, or series-parallel. It is helpful to have the best battery arrangement, so that it is compatible and suitable to the required source voltage and capacities. The implementation of this process is figured by Figure 5, 6, 7 Figure 5 describes parallel circuit or arrangement, Figure 6 describes series circuit or arrangement, Figure 7 describes series-parallel circuit or arrangement. These processes based on the concept of in series circuit the current that pass through each component that put in series is basically the same, or it can be said \( i_1 = i_2 = i_3 \); and the voltage of each component that put in series is different, or it can be said \( V_1 \neq V_2 \neq V_3 \); and vice versa to the parallel circuit.

3. Results and Discussion
This section discusses simulation results of each battery configuration using a mathematical model of lead-acid battery explained above. The simulation runs by using MATLAB/Simulink. Each configuration - series, parallel, and series-parallel - is done to study the behaviour of each battery configuration. Its behaviour will be figured by graphic of SoC (and \( I_{\text{battery}} \)) over time and battery voltage (\( V_{\text{out}} \)) over time. In this simulation, it also simulates a different number of batteries (described as \( n \)) for each configuration to show the effect of battery number to the behaviour of each configuration. The simulation will use time sampling of 250 millisecond and the initial SoC is 1 (or 100%).

![Figure 8. In Series Circuit with \( n = 5 \) (a) SoC and \( I_{\text{battery}} \) (b) \( V_{\text{out}} \)](image)

![Figure 9. In Series Circuit with \( n = 15 \) (a) SoC and \( I_{\text{battery}} \) (b) \( V_{\text{out}} \)](image)
From figures above, SoCs will decrease slowly to 0 depending on the currents flow from the battery and if the current in simulation is increased, the SoC will be empty or going down to 0 faster as well. This behaviour is as stated in Eq. (6) that the higher the $I_{\text{batt}}$, the higher the reduced value of SoC, thus, it makes the SoC value down to zero faster, this phenomenon can be called a discharge process. During the discharge process, each cell shares different current due to the difference of internal resistance or polarization effects. Hence, the SoC of each cell changes with different rates and at the end of discharge, when cells together reach the lower limit (6 Volt, 0% of SoC).

The $n$ over here is the arranged number of cells so that we can compare between series, parallel, and series-parallel circuits based on the number of $n$. The influence of $n$ to the output of simulation is because the number of batteries will affect the mathematical equation of each battery configuration, thus, each $n$ number of batteries will give different output of simulation. As the figures show, the more $n$-series the more voltage we get, the more $n$-parallel the more ampere we get, and the more $n$-parallel-series the more voltage and ampere we get. In [7] explained that modules can be parallel connected to obtain a higher capacity or serially connected to create battery strings with higher voltage. In turns, battery strings can be parallel connected to increase the capacity. However, from the results of simulation the optimum result is on series circuits or parallel circuits, it’s slightly different with [12] the best connection topology is the series connection of parallel modules of cells provided.

4. Conclusion

From the result section, it could be seen that the behavior of lead-acid battery is divided into three kinds of type, the series circuit would make the $V_{\text{out}}$ larger, the parallel circuit would make the $I_{\text{batter}}$ larger, and the parallel-series one would make the $V_{\text{out}}$ and $I_{\text{batter}}$ larger. We can also conclude that, if we have an EV target with $V_{\text{out}} = 400$ Volt and the Capacity $= 100$ Ah, the one of ways we can execute is arranging batteries and make them in 400 series since $n = 15$ produce around 13 V so then we need that value to make it comes true.
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