Abstract. The SU(2) Yang-Mills gluon and ghost Green-functions are studied in Landau gauge by means of lattice gauge simulations. A focal point is their low energy behavior since in particular the Kugo-Ojima confinement criterion relates a diverging ghost form factor at vanishing momentum transfer to the confining capabilities of the theory. This divergence is verified by numerical simulations. Removing the confining vortices from the lattice ensembles converts SU(2) Yang-Mills theory in a non-confining theory. It is shown that in this modified theory the divergence of the ghost propagator disappears.

1. Introduction

The understanding of confinement of the fundamental degrees of freedom, the quarks and the gluons, is still one of the big challenges of QCD. In the case of heavy quarks, a signal of quark confinement can be deduced from the static quark anti-quark potential [1], which is linearly rising at low temperatures. There is only an indirect evidence for the confinement of gluons: there is no gluonic contribution to the vacuum energy density for temperatures below the critical one, while above the critical temperature the gluonic blackbody radiation dominates [2]. In the case of dynamical quarks with a small but non-vanishing quark current mass a precise definition of quark confinement is cumbersome. For these reason, a litmus paper which signals the absence of colored states from physical correlation functions would be highly desirable.

In the past, it has been argued that the low energy behavior of QCD Green-functions in Landau gauge encodes the information on confinement [3, 4]: Zwanziger argued that in the confinement phase the gauge field configurations which are relevant in the thermodynamic limit are restricted to the Gribov horizon [4]. In this case, a ghost propagator which diverges at zero momentum transfer reports that the above horizon condition is satisfied.
In the framework of the BRST quantization, Kugo and Ojima proposed a criterion which signals that the physical subspace only consists of color singlet states \([4]\). The general version of this criterion involves a certain Green-function which approaches \(-1\) in the zero momentum limit. Recent lattice investigations which challenge this general version can be found in \([5]\). It is easier from a technical point of view to test the criterion in Landau gauge. In this gauge, this criterion is fulfilled if the ghost form factor is singular at zero momentum \([6]\). It therefore coincides with Zwanziger’s horizon condition. In the derivation of the Kugo-Ojima criterion, one has assumed that a BRST charge operator is uniquely defined for the whole configuration space. At the present stage of investigations, this assumption is unjustified due to the presence of Gribov ambiguities \([1]\).

In order to explore the relation between the infrared behavior of Yang-Mills Green-functions and confinement, the vortex picture of confinement is a convenient tool at our disposal. Removing the confining vortices \([7]\) (see below for details) from the lattice configurations turns the Yang-Mills theory into a (presumably non-local) gauge theory which does not confine quarks.

In this paper, I will compare the gluon and ghost form factors of the pure SU(2) gauge theory with those of the modified (non-confining) theory. Firstly, this will provide information on the signature of confinement in these correlations functions. Secondly, since these Green-functions are important ingredients for hadron phenomenology \([8]\), a knowledge of the vortex impact on these Green-functions will help to understand the role of the confining vortices in hadron physics.

2. Approaching low energy Yang-Mills theory

2.1. THE VORTEX PICTURE

The basic idea to get insights into the mechanism of quark confinement is to project the SU(2) lattice configurations onto fields belonging to a simpler gauge group such as \(Z_2\). The \((Z_2\text{-gauge invariant})\) degrees of freedom of the latter theory are closed surfaces in four space time dimensions. Equivalently, at a given time slice these surfaces can be viewed as closed strings in the spatial hypercube, i.e. the so-called center vortices. The difficulty is to find the particular definition of these objects which ensures that they are sensible degrees of freedom in the continuum limit. A progress in this direction was made in the recent past \([7]\): a new gauge, the so-called

\[1\] I thank Pierre Van Baal and Maarten Golterman for helpful discussions.
Maximal Center Gauge (MCG) was invented. After MCG gauge fixing, the vortex degrees of freedom are defined by projecting full configurations onto vortex ones. Subsequently, a tight relation of these vortices to the physics of confinement was established: the MCG vortex ensembles reproduce the confining quark potential, while a removal (see e.g. [9]) of these vortices from the lattice ensembles results in a Coulomb type potential. Using this particular gauge, it turned out that the vortices are indeed sensible degrees of freedom in the continuum limit [10]. In addition, an intriguing picture of the deconfinement phase transition is available, which appears as a vortex de-percolation transition [11].

Here, I will use the lattice ensembles from which the MCG vortices have been removed as a model theory which does not show quark confinement. This theory which is defined by the modified lattice ensembles possesses the following properties: (i) it is SU(2) gauge invariant; (ii) it is presumably non-local, since non-local field configurations have been removed from a local theory; (iii) since the objects which have been removed exist on a physical length scale, the renormalization group flow towards the continuum limit is the same as the one of the full theory.

2.2. GLUON AND GHOST FORM FACTORS

In the following, the renormalized gluon and ghost propagators, \( D^{ab}_{\mu\nu}(p) \) and \( G^{ab}(p) \), are parameterized in Landau gauge by

\[
D^{ab}_{\mu\nu}(p) = \delta^{ab} P_{\mu\nu} \frac{F(p^2, \mu^2)}{p^2}, \quad G^{ab}(p) = \delta^{ab} \frac{G(p^2, \mu^2)}{p^2}, \quad (1)
\]

where \( p \) is the momentum transfer, \( P_{\mu\nu} \) is the transverse projector and \( \mu \) the renormalization point where \( F(\mu^2, \mu^2) = 1, G(\mu^2, \mu^2) = 1 \). By means of a truncated set of Dyson-Schwinger equations, the form factors in (1) have only recently been addressed [12, 13]. It was pointed out that, at least for a certain truncation scheme, the gluon and ghost form factors satisfy simple scaling laws in the infra-red momentum range, in particular (for \( p^2 \ll (1 \text{ GeV})^2 \))

\[
F(p^2, \mu^2) \propto \left[ \frac{p^2}{\mu^2} \right]^{2\kappa}, \quad G(p^2, \mu^2) \propto \left[ \frac{p^2}{\mu^2} \right]^{-\kappa}. \quad (2)
\]

Depending on the truncation of the Dyson tower of equations, one finds \( \kappa \) ranging from 0.5 to 0.7 [14, 15]. These findings match with Zwanziger’s horizon condition: the ghost form factor is singular in the limit \( p \to 0 \).

Of great importance for phenomenological purposes is the running coupling strength \( \alpha(p^2) \). This strength directly enters the quark DSE [10], and can be interpreted as an effective interaction strength between quarks.
Surprisingly, the prediction that the running coupling strength in Landau gauge \[12, 13\]
\[
\alpha(p^2) = \frac{\tilde{Z}_1^2(\mu^2, \Lambda)}{Z_1^2(p^2, \Lambda)} \alpha(\mu^2) F(p^2, \mu^2) G^2(p^2, \mu^2), \tag{3}
\]
approaches a constant in the limit \(p^2 \to 0\) is independent of the truncation and approximations used in \[12, 13\]. In the DSE approach the ghost-gluon vertex renormalization constant \(\tilde{Z}_1\) is set to unity. This is assumed since \(\tilde{Z}_1\) is finite at least to all orders of perturbation theory \[18\].

Figure 1. The gluon form factor (left panel) and the running coupling strength (right panel) as function of the momentum transfer; pure SU(2) gauge theory.

3. Numerical results

3.1. PURE SU(2) THEORY

The simulations were carried out on a \(16^3 \times 32\) lattice for \(\beta\) values ranging from 2.1 to 2.5. Physical units are obtained by eliminating the lattice spacing \(a\) using the scaling relation
\[
\sigma a^2(\beta) = 0.12 \exp \left\{ -\frac{6\pi^2}{11} (\beta - 2.3) \right\}, \tag{4}
\]
where I used the string tension \(\sigma = (440\text{MeV})^2\) as a reference scale. The form factors were directly calculated from appropriate correlation functions (rather than calculating the propagator times \(p^2\)). This method efficiently suppresses the statistical noise \[9\]. For a proper definition of the gluon
fields see [9], and for the definition of the ghost propagator see [17]. The Landau gauge condition was implemented using a simulated annealing algorithm [17].

Initially, I obtain the un-renormalized form factors $F_B(p^2, \beta)$, $G_B(p^2, \beta)$ as function of the momentum in physical units. Note that in view of (4) $\beta \propto \ln(\Lambda/\sqrt{\sigma})$ encodes the logarithmic dependence on the cutoff $\Lambda = \pi/a$. The desired renormalized form factors are obtained via multiplicative renormalization, i.e.

$$F(p^2, \mu^2) = Z_3^{-1}(\mu^2, \beta) F_B(p^2, \beta), \quad G(p^2, \mu^2) = Z_3^{-1}(\mu^2, \beta) G_B(p^2, \beta).$$

(5)

In the figures, the overall normalizations of the gluon and the ghost form factor, respectively, were arbitrarily chosen.

$$\text{SU(2), 16}^3\times32$$

Figure 2. The ghost form factor as function of the momentum transfer; pure SU(2) gauge theory.

The ghost-gluon vertex renormalization constant $\tilde{Z}_1$ can be obtained by demanding

$$\tilde{Z}_1^{-2}(\mu^2, \beta) \alpha_0 F_B(p^2, \beta) G_B^2(p^2, \beta) \to \text{finite}, \quad \forall p,$$

(6)

where $\alpha_0 \propto 1/\beta$ is the bare gauge coupling constant squared. In view of [18], one expects that $\tilde{Z}_1(\mu^2, \beta) \to \text{constant}$ in the lattice calculations when the continuum limit is reached $\beta \to \infty$. In practical lattice calculations using the Wilson action, it turned out that a significant dependence of $\tilde{Z}_1$ on $\beta$ is present for $\beta < 2.3$. My lattice results, however, indicate that $\tilde{Z}_1$ indeed approaches a constant for large values of $\beta$. A detailed analysis of these issues is work in progress [19].
A recent comprehensive study of the gluon propagator can be found in [20]. My numerical results for the pure SU(2) theory are shown in figure 1 and 2. At high energies the lattice data for the form factors nicely reproduce the predictions from perturbation theory and, in particular, the correct anomalous dimensions. Most important, a clear signal of a diverging ghost propagator for the momentum \( p \to 0 \) is obtained (see figure 2).

Figure 1 also shows the running coupling constant. The red solid line is the one loop perturbative prediction which diverges at \( \Lambda_{QCD} \), the Landau pole position. Comparing this 1-loop fit with the lattice data, it is possible to estimate the scale \( \Lambda_{QCD} \) of the one loop level which turns out to be roughly 1 GeV.

3.2. THE MODIFIED NON-CONFINING THEORY

![Figure 3](attachment:image.png)

**Figure 3.** The gluon form factor (left panel) and the running coupling strength (right panel) as function of the momentum transfer; modified non-confining SU(2) theory.

In order to construct the modified non-confining theory, I firstly implemented the maximal center gauge with the help of the procedure outlined in [7]. The \( Z_\mu(x) \) center fields are constructed from the full link configuration \( U_\mu(x) \) by center projection. The link variables \( U'_\mu(x) \) of the modified theory are subsequently defined by \( U'_\mu(x) = Z_\mu(x) U_\mu(x) \). When calculating the static quark anti-quark potential using the modified configurations, one finds a Coulomb type of behavior.

The modified configurations are subsequently subjected to the Landau gauge fixing procedure. Employing the Landau gauge fixed ensembles, one obtains the gluon and ghost form factor and the running coupling of the modified theory from the usual correlation functions. This procedure was
Figure 4. The ghost (right panel) form factor as function of the momentum transfer; modified non-confining SU(2) theory.

carried out at several $\beta$ values. The result is shown in figures 3 and 4. The solid lines are the fits to the data points obtained in pure SU(2) theory. One observes that the interaction strength in the intermediate momentum range is drastically reduced when the vortices have been removed. In addition, one observes that divergence of the ghost form factor in the infrared limit disappears. This indicates that in the Zwanziger picture of confinement the removal of the vortices shifts the relevant configurations away from the Gribov horizon: ghost and gluon form factors only acquire moderate corrections to the free field limit. Since the SU(2) scaling behavior also applies for the case of the modified theory, one can use (4) to remove the lattice spacing in favor of a physical energy scale: indeed, one observes that data points for the gluon and ghost form factor fall on top of a single curve also in the case of the modified theory.

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