Neutrino masses and family symmetries

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Abstract

The hypothesis of an additional abelian symmetry acting in a different way on the three families of leptons leads to interesting predictions in the neutrino sector. Contrary to what happens in most seesaw models, the structures of the Dirac and the Majorana matrices are determined without any ansatz, and the neutrino masses and mixing angles are fixed once the lepton charges under the family symmetry have been chosen. Two explicit models using this idea are presented.
1 Introduction

The problem of whether the neutrinos are massive or not is fundamental both for theoretical and phenomenological reasons.

From a theoretical point of view, despite the absence of any experimental evidence for nonzero neutrino masses (the present upper bounds are $m_{\nu_e} < 5.1 \text{ eV}$, $m_{\nu_{\mu}} < 160 \text{ keV}$, $m_{\nu_{\tau}} < 24 \text{ MeV}$), there is no reason to expect the neutrinos to be massless. Indeed, while the photon mass is protected by the $U(1)_{\text{em}}$ gauge symmetry, neutrino masses are not forbidden by any fundamental symmetry. They are only protected by lepton number symmetry, which is an accidental global symmetry of the Standard Model. Now, if the neutrinos are massive, the rather unnatural suppression of their masses relative to the quarks and charged leptons of the same family has to be explained.

On the phenomenological side, massive neutrinos can oscillate from one flavour to another, and this phenomenon could account for the experimental data on solar and atmospheric neutrinos, as well as the recent LSND results. Furthermore, a neutrino with mass in the $1 - 10 \text{ eV}$ range would be a good candidate for hot dark matter, and could solve several cosmological problems, such as structure formation or cosmic microwave background anisotropies.

2 Models of neutrino masses

2.1 Generalities

Various models have been proposed for neutrino mass. All of them need an extension of the particle content of the Standard Model. A Dirac mass term ($\mathcal{L}_m = -m \bar{\nu}_L \nu_R + h.c.$) requires the existence of a right-handed ($RH$) neutrino $\nu_R$ in addition to the standard left-handed ($LH$) neutrino $\nu_L$. A tree-level Majorana mass term ($\mathcal{L}_m = -1/2 m \bar{\nu}_L \nu_R^c + h.c.$) involves a transition from a $I_W = -1/2$ state ($\nu_R^c$) to a $I_W = +1/2$ state ($\nu_L$), and must therefore originate from a Yukawa coupling to a weak Higgs triplet (Gelmini-Roncadelli model). Other models appeal to a particular mechanism to generate a neutrino mass. In the seesaw mechanism, a small Majorana mass for the standard neutrino is induced from heavy $RH$ neutrino exchange. In charged Higgs models, a small Majorana mass is generated from loop diagrams involving charged Higgs bosons.

Among these models, the most popular one is the seesaw mechanism, because it naturally leads to a very small neutrino mass. Let us illustrate this in the one-family case. The particle content of the Standard Model is extended to include, in addition to the ordinary $LH$ neutrino $\nu_L$, a $RH$ neutrino $N_R$. Such Standard Model singlets are present in numerous extensions of the Standard Model (like $SO(10)$ GUT’s or string models). The general neutrino mass term

4Moreover, lepton number violation occurs in numerous extensions of the Standard Model.
one considers has the following form:

\[-\frac{1}{2}(\bar{\nu}_L N_L^c)(0 \quad m \quad M)(\nu_R^c \quad N_R^c) + h.c. \quad (1)\]

The \(\Delta I_W = 1\) entry of the mass matrix is zero because one assumes that there is no Higgs triplet, hence it is impossible to write a Majorana mass term for \(\nu_L\).

The \(\Delta I_W = 1/2\) entry is protected by the electroweak symmetry, so the Dirac mass \(m\) is expected to be of the order of the breaking scale \(M_{\text{weak}} = 246 \text{ GeV}\).

On the contrary, the \(\Delta I_W = 0\) entry is not protected by any symmetry, therefore the Majorana mass for \(N_R\) can be very large (typically \(M \sim 10^{13} - 10^{14} \text{ GeV}\) in realistic seesaw models). With this hierarchy between the mass matrix entries, the diagonalization leads to a hierarchical mass spectrum:

\[m_1 \simeq \frac{m^2}{M} \quad m_2 \simeq M \quad (2)\]

and a small mixing angle between the two mass eigenstates:

\[\tan \theta \simeq \frac{m}{M} \quad (3)\]

Thus, we end up in a natural way with a very light neutrino, with a mass \(m_1\) far below the weak scale, and a heavy neutrino. Since the mixing angle is small, the light neutrino is mainly the standard \(\nu_L\).

It is interesting to note that the presence of a zero in the (1,1) entry, which is due to a gauge symmetry, provides us with a relation between the mass eigenvalues and the mixing angle. Diagonalizing the mass matrix:

\[
\begin{pmatrix}
0 & m \\
m & M
\end{pmatrix}
= \begin{pmatrix}
cos \theta & sin \theta \\
-sin \theta & cos \theta
\end{pmatrix}
\begin{pmatrix}
-m_1 & 0 \\
0 & m_2
\end{pmatrix}
\begin{pmatrix}
cos \theta & -sin \theta \\
sin \theta & cos \theta
\end{pmatrix}
\]

and writing that the (1,1) entry is zero:

\[0 = -m_1 \cos^2 \theta + m_2 \sin^2 \theta \]

we obtain the mass-angle relation:

\[\tan \theta = \sqrt{\frac{m_1}{m_2}} \quad (4)\]

This suggests that symmetries may play a crucial role in constraining the neutrino mass and mixing pattern.

### 2.2 Explicit seesaw models

Let us now see how the seesaw mechanism is implemented in usual models. With one right-handed neutrino per family, the neutrino mass matrix is a 6x6 matrix:

\[
\begin{pmatrix}
0 & \mathcal{M}_D \\
\mathcal{M}_D^T & \mathcal{M}_M
\end{pmatrix}
\]  

(5)
The masses and mixing angles of the light eigenstates are obtained from the diagonalization of the light neutrino mass matrix
\[
\mathcal{M}_\nu = -\mathcal{M}_D \mathcal{M}_M^{-1} \mathcal{M}_D^T
\]
\[
= R_\nu \begin{pmatrix}
  m_{\nu_e} & 0 & 0 \\
  0 & m_{\nu_\mu} & 0 \\
  0 & 0 & m_{\nu_\tau}
\end{pmatrix} R_\nu^T
\]
(6)

Note that the mixing angles relevant for neutrino oscillations are given by the analog of the CKM matrix, which also involves the charged lepton sector diagonalization matrix \( R_e \):
\[
V_L = R_\nu R_e^T
\]
(7)

The natural scale of the Dirac matrix \( \mathcal{M}_D \) is \( M_{\text{weak}} \), whereas the entries of the Majorana matrix \( \mathcal{M}_M \), being not constrained by any symmetry, are expected to be much larger than \( M_{\text{weak}} \). Apart from these restrictions, the entries of both the Dirac and the Majorana matrices are free parameters, and one has to make a specific ansatz in order to constrain the neutrino mass spectrum.

It is usually assumed that the Dirac mass matrix has the same structure than the up quark mass matrix \( \mathcal{M}_D \sim \mathcal{M}_U \). For the Majorana matrix, however, no such simplifying assumption can be done, and it is necessary to choose a specific structure. Various ansätze for \( \mathcal{M}_M \) have been studied in the literature: degenerate (all eigenvalues are equal), hierarchical, democratic (all entries are 1). It follows that the neutrino spectrum of a given model depends on the ansatz that has been chosen, which is not very satisfactory.

This problem can be evaded if one assumes that the structures of the Dirac and the Majorana matrices are determined by a symmetry. This symmetry has to act in a different way on the three neutrino families, otherwise the matrices would be unconstrained. Such a symmetry is called a family symmetry. This approach has proven to be successful in the quark sector, where, following the original idea by Froggatt and Nielsen, several groups have shown that an abelian family symmetry can reproduce the observed mass and mixing hierarchy.

3 Fermion masses and family symmmmetry

3.1 Quark sector

First of all, let us stress the motivations for introducing a new symmetry. The experimental data show a strong hierarchy between the fermion masses, e.g.\(^5\)This arises naturally in Standard Model extensions with a quark/lepton symmetry, like the SO(10) GUT.

\(^6\)Unless one envisages the case of almost degenerate neutrinos.
$m_u \ll m_c \ll m_t$ in the up quark sector. The up quark mass matrix can then be written, in first approximation:

$$\frac{M_u}{m_t} \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

with small corrections of order $\mathcal{O}(m_u/m_t)$ and $\mathcal{O}(m_c/m_t)$. Now the zero entries can be interpreted as zeroes induced by a symmetry (the corresponding Yukawa couplings being forbidden by the symmetry), and the small corrections as arising through the breaking of this symmetry.

Let us now see how this scenario can be realized with an abelian family symmetry. We extend the gauge group of the Minimal Supersymmetric Standard Model (MSSM) with a family symmetry $U(1)_X$ ($X$ denotes the conserved charge associated with the symmetry). Each Yukawa coupling $Q_i \bar{U}_j H_u$ then carries a $X$-charge $n_{ij}$, which is simply the sum of the $X$-charges of the fields entering the coupling: $n_{ij} = X_{Q_i} + X_{\bar{U}_j} + X_{H_u}$ ($i$ and $j$ are generation indices, $Q_i$ is the quark doublet of the $i$th generation, $\bar{U}_j$ the quark singlet of the $j$th generation, and $H_u$ the Higgs doublet that gives a mass to the up quarks). If $n_{ij} \neq 0$, the coupling is forbidden by $U(1)_X$, and the corresponding mass matrix entry is zero. When $n_{33} = 0$ and $n_{ij} \neq 0$ for $(i, j) \neq (3, 3)$, only the top quark coupling is allowed, and the up quark mass matrix has the form (8). The other Yukawa couplings are then generated from non-renormalizable interactions involving a singlet field $\theta$ with $X$-charge $X_\theta = -1$:

$$Q_i \bar{U}_j H_u \left( \frac{\theta}{M} \right)^{n_{ij}}$$

where $M$ is a large scale characteristic of the underlying theory (typically $M \sim M_{Planck}$ or $M_{GUT}$). When the MSSM singlet $\theta$ acquires a vacuum expectation value, which breaks spontaneously $U(1)_X$, effective Yukawa couplings are generated:

$$Y_{ij} \sim \left( \frac{< \theta >}{M} \right)^{n_{ij}}$$

with their orders of magnitude fixed by their charges under $U(1)_X$. It follows that the structure of the mass matrix is determined by the family symmetry (we use the notation $\epsilon = < \theta > / M$):

$$\frac{M_u}{m_t} \sim \begin{pmatrix} \epsilon^{n_{11}} & \epsilon^{n_{12}} & \epsilon^{n_{13}} \\ \epsilon^{n_{21}} & \epsilon^{n_{22}} & \epsilon^{n_{23}} \\ \epsilon^{n_{31}} & \epsilon^{n_{32}} & 1 \end{pmatrix}$$

where only the order of magnitude of each entry is given. Since $U(1)_X$ is broken below the scale $M$, $\epsilon$ is a small parameter (typically $\epsilon \sim 0.1$), and a hierarchy

\footnote{For convenience, we will assume that $n_{ij} \geq 0$.}
between Yukawa couplings naturally appears. The diagonalization of $M_u$ and $M_d$ (which is obtained in the same way than $M_u$) then relates the mass and mixing hierarchy of the quarks to their charges under $U(1)_X$.

### 3.2 Lepton sector

If the lepton fields $L_i, \bar{E}_i$ and $\bar{N}_i$ carry $X$-charge, the previous mechanism also works for the lepton mass matrices $M_e, M_D$ and $M_M$. In particular, the structure of the Dirac and the Majorana matrices is determined by the family symmetry without any ansatz.

More precisely, $M_D$ and $M_M$ are generated from higher order operators of the type:

$$L_i \bar{N}_j H_u \left( \frac{\theta}{M} \right)^{p_{ij}} \quad p_{ij} = X_{L_i} + X_{\bar{N}_j} + X_{H_u} \quad (11)$$

for the Dirac matrix and

$$M \bar{N}_i \bar{N}_j \left( \frac{\theta}{M} \right)^{q_{ij}} \quad q_{ij} = X_{\bar{N}_i} + X_{\bar{N}_j} \quad (12)$$

for the Majorana matrix. After breaking of $U(1)_X$, one obtains:

$$(M_D)_{ij} \sim m \left( \frac{\langle \theta \rangle}{M} \right)^{p_{ij}} \quad (M_M)_{ij} \sim M \left( \frac{\langle \theta \rangle}{M} \right)^{q_{ij}} \quad (13)$$

The structures of the Dirac and the Majorana matrices, and consequently the neutrino masses and mixing angles, are determined by the neutrino charges under $U(1)_X$. No ansatz is required.

Several groups have studied neutrino mass models based on a family symmetry \[8, 9, 10\]. In the following, we present two of them.

### 4 Examples of neutrino mass models with a $U(1)$ family symmetry

#### 4.1 Model 1

The assumptions of this model \[10\] are the following: (a) the anomalies of the horizontal $U(1)_X$ are compensated for by an appropriate mechanism (Green-Schwarz); (b) the dominant entry in each mass matrix is the (3,3) entry; (c) the $X$-charges of all mass terms are positive ($p_{ij} \geq 0$, $q_{ij} \geq 0$). (b) and (c) allow to make a simple analysis. Note that $M_D$ is not assumed to be symmetric, which gives us more liberty in the choice of the parameters of the $X$-charge.

\*For $p_{ij} \geq 0$ and $q_{ij} \geq 0$. 

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With these assumptions, the light neutrino masses are:

\[ m_{\nu_e} \sim \frac{m_3^2}{M_3} \epsilon^{2(X_{L_1}-X_{L_3})} \]

\[ m_{\nu_\mu} \sim \frac{m_3^2}{M_3} \epsilon^{2(X_{L_2}-X_{L_3})} \]  \hspace{1cm} (14)

\[ m_{\nu_\tau} \sim \frac{m_3^2}{M_3} \]

The \( \nu_\tau \) mass is given by the usual seesaw formula (\( M_3 \) is the mass of the heaviest \( RH \) neutrino, \( m_3 \) the largest Dirac mass), whereas the other neutrino masses are suppressed relative to \( m_{\nu_\tau} \) by powers of the small breaking parameter \( \epsilon \).

Note that the hierarchy depends only on the \( X \)-charges of the lepton doublets \( L_i \). The lepton mixing matrix is:

\[
V_L \sim R_\nu \sim \begin{pmatrix}
1 & \epsilon^{[X_{L_1}-X_{L_2}]} & \epsilon^{[X_{L_1}-X_{L_3}]} \\
\epsilon^{[X_{L_1}-X_{L_2}]} & 1 & \epsilon^{[X_{L_2}-X_{L_3}]} \\
\epsilon^{[X_{L_1}-X_{L_3}]} & \epsilon^{[X_{L_2}-X_{L_3}]} & 1
\end{pmatrix}
\]  \hspace{1cm} (15)

\( V_L \) has the same structure than \( R_\nu \) because, due to the assumptions of the model, the diagonalizing matrices for the neutrino masses (\( R_\nu \)) and the charged lepton masses (\( R_e \)) have the same structure.

Note that the light neutrino spectrum would not have been affected if, instead of adding one \( RH \) neutrino per family, we had introduced an arbitrary number of such heavy fields.

This model has several remarkable features. First, it is worth noting that the neutrino mass and mixing hierarchies do not depend on the particular form of the Majorana matrix. This is a great difference with most seesaw models. The reason for this is that the dependences of \( M_D \) and \( M_M \) on the heavy neutrino charges compensate for each other in the matrix \( M_\nu \). Secondly, the mass spectrum obtained is naturally hierarchical\(^9\) without hierarchy inversion: \( m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau} \). Finally, the mixing angles and the mass ratios are usually related by:

\[ \sin^2 \theta_{ij} \sim \frac{m_{\nu_i}}{m_{\nu_j}} \]  \hspace{1cm} (16)

These relations, which generalize \( (3) \), are common to numerous seesaw models. They imply \( V_{e\nu_e}V_{\mu\nu_\mu} \sim V_{e\nu_\mu} \) in lepton charged current, in analogy with \( V_{us}V_{cb} \sim V_{ub} \) in quark charged current.

The experimental data on solar neutrinos and atmospheric neutrinos put constraints on the parameters of the model. For example, if one wants to explain

\(^9\)Mass degeneracies are not excluded, but in this case the model is less predictive, since the mass difference between almost degenerate neutrinos cannot be related to the lepton \( X \)-charges.
simultaneously the solar neutrino data by MSW $\nu_e \to \nu_\mu$ transitions, and the atmospheric neutrino data by $\nu_\mu \to \nu_\tau$ oscillations, one must choose\textsuperscript{10}
\begin{align}
X_{L_1} - X_{L_3} &= 3 \\
X_{L_2} - X_{L_3} &= 1
\end{align}
(17)
which leads to the following spectrum:
\begin{align}
\begin{cases}
m_{\nu_e} &\sim 10^{-5} \text{ eV} \\
m_{\nu_\mu} &\sim 5.10^{-3} \text{ eV} \\
m_{\nu_\tau} &\sim 0.1 \text{ eV}
\end{cases}
\begin{cases}
\sin^2 2\theta_{e\mu} &\simeq 2.10^{-3} - 4.10^{-2} \\
\sin^2 2\theta_{\mu\tau} &\simeq 5.10^{-2} - 0.8
\end{cases}
\end{align}
(18)
The uncertainties in the mixing angles are due to the fact that the mass matrix entries are determined by the family symmetry up to a factor of order one. Note that, in order to avoid fine-tuning, we have only addressed the hierarchical case, which makes it quite difficult to obtain a large mixing angle [see \textsuperscript{13}], as required by the atmospheric neutrino data. Furthermore, the tau neutrino is too light to be an interesting candidate for dark matter. However, if one ignores the atmospheric neutrino problem, it is possible to obtain a relevant $\nu_\tau$ for cosmology and to explain the solar neutrino data at once.

### 4.2 Model 2

The assumptions of this model\textsuperscript{8}, which was proposed first, are quite different from the previous one: (a) all mass matrices are symmetric\textsuperscript{9}, which reduces the number of independent parameters; (b) the $X$-charges of the mass terms can be negative, and the existence of a pair $(\theta, \bar{\theta})$ of singlets with opposite $X$-charges is assumed; (c) the heavy neutrino Majorana masses are generated from the coupling to a singlet Higgs boson $\Sigma$ with charge $X_\Sigma$ [$<\Sigma > N_i N_j$], which gives rise to a discrete spectrum of possible Majorana matrices, depending on $X_\Sigma$.

With (a) and (b), the Dirac matrix takes the form:
\[ M_D \sim \begin{pmatrix} \epsilon |p_{11}| & \epsilon |p_{12}| & \epsilon |p_{13}| \\ \epsilon |p_{12}| & \epsilon |p_{22}| & \epsilon |p_{23}| \\ \epsilon |p_{13}| & \epsilon |p_{23}| & 1 \end{pmatrix} \] (19)
(c) implies that, for a given $M_D$ (corresponding to a given assignment of lepton $X$-charges), eg
\[ M_D \sim \begin{pmatrix} \epsilon^6 & \epsilon^8 \\ \epsilon^4 & \epsilon^2 \\ \epsilon^8 & \epsilon^2 \end{pmatrix} \] (20)
several $M_M$ are possible, depending on $X_\Sigma$. As a consequence, the light neutrino spectrum depends on the value of $X_\Sigma$. This is illustrated by the following two

\textsuperscript{10}We only address the case of a hierarchical mass spectrum: $m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau}$.
\textsuperscript{11}This arises naturally in left-right symmetric GUT’s, like $SU(3)^3$ or $E_6$. 

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examples:

\[ X_\Sigma = X_{H_u} \quad M^{\text{diag}}_\nu \sim \begin{pmatrix} \epsilon^{11} & 0 & 0 \\ 0 & \epsilon^7 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R_\nu \sim \begin{pmatrix} 1 & \epsilon^3 & \epsilon^5 \\ \epsilon^3 & 1 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & 1 \end{pmatrix} \] (21)

\[ X_\Sigma = \frac{3}{2} X_{H_u} \quad M^{\text{diag}}_\nu \sim \begin{pmatrix} \epsilon^{11} & 0 & 0 \\ 0 & \epsilon^5 & 0 \\ 0 & 0 & \epsilon^{-1} \end{pmatrix} \quad R_\nu \sim \begin{pmatrix} 1 & \epsilon^3 & \epsilon^7 \\ \epsilon^3 & 1 & \epsilon^2 \\ \epsilon^7 & \epsilon^2 & 1 \end{pmatrix} \] (22)

Thus, in this model, the neutrino mass ratios and mixing angles are determined by the lepton X-charges up to a discrete ambiguity\(^{12}\). The mass spectrum thus obtained is always hierarchical, and mass degeneracy requires fine-tuning. Note that, due to assumption (b), the mass-angle relation \(^{10}\) is not automatically satisfied. Furthermore, the matrices that diagonalize the charged lepton and neutrino mass matrices do not have the same structure, hence the mixing angles relevant for neutrino oscillations are not simply given by \(R_\nu\).

Finally, the model is able to reproduce the experimental neutrino data. \(m_\nu\) is in the interesting range for cosmology \((1 - 10 \text{ GeV})\) for reasonable values of \(\langle \Sigma \rangle\). The small angle MSW solution to the solar neutrino problem can be accommodated. However, a large mixing angle, as required by the atmospheric neutrino data, cannot be obtained without fine-tuning.

5 Conclusion

We have presented two neutrino mass models based on an abelian family symmetry. The great advantage of such models is that the structure of the Dirac and the Majorana matrices is entirely determined by the symmetry, therefore the neutrino masses and mixing angles are predicted without any ansatz. Furthermore, the fact that the same symmetry is able to explain the observed fermion mass hierarchy and simultaneously constrains the neutrino spectrum sets an interesting connection between two fundamental problems in particle physics. Unfortunately, the lepton X-charges are constrained, but not fully determined by the model, which reduces its predictive power. Moreover, it is difficult to understand mass degeneracies as well as mixing angles of order one, which are necessary to account for all neutrino data.

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