Spectral Clustering Method and Its Application

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ABSTRACT

Despite many empirical successes of spectral clustering methods which use eigenvectors of matrices derived from the data, this paper studies automatic segmentation of multiple motions or patterns from the tracked feature points. We propose an affinity matrix definition algorithm and suggest upper bounds together with a data-driven procedure for choosing automatically the optimal cluster number for the spectral clustering. Our approach is applied to the video benchmark databases and shows good experimental results on a number of clustering problems, such as motion pattern segmentation.

INTRODUCTION

In recent years, spectral clustering has become one of the most popular modern clustering algorithms. It can be solved efficiently by standard linear algebra software and very often outperforms traditional clustering algorithms such as the k-means algorithm. The success of spectral clustering is mainly based on the fact that it does not make strong assumptions on the form of the clusters. Moreover, spectral clustering can be implemented even for large data sets, as long as we make sure that the similarity graph is sparse.

Spectral clustering goes back to Donath [1], who suggested to construct graph partitions based on eigenvectors of the adjacency matrix. Fiedler [2] discovered that bi-partitions of a graph are closely connected with the second eigenvector of the graph Laplacian. A nice overview over the history of spectral clustering can be found in [3-4]. In the machine learning community, spectral clustering has been made popular by the works of Shi [5]. Subsequently, spectral clustering has been
extended to many non-standard settings, for example spectral clustering with additional side information [6] connections between spectral clustering and the weighted kernel k-means algorithm [7]. In the context of motion segmentation, [8-10] combined factorization methods with spectral clustering by building the affinity matrix. The Local Subspace Affinity (LSA) method [11] first estimated local subspaces and then uses spectral clustering with an affinity based on the principal angles between the local subspaces to segment the data. In Spectral Curvature Clustering (SCC) [12], the affinity was defined through the curvature of the spaces generated by all combinations of \( d + 1 \) points for \( d \)-dimensional subspaces. Clustering is one of the most widely used techniques for exploratory data analysis [13-15], with applications ranging from statistics, computer science, biology to social sciences or psychology.

**PROPOSED METHOD**

The paper focuses on how to segment the tracked feature points into different groups on the basis of their motion or position. Spectral clustering cannot serve as a “black box algorithm” which automatically detects the correct clusters in any given data set. But it can be considered as a powerful tool which can produce good results if applied with care. We present a method for calculating the affinity matrix and estimating the optimal cluster number for spectral clustering. Some of the choices for the implementation are motivated by simplicity and efficacy.

**Feature Points Gotten**

We use the KLT method to get the tracked feature points and Brox[16] method to obtain the optical flow field. They will help build the affinity matrix. Figure 1 depicts an example of a flow field.

![Figure 1. Example of KLT feature points and Flow field.](image)

**Self-tuning Spectral Clustering**

Our goal is to find labels for all points and group them. We assume that the motion pattern number is unknown.

**AFFINITY MATRIX DEFINITION**

Suppose there are \( T \) frames in a video stream and \( N \) points. Considering the fact that the points belonging to different groups can be close to each other and have a
large affinity, we propose an improved affinity matrix which discriminates the distance and movement direction between different groups. It is defined by

$$A_{ij} = C + \gamma \cdot \left( \frac{x_i^T x_j}{\|x_i\|_2\|x_j\|_2} \right)^{\beta y} \quad i \neq j$$  \hspace{1cm} (1)$$

$$\gamma = \text{sgn} \left( \frac{u_i^T v_j}{\|u_i\|_2\|v_j\|_2} \right)$$  \hspace{1cm} (2)$$

where $x_i, x_j$ are two vectors. $\beta (\beta > 0)$ is the parameter that tunes the sharpness of the affinity between two points. $u_i$ and $v_j$ are the optical flow vectors at $x_i$ and $x_j$. $C$ is a constant that keeps each element in $A_{ij}$ as non-negative. $C$ is easy to solve. We will transform (1) into (3) if the objects are stationary and have no movement trajectory.

$$A_{ij} = \left( \frac{x_i^T x_j}{\|x_i\|_2\|x_j\|_2} \right)^{\beta y} \quad i \neq j$$  \hspace{1cm} (3)$$

**CLUSTER NUMBER ESTIMATION**

Few studies have been conducted to provide a method of automatically setting it. In this section, we suggest an approach to discovering the number of clusters.

Considering subspace $S_{c_1}$, which contains all the points of the same group as $x_i$, we define the intercluster separability, $S_{S_{c_1} S_{c_2}}$, between $x_i$ in subspaces $S_{c_1}$ and $x_j$ in $S_{c_2}$:

$$S_{S_{c_1} S_{c_2}} = \max_{x_i \in S_{c_1}} \left( \max_{x_j \in S_{c_2}} (A_{ij}) \right)$$  \hspace{1cm} (4)$$

We additionally define the intracluster cohesiveness, $C_{S_{c_n}}$, of the data for subspace $S_{c_n}$ as

$$C_{S_{c_n}} = \min_{l \in I_c} \left( \max_{j \in I_c} (A_{ij}) \right)$$  \hspace{1cm} (5)$$

$I_c = \{ i | x_i \in S_{c_n} \}$ is the set of indexes of the points sampled from $S_{c_n}$. It is evident that each point $x_i$ has a higher connection with its own group than with the others. Suppose that the largest possible group number is $K$. We choose $k$ as the final cluster number calculated by the following equation.

$$k = \arg\max \left\{ k \left| \frac{\sum_{i=1}^{k} S_{S_{c_1} S_{c_2}}}{\sum_{i=1}^{k} \sum_{j=1, j \neq i}^{k} S_{S_{c_1} S_{c_2}}} \right| (k = 1, \cdots, K) \right\}$$  \hspace{1cm} (6)$$

Here $\overline{C_{S_{c_1}}}$ and $\overline{S_{S_{c_1} S_{c_2}}}$ are the average value from $C_{S_{c_n}}$ to 110% of $C_{S_{c_n}}$ and the average of the top 10% of $A_{ij}$ separately.
EXPERIMENTAL RESULTS

In this study, we test the quality of this algorithm on real video data. Some of the images are blurred. Figure 3 shows the motion pattern segmentation results with our proposed algorithm.

![Motion segmentation results from the proposed method.](image1)

We also test the quality of this algorithm on the stationary data artificially synthesized using Eq.(3) in the following.

![Clustering result tested by stationary data.](image2)

We determine that $\beta$ is different when achieving the respective best results among different videos. Most of the better results are obtained when $\beta$ is set to be 0.1-0.4.

CONCLUSIONS

In this paper, we propose a spectral clustering method. Compared to the SC[17] algorithm, the proposed method presents a different strategy for selecting the best cluster number. The SC finds the number determined by the so-called relative gap,
which is related to the eigenvalues of a Laplacian matrix $L$. However, the relative gap is not very effective when the noise level is large.

Some of the choices we make in our implementation are motivated by experiments. The affinity matrix $A$ may be better estimated by a method that relies on more highly informative local statistics. Similarly, other reasonable cost functions may exist.

We conduct experiments on a set of videos depicting real-world crowd scenes. The results indicate that the proposed approach is effective in clustering.

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