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Cite as: AIP Advances 10, 035207 (2020); https://doi.org/10.1063/1.5143927
Submitted: 02 January 2020. Accepted: 19 February 2020. Published Online: 05 March 2020

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AVS Quantum Science
Co-Published by
AIP Publishing

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ABSTRACT
This paper aims to propose a tangential contact model of a lap joint interface with non-Gaussian surfaces. Relying on the full-stick contact condition, the elastic–plastic deformation of a single asperity and the penetration-dependent friction coefficient are considered in this model. The Johnson system is utilized to generate non-Gaussian asperity height distributions. Furthermore, the physical asperity model and the phenomenological Iwan model are combined to obtain a continuous and convergent Iwan solution by the dimensional analysis method. The initial tangential stiffness, the tangential force required for gross slip, and the slip index of lap joints reveal the hysteresis loop shape, describing the tangential response completely and uniquely, and the first two parameters are proved to own statistical characteristics. Next, the effects of topography parameters on the initial tangential stiffness, the tangential force/displacement required for gross slip, and the slip index are analyzed. Comparisons among the proposed model, the published models, and the published experimental results have also been made. The proposed model is shown to be consistent with the experimental results when the tangential load is insufficient to cause gross slip, while an error is produced when gross slip happens. Additionally, the error could be reduced in the calibrated model.

I. INTRODUCTION
The lap-type joint, with complex mechanisms and strong non-linear characteristics, is a typical assembly type and influenced by many factors. Lap joints contribute to most of the total stiffness and damping of mechanical structures, and many scholars have been exploring their dynamic tangential response. All quasi-static loaded lap joints undergo fretting damage, which is currently considered to be one of the major causes of industrial material failure, if a source of vibration is present.

According to the experimental observation and theoretical analysis, Vingsbo and Söderberg and Fouvry et al. have classified fretting regimes of two surfaces into four contact states: Stick, partial slip, gross slip, and reciprocating sliding. A fretting map is a diagram directly representing the relevant regimes under the combined normal and tangential loading. The parameter slip index was subsequently introduced by Varenberg et al. to quantitatively distinguish fretting regimes. Depending on loading conditions, fretting damage can be caused by surface fatigue, including crack nucleation and crack propagation, and wear induced by third body transformation. Consequently, fretting damage types such as cracking and wear can be depicted in the fretting map, as shown in Fig. 1. It shows that the evolution of the fretting damage is closely related to the loading conditions of the contact system. Furthermore, since the fretting damage is responsible for a significant drop in the service lifetime of engineering structures, the prediction of fretting regimes naturally becomes a significant and challenging task for a tangential contact system.

Although the fretting map and the slip index can be used as tools to identify fretting regimes, no universal constitutive model that decides the shape of the fretting map and the magnitude of the slip index has been formulated due to the complexity of roughness, partial slip, adhesion, wear, and lubrication in a contact system. Nevertheless, researchers have long attempted to build some
The classical physical tangential contact model is Mindlin’s partial slip solution, which can be expanded to model the cyclic tangential load contact. With Hertz theory and Coulomb friction law integrated, the Mindlin solution becomes an elegant analytical solution but only effective in the case of elastic contact. To extend the application scope of the Mindlin model, Ödfalk and Vingsbo\(^{12}\) and Fujimoto et al.\(^{13}\) broke up the ideally elastic contact assumption to model the elastic and plastic tangential contact. The Ödfalk model, which employs experimentally determined parameters, is a phenomenological model, and the Fujimoto model is essentially an elastic–then–plastic model inevitably failing to illustrate an elastoplastic contact. Subsequently, Brizmer et al.\(^{14}\) used the von Mises yielding criterion and full-slip condition (where further relative displacement of points engaged in spherical contact is prevented) to simplify and improve the elastic–plastic contact. The finite element and numerical fitting methods collectively contribute to their model. All these models are basic dynamic tangential contact models based on the single-asperity contact. Incorporating single-asperity models into multi-asperity contacts, Cohen et al.\(^{15}\), Eriten et al.\(^{16}\), and Zhan and Huang\(^{17}\) proposed useful tangential contact models for rough surfaces. In addition, since Iwan elements possess the physical qualities of elasticity and plasticity associated with joint mechanics, the Iwan model also appears to be a natural candidate for a dynamic tangential contact system.\(^{18,19}\)

As a result of good performance in numerical computability and certain rationality in engineering applications of Gaussian distribution, it has been assumed to represent mostly surface height distributions of nominally flat rough surfaces. Nevertheless, the surface height distributions are non-Gaussian for most manufacturing processes.\(^{20,21}\) For eliminating this contradiction, the Weibull,\(^{22}\) Pearson,\(^{23}\) and Johnson\(^{24}\) distributions have been utilized to characterize non-Gaussian surfaces in normal rough contact models. Although many articles have been published on the effects of non-Gaussian surfaces on normal contact systems, few have examined their influence on tangential contact systems, primarily because the tangential contact system itself is too complicated to be accurately captured.\(^{25}\)

This paper presents a dynamic tangential contact model of lap joints with non-Gaussian surfaces. The Johnson system is utilized to generate non-Gaussian distributions, and the elastic–plastic deformation of asperities based on the full-slip contact condition is considered. Employing dimensional analysis, the normal single-asperity contact model with a penetration-dependent friction coefficient is incorporated into the Coulomb friction law, to obtain a new solution of the Iwan model. A constitutive model of the tangential response for lap joints under cyclic tangential loads is then constructed, while in this model all the parameters are physical parameters rather than numerical-fitting or experimental-calibrated. Furthermore, the effects of topography parameters on tangential responses of lap joints are investigated, and comparisons of predicted fretting loops among the proposed model, the published models, and the published experimental results are conducted.

II. MODEL DERIVATION

A lap joint interface can be simplified to consist of two nominally flat surfaces subjected to a normal load \(N\), as shown in Fig. 2(a). Then, the mating surfaces come into contact at asperity summits. According to the method of Greenwood and Williamson,\(^{26}\) the lap joint interface is reduced to that between a rigid smooth surface and an equivalent nominally flat one, as shown in Fig. 2(b). Figure 2(b) also shows geometrical parameters of the equivalent surface: asperity radius \(R\), asperity height \(z\), normal interference \(\omega\), and separation based on asperity heights \(d\).

A. Characterization of fretting loop and non-Gaussian surface

When an oscillating tangential load \(T\) with a small amplitude is applied to the lap interface by a controllable tangential force or displacement, a relative cyclic motion occurs at the interface, which is called fretting.\(^{27}\) In this case, a force–displacement hysteresis loop is obtained by analyzing the tangential response of the lap interface as shown in the gray region of Fig. 3(a). A typical fretting
system includes an initial loading, unloading, and reloading. The slip amplitude $A_s$ (aperture of fretting loop), the imposed tangential displacement/force amplitude $A_r/A_T$, and the initial tangential stiffness $K_{IT}$ of the lap joint interface can all be expressed in the loop. Since the “small amplitude” is only a qualitative description, it is difficult to distinguish the specific fretting regime with this description. For representing the contact states of lap joints, more accurately, Varendberg et al. have redefined the fretting behavior and proposed a quantitative measure named the slip index $\delta$,

$$\delta = \frac{A_s K_{IT}}{N},$$  \hspace{1cm} (1)

where the graphical representation of each parameter is shown in Fig. 3(a). When the fretting loop is enveloped by a parallelogram in Fig. 3(a), once the side slope $K_{IT}$, aperture $2A_s$-related base, and height $2A_T$ of this parallelogram are determined, the shape of the loop can be uniquely identified. $A_s$ is a governing parameter for the slip ratio $A_s/A_w$, which sheds light on the contact state of lap joints, and can be replaced by $\delta$. Therefore, the dynamic tangential response of the lap joint interface can be characterized directly by $K_{IT}$, $A_T$, and $\delta$. Figure 3(b) illustrates the correspondence between $\delta$ and fretting regimes. The lap joint interface is in a transition state from the former contact state to the latter one, when the corresponding value of $\delta$ is not obtained in Fig. 3(b). In addition, the energy dissipation per loading cycle $D$ can be displayed by the area of the fretting loop. When the lap interface is in stick, the shape of the loop is a line meaning no energy dissipation. When in gross slip and reciprocating sliding, the energy dissipation is large, implying that wear is severe.

Surface features can be characterized by four topography parameters: mean of asperity heights, standard deviation of asperity heights $\sigma_h$, skewness $Sk$, and kurtosis $Ku$. Although many functions can be used to describe non-Gaussian surfaces, considering that all skewness–kurtosis planes should be covered and as few expressions as possible should be used, the Johnson translation system is selected to generate non-Gaussian surfaces. According to the Johnson system, the probability density function (PDF) of asperity heights of non-Gaussian surfaces is as follows:

$$\varphi(z) = \left\{ \varphi \right\} = \frac{1}{\sqrt{2\pi\lambda}} \frac{1}{\lambda} h'(y) \exp\left[ -\frac{1}{2} \left[ \frac{y + \phi h(y)}{\lambda} \right]^2 \right],$$  \hspace{1cm} (2)

where $y$, $\phi$ decides the shape of the distribution curve of asperity heights $z$, $\lambda$ is a scale factor, and $\xi$ is a location factor. These four parameters are determined by the skewness–kurtosis value. $h(y)$ also depends on the skewness–kurtosis value and has three variants as follows:

The lognormal system $S_L : h(y) = \ln(y)$,

The unbounded system $S_U : h(y) = \sinh^{-1}(y)$, \hspace{1cm} (3)

The bounded system $S_B : h(y) = \ln[y/(1-y)]$.

The mean of asperity heights is zero in this paper. The method of deciding the PDF can be found in Refs. 24 and 28. Then, the randomly generated two-dimensional non-Gaussian surfaces are shown in Fig. 4.

B. Population density function of the Iwan model

After the exploration of predecessors,\textsuperscript{19,29,30} the Iwan model becomes a promising nonlinear model for the dynamic tangential response of a lap joint interface. As shown in Fig. 2(c), researching the contact problem of lap joints can be transferred into the Iwan model. Each asperity is equivalent to a Jenkins element consisting of a spring and a damper. The Jenkins element is assumed to have a uniform spring stiffness $K$ and a random yield force $q_i$. When the tangential force $T$ is imposed on the interface, the overall tangential displacement is $u$, and the sliding displacement of the damper is $x$. Thus, when the lap joint interface is initially loaded, a set of canonical mathematical formulations can be used to indicate the constitutive relationship (force–displacement relationship in this paper) of the tangential contact system.

$$T(u) = \int_0^\infty k\rho(q)[u - x(q)]dq \quad u > 0,$$

$$u - x(q) = \begin{cases} u, & 0 \leq u < q/k \\ q/k, & u \geq q/k, \end{cases}$$  \hspace{1cm} (4)
where \( \rho(q) \) represents the population density function of Jenkins element’s yield force \( q \), and \( \alpha \) refers to the corresponding tangential velocity.

The imposed tangential force amplitude \( A_T \) is usually known and can be treated as an environmental parameter. It is not important here to directly discuss the effects of topography parameters on \( A_T \). Thus, compared with \( A_T \), the threshold gross slip force \( T_{GS} \) (tangential force required for gross slip) is more suitable for identifying the tangential contact system of lap joints. From Eq. (4), the tangential stiffness \( K_T \) of the lap interface at initial loading is calculated as follows:

\[
K_T(u) = \frac{\partial T}{\partial u} = \int_{-\infty}^{\infty} k\rho(q)dq. \tag{5}
\]

Observing Eq. (5), it can be concluded that the tangential stiffness at the initial loading is a multiple of the area enclosed by the tail curve of the population density function of Jenkins elements and the horizontal axis [Fig. 5(a)]. Furthermore, the tangential stiffness decreases with the increase in the tangential displacement, implying stiffness degradation. Then, according to practical experience, the tangential stiffness will not be less than zero, and the lap interface will fail under a sufficient tangential load, meaning zero tangential stiffness. Therefore, it can be predicted that the value of population density \( \rho \) will always be above the horizontal axis and eventually converge to this axis with a starting convergence point \( q_{GS} \) as shown in Fig. 5(a). From Eqs. (4) and (5), the following relationship can be deduced

FIG. 4. Two-dimensional topography profile of non-Gaussian surfaces. (a)–(e) have \( Ku = 4 \) at different skewness values: (a) \( Sk = -1 \), (b) \( Sk = -0.5 \), (c) \( Sk = 0 \), (d) \( Sk = 0.5 \), (e) \( Sk = 1 \). (f)–(j) have \( Sk = 0 \) at different kurtosis values: (f) \( Ku = 2 \), (g) \( Ku = 3 \), (h) \( Ku = 6 \), (i) \( Ku = 10 \), (j) \( Ku = 20 \).
where $A_n$ be the nominal contact area and $\eta$ be the area density of asperities. Because $\int_0^\infty \rho(q)/(A_n\eta)\,dq = 1$, $\rho(A_n\eta)$ is defined as the PDF of the yield force of the Iwan model. Consequently, $K_{IT}$ and $T_{GS}$ possess statistical significance: $T_{GS}/(A_n\eta)$ is the expectation of the yield force and $K_{IT}/(A_n\eta)$ is $k$ times the probability that the yield force is greater than zero. $K_{IT} = A_n\eta k$ explains that the initial tangential stiffness is related to the normal load, independent of the tangential load and is the intrinsic characteristic of the lap joint interface.

Segalman $^9$ solved the population density function of the Jenkins elements by using the linear relationship of force–displacement and the energy dissipation law of Goodman under a small tangential load. Nevertheless, the solution obtained by this method is discontinuous and has four to six parameters that need to be determined by the experiment. It is a typical phenomenological solution. Inspired by Segalman, this paper considers the calculation of the population density based on the state of gross slip, which yields positive results in engineering applications. Many studies $^{13-15}$ have shown that this coefficient is penetration-dependent. Therefore, the penetration-dependent friction coefficient of each asperity is considered in the proposed model. The tangential force of gross slip is the resultant force of the single-asperity tangential force

$$T_{GS} = A_n\eta \int_0^\infty \mu(\omega)p(\omega)\varphi(\omega + d)\,d\omega,$$

where $\mu(\omega)$ is the friction coefficient of a single asperity, $p(\omega)$ is the normal load of a single asperity, and $\varphi(\omega + d)$ is the PDF of asperity heights. When solving the population density through incorporating Eqs. (6) and (7), the dimensional analysis should be carried out first. The dimension of the integral variable in Eq. (6) is the force, while that in Eq. (7) is the length. Therefore, it is necessary to convert Eq. (6) into an expression with the integral variable of length dimension by the substitution method. Let $q = q/k$, $\tilde{\rho}(q) = k^2\rho(q)$ and $\tilde{x}(q) = x(q)$, where $q$ is defined as the yield displacement of Jenkins elements, which has the length dimension. The corresponding tangential response and the interface parameters during initial loading become

$$T(u) = \int_0^\infty \tilde{\rho}(q)\{u - \tilde{x}(q)\}\,dq,$$

$$K_{IT} = \int_0^\infty \tilde{\rho}(q)d\tilde{q}; K_{IT} = \int_0^\infty \tilde{\rho}(q)d\tilde{q},$$

$$T_{GS} = \int_0^\infty \tilde{\rho}(q)d\tilde{q}.$$

Accordingly, the population density function of the yield displacement $\tilde{\rho}(q)$ can directly remark $K_{IT}$ and the threshold gross slip displacement $\tilde{q}_{GS} = q_{GS}/k$, which represents the imposed displacement required for gross slip, and be depicted in Fig. 5(b). Write Eq. (7) in the symmetric form of Eq. (10) and then incorporate the above two equations to obtain

$$\int_0^\infty \tilde{\rho}(q)d\tilde{q} = \int_0^\infty \omega(A_n\eta p(\omega)p(\omega + d)/\omega)\,d\omega.$$

From Eq. (11), a solution for the population density function of the yield displacement is as follows:

$$\tilde{\rho}(q) = A_n\eta \mu(q)p(q)\varphi(q + d)/\tilde{q},$$

where $\mu(q)$ and $p(q)$ are prerequisites of the solution, and $d$ is determined by the normal equilibrium equation Eq. (13) as

$$N(d) = A_n\eta \int_0^\infty p(\omega)\varphi(\omega + d)d\omega.$$

Compared with the tangential response, the research on the normal response of a single asperity is more comprehensive. Hertz
first gave the classical elastic normal load-penetration model.\textsuperscript{25} Chang et al.\textsuperscript{34} (CEB model) added plastic contact to the Hertz theory. Zhao et al.\textsuperscript{26} (ZMC model) then considered the elastic-plastic deformation of asperities to make the asperity model closer to the fact. Subsequently, Brizmer et al.\textsuperscript{44} (BKE model) accurately simulated the single-asperity normal response under elastic-plastic contact by the finite element method. The BKE model can still capture the elastic–plastic deformation of a single asperity well with relaxation of the assumptions of Hertz, CEB, and ZMC models, and the normal load-penetration relationship can be represented by a continuous two-piecewise function, which can reduce the number of pieces of the subsequent tangential force expression. Thus, it is pertinent to select the BKE model to calculate the normal load of a single asperity $p(\omega)$ in this paper.

According to the above analysis, the population density function of the Iwan yield force is as follows:

$$\rho(q) = \frac{1}{k} \frac{1}{q^2} \left( \frac{q}{k}\right)^3 \left( 1 - \exp \left( \frac{\omega}{\delta} - \frac{q}{k}\right) \right) \left( \frac{q}{k} + d \right), \quad q \leq \omega, \delta$$

and

$$k = K_{Iw}/(A_\omega \eta) = \int_0^\infty \rho(q) dq/(A_\omega \eta),$$

where $E$ is the equivalent Young’s modulus and $\omega_c$ is the critical normal interference at the inception of plastic deformation.\textsuperscript{25} $\bar{\omega}$, $\bar{\delta}$, and $\alpha$ are dimensionless parameters, which can be found in Eq. (A1). The friction coefficient $\mu$ is shown in Eq. (A2).

### C. Constitutive equation for tangential response of lap joints

When the lap joint interface is initially loaded, the tangential constitutive relationship is determined by Eqs. (4) and (14) and is a continuous piecewise function.

$$T_{ul}(u) = \left\{ \begin{array}{ll}
\frac{4}{3} \frac{\bar{l}}{K^{3/2}} \frac{A_\omega}{E} \frac{R^{1/2}}{k^{5/2}} & \int_0^{k\bar{u}} \frac{\mu(q/k)}{q^{3/2}} \varphi(q/k + d) dq \\
+ k u \int_{k\bar{u}}^{\infty} \frac{\mu(q/k)}{q^{3/2}} \varphi(q/k + d) dq & \\
+ k u \int_{k\bar{u}}^{\infty} \frac{\mu(q/k)}{q^{3/2}} \left( 1 - \exp \left( \frac{(\omega, \bar{\delta})^a}{\omega, \bar{\delta} - (q/k)^a} \right) \right) \varphi(q/k + d) dq & \\
+ k u \int_{k\bar{u}}^{\infty} \frac{\mu(q/k)}{q^{3/2}} \left( 1 - \exp \left( \frac{(\omega, \bar{\delta})^a}{\omega, \bar{\delta} - (q/k)^a} \right) \right) \varphi(q/k + d) dq & \\
\end{array} \right. \quad (15)$$

The Masing hypothesis that can be represented by the Iwan model is a classical model exhibiting hysteretic loops.\textsuperscript{3} In the proposed model, the constitutive equation of the tangential response of the lap joint interface under unloading and reloading is expressed by the Masing hypothesis as

$$\begin{align*}
T_{ul}(u) &= T(A_\omega) - 2T((A_\omega - u)/2) \quad \text{when unloading}, \\
T_{rl}(u) &= 2T((A_\omega + u)/2) - T(A_\omega) \quad \text{when reloading}. \quad (16)
\end{align*}$$

where $T_{ul}(u)$ and $T_{rl}(u)$ represent the tangential forces during unloading and reloading, respectively. It can be seen from Eqs. (15) and (16) that topography, material, and geometric parameters, which indicate physical properties of the lap joint interface, are used to construct this constitutive model. Since without numerical-fitting or experimental-calibrated parameters, the proposed model is practical and effective.

### III. NUMERICAL RESULTS

The proposed model is now used to simulate the tangential response of the lap joint interface with non-Gaussian surfaces. Following Ref. 16, the lap joint interface is composed of steel on steel surfaces, and the material and geometric parameters are as follows: Young’s modulus $E_1 = E_2 = 200$ GPa, Poisson’s ratio $\nu_1 = \nu_2 = 0.24$, hardness $H_1 = H_2 = 5.825$ GPa, shear modulus $G_1 = G_2 = 80.7$ GPa, and nominal contact area $A_\eta = 156$ mm$^2$. The plasticity index $\eta$ and roughness constant $\beta$ can be solved through Eq. (17),\textsuperscript{25} where $H$ is the hardness of softer material, $K = 0.6$ is the material hardness factor, $\sigma$ is the standard deviation of surface heights, and $\eta$ is the asperity density of asperities. In this paper, material parameters are not considered to be variables. Accordingly, the plasticity index $\eta$ is confirmed by $\sigma$ and $R_c$ and can be regarded as a topography parameter. Surface roughness parameters are obtained from Refs. 16, 22, and 23, as shown in Table 1. Additionally, the standard deviation of

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**Table 1**

| Parameter | Value |
|-----------|-------|
| $\sigma$  |       |
| $R_c$     |       |
TABLE I. Equivalent roughness parameters and plasticity indices.

| $\sigma / R$ | $\sigma$ (\(\mu m\)) | $\beta$ | $\psi$ |
|--------------|-----------------------|-------|-------|
| $4.40 \times 10^{-4}$ | 0.0011 | 0.0440 | 0.4 |
| $2.50 \times 10^{-3}$ | 0.1147 | 0.0405 | 0.9 |
| $8.67 \times 10^{-3}$ | 0.1457 | 0.0565 | 2.0 |
| $8.88 \times 10^{-2}$ | 2.6770 | 0.0235 | 4.3 |

The asperity heights is $\sigma_a = \sqrt{1 - (3.717 \times 10^{-4})/\beta^2}\sigma$.

\[
\psi = \frac{2E}{\pi KH} \left(\frac{\sigma}{R}\right)^{1/2} \left(1 - \frac{3.717 \times 10^{-4}}{\beta^2}\right)^{1/4},
\]

\[
\beta = \sigma R \eta. \tag{17}
\]

A. PDF of the Iwan model and threshold gross slip force

Before analyzing the effect of non-Gaussian surfaces on the tangential response, it is necessary to clarify the influence of non-Gaussian surfaces on the phenomenological parameter $\rho$, which is heuristic for this research. Figures 6 and 7, respectively, show the effect of the skewness and kurtosis on the PDF of the Iwan yield force $\rho(A_n\eta)$ under different plasticity indices. Observing the Scientific Notation index of the ordinate axis in Figs. 6 and 7, the yield force distribution of Jenkins elements is found to be concentrated and small when $\psi$ is small but dispersed when $\psi$ is large. Nevertheless, the concentration degree of the yield force distribution does not decrease monotonously with the increase of $\psi$. In addition, as shown in Fig. 6, the performance of skewness is complex: Under the negative skewness, the distribution of the Iwan yield force obviously changes. The concentration of the distribution increases with the increase in skewness; thus, the expectation of the yield force decreases, leading to a decrease in the threshold gross slip force $T_{GS}$. Under the positive skewness, the concentration of the yield force distribution increases first and then decreases with the increase in skewness, and the increasing trend is inapparent. Then, the expectation of the yield force decreases first and then increases, leading $T_{GS}$ to also decrease and then increase, where the decreasing trend is inapparent. From Fig. 7, it can be concluded that the larger the kurtosis is, the more concentrated the Iwan yield force distribution is, and the smaller the expectation of Iwan yield force and $T_{GS}$ are.

B. Initial tangential stiffness

Both the tangential force and displacement belong to the category of tangential load, and the tangential displacement is shown directly in Fig. 5. As a result, the threshold gross slip displacement $\tilde{q}_{GS}$ is also chosen for a quantitative study in this section, in addition to the initial tangential stiffness $K_{IT}$.

FIG. 6. Effects of skewness on the PDF of yield force of the Iwan model at a preload $N = 300$ N under different plasticity indices: (a) $\psi = 0.4$. (b) $\psi = 0.9$. (c) $\psi = 2.0$. (d) $\psi = 4.3$. 

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Figures 8 and 9, respectively, describe the effect of the skewness and kurtosis on the population density function of the Iwan yield displacement under different plasticity indices. Based on the description in Fig. 5, the area enclosed by the population density curve and the horizontal axis in Figs. 8 and 9 is $K_{IT}$, and the corresponding value when the density curve starts to approach the horizontal axis is $\tilde{q}_{GS}$. Observing the area and the scientific notation index of ordinate axis, it is concluded that $K_{IT}$ tends to decrease with the increase in the plasticity index. This is because the friction coefficient of the proposed model learns from the BKE model, which is based on the full-stick contact condition to avoid the unrealistic zero friction. The full-stick condition states that the “junction” at the contact region can continue to grow after the plastic deformation occurs until the tangential stiffness becomes zero. Therefore, when the plasticity index is small, asperities are basically elastic, the junctions are difficult to shear theoretically, and $K_{IT}$ is large; conversely, when the plastic index is large, asperities are basically plastic, and $K_{IT}$ is small.

Figure 9 shows that the initial tangential stiffness $K_{IT}$ decreases with the increase in kurtosis because the kurtosis reflects the leptokurtic of the PDF of asperity heights: The larger the kurtosis is, the sharper the distribution curve is and the larger the proportion of low asperities is [see Figs. 4(f)–4(j)], while the normal load tends to be borne in the relatively smaller proportion of high asperities. Therefore, when a contact occurs on a lap joint interface with larger kurtosis, it will cause fewer contact asperities and more plastic deformations of high asperities, resulting in a smaller $K_{IT}$. At $Ku \leq 6$, $K_{IT}$ obviously varies. As shown in Fig. 8, the effect of skewness on $K_{IT}$ is non-monotonous: Under the negative skewness, $K_{IT}$ decreases with the increase in skewness, and the change is drastic; under the positive skewness, $K_{IT}$ decreases first and then increases with the increase in skewness, and the change range is smaller than that under the negative skewness. This is because the skewness reflects the symmetry of the PDF of asperity heights, which can only indirectly influence the proportion of high asperities, and the influence is non-monotonous. At $Sk = -1$, the height of positive asperities is relatively low [see Fig. 4(a)]. Normal loads are borne by a considerable number of low asperities, and $K_{IT}$ is large. When $Sk \leq 0$, the height distribution of positive asperities fluctuates markedly and becomes more dispersed with the increase in skewness. Therefore, the proportion of high asperities increases, and $K_{IT}$ decreases with the increase in skewness. When $Sk > 0$, the height distribution of negative asperities varies intensely and that of positive asperities varies conservatively, which influence $K_{IT}$ non-monotonously. The tangential stiffness indicates the ability of the interface to resist gross slip, so the relationship between $K_{IT}$ and the tangential force required for gross slip $T_{GS}$ is covariant: That is, $K_{IT}$ and $T_{GS}$ have the same tendency with the changeable topography parameters. When the skewness and kurtosis change, the trend of $K_{IT}$ is the same as the trend...
FIG. 8. Effects of skewness on initial tangential stiffness at a preload $N = 300$ N under different plasticity indices: (a) $\psi = 0.4$, (b) $\psi = 0.9$, (c) $\psi = 2.0$, (d) $\psi = 4.3$.

FIG. 9. Effects of kurtosis on initial tangential stiffness at a preload $N = 300$ N under different plasticity indices: (a) $\psi = 0.4$, (b) $\psi = 0.9$, (c) $\psi = 2.0$, (d) $\psi = 4.3$. 
of $T_{GS}$ in Sec. III A, which matches the physical meaning of the tangential stiffness.

In each inset of Fig. 10, the vertical axis represents the threshold gross slip displacements $\tilde{q}_{GS}$, the upper horizontal axis represents the kurtosis, and the lower horizontal axis represents the skewness. The blue and red curves, respectively, present the skewness-displacement and kurtosis-displacement relationships. When the plasticity index is small, $\tilde{q}_{GS}$ is small; when the plasticity index is large, $\tilde{q}_{GS}$ is large. $\tilde{q}_{GS}$ increases first and then decreases with the increase in skewness and increases with the increase in kurtosis. Thus, the plasticity index, skewness and kurtosis have opposite effects on $K_{IT}$ and $\tilde{q}_{GS}$. $K_{IT}$ and $\tilde{q}_{GS}$ are a pair of anti-variant parameters, and the increase of the initial tangential stiffness of the lap joint interface will lead to the decrease of the imposed displacement required for gross slip.

C. Slip index

For revealing the characteristics of a tangential contact system imposed by an oscillating tangential load, it is necessary to add the slip index $\delta$ to specify the contact state of the lap joint interface, as described in Sec. II A. Here, the amplitude of the tangential load is controlled by the displacement amplitude, and the change of the slip index is analyzed. To explore the relationship between the amplitude of the tangential displacement required for gross slip and the surface roughness, $3\sigma$ is chosen as the amplitude of the imposed tangential displacement of the lap joint interface. The preload is still set to 300 N, and the slip indices of the lap joint interface under given conditions are then obtained from Eq. (1).

Figure 11 visually and quantitatively describes the influence of non-Gaussian surfaces on the slip index of the lap joint interface. The purple curve represents the skewness–slip index relationship and the green curve represents the kurtosis–slip index relationship. According to Sec. II A, the horizontal lines $\delta = 11$ and $\delta = 0.8$ in Fig. 11 represent the critical slip indices of the lap joint interface entering reciprocating sliding and gross slip under cyclic tangential loads, respectively. Comparing the variation ranges of curves in Fig. 11, it is clear that the effect of skewness and kurtosis on the slip index is more obvious in a small plasticity index than in a large plasticity index. Observing the trend of the curves, it can be concluded that: The slip index decreases first and then increases with the increase in skewness and decreases dramatically and monotonically with the negative skewness. The slip index decreases monotonically with the increase in kurtosis and fluctuates dramatically at $Ku \leq 6$. Different slip indices represent different
FIG. 11. Influence of non-Gaussian surfaces on slip index at $N = 300$ N and $A_\nu = 3\sigma$ under different plasticity indices: (a) $\psi = 0.4$. (b) $\psi = 0.9$. (c) $\psi = 2.0$. (d) $\psi = 4.3$.

fretting regimes, causing the fretting damage types of the lap joint interface to also be different. Therefore, the skewness and kurtosis should be considered as two significant parameters, whether for the prediction of the lap joint operation or for the selection of the lap joints according to the engineering environment.

Additionally, another important phenomenon can be observed: All curves for different skewness and kurtosis are above the line $\delta = 0.8$ in Fig. 11. Because this paper focuses on the effect of the surface topography on the tangential response, the effect of normal load has not been considered. In fact, depending on the numerical results of the proposed model, the value of the slip index still exceeds 0.8 when the normal load varies between 50 N and 1000 N. This means that when the imposed tangential displacement amplitude reaches $3\sigma$, most of the lap joint interfaces in engineering will be in gross slip or reciprocating sliding. The research of Fujimoto et al., Fouvry and Eriten et al. on the tangential responses of plane contacts also validate this conclusion.

IV. EXPERIMENTAL COMPARISONS AND DISCUSSIONS

Eriten et al. established a constitutive model of the tangential response of bolted lap joints [the EPB model, see Eq. (A3)], and carried out experimental verification. In these experiments, the lap joint interface consists of milling steel on steel surfaces, including four groups of tangential response experiments under different normal loads. The material and geometric parameters are shown in Sec. III, and the roughness parameters are in the last line of Table I. The experimental force–displacement data and the calculated data of the EPB model are shown in Fig. 12. Zhan and Huang also developed a constitutive model of lap joints [the ZH model, see Eq. (A4)]. To comprehensively assess the validity of the proposed model, the numerical results of the ZH model are depicted in Fig. 12. The numerical results of the proposed, EPB, and ZH models in Fig. 12 are all based on the Gaussian surface.

From Fig. 12, in general, the proposed model can better represent the experimental tangential behavior than the EPB and ZH models, especially when simulating the initial tangential stiffness of the lap joint interface. The EPB model uses Mindlin partial slip solution to describe the tangential response of lap joints, naturally replicating Mindlin’s hypothesis, which proposes that the two surfaces in contact are purely elastic. However, in the experiments of Eriten et al., the plasticity index $\psi = 4.3$ is large, meaning that most of the asperities of lap joints are in the plastic deformation even under very small external loads. As a result, the EPB model, which employs the elastic contact assumption to mimic the plastic contact
FIG. 12. Comparison of experimental tangential responses from bolted lap joints with numerical results of the proposed, EPB, and ZH models based on Gaussian surfaces at different preloads: (a) $N = 234$ N. (b) $N = 331$ N. (c) $N = 526$ N. (d) $N = 721$ N.

behavior, is bound to have pitfalls, leading to a larger initial tangential stiffness. The ZH model, which applies the bilinear formulation of Fujimoto, considers that even if the plastic deformation occurs, the asperity will undergo a partial slip process until the tangential force reaches the Coulomb friction. This assumption implies that even if the plastic deformation occurs and even the contact center is surrounded completely by the plastic zone, the asperity can still bear additional tangential force, resulting in a larger initial tangential stiffness. Moreover, combined with the experimental data of the initial tangential stiffness in Eriten et al., the numerical results of the proposed model are quantitatively observed in Fig. 13(a). The calculated date of four groups of skewness–kurtosis values in Fig. 13(a) can all better match with the experimental data under partial preloads, and the corresponding skewness–kurtosis values are in line with the range of these values for a milling surface in Ref. 21 (${Sk \in [0,0.6] \cap Ku \in [1.5,4.3]} \cup {Sk \in [-1,0] \cap Ku \in [1.5,7]}$).

Based on the initial tangential stiffness obtained, the fretting regime of the lap joint interface subjected to oscillating tangential loads can be evaluated. In Fig. 12, the experimental fretting regimes and those predicted by the proposed model are shown in Table II, indicating that the model can successfully predict the fretting regime of the lap joint interface.

However, the proposed model predicts that the tangential force amplitude under the given tangential displacement amplitude is smaller than that represented in the experimental data. Especially when $N = 331$ N, the lap joint interface is in gross slip and the deviation reaches 20%. The tangential force amplitude is related to the initial tangential stiffness when the load is insufficient to cause gross slip at the lap interface and to the friction coefficient when gross slip occurs. If only the tangential force amplitude is considered, the ZH model performs the best. This is because the ZH model assumes that the friction coefficients of surfaces in contact are equal everywhere, and the friction coefficient is the measured value of the gross slip interface. Both the proposed model and the EPB model assume that the friction coefficient is related to the normal interference of a single asperity, implying that when gross slip occurs, the friction coefficients of the two models tend to reflect the mean value of the entire fretting cycle $\mu_{m}$, which can be obtained from Eq. (18). When $N = 331$ N, the mean friction coefficient predicted by the model is 0.344, while the experimental friction coefficient is 0.508, a deviation of 30%. Therefore, it is conceivable that the friction coefficient contributes significantly to the above deviation. When the joint interface is in stick or partial slip, a matched tangential force amplitude can be obtained within a reasonable skewness–kurtosis value, and a further
revision for the model is of little significance. In view of this, when gross slip occurs at the joint interface, we attempted to calibrate the model by using the experimental friction coefficient $\mu_e$, instead of the penetration-related friction coefficient $\mu(\omega)$. The calibrated fretting loops are shown in Fig. 14. The calibrated model can accurately capture the experimental date of the tangential force amplitude. In addition, the initial tangential stiffness of the calibrated model is shown in Fig. 13(b). Compared with Fig. 13(a), the calibrated model is inferior to the proposed model in terms of mirroring the initial tangential stiffness. However, considering the accuracy of conveying the tangential force amplitude of gross slip, the calibrated model is still a suitable choice,

### TABLE II. Slip index and fretting regime.

| Preload $N$ | Experiments | The proposed model |
|-------------|--------------|--------------------|
| 234 N       | 3.1402 Gross slip | 2.8945 Gross slip |
| 331 N       | 1.7147 Gross slip | 1.7327 Gross slip |
| 526 N       | 0.7177 Partial slip | 0.7869 Partial slip |
| 721 N       | 0.3959 Stick | 0.4448 Stick |

FIG. 13. Comparison of the experimental initial tangential stiffness with the results of the (a) proposed and (b) calibrated models based on non-Gaussian surfaces.

FIG. 14. Simulation results of the calibrated model based on the Gaussian surface at (a) $N = 234$ N and (b) $N = 331$ N.
\[ \mu_m = \frac{D}{4NA_s}. \]  

Clearly, when the lap joint interface is in gross slip, the experimental data of the tangential force amplitude can be captured without calibration by choosing the appropriate skewness–kurtosis value. However, when the fitting effect of the tangential force amplitude is good, the initial tangential stiffness will mismatch significantly. Therefore, it is difficult to consider both the initial tangential stiffness and the tangential force amplitude only by adjusting the skewness–kurtosis value when the proposed model simulates the tangential response of the lap joint interface in gross slip.

V. CONCLUSIONS

In this paper, inspired by Segalman’s solution to the Iwan model, a new solution for the Iwan model is obtained by the dimensional analysis based on a large tangential incentive. Considering the influence of non-Gaussian surfaces, the Iwan solution is employed to establish the dynamic tangential contact system of the lap joint interface. The validity of the proposed model is assessed by comparison with the published experimental results. According to the above analysis, the main conclusions are as follows:

(1) The tangential response of a lap joint interface can be characterized by the initial tangential stiffness \( K_{IT} \), the threshold gross slip force \( T_{GS} \), and the slip index \( \delta \). The yield displacement of the Jenkins elements \( \bar{q} \) is defined by the dimensional transformation method. \( T_{GS} \) is proportional to the expectation of the Iwan yield force, and \( K_{IT} \) is proportional to the probability that the Iwan yield force is greater than zero.

(2) The initial tangential stiffness \( K_{IT} \) tends to decrease with the increase in the plasticity index. Under the negative skewness, \( K_{IT} \) decreases with the increase in skewness, and the change is drastic; under the positive skewness, \( K_{IT} \) decreases and then increases as the skewness increases. \( K_{IT} \) decreases with the increase in kurtosis and varies substantially when \( Ku < 6 \). When the skewness and kurtosis vary, the change in the trend of \( K_{IT} \) is the same as the trend of \( T_{GS} \), but opposite to that of the threshold gross slip displacement \( \bar{q}_{GS} \).

(3) The skewness and kurtosis have more obvious influence on the slip index \( \delta \) in the large plasticity index. \( \delta \) decreases first and then increases with the increase in skewness and decreases dramatically and monotonously in the negative skewness; \( \delta \) decreases monotonously with the increase in kurtosis and fluctuates dramatically at \( Ku \leq 6 \). \( \delta \) can be regarded as the maximum amplitude of the imposed tangential displacement required for gross slip of the lap joint interface, helping engineers predict the fretting regime and preselect the operating stroke of lap joints.

(4) When the lap interface is in gross slip, it is difficult for the proposed model to determine the matched maximum tangential force without sacrificing the accuracy of the initial tangential stiffness. In gross slip, the calibrated model can capture the tangential force amplitude of the tangential response with a slight loss of the tangential stiffness, which is still a suitable choice.

This study enriches and improves knowledge of the influence of non-Gaussian surfaces on the tangential behavior of the lap joint interface, which is valuable for the optimal design and the dynamic response analysis of the lap joint interface in engineering applications. However, the following factors will lead to the discrepancy between the research in this paper and the experimental results: (1) surface wear will cause dynamic changes in the skewness–kurtosis value, and a discrepancy will occur when the uniform skewness–kurtosis value, instead of the dynamic value of the entire fretting process, is used; (2) the interaction between asperities is neglected; and (3) the stress concentration at bolts of lap joints is not considered. Therefore, the proposed and the calibrated models cannot replicate the tangential response of the lap joint interface. In future work, more comprehensive research on a dynamic tangential contact system of lap joints should be conducted.

ACKNOWLEDGMENTS

This research was supported by the National Natural Science Foundation of China (Grant No. 51575190). The authors would also like to appreciate the reviewers, associate editor, and editor for their suggestions.

APPENDIX: CONSTITUTIVE RELATIONSHIP

This section gives more details about the BKE, EPB, and ZH models.

The BKE model (the normal force–penetration relationship and the friction coefficient)

\[ p_{BKE}(\omega) = \frac{l_c}{\delta_c^{1/2}} \left[ \frac{4}{3} ER^{1/2} \omega^{3/2} \left( 1 - \exp \left( \frac{(\omega, \delta_c)^n}{(\omega, \delta_c)^n - \omega_0^2} \right) \right) \right], \]

\[ \mu_{BKE}(\omega) = 0.27 \coth \left( 0.27 \left( \frac{\omega}{\omega_c} \right)^{0.46} \right), \]  

where \( l_c, \delta_c, \alpha \) are functions of Poisson’s ratio \( \nu \),

\[ l_c = 8.88\nu - 10.13(\nu^2 + 0.089), \]

\[ \delta_c = 6.82\nu - 7.83\nu^2 + 0.0586, \quad \alpha = 0.174 + 0.08\nu. \]

The EPB model (the backbone curve for fretting loops)

\[ T_{EPB}(u) = \frac{4}{3} A_n E \omega_c^{3/2} R^{1/2} \left[ \int_0^u \mu(\omega) f(\varphi + d) d\omega \times \left\{ 1 - \left( 1 - \frac{4Gu}{(\mu_0 E \omega)} \right)^{3/2} \right\} \varphi(\varphi + d) d\varphi \right] \]  

\[ (A3) \]
where the penetration-related friction coefficient $\mu(\omega)$, the normalized asperity contact force $f$, and the critical penetration for stick-slip $\omega_c$ can be found in Ref. 16. $G$ is the equivalent shear modulus.

The ZH model (the backbone curve for fretting loops)

$$T_{234}(u) = A_uE\eta$$

$$\times \left\{ \int_0^{\omega_c} \frac{4\mu R}{3\mu R^{1/2}} \omega^{3/2} \varphi(\omega + d)d\omega \\
+ \int_{\omega_c}^{\omega} \frac{4\mu R}{3\mu R^{1/2}} \omega^{3/2} \left[ 1 - \left( 1 - \frac{4\mu u}{\mu E\omega} \right)^{3/2} \right] \varphi(\omega + d)d\omega \\
+ \int_{\omega_c}^{\omega} \frac{\mu RKH/E(2\omega - \omega_c)}{\varphi(\omega + d)d\omega} \\
+ \int_{\omega_c}^{\infty} 2\pi GR^{1/2}u/E(2\omega - \omega_c)^{1/2} \varphi(\omega + d)d\omega \right\}$$

(A4)

where the constant friction coefficient $\mu$, the critical penetration for stick-slip under elastic contact $\omega_c$, and that under plastic contact $\omega_p$ are shown in Ref. 17.

**DATA AVAILABILITY**

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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