Has the Photon an Anomalous Magnetic Moment?

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Due to its interaction with the virtual electron-positron field in vacuum, the photon exhibits a nonzero anomalous magnetic moment whenever it has a nonzero transversum vector component to an external constant magnetic field. At low and high frequencies this anomalous magnetic moment behaves as paramagnetic, and at energies near the first threshold of pair creation it has a maximum value greater than twice the electron anomalous magnetic moment. These results might be interesting in an astrophysical and cosmological context.

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It was shown by Schwinger[1] in 1951 that electrons get an anomalous magnetic moment $\mu' = \alpha/2\pi\mu_B$ (being $\mu_B = e\hbar/2m_0c$ the Bohr magneton) due to radiative corrections in quantum electrodynamics (QED), that is, due to the interaction of the electron with the background virtual photons and electron-positron pairs. We want to show that also, due to the interaction with the virtual quanta of vacuum, an anomalous photon magnetic moment arises. It is obtained from the expression for the photon self-energy in a magnetic field, calculated by Shabad[2,3] in an external constant magnetic field $\Pi_{\mu\nu}(x, x')|^{ext}$ by starting from the electron-positron Green function in the Furry picture, and by using the Schwinger proper time method. The expression obtained was used by Shabad[3] to investigate the photon dispersion equation in vacuum in presence of an external magnetic field. It was found a strong deviation from the light cone curve near the energy thresholds for pair creation, which suggests that the photon propagation behavior in the external classical magnetic field is strongly influenced by the virtual electron-positron pairs of vacuum near these thresholds, showing a behavior similar to that of a massive particle. These phenomena become especially significant near the critical field $B_c = m_0^2/\alpha \sim 4.1 \cdot 10^{13}$ Gauss, where $m_0$, $e$ are respectively the electron mass and charge.

The photon magnetic moment might have astrophysical and cosmological consequences. For instance, photons passing by a strongly magnetized star, would experience an additional shift to the usual gravitational one produced by the star mass.

In presence of an external field the current vector is non vanishing $j(x)_\mu = i e Tr \gamma_\mu G(x, x| A^{ext}) \neq 0$, where $G(x, x'| A^{ext})$ is the electron-positron Green’s function in the external field. By calling the total electromagnetic field by $A_\mu = A_\mu^{ext} + A_\mu$, the QED Schwinger-Dyson equation for the photon field $A_\mu(x)$, propagating in the external field $A_\mu^{ext}$ is

$$[\square \eta_{\mu\nu} - \partial_\mu \partial_\nu] A^\nu(x) + \int \Pi_{\mu\nu}(x, x'| A^{ext}) A^\nu(x') d^4x' = 0,$$

(1)

where $\mu, \nu = 1, 2, 3, 4$. The expression (1) is actually the set of Maxwell equations in a neutral polarized vacuum, where the second term corresponds to the approximation of the four-current linear in $A_\mu$, where the coefficient is the polarization operator $\delta_{\mu\nu}(x)/\delta A_\nu^{ext}(x')|_{A^{ext}} = \Pi_{\mu\nu}(x, x'| A^{ext})$. The external (constant and homogeneous) classical magnetic field is described by $A_\mu^{ext}(x) = 1/2 F^{ext} x^\mu$, where the electromagnetic field tensor $F^{ext}$ is its dual pseudotensor.

To understand what follows it is necessary to recall some basic results developed in refs. 2,3. The presence of the constant magnetic field creates, in addition to the photon momentum four-vector $C_\mu^4 = k_\mu$, three other orthogonal four-vectors which we write as four-dimensional transverse $k_i^4 = C_i^{4i} = 0$ for $i = 1, 2, 3$. These are $C_{\mu}^1 = k_\mu F_\mu^2 k^3 - k_i (k F_\mu^2 k_i), C_{\mu}^2 = F_\mu^3 k^1, C_{\mu}^3 = F_\mu^4 k^2, (C_{\mu}^{1234}) = 0$. We have $C_\mu^4 C_\nu^4 = k_\mu k_\nu = 0$ on the light cone. One gets from these four-vectors three basic independent scalars $k^2, k F_\mu^2 k, k F_\mu^3 k$, which in addition to the field invariant $F = \frac{1}{4} F_\mu^\nu F_\nu^\mu = \frac{1}{2} B^2$, are a set of four basic scalars of our problem.

In momentum space it can be written the eigenvalue equation

$$\Pi_{\mu\nu}(k, k' A^{ext}) = \sum_i \pi^{(i)}_{n,n'} a^{(i)\mu} a^{(i)\nu}/(a^{(i)\mu} a^{(i)\nu}) \quad (2)$$

In correspondence to each eigenvalue $\pi^{(i)}_{n,n'}$, $i = 1, 2, 3$ there is an eigenvector $a^{(i)\mu}$. The set $a^{(i)\mu}$ is obtained by simply normalizing the set of four vectors $C_\mu^4$. ($C_\mu^4 = k_i$) leads to a vanishing eigenvalue due to the four-dimensional transversality property $\Pi_{\mu\nu}(k, k' A^{ext}) k_\mu = 0$. The solution of the equation of motion (1) can be
written as a superposition of eigenwaves given by

$$A_\mu(k) = \sum_{j=1}^{4} \delta(k^2 - n_j^2) a^j_\mu(k)$$

(3)

By considering $a^j_\mu(x)$ as the electromagnetic four vector describing the eigenmodes, it is easy to obtain the corresponding electric and magnetic fields of each mode $e^{j}(i) = \partial \varphi_0 / \partial x^j - \partial \varphi_0 / \partial x^0$, $h^{j}(i) = \nabla \times \varphi^{j}(i)$ (see [3]).

From now on we specialize in a frame in which $x_3 \parallel B$. Then $kF^2k/2F = -k^2_\perp$ and we name $z_1 = k^2 + kF^2k/2F = k^2_\parallel - \omega^2$. The previous results (see [3]) indicate the existence of three dispersion equations with the following structure

$$k^2 = \pi^{(i)}(z_1, k^2_\perp, eB), i = 1, 2, 3$$

(4)

The eigenvalues $\pi^{(i)}$ contain only even functions of the external field through the scalars $kF^2k$, $kF^2\omega$, and $e\sqrt{2F} = eB$ and can be expressed as a functional expansion in series of even powers of the product $eA^\mu_{ext} \Box$.

One can solve (4) for $z_1$ in terms of $k^2_\perp$. It results

$$\omega^2 = |k|^2 + f_1(k^2_\perp, B)$$

(5)

The term $f_1$ contains the interaction of the photon with the virtual $e^\pm$ pairs in the external field in terms of the variables $k^2_\perp, B$. As it is shown in [4], it makes the photon dispersion equation to have a drastic departure from the light cone curve near the energy thresholds for free pair creation.

We are thus in conditions to define an anomalous magnetic moment of the photon as $\mu_\gamma = -\partial \omega / \partial B$. Then $\mu_\gamma$ is a function of $B$. For weak fields ($B \ll B_c$), and frequencies small enough (see below), the function $f_1$ can be written as linear in $B$, the resulting dispersion law being then

$$\omega = |k| - \mu_\gamma B$$

(6)

The first term corresponds to the light cone equation, which is modified by the second, which contains the contribution of the photon magnetic moment.

The gauge invariance property $\pi^{(i)}(0, 0) = 0$ implies that the function $f_1(k^2_\perp, B)$ vanishes when $k^2_\perp = 0$. This means that, due to gauge invariance, when the propagation is parallel to $B$, $\mu_\gamma$ vanishes. Thus, in every mode of propagation $\mu_\gamma = 0$ if $k_\perp = 0$. Therefore the photon magnetic moment depends essentially on the perpendicular momentum component and this determines the optical properties of the quantum vacuum in presence of $B$.

As a result, the problem of the propagation of light in empty space, in presence of an external magnetic field is similar to the problem of the dispersion of light in an anisotropic medium, where the role of the medium is played by the polarized vacuum in the external magnetic field. An anisotropy is created by the preferred direction in space along $B$. Therefore, the refraction index $n^{(i)} = |k|/\omega$ in mode $i$ is given in the case in which the approximate expression [8] is valid as

$$n^{(i)} = 1 + \frac{\mu_\gamma B}{|k|}$$

(7)

For parallel propagation, $k_\perp = 0$, for any mode it is obviously $n_1 = 1$.

In [2, 3] (see also [3]) it was shown that $\Pi_{\mu\nu}$ has singularities starting the value $z_1 = -4m^2_0$, which is the first threshold for pair creation, corresponding to Landau quantum numbers $n = n' = 0$. Other pair creation thresholds are given by $k^2_\perp = m^2_0(1 + 2nB/B_c)^{1/2} + (1 + 2n'B/B_c)^{1/2}|^2$, with the electron and positron in excited Landau levels $n, n' \neq 0$. In what follows we will work in the transparency region, that is, out from the region for absorption due to the pair creation i.e., $\omega^2 - k^2_\parallel \leq k^2_\perp$ (i.e., within the kinematic domain, where $\pi_{1,2,3}$ are real). We will be interested in two limits, i.e., when its energy is near the first pair creation threshold energy (and the magnetic field $B \sim B_c$), and when it is much smaller than it, $4m^2_0 \gg \omega^2$ and small fields ($B \ll B_c$), in the one loop approximation. Below the first threshold the eigenvalues corresponding to the first and third modes do not contribute, whereas the second mode it is shown in [4], by using the formalism developed in [2, 3] that the eigenvalue near this threshold is in that limit

$$\pi_2 = -\frac{2\mu'B}{m_0} \left[ z_1 \exp \left( -\frac{k^2_\perp}{2eB} \right) \right]$$

(8)

Here $\mu' = (\alpha/2\pi)\mu_B$ is the anomalous magnetic moment of the electron. The exponential factor in (8) plays a very important role. If $2eB < k^2_\perp$, it would make the exponential factor negligible small and in the limit $B \to 0$, it vanishes (as well as $\mu_\gamma$, below). In the opposite case, if $k^2_\perp \ll 2eB$, the exponential is of order unity. By considering the last assumption and the case of transversal propagation ($k_\parallel = 0$), the dispersion equation for the second mode has the solution

$$\omega^2 = k^2_\parallel + k^2_\perp \left( 1 + \frac{2\mu'B}{m_0} \right)^{-1}$$

(9)

from which it results that the photon energy can be expressed approximately as a linear function of the external field $B$, as indicated in [8], where

$$\mu^{(2)}_\gamma = \frac{\mu'k^2}{m_0\omega}$$

(10)

Notice that, as pointed out above, in the limit $B = 0$, if $k_\perp \neq 0$, then $\mu^{(2)}_\gamma$ vanishes. By considering transversal propagation and $\omega \simeq k_\perp$ one can write

$$\mu^{(2)}_\gamma = \frac{\mu'|k_\perp|}{m_0}$$

(11)
As $\mu_\gamma^{(2)} > 0$, the magnetic moment is paramagnetic, which is to be expected since vacuum in a magnetic field behaves as paramagnetic $^2$. For photon energies $\omega \sim 10^{-6} m_0$ and $B \sim 10^4$G, we have $\mu_\gamma \sim 10^{-6} \mu'$. In a more exact approximation we must take into account the contribution from higher Landau quantum numbers.

We will be interested now on the photon magnetic moment in the region near the thresholds, and for fields $B \lesssim B_c$. The eigenvalues of the modes can be written approximately $^3$ as

$$\pi_{n,n'}^{(i)} \approx -2\pi \phi_{n,n'}^{(i)}/|\Lambda| \quad (12)$$

with $|\Lambda| = ((k_{1}^r)^2 - k_{1}^{\prime r})(k_{1}^r - \omega^2 + k_{1}^{\prime r})^{1/2}$ with $k_{1}^{\prime r} = m_0^2(1+2nB/B_c)^{1/2} - (1+2n'B/B_c)^{1/2})^2$, is the squared threshold energy for excitation between Landau levels $n, n'$ of an electron or positron. The functions $\phi_{n,n'}^{(i)}$ are expressed in terms of Laguerre functions of the variable $k_{1}^r/2eB$.

In the vicinity of the first resonance $n = n' = 0$ and considering $k_{1,2} \neq 0$ and $k_{2,2} \neq 0$, according to $^2$ the physical eigenwaves are described by the second and third modes, but only the second mode has a singular behavior near the threshold and the function $\phi_{n,n'}^{(2)}$ has the structure

$$\phi_{0,0}^{(2)} \approx -\frac{2\alpha eBm_0^2}{\pi} \exp\left(-\frac{k_{1}^{2}m_0^2}{2\epsilon B}\right) \quad (13)$$

In this case $k_{1}^{r} = 0$ and $k_{1}^{r} = 4m_0^2$ is the threshold energy.

By using the approximation given by $^{12}$ the dispersion equation $^{12}$ is turned into a cubic equation in the variable $z_1$ that can be solved by applying the Cardano formula. We will refer in the following to $^{12}$ as the real solution of this equation.

We should define the functions $m_n = (k_{1,2}^r + k_{1,2}^{\prime r})/2$, $m_{n'} = (k_{1,2}^r - k_{1,2}^{\prime r})/2$ and $\Lambda^* = 4m_n m_{n'} (k_{1,2}^r - k_{1,2}^{\prime r})$ to simplify the form of the solutions $^{5}$ of the equation $^{12}$. The functions $f_i$ are dependent on $k_{1,2,2}^r, k_{1,2,2}^{\prime r}, B$, and are

$$f_1 = \frac{1}{3} \left[ 2k_{1,2}^r + k_{1,2}^{\prime r} + \frac{\Lambda^*}{(k_{1,2}^r - k_{1,2}^{\prime r})^2} \right]$$

$$f_2 = \frac{1}{3} \left[ 2k_{1,2}^r - k_{1,2}^{\prime r} + \frac{\Lambda^*}{(k_{1,2}^r - k_{1,2}^{\prime r})^2} \right]$$

$$f_3 = \frac{1}{3} \left[ -2k_{1,2}^r + k_{1,2}^{\prime r} + \frac{\Lambda^*}{(k_{1,2}^r - k_{1,2}^{\prime r})^2} \right]$$

where $D = 6\pi \sqrt{G} - \Lambda^* m_0^2 + 54\pi^2 \phi_{n,n'}^{(2)} (k_{1,2}^r - k_{1,2}^{\prime r})^2$ with

$$G = \Lambda^* + 27\pi^2 \phi_{n,n'}^{(2)} (k_{1,2}^r - k_{1,2}^{\prime r})^2$$

Besides $^{12}$, there are two other solutions of the above-mentioned cubic equation resulting from the substitution of $^{12}$ in $^{12}$. These are complex solutions and are located in the second sheet of the complex plane of the variable $z_1 = \omega - k_{1,2}^r$ but they are not interesting to us in the present context.

Now the magnetic moment of the photon can be calculated by taking the implicit derivative $\partial \omega/\partial B$ in the dispersion equation. From $^{12}$ and $^{12}$ it is obtained that

$$\mu_\gamma^{(i)} = \frac{\pi}{\omega(|\Lambda|^3 - 4\pi \phi_{n,n'}^{(i)} m_n m_{n'})} \left[ \phi_{n,n'}^{(i)} \left( A \frac{\partial m_n}{\partial B} + Q \frac{\partial m_{n'}}{\partial B} \right) - \Lambda^* \frac{\partial \phi_{n,n'}^{(i)}}{\partial B} \right] \quad (15)$$

with $A = 4m_n[z_1 + (m_n + m_{n'}) (3m_n + m_{n'})]$ and $Q = 4m_n[z_1 + (m_n + m_{n'}) (m_n + 3m_{n'})]$.

In the vicinity of the first threshold $k_{1,2}^{r} = 0$, $k_{1,2}^{r} = 4m_0^2$ and $\partial m_n/\partial B = 0$ when $n = 0$, therefore for the second mode the photon magnetic moment is given by

$$\mu_\gamma^{(2)} = \frac{\alpha m_0^2 (4m_0^2 + z_1) \exp\left(-\frac{k_{1,2}^r}{2\epsilon B}\right)}{\omega B_c \left[ (4m_0^2 + z_1)^{3/2} + \alpha m_0^2 \exp\left(-\frac{k_{1,2}^r}{2\epsilon B}\right) \right]} \left( 1 + \frac{k_{1,2}^r}{2\epsilon B} \right) \quad (16)$$

It is easy to show that this function has a maximum near the threshold. If we consider $\omega$ near $2m_0$, the function $\mu_\gamma^{(2)} = f(X)$, where $X = \sqrt{4m_0^2 - \omega^2}$ has a maximum for $X = \pi \phi_{00}^{(2)}/m_0^{1/3}$, which is very close to the
threshold.

Thus, near the first threshold and in the second mode of propagation the expression (14) has a maximum value when $k_1^2 \simeq k_1^2$. Therefore in a vicinity of the first pair creation threshold the magnetic moment of the photon has a resonance peak which is positive, indicating a paramagnetic behavior, and its value is given by

$$\mu^{(2)}_\gamma = \frac{m_0^2 (B + 2B_c)}{3m_\gamma B^2} \left[ 2\alpha \frac{B}{B_c} \exp \left( -\frac{2B_c}{B} \right) \right]^{2/3}$$

(17)

Obviously, (17) would vanish also for $B \to 0$. The maximum of (17) is given numerically by

$$\mu^{(2)}_\gamma \approx 3\mu' \left( \frac{1}{2\alpha} \right)^{1/3} \approx 12.85\mu'$$

(18)

Thus, the maximum value achieved by the photon magnetic moment under the assumed conditions is larger than twice the anomalous magnetic moment of the electron.

In (17) we introduced the quantity $m_\gamma$ which has meaning near the thresholds, and which could be named as the "dynamical mass" of the photon in presence of a strong magnetic field, which is defined by the equation

$$m^{(2)}_\gamma = \sqrt{4m_0^2 - m_0^2} \left[ 2\alpha \frac{B}{B_c} \exp \left( -\frac{2B_c}{B} \right) \right]^{2/3}$$

(19)

The "dynamical mass" accounts for the fact that the massless photon coexists with the massive pair near the thresholds, leading to a behavior very similar to that of a neutral massive vector particle bearing a magnetic moment. However, it does not violate gauge invariance since the condition $\Pi_{\mu\nu}(0, 0, B) = 0$ is preserved. The idea of a photon mass has been introduced previously, for instance in ref. [8], in a regime different from ours, in which $k_\parallel \gg 4m^2$.

We conclude, thus, that for photons in a strong magnetic field a nonzero magnetic moment arises, which is paramagnetic, and has a maximum near the first threshold of pair creation. These results may have several interesting consequences. For instance, if we consider a photon beam of density $n_\gamma$, it carries a magnetization $M = n_\gamma \mu^{(2)}_\gamma$ which contributes to increasing the field $B$ to $B' = B + 4\pi M$. Through this mechanism, the radiation field might contribute to the increase of the external field.

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