THE MASS OF THE HEAVY AXION $\eta_6$

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Abstract

If electroweak dynamical symmetry breaking is due to a chiral condensate of color sextet quarks, dynamics analogous to “walking technicolor” will enhance the condensate by orders of magnitude compared to the electroweak chiral scale. This enhancement compensates for the exponential suppression of electroweak scale color instanton interactions. As a result the $\eta_6$ axion can naturally acquire an electroweak scale mass.

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The existence of a massive color sextet quark sector could be an essential factor in the very high-energy consistency of QCD \[1\] and in the dynamics of the electroweak sector \[2, 3\]. In particular, low-energy Strong CP conservation in the normal triplet quark sector can be directly due to a sextet quark axion state\[3\], the $\eta_6$. If this is the case the QCD interactions of the sextet sector are not CP conserving. As a result the $\eta_6$ has large CP violating couplings to the electroweak sector that could be responsible for its production at LEP as an intermediate state in $Z^0 \rightarrow \gamma\gamma + \mu^+\mu^-$ events \[3\]. Mixing of the color triplet and sextet sectors could also underlie CP violation in general.

At first sight\[4\], the $\eta_6$ is a conventional Peccei-Quinn axion\[5\]. However, because of its higher color constituents, color instanton interactions provide an additional contribution to its mass \[6\] not envisaged in the original Peccei-Quinn mechanism. It is very important to understand how large this contribution can be. In this paper we shall outline how the dynamics\[4\] of “walking technicolor”(WTC), as applicable to sextet electroweak symmetry breaking, can compensate for the normal suppression of instanton interactions. As a result it is natural to expect the $\eta_6$ to have an electroweak scale mass.

We shall suppose that the entry of a flavor doublet (U,D) of color sextet quarks into QCD above the electroweak scale can be described by an effective $\beta$-function. If we write

$$\beta(\alpha) = -\beta_0 \alpha^2(q)/2\pi - \beta_1 \alpha^3(q)/8\pi^2 + \ldots \quad (1)$$

then for six color triplet flavors the normal two-loop calculation gives

$$\beta_0 = 11 - 2n_f/3 = 7, \quad \beta_1 = 102 - 38n_f/3 = 26 \quad (2)$$

whereas when the two sextet flavors are included we obtain\[8\]

$$\beta_0 = 7 - 4T(R)n_f^6/3 = 7 - 4(\frac{5}{2})2/3 = 1/3, \quad (3)$$

and

$$\beta_1 = 26 - 20T(R)n_f^6 - 4C_2(R)T(R)n_f^6 = 26 - 100 - 66\frac{2}{3} = -140\frac{2}{3} \quad (4)$$

where we have used $T(R) = 5/2$ and $C_2(R) = 10/3$ for sextet quarks.

The resulting $\beta$-function, $\beta^{(6)}$, for six flavors of light triplet quarks (we ignore subtleties associated with a heavy top quark) is shown in Fig. 1(a) and compared, in Fig. 1(b),
with $\beta^{(6,2)}$, - the $\beta$-function obtained with the sextet quarks added. Noting the greatly expanded vertical scale in Fig. 1(b), it is clear that as soon as the sextet sector enters the theory the evolution of $\alpha_s$ essentially comes to a halt. If, as we assume, the theory nevertheless evolves smoothly, but very slowly, into the small coupling asymptotically-free region then a natural way to connect the evolution before and after the sextet sector enters is via a $\beta$-function of the form shown in Fig. 2. Such an evolution (presumably) provides an oversimplified picture of the physics involved but will allow us to give an order of magnitude discussion of quantities which we hope is not too unrealistic. We assume, therefore, that above the electroweak scale the $\beta$-function can be taken to be a small (almost) momentum independent constant, which we denote as $\beta_c$. This provides the essential prerequisite for the application of WTC dynamics.

A major ingredient of WTC, in addition to the existence of a small $\beta_c$, is the assumption that the linearised Dyson-Schwinger “gap equation” for the quark dynamical mass $\Sigma(p)$, gives a semi-quantitive description of the dynamics of chiral symmetry breaking. The gap equation has a solution corresponding to spontaneous chiral symmetry breaking for $\alpha_s \geq \alpha_c$, where $\alpha_c$ is determined by an equation of the form

$$C_2(R)\alpha_c = \text{constant}$$

(5)

$C_2(R)$ is the Casimir operator already referred to above and, since $C_2$ for sextet quarks is $5/2$ the corresponding triplet Casimir, (5) is consistent with the sextet chiral scale being the electroweak scale provided that $\alpha_s$ evolves logarithmically from the triplet chiral scale up to this scale, in the usual manner.

If $\alpha_s$ is momentum independent above the electroweak scale, as will be approximately the case if $\beta_c$ is small enough, then the solution of the gap equation for $\alpha_s \sim \alpha_c$ has the form

$$\Sigma(p) \sim \mu^2 (p)^{-1}$$

(6)

where $\mu$ is determined at the electroweak scale and should be essentially the sextet quark constituent quark mass i.e. $\mu \geq 300$ GeV. When this behavior is inserted into the perturbative formula for the high-momentum component of the sextet condensate $\langle Q\bar{Q} \rangle$ we obtain a contribution

$$\langle Q\bar{Q} \rangle \sim \int dp \ p \Sigma(p) \sim \mu^2 \Lambda$$

(7)
where \( \Lambda \) is the upper cut-off on the integral. In contrast, the corresponding perturbative formula\([7, 9]\) for the chiral constant \( F_{\pi} \) (which determines the \( W \) and \( Z \) masses) gives a high-momentum component

\[
F_{\pi}^2 \sim \int dp \ p^{-1} \Sigma^2(p) + \ldots \tag{8}
\]

which is not enhanced by the behavior (6).

The upper cut-off \( \Lambda \) should naturally be of the same order as the scale \( M \) at which new (unification) physics appears. However, it could be greater if the new physics is asymptotically-free and does not significantly change the evolution of \( \alpha_s \). The new physics must provide four-fermion triplet/sextet couplings in particular, which then provide triplet quark masses with the order of magnitude

\[
m_q \sim \langle Q\bar{Q} \rangle / M^2 \sim \mu^2 \Lambda / M^2 \tag{9}
\]

To be sure that flavor-changing neutral currents are suppressed we must certainly take\([7]\) \( M \geq 300 \) TeV. However, if \( M = 300 \) TeV, then from (8) we see that to obtain a mass \( \sim 10-100 \) Mev for a triplet quark, we should have

\[
\langle Q\bar{Q} \rangle \sim 10^{10} - 10^{11} (GeV)^3 \tag{10}
\]

If we take \( \mu^2 \sim 10^5 (GeV)^2 \), we must then have

\[
\Lambda \sim 10^5 - 10^6 GeV = 100 - 1,000 TeV. \tag{11}
\]

so that \( \Lambda \sim M \) is clearly possible. Equations (5) - (11) simply illustrate the essence of WTC, i.e. if the gauge coupling starts to “walk” immediately above the chiral scale, then the chiral condensate can be enhanced by orders of magnitude relative to the chiral scale. This allows reasonable fermion masses to be generated without accompanying problems from flavor-changing processes. As we now describe, the instanton interactions contributing to the \( \eta_6 \) mass are also enhanced.

The \( \eta_6 \) is a Goldstone boson associated with the axial U(1) symmetry orthogonal to that broken by color instantons. As such it is an axion which remains massless until four-fermion \( qq\bar{q}\bar{Q} \) couplings are added to the theory. Such couplings then combine with
the triplet/sextet instanton interactions and the sextet condensate to produce an $\eta_6$ mass. The one instanton contribution to this mass is illustrated in Fig. 3.

Because $T(r)$ is 5/2 for sextet quarks and 1/2 for triplet quarks, the axial U(1) current that is conserved before the addition of four-fermion couplings is

$$J^A = 5J^{3A} - J^{6A}$$

where $J^{3A}$ and $J^{6A}$ are respectively the triplet and sextet U(1) currents. As a result the one instanton interaction appearing in Fig. 3 involves five sextet quarks (and antiquarks) of each flavor and just one triplet quark (and antiquark) of each flavor. It is therefore a very high-order fermion interaction of the form

$$(U\bar{U})^5(D\bar{D})^5u\bar{u}d\bar{d}s\bar{s}c\bar{c}b\bar{b}t\bar{t}$$

This interaction is scaled by a factor $(P_I)^{-44}$, where $P_I$ is the momentum scale corresponding to the size of the instanton, and a non-perturbative exponential suppression factor involving the instanton action. Since instanton interactions above the electroweak scale are well-defined (because of the absence of renormalons) we take the minimum allowed value for $P_I$ to be the electroweak scale i.e. $P_I \sim 100$ GeV. (To be really sure that we do not induce interactions violating the established success of perturbative QCD we should probably take $P_I$ slightly larger. However, our estimates will be sufficiently crude that this will not be significant. Also we shall overestimate other suppression factors, for example by underestimating $\alpha_s$.)

Given our assumption that the scale of the instanton interaction is the electroweak scale, we will take $\alpha_s \sim 10^{-1}$. In this case the non-perturbative factor for the one instanton interaction is

$$[2\pi/\alpha_s]^6 \exp[-2\pi/\alpha_s] \sim [60]^6 \exp[-60] \sim (10)^{-15}$$

which, of course, provides a large suppression of the interaction.

It is crucial that the four-fermion couplings and sextet condensate attached to the triplet quark legs of the instanton interaction combine to produce a mass factor $m_q$ for each triplet quark (together with a factor of $P_I^2$ for each of the loop integrals involved). The result is an overall phase factor of $\bar{\theta} = \theta + \arg \det m^{(3)}$, where $\theta$ is the usual topological parameter and $m^{(3)}$ is the triplet quark mass matrix. This allows a mass to be generated which does not disturb the CP conserving minimum of the axion action at $\bar{\theta} = 0$. However, the factor
of \(\det m^{(3)}\) (scaled by \((P_I)^6\)) also provides a significant suppression of the interaction. If we take, in order of magnitude, \(m_u \sim m_d \sim 10^{-2}\) GeV, \(m_s \sim 10^{-1}\) GeV, \(m_c \sim 1\) GeV, \(m_b \sim 10\) GeV, and \(m_t \sim 100\) GeV, we can estimate this suppression as

\[
\frac{(\det m_3)}{(100\text{ GeV})^6} \sim (10^{-2})^2(10^{-1})(10)(10^2) / (10^2)^6 \sim (10)^{-14}
\]

which is a comparable suppression to (14). Both (14) and (13) are automatically present in electroweak scale instanton interactions and provide the usual reason for regarding such interactions as completely negligible.

Now we come to the role of the sextet condensate in compensating for the suppression factors (14) and (15). If we use the sextet dynamical mass \(\Sigma(p)\) to tie together sextet quark legs of the instanton interaction, then the propagators present simply give the high momentum integral (7) and so produce the condensate as an overall multiplicative factor, as illustrated in Fig. 3. As a result, if the condensate enhancement is as large as (14), the combined suppression of (14) and (15) is overcome. The higher color of the sextet quarks is vital here since it is responsible for the large power of the condensate in the single instanton interaction. Indeed, in the \(\eta_6\) mass, we have an enhancement factor of \(\langle \bar{Q}Q \rangle^8\) which, using (10), and including the remaining factor of \((P_I)^{-26}\), gives

\[
\frac{\langle \bar{Q}Q \rangle^8}{(100\text{ GeV})^{26}} \sim (10^{10} - 10^{11})^8 / (10)^{52} \sim (10)^{28} - (10)^{36}
\]

and clearly this can be more than sufficient to compensate for the combined suppression due to (14) and (15). Of course, our estimates are very crude and the numbers could vary considerably from those we have used. However, that the condensate enhancement can reasonably be expected to enhance the instanton interaction from a very weak to a strong interaction is surely well illustrated by the above discussion.

If the condensate is large enough to produce a strong one instanton interaction then clearly multi-instanton interactions will also be strong and we will have a highly non-perturbative theory. Indeed since renormalons are probably absent\(^\text{[1]}\) in the full triplet plus sextet version of QCD, instantons provide the only non-perturbative aspect of the theory. It could be, therefore, that the large sextet condensate is necessary to enhance these interactions sufficiently to produce a “non-perturbative”, confining, solution of the theory at the electroweak scale which is able to match with a low-energy confined triplet sector.
That the instanton interactions are enhanced presumably invalidates the starting assumption of WTC that the dynamics of chiral symmetry breaking is well-described by the linearised Dyson-Schwinger equation. However, from our point of view, this assumption is simply a way of selecting a set of interactions that can produce the dynamics we are proposing. We acknowledge that a full description of the dynamics is surely much more complicated than we have described but, as we stated above, we hope that our order of magnitude discussion retains some substance.

If the instanton interactions provide the major dynamics of the theory then the $\eta_6$ mass should automatically inherit the main dynamical scale of the theory. In this case an electroweak scale mass of, say, 60 GeV would be quite natural. We should also note that, in principle at least, the overall magnitude of the four-fermion triplet/sextet couplings can be regarded as a parameter which can be smoothly varied from zero to its physical value. During this variation the $\eta_6$ mass will go from zero to its physical value. Since the axion status of the $\eta_6$ will be preserved throughout the variation, it is clear that Strong CP will be strictly conserved by the triplet quark sector as masses are induced (by the four-fermion couplings) and that this will continue to be the case even if the $\eta_6$ mass is very large.

It seems therefore that the $\eta_6$ is a good candidate for the new massive state that might have been observed\cite{3} at LEP - via its two photon decay mode in particular. It is important to emphasize in this context that the underlying dynamics of the sextet sector, which couples the $\eta_6$ to the $W^+$, $W^-$ and $Z^0$ is novel in that it involves enhanced instanton interactions. This is why the operators involved in the decay modes are, perhaps, unexpected. Indeed papers have been written, for example \cite{10}, arguing that a new scalar particle interpretation of the LEP events is implausible because the operators that could be involved would also appear in other experiments - where they can clearly be ruled out. In fact there is no inconsistency between the candidate LEP events and other experiments if the events are regarded as $\eta_6$ production. In the classification (and words) of [10], the operator involved is of the “$O_2$ - type” which “only contain $Z$ and $W$ gauge bosons and ... this fact prevents us from ruling them out with current experimental data”.

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Figure Captions

Fig. 1 (a) The $\beta$-function $\beta^{(6)}$, for QCD with six light flavors of color triplet quarks, and (b) comparison of $\beta^{(6,2)}$, the $\beta$-function with two color sextet flavors added, with $\beta^{(6)}$.

Fig. 2 The effective $\beta$-function describing the evolution as the sextet sector enters the theory.

Fig. 3 The one instanton interaction contributing to the $\eta_6$ mass.