Wave load on a navigation lock sliding gate

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Abstract

The wave load on a navigation lock sliding gate equipped with a ballast tank is investigated assuming the linear wave theory is valid and using the eigenfunction expansion matching method to solve the governing equations. The correctness and the limits of the solution have been checked by comparing the results with those of a numerical code which solve the full non linear Navier-Stokes equations. The results relating to the effect of the geometrical parameters and wave characteristics on the load acting on the gate are presented and discussed.

The ballast tank is found to increase the complexity of the phenomenon in relation to the wave interaction with the gate. The results indicate that the peak value of the vertical force occurs for wave numbers mostly dependent on the tank length and on the tank position along the depth, while the thickness has a smaller influence. The ballast tank has also a significant effect on the horizontal dynamic force acting on the gate, which vanishes when the wave number takes particular values. Finally, the moment applied to the gate shows a dependency on the geometrical and hydrodynamic parameters similar to that of the forces.

Keywords: wave load, lock gate, eigenfunction expansion matching method,

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1. Introduction

Lock gates are essential parts of a navigation lock, since they allow for the retention of water and the locking of vessels to a higher or lower level. Several types of lock gates are used around the world: mitre gates, single pivot gates, standing tainter gates, rolling gates, sliding gates, lift gates, etc.

Rolling and sliding gates for navigation locks are usually equipped with a ballast tank, which allows the load on the roller carriages to be reduced in order to improve the maneuverability. The presence of this element may produce some disadvantages, among which is the increase of vertical force due to waves which may have negative effects especially during the gate movement.

Such a problem recently occurred in the seaside gate of the navigation lock realized at the Malamocco inlet of the Venice lagoon which was designed to allow the access to the Port of Venice during the operational period of the flood control system Mo.S.E. This sliding gate is characterized by a vertical plate on the lagoon side and a rack with a ballast tank on the sea side. During a storm in 2015 the gate swung vertically due to the action of waves, characterized by an incident wave height and mean period approximately equal to 1 m and 8 s respectively. Ignoring the presence of the rack, as it provides only a small contribution to the hydrodynamic forces, this gate may be schematized as a totally immersed parallelepiped and a vertical wall adjacent to it. The actions that waves exert on a similar structure were not already analyzed in detail and no formulas or diagrams had been provided that could be used for engineering purposes.
The vertical force induced by waves on horizontal deck has been widely investigated but only for the case in which the horizontal deck is not immersed. Kaplan and Silbert (1976), Kaplan (1992) and Kaplan et al. (1995) investigated the wave forces acting on flat decks and horizontal beams on offshore platforms. They developed a semi-analytic model for the evaluation of time history wave loads on horizontal decks. Shih and Anastasiou (1992) and Toumazis et al. (1989) analyzed wave-induced forces and pressures on horizontal platform decks at small scales. Bea et al. (2001) proposed a semi-empirical method which accounts for the dynamic amplification of slamming due to dynamic response of structural elements. Tirindelli et al. (2002) and McConnell et al. (2003) and McConnell et al. (2004) measured the wave loads on deck and beam elements in a series of 2-dimensional physical model tests. These measurements were carried out to explore the process of wave loading, with the aim of developing improved predictions. Cuomo et al. (2003) and Cuomo (2005) developed a new method for the analysis of non-stationary time-history loads which is based on wavelet transform.

The influence of a vertical wall next to a deck near the coastline was recently investigated by Kisacik et al. (2012a), Kisacik et al. (2012b) and Kisacik et al. (2014) who analyzed the load conditions due to wave impacts on a vertical structure with an overhanging horizontal cantilever slab.

Recently, Guo et al. (2015) analyzed the wave force acting on a semi-submerged deck by using the potential flow approach. Such an approach is a classic method to analyze the wave properties and wave force for structures under gravity waves, such as elastic floating plates (Wu et al., 1995), a group of submerged horizontal plates (Wang and Shen, 1999), and two layers of
horizontal thick plates (Liu et al., 2009). The approach has also been used by Mei and Black (1969) and Black et al. (1971) to investigate the scattering phenomenon and the wave force acting on submerged rectangular objects.

A configuration similar to the one considered in this paper was analyzed by Wu et al. (1998) and Liu et al. (2007) in order to evaluate the performance of a breakwater equipped with a submerged horizontal porous plate. However, such studies were focused on the wave reflection of the breakwater and only a thin plate was considered.

In the present paper the wave load acting on a navigation lock gate, equipped with a prism-shaped tank connected to it at a certain height from the bottom, is studied by means of an analytical solution of the wave field obtained assuming the linear wave theory is valid and using the eigenfunction expansion method to solve the boundary value problem. Since the solution is based on the linear wave theory it cannot reproduce the nonlinear effect of wave propagation. Therefore, in order to evaluate the correctness and the limits of the theoretical model when the nonlinearities may be important, the results have been compared with those obtained by means of the numerical integration of the Navier-Stokes Equations (NSE). In section 2 the mathematical formulation of the boundary value problem is illustrated and the eigenfunction expansion method is used to determine the analytical solution. In section 3 a validation of the model is presented along with a discussion about its limitations. In section 4 the analytical solution is used to analyze the wave force induced on the gate for different geometrical configurations and hydrodynamic conditions. Finally, in section 5 the conclusions are drawn.
2. Formulation of the problem and analytical solution

The flow generated by a progressive water wave propagating towards a navigation lock is here considered. Figure 1 shows a sketch of the problem.

Figure 1: Sketch of the problem.

The origin of the reference system is placed at the intersection between the still water level and the vertical wall. The $x$ axis points in the direction of the incoming waves, while the $z$ axis points upwards. It is assumed that the waves propagate in the direction orthogonal to the gate and that the latter have constant geometrical characteristics along the $y$ axis of the reference system. Therefore, considering a structure infinitely extended in the $y$ direction, the flow is two dimensional. It is assumed that the incoming wave is characterized by an amplitude $H/2$ much smaller than the wavelength $L$ such that the linear potential wave theory can be applied (Mei et al., 2005). Denoting the velocity potential by the symbol $\Phi$, the mathematical problem is posed by the Laplace equation plus appropriate boundary condition as reported below:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0,$$

(1)
\[
\frac{\partial \Phi}{\partial n} = 0 \quad \text{at the rigid boundaries,} \tag{2}
\]
\[
\frac{\partial^2 \Phi}{\partial t^2} + g \frac{\partial \Phi}{\partial z} = 0 \quad \text{at } z=0, \tag{3}
\]
where \( g \) is the gravity acceleration, \( t \) is the time and \( n \) is the unit vector orthogonal to the boundary. Assuming that the incoming wave is sinusoidal with angular frequency \( \sigma \), the potential can be written as \( \Phi = Re(\phi e^{i\sigma t}) \) where \( i \) is the imaginary unit and \( \phi \) is a new potential function depending on \( x \) and \( z \). In terms of \( \phi \), the governing equations 1-3 can be written as follows

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0, \tag{4}
\]
\[
\frac{\partial \phi}{\partial n} = 0 \quad \text{at the rigid boundaries}, \tag{5}
\]
\[
-\sigma^2 \phi + g \frac{\partial \phi}{\partial z} = 0 \quad \text{at } z=0, \tag{6}
\]

After \( \phi \) has been determined, the pressure \( p \) is computed as follows:

\[
p = -Re(i \sigma \Phi e^{i\sigma t}). \tag{7}
\]

The mathematical problem posed by equations 4-6 has been solved by means of the eigenfunctions expansion matching method. The fluid domain has been divided into three regions as shown in Figure 1. Region \( \Pi_1 \) extends from \( x = -b \) up to \( x = -\infty \), region \( \Pi_2 \) is located below the ballast tank while region \( \Pi_3 \) is located above the ballast tank.

In region \( \Pi_1 \) the potential \( \phi \) is given by the sum of three terms as follows

\[
\phi_1 = \frac{H g}{2 \sigma} \frac{cosh[k_1(z + h)]}{cosh[k_1h]} e^{-i k_1 x} + A_{11} \cosh[k_1(z + h)] e^{i k_1 x} + 
\]
\[
+ \sum_{n=2}^{N} A_{1n} \cos[k_n(y + h)] e^{k_n x} \tag{8}
\]
The first term is the potential of the incoming wave, the second term is the potential due to the wave reflected by the gate and finally the third term is the potential of vanishing wave modes. The wavenumbers \(k_n\) satisfy the following dispersion relations:

\[
\sigma^2 = gk_n \tanh(k_nh) \quad n = 1 \tag{9}
\]

\[
\sigma^2 = -gk_n \tan(k_nh) \quad n > 1 \tag{10}
\]

It is worth to highlighting that equation 10 has infinite \(k_n\) solutions. In region \(\Pi_2\) the potential \(\phi\) can be written as follows:

\[
\phi_2 = \sum_{n=1}^{N} A_{2n} \cos[\beta_n(z+h)] \cosh[\beta_n x], \tag{11}
\]

where the wavenumbers \(\beta_n\) are given by the relations:

\[
\beta_n = \frac{(n - 1)\pi}{h_1} \tag{12}
\]

where \(h_1 = h - (d + s)\) (see Figure 1).

In region \(\Pi_3\) the potential has the following expression:

\[
\phi_3 = A_{31} \cosh[\lambda_1(z + d)] \cos[\lambda_1 x] + \sum_{n=2}^{N} A_{3n} \cos[\lambda_n(z + d)] \cosh[\lambda_n x] \tag{13}
\]

where the wavenumbers \(\lambda_n\) satisfy the dispersion relations:

\[
\sigma^2 = g\lambda_i \tanh(\lambda_id) \quad i = 1 \tag{14}
\]

\[
\sigma^2 = -g\lambda_i \tan(\lambda_id) \quad i > 1 \tag{15}
\]

The potentials \(\phi_1, \phi_2\) and \(\phi_3\) given by equations 8, 11 and 13 respectively already satisfy the boundary conditions at the rigid walls (eq. 5), except at the vertical face of the ballast tank, and at the free surface (eq. 6).
The coefficient $A_{ji}$, that appear in equations 8, 11 and 13, are expansion coefficients whose values must be determined by matching the solutions pertinent to the three regions along the strait line $x = -b$ shown in Figure 1. At $x = -b$ the matching of the velocity components in the $x$ direction is imposed by means of the following equations:

$$\frac{\partial \phi_1}{\partial x} = \frac{\partial \phi_2}{\partial x} \quad -h \leq z \leq -(d + s) \quad (16)$$
$$\frac{\partial \phi_1}{\partial x} = 0 \quad -(d + s) < z < -d \quad (17)$$
$$\frac{\partial \phi_1}{\partial x} = \frac{\partial \phi_3}{\partial x} \quad -(h_1) \leq z \leq 0 \quad (18)$$

Multiplying equations 16-18 by $\cos[k_i(z + h)]$ (by $\cosh[k_i(z + h)]$ for $i = 1$), integrating over the respective domains of definition and adding together all the equations we obtain

$$\int_{-h}^{0} \frac{\partial \phi_1}{\partial x} \cos[k_i(z + h)]dz = \int_{-h}^{-(d+s)} \frac{\partial \phi_2}{\partial x} \cos[k_i(z + h)]dz + \int_{-d}^{0} \frac{\partial \phi_3}{\partial x} \cos[k_i(z + h)]dz \quad (19)$$

Letting $i$ vary from 1 to $N$ and making use of the orthogonal properties of the functions $\cos[k_i(z + h)]$, $N$ equations for the unknown coefficients are obtained. The left hand side of the $ith$ equation 19 contains the unknown coefficient $A_{1i}$. In addition, for $i = 1$ the left hand side contains a known term due to the incoming wave. On right hand side of 19 all the unknown coefficients $A_{2i}$ and $A_{3i}$ appear.

The matching of the solutions between the regions $\Pi_1$ and $\Pi_2$ and between the regions $\Pi_1$ and $\Pi_3$ at $x = -b$ is expressed by the following equations:

$$\phi_2 = \phi_1 \quad -h \leq z \leq -(d + s) \quad (20)$$
$$\phi_3 = \phi_1 \quad -d \leq z \leq 0. \quad (21)$$
Equation 20 is multiplied by $\cos[\beta_n(z + h)]$ and equation 21 by $\cos[\lambda_i(z + d)]$ (by $\cosh[\lambda_i(z + d)]$ for $i = 1$), then both equations are integrated over the respective domains of definition as follows,

$$
\int_{-h}^{-(d+s)} \phi_2 \cos[\beta_i(z + h)] dz = \int_{-h}^{-(d+s)} \phi_1 \cos[\beta_i(z + h)] dz
$$

(22)

$$
\int_{-h_1}^{0} \phi_3 \cos[\lambda_i(z + h)] dz = \int_{-h_1}^{0} \phi_1 \cos[\lambda_i(z + h)] dz
$$

(23)

Once again, making use of the orthogonal properties of the functions $\cos[\beta_i(z + h)]$ and $\cos[\lambda_i(z + h)]$ and letting $i$ vary from 1 to $N$, equations 22 and 23 produce $N$ equations each. The terms on the left hand side of equations 22 and 23 contain unknown coefficients only while on the right hand side there are both unknown coefficients and known terms.

Overall, equations 19, 22 and 23 consist in a system of $3N$ equations whose solution provides the values of the $3N$ unknown coefficients. The solution of the linear systems has been carried out by means of the Gauss elimination approach.

After the values of the coefficients have been determined the hydrodynamic force acting on the lower and upper face of the ballast tank can be computed.

By using equation 7, the following expression for the force acting on the lower face of the ballast tank is obtained:

$$
f_{v2} = -i\sigma_2 A_{21} \cos[\beta_n h_1] b - i\sigma_2 \sum_{n=2}^{N} A_{2n} \frac{1}{\beta_n} \cos[\beta_n h_1] \sinh[\beta_n b]
$$

(24)

Analogously, the solution in region $\Pi_3$, provides the following expression for the force acting on the upper face of the ballast tank:
\[ f_{v3} = i\sigma \varrho A_{31} \frac{\sin(\lambda_1 b)}{\lambda_1} + i\sigma \varrho \sum_{n=2}^{N} A_{3n} \frac{\sinh(\lambda_n b)}{\lambda_n} \] (25)

The horizontal force is the sum of the following components: the force acting on the surface that lies below the ballast tank denoted as \( f_{h2} \), the force acting on the vertical face of the ballast tank denoted as \( f_{h1} \) and the force acting on the surface that lies above the ballast tank, denoted as \( f_{h3} \).

These forces have the following expressions:

\[ f_{h2} = -i\sigma \varrho A_{21} h_1 \] (26)

\[ f_{h1} = -i\varrho \frac{1}{k_1} \left( \sinh[k_1(h - d)] - \sinh[k_1 h_1] \right) \left( q \frac{H}{2} \frac{e^{ik_1 b}}{\cosh[k_1 h]} + \sigma A_{11} e^{-ik_1 b} \right) \]

\[ -i\sigma \varrho \sum_{n=2}^{N} A_{1n} \frac{e^{-k_n b}}{k_n} (\sin[k_n(h - d)] - \sin[k_n h_1]) \] (27)

\[ f_{h3} = -i\sigma \varrho A_{31} \frac{\sinh[\lambda_1 d]}{\lambda_1} - i\sigma \varrho \sum_{n=2}^{N} A_{3n} \frac{\sinh[\lambda_n d]}{\lambda_n} \] (28)

When analyzing the gate stability under the action of the wave load, the moment produced by the pressure acting on the gate must also be considered. In the specific case, such moments can be distinguished into moments due to the pressure acting on the vertical walls and moments due to the pressure acting on the horizontal walls. The latter walls consist in the lower and upper face of the ballast tank. During the course of this work it has been
observed that under conditions of interest to engineering applications the
moment due to the wave load acting on the vertical walls is very close to
that produced in the absence of the ballast tank. Hence this moment is not
here presented. On the other hand the moments due to the dynamic pressure
acting on the lower and upper face of the ballast tank are here discussed in
view of their importance as concern the gate stability and also because these
moments depend on the geometrical and flow parameters in a complex way.
The moments are computed with respect to the point of coordinate \( x = 0 \)
and \( z = -h \). In the following the moment due to the pressure acting on the
lower face of the ballast tank is denoted as \( M_2 \), while the moment due to the
pressure acting on the upper face is denoted as \( M_3 \):

\[
M_2 = -i \varrho \sigma A_{21} \frac{b^2}{2} + i \varrho \sigma \sum_{n=2}^{N} A_{2n} \cos[\beta_n h_1] \left[ \frac{1}{\beta_n^2} (\cosh(\beta_n b) - 1) - \frac{\sinh(\beta_n b)}{\beta_n} \right] \tag{29}
\]

\[
M_3 = -i \varrho \sigma A_{31} \left[ \frac{1}{\lambda_1^2} (1 - \cos(\lambda_1 b)b) - \frac{\sin(\lambda_1 b)b}{\lambda_1} \right] + \sum_{n=2}^{N} \left[ \frac{1}{\lambda_n^2} (\cosh(\lambda_n b) - 1) - \frac{\sinh(\lambda_n b)}{\lambda_n} \right] \tag{30}
\]

In the previous equations the number \( N \) of eigenfunctions that appear in
the summation has been fixed to 100. Computations carried out by fixing
\( N = 110 \) provide results with negligible differences with respect to those
obtained by fixing \( N = 100 \).
3. Validation and limits of the model

In order to validate the model, a first comparison was made with the results of Wu et al. (1998). Figure 2 shows the amplitude of the dimensionless vertical force \( |f_v^*| = |f_v2 + f_v3|/(\rho \times g \times H \times b) \) versus \( b/L \), for \( d/h = 0.2 \), \( h/L = 0.25 \) and \( s/h = 0 \) reported by Wu et al. (1998) and that computed by means of the present model. It can be observed that the two models provide indistinguishable results the one from each other.

![Figure 2: Comparison between the dimensionless amplitude of the vertical force reported by Wu et al. (1998) and that computed by the present model for \( d/h = 0.2 \), \( h/L = 0.25 \) and \( s/h = 0 \).](image)

Since the present model is based on the linear wave theory, it may fail when waves of no small amplitude are considered for which non linear effects can be important. In order to gain insights about the model error when wave nonlinearities are important, a comparison of the model results with those provided by the numerical integration of the Navier-Stokes equations (NSE)
has been carried out. More specifically, the integration of the NSE has been carried out by means of the commercial software Flow-3D, produced by Flow Science Inc. Flow-3D solves the Navier Stokes Equations (NSE) even in the presence of a free surface. The numerical approach is based on a finite volume method on a structured Cartesian grid. The boundary conditions at the free surface are introduced by means of the Volume of Fluid (VOF) condition (Hirt and Nichols, 1981). In the present study the flow is 2D, therefore the NSE have been solved by using a 2D structured Cartesian grid. In order to optimize the spatial distribution of the mesh size the flow domain was divided into four subdomains (see Figure 3) and in each of them a square grid was adopted. The dimensionless resolution is $\Delta x/h = \Delta z/h = 0.0168$, 0.0084, 0.0042, and 0.0021 in subdomains A, B, C, D, respectively. To check if the results change when the mesh size is decreased we carried out a numerical simulation with a further subdomains around the ballast tank adopting a mesh size equals to $\Delta x/h = \Delta z/h = 0.00105$, but this did not show significant change in the magnitude of the force on the gate. In all the cases the minimum mesh size was the one of subdomain adjacent to the ballast tank and it is worth pointing out that this was too large to reproduce the boundary layer along the surface of the gate and of the ballast tank. Indeed the boundary has a thickness of the order of 1 cm while the minimum mesh size is approximately 3 cm. Reducing the mesh size in order to describe the boundary layer would lead to the adoption of a huge grid and to an uneconomical computation. An idea about the requirements of the numerical grid to describe the boundary layer can be obtained through the $\beta$ parameter given by the equation $\beta = s^2/(\nu T)$, where $s$ is the thickness
of the ballast tank and \( \nu \) is the fluid viscosity. This parameter, along with the Keulegan Carpenter number and the Reynolds number, is usually considered in studies of oscillatory flow around rigid bodies (see Scandura et al., 2009). The square root of \( \beta \) is the ratio between the thickness of the ballast tank \( s \) and the Stokes layer thickness \( \sqrt{\nu T} \). Thus, it provides information on the grid size necessary to solve the boundary layer around the body. In the present numerical study, where a case that may occur in practical application is considered (\( s \sim O(1) \) and \( T \sim O(10) \)), the value of \( \beta \) is of order \( 10^5 - 10^6 \). This is a very large value for \( \beta \) and prevents the possibility of simulating the boundary layer around the ballast tank. Indeed previous numerical studies on oscillatory flow around bodies have considered \( \beta \) parameters of several orders of magnitude smaller than the previous one (e.g. Zao and Cheng, 2014).

One of the consequences of not simulating the viscous boundary layer is that even the vortex shedding phenomenon cannot be simulated. Even though the forces arising from the viscous boundary layer are generally small and can be neglected, the same is not valid for the forces induced by the vortex shedding. Thus, it is relevant to provide an explanation of why the numerical simulations provide reliable results regarding the force acting on the ballast tank even though the vortex shedding is not correctly reproduced. The justification is based on the Keulegan Carpenter number \( KC \) which is given by the expression \( KC = u_0 T/s \), where \( u_0 \) is the undisturbed amplitude of the velocity oscillations near to the ballast tank. This dimensionless number is the ratio between the fluid particles displacement and the size of the ballast tank. When the \( KC \) number is smaller than 1, the shed vortices are weak and move along a very limited portion of the body, thus they provide a small
contribution to the total force. On the basis of the wave characteristics here considered, close to the ballast tank \( u_0 \approx 0.2 \text{ m/s} \), hence \( KC < 1 \). Then the vortex shedding provides a very small contribution to the force acting on the ballast tank, which is instead dominated by the inertia force, related to fluid acceleration. In summary for the present set of parameters even numerical simulations that do not capture the boundary layer provide reliable results as concerns the hydrodynamic force.

![Figure 3: Sketch of domain adopted for CFD computation.](image)

Figure 4 shows the comparison between the results obtained by means of the analytical model and those provided by the numerical solution of the NSE for a fixed geometry and different wave characteristics. The comparison was carried out by fixing \( d/h = 0.327, \ b/h = 0.269, \) and \( s/h = 0.236 \). Two parameters mainly affect the wave non-linearity i.e.: the wave number \( kh = 2\pi h/L \) and the wave slope \( H/L \). The comparison shows that the present model reproduces quite well the vertical force induced on the ballast tank for waves characterized by small non-linearity (\( kh = 1.761 \) and \( H/L = 0.009 \)). For a fixed value of \( kh \) the agreement between the peaks of
the wave force worsens when $H/L$ increases. In particular when the non-linearity increases the model underestimates the positive peak value of the dimensionless vertical force and overestimates the negative peak. The maximum underestimation is equal to approximately 19% for $kh = 0.859$ and $H/L = 0.019$ while the maximum overestimation of the negative peak is approximately 14% for $kh = 1.310$ and $H/L = 0.019$. It can be also observed that for small $kh$, increasing $H/L$ the phase at which the negative peak occurs shift back while the phase of the positive peak shift forward.

Although Figure 4 shows some discrepancies between model predictions and the NSE solution the theoretical model is appropriate for carrying out a detailed investigation of the wave load on the gate as these discrepancies are acceptable in the range of the parameters investigated. This is particularly advantageous as such a kind of investigation would involve prohibitive computational costs if carried out by means of integration of the NSE.

4. Discussion of model results

The dimensionless amplitude of the vertical force $|f^*_v|$ acting on the ballast tank, is shown in Figure 5 versus $kh$ for several values of the ratios $d/h$, $s/h$, and $b/h$. For all the analyzed conditions a peak value of $|f^*_v|$ is detected. The results show a decrease of $|f^*_v|$ when the relative depth $d/h$ of the upper face of the ballast tank increases. The peak value of $|f^*_v|$ occurs for value of $kh$ mostly dependent on $b/h$ and $d/h$, while $s/h$ has a small effect in the range of values here considered.

Generally, the lower the ratio $b/h$, the higher the value $kh$ at which the maximum of the dimensionless force is detected. This can be explained by
Figure 4: Comparison between the force acting on the ballast tank computed by means of the numerical integration of the NSE and that computed by means of the present model: $d/h = 0.327$, $b/h = 0.269$ and $s/h = 0.236$.

noting that in order to have a strong interaction between waves and ballast tank the relation $L \sim \xi b$ must be satisfied, where $\xi$ is a coefficient larger than 1 but not much larger than 1. Indeed, for $L >> b$ the wave sees the gate as a plane wall, therefore the vertical force is very small. This can be checked by noting the ratio $b/L$ is proportional to $kh \times b/h$ and observing that in Figure 5 when this ratio tends to zero the vertical force is very small.
Figure 5: Dimensionless amplitude of the vertical force versus the relative depth $kh$.

The case $L << b$ is out of practical applications as in general $b$ is of the order of a meter and $L$ is of the order of ten meters. However, assuming
that the condition $L << b$ is true, it is expected that the pressure over the ballast tank oscillates several times around zero such that the overall contribution to the force becomes small or even vanishes for particular values of the parameters. Indeed, the condition $L << b$ can be written as $b/L = (kh/2\pi) \times b/h >> 1$ and it can be observed in Figure 5 that when $kh$ and $b/h$ are both large the force is small. Therefore, being $L \sim \xi b$ we immediately get $kh \sim 1/(\xi b/h)$ which shows that when $b/h$ increases the value at which the peak of the force occurs shift at lower values as shown in Figure 5.

The condition $L > d$, in general necessary to have interaction between waves and ballast tank, can be written as $kh < 1/(d/h)$. In much of the cases shown in figure 5 the maximum of the force occurs for values of $kh$ that are significantly smaller than $1/(d/h)$. Therefore in these cases it is not necessary that when $d/h$ increases the value of $kh$, at which the maximum occurs, decreases. However, for small values of $b/h$ it can be observed in Figure 5 that the maximum is found at large values of $kh$ which are close to the limits imposed by the relation $kh < 1/(d/h)$. Therefore in these cases when $d/h$ increase the maximum shift at lower values of $kh$.

The influence of the relative thickness of the ballast tank $s/h$ is smaller than those related to other geometrical parameters. Indeed, the results show that a variation of one order of magnitude in the ratio $s/h$ can produce a difference of $|f_{v'}|$ in the range 15-30%.

Figures 6-8 summarize results that can be useful for engineering application: for a fixed geometrical configuration the peak value of the dimensionless vertical force acting on the gate and the wave number at which it is detected are reported. The figures show that the value of $kh$ at which the peak value
of $|f_v^*|$ is detected increases with $d/h$ for large value of $b/h$ while it decrease for small value of $b/h$. Comparing figures 6, 7 and 8 it can be observed that the peak value of the force increases with $s/h$ while the value of $kh$ at which such a peak occurs shifts at lower values.

Figure 6: Maximum value of the dimensionless amplitude of vertical force for $s/h = 0.00$: a) dimensionless value of the vertical force; b) value of $kh$ at which the peak is detected.

Figure 7: Maximum value of the dimensionless amplitude of vertical force for $s/h = 0.11$: a) dimensionless value of the vertical force; b) value of $kh$ at which the peak is detected.
As regards the dimensionless horizontal dynamic force $|f_h^*| = |f_{h1} + f_{h2} + f_{h3}|/(\rho \times g \times H \times b)$ acting on the gate, it is significantly affected by the ballast tank. Indeed, in Figure 9, where the amplitude of the dimensionless horizontal force acting on the gate is reported as a function of $kh$ and for different value of $b/h$, it can be observed that for $d/h < 0.37$ the amplitude of the force vanishes when $kh$ takes particular values, while this results is not observed for $b/h = 0$ (absence of ballast tank). Furthermore, it can be observed that the presence of the ballast tank mostly produces a decrease of $|f_h^*|$ with respect to the case of a vertical wall.

In order to explain why the horizontal dynamic force may vanish, it is useful to analyze separately the force acting on the upper and lower portions of the gate. Figure 10 shows that the horizontal force $|f_{h3}^*| = |f_{h3}|/(\rho \times g \times H \times b)$ acting on the upper gate portion never vanishes and that it is only slightly affected by the parameters $d/h$, $s/h$, and $b/h$ for large value of $kh$.

On the contrary, the horizontal force acting on the lower gate portion
\[ f_{h2}^* = \frac{|f_{h2}|}{(\rho \times g \times H \times b)} \]

shown in Figure 11, has a much larger oscillating behavior and vanishes when \( kh \) takes particular values. Note that according to equation (26) these values are those for which the coefficient \( A_{21} \) vanishes. Indeed, the other coefficients do not contribute to the force because their associated eigenfunctions have a purely oscillating behaviour. The correctness of this result was checked by means of the numerical solution of the NSE (see Figure 12) by investigating the behaviour of the force in a neighborhood of the value of \( kh \) that according to the model provides a vanishing value of the force. It can be observed that there is a reasonable agreement between the model and the numerical simulations. The differences may be attributed to the wave non linearities which are not taken into account in the model.

Vanishing of \( f_{h2}^* \) corresponds to a 180 degree variation of phase oscillation. Therefore, since the force acting on the upper portion does not change phase, when adding together all the contributions to the horizontal force the sum may vanishes for certain values of \( kh \).

As regards the moment of the vertical forces, Figure 13 shows the amplitude of the dimensionless momentum acting on the ballast tank \( m_v^* \) = \[ m_v^*/(\rho \times g \times H \times b^2) \] with respect to the foot of the gate, for several values of the ratios \( d/h, s/h, \) and \( b/h \). A decrease of \( m_v^* \) is observed when the relative depth \( d/h \) of the upper face of the ballast tank increases. Furthermore, the peak value of \( m_v^* \) occurs for values of \( kh \) mostly dependent on \( b/h \) and \( d/h \). The parameter \( s/h \) has a smaller effect which becomes more significant for small values of \( b/h \).

Figures 14-16 show, for a fixed geometrical configuration, the peak value
Figure 9: Dimensionless amplitude of the horizontal force acting on the whole gate versus the relative depth $kh$. 
Figure 10: Dimensionless amplitude of the horizontal force acting on the upper gate portion versus the relative depth $kh$. 
Figure 11: Dimensionless amplitude of the horizontal force acting on the lower gate portion versus the relative depth $kh$. 

\[ |F_{h2}| \]
Figure 12: Dimensionless amplitude of the horizontal force acting on the lower gate portion versus the relative depth $kh$. Comparison with Flow-3D results: $d/h = 0.37$, $b/h = 0.314$ and $s/h = 0$.

of $|m^*_v|$ and the wave number at which it is detected. The results repeat the same behavior described for the vertical force acting on the ballast tank.

Finally, as reported in section 2 the analyses showed that in the range of the parameters of engineering interest, the moment of the horizontal force is always smaller than that acting on the gate without ballast tank.

5. Conclusions

The wave load on a navigation lock sliding gate equipped with a ballast tank has been investigated by means of an analytical solution based on the linear wave theory. In addition for several different values of the parameters the wave loads have been calculated by means of numerical integration of the Navier-Stokes equations and the results have been used to gain insights into the importance of the flow nonlinearities and into the accuracy of the
Figure 13: Dimensionless amplitude of the momentum due to the vertical forces versus the relative depth $kh$. 

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Figure 13: Dimensionless amplitude of the moment
Figure 14: Maximum value of the dimensionless amplitude of moment of vertical force for \( s/h = 0.00 \): a) dimensionless value of the vertical force; b) value of \( kh \) at which the peak is detected.

Figure 15: Maximum value of the dimensionless amplitude of moment of vertical force for \( s/h = 0.11 \): a) dimensionless value of the vertical force; b) value of \( kh \) at which the peak is detected.

analytical solution. It has been shown that because of the nonlinearities the analytical solution is affected by an error of the order of 10% in the range of the parameters investigated. This is reasonable for engineering application
Figure 16: Maximum value of the dimensionless amplitude of moment of vertical force for $s/h = 0.22$: a) dimensionless value of the vertical force; b) value of $kh$ at which the peak is detected.

and justify the use of the analytical solution for the computation of the wave load. The results show that the presence of the ballast tank produces both significant vertical force and an important alteration of the horizontal dynamic load.

The peak values of the dimensionless amplitude of the vertical forces $|f_v^*|$ and of its moment $|m_v^*|$ occur for values of $kh$ mostly dependent on the relative length $b/h$ and depth $d/h$. As concerns the effect of the relative thickness $s/h$ of the ballast tank it does not have an important effect when $b/h$ is large while it becomes more important for smaller values of $b/h$. Generally, the lower the ratio $b/h$, the higher the value of $kh$ at which the maximum of $|f_v^*|$ and $|m_v^*|$ are detected and the lower are $|f_v^*|$ and $|m_v^*|$. An increase of the relative depth $d/h$ of the upper edge of the ballast tank, for large $b/h$ in addition to reducing the peak force, produces an increase of the value of $kh$ at which such a peak is detected while the opposite occurs for small $b/h$. 

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The proposed analytical solution also shows that the ballast tank produces a significant variation of the dimensionless horizontal dynamic force acting on the gate, which could vanish for particular values of \( kh \). Such behavior is due to the fact that the oscillations of the force acting on the lower portion of the gate may become 180 degrees out of phase with respect to the forces acting on the upper portion and on the vertical face of the ballast tank. Therefore the total force may vanish.

Finally, the relation between the amplitude of the force acting on the gate and the geometrical parameters is summarized in diagrams that can be useful for engineering purpose.

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