Photon-induced molecular implementation of a quantized magnetic flux photoelectron

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Keywords: magnetic flux molecular resonance, quantized magnetic flux photoelectron quenching, half cycle wave electromagnetic regime, fundamental magnetic flux quantum bound potential, half cycle wave molecular polarizability, molecular ion core inertial threshold, entropic, position-momentum, and time-energy uncertainty transfer in the photon-induced implementation of a fundamental magnetic flux.

Abstract
A fundamental magnetic flux quantum can be implemented into a free rotating molecule when the interacting molecular electron experiences the maximum possible intrinsic energy uncertainty of a gaussian, transform-limited half-cycle optical photon wavepacket. A magnetic flux resonance condition can be defined at this limit, with photoionization quenching, and the excited molecular electron is drawn into a Rydberg-like spherical surface where a 3D-diffraction pattern is oscillating at the minimum of a bound potential around a primary formed closed electronic loop. The induced rotational motion of the molecular ion core is initiated at the threshold of a robust inertial effect and the dissipated information entropy is the lowest allowed. The integrated quantum possibilities occur in the process as structural properties of a quantized magnetic flux implementation threshold.

1. Introduction

The quantized nature of the magnetic flux has been observed experimentally more than a half century ago. Magnetic flux trapped in hollow superconducting cylinders has provided in two independent experiments [1, 2] the very first evidence for the existence of distinct values in the magnetic flux. These values occurred as multiples of a fundamental unit \( \Phi_0 = h/2e \), the same for any observed superconducting loop. This physical effect was predicted earlier by London [3] who used initially a phenomenological model in the description of the electromagnetic properties of superconductors. The presence of a quantum structure has been then suggested [4] at a macroscopic scale and Ginzburg and Landau [5] have further considered the space variation of the corresponding electronic density. At the scale of a single molecule the photon-induced magnetic flux must be also quantized. As the induced magnetic moment of the interacting molecular electron is characterized by the Landé \( g_e \) factor, and this factor contains the one loop quantum mechanical correction \([6]\) relative to the \( g_D = -2 \) Dirac value \([7]\), we may naturally ask whether a well defined internal structure is present into a photon-induced magnetic flux quantum. Since the fine structure constant \( \alpha \) characterizes the strength of the coupling of an elementary electric charge with an electromagnetic field \([8]\), the dimensionless Schwinger–Dirac ratio \( \hat{g}_e = g_e / g_D = 1 + \alpha/2\pi \) is expected to play a fundamental key role in such a quantized magnetic flux implementation process.

The interaction of a photon wavepacket of well defined energy with a single molecule can provide under resonance conditions an energy and angular momentum transfer. With the well defined oscillation of the photon’s electric field the interacting molecular electron accumulates the transferred energy whereas the overall effect of the oscillating induced magnetic field vector occurs only as an average effect, whose magnitude is reduced better and better down to a zero value. The very first experimental evidence for this effect was the issue of the photoelectric effect \([9, 10]\). An excited bound electron can be ejected out of a metal surface and propagate in space as a free photoelectron of well defined kinetic energy if a characteristic threshold frequency is attained. The observed threshold then remains insensitive to the incident light intensity or the time interval of the underlying single-photon bound-electron interaction process. There is no any noticeable magnetic field effect in...
this excitation process. For linearly polarized light, the direction of distribution of the ejected photoelectrons peaks always in the direction of the oscillating electric field.

A peculiar limiting situation can be attained however if the electric field of the photon’s wave packet is constrained to perform a unique, half-cycle oscillation. The corresponding induced magnetic field then exhibits a fixed polarity in space during the time duration $\Delta t_0 = \Delta t_{0_{\mathrm{FWHM}}}$ of the photon- molecule interaction process; for an optical photon wave packet of a Gaussian, Transform-Limited, Half-Cycle Wavepacket (GTL-HCW), the interaction will consist of a molecular electron experiencing, both, the sharp and monotonous time increase of the electric field, and the fixed space polarity of an induced uniform magnetic field. Such a limiting situation is first characterized by the peculiar property of a GTL-HCW optical photon wave packet whose center frequency $\omega_0$ is badly defined. This is because the intrinsic uncertainty $\Delta\omega_0$ is larger than the unrestrained value of $\omega_0$. The intrinsic energy uncertainty $\Delta E_0 = h\Delta\omega_0$ can thus overlap coherently one or even more electronic threshold energies $\epsilon_{\mathrm{th}}^0$ in a specific molecular system. When the interaction is performed in the presence of two indistinguishable entangled electronic states, the joint effect of the GTL-HCW electric and magnetic fields results into, both, the spatial separation of the excited molecular electron from the molecular ion core and, more importantly, the indistinguishability of the electric charge position along a photon- induced closed electronic loop. The formation of this loop is then performed in the presence of a magnetic flux amount transversing a well defined surface. Since at the scale of the interacting molecule the induced magnetic flux and the transferred angular momentum are allowed to occur only as quantized quantities, the photon- induced implementation of a single, Fundamental Magnetic Flux Quantum (FMFQ) must exhibit the property of a FMFQ ionization quenching; i.e., with the increasing peak power of a GTL- HCW absorbed photon the excited molecular electron carrying a quantized amount of magnetic flux can be drawn only into the lower and lower Rydberg molecular states. A requisite condition for such a Molecule Fixed Frame (MFF) space confinement of a molecular electron is the FMFQ resonance condition: $E_{\mathrm{th}}^0 [eV] \tau_{\mathrm{coh}}^0 [fs] = \Phi_0 [Wb]$. The time interval $\tau_{\mathrm{coh}}^0 = 2\Delta t_0$ is the coherence time interval of the GTL-HCW pulse, where the FWHM width $\Delta t_0$ is fixed in terms of a preselected Two Entangled Electronic States (TEES) threshold energy $E_{\mathrm{th}}^0$ and, the constant $\Phi_0 = h/\epsilon$ represents the fundamental magnetic flux quantum. The FMFQ- photoelectron quenching is then realized in the presence of a GTL-HCW entropic uncertainty [11] $S_{\Delta\omega_0} + S_{\Delta\omega_{0}}$ is the maximum possible. The time- energy uncertainty relation expressed in terms of the appropriate units of $fs$ and $eV$ reveals indeed the prominence of the temporal and spectral Shannon entropies into the photon- induced implementation of a fundamental magnetic flux quantum and thus, the emerging and synergetic overall effect of the GTL-HCW photon’s magnetic field at the TEES threshold. We observe the fundamental property: $\ln [\Delta E_0 [eV] \tau_{\mathrm{coh}}^0 [fs] / \Phi_0 [Wb] ] = S_{\Delta\omega_0} - S_{\Delta\omega_{0}}$. The initial indeterminacy of the center frequency $\omega_0$ can be completely removed in the presence of the FMFQ resonance condition. This is because the selected TEES- threshold frequency $\omega_0^0$ becomes the operating value of $\omega_0$. We may see in the following that the trapped FMFQ- photoelectron can evolve in time only as a diffracted bound molecular electron oscillating over a well defined Rydberg spherical surface around a proteiform closed electronic loop. Given that this electronic contour is defined by the photoabsorption of a quantized magnetic flux amount, the position-momentum space and, the time-energy uncertainties cannot operate separately. They can be defined in the interaction process only by means of the synergetic effect of a position- momentum and time-energy uncertainty relation: $[\sigma_{x\tau}^0, \sigma_{p\tau}^0]_{\min} \times (\Delta E_0[eV] \Delta t_{0} [fs]) = \frac{1}{2} \left[ E_{\mathrm{th}}^0 \right]^2 \Phi_0 [Wb]$. This is a direct manifestation of the presence of a FMFQ photoelectron captured at the minimum of a bound potential. Such an issue occurs as a natural consequence of the transferred entropic uncertainty in the time and frequency spaces where the paired configuration of the $l = \pm 2, \pm 3, \ldots$ harmonics of the imposed FMFQ resonance condition contribute exclusively in the formation of the minimum of a bound potential. The maximum possible amount of the dimensionless energy uncertainty $\Delta E_0/E_{\mathrm{th}}^0 = \exp (S_{\Delta\omega_0} - S_{\Delta\omega_{0}})$ can be transferred into the molecule during the formation of a proteiform electronic loop only as the conjugate quantity of the photon- induced position- momentum uncertainty $[\sigma_{x\tau}^0, \sigma_{p\tau}^0/\theta]_{\min}$. The geometric mean of these two conjugate dimensionless quantities matches the structure constant $\hat{g}_c$.

2. GTL-HCW photoelectromagnetic effect: a semi-classical approach of the FMFQ-ionization quenching

The dimensionless electric field $E_{\Delta\omega_0}(\omega)/\epsilon_{\mathrm{th}}^0$ of a GTL linearly polarized laser beam exhibits the time envelope: $\exp(-\left( (t/\tau_{\mathrm{coh}}^0) \right)^2 / \Var \left( |\psi|^2 \right)^2$ where the variance $\Var \left( |\psi|^2 \right) = 1/\Delta\omega_0 \tau_{\mathrm{coh}}^0$ is assigned to the corresponding presence probability distribution $|\psi_{\Delta\omega_0}(t)|^2$. It represents equally well the variance of the distribution $|\psi_{\Delta\omega_0}(\omega)|^2$ in the frequency domain. The dimensionless constant $\hat{Q}_0 = \pi \Var \left( |\psi|^2 \right)$ is an invariant point in these two conjugate spaces and, the time- frequency uncertainty relation $\Delta\omega_0 \tau_{\mathrm{coh}}^0 = 8 \ln 2$ involved in these two distributions, implies the presence of a coherence time interval $\tau_{\mathrm{coh}}^0 = 2\Delta t_0$. It is then relevant that, if we
introduce the appropriate units of eV and fs for the energy and time into the GTL- photon- molecule interaction, the invariant point casts into the following dimensionless interaction constant:

\[ \hat{Q}_0^{-1} = \frac{\Delta E_0 \epsilon [eV] \tau_{coh}[fs]}{\phi_0[f/Wb]} \]

The dimensionless constant \( \Phi_0[f/Wb] = 2.067834 \) defines the fundamental magnetic flux quantum \( \Phi_0 = h/e \). We have indeed: \( \Phi_0[f/Wb] = \pi \tilde{C}_0 \) with \( \tilde{C}_0 = \hbar / [eV \cdot fs] = 0.658212 \). It is worth remarking that \( \hat{Q}_0^{-1} \) can be identified with the ratio \( \Delta \omega_0 \tau_{coh} / \omega_0 \tau_{coh}^0 \) if the dimensionless quantity \( \omega_0 \tau_{coh}^0 \) approaches the FMFQ phase value \( \phi_0^0 = \pi \). That is, when the badly defined center frequency \( \omega_0 < \Delta \omega_0 \) verifies the relation:

\[ \omega_0 \tau_{coh}^0 = \omega_0 \tau_{coh} = \phi_0^0 \]. The frequency \( \omega_0^0 \) can be in fact any electronic transition threshold frequency overlapped coherently by the intrinsic frequency uncertainty \( \Delta \omega_0 > \omega_0^0 \). A requisite condition for this is the absorption of a GTL- HCW optical photon wavepacket. The FWHM width \( \Delta \omega_0 \) can be fixed for instance from the presence of a specific threshold frequency \( \omega_0 \) if we impose into the GTL- HCW photon- molecule interaction the FMFQ-resonance condition:

\[ \tau_{coh}^0 \hat{Q}_0 \epsilon / e = \Phi_0 \]. The quantized nature of the magnetic flux and angular momentum will then remove entirely the initial indeterminacy of the center frequency \( \omega_0 \). It is also worth noting that the imaginary part of the GTL- HCW electric field exhibits in the time envelope a single and monotonous time increase. The corresponding induced magnetic field of the GTL- HCW photon wave packet can thus exhibit in space only a uniform magnetic field of fixed spatial polarity. The joint effect of such a peculiar pair of electric and magnetic fields is therefore expected to result into a distant closed electronic loop, formed about a hydrogen-like electronic ion core, where the excited molecular electron experiences the presence of a TEES system. The underlying issue of the temporal \( S_{\Delta \omega_0} = - \int_{-\infty}^{+\infty} |\psi_{\Delta \omega_0}(t)|^2 \ln \left| \psi_{\Delta \omega_0}(t)^2 \right| dt \) and spectral \( S_{\Delta \omega_0} = - \int_{-\infty}^{+\infty} |\psi_{\Delta \omega_0}(\omega)|^2 \ln \left| \psi_{\Delta \omega_0}(\omega)^2 \right| d\omega \) Shannon entropies [12] contains relevant information into this peculiar limit; i.e., given that the two conjugate fixed length scales \( l_{\Delta \omega_0} \) and \( \omega_{\Delta \omega_0} \) (which ensure homogeneity) must satisfy the relation \( l_{\Delta \omega_0} / \omega_{\Delta \omega_0} = 1 \), we have the freedom to define the length scales in terms of the dimensionless key quantities \( Q_0, \omega_0 \Lambda \omega_0 \), and \( \omega_0^0 \Lambda \omega_0 \). Doing so, we can write:

\[ l_{\Delta \omega_0} = \sqrt{Q_0 / \Delta \omega_0 \Lambda \omega_0} \Lambda \omega_0 \] and \( l_{\Delta \omega_0} = \sqrt{Q_0 / \omega_0 \Lambda \omega_0} \Lambda \omega_0 \). At the GTL- HCW limit the FMFQ- resonance condition then unveils an intrinsic property of the temporal and spectral Shannon entropies. We readily obtain, both, the minimum allowed entropic uncertainty \( S_{\Delta \omega_0} + S_{\Delta \omega_0} = \ln(\pi e) \) in the time and frequency spaces, and the corresponding maximum possible difference \( S_{\Delta \omega_0} - S_{\Delta \omega_0} = \ln \left| \Delta E_0 [eV] \tau_{coh}^0 [fs] / \Phi_0 [f/Wb] \right| \). This maximized entropic uncertainty difference is the logarithm of the interaction constant of equation (1). \( S_{\Delta \omega_0} + S_{\Delta \omega_0} \) and \( S_{\Delta \omega_0} - S_{\Delta \omega_0} \) therefore stand in the interaction process as the fundamental intrinsic properties of the GTL- HCW optical photon wavepacket. Such a photon’s behaviour is neither of wave nor of corpuscular nature. The interacting molecular electron experiences an electromagnetic cyclotrophic regime where the key interaction parameters are the transferred intrinsic uncertainties.

1. We have drawn in figure 1 the imaginary part of \( \mathcal{E}_{\Delta \omega_0}(t / \tau_{coh}) \mathcal{E}^*_0 \mathcal{E} = \exp[-i \hat{Q}_0^{-1} (t / \tau_{coh})^2 - \bar{\hat{Q}} (t / \tau_{coh})] \) as a function of the dimensionless time variable \( t / \tau_{coh} \) when the photon- induced magnetic flux \( \hat{Q}_0 = [E_0 \tau_{coh}^0 / \epsilon] / \Phi_0 \) (measured in units of the fundamental magnetic flux quantum) takes the values: \( \hat{Q}_0 = 1/4, 1 \) and 4. These representative values are associated with a common overall interaction time interval \( \tau_{int}^0 = 4 \Delta \omega_0 \). The badly defined energy \( E_0 = h \omega_0 \) exhibits the peculiar GTL- HCW behaviour, with the maximum possible amplitude and the lowest allowed phase difference \( \Delta \phi_0 = 2 \pi \) around a closed electronic loop, only if \( \hat{Q}_0 = \Phi_0 = 1 \). That is, in the presence of a GTL- HCW phase coherence interval:

\[ \omega_0 \tau_{coh}^0 = \pi \] which defines the magnetic flux resonance condition when the value of \( \omega_0 \) approaches the threshold frequency \( \omega_0^0 \) of a two entangled electronic states system. The photon-molecule interaction is then performed in the presence of two well defined interaction constants: \( \hat{Q}_0 = \exp[S_{\Delta \omega_0} - S_{\Delta \omega_0}] \) and \( \hat{C}_0 = \Phi_0[f/Wb] / \phi_0^0 = \hbar / [eV \cdot fs] \). The interacting molecular electron experiences the monotonous time increase of the electric field:

\[ \mathcal{E}_{\Delta \omega_0}(t / \tau_{coh}) \mathcal{E}^* = \exp[-(\pi / \hat{Q}_0) (t / \tau_{coh})^2 - \bar{\hat{Q}} (t / \tau_{coh})] \] and the space fixed polarity of the induced magnetic field provides to the position of the electric charge a nonlocal behaviour around a closed electronic loop. Such a Molecule Fixed Frame (MFF) confined electron must also give rise (during the short interaction time interval \( \tau_{int}^0 \)) to a position- momentum uncertainty dependence of a quantum particle trapped into a one dimensional box. The case \( \hat{Q}_0 = \Phi_0 / 4 \) is associated with the conditions:

\[ \omega_0 \tau_{coh}^0 < \Delta \phi_0 \) and \( E_0 \tau_{coh}^0 / \epsilon < \Phi_0 \). It is not allowed to provide any stable closed electronic loop.
Let us further remark that the value $\hat{\Phi} = 4$ in figure 1 is associated with multiple oscillations of the electric field. That is, the overall induced magnetic field is averaged in practice down to a zero value. The predominance of the interacting electric field is therefore enhanced and the excited molecular electron ends as an ejected photoelectron to quit the molecular ion core. We may see in the following that the FMFQ- photoelectron is captured into a TEES bound potential at the fundamental state.

The photon is a zero rest mass particle. It is not localizable. Let us then assume that our interaction is operating value of the center frequency $\omega_0$ of a GTL-HCW optical photon wave packet is the electronic threshold frequency $\omega_0$ of a FMFQ preselected two entangled electronic states system. The transferred energy uncertainty $\Delta E$ overlaps coherently two indistinguishable electronic states and the joint effect of the electric and magnetic fields then gives rise to the formation of a 1D proteiform electronic loop carrying a fundamental magnetic flux quantum $\Phi_0$. The nonlocality property of the electric charge along a closed electronic loop of diameter $D_0$ is responsible for a quantized magnetic flux photoionization quenching law:

$$E_0^0 \Delta_0^0 = \sqrt{\kappa/\mu_0} \Phi_0$$

where the underlying magnetic flux property $(\Delta E_0/\mu_0 \omega_0(\omega_0)) \times \exp(-\Delta_0^0 - S_{\Delta_0}) = \Phi_0/[\Phi_0]$ rules the transfer of the temporal and spectral Shannon entropies into the time and frequency spaces. The interaction is performed in presence of two interaction constants $\hat{Q}_0$ and $\hat{C}_0$ and the time dependence law $[\pi/\tau \delta_{c_0}] \sin^2[\pi/\tau \delta_{c_0}]$ (large dot points) matches rigorously well the single and monotonous time increase of $\text{Im} \left( E_{\Delta_0}(t/\tau_{c_0})/E_0^0 \right)$.

Let us consider a semi-classical approach where the distant molecular electron experiences the cyclotropic regime of an extension of this loop relative to the dimension of the positively charged molecular ion core, we will therefore contains an intrinsic uncertainty as large as $\Delta_0$. That is, the overall induced magnetic field $E_0^0 \Delta_0^0$ is averaged in practice down to a zero value. The predominance of the interacting electric field is therefore enhanced and the excited molecular electron experiences the cyclotropic regime of an optical GTL-HCW photon wave packet. The quantum mechanical description of this process will be developed in section 4.

2. The photon is a zero rest mass particle. It is not localizable. Let us then assume that our interaction is centered at time $t = 0$ and the interacting molecule is located at the position $x = 0$. We introduce the dimensionless time variable $\hat{t} = t/\Delta_0^0$ which can be then identified with the dimensionless propagation variable $\hat{x} = x/\Delta x_0$. We set for instance: $x = ct$ and $\Delta x_0 = c \Delta t_0$. The position of the interacting molecule therefore contains an intrinsic uncertainty as large as $\Delta x_0$. The electric field (linearly polarized along the z-axis) of a GTL-HCW photon wave packet is written:

$$[\mathcal{E}_{\Delta z}(\hat{x})]_{\Phi_0} = E_0^0 \exp \left[ -\frac{\pi}{4} \hat{Q}_0 \right] \exp \left[ -\frac{1}{2} \Delta_0^0/\Delta t_0 \left( \hat{x} - i \hat{Q}_0 \right)^2 \right] \varphi_z$$

The index $\Phi_0$ indicates the presence of the FMFQ- resonance condition. That is, the FWHM width $\Delta_0^0$ has been prepared in terms of some specific electronic threshold transition frequency $\omega_0^0$. In absence of external charges and currents the Maxwell equations, where the electric and magnetic fields stand as wave functions, first provide: $[\partial E_{\Delta z}(\hat{x})/\partial \hat{x}]_{\Phi_0} = [\partial E_{\Delta z}(\hat{x})/\partial \hat{z}]_{\Phi_0}$. The average induced magnetic field $\mathcal{E}_{\Delta z}^{\text{ind}} = B_{\Delta z}^{\text{ind}} \varphi_z$, then casts into the following expression:

![Figure 1. Cycloidal trend of the GTL-HCW limit. In presence of the magnetic flux resonance condition $E_0^0 \Delta_0^0/\omega_0 = \Phi_0$, the operating value of the center frequency $\omega_0$ of a GTL-HCW optical photon wave packet is the electronic threshold frequency $\omega_0^0$ of a FMFQ preselected two entangled electronic states system. The transferred energy uncertainty $\Delta E$ overlaps coherently two indistinguishable electronic states and the joint effect of the electric and magnetic fields then gives rise to the formation of a 1D proteiform electronic loop carrying a fundamental magnetic flux quantum $\Phi_0$. The nonlocality property of the electric charge along a closed electronic loop of diameter $D_0$ is responsible for a quantized magnetic flux photoionization quenching law: $E_0^0 \Delta_0^0 = \sqrt{\kappa/\mu_0} \Phi_0$, where the underlying magnetic flux property $(\Delta E_0/\mu_0 \omega_0(\omega_0)) \times \exp(-\Delta_0^0 - S_{\Delta_0^0}) = \Phi_0/[\Phi_0]$ rules the transfer of the temporal and spectral Shannon entropies into the time and frequency spaces. The interaction is performed in presence of two interaction constants $\hat{Q}_0$ and $\hat{C}_0$ and the time dependence law $[\pi/\tau \delta_{c_0}] \sin^2[\pi/\tau \delta_{c_0}]$ (large dot points) matches rigorously well the single and monotonous time increase of $\text{Im} \left( E_{\Delta z}(t/\tau_{c_0})/E_0^0 \right)$.

$$E_0^0 \Delta_0^0/\omega_0 = \Phi_0$$

$$E_0^0 \Delta_0^0 = \sqrt{\kappa/\mu_0} \Phi_0$$

$\Delta_0^0$ of a GTL-HCW optical photon wave packet is the electronic threshold frequency $\omega_0^0$ of a FMFQ preselected two entangled electronic states system. The transferred energy uncertainty $\Delta E$ overlaps coherently two indistinguishable electronic states and the joint effect of the electric and magnetic fields then gives rise to the formation of a 1D proteiform electronic loop carrying a fundamental magnetic flux quantum $\Phi_0$. The nonlocality property of the electric charge along a closed electronic loop of diameter $D_0$ is responsible for a quantized magnetic flux photoionization quenching law: $E_0^0 \Delta_0^0 = \sqrt{\kappa/\mu_0} \Phi_0$, where the underlying magnetic flux property $(\Delta E_0/\mu_0 \omega_0(\omega_0)) \times \exp(-\Delta_0^0 - S_{\Delta_0^0}) = \Phi_0/[\Phi_0]$ rules the transfer of the temporal and spectral Shannon entropies into the time and frequency spaces. The interaction is performed in presence of two interaction constants $\hat{Q}_0$ and $\hat{C}_0$ and the time dependence law $[\pi/\tau \delta_{c_0}] \sin^2[\pi/\tau \delta_{c_0}]$ (large dot points) matches rigorously well the single and monotonous time increase of $\text{Im} \left( E_{\Delta z}(t/\tau_{c_0})/E_0^0 \right)$.
\[ B^{\text{ind}}_{\Delta \lambda i} = \int_0^{\chi_{\text{coh}}^0} \left[ \frac{\partial B_{\Delta \lambda i}(\chi)}{\partial \chi} \right] d\chi = -\frac{\mathcal{E}_0^0}{c} [1 + \exp(-\Delta \omega_0 \tau_{\text{coh}})] \]  

We have introduced the space coherence length \( \chi_{\text{coh}}^0 = c \tau_{\text{coh}}^0 \). The upper integral bound shows that the interaction is delimited by the dimensionless constant: \( \chi_{\text{coh}}^0 = \tau_{\text{coh}}^0 = |g_0| \). We will make a systematic use of properly defined dimensionless quantities \( \mathcal{D} \) all over the development of the present work. Since the value of \( \exp(-\Delta \omega_0 \tau_{\text{coh}}) = [\chi_{\text{coh}}^0]^3 \) in equation (3) is small compared to unity we can assume into the semi-classical description the presence of a half-cycle wave plane behaviour. For the optical energies \( E_{eg}^0 \) the interacting molecular electron experiences indeed (at the scale of a single molecule) a uniform magnetic field of fixed space polarity. Although the energy formula of Rydberg series is a result of a hydrogen-like atom structure, Rydberg states [13, 14] are also present in molecules which can then attain diameters considerably larger than the typical diameter \( D_0^0 \approx 10^{-9} \) m of a molecule. At the Rydberg limit, any isolated neutral molecule behaves like a hydrogen-like atom.

The joint effect of the GTL-HCW electric and magnetic fields on the excited molecular electron and the infinite mass hydrogen-like ion core involves the presence of quantized constraints. First, the induced magnetic field effect in presence of the quantized nature of both, the angular momentum and the photon-induced magnetic flux, must satisfy the following two constraints:

\[ \Phi_0 = \pi [R_{eg}^0]^2 |B^{\text{ind}}_{\Delta \lambda i}|; \quad R_{eg}^0 = \left[ n_{eg}^0 \right]^2 \frac{\hbar}{m_{eg}c\alpha} \]  

(4a)

\( R_{eg}^0 \) is the radius of the induced electronic loop and \( n_{eg}^0 \) is the quantum number of the corresponding Rydberg state. Given the wave plane behaviour of the GTL-HCW optical photon wave packet at the scale of the interacting molecule, we can further relate the spot area \( \pi [R_{eg}^0]^2 \) to the interaction peak power \( I_{\Delta \lambda i}^0 = \frac{1}{2}cc_{eg}^0 |E_{eg}^0|^2 \) (responsible for the spatial separation of the excited molecular electron and the infinite mass ion core) as follows:

\[ I_{\Delta \lambda i}^0 = \frac{E_{eg}^0}{\pi [R_{eg}^0]^2} \Delta \lambda_0; \quad \frac{E_{eg}^0}{c} \tau_{\text{coh}}^0 = \Phi_0 \]  

(4b)

We might first ask whether the increasing peak power of an absorbed GTL-HCW photon wave packet under the FMFQ resonance condition results into an ejected photoelectron or instead, it can only provide a photon-induced proteiform circular electronic loop trapped into a bound state. The synergetic effect of the GTL-HCW electric and magnetic fields provides for instance the following relations:

\[ |B^{\text{ind}}_{\Delta \lambda i}| \Delta \lambda_0 = \eta \]  

(5a)

\[ I_{\Delta \lambda i}^0 [\Delta \lambda_0]^4 = \frac{\hbar \kappa}{c^2} \]  

(5b)

\( \eta = \mu_0 c \) is the free space impedance and \( \eta \kappa \) can be thus observed as a free space magnetic flux coupling constant. \( \kappa \) is the fine structure constant (See footnote 1). The quantities \( |B^{\text{ind}}_{\Delta \lambda i}| \) and \( [\Delta \lambda_0]^4 \) are conjugate each other and, this is also the case for the quantities \( I_{\Delta \lambda i}^0 \) and \( [\Delta \lambda_0]^4 \). We may observe in equation (5) a better and better decreasing intrinsic uncertainty \( \Delta \lambda_0 = c \Delta \lambda_0 \) for the spatial dispersion of the excited molecular electron wavefunction when the photon’s average induced magnetic field \( |B^{\text{ind}}_{\Delta \lambda i}| \) becomes more and more important. Such an overall effect, written in equation (5.1) in terms of the interaction peak power, follows a fourth power law. It is then worth noting that the GTL-HCW time dependence, drawn in figure 1 with large dot points, exhibits equally well a singularly simple fourth power law. Namely: \( \Delta \chi_{\Delta \lambda i} (t/t_{\text{ext}}^0) / \Delta \chi_{\Delta \lambda i}^0 = [\Delta \phi_{\Delta \lambda i} (t/t_{\text{ext}}^0)] \sin^4[\Delta \phi_{\Delta \lambda i} (t/t_{\text{ext}}^0)] \). The increasing peak power of a GTL-HCW photon wave packet under the FMFQ resonance condition therefore reduces drastically the effective value of \( \Delta \chi_{\Delta \lambda i} \). Instead of ionization, the excited molecular electron is drawn into the lower and lower Rydberg states. We will call this shrinking process: FMFQ syriknosis effect. The diameter \( D_{eg}^0 \) of the proteiform circular loop is fixed exclusively by the TEES-threshold energy \( E_{eg}^0 \) so that the conjugate quantities \( E_{eg}^0 \) and \( D_{eg}^0 \) counterbalance the presence of a quantized fundamental magnetic flux quantum:

\[ E_{eg}^0 D_{eg}^0 = \sqrt{\frac{\hbar c}{\mu_0 \Phi_0}} \]  

(6a)

A preionized TEES-threshold energy provides a low Rydberg state whereas a low TEES-threshold energy draws the excited molecular electron into a high Rydberg state. The quantum number \( n_{eg}^0 \) obeys indeed the following relation:
The constant $\sqrt{4\pi\alpha}$ is the elementary electric charge $e_{\text{env}}$ measured in natural (Lorentz–Heaviside) units [15]. The Chirelson’s bound constant [16] $C_{\text{QC}}^{\text{max}} = \sqrt{2}$ is a manifestation of the photon-induced nonlocal behaviour of the excited molecular electron and its maximum quantum correlation with the molecular ion core. We may see indeed in the following that a nonlocal structure is present into the photon- induced TEES system. The subsequent rotational motion of the ion core and the induced dynamics of the excited molecular electron are both associated with a photon-induced diffraction pattern.

The TEES- structure involves the presence of the key interaction constants $\hat{Q}_0$ and $\hat{C}_0$. Let us remind that the threshold energy condition $E_{eg}^0 < \Delta E_0$ defines a two entangled electronic state system which is intimately related with the cyclotrophic effect of the GTR- HCW electric and magnetic fields. We must therefore characterize, beside the displacement from equilibrium of the molecular electron’s charge, also, the simultaneous presence of a photon- induced fundamental magnetic flux quantum. Such a badly defined electric dipole can be conceptually represented by the expression: $\Delta \mu_{eg}^0 = e R_{eg}^0$, i.e., there is no any preferred space orientation for this electric dipole vector within the plane $\{ \hat{e}_x, \hat{e}_z \}$ containing the proteiform photon- induced electronic loop. Equation (6a) then provides: $\Delta \mu_{eg}^0 = q_p \Delta \omega_0$ where, the proportionality constant $q_p = e/\sqrt{4\pi\alpha}$ is the Planck electric charge [17]. We can now readily associate the intrinsic indeterminacy in the space orientation of the induced electric dipole $\Delta \mu_{eg}^0$ with the presence of a delocalized excited molecular electron which, for reasons of symmetry, is not allowed to quit the ion core and propagate in space as a free photoelectron. It is trapped instead as a FMFQ photoelectron into a Rydberg state de.

3. Photon-induced nonlocal TEES structure

In the presence of the FMFQ resonance condition a photon- induced nonlocal TEES structure is implemented into the free rotating molecule. The spectral domain $\Delta E_{\text{N}_0}$ of the TEES system shown in figure 2 contains the first $N_0$ rotational states of a rigid rotor [18] which then belong in an indistinguishable way to both entangled electronic states $|g\rangle$ and $|e\rangle$. These states can thus evolve in time only as a coherent superposition of $\Omega_{\text{N}_0} = N_0(N_0 - 1)/2$ spectral lines in absorption and emission. The indistinguishability of the entangled electronic states $|g\rangle$ and $|e\rangle$ then results in the presence of $\Omega_{\text{N}_0} = I_0 \Delta \omega_{\text{N}_0}/\hbar$ closed magnetic field lines. Given the quantized nature of the magnetic flux and the presence of a FMFQ resonance condition, these magnetic field lines can only operate as the harmonics ±$h \Phi_0$ of the fundamental magnetic flux quantum $\Phi_0$. Equation (6a) first shows that the principal $l = 1$ closed electronic loop carries a fundamental magnetic flux quantum fixed exclusively by the TEES threshold energy $\Delta E_{eg}^0$. The residual closed magnetic field lines of the indistinguishable rotational states are then expected to operate in the TEES system as equiprobable magnetic flux vortices of opposed helicity condensing into a minimized magnetic flux fluctuation. We may further observe that the TEES structure is the issue of the transferred Shannon entropy in the time and frequency spaces and that of the implemented energy uncertainty. Let us remark for instance that the maximum possible energy uncertainty $\Delta E_0$ and the corresponding threshold energy $E_{eg}^0$ (which defines the FMFQ resonance condition) are closely related through the relation: $\Delta E_0 = \ln(E_{eg}^0/E_{eg}^0)$. Moreover, the corresponding spectral domain $\Delta E_{\text{N}_0}$ obeys the property: $I_0 \Delta E_{\text{N}_0} = h^2 \Omega_{\text{N}_0}$ where $I_0$ denotes the moment of inertia of the interacting molecule. We may see
in the following that these physical quantities define rigorously well the photon-induced nonlocal structure of the TEES system pictured in figure 2.

1. Since the TEES structure is carried by a free rotating molecule, an induced fundamental angular frequency \( \omega_{\text{rot}}^{\text{coh}} \) must be present in the subsequent rotational motion. This motion is shared equally well by the two entangled electronic states and, also, by their \( N_0 \) indistinguishable rigid rotor rotational states. Any frequency in the TEES system must occur as a harmonic frequency of some fundamental frequency \( \omega_{\text{rot}} \). The indistinguishable spectral lines in absorption and emission [18] must then obey the following relations: \( \omega_{\text{rot}}^n = (n + 1) \omega_{\text{rot}}^{\text{coh}} \) and \( \omega_{\text{rot}}^n = n \omega_{\text{rot}}^{\text{coh}} \). Since the \( N_0 \) rotational states are overlapped coherently by the maximum possible energy uncertainty \( \Delta E_0 \), we must impose the following nonlocal condition:

\[
\sum_{n=1}^{N_0-1} \hbar \omega_n^n = \sum_{n=0}^{N_0-2} \hbar \omega_n^n = \Delta E_0
\]

Doing so, we obtain:

\[
\omega_{\text{rot}}^{\text{coh}} = \frac{\Delta \omega_0}{\Omega_{N_0}} = \frac{\hbar}{I_0} \left( 1 - \frac{1}{Q_0} \right) = \frac{\hbar}{I_0} \sum_{j=0}^{\infty} \exp[-i(\Delta_0 - \Delta_0^2)]
\]

(8a)

\( \omega_{\text{rot}}^{\text{coh}} \) must be observed as the upper bound of the induced angular frequency of the molecular ion core. Let us say, \( |I_0 \omega_{\text{rot}}^{\text{coh}} / \hbar | < 1 \) with \( B \) the rotational constant. The index \( I \) runs over the infinite-like number \( \Omega_{N_0} \) of closed magnetic field lines experienced by the excited molecular electron. The temporal-spectral Shannon entropy difference \( \Delta S_0 = S_{\Delta 0} - S_{\Delta 0\text{max}} = \ln[\Delta E_0 / E_{\text{rot}}^{\text{coh}}] \) stands in the interaction process as the ending value of the entropic uncertainty transfer and, this is equally well the ending step of the energy uncertainty transfer. That is to say, the photoionization quenching observed in equations (6a)–(6b) must be necessarily associated with the presence of a bound potential formed from the transferred entropic and energy uncertainties into a TEES system. It is indeed of relevant importance to introduce under the above conditions a temporal-spectral Shannon entropy variable: \( \tilde{s} = s - \Delta S_0 = \ln[\Delta E / \Delta E_0] \) which increases from \(-\infty\) to \(0\) as \( \Delta E \) goes from \(0\) up to \(\Delta E_0 \). Given that the advent of the transferred amount of the dimensionless energy uncertainty \( \theta_0^{(0)} \) occurs with the ending step, \( q_0^{(0)}(\Delta S_0) = [I_0 \omega_{\text{rot}} / \hbar]_{\text{max}} [S_{\Delta 0} - S_{\Delta 0\text{max}}] \), we introduce the following entropic plus time-energy uncertainty transfer function:

\[
\hat{f}^{(0)}_{\text{TEES}}(\tilde{s}) = q_0^{(0)}(\Delta S_0) = \left[ \frac{\Delta E}{\Delta E_0} + \frac{(\Delta E / \Delta E_0) \ln(\Delta E / \Delta E_0)}{1 - (\Delta E / \Delta E_0)} \right]
\]

(9a)

The function \( \hat{f}^{(0)}_{\text{TEES}}(\tilde{s}) \) is expected to contain all the necessary information that can be known about the photoionization quenching when the constraint of the FMFQ resonance condition is present. We have introduced in equation (9a) the TEES generic function \( q_0^{(0)}(\Delta S_0) \) where the variable \( s = \ln(\Delta E / E_{\text{rot}}^{\text{coh}}) \) spans the domain \([-\infty, S_{\Delta 0} - S_{\Delta 0\text{max}}]\). Namely, \( q_0^{(0)}(\Delta S_0) \) stands as the characteristic power series of a multiplicative sequence.
\[ q_{\text{TEES}}^{(0)}(s) = \frac{s}{1 - e^{-s}} = -\frac{\theta_{\text{TEES}}^{(0)}(s) \ln \theta_{\text{TEES}}^{(0)}(s)}{1 - \theta_{\text{TEES}}^{(0)}(s)} \] (9b)

The nonnegative values of the function \( \theta_{\text{TEES}}^{(0)}(s) = e^s \) therefore span the domain \([0, \hat{Q}_0^{-1}]\) where the upper bound is fixed by the interaction constant \( \hat{Q}_0^{-1} = \exp[S_{\Delta t_0} - S_{\Delta w_0}]\) of equation (1). It is of crucial importance to notice the fact that this constant verifies the fundamental property:\[ \hat{Q}_0^{-1} [\hat{\sigma}_0 \hat{\rho}_0]_{\text{min}} = [\hat{g}_e]_0^2 \] with 
\[ [\hat{\sigma}_0, \hat{\rho}_0]_0 = \sqrt{[\zeta(2) k^2 - 1]/2} \quad \text{the position- momentum uncertainty relation of a quantum particle confined in a one dimensional box.} \] \[ \zeta(2) = 2 \pi \] the electron's Schrödinger- Dirac ratio \([6, 7, 20]\). The angular momentum state \([\hat{\sigma}_0, \hat{\rho}_0]_{\text{min}} = [\hat{\sigma}_0, \hat{\rho}_0]_0 \) therefore denotes the fundamental angular momentum state of a 1D confined FMFQ molecular photoelectron whose magnetic moment is characterized by the fundamental constant \( \hat{g}_e \). We may see in the following that the TEES generic function \( q_{\text{TEES}}^{(0)}(s) \) contains relevant information on the photon- induced rotational motion of the ion core and, also, in the position- momentum and time- energy uncertainties of the trapped FMFQ molecular photoelectron. The intrinsic uncertainties in the position- momentum and the time-energy experienced by the excited molecular electron can not be separately measured. This is because these uncertainties are intimately related with the transferred entropic uncertainty in the time and frequency spaces.

2. Let us first illustrate a hidden intrinsic property of equations (8a)–(8b):

\[ S_{\Delta t_0} - S_{\Delta w_0} = \ln \left[ \sum_{n=0}^{N_0-2} \left( E_n^{\text{coh}} / E_n^{\text{me}} \right) \right] = \ln \left[ \sum_{n=1}^{N_0-1} \left( E_n^{\text{coh}} / E_n^{\text{me}} \right) \right] \] (10a)

Equation (10a) illustrates the fact that the ending value of the (maximum possible) entropic uncertainty difference \( \Delta S_0 \) is intimately related with the induced coherent superposition of the \( N_0 \) rotational states of the TEES system. The indistinguishable lines in absorption and emission are equiprobable and, the upper bound of the induced angular frequency introduced in equation (8b) exhibits the following property:

\[ \frac{I_{v_0}^{\text{coh}}(\omega)}{h} = \frac{q_{\text{TEES}}^{(0)}(S_{\Delta t_0} - S_{\Delta w_0})}{S_{\Delta t_0} - S_{\Delta w_0}} = \frac{\Delta E_0 [eV]}{\Delta E_0 [eV]} \frac{\theta_{\text{TEES}}^{(0)}(s)}{\theta_{\text{TEES}}^{(0)}(s)} \] (10b)

Equation (10b) further reveals the inertial effect of the molecular ion core. This effect emerges naturally from the function \( q_{\text{TEES}}^{(0)}(s) \) and the properties of the ending value \( S_{\Delta t_0} - S_{\Delta w_0} \) of the variable \( s \). The very first fundamental property of this function concerns indeed the transfer of the entropic uncertainty in presence of the inertial effect of the molecular ion core; this effect occurs as an integral action of the TEES generic function \( q_{\text{TEES}}^{(0)}(s) \) as follows:

\[ S_{\Delta t_0} + S_{\Delta w_0} \sim \int_{-\infty}^{\infty} [S_{\Delta w_0} - S_{\Delta w_0}] \hat{g}_e q_{\text{TEES}}^{(0)}(s) \, ds \sim \int_{-\infty}^{1/2} \frac{\nu^{\text{coh}}}{2 B} \left[ \frac{\Delta E_0 (s)}{\Delta E_0 (s)} \right]_{\text{min}} \] (10c)

The lower bound of equation (10c) must be identified with \( \left[ I_{v_0}^{\text{coh}} / 2 B \right]_{\text{min}}^2 \). The rotational motion is initiated together with an entropic uncertainty and angular momentum transfer, at the GTL-HCW limit where both, the position and impulse distributions still allow for a quantitative representation. Since the transferred angular momentum is performed in the presence of an observable associated with an angular position, the corresponding uncertainty relation must obey the inequality: \( \Delta \theta_0 \Delta \theta_0 \geq \hbar / 2 \). The index \( '0' \) is assigned here to the angular momentum component measured along the induced magnetic field \( \vec{B}^{\text{ind}} \). Moreover, the transferred entropic uncertainty \( S_{\Delta t_0} + S_{\Delta w_0} \) is the minimum allowed and the upper bound is only 0.1% greater than \( S_{\Delta t_0} + S_{\Delta w_0} \). Since the induced rotational motion obeys the property: \( I_{v_0}^{\text{coh}} \geq \hbar \theta_{\text{min}}^0 \), the constant \( \hbar \hat{\sigma}_0^0 = \hbar / 2 (S_{\Delta t_0} + S_{\Delta w_0})^2 \) occurs in the interaction process as a “molecular ion core inertial threshold.” The threshold uncertainty state \( [\Delta \theta_0 \Delta \theta_0]_{\text{min}} = \hbar / 2 \) explains the presence of the factor \( \hbar / 2 \) into the ion core inertial threshold. We may conclude by saying that the lower and upper bounds of equation (10c) define an average inertial effect: \( \Delta \theta_0^0(\omega) = \hbar \sqrt{\hat{\sigma}_0^0 / (1 - \hat{Q}_0)} \) confined within a singularly sharp inertial window. As expected, the induced angular frequency \( \omega^{\text{coh}} \) is the lowest allowed, and \( (I_{v_0}^{\text{coh}} / h) \) depends exclusively on the fundamental unit of angular momentum and the transferred temporal and spectral Shannon entropies of the absorbed GTL- HCW optical photon wavepacket. The explicit dependence of \( \omega^{\text{coh}} \) on the photon- induced fundamental state will be considered in more details at the end of this work.

3. The structural properties of the induced TEES system emerge naturally from the TEES function \( q_{\text{TEES}}^{(0)}(s) \). We can ask for instance; how the entropic and energy uncertainties are related to the position- momentum uncertainty state of the destabilized excited molecular electron along the 1D proteiform photon- induced electronic loop? The very first requisite condition for this should be the confinement of a FMFQ photoelectron into a one dimensional box. The TEES function \( q_{\text{TEES}}^{(0)}(s) \) provides indeed the Riemann zeta [19] integral:
The overall constant \( \int_{-\infty}^{0} \tilde{F}^{(0)}_{\text{TEES}}(\hat{\mathcal{S}}) \, d\hat{s} \) is pure imaginary; this constant relates intrinsically the lowest allowed uncertainty state of the space orientation of the rotationally frozen molecular ion core to the lowest allowed photon-induced position-momentum uncertainty of the excited molecular electron along the 1D protein domain. Given that the imaginary unit is the generator of rotations in the complex plane, the angles \( \beta \) and \( \beta' \) of figure 2 must rotate in a plane with opposed helicities so that their initial orientation \( \beta_0 \) is kept well defined in time and becomes a constant of motion (see this for this in the quantum mechanical description of the process). The function \( \tilde{F}^{(0)}_{\text{TEES}}(\hat{\mathcal{S}}) \) contains relevant information.

We may observe in figure 3 the presence of a photon-induced FMFQ potential where the interaction constants \( \tilde{Q}_0 \) and \( \tilde{C}_0 \) of equations (1) and (7) emerge naturally as the fundamental key dimensionless constants of the FMFQ implementation process. The properties of the bound potential reflect the minimized energy, entropic uncertainty, and angular momentum state of the trapped FMFQ photoelectron. They are all defined together with the formation of the coherent superposition of the TEES rotational states. These properties are characterized by the following three expressions:

\[
\Delta \hat{S}_{\text{FWHD}}^0 = i \hat{Q}_0^{-1} \frac{\Delta \hat{H}_{\text{FWHD}}}{\hat{E}_{\text{FWHD}}} = \frac{1}{\hat{Q}_0} \frac{\Delta \hat{H}_{\text{FWHD}}}{\hat{E}_{\text{FWHD}}}
\]

\[
\int [\Delta E / \Delta E_0 ] \ln \left( \frac{\Delta \hat{H}_{\text{FWHD}}}{\hat{E}_{\text{FWHD}}} \right)
\]

\[
\Delta \hat{S}_{\text{FWHD}}^0 = \frac{1}{\hat{Q}_0} \frac{\Delta \hat{H}_{\text{FWHD}}}{\hat{E}_{\text{FWHD}}}
\]

Figure 3. FMFQ bound potential. During the interaction time interval \([-\delta_{\text{FWHD}}, +\delta_{\text{FWHD}}]\) the molecular electron experiences the transferred energy uncertainty \( \Delta E \) increasing from 0 up to the ending value \( \Delta E_{\text{FWHD}} \). The variable \( s = \ln(\Delta E / \Delta E_0) \) then spans the domain \([-\infty, \Delta S_{\text{FWHD}} - \Delta S_{\text{FWHD}}]\), where \( \Delta S_{\text{FWHD}} \) defines into the time and frequency spaces the time-energy and entropic uncertainty transfer variable: \( s = \ln(\Delta E / \Delta E_0) \). The captured FMFQ photoelectron ends into a bound potential \( \tilde{F}^{(0)}_{\text{TEES}}(\hat{\mathcal{S}}) \) where the intrinsic uncertainty transfer process implies the presence of a fundamental magnetic flux quantum; i.e., \( \Phi_{\text{FWHD}} / \sqrt{2E_{\text{FWHD}}} = \Delta E_{\text{FWHD}} / \hat{E}_{\text{FWHD}} \). The minimum of the potential is fixed by the FMFQ photoelectron’s position-momentum uncertainty \( [\sigma_{\mathcal{S}} \sigma_{\mathcal{E}}]_{\text{min}} = \frac{\Phi_{\text{FWHD}}}{\sqrt{2E_{\text{FWHD}}} \Delta S_{\text{FWHD}}} \) along a 1D electronic loop where the quantized magnetic flux obeys the bound potential depth law: \( \Phi_{\text{FWHD}} = \Phi_{\text{FWHD}}^{\text{coh}} = \frac{\Phi_{\text{FWHD}}}{\sqrt{2E_{\text{FWHD}}} \Delta S_{\text{FWHD}}} \). The fundamental state \( [\sigma_{\mathcal{S}} \sigma_{\mathcal{E}}]_{\text{min}} \) and the Full Width at Half Dipp \( \Delta S_{\text{FWHD}}^0 \) of this potential then provide the explicit interdependence of the key interaction constants \( \tilde{Q}_0 \) and \( \tilde{C}_0 \). This is, \( \tilde{Q}_0 \left[ \sigma_{\mathcal{S}} \sigma_{\mathcal{E}} \right]_{\text{min}} = \frac{\Phi_{\text{FWHD}}}{\sqrt{2E_{\text{FWHD}}} \Delta S_{\text{FWHD}}} \).
Since \( \hat{s} = \ln(\Delta E / \Delta E_0) \), and thus \( [\Delta E / \Delta E_0]_{\text{min}} = \exp[\hat{s}] \), the energy uncertainty observed at the minimum of the potential can be readily associated with the presence of the invariant point \( \hat{Q}_0 \) in the time and frequency spaces; i.e., \( \sqrt{[\Delta E / \Delta E_0]_{\text{min}}} \) is drawn into the minimized value of \( \pi \text{Var} (|\psi|^2) \). Equations (12a)--(12c) further reveal the presence of several well defined overall properties; i.e., the geometric mean of \( \Delta s_{\text{FWHD}} \) and \( s_{\text{min}}^{0} \) matches the interaction constant \( \hat{Q}_0^{-1} \) of equation (1) and, the geometric mean of the dimensionless position-momentum uncertainty \( [\hat{s}_x, \hat{s}_p]_{\text{min}} \) and the interaction constant \( \hat{Q}_0^{-1} \) matches the Schwinger’s correction constant \( [6–8] \alpha / \Delta \phi_0 \). More precisely:

\[
\sqrt{[\hat{s}_x, \hat{s}_p]_{\text{min}} \hat{Q}_0^{-1}} = \frac{\Delta E_0 [eV] \sigma_0 \tau_0 [fs]}{\Phi_0 [Wb]} \tag{13a}
\]

\[
\sqrt{[\text{FWHD}]} = \frac{\Delta E_0 [eV] \sigma_0 \tau_0 [fs]}{\Phi_0 [Wb]} \tag{13b}
\]

The joint effect of the time-energy and position-momentum uncertainties in the photon-induced FMFQ loop therefore occurs as a necessary condition for the formation of the minimum \( s_{\text{min}}^{0} \) into the potential of figure 3. The minimum of this bound potential can be also apprehended by the presence of the lowest allowed phase difference \( \Delta \phi_0 = \omega_0 \tau_0 \int_{\text{int}} = k_0 s_{\text{FWHD}} \) around a closed electronic loop; i.e., we observe the fundamental state property: \( \Phi_0 s_{\text{min}}^{0} = \Delta \phi_0 \). This minimized state is consistent with the Aharonov–Bohm effect [21] which has been observed in the past, more generally, in superconducting and non-superconducting systems as well. The underlying mechanism of this effect is the coupling of the electromagnetic potential with the complex phase of the charged particle’s wavefunction. The phase of the complex quantum mechanical wavefunction is then allowed to change continuously from some value \( \phi_0 \) to the value \( \phi_0 + \Delta \phi_0 \) as one goes around the loop and ends into the same point. The value \( l = 1 \) is therefore associated with the lowest allowed phase difference along a stable closed electronic loop. We may see in the following that the magnetic flux modes \(|l| \geq 2\) can only result in a minimized magnetic flux fluctuation at the minimum of a bound potential. Furthermore, given that the GTL-HCW photon-molecule interaction is performed in the presence of the constraints: \( \Delta \omega_0 \Delta f_0 = 4 \ln 2 \) and \( 0 < \omega_0 < \Delta \omega_0 \), and the center frequency \( \omega_0 \) is only badly defined in the interaction process, we can introduce the following average amount of implemented information entropy:

\[
\langle \hat{H}_{\text{HCMW}}^{\text{GT}} \rangle = \frac{1}{\Delta \omega_0} \int_{\omega_0}^{\Delta \omega_0} \omega_0 \log \left( \frac{\omega_0}{\Delta \omega_0} \right) d\omega_0 = \frac{\hat{C}_0}{\Delta E_0 [eV] \Delta \phi_0 [fs]} \tag{14a}
\]

It is then clear that the presence of the minimum \( s_{\text{min}}^{0} \) and the presence of the average information entropy \( \langle \hat{H}_{\text{HCMW}}^{\text{GT}} \rangle \) into the photon-induced FMFQ potential occur together as a simple manifestation of the presence of the Schwinger–Dirac fundamental constant \( \hat{g}_e \), into the implemented fundamental magnetic quantum flux. We observe for instance the following dimensionless circular cross section of radius \( \hat{g}_e \) times the implemented amount of average information entropy:

\[
[\pi \hat{g}_e] \langle \hat{H}_{\text{HCMW}}^{\text{GT}} \rangle = s_{\text{min}}^{0} \tag{14b}
\]

Equations (13b) and (14b) are in fact the issues of one and the same dimensionless law:

\[
[\hat{s}_x, \hat{s}_p]_{\text{min}} \Delta \phi_0 \int_{\text{int}} = \langle \hat{g}_e \rangle. \tag{14c}
\]

At the minimum of the TEES bound potential we therefore observe the presence of the syriknosis threshold constant: \( C_0 = \hat{s}_{\text{min}}^{0} \hat{P}_{\text{TEES}}(\hat{s}) \).

Since the appropriate units of energy and time in the interaction process are the eV and fs, and \( E_{\text{gap}}^{0} [eV] \) and \( \tau_0 [fs] \) are conjugate each other, the FMFQ syriknosis threshold constant \( \hat{C}_0 \) of equation (7) allows one to write the following syriknosis threshold criterion:

\[
E_{\text{gap}}^{0} [eV] \geq 1; \quad \tau_0 [fs] \leq \hat{C}_0 \Delta \phi_0 \tag{15a}
\]

The FMFQ resonance condition applied to the NO \((A^2 \Sigma^+ \rightarrow X^2 \Pi_{1/2})\) electronic transition [18] (where the TEES threshold energy is \( E_{\text{gap}}^{0} \approx 5.46 \text{eV} \) demands a GTL-HCW time width \( \Delta t_0 \approx 0.19 \text{fs} \). The paired syriknosis threshold criterion of equation (15a) is therefore fulfilled and equation (6b) then provides:
$$n^0_{eg} = 60 < \alpha^{-1}.$$ The diameter $D^0_{eg}$ of the photon-induced magnetic flux quantum $\Phi_0$ is given by $D^0_{eg} \approx 376 \ D^0_{fl}$ with $D^0_{fl} \approx 10^{-9}$ m the typical size of an interacting molecule and, the absorbed energy is therefore stored into a quasi-classical electronic loop.

b) For the preionized TEES threshold NO ($L = X$) we have $E^0_{eg} \geq 14.6$ eV $> 1$ eV and the FMFQ resonance condition demands a GTL-HCW time width $\Delta t_0 \approx 0.07$ fs. The paired syriknosis threshold criterion of equation (15a) is also fulfilled. We obtain: $n^0_{eg} = 36 < \alpha^{-1}$. The preionized state provides a lower Rydberg state than that fixed by a lower TEES threshold energy. Instead of an ejected photoelectron, the diameter $D^0_{eg}$ of the photon-induced magnetic flux quantum $\Phi_0$ now reduces into the value $D^0_{eg} \approx 139 \ D^0_{fl}$.

c) For the multiplet transition NO ($\chi^2_1 \Pi_{1/2} - \chi^2_1 \Pi_{3/2}$) where $|A| = 124$ cm$^{-1}$, the FMFQ resonance condition needs $\Delta t_0 \leq 68.9$ fs. We have: $r^0_{coh}[\hat{S}] / \pi = 43.9 \gg \hat{C}_0$ and $E^0_{eg}[eV] \approx 0.015 < 1$. Neither of the two FMFQ condition criteria is fulfilled. This results into ionization since: $D^0_{eg} \approx 136$ 516 $D^0_{fl}$ and $n^0_{eg} = 1180 \gg \alpha^{-1}$.

d) Let us finally consider the lowest TEES threshold Li$_2$ ($\chi^2_2 \Sigma_+^+ - \chi^2_2 \Sigma_+^+$) of the lithium dimer molecule where [18] $\rho^0_{eg} \approx 14.068$ cm$^{-1}$. The requisite value of the GTL-HCW time width is now $\Delta t_0 \approx 0.59$ fs. We have again: $r^0_{coh}[\hat{S}] / \pi = 0.38 < \hat{C}_0$ and $E^0_{eg}[eV] \approx 1.75 > 1$. The syriknosis threshold criterion is still valid. We obtain $n^0_{eg} = 105 < \alpha^{-1}$ and, the diameter of the photon-induced magnetic flux quantum is as large as $D^0_{eg} \approx 1169 \ D^0_{fl}$. The ionization quenching pertains even for the lowest TEES threshold energy.

5. Let us finally provide a physical interpretation for the presence of the TEES generic functions $q^{(0)}_{TEES}(s)$ and $\theta^{(0)}_{TEES}(s)$. The syriknosis threshold criterion of equation (15a) written in terms of the two interaction constants $\hat{Q}_0$ and $\hat{C}_0$ is: $\Delta E_0[eV] \geq \hat{Q}_0^{-1}$ and $r^0_{int}[\hat{S}] / \Delta \phi_0 \leq \hat{C}_0$. Provided that the first condition is fulfilled, and $\Delta E_0$ overlaps coherently some electronic transition $E^0_{eg}$, the choice of the time width $r^0_{coh}$ from the relation $E^0_{eg} \ r^0_{coh} / e = \Theta_0$ will necessarily satisfy the second condition. The crossed equality is in fact the FMFQ resonance condition: $r^0_{coh}[\hat{S}] = \Delta \phi_0$ where $r^0_{int}$ stands therein as the overall interaction time interval. If we further specify the time-energy uncertainty relation in terms of the appropriate physical units, the presence of the fundamental magnetic flux quantum into the photon-induced TEES system occurs again as a requisite condition; i.e., the quantized quantity $\Phi_0[Wb]$ occurs in the interaction process as the proportionality constant of the coherent superposition of the indistinguishable TEES rotational lines in absorption and emission:

$$\Delta E_0[eV] \ r^0_{int}[\hat{S}] = \Phi_0[Wb] \left[ \sum_{n=0}^{N_n} \left( \frac{E^0_n + E^0_{s,n+1}}{E^0_{eg}} \right) \right]$$

(15b)

The excited molecular electron is therefore drawn into the TEES potential of figure 3 at the lowest allowed state and can evolve in time over a Rydberg surface bound state only as a diffracted photoelectron. It is striking and with than that superposition of the indistinguishable TEES rotational lines in absorption and emission is the typical size of an interacting molecule and, the absorbed energy is therefore stored into a quasi-classical electronic loop.

$$\Delta E_0[eV] \ r^0_{int}[\hat{S}] = \Phi_0[Wb] \left[ \sum_{n=0}^{N_n} \left( \frac{E^0_n + E^0_{s,n+1}}{E^0_{eg}} \right) \right]$$

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The excited molecular electron is therefore drawn into the TEES potential of figure 3 at the lowest allowed state and can evolve in time over a Rydberg surface bound state only as a diffracted photoelectron. It is striking and with than that superposition of the indistinguishable TEES rotational lines in absorption and emission is the typical size of an interacting molecule and, the absorbed energy is therefore stored into a quasi-classical electronic loop.
4. Quantum mechanical description of the photon-induced FMFQ-implementation

The GTL-HCW photon-molecule interaction is initiated in the presence of the constraints: \( \Delta \omega_0 \Delta t_0 = 4 \ln 2 \) and \( 0 \leq \omega_0 \leq \Delta \omega_0 \). The badly defined center frequency \( \omega_0 \) of the photon wavepacket then occurs with the maximum possible intrinsic uncertainty \( \Delta \omega_0 \) and, the interaction process contains a average amount of information entropy \( S_{H-CW}^{\text{GTL}} \). We must then account for the fact that the interacting molecular electron experiences transition frequencies \( \omega_{eg} \) with an intrinsic uncertainty as large as \( \Delta \omega_0 \). Let us say, the molecular electron experiences the complex frequencies \( \tilde{\omega}_{eg} = \omega_{eg} + i \Delta \omega_{eg} \) instead of a well defined transition frequency \( \omega_{eg} \). We will thus consider the presence of complex levels \( \tilde{\omega}_{eg} = \omega_{eg} + i \Delta \omega_{eg} / 2 \) and \( \tilde{\omega}_{eg} = \omega_{eg} + i \Delta \omega_{eg} / 2 \), so that \( \tilde{\omega}_{eg} = \tilde{\omega}_{eg}^{*} \). Let us then assume in the time-dependent uniform electric field the presence of complex frequency components \( \tilde{\omega} = \omega + i \Delta \omega_0 \). Namely, \( \mathcal{E}_{Eg}^0(\omega_0, \omega, t) = \mathcal{E}_{Eg}^0(\omega_0, \omega) \exp[i\tilde{\omega}t] \), where the Fourier transform \( \mathcal{E}_{Eg}^0(\omega_0, \omega) = \mathcal{F}[\mathcal{E}_{Eg}^0(\omega_0, t)] \) represents the electric field component in frequency space \( \omega \).

\( \mathcal{E}_{Eg}^0(\omega_0, \omega) = \mathcal{E}_{Eg}^0 \exp[-2i\Delta \omega_{eg} \Delta t_0 \tilde{\omega}^2] \exp[i\tilde{\omega}t] \) is the \( \rho \) – polarized uniform electric field and, as previously, \( \tilde{\omega} = t / \Delta t_0 \) is the dimensionless time variable measured in units of \( \Delta t_0 = \Delta t_{\text{FWHM}}^{\text{GTL-HCW}} \).

1. We need to precise the expectation value \( \langle \Psi | \mu_{\rho} | \Psi \rangle \) of the photon-molecule interaction at the GTL-HCW limit from the time-dependent Schrödinger equation [23]:

\[
\frac{i \hbar}{\partial t} |\Psi\rangle_t = \left[ H_0 - \sum_{p} \mu_{p} \mathcal{E}_{\Delta \omega_{p}}(\omega_0, \omega) e^{i\tilde{\omega}t} \right] |\Psi\rangle_t \tag{16a}
\]

\[
|\Psi\rangle_t = c_{g}(t) \exp[-i\tilde{\omega}_{eg} t] |\psi\rangle + \sum_{eg \in c} c_{g}(t) \exp[-i\tilde{\omega}_{eg} t] |\psi\rangle \tag{16b}
\]

\( |\Psi\rangle_t \) is the time-dependent vector state accounting for the presence of the intrinsic uncertainty \( \Delta \omega_0 \) in the interaction process. The expectation value will then remain a real quantity provided that the property \( \langle \Psi | \mu_{\rho} | \Psi \rangle \) is the dimensionless time variable measured in units of \( \Delta t_0 = \Delta t_{\text{FWHM}}^{\text{GTL-HCW}} \).

2. The notation \( \langle \psi | \mu_{\rho} | \psi \rangle \) indicates the presence of a summation performed over \( \omega \), and \( \rho \) can contribute in the interaction process. This is the case for instance when the time interval \( \tau_{\text{coh}} = 2 \Delta t_0 \) is fixed from the FMFQ resonance condition: \( \mathcal{E}_{eg}^0 \tau_{\text{coh}} / \hbar \equiv l \Phi_0 \) with \( l = 1 \). The notation \( \rho \) indicates the presence of a summation performed over \( \rho \) coherently excited rotational states in a vibrationally-frozen transition \( \psi = \psi_0 = 0 \). Provided that \( \Delta E_0 > \mathcal{E}_{eg}^0 \), this summation involves \( N_{\rho} \) rotational states shared in an indistinguishable by both entangled electronic states. We first obtain,

\[
c_{g}(t)^{\rho} c_{g}(t) = -\frac{1}{2 \hbar} \sum_{p} \mu_{eg}^{p} \mathcal{E}_{\Delta \omega_{p}}^{*}(\omega_0, \omega) \left[ \int_{-\infty}^{t} c_{g}(t') c_{g}(t')^{*} \exp[i\tilde{\omega}_{eg} + \omega] t' dt' \right] \tag{17a}
\]

For the \( t = 0 \) centered electric field the interaction ends at the instant time \( t = \tau_{\text{coh}} \). We will therefore consider the integral issue of equation (17a) in the time domain \( t \in [0, \tau_{\text{coh}}] \). Given that during this time interval the interaction occurs in the presence of two entangled electronic states, we can replace (within a global phase) the density probability-like quantity \( c_{g}(t') c_{g}(t)^{\rho} \) by the value \( |c_{g}(t')|^2 = |c_{g}(t)|^2 = 1 / 2 \). Doing so, we obtain:

\[
\langle \Psi | \mu_{\rho} | \Psi \rangle_t = \sum_{p} \sqrt{\frac{1}{2 \hbar} \sum_{\rho \in c} \mu_{eg}^{p} \mathcal{E}_{\Delta \omega_{p}}^{*}(\omega_0, \omega) \exp[i\tilde{\omega} t] + c.c. \tag{17b}
\]

The quantity between parentheses must be identified with a polarizability tensor [24] accounting for the \( \omega \) contributions, each contribution containing an intrinsic uncertainty \( \Delta \omega_0 \). We can say equivalently, for a well defined electronic threshold transition \( \omega_{eg}^{0} \) experiencing the uncertainty \( \Delta \omega_0 \) of the \( \omega \) frequency components. Given that both contributions \( \omega \) and \( -\omega \) are equally probable in the TEES system, the polarizability must remain unchanged if we permute the indices \( g \) and \( e \), change the sign of all frequencies and polarizations, and reverse the time. Such a symmetrized polarizability tensor for a fixed space orientation \( \beta_0 \) of the molecular axis, is the following:

\[
\beta_{\Delta \omega_{p}}^{\rho}(\omega_0, \omega) = \frac{1}{2 \hbar} \sum_{\rho \in c} \left[ \frac{\mu_{eg}^{p} \mu_{eg}^{p}}{\tilde{\omega}_{eg} + \omega} + \frac{\mu_{eg}^{p} \mu_{eg}^{p}}{\tilde{\omega}_{eg}^{*} - \omega} \right] \tag{18a}
\]

The notation \( \mu_{eg}^{p} \mu_{eg}^{p} \beta_{0} \) for the matrix element [25] \( \mu_{eg}^{p} \mu_{eg}^{p} = \langle g | \mu_{\rho} | e \rangle \langle e | \mu_{\rho} | g \rangle \) will remind us that we must omit to average over the space fixed orientation \( \beta_0 \). Only the Laboratory Fixed Frame (LFF) and Molecule

\footnote{The normalization of the Fourier transform of the Gaussian distributions consistent with the Shannon- Hirschman bound entropy \( S_{H-CW} = S_{H} = \ln(e\pi) / \pi \). For \( f(t) = (2\pi)^{-1/2} \int_{-\infty}^{0} f(t) \exp[-i\tilde{\omega} t] \).}
Fixed Frame (MFF) azimuthal angles \([26]\) must be averaged. The induced electric dipole component 
\[ \mu_p(\beta_0; \omega_0, \omega, t) \] can be then determined from the following complex expression:
\[ \mu_p(\beta_0; \omega_0, \omega, t) = \sum_p \alpha_{\Delta\omega_0}^{pp'}(\beta_0, \omega) \xi_p(\omega_0, \omega) \exp(i\omega t) \] where \( \alpha_{\Delta\omega_0}^{pp'}(\beta_0, \omega) \) is the following polarizability tensor:
\[ \alpha_{\Delta\omega_0}^{pp'}(\beta_0, \omega) = \frac{1}{2\hbar} \sum_p \xi_p(\omega_0, \omega) \xi_p^{\prime}(\omega_0, \omega) \exp(i\omega t) \]
\[ + \frac{1}{2\hbar} \sum_p \xi_p(\omega_0, \omega) \xi_p^{\prime}(\omega_0, \omega) \exp(i\omega t) \] (18b)

For symmetric-top rotational wave functions \([26]\) \( |MK\rangle = \sqrt{(2J + 1)/8\pi^2} D_{ijk}^{J} (\alpha, \beta, \gamma)^{P} \), we finally obtain:
\[ \alpha_{\Delta\omega_0}^{pp'}(\beta_0, \omega) = (-1)^{p} \xi_p^{K} K_{p}^{K} \xi_p^{K} K_{p}^{K} \exp(\frac{E_0^{K} - E_0^{K}}{\hbar \omega + i\Delta E^0}) \] (19a)

The global factor \( C_{00}^{N_0}(\beta) \) is a coherent superposition of \( N_0 \) rigid rotor autocorrelation functions. It provides the following diffraction pattern:
\[ C_{00}^{N_0}(\beta) = \frac{1}{N_0} \sum_{n=0}^{N_0-1} (2n + 1) d_{00}^{(n)}(\beta) * d_{00}^{(n)}(-\beta) = \cos \left[ \left( N_0 - 1 \right) \frac{\beta}{2} \right] \sin \left[ \frac{N_0 \beta}{2} \right] \] (19b)

The asterisk in equation (19b) denotes a convolution product \([d_{00}^{(n)}(\beta) * d_{00}^{(n)}(-\beta)]\) performed over a unit circle. Since the LFF angles \( \beta \) and \( \beta' \) cannot be defined separately in the photon- induced TEES system, we have introduced the variables: \( \beta_{0} = \frac{1}{2}(\beta + \beta') \) and \( \tilde{\beta} = \beta - \beta \). The coherent excitation of the \( N_0 > 1 \) rotational states is therefore associated with an initial space fixed orientation \( \beta_0 \) and the subsequent ion core rotational motion \( \tilde{\beta}(t) = \omega_{\text{co}}^{\text{coh}} t \) at the angular frequency \( \omega_{\text{co}}^{\text{coh}} \). Any frequency in the TEES system must be in fact a harmonic frequency of some fundamental angular frequency \( \omega_{\text{rot}}^{\text{coh}} \). Provided that a maximum quantum correlation must be present between the induced dynamics of the ion core and the dynamics of the excited molecular electron, we can expect equally well the presence of a diffraction pattern into the induced dynamics of the excited molecular electron. We may see at the end of this work that this pattern is defined equally well by the (commonly shared) variable \( \tilde{\beta} \). We may finally observe in figure 2 that the initial orientation \( \beta_0 \) can be associated with a fixed space orientation only if the angles \( \beta \) and \( \beta' \) of the two entangled electronic states are drawn to rotate in a plane with opposed helicities.

2. The induced electric dipole observed at the time instant \( t \in [0, \tau_{\text{coh}}] \), where the GTL-HCW electromagnetic field is still operating in the formation of the TEES-system, can be characterized by the following, frequency averaged, GTL-HCW photon-molecule dipole interaction:
\[ M_p(\beta_0; \omega_0)_t = \frac{1}{\Delta \omega_0} \int_{-\infty}^{+\infty} \frac{\mu_p(\beta_0; \omega_0, \omega, t) \xi_p(\omega_0, \omega) \exp(-\frac{\Delta \omega_0 t}{\omega_0})}{\Delta E_0} d\omega \] (20a)

Performing the integration over the complex variable \( \omega = -i \Delta \omega_0 \) along a closed contour, in the lower half complex plane where \( +\omega_0^{0} \) and \( -\omega_0^{0} \) stand on the real axis \( \omega \) as two singularities, we obtain the following induced dynamics at the time instant \( t \in [0, \tau_{\text{coh}}] \):
\[ \sum_{i=1}^{N_0} M_p(\beta_0; \omega_0)_t = \frac{2\pi \omega_0^{0}}{\omega_0} \exp(-\frac{\Delta \omega_0 t}{\omega_0}) \]
\[ \times \left[ d_{p, K_p^{K}}^{(l)}(\beta_0) \right] \left[ N_0 C_{00}^{N_0}(\beta) \right] \left\{ \sum_{i=1}^{N_0} \int_{-\infty}^{+\infty} \frac{S_{M}^{\text{MFT}}(\bar{t}, \bar{t}') d\bar{t}'}{\omega_0^{0}} \right\} \] (20b)

Given the cyclotropism of the GTL-HCW electric and magnetic fields and the quantized nature of the photon- induced magnetic flux, we have imposed to the interaction process the magnetic flux resonance condition: \( E_0^{0} = \omega_{\text{co}}^{\text{coh}} / e = \Phi_0 \) where, \( \omega_{\text{co}}^{\text{coh}} = 2\Delta \omega_0 \) is the coherence time interval and, \( \Phi_0 = (\pi / e) \) is the fundamental magnetic flux quantum. This is the Fundamental Magnetic Flux Quantum (FMFQ) resonance condition. The temporal width \( \Delta \omega_0 \) is fixed from a preselected TEES threshold frequency \( \omega_0^{0} \) and the underlying resonance condition is defined by the relation: \( \omega_{\text{co}}^{\text{coh}} = \Delta \omega_0 \). Given that \( \omega_{\text{co}}^{\text{coh}} = (\pi - \Delta \omega_0), \) with \( \Delta \omega_0 = 2\pi \) the lowest allowed phase difference around a closed electronic loop, the possible values of the initially badly defined center frequency \( \omega_0 \) are the harmonic frequencies \( \omega_{\text{co}}^{\text{coh}} \) of the TEES threshold frequency. Since \( \Phi_0 \) is the
The initial orientation $\beta_0$ of the rotationally-frozen ion core, the TEES threshold and the polarization $\rho$ of the photon wavepacket, first define a global factor $\left[ \frac{\rho}{\kappa_\rho}, \beta_0 \right]$. Only this factor contains the polarization dependence. The integrant $\tilde{S}_{\text{MFT}}(t', \delta')$ is associated instead with the interacting molecular electron exclusively. It is independent of the polarization state $\rho$. This peculiar aspect can be apprehended by the limiting situation of an induced magnetic field that cannot be averaged down to a zero value. Either for linear or circular polarization, the joint effect of the GTL-HCW electric and magnetic fields into the dynamics of an interacting molecular electron is the formation of a quasi-classical closed electronic loop drawn away from the molecular ion core.

The integrant $\tilde{S}_{\text{MFT}}(t, \delta')$ of equation (20b) defines the following Magnetic Flux Transfer (MFT) function:

$$
\tilde{S}_{\text{MFT}}(t, \delta') = \frac{\gamma_{\text{coh}}^0}{\tau_{\text{coh}}} \exp\left[-\frac{i}{2} \Delta \omega_0 \Delta t_0 \delta^2 \right] \sin[\omega_0 \Delta t_0 (\delta' - \delta + 1)] \exp[i\omega_0^0 \Delta t_0 (\delta' + 1)]
$$

We have made again use of the dimensionless time variables: $\hat{t} = t / \Delta t_0$ and $\hat{\delta}' = \delta' / \Delta t_0$ and, as also all along this work, $\Delta \omega_0 \Delta t_0$ represents the constant parameter $4 \ln(2)$ at the GTL-HCW limit. The Integrated Magnetic Flux (IMF) transfer function: $\tilde{S}_{\text{IMF}}(\hat{t}) = \int_{-\infty}^{+\infty} \tilde{S}_{\text{MFT}}(t, \delta') \, dt'$ then depends on the considered time instant $\hat{t}$ and the operating value of the dimensionless interaction parameter $\omega_0^0 \Delta t_0$. We observe the following integrated issue:

$$
\int_{-\infty}^{+\infty} \text{Im} \left[ \tilde{g}_e \tilde{S}_{\text{MFT}}(0, \delta') \right] \, d\delta' = \sqrt{\frac{\sigma_{\text{scal}} \sigma_{\text{pa}}}{h}} \frac{\tau_{\text{coh}}}{\Delta \omega_0}
$$

Let us first remark that the overall effect of the $l \rightarrow \delta$ contribution in the above integral issue decreases drastically with the increasing values of $l$. Moreover, summing equation (21b) over all positive integers $l \geq 1$ we further observe that the $l \rightarrow \delta$ contributions are canceled each other; i.e., the $l \rightarrow \delta$ contributions vanish and only the first of these two terms of the $l \rightarrow \delta$ contribution can survive in the coherent superposition. The same arguments hold for the MFT-function $\tilde{S}_{\text{MFT}}(0, \delta')$. Such a paired configuration results necessarily into a minimum which is in fact the minimum of the bound potential of figure 3:

$$
\sum_{l = 1}^{\infty} \int_{-\infty}^{+\infty} \tilde{g}_e \text{Im} \left[ \tilde{S}_{\text{MFT}}(0, \delta') + \tilde{S}_{\text{MFT}}(0, \delta') \right] \, d\delta' = C_{\text{MFT}} \frac{\sigma_{\text{scal}} \sigma_{\text{pa}}}{h} \frac{\tau_{\text{coh}}}{\Delta \omega_0}
$$

In the presence of the FMFQ resonance condition only the magnetic flux quantum $\Phi_0$ is well defined while the minimum of the bound potential occurs as a coherent superposition of the integral of the IMF-function. The above properties suggest the presence of a pairing process in the formation of the magnetic flux quanta: $\pm 2\Phi_0$, $\pm 3\Phi_0 \cdots$. The presence of the large $|l|$ values can be then understood by the formation of a distant quasi-classical circular electronic loop. Furthermore, since these pairs exhibit opposite helicities with the subsequent rotation of the angles $\beta$ and $\gamma$ (see this for figure 2), their overall magnetic flux is naturally assigned to the presence of a minimized magnetic flux fluctuation strength at the minimum of a bound potential. The contribution of the $|l| \geq 2$ harmonics of the FMFQ resonance condition into the TEES potential emerged naturally from the $\zeta(\delta_{\text{coh}})$-dependence of the position- momentum uncertainty $[\hat{x}_e, \hat{p}_e]_{\text{min}} = [\hat{x}_e, \hat{p}_e]_{\text{min}}$. The constant $\zeta(\delta_{\text{coh}})$ is the integral issue of the TEES function $\delta_{\text{coh}}(\delta)$.

The $|l| \geq 2$ contributions of the off-entanglement condition $\omega_0^0 > \Delta \omega_0$ obey the following close relation:

$$
C_{\text{MFT}} \frac{\sum_{l = 2}^{\infty} \Delta \omega_0^0 \Delta \omega_0^0}{\left( \omega_0^0 \right)^2} = C_{\text{MFT}} \frac{\sum_{l = 2}^{\infty} \Delta \omega_0 \Delta \omega_0}{\left( \omega_0 \right)^2} = \frac{\sigma_{\text{scal}} \sigma_{\text{pa}}}{h} \frac{\tau_{\text{coh}}}{\Delta \omega_0}
$$

Given the TEES conditions: $\omega_0^0 = (n + 1) \omega_0^\rho$, $\omega_0^\rho = n \omega_0^\rho$, and $\omega_0^\rho - \omega_0^\rho = \Delta \omega_0 / \delta_{\text{coh}}$ of figure 2, we may readily observe that, independently of the specific value of $N_0 \gg 1$, the application of the FMFQ resonance condition into equation (22b) will always provide the minimum $|\delta_{\text{coh}}| = |\sigma_{\text{scal}} \sigma_{\text{pa}}| / \left( \Delta \omega_0 \Delta \omega_0 \right)_{\text{min}}$. Moreover, inserting equation (14b) into equation (22b) we obtain again the fundamental state property of equation (13b). Since the lines in absorption and emission of the superposed rotational states belong in an indistinguishable way to both entangled electronic states, they can operate only as the harmonics of the TEES threshold frequency. As expected, the minimum of the bound potential is insensitive to the specific value of $N_0$ because the closed magnetic field lines cannot be defined separately. They can only be superposed to the $l = 1$ principal FMFQ loop.
The quantized nature of the induced magnetic flux then removes entirely the oscillatory regime of the electronic susceptibility, and the absorbed GTL-HCW photon wavepacket ends into a bound potential as a captured FMFQ photoelectron. With the advent of the ending value $\Delta \omega_0 - \Delta \omega_{\text{coh}}$ into the time and frequency spaces, the entropic, time and energy uncertainty transfer is then realized in presence of a magnetic flux quantum obeying the GTL-HCW magnetic flux property: $\Delta E_{\text{F}}(\nu) = \Phi_0 / \exp(\Delta \omega_0 - \Delta \omega_{\text{coh}})$. (b) The time width $\Delta t_{\text{FMFT}}$ of the FMFQ dipp of the captured FMFQ photoelectron unveils the presence of a photon-induced TEES structure at a fundamental state where the dissipated information entropy $\hat{S}^0_{\text{coh}}$ measured in Shannons is the lowest allowed: $\hat{S}^0_{\text{coh}} \Phi_0(Wb) = |t_{\text{coh}}^0| / \sqrt{\omega_0^0 / 2\hbar}$. The coherent superposition of the $\Omega_{\text{coh}}$ paired magnetic field lines experienced by the excited molecular electron provides a 3D diffraction pattern (dot curve) which matches the underlying dynamics of the FMFQ dipp with excellent agreement.

as pairs of magnetic flux vortices of opposed helicity which then condense into the minimum of the FMFQ photoelectron’s potential. The key function in the photon-induced implementation of a fundamental magnetic flux quantum is therefore the function: $\hat{S}^0_{\text{FMFT}}(\hat{t}, \hat{t}')$. Under the FMFQ resonance condition this function shows the property: $\hat{S}_1^0(\hat{t}^0_{\text{coh}}) = \exp(\hat{S}_1^0(\hat{t}^0_{\text{coh}})) \hat{\kappa}$, and, at the end of the interaction process we have the operating values: $\omega^0_{\text{eg}} = \omega^0_{\text{eg}} + \Delta \omega_0$ and $\hat{t}_{\text{coh}} = \hat{t}_{\text{coh}}^0$. If the ending step is written in terms of the dimensionless quantities $\hat{\Delta} \omega_{\text{end}} = \omega^0_{\text{eg}} / \Delta \omega_0$ and $\hat{t}_{\text{coh}} = \hat{t}_{\text{coh}}^0$, we may readily observe the integral issue of the average information entropy of equation (15): $[\hat{\Delta} \omega_{\text{end}} \hat{t}_{\text{coh}} / \hat{t}_{\text{coh}}] = \langle \hat{I}_{\text{GTL}} \rangle$.

3. We have drawn in figure 4(a) both, the MFT-function $\Im \hat{S}_1^0(\hat{t}^0_{\text{coh}}, \hat{t}') / \hat{\kappa}$ of equation (21a) (solid dense line) for which the initial indeterminacy of $\omega_0$ is entirely removed when the FMFQ resonance condition is operating, as well as the overall integral action: $\int_{-\infty}^{\infty} \Im \hat{S}_1^0(\hat{t}^0_{\text{coh}}, \hat{t}') / \hat{\kappa} \mu_{\text{coh}} d(\hat{t}')$ (thick gray line) for which the dimensionless variable $\omega^0_{\text{eg}} / \hat{t}_{\text{coh}}$ is not submitted any longer to the constraint of the FMFQ resonance condition.

When the oscillatory regime of the interacting electric field collapses into the GTL-HCW domain $\Delta \omega_0 = \omega^0_{\text{eg}} / \hat{t}_{\text{coh}}$ (delimited in figure 4(a) by the two dashed vertical lines) with the advent of the GTL-HCW limit, and the induced magnetic field cannot be averaged any longer down to a zero value, the cyclotropic effect (see footnote 2) of the FMFQ resonance condition becomes prominent; i.e., the enhancement of the FMFQ-dipp shown in figure 4(a) is due exclusively to the synergetic effect of the GTL-HCW electric and magnetic fields. This is performed in the presence of a maximum quantum correlation established between the rotating ion core and the confined FMFQ photoelectron. This fact can be easily verified if we assume that the excited molecular electron experiences an infinite-like number $\Omega_{\text{coh}}$ of closed magnetic field lines. Their coherent superposition results into the following 3D diffraction pattern:

$$\left[ \frac{\hbar \exp(i\hat{\kappa})}{\hat{t}_{\text{coh}} \Delta \omega_{\text{coh}}} \sum_{l=0}^{\Omega_{\text{coh}}} (2l + 1) d_{\text{coh}}^0(\omega_{\text{coh}}) t^0_{\text{coh}} \hat{t}_{\text{coh}}^0 (\omega_{\text{coh}} t^0_{\text{coh}} t^0_{\text{coh}}) \right]^3$$

The pattern: $\Im \hat{S}_1^0(\hat{t}^0_{\text{coh}}, \hat{t}') / \hat{\kappa}$ of equation (23) is extracted solely from the 1D diffraction pattern of the molecular ion core. We then (locally) observe the excitation dynamics law: $[- \cos(\Phi_0(Wb) / \hat{\kappa} \Delta \omega_{\text{coh}} t^0_{\text{coh}} t^0_{\text{coh}}) \hat{\kappa}])$ which can be identified with the FMFQ-dipp $\Im \hat{S}_1^0(\hat{t}^0_{\text{coh}}, \hat{t}') / \hat{\kappa}$ of figure 4(a). Their excellent agreement within the interaction time interval can be observed in more detail in figure 4(b). These two distinct dimensionless time distributions have been drawn respectively as a solid and a dot line. We can thus conclude by saying that, in the presence of the FMFQ resonance condition $E_{\text{eg}} t_{\text{coh}} / \epsilon = \Phi_0$, the time dependence of the FMFQ photoabsorption process follows the law...
\[\exp(i\omega_0 t) \sin\left(\frac{\Delta}{\omega_0} + \frac{1}{\omega_0} t\right)\]. The underlying photon-induced coherent superposition in the TEES system is then responsible for the subsequent time recurrences of the photon-induced diffraction pattern. Similarly to the two-slit diffraction pattern distributed over a plane surface, the overlapping rigid rotator states of the two entangled electronic states cancel each other for some space orientations whereas they reinforce in other ones, causing in this way the emergence of a diffraction pattern oscillating over a Rydberg spherical surface around a distant proteiform circular electronic loop centered at the rotating molecular ion core.

4. Given that a physical process demands the dissipation of at least 1 Sh of information entropy, let us further introduce the Shannon-threshold amplitude: \(\langle f(\tilde{t}')\rangle_{\text{Sh}} = \frac{1}{\ln 2} \int_{t'}^{t'} f(\tilde{t}') \, d\tilde{t}'\) and account for such a threshold dissipative process, the effect of the sharp inertial window of the robust moment of inertia of the ion core. The presence of an infinite-like moment of inertia relative to the interacting molecular electron can be first observed as a least integral action of the MFT-function; i.e., this action occurs in the presence of the truncation constant \(\delta_{\text{min}} = I_{\text{rot}}/\hbar\) and the fundamental thresholds \(\ln 2\) and \(S_{\Delta\omega_0} + S_{\Delta\omega_t}\) as follows:

\[
\sqrt{\langle \hat{E}_{\text{HCW}} \rangle_{\text{MFT}}} \int_{t'}^{t'} \ln 2 \text{Im} [\hat{z}_{\text{MFT}} (\tilde{t}', \tilde{t})] \, d\tilde{t}' = \left[\delta_{\text{min}} \sigma_{p_0}\right]_{\text{min}} \int_{t'}^{t'} \ln 2 \text{Im} [\hat{z}_{\text{MFT}} (\tilde{t}', \tilde{t})] \, d\tilde{t}'
\]

\[\int_{t'}^{t'} \ln 2 \text{Im} [\hat{z}_{\text{MFT}} (\tilde{t}', \tilde{t})] \, d\tilde{t}' = \exp[i\omega_0 t] \sqrt{\delta_{\text{min}}^{\text{MFT}}}
\]

We can thus write the following integral action balance at the minimum of the FMFQ potential:

\[
\langle \text{Im} [\hat{g}_{\text{MFT}} (\tilde{t}', \tilde{t})] \rangle_{\text{Sh}} = \hat{g}_{\text{MFT}} |\tilde{t}_0| \int_{t'}^{t'} \ln 2 \text{Im} [\hat{z}_{\text{MFT}} (\tilde{t}', \tilde{t})] \, d\tilde{t}' = \exp[i\omega_0 t] \sqrt{\delta_{\text{min}}^{\text{MFT}}}
\]

Equations (25a)–(25b) show that, on average, the dissipated amount of information entropy measured in Shannons is: \(\delta_{\text{min}}^{\text{MFT}} = \langle \text{Im} [\hat{g}_{\text{MFT}} (\tilde{t}', \tilde{t})] \rangle_{\text{Sh}}^{\text{MFT}} = 1.000 094\). This is the lowest allowed threshold because:

\[
\delta_{\text{min}}^{\text{MFT}} \hat{E}_t (\text{FMFQ}) = \sqrt{\langle \text{Im} [\hat{g}_{\text{MFT}} (\tilde{t}', \tilde{t})] \rangle_{\text{Sh}}^{\text{MFT}}} = \left[\delta_{\text{min}} \sigma_{p_0}\right]_{\text{min}} \ln 2 \text{Im} [\hat{z}_{\text{MFT}} (\tilde{t}', \tilde{t})] \, d\tilde{t}'\]

It is also worth noting that this threshold value \(\delta_{\text{min}}^{\text{MFT}}\) occurs in the process as a structural property of the FMFQ implementation threshold. We can now obtain the dimensionless time width \(\Delta_{\text{FMFQ}}^{\text{MFT}} = \Delta_{\text{FMFQ}}^{\text{MFT}} / \tau_{\text{coh}}\) of figures 4(a)–(b) from the underlying time-energy uncertainty threshold of the FMFQ implementation process; it is remarkable that this time width can be defined at the threshold of the molecular ion core inertia by the presence of the maximum possible intrinsic energy uncertainty and the lowest allowed amount of dissipated information entropy. Namely:

\[
\Delta_{\text{EF}} \Delta_{\text{FMFQ}}^{\text{MFT}} = I_{\text{rot}}/\hbar \langle \text{Im} [\hat{z}_{\text{MFT}} (\tilde{t}', \tilde{t})] \rangle_{\text{Sh}}^{\text{MFT}}
\]

Using the appropriate units of time, energy, and angular momentum, we may readily observe the ‘synergetic effect of the time-energy and position-momentum uncertainties’. More specifically:

\[
(\Delta_{\text{EF}} [eV] / \Delta_{\text{FMFQ}}^{\text{MFT}} [\tilde{t}]) \times \left[\delta_{\text{min}} \sigma_{p_0}\right]_{\text{min}} \text{represents the overall threshold } \tilde{t} \text{ of the interaction process. This lowest allowed limit accounts for the effects of the threshold of the FMFQ ionization quenching, the threshold of the moment of inertia of the molecular ion core and, the requisite amount of information entropy dissipated during the interaction time interval. The presence of the invariant point } \tilde{t} = \exp[-W_0(\hat{E}_t^{\text{MFT}})] \text{ at this overall threshold is a direct manifest of the transferred GTL-HCW Shannon entropy into the time and frequency spaces.}

We can finally deduce from equation (26a) (within the 6 digit precision of our numerical constants) the dimensionless time width \(\Delta_{\text{FMFQ}}^{\text{MFT}} = \Delta_{\text{FMFQ}}^{\text{MFT}} / \tau_{\text{coh}}\) of figures 4(a)–(b). This width unveiled the peculiar aspect of the GTL-HCW photon-molecule interaction; i.e., the photon-induced fundamental state is defined by the intrinsic uncertainties of the interaction process exclusively:

\[
\Delta_{\text{FMFQ}}^{\text{MFT}} = \left[\ln 2 \right]_{\text{max}} |\delta_0^{\text{MFT}} |\left[\frac{\Delta_{\Delta\omega_t} + S_{\Delta\omega_0} / \Delta\omega_t}{\Delta\omega_t / \Delta\omega_0}ight]^{1/2}
\]

We can now observe the integral issue of the width of the FMFQ dip in the interaction process as a natural consequence of the transferred average amount of information entropy \(\langle \hat{g}_t^{\text{GTL}} \rangle)

\[
\langle \hat{g}_t^{\text{GTL}} \rangle \Delta_{\text{FMFQ}}^{\text{MFT}} = \left[\delta_{\text{min}} \sigma_{p_0}\right]_{\text{min}} |S_{\Delta\omega_0} + S_{\Delta\omega_t} / \Delta\omega_0 \Delta\omega_t |^{1/2}
\]

5. Since the photon-induced magnetic moment is necessarily characterized by the presence of the fundamental physical constant \(\hat{g}_{\text{MFT}}\), and the excited molecular electron experiences an infinite-like number
of closed magnetic field lines of opposite helicities superposed to the (principal) $l = 1$ TEES-threshold electronic loop, a nonlocal internal structure must be present in the photon-induced TEES system; i.e., the constant $\hat{g}_e$ occurred in equation (13b) as the geometric mean of the nonlocal quantities $[\hat{\sigma}_{\Delta\hat{A}}\hat{\sigma}_{\Delta\min}]^{-1}$ and $\hat{Q}_0^{-1}$ while the ending value of the TEES generic function $q^{(0)}_{\text{TEES}}(s)$ and equations (12α)–(12c) and (13α)–(13b) have defined in the process the fundamental properties of the FMFQ photoelectron’s potential. We can now provide further evidence for the presence of the photon- induced TEES nonlocal structure from equations (21α) and (24b). These equations exhibit the following integral relation:

$$\int_{-\infty}^{+\infty} \text{Im} \left( \frac{\hat{S}_{\text{MFT}}}{\hat{\Delta}_{\text{coh}}} (\hat{\tau}_{\text{coh}}^0 \hat{t}') \right) \hat{b}_0 \hat{d}t' = \sqrt{\hat{G}_{\hat{b}_0}} \int_{-\infty}^{+\infty} \text{Im} \left( \frac{\hat{S}_{\text{MFT}}}{\hat{\Delta}_{\text{coh}}} (\hat{\tau}_{\text{coh}}^0 \hat{t}') \right) \hat{b}_0 \hat{d}t'$$  \hspace{1cm} (27a)

$$\hat{G}_{\hat{b}_0} = \hat{Q}_0 \left[ 1 + \exp(-\pi \hat{Q}_0) \right]^{1/2} \left[ \frac{\Delta \omega_0 \Delta f_0}{\Delta \omega_0 + \Delta f_0} \right]$$  \hspace{1cm} (27b)

The Magnetic Flux Aperture (MFA) amplitude $\sqrt{\hat{G}_{\hat{b}_0}}$ occurs in equation (27a) as an above- threshold overall effect of the integral action of the MFT function.

Let us remind that the overall effect of the magnetic flux implementation occurred in equation (15c) as the issue of the overall interaction constant $P_{\text{GTL}}^{\text{HCW}} = \hat{Q}_0^{-1} \text{Im} \hat{Q}_0^{-1}$; i.e., we have:

$$P_{\text{GTL}}^{\text{HCW}} \hat{Q}_0 = (S_{\Delta\hat{A}} + S_{\Delta\min}) \left( \Delta E_0 \hat{\tau}_{\text{coh}} / \hat{f} \right).$$

The property $W_{0}(P_{GTL}^{HCW}) = S_{\Delta\hat{A}} - S_{\Delta\min}$ is also an overall property of the GTL-HCW photon-molecule interaction process. Given that the magnetic flux aperture is fixed by the advent of the ending value $S_{\Delta\hat{A}} - S_{\Delta\min}$ and $|s| \leq \Delta S_{\Delta\hat{A}} < \pi$ [19], the holomorphic function $q^{(0)}_{\text{TEES}}(s)$ can be readily developed in terms of the Bernoulli numbers [19, 27]. That is, $q^{(0)}_{\text{TEES}}(S_{\Delta\hat{A}} - S_{\Delta\min}) = \sum_{k=0}^{\infty} (-1)^k \frac{B_k}{k!} (S_{\Delta\hat{A}} - S_{\Delta\min})^k$. Note that the Bernoulli numbers can be expressed in terms of the Riemann zeta functions $\zeta(s)$ [19]; these transcendental numbers arise naturally in the higher order terms of the electron’s gyromagnetic ratio. They constitute a somehow general feature of the higher order calculations in perturbative quantum field theory. It is then remarkable that the MFA- amplitude can be defined in terms of the fourth power term of the TEES generic function; i.e.,

$$\sqrt{\hat{G}_{\hat{b}_0}} = 0.999 \text{ for } 999 \text{ to } 999 \text{ exp} \left[ \frac{B_k}{4!} \left( W_{0}(P_{GTL}^{HCW}) \right)^k \right].$$  \hspace{1cm} (28a)

The overall effect of the magnetic flux aperture can be thus defined solely by the unique interaction parameter $P_{GTL}^{HCW}$ within a 7-digit precision. It is also remarkable that the effect of $W_{0}(P_{GTL}^{HCW})$ reveals the presence of the fundamental constant (See footnote 1) $\hat{g}_e^{\exp} = 1.001 \text{ 159 652}$ (experimental value of the 2018 codata electron’s $\hat{g}_e$ – factor). We observe the following overall property for the magnetic flux aperture: $\hat{G}_{\hat{b}_0} = \exp \left[ \hat{b}_0 \hat{b}_1 \hat{b}_2 \hat{b}_3 \right] W_{0}(P_{GTL}^{HCW})^4 = 0.999 \text{ for } 999 \text{ exp} \left[ \hat{g}_e^{\exp} \right]^4$. It follows that the GTL-HCW photon-induced overall property $\left[ \hat{G}_{\hat{b}_0} \right]^4 = \hat{g}_e^{\exp}$ is intimately related with the fourth power law: $\left[ \Delta \omega_0 \left( \hat{t}/\tau_{\text{coh}}^0 \right) \text{sin} \chi \left[ \Delta \omega_0 \left( \hat{t}/\tau_{\text{coh}}^0 \right) \right] \right] = \text{Im} \left[ \hat{S}_{\Delta\hat{A}} \left( \hat{t}/\tau_{\text{coh}}^0 \right) / \left( \hat{f} / \hat{w} \right) \right]$ observed initially in figure 1. Such a sharp overall effect at the FMFQ implementation threshold can be understood by the robust effect of the moment of inertia of the molecular ion core. Given that $I_{\Delta\omega_0} = \hat{b}_0 \hat{b}_1 \hat{b}_2 \hat{b}_3 \hat{b}_4$ represents the inertial threshold, and $\left[ I_{\Delta\omega_0} \right] \hat{b}_0$ contains the average effect of the sharp inertial window, we can write the following compact expression:

$$\left[ I_{\Delta\omega_0} \right] \hat{b}_0 = \hat{g}_e^{\exp} \hat{G}_{\hat{b}_0} \left[ \hat{S}_{\Delta\hat{A}} + S_{\Delta\min} \right]$$  \hspace{1cm} (28a)

Equation (28a) suggests that the sharp inertial window of the moment of inertia of the molecular ion core is responsible for the magnetic flux aperture of the captured FMFQ photoelectron and the dissipated amount of information entropy. We finally obtain the induced rotational motion of the molecular ion core in terms of the rotational constant $B$ and the minimum $\hat{b}_0$ of the bound potential of figure 3:

$$\left[ I_{\Delta\omega_0} \right] \hat{b}_0 = \hat{g}_e^{\exp} \hat{G}_{\hat{b}_0} \left[ \hat{S}_{\Delta\hat{A}} + S_{\Delta\min} \right]$$  \hspace{1cm} (28b)

6. Our very last request will concern the simultaneous presence of the magnetic flux quanta $\pm 2\hat{b}_0 \pm 3\hat{b}_0 \cdots$. They must be necessarily associated with the presence of the large angular momenta $\pm 2\hat{b}_0 \pm 3\hat{b}_0 \cdots$ whose paired contribution minimizes the magnetic flux fluctuation strength. They are formed together with the proteiform FMFQ loop and condense into the minimum $\hat{b}_0$ of the bound potential $P_{GTL}^{HCW}(s)$ of figure 3 when the variable $\hat{S} = \hat{S} - \Delta S_0 = \text{Im}(\Delta E/\Delta f_0)$ spans the domain $[-\infty, 0]$. The average effect of the sharp inertial window written in terms of the temporal and spectral Shannon entropies of the absorbed GTL-HCW photon wavepacket provides indeed the following fundamental state relation:
\[ h \left( 1 - \exp[-W_0(p_{0\text{FMFQ}}^{\text{HCW}})] \sum_{n=0}^{\infty} \frac{I_n \omega_n \vartheta_n}{I \hbar} \right)^{1/2} = \left( S_{\Delta\nu_0} + S_{\Delta\nu_0} \right) (\sigma_{\nu_0} \sigma_p) \text{min} \]  

(28c)

It is then worth to closure with the requisite presence of the photon-induced fundamental position-momentum uncertainty state of the FMFQ photoelectron along the distant 1D probeformed circular loop. If this state is written in terms of the well defined implemented magnetic flux quantum and the appropriate units of time and energy, it provides: \( C_0 \left[ \vartheta_\nu \right] \sigma_{\nu_0} \text{min} = \left[ \vartheta_\nu \right] \sigma_{\nu_0}^2 \Phi_0 / |FWB| \) with \( |\psi|^2 \left[ \vartheta_\nu \right] \text{Var} \{ |\psi|^2 \} \). We find back the variance (invariant point in the time and frequency spaces) of the GTL-HCW electric field as an intrinsic property of the minimum of the bound potential of figure 3. This is not surprising at all because the underlying temporal \( S_{\Delta\nu_0} \) and spectral \( S_{\Delta\nu_0} \) Shannon entropies (introduced at the very beginning of this work) are the integral issues of the key definitions: \( -|\psi_{\Delta\nu_0}(\nu)|^2 \text{Var} \{ |\psi_{\Delta\nu_0}(\nu)|^2 \} \text{Var} \{ \nu \} \) and \( -|\psi_{\Delta\nu_0}(\nu)|^2 \text{Var} \{ |\psi_{\Delta\nu_0}(\nu)|^2 \} \text{Var} \{ \nu \} \).

Their conjugate fixed length scales are respectively: \( l_{\Delta\nu_0} = \vartheta_\nu \Delta\nu_0 \) and \( l_{\Delta\nu_0} = \vartheta_\nu \Delta\nu_0 \) and, the maximized difference \( \Delta S_0 = \ln[\Delta E/E_0 \text{max}] \) must satisfy at the end of the transfer process the overall effect of equations (15b)–(15c) and (28a)–(28c). As expected, for a given entropy in the time and frequency spaces, the underlying Gaussian distributions have minimized the bound state of the confined FMFQ photoelectron. This is done together with the minimized variance of the presence probability distributions \( |\psi_{\Delta\nu_0}(\nu)|^2 \) and \( |\psi_{\Delta\nu_0}(\nu)|^2 \)quared. Namely: \( \phi_0 \text{Var} \{ |\psi|^2 \} = \exp[-\Delta S_0] \). The fundamental electric charge of the captured photoelectron then exhibits the fundamental property: \( [\sigma_{\nu_0} \sigma_p] / \epsilon \text{min} = \left[ \vartheta_\nu \right] \sigma_{\nu_0} \Phi_0 \). This is a requisite condition.

5. Quantized magnetic flux molecular Rydberg exciton

1. We have previously found that the captured FMFQ photoelectron ends into the minimum of a bound potential. It carries a fundamental magnetic flux quantum and can oscillate in time over a Rydberg spherical surface around a distant quasi-classical closed electronic loop. The synergetic effect of the GTL-HCW monotonically increasing electric filed and the related induced uniform magnetic field of fixed space polarity resulted into a photon-induced minimum uncertainty state. A minimum uncertainty state is indeed a coherent state intimately related with the state of a quantized electromagnetic field. The quantized issue of the implemented fundamental magnetic flux quantum is therefore drawn into a classical trajectory wherefrom can evolve in the course of time as a diffracted FMFQ molecular Rydberg photoelectron. The binding energy \( E_{\text{BE}}^{\text{FMFQ}} \) of this molecular system is given by the following expression:

\[ \sqrt{\frac{|e|_{\text{BE}}^2}{E_{\text{BE}}^0}} = \sqrt{\frac{\frac{R_y^2}{m_e c^2} \text{max}}{C_{\text{QC}}^2}} \]  

(29)

\( R_y = \hbar R_\infty = 13.6 \text{ eV} \) is the Rydberg unit of energy. Given that we are dealing exclusively with the relevant dimensionless quantities of the \( \Phi_0 \)-implementation process, we have introduced the fundamental electric charge \( |e|_{\text{BE}}^2 = \sqrt{4\pi\alpha} \) measured in natural units. For the representative case of the preselected TEES system NO (\( A^1\Sigma^+_u \rightarrow X^1\Pi_1 \)) the FMFQ resonance condition provides: \( E_{\text{BE}}^{\text{FMFQ}} \approx -0.004 \text{ eV} \). The preionized TEES system NO (\( L = \chi \)) provides instead: \( E_{\text{BE}}^{\text{FMFQ}} \approx -0.001 \text{ eV} \). These values reflect the Wannier-Mott type large excitons observed in semiconductor crystals and liquid xenon. If we apply now the FMFQ resonance condition into the TEES system of a lithium dimer molecule \( Li_2(A^1\Sigma^+_u \rightarrow X^1\Sigma^+_g) \), we find: \( E_{\text{BE}}^{\text{FMFQ}} \approx -0.001 \text{ eV} \). We are in presence of a quasi-outmost FMFQ molecular exciton. The FMFQ syrnikos threshold is found to be about for instance with the limiting value: \( E_{\text{BE}}^{\text{FMFQ}} \approx -R_y \alpha^2 \approx 0.000 \text{ 72 eV} \). The captured FMFQ photoelectron exhibits a remarkably vast static electric dipole moment; i.e., we can represent the FMFQ electric dipole by the expression: \( \Delta \mu_{\text{eq}}^0 = q_\nu \Delta \xi_{\text{eq}}^0 \). That is, \( \Delta \mu_{\text{eq}}^0 \approx 1.04 \text{ eμ } \). The appropriate unit of the FMFQ electric dipole is the eμ. It is important to notice that there is no any preferred well defined space orientation for the FMFQ electric dipole. This is because the electric charge is first delocalized around a closed electronic loop and then diffracts over the corresponding Rydberg spherical surface. In contrast to the case of semiconductors where the close proximity of the paired electron-hole can only provide short lifetimes at the nanosecond scale, the lifetimes of the outermost FMFQ molecular excitons are expected to be much more longer. It may be then worth to provide some comments of general interest: Given that the molecular excitons can interact with photons and provide polaritons, we may first ask whether two or more FMFQ dressed excitons can combine to form FMFQ multiexciton systems. This can be realized for instance when a weak dipole- dipole attractive potential is present. When a molecule absorbs a GTL-HCW photon wave packet, it converts the energy uncertainty \( \Delta E_0 \) into the form of a FMFQ molecular exciton carrying, both, a quantized amount of energy and a quantized amount of magnetic flux. A FMFQ molecular exciton can then jump within a given molecular alignment from a donor molecule to an adjacent acceptor molecule by means of a near-field dipole-dipole interaction. We can
determine in this way how a quantized amount of energy carrying a quantized amount of magnetic flux can flow at the molecular scale. The FMFQ molecular exciton can be used in quantum devices as the constituent element of the transport of a quantized amount of magnetic flux of a preselected amount of Rydberg energy. On the other hand, since the FMFQ molecular exciton is an integer-spin quantum particle the presence of a repulsive interaction could result into the presence of a Bose-Einstein condensed state. Our minimum uncertainty coherent state induced into a single molecular constituent (written in terms of the appropriate physical units of the interaction process) obeys the following fundamental equation:

\[
\left| x_{\text{min}} \right| \left( \frac{\Delta x_0}{\Delta t_0} + \frac{\Delta x_0}{\Delta t_0} \right)^2 \left( \frac{\Delta E_{0}[eV]}{\Phi_0[Wb]} \right)^3 \left[ \frac{\Delta E_{0}[eV]}{\Phi_0[Wb]} \right] = \left[ \frac{\Delta x_0}{\Delta t_0} + \frac{\Delta x_0}{\Delta t_0} \right]^2 - \left[ \frac{\Delta x_0}{\Delta t_0} + \frac{\Delta x_0}{\Delta t_0} \right]^2
\]

\[
\left( \hat{K}_{\Phi_0} \right)^3
\]

\( \hat{K}_{\Phi_0} = 2\pi n / G\mu^2 \) is the free-space FMFQ coupling constant. We may see in the following that this constant contains relevant information on the entropic uncertainty threshold of the integrated magnetic flux and, also, provides a physical interpretation to the requisite presence of an imaginary part into the complex propagation variable \( \hat{x} = (x/\Delta x_0) + i (-\hat{Q}_0) \) of the GTL-HCW photon wave function \( \left[ \hat{E}_{\Delta x}(\hat{x}) \right]_{\Phi_0} \).

2. A minimum uncertainty state which refers to a coherent state of a quantized electromagnetic field has been first studied by Schrödinger [28]. A complete quantum description of optical coherence phenomena related to electromagnetic fields has been then developed almost four decades later [29]. Coherent fields states and photon wave functions have been further investigated in the past [30, 31]. It has been shown that the photon wave functions, the quantized fields wave packets and their underlying optical coherence are intimately related [32]. The photon wave function description occurred as an equivalent method for describing states and dynamics of quantum electromagnetic fields. The imaginary part of the photon wave function has been found in particular to play the role of a magnetic induction field [32]. It is then important to remark that the overall effect of the imaginary part of the complex propagation variable \( \hat{x} \) of the GTL-HCW photon wave function \( \left[ \hat{E}_{\Delta x}(\hat{x}) \right]_{\Phi_0} \) introduced in equation (2) unveils equally well the fact that, in the \( \Phi_0 \)-implementation process a free space magnetic flux coupling constant \( \hat{K}_{\Phi_0} \) is involved.

Under the magnetic flux resonance condition and in the absence of external charges and currents the Maxwell equations have provided the following free space magnetic induction property: \( B_{\Delta x}^{\text{ind}} [G] \right| \Delta x_0 \right| \right|^2 = \eta e \). Using the appropriate units of our process we can write immediately:

\[
\int_{\Phi_0}^{\Phi_0} \left( 0, \right) d\Phi_0 = \frac{\Delta x_0}{\Delta t_0} + \frac{\Delta x_0}{\Delta t_0} - \frac{\Delta x_0}{\Delta t_0} \left( \frac{\Delta x_0}{\Delta t_0} + \frac{\Delta x_0}{\Delta t_0} \right)^2
\]

\( \hat{K}_{\Phi_0} = \hat{K}_{\Phi_0} \) is the free-space FMFQ coupling constant. We may see in the following that this constant contains relevant information on the entropic uncertainty threshold of the integrated magnetic flux and, also, provides a physical interpretation to the requisite presence of an imaginary part into the complex propagation variable \( \hat{x} = (x/\Delta x_0) + i (-\hat{Q}_0) \) of the GTL-HCW photon wave function \( \left[ \hat{E}_{\Delta x}(\hat{x}) \right]_{\Phi_0} \).

The presence of the entropic uncertainty threshold into the integrated magnetic flux transfer emerged naturally from the requisite presence of the dimensionless magnetic flux coupling constant \( \hat{K}_{\Phi_0} \). Independently of the energy of the absorbed GTL-HCW photon wave packet, the unique, and common to the time and frequency spaces parameter: \( \hat{Q}_0 = \exp \left[ - W_0(P_{\text{GTG}}) \right] \) is self-adapted in the description of the implemented coherent
state. The 1D position- momentum state: \(\sigma_{\alpha}\sigma_{\beta} = \hbar \hat{Q}_0 \hat{P}_0\) induced along the primary formed closed electronic loop is also relevant for the intuitive understanding of the overall effect of the Fourier transform invariant point \(\hat{Q}_0\). We can conclude with the following simple observation made on the overall effect of the constant \(K_{\Phi_0}\); the infinite- like moment of inertia of the molecular ion core provides a singularly sharp inertial window associated with the lower and upper bound integral actions of the magnetic flux transfer: 

\[
[S_1^{\text{BF}}(0)^{(0)}]^{(0)} = \int_{-\infty}^{+\infty} \frac{d^2 f}{d^2 t} \text{Im} [S_1^{\text{MF}}(0, t', t)] \, dt' \text{ and } [S_1^{\text{BM}}(0)]^{(0)} = \int_{-\infty}^{+\infty} \text{Im} [S_1^{\text{MF}}(0, t')] \, dt'.
\]

The overall effect of the integrated magnetic flux transfer into the formation of the fundamental magnetic flux quantum then unveils the presence of the 3D magnetic flux coupling constant; i.e., 

\[
\sqrt{[S_1^{\text{BM}}(0)^{(0)}][S_1^{\text{BF}}(0)^{(0)}]} = [K_{\Phi_0}]^\frac{1}{2}. \text{ This is the geometric mean of the lower and upper bound magnetic flux integral actions.}
\]

### 6. Discussion and concluding remarks

A quantized magnetic flux resonance can be induced into a free-rotating molecule if the magnetic field vector of a photon wavepacket exhibits a fixed space polarity during the photon- molecule interaction process. This is the case for instance for the absorption of a GTL-HCW optical photon wavepacket. The key aspects of this process then emerge naturally from the presence of a maximum possible energy uncertainty \(\Delta E_0\), overlapping coherently two or more distinct electronic states, and the transfer of the corresponding entropic uncertainty defined in the time and frequency spaces. The synergetic effect of the electric and magnetic fields will first result into the formation of a TEES system and the subsequent photon-induced implementation of a fundamental magnetic flux quantum \(\Phi_0\). A requisite condition for the space confinement of such a molecular photoelectron is the FMFQ resonance condition. The FWHM time width \(\Delta t_0\) of the photon wave packet must be fixed in terms of a preselected TEES threshold energy \(E_{\text{sg}}^0\) where, within the experimental uncertainty, the observed results remain independent of the appropriate units of \(fs\) and \(eV\) we can readily observe the prominence of the temporal and spectral Shannon entropies into the photon-induced implementation of a fundamental magnetic flux quantum. The initial indeterminacy of the center frequency \(\omega_0\) of the GTL-HCW photon wavepacket is entirely removed in presence of the FMFQ resonance condition. This is because the selected TEES threshold frequency \(\omega_{\text{sg}}^0\) becomes the operating value of \(\omega_0\). The induced rotational motion of the molecular ion core is then defined from the coherent superposition of a large number of rotational states shared in an indistinguishable way by the two entangled electronic states. We have shown in this work that a photon- induced magnetic flux internal structure is present into the TEES system. The structural properties occur as integrated quantum possibilities and reveal the fundamental physical laws relying on the dissipated information entropy, the entropic uncertainty transfer, and the photon-induced position- momentum uncertainty of a confined molecular photoelectron carrying a fundamental magnetic flux quantum. The space confinement of the molecular electron into the frame of a rotating molecular ion core has been found to follow the dynamics of a diffraction pattern oscillating over a Rydberg spherical surface around a primary formed quasi-classical circular electronic loop.

It is then instructive to provide an analogy with the case of a macroscopic electronic loop formed incoherently in a Penning trap [33, 34]. When a charged particle experiences the presence of a static and uniform magnetic field and the quadrupole electric field of a Penning trap, it is drawn into a state of 3D space confinement. The trapped particle then follows a composite motion consisting of three distinct harmonic oscillations [35, 36]; a slow magnetron motion of frequency \(\omega_{\text{mg}}\) evolving in time over a plane transverse to the uniform magnetic field, a fast reduced- cyclotron motion of frequency \(\omega_{\text{c}}\) superposed in the same plane, and an axial harmonic motion of frequency \(\omega_{\text{a}}\) oscillating along the axis of the applied magnetic field. Although the axial motion depends explicitly on the size and shape of the electrode structure, the dimension of the Penning trap cannot affect the intrinsic properties of the 3D confinement process [35]. This is also the case for the quantized Hall effect [37] where, within the experimental uncertainty, the observed results remain independent of the specific material or the improved and growing techniques of the experimental devices [38]. The quantum Hall effect is not affected by structure irregularities present in a semiconductor like impurities or interface effects. It depends exclusively on fundamental constants. Experiments performed with single charged particles have provided in the past the value of the corresponding \(g\) factor as a simple measurement of a frequency ratio. In particular, for the fundamental electric charge \(e\), we have the free- electron resonance condition:

\[g_e = \omega_e / \omega_{\text{c}}\]

where, \(\omega_e\) is the spin precession (Larmor) frequency, and the free- cyclotron (Lorentz-force) frequency \(\omega_{\text{c}}\) matches the Penning trap invariance relation [35]: \(\omega_{\text{c}}^2 = \omega_e^2 + \omega_{\text{a}}^2 + \omega_{\text{c}}^2\). Although the effective value of \(g_e\) has been found to differ from the expected Dirac value \(g_D = -2\) [7] only by a small fraction, this small discrepancy was the precursor of the singular success [39, 40] of the QED theory. The Schwinger-Dirac ratio \(g_e / g_D = 1 + \alpha / 2\pi\) contains in fact the one loop quantum mechanical correction and occurs in the FMFQ implementation process as the key interaction constant; i.e., in the presence of an energy uncertainty.
transfer \(0 \leq \Delta E \leq \Delta E_0\) made with a dimensionless variable \(\hat{s}^0 = \ln (\Delta E/\Delta E_0)\), the trapped FMFQ photoelectron is drawn into the minimum \(|\hat{s}^0_{\text{min}}|\) of a bound potential defined exclusively from the photon-induced position-momentum uncertainty state of a confined quantum particle. This minimum obeys the threshold law: \(\hbar/2|\hat{s}^0_{\text{min}}| = \left[\sigma_\hbar \sigma_\rho \right]_{\text{min}}\) and, the time-energy uncertainty in the proteiform electronic loop has been found to obey the irreducible fundamental relation: \(|\hat{s}^0_{\text{min}}| (\Delta E_0[\text{eV}] \Delta t_0[\text{fs}] / \Phi_0[\text{Wb}])^{1/2} = \hat{\xi}.\) The position-momentum and time-energy uncertainties of a molecular electron carrying a magnetic flux quantum cannot operate separately. Let us remind to this regard the fact that, although the magnetic flux confined in macroscopic superconducting loop/cylinder systems has provided experimental evidence [1, 2, 41] for the quantized values of the magnetic flux, more generally, this phenomenon is a consequence of the Aharonov-Bohm effect [21] which can be observed equally well in non-superconducting systems.

We have shown in the present work that an excited molecular electron experiencing the impulse electromagnetic field of a GTL—HCW photon wave packet can be also confined into the rotating frame of a molecular ion core. The unique and monotonous time increase of the electric field of a GTL—HCW interacting photon induces a uniform magnetic field of fixed space polarity and the joint effect of these fields can result only into the formation of a proteiform closed electronic loop responsible for a FMFQ photoionization quenching. A very first analogy of such a MFF-3D confinement of an electric charge with the incoherent 3D macroscopic confinement of a charge in a Penning trap provides valuable information. The axial harmonic component in the Penning trap can be first associated with the (moment of inertia depended) induced angular frequency of the ion core, and the slow magnetron motion with the prior formation of a closed electronic loop carrying a fundamental magnetic flux quantum \(\Phi_0\). Due to the implemented TEES coherence into a single free-rotating molecule, the analogy with the fast reduced—cyclotron motion is much less intuitive; i.e., the excited molecular electron cannot be observed during the impulse GTL—HCW—photon—molecule interaction as a point-like particle. The interaction can be realized only in presence of a photon—induced TEES system because the intrinsic uncertainty \(\Delta E_0\) in the energy of the absorbed photon overlaps coherently the threshold electronic transition energy \(E^0_{\text{ss}}\) of a two electronic state entangled system. It is impossible to precisely determine the position and energy of the excited molecular electron along the photon-induced circular loop. The electric charge of the interacting molecular electron is equally well present over the circular path of a closed loop because of the non-locality property of the quantum entanglement. Instead of the spiral fast cyclotron motion of a point-like charged particle in a Penning trap, where the point to point well defined electric and magnetic fields are operating, we have the simultaneous formation of an infinite—like number of equiprobable pairs of magnetic flux loops of opposed helicity superposed to the circular loop of the well defined fundamental magnetic flux quantum. Their presence at the minimum of a bound potential is due exclusively to the minimized effect of the \(|l| \geq 2\) harmonics of the FMFQ resonance. Such a paired configuration of harmonic frequencies can only condense into the minimum of a bound potential, which is then responsible for the photoionization quenching of the well defined magnetic flux quantum \(\Phi_0\). The FMFQ implementation process is performed with the lowest allowed amount of dissipated information entropy at the threshold of a bound potential following the principle of least action.

Although the present contribution may be of fundamental interest in Physics, it may as well target a wider audience in the field of Chemistry, Quantum Information and Quantum Computing. Given that the photon’s wave—particle duality is found to be challenged by the transfer of the fundamental quantum intrinsic uncertainties into a free rotating molecule, the magnetic flux resonance condition unveils new aspects in the light—matter interaction.

Acknowledgments

I would like to thank, A Mainos for the technical support on this manuscript, and J Baudon and M Ducloy for useful suggestions.

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References

[1] Deaver BS Jr and Fairbank W M 1961 Phys. Rev. Lett. 7 43
[2] Doll R and Näbauer M 1961 Phys. Rev. Lett. 7 51
[3] London F 1948 Phys. Rev. 74 562
[4] London F 1950 Superfluids vol 1 (New York: Wiley)
[5] Ginzburg V L and Landau L D 1950 J. Exp. Theor. Phys. (U.S.S.R) 20 1064
[6] Schwinger J 1948 Phys. Rev. 73 416
Schwinger J 1949 Phys. Rev. 76 790
[7] Dirac P A M 1928 Proc. Roy. Soc. London, Series A 117 610
[8] Sommerfeld A 1916 Ann. Phys. 51 125–67
[9] Einstein A 1905 Ann. Phys. 17 132
[10] Millikan R A 1914 Phys. Rev. 4 73
Millikan R A 1916 Phys. Rev. 7 362
[11] Hirschman I I 1957 American Journal of Mathematics 79 152
[12] Shannon C 1948 A mathematical theory of communication Bell Syst. Tech. J. 27 379
Shannon C 1948 A mathematical theory of communication Bell Syst. Tech. J. 27 623
[13] Gallagher T 1994 Rydberg Atoms (Cambridge, UK: Cambridge Univ. Press)
[14] Alber G and Zoller P 1991 Phys. Rep. Rev. Sect. Phys. Lett. 199 232–80
[15] Tomlin K 1999 Natural systems of units: to the centenary anniversary of the planck system Proc. of the XXII Workshop on High Energy Physics and Field Theory p 287
[16] Butcher P N 1965 Nonlinear Optical Phenomena (Columbus OH: Ohio State University, Engineering Publications)
[17] Planck M 1899 Sitzungsberichte d. Preuss. Akad. Wiss. 5 440
[18] Herzberg G 1957 Spectra of Diatomic Molecules (Princeton, NJ: Van Nostrand)
[19] Edwards H M 1974 Riemann’s Zeta Function (New York: Academic)
[20] Sommerfield C M 1958 Ann. Phys. 5 26
[21] Aharonov Y and Bohm D 1959 Phys. Rev. 115 485
[22] Corless R M et al 1996 On the Lambert W function Adv. in Comp. Math. vol 5 (Berlin: Springer)
[23] Schrödinger E 1926 Phys. Review 28 1049
[24] Butcher P N 1965 Nonlinear Optical Phenomena (Columbus OH: Ohio State University, Engineering Publications)
[25] Hougen J T 1970 Monograph N° 135 (Washington, DC: N.B.S)
[26] Edmonds A R 1960 Angular Momentum in Quantum Mechanics (Princeton, NJ: Princeton University Press)
[27] Abramowitz M and Stegun C A 1972 Handbook of Mathematical Functions (New York: Dover)
[28] Schrödinger E 1926 Naturwissenschaften 14 664
[29] Glauber R 1963 Phys. Rev. 130 2529
[30] Titulaer U M and Glauber R J 1966 Phys. Rev. 145 1041
[31] Sipe J E 1995 Phys. Rev. A 52 1875
[32] Smith B J and Raymer M G 2007 New J. Phys. 9 414
[33] Wineland D, Ekstrom P and Dehmelt H 1973 Phys. Rev. Lett. 31 1279
[34] Dehmelt H 1990 Rev. Mod. Phys. 62 525
Dehmelt H 1989 The Nobel Prize in Physics www.nobelprize.org
[35] Brown L S and Gabrielse G 1982 Phys. Rev. A 25 2423
[36] Odom B, Hanneke D, d’Urso B and Gabrielse G 2006 Phys. Rev. Lett. 97 030801
[37] von Klitzing K 1978 The quantized Hall effect Rev. Mod. Phys. 58 519
[38] Delahaye F et al 1986 Metrologia 22 105
[39] Gabrielse G and Hanneke D 2006 CERN CERN-2000-046, 46 35
[40] Gabrielse G, Hanneke D, Kimoshita T, Nio M and Odom B
[41] Bardeen J, Cooper L N and Schrieffer J R 1957 Phys. Rev. 108 1175