Establishing a $\nu_{\mu,\tau}$ Component in the Solar Neutrino Flux

André de Gouvêa and Hitoshi Murayama

Department of Physics, University of California
Berkeley, CA 94720

and

Theoretical Physics Group, Lawrence Berkeley National Laboratory
Berkeley, CA 94720

Abstract

We point out that the recoil electron kinetic energy spectra in the $\nu$-$e$ elastic scattering are different for incident $\nu_e$ or $\nu_{\mu,\tau}$, and hence one can in principle establish the existence of the $\nu_{\mu,\tau}$ component in the solar neutrino flux by fitting the shape of the spectrum. This would be a new model-independent test of the solar neutrino oscillation in a single experiment, free from astrophysical and nuclear physics uncertainties. For the $^7$Be neutrinos, it is possible to determine the $\nu_{\mu,\tau}$ component at BOREXINO or KamLAND, if the background is sufficiently low. Note that this effect is different from the distortion in the incident neutrino energy spectrum, which has been discussed in the literature.

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1 Introduction

The solar neutrino problem, the fact that the detected neutrino flux from the Sun is less than the predicted flux, has been known for decades since the pioneering work of R. Davis in the Homestake mine [1]. Since then, substantial progress has been made. The Kamiokande collaboration confirmed that the neutrinos are indeed coming from the Sun in a real-time experiment with directional capability [2]. Both the Kamiokande and Super-Kamiokande collaborations [3] also reported a depletion of the predicted flux. The GALLEX and SAGE experiments, which are sensitive to the (dominant) $pp$ component of the solar neutrino flux [4], directly related to the solar luminosity, also found a depletion of the predicted flux. Without relying on the standard solar model calculations, one can conclude from the data that the electron neutrino flux from the $^7$Be + $e^-$ → $^7$Li + $\nu_e$ is almost totally depleted (see, e.g., [5]). Furthermore, the credibility of the standard solar model calculations has been verified by their agreement with the helioseismology data at better than one percent level [6]. These facts amount to strong evidence of new physics in the neutrino sector, in particular neutrino oscillations.

Even though the evidence for a “real” (solar model independent) solar neutrino problem is very strong, it is not yet completely established. First, one needs to rely on (at least) two experiments to conclude that there is, model independently, a problem. It would be far more convincing if one could see a signal of neutrino oscillations in a single experiment. Second, all of the experiments have been of the disappearance type, where one sees a depletion of the predicted flux. Given the difficulty of neutrino experiments and of theoretical calculations of nuclear cross sections, an appearance experiment would be much more convincing evidence of neutrino oscillations.

The SNO experiment [7] will go a long way towards resolving the issues raised above. It is designed to measure the solar $^8$B neutrino flux via the charged-current (CC) reaction ($\nu_e + d \rightarrow e^- + p + p$) and the neutral-current (NC) reaction ($\nu_i + d \rightarrow \nu_i + n + p, i = e, \mu, \tau$). Assuming both of these processes can be well understood, a difference between the two measured fluxes would imply that there are neutrinos in the solar neutrino flux which are not of the electron type; one may even call this an appearance experiment of $\nu_{\mu,\tau}$. There is also an additional oscillation signature in the possible distortion of the neutrino energy spectrum. However, if for some astrophysical and/or nuclear-physics reason the $^8$B neutrino flux is lower than currently predicted,
Another possible concern is that the measurement of the CC/NC ratio involves the separate calibration of the efficiencies in the CC and NC processes.

On the other hand, if the current data are correct and the solar neutrinos indeed oscillate (even with an arbitrary $^8$B flux), there must be neutrinos other than $\nu_e$ in the $^7$Be neutrino flux, and their detection would be an unambiguous signal of neutrino oscillations. The $^7$Be neutrinos will be studied using $\nu$-$e$ elastic scattering at BOREXINO [9], and possibly also at KamLAND [10], if the background from natural radioactivity can be sufficiently suppressed.

In this letter, we study the prospect of establishing the $\nu_{\mu,\tau}$ component of the solar neutrino flux in a completely solar model-independent analysis. We point out that the recoil electron kinetic energy spectrum is different for $\nu_e$ and $\nu_{\mu,\tau}$. By fitting the shape of the electron energy spectrum, one can determine the fraction of $\nu_{\mu,\tau}$ in the solar neutrino flux, without relying on the predicted neutrino flux from the standard solar model. We discuss both the $^7$Be neutrinos at BOREXINO or KamLAND and the $^8$B neutrinos at Super-Kamiokande or SNO. This type of model-independent study seems to be difficult with the $^8$B neutrinos at Super-Kamiokande or SNO, but BOREXINO or KamLAND should have enough statistics to do the analysis with the $^7$Be neutrinos.

The sensitivity to the $\nu_{\mu,\tau}$ component is a strong function of the $\nu_e$ survival probability. In the parameter range of the small-angle MSW solution, one can see the $\nu_{\mu,\tau}$ component of the $^7$Be neutrino flux at more than 95% confidence level with two years of BOREXINO running, if the background is sufficiently small. Under the same conditions, the sensitivity at KamLAND would be even greater.

2 Electron Recoil Energy Spectra

The differential cross-section for elastic $\nu_i$-$e$ scattering ($i = e, \mu, \tau$) is known [11]:

$$\frac{d\sigma_i}{dy} = \frac{2G_F^2 m_e E_\nu}{\pi} \left[ g_L^2 + g_R^2 (1 - y)^2 - \frac{g_L g_R m_e}{E_\nu} y \right],$$

(1)

Another possible problem with SNO is that the CC/NC ratio does not differ from unity if the oscillation is into a sterile neutrino. We will not consider this possibility in this letter, because a sterile neutrino is theoretically not very natural (see, however, [8]).
where \( y = T/E_\nu \), \( g_L = \sin^2 \theta_W \pm 1/2 \) and \( g_R = \sin^2 \theta_W \). \( T = E_e - m_e \) is the recoil electron kinetic energy and \( E_\nu \) is the incoming neutrino energy in the lab frame. From the kinematics, \( y \) is related to the recoil electron scattering angle \( \theta \) by

\[
y = \frac{m_e}{E_\nu} \left( \frac{2 \cos^2 \theta}{(1 + m_e/E_\nu)^2 - \cos^2 \theta} \right),
\]

and ranges from \( y_{\text{min}} = T_{\text{threshold}}/E_\nu \) to \( y_{\text{max}} = (1 + m_e/(2E_\nu))^{-1} \). The sign in the definition of \( g_L \) depends on the flavor of the incoming neutrino: it is plus for \( i = e \) and minus for \( i = a \equiv \mu, \tau \) (a for active).

In the presence of oscillations, the \( y \) distribution is

\[
\frac{d\sigma_P}{dy} = P \frac{d\sigma_a}{dy} + (1 - P) \frac{d\sigma_e}{dy},
\]

where \( P \) is the oscillation probability for \( \nu_e \rightarrow \nu_a \). Note that \( d\sigma_P/dy = d\sigma_e(\sigma_a)/dy \) for \( P = 0, (1) \).

To illustrate the difference in the recoil electron kinetic energy spectra for different incoming neutrinos, we plot in Fig. 1 spectra for two neutrino energies, \( E_\nu = 10 \text{ MeV} \) (for \( ^8\text{B} \) neutrinos) and \( E_\nu = .862 \text{ MeV} \) (for \( ^7\text{Be} \) neutrinos). The curves are all normalized to unit area such that their shapes can be compared. The \( \nu_e \) vs \( \nu_a \) difference is more prominent at higher energies, but is not negligible even for the \( ^7\text{Be} \) energy.

The central idea of this letter is the following. One should fit the recoil electron kinetic energy spectrum with an arbitrary normalization, both for \( \nu_e \) and \( \nu_a \). The presence of a non-zero component of \( \nu_a-e \) scattering is the evidence of neutrino oscillations. This test does not depend on the theoretical prediction of the neutrino flux, and hence is independent of solar model and nuclear physics calculations. It can be regarded as an “appearance” experiment of \( \nu_{\mu,\tau} \), similar to the SNO experiment. The rest of this letter is devoted to discussing under what conditions such a test can be performed.

### 3 \( ^7\text{Be} \) Neutrinos

We will analyze the recoil electron kinetic energy distributions for the case of solar neutrinos produced by the electron capture reaction \( ^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu_e \). Because of the 2 \( \rightarrow \) 2 kinematics the neutrinos are mono-energetic,
Figure 1: Shape of the recoil electron kinetic energy spectrum from the reaction $\nu_i + e^- \rightarrow \nu_i + e^-$ as a function of $y = T/E_{\nu}$ for $i = e, a$ and (A) $E_{\nu} = 0.862$ MeV or (B) $E_{\nu} = 10$ MeV. All curves are normalized to unit area.

which greatly simplifies our analyses. We mostly focus on BOREXINO, because it is the only approved experiment which will specifically study the $^7$Be neutrinos. We do comment on a possible statistically superior sample from KamLAND. We find that BOREXINO can in principle show the existence of a $\nu_a$ component in the $^7$Be solar neutrino flux at the two-sigma level after two years of running, if the background is negligible.

Following the idea presented in the previous section, we will not rely on the overall rate of the scattering process, which depends on the theoretical prediction of the flux. To be completely model-independent, we use only the shape of the recoil electron kinetic energy spectrum by allowing the normalization to float in the fit. When discussing the sensitivity of BOREXINO

As a matter of fact, there are two discrete neutrino energies, due to two different final states for the $^7$Li nucleus, namely $E_{\nu} = 0.862, 0.383$ MeV (branching fractions 90% vs 10%). We focus only on the higher energy value because the lower energy one does not produce recoil electron energies above the BOREXINO threshold of 250 keV.
or KamLAND, however, we do need to use some expected neutrino flux; for this purpose, we use the Standard Solar Model (BP95) [12], plus the effect of neutrino oscillations.

The simulated “data” sample will consist of ten y bins,‡ with the number of events in the k-th bin given by

\[ N_k = \frac{N_T}{\sigma_{e,T}} \int_{y_{k-1}}^{y_k} \frac{d\sigma_P}{dy}, \]  \hspace{1cm} (4)

where \( y_k = y_{\text{min}} + (y_{\text{max}} - y_{\text{min}})k/10 \) and \( \sigma_{i,T} = \int_{y_{\text{min}}}^{y_{\text{max}}} \frac{d\sigma_i}{dy} \), for \( i = e, a, P \). We take the detection threshold energy to be 250 keV (i.e., \( y_{\text{min}} = 0.290 \)) for BOREXINO, which is limited by the \(^{14}\text{C}\) radioactivity background. Note that for the BOREXINO y range, \( \sigma_{a,T}/\sigma_{e,T} = 0.213 \). \( N_T = N_{\text{SSM}} = 55 \times \# \text{days} \) is the number of events predicted by the Standard Solar Model for BOREXINO. In the upcoming analysis, we will only consider statistical uncertainties, and no background.

A two-parameter \( \chi^2 \) fit of the “data” events was performed, by varying both \( N_T \) and \( P \) (two parameters). This is equivalent to fitting the data to a linear combination of the \( \nu_e - e \) and \( \nu_a - e \) differential scattering cross sections with arbitrary normalizations (two parameters). Fig. 2(A) shows the extracted \( P_{\text{measured}} \) as a function of \( P_{\text{input}} \) for two years of BOREXINO running. A nonzero value of \( P_{\text{measured}} \) implies the presence of \( \nu_e \to \nu_a \) oscillations.

The analysis indicates that, for two years of BOREXINO running, the active neutrino component can be seen at the one-sigma level if \( P \gtrsim 0.7 \). For \( P \approx 1 \) active neutrino oscillations would yield more than a two-sigma effect. That is the case for the so-called small angle MSW solution, which predicts \( P \approx 0.999 \). On the other hand, the so-called large angle MSW solution predicts \( P \approx 0.50 \), and the vacuum oscillations (the “just-so” solution) \( P \approx 0.55 \) [13].

A different type of analysis can be performed, with very similar results. This different analysis might prove to be useful in order to deal with the background, if it is not negligible. The integrated observable \( A_1 \) is defined by

\[ N_{\text{obs}}A_1 = -\sum_{k=1}^{10} \left( \frac{y_{k-1} + y_k}{2} - \frac{(y_{\text{min}} + y_{\text{max}})}{2} \right) N_k, \]  \hspace{1cm} (5)

\( \dagger \) The number of bins is chosen such that the bin size is roughly the same as the energy resolution of BOREXINO, so that we do not need to smear the energies.
Figure 2: Measured oscillation probability as a function of the input oscillation probability, in the $\nu_e \rightarrow \nu_{\alpha}$ scenario. See text for details. The error bars represent one-sigma statistical uncertainties only. We assume two years of (A) BOREXINO or (B) KamLAND running.

$N_{\text{obs}}$ is the number of observed events $N_{\text{SSM}} \sigma_{P,T}/\sigma_{e,T}$, and the sub(super)script 1 refers to the degree of the polynomial multiplying the data. In the absence of active neutrinos in the solar flux, $A_1 = 5.79 \times 10^{-3}$. Note that $A_1$ is defined in such a way that the contribution of any background with a flat $y$ distribution cancels.

In Fig. 3(A) we plot $A_1$ as a function of $P_{\text{input}}$, for the same conditions considered in the two-parameter fit. The results are very similar to the ones obtained earlier, as expected.

Even though the BOREXINO experiment should have enough statistics for a model-independent test of the $\nu_{\alpha}$ component in the solar $^{7}\text{Be}$ flux, the experimental effort will still be very challenging. The main concern is radioactive background from Rn, U and Th. An accurate energy calibration is also crucial. Our simple analysis is valid only when the background is sufficiently small in the signal range. If the background turns out to be significant, one can still use the recoil electron kinetic energy spectrum if (1)
Figure 3: $A_1$ as a function of the input oscillation probability, in the $\nu_e \rightarrow \nu_\alpha$ scenario. See text for the definition of $A_1$. The horizontal line indicates the value of $A_1$ when there are no active neutrinos (other than $\nu_e$) in the $^7$Be flux. The error bars represent one-sigma statistical uncertainties only. We assume two years of (A) BOREXINO or (B) KamLAND running.

the background can be reliably subtracted and (2) the statistical significance can be kept after the background subtraction.

The first assumption is rather difficult to justify. The Counting Test Facility at BOREXINO demonstrated that the background can be suppressed down to an extremely low level [4], but it was not possible to prove that it can be suppressed to the required level because the background was so low that it could not be studied! Even if the required level is achieved with the full-scale detector, understanding the energy spectrum of the background would require a challenging calibration procedure.

The validity of the second assumption, of course, depends on the level of the background. It would be extremely valuable if KamLAND could also achieve the radio-purity planned for BOREXINO, so that it can also study the recoil electron energy spectrum from the $^7$Be solar neutrinos, but with a larger fiducial volume. For comparison, the same plots as Figs. 2(A) and
are shown in Figs. 2(B) and 3(B), for two years of KamLAND running. We assume the BP95 estimate of 466 KamLAND events per day for a 1 kt fiducial volume.

4 $^8$B Neutrino

The difference in the recoil electron kinetic energy spectra between incident $\nu_e$ and $\nu_\alpha$ is more prominent for $^8$B neutrinos than for lower energy neutrinos such as the $^7$Be neutrinos (see Fig. 1). The main complication with the $^8$B neutrinos is that, unlike the $^7$Be neutrinos, they have a continuous spectrum. The Super-Kamiokande experiment has measured the recoil electron energy spectrum from $\nu_i$-$e$ elastic scattering [14], which is a convolution of the neutrino energy spectrum and the $y$ distribution discussed in Section 2. If the spectrum is not consistent with expectations, it indicates either that (1) the neutrino energy spectrum is not the expected one, possibly due to unknown nuclear-physics uncertainties in the $^8$B beta spectrum (see, however, [15]), (2) the neutrino energy spectrum is distorted due to an energy dependent neutrino oscillation, (3) there is some fraction of $\nu_{\mu,\tau}$ in the flux, which yields a different $y$ distribution, or (4) a combination of them. The aim of this letter is to identify the possibility (3).

The identification of (3) is, in principle, possible. If one measures both the electron recoil energy and the recoil angle (which is an observable because we know the direction of the Sun at the time of the event in a real-time experiment) it is easy to solve the kinematics and calculate both the incident neutrino energy $E_\nu$ and $y$. Then one can select events with some specific value of $E_\nu$ and study the $y$ distribution.

This program, unfortunately, cannot be done at Super-Kamiokande. The main reason is that the recoil angle distribution to too forward-peaked, $\cos^2 \theta \gtrsim 0.9$ from Eq. 4, while the angular resolution is $25^\circ$ to $35^\circ$ in the relevant energy range [16]. The strong forward peak happens because of the high energy threshold for the recoil electron. Large Time Projection Chamber (TPC) experiments, such as ICARUS [17] or HELLAZ [18] might have enough angular and recoil energy resolution to attempt such a program; indeed, HELLAZ quotes a 35mrad ($\sim 2^\circ$) angular resolution and a 3% $T$ resolution, which is enough for our purposes. However, their statistics is very limited (O(1) events per day) and a positive result would require too long a running time.
SNO studies the recoil electron energy from the charged current reaction $\nu_e + d \rightarrow e^- + p + p$, where the energy of the electron is approximately $T = E_\nu + (m_n - m_p - m_e) - B$, where $B = 2.2$ MeV is the deuteron binding energy, when the kinetic energy of the recoil protons is neglected. The measurement of this recoil electron energy spectrum does not reflect the $y$ distribution discussed in Section 2, but rather the neutrino energy spectrum. This is, of course, a very valuable information in order to study the distortion of the neutrino energy spectrum due to oscillations. This is, however, not the effect we wished to study in this letter.

In principle, one can also try to deconvolute the recoil electron energy spectrum at Super-Kamiokande using the measured neutrino energy spectrum from SNO and then determine the presence of a $\nu_a$ component in the $^8$B flux via the methods presented in the previous sections. As a matter of fact, the SNO experiment itself could also use the elastic scattering part of its signal to do this analysis. In principle, SNO could establish active neutrino oscillations even without its neutron capture capabilities. This would, however, require a large elastic scattering sample and hence a very long running time.

5 Conclusion

It seems promising to try to establish neutrino oscillations by analyzing the recoil electron kinetic energy spectrum in the case of $^7$Be neutrinos. In particular we have shown that, in the case of negligible background, two years of BOREXINO running should be enough to determine the presence of a $\nu_{\mu,\tau}$ component in the solar neutrino flux model-independently if $P(\nu_e \rightarrow \nu_a) \sim 1$. Under the same conditions, KamLAND is capable of obtaining even more significant results. We emphasize that this effect is unrelated to the distortion of the incident neutrino energy spectrum, which has been thoroughly discussed in the literature.

It is certainly not clear that the background will be negligible. Unfortunately we cannot simulate its effects clearly. Instead, we chose to define two different methods of establishing active neutrino oscillations. We believe that the background will behave differently under the two methods, and therefore be more readily extracted. Another crucial issue is, of course, the energy calibration. It is clear that a more thorough analysis can only be performed by detailed simulations of the detectors in question (and by the experiments...
themselves!), which is beyond the scope of our letter.

Finally, the situation with the $^8$B neutrinos is much less clear, in part due to their continuous energy spectrum. It is hard to disentangle distortions in the neutrino energy spectrum, possibly due to oscillations, from changes in the recoil electron energy spectrum due to a $\nu_{\mu,\tau}$ component in the solar flux. The TPC appears to be the right technology for this purpose, even though the currently proposed TPC-based experiments, ICARUS and HELLAZ, will not have enough statistics.

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