Generic Construction of the Standard Model Gauge Group and Matter Representations in F-theory

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We describe general classes of 6D and 4D F-theory models with gauge group \((\text{SU}(3) \times \text{SU}(2) \times \text{U}(1))/\mathbb{Z}_6\). We prove that this set of constructions gives all possible consistent 6D supergravity theories with no tensor multiplets having this gauge group and the corresponding generic matter representations, which include those of the MSSM. We expect, though do not prove, that these models are similarly generic for 6D theories with tensor multiplets and for 4D \(\mathcal{N} = 1\) supergravity theories. The largest class of these constructions come from deforming an underlying geometry with gauge symmetry \(\text{SU}(4) \times \text{SU}(3) \times \text{SU}(2)\).

1. Introduction

Since the early days of string theory, ongoing efforts have been made to construct vacuum solutions that match some or all of the features of the observed Standard Model of particle physics. A tremendous range of such models has been produced over the years, using a variety of geometric and algebraic techniques, and there is increasing evidence that string theory can describe a large number of solutions with Standard Model structure (see, e.g., [1, 2]). In recent years, F-theory\(^{[3–5]}\) has emerged as a powerful framework for studying the global structure of the space of string vacua. In six dimensions, the connected space of F-theory vacua reproduces much of the structure of the moduli space of string vacua. In six dimensions, the connected space of F-theory vacua reproduces much of the structure of the moduli space of string vacua. In six dimensions, the connected space of F-theory vacua reproduces much of the structure of the moduli space of string vacua. In six dimensions, the connected space of F-theory vacua reproduces much of the structure of the moduli space of string vacua.

We proceed by first finding a complete solution of the 6D anomaly equations for generic matter content for given anomaly coefficients. In simple cases, those that appear on the moduli space branch of largest dimension match naturally to the simplest singularity types encountered in F-theory constructions of the given gauge groups and the underlying geometric moduli of F-theory models, so from the point of view of F-theory, this notion of genericity naturally extends to four dimensional theories.

With this definition, the matter content of the Standard Model is not generic for the gauge group \(\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)\), but it is generic for the group with global structure \((\text{SU}(3) \times \text{SU}(2) \times \text{U}(1))/\mathbb{Z}_6\). Thus, in this note we focus on the latter gauge group. We proceed by first finding a complete solution of the 6D anomaly equations for the generic matter content of a theory with global gauge group \((\text{SU}(3) \times \text{SU}(2) \times \text{U}(1))/\mathbb{Z}_6\) (Section 2), and then describing how these models are realized in 4D F-theory and generalizing the construction to 4D F-theory (Section 3).

There are a number of ways in which the Standard Model could, in principle, arise in the framework of F-theory. One approach, which has been studied extensively, is to realize the gauge group of the Standard Model through flux breaking of a (geometrically tuned) GUT such as \(\text{SU}(5)\)\(^{[6–9]}\) (see [10] for a review). A second approach is based on the idea that all or part of the Standard Model gauge group may be a generic (i.e., supersymmetrically “non-Higgsable”)\(^{[11,12]}\) feature in the chosen geometry, either directly as explored in [13] or through flux breaking of a larger non-Higgsable GUT group such as \(E_6\)^{[14,15]} A third approach, and the one we focus on here, is that the gauge group may itself be a tuned feature of the geometry. This approach was used, for example, in [16–19]; more recently, Cvetic, Halverson, Lin, Liu, and Tian described a large set of models where the Standard Model gauge group and matter content are tuned over a weak Fano base.\(^{[20]}\) The work in this note gives a general and comprehensive picture of the generic classes of models that realize this gauge group and corresponding matter representations through a tuned geometry in the F-theory context; the models studied in [20] are special cases of those described here.

A key concept for our analysis is the notion of “generic” matter types for a given gauge group. As described in [21], this idea can be made rigorous in the context of 6D supergravity by defining the generic matter representations for a given gauge group as those that appear on the moduli space branch of largest dimension, for fixed and small anomaly coefficients. In simple cases, the number of generic matter fields defined in this way matches the number of anomaly constraints, so there is a unique solution for generic matter content for given anomaly coefficients. Furthermore, the generic matter types defined in this way in six dimensions match naturally to the simplest singularity types encountered in F-theory constructions of the given gauge groups and the underlying geometric moduli of F-theory models, so from the point of view of F-theory, this notion of genericity naturally extends to four dimensional theories.
Table 1. Generic matter representations (not including conjugates) charged under the gauge group \((SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6\), which include all the charged MSSM multiplets. Multiplicities given are for the generic matter solution of the 6D anomaly equations, with \(\beta, X, Y\) defined in the text. The dot product between anomaly coefficients uses the signature-(1, 1), inner product defined in a 6D supergravity theory with \(T\) tensor fields.

| Generic Matter | Multiplicity | MSSM Multiplet |
|----------------|--------------|----------------|
| \(1/6\)        | \(b_3 \cdot b_2\) | \(Q\)          |
| \(1/3\)        | \(b_3 \cdot (\beta - 2a)\) | \(U^c\)        |
| \(1_{-1/3}\)   | \(b_3 \cdot X\) | \(D^c\)        |
| \(1_{-4/3}\)   | \(b_3 \cdot Y\) | \(L, H_u, H_d\) |
| \(1_{1/2}\)    | \(b_2 \cdot (X + \beta - a)\) | \(E^c\)        |
| \(1_{1/2}\)    | \(b_2 \cdot Y\) | \(\tilde{H}_u, \tilde{H}_d\) |
| \((1, 1)_1\)   | \((b_3 + b_2 + 2\beta) \cdot X - a \cdot b_2\) | \(E^c\)        |
| \((1, 1)_2\)   | \(\beta \cdot Y\) | \(\tilde{E}^c\) |
| \((\text{Adj}_1)_{1/2}\) | \(1 + b_1 \cdot (b_3 + a)/2\) | \(E^c\)        |
| \((1, \text{Adj})_{1/2}\) | \(1 + b_2 \cdot (b_3 + a)/2\) | \(E^c\)        |

other forthcoming work, we will describe more details of this framework, both for the gauge group \(SU(5) \times U(1)\) and its quotient, and for the Standard Model gauge group.

2. SM Gauge Group and Matter in 6D Supergravity

2.1. Solutions of 6D Anomaly Equations

As described in [21], for the gauge group \((SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6\) there are ten generic charged matter fields, listed in Table 1. The 6D gauge, gravitational, and mixed anomaly equations\(^{[22, 23]}\) give ten constraints on the multiplicities of these fields; the left-hand side of the anomaly equations are inner products \(a \cdot b, b \cdot b\) of the anomaly coefficients \(a, b_1, b_2, b_3\) associated with gravity and the three gauge factors, respectively (using the notation of \([21, 24]\)). The ten anomaly equations restricted to generic matter fields are an invertible system of linear equations, so we can simply write the general solution of these equations for the multiplicities of each of the fields in terms of the various anomaly coefficient products, as listed in Table 1. We find it convenient to use the following relations to define the quantities \(\beta, X, Y\):

\[
\begin{align*}
\beta &= \frac{4}{3} b_3 + \frac{3}{2} b_2 + 2\beta, \\
X &= -8a - 4b_3 - 3b_2 - 2\beta, \\
Y &= a + b_1 + b_2 + \beta.
\end{align*}
\]

(1)

2.2. \(T = 0\) and Two Classes of Solutions

We now analyze the resulting matter spectra, motivated by the case with no tensor multiplets \((T = 0)\), for which the anomaly coefficients \(b_2, b_3\) are integers, \(a = -3\), and the inner product is simply multiplication. For nontrivial gauge groups \(SU(3)\) and \(SU(2)\) with good kinetic terms we must have

\[
b_2, b_3 > 0.
\]

(2)

At \(T = 0\), this implies that \(X\) and \(Y\) must be non-negative for non-negative spectra, so we have

\[
4b_1 + 3b_2 + 2\beta \leq -8a.
\]

(3)

and

\[
b_1 + b_2 + \beta \geq -a
\]

(4)

The total number of \(T = 0\) models satisfying these constraints is 98. We can now define two classes of solutions, which cover all \(T = 0\) solutions:

(A) \(\beta \geq 0\). This gives a three-parameter family of models, parametrized by \(b_1, b_2, \beta\) subject to the constraints (3) and (4); there are 71 such models for \(T = 0\). These models precisely correspond to the Higgsing of a 6D supergravity model with gauge group \((SU(4) \times SU(3) \times SU(2))\) on a triplet of bifundamental fields (one between each pair of gauge factors) in such a way as to preserve the symmetry \((SU(3) \times SU(2)) \times U(1))/\mathbb{Z}_6\), where the gauge factors have the anomaly coefficients \(B_3 = b_1, B_2 = b_2, B_1 = \beta\).

(B) \(Y = 0\). In this case we can have \(\beta < 0\), but we must have

\[
2a \leq \beta = -a - b_1 - b_2.
\]

(5)

This gives a two-parameter family of models parametrized by \(b_1, b_2\); there are 30 such models for \(T = 0\). Three of these models also fit into class (A). All but three of the models in class (B) correspond to Pati–Salam models, in which we Higgs a 6D supergravity model with gauge group \((SU(4) \times SU(3) \times SU(2))\) on a pair of bifundamental fields, where the gauge factors have anomaly coefficients \(B_3 = b_1, B_2 = b_2, B_1 = -a\). The three exceptional cases without Pati–Salam descriptions are those that would have \(B_1 > 0\), and also do not have a description in class (A). Note that the class (B) model with \(b_1 = b_2 = -a\) can also arise from Higgsing an \(SU(5)\) theory with \(B_3 = -a\) on an adjoint representation.

The classes (A) and (B) of models with \(\beta \geq 0\) and \(Y = 0\), respectively, generalize naturally to theories with arbitrary numbers of tensor multiplets \(T > 0\), where \(\beta \geq 0\) now represents the condition that \(\beta\) lies in the positivity cone, and the condition (2) becomes the statement that \(b_2, b_3\) must be nonzero and lie in the positivity cone. Models of class (A) that satisfy condition (3) can again be interpreted as Higgsed \(SU(4) \times SU(3) \times SU(2)\) models; similarly, a subset of models of class (B) can be realized as Higgsed Pati–Salam models.

We note, however, that the story is more subtle when \(T > 0\) since, for example, \(b_1 \cdot X \geq 0\) with \(b_1\) in the positivity cone of the theory does not necessarily imply that \(X\) is in the positivity cone; similarly, when \(T > 0\) one can have vectors \(\beta \cdot Y \geq 0\) with \(\beta\) outside the positivity cone and \(Y\) nonzero. In general, constraints on inner products are weaker than the corresponding constraints on the factors, as discussed in [21] in the context of the simple cases of theories with \(U(1)\) and \(SU(2)\) gauge groups.
Note that both classes of models in general contain hypermultiplet matter in the adjoint representations of the gauge factors SU(3), SU(2), and the models of class (A) also generally have three other types of matter fields not found in the MSSM.

Note also that as the gauge group becomes more complicated, non-generic matter representations may become more relevant. In particular, for the gauge group \( (SU(3) \times SU(2) \times U(1)) / Z_\delta \), the representation \( [\mathfrak{b}, \mathfrak{b}, 0]_{B_\delta} \) is non-generic, but appears in theories coming from Higgsing SU(5) models where the SU(5) starts with more than one adjoint (i.e., \( B_\delta > -a \)). While in general the anomaly coefficients must be larger to realize models that contain such a non-generic representation, such models may also be of interest to study.

In summary, we have defined two classes of models with gauge group \((SU(3) \times SU(2) \times U(1))/Z_\delta\) and generic matter, which in the \( T = 0 \) case describe all solutions of the 6D supergravity anomaly equations.

### 3. F-theory Construction of the Standard Model Gauge Group and Generic Matter Representations

#### 3.1. F-theory Constructions at \( T = 0 \)

We now consider how the supergravity models with gauge group \((SU(3) \times SU(2) \times U(1))/Z_\delta\) and generic matter spectra given by Table 1 can be realized in F-theory. We begin by considering the 6D \( T = 0 \) models.

As stated above, there are 71 combinations of \( b_\beta, b_\lambda \geq 0, \beta \geq 0 \) that satisfy the constraints (3) and (4). It is straightforward to confirm that these are precisely the combinations of divisor classes on \( \mathbb{P}^2 \) that allow for a Tate tuning of \((SU(4) \times SU(3) \times SU(2)) / (SU(3) \times SU(2) \times U(1))/Z_\delta\) on the corresponding divisor classes \( B_\lambda = b_\lambda, B_\beta = b_\beta, B_\gamma = \beta \). Since there are no obstructions to flat directions in the 6D theory, the Higgsing to the desired \((SU(3) \times SU(2) \times U(1))/Z_\delta\) group can always be carried out. Thus, there is an F-theory construction of all the models in class (A).

The models of class (B) precisely correspond to the construction of F-theory models with toric fiber \( F_{11} \) described in [27]; the global structure of such F-theory models with a gauge group that is a quotient of a product of nonabelian and abelian factors was analyzed in [28, 29]. The matter multiplicities computed in [27] for models with the \( F_{11} \) fiber can be matched to the multiplicities of Table 1 through \( b_\lambda = S_\delta, b_\beta = S_\delta - S_\gamma - K_\delta \). The allowed values for the F-theory models precisely match those of the supergravity models, giving 30 possible distinct models, so again there is an F-theory construction of all the models in class (B). For all but three of the models of class (B), there is also a construction through the Higgsing of a Pati–Salam model, again starting with a Tate tuning. This Higgsing can be related in the toric language to deformations that give fiber \( F_{11} \) by removing a ray from the fiber \( F_{11} \), which as pointed out in [27] generally describes a Pati–Salam model.

We have thus shown that there is an F-theory realization for all possible consistent 6D supergravity theories with \( T = 0 \), gauge group \((SU(3) \times SU(2) \times U(1))/Z_\delta\), and matter in the set of generic matter representations for this group as tabulated in Table 1.

#### 3.2. Larger \( T \)

More generally, the classes of F-theory models used to realize the 6D supergravity models with gauge group \((SU(3) \times SU(2) \times U(1))/Z_\delta\) and generic matter can be constructed over a general F-theory base \( B_T \), giving a theory with \( T = h^{1,1}(B_T) + 1 \) tensor multiplets. It is natural to speculate that such models, for class (A) comprising the set of models arising from Higgsing a theory with gauge group \((SU(4) \times SU(3) \times SU(2)) / (SU(3) \times SU(2) \times U(1))/Z_\delta\) on three bifundamentals, and for class (B) comprising the set of models with fiber \( F_{11} \) as described in [27], may give generic constructions at arbitrary \( T \) of the set of possible F-theory models with gauge group \((SU(3) \times SU(2) \times U(1))/Z_\delta\) and generic matter. For the models of class (B), indeed the construction of [27] essentially gives a parametrized Weierstrass model, analogous to a Tate model for tuning an SU(N) theory or the Morrison–Park model for tuning a U(1) theory with generic matter. While we leave the explicit construction of a Weierstrass model for the general theory of class (A) to further work [31], it is clear that for an arbitrary number of tensor multiplets and a general F-theory base with canonical class \( K \), the constraints (3) and (4), where \( a = K \), are precisely the constraints on the existence of a Tate model for the necessary SU(4) × SU(3) × SU(2) theory (in the absence of other gauge factors).

There is an additional subtlety that at larger values of \( T \), there can be non-generic constructions that nonetheless only give rise to generic matter; we do not discuss such constructions here, but they may provide additional exotic models that nonetheless only contain the generic \((SU(3) \times SU(2) \times U(1))/Z_\delta\) matter.

All of these constructions can be carried out over an arbitrary weak Fano base, as discussed for certain models of class (B) in [20]. Note however that, for all these constructions, complications can ensue when there are non-Higgsable clusters, which correspond to an anti-canonical divisor \( -K \) in the F-theory base geometry that contains a rigid component, giving additional gauge factors that may intersect the divisors carrying the \((SU(3) \times SU(2) \times U(1))/Z_\delta\) gauge group. The clearest situations in which models can be understood under these circumstances are those in which the divisors \( b_\lambda, b_\beta, b_\gamma \) and \( \beta \) for class (A) models do not contain or intersect any of the divisors carrying non-Higgsable clusters, so that the additional gauge and matter factors act like decoupled dark matter sectors, even after unHiggsing to the enhanced group. This simple kind of circumstance only arises for models of class (A), however, since the condition \( Y = 0 \) implies that any divisor supporting a non-Higgsable gauge component must be contained in one of the divisors \( b_\lambda, b_\beta = -4K - 2b_\lambda - b_\gamma \) associated with the gauge factors of the unHiggsed Pati–Salam model. For a more detailed analysis of these issues see [31].

#### 3.3. 4D F-theory Models

We can now generalize the F-theory constructions of class (A) and class (B) models described for 6D theories to 4D F-theory. For 4D models, the anomaly coefficient products for localized matter fields appearing in Table 1 are replaced with the curve given by the intersection of the corresponding divisor classes, so that these
entries in the table describe the matter curve supporting matter in the appropriate representation. (Note that the same replacement does not apply for the adjoint representations, as these are not supported locally on curves.)

For 4D models, the geometry of the solutions of class (B) are again constructed using the fiber $F_{11}$ as described in [27]. For the models of class (A), the starting geometry will again be a deformation of the $SU(4) \times SU(3) \times SU(2)$ Tate model geometry. Even without an explicit Weierstrass description, the geometry of the matter curves is still encoded in Table 1.

Just as, for example, the Tate model describes generic F-theory constructions with an $SU(N)$ gauge group and generic matter in 4D as well as in 6D, it is natural to hypothesize that the two classes of F-theory constructions identified here will also give the general F-theory constructions of models with gauge group $(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$ and the generic matter representations listed in Table 1 in both 6D and 4D.

The class of models recently studied in [20] correspond to the 4D models of class (B) with the specific choice $b_1 = b_2 = -K$ over a weak Fano base. Note that, as mentioned above, this also corresponds to the class of models that are reached by Higgsing an SU(5) gauge theory tuned on the divisor $-K$. The divisor classes associated with the gauge factors are the same in these models, and allow for gauge coupling unification. On the other hand, for most models in class (A), the divisors supporting the three gauge factors are distinct and independent, so we do not expect gauge coupling unification in those models.

As for six-dimensional models, all the constructions described here can be carried out over any weak Fano base, and one can in principle try to tune a Standard Model gauge group in class (A) over any complex threefold base that supports an elliptic Calabi–Yau fourfold. Recent studies suggest that the number of F-theory constructions of models with gauge group $(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$ and the generic matter representations listed in Table 1 can be carried out over any weak Fano base, and one can most models in class (A), the divisorssupporting the three gauge and allow for gauge coupling unification. On the other hand, for most models in class (A), the divisors supporting the three gauge factors are distinct and independent, so we do not expect gauge coupling unification in those models.

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4. Outlook

We have shown that in 6D supergravity without tensor multiplets, two simple classes of F-theory models combine to give all anomaly-free models with gauge group $(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$ and the generic matter multiplets listed in Table 1. By analogy with other known F-theory constructions, it is natural to conjecture that these two classes of models will give the generic constructions of both 4D and 6D models with geometrically tuned gauge group $(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$.

One of the most striking features of this analysis is that we have found that the largest set of F-theory constructions that share the Standard Model gauge group and matter representations live in a three-parameter family of models that come from a deformation of $SU(4) \times SU(3) \times SU(2)$ theory. If this is true in four dimensions as well as in six dimensions with minimal supersymmetry, it is possible that this structure may even be relevant in analyzing Standard Model constructions from string theory in vacuo without supersymmetry.

This analysis also highlights the significance of the second class of models encountered, which corresponds to the $F_{33}$ fiber constructions analyzed in [27]. While the first class (A) gives a larger, three-parameter family of models and is compatible with a wider range of bases including those with possible (supersymmetrically) non-Higgsable hidden dark matter sectors, the second class (B) contains fewer potentially nonzero matter multiplicities for non-MSSM matter and many models in this class have a more explicit description through toric geometry.

The focus of this note has been on the underlying geometry of generic F-theory constructions of the Standard Model. While in six dimensions, this geometry essentially completely determines the physics, for four dimensional models the geometry is just a starting point; fluxes must be included to produce chiral matter, and many other effects must be considered. The classes of models described in this note with the Standard Model gauge group and matter content would be promising to explore further for realistic four-dimensional physics models from F-theory, keeping in mind that all the models described here essentially have a completely tuned gauge group, and are disjoint from other possible constructions that use non-Higgsable components in the gauge group or rely on flux breaking of a GUT.

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Conflict of Interest

The authors have declared no conflict of interest.

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