ABSTRACT

The binary pulsar J0737-3039 is the only known system having two observable pulsars, thus offering a unique laboratory to test general relativity and explore pulsar physics. Based on the low eccentricity and the position within the galactic plane, Piran & Shaviv (2004, 2005) argued that pulsar B had a non-standard formation scenario with little or no mass ejection. They have also predicted that the system would have a very slow proper motion. Pulsar timing measurements (Kramer et al. 2006; Deller et al. 2009) confirmed this prediction. The recent observations of the alignment between the spin of pulsar A and the binary orbit is also in agreement with this scenario. Detailed simulations of the formation process of pulsar B enable us to show that its progenitor, just before the collapse, was a massive O-Ne-Mg white dwarf surrounded by a tenuous, 0.1-0.16 $M_\odot$, envelope. This envelope was ejected when the white dwarf collapsed to form a neutron star. Pulsar B was born as a slow rotator (spin period $\sim 1$ s) and a kick received when the pulsar formed changed its spin direction to the current one. This realization sheds light on the angular momentum evolution of the progenitor star, a process which is strongly affected by interaction with the binary companion. The slow proper motion of the system also implies that the system must have undergone a phase of mass transfer in which Star A shed a significant fraction of its mass onto B.

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discovery, it was suggested that the progenitor of pulsar B must have been a $\sim (2.1 \div 2.3) M_\odot$ He star \citep{Dewi2004, Willems2004}. The large mass loss in the SN leading to the birth of the pulsar would have given a large kick, $\sim 150$ km/s, keeping the system in a low eccentricity orbit. However, \citep{Piran2004, Piran2005} used the small orbital eccentricity of the system and the small height from the galactic plane, to infer that pulsar B must have formed with a small mass ejection (less than 0.1 $M_\odot$) and no kick velocity. They have predicted that the system should have a very low proper motion, $v_{\text{cm,}i} < 15$ km/s. This was later confirmed through pulsar timing measurements \citep{Kramer2006, Deller2009}, showing that the system has a proper motion of $10 \pm 1$ km/s. This value is consistent with the random stellar motion in the galaxy and thus provides an upper limit on the actual kick velocity, $\Delta v_{\text{cm,}i}$, obtained during the collapse.

The confirmed low proper motion implies that pulsar B must have formed with little mass ejection, making it the first neutron star (NS) known to have been formed in the collapse of an unstable white dwarf (WD). This could not have been through accretion induced collapse, because its companion was already a NS and could not provide matter for accretion. A WD dwarf could become unstable either through cooling, if it was above the Chandrasekhar mass, or, as suggested four decades ago \citep{Finzi1967}, through $e$-capture changing the average number of electrons in an O-Ne-Mg WD.

Besides being the best laboratory for general relativity \citep{Kramer2006}, and offering the first evidence for a new channel for neutron star formation, more can be learned, from this system, on its earlier evolutionary phases. Remarkably, the slow proper motion of the system also constraints the mass ejection that took place when the first pulsar, Pulsar A, formed. When combined with the unique measurement of the spin direction of both pulsars it enables us us to clarify some fundamental issues in the evolutionary path of the progenitor system.

We use the following dynamic parameters of the system \citep{Kramer2006}: the pulsars’s masses $M_A = 1.34$ $M_\odot$ and $M_B = 1.25$ $M_\odot$ of the pulsars, the eccentricity of the orbit, $e = 0.0878$, its period $P = 0.102$ d ($\sim 2.45$ hr) and the C.M. proper motion $v_\perp = (10 \pm 1)$ km/s. The height of the system above the galactic plane is $h = 50$ pc \citep{Piran2004}. We also use the age of pulsar B, estimated to be $\leq 50$ Myr, the long spin period (2.7 sec) of Pulsar B \citep{Kramer2006} and the inclinations of the spins of pulsar A and B relative to the orbital angular momentum, $\sim 3.2^\circ$ \citep{Ferdman2013} and $\sim 130^\circ$ \citep{Lyutikov2005, Breton2008, Farr2011}, respectively. We find it remarkable how much information one can obtain on the evolution of the system just from

\footnote{The subscript $\perp$ indicates throughout components of the velocity orthogonal to our line of sight.}
We begin in §2 using the kinematics of the system to limit the mass of pulsar B’s progenitor and from this to obtain limits on its progenitor’s structure and on the collapse that formed the NS. In §3 we show that the slow proper motion also limits the mass of pulsar A’s progenitor just before the first SN took place, to be smaller than the mass of pulsar B’s progenitor at that time. As initially A must have been more massive this provides direct evidence for a significant mass loss of the progenitor of pulsar A. In §4 we turn to the late mass transfer between pulsar B’s progenitor and pulsar A, showing that this mass transfer is consistent with the spin and the mass of this pulsar. Finally, in §5 we examine the orientations of the spins of the two pulsars. We conclude and summarize our results in §5.

2. Pulsar B’s progenitor

2.1. Kinematic considerations: limits on the progenitor’s mass

We begin by obtaining the probability distribution function for the progenitor mass and kick velocity given the additional constraints we know today about the proper motion of the system and the alignment between pulsar A’s spin and the orbital angular momentum. Thus, we repeat our previous analysis (Piran & Shaviv 2005), obtaining more stringent limits.

In short, we generate many random realizations of the binary system, with different initial masses and different kick velocities. We assume for the kick velocity distribution a prior of equal probability in logarithmic space. We then evolve the system and check how many realizations with different initial parameters generate systems which are consistent with the present observations. Fig. 1 depicts the results with different constraints added, namely, the location within 50pc of the galactic plane, the low orbital eccentricity and the small observed proper motion. Finally, the constraint arising from the alignment of the spin of pulsar A with the orbital angular momentum vector is included: Pulsar A’s ms spin was already aligned with the orbit before the formation of pulsar B, having been spun up by accretion, and a significant kick would have destroyed this alignment.

We find that the most likely progenitor mass of pulsar B, $m_B$, was in the range $1.47 \pm 1.53$ M$_\odot$, corresponding to a confidence level of 68%. The associated kick velocity is $\Delta v_{\text{cm}} \leq 20$ km/s. Values of the progenitor mass up to $\lesssim 1.8$ M$_\odot$ lie within the 97% probability curve, with associated kick velocities $\lesssim 30$ km/s. Finally, a low probability tail extends up to masses $\lesssim 2.1$ M$_\odot$ and velocities $\lesssim 70$ km/s. The low-mass branch in Fig. 1 includes very low progenitor masses, $\sim (1.25 \div 1.5)$ M$_\odot$, and large kick velocities, $> (30 \div 100)$ km/s.
Although kinematically possible, this combination is highly unlikely as it requires that the
ejected mass had very large asymmetry and we disregard it as spurious.

These values can be understood as follows. If the explosion that accompanied the
collapse had been spherically symmetric, with negligible mass ejection, then the orbital
eccentricity of the system would be simply related to the ejected mass by (van den Heuvel
2010) \( e = M_{\text{ej}} / (M_A + M_B) \). Adopting the gravitational wave-driven evolution of orbital
eccentricity (Enoki & Nagashima 2007) and a lifetime equal to the spindown age of pulsar
B, the current eccentricity \( e \approx 0.088 \) of PSR J0737-3039 can be traced back to a maximum
\( e \lesssim 0.11 \) right after the formation of pulsar B. By virtue of the above relation, this implies a
maximal mass of \( \sim 1.53 \, M_\odot \) for the immediate progenitor of pulsar B while the minimum,
\( \sim 1.47 \, M_\odot \), is derived using the current orbital eccentricity.

### 2.2. Physical mechanism for pulsar B’s formation

The progenitor mass of pulsar B is less than \( 2.1 \, M_\odot \) at the 99% confidence level. This is
less than the minimal progenitor mass required to trigger a core-collapse SN (Smartt 2009;
Burrows 2013). It is even below the minimal mass of the He core \( \sim (2.1 - 2.4) \, M_\odot \) needed
to ignite O and Ne burning thus leading to a standard iron-core collapse (Nomoto 1987;
Dewi et al. 2002; Willems & Kalogera 2004). The only remaining plausible scenario is thus
that pulsar B originated from a marginally unstable WD-like degenerate core, which has
undergone a gravitational collapse. One possibility for such a collapse is an \( e \)-capture SN
(Miyaji et al. 1980; Nomoto 1984), or the collapse of a hot WD that was initially slightly
above the Chandrasekhar limit (Piran & Shaviv 2004, 2005). The hot degenerate core might
have become unstable because of cooling or, e.g., by accumulating ashes from shell burning
at the base of a tenuous envelope (like a stripped down post-AGB star) and then passing
the Chandrasekhar limit. We will frequently refer to an \( e \)-capture SN, because this specific
case was mostly discussed in the literature. However, the above possibilities stand on equal
footing, hence the gravitational collapse of an unstable WD-like degenerate core should
always be understood.

An \( e \)-capture SN (or, more generally, a marginally stable WD) is expected as the end
result of the evolution of stars in a relatively narrow range of masses, between 8 and 10 \( M_\odot \)
(Nomoto 1984). Evolutionary arguments suggest that \( e \)-capture SN can preferentially (or
only) occur in binary systems, and involve progenitors with masses up to \( \sim 12 \, M_\odot \), due to
the critical effect that the onset of mass transfer can have on the evolution of the stellar
core (Podsiadlowski et al. 2004; van den Heuvel 2010; Ibeling & Heger 2013). This issue is,
however, not yet settled and our arguments do not depend specifically on this assumption.
Independent of the exact channel through which the unstable WD was formed, its collapse released the binding energy of the nascent NS via neutrinos. This contributes to the mass difference between the progenitor star and the resulting NS, thus naturally explaining the relatively small mass (1.25 M⊙) of Pulsar B. The derived mass of the progenitor, (1.47-1.53) M⊙, well exceeds the maximum WD mass and a reasonable neutrino mass loss, thus implying that some mass ejection occurred in the formation of pulsar B, besides the release of the binding energy. Precisely how much mass was lost depends on the NS binding energy, which is determined by the - yet unknown - NS equation of state (EOS), but it is reasonable to guess that the envelope was ejected while the WD collapsed to form the NS.

2.3. The progenitor’s structure

Simple parametric fits to the NS bulk properties can approximate well the results of a large class of EOSs [Lattimer & Prakash 2001; Lattimer 2010]. Particularly simple is the relation between a NS binding energy, \( E_{\text{bind}} \), and its compactness, \( \beta = GM_B/(Rc^2) \), namely \( E_{\text{bind}} \simeq 0.6M_B\beta/(1 - 0.5\beta) \). Currently preferred EOSs indicate NS radii 10 Km \( \lesssim R \lesssim 12 \) km [Özel et al. 2012], corresponding to \( \beta = (0.125 - 0.185) \) and \( 0.12 < E_{\text{bind}}/M_\odot < 0.15 \). The resulting ejected mass \( M_{\text{ej}} \simeq (0.07 - 0.16) M_\odot \). Note that, considering the whole range of possible EOSs, radii between 10 and 15 Km would be allowed for a 1.25 M⊙ NS (see e.g. Fig. 4 in Lattimer 2010). This would give slightly wider ranges for the physical parameters, in particular, \( M_{\text{ej}} \simeq (0.07 - 0.18) M_\odot \). The limits on the ejected mass, on the NS radius and on its binding energy, that can be obtained by our kinematic results, are summarized in Fig. 2.

The Chandrasekhar mass for a zero-temperature O-Ne-Mg WD is [Hamada & Salpeter 1961] \( \approx 1.36M_\odot \). This critical mass is slightly modified by finite temperature effects [Timmes et al. 1996], leading to a typical value of \( M_c \gtrsim 1.37 M_\odot \) at \( T \lesssim 10^9 \) K. The temperature of the WD must have been between \( \sim \) a few \( \times 10^8 \) K, to allow the growth of the degenerate core through C-burning, and \( \lesssim 10^9 \) K, because further nuclear burnings - that would lead to a standard core-collapse - could not be ignited. We thus conclude that the immediate progenitor of pulsar B contained a hot, degenerate, O-Ne-Mg core with a mass \( \sim 1.37 M_\odot \) surrounded by a non-degenerate envelope of lighter elements, with \( M_{\text{env}} \sim (0.10 - 0.16) M_\odot \), in hydrostatic and thermal equilibrium. This envelope is eventually ejected in the explosion while, with our

\[ \text{If the age of Pulsar B is comparable to its current spindown age, then the mass of its progenitor will be close to the maximum allowed value, } m_B \approx 1.53 M_\odot, \text{ and the corresponding ejected mass range becomes } M_{\text{ej}} \simeq (0.13 - 0.18) M_\odot. \]
assumed mass, the collapsing WD would release a binding energy $E_{\text{bind}} = 0.12 \, M_\odot$, leading to a final NS mass of $1.25 \, M_\odot$. This corresponds to a NS radius of 12 km, perfectly in line with best current estimates (cfr. Fig. 2).

To explore the progenitor’s structure we use a simple toy model. We approximate the (smooth) transition region between the two zones as a sharp interface, located where the gas plus radiation pressure becomes comparable to (more than half of) the electron degeneracy pressure. This determines uniquely the temperature, density and pressure ($T_b$, $\rho_b$ and $P_b$) at the base of the envelope. The envelope’s structure is then obtained by solving jointly the equations of hydrostatic equilibrium and radiative transport, with opacity dominated by electron scattering. In this simple model the outer radius is derived in terms of the inner radius ($R_b$) and the pressure scale height at the base of the envelope, $\phi = 4P_b/(\rho_b g_b R_b)$, such that $R_{\text{out}} = R_b/(1 - \phi)$. The total envelope mass is also determined in terms of these quantities,

$$M_{\text{env}} = 4\pi \rho_b (R_b/\phi)^3 \left[-\ln(1 - \phi) - \phi - (\phi^2/2) - (\phi^3/3)\right]. \tag{1}$$

Fixing the total envelope mass, we can thus determine analytically the required values of $R_b$ and $\phi$, hence $R_{\text{out}}$, as a function of $T_b$. Fig. 3 depicts the inner and outer radii of the envelope as a function of $T_b$, for two different values of the envelope mass, corresponding to the minimal and maximal values of the progenitor mass derived from the kinematic constraints (1.47 and 1.53 $M_\odot$ respectively). At smaller temperatures $R_b$ seems implausibly large for a critical-mass WD; for $T \geq 5 \times 10^8$ K, the core radius is less than twice the radius of a zero-temperature WD ($\sim 1.4 \times 10^8$ cm), roughly consistent with the (non-negligible) effects on radius due to partial degeneracy at such high T. The corresponding values of $R_{\text{out}}$ are $\sim$ twice as large as the inner radius, with a typical size $\sim 0.01 \, R_\odot$.

Finally, the surface luminosity of the WD is determined as a function of $T_b$ as $L_{\text{WD}} = L_{\text{Edd}}(P_{b,\text{rad}}/P_b)$, where $L_{\text{Edd}} = 4\pi GMc/\kappa$ is the Eddington luminosity, $(P_{b,\text{rad}})$ the radiation pressure and $(P_b)$ the total pressure at the base of the envelope. The implied surface temperature and optical luminosity as functions of $T_b$ are shown, in the right panel of Fig. 3, for the case $M_{\text{env}} = 0.10 \, M_\odot$. The system was rather dim in the optical band, with a typical luminosity peak of $\sim 0.15 \, L_\odot$. Due to its high temperature it was, however, much brighter than a typical WD in the UV, where most of the emission occurred, with an estimated luminosity $\sim 10^{36}$ erg s$^{-1}$.

Our approximation doesn’t include important effects, such as neutrino cooling and heat conduction in the outer layers. Furthermore, we treat the transition region in a crude way. More accurate results would require numerical modeling of these effects. However, we don’t
expect these effects to change qualitatively our conclusions.

3. Kinematic considerations: the first SN and pulsar A progenitor’s mass

Remarkably, the slow proper motion of the system enables us also to put an interesting limit on $m_A$, the mass of the stellar progenitor of pulsar A just before it collapsed. Let $\tilde{m}_B$ be the mass of the star B when the first SN took place. Mass ejection in this first SN results in a recoil velocity of the center-of-mass (Piran & Shaviv 2004):

$$\Delta v_{\text{cm}} = \frac{M_{\text{tot},i} - M_{\text{tot},f}}{M_{\text{tot},i}} \frac{\tilde{m}_B}{M_{\text{tot},i}} v_{\text{rel},i},$$

(2)

where $M_{\text{tot},i} \equiv \tilde{m}_B + m_A$ and $M_{\text{tot},f} \equiv \tilde{m}_B + M_A$, represent the total mass in the binary before and after the first SN, respectively. $v_{\text{rel},i} = GM_{\text{tot},i}/a_i$ is the relative (orbital) velocity prior to the SN (assuming a circular orbit) and $a_i$ is the corresponding orbital separation.

The observational constraint $|\Delta v_{\text{cm}} \sin \theta_{\text{los}}| \leq v_{\text{obs},\perp}$, where $\theta_{\text{los}}$ is the angle between the direction of $v_{\text{rel},i}$ and our line-of-sight, limit the amount of mass that could have been ejected in the first SN. This in turn puts an upper limit on the total mass of the binary before the first SN went off, $M_{\text{tot},i}^{(\text{max})}$, which is a function of the parameters $(\tilde{m}_B, \sin \theta_{\text{los}}; a_i, v_{\text{obs},\perp})$. This upper limit also constrains the mass ratio in the binary at the moment of the first SN. To see this let’s impose the condition $m_A^{(\text{max})} > \tilde{m}_B$, which of course implies $M_{\text{tot},i}^{(\text{max})} > 2 \tilde{m}_B$. We obtain:

$$\sin^2 \theta_{\text{los}} < 0.224 \left( \frac{a_i/\text{AU}}{\tilde{m}_B/M_\odot} \right) \left( \frac{v_{\text{obs},\perp}}{10 \text{ km/s}} \right)^2 \left[ \frac{\tilde{m}_B + M_A}{\tilde{m}_B - M_A} \right]^2,$$

(3)

which holds only if the center-of-mass of the system obtained a recoil velocity that was directed very close to our line of sight. When the angle $\theta_{\text{los}}$ is drawn from a random distribution, the probability that $\theta_{\text{los}} \leq \theta_{\text{max}}$ is simply $P(\theta_{\text{los}} \leq \theta_{\text{max}}) = 1 - \cos \theta_{\text{max}}$. Combining this with eq. 3 the conditional probability $P(M_{\text{tot},i} > 2 \tilde{m}_B \mid \tilde{m}_B)$ can be derived, for fixed values of the parameters $(v_{\text{obs},\perp}, a_i, M_A)$. Finally, assuming a flat prior for the values of $\tilde{m}_B$ between 8 and 12 solar masses, the conditional probability can be marginalized over $\tilde{m}_B$ to derive that the total probability $P(M_{\text{tot},i}^{(\text{max})} > 2 \tilde{m}_B) \approx 0.02$. The condition $m_A > \tilde{m}_B$ that requires that the center-of-mass recoil velocity was pointed less than $\approx 12^\circ$ away from our line of sight (\(\approx 2\%\) probability) is highly fine-tuned. We conclude that when the progenitor star of pulsar A exploded, it must have been less massive than its companion.

This conclusion implies that a phase of significant mass transfer must have occurred prior to the first SN. This phase reversed the original mass ratio, at the same time bringing

\[\text{This is different from, } i.e. \text{ larger than } m_B, \text{ the mass of star B right before it collapsed.}\]
the progenitor of pulsar B to the right mass range for producing the second NS. The low proper motion of PSR J0737 thus provides evidence of this early binary evolutionary phase, based on the dynamical properties of its late descendants.

4. Mass transfer in later evolutionary phases

Based on the measurement of a very low proper motion of PSR J0737-3039, on its low galactic latitude and the near alignment of pulsar A’s spin with the normal to the orbital plane, we concluded that pulsar B was formed in the collapse of an unstable WD-like degenerate core, thus providing evidence for a new formation channel for NSs. We also argued that the slow proper motion of the system represents by itself a (first) dynamical evidence for a phase of mass transfer between the stellar progenitors, prior to the formation of the first NS in the system. The dynamic properties of the double pulsar also give interesting indications about later stages in the evolution of the progenitor system.

4.1. Common Envelope and Roche Lobe Overflow

With a total mass of \( \sim 2.6 \, M_\odot \) and a period of \( \sim 2.45 \) hrs, the current orbital separation of the two NSs, \( a \lesssim 9 \times 10^{10} \) cm, is only slightly larger than the solar radius. As such it could not accommodate the progenitor stars of the two pulsars, that must have been both much larger than our sun. Significant orbital shrinkage must have occurred before the formation of pulsar B, implying that the binary went through a phase of common-envelope (CE) after the formation of pulsar A. The CE phase stripped the envelope of the \( \sim (8 - 12) \, M_\odot \) star and eventually left pulsar A orbiting around an \( \sim (3.5 - 6) \, M_\odot \) He star in a close binary. At the end of core He burning, the He star left the main sequence and a further stage of Roche-lobe overflow (RLOF) occurred in this system (cf. Dewi & Pols 2003; Dewi & van den Heuvel 2004). Depending on the mass of the He star, this latter stage is expected to last \( \sim 10^3 \div 10^5 \) yrs, lower mass stars corresponding to a longer (nuclear) evolutionary timescale (Dewi et al. 2002; Dewi & van den Heuvel 2004). When mass transfer ends most of the He envelope is lost, leaving behind the degenerate O-Ne-Mg core surrounded by a tenous layer of material.

4.2. Acceleration of the millisecond pulsar

The fast spin (23 ms) of pulsar A and its alignment with the normal to the orbital plane are a natural consequence of the final phase of RLOF accretion in the above scenario.
Mass flowing through the inner Lagrangian point will settle into a disk and feed angular momentum to the NS at the rate \( \dot{M} (GM_A R_a)^{1/2} \). Here \( \dot{M} \) is the mass accretion rate, \( M_A \) is the mass of Pulsar A and \( R_a \) the magnetospheric (or Alfvén) radius, where the Keplerian flow breaks down and matter co-rotates with the magnetosphere. The latter can be written as (Patruno et al. 2012) \( R_a = 26.8 \text{ km} \xi (B_9 R_6^3 (\dot{M}/M_{\text{E,He}})^{-2/7} (M_A/M_\odot)^{-1/7} \), where \( \xi \sim (0.3 \div 1) \) parametrizes the relative thickness of the transition layer between Keplerian flow and corotation (Psaltis & Chakrabarty 1999), \( \dot{M}_{\text{E,He}} \approx 3 \times 10^{-8} M_\odot \text{ yr}^{-1} \) is the Eddington limit for He and \( Q_x \equiv (Q/10^9) \) [c.g.s. units] for any quantity \( Q \). The current angular momentum of pulsar A is \( \approx 2 \times 10^{47} \text{ g cm}^2 \text{ s}^{-1} (M_A/M_\odot) R_A^6 \), where the NS moment of inertia \( I = k_A^2 M_A R^2 \) and we took \( k_A^2 = 1/3 \). This implies that in order to spin up the NS to its current 23 ms period, the accretion phase must have lasted \( \tau_{\text{acc}} \approx (1.2 \div 2.1) \times 10^5 \text{ yrs} \), during which \( \sim (3.6 \div 6.3) \times 10^{-3} M_\odot \) were accreted. This small amount of mass is consistent with the rather low mass of pulsar A.

The value of \( \tau_{\text{acc}} \) derived here is somewhat longer than previous estimates for He stars that are in the \((4.5 \div 6) M_\odot \) range (Dewi et al. 2002; Dewi & van den Heuvel 2004). Those estimates, however, assumed that pulsar B was formed in a core collapse SN, hence the degenerate core of the He star had to exceed \( \sim 2.1 M_\odot \). Our result is qualitatively more in line with the progenitor He star having been less massive, tentatively \( \sim (3 \div 4) M_\odot \), thus developing a lighter core that could not trigger a core-collapse SN.

The weak magnetic field of pulsar A \( (\sim 7 \times 10^9 \text{ G}) \) also fits the picture of accretion-induced decay of NS exterior fields. Empirically, the field evolution is found to scale with the accreted mass as \( B(t) = B_0/(1 + \Delta M/k_m) \), with the observationally determined constant \( k_m \approx 1.25 \times 10^{-5} M_\odot \) (Dewi & van den Heuvel 2004; Francischelli et al. 2002). In this picture, pulsar A’s field would have decayed by a factor \( \sim (300 \div 500) \), making its original value consistent with those typical of NSs.

5. The spin of pulsar B

Another remarkable property of PSR J0737-3039 is that, while pulsar A’s spin is aligned with the orbit, pulsar B’s spin appears to be misaligned at 130°. We turn now to identify the mechanism that produced this misalignment by considering the possible evolutionary steps of Pulsar B’s progenitor up to the final collapse. A critical process that controls the spin of the NS is the earlier coupling between the stellar core and the envelope. This coupling determines the spin of the WD just before its collapse and hence it influences the NS spin. We thus consider this issue before addressing the question of the origin of pulsar B’s misaligned spin.
As discussed in § 4.1, Pulsar B’s progenitor experienced at least two phases of intense mass loss during which a large amount of angular momentum was removed from its envelope. If the core were effectively coupled it would have also been affected by the external torque, loosing its angular momentum. The tidal field of Pulsar A would also tend to synchronize (and align) the rotation of the envelope of star B to the orbit, particularly during RLOF \cite{Zahn1977}. If the (radiative) core were effectively coupled to the envelope, then a WD with a spin period of the order of the orbital period would result. A decoupled core, on the other hand, would retain its original angular momentum and the final orientation (and magnitude) of the WD spin would only depend on initial conditions.

The low degree of differential rotation found in the core of the sun and of some massive stars \cite{Spruit1998} and references therein, provides observational indications for a rather generic coupling between core and envelope \cite{Maeder2005, Charbonnel2003}. The slow measured spins of isolated WDs give independent observational support for the occurrence of strong losses of angular momentum in the cores of their stellar progenitors \cite{Kawaler2004, Kawaler2005, Langer2007, Suijs2008, Charpinet2000}. These findings have in turn stimulated theoretical investigations on the possible mechanisms that enforce efficient coupling between core and envelope in massive stars \cite{Zahn1992, Talon2003}. One specific mechanism that received particular attention invokes the effect of internal magnetic stresses \cite{Spruit1998}, as these are generally amplified by significant differential rotation and efficiently oppose its growth. Convective motions are expected to develop in the transition region between the (radiative) core and the envelope of the He star \cite{Dewi2002, Ivanova2003}, and these might also play a significant role in the synchronization of the core \cite{Zahn1997}. Overall it seems that there is observational evidence and theoretical reasoning to expect that the core and the envelope are coupled. Still as this is not certain \cite{Corsico2011, Beck2012}, we explore in the following both possibilities.

Returning to the spin misalignment of pulsar B we note that it can be explained in one of two possible ways: 1) Pulsar B progenitor’s spin was misaligned relative to the orbital plane before the collapse, and the spin didn’t change when the NS was formed; 2) Pulsar B progenitor’s spin was aligned with the orbital plane, and the spin direction was changed during the NS formation process. We turn now to consider each one of these scenarios.

**Scenario 1:** In this scenario pulsar B progenitor’s spin was misaligned relative to the orbital plane before the collapse and the current spin direction of pulsar B was inherited from its progenitor star. The final collapse of the unstable WD did not affect it. It is reasonable to assume that, in the original binary star system, the two spins were aligned with the orbital angular momentum, \( L_{\text{orb}} \). Depending on the coupling between the core and the envelope,
there are two moments in the subsequent evolution of the system when the spin vector of B could have been tilted and we consider the two cases separately.

**Strong coupling between the stellar core and the envelope**: The coupling implies that the spin of the core was aligned with the stellar spin, which in turn, was aligned with the angular momentum of the orbit. In this case the tilt could have resulted during the ejection of the envelope accompanying the formation of the WD. It was suggested that this process could be responsible for the observed distribution of natal spin periods and kick velocities of isolated WDs, respectively $\sim (0.15 - \text{a few})$ days and $\sim (1 - \text{a few})$ km/s (Spruit 1998; Davis et al. 2006; Heyl 2007).

In isolated WDs the envelope is ejected in a $\sim 10^4 \text{yr}$-long super-wind phase that sets in when the progenitor star reaches the AGB phase (Spruit 1998). However, the onset of mass transfer onto Pulsar A might have avoided precisely this phase in the progenitor of Pulsar B, changing the fate of its degenerate core (Podsiadlowski et al. 2004; van den Heuvel 2010). In fact, the unstable WD was formed in the core of the He star while its envelope was ejected via RLOF, and the ensuing mass transfer phase. It is this different physical situation that we address now.

The ejected mass flows through the inner Lagrangian point towards the accreting neutron star, mostly in the radial direction. While a small degree of anisotropy of the flow would possibly result in a series of random kicks, like in the case of AGB super-winds (Spruit 1998), two major differences with the latter case must be stressed. On the one hand, RLOF is expected to be much smoother than the highly variable and inhomogeneous winds of AGB stars, thus leading to smaller individual kicks (if any). On the other hand, the Roche radius limits the size of the mass donor to be $\lesssim R_\odot$, much less than the radius of a typical AGB star ($> 10^2 R_\odot$). This strongly reduces the “lever arm” of the kicks, hence their effectiveness. Adopting the same scaling law as for AGB winds, the total angular momentum deposited onto the WD at the end of the mass transfer by fall back of the residual envelope material is given by (Spruit 1998):

$$L_F \sim \frac{\epsilon}{\sqrt{N}} R_\star c_s M_{\text{env},0}^{1/2} M_{\text{env},F}^{1/2} \sqrt{3 \frac{k_s^2}{2}}$$

$$\approx 3 \times 10^{43} \frac{\text{g cm}^2}{\text{s}} \epsilon_{-2} \left[ \left( \frac{M_{\text{env},0}}{2 M_\odot} \right) \left( \frac{M_{\text{env},F}}{0.1 M_\odot} \right) \left( \frac{R_{RL}}{R_\odot} \right)^3 \left( \frac{10^5 \text{yrs}}{\tau_{\text{acc}}} \right) \right]^{1/2} T_{s,4}^{1/4} \cdot \quad (4)$$

In the above $M_{\text{env},0}$ is the envelope mass at the beginning of mass transfer, $M_{\text{env},F}$ its residual mass at the end of mass transfer - normalized to the envelope mass estimated in sec. 2.3 - and $T_{\text{surf}}$ is the surface temperature of the expanded star. The degree of anisotropy of the flow, $\epsilon_{-2}$, was normalized to $10^{-2}$ and the velocity of the ejected material was assumed to
be the local sound speed at the stellar surface. This estimate shows that this mechanism fails to provide to the WD enough *misaligned* angular momentum to account for pulsar B’s current spin. As such, this explanation should also be ruled out.

**No coupling between the stellar core and the envelope:** If the core of pulsar B’s progenitor was never coupled to the envelope it would have maintained its original spin direction. If the orbital plane changed abruptly by the kick imparted when the first SN exploded, then after the collapse, the spin of pulsar B would eventually reveal the original orientation of the orbital plane. Accretion onto pulsar A would, on the other hand, lead to the alignment of its spin with the new orbital plane.

The angular momentum that is transferred by the natal kick to the orbit is $\Delta L_{\text{kick}} = M_A v_{\text{kick}} \times a_1$, where $a_1 = a \tilde{m}_B/M_{\text{tot,f}}$ is the distance of the newly formed NS (Pulsar A) from the center of mass. To affect the orientation of the orbital plane, this should exceed the original angular momentum, $L_{\text{orb}} = (m_A \tilde{m}_B/M_{\text{tot,i}}) a_1 \times v_{\text{orb,i}}$. The condition $\Delta L_{\text{kick}} > L_{\text{orb}}$ ultimately requires that

$$a_1 > \left( \frac{m_A}{M_A} \right)^2 \left( \frac{M_{\text{tot,f}}}{M_{\text{tot,i}}} \right) \frac{G M_{\text{tot,f}}}{v_{\text{kick}}^2} \approx 4 \, \text{AU} \left( \frac{m_A}{6 \, M_\odot} \right)^2 \left( \frac{M_{\text{tot,f}}}{9.35 \, M_\odot} \right) \left( \frac{v_{\text{kick}}}{200 \, \text{km/s}} \right)^{-2} \left( \frac{M_{\text{tot,f}}}{M_{\text{tot,i}}} \right).$$

This is $\gtrsim 800 \, R_\odot$ and it exceeds the maximal radius to which an $M < 12 \, M_\odot$ star is expected to expand on the giant branch. Since the system went through a CE phase, pulsar A must have been within the companion’s envelope at some stage of the evolution. Hence, constraint (5) argues against the viability of this scenario.

This conclusion is further strengthened if we account for the actual angle between the spin of pulsar B and the normal to the orbital plane, $\pi/2 + \beta \approx 130^\circ$. In this scenario this corresponds to the angle by which $L_o$ was tilted by the kick. In the sum $L_F = L_o + \Delta L_{\text{kick}}$, the vector $\Delta L_{\text{kick}}$ must be tilted away from $L_o$ by an angle $\gamma > \beta$, which implies that only $\lesssim 15\%$ of the possible random orientations of $\Delta L_{\text{kick}}$ are allowed. After some manipulation, we derive the following condition on the modulus of $\Delta L_{\text{kick}}$

$$| \Delta L_{\text{kick}} | = | L_o | \left( \frac{1}{1 - \sin 2\gamma} \right)^{1/2} = | L_o | \cdot \hat{g}(\gamma)$$

where $\beta \approx 40^\circ$ was used and $40^\circ < \gamma < 90^\circ$. In eq. (5), the rhs should thus be multiplied by the coefficient $[\hat{g}(\gamma)]^2 > 1$. In particular, $\hat{g}(\gamma)^2 > 2$ for $40^\circ < \gamma \lesssim 80^\circ$, making the required orbital separation impossibly large. This range of $\gamma$ leaves out only $\approx 5\%$ of the orientations allowed by $\gamma > \beta$, and less than $\approx 1\%$ of all the possible random orientations of $\Delta L_{\text{kick}}$. We conclude that this scenario is ruled out: a natal kick to pulsar A would not have been able to tilt the orbital plane by the required amount.
We conclude that no known mechanism could have tilted the spin of the progenitor star before the formation of the WD, whether its core was coupled or not to the envelope. Therefore the current spin direction of pulsar B must have been set during the collapse. This is scenario 2).

**Scenario 2: The spin direction was changed during the NS formation process.** In order for the spin direction to change significantly, the collapse must have imparted to the nascent NS angular momentum in excess of that available to the progenitor. The current spin of pulsar B provides a lower limit of \( \simeq 2 \times 10^{45} \, \text{g cm}^2 \, \text{s}^{-1} \) to the required angular momentum. However, the low proper motion of the binary also puts a robust upper limit on the angular momentum that could have been imparted to the NS.

An arbitrarily oriented, off-center kick could impart spin angular momentum to the nascent NS. Given that a linear momentum \( M_{\text{tot}} \Delta v_{\text{cm}} \) would also be transferred to the binary, with \( M_{\text{tot}} = M_A + M_B \), the maximum spin that can be given to pulsar B in this way is \( \Delta L \approx R_{\text{kick}} \Delta v_{\text{cm}} M_{\text{tot}} \), where \( R_{\text{kick}} \) is the distance from pulsar B’s rotation axis at which the kick is imposed. Adopting the current upper limit on \( \Delta v_{\text{cm}} \) we obtain:

\[
\Delta L \approx 5.2 \times 10^{45} \, \text{g cm/s} \left( \frac{R_{\text{kick}}}{R_B} \right) \left( \frac{\Delta v_{\text{cm}}}{10 \, \text{km/s}} \right) \left( \frac{R_B}{10 \, \text{km}} \right) \left( \frac{M_{\text{tot}}}{2.6 \, M_\odot} \right). \tag{7}
\]

This modest amount of angular momentum is sufficient to account for pulsar B’s tilted spin if the progenitor WD was slowly rotating, and if \( R_{\text{kick}} \gtrsim (1/2) R_B (\Delta v_{\text{cm}}/10 \, \text{km/s})^{-1} \). This shows that the putative kick would have to be significantly off-centered to match the current rotation of Pulsar B. Actually, the maximal angular momentum allowed by eq. (7) exceeds the current angular momentum of Pulsar B only by a factor of a few\(^4\).

Assuming that the maximum angular momentum given by the kick is roughly twice the value obtained in eq. (7), we can derive an upper limit to the allowed spin rate of the WD progenitor. Upon writing \( L_{\text{WD}} = I_{\text{WD}} \Omega_{\text{WD}} \) we obtain

\[
\Omega_{\text{WD}} < 4.5 \times 10^{-4} \, \text{rad/s} \left( \frac{\Delta v_{\text{cm}}}{10 \, \text{km/s}} \right) \left( \frac{R_B}{10 \, \text{km}} \right) \left( \frac{M_{\text{WD}}}{M_{\odot}} \right)^{-1} \left( \frac{R_{\text{WD}}}{2 \times 10^8 \, \text{cm}} \right)^{-2} \left( \frac{M_{\text{tot}}}{2.6 M_\odot} \right) \tag{8}
\]

which corresponds to a minimum spin period of \( \sim 3.1 \, \text{hrs} \sim 0.13 \, \text{d} \). The fact that this limit is reminiscent of the orbital period of the system prior to the second SN, suggests that tidal synchronization and alignment of star B’s spin might have played an essential role in setting

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\(^4\)Under the reasonable assumption that \( R_{\text{kick}} \) does not exceed \( R_{\text{NS}} \) by a large factor.

\(^5\)See below.
the final magnitude and orientation of the angular momentum of the unstable WD.

An off-center kick could be part of the collapse itself if the outgoing neutrino flux in the early convective phase of the proto-NS had some small, yet finite, degree of anisotropy (Keil et al. 1996; Spruit & Phinney 1998; Burrows 2013). The kick due to neutrino anisotropies would naturally be applied at the neutrinosphere, at a radius $R_\nu = f_\nu R_{NS}$, with $f_\nu \sim$ a few. The resulting angular momentum would be enough to account for pulsar B’s current spin if $\Delta v_{\text{cm}} \approx 2 \left( f_\nu / 2 \right)^{-1}$ km/s (eq. 7). It is interesting to note that these numbers well match the kick estimates based on the current picture of neutrino-driven proto-NS convection (Janka & Mueller 1994; Spruit & Phinney 1998; Janka 2012; Burrows 2013), in which the neutrino anisotropy is only $\sim$ a few % on the scale of convective cells. This further supports the overall consistency of this scenario.

We conclude that the tilted spin of Pulsar B can be well explained by an off-centered kick imparted to the proto-NS during the collapse, most likely due to a small degree of anisotropy in the outgoing neutrino flux. Independent of the actual cause of the kick, this scenario has two major implications: $i$) the NS was born with a relatively slow rotation, with an initial spin period $\gtrsim 0.5$ s corresponding to $f_\nu = 2$ in eq. (7). The NS received a mild kick which changed its original spin direction and imparted a slow proper motion to the binary; $ii$) The progenitor WD retained only a small amount of its original angular momentum, such that the mild kick could affect the spin of the nascent NS. This supports the occurrence of a phase of effective core-envelope coupling in the progenitor (He) star.

Conclusions

We have considered the peculiar dynamic properties of the binary pulsar PSR J0737-3039. These dynamic properties pose strong constraints on the evolutionary path that has led to the formation of this system and lead to a well determined evolution. Unlike the “first” double pulsar PSR 1913+16, whose observed masses and orbital parameters can be explained as arising from standard core collapses of two massive stars (Burrows & Woosley 1986), the dynamical parameters here rule out this “standard scenario” and pose very stringent constraints leading to a unique evolutionary path. A summary of the main evolutionary steps followed by this binary, since when it hosted massive stars up to when Pulsar B was

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The limit given by Eq. 8 also well matches the fast-rotation end in the distribution of spin periods of isolated WDs (Heyl 2007). This distribution is thought to be caused by a combination of strong mass losses and efficient core-envelope coupling in AGB stars (Spruit 1998). This condition might well have occurred in the evolution of Pulsar B’s progenitor despite the very different environment.
formed, is depicted in Fig. 4. Also indicated in this figure are the corresponding changes in the orientation of the stellar spins (and of the orbital plane). We now summarize the main findings:

The measurement of the proper motion of the system, $\Delta v_{\text{cm},\perp} = 10 \pm 1 \text{ km/s}$, confirms with a higher significance that pulsar B must have formed in the collapse of a WD-like degenerate core with a tenuous envelope, with the ejection of $\sim (0.10 - 0.16) M_\odot$. The newborn NS received a very small kick (Piran & Shaviv 2005). The (near) alignment of pulsar A’s spin with the orbital plane is consistent with this expectation.

The slow proper motion of the system can also constrain the mass of the stellar progenitor of pulsar A. A significant ejection of mass during the first SN would have given a large C.M. velocity to the system. We find that, at the time of its collapse, the progenitor of pulsar A must have been the least massive star in the binary. Since it must have been the more massive one initially, we can deduce, from the dynamics of the double NS system, the occurrence of an early phase of mass transfer in the progenitor stellar binary.

The misaligned spin of pulsar B could only be produced during the collapse of the WD and the formation of pulsar B. The low kick velocity received by pulsar B strongly limits the angular momentum that could be given to the nascent NS. This is found to be sufficient to misalign its spin axis, provided that Pulsar B inherited little angular momentum from the progenitor WD, and that the kick was applied at the NS surface or slightly beyond it, e.g. at the neutrinosphere.

The tilted spin of pulsar B can be viewed as an interesting opportunity to test the physical properties of the hot proto-NS. Having ruled out all other alternatives, the tilt is found to be consistent with the occurrence of a short-lived convective phase. During this phase the huge neutrino flux is released with a small ($\sim 1\%$) randomly oriented anisotropy.

The required slow rotation of the WD progenitor corresponds to a minimum spin period of $\sim 3$ hrs. If the core of the He star, where the WD was eventually formed, was coupled to the envelope during the $\sim 10^5$ yrs of RLOF prior to collapse, then an even slower rotation could be expected. If, on the other hand, the coupling was not effective the core would retain all the aligned angular momentum of the stellar progenitor, eventually forming a fast rotating WD with an aligned spin. In this case, an off-center kick applied to the proto-NS would not have been able to affect the spin direction. From this perspective, the tilted spin of pulsar B could be viewed as an indirect indication of an efficient core-envelope coupling in the progenitor star.

A special role in setting the WD spin could be played by the tidal influence of the NS companion, during the $\sim 10^5$ yr long RLOF phase. If sufficiently extended convective
regions developed in the He-star during that stage, or if strong magnetic stresses developed across the core-envelope boundary, then efficient synchronization of the stellar core might have occurred. In this case the spin rate of the hot WD would have been set to the $\gtrsim 3 \text{ hr}$ orbital period, meeting the minimal requirement of eq. 8. This very intriguing possibility can be confirmed by detailed evolutionary calculations able to follow the He star structure during different nuclear burning stages.

To conclude we have found that the progenitor of Pulsar B within the binary system PSR J0737-3039 was a hot degenerate core with a tenuous envelope of $0.1 - 0.16 M_\odot$. The envelope was ejected during the collapse and the small kick imparted, most likely by the asymmetry in the escaping neutrinos, gave the pulsar its unique spin axis. Moreover, before the collapse the degenerate core was slowly rotating. The overall slow proper motion of the system also implies that the progenitor of Pulsar A was the less massive star in the system, just before it collapsed. This demonstrates the existence of an early phase of mass transfer within the system.

These conclusions have several interesting implications. First, this is a unique new channel of neutron star formation that was not previously described in the literature. Its existence should be considered in any population synthesis calculations that attempt to estimate the rate of neutron star binary mergers. One should also explore evolutionary tracks that would lead the progenitor star to this stage. Given the small mass of the envelope the collapse was not accompanied by any kind of known supernovae. It might have produced a very faint transient. Given the great interest in transients nowadays it is worthwhile to try estimating the observed signal and search for such events. Finally the rotational history of the progenitor provides evidence for a sufficient coupling between the core and the envelope at an earlier stage, possibly resolving the controversy concerning core-envelope coupling.

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Fig. 1.— The probability that a binary system will end up as observed, given a constant prior in progenitor mass and in \( \log(v_{\text{kick}}) \). This probability is obtained by imposing that (i) the binary should be within 50pc from the galactic plane; (ii) with an eccentricity between 0.088 and 0.14, given that 50 Myrs before, the progenitor system had a circular orbit, (iii) star B had a progenitor mass \( m_B \), and it obtained a random kick velocity of size \( v_{\text{kick}} \); (iv) the observed proper motion today should be \( \leq 10 \pm 1 \) km/s; (v) the pulsar A / orbit alignment of 3.7°. The standard core-collapse solution of a He star requires \( m_B \) larger than about 2.1\( M_\odot \). With a fine tuned kick velocity, this type of a solution is ruled out at the 99% c.l. Solutions with \( m_B \) between 1.47 and 1.53 \( M_\odot \) (and kick velocities smaller than \( \sim 20 \) km/s) are kinematically more favorable. Lower mass solutions with high kick velocities are kinematically plausible, but require unrealistically large kicks given the small amount of mass ejected.
Fig. 2.— Schematic representation of the structure of Pulsar B’s progenitor (right-hand side of the figure) and the NS structure (left-hand side). The gravitational mass ($M_B$) of Pulsar B is indicated by the horizontal line in the left-hand side of the figure. The shaded curve above it represents the sum of gravitational mass+binding energy ($M_B + E_{\text{bind}}$) of Pulsar B, where the binding energy is given in terms of the NS compactness (thus, its radius) by the expression provided in the text. The dashed vertical lines identify the minimum and maximum radius of a 1.25 $M_\odot$ NS consistent with currently proposed NS EOSs (cfr. Lattimer & Prakash 2001). On the right-hand side of the figure the horizontal line at 1.37 $M_\odot$ indicates our “chosen” value for the unstable, collapsing WD. The kinematically constrained progenitor mass range is indicated by the shaded strip above, and the (0.1-0.16) $M_\odot$ gap indicates the possible envelope mass range at the moment of the collapse. Assuming that Pulsar B was formed in the collapse of a hot WD-like core with a mass of 1.37 $M_\odot$, the corresponding binding energy is $\approx 0.12$ $M_\odot$. This is indicated by the continuous lines on the left-hand side of the figure, and corresponds to a NS radius of $\approx 12$ km.
Fig. 3.— *Left Panel:* The core radius ($R_b$) and the envelope outer radius ($R_{\text{out}}$) as a function of the temperature at the core/envelope border ($T_b$) for a hot, degenerate core surrounded by a non-degenerate envelope. Two curves are shown for both radii, corresponding to the minimum and maximum value of the envelope mass, as derived in the text. *Right Panel:* The temperature at the surface of the envelope ($T_{\text{out}}$) and the corresponding luminosity in the optical band (4000-7000) Å, in units of $0.1 \, \text{L}_\odot$, for the case with $M_{\text{env}} = 0.1 \, \text{M}_\odot$. The temperature scale is reported on the left $y$-axis, while the luminosity scale is reported on the right $y$-axis.
Fig. 4.— The evolutionary path of the binary: 
a) Main sequence phase: Both stars are more massive than 8 solar masses. A is more massive than B. b) Star A becomes a red giant and transfers mass to B via Roche lobe overflow. Most of the mass of A is lost at this stage. Towards the end of this phase star A contains no more than 4-5 solar masses and possibly significantly less. c) Star A collapses ejecting at most three solar masses. During the collapse the system receives only a mild kick, because of the large mass of star B. This kick may change the spin of star A and, to a limited extent, the orbital spin of the system. d) The neutron star A and the star B form a common envelope phase during which the orbit shrinks significantly and B looses a significant amount of mass. e) Star B continues to shed mass. The accretion spins up the neutron star A and reduces its magnetic field. A becomes a millisecond pulsar and its spin is aligned with the orbital spin. f) Having lost a significant fraction of its mass, B becomes a slowly rotating white dwarf with a tenuous envelope. Both spins are parallel to the orbit’s angular momentum. g) The white dwarf B collapses to a neutron star emitting neutrinos and ejecting the envelope. The ejected neutrinos give a kick to B changing its spin but causing only an insignificant change to the orbit or to the center of mass motion.