**Electt: running auditable and verifiable elections in untrusted environments**

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We present a system for running auditable and verifiable elections in untrusted environments. Votes are anonymous since the order of candidates on a ballot sheet is random. Tellers see only the position of the candidate. Voters can check their vote. An election is auditable using blockchain log. Threshold-encryption, which is used to implement the quorum, prevents a deadlock from occurring if a minority of candidates or observers tries to sabotage the election. Candidates and observers can indicate that the election was free and fair by exposing their keys, which are used by the system to decrypt each vote. Ballot sheets are encrypted by onion-routing, which has a layer with the key of the election instance, so it’s impossible for a quorum to decode the results before they have announced their decision by exposing their keys. A register of voters ensures that only verified voters can vote without compromising their identity. If there any doubts about the identity of a voter, their vote can be excluded from the election, if a quorum agrees. This system is designed to scale from one instance to a distributed system that runs over an unlimited number of instances, which can be achieved using cloud instances or smartphones belonging to voters or tellers.

1 Introduction

This document describes Electt, which is a system for running auditable and verifiable elections or referenda in untrusted environments. It supports elections using either printed or digital ballot sheets, and elections using both types of ballot sheet. An election has one or more constituencies with different candidates and settings.

Votes are kept secret by randomizing the order of the candidates on each ballot sheet and by encrypting votes. Voters can also choose their preferred order of candidates, and they can check the system to see that their vote has been registered correctly. Further, it is possible to run an election in which votes must be confirmed by the voter on the system after the vote has been registered.

This system ensures that the results are verifiable using a blockchain-like election log. Candidates and observers can indicate their acceptance that the election was free and fair by publishing their private keys, which are then used by the system to decrypt each ballot sheet.

An election is auditable since the whole election log is stored on multiple servers and devices, which are automatically synchronized so that the log on each device contains exactly the same transactions. Each record contains the ID of its parent. The ID for a record is the value from a cryptographic hash function based on what is in the record. This log is publicly available.

The system uses a mix of onion routing and threshold encryption without a trusted dealer to implement a quorum. This prevents a deadlock from occurring if a minority of candidates or observers tries to sabotage the election.
The system runs on several identical servers, called election instances. Each instance has its own set of keys. When encrypting votes using onion routing, the final layer is encrypted with the public key of the election instances, which makes it impossible for a quorum of candidates or observers to decode the results of an election before they have announced their decision to accept or reject the results. Thus, this decision is made by them before they know the results.

The system uses a register of voters to ensure that only verified voters can vote without compromising their identity. If there are any doubts about the identity of a voter, their vote can be excluded from the election, if a quorum agrees.

This system is designed to scale from one instance to a distributed system that runs over an unlimited number of instances so that there is no single point of failure.

This system is also designed to support multiple types of elections in which voters can cast their votes using: 1. paper and pencil, 2. smartphones or computers via the internet, 3. special machines at polling stations, or 4. any combination of these.

This document does not specify the cryptosystem to be used, but does give some requirements for compatible cryptosystems. Thus, a single election may use any combination of acceptable cryptographic techniques.

2 Related Work

There has been extensive research on different election methods, election protocols and related topics.

2.1 End-to-end encryption

For ballots, there has been a significant amount of research on end-to-end encryption between the voter and the authority that is responsible for counting the votes.

The first election method to use end-to-end encryption may be the one proposed by Chaum in 1981 [1]. He utilized anonymous channels that are now called mix nets or mix networks [2]. Mix nets are very common and many protocols have been proposed [2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. This method could be used by a voting machine in offline voting or by a device used by someone to vote. For example, Belenos [21] is built upon Helios [22] and is closely related to Benaloh’s simple verifiable voting protocol [23], which itself was partially inspired by the Sako–Kilian mix net [24].

Other methods are based on blind signatures [25, 26, 27, 28, 29, 30, 31, 32, 33, 34]. In this approach, a teller can sign message content (such as a vote) without being able to see the content. This method has several issues [35, 36], which could compromise the entire election or compromise votes cast at particular polling stations. Moreover, for this system to work, a voter must be able to encrypt their vote, which means that it is not applicable for elections with physical polling stations where votes are cast using paper and pencil.

The use of a signing authority could be avoided with ring signatures. However, the challenge is then to prevent voters from voting twice, which can be solved with linkable ring signatures. Many protocols have been proposed [37, 38, 39, 40, 41, 42], in which votes signed by the same voter are correlated without identifying the voter.

Other schemes are based on homomorphic encryption, which was proposed more than 30 years ago [43, 44]. The efficiency has since been improved by many protocols [45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62]. Receipt-free methods [63] are based on sharing a secret [64, 65, 66, 67, 68, 69]. For example, DEMOS-2 [70], CHVote [71], Norwegian vote [72], Neuchâtel’s cast-as-intended [73], Selene [74], and k-out-of-l voting [75, 76] are based on the ElGamal encryption scheme, which operates on a cyclic group with the assumption that a symmetric external Diffie–Hellman key exchange on these groups is impossible or at least hard enough [77].
Some protocols exploit features of physical ballots:

- **Prêt à Voter** [78, 79, 80, 81, 82] randomizes the order of candidates on each ballot sheet. In November 2014, a state election in Victoria, Australia, used such a method called vVote [83]. A multi-authority implementation [84] uses two separate printers [85] to print each ballot sheet without jeopardizing privacy.

- **PunchScan** [86] and Scantegrity II [87] give voters a confirmation code after they vote.

- **ThreeBallot** [88] preserves voting secrecy while verifying the identity of voters by giving each voter three ballot sheets. Voters are verified because they have to choose one of the three ballot sheets, but they must still trust the authority. The divisible voting scheme [89] is very similar but prints as many ballot sheets as the voter would like.

- **SplitBallot** [90] uses different ballot sheets to split the trust between two parties, creating a sort of conflict of interest.

Another way to achieve end-to-end encryption is to use a trusted random number generator. Bingo voting [91] uses a random number list. The number of entries in the list is the number of candidates times the number of voters. At least one approach [92] has been proposed that eliminates the dependency on the trusted random number generator.

The system proposed inherits and combines ideas from some of these methods. It uses a type of mix net with onion routing [93] based on threshold encryption without a trusted dealer [94] to encrypt ballot sheets. Each layer of the onion routing is encrypted using the key of a quorum, which is implemented using a threshold cryptosystem. The final layer is the quorum of election instances, which prevents a quorum of candidates or observers (or both) from independently decrypting the result. A ballot sheet can then be decrypted only with the agreement of each quorum.

### 2.2 Auditable logs

The first system with an auditable log was VoteBox in 2009 [95]. Bitcoin was introduced in 2008 [96]. It uses blockchains, which are based on the work of Haber et al. [97, 98, 99]. There is no proof that VoteBox was based on blockchain technology or whether the techniques used were developed independently. Various protocols [40, 39, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110], including the proposed system, have used auditable logs to ensure that voting is fair and auditable.

### 2.3 Distributing the power

The idea that the power held by a government during a ballot is a threat was proposed in 1986 [44] and legitimized as a privacy concern. To the best of our knowledge, only a limited number of researchers have considered this concern [111, 47, 112]. All of them have suggested splitting voters when more than one election is held on the same day.

The e-voting system used in Estonia [113] does not protect voter privacy or distribute the power. In elections, all votes are encrypted using the election system’s key, signed by the key that is embedded into a voter’s national ID card and then sent to the election authority to be registered in the system. No cryptographic technology is used, such as mix nets, to protect voter privacy. This approach is very similar to a masked ballot [114], its improvement a doubly-masked-ballot [115], and other approaches that use a voter ID card that is distributed before the election [116, 117], confirmation codes [118], national ID card, or a voter-entered secret [119].

One approach [120] uses majority judgments and homomorphic encryption, but it is a new form of voting system. Here, we are interested in implementations based on existing electoral systems.
ADDER \cite{121} relies on a single polynomial key that is shared between several authorities. This approach may fail if one of the authorities loses their part of the key. Moreover, this system does not fully distribute power so cannot prevent collusion.

The proposed system inherits many of these ideas. Since any organization responsible for an election may be corrupt, the proposed system transfers power to a number of quorums, each of which has a layer in the onion routing, which, to the best of our knowledge, has not been done before. Thus, a minority can protect their voice because the quorums are based on a majority of candidates and possibly, independent observers, and election instances. To prevent collusion between candidates, votes are decrypted only after each quorum has agreed that the election was free and fair. The final layer in the onion routing is encrypted with the key of the election instances. Thus, it is impossible for a quorum of candidates or observers (or both) to decrypt the votes before they have announced their decision. Moreover, everything is auditable via the log.

It is assumed that traditional pencil and paper systems are less trustworthy \cite{122} than digital systems based on cryptography, though the challenge remains of convincing the various stakeholders of the trustworthiness of such systems \cite[p. 106]{123}.

3 Overview of the system

3.1 Threshold encryption and quorums

Ballot sheets are encrypted using onion routing in which each layer is encrypted using a group key. The system uses asymmetric threshold cryptography without a trusted dealer. Each group private key (Section 3.2) has \( n \) parts and a threshold of at least \( t \) parts must be available for the message to be decrypted, where \( t \leq n \). Each part of the private key is generated by a different entity (candidate, observer, or instance), which keeps that part secret. Thus, a trusted dealer is not required to create the private keys or any part of the private keys.

The threshold acts as a quorum. For each constituency, the onion routing has two or three layers: 1. Candidates, if any 2. Observers, if any 3. Election instances. There must be either candidates or observers, or both.

The threshold \( t \) for decryption is no less than \( 1/2 \) the number of entities in a layer \( n \) rounded up to a whole number, plus one if the number of entities is even: \( t = \text{floor}(n/2) + 1 \).

Thus, for each constituency, there is quorum for a majority of candidates, a quorum for a majority of observers, and a quorum for a majority of election instances.

The quorums of candidates and observers have several roles:

1. To ensure that the election has been set up correctly
2. To exclude certain ballot sheets registered at particular polling stations, by particular tellers, or belonging to particular voters
3. To ensure that the election is free and fair

For each of these, the candidates and observers can agree, disagree, or abstain.

3.2 Cryptographic key pairs

Each key pair has:

1. A public key that is used to encrypt messages and in verification messages.
2. A private key that is used to decrypt messages and for signing messages.

This system has three types of key pairs:
1. Entity keys that are used to sign messages inserted by an entity into the log. These keys have \( t = n = 1 \).

2. Group encryption keys that are used to decrypt a layer of the onion routing. These keys have \( t \geq 1 \) and \( n \geq 1 \), and each part is held by a different entity.

3. Group signing keys that are used to sign identity strings. These keys also have \( t \geq 1 \) and \( n \geq 1 \), and again each part is held by a different entity.

The system has the following groups:

1. Each layer of the onion routing (candidates, observers, and election instances)

2. Polling station instances

3. Election instances

The keys may be stored in any format, such as: 1. in memory or in one or more files on a device, 2. in a dedicated device like a physical token, 3. encoded and printed in some format like a barcode.

Whatever methods of storing private keys are implemented: 1. It must be possible to erase them as toxic waste from memory or from the physical token. 2. It must be possible to identify the owner of a key used to generate a signature. 3. It must be possible to identify the owner of each part of a private key. 4. Further, each signature should be detached from the message, which means that the signature should not contain the original message.

Announcing an entity’s key means inserting a message into the log containing the entity’s public key. The ID of this message is the key’s ID. The announcement also contains the key type, that is, what type of entity the key is for.

A group is a set of candidates, observers, or instances. To announce a group public key, a message is inserted into the log by each group member. This message contains a list of member IDs, whether it is an encryption or a signing key, an ID that identifies the part of the election instance key used, and the values of \( t \) and \( n \) for this key. This messages must be signed individually by the member’s key. The number of members in the group is \( n \). The group’s public key cannot be used until all members of the group have separately announced it. This group key also cannot be used if the values of \( t \) or \( n \) are invalid, or if \( t \) and \( n \) are not the same in all announcements.

If the cryptographic system implemented requires additional messages between members of the group to generate the key, then these messages are not stored in the election log.

Revoking an entity’s key means inserting a message into the log that contains the key’s ID. It is signed by the key that is being revoked. This key is then no longer valid and any subsequent messages signed by it will be ignored. The corresponding private key must be erased after this message has been created.

To revoke a group key, at least \( t \) messages, each from a different group member, that are signed by their entity’s key must be inserted into the log.

A group’s private key may have parts that can be extracted from where they are stored and parts that cannot be extracted (i.e., because they are on a physical device). To expose a group’s private key, the parts that can be extracted must be inserted into the log as messages signed by the relevant entity’s key. For the parts that cannot be extracted, a message is inserted that asserts that the physical device has been transferred to another entity. This entity must insert a message to confirm that the part has been received and that it is managing the device.

3.3 The log

The log is managed by software called the log manager. This software runs on election instances, polling station instances, and the bootstrap server. The log manager and the log can also be implemented on websites and elsewhere, such as on a voter’s device.
The log is available to every party involved in an election, including candidates, observers, voters, etc. In a general election, the log is available to everyone.

This log is a full archive of an election and contains all the transactions for it. The log can be stored as a legal record of the election.

The log is a chain of blocks. Each block contains:

- block ID
- parent block ID
- block messages

The ID of each block is the value from a cryptographic hash function based on the parent block ID and the IDs of all the messages in the block.

A block is a list of messages. Each message has

- message ID
- message type
- digital signature
- message content

The ID of each message is the value from a cryptographic hash function based on the parent block ID of the block that contains the message and on a signature. A message is signed using the key of the entity that is inserting the message into the log. The message signature is based on the message type and message content. A message is valid if it contains a verified signature and its ID is valid for its parent block.

Valid messages are sent to the log manager. There can be one or more managers. They each have a buffer of new messages that are to be added to the log. Periodically, the messages in the buffer are formed into a block and deleted from the buffer. A block is valid only if all the messages within it are valid and its ID is valid for its parent block. When a manager creates a new block, it sends the block to all the managers it knows about, which are called its neighbors. A block is inserted into the blockchain only if a majority of neighbors accepts it. A majority is defined as $\frac{1}{2}$ the number of neighbors rounded up to a whole number plus one, but at most the number of neighbors. If a block cannot be added, the messages are returned to the buffer.

For each entity, the log may contain one or more messages that are related to that entity. The message ID of the first message for the entity is used as the ID of this entity. Usually the first message announces the entity’s public key, which is used to verify the entity’s signature, but if an entity does not have its own key pair, like when registering a vote, the ID of the message announcing the entity should be used as the ID for the entity.

### 3.4 Trust tree

Initially, the log is empty. The first block contains only one message, which announces the public key that is used to verify the signature for this message. This is called a self-signed message. This key is automatically trusted and it is called the bootstrap key.

When a public key is announced using a trusted key, then it becomes a trusted key itself. For example, if the system trusts key A, then it will trust each key added using A, and all keys added using these other keys, etc. This results in a trust tree. Any messages signed by an untrusted or unknown key are ignored.

Each announced key has a limited ability to insert messages and announce new keys. This limitation depends on the key type. A bootstrap key can announce candidates, observers, election
instances, and polling station instances, and it can create constituencies and the election. A polling station instance key can announce tellers, etc. If a key is announced by the wrong type of key, it will not be accepted as a trusted key and all messages signed by it will be ignored.

A trusted key can revoke trust from itself or any key in its trust tree. This is done by adding a revoke message to the log. This message has a flag to indicate if only one key or the full tree is being revoked.

If a message revokes only one key, then all keys announced by that key become orphans. They become the root of their own trust trees. Keys in such a tree can be revoked only by the orphan key’s tree.

3.5 Bootstrapping

A new election is bootstrapped by an owner via a temporary process on the bootstrap server that creates the bootstrap key, which is self-signed.

When an election has been fully set up and all voters have been registered, the bootstrap key is revoked, and all subsequent messages signed by it are ignored. When it is revoked, all the keys that have been announced by it all become orphans and independent. The bootstrap stage is then finished, and the bootstrap server and the owner are no longer involved in the election.

The bootstrap key is revoked before the election is approved and before \( T_a \) (Section 3.6).

3.6 Creating an election

An election is created by someone referred to as the owner using the bootstrap key. If the election requires additional parameters, such as its name, these are included in its announcement message. An election can have the following parameters:

- timestamp when it is created
- list of election instances
- list of constituencies
- \( T_s \), voting start datetime
- \( T_e \), voting end deadline
- \( T_r \), deadline by which all voters should be registered
- \( T_a \), deadline by which a quorum can accept that the election has been set up correctly
- \( T_d \), deadline by which disputes can be created and acted upon
- \( T_c \), deadline by which the quorum can accept that the election is free and fair

These parameters cannot be changed once the election has been approved.

The minimum requirements for an election are:

- at least one constituency
- at least one election instance
- \( T_s \), which can be now or in the past (a date in the past means that voting starts as soon as the election is approved)
- voting end deadline
• all deadlines should be defined and should satisfy:

\[ T_r < T_a < T_e < T_d < T_c \]

When the election is announced, the election instances create and announce the group signing and encryption keys (Section 3.1).

An election can be approved only when all the candidates and observers in each constituency have announced their group encryption key.

### 3.7 Creating a constituency

An election has one or more constituencies. A referendum has one constituency. In an election, candidates stand in constituencies, which return a number of representatives. In an election for the officers of an organization, each post (president, treasurer, secretary, etc.) is a constituency. A voter may be allowed to vote in one or more constituencies.

Constituencies are created by the owner using the bootstrap key. If additional parameters are required for a constituency, such as a name, these are included in the announcement.

A constituency can have the following parameters:

• list of candidates or, for a ballot asking a question, the list of options (or answers)
• list of polling stations
• list of observers
• list of mandatory identity providers for voters
• list of parameters for identity providers
• number of candidates in the quorum
• number of observers in the quorum
• number of representatives to be elected
• whether it supports write-in candidates or write-in referendum options
• whether it supports “none of the above”
• whether confirmation of a vote is mandatory or optional, or whether votes are not confirmed at all
• weight of each ballot sheet:
  – null, which means that the weight is specified individually for each voter
  – any positive non-zero real number
• voting system (see below)
• the overall maximum number of votes (see below)

A vote means each selection of a candidate or a referendum option on a ballot sheet. There may be multiple votes for a ballot sheet.

A voting system has the following options for each ballot sheet:

• minimum number of votes (natural non-zero number)
• maximum number of votes (natural number, greater than or equal to the minimum)
• whether votes are ordered (as in systems like STV)
• if the maximum number of votes is more than one, whether a voter is allowed to vote for the same candidate or option multiple times
• whether each vote has the same weight or whether the weight is distributed evenly over the votes

The overall maximum number of votes can be:
• Overall for the constituency: When it is reached, any subsequent votes are ignored.
• Per polling station: When it is reached, any subsequent votes are ignored for that polling station.
• For particular polling stations: When it is reached, any subsequent votes are ignored for that polling station (this maximum may be higher than the per polling station limit but cannot be higher than the overall maximum).

If a constituency does not have a specified maximum number of votes, then the number of votes cast in that constituency cannot be more than the number of voters registered to vote in it.

The minimum requirements for constituencies are:
• The minimum number of candidates or options is one of: 1. at least two candidates or options, so that an election with one candidate or one option is invalid, 2. one or more candidates or options with “none of the above” enabled, or 3. zero or more candidates or options with write-in candidates or options enabled
• at least one polling station
• at least one mandatory identity provider

Observers are mandatory for a referendum with no candidates but are optional for an election, which may be run without them.

As soon as a constituency is announced and before the election is approved, all relevant candidates and observers should create and announce the quorum (Section 3.7).

3.8 Creating candidates and observers
Candidates and observers are created in a similar way to each other:
1. Create a new key pair for the entity.
2. Send a request to announce the entity’s public key to the owner.
3. Wait until the public key is added to the log.
4. Announce the entity with a message signed by its key and include its name. If an election requires additional parameters for the entity, these are included in this message.

If an entity contains a mistake or its private key has been compromised, it can be destroyed to prevent future usage. This is done by adding a revoke message to the log for the entity’s key, signed by the owner’s key or the entity itself.
3.9 Creating instances for the election and polling stations

The election and each polling station can run on one or more instances. An instance is a software representation of the election system or polling station plus all the relevant data. The instances are identical to each other, which achieves fault tolerance through redundancy.

Management of the instances is fully automated, and under the control of an impartial operator who is independent of the candidates and observers.

An instance may be in the cloud or on a local server. The private key of a local server is stored on a physical token, which is held securely by the instance operator. The private key of a cloud instance is also held securely by the instance operator and may be stored on a physical token.

If candidates and observers have a physical token that stores their private key, they can take it to any election instance to get that token used, which allows them to expose their key.

An instance is created as follows:

1. Create a new key pair for the instance.
2. Send a request to announce the entity’s public key to the owner.
3. Wait until the key is added to the log.
4. Announce the instance with a message signed by its key and include its name and its type. If an election requires additional parameters for the instance, these are included in this message.

If an instance contains a mistake or its private key has been compromised, it can be revoked to prevent future usage. This is done by adding a revoke message to the log for the instance’s key signed by the owner or the instance itself.

3.10 Creating polling stations

A polling station does not have its own key pair. Instead, it uses any of the keys of its polling station instances.

Polling stations are created as follows:

1. The polling station is announced by the election owner in a message signed by the bootstrap key. The announcement includes the name of the polling station and a list of polling station instances. If an election requires additional parameters for the polling station, these are included in this message.
2. Wait until the announcement is added to the log.

If the data for a polling station contains a mistake, the polling station can be destroyed to prevent further usage. This is done by adding a revoke message to the log for the polling station, signed by the owner.

A polling station instance can belong to only one polling station. If a polling station is destroyed, its instances can be reused by a new polling station.

As soon as a polling station is announced, its instances then create a group signing key and they separately announce it (Section 3.1).

3.11 Identity providers

The system supports different types of identity provider:

- An official, such as a civil servant of the government running the election or an officer of an organization running an election
• All the tellers collectively comprise a single identity provider

• Software that is not part of this system, for example, an official authorization service

Different elections use different methods to verify someone’s identity. When a person registers to vote, their identity can be checked by one or more identity providers. Each identity provider generates a unique reference for each voter as follows:

• If the person is registering for a postal vote, then their name and address can be used as the unique reference.

• If the person is presenting their passport, ID card or other ID in person, then the number from the ID is the unique reference.

• Software can be used to generate a unique reference, such as through the SSO or SAML protocols.

The unique reference for a person for an ID provider is fixed, regardless of which software instance or which employee within an identity provider generated the unique reference.

A voter may be identified by any identity provider but must be identified by all mandatory identity providers. They can get their identity verified in different ways, and for each method, an identity string is generated for each relevant election or polling station instance. An identity string is generated as follows:

1. The ID of the constituency, the ID of the identity provider and the unique reference generated by the identity provider are concatenated into a single string.

2. This string is signed by the group signing key of a relevant entity, such as a polling station or election instance.

3. The final identity string is this digital signature.

Each identity provider has:

• ID, which is the ID of the message announcing the identity provider

• list of polling stations, which may be empty

• other data, such as the name of the identity provider, as required by the law

If the list of polling stations is not empty, then the identity provider can verify identities only for people voting at those polling stations. Otherwise, it is a global provider and can be used only by election instances.

An identity provider is created as follows:

1. Announce the identity provider using the bootstrap key.

2. Wait until the announcement is added to the log.

If an identity provider contains a mistake it can be revoked to prevent future usage. This is done by adding a revoke message to the log for the identity provider’s ID signed by the bootstrap key.

An identity provider can be registered or revoked only after the election has been created and before $T_r$.

If an identity provider is revoked, all voters with identity strings from this identity provider are revoked.
3.12 Voters

The system maintains a register of voters, i.e., those people who are allowed to vote. The register also lists the polling stations at which a voter may cast their vote.

Each voter has:

- ID, which is the ID of the message announcing the voter
- vote weight (a brief description of how this is used appears in Section 3.7); this weight is ignored if the constituency has a non-null weight
- constituency ID
- a list of identity strings

A voter is created as follows:

1. Create identity strings for each identity provider for the voter’s constituency.
2. Announce the voter using the bootstrap key.
3. Wait until the announcement is added to the log.

Voters can be registered or revoked only after the election has been created and before $T_r$.

If a voter identity string contains a mistake, it can be revoked to prevent future usage. This is done by adding a revoke message to the log for the voter’s ID signed by the bootstrap key.

3.13 Approving the election

Once the election has been created and all voters are registered, the election next needs to be approved, which confirms that it has been set up correctly. The candidates and observers in each constituency do this by individually:

- Creating a message saying that they agree that the election has been set up correctly.
- Creating a message saying that they disagree that the election has been set up correctly.
- Abstaining by not creating a message.

Each of these messages contains a timestamp and signature created by the group signing key for the constituency.

If a candidate or an observer is a member of more than one constituency, they can vote in each of them to approve the election or not.

Only the first message inserted by a candidate or observer in each constituency, after the owner has revoked their key, is counted. All subsequent messages are ignored, as are all messages inserted before the owner revoked their key.

If the quorums in all constituencies agree that the election has been created correctly and if the members of all constituencies have announced their group keys, then the election will proceed. This can be announced by any election instance with a message that contains a timestamp. If the log has more than one such message, only the first is accepted; the subsequent messages are ignored.

If the quorum in any constituency disagrees that the election has been set up correctly or if a quorum is not reached by the deadline for any constituency, then the election will not proceed. All election instances will revoke their keys, and all subsequent messages will be ignored.

If the log contains an election-created message but at least one key has been revoked by an election instance, the log has been compromised and this election cannot proceed.
If the bootstrap key has not been revoked by $T_a$ for the election, then the election will not proceed. Once the election has been approved in this way, the bootstrap stage is finished. The election parameters can no longer be changed. In addition, candidates, observers, polling stations, constituencies, identity providers and voters can no longer be added or revoked.

3.14 Tellers

Each physical polling station requires at least one teller, who registers the votes cast in person. For a virtual polling station, the tellers are software instances. For postal votes, the teller may be an official of the organization running the election or may be software, etc.

A teller is created for a polling station as follows:

1. Create a new key pair for the teller.
2. One of the polling station’s instances announces the teller’s key.
3. Wait until the public key is added to the log.
4. Announce the teller with a message signed by its key and include its name. If an election requires additional parameters for tellers, these are included in this message.

If a teller contains a mistake or its private key has been compromised, it can be revoked to prevent future usage. This is done by adding a revoke message to the log for the teller’s key signed by the polling station instance’s key or the teller’s key. This does not affect the signature of any voter ID or any votes already registered by the teller. However, the teller can no longer register new votes, and all voter IDs that have been signed by the teller’s key but who have not yet registered a vote will be ignored.

Tellers can be created or destroyed from when the election is approved until voting is finished.

3.15 Ballot sheets

In this system, each voter can choose to have a paper ballot sheet, an electronic ballot sheet or a mixed ballot sheet. A paper ballot sheet has two parts:

- A private part that lists the candidates in some order, the unique ballot sheet ID, and the ballot sheet’s private key.

- A public part on which the vote is made. It does not have the candidates’ names or anything that may be used to identify the candidates, and it does not indicate whether the vote has to be confirmed. It contains only the ballot sheet ID, which may be encoded as a barcode.

The paper version is easy to tear apart so that the private part can be kept secret.

A mixed ballot sheet has the list of candidates in electronic form, but the public part is printed out.

A ballot sheet can be created by facilities at a polling station (computer or polling machine and a printer) or online. A voter can use any device with an internet connection, such as a smartphone or computer. If the voter uses their own device, the private part of the ballot sheet is generated by that device, and stored only on this device.

Each voter ID can be used in only one constituency. Since ballot sheets are for only one voter ID, then if a voter can vote in more than one constituency, a separate ballot sheet (and voter ID) is created for each.

The payload for a ballot sheet contains:

- whether this vote should be confirmed
• list of candidates or options, as they appear on the ballot sheet

• signature for both fields using the ballot sheet’s key

The payload of each ballot sheet is encrypted. The payload is used to map the vote cast (which is just the position of a candidate or option on the ballot sheet) with the list of candidates (or options).

A ballot sheet is created as follows:

1. A key pair is generated for the ballot sheet, which is used only to confirm or revoke a vote.

2. Announce the ballot sheet’s key with a polling station instance’s key.

3. The voter can choose to order the candidates randomly or choose an order. They can choose to omit some candidates so that the list is shorter. They can also add the names of any write-in candidates. They can also add null names to pad out the list of candidates. The number of candidates on a ballot sheet must be at least the number of representatives returned.

4. The ballot sheet is announced with a message containing its encrypted payload and the constituency ID.

5. A ballot sheet can be printed out if required.

For a constituency, a voter can generate as many ballot sheets as they like at different polling stations, but they can use only one, which can be any of them.

3.16 Encrypting ballot sheets

This system uses both onion routing and threshold encryption to encrypt a ballot sheet’s payload.

In onion routing, a message is encapsulated in several layers (or hops), analogous to the layers of an onion. Each layer is encrypted with a different public key, so the message can be fully decrypted only by using all the private keys in the right order. If any one key is unavailable or they are used in the wrong order, then the message cannot be decrypted. This ordered list of keys is called a path.

Each path contains the keys for all layers for a constituency: candidates, observers, and the election instance. This path is generally unique for each constituency and has at least two hops: 1. the election instance 2. and candidates or observers. If a constituency contains candidates and observers, the path has three hops.

When a ballot sheet is created, the payload is encrypted on the device that generated it.

3.17 Casting a vote

When a person votes, their identity is first checked by a physical or virtual teller from any polling station for this constituency whose key has not been revoked. The teller will generate an identity string for each mandatory identity provider for this constituency. If there are no mandatory identity providers, the teller will generate an identity string for an optional identity provider using the ID provided by the voter. If all these generated identity strings match those on the register for the voter, then their identity has been confirmed and they are given a voter ID signed by the teller’s key. This voter ID might be in physical form (printed out) or in digital form.

To ensure their privacy and to exclude any possibility that a teller can track their real identity using the voter ID, a voter can use two different tellers to register their vote: 1. the first to obtain a voter ID and 2. the second to register their vote.

A vote is made using the public part of a ballot sheet. The vote is marked on it, either physically or electronically, depending on the voting system used (an X, with 1, 2, 3, . . . , etc.). Votes can be cast in person or by post, email, SMS, phone, or online.
The teller will then register the vote by creating a message based on information from only the public part of the ballot sheet and using the voter ID. The message is signed by the teller and contains:

- ballot sheet ID
- positions of selected candidates, with a number indicating the preference order if relevant or distributed weight
- voter ID and teller’s signature
- signature of teller who obtained this ID

If a vote is registered for the same ballot sheet or for the same voter more than once, the system will use only the first. If a vote is registered by a teller in a polling station that is not in the voter’s constituency, the vote will be ignored.

A voter can check their vote online using the ballot sheet ID or their voter ID. Until the ballot sheet is decrypted, the information shown is only the voter ID and the positions in that ballot sheet of the candidates who have been voted for, with a number indicating the preference order, if relevant.

When a ballot sheet is created, the voter may specify that they would like to confirm their vote, or this may be mandatory for a constituency. In this case, their vote will be counted only if the log contains a confirmation from the voter and it has not been revoked. Only the first confirmation or revoke message will be used. If a voter confirms or revokes a vote for a ballot sheet that does not need to be confirmed, this message will be ignored.

If the teller registers the vote incorrectly, the teller can revoke it. However, the teller will be unable to register the vote again for either this ballot sheet or the voter.

3.18 End of voting

Voting ends at $T_e$ or when all voters in all constituencies have voted, whichever occurs first.

The parts of each group signing key are held by election and polling station instances until voting is finished, after which they are erased as toxic waste. If an instance cannot erase its part of a group signing key, then it should treat the part as if it has been compromised. If the number of parts compromised for a group is $\geq t$, then all votes cast by voters with an identity string signed by the group should be excluded from the election via the dispute mechanism. These ballot sheets will not be decrypted.

All election instances will insert a voting-finished message with a timestamp into the log to prevent further votes from being registered.

However, at the deadline, some polling station instances may still have votes in their buffer. These votes will then be inserted into the log. Such votes will be counted provided that they were registered (i.e., inserted into the buffer) before the deadline. The polling station instance will then confirm that the election has finished by inserting a voting-finished message with a timestamp to prevent further voting.

The voting-finished messages have flags that confirm that the instance has erased its part of the group signing key, or not.

3.19 Disputes: Excluding a polling station, teller, or voter

In some situations, all the votes registered by a particular teller, at a particular polling station, or by a voter may need to be excluded. This process can be started only after at least one voting-finished message has been inserted into the log and before $T_d$.

In that period, any candidate or observer can insert a message into the log for their constituency containing:
• a timestamp
• free text explaining why the exclusion is necessary
• a list of polling stations that should be excluded, a list of tellers that should be excluded, or a list of voters that should be excluded, or any combination of these
• whether the exclusion is due to a leak of private data from a polling station instance and the number of suspected instances

After this message has been inserted into the log, the system will wait for the decision of the candidates and observers. The decision is to agree, disagree, or abstain, which is inserted as a message with a timestamp into the log.

If any observer or candidate exposes or revokes their key before they have given their decision on whether to exclude these votes, then the system will consider that:
• they agree that the votes should be excluded if they have exposed their key
• they disagree that the votes should be excluded if they have revoked their key

The relevant votes will be excluded only if a quorum agrees within $T_d$ that they should be excluded. If the votes from a polling station are excluded, all further messages from its instances are ignored. If the votes from a polling station are excluded due to a leak of private data and if the number of suspected instances $\geq t$, then the votes cast for all voters that contain identity strings for that polling station are also excluded.

This stage is not finished until each non-excluded polling station instance has confirmed that it has inserted all votes in its buffer into the log and that it has destroyed its part of the group signing key. If this message cannot be added by a polling station, then to bypass this deadlock, the quorum should exclude it as if there were a leak of private data. Otherwise this would be considered as a disagreement by the quorums in all related constituencies when deciding whether the election was free and fair.

3.20 Free and fair

Once voting has stopped and all disputes about excluding votes are settled, the candidates and observers must confirm that the voting was free and fair. Each member of the quorum in each constituency can:
• Agree that voting was free and fair by exposing their part of the group encryption key.
• Disagree that voting was free and fair by revoking their part of the group encryption key.
• Abstain by not sending a message.

All of these messages have a timestamp.
If a key is not exposed before the deadline for this decision, then the system will consider that this candidate or observer has abstained.
For each candidate or observer, only the first message in the log is counted. All other such messages are ignored.
The entire election is not considered to be free and fair if any of the following conditions apply:

1. If $\geq t$ election instances have not confirmed that they have destroyed their parts of the group signing key by the voting end deadline.
2. If for a polling station: a) it has not been excluded by the dispute mechanism, b) its instances have not confirmed that it has inserted all the registered votes from its buffer into the log, and c) \( \geq t \) instances have not confirmed that its part of the group signing key has been erased by \( T_d \).

3. If the quorum in any constituency disagrees that the election was free and fair.

4. If a quorum is not reached by the free-and-fair deadline for any constituency.

If the election was not free and fair, then each election instance will destroy its part of the group encryption key. It will insert a message indicating that the election has been rejected and confirm that it has destroyed its part of the group encryption key. Because of the encryption method, it is then impossible for the ballot sheet payloads to be decrypted, so that the result can no longer be calculated.

### 3.21 Calculating the results

However, if the election was free and fair, then the result will be calculated as follows. All election instances that have access to a quorum of exposed keys will insert a message indicating that the election was free and fair and will then decrypt the ballot sheets. Once a ballot sheet has been decrypted by an election instance, it will not be decrypted again by another instance. In this way, extracting the ballot sheets can be shared among election instances.

As soon as all non-excluded ballot sheets have been decrypted, all election instances should erase their part of the group encryption key as toxic waste.

If the log contains at least one message from an election instance asserting that the election was free and fair, all rejected messages should be ignored.

A valid vote

- has a valid signature
- has a valid voter ID and a valid signature for the voter ID
- is the first vote for a voter
- has not been revoked and has been confirmed, if necessary
- has not been excluded
- was registered between \( \max(T_s, T_a) \) and \( T_e \).

For each non-excluded vote, the system will insert a message containing:

- ballot sheet ID
- decrypted ballot sheet payload

An election has a maximum number of votes. Any votes registered after the maximum has been reached are ignored and the related ballot sheets will not be decrypted. Similarly, a polling station has a maximum number of votes and related ballot sheets beyond that maximum are ignored.

Using only valid votes that are not being ignored, all election instances will calculate the number of votes for each candidate or option in each constituency based on the voting system, the vote weights, etc., and taking into account write-in candidates or options.

When each election instance has calculated the results, it will insert them as a message into the log and then destroy its key as toxic waste.
4 Threat modeling

4.1 Election commission

This system eliminates the need for an organization that controls all aspects of an election, like an election commission, by splitting its power into different independent roles:

- The owner, who creates the election and does all the necessary administration.

- A quorum of members, who ensure that the election has been set up correctly and that voting was free and fair. A quorum can exclude some votes via the dispute mechanism.

- Election instances, which are automated systems. They prevent a quorum from making decisions about the election behind closed doors. This software is managed by operators only.

- Operators, who are not involved in the election as candidates, observers, or owners. They are technicians who manage an election instance during the election.

- Tellers, who verify the identity of voters.

If a political party or other group wishes to manipulate the election, they need access to more than half of the private keys of the candidates and observers. This may include the keys of their opponents. They also need a majority of the private keys of election instances, but these keys are held securely and only operators have access to them. An election instance key may be contained within a physical token, so it would have to be removed from the instance, though instances are publicly monitored. Moreover, the software on the election instance will generate an alert to indicate that the key is not present. However, it is possible that a majority of candidates and observers are under the control of some organization, such as a political party. This type of threat cannot be eliminated.

This system uses threshold encryption without a trusted dealer. Thus, private keys are not generated by or shared with a trusted dealer. This eliminates the risk that a secret required for decryption is copied before a quorum reaches a decision on whether the election was free and fair.

The software installed on instances and the firmware for physical tokens will be available as open-source software for a reasonable amount of time before an election. This allows independent researchers to check that the software does not have a backdoor or predefined keys.

4.2 Protecting voter identities

Each voter’s identity is protected because the system uses a group signing key to generate an identity string for the voter’s unique reference from each identity provider. Moreover, each identity string is a signature that is generated by a key that is identifiable. All identity strings are generated before the election is approved by the quorum.

A brute-force attack to determine the identities of all voters associated with an instance would need to steal the keys from a majority of instances in the group that was used to create the identity strings. Some of these keys are stored only inside a token. All polling station and election instances will erase the private keys that they use for signing from the token or their memory as soon as voting is complete.

Even if an attack is successful, the attackers will get only the unique references provided by the identity providers, which safeguards the privacy of voters.

If there are any doubts or suspicions about whether private data have been leaked from any instance, the quorum can use the dispute mechanism to exclude from the election the relevant registered votes. This protects the real votes of people associated with particular voter IDs from a brute-force attack.
Thus, all polling station and election instances will erase as toxic waste the secrets that were used to create identity strings. This makes it impossible to calculate the results of the election until all votes have been excluded for voter IDs that contain identity strings from instances that have not erased their secrets.

A voter ID can be used in only a single constituency. Thus, it is impossible to identify a voter by comparing how they voted in different constituencies in a multiple-constituency election.

Moreover, voters can choose to get a voter ID from one teller and then register their vote with either a different teller or a polling station that supports online voting. Thus, the tellers do not know how they voted.

4.3 Coercion and buying votes

As with any remote method of casting votes, the system is unable to prevent someone from stealing someone else’s vote.

A voter can print or generate as many ballot sheets as they want, but only the first vote registered in the system will be counted. This allows someone to appear to comply with coercive demands at no risk by voting twice, first as their intended vote and secondly to comply with the coercion. Similarly, a voter can cast a vote using a friend’s voter ID to prevent their real voter IDs from being exposed.

Because voting records are encrypted, it is very difficult to determine how a voter voted. Once a ballot sheet has been decrypted, however, someone can compare the public part with the private part to determine which candidate was voted for. However, it is still not possible to match the real person with their voter ID.

4.4 Deniable votes

All voting systems should ensure that votes are deniable, which this system achieves by breaking the link between a person and a voter. Each identity provider returns a unique reference for each person and the system uses the instances’ group private key to generate an identity string based on the unique reference. All instances erase their group private keys as soon as voting is complete, which breaks the link between the person and the voter via the identity providers. Thus, all votes are deniable because they are not directly linked to the person who voted.

This system does not use timestamps or time-related fields when recording votes to prevent an attack based on assumptions about who voted by spying on a polling station.

Messages recording a vote are sent to the log in batches by the polling station instances. So, there is an unpredictable delay before a message is sent to the log. Thus, it is hard to identify which voter cast which vote based on when the vote was registered.

4.5 Identity provider fraud

Voters can be verified using more than one identity provider. This prevents collisions and protects against a fraudulent identity provider. However, the system is vulnerable if all identity providers return incorrect verification results for a person due to fraud, censorship, or similar.

4.6 Cracking votes

Each ballot sheet contains an unspecified number of candidates or options in an unspecified order, neither of which can be predicted.

For example, the order of candidates can be randomized using a random number generator on a voter’s trusted device or by a device at a polling station. The order could be selected by the voter. A voter can remove one or more candidates from a ballot sheet (provided enough remain for the voter to be able to cast all their votes). Finally, a voter can add null candidates. These cannot be voted
for, but affect the numbering of candidates on the ballot sheet. All of these measures increase the
entropy of the payload.

The entropy is further increased because a ballot sheet does not contain any information about
where it was generated or how many candidates it contains. This makes an attack based on predicting
random numbers much more difficult (if even feasible at all).

4.7 Excluding inconvenient polling stations

In many elections, there are exit polls, which can be used to predict the result of the election.

If a candidate expects that the results from a particular polling station will not be favorable to
them, they cannot exclude any votes from that polling station unless they have the support of a
quorum, which comprises a majority of candidates and observers for the constituency.

They can challenge the votes cast at a polling station if they suspect fraud, but excluding these
votes requires a quorum. Such challenges are out of scope for this system.

4.8 Predicting the result of an election

It is impossible to predict the result by analyzing the log, since only the position of a candidate
on a ballot sheet is recorded, the list of candidates is different for each ballot sheet, and the list of
candidates is encrypted.

Each ballot sheet has its own key pair, but this key pair is used only to confirm or revoke votes
cast for this ballot sheet. It cannot be used to decrypt a ballot sheet’s payload.

If the private part of a ballot sheet is intercepted (e.g., by finding a paper version in a bin), then
it is possible to compare the vote in the log with this private part. However, the eavesdropper does
not know whether the vote has to be confirmed, and if so, whether it has been confirmed or revoked.
The eavesdropper also does not if this was a second vote by the voter, which would be ignored.

4.9 Tampering with the log

The log is a blockchain. Each block in the chain and the messages in a block contain the ID of
the parent block. Changing a block changes all subsequent message IDs, since these are based on a
hash of the previous block. Moreover, message ID are used as entity IDs, and changing the IDs for
all entities would change all the signatures in the subsequent blocks. Thus, changing even a single
message in the blockchain would mean regenerating all subsequent signatures, which would require
access to all the private keys. That seems almost impossible to achieve.

Finally, the blockchain is distributed. Even if the log is changed, some voters will know their voter
ID and some will also have a copy of the log. Such a voter can easily prove that someone tampered
with the log.

5 Conclusion

This system eliminates the need for an organization that is responsible for all aspects of an election,
like an election commission. Such an organization would need to be trusted by everyone. The power,
instead, is transferred to the quorum and to the election software through the election log. The
software is controlled by operators who are independent of the candidates and observers and have no
other role in the election. The operators have no real power and cannot influence the results of the
election. The decision of the quorum is based on information that is available to everybody.

This system allows each voter to control how their vote will be accounted for. Votes can be excluded
only if a quorum agrees.
This system does not require any changes to the voting systems we currently use (first past the post, STV, etc.) and it supports all types of election, from small and simple votes by corporate stockholders to referenda to large and complex elections with many constituencies, such as UK and US general elections. The voting rules can be customized for each constituency.

Candidates must accept that the election was free and fair before the result can be calculated, so that it is unreasonable for them to claim it was not free or not fair after the results have been announced.

Expensive voting machines do not need to be installed at each polling station since ballot sheets can be created using a cloud polling station, a smartphone belonging to the polling station, or a voter’s personal smartphone.

This system realizes all of the expected requirements of a voting platform, such as deniable votes and voter anonymity. Potentially, an election can be run more cheaply than currently, and it be more transparent and have a higher degree of trust.

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**Glossary**

**ballot sheet** A list of candidates standing for election and used by a voter to cast their vote; can be in paper or digital form, or mixed.

**candidate** A person standing for election to become a representative for their constituency; candidates are eligible for the quorum (not the usual meaning of quorum).

**constituency** *(also known as a legislative, election or voting district, or a division)* An area in which candidates stand to become the representative for that area; a constituency may have several polling stations.

**dispute** A process in which a quorum can challenge some votes; if the challenge is successful, the votes are excluded from the count.

**election** *(also known as a ballot)* A process in which a group of voters cast votes for candidates; the candidates with the most votes become the representatives for the constituency. For simplicity, the word “election” is also use for a referendum.

**election instance** A software representation of the election.

**identity provider** A software app or a person who generates a unique reference for each voter, which is incorporated into the voter’s identity string.

**instance** A software platform that is a representation of an election or a polling station and its data. Each such entity may have more than one instance. They are all identical to each other.

**key** A public key or a private key for an entity. Together, they comprise the key pair.

**key pair** Asymmetric cryptographic key pair or pairs, which includes a public key and a private key.
log (also known as the election log) A full record of the election transactions that is available to at least all parties involved in the election. It may be publicly available. It is organized as a blockchain of messages.

message A record of a transaction in the log, such as the creation of an entity.

none of the above An option in a ballot to reject all candidates or all options.

observer A person or other legal entity who monitors the ballot to ensure it is free and fair. Observers are eligible for the quorum.

operator (also known as an election operator) A person or other legal entity who manages an election instance or a polling station instance. They must have no other role in the election.

option One of the choices presented to voters in a referendum.

owner (also known as the election owner) A person or other legal entity who sets up the election. They are responsible for creating all the software elements of the election (election instances, etc.).

polling station A physical polling station is a place where voters can create a new ballot sheet and cast their vote. An online or virtual polling station is a piece of software, such as a website or app, which voters can use to create ballot sheets and cast votes.

polling station instance A software representation of a polling station.

quorum (also known as an election quorum) In this voting system, a quorum is the minimum number of candidates plus the minimum number of observers in each constituency needed to agree (or not) that the election has been correctly set up and that voting was free and fair. Further, a quorum can choose to exclude certain votes.

referendum A ballot without candidates where voters choose between one of several options (often, only two). The quorum comprises only observers.

registered voter A person for whom it has been verified that they are allowed to vote in the election and who have been registered to do so.

teller A person or software app that verifies a voter’s identity and registers their vote.

timestamp The difference, measured in seconds, between when a block was created and midnight on 1 January 1970 UTC.

voter A person who has a right to vote in the election.

voter identity string A unique string generated for a voter when they register to vote, which is used to confirm their identity when they vote.

write-in (also known as a write-in candidate, write-in referendum option or write-in option) Voters can add the names of candidates or options to their ballot sheet so that they can vote for them.
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