Totally asymmetric simple exclusion process with two consecutive ramps

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Abstract. In this paper, the effect of totally asymmetric simple exclusion process (TASEP) with two consecutive ramps (in the bulk far away boundaries) has been investigated. The phase diagram changes with the off-ramp rate \( p \) and on-ramp rate \( q \) have been given. In addition, the main boundary expressions are also obtained by mean field approximation (MFA). The LD (Low Density)/LD and HD (High Density)/HD phases still vanish with \( p > 1/2 \) and \( q > 1/2 \), respectively. In particular, the MC/MC phase exists in the system with the relationship \( p/(1+2p) < q < p/(1-2p) \). Specially, when \( p \) and \( q \) satisfy \( p > 1/2 \) and \( q > 1/2 \), the phase diagram is divided into four intervals with same area by the LD/MC (Maximum Current), LD/HD, MC/MC and MC/HD phases.

1. Introduction
As an interacting particle model, totally asymmetric simple exclusion process (TASEP) has been employed to explore the particle traffic in different geometric construction [1-6]. Recently, the effective rates approximation of the phase structure of the system [7], the phase structure of networks with bifurcation and merging points [8, 9] and the effective rates approximation has been used in a number of subsequent works on TASEP on networks with junctions [10] have been studied. These methods are used in our study. Ramp is a typical geometry which includes on-ramp and off-ramp two types. Generally, they are used to describe the traffic of molecular motor attaching to and detaching from the lattice in biophysics, respectively [11-17]. Recently, on-ramp and off-ramp on one-dimensional single lane have been investigated, a phase coexistence of the low-density (LD) and high-density (HD) were included in the phase diagram [12]. More recently, on-ramp with zoned inhomogeneity has been studied; the maximum current (MC)/maximum current (MC) phase can exist in the system. In addition, the LD/MC phase would vanish with the fixed on-ramp \( q \) (\( q < 0.5 \)) and decreasing hopping rate \( p \) (from 0.8 to 0.3) [18]. Subsequently, off-ramp on the boundaries [19] and on-ramp with constrained resources were analyzed [20]. However, to the best of our knowledge, there were only two earlier studies of detachment and attachment on a single lane [13, 17], but they are not neighboring. Therefore, the TASEP model with two consecutive ramps will be discussed in this paper. The phase diagrams are affected by the relationship of \( p \) and \( q \).

2. The model and theoretical calculation
Our studying TASEP model consists of two consecutive ramps (off-ramp and on-ramp) and \( L \) sites (\( L \) is an even number), as demonstrated in Figure 1(a). It is noteworthy that both of the ramps are in the
bulk of the system and are neighboring as well. This study can give the change rule of the phase diagram under the condition of two consecutive ramps. In addition, the results can be used to control the current of the system. On the off-ramp site \((k_1 = L/2)\) and the on-ramp site \((k_2 = L/2+1)\), particles detach from and attach to lattice irreversible with \(p\) and \(q\), respectively. To simplify the analysis, the model is divided into two segments (segment 1 and segment 2) and which each of them have the entering rate \(\alpha (a_{\text{eff}})\) and leaving rate \(\beta_{\text{eff}} (\beta)\), as illustrated in Figure 1 (b).

![Figure 1. Sketch map of TASEP with two consecutive ramps.](image)

When the parameter \(p\) (or \(q\)) is equal to 0, our model will be similar to Reference [16] (or Reference [11]). Therefore, the effective leaving rate \(\beta_{\text{eff}}\) and \(\alpha_{\text{eff}}\) for segment 1 and segment 2 can be expressed as follow

\[
\beta_{\text{eff}} = p + (1 - \rho_{k+1}), \quad \alpha_{\text{eff}} = q + \rho_k
\]

where the density of site \(k\) and \(k + 1\) can be described by \(\rho_k\) and \(\rho_{k+1}\), respectively.

To understand the effect of two consecutive ramps on the phase diagrams, the mean field approximation (MFA) will be used in our analysis [21]. There are eight possible stationary phases in the system by theoretical calculations and the topology dependence of phase diagram on \(p\) and \(q\) has been illustrated in Figure 2. There are four lines which divides the topology dependence of phase diagram into eight regions [namely, (1), (2), (3)……, (8)]. Based on the different combinations of \(p\) and \(q\), the different phases are determined in the diagram, as shown in Table 1, the symbols ○ and × are used to express the possible phases existing and vanishing, correspondingly.

|       | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|
| LD/LD | ×   | ×   | ×   | ○   | ○   | ○   | ○   | ○   |
| LD/HD | ○   | ○   | ○   | ○   | ○   | ○   | ○   | ○   |
| LD/MC | ○   | ○   | ○   | ○   | ○   | ×   | ×   | ×   |
| HD/HD | ○   | ○   | ×   | ○   | ×   | ○   | ×   | ×   |
| HD/MC | ○   | ×   | ×   | ○   | ×   | ×   | ×   | ×   |
| MC/LD | ×   | ×   | ×   | ×   | ×   | ○   | ○   | ○   |
| MC/HD | ×   | ○   | ○   | ×   | ○   | ○   | ○   | ○   |
| MC/MC | ×   | ×   | ○   | ×   | ○   | ×   | ×   | ×   |

According to Table1 and Figure 2, we find the phase diagrams are divided into three types by line 1 (namely, \(q = \frac{p}{1-2p}\)) and line 2 (namely, \(q = \frac{p}{1+2p}\)). For the first type, the parameters \(p\) and \(q\) satisfy \(q > \frac{p}{1-2p}\), \(p < \frac{1}{3}\), and the HD/MC stationary phase exists in the system [the regions (1) and (4) of Figure 2]. With fixed \(p\) (\(p = 0.1\)), the LD/LD stationary phase will vanish when \(q\) changes from 0.4 to
0.7, as illustrated in Figure 3. The areas of the HD/MC and HD/HD stationary phases expand with the vertical boundary moving leftward and the boundary condition is described as

\[ p < \frac{1}{3}, q > \frac{p}{1-2p}, \alpha < \frac{(1+p+q)-(q-p)(q-p+2)}{2}, \frac{1-\sqrt{1+4(p-\alpha)(1+q-\alpha)}}{2} \leq \beta \]  

(2)

Figure 2. Topology dependence of phase diagrams on \( p \) and \( q \).

The result indicates that the large on-ramp rate not only reduces the particles entering the segment 2 from the last site \( (k_1) \) of the segment 1 but also enhances the amount of particles attaching the segment 2 from the first site \( (k_2) \) of the segment 2. It leads to particles always moving to the congested state, namely HD condition.

Figure 3. The phase diagram of the first type. (a) \( p = 0.1, q = 0.4 \); (b) \( p = 0.1, q = 0.7 \). Symbols and lines are from simulation and theoretical analysis, respectively.

For the second type, the MC/LD stationary phase occurs in the system under the conditions \( q < \frac{p}{1+2p}, p \leq 1 \) [the regions (7) and (8) of Figure 2]. With \( q = 0.2 \) and \( p \) from 0.4 to 0.7, the phase diagram does not consist of the HD/HD stationary phase and there are only four stationary phases (namely, LD/LD, LD/HD, MC/LD and MC/HD phases), as illustrated in Figure 4. The scopes of the LD/LD and MC/LD stationary phases enlarge with the horizontal boundary moving downward and the boundary condition is expressed as

\[ p < 1, q < \frac{p}{1+2p}, \alpha < \frac{1}{2}, \beta > \frac{(1+p+q)-(1+q-p)^2-4\alpha(1-\alpha)}{2} \]  

(3)

It is obvious that enhancing the off-ramp rate of site \( k_i \) leads to more particles detaching from the lattice, which improves the exiting rate of particles in the segment 1. In addition, the off-ramp reduces
the number of particles entering the segment 2 from segment 1. Therefore, particles always move in the free flow and maximum current in the segment 1. Simultaneously, the probability of the congested state in the segment 2 reduces.

![Phase Diagram](image)

**Figure 4.** The phase diagram of the second type. (a) \( p = 0.4, q = 0.2 \); (b) \( p = 0.7, q = 0.2 \).

For the last type, the MC/MC stationary phase will exist in the system with the relationship of \( \frac{p}{1+2p} < q < \frac{p}{1-2p} \) [the regions (2), (3), (5) and (6) of Figure 2]. For these four regions, the areas of the MC/MC stationary phase are equal (see Figure 5) and the existence conditions of this phase are given as

\[
\alpha > \frac{1}{2}, \beta > \frac{1}{2}
\]  

(4)

However, these four regions are divided into two cases (case A and B) by line 3 \( (q = 1/2) \), as shown in Figure 2. For case A, it includes regions (2) and (3), and the LD/LD stationary phase vanishes, which can be contributed to on-ramp rate \( q \) more than 1/2 \( (q > 1/2) \) such as Reference [7]. Furthermore, when the off-ramp rate \( p \) is enough large (namely, \( p > 1/2 \)), the HD/HD stationary phase vanishes like Reference [12], as demonstrated in Figure 5 (a) and (b). Interestingly, these four existing phases have the same areas [see Figure 5 (b)].

In case B, the HD/HD stationary phase vanishes with \( p > 1/2 \). However, different from case A, the LD/LD stationary phase exists in the system [see Figure 5 (c) and (d)] and the existing conditions satisfy

\[
p < 1, \quad \frac{p}{1+2p} < q < \frac{1}{2},
\]

\[
\alpha < \frac{1 - \sqrt{2q - 2p + 4pq}}{2}, \beta > \frac{(1 + p + q) - \sqrt{(1 + p - q)^2 - 4\alpha(1-\alpha)}}{2}.
\]

(5)

Our results all essentially are based on the MFA which ignores the density correlations on sites \( k_1 \) and \( k_2 \). The existing conditions of possible stationary phases in TASEP with two consecutive ramps is derived by MFA, and it can be generalized to several known results of TASEP with single on-ramp (or off-ramp) models. The vanishing conditions of LD/LD and HD/HD phases in the system depend on the value of parameters \( p \) and \( q \).

Our main results, equations (2)-(5), are described by the relationships of \( p \) and \( q \). Moreover, an important result of our investigation is that the MC/MC phase can exist in the system and we give the existence condition \( \frac{p}{1+2p} < q < \frac{p}{1-2p} \). Specially, the phase diagram will include only four stationary phases with the same area under the relationship \( p > 1/2 \) and \( q > 1/2 \).
Figure 5. The phase diagram of the third type. (a) $p = 0.4$, $q = 0.7$; (b) $p = 0.7$, $q = 0.7$; (c) $p = 0.4$, $q = 0.4$; (d) $p = 0.7$, $q = 0.4$.

Figure 6. Density profiles for Figure 3 (a). (a) the HD/MC phase; (b) the LD/HD phase; (c) the LD/MC phase; (d) the LD/LD phase; (e) the HD/HD phase.
3. Summary
In this paper, the TASEP model with two consecutive ramps has been analyzed. By using mean field theory and computer simulation, the eight phase diagrams which are divided in three types have been obtained according to the relationship of the parameters $p$ and $q$.

When the parameters $p$ and $q$ satisfy $q > \frac{p}{1 - 2p}, p < \frac{1}{3}$, the diagrams are in the first type, which the HD/MC phase exists in the system. The regions of the HD/MC and HD/HD phases expand and the LD/LD phase vanishes. It implies that the traffic congestion occurs often.

Then the parameters $p$ and $q$ satisfy $q < \frac{p}{1 + 2p}, p \leq 1$, the diagrams are in the second type, which the MC/LD phase exists in the system. The regions of the LD/LD and MC/LD phases expand and the HD/HD phase does not exist in the system. The result shows that the probability of traffic congestion reduced.

When the parameter $p$ and $q$ satisfy $\frac{p}{1 + 2p} < q < \frac{p}{1 - 2p}$, the diagrams are in the third type. The existence conditions of the LD/LD and HD/HD phases are determined by the values of $p$ and $q$. Note that, the MC/MC phase always exists and its region will not change. Specially, the phase diagram will include only four stationary phases with the same region under the relationship $p > 1/2$ and $q > 1/2$.

For verifying the validity of our theoretical calculations due to obtaining from mean field approximation, the Monte Carlo simulations are executed, as shown in Figure 6. Here, the theoretical and simulation results are described by lines and symbols, respectively. The results imply they are agreement.

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References
[1] Chowdhury D 2013 Phys. Rep. 529 1
[2] Appert-Rolland C, Ebbinghaus M and Santen L 2015 Phys. Rep. 593 1
[3] Tailleur J, Evans M R and Kafri Y 2009 Phys. Rev. Lett. 102 118109
[4] Melbinger A, Reese L and Frey E 2012 Phys. Rev. Lett. 108 258104
[5] Johann D, Erlenkämper C and Kruse K 2012 Phys. Rev. Lett. 108 258103
[6] Teimouri H, Kolomeisky A B and Mehrabiani K 2015 J. Phys. A 48 065001
[7] Brankov J, Pesheva N and Bunzarova N 2004 Phys. Rev. E 69 066128
[8] Gier J D and Nienhuis B 1999 Phys. Rev. E 59 4899
[9] Evans M R, Rajewsky N and Speer E R 1999 J. Stat. Phys. 95 45
[10] Pronina E and Kolomeisky A B 2005 J. Stat. Mech.: Theor. Exp. 2005 P07010
[11] Xiao S, Liu M Z and Shang J 2012 Mod. Phys. Lett. B 26 1150036
[12] Parmeggiani A, Franosch T and Frey E 2003 Phys. Rev. Lett. 90 137
[13] Xiao S, Liu M Z and Shang J 2012 Chin. Phys. B 21 218
[14] Lipowsky R, Klumpp S and Nieuwenhuizen T 2001 Phys. Rev. Lett. 87 108101
[15] Yang X, Qiu K and Zhang W 2007 Physica A 379 595
[16] Mirin N and Kolomeisky A B 2003 J. Stat. Phys. 110 811
[17] Chen X, Zhang Y, Liu Y and Xiao S 2016 Indian J. Phys. 91 1
[18] Xiao S and Bai J Y 2013 Mod. Phys. Lett. B 27 1
[19] Xiao S, Wu S, Tang L, Q, Zheng D S and Shang J 2012 Mod. Phys. Lett. B 26 1275
[20] Liu Y, Xiao W, Dong P, Zhang Y and Xiao S 2016 Renew. Sust. Energ. Rev. 62 815
[21] Derrida B, Domany E and Mukamel D 1992 J. Stat. Phys. 69 667