GAUGE INVARIANCE AND RENORMALIZATION-GROUP EFFECTS IN TRANSVERSE-MOMENTUM DEPENDENT PARTON DISTRIBUTION FUNCTIONS

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Abstract

A range of issues pertaining to the use of Wilson lines in integrated and transverse-momentum dependent (TMD) parton distribution functions (PDF) is discussed. The relation between gauge invariance and the renormalization properties of the Wilson-line integrals is given particular attention. Using an anomalous-dimensions based analysis in the light-cone gauge, a generalized definition of the TMD PDFs is proposed, which employs a cusped Wilson line, and contains an intrinsic “Coulomb-like” phase.

Introduction. Various calculations in the last few years have addressed TMD PDFs, among others those in which a previously overlooked transverse gauge link was proposed [1, 2, 3]. The sustained interest in integrated and unintegrated (TMD) PDFs lies in the fact that they encapsulate the nonperturbative quark dynamics of confinement and hence in their potential use in phenomenological applications to be compared with experimental data. But while integrated PDFs can be defined in a gauge-invariant way that is compatible with factorization theorems, the definition of TMD PDFs faces serious problems related to specific light-cone divergences (see, e.g., [4, 5]). These so-called rapidity divergences [6] are related to lightlike Wilson lines (or the use of the light-cone gauge $A^+ = 0$) [7, 8] and cannot be cured by ordinary ultraviolet (UV) renormalization alone. In addition, in order to recover the result found in the Feynman gauge, the advanced boundary condition has to be adopted to make the transverse gauge link reduce to unity [2].

The basic statement of the presented work [9] is this: In order to define an unintegrated PDF that preserves gauge invariance under the proviso of collinear factorization and multiplicative renormalizability, we shift our attention from the Wilson lines to their anomalous dimensions within the $\overline{\text{MS}}$ scheme. We will provide concrete arguments that the appropriate contour which goes through light-cone infinity is a cusped one. To compensate the associated anomalous dimension, we introduce into the definition of the TMD PDF a soft counter term (in the sense of Collins and Hautmann [10, 11, 12]) which generates the same anomalous dimension but with opposite sign. Hence, the total TMD PDF expression has the same one-loop anomalous dimension as the one that would involve a

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straight lightlike line between the quark operators. Note, however, that such a gauge contour cannot be adopted because the gluons originating from this would not be collinear with the struck quark and hence they would cause a mismatch in the gluon rapidities.

To substantiate our arguments, we write the standard expression for the TMD PDF \[^6\] for a quark-to-quark distribution, supplemented by a transverse gauge link \[^2\]:

\[
\begin{align*}
  f_{q/q}(x, k_\perp) &= \frac{1}{2} \int \frac{d\xi^+ d^2\xi_\perp}{2\pi(2\pi)^2} e^{-ik^+\xi^++ik_\perp\cdot\xi_\perp} \langle q(p)|\bar{\psi}(\xi^-, \xi_\perp)|\bar{\psi}(\xi^-+i\Delta, \xi_\perp)\rangle \\
  &\quad \times \langle 0^-\gamma^+|q(p)\rangle |_{\xi^+=0} ,
\end{align*}
\]

where the gauge links are defined according to

\[
  [\infty^-, z^-; z_\perp; z_\perp] \equiv \mathcal{P}e^{ig \int_0^\infty d\tau n^\mu A_\mu(z+n\tau)} , \quad [\infty^-, \infty_\perp; \infty^-, \xi_\perp] \equiv \mathcal{P}e^{ig \int_0^\infty d\tau l_\perp A_\perp(\xi_\perp+l_\perp\tau)}
\]

with analogous expressions for the other gauge links and where \(l_\perp\) represents an arbitrary vector in the transverse direction and \(\mathcal{P}\) denotes path ordering.

Within the Collins-Soper approach \[^6\] \((n^2 \neq 0)\), the anomalous dimension of \(f_{q/q}(x, k_\perp)\) is \[^13\]

\[
  \gamma_{CS} = \frac{1}{2} \mu \frac{d}{d\mu} \ln Z_f(\mu, \alpha_s; \epsilon) = \frac{3}{4} \frac{\alpha_s}{\pi} C_F + O(\alpha_s^2) = \gamma_{\text{smooth}} ,
\]

where \(Z_f\) is the renormalization constant of \(f_{q/q}(x, k_\perp)\) in the \(\overline{\text{MS}}\) scheme. Recall that all smooth contours off the light cone in the transverse direction give rise to the same anomalous dimension due to the endpoints of the so-called connector insertion \[^14\].

Figure 1 shows the one-loop diagrams, contributing to \(f_{q/q}(x, k_\perp)\) in the light-cone (LC) gauge \((A \cdot n^-) = 0, (n^-)^2 = 0\). The poles \(1/q^+\) of the gluon propagator

\[
  D_{\mu\nu}^{LC}(q) = \frac{1}{q^2} \left( \eta_{\mu\nu} - \frac{q_\mu n_\nu + q_\nu n_\mu}{[q^+]} \right) ,
\]

are regularized by \(1/[q^+] = 1/(q^+ \pm i\Delta)\), where \(\Delta\) is small but finite. In addition to the standard UV renormalization terms, one has UV divergent contributions from diagrams (a) and (d) stemming from the \(p^+\)-dependent term in

\[
  \Sigma_{\text{UV}}^{LC}(\alpha_s, \epsilon) = \frac{\alpha_s}{\pi} C_F 2 \left[ \frac{1}{\epsilon} \left( \frac{3}{4} + \ln \frac{\Delta}{p^+} \right) - \gamma_E + \ln 4\pi \right] .
\]

Noting that the contribution associated with the transverse gauge link at infinity (diagram Fig. 1(d)) exactly cancels against the term entailed by the adopted pole prescription in

![Figure 1: One-loop gluon contributions to the UV-divergences of the TMD PDF. Double lines denote gauge links. Diagrams (b) and (c) are absent in the light-cone gauge.](image-url)
the gluon propagator, we find for the corresponding anomalous dimension

$$\gamma_{\text{LC}} = \frac{\alpha_s}{\pi} C_F \left( \frac{3}{4} + \ln \frac{\Delta}{p^+} \right) = \gamma_{\text{smooth}} - \delta \gamma \ .$$  \hspace{1cm} (6)

Here $\delta \gamma$ is the term induced by the additional divergence that has to be compensated by a suitable redefinition of the TMD PDF. It is important to realize that $p^+ = (p \cdot n^-) \sim \cosh \chi$ defines an angle $\chi$ between the direction of the quark momentum $p_\mu$ and the lightlike vector $n^-$ with $\ln p^+ \rightarrow \chi, \chi \rightarrow \infty$. Hence, the “defect” of the anomalous dimension, $\delta \gamma$, can be identified with the well-known cusp anomalous dimension [15]

$$\gamma_{\text{cusp}}(\alpha_s, \chi) = \alpha_s C_F \left( \chi \coth \chi - 1 \right), \quad \frac{d}{d \ln p^+} \delta \gamma = \lim_{\chi \to \infty} \frac{d}{d \chi} \gamma_{\text{cusp}}(\alpha_s, \chi) = \frac{\alpha_s}{\pi} C_F \ .$$ \hspace{1cm} (7)

Applying renormalization techniques for contour-dependent composite operators [16, 15, 17] in order to treat angle-dependent singularities, we introduce a compensatory soft term

$$R \equiv \Phi(p^+, n^-|0)\Phi^\dagger(p^+, n^-|\xi), \quad \Phi(p^+, n^-|\xi) = \left\langle 0 \left| \mathcal{P} \exp \left\{ ig \int_{\Gamma_{\text{cusp}}} d\zeta^\mu A^a_\mu(\xi + \zeta) \right\} \right| 0 \right\rangle \hspace{1cm} (8)$$

and evaluate it along the cusp contour $\Gamma_{\text{cusp}}$, illustrated in Fig. 2, which is defined by ($n^-_\mu$ is the minus light-cone vector)

$$\Gamma_{\text{cusp}} : \quad \zeta_\mu = \{|p^+_\mu s^, \, -\infty < s < 0\} \cup \{n^-_\mu s', \, 0 < s' < \infty\} \cup \{|l_\perp \tau, \, 0 < \tau < \infty\} \ .$$ \hspace{1cm} (9)

The one-loop gluon virtual corrections contributing to the UV divergences of $R$ are given by

$$\Sigma_{\text{UV}}^R = -\frac{\alpha_s}{\pi} C_F 2 \left( \frac{1}{\epsilon} \ln \frac{\Delta}{p^+} - \gamma_E + \ln 4 \pi \right) \ .$$ \hspace{1cm} (10)

This expression is equal, but with opposite sign, to the unwanted term in the UV singularity, related to the cusped contour, calculated before. This result enables us to redefine the conventional TMD PDF as follows:

$$f_{q/q}^{\text{mod}}(x, k_\perp) = \frac{1}{2} \int \frac{d\xi d^2\xi_\perp}{2 \pi^2 (2\pi)^2} e^{-i k^+ \xi^+ + i k_\perp \cdot \xi_\perp} \left( \Phi(p^+, n^-|0)\Phi^\dagger(p^+, n^-|\xi^-)\Phi(p^+, n^-|\xi^-) \right)$$

$$\times \left[ [-\infty, \xi_\perp; [0, -\infty; \infty], 0_\perp] \gamma^+ [\infty, -\infty; \xi_\perp, 0_\perp] \Phi(p^+, n^-|0) \Phi^\dagger(p^+, n^-|\xi^-) \right] \ ,$$ \hspace{1cm} (11)
The renormalization of \( f_{\text{ren}}(x, k_{\perp}) = Z_{\text{mod}}^{\text{ren}}(\alpha_s, \epsilon) f_{\text{mod}}(x, k_{\perp}, \epsilon) \) yields the renormalization constant

\[
Z_{f}^{\text{mod}} = 1 + \frac{\alpha_s}{4\pi} C_F \frac{2}{\epsilon} \left( -3 - 4 \ln \frac{\Delta}{p^+} + 4 \ln \frac{\Delta}{p^+} \right) = 1 - \frac{3\alpha_s}{4\pi} C_F \frac{2}{\epsilon} .
\]  

which in turn provides the anomalous dimension

\[
\gamma_{f}^{\text{mod}} = \frac{1}{2} \frac{d}{d\mu} \ln Z_{f}^{\text{mod}}(\mu, \alpha_s, \epsilon) = \frac{3\alpha_s}{4\pi} C_F + O(\alpha_s^2) = \gamma_{\text{smooth}} .
\]  

To conclude, the soft counter term can be considered [9] as that part of the TMD PDF which accumulates the residual effects of the primordial separation of two oppositely color-charged particles, created at light-cone infinity and being unrelated to the existence of external color sources, thus corresponding to an “intrinsic Coulomb phase” that keeps track of the full gauge history of the colored quarks [18, 19].

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