Identifying teachers’ approach in assessing students’ understanding on derivative: SPUR perspective

To cite this article: R Desfitri and L Vermana 2019 J. Phys.: Conf. Ser. 1157 042114

View the article online for updates and enhancements.
Identifying teachers’ approach in assessing students’ understanding on derivative: SPUR perspective

R Desfitri* and I Vermana
Program Studi Pendidikan Matematika, Universitas Bung Hatta, Jalan Sumatera Ulakkarang, Padang, 25133, Indonesia

*rdesfitri@bunghatta.ac.id

Abstract. Students’ understanding on mathematical knowledge could be assessed by multidimensional perspective of understanding; Skills, Properties, Uses, and Representations (SPUR) or which was also known as SPUR approach. This study was focused on identifying teachers’ way in delivering tasks to assess their students’ understanding on mathematical content knowledge. The participants of the study were high school teachers who taught derivative at grade eleven. Assignments that teachers gave to their students were analyzed to see what dimensional perspectives were used to figure out students’ ability on related concepts in derivative. Based on their classroom observation and interviews, majority of teachers tended to uses skills and properties dimensional perspectives rather than uses and representations. Others supporting documents showed that even though teachers recognized that various kind of problems needed to help students understand concepts better, they tended to uses text book based problems rather than generating problems by themselves. The study showed that majority of teachers were not aware of multidimensional perspective on understanding (skills, properties, uses, and representations) to assess their students’ mathematical knowledge.

1. Introduction
Calculus is one of the greatest achievements of human intellect. Inspired by problems in astronomy, Newton and Leibniz developed the idea of calculus about 300 years ago. Since then, each century has demonstrated the power of calculus to illuminate questions in mathematics, physical sciences, engineering, social sciences and biological sciences [1]. In calculus reform, Mumford in [2] suggested to distinguish three types of pedagogical methods, memorization and drill, use of many examples, and presentation of the underlying logic theory to make it as airtight legal case.

Differentiation may be expressed in many different ways, but usually lead to similar explanation, which refers to a technique for analyzing the function change, or a process of looking at the way a function changes from one point to another. If any particular function is given, it is possible to find out what it looks like when it is graphed, and how rapidly the function changed could be measured, particularly measuring the changes at any point. The derivative, on the other hand, may be interpreted as the slope of curve, and physically as rate of change [1]. Subject of derivative is often used to explain the concept of tangent slope, velocity, and acceleration in physics, reaction rate in chemistry, and marginal in economics.

Several studies on students’ understanding on the concept of elementary calculus have been conducted. Orton in [3] found that students had a little understanding as well as some fundamental misconceptions about derivative. Similar to this, Heid in [4] figured out that for many students, the
notion of derivative as a rate of change, or second derivative as a measurement of concavity, falls quickly, with disuse because students learned to rely on memorized procedures for doing exercises rather than understanding the concepts.

2. Assessing students’ work by SPUR approach

Many research found that the majority of students, either at university or secondary school levels, faced many difficulties in understanding derivative. Based on our investigation, students had trouble because they did not have a good conceptual understanding of the topics, and at the same time, they more likely did not have enough prerequisite content knowledge on elementary algebra, so that they have a very little idea about material they were given. When they were asked to solve derivative problems, they focused on the procedural of finding answer regardless the understanding of the derivative concepts.

Assessment is a critical part of teaching and learning process in classroom mathematics teachers need to pay attention on the development of assessment and be equipped with the necessary knowledge and skills in implementing various assessment practices [5]. Similarly, Mok in [6] noted that the essence of assessment for learning is that assessment can be used to inform as well as to support further learning.

Traditionally, the assessment of problem solving in classroom has focused on the product rather than the process of problem solving [7]. However, many mathematics educators have recognized the important of using multiple perspectives to assess the learning of mathematical content [8]. Similarly, Freudental in [9] considered different ways in which topic might be used and how those different perspectives lead to understandings.

Some mathematicians in different periods like Thompson and Senk as well as Usiskin had introduced one approach to a multi-dimensional perspective on understanding which is known by acronym SPUR which stands for Skill, Properties, uses, and Representation [8].

In analyzing the task teachers gave to their students, and the way the teachers assess their students’ work, this study used SPUR approach as multidimensional perspective to understanding. Thompson and Kaur in [8] described the SPUR approach as rewritten in the Table 1.

| Dimensional Perspective | Feature |
|------------------------|---------|
| Skill                  | Represents the procedures that students should master with fluency; they range from application of standard algorithm to discovery or invention of algorithm, including procedures with technology |
| Properties             | Principles underlying the mathematics, ranging from the naming of properties used to justify conclusions to derivations and proofs |
| Uses                   | Applications of the concepts to the real world or to other concepts in mathematics and range from routine world problems to the development and use of mathematical models |
| Representation         | Graphs, pictures, and other visual descriptions of the concepts, including standard representations of the concepts and relations to the discovery of new ways to represent concepts |

Table 1 showed that students who have good understanding of mathematics should possess understanding in each of the dimension [8]. If assessment consistently measure students’ achievement in only one dimension, then teachers might have misconceived view of their students’ understanding. Teachers need to conduct assessments in which are able to measure understanding in all four dimensions, so that they can explore students’ weakness and strength in their students’ knowledge of the related concepts.

The main objectives of this study were to figure out how teachers give tasks and how they assess their students’ work. The study was motivated by two things: good task or good question was needed to
know students’ understanding on the concepts; and multi-dimensional perspective on understanding could help teachers assess their students’ work to describe students’ understanding on mathematical knowledge. The purpose of this study was two fold: i) to figure out how teachers gave task and how they assess their students works in teaching derivative; and ii) to figure out whether teachers aware of using multi-dimensional perspective on understanding when giving task to assess students’ understanding.

3. Experimental method
This research was a descriptive research. Participants of this study were high school mathematics teachers who taught derivatives in their classrooms. Ten teachers were selected from various schools in Padang city. One term of tasks of each teacher were observed and analyzed. Samples of task they gave to their students and how they assessed were analyzed based on a multidimensional perspective on understanding to assess students’ works which was known as SPUR approach. Each task was look down to figure out what dimensional perspective used by teacher to assess students’ understanding: i) skill, which consisted of standard algorithm and procedures that students should master; ii) principle, related to properties to justifies conclusion as well as proofs; iii) uses which emphasized the applications of the concepts to real world, world problems, uses of mathematical models; and iv) representations which could be expressed in graph, pictures, or other visual, standard representation of the concept.

In order to collect more relevant data, some classroom observations and interviews with teachers were also conducted. Teachers were asked about what type of questions or problems they give to their students and do they commonly looked out students’ reasoning in answering the problems given.

4. Result and discussion
This part discussed how the form of tasks given to the students and how teachers assessed them. Two examples of tasks from each teacher were selected and analyzed, so there were 20 problems were picked up. Since every problem may not only consist of one dimensional perspective, then problems can be analyzed and categorized by how many percent of problems given by teachers related to each of dimensional perspective on understanding. This can be described by Table 2.

| Number of Dimensional Perspective | Number of problems | Percentage |
|-----------------------------------|--------------------|------------|
| 1 dimension                       | 8                  | 40         |
| 2 dimensions                      | 6                  | 30         |
| 3 dimensions                      | 5                  | 25         |
| 4 dimensions                      | 2                  | 10         |

Table 2 described that teachers unlikely used multidimensional perspectives to assess students understanding. Teacher tended to assess students’ knowledge on mathematics through single dimensional perspective to understanding that was classified as skills. Some problems that were constructed might be able to assess students; understanding through different lens in form of combination of two or three dimensional perspectives. However, based on 20 problems analyzed, only two out of 20 problems could assess students’ knowledge from all dimensional perspective of understanding: skills, properties, uses, and dimension.

Since each problem might have multidimensional perspective to assess students’ knowledge, distribution of multidimensional perspective to understanding to assess students’ knowledge was described in the following table.
Table 3. Problems given by teachers related to dimensional perspective on understanding based on SPUR approach.

| Dimensional Perspective | Number of problems (out of 20) | Percentage |
|-------------------------|--------------------------------|------------|
| Skill                   | 20                             | 100        |
| Properties              | 10                             | 50         |
| Uses                    | 4                              | 20         |
| Representation          | 3                              | 13         |

Table 3 above showed that from 20 selected problems, if analyzed from SPUR perspective, all problems assessed students’ skills, that was dimensional perspective to understanding which was about procedure as well as standard algorithm. Others dimensional perspectives were not often used to assess students’ knowledge related to topics being taught. From the data, it was found out that one of teachers consistently gave problems in forms of exercises that only required skills, because those two problems only concerned about standard algorithm and procedure to solve problems.

Regarding to problems and assessment from teachers, some of those problems were just kind of matter of practices. The following were examples of problems that have been constructed by teachers:

4.1. Problem #1. Find the derivative of a function

From this problem, the students’ understanding of students on derivative can only be seen from dimension of skills, because it only included standard algorithm or procedures. It might also be a bit dimension of property, if students were asked to determine the rules related to derivative of quadratic function related to problem given. However, in case of assessment, the teacher assessed students’ work by focusing on procedural, little care about reason why student should do particular procedures. Student answer was in such case, a particular student received full mark. However, when he was later asked how he got the answer, surprisingly, he replied:

“I moved the degree and put it back into coefficient. So I took the number represented degree and I added put it back in front of the variable”.

It meant that student was working on something without knowing what actually happened. His skill implied that he has reached the true answer. So, based on SPUR approach, this problem can assess students’ skill. However, from others dimensional perspective such as properties perspective, he has yet to understand how to find derivative of the function and he had no idea about the rules of finding derivative of function.

4.2. Problem #2. Given a function Find the first and the second derivative of the function

This problem only required students’ understanding regarding to skill and procedures. The problem could not be used to assess students’ ability on benefit or uses of derivative in life. Yet, problem could not describe students’ capability in representing the function and its derivatives in the forms of graphs or other representations. So from SPUR perspective, this problem could only assess students’ achievement from only skill perspective.

4.3. Problem #3. Given a function find the first and the second derivative of the function. Sketch the graph of the function, and explain your steps!

This problem already contained skill dimension. Problem also required students were aware of derivative rules, and representation of derivative that was presented in graph. From this problem, it would also be possible for teacher to get to know about students’ awareness about position of graph since this function would not intersect the X-axis. Even though skills, properties, and representation dimensions were used in this problem, the dimension of uses was still absent.
4.4. Problem #4. Sketch graph of the function by using its derivative, find its local maximum and local minimum

This problem has consisted of several dimensional perspectives on understanding to assess students’ mathematical understanding. Teacher could assess students’ skills and properties to find its derivative. At the same time, teacher could also assess from representative dimension from represented graph, as well as to assess from uses perspective since students needed to find its maximum and minimum values of the function.

4.5. Problem #5. For the function graph the function and determine the different slope of the function

This problem has already consisted of representation aspect, however, students would not be able to determine the slope of the function if they did not do same algorithm. Students might also need some rules to find the positions of function. So, it is possible for teachers to assess students’ skill as well as their ability in properties and representation at the same time. This only dimensional perspective was missed in this problem was ‘uses’ of the topic, since the application of this function was not discussed.

Beside problems that were usually found as ‘regular’ or very common to be given to students, one interesting fact was found in a problem constructed by a teacher as the following,

4.6. Problem #6. From derivative function find as much information or fact as you have to explain from its derivative function (such as slope, local maximum or minimum value, the original function, its intersect with axes)!

This problem was kind of an open-ended problem, so all dimensional perspective to understanding might be used to assess students’ knowledge. It has consisted of several dimensional perspectives on understanding to assess students’ mathematical knowledge.

From classroom observations, teachers deliver problems to students mainly in form of manipulating symbols or formulae rather than how understanding the meaning of concepts comprehensively. One of the reasons was that teachers prefer to assess students’ works in simple way. Teachers argued that assessing students’ answers that reasoning is concluded would be more time consuming while they had less time to do. Teachers also argued assessing students’ reasoning was not always easy to apply.

Teachers’ problems in assessing might also related to teachers’ ability in constructing mathematical problems that pre-service teachers also faced it. Desfitri in [10] discussed that it was occasionally found that there were hidden errors on questions tasks assigned by pre-service teachers which might lead their students not to be able to reach a logical or correct answers.

Teachers’ tend to provide problems without asking students’ reasoning should also be changed. It would be better when asking students to solve problems, students were required to show their problem solving process on their paper and add some explanation why they solved problems on that way.

Even though not every problem which was constructed to assess students’ knowledge had to have all four dimensional perspectives to understanding, combining two or three of them would help teachers as well as students in enhancing and monitoring teaching and learning progress. Since students’ mistake originated from lack of understanding about fundamental concept of derivatives and its geometric representation, at the same time they had a little idea about the necessity and the uses of derivatives, it would be teachers’ responsibility to provide problems such that students would be aware of properties underlying the concepts, as well as application of the concepts to the real worlds.

5. Conclusion

Mostly teachers tended to give problems to their students in form of exercises which were much related only to standard algorithms and procedures, or based on the sequences of works, but no reason why it was done. This meant that teachers assess students’ knowledge was still limited to skill, yet shown that other dimensional perspectives were not included. There were not many teachers gave problems to be solved by their students related to applications or its usefulness in daily life. In other words, not many teachers included the ‘uses’ perspective in assessing students’ knowledge. Majority of problems also
consisted of only one type of dimensional perspectives, while one teacher tried to give an open-ended problem so that it could assess students’ knowledge from all dimensional perspectives from SPUR approach. Majority of teachers were not aware of multidimensional perspective on understanding (skills, properties, uses, and representations) to assess their students’ mathematical knowledge.

References
[1] Hughes D H 1998 Calculus, Single and Multi Variables 2nd Ed (New York: John Willey and Son Inc)
[2] Mumford D 1997 Calculus Reform – for the Million Notices of the AMS 44 (5) 559-563
[3] Orton 1983 Students’ understanding of differentiation Educational Studies in Mathematics14 pp 235-250
[4] Heid K M 1988 Resequencing skills and concepts in applied calculus using the computer as a tool J. Research in Mathematics Education19 pp 3-25
[5] Yeo K K J 2011 Implementing alternative assessment in the lower primary mathematics classroom Assessment in Mathematics Classroom Kaur B and Yoong W (Singapore: World Scientific) chapter 6 pp 113-130
[6] Mok M M C 2011 The assessment for, of and as learning in mathematics: the application of SLOA Assessment in Mathematics Classroom Kaur B and Yoong W (Singapore: World Scientific) chapter 9 pp 187-215
[7] Lam T T, Seng Q K, Hoong L Y, Dindyal J, and Guan T E 2011 Assessing problem solving in the mathematics curriculum: a new approach Assessment in mathematics Curriculum Kaur B and Yoong W (Singapore: World Scientific) chapter 3 pp 33-66
[8] Thompson D R and Kaur B 2011 Using a multidimensional approach to understanding to assess students’ mathematical knowledge Assessment in mathematics Curriculum Kaur B and Yoong W (Singapore: World Scientific) chapter 2 pp 17-31
[9] Freudental H 1983 Didactical Phenomenology of Mathematics Structure (the Netherland: Reidel)
[10] Desfiri R 2018 Pre-service teachers’ challenges in presenting mathematical problems J. Physics: Conference Series 948 012035