Minimum Interior Temperature for Solid Objects Implied by Collapse Models

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Heating induced by the noise postulated in wave function collapse models leads to a lower bound to the temperature of solid objects. For the noise parameter values $\lambda = 10^{-8}\,\text{s}^{-1}$ and $r_C = 10^{-5}\,\text{cm}$, which were suggested to make latent image formation an indicator of wave function collapse and which are consistent with the recent experiment of Vinante et al., the effect may be observable.

For metals, where the heat conductivity is proportional to the temperature at low temperatures, the lower bound (specifically for RRR=30 copper) is $\sim 5 \times 10^{-11}(L/r_C)\,\text{K}$, with $L$ the size of the object. For the thermal insulator Torlon 4203, the comparable lower bound is $\sim 3 \times 10^{-6}(L/r_c)^{0.63}\,\text{K}$. We first give a rough estimate for a cubical metal solid, and then give an exact solution of the heat transfer problem for a sphere.

There is increasing interest in testing wave function collapse models, by searching for effects associated with the small noise which drives wave function collapse when nonlinearly coupled in the Schrödinger equation. The original proposals for the noise coupling strength were so small that devising suitable experiments was problematic, but the situation has changed with the suggestion that latent image formation, such as deposition of a developable track in an emulsion or in an etched track detector, already constitutes a measurement embodying wave function collapse. A recent cantilever experiment of Vinante et al. has set bounds consistent with the enhanced parameters suggested in [1], and reports a possible noise signal. Thus, it is timely to consider other experiments which could detect or rule out a noise coupling with the strength suggested by [2].

For a body comprised of a group of particles of total mass $M$, the secular center-of-mass energy gain is given by the formula

$$\frac{dE}{dt} = \frac{3}{4} \frac{\hbar^2}{r^2} \frac{M}{m_N^2},$$

with $m_N$ the nucleon mass. For a body of dimensions $L$ larger than the correlation length $r_C$, the different groups of particles will have independent center of mass motions, and so the energy gain from Eq. (1) will take the form of thermal energy. With the stipulation that we will only be considering bodies of uniform mass density $\rho$ with dimensions $L >> r_C$, we rewrite Eq. (1) as a
formula for the energy deposition rate per $Q$ unit volume given as

$$Q = \frac{3}{4} \frac{\hbar^2}{r_c^2} \frac{\rho}{m_N^2}. \quad (2)$$

For an initial estimate, consider a solid metal cube of side length $L$, with heat conductivity $k(T)$, which at temperatures $T$ below $\sim 10$ K obeys the linear law $k(T) = k_0 T$. The specific example that we shall use for an estimate is medium purity Residual Resistivity Ratio (RRR) = 30 copper, for which $k_0 \simeq 45$ W/(m K$^2$) = 45 Joule/(m K$^2$ s). Assuming the body has surface temperature $T_s = 0$ and central temperature $T_c$, the rate at which energy is transported out by conduction is approximately

$$E_{\text{out}} = 6L^2(k_0 T_c/2)(T_c/L) = 3Lk_0 T_c^2. \quad (3)$$

At equilibrium, this must balance the rate of noise-induced heating given by

$$E_{\text{in}} = QL^3. \quad (4)$$

Equilibrium can be attained in a reasonably short time since the thermal diffusivity of metals, which is proportional to the ratio of the heat conductivity $k$ to the specific heat capacity $c_p$ times the density, increases as the temperature decreases [6], because $c_p$ decreases more rapidly with decreasing temperature than does $k$. Equating $E_{\text{out}}$ with $E_{\text{in}}$ gives a formula for the central temperature $T_c$,

$$T_c = \frac{\theta L}{r_c K}, \quad (5)$$

with $\theta$ the dimensionless number

$$\theta = \left[ \frac{\lambda \hbar^2 \rho}{4(k_0 K^2 m_N^2)} \right]^{1/2}. \quad (6)$$

Taking the values (for RRR=30 copper)

$$\lambda = 10^{-8} \text{s}^{-1},$$
$$\hbar = 1.1 \times 10^{-34} \text{ Joule \ s},$$
$$\rho = 9 \text{ gm cm}^{-3} = 5.0 \times 10^{27} \text{ (MeV}/c^2) \text{ cm}^{-3},$$
$$k_0 K^2 = 45 \text{ Joule/}(\text{m} \text{ s}),$$
$$m_N = 940 \text{ MeV}/c^2,$$
$$c = 3 \times 10^{10} \text{ cm/s},$$
$$1 \text{ Joule} = 6.2 \times 10^{12} \text{ MeV}.$$
we find

\[ \theta \simeq 4.6 \times 10^{-11} \]  

(8)

For \( L = 1 \text{m} \), Eq. (5) gives \( T_c \sim 5 \times 10^{-4} \text{ K} \). For comparison, 400 kg of copper has been cooled to \( 6 \times 10^{-3} \text{ K} \), not constraining the \( \lambda \) value used in this \( T_c \) estimate.

A spin temperature of \( 0.1 \times 10^{-5} \text{ K} \) and a lattice temperature of \( 6 \times 10^{-5} \text{ K} \) have been reported for a \( 0.4 \times 4 \times 25 \text{ mm}^3 \) rhodium single crystal. Taking the smallest dimension 0.4 mm as the relevant \( L \) for an estimate, Eq. (5) gives \( T_c \simeq 4 \times 10^{-7} \text{ K} \). Thus assuming that this estimate should be compared to the lattice temperature and not the spin temperature (an assumption that deserves further study), the rhodium experiment also does not give a constraint on the noise coupling parameter \( \lambda \). Cooling of a drum-shaped aluminum membrane 20 microns wide and 100 nm thick has been reported to the temperature \( 4 \times 10^{-4} \text{ K} \). Again taking the minimum dimension, which in this case is \( \sim r_c \), as the relevant \( L \), this experiment also does not give a constraint on \( \lambda \).

To give a more precise estimate of \( T_c \) and of the temperature variation throughout the volume of a solid body, we consider the simplest case of a sphere of radius \( L \) with thermal conductivity \( k(T) \) and heating rate \( Q \). At a given distance \( R \) from the center of the sphere, the energy transport rate thorough the spherical surface of radius \( R \) is equal to

\[ E_{\text{out}} = -4\pi R^2 k(T) \frac{dT}{dR} \]  

(9)

which at equilibrium must balance the heating rate of the volume within radius \( R \),

\[ E_{\text{in}} = \frac{4\pi}{3} R^3 Q \]  

(10)

giving the differential equation

\[ -k(T) \frac{dT}{dR} = \frac{1}{3} R Q \]  

(11)

Integrating from the center of the sphere at radius 0 to radius \( R \), this gives

\[ -\int_{T_c}^{T} k(u) du = \frac{R^2 Q}{6} \]  

(12)

For \( k(u) = \hat{k}_0 u^\beta \), this becomes

\[ -\frac{\hat{k}_0}{1 + \beta} (T^{1+\beta} - T_c^{1+\beta}) = \frac{R^2 Q}{6} \]  

(13)
If we assume a boundary condition $T = T_s$ at the outer surface of the sphere at $R = L$, Eq. (13) implies that

$$ (T_c^{1+\beta} - T_s^{1+\beta}) = \frac{1 + \beta L^2 Q}{k_0} \frac{1}{6} , $$

which gives the inequality

$$ T_c = \left[ T_s^{1+\beta} + \frac{1 + \beta L^2 Q}{k_0} \frac{1}{6} \right]^{1/(1+\beta)} \geq \left[ \frac{1 + \beta L^2 Q}{k_0} \frac{1}{6} \right]^{1/(1+\beta)} , $$

so that on inserting Eq. (2) we get

$$ T_c \geq \left[ \frac{1 + \beta \frac{\lambda h^2 \rho}{(k_0 K^{1+\beta}) 8 m^2 N}}{L/r_c} \right]^{1/(1+\beta)} \left( \frac{L}{r_c} \right)^{2/(1+\beta)} K . $$

Specializing to a metal with $\beta = 1$ and $k_0 = k_0$, Eq. (16) reproduces Eqs. (5) and (6). For the thermal insulator Torlon 4203 [10], with density $1.42$ gm cm$^{-3}$ and with $k(T) = 6.13 \times 10^{-3}(T/K)^{2.18}$ W/(m K), so that $\beta = 2.18$ and $\hat{k}_0 K^{1+\beta} = 6.13 \times 10^{-3}$, Eq. (15) gives $T_c \geq 3.4 \times 10^{-6}(L/r_c)^{0.63}$ K. For example, for a Torlon sphere of radius $L = 50$ cm, the central temperature $T_c = 5.6 \times 10^{-2}$ K, nearly a factor of 10 bigger than the temperature attained in [7] using the CUORE experiment cryostat.\(^1\) This shows that with thermal insulating material arranged in a compact geometry, such as a sphere or cube, the effect we are proposing could be detected using the CUORE cryostat. In the actual running of the CUORE experiment, the bolometers consisted of $5 \times 5 \times 5$ cm$^3$ cubes of TeO$_2$ stacked in 19 towers, so all (except those at the ends) are in the same geometry relative to the cryostat. With this configuration of material, one would not expect to see a dramatic difference between internal and surface temperatures.

Equations (12)–(14) can be used to give the temperature profile in the sphere as a function of radius $R$. For a cylinder of infinite length, the same equations apply with the substitution $Q \to 3Q/2$, with radii now referring to the cylinder. To get the temperature profile for other geometries of interest, such as an ellipsoid of revolution, a cylinder of finite length, or a rectangular parallelepiped, one must solve the nonlinear differential equation governing thermal equilibrium

$$ - \nabla \cdot (k(T) \nabla T) = Q $$

with suitable boundary conditions. When $k(T) = \hat{k}_0 T^\beta$, using $T^\beta \nabla T = (1 + \beta)^{-1} \nabla T^{1+\beta}$, Eq. (17) becomes the Poisson equation

$$ \nabla^2 T^{1+\beta} + \frac{(1 + \beta)Q}{\hat{k}_0} = 0 , $$

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\(^1\) Taking $\lambda = 10^{-7.7}$ s$^{-1}$ as suggested in [4], instead of the nominal value $\lambda = 10^{-8}$ s$^{-1}$ used in our estimates, increases this by a factor $10^{0.3/3.18} = 1.24$ to $T_c = 7.0 \times 10^{-2}$ K.
which can be solved by standard methods \[11\].

To conclude, the decrease in thermal conductivity at low temperatures for both metals and thermal insulators results in a “trapped heat” phenomenon, in which the noise-induced heating associated with collapse models results in lower bounds on the internal temperature of solid objects. The fact that these lower bounds scale up with increasing $L/r_c$ may make experiments to search for them feasible. Clearly measuring the central temperature $T_c$ of a solid object without disturbing its thermal equilibrium will be a technical challenge. For larger objects, sensors with fine wire leads could be used. For smaller objects, central temperatures could be probed using a small hole from the exterior to the central region, through which molecules of an evaporative medium placed at the center can pass, or through which a laser beam can be directed to detect the state of molecular motion at the center. In designing experiments, it will be important to make sure that the volumetric heating per unit mass from radioactivity and particle penetration is significantly less than

\[
\frac{dE}{dt \, dM} = \frac{3}{4} \frac{\hbar^2}{L_c^2} \frac{1}{m_N^2} \simeq 20 \text{MeV gm s} \quad ,
\]

where we have again used the parameter values of Eq. \[7\]. This brings up an important caution underlying our analysis, which is that we have assumed that the energy production rate of Eqs. \[1\] and \[19\] applies to laboratory scale objects. As already noted in \[1\], this assumption breaks down when applied to the Earth’s heat flow, where assuming an energy production rate of Eq. \[19\] throughout the interior of the Earth leads to an internal heat production roughly three orders of magnitude larger than what is observed. However, as also noted in \[1\], when the effects of dissipation are included, as in the model of Bassi, Ippoliti, and Vachini \[12\], the rate of heat production can vanish at large times where a limiting temperature is reached. For example, with the parameters of \[12\], a limiting temperature of 0.1 K is reached on a time scale of billions of years, indicating a current noise-induced Earth heat production rate much smaller than given by naive application of Eq. \[19\]. Further study of this issue is warranted.

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