$B_s \to f_0(980)$ form factors and $B_s$ decays into $f_0(980)$

Pietro Colangelo, Fulvia De Fazio and Wei Wang
Instituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy

We compute the $B_s \to f_0(980)$ transition form factors using light-cone QCD sum rules at leading order in the strong coupling constant, and also including an estimate of next-to-leading order corrections. We use the results to predict the branching fractions of the rare decay modes $B_s \to f_0 \ell^+ \ell^-$ and $B_s \to f_0 \nu \bar{\nu}$, which turn out to be $\mathcal{O}(10^{-7})$ ($B_s \to f_0(980)\ell^+ \ell^-$, with $\ell = e, \mu$), $\mathcal{O}(10^{-8})$ ($B_s \to f_0(980)\tau^+ \tau^-$) and $\mathcal{O}(10^{-9})$ ($B_s \to f_0(980)\nu \bar{\nu}$). We also predict the branching ratio of $B_s \to J/\psi f_0(980)$ decay under the factorization assumption, and discuss the role of this channel for the determination of the $B_s$ mixing phase compared to the golden mode $B_s \to J/\psi \phi$. As a last application, we consider $D_s \to f_0$ form factors, providing a determination of the branching ratio of $D_s \to f_0 e^+ \nu_e$.

PACS numbers: 13.20.He, 13.20.Fc, 13.25.Hw

1. INTRODUCTION

Theoretical and experimental efforts aimed at disclosing physics beyond the standard model (SM) proceed in several directions. Among these, there is the study of rare processes which are induced only at loop level in the SM and are therefore sensitive to new physics (NP) contributions which may potentially enhance their small ($<10^{-5}$) branching ratios. Another testing ground is the precise study of CP violation. It has been realized that the amount of CP violation within the SM is too small to explain the observed baryon asymmetry of the Universe, a conclusion confirmed by recent analyses. Since the only source of CP violation in the SM is the complex phase of the Cabibbo Kobayashi Maskawa (CKM) mixing matrix, the determination of the elements of this matrix and of their relative phases is of primary importance, in order to disentangle sources of additional contributions to CP violation. As well known, the task is afforded through the study of the so called unitarity triangles, the graphical representations of the conditions stemming from unitarity of the CKM matrix. The most studied triangle is the one which relates the CKM elements involved in $B$ decays. Direct and indirect determinations of its sides and angles lead to a picture of CP violation coherent with the SM description. Also in this case, investigation of effects predicted to be small in the SM is a promising strategy to reveal new physics.

$B_s$ mesons provide the possibility to search for new physics scenarios exploiting both the strategies outlined above. On the one hand, rare $B_s$ decays induced by $b \to s$ transition are suppressed in the SM, as with all decay modes governed by such a transition, and new physics effects may enhance their branching fractions. For example, it has been shown that, in presence of a single universal extra dimension compactified on a circle with radius $R$, the rates of $B_s \to \phi \nu \bar{\nu}$, $B_s \to \eta'^{0} \ell^+ \ell^-$ and $B_s \to \eta^{(0)} \nu \bar{\nu}$ are enhanced when $R^{-1}$ decreases, while the opposite happens in the case of $B_s \to \phi \gamma$, which has a smaller branching fraction with respect to SM for small values of $R^{-1}$.

On the other hand, the analysis of the unitarity triangle of CKM elements relevant for $B_s$ decays is an important test of the SM description of CP violation. The triangle is defined by the relation

$$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0.$$  

One of its angles, $\beta_s$, defined as $\beta_s = \text{Arg} \left( \frac{V_{us} V_{ub}^*}{V_{cs} V_{cb}^*} \right)$, is half of the phase of $B_s - \bar{B}_s$ mixing, and is predicted to be tiny in the SM: $\beta_s \simeq 0.019$ rad. Recent data obtained by the CDF and D0 Collaborations, based on the angular analysis of $B_s \to J/\psi \phi$, indicate much larger values, although with sizable uncertainties, so that the precise measurement of $\beta_s$ represents one of the priorities in the physics programs at the hadron colliders and at the $B$ factories operating at the $\Upsilon(5S)$ peak.

In this paper we consider $B_s$ decays in both respects. We compute the $B_s \to f_0(980)$ form factors using light-cone QCD sum rules (LCSR) at the leading order in the strong coupling constant (Sect. 2) and including an estimate of next-to-leading (NLO) corrections (Sect. 3). In Sect. 4 we use the results to predict the branching fractions of the rare decay modes $B_s \to f_0 \ell^+ \ell^-$ and $B_s \to f_0 \nu \bar{\nu}$ in the SM. The form factors are also a necessary ingredient to study the nonleptonic mode $B_s \to J/\psi f_0$ which, together with $B_s \to J/\psi \phi$, permits one to access the phase $\beta_s$. Our predictions for this mode are collected in Sect. 5. As a byproduct of the calculation, we explore the $D_s \to f_0 e^+ \nu_e$ decay channel, the branching ratio of which has been recently measured by the CLEO Collaboration. Conclusions are presented in the last section.

1 Hereafter, we use $f_0$ to denote the $f_0(980)$ meson.
We describe the calculation of the three functions $F_1$, the auxiliary form factors $F_0$ for the sake of the calculation, it is convenient to define the form factors $F_1$, $F_0$, and $F_T$ using the method of light-cone QCD sum rules. For the sake of the calculation, it is convenient to define the auxiliary form factors $f_+$ and $f_-$.

\[
\langle f_0(p_f) | \bar{s} \gamma_\mu \gamma_5 b | \mathcal{B}_s(p_B) \rangle = -i \left\{ F_1(q^2) P_\mu - \frac{m_B^2 - m_f^2}{q^2} q_\mu \right\},
\]

\[
\text{where } P = p_B + p_f, \text{ and } q = p_B - p_f. \text{ In this section we describe the calculation of the three functions } F_1, F_0 \text{ and } F_T \text{ using the method of light-cone QCD sum rules.}
\]

The leading twist and a few subleading twist LCDA give the dominant contribution, while higher twist terms are power suppressed. The LCSR approach has been successfully applied to compute the hadronic parameters in QCD. Equating the two representations provides one with a sum rule suitable to derive the form factors.

The hadronic representation of the correlation function in QCD consists in the contribution of the $B_s$ meson and of the higher resonances and the continuum of states $h$. In a one-resonance+continuum representation, the correlation function can be written as

\[
\Pi^{\text{QCD}}(p_f, q) = \frac{1}{\pi} \int_{(m_b + m_s)^2}^{\infty} ds \frac{\text{Im} \Pi^{\text{QCD}}(s, q^2)}{s - (p_f + q)^2}.
\]

Using global quark-hadron duality, the integral in (11) can be identified with the corresponding quantity in the QCD representation (11):

\[
\int_{s_0}^{\infty} ds \frac{\rho^h(s, q^2)}{s - (p_f + q)^2} = \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im} \Pi^{\text{QCD}}(s, q^2)}{s - (p_f + q)^2}.
\]
A Borel transformation of the hadronic and of the QCD expressions of the correlation function is carried out, defined as:

$$B[\mathcal{F}(Q^2)] = \lim_{Q^2 \to \infty, n \to \infty} \frac{1}{(n-1)!} (-Q^2)^n \left( \frac{d}{dQ^2} \right)^n \mathcal{F}(Q^2)$$

where $\mathcal{F}$ is a function of $Q^2 = -q^2$ and $M^2$ is the Borel parameter, so that

$$B \left[ \frac{1}{(s+Q^2)^n} \right] = \frac{\exp(-s/M^2)}{(M^2)^n (n-1)!}.$$  \hspace{1cm} (14)

This operation improves the convergence of the OPE series by factorials of $n$ and, for suitably chosen values of $M^2$, enhances the contribution of the low lying states to the correlation function.

Applying the transformation to both $\Pi^{\text{HAD}}$ and $\Pi^{\text{QCD}}$ we obtain the sum rule

$$\langle f_0(p_{f_0}) | j_{\lambda} | \overline{T}_s(p_{B_s}) \rangle \langle \overline{T}_s(p_{B_s}) | j_{\lambda'} | 0 \rangle \exp \left[ -\frac{m_{B_s}^2}{M^2} \right] = \frac{1}{\pi} \int_{(m_{\pi}+m_s)^2}^{s_0} ds \exp[-s/M^2] \text{Im} \Pi^{\text{QCD}}(s, q^2),$$

where $p_{B_s} = p_{f_0} + q$. Eq.(15) allows to derive the sum rules for $f_+, f_-$ and $F_T$, choosing either the current $j_{\lambda_1} = J^{5}_{\mu}$ or the current $j_{\lambda_1} = J^{5T}_{\mu}$.

The calculation of $\Pi^{\text{QCD}}$ is based on the expansion of the T-product in (6) near the light-cone, which produces matrix elements of non-local quark-gluon operators. In the description of $f_0$ as a $s\bar{s}$ state modified by some hadronic dressing [14], these can be defined in terms of $f_0$ light-cone distribution amplitudes of increasing twist:

$$\langle f_0(p_{f_0}) | \bar{s}(x) \gamma_\mu s(0) | 0 \rangle = \tilde{f}_{0\mu} p_{f_0} \int_0^1 du \epsilon^{\mu\nu\rho\sigma} \Phi_{f_0}(u),$$

$$\langle f_0(p_{f_0}) | \bar{s}(x) s(0) | 0 \rangle = m_{f_0} \tilde{f}_{0} \int_0^1 du \epsilon^{\mu\nu\rho\sigma} \Phi_{f_0}(u),$$

$$\langle f_0(p_{f_0}) | \bar{s}(x) \sigma_{\mu\nu} s(0) | 0 \rangle = -\frac{m_{f_0}}{6} \tilde{f}_{0} (p_{f_0} x_{\nu}-p_{f_0} x_{\mu})$$

$$\times \int_0^1 du \epsilon^{\mu\nu\rho\sigma} \Phi_{f_0}^\sigma(u),$$

where the LCDA $\Phi_{f_0}$ is twist-2, and the other two are twist-3, and are normalized as

$$\int_0^1 du \Phi_{f_0}(u) = 0, \int_0^1 du \Phi_{f_0}^\sigma(u) = \int_0^1 du \Phi_{f_0}^\nu(u) = 1.$$ \hspace{1cm} (17)

In terms of these LCDA, the sum rules for the three form factors read:

$$f_+(q^2) = \frac{m_{B_s} + m_s}{2m_{B_s}^2 f_{B_s}} \tilde{f}_{f_0} \exp \left[ \frac{m_{B_s}^2}{M^2} \right] \left\{ \int_0^1 du \exp \left[ \frac{-m_{B_s}^2 + u m_{J_0}^2 - "u" q^2}{u M^2} \right] \left[ -m_{f_0} \Phi_{f_0}(u) + u m_{f_0} \Phi_{f_0}^\sigma(u) + \frac{1}{3} m_{f_0} \Phi_{f_0}^\nu(u) \right] ight\},$$

$$f_-(q^2) = \frac{m_{B_s} + m_s}{2m_{B_s}^2 f_{B_s}} \tilde{f}_{f_0} \exp \left[ \frac{m_{B_s}^2}{M^2} \right] \left\{ \int_0^1 du \exp \left[ \frac{-m_{B_s}^2 + u m_{J_0}^2 - "u" q^2}{u M^2} \right] \left[ m_{f_0} \Phi_{f_0}(u) + (2-u) m_{f_0} \Phi_{f_0}^\sigma(u) \right] ight\},$$

$$F_T(q^2) = \frac{(m_{B_s} + m_{f_0})(m_{B_s} + m_s)}{m_{B_s}^2 f_{B_s}} \tilde{f}_{f_0} \exp \left[ \frac{m_{B_s}^2}{M^2} \right] \left\{ \int_0^1 du \exp \left[ \frac{-(m_{B_s}^2 - "u" q^2 + u m_{J_0}^2)}{u M^2} \right] \left[ \Phi_{f_0}(u) \frac{2}{m_{f_0}} + \frac{m_{f_0} \Phi_{f_0}^\sigma(u)}{6 u M^2} \right] \right\},$$

where

$$u_0 = \frac{m_{f_0}^2 + q^2 - s_0 + \sqrt{(m_{f_0}^2 + q^2 - s_0)^2 + 4 m_{f_0}^2 (m_{f_0}^2 - q^2)}}{2 m_{f_0}^2}.$$ \hspace{1cm} (21)
Our formulae can be compared to the ones for the $B$-to-scalar meson form factors in Ref. [18], where the case of the meson $a_0$ is considered. We find differences in the expression of the form factor $f_+$. 

III. NUMERICAL RESULTS AND DISCUSSIONS

A. Leading order results

Based on the conformal spin invariance, the LCDA can be expanded in terms of Gegenbauer polynomials $C_n^{3/2}$. The expansion of the twist-2 LCDA $\Phi_{f_0}(u)$ reads:

$$\Phi_{f_0}(u) = 6u(1-u) \left\{ B_0 + \sum_{n=1} C_n^{3/2} (2u - 1) \right\}. \quad (22)$$

Due to the charge conjugation invariance, all even Gegenbauer moments of $\Phi_{f_0}(u)$ vanish, so that $B_{2m} = 0$ for $m = 0, 1, \ldots$ in (22); as for the odd moments, we include only the first one, using the value of the coefficient $B_1 = -0.78 \pm 0.08$ fixed in ref. [17]. For the twist-3 LCDA, due to the lack of knowledge about their moments, we use the asymptotic form, i.e. the first term of the Gegenbauer expansion,

$$\Phi^\sigma_{f_0}(u) = 1, \quad \Phi^\sigma_{f_0}(u) = 6u(1-u). \quad (23)$$

Let us quote the numerical values of the other physical parameters. The meson masses are fixed to the PDG values $m_{B_s} = 5.366$ GeV and $m_{f_0} = 0.98$ GeV [19], while for quark masses we use $m_b = 4.8$ GeV and $m_s = 0.14$ GeV [19, 20]. As for the decay constants, we use $f_{B_s} = (0.231 \pm 0.015)$ GeV [21] and $f_{f_0} = (0.18 \pm 0.015)$ GeV [14]. The threshold $s_0$ is fixed to $s_0 = (34 \pm 2)$ GeV$^2$, which should correspond to the mass squared of the first radial excitation of $B_s$.

With these numerical inputs, the sum rules [18]-[20] provide us with the form factors for each value of $q^2$ as a function of the Borel parameter. The result is obtained requiring stability against variations of $M^2$.

In Fig. 1 we show the dependence of the form factors at $q^2 = 0$ on the Borel parameter $M^2$. We observe stability when $M^2 > 6$ GeV$^2$, and we fix $M^2 = (8 \pm 2)$ GeV$^2$. To describe the form factors in the whole kinematically accessible $q^2$ region, we adopt the parameterization

$$F_i(q^2) = \frac{F_i(0)}{1 - a_i q^2 / m_{B_s}^2 + b_i (q^2 / m_{B_s}^2)^2}, \quad (24)$$

where $F_i$ denotes any function among $F_{1,0,T}$. The parameters $a_i, b_i$ are obtained through fitting the form factors in the small $q^2$ region (we choose $0 < q^2 < 15$ GeV$^2$); the results for $F_i(0)$, $a_i$ and $b_i$ are collected in Table I and the $q^2$ dependence is depicted in Fig. 2. The uncertainties in the results reflect those of the input parameters. In Table II we also report the values of the form factors at zero-recoil ($q^2_{\text{max}}$) which are derived using the expression in Eq. (24).

The results in Table II show that the parameters $a_i$ and $b_i$ determining the $q^2$ dependence are close to each other in the case of $F_1$ and $F_T$. The reason is the following. In the heavy-quark limit and in the large energy (LE) limit of the recoiled meson, the three $B_s \to f_0$ form factors can be related to a single universal function $\xi_f$ which is specific for $f_0$ and does not depend on the Dirac structure of the current appearing in the definition of the various matrix elements, such as those in Eqs. (23) [22]. When the energy $E$ of the light meson in the the final state is large, such relations read as

$$\frac{m_{B_s}}{m_{B_s} + m_{f_0}} F_T(q^2) = F_i(q^2) = \frac{m_{B_s}}{2E} F_0(q^2), \quad (25)$$

where, neglecting $m_{f_0}^2$ but keeping $m_{B_s}$ in the kinematical factors, $E$ is related to $q^2$:

$$q^2 = m_{B_s}^2 - 2m_{B_s} E. \quad (26)$$

The first equality in Eq. (25) shows that the large energy limit predicts that $F_1$ and $F_T$ have the same $q^2$ dependence. For the shape parameters of $F_0$, one can obtain two relations through the second equality:

$$a_0 = -1 + a_1, \quad b_0 = 1 - a_1 + b_1. \quad (27)$$

Figure 1: Dependence on the Borel parameter $M^2$ of the $B_s \to f_0$ form factors at $q^2 = 0$: $F_1(0) = F_0(0)$ (upper panel) and $F_T(0)$ (lower panel).
Table I: Parameters of the $B_s \to f_0$ form factors by LCSR at the leading order. The values of $F_i(q^2_{\text{max}})$ are evaluated through Eq. (24).

| $F_i(q^2 = 0)$ | $a_i$ | $b_i$ | $F_i(q^2_{\text{max}})$ |
|----------------|-------|-------|-------------------------|
| $F_1$          | 0.185 ± 0.029 | 1.44 ±0.13 | 0.59 ±0.07 | 0.614 ±0.158 |
| $F_0$          | 0.185 ± 0.029 | 0.47 ±0.09 | 0.01 ±0.06 | 0.268 ±0.038 |
| $F_T$          | 0.228 ± 0.036 | 1.42 ±0.10 | 0.60 ±0.05 | 0.714 ±0.126 |

Table II: $B_s \to f_0(980)$ transition form factors obtained including an estimate of next-to-leading order corrections (see text).

| $F_i(q^2 = 0)$ | $a_i$ | $b_i$ |
|----------------|-------|-------|
| $F_1$          | 0.238 ± 0.036 | 1.50 ±0.13 | 0.58 ±0.09 |
| $F_0$          | 0.238 ± 0.036 | 0.53 ±0.14 | 0.10 ±0.08 |
| $F_T$          | 0.308 ± 0.049 | 1.46 ±0.14 | 0.58 ±0.07 |

B. Estimate of the next-to-leading order corrections

In order to provide an estimate of next-to-leading order effects in the determination of the $B_s \to f_0$ form factors, it is worth comparing this case to the calculation of $B \to \pi$ form factors. In $B \to \pi$ transition, both the light quarks and the light $\pi$ meson have small masses which can be safely neglected, while the strange quark and the scalar meson $f_0$ masses may induce sizable effects. Another observation is that, neglecting the quark masses, the Lorentz structures of pion and $f_0$ matrix elements differ by a minus sign in terms proportional to the twist-2 LCDA. Finally, contributions from the twist-3 LCDA in $B \to \pi$ transition are characterized by the chiral scale parameter $\mu_\pi$, while in $B_s \to f_0$ they are proportional to the mass of $f_0$.

In LCSR, NLO corrections to $B \to \pi$ form factors have been studied by two groups, while the complete expressions for the NLO corrections to $B_s \to f_0$ form factors are not known at present. The expressions relevant for $B \to \pi$ form factors given in Ref. [24] can be used to estimate the radiative corrections in the case $B_s \to f_0$, keeping in mind the three differences above. We first consider the changes to the leading order result due to the different treatment of quark and hadron masses. Setting the quark mass $m_q$ to zero, the values of the form factors are reduced by about 3%. The mass of $f_0$, the analogous of the pion mass $m_\pi$ and the chiral scale parameter $\mu_\pi$, cannot be put to zero, as this would smear all terms from twist-3 LCDA: we set the mass square of $f_0$ to be zero keeping the linear terms in the form factors, obtaining an enhancement of the form factors by about 3%. After that, evolving all the scale-dependent parameters to a scale of about the Borel mass, $\mu \simeq 3$ GeV, we find that the leading order contributions are furtherly enhanced, obtaining the central values: $F_i(0) = F_i(0) = 0.216$, $a_1 = 1.50$, $b_1 = 0.58$, $a_0 = 0.216$, $b_0 = 0.53$ and $F_T(0) = 0.262$, $a_T = 1.46$, $b_T = 0.58$. Then, radiative corrections to twist-2 and twist-3 LCDA are also found to be rather small, the $B_s \to f_0$ form factors being changed to $F_i(0) = F_i(0) = 0.238$ and $F_T(0) = 0.308$. The resulting values, with the inclusion the uncertainty due to the input parameters, are collected in Table II. They are also used in the phenomenological analysis, keeping in mind, however, that the procedure used in their determination must be considered as only approximate.

Before closing this section, it is worth mentioning...
Table III: $B_s \to f_0(980)$ form factors at $q^2 = 0$. Results evaluated by CLFD/DR [23], PQCD [24] and QCDSR [27] approaches are collected for a comparison.

| CLFD/DR | PQCD | QCDSR | This work |
|---------|------|-------|-----------|
| $F_1(0)$ | 0.49/0.30 $^a$ | 0.35 $^{+0.09}_{-0.07}$ | 0.12 $\pm 0.03$ | 0.185 $\pm 0.029$ |
| $F_2(0)$ | 0.40 $^{+0.10}_{-0.08}$ | -0.08 $\pm 0.02$ | 0.228 $\pm 0.036$ |

$^a$using $f_{B_s} = 0.259$ GeV
$^b$using $f_0 = 0.37$ GeV
$^c$using $f_{f_0} = 0.37$ GeV and $f_{B_s} = 0.209$ GeV.

that the $B_s \to f_0(980)$ form factors have been computed by other approaches: the method based on covariant light-front dynamics (CLFD) and dispersion relation (DR) [23], the perturbative QCD approach (PQCD) [24], short-distance QCD sum rules (QCDSR) [27]. The results are collected in Table III. The form factors by PQCD are proportional to the $f_0$ decay constant, while those by short-distance QCD sum rules are proportional to the inverse of this constant. Thus, a larger decay constant, $f_{f_0} = 0.37$ GeV as reported and used in [17], gives larger form factors in the PQCD approach and smaller ones in QCDSR with respect to ours. Taking into account the difference in the decay constant, the results in Refs. [26-27] are consistent with ours, while the two results in Ref. [25] are sensibly larger.

IV. PHENOMENOLOGICAL APPLICATIONS

A. Semileptonic $B_s \to f_0 \ell^+\ell^-$ and $B_s \to f_0\nu\bar{\nu}$ decays

As a first application of our study, we predict the branching ratios of the decays $B_s \to f_0 \ell^+\ell^-$ and $B_s \to f_0\nu\bar{\nu}$, processes which, being induced by the flavor-changing neutral current transition $b \to s$, are potentially important for detecting new physics effects.

The SM $\Delta B = 1$, $\Delta S = -1$ effective Hamiltonian describing the transition $b \to s \ell^+\ell^-$ can be expressed in terms of a set of local operators:

$$ H_{b\to s\ell^+\ell^-} = -4 \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu), $$

(28)

$$ G_F = 1.166 \times 10^{-5} \text{GeV}^{-2} $$

being the Fermi constant and $V_{ij}$ the elements of the CKM mixing matrix (since the ratio $V_{tb} V_{ts}^* / V_{ub} V_{us}^*$ is $O(10^{-2})$, we neglect terms proportional to $V_{ub} V_{us}^*$). The operators $O_i$ are written in terms of quark and gluon fields:

$$ O_1 = (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha}) (\bar{c}_{L\beta}\gamma^\mu c_{L\beta}), $$

$$ O_2 = (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha}) (\bar{c}_{L\beta}s_{L\beta}), $$

$$ O_3 = (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha}) ((\bar{u}_{L\alpha}\gamma^\mu u_{L\alpha}) + \cdots + (\bar{b}_{L\beta}\gamma^\mu b_{L\beta})), $$

$$ O_4 = (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha}) ((\bar{u}_{L\alpha}\gamma^\mu u_{L\alpha}) + \cdots + (\bar{b}_{L\beta}\gamma^\mu b_{L\beta})), $$

$$ O_5 = (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha}) ((\bar{b}_{L\alpha}\gamma^\mu b_{L\alpha}) + \cdots + (\bar{b}_{L\beta}\gamma^\mu b_{L\beta})), $$

$$ O_6 = (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha}) ((\bar{b}_{L\alpha}\gamma^\mu b_{L\alpha}) + \cdots + (\bar{b}_{L\beta}\gamma^\mu b_{L\beta})), $$

$$ O_7 = \frac{e}{16\pi^2} (m_b \bar{s}_{L\alpha}\gamma^\mu b_{L\alpha} + m_s \bar{s}_{R\alpha}\gamma^\mu b_{L\alpha}) F_{\mu\nu}, $$

$$ O_8 = \frac{g_s}{16\pi^2} (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha}) \bar{\ell}_\mu \ell, $$

$$ O_{10} = \frac{e^2}{16\pi^2} (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha}) \bar{\ell}_\mu \gamma_5 \ell, $$

(29)

with $\alpha, \beta$ color indices, $b_{R,L} = \frac{1 \pm \gamma_5}{2} b$, and $\sigma^{\mu\nu} = \frac{i}{2} \varepsilon^{\mu\nu\lambda\gamma} e^\gamma b_{R,L}$ and $g_s$ are the electromagnetic and the strong coupling constant, respectively, and $F_{\mu\nu}$ and $G_{\mu\nu}^s$ in $O_7$ and $O_8$ denote the electromagnetic and the gluonic field strength tensor. $O_1$ and $O_2$ are current-current operators, $O_3$, ..., $O_6$ QCD penguin operators, $O_7$ and $O_8$ magnetic penguin operators, $O_9$ and $O_{10}$ semileptonic electroweak penguin operators. The Wilson coefficients in [28] are known at NNLO in the Standard Model [28]. The operators $O_1$ and $O_2$ contribute to the the final state with a lepton pair through a $c\bar{c}$ contribution that can give rise to charmonium resonances $J/\psi, \psi(2S), \cdots$, resonant term which can be subtracted by appropriate kinematical cuts around the resonance masses. The Wilson coefficients $C_3 - C_6$ are small, hence the contribution of only the operators $O_7$, $O_8$ and $O_{10}$ can be kept for the description of the $b \to s \ell^+\ell^-$ transition. In our study we use a modification of the Wilson coefficient $C_7: C_7^{eff}$, which is a renormalization scheme independent combination of $C_7, C_8$ and $C_2$, given by a formula that can be found, e.g., in [29].

The $B_s$ and $f_0$ matrix elements of the operators in [29] can be written in terms of form factors, so that the differential decay width of $B_s \to f_0 \ell^+\ell^-$ reads:

$$ \frac{d\Gamma(B_s \to f_0 \ell^+\ell^-)}{dq^2} = $$

$$ \frac{G_F^2 a_{em}|V_{tb}|^2|V_{ts}|^2 \sqrt{\lambda}}{512 m_{B_s}^3 \pi^3} \sqrt{q^2 - 4m_{f_0}^2} \frac{1}{q^2} \frac{3q^2}{12} $$

$$ \times \left[ 6m_{f_0}^2 C_{10}^2 (m_{B_s}^2 - m_{f_0}^2)^2 F_0^2(q^2) \right]^2 $$

$$ + (q^2 + 2m_{f_0}^2) \lambda |C_9 F_1(q^2) + 2C_7 (m_{b,s} - m_{f_0}) F_7^2(q^2) |^2 $$

$$ + |C_{10}^2 (q^2 - 4m_{f_0}^2) \lambda F_1^2(q^2)|^2, $$

(30)
with \( \lambda = \chi(m_B^2, m_{f_0}^2, q^2) = (m_B^2 - q^2 - m_{f_0}^2)^2 - 4m_{f_0}^2q^2 \), 
\( \alpha_{em} = 1/137 \) the fine structure constant and \( m_\ell \) the lepton mass.

Analogously, the SM effective Hamiltonian for \( b \to s\nu\bar{\nu} \),

\[
H_{b\to s\nu\bar{\nu}} = \frac{G_F}{\sqrt{2} \sin^2(\theta_W)} V_{tb} V_{ts}^* \eta \chi(x_t) O_L \equiv C_L O_L ,
\]

includes the operator

\[
O_L = (\bar{s} \gamma\mu (1-\tau_3) b) \bar{\nu} \gamma\mu (1-\tau_3) \nu .
\]

\( \theta_W \) is the Weinberg angle; the function \( X(x_t) \) in the numerical calculation we use \( x_t = \frac{m_t^2}{m_W^2} \). with \( m_t \) the top quark mass and \( m_W \) the W mass has been computed in \([30]\) and \([31, 32]\), while the QCD factor \( \eta_{\chi} \) is close to one \([31, 33]\), so that one can use \( \eta_{\chi} = 1 \). From this effective Hamiltonian, the differential decay width

\[
\frac{d\Gamma(\bar{B}_s \to f_0\nu\bar{\nu})}{dq^2} = \frac{3|C_L|^2 \lambda^{3/2}(m_{B_s}^2, m_{f_0}^2, q^2)}{96m_{B_s}^2 \pi^3} |F_1(q^2)|^2
\]

can be obtained.

In the numerical calculation we use

\[
C_7 = -0.30137, \quad C_9 = 4.1696, \quad C_{10} = -4.46418, \quad C_L = 2.62 \times 10^{-9},
\]

together with \( V_{ts} = 0.0387 \) and \( V_{tb} = 0.999 \). Using these inputs and \( \tau(B_s) = 1.47 \) ps \([19]\) we find:

\[
\Re\langle \bar{B}_s \to f_0 \bar{\ell}\ell^- \rangle = (9.5^{+3.1}_{-2.0}) \times 10^{-8} \\
\Re\langle \bar{B}_s \to f_0 \tau^+ \tau^- \rangle = (1.1^{+0.4}_{-0.3}) \times 10^{-8} \]

(34)

\[
\Re\langle \bar{B}_s \to f_0 \nu\bar{\nu} \rangle = (8.7^{+2.8}_{-2.4}) \times 10^{-7}
\]

with \( \ell = e, \mu \). Our estimate of the NLO effects in the form factors modifies the branching ratios to \( \Re\langle \bar{B}_s \to f_0 \bar{\ell}\ell^- \rangle = (16.7 \pm 6.1) \times 10^{-8} \), \( \Re\langle \bar{B}_s \to f_0 \tau^+ \tau^- \rangle = (2.7 \pm 1.3) \times 10^{-8} \), and \( \Re\langle \bar{B}_s \to f_0 \nu\bar{\nu} \rangle = (15.2 \pm 5.6) \times 10^{-7} \). These decay modes are therefore accessible at the LHCb experiment at the CERN Large Hadron Collider and at a Super B factory operating at the \( \Upsilon(5S) \) peak.

**B. Nonleptonic \( B_s \to J/\psi f_0 \) transition**

The study of CP violation and the measurement of the CKM angles mainly proceed through the measurement of nonleptonic decay modes. In the \( B_s \) sector, the channel \( B_s \to J/\psi f_0 \) is the golden mode to investigate CP violation, and from the analysis of this mode the CDF \([2]\) and D0 \([4]\) Collaborations at the Fermilab Tevatron have obtained values of the \( B_s \) mixing phase \( \phi_s = -2\beta_s \) much larger than predicted in the SM, modulo a large experimental uncertainty. If confirmed, this measurement would indicate physics beyond SM. It is of prime importance to consider other processes allowing to access \( \beta_s \), namely \( B_s \to J/\psi \eta, J/\psi \eta' \) and \( J/\psi f_9(980) \) in which the final state is a CP eigenstate and no angular analysis is required to disentangle the various CP components, as needed for \( B_s \to J/\psi \phi \). However, the reconstruction of \( B_s \) modes into \( \eta \) and \( \eta' \) is experimentally challenging, since the subsequent \( \eta \) or \( \eta' \) decays involve photons in the final state. The case of \( f_9(980) \) essentially decays to \( \pi^+\pi^- \) and to \( 2\pi^0 \) (the decay to \( K^+K^- \) has also been seen) \([17]\). Theoretical predictions of \( B_s \to J/\psi f_9(980) \) are therefore of great importance.

The quantitative description of nonleptonic decays is very challenging. The theoretical framework to study such decays is based on the operator product expansion and renormalization group methods, which allow to write an effective Hamiltonian as in the case of the modes considered in the previous section. However, now one has to consider hadronic matrix elements \( \langle J/\psi f_9 |O_i |B_s \rangle \) with \( O_i \) four-quark operators, the calculation of which is a nontrivial task. One of the strategies which has been exploited is the naive factorization \([34]\), in which such quantities are replaced by products of matrix elements of the weak currents appearing in each one of the operators of the effective Hamiltonian relative to the considered process. These objects are expressed in terms of meson decay constants and hadronic form factors. This procedure is affected by several drawbacks, and various refinements have been proposed. It has been shown that a theoretical justification of naive factorization in the case of \( B \) decays can be found in the heavy quark limit \( m_b \to \infty \) only in a limited class of processes \([33]\). One can consider the so called generalized factorization approach, in which the Wilson coefficients (or appropriate combinations of them) appearing in the factorized amplitudes are regarded as effective parameters to be fixed from experiment, a procedure adopted in the following.

Using the factorization ansatz, the decay amplitude \( B_s(p_{B_s}) \to J/\psi(p_\psi, \epsilon)f_9(p_{f_9}) \) (\( \epsilon \) being the \( J/\psi \) polarization vector, \( p_{p_B}, p_\psi, p_{f_9} \) the momenta of the three particles) is given as

\[
A(\bar{B}_s \to J/\psi f_9) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* a_2 m_\psi f_{J/\psi} \\
F_1^{f_9 \to f_0}(m_{J/\psi}^2)(2(\epsilon^* \cdot p_{B_s})) ;
\]

\( f_{J/\psi} \) is the \( J/\psi \) decay constant, determined from the \( J/\psi \to e^+e^- \) decay width \([19]\): \( f_{J/\psi} = (416.3 \pm 5.3) \) MeV. The factor \( a_2 \) is a combination of Wilson coefficients which can be extracted from the \( B \to J/\psi K \) decays, under the assumption that \( a_2 \) is the same in the two processes. For these decays the branching ratios are known \([19]\):

\[
\Re\langle B^- \to J/\psi K^- \rangle = (1.007 \pm 0.035) \times 10^{-3} , \\
\Re\langle B^0 \to J/\psi K^0 \rangle = (8.71 \pm 0.32) \times 10^{-4} .
\]

In order to extract \( a_2 \), the form factor \( F_1^{f_9 \to K} \) is required. We use two different parameterizations, obtained
by short-distance (CDSS) \[36\] and light-cone QCD sum rules (BZ) \[23\]. The result is different for the two sets of form factors, while there is almost no difference whether we use the charged or the neutral channel:

\[ |d_2^{B \to J/\psi K,(CDSS)}| = 0.394^{+0.053}_{-0.041}, \]

\[ |d_2^{B \to J/\psi K,(BZ)}| = 0.25 \pm 0.03. \quad (37) \]

To be conservative with the hadronic uncertainty, we use the average value \( \alpha_2 = 0.32 \pm 0.11 \) of the two values to compute \( \mathcal{B}(B_s \to J/\psi f_0) \). Using \( V_{cb} = 0.0412, V_{cs} = 0.997 \) \[19\] and our LO prediction for the \( B_s \to f_0 \) form factors, we obtain

\[ \mathcal{B}(B_s \to J/\psi f_0) = (3.1 \pm 2.4) \times 10^{-4} \quad (38) \]

where the two results correspond to the \( B_s \to f_0 \) form factor evaluated at the leading order or not. \( A_1^{B_s \to f_0} \) \( A_2^{B_s \to f_0} \) are among the \( B_s \to \phi \) transition form factors and are taken from Ref. \[37\]. In Ref. \[8\] it was suggested that the ratio \( R_{f_0/\phi}^{B_s} \) can be inferred from the ratio of \( D_s \) decay widths to \( f_0 \pi^+ \) and \( \phi \pi^+ \), obtaining \( R_{f_0/\phi}^{D_s} \simeq 0.2 - 0.3 \), which is compatible with our result \[39\].

Another relation has been also proposed in \[8\] connecting \( R_{f_0/\phi}^{B_s} \) to a different observable in \( D_s \) decays:

\[ R_{f_0/\phi}^{B_s} \simeq R_{f_0/\phi}^{D_s} = \frac{\frac{d\mathcal{B}}{dq^2}(D_s^+ \to f_0 e^+\nu, f_0 \to \pi^+\pi^-)|_{q^2=0}}{\frac{d\mathcal{B}}{dq^2}(D_s^+ \to \phi e^+\nu, \phi \to K^+K^-)|_{q^2=0}}. \quad (40) \]

For this quantity the CLEO Collaboration has recently provided a measurement: \( R_{f_0/\phi}^{D_s} = (0.42 \pm 0.11) \) \[10\] which is larger than our \[39\].

All the above considerations show that the mode \( B_s \to J/\psi f_0 \) must be used, together with the golden mode \( B_s \to J/\psi \phi \), to measure the \( B_s \) mixing phase, mainly because it provides us with a large number of events and does not require an angular analysis to separate different CP components of the final state. This is also the case of modes in which \( J/\psi \) is replaced by a spin 0 charmonium state, such as \( \chi_{c0} \), modulo the difficulty of the \( \chi_{c0} \) reconstruction. \( B_s \to \chi_{c0} \phi \) will provide a side-check when the number of accumulated data will increase. Although \( B_s \to \chi_{c0} \phi \) is a suppressed channel in naive factorization, its branching fraction may not be small due to the intermediate rescattering mechanism \[38\] or because of

while, including our estimate of NLO corrections, the branching fraction is \( \mathcal{B}(B_s \to J/\psi f_0) = (5.3 \pm 3.9) \times 10^{-4} \). The rate of \( B_s \to J/\psi f_0 \) is large enough to permit a measurement; notice that the branching fraction of \( B_s \to J/\psi \phi \) is \( \mathcal{B}(B_s \to J/\psi \phi) = (1.3 \pm 0.4) \times 10^{-3} \) \[19\].

To gain a better insight on this point, it is interesting to compare these results to the branching fraction of \( B_s \to J/\psi \phi L \) (\( L \) denotes a longitudinally polarized meson) computed in the factorization approach. Neglecting the mass difference between \( \phi \) and \( f_0 \) in the phase space, the ratio of the branching fractions of the two modes can be written in terms of form factor combinations:

\[ R_{f_0/\phi}^{B_s} = \frac{\mathcal{B}(B_s \to J/\psi f_0)}{\mathcal{B}(B_s \to J/\psi \phi)} = \left( \frac{F_{1f_0}(m_{B_s}^2,m_{B_s}^2,m_{f_0}^2)}{F_{2f_0}(m_{B_s}^2,m_{B_s}^2,m_{f_0}^2)} \right)^2 = 0.13 \pm 0.06 \quad (39) \]



\[ \mathcal{B}(B^0 \to \chi_{c0} \bar{K}^0) = (1.7 \pm 0.3 \pm 0.2) \times 10^{-4}, \]

\[ \mathcal{B}(B^- \to \chi_{c0} K^-) = (1.4 \pm 0.5 \pm 0.2) \times 10^{-4} < 2.1 \times 10^{-4} \quad (90\% \, CL) \quad (41) \]

and, on the basis of SU(3)$_F$ symmetry, the branching fraction of \( B_s \to \chi_{c0} \phi \) should be similar.

![Figure 3: Dependence of the $D_s \to f_0$ form factors at $q^2 = 0$ $F_1(0) = F_0(0)$ on the Borel parameter $M^2$.](image-url)
V. DECAY $D_s \rightarrow f_0 e^+ \nu$

By a suitable change of parameters in the sum rules in Section II, also the $D_s \rightarrow f_0$ form factors can be computed and the branching ratio of the semileptonic decay $D_s \rightarrow f_0 e^+ \nu$ can be predicted. We use $m_c = 1.4$ GeV and $\tau(D_s) = 0.5$ ps [49]; the threshold parameter is fixed to $s_0^{D_s} = (6.5 \pm 1.0)$ GeV$^2$. For the $D_s$ decay constant we use the value quoted by the Heavy Flavor Averaging Group: $f_{D_s} = (256.9 \pm 6.8)$ MeV [41]. The Borel parameter can be fixed requiring stability of the sum rule result with respect to $M^2$ variations. In Fig. 3 we plot $F_1^{D_s\rightarrow f_0}(0)$ versus $M^2$; the stability window is selected in the range $M^2 = (5 \pm 1)$ GeV$^2$. We find

$$F_1^{D_s\rightarrow f_0}(0) = F_0^{D_s\rightarrow f_0}(0) = 0.30 \pm 0.03. \quad (42)$$

The $q^2$ dependence of the two form factors is displayed in Fig. 4. The value of $F_1^{D_s\rightarrow f_0}(0)$ is much smaller than in the $D \rightarrow K$ case, for which the light-cone sum rule prediction is: $F_1^{D \rightarrow K}(0) = 0.75^{+0.11}_{-0.08}$ [42]. We can understand this difference noticing that contribution of the $f_0$ twist-2 LCDA in $D_s \rightarrow f_0$ transition is small due to the different shape of the twist-2 $f_0$ distribution amplitude with respect to the case of $K$. The two LCDA are plotted in Fig. 5, where the position of the parameter $u_0$, defined in Eq. (21), is also displayed (upper panel). The situation can be compared to the $B_s \rightarrow f_0$ case, shown in the lower panel of the figure. Since the LCDA is integrated in the range $[u_0, 1]$, one can see that, in the case of $f_0(980)$, the integral of the distribution amplitude gets two opposite contributions which tend to cancel each other, due to the presence of a zero in the DA. The zero is not present in the kaon DA, so that the integrated DA gives a much larger contribution. In the case of $B_s$, the position of the parameter $u_0$ is such that the zero of the DA is not included in the integration region, so that no sizable difference is expected between the $f_0$ and the kaon cases. This argument explains also why, compared with the results of other approaches, our outcome are smaller. This can be noticed in Table IV, where we compare our results for the $D_s \rightarrow f_0$ form factors with other estimates [25, 44, 45].

![Figure 4: $q^2$ dependence of the $D_s \rightarrow f_0$ form factors.](image)

![Figure 5: Shape of the twist-2 LCDA: $-\Phi_{f_0} = -6u(1-u)B_1 C_1^2(2u-1)$ (dashed) and $\Phi_K$ (solid) taken from [43]. In the upper panel, the red line denotes the position of $u_0^{D_s} = 0.334$ fixed for the $D_s \rightarrow f_0$ transition, while in the lower panel the red line corresponds to the position of $u_0^{D_s} = 0.864$ at $q^2 = 0$ in $B_s \rightarrow f_0$ transition.](image)

| CLF/D | QCDSR | CLFQM | This work |
|-------|-------|-------|-----------|
| $F_1(0)$ | $0.45/0.46$ | $1.7(0.27 \pm 0.02)$ | $0.434$ |
| $F_0(0)$ | $30 \pm 0.03$ | | $0.30 \pm 0.03$ |

$^a$using $f_{D_s} = 0.274$ GeV
$^b$using $f_{D_s} = 0.22 \pm 0.02$ GeV; by using different input parameters two results are obtained, the first one in parentheses, the second one 1.7 times larger.
differential decay rate
$$\frac{d\Gamma(D_s \to f_0 e^+ \nu)}{dq^2} = \frac{G_F^2 V_{cb}^2 \lambda^{3/2}(m^2_{D_s}, m^2_{f_0}, q^2)}{192 m^3_{D_s} \pi^3} |F_1(q^2)|^2$$

(43)

where the lepton mass is neglected. Since in $D_s \to f_0 e^+ \nu$ the kinematically accessible $q^2$ range is limited, the applicable region for LCSR is narrow. One can fit the form factors in the spacelike region, for example $-2 \text{GeV}^2 < q^2 < 0$, and then extrapolate to the timelike region. However, the result of the extrapolation strongly depends on the choice of the fitting region. Moreover, looking at Fig. 4, one can notice that the $q^2$ dependence of $F_1$ and $F_0$ is mild. In view of this, we use a constant form factor $F_1(q^2) = F_1(0)$ to compute the branching ratio of $D_s \to f_0 e^+ \nu$; the result varies less than 10% including the $q^2$ dependence according to different fitting formulae. The obtained branching fraction is

$$\mathcal{BR}(D_s \to f_0 e^+ \nu) = (2.0^{+0.5}_{-0.4}) \times 10^{-3}.$$  

(44)

The modification due to radiative corrections can be estimated as in the case of $B_s \to f_0$, finding $F_1(0)/F_0(0) = 0.29^{+0.05}_{-0.04}$. Let us consider the available experimental data. The CLEO Collaboration has measured the product of branching fractions

$$\mathcal{BR}(D_s \to f_0(980) e^+ \nu) \times \mathcal{BR}(f_0 \to \pi^+ \pi^-) = (0.20 \pm 0.03 \pm 0.01) \times 10^{-2},$$

(45)

updating a previous determination.

$$\mathcal{BR}(D_s \to f_0 e^+ \nu) \times \mathcal{BR}(f_0 \to \pi^+ \pi^-) = (0.13 \pm 0.04 \pm 0.01) \times 10^{-2}.$$  

(46)

Using experimental data provided by the BES Collaboration studying the processes $\chi_{c0} \to f_0(980) f_0(980) \to \pi^+ \pi^- K^+ K^- \text{ and } \chi_{c0} \to f_0(980) f_0(980) \to K^+ K^- K^+ K^-$

$$\mathcal{BR}(f_0 \to \pi^+ \pi^-) = (50^{+7}_{-9}) \times 10^{-2}$$

(47)

which, combined with (45), gives

$$\mathcal{BR}(D_s \to f_0 e^+ \nu) = (4.0 \pm 0.6 \pm 0.6) \times 10^{-3},$$

(48)

marginally consistent with our (44).

VI. CONCLUSIONS

We have computed the $B_s \to f_0$ transition form factors using light-cone QCD sum rules at leading order in the strong coupling constant, and also estimating the size of NLO corrections. The resulting form factors permit to predict the rates of $B_s \to f_0 e^+ e^-$ and $B_s \to f_0 \nu \bar{\nu}$ decays, finding branching ratios accessible at future machines, like a Super B factory, and at the LHCb experiment at CERN. The branching ratio of $B_s \to J/\psi f_0$ can be predicted under the factorization assumption: we find $\mathcal{BR}(B_s \to J/\psi f_0)/\mathcal{BR}(B_s \to J/\psi \phi) \sim 0.2 - 0.3$, thus the $B_s \to J/\psi f_0$ channel can be considered another promising mode to access the $B_s - \bar{B}_s$ mixing phase. We have also investigated the $D_s \to f_0 e^+ \nu$ decay channel by the LCSR approach and compared the results to recent measurements.

Acknowledgments

WW thanks Yu-Ming Wang for useful discussions. This work was supported in part by the EU Contract No. MRTN-CT-2006-035482, "FLAVIAnet".

[1] P. Ball et al., arXiv:hep-ph/0003238.
[2] A. D. Sakharov, Pis'ma Zh. Eksp. Teor. Fiz. 5, 32 (1967) [JETP Lett. 5, 24 (1967) S.PUAN,34,392-393.1991 UFNAA,161,61-64.1991]].
[3] P. Huet and E. Sather, Phys. Rev. D 51, 379 (1995).
[4] P. Colangelo, F. De Fazio, R. Ferrandes and T. N. Pham, Phys. Rev. D 77, 055019 (2008); M. V. Carlucci, P. Colangelo and F. De Fazio, Phys. Rev. D 80, 055023 (2009).
[5] T. Aaltenon et al. [CDF Collaboration], Phys. Rev. Lett. 100, 161802 (2008).
[6] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 101, 241801 (2008).
[7] M. Artuso et al., Eur. Phys. J. C 57, 309 (2008); M. Bona et al., arXiv:0709.0451 [hep-ex].
[8] S. Stone and L. Zhang, Phys. Rev. D 79, 074024 (2009); S. Stone and L. Zhang, arXiv:0909.5442 [hep-ex].
[9] J. Yelton et al. [CLEO Collaboration], Phys. Rev. D 80, 052007 (2009).
[10] K. M. Ecklund et al. [CLEO Collaboration], Phys. Rev. D 80, 052009 (2009).
[11] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979); Nucl. Phys. B 147, 448 (11979).
[12] N. S. Craigie and J. Stern, Nucl. Phys. B 216, 209 (1983); V. M. Braun and I. E. Filyanov, Z. Phys. C 44, 157 (1989); V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. B 345 (1990) 137; V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Ruckl, Phys. Rev. D 51, 6177 (1995).
[13] For a review see P. Colangelo and A. Khodjamirian, in “At the Frontier of Particle Physics / Handbook of QCD”, ed. by M. Shifman (World Scientific, Singapore, 2001), vol. 3, 1495-1576 (arXiv:hep-ph/0001075).
[14] F. De Fazio and M. R. Pennington, Phys. Lett. B 521, 15 (2001).
[15] I. Bediaga, F. S. Navarra and M. Nielsen, Phys. Lett. B 579, 59 (2004).
[16] H. Y. Cheng and K. C. Yang, Phys. Rev. D 71, 054020 (2005).
[17] H. Y. Cheng, C. K. Chu and K. C. Yang, Phys. Rev. D 73, 014017 (2006).
[18] Y. M. Wang, M. J. Aslam and C. D. Lu, Phys. Rev. D 78, 014006 (2008).
[19] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667, 1 (2008).
[20] P. Colangelo, F. De Fazio, G. Nardulli and N. Paver, Phys. Lett. B 408, 340 (1997).
[21] E. Gamiz, C. T. H. Davies, G. P. Lepage, J. Shigemitsu and M. Wingate [HPQCD Collaboration], Phys. Rev. D 80, 014503 (2009).
[22] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D 60, 014001 (1999).
[23] P. Ball and R. Zwicky, Phys. Rev. D 71, 014029 (2005).
[24] T. Inami and C. S. Lim, Prog. Theor. Phys. 15, 294 (1981) [Erratum-ibid. 65, 1772 (1981)].
[25] G. Buchalla and A. J. Buras, Nucl. Phys. B 400, 225 (1993); G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).
[26] M. Misakian and J. Urban, Phys. Lett. B 451, 161 (1999).
[27] Y. M. Wang, M. J. Aslam and C. D. Lu, Phys. Rev. D 78, 014006 (2008).
[28] C. Bobeth, M. Misiak and J. Urban, Nucl. Phys. B 574, 291 (2000); H. M. Asatrian, H. M. Asatrian, C. Greub and M. Walker, Phys. Lett. B 579, 162 (2001); Phys. Rev. D 65, 074004 (2002); Phys. Rev. D 66, 034009 (2002); H. M. Asatrian, K. Bieri, C. Greub and A. Hovhannisyan, Phys. Rev. D 66, 094013 (2002); A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, Nucl. Phys. B 648, 254 (2003); A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, Nucl. Phys. B 685, 351 (2004); C. Bobeth, P. Gambino, M. Gorbahn and U. Haisch, JHEP 0404, 071 (2004).
[29] P. Colangelo, F. De Fazio, R. Ferrandes and T. N. Pham, Phys. Rev. D 73, 115006 (2006).
[30] T. Inami and C. S. Lim, Prog. Theor. Phys. 65, 297 (1981) [Erratum-ibid. 65, 1772 (1981)].
[31] G. Buchalla and A. J. Buras, Nucl. Phys. B 400, 225 (1993); G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).
[32] M. Misakian and J. Urban, Phys. Lett. B 451, 161 (1999).
[33] G. Buchalla and A. J. Buras, Nucl. Phys. B 548, 309 (1999).
[34] For a review see: M. Neubert and B. Stech, Adv. Ser. Direct. High Energy Phys. 15, 294 (1998).
[35] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 591, 313 (2000).
[36] P. Colangelo, F. De Fazio, P. Santorelli and E. Scrimieri, Phys. Rev. D 53, 3672 (1996) [Erratum-ibid. D 57, 3186 (1998)].
[37] P. Ball and R. Zwicky, Phys. Rev. D 71, 014029 (2005).
[38] P. Colangelo, F. De Fazio and T. N. Pham, Phys. Lett. B 542, 71 (2002).
[39] T. N. Pham and G. h. Zhu, Phys. Lett. B 619, 313 (2005); C. Meng, Y. J. Gao and K. T. Chao, Commun. Theor. Phys. 48, 885 (2007); C. H. Chen and H. N. Li, Phys. Rev. D 71, 114008 (2005); M. Beneke and L. Vernazza, Nucl. Phys. B 811, 155 (2009).
[40] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 78, 091101 (2008).
[41] Heavy Flavor Averaging Group, www.slac.stanford.edu/xorg/hfag/.
[42] A. Khodjamirian, C. Klein, T. Mannel and N. Offen, Phys. Rev. D 80, 114005 (2009).
[43] P. Ball, V. M. Braun and A. Lenz, JHEP 0605, 004 (2006).
[44] T. M. Aliev and M. Savci, arXiv:hep-ph/0701108.
[45] H. W. Ke, X. Q. Li and Z. T. Wei, Phys. Rev. D 80, 074030 (2009).
[46] M. Ablikim et al. [BES Collaboration], Phys. Rev. D 70, 092002 (2004); Phys. Rev. D 72, 092002 (2005).