Constraints on alternate universes:
stars and habitable planets with
different fundamental constants

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Abstract. This paper develops constraints on the values of the fundamental constants that allow universes to be habitable. We focus on the fine structure constant $\alpha$ and the gravitational structure constant $\alpha_{G}$, and find the region in the $\alpha$-$\alpha_{G}$ plane that supports working stars and habitable planets. This work is motivated, in part, by the possibility that different versions of the laws of physics could be realized within other universes. The following constraints are enforced: [A] long-lived stable nuclear burning stars exist, [B] planetary surface temperatures are hot enough to support chemical reactions, [C] stellar lifetimes are long enough to allow biological evolution, [D] planets are massive enough to maintain atmospheres, [E] planets are small enough in mass to remain non-degenerate, [F] planets are massive enough to support sufficiently complex biospheres, [G] planets are smaller in mass than their host stars, and [H] stars are smaller in mass than their host galaxies. This paper delineates the portion of the $\alpha$-$\alpha_{G}$ plane that satisfies all of these constraints. The results indicate that viable universes — with working stars and habitable planets — can exist within a parameter space where the structure constants $\alpha$ and $\alpha_{G}$ vary by several orders of magnitude. These constraints also provide upper bounds on the structure constants $(\alpha, \alpha_{G})$ and their ratio. We find the limit $\alpha_{G}/\alpha \lesssim 10^{-34}$, which shows that habitable universes must have a large hierarchy between the strengths of the gravitational force and the electromagnetic force.

Keywords: stars, star formation

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1 Introduction

A long standing problem is that the laws of physics are described by a collection of fundamental constants, but we have no definitive explanation for how the measured values of these constants are determined [1, 2]. One partial explanation is provided by the possible existence of other universes [3, 4], where these separate regions of space-time could display variations in the laws of physics, and hence variations in the fundamental constants. In this scenario, the values of the constants in a given universe are drawn from a set of underlying probability distributions. Our universe represents one particular realization, i.e., one choice for the values of the constants. Unfortunately, we do not have a theory for predicting the form of the underlying probability distributions. In fact, there is not even a general consensus on what fundamental parameters should be allowed to vary from universe to universe (for example, compare the various suggestions presented in [5–11]).

An important issue is that the fundamental constants in our universe have the proper values to allow life to develop. On the other hand, different values could result in a lifeless universe, one with no observers [5, 12, 13]. The necessity for us to live in a universe with observers thus provides a partial explanation for why the fundamental constants have their experimentally measured values. In order to make this type of explanation more complete, however, we need to know [i] what variations of the laws of physics are possible, [ii] the probability distributions for the allowed variations, and [iii] what subset of the possible
universes allow for observers. Many authors have suggested that the universe is “fine-tuned” for the development of life, i.e., that relatively small changes in the laws of physics would preclude the development of observers [5, 6, 13]. However, the definition of what constitutes fine-tuning is not the same for all authors and remains unsettled (a detailed discussion is given in [14]).

One basic step toward a resolution of the aforementioned issues is to determine what values of the fundamental constants allow for the existence of astrophysical structures, such as planets, stars, and galaxies. The goal of this paper is to provide a partial answer. Specifically, we consider constraints placed on a subset of the fundamental constants by requiring that a universe support the existence of both working stars and habitable planets. Parts of this issue have been addressed previously using a variety of approaches [5–10, 13]. In addition, previous work has considered the formation and structure of galaxies in relation to habitable planets [15, 16]. As a rule, however, these previous papers adopt a general approach, e.g., by considering a wide range of parameter space and relying on order of magnitude arguments. In contrast, this paper uses solutions to the equations of stellar structure (following the formalism of [17]) to estimate the range of allowed stellar masses, stellar lifetimes, and stellar surface temperatures across the range of parameter space. This paper thus extends previous work by presenting a more detailed treatment of stellar structure. Finally, we note that the consideration of universes with different values of the fundamental constants is related to the problem of time variations of the constants in our universe [18].

It will be useful to work in terms of the structure constants defined through the relations

\[ \alpha = \frac{e^2}{\hbar c} \quad \text{and} \quad \alpha_G = \frac{G m_p^2}{\hbar c}, \]

(1.1)

where \( m_p \) is the proton mass. In our universe, with standard values of the fundamental constants, these parameters have the values \( \alpha \approx 1/137 \) and \( \alpha_G \approx 5.91 \times 10^{-39} \). In this paper, we consider a parameter space in which these structure constants are allowed to vary by ten orders of magnitude in either direction.

The weakness of gravity relative to the other forces (small values of \( \alpha_G/\alpha \)) is an important aspect of the hierarchy problem in particle physics, where it is more natural for the strengths of the forces to be comparable. On the other hand, a small value of the ratio \( \alpha_G/\alpha \) is necessary for stars to exist [17], and for stars to have sufficiently long lifetimes and hot surface temperatures (see section 2). Additional constraints on the ratio \( \alpha_G/\alpha \) arise from the ordering of mass scales of planets, stars, and galaxies (as emphasized in earlier work [5, 19], and constrained further in section 3). This work thus shows that habitable universes must display an enormous hierarchy of force strengths.

For future reference, we also define the fundamental stellar mass scale \( M_0 \) according to

\[ M_0 \equiv \alpha_G^{-3/2} m_p \left( \frac{\hbar c}{G} \right)^{3/2} \approx 3.7 \times 10^{33} \text{g} \approx 1.85 M_\odot, \]

(1.2)

where the numerical values correspond to our universe. Stellar masses are roughly comparable to this benchmark scale, in our universe [20] and others [17]. Note that the mass scale \( M_0 \) is equivalent to the Chandrasekhar mass [21] with all of the numerical constants set to unity.

Constraints on habitable planets can be divided into two conceptually different categories. The first of class involves the stellar properties associated with habitable planets. In addition to the need for viable stellar structure solutions (working stars), we also require that
the host stars live long enough for biological evolution to occur and have surface temperatures high enough to drive chemical reactions. Note that the relevant time scales and energy levels depend on the fine structure constant \( \alpha \). These stellar constraints are considered in section 2. The second class of constraints concerns the relative ordering of the mass scales involved in producing and maintaining habitable planets. Some of these considerations involve properties of the planets themselves, including the requirement that the planets are massive enough to support a biosphere and to retain an atmosphere, but not so massive as to become degenerate. In addition, we require that planets are smaller than their host stars and that stars are smaller than their host galaxies. These issues are addressed in section 3. The paper concludes in section 4 with a summary of the results and a discussion of their implications. In order for a universe to have working stars and habitable planets, the structure constants cannot vary by more than a few orders of magnitude from their measured values.

2 Constraints from stellar properties

2.1 Stellar structure solutions and the existence of stars

To evaluate habitability constraints that depend on stellar properties, we need a working model for stars in other universes (with different values for the fundamental constants). Toward this end, we use the semi-analytical model of [17]. This treatment solves the standard equations of stellar structure [20–23], but makes a number of simplifying assumptions in order to obtain semi-analytic results: first, the physical structure of the star is taken to be that of a polytrope (with index denoted as \( n \)). Another simplification is that only a single chain of nuclear reactions, characterized by a single nuclear reaction rate, is considered. The resulting model reproduces stellar properties in our universe (including luminosity \( L^* \), temperature \( T^* \), and radius \( R^* \)) to within a factor of \( \sim 2 \), while the stellar mass \( M^* \) varies by a factor of \( \sim 1000 \) and the luminosity varies by a factor of \( \sim 10^{10} \). Although approximate, the resulting stellar structure model is robust enough to provide solutions across a parameter space where \( \alpha \) and \( \alpha_G \) vary by ten orders of magnitude in either direction from their values in our universe.

In this model, the central temperature of the star is given by the solution to the equation

\[
\Theta_c I(\Theta_c) T_c^3 = \frac{(4\pi)^3 a c}{3 \beta \mu_0 C} \left( \frac{M^*}{\mu_0} \right)^4 \left( \frac{G}{(n+1)R} \right)^7,
\]

(2.1)

where \( \Theta_c \) is related to the central temperature \( T_c \) through the expression

\[
\Theta_c = \left( \frac{E_G}{4kT_c} \right)^{1/3} \quad \text{where} \quad E_G = \pi^2 \alpha^2 m_p c^2,
\]

(2.2)

and where the Gamow energy \( E_G \approx 493 \text{keV} \) for hydrogen fusion in our universe. In equation (2.1), \( \mu_0 \) and \( \beta \) are dimensionless parameters of order unity; they are determined by the mass and luminosity integrals over the structure of the star, as characterized by the polytropic index \( n \). The parameter \( R \) is the gas constant that appears in the ideal gas law, and \( \kappa_0 \) is the benchmark value of the stellar opacity. Finally, the composite parameter \( C \) determines the nuclear reaction rate. Note that \( C \) depends on the mass of the reacting particles, their charges, the mean energy generated per nuclear reaction, and the ratio of the overall coefficient of the nuclear cross section to the fine structure constant (see [17]).

\footnote{The expression for the Gamow energy \( E_G \) assumes equal mass reacting particles with unit charge.}
the sake of definiteness, we assume here that $C$ is constant, while $\alpha$ and $\alpha_G$ are allowed to vary; additional variations should be considered in future work.

The function $I(\Theta_c)$ is defined by the integral of the luminosity density over the stellar volume, i.e.,

$$I(\Theta_c) = \int_0^{\xi_c} \xi^2 f^2 \Theta^2 \exp[-3\Theta] \, d\xi,$$

where $\Theta = \Theta_c f^{-1/3}(\xi)$, and where $f(\xi)$ is the solution to the Lane-Emden equation [20–23]. The function $I(\Theta_c)$ can be approximated by a fitting function of the form

$$\Theta_c I(\Theta_c) = B \Theta_c^b \exp[-3\Theta_c].$$

For polytropic index $n = 3/2$, the fitting parameters have values $B = 0.833$ and $b = 2.30$.\footnote{We can also use the full numerical solution to the integral in equation (2.3), but the results are the same.}

The solution for the central temperature can be written in the alternate form

$$I(\Theta_c)\Theta_c^{-8} = \frac{2^{12}\pi^5}{45} \frac{1}{\beta \kappa_0 C E G h^3 \mu_0} \left( \frac{M_*}{\mu_0} \right)^4 \left( \frac{G \langle m \rangle}{n+1} \right)^7 ,$$

where $\langle m \rangle$ is the mean mass of the particles that make up the star. Note that the right hand side of the equation is dimensionless. For the typical parameter values in our universe, the right hand side of this equation has a value of approximately $10^{-9}$.

With the central temperature $\Theta_c$ determined through equation (2.5), the equations of stellar structure specify the remaining the properties of the star. The stellar radius $R_*$ is given by

$$R_* = \frac{G M_* \langle m \rangle}{k T_c} \frac{\xi_*}{(n+1)\mu_0},$$

where $\xi_*$ is the dimensionless radius of the star (and is of order unity). The stellar luminosity $L_*$ takes the form

$$L_* = \frac{16\pi^4}{15} \frac{1}{h^3 c^2 \beta \kappa_0 \Theta_c} \left( \frac{M_*}{\mu_0} \right)^3 \left( \frac{G \langle m \rangle}{n+1} \right)^4 .$$

The photospheric temperature $T_*$ of the star is then determined from the outer boundary condition so that

$$T_* = \left( \frac{L_*}{4\pi R_*^2 \sigma_{sb}} \right)^{1/4},$$

where $\sigma_{sb}$ is the Stefan-Boltzmann constant.

2.2 Minimum stellar temperatures

The surface temperature $T_P$ of a planet is determined by balancing the heating from the central star and the radiated heat of the planet. Using the simplest treatment we obtain

$$\sigma_{sb} T_P^4 = f_T \frac{L_*}{16\pi d^2},$$

where $f_T$ is an efficiency factor that takes into account both the radiation reflected away from the planet and the heat retained by the atmosphere. Here we assume that the planetary orbit is circular with radius $d$. The temperature $T_B$ required to drive chemical reactions, and hence support biological operations, is derived in section 3.2; following equation (3.9), we
write this temperature in the form $kT_B = \epsilon \alpha^2 m_e c^2$, where the efficiency factor $\epsilon \sim 10^{-3}$ for terrestrial chemical reactions. If we then require that the planet is warm enough to support life, $T_P \geq T_B$, and use the fact that the orbital radius must exceed the stellar radius, $d \geq R_*$, we obtain the constraint
\[ \frac{L_*}{R_*^2} \gtrsim \frac{16\pi \sigma_{sh} \left( \frac{\epsilon \alpha^2 m_e c^2}{k} \right)^4}{f_T}. \] (2.10)

If we scale to values in our universe, where $T_B \sim 300 \text{ K}$, we obtain the requirement
\[ \frac{L_*}{R_*^2} \gtrsim 2.3 \times 10^7 \text{erg sec}^{-1} \text{cm}^{-2} \left( \frac{\alpha}{\alpha_0} \right)^8. \] (2.11)

Given the solutions for the stellar luminosity and stellar radius found in the previous subsection, the ratio $L_* / R_*^2$ is given by
\[ L_* / R_*^2 = \frac{16\pi^4}{15} \frac{1}{\hbar c^2 \beta \kappa_\infty \Omega_c} \left( \frac{M_*}{\mu_0} \right) \frac{\left( G \langle m \rangle \right)^2}{n + 1} \left( \frac{kT_c}{\xi_c} \right)^2. \] (2.12)

The right hand side of this equation is an increasing function of stellar mass. In order to satisfy the constraint for habitability, we require that the ratio $L_* / R_*^2$ is larger than the lower bound given in equation (2.10). A necessary condition is thus that the maximum value of the ratio $L_* / R_*^2$ must be larger than this lower bound, which implies that the following constraint must be met
\[ \frac{\pi^4}{15} \frac{1}{\hbar^3 c^2 \beta \kappa_0 \Theta_c} \left( \frac{M_*}{\mu_0} \right)_{\text{max}} \frac{\left( G \langle m \rangle \right)^2}{n + 1} \left( \frac{E_G}{\xi_c} \right)^2 > \frac{16\pi \sigma_{sh}}{f_T} \left( \frac{\epsilon \alpha^2 m_e c^2}{k} \right)^4. \] (2.13)

To move forward, we need to determine the maximum stellar mass for a given set of fundamental constants. As the mass of a star increases, the fraction of its internal pressure that is provided by radiation pressure (instead of gas pressure) increases. Let $f_g$ denote the fraction of the pressure provided by the ideal gas law, so that $(1 - f_g)$ is the fraction provided by radiation. The star becomes unstable when the radiation pressure dominates \cite{20}; here we equate the maximum stellar mass with that for which the fraction has a critical value $f_g \approx 1/2$. The maximum stellar mass is then given by the expression
\[ M_{\text{max}} = \left( \frac{18\sqrt{5}}{\pi^{3/2}} \right) \left( \frac{1 - f_g}{f_g^4} \right)^{1/2} \left( \frac{m_p}{\langle m \rangle} \right)^2 M_0 \approx 50 M_0, \] (2.14)

where $M_0$ is the fundamental stellar mass scale defined by equation (1.2).

Next we want to substitute the maximum mass scale (equation (2.14)) into the stellar structure solution for the central temperature (from equation (2.5)),
\[ I(\Theta_c) \Theta_c^{-8} = \frac{2^{12} \pi^5}{45} \frac{\hbar c^4}{\beta \kappa_0 \epsilon \beta c^4 \left( m_p \right)} \frac{\left( G \langle m \rangle \right)^7}{(n + 1)^2} \frac{\left( 50 \right)^4}{\mu_0^4} \left( \frac{G \langle m \rangle}{n + 1} \right)^2 m_p^{-8}, \] (2.15)

as well as the constraint on the planetary surface temperature (from equation (2.13)),
\[ \frac{\pi^3}{15} \frac{1}{\hbar \beta \kappa_0 \Theta_c} \left( \frac{G}{\hbar c} \right)^{1/2} \left( \frac{50 \langle m \rangle}{\mu_0} \right)^2 \left( \frac{G \langle m \rangle}{m_p (n + 1)} \right)^2 \left( \frac{E_G}{4 \xi_c} \right)^2 > \frac{\sigma_{sh}}{f_T} \left( \frac{\epsilon \alpha^2 m_e c^2}{k} \right)^4. \] (2.16)
Now we can simplify the expressions further. Let \( \langle m \rangle = m_p, \) \( n = 3/2, \) and use the definition of \( E_G, \) so that the central temperature is given by
\[
I(\Theta_c)\Theta_c^{-8} = \frac{2^{23}\pi^5}{9} \frac{\hbar^3 c^4}{\beta\kappa_0 C E_G} \frac{G}{\mu_0^4} m_p^{-1}
\]
and the constraint takes the form
\[
\frac{\pi^3}{30} \frac{E_G^2}{\hbar c} \left( \frac{G}{\kappa_0 C} \right)^{1/2} \left( \frac{1}{\beta\mu_0^2} \right) > \frac{\sigma_{eb}}{f} \left( \frac{e\alpha^2 m_e c^2}{k} \right)^4.
\]
This constraint on the fundamental constants is required for stars to have surface temperatures hot enough to support viable biospheres.

2.3 Minimum stellar lifetimes

For a universe to be habitable, at least some of its stars must live long enough for biological evolution to take place. Because the lowest mass stars live the longest, so we can derive a constraint on the fundamental parameters by considering the smallest possible stars. Previous work \([17, 20]\) shows that the minimum mass necessary to sustain nuclear fusion can be written in the form
\[
M_{\ast\text{min}} = 6(3\pi)^{1/2} \left( \frac{4}{5} \right)^{3/4} \left( \frac{kT_{\text{nuc}}}{m_e c^2} \right)^{3/4} M_0,
\]
where \( M_0 \) is the fundamental stellar mass scale given by equation (1.2). If we invert equation (2.19), it determines the maximum temperature \( T_{\text{nuc}} \) that can be obtained with a star of a given mass, where this temperature is an increasing function of stellar mass. By using the minimum stellar mass from equation (2.19) to specify the mass in equation (2.5), we obtain the minimum value of the stellar ignition temperature. This central temperature, or equivalently the value of \( \Theta_c, \) is determined by solving the following equation
\[
\Theta_c I(\Theta_c) = \left( \frac{2^{23}\pi^7 3^4}{5^{11}} \right) \left( \frac{\hbar^3}{c^2} \right) \left( \frac{1}{\beta\mu_0^4} \right) \left( \frac{1}{m_p m_e^3} \right) \left( \frac{G}{\kappa_0 C} \right).
\]
The parameters on the right hand side of the equation have been grouped to include pure numbers, constants that set units, dimensionless quantities from the polytropic solution, particle masses, and finally the stellar parameters that depend on the fundamental constants. In the context of this paper, these latter quantities can vary from universe to universe. Note that this expression has been simplified by setting \( \langle m \rangle = m_{\text{ion}} = m_p \) and by using the polytropic index \( n = 3/2, \) allowing other choices for the particle masses and the polytropic index leads to the right hand side of equation (2.20) changing by a factor of order unity, whereas we vary \((G/\kappa_0)\) (equivalently, \( \alpha \) and \( \alpha_G \)) by many orders of magnitude.

The stellar lifetime \( t_\ast \) can be written in the form
\[
t_\ast = \frac{f_c E M_\ast c^2}{L_\ast} = \frac{9375}{256\pi^4} f_c \frac{\epsilon}{\kappa_0^4} \frac{\beta^3 m_p^3 \Theta_c M_\ast^{-2} (G\langle m \rangle)^{-4}}{265\pi^4 f_c \hbar^3 c^4 \beta^3 \mu_0^3 \kappa_0 \Theta_c M_\ast^{-2} (G\langle m \rangle)^{-4}},
\]
where \( f_c \) is the fraction of the stellar fuel that is available for fusion and \( \epsilon \) is the efficiency of nuclear fuel conversion (where \( \epsilon \approx 0.007 \) in our universe). Solar-type stars have access to a fraction \( f_c \approx 0.1 \) of their nuclear fuel during their main-sequence phase, whereas smaller stars have larger \( f_c \) \([24, 25]\).
We want to compare the stellar lifetime to the time scale for atomic reactions, where this latter quantity is given by
\[ t_A = \frac{\hbar}{\alpha^2 m_e c^2}. \] (2.22)

In our universe, this atomic time scale has the value \( t_A \sim 2 \times 10^{-17} \). In comparison, in order for biological evolution to develop complex life forms (observers) on Earth, the required time scale was of order 1 Gyr, which corresponds to \( \sim 10^{33} \) ticks of the atomic clock. Unfortunately, we currently have a sample size of one for the specification of the time required for biological evolution; we are thus left with enormous uncertainty. Suppose, for example, that the time necessary for the development of life has a wide distribution. In this case, it could be possible that (i) the probability of life originating within 1 Gyr could be low, but that (ii) the minimum time required for life to develop (anywhere) could sometimes be much less than 1 Gyr [26, 27]. As a result, the fiducial time scale of 1 Gyr, while appropriate for life on Earth, does not represent a definitive limit. Given these uncertainties, we consider a range of values, but use \( 10^{33} \) atomic time scales as the center of the allowed range (see below).

In general, the ratio of the stellar lifetime to the atomic time scale takes the form
\[ \frac{t_\ast}{t_A} = \frac{9375}{256 \pi^4} f_c \mathcal{E} h^2 c^6 \beta \mu_0^3 \alpha^2 m_e \Theta_c M_\ast^{-2} (G\langle m \rangle)^{-4}. \] (2.23)

In other universes, the largest possible value of this ratio, corresponding to the smallest, long-lived stars, is thus given by
\[ \left( \frac{t_\ast}{t_A} \right)_{\text{max}} = \left( \frac{5^{13/2}}{9 \pi^8 2^{10}} \right) \left( \frac{c^4}{\hbar} \right) \left( f_c \mathcal{E} \beta \mu_0^3 \right) \left( \frac{m_e^{5/2} m_p^{5/2}}{(m)^4} \right) \left( \frac{\kappa_0}{G \alpha} \right) \Theta_c^{1/2}, \] (2.24)

where we have grouped the various factors as before. Note that equation (2.24), as written, depends on the temperature parameter \( \Theta_c \), which is specified via equation (2.20). We can thus combine equations (2.20) and (2.24) to solve for the ratio of time scales, and set it equal to the minimum required for life to develop (here we use \( t_\ast/t_A > 10^{33} \) as described above).

### 2.4 Summary of constraints from stellar structure

The results of this section are summarized in figure 1, which shows the allowed plane of parameter space for the structure constants \( \alpha \) and \( \alpha_G \). The location of our universe in the diagram is marked by the star symbol. As outlined below, the figure includes the constraints on parameter space determined from considerations of stellar structure (compare with figure 2, which shows the analogous constraints from planetary considerations).

The first requirement is that working stars exist. The area below the black curve represents the region for which long-lived stable stellar configurations can sustain nuclear fusion. Note that stars can fail to exist for two conceptually different reasons: if the fine-structure constant \( \alpha \) is too large, then nuclear reactions are suppressed and stars fail to generate nuclear power. When this condition occurs, the minimum stellar mass (from equation (2.19)) becomes larger than the maximum stellar mass (from equation (2.14)). On the other hand, if \( \alpha \) is too small, then stable stellar configurations cannot exist (equation (2.5) has no solution). Further, both of the aforementioned constraints depend on \( \alpha_G \). For sufficiently strong gravity (large \( \alpha_G \)), the range of \( \alpha \) that supports working stars shrinks to zero.

We also require that the stellar photospheric temperature (equation (2.8)) is larger than the temperature required for a working biosphere (equation (3.9)). This condition
Figure 1. Allowed plane of parameter space for varying structure constants $\alpha$ and $\alpha_G$, subject to constraints on stellar properties. Both parameters are scaled to the values in our universe. The region under the solid black curve delineates the parameter space that allows stable long-lived stars to exist (from [17]). In order to host habitable planets, stars must have surface temperatures higher than the value required for chemical reactions, where the viable parameter space falls to the upper left of the solid blue line. Stars must also live long enough to allow for biological evolution. The viable parameter space falls to the lower right of the red curves, which are plotted for the required number of atomic time scales varying from $10^{32}$ (top) to $10^{34}$ (bottom). The star symbol denotes the position of our universe in the diagram.

(see section 2.2) is marked by the steeply rising blue line in figure 1. The viable regime of parameter space falls to the (upper) left of the line. To obtain this particular curve, we require the surface temperature of the star to be larger than the benchmark value $T_B = 300$ K $(\alpha/\alpha_0)^2$. Although we expect this scaling with $\alpha$ to hold, the exact value of the coefficient for $T_B$ is not known, so that we should consider a range of possible temperatures. In practice, however, the constraint has such sensitive dependence on $T_B$ that the effect of including a range of values only serves to add width to the line shown in figure 1.

Next we require that the stars live enough enough to allow for biological evolution to take place. To invoke this constraint, we start with the requirement that stars live for at least 1 Gyr in our universe, where this constraint corresponds to $10^{33}$ atomic time scales (allowing for the variation in chemical time scales due to varying $\alpha$). Since the number of required atomic time scales is not known, we consider time scales both larger and smaller by an order of magnitude. The resulting three curves are shown in red in figure 1, where the allowed region of parameter space falls below the curves.
Each point of parameter space allows for a range of stellar masses. To determine the constraints shown in figure 1, we have used the minimum stellar mass to evaluate the lifetime constraint and the maximum stellar mass to evaluate the temperature constraint. One might worry that the region of allowed parameter space could be smaller: the long-lived small stars might not have high enough surface temperatures to maintain biospheres and/or the hot large stars might not live long enough to allow for biological evolution. However, numerical exploration shows that this complication does not reduce the allowed region of parameter space. On the right side of the plane, large values of \( \alpha \) lead to high mass stars becoming too cool; the same (large) values of \( \alpha \) allow the stellar lifetimes to be long enough. On the left side of the plane, small values of \( \alpha \) lead to low-mass stars burning too quickly; these same (small) values of \( \alpha \) allow for sufficiently hot stellar photospheres.

The remaining region of allowed parameter space shown in figure 1 is relatively large. More specifically, if we fix the gravitational constant \( \alpha_G \) to the value in our universe, the allowed range for the fine structure constant \( \alpha \) spans about an order of magnitude in either direction. The constraints from stellar lifetimes and stellar temperatures are thus significant: if we only require working stars, the range of \( \alpha \) (black curve) extends roughly two orders of magnitude in either direction. The allowed range in \( \alpha \) becomes much wider for weaker gravity, but disappears altogether if gravity is stronger by a factor of \( \sim 1000 \) (compared to our universe). In addition, the allowed range of \( \alpha_G \) is not bounded from below: as gravity becomes weaker, stars can still operate, but they require increasingly larger masses. At some sufficiently small value of \( \alpha_G \), we expect the required large stellar masses to become problematic (see section 3), but the issue is one of star formation (and galactic considerations) rather than stellar structure.

3 Constraints from ordering of mass scales

3.1 Maximum planet masses from degeneracy

A planet must be supported primarily by electromagnetic forces. As a result, the existence of planets requires that the electromagnetic self-energy of a body exceeds the energy due to self-gravity [28]. The gravitational energy is given by the expression

\[
E_g = -f_n \frac{GM_P^2}{R_P},
\]

where \( M_P \) is the planet mass and \( R_P \) is the planet radius. The dimensionless constant \( f_n \) is of order unity and depends on the density distribution within the planet. If we model the planet as a polytrope, then \( f_n = 3/(5 - n) \), where \( n \) is the polytropic index [21]. For an \( n = 1 \) polytrope, which provides a reasonably good model for large planets, we thus obtain \( f_n = 3/4 \). The electromagnetic energy is given by the expression

\[
E_{em} = N \frac{e^2}{\ell},
\]

where \( N \) is the number of atoms in the planet, \( e \) is the charge, and \( \ell \) is the effective distance between charges. On average, the distance \( \ell \) is given in terms of the mean number density,

\[
\ell = \langle n \rangle^{-1/3} \quad \text{where} \quad \langle n \rangle = \frac{3N}{4\pi R_P^3}.
\]
Combining the above equations gives us an approximate expression for the electromagnetic energy

\[ E_{\text{em}} = N^{4/3} e^2 \left( \frac{3}{4\pi} \right)^{1/3} R_P^{-1}. \]  

(3.4)

In order for the electromagnetic self-energy to exceed the gravitational energy, \( E_{\text{em}} > |E_g| \), the following constraint must be satisfied:

\[ N^{4/3} e^2 \left( \frac{3}{4\pi} \right)^{1/3} > f_n G M_P^2. \]  

(3.5)

Next we assume that the planet is made of a single type of atom of mass \( A m_p \) so that \( M_P = N A m_p \).

(3.6)

The constraint then simplifies to the form

\[ N < \left( \frac{e^3}{G^{3/2} A^3 m_p^3} \right) \left( \frac{3}{4\pi f_n^3} \right)^{1/2} = \left( \frac{\alpha}{A^2 A_G} \right)^{3/2} \left( \frac{3}{4\pi f_n^3} \right)^{1/2}. \]  

(3.7)

### 3.2 Minimum planet masses from atmosphere retention

In this section we derive a lower limit on planetary masses by requiring that the surface gravity is strong enough to hold on to an atmosphere [13, 29, 30]. In order for a planet to support chemical reactions, its surface temperature cannot be too small. Chemistry takes place on the scale of atoms, where the energy levels are given by

\[ E_n = -a \frac{m_e c^2 \alpha^2}{2 n_r^2}, \]  

(3.8)

where \( a = 1 \) for Hydrogen and \( n_r \) is the radial quantum number. The constraint on the surface temperature of the planet can then be written in the form

\[ kT > kT_B = \epsilon m_e c^2 \alpha^2, \]  

(3.9)

where the dimensionless parameter \( \epsilon \) incorporates any additional uncertainties. Based on terrestrial chemistry, we expect \( \epsilon \sim 0.001 \). In order for air molecules to remain bound to the planetary surface, so that the atmosphere does not evaporate quickly, the temperature must be less than the gravitational binding energy, i.e.,

\[ kT < \frac{G M_P \mu_a}{R_P}, \]  

(3.10)

where \( \mu_a = A_a m_p \) is the mass of an air molecule. Combining the two constraints from above, we find

\[ \epsilon m_e c^2 \alpha^2 < \frac{G M_P \mu_a}{R_P}. \]  

(3.11)

To go further we need to estimate the planetary radius. Here we assume that the atoms are close-packed and have radius given by the Bohr radius

\[ a_0 = \frac{\hbar}{m_e c \alpha}. \]  

(3.12)
The number of atoms in the planet is then given by

\[ N = \frac{R_P^3}{a_0^3} \quad \text{or} \quad R_P = N^{1/3} a_0 = \frac{N^{1/3} \hbar}{m_e c \alpha}. \quad (3.13) \]

Using this result in equation (3.11), we derive a lower bound on the number of atoms in the planet

\[ N > \left( \frac{\epsilon}{AA_a} \right)^{3/2} \left( \frac{\alpha}{\alpha_G} \right)^{3/2}. \quad (3.14) \]

Note that if we combine this lower bound on \( N \) with the upper bound derived in the previous section, we obtain the combined constraint

\[ \left( \frac{\epsilon}{AA_a} \right)^{3/2} \left( \frac{\alpha}{\alpha_G} \right)^{3/2} < N < \left( \frac{\alpha}{A^2 \alpha_G} \right)^{3/2} \left( \frac{3}{4 \pi f_n^2} \right)^{1/2}. \quad (3.15) \]

Since both sides of the expression depend on the structure constants in the same way, this constraint reduces to the form

\[ \epsilon < \frac{A_a}{A f_n} \left( \frac{3}{4 \pi} \right)^{1/3}. \quad (3.16) \]

We expect the right hand side to be of order unity, whereas \( \epsilon \sim 0.001 \), so that this constraint is generally satisfied.

### 3.3 Minimum planet masses from biosphere complexity

Another lower limit on planet masses arises from the requirement that planets must be large enough to support a biosphere. In approximate terms, life can be described as a physical process that requires information, and a certain minimum amount of information must be processed for a planet to support life [31]. In order to function, for example, a human being requires a minimum information of roughly \( Q_1 \approx 10^{23} \) bits, whereas the human species as a whole requires approximately \( Q_T \approx 10^{33} \) bits. A fully functioning biosphere is thus expected to have some minimum value \( Q_B \) (where we expect \( Q_B > Q_T \)). Our own biosphere is estimated to have a mass of 500 to 800 billion tons of carbon, which corresponds to about \( 4 \times 10^{40} \) particles. We thus write the minimum size of a biosphere in the form

\[ Q_B = f_B 10^{40} \text{bits}, \quad (3.17) \]

where the dimensionless parameter \( f_B \) encapsulates the uncertainties in this quantity. The mass of the planet should thus be large enough so that its information content far exceeds this benchmark value.\(^2\) As a result, the minimum number of particles \( N_{\text{min}} \) in a potentially habitable planet is expected to obey the ordering \( N_{\text{min}} \gg Q_B = f_B 10^{40} \). For the sake of definiteness, we define

\[ N_{\text{min}} \equiv f_{\text{bio}} 10^{40}, \quad (3.18) \]

where the dimensionless parameter \( f_{\text{bio}} \) is much larger than unity. For example, if the minimum size of a habitable planet is comparable to Earth, then \( f_{\text{bio}} \sim 10^{11} \). This requirement, in conjunction with equation (3.7), places a constraint on the structure constants,

\[ \frac{\alpha}{\alpha_G} > N_{\text{min}}^{2/3} A^2 f_n \left( \frac{4 \pi}{3} \right)^{1/3} \approx 5 \times 10^{28} f_{\text{bio}}^{2/3}, \quad (3.19) \]

\(^2\)Note that this number of particles corresponds to the information content of the entire structure of the biosphere. The blueprint required to reproduce the biosphere, as encoded in the DNA base-pairs in all living cells, would be significantly smaller.
or, alternately,
\[
\frac{\alpha G}{\alpha G_0} \lesssim 2.5 \times 10^7 \frac{\alpha}{\alpha_0} f_{bio}^{-2/3}.
\]
Keep in mind that \( f_{bio} \gg 1 \).

### 3.4 Considerations of stellar and galactic masses

In this section we consider the constraints that planets should be smaller in mass than their parental stars, and that stars are smaller in mass than their host galaxies. It is not known if this ordering of mass scales is strictly necessary for a universe to be habitable. If this ordering is violated, however, the planets (being more massive than stars) are likely to undergo nuclear fusion and thereby become uninhabitable. Moreover, the formation of stars (within galaxies) and the formation of planets (within the disks associated with forming stars) would be problematic.

As shown previously [17, 20], stars have typical mass scales given by
\[
M_0 = m_p \alpha^{-3/2} G
\]
(see also equation (1.2) and section 2.1), whereas the above considerations indicate that planets have mass scales
\[
M_0 P = m_p (\alpha/\alpha G)^{3/2}
\]
(equation (3.15); see also [13]). If we require that planets are less massive than their host stars, we obtain a constraint of the form
\[
\alpha \lesssim 1 \quad \text{or} \quad \frac{\alpha}{\alpha_0} \lesssim 137.
\]
(3.21)

In addition to being smaller in mass than its host star, the planet must have enough constituent particles to support a sufficiently complex biosphere (section 3.3). The coupled requirements that \( N_P > N_{\text{min}} \) (equations (3.18)–(3.20)) and \( \alpha < 1 \) (equation (3.21)) implies an upper limit on the gravitational constant, i.e.,
\[
N_P > N_{\text{min}} \quad \text{or} \quad \frac{\alpha G}{\alpha G_0} \lesssim 3.38 \times 10^9 f_{bio}^{-2/3},
\]
(3.22)
where the factor \( f_{bio} \sim 1 \) if the planet has the same number of particles as our biosphere and \( f_{bio} \sim 10^{11} \) if the planet has the same number of particles as Earth. Taken together, equations (3.21) and (3.22) imply that the structure constants \((\alpha, \alpha G)\) cannot be larger than those in our universe by more than a few orders of magnitude.

Another constraint is provided by the requirement that stars should be smaller in mass than their host galaxies. With the opposite ordering of mass scales, star formation would be difficult. Unfortunately, the masses of galaxies in other possible universes could vary enormously. Even within our universe, a number of different physical processes play a role in determining the masses of galaxies. Nonetheless, we can derive an expression for galactic masses, analogous to equation (1.2) for stars, through considerations of galaxy formation.

During the process of galaxy formation, the mass scale \( M_{eq} \) contained in the cosmological horizon at the time of matter domination plays an important role [33–35]. In brief, growth on all scales is suppressed earlier during the radiation dominated era, but perturbations with masses \( M < M_{eq} \) begin to grow after the epoch of matter domination. Perturbations on even larger mass scales \( M > M_{eq} \) can grow later, but their development is delayed, and their eventual growth could be compromised if the universe contains enough dark energy [9]. Although perturbations on all scales \( M < M_{eq} \) grow after the epoch of matter domination, several complications arise: the virialization time is somewhat shorter for the smaller dark matter halos, so they reach nonlinearity first; the smaller halos collide and merge in a complex tree of interactions; finally, growth is suppressed for the smallest mass scales below the
baryonic Jeans mass. In our universe, for example, these considerations conspire to produce a cumulative mass distribution of galactic halos that increases (slowly) up to a mass roughly comparable to $M_{\text{eq}}$ (e.g., see figure 3.2 from [36]). We can thus consider $M_{\text{eq}}$ as one proxy for galactic masses in other universes.

The temperature at the epoch of matter domination is given by

$$kT_{\text{eq}} = \eta (m_p c^2) \frac{\Omega_M}{\Omega_b},$$

(3.23)

where $\eta$ is the baryon-to-photon ratio, $\Omega_M$ is the relative energy density in dark matter, and $\Omega_b$ is the relative energy density in baryons (note that we have ignored the contribution of neutrinos in this simple treatment). The mass scale of the horizon at this epoch is thus given approximately by

$$M_{\text{eq}} \approx \left( \frac{5}{\pi} \right)^{1/2} \frac{3}{64 \pi} \alpha G^2 m_p \left( \frac{m_p c^2}{kT_{\text{eq}}} \right)^{2} \approx \frac{M_0}{64 \eta^2} \left( \frac{\Omega_b}{\Omega_M} \right)^{2},$$

(3.24)

where $M_0$ is the stellar mass scale from equation (1.2). In our universe, the baryon-to-photon ratio $\eta \sim 10^{-9}$, so that this mass scale is larger than the stellar mass scale by a factor of about $10^{15}$. In any case, the mass scale $M_{\text{eq}}$ scales linearly with $M_0$, where the coefficient depends on cosmological parameters ($\eta, \Omega_M, \Omega_b$), and not on the structure constants ($\alpha, \alpha_G$). As a result, the mass scale $M_{\text{eq}}$ will always be much greater than the stellar scale $M_0$ as long as $\eta \ll 1$, so that we do not obtain an additional constraint.

Although galactic halos can in principle form with mass scales $M \sim M_{\text{eq}}$, and with even larger masses at later epochs, the resulting structures will not always produce stars. In order for a galaxy to form stars, and thereby become habitable, the baryonic gas must cool on sufficiently rapid time scales [33–35]. The requirement that the gas cooling time is comparable to the free-fall collapse time for cosmological structures can be used to specify a characteristic mass scale for galaxies in terms of the structure parameters [34]. The result for $M_{\text{gal}}$ can be written in the form

$$M_{\text{gal}} = \alpha G^2 \left( \frac{m_p}{m_e} \right)^{1/2} m_p = M_0 \alpha G^{-1/2} \left( \frac{m_p}{m_e} \right)^{1/2},$$

(3.25)

where $M_0$ is the characteristic stellar mass. For our universe, the mass scale of equation (3.25) is larger than the stellar mass scale by a factor of $\sim 10^{10}$, which makes $M_{\text{gal}}$ comparable in mass to an ordinary galaxy. This mass scale is often used as characteristic galactic mass [10, 13]. However, one should keep in mind that galaxies span a wide range of masses, from about $10^7$ to $10^{13} M_\odot$, i.e., a range of a factor of $f_{\text{gal}} \sim 1000$ on either side of the scale given by equation (3.25). The constraint that stars are smaller in mass than their host galaxies thus takes the form

$$\alpha G \lesssim f_{\text{gal}}^2 \alpha G 10 \left( \frac{m_p}{m_e} \right) \quad \text{or} \quad \frac{\alpha G}{\alpha G_0} \lesssim f_{\text{gal}}^2 (1.33 \times 10^{20}) \left( \frac{\alpha}{\alpha_0} \right)^{10}.$$  

(3.26)

At first glance, this constraint might not seem restrictive. However, with the large exponent for $\alpha$, the fine structure constant can only decrease by a factor of $\sim 100$ before the relevant parameter space starts to shrink.
Figure 2. Allowed plane of parameter space for the structure constants $\alpha$ and $\alpha_G$, where both parameters are scaled to the values in our universe. The constraints shown here arise from the ordering of astrophysical mass scales: planets large enough to support a complex biosphere and small enough to be non-degenerate must fall below the diagonal magenta lines. For planets to be less massive than their host stars, the values of $\alpha$ must fall to the left of the vertical green lines. For stars to be less massive than their host galaxies, the value of $\alpha$ must lie to the right of the cyan lines. For planets to be less massive than their host stars and sufficiently complex to support a biosphere, the value of $\alpha_G$ must fall below the black horizontal dashed lines. Constraints are shown for a range of values (see text). The star symbol denotes the position of our universe in the diagram.

3.5 Summary of constraints from mass scales

The constraints of this section are summarized in figure 2, which shows the plane of parameter space for the structure constants $\alpha$ and $\alpha_G$ (scaled to the values in our universe). The location of our universe in the diagram is marked by the star symbol. This figure contains four types of boundaries to the parameter space, as outlined below.

We first invoke the constraint that planets must be large enough in mass to support a sufficiently complex biosphere and simultaneously small enough to remain non-degenerate (see equation (3.20)). Figure 2 shows three choices for the minimum size of a planet: the top diagonal magenta line corresponds to the requirement that the planet contains the information content of our biosphere; a viable planet must be at least as large as the biosphere it supports, so this boundary in parameter space is overly generous. The bottom diagonal magenta line corresponds to the requirement that the planet must be as large as the Earth (in particle number); since our planet is clearly large enough to support a biosphere, this
boundary is too restrictive. The minimum planet size must fall between the previous two choices. Given the large parameter space, and the uncertainties, a good benchmark value is provided by the geometric mean of the two previously defined scales. This estimate implies that the planet must be larger than a moon-sized body; the corresponding boundary is marked by the central diagonal magenta line in the figure.

Planets must also be smaller than their host stars, where this requirement implies that the fine structure constant $\alpha$ is bounded from above (equation (3.21)). These constraints are indicated by the vertical green lines for $\alpha < 1/10, 1,$ and $10$. By combining the previous constraints, we obtain the horizontal dashed lines near the top of figure 2: here we require that a potentially habitable planet must be less massive than its host star (so that $\alpha \lesssim 1$), but large enough to support a biosphere (see equation (3.22)). The three lines shown in the figure correspond to constraints requiring that the planet has the information content of our biosphere (top line), the entire Earth (bottom line), and the geometric mean (center line).

Similarly, stars must have smaller masses than their host galaxies. Here we require galactic structures to cool on sufficiently short time scales, so that they can form stars (see equation (3.26)). The two diagonal cyan lines in figure 2 define the resulting allowed range of parameter space (which lies to the right of the curves). Results are shown for two choices of the dimensionless parameter $f_{\text{gal}} = 1$ and 1000; these values correspond to maximum galactic masses in our universe of $\sim 10^{10} M_\odot$ and $\sim 10^{13} M_\odot$, respectively.

The surviving region of parameter space in the $\alpha$-$\alpha_G$ plane is relatively large: the width of the region corresponds to a range in $\alpha$ spanning 4 to 7 orders of magnitude. For the portion of parameter space shown in figure 2, the height of the allowed region corresponds to a range in gravitational constant, equivalently $\alpha_G$, spanning 8 to 10 orders of magnitude — but note that no lower limit on $\alpha_G$ exists. This allowed region of parameter space is "large" in the sense that both structure parameters can vary by many orders of magnitude. Nonetheless, several caveats should be kept in mind: although the parameter space is presented here in terms of the quantities $\log_{10}[\alpha]$ and $\log_{10}[\alpha_G]$, it is not known if the logarithm of $(\alpha, \alpha_G)$ represents the proper weighting. In fact, the underlying probability distributions for these structure parameters remain unknown. These (unspecified) probability distributions must be convolved with the allowed region of parameter space in order to make a full assessment of the likelihood of obtaining habitable systems.

4 Conclusions

4.1 Summary of results

This paper has developed a number of constraints that limit the parameter space for universes that support both working stars and habitable planets. Previous work shows that stars can exist over a reasonably wide portion of the $\alpha$-$\alpha_G$ plane of parameters [17]. For small values of $\alpha$, stars fail to exist because of the absence of stable nuclear burning configurations; for large values of $\alpha$, stars fail to exist because the allowed range of stellar masses shrinks to zero. This work presents a number of additional constraints that reduce the viable region of the $\alpha$-$\alpha_G$ plane, including considerations of stellar structure (section 2 and figure 1), and planetary properties (section 3 and figure 2). Using characteristic values to evaluate each constraint, figure 3 shows the remaining portion of the $\alpha$-$\alpha_G$ parameter space. The area below the heavy black curve represents the region for which working stars can exist. The other bounds can be summarized as follows:
Figure 3. Allowed plane of parameter space for the structure constants $\alpha$ and $\alpha_G$. The shaded region delineates the portion of the plane that remains after enforcing the constraints from this paper. The black curve shows the requirement that stable stellar configurations exist. The blue curve shows the requirement that the stellar temperature is high enough to allow habitable planets. The red curve shows the constraint that stars live long enough for biological evolution to occur ($10^{33}$ atomic time scales). For planets to be smaller than stars, the fine structure constant $\alpha$ must lie to the left of the vertical green line. For stars to be smaller than their host galaxies, $\alpha$ must fall to the right of the cyan curve. For planets to carry enough information content to support a biosphere, and remain non-degenerate, the parameters must fall below the diagonal magenta line.

The requirement that stars have a sufficiently long life span can be measured by the ratio of the stellar lifetime to the chemical (atomic) time scale. Biological evolution requires a minimum number of ticks of this atomic clock. Although the exact value remains unknown, we use $10^{33}$ as a working estimate, which corresponds to a time of $\sim 1$ Gyr in our universe; we also use the smallest (longest-lived) stars to evaluate the constraint. This requirement reduces the allowed range of parameter space by eliminating large values of $\alpha_G$ and small values of $\alpha$; the allowed region falls below the red curve in figure 3.

We also enforce the constraint that planets can maintain a habitable temperature. This condition is equivalent to requiring that the stellar surface temperature is higher than the temperature necessary to drive chemical reactions, i.e., the habitable zone must lie outside the star. Here we use the largest (hottest) stars to evaluate the bound. This constraint reduces the allowed parameter space by eliminating large values of $\alpha$. The constraint varies slowly with the strength of gravity and is more stringent for smaller values of $\alpha_G$; the allowed region of parameter space falls above (to the left of) the blue curve in figure 3.
Planets must be large enough in mass to hold onto an atmosphere and small enough in mass to remain non-degenerate. We have shown that these requirements scale with the fundamental constants in the same way and are generally satisfied. In addition, planets must be large enough, in particle number, to support a biosphere of sufficient complexity. If we require the biosphere to be as complex as that of our own, then the corresponding constraint is less restrictive than the requirement of working stars. Even if we require that the biosphere — and hence the planet — has the complexity of earth, very little of the allowed parameter space is eliminated. The geometric mean of these extremes in shown as the diagonal magenta line in figure 3. Although this consideration reduces the allowed parameter space, the need for sufficiently long stellar lifetimes provides a roughly parallel but stronger constraint.

The requirement that stars are more massive than planets implies an upper limit on the fine structure constant \( \alpha \lesssim 1 \) (equivalently, \( \alpha / \alpha_0 \lesssim 137 \)), as marked by the vertical green line in figure 3. This constraint is comparable to that required for stars to exist: if \( \alpha \) is too large, then electrostatic repulsion effectively shuts down nuclear reactions in stars. The requirement of a sufficiently high photospheric temperature (blue curve) provides an even stronger limit. We thus have three independent reasons to disfavor universes with large values of the fine structure constant. All three considerations require that \( \alpha / \alpha_0 \lesssim 100 \) or \( \alpha \lesssim 1 \).

Extremely weak gravity, corresponding to small values of \( \alpha G \), leads to extremely massive stars, essentially because stellar masses scale like \( M_0 \sim \alpha^{-3/2} G \). Additional constraints are provided by the requirement that stars cannot be too massive. In order for galaxies to form stars, gas must cool efficiently, and this requirement defines a characteristic mass scale \( M_{\text{gal}} \) for operational galaxies (equation (3.25)). If we take the maximum galactic mass to be \( 1000M_{\text{gal}} \), the constraint that stars are less massive than their host galaxies is delineated by the cyan curve in figure 3.

The combined constraints outlined above also provide global bounds on the structure parameters \( \alpha \) and \( \alpha G \). Specifically, the allowed region shown in figure 3 indicates that \( \alpha / \alpha_0 \lesssim 37 \) and \( \alpha G / \alpha G_0 \lesssim 3700 \). A related quantity of interest is an upper bound on the ratio \( \alpha G / \alpha \) of the structure parameters. For the viable parameter space shown in figure 3, this ratio has a maximum value given by

\[
\frac{\alpha G / \alpha G_0}{\alpha / \alpha_0} \lesssim 130 \quad \text{or} \quad \frac{\alpha G}{\alpha} \lesssim 10^{-34}.
\]

This result shows that any universe with working stars and habitable planets must have a large hierarchy between the strengths of its forces, i.e., the values of \( \alpha \) and \( \alpha G \).

The bounds on the structure parameters described above were derived from a numerical evaluation of the coupled constraints of this paper (see also [5, 19]). For completeness, we present a simpler analytic approach to this problem in appendix A. The resulting bounds are somewhat less constraining than those found above, but they can be expressed in terms of simple analytic expressions. These bounds indicate that \( \alpha / \alpha_0 \lesssim 574, \alpha G / \alpha G_0 \lesssim 2 \times 10^6 \), and \( \alpha G / \alpha \lesssim 2 \times 10^{-31} \).

### 4.2 Discussion

Even with all of the constraints considered in this paper, the region of allowed parameter space for the structure constants is still relatively large. Specifically, if we work in terms of the \( \alpha - \alpha G \) plane using logarithmic units and the range shown in figures 1 and 2, then the region that allows for working stars corresponds to about one fourth of the original plane [17]. The additional constraints considered in this paper reduce the allowed region of the plane by another factor of \( \sim \)two. Keep in mind, however, that this work only delineates the region of
the plane that allows for both working stars and habitable planets (using a particular set of constraints). Whether or not this region is “small” or “large”, or if it provides evidence for or against fine-tuning, depends on unknown quantities: in particular, the proper measure of the parameter space requires knowledge of the underlying probability distribution from which universes select their values of the structure constants, in this case $\alpha$ and $\alpha_G$. Nonetheless, the constants can vary by significant factors (a few orders of magnitude) and still allow a universe to remain viable. We stress that additional constraints would lead to further reduction of the allowed parameter space: for a sufficiently large and restrictive set of constraints, the allowed parameter space must collapse to the neighborhood of the point representing our universe.

As shown in figure 3, this paper presents three upper bounds on $\alpha$: planets must be smaller than stars (green line), stars must burn nuclear fuel (black curve), and photospheres must be hot enough (blue curve). Two additional constraints can also be considered: first, in order for chemistry to operate properly, the fine structure constant must be bounded from above. If $\alpha$ is too large, then the innermost electrons in atoms become relativistic and are subject to capture. As a rough approximation, in order to get $N$ different chemical species, one needs $\alpha < 1/N$; otherwise, the universe in question would lose most of its periodic table [37]. Second, in order for the proton to exist an an allowed bound state of quarks, the fine structure constant is bounded from above according to $\alpha < (m_d - m_u)/141\text{MeV} \approx 1/56$, where $m_u$ and $m_d$ are the quark masses [6, 38]. These considerations thus provide two more reasons to disfavor universes with large $\alpha$. In other words, the fine structure constant is confined to the regime $\alpha \ll 1$, so that electromagnetism must be perturbative.

Another interesting result emerges from the considerations of this paper: the strongest limits on the existence of habitable planets arise from constraints on stellar properties (in particular, sufficiently long lifetimes and hot surface temperatures) rather than constraints on the properties of planets themselves. The bounds arising from planetary considerations, and ordering of mass scales, are roughly parallel to those arising from stellar considerations, but are somewhat weaker (see figure 3). This finding suggests that stars are the key element in determining the potential habitability of a universe.

On one hand, this paper extends previous constraints on the range of allowed values for the fundamental constants. On the other hand, the treatment is specific to the question of planetary habitability and the accompanying constraints on stellar properties. In the future, this work should be extended in several directions: although stars and planets can exist within the range of parameter space found here, we have not yet shown that such bodies are readily formed in these alternate universes. Small values of $\alpha$ lead to inefficient cooling, which can inhibit star formation; small values of $\alpha_G$ lead to large stellar masses, which also inhibit star formation unless galaxies are correspondingly larger. Although we have included simple considerations of galactic cooling (equation (3.25)), future work should place these formation constraints within a broader astrophysical context.

The results presented here are obtained using relatively simple models for stellar structure, as well as for planetary and galactic considerations. A more detailed treatment should be carried out in the future. The current stellar structure calculations are also limited to the hydrogen burning phase, i.e., we consider the fusion of only one nuclear species. The production of heavy elements, including carbon and oxygen, place additional constraints on the fundamental constants, and thereby narrow the allowed range of parameters.

Another line of inquiry is to consider additional possible variations of the fundamental constants. This paper has focused on $\alpha$ and $\alpha_G$, but the strength of the nuclear forces, the masses of the fundamental particles, and/or the cosmological parameters could also vary in
other universes \cite{5,6,9,11,39}. For example, if the strong force were sufficiently weak, no bound nuclei would exist, and no combination of the other constants would allow for long lived, stable, nuclear burning stars. On the other hand, in a complete theory — not yet available — variations in the fundamental parameters could be coupled, so that changes in one constant are not independent from changes in another. In the context of this paper, this latter constraint would imply that the allowed parameter space traces out a particular curve through the $\alpha - \alpha G$ plane shown in the figures. As one example, equation (54) of ref. \cite{12} suggests that $\alpha^{-1} \sim \log \alpha G^{-1}$, which would imply a nearly vertical path through the plane (see also \cite{40}). Notice also that the strengths of the weak and strong forces, and hence the composite nuclear parameter $C$, would also vary in such a coupled theory. In any case, however, the region of parameter space that allows for viable universes does not seem to be prohibitively small, and much more work must be done to delineate its boundaries.

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A Global bounds on the structure constants

This appendix derives upper bounds on the fine structure constant $\alpha$, the gravitational constant $G$ (equivalently $\alpha G$), and on the ratio of force strengths $\alpha G/\alpha$. The results from the main text (see figure 3) follow from numerical evaluation of all of the coupled constraints on the parameter space. The bounds presented here are derived from simple analytic considerations, but are weaker.

A.1 Definitions and dimensionless constraints

We start by writing all of the constraints in dimensionless form. The stellar mass can be written in terms of the fundamental stellar mass scale $M_0$ so that

$$M_* = XM_0 = X \alpha^{-3/2} \frac{G m_p}{c^2}. \quad (A.1)$$

The fine structure constant and the gravitational constant are written in terms of dimensionless factors according to

$$\alpha_{\text{univ}} \equiv a \alpha \quad \text{and} \quad G_{\text{univ}} \equiv g G, \quad (A.2)$$

where the un-subscripted quantities correspond to the values in our universe. Since we keep the particle masses constant, the second expression is equivalent to $\alpha_{G\text{univ}} = g \alpha G$.

The equation that sets the central temperature of the star necessary for a long-lived stable configuration can then be written in the form

$$I(\Theta_c) \Theta_c^{-8} = AX^4 g a^{-8}, \quad (A.3)$$

where the dimensionless constant $A$ is given by

$$A = \left( \frac{2^{19} \pi^5}{9 \cdot 3^8} \right) \left( \frac{1}{\beta \mu_0^4} \right) \left( \frac{\hbar^3 c^4}{E_G^2 m_p} \right) \left( \frac{G}{\kappa q C} \right) \approx 5.23 \times 10^{-9}. \quad (A.4)$$
The condition that stars have a minimum temperature can be written
\[ BXg^{1/2} > a^6\Theta_c^7, \]  
(A.5)
where the dimensionless constant \( B \) is given by
\[ B = \left( \frac{\pi}{25} \right) \left( \frac{1}{\beta\mu_0\xi^3} \right) \left( \frac{E_G^2}{\kappa_0} \right) \left( \frac{G}{hc} \right)^{1/2} \left( \frac{(hc)^2}{(e\alpha^2m_e^2)^4} \right)^{1/2} \approx 5.70 \times 10^{10}. \]  
(A.6)

The condition that stars have a sufficiently long lifetime takes the form
\[ Ca^4\Theta_c > gX^2, \]  
(A.7)
where the constant \( C \) is given by
\[ C = \left( \frac{9375}{256\pi^4} \right) \left( f_c\epsilon\beta\mu_0^3 \right) \left( \frac{m_e^3}{h} \right) \left( \frac{\kappa_0\alpha^2}{G} \right) \frac{1}{N_{\text{life}}} \approx 0.586, \]  
(A.8)
where \( N_{\text{life}} \) is the number of atomic time scales required for a functioning biosphere (and where we use \( N_{\text{life}} = 10^{33} \) to obtain the numerical value).

The maximum allowed value of the stellar mass defines a maximum value of the parameter \( X \), i.e.,
\[ X \leq X_{\text{max}} \approx 50. \]  
(A.9)

Finally, the minimum stellar mass can be written in terms of the minimum value of \( X \) such that
\[ X \geq X_{\text{min}} = Da^{3/2}\Theta_c^{-9/4}, \]  
(A.10)
where the constant \( D \) is defined by
\[ D = 6 (3\pi)^{1/2} \left( \frac{\pi^2 m_p}{5m_e} \right)^{3/4} \alpha^{3/2} \approx 5.36. \]  
(A.11)

### A.2 Upper bound on the fine structure constant

The bounds for a minimum stellar temperature (equation (A.5)) and a minimum stellar lifetime (equation (A.7)) can be combined and written in the form
\[ Ca^4\Theta_c > gX^2 > a^{12}\Theta_c^{14}B^{-2}. \]  
(A.12)

The outer parts of the combined inequality thus imply the constraint
\[ CB^2 > a^8\Theta_c^{13}. \]  
(A.13)

In order for the stellar structure equation (A.3) to have a valid solution, the temperature parameter \( \Theta_c \) is bounded from below, i.e.,
\[ \Theta_c > (\Theta_c)_{\text{min}} \approx 0.869. \]  
(A.14)

We thus obtain the bound
\[ a < (CB^2)^{1/8} (\Theta_c)_{\text{min}}^{-13/8} \approx 574. \]  
(A.15)
A.3 Upper bound on the gravitational constant and ratio of force strengths

Next we derive an upper limit on the gravitational structure constant $\alpha_G$ along with an upper limit on the ratio $\alpha_G/\alpha$. If we combine the stellar structure equation (A.3) with the minimum value of the stellar mass parameter from equation (A.10), we obtain the inequality

$$I(\Theta_c)\Theta_c^{-8} \geq AX_{\text{min}}^4ga^{-8} = AD^4a^6\Theta_c^{-9}ga^{-8},$$

which can be rewritten in the form

$$\Theta_c I(\Theta_c) \geq AD^4ga^{-2}.$$  \hspace{1cm}  (A.17)

We also require $X_{\text{min}} \leq X_{\text{max}}$, where this condition can be used to obtain a bound on $a$, i.e.,

$$a^2 \leq 50^{1/3}D^{-4/3}\Theta_c^3.$$  \hspace{1cm}  (A.18)

Combining the previous two equations then yields the inequality

$$\Theta_c I(\Theta_c) \geq AD^4ga^{-2} \geq AD^4g50^{-4/3}D^{1/3}\Theta_c^{-3},$$

which can be rewritten in the form

$$g \leq A^{-1}D^{-16/3}50^{2/3}\left[\Theta_c I(\Theta_c)\right]_{\text{max}} \approx 4.5 \times 10^6 \left[\Theta_c I(\Theta_c)\right]_{\text{max}}.$$  \hspace{1cm}  (A.20)

Note that we have replaced the value of the function $\Theta_c I(\Theta_c)$ with is maximum value.

Similarly, we can make an analogous argument to find a limit on the ratio $g/a$, which results in the upper bound

$$\frac{g}{a} \leq A^{-1}D^{-14/3}50^{-2/3}\left[\Theta_c^5 I(\Theta_c)\right]_{\text{max}} \approx 10^6 \left[\Theta_c^5 I(\Theta_c)\right]_{\text{max}}.$$  \hspace{1cm}  (A.21)

Using the definition (2.3) of the integral function $I(\Theta_c)$, we can find a bound on the function of the form

$$I(\Theta_c) < J_0 \Theta_c^2 \exp[-3\Theta_c],$$

where $J_0$ is given by the integral

$$J_0 = \int_0^{\xi^*} \xi^2 d\xi f^{2n-2/3},$$

where $f(\xi)$ is the solution to the Lane-Emden equation for polytropic index $n$. Note that we can also write the expression for $J_0$ in the form

$$J_0 = \int_0^{\xi^*} \xi^2 d\xi f^n \left[f^{n-2/3}\right].$$

As long as the polytropic index $n > 2/3$, the factor in square brackets is less than unity, whereas the remaining part of the expression is just $\mu_0$, so that we obtain the bound

$$J_0 < \mu_0.$$  \hspace{1cm}  (A.25)

Given the upper limit on $I(\Theta_c)$, we can find an upper limit on functions of the form

$$F(\Theta_c) = \Theta_c^k I(\Theta_c),$$

which is bounded by

$$F \leq F_{\text{max}} < \mu_0 \left(\frac{k+2}{3}\right)^{k+2} \exp[-(k+2)].$$  \hspace{1cm}  (A.27)

Using this result to evaluate the bounds of equations (A.20) and (A.21), we find the limits

$$g \lesssim 1.9 \times 10^6 \quad \text{and} \quad \frac{g}{a} \lesssim 1.9 \times 10^5.$$  \hspace{1cm}  (A.28)
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