Δ–Baryon Electromagnetic Form Factors in Lattice QCD

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Vienna, Preprint ESI 2118 (2009)  February 25, 2009

Supported by the Austrian Federal Ministry of Education, Science and Culture
Available via http://www.esi.ac.at
We develop techniques to calculate the four Δ electromagnetic form factors using lattice QCD, with particular emphasis on the sub-dominant electric quadrupole form factor that probes deformation of the Δ. Results are presented for pion masses down to approximately 350 MeV for three cases: quenched QCD, two flavors of dynamical Wilson quarks, and three flavors of quarks described by a mixed action combining domain wall valence quarks and dynamical staggered sea quarks. The magnetic moment of the Δ is chirally extrapolated to the physical point and the Δ charge density distributions are discussed.

PACS numbers: 11.15.Ha, 12.38.Gc, 12.38.Aw, 12.38.-t, 14.70.Dj

Lattice Quantum Chromodynamics (QCD) provides a well-defined framework to directly calculate hadron form factors from the fundamental theory of strong interactions. Form factors characterize the internal structure of hadrons, including their magnetic moment, their size, and their charge density distribution. Since the Δ(1232) decays strongly, experiments [1, 2] to measure its form factor are harder and yield less precise results than for nucleons [3, 4]. In this work, we compute Δ form factors using lattice QCD more accurately than can be currently obtained from experiment.

A primary motivation for this work is to understand the role of deformation in baryon structure: whether any of the low-lying baryons have deformed intrinsic states and if so, why. Thus, a major achievement of this work is the development of lattice methods with sufficient precision to show, for the first time, that the electric quadrupole form factor is non-zero and hence the Δ has a non-vanishing quadrupole moment and an associated deformed shape. Unlike the Δ, the spin-1/2 nucleon cannot have a quadrupole moment, so the experiment of choice to explore its deformation has been measurement of the nucleon to Δ electric and Coulomb quadrupole transition form factors. Major experiments [5–7] have shown that these transition form factors are indeed non-zero, confirming the presence of deformation in either the nucleon, Δ, or both [8, 9], and lattice QCD yields comparable non-zero results [10, 11]. Our new calculation of the Δ quadrupole form factor, coupled with the nucleon to Δ transition form factors, should shed light on the deformation of the nucleon.

In order to evaluate the Δ electromagnetic (EM) form factors to the required accuracy, we isolate the two dominant form factors and the sub-dominant electric quadrupole form factor. This is particularly crucial for the latter since it can be extracted with greater precision, although it increases the computational cost. Our techniques are first tested in quenched QCD [12]. We then calculate form factors using two degenerate flavors of dynamical Wilson fermions, denoted by , with pion masses in the range of 700 MeV to 380 MeV [13, 14]. Finally, we use a mixed action with chirally symmetric domain wall valence quarks and staggered sea quarks with two degenerate light flavors and one strange flavor [15], denoted by , at a pion mass of 353 MeV. Using the results obtained with dynamical quarks, we extrapolate the magnetic moment to the physical point. We extract quark charge distributions in the Δ, and discuss their quadrupole moment.

The Δ matrix element \( \langle \Delta(p_f,s_f)|J^\mu_{\text{EM}}|\Delta(p_i,s_i) \rangle \), where \( J^\mu_{\text{EM}} \) is the electromagnetic current, can be parameterized in terms of four multipole form factors that depend only on the momentum transfer \( q^2 \equiv -Q^2 = (p_f - p_i)^2 \) [16]. The decomposition for the on shell \( \gamma^* \Delta \Delta \) matrix element is given by

\[
\langle \Delta(p_f,s_f)|J^\mu_{\text{EM}}|\Delta(p_i,s_i) \rangle = \mathcal{A} \bar{u}_{\mathcal{E}}(p_f,s_f)\mathcal{O}^\sigma_{\nu\tau}u_{\mathcal{V}}(p_i,s_i)
\]

\[
\mathcal{O}^\sigma_{\nu\tau} = -g^{\sigma\tau}\left[a_1(q^2)\gamma^\mu + \frac{a_2(q^2)}{2m_\Delta}(p^\mu_f + p^\mu_i)\right] - \frac{q^\mu}{4m_\Delta}\left[c_1(q^2)\gamma^\nu + \frac{c_2(q^2)}{2m_\Delta}(p^\nu_f + p^\nu_i)\right],
\]

where \( a_1(q^2), a_2(q^2), c_1(q^2), \) and \( c_2(q^2) \) are known linear combinations of the electric charge form factor \( G_{E0}(q^2) \), the magnetic dipole form factor \( G_{M1}(q^2) \), the electric quadrupole form factor \( G_{E2}(q^2) \), and the magnetic octupole form factor \( G_{M3}(q^2) \) [17], and \( \mathcal{A} \) is a known factor depending on the normalization of hadron states. These form factors can be extracted from correlation functions calculated in lattice QCD [17]. We calculate in Euclidean time the two- and three-point correlation functions in a frame where the final state Δ is at rest:

\[
G(t,q) = \sum_{x_f} \sum_{j=1}^{3} e^{-i\bar{x}_f \cdot q} T^{\alpha\beta}_{\alpha\beta}(x_f) J_{\alpha}(0)
\]

\[
G_{\mu\nu}(\Gamma_{\nu},t,q) = \sum_{x_f} e^{i\bar{x}_f \cdot \bar{q}} \Gamma^{\nu}_{\alpha\beta}(x_f) j^\mu(x) J_{\alpha}(0),
\]
where $j^\mu$ is the electromagnetic current on the lattice, $J$ and $\mathcal{J}$ are the $\Delta^+$ interpolating fields constructed from smeared quarks [12], $\Gamma_4 = \frac{i}{2}(1 + \gamma^4)$, and $\Gamma_k = i\gamma^5\gamma^k$.

The form factors can then be extracted from ratios of three- and two-point functions in which unknown normalization constants and the leading time dependence cancel

$$R^\mu_\tau = \frac{G_\mu_\tau(\Gamma, t, \vec{q})}{G(t, \vec{0})} \sqrt{\frac{G(t_f - t, \vec{p}_f)G(t, \vec{0})G(t_f, \vec{0})}{G(t_f - t, \vec{0})G(t, \vec{p}_f)G(t_f, \vec{p}_f)}}.$$  \hspace{1cm} (3)

For sufficiently large $t_f - t$ and $t - t_i$, this ratio exhibits a plateau $R(\Gamma, t, \vec{q}) \rightarrow \Pi(\Gamma, \vec{q})$, from which the form factors are extracted, and we use the particular combinations

$$\sum_{k=1}^{3} \Pi_{kk}^\mu(\Gamma^4, \vec{q}) = K_1 G_{E0}(Q^2) + K_2 G_{E2}(Q^2)$$  \hspace{1cm} (4)

$$\sum_{j,k,l=1}^{3} \epsilon_{jkl} \Pi_{kk}^\mu(\Gamma^4, \vec{q}) = K_3 G_{M1}(Q^2)$$  \hspace{1cm} (5)

$$\sum_{j,k,l}^{3} \epsilon_{jkl} \Pi_{kk}^\mu(\Gamma^4, \vec{q}) = K_4 G_{E2}(Q^2).$$  \hspace{1cm} (6)

The connected part of each combination of three-point functions can be calculated efficiently using the method of sequential inversions [18]. At present, it is not yet computationally feasible to calculate the small corrections arising from disconnected diagrams. The known kinematical coefficients $K_1, K_2, K_3, K_4$ are functions of the $\Delta$ mass and energy as well as of $\mu$ and $\vec{q}$. The combinations above are chosen such that all possible directions of $\mu$ and $\vec{q}$ contribute symmetrically to the form factors at a given $Q^2$ [19]. The over-constrained system of Eqs. (4-6) is solved by a least-squares analysis, and $G_{E2}(Q^2)$ can also be isolated separately from Eq. (6).

The details of the simulations are summarized in Table I. In each case, the separation between the final and initial time is $t_f - t_i \gtrsim 1$ fm and Gaussian smearing is applied to both source and sink to produce adequate plateaus by suppressing contamination from higher states having the quantum numbers of the $\Delta(1232)$. For the mixed-action calculation, the domain-wall valence quark mass was chosen to reproduce the lightest pion mass obtained using $N_F = 2 + 1$ improved staggered quarks [19, 20].

The results for $G_{E0}(Q^2)$ are shown in Fig. 1 as a function of $Q^2$ at the lightest pion mass for each of the three actions. For Wilson fermions, we use the conserved lattice current requiring no renormalization. The local current is used for the mixed action, and the renormalization constant, $Z_{V} = 1.0992(32)$, is determined by the condition that $G_{E0}(0)$ equals the charge of the $\Delta$ in units of $e$. As can be seen, all three calculations yield consistent results. The momentum dependence of the charge form factor is described well by a dipole form

$$G_{E0}(Q^2) = 1/(1 + \frac{Q^2}{\Lambda_{E0}^2})^2.$$  \hspace{1cm} (7)

To compare the slopes at $Q^2 = 0$, we follow convention and show in Table I the so-called “rms radius” $\langle r^2 \rangle = -\frac{\partial}{\partial Q^2} G_{E0}(Q^2) \bigg|_{Q^2=0}$.

The momentum dependence of $G_{M1}(Q^2)$ is displayed in Fig. 2. To extract the magnetic moment, an extrapolation to zero momentum transfer is necessary. Both an exponential form, $G_{M1}e^{-Q^2/\Lambda_{M1}^2}$, and a dipole describe the $Q^2$-dependence well, and we adopt the exponential form because of its faster decay at large $Q^2$, in accord with perturbative arguments. The larger spatial volume for the quenched and mixed action cases yields smaller and more densely spaced values of the lattice momenta and correspondingly more precise determination of the form factor than for the smaller volume used with dynamical Wilson fermions. In Fig. 2, we show the best exponential fit and error band for the mixed action and quenched results. As can be seen, results in the quenched theory and for $N_F = 2$ Wilson fermions are within the error band. The magnetic moment in nuclear units is given by $\mu_{\Delta} = G_{M1}(0)e/(2m_{\Delta})$, where $m_{\Delta}$ is the $\Delta$ mass measured on the lattice and $G_{M1}(0)$ is from the exponential fits. In Table I we give the values of the $\Delta^+$ magnetic moment in nuclear magnetons $\mu_e/(2M_N)$, with $M_N$ the physical nucleon mass. The magnetic
moments of the $\Delta^+$ and $\Delta^{++}$ are accessible to experiments [1, 2], which presently suffer from large uncertainties. The magnetic moment as a function of $m_\pi^2$ is shown in Fig. 3, together with a chiral extrapolation to the physical point [22], which lies within the broad error band $\mu_{\Delta^+} = 2.7^{+1.0}_{-1.3}(\text{stat.}) \pm 1.5(\text{syst.}) \pm 3.0(\text{theory}) \mu_N$ [1]. The $\Delta$ moments using an approach similar to ours are calculated only in the quenched approximation [17, 23, 24]. Our magnetic moment results agree with recent background field calculations using dynamical improved Wilson fermions [25], which supersede previous quenched background field results [26]. The spatial length $L_s$ of our lattices satisfies $L_s m_\pi > 4$ in all cases except at the lightest pion mass with $N_f = 2$ Wilson fermions, for which $L_s m_\pi = 3.6$. For that point, the magnetic moment falls slightly below the error band, consistent with the fact that Ref. [25] shows that finite volume effects decrease the magnetic moment.

The electric quadrupole form factor is particularly interesting because it can be related to the shape of a hadron, and lattice calculations for each of the three accelerations are shown in Fig. 4 with exponential fits for the quenched and mixed action cases. Just as the electric form factor for a spin 1/2 nucleon can be expressed precisely as the transverse Fourier transform of the transverse quark charge density in the infinite momentum frame [27], a proper field-theoretic interpretation of the shape of the $\Delta(1232)$ can be obtained by considering the quark transverse charge densities in this frame [28–30]. With respect to the direction of the average baryon momentum $P$, the transverse charge density in a spin-3/2 state with transverse polarization $s_\perp$ is defined as:

$$\rho_{T s_\perp}(\vec{b}) \equiv \int \frac{d^2 q_{\perp}}{(2\pi)^2} e^{-i \vec{q}_{\perp} \cdot \vec{b}} \frac{1}{2P^+} \langle P^+, \vec{q}_{\perp}/2, s_\perp | J^{+}(0) | P^+, -\vec{q}_{\perp}/2, s_\perp \rangle,$$

where the photon transverse momentum $\vec{q}_{\perp}$ satisfies $\vec{q}_{\perp}^2 = Q^2$, $J^+ \equiv J^0 + J^3$, and $\vec{b}$ specifies the quark position in the $xy$-plane relative to the $\Delta$ center of mass. Choosing the $\Delta$ transverse spin vector along the $x$-axis, the quadrupole moment of this two-dimensional charge distribution is defined as [19]:

$$Q_{s_\perp}^{\Delta} = e \int d^2 \vec{b} (\vec{b}_x^2 - \vec{b}_y^2) \rho_{T s_\perp}^{\Delta}(\vec{b}).$$

In terms of the $\Delta$ EM form factors [19],

$$Q_{s_\perp}^{\Delta} = \frac{1}{2} \left( 2 |G_{M1}(0) - 3e_\Delta| + |G_{E2}(0) + 3e_\Delta| \right) \frac{e}{M_\Delta}.$$
In summary, a formalism for the accurate evaluation of the \( \Delta \) electromagnetic form factors as functions of \( q^2 \) has been developed and used in quenched QCD and full QCD with \( N_F = 2 \) and 2+1 flavors. The charge radius and magnetic dipole moment were determined as a function of \( m^2_\pi \) and the dipole moment was chirally extrapolated to the physical point. The electric quadrupole form factor was evaluated for the first time with sufficient accuracy to distinguish it from zero. The lattice calculations show that the quark density in a \( \Delta^+ \) of transverse spin projection +3/2 is elongated along the spin axis.

Acknowledgments

This work is supported in part by the Cyprus Research Promotion Foundation (RPF) under contract ΠΕΝΕΚ/ΕΝΙΣΧ/0505-39, the EU Integrated Infrastructure Initiative Hadron Physics (I3HP) under contract RI3-CT-2004-506078 and the U.S. Department of Energy (D.O.E.) Office of Nuclear Physics under contracts DE-FG02-94ER40818 and DE-FG02-04ER41302. This research used computational resources provided by RFP under contract EPYAN/0506/08, the National Energy Research Scientific Computing Center supported by the Office of Science of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and the MIT Blue Gene computer under grant DE-FG02-05ER25681. Dynamical staggered quark configurations and forward domain wall quark propagators were provided by the MILC and LHPC collaborations respectively.

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