Study of Kaon Decay to Two Pions

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The weak decay of the kaon to two pions is studied within the NJL Model. Using the standard effective weak Hamiltonian, both the decay amplitude arising from an intermediate state $\sigma$ meson and the direct decay amplitude are calculated. The effect of final state interactions is also included. When the matching scale is chosen such that the decay amplitude with isospin $I = 2$ is close to its experimental value, the $\sigma$ meson contributes more than 85% of the total $I = 0$ amplitude. This supports recent suggestions that the $\sigma$ meson should play a vital role in explaining the $\Delta I = 1/2$ rule in this system.

Keywords: kaon decay, intermediate sigma meson state, final state interaction, NJL model

I. INTRODUCTION

The $\Delta I = 1/2$ rule [1, 2], notably in the $K \to \pi\pi$ decay, is one of the major outstanding challenges to our understanding of the hadronic weak interaction. It has therefore been studied with many different theoretical methods [3–21]. In recent years these efforts have been extended to include lattice QCD studies, with recent results reported in Ref. [3] and Refs. [22, 23], the latter focussing on decays into the isospin $I = 2$ channel.

Amongst many quark model studies devoted to this problem, we note that in Ref. [4] the authors calculated the matrix elements up to $O(p^4)$ within the framework of the chiral quark model. Using chiral perturbation theory, Kambor et al. [5, 7] studied the kaon decays to one loop order within SU(3). Again, within SU(3) chiral perturbation theory, the effect of isospin breaking was included and one-loop results reported in Ref. [12]. Bijnen et al. [8] studied the kaon decays to one loop order within SU(2) chiral perturbation theory. NLO contributions were considered within the large $N_c$ approach in Refs. [9, 13]. The potentially important role of the trace anomaly in weak $K$-decays, especially in regard to the $\Delta I = 1/2$ rule, was discussed in Ref. [13].

The possible role of the charm quark in generating the observed enhancement was discussed in Ref. [14], with the authors presenting there the first results from lattice simulations in the SU(4) flavor limit. In Ref. [15] the authors studied the problem within the framework of a dual 5-dimensional holographic QCD model. The possible effect of “new physics”, specifically the effect of introducing a heavy colorless $Z'$ gauge boson, was discussed by Buras et al. [15].

In a recent report [21], Buras summarized a study of this rule based on the dual representation of QCD using the large $N_c$ expansion. The Wilson coefficients and hadronic matrix elements were evaluated at different energy scales, $\mu$, in the early large $N_c$ studies, and thus the calculated value of $A_0$ was only about 10% of the experimental one. By evaluating the Wilson coefficients and hadronic matrix elements at the same energy scale, the discrepancy was decreased by about 40%. Moreover, the introduction of QCD penguin operators further decreased the initial discrepancy.

The effect of final state interactions (FSI) was studied in various ways in Refs. [24–30]. For example, in Ref. [24] the authors directly calculated the relevant Feynman diagrams for the meson rescattering corrections in chiral perturbation theory. The Omnès approach, which is based on dispersion relations, was used in Refs. [25–28], while in Refs. [29, 30] the effect of FSI was evaluated within potential models.

Of particular interest to us is the recent work by Crewther and Tunstall [31–33], which examined the proposal that the $\Delta I = 1/2$ rule might be resolved if QCD were to have an infrared fixed point. This suggested that the $\sigma$ meson would play an especially important role. While the existence of the $\sigma$ meson has been controversial for decades, there is now convincing evidence of a pole in the $\pi - \pi$ scattering amplitude with a mass similar to that of the kaon, albeit with a very large width. Given that there is a known scalar resonance nearly degenerate with the kaon, it is clear that such a state may well play a significant role in the $K \to 2\pi$ decay. With this motivation, we use the NJL model, together with the familiar operator product formulation of the non-leptonic weak interaction, to make an explicit calculation of the role of the $\sigma$ meson in the decay $K \to 2\pi$, with the aim of clarifying its role in the $\Delta I = 1/2$ rule. Section II gives details of the calculation of the $\sigma$ contribution, while the direct decay to pions is found in sect. III. The numerical results and discussion are given in sect. IV.
II. CALCULATION OF KAON DECAY
INCLUDING THE $\sigma$ MESON

Following the standard conventions we label the $K$ decay to two pions with isospin zero as $A_0$ and with isospin two as $A_2$ \[22\].

$$A_I \equiv \frac{1}{\sqrt{2}} \langle (\pi\pi)_I | K^0 \rangle, \quad I = 0, 2. \quad (1)$$

As explained earlier, for the former we calculate the contribution from two different mechanisms; first, the weak transition from $K$ to a $\sigma$ meson followed by the decay of the $\sigma$ to two pions and second, the direct decay to two pions. For $A_2$ only the latter path is available.

For the first contribution to $A_0$, illustrated in Fig. 1 we write:

\[
\begin{array}{c}
\text{\includegraphics[width=0.5\textwidth]{fig1.png}}
\end{array}
\]

**FIG. 1:** Contribution of $\sigma$ meson to $K \rightarrow \pi\pi$.

$$A_0^\sigma = -\frac{\sqrt{3}}{2} g_{K\sigma} \times \Delta_\sigma \times \gamma \left( \frac{m_K^2}{2} - m_\pi^2 \right), \quad (2)$$

where $g_{K\sigma}$ is the coupling for the $K\sigma$ transition, $\Delta_\sigma$ is the propagator of the $\sigma$ meson and $\gamma$ is the $\sigma\pi\pi$ coupling $\[34\].$

$$\mathcal{L}_{K\sigma} = g_{K\sigma} K_\sigma^0 \sigma = \frac{g_{K\sigma}}{\sqrt{2}} K^0_\sigma + \frac{g_{K\sigma}}{\sqrt{2}} K^0_\sigma, \quad (3)$$

$$\mathcal{L}_{\sigma\pi\pi} = -\frac{\gamma}{\sqrt{2}} \bar{\sigma} \partial_\mu \pi^\mu \pi \quad (4)$$

and we have neglected the effect of CP-violation.

We employ the NJL model with proper time regularization to describe the structure of these mesons. The coupling of the $\sigma$ to the pions is determined from its pole position. Finally, the effective Hamiltonian describing the non-leptonic weak interaction is obtained using the standard operator product expansion. We now briefly summarise each of these parts of the calculation.

A. NJL model

Our work includes the SU(3)-flavour NJL formalism. After Fierz transformation, the Lagrangian density can be written in the meson channels. In this form the contributions from the different types of meson can be read directly $\[35\] [36\]. Here we are just concerned with the scalar ($\sigma$-meson) and pseudoscalar (pion and kaon) channels of the Lagrangian density after Fierz transformations, given by:

$$\mathcal{L}_{NJL}^\sigma = \frac{1}{2} G_\pi \left[ \frac{2}{3} \left( \bar{\psi} \psi \right)^2 + \left( \bar{\psi} \lambda \psi \right)^2 - \frac{2}{3} \left( \bar{\psi} \gamma_5 \psi \right)^2 \right]. \quad (5)$$

**FIG. 2:** Diagrammatic representation of the inhomogeneous Bethe-Salpeter Equation for the different quark-antiquark bound states (mesons) of total 4-momentum $q$.

where the eight Gell-Mann SU(3)-flavor matrices are represented as $\lambda$.

Since NJL is an effective model, it needs to be regularized. We chose the proper-time regularization scheme because it has the property of simulating quark confinement $\[37\] [39\]$, in the sense that we do not have thresholds for decays of the color singlets. We achieve this by including an infrared cutoff, $\Lambda IR$, that, in analogy with QCD, takes the role of the QCD-scale parameter $\Lambda_{QCD}$. Therefore, we set $\Lambda IR = \Lambda_{QCD} = 0.24$ GeV. The regularization of the loop integrals includes an ultraviolet cutoff, $\Lambda UV$.

With $\mathcal{L}_{NJL}^{\sigma\pi\pi}$ we follow the standard method of solving the Bethe-Salpeter equations (BSE) for the quark-antiquark bound states (mesons) $\[35\] [36\]$. The diagram describing this BSE in the NJL model is shown in Fig. 2 and its solutions are given by the following reduced t-matrices:

$$T_j (q) = \frac{-2iG_\sigma}{1 + 2G_\sigma \Pi_j (q^2)}. \quad (6)$$

Here, the polarization, $\Pi_j (q^2)$, represents the quark-antiquark loops that appear in the diagram for the BSE ($j = \sigma$-meson, pion or kaon). Their analytic expressions are

$$\Pi_\sigma = 6i \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ S_{q_1} (k) S_{\bar{q}_2} (k + q) \right] \quad (7)$$

and

$$\Pi_{\pi(K)} = 6i \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \gamma_5 S_{q_1} (k) \gamma_5 S_{\bar{q}_2} (k + q) \right] \quad (8)$$

where $\text{Tr}$ is a trace in Lorentz indices (the traces over color and flavour having already been taken) and $S_i$ are the constituent quark propagators. For the $\sigma$ and pion the two propagators contain the same light quark masses whereas for the kaon case their masses are different.

The pole position of $T_j (q)$ corresponds to the mass of each of the mesons, ($j$), which is evident if one examines the expression for $T_j (q)$ in pole approximation $\[35\]$

$$T_j (q) \rightarrow -\frac{ig_j^2}{q^2 - M_j^2}, \quad (9)$$

where $g_j$ is the effective quark-meson coupling, given by

$$g_j^2 = \left( \frac{\partial \Pi_j}{\partial q^2} \right)^{-1} \bigg|_{q^2 = M_j^2}. \quad (10)$$
Here we assume degenerate masses for the constituent light quarks \((m_l = m_u = m_d)\). Since our treatment of the \(\sigma\)-meson follows the model described in Ref. [40] and summarised in Sec. II D, we adjust \(m_l\) to reproduce its “bare” mass \((M_{\sigma}^{(0)})\). In addition, \(G_\pi\) is fit to reproduce the pion mass. Finally \(\Lambda_{\text{UV}}\) is fit to ensure the correct kaon mass. A summary of the parameters used here is given in Table I.

The complication associated with such a model, when one needs to match to operators that are defined at some renormalization scale, is that the scale associated with a valence-dominated quark model is typically quite low. For example, extensive studies of parton distribution functions within the NJL model \([41-43]\) (as well as other valence-dominated quark models \([44, 45]\)) typically lead to a matching scale of order 0.4-0.5 GeV. This is rather low and one therefore needs to check the reliability of the effective weak couplings at such a scale. We address this below.

### B. Effective weak Hamiltonian

Here we need the \(\Delta S = 1\) effective Lagrangian of the electroweak interaction \([46]\)

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{6} \left( z_i(\mu) - \frac{V_{us}^* V_{ud}}{V_{ts}^* V_{td}} y_i(\mu) \right) Q_i ,
\]

where \(V_{xy}\) is the relevant CKM matrix element, \(G_F\) is the Fermi coupling constant and the four-quark operators, \(Q_i\), are:

\[
\begin{align*}
Q_1 &= \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) u_\beta \bar{u}_\beta \gamma^\mu (1 - \gamma_5) d_\alpha , \\
Q_2 &= \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) u_\alpha \bar{u}_\beta \gamma^\mu (1 - \gamma_5) d_\beta , \\
Q_3 &= \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) d_\alpha \bar{q}_\beta \gamma^\mu (1 - \gamma_5) q_\beta , \\
Q_4 &= \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) d_\beta \bar{q}_\beta \gamma^\mu (1 - \gamma_5) q_\alpha , \\
Q_5 &= \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) d_\alpha \bar{q}_\beta \gamma^\mu (1 - \gamma_5) q_\beta , \\
Q_6 &= \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) d_\beta \bar{q}_\beta \gamma^\mu (1 - \gamma_5) q_\alpha .
\end{align*}
\]

The Wilson coefficients, \(z_i(\mu)\) and \(y_i(\mu)\), have been calculated up to the next to leading order using perturbative QCD \([46]\). Since \(V_{us}^* V_{ud}/V_{ts}^* V_{td}\) is relatively small, we will only keep the contribution of the terms with \(z_i\).

In order to investigate the potential model dependence in matching the renormalization group scale of the operators to the NJL model, in Fig. 3 we show the variation of the coefficients \(z_i(\mu)\) as \(\mu\) varies from 700 to 450 MeV. We can see that these Wilson coefficients vary particularly quickly as \(\mu\) drops below 490 MeV and clearly, if one wants reliable results, one should not choose a scale far below this limit.

![Fig. 3: Wilson Coefficients, \(z_i(\mu)\).](image-url)

\[\text{FIG. 3: Wilson Coefficients, } z_i(\mu) \text{. The abscissa represents the energy scale, } \mu, \text{ in units of MeV.}\]

### C. Coupling for the \(K\sigma\) transition

With the Wilson coefficients and the NJL model explained, we can proceed with the calculation of the weak \(K\) to \(\sigma\) transition amplitude, \(g_{K\sigma}\)

\[
g_{K\sigma} = \sqrt{2} \langle \sigma | H_{\text{eff}} | K^0 \rangle ,
\]

as illustrated in Fig. 4. The corresponding matrix elements are evaluated with dimensional regularization using modified minimal subtraction in order to be consistent with the relevant Wilson coefficients, \(z_i(\mu)\). We find that the contributions of \(Q_1 \sim Q_4\) to \(g_{K\sigma}\) vanish, with only \(Q_5\) and \(Q_6\) contributing to \(g_{K\sigma}\) in our results. In a more sophisticated model, where the masses of the constituent quarks and the couplings between the mesons and quark pairs were momentum dependent, the operators \(Q_1\) to \(Q_4\) would also contribute to \(g_{K\sigma}\). The full expressions for the \(K\sigma\) transition amplitude are given in Appendix A.

![Fig. 4: Illustration of the \(K\sigma\) transition, where the solid line with an \(s\) represents the \(s\) quark and the other solid lines represent \(u\) or \(d\) quarks.](image-url)
TABLE II: Parameter sets describing the $\sigma$ meson, including the "bare" mass, $M_{\sigma}^{(0)}$, the $\sigma-\pi\pi$ coupling, $(\gamma)$, as well as the $\sigma$ propagator, $(\Delta_{\sigma})$, evaluated at the kaon mass. The masses and cut-off are in GeV, $\Delta_{\sigma}$ in GeV$^{-2}$ and $\gamma_0 = 6.416$ GeV$^{-1}$.

| $\Lambda$ | $M_{\sigma}^{(0)}$ | $\gamma \times \gamma_0$ | $\Delta_{\sigma}(M_K)$ |
|----------|------------------|------------------|------------------|
| 0.320    | 0.563            | 4.53             | 7.476 - 8.715i   |
| 0.330    | 0.600            | 4.60             | 4.222 - 7.102i   |
| 0.340    | 0.640            | 4.70             | 2.604 - 5.571i   |

D. $\sigma$ model

The problem we are attacking requires an intermediate $\sigma$ state, therefore we need a model that describes the propagator of this meson. In Ref. [40] the authors proposed a $\sigma$ model, which could be used to calculate the propagator for real values of the energy (corresponding to physical $\pi\pi$ scattering), while ensuring the correct position of the resonance, corresponding to the $\sigma$-meson, in the complex energy plane.

This $\sigma$-model is divided into two parts: one where the "bare" mass of the $\sigma$ is computed from the NJL model (using the formalism of Sec. II A); and a second part that includes its correct pole position in the second Riemann sheet, computed from first principles by Caprini, Colangelo and Leutwyler [47]. This was achieved by including leading order pion loop corrections to the $\sigma$ propagator. These were described by the self energy $\Sigma_{\pi\pi}$, depicted in Fig. 5. In general, the $\sigma$ propagator takes the form

$$\Delta_{\sigma} = \frac{i}{p^2 - (M_{\sigma}^{(0)})^2 + \Sigma_{\pi\pi}^{2}}. \quad (19)$$

Since $\Sigma_{\pi\pi}$ comes from an effective interaction between the pions and the $\sigma$ it needs to be regularized. We employ the original method, including a dipole cut-off with mass $\Lambda$ [40].

Changing $M_{\sigma}^{(0)}$ and keeping the pole position implies variations in the $\sigma$ to $\pi\pi$ coupling, $\gamma$, as well as the regularization parameter, $\Lambda$. In Table II, we show three parameter sets that were originally presented in Ref. [40], along with the value of the propagator evaluated at the mass of the kaon.

FIG. 5: Diagramatic representation of the $\sigma$ meson self energy $\Sigma_{\pi\pi}$.

III. DIRECT DECAY TO PIONS

The second mechanism contributing to the decay $K \to \pi\pi$ proceeds directly to two pions, as illustrated in Fig. 6(a). Since the Wilson coefficients $z_1$ and $z_2$ are much larger than others, we only consider the contributions of $Q_1$ and $Q_2$. Once again the diagrams are calculated with dimensional regularization and modified minimal subtraction. After calculation, we find that only Fig. 6(a) contributes to our results in the NJL model.

We denote the amplitudes corresponding to diagrams Fig. 6 without the contribution of the final state interaction as $A_{I2}^{D,0}$. In this case we should also consider the effect of final state interactions(FSI), which we treat using the method of Refs. [23, 24].

$$A_{I}^{D} = A_{I2}^{D,0} \times F_{I}(m_{K}^2)$$

$$\approx A_{I2}^{D,0} \times \exp\left(\frac{m_{K}^{2}}{\pi} \int_{4m_{K}^{2}}^{\infty} \delta\left(s^{'}\right) \frac{ds^{'}r}{r} \right)$$

$$\approx A_{I2}^{D,0} \times \exp\left(\frac{m_{K}^{2}}{\pi} \int_{1.0 GeV}^{m_{K}^{2}} \delta\left(s^{'}\right) \frac{ds^{'}r}{r} \right), \quad (20)$$

where $\delta\left(s^{'}\right)$ is the phase shift for pion-pion scattering with isospin $I$ and we take the values of $\delta\left(s^{'}\right)$ from Ref. [15]. This yields the result:

$$F_{0}(m_{K}^{2}) \approx 1.4e^{-40^o}, \quad F_{2}(m_{K}^{2}) \approx 0.94e^{-i7.2^o}. \quad (21)$$

FIG. 6: The possible diagrams for the direct weak decay. The solid line with $s$ represents a quark, other solid line represents a or $d$ quark. For the operator $Q_1$ and $Q_2$, only Fig. 6(a) contributes in the NJL model.
IV. NUMERICAL RESULTS AND DISCUSSION

As we have explained, in this work $A_0$ contains two contributions, the first, $A^\sigma_0$, involving the coupling to the $\sigma$ meson and the second, $A^D_0$, involving the direct decay to pions. Since, in the NJL model, $A^\sigma_0$ involves the weak operators $Q_5$ and $Q_6$, while $A^D_0$ involves $Q_1$ and $Q_2$, their contributions can be added with no worry about double counting. The phase shift of $A^\sigma_0$ resulting from the propagator of the $\sigma$ approximately reproduces the physical $\pi\pi$ phase shift appearing in $A^D_0$, so we can set $|A_0| \approx |A^\sigma_0| + |A^D_0|$.

We list the $K$-$\sigma$ coupling and the decay amplitudes, as a function of the matching scale, $\mu$, and the dipole cut-off $\Lambda$ in Table III. From this Table one sees that the decay amplitudes are rather more sensitive to $\mu$ than $\Lambda$.

| $\mu$ | 0.490 | 0.492 | 0.494 |
|-------|-------|-------|-------|
| $\Lambda$ | 0.32 | 0.33 | 0.33 |
| $|g_{K\sigma}|$ | 1327 1951 2714 | 2146 3070 4124 | 6383 8662 10824 |
| $|A^\sigma_0|$ | 57 61 65 | 93 96 98 | 276 272 258 |
| $|A^D_0|$ | 113 126 139 | 79 85 89 | 39 40 39 |
| $|A_0|$ | 54 62 70 | 41 46 52 | 19 20 20 |
| $|A^\sigma_0| + |A^D_0|$ | 170 187 203 | 172 181 187 | 314 311 297 |
| $\frac{|A_0| + |A^\sigma_0|}{|A^D_0|}$ | 3.2 3.0 2.9 | 4.2 3.9 3.6 | 16 16 15 |

In Refs. [9, 21], the authors used the MOM scheme to evolve the Wilson coefficients and hadronic matrix elements to the same energy scale. In order to match the energy scales, the Wilson coefficients were evolved from $\mu = O(M_W)$ to $\mu = O(0.6 \sim 1$ GeV) in the quark-gluon picture, while the hadronic matrix elements were evolved from $\mu = O(M_Z)$ to the same scale $\mu = O(0.6 \sim 1$ GeV) in the meson picture. $|A_0|/|A^2|$ was found to lie in the range $12.5\sim14.9$ as $\mu$ varied from $0.6\sim1$ GeV, if only the contributions from $Q_1$ and $Q_2$ were included.

We notice that the $\mu$ dependence of their results was smaller than what we have found. Although the Dirac traces and loop integrals in the hadronic matrix elements were evaluated with dimensional regularization and modified minimal subtraction (as were the Wilson coefficients in our approach) the possible $\mu$-dependence of the constituent quark masses and the couplings between mesons and quarks do not naturally appear in the NJL model. Rather the model is assumed to represent QCD at a scale at which the gluons are effectively frozen out as degrees of freedom and valence quarks interacting through a chiral effective Lagrangian dominate the dynamics. Thus the best one can do is to match the scale of the effective weak Hamiltonian to the scale at which the NJL model best matches experiment, which seems to be around $0.4 \sim 0.5$ GeV.

We note that, in addition to the processes included here, there are also diagrams which are disconnected if the gluon lines are removed (usually just called disconnected diagrams for short). While such disconnected diagrams can contribute to $A_0$, they do not naturally appear within the NJL model and we omit them here. Since $A^D_0$ is not contributed by the disconnected diagrams, we use it to fix the energy scale $\mu$.

As we already noted earlier, in order that the evolution of the Wilson coefficients is under control, the matching scale, $\mu$, should not be lower than about 490 MeV. This creates some tension as the scale associated with the NJL model, when matching to phenomenological parton distribution functions, tends to be nearer 400-450 MeV. Fortunately, we see from Table III that if we choose $\mu$ to be in the range $0.490 \sim 0.494$ GeV, $A^2$ (which does not involve the $\sigma$ meson) actually lies very close to its experimental value, 14.8 eV. We therefore choose this range for $\mu$ in order to calculate $A_0$.

From Table III one notices that $|g_{K\sigma}|$ increases by as much as $60\% \sim 70\%$ while $A^\sigma_0$ decreases about only $10\%$ as $\Lambda$ is increased from 0.32 GeV to 0.34 GeV for $\mu$ lying in the range $(0.490,0.494)$ GeV. This is mainly because $A^\sigma_0 \propto g_{K\sigma} \Delta_{\sigma}$, and the variation of the $K$-$\sigma$ coupling and the propagator of the $\sigma$ meson compensate each other.

With $\mu$ fixed in the range where the empirical value of $A_0$ is reproduced, one notices that $|A^\sigma_0| + |A^D_0|$ lies in the range $320 \sim 370$ eV, which is close to the experimental value of 332 eV. In view of the uncertainties in matching the model scale to the scale of the weak effective Hamiltonian, it is unrealistic to expect to obtain a more precise result. Nevertheless, our calculation does
confirm that the $\sigma$ meson does indeed play an important role in $A_0$, since it contributes more than 85% of the final value. The direct decay process contributes a mere 10%.

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Appendix A: expressions for the $K\sigma$ transition coupling

One can obtain the $K\sigma$ transition coupling $g_{K\sigma}$ with Eq. (11), Eq. (18) and the matrix elements of $Q_i$. The matrix elements of $Q_1$ to $Q_4$ vanish in NJL model, and those of $Q_5$ and $Q_6$ are expressed as

$$
\langle \sigma | Q^5 | K^0 \rangle = \frac{1}{3} \langle \sigma | Q^6 | K^0 \rangle
$$

$$
= \sqrt{2}g_{K\sigma} \times
\left\{ 48 \left[ (2m^2 - p^2)J^F_0(m_d, m_d, p^2) + 2p^2J^F_0(m_d, m_d, p^2) + 8J^F_2(m_d, m_d, p^2) \right]
- \left[ m_dm_s J^F_0(m_d, m_s, p^2) - p^2 J^F_11(m_d, m_s, p^2) - p^2 J^F_21(m_d, m_s, p^2) - 4J^F_22(m_d, m_s, p^2) \right]
+ \frac{3}{2\pi^2} (6m_d^2 - p^2) \left[ m_dm_s J^F_0(m_d, m_s, p^2) - p^2 J^F_11(m_d, m_s, p^2) - p^2 J^F_21(m_d, m_s, p^2) - 4J^F_22(m_d, m_s, p^2) \right]
- \frac{3}{2\pi^2} (2m_d^2 - 2m_dm_s + 2m_s^2 - p^2)
\times \left[ m_d^2 J^F_0(m_d, m_d, p^2) + p^2 J^F_11(m_d, m_d, p^2) + p^2 J^F_21(m_d, m_d, p^2) + 4J^F_22(m_d, m_d, p^2) \right] \right\}.
$$

(A1)

The second and third terms in the brace will exist only with the dimension regularization, and they will vanish if using other regularization methods such as proper-time regularization. $J^F(m, M, p^2)$ are defined by the following integrals

$$
i \int \frac{d^4q}{(2\pi)^4} \frac{\{1, q^\alpha, q^\beta\}}{(q^2 - m^2 + i\epsilon)((q - p)^2 - M^2 + i\epsilon)} = \{J^F_0, -p^\alpha J^F_11, p^\alpha p^\beta J^F_21 + g^{\alpha\beta} \frac{4}{D} J^F_22\}(m, M, p^2).
$$

(A2)

In the dimensional regularization, $J^F(m, M, p^2)$ can be expressed as

$$
J^F_0(m, M, p^2) = \frac{1}{16\pi^2} [2L + 1 + J_1^0(m, M, p^2)],
$$

(A3)

$$
J^F_11(m, M, p^2) = \frac{1}{2} \frac{J^0_0(m)}{p^2} - \frac{J^0_0(M)}{p^2} - \frac{m^2 - M^2 + p^2}{p^2} J^F_0(m, M, p^2),
$$

(A4)

$$
J^F_21(m, M, p^2) = \frac{1}{p^2} [J^0_0(M) + m^2 J^0_0(m, M, p^2) - 4J^F_22(m, M, p^2)],
$$

(A5)

$$
J^F_22(m, M, p^2) = \frac{D}{4(D - 1)} \left[ J^0_0(m) + m^2 J^0_0(m, M, p^2) + \frac{m^2 - M^2 + p^2}{2} J^F_0(m, M, p^2) - \frac{1}{2} J^0_0(M) \right],
$$

(A6)

and the definitions of the helping functions $J^i_1$ are

$$
J^i_0(m) = \frac{m^2}{16\pi^2} (2L + \log \frac{m^2}{\mu^2}),
$$

(A7)

$$
J^i_1(m, M, p^2) = -2 + \ln \left( \frac{p^2}{\mu^2} \right) + X_+ \ln |a + X_+^2| - X_- \ln |a + X_-^2|.
$$

(A8)

$$
\begin{align*}
& \left\{ \begin{array}{ll}
2\sqrt{-a}(\operatorname{arctanh} \frac{X_+}{\sqrt{-a}} - \operatorname{arctanh} \frac{X_-}{\sqrt{-a}}) & p^2 < 0 \\
\sqrt{|a|} \ln Y & 0 < p^2 \leq (m - M)^2 \\
2\sqrt{a}(\operatorname{arctan} \frac{X_+}{\sqrt{a}} - \operatorname{arctan} \frac{X_-}{\sqrt{a}}) & (m - M)^2 < p^2 \leq (m + M)^2 \\
\sqrt{|a|} \ln Y & p^2 > (m - M)^2
\end{array} \right.
\end{align*}
$$

where

$$
a = \frac{m^2}{p^2} - \frac{1}{4} \left( \frac{M^2}{p^2} - \frac{m^2}{p^2} - 1 \right)^2,
$$

$$
X_+ = \frac{1}{2} \left( \frac{M^2}{p^2} - \frac{m^2}{p^2} + 1 \right),
$$

$$
X_- = \frac{1}{2} \left( \frac{M^2}{p^2} - \frac{m^2}{p^2} - 1 \right).
$$
\[ Y = \left| \frac{1 + X_+/\sqrt{|a|}}{1 - X_+/\sqrt{|a|}} \right| \left| \frac{1 - X_-/\sqrt{|a|}}{1 + X_-/\sqrt{|a|}} \right|, \]
\[ L = -\frac{1}{2} + \left\{ \frac{1}{D - 4} + \frac{1}{2}(\gamma_E - \ln 4\pi) \right\}. \]