Ethnomathematics for congruence concept: a didactical design in a mathematics classroom

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Abstract. The purpose of this study is to design mathematics learning with a realistic mathematics-based ethnomathematics approach with the concepts of congruence in junior high school students by didactic design research’s stage. Didactic design is based on identifying student learning obstacles in the learning process barriers to student learning primarily through group discussion forums with secondary school teachers in Semarang. The research method is qualitative in the form of didactic design research using techniques triangulation (combining participatory observation, interviews with teachers, questionnaires, and document studies). The identification results of students’ learning difficulties are used to design learning and teaching materials. Teaching material is based on ethnomathematics and uses a realistic mathematical approach. The results of the validation show that teaching materials are appropriate to use. The material aspects and DDR receive input from the validator, primarily related to the use of realistic and ethnomathematics learning approaches. For the overall ranking scale, the validator generally gives a good rating. The revised results were tested on a small scale and large scale, with the average trial results showing excellent results. So, the instructional materials designed could be used to overcome the obstacles of student learning in the material congruence of flat fields.

1. Introduction
Learning realistic mathematics (RME) was first coined by a Dutch mathematician named Hans Freudenthal. Freudenthal believes that students should not be seen as passive mathematics that has already been made or processed [1]. Prahmana [2] stated that several learning approaches use real-world contexts, one of which is the Realistic Mathematics Education approach. In learning mathematics, students should not be taught to memorize but to understand concepts so that students’ mathematical abilities can develop [3]. In learning RME, students will experience a conceptual mathematical process, in which student understanding will be mathematized by learning done by students themselves. Gravemeijer [3] described this mathematization as a process of rediscovery, as seen in figure 1.
In horizontal mathematical, students try to parse and solve contextual problems using their way. In this case, the results of working on the problems between one student with other students may be different because every student has a different understanding. While in vertical mathematical students are also faced with contextual problems, but in the long run, students can develop certain procedures to solve similar problems without having to use context assistance [1].

Didactical Design Research (DDR) is a research method that places the design process as an important part, serves to design or develop an intervention to solve complex problems in the education field and to develop knowledge about the characteristics of interventions carried out and the process of designing or developing interventions these [4]. DDR research consists of several stages [5], namely: first, a didactic situation analysis before learning (prospective analysis) in the form of a Didactic Hypothesis Design including ADP (Anticipating Pedagogical Didactic). The second, metapedididactical analysis. Third, retrospective analysis is an analysis that links the results of the didactic situation analysis of the hypothesis with the results of the metapedididactical analysis.

The things done in the prospective analysis stage are 1) Analyze the objectives to be achieved, for example, learning objectives; 2) determine and determine the initial conditions of the study; 3) discuss the conjecture of local instructional theory that will be developed; 4) determine class characteristics and the role of the teacher and 5) set-theoretical goals to be achieved through research [5]. The metapedididactical analysis stage can be called a stage where a teacher implements a design that has been designed at the prospective analysis stage, or it can also be interpreted as an experimental stage for a teacher in applying the didactic design has been made. Also, the researcher can do the test, so it can be known whether the design work or not.

Ethnomathematics can be interpreted as a study of the relationship between mathematics and related socio-cultural backgrounds that show how mathematics is produced, transferred, distributed, and specialized in diverse cultural systems [6]. Ethnomathematics is the study of mathematical ideas of different social groups. Generally, these ideas come from their daily activities, which can be related to the seven ethnographic frameworks above. Ethnomathematics involves a mathematical formula that is developed and used in everyday life by members of the community group [7]. Ethnomathematics and other similar approaches such as Realistic Mathematics Education (RME) are trying to “reconcile” mathematics that is often associated with abstract objects into the realm so that it can be applicable for students to solve encountered problem in everyday life [8]. Besides linking education with culture, mathematics learning must be based on experience, in line with Freudenthal’s view that emphasizes mathematics as a human activity, which is called the Realistic Mathematics Education (RME) approach. The RME’s principle is student involvement in mathematics must begin in a meaningful context. It is essential to point out the “realistic” aspects of RME not only because it has to do with real-world contexts, but is related to the emphasis that RME places students’ problems on imaginable situations [9].
Learning barriers are caused by three factors, namely ontogenic barriers meaning learning barriers caused by students’ lack of mentality in dealing with the learning process [10]. The next obstacle is the didactic obstacle or the obstacle caused by the teaching of the teacher (less comprehensive learning process). And finally, the epistemology barrier means the obstacle caused by students’ knowledge that has a limited application context. There are three types of obstacle learning experienced by students [11]. An epistemological obstacle is a learning barrier caused by understanding students’ concepts limited to a context. A study on the causes of the emergence of the learning obstacle can be done by observing the interrelationship between the didactic triangle components (teacher, students, and material). The implication is that the didactic design created by the teacher must make students experience their learning process to obtain learning objectives and make representations for the understanding they have gained [12]. An epistemological learning obstacle is someone’s knowledge, which is only limited to certain contexts. If the person is exposed to a different context, the knowledge they have can not be used, or he has difficulty using it [13]. Didactic design is defined as the design of the learning environment and a series of instructions that are informed through analysis of a particular topic as a point of attention, and are framed in a specific subject area [14]. This research discussed the learning barrier about the congruent topic in junior high school.

2. Methods
The research method used in this study is qualitative in the form of didactical design research. It’s a study using triangulation data collection techniques. The triangulation technique was chosen by combining participatory observation, interviews with teachers, questionnaires, and document studies. Learning obstacles identification to determine student learning difficulties. Focus Group Discussion (FGD) was conducted with junior high school mathematics teachers who are members of the Mathematics Teachers Association (MGMP) in Semarang. Interview conducted on Mathematics Teachers of grade 9 and several students’ classes’ grades 9.

The next stage is making learning plans and teaching materials according to students’ difficulties in learning based on the identification of learning obstacles. The draft of the finished teaching materials is then carried out a validation process (material and language). The validation stage conduct by a material expert, linguist, and DDR expert and has finished repairing. Then, the product is tested with a small group trial consist of 10 students of grade 10 per school. This is done to test the readability of the teaching material that has been made. The readability test is conducted on students who have studied material congruence and similarity so that they take grade 10 because this textbook will be used for grade 9 junior high school. Whereas the large group trial consisted of 32 students from 9 grade from three schools representing each sub-district of Semarang, 32 students of grade 9 form SMP N 2 Semarang, 32 students of grade 9 form SMP N 10 Semarang, and 32 students of grade 9 form SMP N 7 Semarang City.

3. Result and discussion
The result and discussion of this research, there are several important points, including the identification of learning barriers, processing data collection, and the design of the didactic design of teaching materials. The following is an explanation of the results and discussion:

3.1. Identification of learning barriers
The didactic design making process begins with the identification of learning barriers to students. From the test results obtained, the results of student difficulties on congruence material in groups in five forms of difficulty. First, students have difficulty distinguishing two congruent objects. Second, students have difficulty distinguishing two congruent objects. Third, in determining the corresponding sides and the corresponding angle, students are often confused and incorrectly determine the ratio of the corresponding sides. Fourth, in the case of comparison of two pictures students, are still often wrong with the border on the photo frame, so when given a problem, it is always wrong because the border of the picture frame is not counted. Fifth, the concepts that have been taught in schools often
change with the existence of a “smart solution” in the place of a tutoring institution attended by students. So the teacher needs to repeat the explanation of the concept to students. Examples of forms of student difficulties, as presented in figure 2.

![Figure 2. Example of student difficulties from identification results.](image)

The interaction of geometry between students and teachers in the classroom creates new challenges because many obstacles can occur in learning geometry [15]. Learning difficulties of geometry, especially in the congruence concept, are students do not understand the difference between the congruence of a two-dimensional shape, and students difficult to imagine that triangle congruence conditions can apply to all types of triangles. Difficulties experienced by students, especially in understanding the “side” and “angle.” Usually, students cannot distinguish that two shapes are congruent. The implication, the concept of congruence can be a learning obstacle on congruence material. The results of the analysis of students’ learning barriers were obtained that students’ mistakes in solving mathematical problems caused not only from students themselves but from factors outside the student. Students make the mistakes in answering didactic congruence problems. This means that students can provide solutions to congruence problems, but the students’ answers are less precise. It is caused by students’ learning barriers, namely ontogenic obstacles, didactic obstacles, and epistemological obstacles. Thus learning the concept of congruence in the classroom does not only pay attention to the theories that are standard as written in textbooks in school. Teachers and students need to modify the presentation of plane geometry in the classroom, the presentation of congruence theorems. So that the presentation of geometric shapes more and more varied. As a result, students are rich in congruence concepts.

3.2. Data collection and processing
After identifying students’ learning barriers, then collect and process data that supports the development of teaching materials. Sources or references for the development of teaching materials are obtained from relevant sources.

3.3. The design of the didactic design of teaching materials
At this stage, the product design is developed into a teaching material in the form of a didactic design module. Constructing is arranged using the Indonesian language that is easily understood by students with steps that facilitate students in understanding the material in the learning process.

The cover of teaching materials consists of the front cover and the back cover. The front cover consists of didactic design module writing, module title, class, and author’s name. The picture chosen on the cover is adapted to the learning approach used in compiling teaching materials, namely the RME approach based on ethnomathematics, which is adjusted to the culture of Semarang City. The cover image in the form of a typical Semarang city is packaged in an attractive picture. The cover page layout is structured in such a way as to attract the attention of students. So, it is expected by seeing an interesting and unique cover that will motivate students to learn this module.
3.4. Validation stage of didactic design of teaching materials
Material expert validation aims to test the completeness of the material, the truth of the material, the systematic matter, and the truth of the phenomenon. The results of material validation data in stage 1 and stage 2 can be seen in table 1. The results of the material validation on stage 1 obtained the average 3.42 with “Good” criteria from the first expert, and 4.08 with “Good” criteria from the second expert. In stage 2, the average of validation from the first expert is 3.5 with “Good” criteria, and from the second expert is 4.67 with “Very Good” criteria.

Table 1. Expert validation of material aspects.

| Indicator | Expert 1 | Expert 2 |
|-----------|----------|----------|
|           | Step 1   | Step 2   | Step 1 | Step 2 |
| 1         | 3        | 3        | 4      | 5      |
| 2         | 3        | 3        | 5      | 5      |
| 3         | 4        | 4        | 4      | 4      |
| 4         | 4        | 4        | 4      | 4      |
| 5         | 3        | 4        | 4      | 5      |
| 6         | 3        | 2        | 4      | 5      |
| 7         | 3        | 4        | 4      | 5      |
| 8         | 4        | 4        | 4      | 5      |
| 9         | 4        | 3        | 4      | 4      |
| 10        | 3        | 4        | 4      | 4      |
| 11        | 3        | 3        | 4      | 5      |
| 12        | 4        | 4        | 4      | 5      |
| Total     | 41       | 42       | 49     | 56     |
| Mean      | 3.42     | 3.50     | 4.08   | 4.67   |
| Criteria  | Good     | Good     | Good   | Very Good |

3.5. Results of language expert validation
Validation of linguists aims to test the language of didactic design teaching materials. The results of Language validation data in stage 1 and stage 2 can be seen in table 2. The results of the validation of linguists get the results of the “Good” category for validator 1 and validator 2 in stage 1 and stage 2.

Table 2. Expert validation of language aspects.

| Indikator | Expert 1 | Expert 2 |
|-----------|----------|----------|
|           | Step 1   | Step 2   | Step 1 | Step 2 |
| 1         | 3        | 5        | 4      | 4      |
| 2         | 4        | 4        | 4      | 5      |
| 3         | 4        | 4        | 4      | 4      |
| 4         | 3        | 4        | 4      | 4      |
| 5         | 3        | 3        | 3      | 4      |
| 6         | 4        | 4        | 4      | 4      |
| 7         | 3        | 4        | 3      | 4      |
| Total     | 24       | 28       | 26     | 29     |
| Mean      | 3.43     | 4.00     | 3.71   | 4.14   |
| Criteria  | Good     | Good     | Good   | Good   |
3.6. Results of didactic design expert validation
DDR expert validation aims to test whether teaching materials are following didactic design steps. The results of DDR expert validation data in stage 1 and stage 2 can be seen in table 3. The results of DDR expert validation obtained the category “Good” for all validators, both stage 1 and stage 2.

| Indicator | Expert 1 | Expert 2 |
|-----------|----------|----------|
|           | Step 1   | Step 2   | Step 1   | Step 2 |
| 1         | 3        | 4        | 3        | 4      |
| 2         | 4        | 4        | 4        | 4      |
| 3         | 4        | 4        | 3        | 4      |
| 4         | 4        | 4        | 4        | 4      |
| 5         | 3        | 4        | 3        | 4      |
| 6         | 3        | 5        | 4        | 5      |
| 7         | 3        | 4        | 2        | 4      |
| 8         | 4        | 4        | 4        | 4      |
| 9         | 3        | 3        | 3        | 4      |
| 10        | 3        | 3        | 4        | 4      |
| Total     | 34       | 39       | 34       | 41     |
| Mean      | 3.40     | 3.90     | 3.40     | 4.10   |
| Criteria  | Good     | Good     | Good     | Good   |

3.7. Revision design product
At this stage, the product design was validated through the assessment of material experts, education practitioners, linguists, and DDR experts. The researcher revised the product design that was developed based on expert input. The following are inputs from expert assessments and follow-up from experts. Material expert, after validating the expert material assessment, a revision of the previous material was carried out, which was to emphasize more on ethnomathematics by adding more examples of problems related to the culture and history of Semarang City. From this input, the researcher conducted a follow-up following the advice of the material expert. This revision had reached the teaching material (module) to be used. Linguists after validating linguists, the researcher revised the previous material, namely improving the use of punctuation, Enhanced Spelling (EYD), and sentence structure. The follow-up is done by fixing everything following the input. So the design teaching materials didactically better than before until finally. The teaching materials (modules) are feasible to use. Input is obtained after validating the DDR expert. First, the cover of teaching materials can be improved by adding the logo of institution as a funder, if any. The two teaching materials that are made must be based on student obstacle learning for all levels and abilities so that the teaching material can be used for all students of 9th Junior High School in Semarang City. Thus the authors revise what the reviewers have input until the teaching material is feasible to use.

3.8. Product trial
In small group trials in addition to testing the readability of whether the language used is under the language of junior high school students. It is intended so that students can more easily understand the material in teaching materials if the language uses the language of junior high school students. In this small-scale trial also conducted a test of student interest in the display of teaching materials that have been made by giving questionnaires to students that contain statements about the content and appearance of teaching materials. The sample students were taken heterogeneously, who had taken congruence material based on ability in class and gender. Then students were given a questionnaire to assess the content and appearance of teaching materials. The results of students’ responses got the average 4.26 with “Good” criteria at SMAN 1 Semarang and 3.92 with “Good” criteria at SMA Indonesia Institute. This means that didactic design teaching materials developed by researchers have
interesting criteria to be used as aids in teaching and learning activities on congruence material for 9th grade of junior high school.

The field trial was conducted to convince the data and to know the broad attractiveness of the product. Respondents in this large group test numbered 32 students of SMPN 2 Semarang, 32 students of SMPN 7 Semarang, and 32 students of SMPN 10 Semarang by giving questionnaires to find out students’ responses to the attractiveness of the contents and materials of teaching materials for learning, as presented in Table 4. Test results field trials at SMPN 2 Semarang obtained an average of 4.304 with the interpretation criteria achieved, namely “Very Good” the results of field trials at SMPN 7 Semarang obtained an average of 4.29 with the interpretation criteria achieved, namely “Good.” And the results of field trials at SMPN 10 Semarang received an average of 4.3 with the interpretation criteria achieved, which is “very good.” This means that didactic design teaching materials (modules) developed by researchers have interesting criteria to be used as aids in learning activities teaching on two –dimensional shape material for 9th grade of Junior High School.

3.9. Product revision

After conducting a small group trial and a large group trial to find out the attractiveness of teaching materials (modules) of didactic design with the RME approach based on ethnomathematics in material continuity and congruence, the product is said to be interesting. After undergoing a two-step revision carried out by two material expert validators, two linguists, and two DDR experts. In this study, the revision stage is the last. The revised teaching material will be a learning media that has met the standards in terms of the material. The standards include the suitability of the material, the accuracy of the material, helping in overcoming the obstacles of student learning, especially congruence material, the feasibility of presenting teaching materials, the feasibility of language in teaching materials.

4. Conclusion

Student mistakes in answering congruence problems are didactic. It is caused by students’ learning barriers, namely ontogenic obstacles, didactic obstacles, and epistemological obstacles. The resulting learning design to overcome the learning barriers in this study shows that teaching materials are ready to be used to help students’ learning processes congruence material with a mathematical and realistic mathematical approach. This feasibility is seen from the results of the validation of the material aspects, language aspects, and aspects of DDR. Mathematical and ethnomathematics realistically approaches are the primary concern in the teaching material of the results of this study, thus contributing to facilitate students in the learning process.

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