Analysis of Stability Margin of Dynamic Inverse Control Law for Flying Wing UAV

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Abstract: The longitudinal and directional stability of flying wing UAV is poor, the coupling of triaxial force and torque is serious, and the control surface is special, so the flight control is difficult and the control law is complex. Taking a flying wing UAV as an example, the control laws of track tracking based on dynamic inverse control method and control allocation are designed. The applicability of traditional stability margin excitation method based on signal generator and pilot’s manual control is analyzed, and a stability margin excitation method based on three-a2xis control decoupling is proposed, which is verified by simulation. The results show that the stability margin of the designed three-axis dynamic inverse control law system meets the requirements, and the stability margin excitation method based on three-axis decoupling is suitable for the stability margin analysis of three-axis coupled UAV.

1. Introduction
The three-axis control coupling of flying wing aircraft is serious, and the control law is complex\cite{1-2}, so it is necessary to focus on the stability margin test during scientific research and flight test. On the one hand, the stability margin of the aircraft can be calculated in quasi real time through the flight test data, which is conducive to the state monitoring of the aircraft during the flight test, ensuring the flight test safety during the envelope expansion, and reducing the flight test risk; on the other hand, the flight test of stability margin can provide the flight test data to the designers for the verification and optimization of the aircraft flight quality\cite{3} and flight control system.

For the stability margin flight test of fly by wire flight control system at home and abroad, the closed-loop time response of the aircraft is obtained by sweeping frequency excitation, and then the open-loop frequency domain characteristics of the aircraft are obtained by time-frequency transformation, so as to obtain the amplitude margin and phase margin of the actual system. A large number of stability margin flight tests have been carried out, and the sweeping frequency signal is mainly generated by the pilot’s manual control or FES (flutter excitation system), the waveform of longitudinal and transverse sweep signals generated by signal generator and manual control is poor, which affects the results of data processing, while it is difficult to generate stable sweep signals by pedal control. The sweep signals generated by FES signal generator are standard, and three-axis sweep can be realized, but there is a risk of flight test when the signal generator fails. At present, in the stability margin flight test, when the aircraft reaches the predetermined flight state point, the pilot uses the joystick to generate the pitch and roll frequency conversion sine excitation signals, and uses the signal generator to apply the frequency
conversion sine signal to the heading, so as to complete the three-axis stability margin flight test. Western countries have carried out stability margin test flight on manned aircraft, UAV and scaled model. NASA has developed a case of quasi real-time stability analysis technology for X-29A aircraft, including manual frequency sweep and quasi real-time stability analysis. McDonnell Douglas incorporated sweep excitation into the automation system used in the X-36 flight test, and completed the stability margin flight test by using flight control autonomous excitation. After that, in X-43A and NF-15b intelligent flight control system aircraft, the multi frequency sine excitation automatic injection technology is used to complete the stability analysis of multiple axes[4]. Boeing company has developed the aircraft management system (VMS) running on the flight control computer for X48b scaled verification aircraft. The VMS partially includes an internal loop control system based on dynamic inversion, and a variety of optional internal excitations, including multiple injection points and waveforms, are embedded in the VMS.

Taking a flying wing UAV as an example, the inner loop and outer loop control systems based on dynamic inversion are designed. The applicability of the traditional stability margin excitation method based on flight signal generator and pilot’s manual control is analyzed, and a stability margin excitation method based on three-axis decoupling is proposed, which is verified by simulation.

2. Research objects
As shown in Figure 1, the flying wing UAV studied in this paper has 7 control surfaces, namely split rudder (δ3l, δ3r), inner lift aileron (δ1l, δ1r), outer lift aileron (δ2l, δ2r), δ4 are beaver tail shaped auxiliary pitching control surfaces. According to the spatial characteristics of the trailing edge of the flying wing aircraft and the requirements of the three-axis control efficiency, the redundant configuration of the control surface is adopted, and the lift aileron is used as the redundant control surface in the vertical and horizontal directions[5].

3. Design of control law

3.1. Scheme of control law
According to the requirements of aircraft flight quality evaluation, the flight control system is simplified. The whole flight control system can be divided into three main modes: command generation, command calculation and control distribution. Among them, the input of the command generation module is the control command given by the pilot or the control system[6]. Firstly, the desired command yrm and its derivative ẏrm are generated through the reference model, and then they are combined to be control input v according to certain rules; the command v solving module calculates the pseudo control input for the input v. The input of the control allocation module is the desired three-axis control torque M, and the output is the deflection of each control surface u. The overall structure is shown in Figure 2.
Among them, the input of the system is the control command. According to the requirements of the flight control system, the pilot's command, attitude or trajectory control command can be used, and the output command to the aircraft is the control surface deflection command. The following will discuss the above three modules respectively.

3.2. Command generation

The trajectory angle command configuration pays more attention to the flight trajectory of the aircraft, and its system input commands are flight speed and flight speed $V_c$, longitudinal track angle $\gamma_c$, heading track angle $\chi_c$.

3.2.1. Expected command generation. It is expected that through the design of flight control system, the dynamic model between the response of velocity vector $[V_c \gamma_c \chi_c]^T$ is confirmed by online debugging.

So the dynamic equation of the desired velocity vector is as follows:

$$
\begin{align*}
\dot{V}_c &= k_v (V_c - V) \\
\dot{\gamma}_c &= k_\gamma (\gamma_c - \gamma) \\
\dot{\chi}_c &= k_\chi (\chi_c - \chi)
\end{align*}
$$

(1)

Where: $V$ is flight velocity, $\gamma$ is longitudinal track angle, $\chi$ is directional track angle; subscript $c$ is the required command; $\omega$ is the frequency band or gain of the velocity vector $[k_v, k_\gamma, k_\chi]^T$ is confirmed by online debugging.

3.2.2. Maneuver command generation. The dynamic equation of aircraft centroid is

$$
\begin{align*}
MV \dot{\alpha} &= T \cos \alpha - D - Mg \sin \gamma \\
MV \dot{\gamma} &= T \sin \alpha \sin \mu + L \sin \mu \\
MV \dot{\chi} &= T \sin \alpha \cos \mu + L \cos \mu - Mg \cos \gamma
\end{align*}
$$

(2)

The thrust command can be calculated from the above equation $T_c$. And air flow angle command $[\alpha_c, \mu_c]^T$, and set sideslip angle command $\beta_c$ as zero.

3.2.3. Expected characteristics of air flow angle. In order to ensure the fast response of the trajectory command, the expected response of the air flow angle is the same as that of the command[7].

Therefore, the dynamic equation of the expected flow angle is as follows:

$$
\begin{align*}
\dot{\alpha}_c &= k_\alpha (\alpha_c - \alpha) \\
\dot{\beta}_c &= k_\beta (\beta_c - \beta) \\
\dot{\mu}_c &= k_\mu (\mu_c - \mu)
\end{align*}
$$

(3)

$[k_\alpha, k_\beta, k_\mu]^T$ is confirmed by online debugging.

3.3. Command calculation

The task of the command calculation module is to obtain the three-axis control torque required to complete the given control command through the dynamic inverse calculation according to the pseudo control input.

If the curvature and rotation of the earth are ignored, the influence of wind is not considered, and the aircraft is regarded as a rigid body, the dynamic relationship between the angular rate of rotation and the angle of attack, the sideslip angle, the roll angle of the airflow axis and its derivatives is as
follows

\[
\begin{bmatrix}
\dot{\alpha} \\
\dot{\beta} \\
\dot{\mu}
\end{bmatrix}
= \mathbf{g}^{-1}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
+ \mathbf{f}
\]

(4)

From equation (4), it can be obtained that:

\[
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
= \mathbf{g}^{-1}
\begin{bmatrix}
\dot{\alpha} \\
\dot{\beta} \\
\dot{\mu}
\end{bmatrix}
- \mathbf{f}
\]

(5)

In addition, by deriving from equation (4), we can get the following results:

\[
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix} = \mathbf{g}^{-1}
\begin{bmatrix}
\ddot{\alpha} \\
\ddot{\beta} \\
\ddot{\mu}
\end{bmatrix}
- \mathbf{g}'
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\]

(6)

By further transforming equation (6), it is obtained that:

\[
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
= \mathbf{g}^{-1}
\begin{bmatrix}
\ddot{\alpha} \\
\ddot{\beta} \\
\ddot{\mu}
\end{bmatrix}
- \mathbf{g}'
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\]

(7)

Substituting equation (6) into equation (7) yields:

\[
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
= \mathbf{g}^{-1}
\begin{bmatrix}
\ddot{\alpha} \\
\ddot{\beta} \\
\ddot{\mu}
\end{bmatrix}
- \mathbf{g}'
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\]

(8)

According to the six degree of freedom rigid body motion equation of the aircraft, it is assumed that its mass is symmetrically distributed according to the plane of symmetry.

External torque of body coordinate is

\[
\begin{bmatrix}
L \\
M \\
N
\end{bmatrix}^T
\]

\[
L = I_x \dot{\psi}c + (I_z - I_y)qr - I_z \dot{\phi} (pq + \dot{r})
\]

\[
M = I_y \dot{\phi}c + (I_x - I_z)rp + I_z \dot{\phi} (p^2 - r^2)
\]

\[
N = I_z \dot{\phi} (pq - I_z) - \dot{\phi} (r^2 - p^2)
\]

(9)

Where: \(L\) is the rolling moment, \(M\) is the pitching moment, \(N\) is the yaw moment, \(I_x, I_y, I_z, I_{xz}\) is the moment of inertia. The resultant moment \(L_b, M_b, N_b\), which are formed by the engine block are mainly composed of stabilizing moment and damping moment, can be obtained from the following formula:

\[
\begin{bmatrix}
L_b \\
M_b \\
N_b
\end{bmatrix} = \frac{qSb}{2V} \begin{bmatrix}
(C_{f\beta} + C_{lp} \frac{pb}{2V} + C_{lr} \frac{rb}{2V}) \\
(C_{n\alpha} + C_{mq} \frac{qc}{2V}) \\
(C_{n\beta} + C_{np} \frac{pb}{2V} + C_{nr} \frac{rb}{2V})
\end{bmatrix}
\]

(10)

The required three-axis control torque can be obtained by subtracting the stability torque and damping torque from the total aircraft torque \([L_c, M_c, N_c]^T\):
In the optimization method, linear programming, quadratic programming and nonlinear programming are used to realize the allocation of multiple rudders.

The optimization objectives are described as follows:

$$\min f(u)$$

The constraints are described as follows:

$$\begin{cases}
g(u) = M_d \\
u_{\text{min}} \leq u \leq u_{\text{max}}
\end{cases}$$

Where, $f(u)$ is the optimization objective, $g(u)$ is the control function of the control surface. The equality constraint defines the solution space of the control vector, while the inequality constraint reflects the position limitation of the control surface.

### 4. Stability margin excitation method

For the flying wing UAV, there are many control surfaces, many of which are not the traditional aileron, elevator and rudder, but the multi rudder coupling control. The results obtained by the conventional stability margin excitation method can not fully reflect the stability characteristics of the multi loop system after disturbance. The flight control architecture of flying wing UAV is shown in Figure 3. In the design of conventional control law, the three-axis torque is used as the input and output, and the control law architecture and control parameters are adjusted according to the root locus and bode diagram. Figure 3 shows the position of the excitation signal relative to the closed-loop system. The injection point of the excitation signal is just in front of the control surface distribution subsystem. In this position, the dimension of the controller is reduced to three channels, and the excitation corresponding to the three main axes corresponds to the roll, pitch and heading excitation commands respectively. This position can not only decouple the three-axis control, but also reduce the potential effect of the additional feedback channel of the aircraft platform on the open-loop transfer function.

### 5. Analysis by simulation

#### 5.1. Flight simulation.

In order to show the design effect of the flight control system, take the cruise state of flying wing UAV ($H = 20000m$, $Ma = 0.6$) as an example, select the aircraft to complete the constant speed circling and climbing action for simulation calculation, the control input is the speed vector $\begin{bmatrix} V_c & \gamma_c & \chi_c \end{bmatrix}^T$, and the simulation results are shown in Fig. 4 and Fig. 5.

Figure 4 shows the speed tracking results. It can be seen that the aircraft speed is almost unchanged,
and the track inclination and track deflection angle are more accurate, and can finally reach the steady state.

Figure 5 shows the response result of air flow angle. In order to realize the track command, the angle of attack of the aircraft increases at the beginning of maneuver to provide the required lift to make the aircraft climb. At the same time, the aircraft rolls right to make the lift direction point to the turning direction, and the sideslip angle is very small throughout the whole process.

5.2. Stability margin analysis.

Based on the linear sweep signal, the longitudinal and transverse stability margins are excited. Longitudinal time domain response curves and the stability margin are shown in Figure 6 and Figure 7. Lateral domain response curves and the stability margin are shown in Figure 8 and Figure 9. Directional domain response curves and the stability margin are shown in Figure 10 and Figure 11. It can be seen that the designed excitation method can effectively complete the excitation of the three-axis stability margin characteristics of the flying wing UAV, and the three-axis amplitude margin of the control system is greater than 11db, the relative margin is more than 50 degree, which meets the design requirements of the control system.

Figure 4. response of velocity, track angle

Figure 5. Response of flow angle

Figure 6. Longitudinal response under excitation

Figure 7. Longitudinal stability margin
6. Conclusion
Aiming at the characteristics of multi rudder control coupling of flying wing UAV, the control law based on dynamic inversion and the control allocation method based on nonlinear optimization are completed. In order to verify the stability margin of control system, a control decoupling stability margin excitation method is proposed, and the dynamic inversion stability margin analysis of flying wing UAV is completed. The research idea of this paper can provide reference for multi rudder control system. This paper provides a reference for stability margin analysis and flight test verification of coupled UAV control system.

Reference
[1] Wang Yankui; Tang Xiangxi; Li Tao. Lateral Stability and Control of a Flying Wing Configuration Aircraft[J]. Journal of physics, 2020.
[2] Gloria Stenfelt, Ulf Ringertz. Lateral Stability and Control of a Tailless Aircraft Configuration[J]. Journal of aircraft, 2012.
[3] Baumann E. Tailored excitation for frequency response measurement applied to the X-43A Flight Vehicle. 44th AIAA Aerospace Sciences:Meeting and Exhibit. AIAA 2006—638.
[4] Liu Chaojun, Wu Xinlong, Jing Li, et al. Research on parameter identification technology of longitudinal short period equivalent system of flying wing aircraft [J]. Aeronautical Science and technology, 2017.
[5] Han Ying Hua, fan Yan Ming. UAV automatic landing control system based on nonlinear dynamic inversion [J]. Acta Aeronautica Sinica, 2008.
[6] Zhang yubai, Xiao Chengfang, Zou Junjun, Weng Xuexue. Control law design of flying wing UAV [J]. Measurement and control technology, 2021.
[7] Yang Yunjie;Wang Xiangyang. Robust proportional incremental nonlinear dynamic inversion control of a flying-wing tailsitter. Journal of Aerospace Engineering, 2020.