Gaussian vortex beam modeling for multiplexing in data communication using OAM

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Abstract. Optical vortices carrying orbital angular momentum (OAM) are being explored for improving the overall aggregate capacity along with the spectrum efficacy of data communications systems. The OAM carrying vortex beams provide an additional degree in space division multiplexing consistent with other physical dimensions like amplitude, frequency/wavelength, polarization & phase, leading towards continual increase of the overall transmission capacity along with the spectral efficacy through N-dimensional multiplexing.OAM has unlimited modes which are intrinsically orthogonal thus rendering OAM carrying beams to be efficiently multiplexed and subsequently demultiplexed. In this paper we simulate and study the intensity profiles and phase structures of Gaussian vortex beams. The clockwise and anti-clockwise spiraling of the phase structures for mutually opposite values of the topological charge of OAM and also diverging pattern of the phase structure along the direction of propagation are simulated.

Keywords: optical vortex, orbital angular momentum (OAM), Gaussian vortex beam, OAM multiplexing.

1. Introduction

The exponential increase in data flow has resulted in constantly increasing thirst to achieve high channel capacity along with effective use of spectrum in communication systems [1]. The optical communications community is concerned of achieving higher channel capacity[2]. 1948, Shannon derived the formula for the capacity of the band limited power limited, additive white Gaussian, waveform channel .According to this Shannon’s theorem, channel capacity of power and band limited Gaussian channel can be expressed as

\[ C = W \log_2 (1 + \frac{P}{N_0 B}) = W \log_2 (1 + \text{SNR}) \]  

Where C is the channel capacity, W gives the channel bandwidth, P = signal power, N0 = spectral density of noise power or noise power per unit of bandwidth, N0B = total noise power.

C is the basic limit on the rate of reliable communication for Gaussian channel which is power limited and band limited.
In order to increase channel capacity and spectral performance, various multilevel modulation techniques and multiplexing techniques have been used.

Multiplexing independent data streams is a definite way of increasing the capacity in data communication systems. Different wavelengths or polarizations or spatial structures can be used to carry independent channels of data[3, 4].

Mode division multiplexing (MDM) is a particular case of space division multiplexing (SDM)[2,5]. In MDM, an independent data channel is carried by each mode and efficient (de)multiplexing of the modes results from their orthogonality. The orthogonality of the modes also results in low inter-modal crosstalk.

Orbital angular momentum (OAM) is one amongst the orthogonal basis sets for MDM[2]. Movements of objects result in different kinds of momentums. Linear momentum is carried by an object which moves in straight line. Similarly, angular momentum is imparted when an object spins (spin angular momentum, SAM) or when it orbits around an axis (orbital angular momentum, OAM). A beam of light may possess these two momentums. In paraxial approximation, the rotation of the electric field around the axis of the beam corresponds to circular polarization results in the light beam carrying SAM. Whereas spiralling of the wave vector around the beam axis, resulting in helical phase fronts leads to the light beam that carries OAM. OAM results from the “twist” of helical phase fronts [2,6,7, 8]. Different number of twists of the helical phase fronts leads to different states of OAM beams. OAM beams propagate coaxially and are orthogonal [2].

These orthogonal coaxially propagating OAM beams are potentially useful to improve the optical communications systems performance. In particular, OAM states may be utilized as a separate dimension in MDM [9]. Significantly, OAM multiplexing is independent to wavelength multiplexing or polarization multiplexing. Thus, the channel capacity can be enhanced using various combinations of these different multiplexing techniques. Since the OAM states or modes are circularly symmetric OAM can have some benefits in their implementation, relative to other MDM approaches, which makes it appropriate for technologies of several optical components. [2,10].

In 1992, Allen et al. showed that beams of light having spiral phase fronts involving an azimuthal phase term \(\exp(\imath l\phi)\), possess an OAM and has a per photon value of \(\hbar\) (wherein “\(l\)” represents the topological charge; \(\hbar\) : the reduced Plank’s constant; \(\hbar = \hbar / 2\pi\); \(\hbar\) being the plank’s constant whereas \(\phi\) represents the azimuthal angle)[11,12]. OAM, unlike SAM which can have only two states, can in principle have infinite orthogonal states [6, 11].

A beam with a value “\(l\)” will have “\(l\)” intertwined helical phase fronts. If a circle is traced around the axis of the beam, “\(l\)” corresponds to number of phase shifts of \(2\pi\) along that circle. Being an integer, the value of \(l\) can be positive/negative or even a zero. Positive/negative values correspond to clockwise/anticlockwise phase helices whereas zero corresponds to a Gaussian beam [2,13].

A point of an optical field’s zero intensity is an optical vortex. It also describes light beam twisted like a corkscrew around its travelling axis and thus that has null in it [14].

The organization of the remaining paper is: The Vortex beam section presents the equations for Gaussian and Gaussian vortex beams along with the explanation of the terms in the equations. The simulation results and explanations section present the simulation results and explanations in some detail. The discussion and conclusion section presents a brief discussion on ongoing work using OAM carrying vortex beams in the field of communications and challenges therein and summaries the conclusion.
2. The Vortex Beam:
A monochromatic light beam whose transverse plane amplitude distribution is represented by a Gaussian constitutes a Gaussian beam. An expression for a Gaussian beam’s electrical field can be [15, 16]

\[ E(x,y,z) = E_0 \frac{w_0}{w(z)} \exp \left( \frac{x^2 + y^2}{w(z)^2} i (kz + k \frac{x^2 + y^2}{2R(z)} - \psi(z)) \right) \]

Where:

- \( E_0 \) gives the amplitude of Electrical field at origin,
- \( w_0 = w(0) \) gives radius at the beam waist,
- \( w(z) \) stands for the radius where value of amplitude is 1/e of its value at beam axis,
- \( k = \frac{2\pi n}{\lambda} \) (units: radians per meter)
- \( R(z) \): radius of curvature of wavefront at \( z \),
- \( \psi(z) = \arctan \left( \frac{Z}{Z_R} \right) \)

The expression for a Gaussian vortex beam’s electrical field can be [16, 17]:

\[ E(x,y,z,t) = E_0 \frac{w_0}{w(z)} \sqrt{\frac{x^2 + y^2}{w(z)^2}} \left| l \right| \exp \left( \frac{-x^2 + y^2}{w(z)^2} \right) \exp \left[ i \phi(x, y, z, t) \right] \]

where [16, 17]:

\[ \phi(x, y, z, t) = -\left( |l| + 1 \right) \arctan \left( \frac{2x}{k w_0} \right) + \frac{k(x^2 + y^2)}{2R(z)} + l \arctan \left( \frac{y}{x} \right) + k z - \omega t \]

Where:

- \( l \) stands for the topological charge,

The beams spot radius at position \( z \) is [16, 17]:

\[ w(z) = w_0 \sqrt{1 + \left( \frac{z}{Z_0} \right)^2} \]

\[ R(z) = Z \left( \frac{Z}{Z_0} + \frac{Z_0}{Z} \right) \]

where \( Z_0 \) is the Rayleigh range: \( Z_0 = \frac{1}{2} k \omega^2 = \frac{\pi \omega^2 n}{\lambda} \).
3. Simulation Results and Explanation:

Integrating equations (4), (5) and (6) in equation (3), with \( \lambda = 1550 \) nm, and using equations 2 and 3 we have simulated the Gaussian beam’s intensity profile and the Gaussian vortex beam’s intensity profile, respectively using MATLAB program (figure 1 and figure 2).

![Figure 1: Intensity profile of Gaussian beam (a) 3-d view and (b) x-y view.](image1.png)

![Figure 2: Intensity profile of Gaussian vortex beam (a) 3-d view and (b) x-y view.](image2.png)
It is difficult to imagine a twisting, helical beam of light carrying OAM, but it does create a clear noticeable effect. A conventional beam of light has an intensity profile that is brightest at the center. A light beam carrying OAM due to its twisted phase fronts resulting in the phase singularity at beam center produces a doughnut like or ring-like intensity profile. The helical beam’s cross section has a ring-like shape with a hole at the center. This hole represents the optical vortex at the axis of the beam. The beam’s center is full of miniwaves with every possible phase and it is very likely that a miniwave at its peak will overlap with a one at its trough. This results in destructive interference producing a dark center at the beam’s axis [18].

Our simulation results clearly show this ring like shape intensity profile with a dark center (figure 2).

3.1 The phase structure of a vortex beam

A light beam to possess OAM must have a certain kind of phase front. A phase front can be visualized by looking at the cross section of a light beam. The beam can be considered as a collection of a very large number of “miniwaves.” A beam has a flat phase front if all the miniwaves in the entire cross section at a point along direction of propagation of a beam have the same phase value, either all are at their crests or troughs or somewhere in between these extremes [18].

Whereas in the beams that carry OAM, the miniwaves in the given cross section at a point in the propagation direction are not having uniform phase. In such beams each miniwaves phase depends on its angular position around beam’s center. The phase of the miniwaves either would steadily increase or decrease if a circle is traced around the center of the beam [18].

Every miniwaves oscillates steadily in such beams, however the time sequence at which each of these miniwaves peaks results in the twist in the phase front. These twists in the phase fronts describe a helix as the beam propagates [18].

The Gaussian vortex beams simulated phase structures with \( l = +2, +4, -2, -4, +8 \) and \(-8\) obtained in the simulations of equation 3 are as shown in figure 3. It is observed that when “\( l \)” values are mutual opposite numbers the phase pattern spirals in the opposite directions.
Figure 3 the simulated phase structures of the Gaussian vortex beams for $l = +2(a), -2(b), +4(c), -4(d)$, $+8(e)$ & $-8(f)$. 
3.2 Diversion of Gaussian Vortex Beam

OAM multiplexing definitely leads towards improvement in the channel capacity along with effective use of spectrum in communication systems. However, difficulties are encountered when developing communication systems using OAM carrying beams. One amongst the challenges is the divergence of the OAM beams. OAM beam experiences divergence while its propagation through free space. As “l” of OAM increases, divergence increases [10].

It has been shown that the OAM beam diverges in the far-field. Divergence is proportional to square root of “l” when the beam waist is kept constant for all l values. Whereas, divergence is proportional to “l” when the beams rms radius is kept constant [19].

The effect of beam divergence is the increased power loss for a receiver aperture of limited size. Recovery of the signal power and phase information simultaneously is of prime importance for data recovery from the multiplexed beams. Signal power recovery is necessary to assure an adequate signal to noise ratio (SNR) [2,20]. For data recovery while the phase information allows multiplexed beams to be efficiently separated [21]. Unfortunately, for an OAM beam, the total phase shift per unit of area is highest at the beam core, where the signal strength is also lower. [21, 22, 23]. Trade off-exists between beams intensity and phase information. To ensure high SNR and low intermodal crosstalk, this trade-off needs to be considered carefully[5].

We have simulated the Gaussian vortex beam’s phase structure for a particular “l” value at different ‘z’ values along the direction of propagation. Figure 4 shows the simulated phase pattern of Gaussian vortex beam for same l value (l=+4) at different distances (z=1m, z=10m, z=15m and z=20m) along the propagation direction. The vortex beams divergence along the propagation direction is clearly seen from figure 4.

Figure 4: The Gaussian vortex beams simulated phase structure for l = +4 at z =1m(a), z =5m(b), z =10m(c) and z =15m(d).
Theoretically, there is infinite number of eigenstates of OAM carried by optical vortex beams. These infinite OAM modes, in principle, and orthogonality of OAM beams forms the basis of OAM multiplexing technique [10]. In OAM multiplexing, OAM carrying beams of the same frequency/wavelength are multiplexed and transmitted together and due to their inherent orthogonality are demultiplexed at the receiver.

The simulated phase structures of Gaussian vortex beams of four different “l” values at the same distance in the direction of propagation are shown in figure 5.

![Simulated phase structures of Gaussian vortex beams](image)

Figure 5: The Gaussian vortex beams simulated phase structure for $l = +2(a)$, $+4(b)$, $+6(c)$ & $+8(d)$ at $z = 1 m$.

4. Discussion and Conclusion:

OAM has been extensively studied in recent years, especially in the field of communication, along with many other fields of application, like optical trapping & manipulation, imaging, astronomy and processing of quantum information [16]. Studies have shown that OAM multiplexing can be effectively used to achieve higher spectral efficiencies.

SDM involves multiplexing of many beams of unique spatial pattern. A subset of SDM is MDM. Use of orthogonal set of OAM beams is an approach of achieving MDM [5].
Inherent orthogonality and unbounded states (due to unbounded values of topological charge “l”), in principle are the two distinct properties of OAM using which OAM modulation and OAM multiplexing techniques are being developed[24].

OAM multiplexing as a method of mode-division multiplexing (MDM) may be consistent with various modulation formats of modulation and with techniques of multiplexing involving use of different frequencies/wavelengths or polarizations. Thus, enabling an additional enhancement of the communication system’s capability from an additional aspect[10].

The usage of OAM carrying beams in optical (free space and fiber), radio and acoustic communications is being explored [10].

Using OAM multiplexing a very high overall capacity along with high spectral efficiencies are reported. A high aggregate capacity of transmission of 1.036 Pbits/ s along with 112.6 bits/s/Hertz spectrum efficiency have been successfully demonstrated using n-dimensional multiplexing in free-space[1,24].

Beam divergence and atmospheric turbulence are amongst the major challenges in the design of a communication link using OAM carrying beams in free space. Several methods have been suggested to minimize the impacts of divergence and atmospheric turbulence [10].

Combinations of data reduction techniques, error correcting codes, MIMO systems are explored to enhance the channel capacity [25]. Use of MIMO technology and OAM multiplexing simultaneously is experimentally demonstrated to improve the system performance [26].

The field of OAM-based optical fiber communication is also explored by the research community [10]. Mode crosstalk, nonlinearity, dispersion and losses could restrict the propagation distance and transmission efficiency in fiber-based communications using OAM. The mode purity can be degraded by the imperfections caused during the fabrication of the fiber [24]. Presently, the major focus of research is on designing of fibers supporting reliable transmission of several OAM modes [10].

In radio communications, the use of OAM is looked upon to provide an answer to the issue of bandwidth shortage. OAM-based technology is also being explored in the field of acoustic communications. Thus, considering the potential gains of OAM in communications, development of theoretical analysis as well as experimental verifications is expected to continue in near future [10].

In this study, the intensity and phase structures of the Gaussian and the Gaussian vortex beam were simulated using MATLAB. Some basic characteristics of the vortex beam propagation were obtained by simulations. The results of the simulations show the vortex beam’s ring-shaped intensity profile and its phase structures, the spiraling (clockwise and counterclockwise) of the phase structures for positive and negative values of the topological charges. Gaussian vortex beam’s phase structures for a certain topological charge “l” at different z values along the direction of propagation and phase structures of Gaussian vortex beams of different “l” values at given z were simulated.

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