Capacity Planning in Stable Matching: An Application to School Choice

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Centralized mechanisms are becoming the standard approach to solve several assignment problems. Examples include the allocation of students to schools (school choice), high-school graduates to colleges, residents to hospitals and refugees to cities. In most of these markets, a desirable property of the assignment is stability, which guarantees that no pair of agents has incentive to circumvent the matching. Using school choice as our matching market application, we introduce the problem of jointly allocating a school capacity expansion and finding the best stable matching for the students in the expanded market. We analyze theoretically the problem, focusing on the trade-off behind the multiplicity of student-optimal assignments, and the problem complexity. Since the theoretical intractability of the problem precludes the adaptation of classical approaches to solve it efficiently, we generalize existent mathematical programming formulations of stability constraints to our setting. These generalizations result in integer quadratically-constrained programs, which are computationally hard to solve. In addition, we propose a novel mixed-integer linear programming formulation that is exponentially-large on the problem size. We show that the stability constraints can be separated in linear time, leading to an effective cutting-plane method. We evaluate the performance of our approaches in a detailed computational study, and we find that our cutting-plane method outperforms mixed-integer programming solvers applied to existent formulations extended to our problem setting. We also propose two heuristics that are effective for large instances of the problem. Finally, we use the Chilean school choice system data to demonstrate the impact of capacity planning under stability conditions. Our results show that each additional school seat can benefit multiple students. On the one hand, we can focus on access by prioritizing extra seats that benefit previously unassigned students; on the other hand, we can focus on merit by allocating extra seats that benefit several students via chains of improvement. These insights empower the decision-maker in tuning the matching algorithm to provide a fair application-oriented solution.

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