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A six compartments with time-delay model SHIQRD for the COVID-19 pandemic in India: During lockdown and beyond

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Abstract Dynamics of COVID-19 outbreak in India are different from other countries for her huge population and different administrative approaches. This paper presents a six compartments (SHIQRD) model with different time delays between them, to analyse and forecast the COVID-19 pandemic in India. The model introduces separate compartments for ‘home-quarantined’, ‘quarantined infectious’, and ‘undetected infectious’ pertaining to the Indian scenario. It also incorporates incubation time, sample testing time, and recovery window as time delays between blocks. With the proposed model, reproduction number of different phases during lock-down are evaluated. Besides, dynamics of pandemic for the next one year are also explored.

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1. Introduction

Since the first case reported in Wuhan, China, in Dec 2019, SARS-CoV-2, commonly called as COVID-19, has spread over 213 countries/territories. The pandemic has infected more than 7.5 million people and is responsible for 425 thousand deaths as on 12.06.20 [1]. The impact of COVID-19 in society, economy, innovation and day-to-day life can only be compared with the impact of the last world war in recent history. While the development of vaccines or any anti-viral drugs for COVID-19 are still in progress, administrations have adopted different measures to limit its spread. A proper model for the dynamics of COVID-19 outbreaks would be an important tool in this regard: to analyse the success of these measures and to determine an optimal approach. A compartmental model, based on a set of ordinary differential equations, has been so far the most popular approach to represent an infectious disease. Such model was first proposed by Kermack-McKendrick in 1927 which included three fundamental compartments: Susceptible (S), Infectious (I) and Recovered (R) [2]. Over the time, there have been many modifications on this basic SIR model namely, SIS, SEIR (E: Exposed), MSIR, MSEIR, for different infectious diseases like Malaria, AIDS, Chikungunya, Dengue for whom vaccinations have not yet discovered [3].
Since the outbreak of COVID-19, data analysts and epidemiologists have reported several such models on it [4,5]. A generic stability analysis of SEIR model for COVID-19 using observability matrix and Kalman filter is presented in [6]. As the outbreak started from Wuhan, initial researches mainly focused on Wuhan [7–9] or China [10]. The main objective of these studies was to identify the reproduction number ($R_0$), the case fatality rate (CFR) and the peak infection rate of the pandemic. The SEIR based model by Kucharski et al. reported that the $R_0$ value in Wuhan had reduced from 2.35 to 1.05 by the end of January, 2020, but, the outbreak was likely to spread pan China. Another SEIR model reported by Chen et al., included BHDP network and presented a comparison between $R_0$ values of MERS and COVID-19 [11]. Nadirou et al. modified the traditional SEIR block to incorporate a new compartment, i.e., Q (quarantined) [8] and Ivorra et al. took into account the presence of undetected infectious in the modelling [9].

As the pandemic set into western countries like, Italy, Spain, France, UK and USA, researchers came upon with similar SIR/SEIR based models for these countries as well [12–15]. However, China-based simple SIR/SEIR models failed for these countries for not incorporating country specific socio-political scenarios. The same shortcomings can also be observed in works reported for Indian scenario [16–19]. Not unlike others, these referred works too followed the basic SEIR model, whereas, the actual situation in India is far more complicated.

First of all, India has a gigantic population of 1.38 billion, which is only comparable to China (1.44 billion); the nearest next is USA whose population is only 0.33 billion [1]. We observed how the peak infection rate for USA (36 k/day) had been much higher than that of Italy (6.6 k/day) or Spain (8.2 k/day) owing to its larger population [1]. The fact put India even in a more severe jeopardy. Second, India has a different socio-political structure than China. It is more like the USA and it would not be possible for the Govt. of India to exercise very strict restriction on common mass. Third, the method of battlement is quite different in India. While infected are quarantined in a hospital, uninfected persons who were in contact with the infected ones are also put into home quarantine. This particular class needs a separate compartment, called ‘Home Quarantine’ of uninfected persons. Fourth is the undetected infectious. There is a high possibility of having a large number of undetected infectious in India due to India’s large geographic terrain, lack of awareness and social stigma in common people, and asymptotic nature of infection in many cases [20]. The undetected infectious is a major factor in the COVID-19 pandemic in India and need to be dealt explicitly in the model. Fifth, India practised four consecutive phases of pan-India lock-down from 25th March to 31st May, but, with passing phases more relaxation on movement and activities have been allowed to balance between COVID-19 outbreak and a weakening economy. Finally, more than fifty millions of ‘migrant workers’ of India became jobless and stuck away from their home during first three phases of lockdown. They were relocated to their own places during the fourth phase of lockdown by many ‘Shramik (worker) Special train’ [21] which brought more population of the country to susceptible state.

Therefore, any mathematical model for COVID-19 outbreak in India need to include above features of Indian scenario. In absence of that, the forecasting will be either too high [16] or too less [17] compared to reality. This paper, thus, presents a 6 compartment based model (SHIQRD) with time delay features to represent the Indian scenario more accurately. The model is described in Section 2. Statistical estimations of model parameters are discussed in Section 3. Evaluation of contact rate and analysis of lock-down phases are given in Section 4. Forecasting of pandemic phase in next one year under different case studies are presented in Section 5. The paper concludes in Section 6.

2. The proposed 6 compartments (SIQHRD) model for India

The proposed SIQHRD model contains 6 basic compartments (Fig. 1) whose dynamics are represented in discrete domain by a set of recurrence equations. This will make the model more compatible with computer programming and available data (data are available on per day basis [22]). The nomenclatures of the model are described below:

- **S(k)** = Susceptible: The population at the beginning of the $k^{th}$ day which is not yet infected and also not in home-quarantine.
- **H(k)** = Home-quarantined: The population at the beginning of the $k^{th}$ day which is not infected but is in home-quarantine mode. After another $k_H$ days, it will return back to the S block.
- **I(k)** = Undetected Infectious: The population at the beginning of the $k^{th}$ day which is infectious but not yet detected. Persons belong to this block may be detected in future or may not. Those who will not be detected at all, will either recover or die without going through any quarantine.
- **Q(k)** = Quarantined Infectious or Active Quarantine: This block refers to the infectious population on the $k^{th}$ day who are detected by medical officials and are put into quarantine/isolation in hospital.
- **R(k)** = Recovered: It refers to total recovered population up to the $k^{th}$ day. It is basically a summation of two sub-compartments: $R_I(k)$ and $R_Q(k)$. $R_I(k)$ refers to the recovered population whose infection were never detected and $R_Q(k)$ are the recovered ones coming from the Q block.
- **D(k)** = Deceased: It refers to total deceased population, resulted by the COVID-19 infection up to the $k^{th}$ day. It is also a summation of two sub-compartments: $D_I(k)$ and $D_Q(k)$. $D_I(k)$ refers to the deceased population whose infection were never detected and $D_Q(k)$ are the deceased ones from the detected infectious.

- **p(k)** = The probability of a susceptible to come in contact with an infectious on the day k. From the fundamental theory of probability, we observe that, $p(k) = \frac{S(k)C(k)}{N(k)}$.

$T(k)$ = The model population or the total population of all compartments together, at the beginning of the day $k$. $T(k)$ is not a constant value. It is increasing at the same rate of India’s current growth rate = birth rate $B(k)$– natural death rate $D(k)$. India’s current growth rate is 0.99% per year [1].

$x$ = Contact rate, i.e., the number of persons who come in contact to an infectious per day.

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1. The model population $T(k)$ may be equal to the country population $C(k)$ or may not, depending upon the case we are studying. This point will be elaborated Section 3 and 4.
\( \beta = \) Infection rate, i.e., the fraction of contacted persons that gets infection per day.

\( \eta = \) Detection rate / Quarantine ratio, i.e., the fraction of infectious person that gets detected.

\( \mu = \) Home Quarantine multiplier, i.e., the number of uninfected persons who remain in home-quarantine for being contacted with an infectious.

\( \delta = \) Fatality rate of the infected population.

\( \Delta R_h(k), \Delta R_q(k) = \) The number of persons that recovered at the day \( k \) from I and Q, respectively.

\( \Delta D_h(k) , \Delta D_q(k) = \) The number of persons that died at the day \( k \) from I and Q, respectively.

\( k_H = \) The duration of the home-quarantine (days).

\( k_q = \) The average number of days elapsed between a person getting infection and getting quarantined. It is approximated as the incubation period.

\( k_d = \) The average number of days between the day of quarantine and the day of death.

2.1. Process description and describing function

Let us, on the \( k^{th} \) day, total \( p(k) \) number of persons come in contact with the infectious population. Out of them, only \( \beta \) fraction gets infected and moves to the I compartment. This factor depends on the measure of social distancing, habits of personal protection (use of mask, sanitizer etc.), average immunity level of the group, possible mutation of the virus in India, population density of the location and the age-sex distribution pyramid. After \( k_q \) days, symptoms become evident in an infected person. Consequently, he/she is quarantined by health officials. However, there are several reports of asymptomatic cases [20]. Besides, not everyone (showing symptoms) is reporting to health officials due to his/her lack of awareness and social stigma. Therefore, only \( \eta \) fraction of the infected population gets quarantined. We also incorporate a time delay here. The person who is infected on the \((k - k_q)^{th}\) day, moves to the Q block on the \( k^{th} \) day.

At the same time, those who are in close contact with the detected infectious, like family members, personal helps, drivers move to the ‘home-quarantine mode’ for next \( k_H \) days. In this way, a certain number of persons who are not infected, are temporarily removed from the S compartment and are put to the H compartment for said \( k_H \) days. It is a special event pertaining to Indian scenario and one of the key features of the proposed model. We take, on average, \( \mu \) number of persons per infectious, moves to the H block on the day \( k \).

On the other hand, entities of the I block, who will never be quarantined (undetected infectious), will either recover or die. We take that on the \( k^{th} \) day, \( \Delta R_h(k) \) is the number of recovered persons from the I compartment who move to the \( R_h \) sub-compartment. Whereas, \( \Delta D_h(k) \) is the number of deceased from the I compartment who move to the \( D_h \) sub-compartment. Similarly, on the day \( k \), \( \Delta R_q(k) \) and \( \Delta D_q(k) \) move from the Q compartment to \( R_q \) and \( D_q \) sub-compartment, respectively. Hence, the process so far can be described by following discrete equations:

\[
S(k + 1) = S(k) - \beta S(k) I(k - k_q) + \mu \eta \beta S(k - k_q) + \eta \beta S(k - k_q) - \Delta S(k),
\]

\[
H(k + 1) = H(k) + \mu \eta \beta I(k - k_q) - \mu \beta (k - k_q) - \eta \beta (k - k_q) - \Delta H(k),
\]

\[
I(k + 1) = I(k) + \beta S(k) I(k - k_q) - \eta \beta (k - k_q) - \Delta R_h(k) - \Delta D_h(k),
\]

\[
Q(k + 1) = Q(k) + \eta \beta (k - k_q) - \Delta R_q(k) - \Delta D_q(k),
\]

where, \( p(k) = \frac{\alpha S(k)(k)(k)}{T(k)} \).

It should be mentioned here that, \( D_q(k) \), the total number of natural deaths per day is ideally distributed over blocks S, H, I, Q and R, but, for simplicity we deduct the entire number only from the S block without losing much accuracy.

The recovery of COVID-19 patients, however, follows a unique pattern. In traditional model of SIR, a typical recovery time (say, \( t_r \)) is adopted and its inverse \((1/t_r)\) is used as the recovery rate. This means that the patient can get recovered on any day between 0 to \( k \), after getting infected, with equal probability. However, this is not the case for a COVID-19 patient, especially, when he/she is quarantined in a hospital. In India, patients are treated in hospitals for minimum \( k_{t_1} \) days to maximum \( k_{t_2} \) days. Therefore, we here introduce the concept of a ‘recovery window’. We assume that patients are getting moved from the I compartment to the \( R_q \) sub-compartment, with equal probabilities, between \( k_{t_1} \) and \( k_{t_2} \) days. So, we write,

\[
\Delta R_q(k) = \frac{1}{k_{t_2} - k_{t_1}} \sum_{k_{t_1}}^{k_{t_2}} Q(k - k_{t_q}) - \Delta D_q(k - k_{t_q}).
\]

As no drug has yet been discovered for the infection and no supporting data is available for undetected infectious, we assume that the pattern of recovery of an undetected infectious is same as the detected ones. So, we can write,

\[
\Delta R_q(k) = \frac{1}{k_{t_2} - k_{t_1}} \sum_{k_{t_1}}^{k_{t_2}} Q(k - k_{t_q}) - \Delta D_q(k - k_{t_q}).
\]
\Delta R_t(k) = \frac{1}{k_{sI2} - k_{sI1}} \sum_{i=r_{I1}}^{k_{sI2}} I(k - k_{sI1}) - \Delta D_t(k - k_{sI1}) - \eta \beta \delta \eta \frac{k_{Q} - k_{D}}{k_{Q}}. 

(6)

Here, \(k_{sI1} = k_{sI} + k_{Q}\) and \(k_{sI2} = k_{sI} + k_{Q}\).

For the similar reason, we assume that the fatality rate \(\delta\) is also same for both detected and undetected cases with an average delay of \(k_D\) between the day of quarantine and the day of death. Hence,

\[
\Delta D_Q = \frac{\eta \beta p(k - k_D)}{D_t} \quad (7)
\]

\[
\Delta D_t = \delta (1 - \eta) \eta \frac{k_{Q} - k_{D}}{k_{Q}} \quad (8)
\]

Finally, we can write,

\[
R_t(k + 1) = R_t(k) + \Delta R_t(k) \quad (9)
\]

\[
R_Q(k + 1) = R_Q(k) + \Delta D_Q(k) \quad (10)
\]

\[
D_t(k + 1) = D_t(k) + \Delta D_t(k) \quad (11)
\]

\[
D_Q(k + 1) = D_Q(k) + \Delta D_Q(k) \quad (12)
\]

\[
R(k + 1) = R_t(k + 1) + R_Q(k + 1) \quad (13)
\]

\[
D(k + 1) = D_t(k + 1) + D_Q(k + 1) \quad (14)
\]

2.2. Assumptions

1. The population is homogeneous in nature.
2. The rate of infection is same irrespective of sex, age, ethnicity of people, climate, or geographic location.
3. Recovery is equally probable (rectangular distribution) over a certain period of time: \([r_{I1}, r_{I2}]\) (detected infectious) and \([r_{Q1}, r_{Q2}]\) (undetected infectious).
4. The rate of recovery is identical for both detected and undetected cases.
5. The rate of death is identical for both detected and undetected cases.
6. There will be no reinfection for the recovered and the recovered will be removed from the susceptible forever.

3. Evaluation of model coefficients and delay parameters

As per above equations (Eq. (1)–(14)), the proposed model has five rates or ratio parameters: \(x, \beta, \eta, \delta,\) and \(\mu\) and five time delay parameters, \(k_Q, k_t, k_{sI}, k_{sI2},\) and \(k_D\). For the successful execution of the model one need to evaluate or estimate these parameters as accurately as possible.

Estimation of \(\beta\). Infection rate or \(\beta\) is estimated in this work from the cumulative data of ‘sample tested’ (Let us denote it as \(A_C(k)\)). Per day data of ‘sample tested’ are obtained from [22,23]. First, correlation coefficients between \(Q_C(k)\) and \(A_C(k - k_T)\) are evaluated for different values of \(k_T\). \(Q_C(k)\) is the cumulation of all the quarantined persons till the day \(k\). \(k_T\) is the average time delay to get the test result. It is found that the value of correlation coefficient is maximum when \(k_T = 2\). Based on this, we write

\[
Q_C(k) = \beta A_C(k - 2) + \text{offset} \quad (15)
\]

The value of \(\beta\), as evaluated by Eq. (15), is plotted in Fig. 2a for the period April 10 to May 31, 2020. During that period, the mean rate of infection (\(\beta\)) is found to be 4.08%. Corresponding 95% confidence intervals are \([3.35\%, 4.81\%]\) and normalized standard deviation (SD) is 0.09. We see that the value of \(\beta\) gradually decreased during the lock-down phase L2. It reflects increasing social awareness and growing habit of personal protection. However, beyond May 1, 2020, \(\beta\) again started to increase. This can be explained by the event of mass movement of ‘migrant workers’. India commenced her Shramik Special Train Service from the 1st May. [21].

We extrapolated the trend of L2 over the last two phases to assess what \(\beta\) value would be if migrant workers had not been moved. In that case it is found to be 3.2% on June 19. Values of \(\beta\) evaluated here are further used to determine the reproduction number in Section 4 and to forecast the pandemic pattern in Section 5.

Estimation of fatality rate \(\delta\) and \(k_D\). Fatality rate is also estimated stochastically using the time-delay based relation between \(D_Q(k)\) and \(Q_C(k)\) as follows:

\[
D_Q(k) = \delta Q_C(k - (k_D - k_Q)) + \text{offset} \quad (16)
\]

\[
\Rightarrow \Delta D_Q(k) = \delta Q(k - (k_D - k_Q)) \quad (17)
\]

In this case, the correlation coefficient between \(D_Q(k)\) and \(Q_C(k)\) is maximum when,

\[
(k_D - k_Q) = 0 \Rightarrow k_D = k_Q \quad (18)
\]

The value of \(\delta\) estimated by the above method, is presented in Fig. 2b. We observe that the fatality rate gets apparently stabilized from April 1, 2020. During that period, the mean fatality rate is 3.2%. The 95% confidence intervals are \([2.81\%, 3.59\%]\) with normalized SD = 0.06. This value of \(\delta\) is used later in Section 5.

Value of \(k_Q, k_{sI}, k_{sI2},\) and \(k_D\). The \(k_Q\) is approximated to the incubation time which is taken 5 days in this work [17]. \(k_{sI}\) is the number of days in home- quarantine and it is 14 days as per most of the states in India [24]. \(k_{sI1}\), and \(k_{sI2}\) are determined stochastically by evaluating maximum correlation coefficients and minimum RMS errors. They are found to be 9 days and 23 days, respectively.

Therefore, at the end of this section, only three parameters remain unknown, i.e., \(x, \eta\) and \(\mu\). Next, we implement the model using the describing equations in MATLAB 2018a. From that, we derive the value of \(x\) based on some judicially assumed values of \(\eta\) and \(\mu\). This will be discussed elaborately in the next section.

4. Contact rate, reproduction number \((R_0)\) and effects of lockdown

India went through four consecutive phases of lock-down of total 68 days: Phase 1: March 25 to April 14 (21 days), Phase 2: April 15 to May 3 (19 days), Phase 3: May 4 to May 17 (14 days) and Phase 4: May 18 to May 31 (14 days). Where there was a complete shut-down apart from a few emergency services in the first two phases of lock-down, some secondary services like stand-alone shops, single rider bikes were allowed in the Phase 3 and Phase 4 for some specific durations in daytime. In this section, we shall discuss how the effect of lockdown changed the dynamics of COVID-19 pandemic in India. For that we acquired necessary data from www.COVID19India.org [22] and used the proposed model to determine the \(x\) value (contact rate) and \(R_0\) (reproduction number). In the said source data are available from March 14, 2020. The duration of March 14 to March 24 (10 days) is here denoted as L0,
and the said lock-down phases are denoted as L1, L2, L3 and L4, respectively.

In simulations, we take approximately 15% of Indian population (~207 million) comes within the scope of the model population \( T(k) \). Most part of it belongs to the susceptible class at the start of the L1 phase. It was assumed based on the district data of ‘detected infectious’ at that time. We also assume that during the lock-down phases model population \( T(k) \) grows at the rate of India’s natural growth rate; no new fraction of India’s population (\( C(k) \)) enters into the \( T(k) \).

As the value of \( \eta \) is not known to us, we here present four case studies: (I) \( \eta = 1/2 \) (II) \( \eta = 2/3 \) (III) \( \eta = 4/5 \) and (IV) a gradual increase in the \( \eta \) value from \( 1/2 \) to \( 2/3 \). The fourth case is justified by the fact that over the time, the Govt. of India has adopted many measures to track all infected persons as much as and as soon as possible. Several social awareness programs by different governmental and non-governmental bodies, thermal screening at hotspots and implementation of ‘Aarogya Setu App’ are some of such initiatives. Table 1 presents values of \( x, R_0 \) and numbers of active and total undetected infectious for these four cases, as on 31st May (as per the proposed model). The \( \mu = 10 \) is chosen for these case studies. The reproduction rate \( R_0 \) is found using the following formula:

\[
R_0 = \frac{2\beta\left(\eta k_Q + (1-\eta)\left(\frac{k_{IH} + k_{HR}}{2}\right)\right)}{c}
\]  

(19)

The Table 1 shows that the basic reproduction number, \( R_0 \) can be as high as 3.99 before lock-down starts (L0), depending upon the value of \( \eta \). However, for any value of \( \eta, R_0 \) decreases with consecutive lock-down phases. In literature, we have found that the reported reproduction rate for China is 2.35 to 1.05 [7], 3.58 to 2.2 [11] and 4.2732 [9]; in [9] cases of undetected infectious are considered. We see that the range of \( R_0 \) determined here, belongs to the same range that is reported for China.

The sensitivity of \( R_0 \) with respect to \( \mu \) is, however, almost negligible. We can see that for a wide variation of \( \mu \) from 5 to 20, \( R_0 \) is practically unaltered, viz. Table 2.

To understand the impact of each stage of the lock-down, we evaluate possible trends of ‘active quarantined’ \( (Q(k)) \) and ‘deceased from quarantine’ \( (D_Q(k)) \) using the proposed model for the cases where we would have only one, two, or three lock-down phases. Estimated trends are plotted in Fig. 3 for the time period from March 20, 2020 to May 29, 2020 in comparison with actual data. The trends are estimated by using the contact rate of the last availed lock-down phase and with varying \( \eta \). It is evident from these plots that if only L1 phase was adopted, total number of persons in quarantine on May 31 would be 250 k instead of 85 k (actual). The number of deceased would also be increased by 2.5 times. These trends establish the importance of 4 phases of lock-down in India.

5. India beyond lock-down: Prediction and possibilities

Despite the success of lock-down phases in reducing the counts of infectious and deceased, the country had to move on to the ‘unlocked phase’ on June 1, 2020. It is now more important for administration to predict accurately the upcoming surge in ‘quarantined’ population and to keep necessary actions to reduce the peak as much as possible without going into a re-lock-down.

The most crucial factor of the unlock phase is that the total model population \( T(k) \) will not remain constant. India being a large country (the 7th in the world) with huge population, initially its entire population was not considered under this epidemic model. Based on the district level data of infected persons (as on March 25), we assumed that only 15% of India’s population as the model population. We also assumed that this percentage would remain constant through out the lock-down phases. However, as the lock-down has been lifted, population of more and more area shall come into the susceptible domain. The phenomena is already evident in the pattern of infection rate \( \beta \). As mentioned in Section 3, there was a gradual increase in \( \beta \) value from May 1, viz. Fig. 2a. This was because India had started to move her migrant labourer throughout the country from that time and it basically had brought new populations into the susceptible class. As we had chosen a ‘constant model population’ (except normal growth), this indirectly increased the infection rate.

However, a more accurate approach would be if we include a feature in the proposed model where every day a new fraction of country population \( C(k) \) enters into the S block. This can be incorporated by rewriting Eq. (1) as follows,

\[
S(k+1) = S(k) - \beta p(k) - \mu b(k) - k_Q + \mu b(k) - k_H - k_H + B(k) - D_s(k) + s(k),
\]

(20)

where \( s'(k) = \gamma \frac{N(t)(C(k) - S(k))}{(9/10)} \) and \( \gamma \) is the coefficient of increment.

![Fig. 2](image-url) (a) Estimated infection rate (\( \beta \)) and (b) fatality rate (\( \delta \)) at different phases of lock-down (L1, L2, L3, L4) in India (L0: pre-lock-down phase).
This modification in the traditional compartmental model to emulate the Indian scenario is one of the key contributions of this work. Other reported models took a fixed population for $T(k)$ throughout the time-line, but, that should not be the case for COVID-19 pandemic in India. At present, it is difficult to estimate the value of $c$ accurately. The increasing trend of $b$ during phase L4, yields $c / C_0 = 0.01$. We, thus, study the forecasting model for different values of $c$ from 0.001 to 0.02. Fig. 4a shows how model population may increase in the next one year for different values of $c$.

5.1. Trends of $Q(k)$ for different values of $c$

In Section 3 we have estimated that $\beta = 0.0443$, and the mean $\delta = 0.032$, as on 31.05.2020. In Section 4 we found that $z = 3.74$ and $R_0 = 1.19$ if we choose a varying $\eta$ case. Assuming that these values of $x, \beta$ and $\eta$ will remain same for thenext one year, we derive the trends of population of different blocks using the proposed model. The trend of $Q(k)$ is presented in Fig. 4b, for three different values of $c$: 0.001, 0.005, and 0.010. It shows that at current rate of $c$ (0.010), the $Q(k)$ shall attain a peak of 9.3 million on Feb 17, 2021. Moreover, this will results in 620 thousands hospitalizations per day at its peak. This will impart a tremendous load on India’s health ser-
vices. Even, the increment coefficient $c$ is reduced to 1/10th of its current rate, the peak value of $Q(k)$ will be 2.95 million occurring on Nov 3, 2020 and the peak rate of hospitalization will be 324 thousands/day. Nevertheless, as per the current dataset, $c$ value of that low is quite unlikely. Therefore, one needs to explore other possibilities to contain the epidemic. The only option is to reduce $R_0$.

### Table 1

Estimated values of $x$ (contact rate), $R_0$, and number of active and total undetected infectious as per the proposed model (with $\mu = 10$) at different phases of the lock-down.

| Phase       | $\eta = 0.50$ | $\eta = 0.67$ | $\eta = 0.80$ | Varying $\eta$ |
|-------------|---------------|---------------|---------------|----------------|
|             | $x$    | $R_0$ | $x$    | $R_0$ | $x$    | $R_0$ | $x$    | $R_0$ |
| Phase L0    | 7.14  | 3.99  | 7.47  | 3.32  | 7.79  | 2.74  | 7.14  | 3.99  |
| Phase L1    | 4.59  | 2.60  | 5.12  | 2.30  | 5.65  | 2.01  | 4.45  | 2.01  |
| Phase L2    | 3.22  | 1.64  | 3.81  | 1.55  | 4.48  | 1.44  | 3.80  | 1.34  |
| Phase L3    | 3.13  | 1.52  | 3.74  | 1.44  | 4.43  | 1.35  | 4.07  | 1.25  |
| Phase L4    | 2.67  | 1.44  | 3.20  | 1.38  | 3.81  | 1.30  | 3.75  | 1.21  |
| Active Undetected (31.5.20) | 187634 | 116537 | 80956 | 75527 |
| Total Undetected (31.5.20)   | 281317 | 163393 | 104378 | 107183 |

### Table 2

Variation of $R_0$ as per the proposed model for different values of $\mu$ with varying $\eta$.

| Phase       | $\mu = 5$ | $\mu = 10$ | $\mu = 20$ | $\mu = 40$ |
|-------------|-----------|------------|------------|------------|
| Phase L0    | 3.99      | 3.99       | 3.99       | 3.99       |
| Phase L1    | 2.01      | 2.01       | 2.01       | 2.01       |
| Phase L2    | 1.34      | 1.34       | 1.35       | 1.35       |
| Phase L3    | 1.25      | 1.25       | 1.25       | 1.26       |
| Phase L4    | 1.19      | 1.19       | 1.20       | 1.19       |

![Fig. 3](image-url) Trends of total numbers of (a) active quarantined persons and (b) deceased from the quarantined compartment, if the consecutive phases of lock-down were not implemented.
As per Eq. (19), \( R_0 = f(x, \beta, \eta, k_Q, k_{in}, k_{it}) \). Out of these, the last two variables are related to undetected cases, hence, cannot be controlled. The \( a \) value cannot be expected to be lessened any further now that the lockdown is over. This leaves only three parameters: \( b \), \( g \) and \( k_Q \). Sensitivities of \( R_0 \) with respect to them can be written as:

\[
\frac{\partial R_0}{\partial \eta} = \frac{\eta (k_Q - 0.5(k_{in} + k_{it}))}{k_{in} + 0.5(k_{in} + k_{it})(1 - \eta)} \tag{21}
\]

\[
\frac{\partial R_0}{\partial \beta} = 1 \tag{22}
\]

\[
\frac{\partial R_0}{\partial k_Q} = \frac{\eta k_Q}{k_{in} + 0.5(k_{in} + k_{it})(1 - \eta)} \tag{23}
\]

As per present values of \( \eta, k_Q, k_{in} \) and \( k_{it} \), we get, \( \frac{\partial R_0}{\partial \eta} = -1.74 \), \( \frac{\partial R_0}{\partial \beta} = +1 \) and \( \frac{\partial R_0}{\partial k_Q} = +0.54 \). It means that the \( R_0 \) is most sensitive to \( \eta \), followed by \( \beta \).

5.2. An increase in \( \eta \) and decrease in \( \beta \)

It is found that if \( \eta \) is increased from 0.83 to 0.9, the peak value of \( Q(k) \) becomes much less. For, \( \gamma = 0.001 \), it is only 829 thousand instead of 2.93 million. Besides, the peak also occurs at a later date. For \( \gamma = 0.001 \), the peak occurs on Feb 18, 2021. For \( \gamma = 0.005 \) or 0.010, the peak does not even occur within the next one year. The situation could be made better by increasing the detection rate further. We see, at \( \eta = 0.93 \), the \( Q(k) \) peak value is much less, only 230 thousands and occurring at a much earlier date, around July 2, 2020 for any value of \( \gamma \). The peak rate of hospitalization is also reduced to 15.5 thousands per day (see Fig. 5).

The value of \( R_0 \) for \( \eta = 0.90 \) and 0.93 is 1.093 and 1.014, respectively. \( R_0 \) can also be reduced by reducing \( \beta \) instead of \( \eta \). The trend of \( Q(k) \), shown in Fig. 6a is for \( \eta = 0.83 \) and \( \beta = 0.0354 \) (\( R_0 = 1.015 \)). In this case, the peak value of active quarantine is even less, 126 thousands only and the peak would occur on June 15, 2020. Comparing the last two cases, we see that the peak of \( Q(k) \) is not solely dependent on \( R_0 \). If similar \( R_0 \) value is attained by a lower \( \beta \), a lower peak occurs at an earlier date. This is due to the non-linear nature of the proposed model which includes the feature of increasing model population unlike other reported models [9,11,15]

5.3. A possible scenario: Forecast for the next one year

However, as the lockdown is lifted, the contact rate is now bound to be increased. Average contact rate \( (x) \) was 5.54 at the beginning of the first lockdown. The unlocking phase in India spans over the next two months. We, thus, assume that the value of \( x \) will rise from the current value of 3.74 to 5.54...
within these two months. Besides, we have considered effects of events like ‘migrating worker movement’ by adopting an ‘increasing population’ model, so, we assume that the contact rate $\beta$ will now follow the decreasing trend as shown in Fig. 2a. The trend yields that the infection rate will get stabilized around $0.03$ by the next two months. With these two values $\gamma (5.56)$ and $\beta (0.03)$, the $R_0$ will be $1.0$ when $\eta$ will be $0.938$. Therefore, to contain the infection rate India needs to increase the detection rate close to this value. Based on the analysis presented so far, we here predict that $\eta$ will be increased from $0.83$ and will be stabilized at $0.95$ in the next two months. This is found from its exponential trend, $(1 - e^{-t/\tau})$, with time constant $\tau = 12$ days. Under this prediction, the peak of $Q(k)$ will be $332–342$ thousands depending upon the value of $\gamma$ and will occur on July 20–21, 2020, viz. Fig. 6b. The maximum rate of hospitalization will be approximately $22627–23092$ per day. Trends for total undetected infectious ($I_C(k)$), recovered ($R_I(k)$) and $R_Q(k)$, separately),

![Fig. 6 Trends of $Q(k)$ for (a) $\beta = 0.353$, (b) the foretasted situation (Section 5.3).](image1)

![Fig. 7 Cumulative and active population of compartments, predicted for the next 1 year.](image2)

![Fig. 8 Validation of Forecast: A comparison between predicted and actual data of total numbers of detected infectious, recovered, and deceased, from June 6–13, 2020.](image3)
A six compartments with time-delay model SHIQRD for the COVID-19

| Table 3 | Predicted parameters of forecast for different values of $\eta$ with $\gamma = 0.005 - 0.020$. |
|--------|-------------------------------------------------------------------------------------------------|
| $\eta = 0.935$ | $\eta = 0.950$ | $\eta = 0.965$ |
| Total Detected Infectious (thousand) | 5371–6696 | 2798–3087 | 1859–1952 |
| Total Detected Death (thousand) | 171–213 | 88.6–97.9 | 58.6–61.5 |
| Peak Quarantine (thousand) | 376–391 | 335–343 | 309–314 |
| Date of Peak Quarantine | July 29-Aug 3 | July 20–21 | July 15 |
| Peak Quarantine Rate (per day) | 25221–26169 | 22627–23092 | 21061–21361 |
| Quarantined on 31.05.21 (thousand) | 100–168 | 6.1–9.7 | 0 |
| Quarantine Rate on 31.05.21 (per day) | 6450–10869 | 322–542 | 0 |
| Total Undetected Infectious (thousand) | 488–604 | 234–251 | 159–161 |
| Total Undetected Death (thousand) | 13.7–16.6 | 6.6–7.1 | 4.2–4.3 |

deceased ($D_I(k)$ and $D_Q(k)$) and current home-quarantined ($H(k)$), related to this prediction are shown in Fig. 7a–7b. This prediction shows that there will be active quarantine of 6.1–9.7 thousands people even after 1 year. However, the per day entry to hospital (quarantine) shall be reduced to 322–542 by May, 2021. Fig. 8 shows that the actual values of detected infectious, recovered, and deceased are closely matching the prediction on the first eight days of forecast (June 6 to June 13, 2020).

Nevertheless, the prediction is strongly dependent on the steady state value of $\eta$. Table 3 shows how predicted parameters can vary if steady state values of $\eta$ will vary from 0.935 to 0.965.

6. Conclusion

This paper presents a six compartments model, named as SHIQRD model to describe the dynamics of COVID-19 pandemic in India. The key features of the model are— (i) inclusion of various factors, like, ‘undetected infectious’ population, corresponding recovery and death, uninfected ‘home-quarantined’ population, ‘movement of migrating workers’ into the analysis, (ii) introduction of time delay based population movement incorporating incubation time, death time, ‘recovery time window’, (iii) separate approach pertaining to lockdown and unlocked phase—introduction of time variant model population for the unlocked phase, and (iv) discrete recursion equations to solve the delay dynamics. Apart from the introduction of such novel model, the paper also (i) determines the infection rate ($\beta$), death rate ($\delta$), recovery time window and reproduction number ($R_0$) for the pandemic in India. (ii) It analyses $R_0$ in terms of contact rate $\alpha$, $\beta$ and detection rate $\gamma$ and measure sensitivities of $R_0$ with respect to these factors. The presented work shows how the $R_0$ and $\alpha$ decreased with each passing phase of lockdown and how lockdown has limited the number of active cases and death toll till date.

The paper has also explored several possibilities to understand how India can converge the pandemic growth in upcoming days. It shows that the growth containment is mostly dependent on the detection rate. Based on that, it discusses a typical scenario which forecasts that the peak of the pandemic may occur around July 20, when almost 335–343 thousands people will be in active quarantine. The peak rate of hospitalization will be approximately 23 thousands per day. By May 31, 2021, this rate will get reduced to 322–542 per day. Approximately 2.8–3.1 million people will be infected in total, but, the good news is 2.7–3 million people will also be recovered by that time.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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