Ideal gas with a varying (negative absolute) temperature: An alternative to dark energy?

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Abstract

The present work is an attempt to investigate whether the evolutionary history of the Universe from the offset of inflation can be described by assuming the cosmic fluid to be an ideal gas with a specific gas constant but a varying negative absolute temperature (NAT). The motivation of this work is to search for an alternative to the "exotic" and "supernatural" dark energy (DE). In fact, the NAT works as an "effective quintessence" and there is need to deal neither with exotic matter like DE nor with modified gravity theories. For the sake of completeness, we release some clarifications on NATs in Section 3 of the paper.

Keywords: Negative absolute temperature; Cosmic acceleration; Ideal gas law; Dark energy

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1 Introduction

Since the intriguing discovery of the late time cosmic acceleration in 1998, which was expected by the Indian physicist B. Sidharth \[\Pi\] before the cosmological observations of the High-Z Supernova Search Team \[\Pi\] and of the Supernova...
Cosmology Project \[3\], there have been rigorous attempts to incorporate this unexpected observations into standard cosmology. This is the famous DE issue, which is a challenging problem in cosmology. The simplest and the widely accepted form of DE is the so-called cosmological constant, which arises from a historical idea of Einstein in a different context \[4\]. However, it is plagued by the coincidence \[5\] and the cosmological constant \[6\] problems. As a consequence, alternative ways have been proposed such as modified gravity theories \[7 - 9\], inhomogeneous cosmological models \[10, 11\] etc. But, again, each one of them comes with their own disadvantages. Other models such those of particle creation have also been investigated in the past \[12\] as well as recent times \[13\]. The advantage of these models is that, in order to explain the evolutionary history of the cosmos, one needs neither any exotic matter nor any modification of Einstein’s general theory of relativity (GTR). Moreover, such models seem to be thermodynamically motivated. Nevertheless, the exact form of the particle creation rate have still not been identified. Thus, the research in this particular field has been largely phenomenological.

For the sake of completeness we recall that there are also some new attempts introducing new origins for the DE problem. The probable non-extensive features of spacetime \[27–29\], and the tendency of spacetime to couple with matter in a non-minimal way \[30\] are some of these attempts. Moreover, the negative temperature of cosmic fluid can also be obtained as the direct result of solving the Friedmann and Thermodynamics equations simultaneously \[31\].

Another key point of the current cosmological tapestry is the inflationary era, i.e. the idea that after the initial singularity the Universe was led by a very fast phase of expansion, which was due to a big negative pressure \[14, 15\].

In this paper we attempt to investigate whether the evolutionary history of the Universe from the offset of inflation can be described by assuming the cosmic fluid to be an ideal gas with a specific gas constant but a varying NAT. In this regard, on one hand it is worthwhile to mention that NAT has been considered in the context of Cosmology before \[16\]. On the other hand, we also stress that the assumption that the cosmic fluid should be an ideal gas at the end of the inflationary era has been used in \[17\] while in \[18\] it has been shown that an ideal gas cosmological solution leads to a universal accelerated expansion which seems consistent with the supernova observations. Thus, the most important motivation behind this work is that negative absolute temperature gives rise to negative pressures which can therefore behave as DE and lead the Universe to accelerate.

2 The ideal gas model with negative absolute temperature

Let us consider a flat Friedmann-Lemaitre-Robertson-Walker (FLRW) Universe with the line element (we work with \(c = 1\) in the following) \[19\]

\[
ds^2 = -dt^2 + a^2(t) \left[ dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right].
\]  

(1)
The FLRW field equations are

$$H^2 = \frac{8\pi G}{3} \rho \quad \text{and} \quad \dot{H} = -4\pi G(\rho + p), \quad (2)$$

from which one can derive the conservation equation as

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (3)$$

In Eqs. (2) and (3), $a(t)$ is the scale factor of the Universe, $H = \frac{\dot{a}(t)}{a(t)}$ is the Hubble parameter, $\rho$ is the total energy density of the cosmic fluid, and $p$ is the pressure of the fluid. The ideal gas law is well known and reads

$$pV = nRT, \quad (4)$$

where $p$, $V$, and $T$ are the pressure, volume and temperature of the gas respectively, $n$ denotes the amount of substance in the gas, often known as the number of moles, and $R$ is the universal gas constant. Now, if $m$ and $M$ denote the mass and the molar mass of the gas respectively, and $\rho$ is the density, then we have

$$V = \frac{m}{\rho} \quad \text{and} \quad n = \frac{m}{M}, \quad (5)$$

and the ideal gas law in Eq. (4) can be rewritten as

$$p = R^* T \rho, \quad (6)$$

which we shall call the ideal gas equation of state. Here $R^* = \frac{R}{M}$ is known as the specific gas constant. Notice that the Universe undergoes acceleration or deceleration according as $T < -\frac{1}{3R^*}$ or $T > \frac{1}{3R^*}$. This hints at a remarkable connection between NAT and cosmic acceleration. NAT (or Kelvin temperatures) are interesting and somewhat unusual, but not impossible or paradoxical [20 - 24]. They are related to the concept of population inversion in statistical physics [20 - 22]. The population inversion is obtained by a process called optical pumping, which is a way of imparting energy to the working substance of a laser in order to transfer the atoms to excited states. Systems with a NAT will decrease in entropy as one adds energy to the system [23]. Most familiar systems cannot achieve NATs because adding energy always increases their entropy. The possibility of decreasing in entropy with increasing energy requires the system to "saturate" in entropy [2], with the number of high energy states being small. These kinds of systems, bounded by a maximum amount of energy, are generally forbidden in classical physics. Thus, NAT is a strictly quantum phenomenon. Hence, under special conditions (high-energy states are more occupied than low-energy states), NAT are possible [24]. These are states existing in localized systems with finite, discrete spectra, which can be prepared

\[1\text{In SI units it is } R = 8.314 \text{ } J/\text{mol.K.}\]

\[2\text{This implies that systems with NATs can never achieve thermodynamic equilibrium.}\]
for motional degrees of freedom [24]. An intriguing example is the creation of an attractively interacting ensemble of ultracold bosons at NAT that is stable against collapse for arbitrary atom numbers [24]. Another remarkable fact is that NATs are hotter than all positive temperatures, even hotter than infinite temperature (a proof is given in Appendix A of this paper).

Now, assuming no non-gravitational interaction, the temperature falls [25, 26] with expansion as $1/a$ for relativistic fluids such as photons (radiation) and as $1/a^2$ for non-relativistic fluids like dust and cold dark matter. These expressions get only slightly modified even if there is any interaction because such an interaction is expected to be of very small strength. Thus, one can write the expression for the temperature (in relativistic and non-relativistic eras) in a compact form as

$$T_{r,nr} = T_{\delta_r,\delta_{nr}} a^{-(1+\delta)},$$

(7)

where $\delta$ assumes the values 0 and 1 for relativistic and non-relativistic eras respectively, and $T_{\delta_r}$ ($T_{\delta_{nr}}$) represents the proportionality constant in relativistic (non-relativistic) era.

Putting Eq. (7) into Eq. (6) and using the conservation equation (3), one gets the solution for $\rho$ (in relativistic and non-relativistic eras) as

$$\rho_{r,nr} = \rho_{\delta_r,\delta_{nr}} a^{-3} \exp \left\{ 3 R^* T_{\delta_r,\delta_{nr}} (1 + \delta) a^{-(1+\delta)} \right\},$$

(8)

where $\rho_{\delta_r}$ ($\rho_{\delta_{nr}}$) is the constant of integration corresponding to relativistic (non-relativistic) era. Note that $\rho_r \to \infty$ ($\rho_{nr} \to 0$) as $a \to 0$ ($a \to \infty$). The deceleration parameter for this model in relativistic and non-relativistic eras is given by

$$q_{r,nr} = -\left( \frac{H}{H^2} \right)_{r,nr} - 1$$

= \frac{1}{2} \left[ 1 + 3 R^* T_{\delta_r,\delta_{nr}} a^{-(1+\delta)} \right].$$

(9)

(10)

In terms of the redshift $z$, the above two expressions respectively become

$$\rho_{r,nr}(z) = \rho_{\delta_r,\delta_{nr}} (1 + z)^3 \exp \left\{ 3 R^* T_{\delta_r,\delta_{nr}} (1 + z)^{(1+\delta)} \right\},$$

(11)

$$q_{r,nr}(z) = \frac{1}{2} \left[ 1 + 3 R^* T_{\delta_r,\delta_{nr}} (1 + z)^{(1+\delta)} \right].$$

(12)

Let us consider $z_t$ as being the redshift at which the Universe transits from relativistic era to non-relativistic era. Then from Eq. (7), we obtain $T_r = T_{\delta_r}(1 + z_t)$ and $T_{nr} = T_{\delta_{nr}}(1 + z_t)^2$. Taking the ratio of these two equations and noting that $T_r = T_{nr}$ at $z = z_t$, we get

$$z_t = \frac{T_{\delta_r}}{T_{\delta_{nr}}} - 1.$$
Then, the energy density and the deceleration parameter can be separately expressed in the two eras as

$$
\rho(z) = \begin{cases} 
\rho_{\delta r}'(1 + z)^3\exp[3R^*T_{\delta nr}(1 + z_t)(1 + z)] & \text{(relativistic era)} \\
\rho_{\delta nr}(1 + z)^3\exp\left[\frac{3}{2}R^*T_{\delta nr}(1 + z)^2\right] & \text{(non-relativistic era)} 
\end{cases}
$$

(14)

with

$$
\rho_{\delta r}' = \rho_{\delta nr}\exp\left[-\frac{3}{2}R^*T_{\delta nr}(1 + z_t)^2\right],
$$

(15)

and

$$
q(z) = \begin{cases} 
\frac{1}{2}\left\{1 + 3R^*T_{\delta nr}(1 + z_t)(1 + z)\right\} & \text{(relativistic era)} \\
\frac{1}{2}\left\{1 + 3R^*T_{\delta nr}(1 + z)^2\right\} & \text{(non-relativistic era)} 
\end{cases}
$$

(16)

respectively.

Now, if $q_t$ ($\rho_t$) is the value assumed by the deceleration parameter (energy density) at $z_t$, then, from either of the expressions in Eq. (16), one gets

$$
R^*T_{\delta nr} = \frac{2q_t - 1}{3(1 + z_t)^2},
$$

(17)

and, by substituting in either of the expressions in Eq. (14), one obtains

$$
\rho_{\delta nr} = \rho_t(1 + z_t)^{-3}\exp\left[-\frac{1}{2}(2q_t - 1)\right].
$$

(18)

Thus, on one hand, one can determine the values of the unknown constants $T_{\delta nr}$ and $\rho_{\delta nr}$ once $M$ is known. On the other hand, $T_{\delta r}$ can be obtained from Eq. (13). Consequently, $\rho(z)$ and $q(z)$ can be exactly determined. Furthermore, it is evident from the second expression in Eq. (16) that $q(z)$ vanishes for two values of $z$:

$$
viz. \, z = -1 \pm \sqrt{\frac{1 + z_t}{1 - 2q_t}},
$$

(19)

provided $0 < q_t < \frac{1}{2}$. In this regard, one also notes the following points:

- $T_{\delta nr}$ is negative which implies a NAT throughout the non-relativistic era.

- From Eq. (13), we observe that now $T_{\delta r}$ should assume a (negative) value less than $T_{\delta nr}$ in order to have a positive redshift.

The redshift $z = -1 + \sqrt{\frac{1 + z_t}{1 - 2q_t}}$ may correspond to the transition of the Universe from the decelerating matter dominated phase to the presently observed late time accelerating phase. However, the other value of $z$ is smaller than $-1$ and, in turn, irrelevant in the context of cosmic evolution.

Thus, in a certain sense the NAT works like an “effective quintessence”. In fact, one can introduce an “effective quintessential scalar field” $\phi(z)$ starting from
\[ \phi'(z) \equiv \frac{d\phi(z)}{dz} = \begin{cases} \frac{-1}{1+z} \sqrt{1 + \frac{2q-1}{3(1+z^2)}(1+z)} & \text{(relativistic era)} \\ \frac{-1}{1+z} \sqrt{1 + \frac{2q-1}{3(1+z^2)}(1+z)^2} & \text{(non-relativistic era)} \end{cases} \]

which implies

\[ \phi(z) = \begin{cases} \ln \left( \sqrt{\frac{1 + \frac{2q-1}{3(1+z^2)}(1+z)^2}{1 + \frac{2q-1}{3(1+z^2)}(1+z) - 1}} \right) - 2 \sqrt{1 + \frac{2q-1}{3(1+z^2)}(1+z)} + C_1 & \text{(relativistic era)} \\ \tanh^{-1} \left( \sqrt{\frac{1 - \frac{2q-1}{3(1+z^2)}(1+z)}{1 + \frac{2q-1}{3(1+z^2)}(1+z)^2}} \right) - \sqrt{1 + \frac{2q-1}{3(1+z^2)}(1+z)^2} + C_2 & \text{(non-relativistic era)} \end{cases} \]

where \(C_1\) and \(C_2\) are arbitrary constants of integration. The effective quintessential scalar field of Eq. (21) is associated to an “effective potential”

\[ V(\phi(z)) = \begin{cases} \frac{1}{2} \rho(t)(1+z)^3 \exp(1-2q)(1+z)^3 \exp \left( \frac{2q-1}{2(1+z^2)}(1+z) \right) \left\{ 1 - \frac{2q-1}{3(1+z^2)}(1+z) \right\} & \text{(relativistic era)} \\ \frac{1}{2} \rho(t)(1+z)^3 \exp \left( \frac{1}{2} - q \right)(1+z)^3 \exp \left( \frac{2q-1}{2(1+z^2)}(1+z)^2 \right) \left\{ 1 - \frac{2q-1}{3(1+z^2)}(1+z)^2 \right\} & \text{(non-relativistic era)} \end{cases} \]

Then, one gets

\[ \rho(z) = \frac{1}{2} \dot{\phi}^2 + V(\phi), \]

with the associated Lagrangian

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi). \]

3 Some clarifications about the negative absolute temperature

The negative temperature is the result of using the Boltzmann theorem in order to calculate the system entropy [32]. Only systems whose energy spectrum is bounded above can reach negative temperatures [32]. In this paper we use a classical gas in which energy spectrum has not any restriction. Therefore, the attentive reader could ask the following legitimate questions [33]: What is the origin of this negative temperature? Indeed, what is the maximum bound of energy in this model? Can one consider the maximum value of observable mass of universe in each cosmic era as the upper bound for energy at that era? It
is indeed possible to obtain a maximum energy/mass bound in both relativistic and non-relativistic eras of our model. Since $E = \frac{4}{3} \pi (ar)^3 \rho$ and $T_\delta$, $T_{\delta r}$ are both negative, one gets

$$E_r = \frac{4}{3} \pi (ar)^3 \rho_r = 4 \pi a^3 \rho_\delta, a^{-3} \exp \left[ 3 R^* T_\delta, a^{-1} \right] < V \rho_\delta$$

for relativistic era, and

$$E_{nr} = \frac{4}{3} \pi (ar)^3 \rho_{nr} = \frac{4}{3} \pi a^3 \rho_{\delta r}, a^{-3} \exp \left[ \frac{3}{2} R^* T_{\delta r}, a^{-2} \right] < V \rho_{\delta r}$$

for non-relativistic era. The upper bounds on $E_r$ and $E_{nr}$ can be exactly determined once the volume $V$ and the coefficients $\rho_\delta$, $\rho_{\delta r}$ are known.

It is also important clarify what negative absolute temperature means in physical terms [33]. Let us start with an analogy. In order to boil water, it is required to add energy to it. During the process of heating up, the water molecules gradually increase their kinetic energy and move with faster average velocities. However, the individual molecules possess different kinetic energies, ranging from very slow to very fast. In other words, low energy states are more probable in thermal equilibrium as compared to high energy states which means that only few particles have very fast velocities. This is referred to as Boltzmann distribution in physics. When particles achieve negative absolute temperatures, then Boltzmann distribution undergoes an inversion, i.e., most particles possess large energies, while a few have small energies. As a consequence, a physical system having negative temperature scale is hotter than any system with a positive temperature. When a physical system having negative temperature comes in touch with a physical system having positive temperature heat flows from the negative temperature system to the positive temperature system [34, 35]. This could appear as being a paradox, but the problem is solved if one discusses temperature through the thermodynamic rigorous definition of trade-off between energy and entropy. In that case, the more fundamental quantity is the reciprocal of the temperature, i.e. the thermodynamic beta. Hence, a system having positive temperature increases in entropy if one adds energy to the system. Instead, a system with having negative temperature decreases in entropy when one adds energy to the system [23].

4 Concluding Remarks

In the approach of this work the cosmic fluid has been considered as an ideal gas with a specific gas constant $R^*$ (to be fixed by observations) but a varying NAT ($T_r$ in the relativistic era and $T_{nr}$ in the non-relativistic era). Depending on whether the temperature is greater (less) than $-\frac{1}{3R^*}$, the Universe undergoes deceleration (acceleration). Hence, the cosmic evolution from the offset of inflation to the present accelerating phase can indeed be described by considering an ideal gas as the cosmic fluid. In a certain sense, the NAT works like an “effective quintessence”. As a consequence, it is necessary to deal neither with
exotic matter like DE nor with modified gravity theories. Thus, the quantum nature of the NAT might play an important role in studying the evolutionary history of the Universe.

We observe that $q$ in Eq. (16) gives one transition at a redshift $z_p = -\frac{1}{3R^* T_{b_{nr}}(1+z_t)} - 1$ in the relativistic era which may correspond to the transition of the early Universe from inflation to deceleration. In principle, one can determine a lower bound on $T_{b_{nr}}$ by imposing the condition $z_p > z_t$, which gives

$$T_{b_{nr}} > -\frac{1}{3R^*(1+z_t)^2}.$$  

Finally, if one again recalls the previously cited issue that NAT states have been demonstrated in localized systems with finite, discrete spectra, preparing a NAT state for motional degrees of freedom [24], one could shown that our atomic system is stable, even though the atoms strongly attract each other, that means they want to collapse but cannot due to being at NAT state. Thus, NAT implies negative pressures and open up new parameter regimes for cold atoms, enabling fundamentally new many-body states [24]. The GTR shows that our Universe as a whole is also not collapsing under the attractive force of gravity [19], but it cannot explain that the Universe undergoes an accelerated expansion. DE, which is believed to possess a huge negative pressure, has been introduced to describe this effect. Therefore, whether this negative pressure arises due to a negative fluid temperature remains to be seen, and this is the same conclusion of [24]. It seems that NATs might play an important role in the dynamics of our Universe and whether further investigations reveal a deep connection between the nature of DE and the temperature of the cosmic fluid remains to be seen.

For the sake of completeness, some clarifications on NATs have been discussed in Section 3 of this work.

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Appendix A

**Proposition:** Negative absolute temperatures are hotter than positive absolute temperatures.

**Proof:** Consider two bodies (1 \& 2) at different temperatures ($T_1$ \& $T_2$) in contact with one another. Suppose there is a transfer of a small amount of heat $Q$ from body 1 to body 2, which changes the entropy of body 1 by $-Q/T_1$ and that of body 2 by $Q/T_2$, so that the total change in entropy is

$$dS = Q \left( \frac{1}{T_2} - \frac{1}{T_1} \right).$$

The above quantity must be positive according to the second law. Now, if $T_1 < 0$ and $T_2 > 0$, then $\frac{1}{T_2} - \frac{1}{T_1} > 0$, which implies that body 1 (with a negative temperature) can transfer heat to body 2 (with a positive temperature), but not the other way around. Hence, negative absolute temperatures are hotter than positive absolute temperatures.