Derivation of drag calculation model of rocket sled water brake

J Xiao¹,², WW Zhang¹, Q Xue¹, W B Gao¹ and L R Zhang¹
¹Huayin Ordence Test Center, Huayin,714200, China

E-mail: 50310644@qq.com

Abstract. The calculation model of water resistance is derived by using fluid resistance theory. Using the water resistance calculation model, the braking distance calculation model is derived, and the braking distance of the two two-track sled vehicles is estimated. The pre-estimated water resistance, speed, and braking distance of the sled vehicle are in accordance with the actual external measurement results. The correctness of the water resistance calculation model is verified.

1. Introduction
Double-track sled has a wide range of application in scientific research and test. It is a kind of recoverable rocket sled test mode [1,2]. At present, the most widely used is water brake recovery. In the study of hydraulic brake resistance at home and abroad[3], the resistance caused by the change of water momentum is generally considered, while the impact resistance caused by the impact of hydraulic brake on water surface at different altitudes is neglected, resulting in a large deviation between the calculated results and the actual results. At present, there is a lack of accurate and simple drag calculation model of rocket sled water brake.

The drag calculation model of rocket sled water brake directly influences the calculation results of water brake resistance which directly affects the safety and success of test. Based on the fluid resistance theory[4,5], a drag calculation model of rocket sled water brake is derived. Using the water resistance calculation model, the braking distance calculation equation is derived, and the braking distance of two two-track sled vehicles is estimated. The pre-estimated water resistance, speed, and sled braking distance are consistent with the actual external measurement results. The correctness of the drag calculation model of rocket sled water brake is examined.

2. Drag calculation model
When the water brake (as shown in figure1) enters the flume, the water resistance mainly depends on the components immersed in the water. So water brake head-on collision resistance and differential pressure resistance of water brake are the major parts of the water brake resistance.

The vertical section of the immersed fluid of the water brake is perpendicular to the direction of the movement of the water brake. The contribution of the immersed fluid of the water brake to the resistance of the water brake is head-on collision resistance and differential pressure resistance by using the method of hydrodynamic resistance calculation.
2.1. Water brake head-on collision resistance

The water brake is regarded as a rigid body and the deformation of it under water pressure is not considered [6]. The water brake is not moving, and the water flows from the opposite direction to the water brake section at the current rate. The movement of the water brake is positive [7]. The water flows to the water brake section. The mass of water flowing to the water brake section in a small amount of time:

$$ dm = \rho_{\text{water}} S_{\text{water}} v dt $$

(1)

In the formula: $dm$ is the mass of water flowing into the water brake surface during the $dt$ time. It is actually the quality of water encountered by the brake section during the $dt$ time. After the water flow reaches the brake section, the speed is reduced from $-v$ to 0. From the momentum theorem, the water momentum flowing to the brake section decreases in a small amount of time to:

$$ dp = dm (-v - 0) = -vdm $$

(2)

Surface collision resistance applied by water to the water brake section $F_R$:

$$ F_R = \frac{dp}{dt} = -vdm / dt = -\rho_{\text{water}} S_{\text{water}} v^2 dt / dt = -\rho_{\text{water}} S_{\text{water}} v^2 $$

(3)

In the type, $s$ is the area where the water brake is immersed in water.

2.2. Differential pressure resistance of water brake

The water brake moves in the water and is also subjected to $F_p$, which is the differential pressure resistance along the water in the direction of movement [8, 9]. In front of the water brake section, because the water is stationary, the pressure of the water ($p_1$) is static pressure. Behind the water brake section, because the water brake moves forward, the water behind the water brake section flows with the water brake, and the flow rate is equal to the speed ($v$) of the water brake [10]. Therefore, the pressure behind the water brake is the flow pressure ($p_2$):

$$ p_2 = p_1 - \rho_{\text{water}} v^2 / 2 $$

(4)

So the pressure difference resistance on the front and rear surfaces of the water brake:

$$ F_p = p_1 S_{\text{water}} - p_2 S_{\text{water}} = -\rho_{\text{water}} S_{\text{water}} v^2 / 2 $$

(5)
From this we can see that the water resistance generated by the body of the water brake after entering the water:

\[ f_{\text{physical water resistance}} = F_R + F_p = -\rho_{\text{water}} s_{\text{water}} v^2 - \rho_{\text{water}} s_{\text{water}} v^2/2 = -3\rho_{\text{water}} s_{\text{water}} v^2/2 \]  

(6)

2.3. Hydro momentum resistance

The principle of water brake device is to change the direction of incoming flow speed after the water flow into the water brake device rising to a certain height [10,11]. The water momentum is converted to the water brake momentum, and the water brake resistance is generated through the momentum exchange, thus the sled deceleration braking is realized. When there is a hydrodynamic energy exchange in water brakes, water resistance also includes hydrodynamic energy exchange resistance:

\[ f_{\text{Water kinetic energy resistance}} = (1 - R_v \cos \theta) A_w \rho_w v^2 \]  

(7)

In this expression, \( \rho_w \) is the density of water (kg/m\(^3\)); \( R_v \) is the ratio of outlet velocity to inlet velocity for water brakes; \( A_w \) is the cross area (m\(^2\)) of flow inlet; \( \theta \) is the angle of entry and exit.

3. Brake distance calculation model

Let the mass of the sled be \( M \), and the initial speed of the sled is \( v \). Water Brake immersion in the tank cross-sectional area is \( s \). When the sled enters the water, the thrust of the rocket engine is zero. The running resistance of sled is air resistance \((C_D S \rho_{\text{air}} v^2 / 2)\), friction \((uMg)\) and hydraulic braking resistance \((3\rho_{\text{water}} s_{\text{water}} v^2/2)\). In a small period of time, sledding can be regarded as uniformly accelerated motion. Because the change of speed is very small, the water brake resistance related to speed can be regarded as the velocity correlation of the last moment. According to the motion equation of the sled, the sled enters the water at a small time \((t_1)\), then the speed is \( v_1 \):

\[ v_1 = v - (3\rho_{\text{water}} s_{\text{water}} v^2/2 + C_D S \rho_{\text{air}} v^2 / 2 + uMg) \times (t_1 - 0) / M \]  

(8)

In the form, \( u \) is friction coefficient between slipper and track. \( g \) is gravity acceleration. \( C_D \) is air resistance coefficient of sled.

When the sled speed is changing from \( v \) to \( v_1 \), and the sled moves at a uniform acceleration[11,12]. So the braking distance is:

\[ l_1 = (v_1 + v) \times (t_1 - 0) / 2 \]  

(9)

As the sled enters water, the speed of sled is \( v_1 \). After a small time, speed is \( v_2 \), then:

\[ v_2 = v_1 - (3\rho_{\text{water}} s_{\text{water}} v_1^2/2 + C_D S \rho_{\text{air}} v_1^2 / 2 + uMg) \times (t_2 - t_1) / M \]  

(10)

The speed of the sled is from \( v_1 \rightarrow v_2 \), braking distance is:

\[ l_2 = (v_2 + v_1) \times (t_2 - t_1) / 2 \]  

(11)

After many tiny moments, at \( t_n \), the speed of the sled is \( v_{n-1} \rightarrow v_n = 0 \), and the braking distance is \( l_n \):

\[ l_n = (v_{n-1} + v_n) \times (t_n - t_{n-1}) / 2 \]  

(12)

So when the sled brake ends, the braking distance is \( l \):
$$l = l_1 + l_2 + \ldots + l_n$$  \hspace{0.5cm} (13)

Through the calculation model of braking distance, we can see that if the calculation model of water resistance is inaccurate, it will be difficult to calculate the braking distance accurately. So we can judge whether the calculation of water resistance is accurate by verifying the accuracy of braking distance calculation.

4. Model applications

The test water resistance and calculation water resistance which is obtained by using the derived water resistance calculation model are opposite as shown as figure 2. The test water resistance is basically in agreement with water resistance by calculation model for two dynamic tests. The maximum error of drag is less than 10%.

![Figure 2. Comparison of water resistance. (a) Water resistance for the sled for test 1\textsuperscript{st} and (b) Water resistance for the sled for test 2\textsuperscript{nd}.](image)

During the two double-track sled vehicle tests, the speed after sled entering water by test is basically in agreement with speed by calculation model for two dynamic tests (figure 3). The whole speed error of two tests is less than 20m/s.

![Figure 3. Comparison of speed after entering water. (a) Speed for the sled for test 1\textsuperscript{st} and (b) Speed for test 2\textsuperscript{nd}.](image)

During the two double-track sled vehicle tests, the braking distance by test is basically in agreement with speed by calculation model for two dynamic tests (figure 4). The error of braking distance for two tests is less than 30m.
Based on the theory of fluid resistance, the calculation model of rocket sled water brake is derived, and the calculation equation of braking distance is derived. The water resistance, speed and braking distance of the sled were calculated in accordance with the results of external measurement. The pre-estimated speed change of sled after entering the water is basically consistent with the actual external measurement speed. For test 1st, it is estimated that the braking distance of the sled is 175m and the actual braking distance is 198m. For test 2nd, it is estimated that the braking distance of the sled is 280m, and the actual braking distance is 285m. The braking distance is not much different. The correctness of drag calculation model is verified.

5. Conclusion

Based on the theory of fluid resistance, drag calculation model of rocket sled water brake is derived, and the calculation equation of braking distance is derived. The water resistance, speed and braking distance of the sled were calculated in accordance with the results of external measurement. The specific conclusions are as follows:

- Using the fluid resistance theory, the calculation method of water resistance is derived accurately and simply.
- The calculation equation of braking distance derived by water resistance calculation method is accurate and reliable.

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