Some Aspects of Pre Big Bang Cosmology

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Abstract
This is a summary of a course given at the Fourth Mexican School on Gravitation and Mathematical Physics on some aspects of PBB cosmology. After introductory remarks the lectures concentrate on some amusing consequences derived from the symmetries of the string theory with respect to such classical concepts as isotropy and homogeneity. The extra dimensions and the symmetries of the M theory are further applied to show that the classical singularities might be just physically irrelevant. In the final lecture a model universe is “produced” from “almost nothing” and it is argued that initial plane waves are thermodinamically natural state for the universe to emerge from.

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Introduction

First I would like to thank the Organizers for giving us the opportunity to give lecture Course in such a beautiful surrounding.

As far as my course is concerned, I will be concentrating on our recent works in String Cosmology done mostly in collaboration with Kerstin Kunze and Miguel Vázquez-Mozo.

I will first briefly review some general aspects of String Cosmology, and in the following days we will touch the Initial Conditions, and the Singularity Problem. Most of the References may be found in a nicely organized M. Gasperini WEB page: [http://www.to.infn.it/~gasperin](http://www.to.infn.it/~gasperin).

The central problems of modern theoretical cosmology are: the initial conditions, the singularity problem and the dimensionality of the universe. The first to begin modern discussions of the initial conditions problem was Ch. Misner [1] in his Chaotic Cosmology Program. The idea was to allow the universe to start from an arbitrary complex initial state and to identify the mechanism to smoothen out the irregularities, to finally explain as to why the universe is homogeneous and isotropic to such a high extent. Unfortunately, even if one focuses on homogeneous but anisotropic cosmological models, the so-called Bianchi models, only those which include the FRW models as particular cases would isotropise, and these we say are of measure 0.

Inflation [2], somehow brought Misner’s idea back to life, in the sense that perturbations around FRW backgrounds were to disappear if the universe to undergo a period of accelerated expansion. Nevertheless, generically non-linear inhomogeneities such as strong shock primordial waves etc remain difficult to deal with.

A solution to the i.c. problem may come from rather different a direction, such as thermodynamics, for example, if some idea as to how to assign entropy to gravitational field were available. In the 70’s Penrose [3] put forward a speculation that one should assign a 0 gravitational entropy to geometries with 0 Weyl tensor, so that the universe must have been FRW to start with. Yet, this notion of gravitational entropy is apparently unrelated to the notion of entropy even in the cases we know of, such as Black Holes, or entropy for high frequency gravity waves etc. Moreover, the idea of assigning 0 entropy to FRW geometries is influenced strongly by a classical view on the initial state of the universe. Taking for example “stringy” approach to the early universe one would have rather thought that the background geometry should represent a target space of some exact Conformal Field Theory, and FRW doesn’t seem to be an exact CFT. Also there is rather a different physical view on matters such as isotropy and homogeneity in the context of the string theory. I will be arguing later that certain kinds of inhomogeneities and anisotropies do not really matter when symmetries of the low energy string theory are considered—thus it could be that our notions of isometries must be changed.

The PBB [4] cosmology is rather different. First, it pushes the i.c. into the “cold radiative” past where physics is believed to be known to us. The hot regime, near the
singularity, now becomes the intermediate phase and it is believed that the symmetries of the non-perturbative string theory may help to deal with the singularity. To summarize the PBB scenario in one paragraph:

The universe starts as a perturbation in an otherwise flat background \[3, 12, 16, 14\], this perturbation collapses from the point of view of an Einstein observer. It expands and inflates in the so-called String frame. The inflation drives the universe towards the “intermediate” singularity meanwhile getting rid of all the inhomogeneities and anisotropies and the string symmetries act to squeeze the universe unharmful through the hot regime. Finally, emerges the observed universe.

The inflation in the PBB universe is driven by the kinetic energy of massless dilaton field which is the fundamental ingredient of any string theory, and there are substantial differences between the standard inflation and the PBB phase. In standard inflation the i.c. are prescribed in the regime one knows very little about physics, the initial curvature scale may be very large and remains constant or decreases upon expansion, on the other hand, the initial coupling is arbitrarily weak in PBB, the curvature grows while the initial curvature is practically 0 \[14\].

Now, people often talk about String Cosmology when talking about PBB. What does one really mean when String Cosmology is quoted? Basically we lack a complete non-perturbative string theory valid at Plank time. So really the game is restricted to the energies \(\sim 10^{19} \text{ GeV} \sim GUT \sim 10^{16} \text{ GeV}\). Therefore, one may write the low energy effective action for the theory which reduces to GR + a bunch of extra massless fields depending on the type of the string theory. The questions, however, the cosmologists ask remain the same: why the universe is homogeneous, what happens with initial singularity, can we solve the horizon and flatness problem, and... does the universe we speculate about corresponds to the one we observe?

On general grounds, the massless bosonic sector of superstrings includes the gravitational field \(G_{\mu\nu}\), the dilaton \(\phi\), with vacuum expectation value determining the string coupling constant, and the antisymmetric rank-two tensor \(B_{\mu\nu}^{(1)}\). There are more massless fields depending on the particular superstring model. The lowest order effective action for the massless fields can be written as

\[
S_{\text{eff}} = \frac{1}{(\alpha')^\frac{D-2}{2}} \int d^D x \sqrt{G} e^{-2\phi} \left[ R + 4(\partial \phi)^2 - \frac{1}{12} (H^{(1)})^2 \right] + S_{\text{md}},
\]

where \(H^{(1)} = dB^{(1)}\) is the field strength associated with the NS-NS two-form and \(S_{\text{md}}\) is a model-dependent part which includes other massless degrees of freedom.

When \(D < 10\) some of these massless fields correspond to gauge and moduli fields associated with the specific compactification chosen, and the dilaton \(\phi\) appearing in the above equation is related to the ten-dimensional dilaton \(\phi_{10}\) by \(2\phi = 2\phi_{10} - \log V_{10-D}\), where \(V_{10-D}\) is the volume of the internal manifold measured in units of \(\sqrt{\alpha'}\).

Let us restrict the attention to generic degrees of freedom, leaving aside the internal components of the ten-dimensional fields. In the heterotic string case, \(S_{\text{md}}\)
contains the Yang-Mills action for the background gauge fields \( A^a_\mu \). In the case of the model-dependent part of the type-IIB superstring things are more involved; among the massless degrees of freedom in the R-R sector we find, along with a pseudo-scalar \( \chi \) (the axion) and a rank-two antisymmetric tensor \( B^{(2)}_{\mu\nu} \), a rank four self-dual form \( A_{\mu\nu\sigma\lambda}^{sd} \). The presence of this self-dual form spoils the covariance of the effective action for the massless fields, since there is no way of imposing the self-duality condition in a generally covariant way. But, if we set \( A^{sd} \) to zero, we can write a covariant action for the remaining fields with

\[
S_{\text{IIB md}} = -\frac{1}{(\alpha')^{D-2}} \int d^Dx \sqrt{G} \left[ \frac{1}{2} (\partial \chi)^2 + \frac{1}{12} (H^{(1)} \chi + H^{(2)})^2 \right]
\]

Notice that the R-R fields do not couple directly to the dilaton. Thus, the lower dimensional \((D < 10)\) R-R fields \( \chi \) and \( B^{(2)}_{\mu\nu} \) are obtained from the ten-dimensional ones through \( \chi = \sqrt{V_{10-D}} \chi_{10} \) and \( B^{(2)} = \sqrt{V_{10-D}} B^{(2)}_{10} \).

By combining the dilaton and the axion of the ten-dimensional type-IIB superstring into a single complex field \( \lambda = \chi_{10} + i e^{-\phi_{10}} \), it is possible to check that the bosonic effective action is invariant under the \( SL(2,R) \) transformation \( \lambda \rightarrow (a\lambda + b)/(c\lambda + d), \ G_{\mu\nu} \rightarrow |c\lambda + d| G_{\mu\nu} \). Writing this transformation in terms of four-dimensional fields \((D = 4)\) we find that the four-dimensional fields acquire an “anomalous weight” due to the fact that \( V_6 \), as measured in the string frame, does transform under \( SL(2,R) \) (see the 3rd ref. of [5]):

\[
\begin{align*}
\chi'_4 &= \frac{bd + ae e^{-2\phi_4}}{[(c\chi_4 + d)^2 + c^2 e^{-2\phi_4}]} + \frac{ac^2 + (ad + bc)\chi_4}{[(c\chi_4 + d)^2 + c^2 e^{-2\phi_4}]}, \\
\phi'_4 &= \frac{e^{-\phi_4}}{[(c\chi_4 + d)^2 + c^2 e^{-2\phi_4}]}, \\
H^{(1)'} &= d H^{(1)} - c H^{(2)}, \\
H^{(2)'} &= [(c\chi_4 + d)^2 + c^2 e^{-2\phi_4}][(-b H^{(1)} + a H^{(2)}), \\
G'_{\mu\nu} &= [(c\chi_4 + d)^2 + c^2 e^{-2\phi_4}] G_{\mu\nu}
\end{align*}
\]

## 2 T-Duality

The T-duality symmetry is probably one of the most interesting consequences of the string theory. We often hear people say that the transformation \( R \rightarrow 1/R \), where \( R \) is the string compactification scale, leaves the energy spectrum invariant. The existence of a compact direction is essential for this symmetry. I will not go here into great details as to how this comes, just will make several remarks as applied to cosmology.

Suppose we probe by strings a 10 dimensional geometry, using two different string theories. The topology of the 10 dimensional space is \( R^9 \times S^1 \). The T-duality
identifies these two string theories and, if $\alpha' = \frac{1}{2\pi T}$, where $T$ is the tension in both the theories, then the compactification radii satisfy $R_1 R_2 = \alpha'$. Note that if one of the radii shrinks to 0 the other diverges.

Compactification on a circle implies quantization of momenta which now comes in quanta of $1/R$. These momenta appear as masses for states which are massless in higher dimensions. In addition there exists yet another excitation type, the windings. If $m$ is the number the string winds around the circle, the energy of these excitations is $m \times 2\pi RT = m R / \alpha'$, therefore if the compactification radii are interchanged at the same time as the momenta are interchanged with windings, the energy spectrum of the theory remains invariant. Now, this is what happens on the level of the string theory. To relate this to the background spacetime where the strings propagate, one must take into account that the strings propagate on a 10 dimensional target space, and in turn, the target space must satisfy that the so called $\beta$ functions are to vanish (something roughly similar to Einstein equations, see for example [20]).

If the target space happens to have a compact Killing direction then the actions we where talking about above are invariant under the following “duality” transformation: taking adapted coordinates in which $x^0$ denotes the coordinate along the Killing vector chosen to dualize, we find new values for $(G_{\mu\nu}, \phi, B_{\mu
u}^{(1)})$

\[
\begin{align*}
\tilde{g}_{00} &= g_{00}^{-1}, \quad \tilde{g}_{0i} = g_{00}^{-1} B_{0i} \\
\tilde{g}_{ij} &= g_{ij} - g_{00}^{-1} (g_{i0} g_{0j} - B_{i0} B_{0j}) \\
\tilde{B}_{ij} &= B_{ij} - g_{00}^{-1} (g_{i0} B_{0j} - B_{i0} g_{0j}) \\
\tilde{B}_{0i} &= g_{00}^{-1} g_{0i} \quad \tilde{\phi} = \phi - 1/2 \log g_{00}
\end{align*}
\]

The PBB cosmological models are based on the so-called scale factor duality which uses the T-duality symmetry of the string theory to invert the scale of expansion as in $\tilde{g}_{00} = g_{00}^{-1}$. In the case of FRW model, one may invert the overall scale factor due to isotropy of the model. What happens, then, is that one may have a prolonged inflationary period in the inverted-scale model, while the standard model is decelerating.

**Exercise 1**

Start with the solution of spatially flat FRW model driven by a massless scalar field. Write it both in Einstein and in the String frame. Use the T-duality transformations to obtain the model with an inverted scale factor. What is the metric in the Einstein frame? What happens with the scalar field? Find the kinematical quantities of the model (acceleration, curvature scale etc). What happens if you dualise this model just wrt a single Killing direction, say $\partial_x$? Are the T-duality transformations invariant under diffeomorphism?
Excercise 2

Start with a 4 dimensional model

\[ ds^2 = -\left(\frac{2\nu}{t} - k\right)^{-\alpha} dt^2 + \left(\frac{2\nu}{t} - k\right)^{\beta} dx^2 + \left(\frac{2\nu}{t} - k\right)^{1-\alpha} t^2 d\theta^2 + f_k(\theta)^2 d\phi^2 \]

coupled to a dilaton field

\[ \phi = \frac{1}{4}(\beta - \alpha) \log \left(\frac{2\nu}{t} - k\right). \]

This is an exact solution of dilaton gravity provided the constants \(\alpha\) and \(\beta\) satisfy

\[ \alpha^2 + \beta^2 = 2. \]

Here \(k\) is the spatial curvature \((k = -1, 0, 1)\) and

\[ f_k(\theta) = \begin{cases} 
\sin \theta & k = 1 \\
\theta & k = 0 \\
\sinh \theta & k = -1
\end{cases} . \]

Identify all the Killing vectors. Show that only dualising wrt \(\xi_4 = \partial_x\) keeps all isometries intact. Dualising wrt \(\xi_3 = \partial_{\phi}\), for example, the homogeneity is lost, yet the string theory remains invariant. This signals that the homogeneity, and isotropy are not fundamental concepts of string cosmology. Identify the Schwarzschild solution among the above (it has a constant scalar field) and speculate about the possible fate of the black hole applying the duality transformation.

There are some subtle differences between scale factor duality and T-duality. The inversion of one or more scale factors in the metric (the scale factor duality) leaves the low-energy equations of motion invariant. T-duality, is rather stronger a tool, it is a transformation of the metric, not necessarily only of the scale factors, leading to a new solution of the low-energy field equations which corresponds to an equivalent string theory. In the metric of Exercise(2) we have four Killing vectors, and the degree of homogeneity may be reduced by just performing a T-duality transformation along \(\phi\) direction. Consequently, for this model described by the line element with a cyclic \(x\) coordinate, the only T-duality transformation leaving the spatial symmetry group intact is the one performed along the compactified isometric direction \(x\). In this case it is formally identical to the scale factor duality of the metric. For scale factor duality, however, there is no need to impose the compactness of the \(x\)-coordinate since the “dual” model does not necessarily have to be equivalent to the original one as in complete string theories.

We have just only started to play the game and already on this level see that there are some interesting aspects to string cosmology, for if for example the symmetries of the string theory are used as a guide, we may conclude that two quite inequivalent
backgrounds may host equivalent string theories and thus from the point of view of string propagation be indistinguishable. Strings propagating on two backgrounds related by the T-duality transformation would not notice the difference and wouldn’t care of such aspects as homogeneity or isotropy.

Now, we have seen that the inversion of the scale factor is permitted by the string symmetries. But what happens at $t = 0$? there are different approaches to this problem, known in the literature as an “exit problem”. In the next section I will try to summarize our work on this problem.

3 Regularisation of Singularities

We all believe that the emergence of s-t singularities in GR suggests the breakdown of the theory at natural scales of the Theory. At these scales one expects the quantum corrections to take a leading role and save the situation. While String/M theories are the best candidates to quantize gravity, near the singularity one can not use the perturbative approach, but rather a full-fledged M theory. Unfortunately we lack such a theory. Different approaches to regularize singularity and to find a way out to solve the exit problem are discussed in the papers quoted in [8].

One of the ways of dealing with the problem, might be more qualitative an approach, trying to use the symmetries and dualities of the M (c.f. [1]) theory to map singular backgrounds to nonsingular ones. In plain words, one may formulate dualities in terms of physically equivalent vacua of the theory. Imagine one has two backgrounds A and B, where every physical quantity of A may be written in terms of quantities of B. Now, if A is singular but B is not, we can say that the strings don’t care, and the singularity of A is just irrelevant. This is to say that we were using the wrong degrees of freedom, or wrong “tools” to describe the low energy limit of the theory.

Another hint that different degrees of freedom might be relevant comes already from the following simple [10] example. Imagine there is a hidden extra dimension in the theory and take the scalar field spatially flat FRW cosmology in four dimensions. It is of course singular at the “Bang”. Now, imagine this scalar field is just a “trace” of an extra dimension. The 5 dim. lifted cosmology has a nonsingular scalar curvature. This may hint that the 4 dim. singularity is just an artifact of integrating the degrees of freedom associated with the extra dimension. The extra dimensions come naturally, or I would rather say are must, if one considers S/M theory. So, let us try the two things:

1) Use the extra dimensions.
2) Use the M-theory symmetries, to get some info about singularities.

Our starting point would be a 4-D cosmology with a bunch of massless minimally coupled scalar fields (for more details see [11]). Imagine we have designed an algorithm to construct such solutions. (One can do it for quite a general class of models).
We will concentrate on homogeneous scalar fields \( \varphi_i(t) = q_i \varphi_0(t) \). Just think of these fields as all having the same functional dependence, but different amplitudes.

The field equations are, of course, invariant under \( O(N) \) rotation between the fields. To be even more specific, let us look at the inhomogeneous generalization of spatially open FRW cosmology with \( N \) scalar fields.

\[
\begin{align*}
 ds^2 &= (\sinh 2t)^{\frac{1}{2}(3\lambda - 1)}(\cosh 4t - \cosh 4z)^{\frac{2}{3}(1 - \lambda)}(-dt^2 + dz^2) \\
 &\quad + \frac{1}{2} \sinh 2t \sinh 2z \left( \tanh z \, dx^2 + \cothanh z \, dy^2 \right)
\end{align*}
\]

Here \( \varphi_0 = \sqrt{3}/2 \log \tan(t) \), and \( \lambda \) is a sum of the squares of the amplitudes of the scalar fields and is an \( O(N) \) invariant. The expression is written for generic \( \lambda \), but in particular case \( \lambda = 1 \) the model is just an open FRW solution (though in unusual co-ordinates) coupled to \( N \) scalar fields \( \phi_i \sim p_i \log \tan(t) \).

Let us now “lift” this solution to 4+N dimensions and see what happens.

The 4+N dimensional metric is:

\[
\begin{align*}
 ds^2_{4+N} &= 2(\sinh t)^{1-\sum_{i=1}^N p_i}(\cosh t)^{1+\sum_{i=1}^N p_i}(-dt^2 + dz^2 + \sinh^2 z \, dx^2 + \cosh^2 z \, dy^2) \\
 &\quad + \sum_{i=1}^N \tanh^{2p_i} t (dw^i)^2
\end{align*}
\]

The curvature invariant for the metric:

\[
R_{abcd}R^{abcd} \sim C(p_i)t^{2(S-3)} + O[t^{2(S-2)}]
\]

where we have written \( S = \sum_{i=1}^N p_i \) and \( C(p_i) \) is defined by

\[
C(p_i) = \frac{3}{16}(S - 1)^2(S^2 - 2S + 5) + \sum_{i=1}^N p_i^2[3 + p_i^2 + (S - 3)(p_i + S)] + \sum_{i<j}^N p_i p_j
\]

The remarkable point about all this is that the Kretchman scalar \( R_{abcd}R^{abcd} \) is regular whenever the condition \( \lambda = 1 \) is satisfied. Thus, it may really be the case that the singularity in 4 dimensions is just an artifact of integrating the extra degrees of freedom encoded in higher dimensions.

Let us now move forward and specify the number of extra dimensions to 7. In the 11 dimensional geometry we can parametrize the moduli metric with the numbers \( p_i \), which must satisfy the following constraint:

\[
I = \sum_{i=1}^7 p_i^2 + \frac{2}{3} \sum_{i\neq j} p_i p_j
\]

Now, we want to think of these solutions as representing vacua of the M theory. Therefore we can look now at the \( U \) duality group of M compactified on \((S^1)^7 \). It
happens that this group is generated by the so-called $\frac{2}{3}$ transformation which permutes the
tuple $(p_1, \ldots, p_7) \rightarrow \left( p_1 - \frac{2s}{3}, p_2 - \frac{2s}{3}, p_3 - \frac{2s}{3}, p_3 + \frac{s}{3}, \ldots, p_7 + \frac{s}{3} \right)$

with $s = p_1 + p_2 + p_3$.

These transformations map vacuum solutions into vacuum solutions and one can
check that singular solutions may be transformed by this transformation into no-
singular ones. The U-duality transformations are conjectured to be an exact symme-
try of the M-theory, and the fact that the two backgrounds, one singular, and one
regular are connected via a U-duality transformation may indicate, as in the case
of homogeneity and isotropy, that certain kinds of singularities do not matter. The
results presented here is not a rigorous study, of course, nevertheless one can get an
idea of what might happen near the singularity if the ideas of string cosmology apply.

Finally, I would like to present you with a scenario of a PBB universe where one
starts with a small perturbation which collapses, inflates, gets rid of all its irregulari-
ties before approaching the intermediate singularity and after exiting approaches the
standard model.

4 A Universe from “Almost Nothing”

It is more or less accepted that to solve the flatness/horizon problem in cosmology
one must invoke the idea of accelerated expansion. There are two possibilities which
provide us with accelerated behaviour of the scale factor of the universe:

1) The standard inflation
2) The PBB scenario.

In the PBB picture one starts with the weakly coupled regime of the string theory
and its dynamics is controlled by the low energy effective action. This is in contrast
with the standard inflation where one starts with physics on Plank scale and there
is no way to extract a “decent” Lagrangean description for the inflaton field from a
fundamental theory. Moreover, the mere existence of the dilaton coming from the
string theory is quite “unhealthy” for standard picture of inflation.

To address the problem of initial conditions in PBB scenario, Bounanno, Damour
and Veneziano (BDV) [12] have formulated the basic postulate of “asymptotic past
triviality” identifying the initial state of the universe with the generic perturbative
solution of the tree-level low energy effective action. In this picture the initial state
consists of a bath of gravitational and dilatonic waves, some of which could have
collapsed leading/modulo solution to exit problem to an FRW universe. This picture
generalizes and makes more concrete previous studies of inhomogeneous versions of
PBB cosmology [13], [15].

However, within this generic approach one may perfectly well formulate and iden-
tify the problem, yet there are difficulties in resolving it. What we proposed [16],
[17] is to probe the model with reduced difficulties, starting with strictly plane waves.
Recently this work was generalized to higher dimensions and extra fields in [19]. In this picture one solves the collapse/inflation problem analytically relating the Kasner exponents near the singularity with the initial data for the dilaton and gravity waves. Once we do so, the path is free to ask some phenomenological questions as to the conditions for successful inflation, or entropy generation in these models.

We will be imposing the two BDV conditions in the distant past:
1) The string theory must be weakly coupled
   \( \exp(\varphi/2 \ll 1) \)
2) The curvature is small in string units.

To simplify the things we start with the graviton-dilaton system.

\[
S = \int d^4x \sqrt{-g} e^{-\phi} \left( R + g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi \right)
\]

Furthermore, we will assume throughout that the extra six spatial dimensions are compactified in some internal appropriate manifold considered to be non-dynamical.

The gravity wave being linearly polarized is specified by a single function \( \psi(r, s) \) and the scalar wave by \( \phi(r, s) \), where \( r \) and \( s \) are null coordinates \( r = t - z \) and \( s = t + z \).

Both waves satisfy the same linear Euler Darboux equation in the interaction region (c.f. [18])

\[
\begin{align*}
\psi_{,rs} + \frac{1}{2(r + s)}(\psi_{,r} + \psi_{,s}) &= 0 \\
\phi_{,rs} + \frac{1}{2(r + s)}(\phi_{,r} + \phi_{,s}) &= 0
\end{align*}
\]

These two equations, together with the initial data on the null boundaries of the interaction region \( \{\psi(r, 1), \psi(1, s)\} \) and \( \{\phi(r, 1), \phi(1, s)\} \) pose a well defined initial value problem. Both \( \psi(r, s) \) and \( \phi(r, s) \) are \( C^1 \) (and piecewise \( C^2 \)) functions.

Now, you may analytically integrate the solutions into the interaction region, and work out the Kasner exponents in terms of the incoming initial data.

The line element near the singularity is

\[
ds^2 = \xi^a(z)(-d\xi^2 + dz^2) + \xi^{1+\epsilon(z)}dx^2 + \xi^{1-\epsilon(z)}dy^2
\]

where \( a(z) \equiv \frac{1}{2}[\epsilon^2(z) + \varphi^2(z) - 1] \)

with

\[
\epsilon(z) = \frac{1}{\pi \sqrt{1 + z}} \int_z^1 ds \left[ (1 + s)^{\frac{1}{2}} \psi(1, s) \right]_{,s} \left( \frac{s + 1}{s - z} \right)^{\frac{1}{2}}
\]

\[
+ \frac{1}{\pi \sqrt{1 - z}} \int_{-z}^1 dr \left[ (1 + r)^{\frac{1}{2}} \psi(r, 1) \right]_{,r} \left( \frac{r + 1}{r + z} \right)^{\frac{1}{2}},
\]
and a similar expression for the dilaton

\[
\varphi(z) = \frac{1}{\pi \sqrt{1 + z}} \int_{s}^{1} ds \left[ (1 + s)^{\frac{1}{2}} \phi(1, s) \right]_{s} \left( \frac{s + 1}{s - z} \right)^{\frac{1}{2}} + \frac{1}{\pi \sqrt{1 - z}} \int_{-z}^{1} dr \left[ (1 + r)^{\frac{1}{2}} \phi(r, 1) \right]_{r} \left( \frac{r + 1}{r + z} \right)^{\frac{1}{2}}.
\]

The \( \epsilon \) and \( \varphi \) are the source functions expressed in terms of the gravitational and scalar field respectively. So, for example, we may ask which are the models undergoing the PBB inflation among all possible solutions parametrized by \( \epsilon \) and \( \varphi \), and the answer is “the piece” of cake in the plane \( \epsilon \) and \( \varphi \), meaning that the initial data leading to PBB is dense.

If the incoming data is weak (\( \epsilon \) close to 0 and \( \varphi \) around \(-1\)) the PBB inflation takes place. The fact that the inflationary data is dense one may say that PBB is stable a feature of this collision.

How weak should the waves be to solve horizon/flatness problem?

One usually defines the \( Z \) factor which is a ratio of co-moving Hubble radius at the time when the Dilaton Driven Inflation starts to the Hubble radius at the time the picture breaks down either due to the strong coupling or to the strong curvature. One may work that the strong curvature regime, and therefore the loop corrections are reached before the corrections in \( \alpha' \) become important. Expressing the conditions of successful inflation in terms of the focusing lengths of the waves one finds:

\[
L_{1}L_{2} \gtrsim e^{60(\beta + 2) - \alpha_{\min}} \ell_{pl}^2.
\]

Here \( L_{1} \) and \( L_{2} \) are the focal lengths of the waves. Therefore the waves must be extremely weak to solve the flatness and horizon problems. But this is nice, since this is what one expects anyway, if one has in mind a picture of these waves as small perturbations on an otherwise flat background.

4.1 Entropy Generation in a Wave-Collision-Induced PBB

One of the interesting questions in this scenario is the way the entropy is generated in the collision. First, let me argue that the entropy in the plane wave region is 0 (both the matter and gravitational). Imagine you want to identify the states of 0 gravitational entropy. First to mind comes Flat Space Time. Then? Penrose suggested FRW spacetime due to 0 Weyl tensor. But if we take the string theories as a guide the FRW doesn’t fit that well. Plane waves on the contrary represent the most simple exact string backgrounds (1): all the curvature invariants vanish. So (2) if one would try to describe the gravitational entropy with a homogeneous
function vanishing at the origin (not just the Weyl tensor) one would choose the plane waves as 0 entropy.

The 3rd point is due to triviality of plane wave s-t with respect to vacuum polarization [21]: no particle creation! One usually relates the quantum particle production (gravitons) to gravitational entropy-no such production takes place in the vicinity of plane wave, stressing again the trivial entropy content.

What happens then, when two such waves collide? The interaction region of the two incoming waves can be divided into three regions. Just after the collision takes place at $t_i$ the matter content of the universe can be satisfactorily described in terms of a superposition of the two incoming non-interacting null fluids. In this regime the evolution is approximately adiabatic and almost no entropy is produced. This almost linear regime comes to an end as soon as the gravitational nonlinearities take over and the collision enters an intermediate phase where the dynamics of the universe is dominated by both velocities and spatial gradients. Now the evolution is no longer adiabatic and matter entropy is generated. In this regime the matter content of the universe cannot be described in terms of a perfect fluid equation of state [22] and some effective macroscopic description of the fluid, such as an anisotropic fluid or some other phenomenological stress-energy tensor, should be invoked.

The production of entropy will stop at the moment the velocities begin to dominate over spatial gradients and the evolution becomes adiabatic again, the matter now being described by a perfect fluid with stiff equation of state $p = \rho$. The transition to an adiabatic phase happen either before or at $t_K$ when the universe enters the Kasner phase. From that moment on no further entropy is produced up to the end of DDI. The relative duration of these three regimes crucially depends on the strength of the incoming waves and on the initial data. It is straightforward to show that for both gravitational and scalar waves the ratio of spatial gradients versus time derivatives dies off as $1/\sqrt{L_1 L_2}$ and therefore it takes the universe a time of order $\sqrt{L_1 L_2}$ before the Kasner-like regime is reached, fixing the duration of the nonadiabatic phase.

As discussed above, all the entropy is generated in the intermediate phase between the adiabatic and Kasner regimes. On the other hand, in the Kasner regime the total entropy generated in this intermediate region is carried adiabatically by the perfect stiff fluid represented by the dilaton field $\phi(t)$. If we consider a generic barotropic equation of state, $p = (\gamma - 1)\rho$, the first law of thermodynamics implies that the energy density $\rho$ and entropy density $s \equiv S/V$ evolve with the temperature $T$ as

$$
\rho = \sigma T^{\gamma-1},
$$

$$
s = \gamma \sigma T^{\frac{\gamma-1}{\gamma}},
$$

where the temperature is given by the usual relation $T^{-1} = (\partial s/\partial \rho)_V$. Here $\sigma$ is a dimensionful constant which in the particular case of radiation ($\gamma = 4/3$) is just the Stefan-Boltzmann constant. Using these equations we easily find that for a stiff perfect fluid ($\gamma = 2$) the entropy density $s$ can be expressed in terms of the energy.
density as

\[ s = 2\sqrt{\sigma \rho}. \]

It is important to stress that \( \sigma \) is a parameter that gives the entropy content of, and thus the number of degrees of freedom that we associate with, the effective perfect fluid. In principle it could be computed provided a microscopical description of the fluid is available. However in the case at hand, the stiff perfect fluid is just an effective description of the classical dilaton field in the Kasner regime. Since the evolution in the DDI phase is adiabatic, \( \sigma \) can be seen as a phenomenological parameter that measures the amount of entropy generated during the intermediate region where the dilaton field should be described by an imperfect effective fluid. This effective description of the dilaton condensate, and the entropy generated, would then depend on a number of phenomenological parameters.

We can now give an expression for the total entropy inside a Hubble volume at the beginning of DDI in terms of the initial data. The energy density carried by the dilaton field in the Kasner epoch is given by

\[ \rho(t) = \frac{\dot{\phi}^2}{4\ell_{Pl}^2} = \frac{\beta^2}{4\ell_{Pl}^2 t^2}. \]

Thus, the total entropy inside a Hubble volume at the beginning of the Kasner regime can be written in terms of the Kasner exponents and the source function for the dilaton as

\[ S_H(t_K) = \frac{t_K^2}{\ell_{Pl}^2} \sqrt{\sigma \ell_{Pl}^2} \frac{\beta}{\alpha_1 \alpha_2 \alpha_3}. \]

Now we can write \( t_K = \eta \sqrt{L_1 L_2} \) with \( 0 < \eta \lesssim 1 \), so finally we arrive at

\[ S_H(t_K) = \left[ \eta^2 \sqrt{\sigma \ell_{Pl}^2} \frac{\beta}{\alpha_1 \alpha_2 \alpha_3} \right] \frac{L_1 L_2}{\ell_{Pl}^2} \equiv \frac{\kappa L_1 L_2}{\ell_{Pl}^2}. \]

It is interesting to notice that this scaling for the entropy in the Hubble volume at the beginning of DDI can be also retrieved by considering a particular example when, as a result of the wave collision, a space-time locally isometric to that of a Schwarzschild black hole is produced in the interaction region. In that case the focal lengths of the incoming waves are related with the mass of the black hole by \[ M = \sqrt{L_1 L_2}/\ell_{Pl}^2. \] Now we can write the Bekenstein-Hawking entropy of the black hole \( S = 4\pi L_1 L_2/\ell_{Pl}^2 \) in terms of the focal lengths of the incoming waves as \( S = 4\pi L_1 L_2/\ell_{Pl}^2 \).

This analogy is further supported by the fact that the temperature of the created quantum particles in both the black hole and the colliding wave space-time scales like \[ T \sim 1/M \] and \[ T \sim 1/\sqrt{L_1 L_2} \] respectively, as well as by the similarities between the thermodynamics of black holes and stiff fluids \[ [23] \]. It is important to notice that this scaling of the temperature of the created particles with the focal lengths implies
that, whenever enough inflation occurs, the contribution of these particles to the total entropy is negligible.

One may have thought, in principle, that it is possible to avoid the entropy production before the DDI by just taking a solution in the interaction region for which the evolution is globally adiabatic. The simplest possibility for such a solution would be a Bianchi I metric for which the Kasner regime extends all the way back to the null boundaries. This, however, should be immediately discarded due to the constraints posed by the boundary conditions in the colliding wave problem. Or, put in terms of the null data, it is not possible to choose the initial data on the null boundaries in such a way that the metric is globally of Bianchi I type in the whole interaction region and at the same time $C^1$ and piecewise $C^2$ across the boundary.

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