DECOMPOSITION STRATEGIES AND MULTI-SHOT ASP SOLVING FOR JOB-SHOP SCHEDULING

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ABSTRACT. The Job-shop Scheduling Problem (JSP) is a well-known and challenging combinatorial optimization problem in which tasks sharing a machine are to be arranged in a sequence such that encompassing jobs can be completed as early as possible. In this paper, we investigate problem decomposition into time windows whose operations can be successively scheduled and optimized by means of multi-shot Answer Set Programming (ASP) solving. From a computational perspective, decomposition aims to split highly complex scheduling tasks into better manageable subproblems with a balanced number of operations such that good-quality or even optimal partial solutions can be reliably found in a small fraction of runtime. We devise and investigate a variety of decomposition strategies in terms of the number and size of time windows as well as heuristics for choosing their operations. Moreover, we incorporate time window overlapping and compression techniques into the iterative scheduling process to counteract optimization limitations due to the restriction to window-wise partial schedules. Our experiments on different JSP benchmark sets show that successive optimization by multi-shot ASP solving leads to substantially better schedules within tight runtime limits than single-shot optimization on the full problem. In particular, we find that decomposing initial solutions obtained with proficient heuristic methods into time windows leads to improved solution quality.

1. INTRODUCTION

Effective scheduling methods are essential for complex manufacturing and transportation systems, where allocating and performing diverse tasks within resource capacity limits is one of the most critical challenges for production management [UW00]. The Job-shop Scheduling Problem (JSP) [Bak74, Tai93] constitutes a well-known mathematical abstraction of industrial production scheduling in which operations need to be processed by machines such that a given objective, like the makespan for completing all jobs or their tardiness w.r.t. deadlines, is minimized. Finding optimal JSP solutions, determined by a sequence of operations for each machine, is an NP-hard combinatorial problem [GJS76, LKB77, LHW08]. Therefore, optimal schedules and termination guarantees can be extremely challenging or even unreachable for complete optimization methods, already for moderately sized instances. For example, it took about 20 years to find a (provably) optimal solution for an instance called FT10 with 10 jobs [ABZ88, ZW10], each consisting of a sequence of 10 operations to be processed by 10 machines.

Roughly speaking, the optimization approaches to JSP can be classified into exact and approximation methods [XSRH22]. The former guarantee optimal schedules upon termination of their...
complete optimization algorithms, while the latter aim at finding good-quality schedules without exhaus-
tively traversing the search space. In view of the combinatorial explosion faced when the instance size grows, exact methods manage to guarantee optimal solutions in limited time for reasonably small or extraordinarily simple JSP instances only. This makes approximation methods attractive whenever termination guarantees are beyond reach and the optimization task turns into finding the best possible solution within a tight runtime limit. Considering those instances in Taillard’s benchmark set [Tai93] whose optima are known, the optimal schedules were actually found by approximation methods, and theoretical lower bounds rather than exhaustive search could be applied to exclude the existence of any better solution.

In this paper, we combine and significantly extend our studies [EGS22, ESG22] on problem decomposition into time windows and successive schedule optimization through an extension of Answer Set Programming (ASP) [Lif19] with Difference Logic (DL) constraints [GKK+16] and multi-shot solving [GKKS19] supported by the clingo[DL] system [JKO+17]. The goal of the decomposition is to split highly complex scheduling tasks into balanced portions for which partial schedules of good quality can be reliably found within tight runtime limits. Then, the partial schedules are merged into a global solution of significantly better quality than obtainable in similar runtime with single-shot optimization on the full problem. We address computational efficiency as well as solution quality by devising and empirically assessing decomposition strategies regarding the size of time windows and heuristics for selecting their operations.

1.1. Related Work. While decision versions of scheduling problems can be successfully modeled and solved by means of ASP modulo DL, implemented by clingo[DL] on top of the (multi-shot) ASP system clingo [GKKS19], the optimization capacities of clingo[DL] come to their limits on moderately sized yet highly combinatorial JSP instances [EG20], for some of which optimal solutions are so far unknown [SS18]. Successful applications in areas beyond JSP include, e.g., industrial printing [Bal11], team-building [RGA+12], matchmaking [GGSS13], shift design [AGM+16], course timetabling [BIK+19], workload smoothing [SS20], and medical treatment planning [DGG+21], pointing out the general attractiveness of ASP for modeling and solving scheduling problems.

In real-world production scheduling, the number of operations to process can easily go into tens of thousands [DCT22, KHKM20, KTK+21], which exceeds exact optimization capacities even of state-of-the-art solvers for ASP, Mixed Integer Programming (MIP), or Constraint Programming (CP) [DFH21, FSE21, SYXQ22]. Hence, more efficient approaches to approximate good-quality schedules instead of striving for optimal solutions have attracted broad research interest. On the one hand, respective methods include greedy and local search techniques such as dispatching rules [BPH82], shifting bottleneck [ABZ88] and genetic algorithms [PMC08]. On the other hand, problem decomposition strategies based on a rolling horizon [Sin01, LHW08] or bottleneck operations [ZW10, ZLC+14] have been proposed to partition large-scale instances into better manageable subproblems, where no single strategy strictly dominates in minimizing the tardiness w.r.t. deadlines [OU12].

1.2. Main Contributions. The contributions of our work going beyond the studies in [EGS22, ESG22] are:

- We develop a comprehensive multi-shot ASP modulo DL framework for JSP solving and significantly extend the preliminary presentation in [EGS22]. In addition to a decomposition strategy based on the earliest starting times of operations, we describe how their remaining processing
times can be used instead and further detail the ASP encoding of two-layered decomposition approaches, combining either basic strategy with the consideration of bottleneck machines.

- Since a decomposition into time windows may be incompatible with the optimal sequences of operations sharing a machine, we incorporate overlapping time windows into the iterative scheduling process to offer chances for revising “decomposition mistakes”. Moreover, the makespan objective, which we apply to optimize (partial) schedules, tolerates unnecessary idle times of machines as long as they do not yield a greater scheduling horizon. Hence, we model a compression strategy in ASP to postprocess partial schedules by reassigning operations to earlier idle slots available on their machines. The ASP programs providing declarative implementations of the overlapping and compression techniques have not been elaborated before.

- Considering that static properties of JSP instances, describing the jobs, their operations, and available machines, merely provide rough information for guiding problem decomposition by basic features [EGS22] or clustering methods [ESG22], we further explore greedy search methods given by the First-In-First-Out, Most-Total-Work-Remaining, and Reinforcement Learning dispatching heuristics proposed in [TGS21] to assign operations to time windows based on the greedy solutions.

- We experimentally evaluate decomposition strategies varying the number of operations per time window as well as the overlapping and compression techniques to apply during multi-shot ASP modulo DL solving on Taillard’s and Demirkol’s well-known JSP benchmark sets [Tai93, DMU98]. Our experiments show that the successive optimization of time windows leads to substantially better schedules within tight runtime limits than single-shot ASP modulo DL optimization on the full problem. In particular, taking greedy solutions obtained with the aforementioned dispatching heuristics as basis for problem decomposition yields improved solution quality. To assess the scalability of our multi-shot ASP modulo DL methods, we extend the scope to a benchmark set of industrial-size JSP instances due to Da Col and Teppan [DCT22], and further contrast the obtained results with the state of the art established by high-performance CP systems.

The paper is organized as follows. Section 2 briefly introduces ASP along with the relevant extensions of multi-shot solving and DL constraints. In Section 3, we present our successive optimization approach, incorporating ASP programs encoding problem decomposition or iterative scheduling based on time windows, respectively, as well as a decomposition scheme by constrained clustering. Section 4 provides experimental results on JSP benchmark sets, assessing different decomposition strategies along with the impact of overlapping and compression techniques. Conclusions and future work are discussed in Section 5.

2. Preliminaries

Answer Set Programming (ASP) [Lif19] is a knowledge representation and reasoning paradigm geared for the effective modeling and solving of combinatorial (optimization) problems. A (first-order) ASP program consists of rules of the form $h ::= b_1, \ldots, b_n$, in which the head $h$ is an atom $p(t_1, \ldots, t_m)$ or a choice $\{p(t_1, \ldots, t_m)\}$ and each body literal $b_i$ is an atom $p(t_1, \ldots, t_m)$, possibly preceded by the default negation connective not and/or followed by a condition : $c_1, \ldots, c_l$, a built-in comparison $t_1 \circ t_2$ with $\circ \in \{<, \leq, =, !>, =, >\}$, or an aggregate $t_0 = \#count(t_1, \ldots, t_m : c_1, \ldots, c_l)$. Each $t_j$ denotes a term, i.e., a constant, variable, tuple, or arithmetic expression, and each element $c_k$ of a condition is an atom that may be preceded by not or a built-in comparison. Roughly speaking, an ASP program is a shorthand for its ground instantiation, obtainable by substituting variables with all of the available constants and evaluating arithmetic expressions, and the semantics is given by answer sets, i.e., sets of (true) ground atoms such that all rules of the ground instantiation are satisfied and allow for deriving each of the ground atoms by some rule whose body
is satisfied. The syntax of the considered ASP programs is a fragment of the modeling languages described in [CFG+20, GHK+15], the ground instantiation process is detailed in [KS23], and the answer set semantics is further elaborated in [GHK+15, Lif19].

For example, the ASP program

\begin{verbatim}
{p(1..2)}.
p(0) :- not p(X) : X = 1..2.
p(X) :- X = #count{Y : p(Y), Y < 2}.
\end{verbatim}

with the variables \(x\) and \(y\), the arithmetic expression \(1..2\) standing for the integer interval \([1, 2]\), and the built-in comparisons \(x = 1..2\) and \(y < 2\) is a shorthand for the following ground instantiation:

\begin{verbatim}
{p(1)}.
{p(2)}.
p(0) :- not p(1) : 1 = 1; not p(2) : 2 = 2.
p(0) :- 0 = #count{0 : p(0), 0 < 2;
1 : p(1), 1 < 2;
2 : p(2), 2 < 2}.
p(1) :- 1 = #count{0 : p(0), 0 < 2;
1 : p(1), 1 < 2;
2 : p(2), 2 < 2}.
p(2) :- 2 = #count{0 : p(0), 0 < 2;
1 : p(1), 1 < 2;
2 : p(2), 2 < 2}.
\end{verbatim}

The first two rules with the choices \(\{p(1)\}\) and \(\{p(2)\}\) express that each of the atoms \(p(1)\) and \(p(2)\) may, but does not necessarily have to be true. As the built-in comparisons in the conditions : \(1 = 1\) and : \(2 = 2\) of the third rule hold, this rule derives the atom \(p(0)\) when both \(p(1)\) and \(p(2)\) are false. In that case, the aggregate in the body of the fifth rule

\begin{verbatim}
p(1) :- 1 = #count{0 : p(0), 0 < 2;
1 : p(1), 1 < 2;
2 : p(2), 2 < 2}.
\end{verbatim}

holds, while the head atom \(p(1)\) is false, so that the rule is not satisfied. This in turn means that at least one of the atoms \(p(1)\) and \(p(2)\) needs to be true, where having \(p(2)\) alone contradicts with the fourth rule:

\begin{verbatim}
p(0) :- 0 = #count{0 : p(0), 0 < 2;
1 : p(1), 1 < 2;
2 : p(2), 2 < 2}.
\end{verbatim}

If \(p(0)\) is false, the aggregate in the body holds and the rule is not satisfied. On the other hand, the condition : \(p(0), 0 < 2\) applies in case \(p(0)\) is true, so that the rule body is not satisfied and \(p(0)\) turns out to be underivable. Unlike that, the aggregate does not hold when \(p(1)\) is true, which makes the atom \(p(0)\) underivable and \(p(2)\) optional, considering that the satisfaction of the rules

\begin{verbatim}
p(1) :- 1 = #count{0 : p(0), 0 < 2;
1 : p(1), 1 < 2;
2 : p(2), 2 < 2}.
p(2) :- 2 = #count{0 : p(0), 0 < 2;
1 : p(1), 1 < 2;
2 : p(2), 2 < 2}.
\end{verbatim}
is readily established once \( p(0) \) is false. Hence, we obtain the two answer sets \( \{ p(1) \} \) and \( \{ p(1), p(2) \} \), which satisfy all ground rules, where \( p(1) \) and \( p(2) \) are also derivable in view of the choice rules \( \{ p(1) \} \) and \( \{ p(2) \} \).

Multi-shot ASP solving [GKKS19] allows for iterative reasoning processes by controlling and interleaving the grounding and search phases of a stateful ASP system. For referring to a collection of rules to instantiate, the input language of clingo supports \( \#program \ name(c) \) directives, where \( name \) denotes a subprogram comprising the rules below such a directive and the parameter \( c \) is a placeholder for some value, e.g., the current time step in case of a planning problem, supplied upon instantiating the subprogram. Moreover, \( \#external \ h : b_1, \ldots, b_n \) statements are formed similar to rules yet declare an atom \( h \) as external when the body is satisfied: such an external atom can be freely set to true or false by means of the Python interface of clingo, so that rules including it in the body can be selectively (de)activated to direct the search.

ASP modulo DL integrates DL constraints [CM06], i.e., expressions written as \( \&\text{diff}\{t_1 - t_2\} \leq t_3 \), in the head of rules. With the exception of the constant 0, which denotes the number zero, the terms \( t_1 \) and \( t_2 \) represent DL variables that can be assigned any integer value. However, the difference \( t_1 - t_2 \) must not exceed the integer constant \( t_3 \) if the body of a rule with the DL constraint in the head is satisfied. That is, the DL constraints asserted by rules whose body is satisfied restrict the feasible values for DL variables, and the clingo[DL] system extends clingo by assuring the consistency of DL constraints imposed by an answer set. If these DL constraints are satisfiable, a canonical assignment of smallest feasible integer values to DL variables can be determined in polynomial time and is output together with the answer set.

3. MULTI-SHOT JSP SOLVING

This section describes our successive optimization approach to JSP solving by means of multi-shot ASP with clingo[DL]. We start with specifying the fact format for JSP instances, then detail several problem decomposition strategies, including simple approaches based on static properties like the earliest starting times of operations as well as more elaborate feature extraction and constrained clustering schemes, present our ASP encoding with DL constraints for optimizing the makespan of partial schedules, and finally outline the iterative scheduling process along with the incorporation of time window overlapping and compression techniques.

3.1. Problem Instance. Each job in a JSP instance is a sequence of operations with associated machines and processing times. Corresponding facts for an example instance with three jobs and three machines are displayed in Listing 1. An atom of the form \( \text{operation}(j, s, m, p) \) denotes that the step \( s \) of job \( j \) needs to be processed by machine \( m \) for \( p \) time units. For example, the second operation of job 3 has a processing time of 3 time units on machine 1, as specified by the fact \( \text{operation}(3, 2, 1, 3) \). The operation cannot be performed before the first operation of job 3 is completed, and its execution must not intersect with the first operation of job 1 or the second operation of job 2, which need to be processed by machine 1 as well. That is, a schedule for the
example instance must determine a sequence in which to process the three mentioned operations on machine 1, and likewise for operations sharing machine 2 or 3, respectively.

Figure 1 depicts a schedule with the optimal makespan, i.e., the latest completion time of any job/operation, for the JSP instance from Listing 1. The J-S pairs in horizontal bars indicated for the machines 1, 2, and 3 identify operations by their job J and step number S. For each machine, observe that the bars for operations it processes do not intersect, so that the operations are performed in sequential order. Moreover, operations belonging to the same job are scheduled one after another. For example, the second operation of job 3 is started after the completion of its predecessor operation at time 9, regardless of the availability of machine 1 from time 3 on. As the precedence of operations within their jobs must be respected and the sum of processing times for operations of job 3 matches the makespan 20, it is impossible to reduce the scheduling horizon any further, which yields that the schedule shown in Figure 1 is optimal.

3.2. Problem Decomposition. Since JSP instances are highly combinatorial and the ground representation size can also become problematic for large real-world scheduling problems, achieving scalability of complete optimization methods necessitates problem decomposition. In order to enable a successive extension of good-quality partial schedules to a global solution, we consider strategies for partitioning the operations of JSP instances into balanced time windows, each comprising an equal number of operations such that their precedence within jobs is respected. In the following, we first detail problem decomposition based on the earliest starting times of operations, and then outline further strategies that can be encoded by stratified ASP programs having a unique answer set [Prz88] as well.

Our encoding for Job-based Earliest Starting Time (J-EST) decomposition in Listing 2 takes a JSP instance specified by facts over operation/4 as input. In addition, a constant n, set to the default value 2 in line 1, determines the number of time windows into which the given operations shall be split. As we aim at time windows of (roughly) similar size, the target number of operations per time window is in line 2 calculated by \[ \left\lceil \frac{N}{n} \right\rceil \], where N is the total number of operations. For example, we obtain width(5) for partitioning the nine operations of the JSP instance in Listing 1 into two time windows.

The rules in lines 4 and 5 encode the J-EST calculation per operation of a job, given by the sum of processing times for predecessor operations belonging to the same job. This yields, e.g., \( \text{est}(3, 1, 9, 0) \), \( \text{est}(3, 2, 3, 9) \), and \( \text{est}(3, 3, 8, 12) \) for the three operations of job 3 in our example instance, where the third argument of an atom over est/4 provides the processing time and the fourth the earliest starting time of an operation. Note that the obtained earliest starting times match the first feasible time points for scheduling operations and do thus constitute an optimistic estimation of when to process the operations.

With the earliest starting times of operations at hand, the rule in lines 7-8 determines a total order of operations in terms of consecutive indexes ranging from 0. That is, each operation is mapped to the number of operations with (i) a smaller earliest starting time, (ii) the same earliest starting time
and shorter processing time, or (iii) a smaller job identifier as tie-breaker in case of identical earliest starting and processing times. For the example JSP instance in Listing 1, we obtain the indexes 0 to 2 for the first operations of the three jobs, the indexes 3 and 4 for the second operation of job 1 or 2, respectively, in view of their earliest starting times 3 and 4, and indexes from 5 to 8 for the remaining operations. A relevant condition that is guaranteed by such a total order is that indexes increase according to the precedence of operations within their jobs, given that the earliest starting times grow along the sequence of operations in a job.

The last rule in line 10 inspects the total order of operations to partition them into time windows of the size \( W \) determined by \( \text{width}(W) \), where only the last time window may possibly include fewer operations in case the split is uneven. As the ASP program encoding problem decomposition is stratified, its ground instantiation can be simplified to (derived) facts, as shown in Listing 3 for our example instance. Time window numbers from 1 to \( n = 2 \) are given by the third argument of atoms over \( \text{window}/3 \), so that the second operation of job 3 and the third operation of each job form the time window 2, while time window 1 consists of the five other operations.

In addition to J-EST decomposition, we have devised a similar ASP program for \textit{Job-based Most Total Work Remaining (J-MTWR)} decomposition, where the total order of operations is decreasing by the sum of processing times for an operation and its successors in a job. The J-MTWR strategy is encoded by replacing the rules in lines 4-8 of Listing 2 by:

\begin{verbatim}
  rpt(J,S,P) :- operation(J,S,M,P), not operation(J,S + 1,_,_).
  rpt(J,S,P + T) :- operation(J,S,M,P), rpt(J,S + 1,T).

  index(J,S,N) :- rpt(J,S,T),
  N = #count{J’,S’ : rpt(J’,S’,T’), (T’,J’) < (T,J)}.  
\end{verbatim}

For example, we obtain the J-MTWR values 7, 12, and 20, represented by the atoms \( \text{rpt}(1,1,7) \), \( \text{rpt}(2,1,12) \), and \( \text{rpt}(3,1,20) \), for the first operation of job 1, 2, or 3, respectively, for the JSP instance in Listing 1, matching the time for executing all three operations of each job. Hence, the first operation of job 3 is considered as the most important and is associated with the index 0, and one can check that all three operations of job 3 together with the first two operations of job 2 form

\begin{verbatim}
window(1,1,1). window(2,1,1). window(3,1,1). window(1,2,1). window(2,2,1). window(3,2,2). window(1,3,2). window(2,3,2). window(3,3,2).
\end{verbatim}
the first of \( n = 2 \) time windows, so that the obtained decomposition varies from the time windows of J-EST in Listing 3. However, as J-MTWR values are decreasing along the sequence of operations in a job, the resulting operation indexes and time windows also respect the precedence of operations.

Beyond partitioning operations in a purely Job-based fashion, we have encoded Machine-based decompositions M-EST and M-MTRW in which an operation from a bottleneck machine with the greatest sum of processing times for yet unordered operations is of highest priority. To this end, the encoding in Listing 4 builds on the atoms over \( \text{index/3} \) as obtained with either the J-EST or J-MTWR strategy, but then it determines a bottleneck machine from which the next operation in the underlying Job-based priority is picked and inserted into the total order of operations to be used for decomposition into time windows. As long as some operation is yet unordered, the rules in lines 10-16 group the operations into those that still need to be inserted or not, indicated by atoms over \( \text{todo/4} \) or \( \text{done/4} \), respectively, whose third argument provides the Job-based index of an operation and the fourth an iteration counter for determining a bottleneck machine w.r.t. the processing times for still unordered operations. For our example instance in Listing 1, the sum of processing times for operations is 12, 15, or again 12, respectively, for the machines 1, 2, and 3.

In the first iteration, where all operations are yet unordered, the rules in lines 18-25 that sum up the processing times per machine, yield the loads in terms of the atoms \( \text{load}(1,1,12,1) \), \( \text{load}(2,1,15,1) \), and \( \text{load}(3,1,12,1) \) with the respective machine as the first argument. Then, the rules in lines 27-29 traverse all machines to determine the one with the highest remaining load, which in the first iteration leads to the atoms \( \text{high}(3,3,12,1) \), \( \text{high}(2,2,15,1) \), and \( \text{high}(1,2,15,1) \) with the last one providing the outcome that some operation processed by the bottleneck machine 2 is to be inserted into the total order of operations next. The rules in lines 31-34 inspect operations in the order of their Job-based indexes, skipping over those that are already inserted or not processed by the bottleneck machine of the current iteration. This yields the atom \( \text{screen}(1,1) \) along with \( \text{select}(2,1,1,1) \) by the rule in lines 36-37, expressing that the first operation of job 2 with the J-EST/J-MTWR index 1 (instead of the first operation of job 1 or 3, respectively, which with index 0 come first according to the J-EST and J-MTWR orderings) is to be inserted in the first iteration. In general, this may lead to the choice of an operation such that some of its predecessor operations processed by other machines are yet unordered, and the rules in lines 38-39 indicate all of them by derived atoms over \( \text{insert/4} \), while the first iteration for our example yields \( \text{insert}(2,1,1,1) \) only. The final positions for operations to insert into the Machine-based order are then determined by the rules in lines 44-47, which derive the atom \( \text{result}(2,1,0) \) to assign the first operation of job 2 the highest priority in the total order of operations.

Remaining operations are further associated with the positions 1 to 8 in the third argument of atoms over \( \text{result/3} \) obtained in the following iterations, and the corresponding decomposition into time windows is as with Job-based decomposition strategies represented by atoms over \( \text{window/3} \), derived by the rule in line 49. That is, our stratified ASP program in Listing 4 for (re)ordering operations based on bottleneck machines can be transparently combined with any initial total order of operations given in terms of atoms over \( \text{index/3} \), so that this encoding is sufficient to switch from both the J-EST and J-MTWR decomposition strategies to M-EST or M-MTWR, respectively.

3.3. Feature Extraction. While the static properties considered above constitute easily applicable yet very rough decomposition criteria, we now turn to more elaborate features providing the basis for constrained clustering. In fact, the application of machine learning methods to scheduling problems requires a careful selection of data describing the hidden dependencies between operations of different jobs [IOB12, NSR+19]. Clustering methods, which we apply in our approach, are no exception to this. That is, a clustering method requires an informative set of features characterizing
Listing 4. Machine-based decomposition encoding

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the jobs, their operations, and machines of a JSP instance to identify patterns resulting in a beneficial
decomposition of the operations into time windows.

Heuristic methods suggested in the literature [KT00, HCZ02, SM10, IOB12, AS14, NSR+19] characterize instances of scheduling problems based on the following features: priority, processing
time, remaining processing time, machine load, and sequence position. Most of these approaches
convert the quantitative feature values into qualitative attributes in order to obtain generic dispatching
rules that remain applicable to instances of different size, while we propose a clustering method that
can be applied to feature values directly and does not require any problem-specific transformations. However, our clustering method for JSP instance decomposition requires all features to have numerical values, which permit the calculation of distance measures for estimating (dis)similarities between operations.

In detail, we consider the following features of jobs, operations, and machines:

**Operation (OP):** is the ordinal value for the position of an operation in its job.

**Processing Time (PT):** is the time for executing an operation on its machine.

**Remaining Processing Time (RPT):** provides the total processing time for pending operations until the completion of a job. For example, Table 1 lists \( RPT \) values for operations of the example instance in Listing 1. The job 1 consists of 3 operations with a total processing time of 7 time units, which matches \( RPT \) for its first operation \( O_{1,1} \). The \( RPT \) for \( O_{1,2} \) is obtained by subtracting the processing time of \( O_{1,1} \), i.e., \( 7 - 3 = 4 \), and it corresponds to the processing time 1 for the last operation \( O_{1,3} \) of job 1.

**Time Length of a Job (TLJ):** is the total processing time for operations of a job, which coincides with the \( RPT \) value of the job’s first operation and is more coarse-grained than the operation-specific \( RPT \) feature.

**Earliest Starting Time (EST):** represents the earliest possible time for executing an operation, given by the total processing time for the predecessor operations in its job. For the first operation of each job, the \( EST \) value defaults to 0.

**Machine Load (ML):** is a property describing how much time it takes to execute the operations assigned to a machine. Initially, \( ML \) corresponds to the total processing time for all operations to be executed by a machine. Then the assumption is that the operations are processed in increasing order of their \( EST \) values, and \( ML \) is thus reduced by the processing times of preceding operations. For example, the \( EST \) for the operations \( O_{3,1}, O_{1,3}, \) and \( O_{2,3} \) assigned to machine 3 is 0, 6, or 10, respectively. Proceeding in this execution order, \( ML \) is the total processing time 12 for \( O_{3,1} \), reduced by the processing time of \( O_{3,1} \) to 12 − 9 = 3 for \( O_{1,3} \), and then we obtain the processing time 2 for the last operation \( O_{2,3} \).

**Starting Time (ST):** is a family of features providing the starting times of operations obtained by scheduling them with heuristic search methods based on the greedy algorithm shown in Figure 2. Its idea is to maintain a set \( R \) of pairs \((O_{j,s}, r)\) such that an operation \( O_{j,s} \) represents the step \( s \) of a job \( j \) to be processed next and \( r \) stands for the release time of \( O_{j,s} \), i.e., the predecessor operation \( O_{j,s-1} \) (if any) is completed and the machine of \( O_{j,s} \) is available. Greedy allocation then proceeds by these release times, where the smallest time is denoted by \( t \) in step 1 of Figure 2. The crucial component of the algorithm is the selection heuristic in step 2, picking the next operation \( O_{j,s} \) to allocate at time \( t \).

In our work, we consider Earliest Starting Time (\( ST_{EST} \)), First-In-First-Out (\( ST_{FIFO} \)), and Most Total Work Remaining (\( ST_{MTWR} \)) as heuristics for the greedy operation allocation; see [JRS99] for an overview of such techniques. In the case of \( ST_{FIFO} \), the algorithm selects an operation waiting longest for its machine to become available. For example, as indicated in Table 1, the first operations \( O_{1,1}, O_{2,1}, \) and \( O_{3,1} \) are allocated at time 0 (on different machines), then \( O_{1,2} \) waits 1 time unit for its machine 2 to get available and starts at time 4, and \( O_{1,3} \) waits from time 7 for machine 3 and is started at time 9. While \( O_{2,2} \) is started on machine 1 at time 4, so that \( O_{2,3} \) is started on machine 3 at time 10, \( O_{3,2} \) needs to wait 1 time unit to start on its machine 1 at time 10, and then \( O_{3,3} \) is started on machine 2 at time 13. If the processing time of \( O_{3,1} \) were 10 instead of 9, \( ST_{FIFO} \) would have the choice between allocating either \( O_{1,3} \) or \( O_{2,3} \) on machine 3 at time 10. In view of the longer waiting time of \( O_{1,3} \), it gets selected first and \( O_{2,3} \) comes after its completion, which is how \( ST_{FIFO} \) aims at
Starting with \( R := \{(O_{j,1}, 0) \mid j \text{ is a job with first step 1}\}, S := \emptyset, \) and \( T := \emptyset, \) while \( R \neq \emptyset: \)

1. \( t := \min\{r \mid (O_{j,s}, r) \in R\} \)
2. SELECT \((O_{j,s}, t) \in R\) and set \( R := R \setminus \{(O_{j,s}, t)\}\)
3. If NOOP, then \( S := S \cup \{O_{j,s}\}, \) else:
   a. \( T := T \cup \{(O_{j,s}, t)\} \)
   b. \( u := t + p \) for the processing time \( p \) of \( O_{j,s} \) on machine \( m \)
   c. For each \((O'_{j',s'}, r) \in R\) such that \( O'_{j',s'} \) is processed by \( m, \) set \( R := (R \setminus \{(O'_{j',s'}, r)\}) \cup \{(O'_{j',s'}, \max\{\{r, u\}\})\} \)
   d. For each \((O'_{j',s'}, u) \in S\) such that \( O'_{j',s'} \) is processed by \( m, \) set \( R := R \cup \{(O'_{j',s'}, u)\} \) and \( S := S \setminus \{O'_{j',s'}\} \)
   e. If there is a step \( s + 1 \) of job \( j \) to be processed by machine \( m', \) then
      \( R := R \cup \{(O_{j,s}, t') \cup \{t' + p' \mid (O'_{j',s'}, t') \in T \text{ with processing time } p' \text{ on machine } m'\})\} \)

If \( S = \emptyset, \) then return \( T \)

**Figure 2.** Basic greedy operation allocation algorithm for computing a heuristic schedule

With the \( ST_{EST} \) and \( ST_{MTWR} \) heuristics, the greedy algorithm’s selection of the next operation to allocate is based on smaller \( EST \) or greater \( RPT \) values, respectively.

The implementation of greedy heuristics \( [TGS21] \) utilize never performs a noop in step 3 of Figure 2 for \( ST_{EST}, \) \( ST_{FIFO}, \) or \( ST_{MTWR}, \) yet includes this option for dispatching by Reinforcement Learning, which we compare in Section 4. When allocating a selected operation \( O_{j,s}, \) it is added to the heuristic schedule \( T \) in step 3a, and its completion time \( u, \) calculated in step 3b, is used to update the release times of operations processed by the same machine as \( O_{j,s} \) in step 3c. If there is a successor operation \( O_{j,s+1}, \) step 3e takes it to replace \( O_{j,s} \) in \( R, \) where the release time depends on the completion of \( O_{j,s} \) at time \( u \) and machine availability. Without noops, the final heuristic schedule \( T, \) such as the one for \( ST_{FIFO} \) in

**Table 1.** Part of the features extracted for the example JSP instance in Listing 1

| Operation | RPT | EST | ML | \( ST_{FIFO} \) | \( WT_{FIFO} \) |
|-----------|-----|-----|----|----------------|-----------------|
| \( O_{1,1} \) | 7   | 0   | 12 | 0              | 0               |
| \( O_{1,2} \) | 4   | 3   | 11 | 4              | 1               |
| \( O_{1,3} \) | 1   | 6   | 3  | 9              | 2               |
| \( O_{2,1} \) | 12  | 0   | 15 | 0              | 0               |
| \( O_{2,2} \) | 8   | 4   | 9  | 4              | 0               |
| \( O_{2,3} \) | 2   | 10  | 2  | 10             | 0               |
| \( O_{3,1} \) | 20  | 0   | 12 | 0              | 0               |
| \( O_{3,2} \) | 11  | 9   | 3  | 10             | 1               |
| \( O_{3,3} \) | 8   | 12  | 8  | 13             | 0               |
Table 1, is always complete, while skipped operations are otherwise reconsidered in step 3d after some allocation on their machine.

**Waiting Time (WT):** is also a family of features, where variants denoted by WT\_EST, WT\_FIFO, and WT\_MTWR rely on schedules obtained with the corresponding ST heuristic, i.e., ST\_EST, ST\_FIFO, or ST\_MTWR. Given a schedule computed by the greedy algorithm, the waiting time of an operation is determined by the difference between its starting time and the time of completing the predecessor operation, or simply the starting time for the first operation of each job. For instance, the starting times with ST\_FIFO listed in Table 1 yield the waiting times given in the WT\_FIFO column. In fact, $O_{1,2}$ and $O_{1,3}$ wait for 1 or 2 time units, respectively, for their machines 2 and 3 to get available, and $O_{3,2}$ also needs to wait 1 time unit for the completion of $O_{2,2}$ before its execution by machine 1.

We extract all of the features described above from a given JSP instance and can thus use them as inputs to our decomposition method by means of constrained clustering presented in the following.

### 3.4. Constrained Clustering

Clustering algorithms are unsupervised learning methods whose goal is to partition a set of data objects into (disjoint) clusters such that each cluster gathers objects of high similarity, i.e., operations belonging to the same time window in our case. Such similarity is determined by some measure, e.g., Euclidean distance, based on features of each object, like the features of operations described in the previous section. However, the direct application of common clustering algorithms, such as K-Means [For65], to scheduling problems is impractical since the partitioning does not take the sequence of operations in a job into account. For instance, a clustering algorithm may put $O_{1,1}$ and $O_{1,3}$ into the same and $O_{1,2}$ into another cluster, in which case the sequence of time windows becomes inconsistent and no compatible schedule exists.

We thus propose a constrained clustering algorithm that preserves sequences of operations by considering their order in the assignment to clusters. That is, the predecessors of an operation to be put into the $n$th cluster must be assigned within the clusters 1, . . . , $n$. Also considering that our approach involves cluster-wise combinatorial optimization, the generation of large clusters risks to deteriorate the solving performance significantly. In the extreme case, all operations could be put into a single cluster representing the entire problem instance. Hence, in addition to the similarity of operations, our decomposition method also aims at balancing the number of operations per cluster.

Algorithm 1 provides a pseudocode description of our constrained clustering algorithm. Similar to K-Means, we assume that the algorithm gets the target number of clusters into which the operations shall be partitioned as input. The cluster capacity, used for balancing the operations per cluster, is then obtained by dividing the total number of operations by the number of clusters. Moreover, the clustering algorithm takes care of generating one initial centroid per cluster, given by randomly selected operations that are compatible with the precedence relation. For example, when each job consists of 15 operations and the target number of clusters is 3, the first centroid will be an operation at the first to fifth place of its job, the second an operation from place six to ten, and the third an operation at the eleventh or later place. In order to populate each cluster, the algorithm inspects features to determine the Euclidean distance of each yet unassigned operation to the centroid of the current cluster and assigns the nearest operation to the cluster. To also preserve the precedence between operations, we additionally include any yet unassigned predecessor operations in the current cluster, and then update its centroid with the features of newly assigned operations. Whenever the cluster capacity is reached, the algorithm proceeds to the next cluster, and this decomposition process continues until all operations are assigned to clusters.
Algorithm 1 Constrained Clustering Algorithm

**Input:** operations, num_clusters

$cluster\_capacity \leftarrow \left\lceil \frac{|\text{operations}|}{num\_clusters} \right\rceil$

Generate num_clusters many centroids

for $n = 1$ to num_clusters do

$clusters[n] \leftarrow \emptyset$

$current\_capacity \leftarrow cluster\_capacity$

while $0 < current\_capacity$ do

Calculate distance between data objects and $n$th centroid $\triangleright$ Using Euclidean distance

$O_{i,j} \leftarrow$ Nearest data object from operations

repeat

$current\_capacity \leftarrow current\_capacity - 1$

operations $\leftarrow$ operations $\setminus \{O_{i,j}\}$

$clusters[n] \leftarrow clusters[n] \cup \{O_{i,j}\}$ $\triangleright$ Assigning operation $O_{i,j}$ to $n$th TW

$j \leftarrow j - 1$

until $O_{i,j} \notin operations$ $\triangleright$ Satisfying the precedence constraint

Update the $n$th centroid

end while

end for

In order to identify the most promising features for distance calculation among those introduced in the previous subsection, we suggest the following forward selection principle: start with a small set of features, perform decomposition by Algorithm 1 with several seeds to generate the initial centroids, and iteratively solve JSP instances with the obtained time windows. Then, evaluate the possible extensions by one more feature, compare the quality of resulting schedules, and pick the best extended set of features. This process continues until either (i) all features are selected or (ii) any extension by another feature leads to solutions of lower average quality.

3.5. **Problem Encoding.** Given a JSP instance as in Listing 1 along with facts like those in Listing 3 providing a decomposition into time windows, the idea of successive schedule optimization is to consider time windows one after the other and gradually extend a partial schedule that fixes the operations of previous time windows. In this process, we adopt the makespan as optimization objective for scheduling the operations of each time window, thus applying the rule of thumb that small scheduling horizons for partial schedules are likely to lead towards a global solution with short makespan. While we use DL variables to compactly represent the starting times of operations to schedule, we assume that a partial schedule for the operations of previous time windows is reified in terms of additional input facts of the form $\text{start}((j, s), t, w)$, where $t$ is the starting time scheduled for the step $s$ of job $j$ at the previous time window indicated by $w$.

The $\text{step}(w)$ subprogram until line 29 in Listing 5 constitutes the central part of our multi-shot ASP modulo DL encoding, whose parameter $w$ stands for consecutive integers from 1 identifying time windows to schedule. Auxiliary atoms of the form $\text{use}(m, w', w)$, supplied by the rules in lines 3 and 5, indicate the latest time window $1 \leq w' \leq w$ including some operation that needs to be processed by machine $m$. The next rule in lines 7-12 identifies pairs $(j_1, s_1)$ and $(j_2, s_2)$ of operations sharing the same machine $m$, where $(j_2, s_2)$ belongs to the time window $w$ and $(j_1, s_1)$ is either (i) contained in the latest time window $1 \leq w' < w$ indicated by $\text{use}(m, w', w - 1)$ or (ii) also part of the time window $w$, in which case $j_1 < j_2$ establishes an asymmetric representation for
#program step(w).
use(M,w,w) :- operation(J,S,M,P), window(J,S,w).
use(M,W,w) :- use(M,W,w - 1), not window(J,S,w) : operation(J,S,M,P).

share((J1,S1),(J2,S2),P1,P2,X,w) :- operation(J1,S1,M,P1),
                operation(J2,S2,M,P2),
                window(J1,S1,W),
                window(J2,S2,w),
                (W,J1) < (w,J2),
                use(M,W,w - X), X = 0..1.

order(O1,O2,P1,w) :- share(O1,O2,P1,P2,1,w).
order((J,S1),(J,S2),P,w) :- operation(J,S1,M,P), window(J,S2,w),
                                S1 = S2 - 1.

(order(O1,O2,P1,w)) :- share(O1,O2,P1,P2,0,w).
order(O2,P1,w) :- share(O1,O2,P1,P2,0,w),
                  not order(O1,O2,P1,w).

&diff{O - 0} <= T :- start(O,T,w - 1).
&diff{0 - O} <= T :- start(O,T,w - 1).
&diff{0 - (J,1)} <= 0 :- window(J,1,w).
&diff{0 - O2} <= -P :- order(O1,O2,P1,w).
&diff{(J,S) - makespan} <= -P :- operation(J,S,M,P), window(J,S,w),
                                    not window(J,S + 1,w).

#program optimize(m).
#external horizon(m).

&diff{makespan - 0} <= m :- horizon(m).

LISTING 5. Multi-shot ASP modulo DL encoding

the pair of operations in derived atoms share((j1,s1), (j2,s2), p1, p2, x, w). If the flag x = 1 signals that (j1,s1) belongs to a previous time window w', the rule in line 14 derives the atom order((j1,s1), (j2,s2), p1, w) to express that (j1,s1) needs to be completed before performing (j2,s2), i.e., the execution order must comply with the decomposition into time windows. The rule in lines 15-16 yields a similar atom when (j1,s1) is the predecessor operation s1 = s2 - 1 of (j2,s2) in the same job j1 = j2. In contrast to the cases in which (j1,s1) must be processed before (j2,s2), the choice rule in line 18 allows for performing two operations sharing a machine in the lexicographic order of their jobs if the operations belong to the same time window. In case the atom representing execution in lexicographic order is not chosen, the rule in lines 19-20 derives an atom expressing the inverse, given that the operations must not intersect and some sequence has to be determined. Note that the rules in lines 18-20 allow for choosing an execution order between two
operations \((j_1, s_1)\) and \((j_2, s_2)\) only if they belong to the same time window \(w\) but distinct jobs \(j_1 < j_2\), while the rules in lines 14-16 handle pairs of operations whose execution order is fixed.

Given that the rules up to line 20 yield atoms of the form \(\text{order}(o_1, o_2, p_1, w)\), expressing the hard requirement (or choice) to perform an operation \(o_1\) with processing time \(p_1\) before an operation \(o_2\) that belongs to time window \(w\), the remaining rules assert corresponding DL constraints. To begin with, the starting times of operations \(o\) from the previous time window \(w - 1\) (if any) are in the lines 22 and 23 fixed by restricting them from above or below, respectively, to the value \(t\) in an (optimized) partial schedule for time window \(w - 1\), as supplied by reified facts \(\text{start}(o, t, w - 1)\).

In line 25, the lower bound 0 is asserted for the starting time of the first operation of some job included in the time window \(w\) for which a partial schedule is to be determined next. In addition, DL constraints reflecting the order of performing operations are imposed in line 26, which concerns operations sharing a machine as well as successor operations within jobs. Since such constraints trace the sequence of operations in a job, they establish the earliest starting time, considered for problem decomposition in Section 3.2, as lower bound for scheduling an operation, and the execution order on the machine processing the operation can increase its starting time further. The last rule of the \(\text{step}(w)\) subprogram in lines 28-29 asserts that the value for the DL variable \(\text{makespan}\) cannot be less than the completion time of any operation of the time window \(w\). As a consequence, the least feasible \(\text{makespan}\) value provides the scheduling horizon of a partial schedule for operations of time windows up to \(w\).

The task of optimizing the horizon of a (partial) schedule means choosing an execution order of operations of the latest time window sharing some machine such that the value for the \(\text{makespan}\) variable is minimized. In single-shot ASP modulo DL solving with \text{clingo}[DL], such minimization can be accomplished via the command-line option --minimize-variable=\(\text{makespan}\). This option, however, is implemented by means of a fixed control loop that cannot be combined with (other) multi-shot solving methods. For the successive optimization based on time windows, where the scheduling horizon gradually increases, we thus require a dedicated treatment of DL constraints limiting the value for \(\text{makespan}\). To this end, the \(\text{optimize}(m)\) subprogram below line 31 declares an external atom \(\text{horizon}(m)\) for controlling whether a DL constraint asserted in line 34 is active and limits the \(\text{makespan}\) value to an integer supplied for the parameter \(m\).

### 3.6. Iterative Scheduling.

The main steps of our control loop for successive schedule optimization by multi-shot ASP modulo DL solving, implemented by means of the Python interface of \text{clingo}[DL], are displayed in Figure 3 and further detailed in the following. When launching the optimization process for a new time window, any instances of the \(\text{horizon}(m)\) atom introduced before are set to false in step 1 for making sure that some (partial) schedule \(X\) is feasible. No such atoms have been introduced yet for the first time window \(w = 1\), where the static (default) subprogram called \(\text{base}\), supplying a JSP instance along with its decomposition in terms of facts over window/3, and the \(\text{step}(1)\) subprogram for operations of the first time window are instantiated in steps 2 and 3.

Once some schedule \(X\) with a horizon \(h + 1\) is found in step 4a, the step 4b consists of instantiating the subprogram \(\text{optimize}(h)\) on demand, i.e., in case \(h\) has not been passed as a value for \(m\) before, and the corresponding \(\text{horizon}(h)\) atom is set to true in step 4c for activating the DL constraint limiting the admitted scheduling horizon to \(h\). The step 4 of successively reducing the horizon \(h\) in order to find better partial schedules stops when the imposed \(\text{makespan}\) value turns out to be infeasible, meaning that an optimal partial schedule has been found. As already mentioned above, the introduced instances of the external \(\text{horizon}(m)\) atom are then set to false in step 1 on the next iteration, and the successive optimization proceeds by in steps 2 and 3 instantiating the \(\text{step}(w + 1)\) subprogram for the next time window \(w + 1\) (if any) and also supplying the determined
Starting with $H := \emptyset$, for $w := 1$ to number $n$ of time windows:

1. For each $h \in H$, set $\text{horizon}(h)$ to false
2. If $w = 1$, then $P := \{\text{base}\}$; else $P := \{\text{start}((j, s), t, w - 1), (j, s) = t \} \subseteq X$
3. Ground $P \cup \{\text{step}(w)\}$
4. While there is some answer set $X$ and the time limit per time window is not reached:
   a. $h := t - 1$, where $(\text{makespan} = t) \in X$
   b. If $h \notin H$, then $H := H \cup \{h\}$ and ground $\{\text{optimize}(h)\}$
   c. Set $\text{horizon}(h)$ to true

**Figure 3.** Control loop for successive schedule optimization by multi-shot ASP modulo DL solving

starting times of operations from time window $w$ by reified facts. The described control loop for successively extending good-quality partial schedules to a global solution can be configured with a time limit, included as secondary stopping condition in step 4, to restrict the optimization efforts per time window and thus make sure that the iterative scheduling progresses.

For illustrating some phenomena going along with problem decomposition and iterative scheduling, let us inspect the schedule in Figure 4 that can be obtained with the decomposition into windows given in Listing 3. The separation between the two time windows is indicated by bold double lines marking the completion of the latest operation of the first time window on each of the three machines. Notably, the partial schedule for the first time window as well as its extension to the second time window are optimal in terms of their respective makespan. However, the obtained global solution has a makespan of 21 rather than just 20 as for the optimal schedule in Figure 1. The reason is that the second operation of job 3 would need to be scheduled before the second operation of job 2 on machine 1, while the decomposition into time windows dictates the inverse order and necessitates the later completion of job 3. Given the available buffer for scheduling operations with comparably short processing times on machine 3, the third operation of job 1 can be performed after the third operation of job 2 without deteriorating the makespan, yet introducing an unnecessary idle time from 9 to 10 on machine 3 that could be avoided by choosing the inverse execution order. Even though this may seem negligible for the example instance at hand, idle slots can potentially propagate when a partial schedule gets extended to later time windows.

In order to counteract limitations of window-wise successive optimization due to “decomposition mistakes” as well as unnecessary idle times that do not directly affect the makespan, we have devised two additional techniques that can be incorporated into the iterative scheduling process. The first extension is time window overlapping, where a configurable percentage of the operations per time window can still be rescheduled when proceeding to the next time window. To this end, an (optimized)

**Figure 4.** Decomposed schedule for the example JSP instance in Listing 1
partial schedule is postprocessed and the configured number of operations to overlap are picked in
decreasing order of starting times, as encoded by the following stratified ASP program:

\[
\text{current}((J,S),T,P,W) :- \text{operation}(J,S,M,P), \text{start}((J,S),T,W), \\
\quad \text{window}(J,S,W).
\]

\[
\text{current}((J,S),T,P,W) :- \text{operation}(J,S,M,P), \text{start}((J,S),T,W), \\
\quad \text{overlap}((J,S),W-1).
\]

\[
\text{inverse}(O,N,W) :- \text{current}(O,T,P,W), \\
\quad N = \#\text{count}(O' : \text{current}(O',T',P',W), (T,P,O) < (T',P',O')).
\]

\[
\text{overlap}(O,W) :- \text{inverse}(O,N,W), \text{portion}(C), N < C.
\]

In addition to facts specifying a JSP instance and time windows like in Listings 1 and 3, the inputs
consist of reified facts of the forms \(\text{start}(o,t,w)\) and \(\text{overlap}(o,w-1)\), giving the starting
times \(t\) of operations \(o\) in a partial schedule for time window \(w\) along with the overlapped operations
picked for the previous time window \(w-1\) (if any), as well as a fact of the form \(\text{portion}(c)\),
whose argument \(c\) provides the absolute number of operations to reschedule per time window.
Then, (derived) facts over \(\text{current}/4\) indicate the operations that were scheduled for the time
window \(w\), including those belonging to the overlap from time window \(w-1\), together with their
starting and processing times. The latter are considered to determine (derived) facts of the form
\(\text{inverse}(o,n,w)\). such that indexes \(n\) ranging from 0 are decreasing by starting times, where
processing times and operation identifiers serve as tie-breakers. If \(n < c\) applies, the respective
operation \(o\) belongs to the \(c\) operations with the latest starting times for the time window \(w\) and is
taken as overlap represented in the form \(\text{overlap}(o,w)\). For example, if one operation, matching
20% of the size 5 of time windows, from the first time window is taken as overlap for the decomposed
schedule in Figure 4, the second operation of job 2 with the latest starting time 4 (and processing
time 6) is chosen in view of the derived atoms \(\text{inverse}(2,2,4,1)\) and \(\text{overlap}(2,2,1)\),
which then enables its processing after the second operation of job 3 by rescheduling together with
operations of the second time window.

The addition of overlapping operations to the encoding in Listing 5 requires handling them
similar to operations of the time window to schedule, i.e., enabling the choice of an execution order
by the rules in lines 18-20 rather than fixing the order by the rule in line 14. Given that overlapped
operations \(o\) for a time window \(w-1\) are supplied by reified facts of the form \(\text{overlap}(o,w-1)\),
augmenting the \(\text{step}(w)\) subprogram in Listing 5 with the rules

\[
\text{share}((J1,S1),(J2,S2),P1,P2,0,w) :- \text{operation}(J1,S1,M,P1), \\
\quad \text{operation}(J2,S2,M,P2), \\
\quad \text{overlap}((J1,S1),w-1), \\
\quad \text{window}(J2,S2,w).
\]

\[
\text{share}((J1,S1),(J2,S2),P1,P2,1,w) :- \text{operation}(J1,S1,M,P1), \\
\quad \text{operation}(J2,S2,M,P2), \\
\quad \text{overlap}((J1,S1),w-2), \\
\quad \text{window}(J2,S2,w), \\
\quad \text{not overlap}((J1,S1),w-1).
\]

\[
\text{share}((J1,S1),(J2,S2),P1,P2,1,w) :- \text{operation}(J1,S1,M,P1), \\
\quad \text{operation}(J2,S2,M,P2), \\
\quad \text{overlap}((J1,S1),w-2), \\
\quad \text{overlap}((J2,S2),w-1), \\
\quad \text{not overlap}((J1,S1),w-1).
\]
as well as the additional body literals not overlap((J1, S1), w-1) or not overlap(O, w-1), respectively, for the rules in lines 7-12, 22, and 23 enables the rescheduling of overlapped operations together with the next time window, or fixes the execution order and starting times in case of operations that were but are no longer overlapping.

As the second extension, we can make use of the stratified ASP program in Listing 6 to postprocess an (optimized) partial schedule by inspecting operations of the latest time window in the order of starting times whether idle slots on their machines allow for an earlier execution. To this end, the encoded greedy compression approach traverses the operations of the latest time window in (any total) non-decreasing order of starting times and accomplishes the following step for each operation: remove the operation from the schedule and reinsert it at the earliest time after the completion of its predecessor operation (if any) such that the machine executing the operation is available for the operation’s processing time. In the worst case, an operation is reinserted into the partial schedule at its original starting time, so that the postprocessing never deteriorates the makespan.

In more detail, the rules in lines 1-7 distinguish the operations scheduled for the current time window from those whose starting times have already been fixed before. An order to traverse the former one by one according to their starting times is determined by the rules in lines 9-11. Beforehand, the time intervals [t, u] at which machines m process previously scheduled operations are represented by atoms of the form occupied(m, t, u, n, 0), derived by the rules in lines 13-15, where occupied(m, 0, 0, 0, 0) is included as base case and the execution of the n-th operation in the order of starting times on machine m is indicated otherwise. The last interval n at which a machine m is processing some previously scheduled operation is pointed out by an finished(m, n, 0) atom provided by the rule in lines 24-25. Moreover, the rules in lines 31-34 yield atoms of the form released((j, s, m, p, r, w, i), where (j, s) is traversed as i-th operation during compression and r is the completion time of its predecessor operation, which may itself result from a move to an earlier idle slot, or 0 if (j, 1) is the first operation of its job j. The earliest starting time given by r is inspected by the rules in lines 36-40, which iterate over the time intervals [t, u] at which the machine m processing (j, s) is occupied and stop with an atom consider((j, s, m, p, r, t, u, w, n, i) at the first such [t, u] interval after which (j, s) can be reinserted into the partial schedule. If this n-th time interval is the last at which machine m is processing some previously scheduled operation, the rule in lines 42-44 applies, or the rule in lines 45-48 otherwise, and in either case (j, s) is reinserted at the starting time r' = max{r, u}, as expressed by an atom reinsert((j, s, m, r', r' + p, w, n, i). The resulting partial schedule incorporates [r', r' + p] as the (n + 1)-th time interval at which the machine m is occupied, where the rules in lines 16-20 and 26-27 update the intervals for m, while the rules in lines 21-22 and 28-29 reflect that the operations processed by other machines remain unchanged. Finally, the rules in lines 50-51 provide the compressed partial schedule in terms of atoms of the form start'((j, s), t, w), thus resembling the input facts start((j, s), t, w).

As mentioned above, the updated starting times resulting from the compression are earlier or the same as the original starting times calculated in the actual makespan optimization. Hence, the latter can be safely exchanged by mapping start'((o, t, w) atoms back to start((o, t, w) for proceeding with the iterative scheduling process, i.e., determining operations to overlap and moving on to the next time window. For example, when compressing the decomposed schedule in Figure 4, the starting time 12 of the third operation of job 1 is turned into 9 to fill the idle slot available on machine 3, which is reflected by start'((1, 3), 9, 2) in the answer set obtained with the encoding in Listing 6. Then, start((1, 3), 12, 2) can be replaced by start((1, 3), 9, 2) to make machine 3 available from time 12 instead of 13 in case there were a third time window with operations to be processed by machine 3.
compress(J,S,M,P,T,W) :- operation(J,S,M,P), start((J,S),T,W),
                   window(J,S,W).
compress(J,S,M,P,T,W) :- operation(J,S,M,P), start((J,S),T,W),
                   overlap((J,S),W - 1).
constant(J,S,M,T,T + P,W) :- operation(J,S,M,P), start((J,S),T,W),
                           not compress(J,S,M,P,T,W).
traverse(J,S,M,P,W,I) :- compress(J,S,M,P,T,W),
                        I = #count{J',S' : compress(J',S',M',P',T',W), (T',J')<(T,J)}.
traverse(I) :- traverse(J,S,M,P,W,I).
occupied(M,0,0,0,0) :- traverse(0), operation(J,S,M,P).
occupied(M,T,U,N,0) :- traverse(0), constant(J,S,M,T,U,W),
                      N = #count{J',S' : constant(J',S',M',T',U',W), T' <= T}.
occupied(M,T,U,N,I) :- traverse(I), reinsert(J,S,M,T,U,W,N-1,I-1).
occupied(M,T,U,N,I) :- traverse(I), occupied(M,T,U,N',I - 1),
                      reinsert(J,S,M,T',U',W,N'',I - 1),
                      D = N' - N'', E = (D + 1 - |D - 1|) / 2,
                      N = N' + (E + |E|) / 2.
occupied(M,T,U,N,I) :- traverse(I), occupied(M,T,U,N',I - 1),
                      reinsert(J,S,M',T',U',W,N',I - 1), M' != M.
finished(M,N,0) :- occupied(M,T,U,N,0),
                N = #count{J',S' : constant(J',S',M',T',U',W)}.
finished(M,N,I) :- traverse(I), finished(M,N - 1,I - 1),
                reinsert(J,S,M,T,U,W,N','I - 1).
finished(M,N,I) :- traverse(I), finished(M,N,I - 1),
                reinsert(J,S,M',T,U,W,N',I - 1), M' != M.
released(J,1,M,P,0,W,I) :- traverse(J,1,M,P,W,I).
released(J,S,M,P,T' + P',W,I) :- traverse(J,S,M,P,W,I),
                               start'((J,S - 1),T',W),
                               operation(J,S - 1,M',P').
consider(J,S,M,P,R,T,U,W,0,I) :- released(J,S,M,P,R,W,I),
                                  occupied(M,T,U,0,I).
consider(J,S,M,P,R,T,U,W,N,I) :- consider(J,S,M,P,R,T',U',W,N-1,I),
                                   occupied(M,T,U,N,I),
                                   T < (R + U' + |R - U'|) / 2 + P.
reinsert(J,S,M,R',R' + P,W,N,I) :- consider(J,S,M,P,R,T,U,W,N,I),
                                   finished(M,N,I),
                                   R' = (R + U + |R - U|) / 2.
reinsert(J,S,M,R',R' + P,W,N,I) :- consider(J,S,M,P,R,T,U,W,N,I),
                                   occupied(M,T',U',N + 1,I),
                                   R' = (R + U + |R - U|) / 2,
                                   R' + P <= T'.
start'((J,S),T,W) :- constant(J,S,M,T,U,W).
start'((J,S),T,W) :- reinsert(J,S,M,T,U,W,N,I).

Listing 6. Schedule compression encoding
4. Experiments

For evaluating our multi-shot ASP modulo DL approach to JSP solving, we ran experiments on JSP benchmark sets due to Taillard and Demirkol [Tai93, DMU98], each including ten instances with $50 \times 15$ jobs and machines. The instances are generated such that each job consists of 15 operations, where the sequence of machines processing the operations varies from job to job. In [ESG22], we focussed on these instances as well because they yield representative relative results among different decomposition strategies, while their uniform size helps for configuring an appropriate number of time windows to use with every decomposition strategy. For scalability experiments in the second part of this section, we additionally run selected decomposition strategies on a benchmark set of industrial-size JSP instances due to Da Col and Teppan [DCT22].

In our experiments, we assess the decomposition strategies presented in Sections 3.2 and 3.4 with different numbers (and sizes) of time windows as well as the impact of the time window overlapping and compression techniques described in Section 3.6. For the comparability of results between runs with a different number of time windows and respective optimization subproblems addressed by multi-shot solving, we divide the total runtime limit of 1000 seconds for clingo [DL] (version 1.3.0) by the number of time windows to evenly spend optimization efforts on subproblems, where all experiments have been run on an Intel® Core™ i7-8650U CPU Dell Latitude 5590 machine under Windows 10.

Our first comparison, shown in Table 2, concerns the Machine-based Earliest Starting Time (M-EST) decomposition strategy, which turned out as most effective among the considered static decomposition strategies, with varying numbers of time windows in separate columns and compression excluded in the upper or included in the lower part, respectively. For both benchmark sets, we report the average makespan over the ten instances, where smaller values indicate better schedules, the average runtime of clingo [DL], and the average number of interrupted optimization processes on subproblems, where the best partial schedule obtained in time is taken to progress with the iterative

| Instances | Average | 1        | 2        | 3        | 4        | 5        | 6        | 10       |
|-----------|---------|----------|----------|----------|----------|----------|----------|----------|
| Taillard  | Makespan| 3542.1   | 3149.7   | **3083.8** | 3110.7   | 3213.6   | 3225.3   | 3524.9   |
|           | Time    | 1000.0   | 1000.0   | 1000.0   | 934.4    | 615.5    | 243.8    | 18.5     |
|           | Interrupts| 1.0      | 2.0      | 3.0      | 3.6      | 2.7      | 1.0      | 0.1      |
| Demirkol  | Makespan| 8589.8   | 7436.1   | **7283.2** | 7327.8   | 7548.0   | 7727.4   | 8912.7   |
|           | Time    | 1000.0   | 1000.0   | 1000.0   | 882.4    | 640.2    | 486.8    | 68.7     |
|           | Interrupts| 1.0      | 2.0      | 3.0      | 3.4      | 2.8      | 2.4      | 0.5      |
| Taillard  | Makespan| 3542.1   | 3068.6   | 3015.9   | **3010.3** | 3040.7   | 3058.2   | 3151.6   |
|           | Time    | 1000.0   | 1000.0   | 1000.0   | 907.5    | 573.8    | 274.3    | 18.6     |
|           | Interrupts| 1.0      | 2.0      | 3.0      | 3.4      | 2.5      | 1.4      | 0.1      |
| Demirkol  | Makespan| 8589.8   | 7138.2   | 6734.4   | 6727.2   | **6698.6** | 6840.9   | 7063.4   |
|           | Time    | 1000.0   | 1000.0   | 970.5    | 826.9    | 541.5    | 307.8    | 35.0     |
|           | Interrupts| 1.0      | 2.0      | 2.9      | 3.1      | 2.2      | 1.4      | 0.1      |

1The JSP benchmark sets and our implementation are available at: https://github.com/prosysscience/Job-Shop-Scheduling
Table 3. Comparison of different static decomposition strategies

| Strategy   | Makespan | Taillard Time | Interrupts | Demirkol Makespan | Demirkol Time | Interrupts |
|------------|----------|---------------|------------|-------------------|---------------|------------|
| J-EST      | 3492.1   | 1000.0        | 3.0        | 7789.3            | 1000.0        | 3.0        |
| J-MTWR     | 3440.2   | 1000.0        | 3.0        | 7738.2            | 1000.0        | 3.0        |
| M-EST      | **3015.9** | 1000.0       | 3.0        | **6734.4**        | 970.5         | 2.9        |
| M-MTWR     | 3060.3   | 1000.0        | 3.0        | 6861.5            | 1000.0        | 3.0        |
| Clustering | 3256.4   | 1000.0        | 3.0        | 7174.6            | 1000.0        | 3.0        |

scheduling. We gradually increase the number of time windows from 1 to 6 and additionally include results for 10 time windows to outline the trend of degrading solution quality when the partition into time windows becomes too fragmented. In fact, the shortest average makespans, highlighted in boldface, are obtained with problem decomposition into 3, 4, or 5 time windows, each consisting of 750/3 = 250, 750/4 ≈ 187, or 750/5 = 150 operations, respectively. We observe that the average makespans with compression in the lower part of Table 2 are substantially shorter, apart from the indifferent single-shot optimization with 1 time window only. The latter represents global optimization on the full problem, and decompositions into more than one time window clearly improve the solution quality. These advantages are not surprising, considering that the JSP instances are highly combinatorial [SS18] and each global optimization run times out with a more or less optimized solution. Since the compression technique consistently improves the solution quality, we commit to it in the comparisons addressed below. We also fix the number of time windows to 3 from now on in order to reduce variable parameters, while other numbers may occasionally be better.

In Table 3, we compare the previously considered M-EST decomposition to the Machine-based Most Total Work Remaining (M-MTWR) strategy, their Job-based versions J-EST and J-MTWR, as well as constrained clustering using the best feature set among those investigated in [ESG22] for each instance. The considerably increased average makespans in the first two rows clearly indicate that time windows determined with Job-based decomposition strategies are less adequate than those investigating bottleneck machines in the first place, and then picking their operations based on the smallest EST or greatest MTWR value as a secondary criterion. While problem decomposition by constrained clustering also improves over the Job-based J-EST and J-MTWR strategies, it remains behind M-EST and M-MTWR, whose differences in average makespan are less pronounced. This observation suggests that bottleneck machines should play the most prominent role in decomposition and scheduling heuristics, while other features provide secondary criteria for picking operations.

While the decomposition strategies considered so far were based on static properties, Table 4 provides results for decomposition based on the starting times of operations in schedules obtained with greedy search methods using First-In-First-Out (FIFO), Most-Total-Work-Remaining (MTWR), and Reinforcement Learning (RL) [TGS21] as heuristics for selecting the next operation to allocate in step 2 of Figure 2. The job features explored by the RL policy, namely, the remaining processing time (RPT) and the waiting time (WT), combine the FIFO and MTWR criteria for selecting operations. Going beyond the basic FIFO and MTWR heuristics, the RL policy includes flexibility to perform noops in step 3 of Figure 2, meaning that a machine can be kept free for operations getting released later on, which enables global solutions that are less greedy than operation allocation by FIFO or MTWR. We observe that the average makespan of schedules for FIFO and MTWR is larger than with the best static strategy M-EST, while the latter is outperformed by RL. Hence, we conclude...
that a machine learning approach should be used to generate initial schedules, which can then be decomposed into time windows by taking operations in the order of their starting times and further optimizing the time windows one after the other. Moreover, our preliminary experiments with static decomposition strategies suggested that overlapping techniques can be beneficial, yet we obtain negative results for the RL decomposition in Table 5. In fact, the RL policy is (re)trained on each instance and then manages to generate a good-quality schedule that is all but easy to improve further, which is a likely reason that the possibility to reschedule overlapping operations does not yield solutions of better quality.

In Table 6, we additionally compare the average makespan of schedules obtained with single- and multi-shot ASP modulo DL solving, the latter taking RL solutions for problem decomposition, to the (global) solution quality achieved by the state-of-the-art CP systems CP-Optimizer (version 22.1.1) [LRSV18] and OR-Tools (version 9.5) [PDG23]. We use CP encodings of JSP supplied by the CP system developers [ILO, PF], set the runtime limit to 1000 seconds, as also taken for clingo [DL], and indicate the distance between the average makespan of obtained schedules and the optima known for Taillard’s or within ranges for Demirkol’s instances. We observe that CP-Optimizer, which succeeds to find optimal schedules and terminates its runs before the time limit for Taillard’s instances, as well as OR-Tools have an edge on our single- and multi-shot ASP modulo DL approaches. This can be explained by the availability of interval variables and global constraints in the CP encodings, which turn out to be particularly effective in modeling the non-intersecting execution of operations on each machine. However, the RL decomposition and multi-shot optimization help clingo [DL] to come substantially closer than single-shot ASP modulo DL solving on the full problem.

The impressive performance of CP systems for JSP solving is also confirmed by the experimental observations made in [DCT22, TGS21], and the JSP instances with $50 \times 15$ jobs and machines are small enough for them to find good-quality or even optimal solutions in limited time. For further assessing the scaling behavior, we extend the scope to industrial-size instances due to Da Col and Teppan [DCT22] with $100 \times 100$ jobs and machines. As these instances were not considered in [TGS21], initial schedules of the RL policy are unavailable, and we apply the J-EST, J-MTWR, M-EST, and M-MTWR strategies for problem decomposition into 30 time windows comprising

### Table 4. Comparison of FIFO, MTWR, and RL decomposition

| Strategy | Taillard | Demirkol |
|----------|----------|----------|
|          | Makespan | Time     | Interrupts | Makespan | Time     | Interrupts |
| FIFO     | 3257.1   | 1000.0   | 3.0        | 7880.1   | 1000.0   | 3.0        |
| MTWR     | 3234.1   | 1000.0   | 3.0        | 7109.6   | 1000.0   | 3.0        |
| RL       | 2943.5   | 1000.0   | 3.0        | 6458.3   | 1000.0   | 3.0        |

### Table 5. Comparison of time window overlapping techniques using RL decomposition

| Overlap | Makespan | Taillard | Demirkol |
|---------|----------|----------|----------|
|         |          | Time     | Interrupts | Makespan | Time     | Interrupts |
| 0%      | 2943.5   | 1000.0   | 3.0        | 6458.3   | 1000.0   | 3.0        |
| 10%     | 2952.9   | 1000.0   | 3.0        | 6486.9   | 1000.0   | 3.0        |
| 20%     | 2954.9   | 1000.0   | 3.0        | 6563.6   | 1000.0   | 3.0        |
| 30%     | 2969.0   | 1000.0   | 3.0        | 6621.7   | 1000.0   | 3.0        |
TABLE 6. Comparison of single- and multi-shot ASP modulo DL solving approaches to CP-Optimizer and OR-Tools

| Strategy    | Taillard          | Demirkol         |
|-------------|-------------------|------------------|
|             | Makespan | Distance | Makespan | Distance |
| Optima      | 2773.8    | 0.0       | 5894.3–5950.6 | 0.0      |
| CP-Optimizer| 2773.8    | 0.0       | **6211.5** | 260.9–317.2 |
| OR-Tools    | 2778.3    | 4.5       | 6293.3    | 342.7–399.0 |
| Multi-shot  | 2943.5    | 169.7     | 6458.3    | 507.7–564.0 |
| Single-shot | 3542.1    | 768.3     | 8589.8    | 2639.6–2695.5 |

10000/30 ≈ 333 operations each. For reference, Table 7 reports the average makespans obtained by running the single-core configurations of CP-Optimizer and OR-Tools for 6 hours on a 2 GHz AMD EPYC 7551P 32 Cores CPU machine [DCT22], where CP-Optimizer has a significant edge on OR-Tools thanks to its memory-efficient implementation of the global constraint establishing non-intersecting operation execution for machines.

Accordingly, we also impose a total runtime limit of 6 hours, divided into up to 21600/30 = 720 seconds per time window. As a general trend, we see that the solution quality achieved by our decomposition strategies lies in-between CP-Optimizer and OR-Tools, with some advantages for the Job- and Machine-based MTWR strategies. That is, the previously encountered gap to OR-Tools on moderately sized instances is made up by better multi-shot optimization performance of clingo[DL] on the downscaled subproblems. The latter can also be observed on the average runtimes given in the third column of Table 7, indicating that all decomposition strategies lead to optimal partial schedules for more than half of the time windows and take only about 3 of the 6 hours available in total. Clearly, this performance improvement comes to the cost of a reduced overall solution quality, as the decomposition into 30 time windows constrains the potential execution order of operations. Hence, the good-quality schedules found by CP-Optimizer remain unmatched for the large-scale instances due to Da Col and Teppan.

TABLE 7. Comparison of different static decomposition strategies to CP-Optimizer and OR-Tools

| Strategy     | Da Col & Teppan |
|--------------|-----------------|
|              | Makespan | Time |
| CP-Optimizer | **80565.5** | 21600.0 |
| OR-Tools     | 114476.2   | 21600.0 |
| J-EST        | 103260.0   | 10615.8 |
| J-MTWR       | 100120.9   | 10464.8 |
| M-EST        | 105925.9   | 10327.4 |
| M-MTWR       | 100793.2   | 10503.9 |
5. Conclusions

Our work develops multi-shot ASP modulo DL methods for JSP solving by means of problem decomposition into balanced time windows, which respect the operation precedence within jobs, along with successive makespan minimization for extending partial schedules to a global solution. Several simple static decomposition strategies (denoted by J-EST, J-MTWR, M-EST, and M-MTWR) can be directly encoded in ASP, which provides declarative means to split highly combinatorial JSP instances into better manageable subproblems, leading to substantially better schedules than tackling the full problem at once with single-shot ASP modulo DL optimization. Our experiments with different decomposition strategies show that (i) the consideration of bottleneck machines is beneficial when problem decomposition is based on simple static properties, (ii) performing compression after optimizing each time window improves the quality of global solutions, (iii) decomposing and further optimizing schedules generated by greedy search methods is preferable to static decomposition strategies, even when the latter incorporate clustering w.r.t. a variety of features, and (iv) time window overlapping does not improve the solution quality further when the problem decomposition process incorporates proficient heuristic methods, where RL policies turn out to be particularly successful. No matter which decomposition strategy is applied, for runtime limits of \(5\) to \(12\) minutes per time window, a few hundred operations in each time window, namely, between \(250\) and \(350\) operations, turned out to be adequate for achieving a robust performance of optimizing partial schedules by \textit{clingo}[DL].

The main focus of our work is on investigating how problem decomposition and multi-shot optimization can be exploited to reliably find good-quality solutions for JSP instances on which single-shot ASP modulo DL solving fails. Simple static decomposition strategies, especially the four we encoded in ASP, should thus be understood as tools for partitioning the operations of JSP instances into time windows, yet without claiming that these decomposition strategies would be very elaborate or highly likely to come close to global optima. The experimental finding that the RL decomposition performs best indicates that applying heuristic search methods to generate initial solutions that can then be optimized further is a promising approach. This matches the principle of Large Neighborhood Search [PR10], where a dedicated framework has recently been introduced for ASP (modulo DL) [EGR+22], and in our case the subproblems based on time windows constitute the neighborhood of an RL solution.

The comparison with the state-of-the-art CP systems CP-Optimizer and OR-Tools yields that global optimization by the latter is still ahead of our multi-shot ASP modulo DL methods on moderately sized JSP instances including 750 operations to be scheduled. In fact, CP-Optimizer and OR-Tools take advantage of dedicated interval variables and global constraints, designed specifically for scheduling problems, in their CP encodings. However, for industrial-size instances with 10000 operations, OR-Tools does not scale well anymore, and the problem decomposition promotes our multi-shot ASP modulo DL solving approach, while the memory-efficient global constraint implementation of CP-Optimizer remains unmatched. We conclude that modeling scheduling problems in terms of the global constraints supplied by CP systems, which also outperform Mixed Integer Programming approaches [KB16], is likely to yield better single-shot optimization performance than the plain difference constraints of \textit{clingo}[DL]. The first-order modeling language and multi-shot solving support available in ASP modulo DL are nonetheless advantageous for prototyping a variety of decomposition strategies, while adopting CP systems for the window-wise successive optimization of partial schedules would require dedicated implementations that are beyond the scope of this paper.

In ongoing and future work, we aim to extend our hybrid ASP solving approach to more complex and realistic scheduling scenarios, where features like (partially) flexible and multi-resource
allocation, preemptive maintenance, sequence-dependent setups, as well as batching and cascading operation execution are important. As a particular starting point, in [AEG23, EAG23] we prototypically address scheduling in the context of the SMT2020 semiconductor manufacturing setting [KHKM20], whose manufacturing processes involve jobs with hundreds instead of tens of operations and large-scale factories with thousands instead of hundreds of machines. The resulting problem size and complexity make partitioning into subproblems unavoidable, and beyond the time-wise decomposition of jobs, elaborate schemes to focus scheduling on subproblems concerning specific functionalities of machines and operations can be beneficial.

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REFERENCES

[ABZ88] Joseph Adams, Egon Balas, and Daniel Zawack. The shifting bottleneck procedure for job shop scheduling. Management Science, 34(3):391–401, 1988.
[AGM^+16] Michael Abseher, Martin Gebser, Nysret Musliu, Torsten Schaub, and Stefan Woltran. Shift design with answer set programming. Fundam. Informaticae, 147(1):1–25, 2016.
[Bal11] Marcello Balduccini. Industrial-size scheduling with ASP+CP. In LPNMR, volume 6645 of Lecture Notes in Computer Science, pages 284–296. Springer, 2011.
[Bak74] Kenneth Baker. Introduction to Sequencing and Scheduling. John Wiley & Sons, 1974.
[BIK^+19] Mutsunori Banbara, Katsumi Inoue, Benjamin Kaufmann, Tenda Okimoto, Torsten Schaub, Takehide Soh, Naoyuki Tamura, and Philipp Wanko. teaspoon: Solving the curriculum-based course timetabling problems with answer set programming. Ann. Oper. Res., 275(1):3–37, 2019.
[BPH82] John Blackstone, Don Phillips, and Gary Hogg. A state-of-the-art survey of dispatching rules for manufacturing job shop operations. International Journal of Production Research, 20(1):27–45, 1982.
[CFG^+20] Francesco Calimeri, Wolfgang Faber, Martin Gebser, Giovambattista Ianni, Roland Kaminski, Thomas Krennwallner, Nicola Leone, Marco Marat, Francesco Ricca, and Torsten Schaub. ASP-Core-2 input language format. Theory Pract. Log. Program., 20(2):294–309, 2020.
[CM06] Scott Cotton and Oded Maler. Fast and flexible difference constraint propagation for DPLL(T). In SAT, volume 4121 of Lecture Notes in Computer Science, pages 170–183. Springer, 2006.
[DCT22] Giacomo Da Col and Erich C. Teppan. Industrial-size job shop scheduling with constraint programming. Operations Research Perspectives, 9:100249, 2022.
[DFH21] Fatemeh Daneshamooz, Parviz Fattahi, and Seyed M. H. Hosseini. Mathematical modeling and two efficient branch and bound algorithms for job shop scheduling problem followed by an assembly stage. Kybernetes, 50(12):3222–3245, 2021.
[DMU98] Ebru Demirkol, Sanjay Mehta, and Reha Uzsoy. Benchmarks for shop scheduling problems. Eur. J. Oper. Res., 109(1):137–141, 1998.
[DGG^+21] Carmine Dodaro, Giuseppe Galatà, Andrea Grioni, Marco Marat, Marco Mochi, and Ivan Porro. An ASP-based solution to the chemotherapy treatment scheduling problem. Theory Pract. Log. Program., 21(6):835–851, 2021.
[EAG23] Mohammed M. S. El-Kholany, Ramsha Ali, and Martin Gebser. Hybrid ASP-based multi-objective scheduling of semiconductor manufacturing processes. In JELIA, volume 14281 of Lecture Notes in Computer Science, pages 243–252. Springer, 2023.
DECOMPOSITION STRATEGIES AND MULTI-SHOT ASP SOLVING FOR JOB-SHOP SCHEDULING

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[1] Philippe Laborie, Jerome Rogerie, Paul Shaw, and Petr Vilím. IBM ILOG CP optimizer for scheduling: 20+ years of scheduling with constraints at IBM/ILOG. Constraints, 23(2):210–250, 2018.

[2] Mohammad M. Nasiri, Sadegh Salehi, Ali Rahbari, Navid Salmanzadeh Meydani, and Mojtaba Abdollahi. A data mining approach for population-based methods to solve the JSSP. Soft Comput., 23(21):11107–11122, 2019.

[3] Irfan Ovacik and Reha Uzsoy. Decomposition Methods for Complex Factory Scheduling Problems. Springer, 2012.

[4] Laurent Perron, Frédéric Díüier, and Steven Gay. The CP-SAT-LP solver (invited talk). In CP, volume 280 of LIPIcs, pages 3:1–3:2. Schloss Dagstuhl, 2023.

[5] Laurent Perron and Vincent Furnon. OR-Tools. URL: https://developers.google.com/optimization/.

[6] Ferdinando Pezzella, Gianluca Morganti, and Giampiero Ciaschetti. A genetic algorithm for the flexible job-shop scheduling problem. Comput. Oper. Res., 35(10):3202–3212, 2008.

[7] David Pisinger and Stefan Ropke. Large neighborhood search. In Handbook of Metaheuristics, volume 146 of International Series in Operations Research & Management Science, pages 399–419. Springer, 2010.

[8] Teodor C. Przymusinski. On the declarative semantics of deductive databases and logic programs. In Foundations of Deductive Databases and Logic Programming, pages 193–216. Morgan Kaufmann, 1988.

[9] Francesco Ricca, Giovanni Grasso, Mario Alviano, Marco Manna, Vincenzino Lio, Salvatore Iiritano, and Nicola Leone. Team-building with answer set programming in the Gioia-Tauro seaport. Theory Pract. Log. Program., 12(3):361–381, 2012.

[10] Marcos Singer. Decomposition methods for large job shops. Comput. Oper. Res., 28(3):193–207, 2001.

[11] Atif Shahzad and Nasser Mebarki. Discovering dispatching rules for job shop scheduling problem through data mining. In MOSIM, pages 10–12, 2010.

[12] Oleg V. Shylo and Hesam Shams. Boosting binary optimization via binary classification: A case study of job shop scheduling. CoRR, abs/1808.10813, 2018.

[13] Orkunt Sabuncu and Mehmet C. Sımsek. Solving assembly line workload smoothing problem via answer set programming. In ICLP Workshops, volume 2678 of CEUR Workshop Proceedings. CEUR-WS.org, 2020.

[14] Gangquan Shi, Zhouwang Yang, Yang Xu, and Yuchen Quan. Solving the integrated process planning and scheduling problem using an enhanced constraint programming-based approach. Int. J. Prod. Res., 60(18):5505–5522, 2022.

[15] Éric D. Taillard. Benchmarks for basic scheduling problems. European Journal of Operational Research, 64(2):278–285, 1993.

[16] Pierre Tassel, Martin Gebser, and Konstantin Schekotihin. A reinforcement learning environment for job-shop scheduling. In PRL, pages 21:1–21:9, 2021. URL: https://prl-theworkshop.github.io/prl2021/papers/PRL2021_paper_9.pdf.

[17] Reha Uzsoy and Cheng-Shuo Wang. Performance of decomposition procedures for job shop scheduling problems with bottleneck machines. International Journal of Production Research, 38(6):1271–1286, 2000.

[18] Hegen Xiong, Shuangyuan Shi, Danni Ren, and Jinjin Hu. A survey of job shop scheduling problem: The types and models. Comput. Oper. Res., 142:105731, 2022.

[19] Yingni Zhai, Changjun Liu, Wei Chu, Ruifeng Guo, and Cunliang Liu. A decomposition heuristics based on multi-bottleneck machines for large-scale job shop scheduling problems. Journal of Industrial Engineering and Management, 7(5):1397–1414, 2014.

[20] Rui Zhang and Cheng Wu. A hybrid approach to large-scale job shop scheduling. Appl. Intell., 32(1):47–59, 2010.