Glueballs and Their Kaluza-Klein Cousins

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Abstract

Spectra of glueball masses in non-supersymmetric Yang-Mills theory in three and four dimensions have recently been computed using the conjectured duality between superstring theory and large $N$ gauge theory. The Kaluza-Klein states of supergravity do not correspond to any states in the Yang-Mills theory and therefore should decouple in the continuum limit. On the other hand, in the supergravity limit $g_{YM}^2 N \to \infty$, we find that the masses of the Kaluza-Klein states are comparable to those of the glueballs. We also show that the leading $(g_{YM}^2 N)^{-1}$ corrections do not make these states heavier than the glueballs. Therefore, the decoupling of the Kaluza-Klein states is not evident to this order.
1 Introduction

Spectra of glueball masses in non-supersymmetric Yang-Mills theory in three and four dimensions have recently been calculated [1] using the conjectured duality between string theory and large $N$ gauge theory [2–5]. The results are apparently in good numerical agreement with available lattice gauge theory data, although a direct comparison may be somewhat subtle, since the supergravity computation is expected to be valid for large ultraviolet coupling $\lambda = g^2 \frac{N}{YM}$, whereas we expect that QCD in the continuum limit is realized for $\lambda \rightarrow 0$ [5, 6]. As explained in [6, 1], the supergravity computation at $\lambda \gg 1$ gives the glueball masses in units of the fixed ultraviolet cutoff $\Lambda_{UV}$. For finite $\lambda$, the glueball mass $M$ is expected to be a function of the form

$$M^2 = F(\lambda)\Lambda_{UV}^2.$$  \hspace{1cm} (1.1)

In the continuum limit $\Lambda_{UV} \rightarrow \infty$, $M$ should remain finite and of order $\Lambda_{QCD}$. This would require $F(\lambda) \rightarrow 0$ as $\lambda \rightarrow 0$. In [1], the leading string theory corrections to the masses were computed and shown to be negative and of order $\lambda^{-3/2}$, in accordance with expectation.

Witten has proposed [5] that three-dimensional pure QCD is dual to type IIB string theory on the product of an AdS$_5$ black hole and $S^5$. This proposal requires that certain states in string theory decouple in the continuum limit $\lambda \rightarrow 0$. One class of such states are Kaluza-Klein excitations on $S^5$. The supergravity fields on the AdS$_5$ black hole $\times$ $S^5$ can be classified by decomposing them into spherical harmonics (the Kaluza-Klein modes) on $S^5$ [7, 8]. They fall into irreducible representations of the isometry group $SO(6)$ of $S^5$, which is the $R$-symmetry of the four-dimensional $\mathcal{N} = 4$ supersymmetric gauge theory from which QCD$_3$ is obtained by compactification on a circle. Consequently, only $SO(6)$ singlet states should correspond to physical states in QCD$_3$ in the continuum limit. These are the glueball states studied in [1]. However, we find that, in the supergravity limit, masses of the $SO(6)$ non-singlet states are of the same order as the $SO(6)$ singlet states. Since these states should decouple in the limit $\lambda \rightarrow 0$, it was speculated in [1] that the string theory corrections should make the non-singlet states heavier than the singlet states.

The purpose of this paper is to test this idea. We compute the masses of the $SO(6)$ non-singlet states coming from the Kaluza-Klein excitations of the dilaton in ten dimensions. We find the masses in the supergravity limit to be of the same order as those of the $SO(6)$ singlet states. We then calculate the leading string theory corrections to the masses. We find that the leading corrections do not make the Kaluza-Klein states heavier than the
Therefore, the decoupling of the Kaluza-Klein states is not evident to this order. This suggests that the quantitative agreement between the glueball masses from supergravity and the lattice gauge theory data should be taken with a grain of salt.

2 The Supergravity Limit

We calculate the masses of the Kaluza-Klein states following the analysis of [1]. According to [5], QCD$_3$ is dual to type IIB superstring theory on the AdS$_5$ black hole $\times$ S$^5$ geometry given by

$$\frac{dx^2}{l_s^2\sqrt{4\pi g_{YM}^2 N}} = \frac{d\rho^2}{(\rho^2 - \frac{b^4}{\rho^2})} + \left(\rho^2 - \frac{b^4}{\rho^2}\right) d\tau^2 + \rho^2 \sum_{i=1}^3 dx^2_i + d\Omega_5^2,$$  \hspace{1cm} (2.1)

where $d\Omega_5$ is the line element on the unit S$^5$ and $l_s$ is the string length. The horizon of the black hole is located at $\rho = b$. In order for the geometry to be regular at the horizon, the coordinate $\tau$ must be periodic with period $2\pi R$, where $R = (2b)^{-1}$. The inverse radius $R^{-1}$ serves as the ultraviolet cutoff of QCD$_3$; namely, $\Lambda_{UV} = (2R)^{-1} = b$.

To compute the mass of an $SO(6)$ non-singlet state, we express the dilaton field $\Phi$ as

$$\Phi = f_0(\rho)e^{ikx}Y_l(\Omega_5),$$  \hspace{1cm} (2.2)

where $Y_l(\Omega_5)$ is the $l$-th spherical harmonic on S$^5$, and solve the dilaton equation in the geometry (2.1). This equation reduces to a second-order ordinary differential equation for $f_0(\rho)$; in units in which $b = 1$,

$$\rho^{-1} \frac{d}{d\rho} \left( (\rho^4 - 1) \rho \frac{df_0}{d\rho} \right) - (k_0^2 + l(l+4)\rho^2) f_0 = 0.$$  \hspace{1cm} (2.3)

The mass in three dimensions is equal to $-k_0^2$ [5]. Since the geometry (2.1) is smooth everywhere, we require that $f_0(\rho)$ be regular everywhere, and in particular at $\rho = \infty$ and at the horizon $\rho = 1$. The equation admits a regular solution $f_0(\rho)$ for discrete values of $k_0^2$. This determines the mass spectrum.

As in [1], we determine $M^2 = -k_0^2$ numerically by the shooting method. We first solve the differential equation (2.3) as an asymptotic expansion in $\rho^{-2}$ and compute the first few terms in the expansion. We then numerically integrate the equation, with boundary conditions derived from the asymptotic expansion imposed at a sufficiently large value of $\rho$ ($\rho \gg k_0^2$). The solution must be regular at the horizon $\rho = 1$. This requirement determines the spectrum of $k_0^2$. In the numerical evaluation, we find it convenient to set the boundary condition to be $f_0' = 0$ at the horizon. As we will show in the Appendix,
this shooting method can be used to compute $k_0^2$ and the wavefunction $f_0(\rho)$ to arbitrarily high precision. The results of the numerical work are listed in Table 1. As expected, the masses are all of the order of the ultraviolet cutoff $\Lambda_{UV} = b$.

| $l$ | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|-----|----|----|----|----|----|----|----|----|
| $M_{l}^2$ | 11.59 | 19.43 | 29.26 | 41.10 | 54.93 | 70.76 | 88.60 | 108.4 |
| $M_{l}^2$ | 34.53 | 48.07 | 63.60 | 81.11 | 100.6 | 122.1 | 145.6 | 171.1 |
| $M_{l}^{*2}$ | 68.98 | 88.24 | 109.5 | 132.7 | 157.9 | 185.1 | 214.3 | 245.5 |

Table 1: $3d (\text{Mass})^2$ of the $l$-th Kaluza-Klein modes on $S^5$ and their excited states, in units of $b^2$

3 Leading String Theory Corrections

Witten’s proposal requires that the Kaluza-Klein states decouple in the continuum limit $\lambda \to 0$. Here we examine whether this effect is evident from the leading string theory corrections.

According to [9], the leading $\alpha' = (4\pi g_{YM}^2 N)^{-1/2}$ correction to the AdS$_5$ black hole metric is

$$\frac{ds^2}{l_s^2 \sqrt{4\pi g_{YM}^2 N}} = (1 + \delta_1) \frac{d\rho^2}{(\rho^2 - \frac{b^4}{\rho^2})} + (1 + \delta_2) \left( \rho^2 - \frac{b^4}{\rho^2} \right) d\tau^2 + \rho^2 \sum_{i=1}^{3} dx_i^2 + d\Omega_5^2, \tag{3.1}$$

where

$$\delta_1 = +15\gamma \left( \frac{b^4}{\rho^4} + \frac{b^8}{\rho^8} - \frac{19 b^{12}}{\rho^{12}} \right),$$

$$\delta_2 = -15\gamma \left( \frac{b^4}{\rho^4} + \frac{b^8}{\rho^8} - \frac{3 b^{12}}{\rho^{12}} \right), \tag{3.2}$$

and $\gamma = \frac{1}{8} \zeta (3) \alpha'^3$. In this geometry, the dilaton is no longer constant, but is given by

$$\Phi_0 = -\frac{45}{8} \gamma \left( \frac{b^4}{\rho^4} + \frac{b^8}{2\rho^8} + \frac{b^{12}}{3\rho^{12}} \right). \tag{3.3}$$

There is also a correction to the ten-dimensional dilaton action [10, 11],

$$I_{dilaton} = -\frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{g} \left[ -\frac{1}{2} g^{\mu\nu} \partial_{\mu} \Phi \partial_{\nu} \Phi + \gamma e^{-\frac{2}{3} \Phi} W \right], \tag{3.4}$$

where $W$ is given in terms of the Weyl tensor. In our background and in units where $b = 1$, $W = 180/\rho^{16}$. The relation between the location of the horizon $\rho = b$ and the
periodicity $2\pi R$ of $\tau$ is also modified to

$$R = \left(1 - \frac{15}{8}\zeta(3)\alpha'^3 + \cdots\right)\frac{1}{2b}. \quad (3.5)$$

It is the inverse radius $R^{-1}$ that serves as the ultraviolet cutoff of QCD$_3$.

To solve the dilaton wave equation in the $\alpha'$-corrected geometry (3.1), we write

$$\Phi = \Phi_0 + f(\rho)e^{ikx}Y_l(\Omega_5), \quad (3.6)$$

where $\Phi_0$ is the dilaton background given by (3.3), and expand $f(\rho)$ and $k^2$ in $\gamma$ as

$$f(\rho) = f_0(\rho) + \gamma h(\rho), \quad k^2 = k_0^2 + \gamma\delta k^2. \quad (3.7)$$

Here $f_0(\rho)$ obeys the lowest order equation (2.3) and is a numerically given function, and $k_0^2$ is likewise determined from (2.3). The second-order differential equation obtained from the action (3.4) in the background metric (3.1) and dilaton field (3.3) is, in units in which $b = 1$,

$$\rho^{-1}\frac{d}{d\rho}\left(\rho^4 - 1\rho\frac{dh}{d\rho}\right) - \left(k_0^2 + l(l + 4)\rho^2\right)h =$$

$$= (75 - 240\rho^{-8} + 165\rho^{-12})\frac{d^2f_0}{d\rho^2}$$

$$+ (75 + 1680\rho^{-8} - 1815\rho^{-12})\rho^{-1}\frac{df_0}{d\rho}$$

$$+ (\delta k^2 - 120(k_0^2 + l(l + 4)\rho^2)\rho^{-12} - 405\rho^{-14})f_0(\rho). \quad (3.8)$$

With $f_0(\rho)$ and $k_0^2$ given, one may regard this as an inhomogeneous version of the equation (2.3). We solve this equation for $h(\rho)$ and $\delta k^2$.

We are now ready to present our results. Let us denote the lowest mass of the $l$-th Kaluza-Klein state by $M_l$. In units of the ultraviolet cutoff $\Lambda_{UV} = (2R)^{-1}$, with $R$ given by (3.5), we find

$$M_0^2 = 11.59 \times (1 - 2.78\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2$$

$$M_1^2 = 19.43 \times (1 - 2.66\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2$$

$$M_2^2 = 29.26 \times (1 - 2.62\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2$$

$$M_3^2 = 41.10 \times (1 - 2.61\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2$$

$$M_4^2 = 54.93 \times (1 - 2.63\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2$$

$$M_5^2 = 70.76 \times (1 - 2.66\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2$$

$$M_6^2 = 88.60 \times (1 - 2.69\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2$$

$$M_7^2 = 108.4 \times (1 - 2.72\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2. \quad (3.9)$$
Similar behavior is observed for the excited levels of each Kaluza-Klein state.

Thus the corrections do not make the Kaluza-Klein states heavier than the glueballs, and the decoupling of the Kaluza-Klein states is not evident to this order. According to Maldacena’s duality, the $\lambda^{-1/2}$ expansion of the gauge theory corresponds to the $\alpha'$-expansion of the two-dimensional sigma model with the AdS$_5$ black hole $\times$ S$^5$ as its target space. It is possible that the decoupling of the Kaluza-Klein states takes place only non-perturbatively in the sigma model.

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Appendix: The Boundary Condition at the Horizon

In this appendix, we show that the boundary condition at the horizon $\rho = b$ used in the shooting method [1] is consistent, and that the eigenvalue $k^2$ and the wavefunction $f(\rho)$ can be evaluated to an arbitrarily high precision using this method.

In the neighborhood of $\rho = b$, the dilaton wave equation takes the form

$$\partial_\rho (\rho - b) \partial_\rho f(\rho) + \cdots = 0.$$  \hfill (3.10)

Its general solution is of the form

$$f(\rho) = c_1 \left[ 1 + \alpha (\rho - b) + \cdots \right] + c_2 \left[ \log(\rho - b) + \cdots \right]$$  \hfill (3.11)

with arbitrary coefficients $c_{1,2}$ (the constant $\alpha$ is determined by the wave equation and is in general non-zero). The regularity of the dilaton field requires $c_2 = 0$. In the shooting method, we numerically integrate the differential equation starting from a sufficiently large value of $\rho$ down to the horizon. For generic $k^2$, the function thus obtained, when expanded as in (3.11), would have $c_2 \neq 0$. The task is to adjust $k^2$ so that $c_2 = 0$. 

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Since \( f(\rho) \) is divergent at \( \rho = b \) for generic \( k^2 \), it is numerically difficult to impose the boundary condition directly at \( \rho = b \). Instead, in [1] and in this paper, we required \( f' = 0 \) at \( \rho = b + \epsilon \) for a given small \( \epsilon \) (for example, \( \epsilon = 0.0000001b \) in this paper). By (3.11), this condition implies

\[
c_2 = -c_1 \alpha \epsilon + \cdots.
\] (3.12)

Therefore, \( c_2 \) can be made arbitrarily small by adjusting \( \epsilon \). This justifies the numerical method used in [1] and in this paper.

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