Coarsening Dynamics in a Two-dimensional XY model with Hamiltonian Dynamics

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We investigate the coarsening dynamics in the two-dimensional Hamiltonian XY model on a square lattice, beginning with a random state with a specified potential energy and zero kinetic energy. Coarsening of the system proceeds via an increase in the kinetic energy and a decrease in the potential energy, with the total energy being conserved. We find that the coarsening dynamics exhibits a consistently superdiffusive growth of a characteristic length scale as \( L(t) \sim t^{1/2} \) with \( 1/2 > 1/2 \) (ranging from 0.54 to 0.57 for typical values of the energy in the coarsening region). Also, the number of point defects (vortices and antivortices) decreases as \( N_V(t) \sim t^{-1/2} \), with a typical exponent of \( \phi_V \approx 0.88 \), which shows deviations from the energy-scaling relation. The spin autocorrelation function exhibits a peculiar time dependence with non-power law behavior that can be fitted well by an exponential of logarithmic power in time. We argue that the conservation of the total Josephson (angular) momentum plays a crucial role for these novel features of coarsening in the Hamiltonian XY model.

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I. INTRODUCTION

In recent years, there have been research efforts toward investigating the statistical mechanical behavior of lattice spin systems by solving directly the Hamiltonian equation of motion associated with the lattice spin system of interest. Especially, equilibrium phase transitions have been found to be associated with the existence of topological change in the dynamics, as manifested in the distributions of Lyapunov exponents.

In relation to these interests, we here investigate the coarsening dynamics of the XY model on a square lattice based on Hamiltonian dynamics. In conventional studies of phase ordering and coarsening dynamics in a statistical system, the system is quenched from a random disordered state to a low-temperature state below the transition temperature. In computational studies, one employs Monte Carlo or Langevin dynamics methods for simulating the coarsening processes. In typical situations, the average length scale \( L \) of ordered domains grows in time as a power law \( L(t) \sim t^{1/2} \), where the growth exponent \( 1/2 \) depends on the dimension of the space and the dimension of the relevant order parameter, in addition to the conserved or nonconserved nature of the latter in the relaxation dynamics. Usually, characteristic topological defects, such as point vortices or domain walls, are generated in the initial disordered state, depending on the dimension of the order parameter, and the annihilation of these defects gives the main mechanism of coarsening and phase ordering in the system. The observed self-similarity of these coarsening systems at different time instants is usually represented by the so-called dynamic scaling hypothesis of the equal-time spatial correlation function of the order parameter. In equilibrium, the XY model exhibits a Kosterlitz-Thouless (KT) transition at \( T_{KT} \) due to the unbinding of vortex-antivortex pairs. Below \( T_{KT} \), the system has a quasi-ordered phase that is characterized by a power-law decay of the order parameter correlation function for long distances. The critical exponent governing the power-law decay decreases continuously down to zero temperature: the system is critical at equilibrium for all temperatures below \( T_{KT} \).

After some efforts of various groups, as well as some controversies, it is now agreed that, in the phase ordering dynamics of the XY model via Monte Carlo simulations or Langevin dynamics methods, the growing length scale exhibits a logarithmic correction to diffusive growth as \( L_{MC}(t) \sim t/\log t \). Here, the logarithmic correction can be attributed to a logarithmic divergence (in the system size) of the effective friction constant of a moving vortex in the dissipative dynamics with a non-conserving order parameter. In a related simulational work on the coarsening dynamics of superconducting juction arrays based on the dynamics of the resistively-shunted junction (RSJ) model, it was revealed that there is no logarithmic correction. This could be understood in terms of a finite friction constant of a moving vortex in the limit of large system size, which is due to the particular type of dissipative coupling in Josephson junction arrays.

One of the questions we would like to address in this work is whether we would find similar features as mentioned above in the coarsening of the XY model based on Hamiltonian dynamics. Unlike the case of conventional coarsening based on dissipative dynamics, almost no works exist on coarsening via Hamiltonian dynamics.
We find also that the relaxation of the potential energy exhibits a power-law decay \( \Delta U \equiv U(t) - U_\infty \sim t^{-\phi_U} \) toward a long-time equilibrium value, with the exponents taking values approximately in the range 0.86 < \( \phi_U < 0.90 \) (in typical cases, \( \phi_U \approx 0.87 \sim 0.88 \)). This shows a considerable deviation from the energy-scaling relation relating the excess energy and the growing length scale. Another interesting feature of the coarsening dynamics in the Hamiltonian XY model is that the spin autocorrelation exhibits a peculiar non-power-law behavior, which can be reasonably well fitted by \( A(t) \sim A_0 \exp[-b(\ln(t))^{\gamma}] \), with \( b \) ranging from 0.24 to 0.4 and the exponent \( \gamma \) from 1.5 to 1.7.

In order to understand the microscopic mechanism of these coarsening features, including the notable superdiffusive growth exponent, that are quite distinct from the cases of purely dissipative dynamics, we note that one conserved quantity exists in this Hamiltonian model system. That is, the total angular momentum (or Josephson angular momentum) is conserved in the Hamiltonian dynamics. Due to the conservation of angular momentum, propagating spin wave modes appear. We strongly believe that the existence of propagating spin wave modes due to the conservation of angular momentum is the main cause of the superdiffusive nature of the coarsening and of the other unique features in this system. In order to obtain an effective friction constant of a vortex, we could apply a collective variable approach. However, this method gave us a vanishing effective friction constant to the lowest order in the velocity of a vortex. Higher-order friction can be argued to increase the growth exponent 1/\( z \) from the diffusive value of 1/2. Further work is necessary to reach a full understanding of the main features of the coarsening in the Hamiltonian XY model in two dimensions.

II. THE HAMILTONIAN XY MODEL AND SIMULATION METHODS

The potential energy function of the hard-spin XY model on a square lattice is given by

\[
U = J \sum_{<ij>} (1 - \cos(\theta_i - \theta_j)),
\]

where \( J \) is the interaction strength, \( \theta_i \) is the phase angle of the spin at site \( i \) and the sum is over nearest neighbor pairs. The corresponding Hamiltonian dynamic equation can be obtained by adding the kinetic term:

\[
H = \sum_{i} \frac{m_i^2}{2} + J \sum_{<ij>} (1 - \cos(\theta_i - \theta_j)),
\]

where \( m_i = \dot{\theta}_i \) represents the canonical momentum associated with the angular variable \( \theta_i \) of planar spins at...
site $i$. This model appears naturally in easy-plane ferromagnets or superfluid helium with the conserved variable $m$ corresponding to the $z$-component of the spins or the density of the superfluid respectively in the limit of negligible thermal noise (e.g., negligible phonon effect).

Previous works have shown that a Kosterlitz-Thouless (KT) transition occurs at the value of potential energy around $U_{KT} \approx 1.0$. Our computational scheme in this work performs Newtonian dynamic simulations based on the above Hamiltonian with initial states that take zero kinetic energy and specified potential energy values but are otherwise random. Specified potential energy values are chosen from the region that corresponds to below or near the KT transition energy ($U_{KT} \approx 1.0$).

Unlike the case of dissipative systems with Langevin dynamics or Monte Carlo dynamics where dissipative relaxation is incorporated in a natural way, here in our case of Hamiltonian dynamics, the total energy (sum of kinetic and potential energy) is conserved. However, since we begin the Hamiltonian dynamics with zero kinetic energy with a given specified potential energy, we expect that the system will evolve in such a way as to increase (on the average) the kinetic energy and decrease the potential energy. Eventually, in the long-time limit, the system will reach a certain kind of equilibrium dynamic state where the average potential energy and the kinetic energy do not vary anymore. Of course, in terms of phase-ordering dynamics in dissipative systems, a system quenched to a symmetry-broken phase will never reach complete equilibrium in a finite time period.

Therefore, we can consider the kinetic part of the system acting as a kind of thermal heat bath that absorbs the "dissipated heat" coming from the decreasing potential energy. We can, thus, investigate the time-dependent spin configuration for the system in terms of spatial ordering and relaxation of vortex numbers, etc., in analogy to the Monte Carlo or Langevin relaxation dynamics. We generate the initial states with specified potential energies in the following manner: To begin with, we take a random spin configuration. Then we employ a Monte Carlo annealing algorithm such that the spin configurations with potential energies that are closer to the target energy are preferentially accepted. In other words, we steer the initial state to some target spin configuration such that we obtain a state whose potential energy is equal to a specific energy; otherwise, the spin configuration is random.

The Hamiltonian equation reads

$$\dot{m}_i = -\frac{\delta U}{\delta \theta_i} = -J \sum_j \sin(\theta_i - \theta_j),$$

$$\dot{\theta}_i = m_i$$

where $j$ denotes the nearest neighbors of site $i$ and $i$ takes values $i = 1, \ldots, N^2$. For simplicity, we set the rotational inertia to be equal to unity and put $J = 1$. Here, we note that, in addition to the conservation of total energy, another conserved quantity, namely the total (Josephson) angular momentum $\sum_i m_i \equiv \sum_i \dot{\theta}_i$ exists; This is due to the invariance of the equations under global translations in the variable $\theta_i$, that is, under $\theta_i \rightarrow \theta_i + \alpha$.

Equations 4 and 5 are numerically integrated in time using a second order velocity-Verlet algorithm with the time integration step of $\Delta t = 0.01$, which conserves the total energy to within a ratio of $10^{-4}$ up to the maximal time integration steps of around $10^6$. Periodic boundary conditions on both lattice directions are employed. The simulations are carried out on square lattices of dimensions up to $N \times N = 1800 \times 1800$ with sample averages of over 20 to 40 different initial configurations. A parallel computation is employed on Linux clusters with from 16 to 32 CPU’s via a domain decomposition method with high parallel efficiency.

The final results are obtained from averages over 10 to 40 different random initial configurations. The main quantities of interest are as follows:

(i) Equal time spatial correlation function of the order parameter,

$$C(r, t) = \frac{1}{N^2} \left< \sum_i \cos(\theta_i(t) - \theta_{i+r}(t)) \right>,$$

where $< \cdots >$ denotes an average over random initial configurations.

(ii) Number of vortices $N_V(t)$ at time $t$, including both positive and negative vortices.

(iii) Time-dependent relaxation of the potential energy, $U(t)$,

$$U(t) \equiv \frac{1}{N^2} \left< \sum_{<i,j>} (1 - \cos(\theta_i(t) - \theta_j(t))) \right>,$$

(iv) Nonequilibrium spin autocorrelation function

$$A(t) \equiv \frac{1}{N^2} \left< \sum_i \cos(\theta_i(0) - \theta_i(t)) \right>.$$  

The results of simulations will be presented in section III.

III. SIMULATION RESULTS AND DISCUSSIONS

Figure 1 shows snapshots of spin configurations for the case of $E = 0.4$ at four different time stages of coarsening. We can see that coarsening occurs via decay of vortices and anti-vortices. Figure 2(a) shows the equal-time spatial correlation for $E = 0.4$. The simplest way to analyze the data is to attempt the so-called critical dynamic scaling

$$C(r, t) = r^{-\eta(E)} g(r/L(t)),$$
with the spatial correlation exponent \( \eta(E) \) and a length scale \( L(t) \) growing typically as \( L(t) \sim t^{1/z} \). In our case, for a given time instant \( t \), we determined \( L(t) \) in such a way that \( r^{-\eta(E)}C(r,t)|_{(r=L(t))} = g(r/L(t))|_{(r=L(t))} = g(1) = 0.4 \). For a suitable choice of \( \eta(E) \), we can obtain a good collapse of the rescaled correlation functions \( g(x) \), at least in the late-time region. Shown in Fig. 2(b) is such an attempt at critical dynamic scaling. We find that the equal-time spatial correlation functions exhibit a reasonable critical dynamic scaling, at least, in the late-time region. However, in the early-time regime, we can see that it is not possible to collapse the correlations with any reasonable values of \( \eta \).

\[
\begin{array}{|c|c|c|c|c|}
\hline
E & \eta (N = 1000) & 1/z (N = 1000) & \eta(N = 1800) & 1/z (N = 1800) \\
\hline
0.1 & 0.046(10) & 0.520(12) & 0.046(10) & 0.520(12) \\
0.2 & 0.065(16) & 0.540(15) & 0.09(2) & 0.549(12) \\
0.3 & 0.104(16) & 0.566(10) & 0.115(18) & 0.555(10) \\
0.4 & 0.112(17) & 0.565(12) & 0.549(13) & 0.549(13) \\
0.5 & 0.120(18) & 0.549(13) & 0.156(21) & 0.560(14) \\
0.6 & 0.135(19) & 0.549(13) & 0.170(22) & 0.565(14) \\
0.7 & 0.235(20) & 0.62(25) & 0.235(20) & 0.62(25) \\
0.8 & 0.235(20) & 0.62(25) & 0.235(20) & 0.62(25) \\
1.0 & 0.235(20) & 0.62(25) & 0.235(20) & 0.62(25) \\
\hline
\end{array}
\]

TABLE I: Correlation exponent \( \eta \) and the growth exponent \( 1/z \).

In the late-time region, we find that the length scale \( L(t) \) grows with an exponent \( 1/z \) that is larger than the diffusive exponent 0.5. For example, in the case of \( E = 0.4 \), we obtain \( 1/z = 0.555 \pm 0.010 \), which is definitely larger than 1/2 (Fig. 2(c), Fig. 2(d), and Table 1). Note that the power law region shows up at \( t > t_P \approx 50 \), which is a considerably late-time stage.

Closely related to this growing length scale is the average separation between vortices, \( L_V(t) \), which can be derived from the decay of the number of vortices, \( N_V(t) \). Shown in Fig. 3(a) are the number of vortices versus time, which behaves as \( N_V(t) = N_V(\infty) \sim t^{-\phi_V} \), where \( N_V(\infty) \) denotes the limiting value of the vortex number at equilibrium. At low temperature, the equilibrium vortex density becomes vanishingly small. We regard \( N_V(\infty) \) as a fitting parameter such that the relaxation of the excess vortex number obeys the best power-law behavior. We find that the relaxation of the vortex number exhibits an excellent power-law behavior except for the early-time region where smooth plateau is seen before crossing over to the late-time power-law region. The exponent \( \phi_V \) takes values in the range from 1.0 to 1.10, depending on the system size and the energy, except for the limit of higher per-site energy. For example, in the case of \( E = 0.4 \) with \( L = 1800 \), we get \( \phi_V \approx 1.055 \). Table 2 and Figure 3(c) shows the \( E \)-dependence of the \( \phi_V \).

Now, for a uniform distribution of vortices, we can expect the average separation between the vortices to satisfy (for negligible \( N_V(\infty) \))

\[
L(t) \sim L_V(t) \sim (N_V(t))^{-1/2} \sim t^{-\phi_V/2}.
\]

Since we have \( L(t) \sim t^{1/z} \), we expect \( 1/z = 0.5 \). The two exponents \( 1/z \) and \( \phi_V/2 \) obtained from simulations satisfy this relation only approximately. To be more precise, systematically a small discrepancy exists such that \( 1/z \) is slightly larger than \( \phi_V/2 \).

We find also that the relaxation of the potential energy exhibits a power law decay \( \Delta U \equiv U(t) - U_\infty \sim t^{-\phi_U} \) toward a long-time equilibrium value, with the exponents taking values approximately in the range \( 0.86 < \phi_U < 0.90 \) as shown in Fig. 3(b) and Fig. 3(c). The value of this exponent is almost independent of the energy \( E \). For the usual dissipative coarsening with a non-conservative order parameter in two dimensions, the excess energy relaxation can be related to the growing length scale \( L(t) \) as

\[
\Delta U \sim L^{-2}(t) \log(L(t)/r_0),
\]

where \( r_0 \) denotes the size of a vortex core. Interestingly, in our model system with Hamiltonian dynamics, we could not show confidently that the simulation results are consistent with the above relation between the energy and the domain growth relation. Further work is necessary in this regard.

Another interesting feature of the coarsening dynamics in the Hamiltonian \( XY \) model is that the spin autocorrelation exhibits a peculiar non-power law behavior that can be reasonably well fitted by

\[
A(t) \sim A_0 \exp[-b(\ln(t))^\gamma],
\]

with \( b \) ranging from 0.24 to 0.4 and the exponent \( \gamma \) from 1.5 to 1.7. One typical example is shown in Fig. 4 for the case of \( E = 0.4 \) with \( L = 1800 \), which shows an excellent fit to the above functional form with \( b = 0.242 \) and \( \gamma \approx 1.67 \). Note that, in the limit of \( \gamma = 1 \), \( A(t) \) reduces to a power law of \( A(t) \sim A_0 t^{-b} \). The fitted values of \( \gamma \) (ranging from 1.5 to 1.7) implies that \( A(t) \) exhibits a considerable deviation from a power-law behavior. To the best of our knowledge, the Hamiltonian \( XY \) model...
on a square lattice is the first system in which the
autocorrelation function exhibits a functional form with an
exponential of a logarithmic power.

Since we have no theories available on the coarsening
of the Hamiltonian $XY$ model in two dimensions, we may
resort to phenomenological approaches for the explanation
of these results on the phase ordering of the Hamil-
tonian $XY$ model, especially the relationship between the
characteristic growth exponents and other exponents ob-
tained from our simulations.

To begin with, let us note that a strong contrast ex-
ists between the above results of Hamiltonian coarsening
and the conventional dissipative coarsening (via Monte
Carlo or Langevin dynamics) in the $XY$ model. The
first thing to recall is that the growth law of the usual
$XY$ model with Monte Carlo dynamics (on nonconserved
order parameter) follows $L(t) \sim (t/\ln(t))^{3/2}$. In con-
trast, here, we observe in the Hamiltonian $XY$ model, a
growth law with the exponent $1/z$ being greater than
the exponent for the diffusive growth of $1/z = 1/2$.
Also, the vortex number density exhibits a relaxation
$N_V - N_V(\infty) \sim t^{-\phi_V}$ with $\phi_V$ greater than 1.0.

As we mentioned earlier, the system is expected to de-
crease in its total potential energy via a corresponding
increase of kinetic energy. This kinetic energy part is
thought to consist of spin wave oscillations (superposed
upon vortices or anti-vortices). We expect growth laws of
length scales to be determined by the effective dynam-
ics of vortices and anti-vortices and by the statistics of
vortex-antivortex annihilation.

Even though our model system is very simple in its
equations, we find it not very easy to find an effective
vortex dynamics for a given total energy of the system.
One way may be to find a Langevin-type model system,
which is expected to be equivalent to our microcanoni-

cal Hamiltonian system in the thermodynamic limit, and
then to investigate the coarsening dynamics of the equiv-

alent model.

Noting that the equation of motion can be derived from
the Hamiltonian

$$H = \sum_{<ij>} (1 - \cos(\theta_i - \theta_j)) + \sum_i \frac{m^2_i}{2}$$  (13)

with the Poisson bracket $\{\theta_i, m_j\} = g$ (where $g = 1$ here)
and considering that the total sum of $m_i$, i.e., $\sum_i m_i$, is
conserved, we can construct a Langevin equation for $\theta$
and $m$ in the following way: We suppose, to begin with,
that the fluctuating spin waves act as a thermal bath
on the vortices, with the temperature determined by the
equivalent equilibrium temperature corresponding to the
total energy. Thus, we may suppose the effective coarsen-
ing dynamics of the original Hamiltonian $XY$ model sys-
tem to be equivalent to the dissipative dynamics of the
system under thermal noise associated with the equiva-

lent temperature:

$$\dot{\theta}_i = \frac{\delta H}{\delta m_i} - \lambda \frac{\delta H}{\delta \theta_i} + \eta_i(t),$$  (14)

$$\dot{m}_i = -\frac{\delta H}{\delta \theta_i} + \mu \nabla^2 \frac{\delta H}{\delta m_i} + \xi_i(t),$$  (15)

where the Gaussian noises $\eta_i$ and $\xi_i$ satisfy

$$\langle \eta_i(t)\eta_j(t') \rangle = 2k_B T \delta_{ij} \delta(t - t'),$$  (16)

$$\langle \xi_i(t)\xi_j(t') \rangle = 2\mu k_B T \nabla^2 \delta_{ij} \delta(t - t')$$  (17)

at temperature $T$, $k_B$ being the Boltzmann constant.
Here, $\delta_{ij}$ represents the discrete delta function on a two-
dimensional square lattice, and $\nabla^2$ the corresponding
discrete Laplacian reflecting the conservation of the total
Josephson angular momentum, $m$. In the linear limit,
ignoring the vortices, these set of equations reduce to the
Nelson-Fisher model describing low-temperature spin dy-
namics in one and two dimensions.

It is not a trivial matter to prove an equivalence be-
tween the relaxation of the Hamiltonian $XY$ model and
that of the above Langevin equations with reversible
mode-coupling. In order to be sure of the equivalence,
we should assume that the spin dynamics is sufficiently
chaotic so that the spin-wave fluctuations generate fully
Gaussian random noise on the spin dynamics. Beginning
with Eqs. 14 and 15, we may also obtain an effective dy-
namics of a single-vortex excitation under external force.

By assuming a slowly moving vortex, we attempted to
apply a collective variable method to Eqs. 14 and
15 in order to obtain an effective equation of motion for a
single vortex. We obtain from this method an interesting
result that, due to the conservation of $m$, the effective
equation for the slow-velocity limit of a vortex can be written as

$$M \ddot{\mathbf{V}} + \zeta \dot{\mathbf{V}} = \mathbf{F} + \mathbf{F}_{\text{noise}},$$  (18)

with the effective mass $M$ of a vortex behaving as $M \sim
\ln(L)$, and the noise term $F_{\text{noise}}$ being an uncorrelated
Gaussian noise. $\zeta$ represents the effective linear friction
constant of the vortex. $\mathbf{F}$ is an external force, such as
coulomb forces due to other vortices, etc. An interesting
feature is that the dissipation term vanishes to the lowest
order in the velocity of the vortex i.e., $\zeta \sim 0 + c|\mathbf{V}| + \cdots$.
Including the next order terms is a complicated task that
we didn’t actually carry out, but we can attempt a phe-
nomenological argument on the coarsening by assuming
a higher-order dissipation. Here, let us assume a dissi-
pation term that is second order in the velocity of a vor-
tex, and let us consider a vortex-antivortex pair with the
coulomb force varying inversely in the mutual distance
(in two dimensions). Then, a simple argument based on
a force balance between the coulomb force and friction
leads to (neglecting the inertial term in the limit of slow
velocity and at low temperatures)

$$cV^2 = c\left(\frac{dr}{dt}\right)^2 \sim \frac{1}{r},$$  (19)
In this case, the time scale for annihilation of a vortex-antivortex pair with size $R$ goes as $\tau(R) \sim R^{3/2}$. This, in turn, implies that the typical length scale between vortices will grow in time as $L(t) \sim t^{2/3}$, which is definitely superdiffusive. This value of the growth exponent, $2/3$, is a little larger than those exponents obtained from our simulations. This discrepancy might be attributable to the effect of vortex inertia, which diverges logarithmically in the vortex size. Further work is necessary for a detailed quantitative understanding of the growth laws.

We should note that, in our Hamiltonian dynamics simulations, the initial states are not completely random states due to the finite energy constraint. Therefore, the initial states already have some finite correlation length scale, i.e., separation $L_s$ between vortices. These length scales are of order of $3$ to $5$ lattice constants. Therefore, we expect that a finite time scale $\tau_s$ (corresponding to the time scale for a vortex to travel the distance $L_s$) exists after which a scaling region emerges, which is well reproduced in our simulations.

IV. CONCLUSIONS

The initial state that is specially tuned to have a given potential energy (otherwise random) but with zero kinetic energy develops into a late-time coarsening state, where the potential energy slowly decays as a power law with a compensating increase in the kinetic energy. We find that the coarsening dynamics exhibits a characteristic superdiffusive growth, with the exponents being consistently larger than the value for usual diffusive growth of $1/2$. Relaxation of various quantities cannot be understood in the usual framework of the dissipative dynamics of the Ginzburg-Landau Hamiltonian. We believe that these novel features of coarsening dynamics come from conservation of angular momentum, which, in turn, causes an effective nonlinear dissipation of individual vortices.

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FIG. 1: Snapshots of the configuration of the XY spins in a square lattice of size $L = 32$, $E = 0.4$ and time $t$: (a) $t = 0$, (b) $t = 0.01 \cdot 2^7$, (c) $t = 0.01 \cdot 2^{16}$, and (d) $t = 0.01 \cdot 2^{23}$. Vortices and antivortices are denoted by solid squares and empty squares, respectively.
FIG. 2: (a) Spatial behavior of the equal time XY spin correlation function at various time stages in square lattices of size $N = 1800$ at a per-site energy of $E = 0.4$. (b) Scaling collapse of the data in (a) with the appropriate scaling length $L(t)$. (c) Scaling length versus time at various temperatures $E = 0.4$ and $N = 1800, N = 1000$. (d) Growth exponents and $\eta(E)$ versus energy, manifesting superdiffusive growth. Error bars are of the order of the sizes of the symbols or smaller. Lines are only guides to the eyes.
FIG. 3: Relaxation of (a) the vortex number $N_v$ at energy $E = 0.40$ and system size $N = 1800$ and 1000 and of (b) the excess potential energy $\Delta U$ at $E = 0.4$ in the case of size $L = 1800$. In (a), the power-law lines $t^{-1.06}$ and $t^{-1.09}$ are also plotted whereas in (b) a power law line with $\sim t^{-0.88}$ is shown. (c) shows the exponents $\phi_V$ and $\phi_U$ versus energy. Error bars are of the order of the sizes of the symbols. Lines are only guides to the eyes.

FIG. 4: Spin autocorrelation function at energy $E = 0.40$ and (b) the same in a double logarithmic plot. The fitted values are $b \simeq 0.242$ and $\gamma \simeq 1.67$. 