Effects of dephasing on electron localization in a double quantum dot system

C. Dedes, A. F. Terzis and E. Paspalakis

1 Physics Department, School of Natural Sciences, University of Patras, Patras 265 04, Greece
2 Materials Science Department, School of Natural Sciences, University of Patras, Patras 265 04, Greece
3 E-mail: terzis@physics.upatras.gr

Abstract. The influence of dephasing due to the presence of a quantum point contact detector on electron localization in a double quantum dot structure is studied with the use of the reduced density matrix equations. We show that when the electron localization is induced by the presence of a Coulomb charging mechanism, then the localization is quite fragile to the presence of dephasing, even for intermediate values of the dephasing rate. So, the system is rapidly driven to a statistical mixture. However, in the case when the electron localization is achieved by the application of an appropriate AC field the system is more robust to dephasing, even for large values of the dephasing rate. In this case, too, we are led to a statistical mixture of the system for long times, but the dephasing times are significantly enhanced and the effective dephasing rate is effectively slowed down.

1. Introduction

The dynamics of an electron in double quantum dot structure has attracted a lot of attention recently, as this system is one of the main candidates for a solid state quantum bit (qubit). The controlled behaviour of the system is achieved either by manipulation of the internal parameters of the system or by application of external fields. A major obstacle in performing quantum computations is the occurrence of dephasing due to either interaction of the qubit with the environment or due to application of a measurement in the qubit [1]. This effect leads to destruction of the coherent evolution of the system and the density matrix of the system is driven into a statistical mixture. Such a process could also severely affect quantum dot qubits [2, 3, 4].

In this work, we study the effects of dephasing on a double quantum structure due to measurement with a quantum point contact. We apply the model developed by Gurvitz [2], where the appropriate Bloch-type equations for the reduced density matrix are used for the description of the dynamics of populations and coherences in the system. We give special attention to cases where the electron is kept localized in one of the quantum dots in the absence of dephasing. The latter can happen either by introducing a Coulomb charging mechanism [5, 6] or by applying an AC driving field to the quantum dot structure [7, 8, 9, 10, 11]. We show by numerical solution of the appropriate equations of motion that the above two cases give quite different response on dephasing. In both cases, at long times, the system is led to a statistical mixture with equal populations in both quantum dots. However, even in the large dephasing rate regime the application of an appropriate AC driving field could significantly slow down the
dephasing process. This is not the case when only Coulomb charging effects are significant in the system.

2. Theoretical Model and Numerical Results
The system under study is described by a double quantum dot structure with one electron in it which interacts with a quantum point contact detector. The two quantum dots, 1 and 2, are taken to be identical and each quantum dot contains only one energy level which we name \( |\sigma_1\rangle \) and \( |\sigma_2\rangle \). The tunneling coupling coefficient between the two quantum dots is denoted by \( \kappa \), which is assumed positive as this does not change the results of the paper for the chosen initial conditions \([6]\). The quantum point contact performs a measurement on the location of the electron between the two quantum dots. This is done by observation of the change in the (macroscopic) electric current in the quantum point contact.

For small quantum dots the capacitance, \( C \), of the structure is small, so the Coulomb charging energy that is defined by \( E_c = e^2/(2C) \), can be important \([12]\). Indeed, for a quantum dot structure with quite small capacitance, \( C \leq 10^{-16} \text{ F} \), \( E_c \) can be of the order of several meV. Therefore, for quantum dot structures with small capacitance the Coulomb charging effect should be accounted for in the description of the system. Here, we use the approach proposed by Tsukada and co-workers \([5]\) for the description of the Coulomb charging effect.

As proposed by Gurvitz \([2]\) the theoretical analysis of this system can be done by the reduced density matrix equations for the double quantum dot where all quantum point contact states are traced out. We follow the same procedure here for the derivation of the reduced density matrix equations and in addition we include a time-dependent energy mismatch term between the energies of the two quantum dots. The energy mismatch term can be used to describe the effects of interaction of an AC field with the system and the effects of Coulomb charging. These equations read

\[
\frac{d}{dt} \sigma_{11}(t) = i\kappa [\sigma_{12}(t) - \sigma_{21}(t)],
\]

\[
\frac{d}{dt} \sigma_{12}(t) = \frac{i}{\hbar} \Delta \epsilon(t) \sigma_{12}(t) + i\kappa [\sigma_{11}(t) - \sigma_{22}(t)] - \frac{\Gamma_D}{2} \sigma_{12}(t),
\]

with \( \sigma_{21}(t) = \sigma_{12}^*(t) \) and \( \sigma_{22}(t) = 1 - \sigma_{11}(t) \). Here, \( \Gamma_D \) is the dephasing rate due to the existence of the point contact detector. The energy mismatch term \( \Delta \epsilon(t) \) is zero if no external field is applied to the system and if the capacitance of the system is large so that the Coulomb charging effect is negligible. If, now, the effects of Coulomb charging are present, then \( \Delta \epsilon(t) = 2\hbar \Omega [\sigma_{22}(t) - \sigma_{11}(t)] \) with \( \Omega = E_c/(2\hbar) \). Also, the interaction between the quantum dot structure and an external AC field will change the mismatch term to \( \Delta \epsilon(t) = \hbar \Omega_0 \cos(\omega t) \), where \( \Omega_0 \) is the Rabi frequency which determines the strength of the interaction between the quantum dot system with the external field and \( \omega \) is the angular frequency of the AC field.

We first describe the dynamics of the system in the case that no dephasing is present, i.e. for \( \Gamma_D = 0 \). For the initial condition in our study we choose the electron to be initially localized in one of the quantum dots, let’s say the quantum dot 1, i.e. \( \sigma_{11}(t = 0) = 1, \sigma_{22}(t = 0) = 0 \), and \( \sigma_{12}(t = 0) = \sigma_{21}(t = 0) = 0 \). Obviously, in the absence of an AC field driving and with zero Coulomb charging energy the probabilities will read \( \sigma_{11}(t) = \cos^2(\kappa t) \) and \( \sigma_{22}(t) = \sin^2(\kappa t) \). Therefore, the electron will undergo coherent oscillations between the left and right quantum dots with period \( 2\pi/\kappa \).

If we now include the effect of Coulomb charging, then the behaviour of the system is crucially dependent on the value of \( \Omega/(2\kappa) \). If \( \Omega < 2\kappa \) then the system oscillates between the two quantum dots. In the case that \( \Omega \geq 2\kappa \) a self-trapping behaviour is observed, and localization of the electron in the initially occupied quantum dot occurs \([5, 6]\). This localization becomes more pronounced as \( \Omega \) becomes much larger than \( 2\kappa \). On the other hand, if the quantum dot system
interacts only with a high frequency AC field, such that $\omega \gg \kappa$, the effective tunneling coefficient that the system experiences becomes $\kappa J_0(\Omega_0/\omega)$ [7, 8, 9, 10, 11], where $J_0(\cdot)$ is the zeroth order Bessel function. Thus, if $\Omega_0/\omega$ is a root of $J_0$ we obtain dynamical localization of the electron in quantum dot 1, a phenomenon which is also known as coherent destruction of tunneling [7].

In what follows we turn to the study of the effects of dephasing in the system when electron localization phenomena are present. As exact, or useful approximate, analytic solutions of equations (1) and (2) in their generic form do not exist, we present results obtained from the numerical solution of equations (1) and (2). In the numerical calculations the tunneling coefficient is chosen to be $\hbar\kappa = 0.5$ meV.

In Figure 1 we investigate the effect of dephasing on the localization of an electron due to the presence of Coulomb charging energy. We choose $\Omega = 2.5\kappa$ and present results for the probabilities in states $|1\rangle$ and $|2\rangle$ for four values of the dephasing rate $\Gamma_D$. From Fig. 1(a) it is clear that the electron is localized in the absence of dephasing with probability greater than (approximately) 80%. If now the dephasing is included the dynamics of the system is quite different. The presence of dephasing influences crucially the localization of the electron, the electron is rapidly delocalized and the system goes to a mixed state, where for large times $\sigma_{11}(t) = \sigma_{22}(t) = 1/2$ and $\sigma_{12}(t) = \sigma_{21}(t) = 0$.

Figure 1. Time evolution of $\sigma_{11}(t)$ (solid curve) and $\sigma_{22}(t)$ (dashed curve) in the presence of Coulomb charging with $\Omega = 2.5\kappa$ and (a) $\Gamma_D = 0$, (b) $\Gamma_D = \kappa$, (c) $\Gamma_D = 2\kappa$ and (d) $\Gamma_D = 10\kappa$.

Figure 2 displays the effects of dephasing on coherent destruction of tunneling, i.e. in the case when the electron localization in a double quantum dot structure is achieved by the presence of an AC field with specific properties. We choose $\omega = 15\kappa$ and $\Omega_0 = 2.405\omega$ in order to have $\omega \gg k$ and $\Omega_0/\omega$ to be close to the first root of $J_0(\Omega_0/\omega)$. In the absence of dephasing, as it is shown in Fig. 2(a), strong electron localization is achieved in the initially occupied quantum dot. The effect of dephasing is to gradually influence the localization of the electron, as it is shown in Figs. 2(b) - 2(d). In this case, too, a mixed state of the system occurs for long times. However, in contrast to the case of Fig. 1, when we have electron localization by Coulomb charging, the times that the mixed state is created are significantly enhanced by the existence of the external AC field. In other words the existence of an AC field with parameters that obey the relation
\(J_0(\Omega_0/\omega) = 0\) slows down the effective dephasing time of the system and increases the time that the system stays localized in the initially occupied quantum dot.

Figure 2. Time evolution of \(\sigma_{11}(t)\) (solid curve) and \(\sigma_{22}(t)\) (dashed curve) in the presence of an AC field with \(\omega = 15\kappa\), \(\Omega_0 = 2.405\omega\) and (a) \(\Gamma_D = 0\), (b) \(\Gamma_D = \kappa\), (c) \(\Gamma_D = 2\kappa\) and (d) \(\Gamma_D = 10\kappa\).

3. Summary
In summary, we have studied the influence of dephasing due to the presence of a quantum point contact detector on electron localization in a double quantum dot structure. We show that when the electron localization is induced by the effect of self-trapping due to the presence of a Coulomb charging mechanism then, even for intermediate values of dephasing rate, the system is rapidly driven to a statistical mixture. However, in the case when the electron localization is achieved by the application of an appropriate AC field, the system is more robust to dephasing, even for large values of the dephasing rate. The system is driven, in this case too, in a statistical mixture for long times, but the effective dephasing rate is significantly and effectively slowed down. The latter can be potentially useful in the area of quantum computation in nanostructures.

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