Towards Expectation-Maximization by SQL in RDBMS

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ABSTRACT
Integrating machine learning techniques into RDBMSs is an important task since there are many real applications that require modeling (e.g., business intelligence, strategic analysis) as well as querying data in RDBMSs. Without integration, it needs to export the data from RDBMSs to build a model using specialized machine learning toolkits and frameworks, and import the model trained back to RDBMSs for further querying. Such a process is not desirable since it is time-consuming and needs to repeat when data is changed. To support machine learning in RDBMSs, there are proposals that are platform-specific with limited functionalities to support certain modeling. In this paper, we provide an SQL solution that has the potential to support different machine learning modeling. As an example, we study how to support unsupervised probabilistic modeling, that has a wide range of applications in clustering, density estimation and data summarization, and focus on Expectation-Maximization (EM) algorithms, which is a general technique for finding maximum likelihood estimators. To train a model by EM, it needs to update the model parameters by an E-step and an M-step in a while-loop iteratively until it converges to a level controlled by some threshold or repeats a certain number of iterations. To support EM in RDBMSs, we show our answers to the matrix/vectors representations in RDBMSs, the relational algebra operations to support the linear algebra operations required by EM, parameters update by relational algebra, and the support of a while-loop. It is important to note that the SQL'99 recursion cannot be used to handle such a while-loop since the M-step is non-monotonic. In addition, assume that a model has been trained by an EM algorithm, we further design an automatic in-database model maintenance mechanism to maintain the model when the underlying training data changes. We have conducted experimental studies and will report our findings in this paper.

1 INTRODUCTION
Nowadays, integrating advanced data analytical techniques into RDBMSs is an urgent requirement for data integration [19], business intelligence and strategic analysis [14, 41]. Among these techniques that need to be integrated into RDBMSs, machine learning models play a leading role in predictive and estimation tasks. Although many specialized machine learning toolkits and frameworks (e.g., scikit [1] and TensorFlow [5]) are designed and developed, the approaches of building, utilizing and managing machine learning models in RDBMSs still need a comprehensive exploration. First, in most enterprise applications, data are stored in a database system. It is cumbersome and time-consuming of exporting the data from the database system and then feeding it into models, as well as importing the prediction and estimation results back to the database system. Second, it is highly desirable that users can build a model as to query data in RDBMSs, and query their data by exploiting the analysis result of the models trained as a part of query in a seamless similar in RDBMSs. What we need is a flexible way to train/query a machine learning model together with data querying by a high-level query language (e.g., SQL). Third, the data maintained in RDBMSs is supposed to change, and there is more data collected from time to time frequently. The analysis result of the models trained in a machine learning toolkit/framework may be out-dated, which requires to repeat the process of exporting data from RDBMSs followed by importing the model trained into RDBMSs. Given the fact that RDBMSs have the techniques (e.g., trigger) to manage data updating automatically when data changes, a further consideration is how to manage the machine learning models update automatically using the database techniques available.

There are efforts to support machine learning in RDBMSs. Early in-database machine learning is developed based on UDFs or specific libraries like MADlib [28] for PostgreSQL, Oracle Data Mining [50], DB2 Intelligent Miner, etc. On one hand, these functions and libraries can achieve optimized performance. On the other hand, they are platform-specific and have limited functionalities from the high-level syntax to the low-level implementation. It is difficult for database end-users to extend these libraries to support their own models that are not available in the libraries. To fulfill logical and physical data isolation, model-based views [18, 33] are proposed to support classification and regression analysis in database systems. Like regular views, model-based views support querying, materialization, and maintenance strategies. In brief, [18, 33] allow using an ad-hoc create view statement to declare a classification view. In this create view statement, [18] specifies the model by an as...fit...bases clause, and the training data is fed by an SQL query, while [33] specifies a model explicitly with using svm clause, where the features and labels are fed by feature function and labels, respectively. Here, feature function takes database attributes as the input features, and labels are database attributes. Although these approaches provide optimized implementation for classification models, their create view statement is lack of generality and deviating from the regular SQL syntax. In addition, the models supported are limited and implemented in a low-level form in a database system, which makes it difficult for ordinary database end-users to develop new models swiftly. In this work, we demonstrate our SQL recursive
query can define a model-based view in an explicit fashion and can be used to support many machine learning models. Different from [18, 33], we focus on unsupervised models in the application of in-database clustering, density estimation and data summarization.

We take a two-step approach to support machine learning in RDBMSs. The first step is to design in-database machine learning framework based on SQL, in particular SQL recursive queries, to show that SQL has its capability of doing machine learning by SQL in RDBMSs. The second step is to further find an efficient way to support queries for machine learning in RDBMSs. In this paper, we concentrate on the first step, following our previous work to support graph analytics in RDBMSs using SQL recursive queries [57]. The focus of this paper is on how to train machine learning models in RDBMSs, given that the core computations of model training are linear algebra operations and parameter updating.

Consider training a machine learning model. In brief, it has an initial parameter setting for the model, and will update the parameters in a while-loop iteratively until it converges to a level controlled by some threshold or repeats a certain number of iterations. The model trained is the model with the parameters obtained at the end of the while-loop. To do so in RDBMSs, there are several things that need to be handled: the ways to represent matrix/vector in RDBMSs, the relational algebra operations to support the linear algebra operations required, the way to update parameters, and the support of a while-loop. In this paper, we provide our answer to such needs.

The main contributions of this work are summarized below. First, we study in RDBMSs how to support unsupervised probabilistic modeling, that has a wide range of applications in clustering, density estimation and data summarization, and focus on Expectation-Maximization (EM) algorithms [39], which is a general technique for finding maximum likelihood estimators. In EM, the parameters to be trained are means, covariances, and mixing coefficients; there are two main steps in a while-loop, namely, E-step for expectation and M-step for maximization, and the parameters are updated in the while-loop. Second, we discuss how to represent data in RDBMSs in different ways, how to compute E-step and M-step using relational algebra operations (e.g., natural join, group-by and aggregation), how to update parameters using relational algebra operations, and how to support the while-loop using SQL recursive queries. It is worth mentioning that the recursion for EM is a mutual recursion of E-step and M-step. Recall that the E-step is to compute the conditional posterior probability by Bayesian inference, which can be supported by SQL as a monotonic operation, whereas the M-step is to compute and update the parameters of the model given a closed-form updating formula, which cannot be monotonic. This fact suggests that SQL’s recursion cannot be used to support EM, since SQL’s recursion (e.g., recursive with) only supports stratified negation, and therefore cannot support non-monotonic operations. We use XY-stratified [7, 55, 56], and provide an enhanced SQL recursion (e.g., with+), which can handle non-monotonic operations. We have implemented our approach as an SQL layer on top of PostgreSQL, and process our with+ using PostgreSQL. We show how to train a batch of classical statistical models [8], including Gaussian Mixture model, Bernoulli Mixture model, mixture of linear regression, Hidden Markov model, Mixtures of Experts, using the recursive SQL queries. Third, Given a model trained by an EM algorithm, we further design an automatic in-database model maintenance mechanism to maintain the model when the underlying training data changes. Inspired by the online and incremental EM algorithms [37, 42], we show how to obtain the sufficient statistics of the models to achieve the incremental even decremental model updating, without re-building the model using all data. It is worth mentioning that our setting is different. Different from the incremental EM algorithms which are designed to train the model during its iterative processing, we re-train the model by sufficient statistics using partial data being used to build the previous model in addition to the new data. Fourth, we have conducted experimental studies and will report our findings in this paper.

Organization Section 2 discusses the related works. In Section 3, we introduce the preliminaries including the EM algorithm and the requirements to support it in database systems. Then, our solution is sketched in Section 4 and the SQL recursive query and the implementation details are introduced in Section 5. In Section 6, we design a view update mechanism, which is facilitated by triggers. We conduct extensive experimental studies in Section 7 and conclude the paper in Section 8.

2 RELATED WORKS

Our study is closely related to the following research topics:

Machine Learning with SQL There are a board discussion on the approaches to using SQL for ML, which are at different levels of abstraction in the long-term research. In early years, Ordonez et al. presents pure SQL implementation of the EM algorithm in RDBMS, including K-means [46] and Gaussian Mixture Model [48]. Their approach, SQL EM [48], presents three strategies to implement EM in SQL: horizontal, vertical and a hybrid one. However, these implementations cannot support high dimensional data and a large cluster number effectively. Computations involving matrix and vector primitives are expressible in SQL with the aid of UDFs. For example, MAD [16, 28] is a in-database analytics library for matrix and vector operators. Luo et al. [38] extend SimSQL [12], a Hadoop-based relational database system to enable linear algebra computations. Taking the functions manipulating matrix/vector data type as a set of building blocks, a SQL query can support basic machine learning task, e.g., least square regression. Furthermore, UDF and UDA can be used to implement gradient methods [10, 24]. MAD and BISMARCK [24] use python UDF and UDA to support stochastic gradient decent, respectively. To deploy applications of stochastic models and analytics techniques, the monte carlo database system (MCDB) [12, 29] provides stochastic models to be directly used with the data stored in a large database. In MCDB, a UDF called value-generating (VG) function is used to draws samples from databases and bayesian learning can be performed by SQL queries subsequently.

Query Optimization for Machine Learning Using database techniques to improve the efficiency of machine learning application is a research focus currently. These techniques mainly aim at minimizing the computational redundancy incurred by the extra storage for normalized data and sparse data [34, 49] learn linear models over multi-table normalized data, introducing the idea of
learning on factorized database. Morpheus [15], a database middleware, can automatically convert the linear algebra operators of denormalized matrix/vector to normalized data by a set of rewriting rules. [35] utilizes tuple-oriented compression to reduce the data redundancy for mini-batch stochastic gradient descent. In addition, in the ML system design, logical and physical plan optimization techniques, like plan simplification and rewriting [9, 30, 31], operator selection [9], physical operator fusion [9, 23], delta updating rules [44] are widely used to improve the performance of the system. These optimization techniques for linear algebra and matrix calculus provides a large potential to improve the performance of model-based view in different scenarios.

Declarative Language for Machine Learning. Apart from pure SQL, there have been some efforts aimed at building statistical and machine learning applications by a declarative language in database and data processing systems. Microsoft Azure Data Lake Analytics provides an extension of SQL, named U-SQL [32] with a tight C# binding to support distributed machine learning. Similarly, SystemML [9] expresses machine learning algorithms by a simplified R and python like-language, and automatically translates the program into execution plan on top of Spark. BUDS [26] is a language for Bayesian machine learning, specifically, Markov chain simulation, allowing distributed computation on types of sets, maps, vectors and matrices. In addition, Datalog and its extensions [10, 22, 36] are also used to integrate statistical and machine learning into database management systems. In this paper, we focus on SQL query, as it is the most widely-used declarative query language in database. We show that users can build model-based view by SQL recursive query with limited enhancement and the support of vector/matrix data type.

Model-based View in RDBMS. To provide adequate support for modeling data in database system, the abstraction of model-based view is proposed to persist the model in the database and data mining area. MauveDB [18] is an architecture which supports a SQL-based declarative language to define views for regression model and interpolation. Koc et. al. [33] define and maintain statistical model based declarative language to define views for regression model and matrices. In addition, Spark and matrices. In addition, Bayesian machine learning, specifically, Markov chain simulation, allowing distributed computation on types of sets, maps, vectors and matrices. In addition, Datalog and its extensions [10, 22, 36] are also used to integrate statistical and machine learning into database management systems. In this paper, we focus on SQL query, as it is the most widely-used declarative query language in database. We show that users can build model-based view by SQL recursive query with limited enhancement and the support of vector/matrix data type.

3 PRELIMINARIES

In this paper, we focus on unsupervised probabilistic modeling, which has broad applications in clustering, density estimation and data summarization in database and data mining area. Specifically, the unsupervised models aim to reveal the relationship between the observed data and some latent variables by maximizing the data likelihood. The expectation-maximization (EM) algorithm, first introduced in [17], is a general technique for finding maximum likelihood estimators. It has a solid statistical basis, robust to noisy data and its complexity is linear in data size. Here, we use the Gaussian mixture model [8], a widely used model in data mining, pattern recognition, and machine learning, as an example to illustrate the EM algorithm and our approach throughout this paper.

Suppose we have an observed dataset $X = \{x_1, x_2, \cdots, x_n\}$ of $n$ data points where $x_j \in \mathbb{R}^d$. Given $N(x|\mu, \sigma)$ is the probability density function of a Gaussian distribution with mean $\mu \in \mathbb{R}^d$ and covariance $\sigma \in \mathbb{R}^{d \times d}$, the density of Gaussian mixture model is a simple linear super-position of $K$ different Gaussian components in the form of Eq. (1).

$$p(x_i) = \sum_{k=1}^{K} \pi_k N(x_i|\mu_k, \sigma_k)$$

Here, $\pi_k \in \mathbb{R}$ is the mixing coefficient, i.e., the prior of a data point belonging to component $k$ and satisfies $\sum_{k=1}^{K} \pi_k = 1$. To model this dataset $X$ using a mixture of Gaussians, the objective is to maximize the log of the likelihood function in Eq. (2).

$$lnp(X|\pi, \mu, \sigma) = \sum_{i=1}^{n} \ln\left(\sum_{k=1}^{K} \pi_k N(x_i|\mu_k, \sigma_k)\right)$$

Algorithm 1 sketches the EM algorithm for training the Gaussian Mixture Model. First, in line 1-2, the means $\mu_k$, covariances $\sigma_k$ and the mixing coefficients $\pi_k$ of $K$ Gaussian distributions are initialized, and the initial value of the log-likelihood (Eq. (3)) is computed. In the while loop of line 3-7, the Expectation-step (E-step) and Maximization-step (M-step) are executed alternatively. In the E-step, we compute the responsibilities, i.e., the conditional probability that $x_i$ belongs to component $k$, denoted as $p(z_{ik})$ by fixing the parameters based on the Bayes rule in Eq. (3).

$$p(z_{ik}) = \frac{\pi_k N(x_i|\mu_k, \sigma_k)}{\sum_{j=1}^{K} \pi_j N(x_i|\mu_j, \sigma_j)}$$

In M-step, we re-estimate a new set of parameters using the current responsibilities by maximizing the log-likelihood (Eq. (2)) as follows.

$$\mu_k^{new} = \frac{1}{n_k} \sum_{i=1}^{n} p(z_{ik}) x_i$$

$$\sigma_k^{new} = \frac{1}{n_k} \sum_{i=1}^{n} p(z_{ik}) (x_i - \mu_k^{new}) (x_i - \mu_k^{new})^T$$

$$\pi_k^{new} = \frac{n_k}{n}$$

where $n_k = \sum_{i=1}^{n} p(z_{ik})$. At the end of each iteration, the new value of log-likelihood is evaluated and used for checking convergence. The algorithm ends when the log-likelihood converges or a given

| Algorithm 1: EM Algorithm for Mixture Gaussian Model |
|-----------------------------------------------------|
| 1. Initialize the means $\mu$, covariances $\sigma$ and mixing coefficients $\pi$; |
| 2. Compute the initial log-likelihood $L$, $i \leftarrow 0$; |
| 3. while $\Delta L > \epsilon$ or $i < \max$ recursion do |
| 4. E-step: compute the responsibilities $p(z_{ik})$ based on current $\mu$, $\sigma$ and $\pi$ by Eq. (3); |
| 5. M-step: re-estimate $\mu$, $\sigma$ and $\pi$ by Eq. (4)-(6); |
| 6. re-compute the log-likelihood $L$, $i \leftarrow i + 1$; |
| 7. end while |
| 8. return $\mu$, $\sigma$, $\pi$. |
iteration time is reached. In RDBMS, the learnt model, the parameters of $K$ components, can be persisted in a relation of $K$ rows as shown in Table 1(a). Suppose 1-dimensional dataset $X$ as Table 1(b) is given, the posterior probability of $x_i$ belongs to component $k$ can be computed as Table 1(c) and clustering can be conducted by assigning $x_i$ to component with the maximum $p(z_{ik})$.

To fulfill the EM algorithm in database systems, there are several important issues need to be concerned, including (1) the representation and storage of high dimensional data in database, (2) the relation algebra operations used to perform linear algebra computation in EM, (3) the approach for iterative parameter updating, (4) the way to express and control the iteration of EM algorithm, (5) the mechanism to maintain the existing model when underlying data involves.

As an early attempt, Ordonez et al. [48] proposed a SQL implementation, SQLEM. Their implementation is based on two strategies: horizontal and vertical. The horizontal approach organizes the data points as the horizontal representation as shown in Table 2(b), where the relation has $n$ rows and $d$ attributes. The vertical approach organizes the data points as the coordinate representation as shown in Table 2(a), where the relation has $nd$ rows, 3 attributes for the identity of data point and index value. The horizontal approach has efficient performance while the vertical approach has flexible expression power. Thereby, [48] further proposes a hybrid approach, which persists both vertical and horizontal representations simultaneously to achieve a tradeoff. Although it provides a feasible solution for EM algorithm in RDBMS, their approach has some drawbacks. First, the hybrid and vertical approaches only have limited flexibility in expressing linear algebra operations. For the hybrid approach, the users still need to specify the computation for each dimension $d$ in the E-step and for each $k$ in the M-step, leading to many repetitive and complicated expressions in the SQL queries. These queries also limit the supported models. For example, the covariance matrix $\sigma$ must be diagonal. Second, it has not provided an effective parameter updating and iteration control mechanism inside the database system. The SQL queries for one iteration E-step and M-step are wrapped in a while-loop of a host language, i.e., Java, python. This will undermine the overall performance due to large communication and I/O cost. Furthermore, their approach does not support model updating.

4 OUR SOLUTION

In this section, we propose a complete solution to deal with above issues in applying the EM algorithm and building model-based views inside RDBMS.

High Dimensional Representation. Regarding the issue of high dimensional data, different from [48], we adopt the row-major representation, as shown in Table 2(c), which is endorsed by allowing array/vector data type in database. For one thing, this keeps the efficient performance of horizontal representation by reducing I/O cost, especially for dense vectors. For the other, we can use the vector/matrix operations to support complicated linear algebra computation in a concise SQL query. As most RDBMSs have provided the array/vector datatype internally, apart from the build-in array functions, many extended libraries of database [16, 50] also provide additional statistical function and vector/matrix operations for multivariable statistical analysis and basic linear algebra calculus. These high-level abstractions avoid letting end-users specify the arithmetic operations on each dimension of the data point so that serve as a set of building blocks of machine learning algorithms.

Consider computing the means $\mu$ in the M-step (Eq. (4)) with the 3 different representations $X_a$, $X_b$ and $X_c$ in Table 2. Suppose the responsibilities are in relation $R(ID, K, p)$, where $ID$, $K$ and $p$ is the identifier of data point and component, and the value of $p(z_{ik})$. The relational algebra expressions to compute Eq. (4) are shown in Eq. (7)-(9), respectively.

$$M_a \leftarrow \rho(K, C, \text{mean}) (\langle K, C, \Sigma \text{sum}(p\cdot x) \rangle (R \mid R.ID=X_a.ID)) \quad (7)$$

$$M_b \leftarrow \rho(K, d_1, d_2) (\langle K, \Sigma \text{sum}(p\cdot d_1)\cdot \text{sum}(p\cdot d_2) \rangle (R \mid R.ID=X_b.ID)) \quad (8)$$

$$M_c \leftarrow \rho(K, C, \text{mean}) (\langle K, \Sigma \text{sum}(p\cdot x) \rangle (R \mid R.ID=X_c.ID)) \quad (9)$$

We elaborate on these expressions. In Eq. (7) and Eq. (8), the binary operator $\leftarrow$ is the arithmetic multiplication while in Eq. (9), the operator $\cdot$ denotes a scalar-vector multiplication. First, all of these 3 representations need to join $X$ and $R$ on the ID attribute to compute $p(z_{ik})x_i$. The differences lay in the group and aggregation for each component $k$. For the coordinate representation $X_a$ in Eq. (7), apart from $K$, we also need to group the index $C$. For the horizontal representation $X_b$ in Eq. (8), we need to define the computation of each dimension $d$ in the aggregation expression. As the dimension increases, Eq. (7) faces the problem of high I/O cost while Eq. (8) leads to a verbose query. Consider the row-major representation which nesting separate dimension attributes into one vector-type attribute. By introducing the $\cdot$ operator for vector computation, Eq. (4) is expressed in an efficient and clear way (Eq. (9)).

Relational Algebra to Linear Algebra. On the basis of array/vector data type and the derived statistical function and linear algebra operations, the complicated linear algebra computation can be expressed by basic relational algebra operations (selection ($\sigma$), projection ($\Pi$), union ($\cup$), Cartesian product ($\times$), and rename ($\rho$)), together with group-by & aggregation. Let $V$ and $E$ ($E'$) be the
The matrix-vector multiplication (Eq. (10)) is done in two steps. The first step computes all the $\circ$ results by the operation of $\circ$ for every group-by grouping by the attribute $E.F$. Similarly, the matrix-matrix multiplication (Eq. (11)) is done in two steps. The first step computes $\circ$ between a tuple in $E$ and a tuple in $E'$ under the join condition $E.T = E'.T$. The second step is aggregating all the $\circ$ results by the operation of $\circ$ for every group-by grouping by the attribute $E.F$. The second step aggregates all the $\circ$ results by the operation of $\circ$ for every group-by grouping-by the attributes $E.F$ and $E.T$. The formula of re-estimating the means $\mu$ (Eq. (4)) is a matrix-vector multiplication if data is 1-dimensional or a matrix-matrix multiplication otherwise. When high dimensional data is in coordinate representation (Table 2(a)), Eq. (7) is the application of Eq. (11). When high dimensional data is nested as the row-major representation (Table 2(c)), the matrix-matrix multiplication is reduced to matrix-vector multiplication, as shown in Eq. (9).

Re-estimating the covariance/standard deviation $\sigma$ (Eq. (5)) involves the element-wise matrix multiplication if data is 1-dimensional or a tensor-matrix multiplication otherwise. The element-wise matrix multiplication can be expressed by join two matrices on their two indices to compute $E.e \odot E'.e$. An extra aggregation is required to aggregate on each component $k$ as shown in Eq. (11).

$E \odot E' = E.F \odot E'.F \odot E.T \odot E'.T$

Similarly, when $\odot$ and $\oplus$ are vector operation and vector aggregation, Eq. (12) is reduced to high dimensional tensor-matrix multiplication.

**Value Updating.** So far, we still need to deal with parameter update when training the model in multiple iterations. There is a new relational operation, union by update, denoted as $\cup$ and first defined in [57] (Eq. (13)) to address value update in iterative graph computation.

$R \cup A S = (R \ominus (R \ominus (S))) \cup S$

Suppose $t_r$ is a tuple in $R$ and $t_s$ is a tuple in $S$. Different from the conventional union ($\cup$), the union by update updates $t_r$ by $t_s$ if $t_r$ and $t_s$ are identical by some attributes $A$. If $t_r$ does not match any $t_r$ or $t_s$ is merged into the resulting relation. Given the relation of parameters as shown in Table 1(a), the union by update updates the set of old parameters by the set of new parameters if they have the identical component identifier $K$.

**Iterative Evaluation.** In the following, we elaborate on our approach supporting the iterative model training by SQL recursive queries. Over decades, RDBMSs have provided the functionality to support recursive queries, based on SQL’99 [25, 40]. The recursive queries are expressed using with clause in SQL. We introduce the with clause following the discussions given in [27].

**with**

1. from $Q_1$ computed by $\cdots$ (Eq. (9))
2. union all
3. (select $F, T$ from $E$)
when union by update is used, it cannot be used more than once, and cannot be used with other union all together. Here, the computed by statement, as a new feature of enhanced with, allows users to specify how a relation \( R_{ij} \) is computed by a sequence of queries. The queries wrapped in computed by must be non-recursive. In this paper, we mainly use the union by update of the recursive query for parameters update instead of union all. In the following sections, we elaborate on the recursive query for EM and model updating in detail.

5 IMPLEMENTATION DETAILS

We show the details of supporting the model-based view by the ‘algebra + while’ approach, using SQL recursive query. First, we present the relational algebra expressions needed, followed by the enhanced recursive query and our implementation. Second, the queries for model inference are introduced.

5.1 Parameter Estimation

For simplicity, here we consider the training data point \( x_i \) is 1-dimensional scalar. It is natural to extend the query to high dimensional input data when matrix/vector data type and functions are supported by the database system. We represent the input data by a relation \( X(ID, x) \), where \( ID \) is the tuple identifier for data point \( x_i \) and \( x \) is the numeric attribute. The model-based view, which is persisted in the relation GMM(K, pie, mean, cov), where \( K \) is the identifier of the \( k \)-th component, and ‘pie’, ‘mean’, and ‘cov’ denote the corresponding parameters, i.e., mixing coefficients, means and covariances (standard deviations), respectively. The relation representations are shown in Table 1. The following relational algebra expressions describe the E-step (Eq. (14)), M-step (Eq. (15)-(18)), and parameter updating (Eq. (19)) in one iteration.

\[
R \leftarrow p(ID,K,p) \Pi(ID,K,f) (GMM \times X) \tag{14}
\]

\[
N \leftarrow p(K,pie) \left( \sum_{R.ID = X.ID} p \right) X \tag{15}
\]

\[
M \leftarrow p(K,mean) \left( \sum_{R.ID = X.ID} p \cdot px \right) / \sum_{R.ID = X.ID} p \tag{16}
\]

\[
T \leftarrow \Pi(ID,K,pow(x-mean)) (X \times N) \tag{17}
\]

\[
C \leftarrow p(K,cov) K \sum_{p+T} (T \ominus R)_{R.ID = T.ID, R.K = T.K} \tag{18}
\]

\[
GMM \leftarrow p(K,pie,mean,cov) (N \otimes M \otimes C) \tag{19}
\]

First, in Eq. (14), by performing a Cartesian product of GMM and \( X \), each data point is associated with the parameters of each component. The responsibilities are evaluated by applying an analytical function \( f \) to compute the normalized probability density (Eq. (3)) for each tuple, which is the E-step. The resulted relation \( R(ID, K, p) \) is shown in Fig. 1(c). For the M-step, the mixing coefficients ‘pie’ (Eq. (15)), the means ‘mean’ (Eq. (16)) and the covariances ‘cov’ (Eq. (17)-(18)) are re-estimated based on their update formulas in Eq. (4)-(6), respectively. In the end, in Eq. (19), the temporary relations \( N, M \) and \( C \) are joined on attribute \( K \) to merge the parameters. The result is assigned to the recursive relation GMM.

Fig. 3 shows the enhanced with query to support Gaussian Mixture Model by EM algorithm. The recursive relation GMM specifies the parameters of \( K \) Gaussian distributions. In line 3, the initial query loads the initial parameters from relation INIT_PARA. The new parameters are selected by the recursive query (line 5-6) evaluated by the computed by statement and update the recursive relation by union by update w.r.t. the component index \( K \). It wraps the queries to compute E-step and M-step of one iteration EM.

We elaborate on the queries in the computed by statement (line 8-17). Specifically, the query in line 8-10 performs the E-step, as the relational algebra in Eq. (14). Here, norm is the Gaussian (Normal) probability density function of data point \( x \) given the mean and covariance as input. We can use the window function, introduced in SQL ’03 to compute the responsibility by Bayes rule in Eq. (3). In line 9, \( \sum \) over (partition by()) is the window function performing calculation across a set of rows that are related to the current row. As it does not group rows, where each row retains its separate identity, many RDBMSs allow to use it in the recursive query, e.g., PostgreSQL and Oracle. The window function partitions rows of the Cartesian product results in partitions of the same ID and computes the denominator of Eq. (3). In line 11-13, the query computes the means (Eq. (4)) and the mixing coefficients together by a matrix-matrix multiplication due to their common join of \( X \) and \( N \), and computes the covariances (standard deviations), respectively. The resulted relation \( C \) will be merged by joining on the component index \( K \) in line 6.

An acute reader may find that in Fig. 3, the recursive query does not compute and check the convergence of the log-likelihood explicitly. That is because the existing recursive query does not support the functionality of checking value convergence as well as early stopping. However, the depth of recursion can be controlled by maxrecursion clause, which is adapted from SQL Server [3]. The maxrecursion clause can effectively prevent infinite recursion because of infinite fix point, e.g., \( \text{with } R(n) \text{ as (select values(0) union all (select n + 1 from R))} \), a standard SQL'99 recursion. Users can check the convergence after training for a fixed number of recursion and resume the training from current parameters if necessary.

Figure 3: The enhanced recursive SQL for Gaussian Mixtures
The implementation: We sketch how to support recursive queries using the enhanced with in RDBMSs. First, for each subquery $Q_i$ used in $Q$ including those defined by the computed by statement, we construct a local dependency graph $G_i$. The graph $G_i$ constructed must be cycle free. We ensure that it is $XY$-stratified. Second, we create a PSM (Persistent Stored Model) in the recent SQL standard. With PSM, we create a unique procedure $F_Q$ for the recursive query $Q$ to be processed, as illustrated below.

```sql
create procedure $F_Q$ (
    declare $C_1$, $\ldots$, $C_i$, $\ldots$;
    create table $R_{i,j}$ for all tables defined by as in a subquery $Q_i$;
    create SQL statement to compute the initial $R$ by union of all initial subqueries;
    loop
        insert into $R_{i,j}$ select $\ldots$ for every $R_{i,j}$ used in $Q_i$;
        compute condition $C_i$ for each recursive subquery $Q_i$;
        if all $C_i$ for the recursive subqueries are false then exit
        compute the recursive relation $R$ for the current iteration;
        union the current $R$ with the previous $R$ computed;
    end loop
)
```

In the procedure, $F_Q$, first we declare variables $C_1, \ldots, C_i, \ldots$ for every subquery $Q_i$, which are used to check the condition to exit from the looping. Second, we create the temporary tables for the relations defined by as in the computed by statements. Third, we include SQL statements to compute the initial recursive relation $R$. Fourth, we create a looping. In the looping, we generate an insert for $R_{i,j}$, and check whether $Q_i$ is empty. If so, $C_i$ is set to 0, indicating $Q_i$ generates 0 tuple, the loop will be terminated. Otherwise, the recursive relation computed in this iteration will union with the one computed in the previous iteration by either union all or union by update. An extra counter will be maintained in the loop if maxrecursion is used. With the procedure defined, we can run the statements in the procedure $F_Q$ by issuing “call $F_Q$.”

5.2 Model Inference

Once the model is trained by the recursive query in Fig. 3, it can be materialized in a view for online inference. In the phase of inference, users can query the view by SQL to perform clustering, classification and density estimation. Given a batch of data in relation $X$ and a view GMM computed by Fig. 3. The query below computes the posterior probability that the component $K$ generated the data with index $ID$. The query is similar to computing the E-step (Eq. (3)) in line 5-7 of Fig. 3.

```sql
create table $R$ as select $ID$, $K$,
    norm(x, mean, cov) * pie / (sum(norm(x, mean, cov) * pie)
    over (partition by $ID$)) from GMM, $X$
```

Based on relation $R(ID, K, p)$ above, we can further assign the data into $K$ clusters, where $x_i$ is assigned to cluster $k$ if the posterior probability $p(z_{ik})$ is the maximum among the $(p(z_{i1}), \ldots, p(z_{iK}))$. The query below creates a relation CLU(ID, $K$) to persist the clustering result where $ID$ and $K$ are the attributes of data point and its assigned cluster, respectively. It first finds the maximum $p(z_{ik})$ for each data point by a subquery on relation $R$. The result is renamed as $T$ and is joined with $R$ on the condition of $R.ID = T.ID$ and $R.p = T.p$ to find the corresponding $k$.

![Figure 4: Overview of Model Maintenance](image)

![Figure 5: The triggers for incremental update](image)

6 MODEL MAINTENANCE

In this section, we investigate the automatic model/view updating. When the underlying data $X$ changes, a straightforward way is to re-estimate the model over the updated data. However, when only a small portion of the training data are updated, the changes of the corresponding model are slight, it is inefficient to re-estimate the model on-the-fly. Hence, a natural idea is arose that whether we can update existing model by exploring the ‘incremental variant’ of the EM algorithm. And this variant can be maintained by the newly arriving data and a small portion of data extracted from original database. As the statistical model trained by the SQL queries can be represented by its sufficient statistics, the model is updated by maintaining the model and sufficient statistics.

The sufficient statistic is a function of data $X$ that contains all of the information relevant to estimate the model parameters. The sufficient statistics of these data are computed to update the statistics of model. As the model is updated, the statistics of data is also updated followed by the changing of the posterior probability $p(z_{ik})$. This process repeats until the statistics converge. It is worth mentioning that this maintenance mechanism can support all the models with have sufficient statistics. We elaborate the sufficient statistics updating rules in the following.
Suppose the dataset of model $\theta$ is $\{x_1, x_2, \cdots, x_n\}$. Let $s$ be the sufficient statistics of $\theta$, based on the Factorization Theorem [21], we can obtain

$$s = \sum_{i=1}^{n} \sum_{z} p(z|x_i, \theta)\phi(x_i, z)$$

(20)

where $z$ is the unobserved variable, $\phi$ denotes the mapping function from an instance $(x_i, z)$ to the sufficient statistics contributed by $x_i$. The inserted data is $\{x_{n+1}, x_{n+2}, \cdots, x_m\}$. Let the new model for overall data $\{x_1, \cdots, x_n, x_{n+1}, \cdots, x_m\}$ be $\tilde{\theta}$ and the corresponding sufficient statistics be $\tilde{s}$. The difference of $\tilde{s} - s$, denoted as $\Delta s$ is

$$\Delta s = \sum_{i=1}^{n+m} \sum_{z} p(z|x_i, \tilde{\theta})\phi(x_i, z) - \sum_{i=1}^{m} \sum_{z} p(z|x_i, \theta)\phi(x_i, z)$$

(21)

$$= \sum_{i=1}^{n} \sum_{z} [p(z|x_i, \tilde{\theta}) - p(z|x_i, \theta)]\phi(x_i, z)$$

(22)

According to above equations, we observe that the delta part of the sufficient statistics $\Delta s$ consists of two parts: (1) changes of the sufficient statistics for the overall data points $\{x_1, x_2 \cdots x_m\}$ in Eq. (21), and (2) the additional sufficient statistics for the newly inserted data points $\{x_{n+1}, \cdots x_m\}$ in Eq. (22). Consider to retrain a new model $\tilde{\theta}$ over $\{x_1, x_2, \cdots, x_m\}$ in $T$ iterations by taking $\theta$ as the initial parameter, i.e., $\theta^{(0)} = \theta$ and $\theta^{(T)} = \tilde{\theta}$. We have

$$\Delta s = \sum_{i=1}^{n+m} \sum_{z} [p(z|x_i, \theta^{(T)}) - p(z|x_i, \theta^{(0)})]\phi(x_i, z)$$

(23)

$$+ \sum_{i=n+1}^{m} \sum_{z} p(z|x_i, \theta^{(0)})\phi(x_i, z)$$

(24)

$$= \sum_{i=1}^{T} \sum_{j=1}^{n} \sum_{z} [p(z|x_i, \theta^{(j)}) - p(z|x_i, \theta^{(j-1)})]\phi(x_i, z)$$

(25)

$$+ \sum_{i=n+1}^{m} \sum_{z} p(z|x_i, \theta^{(0)})\phi(x_i, z)$$

(26)

Above equations indicts how to compute $\Delta s$. For the inserted data $\{x_{n+1}, \cdots, x_m\}$, the delta can be directly computed by evaluating the original model $\theta^{(0)}$ as Eq. (26). while for original data, the delta can be computed by updating the model $\theta^{(t)}$ iteratively using all the data $\{x_1, x_2 \cdots x_m\}$ as Eq. (25). Since most of the computational cost is concentrated on the iteration of Eq. (25), we use two tricks to approximate the computation. First, we use the stochastic approximation algorithm, where the parameters are updated after the sufficient statistics of each new data point $x_i$ is computed, instead of the full batch dataset. This approach is widely used in many online and incremental EM algorithm variations [37, 43, 51]. The second is discarding the data points which are not likely to change their cluster in the future, as the scaling clustering algorithms adopt for speedup [11]. In other words, due to the slight changes of data and models, only a small portion of the original data points with unstable membership need to be retrained. We discuss our strategy of selecting partial original data in $\{x_1, x_2, \cdots, x_n\}$ for model update. It is a tradeoff between the accuracy of the model and the updating cost. The more data we use to update the model, the more accuracy we can achieve while the larger computation and I/O cost to spend. The data points which are unlikely to change cluster membership can be discarded in the future model update. There are two strategies for the discard, a distance-based and a density-based strategy. For the distance-based strategy, we use Mahalanobis distance [20] to measure the distance between a data point and a distribution. For each data $x_i$, we compute the Mahalanobis distance, $D_k(x_i)$, to the $k$-th component with mean $\mu_k$ and covariance $\sigma_k$.

$$D_k(x_i) = \sqrt{(x_i - \mu_k)^T \sigma_k^{-1} (x_i - \mu_k)}$$

(27)

We can filter the data within a given thresholding radius with any component. Another measurement is the entropy of the posterior probability for data $x_i$ as in Eq. (28), where $p(z_{ik})$ is evaluated by parameter $\theta^{(0)}$. The larger the entropy, the lower the possibility of assigning $x_i$ to any one of the component.

$$E(x_i) = - \sum_{k=1}^{K} p(z_{ik}) \ln p(z_{ik})$$

(28)

It is worth mentioning that the data selection in trigger T1 can be performed offline, i.e., persisting a subset of training data with fixed budget size for model updating in the future. In addition, the sufficient statistics for original model $\theta^{0}$ (line 1 of Algorithm 2) can be precomputed. Those will improve the efficiency of online model maintenance significantly. The alternative fine-grained data selection strategies are beyond the scope of this paper.

Similarly, considering deleting $m$ data points $\{x_{n-m+1}, \cdots x_n\}$ from $\{x_1, x_2 \cdots x_n\}$, the difference of the sufficient statistics, $\Delta s$ is

$$\Delta s = \sum_{i=1}^{T} \sum_{j=1}^{n} \sum_{z} [p(z|x_i, \theta^{(j)}) - p(z|x_i, \theta^{(j-1)})]\phi(x_i, z)$$

(29)

$$- \sum_{i=n+1}^{n+m} \sum_{z} p(z|x_i, \theta^{(0)})\phi(x_i, z)$$

In RDBMSs, the automatic model updating mechanism is enabled by triggers build on the relation of the input data $X$. Fig. 4 illustrates the overview of our model/view updating mechanism. There are three triggers built on the relation of training data $X$, whose definitions are shown in Fig. 5. Before executing the insertion operation, two triggers T1 (line 1-3 in Fig. 5) and T2 (line 4-6 in Fig. 5) prepare the data for model updating in a temporary relation $X'$. Here, T1 performs on each row to select a subset from original data in $\{x_1, x_2, \cdots, x_n\}$ based on a selection criterion. Additionally, T2 inserts all the newly arrived data $\{x_{1+n}, x_{2}, \cdots, x_m\}$ to relation $X'$. After the data preparation finished, another trigger T3 (line 7-9 in Fig. 5) will call a PSM to compute the $\Delta s$ by $X'$. In the PSM, first, the delta of the newly inserted data (Eq. (25)) is computed as used to reinitialize the parameters of the model. Then, $T$ iterations of scanning relation $X'$ is performed. Where in each iteration, $X'$ is randomly shuffled and each data point is used to update the sufficient statistics it contributes as well as the model instantly. The actions of these triggers are transparent to the database users. Finally, we use Gaussian Mixture model an example to illustrate this procedure.
Algorithm 2: MODEL_UPDATE

1: Initialize the original sufficient statistics \( s \) by Eq. (30);
2: \( s_{1k} \leftarrow s_{1k} + \sum_{i=n+1}^{m} p(z_{ik}) x_i; \) \( s_{2k} \leftarrow s_{2k} + \sum_{i=n+1}^{m} p(z_{ik}) x_i x_i^T; \)
3: Update model parameters \( \mu, \pi \) and \( \sigma \) by Eq. (31);
4: for \( t \leftarrow 1 \) to \( T \) do
5: \( \text{for } x_i \in X' \) in random order do
6: \( s_{1k} \leftarrow s_{1k} + p(z_{ik}) x_i - p(z_{ik})^{(t-1)} x_i; \)
\( s_{2k} \leftarrow s_{2k} + p(z_{ik})^{(t)} x_i x_i^T - p(z_{ik})^{(t-1)} x_i x_i^T; \)
7: Update model parameters \( \mu, \pi \) and \( \sigma \) by Eq. (31);
8: end for
9: end for
10: return \( \mu, \sigma, \pi \);

Example 6.1. For Gaussian Mixture model of \( K \) components, the minimal sufficient statistics \( s = (s_{11}, s_{21}, \cdots, s_{1K}, s_{2K}) \), where \( s_{1k} \in \mathbb{R} \) and \( s_{2k} \in \mathbb{R}^{d 	imes d} \) for each \( k \in \{1, \cdots, K\} \) as are below.
\[
s_{1k} = \sum_{i=1}^{n} p(z_{ik}) x_i, \quad s_{2k} = \sum_{i=1}^{n} p(z_{ik}) x_i x_i^T \tag{30}
\]
And the parameter can be computed by the sufficient statistics as shown in Eq. (31).
\[
\mu_k = \frac{1}{n} s_{1k}, \quad \pi_k = \frac{1}{n} \sum_{i=1}^{n} p(z_{ik}) \tag{31}
\]
\[
\sigma_k = \frac{1}{n} s_{2k} - \mu_k \mu_k^T
\]
When an insert command of relation \( X \) is issued, at first, trigger T1 and T2 prepare the temporary relation \( X' \). T3 is triggered followed by the insertion command. The procedure T3 executed is illustrated in Algorithm 2. In line 2, trigger T3 first adds the sufficient statistics of the inserted data into \( s \). Then it further updates \( s \) by performing \( T \) iterations over \( X' \), in each iteration, each data point \( x_i \) is used to update \( s \) as well as the model instantly. Here, \( p(z_{ik})^{(t)} \) is the responsibility of \( x_i \) (Eq. (3)) evaluated in \( t \)-th iteration.

7 Experimental Studies

In this section, we present our experimental studies of supporting model-based view training, inference, and maintenance in RDBMS. We conduct extensive experiments to investigate the following facets:

- compare the performance of our enhanced with and looping control by a host language.
- test the scalability of the recursive queries for different models on synthetic data.
- conduct a case study on market segmentation on retail data.
- validate the efficiency of our model maintenance mechanism.

Experimental Setup: We report our performance studies on a PC with Intel(R) Xeon(R) CPU E5-2697 v3 (2.60GHz) with 96GB RAM running Linux CentOS 7.5 64 bit. We tested the enhanced recursive query on PostgreSQL 10.10 [4]. The statistical function and matrix/vector computation function are supported by Apache MADlib 1.16 [28]. All the queries we tested are evaluated in a single thread PostgreSQL instance.

7.1 with+ vs. Psycopg2

We compare the enhanced with, which translates the recursive SQL query to SQL/PSM with the implementation of using a host language to control the looping, which is adopted in pervious EM implementation [48]. We implement the latter by Psycopg2 [2], a popular PostgreSQL adapter for the python language. Regarding the EM algorithm, the E-step, M-step, and parameter updating are wrapped in a python for-loop, and executed by a cursor alternatively. We compare the running time of this two implementations, i.e., enhanced with and Psycopg2 for training Gaussian Mixture Model by varying the dimension \( d \) of data point (Fig. 6(a)), the scale of the training data \( n \) (Fig. 6(b)), the number of components \( k \) (Fig. 6(c)) and the number of iterations (Fig. 6(d)). The training data is evenly generated from 10 Gaussian distributions.

The evaluated time is the pure query execution time where the costs of database connection, data loading and parameter initialization are excluded. The experiments show that enhanced with outperforms Psycopg2 significantly, not only for multiple iterations in Fig. 6(d) but also for per iteration in Fig. 6(d)-6(c). For one thing, the implementation of Psycopg2 calls the databases multiple times per iteration, incurring much client-server communication and context switch costs. For the other, the issued queries from client to server will be parsed, optimized and planned on-the-fly. These are the general problems of calling SQL queries by any host language. Meanwhile, we implement the hybrid strategy of SQLEM [47] on PostgreSQL. For Gaussian Mixture model, one iteration for 10,000 data points with 10 dimensions fails to terminate within 1 hour. In their implementation, 2K separate SQL queries evaluate the means and variances of \( k \) components respectively, which is a performance bottleneck.

7.2 Experiments on synthetic data

We train Gaussian Mixture model (GMM) [8], mixture of linear regression (MLR) [52] and a neural network model, mixture of experts (MOE) [54] by evaluating SQL recursive queries in PostgreSQL.
Log Likelihood
Time(s)
1000
3000
1200
4000
2000

Figure 7: Scalability Test

(a) k = 8, n = 10000, varying d
(b) k = 8, d = 20, varying n
(c) d = 20, n = 10000, varying k

Figure 8: Convergence

Given the observed dataset as $\{(x_1, y_1), (x_2, y_1), \cdots, (x_n, y_n)\}$, where $x_i \in \mathbb{R}^d$ and $y_i \in \mathbb{R}$, the MLR models the density of $y$ given $x$ as

$$p(y_i|x_i) = \sum_{k=1}^{K} \pi_k \mathcal{N}(y_i|x_i^T \beta_k; \sigma_k)$$

(32)

And the MOE models the density of $y$ given $x$ as

$$p(y_i|x_i) = \sum_{k=1}^{K} g_k(x_i) \mathcal{N}(y_i|x_i^T \beta_k; \sigma_k)$$

(33)

where $\beta_k \in \mathbb{R}^d$ is the parameters of a linear transformer, $\mathcal{N}$ is the probability density function of a Gaussian given mean $x_i^T \beta_k \in \mathbb{R}$ and standard deviation $\sigma_k \in \mathbb{R}$. In Eq. (33), $g_k(x)$ is called the gating function, given by computing the softmax in Eq. (34) where $\theta \in \mathbb{R}^d$ is a set of linear weights on $x_i$.

$$g_k(x_i) = \frac{e^{x_i^T \theta_k}}{\sum_{j=1}^{K} e^{x_i^T \theta_j}}$$

(34)

The intuition behind the gating functions is a set of ‘soft’ learnable weights which determine the mixture of $K$ local models. We adopt the single loop EM algorithm [53] to estimate the parameters of MOE, which uses least square regression to compute the gating network directly. For GMM, the training data is evenly drawn from 10 Gaussian distributions. For MLR and MOE, the training data is generated from 10 linear functions with Gaussian noise. The parameters of the Gaussians and the linear functions are drawn from the uniform distribution $[0, 10]$. And the initial parameters are also randomly drawn from $[0, 10]$.

Fig. 7 displays the training time per iteration of the 3 models by varying the data dimension $d$ (Fig. 7(a)), the scale of the training data $n$ (Fig. 7(b)) and the number of clusters $k$ (Fig. 7(c)). In general, for the 3 models, the training time grows linearly as $n$ and $k$ increase, while the increment of data dimension $d$ has a more remarkable impact on the training time. When increasing $n$ and $k$, the size of intermediate relations, e.g., relation $R$ for computing the responsibilities in Eq. (14) grow linearly. Therefore the training cost grows linearly with regards to $n$ and $k$. However, in the 3 models, we need to deal with $d \times d$ dimensional matrices in the M-step. For GMM, it needs to compute the probability density of the multivariable Gaussians and reestimate the covariance matrices. For MLR and MOE, they need to compute the matrix inversion and least square regression. The training cost grows with regard to the size of matrix. The comparison shows it is still hard to scale high-dimensional analysis in a traditional database system. However, the efficiency can be improved on a parallel/distributed platform and new hardware.

7.3 Case study: market segmentation

We study building model-based view in PostgreSQL for a real application, i.e., market segmentation, which partitions the consumers into sub-groups based on their features to analyze their purchase behavior and identify potential market. The data is collected from an online retailer, containing 541,908 transactions of 4,308 consumers. Based on the RFM model [13] for consumer value analysis, for each consumer, a three dimensional feature of real value including recency, frequency, monetary is extracted. The feature is normalized to a linear relationship between the recency, frequency and monetary is $y$. These two models assume it exists a linear relationship between the recency, frequency and monetary of a consumer. All the initial parameters are randomly drawn from the uniform distribution $[0, 1]$. In Fig. 9, the consumers are segmented into 4 clusters by different colors. The segmentation of GMM (Fig. 9(a)) fits data points by Gaussian distributions of 3 variables, while MLR (Fig. 9(b)) and MOE (Fig. 9(c)) fit them by planes with linearity. The neural model MOE tends to generate a sharper cluster boundary than MLR. Fig. 8 shows the convergence of the log-likelihood of the 3 models during 15 training iterations. In fact, the convergence can be fast and achieved within 5 iterations.
Finally, we test the performance of our trigger-based model updating mechanism. First, we train GMM for 1-dimensional data generated from 2 Gaussian distributions. The original models are trained over 100k, 1M and 10M data points, respectively with 15 iterations. The overall training time is recorded as the ‘batch’ mode training time, which is 54s, 501s and 4,841s respectively. After the model is trained and persisted. We insert 10, 20, 30, 40, 50 data points to the underlying data by varying the budget size of selected data from 0 to 1,000.

Fig. 10 shows the insertion time w.r.t. the budget size of the selected data for the 3 models. The insertion time is the collapsed time from the insert command issuing to the transaction commit, including the cost of data selection with the density-based strategy and computing initial sufficient statistics (line 1 of Algorithm 2). As the number of processed tuples increases, the insertion time grows linearly. Compare to the retraining cost, i.e., the batch training time, it is not always efficient to update the existing model. The choice depends on two factors, the size of overall data points, and the budget size plus insert size, i.e., the numbers of data points to be processed in the updating. The updating mechanism may not be efficient and effective when the overall data size is small or there is a large volume of insertion. That is because, for the batch training mode, computation of parameter evaluation dominates the cost. While for the model updating, since the sufficient statistics and the model are updated when processing each data point, the updating overhead becomes a main overhead. Meanwhile, we notice that the collapsed time of data selection and computing initial sufficient statistics take about 10s, 100s and 1,000s for data size of 100k, 1M and 10M, respectively. Precomputing and persisting these results will benefit for a larger dataset.

In this paper, we focus on testing the efficiency of the approximation for model updating. The convergence and approximation guarantee involves a wide range of research topics in statistical and machine learning area, which we leave it as future investigation.

8 CONCLUSION
Integrating machine learning techniques into database systems facilitates a wide range of applications in industrial and academic fields. In this paper, we focus on supporting EM algorithm in RDBMS. Different from the previous approach, our approach wraps the E-step and M-step in an enhanced SQL recursive query, which is ensured to reach an iterative fix point. The learned model can be materialized as a database view and queries as conventional views. Furthermore, to tackle the slight changes of underlying training data, we propose an automatic view updating mechanism by exploiting the incremental variant of the EM algorithm. The extensive experiments we conducted validate our enhanced recursive query outperforms previous approach significantly and can support multiple mixture models by EM algorithm, as well as the efficiency of the incremental model update. It is worth mentioning that the SQL recursive query is not only suitable for unsupervised learning like EM algorithm, but also has the potential to support supervised learning like classification and regression. And the implementation of the query can
be migrated to parallel and distributed platforms, e.g., Hadoop and Spark, to deploy large scale machine learning applications. These directions and convergence and performance guarantee deserve future explorations.

ACKNOWLEDGEMENT

This work is supported by the Research Grants Council of Hong Kong, China under No. 14203618, No. 14020919 and No. 14025520.

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