Essay of thermodynamic description and optimization of Ranque effect

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Abstract. This paper presents a thermodynamic description of Ranque effect. Including expansion and compression works into energy exchange provides a rational explanation of the observed effect. These works are performed by small portions of gas in polytropic processes. Rotational motion of the gas and presence of non-reversibility in the processes of expansion and compression are responsible for energy separation. A mathematical model with the use of the conservation laws and the polytropic expansion process was created. Optimization calculations for various values of the polytropic efficiency coefficient and the Mach number of hot gas outflow were performed. It has been obtained that the optimal relative values of exit section area and the Mach number of inflowing high-pressure gas are invariant with respect to the values of the parameters mentioned above. Using this result, a practical model for optimizing operation process was elaborated. The efficiency coefficient of expansion work is determined during the optimization process. Using this result a practical technique of operational process has been developed.

1. Introduction
Ranque vortex tube is a cylindrical chamber with a near-axial hole with a cross-section area $F_q$ at one tube end, and with an annular slotted opening near the wall with a cross-section area $F_1$ at another one. Compressed gas with parameters $\{T_e, p_e, M_e\}$ (figure 1) is supplied into the chamber in tangential direction through the swirl nozzle with an inlet cross-section area $F_e$ on its side wall. Ranque discovered that the temperature of gas flowing out from the near-axial hole is considerably lower than the temperature of inflow gas; and the temperature of gas flowing out from the near-axial hole is considerably higher [1]. In spite of many publications available on the effect, there is still no generally recognized physical-mathematical model.

Figure 1. Basic diagram of vortex tube:
1 – smooth vortex tube,
2 – vortex generator for supplying compressed gas,
3 – throttle valve,
4 – outflow of hot gas through annular slot,
5 – diaphragm for cold gas outflow.
2. Briefly about existing approaches to describe Ranque effect

Critical reviews of significant number of works are presented in [2, 3]. The authors of the studies build stationary models of parameters distribution in the near-diaphragm section orthogonal to the rotation axis using the condition of circular motion balance, conditions of isentropic flow when establishing the link between gas parameters at various radii, and equations of ideal gas law. The models differ in distribution functions of tangential velocity depending on the radius. Unfortunately, the models are erroneous, i.e. the isentropic condition is erroneous (there is viscosity and heat exchange). Absence of analytical description of the effect using minimal experimental data stimulates conducting further theoretical and experimental research. In the present work a variant of thermodynamic description of the effect was suggested (without going into structure details of vortex flow in the tube), which also allowed us to perform the optimization procedure.

3. Physical model of Ranque effect

Let us pick out an elementary volume of gas \( \sigma = \delta r \cdot \delta x \cdot r \delta \varphi \) in the cylindrical coordinate system \((r, x, \varphi)\), with \(\{u, v, w\}\) – axial, peripheral, and radial velocity components. Let the radial gas velocity in any elementary volume (mole of fluid [4]) be directed to the axis of rotation. When moving, gas entering the area of lower pressure expands and performs work, then the temperature and density in the elements decrease. The accompanying entropy increase does not allow a particle to return to the previous exergy level. Density decrease aids in further moving the particle toward the rotation axis. Part of the expansion work is spent on compressing the adjacent elementary volumes of gas. The density inside them increases causing their moving away from the rotation axis. The temperature and pressure also increase. This process is accompanied by the entropy increase, which also excludes the possibility of returning the particle to the previous exergy level. Density increase contributes in further moving the particle away from the rotation axis. As a consequence of these motions, cold gas is accumulated in the central area, and hot gas is accumulated at the periphery. As the gas temperature grows driven by compression of portions of gas, the heat flow towards the portions of gas that do the work increases. This lowers the efficiency coefficient of expansion, and, in the limit, determines the maximum temperature of the heating. Thus, the reasons of energetic gas separation during its rotation motion are the processes of expansion and compression in presence of dissipation (creating a complicated structure of interpenetrating flows). The flow is characterized by high values of relative velocities of the particles, which results in occurrence of significant noise effects.

![Figure 2. i-s – diagram of energy gas separation process.](image)

Consider the diagram in figure 2. The point denoted as 0 corresponds to the ambient parameters. It is assumed that the pressure of outflow gases is equal to the pressure \( p_{\infty} \) of the ambient environment. The process of kinetic energy dissipation and flow slowdown due to viscosity and interaction with the wall is represented in the diagram by the trajectory “A–D”. The trajectory “D–C” corresponds to the gas flow from the periphery to the central area. It is described by a sequence of elementary acts of isentropic expansion, isothermal throttling, and isobaric heat supply. The trajectory “D–H”
corresponding to the flow of heated gas is described by the sequence of elementary acts of isentropic compression, isothermal throttling, and isobaric heat removal. The integral compression work is equivalent to the full work of expansion. Energy separation is not possible in the model of ideal gas, since only one trajectory of possible motion of the representative point issues from the point D – there is no trajectory splitting.

4. Conventions and indexes of parameters

- \( m \) is the gas mass flux, \( V \) is the velocity, \( M \) is the Mach Number, \( \text{Re} = \frac{Vc_D}{\nu} \) is the Reynolds number, \( \text{Pr} \) is the Prandtl number, \( p \) is the pressure, \( T \) is the temperature, \( \rho \) is the density, \( i \) is the enthalpy, \( s \) is the entropy, \( \Delta s = \frac{\Delta s}{R} \), \( R \) is the gas constant, \( \gamma \) is the adiabatic index, \( \nu \) is the kinematic coefficient of viscosity, \( F_{ec}, F_{e\square}, F_{c}, F_{\square} \) are the cross-section areas of incoming and outgoing flows corresponding to cold and hot gas, \( F_c = F_{ec} + F_{eh}, \alpha = m_c/m_e = F_{ce}/F_e \) is the mass ratio of cold gas, \( \bar{p}_c = \bar{p}_e, \bar{p}_\square = \bar{p}_e, F_c = \frac{p_c}{p_e} \).

\( D \) is the tube diameter, \( p_\infty = 1 \text{ atm} \) is the ambient pressure, \( \bar{p}_c = \bar{p}_\square = \bar{p}_e = \frac{p_c}{p_e} \).

Indices notation is as follows: \( e \) denotes entry section, \( c \) denotes cold gas, \( h \) denotes hot gas, \( 0 \) denotes stagnated flow, \( \bar{x} = x/x_e \) denotes relative value of any parameter.

5. Mathematical model. Optimizing parameters

Figure 3 shows the scheme of control sections, the gas flow are indicated by the dashed line. Let us write the continuity equations for the flows of hot and cold gas of stationary process supposing that the flow parameters are uniform in control sections

\[
\frac{\bar{F}_c}{\alpha \sqrt{T_c}} \frac{M_c}{M_e} = 1, \quad \frac{\bar{F}_h}{(1-\alpha) \sqrt{T_h}} \frac{M_h}{M_e} = 1, \quad \alpha = m_c/m_e = F_{ce}/F_e.
\]

Table 1. Acceptable values of parameters to be optimized

| Parameter | Minimal value | Maximal value | Location step |
|-----------|---------------|---------------|---------------|
| \( p_e \) | 3.0           | 15.0          | 0.1           |
| \( M_e \) | 0.5           | 1.0           | 0.01          |
| \( M_c \) | 0.5           | 1.0           | 0.01          |
| \( \bar{F}_c \) | 2.0           | 5.0           | 0.01          |
| \( \bar{F}_h \) | 2.0           | 5.0           | 0.01          |

Figure 3. Scheme of control sections. Gas flows are marked by the dashed line.

It is admitted, as in gas turbines, that the dissipation energy is proportional to the change of static enthalpy \( Tds = -\delta_c \cdot \delta T \) owing to irreversible processes. Then, from the equation \( Tds = du + pdv \) in case \( \delta_c = \text{const} \) we obtain

\[
\bar{p}_c = \bar{F}_c \frac{V}{T} t^{(1+\delta_c)}.
\]
Let us define the value $\eta_c = (1 + \delta_c)^{-1}$ as the polytropic efficiency coefficient of gas expansion process in the Ranque tube analogically to the characteristic of gas expansion process in a turbine [5, 6]. The parameter $\eta_c$ is a function of Mach number $M_e$, Reynolds number $Re$, Prandtl number $Pr$, specific heat ratio $\gamma$, geometrical parameters of assembly $G$, direction $\theta$ of the incoming flow velocity vector. It is not obvious today how to obtain theoretically integral estimation $\eta_c$. The way to obtain the experimental estimation is shown below. Energy balance equation

$$\alpha T_c \left( 1 + \frac{\gamma - 1}{2} M_c^2 \right) + (1 - \alpha) T_h \left( 1 + \frac{\gamma - 1}{2} M_h^2 \right) = \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right).$$

(3)

The temperature of gas flow slowdown in control sections

$$T_{0e} = \left( 1 + \frac{\gamma - 1}{2} M_c^2 \right), T_{0e} = T_c \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right), T_{0c min} = \frac{\gamma - 1}{\gamma} \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right).$$

(4)

Efficiency of energy separation is estimated by thermal efficiency coefficient [7]

$$\eta_h = \left( \frac{T_{0e} - T_{0c}}{T_{0e} - T_{0c min}} \right)^{-1}$$

(5)

or by normalized value of refrigeration capacity [3]

$$\eta_{cp} = \left[ m_c c_p \left( T_{0e} - T_{0c} \right) \right] \left[ m_c c_p \left( T_{0e} - T_{0c min} \right) \right]^{-1} = \alpha \eta_h.$$ 

(6)

The values of parameters $\Omega = \{ p_e, M_e, M_c, F_h, F_c \}$ are determined from solutions of optimization problems

$$\max_{\Omega} \eta_{cp}$$

(7)

for equations (1), (2), (3) with condition (7)

$$M_{h min} \leq M_h \leq 1$$

(8)

from the given values of parameters $\eta_c, M_{h min}$. The area of search of $\Omega$ parameters is defined (table 1), $\gamma = 1.4$. When solving the problem (6) the limitation (7) was considered by the method of penalty functions [8], search of the maximum was done by the method of coordinate-wise optimization [9].

| Table 2. Optimal valued $F_c, F_h, M_c, M_e$ at various values $\eta_c$ and $M_{h min}$. |
|-----------------|-----------------|
| Parameter       | Value           |
| $F_c$           | $3.1 \pm 0.1$   |
| $F_h$           | $2.9 \pm 0.1$   |
| $M_c$           | $0.96 \pm 0.04$ |
| $M_e$           | $0.86 \pm 0.03$ |

Figure 4. Optimal values of cooled gas mass ration $\alpha$. 
6. Calculation results
In all variant of the problem (6) $M_{h_{\text{opt}}} = M_{h_{\text{min}}}$ was obtained at the point of maximum of function $\eta_{cp}$. The parameters $\eta_{c}, M_{h_{\text{min}}}$ are basically determining ones in the process of energy separation: more is $\eta_{c}$, more is the work of expansion in case of fixed pressure fall; less is $M_{h_{\text{min}}}$, more is proportion of the gas being cooled $\alpha$ (figure 4) and more is the functional $\eta_{cp_{\text{opt}}}$ (figure 5). A considerable increase of the temperature of the gas being heated (figure 6) is conditioned by increase of refrigeration efficiency $\eta_{cp_{\text{opt}}}$ and decrease of heated gas flux $(1-\alpha)$. In reality, the temperature growth will be limited by increasing the outflowing heat flux toward the ambient cold gas. Figure 7 shows the optimal values of the pressure of the input gas flow $p_{e}$, figure 8 shows the optimal values of relative temperature of the gas cooled $\bar{T}_c$. The numerical results presented illustrate possible values of device characteristics.

![Figure 5](image1.png)  
**Figure 5.** Optimal values of normalized value of refrigeration capacity $\eta_{cp}$.

![Figure 6](image2.png)  
**Figure 6.** Optimal values of heated gas relative temperature $\bar{T}_h$.

![Figure 7](image3.png)  
**Figure 7.** Optimal values of input gas flow pressure $p_{e}$.

![Figure 8](image4.png)  
**Figure 8.** Optimal values of cooled gas relative temperature $\bar{T}_c$.

It is noteworthy that the optimal values of relative areas of pipe exit section, Mach number of incoming flow are practically independent on the values $\eta_{c}, M_{h_{\text{min}}}$ (table 2). For this reason, we can accept mean values of parameters $\bar{F}_c, \bar{F}_h, M_e$ for the design implementation. The value of inlet cross-
section area of the compressed gas $F_c$ is determined by its defined flux. The diameter of diaphragm aperture is $d_c = 2\sqrt{F_cF_c}/\pi$. In [5] a linear relation was obtained between the diaphragm aperture radius and inner diameter of the tube $d_c = \beta D, \beta \approx 0.4$ (based on the experimental results of various authors), from which we obtain the value $D$. Other empirical ratios were also obtained for the value $d_c/D$. The scatter of the values motivates using experimental technology to determine the rational value of $d_c/D$ subject to the conditions $F_c = \text{const}, \bar{F}_h = \text{const}$. The tube length is 9–10 calibers [10] in case of presence of a device for vortex motion damping near the throttle.

7. **Practical evaluation of the optimal mode**

To determine the characteristics realized, it is necessary to measure the values of the parameters $p_{de}, p_c, T_{de}, T_{de}, T_{de}, T_{de}, T_{de}, T_{de}$, and then to calculate all characteristics $p_{de}, T_{de}, M_e, T_e, \alpha, M_c, T_c, T_c, \eta_e, \eta_c, \eta_c, \eta_c, \eta_c$ according to the presented thermodynamic model. Performing this procedure for various values of pressure $p_e$, we can determine the optimal mode.

8. **Conclusion**

A thermodynamic model of Ranque effect has been created. It was obtained that the optimal values of relative areas of the tube outlet sections and the Mach numbers of incoming flow are practically independent on polytropic efficiency coefficient of expansion $\eta_c$ or Mach number of hot gas outflow $M_{hmin}$. A practical technique for optimizing the operation process has been suggested.

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