Antiferromagnetic skyrmion emerging in a ferromagnet with gain

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We present a theoretical mapping to show that a ferromagnet with gain (loss) is equivalent to an antiferromagnet with an equal amount of loss (gain). As an appealing application of this finding, we demonstrate the realization as well as the manipulation of the antiferromagnetic skyrmion, a stable topological quasiparticle not yet observed experimentally, in a ferromagnetic thin film with gain. We also consider ferromagnetic bilayers with balanced gain and loss, and show that the antiferromagnetic skyrmion can be found only in the cases with broken parity-time symmetry phase. Our results pave a way for investigating the emerging antiferromagnetic spintronics and parity-time symmetric magnonics in ferromagnets.

Introduction.—Magnetic skyrmions are topologically protected spin textures which can form in bulk noncentrosymmetric magnets and thin films. They are an active research area in condensed matter physics because of not only the potential for future spintronic applications such as skyrmion racetrack memories and logic devices [1–3], but also the fundamental interest [4]. Until now, skyrmions had been investigated mainly in ferromagnets (FMs) and, in most cases, at low temperature. A recent intense research effort pushed skyrmions into antiferromagnets (AFMs) [5–14] of which elementary excitations fall in the terahertz range. Compared to their FM counterparts, AFM skyrmions have some advantages, such as the insensitivity to stray fields, the immunity of skyrmion Hall effects, the elevated mobility, and the unusual thermal properties, among others. One recent breakthrough toward this direction is the experimental realization of ferrimagnetic skyrmions in GdFeCo films with inhibited skyrmion Hall effect [15, 16]. Because of its intrinsic difficulties, the antiferromagnetic skyrmion, however, is yet to be observed in experiments. An intriguing question one may ask is: can an antiferromagnetic skyrmion survive in simple ferromagnets? Finding the positive answer to the question is the main scheme of this work.

Loss and gain are ubiquitous in nature. All physical systems have a finite decay time due to the presence of dissipation into the environment. Taking magnetic skyrmions as the example, their stabilization relies on not only the Dzyaloshinskii-Moriya interaction (DMI) in chiral magnets [17, 18], but also the magnetic frictions. However, it is the competing gain, such as the external driving field and/or current, that prevents them from staying in the boring ground state. Tantalizing physics under balanced gain and loss has attracted enormous attention and found many great applications in the context of parity-time (PT) symmetry and exceptional points [19] in a broad field of quantum mechanics [20], optics [21–24], acoustics [25, 26], optomechanics [27, 28], electronics [29–33], and very recently in spintronics [34–38] and cavity spintronics [39, 40]. In Ref. [34], Lee, Kottos and Shapiro proposed two coupled macroscopic ferromagnetic layers respecting the PT symmetry: one layer with loss and another one with an equal amount of gain, and discussed their dynamics in the framework of Landau-Lifshitz-Gilbert (LLG) equation [41]. The positive Gilbert damping (loss) in magnets usually comes from the phonon dissipation and the electromagnetic radiation, while the negative one (gain) can be realized by parametric driving and/or spin transfer torque [34, 36–38]. In this work we investigate the properties of microscopic easy-plane “gain” ferromagnets. We map the equation of motion of local magnetic moments to a dissipative one in antiferromagnets, and thus argue their equivalence. Based on this finding, we numerically demonstrate the formation of an antiferromagnetic skyrmion stabilized in single-layer chiral ferromagnets with gain, and study its dynamics driven by spin-polarized electric currents. We also investigate the spin-wave spectrum in PT symmetric bilayer ferromagnets by tuning the balanced loss-gain parameter. It is interesting that the emerging antiferromagnetic skyrmion can only be found when the PT symmetry is broken.

FIG. 1: (a) An easy-plane ferromagnet with gain. The negative-damping torque drives the local magnetic moment \( \mathbf{m} \) away from the parallel state. (b) An easy-axis antiferromagnet with loss. The antiparallel \( \mathbf{n} \) state is stable due to the very presence of the damping torque.

Theoretical mapping.—Without loss of generality, we consider the following Hamiltonian of a ferromagnet in two spatial dimensions (the \( xy \) plane)

\[
\mathcal{H}\{\mathbf{m}_i\} = - \sum_{ij} J_{ij} \mathbf{m}_i \cdot \mathbf{m}_j - \sum_{ij} D_{ij} \hat{\mathbf{t}}_{ij} \cdot (\mathbf{m}_i \times \mathbf{m}_j) + \sum_i K(\mathbf{m}_i \cdot \hat{\mathbf{z}})^2 + \mathcal{H}_{DEN}\{\mathbf{m}_i\},
\]

where \( \mathbf{m}_i \) is the unit spin vector at the \( i \)-th site \( (i, a, i, a) \) with \( i \in \mathbb{Z} \) an arbitrary integer and \( a \) the lattice constant, \( J > 0 \) is the ferromagnetic exchange coupling constant, \( D_{ij} = D \hat{t}_{ij} \times \hat{\mathbf{z}} \).
emergent skyrmions in frustrated magnets. A model of a nanomagnet with a uniaxial anisotropy was used to demonstrate the formation of skyrmions. The model consists of a nanomagnet with a uniaxial anisotropy and a small external magnetic field. The skyrmions are formed due to the interplay between the anisotropy and the external field, and they can be manipulated by applying a spin-polarized electric current.

Emerging AFM skyrmion in ferromagnets. It is a common wisdom that skyrmions cannot stabilize in easy-plane ferromagnets without applying the external magnetic field perpendicular to the plane [42, 44, 45]. We challenge this view by realizing an antiferromagnetic skyrmion in a ferromagnetic thin film with gain. To this end, we numerically solve Eq. (3) with the MuMax3 package [46]. We use materials parameters of Fe$_{0.7}$Co$_{0.3}$Si [42] with the saturation magnetization $M_s = 9.5 \times 10^4$ A m$^{-1}$, the lattice constant $a = 0.5$ nm, the ferromagnetic exchange constant $J = 1.3$ meV, the DMI $D = 0.42$ meV, and the magnetocrystalline anisotropy coefficient $K = 0.12$ meV. We consider a sample of size $30 \times 30 \times 0.5$ nm$^3$ with the free boundary condition and the gain parameter $\alpha = 0.01$.

We start our simulation with a random initial ($t = 0$) magnetization profile [see Fig. 2(a)], which mimics the state of the thermal demagnetization, for instance. At $t = 0.035$ ns, local magnetic moments quickly evolved to an antiparallelly aligned state, as shown in Fig. 2(b). We therefore achieve an antiferromagnetic state in a ferromagnet, with the energy cost $2 \times 60^2 J = 9.4$ eV. We can thus estimate the average power to be as low as 43 nW, which is comparable with the power to excite spin waves in ferromagnetic thin films [47]. However, the skyrmionic spin structure is yet to emerge. In Fig. 2(c) we randomize all spins inside a circle of radius 5 nm in the film center, which can be realized by local heatings. At $t = 0.14$ ns, an AFM skyrmion stabilizes in the spin lattices [see Fig. 2(d) and the inset for the spin profile]. To manipulate the AFM skyrmion motion, we apply an in-plane spin-polarized electric current $j_e = -j_e \hat{z}$ with $j_e = 5.0 \times 10^{11}$ A m$^{-2}$. We find that the AFM skyrmion propagates with a large velocity $3000 \text{ m s}^{-1}$ which is much faster than its ferromagnetic counterpart in the same material [42].

We deduce two compelling applications of our findings.
FIG. 3: (a) Schematic plot of coupled ferromagnetic bilayers (30 × 30×1 nm$^3$) with balanced gain (red layer) and loss (green layer). The equilibrium magnetizations point along $\hat{x}$-direction. (b) Evolution of the eigenfrequencies $\omega_{1,2}$ on the gain-loss parameter $\alpha$ of the $\mathcal{PT}$ symmetric magnetic bilayers for two representative spin-wave modes $k=(\frac{\pi}{30}, \frac{\pi}{30})$ (blue curves) and $\left(\frac{\pi}{60}, \frac{\pi}{60}\right)$ (red curves). (c) Contour plot of the mode dependence of $\alpha_{k}$. Its constituent $m$ and $m'$ representing the spatiotemporal magnetization direction in the layer with gain and the layer with loss, respectively. The equations of motion for the coupled magnetization dynamics read

$$\begin{align*}
\frac{d m}{d t} &= -\gamma m \times \left[H_{\text{eff}} + \lambda J/\mu_{0} M_{s} a^{3}\right] m', \\
\frac{d m'}{d t} &= -\gamma m' \times \left[H'_{\text{eff}} + \lambda J/\mu_{0} M_{s} a^{3}\right] m + \alpha m \times \frac{d m}{d t},
\end{align*}$$

(6)

where $H'_{\text{eff}}$ is identical to $H_{\text{eff}}$ in Eq. (3) by replacing its constituent $m$ with $m'$, and $\lambda > 0$ is the ratio between the NN interlayer and the NN intralayer exchange coupling. Under a combined operation of parity $\mathcal{P}$: $m_+ \leftrightarrow m_-$ and $H_{\text{eff}} \leftrightarrow H'_{\text{eff}}$, and time reversal $\mathcal{T}$: $t \rightarrow -t$, $m_+ \rightarrow -m_-$, $m'_+ \rightarrow -m'_-$, $H_{\text{eff}} \rightarrow -H_{\text{eff}}$, and $H'_{\text{eff}} \rightarrow -H'_{\text{eff}}$, we find that Eqs. (6) are invariant and thus respect the $\mathcal{PT}$ symmetry. To obtain the spin-wave spectrum, we consider a small deviation of both $m$ and $m'$ from their equilibrium direction $\hat{x}$: $m \rightarrow (1, \delta m_{lz}, \delta m_{iz})$ and $m' \rightarrow (1, \delta m'_{lz}, \delta m'_{iz})$ with $|\delta m_{lz}| + |\delta m_{iz}| + |\delta m'_{lz}| + |\delta m'_{iz}| \ll 1$. The eigensolutions of linearized Eqs. (6) have the forms of $\delta m_{lz} = Y e^{i(k_{x}r-\omega t)}$, $\delta m_{iz} = Z e^{i(k_{y}r-\omega t)}$, and $\delta m'_{lz} = Y' e^{i(k_{x}r-\omega t)}$, $\delta m'_{iz} = Z' e^{i(k_{y}r-\omega t)}$ with $r = (i_{x}, i_{y})$ and $k = (k_{x}, k_{y})$ the wave vector of the spin wave. From Eqs. (6), we obtain the equation for the column vector $\Psi(k) = (Y, Z, Y', Z')^{T}$:

$$H(k)\Psi(k) = \omega(k)\Psi(k),$$

(7)

where $H$ is a 4 × 4 matrix

with $\chi_0 = iJ$, $\chi_{1}(k) = 2D\sin k_{x} a$, $\chi_{2}(k) = 2iJ(\cos k_{x} a + \cos k_{y} a - i(k_{x} + k_{y})) - 2iK'$, and $K = K + \mu_{0} M_{s} a^{3}/2$ summing up the easy-plane anisotropy and the demagnetizing energy. The solutions of eigenfrequencies come in pairs $\pm \omega$. Two positive solutions, corresponding to counterclockwise magnetization precession around the ground state along $\hat{x}$-direction, are relevant and can be expressed as

$$\omega_{1,2}(k) = \lambda + 2\zeta(k) \pm \sqrt{\lambda^{2} - 4\zeta^{2}(k)[\lambda + \zeta(k)]}$$

(9)
multiplying \(\gamma J / [(1 + \alpha^2)\mu_0 M_s a^3]\), with \(\zeta = 2 - \cos k_x a - \cos k_y a + (D/J) \sin k_x a\). In deriving the dispersion relation (9), we have dropped the contribution from \(K'\) since we focus on the exchange spin-wave region. For a given \(k\), as the gain and loss parameter \(\alpha\) increases, the two eigenfrequencies approach one another, and at some critical value \(\alpha = \alpha_c\) they coalesce at the exceptional point (EP) and bifurcate into the complex plane [see Fig. 3(b)]. At the EP, the two normal modes coalesce as well. The domain with real eigenfrequencies is termed the exact phase, otherwise it is called the broken phase. From Eq. (9) we can obtain both the critical gain-loss parameter and the critical frequency. Nevertheless, we point out a special region \(-\lambda < \zeta(k) < 0\) in which the \(PT\) symmetry is never broken without considering the nonlinear effect associated with the LLG equations (6). This fact is in contrast to conventional \(PT\) symmetric systems suffering symmetry breaking when the strength of the gain-loss term exceeds a certain critical value [34]. Of course, the nonlinear magnon-magnon interaction complicates this picture and will generate a level broadening of spin-wave eigenmodes [47]; that is to say, magnon modes in this region acquire a finite lifetime. For \(\zeta(k)\) outside \([-\lambda, 0]\), the two critical parameters are given by

\[
\alpha_c(k) = \frac{\lambda}{2 \sqrt{2|\zeta(k)| \lambda + \zeta(k)}}, \quad \omega_c(k) = \frac{\gamma J}{\mu_0 M_s a^3} \left(1 - \frac{2 \zeta(k)}{\lambda + \zeta(k)}\right),
\]

both of which are mode-dependent. Figures 3(c) and (d) show the distribution of \(\alpha_c\) and \(\omega_c\) over the first Brillouin zone, respectively. The center of the white region in Fig. 3(c) does not coincide with the origin, with a downward shift \(\arctan(D/J)\) caused by the DMI.

In the exact phase \(\alpha < \min_{k_x} \alpha_c(k) = \lambda / [2 \sqrt{2|\zeta_0| (\lambda + \zeta_0)}]\), with \(\zeta_0 = 3 + \sqrt{1 + (D/J)^2}\), predictions from the linear spin-wave theory compare well with the full simulation of Eqs. (6) that the steady-state magnetizations in both layers oscillate around the initial misalignment from the \(\hat{x}\) axis without being attenuated or amplified (see Fig. 4).

In the broken phase \(\alpha > \lambda / [2 \sqrt{2|\zeta_0 (\lambda + \zeta_0)}]\) \(\approx 0.012\) for \(\lambda = 0.1\), the linear theory indicates an exponential growth of the spin-wave amplitude, which is associated with the case that the eigenfrequencies (9) have an imaginary part. The induced instability can drive the spin away from its equilibrium direction, but is eventually suppressed by nonlinearities. The situation in the critical phase \(\alpha = \lambda / [2 \sqrt{2|\zeta_0 (\lambda + \zeta_0)}]\) is similar. The linear spin-wave theory gives a linear instead of exponential growth of the wave amplitude, which is the consequence of the EP degeneracy, and it is finally taken over by the intrinsic nonlinearities. The lossy layer thus preserves the in-plane ferromagnetic state to some extent (not shown). However, in the gain layer, we find it interesting that the original in-plane magnetizations along the \(\hat{x}\)-direction evolve to be perpendicular to the plane, i.e., in \(\hat{z}\)-direction, and finally form an AFM skyrmion, as shown in Figs. 5(a)-(d). To speed up the evolution, we set the gain-loss parameter \(\alpha = 0.3\) in numerically simulating Eqs. (6).

**Discussion.**—Negative damping is essential to realize our proposal. Its real world implementation methods are multiformal besides the two approaches introduced above. A recent experiment reported the electric field-induced negative magnetic damping in FM/FE (ferroelectric) heterostructures [49]. In Ref. [50], Wegrowe et al. thoroughly analyzed the spin transfer in an open ferromagnetic layer, and found that the negative damping appears naturally for describing the exchange of spins between the magnetic system and the environment [51–53].

**Conclusion.**—In summary, we uncovered a mapping between a ferromagnet with gain and an antiferromagnetic with an equal amount of loss. In a chiral easy-plane ferromagnet in the presence of gain, we showed the emergence of a stabilized antiferromagnetic skyrmion without applying any external field. In 1D and 2D non-chiral “gain” ferromagnets, we envision the formation of antiferromagnetic domain walls [54] and antiferromagnetic vortices [55], respectively. We also studied the spin-wave spectrum in ferromagnetic bilayers with balanced gain and loss. The magnon dispersion relation and the mode-dependent critical gain-loss parameter were derived. We predicted a spectral region in the first Brillouin zone, in which the \(PT\) symmetry is never broken in the framework of linear spin-wave theory. This is in sharp contrast to the result in macroscopic magnetic structures [34]. We found that the antiferromagnetic skyrmion appears in the “gain” layer only in the cases of broken \(PT\) symmetry phase. The results presented here open a new way to create and manipulate antiferromagnetic solitons in simple ferromagnets, and build a
novel bridge connecting the $\mathcal{PT}$ symmetry to magnonics and skyrmionics.

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