Perfect fluid nature in weakly-interacting magnetized quark matter

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Abstract

We have investigated shear viscosity of quark matter in presence of a strong uniform magnetic field background where Nambu-Jona-Lasinio model has been considered to describe the magneto-thermodynamical properties of the medium. In presence of magnetic field, shear viscosity coefficient gets splitted into different components because of anisotropy in tangential stress of the fluid. Those splitted components normalized by entropy density in fact measure the fluidity of magnetized quark matter. A nearly perfect fluid nature is found even in weakly interacting quark matter at high temperature and high magnetic field zone which is not possible in the case of vanishing magnetic field. Our results indicate that the magnetic field may be regarded as one of the possible sources that can increase the fluidity of quark matter.

1 Introduction

One of the major update in the research of heavy ion collision (HIC) experiments like RHIC and LHC is that the produced medium behaves like a nearly perfect fluid \([1]\), with smallest shear viscosity to entropy density ratio \((\eta/s)\), ever observed in nature. On the other hand, recent progress in the HIC research have speculated that the produced medium may face a high magnetic field \([2]\) in the non-central heavy-ion collisions. The possible space-time dependence of this produced magnetic field has been investigated in Refs. \([3][4][5][6][7]\). A considerable amount of research work has already been performed in understanding the influence of the magnetic field on the QCD phase diagram. See, for example, the review article \([8]\) for recent updates. The modification of the QCD phase diagram in presence of magnetic field is directly related to the corresponding change in the quark condensate and its enhancement with magnetic field is known as
magnetic catalysis (MC) which is quite expected feature in vacuum as well as at finite temperature [9, 10, 11, 12, 13, 14]. However, recent calculations, based on lattice quantum chromodynamics (LQCD) [15, 16] have found inverse magnetic catalysis, whose possibility is also indicated by some effective QCD model calculations [17, 18, 19, 20]. The modifications pertaining to the QCD phase diagram may also have some impact in the transport properties of the medium produced in HIC. In presence of magnetic field, different transport coefficients like shear viscosity [22, 23, 24, 25, 26, 27, 28, 29], bulk viscosity [27, 28, 29, 30, 31] and electrical conductivity [32, 33, 34, 35, 36, 37, 38, 39] of quark matter are calculated in recent times. The simulation of magnetohydrodynamics [40, 41] as well as the transport simulation for an external magnetic field [42] may require these temperature and magnetic field dependent transport coefficients for their future up-gradation.

Among the different transport coefficients, only the shear viscosity is our matter of interest in the present work. We have calculated shear viscosity to entropy density ratio for quark matter, where two flavor Nambu-Jona-Lasinio (NJL) model has been used as a dynamical framework. Among the earlier calculations of shear viscosity for magnetized matter [22, 23, 24, 25, 26, 27, 28, 29], we find that Refs. [22, 23, 24, 25] have not explored its component decomposition, which is explicitly analyzed in Refs. [26, 27, 28, 29]. This component decomposition of shear viscosity due to anisotropy, created by external magnetic field or other sources, is well studied in the direction of gauge gravity duality (See [43, 44] and references therein). Adopting this standard decomposition technique, mainly following Ref. [26], we have estimated different components of shear viscosity, which can be classified into two main components as also found in gauge gravity dual theory [43, 44]. In the context of lower bound restriction in these two components, we get a qualitative mapping with the expectation of gauge gravity dual theory [43, 44]. Based on the estimation of these viscosity components, we have pointed out a possibility of getting a perfect fluid quark matter at high temperature and high magnetic field even in the weakly interacting limit.

The article is organized as follows. In Sec. (2) and (3), the background formalism of NJL model and shear viscosity in presence of magnetic field are briefly addressed. After that, our estimations are discussed in Sec. (4). Finally, Sec. (5) summarizes the main conclusions.

## 2 NJL model in presence of magnetic field

We shall consider here, two flavor (u, d quarks) NJL model with a determinant interaction with the Lagrangian given as [45, 12] 

\[
\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi + G \sum_{a=0}^{3} [(\bar{\psi}\gamma^{a}\psi)^2 + (\bar{\psi}i\gamma^{5}\tau^{a}\psi)^2] + K \left[ det_f \bar{\psi}(1 + \gamma_{5})\psi + det_f \bar{\psi}(1 - \gamma_{5})\psi \right],
\]

where \(\psi = (u, d)^T\) is the doublet of quarks, \(m = (m_u, m_d)\) is the current quark mass with \(m_u = m_d\). The first term is basically the Dirac Lagrangian in presence of an external magnetic field, which we assume to be constant and in
the direction of $z$-axis. For calculational purpose, we shall further choose the gauge such that the corresponding electromagnetic potential is given by $A_{\mu}(x) = (0, 0, Bx, 0)$. The second line is attractive part of the quark anti-quark channel of the Fiertz transformed color current-current interaction. The third line is the 't-Hooft determinant interaction in the flavor space that describes the effects of instantons and is flavor mixing. $\tau^a, a = 0 \cdots 3$ are the U(2) generators in the flavor space. In the absence of magnetic field the interaction is invariant under $SU(2)_L \times SU(2)_R \times U_V(1)$. The second term has an additional $U(1)_A$ symmetry while the t-Hooft term does not have this symmetry and reflects the $U(1)_A$ anomaly of QCD.

The thermodynamic potential corresponding to Eq.(1) can be computed exactly in the same manner as was done previously in Ref. [13], that was done for three flavors in a variational method with an explicit structure for the vacuum with quark anti-quark condensates. The thermodynamic potential is then given as

$$\Omega = \sum_i \Omega_i^0 + \sum_i \Omega_i^{field} + \sum_i \Omega_i^{med}$$

where, $i$ is the flavor index. The vacuum term for $i$-th flavor $\Omega_i^0$ is given as

$$\Omega_i^0 = -\frac{2N_c}{(2\pi)^3} \int dp \sqrt{p^2 + M_i^2} \theta(\Lambda - |p|)$$

$$= -\frac{N_c}{8\pi^2} \left[ \Lambda \sqrt{\Lambda^2 + M_i^2} - \frac{M_i^4}{\sqrt{\Lambda^2 + M_i^2}} \right] \ln \frac{\Lambda}{M_i} + \frac{M_i^4}{\sqrt{\Lambda^2 + M_i^2}} \right]$$

with, $\Lambda$ as the three momentum cutoff associated with the NJL model. The field contribution that arises from the effect of magnetic field on the Dirac vacuum is given by

$$\Omega_i^{field} = -\frac{N_c}{2\pi^2} \sum_i |q_i B|^2 \left[ \zeta'(-1, x_i) \right.$$

$$- \frac{1}{2} \left( x_i^2 - x_i \right) \ln x_i + \frac{1}{4} x_i^2 \right]$$

where we have defined a dimension less quantity, $x_i = M_i^2/2|q_i B|$, i.e. the mass parameter in units of magnetic field and $\zeta'(-1, x) = d\zeta(z, x)/dz|_{z=1}$ is the derivative of the Riemann-Hurwitz $\zeta$ function which is given by

$$\zeta'(-1, x) = \frac{\ln x}{2} \left[ x^2 - x + \frac{1}{6} \right] - \frac{x^2}{4}$$

$$+ x^2 \int_0^\infty \frac{2 \tan^{-1} y + y \ln(1 + y^2)}{e^{2\pi xy} - 1} dy.$$
with the single particle energy in presence of magnetic field \( \omega_n = \sqrt{p_z^2 + 2n|q_i|B + m^2} \).

The condition of a sharp three momentum cutoff translates to a finite number of Landau level summation with \( n_{\text{max}} = \text{Int}\left[ \frac{\Lambda^2}{2|q_i|B} \right] \) when \( p_z = 0 \). Further, for the medium contributions this also leads to a cutoff for the \( |p_z| \) as \( \Lambda' = \sqrt{\Lambda^2 - 2n|q_i|B} \) for a given value of \( n \).

Similarly, in Eq. (2), the quark condensate \( I_{s} = -\langle \bar{\psi}^i \psi^i \rangle \), can be separated into a zero field vacuum term, a finite field dependent term and a medium dependent term as

\[
I_{s} \equiv -\langle \bar{\psi}^i \psi^i \rangle = 2N_c \int_{|p|<\Lambda} dp \frac{M^i}{\sqrt{p^2 + M_i^2}} + \frac{N_c M^i |q^i| B}{(2\pi)^2} \left[ x^i (1 - \ln x^i) + \ln \Gamma(x^i) + \frac{1}{2} \ln \frac{x^i}{2\pi} \right] - \sum_{n=0}^{n_{\text{max}}} \frac{N_c |q^i| B \alpha_n}{(2\pi)^2} \int dp_z \frac{M^i}{\omega_v^{n}} \frac{1}{1 + e^{-\beta \omega_n}} = I_{s\text{vac}} + I_{s\text{field}} + I_{s\text{med}}. \tag{7}
\]

The zero field vacuum contribution, \( I_{s\text{vac}} \), can be analytically calculated using a sharp momentum cutoff \( \Lambda \) and can be written as

\[
I_{s\text{vac}} = \frac{N_c M^i}{2\pi^2} \left[ \Lambda \sqrt{\Lambda^2 + M^2} - M^2 \log \left( \frac{\Lambda + \sqrt{\Lambda^2 + M^2}}{M^i} \right) \right]. \tag{8}
\]

The constituent quark mass \( M^i \) satisfies the gap equation

\[
M_i = m_i + 4GI_{s\text{vac}} + 2K|e^3|I_{s} \tag{9}.
\]

This completes the definitions of all the quantities which are used to describe the thermodynamic potential in Eq. (2).

For numerical evaluations we choose the parameters as in Ref. [45] i.e. we write \( G = (1 - \alpha)G_0 \) and \( K/2 = \alpha G_0 \). The parameter \( \alpha \) controls the strength of the instanton interaction while the value of the quark condensate is determined by the combination of parameters : \( m = 6 \) MeV, the three momentum cut off \( \Lambda = 590 \) MeV and the dimensionless coupling \( G_0 \Lambda^2 = 2.435 \). These values lead to pion mass in vacuum as 140.2 MeV, pion decay constant of 92.6 MeV and quark condensate \( \langle \bar{u}u \rangle = \langle \bar{d}d \rangle = (-241.5 MeV)^3 \), all in reasonable agreement with the experimental values. This also leads to a vacuum constituent quark mass of 400 MeV. Further, in all these calculations we have taken \( \alpha = 0.15 \) as a reasonable value interpolated from \( \eta - \eta' \) splitting within 3-flavor NJL model [45].

Fig. 1(a) shows the constituent quark mass as a function of temperature for different values of magnetic fields. At \( eB = 0 \), masses of u and d quarks exactly coincide (dotted line), while for non-zero \( eB \), they are splitted and their
splitting increases with the magnetic field. Our results reveal the magnetic catalysis in entire temperature range and therefore, transition temperature $T_c$ increases with $B$. Using this $M_Q(T, eB)$, one can calculate entropy density $s$ with the help of a quasi-particle relation:

$$s = \frac{N_c}{\pi^2} \sum_{i=u,d} \sum_{n=0}^{n_{\text{max}}} \alpha_n |q_i| B \int d\omega_n \left[ \frac{k^2}{\omega_n^2} + \omega_n \right] f_0(\omega_n^i),$$

where $f_0(\omega_n^i)$ is Fermi-Dirac distribution function. The temperature dependence of normalized entropy density $s/T^3$ for $eB = 0$ (dotted line), $10m^2_\pi$ (solid line) and $20m^2_\pi$ (dash-dotted line) are shown in Fig. 1(b). We notice that $s$ decreases as $eB$ increases in lower temperature domain but all the curves are merged into its Stefan-Boltzmann (SB) limit at high temperature region.

### 3 Shear viscosity in presence of magnetic field

Let us first take a brief recapitulation of relaxation time approximation (RTA) technique to calculate shear viscosity coefficients of a relativistic fluid in absence of any magnetic field (i.e. $B = 0$), which is elaborately given in Refs. [46, 47]. Then, we will come to its corresponding formalism in presence of the strong magnetic field, well described in Refs. [26, 48].

Total energy-momentum tensor of relativistic fluid, $T^{\mu\nu} = T_0^{\mu\nu} + T_D^{\mu\nu}$ contains ideal part $T_0^{\mu\nu} = -P g^{\mu\nu} + (P + \epsilon) u^\mu u^\nu$ and dissipation part $T_D^{\mu\nu} = \eta U^{\mu\nu}$ (only shear dissipation), where $P$, $\epsilon$, $u^\mu$ are respectively pressure, energy density and four velocity of the fluid. The tensor structure $U^{\mu\nu}$, linked with shear viscosity $\eta$, has a form [47]:

$$U^{\mu\nu} = D^\mu u^\nu + D^\nu u^\mu + \frac{2}{3} \Delta^{\mu\nu} \partial_\sigma u^\sigma$$

with
\[ D^\mu = \partial^\mu - u^\mu u^\nu \partial_\nu, \quad \Delta^{\mu \nu} = u^\mu u^\nu - g^{\mu \nu}. \]  

(11)

Now, in terms of four momentum \( k^\mu = (\omega, \mathbf{k}) \) and thermal distribution function \( f_0 = 1/(e^{\beta \omega} + 1) \) of quark at temperature \( T = 1/\beta \), one can express the total energy-momentum tensor as

\[ T^{\mu \nu} = \int \frac{d^3 k}{(2\pi)^3} \frac{k^\mu k^\nu}{\omega} \{ f_0 + \phi f_0(1 - f_0) \}, \]  

(12)

where, the second term in the curly bracket involving the function \( \phi \) describes the non-equilibrium part for which one can construct the shear dissipative part \( T^{\mu \nu}_{\text{D}} \) of the energy momentum tensor \[17\]. The function \( \phi \) for the same can be written as \( \phi = C k^\mu k^\nu U_{\mu \nu} \). The unknown \( C \) can be obtained as \( C = \frac{\tau_c}{\omega} \beta \) by using the relativistic Boltzmann equation (RBE), where \( \tau_c \) is the relaxation time of the quark in the medium. comparing the coefficients of \( U_{\mu \nu} \) from the dissipative part of the energy-momentum tensor, we finally obtain the expression of shear viscosity coefficient as

\[ \eta = \frac{g \beta}{15} \int \frac{d^3 k}{(2\pi)^3} \frac{k^4}{\omega^2} \tau_c f_0(1 \mp f_0), \]  

(13)

where \( g = 2 \times 2 \times 2 \times 3 \) is an additional input that takes care of the degeneracy factor for 2 flavor (isospin symmetric) quark matter.

Now, let us discuss the effect of external magnetic field on the shear viscosity of the medium. In presence of a constant background magnetic field, the medium can possess five independent components of shear viscosity \[48, 26\] and the dissipative part of the energy-momentum tensor (in three vector notation) can be written as \[48, 26\]

\[ T^{\mu \nu}_{\text{D}} = \int \frac{d^3 k}{(2\pi)^3} \frac{k^\mu k^\nu}{\omega} \delta f, \]  

(14)

where

\[ \delta f = \phi f_0(1 \mp f_0) = \sum_{n=0}^{4} C_n k^i k^j V_{n}^{ij} f_0(1 \mp f_0), \]  

(15)

and

\[ \phi = \sum_{n=0}^{4} C_n k^i k^j V_{n}^{ij} \]  

(16)

is assumed in terms of same tensorial components \( V_{n}^{ij} \). Following further simplification, where \( V_{0}^{ij} \) component vanishes \[26\], one obtains four shear viscosity coefficients instead of five and they are

\[ \eta^{i}_{(n=1,2,3,4)} = \frac{2g_i}{15} \int \frac{d^3 k}{(2\pi)^3} \frac{k^4}{\omega^2} C_{n}^{i}(n=1,2,3,4)f_0(1 \mp f_0), \]  

(17)

where the unknown \( C_n \) again will be determined with the help of the RBE but in two step approximations. Since the magnetic field will destroy the degeneracy of up and down quark masses, therefore, energy \( \omega \), distribution function \( f_0 \) and \( C_n \) in Eq. \[17\] carry the flavor index \( i \). The \( g_i = 2 \times 2 \times 3 \) is degeneracy factor of each flavor.
As a first approximation, the particle relaxation time $\tau_c$ in the RBE is ignored by assuming that the deviation from equilibrium due to the strong magnetic field is much larger than that due to the particle collisions. Therefore, we get a magnetic field induced relaxation time $\tau_i = 1/\omega_i$, where

$$\omega_i = q_i B/\omega^i,$$  

$q_i = +\frac{2}{3}e$, $-\frac{1}{3}e$ for $i = u, d$. (18)

is the synchrotron frequency of quark and this first approximation of RBE provides (26)

$$C_i^1 = C_i^2 = 0,$$  

$$C_i^4 = 2C_i^3 = \tau_i \beta/2\omega_i.$$(19)

Now, in the second approximation, a collisional or thermal width $\Gamma_i = 1/\tau_i$, obeying the inequality $\Gamma_i << \omega_i$ or $\tau_i >> \tau_i$, is considered which leads to the relation (26):

$$C_i^2 = 4C_i^1 = \frac{\Gamma_i}{\omega_i} C_i^1 = \frac{\Gamma_i}{2\omega_i} C_i^4,$$  

$$C_i^3 = 2C_i^4 = \frac{\tau_i \beta}{2\omega_i}.$$ (20)

with $C_i^4 = 2C_i^3 = \frac{\tau_i \beta}{2\omega_i}$. Thus, in presence of constant background magnetic field $B$, the expressions of the four components of the shear viscosity for $i = u/d$ quark are

$$\eta_{i}^2 = 4\eta_{i}^1 = \frac{g_i \beta}{15} \int \frac{d^3 k}{(2\pi)^3} \left[ f_i^0 \{ 1 \mp f_i^0 \} \right]$$  

$$\left( \frac{\Gamma_i}{\omega_i} \right) \left( \frac{1}{\omega_i} \right) \left( \frac{k^2}{\omega^i} \right)^2,$$ (21)

and

$$\eta_{i}^4 = 2\eta_{i}^3 = \frac{g_i \beta}{15} \int \frac{d^3 k}{(2\pi)^3} \left[ f_i^0 \{ 1 \mp f_i^0 \} \right]$$  

$$\left( \frac{1}{\omega_i} \right) \left( \frac{k^2}{\omega^i} \right)^2.$$ (22)

If we compare Eqs. (21), (22) with Eq. (13), then we can get a physical interpretation of these shear viscosity components. If $B = Bz$ then in $x \rightarrow y$ and $y \rightarrow x$ tangential directions, momentum transfer is independent of the particle collisions and will be proportional to the field induced relaxation ($\tau_B = 1/\omega_B$) which is basically inverse of the synchrotron frequency. In other words, rotational motion of the charged particles with corresponding synchrotron frequency provides the required momentum transfer for generating shear stress along $x \rightarrow y$ and $y \rightarrow x$ direction. This strength of shear stress, velocity gradient and its proportional coefficients $\eta_1, \eta_2$ are completely originated due to (strong) magnetic background.

In other possible tangential directions $y \rightarrow z, x \rightarrow z$ and vice versa, both the collisional and rotational energies take part in momentum transfer. Therefore, the fraction $\Gamma_i/\omega_B$ is required for fixing the proportional strength of viscosities $\eta_1$ and $\eta_2$. The corresponding relaxation time for these components becomes $\left[ \left( \frac{\Gamma_i}{\omega_i} \frac{1}{\omega_B} \right) \right]^{-1}$.
4 Results and discussion

Let us focus on the region of applicability of the present formalism of shear viscosity in presence of (strong) magnetic field. At first, the particle relaxation time $\tau_c$ should be less than the life time of the expanding fire ball or the size of the medium produced in heavy ion collision. This upper limit of relaxation time is valid for both with and without magnetic field scenarios as it is related with threshold condition, beyond which system loose its medium identity. Considering roughly 10 fm as a maximum size of the fire ball, we can take $\tau_{c,up} \approx 10$ fm and alternatively lower limit of collisional thermal width $\Gamma_{c,low} \approx 0.0197$ GeV by using the their relation $\tau_{c,up} = 1/\Gamma_{c,low}$. On the other hand, $\tau_c$ should be greater than $\tau_{c,low} = 1/\omega_B = \omega^2/QeQB = (k^2 + m_Q^2)^{1/2}/eQB$, which is basically the inverse of the synchrotron frequency $\omega_B$. So the relaxation time, which we are interested in, should satisfy the inequality (23)

$$\tau_{c,low} < \tau_c < \tau_{c,up} = 10 \text{ fm}$$

or, $\omega_B = \Gamma_{c,up} > \Gamma_c > \Gamma_{c,low} = 0.0197$ GeV. (23)

This is shown in Fig. 2(a) and (b), where solid green line denotes $\tau_{c,up}$ and dashed, dash-dotted lines stand for $\tau_{c,low}$ for u and d quarks respectively. We have taken thermal average of $\tau_{c,low}$ to make it momentum independent. Now, using these limiting values of $\tau_c$ in Eq. (21), we get the corresponding upper and lower limit estimations of total $\eta_2(T, B)$, where total contribution is obtained by adding u and d flavor contributions. Normalizing $\eta_2(T, B)$ by $s(T, B)$, the results for $\tau_{c,low}$ and $\tau_{c,up}$ are shown in Fig. 2(c) and (d). From Fig. 2(d), one can notice that the inequality (23) is valid only above a threshold magnetic field (which in this case is $eB = 3.5m_Q^2$ for $T = 0.225$ GeV).
Interesting point is that $\eta(T, B = 0) \propto \tau_c$ in Eq. (13), while $\eta_2(T, B) \propto 1/\tau_c$ in Eq. (21). Therefore, we get lower limit estimation of $\eta_2(T, B)$ for $\tau_c^{up}$ and vice-versa, which is exactly opposite to $\eta(T, B = 0)$, whose small value is associated with small $\tau_c$ (large $\Gamma_c$) or strongly-coupled system. It indicates that a weakly interacting medium in presence of magnetic field may have a small shear viscosity.

Next in Fig. (3), we have explored the estimations of $\eta_2, \eta_4(T, eB)$ and $\eta_2/s(T, eB)$ for constant values of $\tau_c$ within the inequality (23). We have chosen $\tau_c = 5$ fm and use it in Eqs. (21) and (22) to generate $\eta_2(T, eB)$. One should notice that $\eta_2 \propto 1/\omega_B^2$ and $\eta_4 \propto 1/\omega_B$, which implies that d quark contribution in $\eta_2$ and $\eta_4$ are respectively 4 and 2 times larger than that of u quark in isospin symmetry case. However, our results in Figs. 3(a) and (b) also contain isospin symmetry breaking information due to magnetic field. It is worth mentioning that $\eta_4$ is independent of $\tau_c$ and its values for u and d quarks are opposite because of their opposite circular motions. Owing to the latter property of $\eta_4$, we have presented $|\eta_4|$ in Figs. 3(a) and (b) and total contribution of u and d quark is taken as $|\eta_4^u - \eta_4^d|$. The total $\eta_2$ and $\eta_4$ have been normalized again by the entropy density $s$ as given in Eq. (10). They are plotted against $T$ and $eB$ axes in Figs. 3(c) and (d). By using the same $\tau_c$ in Eq. (13), one can generate $\eta/s$, which are plotted by dotted line in Figs. 3(c) and (d). By comparing with and without magnetic field results, we get an immediate message - the shear viscosity to entropy density ratio of quark matter can be reduced because of an external magnetic field and as we increase the field strength, the ratio approaches towards KSS limit, shown by red horizontal line in Fig. (3).

We know that for a system with massless spin $1/2$ particles, the KSS limit
Figure 4: $T$ dependence of lower bound relaxation time for $eB = 0$ (dotted line) and upper bound relaxation time for $eB = 10m_q^2$ (solid line), $eB = 20m_q^2$ (dash line), where bounds are obtained to achieve KSS limit $\eta/s = 1/(4\pi)$.

$\eta/s = 1/(4\pi)$ gives the lower bound of relaxation time $\tau_c = 3/(2\pi T)$, which will be $\tau_c = 3/(2N_f N_c \pi T)$ for quarks with $N_f$ flavors and $N_c$ colors. In this context, our finite constituent quark mass $M_Q(T, eB = 0)$ in absence of magnetic field will give $\tau_c(T) = s(T, eB = 0)/[4\pi \eta(T, eB = 0)]$, displayed by dotted line in Fig. (4), which follows the massless relation, $\tau_c = 1/(4\pi T)$, only in high $T$ zone. This is expected because of chiral symmetry restoration at high $T$ range. In this without magnetic field picture, this low $\tau_c$ at high $T$, signifies that a strongly coupled system is required to achieve the KSS limits. While in presence of magnetic field, we get a completely opposite picture because $\eta_2 \propto 1/\tau_c$. The solid and dashed lines in Fig. (4) represent $\tau_c(T) = [4\pi \eta_2]/s$ at $eB = 10m_q^2$ and $20m_q^2$ respectively, where we observe that a weakly interacting matter, quantified by high $\tau_c(T, eB)$ can have $\eta/s = 1/(4\pi)$ in the high $T$ and $eB$ zone.

Most exciting fact is that RHIC (partially) and LHC experiments may access this $T-eB$ domain, where a weakly interacting quark matter may also behave as a nearly perfect fluid.

Similar to the calculations of gauge gravity duality [43, 44], we find that parallel component of shear viscosity with respect to applied external magnetic field may not be restricted by KSS bound. We can relate this component with our $\tau_c$ independent components $\eta_{1,4}$. Whereas other $\tau_c$ dependent components $\eta_{1,2}$ may correspond to the perpendicular component, given in Refs. [43, 44], which obey the KSS bound. Following the standard concepts of quantum lower bound, the $\tau_c$ of the medium constituent can not be lower than its de-Broglie wave length and therefore, we expect a corresponding lower limits in $\eta_{1,2}/s$, which may be equivalent roughly with the KSS bound. However, the $\tau_c$ independent ratios $\eta_{1,2}/s$ don’t have any such kind of restriction and we may get $\eta_{1,2}/s < 1/(4\pi)$ for some large values of $eB$ in Eq. (22).
5 Summary

We have studied shear viscosity of quark matter in a uniform magnetic field background, where the medium looses its isotropic property. Due to this anisotropic nature, one can get more than one components of shear viscosity, which are ultimately reduced to two main components $\eta_2$ and $\eta_4$. We know that isotropic shear viscosity $\eta$ in absence of magnetic field is mainly governed by two parts - the phase space and the relaxation time. Here also $\eta_2$ and $\eta_4$ can be casted into the similar structure with phase space and relaxation time parts. The relaxation time of $\eta_4$ is inversely proportional to synchrotron frequency $\omega_B$ and relaxation time of $\eta_2$ is $\Gamma_c/\omega_B^2$, for a small collisional thermal width $\Gamma_c$ of medium constituents (i.e. $\Gamma_c < \omega_B$). We have used the formalism of NJL model in presence of magnetic field to describe the magneto-thermodynamics of quark matter and we get a temperature and magnetic field dependent quark mass, which will enter to the phase space factors of $\eta_2$ and $\eta_4$. Thus, magnetic field dependence enters in both phase space and relaxation time parts. For constant values of $\Gamma_c$, we notice that shear viscosity in presence of magnetic field decreases with the field strength. Based on the results of the present study, we believe that the lower values of (specific) shear viscosity, observed in RHIC experiment, may have some association with this reduction of shear viscosity in the high magnetic field region. Thus the behaviour of the RHIC matter as nearly a perfect fluid can have some connection with the strong magnetic field produced in such collisions. This possibility, raised by present investigation, has to be verified by future studies based on other alternative dynamical framework.

In the present study we have taken the particle relaxation time as a parameter. However, one can calculate this relaxation time from the underlying microscopic theory. For example, in case of zero magnetic field, Refs [49, 50, 51] have obtained the momentum and temperature dependent relaxation time. Our next plan is to enter in the microscopic calculation of relaxation time in presence of magnetic field and then to see its impact on shear viscosity coefficient.

Acknowledgment: SG is financially supported from University Grant Commission (UGC) Dr. D. S. Kothari Post Doctoral Fellowship (India) under grant No. F.4-2/2006 (BSR)/PH/15-16/0060. AM, BC, HM are supported from DAE. SG would like to thank to Pracheta Singha for initial constructive discussion on this work.

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