Tunable multi-channel inverse optomechanically induced transparency

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Abstract

In contrast to the optomechanically induced transparency (OMIT) defined conventionally, the inverse OMIT behaves as coherent absorption of the input lights in the optomechanical systems. We characterize a feasible inverse OMIT in a multi-channel fashion with double-sided optomechanical cavity system coupled to a nearby charged nanomechanical resonator via Coulomb interaction, where two counter-propagating probe lights can be absorbed via one of the channels or even via three channels simultaneously with the assistance of a strong pump light. Under realistic conditions, we demonstrate the experimental feasibility of our model using two slightly different nanomechanical resonators and the possibility of detecting the energy dissipation of the system. In particular, we find that our model turns to be an unilateral inverse OMIT once the two probe lights are different with a relative phase, and in this case we show the possibility to measure the relative phase precisely.

PACS numbers: 42.50.Nn, 78.40.-q, 42.50.Gy

I. INTRODUCTION

Electromagnetically induced transparency (EIT) [1] is caused by quantum interference, creating a narrow transmission window within an absorption line. EIT was first theoretically predicted in three-level atoms [2–8] and then observed in optically opaque strontium vapor [9, 10]. So far, EIT effects have attracted considerable attention both theoretically and experimentally due to relevant optical effects and applications, such as, optical Kerr effect and optical switch [11,12], slow light and quantum memory [13, 15], and quantum interference and vibrational cooling [16, 17]. The key point in realization of the EIT effect is to find a Λ-type level configuration and construct quantum interference. In this context, for some hybrid systems with Λ-type level structures, e.g., metal materials [18], coupled waveguides [19], atom-cavity systems [20], and optomechanical systems [21, 24], the analog of the EIT effects can also be observed.

The analog of the EIT effects in the optomechanics is named the optomechanically induced transparency (OMIT), which was predicted in the pioneering theoretical work [25], and then verified experimentally [26]. Very recently, the slow light was experimentally confirmed in the OMIT system [27], motivated by different proposals [28, 29] based on the OMIT effect. One of the outstanding works is for an inverse OMIT [30] in an optomechanical cavity, i.e., an optomechanical resonator inside a single-mode cavity, which shows that, when two weak counter-propagating probe lights within the narrow transmission window of the OMIT are injected simultaneously, neither of the probe lights can be output from the cavity due to complete absorption by the optomechanics. Therefore, this effect is also named the coherent perfect absorption and has been stretched to two optomechanical cavities coupled to an optomechanical resonator [37], showing the prospect for coherent perfect transmission and beyond. Since both the schemes [33, 36] focus only on the tunable double-channel inverse OMIT, it is natural to consider the possibility for a multi-channel inverse OMIT as well as the corresponding applications.

On the other hand, with an optomechanical cavity coupled to an external nanomechanical resonator (NR) via Coulomb interaction, the single narrow transmission window in the output light is split into two narrower transmission windows with the splitting governed by the Coulomb coupling [37]. This is due to the fact that an additional hybrid energy level is introduced into the original three-level system by the Coulomb coupling between the external NR and the optomechanical resonator. Similarly, the double OMIT effect can also be observed when the optomechanical resonator interacts with a qubit [38]. These observations remind us of the necessity to explore a multi-channel inverse OMIT in the optomechanical system.

In the present work, by considering a double-sided optomechanical cavity (involving a charged NR) coupled to another identical charged NR nearby via Coulomb interaction, we present a multi-channel inverse OMIT and study the energy dissipation of the system through the intracavity photon number and the mechanical excitations of the NRs. For a general case, two charged NRs with different frequencies will also be considered. Besides, provided a relative phase between two probe lights,
the inverse OMIT can only be observed on one side of the optomechanical cavity. We show how to measure this relative phase in our model.

Compared with the inverse OMIT in [33, 36], our idea has significant differences and thus owns different applications. First, our inverse OMIT is generated from a double-OMIT system. It is a multi-channel inverse OMIT with the windows of narrower profiles than the counterpart in [33, 36], and the dissipation of the input probe light can be directly detected by the external NR, without the need of an additional light field as required in [33, 36]. Second, if there is a relative phase between two probe lights, the inverse OMIT is observed only on one side of the optomechanical cavity, which is essentially different from the inverse OMIT in [33, 36]. This unilateral inverse OMIT can not only reduce the experimental difficulty for demonstrating the inverse OMIT effects, but also be very sensitive to the relative phase between the two probe lights. As such, it can be applied to measuring the relative phase. In addition, different from the analog Stokes process in the region of blue detuning [27, 39], our scheme can be achieved by an anti-Stokes process in the region of red detuning. We mention that some interesting characteristics demonstrated below might be helpful for practical applications using optomechanical systems, including the detection of the system dissipation and the relative phase between two probe lights.

The present paper is structured as follows. Sec. II presents the model and the analytical solution to the multi-channel inverse OMIT of an optomechanical system. In Sec. III we characterize the output probe fields, and under some realistic conditions, we explore in Sec. IV the situation of non-identical NRs, the energy dissipation and a possible application in precision measurement. A brief conclusion can be found in the last section.

II. THE MODEL AND SOLUTION

As sketched in Fig. 1, our system consists of a Fabry-Perot (FP) cavity and two charged NRs, i.e., NR$_1$ and NR$_2$. The NR$_1$ is inside the FP cavity formed by two fixed mirrors with finite equal transmissions, and couples to the cavity mode with a radiation pressure. The NR$_1$ also interacts with the NR$_2$ outside the FP cavity via a tunable Coulomb interaction. We suppose that the FP cavity is driven by a strong pump field (frequency $\omega_p$) from the left-hand side of the cavity, and two weak classical probe fields (frequency $\omega_q$) are injected into the cavity from both sides of the cavity. The Hamiltonian in the rotating frame at the pump field frequency $\omega_p$ can be written as

$$H = \hbar(\omega_0 - \omega_p)c\hat{c} + \left(\frac{\hbar^2}{2m_1} + \frac{1}{2}m_1\omega^2_{1q}\right)$$

$$+ \left(\frac{\hbar^2}{2m_2} + \frac{1}{2}m_2\omega^2_{2q}\right) + \hbar\varepsilon_c\hat{c}^\dagger\hat{c} - \hbar\varepsilon_R\hat{c}\hat{c}^\dagger - \hbar\varepsilon_L\hat{c}\hat{c}^\dagger - \hbar\varepsilon_R\hat{c}\hat{c}^\dagger - \hbar\varepsilon_L\hat{c}\hat{c}^\dagger$$

$$+ \hbar\delta\hat{c}\hat{c}^\dagger + \hbar\lambda_0\hat{c}\hat{c}^\dagger + \hbar\lambda_0\hat{c}\hat{c}^\dagger,$$

where the first three terms represent the free parts of the Hamiltonian for the cavity field and the NRs. $c (c^\dagger)$ is the annihilation (creation) operator of the cavity mode at frequency $\omega_0$. The charged NR$_1$ (NR$_2$) owns the frequency $\omega_1$ ($\omega_2$), the effective mass $m_1$ ($m_2$), the position $q_1$ ($q_2$) and the momentum $p_1$ ($p_2$). The next three terms describe the cavity mode driven by a pump field and two probe fields. $\varepsilon_c = \sqrt{2\kappa\varepsilon_p/\hbar\omega_c}$ ($\varepsilon_L(R) = \sqrt{2\kappa\varepsilon_p/\hbar\omega_p}$) is an amplitude of the strong pump (weak probe) field with $\varepsilon_p$ ($\varepsilon_q$) and $\kappa$ being the power of the pump (probe) field and the cavity decay rate, respectively, and $\delta = \omega_p - \omega_c$ is a detuning between the probe field and the pump field. The last two terms include the coupling of the NR$_1$ to the cavity mode via the radiation pressure strength $\gamma_0$ [40], and also the interaction between the NR$_1$ and NR$_2$ with the Coulomb coupling strength $\lambda_0 = C_1C_2\epsilon_{12}/2\pi\hbar\omega_{12}$ [41, 42]. The NR$_1$ (NR$_2$) takes the charge $Q_1 = C_1V_1$ ($Q_2 = -C_2V_2$), with $C_1$ ($C_2$) and $V_1$ ($-V_2$) being the capacitance and the voltage of the bias gates, respectively.

With the annihilation (creation) operator $b_j (b_j^\dagger)$, the position and momentum operators of the NR$_j$ are rewritten as

$$q_j = \sqrt{\frac{\hbar}{2m_j\omega_j}}(b_j + b_j^\dagger), \quad p_j = i\sqrt{\frac{\hbar m_j\omega_j}{2}}(b_j^\dagger - b_j).$$

When the two charged NRs are in near resonance, the
above Hamiltonian under the rotating-wave approximation is reduced to

\[
H' = \hbar (\omega_0 - \omega_c) c^\dagger c + \hbar \omega_1 b_1^\dagger b_1 + \hbar \omega_2 b_2^\dagger b_2
+ \hbar g c^\dagger (b_1 + b_1^\dagger) + i \hbar \varepsilon_c (c^\dagger c - c) + \hbar \lambda (b_1^\dagger b_2 + b_1 b_2^\dagger)
+ i \hbar (c^\dagger \varepsilon_L e^{-i \delta t} - h.c.) + i \hbar (c^\dagger \varepsilon_R e^{-i \delta t} - h.c.),
\]

with \( g = g_0 \sqrt{\hbar/2 m_1 \omega_1} \) and \( \lambda = \frac{k_0}{2} \sqrt{m_1 m_2 \omega_1 \omega_2} \).

Considering the damping and noise terms, the quantum Langevin equations are generated from Eq. (2),

\[
\dot{b}_1 = -i g c^\dagger (c - (\omega_1 + \frac{\gamma_1}{2})) b_1 - i \lambda b_2 + \sqrt{\gamma_1} b_1^{\dagger n},
\dot{b}_2 = -i (\omega_2 + \frac{\gamma_2}{2}) b_2 - i \lambda b_1 + \sqrt{\gamma_2} b_2^{\dagger n},
\dot{c} = -i (\omega_0 - \omega_c) c - i g (b_1 + b_1^\dagger) c + \varepsilon_c
+ (\varepsilon_L + \varepsilon_R) e^{-i \delta t} - 2 \kappa c + \sqrt{2 \kappa} (c_{in} + d_{in}),
\]

where \( \gamma_1 (\gamma_2) \) is the NR1 (NR2) decay rate, \( 2 \kappa \) is the cavity decay rate from the two sides. The quantum Brownian noise \( b_1^{\dagger n} (b_2^{\dagger n}) \) is resulted from the coupling between the NR1 (NR2) and the environment [32]. \( c_{in} (d_{in}) \) is the input quantum noise from the environment [32]. The mean values of the noise terms \( b_1^{\dagger n}, b_2^{\dagger n}, c_{in}, \) and \( d_{in} \) are zero.

To solve the Langevin equations in Eq. (3), we assume that each operator is a mean value plus a small quantum fluctuation, i.e., \( b = \alpha + \delta b \), with \( \delta b = b_1, b_2, \) and \( c, \) and \( \delta \varepsilon \ll |\alpha| \). Inserting them into Eq. (3) and neglecting the second-order smaller terms, we obtain the steady-state mean values of the system as

\[
\begin{align*}
\dot{b}_1 &= -i G^* \delta c - (i \omega_1 + \frac{\gamma_1}{2}) \delta b_1 - i \lambda \delta b_2 + \sqrt{\gamma_1} \delta b_1^{\dagger n}, \\
\dot{b}_2 &= -i (\omega_2 + \frac{\gamma_2}{2}) \delta b_2 - i \lambda \delta b_1 + \sqrt{\gamma_2} \delta b_2^{\dagger n}, \\
\dot{c} &= -i (\Delta + 2 \kappa) \delta c - i G \delta b_1 + (\varepsilon_L + \varepsilon_R) e^{-i \delta t} \\
&+ \sqrt{2 \kappa} (c_{in} + d_{in}),
\end{align*}
\]

with \( G = g c_{in} \) being the effective radiation pressure coupling.

The inverse OMIT effect can be studied by analyzing the mean response of the system to two probe fields in the presence of the pump field. After the input noises of the system are ignored, the mean value equations with the probe fields in Eq. (3) are rewritten as [25, 32],

\[
\begin{align*}
\langle \delta b_1 \rangle &= -i G^* \langle \delta c \rangle - (i \omega_1 + \frac{\gamma_1}{2}) \langle \delta b_1 \rangle - i \lambda \langle \delta b_2 \rangle, \\
\langle \delta b_2 \rangle &= -i (\omega_2 + \frac{\gamma_2}{2}) \langle \delta b_2 \rangle - i \lambda \langle \delta b_1 \rangle, \\
\langle \delta c \rangle &= -i (\Delta + 2 \kappa) \langle \delta c \rangle - i G \langle \delta b_1 \rangle + (\varepsilon_L + \varepsilon_R) e^{-i \delta t},
\end{align*}
\]

Further supposing the solution to Eq. (1) with the following form [32]

\[
\langle \delta o \rangle = \delta o_+ e^{-i \delta t} + \delta o_- e^{i \delta t},
\]

the results for the probe lights are given by

\[
\begin{align*}
\delta b_{1+} &= \frac{-i G^* \langle \delta c \rangle}{2 \kappa + i (\Delta -\delta) + \frac{\lambda^2}{2 \kappa + i (\Delta -\delta -\omega_1 + \omega_2)}}, \\
\delta b_{2+} &= \frac{-i \lambda \langle \delta b_1 \rangle}{2 \kappa + i (\Delta -\delta -\omega_1 + \omega_2)}, \\
\delta c_+ &= \frac{-i (\Delta + 2 \kappa) \langle \delta c \rangle}{2 \kappa + i (\Delta -\delta) + \frac{\lambda^2}{2 \kappa + i (\Delta -\delta -\omega_1 + \omega_2)}},
\end{align*}
\]

with \( A = \frac{\gamma_1}{2} - i (\delta - \omega_1) + \frac{\lambda^2}{2 \kappa + i (\Delta -\delta -\omega_1 + \omega_2)} \).

Our scheme considers a more general situation than in [32], since our results can be reduced to the counterpart in [32] in the absence of the Coulomb coupling. This is also confirmed in the comparison with the output field involving two tunable central frequencies for the inverse OMIT in [32], that our scheme owns three frequencies for the inverse OMIT effect, two of which can be adjusted by the Coulomb interaction.

### III. THE INVERSE OMIT

Based on the solutions above, we present below the multi-channel inverse OMIT in our system with some channels controllable by the driven field due to effective coupling and Coulomb interaction between the NRs.

For simplicity, we first assume two identical charged NRs (\( \omega_1 = \omega_2 = \omega_m \)) and the detuning between the pump field and the cavity mode satisfying \( \Delta \approx \omega_m \). This assumption helps an analytical understanding of characteristics of the model, but changes nothing in the physical essence of the considered system. The assumption will be released later under consideration of realistic condition.

Considering the output fields from the two sides of the cavity by the input-output relations [44]

\[
\varepsilon_{outa} + \varepsilon_{a} e^{-i DT} = 2 \kappa \langle \delta c \rangle, \quad \alpha = R, L,
\]

with \( D = \delta - \omega_m \) being the detuning of the probe field from the cavity resonance frequency, we define the output fields as

\[
\varepsilon_{outa} = \varepsilon_{outa} + e^{-i DT} + \varepsilon_{outa} - e^{i DT}, \quad \alpha = R, L,
\]

where \( \varepsilon_{outa} \) and \( \varepsilon_{outa} \) are the responses at the frequencies \( \omega_p \) and \( 2 \omega_c - \omega_p \) in the original frame.

Using Eqs. (7), (9) and (10), the output fields at the probe frequency \( \omega_p \) are presented as

\[
\varepsilon_{outa} = \frac{2 \kappa \delta c_+ - \varepsilon_{\alpha}}{2 \kappa + i (\Delta -\delta) + \frac{\lambda^2}{2 \kappa + i (\Delta -\delta -\omega_1 + \omega_2)}} - \varepsilon_{\alpha}
\]

with \( \alpha = R, L \).
The zero output fields, i.e., $\varepsilon_{outR^+} = \varepsilon_{outL^+} = 0$, occur under the following conditions

$$\begin{align*}
\varepsilon_L &= \varepsilon_R, \gamma_1 = \gamma_2 = 2\kappa, \\
\lambda^2 &= \frac{1}{2}(G)^2 - \kappa^2.
\end{align*}$$

Thus there are three channels at

$$D_0 = 0,$$

$$D_{\pm} = \pm \sqrt{|G|^2 + \lambda^2 - 3\kappa^2} = \pm \sqrt{\frac{3}{2}|G|^2 - 4\kappa^2},$$

for the coherent perfect absorption, implying that the probe lights cannot be reflected or transmitted from this optomechanical system, but entirely absorbed. This is due to a perfect destructive interference between the two probe lights along opposite directions. The energy of the probe lights will be finally dissipated via the vibrational decay of the NRs and the thermal-photon decay in the optomechanics, as discussed in detail later. As a result, this optomechanical system can be used to realize the multi-channel inverse OMIT (Fig. 2) with the essential prerequisite of the optomechanical normal-mode splitting.

For the detuning cases of $D_{\pm} = \pm \sqrt{\frac{3}{2}|G|^2 - 4\kappa^2}$, the effective coupling rate should follow $|G| \geq \sqrt{\frac{3}{2}\kappa}$. $D_0 = D_{\pm} = 0$ is a special case representing only a single channel involved in the inverse OMIT when $|G| = \sqrt{\frac{3}{2}\kappa}$ and $\lambda = \frac{1}{\sqrt{3}}\kappa$. Considering the general cases with $|G| > \sqrt{\frac{3}{2}\kappa}$, there are three channels as presented in Eq. (13), corresponding to the three injected probe lights at the frequencies $\omega_p = \omega_c + \omega_m$ and $\omega_p = \omega_c + \omega_m + D_{\pm}$ with $D_{\pm} = \pm \sqrt{\frac{3}{2}|G|^2 - 4\kappa^2}$. For example, in the case of $|G| = 2\kappa$ and $\lambda = \kappa$, the inverse OMIT effect can be observed at $D_0 = 0$ and $D_{\pm} = \pm \sqrt{2\kappa}$, corresponding to the three injected probe lights at the frequencies $\omega_p = \omega_c + \omega_m$ and $\omega_p = \omega_c + \omega_m \pm \sqrt{2\kappa}$, respectively. Moreover, these two additional windows become more separate with the increase of both the effective radiation coupling $|G|$ and the corresponding Coulomb coupling $\lambda = \sqrt{\frac{3}{2}|G|^2 - \kappa^2}$, as demonstrated in Fig. 2.

IV. DISCUSSION

A. Multi-channel inverse OMIT with two non-identical NRs

The two identical NRs considered above are theoretically simplified, but rarely existing experimentally. To release this stringent condition, we consider below the multi-channel inverse OMIT with non-identical NRs.

For two charged NRs with different frequencies, as plotted in Fig. 3 the multi-channel inverse OMIT occurs with some window shifts with respect to the case of identical NRs, turning it to be asymmetric for the curves of the normalized probe photon number versus the probe detuning. It can be understood from the bright mechanical mode $b = b_2 \sin \theta + b_1 \cos \theta$ and the dark one $d = b_2 \cos \theta - b_1 \sin \theta$ with $\tan \theta = [(\omega_2 - \omega_1) + \sqrt{4\lambda^2 + (\omega_2 - \omega_1)^2]}/(2\lambda)$, which are the diagonalized orthogonal modes of the two coupled mechanical modes $b_1$ and $b_2$. These bright and dark modes can effectively couple to the optical mode with the strengths $G \cos \theta$ and $G \sin \theta$, respectively. In contrast to the case of $\omega_1 = \omega_2$ with both the bright and dark modes sharing the same coupling strength $G/\sqrt{2}$, the effective couplings for the bright and dark modes are different in the case of $\omega_1 \neq \omega_2$. Different from the symmetric curves in the case of $\omega_1 = \omega_2$, the curves of the normalized probe photon number versus the probe detuning move leftward if $\omega_1 > \omega_2$ and rightward if $\omega_1 < \omega_2$ (see Fig. 3). This implies that the middle channel of this multi-channel inverse OMIT is not always fixed, but variable if we appropriately tune the frequencies of the NRs, as in Fig. 3.

More differences can be found in the discussion below from the comparison between the identical and non-identical NRs.

B. The energy distribution

We analyze below the paths of the energy dissipation during the inverse OMIT process. To identify the thermal dissipation in the inverse OMIT, we calculate the intracavity probe photon number $|\delta c_+|^2$ and the quantum excitation of the thermal phonons $|\delta b_1|^2$ ($|\delta b_2|^2$) in NR1 (NR2).

Using the fluctuation operators in Eq. (8), we obtain the normalized intracavity probe photon number,

$$\frac{4\kappa^2}{|\varepsilon_L|^2 + |\varepsilon_R|^2}|\delta c_+|^2 = 0.5,$$

which is the ratio of the probe photon number $|\delta c_+|^2$.
TABLE I: The relationship among the normalized output probe photon numbers ($\varepsilon_{\text{out}R\pm}$ and $\varepsilon_{\text{out}L\pm}$), the intracavity probe photon numbers ($\frac{4\kappa^2}{|\varepsilon_L|^2+|\varepsilon_R|^2}|\delta b_{1\pm}|^2$), and the mechanical excitations ($\frac{4\kappa^2}{|\varepsilon_L|^2+|\varepsilon_R|^2}|\delta c_{1\pm}|^2$ and the summation) for different effective radiation $G$ and Coulomb coupling strengths $\lambda$ in the inverse OMIT. Part I presents the middle channel $D = D_0$ and part II for the side channels $D = D_\pm$. We consider two non-identical NRs with $\omega_1 = 1.2\kappa$ and $\omega_2 = \kappa$ and Coulomb coupling strengths $\lambda$ for identical NRs and $\lambda = 0$ for non-identical case. The values in parentheses are for the identical case.

| $D/\kappa$ | $G/\kappa$ | $\lambda/\kappa$ | $\varepsilon_{\text{out}R\pm}$ | $\varepsilon_{\text{out}L\pm}$ | $\frac{4\kappa^2}{|\varepsilon_L|^2+|\varepsilon_R|^2}|\delta c_{1\pm}|^2$ | $\frac{4\kappa^2}{|\varepsilon_L|^2+|\varepsilon_R|^2}|\delta b_{1\pm}|^2$ | $\frac{4\kappa^2}{|\varepsilon_L|^2+|\varepsilon_R|^2}|\delta b_{2\pm}|^2$ | $\frac{4\kappa^2}{|\varepsilon_L|^2+|\varepsilon_R|^2}|(|\delta b_{1\pm}|^2 + |\delta b_{2\pm}|^2)$ |
|-----------|-----------|-----------------|----------------|----------------|------------------|------------------|------------------|------------------|
| I         |           |                 |                |                |                  |                  |                  |                  |
| 0.198 (0) | 2         | 1               | 0 (0)          | 0 (0)          | 0.503 (0.5)      | 0.498 (0.5)      | 0.497 (0.5)      | 0.997 (1.0)      |
| 0.140 (0) | 4         | $\sqrt{7}$     | 0 (0)          | 0 (0)          | 0.501 (0.5)      | 0.125 (0.125)    | 0.874 (0.875)    | 0.999 (1.0)      |
| 0.136 (0) | 6         | $\sqrt{17}$    | 0 (0)          | 0 (0)          | 0.518 (0.5)      | 0.006 (0.036)    | 0.976 (0.944)    | 0.932 (1.0)      |
| $+1.415 (\pm \sqrt{2})$ | 2 | 1 | 0 (0) | 0 (0) | 0.502 (0.5) | 0.711 (0.711) | 0.287 (0.287) | 0.998 (1.0) |
| $-1.415 (\pm \sqrt{2})$ | 2 | 1 | 0 (0) | 0 (0) | 0.500 (0.5) | 0.783 (0.783) | 0.217 (0.217) | 0.998 (1.0) |
| II        |           |                 |                |                |                  |                  |                  |                  |
| $+4.029 (+2\sqrt{5})$ | 4 | $\sqrt{7}$ | 0 (0) | 0 (0) | 0.492 (0.5) | 0.742 (0.742) | 0.266 (0.266) | 1.008 (1.0) |
| $-4.358 (-2\sqrt{5})$ | 4 | $\sqrt{7}$ | 0 (0) | 0 (0) | 0.508 (0.5) | 0.757 (0.757) | 0.255 (0.255) | 0.992 (1.0) |
| $+7.227 (+5\sqrt{2})$ | 6 | $\sqrt{17}$ | 0 (0) | 0 (0) | 0.494 (0.5) | 0.748 (0.748) | 0.261 (0.261) | 1.006 (1.0) |
| $-6.943 (-5\sqrt{2})$ | 6 | $\sqrt{17}$ | 0 (0) | 0 (0) | 0.506 (0.5) | 0.754 (0.754) | 0.240 (0.240) | 0.994 (1.0) |

versus the sum of the probe photon numbers $|\frac{4\kappa^2}{|\varepsilon_L|^2+|\varepsilon_R|^2}|\delta b_{1\pm}|^2$ without the coupling field. By a similar way, the corresponding normalized mechanical excitations of the charged NR1 and NR2 for different channels, in units of the sum of the probe photon numbers, are given, respectively, by

$$\frac{4\kappa^2}{|\varepsilon_L|^2+|\varepsilon_R|^2}|\delta b_{1\pm}|^2 = \begin{cases} 2\frac{\kappa^2}{|\varepsilon_L|^2}, & D_0 = 0 \\ 0.75, & D_0 = \pm \sqrt{\frac{3}{2}}|G|^2 - 4\kappa^2 \end{cases},$$

and

$$\frac{4\kappa^2}{|\varepsilon_L|^2+|\varepsilon_R|^2}|\delta b_{2\pm}|^2 = \begin{cases} 1 - 2\kappa^2|\varepsilon_L|^2, & D_0 = 0 \\ 0.25, & D_0 = \pm \sqrt{\frac{1}{2}}|G|^2 - 4\kappa^2 \end{cases}.$$

Eqs. (15) and (16) present independent thermal dissipations for the probe lights with different frequencies. Due to this fact, the multi-channel inverse OMIT can occur simultaneously in the three channels with different dissipations.

From Figs. 4 and Table I, we find in the case of different NRs that, when the inverse OMIT takes place, the sum of the mechanical excitations $\sum_{i=1}^{2}\frac{4\kappa^2}{|\varepsilon_L|^2+|\varepsilon_R|^2}|\delta c_{1\pm}|^2 = 1$ is always double of the intracavity probe photon number $\frac{4\kappa^2}{|\varepsilon_L|^2+|\varepsilon_R|^2}|\delta c_{1\pm}|^2 = 0.5$. This implies that the energy distribution in the two NRs and the cavity field always remains with the ratio 2 : 1. Besides, with
OMIT observed in the left-hand side takes the form input from the right-hand side of the cavity, the inverse increase of $G$ imperfect inverse OMIT.

The difference between the two probe lights in such deviation from the perfect destructive interference. As a difference between the two probe lights, would lead to positive directions [35], any imperfection, such as a phase $D$ with $\omega$ characteristics of our multi-channel inverse OMIT are very satisfies the condition for the inverse OMIT. These characteristics to the identical case, and only the middle channel for two different NRs, there is a small deviation with respect to those opto-mechanical systems in different parameter values; (b) and (d) The phase $\theta$ of the unilateral inverse OMIT involving from both sides of the cavity, the inverse OMIT involving lateral inverse OMIT, the relative phase $\theta$ is found to be monotonously varying with the detuning $D$ within some

increase of $G$ and $\lambda$, the phonon distribution in the two NRs varies in different ways conditional on the channels. For two different NRs, there is a small deviation with respect to the identical case, and only the middle channel satisfies the condition for the inverse OMIT. These characteristics of our multi-channel inverse OMIT are very different from in previous schemes [35, 36].

C. Measurement of the relative phase in an unilateral inverse OMIT

Since the inverse OMIT is resulted from the perfect destructive interference between two probe lights along opposite directions [35], any imperfection, such as a phase difference between the two probe lights, would lead to deviation from the perfect destructive interference. As a result, it is interesting to explore the possibility of detecting the difference between the two probe lights in such imperfect inverse OMIT.

After a relative phase $\theta$ is introduced in the probe light input from the right-hand side of the cavity, the inverse OMIT observed in the left-hand side takes the form

$$\varepsilon_{outL+} = \frac{2k(\varepsilon_L + \varepsilon_R e^{i\theta})}{2k + i(\Delta - \delta) + \frac{\omega^2}{D - i(\delta - \omega_c) + \frac{\omega^2}{D - i(\delta - \omega_2)}}} - \varepsilon_L = 0. \tag{17}$$

In contrast to the same output lights ($\varepsilon_{outL+} = \varepsilon_{outR+}$) from both sides of the cavity, the inverse OMIT involving a relative phase outputs the lights with $\varepsilon_{outL+} \neq \varepsilon_{outR+}$, implying that the inverse OMIT, if occurring, is observed only on one side of the cavity. In such an unilateral inverse OMIT, the relative phase $\theta$ is found to be monotonously varying with the detuning $D$ within some parameter regimes, which can be employed for evaluating $\theta$ (see Fig. 5).

Provided that the strengths of the two probe lights are the same ($\varepsilon_L = \varepsilon_R$), the above equation is reduced to

$$\frac{|G|^2}{D - i(\delta - \omega_1) + \frac{\omega^2}{D - i(\delta - \omega_2)}} - i(\delta - \Delta) = 2k\varepsilon L^2. \tag{18}$$

Straightforward deduction using the relations among $D$, $\delta$ and $\Delta$ yields that, the relative phase $\theta$ is a function of the detuning $D$, as plotted in Fig. 5(b,d), and not all the frequencies of the probe lights can generate the unilateral inverse OMIT effect.

For a precision measurement of $\theta$, choosing qualified regimes of the parameters, e.g., $D/\kappa \in [-1.5, 1.5]$, is necessary to obtain a monotonous change of $\theta$ with $D$. Besides, for the measurement to be more precise, we expect a large change of $D$ for tiny variation of $\theta$, implying a small slope of $\Delta \theta / \Delta D$. As a result, smaller radiation coupling is optimal [see the curves in Fig. 5(b)] with $|G| = 2k, 4\kappa$ and note the lower limit $|G| \geq \sqrt{8/\kappa}$. In comparison with the identical NRs [in Fig. 5(a)], the curves for the non-identical NRs [in Fig. 5(d)] own smaller slopes, implying a better measurement. For example, in the case of $D/\kappa \in [-0.01, 0.01]$, the measurement sensitivity $\Delta D / \Delta \theta$ is 6.3 MHz/rad for the blue curve in Fig. 5(b), and 7.7 MHz/rad for the black curve in Fig. 5(d). So elaborately choosing different NRs can be favorable for enhancing the measurement precision of $\theta$. By numerical simulation of Eq. (18), we find the largest measurement sensitivity $\Delta D / \Delta \theta = 8.3$ MHz/rad at $\omega_2/\omega_1 = 1.346$, since the unilateral inverse OMIT would disappear once $\omega_2/\omega_1 > 1.34$.

V. CONCLUSION

In summary, we have investigated a tunable multi-channel inverse OMIT in the optomechanical system with the assistance of the Coulomb interaction between two charged NRs. Our results have shown both analytically and numerically three channels for perfectly absorbing the input probe fields at different frequencies in such a system, which makes it possible to select a desired frequency of inverse OMIT by adjusting effective radiation coupling rate and the corresponding Coulomb coupling strength. Some applications have been discussed based on the considered model. We believe that study would be useful for further understanding the inverse OMIT and exploring the applications of the inverse OMIT.

We have also noted the opto-mechanical experiments reported recently with NRs coupled by tunable optical coupling [46] and fixed elastic coupling [47]. Replacing the Coulomb coupling by those couplings, our model can immediately apply to those opto-mechanical systems in [46, 47]. In addition, we are also aware of a recent work for a multi-channel inverse OMIT by confining many NRs in a single cavity [53]. The idea is very interesting, but
much more difficult to achieve experimentally than our scheme.

ACKNOWLEDGMENTS

QW thanks Lei-Lei Yan, Yin Xiao and Peng-cheng Ma for their helps in numerical simulation. This work is supported by the National Natural Science Foundation of China (Grants Nos. 91121023, 61378012, 60978009, 11274352, 91421111 and 11304366), the SRFDPHEC (Grant No. 2012407110009), National Fundamental Research Program of China (Grants Nos. 2011CB900200, 2012CB22102 and 2013CB921804), the PCSIRT (Grant No. IRT1243), and China Postdoctoral Science Foundation (Grants Nos. 2013M531771 and 2014T70760).

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