Mass Spectrum of the 3d SU(2) Higgs Model and the Symmetric Electroweak Phase

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We present results for the masses of the low-lying states with quantum numbers $0^{++}$, $2^{++}$ and $1^{--}$ as well as Polyakov line correlations in the Higgs and confinement regions of the 3d SU(2) Higgs model. In the confinement phase we find a dense spectrum of bound states approximately split into two disjoint sectors. One consists of W-balls nearly identical to the glueball spectrum of the pure gauge theory, the other of bound states of scalars.

1. INTRODUCTION

It is by now well established that the electroweak phase transition is of first order for Higgs masses up to $m_H \sim 70$ GeV \cite{footnote:lambda}, and there is strong evidence for a crossover behaviour for $m_H > \sim 80$ GeV \cite{footnote:betaH}. While the problem of the order of the electroweak phase transition thus seems to be solved, some puzzles remain concerning the nature of the symmetric phase. In particular, computing the mass spectrum of the 3d effective theory from gauge-invariant composite operators leads to a picture of a confining symmetric phase with a spectrum of bound states \cite{footnote:betaH}. However, calculations of the full propagators for the Higgs and W bosons in Landau gauge suggest significantly lighter states in the symmetric phase, while they yield similar masses in the Higgs phase \cite{footnote:betaH}.

Here we elaborate on our gauge-invariant computation of the mass spectrum in \cite{footnote:betaH} by paying more attention to the excitation spectrum. In addition to the $0^{++}$ and $1^{--}$ states considered previously, we also compute the $2^{++}$ spectrum and the Polyakov line correlations. The three-dimensional lattice action is

\begin{equation}
S[U, \phi] = \beta_G \sum_p \left(1 - \frac{1}{2} \text{Tr} U_p\right) + \sum_x \left\{ - \beta_H \sum_{\mu} \frac{3}{2} \text{Tr} \left( \phi_x^\dagger U_{\mu x} \phi_{x+\hat{\mu}} \right) + \frac{1}{2} \text{Tr} \left( \phi_x^\dagger \phi_x \right) - 1 \right\} \right)^2 \end{equation}

In this simulation we kept the ratio

\begin{equation}
\frac{\lambda_3}{g_3^2} = \frac{\beta_R \beta_G}{\beta_H^2} = 0.0239
\end{equation}

fixed and chose a point in the Higgs and confinement phase, $\beta_H = 0.3450$ and $\beta_H = 0.3438$, respectively, at $\beta_G = 9$.

2. IMPROVED OPERATORS AND CROSS CORRELATIONS

We measure correlation functions of gauge-invariant operators of three basic types. In the $0^{++}$ and $2^{++}$ channels we consider operators containing only scalar fields, $R \sim \text{Tr}(\phi_x^\dagger \phi_x)$, scalar fields and links, $L_\mu \sim \text{Tr}(\phi_x^\dagger U_{\mu x} \phi_{x+\hat{\mu}})$, or only plaquettes $P \sim \text{Tr} U_p$. In the $1^{--}$ channel there is only one operator type, $V^a_\mu \sim \text{Tr}(\tau^a \phi_x^\dagger U_{\mu x} \phi_{x+\hat{\mu}})$.

In order to improve the projection properties of our operators we employ "smearing” or "blocking” techniques \cite{footnote:betaH}. These consist of constructing non-local, composite field variables $\phi_x^\dagger$, $U^c_{\mu x}$ by covariantly connecting them with their neighbours and then building the operators out of these smeared fields. This procedure can be iterated to create operators of various spatial extensions. These are more sensitive to infrared physics and have a better overlap with states that represent extended particles, such as weakly bound states.
In order to compute the excitation spectrum of states with a given quantum number we keep \( N \) operators of different types and blocking levels and measure cross correlations between all of them. The correlation matrix can then be diagonalised numerically following a variational method. For a given set of operators \( \phi_i \) we find the linear combination that minimises the energy and thus corresponds to the lightest state. The first excitation can be found by repeating this step, but restricted to the subspace \( \{ \phi_i \} \) that is orthogonal to the ground state. This procedure can be continued to higher states, and we end up with a set of \( N \) eigenstates, \( \Phi_i = \sum_{k=1}^{N} a_{ik} \phi_k \). The coefficients \( a_{ik} \) are useful in identifying the contributions of the individual operators used in the simulation to the mass eigenstates.

A detailed discussion of the construction of operators, the blocking techniques adopted and the diagonalisation procedure may be found in [4].

3. THE MASS SPECTRUM

The continuum extrapolation of the lowest \( 0^{++} \) and \( 1^{--} \) mass eigenstates of the spectrum as obtained in [3] is shown in Figs. 1, 2.

In the Higgs phase higher excitations are heavy (the lightest being about twice the mass of the W) and plateaux in the effective masses are hard to identify. This is not surprising since they are expected to be scattering states with relative momentum. In the symmetric phase, on the other hand, there is a dense spectrum of bound states. Their composition may be characterised by considering the contributions of the individual operators to each eigenstate, as shown in Fig. 3 for the \( 0^{++} \) channel at \( \beta_G = 12 \). The pure gauge de-
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ball”, in analogy to the glueballs of pure gauge theory. Fig. 1 also shows preliminary results for a set of $2^{++}$ operators. In this channel the lowest state is composed predominantly of $R$ and $L$ type operators, the following two excited states are mixed, and the third excited state receives $P$ contributions only, thus being interpreted as a $2^{++}$ W-ball. In Table 1 the masses of the W-balls and their excitations are compared with those of the glueballs in the pure gauge theory, from which they differ at the percent level at most.

Table 1

| Purely gluonic excitations in the gauge Higgs and pure gauge systems at $\beta_G = 9$. |
|---------------------------------------------|
| aM$_{[0^+]}$ | 0.751(8) | 0.767(6) |
| aM$_{[0^+]^*}$ | 1.07(2) | 1.08(2) |
| aM$_{[0^+]^{**}}$ | 1.26(3) | 1.27(2) |
| aM$_{[2^+]}$ | 1.19(3) | 1.26(2) |
| aM$_{[2^+]^*}$ | 1.38(4) | 1.50(5) |
| aM$_{[2^+]^{**}}$ | 1.76(10) | 1.77(6) |
| $a\sqrt{\sigma}$ | 0.1582(6) | 0.1616(6) |

4. POLYAKOV LOOPS

Another quantity that may help to clarify the nature of the symmetric phase is the Polyakov line operator whose expectation value vanishes in a confining theory. One would not expect it to be exactly zero in the symmetric phase of the Higgs model because flux tubes between fundamental charges eventually break with increasing separation. At our parameter values in the symmetric phase, however, the VEV of the Polyakov line operator is zero within errors. Hence we may extract a volume-corrected string tension from Polyakov line correlations according to

$$a^2 \sigma = a^2 \sigma_L + \frac{\pi}{6} \frac{1}{L^2}; \quad aM_{pol}(L) = a^2 \sigma_L L; \quad (3)$$

where $L$ is the side length of the lattice. In the symmetric phase we find $aM_{pol}(24) = 0.579(4)$. This yields a string tension $\sim 97\%$ of the one found in the pure gauge theory, c.f. Table 1. In the Higgs phase we estimate $aM_{pol} = 1.2(1)$. Here the Polyakov loop operator exhibits a VEV, no flux tubes exist, and the correlator cannot be related to a string tension. Instead, a weak coupling expansion is possible whose leading term is a two-W exchange diagram. Indeed, the effective mass for the Polyakov line is consistent with twice the W-mass, $aM_W = 0.836(2)$ at $\beta_G = 9$.

5. CONCLUSIONS

To summarise, we have confirmed the approximate decoupling of the pure gauge sector from the Higgs sector in the symmetric phase. The mass spectrum includes a set of W-balls, which is essentially equivalent to the glueball spectrum in pure gauge theory and undisturbed by the presence of matter fields. In addition there are also bound states of scalars, seemingly rather disconnected from the W-ball part of the spectrum. The effective vanishing of the Polyakov loop VEV as well as the value of the string tension are further similarities of the symmetric phase with the pure gauge theory. More work is needed to understand the relation between the gauge-fixed and gauge-invariant calculations in the symmetric phase. It is interesting to speculate whether the same decoupling of glueballs holds in QCD.

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