An Effective Model for Hot Gluodynamics

G.W. Carter\textsuperscript{1}, O. Scavenius\textsuperscript{1}, I.N. Mishustin\textsuperscript{1,2} and P.J. Ellis\textsuperscript{3}

\textsuperscript{1}The Niels Bohr Institute, Blegdamsvej 17
DK–2100 Copenhagen \O, Denmark

\textsuperscript{2}The Kurchatov Institute, Russian Research Center
Moscow 123182, Russia

\textsuperscript{3}School of Physics and Astronomy, University of Minnesota
Minneapolis, MN 55455, USA

We consider an effective Lagrangian containing contributions from glueball and gluon degrees of freedom with a scale-invariant coupling between the two. The thermodynamic potential is calculated taking into account thermal fluctuations of both fields. The glueball mean field dominates at low temperature, while the high temperature phase is governed by low-mass gluon-like excitations. The model shows some similarities to the lattice results in the pure glue sector of QCD. In particular, it exhibits a strong first order phase transition at a critical temperature of approximately 265 MeV when reasonable parameters are taken.

I. INTRODUCTION

The aim of this paper is to non-perturbatively describe the pure glue sector of quantum chromodynamics (QCD) with an effective Lagrangian containing a small number of parameters. In this endeavor we are guided by the requirement that the model reproduce the most important non-perturbative features of QCD. At zero temperature the model should therefore account for the gluon condensate. As the temperature increases through a critical temperature, \( T_c \), a phase transition should occur in which the condensate is melted and gluon-like excitations become the relevant degrees of freedom. This picture is supported by lattice calculations which are quite well established for the pure glue sector \cite{1,2}. Above \( T_c \) the thermodynamic “data” has been fitted by several simple models ranging from one employing massless gluons with a low-momentum cut-off \cite{3} to a model assuming temperature dependence in both the effective gluon mass and the bag constant \cite{4} (see this reference for a more complete review).

In this paper we formulate a simple effective field-theoretical model for constituent gluons which become massive via an interaction with the gluon condensate. The constituent gluons are described by an Abelian vector field, while the gluon condensate is identified with the dynamical glueball field which has a non-zero expectation value. The form of the glueball potential is well known \cite{5}, fixed by the requirement that scale invariance be broken through generation of a trace anomaly as in QCD. Therefore it is surmised that the non-linearities of the actual QCD Lagrangian can be modeled by the glueball potential and the mass-like coupling term. Below we demonstrate that this simple model can indeed reproduce some of the features of QCD.

The plan of this paper is as follows. In Sec. II the effective Lagrangian is written down and the fields are decomposed into mean field and thermally fluctuating parts. Expressions are given for the equations of motion, masses and thermodynamic variables. Since we have to deal with a non-linear potential we introduce a novel method of handling the fluctuations. In Sec. III we discuss the choice of model parameters and present our results. Sec. IV is reserved for conclusions and outlook.

II. THE MODEL

A. Effective Lagrangian

Motivated by the above considerations we write our effective Lagrangian as

\[
\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - U(\phi) - \frac{1}{4} A_{\mu \nu} \cdot A^{\mu \nu} + \frac{1}{2} \partial^{\nu} \phi A_{\mu \nu} - \frac{1}{4} C^2 \phi^2 F_{\mu \nu} \cdot F^{\mu \nu},
\]

where \( \phi \) is the scalar glueball field and \( A_{\mu} \) is the Abelian vector field with strength tensor \( A_{\mu \nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \). The dimension of the vector \( A_{\mu} \) is \textit{a priori} unknown, and we define it to be \( \nu/3 \) so that the effective number of constituent gluon degrees of freedom is \( \nu \). The second term in Eq. (1) is the glueball potential.
\[ U(\phi) = \frac{1}{4} \lambda^2 \phi^4 \ln \left( \frac{\phi^4}{\Lambda^4} \right), \tag{2} \]

where \( \lambda \) is a constant and \( \Lambda \), which is the only dimensionful parameter in the model, defines the vacuum glueball field, \( \phi_0 = \Lambda/e^\frac{4}{7} \). The last term in (2) gives a masslike coupling between the \( \phi \) and \( A_\mu \) fields with a coupling constant \( G \).

Since \( G \) is dimensionless, scale invariance is broken only by the glueball potential, \( U(\phi) \). This potential was constructed to reproduce the trace anomaly of QCD in an effective theory [5]. Indeed, for our Lagrangian the trace of the energy-momentum tensor is

\[ \theta^\mu_\mu = 4U - \phi \frac{dU}{d\phi} = -\lambda^2 \phi^4, \tag{3} \]

so that the vacuum energy density is \( -\frac{1}{4} \lambda^2 \phi_0^4 = -\frac{1}{4} B_0 \). In QCD, the latter is proportional to the gluon condensate and is roughly known from QCD sum rules [6]; see Narison [7] for a more recent value. Therefore a nonzero expectation value for the glueball field \( \langle \phi \rangle \) implies the presence of a gluon condensate. In the following we consider \( \langle \phi \rangle \) to be the order parameter for our study of the thermodynamics and phase structure at finite temperature. The model thus contains only four parameters: \( \nu \), \( B_0 \), \( \phi_0 \) and \( G \). The pure glueball sector of the Lagrangian (1), which we will treat as a special case, has been discussed previously by Agasyan [9].

The Lagrangian (1) is of similar structure to the dual Ginzburg-Landau model [8]. The latter also assumes Abelian dominance, for which lattice calculations offer some support, and considers three scalar magnetic monopole fields, rather than the single glueball field here. Where a quartic potential has been assumed in Ref. [8], we take a logarithmic dominance, for which lattice calculations offer some support, and considers three scalar magnetic monopole fields, \( \nu \), \( B_0 \), \( \phi_0 \) and \( G \). The pure glueball sector of the Lagrangian (1), which we will treat as a special case, has been discussed previously by Agasyan [9].

The field fluctuations are decomposed into plane waves and we take the thermal average of the equations of motion,

where we have defined \( g = G \phi_0 \). The non-trivial point of our treatment is that in addition to the mean field we include the thermal fluctuations of the glueball and gluon fields in a consistent way. To that end we break the glueball into mean field and fluctuating parts, \( \chi = \bar{\chi} + \Delta \) with \( \langle \Delta \rangle = 0 \), where the angle brackets denote a thermal average. In the second of Eqs. (4) we replace \( \chi^2 \) by its thermal average \( \langle \chi^2 \rangle \) so that we can interpret the third term as a mass term; this amounts to imposing the condition \( \partial^\mu A_\mu = 0 \), as appropriate for a vector field with three degrees of freedom.

The field fluctuations are decomposed into plane waves and we take the thermal average of the equations of motion, assuming that the thermal average of the product of glueball and gluon fields can be approximated by the product of their respective thermal averages. This gives the mean-field versions of Eqs. (4) for the glueball field,

\[ 2B_0 \langle \chi^3 \log \chi^2 \rangle = g^2 \bar{\chi} \langle A_\mu \cdot A^\mu \rangle, \tag{5} \]

and the subsequently vanishing mean gluon field, \( \langle A_\mu \rangle = 0 \).

The dispersion relations for the gluon and glueball excitations, including contributions from thermal fluctuations, can be written

\[ e_A^2 = k^2 + m_A^2 \quad ; \quad e_\chi^2 = k^2 + m_\chi^2, \tag{6} \]

where the effective masses are defined to be the thermal average of the second derivative of the potential, \( i.e. \)

\[ \phi_0^2 m_\chi^2 = -\left( \frac{\partial^2 \mathcal{L}}{\partial \Delta^2} \right) = 6B_0 \langle \chi^2 \ln \chi^2 \rangle + 4B_0 \langle \chi^2 + \langle \Delta^2 \rangle \rangle - g^2 \langle A_\mu \cdot A^\mu \rangle \]

\[ m_A^2 = \left( \frac{\partial^2 \mathcal{L}}{\partial A_\mu^2} \right) = g^2 \langle \chi^2 + \langle \Delta^2 \rangle \rangle. \tag{7} \]

This set of equations is closed by expressing the quantities \( \langle A_\mu \cdot A^\mu \rangle \) and \( \langle \Delta^2 \rangle \) in terms of the field quanta distributions. Using standard methods one obtains for the vector and scalar fields, respectively,
\begin{align}
\langle A_\mu \cdot A^\mu \rangle &= -\frac{\nu}{2\pi^2} \int_0^\infty dk \frac{k^2}{e_A} n_B(e_A) \; ; \; \langle \Delta^2 \rangle = \frac{1}{2\pi^2 e_\chi^2} \int_0^\infty dk \frac{k^2}{e_\chi} n_B(e_\chi) ,
\end{align}

where \( n_B(x) = (e^{\beta x} - 1)^{-1} \) is the Bose-Einstein distribution function and \( \beta = 1/T \) is the inverse temperature. The equation for the mean field (8) and the equations for the masses (9) must be solved self-consistently.

### C. Evaluation of the Thermal Fluctuations

Equations (3) and (4) require the thermal average of functions of \( \chi \) which involve a logarithm and these are handled in the following manner. Consider a general function \( f(\chi) \) and Taylor expand the fluctuations so \( \langle f(\chi) \rangle = \sum_0^\infty f^{(n)}(\bar{\chi}) \langle \Delta^n \rangle / n! \), where \( f^{(n)} \) denotes the \( n^{th} \) derivative of the function. Here we are considering the contributions from a single vertex, ignoring loop diagrams with two or more vertices, as is appropriate for a mean field treatment. Simply truncating the Taylor expansion at low order as in Ref. (4) would be appropriate for low temperatures, but would be inadequate for high temperatures where the mean field vanishes and the fluctuations are large. Fortunately we can treat the problem exactly. Taking the thermal average of each possible pair of fields \( \Delta \) for a given \( n \)-point vertex gives \( \langle \Delta^n \rangle = (n - 1)! \langle \Delta^2 \rangle^\frac{n}{2} \) for \( n \) even and zero for \( n \) odd (see Ref. (11) for further discussion). Defining a Gaussian weighting function

\[ P(z) = (2\pi\langle \Delta^2 \rangle)^{-\frac{1}{2}} \exp\left(-\frac{z^2}{2\langle \Delta^2 \rangle}\right) \],

one finds that

\[ \langle f(\chi) \rangle = \int_{-\infty}^{\infty} dz \; P(z) \left( \sum_{n=0}^\infty f^{(n)}(\bar{\chi}) \frac{z^n}{n!} \right) = \int_{-\infty}^{\infty} dz \; P(z) f(\bar{\chi} + z) , \]

where in the last step we have resummed the Taylor series. Thus the fluctuations enter with a Gaussian weighting and Eq. (10) is straightforward to compute for any \( \bar{\chi} \). By performing the series expansion one can see that \( \bar{\chi} = 0 \) is an exact solution of Eq. (8). In this case the integral (10) can be carried out analytically (12) and Eqs. (7) become

\[ \phi_0^2 m_\chi^2 = 6 B_0 \langle \Delta^2 \rangle \ln \alpha \langle \Delta^2 \rangle - g^2 \langle A_\mu \cdot A^\mu \rangle \; ; \; m_\chi^2 = g^2 \langle \Delta^2 \rangle , \]

where \( \ln \alpha = \frac{8}{3} - \gamma - \ln 2 \), with Euler’s constant denoted by \( \gamma \), giving \( \alpha \approx 4.0402 \).

### D. Thermodynamics

The grand canonical potential per unit volume can be written in a straightforward way:

\[ \Omega = \frac{1}{2} B_0 \langle \chi^4 \ln \chi^2 - \frac{1}{2} \rangle + \frac{1}{4} B_0 - \frac{1}{2} \phi_0^2 m_\chi^2 \langle \Delta^2 \rangle - \frac{1}{6} \int_0^\infty dk k^4 \left( \frac{\nu n_B(e_A)}{e_A} + \frac{n_B(e_\chi)}{e_\chi} \right) . \]

Here a constant second term has been added so that \( \Omega = 0 \) at zero temperature and the third term subtracted so as to avoid double counting. Notice that, apart from the constant term, all the quantities are temperature dependent, for instance \( \langle \chi^4 \ln \chi^2 - \frac{1}{2} \rangle \) is evaluated using Eq. (8) which involves the temperature-dependent quantity \( \langle \Delta^2 \rangle \) of Eq. (8).

In order to have consistent thermodynamics \( \Omega \) must be a minimum with respect to variations in the mean field \( \bar{\chi} \). Performing the minimization we indeed find that Eq. (3) is the necessary condition. Thus our equation of motion, our mass equations and the grand potential treat the thermal fluctuations in a coherent and well defined manner. The pressure is simply \( P = -\Omega/V \), and the energy density is easily obtained:

\[ \mathcal{E} = \left( 1 + \beta \frac{\partial}{\partial \beta} \right) \frac{\Omega}{V} = \frac{1}{2} B_0 \langle \chi^4 \ln \chi^2 - \frac{1}{2} \rangle + \frac{1}{4} B_0 - \frac{1}{2} \phi_0^2 m_\chi^2 \langle \Delta^2 \rangle \]

\[ + \frac{1}{2\pi^2} \int_0^\infty dk k^2 \left( \nu e_A n_B(e_A) + e_\chi n_B(e_\chi) \right) . \]

Note that in the case \( \bar{\chi} = 0 \), \( \langle \chi^4 \ln \chi^2 \rangle = 3 \langle \Delta^2 \rangle^2 \), and \( \langle \Delta^2 \rangle = \frac{1}{2} B_0 \langle \chi^4 \ln \chi^2 - \frac{1}{2} \rangle \), where \( \alpha \) is the constant defined previously.
III. RESULTS

A. Choice of Parameters

The model has four free parameters: $\nu$, $B_0$, $\phi_0$, and $G$. The effective number of gluon degrees of freedom, $\nu$, determines the asymptotic behaviour of the equation of state. In order to have $E/T^4 \approx 4.7$ at high temperature, as found on the lattice \[6\], we need to choose $\nu \approx 14$ for gluons. The standard degeneracy for massless gluons in $SU(3)$ is, of course, 16. We will also consider $\nu = 6$ which would roughly correspond to $SU(2)$, as well as the case where gluons are excluded and we have a pure glueball theory ($\nu = 0$). For a given $\nu$ the critical temperature, $T_c$, of the phase transition (see below) is largely determined by the quantity $B_0$. For $SU(3)$ we take $\nu = 14$ and choose a value for $B_0$ of $(391 \text{ MeV})^4$ so as to reproduce the deconfinement temperature found in lattice calculations. However $B_0$ also determines the zero-temperature vacuum gluon condensate and our value must be consistent with independent determinations of this quantity. Our $B_0$ is somewhat larger than the old value of $(340 \text{ MeV})^4$ found by Shifman, Vainshtein and Zakharov \[3\], but in good agreement with the more recent updated average of $(399 \pm 13 \text{ MeV})^4$ given by Narison \[7\]. The magnitude of the vacuum energy density associated with our value of the gluon condensate (bag constant) $\frac{1}{4}B_0 \approx 0.8 \text{ GeVfm}^{-3}$. The third parameter, $\phi_0$, can be fixed by appealing to the vacuum glueball mass, $m_\chi^2 = 4B_0/\phi_0^2$, which follows from Eq. (7) at $T = 0$. For the glueball mass, values in the range 1.5 – 1.7 GeV are suggested by data and by calculations \[13\]; for definiteness we take $m_\chi = 1.7 \text{ GeV}$ following Sexton et al. \[4\]. Finally in order to fix $G$ we assume that the glueball is a loosely bound system of two gluons and therefore choose the effective gluon mass in vacuum, $m_A = G\phi_0 = g$, to be $\frac{1}{2}m_\chi$; the recent study of phenomenological gluon propagators \[15\] suggests that this is a reasonable estimate.

B. Phase Transition

Fig. 1 shows the mean glueball field, $\bar{\chi}$, for the three cases mentioned above. Since $\bar{\chi}$ and the other variables are essentially constant at lower temperatures the abscissa starts at $T = 100 \text{ MeV}$. A most striking feature of the model is that it exhibits a first order phase transition at a critical temperature $T_c$. Here the mean glueball field drops from a value of slightly less than unity to zero, and it remains zero for $T > T_c$. The arrows on the figure indicate where the transition takes place (the remainder of the curves correspond to metastable or unstable branches). The physics behind this is indicated in Fig. 2, where we plot $\Omega/V$ (relative to the value at $\bar{\chi} = 0$) as a function of $\bar{\chi}$ for various temperatures in the vicinity of $T_c$. Below $T_c$ there exists a local minimum of the effective potential at $\bar{\chi} = 0$, but the absolute minimum is at $\bar{\chi} \approx 1$. As the temperature is increased the latter minimum becomes shallower and at $T_c$ it has the same depth as the minimum at $\bar{\chi} = 0$. Thereafter the stable solution corresponds to $\bar{\chi} = 0$ and ultimately the minimum at $\bar{\chi} \approx 1$ disappears.

For the pure glueball case (dotted line in Fig. 1) the transition temperature, $T_c = 490 \text{ MeV}$, is much higher than that estimated by Agasyan \[9\]. This is probably due to the fact that in that work the potential was expanded to low orders, whereas here we have an essentially exact treatment of the thermal fluctuations. When the glue degrees of freedom are included the transition temperature is much reduced and, using reasonable values for the parameters, is found to lie in the neighborhood of the $SU(3)$ lattice result $T_c = 270 \pm 5 \text{ MeV}$ for the pure glue sector of QCD \[6\]. For our chosen parameters the first order transition for the solid curve occurs at 265 MeV. For $\nu = 6$ the dashed curve shows that the critical temperature rises to 340 MeV. To compare with $SU(2)$ one should also take into account the scaling of the gluon condensate with the number of colors. Reducing the condensate by a factor of $\frac{2}{3}$ results in a value of $T_c = 300 \text{ MeV}$. These figures are reasonable in view of lattice calculations which yield a 20% increase in $T_c$ (with considerable error) \[17\] when the number of colors is changed from three to two. However the lattice results indicate a second order transition for $SU(2)$ \[13\], whereas we have a first order transition, although mean field treatments are expected to be inadequate in the neighborhood of critical points.

C. Effective Masses

The ratios of the effective masses to the temperature, $m_\chi^*/T$ and $m_A^*/T$, are displayed in Fig. 3 (the complicated structure of the low temperature metastable region for $\nu = 14$ is not relevant here and for clarity is suppressed in the figures). The masses change little below the critical temperature and so the ratio drops as the temperature increases. Beyond the critical point the masses grow linearly with temperature to a good approximation. Such behavior is expected at very high temperatures where perturbation theory is applicable. For the solid curves ($\nu = 14$) $m_\chi^*/T$ is quite small, 0.18, whereas $m_A^*/T$ is large, 4.5, so that the glueball plays only a minor role beyond the transition.
temperature, as is physically expected. The reason that one inevitably has a large glueball mass follows from Eq. (11). The gluon contribution to \( m^*_B \) (second term) is positive and large at high degeneracy \( \nu \), so in order to have a low glueball mass the first term would have to be negative. This would be only be possible if the argument of the logarithm were less than 1. However for a low-mass glueball with \( m^*_B/T \ll 1 \) the argument of the logarithm \( \alpha(\Delta^2) \sim [T/(1.72\phi_0)]^2 \). So in order to have a negative value for the logarithm for \( T > T_c \), \( \phi_0 \) would have to be substantially increased compared to the chosen value of \( \phi_0 = 180 \text{ MeV} \). This would lead to a reduction of the vacuum glueball mass to an unreasonably low value.

D. Thermodynamics

The pressure and the energy density are shown in Fig. 4, where we plot \( \mathcal{E}/T^4 \) and \( 3P/T^4 \). At low temperatures the energy density and pressure are little changed from the vacuum values. The phase transition temperature is determined by the point at which the pressure curves for the two stable solutions intersect. We note that the model predicts interesting behaviour for the pressure in the vicinity of this critical point. On cooling from high temperatures the system can enter a supercooled metastable phase, which can even have zero pressure. It is conceivable that in relativistic heavy ion collisions such behavior could allow metastable, supercooled droplets of hot gluonic fluid to be produced.

Beyond \( T_c \) the pressure and energy density in Fig. 4 increase rapidly to their asymptotic values. This qualitative behavior for the pressure is in agreement with lattice calculations, although the approach to the asymptotic value is sharper in our model. At high temperatures \( 3P \approx \mathcal{E} \) and the value of these quantities for the solid and dashed curves is very close to that expected for an ideal gas of massless gluons with the appropriate degeneracy, \( \nu = 14 \) or 6, indicating that we have reached the asymptotic regime. One aspect of this model is seemingly in strong disagreement with the lattice data. Namely, at the critical temperature there is a large latent heat and the energy density overshoots the asymptotic value of \( \mathcal{E}/T^4 \), approaching it from above. This is in contrast to the lattice calculations where the latent heat is a factor of 2–3 smaller and \( \mathcal{E}/T^4 \) approaches its asymptotic value from below.

For \( T > T_c \) the behavior can be understood rather simply. The ratio of the glueball mass to temperature, \( m^*_B/T \), is large, while the gluon mass is small and, to a good approximation, can be taken to be zero. This allows the necessary Bose integrals to be approximated easily (see, for example, Ref. [24]). It turns out that only the constant term and the thermal gluon term are numerically significant for the pressure and energy density. Thus, one obtains expressions very similar to those of the bag model

\[
\frac{3P}{T^4} \simeq \frac{\pi^2 \nu}{30} - \frac{3B_0}{4T^4} ; \quad \frac{\mathcal{E}}{T^4} \simeq \frac{\pi^2 \nu}{30} + \frac{B_0}{4T^4} .
\]

Since the pressure is approximately zero at the phase transition, the above equation provides an estimate of the critical temperature \( T_c \approx \{45B_0/(2\pi^2\nu)\}^{1/4} \) which is accurate to 10% or better. The thermodynamics of the model given by Eq. (14) can be contrasted with a power law fit to the lattice results at high temperatures not too close to \( T_c \), which gives

\[
\left( \frac{3P}{T^4} \right)_{\text{latt}} \simeq 4.8 - 5.2 \left( \frac{T_c}{T} \right)^{2.16} ; \quad \left( \frac{\mathcal{E}}{T^4} \right)_{\text{latt}} \simeq 4.7 - 2.04 \left( \frac{T_c}{T} \right)^{3.37} .
\]

Indeed the sign of the deviation of the energy density from the asymptotic value is opposite to the prediction of the bag model. This results in too large a latent heat, as we have mentioned earlier. We remark that the dual Ginzburg-Landau model appears to suffer from similar difficulties.

The qualitative features of our model are not sensitive to variation of the parameters within reasonable limits. For example, the primary effect of varying the glueball-gluon coupling constant \( G \) is to scale the gluon effective mass according to Eq. (14). There is little change in \( T_c \), and above this temperature the thermodynamics is still represented by Eq. (14) since \( m^*_B/T \) remains small (unless \( G \) is made unreasonably large). Similarly, the main effect of altering \( \phi_0 \) is to change the masses; increasing \( \phi_0 \) decreases the glueball mass and increases the gluon mass. This assumes that the constant \( B_0 \) has been fixed to vacuum expectations. If we choose to alter \( B_0 \), it is the vacuum (and low-temperature) glueball mass which must change. This alters the critical temperature according to \( T_c \sim B_0^{1/4} \), as we have mentioned. Above \( T_c \) the value of \( B_0 \) is inconsequential for the masses, while the thermodynamics still follows Eq. (14).

We have also considered introducing an \( (A_\mu \cdot A^\mu)^2 \) term in the Lagrangian, which would be suggested by QCD, but this does not appear to improve the situation. However it is worth noting that the simple replacement \( m^*_A \to m^*_A + m_0^2 \), where \( m_0 \) is a constant mass of order 500 MeV, can alter the predictions. With some modest adjustment of the other parameters, the energy density and pressure above \( T_c \) can be brought into semi-quantitative agreement with the
lattice data. In particular, the energy density no longer overshoots the asymptotic value. This modification, however, is inconsistent with QCD in that the corresponding term in the Lagrangian, $\frac{1}{2} m^2_0 A_\mu \cdot \mathbf{A}_\mu$, introduces a dimensional parameter and consequently gives an unwanted contribution to the trace anomaly. Thus such an addition contradicts our initial approach. It does however strongly suggest that the problem with our model is that above $T_c$ the masses immediately become proportional to the temperature and some important physics has not been accounted for in our description of the masses in the region $T_c < T < 2T_c$.

IV. CONCLUSIONS

In conclusion, we have examined a very simple Lagrangian model for the pure glue sector of QCD and determined its thermal properties using a novel treatment of the thermal fluctuations. In accord with physical expectations, the model shows a phase transition between a low-temperature, glueball-dominated regime and a high-temperature phase dominated by low-mass gluon-like excitations. Sensible parameters yield a transition temperature in agreement with lattice simulations. Beyond $T_c$ the qualitative behavior of the pressure is reasonable, but the energy density in the neighborhood of the phase transition disagrees with the lattice data. This pathology appears to be a result of the low effective gluon mass immediately after the phase transition. So while the thermodynamics are reasonable at low and high temperatures, the lattice results reveal our model to be incomplete just above the critical temperature. Nevertheless, we conclude that our simple Lagrangian indeed captures important non-perturbative features of QCD.

In the future it would be interesting to consider modifications suggested by the color dielectric model which simulates confinement. It would also be worthwhile to consider the inclusion of quark degrees of freedom. Such effective models may be particularly useful for simulations of time-dependent processes and the effects of finite baryon density which are yet intractable on the lattice.

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FIG. 1. The mean glueball field, $\bar{\chi}$, as a function of temperature, $T$. For the dotted curve the gluon field is neglected, while the dashed and solid curves correspond to degeneracies, $\nu$, of 6 and 14 for the gluon field, respectively. The arrows indicate where a phase transition takes place and the thermodynamically stable phase becomes $\bar{\chi} = 0$.

FIG. 2. The grand potential density, $[\Omega - \Omega(\bar{\chi} = 0)]/V$, as a function of the mean glueball field, $\bar{\chi}$, for temperatures, $T$, in the vicinity of the critical temperature $T_c$. 
FIG. 3. The effective masses of the glueball and gluon in units of the temperature, $m^*_\chi/T$ and $m^*_A/T$, as a function of temperature. See caption to Fig. 1 for the meaning of the curves.
FIG. 4. The thermodynamic quantities $\mathcal{E}/T^4$ and $3P/T^4$ as a function of temperature. See caption to Fig. 1 for the meaning of the curves.