Physics-based Neural Networks for Shape from Polarization

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ABSTRACT

How should prior knowledge from physics inform a neural network solution? We study the blending of physics and deep learning in the context of Shape from Polarization (SfP). The classic SfP problem recovers an object’s shape from polarized photographs of the scene. The SfP problem is special because the physical models are only approximate. Previous attempts to solve SfP have been purely model-based, and are susceptible to errors when real-world conditions deviate from the idealized physics. In our solution, there is a subtlety to combining physics and neural networks. Our final solution blends deep learning with synthetic renderings (derived from physics) in the framework of a two-stage encoder. The lessons learned from this exemplary problem foreshadow the future impact of physics-based learning.

1 INTRODUCTION

How can an uncertain physical prior can be blended into a deep learning framework? We address this question by rethinking a classic computer vision problem for which the physics are approximate. The Shape from Polarization (SfP) problem involves the capture of polarized photographs of a scene to estimate the shape of an object. The motivation is easy to grasp: light reflecting off an object has a polarization state that relates to the object’s shape. This problem is interesting because the physics of polarized light reflections are idealized leading to unusual forms of model mismatch. This special uncertainty in the physics-based prior makes it difficult to follow previous strategies of blending priors with deep learning (Che et al. 2018; Chen et al. 2018a; Diamond et al. 2017; Goy et al. 2018a,b; Jin et al. 2017; Karpatne et al. 2017; Le et al. 2017; Li et al. 2018a; Pan et al. 2018; Shi et al. 2018; Stewar and Ermon 2017). Figure 1 is conceptual, but reflects our observation that the suitability of a blending deep learning method is dependent on the robustness of model versus data.

We now expand on the unique uncertainties present in SfP, starting with the ambiguity problem. This problem arises because a linear polarizer cannot distinguish between polarized light that is rotated by π radians. This results in two confounding estimates for the azimuth angle. Previous work has used additional information to constrain the ambiguity problem. For instance, (Smith et al. 2018; Chen et al. 2018a; Diamond et al. 2017; Goy et al. 2018a,b; Jin et al. 2017; Karpatne et al. 2017; Le et al. 2017; Li et al. 2018a; Pan et al. 2018; Shi et al. 2018; Stewar and Ermon 2017). Figure 1 is conceptual, but reflects our observation that the suitability of a blending deep learning method is dependent on the robustness of model versus data.

Another physical challenge in SfP is the refractive problem. SfP requires knowledge of per-pixel refractive indices. Previous work has used hard-coded values to estimate the refractive index of scenes. This leads to a relative shape that is recovered with refractive distortion. Another physical challenge is the noise problem. SfP is ill-conditioned, requiring input images that are relatively noise-free. Ironically, a polarizer reduces the captured light intensity by 50 percent, worsening the effects of Poisson shot noise.

We address these SfP pitfalls by moving away from a physics-only solution, toward the realm of data-driven techniques. A reasonable first attempt could apply vanilla convolutional neural networks (CNN) to the SfP problem. Unfortunately, machine learning alone is not a satisfactory solution. As illustrated in Figure 2, a naive CNN implementation does not work even on the simplest of scenes. In contrast to prior work, we fuse both physics and deep learning in symbiosis. This hybrid approach outperforms previous SfP methods.

1.1 Contributions

In context of prior work in SfP, this paper demonstrates two technical first attempts:

(1) using deep learning techniques to solve the SfP problem; and
Figure 2: Ordinary neural networks are unable to solve complicated model-based problems. Here, we use physics-based neural networks to address the shape from polarization (SfP) problem. SfP is a very unique imaging problem that has significant model-based uncertainty. We study SfP as a test case that highlights the importance of combining physical priors with neural networks.

Figure 3: Summarizing the tradeoffs of our proposed physics-based neural networks (NN) versus physics-only and learning-only approaches.

(2) blending approximate physics into the deep learning approach;

Scope: Because this is only a first attempt at blending SfP with deep learning, the proposed solution is not perfect, particularly when obtaining the shape of objects with mixed reflectivities. However, all prior methods in SfP also fail in this scenario. While our physics-based approach to neural networks does outperform the individual strategy of physics and learning alone, this may just be a first attempt at the problem.

2 RELATED WORK

Polarization cues have been employed previously for different tasks, such as reflectometry estimation (Ghosh et al. 2010), facial geometry reconstruction (Ghosh et al. 2011), dynamic interferometry (Maeda et al. 2018), polarimetric spatially varying surface reflectance functions (SVBRDF) recovery (Baek et al. 2018), and object shape acquisition (Guarrera et al. 2012; Ma et al. 2007; Riviere et al. 2017). This paper sits at the seamline of deep learning and SfP, offering unique performance tradeoffs from prior work. Refer to Figure 3 for an overview.

Shape from polarization infers the shape (usually represented in surface normal) of a surface by observing the correlated changes of image intensity with the polarization information. Changes of polarization information could be captured by rotating a linear polarizer in front of an ordinary camera (Atkinson and Ernst 2018; Wolff 1997) or polarization cameras using a single shot in real time (e.g., PolarM (PolarM polarization camera 2017) camera used in (Yang et al. 2018)). Conventional shape from polarization decodes such information to recover the surface normal up to some ambiguity. If only images with different polarization information are available, heuristic priors such as the surface normals along the boundary and convexity of the objects are employed to remove the ambiguity (Atkinson and Hancock 2006; Miyazaki et al. 2003). Photometric constraints from shape from shading (Mahmoud et al. 2012) and photometric stereo (Atkinson 2017; Drbohlav and Sara 2001; Ngo et al. 2015) complements polarization constraints to make the normal estimates unique. If multi-spectral measurements are available, surface normal and its refractive index could be estimated at the same time (Huynh et al. 2010, 2013). More recently, a joint formulation of shape from shading and shape from polarization in a linear manner is shown to be able to directly estimate the depth of the surface (Smith et al. 2016, 2018; Tozza et al. 2017). This paper is the first attempt at studying deep learning and SfP together.

Polarized 3D involves stronger assumptions than SfP and has different inputs and outputs. Recognizing that SfP alone is a limited technique, the Polarized 3D class of methods integrate shape from polarization with a low resolution depth estimate. This additional constraint allows not just recovery of shape but also a high-quality 3D model. The low resolution depth could be achieved by employing two-view (Atkinson and Hancock 2005; Berger et al. 2017; Miyazaki et al. 2004), three-view (Chen et al. 2018c), multi-view (Cui et al. 2017; Miyazaki et al. 2016) stereo, or even in real time by using a SLAM system (Yang et al. 2018). These depth estimates from geometric methods are not reliable in textureless regions where finding correspondence for triangulation is difficult. Polarimetric cues could be jointly used to improve such unreliable depth estimates to obtain a more complete shape estimation. A depth sensor
such as the Kinect can also provide coarse depth prior to disambiguate the ambiguous normal estimates given by SfP (Kadambi et al. 2015, 2017). The key step that characterizes Polarized 3D is a holistic approach that rethinks both SfP and the depth-normal fusion process. The main limitation of Polarized 3D is the strong requirement that there be a coarse depth map, which is not true for our proposed technique.

Data-driven computational imaging approaches draw much attention in recent years thanks to the powerful modeling ability of deep neural networks. Various types of convolutional neural networks (CNNs) are designed and trained to enable 3D imaging for different types of sensors and measurements. From single photon sensor measurements, a multi-scale denoising and upsampling CNN is proposed to refine depth estimates (Lindell et al. 2018). CNNs also show advantage in solving phase unwrapping, multipath interference, and denoising jointly from the raw time-of-flight measurements (Marco et al. 2017; Su et al. 2018). From multidirectional lighting measurements, a fully-connected network is first proposed to solve photometric stereo for general reflectance with a pre-defined set of light directions (Santo et al. 2017). Then the fully-convolutional network with an order-agnostic max-pooling operation (Chen et al. 2018b) and the observation map invariant to the number and permutation of the images (Ikehata 2018) are concurrently proposed to deal with an arbitrary set of light directions. Normal estimates from photometric stereo can also be learned in an unsupervised manner by minimizing the reconstruction loss (Taniai and Maehara 2018). Other than 3D imaging, deep learning has been used to solve several inverse problems in the field of computational imaging (Satat et al. 2017; Tancik et al. 2018a,b). Separation of shape, reflectance and illumination maps for wild facial images can be achieved with the assistance of CNNs as well (Sengupta et al. 2018). Besides, CNNs also exhibit potentials for modeling SVBRDF of a near-planar surface (Deschaintre et al. 2018; Li et al. 2017, 2018b; Ye et al. 2018), and more complex objects (Li et al. 2018c).

The challenge with existing deep learning frameworks is that they do not leverage the unique physics of polarization.

3 PROPOSED METHOD

In this section, we first introduce some basic knowledge of SfP, and then present our physics-based convolutional neural network architecture. The blending of physics into deep learning helps improve the performance and generalizability of the method.

3.1 Image Formation and Physical Solution

Our objective is to reconstruct surface normals \( \hat{N} \) from a set of polarized images \( \{ I_{\phi_1}, I_{\phi_2}, \ldots, I_{\phi_M} \} \) with different rotations of polarizer angles. For a specific polarizer angle \( \phi_{pol} \), the intensity at a pixel of a captured image follows a sinusoid variation under unpolarized illumination:

\[
I(\phi_{pol}) = I_{\max} \cos(2(\phi_{pol} - \phi)) + I_{\min} 
\]

(1)

where \( \phi \) denotes the phase angle, and \( I_{\min} \) and \( I_{\max} \) are lower and upper bounds for the observed intensity. Equation 1 has a \( \pi \)-ambiguity in context of \( \phi \): two phase angles, with a \( \pi \) shift, will result in the same intensity in the captured images. Based on the phase angle \( \phi \), the azimuth angle \( \varphi \) can be retrieved with \( \frac{\pi}{2} \)-ambiguity as follows (Cui et al. 2017):

\[
\varphi = \begin{cases} 
\phi, & \text{if diffuse reflection dominates} \\
\phi - \frac{\pi}{2}, & \text{if specular reflection dominates}
\end{cases}
\]

(2)

The zenith angle \( \theta \) is related to the degree of polarization \( \rho \), which can be written as:

\[
\rho = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}
\]

(3)

When diffuse reflection is dominant, the degree of polarization can be expressed with the zenith angle \( \theta \) and the refractive index \( n \) as follows (Atkinson and Hancock 2006):

\[
\rho_d = \frac{(n - \frac{1}{n})^2 \sin^2 \theta}{2 + 2n^2 - (n + \frac{1}{n})^2 \sin^2 \theta + 4 \cos \theta \sqrt{n^2 - \sin^2 \theta}}
\]

(4)

The effect of \( n \) is not decisive, and we assume \( n = 1.5 \) throughout the rest of this paper. With this known \( n \), Equation 4 can be rearranged to obtain a close-form estimation of the zenith angle for the diffuse dominant case.

When specular reflection is dominant, the degree of polarization can be written as (Atkinson and Hancock 2006):

\[
\rho_s = \frac{2 \sin^2 \theta \cos \theta \sqrt{n^2 - \sin^2 \theta}}{n^2 - \sin^2 \theta - n^2 \sin^2 \theta + 2 \sin^4 \theta}
\]

(5)

Equation 5 can not be inverted analytically, and solving the zenith angle with numerical interpolation will produce two solutions if there are no additional constraints. For real world objects, specular reflection and diffuse reflection are mixed depending on the surface material of the object. As shown in Figure 5, the ambiguity in the azimuth angle and uncertainty in the zenith angle are fundamental limitations of SfP. Overcoming these limitations through physics-based neural networks is the primary focus of this paper.

3.2 Learning with Physics

Large amounts of labeled data are critical to the success of neural networks. To alleviate the burden of data requirement, one possible method is to blend physical priors during learning. However, it is essentially difficult to use physical information for SfP tasks due to the following reasons: 1. Polarization normals contain ambiguous azimuth angles. 2. Specular reflection and diffuse reflection coexist simultaneously, and determining the proportion of each type is complicated. 3. Polarization normals are usually noisy, especially when the degree of polarization is low. Shifting the azimuth angles by \( \pi \) or \( \frac{\pi}{2} \) could not reconstruct the surface normals properly for noisy images.

Due to the above reasons, regularization from the physical azimuth angle or the physical zenith angle will degrade the network performance and lead to a fragile model. Therefore, instead of using physical solutions as regularization, we directly feed both the polarized images and the ambiguous normal maps into the network, and leave the network to learn how to combine physical solutions with the polarized images effectively. The estimated surface normals can be structured as following:

\[
\hat{N} = f(I_{\phi_1}, I_{\phi_2}, \ldots, I_{\phi_M}, N_{\text{diff}}, N_{\text{spec1}}, N_{\text{spec2}})
\]

(6)
Ambiguous Surface
Polarized Reflections
Polarizer

Ambiguous Surface
Real Surface
Polarized Reflections

Figure 5: SfP lacks a unique solution due to the ambiguity problem. Here, two different surface orientations could result in the same exact polarization signal, represented by dots and hashes. The dots represent polarization out of the plane of the paper and the hashes represent polarization in the same exact polarization signal, represented by ambiguous normals can implicitly direct the network to learn some physical information and serve as a good initialization to improve generalizability.

### 3.3 Network Architecture

Our network structure is illustrated in Fig. 4. It consists of two independent encoders to extract features from polarized images and ambiguous surface normals separately and a common decoder to output surface normal $\hat{N}$. A variation of U-Net (Ronneberger et al. 2015) and LinkNet (Chaurasia and Culurciello 2017) is used to connect encoder block and decoder block at the same hierarchical level. We argue that addition is superior to concatenation when merging feature maps, since it achieves comparable performance, yet requires less memory and computational power in general based on our testing results.

There are 7 encoder blocks to encode the input to a tensor of dimensionality $B \times 1024 \times 2 \times 2$ to guarantee the receptive field, where...
B is the minibatch size. The encoded tensor is then decoded by the same number of decoder blocks to produce the estimated surface normals \( \hat{N} \). An L2-normalization layer is appended after the last decoder block to convert corresponding feature maps into surface normals. Table 1 shows the structure of each encoder and decoder block. Two additional feature extractors containing 3 convolutional layers of kernel size \( 3 \times 3 \) are placed before the first encoder block to prepare feature maps suitable for downsampling purpose. We use convolutional layers with stride of 2 for downsampling, and transposed convolutional layers for upsampling. Batch normalization layers (Ioffe and Szegedy 2015) are inserted after each layer, except the output layer, where batch normalization would cause distortion of the estimated surface normals \( \hat{N} \). After batch normalization, LeakyReLU with a negative slope of 0.1 is used for the activation function.

For the image encoder, pictures captured with a polarizer at angles \( \phi_{pol} \in \{0^\circ, 45^\circ, 90^\circ, 135^\circ\} \) are selected for training and testing. It is sufficient to solve the polarization cues with three values of \( \phi_{pol} \), Nevertheless, we use four values to ensure the robustness over noise. The four polarized images are stacked to form a tensor of dimensionality \( 4 \times H \times W \), where \( H \times W \) is the spatial resolution of polarized images. Our motivation is that, since the relative 3D information from polarization is essentially from the intensity difference between polarized images, it is beneficial for convolutional layers to learn this difference by concatenating images along the channel dimension as input. For the normal encoder, we use the identical architecture for the sake of feature map addition. We use ground truth surface normals to supervise the physics-based neural networks with the cosine similarity loss function:

\[
L_{\text{cosine}} = \frac{1}{W \times H} \sum_{i}^{W} \sum_{j}^{H} (1 - \langle \hat{N}_{ij}, N_{ij} \rangle),
\]

where \( \langle \cdot, \cdot \rangle \) denotes the dot product, \( \hat{N}_{ij} \) is the estimated surface normal at pixel location \((i, j)\), and \( N_{ij} \) is the corresponding ground truth of surface normal. This loss is minimized when \( N_{ij} \) and \( \hat{N}_{ij} \) have identical orientation.

### 4 Dataset and Implementation Details

In what follows, we describe the dataset capture and organization as well as software implementation details, including comparison implementations.

#### 4.1 Dataset

To train the physics-based neural network, polarization images with corresponding normal maps are needed. However, neither synthetic nor real datasets for such a purpose are publicly available. We therefore create the first real and synthetic datasets for data-driven SfP as illustrated in Fig. 7.

**Real dataset:** A camera with a layer of polarizers above the photodiodes (Lucid Vision Phoenix polarization camera 2018) is used to capture four polarized images at angles \( 0^\circ, 45^\circ, 90^\circ \) and \( 135^\circ \) in a single shot. Then a structured light based 3D scanner (SHINING 3D scanner 2018) (with single shot accuracy no more than 0.1 mm, point distance from 0.17 mm to 0.2 mm, and a synchronized turntable for automatically registering scanning from multiple viewpoints) is used to obtain high-quality 3D shapes. Our real data capture setup is shown in Fig. 6. The scanned 3D shapes are aligned from the scanner’s coordinate system to the image coordinate system of the polarization camera by using the shape-to-image alignment method adopted in (Shi et al. 2019). Finally, we compute for surface normals of the aligned shapes by using the Mitsuba renderer (Jakob 2010) as ground truth. In total, we capture 65 sets (with 4 polarized images plus a surface normal map) of real data, and we use 58 sets of them for training and the remaining 7 sets for testing and quantitative evaluation.

**Synthetic dataset:** The scanned real data are not sufficient in terms of scale and lighting variation for training a deep neural network. We further create a synthetic dataset to complement the real one. We use the normal maps provided in (Shi et al. 2019), since they cover a great diversity of geometry from a simple sphere to surfaces with highly delicate structures. Given a normal map, we calculate the its diffuse shading by assuming the Lambertian reflectance and a distant environment map (Debevec 2008), as \( I_0, I_{45}, I_{90}, I_{135} \) are calculated using Equation 1. By using 10 different environment maps on 10 different normal maps, we obtain 100 sets of synthetic data, and all these data are used for training.

#### 4.2 Software Implementation

Our model was implemented in PyTorch (Paszke et al. 2017), and trained for 500 epochs with a batch size of 64. It took 8 hours for the network to converge with a single NVIDIA Titan V GPU. We used Adam optimizer (Kingma and Ba 2014) with default parameters \((\beta_1 = 0.9 \text{ and } \beta_2 = 0.999)\), and the base learning rate was set to be 0.01. The learning rate was multiplied with a factor of 0.8 when loss reached the plateau regions during the training process. We tried both He initialization (He et al. 2015) and Xavier initialization (Glorot and Bengio 2010) on the convolutional weights, and the performance was evaluated according to accuracy and training time.
of Xavier initialization is slightly better. For data augmentation, images patches of size 256 × 256 are randomly cropped during training, and a patch is discarded if its foreground ratio is less than 20%. No random rescaling is used to preserve the original high-resolution details and aspect ratio. The final prediction is the average of 32 shifted input to preserve the accuracy at boundaries of each patch.

4.3 Comparisons to Physics-only SfP

We used a test dataset consisting of scenes that include ball, horse, vase, half painted vase, Christmas, flamingo, rabbit. On this test set, we compared performance between our proposed method and three physics-only methods for SfP: 1. (Smith et al. 2016). 2. (Mahmoud et al. 2012). 3. (Atkinson and Hancock 2006; Miyazaki et al. 2003). The first method recovers the depth map directly, and we only use the diffuse model due to the lack of specular reflection masks. The surface normals are obtained from the estimated depth with bicubic fit. Both the first and the second methods require lighting input, and we use the estimated lighting from the first method during comparison. The second method also requires known albedo, and following convention, we assume an uniform albedo of 1. All the comparison codes were provided by Smith et al. (Smith et al. 2016). Source codes of (Smith et al. 2018; Tozza et al. 2017) are not currently publicly available, therefore we are not able to conduct a fair comparison with these two methods.

5 RESULTS

In this section, we evaluate our model with the presented challenging real-world scene benchmark, and compare it against three physics-only methods for SfP. Mean angular error (MAE) is selected as the metric to quantify the accuracy of the estimated surface normals during comparison.

5.1 Machine Learning Alone is Insufficient (Ball Scene)

As illustrated in Figure 2, a naive approach to deep learning that does not blend physics is insufficient. On one of the simplest scenes possible (a white ping-pong ball), the naive neural network cannot recover accurate surface normals. There is only slight difference between images with different polarized angles, and it is difficult for a naive neural net to learn from these differences with limited number of training samples. The proposed method incorporates multiple physical solutions. Therefore, apart from learning from pure polarized images, which is difficult, the network can also learn from physical solutions, which may be easier. Generalizability of the network is thus improved, and it becomes realistic for the network to predict high-quality normals in this case.

5.2 Choice of Loss Function is Important (Vase Scene)

As shown in Figure 9, the choice of loss function affects both the quantitative error and the recovery of qualitative detail. Use of the

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1https://github.com/waps101/depth-from-polarisation
5.3 Improved Performance on Shiny and Detailed Scene (Horse Scene)

Here, we show improved performance on a relatively shiny scene with surface details. As illustrated in Figure 8, the proposed method of physics-based NN achieves the highest qualitative and quantitative accuracy. Worth noting is that, the result from (Smith et al. 2016) does not perform well on the Horse Scene because the simple hybrid reflection model and spherical harmonics based lighting model are not well satisfied for Horse Scene, and the estimated depth becomes inaccurate, which results in a normal map with a large error.

5.4 Improved Performance in Noise-degraded Environments (Vase Scene)

Here, we show that the physics-based NN approach outperforms physics-only approaches when the signal-to-noise level drops. As illustrated in Figure 10, the input to each of the methods are noisy polarization images. This noise was generated in simulation to mimic low light levels (when shot noise dominates). The proposed physics-based NN approach shows a qualitative and quantitative improvement over the physics-only methods. Our proposed approach of using a physics-based neural network works in low noise levels because of the encoder-decoder architecture. Both polarized images and physical solutions will be downsampled into a condensed feature map by the encoder, and the decoder has to use this condensed feature map to recover the normal map. With limited number of parameters, the network has to learn some intrinsic representation of the input, which gives us the robustness over noise.

5.5 Additional Scenes

Over all tested scenes in the paper, the proposed physics-based neural network outperforms physics-only methods from (Mahmoud et al. 2012; Miyazaki et al. 2003; Smith et al. 2016). In particular, Figure 11 shows that the proposed method recovers surface normals

\(\ell_2\) loss function results in an overall smoothened result, while the \(\ell_1\) shows widening of the ridges in the vase. The cosine loss function is closest to the ground truth and is used in all other scenes from the paper. The success of cosine loss may come from its emphasis on the orientation information. Both \(\ell_1\) and \(\ell_2\) loss will penalize the length of estimated surface normals, however, the normalization layer at the end has already constrained the normal length.
Figure 11: The proposed method has the lowest angular error in recovering normal maps. We compare with SfP papers from (Smith et al. 2016), (Mahmoud et al. 2012) and (Miyazaki et al. 2003). Not shown is the performance from (Atkinson and Hancock 2006), which behaves similarly to (Miyazaki et al. 2003).
that are quantitatively and qualitatively closest to ground truth. The large region-wise anomalies on many of the results from (Miyazaki et al. 2003) are to do with the region-growing constraint on the convexity that is imposed. The method of (Mahmoud et al. 2012) uses shading constraints which require a distant light source, which is not the case for tested scenes. Finally, the results in (Smith et al. 2016) are explained both by the use of 4 polarized images as input (ordinarily the method requires 18), as well as change in the lighting direction.

5.6 **SfP Still Fails on Mixed Material Scenes**

This paper, like other SfP methods, is unable to solve the mixed material problem. This problem occurs when the polarimetric signal is not just due to surface geometry, but also material effects. Figure 12 shows one such scene, consisting of a vase painted with two different styles of paint. While the physics-based NN result has the lowest quantitative error, none of the SfP methods are correct. There is a texture copy artifact at the point where the paints change.

6 **DISCUSSION**

In summary, we have presented a first attempt at blending the physics of SfP with deep learning. This blending is very unique because of the uncertain physics inherent to SfP. This special uncertainty in the physics-based prior motivates our use of a novel, multi-stream encoder, as compared to existing deep learning approaches.

In addition, we report a performance improvement over existing methods to solve SfP. However, there are still open problems. We find that existing SfP methods (including this paper) fail on scenes with mixed reflectivity. It would be interesting to study how material properties could be incorporated into the physics-based NN architecture. Part of the solution may also rely on expanding the training dataset, to include a wider variety of object materials and paints. For these types of computational photography problems, where the capture procedure is labor intensive, it is likely that dataset sizes will be small. This underscores the importance of including physical priors in the network model. With this inclusion, we were able to obtain results from a relatively small dataset size.

The lessons learned in this “Deep Shape from Polarization” study may also apply to a future “Deep Polarized 3D” study. The physics-only family of Polarized 3D techniques benefit from robust integration of surface normals with a depth prior. The state-of-the-art Polarized 3D integration has been performed with a simplistic matrix inversion (Kadambi et al. 2015). A physics-based NN approach might be able to learn this elementary function to potentially obtain state-of-the-art results. Overall, this paper’s results appear to validate the direction of jointly studying deep learning and SfP.

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