Effect of the Length and Diameter of a Cylindrical Capsule on the Pressure Drop in a Horizontal Pipeline

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Abstract. Capsule pipeline research involves the study of the flow in a pipe of a long train of spherical or cylindrical capsules (hollow containers) filled with minerals or other materials including hazardous liquids. The behavior of the capsule train will depend upon the behavior of each capsule in the train and the hydrodynamic influence of one capsule on another. Designers are in need of a general correlation to calculate pressure drop in a capsule pipeline. Researchers, so far, have used rather simplified empirical and semi-empirical correlations for pressure drop calculations, the range and application of which is fairly limited. A mathematical correlation developed for pressure drop in cylindrical capsule of equi-density as its carrying medium is presented here. Based on Computational Fluid Dynamics (CFD) a numerical solution has been obtained from the equations governing the turbulent flow around a concentric cylindrical capsule in a hydraulically smooth pipe section. The diameter of the pipe used in present study is 0.1 m while that of capsules are in the range of 50–80% of the pipe diameter. The investigation was carried out in the practical range of 0.2 ≤Vc≤ 1.6 m/sec. The computationally obtained data set over a wide range of flow conditions have then been used to develop a rigorous model for pressure drop. Using this model the pressure drop along the pipeline can be computed which then can be used to calculate pumping requirements.

1. Introduction
Due to their relatively simple shape, economic design and ease of connecting the capsules to form a train of cargo etc. cylindrical capsules have always been the first choice of the capsule transporting industries throughout the world. Kroonenberg [1] developed a mathematical relationship to predict the pressure drop, and hence the head loss in capsule transporting pipelines carrying cylindrical capsules. For simplicity, only horizontal pipelines were considered. The relationship developed by Kroonenberg was based on the fact that the presence of the solid phase in the pipeline increases the friction factor and hence the pressure drop. Moody’s chart was used to calculate the friction factor.
For liquid phase, Ulusarslan and Teke [2-4] have conducted various experiments on spherical capsules and have developed a model for pressure drop based on the curve fitting techniques which accounts for the increase in the friction factor due to the presence of solid phase in the pipeline. However, the aforementioned methodologies are based on limited data sets. Hence, to develop equations that cover wider range of flow conditions and to check validity of the aforementioned models, a Computational Fluid Dynamics (CFD) based technique has been used in the present study to predict the pressured drop in the capsule transporting pipelines carrying cylindrical capsules. The analysis has been conducted on a wide range of geometric and flow conditions.

2. Numerical Modelling
Commercial CFD package FLUENT has been used to obtain the pressure drop in the capsule carrying pipeline. A hydrodynamically smooth (i.e. $\varepsilon/D = 0$) test section similar to that of Ulusarslan and Teke [2] has been numerically modelled for $L = 1$ m and $D = 0.1$ m. According to Munson and Young [5], the minimum length criterion to obtain a fully developed flow is 50*D; hence an additional pipe length of 100*D has been introduced before the test section. Capsules of various sizes i.e. $d = 0.05, 0.06, 0.07$ and 0.08 m are introduced in the test section. Pressure drop investigations have been carried out in bulk velocities range of $V_b = 0.2–1.6$ m/s. Capsules of length $L_c = 0.1 – 0.5$ m have been used to carry out the analysis. Figure 1 shows the geometrical setup for the case $L_c = 0.5$ m and $d = 0.08$ m. Following assumptions have been made to solve the equations governing the turbulent flow in the capsule carrying pipeline:

- Flow is steady
- Capsule velocity has been taken to be equal to the velocity of water i.e. $V_w = V_c = V$ as suggested by Ulusarslan [4]
- The pressure drop can be computed using a single phase method for the bulk velocity $V_b = V$
- Capsules are made of polypropylene material which has the same density as water i.e. $\rho_w = \rho_c = \rho$
3. Results
The important non-dimensional parameters, as suggested by Ulusarslan [4], in order to develop a semi-empirical model for the calculation of pressure drop in the cylindrical capsule carrying hydraulic pipeline, are Reynolds number (for both water and capsule), capsule to pipe diameter ratio \(k = \frac{d}{D}\) and capsule to pipe length ratio \(a = \frac{L_c}{L}\).

3.1. Pressure Drop in a Hydraulic Pipeline
Figure 2 shows the variations in pressure drop per unit length of the pipe at different Reynolds number of water calculated using Darcy-Weisbach [6] equation and from CFD. It can be clearly seen that CFD predicts the pressure drop in a hydraulic pipeline with reasonable accuracy.

![Figure 2. Comparison of pressure drops in a hydraulic pipeline.](image)

**Figure 1.** Geometrical setup of the Capsules in the Pipeline.
For water flow in a straight pipe, Darcy-Weisbach [6] formulated the following expression for the prediction of pressure drop in the pipe:

$$\left(\frac{\Delta P}{L}\right)_W = \frac{\rho V^2}{2D} * f_W$$  \hspace{1cm} (1)

Where $f_w$ is the Darcy’s friction factor and is computed using Moody’s chart and/or semi-empirical expressions. Figure 3 shows the variations of $f_w$ against $Re_w$ using the data collected from CFD simulations. It can be seen that as the Reynolds number of water increases, the friction factor decreases. Using curve fitting technique as shown in figure 3, the following expression for the friction factor can be developed with an average deviation of less than 0.7% from the CFD data shown:

$$f_W = \frac{0.177}{Re_w^{0.2}}$$  \hspace{1cm} (2)

Where the Reynolds number of water is expressed as:

$$Re_w = \frac{\rho DV}{\mu}$$  \hspace{1cm} (3)

![Figure 3. Variations of friction factor in a hydraulic pipeline.](image)
3.2. Pressure Drop in Capsule Transporting Hydraulic Pipeline

Darcy-Weisbach equation is for single phase flow only and hence cannot be used to calculate the pressure drop in multiphase problems. Kroonenberg [1] states that the pressure drop in a cylindrical capsule transporting hydraulic pipeline is greater than the pressure drop in a hydraulic pipeline in the absence of capsule/s. The reason being the presence of solid phase in the pipe which increases the friction factor expressed in equation (1). In the present study, the friction factor in capsule transporting pipe has been assumed to be the sum of the friction factor for water flow, in the absence of capsule, and the friction factor due to capsule only. To calculate the friction factor due to capsule only, the friction factor due to water is subtracted from the total friction factor which is due to both liquid and solid phases in the flow. Hence, a methodology has been developed to calculate both the friction factors separately.

Figure 4a shows the variations in pressure drop per unit pipe length at different bulk velocities of the flow. The data presented is for different capsule to pipe diameter ratios and for a cylindrical capsule of length 0.1m. The figure depicts that as the bulk velocity of the flow increases, the pressure drop in the pipe increases. This is evident from equation (1) where the square of flow velocity and pressure drop have a proportional relationship. Furthermore, the results show that as the capsule to pipe diameter ratio increases, the pressure drop in the pipe increases. This is due to the fact, as stated by Ulusarslan [4], that as the volumetric concentration of the solid phase in the flow increases, the pressure drop in the pipe increases.
Similar trends can be seen in figures 4b and 4c. Figure 4b corresponds to capsule of length 0.3m and figure 4c corresponds to capsule length of 0.5m. In both the figures, it can be clearly seen that as the bulk velocity of the flow increases, the pressure drop in the pipe increases. Furthermore, as the capsule to pipe diameter ratio increases, the pressure drop in the pipe increases.

It should be noted that as the length of the capsule increases, the pressure drop in the pipe increases. This is in accordance with the justification given by Ulusarslan [4] that the increase in the solid phase concentration in the flow results in the increase of pressure drop in the pipe. For a capsule to pipe diameter ratio of 0.7 and at bulk velocity of 1m/s, the pressure drop per unit pipe length for 0.1m, 0.3m and 0.5m long cylindrical capsule is 1490 Pa/m, 1690 Pa/m and 1883 Pa/m respectively. Hence, there is an increase of 11.8% and 20.8% in the pressure drop values for 0.3m and 0.5m long capsule with respect to 0.1m long capsule.
As discussed earlier, the mathematical form of the correlation for capsule-water flow in a horizontal pipeline can be expressed as:

\[
\left( \frac{\Delta P}{L} \right)_m = \frac{\rho v^2}{2D} \cdot f_m
\]  \hspace{1cm} (4)

Where \( f_m \) is the friction factor for the mixture of capsule and water in the pipeline. It can be expressed as the summation of the friction factor due to water alone and the friction factor due to capsule alone.

\[
f_m = f_w + f_c
\]  \hspace{1cm} (5)

Figure 4. Variations in pressure drop against bulk velocities of the flow for cylindrical capsules (a) \( L_c = 0.1 \text{m} \) (b) \( L_c = 0.3 \text{m} \) (c) \( L_c = 0.5 \text{m} \).
The steps involved in the methodology developed are:

1. Calculate the pressure drop i.e. $\left( \frac{\Delta p}{L} \right)_m$ from CFD

2. Divide this pressure drop by $\frac{\rho V^2}{2D}$. This will give the value of $f_m$

3. Subtract $f_w$ from $f_m$ to get the value of $f_c$

In order to develop a semi-empirical model for the calculation of pressure drop in a cylindrical capsule transporting horizontal hydraulic pipeline, the curve fitting technique, as discussed earlier, has been applied for $f_c$ data calculated using the above steps. Figures 5a, b and c show the variations in $f_c$ at different Reynolds number of the capsule.

(a) Friction factor for $L_c = 0.1 \text{m}$. 
Figures 5a, b and c depict the same trend as observed for the friction factor due to liquid phase alone (figure 3). The results show that as the Reynolds number of the capsule increases, the friction factor due to solid phase alone decreases. Furthermore, as the capsule to pipe diameter ratio increases, the friction factor due to solid phase in the flow increases.

![Graph showing friction factor for different capsule to pipe length ratios](image)

(b) Friction factor for \( L_c = 0.3 \text{m} \).

Figures 5a, b and c also show that as the length of the capsule increases, the friction factor due to solid phase increases. Using the curve fitting technique, the correlation for the friction factor due to the capsule in the pipe can be expressed as follows with an average deviation of less than 5% (shown in figure 6) from the data shown in the plots:

\[
f_c = (0.012a - 0.004) \times \frac{e^{(9k)}}{Re_c^{0.2}}
\]

(6)

Where \( a \) and \( k \) are the important non-dimensional parameters as discussed earlier. \( a \) is the capsule to pipe length ratio and \( k \) is the capsule to pipe diameter ratio. The Reynolds number of capsules is calculated by:

\[
Re_c = \frac{\rho d V}{\mu}
\]

(7)

Where \( d \) is the diameter of the cylindrical capsule. Substituting equation (5) into equation (4), we get:
Where $f_w$ and $f_c$ are expressed in equations (2) and (6).

\[
\left(\frac{\Delta P}{L}\right)_m = \frac{\rho V^2}{2D} \times (f_w + f_c) \tag{8}
\]

Figure 5. Variations in friction factor due to capsule at different capsule reynolds numbers (a) $L_c = 0.1m$ (b) $L_c = 0.3m$ (c) $L_c = 0.5m$. 
4. Conclusions
A rigorous semi-empirical mathematical model for the calculation of pressure drop per unit pipe length in a cylindrical capsule transporting hydraulic pipeline has been developed. The results depict that the pressure drop in capsule transporting horizontal pipelines is dependent on the volumetric concentration of the solid phase and the bulk velocity. A methodology for the calculation of the friction factor in the pipeline due to the presence of the solid phase alone has also been presented.

References
[1] H. H. van den Kroonenberg, “A mathematical model for concentric horizontal capsule transport”, Canadian Journal of Chemical Engineering, vol. 56, pp. 538, 1978
[2] D. Ulusarslan, and I. Teke, “An experimental investigation of the capsule velocity, concentration rate and the spacing between the capsules for spherical capsule train flow in a horizontal circular pipe,” Journal of Powder Technology, vol. 159, pp. 27-34, July 2005
[3] D. Ulusarslan, “Effect of Capsule Density and Concentration on Pressure Drops of Spherical Capsule Train Conveyed by Water,” Journal of Fluids Engineering, vol. 132, pp. 11304-1, Jan.

Figure 6. Variations in friction factor due to the capsule for Lc=0.1m and k=0.5.
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[4] D. Ulusarslan and I. Teke, “Relation between the Friction Coefficient and Re Number for Spherical Capsule Train-Water Flow in Horizontal Pipes”, Journal of Particulate Science and Technology, vol. 27, pp. 488-495, 2009

[5] B. R. Munson, and D. F. Young, Fundamentals of Fluid Mechanics, 4th ed., USA: John-Wiley & Sons Inc., 2002

[6] J. Weisbach, “Lehrbuch der Ingenieur- und Maschinen-Mechanik,” Theoretische Mechanik, Vieweg und Sohn, Braunschweig, vol. 1, 1845