Reconfiguring Independent Sets in Cographs

Marthe Bonamy
Nicolas Bousquet

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Independent Set Reconfiguration

\[ \text{TAR} \]

\[ k(G) = 1 \]
Independent Set Reconfiguration

$\text{TAR}_k(G) = 1$
**Independent Set Reconfiguration**

- **TAR**

\[ k(G) = 1 \]
Independent Set Reconfiguration

Reconfiguration Graph

Solutions // Vertices. Closest solutions // Neighbors.

$\text{TAR}_k (G) = 1$
Independent Set Reconfiguration $\Rightarrow$ Reconfiguration Graph

Solutions // Vertices. Closest solutions // Neighbors.

$TAR_k(G)$

$k = 1$
Independent Set Reconfiguration $\Rightarrow$ Reconfiguration Graph

Solutions // Vertices. Closest solutions // Neighbors.

$TAR_k(G)$

$k = 2$
Two solutions:
- In the same connected component?
- What distance between them?
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Reconfiguration graph:
- Connected?
- Maximal diameter of a connected component?
Reconfiguration Graphs

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  - In the same connected component?
  - What distance between them?

- **Reconfiguration graph:**
  - Connected?
  - Maximal diameter of a connected component?

Colorings, Dominating sets, Vertex covers...
Reconfiguration Graphs

Two solutions:
- In the same connected component?
- What distance between them?

Reconfiguration graph:
- Connected?
- Maximal diameter of a connected component?

Colorings, Dominating sets, Vertex covers...
Token Addition & Removal, Token Jumping, Token Sliding...
Theorem (Hearn, Demaine ’05, Kamiński, Medvedev, Milanič ’12)

\[ G \text{ known to be perfect or subcubic planar:} \]
\[ \text{Are } \alpha, \beta \text{ in the same connected component of } TAR_k(G)? \]
\[ \text{PSPACE-complete.} \]
State of the Art

Theorem (Hearn, Demaine ’05, Kamiński, Medvedev, Milanič ’12)

$G$ known to be **perfect** or **subcubic planar**:
Are $\alpha, \beta$ in the **same connected component** of $\text{TAR}_k(G)$?
**PSPACE-complete.**

Efficient algorithms for:
- claw-free graphs,
- line graphs,
- chordal graphs...
Cographs: $P_4$-free graphs.
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Our Results

**Theorem (Bonsma ’14)**

\[ G \text{ cograph, } \alpha, \beta \in TAR_k(G) \Rightarrow \text{Decide in } O(n^2) \text{ whether } \alpha \text{ and } \beta \text{ in the same connected component.} \]

**Question (Bonsma ’14)**

\[ G \text{ cograph } \Rightarrow \text{Decide in } \text{Poly}(n) \text{ whether } TAR_k(G) \text{ is connected.} \]
Our Results

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**Theorem (B., Bousquet ’14+)**

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\[ G \text{ cograph } \Rightarrow \text{Decide in } O(n^3) \text{ whether } TAR_k(G) \text{ is connected.} \]
Proof

Take your favorite $G$ and $k$. 

\[ \text{Maximal stable set in } G \setminus (B \cup N(B)) \text{ of size } k - \alpha(B) \leq k + \alpha(B) - 1 \]
Proof

- Take your favorite $G$ and $k$.
- Build the decomposition tree in $O(n)$. 

\begin{itemize}
  \item Pick good and bad sides.
  \item Maximal stable sets $\iff$ "Stable-searches".
  \item Find bad side $B$ with smallest $\alpha(B)$.
  \item Maximal stable set in $G \setminus (B \cup N(B))$ of size $k - \alpha(B) \leq k + \alpha(B) - 1$.
\end{itemize}
Proof

- Take your favorite $G$ and $k$.
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Maximal stable sets $\iff$ "Stable-searches".

Maximal stable set in $G \setminus (B \cup N(B))$ of size $k - \alpha(B) \leq \alpha(D) \leq k + \alpha(B) - 1$?
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Conclusion

Question (Bonsma’14)

\( G \text{ cograph}, \alpha, \beta \in TAR_k(G) \Rightarrow \) Decide in \( \text{Poly}(n) \) whether \( \alpha \) and \( \beta \) at distance at most \( \ell \).
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Thanks for your attention!