Polarization of high-energy electrons traversing a laser beam

G.L. Kotkin\textsuperscript{a)}, H. Perlt\textsuperscript{b)}\textsuperscript{†} and V.G. Serbo\textsuperscript{a)}\textsuperscript{‡}

\textsuperscript{a)}Novosibirsk State University, 630090, Novosibirsk, Russia
\textsuperscript{b)}Institut für Theoretische Physik, Leipzig University, 04109, Leipzig, BRD

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Abstract

When polarized electrons traverse a region where the laser light is focused their polarization varies even if their energy and direction of motion are not changed. This effect is due to interference of the incoming electron wave and an electron wave scattered at zero angle. Equations are obtained which determine the variation of the electron density matrix, and their solutions are given. The change in the electron polarization depends not only on the Compton cross section but on the real part of the forward Compton amplitude as well. It should be taken into account, for example, in simulations of the $e \to \gamma$ conversion for future $\gamma\gamma$ colliders.

1 Introduction

Let an electron beam traverse a region where the laser light is focused. It is well known that the energies of these electrons as well as their polarizations are varied due to Compton scattering. The polarization of the scattered electrons in dependence on the spin states of the colliding electrons and photons was discussed in Refs. [1, 2, 3].

However, when the electron passes the laser beam the polarization varies also for those electrons which conserve their energies and directions of motion. In the present paper we calculate the polarization of those undeflected electrons. To our knowledge this variation of polarization has been considered in previous papers on this subject as a consequence of the fact that electrons with different values of their helicities are knocked out of the beam differently\textsuperscript{1}. In the present paper we use a more general approach. We consider the variation in polarization of the undeflected electrons as the result of interference of the incoming electron wave and the wave scattered at zero angle. This approach allows to reproduce the previous results, but we found that they are not complete. The reason is that previous authors consider the electron state as a mixture of the incoherent helicity

\textsuperscript{*}Electronic address: kotkin@phys.nsu.nsk.ru
\textsuperscript{†}Electronic address: perlt@tph204.physik.uni-leipzig.de
\textsuperscript{‡}Electronic address: serbo@math.nsc.ru

\textsuperscript{1}For example, this effect was considered as the base for obtaining the polarized electron beams in Ref. [4].
states. However, for electrons possessing a transverse polarization there is a defined phase relation between the helicity states. Moreover, the coherence of helicity states is not destroyed in the scattering process which leads to the rotation of the transverse component of the electron polarization vector as well as to variation in its absolute value. This phenomenon is similar to the well-known effect of the rotation of the neutron spin in the polarized target.

The topic of this paper is to be seen in connection with projects of $\gamma\gamma$ colliders which are under development now (see Refs. [6, 7, 8, 9]). In these projects it is suggested to obtain the required high-energy $\gamma$-quanta by backward Compton scattering of laser light on the electron beam of a linear collider. The spectrum of the obtained $\gamma$-quanta depends very strongly on the electron polarization, therefore, it is necessary to take into account the variation in polarization of the undeflected electrons in simulations of the $e \rightarrow \gamma$ conversion as well as of the $\gamma\gamma$ luminosity.

In the next section we derive the equations for components of the electron polarization vector. They are suited to simulation of the $e \rightarrow \gamma$ conversion at the $\gamma\gamma$ colliders. These equations include not only the Compton cross sections (which are proportional to the imaginary parts of the corresponding forward amplitudes) but the real parts of the same amplitudes as well. In the last section we give solutions to these equations, compare with previous results and discuss a possible scheme of simulation.

Note, that with the growth of intensity of laser flash it is necessary to take into account the effects of the intense electromagnetic fields (see Refs. [10, 11] and literature therein) which we neglect in present paper.

# Equations for the electron polarization vector

In this section we derive the equations for the electron polarization vector based on the method given in Ref. [12] where it was used to solve the analogous problem of polarization of $\gamma$-quanta traversing a laser bunch.

Let us consider the head-on collision of electrons with the bunch of laser photons. We choose the z axis along the momenta of electrons. The polarization state of an electron is described by the polarization vector $\zeta$. The electron density matrix in the helicity basis ($\lambda, \lambda' = \pm \frac{1}{2}$) has the form

$$\rho_{\lambda\lambda'}^e = \frac{1}{2} \begin{pmatrix} 1 + \zeta_z & \zeta_x - i\zeta_y \\ \zeta_x + i\zeta_y & 1 - \zeta_z \end{pmatrix}.$$  \hfill (1)

For a laser photon the density matrix is described by the following parameters: the degree of the circular polarization $P_c$, the degree of the linear polarization $P_l$ and the angle $\gamma$ of the linear polarization direction. In the helicity basis ($\lambda, \lambda' = \pm 1$) this matrix has the form (see, for example, Ref. [13] §8):

$$\rho_{\lambda\lambda'}^L = \frac{1}{2} \begin{pmatrix} 1 + P_c & -P_l e^{-2i\gamma} \\ -P_l e^{2i\gamma} & 1 - P_c \end{pmatrix}.$$  \hfill (2)

We will use also a compact expression describing the polarization of the electron and photon

$$\rho_{\lambda\lambda'} = \rho_{\lambda_1\lambda_1'}^e \rho_{\lambda_2\lambda_2'}^L.$$  \hfill (3)
In the following we derive equations for components of the polarization vector of an electron traversing a laser bunch. As is well known the variations in intensity and polarization of the wave passing through a medium are due to interference the incoming wave with the wave scattered at zero angle. Let the incoming wave have the form

\[ A_\Lambda e^{ikz}. \]

Here the amplitude \( A_\Lambda \) describes the polarization state of the electron with the energy \( E \) and the laser photon with the energy \( \hbar \omega \ll E \), the wave vector being \( k = \sqrt{E^2 - m^2c^4}/(\hbar c) \). When the wave passes through a “target” layer of a thickness \( dz \) and density \( n_L \) a forward scattered wave appears

\[
f_{\Lambda\Lambda'} A_{\Lambda'} 2n_L dz \int \frac{e^{ikr}}{r} dx dy = \frac{2\pi i}{k} f_{\Lambda\Lambda'} A_{\Lambda'} 2n_L dz e^{ikz} = e^{ikz} dA_\Lambda, \tag{4}\]

where \( f_{\Lambda\Lambda'} \) is the forward amplitude for the process of the Compton scattering. The factor 2 in front of \( n_L \) is due to relative motion of the electrons and the “target”.

The matrix \( \rho_{\Lambda\Lambda'} \) from Eq. (3) is expressed via the product of \( A_\Lambda \):

\[
\rho_{\Lambda\Lambda'} = \frac{\langle A_\Lambda A_{\Lambda'}^* \rangle}{B}, \quad B = \langle A_\Lambda A_\Lambda^* \rangle, \tag{5}\]

where \( \langle \ldots \rangle \) denotes a statistical averaging. The quantity \( B \) is proportional to the number of electrons \( N_e \). When the electron wave passes through the layer of thickness \( dz \) its relative variation in intensity is equal to

\[
\frac{dN_e}{N_e} = \frac{dB}{B} = \frac{2}{B} \Re \langle (dA_\Lambda) A_\Lambda^* \rangle = -\frac{4\pi}{k} \Im (f_{\Lambda\Lambda'} \rho_{\Lambda\Lambda'}) 2n_L dz. \tag{6}\]

This equation can be cast into the form

\[
dN_e = -\sigma_C 2n_L dz N_e, \tag{7}\]

where

\[
\sigma_C = \frac{4\pi}{k} \Im (f_{\Lambda\Lambda'} \rho_{\Lambda\Lambda'}) \tag{8}\]

is the total Compton cross section (optical theorem). Analogously,

\[
d\rho_{\Lambda\Lambda'} = d \frac{\langle A_\Lambda A_{\Lambda'}^* \rangle}{B} = \frac{2\pi i}{k} (f_{\Lambda\Lambda'} \rho_{\Lambda\Lambda'} - f_{\Lambda\Lambda'}^* \rho_{\Lambda\Lambda'}) 2n_L dz - \rho_{\Lambda\Lambda'} \frac{dB}{B}. \tag{9}\]

The scattering amplitudes \( f_{\Lambda\Lambda'} \) can be expressed by the relativistic helicity amplitudes \( M_{\lambda_1 \lambda_2 x_1 x_2} \)

\[
f_{\Lambda\Lambda'} \equiv f_{\lambda_1 \lambda_2 x_1 x_2} = \frac{k \lambda_C^2}{4\pi x} M_{\lambda_1 \lambda_2 x_1 x_2},
\]

\[
\lambda_C = \frac{\hbar}{mc}, \quad x = \frac{s - 4m^2c^4}{m^2c^4} = \frac{2(E + \hbar k c) \hbar \omega_L}{m^2c^4}, \tag{10}\]

where \( m \) is the electron mass, \( \lambda_C \) is the electron Compton wavelength and \( s \) is the square of the total energy of the electron and the laser photon in their centre-of-mass system.
Among the four independent helicity amplitudes for the Compton process only two are not equal zero for the forward scattering (compare \[13\] §70):

\[ M_{+++} = M_{---}, \quad M_{+-+} = M_{-++}. \]

According to the optical theorem, the imaginary parts of these amplitudes are connected with the Compton cross sections \(\sigma_{++}\) and \(\sigma_{+-}\) for collisions of electrons and photons with helicities \(\lambda_1 = +\frac{1}{2}, \lambda_2 = +1\) and \(\lambda_1 = +\frac{1}{2}, \lambda_2 = -1\), respectively:

\[ \sigma_{++} = \frac{\lambda_2^2}{x} \text{Im} M_{+++}, \quad \sigma_{+-} = \frac{\lambda_2^2}{x} \text{Im} M_{-++}. \] (11)

Instead of these two amplitudes, it is convenient to use their half-sum and half-difference. The imaginary part of the first quantity is related to the Compton cross section for unpolarized particles

\[ \sigma_{np} = \frac{1}{2}(\sigma_{++} + \sigma_{+-}) = \pi r_e^2 I_{np}, \] (12)

where \(r_e = e^2/(mc^2)\) is the classical electron radius.

Further, instead of half-difference we will use real dimensionless quantities \(R\) and \(I\) proportional to the real and imaginary parts of half-difference of the forward scattering amplitudes, correspondingly

\[ \pi r_e^2 (R + iI) = \frac{\lambda_2^2}{x} \frac{1}{2} (M_{+++} - M_{-++}). \] (13)

Note that

\[ \pi r_e^2 I = \frac{1}{2}(\sigma_{++} - \sigma_{+-}). \] (14)

By substituting Eqs. (3), (10)-(13) into Eqs. (3) and (13) and after summing up over the polarization states of the final photons, we obtain equations for the components of the electron polarization vector. To write down these equations it is convenient to introduce the quantity \(t\) via the relation

\[ dt(z) = 2\pi r_e^2 n_L dz. \] (15)

\(dt\) is called the reduced optical thickness of the layer \(dz\). Then

\[ \frac{d\zeta_x}{dt} = (R\zeta_y + I\zeta_x \zeta_y) P_c, \]
\[ \frac{d\zeta_y}{dt} = (-R\zeta_x + I\zeta_x \zeta_y) P_c, \] (16)
\[ \frac{d\zeta_z}{dt} = -I(1 - \zeta_z^2) P_c. \]

Let us note that these equations do not depend on the degree of the linear polarization of the laser photon \(P_l\). Integrating this system of equations one obtains the dependence of the electron polarization vector on the reduced optical thickness \(t\).

With theses results one calculates the \(t\)-dependence of cross section \(\sigma_C\)

\[ \sigma_C = \pi r_e^2 (I_{np} + \zeta_z P_c I), \] (17)
and then the number of electrons $N_e(t)$ from Eq. (7).

The forward scattering amplitudes and, therefore, the quantities $I_{np}$, $R$ and $I$ depend on the single variable $x$ (10) only. Let us write down formulas for $I_{np}$ and $I$ (see, for example, Ref. [1]):

$$
\sigma_{np} = \pi r_e^2 I_{np} = \frac{2\pi r_e^2}{x} \left[ \left( 1 - \frac{4}{x} - \frac{8}{x^2} \right) \ln (x + 1) + \frac{1}{2} + \frac{8}{x} - \frac{1}{2(x+1)^2} \right],
$$

$$
\pi r_e^2 I = \frac{1}{2}(\sigma^{++} - \sigma^{+-}) = \frac{2\pi r_e^2}{x} \left[ \left( 1 + \frac{2}{x} \right) \ln (x + 1) - \frac{5}{2} + \frac{1}{x+1} - \frac{1}{2(x+1)^2} \right].
$$

(18)

The Compton forward amplitude in the Born approximation ($O(\alpha)$) is equal to

$$
M_{\lambda_1\lambda_2 \lambda'_1 \lambda'_2} = -8\pi \alpha \delta_{\lambda_1 \lambda'_1} \delta_{\lambda_2 \lambda'_2}
$$

(19)

and, therefore, the difference $M_{++++} - M_{+-+-}$ is equal to zero in this approximation.

In order $O(\alpha^2)$ the quantity $R$ is given in Ref. [11] as

$$
R(x) = \frac{2}{\pi} \int_0^x \ln \left| \frac{1 + t}{t} \right| dt
$$

(20)

where

$$
F(x) = \int_0^x \ln \left| \frac{1 + t}{t} \right| dt
$$

(21)

is the Spence function. We have checked that the same function $R(x)$ can be obtained by the following dispersion relations:

$$
R(x) = \frac{2}{\pi} \mathcal{P} \int_0^\infty \frac{x' I(x')}{x'^2 - x^2} dx',
$$

(22)

where $\mathcal{P}$ means the principal value of the integral.

Let us give some particular values of the function $R(x)$:

$$
R(x) = \frac{x^2}{\pi} \left( \frac{10}{3} \ln \frac{1}{x} - \frac{37}{18} \right) \quad x \ll 1, \quad R(x) = \frac{\pi}{x} \quad x \gg 1,
$$

$$
R(1) = \frac{\pi^2 - 6}{6\pi} = 0.205, \quad R(4.8) = 0.223.
$$

(23)

The functions $R$ and $I$ are plotted in Fig. 1.
3 Discussion

1. The solution of Eqs. (17) has the form:

\[
\begin{align*}
\zeta_x &= \frac{\zeta_x^0 \cos \varphi + \zeta_y^0 \sin \varphi}{\cosh \tau - \zeta_z^0 \sinh \tau}, \\
\zeta_y &= -\frac{\zeta_x^0 \sin \varphi + \zeta_y^0 \cos \varphi}{\cosh \tau - \zeta_z^0 \sinh \tau}, \\
\zeta_z &= \frac{\zeta_z^0 \cosh \tau - \sinh \tau}{\cosh \tau - \zeta_z^0 \sinh \tau},
\end{align*}
\]  

where \(\varphi = \frac{P_c}{c} R t, \quad \tau = \frac{P_c}{c} I t,\) \((24)\)

and \(\zeta_x^0, \zeta_y^0, \zeta_z^0\) are the initial values of the components of the electron polarization vector.

If we introduce the auxiliary quantities \(\zeta_x^0 \perp, \zeta_y^0, \zeta_z^0\) defined as

\[
\begin{align*}
\zeta_x^0 &= \zeta_x \perp \cos \varphi_0, \\
\zeta_y^0 &= \zeta_x \perp \sin \varphi_0, \\
\zeta_z^0 &= \tanh \tau_0, \quad \zeta_x \perp (\tau) = \frac{\zeta_x \perp \cosh \tau_0}{\cosh (\tau_0 - \tau)}
\end{align*}
\]  

then Eqs. (25) can be cast into a form convenient for the analysis

\[
\begin{align*}
\zeta_x &= \zeta_x \perp \cos (\varphi_0 - \varphi), \\
\zeta_y &= \zeta_x \perp \sin (\varphi_0 - \varphi), \\
\zeta_z &= \zeta_z^0 \tanh (\tau_0 - \tau).
\end{align*}
\]  

It is seen from these solutions that the transverse electron polarization \((\zeta_x, \zeta_y)\) rotates on the angle \((\varphi_0 - \varphi)\) with increasing optical thickness \(t\). Its magnitude varies nonmonotonically and tends to zero at large \(t\). The longitudinal component \(\zeta_z\) varies monotonically and tends to \(\pm 1\) at \(\tau \rightarrow \pm \infty\).

Let us consider now the number of the undeflected electrons \(N_e(t)\). Substituting Eqs. (25) and (17) into (17), we obtain

\[
N_e = N_e^0 (\cosh \tau - \zeta_z^0 \sinh \tau) e^{-I_{np} t},
\]  

where \(N_e^0\) is the number of the incoming electrons.

Therefore, the characteristic scale of the reduced optical thickness \(t\) for the number of electrons it is determined by the quantity \(1/I_{np}\). This should be compared to the corresponding scale for the variation of the electron polarization given by \(1/|P_c I|\). The relation between these scales is given by the spin asymmetry

\[
A(x) = \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}} = \frac{I}{I_{nl}},
\]  

which is shown in Fig. 2. In the region \(x < 5\) (which is the most interesting from the experimental point of view) the spin asymmetry is small, it does not exceed 10% \(|A(x)| < 0.09\) at \(x < 5\).

In a more wide region \(x < 20\) the quantity \(A(x)\) does not exceed 30%.

\footnote{We restrict ourselves to the case when the degree of the circular polarization \(P_c\) is constant inside the laser flash.}
2. If one is interested only in the mean electron helicity, one can use a simple alternative approach. Instead of the variables $N_e$ and $\zeta_z$ one can introduce two new variables

$$N_\pm = \frac{1}{2}(1 \pm \zeta_z)N_e,$$  

which are the numbers of electrons with different signs of helicity. For these quantities the following equations are valid

$$dN_\pm = -N_\pm 2n_L \sigma_\pm dz = -N_\pm \frac{\sigma_\pm}{\pi r_e^2} dt,$$  

where $\sigma_\pm$ are the total cross sections for the scattering of electrons with helicities $\lambda = \pm 1/2$ off the laser photons (taking into account the laser polarization)

$$\sigma_\pm = \pi r_e^2 (I_{np} \pm P c I).$$

Eqs. (31)–(33) are equivalent to Eqs. (7), (8) and the last equation of the system (17). Let us stress that the transverse components of the electron polarization do not appear in Eqs. (32).

The described approach could be interpreted as follows: (i) the electron bunch is a mixture of electrons with helicities equal to $+1/2$ and $-1/2$; (ii) the numbers of electrons with given helicity change independently; (iii) the mean value of the longitudinal polarization is determined as $\zeta_z = (N_+ - N_-)/(N_+ + N_-)$. Formula (28) for the number of the undeflected electrons can be transformed into a form corresponding to Eq. (32)

$$N_e = N^0_+ e^{-(I_{np} + P c I)t} + N^0_- e^{-(I_{np} - P c I)t}.$$  

This approach is satisfactory if one is interested in the mean value of the electron helicity in the beam only. Besides, the averaged beam value of the transverse electron polarization is equal to zero if the bunch has axial symmetry. The calculations in Ref. [4] have been performed according to the approach outlined in this topic assuming this special assumption.

3. However, this approach is not quite correct since it only takes into account quantities related to the diagonal elements of electron density matrix: $N_\pm = N_e \rho_{\pm \pm}$, but it does not take into account the components of the electron transverse polarization related to the off-diagonal elements $\rho_{+-}$ and $\rho_{-+}$ of the same matrix. The inclusion of the transverse electron polarization is necessary, for example, for the laser conversion of the electron beam into the high-energy $\gamma$ beam when multiple Compton scattering occurs and when the $e \rightarrow \gamma$ conversion region is situated at some distance from the interaction point. In this case the transverse electron polarization appears after the first collision of an electron with a laser photon (even if this polarization is absent before the collision) and affects the angular distribution of the produced $\gamma$-quanta. The electron polarization (the longitudinal as well as the transverse one) is varied on the rest path through the laser bunch. Consequently, an approach which treats the electron bunch as an independent mixture of electrons with helicities $\lambda = +1/2$ and $\lambda = -1/2$, is incorrect for this problem.

The transverse electron polarization was not taken into account, for example, in Ref. [8]. However, the resulting inaccuracy seems to be less than 10 % as it was noted in (30).

4. In conclusion, the variation in polarization for electrons, which do not scatter, should be taken into account in simulations of the conversion process. It can be realized in the following way:
The electron state is defined by the current values of its energy $\varepsilon$, the direction of its momentum $z$ and its mean polarization vector $\zeta$. The probability to scatter on the path $dz$ is equal to $dw = \sigma(\varepsilon, \zeta) 2n_L dz$, where $\sigma(\varepsilon, \zeta)$ is the total cross section of the Compton scattering process. Then, as usual, one can simulate whether the scattering takes place on this path $dz$ comparing $dw$ with the random number in the interval $(0;1)$.

If the scattering takes place, one has to simulate the polar and azimuthal angles using the differential cross section. It allows to calculate the energy of the scattered $\gamma$-quantum and its Stokes parameters. A new value of the electron polarization vector $\zeta'$ can also be calculated using the known formulas (see Ref. [3]).

If scattering does not take place, nevertheless, one has to change the electron polarization vector in accordance with Eqs. (17). It is the essence of the present paper.

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**Figure captions**

**Fig. 1.** The real ($R$) and imaginary ($I$) parts of the Compton scattering amplitudes at zero angle (see Eq. (13)) as functions of the parameter $x = 4E\hbar \omega_L/(m^2 c^4)$.

**Fig. 2.** The spin asymmetry $A(x) = (\sigma_{++} - \sigma_{+-})/ (\sigma_{++} + \sigma_{+-})$ for the Compton scattering function of the parameter $x = 4E\hbar \omega_L/(m^2 c^4)$. 
