Backward stochastic differential equations with stopping time as time horizon

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Abstract

In this paper, we introduce a new method for study on backward stochastic differential equations with stopping time as time horizon. And using this, we show that some results on backward stochastic differential equations with constant time horizon are generalized to the case of random time horizon.

Keywords: BSDE (backward stochastic differential equation), random time horizon, measure solution

1 Introduction

Backward stochastic differential equations (BSDEs) were first introduced by E. Paradoix and S. Peng in 1990 [2]. Since then BSDEs have been widely used in mathematical finance and partial differential equations (PDEs). Many pricing problems can be written in terms of linear BSDEs, or non-linear BSDEs when portfolios constraints are taken into account as in El Karoui, Peng and Quenez [10]. And numerous results show the intimate relationship between BSDEs and PDEs, which suggests that existence and uniqueness results which can be obtained on one side should have their counterparts on the other side. ([13,14, 15])

Many mathematicians have worked to improve the existence/uniqueness conditions of a solution for BSDEs in connection with the specific applications. ([6, 7, 12])

Most of those works are concerned with the case of constant time horizon.

But in many applications we encounter the case of random time horizon. For example M.Marcus and L. Veron show that the solution to PDE: \(-\Delta u + u|u|^q = 0\) are related to the BSDE: \(Y_t = \xi - \int_t^\tau Y_r |Y_r|^q dr - \int_t^\tau Z_r dW_r\). Here the time horizon \(\tau\) is a stopping time.

And in finance, there are many cases when the time horizon is not constant but random. There are many research papers about the case of random time horizon in connections with applications. ([9,11])

But those works on the case of random time horizon is limited to their specific cases and in many papers additional efforts are put to generalize their results to the case of...
random time horizon. In this paper, we suggest a general map from one side to the other side in section 2(Theorem 2.1) and show some results using this as examples in section 3. We note that our work is devoted to the case of stopping time and it will meet the most of applications.

2 Map from the case of constant time horizon to the case of random time horizon

In this section, we suggest a general map from the case of constant time horizon to the case of random time horizon.

Let \((W_t)_{t\geq 0}\) be a standard one dimensional Brownian motion on a complete probability space \((\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0})\).

We will use the following notations:

\[
\mathcal{L}^{loc}_2 := \{\Phi = \{\Phi(t)\}_{t\geq 0} : \text{real valued measurable processes adapted to the filtration } (\mathcal{F}_t)_{t\geq 0} \text{ such that } \forall T > 0, \int_0^T \Phi^2(t, \omega) dt < \infty \text{ a.s.}\}\]

\[
\mathcal{M}^{loc}_2 := \{X = \{X(t)\}_{t\geq 0} : \text{locally square integrable } (\mathcal{F}_t)-\text{martingales such that } X_0 = 0 \text{ a.s.}\}\]

\[
\mathcal{M}^{c,loc}_2 := \{X \in \mathcal{M}^{loc}_2 : t \rightarrow X_t \text{ is continuous a.s.}\}\]

**Definition 2.1** (Time Change). Let \((\Phi(t, \omega))_{t\geq 0}\) be a continuous real valued process on \((\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F})\) which satisfies following conditions.

(i) \((\Phi(t, \omega))\) is adapted to \((\mathcal{F}_t)_{t\geq 0}\)

(ii) \(t \rightarrow \phi(t)\) is strictly increasing a.s.

(iii) \(\lim_{t \to \infty} \phi_t = \infty \text{ a.s.}\)

(iv) \(\phi(0) = 0 \text{ a.s.}\)

Then we call \(\phi\) time change on \((\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F})\).

**Example 2.1.** If \(\tau\) is a \((\mathcal{F}_t)-\text{stopping time}, \phi(t, \omega) = \frac{t}{\tau}, t \in [0, \infty]\) is a time change on \((\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F})\).

From the definition, if \(\phi\) is a time change, there exist an inverse function almost surely and we have an inverse process \(\phi^{-1}(t, \omega)\). Then \(\phi^{-1}_t(\omega)\) is a stopping time for all \(t\) because 
\[
\{\phi^{-1}_t(\omega) \leq s\} = \{t \leq \phi_s(\omega)\} \in \mathcal{F}_s.
\]

And if \(X = \{X_t\}\) is a continuous \((\mathcal{F}_t)-\text{adapted process, then } T^\phi X = ((T^\phi X)(t)) = (X(\phi^{-1}_t))\) is a continuous \((\mathcal{F}_{\phi^{-1}_t})-\text{adapted process. Given a process of time change } \phi,\) we define a new reference family \((\tilde{\mathcal{F}}_t)\) by \(\tilde{\mathcal{F}}_t = (\mathcal{F}_{\phi^{-1}_t}), t \in [0, \infty)\) . The class \(\mathcal{M}^{c,loc}_2\) with
respect to $\tilde{F}_t$ is denoted by $\tilde{\mathcal{M}}_2^{\text{loc}}$. By Doobs optional sampling theorem if $M \in \mathcal{M}_2^{\text{loc}}$ then $\tilde{M} \in \mathcal{M}_2^{\text{loc}}$. And

$$X, Y \in \mathcal{M}_2^{\text{loc}} \Rightarrow <X_{\phi^{-1}}, Y_{\phi^{-1}} > = <X, Y>_{\phi^{-1}}$$

So we can see that the characters of process are preserved in time change.

**Lemma 2.1.** If $\phi(t, \omega)$ is a time change on $(\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F})$, $\phi^{-1}(t, \omega)$ is a time change on $(\Omega, \mathcal{F}, \mathbb{P}, \tilde{\mathbb{F}} = (\tilde{\mathcal{F}}_t)_{t \geq 0})$.

Now we show how the stochastic integral changes in time change.

**Proposition 2.1** ([1]). Let $M \in \mathcal{M}_2^{\text{loc}}$ and $\Phi \in \mathcal{L}_2^{\text{loc}}(M)$. Then $X = I^M(\Phi) := \int_0^t \Phi(s)dM_s$ is characterized as the unique $X \in \mathcal{M}_2^{\text{loc}}$ such that

$$<X, N>(t) = \int_0^t \Phi(u)d <M, N>(u)$$

for every $N \in \mathcal{M}_2^{\text{loc}}$ and all $t \geq 0$.

**Lemma 2.2.** Let $\xi$ be a nonnegative random variable on $(\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F})$, $\phi(t, \omega)$ be a $C^1$ time change on $(\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F})$ and $(X_t)_{t \geq 0} \in \mathcal{L}_2^{\text{loc}}(M)$.

Then $(\tilde{X}_t)_{t \geq 0} := (X_{\phi^{-1}(t)})_{t \geq 0} \in \mathcal{L}_2^{\text{loc}}(\tilde{M})$ and

$$\int_0^\xi X_s dM_s = \int_0^{\phi(\xi)} \tilde{X}_s d\tilde{M}_s \ a.s.$$  

**Proof.** As we mentioned above, $M \in \mathcal{M}_2^{\text{loc}}$ implies that $\tilde{M} \in \tilde{\mathcal{M}}_2^{\text{loc}}$ and

$$\tilde{I} := I^{\tilde{M}}(\tilde{X}_s) = \int_0^{\phi(\xi)} \tilde{X}_s d\tilde{M}_s \in \tilde{\mathcal{M}}_2^{\text{loc}}$$

And

$$<\tilde{I}, \tilde{N}>(t) = <I, N>(\phi^{-1}(t)) =
\int_0^{\phi^{-1}(t)} X_s d <M, N>(s) =
\int_0^t X_{\phi^{-1}(u)} d <M, N>(\phi^{-1}(u)) = \int_0^t \tilde{X}_u d <\tilde{M}, \tilde{N}>(u)$$

for all $\tilde{N} \in \tilde{\mathcal{M}}_2^{\text{loc}}$ and $t \geq 0$.

So from Proposition 2.1 we have the result.  

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**Corollary 2.1.** Let \( \eta \) be another nonnegative random variable on \((\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F})\). Then we have
\[
\int_{\eta}^{\xi} X_s dM_s = \int_{\phi(n)}^{\phi(\xi)} \tilde{X}_s d\tilde{M}_s \quad \text{a.s.}
\]

Now we state our main result which shows the relationship between the case of constant time horizon and the case of random time horizon.

**Theorem 2.1.** Let \((Y_t, Z_t)\) be a solution of the following BSDE defined on \((\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t))\)
\[
Y_t = \xi - \int_{t}^{\tau} Z_s dW_s + \int_{t}^{\tau} f(s, Y_s, Z_s) ds, \quad 0 \leq t \leq \tau \tag{2.1}
\]
where \(\tau\) is \((\mathcal{F}_t)_{t \geq 0}\)-stopping time such that \(0 \leq \tau < \infty\) a.s. and \(\xi \in \mathcal{F}_\tau\). Then there exist a \(C^1\) time change \(\phi(t, \omega)\) on \((\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t))\) such that
\[
(y_t, z_t) := (Y_{\phi^{-1}(t)}, Z_{\phi^{-1}(t)}(\phi(t)^{-\frac{1}{2}}))
\]
is a solution of the following BSDE defined on \((\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t))\).
\[
y_t = \xi - \int_{t}^{1} z_s d\tilde{W}_s + \int_{t}^{1} f(\phi^{-1}(s), Y_{\phi^{-1}(s)}, Z_{\phi^{-1}(s)}(\phi(s)^{-\frac{1}{2}})) \phi'(s)^{-1} ds, \quad 0 \leq t \leq 1 \tag{2.2}
\]
where \((\tilde{W}_t)_{t \geq 0}\) is a Wiener process on \((\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t))\).

**Proof.** Let’s take a \(C^1\) time change on \((\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t))\) such that \(\phi(\tau, \omega) = 1\). (See Example 2.1)

Now we’ll prove the result for this \(\phi\).

From the definition
\[
y_t = Y_{\phi^{-1}(t)} = \xi - \int_{\phi^{-1}(t)}^{\tau} Z_s dW_s + \int_{\phi^{-1}(t)}^{\tau} f(s, Y_s, Z_s) ds
\]
And from Lemma 2.2
\[
\int_{\phi^{-1}(t)}^{\tau} Z_s dW_s = \int_{\phi(\phi^{-1}(t))}^{\phi(\tau)} Z_{\phi^{-1}(s)} dW_{\phi^{-1}(s)} = \int_{t}^{1} Z_{\phi^{-1}(s)} dW_{\phi^{-1}(s)}
\]
and it’s clear that
\[
\int_{\phi^{-1}(t)}^{\tau} f(s, Y_s, Z_s) ds = \int_{t}^{1} f(\phi^{-1}(s), Y_{\phi^{-1}(s)}, Z_{\phi^{-1}(s)}) d\phi^{-1}(s)
\]
So we have
\[
y_t = \xi - \int_{t}^{1} Z_{\phi^{-1}(s)} dW_{\phi^{-1}(s)} + \int_{t}^{1} f(\phi^{-1}(s), Y_{\phi^{-1}(s)}, Z_{\phi^{-1}(s)}) d\phi^{-1}(s)
\]
Now let \( h(s, \omega) := \left( \frac{d\phi^{-1}(s)}{ds} \right)^{-\frac{1}{2}} \) and \( \tilde{W}_t := \int_0^t h(s, \omega) dW_{\phi^{-1}(s)} \), then \( \tilde{W}_t \) is a \((\tilde{\mathcal{F}}_t)\)-Brownian motion.

In fact, if we let \( M_t := W_{\phi^{-1}(t)} \), then as we have seen before, \( M_t \in \tilde{\mathcal{M}}^{c,loc}_2 \) and so \( \tilde{W}_t \in \tilde{\mathcal{M}}^{c,loc}_2 \).

And since \( \langle M \rangle_t = \phi^{-1}(t) \),

\[
\langle \tilde{W} \rangle_t = \int_0^t h^2(s, \omega)d\langle M \rangle_s = \int_0^t h^2(s, \omega)h^{-2}(s, \omega)ds = t
\]

So we have

\[
y_t = \xi - \int_t^1 Z_{\phi^{-1}(s)}h^{-1}(s, \omega)dW_s + \int_t^1 f(\phi^{-1}(s), Y_{\phi^{-1}(s)}, Z_{\phi^{-1}(s)})h(s, \omega)^{-2}ds
\]

Since \( z_t = Z_{\phi^{-1}(t)}h^{-1}(t, \omega) \), we have

\[
y_t = \xi - \int_t^1 z_sd\tilde{W}_s + \int_t^1 f(\phi^{-1}(s), y_s, z_s, \phi'(s)\phi^{-1}(s))dW_s
\]

And \( y_t, z_t \) are \((\tilde{\mathcal{F}}_t)_{t \geq 0}\) adapted because \( Y_t, Z_t \) are \((\mathcal{F}_t)_{t \geq 0}\) adapted and \( h(t, \omega) \) is \((\tilde{\mathcal{F}}_t)_{t \geq 0}\) adapted.

So \( (y_t, z_t) \) is a solution to the BSDE with constant time horizon defined on \((\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t))\).

\[\square\]

**Remark 1.** Note that the transformation of Brownian motion is given as following.

\[
\begin{aligned}
d\tilde{W}_s &= \phi'(s)^{-\frac{1}{2}}dW_{\phi^{-1}(s)} \\
dW_s &= \phi'(\phi(t))^{-\frac{1}{2}}d\tilde{W}_{\phi^{-1}(s)}
\end{aligned}
\]

**Remark 2.** From now on, we’ll use the notation \( t \in [0, \tau] \) to represent the set \( \{ (\omega, t) \in \Omega \times [0, \infty) \} \).

**Remark 3.** From Lemma 2.1, we can consider the inverse mapping. That is, if \((y_t, z_t)\) is a solution of (2.2), then \((Y_t, Z_t) = (y_{\phi(t)}, z_{\phi(t)}\phi'(\phi(t))^{-\frac{1}{2}})\) is a solution of (2.1). So in some sense, we can say that the two equations (2.1) and (2.2) are equivalent and we can get results for one equation from the results for the other one.

**Example 2.2.** Recently there has been much work on quadratic BSDEs in connections with partial differential equations. A typical and very simple quadratic BSDE is

\[
Y_t = \xi - \int_t^T Z_s dW_s + \int_t^T \alpha Z_s^2 ds
\]

If we replace the time horizon with stopping time \( \tau \), we have

\[
Y_t = \xi - \int_t^\tau Z_s dW_s + \int_t^\tau \alpha Z_s^2 ds
\]
Here if we apply the transformation in the theorem we have
\[ y_t = \xi - \int_t^1 z_s d\tilde{W}_s + \int_t^1 \alpha z_s^2 ds \]
and it’s just the same as the original one with constant time horizon. From this example we can guess that there isn’t any difference between the two equations.

3 Some Results

In this section, we generalize some results known for the case of constant time horizon to the case of random time horizon using our method.

3.1 Formula for the solution of linear BSDE

The linear BSDEs are very important in mathematical finance and there exists an explicit formula for the solution in the case of constant time horizon. We generalize this formula to the case of random time horizon.

In this section we assume that the random time horizon \( \tau \) is bounded by real number \( T > 0 \) and define the following spaces:

- \( \mathcal{P}_n \): the set of \( \mathcal{F}_t \) - measurable, \( \mathbb{R}^n \) - valued processes on \( \Omega \times [0, T] \)
- \( \mathcal{L}_2^0(\mathcal{F}_t) = \{ \eta : \mathcal{F}_t \text{-measurable random } \mathbb{R}^n \text{-valued variable such that } E[|\eta|^2] < \infty \} \)
- \( \mathcal{S}_2^n = \{ \varphi \in \mathcal{P}_n \text{ with continuous paths such that } E[|\sup_{t \leq T} |\varphi_t|^2] < \infty \} \)
- \( \mathcal{H}_2^1(0, T) = \{ Z \in \mathcal{P}_n \mid E[\int_0^T |Z_s|^2 ds] < \infty \} \)
- \( \mathcal{H}_1^2(0, T) = \{ Z \in \mathcal{P}_n \mid E[(\int_0^T |Z_s|^2 ds)^{\frac{1}{2}}] < \infty \} \)

**Theorem 3.1** (El Karoui, N., Peng S., Quenez M.C.[10]).

Let \( (\beta, \mu) \) be a bounded \( (\mathbb{R}, \mathbb{R}^d) \) - valued progressively measurable process, \( \varphi \) be an element of \( \mathcal{H}_1^2(0, T) \) and \( \xi \in \mathcal{L}_2^2(0, T) \).

We consider the following linear BSDE:

\[
\begin{cases}
-dY_t = (\varphi_t + Y_t \beta_t + Z_t \mu_t) dt - Z_t dW_t \\
Y_T = \xi
\end{cases}
\]

This equation has a unique solution \( (Y, Z) \in \mathcal{S}_2^2(0, T) \times \mathcal{H}_1^2(0, T) \) and \( Y \) is given explicitly by

\[ Y_t = E\left[ \xi \Gamma_{t,T} + \int_t^T \Gamma_{t,s} \varphi_s ds \mid \mathcal{F}_t \right] \]

where \( (\Gamma_{t,s})_{s \geq t} \) is the adjoint process defined by the forward linear SDE

\[
\begin{cases}
d\Gamma_{t,s} = \Gamma_{t,s}(\beta_s ds + \mu_s dW_s) \\
\Gamma_{t,t} = 1
\end{cases}
\]
Theorem 3.2. Let $(\beta, \mu)$ be a bounded $(\mathbb{R}, \mathbb{R}^d)$-valued progressively measurable process, $\varphi$ be an element of $\mathcal{H}_2^2(0, T)$, $\tau$ be a $(\mathcal{F}_t)$-stopping time bounded by $T > 0$ and $\xi \in L_1^2(0, T)$. We consider the following linear BSDE:

\[
\begin{cases}
  -dY_t = (\varphi_t + Y_t \beta_t + Z_t \mu_t)dt - Z_t dW_t \\
  Y_\tau = \xi
\end{cases}
\]

This equation has a unique solution $(Y, Z) \in S_2^2(0, T) \times \mathcal{H}_d^2(0, T)$ and $Y$ is given explicitly by

\[
Y_t = E \left[ \xi \Gamma_{t, \tau} + \int_t^\tau \Gamma_{t, s} \varphi_s ds \mid \mathcal{F}_t \right]
\]

where $(\Gamma_{t, s})_{s \geq t}$ is the adjoint process defined by the forward linear SDE

\[
\begin{cases}
  d\Gamma_{t, s} = \Gamma_{t, s}(\beta_s ds + \mu_s dW_s) \\
  \Gamma_{t, t} = 1
\end{cases}
\]

Proof. If we set $\phi(t, \omega) := \xi$ and apply the transformation in Theorem 2.1, then we have

\[
y_t = \xi + \int_t^1 [\varphi_{st} + \beta_{st} y_s + \mu_{st} z_s \frac{1}{\sqrt{\tau - s}}] \tau ds - \int_t^1 z_s d\tilde{W}_s
\]

Then from the above theorem we get the explicit solution

\[
y_t = E \left[ \xi \tilde{\Gamma}_{t, \tau} + \int_t^1 \tilde{\Gamma}_{t, s} \varphi_s ds \mid \tilde{\mathcal{F}}_t \right]
\]

And from Remark 2.2, the solution of the original equation is given by

\[
Y_t = E \left[ \xi \Gamma_{t, \tau} + \int_t^\tau \Gamma_{t, s} \varphi_s ds \mid \mathcal{F}_t \right]
\]

where $(\Gamma_{t, s})_{s \geq t}$ is the adjoint process defined by the forward linear SDE

\[
\begin{cases}
  d\Gamma_{t, s} = \Gamma_{t, s}(\beta_s ds + \mu_s dW_s) \\
  \Gamma_{t, t} = 1
\end{cases}
\]

3.2 Measure Solution

Measure solutions of BSDE were introduced by S. Ankirchner, et al.[16] and they prove the existence of measure solution for the case of constant time horizon under some constraints. Using our method, we can generalize the result to the case of random time horizon. The generalization is not so difficult and we do not note here. We just mention the result for the case of stopping time horizon.
Let $\tau$ be a $(\mathcal{F}_t)$—stopping time bounded by $T > 0$, $\xi$ be a bounded $\mathcal{F}_\tau$—measurable variable, $f: \Omega \times [0, T] \times \mathbb{R} \rightarrow \mathbb{R}$ be a measurable function such that $f(\cdot, \cdot, z)$ is predictable for all $z \in \mathbb{R}$ and satisfies following assumption.

**Assumption (H):**

(i) $\forall z \in \mathbb{R}, f(s, z) = f(\cdot, s, z)$ is adapted.

(ii) $\forall s \in [0, T], g(s, z) = \frac{f(s, z)}{z}, z \in \mathbb{R}$ is continuous in $z$

(iii) $\forall s \in [0, T], \forall z \in \mathbb{R}, |f(s, z)| \leq c(1 + z^2)$

(iv) there exists $\varepsilon > 0$ and a predictable process $(\psi_s)_{s \geq 0}$ such that $\int_0^\tau \psi_s dW_s$ is a BMO-martingale and for every $|z| \leq \varepsilon$, $|g(s, z)| \leq \psi_s$.

Let $\xi$ be an $\mathcal{F}_\tau$—measurable random variable and consider a BSDE

$$Y_t = \xi - \int_t^\tau Z_s dW_s + \int_t^\tau f(s, Z_s) ds \quad t \in [0, \tau]$$

**Theorem 3.3.** There exists a probability measure $\mathbb{Q}$ equivalent to $\mathbb{P}$ and an adapted process $Z$ such that $E(\int_0^\tau Z_s^2 ds)^{\frac{1}{2}} < \infty$ such that, setting

$$R = \exp\left(\int_0^\tau g(s, Z_s) dW_s - \frac{1}{2} \int_0^\tau g(s, Z_s)^2 ds\right) \mathbb{Q} = W - \int_0^\tau g(s, Z_s) ds$$

we have $\mathbb{Q} = R \cdot \mathbb{P}$ and such that the pair $(Y, Z)$ defined by

$$Y = E^\mathbb{Q}(\xi \mid \mathcal{F}_\cdot) = \int_0^\tau Z_s dW_s^\mathbb{Q}$$

solves the BSDE

$$Y_t = \xi - \int_t^\tau Z_s dW_s + \int_t^\tau f(s, Z_s) ds \quad t \in [0, \tau]$$

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