2-Terminal Coaxial Capacitance Measurements and Cable Corrections

F. A. Silveira*

Instituto Nacional de Metrologia, Qualidade e Tecnologia – Inmetro,
Avenida N. S. das Graças 50, 25250-020 D. Caxias RJ, Brazil

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Coaxial 2-terminal bridge comparison is a widely used metrological method in the dissemination of value of capacitance up to 1000 pF and 10⁴ rad/s, in this range, capacitance can be measured with uncertainties as small as few parts in 10⁷; however, such measurements must take into account the corrections due to the impedance of cables, connectors and ports, which can typically amount up to 10⁻⁶ μF/F – and maybe larger. In this work, we report on how correction factors in 2-terminal pair coaxial bridge capacitance measurements are presently measured and calculated at Inmetro.

Keywords: Capacitance standards, coaxial bridges measurements.

I. INTRODUCTION

The Laboratory of Electrical Standards at Inmetro (Lampe) guarantees the traceability of the capacitance unit between 1 and 1000 pF, at frequencies from 400 to 10⁴ rad/s, through a chain of comparisons of capacitors in a 2-terminal pair (2T) coaxial bridge [1, 2]. This system is very stable, and went through but small modifications of its original project ever since it started operating in 2005 [3]. With this bridge, Lampe compares capacitors with uncertainties smaller than a few parts in 10⁷. In this range, corrections due to cabling are not always negligible, and must be considered.

![Diagram of 2T coaxial impedance bridge at 1:1 setting.](image)

**FIG. 1: 2T coaxial impedance bridge at 1:1 setting.**

In this context, correction assessment means determining the relation between the real and apparent impedances of the object in some specific measurement setup. The apparent impedance of a standard is expected to be a combination of its real impedance value and the shunt admittances of its ports and the cables that couple the standard to the measurement system. In our case, we seek to find the corrections to the capacitance measurements of an object capacitor that’s compared to a known standard capacitor in a coaxial 2T impedance bridge of the type shown in Fig. 1.

In this Figure, bold lines represent the outer conductors of the coaxial cables and the chassis of the circuit components; the thin lines represent the coax inner conductors.

Figure 1 illustrates the coaxial nature of the circuit and its main inductive voltage divider (IVD), the standard and object admittances A_S and A_X; the auxiliary ground level (Wagner) balance arms, C_W and R_W [3]: and the in-phase injection IVD and injection admittance A_P. The five double-bars that appear next to some of the cables represent coaxial chokes, and are meant to equalise the inner and outer currents of the choked cables [4].

In 2T bridge measurements, a fixed ratio of the main IVD is chosen, usually appropriate to decadic comparisons of the type 1:1, 1:10 or 1:100 between the capacitances. Then the injection IVD is adjusted until balance is attained, by null readings at the detector D. This balance is subject to one main constraint, though: the low potencial of the impedances are kept at the ground potencial through the current injected by C_W, R_W [3]. The final balance is attained after a few iterated adjustments of C_W, R_W and the injection IVD ratio [1, 3].

At bridge balance, the in-phase circuit admittances in Fig. 1 relate through the equation

\[ C'_X = \frac{1 - r}{r} C'_S + \frac{\alpha C_P}{r} \]

where C'_S,X are the apparent capacitances (related through corrections to the real value of the capacitances C_S,X) being compared, C_P is the capacitance component of A_P, α is the ratio of the in-phase injection IVD [2], and r is the ratio of the main IVD. Common r settings at the measurement chain in Lampe are r = 1/11 (1:10 comparisons) and r = 1/2 (1:1 comparisons, shown in Fig. 1).

It worths adding that, in practice, there must be a second injection system, in parallel to A_P and the in-phase injection IVD, that’s not shown in Fig 1. That system, usually called quadrature injection, has an injection admittance that’s predominantly conductive, and is meant to balance the small dissipative components of the standards [1, 2].

This work is organized as follows. In Sec. II we present the general lumped parameter model for 2T measurements of a combined impedance formed by the standard and its cables and ports, and derive a way of measuring corrections. In Sec. III we explore this model in a form suitable to treating capacitance 2T measurements, and in Sec IV we present some measurement results. Finally, we comment on these results and draw some conclusions at Sec. V.

*Email address: fsilveira@inmetro.gov.br
II. EQUIVALENT CIRCUIT MODEL

Figure 2 shows a convenient model in ac practice to represent the real impedance $Z$, and the equivalent admittance of cables and connectors that link it to the measurement circuit. In this model, the standard impedance $Z$ is represented with its high and low coaxial ports at B and C, and their respective shunt admittances $Y_B$ and $Y_C$. Cables connecting these ports to the external circuit, and their equivalent immittances $Z_1$, $Y_1$ and $Z_2$, $Y_2$, are represented by the meshes at A and D.

![FIG. 2: Model of 2T measurement of impedance $Z$.](image)

We associate to each independent loop a clockwise loop current, numbered from $I_1$ to $I_5$ (only $I_4$ and $I_5$ are shown in Fig. 2) and we apply the Kirchoff laws to find the relation between the real impedance $Z$ and the apparent impedance $Z'$. The latter is defined by $Z' = e_1/I_5$, and under the Wagner constraint ($e_5 = 0$) the loop equations read

$$I_1 \left( \frac{Z_1}{2} + \frac{1}{Y_1} \right) - I_2 \frac{Z_1}{Y_1} - Z'I_5 = 0$$

(2)

$$I_1 \frac{1}{Y_1} - I_2 \left( \frac{Z_1}{2} + \frac{1}{Y_1} + \frac{1}{Y_B} \right) + I_3 \frac{1}{Y_B} = 0$$

(3)

$$I_2 \frac{1}{Y_B} - I_3 \left( Z + \frac{1}{Y_B} + \frac{1}{Y_C} \right) + I_4 \frac{1}{Y_C} = 0$$

(4)

$$I_3 \frac{1}{Y_C} - I_4 \left( \frac{Z_2}{2} + \frac{1}{Y_C} + \frac{1}{Y_2} \right) + I_5 \frac{1}{Y_2} = 0$$

(5)

$$I_4 \frac{1}{Y_2} - I_5 \left( \frac{Z_2}{2} + \frac{1}{Y_2} \right) = 0$$

(6)

By the time of the elaboration of this report, measurements performed with a digital RLC bridge on the coax cables used in 2T bridge calibration services, and on one of our 100 pF standards, resulted (within combined standard deviations) $L_1 = (0.2836 \pm 0.0003) \mu H$, $C_1 = (72.19 \pm 0.05) \text{ pF}$ and $C_6 = (96.588 \pm 0.004) \text{ pF}$ at $10^4 \text{ rad/s}$. Then we may safely approximate $Z_{1,2} \sim 10^{-3} \Omega$ and $Y_{B,C} \approx Y_{1,2} \lesssim 10^{-6} \text{ S}$, and contributions to $Z'$ of powers higher than the product of these two quantities are expected to be or order $10^{-9}$ or smaller, and are considered negligible.

Solving the simultaneous Eqs. 2 to 6 is but a tedious calculation, which can be carried out by a number of methods. After collecting to the lower order, the first seven terms of the solution to this system are

$$Z' = Z \left[ 1 + \frac{Z_1 Y_1}{2} + Z_1 Y_B + \frac{Z_1}{Z} + \frac{Z_2 Y_2}{2} + Z_2 Y_C + \frac{Z_2}{Z} \right]$$

(7)

As we can expect from the ideal limit $Y_{1,2,B,C} \rightarrow 0$ of Fig. 2, $Z_{1,2}$ add directly to $Z$, and can be disregarded unless $Z$ is much larger than $Z_{1,2}$. Products like $Z_1 Y_1$ or $Z_2 Y_C$ come from earlier obvious series associations of $Z_{1,2}$ and $Y_{1,2,B}$, and of $Z_{2,3}$ and $Y_{2,3,5}$, that act as loaded voltage dividers cascaded along the current path. And although such terms can be $10^6$ times smaller than terms like $Z_{1,2}/Z$, they certainly can’t always be considered negligible when uncertainties are smaller than parts in $10^5$, as is the case.

This solution differs from the results obtained through a heuristic method applied over the same model in [1] that omits the last two terms in Eq. 7, and from the results in [2], that uses a less general, simpler model.

III. CORRECTIONS TO CAPACITANCE MEASUREMENTS

In the model of Fig. 2, the cable immittances have the general form $Z_{1,2} = R_{1,2} + j \omega L_{1,2}$ (lossy inductances) and $Y_{1,2,B,C} = j \omega C_{1,2,B,C}$ (pure capacitances), where $R_{1,2}$, $L_{1,2}$ and $C_{1,2}$ stand for the series short-circuit resistances and inductances, and the parallel open-circuit capacitances of the high and low cables; and $C_{B,C}$ are the open-circuit capacitances of the high and low ports of the standards, at the work angular frequency $\omega$. As we limit ourselves to the measurement of standard capacitors, we replace $Z^{-1}, Z'^{-1}$ by the pure capacitances $j \omega C$ and $j \omega C'$, and, by operating these substitutions on Eq. 7, we obtain

$$\frac{1}{C'} = \frac{1}{C} \left[ 1 - \frac{\omega^2}{2} \left( L_1 C_1 + L_2 C_2 \right) - \omega^2 \left( L_1 C_B + L_2 C_C \right) 
- \omega^2 \left( L_1 + L_2 \right) C + \frac{j \omega}{2} \left( R_1 C_1 + R_2 C_2 \right) 
+ j \omega \left( R_1 C_B + R_2 C_C \right) + j \omega \left( R_1 + R_2 \right) C \right]$$

(8)

As long as, for the purpose of correction calculations, $Z^{-1}, Z'^{-1}$ are considered pure susceptances from the start, the imaginary component of this equation has no physical meaning, as it would imply a nonexistent constraint on the values of $C$, $R_{1,2}$ and $C_{1,2,B,C}$. Solving the real part of this equation for the real capacitance $C'$, we obtain

$$C' = C \left[ 1 - \frac{\omega^2}{2} \left( L_1 C_1 + L_2 C_2 \right) - \left( L_1 C_B + L_2 C_C \right) \right]$$

(9)

In Lampe, our main capacitance standards are of gas-dielectric General Radio Co. models 1404A, 1404B and 1404C (10 to 1000 pF); and fused silica Andeen-Hagerling model AH11A (1 and 100 pF). Typically, $C_{B,C} \sim 100 \text{ pF}$ for these standards; and $L_{1,2} \sim 0.3 \mu H$ and $C_{1,2} \sim 100 \text{ pF}$ for a metre of good quality, commercially available RG-58 coaxial cable, as results in the Section [1] show.

With these figures, at $10^4 \text{ rad/s}$, therefore, $\omega^2 C' \left( L_1 + L_2 \right)$ ranges from $10^{-10}$ to $10^{-7}$ for capacitances with nominal value between 1 and 1000 pF, and we may approximate Eq. 9 up to first order, to

$$C' = C' \left( 1 - \Delta \right)$$

(10)

where

$$\Delta = \omega^2 \left[ \frac{1}{2} \left( L_1 C_1 + L_2 C_2 \right) + \left( L_1 C_B + L_2 C_C \right) 
+ \left( L_1 + L_2 \right) C' \right]$$

(11)
This makes sense, as we expect $C' > C$, that is, shunt admittances of cables and input ports add up to the real capacitance. $\Delta$ is the correction to capacitance, and, at $10^8$ rad/s, it is expected to range from $10^{-9}$ to $10^{-7}$ for capacitances with nominal value between 1 and 1000 pF.

IV. MEASUREMENT AND APPLICATION OF CORRECTIONS

We must determine corrections before carrying the actual 2T bridge comparison. To do this, according to Eq. [1] for each capacitor standard, we must carry an independent set of seven different measurements, over the high and low cables, and over the standard and its high and low ports, namely: 1- the capacitance $C$ of the standard; 2- the open-circuit capacitance $C_1$ of the high cable; 3- the open-circuit capacitance $C_2$ of the low cable; 4- the shunt capacitance $C_B$ of the high port of the standard; 5- the shunt capacitance $C_C$ of the low port of the standard; 6- the short-circuit inductance $L_1$ of the high cable; and finally 7- the short-circuit inductance $L_2$ of the low cable.

TABLE I: Values of $\Delta_X$ for two different units of 100 pF, used as standards in the calibration of 1000 pF units, at frequencies between 2500 and $10^8$ rad/s.

| 100 pF Standards | 0398 Hz | 1592 Hz |
|-------------------|---------|---------|
|                   | $\Delta [10^{-9} \text{ pF}]$ | $U [10^{-9} \text{ pF}]$ | $\Delta [10^{-9} \text{ pF}]$ | $U [10^{-9} \text{ pF}]$ |
| AH11A 1163        | 3.9273  | 0.0009  | 63.0621 | 0.0021 |
| AH11A 1508        | 3.7224  | 0.0010  | 59.8007 | 0.0020 |

TABLE II: Values of $\Delta_X$ for two different units of 1000 pF, at frequencies between 2500 and $10^8$ rad/s.

| 1000 pF Objects | 0398 Hz | 1592 Hz |
|-----------------|---------|---------|
|                 | $\Delta [10^{-9} \text{ pF}]$ | $U [10^{-9} \text{ pF}]$ | $\Delta [10^{-9} \text{ pF}]$ | $U [10^{-9} \text{ pF}]$ |
| 1404A 3077      | 5.2466  | 0.0032  | 84.746  | 0.007  |
| 1404A 3825      | 5.1729  | 0.0031  | 83.541  | 0.007  |

These measurements can be carried to fairly good uncertainties with a digital bridge like the one described in [5], or any other device suitable for some given target uncertainty. It is also important to mention that $C_{B,C}$ (steps 4 and 5) must be calculated from intermediary measurements $c_{B,C}$ of the capacitance of the ports B and C, with their complementary port (respectively, C and B) short-circuited. Then, using the model of Fig. 2 it’s trivial to show that $C_{B,C}$ equals the difference between the measured quantities $c_{B,C} - C$.

When we measure $C_X$ in a 2T bridge ($C_S$ as standard), at bridge balance the apparent capacitances $C'_{S,X}$ relate through Eq. [1] so we adopt a naming convention as follows. May $\Delta_{S,X}$ be, respectively, the corrections to $C_{S,X}$; that is

$$C_S = C'_S (1 - \Delta_S) \tag{12}$$

$$C_X = C'_X (1 - \Delta_X) \tag{13}$$

For each one of the standards, say $C_X$, $\Delta_X$ depends on $C_X$ measured value, on the shunt capacitances $C_B$ and $C_C$ of $C_X$ high and low ports, and on the immittances $\omega L_{1,2}$ and $\omega C_{1,2}$ of the high and low cables connecting $C_X$ to the system. Accordingly, to calculate $\Delta_S$ we must measure $C'_S$, the shunt capacitances $C_B$ and $C_C$ of its ports and the immittances $\omega L_{1,2}$ and $\omega C_{1,2}$ of its cables.

Once all these quantities are measured, and corrections $\Delta_S$ and $\Delta_X$ calculated, all at some frequency $\omega$, we substitute Eqs. [12] and [13] in Eq. [1] so we get, to first order in $\Delta_{S,X}$,

$$C_X = \frac{1 - r}{r} C_S (1 + \Delta_S - \Delta_X) + \frac{\alpha C_P}{r} (1 - \Delta_X) \tag{14}$$

where $C_S$ is the known value of the real capacitance of the standard, and $C_X$ is the real capacitance of the object being compared.

We show in the tables [1] and [11] the values of $\Delta_{S,X}$ for two pairs of 100 and 1000 pF standards, used in 1:10 comparisons at the 2T bridge. The units used as standards in these comparisons are two Andeen-Hagerling AH11A, of nominal value 100 pF, identified in the tables with their serial numbers, 1163 and 1508. The object units are two General Radio Co. 1404A capacitors, of nominal value 1000 pF, identified with the serials 3077 and 3825.

V. COMMENTS ON RESULTS

The main characteristic Tables [1] and [11] illustrate is that corrections don’t significantly vary between standards of the same model and nominal value. This was an expected result for commercial, serially manufactured units, but it is reassuring to quantify it. In practice, then, we can estimate average values for $C_{B,C}$ between units that share same model and nominal value, with negligible loss of information, and add the dispersion of their values to the final uncertainties of these parameters.

Tables [1] and [11] also illustrate the very week deviation of $\Delta$ of the $\omega^2$ dependence predicted in Eq. [11] which reaffirms the consistency of the model in Fig. 2 with but small dependece of the model parameters on $\omega$.

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[1] B. P. Kibble e G. H. Rayner, *Coaxial AC Bridges*, Adam Hilger LTD., Bristol (1984).
[2] Kyriazis, G. A., Vasconcellos, R. T. B., Ogino, L. M., Melcher, J. and Moreno, J., *Design and Construction of a Two-terminal Pair Coaxial Bridge*, analis of VI Semetro, Rio de Janeiro (2005).
[3] K. W. Wagner, Elektrotechel. Z. 32, 1001 (1911). See also H. H. Wolff, *The Review of Scientific Instruments* 30, 1116 (1959).
[4] D. N. Homan, J. Res. NBS, v. 72, n. 2, p. 161 (1968).
[5] Digital RLC Bridge Agilent, [https://www.keysight.com/us/en/product/E4980A/precision-lcr-meter-20-hz-2-mhz.html](https://www.keysight.com/us/en/product/E4980A/precision-lcr-meter-20-hz-2-mhz.html).