Systems and models with anticipation in physics and its applications

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Abstract. Investigations of recent physics processes and real applications of models require the new more and more improved models which should involved new properties. One of such properties is anticipation (that is taking into accounting some advanced effects). It is considered the special kind of advanced systems – namely a strong anticipatory systems introduced by D. Dubois. Some definitions, examples and peculiarities of solutions are described. The main feature is presumable multivaluedness of the solutions. Presumable physical examples of such systems are proposed: self-organization problems; dynamical chaos; synchronization; advanced potentials; structures in micro-, meso- and macro- levels; cellular automata; computing; neural network theory. Also some applications for modeling social, economical, technical and natural systems are described.

1. Introduction

The history of self-organization science has many brilliant achievements: dissipative structures theory; oscillations in distributed systems; auto-waves; deterministic chaos; collapses; synchronization etc. The ordinary differential equations, discrete time equations and partial differential equations with memory effects have been used as the background for such investigations [1].

But recently it has been found that also very interesting are the investigation of the systems with anticipation. The term ‘anticipation’ had been firstly attached to the systems with the intrinsic models for predicting the evolution of the systems and of their environment [2-5] and many others. But now since the works by Daniel M. Dubois (Liege, Belgium) the notion of ‘strong anticipation’ had been introduced and investigated [3-5]. In case of strong anticipation the system haven’t the models for predictions but are self-making with accounting presumable future states of the system. D. Dubois also introduced the notion of incursive and hyper-incursive systems (with presumable multivaluedness of solutions). Also D. Dubois firstly described the elementary single element with hyper-incursion. One of the most important examples of the models with anticipation follows from modeling of large social systems. Some examples of investigations of the systems with anticipation had been described in [6-8]. Remark that one of the most new and interesting properties in such systems is presumable multivaluedness of the solutions (that is existing of some values of the solution at each moment of time). Because of this it is very prospective to consider the properties of multivalued solutions from the point of view of synchronization investigations.

So in the proposed paper the next issues will be represented: The description of strong anticipation following D. Dubois, general example of network of elements with strong anticipation; examples of multivalued solutions in different systems with anticipation and some presumable types of solutions.
behavior; presumable possibilities of manifestation of such kind of solutions and further research problems. Thus the given paper should be considered as the beginning the investigations of new class of systems namely the systems of coupled strong anticipated elements.

2. Strong anticipation

Since the beginning of 90-th in the works by D.Dubois – see [3-5] the idea of strong anticipation had been introduced: “Definition of an incursive discrete strong anticipatory system ...: an incursive discrete system is a system which computes its current state at time \( t \), as a function of its states at past times \( t-3, t-2, t-1 \), present time, \( t \), and even its states at future times \( t+1, t+2, t+3, ... \)

\[
x(t+1) = A(...,x(t-2),x(t-1),x(t),x(t+1),x(t+2),...,p)
\]

where the variable \( x \) at future times \( t+1, t+2, t+3, ... \) is computed in using the equation itself.

Definition of an incursive discrete weak anticipatory system: an incursive discrete system is a system which computes its current state at time \( t \), as a function of its states at past times \( t-3, t-2, t-1 \), present time, \( t \), and even its predicted states at future times \( t+1, t+2, t+3, ... \)

\[
x(t+1) = A(...,x(t-2),x(t-1),x(t),x^*(t+1),x^*(t+2),...,p)
\]

where the variable \( x^* \) at future times \( t+1, t+2, t+3, ... \) are computed in using the predictive model of the system” [5], (Dubois, 2001, p. 447).

Thus as the further research problem in the field of anticipatory systems should be considered the behavior of the system with strong anticipation. The simplest but rather general counterparts for the equations (1) and (2) are the equations for continuous time case and for discrete time case accounting of strong anticipation with single anticipation time.

Remark that the common example for synchronization investigations is the chain of couple maps with strong anticipation on one forward discrete step:

\[
u_i(n+1) = f(\nu_i(n),\nu_i(n+1),\alpha) + \varepsilon g(\{\hat{\nu}_i\},\{\hat{\nu}(n+1)\},\theta)
\]

where \( \nu_i(n) = \nu_i(m) \), \( \gamma \) -time step.

The equations (1) - (3) are rather new mathematical objects for physics and their full considerations are the tasks for further investigations. So here we had described only first results of such objects investigations. We describe the results on complex behavior research for discrete maps, artificial neural networks, cellular automata. The complex branching non-homogeneous in space solutions also are proposed.

3. Examples of models with anticipation

In this subsection we briefly describe for illustration the possibilities of strong anticipation property manifestation in some problems.

3.1. Two-Steps Discrete-Time Anticipatory Models

The proposed model has two features (see details at [9]). The first is two-steps nature (that is passing on two steps ahead at discrete). The second is that the function \( f(x) \) in the model has a piecewise-linear character, and looks like the transition function of neurons in neuronets. Remark that the piecewise character of the nonlinearity usually allows to simplifying mathematical investigations.

We write down the proposed model as follows:
The function \( f(x) \) depends on the parameter \( \alpha \) and has the following expression:

\[
\begin{align*}
    f(x) &= 0, & x &\leq 0 \\
    f(x) &= \alpha \cdot x, & x &\in (0, \frac{\pi}{\alpha}] \\
    f(x) &= 1, & x &> \frac{\pi}{\alpha}
\end{align*}
\]

The software had been developed so that it is possible to visualize some branches of the states of map, but only those, that are not thrown out to infinity. This facilitates the understanding of the processes, which take place in the region of ambiguousness. Namely, at the Figure 1 below we can see branching of solution at discrete time steps. In other calculations we had seen the number of different period of cycles and increasing of the number of cycles in the course of time, that is we had seen a tendency to phenomena which we may titled as the analogues of usual „chaos”.

**Figure 1.** The graph of branching of solution. The parameters values are: \( \alpha = 2, m=0 \). Limitations for visualization of the values of variables are the next: maximal \( m+7 \) and minimal \( m-5 \), without representing the branches, which are thrown out to infinity.

**Figure 2.** Map in state space. The sequence of values of \( (p_t, p_{t+1}) \) is represented for the solution from figure 1.

3.2. Cellular automata with anticipation

Recently we have investigated some kind of such models. With anticipation accounting: game 'Life' with anticipation; movement of pedestrian crowds; general problems [6].
First of all let's make a short definition of cellular automata. The cellular automata are discrete dynamic system which represents set of identical cells connected among them. Cells create a lattice of the cellular automata. Lattices can be different types, differing both on the dimension, and under the form of cells [6, 10].

The local rule for cell k on the $Z^d$ is the transformation $T_k$ which transforms the state $s_k(t)$ in $S$ of cell k at moment t to the state $s_k(t+1)$ in $S$ of the same cell at moment $(t+1)$.

$$S_j(t+1) = G_j(\{s_j(t)\}, R),$$  \hspace{1cm} (5)

where $N_k$ – some neighborhood of cell k on the lattice $Z^d$; $\{s_k(t)\}$ is the set of cell’s states within $N_k$, the transformation $T_k$ result depends only on the states of elements within the neighborhood $N_k$ (locality), $R$ – some parameters for rules which define transformations. The collection of local transformations $T_k$ define the global transformation $G$ on the configuration space $C$ which represents the states of all cells at each time moments.

3.2.1. Game ‘Life’ as example of CA

One of the basic examples of CA is well-known Game ‘Life’ by J.Conway [6, 10]). At a time t let some subset of the cells in the array are living. The living cells at time t+1 are determined by those at time t according to the following evolutionary rules (the details are posed at next section of paper). Remark that anticipation follows to the origin of equations of type:

$$S_i(t+1) = G_i(\{s_i(t)\}, \{s_i(t+1)\}, R),$$  \hspace{1cm} (6)

The main peculiarities of such equation is that such equations may have multivalued solutions (of course in some range of parameters). Next graph presents a case of well-developed solution multivaluedness within the model “LifeA”.

Figure 3. A large number of configurations coexisting in system (additive next state function, $\alpha = 0.5$, initial state is 0000)

Such objects are very interesting and prospective for investigations of synchronizations. Remark that the main peculiarities in such investigated systems are presumable multivaluedness and space non-homogeneity of solutions behavior.

3.3. Neural networks with anticipation
Other very important object of proposed type (with anticipation and discrete time dynamic) is neural networks with anticipation. We already have investigated some such models (see [11]). Here we propose for the understanding of presumable behavior only very short description of calculations result. One of the simplest variant of such models with anticipation accounting has the form (counterpart for Hopfield’s neural networks):

\[
x_j(n+1) = f\left( (1 - \alpha) \sum_{i=1}^{N} w_{ji} x_i(n) + \alpha \sum_{i=1}^{N} w_{ji} x_i(n+1) \right)
\]

(7)

where \( \alpha \) is anticipation parameter. The case of \( \alpha = 0 \) corresponds to the absence of anticipation (with classical neuronet). Below at the picture we propose the example of presumable behavior of the model (7) at different time moments.

Figure 4. Behavior of the multivalued solution.

The numbers at right side of figure corresponds to the discrete time moments. The network had 6 elements. The non-homogeneous in space solution is seen.

Two properties of artificial neural network solutions are represented at the figure 4: multivaluedness and non-homogeneities of elements behavior. Both are rather new and prospective for development of synchronization investigations as in theory as in applications.

3.4. Presumable properties of solutions and further research problems

Here we pose some comments which may be useful also for considering proposed problem. The detailed consideration of such multivalued dynamical systems may constitute the subject of further investigations. Here we only pose some general comments. Because the map acts on the multivalued functions of \([0, 1]\) than the dynamical system is infinite-dimensional. Very important is the asymptotic solutions when \( t \to \infty \). For single-valued case (without anticipation) by A. Sharkovsky and other [12] it has been found the existence of so called relaxation oscillation regimes when the solution tends to the function with finite number of discontinuities at fix length time interval and so-called turbulent solutions when the number of discontinuities tends on fix length interval tends to infinity as \( t \to \infty \). Some limit objects for such solution may have fractal structure. Moreover it has been proved that the limiting regime is multivalued function for which some probability distribution corresponds. And finally very interesting for such problems is the structure of attractors in single-valued case. It had
been found that attractors are infinite-dimensional and contained generalized functions (A. Sharkovsky, Yu. Maistrenko, E. Romanenko). The attractors are with simple dynamics: ‘… the movement on attractors is rather simple – periodical or almost periodical, also the elements of attractor are functions acting from I=[0, 1] into I, may be very complex (for example with cantorian set of multivaluedness points’ [12].

Based on our results it may be proposed some generalizations for cases with accounting of anticipating. At first as we described above the solutions in such case may be multivalued. So the fractal dimension of solutions may be bigger then that in single-valued case. Moreover the behavior of the solutions may be much more complex. Different regimes on the different branches of solutions may exist (some examples of such different branches see in [9]). So each branch may have different stochastic properties, different types of relaxation behavior etc. But may be the most interesting is the common behavior of mixture of the branches in multivalued case. Especially interesting is the problem of limiting behavior as \( t \to \infty \): limit solutions, attractors of such solutions, probability properties of limiting objects. But also the new problems arrows: observability of solutions, selection the single-valued trajectories of the system, its limiting objects complexity and such complexity measures, searching adequate mathematical spaces for considering the problems and solutions. Just the problems of adequate definition of periodic behavior (and moreover of dynamical chaos), Lemeray’s staircase, Poincare’s bifurcation diagram, ergodicity, mixing are interesting.

**Conclusion**

Thus in given paper we propose to consider the problem of complex behavior and self-organization for new class of objects namely for the chains and networks with strong anticipation. The presumable multivaluedness of solutions follows to new interesting properties in the frame of already existing concepts. But also new features can appear (for example non-heterogeneous multivalued solutions) which are very prospect for further investigation. We described only the first results of investigations and only some new presumable forms of research problems. But evident mathematical novelty of proposed problems and presumable great importance in applications (for examples for social systems; computation and signal processing theory; consciousness investigations etc.) follows to needs of further development of such investigations.

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