CERN to Gran Sasso; An ideal distance for superbeam? *
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We use the CP trajectory diagram as a tool for pictorial representation of the genuine CP and the matter effects to explore the possibility of an \textit{in situ} simultaneous measurement of $\delta$ and the sign of $\Delta m^{2}_{13}$. We end up with a low-energy conventional superbeam experiment with a megaton-class water Cherenkov detector and baseline length of about 700 km. A picturesque description of the combined ambiguity which may arise in simultaneous determination of $\theta_{13}$ and the above two quantities is given in terms of CP trajectory diagram.

1. Introduction

Exploring the structure of lepton flavor mixing and the neutrino mass pattern is one of the most challenging goals of contemporary particle physics. Among other things, the least known is the (1-3) sector of the MNS matrix, the angle $\theta_{13}$, the sign of $\Delta m^{2}_{13}$, and the CP violating angle $\delta$. (See e.g., ref. \cite{1}.)

We have explored in a series of papers \cite{2} the features of interplay between the CP phase and matter effect, and tried to develop strategies for measurement of lepton CP violation in long-baseline neutrino oscillation experiments. See also ref. \cite{3} for related works. Along the line of thought, we have introduced recently a powerful tool called “CP trajectory diagram in bi-probability space” which allows us a separate pictorial representation of the genuine CP and the matter effects \cite{4}. We pointed out that an ambiguity exists in determination of these parameters in a correlated way ($\delta - \text{sign of } \Delta m^{2}_{13}$).

We address here the issue of an \textit{in situ} simultaneous determination of $\delta$ and the sign of $\Delta m^{2}_{13}$ in a single experiment. In a companion article \cite{5}, which is a contribution to Proceedings of NuFACT01, a concise summary of the idea of CP trajectory diagram and its use is given and the principle of optimizing beam energy is discussed.

Let us start with a brief remark on the more generic feature of the ambiguity problem.

2. Clover-leaf ambiguity

If the value of $\theta_{13}$ is unknown, there exists another ambiguity in a correlation ($\delta - \theta_{13}$), as pointed out in ref. \cite{6}. Together with the ambiguity we have uncovered, there exists the combined ambiguity which can be as large as four-fold. We now demonstrate that it can be described in a simple picturesque way by using the CP trajectory diagram.

In fig. \cite{7} drawn is the CP trajectory diagram for four values of the mixing parameters which are given in the caption of fig. \cite{7} assuming, for simplicity, neutrino beam with the Gaussian type energy distribution as used in our recent work \cite{8}. The point of fig. \cite{7} is that the four-fold solutions are possible for given oscillation probabilities of $P(\nu) \equiv P(\nu_{\mu} \to \nu_{e})$ and $P(\bar{\nu}) \equiv P(\bar{\nu}_{\mu} \to \bar{\nu}_{e})$, both at about 1.1 \% in fig. \cite{7}. One notices from fig. \cite{7} that the name ”clover-leaf ambiguity” by which the ambiguity is referred at TAUP2001 is quite natural and appealing.

3. Optimal distance for measuring the sign of $\Delta m^{2}_{13}$

Now we turn to the discussion of our original question, i.e., how to resolve the ($\delta - \text{sign of } \Delta m^{2}_{13}$) ambiguity. Since the sign of $\Delta m^{2}_{13}$ can be determined by measuring interference between the vacuum and the matter effects, it is natural to think about neutrino oscillation experiments
which utilize longer baselines. One can think of these possibilities in the context of either
(i) a single detector experiment for in situ simultaneous determination of $\delta$ and the sign of $\Delta m_{12}^2$,
or (ii) a two-detector experiment with second supplemental detector which is primarily devoted for
determination of the sign of $\Delta m_{12}^2$.

The next question to ask is; what is the optimal baseline length for this purpose? It would
be the best situation if we can tune the distance in such a way that the matter effect is relatively
enhanced compared to the genuine CP violating effect. To quantify the request we define the
asymmetry parameter defined by using the ratio
\[ R(P) = \frac{\langle P(\bar{\nu}_e \rightarrow \nu_e) \rangle}{\langle P(\nu_e \rightarrow \bar{\nu}_e) \rangle}, \]
where the probabilities are averaged over Gaussian type energy distributions. By using the ratio
in defining the asymmetry it is insensitive to the values of the mixing parameters, in particular to $\delta$.

In fig. 2, the asymmetry $A(R)$ is plotted for $\delta = \pi/2$ with the same mixing parameters as in fig. 1,
apart from fixing $\sin^2 2\theta_{13}$ to be 0.05. One notices that the asymmetry is large at $L = 600 - 700$ km,
and at $1000 - 1500$ km for $E \sim 1$ GeV. It can be explicitly shown that the asymmetry is indeed
very insensitive to $\delta$. Therefore, the baselines $L \sim 700$ km, and at $\sim 1000$ km look promising.
We take the former option and examine the feature of CP-matter interplay by using the CP trajectory
diagram.

**4. Longer baseline option; single vs. two-detector methods**

In fig. 3, we present (a) CP trajectory diagram in bi-probability plane for neutrino beam
with the Gaussian type energy distribution with averaged energy $\langle E \rangle = 1.5$ GeV and baseline distance
$L = 700$ km, and (b) CP trajectory diagram on number of events plane with the same baseline
for appearance channels $\nu_{\mu} \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$.

One can clearly see in fig. 3a that the two trajectories corresponding to $\Delta m_{12}^2 > 0$ (solid line)
and $\Delta m_{12}^2 < 0$ (dashed line) are well separated with each other. Therefore, it is in principle
possible to carry out an in situ simultaneous measurement of $\delta$ and the sign of $\Delta m_{12}^2$ in such
a baseline length. It is amusing to note that the length just corresponds to either CERN $\rightarrow$ Gran
Sasso, or Fermilab $\rightarrow$ Soudan mine distances.

In fig. 4, we assume a water Cherenkov detector of fiducial volume 0.9 Mton, and 4 MW of
proton beam power which is planned in the JHF experiment in its phase II. We use the narrow
band (NB) 3 GeV beam whose neutrino energy peaks at $E \sim 1.4$ GeV considered in ref. [4],
but with intensity multiplied by factor of 3. It is to mimic the off-axis (OA) beam which is designed
for $E \sim 1.4$ GeV, a bit of higher energy than the

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**Figure 1.** Illustration of the clover-leaf ambiguity in terms of CP trajectory diagram; four solutions exist
for given values of $P(\nu_{\mu} \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ at neutrino energy $\langle E \rangle = 1.5$ GeV
and baseline distance $L = 295$ km. Corresponding values of $\sin^2 2\theta_{13}$ as well as the sign of $\Delta m_{12}^2$
whose absolute value is $3 \times 10^{-3}$ eV$^2$ are indicated in the plot. The remaining mixing
parameters are taken as; $\sin^2 2\theta_{13} = 1.0$, $\Delta m_{12}^2 = 5 \times 10^{-5}$ eV$^2$, $\sin^2 2\theta_{12} = 0.8$, and $\delta = \pi/2$. We take
the matter density as $\rho = 2.72$ g/cm$^3$ and the electron fraction as $Y_e = 0.5$.

**Figure 2.** Asymmetry of the probability ratio defined in Eq. (1) in the text computed for $\delta = \pi/2$. The mixing
parameters are chosen as the same with those of fig. 1 and $\sin^2 2\theta_{13} = 0.05$.
one actually prepared for the JHF experiment. Two (six) years of running is assumed for neutrino (antineutrino) channel. We refer ref. 7 for a detailed explanation of how the computation of number of events is done.

As you see in fig. 3b, the numbers of events are sizable, some $1000 - 2000$, though not gigantic. Resultant 3 $\sigma$ contour is small enough so that simultaneous measurement of $\delta$ and the sign of $\Delta m^2_{31}$ seems feasible, justifying the title of this talk.

If the detector at baseline $L = 700$ km cannot be so massive by some reasons, or if the practical site requires much longer distance, then it can be viewed as a secondary (farthest) detector, assuming that the primary far detector (e.g., Hyper-Kamiokande 5 in the case of the JHF experiment) already exists. In this case, the secondary detector is primarily for measurement of the sign of $\Delta m^2_{31}$ and the requirement of statistics can be relaxed.

Finally, it was pointed out in the talk that while the present tunnel in the Gran Sasso Laboratory is too small to accommodate a megaton water Cherenkov detector, a 180 kton "proto-type" is ready to be created upon filling water into the whole tunnel!

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