Strange Attractors in Multipath propagation: Detection and characterisation.

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(Dated: March 16, 2001)

Abstract

Multipath propagation of radio waves in indoor/outdoor environments shows a highly irregular behavior as a function of time. Typical modeling of this phenomenon assumes the received signal is a stochastic process composed of the superposition of various altered replicas of the transmitted one, their amplitudes and phases being drawn from specific probability densities. We set out to explore the hypothesis of the presence of deterministic chaos in signals propagating inside various buildings at the University of Calgary. The correlation dimension versus embedding dimension saturates to a value between 3 and 4 for various antenna polarizations. The full Lyapunov spectrum calculated contains two positive exponents and yields through the Kaplan-Yorke conjecture the same dimension obtained from the correlation sum. The presence of strange attractors in multipath propagation hints to better ways to predict the behaviour of the signal and better methods to counter the effects of interference. The use of Neural Networks in non-linear prediction will be illustrated in an example and potential applications of same will be highlighted.

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I. INTRODUCTION

Multipath propagation of radio waves in indoor or outdoor environments shows a highly irregular behavior as a function of time [1]. The characterization of radio channels in mobile or in building propagation is important for addressing issues of design, coding, modulation and equalization techniques tailored specifically to combat time and frequency dispersion effects.

Irregular behavior of the received signal has prompted researchers in the past to model the channel with stochastic processes. One of the earliest linear models in this vein is the Turin et al. model [2] in which the impulse response of the channel is written as a superposition of replicas of the transmitted signal delayed and having altered amplitudes and phases.

A number of models exist differing in the numbers of replicas of the signal or in the type of probability distributions from which the amplitudes and phases are drawn. Also, different stochastic processes are used in the generation of delay times of received replicas. The popular choice for the amplitude probability density functions (PDF) are Rayleigh or Rice PDFs depending on whether a weak or strong line of sight propagation exists; nevertheless other PDFs have been used such the lognormal, Nakagami-m or uniform. The phases ought to be drawn from PDFs compatible with the ones selected for the corresponding amplitudes; nevertheless the most popular choice found in the literature is the uniform \([0 - 2\pi]\) distribution. Delay times are usually extracted from either stationary of stationary Poisson processes although in some cases the Weibull PDF is used.

II. A HYPOTHESIS OF DETERMINISTIC CHAOS

Although an assumption of stochastic behavior in mobile or indoor propagation is ubiquitous, in the present work we set out to explore the hypothesis that the indoor communication channel displays deterministically chaotic behavior. This question is important in many respects and the tools to answer it readily exist. These tools are based on the determination of the correlation dimension of the strange attractor associated with the multipath profile considered as a real valued time-series \(x(t)\). Using delay coordinates [3] one forms the \(m\)-dimensional delay vector \(X(t) = [x(t), x(t-T),...x(t-(m-1)T]]\) with delay \(T\) and computes the correlation sum \(C(r)\) which is the ratio of the number of pairs of delay vectors (the
distance between which is less than \( r \) to the total number of pairs. From this, the correlation dimension \( \nu \) is defined as the logarithmic slope of \( C(r) \) versus \( r \) for small \( r \). For a true stochastic process, \( \nu \) increases with \( m \) without showing any saturation. In contrast, for deterministic chaos, \( \nu \) saturates at a value, the next integer greater than which, represent the minimum number of non-linear recursion or differential equations from which it originates. If the profile turns out to be deterministic, the modulation, coding and detection/demodulation techniques ought to be adapted accordingly in order to account for this fact; otherwise one has to rely upon techniques capable of handling stochastic signals. Let us illustrate this by an analysis of multipath measurements we have made in an indoor environment.

III. EXPERIMENT AND CORRELATION DIMENSION ANALYSIS

The propagation environment from which the data are collected are hallways in the Engineering Building at The University of Calgary [4]. The transmitted power was 10 dBm fed into a half wave dipole antenna with a matching balun. The receiving antenna was a cross-polarized dipole array. The co-polarized antennas (CPA) and crosspolarized antennas (XPA) profiles referred to in Figure 1 are from a single measurement run and points from both profiles were obtained in coincident pairs. The terms CPA and XPA simply refer to the relative state of polarisation between the transmit and receive antennas. The receiving hardware was specially developed to measure diversity characteristics and gives an accurate reference between distance from transmitting to receiving antennas and received signal strength. The same measurement procedure was employed as for the arbitrarily polarized data set. We have estimated the correlation dimension for the sets of data: Arbitrarily Polarized Antennas (APA, 6000 data points), co-polarized (CPA, 3800 points) and Cross-polarized antennas (XPA, 3800 points) by the box-counting method of Grassberger and Procaccia [3]. There are a number of limitations and potential pitfalls with correlation dimension estimation that have been discussed by various authors [5]. In addition, the number of operations it takes to estimate \( C(r) \) is \( O(N^2) \) where \( N \) is the number of collected experimental points. Recently Theiler [5] devised a powerful box-assisted correlation sum algorithm based on a lexicographical ordering of the boxes covering the attractor, reducing the number of operations to \( O(N \log(N)) \) and incorporating several test procedures aimed
at avoiding the previous pitfalls. Before we used Theiler’s algorithm, we made some pre-
liminary tests against well known cases. We generated uniform random numbers, Gaussian
random numbers, and numbers $z(n)$ according to the logistic one-dimensional map at the
onset of Chaos: $z(n+1) = a \cdot z(n)(1-z(n))$ with $a = 3.5699456$ and in the fully developed
Chaotic regime at $a=4$. In the first two cases we found $\nu$ approximately equal to $m$ as
expected in purely stochastic series whereas $\nu$ saturated respectively at 0.48 and 0.98 (we
used 2000 points only) for the logistic map indicating the presence of a low dimensional
attractor and deterministic Chaos (the exact correlation dimensions for the logistic map is
0.5 at $a=3.5699456$ and 1 for $a=4$). We tested as well the Hénon two dimensional map,
the Lorenz three dimensional system of non-linear differential equations, the Rössler three
and four dimensional systems as well as an infinite dimensional system, the delay-differential
Mackey-Glass equation whose attractor dimension is tunable with the delay time. The re-
results we found for the various correlation dimensions agreed with all the results known in the
literature to within a few percents. Then we went ahead and examined the $\nu$ vs. $m$ curves
for the three sets of experimental data along with a set of 6000 Rayleigh and band-limited
Rayleigh distributed numbers which constitute prototypic received envelopes. Our results,
in Figure 1 show that, while $\nu \sim m$ for the pure Rayleigh case (with a slope equal to one),
and $\nu \propto m$ for the band limited Rayleigh case (with a slope smaller than one) the $\nu$ vs.
m curves for the three examined experimental sets of data start linearly with $m$ then show
saturation indicating the presence of a low dimensional attractor (whose dimension is about
4 for CPA and XPA data whereas it is slightly above 4 for the APA situation). This finding
is in line with the Ruelle criterion [6] that sets an upper bound on the possible correlation
dimension one can get from any algorithm of the Grassberger-Procaccia type. This upper
bound is set by the available number of data points $N$ in the time series. The dimension
that can be detected should be much smaller than $2 \log_{10}(N)$. Since we used 3800 and 6000
points respectively, the upper bound for the detectable correlation dimension in our case
is about 7.16 to 7.56. We respect this bound since our correlation dimensions are around
4. Nevertheless the presence of Chaos is going to be confirmed through another route, the
spectrum of the Liapunov exponents that will be discussed next.
IV. SPECTRUM OF THE LIAPUNOV EXPONENTS

The spectrum of Liapunov exponents is very important in the study of dynamical systems. If the largest exponent is positive, this is a very strong indication for the presence of Chaos in the time series originating from the dynamical system. The reciprocal of this exponent is the average prediction time of the series and the sum of all the positive exponents (if more than one is detected like in hyper-chaotic systems such as the Rössler four dimensional system of non-linear differential equations or the large delay Mackey-Glass equation) is the Kolmogorov entropy rate of the system. The latter gives a quantitative idea about the information processes going on in the dynamical system. In addition, with the Kaplan-Yorke conjecture, the full spectrum gives the Hausdorff dimension of the strange attractor governing the long time evolution of the dynamical system. We have calculated the Liapunov exponents of the data with four different methods. Firstly, we determined the largest exponent $\lambda_{\text{max}}$ from the exponential separation of initially close points on the attractor and averaging over several thousand iterations. Second, we determined the largest Liapunov exponent from the correlation sum with the help of the relation $C(r) \sim r^\nu \exp(-mT \lambda_{\text{max}})$ valid for large values of the embedding dimension $m$ and small values of $r$. Finally, we determined all exponents with two different methods: the Eckmann et al. method [7] and the Brown et al.’s [8]. Our results for the spectrum of exponents is shown in figures 2 and 3. We tried several embedding delay times $T$ and several approximation degrees for the tangent mapping polynomial (as allowed in the Brown et al. [8] algorithm) without observing major changes in the spectrum. Several time series (Logistic map, Hénon, Lorenz, Rössler and Mackey-Glass) were tested for the sake of comparison to results obtained with the experimental data. In addition, the Liapunov exponents saturate smoothly as they should for large embedding dimension. Then we applied the Kaplan-Yorke conjecture to get the dimension of the attractor: Using the following typical numbers we obtained for the exponents $\lambda_{\text{max}} = \lambda_1 = 18.06, \lambda_2 = 1.88, \lambda_3 = -8.85, \lambda_4 = -24.94, \lambda_5 = -68.80$ and using the formula:

$$D = j + \frac{\sum_i \lambda_i}{|\lambda_{j+1}|} \quad (1)$$

where the summation is over $i=1,2...j$. The $\lambda_i$’s are ordered in a way such that they decrease as $i$ increases. We determine $j$ from the conditions $\sum_i \lambda_i > 0$ and $\lambda_{j+1} + \sum_i \lambda_i < 0$. We get $j=3$ and $D=3.44$ for the strange attractor dimension (called its Liapunov dimension).
The total sum of the Lyapunov exponents is negative (equal to -82.65) as it should be for dissipative systems with a strange attractor. The value of the attractor dimension will be confirmed from the spectrum of singularities or the multifractal spectrum in the next section.

V. MULTIFRACTAL SPECTRUM

The generalized dimension may be used to characterize non-uniform fractals for which there are different scaling exponents for different aspects of the fractal, so-called multifractals. For these, there are two scaling exponents, one generally called $\tau$, for the support of the fractal, and one called $q$, for the measure of bulk of the fractal. In general, $\tau(q) = (q - 1)D_q$, where $D_q$ is the generalized dimension. Multifractals have been employed to characterize multiplicative random processes, turbulence, electrical discharge, diffusion-limited aggregation, and viscous fingering [9]. Multifractals have this in common: there is a non-uniform measure (growth rate, probability, mass) on a fractal support. Besides the exponents, $\tau$ and $q$, and the generalized dimension $D_q$, there is another method for characterizing multifractals. This depends upon the use of the mass exponent $\alpha$, and the multifractal spectrum, $f(\alpha)$ [9]. A graph of the multifractal spectrum explicitly shows the fractal dimension, $f$, of the points in the fractal with the mass exponent (or scaling index), $\alpha$. We have estimated $D_q$ by use of the generalised moments of the correlation sum with a window chosen carefully enough to avoid temporal correlation effects. We have developed a program that computes the generalized correlation sum using a box-assisted method. Our program is based on one written by Theiler [5]. Several modifications had to be made to the straightforward box-assisted correlation sum method. In addition, our program allows for logarithmic scaling with the distance parameter $r$. From a log-log graph of the generalized correlation sums, appropriate scaling regions can be identified, for each order, $q$. Least-squares fits to these scaling regions yields a sequence of generalized correlation dimensions, $D_q$, for values of $q$ between $\pm\infty$. We have found computation of $D_q$ for integers in the interval [-10,10] and $D_{-\infty}$ and $D_{+\infty}$ to be sufficient. From the $D_q$, we calculate the $\tau(q) = (q - 1)D_q$. We then perform a Legendre transform to obtain the $f(\alpha)$ curve. We do this by first fitting a smooth curve (a hyperbola was considered to be adequate) to the $\tau(q)$ curve. With an analytic expression for the $\tau(q)$ curve, we can compute the Legendre transform in closed form. The domain of $f(\alpha)$ may be found from $D_{-\infty}$ and $D_{+\infty}$; we assume that $\alpha$ is confined to this
region, and that \( f(\alpha) \) is 0 at these points. The values of \( f(\alpha) \) for \( D_{-\infty} \leq \alpha \leq D_{+\infty} \) are calculated as \( \min[q\alpha - \tau(q)] \), the minimum being taken over \( q \). We found that this procedure, although complex, corrects for the known numerical sensitivities of the Legendre transform.

We checked that our method for obtaining \( f(\alpha) \) gave the same results as those found in the literature for the Logistic map and the strongly dissipative circle map [9]. The \( f(\alpha) \) curve for the multipath data is shown in Figure 4. It may be seen that the peak value of \( f(\alpha) \), corresponding to the box-counting dimension at \( q=0 \), is about 3.7. This is consistent with our above findings from the correlation dimension and the full spectrum of Liapunov exponents. Our further research in this area concerns the prediction of the received signal intensity, from our above hypothesis of the presence of deterministic chaos.

VI. NON LINEAR PREDICTION

We applied the above findings to the non linear prediction of multipath profiles considering that each point on the envelope of the measured signal is a function of some number of past points in the series, deviating from the traditional wave superposition approach. More precisely, we write:

\[
y(n + 1) = F[y(n), y(n - 1), y(n - 2), y(n - 3)...y(n - m + 1)]
\]  

(2)

with \( F \) an \( m \) dimensional map and \( y(n) \) the value of the signal \( x(t) \) sampled at timestep \( n \). Expressing \( F \) as a sum of sigmoidal and linear functions [10] we determine the unknown weights through the Marquardt least squares minimisation method [11] in order to achieve the best fit to the data. Our results comparing the onestep prediction to the multipath data analysed above are displayed in figure 5. The goodness of fit between the measured and predicted envelopes for a map dimension \( m=5 \) is another indication of the soundness of the approach. This is confirmed in Figure 6 where we display the normalised prediction error versus the embedding dimension. A minimum is observed in the prediction error for an embedding equal to 5 or 6. In Figure 6, we started always from the same initial weights and let the system run through 150 iterations searching for the least squares minimum for 200 data points and later on making 300 one step ahead predictions. For embedding dimension larger or equal to 7 the minimisation procedure stopped because of the presence of zero-pivot in the least-squares matrices. The presence of a minimum in the prediction
error around an embedding equal to 5 or 6 complies again with the value of embedding dimension used previously in the correlation dimension analysis, the Liapunov spectrum and the Kaplan-Yorke conjecture.

VII. DISCUSSION

We stress that although the profiles we examined were found to be chaotic in all three experimental configurations with confirmations from the Liapunov spectrum and non linear prediction studies, indicating that we would be able to describe our data with a set of at most 5 non-linear differential or algebraic equations, investigation in other propagation situations is needed. Nevertheless, in our investigations we observed a significant amount of consistency between the various methods of detecting Chaos and characterising it using embedding dimensions $m$ beyond the minimum $m_{\text{min}}$ required by the Takens theorem ($m_{\text{min}} > 2d + 1$, where $d$ is the dimension of the strange attractor). Assuming the hypothesis of the presence of Chaos in a given multipath profile is firmly established, many avenues become possible. For instance, one might consider devising ways for controlling the signal propagation by altering slightly some accessible system parameter and improving the performance characteristics of the channel [12]. Shaw [13] has introduced the concept that a deterministically chaotic system can generate entropy. The consequences of this observation is important for the design of communication equipment when the channel is a chaotic system. For one, it implies that information at the receiver about the state of the channel is lost at a mean rate given by the Kolmogorov entropy. For another it implies that a channel estimator should be adapted to the mathematical nature of the set of non linear equations describing the channel as shown in the previous paragraph. Our studies up to this date have shown that this approach is valid in an indoor situation but not in an outdoor one. This might be due to the confined geometry one encounters inside buildings and the boundary conditions for the electromagnetic fields leading to a low dimensional system of non linear equations giving birth to the observed chaotic behaviour. Our studies in this direction are in progress.

Acknowledgements
We thank James Theiler, Jean-Pierre Eckmann and Reggie Brown for sending us their computer programs and correspondence, as well as Halbert White for some unpublished material. C.T. thanks Sunit Lohtia and Bin Tan for their friendly help with the manuscript.

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Figure Captions

Fig. 1: Correlation dimensions vs embedding dimension: Full squares are for Rayleigh distributed points; full triangles are for handlimited Rayleigh distributed points. Full diamonds are for experimental results in the XPA case whereas open diamonds correspond to the CPA case and open squares to the APA case.

Fig. 2: Liapunov exponent spectrum from Eckmann et al. [7] method versus embedding dimension for the APA data (since APA data consist of the largest number of points, 6000).

Fig. 3: Liapunov exponent spectrum from Brown et al. method [8] for the same data as those of Fig.2. A linear tangent mapping is used to fit the dynamics. A very similar spectrum is obtained for a second order polynomial.

Fig. 4: Spectrum of generalised dimensions $f(\alpha)$ versus $\alpha$ for the APA data used in fig.2. The value at the maximum of $f(\alpha)$ corresponding to the Hausdorff dimension of the strange attractor agrees with the minimum bound obtained from fig.1 and with the Kaplan-Yorke conjecture (see text). The spectrum is obtained through embedding in 10 dimensions. This happens to be enough, given the values obtained for the various generalised dimensions.

Fig. 5: Measured envelope (APA data used in Fig.2 continuous curve) and its one step prediction (dashed curve) from the Neural Network fit to the five dimensional map $F$ (eq.2). The training is over the first 200 first points.

Fig. 6: Normalised prediction error versus embedding. Starting from the same initial weights, we trained the neural network, for a given embedding, over the first 200 points with a Marquardt minimisation standard deviation parameter equal to 0.01 and total number of 150 iterations. Once, the parameters at the minimum error are found we made a one-step ahead prediction over the next 300 points and calculated the resulting squared error divided by the total of points. One sees a minimum for an embedding dimension around 5 or 6. For an embedding equal to 7 or larger, a large error or no convergence (null pivot encountered in the least square error matrices) were observed.