Replication and Evolution of Quantum Species

Arun K. Pati
Institute of Physics, Sainik School Post,
Bhubaneswar-751005, India
and
1 School of Informatics,
University of Wales, Bangor LL 57 1UT, United Kingdom

April 1, 2022

Abstract

We dwell upon the physicist’s conception of ‘life’ since Schrödinger and Wigner through to the modern-day language of living systems in the light of quantum information. We discuss some basic features of a living system such as ordinary replication and evolution in terms of quantum bio-information. We also discuss the principle of no-culling of living replicas. We show that in a collection of identical species there can be no entanglement between one of the mutated copies and the rest of the species in a closed universe. Even though these discussions revolve around ‘artificial life’ they may still be applicable in real biological systems under suitable conditions.

1 Introduction

The desire to understand life in a dynamical manner is as old as modern civilization [1, 2, 3]. The question that has bothered some of the best minds of the last several centuries is: Can the dynamics of living systems be truly different to that of the inanimate objects? The later objects in the macroscopic world are described by Newtonian laws of motion which are completely deterministic. Although inanimate systems have order at the macroscopic level, they may nevertheless be disordered at the microscopic level. However, all living systems display orderliness from the cell level through to the level of DNA. As Schrödinger has beautifully paraphrased in his classic work: “In biology we are faced with an entirely different situation. A single group of atoms existing only in one copy produces orderly events, marvelously tuned in with each other and with the environment according to most subtle laws” [2]. Order in a system can arise in two ways, one possibility is ‘order from disorder’ and the other is ‘order from order’. How can such orderly events arise in living systems? Many would support the view that life emerges by ‘order from chaos’ and many others support that ‘order comes from order’. Schrödinger had firmly believed that the behavior of living systems is based on the ‘order-from-order’ principle.

But what mechanics governs living systems? Does it have to be classical mechanics or the ultimate theory of nature such as quantum mechanics? No one knows for sure, because no one knows what the word ‘life’ really means. There have been many attempts to define
life in a mathematical abstract manner [4]. But it is evidently clear that there is some special quality of a living system, which is called ‘life’ [2, 3, 4, 5]. All we know is certain features of living systems that distinguish them from non-living objects. For example, there is no doubt that a living system can replicate, evolve, and moreover it can self-reproduce, self-repair and so on, which are not generally seen in non-living systems. In non-living systems one can observe phenomena of replication and evolution. Replication of non-living objects (such as a piece of paper containing some information) or a living object (such as a cell) is a process where an input is fed through a suitable device and at the output one obtains two identical copies. Evolution of non-living or living systems involves dynamical interaction where the initial structure can evolve to a new structure consistent with physical laws.

There is growing thought among physicists [2, 6] and biologists [7] that quantum mechanics might play an important role in living systems. McFadden has the view that ultimately the origin of life will be answered by quantum mechanics [7]. To cite a few others, Frolich has suggested that the action of enzymes in biological systems would be understood from quantum mechanical principles [8]. Home and Chattopadhyaya have proposed a way to describe quantum mechanical measurement process in a DNA molecule, which may be in a superposition of mutational states [9]. McFadden and Al-Khalili have argued that during cell mutation quantum mechanics may play an important role and entanglement between the mutational state and the environment may enhance the probability of mutation [10]. Patel has proposed that a quantum algorithm may be at work in genetic evolution [11], though skepticism prevails on how quantum coherence can be maintained in a DNA molecule.

Notably, Wigner was baffled by the fact that “there are organisms with certain structures which, if brought into contact with certain nutrient materials multiply in order to produce identical structures to that of the original one” [12]. So he addressed the following question: Can laws of physics describe living matter? Especially, he had first asked the question of replication of quantum states, which in the language of quantum information is called ‘cloning’ of quantum states 1. However, he could not come to a doctrine such as the ‘no-cloning’ principle [13, 14]. He purports to argue that in quantum world it is infinitely unlikely that such a replicating machine could exist [12]. It may be remarked that the process of replication and self-replication are different games altogether. In the replication process any input is supposed to be duplicated — whereas, in self-replication, the state along with its program would be duplicated. That is, by furnishing a suitable description of the program, the machine will construct a copy of the input species as well as a copy of the instruction. In replication or cloning, one does not need to produce a copy of the program. The question of self-replication in a mechanical context was asked by von Neumann [15]. He had explicitly shown that in the classical world there exists a machine called a ‘universal constructor’. If it is provided with its complete specification then it can self-reproduce. Fifty years later, it was asked if one can design a quantum mechanical universal constructor that can self-reproduce any arbitrary state along with its program. It is found that there cannot exist an all-purpose universal quantum constructor in a closed universe with finite resource [16]. If one studies emergence of life from a point of view of game theory then interesting quantum effects can be incorporated. Flitney and Abbott have recently studied Conway’s game of life in a semi-quantum mechanical context [17].

In this paper, we discuss various features of a living system such as the process of

1Though the title of Wigner’s paper in [12] says that it addresses the question of a ‘self-reproducing’ unit, it actually addresses the question of ‘reproducing’ unit only.
replication, culling of identical replicas, and entanglement between mutated and the rest of the replicas from quantum information point of view. In Section 2, we discuss the original version of Wigner’s replication machine. In Section 3, we discuss how Wigner’s replication process could be ruled out simply using the linearity or unitarity of quantum theory. Also we will mention how one can have replication of quantum species states in a stochastic manner with a certain probability of success. In Section 4, we discuss the no-culling principle for living replicas. In Section 5, we will discuss a paradox of how entangling several copies of a living organism with one of its mutated versions nullifies the interaction. Then, we will show why it is not possible to entangle one copy with a mutated copy of a living state. Then our conclusion follows in the Section 6. It may be mentioned that we are not talking of ‘real’ biological systems in their full detail but rather some kind of ‘artificial’ living system in the quantum mechanical sense. The main motivation here is to discuss and contrast this virtual ‘world of quantum mechanical life’ with real biology. Though some ideas may still apply to real biological systems, whether one can apply all these ideas to real bio-systems will depend on various other factors, such as decoherence time at the molecular level.

2 Wigner’s Replication Machine

In the discussion of replicating machines Wigner assumed that there be a ‘living state’ $|\psi\rangle$ which is given in a quantum mechanical sense in a finite dimensional Hilbert space $\mathcal{H}^N$. This may store some ‘bio-information’ which may undergo the replication process. Further, there exists at least one nutrient state $|w\rangle \in \mathcal{H}^{NR}$ which will allow the organism to replicate onto it. Therefore, the initial state of the system, namely, the organism and nutrient is given by

$$|\Psi_i\rangle = |\psi\rangle |w\rangle.$$  

After the replication process the combined state would be given by

$$|\Psi_i\rangle \rightarrow |\Psi_f\rangle = U|\Psi_i\rangle = |\psi\rangle |\psi\rangle |r\rangle,$$  

where two organisms have been produced in the same state $|\psi\rangle$ and $|r\rangle$ would be the rejected part of the nutrient state that may belong to a Hilbert space of dimension $R$. The interaction is unitary, which means that there is some Hamiltonian underlying this process. It was assumed that the Hamiltonian that governs the behavior of such a complex organism would be a random symmetric matrix since nothing much can be known about it. If such a unitary interaction occurs between the organism and the nutrient, then in terms of its components (with respect to some chosen basis) in their respective Hilbert spaces, the replication process can be described by

$$\psi_k \psi_l r_{\mu} = \sum_{k' l' \mu'} \langle k l \mu | U | k' l' \mu' \rangle \psi_{k'} w_{l' \mu'}.$$  

The question Wigner posed is: Given an interaction, described by a unitary operator $U$, is it possible to find $N$ numbers $\psi_k$, which together with suitably chosen $R$ numbers $r_{\mu}$ and $NR$ numbers of $w_{l' \mu'}$ that satisfies (3)? He argued that since (3) must be valid for any $k, l$ and $\mu$, there are a total of $2N^2R$ real equations. On the other hand there are only $2(N + R + NR)$ real unknowns. Since the number of unknowns are very much fewer than the equations, it is not possible to satisfy all the equations simultaneously.
number of equations, proclaimed in his words, it would be a miracle if (3) would be satisfied. This was Wigner’s reasoning that it is infinitely unlikely that there can be a replicating machine for a living organism in the quantum mechanical sense.

In his thought provoking article, he went on providing further arguments to support his claim. However, he assumed something erroneous in between. He said that if $|k\rangle$ represents a ‘living state’, then any linear superposition of such basis living states would also represent a living state. This means that a state of the type $|\psi\rangle = \sum_k \psi_k |k\rangle$ would also be a replicating state. But now we know that it is precisely this linearity of superposition that prohibits the replication of a single quantum state [13, 14].

### 3 Replication of Quantum Species

In the language of quantum information theory, it is now well known that an arbitrary quantum state cannot be replicated—called the ‘no-cloning’ principle. This is due to the seminal work of Wootters-Zurek [13] and Dieks [14]. Let us see how Wigner’s replicating machine for a species can be ruled out simply based on the linearity of the quantum theory. Here a quantum species and organism will be used in the same sense which means possibly a smallest unit of a living system that represents either ‘artificial’ or ‘real’ life.

The question is can there be a process such that the initial state of the organism and nutrient given by

$$|\Psi_i\rangle = |\psi\rangle |w\rangle$$

(4)

can be transformed to

$$|\Psi_f\rangle = |\psi\rangle |\psi\rangle |r\rangle,$$

(5)

where $|r\rangle$ is rejected part of the nutrient state?

Suppose that there is a process where one can replicate orthogonal basis states

$$|k\rangle |w\rangle \rightarrow |k\rangle |k\rangle |r_k\rangle,$$

(6)

where $|r_k\rangle$ is the final nutrient state when the living state is $|k\rangle$. If we take an arbitrary state $|\psi\rangle = \sum_k \psi_k |k\rangle$ which is a linear superposition of orthogonal states, then by the linearity of quantum theory we will have

$$|\psi\rangle |w\rangle = \sum_k \psi_k |k\rangle |w\rangle \rightarrow \sum_k \psi_k |k\rangle |k\rangle |r_k\rangle.$$  

(7)

However, the actual replication of a living organism would be given by

$$|\psi\rangle |\psi\rangle |r\rangle = \sum_{kl} \psi_k \psi_l |k\rangle |l\rangle |r\rangle.$$  

(8)

Under no circumstance can (7) and (8) be identical. The simplest reason is, in general, that (7) is an entangled state of the original and the nutrient state, whereas (8) is not an entangled state. It can be seen from (7) that the individual state of the original or the replica is a mixed state (after tracing out the other two subsystems) whereas in the ideal case the original or the replica is a pure state. Therefore, we have failed to replicate an arbitrary living system in a quantum mechanical sense.
Also one cannot replicate any two non-orthogonal states by a unitary process [18]. Non-orthogonality of two states physically would mean that there is a finite overlap between them, i.e., they are not completely distinct. In the biological world this would probably mean two species described by non-orthogonal state have some similarity in their information content, behavior, or qualities. Let us see how to rule out Wigner’s replicating machine for two non-orthogonal states by a physical interaction (whether it is described by a random Hamiltonian or not does not really matter). Suppose it is possible to satisfy the following two equations for two species represented by two non-orthogonal states:

\[ |\psi_1\rangle|w\rangle \rightarrow |\psi_1\rangle|\psi_1\rangle|r_1\rangle \]
\[ |\psi_2\rangle|w\rangle \rightarrow |\psi_2\rangle|\psi_2\rangle|r_2\rangle. \tag{9} \]

Then it must preserve the inner product. This implies that we have

\[ \langle \psi_1 | \psi_2 \rangle = \langle \psi_1 | \psi_2 \rangle^2 \langle r_1 | r_2 \rangle. \tag{10} \]

But this can never be satisfied by a unitary process. This is so, because (10) implies that \(|\langle \psi_1 | \psi_2 \rangle| \leq |\langle \psi_1 | \psi_2 \rangle|^2\), using the condition that \(|\langle r_1 | r_2 \rangle| \leq 1\) which implies a contradiction. However, for two orthogonal species states (10) can hold, since we have \(\langle \psi_1 | \psi_2 \rangle = 0\). This is consistent with the understanding that we store classical information in the orthogonal states and the former can be perfectly copied so also the later.

Schrödinger, Wigner and probably others too have suspected that quantum mechanics would not permit the accurate replication of biological information. We now understand that it is indeed true. However, biological organisms do replicate information. The explanation for this might therefore be because (i) they transcend the laws of quantum world or (ii) biological systems operate in the classical regime, where accurate information replication is possible. The one that will strike most readers is the second explanation. This may be an appealing reason why we can clone a sheep or a pig (a macroscopic living object) but we cannot clone the fundamental building blocks that constitute them (such as an electron or proton) [19]. If a living organism is macroscopically large and distinguishable then it is like a classical state, and hence one can clone it perfectly. In this sense the quantum mechanical living states which have undergone decoherence due to interaction with the environment are useful for making perfect clones.

And if there is some error during replication then that will be very small. For example, for an\( E. coli\) cell the replication error is so low that, on average an error is made only once every thousand generations. One may ask whether this violates the no-cloning principle? The answer would be no, because if it really violates no-cloning then that would be seen in one generation, i.e., there should be error in each generation. However, if quantum superposition does prevail at the level of living organism, and they undergo replication process, then there is something beyond linearity of quantum mechanics that may be at the root. That would mean a modification of quantum mechanics to take into account nonlinearity and even the process of superluminal signaling in the realm of biological systems.

Another possibility for replication of living organisms would be through a cyclic evolution of the state. In the quantum world, if the initial state of a system is \(|\psi(0)\rangle = |k\rangle\) then it can evolve into a linear superposition of basis states such as \(\sum_k \psi_k(t) |k\rangle\) and after a certain time period \(t = T\) the state can come back to its original state, i.e., \(|\psi(T)\rangle = |\psi(0)\rangle = |k\rangle\), up to an overall phase. Suppose, the useful information (which is classical) is stored in
the state $|k\rangle$, then the state can be copied at times $t = 0, T, 2T \ldots$ and so on. However, at any time $0 < t < T$ the state is in an arbitrary superposition and hence cannot be replicated. In the same spirit, if the living organism would be in a quantum superposition of various possible states at intermediate times then it cannot undergo replication, but it can do so periodically. This is solely possible due to cyclic evolution of information from being ‘classical’ to ‘quantum’ and back.

It is now clear that no unitary transformation can accurately replicate non-orthogonal quantum states. So can non-unitary transformations such as the measurement process help in the replication process? The process which is ruled out by linearity alone is replication of linearly dependent living states (i.e. if we could clone $|k\rangle$ then we could not have cloned $|\psi\rangle = \sum_k \psi_k |k\rangle$). Linear transformations include unitary as well as measurement operations. So an arbitrary state cannot be cloned even by invoking a non-unitary transformation. But replication is possible for linearly independent non-orthogonal species states $\{|\psi_i\rangle\}$ with a certain probability of success [20]. Therefore, by invoking unitary and measurement processes together, it is possible to generate replicas in a probabilistic manner. The probabilistic replicator of Wigner, in the light of Duan-Guo [20] will consist of the original, nutrient and a measuring apparatus. The transformation would be given by

$$|\psi_i\rangle|w\rangle|P_0\rangle \rightarrow \sqrt{p_i}|\psi_i\rangle|r_i\rangle|P_1\rangle + \sqrt{1-p_i}|\Phi_i\rangle,$$

where $p_i$ is the probability of success that the replicator works, $|P_0\rangle$ and $|P_1\rangle$ are the initial and final probe states, and $|\Phi_i\rangle$ is some junk state of the combined living organism, nutrient and probe. Ideally, here, a junk state would mean a state that has no information about the original species. By performing a measurement on the probe state the rhs of (11) undergoes a collapse process. If the outcome is $|P_1\rangle$, then there are two replicas with a probability $p_i$. However, if the outcome is different than $|P_1\rangle$ then it is a failure. The success probability of replication process of two non-orthogonal species $|\psi_i\rangle$ and $|\psi_j\rangle$ is bounded by

$$p_{ij} = \frac{1}{2}(p_i + p_j) \leq \frac{1}{1 + |\langle \psi_i | \psi_j \rangle|}.$$ 

Obviously, this shows that the probability of replication cannot be a hundred percent. It is even possible to create a linear superposition of a different number of replicas of living states by similar stochastic processes [21] of which transformation (11) may be a special case.

4 Culling of Living Replicas

Suppose we have a collection of replicas of a living organism. Can there be a process which can remove a few of the replicas? i.e. take away the information content and make the living states some nutrient states for other species in the universe. Similar to the no-cloning ordinance in the quantum world, we have another principle called ‘no-deletion’ [22]. This tells us that if we are given two or more copies of a quantum object then it is impossible to delete the information of one copy by a physical transformation. Here the meaning of the word ‘deletion’ of a copy from two copies means designing a physical operation that transforms one of the copies to a blank state completely (analogous to a blank paper) which has no original information at all. In living systems, this would mean ‘culling’ of replicas of
a organism, which must be impossible. This would be called a ‘no-culling’ principle in the
quantum biological world. Let us imagine that we have two copies of a living organism
$|\psi\rangle|\psi\rangle$. The culling process will take two copies of the organism and an ancillary state and
transform them as

$$|\psi\rangle|\psi\rangle|r\rangle \rightarrow |\psi\rangle|w\rangle.$$  \hspace{1cm} (13)

However, by linearity of quantum mechanical evolution one can show that the above
process cannot exist. Consider the culling process of orthogonal living states

$$|k\rangle|k\rangle|r\rangle \rightarrow |k\rangle|w_k\rangle.$$  \hspace{1cm} (14)

If we have two copies of an arbitrary species $|\psi\rangle = \sum_k \psi_k|k\rangle$ then by the linearity of
quantum theory we have

$$|\psi\rangle|\psi\rangle|r\rangle = \sum_{kl} \psi_k\psi_l|k\rangle|l\rangle|r\rangle + \sum_{k\neq l} \psi_k\psi_l|k\rangle|l\rangle|r\rangle \rightarrow \sum_k \psi_k^2|k\rangle|w_k\rangle + \sum_{k\neq l} \psi_k\psi_l|\Phi_k\rangle.$$  \hspace{1cm} (15)

However, the resulting state given in (15) is not the one that we desire in the culling
process which is given in rhs of (13). In rhs of (13) we have $\sum_k \psi_k|k\rangle|w\rangle$. Hence, one cannot
perform culling of living replicas by a linear operation. The only option is to move the copy
to the ancillary state without culling it perfectly.

The ‘no-cloning’ and ‘no-culling’ theorems are based on the linearity of quantum me-
chanical evolution. Also based on unitarity only one can show that it is impossible to cull
a copy from two copies of non-orthogonal states. Recently, Jozsa has proved a much
stronger result known as the ‘stronger no-cloning’ theorem, based on full unitarity of the
evolution of system and ancilla. A simple statement is as follows: If $\{|\psi_k\rangle\}$ is a finite
collection of pure states, containing no orthogonal pairs of states and $|a_k\rangle$ be any other set
of states, then a physical operation taking

$$|\psi_k\rangle|a_k\rangle \rightarrow |\psi_k\rangle|\psi_k\rangle$$  \hspace{1cm} (16)
is possible if and only if there is a physical operation taking

$$|a_k\rangle \rightarrow |\psi_k\rangle.$$  \hspace{1cm} (17)

That is to say, in order to clone a quantum state one needs to supply full information in
the ancillary system. In his proof, the ‘other set of states’ could be a mixed state also. But

\footnote{In a private communication J. P. Dowling has suggested to S. L. Braunstein that the ‘no-deletion
principle’ may be called ‘no-culling principle’. After recalling this, now, I feel this term may suit better for
quantum mechanical living organisms.}
for general readers we have stated a simplified form of his theorem. Similarly, he has given
a simple proof of a quantum no-deletion principle for non-orthogonal states.

As Jozsa has emphasized, to make a replica one must supply a clone from somewhere
and to delete a replica one must move the information to somewhere. Hence, the ‘no-
replicating’ and ‘no-culling’ principles tell us that there is something robust about quantum
information. We can as well say that there is some permanence associated with quantum
information stored in a ‘artificial life’ system. In the light of these thoughts, it is very
important to realise that in a living organism we do not yet understand how the hereditary
information is passed from one generation to another without being altered. As Schrödinger
has put, there is some permanence in the hereditary information [2]. It may happen that if
the hereditary information is actually quantum mechanical in nature then the ‘no-culling’
principle may be at work in explaining some of these features. This may strike many
readers as something extraordinary, since the decoherence time scale is far less than one
second whereas evolutionary time scales are measured in thousand of generations or more.
A possible resolution may be that nature knows how to protect the coherence of hereditary
information by encoding it in a larger Hilbert space in a sequence of time steps by some
means of self-error correcting codes, so that ‘no-culling’ principle works at each time step
(less than the decoherence time) and its accumulated effect is seen from generation to
generation. However, this will again depend on how well the information is protected from
strong decoherence associated with complex biological systems open to their environment.
This will deserve a separate study on its own and we do not intend to do that here.

In real biological evolution, culling of a macroscopic species is a classical process that
takes place in an open system in which decoherence is very strong and rapid. This is
again consistent with the fact that culling of classical information is possible accurately.
Although information culling at the quantum level may resemble biological death, the later
is a different process altogether and should not be confused with the former process.

5 Evolution and Correlation in Species

In living systems, another important property is the interdependency between different
species. Different species could arise from mutation of some copies in the natural process of
evolution of the quantum species. Here we discuss the question of evolution of a collection
of species and if there can be strong correlations between one of the mutated copies and the
rest. We are working within a model where there are only identical replicas at our disposal
in a closed universe. If there is some interaction, it is possible that one of the copies can
undergo evolution leading to mutation described by $|\psi_1\rangle|\psi_1\rangle \rightarrow |\psi_1\rangle U|\psi_1\rangle = |\psi'_1\rangle|\psi'_1\rangle$ or it could be $|\psi_1\rangle|\psi_1\rangle \rightarrow U|\psi_1\rangle|\psi_1\rangle = |\psi'_1\rangle|\psi'_1\rangle$, where $|\psi'_1\rangle = U|\psi_1\rangle$ is the mutated copy.

Now we are looking for an interaction between two copies leading to the evolution of
one, yet unknowable to us which one has actually undergone mutation. This means that
the first copy could have evolved or the second copy could have. In other words, one of the
evolved copies has been entangled with the other copy. Specifically, we define the process
to be a unitary evolution that takes two copies of a living organism $|\psi\rangle|\psi\rangle$ and transforms
them as

$$|\psi\rangle|\psi\rangle \rightarrow N(\psi)[|\psi\rangle U|\psi\rangle + U|\psi\rangle|\psi\rangle],$$

where $N(\psi) = 1/\sqrt{1 + |\langle\psi|U|\psi\rangle|^2}$ is the normalisation constant and $U$ is the local unitary
operator that causes evolution of one of the species. Can such an indistinguishable evolution of one of the species occur in nature?

We will show that in a closed universe if there are an infinite number of replicas, then there is no interaction between them that can cause an evolution of one and entanglement between the evolved copy and the rest in a deterministic manner.

5.1 A paradox of mutation and entanglement

We will argue that by entangling one of the mutated copies with the rest of the species, we are increasing the overlap between the initial and final living states (compared to what would be the overlap between the initial and the final unentangled states). It turns out that if we start with \( M \) copies of a quantum species, then in the limit of infinite number of copies (where information has essentially become classical), the overlap between the initial and the final states become one. This implies that the effect (namely, the entangling of evolved copy with other copies) nullifies its cause (namely, the interaction). But how can any effect in nature nullify its cause? This is a paradox that we resolve in the following.

To see this interesting effect let us start with \( M \) copies of a quantum species and suppose that we are able to switch on an interaction between them such that we have the following transformation

\[
|\Psi_i\rangle = |\psi\rangle^\otimes M \rightarrow N^{(M)}(\psi)[|\psi\rangle|\psi\rangle \cdots U|\psi\rangle + |\psi\rangle|\psi\rangle \cdots U|\psi\rangle|\psi\rangle] = |\Psi_f^E\rangle,
\]

where \( N^{(M)}(\psi) = 1/\sqrt{M + M(M - 1)|\langle \psi|U|\psi\rangle|^2} \) is the normalisation constant and \( U \) is the unitary evolution that causes mutation of one of the copy. If such an evolution is possible, then the overlap between the initial and final entangled states is given by

\[
|\langle \Psi_i|\Psi_f^E\rangle|^2 = \frac{M|\langle \psi|U|\psi\rangle|^2}{1 + (M - 1)|\langle \psi|U|\psi\rangle|^2}.
\]

However, if there is no entanglement between the mutated copy and the rest, then any one of the species will evolve in time and we will have

\[
|\Psi_i\rangle = |\psi\rangle^\otimes M \rightarrow |\psi\rangle|\psi\rangle \cdots U|\psi\rangle = |\Psi_f^{UE}\rangle.
\]

In the above we have assumed that the last copy evolves, but the argument holds if any other copy evolves in time. Therefore, the overlap between the initial and the final state is given by

\[
|\langle \Psi_i|\Psi_f^{UE}\rangle|^2 = |\langle \psi|U|\psi\rangle|^2.
\]

Now for any \( M \geq 2 \), we have \( |\langle \Psi_i|\Psi_f^{UE}\rangle|^2 \geq |\langle \psi|U|\psi\rangle|^2 \), with equality sign holding when \( |\langle \psi|U|\psi\rangle| = 0 \). This shows that after the interaction, the entangling process leads to increase of the overlap between the initial and the final state. The crucial factor that make these states more indistinguishable is \( \frac{M|\langle \psi|U|\psi\rangle|^2}{1 + (M - 1)|\langle \psi|U|\psi\rangle|^2} \), which is greater than unity for \( M \geq 2 \). Interestingly, if we have infinite number of copies of a living species, then the overlap becomes

\[
\lim_{M \to \infty}|\langle \Psi_i|\Psi_f^E\rangle|^2 = \lim_{M \to \infty} \frac{M|\langle \psi|U|\psi\rangle|^2}{1 + (M - 1)|\langle \psi|U|\psi\rangle|^2} \to 1.
\]
for all admissible $|\psi\rangle$ and $U$. This shows that if we start with an ensemble of identical prepared quantum species, then the interaction leading to entanglement between one of the mutated copies and the rest almost ceases the evolution, i.e. as if the system has not evolved at all. It is an effect that nullifies its cause. So what prohibits such a strange phenomenon in the quantum world?

### 5.2 Impossibility of entangling a species with its mutated replica

The explanation for this paradox lies in the fact that a mutated copy cannot be entangled with the rest of the species in the quantum mechanical sense. We can illustrate this limitation for two copies of a quantum species and then it holds for any number of copies. If the above process exists for two arbitrary non-orthogonal states, then for two copies of the quantum systems in the states $|\psi\rangle$ and $|\phi\rangle$, we have

\[
|\psi|\psi\rangle \rightarrow N(\psi)[|\psi\rangle U|\psi\rangle + U|\psi\rangle |\psi\rangle], \\
|\phi|\phi\rangle \rightarrow N(\phi)[|\phi\rangle U|\phi\rangle + U|\phi\rangle |\phi\rangle],
\]

where $N(\phi) = 1/\sqrt{2(1 + |\langle \phi | U |\phi\rangle|^2)}$ is the normalisation constant and $U$ is the same as before. If such a quantum mechanical process exists, then the unitarity of the interaction would mean we must have the following requirement

\[
\langle \psi|\phi\rangle^2 = 2N(\psi)N(\phi)[|\psi\rangle|\phi\rangle^2 + \langle \psi|U|\phi\rangle\langle \psi|U^\dagger|\phi\rangle].
\]

(24)

However, the above equation cannot hold for two non-orthogonal quantum species. Hence, there can be no physical interaction (i.e. unitary process) that can entangle one of the mutated copies with the original one in a deterministic manner.

What is more surprising is that even two orthogonal species cannot satisfy the above condition. When $\langle \psi|\phi\rangle = 0$, still there are two non-zero terms $\langle \psi|U|\phi\rangle$ and $\langle \psi|U^\dagger|\phi\rangle$. To illustrate this point better, consider two orthogonal states of a species. Let $|\psi\rangle = |0\rangle$ and $|\phi\rangle = |1\rangle$, then we have the interaction between copies leading to

\[
|0\rangle|0\rangle \rightarrow N(0)[|0\rangle U|0\rangle + U|0\rangle |0\rangle], \\
|1\rangle|1\rangle \rightarrow N(1)[|1\rangle U|1\rangle + U|1\rangle |1\rangle]
\]

(25)

where we have taken $U|0\rangle = a|0\rangle + b|1\rangle$ and $U|1\rangle = a^*|1\rangle - b^*|0\rangle$. In this case $\langle \psi|U|\phi\rangle\langle \psi|U^\dagger|\phi\rangle = -b^2$ and hence (25) cannot be satisfied.

Since species in orthogonal states carry classical information, this shows that classical information cannot be entangled with its mutated counterpart in a deterministic manner. However, if we allow an ancillary system and a non-unitary operation then it will be possible to satisfy (23) and (25) in a probabilistic manner.

### 6 Conclusion

In this paper we have addressed various quantum mechanical processes which are basic to life but we limit ourselves to an ‘artificial living system’ with possible implication to
issues in biological systems. We all know that real biological systems (as with any meso or macroscopic systems) are in contact with the environment, thus causing decoherence. Because of this contact, such systems can therefore show order even though the overall disorder of the Universe increases. In other words, they do not violate Second Law of Thermodynamics since they are just a subsystem. With a suitable amount of interaction with the outside world (via exchange of energy, negentropy) life can possibly emerge. One may argue that apparent order can possibly emerge from Second Law, so there is no real mystery in that sense, when we realize that overall disorder (entropy) is increasing. However, here we have analysed such biological effects from the perspective of a closed quantum system. Therefore, we do not enter into the discussion of Second Law, nor exchange of energy with the environment, which would surely play an important role. Future investigation can throw more light on the question of replication, culling and correlation with reference to Second Law of Thermodynamics when one takes into account non-unitary evolutions in an open universe.

Merging of ideas from information theory and quantum theory has brought a revolution and opened up a new field called quantum information science [25]. Similarly, ideas from biology and quantum information may lead to a new discipline called quantum bio-information theory. Here, we have tried to bring out a physicist’s approach to the problem of life and some of its properties. Schrödinger, one of the founders of quantum mechanics, had initiated the question on how physical and chemical laws can account for orderly and lawful events within a living organism. We discussed Wigner’s replication machine for a living state in the quantum mechanical sense. However, it should be remarked that we have not dealt with real biological systems in their full detail, rather we have worked within a model of an ‘artificial life system’ where species are described by quantum mechanical amplitudes. Nevertheless, one can make some contact with ‘real’ biological systems. We elaborated the ‘no-cloning’ and the ‘no-culling’ principles in the language of quantum bio-information. Impossibility of the replication and culling of quantum organism provide ‘permanence’ to the information content. We have shown that there cannot be any physical interaction that can entangle a mutated quantum copy with the rest in a deterministic manner. It is hoped that these discussions will be interesting to those who wish to understand artificial or real ‘life’ from a quantum information theoretic angle and could have some implication in quantum biology.

Acknowledgment: I wish to thank C. Fuch for bringing the paper of E. P. Wigner which has been a source of inspiration. Also I thank D. Abbott for his encouragement throughout this project.

References

[1] W. M. Elsasser, The Physical Foundation of Biology, Pergamon Press, London, (1958).

[2] E. Schrödinger, What is life?, Cambridge University Press, London, (1944).

[3] P. C. W. Davis, The Cosmic Blueprint, Penguin, London (1995).

[4] See for example: G. J. Chaitin, To a mathematical definition of life, ACM SICACT News 4 (1970) 12-18.

[5] C. Adami, On modeling life, Artificial Life 1 (1995) 129-138.
[6] R. Penrose, *Shadows of the Mind*, Oxford University Press, Oxford, (1994).

[7] J. McFadden, *Quantum Evolution*, Harper Collins, London (2000).

[8] H. Frolich, *Long range coherence and the action of enzymes*, *Nature* 228 (1970) 1093.

[9] D. Home and R. Chattopadhyaya, *DNA molecular cousin of Schrödinger’s cat: a curious example of quantum measurement*, *Phys. Rev. Lett.* 76 (1996) 2836-2839.

[10] J. McFadden and J. Al-Khalili, *A quantum mechanical model of adaptive mutation*, *BioSystems* 50 (1999) 203-211.

[11] A. Patel, *Why genetic information processing could have a quantum basis*, *J. of Biotechnology* 26 (2001) 145-151.

[12] E. P. Wigner, *The Probability of the Existence of a Self-Reproducing unit*, The Logic of Personal Knowledge: Essays Presented to Michael Polany on his Seventieth Birthday, (Routledge & Kegan Paul, London) 231-238 (1961).

[13] W. K. Wootters and W. H. Zurek, *A single quantum cannot be cloned*, *Nature* 299 (1982) 802-803.

[14] D. Dieks, *Communication by EPR devices*, *Phys. Lett. A* 92 (1982) 271-272.

[15] J. von Neumann, *The Theory of Self-Replicating Automata*. University of Illinois Press, Urbana, IL (1966) (work by J. von Neumann in 1952-53).

[16] A. K. Pati and S. L. Braunstein, *Quantum mechanical universal constructor*, quant-ph/0303124 (2003); C. Seife, *Science* 300 (2003) 884.

[17] A. P. Flitney and D. Abbott, *A semi-quantum version of the game of life*, quant-ph/0208149 (2002).

[18] H. P. Yuen, *Amplification of quantum states and noiseless photon amplifiers*, *Phys. Lett. A* 113 (1986) 405-407.

[19] W. H. Zurek, *Quantum cloning: Schrödinger’s sheep*, *Nature* 404 (2000) 130.

[20] L. M. Duan and G. C. Guo, *Probabilistic cloning and identification of linearly independent states*, *Phys. Rev. Lett.* 80 (1998) 4999-5002.

[21] A. K. Pati, *Quantum superposition of multiple clones and the novel cloning machine*, *Phys. Rev. Lett.* 83 (1999) 2849-2852.

[22] A. K. Pati and S. L. Braunstein, *Impossibility of deleting an unknown quantum state*, *Nature* 404 (2000) 164.

[23] A. K. Pati and S. L. Braunstein, *Quantum no-deleting principle and some of its implications*, quant-ph/0007121 (2000).

[24] R. Jozsa, R. *A stronger no-cloning theorem*. quant-ph/0204153 (2002).

[25] M. A. Nielsen and I. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, Cambridge (2000).