THE ORIGIN OF CARBON ENHANCEMENT AND THE INITIAL MASS FUNCTION OF EXTREMELY METAL-POOR STARS IN THE GALACTIC HALO

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Received 2006 March 3; accepted 2006 October 21

ABSTRACT

It is known that the carbon-enhanced, extremely metal-poor (CEMP) stars constitute a substantial proportion of the extremely metal-poor (EMP) stars of the Galactic halo, and a by far larger proportion than CH stars among Population II stars. We investigate their origin by taking into account an additional evolutionary path to the surface carbon enrichment, triggered by hydrogen engulfment by the helium flash convection, in EMP stars with \( [\text{Fe}/\text{H}] \lesssim -2.5 \). This process is distinct from the third dredge-up operating in more metal-rich stars and in EMP stars. In binary systems of EMP stars, the secondary stars become CEMP stars through mass transfer from the low- and intermediate-mass primary stars that have developed the surface carbon enrichment. Our binary scenario can predict the variations in the abundances not only for carbon but also for nitrogen and \( s \)-process elements and can reasonably explain the observed properties such as the stellar distributions of the carbon abundances, the binary periods, and the evolutionary stages. Furthermore, from the observed frequencies of CEMP stars with and without \( s \)-process enrichment, we demonstrate that the initial mass function of EMP stars needed gives the mean mass \( \sim 10 \, M_\odot \) under the reasonable assumptions for the distributions of orbital separations and mass ratios of the binary components. This also indicates that the currently observed EMP stars were exclusively born as the secondary members of binaries, making up \( \sim 10\% \) of EMP binary systems, with mass \( \sim 10^8 \, M_\odot \) in total; in addition to CEMP stars with white dwarf companions, a significant fraction of them have experienced supernova explosions of their companions. We discuss the implications of the present results for the formation of the Galactic halo.

Subject headings: Galaxy: halo — stars: abundances — stars: carbon — stars: evolution — stars: luminosity function, mass function

Online material: machine-readable tables

1 INTRODUCTION

Both the HK (Beers et al. 1992) and Hamburg/ESO (HES) surveys (Christlieb et al. 2001) have shown that a large fraction (\( \sim 20\% - 25\% \)) of EMP stars \( [\text{Fe}/\text{H}] \lesssim -2.5 \) exhibit enhancement of the surface carbon abundance (Rossi et al. 1999; Beers 1999; Norris et al. 2001; Christlieb 2003; see also Beers & Christlieb 2005). This forms a striking contrast to the fact that more metal-rich CH stars, characterized by enhanced CH bands in the spectra, account for only \( \sim 1\% \) of the other Population II stars (Tomkin et al. 1989; Luck & Bond 1991). Figure 1 shows the observed carbon enrichment against the metallicity for the stars whose abundances are available from analysis of high-dispersion spectroscopy. This figure contains 34 CEMP stars for \( [\text{Fe}/\text{H}] \lesssim -2.5 \), and we can readily see the prominence of CEMP stars among EMP stars even though the sample may not be free from bias. In addition, various peculiar aspects have been revealed: (1) CEMP stars are divided into two groups, one (CEMP-s) with the enhancement of \( s \)-process elements up to \( \sim 2 - 3 \, \text{dex} \) and the other (CEMP-no) with normal \( s \)-process element abundances (Norris et al. 1997b; Aoki et al. 2002a; Ryan et al. 2005; see also Fig. 7 below); (2) a fairly large group of stars display a great enrichment of nitrogen relative to carbon (Norris et al. 2002; Spite et al. 2005); and (3) several stars show a significant enhancement of \( r \)-process elements (Sneden et al. 1996) with large variations in degree up to a factor of \( \sim 100 \) or more (Honda et al. 2004).

These features may arise from the peculiarities in the structure and evolution of stars due to very low metallicity and also from their formation processes, characteristic of the early stages of our Galaxy. Theoretically, it has been known that low-mass stars of metallicity \( [\text{Fe}/\text{H}] \lesssim -2.5 \) dredge up carbon to the surface by a mechanism different from that in the stars of the metal-rich populations (Fujimoto et al. 1990, 2000). This mechanism, which is called helium flash-driven deep mixing (He-FDDM; see § 5), is proposed to explain the observed features of CEMP stars. Because of this theoretical property, as well as the observational characteristics mentioned, we call stars of \( [\text{Fe}/\text{H}] \lesssim -2.5 \) EMP stars in the following and distinguish them from Population II stars of \( [\text{Fe}/\text{H}] \gtrsim -2.5 \).

There is a well-established scenario for the origin of carbon-rich stars among the Population I and II stars. According to Iben (1975a) and Iben & Renzini (1983), a Population I or II star enhances the carbon abundance in the hydrogen-rich envelope by the third dredge-up (TDU) when it evolves to the asymptotic giant branch (AGB). Then the star loses its envelope by the stellar wind, eventually turning into a white dwarf. If this star belongs to a binary system, the companion star, less massive than this AGB star initially, is exposed to and accretes the wind matter enriched with carbon, and changes its surface composition to exhibit the carbon enrichment. As a result, a carbon star remains with an unseen companion star (a white dwarf). This is supported by the results of spectroscopic observations that all CH stars belong...
to binary systems with unseen companions (McClure 1984; McClure & Woodsworth 1990).

We propose that all CEMP stars are also made in binaries as CH stars are, but with carbon dredged up by He-FDDM, as well as by the TDU. A statistical analysis of CEMP stars by Lucatello et al. (2005b) suggests that most of these stars belong to multiple systems, supporting the argument that CEMP stars are metal-poor analogs of CH stars. In fact, if plotted as a function of the surface carbon enhancement ([C/H]), CEMP dwarfs and CEMP giants have distinct distributions, as shown in Figure 2. This indicates that these stars have suffered from external pollution and hence changed their surface carbon abundances according to the depths of their surface convective zones. CH stars may share a similar feature, but because of their large metallicity, the giants can hardly display the strong feature of CH lines since the dilution of surface carbon by deepening surface convection may reduce the surface abundance below C/O < 1. In fact we see in Figure 1 that CH giants are predominant at lower metallicities while at higher metallicities, Ba stars, which are giant counterparts of CH stars with C/O < 1, are abundant.

However, some differences are discernible between CEMP and CH stars. The proportion of CEMP to EMP stars is significantly larger than that of CH to Population II stars, as stated above. CH and CEMP stars have different distributions with respect to their orbital separations, as shown in Figure 3, although the statistics may not be sufficient for CEMP stars with large separations. In addition, some CEMP stars do not exhibit the enrichment of s-process elements, while all the CH stars show fairly large enrichments. The issue of these two subclasses, CEMP and CEMP-no s, is raised by Ryan et al. (2005). There has been no explanation for where these features come from.
The first purpose of this paper is to investigate the observational characteristics of CEMP stars by compiling data from the literature and to explain their origins by applying the binary scenario discussed above. The features of the CEMP star population have been discussed only from the observational viewpoint by Lucatello et al. (2005b) and by Ryan et al. (2005). Relying on the current understanding of stellar evolution, we present an explanation of the origins of both subclasses, CEMP-s and CEMP-nox, and also of the stars enriched with nitrogen but with little carbon. These subclasses are shown to be divided by the mass of their primary stars.

Second, we examine the binary scenario from the viewpoint of the evolution of binary systems with mass transfer taken into account. We corroborate our interpretation by comparing theoretical predictions with the observed properties of CEMP stars and by analyzing the metallicity dependencies through comparison with CH stars; we deal with the relative frequencies, the correlation of orbital period and carbon enrichment, and the distribution of evolutionary stages.

Finally, we inquire into the initial mass function (IMF) of EMP binaries on the basis of our binary scenario. The observed properties of EMP stars are thought to give information on the formation history and the early evolution of the Galaxy. From the viewpoint of chemical evolution, the richness of CEMP stars are linked with the IMF peaking at the intermediate stellar mass range (e.g., 4–8 M☉; Abia et al. 2001). On the other hand, Lucatello et al. (2005a) discuss the observed frequency of CEMP-s stars from a binary scenario similar to ours, arguing for an IMF shifted slightly to higher mass as compared with those of metal-rich populations, such as that expressed in a lognormal form peaking at 0.79 M☉. These studies, however, seem not to properly take into account the recent progress in understanding the evolution of currently observed EMP stars, and in particular, the modifications of surface characteristics during their long lives. Lucatello et al. (2005a) ignore the differences in the evolutionary paths to the carbon enhancement across the metallicity [Fe/H] ≃ −2.5, theoretically predicted as stated above. Furthermore, they deal only with CEMP-s stars, although it is desirable to put all other constituents of EMP stars into perspective as well. In this work, we apply the current best knowledge of the evolution of metal-poor stars and consider not only CEMP-s stars but also CEMP-nox stars and other subclasses.

Another important factor for the binary scenario is the assumption of the mass ratio distribution between the component stars, for which a consensus has not been reached neither observationally nor theoretically (e.g., see Mazeh et al. 2003 and references therein). In this paper, we assume one plausible assumption, although different from that assumed by Lucatello et al. (2005a; see § 7), and investigate the consequences.

The IMF derived in this work will give a new insight into the nature of EMP stars and their roles in the early evolution of our Galaxy and the formation of the Galactic halo. One proven feature of EMP stars is their scarcity compared with stars of more metal-rich populations. Bond (1981) has raised the issue of a “G dwarf” problem in the Galactic halo on the basis of the fact that the number of stars decreases sharply with decreasing metallicity below [Fe/H] ≃ −1.7 and no stars were found below [Fe/H] ≃ −2.6 at that time. Thanks to the recent large-scale HK and HES surveys, the number of known EMP stars has greatly increased, and yet, it is true that they remain by far fewer as compared with other constituent, Population II stars in the Galactic halo. The HES survey identifies ≲200 stars with [Fe/H] < −3.0 in the magnitude range of 10 ≤ B ≤ 17.5 among ∼40% of candidates selected from the fields of 8225 deg² (Beers & Christlieb 2005). Accordingly, EMP stars with [Fe/H] < −3.0 may be only a tiny fraction of stars, much fewer as compared with the stars in the globular clusters, ancillary constituents of Galactic halo. We also address this problem.

This paper is organized as follows. We start by reviewing the mechanisms for enhancing the carbon abundance in the stellar envelope of EMP stars in § 2. Then we formulate our binary scenario in §§ 3 and 4; the origin of abundance anomalies observed among EMP stars is identified in § 3 on the basis of evolutionary models, and the parameters of binary systems are specified in § 4 from the evolution of binary systems, taking into account the stellar wind from the primary and the subsequent accretion by the secondary. In § 5, the predictions of our binary hypothesis are compared with observations of particular CEMP stars for the period–carbon enhancement relations. In § 6, we investigate our binary scenario for CEMP-s stars, whose origin is well-defined theoretically and which deserve a separate statistical study because of their relative richness as compared with other EMP stars. In § 7, we deal with both subclasses, CEMP-s and CEMP-nox stars, to derive the IMF of EMP stars and to examine its implications. Finally, § 8 concludes the paper.

2. EVOLUTIONARY CHARACTERISTICS OF EMP STARS

A basic distinction in the evolution between the EMP stars and the more metal-rich (Population I and II) stars is the engulfment of hydrogen by the helium convection during the helium flash in the core or in the shell. Consequently, the low- and intermediate-mass EMP stars can enrich the surface with the nuclear products of helium burning through He-FDDM, triggered by the hydrogen mixing in addition to the TDU, as in the metal-rich populations. In this section, we review the evolution of EMP stars in relation to the carbon enhancement, as well as to the s-process nucleosynthesis, and present a revision of the general picture of their evolution, originally formulated by Fujimoto et al. (2000) as a function of initial mass and metallicity.

2.1. Evolutionary Paths to Surface Carbon Enhancement

Fujimoto et al. (1990) first show that the hydrogen mixing occurs during the helium flash in the core of a metal-free (Population III) star of mass 1 M☉ at the RGB tip, which is consequent upon a low entropy of the hydrogen-burning shell, realized by low CNO abundances. Hollowell et al. (1990) compute the progress of subsequent He-FDDM on the red giant branch (He-FDDM-R) in detail and demonstrate that it enhances the surface abundance of carbon and nitrogen. A number of subsequent works obtained consistent results for He-FDDM-R (Schlattl et al. 2001, 2002; Weiss et al. 2004; Picardi et al. 2004). This mechanism works for stars of very low metallicity ([Fe/H] ≲ −4.5; Fujimoto et al. 1995) and of low mass (M ≤ 1.1 M☉ for Z = 0, Suda et al. 2004; M ≤ 1.2 M☉ for [Fe/H] ≃ −5.3, T. Suda & M. Y. Fujimoto 2007, in preparation).

For the stars that do not undergo He-FDDM-R, Fujimoto et al. (2000) show that similar events happen during helium shell flashes in the early stage of the thermally pulsating AGB (TP-AGB) phase. He-FDDM on the AGB (He-FDDM-A) occurs in a star with larger metallicity ([Fe/H] ≲ −2.5) and of larger mass (M ≤ 3 M☉). Cassisi et al. (1996) report the hydrogen mixing into helium convection during a helium shell flash for a star of M = 0.8 M☉ and log Z = −10; despite the sufficiently low metallicity, their model star has skipped He-FDDM-R, presumably owing to the difference in the input physics affecting the evolution leading to the main He flash at the RGB tip. The progress of He-FDDM-A is investigated by a few other authors (Iwamoto et al. 2004, a 2 M☉ star of [Fe/H] = −2.7; Straniero et al. 2004, a 1.5 M☉ star of Z = 5 × 10⁻³), and all these results are consistent with Fujimoto et al. (2000).
Figure 4 schematically illustrates the progress of He-FDDM during core flash on the RGB tip (top) and He-FDDM during shell flashes on the AGB (bottom). The physical processes are quite similar, and yet they may differ according to the rate of hydrogen mixing when the helium convection first makes contact with the hydrogen-containing layer (Sweigart 1974). If the mixing rate of hydrogen is sufficiently large, the hydrogen engulfment causes the split of the convective zone and triggers He-FDDM. For the core helium flash, this is the case for the first flash because the core flash starts with a sufficient strength, as shown in Figure 4 (He-FDDM-R). For the helium shell flashes on the AGB, on the contrary, the hydrogen mixing takes place while the shell flashes grow in strength along with the cooling of the core, and hence starts with a weak flash that cannot bring about the splitting of the helium convective zone. After repeating the weak hydrogen mixing as it recurs, the shell flash finally grows strong enough to trigger He-FDDM, as shown in Figure 4 (He-FDDM-A). During the He-FDDM(-R and -A), the mixed hydrogen is carried down, burns in the middle of the helium convective zone, and ignites a hydrogen shell flash that splits the convection zone into lower and upper ones, driven by the helium and hydrogen burning, respectively. In the decay phase of the hydrogen shell flash, the flash convection retreats in the upper zone, and the shell occupied by the flash convection expands due to the heat deposited during the hydrogen flash and the helium flash. Upon the shell expansion, the surface convective zone deepens in mass and penetrates down into...
the shells formerly occupied by the hydrogen flash convection. As a result, the nuclear products of the helium flashes processed by the hydrogen shell flash are brought up to the surface. He-FDDM enriches the surface not only with carbon but also with nitrogen up to $N/C \leq 1$.

In addition to He-FDDM, EMP stars undergo the TDU as the stars of metal-rich populations do, except for the very low metallicity. There is a lower mass limit to the stars that undergo the TDU; Lattanzio (1986, 1987) shows that the TDU operates in a Population II star of mass $1.5 M_\odot$. A trend is recognized that the TDU occurs in less massive stars for lower metallicity (Iben 1983), although the efficiency of dredge-up depends on the treatment of convection (see Karakas et al. 2002 and references therein). Furthermore, our low-mass models may no longer remain metal-poor as far as CN elements are concerned because of previous He-FDDM. Thus, we can well set the lower mass limit for the TDU to occur at $\sim 1.5 M_\odot$, although this assumption has little to do with the following discussion. This implies that stars in the middle mass range between $\sim 1.5$ and $\sim 3.5 M_\odot$ experience both, first He-FDDM and then the TDU (Iwamoto et al. 2004; Straniero et al. 2004). It is also known that in massive AGB stars, the temperature at the bottom of the convective envelope reaches high enough to convert carbon into nitrogen and even to reduce the C/O ratio in the envelope below unity (Boothroyd et al. 1993), while the total abundance of CN elements increases as carbon is dredged up via TDU. We take the lower mass limit to this hot bottom burning (HBB) to be $\sim 5 M_\odot$, although it may depend on the metallicity and on the assumption about the efficiency of convective heat transport (see, e.g., Ventura et al. 2001).

2.2. $s$-Process Nucleosynthesis in EMP Stars

The hydrogen mixing may also promote the $s$-process nucleosynthesis in the helium convective zone. The engulfed hydrogen is carried inward and captured by $^{12}$C in the middle of the convective zone, and $^{13}$C, thus produced, is mixed further inward to release neutrons through the $^{13}$C($\alpha$, $n$)$^{16}$O reaction (Iwamoto et al. 2004; Suda et al. 2004). Thus, He-FDDM brings about the surface enrichment with $s$-process elements, as illustrated in the bottom panel of Figure 4. During the weak hydrogen mixing without the splitting of the convective zone, the synthesized $s$-process elements are spread all over the helium convective zone and will be involved again in the helium convective zones during the subsequent helium shell flashes. Eventually as the shell flash grows strong enough to trigger He-FDDM-A, $s$-process elements are dredged up to the surface along with carbon and nitrogen. For massive stars with $M \gtrsim 1.5 M_\odot$, some $s$-process elements, stored in the helium convective zone, are dredged up during the following TDU, although their abundances suffer dilution by matter newly added to the helium core due to quiescent hydrogen shell burning and then are incorporated into the flash convection. On the other hand, He-FDDM-R may bring few $s$-process elements to the surface since the convective zone splits for the first hydrogen engulfment, as shown in the top panel, although some $^{13}$C is carried down into the lower helium convective zone before the splitting and gives rise to $s$-process nucleosynthesis.

Another site for the $s$-process nucleosynthesis with the same reaction $^{13}$C($\alpha$, $n$)$^{16}$O as the neutron source but attendant on the TDU in AGB stars has been proposed; it relies on the possible formation of a thin layer containing $^{13}$C, i.e., the $^{13}$C pocket, in the top of the helium zone during the dredge-up phase, presumably by semiconvection in the carbon-rich helium zone (Iben & Renzini 1982a, 1982b) and/or by convective overshooting across the bottom of the surface convective zone. It is found that the $^{13}$C in the $^{13}$C pocket burns during the interpulse phase and the $s$-process nucleosynthesis occurs under radiative conditions before the subsequent helium shell flash is ignited (Straniero et al. 1995; Gallino et al. 1998). This radiative $^{13}$C-burning model is argued to work for the metal-rich populations from comparison with solar relative abundances of $s$-process elements. In particular, a tendency is proposed for larger production of heavier elements for lower metallicity since the number of neutrons available per seed nucleus may increase with decreasing metallicity (Busso et al. 1995; for a review see Busso et al. 1999). From this radiative $^{13}$C model, therefore, we can expect that EMP stars exhibit very large ratios between the heavy to main $s$-process elements that are much larger than those in stars of metal-rich populations. In actuality, however, existing observations indicate that this is not necessarily true; rather, an opposite trend of decreasing the Pl/Ba ratio for smaller metallicity [$Fe/H$] $\simeq -2.5$ is reported (Aoki et al. 2000; see also Lucatello et al. 2003). Figure 5 shows the ratios of heavy to main $s$-process elements, compiled from the literature, as a function of the metallicity. A break in the variation of the Pb/Ba ratio with metallicity is discernible near $[Fe/H] \simeq -2.5$; this is particularly evident if we note that two stars with the largest Pb/Ba ratios are among the binaries with the three shortest periods and hence are likely to have suffered from extensive mixing during the common envelope evolution (Lucatello et al. 2003).

The above fact can be interpreted as evidence that the efficiency of the $^{13}$C pocket decreases and/or varies in EMP stars (Ryan et al. 2001; R. Gallino et al. 2006, private communication) or even as evidence that the radiative $^{13}$C burning will not work for metallicities below $[Fe/H] \lesssim -2.5$ (Suda et al. 2004). The overshooting from the bottom of the convective envelope may be conceivable in some way or other, but we have not yet a reliable theory for supporting the formation mechanism of the $^{13}$C pocket and, in particular, for predicting the variations with the metallicity. In addition, for metallicities $[Fe/H] \lesssim -2.5$, the $s$-process nucleosynthesis occurs in the helium convective zone triggered by hydrogen mixing within the current standard framework of stellar structure, in which mixed $^{13}$C is diluted over the entire helium convective zone and will not necessarily give such large numbers of

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**Fig. 5.**—Relative abundances between the main $s$-process elements, Ba or La, and the heavy $s$-process elements, Pb, plotted against the metallicity for CEMP and CH stars. Double circles denote those with orbital periods observed to date; points with names attached denote three stars with the shortest periods, which are likely to have experienced common envelope evolution. The dashed line denotes the prediction from the radiative $^{13}$C-burning model by Busso et al. (1999).
The above discussion leads to a general picture of carbon and $s$-process enhancement in EMP stars, as illustrated in Figure 6. It is an up-to-date version of the dependences of the He-FDDM and the TDU on the initial stellar mass and metallicity, formulated by Fujimoto et al. (2000).

**Case I.**—Low-mass stars of $[\text{Fe/H}] \leq -4.5$ in the mass range of $M \leq 1.1 M_\odot$ for $Z = 0$ and $M \leq 1.2 M_\odot$ for $[\text{Fe/H}] \simeq -5.3$ (Suda et al. 2004; T. Suda & M. Y. Fujimoto 2007, in preparation) undergo He-FDDM-R at the helium core flash. Once He-FDDM-R occurs, the surface composition becomes $[\text{C/H}] \sim 0$ and $N/C \sim 1$. These stars may exhibit little enhancement of $s$-process elements.

**Cases II and II'.**—For stars located in the region of $[\text{Fe/H}] \simeq -2.5$ and $M \lesssim 3.5 M_\odot$ in Figure 6, but excluding the region of Case I, He-FDDM-A occurs when they evolve into the early phase of the TP-AGB. The surface abundance of carbon reaches as large as $[\text{C/H}] \sim 0$ for stars with the smallest masses, while it decreases for stars of larger masses because of larger envelope mass and also smaller mass in the flash convective zone. He-FDDM-A ceases after the CN abundances in the envelope exceed $[\text{C} + \text{N}/\text{H}] \simeq -2.5$, and the subsequent evolution is divided by the occurrence of the TDU: case II of $M < 1.5 M_\odot$ is that no further mixing takes place, and case II' of $M \gtrsim 1.5 M_\odot$ is that the TDU follows and brings C synthesized by helium burning to the surface to reduce the N/C ratio. The $s$-process elements, synthesized via the convective $^{13}$C burning, are dredged up by He-FDDM-A for both cases, and in addition, by the subsequent TDU for case II'.

**Case III.**—The stars more massive than $\sim 3.5 M_\odot$ and with the smallest metallicities will undergo neither He-FDDM nor TDU, although the latter point is a matter of controversy because of the efficiency of overshooting (Suda et al. 2004).

**Cases IV and IV'.**—The stars in these region undergo the TDU but not He-FDDM. Cases IV and IV' are distinguished by the surface enrichment of $s$-process elements, i.e., case IV' is of lower metallicity without $s$-process enhancement, and case IV is of higher metallicity with $s$-process enhancement. The difference is ascribed to the efficiency of the radiative $^{13}$C burning. The TDU dredges up carbon, but it may be converted into nitrogen via the HBB in the envelope for massive stars of $M \gtrsim 5 M_\odot$; thus, the nitrogen abundance may vary with the stellar mass from no enhancement to large enhancements, even exceeding the carbon abundance and reaching up to the equilibrium value of CN (or CNO) cycles ($N/C \sim 30$–16 in the range of $T \sim 5 \times 10^9$–$10^9$ K).

In summary, the surface carbon enhancement in EMP stars is brought about by He-FDDM for the initial mass $M \leq 1.5 M_\odot$, by both He-FDDM and the TDU for $1.5 M_\odot \lesssim M \lesssim 3.5 M_\odot$, and only by the TDU for $M \gtrsim 3.5 M_\odot$. One of the distinguishing features of He-FDDM, as compared with the TDU, is the surface enrichment of nitrogen at the same time as that of carbon; for cases I and II, the surface abundance ratio results are $N/C \simeq 1$ to $1/5$ (Fujimoto et al. 2000), while for case II', it decreases as carbon is enriched by the TDU. For case IV, the nitrogen abundance may range from no enhancement to a large value corresponding to the equilibrium in CN (or CNO) cycles. As for $s$-process elements, our scenario predicts the surface enrichment due to the convective $^{13}$C burning for cases II and II', and postulates no or little enhancement for case IV'. This conjecture predicts the variations in...
We select a CEMP star as having...process elements in CEMP stars.

3. BINARY SCENARIO I: THE ORIGINS OF CEMP AND RELATED EMP STARS

If EMP stars belong to a close binary system, the surface characteristic abundances that the evolved primary star has developed can be imprinted onto the secondary star through the mass transfer. A CEMP star, thus produced, may keep a record of the primary star in its surface characteristics. In this section, we survey the observed characteristics of CEMP stars and other related EMP stars, and discuss their origins, in the light of the modifications of the abundances of nitrogen and of s-process elements in CEMP stars as well.

3.1 THE ORIGINS OF CEMP STARS

TABLE 1
OBSERVATIONAL SAMPLE OF CEMP STARS FROM THE LITERATURE

| Object | Period (days) | e | log g | [Fe/H] | [C/Fe] | [N/Fe] | [O/Fe] | [Sr/Fe] | [Ba/Fe] | [Pb/Fe] | References |
|--------|--------------|---|------|-------|--------|--------|--------|--------|--------|--------|------------|
| HE 0024–2523 | 3.14 | 0 | 4.3 | -2.72 | 2.6 | 2.1 | 0.40 | 0.34 | 1.46 | 3.3 | 1 |
| G77-61 | 245 | 0 | 5.05 | -4.03 | 2.6 | 2.6 | 0 | <1 | 2.17 | 3.55 | 3, 4, 5 |
| CS 29497-030 | 342 | 0 | 4.1 | -2.57 | 2.30 | 2.12 | 1.48 | 0.84 | 2.17 | 3.55 | 3, 4, 5 |
| CS 22948-027 | 426.5 | 0.2 | 1.8 | -2.47 | 2.43 | 1.75 | 0.90 | 2.26 | 2.72 | 6, 7, 8 |
| CS 22942-019 | 2800 | 0.1 | 2.4 | -2.64 | 2.0 | 0.8 | 1.7 | 1.92 | <1.6 | 8, 9, 10 |
| CS 22957-027 | 3125 | 0.45 | 2.4 | -3.11 | 2.4 | 1.6 | -0.56 | -1.23 | 8, 9, 10 |
| CS 29497-034 | 4130 | 0.02 | 1.8 | -2.90 | 2.63 | 2.38 | 1.00 | 2.03 | 2.95 | 7 |
| LP 625-44 | >12 yr | 2.5 | -2.72 | 2.25 | 0.95 | 1.85 | 1.32 | 2.81 | 2.55 | 11, 12, 13 |
| CS 22892-052 | 127.8? | ? | 1.6 | -3.03 | 0.89 | 0.71 | 0.72 | 0.44 | 0.92 | 1.2 | 8, 13, 14, 15, 16 |
| CS 30301-015 | Binary | 0.8 | -2.64 | 1.6 | 1.7 | 0.3 | 1.45 | 1.7 | 9, 10 |
| CS 29526-110 | Binary | 3.2 | -2.38 | 2.2 | 1.4 | 0.88 | 2.11 | 3.3 | 9, 10 |
| HE 2148-1247 | Binary | 3.9 | -2.3 | 1.91 | 1.65 | 0.76 | 2.36 | 3.12 | 17 |
| CS 22877-001 | Binary? | 2.2 | -2.85 | 1.0 | 0.0 | -0.12 | -0.49 | 16, 18, 19 |
| CS 22183-015 | Binary? | 2.5 | -2.85 | 2.34 | ... | 2.09 | 3.17 | 19, 20 |
| CS 22949-027 | 1.3 | 3.8 | ? | 0.14 | 0.16 | ... | 2.82 | 24 |
| CS 30162-012 | 4.5 | -2.55 | 2.1 | 1.2 | 0.3 | 1.98 | 2.4 | 9, 10 |
| HD 187216 | 0.4 | -2.48 | 1.3 | 0.2 | 0.2 | 2.5 | 26 |
| CS 30162-050 | 0.3 | -2.31 | 2 | 1.2 | 0.91 | 2.30 | 2.9 | 9, 10, 27 |
| CS 22896-027 | 3.7 | -2.26 | 2.2 | 0.9 | 0.92 | 2.23 | 2.84 | 9, 10 |
| HD 196994 | 1.8 | -2.25 | 1.32 | 1.3 | 0.84 | 1.10 | 1.9 | 9, 10 |

Notes.—We select a CEMP star as having [Fe/H] ≤ -2.5 and [C/Fe] ≥ 0.5 from the literature. Also included are a N-rich star with [Fe/H] ≤ -2.2 and [C/Fe] ≥ 0.5. Table 1 is also available in machine-readable form in the electronic edition of the Astrophysical Journal.

a CS 31062-012 = LP 706-7.
b Radial velocity variations are observed but not enough to estimate the period.

References.—(1) Lucatello et al. 2003; (2) Plez & Cohen 2005; (3) Sivarani et al. 2004; (4) Ivans et al. 2005; (5) Sneden et al. 2003; (6) Aoki et al. 2002a; (7) Barbuy et al. 2005; (8) Preston & Sneden 2001; (9) Aoki et al. 2002d; (10) Aoki et al. 2002e; (11) Aoki et al. 2002f; (12) Aoki et al. 2002g; (13) Norris et al. 1997a; (14) Honda et al. 2004; (15) Spite et al. 2005; (16) Sneden et al. 2003; (17) Cohen et al. 2003; (18) Giridhar et al. 2001; (19) Tsangarides et al. 2004; (20) Johnson & Bolte 2002; (21) Frebel et al. 2005; (22) Christlieb et al. 2002; (23) Christlieb et al. 2004; (24) Cohen et al. 2004; (25) McWilliam et al. 1995; (26) Kimper & Jorgensen 1994; (27) Johnson & Bolte 2004; (28) Depagne et al. 2002; (29) Norris et al. 2001; (30) Aoki et al. 2005.

We compile the characteristics of CEMP and CH stars from the literature, as summarized in Tables 1 and 2, respectively. Our criterion of CEMP stars is that [Fe/H] ≤ -2.5 and [C/Fe] ≥ 0.5, but we include those with slightly larger metallicity that show the nitrogen enhancement ([N/Fe] ≥ 0.5) while taking into account possible errors (~0.25 dex) in the abundance determination. We have 34 CEMP stars in total (see Fig. 1) and list their available data for the orbital parameters, the gravity, the metallicity, and the abundances of C, N, O, and s-process elements (Sr, Ba, and Pb) relative to iron; seven stars have the orbital periods derived with...
| Object | Binarity | Period (days) | [Fe/H] | [C/H] | [C/Fe] | [Ba/Fe] | Class | References |
|--------|----------|---------------|--------|-------|--------|---------|-------|------------|
| HD 77247 | Y | 80.55 | ... | ... | ... | ... | Ba | 1 |
| HD 46407 | Y | 458.6 | -0.42 | 0.11 | 0.53 | 1.39 | Ba | 1, 2 |
| HD 199939 | Y | 564.9 | ... | ... | ... | ... | Ba | 1 |
| HD 58368 | Y | 672.7 | ... | ... | ... | ... | Ba | 1 |
| HD 31987 | Y | 1066.4 | ... | ... | ... | ... | Ba | 1 |
| HD 223617 | Y | 1301 | ... | ... | ... | ... | Ba | 1 |
| NGC 2420-X | Y | 1402 | -0.7 | ... | ... | ... | Ba | 1, 3 |
| HD 13611 | Y | 1642.1 | ... | ... | ... | ... | Ba | 4, 5 |
| HD 101013 | Y | 1710.9 | ... | ... | ... | ... | Ba | 1 |
| HD 49641 | Y | 1768 | ... | ... | ... | ... | Ba | 1 |
| HD 16458 | Y | 2018 | ... | ... | ... | ... | Ba | 1 |
| HD 204075 | Y | 2422 | ... | ... | ... | ... | Ba | 1 |
| HD 16458 | Y | 2018 | ... | ... | ... | ... | Ba | 1 |
| HD 204075 | Y | 2422 | ... | ... | ... | ... | Ba | 1 |
| HD 178717 | Y | 2866 | 0.21 | 0.21 | 0.7 | Ba | 1, 3, 6 |
| HD 199394 | Y | 4390 | ... | ... | ... | ... | Ba | 1 |
| HD 196673 | Y | 4000 | ... | ... | ... | ... | Ba | 1 |
| HD 178717 | Y | 2866 | 0.21 | 0.21 | 0.7 | Ba | 1, 3, 6 |
| HD 199394 | Y | 4390 | ... | ... | ... | ... | Ba | 1 |
| HD 139195 | Y | 5324 | ... | ... | ... | ... | Ba | 5, 7 |
| HD 202190 | Y | 6489 | ... | ... | ... | ... | Ba | 5, 7 |
| HD 5825 | ... | ... | -0.7 | -0.85 | -0.15 | ... | Ba | 8 |
| HD 4084 | ... | ... | -0.7 | -0.58 | 0.12 | ... | Ba | 8 |
| HD 15589 | ... | ... | -0.6 | -0.45 | 0.25 | ... | Ba | 8 |
| HD 27271 | ... | ... | -0.5 | -0.35 | 0.15 | ... | Ba | 8 |
| HD 44896 | ... | ... | -0.4 | 0.36 | 0.76 | 0.82 | Ba | 3, 6 |
| HD 164979 | N | ... | -0.33 | -0.38 | -0.05 | ... | Ba | 1, 9 |
| HD 83548 | ... | ... | -0.33 | 0.04 | 0.37 | 0.49 | Ba | 10 |
| HD 65699 | ... | ... | -0.3 | -0.45 | -0.15 | ... | Ba | 8 |
| HR 774 | ... | ... | -0.3 | 0.31 | 0.61 | 1.03 | Ba | 3, 6 |
| HD 116713 | ... | ... | -0.29 | 0.27 | 0.56 | 1.62 | Ba | 2 |
| HD 109061 | ... | ... | -0.21 | -0.32 | 0.11 | 0.6 | Ba | 3 |
| HD 95345 | ... | ... | -0.2 | -0.45 | -0.25 | ... | Ba | 8 |
| HD 65966 | ... | ... | -0.2 | -0.17 | 0.03 | 0.49 | Ba | 10 |
| HD 60197 | ... | ... | -0.2 | 0.21 | 0.41 | 0.57 | Ba | 3, 6 |
| ζ Cyg | ... | ... | -0.1 | 0.4 | 0.5 | 0.9 | Ba | 3, 11 |
| HD 92626 | ... | ... | 0.04 | 0.68 | 0.64 | ... | Ba | 3 |
| HD 89638 | ... | ... | 0.1 | 0.1 | 0.5 | 0.5 | Ba | 3 |
| ζ Cap | ... | ... | 0.1 | 0.13 | 0.03 | 1 | Ba | 3, 12 |
| HD 121447 | ... | ... | 0.1 | 0.31 | 0.21 | 0.57 | Ba | 3, 6 |
| HD 100012 | ... | ... | 0.33 | 0.67 | 0.74 | 0.28 | Ba | 3 |
| +42 2173 | Y | 328.3 | ... | ... | ... | ... | Ba | 8 |
| +20 29621 | Y | 407.4 | -0.9 | 0.11 | 1.01 | ... | CH | 1, 13 |
| +08 2654A | Y | 571.1 | ... | ... | ... | ... | CH | 1 |
| HD 5223 | Y | 755.2 | -2.06 | -0.49 | 1.57 | 1.82 | CH | 1, 15 |
| HD 224959 | Y | 1273 | -1.6 | 0.11 | 1.71 | ... | CH | 1, 13 |
| HD 198269 | Y | 1295 | -1.4 | -0.49 | 0.91 | 2.07 | CH | 1, 13, 14 |
| HD 135148 | Y | 1416 | -1.88 | -0.1 | 1.78 | ... | CH | 1, 15 |
| HD 201626 | Y | 1465 | -1.3 | 0.01 | 1.31 | ... | CH | 1, 13 |
| HD 30443 | Y | 2954 | ... | ... | ... | ... | CH | 1 |
| HD 187861 | Y | ... | -1.65 | 0.41 | 2.06 | ... | CH | 13 |
| −38 2151 | Y | ... | -1.4 | -0.39 | 1.01 | ... | CH | 13 |
| HD 189711 | Y | ... | -1.15 | 0.80 | 1.95 | 1.80 | CH | 16 |
| HD 42272 | Y | ... | -1.10 | 1.21 | 2.31 | 1.65 | CH | 16 |
| HD 197604 | Y | ... | -0.90 | 0.85 | 1.75 | 1.80 | CH | 16 |
| HD 25408 | Y | ... | -0.82 | 0.70 | 1.52 | 1.47 | CH | 16 |
| HD 59643 | Y | ... | -0.70 | 0.58 | 1.28 | 2.95 | CH | 16 |
| HD 26 | Y | ... | -0.44 | 0.01 | 0.45 | ... | CH | 1, 13, 17 |
| HD 104340 | Y | ... | -1.15 | -0.62 | 0.53 | -0.83 | MDBa | 3 |
| HD 204613 | Y | 878 | -0.35 | 0.52 | 0.87 | 0.56 | SGCH | 17, 18, 19 |
| HD 89948 | Y | 1153 | -0.27 | 0.47 | 0.74 | 0.83 | SGCH | 17, 18, 19 |
| HD 122202 | Y | 1290 | -0.1 | ... | ... | ... | SGCH | 18 |
| HD 202020 | Y | 1796 | -0.2 | ... | ... | ... | SGCH | 18 |
| HD 227847 | Y | 2046 | -0.1 | ... | ... | ... | SGCH | 18 |
| HD 216219 | Y | 3871 | -0.32 | 0.63 | 0.95 | 0.89 | SGCH | 17, 18, 19 |
| HD 11377 | Y | 4140 | -0.05 | 0.32 | 0.37 | 0.03 | SGCH | 17, 18, 19 |
| HD 4395 | Y | >6200 | -0.33 | 0.2 | 0.53 | 0.56 | SGCH | 17, 18, 19 |
the maximum $P = 11.3$ yr, and seven more stars are suspected of binarity from variations in radial velocity. As for CH stars, we compile 85 stars by the criterion that [Fe/H] $> -2.5$; in addition to those classified as CH stars in the literature, we include three blue stragglers showing strong CH lines and also Ba stars that are thought to be giant counterparts of subgiant CH stars with C/O $< 1$ (e.g., Luck & Bond 1991). Thirty-eight of these stars have the orbital periods determined (two stars with lower bounds). Note that CH stars are divided into two groups, giant and subgiant CH stars, the latter of which includes both dwarfs and subgiants (Bond 1974).

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### 3.1. Subclassification of CEMP Stars

Figure 7 shows the observed relationship of the enrichment between Ba and N among CEMP stars. As already noted by Aoki et al. (2002a; see also Ryan et al. 2005), they are rather clearly divided into two groups of [Ba/Fe] $\geq 1$ and [Ba/Fe] $\leq 0$; among the exceptions is CS 22892-052, which is enriched with $\alpha$-process elements (Sneden et al. 1996).

All CEMP stars with Ba enrichment ([Ba/Fe] $\geq 1$), which are defined as CEMP-$\alpha$, are enriched with nitrogen ([N/Fe] $\geq 1$). The relative abundance to carbon lies in the range of $0.01 \leq N/C \leq 1$ and is consistent with the theoretical predictions of cases II and II'; the largest ratios may result from He-FDDM in case II, and the smaller ratios are explicable in terms of the increase of carbon via the TDU in case II'.

On the other hand, CEMP stars without Ba enrichment ([Ba/Fe] $< 0$), which are defined as CEMP-no, exhibit still larger variations in the nitrogen abundances, ranging from no enhancement $[N/Fe] \approx 0$ (CS 22877-001) to much larger enhancements with $N/C \approx 10$ (CS 22949-037). The range of enrichment fits well case IV$^*$ evolution in our scenario with and without HBB. It is difficult to identify a site of nucleosynthesis in the stars other than HBs that can realize such large N/C ratios near to the equilibrium values in the CN cycles. As a class, CEMP-no stars have carbon enhancements smaller than CEMP-$\alpha$ stars, as pointed out by Aoki et al. (2002a, also see Fig. 7). This is attributable to the dilution of dredged-up carbon in the envelope of the primary star, which is

![Fig. 7.—Enhancement of s-process elements [Ba/Fe], plotted against the enrichment of carbon (open symbols) and of nitrogen (filled symbols) for CEMP stars. Squares, circles, and triangles denote giants, subgiants, and dwarfs, respectively; diamonds plot three mixed stars enriched with nitrogen but not with carbon for comparison.](image-url)
more massive in case IV than in cases II and II', and also to the transformation of carbon into nitrogen by HBB.

Some stars in the CEMP-nos subclass have large oxygen abundances comparable to, or even larger than, the carbon abundance. In the binary scenario, the enhancement of oxygen exceeding about a tenth of the carbon abundance has to be attributed to the pristine abundance (cf. Suda et al. 2004), which can well be explained in terms of the general tendency of [O/Fe] to increase with decreasing metallicity and its extension to EMP stars (Israel et al. 1998; Boesgaard et al. 1999; but see also Garcia Perez et al. 2006) (see Appendix A.1).

Among EMP stars, a class of stars displays large nitrogen enrichment with [C + N/Fe] > 0 but only weak or little carbon enhancement with the ratio C/O < 1; these are classified as “mixed” stars by Spite et al. (2005). We propose that these stars are made by the same mechanism as CEMP-nos stars but that carbon was converted into nitrogen by HBB in more massive primary stars than those of CEMP-nos stars (Appendix A.2). In our compilation, those with r-process element enhancements such as CS 22892-052 can also be included in the subclass of CEMP-nos since the Ba enrichment can be attributed to the r-process nucleosynthesis (Sneden et al. 1996). We inquire into their origin below since the Ba enrichment can be attributed to the surface carbon enhancement. The second one is that the secondary component is set at 0.8 M⊙ in mass limit to HBB in the envelope (Ventura et al. 2001). Models of [Fe/H] = −3 in shaded zones become CEMP via RLO or via wind accretion, and models of [Fe/H] = −1 in hatched zones become CH stars via RLO or via wind accretion.

4. BINARY SCENARIO II: BINARY PARAMETERS OF CEMP STARS

In this section, we discuss the binary parameters under which the secondary star can evolve to CEMP stars through mass transfer. The first condition is that the primary star is allowed to develop the surface carbon enhancement. The second one is that the binary system can enrich the secondary star sufficiently with the envelope mass ejected from the primary stars. These conditions set lower and upper bounds to the binary separations, respectively, for a given set of masses of the primary and secondary stars, as discussed below. We can derive the relation of the carbon abundance of the secondary star to the mass of the primary star and the orbital separations (periods), which can be compared with the observations.

4.1. Roche Lobe Overflow

We start with the discussion of the first condition. In close binary systems, the primary star expands as it evolves and sometimes fills its Roche lobe to lose the envelope through mass transfer to the secondary and/or the mass outflow from the systems (e.g., Iben & Tutukov 1985). If this Roche lobe overflow (RLO) happens before the star becomes a carbon-enhanced star, then this system cannot yield a CEMP or CH star. In order to yield a CEMP-s star, therefore, the initial size of the Roche lobe has to be greater than the stellar size when the primary star for M ≤ 3.5 M⊙ starts the He-FDDM. The same is true for a CEMP-nos star and for a CH star but with the stellar size when the TDU starts. Since the volume of the Roche lobe is given as a function of the mass ratio M2/M1 and the separation A (Paczynski 1971), the lower bounds, AHe-FDDM and ATDU, to the initial binary separation necessary to accommodate the primary stars when He-FDDM and TDU start, respectively, can be derived from the radii, RHe-FDDM and RTDU, at the stage of He-FDDM and TDU as

\[ A_{\text{He-FDDM}} (\text{or } A_{\text{TDU}}) = \frac{R_{\text{He-FDDM}} (\text{or } R_{\text{TDU}})}{0.38 + 0.2 \log (M_1/M_2)}. \]  

Figure 8 shows these lower bounds on the diagram of the initial stellar mass of the primary star and the binary separation for the stellar models taken from existing computations (Iben 1975a; Lattanzio 1986; Mowlavi 1999; Marigo 2002). The initial orbital radius needs to be greater than ~0.2–1 AU for a CEMP-s star and greater than ~1 AU for a CH and a CEMP-nos star.

After the primary has developed the surface carbon enrichment, the RLO may occur on the AGB before the primary evolves to a white dwarf. In this case, the binary will undergo common envelope evolution and shrink in size to be a short-period system, or even coalesce in some cases. These systems can be distinguished from CH and CEMP stars produced through the accretion process in the wind from the primary star in the binary of wider initial separation as proposed by Boffin & Jorissen (1988) and discussed below.

4.2. Wind Accretion from AGB Stars

The second condition, i.e., an upper bound to the binary separation, results from the wind accretion. AGB stars in a binary lose their envelopes through stellar wind (Reimers 1975; Gail & Sedlmayr 1987) if the separations are sufficiently large to avoid
RLOs. Then the secondary stars may accrete carbon-rich material in this wind to become carbon-enhanced stars. The process for the secondary stars to accumulate the gas from the wind can well be approximated to the Bondi accretion (Bondi & Hoyle 1944), and the cross section, \( \sigma_{\text{acc}} \), can be evaluated from the following formula,

\[
\sigma_{\text{acc}} = \pi \left( \frac{2GM_{\ast}}{v_{\text{rel}}} \right)^2,
\]

where \( G \) is the gravitational constant, \( M_{\ast} \) is the mass of the secondary star, and \( v_{\text{rel}} \) is the velocity of the wind relative to the secondary. Thus, the mass accretion rate \( dM_{\ast}/dt \) of the secondary star is related to the mass-loss rate \( dM_{\ast}/dt \) of the primary as

\[
\frac{dM_{\ast}(t)}{dt} = - \frac{G^2M_{\ast}^2(t)}{A(t)^2v_{\text{rel}}^4(t)} \frac{dM_{\ast}(t)}{dt},
\]

where the separation \( A(t) \) between the two stars has been introduced as a function of time, \( t \), and \( v_{\text{wind}} \) is the wind velocity with respect to the primary star. The relative speed of the secondary star to the stellar wind is given by

\[
v_{\text{rel}} = \sqrt{\frac{v_{\text{wind}}^2 + |r_1 - r_2|^2}{2}},
\]

with the orbital velocities, \( r_1 \) and \( r_2 \) of the primary and secondary stars, respectively. The mass accreted by the secondary star depends on the separation \( A(t) \) and the relative velocity \( v_{\text{rel}}(t) \), which may vary during the mass transfer along with the masses \( M_1(t) \) and \( M_2(t) \).

Suppose that the primary and secondary stars each rotate on a circular orbit around the center of mass while the primary loses mass at a constant rate \( dM_{\ast}/dt \). The basic equations governing the evolution of a binary system are then

\[
M_1(t)v_1^2(t)/r_1(t) = M_2(t)v_2^2(t)/r_2(t) = GM_1(t)M_2(t)/A^2(t),
\]

\[
M_1(t)r_1(t) - M_2(t)r_2(t) = 0,
\]

\[
r_1(t) + r_2(t) = A(t).
\]

Here \( r_i \) denotes the radius of the orbit of star \( i \), and \( v_i \) denotes the velocity. The subscript \( i = 1 \) refers to the values of the primary star, and \( i = 2 \) refers to those of the secondary star. The timescale for the change of \( A(t) \) is assumed to be much longer than the orbital period.

If Jeans’ theorem is applied, the relation between the separation and the mass of the binary system is given by

\[
A(t)[M_1(t) + M_2(t)] = \lambda,
\]

where \( \lambda \) is a constant, given by the initial conditions at time \( t_{\text{fm}} \). This implies that the wind is assumed to carry the angular momentum away from the system without transferring it to the other material. By expressing \( A(t) \) and \( v_{\text{rel}}(t) \) in terms of the stellar masses and the wind velocity, equation (3) is reduced to

\[
\frac{dM_{\ast}(t)}{dM_1(t)} = - \frac{M_{\ast}(t)}{M_1(t) + M_2(t)} \left[ \frac{\mu}{M_1(t) + M_2(t)} \right]^{-1} \times \left[ 1 + \left( \frac{\mu}{M_1(t) + M_2(t)} \right)^2 \right]^{-3/2},
\]

where \( \mu \) is a constant, defined as

\[
\mu = \sqrt{\frac{\lambda v_{\text{wind}}^2}{G}} = \sqrt{2M_1(M_1 + M_2)(A/R_1)(v_{\text{wind}}/v_{\text{esc}})}.
\]

In the second line, we normalize the wind velocity with respect to the escape velocity, \( v_{\text{esc}} \), from the surface of the primary star \([=(2GM_1/R_1)^{1/2}] \), where \( R_1 \) is the radius of the primary star. If \( A \gg R_1 \), then \( \mu(M_1 + M_2) \gg 1 \) (since \( v_{\text{wind}} \sim v_{\text{esc}} \)) so that the variation of \( M_2 \) is much smaller than that of \( M_1 \). We can neglect the variation of \( M_2 \) in the right-hand side and integrate equation (9) until the primary becomes a white dwarf at time \( t_{\text{fm}} \) to find the accreted mass, \( M_{\text{acc}} \), on the secondary star:

\[
M_{\text{acc}} = \left[ \frac{M_2^2}{\mu} \left( \frac{M_1 + M_2}{\mu^2 + (M_1 + M_2)^2} \right) \right]^{M_1=M_2(t_{\text{fm}})}_{M_1=M_1(t_{\text{fm}})}.
\]

The relation between the initial stellar mass, \( M_1(t_{\text{fm}}) \), and the mass, \( M_{\text{acc}}(t_{\text{fm}}) \), of the C+O core for metal-poor stars can well be approximated to

\[
M_1(t_{\text{fm}}) = \max \{0.54 + 0.073M_1(t_{\text{fm}}), \min[0.29 + 0.178M_1(t_{\text{fm}}), 0.65 + 0.062M_1(t_{\text{fm}})]\},
\]

for the metallicity of \( Z_0 = 0.001 \), given by Han et al. (1994).

4.3. Dilution Due to Surface Convection

Suppose that matter of constant carbon abundance, \( X_{\text{C},1} \), is accreted onto the secondary of initial carbon abundance \( X_{\text{C},2,0} \). If the mass in the surface convection, \( M_{\text{acc},d} \), remains constant through mass accretion, then the variation of the surface carbon abundance, \( X_{\text{C},2} \), of the secondary star can be described by

\[
\frac{dX_{\text{C},2}}{dt} = \frac{X_{\text{C},1} - X_{\text{C},2} - dM_2}{M_{\text{acc},d}}.
\]

The integration yields \( X_{\text{C},2} \) as a function of the accreted mass, \( M_{\text{acc}} \), as

\[
X_{\text{C},2} = X_{\text{C},2,0} + (X_{\text{C},1} - X_{\text{C},2,0})[1 - \exp(-M_{\text{acc}}/M_{\text{acc},d})].
\]

The mass transfer is likely to occur while the secondary star resides on the main sequence with a shallow surface convective zone. Then the surface abundance suffers an additional dilution owing to the deepening of surface convection as the secondary later evolves to the red giant branch (RGB). For the metallicity of \( [\text{Fe}/\text{H}] \lesssim -3 \), a star of mass \( M_2 \approx 0.8 M_\odot \) has the surface convection of mass \( M_{\text{acc},d} \approx 0.003 M_\odot \), on the main sequence. The surface convection, although decreasing as the hydrogen abundance decreases in the center, begins to deepen rapidly as the shell burning starts and attains its deepest reach of \( M_{\text{acc},d} \approx 0.20 - 0.35 M_\odot \) in mass near the base of the RGB when the luminosity \( L \approx \) several \( \times 10 L_\odot \) (Fujimoto et al. 1995; Suda et al. 1999).
2004). At the base of the RGB, therefore, the surface carbon abundance reduces to

\[
X_{C,2} = X_{C,2,0}(1 - M_{\text{acc}}/M_{\text{scz},g}) + X_{C,1}(M_{\text{acc}}/M_{\text{scz},g})
\] (15)

(here we assume \(M_{\text{scz},g} > M_{\text{acc}} + M_{\text{scz},\text{ch}}\)). After that, it remains constant along the giant branch. As a corollary, if carbon is brought by external pollution, the \([\text{C}/\text{H}]\) value decreases up to the maximum of 2 dex when an EMP star evolves to a giant.

The surface carbon abundance of secondary stars resulting from the wind accretion is given as a function of the initial masses of the component stars and the initial binary separation from equations (11) and equation (14) or (15) for dwarfs or giants, respectively. These relations can be transferred to the upper bound, \(A_M(M_1, [\text{C}/\text{H}], M_{\text{scz}})\), to the initial binary separation that allows the binary system with the primary star of mass \(M_1\) to enrich the secondary star of mass \(M_2\) that has the surface convection of mass \(M_{\text{scz}}\) with carbon above \([\text{C}/\text{H}]\) through the wind accretion. In Figure 8, we plot the upper bounds of initial binary separation that give the carbon enhancement of \([\text{C}/\text{H}]\) = −1, −2, and −3 for the secondary giants with the surface convection \(M_{\text{scz}} = 0.35 M_0\) (\(M_2 = 0.8 M_0\)). The carbon enhancement necessary for CEMP and CH stars can be taken to be \([\text{C}/\text{H}] > -3\) and −1, respectively. For such a large initial separation, since the orbital velocity is much smaller than the wind velocity, the cross section of Bondi-Hoyle accretion reduces to a constant, \(\sigma_{\text{acc}} = \pi(GM_2/v_{\text{wind}})^2\), and hence, we can approximate

\[
A_M(M_1, [\text{C}/\text{H}], M_{\text{scz}}) = \left\{ [M_1 - M_1(t_{\text{fin}})]\sigma_{\text{acc}}/(4\pi10^{[\text{C}/\text{H}]-[\text{C}/\text{H}]_0}M_{\text{scz}}) \right\}^{1/2}. \quad (16)
\]

The upper bound of separation increases in inverse proportion nearly to the second power of the carbon enrichment of the secondary star, and hence, CEMP stars can have much wider separations than CH stars. The bound also depends on the carbon abundance, \([\text{C}/\text{H}]_1\), in the wind. For TDU, the latter is currently subject to uncertainties because of the lack of a reliable theory for the mass loss, and yet, we can well assume that \([\text{C}/\text{H}]_1 \approx 0\) or so.

We have formulated our binary scenario as a function of the mass ratios and the binary separations, as illustrated in Figure 8. In §§ 5−7, we examine the validity of our interpretation and binary scenario first for the relationship between the orbital period and carbon enhancement, second for the statistical properties of CEMP-s stars, and finally for the relative frequencies of CEMP-s and CEMP-nos stars, and the IMF of EMP stars.

5. PERIOD−CARBON ENHANCEMENT RELATION

We can deduce the relations between the carbon abundance of the secondary star and the orbital period from our modeling in § 4. The accreted mass, \(M_{\text{acc}}\), onto the secondary star can be estimated from equation (11) as a function of the initial masses of the component stars, \(M_1(t_{\text{fin}})\) and \(M_2(t_{\text{fin}})\), and the initial separation, \(A_{\text{fin}}\). The surface chemical compositions of the secondary stars can then be evaluated with the mixing in the surface convective zone of the secondary star taken into account from equation (14) or (15). On the other hand, as a kinematic feature, the orbital period after the primary star becomes a white dwarf is calculated from the final separation \(A(t_{\text{fin}})\) in equation (8) and the final mass \(M_1(t_{\text{fin}})\) of the primary star in equation (12). These are observable and can be used to test the present scenario, as in Boffin & Zac's (1994), who discuss the relationship between the overabundances of s-process elements and the orbital periods for barium stars.

In deriving the relationship between the accreted mass and the orbital period, we have applied Jeans’ theorem. In actuality, however, the wind material is thought to carry an additional orbital angular momentum unless the wind velocity \(v_{\text{wind}}\) is much larger than the orbital velocity \(v_{\text{orb}} = (v_1 - v_2)\). In fact, hydrodynamic calculations of wind accretion (e.g., Nagae et al. 2004; Jahanara et al. 2005) have shown that the orbit shrinks if \(v_{\text{wind}} \leq (1.5−1.7)v_{\text{orb}}\) for the mass ratio of \(q(M_2/M_1) = \sim 1\) to 3/3 (see also Hachisu et al. 1999), which can correspond in our scenario to

\[
A \leq 5−7(1 + q)(M_1/M_0)\left(\frac{v_{\text{wind}}}{20 \text{ km s}^{-1}}\right)^{-2} \text{AU}, \quad (17)
\]

\[
P \leq 11−16(1 + q)(M_1/M_0)\left(\frac{v_{\text{wind}}}{20 \text{ km s}^{-1}}\right)^{-3} \text{yr.} \quad (18)
\]

Jeans’ theorem holds good for the wider binaries of \(v_{\text{wind}} \geq (1.8−1.9)v_{\text{orb}}\), while the flow structure resembles the RLO for the wind velocity of \(v_{\text{wind}} < 0.4v_{\text{orb}}\).

Figure 9 plots the relations between the surface carbon abundance \([\text{C}/\text{H}]\) of secondary stars and the orbital period, calculated under the assumption of Jeans’ theorem with the primary stars of mass 1.5 and 3 \(M_0\). The right panel is for the binary systems of \([\text{Fe}/\text{H}] > -2.5\) whose primary stars have undergone the carbon enhancement by the TDU and can be compared with the observations for CH stars. Here we assume the carbon abundance in the wind matter to be \([\text{C}/\text{H}]_0 = 0.5\) and the wind velocity to be \(v_{\text{wind}} = 20 \text{ km s}^{-1}\). A subgiant has a smaller surface convective zone (by a factor of 30 in mass) than a red giant star, which yields larger carbon enhancement with the same binary parameters. For the systems of short period \(P \leq \sim 10 \text{ yr}\), the subgiants tend to have the same surface abundances as the wind. On the other hand, the giants display a carbon abundance smaller than the wind even for the smallest separations and decreases for larger separations nearly as \([\text{C}/\text{Fe}] \sim -4/3 \log P\). These tendencies agree with the observations that the subgiants tend to have larger carbon abundances than the giants; the exceptions are the blue stragglers of the smallest metallicity, \([\text{Fe}/\text{H}] < -2\), which can be attributed to the difference in the carbon abundances reached in the primary stars, as discussed below. For most subgiant CH stars, the observed carbon abundances are larger than the solar value, which indicates that the carbon abundance in the wind is greater than the solar value (up to \([\text{C}/\text{H}]_0 \approx 0.5\)). The observed scatter can be taken as reflecting the different efficiencies of the TDU in bringing carbon into the envelopes of primary stars and/or be attributed to the gravitational settling of accreted carbon in the envelope of secondary stars. Weiss et al. (2000) show that the heavy pollutants settle down to decrease their surface abundance by a factor up to \(\sim 10\) in 10 Gyr, which may partly explain the spread of surface carbon abundances, observed from CH subdwarfs and also CEMP stars in Figure 2. The gravitational settling has little to do with the shell where the hydrogen shell-burning happens, and hence, the accreted matter may affect the evolution of the secondary to the helium core flash (Fujimoto et al. 1995).

As for the separation (or the period), the observations lies well below the predictions based on Jeans’ theorem. Since most periods of the observed systems fall in the range of equation (18) or shorter, they are thought to have suffered orbital shrinkage during the mass transfer episode. In order to see the effects of additional
angular momentum loss, we plot the relations under the assumption of constant separation in the figure (dashed lines); they end with smaller final separations and larger accreted masses compared to the relations based on Jeans’ theorem. The observed points mostly lie near the shortest ends of the separation necessary to contain the primary stars within the Roche lobe at the beginning of the TDU. Furthermore, some, including the two blue stragglers, have observed periods even shorter than the latter, which suggests that these systems have experienced a large decrease in separation during the common envelope phase.

The left panel shows the results for CEMP stars, for which the carbon abundance of the wind is taken to be \( \frac{C}{H} = 0 \). Since the radius of the EMP primary at He-FDDM is smaller than the radius of the Population II primary at the TDU (see Fig. 8), the binary separation can be smaller for CEMP stars than CH stars. CEMP stars of short period \( P \leq 10 \) yr must also have suffered decreasing separations during the mass transfer episode. If the shrinkage is taken into account, the relation calculated from the wind accretion scenario gives a reasonable description of the observed trend of CEMP stars, similar to that of CH stars, discussed above. It is worth noting that the CEMP-nos star, CS 22957-027 (\( \frac{C}{H} = -7.1 \) and \( P = 3, 125 \) days) is also among the stars showing reasonable agreement with the theoretical prediction. A dwarf, HE 0024–2523, with the shortest period and possibly the subgiant star CS 29497-030 (names attached in the figure) have undergone RLO to accrete carbon-rich matter from the primary stars. The eccentricities of these stars are nearly zero, which is also expected from RLO, although the detailed modeling of their evolution awaits future studies with the interaction between the stellar wind and the orbital motion properly taken into account. The other dwarf, G77–61, with the second shortest period, deserves comment because of the rather weak carbon enrichment of \( \frac{C}{H} \approx -1.4 \) (Plez & Cohen 2005). It has the mass \( \sim 0.3 M_\odot \) (Dearborn et al. 1986) and is likely to be wholly convective. In addition, the cross section of the star for the accretion is a factor of \( \sim 7 \) smaller than that of a red giant CEMP star (see eq. [2]), and hence, the accreted mass of carbon-rich material might be smaller by the same factor. Even if we take into account these two factors that can lead to a relatively low value of \( \frac{C}{H} \), our model has difficulty reproduc-

![Diagram](image_url)

Fig. 9.—Orbital periods vs. surface carbon abundance \( \frac{C}{H} \) for CEMP stars (left) and for CH stars (right). The theoretical predictions from the wind accretion model described in § 4.2 are plotted under the assumption of Jeans’ theorem (solid lines) and of constant separations (dashed lines) for giants (left-hand side) and for dwarfs and subgiants (right-hand side) with the primary masses of 1.5 \( M_\odot \) (thin lines) and 3.0 \( M_\odot \) (thick lines). The mass of the secondary star is set at 0.8 \( M_\odot \), and the mass in the surface convection is taken to be 0.0035 and 0.35 \( M_\odot \) for CEMP dwarfs and giants, respectively, and 0.01 and 0.35 \( M_\odot \) for subgiant and giant CH stars, respectively. The carbon abundance in the wind is assumed to be \( \frac{C}{H} = 0 \) for CEMP stars and \( \frac{C}{H} = 0.5 \) for CH stars. Two dash-dotted lines in the left panel plot the relations for a wholly convective secondary star of mass 0.3 \( M_\odot \). Symbols denote observational data for CEMP giants (filled squares), subgiants (filled circle), and dwarfs (filled triangles) in the left panel and for CH stars (open squares), subgiant CH stars (open circles), and field blue stragglers (open diamonds) in the right panel.

6. STATISTICAL FEATURES OF CEMP-\( s \) STARS

We have proved that our binary scenario gives a reasonable description of the observed characteristics for the particular CEMP stars with known binary periods in comparisons with CH stars. Now we turn to the statistics of CEMP stars to demonstrate the validity of our binary scenario and to investigate insights into their properties, gained from our binary scenario, in § 6.1 and 6.2. We first deal with CEMP-\( s \) stars of cases II and II', which are well delineated as a subclass, both observationally and theoretically by the large enrichments not only of carbon and nitrogen but also of the \( s \)-process elements. We then demonstrate that constraints can be drawn on the IMF and the nature of EMP binaries, taking both CEMP-\( s \) and CEMP-nos stars into account.

6.1. Frequency in Comparison with CH Stars

On the basis of our scenario, we can estimate the fractions that CH and CEMP-\( s \) stars occupy among the Population II and EMP stars, respectively. The binary parameters that the secondary stars can evolve to CEMP-\( s \) and CH stars are depicted on the diagram...
of initial mass of primary stars and binary separation in Figure 8. For CEMP-s stars, we can take the lower bound to the initial mass of the primary star to be $M_1(t_{in}) = 0.8 \, M_\odot$ and assume that the primary stars more massive than this mass have already ejected their carbon-rich envelope and evolved to white dwarfs. The upper bound to the primary initial mass for CEMP-s stars is set at $M_{He-FDDM} \simeq 3.5 \, M_\odot$ by the condition for He-FDDM to occur. For CH stars, on the other hand, the lower and upper bounds are set by the lower mass limit to the TDU at $M_{TDU} \simeq 1.5 \, M_\odot$ and by the HBB, which converts carbon into nitrogen in the surface convection, at $M_{HBB} \simeq 5 \, M_\odot$, respectively. As for the initial binary separation, the first condition is that the primary star is allowed to develop carbon enhancement and sets the lower bounds, $A_{He-FDDM}$ and $A_{TDU}$, for CEMP-s and CH stars, respectively. The second condition is to enrich sufficiently the carbon abundance of the secondary star through the wind accretion, which imposes the upper bounds, $A_M(M_1, [C/H], M_{acc})$. The carbon enhancement necessary for CEMP and CH stars can be taken to be $[C/H] > -3$ and $-1$, respectively. Consequently, CEMP-s stars occupy a much larger parameter space than CH stars, as seen from Figure 8.

In order to evaluate the relative frequencies of CEMP-s and CH stars numerically, we have to specify the IMF $\phi_{CEMP-s}$ and $\phi_{CH}$, are then proportional to the following integrals:

$$
\phi_{CEMP-s} \propto \int_{M_{HBB}}^{M_{TDU}} \frac{n(M_2/m)}{m} \, dm \times \int_{A_{He-FDDM}}^{A_{TDU}} f(P) \, dP \, da,
$$

$$
\phi_{CH} \propto \int_{M_{HBB}}^{M_{TDU}} \frac{n(M_2/m)}{m} \, dm \times \int_{A_{TDU}}^{M_{TDU}} f(P) \, dP \, da.
$$

As for the IMF for the primary stars of mass $> 0.8 \, M_\odot$, we can here assume Salpeter’s power law,

$$
\xi(m) \propto m^{-2.35}.
$$

We can also assume a lognormal form as in § 7 (see eq. [36]) below. For the binary periods, we can adopt the distribution that has been derived from the observations of local stars,

$$
f(P) \propto \frac{1}{P} \exp \left[ \frac{-(\log P - 4.8)^2}{2 \times 2.3^2} \right]
$$

(Duquennoy & Mayor 1991), where $P$ is the period in units of days.

On the contrary, as for the mass ratio distribution, no consensus has been achieved. For the nearby G dwarf binary samples, it is shown that the distribution function can fit well by a Gaussian form, centered at $q = 0.23$ with a rather large dispersion of $\sim 0.42$, or could be even flat for $q < 0.23$. It is also pointed out, however, that the same mass function as for the single stars in Millar and Scalo law is admissible for the secondary stars because of large uncertainty in the mass ratio below $q \leq 0.1$ (Duquennoy & Mayor 1991). Even with the recent studies it still remains an open question. On one hand, it is reported that the distribution has a small peak at $q \sim 0.8$ and rises as the mass ratio decreases to $q \lesssim 0.2$ with a drop at $q < 0.2$ for nearby spectroscopic binaries (Goldberg et al. 2003) or that it has a broad shallow peak between $q \sim 0.2$ and $\sim 0.7$ and a sharp peak at $q > 0.8$ for nearby F7–K stars (Hallwachs et al. 2003). On the other hand, Mazeh et al. (2003) argue that the mass ratio distribution is approximately constant over the range $q = 1.0–0.3$ and a constant distribution cannot be ruled out at lower $q$ for main-sequence binaries as measured by infrared spectroscopy. In particular, Goldberg et al. (2003) find that the halo binaries have a flatter distribution than the disk binaries. For such low-mass binaries, the mass ratio distribution is subject to large uncertainties in the range of mass ratio below $q \sim 0.1$ since the main sequence does not exist for $M < 0.08 \, M_\odot$. For massive star binaries with O and B-type primaries, in contrast, the mass function of the secondary stars is found to extend into the very small mass ratio $q < 0.1$ (Abt 1983). In the present study, therefore, we can well assume a uniform distribution of mass ratio of binary systems for simplicity:

$$
n(q) = \text{const. for } 0 < q \leq 1.
$$

The resulting mass functions of CEMP and CH stars depend only weakly on the secondary mass $M_2$ through the upper and lower boundary values of integral with respect to the separation. This makes a sharp contrast with the case when the same single-star mass function as the primary stars is assumed for the secondary stars, as was done by Lucatello et al. (2005a). We can examine the validity of our assumption from the comparisons with the observed properties of CEMP and CH stars, as discussed below.

The observed flux-limited samples of CEMP-s and CH stars are dominated by those that have masses near the upper end and are most luminous. Putting $M_2 = 0.8 \, M_\odot$ and evaluating the double integrals with respect to the period and primary mass in equations (19) and (20) numerically with $[C/H] = -3$ and $-1$, respectively, for CEMP-s and CH stars, we find that

$$
\frac{\phi_{CEMP-s}}{\phi_{CH}} \simeq 14.
$$

This number represents the ratio between the frequencies of CEMP-s and CH stars among the stars of corresponding metallicity that are born in binaries. Therefore, it gives a reasonable account of the observed much larger frequency of CEMP stars among EMP stars as compared with the frequency of CH stars among Population II stars, unless the fraction of EMP stars born in binaries is significantly smaller than those of the more metal-rich populations. In evaluating the ratio, we have applied Salpeter’s IMF to EMP stars, as well as to CH stars, although there is no guarantee that both obey the same IMF. Nevertheless, this conclusion will be weakly affected since the large ratio arises essentially from the difference in the parameter space in which EMP and Population II stars can develop the carbon enhancement, as seen from Figure 8. We discuss the influence of different IMFs in § 7.

### 6.2. Period Distributions in Comparison with CH Stars

Since CEMP-s and CH stars stem from the binary systems of different parameters in our binary scenario, they should differ in the distribution of the orbital separation (or the period). The relative frequencies of the initial orbital separation (or the initial orbital period) can also be predicted from the integration of equations (19) and (20) with respect to the primary mass for CEMP and CH stars, respectively. The derived relative frequencies are
plotted in Figure 3 and compared with the observations. Here, we take into account both giants and dwarfs, and a shoulder on the right-hand side of each curve comes from the contribution of dwarfs.

For CH stars, our binary scenario predicts the initial separations (or periods) restricted to rather narrow range between ~3 and 30 AU (~several to 100 yr). The observed periods also fall in a narrow range of separations (or orbital periods) in agreement with the prediction but are shifted toward smaller separations (or shorter periods) by a factor of several on average. This shrinkage of binary systems is consistent with the argument in § 5, based on equation (18), which anticipates that the wind may carry away a larger amount of angular momentum than is specific to the orbital motion of the primary star in interaction with the secondary motion.

In contrast, CEMP-s stars are predicted to stem from binaries with a much larger range of initial separations (or initial periods); for those with [C/H] > −3, the initial parameters range from 0.3 AU (less than 1 yr) to more than 1000 AU (~10,000 yr) with the central values near ~30 AU (~100 yr). All the observed orbital periods of CEMP-s stars are less than ~10 yr because of the limited span of observation time to date and fall within the shorter half of the initial period range, as predicted from our scenario. For such shorter periods, the binary systems must have suffered from shrinkage during the mass-loss event from the primaries. The observations are consistent with this expectation, for which the distribution of observed separations (or periods) is extended toward shorter separations (or periods) than predicted. In addition, we expect from Figures 3 and 8 that more than half the CEMP-s giants and a still larger portion of CEMP-s dwarfs have wider separations (or longer periods) than those observed to date. In this parameter range, Jeans’ theorem tends to hold, and the binary separations (or periods) increase further. So, in the extreme end, it is difficult to detect the orbital motion of binary spectroscopically as a periodic variation of absorption lines.

In summary, the binary scenario is compatible not only with the observed period distribution of CEMP-s stars but also with the lack of detected variations in the radial motion for half of them to date. These results are indifferent to the assumptions of the IMF and of the distribution of mass ratio. They are basically not in agreement with the IMF of extreme metal-poor stars but also with the spatial distributions. We denote by $N_s(IL)$ the surface number density (per unit steradian) of stars with the luminosity $L$ that are accessible with a given limiting flux $f_{lim}$. As for the spatial distribution, we can well assume that the EMP stars obey the density distribution, $\rho(r)$, of stars in the Galactic halo, approximated by

$$\rho(r) \propto r^{-3}$$

as a function of radial distance $r$ from the Galactic center (e.g., see Majewski 1993 and references therein). Since the surveys have been performed exclusively in fields of high Galactic latitude, we can well consider the direction perpendicular to the Galactic disk, which leads to

$$N_s(IL) = \int_0^{\sqrt{L/4\pi f_{lim}}} \rho(r) r^2 dz \propto \int_0^{\sqrt{1/4\pi f_{lim}}} \frac{z^2}{(8^2 + z^2)^{3/2}} dz,$$

(28)

where the distance from the Galactic center to the Sun is taken to be 8 kpc and $z$ denotes the height above the Galactic disk in units of kiloparsecs. This factor becomes important as the stars evolve to be luminous along the giant branch. For the dwarfs with low luminosity, on the other hand, the spatial variations in the density of EMP stars are negligible, and we simply have $N_s(IL) \propto L^{3/2}$.

The size of surface convection, $M_{sz}$, differs by a factor of ~100 (see eq. [16]) between dwarfs and giants, and hence, the upper limit, $A_M(M_1, [C/H], M_{sz})$, to the binary separation of carbon
enhancement for dwarfs and subgiants is 10 times larger than that for giants. Thus, the number ratio of CEMP-s giants to CEMP-s dwarfs and subgiants will be given by \( \psi_{\text{s, gw}}(\psi_{\text{s, dw}} + \psi_{\text{s, sg}}) \), each of which is defined as

\[
\psi_{\text{s, dw}} = \int_{0.08}^{0.8} dM_2 N_2(L(M_2)) \\
\times \int_{0.8}^{M_{\text{bin, EDM}}} dM_1 \xi(M_1) \frac{n(M_2/M_1)}{M_1} \\
\times \int_{A_{\text{min, EDM}}}^{4\mu(M_1, -3, M_{\text{ms}, a, dw})} f(P) \frac{dP}{da} da, \quad (29)
\]

\[
\psi_{\text{s, sg}} = \int_{0.8+\Delta m_a}^{0.8+\Delta m_a + \Delta m_s} dM_2 N_2(L(M_2)) \\
\times \int_{0.8}^{M_{\text{bin, EDM}}} dM_1 \xi(M_1) \frac{n(0.8/M_1)}{M_1} \\
\times \int_{A_{\text{min, EDM}}}^{4\mu(M_1, -3, M_{\text{ms}, a, dw})} f(P) \frac{dP}{da} da, \quad (30)
\]

\[
\psi_{\text{s, gw}} = \int_{0.8+\Delta m_a}^{0.8+\Delta m_a + \Delta m_s} dM_2 N_2(L(M_2)) \\
\times \int_{0.8}^{M_{\text{bin, EDM}}} dM_1 \xi(M_1) \frac{n(0.8/M_1)}{M_1} \\
\times \int_{A_{\text{min, EDM}}}^{4\mu(M_1, -3, M_{\text{ms}, a, si})} f(P) \frac{dP}{da} da. \quad (31)
\]

By adopting the limiting magnitude of 15 mag, we find the resulting proportions of RGB+HB stars, subgiants, and dwarfs at 63%, 10%, and 27%, respectively. In our samples, there are 13 giants, four subgiants, and four dwarfs (62%, 19%, and 19%, respectively) among CEMP-s stars if we define dwarfs as stars with the surface gravity, \( \log g \geq 4.2 \), subgiants as \( 4.2 > \log g \geq 3.5 \), and giants (including HB stars) as \( \log g < 3.5 \), respectively. The above estimate based on the binary scenario gives a reasonable agreement with the current observations.

7. INITIAL MASS FUNCTION AND BINARITY OF EMP STARS

In § 6, we have demonstrated that the statistical features of CEMP-s stars as a subclass are understandable in terms of our binary scenario. In this section, we take up the problems of the observed frequencies of CEMP-s and CEMP-no stars among EMP stars and of the total number of EMP stars in an attempt to inquire into the IMF of EMP stars and to give an insight into their nature.

From our scenario, we can estimate the proportions of CEMP-s and CEMP-no stars to EMP binaries. The number, \( \psi_{\text{bin}} \), of all the EMP stars born in binary systems in a flux-limited sample can be evaluated in the same way as we derived the CEMP-s population in § 6. We can divide the EMP binaries into three categories. In addition to (1) white dwarf binary systems, discussed above, in which the primary stars in the mass range of 0.8 \( M_0 \) < \( M_1 \) \leq \( M_{\text{sp}} \) have already become white dwarfs and the secondary stars still undergo nuclear fusion, we define two more of (2) low-mass binary systems in which both members are in the mass range of \( \leq 0.8 M_0 \) and still alive as nuclear-burning stars and (3) supernova binary systems in which the primary stars in the mass range of \( M_1 > M_{\text{sp}} \) have already exploded as supernovae and the secondary stars still remains in the nuclear-burning stages.

Then the total EMP stars currently observed are given by the sum of these three categories as

\[
\psi_{\text{bin}} = \psi_{\text{WDB}} + \psi_{\text{LMB}} + \psi_{\text{SNB}}, \quad (32)
\]

where

\[
\psi_{\text{WDB}} = \int_{0.08 M_0}^{0.8 M_0} dM_2 N_2(L(M_2)) \int_{0.8 M_0}^{M_{\text{up}}} dM_1 \xi(M_1) \frac{n(q)}{M_1} \\
\times \int_{A_{\text{min, WDB}}(M_1, M_2)}^{4\mu(M_1, -3, M_{\text{ms}, a, si})} f(P) \frac{dP}{da} da, \quad (33)
\]

\[
\psi_{\text{LMB}} = \int_{0.08 M_0}^{0.8 M_0} dM_2 \int_{0.8 M_0}^{M_{\text{up}}} dM_1 N_1(L(M_1)) \xi(M_1) \frac{n(q)}{M_1} \\
\times \int_{A_{\text{min, LMB}}(M_1, M_2)}^{4\mu(M_1, -3, M_{\text{ms}, a, si})} f(P) \frac{dP}{da} da, \quad (34)
\]

\[
\psi_{\text{SNB}} = \int_{0.08 M_0}^{0.8 M_0} dM_2 N_2(L(M_2)) \int_{0.8 M_0}^{M_{\text{up}}} dM_1 \xi(M_1) \frac{n(q)}{M_1} \\
\times \int_{A_{\text{min, SNB}}(M_1, M_2)}^{4\mu(M_1, -3, M_{\text{ms}, a, si})} f(P) \frac{dP}{da} da. \quad (35)
\]

Here \( A_{\text{min, WDB}}(M_1, M_2) \), \( A_{\text{min, LMB}}(M_1, M_2) \), and \( A_{\text{min, SNB}}(M_1, M_2) \) denote the minimum separations that can contain an AGB and a main-sequence star, two main-sequence stars, and a supernova progenitor and a main-sequence star, respectively; and \( A_{\text{cut}} \) denotes the upper bound of binary separation and is taken to be consistent with \( P = 10^{10} \) days given by Duquennoy & Mayor (1991).

The evaluation of these numbers requires the IMF of EMP stars in the whole mass range, and we can well assume a lognormal form of the IMF with the medium mass, \( M_{\text{md}} \), and the variance, \( \Delta M \), as parameters, i.e.,

\[
\xi(m) \propto \frac{1}{m} \exp \left[ -\frac{(\log m - \log M_{\text{md}})^2}{2\Delta M^2} \right]. \quad (36)
\]

For the Galactic spheroid, the IMF is shown to be well approximated by a lognormal form with the parameters of \( M_{\text{md}} = 0.22 M_\odot \) and \( \Delta M = 0.33 \) for low-mass stars (\( m < 1 M_\odot \)) and the Salpeter form for more massive stars (see, e.g., review by Chabrier 2003). EMP stars may not necessarily follow the same IMF as the Galactic spheroid components of larger metallicity. Rather we here treat these two parameters as free and discuss the constraints imposed on them from comparisons with the current observations. As for the binary mass ratio and period, we assume the same distributions as adopted in § 6.

7.1. Frequency of CEMP-s Stars

The population of CEMP-s stars, \( \psi_{\text{CEMP-s}} \), predicted from our binary scenario, is given by the sum of CEMP-s dwarfs, subgiants, and giants (including HB stars) in equations (29)–(31):

\[
\psi_{\text{CEMP-s}} = \psi_{\text{s, dw}} + \psi_{\text{s, sg}} + \psi_{\text{s, gw}}. \quad (37)
\]

We can integrate equations (37) and (32) by using the IMF in equation (36). Figure 10 plots the proportion of CEMP-s stars to EMP binaries, \( \psi_{\text{CEMP-s}}/\psi_{\text{bin}} \), as a function of medium mass, \( M_{\text{md}} \), with a fixed dispersion of \( \Delta M = 0.33 \), assuming simple
lognormal IMFs. If the same IMF as the Galactic spheroid (peaked at 0.22 \( M_\odot \)) is applied, we find

\[
\begin{align*}
\psi_{\text{CEMP-s}} / \psi_{\text{WDB}} & \simeq 0.58, \\
\psi_{\text{CEMP-s}} / \psi_{\text{bin}} & \simeq 0.14.
\end{align*}
\]

The former ratio indicates that more than half the white dwarf binaries have produced CEMP-s stars, which is attributable to the very large coverage in the parameter space of separations and mass ratios, as stated above. Nevertheless, their proportion to the total EMP stars decreases by a factor of \( \sim 4.1 \), owing mainly to the contribution from the low-mass binaries. If we take into account the contribution of the stars born as single, this fraction is too small when compared with the observed very high frequencies of CEMP-s stars (see below). This may be indicative of a more massive IMF for EMP stars. As \( M_{md} \) increases, the frequency of low-mass binaries diminishes, and the CEMP-s proportion increases to attain a maximum of \( \psi_{\text{CEMP-s}} / \psi_{\text{bin}} \simeq 0.37 \) near \( M_{md} \simeq M_{\text{He-FDDM}} \). For a still larger \( M_{md} \), it begins to decrease as the contributions increase from white dwarf binaries with the massive primaries of \( M > M_{\text{He-FDDM}} \) and then from supernova binaries of \( M > M_{\text{SN}} \). In this figure, we also plot the CEMP-s proportion if we assume that the same number of EMP stars are born as single. The contribution of single-born EMP stars may be significant for the IMF of small \( M_{md} \) but decreases rapidly to be negligible for the IMF of large \( M_{md} > M_{\text{He-FDDM}} \).

From the HK and HES observations, it is reported that the proportion of CEMP stars of \([\text{C}/\text{Fe}] > 1\) amounts to 25% in the metallicity range of \([\text{Fe/H}] < -2.5\) (Beers 1999; Rossi et al. 1999; Christlieb 2003; see also Lucatello et al. 2005a, 2005b). They include the contribution from CEMP-no stars, which accounts for \( \sim 25\% \) of CEMP stars (Ryan et al. 2005; Aoki et al. 2007). If allowance is made for the contribution of CEMP-no, then the CEMP-s proportion may be smaller. However, it will be larger if we include CEMP-s stars that have a smaller carbon enrichment of \( [\text{C}/\text{Fe}] > 0.5 \). Recently, Cohen et al. (2005) find the frequency of CEMP stars to be \( 14.4\% \pm 4\% \) for \([\text{Fe/H}] < -2\) and \([\text{C}/\text{Fe}] \geq 1.0 \) in the HES sample by correcting errors in abundance analyses of CEMP stars by means of follow-up spectroscopy. This gives the CEMP-s fraction at \( \sim 10\% \) with the possible contribution from CEMP-no subtracted. However, their definition of CEMP stars is different from ours and may underestimate the CEMP-s fraction since the efficiency of producing the carbon enrichment greatly decreases for larger metallicities of \([\text{Fe/H}] > -2.5\), as discussed above.

Accordingly, we can well adopt the observed proportion of CEMP-s stars among the EMP stars at \( \sim 10\% - 25\% \). As seen from Figure 10, there are two possible solutions with the separate ranges of \( M_{md} \) that predict the proportion of CEMP-s stars compatible with the observations; one is the low-mass IMF with \( M_{md} \simeq 0.6 - 2.5 M_\odot \), and the other is the high-mass IMF with \( M_{md} \simeq 7.5 - 14 M_\odot \). The two solutions are separated by the overproduction of CEMP-s stars from the white dwarf binaries because of the large coverages in the parameter space, as seen above from Figure 8. Although both IMFs produce similar proportions of CEMP-s stars, they predict quite different constituents for other EMP stars. In the case of the low-mass IMF, EMP stars other than CEMP stars are constituted mainly of low-mass stars born as single and of the members of low-mass binaries. In the case of the high-mass IMF, the extant EMP stars were mostly born as low-mass members of binaries, and more than half stem from binaries with a massive primary, which have undergone supernova explosions.

Our result differs from that of Lucatello et al. (2005a), who find only the low-mass solution with \( M_{md} \simeq 0.79 M_\odot \), which arises from a different assumption for the distribution of mass fraction \( n(q) \); we adopt a flat distribution of mass fraction, while they assume that the primary and secondary stars both obey the same IMF as single stars. As stated above, either of these two assumptions is not ruled out according to current research, and yet they entail distinct consequences for the mass spectra of secondary stars. We have a nearly flat mass function for CEMP and CH stars. On the other hand, if we apply their assumption to CH stars, it entails the predominance of low-mass stars down to \( \sim 0.2 M_\odot \) (Chabrier 2003). From the observation of CH stars, however, the contrary is reported: subgiant CH stars are found only in a narrow spectral-type range around G0 but not among dwarfs of late G and K types (Luck & Bond 1982). This lack of low-mass main-sequence CH stars favors our assumption of a flat distribution of mass ratio rather than the same single-star mass function of primary and secondary stars. Luck & Bond (1991) have argued the thick accreted layers as the cause, but it seems difficult to prevent the formation of low-mass CH stars through wind accretion, as seen in § 5. These assumptions are distinguished also in the production of CEMP-no stars, as discussed below.

7.2. Relative Frequencies of CEMP-s and CEMP-no Stars

In our scenario, both CEMP-s and CEMP-no arise from the white dwarf binaries, for which the existence and absence of \( s \)-process element enhancements are separated by the mass of primary components at \( M_1 = M_{\text{He-FDDM}} \). The populations of CEMP-no stars can then be estimated in the similar way to the populations of CEMP-s stars as

\[
\psi_{\text{CEMP-no}} = \psi_{\text{No dw}} + \psi_{\text{No q}} + \psi_{\text{No ag}},
\]

where each term on the right-hand side can be computed by integrating the corresponding equations (29)–(31) for the mass range of \( M_1 \) between \( M_{\text{He-FDDM}} \) and \( M_{\text{ap}} \) and by replacing the
lower bound of separation by $A_{TDU}$. Because of difference in the primary mass, the ratio between $\psi_{\text{CEMP-no}}$ and $\psi_{\text{CEMP-s}}$ depends on the assumed IMF. If we assume the Salpeter IMF for $M > 0.8\ M_\odot$, it reduces to

$$\psi_{\text{CEMP-no}}/\psi_{\text{CEMP-s}} \simeq 1/50,$$

which predicts a negligible fraction of CEMP-no stars as compared with CEMP-s stars. The distribution, $n(q)$, of the mass fraction is unlikely to differ by an order of magnitude in the mass range of concern here, and the distribution, $f(\dot{P})$, of orbital period is also expected not to vary greatly with the primary mass. Accordingly, a larger population of CEMP-no stars is possible only for the IMF shifting toward higher mass.

In Figure 10, we plot the proportion of CEMP-no stars to the EMP binaries and the ratio between the CEMP-no and CEMP-s stars, as a function of $M_{\text{md}}$. As $M_{\text{md}}$ increases and the IMF shifts to be more massive, the CEMP-no proportion first increases and hits a maximum of $\psi_{\text{CEMP-no}}/\psi_{\text{bin}} \simeq 0.17$ near $M_{\text{md}} \simeq M_{\text{ap}}$. The maximum fraction is smaller than that obtained for CEMP-s stars above because of the larger mass ratio between the primary and secondary stars. For a still larger $M_{\text{md}}$, the CEMP-no fraction turns to decrease, and the supernova binaries outnumber the other binaries. The ratio, $\psi_{\text{CEMP-no}}/\psi_{\text{CEMP-s}}$, is a monotonically increasing function of $M_{\text{md}}$; it starts from a very small value for the Salpeter IMF, the increase accelerates for $M_{\text{md}} > M_{\text{He-FDDM}}$ because of the decrease in the CEMP-s proportion, and eventually, the CEMP-no proportion exceeds the CEMP-s proportion for $M_{\text{md}} > 11\ M_\odot$.

The ratio between CEMP-no and CEMP-s stars is reported to be $\simeq 1/3$ if limited to $[\text{C/Fe}] > 1$ (Aoki et al. 2007; Tsangarides et al. 2004). In our compilation of $[\text{C/Fe}] > 0.5$, the ratio between CEMP-no to CEMP-s stars is 13/21 = 0.62, with two stars rich in $r$-process elements included. In the above estimates, the upper mass bound, $M_{\text{ap}}$, of primary stars for the binaries producing CEMP-no stars is set equal to the lower mass limit at which carbon ignites under the nondegenerate condition. Because of uncertainties in modeling of HBB (e.g., due to dependence on the mixing length), however, the boundary may not be clearly delineated from those yielding EMP stars that have low or moderate carbon enhancement but show large nitrogen enrichment. In particular, most of the mixed stars discussed by Spite et al. (2005) are left out from our compilation since we define CEMP stars as $[\text{C/Fe}] > 0.5$. These nitrogen-rich stars could be considered as the counterpart of CEMP-no stars with a more massive and hence processed deeply by HBB. If mixed stars are included, the number of CEMP-no stars amounts to 27 and is comparable with, or even exceeds, the number of CEMP-s stars in our list. Accordingly the current observations suggest the ratio of CEMP-no to CEMP-s stars is in the range of $\psi_{\text{CEMP-no}}/\psi_{\text{CEMP-s}} \simeq 1/3$, and we can well take an upper bound at $-1$. The ratios in this range agree well with those predicted from the high-mass IMF with $M_{\text{md}} \simeq 10\ M_\odot$, one of the two solutions derived above from the CEMP-s proportion. The low-mass IMF is excluded on the basis of current observations since it can predict too small a proportion of CEMP-no stars ($\psi_{\text{CEMP-no}}/\psi_{\text{CEMP-s}} < 0.05$).

7.3. Nature of EMP Stars in Galactic Halo

We have shown that the IMF for EMP binaries can be constrained from the observed characteristics of CEMP-s and CEMP-no stars. The constraints thus derived are summarized in Figure 11 on the diagram of the medium mass $M_{\text{md}}$ and the standard deviation $\Delta_M$, the parameters of the IMF in the lognormal form of equation (36). Solid lines denote the loci of the IMFs that yield CEMP-s stars at constant fractions, and dashed lines the loci of IMFs that produce CEMP-s and CEMP-no at constant ratios. Since the production rate of CEMP-s stars from the white dwarf binaries is very high (~60%), the parameter space near $M_{\text{md}} \simeq M_{\text{He-FDDM}}$ is excluded by the overproduction of CEMP-s as compared with the observation for the dispersion of $\Delta_M < 0.4$. For smaller dispersions, therefore, the allowed parameter space for CEMP-s stars is separated into two parameter spaces with low- and high-mass regimes, converging toward $M_{\text{md}} \simeq 1$ and $6\ M_\odot$, respectively. As $\Delta_M$ increases, the wider range of $M_{\text{md}}$ comes to be compatible with the observation of CEMP-s stars, and the two regimes are connected for a broad IMF as $\Delta_M > 0.4$. On the other hand, the formation of CEMP-no stars at the observed ratios to CEMP-s stars restricts the IMFs to those with large $M_{\text{md}}$ (> $M_{\text{He-FDDM}}$); the allowed range of $M_{\text{md}}$ increases as the IMF grows broader and gives birth to a larger fraction of low-mass stars.

In conclusion, only the IMFs with $M_{\text{md}}$ greater than ~6 $M_\odot$ and increasing with $\Delta_M$, as shaded in Figure 11, are compatible with the observations of both CEMP-s and CEMP-no stars. One of the important consequences is that the currently observed EMP stars have to stem exclusively from the binary systems; Figure 12 exemplifies the mass distributions of primary and secondary stars for a typical IMF derived here. We see that the secondary components extend well into the low-mass regime below 0.8 $M_\odot$, but the primary components have a negligible fraction of low-mass stars. We cannot expect a significant fraction of low-mass stars born as single unless the IMF for single stars greatly differs from that of primary stars. Another remarkable consequence is that a significant fraction of currently observed EMP stars were formed as the member of supernova binaries of $M_1 > M_{\text{ap}}$. For example, for $M_{\text{md}}$ in the range obtained in Figure 10, the proportion of supernova binaries accounts for ~40%–60% of the currently observed EMP stars.
The proportion of low-mass binaries is less than 0.1%, and the contribution to EMP stars is less than 1%. Note that our conclusion is dependent on the assumption of $n(q)$, which is currently very uncertain. In order to explain the origins of not only CEMP-s stars but also CEMP-nos stars, however, the IMFs of EMP stars have to be weighted in the intermediate and larger mass range. Otherwise, one has to seek other formation mechanism(s) for all the CEMP-nos stars, which are utterly unknown to the current theory of stellar evolution and/or nucleosynthesis.

The present results further provide a way of probing into the stellar populations that have left the EMP stars now constituting the Galactic halo. We can estimate the total stellar mass necessary to explain the number of currently observed EMP stars. From the above derived IMF, we expect one low-mass star of $M < 0.8 M_\odot$ out of EMP binaries, and hence of total stellar mass (3/2) $N_{\text{bin}} M_{\text{md}}$ on average if we assume the same flat distribution of mass ratio $[n(q) = 1]$ as above. On the other hand, from recent large-scale surveys, the number of observed EMP stars in the Galactic halo is estimated at $\sim 670$ sr$^{-1}$ for $[\text{Fe/H}] < -2.5$ and with the limiting magnitude of $B \lesssim 17.5$ (Beers & Christlieb 2005; in deriving this number we assume the ratio between EMP stars of $[\text{Fe/H}] < -3$ and $[\text{Fe/H}] < -2.5$ from their Table 3). With this limiting magnitude, giants can be observed up to distances of $\sim 100$ kpc and hence within the whole stellar halo, while dwarfs can be observed only in the neighborhood of $\sim 3$ kpc. Giants are about half the observed EMP stars, as discussed in § 4. By taking into account the mass range of stars on the giant branch, $\Delta M_G = 0.01 M_\odot$, and the flat mass function of EMP stars, we can estimate the total number of EMP stars in the Galactic halo,

$$N_{\text{EMP}} \simeq 670 \times 0.5 \times 4\pi \times 0.8 M_\odot / \Delta M_G \simeq 3.4 \times 10^5.$$ \hspace{1cm} (39)

Thus, the total mass, $M_{[\text{Fe/H}] < -2.5}$, of stars in the mother stellar populations of $[\text{Fe/H}] < -2.5$ that have produced these low-mass EMP stars that survive to date in the Galactic halo amounts to

$$M_{[\text{Fe/H}] < -2.5} \simeq 6 \times 10^7 M_\odot (N_{\text{EMP}} / 3 \times 10^5)(M_{\text{md}} / 10 M_\odot)^2.$$ \hspace{1cm} (40)

The loci of constant mass of $M_{[\text{Fe/H}] < -2.5}$ ($10^7, 10^8,$ and $10^9 M_\odot$), obtained numerically from the equations given in §§ 7.1 and 7.2, are plotted in Figure 11 (dash-dotted lines). The total mass of stars in the mother populations increases with $M_{\text{md}}$, and most of them have exploded as supernovae. Accordingly, the metal production by these erstwhile supernovae can impose an upper bound on the total stellar mass that has been involved in the mother populations and hence an upper bound on the medium mass. If we take the averaged iron yield to be $\sim 0.01$ of the initial stellar mass, then the mother populations can have increased the iron abundance in our Galaxy up to

$$[\text{Fe/H}] \simeq -2 + \log (M_{[\text{Fe/H}] < -2.5}/10^8 M_\odot)$$ \hspace{1cm} (41)

on an averaged basis all over the Galaxy of total (baryon) mass $M \simeq 10^{11} M_\odot$. With the derived high-mass IMF, the total mass of stars of $10^8 M_\odot$ are sufficient to promote the chemical evolution of the Galaxy to the metallicity of Population II. As a consequence, the IMFs with $M_{\text{md}}$ significantly exceeding $\sim 10 M_\odot$ may be excluded by the metal overproduction.

In summary, the EMP stars currently observed originate from a small fraction ($\sim 10\%$ of binary systems in number) of stellar populations of total mass $\sim 10^8 M_\odot$ that have once constituted, or merged into, the Galactic halo. A significant portion of them become CEMP stars, and another significant portion ($\sim 40\%$-$60\%$) have been exposed to the supernova explosion of their companions. In the supernova binaries, the secondary stars are likely to become unbound after the supernova explosions as a result of the sudden reduction of the primary mass. The secondary stars may possibly interact with the envelope matter lost through the wind before being released and also with supernova ejecta from the primary stars.

We conclude this section with two comments on the implications of these supernova binaries for the observed characteristics of EMP stars. Because of the large wind velocity and the expansion velocity of supernovae, only a small fraction of the ejecta can be accreted by the secondary stars, and yet it may influence the surface characteristics for the elements of such small abundances as the $r$-process elements. The $r$-process is most poorly understood among the stellar nucleosynthesis mechanisms, and yet, it can be argued from the solar abundances and the supernova rates that the amount of $r$-process elements ejected per event is of order $M_{r-p} \sim 10^{-4} M_\odot$ on average (Mathews & Cowan 1990; Woosley et al. 1994). Simply assuming the geometrical cross sections for the accretion, we can expect the surface enrichment of $r$-process elements of secondary stars to be as large as

$$[r/\text{Fe}] \simeq 1.3 + \log (M_{r-p}/10^{-5} M_\odot) - 2 \log A(U) - \log (M_{\text{acc}} / 0.35) M_\odot$$ \hspace{1cm} (42)

for the giants of $[\text{Fe/H}] \simeq -3$. EMP stars are known to display variations of $r$-process element abundances with a large range, a factor of $\sim 1000$, $1 < [\text{Eu}/\text{Fe}] < 2$ (Honda et al. 2004). The above estimate can be compatible with the observed enrichments, with the largest from the systems of the smallest separations. In particular, our binary scenario gives a straightforward explanation for the observed large variations in terms of the difference in the binary separation. This new channel of surface pollution is worth future investigation with the interaction between the matter ejected by the supernova explosion and the secondary star taken into account. It may also happen that the secondary stars accrete the envelope mass ejected by wind from primary stars before the supernova and also are polluted through the accretion of the gas shell of supernova remnants after the explosion.
Among the supernova binary systems, the primary stars of $M_{\text{up}} < M_1 \lesssim 11 \, M_\odot$ have been proposed to make a supernova explosion, triggered by electron capture on $^{20}$Ne and oxygen burning in the electron-degenerate $O+Ne$ core (Miyaji et al. 1980). On the other hand, Ritossa et al. (1996) show that the helium layer is dredged up by the surface convection during the carbon shell burning, carbon shell burning is extinguished, and these stars enter into the thermally pulsating (super) AGB (TP-SAGB) phase with the hydrogen and helium double shell burnings. If this is the case, these stars end in $O+Ne$ white dwarfs, ejecting the envelope through wind mass loss, just as AGB stars with a $C+O$ core (Ritossa et al. 1999; Gil-Pons et al. 2005), although the lower efficiency of mass loss may tend to narrow the mass range for stars of lower metallicity. Consequently, some of the binaries with the primary stars in this mass range may produce EMP stars of small carbon abundance but greatly enriched with nitrogen, similar to mixed stars, because of dilution due to the larger envelope mass and of deeper processing by HBB in larger core masses at the onset of the TP-SAGB phase, as in the upper mass end of white dwarf binary systems of $M < M_{\text{up}}$. In addition, because of the very short lifetimes of these massive primaries ($< 10^8$ yr), the secondary stars may suffer from the surface pollution by later accreting interstellar gas, enriched with metals, if their parent clouds persist sufficiently long to be polluted by supernova ejecta of subsequent generations (Suda et al. 2004).

8. CONCLUSIONS AND DISCUSSION

In this paper, we propose that the origins of carbon-enhanced metal-poor (CEMP) stars, currently observed in the Galactic halo, are explained in terms of the evolution of binary systems of extremely metal-poor (EMP) stars of $[\text{Fe/H}] \lesssim -2.5$. Our binary scenario is based on the evolutionary models for both EMP stars and binary mass transfer. We have examined it by comparing its consequences with the observations of the relation of the carbon enrichment to the orbital periods and the statistical properties of CEMP stars, and also through the comparison with their higher metallicity counterparts, CH stars. Finally, we demonstrate that a constraint can be imposed on the initial mass functions (IMFs) of EMP stars and discuss the implications for the nature of EMP stars in the Galactic halo.

The main results are summarized as follows:

1. We present an updated summary of the evolution of EMP stars. The primary stars can develop the surface carbon enhancement via two distinct mechanisms when evolving to the AGB: He flash–driven deep mixing (He-FDDM) in the mass range of $M \lesssim 3.5 \, M_\odot$ and the third dredge-up (TDU) in the mass range of 1.5 $M_\odot \lesssim M \lesssim M_{\text{up}}$, where $M_{\text{up}}$ denotes the upper mass limit of stars that end as white dwarfs. Nitrogen enhancement results either from He-FDDM or from the hot bottom burning (HBB) in the envelope, and $s$-process elements are synthesized in the helium convective zone promoted by hydrogen mixing prior to He-FDDM.

2. The secondary stars accrete part of the carbon-rich envelope, ejected from the AGB primary stars, to be CEMP stars. CEMP-s stars with nitrogen and $s$-process enrichment originate from the EMP binary systems with the primary stars of mass between 0.8 $M_\odot \lesssim M_1 \lesssim 3.5 \, M_\odot$ through He-FDDM. On the other hand, CEMP-no stars without $s$-process enhancement stem from the systems with the primary of mass 3.5 $M_\odot \lesssim M_1 \lesssim M_{\text{up}}$. In the latter case, the abundance of nitrogen, as well as of carbon, varies with the mass of the primary, and nitrogen-rich stars with mild or little carbon enhancement are associated with the massive primaries.

3. Our binary scenario is shown to give reasonable accounts of the observed characteristics and statistical features of CEMP stars. The large fraction of CEMP stars is explained by the broad parameter space of mass range and binary separation of progenitor binary systems as compared with CH stars. In particular, it is shown that CEMP binary systems have orbital periods of a much wider range than CH stars; all the detected orbital periods and the lack of detected variations in the radial velocity from about half the CEMP stars are both consistent with the predictions from the binary scenario and from the time span of monitoring of binarity to date.

4. From the observed frequencies of CEMP-s and CEMP-no stars among EMP stars, we demonstrate that the IMF of EMP stars has to be massive with the medium mass $M_{\text{med}} \gtrsim 6 \, M_\odot$, if approximated to a lognormal form. This also implies that the currently observed EMP stars in the Galactic halo were formed exclusively as the members of binary systems; low-mass EMP stars born as single stars account for a tiny fraction ($\lesssim 1\%$) if the single stars were born at nearly equal numbers with a similar IMF to that of primary stars. Accordingly, in addition to CEMP stars now with white dwarf companions, a significant fraction of EMP stars ($\sim 40\%$–$60\%$) used to have a massive primary star and have been exposed to a supernova explosion before being dismissed from the binary systems. We suggest as a new channel of surface pollution of EMP stars the accretion from stellar wind matter and supernova ejecta of massive companions.

5. From the total number of EMP stars in the Galactic halo, obtained from the recent large-scale surveys, the total mass of stars in their mother stellar populations is estimated to be $\sim 10^8 \, M_\odot (M_{\text{med}}/10 \, M_\odot)^2$. The metal production of supernovae imposes another constraint and can rule out the very high mass IMF, with the medium mass, $M_{\text{med}}$, significantly exceeding 10 $M_\odot$.

The following picture of the Galactic halo emerges from the present study. The Galactic halo once involved stellar populations of metallicity $[\text{Fe/H}] < -2.5$ and of mean mass $\sim 10 \, M_\odot$, which contain as many as $\sim 10^7 (M_{\text{med}}/10 \, M_\odot)$ binary systems in total; about half had primary components of low and intermediate masses that have now evolved to white dwarfs, and the other half had more massive primary components that have ended as supernovae. About 10% of the low-mass members of these binaries survive and become the currently observed EMP stars. The mother stellar populations of EMP stars therefore constitute 0.1% of the baryon mass of our Galaxy ($\sim 10^{11} \, M_\odot$). Supernovae from the massive stars in these populations suffice to raise the metallicity of the whole Galaxy to $\sim 0.01$ solar on average, leading to the formation of Population II objects, observed in the Galactic halo.

Our results indicate that the transition from an IMF dominated by massive stars to an IMF overwhelmed by low-mass stars occurred after the metallicity rises above $[\text{Fe/H}] \sim -2.5$. Our interpretation of the origin of CEMP-no stars, i.e., TDU and HBB, should be similar to that of stars of more metal-rich generations. Therefore, the lack of their correspondences with Population II stars, particularly nitrogen-rich mixed stars, may be attributed to the difference in the IMF, as discussed in § 7. The metallicity at the transition suggested from the present work seems significantly larger than claimed from the studies of the dynamical and thermal evolution of gas clouds under the metal-deficient circumstance. Recent studies tend to argue that the metal and dust cooling can supersede the cooling by hydrogen molecules even for metallicities as small as $[\text{Fe/H}] \sim -5$, which is argued to reduce the Jeans mass and produce the fragments below a solar mass (e.g., see Omukai 2000; Omukai et al. 2005). It is also shown that the sub-solar Jeans masses and fragments can be formed even from gas completely devoid of metals, once gas is heated by shock ionization (Uehara & Inutsuka 2000) either due to the collapse of
massive primordial objects with total (baryon plus dark matter) mass $M \gtrsim 10^8 M_\odot$ or the supernova explosion of the first-generation stars (Machida et al. 2005). In these studies the Jeans mass is directly connected to the masses of formed stars. It is true, however, that the process of fragmentation is still poorly understood and a proper understanding of star formation is yet to be established. It is worth noting here that the globular clusters, which embrace a host of low-mass stars, exist only for metallicities of $[\text{Fe/}H] > -2.5$ in our Galaxy.

One critical assumptions in the present study is the distribution of mass ratio $n(q)$ of binary systems. It is still an open question both observationally and theoretically, as discussed in § 6.1. We postulate a flat mass function of secondary components extending well below a tenth of the mean mass of primary components, which enables us to explain the origins not only of CEMP-s stars but also of CEMP-nos stars within the current standard framework of the theory of stellar evolution. If both binary components are assumed to have the same distribution functions, as done by Lucatello et al. (2005a), we should seek another origin for CEMP-nos stars, presumably elsewhere outside the current theory. In addition, we have pointed out that a flat distribution of mass ratio finds support in the low-mass cutoff reported for CH stars (Luck & Bond 1991).

Another critical assumption in our argument is on the s-process nucleosynthesis in EMP stars. In the present paper, we work out the stellar evolution and nucleosynthesis within the standard framework, which takes into account the thermal convection and chemical diffusion as the mechanisms of material mixing in the stellar interior, with a negligible contribution from convective overshoot. It is true that observations of surface abundance anomalies are not all explicable within this framework. One relevant issue is the formation of the $^{13}$C pocket, which is proposed to work as a neutron source in the metal-rich stars, but cannot be realized within the current standard framework of stellar evolution. Our scenario postulates that it is inefficient and that the s-process nucleosynthesis via radiative $^{13}$C burning is not effective in the stars of metallicity $[\text{Fe/}H] < -2.5$, at least, in the massive AGB stars of $M \gtrsim 3.5 M_\odot$. In actuality, the necessity of the $^{13}$C pocket is claimed from comparisons with the observed distribution of s-process elements in stars of more metal-rich populations, and yet it is treated as a free parameter. The present results may give an insight into the modeling of the $^{13}$C pocket formation if our interpretation of CEMP-nos stars is correct.

As for s-process nucleosynthesis in EMP stars, we can point out that the relevance of our high-mass IMFs to the synthesis, in particular, of the light s-process elements. Most CEMP-nos stars in our samples show larger abundances of strontium than barium ($[\text{Sr/}\text{Ba}] \geq 0$), which is distinct from CEMP-s stars that show larger enrichments of the main and heavy s-process elements relative to the light s-elements. This may be explicable in terms of the difference in the mass of the primary stars and hence in the core mass when the primary stars start TP-AGB evolution. For the primary stars of CEMP-nos stars, the core mass can be sufficiently large, and the temperature in the helium flash convection high enough to burn $^{22}$Ne via $^{22}$Ne$(\alpha, n)^{25}$Mg and to promote s-process nucleosynthesis (Iben 1975b). This neutron capture process is likely to produce mostly light s-process elements and to yield a distribution with the ratio of $[\text{Sr/}\text{Ba}] > 0$, as shown by Truran & Iben (1977) for the very source reaction produces neutron poison, and hence, the available neutrons per seed nuclei are restricted to be rather small. Recently Aoki et al. (2005) report a general tendency of the excess of light over main s-process elements to increase toward lower metallicities of $[\text{Fe/}H] < -2.9$ up to $[\text{Sr/}\text{Ba}] \simeq 1.5$. The helium shell flashes in massive AGB stars can be one candidate for the site of light s-process element synthesis.

As for some CEMP stars, other origins than binary evolution have been proposed; Umeda & Nomoto (2003) construct a usual supernova model with carbon-rich ejecta and propose the second generation of stars formed from gas mixed with the ejecta as a possible scenario for CEMP stars. In particular, for stars with $[\text{Fe/}H] < -5$, recently discovered, the formation of low-mass stars is argued by assuming the high carbon abundance $[\text{C/}H] \simeq -1.3$ as observed from the stars (Bromm & Loeb 2003). Since the carbon yield from Type II supernovae is $\sim 0.2 M_\odot$, however, it is open to question whether the second-generation stars were formed with carbon abundances as large as $[\text{C/}H] \sim -1.2$ to $-1.3$ with such a small amount of carbon ejected. This seems to an apparent variance to the fact that most EMP stars have iron abundances of $[\text{Fe/}H] < -2.5$ even though they are thought to be formed with iron ejecta of mass $\sim 0.1$ to $1 M_\odot$ in a similar way. Furthermore, with the smallest explosion energy, $< 10^{51}$ ergs, assigned to the carbon-rich supernovae, the star formation triggered by supernova (Tsujimoto et al. 1999) is unlikely to work since the shock driven by a supernova dies away before the fragmentation occurs in the swept shell (Machida et al. 2005). Even if the stars of the second generation are not formed, however, our binary scenario can propose an alternative channel of the surface pollution of the low-mass members of the first generation by accreting the wind matter and supernova ejecta. This is worth consideration in future work.

Finally, we comment on the surface pollution by accreting interstellar gas in the parent clouds where these EMP stars were born, discussed by Shigeyama et al. (2003) and by Suda et al. (2004). The latter argue that the surface of EMP dwarfs may be polluted up to $[\text{Fe/}H] \sim -3$ if the parent clouds survive sufficiently long ($\gtrsim 10^9$ yr) and enrich their gases with metals up to metallicity of $[\text{Fe/}H] \simeq -3$ and higher. If the lifetimes of primary stars are shorter than $\sim 10^5$ yr, as in the cases of white dwarf binaries at the massive end and of supernova binaries, EMP stars may have suffered from the accretion of interstellar gas after the mass transfer and hence can disguise their surface abundance with that of interstellar gas. When they evolve to giants, however, the surface abundance is diluted by a factor of $\sim 100$ by the internal matter, accumulated by the mass transfer from the primary stars. Accordingly, some CEMP-nos and mixed stars, and some supernova binaries, may have quite different appearances when they are dwarfs and after they evolve to the giant branch.

The surface pollution by accreting interstellar metal-rich gas has been addressed in relation to the two most iron-deficient, carbon-enhanced stars of $[\text{Fe/}H] < -5$ recently discovered by Christlieb et al. (2002) and by Frebel et al. (2005). Suda et al. (2004) have suggested that these stars are Population III stars that have been polluted by accreting iron-rich interstellar gas and also have become CEMP stars through the mass transfer from the erstwhile AGB companion in the close binaries. If they are really Population III survivors, we can also expect that some Population III stars without carbon enhancement exist. These two stars are thought to undergo He-FDDM, and hence, if the Population III stars were formed under the same IMF as obtained above, then we can expect $\sim 8$ to 20 Population III stars, mostly without carbon enhancement (3 to 12 single stars from the supernova binaries). Since their iron-group elements are due to pollution from metal-rich interstellar gas, these stars have to display surface metallicities of $[\text{Fe/}H] \simeq -3$ to $-4$ (with the gravitational settling taken into account) while they are dwarfs and suffer from the dilution of
surface pollution as they evolve to giants and deepen the surface convection. The surface abundances of these pollutants should be similar to those observed in HE 0107–5240 and HE 1327–2326 for the elements accreted from the interstellar gas, since for metallicities $[\text{Fe/H}] \simeq -2$ and higher, the variations in the abundances tend to be small. We can anticipate detection of such stars even in current compilations of EMP stars with careful investigation.

We very much benefited from discussion with Icko Iben, Jr. and Toshitaka Kajino. This paper is based on one of the author’s (Y. K.) dissertation submitted to Hokkaido University, in partial fulfillment of the requirement for the doctorate. This work has been partially supported by Grants-in-Aid for Scientific Research (15204010, 16540213, and 18104003) from the Japanese Society for the Promotion of Science.

APPENDIX A

AN ATTEMPT TO INTERPRET THE ABUNDANCE ANOMALIES FROM EMP STARS

A1. CEMP-noS WITH OXYGEN ENHANCEMENT

HBB may burn not only carbon but also oxygen into nitrogen in metal-poor stars since the temperature in the bottom of surface convection increases with decreasing metallicity (e.g., Ventura et al. 2001). If this is the case, the abundance of oxygen now observed should be inherent in the secondary stars, mixed and diluted with matter transferred from the primary stars. The carbon abundances larger than the equilibrium ratio to nitrogen may also be attributed to the pristine matter of secondary stars. All CEMP-no stars listed in our sample are giants, and the accreted matter has been mixed in the deep convection. As a corollary, before developing the deep convection, these CEMP-no stars should have a different appearance as dwarfs in that only nitrogen is enhanced; in addition, the surface abundance of dwarfs may also be subject to pollution by accreting interstellar matter after the mass transfer because of the relatively short lifetime of the primary stars, as discussed by Suda et al. (2004). Note that the nitrogen enrichment in CEMP-no stars is different from that observed from some giants in the globular clusters, attendant with the depletion of oxygen, and hence, the deep mixing mechanisms along the giant branch proposed for the latter (e.g., see Suda & Fujimoto 2006) will not be applicable.

A2. NITROGEN-RICH EMP STARS AND THE RELATION TO CEMP-no STARS

Some EMP stars show a large nitrogen enhancement but little or no carbon enhancement. A well-known example is CD $-$38 245, which was thought to be the most metal-poor star until the discovery of HE 0107–5240 ($[\text{Fe/H}] = -4.5$; Bessell & Norris [1984] but revised later to $[\text{Fe/H}] = -3.98$ by Norris et al. [2002]), with abundances of $[\text{C}/\text{Fe}] = 0.0$, $[\text{N}/\text{Fe}] = 1.7$, $[\text{O}/\text{Fe}] = 1.3$, and $[\text{Sr}/\text{Fe}] = [\text{Ba}/\text{Fe}] = -0.5$ (Bessell & Norris 1987; cf. Spite et al. 2005). For comparison, we plot this star in Figure 7 with two more similar stars, CS 22878-101 ($[\text{Fe/H}] = -3.25$) and CS 22952-015 ($[\text{Fe/H}] = -3.43$). These stars may arise from the same mechanism as CEMP-no stars with nitrogen enrichment but may belong to the binaries with a more massive primary and wider separation, yielding greater processing by HBB and smaller enhancement of $[\text{C} + \text{N}/\text{Fe}]$. Spite et al. (2005) observe EMP giants with weak or no carbon enhancement and argue that they are grouped into mixed stars with $[\text{N}/\text{C}] \approx 1$, including the above three stars, and “unmixed” stars of $[\text{N}/\text{Fe}] < 0.5$, both with similar $[\text{C} + \text{N}/\text{Fe}]$. They compile 17 mixed stars, four of which have $[\text{C} + \text{N}/\text{Fe}] > 0.5$, and our list of CEMP-no stars shares one star, CS 22949-037, with $[\text{C}/\text{Fe}] > 0.5$. For the mixed stars, the problem is also reduced to identifying the site(s) for conversion of carbon into nitrogen, and they suggest several possible sites, including HBB in massive AGB stars and hydrogen burning in the envelope of very massive stars. In the latter case, however, we have to seek a way to retain such large abundance ratios of nitrogen to carbon and to oxygen as observed from the mixed stars if these stars were formed from matter polluted with matter ejected by supernova explosions of the massive stars, since the nitrogen-rich matter from their envelopes suffer from dilution due to the mixing with carbon- and/or oxygen-rich ejecta from the inner parts, as well as with the interstellar matter (but see Norris et al. 2002). Instead, the origin of mixed stars can well be understood in terms of the binary evolution of case IV but with a primary of large mass and of large separation.

A3. ABUNDANCE FEATURES OF CS 22892-052 AND RELATION TO ORBITAL PERIOD

CS 22892-052 has metallicity $[\text{C}/\text{H}] \approx -2$ and is observed to show the orbital period of $P = 127$ days. In order to explain its origin in terms of the binary scenario, this star seems to require a small carbon abundance $[\text{C}/\text{H}]_1 \leq -1$ in the wind, as seen from comparison with the plots in Figure 9. The binarity of this system is subject to suspicion, however, since it displays only radial velocity variations of a very low amplitude ($\sim 1\; \text{km\;s}^{-1}$; Preston & Sneden 2001). If this star really belongs to a binary of $P = 127$ days, the low amplitude implies either a significantly small inclination angle of the orbital plane and/or a very small mass ratio. In the latter case, the companion star is likely to be a brown dwarf rather than a white dwarf, and we have to seek the origin of carbon and other heavy elements of this star in another site such as a prior supernova. More specifically, Tsujimoto & Shigeyama (2001) have argued that this star with enhanced abundances of $r$-process elements inherited its heavy elements from the supernova explosion of a $\sim 20\; \text{M}_\odot$ star. On the other hand, this star shows the enhancement and anomalous relative CNO abundances, as seen from Figure 7, and this makes the mass transfer in a binary more likely as far as CN enrichment is concerned.

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