Validation of constitutive models for experimental stress-strain relationship of high-strength steel sheets under uniaxial tension

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Abstract. The constitutive relationship between stress and strain is one of the important issues in numerical simulation of metal forming. For steel, there are two kinds of constitutive models wildly used. One is power law like Ludwik and Swift. The other is saturation models like Voce and Hockett-Sherby. It is necessary to determine the most appropriate model and parameters for newly developed high strength steels. In order to describe the ultra-high strength dual phase and quenching and partitioning steels, uniaxial tensile tests with the aid of digital image correlation techniques were performed to analyse the strain field over the whole gauge section as well as the element close to the necking point. Through experimental data analysis and numerical fitting, the validity of different models have been confirmed. It shows that equation of Hockett-Sherby model has the best fitting accuracy and high strength steels require a model with more parameters. The extrapolated weighting coefficient of combined Swift and Hockett-Sherby models is characterized, which is proved to improve the accuracy of the numerical simulation of forming of ultra-high strength steels based on the conventional tensile tests.

1. Introduction

Along with the requirements of energy-saving, safety and comfort of vehicles, lightweight of car body has become an important trend in automotive industry. For the purpose of lower thickness but same strength level, application of high strength steel is one of the most economical way to reduce the weight of car body with acceptable performance. Though many high strength steels have been developed and applied to automotive industry in recent years, challenges remain for the forming simulation, which constrains the widely application of high strength steels [1,2]. Constitutive relationship, which is one of the most important parameters during metal sheet forming, describes the stress strain behavior under external force of metal sheet. An appropriate constitutive relationship could help to improve the accuracy of forming simulation, which brings much benefit to the application of high strength steels.

As early as in 1909, Ludwik put forward the concept of the strain-hardening exponent (n) [2]. Through theoretical analysis, Hollomon established the exponential equation (n) from experience in
metal tensile test in 1944 [3]. Then more models of constitutive equation describing the relationship of true stress and strain were proposed in literature. For steels, models like Ludwik, Ghosh [4], Hockett-Sherby [5], Swift [6], and Voce [7] have been widely used. Combinations of models such as the Swift-Voce or Swift/Hockett-Sherby models could better optimize the modeling results [7]. The uniaxial tensile testing is the most convenient method to generate stress-strain data of homogeneous deformation stage using rectangular cross-section specimen. Kang et al [8] studied compatibilities of different constitutive equations for the determination of yield and ultimate tensile strengths by tensile true stress vs. true strain curves of 27 metal alloys. To identify the constitutive model of newly developed high strength steel, like dual-phase (DP) steel and quenching and partition (QP) steel, the applicability of each model needs to be scrutinized.

However, the stress vs. strain curve obtained by these methods is limited to a certain strain level due to the diffuse necking in the uniform elongation of the tensile specimens. The equivalent strain of uniform elongation during uniaxial tensile testing is normally much lower than that during metal forming. There is significant difference at larger strains by extrapolating the experimentally obtained stress-strain curve to larger strain values with assuming constitutive equations [9]. There are several experimental methods to obtain stress vs. strain curves up to a larger strain [10,11]; however, more complex experiment equipment and numerical analysis are still required. The strain hardening behavior at these large strains can be better extracted from the tensile tests with some additional non-standard measurements and associated analyses. One method is firstly to measure the current cross-section area at the narrowest section of a neck, obtaining the average axial true strain and then to compute the average axial true stress by applying a correction factor to obtain the effective true stress vs. true strain curves at larger strains [12,13]. Optical full-field measurement with digital image correlation technique is also helpful to measure local strain accurately during tensile testing [14]. Another way is an inverse method based on iteratively correcting an initially assumed post-necking strain hardening behavior by minimizing the difference between FEA and experiment results of force-displacement curve, or total works (real or virtual, external or internal) within the gauge section [15-17].

In this work, 10 typical steels from mild steel to ultra-high strength steel were selected to perform uniaxial tensile tests, extract stress vs. strain data of homogeneous deformation, and evaluate the fitting precision of different models including Ludwick, Swift, Ghosh and Hockett-Sherby. For characterizing the hardening behavior of ultra-high strength steel, digital image correlation techniques were employed to obtain accurate strain data of uniaxial tensile testing, and combinations of models were applied to fitting the experimental true stress vs. true strain data.

2. Experiment materials

| Type | Steel grade | Thickness mm | Rp0.2/ReL MPa | Ae % | Rm MPa | Agt % |
|------|-------------|--------------|----------------|------|--------|-------|
| Mild Steel | DC04 | 0.8 | 141 | 0.269 | 294 | 26.0 |
| HSS IF | HC260Y | 1.0 | 303 | 0.345 | 449 | 19.3 |
| BH | HC220B | 0.7 | 225 | 0.307 | 354 | 22.0 |
| HSLA | HC420LA | 1.5 | 476 | 4.394 | 525 | 15.4 |
| DP | HC340/590DP | 1.5 | 388 | 0.383 | 620 | 18.5 |
| | HC500/780DP | 2.0 | 587 | 0.476 | 825 | 6.30 |
| | HC550/980DP | 1.2 | 667 | 0.530 | 1005 | 8.61 |
| | HC700/980DP | 1.2 | 796 | 0.584 | 1104 | 6.60 |
| QP | HC600/980QP | 1.2 | 672 | 0.525 | 1052 | 16.9 |
| | HC820/1180QP | 1.0 | 881 | 0.618 | 1192 | 12.7 |

Cold-rolled mild steel, high strength interstitial-free steel, bake-hardening steel, high strength low alloy steel, DP and QP steel are widely used in car body manufacturing. 10 grades of steel sheets were
selected from various categories, with nominal thicknesses ranging from 0.7 mm to 2.0 mm. All materials were produced by Baosteel for the application in automotive industry. The basic mechanical properties of these materials are listed in table 1. The steel grade is named according to Baosteel enterprise standards (Q/BQB 408-2018, Q/BQB 418-2018 and Q/BQB 419-2018). Taking HC340/590DP as an example, “HC” stands for high strength steel with cold rolling, and 340/590 is the minimum yield strength and minimum tensile strength respectively.

3. Methodology

Uniaxial tensile testing specimens were processed into the dimensions recommended by ISO 6892-1-2009, as illustrated in figure 1. Wire cut electrical discharge machining was applied to cut the specimen for improving the edge quality. The axial direction of all specimens is along the rolling direction. Sample surface of machining area was polished before testing. The tensile test was operated on a material testing system and the crosshead speed was set to 3 mm/min. Strain was measured using video extensometer by calculating the length change of gauge area. In order to measure full-field strain of DP and QP steel during tensile process, digital image correlation method was used by spraying random speckles on one surface of each specimen. Average axial strain within gauge area (i.e. with a length of 50 mm and a width of 12.5 mm) and local true strains at different segments were calculated respectively.

Figure 1. Dimensions of the uniaxial tensile testing specimens (unit in mm).

Before the onset of diffuse necking, average true stress (σ) and true strain (εT) during homogeneous deformation can be calculated by equations (1)-(3):

\[
\sigma = \sigma_E (1 + \varepsilon_E) \quad (1)
\]

\[
\varepsilon_T = \ln (1 + \varepsilon_E) \quad (2)
\]

\[
\sigma_E = \frac{F}{A_0} \quad (3)
\]

where σE is the engineering stress, F is the tensile force recorded by the tensile testing machine, A0 is the original cross section area of the parallel segment, and εE is the average engineering strain of the gauge segment. By subtracting elastic strain, true plastic strain ε is achieved by equation (4), where E is the elastic modulus.

\[
\varepsilon = \varepsilon_T - \frac{\sigma}{E} \quad (4)
\]

Then true stress vs. true strain data of homogeneous deformation was used to fit different models using the non-linear least squares method with an algorithm of trust-region. Table 2 summarizes corresponding model equations with σ true stress, ε true plastic strain, and α, C1-C4 modeling parameters.

R-square value is describes the goodness of fit, i.e. the ratio of the sum of squares of the regression and the total sum of squares. The value closer to 1 indicates that a more suitable model for the investigating data.
Table 2. Equations of constitutive models.

| Model               | Equation                        |
|---------------------|---------------------------------|
| Ludwik             | $\sigma = C_1 + C_2 e^{C_3 \varepsilon}$ (5) |
| Swift              | $\sigma = C_i (C_2 + \varepsilon) e^{C_3 \varepsilon}$ (6) |
| Ghosh              | $\sigma = C_i (C_2 + \varepsilon) e^{C_3 \varepsilon} - C_4$ (7) |
| Hockett-Sherby     | $\sigma = C_2 - (C_2 - C_1) e^{-C_3 \varepsilon}$ (8) |
| Swift/Hockett-Sherby | $\sigma = (1 - \alpha) \sigma_{Swift} + \alpha \sigma_{Hockett-Sherby}$ (9) |

4. Results and discussion

True stress vs. true plastic strain data before the onset of diffuse necking from tensile testing of 10 steels were extracted to evaluate the fitting precision of different models including Ludwick, Swift, Ghosh and Hockett-Sherby. For further characterization of the hardening behavior of ultra-high strength steels, accurate strain data of HC550/980DP and HC600/980QP was analyzed. By calculating the true strain of the segment close to necking points, more stress vs. strain data beyond the conventional strain calculation method was achieved. Combination of Swift and Hockett-Sherby model was applied to fitting the true stress vs. true plastic strain data and identifying the weighting parameter in the model.

Figure 2 shows the true stress vs. true plastic strain curves of the tested 10 steels and fitting curves for 4 models. Figure 3 shows the analyzed result of goodness of fit of these models for different steels by considering tensile strength and strain hardening exponent.
The results show that all 4 models are capable to describe the constitutive relationship of stress and strain according to data from uniform deformation stage of uniaxial tensile test. Among these models, Hockett-Sherby is the best one, followed by Ghosh and Ludwik. Equation of Hockett-Sherby model with 4 parameters has more degrees of freedom to fit the hardening behavior of steel. By examining the effect of strain hardening exponent on goodness of fit, figure 3 indicates that steel with a larger n value resulting a better fitting result. In comparison with low strength steels and high strength steels (with a tensile strength lower than 780 MPa), showing in figure 3(b), ultra-high strength steel (with a tensile strength higher than 780 MPa), are more complex and require a model with more parameters.

4.1. Model validation at larger strains of DP and QP steels
It has been pointed out that there is a significant difference at larger strains by extrapolating the experimentally obtained stress vs. strain curve to larger strains by assuming constitutive equations. Optical full-field measurements with digital image correlation techniques provide a feasible solution to measure local strain accurately during tensile testing. As illustrated in figure 1, 101 points of the axial section where the maximum longitudinal strain locates were selected, where ‘0’ was the middle point, ‘-50’ and ‘50’ were points at both ends. The distance of adjacent points was about 0.78 mm. The strain history of each pair of points from initial deformation to fracture was obtained and the strain of all points versus time of HC600/980QP is shown in figure 4(b). By analyzing the true strain of the segment close to the final localized necking point within diffuse necking stage that is from the onset of diffuse necking “t3” to the onset of localized necking “t6” in figure 4(b), more stress vs. strain data can be acquired than conventional method by measuring the average strain over the whole gauge. Figure 4(a) shows true stress vs. true plastic strain curve of point near final localized necking point, comparing with data of whole gauge section.
Figure 4. Tensile data of HC600/980QP. (a) Strain distribution at different time; (b) True stress-strain curve using different gauge.

The advantage of the digital image correlation method is that the strain at any location in the gauge section is effective until the fracture of specimen when the digital image correlation method is applied, while the strain measured by extensometer after the necking is not accurate anymore. The onset of localized necking is determined by Hill localized necking theory [18]. Without consideration of stress triaxiality variation within the diffuse necking stage, the data from the onset of diffuse necking to localized necking is able to validate the constitutive model and determine the weighting parameter of the combined model. Figure 5 shows the validation result of HC550/980DP and HC600/980QP, where the parameter of Swift and Hockett-Sherby model was calculated by fitting of data from homogeneous stage. According to equation (9), the weighting parameter $\alpha$ of combination of Swift and Hockett-Sherby model is estimated by the least square method, which is 0.66 and 0.8 for HC550/980DP and HC600/980QP, respectively. The extrapolation of combination model obtained by digital image correlation method can further improve the accuracy of forming simulation for ultra-high strength steels.

Figure 5. Model validation result at larger strains. (a) HC550/980DP; (b) HC600/980QP.

5. Conclusions

- The selected models are capable to describe the constitutive relationship of stress and strain during the homogeneous deformation in uniaxial tensile testing. Hockett-Sherby model is the best, followed by Ghosh and Ludwik models.
- In comparison with the traditional steels, high strength steels and ultra-high strength steels are more complex and require a model with more parameters.
- By analyzing the full strain field with digital image correlation method, the validity of model extrapolating to large strain can be examined, and more accurate model are achieved to
describe the hardening behavior of ultra-high strength steels.

References
[1] Lian C W, Chen X P and Ye Y 2017 Formability of the 3rd generation high strength steel - QP steel based on part application J. Netshape Forming Eng. 9 79-84
[2] Ludwik P 1909 Elemente der Technologischen Mechanik (Berlin Heidelberg: Springer)
[3] Hollomon J H and Member J 1945 Tensile Deformation Metal Technol. 12 268-90
[4] Ghosh A K 1977 Tensile instablity and necking in materials with strain hardening and strain-rate hardening Acta Metall. 25 1413-24
[5] Hockett J E and Sherby O D 1975 Large strain deformation of polycrystalline metals at low homologous temperatures J. Mech. Phy. Solid. 23 87-98
[6] Swift H W 1952 Plastic instability under plane stress J. Mech. Phy. Solid. 1 1-18
[7] Voce E 1948 The relationship between stress and strain for homogeneous deformation J. Instit. Metal 74 537-62
[8] Kang S, Kim Y, Kim K, Kwon D and Kim J 2014 Constitutive equations optimized for determining strengths of metallic alloys Mech. Mater. 73 51-7
[9] Ling Y 1996 Uniaxial true stress-strain after necking AMP J. Technol. 5 37-48
[10] Kessler L and Gerlach J 2006 The impact of materials testing strategies on the determination and calibration of different FEM material models Conf. Proc. IDDRG pp 113-20
[11] Capilla G, Hamasaki H and Yoshida F 2017 Determination of uniaxial large-strain workhardening of high-strength steel sheets from in-plane stretch-bending testing J. Mater. Processing Technol. 243 152-69
[12] Zhang Z L, Deg Rd J and Vik O P S 2001 Determining true stress–strain curve for isotropic and anisotropic materials with rectangular tensile bars: method and verifications Comput. Mater. Sci. 20 77-85
[13] Wang L and Tong W 2015 Identification of post-necking strain hardening behavior of thin sheet metals from image-based surface strain data in uniaxial tension tests Int. J. Solid Struct. 75-76 12-31
[14] Zhu F, Bai P, Zhang J, Lei D and He X 2015 Measurement of true stress–strain curves and evolution of plastic zone of low carbon steel under uniaxial tension using digital image correlation Optics and Lasers in Engineering 65 81-8
[15] Zhao K, Wang L, Chang Y and Yan J 2016 Identification of post-necking stress–strain curve for sheet metals by inverse method Mech. Mater. 92 107-18
[16] Coppieters S, Kim J H, Denys K, Cooreman S and Debruyne D 2017 On complete solutions for the problem of diffuse necking in sheet metal Procedia Eng. 207 2012-7
[17] Tardif N and Kyriakides S 2012 Determination of anisotropy and material hardening for aluminum sheet metal Int. J. Solid Struct. 49 3496-506
[18] Ding L, Lin J, Min J, Pang Z and Ye Y 2013 Necking of Q & P steel during uniaxial tensile test with the aid of DIC technique Chin. J. Mech. Eng. 26 448-53