Analyzing the streamflow-sediment links of three major river basins in India using multifractal theory

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Abstract. This study investigated the cross-correlation between daily streamflow and total suspended sediment (TSS) data of 65 gauging stations located in three major river basins in a multifractal perspective. The novel Multifractal Cross Correlation Analysis (MFCCA) method of cross correlation studies is used to analyze the streamflow-sediment links of Krishna, Godavari and Mahanadi basins. The results showed that for the records of a particular station, the joint persistence of streamflow and TSS is approximately the mean of the persistence of individual series. The streamflow displayed higher persistence than TSS in 60% of the stations while in majority of stations of Godavari basin the trend was opposite. The annual cross correlation is higher than overall cross correlation in majority of stations but at these time scales strength of their association differs with river basin.

Keywords: streamflow, multifractal, sediment, persistence, correlation

1. Introduction
Investigating of streamflow-sediment relationships is a complex problem in hydrology. Streamflow and sediment load relationships are influenced by external factors like climate forcing and internal factors like human activities, catchment characteristics etc., and their influence on the link varies with time scales. Larger temporal scale corresponds to a slower variation of the physical quantity in time (such as the bed elevation changes) while smaller time scale corresponds to rapid variations (such as variations of flow). Such information on multiple temporal scales may lead to a singularly perturbed behaviour and used to justify the decoupled sediment transport models [1]. Few studies have investigated association between sediment load and streamflow in multiple time scales using scanning t-test, F-test and coherency analysis and the possible implications were discussed [2, 3]. Similar studies were helpful in understanding the stable and unstable properties of hydrological processes at different time scales, to identify the possible causes and impact of construction of storage reservoirs etc. To identify such multiple time scale of
variability and for subsequent interpretations spectral analysis procedures are helpful. Wavelet transforms and Hilbert Huang Transforms (HHT) are two popular and recent approaches used for such studies in multiple time scales [2, 4]. Adarsh and Janga reddy [5] used the HHT based Time Dependent Intrinsic Correlation (TDIC) method for analyzing the streamflow-sediment link in selected Indian river basins. Further, Adarsh et al. [6] used multifractal detrended fluctuation analysis (MF DFA) and wavelet transforms conjunctively for analyzing the streamflow and sediment variability of Kallada river in Kerala, India. The determination of Pearson correlation is the fundamental rationale in the estimation of possible dependency between the two candidate variables even in the multiscale approaches like wavelet coherency and TDIC. However, some studies states that this coefficient is not robust and can be misleading if outliers are present, as in real-world data characterized by a high degree of non-linearity and non-stationarity [7]. The Pearson correlation may display the spurious correlations in the presence of trend in non-stationary time series. In this context, the scale dependent link may give a realistic estimate to study the mutual associations between two non-stationary series. Podobnik and Stanley [8] proposed the detrended cross-correlation analysis (DCCA), to investigate power-law cross correlations between two candidate non-stationarity time series in a multifractal framework. Multifractal extension of DCCA (namely MF-DCCA) and its variant namely Multifractal Detrending Moving Average Cross Correlation Analysis (MFXDMA) was also proposed by researchers [9, 10]. Later on Oświęcimka et al. [11] propounded a more generalized version of cross correlation analysis namely Multifractal Cross Correlation Analysis (MFCCA) which can also incorporate the sign of fluctuation function to their generalized moments [12]. DCCA and its variants have successfully been applied to financial, biomedical and meteorological time series [13, 14]. Recently, Wu et al. [15] investigated the cross-correlation between streamflow and sediment of Makou (MK) and Sanshui (SS) stations at the apex of the Pearl River Delta using Multifractal Detrended Cross-Correlation Analysis (MF-DXA). They found that the multifractal response between streamflow and sediment at smaller timescale was characterized by long-range correlations whereas it exhibited random behavior at larger timescale. Dey and Mujumdar [16] applied DCCA method for investigating the rainfall-streamflow link of MPOPEX basins in USA and Cauvery basin in southern India. They used fine resolution (0.25°x0.25°) daily gridded rainfall and station wise daily streamflow data for the studies. The temporal dynamics of the persistence of rainfall and streamflow and their joint behaviour were found to be non-uniform across different time scales. It was found that the contribution of catchment processes influencing the persistence of the streamflow is a function of the catchment area. This study investigates the streamflow-sediment link in three major river basins in India using the novel sign preserved version of multifractal cross-correlation studies. The next section presents the theoretical details of MFCCA method. The details of data used in the study are presented in the section thereafter. Subsequently, results of MFCCA on streamflow-total suspended sediment (TSS) links of the basins are presented along with relevant discussions. The major conclusions drawn from the study are presented in the last section.

2. Multifractal Cross Correlation Analysis (MFCCA)

The different steps involved in MFCCA computational procedure can be described as follows:

For two time series \(x_i\) and \(y_i\) (\(i=1,2,...,N\)); determine the profiles as two new series:

\[
X(j) = \sum_{i=1}^{N} \left[ x_i - \langle x \rangle \right]
\]

and

\[
Y(j) = \sum_{i=1}^{N} \left[ y_i - \langle y \rangle \right]
\]

(1)
\[ Y(j) = \sum_{i=1}^{j} \left[ y_i - \langle y \rangle \right] \]  

(2)

where, \( i = 1, 2, \ldots, N \); \( \langle x \rangle \) and \( \langle y \rangle \) are the mean of the two series.

Each series \( x_i \) and \( y_i \) are divided into \( N_s \) non-overlapping segments both in progressive and retrograde directions, to avoid any omission of time series data at the beginning or end of the series. For each \( 2N_s \) segments, local trend of both series \( x_j \) and \( y_j \) are computed by fitting polynomial of appropriate order \( (m) \). The subtraction of the fitted polynomial from the original segment gives the covariance:

\[ f_{xy}^2(u, s) = \left\{ \frac{1}{s} \sum_{k=1}^{s} \left[ (X((u-1) + k) - p_{x,u}^m(k)) \times (Y((u-1) + k) - p_{y,u}^m(k)) \right] \right\} \]  

(3)

Calculate detrended covariance by summing over all overlapping all segments of length \( n \):

\[ F^q_{xy}(s) = \frac{1}{2N_s} \sum_{u=0}^{2N_s-1} \text{sign}(f_{xy}^2(u, s)) \left| f_{xy}^2(u, s) \right|^{q/2} \]  

(4)

\( F^q_{xy}(s) \) behaves as a power-law function of \( s \) (the scaling behavior), where \( s \) is the segmental sample size:

\[ F^q_{xy}(s) \sim s^{\lambda(q)} \]  

(5)

The scaling exponent \( \lambda(q) \) similar to the generalized Hurst exponent \( h(q) \) in MF-DFA and it can be obtained by observing the slope of log-log plot of \( F(s) \) versus \( s \) by ordinary least squares.

DCCA cross-correlation coefficient is defined as the ratio between the detrended covariance function \( F_{xy} \) and the detrended variance functions \( F_x \) and \( F_y \) [17, 18]

\[ \rho_{xy} = \frac{F^q_{xy}}{\sqrt{F^q_{x} F^q_{y}}} \]  

(6)

Theoretically the value of \( \rho_{xy} \) ranges between \(-1 \leq \rho_{xy} \leq 1 \). The MFCCA analysis facilitate the estimation of scale dependent correlation between two candidate time series, which can provide better insight into the physical association between the variables. It is to be noted that in this study MFCCA is retrieved for the moment order \( q=2 \).

### 3. Study area and Data

In this study long term daily streamflow data of 65 stations located at three major river basins in India were collected from Water Resources Information System (WRIS) India (www.india-wris.nrsc.gov.in) operated by the Central Water Commission (CWC) India, which is one of the most reliable database pertaining to India. The map showing different major river basins are presented in Fig. 1. The data ranging from 1969 to 2016 are considered for the study. For brevity, the maximum and minimum data lengths of the basin along with the maximum and minimum drainage area of stations of different basins, are provided in Table 1.

| Sl No | Basin | Number of stations | Drainage Area (km²) | Data length |
|-------|-------|--------------------|---------------------|-------------|
|       |       |                    | Minimum            | Maximum     |
|       |       |                    |                     | Minimum     | Maximum     |
| 1.    | Krishna | 23                     | 1850               | 251360      | 1095        | 18615       |
| 2.    | Mahanadi | 16                      | 1100               | 124450      | 4015        | 15695       |
| 3.    | Godavari | 26                      | 2500               | 307800      | 1019        | 13111       |
4. Results and Discussion
Multifractal Cross Correlation Analysis (MFCCA) between streamflow and total suspended sediment (TSS) time series was performed for three major basins in India - Mahanadi, Krishna and Godavari, by choosing the moment order -4 to +4, maximum scale as N/2 and minimum scale is selected as more than the length of longest stretch of zero values. From the MFCCA, the individual persistence, joint persistence and cross correlation coefficient at annual scale and the overall correlation are determined for each case. The annual and overall cross correlation coefficient along with Hurst exponents obtained for stations in Mahanadi basin are given in figure 2.

From Fig. 2 it is noticed that in 81% of stations (i.e., 13 out of 16) the persistence of streamflow is more than that of TSS. Except in two cases (at Basantpur and Tikarapara stations), the seasonal correlation was detected. The annual cross correlation coefficient is more than 0.7 at all stations except Kesinga indicating very strong positive correlation between the parameters in the basin.
Figure 2 Hurst exponents of streamflow and TSS data of Mahanadi basin along with the cross correlation. (Hx, Hy, and Hxy refers the scaling exponent of streamflow, TSS and the joint persistence. Stations 1 -16 are Andhiyarkore, Bamnidhi, Baronda Basantpur Ghatora, Jonghra, Kantamal, Sundaragarh, Kesinga, Kurubhata, Manendragarh, Rajim, Rampur, Salebhata, Simga, Tikarapara). (a) Hurst Exponent (b) SF-TSS Cross correlation

Reasonably good overall correlation (>0.4) between the parameters is noted at 14 stations of the basin. The mean value of annual correlation is found to be 0.748 while it is 0.496 for overall data. The correlation plot and multifractal plots of Tikarapara station is presented in Fig. 3. The annual and overall correlation between streamflow and TSS along with scaling exponents of datasets of Krishna basin is given in figure. 4.

Figure 3 Plots of multifractal analysis of data of Tikarapara station along with the variability of cross correlation (a) scaling exponent plot; (b) mass exponent plot; (c) multifractal spectrum; (d) log-log plot of fluctuation function vs scale for q=2; (e) temporal variability of cross correlation coefficient.
Figure 4  Hurst exponents of streamflow and TSS data of Krishna basin along with the cross correlation. H<sub>SF</sub>, H<sub>TSS</sub> and H<sub>ST</sub> refers the scaling exponent of streamflow, TSS and the joint persistence; stations 1-23 are Bagalkot, Bawapuram, Byaladahalli, Cholaghuda, Haralahalli, Honnali, Huvanahedgi, KAgraharam, Karaad, Keesara, Kurundwad, Sarati, Malkhed, Mantralayam, Marol, Pondugala, Yadgir, Warunji, Wadanapalli, Wadakbal, Vijayawada, Takli, Shimoga. From Fig. 4 it is clear that for 14 out of 23 stations, the persistence of streamflow is more than that of TSS. In this case, the joint persistence (with a mean of 0.614) is found to be the average of the individual persistence of streamflow and TSS. Strong annual correlation (>0.7) is noted in 7 cases while it is more than 0.5 in 18 cases. In 9 cases seasonal correlation was also noted and the annual correlation is greater than that of seasonal correlation in these stations. The overall correlation was found to be weak (with a mean of 0.375) and in 5 cases the correlation is found to be more than 0.5. Fig. 5 shows typical plots of multifractal analysis of streamflow and sediment data along with the variability of Malkhed station in Krishna basin.

Figure 5  Plots of multifractal analysis of data of Malkhed station along with the variability of cross correlation (a) Scaling exponent plot; (b) mass exponent plot; (c) multifractal spectrum; (d) log-log plot of fluctuation function vs scale for q=2; (e) temporal variability of cross correlation coefficient

Similar analysis is performed for Godavari basin also and the scaling exponents and correlation estimates are presented in figure 6.
Figure 6 Hurst exponents of streamflow and TSS data of Godavari basin along with the cross correlation (H_x, H_y, and H_xy refers the scaling exponent of streamflow, TSS and the joint persistence; stations 1-16 are Ashti, babli, banni, Basar, Bhatpalli, Bishnur, Dhalegaon, GRBridge, Hivra, jagdalpur, Konta, Satrapur, Tekra, Kumhari, Mancheril, Nandgaon, Nowrangpur, PGBridge, Pathaguden, Pauni, Perur, Polavaram, Purna, Rajegaon, Saigaon, Yelli) (a) Hurst Exponent (b) SF-TSS Cross correlation

From figure 6 it is noted that unlike for other two cases, for majority of the stations in Godavari basin (i.e., 14 out of 26), the persistence of TSS is more than that of streamflow. The persistence is strong and long term for both streamflow and TSS series with a mean of 0.803 and 0.789 respectively. There exists a strong annual correlation between streamflow and TSS in this basin (mean value of 0.702). The annual correlation is greater than 0.5 in 23 cases, out of which in 17 cases the correlation is more than 0.7. The overall correlation was found to be more than 0.5 in 18 cases out of which the association is strong (>0.4) in 4 cases. For the datasets of Bishnur, Bhatpalli and Satrapur stations, both the annual and overall correlation are found to be very weak. It was also noted the seasonal correlation (at 3 month time scale) was also detectable at 16 out of 26 stations and annual correlation was found to be greater than seasonal correlation for data of all stations except Satrapur. At all stations of this basin, the joint persistence is found to be the average of persistence of streamflow and TSS. Fig. 7 shows typical plots of multifractal analysis along with the variability of cross correlation with time scale of Saigaon station in the Godavari basin.

In general, in most of the stations the persistence of streamflow is greater than that of TSS. In Godavari basin, majority of the stations the persistence of TSS is more than that of streamflow. The human interventions and flow regulations might have influenced the persistence and multifractality of streamflow in this basin to a great extent.
The investigation using MFCCA provides the time (scale) dependent information of the association between streamflow and TSS against the unique and traditional linear correlation between them, i.e., eventhough the overall correlation between the two are less, at specific time scale the association could be of considerable magnitude. In 39 stations, seasonal (intra-annual) association between streamflow and TSS are also noticed, among which highest number of stations (16 stations) are located in Godavari basin. This also infers the role of flow regulations in streamflow-TSS links of this basin. Eventhough streamflow-TSS association varies with temporal scales and there is no systematic pattern in this variation for the datasets of different basins. But it is noted that the strength of their association could vary significantly with time scale and their association could significantly depend on the basin and precipitation characteristics.

5. Conclusion
This study used the Multifractal Cross Correlation Analysis (MFCCA) for analysing the streamflow-sediment link in 65 rivers of three major basins in India in a multifractal perspective. From the results it is noted that the streamflow datasets of different river basins displayed multifractality and long term persistence. The streamflow-sediment links of different stations evaluated using MFCCA showed that the joint persistence is nearly the mean of the persistence of individual series. The streamflow displayed higher persistence than TSS in majority of the stations except that in Godavari basin. The annual cross correlation between streamflow and sediment is higher than seasonal and overall cross correlation but the strength of their association differs with river basin.

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