The study of the dynamic model of the distribution of labor resources by region in conflict interaction

I V Zaitseva, O A Malafeyev, A V Zubov, L A Bondarenko and V B Orlov

1 Russian State Hydrometeorological University, Saint Petersburg, Russian Federation
2 Stavropol State Agrarian University, Stavropol, Russian Federation
3 Saint Petersburg University, St. Petersburg, Russian Federation

E-mail: irina.zaitseva.stv@yandex.ru

Abstract. The introduced models turn out to be the development of forces and resources allocation in municipal economy branches if a lot of interests appear in a competitive case. The models combine the reasonable level of adequacy with the practical application in solving the problems. The plans of labour force allocation can be compiled on their basis.

1. Introduction

The process when two competitive companies (participants in economic process) are aimed at solidifying at several interfacing economic regions (market sectors) is considered. To do this they locate their liquid asserts, industrial facilities, labour force and other resources in some regions. The capital productivity ratio from every participant investing labour force in every region is set by marketed law and normally depends on joint allocation of labour force in all the interfacing regions. The process is supposed to be uncapped and sampled time. Every participant is aimed to maximize mean income received at every process step or to maximize its resource share taking part in the regional activity.

2. Game-theoretic model of labour force allocation

We introduce an extend dynamic model (EDM) $D = (X, U, \psi)$ into $X$, where $U = \bigcup_{x \in X} U(x)$ and $U(x)$ are compact metric spaces given for every $x$, $\psi$ –function:

$$x_{i+1} = \psi(x_i, u_i), \psi \in C^1.$$  

We interpret $X$ as a state space of some process $\Pi$. Let consider $U$ as a control set of this process. The process is controlled by two agents $\alpha$ and $\beta$ (but, in any case, there can be any finite quantity), $U_\alpha(x)$, $U_\beta(x)$ – agents control sets at every condition $x$ of the process $\Pi$, they are assumed metric spaces, correspondently, $\rho_{u,\alpha}$, $\rho_{u,\beta}$. Suppose that $U = \bigcup_x U(x)$ and $U(x) = \{ (u_\alpha, u_\beta) ; u_\alpha = u_\alpha(x) \in U_\alpha(x), u_\beta = u_\beta(x) \in U_\beta(x) \}$. We define metric $\rho$ in space $U$ induced by metrics $\rho_x, \rho_{u,\alpha}, \rho_{u,\beta}$ as follows:

$$\rho((x,u_\alpha,u_\beta),(x',u'_\alpha,u'_\beta)) = [\rho_x(x,x'), \rho_{u,\alpha}(u_\alpha,u'_\alpha), \rho_{u,\beta}(u_\beta,u'_\beta)].$$

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

Published under licence by IOP Publishing Ltd
Suppose that the set \( \{(x,u_\alpha,u_\beta) : x \in X, u_\alpha \in U_\alpha(x), u_\beta \in U_\beta(x)\} \) is \( N \)-dimensional differentiable manifold of class \( C^1 \), the space \( X \) \( N \times \)-dimensional, \( U_\alpha(x) \) \( N_\alpha \)-dimensional, \( U_\beta(x) \) \( N_\beta \)-dimensional differentiable manifolds of class \( C^1 \).

Let us define the dynamics of the process \( \Pi \) by recurrence formula. Suppose that \( t \) is a time parameter. The process initial state \( x(t) = x(0) = x_0 \) is selected according to the probability measure \( \mu \) on \( \sigma \)-algebra \( (X, \Sigma) \). Suppose that at \( k \)-step \((t = k)\) the process is in the condition \( x_k \) and the agents chose the controls \( u_{\alpha,k}, u_{\beta,k} \). Then, the process state at \((k+1)\)-step is defined by formula (1) and etc. The sequence of space points \( X\{x_0, x_1, x_2, x_3, \ldots\} \) is called the process path if the following point is built out of the previous one by function \( \psi \).

We consider the game \( \Gamma \) which dynamics is defined by the process \( \Pi \). As the agents strategy we account the mappings of set \( X \) in \( U_\alpha \) for agent \( \alpha \) and in \( U_\beta \) for the agent \( \beta \). \( S_\alpha \) is defined as the set of agent \( \alpha \) strategy, \( S_\beta \) - the set of agent \( \beta \) strategy. The game imputation is considered as the path function built by the abovementioned approach.

The agents’ income functions for the set of the game imputations are succeeded by the indiscrete number functions \( u(x), v(x) \) succeeded at \( X \). Let us assume that the consequence of the process states \( \{x_0, x_1, x_2, x_3, \ldots\} \) is the game imputation received after the agents have implied their strategies: \( s_\alpha \) – the strategy of agent \( \alpha \), \( s_\beta \) – the strategy of agent \( \beta \). Then, the agents’ incomes are defined from the following limited ratios:

\[
\rho_s(\psi, x_0, s_\alpha, s_\beta) = \lim_{t \to T} \frac{1}{T-t_0} \int_{t_0}^t \psi(s) \, ds,
\]

\[
\rho_v(\psi, x_0, s_\alpha, s_\beta) = \lim_{t \to T} \frac{1}{T-t_0} \int_{t_0}^t v(s) \, ds.
\]

Let us consider the mixed extension \( \Gamma \) of game \( \Gamma \). By introducing \( \sigma \) – algebras on the sets of the agents’ number strategies \( S_\alpha \sum_s \), \( S_\beta \sum_s \) and considering the spaces of the mixed strategies of agents \( S_\alpha \), \( S_\beta \) as the sets of all possible probability measures on \( (S_\alpha \sum_s), (S_\beta \sum_s) \). Let us make the spaces metrical. In order to do this we consider the fundamental system \( \{\phi_k\}_0 \) in the space \( C^0(S_\alpha \times R^1) \) with uniform metric, i.e. the denumerable dense set of continuous functions in Banach space \( C^0(S_\alpha \times R^1) \) of continuous functions with uniform norm \( \|p_\alpha\| \leq 1 \). The following number \( s_\alpha \), \( s_\alpha' \) from \( \widetilde{S_\alpha} \) is assumed for every pair:

\[
p_{s_\alpha}(s_\alpha, s_\alpha') = \sum_{k=0}^\infty \frac{1}{2^k} \left| \int_{p_\alpha} \phi_k d\tilde{s}_\alpha - \int_{p_\alpha} \phi_k d\tilde{s}_\alpha' \right|.
\]

It can be shown that the function \( \rho_{s_\alpha}(\tilde{s}_\alpha, \tilde{s}_\alpha') \) defined in the following way will actually be the metrics [1-4]. The metrics \( \rho_{\tilde{s}_\beta} \) is introduced by the same approach in the agent’s \( \beta \) space of mixed strategies. Let us also introduce the metrics \( \rho_{s_\alpha} \) in the mixed strategies situation space \( \tilde{S} = \tilde{S}_\alpha \times \tilde{S}_\beta \). If \( \tilde{s} = (\tilde{s}_\alpha, \tilde{s}_\beta), \tilde{s}' = (\tilde{s}_\alpha', \tilde{s}_\beta) \), then \( \rho_{\tilde{s}}(\tilde{s}, \tilde{s}') = \max \{\rho_{\tilde{s}_\alpha}(\tilde{s}_\alpha, \tilde{s}_\alpha'), \rho_{\tilde{s}_\beta}(\tilde{s}_\beta, \tilde{s}_\beta) \} \). The game introduces in mixed strategies is formally defined as \( \bar{\Gamma} = \{[\alpha \beta], \tilde{S}_\alpha, \tilde{S}_\beta, D, \rho_{\tilde{s}}, \rho_{s_\alpha} \} \), where \( D \) is the abovementioned EDM, \( \rho_{\tilde{s}}(\tilde{s}, \tilde{s}') = \int_{p_\alpha} \rho_{s_\alpha}(\psi, x_0, s_\alpha, s_\beta) d\tilde{s}_\alpha d\tilde{s}_\beta, \rho_{s_\alpha}(\psi, x_0, s_\alpha, s_\beta) = \int_{s_\beta} \rho_{s_\alpha}(\psi, x_0, s_\alpha, s_\beta) d\tilde{s}_\alpha, \rho_{\tilde{s}_\alpha}(\tilde{s}_\alpha, \tilde{s}_\alpha') = \int_{p_\alpha} \rho_{s_\alpha}(\psi, x_0, s_\alpha, s_\beta) d\tilde{s}_\alpha, \rho_{\tilde{s}_\beta}(\tilde{s}_\beta, \tilde{s}_\beta') = \int_{p_\beta} \rho_{s_\beta}(\psi, x_0, s_\alpha, s_\beta) d\tilde{s}_\beta. \)
The Hausdorff metric introduces some additional conditions to the consequence of finite games at the transition from the finite game to the infinite one, different peculiarities that, in general, do not allow considering the equilibrium property appear. Partially, this problem is solved by introducing some additional conditions to the consequence of equilibrium situations in finite games that approximate the initial game. The researches on this problem allow receiving the following result.

Further we need the following result (also see [1], [2], [3], [5], [8]). Theorem 1. There is Nash equilibrium situation in mixed strategies in the game \( \Gamma \) with compact metric spaces of strategies \( S_{\alpha} \) and continuous income functions \( r_{\alpha} \).

The approximation algorithm by the consequence of finite games \( \{ \Gamma_{\alpha} \}_{n=0}^{\infty} \) is suggested to the game \( \Gamma \). There is Nash equilibrium in mixed strategies for every game. In order to follow the consequence of equilibrium situations in mixed strategies \( \{ \tilde{S}^{\alpha} \}_{n=0}^{\infty} \) in game \( \Gamma^{\alpha} \), the conditions are laid down and their ampleness for the equilibrium situation in game’s \( \Gamma \) mixed strategies is proved.

The question about the existence of mathematical decision for the game in its normal form is followed by the urgent question concerning the solution ampleness if the parameters defining the following task are altered.

Suppose that the space \( \gamma \) is generally topological and its point is the game in its normal form \( \Gamma \in \gamma \). The set of the solutions \( e(\Gamma) \neq \emptyset \) for the game \( \Gamma \) being the subset (often compact) of some metric space \((S, d)\) is defined for every game or for the games of the subset \( \gamma' \subset \gamma \). The Hausdorff metric \( \rho \) is introduces for of the compact subsets of the space \( S \) at the set \( K(S) \). The game \( \Gamma \in \gamma' \) is stable if any solution out of the set \( e(\Gamma) \) is altered a little when the game \( \Gamma \) is altered a little in the space \( \gamma' \).

The situation is mainly the following. If there is the open and dense set in the space \( \gamma' \), every game \( \Gamma \) in it is stable.

Let us consider as earlier that \( \Gamma = \langle I = \{ 1, 2, \ldots, n \}, \{ S_{i} \}^{n}_{i=1}, \{ r_{ni} \}^{n}_{i=1} \rangle \) is a coalition-free game of \( n \) people, where \( I \) – the set of players, \( S_{i} \) – the set of the player’s strategy, \( i \) – the compact metric space. \( r_{i} : S_{i} \to R_{i} \) is the real continuous income function of the player \( i \). Let us consider the metric space \((\gamma', \rho)\) for such games:

\[ \Gamma_{\nu} = \langle I = \{ 1, 2, \ldots, n \}, \{ S_{i} \}^{\nu}_{i=1}, \{ r_{ni} \}^{\nu}_{i=1} \rangle. \]

The metrics \( \rho \) is allocated in the following:

\[ \rho(\Gamma_{\nu}, \Gamma_{\nu}) = \max_{s \in S_{i}} \| r_{\nu}(s) - r_{\nu}(s) \|. \]
Definition. The equilibrium situation \( s \in e(\Gamma) \) is called stable if for every positive \( \varepsilon \) exists such a position of \( \delta \) that if \( \Gamma_{\mu} \in \gamma \) is \( \rho(\Gamma_{\mu}, \Gamma_{\mu'}) < \delta \), so there is the situation \( s' \in e(\Gamma_{\mu'}) \) when \( d(s, s') < \varepsilon \). If \( E_{s}^* \) is the set of stable situations for the game equilibrium, then \( \Gamma_{\mu} \) is the stable game when \( E_{s}^* \in e(\Gamma_{\mu}) \).

In the research [6-7] the following theorem is established.

Theorem 2. The Set of Games for Space \( \gamma' \) is Everywhere Dense in \( \gamma' \).

As a sequence, every game of the abovementioned type with nonvacuous set of solutions can be approximated by a stable game. In general, the game space \( \gamma' \) can be divided into the open set of regular (stable) games and the close nondense bifurcation set of unstable games. The restriction of the function diagram projection \( e(\Gamma_{\mu}) \) to this regular set is the finite-sheeting covering on its every component and the number of papers over every component is stable. Every one-valued branch over the component is considered to be the differentiable function that can be taken as the solution for statistical games and used for analyzing any differential game. The competitive distribution process for two types of labour force.

Let us consider two examples of applying the above-mentioned model.

Let’s analyze the sampled time distribution process for two types of labour force (\( \alpha \) and \( \beta \)). The process is analyzed by the simplicial subdivision of two-dimensional sphere. Suppose that the sphere is divided into \( N \) simplexes. The process dynamics is assumed by the following function:

\[
\psi: R_{2N}^2 \rightarrow R_{2N} \psi = \varphi \cdot s,
\]

where \( \varphi \) takes the form:

\[
(\varphi_{\alpha_i} \cdot x_i) = \left[ x_i \cdot (1 + e^{-\sum_{i=1}^{N} c_i x_i - m_x}) - m_x \right] \leq N',
\]

\[
(\varphi_{\beta_i} \cdot x_i) = \left[ x_i \cdot (1 + e^{-\sum_{i=1}^{N} c_i x_i - m_x}) - m_x \right] \geq N',
\]

where \( x \in R_{2N}^2 \) is the process state, \( x_i, i \leq N \) is the value of the first labour force type located on \( i \)-sphere simplex, \( x_i, N < i \leq 2N \) is the value of the second labour force located on \( i \)-sphere simplex.

Let’s consider n-parameter family

\[
\Psi = \left\{ \left. m = (m^x, m^y) : m_x \in (0, 1), m^y \in (0, 1) \right| c = (c_0, c_1, c_2, c_3) \right\},
\]

where \( m = (m^x, m^y) \) is the linear factor of the labour force loss, \( \alpha \) and \( \beta \) correspondently, at one step of the process, \( c_i, i = (0.3) \) are the influence factors of competitive labour force types located on the adjacent simplexes. So, the resource of \( \alpha \) – type that has already been located by that time in \( k \)-simplex (factor \( c_0 \)), the competitive resource located in the same simplex (factor \( c_1 \)), the resources of \( \alpha \)–type located by that time in simplexes adjacent to \( k \) (factor \( c_2 \)) and resources of \( \beta \)–type located by that time in simplexes adjacent to \( k \) (factor \( c_3 \)) influence on the growth of the resource of \( \alpha \)–type in a step of the process allocated in \( k \)–simplex. The growth of the \( \beta \)–type resource undergoes the same influence from the earlier located resources of its and competitive types of labour force; \( B_\alpha \) – the set of simplex indices adjacent to \( i \); \( s = (s_\alpha, s_\beta) \) is the function setting to every resource location in sphere simplexes the other resource location while saving the total resource of every technology, \( s_\alpha \) is the rule.
setting the resource allocation of \( \alpha \)-technology (\( s_\beta \) - \( \beta \) - types) per the joint resource allocation for both technologies.

Thus, the following lemma is established:

Lemma. For any process path function \( \{ x_0, x_1, ..., x_n \} \) such \( k^* \) is found that every path function points starting with \( x_k \) remain to the third power \( [0, R]^{D^N} \), where \( R \) does not depend on the initial point of the path function and choice of function \( s \).

As the result of numerical experiments the following exposition is received.

One and only one of the following expositions is always true for the function \( \psi \):

The exposition \( A \) – function \( \psi \) has three critical points:

the 1st point \((x_1 = 0)\) is an unstable nodal point,

the 2nd point
\[
\left( x^2 \right. \text{ for } i = (0, N - I) x_i = -\frac{\ln m_i}{c_i + 3c_2}, \text{ for } i > N - I, x_i = 0 \right) \text{ is a stable nodal point,}
\]

the 3rd point
\[
\left( x^3 \right. \text{ is that } i = N - I x_i = -\frac{\ln m_i}{c_i + 3c_2}, \text{ for } i = (0, N - I), x_i = 0 \right) \text{ is a saddle point. Its stable manifold is } N \text{-dimensional manifold set by the system of equations: } x_i = 0 \text{ for } i = (0, N - I), \text{ and its unstable manifold is the hyperplane passing through the 2nd and the 3rd critical points and transversal to the stable manifold in the point } x^3.
\]

The exposition \( B \) is the function \( \psi \) that has four critical points:

the 1st point \((x_1 = 0)\) is an unstable nodal point,

the 2nd point
\[
\left( x^2 \right. \text{ for } i = (0, N - I) x_i = -\frac{\ln m_i}{c_i + 3c_2}, \text{ for } i > N - I, x_i = 0 \right) \text{ is a stable nodal point,}
\]

the 3rd point
\[
\left( x^3 \right. \text{ is that } i = N - I x_i = -\frac{\ln m_i}{c_i + 3c_2}, \text{ for } i = (0, N - I), x_i = 0 \right) \text{ is a stable nodal point,}
\]

the 4th point \((x^4)\) is the decision of the equation \( \eta \cdot x = x \) is a saddle point which unstable manifold is a curve joining the 2nd and the 3rd critical points. Its stable manifold is \((2N - 1)\)-dimensional manifold joining the 1st and the 4th critical points and transversal to the unstable manifold in the point \( x^4 \).

The exposition \( C \) is the same as for the exposition \( A \), except the following: the properties of the 3rd critical point from the exposition \( A \) proceed to the 2nd critical point and the properties of the 2nd critical point proceed to the 3rd critical point.

3. Numerical example for model of labour force allocation in regions with infinite time

Nowadays the labour market mostly follows the model of monopolistic competition [9] and has the following structure. Between two poles, the employee, on the one hand, and the consumer (the employer), on the other hand, the chain of mediators has been built:

- “big wholesalers” (in other words, dealers) that are, as a rule, government organizations supplying vacant jobs;
- “small wholesales” are the companies dealing with recruitment.

The employee interacts with other market participant only through “big wholesalers”. The employer, in its turn, also interacts through them or seldom through “small wholesales”.

Let’s consider the labour market per the criterion “price group”. There are four main groups:

- the 1st group includes employees of low quality that are situated on the lower range of retail prices (from 0.10 to 0.25 c.u. per hour),
- the 2nd group includes employees of middle quality and correspond to the second-from-the-bottom range of retail prices (from 0.25 to 0.50 c.u. per hour),
- the 3rd group includes employees of high quality that characterize by the high level of retail prices (from 0.50 to 0.90 c.u. per hour),
- the 4th group is presented by employees of high consumer quality imported to Russia (as a rule, of elite quality) and with the highest level of retail prices (from 0.90 to 1.50 c.u. per hour).

The fractions of the labour market occupied by every analyzing price group are in the following correlation: the 1st price group complies 48%, the 2nd – 27%, the 3rd – 22%, the 4th – 7% from the total labour force volume.

Let us confine the further analysis to the competition among one type of labour market participants, i.e. the employees. Herein, there are two lines of competitive interaction: the 1st is between the foreign and Russian employees, the 2nd is among Russian employees.

Suppose that employees are the supports of two competitive technologies (α and β). Let’s analyze the multistep process where employees are considered to be goods. Every process step is presented as a manufacturing cycle including allocation of capital, manufacture of goods, its disposal and repayment.

Let’s introduce agents’ production functions and suppose that all the factors except current assets are unaltered. Then, the agents’ production functions will be one inner functions:

$$\Pi_{i,j}(\kappa)\forall j/i \in \{\alpha, \beta\}, j = 1,4,$$

corresponding the number of manufactured goods \(j - j^*\), market fraction to the agent’s allocated capital \(k \cdot j - j^*\).

Suppose that the production function allows linear approximation and can be presented as

$$\Pi_{i,j}(\kappa) = c_{i,j} \cdot \kappa, \text{ where } c_{i,j} \text{ is the reciprocal variable of cost-per-unit.} \text{ After agents having allocated their capitals the full volume of product output in every four labour market fractions will be defined.}$$

Let’s correlate the total supply to the vector \(v = (v_1, v_2, v_3, v_4)\) in every four market fractions. Then, using the abovementioned production functions, we define the total supply in \(i\)-fraction by the function:

$$v \cdot (\kappa^\alpha, \kappa^\beta) = \Pi_{1,i}(\kappa^\alpha) + \Pi_{2,i}(\kappa^\beta)$$

where \(\kappa^\alpha\) – the capital provided by the employee \(\alpha\), \(\kappa^\beta\) – the capital provided by the employee \(\beta\), for production and distribution of the goods in market \(i\) - fraction.

Suppose that other price-forming factors are constant, then all the volume of goods will be sold by prices in accordance with the demand curve [9], implicitly defined by functionals:

$$f_i(v, p) = 0$$

where, \(i\) is the index identifying the labour market fraction, \(v\) is the earlier introduced supply volume vector, \(p = (p_1, p_2, p_3, p_4)^T\) is the vector variable presenting retail prices in the 1st, the 2nd, the 3rd and the 4th labour market fractions, correspondently.

It’s appropriate to suppose that the marginal utility for every group of goods is the monotonic decreasing function per the argument “quantity of purchased goods”. Let’s assume as the utility function the functions:

$$g_\kappa(v) = b_\kappa \cdot (1 - e^{-a_\kappa \cdot v})$$

where, \(k\) is the index identifying the group of goods, \(a_\kappa\) is the coefficient of exponential growth, \(b_\kappa\) is the factor, \(v\) is the number of goods in this group purchased by the customer.

Let’s define with \(w_1, w_2\) the volumes of goods in the 1st and the 2nd groups purchased by the customer during the period corresponding to one step of the process. Then, the pair \((w, v)\), \(w = (w_1, w_2)^T, v = (v_1, v_2, v_3, v_4)\) will define its needs this period.
Let’s suppose that during one step of the process the employer has the budget \( b \), calculated by conventional monetary units that he can spend on his needs. Let \( q_1, q_2 \) are retail prices for the goods of the 1st and the 2nd groups, \( p_1, p_2, p_3, p_4 \) are, as earlier, the prices of the 1st, the 2nd, the 3rd and the 4th labour market fractions. Using economic terms [6], the employer forms his consumer basket so that the marginal utilities of all the groups of goods included into the basket are equal. In mathematical terms, the consumer is finding the constrained maximum:

\[
\begin{align*}
\sum_{k=1}^{2} g_k(w_k) + \sum_{k=1}^{4} g_k(v_k) & \to \max, \\
\sum_{k=1}^{2} g_k \cdot w_k + \sum_{k=1}^{4} p_k \cdot v_k & = b, \\
w_k & \geq 0, k = 1, 2, \\
v_k & \geq 0, k = (1, 4)
\end{align*}
\]  

As the objective functional is a convex function and the acceptable region is also convex, this problem can be changed into the equivalent problem of solving simultaneous equations:

\[
\begin{align*}
\frac{\partial g_k(w_k)}{\partial w_k} + \lambda \cdot q_k & = 0, k = 1, 2, \\
\frac{\partial g_k(v_k)}{\partial v_k} + \lambda \cdot p_k & = 0, k = (1, 4), \\
\sum_{k=1}^{2} g_k \cdot w_k + \sum_{k=1}^{4} p_k \cdot v_k & = b.
\end{align*}
\]  

The system \((6)\) sets the demand curves mentioned in \((2)\). Let’s use the above-defined functions \( f_i \) to set the equations of this system:

\[
\begin{align*}
f_i(w, v, q, p, \lambda) &= \frac{\partial g_k(w_k)}{\partial w_k} + \lambda \cdot q_k, k = 1, 2, \\
f_i(w, v, q, p, \lambda) &= \frac{\partial g_k(v_k)}{\partial v_k} + \lambda \cdot p_k, k = (1, 4), \\
f_i(w, v, q, p, \lambda) &= \sum_{k=1}^{2} q_k \cdot w_k + \sum_{k=1}^{4} p_k \cdot v_k - b.
\end{align*}
\]

Thus, the system \((6)\) can be rewritten more compactly as follows:

\[
f(w, v, q, p, \lambda) = 0,
\]

where, \( f(w, v, q, p, \lambda)^T \). Suppose that the prices for the goods of the first two groups \( q = (q_1, q_2) \) and the turnover in every four labour market fractions \( v = (v_1, v_2, v_3, v_4) \) are defined. Then, the system \((6)\) can be solved in relation to the variables: the goods quantity of the 1st and the 2nd groups in the consumer basket \( w = (w_1, w_2) \) and the prices for the goods \( p = (p_1, p_2, p_3, p_4) \). The conditions allow solving the problem can be found in a well-known implicit function theorem [8-9]. Further, let’s analyze the case when the conditions of this theorem are applicable to the system \((6.1)\). Let’s fix the variables \((q_1, q_2)\) as the system parameters. Then, we can consider the function

\[
v \to p_q \cdot v,
\]

received by solving the system \((6.1)\) in relation to the variable \( p = (p_1, p_2, p_3, p_4) \).
Suppose that the agents in every labour market fraction allocate the capital for purchasing goods:

\[
\mathbf{k}^\alpha = (k_1^\alpha, k_2^\alpha, k_3^\alpha, k_4^\alpha)^T, \\
\mathbf{k}^\beta = (k_1^\beta, k_2^\beta, k_3^\beta, k_4^\beta)^T.
\]

Then, the production functions (2) will define the number of employees in every market fraction \( v = v \cdot (\mathbf{k}^\alpha, \mathbf{k}^\beta) \). The sales prices that can be calculated with the function (7): \( \mathbf{p} = \mathbf{p}_q \cdot \mathbf{v} \) will be set on the volumes of goods for sale on the market. Every agent’s profit can be calculated as the product of volumes times price summing for every labour market fractions. And finally, after the complete turnover, the capital stock will be defined as the difference between the revenue and all the taxes and normative payments [10-11].

These functions allow defining the exposure between the invested capital and the capital increased after turnover. Suppose that at the initial phase of \( t \)-step process the agents have the capital \( \mathbf{K}^\alpha(t), \mathbf{K}^\beta(t) \) and invest it into the production allocating it in labour market fractions: \( \mathbf{K}_1^\alpha(t), \mathbf{K}_1^\beta(t) \) of capital units into the production of goods in the lower price group (the 1st market fraction, \( \mathbf{K}_2^\alpha(t), \mathbf{K}_2^\beta(t) \) of capital units into the production of the following price group (the 2nd market fraction) and etc [12-13]. Thus, the vector variables:

\[
\mathbf{k}^\alpha(t) = (k_1^\alpha(t), k_2^\alpha(t), k_3^\alpha(t), k_4^\alpha(t))^T, \\
\mathbf{k}^\beta(t) = (k_1^\beta(t), k_2^\beta(t), k_3^\beta(t), k_4^\beta(t))^T,
\]

allow fulfilling:

\[
\sum_{i=1}^4 k_i^j(t) = K_i^{j(t)}, \quad j = \alpha, \beta.
\]  

After the complete turnover the variables of the agents’ capitals take the values \( \mathbf{K}^\alpha(t+1), \mathbf{K}^\beta(t+1) \), that can be calculated by applying function \( \varphi \) to \( \mathbf{k}^\alpha(t), \mathbf{k}^\beta(t) \):

\[
\mathbf{K}^\alpha(t+1) = \varphi_1(\mathbf{k}^\alpha(t), \mathbf{k}^\beta(t)), \\
\mathbf{K}^\beta(t+1) = \varphi_2(\mathbf{k}^\alpha(t), \mathbf{k}^\beta(t)),
\]

\[
\varphi(k^\alpha(t), k^\beta(t)) = \begin{vmatrix}
\Pi_{1,\alpha}(k_1^\alpha(t)) & \Pi_{1,\beta}(k_1^\beta(t)) & \Pi_{1,\alpha}(k_2^\alpha(t)) & \Pi_{1,\beta}(k_2^\beta(t)) \\
\Pi_{2,\alpha}(k_1^\alpha(t)) & \Pi_{2,\beta}(k_1^\beta(t)) & \Pi_{2,\alpha}(k_3^\alpha(t)) & \Pi_{2,\beta}(k_3^\beta(t)) \\
\Pi_{3,\alpha}(k_1^\alpha(t)) & \Pi_{3,\beta}(k_1^\beta(t)) & \Pi_{3,\alpha}(k_4^\alpha(t)) & \Pi_{3,\beta}(k_4^\beta(t)) \\
\Pi_{4,\alpha}(k_1^\alpha(t)) & \Pi_{4,\beta}(k_1^\beta(t)) & \Pi_{4,\alpha}(k_4^\alpha(t)) & \Pi_{4,\beta}(k_4^\beta(t))
\end{vmatrix},
\]

\[
\times p_z \cdot v(k^\alpha(t), k^\beta(t))
\]

where, \( d \) – the amount of revenue taken for taxes and normative payments, the seed capital invested into the labour force. Due to the labour market situation and rival’s position, both the agents regulate the volume of labour force allocating the current assets among the labour market fractions, thus, trying to increase its volume [14-15].

4. Conclusion

In view of the above-mentioned, the research of the principles applied for the labour force alteration, the assessment of labour force planning and its impact on the regional economic development, the development and justification of methods and procedures used in the labour force management are turned out to be topical.

The concept of economy development based on the analysis considering the assessment and forecasting the labour force level could be formed as the result of this research. The solution of the
complex problems connected with the labour force management refers to the formulation of the new theoretical and methodological approach to the management system. Therefore, it’s required to create the corresponding economic and mathematic modeling apparatus, management and optimization procedures, to determine the quality criteria for transition processes and perspective management laws. The reference model differs by the ability to achieve the qualified take-off and immediate intellectual development of labour force at work getting more and more complex should be taken as a base.

References
[1] Gliksberg I 1963 Further generalization of fixed-point Cacutani theorem about with the application to equilibrium situations in Nash sense. Infinite antagonistic games (M: FIZMATGIS)
[2] Golubitskiy M and Giyemin V 1977 Stable mappings and their peculiarities (Moscow: Mir)
[3] Danford N and Schwartz G 1962 Linear operators General theory Vol. 1 (Moscow: IL)
[4] Cornfield I P, Sinay Y G and Fomin S V 1975 Ergodic theory (Moscow: Nauka)
[5] Rokhlin V A 1979 Selected problems of metric theory for dynamic systems (Moscow: UMN)
[6] Fan Tsi 1963 Minimax theorems. Infinite antagonistic games (Moscow: FIZMATGIZ)
[7] Fisher S, Dornbush R and Shmalenzi R 1995 Economics (Moscow: Delo)
[8] Bondarenko L A, Zubov A V, Orlov V B, Petrova V A and Ugegov N S 2016 Application in practice and optimization of industrial information systems Journal of Theoretical and Applied Information Technology 85 (3) 305–08
[9] Bondarenko L A, Zubov A V, Zubova A F, Zubov S V and Orlov V B 2015 Stability of quasilinear dynamic systems with after effect Biosciences Biotechnology Research Asia 12 (1)
[10] Dikusar V V, Zubov A V and Zubov N V 2010 Structural minimization of stationary control and observation systems Journal of Computer and Systems Sciences International 49 (4) 524–28
[11] Malafeyev O, Zaitseva I, Onishenko V, Zubov A, Bondarenko L, Orlov V, Petrova V and Kirjancen A 2019 Optimal location problem in the transportation network as an investment project: A numerical method AIP Conference Proceedings 2116 450058
[12] Zaitseva I 2019 Numerical method of distribution of labor resources by game-theoretic model AIP Conference Proceedings 2116 450057
[13] Zaitseva I, Malafeyev O, Dolgopolova A, Zhukova V and Vorokhobina Y 2019 Numerical method for computing equilibria in economic system models with labor force AIP Conference Proceedings 2116 450060
[14] Malafeyev O, Zaitseva I, Parfenov A, Strekopytova M and Strekopytov S 2019 Game-theoretical model of cooperation between producers AIP Conference Proceedings 2116 450059
[15] Zaitseva I V, Malafeyev O A, Kostyukov K I, Orlov V B and Yupatova K V 2020 Dynamic programming method in the tasks of optimal labor capital distribution programs IOP Conference Series: Earth and Environmental Science 421(3) 032024