Split-SUSY versus SUSY GUTs

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Abstract

The gauge coupling unification is one of main motivations in the split-SUSY scenario, and the existence of the grand unified theories (GUTs) is assumed. We examine how to realize split-SUSY mass spectrum in the context of GUTs and find that the construction of split-SUSY GUTs is by no means straightforward. With $R$-symmetry breaking sources in the GUT sector, GUT particles play a role of the messengers in the gauge mediation scenario and their contributions to gaugino masses can be sizable. We find the upper bound on soft scalar masses of $\mathcal{O}(10^{10})$ GeV from consistency for constructing the split-SUSY GUT. Also, we discuss the attempt to construct $R$-symmetric GUT models.
1 Introduction

It is well known that the gauge hierarchy problem in the Higgs sector of the standard model (SM) can be solved by introducing supersymmetry (SUSY) [1]. The minimally extended SUSY SM (MSSM) has an elegant feature, namely, the unification of three gauge couplings at the scale $M \sim 2 \times 10^{16}\text{GeV}[2]$, so that the existence of SUSY and also SUSY Grand Unified Theories (GUTs) have been widely believed. Also, SUSY seems to be necessary for the construction of consistent string theories which include quantum gravity. However, experiments have not observed any SUSY particles yet. In addition, the predicted proton-decay by SUSY GUTs has not been observed yet [3]. Then, we might consider the possibility of heavy SUSY particles as one option.

The split supersymmetry (split-SUSY) scenario proposed in Refs.[4, 5] is just the case, in which scalar masses are super-heavy while fermion masses are maintained to be around the weak scale, protected by the $R$ (chiral) symmetry. This split-SUSY soft mass spectrum can be simply obtained by taking decoupling limit of the scalar particles except the SM-like Higgs boson in the MSSM[4, 5]. Therefore, in this scenario, the nature is fine-tuned intrinsically and SUSY is nothing to do with the gauge hierarchy problem (for the related studies, see for examples, Refs.[6]-[13]). Nevertheless, SUSY still plays two important roles. One is for the gauge coupling unification. It has been found that the gauge couplings are successfully unified at the GUT scale even with the split-SUSY soft mass spectrum[4]. The other is to provide the dark matter candidate (neutralino) as usual[4, 5]. In other words, these two facts are main motivations for the split-SUSY scenario.

As discussed in Refs.[4, 5], it is nontrivial to keep the split-SUSY mass spectrum under quantum corrections. First of all, contributions by the anomaly mediation (AMSB)[14, 15] should be concerned. In the split-SUSY scenario, soft SUSY breaking masses for sfermions are super-heavy so that SUSY breaking scale is very high. This means that vacuum expectation value (VEV) of superpotential is also very large in order to obtain a vanishing cosmological constant in normal supergravity. In this case, F-term of the compensating multiplet is very large, and thus the AMSB contributions make gauginos very heavy. To switch them off, the structure of almost no-scale supergravity[16] is necessary[4, 5]. In general, to avoid gauginos obtaining large masses, the $R$-symmetry, which works as the chiral symmetry for gauginos, plays a crucial role. It has been shown that, using (approximate) $R$-invariance in the MSSM and the hidden sectors, the split-SUSY mass spectrum can be realized[4, 5].

However, since the grand unification is one of main motivations of the split-SUSY scenario, we must examine how to realize the split-SUSY mass spectrum in the context of GUTs. In this paper, we address this issue and find that, concerning GUT models, to realize the split-SUSY spectrum is not straightforward. In a simple GUT model, there are some $R$-symmetry breaking sources associated with new GUT particles and their VEVs to break the GUT gauge symmetry down to the SM one. Then, there emerge some large contributions to gaugino masses due to the GUT particles. To avoid these contributions, we must construct the GUTs in which $R$-symmetry breaking effects from $A$-terms, hidden sector fields and the VEV of the superpotential are all small. These considerations lead to the upper bound on soft scalar masses as $\mathcal{O}(10^{10})$ GeV. We also discuss attempt to construct $R$-symmetric GUTs (RGUTs). It seems to be very difficult to obtain a realistic RGUTs maintaining the gauge coupling unification.
2 Radiative corrections from GUT particles

The split-SUSY is essentially the MSSM with the decoupling of scalar fields except for one SM-like Higgs. For the split-SUSY spectrum, we need the huge $F$-term for the heavy scalar masses. We introduce the gauge singlet field, $X$, taking a $F$-term as

$$X = F_X \theta^2. \quad (1)$$

The split-SUSY requires the high scale SUSY breaking as[4, 5].

$$\frac{F_X}{M_p} \equiv \tilde{m} \simeq 10^6 \sim 10^{12} \text{GeV}, \quad (2)$$

where $M_p$ denotes the reduced Planck scale. Scalar soft SUSY breaking masses are obtained as

$$\int d^4\theta \frac{X^\dagger X}{M_p^2} Q^\dagger Q = \tilde{m}^2 |\tilde{Q}|^2, \quad (3)$$

where $Q$ denotes visible sector superfields. The large soft squared masses of sfermions and Higgs are induced from this operator. Also, the so-called $B$-term is induced similarly as

$$\int d^4\theta \frac{X^\dagger X}{M_p^2} (H_u H_d + \text{h.c.}) = \tilde{m}^2 (H_u H_d + \text{h.c.}), \quad (4)$$

where $H_u$ and $H_d$ denote the Higgs doublets. It suggests $B\mu \sim \tilde{m}^2$, which is needed for the structure of the split-SUSY[13].

Since the $F$-term in Eq.(1) is huge, the vanishing cosmological constant requires the large constant superpotential, so that gravitino mass is very large. To avoid large contributions to gaugino masses due to the AMSB, the structure of the almost no-scale supergravity[16] should be incorporated, which leads to a hierarchy between gravitino mass and soft scalar masses, $m_{3/2} \gg \tilde{m}$ (for basic formulas, see, for example, Ref.[13]). Furthermore, note that the superpotential generally induces negative soft scalar mass squareds of $O(0.01 \times m_{3/2}^2)$ through the gravitino one-loop diagram[17]. Thus, as a realistic scenario, we consider the sequestering scenario in extra dimensions, where the huge constant superpotential is located at the different brane spatially separated from the brane on which the GUT sector and the hidden sector field $X$ reside. In this setup, the negative contributions through the gravitino loop diagram are enough suppressed by a factor $(LM_p)^{-1}[17]$ with a large $L$ being the distance between two branes. On the other hand, there is no correction for gaugino masses from this gravitino loop diagram, because of $R$-symmetry[18].

Since the grand unification is one of main motivations of the split-SUSY scenario, we should examine how the split-SUSY mass spectrum can be realized in the context of GUTs. As an example, let us consider the minimal $SU(5)$ GUT. The Higgs superpotential is given by

$$W = \frac{1}{2} Mtr \Sigma^2 + \frac{1}{3} tr \Sigma^3 + \bar{H} \Sigma H + 3MH\bar{H}, \quad (5)$$

where $M \simeq 10^{16}$ GeV is the mass around the GUT scale, $\Sigma$ is the adjoint Higgs, and $H$ and $\bar{H}$ are $5$ and $\bar{5}$ representation Higgs fields, respectively. Here, we have dropped
dimensionless coupling constants, for simplicity. The adjoint Higgs, \( \Sigma \), breaks \( SU(5) \) to the SM gauge group through its VEV,

\[ \langle \Sigma \rangle = \text{diag.}(2, 2, 2, -3, -3)M. \]  

Triplet-doublet (TD) splitting is assumed to be achieved by the fine-tuning.

If \( R \)-charge of \( X \) is zero \( (Q_R(X) = 0) \), soft SUSY breaking \( A \) and \( B \)-terms can be induced as

\[ \mathcal{L}_{soft} = \int d^2 \theta \frac{X}{M_p} \mathcal{W} = \tilde{m}[M_{tr} \Sigma^2 + tr \Sigma^3 + \bar{H}\Sigma H + 3MH\bar{H}], \]  

from Eq.(5), where all fields stand for scalar component. Immediately, we notice that the gluino and bino obtain their masses through 1-loop diagrams shown in Fig.1, where colored-Higgs \((T, \tilde{T})\) and colored-higgsino \((\tilde{T}, \tilde{\bar{T}})\) are running in the loop. Here, the GUT particles play the role of the messenger in the gauge mediated SUSY breaking[19]. In Fig.1, \( M_T \) is the supersymmetric fermion mass for \( T\tilde{T} \), and \( B_T \) is the SUSY breaking \( B \)-term for \( \tilde{T}\tilde{T} \). These are given by \( M_T \simeq M \) and \( B_T \simeq \tilde{m}M \) in the minimal \( SU(5) \) GUT. Therefore, the 1-loop diagram makes masses of gluino and bino huge,

\[ M_{1/2} \simeq \frac{\alpha \tilde{m}M}{4\pi} \sim 10^{-2} \times \tilde{m}. \]  

As for the wino mass, there is a 1-loop diagram shown in Fig.2, where the adjoint Higgs plays a role of the messenger. In the figure, the supersymmetric fermion mass of \( \Sigma \) is denoted as \( M_\Sigma \), and scalar soft mass is denoted as \( B_\Sigma \). Again, these are given by \( M_\Sigma \simeq M \) and \( B_\Sigma \simeq \tilde{m}M \), so that we obtain the same result as Eq.(8). So, the wino also obtains too heavy mass. Above contributions completely destroy the hierarchical structure of the split-SUSY.

This difficulty seems to originate from the fact that \( R \)-symmetry is broken badly by \( F_X \) in Eq.(7). Reminding that we must impose \( R \)-symmetry in order to construct the split-SUSY spectrum, fermion masses \( \ll \) scalar masses, in the usual (non-GUT) MSSM, we should avoid the large scale \( R \)-symmetry breaking. Thus, since the split-SUSY requires the large magnitude of \( F_X \) in Eq.(2), we should take \( Q_R(F_X) = 0 \). This implies \( Q_R(X) = 2 \), so that Eq.(7) is forbidden by the \( R \)-symmetry. Of course, since there is no \( R \)-symmetry in Eq.(5) from the beginning, we can, in general, add any higher dimensional terms with \( X \) in spite of the \( R \)-charge assignment of \( X \). In this paper, we impose \( R \)-symmetry for SUSY breaking \( A \) and \( B \)-terms, the hidden sector field, and higher dimensional terms among
the GUT fields. For superpotential, we may omit some dangerous terms for realization of split-SUSY mass spectrum in the way of so-called “technically natural”, thanks to the non-renormalization theorem in SUSY theories.

In the split-SUSY scenario, the weak-scale \( \mu \)-term is supposed to be realized by the \( R \)-symmetry[4, 5], the magnitude of \( A \)-term in Eq.(7) should be suppressed and be replaced to the weak scale\(^{\dagger} \) as

\[
\mathcal{L}_{\text{soft}} = \int d^2 \theta \delta \tilde{m} \theta^2 \mathcal{W} = \delta \tilde{m} [M_{tr} \Sigma^2 + \lambda tr \Sigma^3 + \bar{H} \Sigma H + 3M H \bar{H}],
\]

where \( \delta \tilde{m} \equiv \mu = \mathcal{O}(100) \) GeV. In this case, scalar masses in Figs.1 and 2 are suppressed as \( B_T \simeq B_\Sigma \simeq \mu M \), so that the 1-loop correction to gaugino masses from the GUT particles is given by

\[
M_{1/2} \simeq \frac{\alpha}{4\pi} \frac{\mu M}{M} \sim 10^{-2} \times \mu,
\]

which is negligibly small.

The problem seems to be solved by imposing \( R \)-symmetry as \( Q_R(X) = 2 \). However, there exist other contributions. The adjoint Higgs and colored Higgs can obtain the soft masses through the \( D \)-term as

\[
\int d^4 \theta \frac{X^\dagger X}{M_p^2} tr \Sigma^2 = \tilde{m}^2 tr \Sigma^2,
\]

\[
\int d^4 \theta \frac{X^\dagger X}{M_p^2} T T = \tilde{m}^2 T T.
\]

These soft masses mean \( B_T \simeq B_\Sigma \simeq \tilde{m}^2 \), which induce gaugino mass corrections as

\[
M_{1/2} \simeq \frac{\alpha}{4\pi} \frac{\tilde{m}^2}{M} \sim 10^{-2} \times \tilde{m} \left( \frac{\tilde{m}}{M} \right).
\]

This implies the existence of the upper bound on scalar mass as \( \tilde{m} \leq 10^{10} \) GeV\(^{\S} \). This problem is originated from the fact that, in the minimal \( SU(5) \) GUT, the \( R \)-symmetry is explicitly broken in the superpotential Eq.(5) from the beginning. Since non-vanishing

\(^{\dagger}\)For example, we introduce an additional field, \( Y \), which is \( Q_R(Y) = -2 \). Then the \( \mu \)-term is given by

\[
\int d^4 \theta \frac{Y^\dagger Y}{M_p^2} (H_u H_d + h.c.) = \int d^2 \theta \delta \tilde{m} (H_u H_d + h.c.).
\]

Assuming \( \langle Y \rangle / M_p \equiv \delta (\ll 1) \), \( A \)-terms in the GUT sector as well as \( \mu \) term is suppressed to the weak scale.

\(^{\S}\)There are not so large phenomenologically suitable parameter region in \( 10^{10} \text{GeV} \leq \tilde{m} \leq 10^{12} \text{GeV} \)[20].
gaugino masses need both SUSY breaking and $R$-symmetry breaking, the GUT scale $R$-symmetry breaking induces the serious problem. This is the essential reason for appearing the upper bound of $\tilde{m}$, when we extend the split-SUSY to the minimal $SU(5)$ GUT.

Here, we should comment on the effect of non-zero $F$-component of $\Sigma$ ($F_\Sigma$). Once non-zero $F_\Sigma$ is developed, $B_T$ and $B_\Sigma$ are produced through Eq.(5). Hence, gauginos obtain their masses as

$$M_{1/2} \simeq \frac{\alpha}{4\pi} \frac{F_\Sigma}{M}.$$  

(14)

There are two possibilities for non-zero $F_\Sigma$ to be produced after SUSY breaking as follows. One is a slight VEV shift by $\langle \Sigma \rangle \simeq M \to M(1 + O(\tilde{m}^2/M^2))$ in the Higgs potential after the soft mass term, $\tilde{m}^2\Sigma^2$, in Eq.(11) is taken into account\(^4\). Then, we find $F_\Sigma = O(\tilde{m}^2)$, and the contribution from Eq.(14) is of the same order of magnitude as the one from Eq.(13). The other possibility is a tad-pole term for $F_\Sigma$ in Eq.(11) with non-zero VEVs for $\langle X \rangle$, $F_X$ and $\langle \Sigma \rangle$. Together with $|F_\Sigma|^2$ from kinetic term, non-zero $F_\Sigma$ is induced through equation of motion for $F_\Sigma^T$ such as

$$F_X \sim \frac{\langle X \rangle^* F_X}{M_p^2} \langle \Sigma \rangle \simeq \delta_X \tilde{m} M,$$  

(15)

where $\delta_X \equiv \langle X \rangle / M_p$. This parameter parameterizes the magnitude of $R$-symmetry breaking by the hidden sector field, since $X$ has $R$-charge $Q_R(X) = 2$. The value of $\delta_X$ must be small enough to reproduce weak-scale gaugino masses. We should notice that the value of $\langle X \rangle$ depends on the hidden sector potential and can be non-zero in general, although we assume $\langle X \rangle = 0$ in Eq.(1).

Here, let us briefly summarize discussions above. For the weak-scale gaugino masses, the weak-scale $A$ and $B$-terms are required in the MSSM sector discussed in Refs.[4, 5]. In addition to this approximate $R$-symmetry, the following two conditions should be imposed in the context of the minimal $SU(5)$ GUT with the split-SUSY structure, (i): $\tilde{m} \leq 10^{10}$ GeV, (ii): $\langle X \rangle \ll M_p$.

| $\tilde{m}$ (GeV) | $\Delta M_{1/2}$ (GeV) | $\frac{\alpha}{4\pi} \frac{\tilde{m}^2}{M}$ | $\delta_X$ | $\langle X \rangle^* \tilde{m} \leq 100$ GeV | $\Delta X \ll 10^{-n}$ |
|------------------|------------------------|------------------------------------------|--------------|---------------------------------|------------------|
| $10^n$           | $10^{2n-18}$           | $\leq 10^{-n}$                          |              |                                 |                  |

Here, $n = 6, 7, \cdots, 12$. The second condition is strongly dependent of the structures of the SUSY breaking hidden sector. We have shown that the split-SUSY GUT requires $\tilde{m} \leq 10^{10}$ GeV and $\delta_X \ll 1$ which means the small $R$-symmetry breaking in the hidden sector.

The magnitude of the $R$-symmetry breaking in the hidden sector parameterized by the magnitude of $\langle X \rangle$ ($\delta_X$ in Eq.(15)), is completely model dependent. When we consider the Polonyi hidden sector, $W = m^2X$, as an example in the framework of the almost no-scale supergravity, we can easily show that $\langle X \rangle = 0$ is obtained by introducing higher order terms in the Kahler potential[5], $\mathcal{K} = X^\dagger X - (X^\dagger X)^2 + \cdots$. In order to obtain $\delta_X \ll 1$, it is crucial that the almost no-scale structure and the sequestering of the constant superpotential mentioned above. In this setup, potentials in the Polonyi sector and the almost no-scale sector with the constant superpotential on the other brane works almost

\(^4\)The contribution from the soft mass, $\mu M tr \Sigma^2$ in Eq.(9), is negligible, since the VEV shift from this soft mass is $\langle \Sigma \rangle \simeq M(1 + O(\mu M/M^2))$, which induces $F_\Sigma = O(M)$, and the gaugino mass correction is given by $M_{1/2} \simeq \frac{\alpha}{4\pi} \mu$. 

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independently, so that the vanishing cosmological constant can be achieved between the positive contribution from the hidden sector and the negative contribution from the almost no-scale sector. Note that if we consider the Polonyi hidden sector in usual supergravity, the vanishing cosmological constant condition requires $X = O(M_p)$.

Before closing this section, we give further comments on our setup in the framework of supergravity. After the GUT symmetry breaking, Eq.(5) leads to $\langle W \rangle = O(M^3)$, so that, in normal supergravity, the huge SUSY breaking scale, $F \simeq M^3/M_p \sim (10^{15}\text{GeV})^2$, is required to achieve the vanishing cosmological constant. Therefore, soft scalar masses become of order $m = F_X/M_p \sim 10^{12}$ GeV, which contradicts against the condition obtained above, $m \leq 10^{10}$ GeV. This difficulty is also originated from the GUT scale $R$-symmetry breaking by $\langle W \rangle$. On the other hand, in the framework of the almost no-scale supergravity, the contribution of $\langle W \rangle$ to vacuum energy is suppressed by the special structure of the no-scale supergravity and we can avoid this problem. However, a large $\langle W \rangle$ induces a large negative contribution to scalar squared masses through the gravitino loop diagram, $m^2 \sim -\frac{0.01}{M^4}$. For $\langle W \rangle = O(M^3)$, this contribution is too large $|m| = O(10^{11})$ GeV to be consistent with the condition we obtained. To avoid this problem, we need to introduce a constant superpotential term, $-M^3$, in the visible sector superpotential in Eq.(5), so as to achieve small VEV of the total superpotential through the fine-tuning.

### 3 $R$-symmetric GUTs

All the difficulties discussed in the previous section originates from the fact that the minimal $SU(5)$ GUT breaks $R$-symmetry explicitly at the GUT scale. It implies that the $R$-symmetry should not be broken at the GUT scale for maintaining weak-scale gaugino masses. In this section, we discuss some attempt to construct RGUTs\footnote{The continuous global $U(1)_R$ symmetry might not be necessary. Even a discrete symmetry $Z_{3R}$ or larger might be useful for preserving weak-scale gaugino masses.}. We consider two explicit models of RGUT as examples.

The first model is the 5D $SU(5)$ GUT in which there is no $\Sigma$ and the GUT gauge group is broken through the boundary condition\cite{22,23,24}. Thus, there is no need to take into account the diagrams of $\Sigma$ in Fig.2. This model possesses the $R$-symmetry, which is broken only by the weak-scale mass parameters, $\mu$-term and gaugino masses. The gauge coupling unification, which is another motivation in the split-SUSY, can be achieved in this setup\cite{23}. We can also neglect the VEV of $\langle W \rangle$, because of the absence of the adjoint field.

As for the colored-Higgs contribution in Fig.1, it is sufficiently suppressed as follows. The $H$ and $\bar{H}$ Higgs fields are in the bulk with $R$-charge of $Q_R(T\bar{T}) = 0$. This is the reason why the value of $\mu$ in the $\mu T\bar{T}$ term in $W$ is suppressed to the weak scale. This $R$-charge assignment allows the soft scalar mass, $m^2 T\bar{T}$, in Eq.(12). On the other hand, the effective fermion mass becomes $M_T \simeq M^2/\mu$ in the diagram in Fig.1, since $T$ and $\bar{T}$ have the compactification (GUT scale) masses with their chiral partner of 5D. It works just like the missing partner models, and the contribution through the diagram of Fig.1 is

\footnote{Generally, the condition of the vanishing cosmological constant in the GUTs tends to need too large SUSY breaking\cite{21}.}
suppressed by an additional factor, \( \mu/M \),
\[
M_{1/2} \simeq \frac{\alpha \mu \tilde{m}^2}{4\pi M M}.
\]
(17)

This is negligibly small.

The split-SUSY in this 5D \( SU(5) \) GUT was considered in Ref.[25]. (The gauge coupling unification is discussed in Ref.[26].) However, since the gauge multiplets reside in the bulk and couple to the radion multiplet, we have to consider a contribution to gaugino masses due to the F-term of the radion \( F_T \), (which is equivalent[27] to the mechanism of the Scherk-Schwarz SUSY breaking[28]). Through non-zero \( F_T \), gauginos obtain masses of order of gravitino mass in the framework of the almost no-scale supergravity, so that we cannot realize the split-SUSY mass spectrum. Thus, some modification of the setup is necessary. The simplest extension might be to consider a six dimensional model with the compactification of extra two dimensions on the orbifold \( (S^1/Z_2)^2 \), and set the 5D GUT sector only on the 5th dimensional direction and the almost no-scale sector in Ref.[13] on the 6th dimensional direction. However, in general, we will not able to assume the sequestering between almost no-scale sector and the 5D GUT sector, because two radion fields corresponding each extra dimensional radius must couple with each other in a six dimensional supergravity. Thus, we may conclude that it is difficult to construct the consistent split-SUSY GUT in extra dimensions.

The second model of the RGUT is the 4D non-minimal \( SU(5) \) GUT[21], in which an additional adjoint superfield, \( \Sigma' \), is introduced. The superpotential is extended as
\[
W = M tr(\Sigma \Sigma') + tr(\Sigma^2 \Sigma') + \bar{H} \Sigma H + M H \bar{H}.
\]
(18)

The \( R \)-charges are \( Q_R(\Sigma) = 0 \), \( Q_R(\Sigma') = 2 \), and \( Q_R(T T) = 2 \). The \( \Sigma' \) has vanishing VEV as \( \langle \Sigma' \rangle = 0 \) in the SUSY vacuum. The VEV of the superpotential also vanishes \( \langle W \rangle = 0 \), so that \( R \)-symmetry preserved at the GUT scale. The TD splitting is assumed to be realized by the fine-tuning\(^\dagger\).

In this model, the soft scalar mass of \( \Sigma^2 \) is \( \tilde{m}^2 \) as Eq.(11), while that of \( \Sigma'^2 \) is suppressed due to the \( R \)-symmetry, since we can not write down the operator of Eq.(11) for \( \Sigma'^2 \). We have shown in the previous section that the \( \mu \)-term should be suppressed to the weak scale by the \( R \)-symmetry. In the same way, the magnitude of the soft mass of \( \Sigma'^2 \) should be suppressed at least \( O(\mu M) \). The soft squared mass term, \( \tilde{m}^2 \Sigma'^2 \), causes the slightly shift of VEV as \( \langle \Sigma \rangle \simeq M \to M(1 + O(\tilde{m}^2/M^2)) \). There is no VEV shift for \( \langle \Sigma' \rangle \). This is because the SUSY vacuum of \( \Sigma' \) exists at the origin, so that the vacuum is not shifted from the origin by the soft SUSY breaking parameters. However, here we assume the possibly maximal shift of \( \langle \Sigma' \rangle = O(\mu) \) concerning to some other effects of SUSY breaking. These VEV-shifts produce \( \langle F_\Sigma \rangle \simeq \mu M \) and \( \langle F_{\Sigma'} \rangle \simeq \tilde{m}^2 \).

At first, we estimate the contributions from Fig.2. The scalar soft mass is \( B_\Sigma \simeq \tilde{m}^2 \) as shown above (Eq.(11)). We can show that \( \langle F_\Sigma \rangle \simeq \tilde{m}^2 \) also induces the scalar mass of \( B_\Sigma \simeq \tilde{m}^2 \). The supersymmetric fermion mass, \( M_\Sigma \simeq \mu \), should be induced through the

\(^\dagger\)The main discussion in Ref.[21] is not the Eq.(18). They considered the extra \( SU(3)_H \) to achieve the TD splitting. We do not take this model, since we would like to consider the simple unification of the three gauge couplings.
$R$-symmetry breaking just like the $\mu$-term. Then the contribution from the 1-loop diagram in Fig.2 becomes

$$M_{1/2} \sim \frac{\alpha}{4\pi} \frac{\mu\tilde{m}^2}{\mu^2} \sim \frac{\alpha}{4\pi} \mu,$$

(19)

which is negligibly small.

Next, we show the contribution from the diagram in Fig.3. The soft squared scalar mass of it should be given by $B_\Sigma \simeq \mu\tilde{m}$ maximally, since $Q_R(\Sigma^2) = 4$ suggests no contribution from Eq.(11). Without $R$-symmetry breaking, this term can not be induced, and it is suppressed at least by $O(\mu\tilde{m})$. The fermion mass of $M_\Sigma \simeq \mu$ is induced through the $R$-symmetry breaking just like the $\mu$-term. Then, the 1-loop diagram from Fig.3 becomes

$$M_{1/2} \simeq \frac{\alpha}{4\pi} \frac{\mu^2 \tilde{m}}{\mu\tilde{m}} \sim \frac{\alpha}{4\pi} \mu,$$

(20)

which is also negligible.

The third is contribution from the diagram in Fig.4. The $R$-symmetry suppresses the SUSY breaking $B$-term as $B_{\Sigma \Sigma'} \simeq \mu M$. $\langle F_\Sigma \rangle \simeq \mu M$ from the VEV shift also induces the same magnitude of $\mu M \Sigma \Sigma'$. Since the supersymmetric fermion mass is $M$, the 1-loop diagram from Fig.4 induces

$$M_{1/2} \simeq \frac{\alpha}{4\pi} \frac{\mu M}{\mu \tilde{m}} \sim \frac{\alpha}{4\pi} \mu,$$

(21)

which is also tiny.

Finally we estimate the contribution from $T, \bar{T}$ in Fig.1. The soft scalar mass Eq.(12) is suppressed by the $R$-symmetry ($Q_R(T\bar{T}) = 2$), which should be estimated to be maximally $O(\mu M)$. The $A$-term suppressed by the $R$-symmetry (Eq.(9)) and the VEV shift induced $\langle F_\Sigma \rangle = O(\mu M)$ also suggest $B_T \simeq \mu M$. Since the supersymmetric fermion mass is $M_T \simeq M$, the 1-loop diagram from Fig.1 induces the same negligible correction as Eq.(21).
Therefore, the 1-loop diagrams contributions from the GUT particles are negligibly small in this model.

The $R$-symmetry breaking from $\langle W \rangle$ is estimated as

$$\langle W \rangle \sim \mu M^2 \sim 10^2 \times (10^{16})^2 \text{ GeV}^3.$$ 

(22)

This is small enough for contributions through the gravitino loop diagram to be negligible.

Although the above model works very well to obtain split-SUSY mass spectrum, unfortunately, colored doublet components of the $\Sigma'$ superfields are light, so that the gauge coupling unification, which is one of the strongest motivations of the split-SUSY, is destroyed. They can get masses through the $R$-symmetry breaking term, $\mu tr \Sigma'^2$, in Eq.(18). This term should naturally be the weak scale. In this case, the VEV shift of $\langle \Sigma' \rangle = \mathcal{O}(\mu)$ might be happened in the direction which breaks color. Therefore, this mass term must be larger than the weak scale. Even if we assume it, the gauge coupling unification does not achieved, unless this mass term is around the GUT scale, which is quit unnatural. In general, there remains the flat direction in RGUT[29], which implies the existence of additional light fields. Thus, unfortunately, we should extend the matter content of the MSSM to reproduce the gauge coupling unification in 4D RGUT. It is nontrivial whether such model can be constructed within the framework of the RGUT models. Even if such a model can be successfully constructed, the original motivation of the split-SUSY scenario will be broken.

4 Summary

Recently, the split-SUSY scenario has been proposed, in which scalar soft masses are all super-heavy while fermion soft masses are around the weak scale. This mass spectrum is obtained by taking decoupling limit of the scalar particles (except the SM like Higgs) in the MSSM. It has been shown that, even with this split-SUSY mass spectrum, the gauge couplings are successfully unified, which implies the existence of the GUT behind the MSSM. Thus, the grand unification is one of main motivations of the split-SUSY scenario.

Although realization of the split-SUSY mass spectrum have been discussed in the context of the MSSM, it is necessary to examine it in the context of GUTs because the existence of the GUT is one of the most important assumptions in the split-SUSY scenario. We have shown that, if we consider the GUT sector, GUT fields play a role of the messengers in the gauge mediated SUSY breaking and induce sizable contributions to gaugino masses, so that the construction of the split-SUSY GUT is not straightforward. We have considered many possible contributions which destroy the split-SUSY mass spectrum and obtained the upper bound on soft scalar masses as $\mathcal{O}(10^{10})$ GeV. Furthermore, noting that dangerous contributions originated from the $R$-symmetry breaking in the GUT sector, we have discussed the attempt to construct $R$-symmetric GUT models in both four dimensions and extra dimensions. Although the $R$-symmetric $SU(5)$ GUT model works well to preserve the split-SUSY mass spectrum, additional light fields (which seems to be associated with $R$-symmetry) appear in the model and the success of the gauge coupling unifications is destroyed. Thus, we need to extended the matter sector to reproduce the gauge coupling unification. Such an extention of the MSSM sector destroys the original motivation of the split-SUSY scenario. The contributions from the GUT particles disturb the realization of
the approximate \( R \)-symmetry in the low energy, and the construction of the split-SUSY spectrum in the context of GUTs is much more nontrivial than that in the MSSM.

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