Research on H\(\infty\) Robust Control Strategy of Wind Turbine Hydraulic Variable Pitch System

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Abstract. The variable pitch system is an important part of the wind turbine. In consideration of the parameter uncertainty and external interference in the hydraulic variable pitch system, this paper proposes a H\(\infty\) robust control strategy for the wind turbine variable pitch system. Firstly, the model of the system is given and the equation of state is obtained. According to the performance requirements, the weighted function which meets the system requirements in the process of the mixed sensitivity optimization is selected, and then the state space realization of the system generalized controlled object is obtained. Then, the H\(\infty\) controller is obtained by solving Riccati equation. Finally, the designed system reduces the steady-state error, and improves the robustness of the system.

Keywords: Wind turbine; Hydraulic variable pitch system; State space implementation; Parameter uncertainty; External disturbance.

1. Introduction
With the emergence of traditional fossil energy and the deterioration of global environment, wind energy, as a kind of clean and pollution-free renewable energy, has attracted wide attention\(^1\). At present, the control strategies for variable pitch wind power generation system are as follows: optimal control based on data mining\(^2\), sliding mode control\(^3\), control based on state observer\(^4\) and so on. H\(\infty\) robust controller is designed to deal with the parameter uncertainties and external disturbances of the variable pitch system of wind turbines. Firstly, this paper simplifies the model of the hydraulic variable pitch system of wind turbines and rewrites the state equation of the nominal object. Then, according to the tracking performance of the variable pitch system, selected the weighting function by using the mixed sensitivity optimization method, and obtained the state space realization of the generalized controlled object. Afterwords, using the robust control toolbox, solve the algebraic Riccati equation, and obtained the H\(\infty\) controller, and the system performance is analyzed. Finally, simulation is conducted in Matlab to verify the effectiveness of the designed system and verify the superiority of H\(\infty\) control.

2. Hydraulic Variable Pitch Control System Model
Hydraulic pitch control system of wind turbine is an automatic control system, including pitch controller, D/A converter, proportional reversing valve, hydraulic cylinder, crank connection structure, displacement sensor, blade and etc\(^5\).
The transfer function of hydraulic variable pitch system can be obtained as:
\[ G_0(s) = \frac{Y(s)}{I(s)} = \frac{K_i}{s^2 + \frac{2 \xi}{\alpha} s + \alpha^2 + K_i K_m} \]  

(1)

\( \alpha \) — hydraulic natural frequency; \( \xi \) — damping ratio of hydraulic system; \( K_m \) — feedback control factor, \( \text{mA/mm} \); \( K_c \) — hydraulic cylinder piston velocity amplification factor, \( \text{m}^2/\text{cm}^2 \cdot \text{s} \), and, \( K_v = \lambda_m / A_m \); \( K_i \) — electro-hydraulic proportional amplification factor, and, \( K_i = K_c / K_b \). The above equation can be simplified into:

\[ G_0(s) = \frac{K_i}{s + K_i K_m} \]  

(2)

3. H∞ Control Method and State Space Realization of Hydraulic Variable Pitch System

3.1. H∞ Control for Hydraulic Variable Pitch System

The figure below shows the standard H∞ control problem:

- \( z \) — evaluation signal output by the variable pitch system;
- \( y \) — actual measured value of variable pitch Angle;
- \( d \) — interference from external environment;
- \( u \) — input voltage of the system;
- \( G(s) \) — generalized controlled object of variable pitch system;
- \( K(s) \) — required variable pitch system controller.

According to the input/output relationship in Fig. 1, the generalized controlled object of hydraulic variable pitch \( G(s) \) can be described by the following state space:

\[ \begin{align*}
\dot{x} &= Ax + B_1d + B_2u \\
z &= C_1x + D_{11}d + D_{12}u \\
y &= C_2x + D_{21}d + D_{22}u
\end{align*} \]  

(3)

Therefore, the closed-loop transfer function \( d \) interfered to the evaluation output \( z \) in Fig. 3 can be expressed as:

\[ H_{\text{cl}}(s) = G_{11}(s) + G_{12}(s)K(s)[I - G_{22}(s)K(s)]^{-1}G_{22}(s). \]  

(4)

Mixed sensitivity problem is one of the most typical problems of H∞ control, when H∞ method is applied to design system, in order to ensure the system robust performance, usually mixed sensitivity design problem can be converted to optimization problems. The following figure shows the standard model of mixed sensitivity problem:

\[ \begin{align*}
W_1z &= z \\
W_2y &= y \\
W_3r &= r
\end{align*} \]
\( W_3 \) is the robust weight of the system; \( e \) is tracking error; \( r \) is external input. The closed-loop transfer functions of \( r \) to \( e, u \) and \( y \) are \( S, R, T \).

The external environment \( d \) interferes with the transfer function \( Z_1, Z_2, Z_3 \) is

\[
\varphi_{zw} = \begin{bmatrix} W_1S \\ W_1R \\ W_1T \end{bmatrix} = \begin{bmatrix} W_1(I + GK)^{-1} \\ W_2K(I + GK)^{-1} \\ W_1KG(I + GK)^{-1} \end{bmatrix}.
\]

(5)

The \( H^\infty \) control design problem of mixed sensitivity for hydraulic variable blade control strategy can be described as: search for true rational function controllers \( K(s) \), enable the hydraulic variable paddle system can maintain stable and rapid adjustment within a certain range, and the norm of the transfer function matrix of the control system can be minimized, \( \|\varphi_{zw}\| < 1 \) \([6-7]\).

By comparing equations (4) and (5), it can be concluded that the generalized controlled object of hydraulic variable pitch system is

\[
\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} d^+ \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -GW_1 \\ W_2 \\ GW_3 \end{bmatrix} \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}^T = \begin{bmatrix} W_1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -GW_1 \\ W_2 \\ GW_3 \end{bmatrix} \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} = P_3 \begin{bmatrix} d \\ u \end{bmatrix}.
\]

(6)

\( P_3 \) is the generalized controlled pair matrix of hydraulic variable pitch system.

3.2. State Space Implementation of Generalized Controlled Objects

In this section, according to the state equation of the nominal object of the hydraulic variable pitch system and the designed three weighting functions, the state space realization of the generalized controlled object of the hydraulic variable pitch system is deduced.

(1) space of hydraulic variable pitch system transfer function \( G(s) \) can be realized as:

\[
G_n = \begin{bmatrix} A_{n0} & B_{n0} \\ C_{n0} & D_{n0} \end{bmatrix} = \begin{bmatrix} -K_u & K_w \\ 0 & K_t \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}.
\]

(7)

(2) The state space of the three weighted functions designed is implemented as:

\[
W_1 = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}, W_2 = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}, W_3 = \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix}.
\]

(8)

(3) The state space realization of hydraulic variable pitch generalized controlled object:

According to the (12) and (13), the state space equation of the generalized controlled object of the system can be written as

\[
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -K_u & 0 & 0 \\ -K_u & A_1 & 0 \\ B_1K_u & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} d^+ + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} u.
\]

\[
\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ y \end{bmatrix} = \begin{bmatrix} -D_uK_u & C_1 & 0 \\ 0 & C_1 & 0 \\ D_uK_u & 0 & C_3 \\ -K_t & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} d^+ + \begin{bmatrix} D_u \\ D_u \\ D_u \end{bmatrix} u.
\]

(9)
4. Selection of Weighting Function and Design of Controller

4.1. Selection of Weighting Function

$W_1$ is a weighting function of the sensitivity function, used for shaping the function $S$. After the simulation comparison, $W_1 = \frac{180}{(25s + 1)}$.

$W_2$ is a weighted function of the complement sensitivity $R$, represents the norm bound of additive perturbation, which is determined by the perturbation range of system parameters and used to constrain the output of the designed hydraulic variable pitch controller. $W_2 = 0.008$.

$W_3$ is the right weighting function, represents the norm bound of multiplicative perturbation, and reflects the requirement of robust stability performance. After the simulation comparison, $W_3 = \frac{(10s + 32)}{320}$

4.2. Design of Controller $K(s)$

According to the equation of state and the weighting function of the system $W_1$, $W_2$, $W_3$, calculate to start after the oar opening process $K_1 = 9.876$, $K_2 = 1.075$, substituted into the system model to obtain the state space of generalized objects. The $H_\infty$ controller based on Riccati equation is used to solve the controller through the robust control toolbox in Matlab.

According to Matlab simulation, it can be concluded that the system can better realize the accurate tracking of the system at this time, with faster response rate and better amplitude margin and phase angle margin. The designed controller is $K(s) = \frac{2196s + 219.6}{S^2 + 28.26S + 1.129}$.

5. Simulation and Analysis

In order to verify the performance of the designed robust controller, the dynamic response curve, root track, Nyquist plots and bode plots of the system are drawn, as shown in Figure 3-6.

![Dynamic response curve](image1)

**Figure 3. Dynamic response curve**

![Root trajectory](image2)

**Figure 4. Root trajectory**

![Nyquist diagram](image3)

**Figure 5. Nyquist diagram**

![Bode diagram](image4)

**Figure 6. Bode diagram**

It can be seen from figure 5 that the rise time of the system is significantly shortened, reaching the steady-state value at 0.4s, and the system has no overshoot during operation, and the dynamic performance of the variable pitch system is obviously improved. According to figure 6, when the system changes from zero to infinity, the root trajectory curves are all located in the left plane of S, and the system's poles are located in the left half axis of S axis, so the system is stable. It can be seen from figure 8 bode that after adding the controller, the gain of low frequency is increased, and the cut-off frequency, phase Angle margin and system bandwidth are all significantly increased. This
shows that the system not only guarantees dynamic performance and tracking accuracy, but also enhances its robustness.

6. Summary
This paper, based on the principle of hydraulic variable pitch system corresponding dynamic model is established, and according to the write the state equations of the nominal system model, and then according to the mixed sensitivity optimization process is analyzed. Studying conforms to the hydraulic system of the three selected weighted function, obtained $H_{\infty}$ hydraulic variable pitch system of generalized controlled object, and then obtained the generalized state space of the controlled object, according to the robust control toolbox for $H_{\infty}$. The controller is simulated on Matlab platform. The designed system not only reduces the steady-state error, but also improves the robustness of the system.

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