Quantum depletion signature of emerging sonic horizons in a quasi-one-dimensional Bose-Einstein condensate

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We consider the quasi-one-dimensional (quasi-1D) model of a sonic black hole in a dilute Bose-Einstein condensate. It is shown that an accurate treatment of the dimensional reduction to quasi-1D leads to a finite condensate quantum depletion even for axially infinite systems, and an intrinsic nonthermality of the black hole radiation spectrum. By calculating the depletion, we derive a first-order many-body signature of the sonic horizon, represented by a distinct peak in the depletion power spectrum. This peak constitutes a readily experimentally accessible tool to detect quantum sonic horizons even when a negligible Hawking radiation flux is created by the black hole.

The discovery by Stephen Hawking that black holes, quantum mechanically, are not black but radiate a thermal spectrum of particles [1, 2] continues to furnish an intriguing milestone in the quest for a unification of quantum mechanics with gravity [3]. While an observation of the Hawking effect with astrophysical black holes is essentially impossible, analogue systems in fluids have enabled it due to its kinematical nature [4–7] and robustness against (most variants of) Lorentz invariance breaking [8–11]. In particular Bose-Einstein condensates have been identified as suitable system to verify an analogue of Hawking’s prediction in a superfluid of very low temperature, and other quantum effects related to sonic horizons see, e.g. [12–24].

An unambiguous confirmation of the quantum Hawking effect was achieved in 2019 by the Steinhauer group [25], see also the more recent experiment [26]. The observation of the density-density correlations as a second-order correlation function signature of the Hawking effect [18, 27, 28], however, still presents, in particular for small Hawking temperatures TH, a formidable task [29, 30].

Current experiments are carried out in (or close to) a quasi-1D flow geometry of strongly elongated condensates, with a primary motivation being to avoid turbulence developing when the condensate flows supersonically. We show that a careful treatment of the radial degree of freedom of excitations leads in a quasi-1D setup to a frequency gap in the Bogoliubov spectrum, which is present even for axially infinite systems. As a consequence of the finite gap, we obtain an intrinsic nonthermality of quasi-1D Hawking radiation, which has a different origin than in the analysis of, e.g., Refs. [31–33].

For dilute Bose-Einstein condensates, which are interacting many-body systems described by Bogoliubov theory, an observable of fundamental relevance is quantum depletion, which is a first-order correlation function. The excitation gap intrinsic to the quasi-1D approach, which we derive, renders the quantum depletion finite (by contrast to an idealized, strictly 1D gas where depletion diverges with system size). We consequently present, to the best of our knowledge, the first calculation of the depletion in inhomogeneous BECs with a sonic spacetime horizon. It is demonstrated that when the sonic horizon just emerges, a distinct peak in the Fourier spectrum of the quantum depletion forms, which constitutes a valuable tool to identify the emergence of quantum Hawking channels even for negligible flux at infinity. We finally suggest an experiment to verify this peak in the power spectrum of depletion, using the Bragg scattering technique of [34], to probe this emergence of a sonic quantum horizon by first-order correlations.

The quasi-1D Bogoliubov potential. — As it will play a crucial role in our analysis, we begin by deriving the additional Bogoliubov potential UH term emerging in the quasi-1D-reduced Bogoliubov de Gennes (BdG) equations [35, 36], which leads to a gap in the excitation spectrum. The latter obtains because condensate and excitations possess different transverse wave functions for strong transverse trapping potentials, cf. Ref. [37].

Let Φ(t, x) be the condensate order parameter of the three-dimensional (3D) gas whose mean-field dynamics is governed by the Gross-Pitaevskii (GP) equation (ℏ = 1)

\[ i\partial_t \Phi = \left( -\nabla^2 / 2m + U + g_3 \Phi^2 \right) \Phi, \tag{1} \]

with contact coupling g3D, and total 3D potential U(x) = U∥(x) + U⊥(x⊥) with the transverse trapping assumed harmonic, U∥(x⊥) = x⊥2 / 2mℓ⊥4. Small perturbations δΦ, i.e., |δΦ|^2 ≪ |Φ|^2, satisfy the BdG equation

\[ i\partial_t \delta \Phi = \left( -\nabla^2 / 2m + U + 2g_3 \Phi^2 \right) \delta \Phi + g_3 \Phi^2 \delta \Phi^*, \tag{2} \]

In the supplement [38], we show that under the adiabatic approximation [39, 40], Eqs. (1) and (2) reduce to

\[ i\partial_t \phi = \left[ -\nabla^2 / 2m + U∥ + \mu_\phi(n) \right] \phi, \tag{3} \]

\[ i\partial_t \psi = \left[ -\nabla^2 / 2m + U∥ + \mu_\psi(n) \right] \psi + g\phi^2 \psi^*, \tag{4} \]
respectively, for the 1D background order parameter \( \phi \) and its 1D perturbations \( \psi \); the background linear density \( n(t, x) \equiv |\phi(t, x)|^2 \). The effective radial chemical potentials are defined by the minima of the functionals

\[
\mu_\phi(n) := \int \frac{d^2 x}{\ell_\perp^2} \phi_\perp^2 \left( -\frac{\nabla^2}{2m} + U_\perp + \frac{n g_{3D}}{\ell_\perp^2} |\phi_\parallel|^2 \right) \phi_\parallel,
\]

\[
\mu_\psi(n) := \int \frac{d^2 x}{\ell_\perp^2} \psi_\perp^2 \left( -\frac{\nabla^2}{2m} + U_\perp + \frac{2 n g_{3D}}{\ell_\perp^2} |\phi_\parallel|^2 \right) \psi_\parallel.
\]

The minimizing functions, \( \phi_\parallel, \psi_\parallel \) are the estimated radial profile for background and perturbations, respectively. Finally, the 1D coupling constant \( g \) in (4) reads \( g = g_{3D} \int d^2 x \phi_\parallel^2 / \ell_\perp^2 \). Note in particular the extra factor 2 in \( \mu_\psi \). It has the important consequence of leading to a frequency gap of the excitation spectrum [38]

\[
\left( \frac{\omega}{ng} \right)^2 = \left( \frac{\xi^2 k^2 + U_B}{1 + \xi^2 k^2 + U_B} \right) + \frac{1}{4},
\]

with \( \xi = 1/\sqrt{ngm} \) being the 1D healing length. Here, \( U_B \) is the (assumed in the above relation constant) quasi-1D Bogoliubov potential, defined by

\[
\frac{1}{2} U_B = \frac{\mu_\psi(n) - \mu_\phi(n)}{gn} - 1.
\]

When \( \psi_\perp \equiv 0, U_B \equiv 0 \), reproducing the standard result. However, in general, \( U_B \neq 0 \), and for \( U_B > 0 \), the spectrum is bounded from below by \( ng[1 + U_B/4]^{1/2} \). When \( U_B < 0 \), the background is unstable. In [38], we work out a toy model showing that this instability may occur even for small deviations of \( \psi_\perp \) from \( \phi_\perp \). To realize a stable experiment, one should have \( U_B > 0 \). In Table I we present numerical estimates.

The analogue model.— We consider a stationary gas flowing in the direction of increasing \( x \) and which is everywhere homogeneous except in a region around \( x = x_H = 0 \) where its density changes (cf. top panel Fig. 1). The region in which the density varies is assumed small in comparison with the gas axial length, which can thus be regarded as a single point. Accordingly, the background solution takes the form \( \phi(t, x) = \exp(-i \mu t + i k x) / \sqrt{n}, \) and \( \sqrt{n}, k > 0 \) are constant everywhere besides \( x = 0 \), where they possess discontinuities. We show in [38] how this configuration is modeled by an extra potential proportional to \( \partial_x \delta(x) \), deriving from a specification of boundary conditions to \( \phi \) [41, 42]. We reserve the subindex “u” to denote parameters in the “upstream” region \( x < 0 \) and “d” for the downstream region, \( x > 0 \). Then the

| \( \ell_\parallel / \xi_{3D} \) | 0.1 | 0.5 | 1 | \( \sqrt{2} \) | 2 |
|-----------------|-----|-----|---|-----|---|
| \( U_B \)       | 0.001 | 0.011 | 0.046 | 0.094 | 0.193 |

TABLE I. \( U_B \) estimates, parametrized in terms of the 3D healing length \( \xi_{3D} = \ell_\parallel / \sqrt{ngm} \); as expected, \( U_B \) approaches zero in the proper 1D limit of infinitely strong trapping.

only constraint imposed on the background solution by the GP equation (3) is due to continuity, \( n_u v_u = n_d v_d \) [38], where \( v = \kappa/m \) is the flow velocity. This model was previously explored in [43], and we adopt it here over the (more constrained) waterfall configuration [44], which is based on a solitonic solution: The latter assumes the conventional quasi-1D approach to be exact and fixes downstream parameters in terms of upstream ones via a particular external potential. The model [43] thus offers more parameter choice freedom, and therefore a greater control on the emergence of the sonic horizon. A further comparable alternative is a jump in the coupling \( g \) instead of in the density [27], which we describe in [38].

The density \( n \) at \( x_H \) making a jump between two different constants corresponds to a change in the local sound velocity \( c = \sqrt{ng/m} \), and a necessary condition for a “dumb hole” [9] to occur is \( v/c < 1 \) for \( x < 0 \) and \( v/c > 1 \) for \( x > 0 \). In terms of the Mach number, defined as \( \mathfrak{m} \equiv v/c \), we thus should have \( \mathfrak{m}_u < 1 < \mathfrak{m}_d \). This configuration is fixed by specifying the parameters \( \{ \mathfrak{m}_u, \mathfrak{m}_u, \mathfrak{m}_d \} \) or equivalently \( \{ \xi_u, m_u, m_d \} \), which determines the experimental parameters \( n_u, v_u, m_d \), recalling that \( v_d \) is fixed by continuity. Note that it is necessary to fix \( \ell_\parallel \) and \( \mu \) to fully characterize \( U_B \). We use \( \{ \xi_u, m_u, m_d, U_B, u \} \) as the independent parameters in our analysis.

In [38] we show how to calculate the Bogoliubov field modes for each frequency \( \omega \). Using \( 1/\xi_u \) as wavevector

![FIG. 1. Top: Illustration of flow configuration. Bottom: Solutions to the Bogoliubov dispersion relation, Eq. (8), with \( m_u = 0.5, m_d = 1.5 \); and, for the sake of illustration, \( U_{B,u} = U_{B,d} = 0.01 \). Left (right) panel depicts the solutions at the upstream (downstream) region. The red dotted curves represent the dispersion when \( U_B > 0 \). The blue points indicate which branch the corresponding channel belongs.](image-url)
units, the possible (scattering and evanescent) channels are given by the dispersion relation

$$\left( \frac{\omega}{n_u g} - \frac{m_u^{1/3}m^2/k}{3} \right)^2 = (k^2 + U_B) \left( \frac{m_u^{2/3}}{m^2/3} + \frac{k^2 + U_B}{4} \right).$$

We again note the important presence of $U_B$, which does not occur, e.g., in [27, 44].

We shall denote the solutions in the upstream region by $k_{in}$, $k_1$ and $k_2$ (with $\text{Im } k_i \leq 0$, $i = 1, 2$), and downstream by $p_{in}$, $p_H$, $p_1$ and $p_2$ (with $\text{Im } p_i \geq 0$, $i = 1, 2$). Each solution in the set $\{k_{in}, p_{in}, p_H\}$ represents a possible incoming signal towards the horizon [38], and $\{k_1, k_2, p_1, p_2\}$ parametrize the various reflected, transmitted, and evanescent channels. Thus we have at most three possible field modes, which will be called “u” modes ($k_{in}$), ordinary “d” modes ($p_{in}$), and Hawking modes ($p_H$). Note that Hawking modes are possible only up to the cutoff frequency $\Omega^{(H)}$, see Fig. 1.

We obtain from nonvanishing $U_B$ that frequency gaps for the u and d modes, hereafter denoted by $\Omega^{(u)}$, $\Omega^{(d)}$, occur, which depend on \{$\xi_u, m_u, m_d, U_B$\}. This diminishes the number of field modes usually predicted in this system, as we cannot have real (i.e., propagating) solutions $k_{in}$ ($p_{in}$) for $\omega < \Omega^{(u)}$ ($\Omega^{(d)}$). Furthermore, it follows from Eq. (8) that $\omega/n_u g \approx (m_u m^3)^{1/3}k^3 \pm \sqrt{U_B} \sqrt{(m_u m^2)^{2/3} + U_B/4}$ for small $k$ and the system is inherently gapped (and thus also regularized) due to $U_B$.

In [38], we show how the various channels are accommodated into field modes which are then used to expand an expansion for the canonical field operator

$$\psi = \sum_{j=u,d} \int_{\Omega^{(j)}}^\infty d\omega \left[ a_j^{(j)} e^{-i\omega t} f^{(j)}(x) + a_j^{(j)} \delta(\omega - \omega') \right]$$

$$+ \int_0^{\Omega^{(H)}} d\omega \left[ a_j^{(H)} e^{-i\omega t} f_j^{(H)}(x) + a_j^{(H)} e^{i\omega t} h_j^{(H)*}(x) \right],$$

where $[a_j^{(j)}, a_j^{(j)}]$ = $\delta_j, \delta(\omega - \omega')$, and the functions $f_j^{(j)}, h_j^{(H)}(x)$ are defined in [38]. The vacuum state in this representation is therefore the state satisfying the kernel condition $a_j^{(j)}|0\rangle = 0$ for all $\omega$ and $j$, which is the state corresponding to zero temperature. Next, we shall see how the above expansion can be used to produce our major result, the intimate relation of an emerging (quantum) sonic horizon and condensate depletion.

**Radiation spectrum.**—For the particular setup leading to Fig. 1, we notice that the Hawking frequency cutoff $\Omega^{(H)}$ is bigger than $\Omega^{(u)}$. However, field modes with $\omega < \Omega^{(u)}$ coming from the downstream region cannot excite propagating channels in the upstream region, for $\text{Im } k_1 < 0$. This implies that if $0 < \Omega^{(H)} < \Omega^{(u)}$, deep into the upstream region, the system behaves as if no Hawking modes were present, leading to a nonradiating family of analogue black holes. Radiation is formally given by the vacuum expectation value of the energy flux density deep into the upstream region [38]

$$S = -\Theta(\Omega^{(H)} - \Omega^{(u)}) \int_{\Omega^{(u)}}^{\Omega^{(H)}} d\omega \frac{a_j^{(H)} e^{-i\omega t} f_j^{(H)}(x) + a_j^{(H)} e^{i\omega t} h_j^{(H)*}(x)}{2\pi},$$

where $S_{k_j}$ is the Hawking mode transmission coefficient (indexed by $p_{in}$), and $\Theta$ is the unit step function.

While even within Einsteinian gravity the folklore that gravitational Hawking radiation is thermal [45], and e.g. “greybody factors” [46] can be neglected, the Hawking view of analogue gravity phenomena, the gravitational wave spectrum description for the radiation we obtain in our quasi-1D setup. In Fig. 2, we show how significant the departure from “thermality” can be.

Note from Eq. (10) that $S = 0$ when $\Omega^{(H)} < \Omega^{(u)}$. The physical interpretation of this regime is that the prospective Hawking particles do not have enough energy to propagate out of the black hole and stay bounded to the black hole interior, leading to a nonradiating black hole. Below, we restrict ourselves to such nonradiating configurations, and demonstrate that even then a distinct hallmark of the quantum horizon can be extracted (see for similar features of the radiating case [38]).

**Quantum depletion.**—Condensate depletion arises from interactions and is a genuine quantum many-body
from Table I corresponds approximately to $\ell$ stream Mach number. All curves are for $U_{B,a} = 0.05$, which from Table I corresponds approximately to $\ell_\perp/\xi_{3D} = 1$, and we have taken the lower bound $U_{B,d} \approx 0.01$.

We again note the important role of the Bogoliubov potential. When $U_B \to 0$, $\Omega^{(u)}\to 0$, and $\delta n \to \infty$, an expected infrared divergence in 1D quantum field theories occurs. This divergence is linked to the infinite axial length of the system, and it can be regularized by introducing frequency cutoffs to the integral in Eq. (11). However, generally it is not possible to a priori relate the system size with the cutoffs, for each field mode is composed of various channels of different wave vectors, dependent on the region the mode originates.

Here we use that, physically, the system is, while laterally strongly confined, still physically 3D. Thus depletion should be finite even for an axially unbounded configuration, suggesting a potential problem is lurking when one passes from 3D to the quasi-1D description. This is however naturally solved by the Bogoliubov potential, which affects the dispersion relation as in Fig. 1, and renders the quantum depletion finite even for an axially infinite quasi-1D system, as shown in Fig. 3.

Two core features of Fig. 3 deserve special attention. We note, in particular, that when $m_u = 0.5$, $U_{B,a} = 0.05$, and $U_{B,d} = 0.01$, Hawking modes exist ($\Omega^{(H)} > 0$) only for $1.066 \lesssim m_d$. From Fig. 3, the curves for $m_d = 1$ and $m_d = 1.05$ show that the overall depletion inside the black hole diminishes as $m_d$ increases towards 1.066, which is expected as the particle density $n$ decreases [48]. However, as soon as Hawking modes emerge, we see that the overall depletion increases, showing that more particles cross the event horizon and leave the black hole. They however stay bounded (nonradiating configuration) for $\Omega^{(H)} < \Omega^{(u)}$, which corresponds to $m_d \leq 1.51$. In Fig. 4 we show the depletion deep inside the black hole.

The second notable feature revealed by our model is that in the presence of Hawking modes an oscillatory pattern of the depletion profile develops. This suggests that it is possible to probe the existence of Hawking modes using the Bragg technique employed by [34]. Denoting Fourier transforms as $\hat{n}(k) = \int dx \exp(-ikx)n(x)$, and similarly for $\delta\hat{n}(k)$, Ref. [34] exploits the fact that $\hat{n}(k)$ is exponentially suppressed for large $k$ in comparison to the polynomial decay of $\delta\hat{n}(k)$. One thus obtains a large $k$ window which is sensitive to depletion. The emergence of the oscillatory pattern in Fig. 3 upon formation of the horizon, then, transforms to a distinct secondary peak in $\delta\hat{n}(k)$ emerging for $|k| > 0$, as shown in Fig. 4.

Conclusion.— We have proposed to employ the first-order correlation measure of a finite quasi-1D quantum depletion to detect quantum sonic horizons in dilute Bose gases, which is a viable tool even when the horizon does not produce a Hawking flux at infinity. We note that depletion is a fundamentally distinct quantum many-body measure to detect sonic horizons and Hawking channels as compared to the one-body correlations proposed in [51], which probe the single-particle momentum distribution. We anticipate that an analysis similar to ours can be carried out for the experimentally currently realized waterfall configuration [25, 44].
rived quasi-1D Bogoliubov potential will give rise to the same type of dispersion relation furnishing the detection of sonic horizons by the spectrum of quantum depletion.

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[1] S. W. Hawking, “Black hole explosions?” Nature 248, 30–31 (1974).
[2] S. W. Hawking, “Particle creation by black holes,” Communications in Mathematical Physics 43, 199–220 (1975).
[3] Steven B. Giddings, “Black holes in the quantum universe,” Phil. Trans. R. Soc. A 377, 20190029 (2019).
[4] W. G. Unruh, “Experimental Black-Hole Evaporation?” Phys. Rev. Lett. 46, 1351–1353 (1981).
[5] M. Visser, “Hawking Radiation without Black Hole Entropy,” Phys. Rev. Lett. 80, 3436–3439 (1998).
[6] M. Visser, “Acoustic black holes: horizons, ergospheres and Hawking radiation,” Classical and Quantum Gravity 15, 1767 (1998).
[7] Carlos Barceló, Stefano Liberati, and Matt Visser, “Analogue Gravity,” Living Reviews in Relativity 14, 3 (2011).
[8] Theodore Jacobson, “Black-hole evaporation and ultrashort distances,” Phys. Rev. D 44, 1731–1739 (1991).
[9] W. G. Unruh, “Sonic analogue of black holes and the effects of high frequencies on black hole evaporation,” Phys. Rev. D 51, 2827–2838 (1995).
[10] S. Corley and T. Jacobson, “Hawking spectrum and high frequency dispersion,” Phys. Rev. D 54, 1568–1586 (1996).
[11] W. G. Unruh and R. Schützhold, “Universality of the Hawking effect,” Phys. Rev. D 71, 024028 (2005).
[12] Steven Corley and Ted Jacobson, “Black hole lasers,” Phys. Rev. D 59, 124011 (1999).
[13] S. Finazzi and R. Parentani, “Black hole lasers in Bose–Einstein condensates,” New Journal of Physics 12, 095015 (2010).
[14] L. J. Garay, J. R. Anglin, J. I. Cirac, and P. Zoller, “Sonic Analog of Gravitational Black Holes in Bose-Einstein Condensates,” Phys. Rev. Lett. 85, 4643–4647 (2000).
[15] Petr O. Fedichev and Uwe R. Fischer, “Gibbons-Hawking Effect in the Sonic de Sitter Space-Time of an Expanding Bose-Einstein-Condensed Gas,” Phys. Rev. Lett. 91, 240407 (2003).
[16] Petr O. Fedichev and Uwe R. Fischer, “Observer dependence for the phonon content of the sound field living on the effective curved space-time background of a Bose-Einstein condensate,” Phys. Rev. D 69, 064021 (2004).
[17] Ralf Schützhold, “Detection Scheme for Acoustic Quantum Radiation in Bose-Einstein Condensates,” Phys. Rev. Lett. 97, 190405 (2006).
[18] Iacopo Carusotto, Serena Fagnocchi, Alessio Recati, Roberto Balbinot, and Alessandro Fabbri, “Numerical observation of Hawking radiation from acoustic black holes in atomic Bose–Einstein condensates,” New Journal of Physics 10, 103001 (2008).
[19] A. Recati, N. Pavloff, and I. Carusotto, “Bogoliubov theory of acoustic Hawking radiation in Bose-Einstein condensates,” Phys. Rev. A 80, 043603 (2009).
[20] Jean Macher and Renaud Parentani, “Black-hole radiation in Bose-Einstein condensates,” Phys. Rev. A 80, 043601 (2009).
[21] Oren Lahav, Amir Itah, Alex Blumkin, Carmit Gordon, Shahar Rinott, Alona Zayats, and J. Steinhauser, “Realization of a Sonic Black Hole Analog in a Bose-Einstein Condensate,” Phys. Rev. Lett. 105, 240401 (2010).
[22] J. Steinhauser, “Observation of quantum Hawking radiation and its entanglement in an analogue black hole,” Nat. Phys. 12, 959–965 (2016).
[23] Cisco Gooding, Steffen Biermann, Sebastian Erne, Jorma Louko, William G. Unruh, Jörg Schmiedmayer, and Silke Weinfurtner, “Interferometric Unruh Detectors for Bose-Einstein Condensates,” Phys. Rev. Lett. 125, 213603 (2020).
[24] Ulf Leonhardt, “Cosmological horizons radiate,” arXiv:2011.09930 [gr-qc].
[25] Juan Ramón Muñoz de Nova, Katrine Golubkov, Victor I. Kolobov, and Jeff Steinhauser, “Observation of thermal Hawking radiation and its temperature in an analogue black hole,” Nature 569, 688–691 (2019).
[26] Victor I. Kolobov, Katrine Golubkov, Juan Ramón Muñoz de Nova, and Jeff Steinhauser, “Observation of stationary spontaneous Hawking radiation and the time evolution of an analogue black hole,” Nature Physics (2021), 10.1038/s41567-020-01076-0.
[27] Roberto Balbinot, Alessandro Fabbri, Serena Fagnocchi, Alessio Recati, and Iacopo Carusotto, “Nonlocal density correlations as a signature of Hawking radiation from acoustic black holes,” Phys. Rev. A 78, 021603 (2008).
[28] J. Steinhauser, “Measuring the entanglement of analogue Hawking radiation by the density-density correlation function,” Phys. Rev. D 92, 024043 (2015).
[29] Ulf Leonhardt, “Questioning the Recent Observation of Quantum Hawking Radiation,” Annalen der Physik 530, 1700114 (2018).
[30] Yi-Hsieh Wang, Ted Jacobson, Mark Edwards, and Charles W. Clark, “Induced density correlations in a sonic black hole condensate,” SciPost Phys. 3, 022 (2017).
[31] Paul R. Anderson, Roberto Balbinot, Alessandro Fabbri, and Renaud Parentani, “Gray-body factor and infrared divergences in 1D BEC acoustic black holes,” Phys. Rev. D 90, 104044 (2014).
[32] Alessandro Fabbri, Roberto Balbinot, and Paul R. Anderson, “Scattering coefficients and gray-body factor for 1D BEC acoustic black holes: Exact results,” Phys. Rev. D 93, 064046 (2016).
[33] M. Isoard and N. Pavloff, “Departing from Thermality of Analogue Hawking Radiation in a Bose-Einstein Condensate,” Phys. Rev. Lett. 124, 060401 (2020).
[34] Raphael Lopes, Christoph Eigen, Nir Navon, David Clément, Robert P. Smith, and Zoran Hadzibabic, “Quantum Depletion of a Homogeneous Bose-Einstein Condensate,” Phys. Rev. Lett. 119, 190404 (2017).
[35] E. Zaremba, “Sound propagation in a cylindrical Bose-condensed gas,” Phys. Rev. A 57, 518–521 (1998).
[36] J. Steinhauser, N. Katz, R. Ozeri, N. Davidson, C. Tozzo, and F. Dalfovo, “Bragg Spectroscopy of the Multibranch Bogoliubov Spectrum of Elongated Bose-Einstein Condensates,” Phys. Rev. Lett. 90, 060404 (2003).
[37] R. Zamora-Zamora, G. A. Domínguez-Castro,
C. Trallero-Giner, R. Paredes, and V. Romero-Rochín, “Validity of Gross–Pitaevskii solutions of harmonically confined BEC gases in reduced dimensions,” *Journal of Physics Communications* **3**, 085003 (2019).

[38] See supplemental material for detailed discussion and derivations.

[39] L. Salasnich, A. Parola, and L. Reatto, “Effective wave equations for the dynamics of cigar-shaped and disk-shaped Bose condensates,” *Phys. Rev. A* **65**, 043614 (2002).

[40] A. Muñoz Mateo and V. Delgado, “Effective mean-field equations for cigar-shaped and disk-shaped Bose-Einstein condensates,” *Phys. Rev. A* **77**, 013617 (2008).

[41] D. J. Griffiths, “Boundary conditions at the derivative of a delta function,” *Journal of Physics A: Mathematical and General* **26**, 2265–2267 (1993).

[42] Marcos Calçada, José T. Lunardi, Luiz A. Manzoni, and Wagner Monteiro, “Distributional approach to point interactions in one-dimensional quantum mechanics,” *Frontiers in Physics* **2**, 23 (2014).

[43] Jonathan Curtis, Gil Refael, and Victor Galitski, “Evanescent modes and step-like acoustic black holes,” *Annals of Physics* **407**, 148–165 (2019).

[44] P.-É. Larré, A. Recati, I. Carusotto, and N. Pavloff, “Quantum fluctuations around black hole horizons in Bose-Einstein condensates,” *Phys. Rev. A* **85**, 013621 (2012).

[45] Robert M. Wald, “The Thermodynamics of Black Holes,” *Living Reviews in Relativity* **4**, 6 (2001).

[46] Don N. Page, “Particle emission rates from a black hole: Massless particles from an uncharged, nonrotating hole,” *Phys. Rev. D* **13**, 198–206 (1976).

[47] Matt Visser, “Thermality of the Hawking flux,” *Journal of High Energy Physics* **2015**, 9 (2015).

[48] L. P. Pitaevskii and S. Stringari, *Bose-Einstein Condensation*, International Series of Monographs on Physics (Clarendon Press, Oxford, 2003).

[49] We verified that throughout our simulations, the relative (local) depletion $\delta n/n$ remains below 10 %, when taking experimentally realistic parameters [38].

[50] S. A. Fulling and S. N. M. Ruijsenaars, “Temperature, periodicity and horizons,” *Physics Reports* **152**, 135–176 (1987).

[51] D. Boiron, A. Fabbri, P.-É. Larré, N. Pavloff, C. I. Westbrook, and P. Ziń, “Quantum Signature of Analog Hawking Radiation in Momentum Space,” *Phys. Rev. Lett.* **115**, 025301 (2015).

[52] Guillaume Vergez, Ionut Danaila, Sylvain Auliac, and Frédric Hecht, “A finite-element toolbox for the stationary Gross-Pitaevskii equation with rotation,” *Computer Physics Communications* **209**, 144–162 (2016).

[53] C. Barceló, L. J. Garay, and G. Jannes, “Two faces of quantum sound,” *Phys. Rev. D* **82**, 044042 (2010).

[54] Cao C. Holanda Ribeiro and Daniel A. Turolla Vanzella, “Analogues of gravity-induced instabilities in anisotropic metamaterials,” *Phys. Rev. Research* **2**, 013281 (2020).
**SUPPLEMENTARY MATERIAL**

**Derivation of the Bogoliubov potential**

We start from the 3D GP equation (1) and the corresponding 3D BdG equation (2), and assume the external potential \( U(x) = U_{||}(x) + U_{\perp}(x_{\perp}) \) with

\[
U_{\perp}(x_{\perp}) = \frac{x_{\perp}^2}{2m}\ell_{\perp}^2.
\]

(S1)

We shall assume that the radial length scale provided by \( \ell_{\perp} \) is sufficiently small as to justify the adiabatic approximation, in which the radial degrees of freedom responds instantaneously to axial changes, as explored in [39, 40]. In simple terms, it consists in neglecting terms with \( \ell_{\perp} \partial_t \) and \( \ell_{\perp} \partial_x \) in the radial profile dynamics, in such a way that the validity of the model is restricted to low energy phenomena, like the one considered in this letter. Moreover, we shall take a step further in this approximation by showing that the radial degrees of freedom of the perturbations \( \delta \Phi \) are predicted to behave in a slightly different manner, which results in novel predictions. Let us start by analyzing the background field \( \Phi \). Under the trapping \( U_{\perp} \), we shall write

\[
\Phi_0(t, \mathbf{x}) = \frac{1}{\ell_{\perp}} \phi_{\perp}[\mathbf{x}_{\perp}; n(t, x)] \phi(t, x),
\]

(S2)

where, for mere convenience, \( \phi_{\perp} \) is normalized as \( \int d^2x_{\perp} |\phi_{\perp}|^2 = \ell_{\perp}^2 \), and the 1D particle density is given by \( n = |\phi|^2 \). Starting from this Ansatz, the adiabatic regime corresponds to the assumption that the trapping potential is strong enough as to force \( \phi_{\perp} \) to respond instantaneously to changes in \( n(t, x) \). Thus, upon its substitution on the 3D GP equation followed by an averaging over the radial degrees of freedom (\( \int d^2x_{\perp} \phi_{\perp}^* / \ell_{\perp} \)) results in the effective 1D GP equation

\[
i\partial_t \phi = \left[ -\frac{\partial_x^2}{2m} + U_{||} + \mu_{\phi}(n) \right] \phi,
\]

(S3)

where the density-dependent chemical potential \( \mu_{\phi}(n) \) is given by the smallest eigenvalue of

\[
\left( -\frac{\nabla_{\perp}^2}{2m} + U_{\perp} + \frac{ng}{\ell_{\perp}^2} |\phi_{\perp}|^2 \right) \phi_{\perp} = \mu_{\phi}(n) \phi_{\perp}.
\]

(S4)

We stress that the common folklore in this analysis then consists in approximating the eigenfunction \( \phi_{\perp} \) by some suitable choice independent of \( n \) and returning to the 1D GP equation, from which the perturbations of \( \phi \), and not \( \Phi \), are calculated. This step, however, leads to ill-defined correlations in some cases, because low energy excitations are highly dependent on it, as shown in the main text. The physical reason behind this is the assumption that the radial profile of the excitations is the same fixed background radial profile, which cannot be as the involved 3D dynamics are different. Fortunately, the same analysis is suitable for the field \( \delta \Phi \). Indeed, we shall assume that

\[
\delta \Phi(t, \mathbf{x}) = \frac{1}{\ell_{\perp}} \psi_{\perp}[\mathbf{x}_{\perp}; (\psi^*/\psi)(t, x)] \psi(t, x),
\]

(S5)

with the normalization \( \int d^2x_{\perp} |\psi_{\perp}|^2 = \ell_{\perp}^2 \), which implies that the smallness of \( \delta \Phi \) in comparison to \( \Phi \) is encapsulated in \( \psi \). We note the strange dependence on the field \( \psi \) is here kept to maintain consistence with the order we are working with, namely, \( |\psi|^2 \) is neglected in the BdG equation in comparison with the dominant contribution \( n \). However, it must depend on \( \psi \)'s phase through \( \psi^*/\psi \) as we shall see, and, as long as we restrict our analysis to low energy phenomena, i.e., small frequencies and wave vectors, we can neglect derivatives of \( \psi_{\perp} \) with respect to \( t \) and \( x \) in comparison with the radial derivatives.

Thus, we obtain, after substitution of our Ansatz in the BdG equation followed by an averaging over the radial degrees of freedom the effective 1D BdG equation

\[
i\partial_t \psi = \left[ -\frac{\partial_x^2}{2m} + U_{||} + \mu_{\psi}(n) \right] \psi + g_{3D}(n) \frac{\partial^2}{\ell_{\perp}^2} \alpha(\psi_{\perp}) \psi^*,
\]

(S6)

where we have defined

\[
\alpha(\psi_{\perp}) = \int d^2x_{\perp} \phi_{\perp}^2 |\psi_{\perp}|^2.
\]

(S7)
and the chemical potential $\mu_\psi(n)$ is the smallest (real) eigenvalue of
\[

\left(-\frac{\nabla^2}{2m} + U_\perp + \frac{2ng|\phi_\perp|^2}{\ell_\perp^2}\right)\psi_\perp + \frac{\psi_\perp^* g \phi_\perp^2}{\ell_\perp^4} \left[\phi_\perp^2 \psi_\perp^* - \alpha(\psi_\perp)\psi_\perp \right] = \mu_\psi(n)\psi_\perp.
\]

(S8)

We note how intricate this second order, integro-differential nonlinear (through $\alpha$) complex equation is. However, for our purposes it can be handled quite easily by using the variational approach under the usual $L^2$ scalar product, for if $\psi_\perp$ is a solution, then by integrating Eq. (S8) as $\int d^2x_\perp \psi_\perp^*$, it follows that $\mu_\psi$ is given by
\[

\mu_\psi(n) = \int \frac{d^2x_\perp}{\ell_\perp^2} \psi_\perp^* \left(-\frac{\nabla^2}{2m} + U_\perp + \frac{2ng|\phi_\perp|^2}{\ell_\perp^2}\right)\psi_\perp,
\]

(S9)

and thus it is real, as anticipated. Therefore, the smallest value of $\mu_\psi$ is bounded from below by the functional in Eq. (S9), which we shall use to estimate the radial profile $\psi_\perp$.

Let us see how subtle are these deviations with a simple example of the perturbations over homogeneous stationary gases. Suppose we prepare the 1D background gas as $\phi(t, x) = \exp(-i\mu t)\sqrt{n}$ with constants $\kappa$ and $n$, in which $n$ represents the particle density. Then the 1D GP equation fixes the axial potential $U_\parallel$ as
\[

U_\parallel = \mu - \mu_\psi(n).
\]

(S10)

As for the Bogoliubov excitations, we shall write $\psi = \exp(-i\mu t)f(t, x)$, and thus it follows from Eqs. (S6) and (S10) that
\[

i\partial_t f = \left[-\frac{\partial^2}{2m} + \frac{gn}{2}U_\parallel\right] f + gn(f + f^*),
\]

(S11)

where we have defined the effective 1D interaction strength $g = g_{3D}\alpha(\psi_\perp)/\ell_\perp^2$, and the Bogoliubov potential
\[

\frac{1}{2}U_B = \frac{\mu_\psi(n) - \mu_\psi(n)}{gn} - 1.
\]

(S12)

We note that when the radial profiles are equal, $U_B$ is zero, as follows from our equations (see [44]). However, if $U_B \neq 0$, it leaves an interesting fingerprint on the Bogoliubov dispersion relation. In fact, for an excitation of frequency $\omega$ and (dimensionless) wave vector $k$, we find that
\[

\frac{\omega}{ng} = \pm \left(\xi^2 k^2 + U_B\right)^{1/2} \left(1 + \frac{\xi^2 k^2 + U_B}{4}\right)^{1/2},
\]

(S13)

where $\xi^2 = 1/mng$ is the 1D healing length. This modified Bogoliubov dispersion relation has two important consequences. If $U_B < 0$, then Eq. (S13) admits complex solutions for $\omega$ for small real wave vectors $k$, characterizing an instability. On the other hand, if $U_B > 0$, this corresponds to the emergence of the frequency gap $\omega_0 = nq[U_B(1+U_B/4)]^{1/2}$, and the dispersion relation does not admit arbitrarily small frequency solutions for small real wave vectors as shown in Fig. S1, which is the main conclusion of our analysis as it is directly related to low energy phenomena. Thus we see that $U_B = 0$ is an unstable equilibrium configuration.

In order to illustrate how sensible is $U_B$ with respect to deviations between the radial profiles, let us consider the model in which $\phi_\perp(x_\perp) = \exp(-x_\perp^2/2\ell_\perp^2)/\sqrt{\pi}$, and
\[

\psi_\perp(x_\perp) = \frac{1}{\sqrt{\pi[1 + 2\beta(1 + \beta)]}} e^{-x_\perp^2/2\ell_\perp^2} \left(1 + \beta \frac{x_\perp^2}{\ell_\perp^2}\right).
\]

(S14)

For this choice of profiles, we have
\[

U_B = \frac{2\beta}{2 + \beta(2 + \beta)} \frac{\xi_{3D}^2}{\ell_\perp^2} \left\{\beta \left(8\pi - 3\frac{\ell_\perp^4}{\xi_{3D}^2}\right) - 2\frac{\ell_\perp^2}{\xi_{3D}^2}\right\}.
\]

(S15)

In Fig. S2 we present a region plot for $U_B > 0$. We see from it that for $\beta < 0$, $U_B > 0$. For instance, if $\ell_\perp/\xi_{3D} = 1$ and $\beta = -0.001$, we have $U_B \sim 0.002$.

The numerical estimates for $U_B$ presented in table I of the main text were simulated with the free c++-based numerical tool described in [52], that is able to solve for the 2D GP equation (S4) groundstate. We have used this solution to simulate the excitation radial profile in Mathematica. We present in Fig. S3 the simulated profiles corresponding to $\ell_\perp/\xi_{3D} = 1$, alongside the groundstate Gaussian for the free 2D harmonic oscillator. The deviations are clearly small, even outside the validity of $\ell_\perp/\xi_{3D} \ll 1$. They are however present and not negligible, as we have verified.
FIG. S1. Modified Bogoliubov dispersion relation for some values of $U_B$.

FIG. S2. Region plot for $U_B > 0$ (shaded region) corresponding to the potential in Eq. (S15).

FIG. S3. Normal sections of the radial profiles graphs, simulated for $\ell_\perp/\xi_{3D} = 1$. The effect of the linear potential term $2(\ell_\perp \phi_\perp/\xi_{3D})^2$ on the curve corresponding to the Bogoliubov excitations is more accentuated, resulting in a relevant deviation from the background profile.

The background solution

The dynamics of the background solution over which our analogue model is studied is ruled by the 1D GP equation (S3), and for the sake of completeness, we shall show in this section how an external potential proportional to $\partial_x \delta(x)$ mimics a boundary at $x = 0$ which produce the desired solution, namely, $\phi = \exp(-i\mu t + i\kappa x)\sqrt{n}$, with
\[ \sqrt{n} = \sqrt{n_u} \Theta(-x) + \sqrt{n_d} \Theta(x), \]  
and similarly for \( \kappa \), where \( \Theta \) is the unit step function. Suppose \( U_\parallel \) is such that
\[ U_\parallel = \alpha_n \partial_x \delta(x) + U_0 \Theta(x). \]  
Then substitution of the solution Ansatz into Eq. (S3) for \( x \neq 0 \) fixes the chemical potential and the constant \( U_0 \) as
\[ \mu = \frac{\kappa_0^2}{2m} + \mu_\phi(n_u), \]  
\[ U_0 = \mu - \frac{\kappa_0^2}{2m} - \mu_\phi(n_d), \]  
in terms of the goal parameters \( \kappa_{u,d}, n_{u,d} \), which are not fixed yet. The delta derivative potential is translated as the boundary conditions the expressions
\[ \sqrt{n} \psi \]  
we have \( \Phi(\kappa, x) \) and the latter giving rise to the continuity condition \( \phi(0^+) = (1 + m\alpha_n)\phi(0^-)/(1 - m\alpha_n), \) \( \partial_x\phi(0^+) = (1 - m\alpha_n)\partial_x\phi(0^-)/(1 + m\alpha_n) \) \[ \text{[41]}, \]  
the former implying that
\[ \alpha_n = \frac{1}{m} \frac{\sqrt{n_d} - \sqrt{n_u}}{\sqrt{n_d} + \sqrt{n_u}}, \]  
and the latter giving rise to the continuity condition \( \kappa_{u} n_{u} = \kappa_{d} n_{d} \).

### Quantization

The canonical quantization of the field \( \psi \) ruled by Eq. (S6) under the steplike horizon background solution is performed by solving for some set of the field modes which “diagonalizes” the theory, and the procedure is similar to known quantization schemes in most aspects \[ \text{[43, 44]} \]. Notwithstanding, we shall fill in the details here, as our approach provides a closed expression for the “scattering coefficients.” In order to set a clear starting point and make our analysis clear, it is very instructive to explore the fact that the 1D BdG equation we are going to quantize follows
\[ \text{approach provides a closed expression for the “scattering coefficients.”} \]

Finally, the field canonically conjugate to \( \psi \), which correspond to the freely fluctuating field, to obtain our 1D effective Lagrangian
\[ \text{L}_{\Phi} = \int d^3x \left( i\Phi^* \partial_t \Phi - \frac{\nabla^2 \Phi^2}{2m} - U|\Phi|^2 - \frac{\delta L}{\delta \Phi^2} \right), \]  
with field equation
\[ i\partial_t \psi = \left( -\frac{\partial^2}{2m} + \frac{\kappa^2}{2m} + gn \left( \frac{U_\parallel}{2} + 1 \right) + \alpha_n \partial_x \delta(x) \right) \psi + g \phi_0^2 \psi^*. \]  
Finally, the field canonically conjugate to \( \psi \) is given by \( \pi = \delta L/\delta (\partial_t \psi) = i\psi^* \), which, upon quantization, satisfy \( \psi(t, x), \pi(t, x') \) \[ i\delta(x - x'). \]

We readily see that \( \psi \) inherits the same discontinuity of the background at \( x = 0 \) caused by the delta derivative potential in Eq. (S22), and it is straightforward to solve for the field modes using as boundary conditions the expressions from the previous section, namely,
\[ \sqrt{n_u} \psi(0^+) = \sqrt{n_d} \psi(0^-), \]  
\[ \sqrt{n_d} \partial_x \psi(0^+) = \sqrt{n_u} \partial_x \psi(0^-). \]  
Note that in terms of the variable \( \zeta = \psi/\sqrt{n} \), these boundary conditions are equivalent to require \( \zeta \) and \( n\partial_x \zeta \) continuous at \( x = 0 \). A possible set of field modes for Eq. (S22) is conveniently found in terms of \( \zeta \) by setting \( \zeta(t, x) = \exp(-i\omega t + i\kappa x) f_\omega(x) + \exp(i\omega t + i\kappa x) h^*_\omega(x) \), in such a way that the auxiliary vector field \( \Psi_\omega = (f_\omega, h_\omega)^t \) is solution of
\[ \left[ \omega \sigma_3 - \frac{n_u g_n U_\parallel}{2} - n_u g_n \delta_n \delta_x \sigma_4 \right] \cdot \Psi_\omega = -\frac{1}{2\sqrt{m}} \partial_x (n\partial_x \Psi_\omega) - iv\partial_x \psi, \]  
\[ \text{[S25]} \]
where \(\sigma_i, i = 1, 2, 3\) are the Pauli matrices, and \(\sigma_4 \equiv 1 + \sigma_1\), where, for simplicity, we have set \(g = g_0\delta g\) and similarly for \(n\). We also have scaled \(ngU_B\) as \(n_\alpha g_\alpha U_B\), leaving to \(U_B\) the dependence on \(x\). Associated to Eq. (S25) is also a scalar product, obtained as follows. If \(\Psi_{\omega'}\) is another solution, by multiplying Eq. (S25) on the left by \(\Psi_{\omega'}\), the equation for \(\Psi_{\omega'}^\dagger\) on the right by \(\Psi_\omega\), and subtracting the resulting equations implies the identity

\[
(\omega' - \omega)n\Psi_{\omega'}^\dagger \cdot \sigma_3 \cdot \Psi_\omega = -\frac{\partial_x}{2m} \left[ n(\partial_x \Psi_{\omega'}^\dagger) \cdot \Psi_\omega - n\Psi_{\omega'}^\dagger \cdot \partial_x \Psi_\omega - 2imn \Psi_{\omega'}^\dagger \cdot \sigma_3 \cdot \Psi_\omega \right],
\]

which, upon integration over \(x\) under Dirichlet boundary conditions far from the boundary at \(x = 0\) produces the orthogonality relation

\[
(\omega' - \omega) \int dx n\Psi_{\omega'}^\dagger \cdot \sigma_3 \cdot \Psi_\omega = 0, \tag{S27}
\]

and identifies the Bogoliubov scalar product \(\langle \Psi_{\omega'} | \Psi_\omega \rangle \equiv \int dx n\Psi_{\omega'}^\dagger \cdot \sigma_3 \cdot \Psi_\omega [53]\). Our analysis here is not concerned in studying quantum fluctuations in the presence of instability [54], which amounts to say that the excitation spectrum is real. Accordingly, the general solution of the distributional equation (S27) is \(\langle \Psi_{\omega'} | \Psi_\omega \rangle \propto \delta(\omega - \omega')\), which is used to normalize the field modes as \(\langle \Psi_{\omega'} | \Psi_\omega \rangle = \pm \delta(\omega - \omega')\), where the minus sign defines the so-called negative norm modes [44]. Furthermore, by letting \(\omega' \to \omega\) in Eq. (S26) shows that the quantity inside the square brackets on the R.H.S. is independent of \(x\), which is commonly linked to current conservation [44].

We shall now explicitly solve for the field modes. It is helpful to work below with dimensionless quantities, where length and wavevector is scaled by \(\xi_u\) and \(1/\xi_u\), respectively, and \(E = \omega/\xi_u g_\alpha n\), such that

\[
\left[ E\sigma_3 - \frac{U_B}{2} - \delta_n \delta_g \sigma_4 \right] \cdot \Psi_\omega = -\frac{1}{2} \left( \partial_x + 2i \frac{m_n}{\delta_n} \sigma_3 \right) \cdot \partial_x \Psi_\omega \tag{S28}
\]

for \(x \neq 0\). Notice that Eq. (S28) possesses simple exponential solutions of the form \(\Psi_\omega(x) = \exp(ikx)\Phi_k^0\), for constant \(\Phi_k^0\), as long as the Bogoliubov dispersion relation

\[
\left( E - \frac{m_n}{\delta_n} k \right)^2 = (k^2 + U_B) \left( \delta_n \delta_g + \frac{k^2 + U_B}{4} \right), \tag{S29}
\]

is verified. Note that this degree four polynomial equation always possesses 4 solutions for a given \(\omega\), with the ones presented in the main text (see Fig. 1) entering the field modes. Moreover, we stress that the corresponding solutions are not field modes, but possible propagating channels that compose the mode, as it must be by definition a solution defined everywhere on the gas. Before presenting the exact solutions, let us discuss their general features. A field mode is indexed by an incoming signal and its reflected and evanescent parts. Outside the black hole (\(\bar{x} < 0\)), the incoming channel is denoted by \(k_n\), which is identified by the positive group velocity \(\xi_u d\omega/dk\). The channel indexed by \(k_2\) is evanescent, with negative imaginary part, and \(k_1\) is propagating or evanescent depending on \(\omega > \Omega^{(u)}\). Inside the black hole, we have two possible incoming channels. The first one is here indexed by \(p_n\) and it is always present for \(\omega > \Omega^{(d)}\). The second one appears when \(\Omega^{(H)} > 0\) and it is denoted by \(p_H\), where \(H\) stands for Hawking. This is the channel giving rising to the Hawking radiation in these systems. We also denote the other channels \(p_1\) and \(p_2\), which may be real or imaginary depending on the downstream parameters. For each incoming signal, the field mode is constructed from the various channels using the boundary conditions discussed earlier in this section, namely, \(\Psi_\omega\) and \(n_\alpha d\omega/dk\) are continuous at \(x = 0\).

We shall present the field modes in a compact manner. The normalization of each channel is not relevant, for the boundary conditions as well as the normalization condition \(\langle \Psi_{\omega'} | \Psi_\omega \rangle = \pm \delta(\omega - \omega')\) uniquely fix the solution up to an overall phase. However, a convenient channel normalization, i.e., for the solutions \(\Phi_k^0\), is

\[
\Phi_k^0 = \left| \frac{k^2 + U_B}{4\pi n V_g(k)(E - m_\omega k/\delta_n)(2E - m_\omega k/\delta_n(k^2 + U_B))} \right|^{1/2} \left( E - m_\omega k/\delta_n - (k^2 + U_B)/2 - \delta_n \delta_g \right), \tag{S30}
\]

where \(V_g(k) = c_t dE/dk\) is the group velocity. This choice amounts to fix the coefficient of the incoming channels as 1 and it is straightforward to verify that for real \(k\),

\[
\frac{1}{m \xi_u} \Phi_k^0 \cdot (k + m_\omega \sigma_3/\delta_n) \cdot \Phi_k^0 = \frac{1}{2\pi n} \text{sgn}[V_g(k)(E - m_\omega k/\delta_n)], \tag{S31}
\]
in terms of the sign function. We denote as

\[
\Psi^{(u)}_\omega = \begin{cases} 
\phi_{\xi}^0 + S^{(u)}_{\xi_1} e^{i k_1 \xi} \phi_{\xi_1}^0 + S^{(u)}_{\xi_2} e^{i k_2 \xi} \phi_{\xi_2}^0, & \bar{\xi} < 0, \\
S^{(u)}_{p_1} e^{i p_1 \xi} \phi_{p_1}^0 + S^{(u)}_{p_2} e^{i p_2 \xi} \phi_{p_2}^0, & \bar{\xi} > 0,
\end{cases}
\] (S32)

the u-modes and

\[
\Psi^{(j)}_\omega = \begin{cases} 
S^{(j)}_{\chi_1} e^{i k_1 \chi} \phi_{\chi_1}^0 + S^{(j)}_{\chi_2} e^{i k_2 \chi} \phi_{\chi_2}^0, & \bar{\chi} < 0, \\
S^{(j)}_{p_1} e^{i p_1 \chi} \phi_{p_1}^0 + S^{(j)}_{p_2} e^{i p_2 \chi} \phi_{p_2}^0, & \bar{\chi} > 0,
\end{cases}
\] (S33)

where \( j = d, H \) and \( \alpha_2 = p_m, \alpha_3 = p_H \), for the modes coming from within the black hole. The scattering coefficients are explicitly given by

\[
\begin{align*}
S^{(j)}_{p_1} &= -\frac{i \phi_{p_1}^{0,0} P_{k_1 k_2} (p_2) \sigma_2 P_{k_1 k_2} (\alpha_j) \phi_{\alpha_j}^{0,0}}{i \phi_{p_1}^{0,0} P_{k_1 k_2} (p_2) \sigma_2 P_{k_1 k_2} (\alpha_j) \phi_{\alpha_j}^{0,0} }, \\
S^{(j)}_{p_2} &= -\frac{i \phi_{p_1}^{0,0} P_{k_1 k_2} (p_2) \sigma_2 P_{k_1 k_2} (\alpha_j) \phi_{\alpha_j}^{0,0}}{i \phi_{p_1}^{0,0} P_{k_1 k_2} (p_2) \sigma_2 P_{k_1 k_2} (\alpha_j) \phi_{\alpha_j}^{0,0} }, \\
S^{(j)}_{k_1} &= \frac{i \phi_{k_1}^{0,0} P_{k_1 p_1 p_2} (k_2) \sigma_2 P_{p_1 p_2} (\alpha_j) \phi_{\alpha_j}^{0,0}}{i \phi_{k_1}^{0,0} P_{k_1 p_1 p_2} (k_2) \sigma_2 P_{p_1 p_2} (\alpha_j) \phi_{\alpha_j}^{0,0} }, \\
S^{(j)}_{k_2} &= \frac{i \phi_{k_1}^{0,0} P_{k_1 p_1 p_2} (k_2) \sigma_2 P_{p_1 p_2} (\alpha_j) \phi_{\alpha_j}^{0,0}}{i \phi_{k_1}^{0,0} P_{k_1 p_1 p_2} (k_2) \sigma_2 P_{p_1 p_2} (\alpha_j) \phi_{\alpha_j}^{0,0} }.
\end{align*}
\] (S34)

where \( t \) stands for (real) transpose, \( \Phi^{0}_m = -\Phi^{0}_m, \Phi^{0}_j = \Phi^{0}_j \) for \( j = 2, 3 \), and we have identified the three-parameter projection matrix

\[
P_{kp}(q) = 1 - \frac{\chi(q) i \phi_{k}^{0,0} \phi_{p}^{0,0} \sigma_2}{\chi(k) i \phi_{p}^{0,0} \phi_{k}^{0,0} \sigma_2} - \frac{\chi(q) i \phi_{p}^{0,0} \phi_{k}^{0,0} \sigma_2}{\chi(p) i \phi_{k}^{0,0} \phi_{p}^{0,0} \sigma_2},
\] (S35)

where \( \chi(k) = nk \).

These modes are normalized as \( \langle \Psi^{(j)}_\omega | \Psi^{(l)}_\omega \rangle = \delta_{ij} \delta(\omega - \omega') \), with \( j, l = u, d, H \), and \( \delta_j = 1 \) for \( j = u, d \) and \(-1 \) otherwise, i.e., the Hawking modes are negative norm modes. Finally, by defining \( f^{(j)}_\omega \equiv \sqrt{\lambda} \exp(i k \chi) \Psi^{(j)}_\omega \), and \( \Psi^{(j)}_\omega \) is the \( i \)-th component of \( \Psi^{(j)}_\omega \), we obtain the quantum field expansion (9) presented in the main text.

**Energy flux**

In this section we shall derive Eq. (10) from the main text. We start from the energy stored on the perturbations calculated from the Lagrangian (S21) to obtain the energy density

\[
\mathcal{H} = \frac{|\partial_x \psi|^2}{2m} + \frac{\kappa^2}{2m} + \frac{\alpha_n \partial_x \delta(x) + g n \left( \frac{U_B}{2} + 1 \right)}{2} \frac{1}{2} |\psi|^2 + \frac{g}{2} \left( \phi_0^2 \psi^* \psi + \phi_0^2 \psi^* \psi \right),
\] (S36)

which is easily shown to satisfy the continuity equation \( \partial_t \mathcal{H} = -\partial_x S \), in terms of the energy flux density

\[
S = -\frac{1}{2m} \left[ \frac{1}{2} \left( \partial_x \phi^* \right) \partial_x \phi + \left( \partial_x \psi^* \right) \partial_x \psi \right].
\] (S37)

Therefore, upon quantization, the vacuum expectation value \( \langle S \rangle \) far from the horizon, and deep into the upstream region gives us the information regarding the energy being expelled from the system. From the quantum field expansion in the main text (Eq. (9), we can show that for \( x \to -\infty \))

\[
\langle S \rangle = \frac{n_u}{m \xi_u} \int_{\Omega^{(u)}} d\omega \left[ (k_m - m_u) |\Phi_{k_m, u}^0|^2 + (k_1 - m_u) |\Phi_{k_1, u}^0|^2 + \Theta(\Omega^{(H)} - \omega) |S^{(H)}_{k_1} |^2 \phi_{k_1}^0 \cdot (k_1 + m_u \sigma_3) \cdot \Phi_{k_1}^0 \right].
\] (S38)

The second term inside the square brackets gives us Eq. (10) in the main text, in view of Eq. (S31). The first two terms cancel after integration, as they give rise to the energy flux when there is no boundary at \( x = 0 \).
Homogeneous density profile analogues

It should be stressed that the quantization performed thus far is not limited to the steplike horizon configuration. Indeed, our quantum field expansion includes not only a possible discontinuity in $g$ at $x = 0$, but also a discontinuity in the trapping length scale $\ell_\perp$ at $x = 0$, which increases the number of possible analogues modeled by the system. In this section we present two of such results for the analogue known as the homogeneous density profile configuration [27], which corresponds to a continuous particle density $n$ (and thus $\kappa$) and $g$ with a jumplike discontinuity at $x = 0$. For the sake of simplicity, we set $U_B$ constant along the gas, which implies that $\ell_\perp$ also possesses a jumplike discontinuity to account for the change in $g$. It turns out that all of our results presented in the main text for the steplike horizon remain qualitatively valid for this case as well, as presented in the following two figures. In particular, we notice

![](image1.png)

FIG. S4. Radiation spectrum for the homogeneous profile analogue. Here we set $m_d = 4m_u = 2$.

![](image2.png)

FIG. S5. Local quantum depletion for the homogeneous profile analogue. We set $U_B = 0.01$ in this simulation.

that the radiation spectrum for this analogue model, in comparison with the steplike horizon model, is more sensible to the nonvanishing of $U_B$, which leads to more noticeable deviations from a thermal spectrum.

Depletion power spectra for various nonradiating and radiating cases

From the depletion in Fig. 3, presented in the main text, we see that as the oscillatory pattern emerges inside the analogue black hole, the oscillation wavelength increases with $m_d$. This, in turn, implies that the position of the secondary peak of the depletion in Fourier space changes, as shown in Fig. S6. When $m_d \rightarrow \infty$, our simulations suggest the secondary peak merges with the main peak at $k = 0$. In Fig. S7, we show local depletion profiles from the same range of Mach numbers.
Nonradiating BHs

Radiating BHs

FIG. S6. Left: power spectrum of the depletion profiles presented in 3 of the main text, for nonradiating BHs. Right: power spectrum for radiating configurations \((m_d > 1.5)\). Recall that the parameters used here are \(U_{b,u} = 0.05, U_{b,d} = 0.01\); the system size is specified by \(-100\xi_u \leq x \leq 100\xi_u\).

FIG. S7. Local quantum depletion for downstream Mach numbers as indicated. Other parameters identical to Fig. S6.

Validity of Bogoliubov approach

We note from Fig. S7 that for \(m_d = 2\), we predict \(\xi_u \delta n\) assuming values near unity inside the black hole, meaning that we must have approximately one depleted particle per unit healing length \(\xi_u\) near these regions. Recall that Bogoliubov theory holds as long as \(\delta n \ll n\) and thus some estimates are in order. We shall use for guidance the experimental downstream density from [25], namely, \(n_d \sim 25 \mu \text{m}^{-1}\), and let us assume as a conservative estimate that \(\xi_u = 1 \mu \text{m}\) for the upstream healing length. Therefore, from Fig. S7, we maximally have \(\delta n/n \sim 0.04\) in the downstream region, supporting the validity of our analysis.