Auxiliary information based maximum generally weighted moving average (AIB-MaxGWMA) control chart

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Abstract. Product quality has a relationship with customer satisfaction. Therefore, the company always strives to maintain the product quality. The effort can be made by the company to maintain product quality is monitoring the production process. One of the tools used to monitor the production process is the control chart. In process monitoring, there are two parameters that are observed in the process that are mean and variance. The univariate process monitoring is only carried out on a study variable and only using the information on the corresponding variable. However, in this study, need to monitor process mean and variance simultaneously on a study variable that requires information on study variable and information on auxiliary variable using Auxiliary Information Based Maximum Generally Weighted Moving Average (AIB-MaxGWMA). Besides that, in this study, compare AIB-MaxGWMA with the MaxGWMA control chart to know the effect of adding an auxiliary variable in process monitoring. The performance of these control chartss is evaluated using out of control Average Run Length (OC ARL), where OC ARL is the average number of samples needed to detect a particular shift. The result of this study is AIB-MaxGWMA control chart has a smaller OC ARL than the MaxGWMA control chart which showed that the AIB-MaxGWMA control chart is more sensitive or faster than MaxGWMA control chart to detect a shift. In further study, we recommended to enhance the performance of the AIB-MaxGWMA control chart by extending the current work to the Auxiliary Information Based Maximum Multivariate Generally Weighted Moving Average (AIB-Max MGWMA) control chart, so it is possible to monitor the mean and variance process simultaneously in multivariate case (at least two quality characteristics).

1. Introduction
One of key of Statistical Process Control (SPC) is to detect shift in parameters of a process as quickly as possible. A control chart is a statistical tool used to monitor the production process. The basic concept of the control chart was first introduced by Shewhart in the 1920s. There are many control charts have been developed to monitor process mean and variance. These control chartss are Cumulative Sum (CUSUM) proposed by Page in 1954 and Exponentially Weighted Moving Average (EWMA) proposed by Robert in 1959—memory type control chart. These control chartss are more sensitive to detect a small shift than Shewhart control chart. The control chart is classified into two categories, the location and dispersion control chart. The location control chart is used to monitor changes in the mean of a process and a dispersion control chart is used to monitor changes in the variance of a process. But, process monitoring is not efficient when using two separate control charts...
(location and dispersion, respectively). This is because we may not know whether a shift occurs either in the process mean or variance or both. In recent years, a few studies have been carried out to monitor mean and variance simultaneously into a single chart. A single control chart was developed called Max chart for monitoring process mean and variance simultaneously by [1]. In other work, a control chart was proposed to detect small shift in process mean and variance named Maximum Exponentially Weighted Moving Average (MaxEWMA) by [2] and a control chart that combined two Generally Weighted Moving Average (GWMA) control chart—one for mean and one for variance—into a single control chart called MaxGWMA control chart was proposed by [3], they showed that MaxGWMA control chart is more sensitive than MaxEWMA control chart.

The control chart used to monitor the mean and variance of one quality characteristic (variable) is univariate control chart. However, in certain cases, the monitoring process mean and variance of a study variable can be done by using the information on the corresponding variable and also information from auxiliary variable, where there is a between study variable and auxiliary variable. A new control chart was constructed by [4] called Auxiliary Information Based Shewhart (AIB Shewhart) control chart using a regression estimator to make better performance of the Shewhart control chart. A suggestion to build Shewhart-type dispersion control chart by [5] using ratio-type variance estimator for phase I quality control. They showed that the AIB dispersion control chart is more powerful than the existing Shewhart unbiased sample variance estimator. A control chart was proposed by [6] to monitor process mean involved an auxiliary variable called Auxiliary Information Based Exponentially Weighted Moving Average (AIB-EWMA) control chart, they showed that AIB-EWMA has a better performance in detecting a small shift in the process mean than EWMA control chart. The addition of an auxiliary variable makes process monitoring better than without using auxiliary variables. This is because the variance of the estimator by using information from the study variable and auxiliary variable is smaller than the variance of the estimator that only used information from the study variable without an auxiliary variable. A control chart was developed by [7] that used to monitor process mean and variance simultaneously by adding an auxiliary variable called Auxiliary Information Based Maximum Exponentially Weighted Moving Average (AIB-MaxEWMA) control chart, he showed that AIB-MaxEWMA control chart has a better performance in detecting shift in process mean and variance than MaxEWMA control chart. For more interesting works for the AIB chart, we refer to [8][9][10].

The performance of control chart can be measured using Average Run Length (ARL). ARL is classified into two categories that are in control ARL (IC ARL) and out of control ARL (OC ARL). When process is said to be in control, the IC ARL should be large to avoid false alarm and when process is said to be out of control, the OC ARL become small to rapidly detect shift in the process. There are many approaches that can used to compute ARL that are Monte Carlo simulation, Markov Chain approach, and Integral Equation.

In this paper, we proposed a new MaxGWMA control chart for simultaneously monitoring process mean and variance of a Normally distributed process using an auxiliary variable called AIB-MaxGWMA control chart. Monte Carlo simulation is used to compute ARL. We show that the AIB-MaxGWMA control chart is more sensitive than the MaxGWMA control chart to detect small shift in the process mean and variance.

The rest of this paper is organized as follows: the MaxGWMA control chart review in section 2, suggest an AIB-MaxGWMA in section 3, the performance comparative studies are in section 4, an illustrative example is given in section 5 to explain the implementation of proposed control chart, and section 6 concludes the paper.

2. The Maximum Generally Weighted Moving Average (MaxGWMA) Control Chart
In this section, we review the MaxGWMA control chart when monitoring process mean and variance of a Normally distributed process. Let $Y$ denote a quality characteristic of a process, assuming that $Y$ is Normally distributed with the mean $\mu + \delta \sigma$ and standard deviation $\delta \sigma$. The process is said to be in
control if \( \delta = 0 \) and \( \theta = 1 \); otherwise, the process is said to be out of control. Let \( Y_j \), where \( i = 1, 2, \ldots, n \) and \( j = 1, 2, \ldots, m \), be the measurements of \( Y \) with sample size \( n \) and \( j \) indexes the sample number. Then \( \bar{Y}_j \) and \( S^2_{Y,j} \) be the sample mean and variance, respectively, where \( \bar{Y}_j = \frac{\sum_{i=1}^{n} Y_{ij}}{n} \) and \( S^2_{Y,j} = \frac{1}{n-1} \sum_{i=1}^{n} [Y_{ij} - \bar{Y}_j]^2 \) are Normal random variables with the mean \( \mu + \delta \sigma \) and variance \( \frac{\theta^2 \sigma^2}{n} \), \( \frac{(n-1)S^2_{Y,j}}{\rho^2 \sigma^2} \) are Chi-Square random variables with degree of freedom \( n-1 \). Note that \( \bar{Y}_j \) and \( S^2_{Y,j} \) are independent random variables for in control process. For an in control process, we consider the following transformation for mean and variance estimator:

\[
U_j = \frac{\bar{Y}_j - \mu_{H_j}}{\sigma / \sqrt{n}} \quad \text{and} \quad V_j = \Phi^{-1} \left\{ F \left[ \frac{(n-1)S^2_{Y,j}}{\rho^2 \sigma^2} \right] \right\} = \Phi^{-1} \left\{ F \left[ h \right] \right\}
\]

where \( \Phi^{-1} (.) \) is the inverse cumulative distribution of standard Normal distribution and \( F (h) \) is the cumulative distribution of Chi-Square distribution with \( n-1 \) degree of freedom. More details on these transformation can be seen in [11]. Note that \( U_j \) and \( V_j \) are standard Normal random variables.

The MaxGWMA control chart combines two GWMA control charts—one for mean and one for variance—into a single control chart. Using \( U_j \) and \( V_j \), consider two GWMA sequences, say \( G_j \) and \( H_j \), respectively, by using the following formulas:

\[
G_j = P(M=1)U_j + P(M=2)U_{j-1} + \ldots + P(M=j)U_1 + P(M > j)G_0
\]

\[
G_j = (\bar{Y}_0 - \bar{Y}_j)U_j + (\bar{Y}_1 - \bar{Y}_j)U_{j-1} + \ldots + (\bar{Y}_{j-1} - \bar{Y}_j)U_1 + \bar{Y}_j G_0
\]

\[
H_j = P(M=1)V_j + P(M=2)V_{j-1} + \ldots + P(M=j)V_1 + P(M > j)H_0
\]

\[
H_j = (\bar{Y}_0 - \bar{Y}_j)V_j + (\bar{Y}_1 - \bar{Y}_j)V_{j-1} + \ldots + (\bar{Y}_{j-1} - \bar{Y}_j)V_1 + \bar{Y}_j H_0
\]

where \( M \) count the number of samples until event A first occur since the previous of event A occur, \( \bar{Y}_j \) is the probability that event A does not occur in the first \( j \) sample. \( G_0 \) and \( H_0 \) is the starting values of GWMA sequences and \( G_0 = H_0 = 0 \) when process is in control. \( G_j \) and \( H_j \) are Normal random variables, that is \( g_j \sim N(0, \sigma^2 G_j) \) and \( h_j \sim N(0, \sigma^2 H_j) \). Because of \( U_j \) and \( V_j \) are independent, so \( G_j \) and \( H_j \) are also independent. \( P(M=1), P(M=2), \ldots \) be the weights of the current sample, the previous sample, \ldots, and the most out-of-data sample, respectively. Therefore, \( P(M > j) \) is weighted with the target value of the process.

For more easy computation, we change \( \bar{Y}_j \) by \( q^\mu_r \) and Eq. 2 and 3 can be write:

\[
G_j = P(M=1)U_j + P(M=2)U_{j-1} + \ldots + P(M=j)U_1 + P(M > j)G_0
\]

\[
G_j = (\bar{Y}_0 - \bar{Y}_j)U_j + (\bar{Y}_1 - \bar{Y}_j)U_{j-1} + \ldots + (\bar{Y}_{j-1} - \bar{Y}_j)U_1 + \bar{Y}_j G_0
\]

\[
H_j = P(M=1)V_j + P(M=2)V_{j-1} + \ldots + P(M=j)V_1 + P(M > j)H_0
\]

\[
H_j = (\bar{Y}_0 - \bar{Y}_j)V_j + (\bar{Y}_1 - \bar{Y}_j)V_{j-1} + \ldots + (\bar{Y}_{j-1} - \bar{Y}_j)V_1 + \bar{Y}_j H_0
\]
\[ H_j = \left(q^r - q^*\right) V_j + \left(q^r - q^{**}\right) V_{j-1} + \ldots + \left(q^{(j-1)r} - q^{*}\right) V_1 + q^r H_0 \]

where \( q = 1 - \lambda \), \( \lambda \) is a smoothing constant and \( \omega \) is adjustment parameter determined by the practitioner.

The plotting statistic of the MaxGWMA control chart given by:

\[ MG_j = \max \left| G_j \right|, H_j \right| \]  \tag{6}

where \( MG_j \) is the maximum of the absolute value of \( G_j \) and \( H_j \). Because \( MG_j \) is non-negative value, control limit for the MaxGWMA control chart only has an upper control limit, say \( UCL \), at \( j \)th sample given by:

\[ UCL_j = E\left(MG_j\right) + L \sqrt{\text{var}(MG_j)} \]  \tag{7}

where \( E(MG) \) is expectation value of \( MG_j \), \( \text{var}(MG_j) \) is variance of \( MG_j \), and \( L \) is the width of the control limit. The process is said to be out of control if \( MG_j \) value goes beyond the control limit [3].

3. The Proposed Control Chart

In this section, we proposed a new control chart named Auxiliary Information Based Maximum Generally Weighted Moving Average (AIB-MaxGWMA) for monitoring process mean and variance simultaneously using the auxiliary variable.

Let \( (Y_i, X_i) \) be a random sample of size \( n \) from in control process, \( (Y_i, X_i) \) have a bivariate Normal distribution, that is \( (Y, X) \sim N_2(\mu_Y, \mu_X, \sigma_Y^2, \sigma_X^2, \rho_{XY}) \), where \( \mu_Y \) and \( \mu_X \) are the mean of \( Y \) and \( X \), respectively, and \( \sigma_Y^2 \) and \( \sigma_X^2 \) are the variance of \( Y \) and \( X \), respectively, and \( \rho_{XY} \) is the correlation between \( Y \) and \( X \). Here, \( Y \) as study variable and \( X \) as auxiliary variable, where \( Y \) and \( X \) correlate.

Let \( (Y_{ij}, X_{ij}) \) a bivariate random sample, where \( i = 1, 2, \ldots, n \) and \( j = 1, 2, \ldots, m \), be the measurements of \( Y \) and \( X \) with sample size \( n \) and \( j \) indexes the sample number. Then \( \bar{Y}_j \) and \( S^2_{Y,j} \) be the sample mean and variance of \( Y \), respectively, where \( \bar{Y}_j = \frac{\sum_{i=1}^{n} Y_{ij}}{n} \) and \( S^2_{Y,j} = \frac{\sum_{i=1}^{n} (Y_{ij} - \bar{Y}_j)^2}{(n-1)} \) and \( \bar{X}_j \) and \( S^2_{X,j} \) be the sample mean and variance of \( X \), respectively, where \( \bar{X}_j = \frac{\sum_{i=1}^{n} X_{ij}}{n} \) and \( S^2_{X,j} = \frac{\sum_{i=1}^{n} (X_{ij} - \bar{X}_j)^2}{(n-1)} \).

When the process is assuming in control, a different estimator made by [8] in the following formula for estimate \( \mu_Y \) using information from an auxiliary variable:

\[ M_{Y,j} = \bar{Y}_j + \rho_{XY} \left( \frac{\sigma_Y}{\sigma_X} \right) \left( \mu_X - \bar{X}_j \right) \]  \tag{8}

with the expected value and variance for \( M_{Y,j} \) given by:

\[ E\left(M_{Y,j}\right) = \mu_Y \] and \( \text{var}(M_{Y,j}) = \frac{1}{n} \sigma_Y^2 \left( 1 - \rho^2_{XY} \right) \]  \tag{9}

Then, the following formula is transformation of estimator \( M_{Y,j} \):

\[ A_{Y,j} = \frac{M_{Y,j} - \mu_Y}{\sigma_Y \sqrt{\left( 1 - \rho^2_{XY} \right) / n}} \]  \tag{10}
where $A_{y,j}$ is a random variable with standard Normal distribution when process said to be in control, that is, $A_{y,j} \sim N(0,1)$. Then, an estimator of $\sigma_y^2$ by using $V_{y,j}$ and $V_{X,j}$, where $V_{x,j} = \Phi^{-1}\left\{F[h,v]\right\} \sim N(0,1)$ developed by [12]. The following $S_{y,j}^{(i)}$ is estimator for $\sigma_y^2$:

$$S_{y,j}^{(i)} = V_{y,j} - \rho_{xy}^* V_{X,j}$$

(11)

where $\rho_{xy}$ is correlation between $V_{y,j}$ and $V_{X,j}$. The expected value and variance of $S_{y,j}^{(i)}$ given by:

$$E(S_{y,j}^{(i)}) = 0$$

and

$$\text{var}(S_{y,j}^{(i)}) = 1 - (\rho_{xy}^*)^2$$

(12)

Then, the following formula is transformation of estimator $S_{y,j}^{(i)}$.

$$B_{y,j} = \frac{S_{y,j}^{(i)} - 0}{\sqrt{1 - (\rho_{xy}^*)^2}}$$

(13)

When the process is said to be in control, $B_{y,j}$ is a random variable with Normal standard distribution, that is, $B_{y,j} \sim N(0,1)$.

Using $A_{y,j}$ and $B_{y,j}$ consider two GWMA sequences, say $A_{y,j}^*$ and $B_{y,j}^*$, respectively, by using the following formulas:

$$A_{y,j}^* = P(M = 1) A_{y,j} + P(M = 2) A_{y,j-1} + \ldots + P(M = j) A_{y,0}; ~ A_{y,0}^* = 0$$

(14)

$$B_{y,j}^* = P(M = 1) B_{y,j} + P(M = 2) B_{y,j-1} + \ldots + P(M = j) B_{y,0}; ~ B_{y,0}^* = 0$$

(15)

where $A_{y,0}$ and $B_{y,0}$ are the starting values. $A_{y,j}^*$ and $B_{y,j}^*$ are independent because $A_{y,j}$ and $B_{y,j}$ are independent. When the process is said to be in control, $A_{y,j}^* \sim N\left(0, \sigma_{A_j}^2\right)$ and $B_{y,j}^* \sim N\left(0, \sigma_{B_j}^2\right)$, where the variance of $A_{y,j}^*$ and $B_{y,j}^*$ is $\sigma_{A_j}^2 = \sigma_{B_j}^2 = \sum_{m=1}^{j} \left[p(M=m)\right]^2$.

The plotting statistic of the AIB-MaxGWMA control chart by using the following formula:

$$\text{AIBMG}_j = \max \left\{ \left| A_{y,j}^* \right|, \left| B_{y,j}^* \right| \right\}$$

(16)

where $\text{AIBMG}_j$ is the maximum of absolute value of $A_{y,j}^*$ and $B_{y,j}^*$. Because $\text{AIBMG}_j$ is non-negative value, control limit for the AIB-MaxGWMA control chart only has an upper control limit, say UCL, at $j$th sample given by:

$$\text{UCL}_j = E\left(\text{AIBMG}_j\right) + L\sqrt{\text{var}\left(\text{AIBMG}_j\right)}$$

(17)

Where $E\left(\text{AIBMG}_j\right)$ is expected value of $\text{AIBMG}_j$ statistic, $\text{var}\left(\text{AIBMG}_j\right)$ is variance of $\text{AIBMG}_j$ statistics, and $L$ is the width of the control limit. The process is said to be out of control if $\text{AIBMG}_j$ value goes beyond the control limit.

4. Evaluation and Performance Comparison

The performance of control chart is evaluated using Average Run Length (ARL). When process is said to be in control, IC ARL will be large to avoid false alarm. When process is said to be out of control, OC ARL needs to be small to rapidly detect the shift in the process mean and variance.

In this study, Monte Carlo simulation is used to compute ARL with determined IC ARL = 137. We simulated the data with mean and variance corresponding to the shift value in mean and variance with different choices of correlation, that is $(\rho_{xy}, \rho_{xy}^*) = (0,0); (0.5,0.22); (0.9,0.78)$.
For example, let \((y_i, x_i)\) is a bivariate Normal random sample with each sample size is 4. Then, a mean changes from \(\mu\) into \(\mu + \delta\sigma\), where \(\delta = 0.0, 0.25, 0.75, 1, 1.25, 1.5\) and standard deviation changes from \(\sigma\) into \(\theta\sigma\), where \(\theta = 0.25, 0.75, 1, 1.25, 1.5\). We use 5000 iteration to compute ARL with the number of run lengths taken is 500. The the width of control limit is \(L = 2.764\) with parameter \(q = 0.9\) and \(\omega = 0.5\).

The AIB-MaxGWMA will idenitc to the MaxGWMA if there is no correlation between \(X\) and \(Y\) or \(\rho_{XY} = 0\). Table 1 showed the OC ARL of the Max-GWMA and AIB-MaxGWMA control charts based on different level of correlation with determined IC ARL = 137.

**Table 1.** OC ARL of MaxGWMA and AIB-MaxGWMA Control Charts based on IC ARL = 137.

| \(\theta\) | \(\rho_{XY}\) | 0   | 0.25 | 0.75 | 1   | 1.25 | 1.5 |
|----------|--------------|-----|------|------|-----|------|-----|
| 0.25     | 0            | 46.570 | 46.070 | 45.952 | 45.916 | 45.332 | 42.662 |
|          | 0.5          | 44.338 | 42.920 | 42.786 | 42.662 | 42.662 | 41.464 |
|          | 0.9          | 8.552  | 8.146  | 7.918  | 7.880  | 7.874  | 7.814  |
| 0.75     | 0            | 110.156 | 105.996 | 105.502 | 105.340 | 102.670 | 99.026 |
|          | 0.5          | 103.522 | 102.982 | 100.978 | 94.240  | 91.224  | 90.782  |
|          | 0.9          | 14.968  | 14.562  | 14.184  | 13.936  | 13.882  | 13.616  |
| 1        | 0            | 136.212 | 130.972 | 129.922 | 122.832 | 125.440 | 122.832 |
|          | 0.5          | 136.944 | 117.496 | 116.092 | 113.262 | 112.520 | 111.534 |
|          | 0.9          | 137.444 | 19.414  | 18.990  | 18.572  | 18.028  | 17.994  |
| 1.25     | 0            | 133.832 | 133.596 | 133.006 | 133.894 | 132.090 | 129.120 |
|          | 0.5          | 117.014 | 114.786 | 114.394 | 114.164 | 114.160 | 110.094 |
|          | 0.9          | 35.034  | 25.562  | 25.228  | 25.162  | 24.710  | 24.402  |
| 1.5      | 0            | 131.036 | 129.792 | 127.674 | 127.608 | 125.690 | 125.688 |
|          | 0.5          | 109.608 | 109.216 | 109.200 | 105.318 | 105.320 | 100.084 |
|          | 0.9          | 33.412  | 33.138  | 32.100  | 32.010  | 31.408  | 26.398  |

From Table 1, it can be seen that the OC ARL is getting smaller with the increasing level of correlation between \(X\) and \(Y\). It means that as the correlation level increase, the control chart is faster or more sensitive to detect shift in the process. We also showed that performance of the AIB-MaxGWMA control chart is better than the MaxGWMA control chart because OC ARL of the AIB-MaxGWMA control chart is smaller in all shift level than OC ARL of the MaxGWMA control chart. In order words, the AIB-MaxGWMA is faster to detect a change in the process. The best performance of the AIB-the MaxGWMA control chart when correlation is 0.9. In addition, if correlation value exceeds 0.9, we believe that the performance of the AIB-MaxGWMA control chart can be enhanced.

5. **An Illustrative Example**

In this study, the data that used to implement the MaxGWMA and AIB-MaxGWMA control charts is drinking water production process with variable used is water turbidity as study variable and water pH as auxiliary variable. Let \((y_i, x_i)\) are bivariate Normal random sample with total 20 samples and each sample size is 3. The the width of control limit is \(L = 2.764\) with \(q = 0.9\) and \(\omega = 0.5\). Using the
bivariate data, the statistic value of the MaxGWMA and AIB-MaxGWMA control charts and their control limit are showed in Table 2 and plotting statistic displayed in Figure 1 and 2.

| Sample | AIB-MaxGWMA Statistic Value | Upper Control Limit (UCL) | MaxGWMA Statistic Value | Upper Control Limit (UCL) |
|--------|------------------------------|---------------------------|-------------------------|--------------------------|
| 1      | 0.181                        | 0.279                     | 0.091                   | 0.279                    |
| 2      | 0.284                        | 0.299                     | 0.102                   | 0.299                    |
| 3      | 0.289                        | 0.309                     | 0.085                   | 0.309                    |
| 4      | 0.430                        | 0.316                     | 0.032                   | 0.316                    |
| 5      | 0.452                        | 0.321                     | 0.091                   | 0.321                    |
| 6      | 0.762                        | 0.324                     | 0.058                   | 0.324                    |
| 7      | 0.628                        | 0.327                     | 0.064                   | 0.327                    |
| 8      | 0.027                        | 0.330                     | 0.032                   | 0.330                    |
| 9      | 0.821                        | 0.332                     | 0.132                   | 0.332                    |
| 10     | 2.432                        | 0.334                     | 0.609                   | 0.334                    |
| 11     | 2.150                        | 0.335                     | 0.503                   | 0.335                    |
| 12     | 0.892                        | 0.337                     | 0.189                   | 0.337                    |
| 13     | 0.464                        | 0.338                     | 0.177                   | 0.338                    |
| 14     | 0.333                        | 0.339                     | 0.046                   | 0.339                    |
| 15     | 0.9247                       | 0.340                     | 0.025                   | 0.340                    |
| 16     | 0.119                        | 0.341                     | 0.034                   | 0.341                    |
| 17     | 0.071                        | 0.342                     | 0.100                   | 0.342                    |
| 18     | 0.581                        | 0.343                     | 0.163                   | 0.343                    |
| 19     | 0.094                        | 0.343                     | 0.099                   | 0.343                    |
| 20     | 0.611                        | 0.344                     | 0.142                   | 0.344                    |

From Table 2, it is known that for the AIB-MaxGWMA control chart, the statistic value goes beyond UCL in sample number 4, 5, 6, 7, 9, 10, 11, 12, 13, 15, 18, and 20, while in the MaxGWMA control chart, only sample number 10 and 11 goes beyond the UCL. It means that the MaxGWMA and AIB-MaxGWMA control charts detect an out of control signal in the process. Figure 1 and 2 displayed plotting statistic of these control charts and their corresponding control limit. Figure 1 displayed plotting statistic of the AIB-MaxGWMA control chart and Figure 2 displayed plotting statistic of the MaxGWMA control chart.
From Figure 1, it is known that in the AIB-MaxGWMA control chart, detect out of control signal since 4th sample while in Figure 2, the MaxGWMA control chart detect out of control signal since 10th sample. It is illustrated that the AIB-MaxGWMA control chart is more sensitive than the MaxGWMA control chart in detecting small shift in the process.

6. Conclusion

This study showed the effect of adding auxiliary variable in monitoring mean and variance simultaneously. It can be seen from the OC ARL comparison of the MaxGWMA and AIB-MaxGWMA control charts. When the level of correlation higher, the OC ARL value is getting smaller, it shown that when the level correlation is higher, the control chart is faster to detect a shift in the process. From the result, the AIB-MaxGWMA has a smaller OC ARL in all mean and variance shifts level and concluded that use of auxiliary variable makes the performance of control chart be better than without using auxiliary variable. In further study, we recommended to enhance the performance of the AIB-MaxGWMA control chart by extend the current work to the Auxiliary Information Based Maximum Multivariate Generally Weighted Moving Average (AIB-Max MGWMA) control chart, so it is possible to monitor the mean and variance process simultaneously in multivariate case (at least two quality characteristics).

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