Chaotic Inflation from Nonlinear Sigma Models in Supergravity

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We present a common solution to the puzzles of the light Higgs or quark masses and the need for a shift symmetry and large field values in high scale chaotic inflation. One way to protect, for example, the Higgs from a large supersymmetric mass term is if it is the Nambu-Goldstone boson (NGB) of a nonlinear sigma model. However, it is well known that nonlinear sigma models (NLSMs) with nontrivial Kähler transformations are problematic to couple to supergravity. An additional field is necessary to make the Kähler potential of the NLSM invariant in supergravity. This field must have a shift symmetry — making it a candidate for the inflaton (or axion). We give an explicit example of such a model for the coset space $SU(3)/SU(2) \times U(1)$, with the Higgs as the NGB, including breaking the inflaton’s shift symmetry and producing a chaotic inflation potential. This construction can also be applied to other models, such as one based on $E_7/SO(10) \times U(1) \times U(1)$ which incorporates the first two generations of (light) quarks as the Nambu-Goldstone multiplets, and has an axion in addition to the inflaton. Along the way we clarify and connect previous work on understanding NLSMs in supergravity and the origin of the extra field (which is the inflaton here), including a connection to Witten-Bagger quantization. This framework has wide applications to model building; a light particle from a NLSM requires, in supergravity, exactly the structure for chaotic inflaton or an axion.

I. INTRODUCTION AND MOTIVATION

Over the past two years there have been several exciting experimental results, which both confirm theories developed long before as well as challenge us to better understand their origin. The discovery of the Higgs boson [1] brings renewed attention to the issue of the apparent lightness of the Higgs mass compared to any UV scale, like the Planck mass. The Higgs is not the only light field we are puzzled over: the lightness (smallness of the Yukawa couplings) of the first two generations of quarks is a longstanding question. More recently, there has been much discussion on the possible discovery of B-modes in the CMB by BICEP2 [2], but which may have possibly been due to dust [3] rather than primordial gravitational waves. However, a large value for the tensor to scalar ratio, $r \sim 0.1$, is still possible. Such a value, or more generally any motivations of models for high scale or large field inflation like chaotic inflation [4], raises the question of how to control higher dimensional operators which will not be suppressed in the inflaton potential. In these models, where do such large field values (of order or greater than the Planck scale) come from, and how are such models consistent?

There are several ways to address these problems, although it is not at all obvious that they could be closely related. Consider first the Higgs mass, which can be generated by a supersymmetric mass term. One requires some way to either generate a mass much smaller than the supersymmetry scale, or else forbid this operator. If the Higgs is a Nambu-Goldstone boson (NGB) of a G/H nonlinear sigma model (NLSM) [5], this would do both of these things: a NGB is massless at first approximation, and cannot have such a mass term. The Higgs mass is then protected until we introduce operators which break G/H. More generally, we can think of any light particle, such as the first two generations of quarks, as a possible NGB (or fermion partner under supersymmetry) from a NLSM.

However, as soon as we consider local supersymmetry, we run into well known problems for coupling a NLSM to supergravity [6]. The reason is that the Kähler potential has a nontrivial transformation,

$$K(\Phi, \Phi^\dagger) \to K(\Phi, \Phi^\dagger) + g(\Phi) + g^\dagger(\Phi^\dagger),$$

with $g$ a holomorphic function. Such functions have no effect in global supersymmetry when integrated over all of superspace, but in local supersymmetry they do not disappear. Here, too, there are several solutions which have been studied in the past [7, 8] (see also [9] and [10] for earlier work). Generally one must consider a noncompact NLSM, $G'/H$, which can be coupled to supergravity. The extension of the original compact manifold necessarily contains a (at least one) new chiral superfield $Z$. It may be surprising that this field must appear in the Kähler potential as $Z + Z^\dagger$, possessing a shift symmetry. In special cases, as in Witten-Bagger models [6], the manifold can be compact and does not require extra fields (instead there is the quantization condition of the Kähler form), and we will discuss how these two cases may possibly be connected.
Thus we see we are led to a possible solution to our second problem of how to forbid higher dimensional operators in the inflaton potential. \( Z \) will have a completely flat direction \((Z-Z^\dagger)\) does not appear in the potential\) which is protected by the shift symmetry. More generally, to embed chaotic inflation in supergravity [11] one typically requires a shift symmetry, whose existence is simply imposed. Here we have an immediate explanation for the origin of this symmetry. In order to have an inflationary potential, though, we will need to break this symmetry. To understand the large initial value of the inflaton in chaotic inflation we will look at Witten-Bagger models which naturally have large field values (larger than the Planck mass). We can then construct phenomenological models, which incorporate this solution to both the Higgs or light quark masses and chaotic inflation problems, in detail.

This paper is organized as follows. In the following section, Section II, we will clarify the difficulties and solutions to coupling a NLSM to supergravity. The key points have been understood in the past, but a coherent picture is essential to this work. Using a \( \mathbb{C}P^1 \) model as our guide, we will relate various proposals for coupling a NLSM to supergravity. Following that we will look at an explicit model, one based on \( SU(3)/SU(2) \times U(1) \) which includes the Higgs as a NGB. In Section III we investigate how to break the shift symmetry and propose a connection to Witten-Bagger models. Again we use \( \mathbb{C}P^1 \) as the prototype before applying to our model with the Higgs. Finally, in Section IV we will discuss the application to other models, such as an \( E_7/SO(10) \times U(1) \times U(1) \) model for two generations of light quarks. This model includes an axion in addition to the inflaton. Finally, we will comment on future directions.

\section{Nonlinear Sigma Models and Supergravity}

Let us demonstrate the difficulties in coupling a NLSM to supergravity by considering the basic \( \mathbb{C}P^1 \cong SU(2)/U(1) \) model. This model is determined completely by its symmetries, which require a Kähler potential of the Fubini-Study form,

\[ K = f_\phi^2 \log \left( 1 + \frac{\phi \phi^*}{f_\phi^2} \right) \]  

where \( \phi \) is the NG multiplet (a chiral superfield\(^1\)) and \( f_\phi \) is the scale of the effective theory (i.e. the decay constant). If we think of \( \mathbb{C}P^1 \) as a manifold it is \( S^2 \), the 2-sphere, and requires two coordinate patches. A Kähler transformation takes us between these patches, with \( \phi \to f_\phi^2/\phi \). Then the Kähler potential transforms as \( K(\phi, \phi^*) \to K(f_\phi^2/\phi, f_\phi^2/\phi^*) \) and

\[ K \left( \frac{f_\phi^2}{\phi}, \frac{f_\phi^2}{\phi^*} \right) = f_\phi^2 \log \left( 1 + \frac{f_\phi^2}{\phi \phi^*} \right) = K(\phi, \phi^*) - f_\phi^2 \log \left( \frac{\phi^*}{f_\phi} \right) \]  

Indeed, we see we have the form \( K \to K + g + g^* \) with \( g \) a holomorphic function. In global supersymmetry then \( g \) drops out when integrating \( \int d^4\theta K \) in the Lagrangian. In local supersymmetry, however, these terms remain and we must think more carefully of how to couple to supergravity.

We can make \( K \) invariant with an additional field, \( Z \), with the right transformation properties [7, 8]. The Kähler potential is then

\[ K = K(\phi, \phi^*) + f_\phi (Z + Z^*), \]  

where we have used \( f_\phi \) for dimensional reasons (the only other scale is the Planck mass, but we will see later these are related). This Kähler potential is incomplete, as \( Z \) does not have a kinetic term without considering higher order terms. We will consider such additional terms below. Under a Kähler transformation we must have

\[ Z \to Z + f_\phi \log \left( \frac{\phi}{f_\phi} \right), \]  

and similarly for \( Z^* \). We see that \( Z \) possesses a shift symmetry and the imaginary part will have no potential with such a symmetry (to build an inflationary model we will of course need to break the shift symmetry).

Now we make an interesting observation: \( Z \) must be charged under the (assumed) linearly realized \( U(1) \) of \( \mathbb{C}P^1 \). Thus \( SU(2) \) is actually completely broken and the additional flat direction is a consequence of this breaking. This can easily be seen by using the Jacobi identity for the \( SU(2) \) generators on the field \( Z \). In order for it to be satisfied, \( Z \) must transform under the \( U(1) \). This requirement for breaking the \( U(1) \) is another understanding for how to couple a NLSM to supergravity [8].

The origin of \( Z \) can also be seen from considering how a nonlinear sigma model comes from a linear sigma model (which we can straightforwardly couple to supergravity) in supersymmetry. It has been long known that supersymmetric NLSMs, with a linear sigma model origin, must come with additional degrees of freedom, called quasi-NGBs [9]. Generically, these quasi-NGBs double the NGB degrees of freedom. However, if one has a special Kähler potential (non-generic), it is possible to realize the NLSM with fewer quasi-NGBs. We will show such a specific model below. With this interpretation, the \( Z \) field has a clear origin and reason for its associated flat direction.

\(^1\) In terms of the homogeneous coordinates of \( \mathbb{C}P^1 \), \( \phi \) is given by their ratio, an affine coordinate. The “conjugate” coordinate is the reciprocal.
A. A Model for a Light Higgs and Chaotic Inflation

As we discussed above, one possible resolution to the puzzle of the lightness of the Higgs mass is if the Higgs is a NGB of some NLSM. In fact, we have previously constructed such a model in [5] (which is an extension of the work in [12]). In this model the Higgs is the NGB of a SU(3)/SU(2)\_L × U(1)\_V NLSM, as it has exactly the right quantum numbers.

However, we know we cannot simply couple such a NLSM to supergravity, as we have seen above. Since the extra field in the construction of [7] must break the U(1) (which is the requirement in the language of [8]), the group structure is actually SU(3)/SU(2). This is equivalent to a U(3)/SU(2) × U(1) NLSM which has a known construction [13] (see also [8]) with an invariant K and a quasi-NGB. We identify the unbroken SU(2) as the weak gauge group of the Standard Model, and the NG superfield is one (the lightest) of the Higgs multiplets, H\_4 or H\_2.

The NG superfield is an SU(2) doublet, labeled (\(\phi_1, \phi_2\)), and the quasi-NGB chiral superfield is Z (which has been dubbed the “novino” in [14]). Defining the matrix \(\xi\) as

\[
\xi \equiv \begin{pmatrix}
    e^{\kappa Z} & 0 \\
    0 & e^{\kappa Z}
\end{pmatrix},
\]

where \(\kappa\) has mass dimension \(-1\) and is related to the scale of the NLSM, the Kähler potential can be written as [13]

\[
K = -F(\det \xi^\dagger \xi),
\]

for a function \(F\), subject to constraints for proper kinetic terms and vacuum.

Under the global U(3) transformation the matrix \(\xi\) transforms as

\[
\xi \to g \xi h^{-1},
\]

where \(g \in U(3)\) and \(h \in SU(2)\). Thus \(\det(\xi^\dagger \xi)\), and therefore \(K\), is invariant under the global U(3) transformations. There is no difficulty then in coupling this model to supergravity.

We can connect directly to the work of [7] and the Kähler potential of the form in eq. (4) by the following field redefinitions. First, we define and write explicitly

\[
x \equiv \det \xi^\dagger \xi = e^{2\kappa(Z+Z^\dagger)} + e^{\kappa(Z+Z^\dagger)} (|\phi_1|^2 + |\phi_2|^2),
\]

which can be rewritten as

\[
x = e^{2\kappa(Z+Z^\dagger)} \left(1 + \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2\right),
\]

with the field redefinition \(\phi' = e^{-\kappa Z} \phi\). Now define

\[
y \equiv \log x = \log(1 + \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2) + 2\kappa(Z+Z^\dagger),
\]

which is exactly the field combination in [7] with Kähler potential

\[
K \left(\log(1 + \phi_1^\dagger \phi_1) + 2\kappa(Z+Z^\dagger)\right) = K(y).
\]

Thus we thus have an exact equivalence between the two different looking models. The group structure is equivalent once one notices that the U(1) is actually broken in [7], and the counting of flat directions is the same. It is clear that Z has a shift symmetry and can play the role of the inflaton. While Z comes from an extended supergravity multiplet in [7], we see that it is equivalent to a quasi-NGB origin as in [8, 13].

III. LARGE FIELD VALUES AND SHIFT SYMMETRY BREAKING

While we now have a model demonstrating how a NLSM in supergravity has the right starting point for inflation, there are two key questions. How do we break the shift symmetry and produce an inflationary potential, and how do we achieve the large field values necessary for chaotic inflation? We will use a simpler CP\(^3\) model to answer these questions, and the extension is straightforward to the SU(3)/SU(2) model above, or other CP\(^k\) models.

To successfully have inflation we must break the shift symmetry of Z and generate a potential for its imaginary part. Perhaps the simplest way to accomplish this is to add a superpotential with a new field, X, and coupling

\[
W = mXZ.
\]

We need \(m \sim 10^{-5} M_p\) for chaotic inflation [11, 15] (see also the recent work of [16]). This breaking is technically natural as we have the shift symmetry as we take \(m \to 0.\)

There are several difficulties in trying to introduce such a potential in a NLSM in supergravity. Besides breaking the shift symmetry of Z, this superpotential will break Kähler invariance and G-invariance due to the properties of Z (which were required to couple to supergravity in the first place). However, G is just a global, nonlinearly realized symmetry and hence a violation is not a serious problem (and motivates such a small value for \(m\)).

A more serious problem is that we need to have very large (in Planck units) field values for chaotic inflation, yet we naturally expect \(f_\phi < M_p\), with \(M_p\) the reduced Planck mass. Therefore it seems desirable to have values of \(f_\phi\) larger than \(M_p\). One of the few such models is

\(^2\) In the SU(3)/SU(2) model a term linear in \(X\) in the superpotential, c\(X\), is allowed by all symmetries. Shifting \(Z\) to cancel this term normally leads to a large linear term in \(K\) which must be small or inflation will not end [15]. By viewing the model written in terms of \(x\) we see this is not a problem: a real shift in \(Z\) is equivalent to an overall factor and can be absorbed, while any imaginary part cancels.
that of Witten and Bagger [6]. Thus we wish to see if there can be a connection to Witten-Bagger theories as a special case of the models we are studying to explain large field values.

Consider if $Z$ is just a chiral superfield, invariant under Kähler and $G$ transformations, but with a shift symmetry. In this case, to be coupled to supergravity, the $\mathbb{C}P^1$ NLSM would have to be a Witten-Bagger theory, with the decay constant constrained to be quantized in units of $M_p$, and thus larger than $M_p$. One gains the benefit of naturally explaining large field values (we expect fields to take values of order the decay constant), but pays the price of losing an origin for the shift symmetry and all interactions in the superpotential (we must have $W = 0$ in a Witten-Bagger model).

We propose a connection between these different types of theories, one where $Z$ and its shift symmetry are put in by hand and another where $Z$ is required by supergravity, by the following Kähler potential with real, dimensionless parameters $a, b$:

$$K = f_\phi^2 \left[ \log \left( 1 + \frac{\phi \phi^*}{f_\phi^2} \right) + \frac{1}{f_\phi} (Z + Z^*) \right]$$

$$+ \frac{a^2}{2} f_\phi^2 \left[ \log \left( 1 + \frac{\phi \phi^*}{f_\phi^2} \right) + \frac{1}{f_\phi} (Z + Z^*) \right]^2$$

$$+ \frac{b}{2} (Z + Z^*)^2 + XX^*. \quad (14)$$

The superpotential is only turned on when it is allowed, namely when $a \neq 0, b = 0$ and the theory is in the Komargodski-Seiberg-Kugo-Yanagida (KSKY) branch. In general, the first two terms could be replaced with some general function of the Kähler- and $G$-invariant quantity in square brackets (subject to appropriate constraints for canonical kinetic terms, etc.). There may also be higher order terms besides what is written above, but these are not relevant for this discussion. When $a = 0, b \neq 0$, we are in a Witten-Bagger theory and the term with coefficient $b$ is a kinetic term for $Z$. We conjecture that the KSKY branch is smoothly connected to the Witten-Bagger branch through the parameters $a$ and $b$. This implies that $f_\phi^2 = 2nM_p^2$, for some integer $n$, everywhere.

We now consider the theory on the KSKY branch as a model for chaotic inflation. From the Kähler potential above with $b = 0$, we can canonically normalize the kinetic term for $Z$ by the field redefinition $aZ \rightarrow Z$. Then the linear term in $Z$ has the dimensionless combination $(Z + Z^*)/af_\phi$, and thus we expect the initial value of $Z$ to be of order $af_\phi$. For chaotic inflation we require this combination to be $O(10)$ in Planck units. Furthermore, this linear term has overall coefficient $f_\phi/a$, which must be less than order one for inflation to end [11]. Thus we need that $f_\phi \sim a$ and with $f_\phi \sim \sqrt{n}$ by Witten-Bagger quantization (in Planck units, and $n \in \mathbb{Z}$), we have that $a \sim \sqrt{n}$. Finally, we see that we only need $n \sim 10$ to satisfy $af_\phi \sim O(10)M_p$, which is very reasonable.

We have now the following picture. The additional field $Z$ comes from considering the general case of coupling to supergravity when $f_\phi$ is not quantized. This is the origin of the inflaton and its shift symmetry (in the limit $m \rightarrow 0$). We then flow to the Witten-Bagger theory: $f_\phi$ must be quantized and larger than $M_p$, explaining the necessary large field values. Here $Z$ is just a field with a shift symmetry and does not transform in any special way to couple the theory to supergravity.

IV. DISCUSSION AND CONCLUSION

We can easily apply the above work to other NLSMs. Consider the $E_7/SU(10) \times U(1)$ NLSM [17] which contains two 16 NG multiplets which can be identified with the light two generations of quarks and leptons. It is interesting that their small Yukawa couplings can be explained in this model as a weak breaking effect. To couple this model to supergravity the two $U(1)$s should be broken [8]. Again, we can identify $Z$ with the inflaton, but we have an additional NG superfield, $Z'$. It is tempting to identify this with the QCD axion multiplet in order to solve the strong CP problem.

If we use the Kähler manifold $G/H = E_7/SU(5) \times U(1)^3$, we have three families of quarks and leptons as the NG multiplets. In this case we should introduce three singlets, $Z, Z'$, and $Z''$ to couple to supergravity. The $E_7$ manifold has three submanifolds, $E_7/E_6 \times U(1), E_6/SO(10) \times U(1), \text{ and } SO(10)/SU(5) \times U(1)$. When we introduce an explicit breaking for $E_7 \rightarrow E_6$, the third family of quarks and leptons will have Yukawa couplings and the third singlet, $Z''$, gets a corresponding mass, and so on for the second generation. The very small Yukawa coupling of the up quark, $\sim 10^{-5}$, would be due to the good $SO(10)$ symmetry. Thus, when we introduce an explicit breaking of $SO(10)$ the up quark has a Yukawa coupling and the singlet $Z$ gets a mass. The lightest singlet $Z$ is the inflaton. Since the Yukawa coupling for the up quark and inflaton mass arise from the $SO(10)$ breaking, it is plausible to have a relation [18] between them,

$$Y_{up} \simeq m_{inflaton}/M_p. \quad (15)$$

It is perhaps surprising that this relation is almost satisfied for chaotic inflation with $m_{inflaton} \gtrsim 10^{13}$ GeV.

It is straightforward to apply the above techniques to other models as well. A NLSM explanation for a light field requires other flat directions, suitable for inflation or axions, once coupled to supergravity. Our phenomenological model which may connect the present KSKY branch to the Witten-Bagger branch may be realized in some higher dimensional theories. The embedding of the Bagger-Witten realization of SUGRA into an

\footnote{We have to assume that the mass term for the axion generated by explicit breaking of the global $E_7$ is negligible.}
ultraviolet-complete theory has been little explored, and we hope that our model may motivate further research in this direction. However, this is beyond the scope of the present paper.

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