Monitoring data quality for telehealth systems in the presence of missing data

Tahir Mahmood,1,2* Philipp Wittenberg,2* Inez Maria Zwetsloot,1* Hailiang Wang,1 and Kwok Leung Tsui1

1Department of System Engineering and Engineering Management, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong
2Department of Mathematics and Statistics, Helmut Schmidt University, Hamburg, Germany

*Corresponding author: i.m.zwetsloot@cityu.edu.hk

Abstract

Quality issue: All-in-one-station-based health monitoring devices are implemented in elder homes in Hong Kong to support the monitoring of vital signs of the elderly. During a pilot study, it was discovered that the systolic blood pressure was incorrectly measured during multiple weeks. A real-time solution was needed to identify future data quality issues as soon as possible.

Initial assessment: Control charts are an effective tool for real-time monitoring and signaling issues (changes) in data. In this study, as in other health care applications, many observations are missing. Few methods are available for monitoring data with missing observations.

Choice of solution: A data quality monitoring method is developed to signal issues with the accuracy of the collected data quickly. This method has the ability to deal with missing observations. A Hotellings $T^2$-squared control chart is selected as the basis for our proposed method.

Implementation: The proposed method is retrospectively validated on a case study with the known measurement error in the systolic blood pressure measurements. The method is able to adequately detect this data quality problem.

Evaluation: The proposed method was integrated in a personalized telehealth monitoring system and prospectively implemented in a second case study. It was found that the proposed scheme supports the control of data quality.

Lessons learned: Data quality is an important issue and control charts are useful for real-time monitoring of data quality. However, these charts must be adjusted to account for missing data that often occur in a health care context.

Key words: data quality, elderly, multivariate control charts, statistical quality control, vital sign monitoring
1 Quality issue

Telehealth is growing due to the fast development of sensor technology. This has enabled the development of relatively cheap and easy-to-use devices for (self-)evaluation of health indicators and well-being. These techniques have the potential to help current elder care providers to track vital signs, detect physiological changes and predict health risks.

A pilot study has been conducted with an all-in-one station-based telehealth device in Hong Kong to help track elder’s vital signs. This telehealth system is designed to provide computer-aided decision support for clinicians and community nurses. It also enables them to easily monitor and analyze an elder’s vital signs and well-being. For more details on this system see [1]. This study involved multiple elder care centers. In each center, elder’s volunteered to have their vital signs measured daily. For approximately three months, trained and qualified research staff visited the centers and assisted the elder’s to accomplish the measurements on each measurement day (usually five or six days a week). Vital signs were measured using a commercial all-in-one station-based telehealth device (TeleMedCare, Health Monitor, TeleMedCare, Sydney, Australia). Data is stored, processed, analyzed and summarized into a report which is given to the participants. The framework of this personalized health monitoring system is shown in the top panel of Figure 1.

![Figure 1. Overview of personalized health monitoring system, before the project (top) and after this project (bottom).](image)

A pilot study was setup to test this system (Figure 1). In one of the centers it was discovered that the systolic blood pressure was incorrectly recorded during multiple weeks. It was felt that a structural method was needed to detect data quality issues as soon as possible in the future. Therefore, a data quality monitoring method was developed to ensure the accuracy of vital signs data collection. This data quality monitoring method is integrated into the personalized health monitoring system, as illustrated in the lower panel of Figure 1. It is implemented on a daily basis with a feedback loop to the data collection device when a signal is observed.

2 Initial assessment

The elder care center where the systolic blood pressure was incorrectly measured (center A), is a typical elder day care center situated in Kowloon, Hong Kong. This pilot study had 24 participants and was conducted in a period from 18.12.2017 to 07.03.2018. For each participant, five vital signs were measured each day: body temperature (BT) in degrees Celsius (°C), systolic and diastolic blood pressure (SBP and DBP) in millimeters of mercury (mm Hg), heart rate (HR) in number of beats per minute and peripheral capillary oxygen saturation (SpO2) in percentages (%). We also include a second center (B) in our study. In center B there were 12 participants and the study ran from 01.03.2018 to 31.05.2018.

Figure 2 gives an impression of the collected data, the panels show the vital signs for all participants in center A (left) and center B (right) over the entire study period. Note that initial data is cleaned by deleting improbable low values (such as HR values of zero).

From Figure 2, it is clear that something changed in the SBP measurements starting from 12.02.2018. Retrospective investigation revealed that the SBP measurement subsystem was accidentally and unknowingly limited to a maximum value of 136. Hence, the collected data was capped for the rest of the study period. This data quality issue affected all study participants in center A.

In industrial applications, measurement system analysis (MSA) is performed to verify the accuracy of the measurement system. MSA studies evaluate the repeatability and reproducibility of the measurement system. A focus is put on the measurement system as a whole; including the measurement device, the people operating the device as well as the environment [2]. A control chart is an associated tool often used to visualize measurement variability. In this paper a data quality monitoring method based on a control chart is proposed to help improve the measurement accuracy of the telehealth device.

The proposed monitoring method is validated on center A and prospectively run on center B to monitor the data quality (for details see section 6).

3 Choice of solution

A control chart is a statistical and visual tool. It is designed to prospectively signal change in data stream quickly. Control charts have been used in health care for many years. A few examples are; [3] who implement a multivariate control chart in a clinical setting; [4] who implement control charts to monitor the quality of cardiac surgery; and [5] who use control charts to monitor the data collection in epidemiologic studies. Overviews of control charts in health care settings can be found in [6–8].

A control chart can also be used to monitor data quality. It may assist in detecting changes in the data collection system as timely as possible. [9] provide a framework for control charts and data quality monitoring.

An error in the measurement system will show up as a change in the level of the vital sign(s). Hence, a change in the level of vital sign(s) should be signaled. A standard method used to monitor multiple variables is the Hotelling’s $T^2$-squared control chart (see for example [3, 10]). As our all-in-one station-based telehealth device records multiple vital signs for each participant we em-
deployed a Hotelling’s $T^2$-squared control chart.

In observational studies missing data is often encountered (cf. [11]) and our study is no exception. We encounter three types of missing data. First, not all measurements are obtained daily. Second, due to the type of elder care centers (day care) not all elders show up everyday for measurements. And third, some elders join the program late or drop out early which resulted into a long streams of missing data at the start or end of the study period, see [12]. Figure 3 displays the varying number of obtained measurements (elders) over time for each indicator and center. This varying sample size shows that we have many missing data (without missing data all sample sizes would be equal to 24 of center A and 12 for center B).

The literature categorizes random missing data into two types: missing at random (MAR) and missing completely at random (MCAR) (for details see [13]). The propensity of the MAR observations is due to the random in-out of the elders and machining problems while observations in the data set are MCAR, when a random event (e.g., typhoon) occurred and it is independent of the machining problems and presence of elders. From the available information, it is assumed that the data is missing at random (MAR) in our study.

Traditional control charts are designed for complete data sets and it is difficult to run them with missing data. A solution to apply them to our incomplete dataset (addressed above as first type of missing data) with varying number of elder’s on each time point (afore-mentioned missing data type two and three) is needed. One way to deal with missing data is to perform data imputation to “fill in” the missing values (see for example [3, 14, 15]). However, for our settings, data imputation is unsuitable because late joining and dropping of elder’s is uncontrollable and the number of late joiners is unknown a priori. Therefore, it is difficult to impute the correct number of missing data streams.

Another way to deal with missing data is to use control charts for variable sample sizes (i.e., varying number of participants per day). Some methods are proposed by [16] for univariate and by [17, 18] for multivariate data. The latter methods assume that the varying sample sizes are known a-priori and that the researcher controls the number of samples. Hence these methods are not applicable here.

To the best of our knowledge, none of the existing control charts for variable sample sizes has the ability to adequately deal with random varying sample sizes in a multivariate setting. Hence, a new method is developed by adapting the Hotelling’s $T^2$-squared control chart to accommodate for missing data and random varying sample size without the need to impute missing data.

4 Method development

For the purpose of method development, each participant is indexed by $k; (k = 1, 2, \ldots, n)$, each vital sign by $j; (j = 1, 2, \ldots, p)$ and each day by $i$. Let $X_{ijk}$ be the measurement of person $k$ for vital sign $j$ on day $i$. Hence, $n$ vectors $X_i \in \mathbb{R}^p$ are obtained each day. Let $\mu$ denote the expectation of $X_i$ and $\Sigma$ the covariance matrix.
In order to setup a control chart, estimates of these unknown parameters are needed. To avoid biased estimates due to outliers in the data (see Figure 2), estimates of the mean vector \( \hat{\mu} \) and the covariance matrix \( \hat{\Sigma} \) are obtained using a robust estimation method. Various robust methods have been evaluated by [19] for the Hotelling’s \( T^2 \)-squared control chart. Here, the orthogonalized Gnanadesikan–Kettenring (OGK) estimation method (cf. [20]) is used, because it provides positive definite and approximately affine equivariant robust estimates. The OGK estimators are obtained using the R-package rrcov [21] with clubbed data.

To monitor the data quality, the average value for each vital sign is computed by averaging over those patients who showed up for measurement on day \( i \). Giving vector \( \bar{X}_i \) of \( p \) elements \( (\bar{X}_{i1}, \ldots, \bar{X}_{ip})' \). Here, \( \bar{X}_{ij} \) is the average of \( n_{ij} \) elders for vital sign \( j \). The number of elders \( n_{ij} \) depends on the day \( i \) and the vital sign \( j \), and the changing \( n_{ij} \) allow us to model the missing data \( (n_{ij} \text{ are displayed in Figure } 3) \). In this study, it is assumed that \( \bar{X}_i \) follows a multivariate normally distributed with a mean vector \( \mu_{\bar{X}_i} \) and covariance matrix \( \Sigma_{\bar{X}_i} \). We confirm this assumption by performing several multivariate normality tests implemented in the R-package MVN [22].

In section 3, we argued that our data is MAR. Under this MAR assumption it follows that \( \mu_{\bar{X}_i} = \mu \) and after some derivation (see Appendix), we also have

\[
\hat{\Sigma}_{\bar{X}_i} = \landau_{\bar{X}_i} \odot \hat{\Sigma}.
\]  

(1)

Here, \( \odot \) denotes element wise matrix multiplication, which is also known as Hadamard multiplication and \( \landau_{\bar{X}_i} \) is a matrix weighting the elements of \( \hat{\Sigma} \) according to the number of paired observations (participants) available for measurement on day \( i \). The matrix \( \landau_{\bar{X}_i} \) is a \( p \times p \) matrix defined as

\[
\landau_{\bar{X}_i} = \left[ \frac{|u_{ij} \cap u_{ij}'|}{n_{ij} r_{ij'}} \right]_{j,j'=1,\ldots,p}.
\]

(2)

Here, \( u_{ij} \) is the set of participants for whom a measurement of vital sign \( j \) on day \( i \) is available and \( n_{ij} = |u_{ij}| \) is the number of elements in \( u_{ij} \), i.e., the number of participants. For example, if on day \( i \) two participants, say 01 and 03, show up to measure vital sign \( j = 2 \) then \( u_{ij} = \{01, 03\} \) and \( n_{ij} = 2 \). Note that under complete observations, it follows that \( n_{ij} = n \) and hence \( \landau_{\bar{X}_i} = [1/n] \).

The design of the Hotelling’s \( T^2 \)-squared control chart is

\[
T^2_i = (\bar{X}_i - \hat{\mu})' \hat{\Sigma}_{\bar{X}_i}^{-1} (\bar{X}_i - \hat{\mu})
\]

(3)

and the control chart signals, when \( T^2_i \) exceeds the control limit (CL). The control limit is obtained by the following simulation procedure:

1. Generate a data set of \( m \times \bar{n} \) vectors \( X_i \sim N(\hat{\mu}, \hat{\Sigma}) \). Here \( m \) represents the number of days of data used to estimate the mean and covariance, and \( \bar{n} \) is defined as the overall average number of elders.

2. Based on this data set, compute the robust mean vector \( \hat{\mu}_{OGK} \) and covariance matrix \( \hat{\Sigma}_{OGK} \) by using the OGK estimation method.

Figure 3. Sample size of vital signs for elders in center A (left panel) and B (right panel).
3. Generate $\bar{n}$ new vectors $X_i \sim N(\bar{\mu}, \hat{\Sigma})$. Compute the average vector $\bar{X}_i$ and the Hotelling’s $T^2$-squared statistic as $T^2_i = (\bar{X}_i - \bar{\mu}_{OGK})^\top \hat{\Sigma}_i^{-1}(\bar{X}_i - \bar{\mu}_{OGK})$.

4. Repeat step 3 for 10,000 times and select the $(1-\alpha)$-th quantile of $T^2$ as control limit $CL$.

Repeat steps 1–4, a large number of times and calculate the final $CL$ as the average of all obtained control limits from step 4. For both center A and B, $m$ equals 19. For center A $\bar{n}$ equals 20 and for center B $\bar{n}$ equals 9. From the simulation, we get $CL = 17.31$ for center A and $CL = 18.59$ for the center B. These $CL$s are obtained by fixing the false alarm rate $\alpha = 0.02$, which implies a false alarm every 50 days on average.

One additional modification to the chart is necessary; whenever a vital sign is not measured for at least 1 participant, one element in $\bar{X}_i$ will be missing. The corresponding row from $(\bar{X}_i - \bar{\mu})$ is then removed, the corresponding row and column from $\Sigma_\bar{X}$ are also removed and finally, the control limit is adjusted for that time instance. For example, in center A, only three vital signs are measured on the last two days of the study (see Figure 3), so a reduced $CL$ is plotted at 13.29.

Whenever $T^2$ exceeds the control limit $CL$, a signal is observed. This signal should be investigated and appropriate corrective action should be taken. Usually, in multivariate control charts, a decomposition method is used to determine which variable(s) are responsible for the signal. For our proposed Hotelling’s $T^2$-squared control chart, the Mason-Young-Tracy (MYT) decomposition method was adopted [23]. In the MYT decomposition, $T^2$ statistics are calculated for all possible subsets of vital signs and plotted against the respective $CL$ to identify the vital sign or combination of vital signs, which give a signal. The $CL$s for all possible combinations of vital signs are also obtained by using the simulation procedure.

Our control chart is designed to detect data quality issues. A signal may show that the SBP measurement subsystem is capped (the reason for this project). However, a signal can also be caused by changes in individual participant’s vital signs. This signal than may point towards the need for medical assistance (rather than measurement system calibration). In the elder care centers, well trained personal takes care of the elders. In this project, we consulted with them after any signal caused by individual vital sign levels. A separate project is being conducted to design a monitoring system for individuals. However, before we can monitor individual’s vital signs for changes in health, we need to ensure that the collected data is accurate. That is where our proposed method comes in.

5 Implementation: retrospectively for center A

Next the proposed method is retrospectively implemented for center A. The first 19 days, from 19.12.2017 to 16.01.2018, are used to obtain the OGK estimates:

$$\hat{\mu} = \begin{pmatrix} \hat{\mu}_{BT} \\ \hat{\mu}_{SBP} \\ \hat{\mu}_{DBP} \\ \hat{\mu}_{HR} \\ \hat{\mu}_{SpO_2} \end{pmatrix} = \begin{pmatrix} 35.93 \\ 131.31 \\ 67.55 \\ 73.38 \\ 97.94 \end{pmatrix}$$

and

$$\hat{\Sigma} = \begin{bmatrix} 0.10 & -0.01 & -0.06 & 0.28 & 0.00 \\ -0.01 & 254.92 & 22.33 & -48.58 & 0.56 \\ -0.06 & 22.33 & 87.13 & 3.54 & -0.04 \\ 0.28 & -48.58 & 3.54 & 130.38 & 5.18 \\ 0.00 & 0.56 & -0.04 & 5.18 & 2.36 \end{bmatrix}.$$ 

Further, the control chart is plotted to monitor the data quality in case A. The Hotelling’s $T^2$-squared statistics are calculated by using Equation (3) and plotted against the $CL = 17.31$ in Figure 4. The following catches the eye:

- The first four signals are observed in the interval from 21.12.2017 to 29.12.2017. The MYT decomposition method identifies all signals as caused by changes in the mean of the DBP measurements.

- The next four signals are observed in the interval from 10.01.2018 to 02.02.2018. MYT decomposition method classifies the 5th, and 7th signal as driven by variation in the DBP and BT measurements. The 6th signal, on 25.01.2018, is due to the variation in SpO2 measurements. The 8th signal, on 02.02.2018, is due to variation in DBP.

- The signals from 20.02.2018 until 02.03.2018 were expected as these signals are due to the capped measurements of SBP. However, the signal is delayed by four days. The MYT decomposition indicates that apart from SBP the BT also influenced the signal.

- In the last two days, two signals are observed. The MYT decomposition indicated a decrease in the means of the BT measurements.

![Figure 4. Hotelling’s $T^2$-squared chart for center A.](image-url)
Overall, the collected data are not stable in this pilot study. Many issues, especially with the SBP measurement are discovered. Hence, it is necessary to implement a data quality monitoring method into the telehealth system (Figure 1). Such a method can help to detect undesirable situation as soon as possible so that appropriate action can be taken. Apart from the monitoring scheme, other actions such as training of staff and calibrated working procedures were also applied to ensure repeatable and reproducible measurements.

6 Evaluation: prospective implementation for center B

To evaluate the usefulness of our developed Hotelling’s $T^2$-squared control chart, we prospectively implement the developed control chart for center B. Thereby, we verify in real-time, whether the telehealth system is collection stable data. The data collection started on 02.03.2018 and the first week of observations are used to estimate the parameters ($\hat{\mu}$ and $\hat{\Sigma}$). A signal was observed on 09.03.2018, after careful investigation and MYT decomposition, it was discovered that the DBP measurements were set to a maximum level on that day. This issue was solved by adjusting the telehealth device.

The data collection was continued in the subsequent weeks and the Hotelling’s $T^2$-squared control chart, based on re-estimated parameters was plotted. This iterative method was continued until $m = 19$ days of data were collected. The OGK estimates for the mean vector and covariance matrix are

$$\hat{\mu} = \begin{pmatrix} \hat{\mu}_{BT} \\ \hat{\mu}_{SBP} \\ \hat{\mu}_{DBP} \\ \hat{\mu}_{HR} \\ \hat{\mu}_{SpO_2} \end{pmatrix} = \begin{pmatrix} 36.83 \\ 133.96 \\ 69.80 \\ 71.11 \\ 96.96 \end{pmatrix}$$

and

$$\hat{\Sigma} = \begin{bmatrix} 0.12 & 0.90 & -0.06 & 0.55 & 0.23 \\ 0.90 & 328.42 & 29.28 & 60.74 & 5.59 \\ -0.06 & 29.28 & 69.99 & 27.13 & -0.39 \\ 0.55 & 60.74 & 27.13 & 184.52 & 0.29 \\ 0.23 & 5.59 & -0.39 & 0.29 & 3.34 \end{bmatrix}.$$  

The Hotelling’s $T^2$-squared control chart for the full study period is displayed in Figure 5.

A second signal was observed on 13.04.2018, after careful investigation the signal can be attributed to a very high HR for a single participant. Another signal was observed on 29.05.2018, the MYT decomposition method identified that the cause for the signal is an unusual high SBP level and an unusual low $SpO_2$ level.

Overall, the implementation of the data quality monitoring method provides a timely indication of data quality issues which may lead to adjustment of the telehealth system and a decrease in the wastage of resources.

7 Discussion and lessons learned

Telehealth provides many opportunities for innovations in health care. However, the accuracy and reliability of measurements may be challenging to guarantee when various people, especially non-medical experts, perform the measurements [24–26]. There are many ways to validate and guarantee data accuracy, such as training people and calibration of the measurement system. In this article, we set forth an additional check to verify the real-time quality of measured data. A Hotelling’s $T^2$-squared control chart is designed, which is modified to deal with MAR data. After testing the proposed method on a case study with known data quality issues, the control chart is prospectively implemented on a second case study. In the second study, a data quality issue is detected after one week, which was timely solved. Hence, a regular focus on the data quality helps to ensure the validity and accuracy of the collected measurements and a quick feedback to the data quality monitoring system is essential to solve problems on-time.

Data monitoring is also important on an individual level and more comprehensive models are needed to deal with the heterogeneity of individuals. An individual monitoring tool can be helpful to detect health changes for each elder separately. However, before this can be done, the data has to be accurate. Our method for data quality monitoring can thus be seen as a first step before monitoring the health individually. Our focus is on handling the variable sample sizes, in our application due to MAR type missing data. Our approach can also be useful for other applications with missing data where monitoring is required.

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References

1. Yu L, Chan WM, Zhao Y et al. Personalized health monitoring system of elderly wellness at the community level in Hong Kong. *IEEE Access* 2018;6:35558–35567. doi:10.1109/ACCESS.2018.2848936.

2. Montgomery DC. Introduction to Statistical Quality Control. *John Wiley & Sons* 2007;7th Edition.

3. Waterhouse M, Smith I, Assareh H et al. Implementation of multivariate control charts in a clinical setting. *Int J Qual Health Care* 2010;22:408–414. doi:10.1093/intqhc/mxq044.

4. Gan FF, Tang X, Zhu Y et al. Monitoring the quality of cardiac surgery based on three or more surgical outcomes using a new variable life-adjusted display. *Int J Qual Health Care* 2017;29:427–432. doi:10.1093/intqhc/mxz033.

5. Harel O, Schisterman EF, Vexler A et al. Monitoring quality control. *Epidemiology* 2008;19:621–627. doi:10.1097/ede.0b013e318176b9b2.

6. Woodall WH. The use of control charts in healthcare and public-health surveillance. *J Qual Technol* 2006;38:89–104. doi:10.1080/00224065.2006.11918593.

7. Thor J, Lundberg J, Ask J et al. Application of statistical process control in healthcare improvement: Systematic review. *Qual Saf Health Care* 2007;16:387–399. doi:10.1136/qshc.2006.022194.

8. Tennant R, Mohammed MA, Coleman JJ et al. Monitoring patients using control charts: A systematic review. *Int J Qual Health Care* 2007;19:187–194. doi:10.1093/intqhc/mzm015.

9. Jones-Farmer LA, Ezell JD, Hazen BT. Applying control chart methods to enhance data quality. *Technometrics* 2014;56:29–41. doi:10.1080/00401706.2013.804437.

10. Rigdon SE and Fricker RD. Health surveillance. In *ICSA Book Series in Statistics*, pages 203–249. Springer International Publishing, 2015. doi:10.1007/978-3-319-18536-1_10.

11. Needham DM, Sinopoli DJ, Dinglas VD et al. Improving data quality control in quality improvement projects. *Int J Qual Health Care* 2009;21:145–150. doi:10.1093/intqhc/mzp005.

12. Celler B, Argha A, Varnfield M et al. Patient adherence to scheduled vital sign measurements during home telemonitoring: Analysis of the intervention arm in a before and after trial. *JMIR Med Inform* 2018;6:e15. doi:10.2196/medinform.9200.

13. Schafer JL, Graham JW. Missing data: Our view of the state of the art. *Psychol Methods* 2002;7:147–177. doi:10.1037/1082-989X.7.2.147.

14. Hebert PL, Taylor LT, Wang JJ et al. Methods for using data abstracted from medical charts to impute longitudinal missing data in a clinical trial. *Value Health* 2011;14:1085–1091. doi:10.1016/j.jval.2011.05.049.

15. Madbuly DF, Maravelakis PE, Mahmoud MA. The effect of methods for handling missing values on the performance of the MEWMA control chart. *Commun Stat Simul Comput* 2013;42:1437–1454. doi:10.1080/03610918.2012.665547.

16. Huang W, Shu L, Woodall WH et al. CUSUM procedures with probability control limits for monitoring processes with variable sample sizes. *IEE Transactions* 2016;48:759–771. doi:10.1109/704817x.2016.1146422.

17. Aparisi F. Hotelling’s $T^2$ control chart with adaptive sample sizes. *Int J Prod Res* 1996;34:2853–2862. doi:10.1080/00207549608905062.

18. Kim K, Reynolds JR, MR. Multivariate monitoring using an MEWMA control chart with unequal sample sizes. *J Qual Technol* 2005;37:267–280. doi:10.22240/804437.

19. Alfaro JL, Ortega JF. A comparison of robust alternatives to hotelling’s $T^2$ control chart. *J Appl Stat* 2009;36:1385–1396. doi:10.1080/02664760902810813.

20. Maronna RA and Zamar RH. Robust estimates of location and dispersion for high-dimensional datasets. *Technometrics* 2002;44:307–317. doi:10.1198/004017002754062.

21. Todorov V, Filzmoser P. An object-oriented framework for multivariate analysis. *J Stat Softw* 2009;32:1–47. doi:10.18637/jss.v032.i03.

22. Korkmaz S, Goksuluk D, Zararsiz G, Mvn: An r package for assessing multivariate normality. *The R Journal* 2014;6:151–162.

23. Mason RL, Tracy ND, Young JC. Decomposition of $T^2$ for multivariate control chart interpretation. *J Qual Technol* 1995;27:99–108. doi:10.22240/804437.

24. Brewster L, Mountain G, Wessels B et al. Patient adherence to scheduled vital sign measurements during home telemonitoring: Analysis of the intervention arm in a before and after trial. *JMIR Med Inform* 2018;6:e15. doi:10.2196/medinform.9200.
Appendix

In this appendix, we provide the derivation of the adjustment factor $\mathcal{V}_i$ as used in equation (1): $\Sigma \bar{X}_i = \mathcal{V}_i \odot \Sigma$. As stated in equation (2), $\mathcal{V}_i$ is equal to

$$\mathcal{V}_i = \left[ \frac{|u_{ij} \cap u_{ij'}|}{n_{ij}n_{ij'}} \right]_{j,j'=1,...,p}.$$

As discussed in section 4, $u_{ij}$ is the set of elders who showed up for measuring their vital sign $j$ on day $i$ and define $n_{ij} = |u_{ij}|$. Recall that the matrix $\Sigma \bar{X}_i$ is the covariance matrix for the random mean vector $\bar{X}_i$ which is defined as

$$\bar{X}_i = \left[ \frac{1}{n_{ij}} \sum_{k \in u_{ij}} X_{ijk} \right]_{j=1,...,p}.$$

Following a similar set-up as [18], we derive the covariance between any two elements $j$ and $j'$ of $\bar{X}_i$ as

$$Cov(\bar{X}_ij, \bar{X}_ij') = Cov \left( \frac{1}{n_{ij}} \sum_{k \in u_{ij}} X_{ijk}, \frac{1}{n_{ij'}} \sum_{k' \in u_{ij'}} X_{ij'k'} \right) = \frac{1}{n_{ij}n_{ij'}} \sum_{k \in u_{ij}} \sum_{k' \in u_{ij'}} Cov(X_{ijk}, X_{ij'k'}) .$$

We assume independence of observations between individual elder, i.e., $Cov(X_{ijk}, X_{ij'k'}) = 0$ whenever $k \neq k'$. Now it follows that

$$\begin{align*}
\Sigma \bar{X}_i & = \left[ \frac{|u_{ij} \cap u_{ij'}|}{n_{ij}n_{ij'}} \sigma_{jj'} \right]_{j,j'=1,...,p} \\
& = \left[ |u_{ij} \cap u_{ij'}| / n_{ij}n_{ij'} \sigma_{jj'} \right]_{j,j'=1,...,p} \\
& = \left[ \frac{|u_{ij} \cap u_{ij'}|}{n_{ij}n_{ij'}} \sigma_{jj'} \right]_{j,j'=1,...,p}.
\end{align*}$$

Alternatively, we can write

$$\Sigma \bar{X}_i = \left[ \frac{|u_{ij} \cap u_{ij'}|}{n_{ij}n_{ij'}} \sigma_{jj'} \right]_{j,j'=1,...,p} .$$

Where $\mathcal{V}_i = |u_{ij} \cap u_{ij'}|/n_{ij}n_{ij'}$, can be interpreted as the weighting matrix having which takes the number of observed data points into account. Hence, the covariance matrix of $\bar{X}_i$ is equal to

$$\Sigma \bar{X}_i = \mathcal{V}_i \odot \Sigma$$

where, $\odot$ is the Hadamard product, which denotes element wise matrix multiplication and $\Sigma$ is the covariance matrix of the individual data.