Real-Time Deployment of a Large-Scale Multi-Quadcopter System (MQS)

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Abstract—This paper presents a continuum mechanics-based approach for real-time deployment (RTD) of a multi-quadcopter system between moving initial and final configurations arbitrarily distributed in a 3-D motion space. The proposed RTD problem is decomposed into spatial planning, temporal planning and acquisition sub-problems. For the spatial planning, the RTD desired coordination is defined by integrating (i) rigid-body rotation, (ii) one-dimensional homogeneous deformation, and (ii) one-dimensional heterogeneous coordination such that necessary conditions for inter-agent collision avoidance between every two quadcopter UAVs are satisfied. By the RTD temporal planning, this paper suffices the inter-agent collision avoidance between every two individual quadcopters, and assures the boundedness of the rotor angular speeds for every individual quadcopter. For the RTD acquisition, each quadcopter modeled by a nonlinear dynamics applies a nonlinear control to stably and safely track the desired RTD trajectory such that the angular speeds of each quadcopter remain bounded and do not exceed a certain upper limit.

Index Terms—Real-Time Deployment, Nonlinear Control Design, Multi-Quadcopter System, and Multi-Agent Coordination.

I. INTRODUCTION

Over the past few decades, multi-agent coordination problems have been extensively studied and found numerous applications in surveillance [1], search and rescue [2], agricultural scouting [3], structural health monitoring [4], and air traffic management [5]. Early work on multi-agent coordination commonly treats a group of vehicles (agents) as particles of a single rigid or deformable body acquiring the desired coordination in a centralized fashion through leaderless [6–8] or leader-follower communication-based [9–12] approaches. More recently, researchers have studied real-time deployment (RTD) of multi-agent systems which is also called optimal mass transport (OMT) in the literature. In the OMT problem, agent coordination is governed by the continuity PDE and assigned by finding the optimal transformation between two arbitrary distributions with an equal mass [13, 14]. The existing OMT work assures convergence of agent deployment from an initial distribution to a target configuration. However, inter-agent collision avoidance may not be necessarily avoided when each individual agent represents an actual vehicle with finite size and nonlinear dynamics. This paper develops a novel continuum-mechanics-based approach for collision-free real-time deployment of multi-vehicle system coordinating between two moving formations in a three-dimensional coordination space, where each vehicle represents a quadcopter modeled by a nonlinear dynamics.

A. Related Work

Early work on OMT was inspired by Schrodinger bridge problem [15–17] which was presented as transformation of the state density function from a reference configuration to a target configuration. Refs. [15–17] study the relation between the Schrodinger bridge problem and OMT problems. Mass transport of linear systems from an initial configuration to an arbitrary target configuration is presented as an energy minimization optimization problem in Ref. [18]. Furthermore, optimal transport of discrete-time linear systems are studied in Refs. [17], [19] where [19] uses linear quadratic Gaussian (LQG) regulation to formulate the OMT problem. This paper offers a continuum-mechanics-based solution to the OMT (RTD) problem which is inspired by the existing work on homogeneous transformation coordination of multi-agent systems presented in the author’s previous work [20, 21]. Because homogeneous transformation is an affine transformation, an n-D homogeneous transformation coordination can be defined as a decentralized leader-follower problem with n + 1 that move independently and for and n-D simplex at any time t and followers acquiring the desired coordination through local communication.

B. Contributions

This paper applies the principles of kinematics of continuum mechanics to define RTD problem between arbitrary moving configurations by combining (i) rigid-body rotation, (ii) 1-D homogeneous transformation, and (iii) 2-D heterogeneous coordination. This decomposition is advantageous since we can formally specify safety conditions, assure inter-agent collision avoidance, and impose the input constraints of individual vehicles in a large-scale RTD problem. In this paper, we consider RTD of a multi-quadcopter system (MQS) and define it as spatial planning, temporal planning, and acquisition sub-problems. For the spatial planning, the RTD paths are determined between two moving configurations such that necessary conditions for inter-agent collision avoidance are provided. The RTD temporal planning determines the reference trajectories of individual quadcopters verifying all safety requirements. For the RTD acquisition, a low-level feedback linearization control is designed for each quadcopter such that the desired RTD trajectories are stably tracked and rotor angular speeds of all quadcopters remain bounded.

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We define unit vectors $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$ as $\hat{e}_1 = [1\ 0\ 0]^T$, $\hat{e}_2 = [0\ 1\ 0]^T$, and $\hat{e}_3 = [0\ 0\ 1]^T$. Therefore, rigid-body displacement, actual position, and global desired positions can be expressed in vector forms by $d = [d_x\ d_y\ d_z]^T$, $r_i = [x_i\ y_i\ z_i]^T$, and $p_i = [x_{i,R}\ y_{i,R}\ z_{i,R}]^T$, respectively.

III. Problem Statement

We consider an MQS consisting of $N$ quadcopters where dynamics of quadcopter $i \in V$ is given by

$$
\begin{align*}
\dot{x}_i &= f(x_i) + g(x_i) u_i, & &\forall i \in V, \\
y_i &= C x_i, & &\forall i \in V.
\end{align*}
$$

In (5), $x_i \in \mathbb{R}^{14}$ is the state vector; actual position $y_i \in \mathbb{R}^4$ is the output vector; $u_i \in \mathbb{R}^8$ is the input vector; $C \in \mathbb{R}^{4 \times 14}$ is constant; $f: \mathbb{R}^{14} \rightarrow \mathbb{R}^{14}$ and $g: \mathbb{R}^{14} \rightarrow \mathbb{R}^{14 \times 8}$ are smooth functions; $x_i$, $u_i$, $f$, $g$, and $C$ are specified in Section V. For every quadcopter $i \in V$, we define the following properties and characteristics:

1) **Agent size $\epsilon$**: Every quadcopter $i \in V$ can be enclosed by a ball of radius $\epsilon$.

2) **Deviation upper bound $\delta$**: This paper assumes that each quadcopter can execute a proper trajectory tracking control such that the norm of tracking error is less than $\delta$ for every quadcopter $i \in V$ at any time $t$.

3) **Quadcopter rotor speed**: Angular speed of rotor $j \in \{1, 2, 3, 4\}$ of quadcopter $i \in V$ is denoted by $\omega_{ij}$. The rotor angular speeds cannot exceed the upper bound $\omega_{\text{max}}$ for every quadcopter $i \in V$.

4) **Admissible control set $U$**: Control input $u_i$, executed by quadcopter $i \in V$, must belong to compact set $U$, i.e. $u_i \in U$, $\forall i \in V$.

We assume that the initial and final configurations of the quadcopter team are arbitrarily distributed in the motion space and defined by sets

$$
\Omega_s = \{a_i,s = u_i,s \hat{e}_1 + v_i,s \hat{e}_2 + w_i,s \hat{e}_3, \forall i \in V\},
$$

$$
\Omega_f = \{a_i,f = u_i,f \hat{e}_1 + v_i,f \hat{e}_2 + w_i,f \hat{e}_3, \forall i \in V\},
$$

where

$$
a_i,s = a_i(t_s), & &\forall i \in V, \\
a_i,f = a_i(t_f), & &\forall i \in V.
$$

Note that $\Omega_s$ and $\Omega_f$ are expressed with respect to the local coordinate system at times $t_s$ and $t_f$, respectively. However, $\gamma_s = \gamma(t_s)$, $\mu_s = \mu(t_s)$, $\gamma_f = \gamma(t_f)$, and $\mu_f = \mu(t_f)$ are assigned based on initial and target positions of the MQS, expressed with respect to the inertial coordinate system, by solving the following optimization problem:
Given above problem setting, the main objective of this paper is to define desired deployment trajectory \( p_i(t) \) and choose \( u_i(t) \), for every quadcopter \( i \in \mathcal{V} \), such that initial condition (7a), final condition (7b), and the following safety conditions are all satisfied:

\[
\sum_{i=1}^{N} \left( ||r_i(t) - r_f(t)|| \right) \geq 2e, \quad \forall t \in [t_s, t_f], \quad (9b)
\]

\[
\sum_{i=1}^{N} \left( ||r_i(t) - p_i(t)|| \right) \leq \delta, \quad \forall t \in [t_s, t_f]. \quad (9c)
\]

Condition (9a) assures that the angular speed of no rotor exceeds \( \sigma_{\text{max}} \). Eq. (9b) specifies the inter-agent avoidance collision between every two quadcopters. Stable tracking condition is formally specified by Eq. (9c).

To assign \( p_i(t) \), for every quadcopter \( i \in \mathcal{V} \), deployment of the MQS from arbitrary initial \( \Omega_s \) to target configuration \( \Omega_f \) is defined by integrating three collective motion modes: (i) rigid-body rotation, (ii) homogeneous coordination, and (iii) heterogeneous coordination. Assuming every quadcopter can satisfy safety condition (9c). Section IV provides guarantee conditions for inter-agent collision avoidance in a large-scale RTD. The RTD planning is complemented with the RTD acquisition in Section VI where we apply a feedback linearization approach to design control \( u_i \), for every quadcopter \( i \in \mathcal{V} \), such that: (i) quadcopter \( i \in \mathcal{V} \) stably tracks the desired trajectory \( p_i(t) \) and safety conditions (9a) and (9c) are both satisfied.

IV. RTD PLANNING

Given the initial and final condition (7a) and (7b), the desired position of quadcopter \( i \in \mathcal{V} \), denoted by:

\[
a_i(t) = u_i(t) \hat{e}_1 \gamma(t, \mu(t)) + v_i(t) \hat{e}_2 \gamma(t, \mu(t)) + w_i(t) \hat{e}_3 \gamma(t, \mu(t)), \quad (10)
\]

is planned under the assumption that the the desired formation of the MQS translates rigidly with constant velocities at the initial time \( t_s \) and final time \( t_f \). This assumption can be satisfied, if:

\[
\hat{a}_i(t_s) = \hat{a}_i(t_f) = 0, \quad \forall i \in \mathcal{V}, \quad (11a)
\]

\[
\hat{a}_i(t_s) = \hat{a}_i(t_f) = 0, \quad \forall i \in \mathcal{V}. \quad (11b)
\]

Therefore, the global desired velocities of the quadcopters satisfy the following initial and final conditions:

\[
p_i(t) = \hat{d}(t) = \text{constant}, \quad \forall i \in \mathcal{V}, \quad t \leq t_s, \quad (12a)
\]

\[
p_i(t) = \hat{d}(t) = \text{constant}, \quad \forall i \in \mathcal{V}, \quad t \geq t_f. \quad (12b)
\]

The RTD problem is spatially planned by combining three collective motion modes: (i) rigid-body rotation, (ii) homogeneous motion along \( \hat{e}_1 \), and (iii) heterogeneous motion in \( \hat{e}_2 - \hat{e}_3 \) plane.

In Sections IV-A, IV-B, and IV-C we use the quintic polynomial function

\[
\sigma(t, t_s, t_f) = 15 \left( \frac{t-t_s}{t_f-t_s} \right)^5 - 16 \left( \frac{t-t_s}{t_f-t_s} \right)^4 + 10 \left( \frac{t-t_s}{t_f-t_s} \right)^3, \quad (13)
\]

for \( t \in [t_s, t_f] \), to define the collective motion modes in an RTD problem. Note that \( \sigma(t, t_s, t_f) = 0 \), \( \sigma(t, t_s, t_f) = 1 \), \( \sigma(t, t_s, t_f) = \sigma(t, t_s, t_f) = 0 \), and \( \sigma(t, t_s, t_f) = \sigma(t, t_s, t_f) = 0 \). Note that \( \sigma(t, t_s, t_f) \) is strictly increasing with respect to \( t \).

A. Rigid-Body Rotation

The orientation of the local coordinate system are assigned by using (2), where rotation matrix \( R_i(t) \), defined based on \( \gamma(t) \) and \( \mu(t) \) at any time \( t \in [t_s, t_f] \), is given in (3). Therefore, rigid-body rotation of the MQS is specified by angles \( \gamma(t) \) and \( \mu(t) \) at any time \( t \in [t_s, t_f] \). Given \( (\gamma_s, \mu_s) \) and \( (\gamma_f, \mu_f) \), we define

\[
\gamma(t) = \gamma_s \left( 1 - \sigma(t, t_s, t_f) \right) + \gamma_f \sigma(t, t_s, t_f), \quad (14a)
\]

\[
\mu(t) = \mu_s \left( 1 - \sigma(t, t_s, t_f) \right) + \mu_f \sigma(t, t_s, t_f), \quad (14b)
\]

for \( t \in [t_s, t_f] \).

B. Homogeneous Transformation Coordination along \( \hat{e}_1 \)

Given angles \( (\gamma_s, \mu_s) \) and \( (\gamma_f, \mu_f) \), assigned by solving (5a) and (5b), the initial and final configurations of the UAVs, denoted by \( \Omega_s \) and \( \Omega_f \), are given by (6a) and (6b), respectively. UAVs can be sorted based on their \( u_{i,s} \) coordinates, along the unit vector \( \hat{e}_{1,s} \), and set \( \mathcal{V} \) be expressed by

\[
\mathcal{V} = \{b_1, \cdots, b_N : u_{b_k,s} < u_{b_{k+1},s}, \quad k = 1, \cdots, N-1\} . \quad (15)
\]

where \( b_k \in \mathcal{V} \) represents a quadcopter whose order number is \( k \) in the initial formation \( \Omega_s \). To assure inter-agent collision avoidance, we require that the order numbers of the quadcopters do not change when they are transforming from \( \Omega_s \) to \( \Omega_f \). Therefore, the RTD planning satisfies the following requirement:

\[
\sum_{k=1}^{N} \left( u_{b_k}(t) < u_{b_{k+1}}(t) \right), \quad \forall t \in [t_s, t_f], \quad \mathcal{V} = \{b_1, \cdots, b_N\}. \quad (16)
\]

Per this requirement, the order numbers of the quadcopters in final configuration are the same as the order numbers of quadcopters in the initial configuration. Therefore, set \( \mathcal{V} \) can be also defined as follows:

\[
\mathcal{V} = \{b_1, \cdots, b_N : u_{b_k,f} < u_{b_{k+1},f}, \quad k = 1, \cdots, N-1\}, \quad (17)
\]

where \( u_{b_k,f} = u_{b_k}(t_f) \).

Definition 1. Set \( \mathcal{V} \) can be expressed as \( \mathcal{V} = \mathcal{L} \cup \mathcal{F} \) where disjoint subsets

\[
\mathcal{L} = \{b_1, b_N\}, \quad \mathcal{L} = \{b_1, b_N\}, \quad (18a)
\]

\[
\mathcal{F} = \{b_2, \cdots, b_{N-1}\}, \quad \mathcal{F} = \{b_2, \cdots, b_{N-1}\}, \quad (18b)
\]

define the leader quadcopters and follower quadcopters, respectively.

For better clarification, consider the RTD example shown in Fig. 1 that illustrates safe coordination of 20 quadcopters.
MQS, we define initial reference weight \( \beta_{i,s} \) for every quadcopter \( i \in \mathcal{V} \) by

\[
\beta_{i,s} = \frac{u_{bN,s} - u_{i,s}}{u_{bN,s} - u_{b1,s}} \in [0,1], \quad (19a)
\]

\[
\beta_{i,f} = \frac{u_{bN,f} - u_{i,f}}{u_{bN,f} - u_{b1,f}} \in [0,1]. \quad (19b)
\]

Per definition of set \( \mathcal{T} \), \( u_{b1,s} < u_{i,s} < u_{bN,s} \) and \( u_{b1,f} < u_{i,f} < u_{bN,f} \). Therefore, \( \beta_{i,s} > 0 \) and \( \beta_{i,f} > 0 \) for every quadcopter \( i \in \mathcal{T} \). Theorem 1 provides guarantee conditions for satisfaction of (16) through specifying initial and final MQS arrangements.

**Theorem 1.** Define

\[
d_{\text{min}} = \min \{ u_{bN,s} - u_{b1,s}, u_{bN,f} - u_{b1,f} \} \quad (20a)
\]

\[
\beta^* = \min_{i,j \in \mathcal{V}, i \neq j} \min \{|\beta_{ij,s}|, |\beta_{ij,f}|\} \quad (20b)
\]

where

\[
\beta_{ij,s} = \beta_{i,s} - \beta_{j,s}, \quad (21a)
\]

\[
\beta_{ij,f} = \beta_{i,f} - \beta_{j,f}. \quad (21b)
\]

Assume every quadcopter \( i \in \mathcal{V} \) can execute a proper control input \( u_i \) such that safety condition (8c) is satisfied when every quadcopter is enclosed by a ball of radius \( \epsilon \). Then, inter-agent collision avoidance between every two quadcopters are avoided, if

\[
d_{\text{min}} \beta^* \geq 2(\delta + \epsilon), \quad (22)
\]

and the \( u_i \) component of local desired position of every quadcopter \( i \in \mathcal{V} \) is defined by

\[
u_i(t) = \begin{cases} (1 - \sigma(t,t_s,t_f)) u_{i,s} + \sigma(t,t_s,t_f) u_{i,f} & i \in \mathcal{L} \\ (1 - \beta_i(t)) u_{b1}(t) + \beta_i(t) u_{bN}(t) & i \in \mathcal{T} \end{cases} \quad (23)
\]

at any time \( t \in [t_s,t_f] \), where

\[
\beta_i(t) = (1 - \sigma(t,t_s,t_f)) \beta_{i,s} + \sigma(t,t_s,t_f) \beta_{i,f} \quad \forall i \in \mathcal{V}. \quad (24)
\]

**Proof.** Quadcopters \( i \) and \( j \) can both be enclosed by two balls with the same radius \( \epsilon \) but different centers located at \( r_i(t) \) and \( r_j(t) \) (\( r_i(t) \) and \( r_j(t) \) are the actual position of quadcopters \( i \) and \( j \) at time \( t \)). If safety conditions (9c), inter-agent collision avoidance can be assured by satisfying the following condition:

\[
\bigcap_{i=1}^{N-1} \bigcap_{i=1}^{N} \left\{ |u_i(t) - u_j(t)| \leq 2(\delta + \epsilon) \right\} \quad \forall t \in [t_s,t_f]. \quad (25)
\]

When \( u_i \) and \( u_j \) coordinates of different quadcopters \( i \) and \( j \) are defined by (23), the following relation holds:

\[
u_i(t) - u_j(t) = (\beta_{i,s} - \beta_{j,s})(u_{bN,s} - u_{b1,s}) \quad (26)
\]

Per Eq. (24), \( \beta_i = (1 - \sigma) \beta_{i,s} + \sigma \beta_{i,f} \) and \( \beta_j = (1 - \sigma) \beta_{j,s} + \sigma \beta_{j,f} \) can be substituted into Eq. (26); Eq. (24) can be rewritten as follows:

\[
u_i(t) - u_j(t) = (\beta_{i,s} + \sigma(t,t_s,t_f) (\beta_{i,f} - \beta_{j,s}) ) (u_{bN} - u_{b1} \quad (27)
\]

Because \( \sigma(t,t_s,t_f) \) is strictly increasing over [\( t_s,t_f \)], the right-hand side of Eq. (27) reaches its minimum value, over [\( t_s,t_f \)], either \( t = t_s \), when \( \sigma = 0 \), or \( t = t_f \), when \( \sigma = 1 \):

\[
\min_{t \in [t_s,t_f]} |\beta_{i,s} + \sigma(t,t_s,t_f) (\beta_{i,f} - \beta_{j,s})| = \min \{|\beta_{i,s}|, |\beta_{i,f}|\} = \beta^*. \quad (28)
\]

\[
\min_{t \in [t_s,t_f]} |u_{bN} - u_{b1}| = \min \{u_{bN} - u_{b1}, u_{bN} - u_{b1}, \ldots, u_{b1} - u_{bN}\} = d_{\text{min}}. \quad (29)
\]

This implies that

\[
\min_{t \in [t_s,t_f]} \left\{ |u_i(t) - u_j(t)| \right\} \geq d_{\text{min}} \beta^*, \quad i \neq j, i,j \in \mathcal{V}. \quad (30)
\]

Therefore, inter-agent collision avoidance (25) is satisfied, if condition (22) holds.
C. Heterogeneous Transformation Coordination in the $\hat{e}_2 - \hat{e}_3$

Evolution of the UAVs in the plane made by $\hat{e}_2$ and $\hat{e}_3$ are defined by

$$
\begin{align*}
\begin{bmatrix} v_l(t) \\ w_l(t) \end{bmatrix} &= (1 - \sigma(t,t_a,t_f)) \begin{bmatrix} v_{l,i} \\ w_{l,i} \end{bmatrix} + \sigma(t,t_a,t_f) \begin{bmatrix} v_{l,f} \\ w_{l,f} \end{bmatrix}, & \forall t \in [t_a,t_f].
\end{align*}
$$

(28)

V. RTD ACQUISITION

We first present the quadcopter dynamics in Section V-A. Then, we design a feedback linearization control in Section V-C so that every quadcopter $i$ can stably track the desired RTD trajectory $p_i(t)$ and safety conditions (9a)-(9e) are all satisfied.

A. Quadcopter Dynamics

This paper models quadcopter $i \in \mathcal{V}$ by dynamics (5) with the state vector $x_i$ and input vector $u_i$, and smooth functions $f$ and $g$ defined as follows:

$$
x_i = \begin{bmatrix} x_i \\ y_i \\ z_i \\ \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \end{bmatrix},
$$

$$
u_i = \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \\ \dot{\phi}_i \\ \dot{\theta}_i \\ \dot{\psi}_i \\ \dot{p}_i \end{bmatrix},
$$

(29a)

$$
f(x_i) = \begin{bmatrix} f \end{bmatrix}^T = \begin{bmatrix} \frac{p}{m_i} \dot{k}_{b,i} - g \hat{e}_3 \end{bmatrix}^T \begin{bmatrix} \phi_i \\ \theta_i \\ \psi_i \end{bmatrix},
$$

(29b)

$$
g(x_i) = \begin{bmatrix} g \end{bmatrix} = \begin{bmatrix} 0_{9 \times 1} \\ 0_{9 \times 1} \\ 0_{9 \times 1} \\ 0_{9 \times 1} \\ 0_{9 \times 1} \\ 0_{9 \times 1} \\ 0_{9 \times 1} \\ 0_{9 \times 1} \end{bmatrix},
$$

(29c)

$$
C = \begin{bmatrix} 0_{3 \times 5} & 0_{3 \times 5} & 0_{3 \times 5} \\ 0_{3 \times 5} & 0_{3 \times 5} & 0_{3 \times 5} \\ 0_{3 \times 5} & 0_{3 \times 5} & 0_{3 \times 5} \end{bmatrix},
$$

(29d)

where $r_i = [x_i \\ y_i \\ z_i]^T$ is the actual position of quadcopter $i \in \mathcal{V}$; $\phi_i$, $\theta_i$, and $\psi_i$ are the roll, pitch, and yaw angles of quadcopter $i \in \mathcal{V}$; $p_i$ is the magnitude of the thrust force of quadcopter $i \in \mathcal{V}$; $m_i$ is the mass of quadcopter $i \in \mathcal{V}$, and $g = 9.81m/s^2$ is the gravity acceleration. Also, unit $\hat{k}_{b,i}$ is the unit vector assigning the direction of the thrust force of quadcopter $i \in \mathcal{V}$.

1) Quadcopters’ Angular Velocities and Accelerations

We use 3-2-1 standard to determine orientation of quadcopter $i \in \mathcal{V}$ at time $t$ with the three rotations shown in Fig. 2. Given roll angle $\phi_i(t)$, pitch angle $\theta_i(t)$, and yaw angle $\psi_i(t)$ and the base vectors of the inertial coordinate system ($\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$), we obtain

$$
\begin{bmatrix} \dot{i}_{1,i} \\ \dot{j}_{1,i} \\ \dot{k}_{1,i} \end{bmatrix}, \begin{bmatrix} \dot{i}_{2,i} \\ \dot{j}_{2,i} \\ \dot{k}_{2,i} \end{bmatrix}, \begin{bmatrix} \dot{i}_{b,i} \\ \dot{j}_{b,i} \\ \dot{k}_{b,i} \end{bmatrix}
$$

as follows:

$$
\begin{align*}
\dot{i}_{1,i} &= L_{\text{Euler}}^T (0,0,\psi_i) \hat{e}_1 = [C_{\psi_i} \quad S_{\psi_i} \quad 0]^T, \\
\dot{j}_{1,i} &= L_{\text{Euler}}^T (0,0,\psi_i) \hat{e}_2 = [-S_{\psi_i} \quad C_{\psi_i} \quad 0]^T, \\
\dot{k}_{1,i} &= L_{\text{Euler}}^T (0,0,\psi_i) \hat{e}_3 = [0 \quad 0 \quad 1]^T,
\end{align*}
$$

(30a)

$$
\begin{align*}
\dot{i}_{2,i} &= L_{\text{Euler}}^T (0,\theta_i,\psi_i) \hat{e}_1 = [C_{\theta_i}C_{\psi_i} \quad C_{\theta_i}S_{\psi_i} \quad -S_{\theta_i}]^T, \\
\dot{j}_{2,i} &= L_{\text{Euler}}^T (0,\theta_i,\psi_i) \hat{e}_2 = [-S_{\theta_i} \quad C_{\theta_i} \quad 0]^T, \\
\dot{k}_{2,i} &= L_{\text{Euler}}^T (0,\theta_i,\psi_i) \hat{e}_3 = [S_{\theta_i}C_{\psi_i} \quad S_{\theta_i}S_{\psi_i} \quad C_{\theta_i}]^T,
\end{align*}
$$

(31a)

$$
\begin{align*}
\dot{i}_{b,i} &= L_{\text{Euler}}^T (\phi_i,\theta_i,\psi_i) \hat{e}_1 = [C_{\phi_i}C_{\psi_i} \quad C_{\phi_i}S_{\psi_i} \quad -S_{\phi_i}]^T, \\
\dot{j}_{b,i} &= L_{\text{Euler}}^T (\phi_i,\theta_i,\psi_i) \hat{e}_2 = [C_{\phi_i}S_{\psi_i} \quad C_{\phi_i}S_{\psi_i} \quad S_{\phi_i}]^T, \\
\dot{k}_{b,i} &= L_{\text{Euler}}^T (\phi_i,\theta_i,\psi_i) \hat{e}_3 = [S_{\phi_i}C_{\psi_i} \quad S_{\phi_i}S_{\psi_i} \quad C_{\phi_i}]^T
\end{align*}
$$

(32a)

The angular velocity of quadcopter $i \in \mathcal{V}$ is then given by

$$
\omega_i = [\omega_{x,i} \quad \omega_{y,i} \quad \omega_{z,i}]^T = \Gamma (\phi_i, \theta_i, \psi_i) \begin{bmatrix} \dot{\phi}_i \\ \dot{\theta}_i \\ \dot{\psi}_i \end{bmatrix}^T,
$$

(33)

where

$$
\Gamma (\phi_i, \theta_i, \psi_i) = \begin{bmatrix} 1 & 0 & -\sin \theta_i \\ 0 & \cos \phi_i & \cos \theta_i \sin \phi_i \\ 0 & -\sin \phi_i & \cos \phi_i \cos \theta_i \end{bmatrix}.
$$

(35)

Angular acceleration of quadcopter $i \in \mathcal{V}$ is obtained by taking the time derivative of the angular velocity vector $\omega_i$ and related to control vector $u_i$ by

$$
\ddot{\omega}_i = B_{1,i} \begin{bmatrix} 0_{3 \times 1} \\ \text{I}_3 \\ u_i \end{bmatrix} + B_{2,i} \dot{u}_i.
$$

(36)

where

$$
B_{1,i} = \begin{bmatrix} 0_{9 \times 1} \quad \text{I}_3 \end{bmatrix}
$$

(37a)

$$
B_{2,i} = \dot{\theta}_i \begin{bmatrix} \dot{k}_{1,i} \times \dot{j}_{1,i} + \dot{\phi}_i \dot{k}_{b,i} \end{bmatrix} \times \dot{i}_{b,i}.
$$

(37b)

The rotational dynamics of quadcopter $i \in \mathcal{V}$ is given by

$$
J_i \ddot{\omega}_i = -\omega_i \times (J_i \omega_i) - J_{r,i} \omega_i \times \omega_i \times k_{b,i} \dot{\theta}_i + T_i
$$

(38)

where

$$
T_i = \tau_{\phi_i} \dot{i}_{b,i} + \tau_{\theta_i} \dot{j}_{b,i} + \tau_{\psi_i} \dot{k}_{b,i} = B_{1,i} \begin{bmatrix} \tau_{\phi_i} \\ \tau_{\theta_i} \\ \tau_{\psi_i} \end{bmatrix}^T
$$

(39)

is the quadcopter torque exerted on quadcopter $i \in \mathcal{V}$.
Fig. 3: Schematic of the plane of quadcopter $i \in \mathcal{V}$ defined by base vectors $\hat{i}_{b,i}$ and $\hat{j}_{b,i}$.

### B. Rotors’ Angular Speeds

The thrust force generated by rotor $j \in \{1, 2, 3, 4\}$ of quadcopter $i \in \mathcal{V}$ is denoted by $p_{ij}$ and defined as follows:

$$ p_{ij} = b \omega^2, \quad \forall i \in \mathcal{V}, \quad j \in \{1, 2, 3, 4\}, $$

where $b > 0$ is the aerodynamic constant. The standard shown in Fig. 3 is used to situate motors of quadcopter $i \in \mathcal{V}$, thus, components of torque $T_i$, exerted on quadcopter $i \in \mathcal{V}$, are obtained as follows:

$$ \tau_{\phi,i} = p_{iA} - p_{i2} = b l \left( \sigma_{13}^2 - \sigma_{12}^2 \right), \quad \forall i \in \mathcal{V}, $$

$$ \tau_{\theta,i} = p_{i3} - p_{i1} = b l \left( \sigma_{13}^2 - \sigma_{11}^2 \right), \quad \forall i \in \mathcal{V}, $$

$$ \tau_{\phi,i} = k \sum_{j=1}^{4} (-1)^j \sigma_{ij}^2, \quad \forall i \in \mathcal{V}, $$

where $k$ is the aerodynamic constant in Eq. (41c). Therefore, the rotors’ angular speeds can be uniquely determined based on the thrust force $(p_i)$ and control torque components $(\tau_{\phi,i}$, $\tau_{\theta,i}$, and $\tau_{\psi,i})$ by

$$ \begin{bmatrix} p_i \\ \tau_{\phi,i} \\ \tau_{\theta,i} \\ \tau_{\psi,i} \end{bmatrix} = \begin{bmatrix} b_i & b_i & b_i & b_i \\ -b_i l_i & 0 & b_i l_i & 0 \\ -b_i l_i & 0 & b_i l_i & 0 \\ -k_i & k_i & -k_i & k_i \end{bmatrix} \begin{bmatrix} \sigma_{11}^2 \\ \sigma_{12}^2 \\ \sigma_{13}^2 \\ \sigma_{14}^2 \end{bmatrix}, \quad \forall i \in \mathcal{V}, $$

(42)

### Proposition 1.

Given $p_{i1}, \phi_i, \theta_i, \psi_i, p_i, \phi_i, \theta_i, \psi_i$, and $u_i = [u_{p,i} \ u_{\phi,i} \ u_{\theta,i} \ u_{\psi,i}]^T$ at time $t \in [t_s, t_f]$, the angular speeds of rotors of quadcopter $i$ are determined by solving the following set of quadratic algebraic equations:

$$ \begin{bmatrix} \sigma_{11}^2 \\ \sigma_{12}^2 \\ \sigma_{13}^2 \\ \sigma_{14}^2 \end{bmatrix} + \begin{bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{14} \end{bmatrix} + \begin{bmatrix} H_{1,i} \\ H_{2,i} \\ H_{3,i} \end{bmatrix} = 0_{4 \times 1}, \quad \forall i \in \mathcal{V}, $$

(43)

where

$$ H_{1,i} = \begin{bmatrix} 1 & \hat{0}_{1 \times 3} \\ 0 & \hat{0}_{3 \times 3} \end{bmatrix}, $$

$$ H_{2,i} = \begin{bmatrix} 0 & \hat{0}_{1 \times 3} \\ \omega_i \times \hat{k}_{b,i} & \omega_i \times \hat{k}_{b,i} \end{bmatrix} $$

(44a)

$$ H_{3,i} = \begin{bmatrix} -1 & \hat{0}_{1 \times 3} \\ 0 & -\hat{0}_{3 \times 3} \end{bmatrix} u_i - \hat{J}_B \hat{B}_{2,i} + \omega_i \times (\hat{J}_i \omega_i), $$

(44b)

$$ \hat{H}_{3,i} = \begin{bmatrix} -1 & \hat{0}_{1 \times 3} \\ 0 & -\hat{0}_{3 \times 3} \end{bmatrix} u_i - \hat{J}_B \hat{B}_{2,i} + \omega_i \times (\hat{J}_i \omega_i). $$

(44c)

**Proof.** By considering Eqs. (39) and (42), Eq. (42) can be rewritten as

$$ [\sigma_{11}^2 \ \ldots \ \sigma_{13}^2]^T = H_{1,i} [p_i \ T_i]^T, \quad \forall i \in \mathcal{V}. $$

By substituting $\omega_i$ from Eq. (39), $T_i$ is obtained as follows:

$$ T_i = B_{1,i} \begin{bmatrix} 0_{3 \times 1} \\ I_3 \end{bmatrix} u_i + B_{2,i}, \quad \forall i \in \mathcal{V}, $$

(45)

where

$$ B_{1,i} = \hat{J}_B \hat{B}_{1,i}, $$

$$ B_{2,i} = \hat{J}_B \hat{B}_{2,i} - \omega_i \times (\hat{J}_i \omega_i) - \omega_i \times \hat{k}_{b,i} \hat{B}_{1,i} = \hat{J}_B \hat{B}_{2,i} + \omega_i \times (\hat{J}_i \omega_i) - H_{2,i} \begin{bmatrix} \sigma_{11} \\ \ldots \ \sigma_{13} \end{bmatrix}^T. $$

(46b)

By substituting $T_i$ obtained in Eq. (45), the angular speeds of the rotors of quadcopter $i \in \mathcal{V}$ are assigned by Eq. (43).

### C. Quadcopter Trajectory Control

In this section, we use the feedback linearization control method to design trajectory control $u_i$ for every quadcopter $i \in \mathcal{V}$. To this end, we provide Definition 3 to formally define Lie derivative before proceeding.

**Definition 3.** Let $y : \mathbb{R}^p \rightarrow \mathbb{R}$ and $f : \mathbb{R}^p \rightarrow \mathbb{R}^p$ be smooth functions. The Lie derivative $y$ with respect to $f$ is defined as follows:

$$ L_f y = \nabla f \cdot y. $$

We define state transformation $z_i = (z_i, y_i)$ given by

$$ z_i = [r_i^T \ \hat{r}_i^T \ \hat{p}_i^T \ \hat{r}_i \ \psi_i \ \psi_i]^T \in \mathbb{R}^{14 \times 1}, \quad \forall i \in \mathcal{V}, $$

(47)

where $z_i$ is updated by the following linear-time-invariant dynamics:

$$ \dot{z}_i = A_{SF} z_i + B_{SF} v_i, \quad \forall i \in \mathcal{V}, $$

and

$$ A_{SF} = \begin{bmatrix} 0_{9 \times 3} & I_3 & 0_{9 \times 1} & 0_{9 \times 1} \\ 0_{3 \times 3} & 0_{9 \times 9} & 0_{9 \times 1} & 0_{9 \times 1} \\ 0_{1 \times 3} & 0_{1 \times 9} & 0 & 1 \\ 0_{1 \times 3} & 0_{1 \times 9} & 0 & 0 \end{bmatrix}, $$

(49a)

$$ B_{SF} = \begin{bmatrix} 0_{9 \times 3} & 0_{9 \times 1} \\ I_3 & 0_{9 \times 1} \\ 0_{1 \times 3} & 0 \\ 0_{1 \times 3} & 1 \end{bmatrix}. $$

(49b)

Here, $v_i$ is related to the control input of quadcopter $i \in \mathcal{V}$, denoted by $u_i$, by

$$ v_i = M_{1,i} u_i + M_{2,i}, $$

(50)
where

$$
M_{1,i} = \begin{bmatrix} L_{g} \frac{L_{4}^{2}}{k} x_{i} & L_{g} \frac{L_{4}^{2}}{k} x_{i} & L_{g} \frac{L_{4}^{2}}{k} x_{i} & L_{g} \frac{L_{4}^{2}}{k} x_{i} \\
L_{g} \frac{L_{4}^{2}}{k} x_{i} & L_{g} \frac{L_{4}^{2}}{k} y_{i} & L_{g} \frac{L_{4}^{2}}{k} y_{i} & L_{g} \frac{L_{4}^{2}}{k} y_{i} \\
L_{g} \frac{L_{4}^{2}}{k} y_{i} & L_{g} \frac{L_{4}^{2}}{k} z_{i} & L_{g} \frac{L_{4}^{2}}{k} z_{i} & L_{g} \frac{L_{4}^{2}}{k} z_{i} \\
L_{g} \frac{L_{4}^{2}}{k} z_{i} & L_{g} \frac{L_{4}^{2}}{k} \psi_{i} & L_{g} \frac{L_{4}^{2}}{k} \psi_{i} & L_{g} \frac{L_{4}^{2}}{k} \psi_{i} 
\end{bmatrix} \in \mathbb{R}^{14 \times 14},
$$

(51a)

$$
M_{2,i} = \begin{bmatrix} L_{4}^{2} x_{i} & L_{4}^{2} y_{i} & L_{4}^{2} z_{i} & L_{4}^{2} \psi_{i} \end{bmatrix}^{T} \in \mathbb{R}^{1 \times 14}.
$$

(51b)

The control design objective is to choose \( u_i \) such that \( y_i \) stably tracks desired output \( y_{i,d} = [p_i^T \quad \psi_{i,d}] \) where \( p_i \) and \( \psi_{i,d} \) are the global desired trajectory and desired yaw angle of quadcopter \( i \in \mathcal{V} \). Without loss of generality, this paper assumes that \( \psi_{i,d} = 0 \) at any time \( t \). To achieve the control objective, we define desired state vector

$$
z_{i,d} = [r_{i,d}^{T} \quad \theta_{i,d}^{T} \quad \psi_{i,d} \quad \hat{\psi}_{i,d}]^{T}, \quad \forall i \in \mathcal{V},
$$

(52)

and choose

$$
v_i = K_i (z_{i,d} - z_i), \quad \forall i \in \mathcal{V},
$$

(53)

such that \( A_{SF} - B_{SF} K_i \) is Hurwitz. Then, the control input of quadcopter \( i \in \mathcal{V} \) is obtained by

$$
u_i = M_{1,i}^{-1} (v_i - M_{2,i}).
$$

(54)

Theorem 2. Assume \( z_{i,d} \), defined by (52), is a bounded input, \( v_i \) is selected by (53), and control gain matrix \( K_i \) is selected such that \( A_{SF} - B_{SF} K_i \) is Hurwitz. Then, there exists a unique \( t_f' > t_s \) such that safety conditions (9a) and (9b) are satisfied by choosing any \( t_f \geq t_f' \).

Proof. By substituting \( v_i \) from Eq. (53), Eq. (48) simplifies to

$$
\dot{z}_i = (A_{SF} - B_{SF} K_i) z_i + B_{SF} K_i z_{i,d}, \quad \forall i \in \mathcal{V}.
$$

(55)

If \( A_{SF} - B_{SF} K_i \) is Hurwitz and \( z_{i,d} \) is bounded, then, dynamics is Bounded Input Bounded Output (BIBO) stable which in turn implies that \( z_i(t) \) remains bounded at any time \( t \). Now, we define \( E_i = [z_{i-d} \quad \psi_{i,d}]^{T} \) as the error, and obtain the following error dynamics:

$$
\dot{E}_i = (A_{SF} - B_{SF} K_i) E_i + \begin{bmatrix} 0_{3 \times 9} & \mathbf{I}_3 \\
\mathbf{0}_{3 \times 2} \end{bmatrix} \bar{p}_i.
$$

(56)

We say that \( \bar{p}_i(t) \to 0 \), if \( (t_f - t_s) \to \infty \). Therefore, there exists a final time \( t_f' \) such that safety condition (9c) is satisfied for every quadcopter \( i \in \mathcal{V} \). Also, \( p_i(t), \hat{p}_i(t), \bar{p}_i(t), \) and \( \bar{p}_i(t) \) are decreased at any time \( t \in [t_s, t_f] \), if \( t_f - t_s \to \infty \). Therefore, there exists a final time \( t_f' \) such that the angular speeds of rotors of every quadcopter \( i \in \mathcal{V} \) satisfy safety condition (9a). Therefore, safety conditions (9a) and (9b) are both satisfied if we choose a final time \( t_f \geq t_f' \), where \( t_f' = \max \{t_f', t_f''\} \).

VI. Simulation Results

We consider evolution of an MQS consisting of 60 quadcopters where quadcopters have the same characteristics and are all modeled by dynamics (5) with \( x_i, u_i, f(x_i), \) and \( g(x_i) \) given in (29). We use the quadcopter parameters presented in Ref. [23] and listed in Table 1 to simulate the real-time deployment coordination of the MQS from an initial formation shown in Fig. 4 to the final configuration shown in Fig. 5. Given the initial and final configurations of the MQS, \( d_{mn} \beta_f = 1.1889 \). Therefore, \( \delta = 0.19 \) assign the upper-bound for the RTD tracking error. For simulation, we assume that every quadcopter can be enclosed by a ball of radius \( \epsilon = 0.40, \) \( \sigma_{max} = 215 \text{ rad/s} \) is the upper limit for the quadcopters’ angular speeds.

We further assume that the MQS moves with velocity \( 10 \text{ m/s} \) before and after RTD is activated, i.e \( d(0) = -d(t_f) = 10 \text{ m/s} \). Fig. 6 plots angular speeds of all quadcopter rotors. As it is seen safety condition (9a) is satisfied for every quadcopter \( i \in \mathcal{V} \). Fig. 7 plots \( x, y, \) and \( z \) components of actual positions of all quadcopters versus time for \( t \in [0, 50] \) s.

| Parameter (\( vi \in \mathcal{V} \)) | Value | Unit |
|----------------------------------|-------|------|
| \( m_i \) | 0.5 | kg |
| \( I_i \) | 0.25 | m |
| \( J_{x,i} \times J_{y,i} \) | 3.357 \times 10^{-5} | kg m^2 |
| \( J_{x,i} \) | 0.0196 | kg m^2 |
| \( J_{y,i} \) | 0.0196 | kg m^2 |
| \( J_{z,i} \) | 0.0264 | kg m^2 |
| \( b_i \) | 3 \times 10^{-3} | N s^2/\text{rad} |
| \( k_i \) | 1.1 \times 10^{-6} | N s^2/\text{rad} |

VII. Conclusion

This paper developed a novel physics-based solution for the real-time deployment of multi-agent systems between arbitrary moving configurations. The proposed approach decomposes the RTD into rigid-bodily rotation, 1-D homogeneous transformation, and 2-D heterogeneous motion. Without loss of generality, we assumed that each agent is a quadcopter modeled by a 14-th order nonlinear dynamics, and applied the feedback linearization control for each quadcopter to stably and safely track the desired RTD trajectory. By choosing a sufficiently-large RTD travel time, we assured that the safety

![Fig. 4: Initial configuration of the quadcopter team forming a cuboid in the motion space.](image)
Fig. 5: Final configuration of the quadcopter team forming a disk in the $x-y$ plane.

Constraints, including bounded rotor speeds conditions and inter-agent collision avoidance, are assured.

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Fig. 6: Angular speeds of rotors 1 through four of all quadcopters. It is seen that the safety constraint (9a) of every quadcopter $i \in \mathcal{V}$ is satisfied where $\omega_{i}^{\text{max}} = 220 \text{ rad/s}$. 

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