Fast optical signal filtering by means of amplitude and phase operators

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Abstract. The article is devoted to a new numerical method of fast optical signal filtering. It is a unique mathematical apparatus of amplitude and phase operators, which has no analogues among the classical methods. The main advantages of the method for optical signal processing are computational simplicity, the versatility of the approach for solving various kinds of problems, and the availability of a geometric and physical interpretation of transformations. The amplitude and phase transformation algorithm, as well as its modifications, are presented in the article. It is shown that the method can be applied both to analog signals and to discrete measurements of a certain signal. The generalization of the operator to the two-dimensional case allows it to be applied to the processing of signals given on a rectangular grid. Proposed algorithm allows to achieve a significant acceleration during solving the problem of signal filtering compared with traditional fast Fourier transform.

1. Introduction

The problems of optical signal processing are relevant for most tasks of optics, such as development of additive systems, distorted signals filtering, processing of optical data streams, research of ultrafast nonlinear processes [1-3]. Fast algorithms are required to achieve high-speed signal processing functions that can operate at the real-time rate, including by means of distributed flows. Different hardware [4-7] and analytical methods [8-10] for solving the problem have been developed. Among the latter, the most common spectral methods based on Fourier transforms [11,12], wavelet transforms [13,14] and other ways of numerical approximation. A new analytical method based on amplitude and phase transformation of signal are presented.

Let us note the features of the proposed approach. The algorithm of extracting harmonics from signals considered below is based on an original mathematical apparatus, which has a number of advantages over the classical approach. In particular, this is the purely arithmetic nature of the method. The operator's action is geometrically and physically interpreted, there are no integral transformations and any other complicated operators, constructions. These conditions make it possible to implement the algorithm by means of any integrated development environment without additional connection of special libraries.
2. The amplitude and phase operator

In the original version [15], the author considered the action of the operator on a signal represented as a sum of trigonometric components

\[ T_n(t) = \tau_1 + \tau_2 + \ldots + \tau_n, \quad \tau_k(t) = a_k \cos kt + b_k \sin kt. \]

Due to the physical interpretation of \( T_n(t) \), it is assumed that the signal and all actions performed on it are real.

Let us define amplitude and phase operator (APO) as transformation of signal in the following way

\[
H_m(T_n, \{X_j\}, \{\lambda_j\}, t) = \sum_{j=1}^{m} X_j \cdot T_n(t - \lambda_j),
\]

where \( X_j, \lambda_j \) are real parameters of APO, positive integer \( m \leq n \) is order of the operator. The definition (1) shows, that APO is addition of finite number of signals, which are similar to initial one and each other, i.e. these similar signals are generated from \( T_n \) by modification its amplitude (multiplication by \( X_j \)) and initial phase (shift by \( \lambda_j \)). The example of signal transformation by the operator and the extraction of one harmonic is shown in figure 1.

Figure 1. The example of signal transformation by means of APO.

Signal filtering requires the extraction of one or the sum of low frequency harmonics up to order \( M \ll n \). Thus, it is necessary to to find such real parameters \( X_j, \lambda_j \) that (excluding constant):

\[
H_m(T_n, \{X_j\}, \{\lambda_j\}, t) = \sum_{k=1}^{M} \tau_{\mu_k}(t),
\]

Thus, the main goal in APO constructing is to find the real parameters of an operator, which, as will be shown below, is a very difficult problem. In addition, it is necessary to adapt the APO to discrete signals presented in the form of a series of measurements at equally spaced points in time.

3. Capabilities of the operator

Following results giving solution of the problem (2) were obtained in [16-18].

- Extraction of one harmonic \( \tau_\mu(t) \) from analog signal. In this case parameters are calculated by direct formulas for order \( n = s \mu - 1 \) with positive integer \( s > 1 \). This result has been generalized to the case of arbitrary order.

- Extraction of one harmonic from discrete signal. Let the signal defined on uniform grid of \( m \) nodes \( t_k \). Then exact values of harmonic \( \tau_\mu \) in points \( t_k \) are calculated by the formula:

\[
\tau_\mu(t_k) = \sum_{j=1}^{k} X_j \cdot T_n(t_k-j+1) + \sum_{j=k+1}^{m} X_j \cdot T_n(t_{m+k-j+1}).
\]
• Extraction of harmonic set. There is numerical algorithm for calculating parameters of problem (2) in cases $M > 1$. In this case, the set of extracted harmonics can be random (figure 2), and the result is obtained by a single APO action, that is, it is not required to extract each harmonic independently. For example, to extract harmonics up to order 10 with $n = 20$, the required APO parameters are given in table 1.

![Figure 2. The extraction of harmonic sets from signal by means of APO.](image)

Table 1. The parameters of APO required to extract harmonics up to order 10 with $n = 20$.

| $j$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|-----|----|----|----|----|----|----|----|----|----|----|
| $X_j$ | 0.77 | 0.77 | 0.77 | 0.77 | 0.77 | 0.77 | 0.77 | 0.77 | 0.77 | 0.77 |
| $j$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| $X'_j$ | 0.77 | 0.77 | 0.76 | 0.76 | 0.76 | 0.76 | 0.74 | 0.74 | 0.73 | 0.73 |
| $j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $\lambda_j$ | 2.98 | 2.98 | 2.69 | 2.69 | 2.39 | 2.39 | 2.1 | 2.1 | 1.79 | 1.79 |
| $j$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| $\lambda'_j$ | 1.5 | 1.5 | 1.2 | 1.2 | 0.91 | 0.91 | 0.6 | 0.6 | 0.34 | 0.34 |

• The above results are generalized to the case of a two-dimensional analog or discrete signal. This allows to process in a similar way, for example, digital images.

4. Conclusion
The main advantages of the AFO method for optical signal processing are computational simplicity, the versatility of the approach for solving various kinds of problems, and the availability of a geometric and physical interpretation of transformations. In addition, it should be noted about acceleration that can be achieved by means of APO in comparison with the classical apparatus of the Fourier transform.

Let us consider discrete signal filtering (smoothing by harmonic components) by means of fast Fourier transform (FFT) [19,20] and APO method and compare the number of required real-valued operations. Estimate of speed will be built for problem of smoothing by $n$ first harmonics from $N$-component signal, moreover, for noise filtering it is necessary $n \ll N$.

As for one-dimensional FFT and inverse FFT, total number of real-value calculations over $3N \log_2 N$ operations, since complex calculations arise during realization of FFT. In case of APO all of calculations
are real-value. Total number of operations is $n(2n - 1)$. For example, when $N = 512$, $n = 25$ APO solves the task 11 times faster than FFT.

A similar estimate for two-dimensional case is $6N^2 \log N$ operations for FFT realization and $2n^2(2n - 1)$ operations for APO algorithm. With the same $n, N$ APO completes the problem 231 times faster than FFT. Thus, the obtained results make the proposed approach attractive for different kinds of fast optical signal processing tasks.

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