Correlation Analysis and Reconstruction of the Geometric Evaluation Indicator System of the Discrete Global Grid

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Abstract: Although a Discrete Global Grid (DGG) is uniform in its initial subdivision, its geometric deformation increases with the level of subdivisions. The Goodchild Criteria are often used to evaluate the quality of DGGs. However, some indicators in these criteria are mutually incompatible and overlap. If the criteria are used directly, the evaluation of the DGGs is inaccurate or unreliable. In this paper, we calculated and analyzed the correlation between the evaluation indicators of the DGG and reconstructed a quality evaluation system of DGGs with independent indicators. Firstly, we classified the Goodchild Criteria into quantitative and qualitative indicators. Then, we calculated the correlation among the quantitative indicators and extracted the independent evaluation factors and related weights of the observed values by factor analysis. After eliminating or merging the incompatible and overlapping quantitative indicators and performing a logical reasoning of the qualitative indicators, we reconstructed a comprehensive evaluation system with independent indicators. Finally, taking the Quaternary Triangular Mesh (QTM) model as an example, we verified the independence of the indicators and the feasibility of the evaluation system. The new indicator system ensures the reliability of the evaluation of DGGs in many fields.

Keywords: Discrete Global Grid; Goodchild Criteria; evaluation indicator system; correlation analysis

1. Introduction

A Discrete Global Grid (DGG) is a spatial structure based on a spherical (or ellipsoidal) surface that can be subdivided infinitely without changing shape and simulates the shape of the Earth’s surface [1,2]. The various resolution grids form a hierarchy with a high degree of regularity and strict transformation relationships among the DGGs [3]. This provides a unified expression mode that can fuse unevenly distributed geographical phenomena at different scales [4,5]. DGGs have been widely used in large-scale spatial data management, decision making, and simulation analysis. For example, they have been used in the spatial data organization, indexing, analysis, and visualization [6–9]; global environmental and soil monitoring models [10,11]; atmospheric numerical simulations and visualizations [12]; ocean numerical simulations and visualizations [13]; place names management or gazetteers [14]; big Earth data observations [12,15]; modeling of offset regions around the locations of Internet of Things (IoT) devices [16]; cartographic generalization and pattern recognition for choropleth map-making [17]; the Unmanned Aerial Vehicle (UAV) data integration and sharing [18]; the Modifiable Areal Unit Problem (MAUP) [19], especially in digital Earth [5,16,20–22], and so on.

Most of the DGGs are generated from five regular polyhedrons (tetrahedron, hexahedron, octahedron, dodecahedron, and icosahedron) to limit the effects of non-uniformity of the latitudinal and longitudinal grid and the singularity of the poles. In the first level, the cells of a DGG are uniform, but their geometric deformation increases with the subdivision level [23]. Because of the non-uniformity of DGGs, there is uncertainty in the spatial
data expression, analysis, or decision-making, which greatly restricts their applications to fields with low accuracy such as data indexing, meteorological, and environmental monitoring [24]. However, large-scale and high-precision applications and comprehensive spatial analyses of the DGGS are not available [25]. Therefore, the application of reliable and credible DGGS needs a quality evaluation system or error control model of the deformation of the spherical (or ellipsoidal) grids [26,27]. From the 1990s, many researchers have discussed the geometric deformation and quality evaluation of DGGS. In the early stages, Goodchild (1994) proposed the first set of quality evaluation criteria of DGGS. These were supplemented and modified by other researchers, and an updated and integrated 14-terms list was provided by Kimerling [28], which was called the Goodchild Criteria (Appendix A). The Goodchild Criteria became a standard for quality evaluation of DGGS, and an ideal description of DGGS. These criteria include domain, area, topology, shape, compactness, edge, midpoints, hierarchy, uniqueness, location, center-point spacing, structure, grids transformation, and spatial resolution. Clarke [29] conducted a qualitative evaluation and analysis of the Goodchild Criteria from 10 terms or aspects, including, for example, universality, authority, succinctness, hierarchy, uniqueness, etc. Clarke also pointed out that the Goodchild Criteria concerned primarily with the geometric features rather than the topological features of the DGG.

Recently, researchers have been interested in the quantitative indicators of the Goodchild Criteria, constructing new quantitative indicators to evaluate and analyze the geometric deformation of DGG [30]. Heikes and Randall picked the midpoints from the Goodchild Criteria (no. 7 in Appendix A) and constructed a method to measure the distance between the midpoint of the arc connecting two adjacent cell centers and the midpoint of the edge between the two cells [31]. They improved an atmospheric process model based on the midpoints indicator and concluded that the midpoints were an extension of the concept of DGGS. White made choices of indicators of area and shape from the Goodchild Criteria [32]. The surface area of the cell and its perimeter were measured to calculate the normalized area and the normalized compactness, respectively, to evaluate the deformation of DGGS. Kimerling considered that area, compactness, and center-points spacing (no. 2, 5, and 11 in Appendix A) were the most important indicators to evaluate the deformation of a DGG [28]. The three indicators were measured by cell surface area, zone spherical compactness (ZSC), and the intrinsic distance of the center points of adjacent cells, respectively. These indicators were compared to represent the deformation features of the DGG. Zhao selected indicators of area and edge (no. 2 and 6 in Appendix A) to assess a variation of the Quaternary Triangular Mesh (QTM) [33]. The cell area and its length were measured as observation data of the two indicators above, which uncovered their convergence and spatial distribution features [34]. Ben chose the edge indicator (no. 6 in Appendix A) and compared the advantages and disadvantages of three equal-area spherical hexagonal grid models using observation data of the cell edge length, which would be used to calculate its perimeter, as well as features of convergence and spatial distribution of the edge indicator [35]. Gregory compared the midpoints and center-points spacing (no. 7 and 11 in Appendix A) using the coefficient of variation to describe their performance and relative merits [36]. Tong selected the indicators of area (no. 2 in Appendix A), edge (no. 6 in Appendix A), cell center radius (distance between the center point of a cell and its edge point, not part of the Goodchild Criteria), and cell center angle (angle between the center point of the cell and the two ends of its edge, also not part of the Goodchild Criteria) [37]. Zhang chose indicators of area (no. 2 in Appendix A), compactness (no. 5 in Appendix A), and internal angle (fuzzy similarity of internal cell angle, not part of the Goodchild Criteria) to assess their convergence and spatial distribution of the DGG [38]. Similarly, Sun and Zhou chose area and its measurement method to study the spatial distribution of the DGG [39]. Recently, area (no. 2 in Appendix A) and its measurement were selected by Zhao, and its features of convergence and spatial distribution for QTM based on “latitude loop” were revealed [40].

In summary, most of the indicators selected for the deformation evaluation of the DGG model are from the Goodchild Criteria. The evaluations are mainly based on single
or several indicators separately. A comprehensive evaluation is still missing because the Goodchild criteria include contradictory and repetitive evaluation indicators, and thus no DGG model can meet all the criteria at the same time [28]. Therefore, if all Goodchild Criteria were directly used to evaluate a DGG model’s quality, the information among different evaluation indicators would overlap and may lead to inaccurate or unreliable results.

In this paper, we will:
• eliminate contradictory and duplicate information,
• calculate and analyze the correlations among the Goodchild Criteria, and
• reconstruct a comprehensive and reliable quality evaluation system of DGGs with independent indicators.

The rest of this paper is organized as follows. Section 2 describes the proposed method, classifies the Goodchild Criteria into qualitative and quantitative evaluation indicators, and establishes the correlation model of the quantitative evaluation indicator. Section 3 presents the test and verification of the research method. Taking the QTM model as an example, it tests the research method for one level and applies it to other subdivision levels for comparison and verification. In Section 4, by analysis and integration of qualitative and quantitative indicators, the quality evaluation indicator system of the DGG is reconstructed. Finally, the paper summarizes the research results and contributions.

2. Methods

The Goodchild Criteria can be sorted into two categories: qualitative evaluation indicators and quantitative evaluation indicators. The former describes categorical variables that can be distinguished with a “yes or no” or “with or without”, and are no. 1 (domain), no. 3 (topology), no. 8 (hierarchy), no. 9 (uniqueness), no. 10 (location accuracy), no. 13 (grids transformation), and no. 14 (spatial resolution). The latter can be calculated quantitatively, i.e., no. 2 (area), no. 4 (shape), no. 5 (compactness), no. 6 (edge), no. 7 (midpoints), no. 11 (center-point spacing), and no. 12 (structure).

The correlations among the quantitative indicators were calculated by factor analysis. Indicators with incompatible and overlapping information were eliminated or merged. The internal dependency relationship was then obtained by naming and interpreting the indicator with expert knowledge, and independent factors were extracted to construct a revised evaluation indicator system. The process is shown in Figure 1, and the steps are as follows:
• The Goodchild Criteria Sorting. The Goodchild Criteria are assigned into two groups: qualitative evaluation indicators and quantitative evaluation indicators. We classify the indicators according to their nature or properties.
• Quantitative Evaluation Indicators. These indicators can be measured and calculated by the formulae of Section 2.1. As a result, indicators correlation matrix is created. With professional knowledge, common factors and their loadings will be extracted from the matrix.
• Elimination and Merging of Indicator Correlation. Qualitative Indicators will be analyzed in Section 4. After quantitative indicators calculation and qualitative indicators analysis, we eliminate and merge the relevant indicators. This process needs to be combined with specific applications.
• Independent Evaluation Indicator Set. The independent indicators set is formed here. From the set, we select single- or multi-indicator to evaluate DGGs.
• Comprehensive Evaluation Model for DGGS. Considering the indicators set, weight of indicators, and other factors, we will construct a comprehensive evaluation model for DGGs.
2.1. Measurement of Quantitative Indicators

The measurement methods of the quantitative indicators were designed first. The coordinates of \( n \) vertices of the grid cell are denoted as \( p_1(x_1, y_1, z_1), \cdots, p_n(x_n, y_n, z_n) \). In spherical coordinates, a point is represented as \( p(\theta, \phi) \) in radians (see Figure 2), where \( \theta \) is latitude \((-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2})\) and \( \phi \) is longitude \((-\pi < \phi \leq \pi)\). Conversion from the
coordinates \( p(\theta, \varphi) \) to the Cartesian coordinates \( p(x, y, z) \) of any point on the Earth were calculated as Formula (1) \[41\]

\[
\begin{align*}
    x &= r \cos \theta \cos \varphi, \\
    y &= r \cos \theta \sin \varphi, \\
    z &= r \sin \theta
\end{align*}
\]  

(1)

where, \( r \) is the radius of the Earth. The values of \( \theta \) and \( \varphi \), if needed, can be calculated from the inverse of Formula (1). Because the domain of \( \tan^{-1} \) is \((-\pi/2, \pi/2)\), the value of \( \varphi \) depends on not only \( \tan^{-1} \) but also where \( x \) and \( y \) are.

![Diagram of a point \( p \) in spherical coordinates (longitude/latitude) and its Cartesian coordinates. Let \( O \) be the sphere center with radius \( r \), and \( p \) be a point on the spherical surface. Its latitude is \( \theta \) and longitude is \( \varphi \). Let \( p' \) be the projection of the point \( p \) on the Equator. In right triangle \( \Delta Op'p \), there is \( Op' = r \times \cos \theta \) and \( z = r \times \sin \theta \). Therefore, we get the formulae \( x = Op' \times \cos \varphi = r \times \cos \theta \times \cos \varphi \) and \( y = Op' \times \sin \varphi = r \times \cos \theta \times \sin \varphi \).](image)

Figure 2. Diagram of a point \( p \) in spherical coordinates (longitude/latitude) and its Cartesian coordinates. Let \( O \) be the sphere center with radius \( r \), and \( p \) be a point on the spherical surface. Its latitude is \( \theta \) and longitude is \( \varphi \). Let \( p' \) be the projection of the point \( p \) on the Equator. In right triangle \( \Delta Op'p \), there is \( Op' = r \times \cos \theta \) and \( z = r \times \sin \theta \). Therefore, we get the formulae \( x = Op' \times \cos \varphi = r \times \cos \theta \times \cos \varphi \) and \( y = Op' \times \sin \varphi = r \times \cos \theta \times \sin \varphi \).

Area of the actual region represented by a grid cell is denoted by \( s \). The calculation was based on the shape of the DGG and the properties of the grid cell edge.

Shape of the grid cell was measured by the Landscape Shape Index. This term comes from landscape ecology and is an indicator that expresses the shape characteristics of the patches in the whole landscape \[42\]. The Landscape Shape Index represents the degree of deviation between the shape of patches and a regular geometric figure or polygon whose area is equal to the patches, such as a square or a circle, and represents the shape complexity of the grid cells instead of patches. The formula of the Landscape Shape Index \( SI \) \[42\] is:

\[
SI = \frac{\omega \sqrt{s}}{P}
\]  

(2)

where, \( s \) is the cell area and \( P \) is its perimeter. Let \( SI \) be 1.0, and \( \omega \) can be calculated. If the regular polygon is a spherical triangle, the triangular landscape shape is used and the
cell is compared with a regular spherical triangle of the same area. Given an equilateral triangle with its side length $l$, its area $s = \frac{\sqrt{3}}{4} l^2$, and its perimeter $P = 3l$. And $\omega$ calculated by Formula (2) is 4.559. Similarly, if the regular polygon is a square with its length $l$ (see Figure 3), the cell is compared with a regular spherical quadrilateral of the same area and $\omega = 4$, and if the regular shape is a circle, the cell is compared with a spherical circle with the same area and $\omega = 3.545$.

![Figure 3](image-url) Diagram of the Landscape Shape Index of a square. The compared regular polygon is a square with the length $l$, its area $s = l^2$, and $P = 4l$. For regular polygon, there is $SI = 1.0$, the parameter $\omega$ will be calculated by Formula (2). The result is $\omega = 4$.

Compactness of the grid cell is the ratio of the surface area of the cell to its circumference. In this paper, compactness $C$ was calculated as the Zone Standardized Compactness (ZSC) [28]. ZSC is given by zone_perimeter/cell_perimeter. Let the surface area of a spherical zone be equal to the zone surface area (see Figure 4), so that the latitude of the bounding parallel is obtained. The perimeter of spherical zone which is the circumference of the bounding parallel can be computed. Substituting and combing equations, we compute $C$ from:

$$C = \frac{\sqrt{4\pi s - s^2/r^2}}{P},$$

where, $s$ is the cell area, $P$ is its perimeter, and $r$ is the radius of the Earth.

Edge, the length of the cell side, is denoted by $e$. Depending on the nature of the grid, the cell sides may be arcs of great circles, small arcs, or even arbitrary curves. If the boundary of a cell is an arc of a great circle, the actual length of the edge is calculated according to Formula (4); if it is a small or a parallel arc, Formula (5) is used. The two endpoints of the cell side are denoted as $p_1(\theta_1, \varphi_1)$ and $p_2(\theta_2, \varphi_2)$ (see Figure 5). If the edge is an arc of a great circle, the length (denoted by $e_{SC}$) is the intrinsic distance [43] between its two endpoints.

$$e_{SC} = 2 \sin^{-1} \frac{\sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}}{2r} r,$$

where, $r$ is the radius of the Earth, and

$$\Delta X = r(\cos \varphi_2 \cos \theta_2 - \cos \varphi_1 \cos \theta_1),$$
$$\Delta Y = r(\cos \varphi_2 \sin \theta_2 - \cos \varphi_1 \sin \theta_1),$$
$$\Delta Z = r(\sin \varphi_2 - \sin \varphi_1)$$
The range of $e_{sc}$ varies according to the scale or subdivision level of the DGG. If the edge is a small arc or a parallel arc, the length (denoted by $e_{sc}$) is:

$$e_{sc} = r|\varphi_2 - \varphi_1| \cos \theta,$$

where, $r$ is the radius of the Earth, and

$$\theta_1 = \theta_2 = \theta$$

Identically, the range of $e_{sc}$ varies according to the scale or subdivision level of the DGG.

![Figure 4](image_url)

**Figure 4.** Diagram of the Zone Standardized Compactness (ZSC) calculation for an octahedron face. The dealt cells are on the globe, and we take the Earth’s sphericity into account. A pole-centered spherical zone of the same area is used as the grid cell for ideal compactness calculation. Let $r$ be the radius of the sphere and $\theta$ be the latitude of the bounding parallel. The surface area of a spherical zone is computed from $Z_{area} = 2\pi r^2 (1 - \sin \theta)$. Let the zone surface area equal to the cell area, and we obtain $C_{\text{ellarea}} = 2\pi r^2 (1 - \sin \theta)$, and $\cos \theta = \sqrt{1 - (1 - C_{\text{ellarea}}/(2\pi r^2))^2}$. The perimeter of the spherical zone is the circumference of the bounding parallel, computed from:

$$Z_{\text{perimeter}} = 2\pi r \cos \theta.$$ Hence, we get $ZSC = \sqrt{4\pi r \times C_{\text{ellarea}} - (\text{cellarea})^2} / C_{\text{ellperimeter}}$. Therefore, Formula (3) is:

$$C = \sqrt{4\pi r \times C_{\text{ellarea}} - (\text{cellarea})^2} / C_{\text{ellperimeter}}.$$

Midpoints are the center-point of a cell boundary [36] and the center-point of the edge between the two cells (no. 7 of Goodchild Criteria). The two cells involved in the indicator must be two adjacent cells, i.e., the faces where the two cells are located intersect along the side; otherwise, the indicator is invalid. This indicator calculates the geodesic distance between the two midpoints, i.e., the intrinsic distance or great circle distance between the two midpoints, denoted by $d_{gm}$, and the calculation formula is shown in Formula (4).
Figure 5. Diagram of the length calculation of an arc. Let O be the sphere center with radius r, and 
P_1, P_2 be two arbitrary points on the arc. Its center angle is \( \beta \), which is an acute angle, and half angle is \( \frac{\beta}{2} \). ON is the angular bisector of the center angle, and N is on the arc. Given the length of \( P_1P_2 \) is \( \beta \times r \), we will calculate \( \frac{\beta}{2} \) in the right triangle \( \triangle OMP_2 \). There is \( \sin \frac{\beta}{2} = \frac{OM_{P_2}}{P_1P_2} = \frac{\sqrt{AX^2 + \Delta Y^2 + \Delta Z^2}}{2r} \), and hence \( \beta = 2 \sin^{-1} \frac{\sqrt{AX^2 + \Delta Y^2 + \Delta Z^2}}{2r} \times r \). Therefore, the length of \( P_1P_2 \) is \( 2 \sin^{-1} \frac{\sqrt{AX^2 + \Delta Y^2 + \Delta Z^2}}{2r} \times r \).

Center-point Spacing, the distance between the center points of adjacent grid cells, includes the distance between cells adjacent by edges and the distance between cells adjacent by angles and is represented by the symbols \( d_e \) and \( d_a \), respectively. The measurement of the adjacent distance indicator is the length of the geodesic between the reference points of adjacent cells, i.e., the intrinsic distance or great circle distance between two reference points. The calculation method is shown in Formula (4). In this paper, the center point of the cell is taken as the reference point, and the center point is taken as the spherical mean of the cell [41], so the distance between adjacent cells is the length of the geodesic between the sphere medians of two adjacent cells.

The spherical mean is a point on the sphere surface, and its location is calculated by Formula (6) [41]. Let the spherical mean of a cell be \( p_c(x_c, y_c, z_c) \), and the vertex coordinates of the cell be \( p_i(x_i, y_i, z_i) \). \( p_c \) is shown in Formula (6):

\[
p_c = \left( \frac{\Sigma x_i}{rl}, \frac{\Sigma y_i}{rl}, \frac{\Sigma z_i}{rl} \right)^	ext{T}
\]

where, \( rl = \sqrt{\Sigma x_i^2 + \Sigma y_i^2 + \Sigma z_i^2} \).

Structure, whether it is regular or arbitrary, is an important attribute of the grid model, and it is the basis for obtaining geocoding and run efficient algorithms of the grid [28]. Because regularity is not easy to quantify, the irregularity indicator used to measure patches in landscape ecology is introduced, i.e., the fractal dimension \( f \) [44]. In normal circumstances, there would be \( \ln s = \frac{2}{f} \ln P + \text{constant} \), and \( f \in [1, 2] \). For the sake of simplicity, let the parameter constant be 0.0. Therefore, the fractal dimension \( f \) is:

\[
f = 2 \frac{\ln P}{\ln s}
\]

where, \( s \) is the cell area and \( P \) is the perimeter of the cell.
2.2. Correlation and Evaluation System of Indicators

We calculated the correlation of the evaluation indicators with observation data. In any layer of the DGG, the number of grid cells is \( n \) and the number of evaluation indicators is \( p \). According to the evaluation indicator calculation formula, \( n \) groups of observation data are obtained, and each group of observation data is denoted by \( X = (X_1, X_2, \cdots, X_p) \).

Standardization of the original observation data \( X \). Due to the different nature of the evaluation indicators, the magnitude or dimension of the obtained observation data is different. Therefore, to ensure the reliability of the results, we standardized the original observation data to eliminate the influence of the order of magnitude or dimension. The standardized observed value of the indicator \([45]\) is calculated as:

\[
x_i = \frac{X_i - \min_{1 \leq j \leq p} \{ X_j \}}{\max_{1 \leq j \leq p} \{ X_j \} - \min_{1 \leq j \leq p} \{ X_j \}},
\]

(8)

Correlation matrix \( R \). For the standardized observation data, we calculated the correlation matrix \([45]\) among the indicator variables \( R = (R_{ij})_{p \times p} \):

\[
R_{ij} = \frac{\sum_{a=1}^{p} (x_{ai} - \bar{x}_i)(x_{aj} - \bar{x}_j)}{\sqrt{\sum_{a=1}^{p} (x_{ai} - \bar{x}_i)^2} \sqrt{\sum_{a=1}^{p} (x_{aj} - \bar{x}_j)^2}}.
\]

(9)

Eigenvalue \( \lambda \) and the cumulative contribution rate \( \tau_k \) of the correlation matrix \( R \). The eigenvalue \( \lambda \) of matrix \( R \) can be obtained by matrix transformation. According to the cumulative contribution rate, the first \( k \) \( (k < p) \) principal components were used to express the information of the model. The cumulative contribution rate \([45]\) of the first \( k \) principal components was calculated as:

\[
\tau_k = \frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{p} \lambda_i},
\]

(10)

Factor loading matrix \( \mu \). In the cumulative contribution rate \( \tau_k \) of the eigenvalues, if the cumulative contribution of the first \( k \) principal components reaches \( 85\% \), i.e., \( \tau_k \geq 85\% \), it indicates that the first \( k \) principal components contain most (i.e., \( 85\% \)) of the information of all the measurement indicators. Taking the first \( k \) principal components as independent factors, we built the factor loading matrix \( \mu \), and the correlation matrix \( R \) is simplified to Formula (11) \([45]\). Clearly, a factor loading \( \mu_{ij} \) can be obtained by \( R_{ij} \) divided by the square root of the corresponding eigenvalue (i.e., \( \sqrt{\lambda_i} \)):

\[
R = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1k} \\ R_{21} & R_{22} & \cdots & R_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ R_{p1} & R_{p2} & \cdots & R_{pk} \end{bmatrix} = \begin{bmatrix} 1 \sqrt{\lambda_1} & \mu_{12} \sqrt{\lambda_2} & \cdots & \mu_{1k} \sqrt{\lambda_k} \\ 1 \sqrt{\lambda_1} & \mu_{22} \sqrt{\lambda_2} & \cdots & \mu_{2k} \sqrt{\lambda_k} \\ \vdots & \vdots & \ddots & \vdots \\ 1 \sqrt{\lambda_1} & \mu_{p2} \sqrt{\lambda_2} & \cdots & \mu_{pk} \sqrt{\lambda_k} \end{bmatrix}
\]

(11)

where, \( 0 < i \leq k \) and \( 0 < j \leq p \). Therefore, factor loading matrix \( \mu \) is obtained.

Indicator analysis and classification. The quantitative indicators were classified according to their correlation. Each indicator was assigned to \( k \) common factors (\( k \) categories) based on their absolute value in the factor loading matrix \( u \). The maximum variance method was used to rotate the factor loading matrix \( u \) so that the matrix had a compact structure and the common factors more clearly expressed the information of the relevant indicators. The common factor loading coefficient may be represented by the factor weight assigned based on expert knowledge.

Restructuring of the evaluation indicator system. The contradictory and repetitive components of the evaluation indicators were eliminated using expert knowledge and the factor naming and interpretation, and merged. In this way, the number of indicators was
decreased to form a new independent and comprehensive evaluation system that served the applications of DGGs.

3. Results
3.1. Selection of the DGG Model

This paper selected a typical DGG model—QTM [33] (see Figure 6)—as an example to study the correlation of the quantitative indicators of the Goodchild Criteria. Because the spherical triangle is the most basic grid unit of many global discrete grid systems, it is simple and can be combined into other shapes of grid systems such as spherical diamonds, pentagons, and hexagons, depending on the needs [22,46]. QTM is a spherical triangular grid structure based on an octahedron. In the initial partition, its vertices occupy the main points of the sphere (including the poles), while the edges coincide with the equator, the Prime Meridian, and/or the 90° or 180° or 270° Meridians. It is easy to determine which face of the spherical octahedron a point is on, and it is easy to transform the coordinates with the latitude and longitude grid [33]. The points are obtained using the Formula (1).

![Figure 6. Schematic of the Quaternary Triangular Mesh (QTM). The solid white lines represent grid cells. The subdivision of the Earth results in denser grid cells, where 2a, 2b, and 2c are the first, second, and third levels of the initial grid of spherical octahedron, respectively. The fourth level (2d) is of an initial surface of a spherical octahedron (i.e., a spherical octahedron face).](image-url)
Due to the global symmetry of the QTM grid model, one of the initial faces of the QTM grid model, i.e., one-eighth sphere surface, was selected as the test area (Figure 6d). The test area was a surface of a spherical octant composed of three spherical points, and their latitude and longitude coordinates were (0°, 0°), (0°, 90°), (90°, 0°), respectively.

Taking the sixth-level grid cells of QTM as an example, the observation data of quantitative indicators in Appendix A were calculated. Here, the hardware and software environments are listed. The processor is Intel (R) Core (TM) i7-3610QM and CPU @2.30 GHz; and RAM is 8.00GB. The OS is Windows 7 64-bit. Mathematical software is IBM SPSS Statistics 23 (Trial) and JASP (Version 0.14.1).

3.2. Calculation Results

Correlation coefficient matrix of the observation data of the indicators. Due to a large amount of observation data of the indicator variables (a total of 4096 groups in the test area), only the statistics calculated in the first step are shown in Table 1. The criteria are calculated with the Formulas (2)–(7).

Table 1. Descriptive statistics of the indicator variable.

| Criterion(Unit) | Mean     | Standard Deviation |
|-----------------|----------|--------------------|
| s(km²)          | 15,326.046| 2634.076           |
| SI              | 0.636    | 0.013              |
| C               | 0.752    | 0.015              |
| e(km)           | 193.838  | 14.962             |
| dm(m)           | 0.276    | 0.043              |
| de(km)          | 204.693  | 15.774             |
| da(km)          | 112.384  | 9.252              |
| f               | 1.132    | 0.002              |

Solving the correlation coefficient matrix $R$. The correlation matrix $R$ of the evaluation indicator is calculated with Formula (9), and $R$ is symmetric, so only the lower triangular matrix is reported (Table 2). The absolute value of the correlation coefficient of some evaluation indicators was greater than 0.7, suggesting strong correlation and information overlap, indicating the need to merge relevant information.

Table 2. Correlation matrix ($R$) of the indicator variables.

|     | s   | SI  | C   | e   | dm  | de  | da  | f   |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| s   | 1.000 |     |     |     |     |     |     |     |
| SI  | 0.436 | 1.000 |     |     |     |     |     |     |
| C   | 0.436 | 1.000 | 1.000 |     |     |     |     |     |
| e   | 0.968 | 0.199 | 0.199 | 1.000 |     |     |     |     |
| dm  | 0.685 | 0.225 | 0.225 | 0.683 | 1.000 |     |     |     |
| de  | 0.921 | 0.223 | 0.223 | 0.945 | 0.719 | 1.000 |     |     |
| da  | 0.901 | 0.245 | 0.245 | 0.916 | 0.760 | 0.959 | 1.000 |     |
| f   | -0.470 | -0.999 | -0.999 | -0.237 | -0.246 | -0.258 | -0.278 | 1.000 |

Solving the eigenvalue $\lambda$ and cumulative contribution rate $\tau_k$ of the coefficient matrix $R$. $\tau_k$ is calculated with Formula (10). The initial eigenvalues of the first two components were 4.986 and 2.454 (Table 3), which, being greater than 1, indicated that the amount of information represented by the component exceeded one evaluation indicator; the initial eigenvalue of the other components, being less than 1, indicated that the amount of information represented by them was less than that of one evaluation indicator. The cumulative contribution rate $\tau_k$ of the first two variances reached 93.004%, which, being greater than 85%, indicated that the first two components contained nearly all the information of the evaluation indicators.
Table 3. Eigenvalues of the correlation matrix.

| CMPT | Initial Eigenvalues | Sum of Squares of the Loading | Rotated Sum of Squares of the Loading |
|------|---------------------|-------------------------------|--------------------------------------|
|      | TOT                 | Proportion | ACUM | TOT | Proportion | ACUM | TOT | Proportion | ACUM |
| 1    | 4.986               | 0.623      | 0.623| 4.986| 0.623      | 0.623| 4.438| 0.555      | 0.555|
| 2    | 2.454               | 0.307      | 0.930| 2.454| 0.307      | 0.930| 3.003| 0.375      | 0.930|
| 3    | 0.417               | 0.521      | 0.982| 0.417| 0.521      | 0.982|      |            |      |
| 4    | 0.106               | 0.133      | 0.995| 0.106| 0.133      | 0.995|      |            |      |
| 5    | 0.035               | 0.433      | 0.9998| 0.035| 0.433      | 0.9998|      |            |      |
| 6    | 0.001               | 0.013      | 0.9999| 0.001| 0.013      | 0.9999|      |            |      |
| 7    | 0.000               | 0.006      | 1.000| 0.000| 0.006      | 1.000|      |            |      |
| 8    | 0.000               | 0.000      | 1.000| 0.000| 0.000      | 1.000|      |            |      |

The scree plot shows a steep slope at the beginning of the curve, corresponding to the first two components (Figure 7), which play a strong role in describing the original information; the end of the curve flattens, indicating that the effect of the other components is weak. Therefore, we selected the first two eigenvalues to evaluate the common factors.

Figure 7. Scree plot.

Calculation of the factor loading matrix $\mu$. The loading matrix of the first two principal components is calculated with Formula (11), (see Table 4). To highlight the role of the common factors, we took the directions of the two principal components as the axis and performed the rotation of the variance maximization factor to obtain the rotated component matrix (Table 5). The spatial relationship between the rotated factors and the two principal component axes is shown in Figure 8.

Table 4. Component loading matrix.

| Variable | Factor1 | Factor2 |
|----------|---------|---------|
| $s$      | 0.958   |         |
| $SI$     | 0.607   | -0.794  |
| $C$      | 0.607   | -0.794  |
| $e$      | 0.918   |         |
| $d_m$    | 0.753   |         |
| $d_c$    | 0.889   | 0.400   |
| $d_o$    | 0.895   | 0.378   |
| $f$      | -0.636  | 0.772   |
Table 5. Rotated component matrix. Note. Applied rotation method is varimax.

| Variable | Factor1 | Factor2 |
|----------|---------|---------|
| s        | 0.922   |         |
| SI       | 0.991   |         |
| C        | 0.991   |         |
| e        | 0.984   |         |
| dm       | 0.802   |         |
| dc       | 0.959   |         |
| ds       | 0.966   |         |
| f        |         | -0.986  |

Figure 8. Rotated factor loading plot of level 6 in space.

The information coverage of the common factors to the evaluation indicator is very high, with the lowest absolute value being 0.802 and the highest being 0.991 (Table 5). It can be seen from Figure 8 that the evaluation indicators are distributed at both ends of the common factor axes. When the evaluation indicator is close to the horizontal axis, its coordinate value on the vertical axis is close to 0; when the evaluation indicator is close to the vertical axis, its coordinate value on the horizontal axis is close to 0.

Indicators allocation and interpretation. The evaluation indicators are related to two independent common factors (Table 6 and Figure 8). Common factor 1 mainly covers four indicators and five variables (area (s), edge (e), midpoints (dm), center-point spacing (dc, ds)), which are all related to the cell area and thus can be called the Area Factor. Common factor 2 mainly covers three indicators and variables (shape (SI), compactness (C), and structure (f)), which are all related to the shape of the grid cell, and thus can be called the Shape Factor. The loadings of Area Factor and Shape Factor to the indicator variables are shown in Figure 9.

Table 6. Reconstruction of the Goodchild Criteria.

| No. | Criterion   | Description                              | Recommended Method | Goodchild Criteria |
|-----|-------------|------------------------------------------|--------------------|--------------------|
| 1   | Domain      | Tiling the globe, completeness           | Octant             | 1                  |
| 2   | Conversion  | Following subdivision rule, consistency  | QTM rules          | 13                 |
| 3   | Hierarchy   | Infinite hierarchy without redundancy     | Recursion          | 8, 14              |
| 4   | Location    | Central point, accuracy                   | Spherical Mean     | 9, 10              |
| 5   | Area        | Area or edge or midpoints or spacing     | Formula (4)        | 2, 6, 7, 11        |
| 6   | Shape       | Shape or compactness or structure         | Formula (3)        | 3, 4, 5, 12        |
Figure 9. Factor loading diagram. The x-axis shows the components, the y-axis is the loading of the common factor to the component, and the labels on the histogram are the corresponding loading values.

Testing and verification. To measure of the fourth-, fifth-, seventh-, and eighth-level of cells of the QTM model, we calculated the evaluation indicator of the cells, and obtained the information coverage of the common factors at each level (Figure 10). The common factors extracted from the evaluation indicator variables have basically the same coverage of the principal components in Figure 9.

Figure 10. Information coverage of the common factors of the indicator variables.

Clustering effect of the evaluation indicators. Figure 11 shows the spatial correlation between the evaluation indicator and the two common factors. The horizontal and vertical coordinates represent the correlation between the evaluation indicator and common
factors 1 and 2, respectively. The stronger the correlation, the closer the absolute value of the coordinates between the indicators. The evaluation indicator near factor 1 is approximately equal to 0 in factor 2, and the evaluation indicator near factor 2 is approximately equal to 0 in factor 1. The clustering characteristics of the indicators in Figure 11 are similar to those in Figure 8.

Figure 11. Cont.
An initial surface of the QTM (the octant) was selected as the experimental area, and deformation characteristics of the quantitative indicators of the Goodchild Criteria and the common factors extracted in this paper were calculated and normalized.

Figure 12 shows the Area Factor evaluation results (area, edge, midpoints, the center-point spacing indicator with an adjacent edge, and center-point spacing indicator with adjacent vertices). The structure of the distribution diagram of each indicator is similar, but the distribution of the values is different, indicating different degrees of correlation.

Figure 11. Factor pattern plot in rotated space.
An initial surface of the QTM (the octant) was selected as the experimental area, and deformation characteristics of the quantitative indicators of the Goodchild Criteria and the common factors extracted in this paper were calculated and normalized. Figure 12 shows the Area Factor evaluation results (area, edge, midpoints, the center-point spacing indicator with an adjacent edge, and center-point spacing indicator with adjacent vertices). The structure of the distribution diagram of each indicator is similar, but the distribution of the values is different, indicating different degrees of correlation.

(a) Area evaluation

(b) Edge evaluation

Figure 12. Cont.
Figure 12. Evaluation of Area Factor and its component indicator. The horizontal coordinate is the longitude (0° ~ 90°), and the vertical coordinate is the latitude (0° ~ 90°).

Figure 13 evaluates the shape factors (shape, compactness, and structure indicator). The structure and the values of the distribution diagrams of the two indicators in Figure

(d) Spacing evaluation (edge)

Figure 12. Cont.
Figure 12. Evaluation of Area Factor and its component indicator. The horizontal coordinate is the longitude (0°~90°), and the vertical coordinate is the latitude (0°~90°).

Figure 13 evaluates the shape factors (shape, compactness, and structure indicator). The structure and the values of the distribution diagrams of the two indicators in Figure 13a,b are similar, suggesting that each indicator has a very strong correlation. Figure 13c and the previous two figures are mutually exclusive in structure and have a negative correlation.

If the two common factors extracted in this paper are combined to evaluate the grid cell together, then a comprehensive evaluation model of multi-factor of the grid cell is constructed. The factor weight is the ratio of the proportion of the rotated sum of squares of the loading to the accumulation of the rotated sum of squares of the loading, and the comprehensive score of the grid cell is calculated as the evaluation results as:

\[
\text{score} = 0.597 \times \text{AreaFactor} + 0.403 \times \text{ShapeFactor},
\]

This paper calculated the comprehensive scores of each grid cell in the above-mentioned experimental area as the evaluation results of the comprehensive evaluation model of DGG. Figure 14 is the effect diagram of the comprehensive factor evaluation model. Figure 14a is the Area Factor evaluation result, Figure 14b is the Shape Factor evaluation result, and Figure 14c shows the comprehensive evaluation result of the Area Factor and the Shape Factor. Figure 14a is like Figure 12, Figure 14b is like Figure 13, and Figure 14c resembles the comprehensive result of Figure 14a,b. The graphic structure conforms to the correlation.
Figure 13. (a) Shape evaluation (b) Compactness evaluation

Figure 13. Cont.
Figure 13. Component indicator evaluation of shape factor. The horizontal coordinate is the longitude (0° ~90°), and the vertical coordinate is the latitude (0° ~90°).

This paper calculated the comprehensive scores of each grid cell in the above-mentioned experimental area as the evaluation results of the comprehensive evaluation model of DGG. Figure 14 is the effect diagram of the comprehensive factor evaluation model. Figure 14a is the Area Factor evaluation result, Figure 14b is the Shape Factor evaluation result, and Figure 14c shows the comprehensive evaluation result of the Area Factor and the Shape Factor. Figure 14a resembles Figure 12, Figure 14b resembles Figure 13, and Figure 14c resembles the comprehensive result of Figures 14a and 14b. The graphic structure conforms to the correlation.
Figure 14. Comprehensive evaluation of grid deformation. The horizontal coordinate is the longitude (0°~90°), and the vertical coordinate is the latitude (0°~90°).
4. Analysis and Discussion

4.1. Qualitative Analysis

The qualitative indicators of the Goodchild Criteria include no. 1, 3, 8, 9, 10, 13, and 14. The details are as follows:

No. 1 is summarized as a spatial domain and indicates whether the grid coverage is the Globe. This indicator does not only clarify the basic research object but also defines the scope of the grid model. This study retains this indicator.

No. 3 is summarized as topology and indicates whether the grid maintains topological consistency. This indicator only requires the relative positional relationship between objects to remain unchanged, but not their shape and size, i.e., the relative positional relationship and number of points, lines, and surfaces remain unchanged. If the shape of the grid cell does not change, then the relationship and number of cell vertices and cell edges must be the same, i.e., the topological relationship among points, lines, and surfaces does not change. If the topological relationship is unchanged, the consistency of shape and size cannot be guaranteed. Therefore, a shape that remains unchanged is a sufficient and unnecessary condition for the topology to remain unchanged. This means that indicator 4 is a sufficient and unnecessary condition of indicator 3. Indicator 3 can be derived from indicator 4. This study merges indicator 3 with 4.

No. 8 is summarized as hierarchy, indicating whether the grid is hierarchical. The hierarchy indicator has two meanings. First, the indicator requires the grid to have a hierarchical relationship, i.e., the grid can be subdivided into different levels. Secondly, the hierarchy requires that the bottom level or sub-grid can form a top-level or the parent grid, i.e., the combination of sub-grid cells overlaps with the parent grid. The hierarchy determines that the grid can be subdivided into more detailed grid cells. This article retains this indicator.

No. 9 is summarized as uniqueness and indicates whether the reference point of the cell is unique. This indicator is universal for all DGGs. The reference point constitutes a point grid, which is located on the surface of the cell and can directly participate in calculations. Whether the reference point is unique is related to the value of the reference point.

No. 10 is summarized as cell positioning and indicates whether the reference point is closest to the center of the cell. The center of the cell is unique. Once the center point is taken as the reference point, the reference point of the grid is unique. If another point is selected as the reference point, the reference point must not be the point closest to the center of the grid cell. Therefore, indicator 10 is a sufficient and unnecessary condition for indicator 9. Accordingly, this article takes the central point as the reference point and merges indicator 9 with 10. The DGG model is a surface model. According to its definition, the Spherical Mean is the center point of the spherical polygon [41], which is also the point of the spherical polygon on the sphere. The Spherical Mean of the sphere as a reference point satisfies both indicator 9 and 10. Therefore, this article takes the Spherical Mean of the cell as the center (i.e., reference) point.

No. 13 is summarized as transformation, i.e., whether the coordinate of the grid can be simply transformed into a longitude–latitude grid. This indicator requires a simple geometric relationship between the grid model cell and the square-like longitude–latitude grid cell. Generally, if the grid model conforms to specific subdivision rules, there must be a geometric relationship between the grid model and the latitude–longitude grid model. Particularly, it is obvious that if the edges of the grid cell and the graticule have the same properties, the transformation must be easy. This indicator belongs to the application category of the grid model, i.e., the grid model has the function of interoperating with the longitude–latitude grid. This indicator is an extension of the concept of the grid model, so this article retains this indicator.

No. 14 is summarized as spatial resolution, i.e., whether the grid has an infinite spatial resolution. This indicator requires that the grid can be infinitely subdivided in the current space. Indicator 8 requires that the grid can be subdivided spatially and does not limit the degree of subdivision. That means that indicator 8 can achieve the requirement of indicator
14 infinite subdivision in space. Under the premise of indicator 1, indicator 8 is a sufficient condition for indicator 14. Therefore, indicators 1 and 8 can replace indicator 14. This article merges indicator 14 with indicators 1 and 8, instead of listing indicator 14 separately.

In summary, from the above analysis, four qualitative indicators of the Goodchild Criteria are extracted as reconstruction indicators or factors, namely domain (no. 1), hierarchy (no. 8), location (no. 10), and grids transformation (no. 13).

4.2. Quantitative Analysis

For the quantitative evaluation of the indicators of the Goodchild Criteria, this paper finds through experiments that when solving the eigenvalues of the relationship matrix, the cumulative contribution value of the two components reaches 93.004% (i.e., greater than 85%). Therefore, it can be considered that the information expressed by the two evaluation factors is enough to evaluate the DGG model, and the loading of each evaluation indicator on the two factors can be calculated. From the loading matrix (Table 4 and Figure 8), it can be found that the common factor 1 mainly covers four indicators and five variables such as area \( (s) \), edge \( (e) \), midpoints \( (d_m) \), center-point spacing \( (d_e, d_d) \). These indicator variables are all related to the area of the cell, and the common factor 1 can be called the Area Factor. Common factor 2 mainly covers three indicators and variables such as shape \( (SI) \), compactness \( (C) \), and structure \( (f) \). These variables are all related to the outline or shape of the grid cell, and the common factor 2 can be called the Shape Factor. The specific analysis is as follows:

Correlation analysis of Area Factor. The evaluation indicators of the Area Factor are all positive contributions to the factor (Figure 9). The loadings on the Area Factor such as indicators of area, edge, midpoints, and center-point spacing are 0.920, 0.967, 0.802, 0.971, and 0.964, respectively. These loadings all reach more than 90%, which is an important indicator of the Area Factor. In addition, the loading of the midpoints indicator on Area Factor is 0.802, which is also more than 80%. According to the geometric properties of the spherical surface, the calculation formula for the area of the spherical triangle is \((\sum A_i - \pi)r^2\), so the area of the spherical triangle has a strong positive correlation with its inner angle. The ratio of the sine of the inner angle to the sine of the corresponding side in a spherical triangle is constant, i.e., \(\frac{\sin A}{\sin R} = \frac{\sin B}{\sin \alpha} = \frac{\sin C}{\sin \beta}\). This shows that the side length of a spherical triangle has a positive correlation with its inner angle, so the side length and area also have a strong positive correlation. Therefore, within the Area Factor, the indicators of grid edge, spacing, and midpoints all have a strong positive correlation with the cell area indicator.

Correlation analysis of Shape Factor. The evaluation indicators of the Shape Factor are indicators of compactness, shape, and structure, and their loadings are 0.991, 0.991, and −0.986, respectively. In terms of absolute value, the loadings of three indicators on the Shape Factor are all more than 90%, and they are important indicators of the Shape Factor. The loadings of the compactness indicator and the shape indicator are positive while the loading of the structure indicator is negative, which shows that their contributions to the Shape Factor are different. In Formula (3) of the compactness indicator, if the area \( A \) of the grid cell is extremely small, \((\frac{r}{\pi})^2 \approx 1\), the ratio is infinitely close to 0, then \(C \approx \frac{3.545\sqrt{2}}{r}\). Comparing this formula with Formula (2), we find that the calculation formula of the compactness indicator and the calculation formula of the shape indicator only differ by a constant multiple. Therefore, they must have a strong positive correlation. Since Formula (2) comes from evaluating the shape characteristics of flat landscape patches, and Formula (3) comes from the calculation of spherical compactness, we regard Formula (3) and Formula (2) as different manifestations of the same formula on a sphere and a plane. From this perspective, indicators of compactness and shape are the same, and there must be a positive correlation (see Figure 9). Fractal dimension is a measure of the irregularity of a complex shape, which reflects the irregular characteristics of a grid cell. ZSC and Shape Index (SI) reflect the regular characteristics of a grid cell. Therefore, the numerical sign of
the fractal dimension in the Shape Factor is opposite to the numerical sign of indicators of compactness and shape, i.e., they have a strong negative correlation.

Through the above calculation and analysis, two common factors (Area Factor, Shape Factor) can be extracted as independent evaluation factors. The indicators of the common factor have a strong correlation, and the indicators between the two common factors are independent (as shown in Figures 9 and 10).

4.3. Evaluation Indicator System Reconstruction and Comparisons

After the above calculation and analysis, the quantitative calculation indicators can be reduced to two, i.e., Area Factor and Shape Factor; the qualitative analysis indicators can be reduced to four, namely Domain, Hierarchy, Location, Transformation. Thus, the quality evaluation indicator system of the DGG can be reconstructed into six independent evaluation indicators (Table 6). By comparing the traditional Goodchild Criteria (column 5 in Table 6), the new indicator system only has six indicators, which are independent, clear, and easy to operate. What is more, these six indicators cover all the information of the Goodchild Criteria and can provide a feasible indicator system for the comprehensive evaluation of the quality of the DGG models. At the same time, in applications in different fields, one or more evaluation indicators related to the application can be selected from the above reconstruction evaluation indicators according to specific actual needs, and the geometric deformation evaluation model of the grid model can be established to ensure the reliability of the application model or application service.

5. Conclusions

In this paper, to overcome the limitations of incompatible and overlapping geometric evaluation criteria (the Goodchild Criteria) of DGGs, the quantitative and qualitative indicators were extracted and reduced by factor analysis and calculation, and by logical reasoning and induction, respectively. A quality evaluation indicator system of the DGG with independent indicators was reconstructed. The main contributions of this study are as follows.

(1) We measured and calculated the correlation among the seven quantitative evaluation indicators of the Goodchild Criteria. According to their correlations, the above indicators were divided into two groups, one includes indicators of area, edge, and midpoints and center-point spacing; the other includes indicators of shape, compactness, and structure. There were strong correlations within each group but no correlations between groups. As a result, the above seven quantitative indicators were classified into two independent evaluation factors, namely Area Factor and Shape Factor.

(2) The qualitative indicators were summarized by logical analysis, and four relatively independent evaluation indicators were extracted. Therefore, an independent quality evaluation indicator system of the DGG was reconstructed, including only six indicators or factors of Domain, Conversion, Hierarchy, Location, Area, Shape. The new indicator system covers all the information of the Goodchild Criteria in a clear, independent, and simple indicator system for the comprehensive quality evaluation of a DGG model. In the end, we took an octant of the QTM model as the test area and the visualization results conform to the correlation conclusions of the above quantitative indicator.

This article mainly used the QTM model as an example to reconstruct the quality evaluation indicator of the DGG. Although the triangle is the most basic shape, it has the main characteristics of other shapes. In future work, we will explore the specificity of the indicators of grid models in various shapes (such as triangular, quadrilateral, or hexagonal grid, etc.) and their influence on the quality evaluation system.

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**Appendix A**

**Table A1.** The Goodchild Criteria.

| No. | Criterion                                                                 | Simplification (Symbol) | Ideal State                  |
|-----|---------------------------------------------------------------------------|--------------------------|------------------------------|
| 1   | Areal cells constitute a complete tiling of the globe,                    | Domain                   | Global tiling                |
|     | exhaustively covering the globe without overlapping.                     |                          |                              |
| 2   | Areal cells have equal areas.                                            | Area ($s$)               | Equal areas                  |
| 3   | Areal cells have the same topology (the same number of edges and vertices) | Topology                 | The same number of edges and vertices |
| 4   | Areal cells are the same shape, ideally a regular spherical polygon with edges that are great circles. | Shape ($SI$)             | Regular spherical polygons   |
| 5   | Areal cells are compact.                                                  | Compactness (C)          | Compact cells                |
| 6   | Edges of cells are straight in a projection                               | Edge ($e$)               | Straight projection edges    |
| 7   | The midpoint of the arc connecting two adjacent cell centers coincides with the midpoint of the edge between the two cells. | Midpoints ($d_{im}$)     | Identical points             |
| 8   | The points and areal cells of the various resolution grids which constitute the grid system form a hierarchy that displays a high degree of regularity. | Hierarchy                | Hierarchical grids           |
| 9   | A single areal cell contains only one grid reference point.               | Uniqueness               | Unique reference point       |
| 10  | Grid reference points are maximally central within areal cells.          | Location                 | Central point                |
| 11  | Grid reference points are equidistant from their neighbors.              | Spacing ($d_{e}, d_{a}$) | Equidistance                 |
| 12  | Grid reference points and areal cells display regularities and other properties that allow them to be addressed efficiently. | Structure ($f$)          | Regular structure            |
| 13  | The grid system has a simple relationship to the latitude-longitude graticule. | Conversion               | Simple conversion            |
| 14  | The grid system contains grids of any arbitrarily defined spatial resolution. | Resolution               | Infinite spatial resolution   |

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