Evaporation of Three Dimensional Black Hole in Quantum Gravity

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ABSTRACT

We discuss an evaporation of (2+1)-dimensional black hole by using quantum gravity holding in the vicinity of the black hole horizon. It is shown that the black hole evaporates at a definite rate by emitting matters through the quantum tunneling effect. A relation of the present formalism to the black hole entropy is briefly commented.

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The canonical formalism provides us with a useful starting point for quantization by Dirac [1]. In particular when applied for general relativity, the dynamics is entirely controlled by constraints [1, 2]. It is well known that their imposition as operator equations on the physical states produces the Wheeler-DeWitt equation [3] although its physical interpretation is sometimes unclear and difficult. Our aim in the present study is to make use of the canonical quantization formalism for a system having a black hole in order to understand the black hole evaporation [4] in three dimensions in the framework of quantum gravity.

Since an announcement by Hawking [4] that black holes are not, after all, completely black, but emit a thermal radiation due to quantum effects, there have been many efforts to extend his semiclassical analysis where the background metric is treated classically, but the matter fields quantum mechanically, to a completely quantum-mechanical analysis. At the moment, it seems to be fair to say that we do not yet have a fully satisfactory and consistent theory of quantum gravity. Recently, however, there appeared one interesting and fully quantum-mechanical approach that is based on both the black-hole minisuperspace model and the canonical quantization formalism, and was applied for understanding of the black hole radiation [5, 6] and the mass inflation in four dimensions [7]. The key observation is that near the black hole horizon the Hamiltonian constraint becomes proportional to the supermomentum constraint (we will later clarify the reason) and, at the same time, has a tractable form by which we can solve the Wheeler-DeWitt equation analytically.

Even if not completely satisfactory, it seems to be very reasonable at least for the present author to consider a theory of quantum gravity in the vicinity of the black hole horizon by the following reasons: it is nowadays thought that some important properties of quantum black holes such as the black hole thermodynamics [8] and the Hawking radiation [4] have an origin of the existence of the horizon [9, 10].

† P.Moniz has independently considered a similar model from a different motivation (private communication).
More recently, Carlip has remarkably derived the entropy of the three-dimensional black hole by counting the microscopic states associated with the horizon [12].

In this article, by means of an extended formalism of the above-mentioned formalism [5-7] we would like to consider an evaporation of (2+1)-dimensional black hole which was recently discovered by Bañados et al. [13] since one expects that questions about quantum black holes can be explored in considerable detail without the unnecessary complications coming from higher spacetime dimensions. Moreover, it is observed that in the Lovelock gravity the two branches of black holes emerge, namely, one for even dimensions, with strong similarities to the Schwarzschild black hole in four dimensions, and another for odd dimensions, with common features with three dimensional black hole [14]. Thus it is of interest to study whether the previous formalism or its extended version is applicable to the present case and leads to a physically meaningful result or not.

Let us start by constructing a canonical formalism of a spherically symmetric system with a black hole in three dimensions. An analogous formalism in four dimensions has already constructed in the Refs.[6, 15].

The three dimensional action that we consider is of the form

\[ S = \int d^3x \sqrt{-g} \left[ \frac{1}{16\pi G} \left( \frac{1}{l^2} R + 2 \right) - \frac{1}{4\pi} g^{\mu\nu} (D_{\mu}\Phi)^{\dagger} D_{\nu}\Phi - \frac{1}{16\pi} F_{\mu\nu}F^{\mu\nu} \right], \]

(1)

where \( l \) is the scale parameter with dimension of length and is related to the cosmological constant by \( l = \frac{1}{\sqrt{-\Lambda}} \), \( \Phi \) is a complex scalar field with the electric charge \( e \),

\[ D_{\mu}\Phi = \partial_{\mu}\Phi + ieA_{\mu}\Phi \]

(2)

is its covariant derivative, \( A_{\mu} \) is the electromagnetic field, and \( F_{\mu\nu} \) is the corre-
sponding field strength as usual given by

\[ F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \]  

(3)

To clarify the three dimensional meaning we put the suffix (3) in front of the metric tensor and the curvature scalar. We will follow the conventions adopted in the MTW textbook [16] and use the natural units \( G = \hbar = c = 1 \). The Greek indices \( \mu, \nu, \ldots \) take 0, 1 and 2, and the Latin indices \( a, b, \ldots \) take 0 and 1. Of course, the inclusion of other matter fields and the surface term in this formalism is straightforward even if we limit ourselves to the action (1) for simplicity.

The most general spherically symmetric assumption for the metric is

\[ ds^2 = (3) g_{\mu \nu} dx^\mu dx^\nu, \]

\[ = g_{ab} dx^a dx^b + \phi^2 d\theta^2, \]  

(4)

where the two dimensional metric \( g_{ab} \) and the radial function \( \phi \) are the function of only the two dimensional coordinates \( x^a \). And the angular variable \( \theta \) takes the value from 0 to \( 2\pi \). For the charged matter and the electromagnetic potential we take the spherical ansatz \( D_\theta \Phi = A_\theta = 0 \). The substitution of these ansatz into (1) and then integration over the angular variable \( \theta \) leads to the following effective action in two dimensions:

\[ S = \frac{1}{8} \int d^2x \sqrt{-g} \phi \left( R + \frac{2}{l^2} \right) - \frac{1}{2} \int d^2x \sqrt{-g} \phi \ g^{ab}(D_a \Phi)^\dagger D_b \Phi \]

\[ - \frac{1}{8} \int d^2x \sqrt{-g} \phi \ F_{ab} F^{ab}. \]  

(5)

Here let us make a brief comment on a curious feature of the gravitational sector in this action. It is easy to show that the dimensional reduction of the Einstein-Hilbert action with the cosmological constant from higher dimensions to two dimensions
under the spherically symmetric ansatz in general leads to

\[ S = \frac{1}{8} \int d^2 x \sqrt{-g} \left[ Y(\phi)R + V(\phi) + Z(\phi)g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi \right], \quad (6) \]

which is a general class of the dilaton gravity. The model in hand, which is obtained by the reduction from three to two dimensions, has an interesting feature \( Z(\phi) = 0 \) and \( Y(\phi) \propto V(\phi) \propto \phi \) which was previously introduced by Jackiew and Teitelboim [17] as a model of gravity in two dimensions since the Einstein-Hilbert action is a surface term in two dimensions. Subsequently, this de Sitter-type action was rewritten in terms of the topological BF theory [18]

\[ S = \int_M TrB F. \quad (7) \]

Since it is known that the topological BF theory is exactly solvable [19-21], this interesting structure of the present model (5) would be of benefit to, for instance, an understanding of the black hole entropy in the future from the following observation. Recall that the geometry of the Euclidean black hole is largely independent of the spacetime dimension except the difference of even and odd dimensions, and then the black hole in \( d \) dimensions has in general the topology of \( R^2 \times S^{d-2} \), and the entropy comes quite generically from the existence of a possible conical singularity in the \( R^2 \) plane [22]. One therefore expects that a detailed analysis of the effective two dimensional theories such as (5) has a possibility of bringing us important informations in the black hole thermodynamics.

Now let us rewrite the action (5) into the first-order ADM form [2]. To do so, we shall foliate the region outside the black hole horizon by \( x^0 = \text{const} \) spacelike hypersurfaces. The appropriate ADM splitting of (1+1)-dimensional spacetime is given by

\[ g_{ab} = \begin{pmatrix} -\alpha^2 + \beta^2 & \beta \\ \beta & \gamma \end{pmatrix}, \quad (8) \]
and the normal unit vector orthogonal to the hypersurface \( x^0 = \text{const} \) becomes

\[ n^a = \left( \frac{1}{\alpha}, -\frac{\beta}{\alpha\gamma} \right). \tag{9} \]

Following the analogous procedure in the Refs.\([6, 15]\), the action (5) can be written as

\[
S = \int d^2x \, L = \int d^2x \left[ \frac{1}{4} \alpha \sqrt{-\gamma} \left( -Kn^a \partial_a \phi + \frac{\alpha'}{\alpha\gamma} \phi' + \frac{1}{\ell^2} \phi \right) + \frac{1}{2} \alpha \sqrt{-\gamma} \phi \left( |n^a D_a \Phi|^2 - \frac{1}{\gamma} |D_1 \Phi|^2 \right) + \frac{1}{4} \alpha \sqrt{-\gamma} \phi E^2 \right], \tag{10}
\]

where

\[
E = \frac{1}{\sqrt{-g}} F_{01} = \frac{1}{\alpha \sqrt{-\gamma}} (\dot{A}_1 - A'_0), \tag{11}
\]

and the trace of the extrinsic curvature \( K = g^{ab} K_{ab} \) is expressed by

\[
K = \frac{\dot{\gamma}}{2\alpha\gamma} - \frac{\beta'}{\alpha\gamma} + \frac{\beta}{2\alpha\gamma^2} \gamma'. \tag{12}
\]

Here \( \frac{\partial}{\partial x^0} = \partial_0 \) and \( \frac{\partial}{\partial x^1} = \partial_1 \) are also denoted by an overdot and a prime, respectively.

By differentiating the action (10) with respect to the canonical variables \( \Phi(\Phi^\dagger), \phi, \gamma \) and \( A_1 \), we have the corresponding conjugate momenta \( p_\Phi(p_{\Phi^\dagger}), p_\phi, p_\gamma \) and \( p_A \)

\[
p_\Phi = \frac{\sqrt{-\gamma}}{2} \phi (n^a D_a \Phi)^\dagger, \tag{13}
\]

\[
p_\phi = -\frac{\sqrt{-\gamma}}{4} K, \tag{14}
\]

\[
p_\gamma = -\frac{1}{8\sqrt{-\gamma}} n^a \partial_a \phi, \tag{15}
\]
\[ p_A = \frac{1}{2} \phi E. \]  

(16)

Then the Hamiltonian \( H \) which is defined as

\[ H = \int dx^1 (p \dot{\Phi} + p \dot{\Phi}^\dagger + p \dot{\gamma} + p A \dot{A}_1 - L) \]

(17)

is expressed by a linear combination of constraints as expected

\[ H = \int dx^1 (\alpha H_0 + \beta H_1 + A_0 H_2), \]

(18)

where

\[ H_0 = \frac{2}{\sqrt{\gamma} \phi} p \phi p \phi^\dagger - 8 \sqrt{\gamma} p \phi p \gamma + \frac{1}{4} \left( \phi' \sqrt{\gamma} \right)' - \frac{\sqrt{\gamma}}{4} \frac{1}{l^2} \phi \]

\[ + \frac{\phi}{2 \sqrt{\gamma}} |D_1 \Phi|^2 + \frac{\sqrt{\gamma}}{\phi} p_A^2. \]

(19)

\[ H_1 = \frac{1}{\gamma} [p \phi D_1 \Phi + p \phi^\dagger (D_1 \Phi)^\dagger] + \frac{1}{\gamma} p \phi \phi' - 2 p \gamma' - \frac{1}{\gamma} p \gamma \gamma', \]

(20)

\[ H_2 = -ie (p \phi - p \phi^\dagger) - p_A'. \]

(21)

Note that \( \alpha, \beta \) and \( A_0 \) are non-dynamical Lagrange multiplier fields.

The action can be cast into the first-order ADM canonical form by the dual Legendre transformation

\[ S = \int dx^0 [\int dx^1 (p \dot{\Phi} + p \dot{\Phi}^\dagger + p \phi \dot{\phi} + p \gamma \dot{\gamma} + p A \dot{A}_1) - H]. \]

(22)

As Regge and Teitelboim pointed out [23], in order to have the correct Hamiltonian which produces the Einstein equations through the Hamilton equations, one has to supplement the surface term to the Hamiltonian (22). It is straightforward to show that if one takes the boundary condition such that a contribution to the surface term from the apparent horizon vanishes the surface term from the spatial infinity produces the ADM mass of the black hole.
We now turn our attention to an application of the canonical formalism constructed in the above for understanding of an evaporation of the (2+1)-dimensional black hole [13] from the viewpoint of quantum gravity. To consider the simplest model of the black hole radiation, let us turn off the electromagnetic field and deal with the neutral scalar field by which the constraint \( H_2 \) generating the \( U(1) \) gauge transformations identically vanishes. Moreover, we shall use an ingoing Vaidya metric to express the black hole radiation. The treatment of the case of the outgoing Vaidya metric can be made in a perfectly similar way.

First of all, let us define the two dimensional coordinate \( x^a \) by

\[
x^a = (x^0, x^1) = (v - r, r),
\]

where the advanced time coordinate is defined as \( v = t + r^* \) with the tortoise coordinate \( dr^* = \frac{dr}{g_{00}} \). Then we shall fix the gauge freedoms corresponding to the two dimensional reparametrization invariances by the gauge conditions

\[
\begin{align*}
g_{ab} &= \begin{pmatrix} -\alpha^2 + \frac{r^2}{\gamma} & \beta \\ \beta & \gamma \end{pmatrix}, \\
&= \begin{pmatrix} -(M + \frac{r^2}{2l}) & 1 + M - \frac{r^2}{2l} \\ 1 + M - \frac{r^2}{2l} & 2 + M - \frac{r^2}{2l} \end{pmatrix},
\end{align*}
\]

where the scale parameter \( l \) and the black hole mass \( M \) are now generally the function of the two dimensional coordinates \( x^a \). Note that we have not fixed the gauge symmetries completely for later convenience though we may usually set the scale function \( l \) to be constant by means of the remaining one gauge freedom. At present this does not cause any trouble since, afterward, we effectively fix this gauge symmetry in making an assumption of dynamical fields near the horizon.

From these equations the two dimensional line element takes a form of the
Vaidya metric corresponding to the three dimensional black hole without rotation

\[ ds^2 = g_{ab} dx^a dx^b, \]
\[ = -(-M + \frac{r^2}{l^2}) dv^2 + 2 dvdr. \]  

(25)

Since we would like to study a dynamical black hole, it is useful to consider the local definition of horizon, i.e., the apparent horizon, instead of the global one, the event horizon. The apparent horizon is then defined as

\[ r_+ = l \sqrt{M}. \]  

(26)

Since we have constructed the canonical formalism of a spherically symmetric system with a black hole in three dimensions, let us perform a canonical quantization. Following Dirac [1], we have to impose the constraints on the states as the operator equations and solve them. However, even in a rather simple setting of the present model, it is quite difficult to solve the constraints’ equations owing to their complicated form. We therefore need some approximation method that retains the important features of the black hole physics. One of such approximations was proposed by Tomimatsu [5]. His critical idea is to solve the Hamiltonian and supermomentum constraints only in the vicinity of the apparent horizon. We will see that this approximation scheme yields to a remarkable simplification in the model at hand.

Near the apparent horizon, we shall make an assumption

\[ \Phi \approx \Phi(v), \phi \approx r, l \approx l(v), M \approx const. \]  

(27)

We shall use \( \approx \) to indicate the equalities which hold approximately near the apparent horizon from now on. In the latter two equations in (27), we have made a specific assumption in three dimensions. The reason is that in three dimensions
or even in general odd dimensions the mass function $M$ is dimensionless so that it is not the mass but the scale function $l$ with dimension of length that plays a key role in describing the black hole properties [13, 14]. Indeed it is valuable to note that (27) is equivalent to an assumption where the radius of the black hole is in itself a dynamical function as seen later explicitly. Incidentally one can prove the above assumption (27) to be consistent with the field equations near the apparent horizon in an analogous way to our previous work [6].

Eq.(24) yields near the apparent horizon (26)

$$\alpha \approx \frac{1}{\sqrt{2}}, \beta \approx 1, \gamma = \frac{1}{\alpha^2} \approx 2,$$

(28)

and the canonical conjugate momenta (13)-(15) are given approximately as

$$p_\Phi \approx \frac{1}{2} \phi \partial_v \Phi,$$

$$p_\phi \approx \frac{1}{8} \frac{M}{l} \partial_v l - \frac{3 \sqrt{M}}{8 l},$$

$$p_\gamma \approx \frac{1}{16}.$$

(29)

Moreover, after a careful calculation the two constraints are proportional to each other

$$\frac{1}{\sqrt{2}} H_0 \approx H_1,$$

$$\approx \frac{2}{\phi} p_\Phi^2 - p_\phi - \frac{3 \sqrt{M}}{8 l}.$$

(30)

At this stage we would like to think of the reason why the Hamiltonian and the supermomentum constraints have become proportional to each other near the horizon since the previous works [5-7] are obscure in this respect. The Hamiltonian constraint $H_0$ and supermomentum constraint $H_1$ generate the time translation and
the spatial displacement, respectively, given by

\[ H_0 : x^0 \rightarrow x^0 + \varepsilon^0, \]
\[ H_1 : x^1 \rightarrow x^1 + \varepsilon^1, \]  
(31)

where \( \varepsilon^a \) is the infinitesimal transformation parameter. From (23), it is shown that

\[ H_0 : v \rightarrow v + \varepsilon^0 + \varepsilon^1, \]
\[ H_1 : r \rightarrow r + \varepsilon^1. \]  
(32)

Note that the time is standing still at the apparent horizon, so that the time translation associated with the transformation parameter \( \varepsilon^0 \) is frozen there. Thus in the coordinates \( (v, r) \) the only nontrivial gauge motion is nothing but the spatial displacement at the horizon. This observation is also certified by the equalities from (18), (28) and (30) that \( \alpha H_0 \approx \beta H_1 \approx H_1. \)

By imposing the constraint (30) as an operator equation on the state, one obtains the Wheeler-DeWitt equation

\[ i \frac{\partial \Psi}{\partial T} = \left( -\frac{\partial^2}{\partial \Phi^2} - \frac{3}{16} M \right) \Psi, \]  
(33)

where we have introduced \( T = \log \left( \frac{1}{\phi} \right)^2 \). This Wheeler-DeWitt equation can be interpreted as the Schrödinger equation with the time \( T \) and the Hamiltonian \( H = p_\Phi^2 - \frac{3}{16} M \) in the superspace.

Now it is easy to find a special solution of the above Wheeler-DeWitt equation by the method of separation of variables. The result is

\[ \Psi = (Be^{i\sqrt{A}\Phi(v)} + Ce^{-i\sqrt{A}\Phi(v)}) \ e^{-i(A-\frac{3}{16}M)T}, \]  
(34)

where \( A, B, \) and \( C \) are integration constants. If one defines an expectation value
<\mathcal{O}> of an operator $\mathcal{O}$ in a rather naively as

$$<\mathcal{O}> = \frac{1}{\int d\Phi |\Psi|^2} \int d\Phi \Psi^* \mathcal{O} \Psi,$$  \tag{35}$$

one can calculate a change rate of the radius of a black hole horizon $<\partial_v r_+>$ through the absorption or the emission of the neutral scalar matters by using (26) and either (29) or (30)

$$<\partial_v r_+> = \frac{16A}{M}.$$  \tag{36}$$

This equation shows the absorption of the external matters by a black hole when one chooses the constant $A$ to be a positive constant, e.g., $\frac{1}{16} k_1^2$. Then the change rate of the radius of a black hole horizon becomes

$$<\partial_v r_+> = \frac{k_1^2}{M},$$  \tag{37}$$

and the physical state is given by

$$\Psi = \left( Be^{i\frac{1}{4}|k_1|\Phi(v)} + Ce^{-i\frac{1}{4}|k_1|\Phi(v)} \right) e^{-i\frac{1}{16}(k_1^2-3M)T}. \tag{38}$$

On the other hand, if one takes the constant $A$ to be a negative constant, e.g., $-\frac{1}{16} k_1^2$, (36) means the Hawking radiation:

$$<\partial_v r_+> = -\frac{k_1^2}{M},$$  \tag{39}$$

with the physical state

$$\Psi = \left( Be^{-i\frac{1}{4}|k_2|\Phi(v)} + Ce^{i\frac{1}{4}|k_2|\Phi(v)} \right) e^{i\frac{1}{16}(k_2^2+3M)T}. \tag{40}$$

Note that if one chooses the boundary condition $C = 0$ in (38) and (40), the physical state (38) represents the scalar wave propagating in black hole from the
exterior region across the horizon while the state (40) is exponentially damping tunneling state in classically forbidden region. These behavior of the physical states seems to be physically plausible with our interpretation that (37) describes the absorption of matters by a black hole, on the other hand, (39) represents the the Hawking radiation through the quantum tunneling effect. Incidentally, it is of interest to comment here that if, instead of (27), we adopt an alternative assumption $\Phi \approx \Phi(v), \phi \approx r, M \approx M(v), \text{and} \ l \approx \text{const.}$ as done in the case of the four dimensional Schwarzschild black hole (precisely speaking, in four dimensions, $\Lambda = -\frac{1}{l^2} = 0$), it is shown that we obtain the same result as (36). This correspondence stems from the mathematical fact that both assumptions share the common Wheeler-DeWitt equation. In this sense, our result is independent of these assumptions.

Now let us consider the physical meaning of the result in the case of the Hawking radiation (39) and (40). Eq.(39) implies that the radius of the black hole horizon gradually decreases at a definite rate by emitting the thermal radiation, and then evaporates completely. However, it is dubious to extrapolate this result literally to the endpoint of evaporation since the physical state (40) fluctuates strongly there owing to the huge $T$ factor. This huge quantum fluctuation might be related to the quantum instability of the black hole singularity in three dimensions [24].

To summarize, in this article we have analysed the black hole radiation in three spacetime dimensions by using the quantum gravity holding in the vicinity of the black hole horizon. We have seen that the black hole radius decreases gradually by emitting the thermal radiation. Here it is worth pointing out an advantage of our formalism compared to the Hawking’s original formulation. In the Hawking’s semiclassical approach the gravitational field is fixed as a classical background and only the matter field is treated to be quantum-mechanical. By contrast, our present formulation is purely quantum-mechanical. We hope that the present formalism may have some implications in the membrane paradigm of quantum black holes in the future.
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