Jet-Inflated Cocoons in Dying Stars: New LIGO-Detectable Gravitational Wave Sources

Ore Gottlieb,1* Hiroki Nagakura,2 Alexander Tchekhovskoy,1 Priyamvada Natarajan,3,4,5 Enrico Ramirez-Ruiz,6 Jonatan Jacquemin-Ide,1 Nick Kaaz,1 Vicky Kalogera1

1Center for Interdisciplinary Exploration & Research in Astrophysics (CIERA), Northwestern University, 1800 Sherman Ave, Evanston, IL 60201, USA
2Division of Science, National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan
3Department of Astronomy, Yale University, 52 Hillhouse Avenue, New Haven, CT 06520, USA
4Department of Physics, Yale University, P.O. Box 208121, New Haven, CT 06520, USA
5Black Hole Initiative, Harvard University, 20 Garden Street, Cambridge MA 02138, USA
6Department of Astronomy and Astrophysics, University of California, Santa Cruz, CA 95064, USA

*Corresponding author. Email: ore@northwestern.edu.

Long Gamma-Ray Bursts (LGRBs), the most powerful events in the Universe, are generated by jets that emerge from dying massive stars. Highly beamed geometry and immense energy make jets promising gravitational wave (GW) sources. However, their sub-Hertz GW emission is outside of ground based GW detectors’ frequency band. Using a 3D general-relativistic magnetohydrodynamic simulation of a dying star, we show that jets inflate a turbulent, energetic bubble-cocoon that emits strong quasi-spherical GW emission within
the ground-based GW interferometer band, 100–600 Hz, over the characteristic jet activity timescale, \( \approx 10 – 100 \) s. Our prediction for the source amplitude makes this the first non-inspiral GW source detectable by current interferometers out to hundreds of Mpc, with \( \approx 0.1 – 3 \) detectable events expected during LIGO/Virgo/Kagra’s observing run O4. These GWs are likely accompanied by detectable energetic core-collapse supernova and cocoon electromagnetic emission, making jetted stellar explosions promising multi-messenger sources.

Core-collapse supernovae (CCSNe) provide a unique opportunity to study the last stages of stellar life-cycles, the synthesis of heavy elements, and the birth of compact objects (1–3). However, the intervening opaque stellar gas limits the prospects for learning about the underlying physics of the explosion mechanism and the compact object environment from electromagnetic signals. Fortuitously, CCSNe produce two extra messengers: neutrinos and gravitational waves (GWs); both carry information from the stellar core to the observer with negligible interference along the way (4–6). Numerical studies (7, 8) showed that CCSNe can be highly asymmetric, giving rise to a substantial time-dependent gravitational quadrupole moment, which generates GW emission. However, with a small fraction \( \left( E \approx 10^{46} \text{ erg} \right) \) of the CCSN energy going in GW production, the ground-bound based interferometers LIGO/Virgo/Kagra (LVK) (9) can detect only nearby \( (\lesssim 1 \text{ Mpc}) \) events at design sensitivity (10).

A special class of CCSNe - collapsars (11) is associated with long-duration gamma-ray bursts (LGRBs), which originate in energetic jets powered by a rapidly rotating newly-formed compact object, a black-hole (BH) or a neutron star. Their enormous power makes LGRB jets promising GW sources. The GW memory effect (12, 13) implies that the GW frequency is inversely proportional to the timescale over which the metric is perturbed. For jets, this timescale is set by the longer of the launching and acceleration timescales (14–20). Thus, the characteristic duration of LGRBs \( (\gtrsim 10 \text{ s}) \) places the GW emission from LGRB jets at the sub-
Hz frequency band, too low for LVK, but potentially detectable by the proposed space-based Decihertz Interferometer GW Observatory (21).

As the jets drill their way out of the collapsing star, they shock the dense stellar material and build a cocoon - a hot and turbulent structure that envelops the jets (Fig. 1) (22–25). The cocoon is generated as long as parts of the jets are moving sub-relativistically inside the star. LGRB jets break out from the star after $t_b \approx 10$ s (26) and spend a comparable amount of time outside of the star before their engine turns off (27). After breakout, jets are expected to stop depositing energy into the cocoon (23, 28), unless they are intermittent or wobbly (29, 30). This implies that the cocoon energy $E_c \approx 10^{51} - 10^{52}$ erg (31) is at least comparable to that of the jet (the energy retained by the jet after breakout), $E_c \gtrsim E_j$.

Figure 1: Jets (light blue) inflate the energetic cocoon (dark blue) inside of a dying massive star (pale yellow). The jets run into and shock against the collapsing star, forming a backflow (yellow arrows). Infalling star runs into and shocks against the backflow (red arrows). The shocked jet and shocked stellar components form the cocoon and turbulently mix inside of it (white arrows).

Here, we show that the cocoon is an attractive new GW source for current detectors. First, the cocoon evolves over shorter timescales than the jets, and, as we demonstrate below, its GW
signal lies within LVK frequency band. Second, relativistic jets are subject to the anti-beaming effect \(^{(32)}\) that inhibits the GW emission within their opening angle, whereas the cocoon can emit GWs in all directions across the sky.

The GW strain can be approximated as

\[
h \approx \frac{2G}{Dc^4} \frac{d^2 Q}{dt^2},
\]

where \(G\) is the gravitational constant, \(c\) is the speed of light, \(D\) is the distance to the source, and \(Q\) is the gravitational quadrupole. From Eq. \([1]\) cocoon-powered GW characteristic strain is \((10)\)

\[
h_{\text{coc}} \approx \frac{4G}{Dc^4} E_c \epsilon \approx 10^{-23} \frac{100 \text{ Mpc}}{100 \text{ Mpc}} \frac{E_c \epsilon}{10^{52} \text{ erg}},
\]

where \(\epsilon\) is the degree of asymmetry of the cocoon, which depends on the viewing angle \(\theta_{\text{obs}}\) (see Fig. \([1]\)).

We carry out a high resolution 3D general-relativistic magnetohydrodynamic (GRMHD) simulation of energetic LGRB jets in a collapsing star by repeating the simulation from \((30)\), but with an increased data output frequency, as needed to resolve the time-variability of the gravitational quadrupole. This simulation follows the jets from the BH for \(\approx 3.1\) s until they reach distance \(\approx 1.5R_*\) and energy \(E_j \approx 2 \times 10^{52} \text{ erg}\), where \(R_*\) is the stellar radius. Fig. \([2]\) depicts the hourglass-shaped cocoon upon breakout from the star (see the full animation and accompanying sonification in \[https://oregottlieb.com/gw.html\]). For observers facing the jet axis (Fig. \([2a]\)), the projected shape of the cocoon is close to circular (\(\epsilon \ll 1\)), significantly suppressing the quadrupole moment. Off-axis observers (Fig. \([2b,c]\)), on the other hand, will see an asymmetric cocoon with an order unity asymmetry \(\epsilon \approx 1\), thus maximizing the observed GW signal strain amplitude in Eq. \([2]\).

Turbulent motions in the jet-cocoon result in a stochastic GW signal, and lead to a broad spectrum of GW frequencies that is challenging to evaluate analytically. However, we can

\[1\)Jets are also subject to instabilities \((29,30)\) that may introduce high frequency GWs. Computing those requires going beyond analytic and numerical modeling that assume continuous and steady axisymmetric jets.
Figure 2: Three-dimensional (3D) rendering of the cocoon upon breakout from the star at different viewing angles. At $\theta_{\text{obs}} = 0^\circ$ (a) the axisymmetric projection results in a weak on-axis signal as the cocoon appears nearly circular, whereas at larger angles the asymmetric shape of the cocoon enables strong GW emission (b,c). The colormap delineates $\log(T^{00}r^2)$ in c.g.s., which is an order of magnitude estimate to the gravitational quadrupole density, where $T^{00}$ is the contravariant energy density component of the stress-energy tensor. See details in supplementary materials in (33).
estimate the main features of the emerging GW spectrum frequency range as follows. The smallest length-scales of the cocoon emerge over the thickness of the shocked region $\Delta r_{\text{sh}} \approx 10^{-2} R_\star \Gamma^{-2} \approx 10^8 \text{ cm}$ \cite{34}, where $\Gamma \approx 3$ is the jet head Lorentz factor inside the star, and $R_\star \approx 10^{11} \text{ cm}$. These shocked regions evolve over $t_{\text{min}} \approx \Delta r_{\text{sh}}/c_s \approx 2 \times 10^{-3} \text{ s}$, where $c_s \approx c/\sqrt{3}$ is the relativistic sound speed, implying the highest GW frequency, $f_{\text{max}} \approx t_{\text{min}}^{-1} \approx 500 \text{ Hz}$. The maximum timescale is the time that the jet energizes the cocoon, $t_{\text{max}} \gtrsim t_b \approx 10 \text{ s}$, thereby setting the minimum GW frequency, $f_{\text{min}} \approx t_{\text{max}}^{-1} \lesssim 0.1 \text{ Hz}$. The cocoon energy is distributed quasi-uniformly in the logarithm of the proper-velocity, $10^{-2.5} \lesssim \Gamma \beta \lesssim 3$ \cite{28,30}. Thus, although various cocoon components evolve on different timescales, they carry comparable amounts of energy, and are expected to result in a flat GW spectrum between $f_{\text{min}}$ and $f_{\text{max}}$.

To compute the GW signal, we post-process $\gtrsim$ petabyte of simulation data output at a high cadence, enabling us to numerically calculate the second derivatives of the gravitational quadrupole moment (see Supplementary Materials in \cite{33})\textsuperscript{2}. Fig. 3a,b shows a 3D mass density rendering of the cocoon after breakout from the star, taken from \cite{30}. Similar to its pre-breakout shape in Fig. 2, the cocoon is asymmetric when observed off-axis (Fig. 3a), and near-axisymmetric when observed on-axis (Fig. 3b). In fact, the strain amplitudes in Fig. 3c,d show that on-axis emission is weaker by 1-2 orders of magnitude than off-axis emission, whose spectrum is roughly flat at $100 \text{ Hz} \lesssim f \lesssim 600 \text{ Hz}$, followed by an exponential cutoff, $h(f) \propto \exp \left[ - (f/600 \text{ Hz})^2 \right]$ (dotted line), consistent with our $f_{\text{max}}$ estimate. The off-axis strain amplitude spectrogram (Fig. 3e) shows that as the cocoon expands ($t \lesssim 1.2 \text{ s}$ in our simulation), the GW emission turns on, intensifies and shifts only slightly toward lower frequencies, as the spectrum does not vary considerably between different regions in the cocoon, owing to mixing. At $\approx 1.2 \text{ s}$, the jets traverse about a third of the star, the cocoon is fully formed, and the signal plateaus in amplitude. The GW emission is expected to last until the jet

\textsuperscript{2}For comparison, in \cite{33} we also calculate the weak GW signal from non-jetted explosions, whose emission is dominated by an expanding accretion shock.
Figure 3: 3D rendering of off-axis (a) and on-axis (b) projections of the cocoon mass density after breakout from the star (star is in the center shown in white), taken from (30). On- and off-axis strain amplitude in time (c) and frequency (d) domains. Normalized amplitude (by maximal off-axis amplitude) spectrograms for a sliding window of 50 ms for (average of $\phi_{\text{obs}} = 0^\circ$ and $\phi_{\text{obs}} = 90^\circ$) off-axis (e) and on-axis (f) observers. The off-axis amplitude is constant in frequency at $100 \text{ Hz} \lesssim f \lesssim 600 \text{ Hz}$, and in time from $\approx 1 \text{ s}$ until the jet shuts off and the cocoon relaxes.
engine shuts off at $t_j$ (longer than our simulation), and the cocoon turbulent motions relax\footnote{GW travel time effects may slightly prolong the signal duration for observers close to the jet axis, see \cite{33}.}. On the other hand, the on-axis GW signal becomes gradually stronger as the jet breaks out from the star (Fig. 3f), owing to a stronger deviation from axisymmetry in the absence of a cocoon confinement by the dense stellar envelope after breakout. Both on- and off-axis signals are qualitatively different from traditional GWs in CCSNe which have a peak frequency that rises over a much shorter GW emission timescale ($\ll 1$ s) \cite{35,36}.

The off-axis GW emission from the energetic cocoon ($E_c \approx 2E_j \approx 4 \times 10^{52}$ erg) is detectable out to distances as far as $\approx 200$ Mpc (Fig. 3f), in agreement with Eq. 2 (to within a factor of two). For on-axis observers the GW detection horizon of such cocoons is much smaller $\approx 10$ Mpc, owing to the low degree of non-axisymmetry which renders a direct on-axis detection unlikely. We use our numerical result to calibrate Eq. 2, assuming a linear scaling of the strain amplitude with the cocoon energy\footnote{We neglect secondary effects on the cocoon-powered GW emission, such as the density profile of the star and turbulent mixing, that may affect the cocoon shape and its GW spectrum, and cause deviations from the linear scaling shown in Eq. 2.}

\begin{equation}
    h_{\text{coc}} \approx 10^{-23} \frac{200 \text{ Mpc}}{D} \frac{E_c}{10^{52} \text{ erg}}.
\end{equation}

Approximating $E_c \approx E_j$ and $\epsilon(\theta_{\text{obs}}) \approx 1$, we can express Eq. 3 via the BH spin $a$, mass $M_{\text{BH}}$ and magnetic field strength on the horizon $B_{\text{BH}}$, assuming that the jet is powered by the Blandford-Znajek mechanism \cite{37,38,39},

\begin{equation}
    h_{\text{coc}} \gtrsim 10^{-23} \frac{200 \text{ Mpc}}{D} \left( \frac{t_j}{10 \text{ s}} \right) \left( \frac{M_{\text{BH}}}{5 M_\odot} \frac{B_{\text{BH}}}{10^{15} \text{ G}} \frac{a}{0.8} \right)^2.
\end{equation}

Therefore, detections of cocoon-powered GWs can help us constrain the properties of the central BH and its environment deep inside the dying star.

To estimate the expected number of detectable GW events in LVK observing run O4, we connect $E_c$ to the observed jet energy via $E_c \approx E_j = \xi E_{j,\text{obs}}$, and consider two types of
jets with opening angle $\theta_j \approx 0.1^{+0.07}_{-0.03}$ rad (40): i) a traditional axisymmetric jet, for which we adopt $\xi = 1$ and conventional local LGRB rate $R_{\text{GRB}} \approx 100 \text{ Gpc}^{-3} \text{ yr}^{-1}$ (41); and ii) a jet wobbling by $\theta_w \approx 0.2$ rad as in our simulations, for which the local GRB rate is an order of magnitude lower (30). When a jet wobbles by $\theta_w \approx 2\theta_j$, it is observed on average only $(\theta_j/(\theta_w + \theta_j))^2 \approx 10\%$ of the time, namely $\xi = 10$. Assuming the $\gamma$-ray energy is $20\%$ of the total jet energy (42), the isotropic equivalent $\gamma$-ray energy is $E_{\gamma,\text{iso}} \approx 0.4\theta_j^{-2}E_{\gamma,\text{obs}}$. The distribution of LGRBs in $E_{\gamma,\text{iso}}$ can be approximated by a power-law (43), which we find to be $N_{\text{LGRBs}} \propto E_{\gamma,\text{iso}}^{-1.5}$. This implies that the most energetic GRBs dominate cocoon-powered GW detections, so that the $E_{\gamma,\text{iso}}$ power-law cutoff, $E_{\gamma,\text{iso},\text{m}}$ sets the expected number of detectable events, see (33).

Assuming the LVK detection threshold of the characteristic strain at the relevant frequencies, $|h_{\text{crit}}| \approx 10^{-22}$ (Fig. 3d) and isotropic GW emission ($\epsilon(\theta_{\text{obs}}) \approx 1$), we use Eq. 3 to show in Fig. 4 the number of detectable cocoon-powered GW events per year in the upcoming LVK observing run O4 for the above two types of jets (33), where the shaded areas depict the standard deviation in $\theta_j$. If jets propagate along a fixed axis, the detection probability (red line) is $\simeq 10\%$. If jets wobble (blue line), then a jet (and the jet-powered cocoon) has more energy for a given $E_{\gamma,\text{iso}}$, since most of its energy ($\approx 90\%$) is beamed away from our line of sight. This dramatically increases the expected number of detectable GW events to a few during observing run O4. Had the latter case been the true rate, then such events might already exist in the data from previous LVK runs. However, this signal was never explicitly searched for, and might be easily overlooked due to its noisy nature compared to inspiral GWs. We emphasize that the large uncertainties in $\theta_j$, the distribution of the most energetic GRBs, and the prompt/afterglow energy ratio (44), in turn introduce large uncertainties in the expected number of detectable GW events.\footnote{Additionally, in our estimate we ignore jets that are choked in the stellar envelope and do not generate a GRB.} Interestingly, the detection rate of quasi-isotropic GWs from cocoons with energies
$E_c < E_{\text{max}}$ constrains the fraction of SN Ib/c progenitors that power LGRBs with $E_c < E_{\text{max}}$, as shown on the right vertical axis in Fig. 4, see (33).

Figure 4: The number of detectable cocoon-powered GW events in one year of LVK observing run O4 as a function of the maximal isotropic equivalent $\gamma$-ray energy $E_{\gamma,\text{iso},\text{m}}$ (the cut-off of the power-law fit to $N_{\text{LGRBs}}(E_{\gamma,\text{iso}})$ distribution), assuming LVK detection threshold of $|h_{\text{crit}}| = 10^{-22}$ and isotropic GW emission. If the jets propagate along a fixed axis, then $\simeq 0.1$ events are expected (red line), whereas if jets wobble (blue line), multiple GW events could be detectable. Variations in jet opening angle (shaded areas) introduce uncertainties into predicted GW detection rate. The detection rate of cocoons with $E_c < E_{\text{max}}$ also indicates the abundance of LGRBs with such cocoons among SNe Ib/c (right vertical axis labels).

LGRBs are likely to be rich multi-messenger events. While a coincident LGRB-GW detection is unlikely due to the weak on-axis GW emission, the GW signal is likely accompanied by a wide range of electromagnetic counterparts powered by the SN explosion and the cocoon: shock breakout in $\gamma$- and X-rays (seconds to minutes), cooling emission and radioactive decay in UV/optical/IR (days to months), and broadband synchrotron (afterglow) emission (days to years) (31, 45). The earliest radiative signal emerges when the cocoon or SN shock wave breaks out from the star, producing a nearly coincident electromagnetic counterpart to the cocoon-

If such phenomenon is common among massive stars (39), then the predicted cocoon-powered GW detection rate is increased significantly, see (33).
powered GWs. However, the shock breakout signal originates in a thin layer, and primarily depends on the breakout shell velocity and structure of the progenitor star, rather than the total explosion energy \((34, 46)\). Thus, although under favorable conditions of viewing angle and progenitor structure it is possible to detect a shock breakout in \(\gamma\)-rays, those signals will typically go unnoticed.

After releasing the shock breakout emission, the stellar shells and the cocoon expand adiabatically and give rise to an optical cooling signal. SNe Ib/c cooling emission lasts weeks and peaks at an absolute magnitude \(M_{\text{AB}} \approx -18\) \((47)\), whereas cocoons with energies of \(E_c \gtrsim 10^{52}\) erg that emit strong GWs, will power even brighter \((M_{\text{AB}} \lesssim -19)\) quasi-isotropic \((\theta_{\text{obs}} \lesssim 1.0\) rad) UV/optical cooling emission on timescales of days \((31, 48)\). The optical emission of the cocoons and SNe is sufficiently long and bright to be detected at all relevant distances of a few hundred Mpc by Zwicky Transient Facility (ZTF) \((49, 50)\) and upcoming Rubin Observatory \((51)\). Furthermore, cocoons with \(E_c \gtrsim 10^{52}\) erg s\(^{-1}\) will explode the entire massive star and may power superluminous supernovae (SLSNe) with \(M_{\text{AB}} \lesssim -21\) \((52)\) over \(\approx\) months timescale.

Finally, the interaction of such energetic cocoons with the circumstellar medium will produce a detectable broadband afterglow, assuming typical ambient densities and standard equipartition parameters \((53)\). The timescale over which the afterglow emission emerges varies from days to years, as it depends on the specific parameters of the system and the observer’s viewing angle, thereby posing a challenge to its detection. However, the early cooling signal that can be detected by a rapid search will enable an early localization of the event and a targeted search for the later multi-band afterglow, which will potentially alleviate the afterglow detection difficulty.

In fact, the ZTF online catalog \((54)\) already contains \(\approx 100\) CCSNe Ib/c/SLSNe that were discovered during LVK observing run O3. 14 of these are SNe Ic-BL: the only confirmed SN type to be associated with LGRBs \((55)\). About 50 of these CCSNe (and 10 SNe Ic-BL)
lie within 170 Mpc of Earth, implying that if their progenitors harbored (off-axis) GRBs with conventional $E_c \lesssim 10^{52}$ erg (31), GWs from their cocoon might conceivably be detectable in LVK run O3 data. It will thus be intriguing to perform a targeted search for GWs from these progenitors.

In this report, we propose and investigate the first non-inspiral GW source that is detectable by LVK out to hundreds of Mpc. Future calculations of the zoo of cocoon-powered GWs for different LGRB progenitors will enable the characterization of amplitude/spectrum to source properties, and will be addressed in a follow-up work. This will enable a more efficient search for GWs from collapsar cocoons, enhance the wealth of information regarding the physical properties of the source to be extracted from the GW signal, and aid follow-up searches for electromagnetic counterparts of these multi-messenger events, ultimately providing a better understanding of the relation between CCSNe and LGRBs.
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**Author Contributions**

OG initiated the study, performed the simulations, conducted the analysis and wrote the manuscript. OG, HN, AT, PN and ER-R discussed the results, their interpretation and implications. OG, HN, AT and JJ discussed the numerical methods. OG, AT, NK and VK discussed the observational prospects. All coauthors provided comments on the manuscript.
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**Competing Interests**

The authors declare no competing interests.

**Data and Materials availability**

All numerical data used in this paper will be shared upon request to the corresponding author.

**Supplementary Materials**

Gravitational wave calculation
Numerical setup
GRB population
GWs from accretion shocks
Fig. S1
References (56-73)
Gravitational wave calculation

When a fluid particle accelerates, it induces a perturbation in the metric, which results in emission of GW radiation. Here, we assume the background metric to be flat since all the GW energy is essentially released far away from the BH. The monopole and dipole gravitational radiation are set to zero by conservation of mass and momentum, respectively. Thus, the lowest non-vanishing multipole of gravitational emission is the quadrupole component. The GW strain can be approximated by the quadrupole formula for the transverse-traceless (TT) gauge metric perturbation

\[ h \approx \frac{2G}{Dc^4} \frac{d^2 Q}{dt^2}, \quad (5) \]

where the gravitational quadrupole is

\[ Q_{ij} = \int T^{00} \left( x_i x_j - \frac{1}{3} \delta_{ij} r^2 \right) d^3 x, \quad (6) \]

\( T^{00} \) is the contravariant energy density component of the stress-energy tensor, and the integration is over equal arrival time surfaces (radii with retarded times that correspond to a given observer time). Eqs. 5 and 6 show that axisymmetric modes and stationary sources do not emit GWs. It then follows that asymmetric, time-evolved energetic objects such as the off-axis cocoon, are most promising in generating detectable GWs.

For a single GW with angular frequency \( \omega \), that without the loss of generality propagates along the \( \hat{z} \)-axis, the plane wave solution has two independent components

\[ h_{ij}(t, z) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cos \left[ \omega \left( t - \frac{z}{c} \right) \right], \quad (7) \]

where

\[ h_+(t) = \frac{G}{Dc^4} \left( \frac{d^2 Q_{xx}}{dt^2} - \frac{d^2 Q_{yy}}{dt^2} \right) ; h_\times(t) = \frac{2G}{Dc^4} \frac{d^2 Q_{xy}}{dt^2} \quad (8) \]
are the two independent polarizations of the plane wave moving along the $\hat{z}$-axis. The GW detector response function to the planar GW is a linear combination of the two polarizations (56) and is given by:

$$h(t) = F_+ h_+(t) + F_\times h_\times(t), \quad (9)$$

where $F_+, F_\times$ are functions of the antenna-pattern of the detector that depend on the sensitivity to each polarization. For an order of magnitude estimate, the GW strain can be approximated as

$$h(t) \approx \sqrt{h_+^2(t) + h_\times^2(t)}. \quad (10)$$

To characterize the sensitivity for detection at a given frequency, one defines the characteristic strain as (57)

$$h_c(f) = 2f |\tilde{h}(f)|, \quad (11)$$

where

$$\tilde{h}(f) = \sqrt{\frac{\tilde{h}_+^2(f) + \tilde{h}_\times^2(f)}{2}} \quad (12)$$

is the Fourier transform of the interferometer response to the dimensionless GW strain.

By using the quadrupole formula, we make the following approximations:

i) To separate the contribution of the cocoon from that of the jet, we integrate the gravitational quadrupole only over plasma with asymptotic proper-velocity smaller than the upper limit of the cocoon velocity, $\Gamma \approx 3$ (28). At these velocities the anti-beaming effect, which attenuates the GW emission of relativistic sources that move toward the observer (32), is small and therefore the quadruple approximation applies. To verify that the exclusion of the jet does not artificially increase the quadruple contribution at the expense of lower moments, we also

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6However, at mildly-relativistic velocities, higher multipoles are no longer negligible, and their contribution might be comparable to that of the quadrupole. Nevertheless, since most of the cocoon energy is at sub-relativistic velocities, our estimates are not expected to change significantly.

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calculate the quadrupole with the jet contribution, and find that the change in the quadrupole is minimal since the jet is less energetic than the cocoon.

ii) The anti-beaming effect is also applicable to the mildly-relativistic part of the cocoon\(^7\) that forms before breakout and expands to an angle of \(\approx 1/\Gamma\). Since the energy in the cocoon is distributed uniformly in log-scale of the velocity from \(\beta \approx 10^{-2.5}\) to \(\Gamma \approx 3\), only a small amount of energy undergoes anti-beaming, and is attenuated for observers at \(\lesssim 0.5\) rad. Thus, anti-beaming in the cocoon may introduce an error of a few tens of percents at low frequencies for observers close to the axis, where the projected quasi-axisymmetric shape of the cocoon leads to weaker GW emission anyway.

iii) Eq.\(^6\) includes an integration over surfaces with equal arrival time, i.e. different fluid elements at the explosion frame (lab time) contribute to the GW emission at different observer times. Our expensive numerical calculation which processes \(\gtrsim 1\) petabytes of data prevents us from performing an accurate calculation with the GW travel time effect. Instead, the calculation is performed in the explosion frame such that it assumes that the lab time is identical to the observer time. Since we obtain a strain amplitude that is roughly constant in space and time, this assumption is reasonable. That is, different regions in the cocoon maintain similar spectrum at all times such that the GW travel time is not anticipated to alter our result considerably. This is especially true for off-axis observers, where the GW travel time difference is on the order of the cocoon width divided by the speed of light, which is much shorter than the GW signal duration. For on-axis observers, the GW travel time difference can be a few seconds, that would moderately prolong the GW emission duration, but should not affect its spectrum qualitatively.

\(^7\)Recently, (20) estimated the GW signal from a GRB jet outflow numerically, however their use of the GW memory expressions assumes a coherent motion and thus is inapplicable to the cocoon.
Numerical setup

We follow (30) who performed high resolution 3D GRMHD simulations of a collapsing star with strong magnetic fields (maximal B-field at the time of the BH collapse is $B \approx 10^{12.5}$ G), that lead to the formation of powerful bipolar jets with $E_j \approx 10^{53}$ erg near a Kerr BH of mass $M_{\text{BH}} = 4.2 M_\odot$ and dimensionless spin $a = 0.8$. The simulations followed the jet head for 18 seconds, until it is well outside the stellar envelope of mass $14 M_\odot$, at $10 R_\star$, where $R_\star = 4 \times 10^{10}$ cm is the stellar radius. Here, we use the code H-AMR (58) to rerun one of their simulations - the configuration with initial jet magnetization of $\sigma_0 = 15$, until the jet head breaks out from the collapsing star and reaches $\sim 1.5$ stellar radii at $t \approx 3.1$ s.

To properly follow the time evolution of the gravitational quadrupole in the simulation, we output data files every $\Delta t = r_g/c \approx 0.02$ ms, where $r_g$ is the BH gravitational radius. This timescale guarantees that the physical processes in the disk are temporally resolved. We verify this by calculating the second derivative with respect to time of the gravitational quadrupole using $\Delta t = 2r_g/c$, and find that the GW signal is independent of the interval choice. We also verify that the quadrupole moment does not change when multiplying the spatial resolution of each axis by 1.5. The integration to $t \gtrsim 3$ s $\approx 150,000$ $r_g/c$ implies that we post-process 150,000 files with more than a petabyte of data.

GRB population

To estimate the number of detectable events in LVK during observing run O4, one needs to evaluate the LGRB distribution in $E_{\gamma,\text{iso}}$. The detected LGRB population can be modeled by a log-normal distribution, whereas the total number of LGRBs can be fit by a power-law that coincides with the log-normal distribution tail (figure 5 in (43)). We define the distribution tail as LGRBs with $E_{\gamma,\text{iso}} > 10^{54}$ erg, since those are less prone to detection bias, and are relevant for
producing detectable GW emission. Then, by fitting a power-law to the log-normal distribution tail, we find \( N_{\text{LGRBs}} \propto E_{\gamma, \text{iso}}^{-1.5} \). Assuming a LVK detection threshold of \( |h_{\text{crit}}| \simeq 10^{-22} \) and using Eq.\( ^3 \) the number of detectable cocoon-powered GW events per year in LVK observing run O4 is (Fig. 4)

\[
N_{\text{O4}}(E_{\gamma, \text{iso}, m}) = \int_0^{N(E_{\gamma, \text{iso}, m})} \left( \frac{\theta_2^2 \xi E_{\gamma, \text{iso}}}{8 \times 10^{52} \text{ erg}} \right)^3 \frac{\mathcal{R}_{\text{GRB}}}{\text{Gpc}^{-3} \text{ yr}^{-1}} f_p(E_{\gamma, \text{iso}}) dN(E_{\gamma, \text{iso}}),
\]

where we assume the \( \gamma \)-ray energy to be 20% of the total jet energy, \( f_p \) is the normalized density function of the \( N_{\text{LGRBs}}(E_{\gamma, \text{iso}}) \) distribution, \( E_{\gamma, \text{iso}, m} \) is the distribution cutoff, and \( \xi = 1 \) if jets propagate along one axis, and \( \xi = 10 \) if they wobble.

The detection rate of the quasi-isotropic GW signal can also be used to estimate the abundance of LGRBs among their progenitors - SNe of type Ib/c, whose rate is \( \mathcal{R}_{\text{SNe Ib/c}} \approx 2.6 \times 10^4 \text{ Gpc}^{-3} \text{ yr}^{-1} \) (59). If the number of detected events with \( E_c < E_{\text{max}} \) in LVK observing run O4 is \( N_{\text{O4}}(E_c < E_{\text{max}}) \), then using Eq.\( ^3 \) the fraction of SNe Ib/c progenitors that power LGRBs with \( E_c < E_{\text{max}} \) can be estimated as (Fig. 4)

\[
\frac{N_{\text{LGRBs}}(E_c < E_{\text{max}})}{N_{\text{SNe Ib/c}}} \approx \frac{N_{\text{O4}}(E_c < E_{\text{max}})}{\mathcal{R}_{\text{SNe Ib/c}}} \left( \frac{E_{\text{max}}}{5 \times 10^{52} \text{ erg}} \right)^3.
\]

Another noteworthy jet populations for production of detectable cocoon-powered GWs are short GRB jets in binary neutron star (BNS) mergers, and jets that fail to break out (and deposit all their energy in the cocoon) either from the BNS ejecta (60) or from the star. (61) estimated that choked jets could be even more abundant than those that successfully break out, based on the GRB duration distribution. However, LGRB jets with a short central engine activity are unlikely to drive the powerful cocoons required for producing a detectable GW signal, as their cocoon structure will be quasi-spherical due to early choking. Alternatively, if jets fail because of the progenitor structure, e.g. it has an extended envelope such as in SN Ib progenitors (62,63), their cocoon could be very powerful, and even give rise to energetic explosions such as fast blue
optical transients (64). In this case, the cocoon-powered GW detection rate can be significantly higher. Short GRBs are $3 - 30$ (depending on whether jets wobble) more abundant than LGRBs (65), but are also about an order of magnitude less energetic (66). Because the detectable GW events are dominated by the most energetic GRBs, it might be too challenging for LVK to detect cocoons that accompany short GRB jets in binary mergers.

**GWs from accretion shocks**

We consider the case of a rapidly rotating star in which the magnetic field is insufficient to power relativistic jets via the Blandford-Znajek (BZ) mechanism. Instead, disk winds energize the expansion of an accretion shock. (39) showed that reducing the magnetic field magnitude, such that the BZ power falls below the one needed to overcome the ram pressure of the falling stellar material, results in an expanding accretion shock. We follow this logic and reduce the magnetic field strength in the star by two orders of magnitude, while keeping the stellar mass, radius, angular momentum and magnetic field geometry identical to the jet simulation. Using a stellar density profile

$$\rho(r) \propto r^{-\alpha} \left(1 - \frac{r}{R_*}\right)^3,$$

we consider two cases of an expanding accretion shock in a stellar core: $\alpha = 0$ (constant core density) and $\alpha = 2$ (steep core density).

We note that the expanding shock is similar to standing accretion shock instabilities (SASI) in CCSNe (where the rotation can be slow), which are also candidates for GW emission (67–73). However, there are two important differences: i) while SASI is driven by the core bounce of the collapse, the energy source of the expanding shock is disk winds; ii) the stagnation of the shock in SASI occurs as the supersonic collapsing plasma cannot be fully accreted onto the BH, forming a shock front, whereas in our simulation the fast rotation forms a disk that keeps energizing the expanding shock at all times.
Eq. 6 dictates that $Q \approx M_{sh} r^2$, where $M_{sh}$ is the mass contained in the shock, which depends on the stellar core density profile. The virial theorem for a gravitationally bound object at radius $r$ suggests that $\dot{Q} \approx Mv^2 \approx GM^2/r$. In our simulation of a star with a steep density profile, the accretion shock expands from the disk size at $r_{sh} \approx 100$ km to $r_{sh} \gtrsim 10^{3.5}$ km, and the mass contained in the shock is $M_{sh} \approx M_{\odot}$. It then follows from Eq. 5 that the strain amplitude of the shock is

$$h_{sh} \approx \frac{r^2_s}{r_{sh}D} \approx 10^{-22} \left( \frac{200 \text{ kpc}}{D} \right) \left( \frac{M_{sh}}{M_{\odot}} \right)^2 \frac{10^{3.5} \text{ km}}{r_{sh}} ,$$

where $r_s$ is the Schwarzschild radius of the shock. The GW frequency is governed by the Keplerian orbit, such that as the shock expands over time, it covers the following frequency range

$$f_{sh} \approx \sqrt{\frac{GM_{BH}}{r^3_{sh}}} \approx 30 - 10^3 \text{ Hz} .$$

Eqs. 16 and 17 show that the expansion velocity of the shock essentially dictates the GW signal evolution. When $\alpha = 0$ ($\alpha = 2$), the shock gradually (rapidly) expands to larger radii, thus the GW spectrum evolves slow (fast) toward lower frequencies.

Fig. S1 depicts the numerically calculated strain amplitudes during the first $\approx 1$ s of the simulations, in time (a) and frequency (b). When $\alpha = 0$, the shock expands slowly to larger radii that contain more mass, so the GW signal intensifies while gradually shifting toward lower frequencies. When $\alpha = 2$, the shock expands rapidly to reach longer orbits at lower frequencies, but with most of the shock mass lies at small radii, the strain amplitude remains largely unchanged. In both cases, the LVK detection horizon is $\approx 100$ kpc, in agreement with Eq. 16 to an order of magnitude, and the spectrum has an exponential cutoff at high frequencies, $h(f) \propto \exp \left[-(f/1.3 \text{ kHz})^2\right]$ (dotted line), consistent with Eq. 17.

We find that most of the symmetry breaking takes place in the disk at a small radius. Thus, when most of the mass is concentrated at the shock front ($\alpha = 0$), the axisymmetric spiral mode $m = 1$ of the shock (see animation in https://oregottlieb.com/transients.
Fig. S1: The strain amplitude in time (a) and in frequency (b) shows that the GW emission from an expanding accretion shock is weaker by $\approx 3$ orders of magnitude than that from the cocoon. The lines for $\theta_{\text{obs}} = 90^\circ$ are averages of observers at $\phi_{\text{obs}} = 0^\circ$ and $\phi_{\text{obs}} = 90^\circ$.

HTML attenuates the on-axis GW signal. By contrast, when most of the mass is around the disk ($\alpha = 2$), then the asymmetric mass distribution leads to a quasi-isotropic GW emission. The magnitude and dependence on the viewing angle are in a rough agreement with the results reported for SASI-powered GWs in CCSNe (67–73). Most of the considered CCSN progenitors maintain a relatively steep density profile, where SASI’s characteristic frequencies are in the range of 20-100 Hz (68), and thus their results resemble those of our $\alpha = 2$ model.