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Concentration inequalities for s-concave measures of dilations of Borel sets and applications.
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The main purpose of the paper is to establish a sharp inequality, conjectured by S. Bobkov in [“Large deviations and isoperimetry over convex probability measures with heavy tails”, Electron. J. Probab. 12, 1072–1100 (2007; Zbl 1148.60011)], comparing the measure of a Borel set in $\mathbb{R}^n$ with an $s$-concave probability measure and the measure of its dilation. Among the $s$-concave probability measures are the log-concave ones ($s = 0$) and thus the Gaussian ones, so that it is expected that they satisfy good concentration inequalities and large and small deviations inequalities. This is indeed the case and these inequalities as well as Kahane-Khintchine type inequalities with positive and negative exponent are deduced. By using a localization theorem in the form given in [M. Fradelizi and O. Guédon, “The extreme points of subsets of $s$-concave probabilities and a geometric localization theorem”, Discrete Comput. Geom. 31, No. 2, 327–335 (2004; Zbl 1056.60017)], the author exactly determines among $s$-concave probability measures $\mu$ on $\mathbb{R}^n$ and among Borel sets $F$ in $\mathbb{R}^n$, with fixed measure $\mu(F)$, what the smallest measure of the $t$-dilation of $F$ (with $t \geq 1$) is. This infimum is reached for a one-dimensional measure which is so-called $s$-affine and $F = [-1, +1]$. In other words, it gives a uniform upper bound for the measure of the complement of the dilation of $F$ in terms of $t$, $s$ and $\mu(F)$.

The paper is organized as follows. In Section 2, the author studies some general properties of dilation and determines its effect on examples. The case of convex sets is treated in Section 2.2, the case of sublevel sets of the seminorm of a vector valued polynomial in Section 2.3 and the case of sublevel sets of a Borel measurable function in Section 2.4. In Section 3, the author deduces distribution and Kahane-Khintchine inequalities for functions of bounded Chebyshev degree. The main tool for the proof is the localization theorem in the form given by M. Fradelizi and O. Guédon in [loc. cit.].

Reviewer: Viktor Ohanyan (Erevan)

MSC:
46B07 Local theory of Banach spaces
46B09 Probabilistic methods in Banach space theory
60B11 Probability theory on linear topological spaces
52A20 Convex sets in $n$ dimensions (including convex hypersurfaces)
26D05 Inequalities for trigonometric functions and polynomials

Keywords:
dilation; large deviations; small deviations; Khintchine type inequalities; sublevel sets

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