Reconfigurable Digital Channelizer Design Using Factored Markov Decision Processes

Adrian Sapio\textsuperscript{1}  
Lin Li\textsuperscript{1}  
Jiahao Wu\textsuperscript{1}  
asapio@umd.edu  
lli12311@umd.edu  
jiahao@umd.edu  

Marilyn Wolf\textsuperscript{2}  
Shuvra S. Bhattacharyya\textsuperscript{1,3}  
wolf@ece.gatech.edu  
ssb@umd.edu  

\textsuperscript{1}University of Maryland, College Park, Maryland, USA  
\textsuperscript{2}Georgia Institute of Technology, Georgia, USA  
\textsuperscript{3}Tampere University of Technology, Finland  

Abstract

In this work, a novel digital channelizer design is developed through the use of a compact, system-level modeling approach. The model efficiently captures key properties of a digital channelizer system and its time-varying operation. The model applies powerful Markov Decision Process (MDP) techniques in new ways for design optimization of reconfigurable channelization processing. The result is a promising methodology for design and implementation of digital channelizers that adapt dynamically to changing use cases and stochastic environments while optimizing simultaneously for multiple conflicting performance goals. The method is used to employ an MDP to generate a runtime reconfiguration policy for a time-varying environment. Through extensive simulations, the robustness of the adaptation is demonstrated in comparison with the prior state of the art.

1 Introduction

Digital channelizers are critical subsystems in wireless communication systems that are employed when a multiplexed signal contains information in different frequency subbands, and the application requires separating one input signal (containing multiple subbands) into one or more output signals (each containing a subset of the input subbands) \cite{22}. This function is commonly required in cognitive radio systems \cite{7}.

In this work, we seek to leverage the reconfiguration capabilities of modern embedded platforms to develop digital channelizers that can better adapt to the environment in which they are operating. Adapting to the environment using an effective \textit{system-level reconfiguration framework (SLRF)} can help these systems operate more effectively — e.g., with improved trade-offs among achievable data rate, latency, and energy efficiency. For this purpose, we apply Markov Decision Processes (MDPs) in novel ways

\textit{This article has been accepted for publication in a future issue of the Journal of Signal Processing Systems, but has not been fully edited. Content may change prior to final publication.}
to make dynamic decisions on maintaining or adapting signal processing configurations during channelizer operation. We propose an MDP-based SLRF to develop dynamic reconfiguration policies for use in stochastic environments in which adaptation of hardware/software configurations for digital channelizer processing is strategic.

While the SLRF techniques are developed in this paper with a specialized focus on digital channelizer implementation, we believe that the underlying MDP techniques are applicable across many other types of embedded signal processing systems (ESIPs). Exploring the generalization of our SLRF for broader classes of ESIPs is therefore a useful direction for future work.

Our MDP-based approach for digital channelizer design optimization results in increased robustness when used to periodically re-optimize the system policy specifically for the external environment it is being used in. This periodic re-optimization can be done completely autonomously by an embedded signal processor, without any need for human-in-the-loop intervention. The information our design optimization methods require is completely observable by the system at runtime.

The remainder of the paper is organized as follows. We provide a cursory review of the history of channelizers and MDPs, and their development in Section 2. In Section 3, we detail the signal processing application and the algorithms involved. In Section 4, we introduce our MDP-based approach and illustrate how it is applied to the signal processing application. We follow that in Section 5 with a summary of the simulations performed and the resulting data and observations that were made. We conclude in Section 6 with a discussion of future work on the use of MDPs in channelizers.

2 Background and Related Work

A digital channelizer can be generalized as having the inputs and outputs shown in Figure 1. Without loss of generality, we represent the inputs and outputs as frame-based vector quantities, with time decomposed into fixed-width slots referred to as frames. The frame arrival rate is constant and the stream of incoming frames is never ending. A channelizer is often a subsystem of a larger signal processing system. For each frame of data, the channelizer is commanded by higher-level elements of the larger signal processing system on a per-frame basis. These higher level elements determine which sub-channels need to be produced and which do not.

An example of such a channelizer framework can be found in the cognitive radio of [17]. In that application, a channelizer is used to isolate sub-bands within some wireless spectrum dynamically. This dynamic behavior involves consuming a wideband signal, and applying digital filters and rate-changing operations to produce an output that contains some subset of the input signal frequencies.

In Figure 1 for each frame \( n \) of data, \( x^{(n)} \) is a length \( N \) complex vector of the wideband input signal. This data is presented to the channelizer alongside \( CR^{(n)} \), a length \( N_C \) binary vector that provides the channelization request for that frame. The channelizer outputs \( N_C \) parallel output data vectors,

\[
y_{\alpha}^{(n)}, \alpha = \{1, 2, \ldots, N_C\}. \tag{1}
\]

Each of these vectors contains a channelized subset of the input.
Good surveys of popular digital channelizer architectures to date are found in [16, 22, 25]. The most common architectures are based on the Cosine Modulated Filter Bank (CMFB), Discrete Fourier Transform Filter Bank (DFTFB) and Per-Channel Filter Bank (PCFB). Aside from these well-established architectures, several other interesting designs for application-specific channelizers can be found in [13, 14, 9, 10].

As illustrated in [7, 1], the channelizer is often one of the most computationally intensive and power consuming blocks of cognitive radio transceivers, mainly due to its need to run at the highest data rates. For this reason, several researchers have sought to create channelizer designs where the key parameters that control the processing (e.g., filter coefficients, data rates, and subchannel masks) are configurable at run-time [5, 6, 8]. We refer to this class of DSP systems as “reconfigurable channelizers”, and point to this active body of DSP research as evidence for the importance of optimizing channelizer processing for exactly what is required, and nothing more. The goal is generally to improve efficiency by increasing processing productivity, while simultaneously decreasing energy consumption.

The body of prior work referenced above provides a number of efficient channelizer designs that can be flexibly configured for different trade-offs. However, this body of work does not address how or when the configurable parameters are changed, nor provide policies for changing them at run-time. In this paper, we develop MDP-based methods to bridge this gap.

Other researchers have sought to use MDPs with similar goals. Wei et al. have demonstrated the effective use of an MDP to control the processing rate of a network router [23]. This work created a Markov model of only the external environment, not the system under control. In contrast, as described above, our proposed SLRF incorporates Markov models of both the controlled system and the external environment, which provides a more comprehensive foundation for dynamic adaptation.

Hsieh et al. [15] devise a scheduling policy that selects among alternative implementations of common functions, such as FFTs. The alternative options accomplish functionally the same operation, but with different execution times, power demands, and hardware requirements. As in our SLRF, Hsieh’s approach uses an algorithm to make reconfiguration decisions based on what requests are placed on the system at runtime. However, in Hsieh’s approach, these requests are converted to a time series signal, smoothed using a moving average filter, and then compared to thresholds in order to derive reconfiguration decisions. The designer must commit to a smoothing
factor on the incoming requests, and effectively assume a-priori some of the resulting dynamics of the system.

Compared to Hsieh’s methods, our SLRF takes a very different approach by transforming both the system and operating environment into stochastic models, which can then be reasoned upon within the framework of MDPs. In contrast to the approach of Hsieh, there are no a-priori trade-offs on the smoothing of incoming requests. Furthermore, instead of condensing the observable data into one-dimensional signals, larger conditional probability tables are maintained. Thus, the algorithms in our SLRF can incorporate more knowledge into the decision framework. By incorporating historical transition probabilities, the MDP is able to infer in real-time whether a new request is likely to be the start of an event that should be acted upon, or is more likely a spurious request that is better ignored. This inference can be performed immediately and without the delay associated with the step response through a smoothing filter.

As described in Section 1, we apply MDPs as a core part of our proposed methodology for reconfigurable channelizer design. A preliminary version of this work was published in [20]. This preliminary version built on the results of [2], where MDPs were demonstrated to be useful tools for controlling resources in computing systems. In our preliminary version [20], we introduced two innovations that significantly enhanced the effectiveness of MDPs for channelizer design optimization. First, we added a mechanism to address hardware/software codesign scenarios that involve multidimensional design objectives and constraints, which are commonly encountered in transceiver system design. This was done through a multidimensional framework for the definition of the MDP rewards function.

Second, we introduced transition states in our MDP formulation to represent intermediate states (between distinct channelizer configurations) in the target system. We applied transition states in scenarios where commanding a state change can result in one or more time steps (frames) where the system is in a non-productive transition mode. Since being in transition from one state to another can result in missing real-time deadlines for processing requests, the control policy must choose carefully when to command a transition, and only seek to do so when the end result will be a net positive for the system in the long run, in spite of any short-term negative effects due to the transition frames. Such incorporation of transition states within our SLRF extends its utility to a broader class of applications, including channelizers, where transitions between productive states must be taken into account for accurate assessment and optimization of dynamic reconfiguration control. To the best of our knowledge, this was the first time that transition states and MDPs have been used together in this way in reconfigurable embedded systems.

In this paper, we build on the preliminary version [20] in three ways. First, we apply a methodology developed in [4] to transform an MDP into a factored MDP. This concept addresses a problem that frequently occurs with MDPs — the number of possible states of the model can be extremely large. As detailed in [21], a major motivation behind factored representations is that some parts of this large state space generally do not depend on each other and that this independence can be exploited to derive a more compact representation of the global state. In our work, factorization serves to reduce the storage size of the MDP model and execution time of the policy generation algorithms. Such advancements are critical enablers for a future direction of this work.
— deploying the modeling framework and policy generation algorithms to the targeted embedded system. When the framework and algorithms are integrated with the application on the embedded platform, they can be used to perform periodic re-optimization of the reconfiguration policies in addition to applying the policies to manage system configurations. To be practical in resource constrained and power constrained embedded environments, the deployed implementations of the modeling framework and policy generation algorithms must be carefully optimized so that they consume minimal amounts of storage and impose minimal computational burden. Our application of factored MDP techniques in this paper is an important step towards these objectives.

Second, we detail the findings of an expanded performance analysis of the proposed methodology. Specifically, we describe a suite of competing control policies and compare them objectively with the MDP based techniques. The results show that the MDP based techniques outperform the alternative schemes in nearly all cases.

Third, we perform a trade-off analysis of the costs and benefits of including transition states in the framework. This exploration details and quantifies the design time modeling costs of transition states in both storage size and execution time. These costs are then contrasted with the benefits in the form of the run-time performance when transition states are included versus when they are not. While transition states were introduced in the preliminary version [20] as a novel technique for MDP-based design of reconfigurable embedded systems, no experimental investigation of their associated trade-offs was provided due to space limitations. In this paper, we provide a more complete presentation of transition states by developing such an experimental study.

3 Reconfigurable Channelizer Design

In this section, we present a reconfigurable digital channelizer design that forms the foundation for our MDP-based, adaptive channelization system, which we present in Section 4 and demonstrate experimentally in Section 5.

Our channelizer system is implemented on the Silicon Labs EFM32GG, a small and low power ARM Cortex M3-based microcontroller. The processor is running on the EFM32 STK3700 development kit, which houses the CPU as well as sophisticated energy monitoring circuitry. For this hardware, a channelizer width of $N_C = 8$ sub-channels is used in an illustrative experiment.

This particular channelizer system is developed with applicability to wireless sensor networks, which impose challenging constraints on energy consumption and resource utilization. However, with its foundation in MDP techniques, our design methodology is not specific to any particular domain of channelization applications. For example, the methodology can be adapted to large scale, high performance channelization scenarios that involve dozens or hundreds of sub-channels that require the use of FPGAs or GPUs to run in real-time. Developing such adaptations for these additional classes of processing platforms is an interesting area for future investigation.

To examine the ability of the system to adapt to its environment, we consider two separate use cases, which we refer to as A and B. Additionally, we create multiple scenarios within those use cases, by varying parameters of the application that are understood to be time-varying. We design two separate channelizers, one ideally suited
Figure 2: DFTFB block diagram, $M = N_C$.

3.1 Polyphase DFT Filter Bank

Use Case A is the application in [17]. In that system, the requests are modeled as i.i.d. (independent and identically distributed) Bernoulli across both the time and sub-channel dimensions. These statistics for the requests mean that there is no opportunity to anticipate the request vector. For such an environment, a sensible option is a filter bank that outputs all subchannels at all times, in the most efficient way possible. For this, we use a Polyphase implementation of the canonical Discrete Fourier Transform Filter Bank (DFTFB) described in [22].

To implement this DSP block, we begin by designing a low pass filter to be used as the “prototype” filter in the filter bank. The filter has a passband width of one eighth of the full spectrum, since there are eight equally spaced channels. The filter coefficients are chosen using the Equiripple FIR design method detailed in [18]. The prototype filter is then shifted in frequency, decomposed into its polyphase components $E_m(z)$, and implemented into the DFTFB, as described in [22]. A block diagram of the derived DFTFB is shown in Figure 2. The resulting magnitude response for each of the 8 outputs is shown in Figure 3.

As can be seen from the magnitude responses of the 8 channelized outputs, this filter bank can simultaneously channelize all of the sub-channels, and thus, we require no tunable parameters for this algorithm. In order to optimize for bursts of communication activity as well as idle time, we give the controller the ability to put the DFTFB in and out of a sleep mode. The DFTFB remains resident in the current configuration, and can be gated on and off very quickly. The gating off of the DFTFB corresponds to its sleep mode.
3.2 Tunable Polyphase Decimation Filter

Use Case B is the Sequential Sensing application in [24], where a channelizer with the same inputs and outputs as Use Case A is required. However, the request statistics imposed on this channelizer are quite different from those in Use Case A. In Use Case B, the channelizer is requested to produce only one output subchannel at a time. One or more frames (usually multiple frames) elapse between requests for different subchannels.

Since only one channel is requested at any given time, we only need a tunable decimation of the input data — i.e., to filter out the unwanted subchannels. For this, we employ a polyphase implementation of an 8-to-1 decimation (DCM) filter and mixer as described in [12], shown in Figure 4.

The operation shown suppresses all but one subchannel out of the incoming signal, and then uses a complex mixer to shift the extracted channel down to be centered at DC. Once centered at DC, a simple decimation of samples gives the resulting output stream. The same filter coefficients used for the prototype low pass filter of the DFTFB can be used in the DCM. Such a DCM design produces the same frequency response per subchannel. Prior to implementation, we utilize the polyphase technique detailed in [12] to reduce the runtime processing requirements further without changing the resulting filtering operation. We refer to the resulting subsystem as a polyphase decimation filter.
Unlike the DFTFB, this configuration does have tunable parameters: the filter coefficients and mixing frequency. Using 8 parameter sets, this algorithm can be modified to select any of the 8 subchannels, effectively being an efficient low-pass, band-pass or high-pass decimation filter. Both the filter coefficients and the amount of frequency shifting are tunable, as shown in the block diagram (Figure 4). The signal is first passed through a digital filter $H_m(z)$, whose coefficients are specific to each channel $m$. Then, the filter output is shifted in frequency by multiplying it with a sinusoidal signal, whose frequency is also specific to each channel $m$. The formula to generate the sinusoidal frequency is the exponential shown in the block diagram. This configuration is also designed to be kept in a sleep mode during periods of idle user activity.

### 3.3 Summary of Processing States and Their Properties

Our MDP framework requires an enumeration of the states that the processing system can be in at any time. Our experimental embedded system has 13 states, which fall in the categories listed in Table 1.

The first row of the table covers the states when the system is in a sleep mode, with either the DCM or DFTFB ready to run. We make the distinction between these as two separate states to allow the model to capture any difference in time that it may take to re-enable the resident and already initialized algorithm out of sleep mode compared to switching to the other algorithm. Further discussion on these delays will be presented in Section 4.3.

The last two states, whose labels are prefixed with "Trans.", are states of being in transition to the DFTFB or DCM, respectively. The time required by the processing system to transition between states is an important detail in this framework. The incorporation of transition states into the MDP is a novel contribution in our work that is intended to take such transition times into account (detailed in Section 4.3). This concept of transition states allows an SLRF to compute decision paths involving transitions that can take multiple time frames to complete.

The third column of the table shows the number of channels provided by the system while in each state. Note that while in transition, the system is consuming power but not producing any channelized data.

The fourth column of the table shows the CPU power consumed by the system in each state. These measurements were performed at design time by putting the processor into test modes created for this purpose. Each test mode loaded a single configuration and iterated at the experimental application’s frame rate. With the processor operating in such a test mode, the Silicon Labs EFM32GG development tools allowed the power consumption of the associated state at the associated frame rate to be measured.

It is clear from Table 1 that the DFTFB is the most productive configuration (producing all 8 subchannels), while being the most power hungry in its ON state. It is also clear from the table that the DCM algorithm represents a less productive configuration (producing only 1 subchannel) compared to the DFTFB, but with the benefit of reduced power consumption. If only one channel is requested for an extended period of time, then a rational controller should select the DCM configuration over the DFTFB during that time in order to conserve power. This means the controller must balance...
Table 1: Categories of processing states and their properties.

| State Category | Num States | Num Channels Provided | Average Power |
|----------------|------------|-----------------------|---------------|
| SLEEP          | 2          | 0                     | 5.36 µW       |
| DCM            | 8          | 1                     | 7.61 mW       |
| DFTFB          | 1          | 8                     | 17.92 mW      |
| Trans. DFTFB   | 1          | 0                     | 10.25 mW      |
| Trans. DCM     | 1          | 0                     | 10.25 mW      |

the short term penalty of a non-productive transition with the long term benefit of the presumably more favorable new state.

It can be seen from Table 1 that the number of channels affects the number of states, and thus, the size of the MDP state space. This has significant implications on the resources required to host an MDP-based control policy on the target system, and ultimately, on the scalability of this approach to channelizers with more than 8 channels. This concept will be explored in detail in Section 4.4.

4 MDP-Based Channelizer Control

In this section, we develop an SLRF for modeling reconfigurable channelizers with the goal of generating run-time control policies that can be steered in terms of multidimensional operational objectives, including latency, throughput, and energy efficiency. The procedure is to first create a Markovian model of the system, and then use an MDP solver to generate a control policy from the developed system model. We emphasize here that the system and the environments that it operates in need not be Markovian or even stochastic in nature, and the Markovian assumptions are made as approximations expressly for the purpose of arriving at the control policy. These assumptions are validated by evaluating the resulting control policy on the real system (not the model) in its intended use case.

The resulting MDP-based dynamically reconfigurable channelizer is illustrated by the block diagram shown in Figure 5. The key feature of this system is that the channelization requests do not have direct control over the processing system. Rather, the channelization requests go only to the MDP-generated run-time control policy, which decides when and how to act on each specific request. The policy determines the best action to take, with the objective of maximizing the long-term average performance rather than solely based on an immediate reward. To make this determination, the policy uses models of the application and processing system characteristics. The policy may decide to reconfigure the processing system immediately if that is assessed as the best decision, or counterintuitively, it may decide to ignore a request that it predicts is a spurious request and would not justify a reconfiguration event.

The key components of the MDP underlying our reconfigurable channelizer system are the 4-tuple $(S, A, STM, R)$, where the components of this 4-tuple are respectively...
referred to as the system state space, action space, state transition matrix (STM), and reward function. The state space \( S \) is defined by enumerating all possible states of the external requests imposed on the processing system (channelization requests), as well as a list of modes that the processing system can be in at any time (reconfiguration states), which were detailed in Section 3. The combination (product) of these two subspaces (external requests and processing modes) yields the state space of the channelizer system.

For the Action Space \( A \), we give the MDP policy control over the reconfiguration decision, as well as selected parameter values within particular configurations. As a result, the action space consists of all the possible configurations and parameter values that can be commanded.

The STM is a stochastic matrix that defines the probability of the next state given the existing state, conditioned on a given action. This matrix is obtained by multiplying together the independent statistics of the external channelization requests with the conditional statistics of the processing system’s state transitions. The statistics of the channelization requests used to generate the STM are given by the following equations.

\[
P(CR_{ji}) = \begin{cases} P_0(CR_j), & i = i_0 \\ P_1(CR_j), & i \neq i_0 \end{cases} 
\]

\[
P_0(CR_j) = (p_{\text{start}})P_D(CR_j) + (1 - p_{\text{start}})1_{\{j=i_0\}} 
\]

\[
P_1(CR_j) = (p_{\text{stop}})1_{\{j=i_0\}} + (1 - p_{\text{stop}})P_D(CR_j) 
\]

\[
P_D(CR_j) = \beta^{\sigma(j)}(1 - \beta)^{N_C - \sigma(j)} 
\]

where \( i_0 \) is the state where no processing requests are incoming (representing periods of inactivity), \( \sigma(j) \) represents the number of requested subchannels in the CR state \( j \), \( \beta \) is a parameter used to simulate various levels of communication activity, and \( p_{\text{start}} \), \( p_{\text{stop}} \) are used to simulate the system entering and exiting periods of inactivity. The statistics of the processing system used to generate the STM are detailed in Section 4.3.
4.1 Multiobjective Rewards

For the reward function $R$, we contribute a methodology for incorporating multidimensional design objectives into an MDP-based channelizer design framework. Given a set $X = \{x_1, x_2, \ldots, x_{N_R}\}$ of $N_R$ evaluation functions for key performance metrics, a reward function $R : (S \times A) \rightarrow \mathbb{R}$ is defined in terms of these metrics for each action in each state. Here, $\mathbb{R}$ denotes the set of real numbers.

Each evaluation function $x_i : (S \times A) \rightarrow \mathbb{R}$ is used to estimate system performance in terms of a specific implementation concern, such as average energy consumption, latency, or throughput. These estimation functions can be formulated at design time by using knowledge of the system and its available configurations, or measured online by supporting instrumentation. The result of each evaluation function $x_i$ is transformed by a mapping $g_i : \mathbb{R} \rightarrow [0, 1]$, which is defined at design time for each metric. These transformations are introduced to normalize the performance metrics in order to allow them to be combined into the single scalar output of $R$. This kind of transformation and combination follows the scalarization approach to multiobjective optimization, as described in [3].

The combination of the transformed results of the evaluation functions are performed by a set of weights $\rho = \{r_1, r_2, \ldots, r_{N_R}\}$, one corresponding to each metric, such that

$$ (r_i \in [0, 1] \text{ for each } i) \text{ and } (1 = \sum_{i=1}^{N_R} r_i). \quad (6) $$

Determining these weights $\rho$ is a design time aspect of our SLRF. The weights are determined once and then continually used to steer any executions of the solver to seek policies that achieve the desired prioritization of metrics in consideration with the observed external environment statistics.

Once the evaluation functions $X$, transformations $\{g_i\}$, and combination weights $\rho$ are determined, the reward function can be evaluated using Equation (7) for any given $s \in S$ and $a \in A$.

$$ R(s, a) = \sum_{i=1}^{N_R} r_i g_i(x_i(s, a)) \quad (7) $$

In our experiments, we define the rewards as follows. First, we define $g_1$ as the normalized rate of successful channelization requests. This can be expressed as $(\eta_r - \eta_d) / N_C$, where $\eta_r$ represents the total number of channelization requests input to the system during a given time interval $\tau$, and $\eta_d$ represents the number of dropped requests (i.e., requests where there is a failure to produce the desired channel) during $\tau$.

We define $g_2$ based on a formulation in [23] for the normalized power savings of an electronic system. Specifically, in order to normalize power consumption and treat it as a form of savings, we measure power consumption ($x_2$) in each state and note the minimum and maximum possible values. Then we transform the power measurement relative to the maximum and minimum power that the system consumes in all of the possible states ($g_2$). The result is shown in Equation (8) and Equation (9).
\[
g_2(x_2(s, a)) = \frac{x_{2, \text{MAX}} - x_2(s, a)}{x_{2, \text{MAX}} - x_{2, \text{MIN}}} \tag{8}
\]

where

\[
x_2(s, a) \equiv \text{Power Consumed}(s, a)
\]

\[
x_{2, \text{MAX}} = \max_{s', a'} \{x_2(s', a')\}
\]

\[
x_{2, \text{MIN}} = \min_{s', a'} \{x_2(s', a')\}. \tag{9}
\]

Note that this definition is consistent with the convention we have defined: the most power hungry state has \(g_2 = 0\) (and thus is the least rewarded), while the least power hungry state has \(g_2 = 1\) (and thus is the most rewarded).

The combination of rewards functions \(g_1\) and \(g_2\) effectively steer the MDP to find policies that are most productive at channelizing the incoming signal as per the channelization requests, while consuming as little power as possible on average.

### 4.2 MDP Solver and Policy

With the definitions and rewards described above, an off-the-shelf MDP solver can be employed to generate a policy that simultaneously seeks to maximize the rate of successful channelization requests while consuming the least energy possible, taking into account both the physical characteristics of the processing system as well as the independent statistics of the operating environment at the current time. In our experiments, we apply the open source solver MDPSOLVE [11] in MATLAB.

The resulting control policy has the form \(f : S \rightarrow A\) — i.e., a mapping from states into actions. This mapping can be implemented as a function or simple lookup table that is invoked or accessed once per frame, respectively. To execute the controller, the incoming request is combined with the current processing system state. The result is then used as an index to lookup the operations involved in the next optimal control action.

In this example application, the total number of states is 3328 and the total number of actions is 13. For these quantities, the action can be encoded into 4 bits and thus 2 encoded actions can be packed into 1 byte of storage. The result is a policy that can be packed into 1.6kB. For our prototype hardware implementation, it was feasible to simply store the policy as a lookup table in RAM and index it to look up the next action.

### 4.3 Transition States

In our design context, the processing system is typically a deterministic, controllable machine, such as a general purpose processor (GPP), programmable digital signal processor (PDSP), field programmable gate array (FPGA) or graphics processing unit (GPU). Our framework assumes that this type of processing system can be modified
or reconfigured through the action decision of the MDP. By definition, in MDP frameworks the system is assumed to transition probabilistically from one state to another as a result of an action decision. This abstract probabilistic transition viewpoint is not immediately amenable to modeling the transitions of a deterministic processing machine. Rather, the resulting state changes in the processing system are better described as a change that is guaranteed to occur but can take some fixed or variable amount of time to complete. Additionally, the change may take longer than a single frame to complete. Some examples of the types of operations typically encountered in this context that must be accounted for are: (1) computation of the schedule for a dataflow graph before being able to execute it, (2) allocation of memory from an operating system heap when initializing algorithms, (3) the block copy of code or data from a slower, larger long-term storage to a smaller, faster location (e.g., page fault), (4) the block copy of code from non-executable regions to executable regions (e.g., overlays), and (5) dynamic full or partial reconfiguration (DPR) of FPGA regions, to name a few.

To assign the required state transition probabilities in this context, suppose that the processing system receives action $w$ in frame $n$ while in state $sp^{(n)} = u$, and that this state/action pair is known to deterministically transition the processing system to a new state $v$ in an amount of time denoted as $T_{u,v|w}$, which need not be an exact multiple of the frame period $T_F$.

If $T_{u,v|w} < T_F$, then the conditional State Transition Matrix for the processing system (SPSTM) is trivially computed by

$$SP_{STM}\begin{cases} 1, & j = v \\ 0, & \text{otherwise} \end{cases}$$

This represents a guaranteed (i.e., with probability 1) transition of the processing system to state $v$ that completes before the start of the next frame.

If, on the other hand, this transition takes longer than $T_F$, we define a new processing system state $m$, which is defined as the state of being in transition from $sp = u$ to $sp = v$. In this case, the conditional SPSTM matrix is calculated by

$$SP_{STM}\begin{cases} 1, & i = j = v \\ 1, & i = u, j = m \\ 1 - c, & i = j = m \\ c, & i = m, j = v \\ 0, & \text{otherwise} \end{cases}$$

where

$$c = \left\lfloor \frac{T_{u,v|w}}{T_F} \right\rfloor^{-1}.$$  

For example, if the processing system transition takes 4.67 frames to complete and the action is held constant until the completion of the transition, then the system will begin transitioning immediately following the triggering action, and will remain in transition for 4.67 frames before arriving at the destination state. In this case, the
conditional transition matrix states that with probability 1, the processing system will transition from the starting state to the transition state in the first frame, and then for each subsequent frame will remain in the transition state with probability 3/4, and will jump to the destination state with probability 1/4. This is exactly how the transition would appear to an agent who naively observes the processing state during just the transition sequence. This agent would observe 3 non-transitions and 1 transition out of 4 trials.

We can model observations during the transition as a Bernoulli random variable, as was done in [2] through the use of Bernoulli trials. Here, we take the two random outcomes as those of remaining in transition and completing the transition. Then the Maximum Likelihood Estimator (MLE) of the Bernoulli parameter can be shown to be exactly as given by Equation [12]. For this reason, the Bernoulli probability mass function is given by the corresponding row of the conditional transition matrix, as expressed in Equation [11]. With knowledge (or an estimate) of the transition time from each state/action pair in the model, the entire set of SPSTM matrices can be populated in this manner.

4.4 Factorization

In this work, the MDP model and solver components are implemented and invoked at design time, in order to generate a control policy that is used at run time. However, an interesting future direction for this work is that of transferring the MDP model and solver to the target system such that the solver can be invoked periodically at run time. The solver can then be applied to dynamically re-optimize the control policy in response to a changing external environment. Working towards this goal, in this section we analyze the target platform resources necessary for embedded deployment of the MDP model and solver. The main aspects of resource utilization that we investigate here are (1) the size of the four MDP constructs \((S,A,STM,R)\) that need to be held in memory, and (2) the execution time of the MDP solver required to generate the control policy.

In this context, we find significant advantages to adopting the Factored MDP approach developed in [4]. In that work, knowledge of the stochastic inter-dependencies between the state space variables are exploited to reduce both the memory requirements and solver execution time. In the remainder of this section, we summarize relevant background on MDP factorization, and present details of our proposed application of factorization techniques to reconfigurable channelizer implementation.

To facilitate the factorization of MDPs, the state \(s \in \mathcal{S}\) is generally described as an instantiation of a discrete multivariate random variable \(Z = (Z_1, Z_2, \ldots, Z_N)\), where each variable \(Z_i\) takes on values in \(\text{DOM}(Z_i)\), and \(\text{DOM}(V)\) represents the set of admissible values of the random variable \(V\). Then a state becomes a set of instantiations of the \(N\) random variables, and can be written as a vector \(z \in \text{DOM}(Z)\). The size of the state space is defined by the cardinality of this set, which we denote as \(|\text{DOM}(Z)|\).

Using this approach, the state space of the channelizer can be represented as:

\[
s = (CR_1, CR_2, \ldots, CR_{N_C}, CF_1, CF_2).
\] (13)
Figure 6: Dynamic Bayesian network representation of the channelizer state space.

Here, $CR_i$ is the channelization request for channel $i$, $CF_1$ is the top-level processing configuration, and $CF_2$ is the processing subconfiguration. The benefit of using this scheme is that it enables the explicit specification of the stochastic inter-dependencies of the variables within the state space. With this in mind, factored MDPs make use of Dynamic Bayesian Network (DBN) diagrams [19] to explicitly define and illustrate these dependencies.

A DBN diagram of the channelizer’s STM when conditioned on an MDP action is shown in Figure 6. Note that the $(CR_1, CR_2, \ldots, CR_N)$ requests are grouped together into a single vector $CR$ for conciseness. A stochastic dependency between two variables in the state space (from one time frame to the next) is denoted via the presence of an arrow between the dependent variables. The absence of an arrow denotes independence. Thus, the diagram shows that the joint probability distribution of the channelization requests is dependent only on the requests in the previous frame, and is independent of the processing configuration. The processing configuration is dependent only on the previous processing configuration (since reconfigurations are not instantaneous). However, this dependency is only on the top-level processing configuration (e.g., DCM, DFTFB, etc.) and not on the subconfiguration (e.g., the filter coefficients).

Knowledge of this underlying stochastic structure within the state space allows for considerable reduction of the size of the data structures required to store the MDP model. We highlight the effect on the largest of these components: the STM. Only the conditional probabilities with respect to the dependent variables need to be stored, rather than with respect to all variables — as would be necessary in an equally sized state space where the underlying stochastic structure is unknown. The factorization made possible by the knowledge is represented in Equation [14]. The rearrangements are made possible through (1) independence between the channelization request and processing configuration, and (2) independence between the channelization request and the MDP action.
\[
p(s'|s,a) = p(cf_1',cf_2'|cr,cf_1,cf_2,a) \\
= p(cf'|cr)p(cf_1',cf_2'|cf_1,a) \quad (14)
\]

The resulting reduction in the number of elements in the STM is shown in Equation 15. This reduction represents a significant savings. Note that the quantity shown is the cardinality of the sets, which is a count of the number of elements regardless of what underlying data type is used for representation in the MDP model and solver algorithms. For example, if the data type used is a 16-bit or 32-bit fixed-point representation, the total storage size would be 2 bytes or 4 bytes per element, respectively.

\[
|S|^2 |A| \gg |\text{DOM}(CR)|^2 + \\
|\text{DOM}(CF_1,CF_2)| |\text{DOM}(CF_1)| |A| 
\]

To evaluate the effectiveness of our MDP-based Reconfigurable Channelizer System (MRCS), we developed a simulation with external requests that follow the statistics of the two use cases — here termed IID for the i.i.d. requests of Use Case A (introduced in Section 3.1), and SEQ for the sequential sensing of Use Case B (introduced in Section 3.2). In the following sections we perform three evaluations. First, we compare the results against those of manually generated policies, that we consider representative of a typical approach used in industry. Second, we compare the results against another method published by researchers. Third, we explore the effectiveness and trade-offs associated with modeling transition states.

5 Results

In order to evaluate the effectiveness of the MDP generated control policy, we created several alternative control policies to compare it against. These are referred to as the “manually generated” policies, and contrasted with the set of “MDP generated” control policies. The manually generated policies were generated through intuitive heuristics, by first defining common sense rules for controlling the system in question, and then translating those rules into code. This represents the traditional method that an embedded software developer would use to create a reconfiguration policy. For the manually generated alternatives, the rules and resulting policies are as follows:

1. DFTFB — This policy keeps the DFTFB algorithm on the chip at all times, and invokes it in all frames regardless of the external requests. This policy was used purely as a starting baseline, as this policy represents the absence of reconfiguration options, using the most productive and processor intensive channelizer available in the system at all times to meet all requests.
2. DFTFB+Sleep — This policy also keeps the DFTFB algorithm on the chip at all times. However, if the number of requested channels is 0, the DFTFB is put into sleep mode. Otherwise, the DFTFB is kept on.

3. DCM+Sleep — This policy keeps the DCM algorithm on the chip at all times. If the number of requested channels is 0, the DCM is put into sleep mode. Otherwise, the DCM is kept on and applied to produce one of the requested channels.

4. DFTFB+DCM+Sleep — This is a set of policies that use both the DFTFB and DCM algorithms. The reconfiguration decision occurs based on how many channels are requested in the upcoming frame. If less than DFT_THRESH channels are requested, the DCM algorithm is used. If more than this threshold are requested, the DFTFB algorithm is used. Additionally, if the number of requested channels is 0, the algorithm that is currently loaded is put into sleep mode. If a reconfiguration is in progress, it is allowed to finish regardless of incoming requests. The DFT_THRESH parameter is varied from 2 to 6, resulting in 5 different control policies.

In order to compare the policies objectively, we created the following experimental setup on the EFM32GG development board. Both channelizer algorithms were implemented in C and stored on the external system FLASH. A MATLAB simulation was created that produced a time series of channelization requests having the statistics described in the two use cases A and B. The time series output of the simulation was translated to a C array and stored on the EFM32GG. A test harness was written on the EFM32GG, which was driven by a periodic timer interrupt. At the interrupt rate, the next channelization request was pulled from the stored vector and that channelization request was then used as an input to our dynamically reconfigurable channelizer system.

This system was implemented in C and executed on the EFM32GG. In order to facilitate an objective comparison of control policies, all of the manually generated policies were stored as Lookup Tables (LUTs) in addition to the MDP generated policies. This allowed both the manually- and MDP-generated policies to be invoked by suitably swapping out the contents of the LUT.

As part of the test harness, we incorporated a small amount of diagnostic code to compute performance objective 1 (productivity) in real-time. This computation was performed by comparing the produced channelizer outputs with the requests. A channelization request that was successfully carried out was labeled a success. Conversely, a request that was not met was labeled a failure (e.g., if the processing system was in a reconfiguration state during a frame with channelization requests in it, or if a configuration was in place that could not produce enough output channels, etc.). The ratio of the successful outcomes to the number of requests was used to compute a success rate, which was used as a measure of system productivity. The measured productivity results were periodically streamed to a laptop computer using the ARM on-chip trace functionality, and EFM32GG Single Wire Output (SWO) port. The streamed output for each case was tabulated and used for comparison.

Metric 2 (CPU power consumption) was measured by using the EFM32GG board’s energy monitoring tools. These development tools allowed a very accurate current
measurement to be taken, showing the exact current drawn by the CPU over time for each control policy. The total current drawn over the total simulation time was used to create a single metric for average power consumption. Thus, a highly repeatable experimental setup was applied, where all experimental settings were kept the same from case to case with the only difference being the control policy being used.

Results of our experiments are summarized in Figure 7. Here, each point in the figure represents the average performance of one policy over the entire simulation. The MDP policies generated by different values of $r_1$ are connected together, illustrating a Pareto front generated by the suite of MDP policies. The manually generated policies are plotted without any connecting lines. If the distance from the origin is used as a scalar metric of performance, the MDP generated policies all outperform or perform equally to the best manually generated policies.

### 5.2 Comparison with mHARP

Next, we compared our MRCS to a competing published method, the Highly Adaptive Reconfiguration Platform (HARP), introduced in [15]. One modification was needed, as the published HARP made decisions purely to optimize energy efficiency. This was inadequate for our setup, as the most energy-efficient result is one where the system never leaves its sleep state. To remedy this, the single metric in HARP was replaced with our multidimensional reward framework (Section 4.1) to construct a useful policy and also to provide a fair comparison between the two methods. We refer to this modified method as multiobjective HARP (mHARP).

For each of the two competing techniques, we created 10 scenarios by varying the Bernoulli parameter in use case A, and another 10 by varying the channel dwell time in use case B. The result is 20 simulations where our method and the baseline method (described below) were allowed to implement and run the optimal control policy for the given use case and external environment. The system characteristics and measurements described in the previous section were used to define the processing system under control. The results from our experiments are summarized in Figures 8 and 9.
for use cases $A$ and $B$, respectively. As previously mentioned, HARP requires a-priori tuning for a given desired system dynamic. In this simulation, we optimized mHARP for power savings. The results show that when tuned in this way, mHARP does well in this metric for all scenarios (producing slightly better performance than our MRCS approach), but greatly sacrifices performance in the success rate for half of the scenarios. Conversely, when we attempted to optimize mHARP for the success rate, we saw large shortcomings in the power savings. In contrast, MRCS involves no a-priori tuning, and optimizes all decision making for each scenario individually without compromises. These results show MRCS to have greater robustness to a wide range of parameters in different applications, all without any human-in-the-loop intervention.

5.3 Trade-offs in Modeling Transition States

An analysis was performed into the effectiveness of modeling processing state transitions, as described in Section 4.3. Although our prototype system did not incur large reconfiguration delays, we anticipate larger delays in our future work as we scale up to larger channelizer applications. Adding transition states to the MDP model has the undesirable effect of increasing the size of the state space, which is known to increase the size of the model’s data structures as well as the execution time of the policy generation algorithms. In order to make informed modeling decisions, it is crucial to understand what is gained at the expense of these costs. With these goals in mind, one of the scenarios of the IID application was selected for exploration, and modified in two ways.

First, the dynamics of the processing system were modified by changing the amount of time that transitions of the top-level reconfigurations would take to complete. This delay was varied between 1 and 5 frames, representative of a range of a small reconfiguration delay to a large delay. Second, two alternative MDP modeling approaches were used and compared: one with the transition states modeled and one without.

The cost of the additional modeling is shown in Table 2. The increase in the size
of the STM is practically negligible, however the increase the solver’s execution time is not. The benefits of this more expensive modeling come at run-time, and are shown in Figure 10. This figure shows the resulting assessment in terms of the performance metrics defined in the previous section.

From this assessment, we see that when transitions are not modeled, the performance of the system (with respect to both metrics) degrades proportionally with the length of the reconfiguration delays. This degradation is attributed to the system spending more time in a non-productive reconfiguration state. In comparison, the MDP that has the transitions modeled does not exhibit this performance degradation. We attribute these results to the fact that the MDP with transition states is able to consider the reconfiguration penalties in its decision criteria, and as a result is more “reluctant” to trigger costly reconfigurations.

Table 2: Modeling costs with and without transition delay modeling.

| Delays Modeled | STM Size [Elements] | Execution Time [Seconds] |
|----------------|---------------------|--------------------------|
| No             | 66020               | 17.2                     |
| Yes            | 66394               | 24.0                     |

6 Conclusions and Future Work

In this work, we have presented a methodology for design and implementation of adaptive digital channelizer systems, and we have demonstrated a novel channelizer design, called the MDP-based reconfigurable channelizer system (MRCS), that is derived using our new methodology. Our methodology and MRCS employ compact, system-level
models based on Markov Decision Processes (MDPs) to generate control policies that optimize the required embedded signal processing tasks in terms of relevant, multidimensional design optimization metrics. Through extensive simulations, we have shown that MRCS outperforms the prior state of the art in terms of robustness to changing applications and scenarios.

Useful directions for future work include adapting our MDP-based, reconfigurable channelizer design methodology to derive dynamically reconfigurable forms of other types or other combinations of channelizer architectures, and generalizing the proposed design methodology to address broader classes of embedded signal processing applications.

One requirement of our SLRF is that the statistics of the external environment and reconfiguration dynamics must be known at design time. In certain applications, this may not be feasible, or they may be time-varying to such a point that a policy generated offline at design time may experience a reduction in effectiveness as these quantities change.

An important area for future exploration is pairing our framework with learning strategies to estimate these statistics at runtime for systems where they are not constant or not known up front. These running estimates could then be used to periodically re-optimize the control policy and keep it performing optimally across time-varying use cases and a time-varying environment.

7 Acknowledgements

This research was sponsored in part by the US National Science Foundation (CNS1514425 and CNS151304).
References

[1] Abu-Al-Saud, W.A., Stuber, G.L.: Efficient wideband channelizer for software radio systems using modulated PR filterbanks. IEEE Transactions on Signal Processing 52(10), 2807–2820 (2004)

[2] Benini, L., Bogliolo, A., Paleologo, G.A., De Micheli, G.: Policy optimization for dynamic power management. IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems 18(6), 742–760 (1999)

[3] Bjornson, B.E., Jorswieck, E.A., Debbah, M., Ottersten, B.: Multiobjective signal processing optimization: The way to balance conflicting metrics in 5G systems. IEEE Signal Processing Magazine 31(6), 14–23 (2014)

[4] Boutilier, C., Dearden, R., Goldszmidt, M.: Exploiting structure in policy construction. In: Proceedings of the International Joint Conference on Artificial Intelligence, pp. 1104–1111 (1995)

[5] Chang, Z., Vinod, A.P., Meher, P.K.: Reconfigurable architectures for low complexity software radio channelizers using hybrid filter banks. In: Proceedings of the IEEE Singapore International Conference on Communication Systems, pp. 1–5 (2006)

[6] Darak, S.J., Gopi, S.K.P., Prasad, V.A., Lai, E.: Low-complexity reconfigurable fast filter bank for multi-standard wireless receivers. IEEE Transactions on Very Large Scale Integration (VLSI) Systems 22(5), 1202–1206 (2014)

[7] Darak, S.J., Vinod, A.P., Mahesh, R., Lai, E.M.K.: A reconfigurable filter bank for uniform and non-uniform channelization in multi-standard wireless communication receivers. In: Proceedings of the International Conference on Telecommunications, pp. 951–956 (2010)

[8] Devi, P.K., Bhuvaneswaran, R.S.: Flexible reconfigurable filter architecture for SDR receiver. In: Proceedings of the Malaysia International Conference on Communications, pp. 265–270 (2013)

[9] Dhabu, S., G., S.K., Vinod, A.P.: A low complexity reconfigurable channel filter based on decimation, interpolation and frequency response masking. In: IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP), pp. 5583–5587. Vancouver, BC, Canada (2013)

[10] Edison, A., James, T.G.: Reconfigurable perfect reconstruction filter bank channelizer for software defined radio. In: IEEE India Conference (INDICON), pp. 1138–1141. Kochi, India (2012)

[11] Fackler, P.L.: MDPSOLVE a MATLAB toolbox for solving Markov decision problems with dynamic programming — user’s guide. Tech. rep., North Carolina State University (2011)
[12] Farzad, B.: Variable bandwidth polyphase filter banks. Master’s thesis, San Diego State University (2014)

[13] Harris, F., Dick, C., Chen, X., Venosa, E.: Wideband 160-channel polyphase filter bank cable TV channeliser. IET Signal Processing 5(4), 325–332 (2011)

[14] Harris, F., Venosa, E., Chen, X., Dick, C., Adams, B.: A novel and efficient multi-resolution channelizer for software defined radio. In: Proceedings of the International Conference on Acoustics, Speech, and Signal Processing, pp. 2649–2653 (2013)

[15] Hsieh, C., Samie, F., Srouji, M.S., Wang, M., Wang, Z., Henkel, J.: Hardware/software co-design for a wireless sensor network platform. In: Proceedings of the International Conference on Hardware/Software Codesign and System Synthesis, pp. 1–10 (2014)

[16] Hu, J., Zuo, Z., Huang, Z., Dong, Z.: Dynamic digital channelizer based on spectrum sensing. PLOS One (2015). URL https://doi.org/10.1371/journal.pone.0136349

[17] Lee, C.S., Chen, W.C., Bhattacharyya, S.S., Lee, T.S.: Dynamic, data-driven spectrum management in cognitive small cell networks. In: Proceedings of the International Conference on Signal Processing and Communication Systems, pp. 1–5. Gold Coast, Australia (2014). URL http://ieeexplore.ieee.org/xplore

[18] Oppenheim, A.V., Schafer, R.W., Buck, J.R.: Discrete-Time Signal Processing, second edn. Prentice Hall (1999)

[19] Russell, S., Norvig, P.: Artificial Intelligence: A Modern Approach, third edn. Pearson (2009)

[20] Sapio, A., Wolf, M., Bhattacharyya, S.S.: Compact modeling and management of reconfiguration in digital channelizer implementation. In: Proceedings of the IEEE Global Conference on Signal and Information Processing, pp. 595–599. Washington, D.C. (2016)

[21] Sigaud, O., Buffet, O. (eds.): Markov Decision Processes in Artificial Intelligence. Wiley (2010)

[22] Vaidyanathan, P.P.: Multirate Systems and Filter Banks. Prentice Hall (1993)

[23] Wei, Y., Wang, X., Guo, F., Hogan, G., Collier, M.: Energy saving local control policy for green reconfigurable routers. In: IEEE International Conference on Communications, pp. 221–225 (2015)

[24] Xu, M., Li, H., Gan, X.: Energy efficient sequential sensing for wideband multi-channel cognitive network. In: IEEE International Conference on Communications, pp. 1–5 (2011)
[25] Zhou, D.: A review of polyphase filter banks and their application. Tech. Rep. AFRL-IF-RS-TR-2006-277, Air Force Research Laboratory, Rome, NY USA (2006)