The long piles interaction with the surrounding and underlying soils, taking into account the linear and nonlinear rheological properties

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Abstract. The article presents the long incompressible pile with a two-layer base interaction problem formulation and solution, taking into account the underlying soil visco-plastic properties by the analytical method. A significant change in the stress-strain state at the base of an incompressible pile when stress changes around the pile side surface is shown. It is shown that in such conditions, the stress under the pile bottom varies significantly over time, which can lead to the pile bearing capacity loss in the long run.

Introduction
Nowadays, large length (up to 100 meters) and large diameters (up to 3 meters) piles are increasingly used in design and construction of foundations on weak, water-saturated clay soils. Load bearing capacity of such piles may be thousand tons. To select the number and length of piles in a pile foundation, it is important to determine the settlement and bearing capacity of a single pile under given geotechnical conditions. Obviously, these piles parameters depend on their length and diameter, the distance between them, as well as the deformation, strength and rheological properties of the surrounding and underlying soils ($G_2 > G_1$, $\phi_1$, $c_1$, $\eta_1$).

The pile interaction with the surrounding and underlying soils at the base under the force applied to the pile head influence causes a complex and non-uniform SSC that can transform in time and space. This paper presents the formulation and solution of the problem of the interaction of a long incompressible single pile with the surrounding and underlying soils, taking into account their linear and visco-plastic properties. It is shown that the force applied to the pile head can increasingly be transmitted to the pile lower end with time, which can lead to its bearing capacity loss.

1. Statement of the problem. Source equations
As a calculation for solving the problem, we will take a calculated geo-mechanical model in the form of a ground thick-walled cylinder of finite size ($2b$, $L=l+l'$), containing a pile of length $l$ and diameter $d=2a$ (Figure 1a), assuming that the pile step is determined by the soil cylinder diameter $2b$ with its staggered arrangement (Figure 1b).

From the design scheme (Figure 1a) it follows that an equilibrium condition of the form takes place:
\[ N = T + R \quad \text{or} \quad \sigma_N = \sigma_R + \tau_a \cdot 2\pi(l/a), \quad (1) \]

moreover, when \( r = a \)

\[ \tau_a = T / 2\pi a l, \quad (2) \]

with \( a < r < b \)

\[ \tau_r = \tau_a \frac{a}{r}. \quad (3) \]

![Design scheme of a long pile with the surrounding \( (G_1) \) and underlying \( (G_2 > G_1) \) soils (a) and the piles layout in the plan in a staggered manner (b)](image)

**Figure 1.** Design scheme of a long pile with the surrounding \( (G_1) \) and underlying \( (G_2 > G_1) \) soils (a) and the piles layout in the plan in a staggered manner (b)

The soil precipitation from the friction forces on the lateral surface can be determined by the angular deformation

\[ S = -\int \gamma(r) dr + C, \quad (4) \]

where \( \gamma(r) = \frac{\tau_c}{G_i} \).

### 2. Solution in linear formulation

Integration (3) with consideration of (2) and (4) in the range from \( a \) to \( b \) gives the maximum pile settlement:

\[ S(a) = \frac{a \cdot \tau_a}{G_1} \ln(b/a). \quad (5) \]

The pile settlement at its lower end can be determined on the basis of the well-known formula for pressing a rigid round stamp into the elastic medium in the form of \([1,2,6]\):
\[ S_0 = \frac{\pi a(1-v_2) \cdot \sigma_0^2 G_1}{4G_2} K(I), \]

where \( \sigma_0 \) is the stress under the lower end of the pile, \( K(I) \) is the coefficient taking into account the influence of the stamp depth.

Comparing (5) and (6), we obtain the connection between \( \tau_a \) and \( \sigma_o \), i.e. we get

\[ \tau_a = \frac{\pi \left(1-v_2\right)}{4G_2} \frac{\sigma_0}{\ln(b/a)} \cdot K(I) G_1. \]

From the pile balance condition (1) it follows that

\[ \sigma_N = \sigma_0 + \frac{2l}{a} \tau_a; \quad \tau_a = \left(\sigma_N - \sigma_0\right) \frac{a}{2l}, \]

where \( \sigma_N = N / \pi a^2 \).

Substituting the value from (9) into (10), we get

\[ \sigma_N = \sigma_0 \left(\frac{\pi \left(1-v_2\right)}{4G_2} \cdot \frac{G_1}{\ln(b/a)} \cdot \frac{2l}{a} + 1\right) = \sigma_0 \cdot A \quad \text{or} \quad \sigma_0 = \frac{\sigma_N}{A}. \]

Accordingly, it follows from (9) that

\[ \tau_a = \left(\sigma_N - \sigma_0\right) \frac{a}{2l} = \left(\sigma_N - \frac{\sigma_N}{A}\right) \frac{a}{2l} = \frac{\sigma_N \left(A-1\right)}{A} \frac{a}{2c} = \sigma_N \frac{A^*}{A} \frac{a}{2l}. \]

To determine the pile settlement from the force \( N \) action, it suffices to use the formulas (6) and (9). Then we get:

\[ S_0 = \frac{\pi \left(1-v_2\right)}{4G_2} \cdot \frac{G_1}{\ln(b/a)} \cdot \frac{\sigma_N^2}{A}. \]

To determine the bearing capacity of the pile in the first approximation, it is sufficient to use the formula to determine the degree of approximation to the initial critical load \( \eta = P / P_{cr} \). For the case of a distributed load on the area of the circle \( b=2a \) on a soil base, it is possible to use the formula [3,4]:

\[ P_{cr} = \gamma \cdot d + \frac{2\gamma d \cdot \sin \phi_2 + 2c_2 \cdot \cos \phi_2}{1-2\nu_2}. \]

where \( \phi_2, c_2, \nu_2 \) are the parameters of the underlying soil, \( \gamma \) is the specific weight of the overlying soil, \( d \) is the depth from the surface.

Note that this expression differs from the Puzyrevsky formula for determining the initial critical load for a distributed load over a bandwidth \( b=2a \) (plane strain problem).

### 3. Solution in nonlinear rheological formulation

In this case, to determine the SSC of soils around a long pile and at its base we take the following Bingham-Shvedov-Maslov viscous-plastic model as a calculation for the surrounding soil:

\[ \dot{\gamma} = \frac{\tau - \tau^*}{\eta_1} + \frac{\dot{\tau}}{G_1}, \]

where \( \tau^* = \sigma_m \tan \phi + c, \eta_1 \) is the soil viscosity, \( \sigma_m \approx \gamma \cdot 1/2 = \text{const} \).
This elastic-visco-plastic problem solution will be considered at speeds, assuming that at the initial moment of the SSC the “pile-surrounding and underlying soils” system corresponds to a linear solution (see section 1 of this work).

The speed of pile settlement and the surrounding soil in this case from the action of friction forces \( T = \tau_a 2\pi al \) can be determined in accordance with (3) with regard to (13) by the integral of the form

\[
\dot{S} = -\int_{a}^{b} \left( \frac{\tau - \tau^*}{\eta_l} + \frac{\dot{\tau}}{G_l} \right) dr,
\]

where \( \dot{\tau} = \frac{\tau_a}{r} \). Then we get

\[
\dot{S} = \frac{a \cdot \tau_a}{\eta_l} \ln(b/a) + \frac{\tau^*}{\eta_l} (b-a) .
\]

From (12) it follows that

\[
\dot{S}_0 = \frac{\pi a (1-v_2) \cdot \sigma_0 K_l}{4 G_2}.
\]

From the pile speed equality condition from the action of friction forces \( T = \tau_a 2\pi al \) and the frontal resistance \( R = \pi a^2 \sigma_0 \), it follows that values of \( \dot{S} \) by (15) and (16) are equal. Then we get

\[
\left( \frac{a \cdot \tau_a}{\eta_l} + \frac{\dot{\tau}}{G_l} \right) \ln(b/a) + \frac{\tau^*}{\eta_l} (b-a) = \frac{\pi a (1-v_2) \cdot \sigma_0}{4 G_2}.
\]

From the equilibrium condition (1) it follows that for \( N=\text{const} \) \( \ddot{N} = 0 \). Then

\[
\ddot{T} = -\dot{R} \Rightarrow \sigma_0 = -\frac{2l}{a} \dot{\tau}_a .
\]

Substituting this value of \( \sigma_0 \) into (17), we get

\[
\dot{\tau}_a + \tau_a \frac{G_l}{\eta_l} \frac{\tau^*}{G_l} - \frac{b-a}{a \ln(b/a)} = -\frac{\pi (1-v_2)}{4 G_2} \frac{G_l}{\eta_l} \frac{a \dot{\tau}_a}{2l} .
\]

After some transformations we get

\[
\dot{\tau}_a (1 + B) + \tau_a \frac{G_l}{\eta_l} \frac{\tau^*}{G_l} - \frac{G_l (b-a)}{a \ln(b/a)} = 0 .
\]

where \( B = -\frac{\pi (1-v_2)}{4 G_2} \frac{G_l}{\eta_l} \frac{2l}{a \ln(b/a)} \).

And finally

\[
\dot{\tau}_a + \tau_a P = Q ,
\]

where \( P = \frac{G_l}{\eta_l} \frac{1}{1 + B} ; \quad Q = \frac{\tau^*}{\eta_l} \frac{G_l (b-a)}{a \ln(b/a)} \frac{1}{1 + B} .
\]

Solution (21) is known and has the form:

\[
\tau_a(t) = e^{-P \dot{t}} \left( \int Q e^{P \dot{t}} + C \right) .
\]

Then we get
\[
\tau_a(t) = \frac{Q}{P} + Ce^{-Pt}.
\] (23)

When \(t=0\) and \(\tau_a(t) = \frac{a}{2l} \sigma_N (A-1) / A = \frac{Q}{P} + C\) we get

\[
C = \frac{a}{2l} \sigma_N (A-1) / A - \frac{Q}{P}.
\] (24)

And finally

\[
\tau_a(t) = \frac{Q}{P} (1 - e^{-Pt}) + \frac{a}{2l} \sigma_N \frac{A-1}{A} e^{-Pt}.
\] (25)

It follows that when \(t \to \infty\)

\[
\tau_a(t = \infty) = \frac{Q}{P} = \frac{\tau}{a \cdot \ln (b/a)}.
\] (26)

From the equilibrium condition (8) it follows that

\[
\sigma_0(t) = \sigma_N - \frac{2l}{a} \tau_a(t).
\] (27)

In view of (25), it follows that

\[
\sigma_0(t) = \sigma_N - \frac{2l}{a} \left( \frac{Q}{P} (1 - e^{-Pt}) + \frac{a}{2l} \sigma_N \frac{A-1}{A} e^{-Pt} \right).
\] (28)

After some transformations we get

\[
\sigma_0(t) = \left( \sigma_N - \frac{2l}{a} \frac{Q}{P} \right) - \left( \sigma_N \frac{A-1}{A} + \frac{2l Q}{a \cdot P} \right) e^{-Pt}.
\] (29)

When \(t=0\) and \(t \to \infty\) \(\sigma_0(t)\) with, respectively, is

\[
\sigma_0 = \sigma_N / A \quad \text{and} \quad \sigma_0(t) = \sigma_N - \frac{2l}{a} \frac{Q}{P}.
\] (30)

Comparing the value \(\sigma_0(\infty)\) with the initial critical load, we determine the approximation degree to the initial critical load \(P_{nc}\) (12).

We will determine the precipitation of the pile in time under the condition \(\sigma_0 < P_{nc}\) on the basis of (12), i.e. we get

\[
S(t) = \frac{\pi (1 - \nu_2)}{4G_2} \cdot \frac{G_1 K_1}{\ln (b/a)} \cdot \sigma_0(t),
\] (31)

where \(\sigma_0(t)\) is determined by (21).

Determine the dependence of the voltage under the pile lower end in time \(\sigma_0(t)\) and \(S_0(t)\) at different viscosity values. The solution results (29) with different values \(\eta_1\) are given in Fig. 2. It can be seen that the soil viscosity has a significant effect on the stresses’ development under the lower end of the pile in time.
Thus, the problem is completely solved. The sediment and stress at the lower end of the pile are determined. A significant dependence of these parameters on the deformation and strength properties of soils, as well as on the coefficient of viscosity of the soil $\eta_1$, is shown.

Summary
1. When a long incompressible pile interacts with the surrounding and underlying linearly deformable soils, the force applied to the pile head is distributed between its lateral surface and the lower end proportional to the ratio of the stiffnesses of the surrounding and underlying soils ($G_2/G_1$). With increasing $G_2$, the voltage $\sigma_0$ tends to $\sigma_N$.

2. Accounting for the elastic-visco-plastic properties of the surrounding soil pile and the linear properties of the underlying soil leads to the distribution of the force applied to the pile head between the side surface of the pile and its lower end in time. Moreover, the voltage at the lower end of the pile from the initial zero value increases with time to a stabilized value, depending on the structural strength $\tau^*$ and viscosity $\eta_1$ of the soil.

3. The initial critical load $P_{ns}$ under the lower end of the pile, due to the onset of limiting equilibrium on the contour of the circle $r=2b$, depends significantly on the strength parameters of the underlying soil ($\varphi_2, c_2$), Poisson’s ratio $\nu_2$ and the depth of the lower end of the pile $d$.

4. The pile draft also develops slowly, in accordance with the visco-plastic resistance of the surrounding soil, and stabilizes to a certain value.

5. The problem-solving results considered in this paper can be used in the design of a pile-slab foundation to determine the piles number and pitch.

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