Interference analysis of wells in oil and gas reservoir with bottom water

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Abstract. A mathematical model of the inflow and algorithm for calculating the pressure field for interfering wells in a complex oil and gas reservoir with bottom water were developed in this paper. The interference model explores well interaction and when the operating mode of one of them is changing it allows to evaluate the change of bottom hole pressure in the reacting wells. While using the developed model it is possible to calculate a pressure field for a group of wells, to simulate the process of pulse-code well testing and also to specify the boundary conditions for more detailed inflow models. All the results were successfully tested on real data from one of the Eastern Siberia fields.

1. Introduction

Recently, there is an increasing tendency to complicate a process of hydrocarbon production, associated both with the properties of reservoir rocks and hydrocarbons. Modeling of reservoir with a complex structure of pore space is characterized by a high indeterminacy of the input parameters, due to both the insufficiency of initial information and the excessive complexity of the physical processes described in the models. In such cases it is important to create the methods for quick calculations adapted to the specified characteristics of the hydrocarbon system and based on simplified analytical or numerical models.

The main objectives of this research are creating of the mathematical model of inflow to a group of interfering wells in a complex reservoir with bottom water and a gas cap and its testing on real data for one of the Eastern Siberia oil fields. It should be noted that by the interference model it is possible not only to evaluate the pressure behavior of piezometric wells and to build a pressure field for the simultaneous operation of a group of wells, but also to simulate the process of pulse-code well testing and correct the boundary conditions for more detailed inflow models.

2. Mathematical model of well interference

The well interference consists in fact that while starting, stopping or changing the operating modes of one of the wells, the behavior of other wells in the same reservoir changes too [2].

The problem of calculating the pressure field during the simultaneously operation of a group of wells in a massive oil and gas reservoir with bottom water is considered. To solve it, we will use a model of inflow to a point sink, which was developed by the authors earlier. The linearity of this model makes it possible to use the principle of superposition to account for well interference.

A schematic model of the problem under consideration is characterized by the following features: it is believed that oil is contained in limited areas with sufficiently high porosity, which allows to ignore
capillary forces. Hence, we can assume that there is a gravitational fluid segregation. The densities of oil, water and gas \( \rho_0, \rho_w, \rho_g \) and porosity \( m \) are assumed to be linearly dependent on pressure [2, 3]:

\[
\begin{align*}
\rho_0 &= \rho_0^0 (1 + c_o (p_o - p_o^0)), \\
\rho_w &= \rho_w^0 (1 + c_w (p_o - p_o^0)), \\
\rho_g &= \rho_g^0 (1 + c_g (p_o - p_o^0)), \\
m &= m^0 (1 + c_r (p_o - p_o^0)).
\end{align*}
\]

Here \( c_o, c_w, c_g \) are the compressibility of phases, \( p_o \) – oil pressure, upper index means parameters value at the initial conditions.

The mathematical model of the inflow can be described by the system of mass balance equations under conditions of gravitational equilibrium. These equations can be solve for oil pressure at a given level (for example, horizontal wellbore depth) \( p_o(x, y, t) \) and the shapes of the interface - oil-water (WOC) \( z_{ow}(x, y, t) \) and gas-oil contacts (GOC) \( z_{og}(x, y, t) \). Also, it is important to study the water and gas coning and determine the conditions for the stability of contacts [4, 5]. However, nonequilibrium effects in the reservoir, including the breakthrough of cones, occur only in the vicinity of the wells. While studying the interaction of wells, we are considering a large-scale approximation, for which such effects are insignificant. Therefore, while modeling interference, the use of the equilibrium model is quite reasonable.

The shape of the contacts in large-scale calculation can be determined by the following equations:

\[
\begin{align*}
\alpha_w &= WOC velocity, \\
\alpha_g &= GOC velocity,
\end{align*}
\]

Substitution of these expressions in the equation of oil balance gives the following nonlinear equation:

\[
\begin{align*}
div \left[ \frac{\rho_0^0 k}{\mu_o} (1 + c_o (p_o - p_o^0)) \nabla p_o (z_{ow} - z_{og}) \right] &= \frac{\partial}{\partial t} \left[ m^0 \rho_o^0 (1 + (c_o + c_r) (p_o - p_o^0)) (z_{ow} - z_{og}) \right],
\end{align*}
\]

Subsequently this equation can be linearized [1, 2].

The solution of equation (3) for the point sink and the principle of superposition which is valid for linear equations have been giving us a possibility to estimate the well interference and calculate the pressure field for simultaneously operations of wells group. It is advisable to note that in a large-scale approximation any well will behave as a point sink. In this regard, the following problem is the basis of the proposed well interference estimation algorithm.

Consider a group of wells in the plane \( Ox \). It is assumed that these wells are located inside the rectangle area with impermeable borders. Its dimensions can be set arbitrarily, but in such way that all wells have to be inside this area and its center coincided with geometric center of analyzed well group (Figure1).
The impermeability of the area boundaries is simulated by multiple reflection of all sinks, relative to each area boundary. The accuracy of the solution will depend on the selected number of reflections. Ultimately, the use of reflection and superposition methods [2] gives the following expression for calculating the pressure field for interfering wells:

\[
p_{o}(x, y, t) = P_{0} + \sum_{i=1}^{n} Q_{i}(t) E_{i} \left( -\frac{(x-x_{i})^{2} + (y-y_{i})^{2}}{4\chi t} \right) + \sum_{j=1}^{\infty} \sum_{i=1}^{n} Q_{i}(t) E_{i} \left( -\frac{(x-x_{ijLR})^{2} + (y-y_{ij})^{2}}{4\chi t} \right) + \sum_{j=1}^{\infty} \sum_{i=1}^{n} Q_{i}(t) E_{i} \left( -\frac{(x-x_{ijUD})^{2} + (y-y_{ij})^{2}}{4\chi t} \right) + \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{n} Q_{i}(t) E_{i} \left( -\frac{(x-x_{ijUD})^{2} + (y-y_{ijUD})^{2}}{4\chi t} \right)
\]  

Here values of \( P_{0} \) and \( Q_{i}(t) \) are determined by the initial pressure and flow rates of the wells, \( n \) - number of wells in the considered area, \( x_{i}, y_{i} \) – its coordinates, \( x_{ijLR} \) and \( y_{ijUD} \) – coordinates of imaginary wells reflected from the left and right and, respectively, upper and lower boundaries of the considered area, \( j, k \) are the reflection numbers.

The proposed interference model is quite useful in practice. First, the calculated pressure distribution allows to correct the boundary conditions for a single well fine-scale model. Secondly, the model can be used to estimate the reservoir parameters and to evaluate the response of each well disturbances in the simultaneous operations of several wells. Third, while using this interference model, it is possible to successfully solve the problems of well testing.

3. Simulation results

The model presented in this paper was successfully tested on real data of one of the Eastern Siberia oil fields. Adjusting the parameters of the flow area, a relatively small deviation value from the actual data was achieved. The correct matching of reservoir characteristics allowed to repeat the trend of wells pressure behavior despite many assumptions of the presented model.
One example of piezometry process is demonstrated. The process of pulse-code piezometry includes series of short-term and small changes in the flow rate (pulses), which lead to changes in the measured pressures. Since the pulses are short-lived, pressure changes are small too.

Figure 2 shows the simulation results of piezometry modeling for one reacting and four production wells. Green color in the figure shows the actual measured pressure change in the reacting well R1 when starting and stopping production wells P1, P2, P3, P4 (other curves). The changes obtained by interference model calculation are represented in black color in the figure. The graph demonstrates how the pressure in the observation well responds to the start and stop of the production wells. It can be seen that the interference model made it possible to successfully reproduce the reaction to all disturbances and repeat the actual trend with enough quality.

![Figure 2](image_url)

**Figure 2.** Adaptation of the interference model for the reacting well R1 (black color - calculation with noise filtering with a moving average, green color – actual data) when changing the flow rates of production wells P1 (blue), P2 (red), P3 (green), P4 (purple).

4. **Conclusion**
The mathematical model of well interference in complex oil and gas reservoir with bottom water was constructed in this research. All developed algorithms were implemented as a program in FORTRAN. During programming, all the computational complexity of the resulting algorithms was taken into account and methods that speed up the calculation, including parallel, were applied. The model has been successfully tested and adapted to real data of one of the Siberian oil fields.
5. References

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