Generation of multicomponent atomic Schrödinger cat states of up to 20 qubits

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Multipartite entangled states are crucial for numerous applications in quantum information science. However, the generation and verification of multipartite entanglement on fully controllable and scalable quantum platforms remains an outstanding challenge. We report the deterministic generation of an 18-qubit Greenberger-Horne-Zeilinger (GHZ) state and multicomponent atomic Schrödinger cat states of up to 20 qubits on a quantum processor, which features 20 superconducting qubits, also referred to as artificial atoms, interconnected by a bus resonator. By engineering a one-axis twisting Hamiltonian, the system of qubits, once initialized, coherently evolves to multicomponent atomic Schrödinger cat states—that is, superpositions of atomic coherent states including the GHZ state—at specific time intervals as expected. Our approach on a solid-state platform should not only stimulate interest in exploring the fundamental physics of quantum many-body systems, but also enable the development of applications in practical quantum metrology and quantum information processing.

The capability of entangling multiple particles is central to fundamental tests of quantum theory (1) and represents a key prerequisite for quantum information processing. There exist various kinds of multipartite entangled states, among which the Greenberger-Horne-Zeilinger (GHZ) states (i.e., the two-component atomic Schrödinger cat states) are particularly appealing (2–4). These states play a key role in quantum-based technologies, including open-destination quantum teleportation (5), concatenated error-correcting codes (6), quantum simulation (7), and high-precision spectroscopy (8). In principle, the number of particles that can be deterministically entangled on a quantum processor is a benchmark of its capability in processing quantum information. However, it is difficult to scale up this number because the conventional step-by-step gate methods require long control sequences, which increase exposure to perturbing noise. A shortcut is to realize the free evolution under a nonlinear Hamiltonian with, for example, one-axis twisting, and the system of qubits initialized in an atomic coherent state is predicted to evolve to squeezed spin states (9) and then to the multicomponent atomic Schrödinger cat states (3)—that is, superpositions of atomic coherent states including the GHZ state (10).

Multipartite entanglement on several physical platforms suitable for quantum information processing has been achieved in a series of experiments (11–23). Some of these experiments have involved local detections of only the subsystems (12, 15). Multipartite entanglement—in particular, the GHZ state, which possesses global entanglement—would be better characterized by synchronized detections of all system parties and was achieved with 14 trapped ions (16), 12 photons (17), 18 photonic qubits exploiting three degrees of freedom of six photons (18), and 12 superconducting qubits (19). Across the different physical platforms, the GHZ state fidelity \( F > 0.5 \) verifies the existence of genuine multipartite entanglement (24).

Here, we developed a 20-qubit superconducting quantum processor that features all-to-all connectivity and programmable qubit-qubit couplings. We used this processor to engineer a one-axis twisting Hamiltonian and steer the system of qubits to squeezed spin states, and then to an oversqueezed regime where multicomponent atomic Schrödinger cat states, including a GHZ state, sequentially appear. Nonclassicality of the five-component cat state is indicated by the negative values in the experimentally obtained spin Wigner function. The GHZ state can be characterized by just two diagonal and two off-diagonal elements in its density matrix, on the basis of which we measure a state fidelity \( F = 0.525 \pm 0.005 \) for 18 qubits, which confirms genuine 18-partite entanglement (24).

We note that independent experiments creating the 18-qubit GHZ state with fixed-frequency superconducting qubits (25) and the 20-qubit GHZ state with Rydberg atoms (26) were recently demonstrated.

Our superconducting quantum processor (Fig. 1A) consists of 20 frequency-tunable transmon qubits, labeled as \( Q_j \) for \( j = 1 \) to 20, surrounding a central coplanar waveguide bus resonator (B) whose resonant frequency is fixed at \( \omega_0/2\pi = 5.51 \) GHz. Qubit-resonator (\( Q_j \)-B) coupling strengths \( g_j \) are designed to be uniform, and measured \( g_j/2\pi \) values range from 24.1 to 30.1 MHz. Qubits are detected through their respective readout resonators (Fig. 1B) using impedance-matched Josephson parametric amplifiers (JPAs) and an optimized arrangement of the qubit frequencies for measurement, \( \omega_q/m \), to enhance the signal-to-noise ratio.

All qubits are individually tunable with high flexibility, which can be demonstrated by measuring \( Q_0 \)’s swap spectroscopy (Fig. 1C) while the other 19 qubits are equally spaced in frequency around the resonator B. Typical qubit energy relaxation times, \( T_1 \), are in the range of 20 to 50 \( \mu s \), and the dephasing times for standalone qubits, \( T_2 \), are around 2 \( \mu s \) (27). With a proper arrangement of the qubit idle frequencies, \( \omega_0 \), where qubit initializations and single-qubit rotations are applied, fidelity values of the simultaneous single-qubit \( \pi/2 \) rotational gates used in the GHZ experiment are all above 0.99 as estimated by quantum state tomography and simultaneous randomized benchmarking (27).

With each of the 20 qubits being addressable, the system Hamiltonian is

\[
H = \frac{\hbar}{2} \omega_0 a^\dagger a + \sum_{j=1}^{20} \left[\omega_j(t)\left|1_j\right\rangle\left\langle 1_j\right| + g_j(s_j^a a^\dagger + s_j^a a^\dagger a^\dagger + s_j^a a^\dagger a^\dagger + s_j^a a^\dagger a^\dagger)\right] + \sum_{j=1}^{20} \lambda_{j,j+1}^\alpha s_j^\alpha s_{j+1}^\alpha + s_j^\alpha s_{j+1}^\alpha
\]

where \( \omega_0(t) > g_j \) is dynamically tunable, \( s_j^\alpha \) (\( s_j^\alpha \) is the raising (lowering) operator of \( Q_j \), \( a^\dagger \) (\( a \) is the creation (annihilation) operator of B, and \( \lambda_{j,j+1}^\alpha \) describes the cross-talk couplings between neighboring qubits (subscripts in \( \lambda_{j,j+1}^\alpha \) run cyclically from 1 to 20). Note that there may exist qubit-qubit cross-talk couplings beyond neighboring pairs (27), which are not included in Eq. 1.

We can selectively entangle \( N \) of the 20 qubits by detuning the selected qubits from the resonator by the same amount \( \Delta (\geq g_j) \), with the other qubits being far off-resonant. When the resonator B is initially in vacuum, the effective Hamiltonian for these \( N \) qubits, labeled by \( Q_j \).
with $j$ going from 1 to $N$, in the frame rotating at the detuned qubit frequency is

$$H_2 = \sum_{(j,k)=N} \frac{g_{jk}\hbar}{\Delta} (\sigma_j^x \sigma_k^x + \sigma_j^y \sigma_k^y)$$

$$+ \sum_{j=1}^{N} \frac{g_{jj}\hbar}{\Delta} |j\rangle\langle j|$$

$$+ \sum_{j=1}^{N} \lambda_{j,j+1}\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y$$

(3, 10), where $(j,k)$ takes all possible pairs within the $N$ qubits, and subscripts in $\lambda_{j,j+1}$ run cyclically from 1 to $N$.

The scenario of a system of $N$ identical two-level atoms interacting collectively and dispersively with a single-mode electromagnetic field in a cavity has been theoretically investigated (3, 10). Experimentally we position the qubits 330 MHz below $\omega_{0q}/2\pi$ for all the effective qubit-qubit couplings (190 terms) in the first summation of Eq. 2, $|g_{jk}/(2\pi\Delta)|$, to be $-2$ MHz while the few (<20 terms) neighboring couplings $\lambda_{j,j+1}/2\pi$ are from 0.5 to 1 MHz. Therefore, we can ignore the effect of qubit-qubit cross-talk couplings and assume that couplings within all qubit pairs are approximately equal. We emphasize that the effect of cross-talk couplings and other factors limiting the GHZ state fidelity have been considered by numerical simulations (27), and we find decent agreement between our experimental results and the simplified theoretical treatment in (3, 10).

With uniform couplings noted as $\lambda = g_{jk}\hbar/\Delta$, we now apply the spin representation of qubit states and define the collective spin operators $S'_z = \sum_j \sigma_j^z$, $S'_x = \sum_j \sigma_j^x$, and $S'_y = \sum_j \sigma_j^y$. The term $\sum_{j} \lambda_{j,j+1}\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y$ in Eq. 2 is then transformed to $\lambda S'_z S'_z - \lambda S'_z S'_y$ (ignoring lower-order terms), which is the one-axis twisting Hamiltonian. By initializing the $N$ qubits identically so that each individual qubit points to the same direction represented by the angles $(\theta, \phi)$ in its Bloch sphere, we write the wave function of the atomic (spin) coherent state as

$$|\psi(0)\rangle = |\theta, \phi\rangle$$

$$= \otimes_{j=1}^{N}\left[ \cos \left( \frac{\theta}{2} \right) |0\rangle_j + \sin \left( \frac{\theta}{2} \right) \exp(\ii \phi)|1\rangle_j \right]$$

(3)

Here we apply the phase space representation for a collective spin system, because visualizing many-body quantum states on the Bloch sphere is useful to gain an intuition about the evolution of the states during the dynamics and collective rotations. At $t = \pi/(2|\lambda|)$, where $m$ is an integer no less than 2, $|\psi(0)\rangle$ evolves to a superposition of multiple atomic coherent states—that is, it becomes an atomic Schrödinger cat state (3). In particular, at $t = \pi/(2|\lambda|)$, it evolves to a two-component cat state: the $N$-qubit GHZ state,

$$|\psi(t)\rangle = \exp(-\ii HT)|0, \phi\rangle$$

$$= \frac{\exp(-\ii(N - 0.5\pi/2))}{\sqrt{2}}$$

$$|0, \phi - \frac{N - 1}{2}\pi\rangle + \exp(-\ii \phi/2)|0, \phi - \frac{N - 3}{2}\pi\rangle$$

(4)

In the pulse sequence for generating and characterizing the $N$-qubit GHZ state (Fig. 2A), we start with initializing each of the $N$ qubits in $|0\rangle(\phi=\pi/2)$, which collectively corresponds to an atomic coherent state $|\theta/2, -\pi/2\rangle$ in the $|0\rangle$ notation, by applying a $\pi/2$ rotation at each
Fig. 2. GHZ states of up to 18 qubits. (A) Pulse sequence for generating and characterizing the N-qubit GHZ state. (B) N-qubit GHZ parity oscillations. For each data point (blue circles), we repeat the pulse sequence about 30 × 2₁⁰ times to find the raw 2⁵⁰ probabilities and then apply readout corrections to eliminate the measurement errors (2⁷), after which we use maximum likelihood estimation to validate the occupational probabilities and calculate the parity value (P). To estimate error bars, we divide the complete dataset into subgroups, each containing about 5 × 2¹⁰ samplings, and the error bars correspond to the standard deviations of those calculated from these subgroups. Red lines are sinusoid fits, with the fringe frequencies, w, these qubits back to their respective idle frequencies, Ωq, for further operations if necessary, and then to their respective measurement frequencies, Ωm, for readout. We note that during the frequency-tuning process, qubits may gain different dynamical phases; that is, the ξ-y axes rotate differently in the equator planes for different qubits, which can be determined by a separate phase-tracking measurement followed by an optimization procedure (2⁷).

The resulting GHZ state is a superposition of |π/2, -Nπ/2⟩ and |π/2, -(N - 2)π/2⟩ in the collective spin representation, which can be transformed to a superposition of the N qubits all in |0⟩ and those all in |1⟩ by applying to each qubit a π/2 rotation around its x (N odd) or y (N even) axis. After such a transformation (sinusoids in zone III of Fig. 2A), the state is written as |00...0⟩ + exp(φ)|11...1⟩/√2, where φ = π/2 for uniform couplings. The diagonal elements of the GHZ density matrix ρ₀₀...₀ and ρ₁₁...₁ can be directly probed (2⁷).

The off-diagonal elements ρ₀₀...₀₁₁...₁ and ρ₁₁...₁₀₀...₀ are obtained by measuring the parity oscillations, defined as the expectation value of the operator "P(γ) = cos(γ)σ_{z, q} + sin(γ)σ_{x, q}" , which is given by "P(γ) = 2[ρ₀₀...₀₁₁...₁ cos(Nγ + φ) for the above-mentioned GHZ state (16). Experimentally we apply to each qubit a rotation (sinusoids in zone IV of Fig. 2A); these rotations bring the axis defined by the operator "P(γ) "[i.e., the direction represented by the angles (π/2, π/2 - γ) in each qubit’s Bloch sphere] to the z axis, followed by simultaneous qubit readout. Repeating each state generation and measurement pulse sequence multiple times yields 2⁵⁰ probabilities (P₀₀...₀, P₀₀...₁,..., P₁₁...₁), and the parity is calculated as ⟨P⟩ = P_even - P_odd with P_even (P_odd) corresponding to the summation of all those probabilities with even (odd) numbers of qubits in |1⟩. The amplitude of the oscillation patterns of "P(γ) " gives |ρ₀₀...₀₁₁...₁⟩ (Fig. 2B). Using values of ρ₀₀...₀, ρ₁₁...₁, and |ρ₀₀...₀₁₁...₁⟩, N-qubit GHZ state fidelities F are calculated as 0.817 ± 0.007 (N = 10), 0.775 ± 0.011 (N = 12), 0.655 ± 0.009 (N = 14), 0.579 ± 0.007 (N = 16), 0.549 ± 0.006 (N = 17), and 0.525 ± 0.005 (N = 18), all confirming genuine multipartite entanglement with F > 0.5 (2⁴).

Furthermore, during the dynamics under the one-axis twisting Hamiltonian (3), we take snapshots of the system with up to 20 qubits by measuring the quasi-distribution Q function Ω(0, Ψ) = ⟨0, Ψ|Ω(0, Ψ)|0⟩ (Fig. 3B), where Ω(0) is the evolving multiqubit density matrix, in comparison with numerical simulations (Fig. 3A). We observe the squeezed spin regime at the beginning (~15 ns), and the atomic Schrödinger cat states (which are superpositions of |m = 5, 4, 3, and 2 atomic coherent states) at t_m = 72, 91, 123, and 187 ns, respectively. For an m-component atomic Schrödinger cat state of N qubits, the overlap between two adjacent components is |cos(π/m)|2N. Therefore, to observe superpositions with more components, one needs to increase N to reduce the overlap. We note that superpositions of up to four coherent states have been previously observed in cold atoms and superconducting cavities (2⁸, 2⁹). Here, we observe the five-component atomic Schrödinger cat state of 20 qubits. In Fig. 3C, we plot its sliced Wigner function W(θ, φ) = Tr|υ⟩⟨υ|Ω(θ, φ)|11...1⟩ (30), where Ω = i∫F_1(1 - √3s_{y, j}) and Ω(θ, φ) rotates each qubit state by an angle θ around the axis, which has an angle φ + π/2 with respect to the x axis in the equator plane (2⁷). Nonclassicality of the state is indicated by the negative values of the sliced spin Wigner function (Fig. 3C). Because there is no straightforward witness measure for a cat state with a component number of more than 2 (31), the next step would be to find an efficient method to prove genuine entanglement in the experimentally generated multicomponent cat states.

Our experiment demonstrates a superconducting quantum processor featuring 20 individually addressable qubits and programmable qubit-qubit couplings. We deterministically generate the 18-qubit GHZ state and multicomponent atomic Schrödinger cat states of up to 20 qubits by engineering a one-axis twisting Hamiltonian. The high controllability and efficiency of our superconducting quantum processor indicate the potential of an all-to-all connected circuit architecture for exploring profound quantum many-body physics, and also for applications in practical quantum metrology and quantum information processing.

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**Fig. 3.** Multicomponent atomic Schrödinger cat states of 20 qubits generated during the dynamics at Δ/2π ≈ −450 MHz. (A) Numerical simulations of the quasi-distribution Q function in the spherical polar plots at specific time intervals according to Eq. 1 without decoherence, after the qubits are initialized in an atomic coherent state |π/2, −π/2⟩. (B) Experimental Q_{exp}(θ, φ) at time intervals as shown. Additional single-qubit dynamical phases are numerically added to rotate the plots in (A) for a better visual match with those in (B). The difference in the time steps between (A) and (B) may be due to various factors such as uncertainties in some device parameters including Δ and g_{ij}, imperfection in the experimental pulse sequences, and the existence of the small qubit-qubit cross-talk coupling terms beyond neighboring pairs (27). (C) Plots of the sliced spin Wigner function (27, 30) for the five-component cat state using the numerical simulation result at 65 ns in (A) and the experimental data at 72 ns in (B), where the negative values (blue regions) indicate the nonclassical nature of the state. Colors are set to be slightly transparent in (A) and (B), but not in (C) for visual clarity.