Identification of modal parameters of response signals in the white noise excitation with harmonics based on cepstrum

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Abstract. Operational Modal Analysis (OMA) makes up for the shortcomings of the traditional experimental modal analysis based on known excitation-response and only needs to identify the structure's modal parameters based on the vibration response signal of the structure. The OMA method is mostly based on white noise or random excitation. However, for rotating parts, the excitation is often not ideal white noise, and there are standard harmonic components caused by periodic signals. Without processing the harmonics, using the working modal analysis technology to identify the modal parameters is easy to produce false modal information. In order to study the recognition of modal parameters with harmonic excitation, in this paper, the three-free spring-mass-damping model is used as the research object to apply white noise excitation with harmonics, and the method of cepstrum editing is used to remove the harmonics. The Covariance-Driven Stochastic Subspace Identification (SSI-COV) method is used to obtain the structural modal parameters. The simulation results show that using the cepstrum editing method to filter out the harmonic interference components before using the working modal analysis technology to identify the modal parameters can effectively obtain the real modal information.

1. Introduction
Modal analysis is the process of analyzing the structure's inherent characteristics or modal parameters, such as the structure's natural frequency, damping ratio, and modal shape, through the use of vibration system theory. The computational modal analysis uses the finite element method to discretize the vibration structure, establish the mathematical model of the eigenvalue problem, and use various approximate methods to solve the eigenvalues and eigenvectors of the system and obtain modal information such as the natural frequency of the structure. Experimental modal analysis can be divided into the traditional experimental modal analysis (Experimental Modal Analysis, EMA) and operating modal analysis (Operational Modal Analysis, OMA)[1] according to whether the excitation method is measurable. EMA technology obtains the excitation and response signals of the structure through experiments, establishes the input-output model of the system, and obtains the structure's modal parameters by using the frequency response function. However, the OMA technology does not need to collect excitation signals; only the response signals are needed to identify modal parameters. In recent years, the OMA technology based solely on the response of unknown incentives or environmental incentives have attracted more and more attention at home and abroad and has been widely used in machinery, civil engineering, and other fields. The need for working modal analysis technology based only on response signals first appeared in the field of civil engineering because the use of force hammers or vibration exciters to excite large structures such as bridges and buildings, the vibration signals obtained are far greater than those in the environment It is difficult and costly under incentives.
When the vibration signal caused by the harmonic excitation part is identified in the modal parameter, the resonance peak generated will often be the same. Resonant frequencies with similar natural frequencies are mistaken for the natural frequencies of the structure, which can quickly generate false modal information, which leads to a decrease in the robustness of modal parameter identification [2]. For example, in the cutting conditions of machine tools, false modal information cannot provide accurate dynamic parameters, affecting the prediction of chatter boundaries. Therefore, the harmonic interference generated during modal parameter identification of rotating machinery in the machining state is a problem that needs to be solved. To solve the problem of operating modal analysis in the presence of harmonics, we can see a variety of methods to solve harmonic interference, which is generally divided into two categories [3]: The first type is to deal with the problem of harmonic interference in the presence of harmonics. The operating modal analysis method is improved; the second type eliminates the harmonic signal before the operating modal analysis. Brincker [4] proposed in 2000 that the use of statistical characteristics to find that the probability density curves of the harmonic excitation signal and the pure white noise excitation signal are different can be used to detect the harmonics. Modak [5] uses the random decrement method to remove the false modal information caused by harmonics. However, when the harmonic frequency is close to the system's natural frequency, the method based on statistical characteristics cannot achieve harmonic removal. Preprocessing the signal before OMA processing is called signal preprocessing technology, such as the time synchronization averaging method, non-parameter removal method, cepstrum editing method, etc. Compared with the previous two methods, cepstrum editing is the most direct and lower cost.

In this paper, the object adopted is a multi-degree-of-freedom vibration system. The primary purpose is to eliminate the harmonic components in random excitation through cepstrum analysis, and use the random subspace method based on covariance drive to identify the modal parameters of the system [6], and obtain the inverted Spectrum analysis can effectively remove the harmonic part and avoid false modal information when performing working modal analysis.

2. Cepstral properties

The formation process of the cepstrum is shown in Figure 1.

![Cepstrum formation process](image)

The cepstrum is divided into complex cepstrum and real cepstrum. For the time domain signal, the complex cepstrum is defined as [7]:

\[
C_f(\tau) = 3^{1/2} \left[ \log \left[ F(f) \right] \right] = 3^{1/2} \left[ \ln \left( A(f) \right) + j\phi(f) \right]
\] (1)

In equation (1), \(F(f)\) Represents the frequency spectrum of the time domain signal, \(A(f)\) is the amplitude spectrum of \(F(f)\), \(\tau\) Represents the frequency of the complex cepstrum. The convolution relationship can be converted into an additive form through the cepstrum operation:

\[
y(t) = f(t) * h(t)
\] (2)

In equation (2), \(y(t)\) represents the response signal of a linear system, \(f(t)\) represents the convolution of the input signal, \(h(t)\) represents the impulse response function of the transmission path. Perform Fourier transform on the response signal to get the complex spectrum of the response signal.

\[
Y(f) = F(f) \cdot H(f)
\] (3)

Logarithmic transformation of the complex frequency spectrum and inverse Fourier transformation can get the response signal's complex cepstrum.

\[
\hat{C}_f(\tau) = \hat{C}_f(\tau) + \hat{C}_s(\tau)
\] (4)
Take the logarithm of the amplitude spectrum of the input signal and then perform the inverse Fourier transform to obtain the signal’s real cepstrum. In the cepstrum domain, the cepstrum of the system’s vibration signal can be expressed as the sum of the cepstrum of the input signal and its impulse response function.

Cepstrum has the characteristics of homomorphic processing. Homomorphic processing is a method that tries to transform nonlinear problems into linear problems for processing. It can separate two signals synthesized by multiplication or convolution. Since the cepstrum of a periodic signal has periodic spikes, the periodic signal components can be determined according to the position of the peak in the cepstrum of the vibration signal. The cepstrum of the random excitation signal is random, and there is no peak. From this feature, the periodic signal can be judged, and the frequency of the periodic signal can be estimated. Periodic signals and random signals can be separated through the cepstrum editing process to eliminate harmonic interference. The editing process of the cepstrum is shown in Figure 2.

3. Simulation analysis
To verify the effectiveness of the above method, MATLAB is used to carry out numerical simulation of the three-degree-of-freedom linear time-invariant vibration system. In Figure 3.

According to Newton’s second law, the vibration system’s differential equation of motion is written in matrix form.

$$
\begin{bmatrix}
    m_1 & 0 & 0 \\
    0 & m_2 & 0 \\
    0 & 0 & m_3
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_1 \\
    \ddot{x}_2 \\
    \ddot{x}_3
\end{bmatrix}
+
\begin{bmatrix}
    c_1 + c_2 & -c_2 & 0 \\
    -c_2 & c_2 + c_3 & -c_3 \\
    0 & -c_3 & c_3 + c_4
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_1 \\
    \dot{x}_2 \\
    \dot{x}_3
\end{bmatrix}
+
\begin{bmatrix}
    k_1 + k_2 & -k_2 & 0 \\
    -k_2 & k_2 + k_3 & -k_3 \\
    0 & -k_3 & k_3 + k_4
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3
\end{bmatrix}
=
\begin{bmatrix}
    f_1 \\
    f_2 \\
    f_3
\end{bmatrix}
$$

Assuming that the damping matrix satisfies the condition of proportional damping: $[C] = \alpha[M] + \beta[K]$, $\alpha$, $\beta$ is a real number. The mass of the system is $m_1 = m_2 = m_3 = 1kg$, The damping of the system is $c_1 = c_2 = c_3 = 0.1N \cdot s \cdot m^{-1}$, The stiffness of the system is $k_1 = k_2 = k_3 = 10000N \cdot m^{-1}$. The
The eigenvalue method is used to obtain the theoretical values of the vibration system's modal parameters, as shown in Table 1.

Table 1. Theoretical values of modal parameters of a three-degree-of-freedom linear time-invariant vibration system.

| Mode order | Natural frequency(Hz) | Damping ratio(%) |
|------------|-----------------------|-----------------|
| 1          | 12.181                | 0.038           |
| 2          | 22.508                | 0.071           |
| 3          | 29.408                | 0.092           |

When applying a random excitation containing the first harmonic to the mass of the system, set the sampling frequency to 100Hz, and the sampling time to 100s, and use the state space method to set the corresponding parameters to obtain the system acceleration response signal in Simulink simulation, as shown in Figure 4. The excitation signal is \( x(t) = 0.1\sin(10\pi t) + 0.001\text{randn}(t) \).

![Figure 4. Time domain response signal](image1)

![Figure 5. Power density spectrum](image2)

In Figure 4, the harmonic part is not visible, so the cepstrum signal diagram of the response signal is calculated. Figure 5 shows the power spectral density function of the response signal. There is a peak at 5Hz. The direct use of the response signal for modal parameter identification will lead to false modes. Therefore, when identifying the modal parameters, harmonics need to be eliminated. Take the acceleration response signal of the first layer and find the actual cepstrum to get Figure 6.

![Figure 6. Cepstrum of a response signal](image3)

According to the periodic signal's cepstrum's signal characteristics, it can be seen in the figure that there is a spike corresponding to the harmonic frequency at 0.2s, and the harmonic excitation causes the spike. The Pap filter is used to remove the harmonic components at the spikes.

It can be seen in the power spectral density diagram of the response signal shown in Figure 6 that the harmonic components are effectively removed. Finally, the covariance-driven random subspace method in the working modal analysis is used to identify the modal parameters of the response signal, and the
modal parameters of the vibration system are obtained and compared with the theoretical values. Table 2 shows

| Mode order | Theoretical values. | SSI-COV |
|------------|---------------------|---------|
|            | $f$ (Hz) | $\xi$ (%) | $f$ (Hz) | $\xi$ (%) |
| 1          | 12.181  | 0.038      | 12.178  | 0.043      |
| 2          | 22.508  | 0.071      | 22.507  | 0.009      |
| 3          | 29.408  | 0.092      | 29.408  | 0.104      |

According to the above simulation results, it is concluded that the random excitation obtained after removing the harmonics can effectively identify the modal parameters of vibration through the SSI-COV method. For the response signal obtained from the white noise excitation with harmonics, in this case, it is very convenient to use the method proposed in this paper to complete the modal parameter identification.

4. Conclusion
This paper studies the modal parameter identification with harmonic excitation uses cepstrum editing to remove the harmonic part of the vibration response signal and uses the covariance-driven random subspace method for modal parameter identification. The nature of the cepstrum and the process of cepstrum analysis are introduced in detail. The advantages of this harmonic removal method are high robustness and low cost. After removing the harmonic part, the random subspace method based on random excitation is used for modal parameter identification, which effectively obtains the vibration system's real modal information. The simulation results prove the validity of the working modal analysis based on cepstrum editing. Therefore, it has good engineering application value.

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