Abstract

We present some informal remarks on aspects of relativistic quantum computing.

The continuing miniaturization of electronic circuitry has forced designers of computer chips to deal with all of the complications that are entailed in using Quantum Mechanics. Meanwhile, physicists have identified dramatic new possibilities for increases in computer performance by making use of such uniquely quantum effects as superposition, the entanglement of states, and the collapse of wave functions during measurement. Quantum computation and quantum information theory have now become exciting new fields of research. As circuits become yet smaller and the demand for higher clock speeds continues to escalate, relativistic effects, as well as quantum effects, will become important in computing.

The energy-time uncertainty relation in Quantum Mechanics places a number of interesting bounds on the performance of computers. We shall show that as faster computer circuits are developed, a series of thresholds is encountered, each requiring major changes of engineering approach. Eventually, in order to design the very fastest computer circuits, engineers will be forced to deal with the full machinery of Relativistic Quantum Field Theory, which is today used mainly by high energy physicists who study relativistic particle collisions.

Let us pause to think back to the level of technology available to engineers in the early part of the twentieth century at the time that Einstein proposed his theory of the specific heat of solids. Just after his theory was proposed,
the internal states of solids were known to be astronomical in number, multi-
particle, quantized, and generally evanescent in character. Those internal
states could be manipulated only in the crudest way with the equipment
available at that time. Physicists and engineers of that period evidently did
not anticipate that by the last quarter of the century, the control of internal
states of materials would be at the core of all modern technology. In fact,
given some of the well known and laughable negative predictions made by
physicists at various times past, it seems possible that many physicists would
have dismissed out of hand the possibility of electronic micro-circuitry.

Today we know that relativistic collisions produce complex, non-linear,
multi-particle, quantized states that are highly evanescent and individually
inaccessible to our equipment. We suggest that is not unreasonable to suppose
that one day these relativistic states will be technologically significant in new
ways.

Uncertainty Principle Limitations

It is common in classical computer engineering to associate a bit with an
energy state. For example, a 1 might be represented by a high voltage and a
0 by a low voltage. As long as it is necessary to distinguish amongst states
with distinct values of energy in order to perform logical operations, then the
energy-time uncertainty relation dictates that there be a minimum energy
associated with any given clock rate. For a clock frequency, $f$, the minimum
energy, $\Delta E$, required by the uncertainty relation is, (in atomic units),

$$ f = \Delta E $$

Specifically, one electron-volt corresponds to a frequency of $1.5 \times 10^{15}$
Hertz, or 1.5 peta-Hertz, which is a million-fold faster than today’s machines.

This relationship is quite general and applies to any form of energy when
used to distinguish bits; light, electric charge separation, electron spin in a
magnetic field, etc. Moreover, it applies to any mechanism that uses energy
to distinguish bits, whether it be a CPU, a memory device, or communication
line.

For the same reasons, the power flowing through a logic circuit like the
ones described above, must increase with the square of the clock frequency.
We note that nothing dictates that the power flowing through the circuit
must be wasted, but it must flow for the circuit to do its job within the
restrictions of the uncertainty relation, and this quadratic increase of power
with frequency will become very burdensome.

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1Peta-Hertz equals $10^{15}$ Hertz, Exa-Hertz equals $10^{18}$ Hertz, Zetta-Hertz equals $10^{21}$ Hertz, and
Yotta-Hertz equals $10^{24}$ Hertz.

Femto-second equals $10^{-15} \text{s}$, Atto-second equals $10^{-18} \text{s}$, Zepto-second equals $10^{-21} \text{s}$, and Yocto-
second equals $10^{-24} \text{s}$. 
As we shall discuss, below, the two simple and fundamental, equations (1) and (2), when taken together with the properties of matter and radiation, determine the shape of the future of both classical and quantum computing.

A quantum circuit that processes an input string, would read in qubits, which are two component superposition states, one by one. Suppose that the engineering choice is made to limit each qubit to only one of two energy eigenstates, which differ in energy by $E$. That is, each qubit carries only one bit of information. In this case, Eq. (1) gives us the maximum frequency of this particular quantum circuit.

It has been known for more than twenty years that a single qubit cannot carry more information than the single bit just discussed. For example, if we set the qubit to be a superposition of the two eigenstates, then there are up to three real parameters that describe the mixing. But a single measurement of the qubit will result in one or the other of the eigenvalues and can yield essentially no information about the mixing angles.

The Future of Computers

With the limitations imposed upon computers by the uncertainty relation in mind, let us explore the future of this technology.

Let us begin with a simple application of Eq. (1). If all of the properties of semiconductors, which make them so useful, are to be available, then the energy of the electron state should be less than the energy gap, which might typically be one electron-volt. This implies the existence of a sort of barrier at the frequency corresponding to one electron-volt, i.e. 1.5 peta-Hertz. Above that frequency, unique semiconductor properties start to become unavailable. This barrier, like the sound barrier, is in some sense illusory, entailing no problem of principle. Surpassing the barrier simply requires changing the technology.

As another example, suppose that bits are distinguished using the energy of orientation of a spin relative to a magnetic field. Then for a given frequency, the magnetic field strength must be large enough so that the magnetic spin energy, (which is proportional to the magnetic field strength) is at least as large as the frequency. It follows that the energy stored in the magnetic field, (which goes like the square of the field strength), must increase with the square of the frequency, as does the power passing through the spin states. Thus, in this example, both the (stored) energy of the computer mechanism and the power passing through its circuits increase like the square of the frequency.

As a part of the trend toward higher clock frequencies, circuit engineers are reducing the sizes of the elemental circuit elements. This means that they must deal with individual electrons, whose energies are increasing with the frequency. The smaller circuits also encounter all of the well known problems
associated with controlling quantum particles.

As we have said, as the desired clock frequency increases, the minimum energy required to represent information increases in proportion. However, quantized systems will, in general, have energy levels spaced closer than this minimum, thus and these levels cannot be used to distinguish information. Thus, as clock speed increases, there is a decrease in the number of energy states available for representing information. For example, the hydrogen atom has an infinite number of levels but only some of them could be used to store information if one wants to get out answers in a finite time. One possible way to release more levels for use is to engineer the over-all energy of the circuit elements yet higher; that is, choose quantum systems of greater total energy.

As a final example of the importance of the uncertainty relation in computing, we note that it has recently been suggested that spin-spin interactions in semiconductors may be useful in quantum computing. If spin domains are used to store bits, then the clock period will be limited by the light travel time across the domains. If individual spin-spin interactions are used, in the interest of speed, then the energy of interaction is of the order of \(10^{-4}\) eV and the limiting frequency is only about 150 GHz.

Relativistic Effects

As engineers respond to the limitations discussed above, they must push the maximum operating energy of circuit elements higher in order to facilitate higher clock frequencies demanded by users. If the rapid progress in clock speed proceeds as in recent years, then relativistic effects will appear shortly. Once the limitations entailed in the uncertainty relation become important, then, as mentioned above, the power throughput will increase quadratically with frequency. Therefore, computers of the future may become high power, relativistic devices similar to particle accelerators.

While, engineering practice has included classical relativistic electronic effects ever since the introduction of radar, the combination of relativistic and quantum effects in logic circuits will probably come as a rude surprise to engineers who will have to begin using relativistic quantum mechanics or quantum field theory.

One relativistic effect, which will ultimately turn out to be very important in computing, is the impossibility of localizing an electron to a volume smaller than that characterized by electron’s Compton wavelength. An important implication of this, which is discussed below, is that the time required for light to cross this small distance, which is about one zepto-second, represents a minimum time for a logical process involving electrons, and a maximum operating frequency measured in zetta-Hertz. Furthermore, at zetta-Hertz frequencies, there is another significant relativistic effect. At a frequency of a few zetta-Hertz the energy-time relationship implies that the voltage in the single electron logic circuits will exceed the threshold for the production of
electron-positron pairs. The positrons will quickly annihilate with environmental electrons, producing 0.5 MeV gamma rays.

In semiconductor circuits, the annihilation of electron-hole pairs produces electromagnetic radiation and that has not inhibited the development of practical circuits - in fact, the radiation is often quite useful. Similarly, the production of gamma rays may not inhibit the development of high energy circuits.

The limitation upon the localization of electrons is, however, a barrier of principle. Present day concepts of computer circuitry, based, as they are on electrons, and extrapolated to high frequency and single particle circuits, simply will not work above the zetta-Hertz frequency range.

**Power Requirements**

Based upon the discussion so far, we can estimate, very roughly, the power consumption of a typical relativistic quantum computer.

Let us suppose we want to have a machine that operates on a clock cycle of one zepto-second and which represents each bit by a single electron. From the uncertainty relation, the electron kinetic energy must be at least one MeV. If the through-put is one kilo-bit wide, then the machine has a throughput of one yotta-bit per second. This might be a nice piece of equipment to look forward to owning, because it is about fourteen orders of magnitude faster than the 32 bit, 500 MHz machine that may now be sitting on the reader’s desk. (Note, for example, that a computation that takes just one second at a rate of one yotta-bit/second would require thirty million years to complete on your desk machine.)

For simplicity in estimating the power requirements, let us set aside the possibility of reversibility and of recycling energy from cycle to cycle. The elementary circuit process involves energizing one thousand electrons and making a logical operation on them in one clock cycle, then repeating the process during the next cycle. At the uncertainty limit, the power is just the number of electrons operating in parallel, times the energy of each electron, times the frequency. The power required in this example is about one hundred billion watts! Considering that almost all of that comes out of the machine as gamma rays, it might not be a desirable replacement for the machine on your desk, after all.

The power estimated, above, is only the power for a single tier of the most elementary circuit elements. To estimate the power of the entire computer, the power, above, must be multiplied by the number of elementary circuit elements in the machine, which is generally a large number.

**Ultimate Speed Computer**

The question naturally arises as to whether there is any ultimate physics limit to the speed of logic circuitry, or whether we can build as fast a computer as we like if we are clever enough and are willing to commit unlimited power.
There is no fundamental unit of time in physics. That implies that there is no known upper limit to how fast an event can take place. However, at a fundamental level, there is a fastest process.

The fastest fundamental process now known to physics, is the formation or disintegration of the $Z^0$ boson. The lifetime of the $Z^0$ is about one hundredth of a yocto-second, which corresponds to a frequency of one hundred yotta-Hertz. Therefore, assuming that no circuit can operate faster than the fastest fundamental process, there is an ultimate limit to computation speed. However, as emphasized earlier, due to relativistic effects, electrons cannot be measured in times less than about one zepto-second. Therefore the ultimate clock speed for an electronic circuit would be in the range of zetta-Hertz, which was used in the example in the previous section.

These same limitations apply to digital communications. The fastest processes that can be sustained using electrons are in the zetta-Hertz range.

**How Soon Will We Reach These Limits?**

Computer clock speed has been increasing dramatically in recent decades and may well fit the exponential progress model. Early in this century data communication was accomplished by using Morse code and by using manual keyboard entry into hand cranked mechanical calculators. Today, the same functions are performed by giga-Hertz or tera-Hertz data communication lines. This represents an increase of ten or twelve orders of magnitude in speed over eight or nine decades. For discussion purposes, let us take the average figure to be an order of magnitude increase in clock speed, or equivalently, operating frequency, every ten years. Is it plausible to suppose that progress in computer speed will continue at this pace?

At each stage of development, the newest computers are applied to the problem of designing better and faster equipment and for improving and controlling the manufacturing processes. For example, enormous progress has been made in recent years in simulating the properties of materials on the computer; and this accelerates the development of materials to be used in the manufacture of better equipment. At each stage of computer development, the existing computers provide both tools for research and design and provide components for yet more sophisticated machines. Additionally, all of the new applications have opened up new markets causing an influx of capital into development of new technology. It is thus, not implausible to speculate that the rate of increase of clock speed is proportional to clock speed.

Let us suppose, then, that computer clock speeds are in an exponentially increasing stage of development, and assume that the same pace, of an order of magnitude per decade, will continue indefinitely. From today’s giga-Hertz machines to peta-Hertz machines will take six decades. That is about all of the time available to use today’s semiconductor and quantum dot technology. With the same extrapolation in another six decades, around the year 2120,
the zetta-Hertz barrier will be encountered. In some sense, that will mark the end of the era of electronics.

Conclusions

Quantum Mechanics, together with elementary properties of matter and radiation, determine the future of computers and computation. The energy-time uncertainty relation, together with strong engineering trends, implies the ultimate computer will have to incorporate the principles of relativistic quantum physics.

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Note Added: After completing this work, we became aware of a paper by Lloyd [1], which also addresses physics limitations of the ultimate computer.
REFERENCES

[1] Seth Lloyd, Physical Limits to Computation, quant-ph/9908043 v2, 16 August 1999.