Estimation of covariance matrix using multi-response local polynomial estimator for designing children growth charts: A theoretically discussion

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Abstract. In statistical analysis for instance regression analysis we always be faced a estimation problem of regression function which draws relationship between variables in the regression model. In the real cases we frequently meet the relationship between one or more response variables and one or more predictor variables where there are correlations between responses that is called a multi-response regression model. There are two approaches to estimate the multi-response regression model, i.e., parametric and nonparametric. One of estimators in nonparametric regression model is local polynomial estimator for estimating the regression function. Since there are correlations between responses then in the estimating of regression function we need a weight matrix that is to be inverse of covariance matrix of error. Therefore, the main objective of this research is to estimate covariance matrix of error by using multi-response local polynomial estimator. The result of this research is a covariance matrix estimator that is in the future can be used to design children growth charts.

1. Introduction
In statistical modelling there are two approaches for estimating the regression function, i.e., parametric and nonparametric. The nonparametric approach gives more flexibility for the form of the regression function. Many researchers have studied nonparametric regression, i.e., spline estimator by [1-4], local linear estimator by [5-8], and local polynomial estimators with one response and one predictor variables for polynomial order one called as local linear have been discussed by [9] who applied it to spatial regression. Next, [10] have researched local linear estimator and stated that it is better than kernel estimator because it has mean square prediction least than that kernel estimator. Furthermore, [11] applied local polynomial estimator to time series data, and [12] used local polynomial estimator to estimate esophageal pressure on gastroesophageal reflux disease.

In the widely field we frequently be faced cases involving regression models with more than one response variables and there are correlation between responses. So, these regression function problems must be solved by multi-response nonparametric regression or multi-response semiparametric regression. For example [13] used smoothing spline estimator for biresponse that is applied to hormone data; [14-17] have discussed the multi-response nonparametric regression by using spline

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estimator and [18-20] have studied biresponse local linear regression. Meanwhile, multi-response semiparametric regression in some cases by using spline estimator and local linear estimator have discussed by [21-22] and [23], respectively.

One of real cases that involve regression model with more than one response variables and there are correlations between responses is children growth model [19], [20], [23]. According to [24] children up to one year old grows up quickly and then goes down slowly as long as going up of children’s age, so that locally model approach is more suitable for this case. Next, [25] proposed quantile polynomial parametric regression approach to design children growth chart. But, the growth chart given by [25] was too smooth, so that it did not represent the data which has locally pattern and its pattern change is very sharp. In addition, all researchers who have mentioned previously, discussed estimation the regression functions of the nonparametric and semiparametric regression models by assuming the covariance matrix is known, so that the covariance matrix is not need to be estimated.

If the covariance matrix is unknown then it must be estimated from the data. Therefore, in this paper we give estimating method of covariance matrix by using multi-response local polynomial estimator.

2. Method
To estimate children growth chart based on anthropometric measures, i.e., weight and height, we use multi-response model by weighted least squares method because there is significant correlation between weight and height of children. Therefore, for estimating the children growth charts we estimate the charts simultaneously and need the weight matrix, i.e., inverse of covariance matrix of error. In next section, we give results and discussion about estimations of the regression function and the covariance matrix methods by using multi-response local polynomial estimator.

3. Results and discussion
3.1. Estimation of Regression function by using multi-response local polynomial estimator
Suppose that the paired observations \((t_i, y_{i1}, y_{i2}, \ldots, y_{ip})\), \(i = 1, 2, \ldots, n\) follows the following regression model:

\[
y_i = f(t_i) + \epsilon_i, \quad r = 1, 2, \ldots, p
\]

where random error \(\epsilon_i\) follows the following assumptions:

\[
E(\epsilon_i) = 0, \quad \text{and} \quad E(\epsilon_i^2) = \sigma_i^2, E(\epsilon_i, \epsilon_{i'}) = \begin{cases} \sigma_{(i,i')}, & i = i' \\ 0, & i \neq i' \end{cases}
\]

for \(r \neq s\), \(r = s = 1, 2, \ldots, p\).

Next, based on \(i\)-th observation and for \(p\) responses, we can elaborate equation (1) as follows:

\[
\begin{aligned}
y_{i1} &= f_1(t_i) + \epsilon_{i1} \\
y_{i2} &= f_2(t_i) + \epsilon_{i2} \\
&\vdots \\
y_{ip} &= f_p(t_i) + \epsilon_{ip}
\end{aligned}
\]

Therefore, we can express equation (3) in the following matrix notation:

\[
\begin{pmatrix}
y_{i1} \\
y_{i2} \\
\vdots \\
y_{ip}
\end{pmatrix} = \begin{pmatrix}
f_1(t_i) \\
f_2(t_i) \\
\vdots \\
f_p(t_i)
\end{pmatrix} + \begin{pmatrix}
\epsilon_{i1} \\
\epsilon_{i2} \\
\vdots \\
\epsilon_{ip}
\end{pmatrix}
\]

where \(y_i = (y_{i1}, y_{i2}, \ldots, y_{ip})'\) and \(f(t_i) = (f_1(t_i), f_2(t_i), \ldots, f_p(t_i))'\) is unknown smooth function.

Further, if we elaborate every response in equation (4) for every observation, then equation (4) can be written as follows:

\[
y = f + \epsilon.
\]
where \( E(\varepsilon) = 0, \ Var(\varepsilon) = V \), \( y = (y_1, y_2, \ldots, y_p)' \), \( f = (f_1, f_2, \ldots, f_p)' \), \( \varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_p)' \), 
\( \varepsilon_e = (\varepsilon_{e1}, \varepsilon_{e2}, \ldots, \varepsilon_{en})' \), \( y_e = (y_{e1}, y_{e2}, \ldots, y_{en})' \), and \( f_e = (f_e(t_1), f_e(t_2), \ldots, f_e(t_n))' \).

In the model given in (5), there are correlations between responses, so that to get the estimated model by using multi-response local polynomial estimator we must use weights either kernel function or covariance matrix. Note that, statistically, the weight matrix is to be inverse of covariance matrix of error vector \( \varepsilon \). If we note the covariance matrix as \( \mathbf{V} \), then we can determine \( \mathbf{V} \) as follows:

\[
\mathbf{V} = \text{Var}(\varepsilon) = E(\varepsilon - E(\varepsilon))(\varepsilon - E(\varepsilon))' = E(\varepsilon\varepsilon')
\]

Next, by considering the assumptions in (2), we obtain the covariance matrix \( \mathbf{V} \) as follows:

\[
\mathbf{V} = E \left( \begin{bmatrix}
\varepsilon_{11}^2 & \varepsilon_{11}\varepsilon_{12} & \ldots & \varepsilon_{11}\varepsilon_{1n} \\
\varepsilon_{12}\varepsilon_{11} & \varepsilon_{12}^2 & \ldots & \varepsilon_{12}\varepsilon_{1n} \\
\vdots & \vdots & \ddots & \vdots \\
\varepsilon_{1n}\varepsilon_{11} & \varepsilon_{1n}\varepsilon_{12} & \ldots & \varepsilon_{1n}^2
\end{bmatrix}
\right)
\]

Furthermore, by applying Taylor expansion we get \( f_r(t), r = 1, 2, \ldots, p \) as follows:

\[
f_r(t) = \sum_{j=0}^{d_r} \frac{f_r^{(j)}(t_0)}{j!} (t-t_0)^j = \sum_{j=0}^{d_r} \beta_{rj}(t_0)(t-t_0)^j, \quad r = 1, 2, \ldots, p
\]

where \( d_r \) is order of polynomial, \( \beta_{rj}(t_0) = \frac{f_r^{(j)}(t_0)}{j!} \) and \( t \in (t_0-h, t_0+h) \).

We can express equation (7) in the following matrix notation:

\[
f_r(t) = X^*_r \beta_r(t_0)
\]

where \( X^*_r = \text{diag}(x_{r1}, x_{r2}, \ldots, x_{rp}) \), \( x_{r0} = \begin{pmatrix} (t-t_0) & \cdots & (t-t_0)^{d_r} \end{pmatrix} \), \( r = 1, 2, \ldots, p \).
\[ \beta(t_0) = \left( \beta_1(t_0) \quad \beta_2(t_0) \quad \cdots \quad \beta_p(t_0) \right)^\prime; \quad \text{and} \quad \beta_r(t_0) = \left( \beta_{r0}(t_0) \quad \beta_{r1}(t_0) \quad \cdots \quad \beta_{rl}(t_0) \right)^\prime. \]

Based on equation (7) and by using \( n \) samples of paired observation \((t_i, y_{i1}, y_{i2}, \ldots, y_{ip})\), we can write equation (8) as follows:

\[ f(t) = X_0 \beta(t_0) \]

where \( f(t) = (f_1(t), f_2(t), \ldots, f_p(t))^\prime; \quad f_r(t) = (f_{r1}(t_1), f_{r2}(t_2), \ldots, f_{rn}(t_n))^\prime; \)

\[ X_0 = \text{diag}(X_{t0}, X_{2t0}, \ldots, X_{pt0}) \quad \text{and} \quad X_{r0} = \begin{bmatrix} 1 & (t_1 - t_0) & \cdots & (t_1 - t_0)^d_r \\ 1 & (t_2 - t_0) & \cdots & (t_2 - t_0)^d_r \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (t_n - t_0) & \cdots & (t_n - t_0)^d_r \end{bmatrix}. \]

Based on (9) we can write equation (1) as follows:

\[ Y = X_0 \beta(t_0) + \varepsilon \]

where \( Y = (y_1, y_2, \ldots, y_p)^\prime \) and \( y_r = (y_{r1}, y_{r2}, \ldots, y_{rn})^\prime. \)

In estimating the regression function in model (1) based on multireponse local polynomial, we use weighted least squared optimization as follows:

\[ \text{Min}_\beta Q(\beta(t_0)) = \text{Min}_\beta(Y - X_0 \beta(t_0))^\prime V^{-1} K_h(t_0) (Y - X_0 \beta(t_0)) \]

where \( K_h(t_0) = \text{diag}(K_{h1}(t_0), K_{h2}(t_0), \ldots, K_{hp}(t_0)) \)

\[ K_{hr}(t_0) = \left( K_{hr}(t_1 - t_0), K_{hr}(t_2 - t_0), \ldots, K_{hr}(t_n - t_0) \right)^\prime \]

and matrix \( V \) is block diagonal matrix of error given in (6) so it is invertible matrix. So, we get solution of (11) as follows:

\[ \hat{\beta}(t_0) = (X_0^\prime V^{-1} K_h(t_0) X_0)^{-1} X_0^\prime V^{-1} K_h(t_0) Y. \]

Based on (8) and (12) we obtain the estimated of regression function \( \hat{f}(t) \) as follows:

\[ \hat{f}(t) = X_0^\prime \hat{\beta}(t_0) = X_0^\prime \left( X_0^\prime V^{-1} K_h(t_0) X_0 \right)^{-1} X_0^\prime V^{-1} K_h(t_0) Y, \quad t \in (t_0 - h, t_0 + h) \]

Therefore, for \( t = t_0 \) we have \( \hat{f}(t_0) \) as follows:

\[ \hat{f}(t_0) = e \left( X_0^\prime V^{-1} K_h(t_0) X_0 \right)^{-1} X_0^\prime V^{-1} K_h(t_0) Y \]

where \( e = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_p)^\prime \) and \( e_r (r = 1, 2, \ldots, p) \) is row matrix with dimension \( \left( p + \sum_{r=1}^p d_r \right) \), and \( d_r \) is polynomial order of \( r \)-th response. For \( p = 1 \), \( \varepsilon_1 \) equals to 1 at the first element, and equals to zero elsewhere. Next for \( p \geq 2 \), \( \varepsilon_r \) equals to 1 at \( \left( \sum_{r=1}^p d_r \right) + 1 \)-th element, and equals to zero elsewhere. Finally, based on multi-response local polynomial estimator we obtain estimation of regression function \( \tilde{f}(t) = \left( \tilde{f}(t_1), \tilde{f}(t_2), \ldots, \tilde{f}(t_n) \right)^\prime \) as follows:

\[ \tilde{f}(t) = A(h)Y \]

\[ \text{(14)} \]
where $A(h) = (A_h(t_1), A_h(t_2), \ldots, A_h(t_n))'$ and $A_h(t_i) = e \left( X'_h V^{-1} K_h(t_i) X_i \right)^{-1} X'_h V^{-1} K_h(t_i)$, $i = 1, 2, \ldots, n$.

3.2. Estimation of covariance matrix by using multi-response local polynomial estimator

In equation (13), $\hat{f}(t_0)$ contains unknown covariance matrix $V$ which must be estimated. In this section we discuss estimation of covariance matrix. To obtain estimated covariance matrix, firstly we consider the following optimization:

$$\min_{\beta'_r(t_0)} Q'(t_0) = \min\left( Y'_r - X_{n_0} \beta'_r(t_0) \right)' K_h(t_0) \left( Y'_r - X_{n_0} \beta'_r(t_0) \right)$$

So, we get solution of (15) as follows:

$$\hat{\beta}'_r(t_0) = \left( X'_{n_0} K_h(t_0) X_{n_0} \right)^{-1} X'_{n_0} K_h(t_0) Y'_r$$

and

$$E(Y'_r) = X_{n_0} \hat{\beta}'_r(t_0) = X_{n_0} \left( X'_{n_0} K_h(t_0) X_{n_0} \right)^{-1} X'_{n_0} K_h(t_0) Y'_r = A_{n_0} Y'_r$$

where $A_{n_0} = X_{n_0} \left( X'_{n_0} K_h(t_0) X_{n_0} \right)^{-1} X'_{n_0} K_h(t_0)$.

By assuming $E(\varepsilon'_{n_0}) = 0$, $\text{Var}(\varepsilon'_{n_0}) = \sigma^2_{n_0}$, we estimate $\sigma^2_{n_0}$ that is obtained from $E\left[ \varepsilon'_{n_0} \varepsilon'_{n_0} \right]$ as follows:

$$E(\hat{\sigma}^2_{n_0}) = E\left[ \varepsilon'_{n_0} \varepsilon'_{n_0} \right] = E\left[ \left( Y'_r - E(Y'_r) \right)' \left( Y'_r - E(Y'_r) \right) \right]$$

$$= E\left[ \left( Y'_r - A_{n_0} Y'_r \right)' \left( Y'_r - A_{n_0} Y'_r \right) \right] = E\left[ Y'_r (I - A_{n_0})' (I - A_{n_0}) Y'_r \right]$$

$$= E\left[ Y'_r Q_{(n_0)} Y'_r \right]$$

where $Q_{(n_0)} = (I - A_{n_0})' (I - A_{n_0})$.

Further, equation can be written as follows:

$$E(\hat{\sigma}^2_{n_0}) = \text{tr} \left( Q_{(n_0)} \sigma^2_{n_0} I_n \right) + (A_{n_0} Y'_r)' Q_{(n_0)} A_{n_0} Y'_r = \sigma^2_{n_0} \text{tr} \left( Q_{(n_0)} \right) + (A_{n_0} Y'_r)' Q_{(n_0)} A_{n_0} Y'_r$$

$$= \sigma^2_{n_0} \text{tr} \left( Q_{(n_0)} \right) + Y'_r A_{n_0}' Q_{(n_0)} A_{n_0} Y'_r = \sigma^2_{n_0} \text{tr} \left( Q_{(n_0)} \right) + Y'_r A_{n_0}' (I - A_{n_0}) A_{n_0} Y'_r$$

$$= \sigma^2_{n_0} \text{tr} \left( Q_{(n_0)} \right) + Y'_r A_{n_0}' (I - A_{n_0}) A_{n_0} Y'_r$$

Since matrix $A_{n_0}$ is idempotent, i.e.,

$$A_{n_0} A_{n_0} = X_{n_0} \left( X'_{n_0} K_h(t_0) X_{n_0} \right)^{-1} X_{n_0} K_h(t_0) X_{n_0} \left( X'_{n_0} K_h(t_0) X_{n_0} \right)^{-1} X_{n_0} K_h(t_0) = A_{n_0}$$

then by considering equation (17), we can write equation (18) as follows:
\[ E(\hat{\sigma}^2_{t_0}) = \sigma^2_{t_0} tr\left(\mathbf{Q}_{(rr)0}\right) + Y' A_{t_0} \left( I - A_{t_0} \right)' \left( A_{t_0} - A_{t_0} \right) Y \]

\[ = \sigma^2_{t_0} tr\left(\mathbf{Q}_{(rr)0}\right) \]

So, we obtain \( \hat{\sigma}^2_{t_0} = \frac{Y' \mathbf{Q}_{(rr)_0} Y}{tr(\mathbf{Q}_{(rr)_0})} \)

On the other hand, we have:

\[ E(\hat{\sigma}^2_{(rr)0}) = E\left[ \left( \mathbf{e}_{t_0}^* \mathbf{e}_{t_0} \right)' \right] = E\left[ \left( \mathbf{Y} - E(\mathbf{Y}) \right)' \left( \mathbf{Y} - E(\mathbf{Y}) \right) \right] = E\left[ Y' \left( I - A_{t_0} \right)' \left( I - A_{t_0} \right) Y \right] \]

\[ = E\left[ Y' \mathbf{Q}_{(rr)_0} Y \right] \]

where \( \mathbf{Q}_{(rr)_0} = \left( I - A_{t_0} \right)' \left( I - A_{t_0} \right) \).

Therefore, we get:

\[ E(\hat{\sigma}^2_{(rr)0}) = tr\left(\mathbf{Q}_{(rr)_0} \sigma_{(rr)_0} \mathbf{I}_m \right) + \left( A_{t_0} Y \right)' \mathbf{Q}_{(rr)_0} \left( A_{t_0} Y \right) \]

\[ = \sigma_{(rr)_0} tr\left(\mathbf{Q}_{(rr)_0} \right) + Y' A_{t_0} \left( I - A_{t_0} \right)' \left( A_{t_0} - A_{t_0} A_{t_0} \right) Y \]

\[ = \sigma_{(rr)_0} tr\left(\mathbf{Q}_{(rr)_0} \right) \]

Thus, we obtain:

\[ \hat{\sigma}^2_{(rr)_0} = \frac{Y' \mathbf{Q}_{(rr)_0} Y}{tr(\mathbf{Q}_{(rr)_0})} = \frac{Y' \left( I - A_{t_0} \right)' \left( I - A_{t_0} \right) Y}{tr\left( \left( I - A_{t_0} \right)' \left( I - A_{t_0} \right) \right)} \]

Next, for \( t_0 = t_i \), \( i = 1, 2, ..., n \), we have:

\[ \hat{\sigma}^2_{t_i} = \frac{Y' \mathbf{Q}_{(rr)(t_i)} Y}{tr(\mathbf{Q}_{(rr)(t_i)})} = \frac{Y' \left( I - A_{t_i} \right)' \left( I - A_{t_i} \right) Y}{tr\left( \left( I - A_{t_i} \right)' \left( I - A_{t_i} \right) \right)} \]

and for \( r \neq s \), we have:

\[ \hat{\sigma}^2_{(rs)} = \frac{Y' \mathbf{Q}_{(rs)} Y}{tr(\mathbf{Q}_{(rs)})} = \frac{Y' \left( I - A_{t_i} \right)' \left( I - A_{t_s} \right) Y}{tr\left( \left( I - A_{t_s} \right)' \left( I - A_{t_s} \right) \right)} \]

where \( r = s = 1, 2, 3, ..., p \), \( A_{t_i} = X_{t_i} \left( X_{t_i}' K_{rh} (t_i) X_{t_i} \right)^{-1} X_{t_i} K_{rh} (t_i) \).

Therefore, we obtain estimation of covariance matrix \( \mathbf{V} \) as follows:
\[ \hat{V} = \begin{bmatrix} \hat{\Sigma}_1 & \hat{\Sigma}_{12} & \cdots & \hat{\Sigma}_{1p} \\ \hat{\Sigma}_{21} & \hat{\Sigma}_2 & \cdots & \hat{\Sigma}_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\Sigma}_{p1} & \hat{\Sigma}_{p2} & \cdots & \hat{\Sigma}_p \end{bmatrix} \]

(19)

where

\[ \hat{\Sigma}_r = \text{diag} \left( \hat{\sigma}_{1r}^2, \hat{\sigma}_{2r}^2, \ldots, \hat{\sigma}_{nr}^2 \right) = \text{diag} \left( \frac{Y'Q_{(ir)r_1}Y}{\text{tr} \left( Q_{(ir)r_1} \right)}, \frac{Y'Q_{(ir)r_2}Y}{\text{tr} \left( Q_{(ir)r_2} \right)}, \ldots, \frac{Y'Q_{(ir)r_n}Y}{\text{tr} \left( Q_{(ir)r_n} \right)} \right) \]

\[ \hat{\Sigma}_{rs} = \hat{\Sigma}_{sr} = \text{diag} \left( \hat{\sigma}_{(ir)s_1}^2, \hat{\sigma}_{(ir)s_2}^2, \ldots, \hat{\sigma}_{(ir)s_p}^2 \right) = \text{diag} \left( \frac{Y'Q_{(ir)s_1}Y}{\text{tr} \left( Q_{(ir)s_1} \right)}, \frac{Y'Q_{(ir)s_2}Y}{\text{tr} \left( Q_{(ir)s_2} \right)}, \ldots, \frac{Y'Q_{(ir)s_p}Y}{\text{tr} \left( Q_{(ir)s_p} \right)} \right) \]

and \( r = s = 1, 2, 3, \ldots, p. \)

3. Conclusion

For designing the children growth charts, we estimate them simultaneously and need the weight matrix, i.e., inverse of covariance matrix of error. By using multi-reponse local polynomial, we obtain the estimated regression function of the model as given in equation (14), and we obtain the estimated covariance matrix as given in equation (19). Furthermore, in the next our research, the children growth charts can be used for assessing nutritional status of children.

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