Research on Chaos Characteristics of Nonlinear Systems with Fractional Exponential

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Abstract. In this paper, a chaotic system with fractional exponential term is proposed according to the classical chaotic system. The bifurcation diagram, LE and attractor phase diagram of the system are analyzed by Matlab software. The numerical simulation results show that the system has rich dynamic characteristics, which provides a theoretical basis for the parameter selection of the system when applied to image encryption.

1. Introduction

Chaos is a common form of motion in nature. It is also a seemingly random phenomenon in the deterministic system and has long-term dependence on initial values. In the 1960s, Lorenz system, the first chaotic system, was first proposed by Lorenz, a famous meteorologist from Massachusetts Institute of Technology. Since then, many scholars have taken great interest in this field. In 1999, Professor Chen Guanrong of City University of Hong Kong proposed Chen system, a dual system with Lorenz system, when studying chaos anti-control. In 2002, Professor Lv Jinhui and Professor Chen Guanrong of the Chinese Academy of Sciences proposed a new kind of chaotic system-Lü system [3]; In 2004, Professor Liu Chongxin of Xi’an Jiaotong University and others proposed Liu system [4] and extended Liu system. Since then, new chaotic systems have been proposed and constructed by scientists [5-7].

In 2004, T et al. removed a linear term from Lorenz system and proposed a new chaotic system called T chaotic system [5]. Wang Zhen and others have done a series of researches and analyses on the T-chaotic system, such as the basic dynamic analysis of the three-dimensional T-chaotic system through phase diagram, bifurcation and Lyapunov exponent in the reference [6], and have carried out adaptive synchronization control and circuit simulation to realize its control method. Ref [7] proves the existence of heteroclinic orbits of the system by numerical calculation, and uses backstepping control method to control T chaotic system. Ref [8] calculates and numerically simulates the periodic orbits of T chaotic systems, further illustrating the existence of chaos. Lei tengfei and others aimed at t-chaotic system with exponential term [9] and carried out basic dynamics research on t-chaotic system with exponential term. Ref [10] uses predictor-corrector method to draw and analyze the phase diagram of fractional order T chaotic system. The above documents have done some research and exploration on the basis of T chaotic system. In the transitional T chaotic system, the nonlinearity is increased, and the power of the linear term is changed into a fraction. However, there is no research on this kind of system in the existing literature. Therefore, the research on its dynamic behavior provides a new idea for the research of critical systems and practical engineering.
2. Chaotic System with fractional exponential

Chaotic System with fractional exponential is constructed on the basis of the chaotic system in reference [4]. The dynamic equation is as follows:

\[
\begin{align*}
\dot{x} &= a(y - x) + yz \\
\dot{y} &= cx - xz \\
\dot{z} &= xy - bz^k
\end{align*}
\]  

(1)

Where \(x, y, z\) is variable and \(a, b, c, k\) is parameter of system. When \(a = 8, b = 3, c = 13, k = 0.6\), the system (2) is numerically simulated by Matlab, the chaotic attractor of system (2) is shown in figure 1. Through numerical calculation, the Lyapunov exponent of system (1) when system \(k = 1\) and \(k = 0.6\) is given as shown in figure 2. Obviously, the maximum LE when \(k = 0.6\) is greater than the maximum LE when \(k = 1\).

![Phase diagram of \(x - z\)](image1)

![Phase diagram of \(y - z\)](image2)

Figure 1. The chaotic attractor of system

![Lyapunov exponent spectrum of the system with \(k = 0.6\)](image3)

![Lyapunov exponent spectrum of the system with \(k = 1\)](image4)

Figure 2. Lyapunov exponent spectrum of the system

3. Nonlinear Characteristic Analysis

The influence of parameters on the system is generally described by bifurcation diagram and Lyapunov exponent (LE) variation rule under parameter variation. Therefore, the bifurcation diagram and Lyapunov exponent of the system are analyzed below.
When parameter $k$ changes, $k \in [0.6, 1.6]$ the bifurcation diagram and Lyapunov exponent spectrum of chaotic system (1) are shown in figure 3. It can be seen that the system $k \in [0.6, 1.2]$ is in chaotic state and other regions are in periodic state. The maximum Lyapunov exponent corresponding to the chaotic state region is greater than 0, the maximum Lyapunov exponent corresponding to the periodic state is not greater than 0, and the phase diagram of the chaotic state and the period is shown in figure 4(a)(b).

(a) Bifurcation diagram of $x$

(b) Lyapunov exponent spectrum

Figure 3. Bifurcation diagram and Lyapunov exponent spectrum of system (1) about $x$ with $k$ changes

When parameter $a$ changes, $a \in [2, 10]$ the bifurcation diagram and Lyapunov exponent spectrum of chaotic system (1) are shown in figure 5. It can be seen that the systems $a \in [2, 4]$ and $a=9$ are in non-chaotic states, while other regions are in chaotic states. The maximum Lyapunov exponent corresponding to the chaotic state region is greater than 0, the maximum Lyapunov exponent corresponding to the periodic state is not greater than 0.

(a) $a=1$

(b) $a=1.5$

Figure 4. Phase diagrams of the system with parameter $k$ changes
4. Conclusion

In this paper, for a new kind of autonomous chaotic system, Matlab is used to carry out numerical simulation of the system. By adjusting the parameters of the three-dimensional autonomous system, the periodic and chaotic motion of the system is realized, and even more complex results are achieved. Numerical simulation verifies the rich dynamic characteristics of the system. The obtained results provide reference value for practical engineering applications such as chaotic cryptography, chaotic detection and electromechanical coupling control of the system.

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