MAXIMAL REGULARITY AND GLOBAL EXISTENCE OF SOLUTIONS TO A QUASILINEAR THERMOELASTIC PLATE SYSTEM

IRENA LASIECKA
Department of Mathematics
University of Virginia
Charlottesville, VA 22903, USA

MATHIAS WILKE
Institut für Mathematik
Martin-Luther-Universität Halle-Wittenberg
06099 Halle, Germany

Dedicated to Jerry Goldstein on the occasion of his 70th birthday.

Abstract. We consider a quasilinear PDE system which models nonlinear vibrations of a thermoelastic plate defined on a bounded domain in $\mathbb{R}^n$. Well-posedness of solutions reconstructing maximal parabolic regularity in nonlinear thermoelastic plate is established. In addition, exponential decay rates for strong solutions are also shown.

1. Introduction

In this paper we study the existence and exponential stability of solutions to a quasilinear system arising in the modeling of nonlinear thermoelastic plates. The mathematical analysis of thermoelastic systems has attracted a lot of attention over the years. An array of new and fundamental results in the area of wellposedness and stability of solutions to both linear and nonlinear thermoelasticity have been contributed to the field (see [10, 11, 23, 15, 16, 48, 49] and references therein).

The focus of this paper is on thermoelastic plates and associated uniform stability issues. This particular class of problems has received considerable attention in recent years, particularly in the context of some new developments in control theory. Questions such as exponential stability, controllability, observability, unique continuation have been asked and partially answered for both linear and nonlinear plates (see [28] and references therein). While there is at present a vast literature dealing with well-posedness and stability of linear and semilinear thermoelastic equations (see above), the treatment of quasilinear and fully nonlinear models defined on multidimensional domains is much more subtle and requires different mathematical approaches. This paper deals with global and smooth solutions defined for small initial data.

2010 Mathematics Subject Classification. Primary: 74F05; Secondary: 35B30, 35B40, 74H40.

Key words and phrases. Quasilinear thermoelastic plates, existence and uniqueness of strong solutions, maximal regularity, exponential decay.

The research of I. Lasiecka has been partially supported by DMS-NSF Grant Nr 0606882. M. Wilke expresses his thanks for hospitality to the Department for Mathematics at the University of Virginia.
The equations we consider arise from a model that takes into account the coupling between elastic, magnetic and thermal fields in a nonlinear elastic plate model (see [1, 9, 22, 39, 19]). In non-dimensional form, the equations we consider are given below in (1)-(3). The nonlinearity arises from the nature of the magnetoelastic material, owing to a nonlinear dependence between the intensities of the deformation and stress. We also assume that the material nonlinearity is cubic, as in the original plate model [19]. However, the arguments provided depend neither on structure of nonlinearity nor on the order near the origin. We put this generalization in evidence by considering a more general system under the sole Assumption 1 (see below).

Let $\Omega$ be a bounded domain of $\mathbb{R}^n$, $n \in \mathbb{N}$, with boundary $\partial \Omega \in C^2$. Consider the system

\begin{align*}
(1) & \begin{cases}
W_{tt} + \Delta^2 W - \Delta \Theta + a \Delta((\Delta W)^3) = 0 \\
\Theta_t - \Delta \Theta + \Delta W_t = 0
\end{cases} \quad \text{in } \Omega \times (0, T) \\
(2) & W = \Delta W = \Theta = 0 \quad \text{on } \partial \Omega \times (0, T) \quad \text{(Boundary Conditions)} \\
(3) & \begin{cases}
W(x, 0) = f(x) \quad (x \in \Omega) \\
W_t(x, 0) = g(x) \quad (x \in \Omega) \\
\Theta(x, 0) = h(x) \quad (x \in \Omega)
\end{cases} \quad \text{(Initial Conditions)}.
\end{align*}

We assume that the material constant $a$ is positive.

In fact, in what follows we will be able to obtain results for a more general version of equation (1), where the cubic nonlinearity is replaced by a more general nonlinear function of superlinear growth. More specifically, we consider

\begin{align*}
(4) & \begin{cases}
W_{tt} + \Delta^2 W - \Delta \Theta + a \Delta(\phi(\Delta W)) = 0 \\
\Theta_t - \Delta \Theta + \Delta W_t = 0
\end{cases} \quad \text{in } \Omega \times (0, T)
\end{align*}

where the function $\phi$ satisfies:

**Assumption 1**: $\phi \in C^3(\mathbb{R})$, $\phi(0) = \phi'(0) = \phi''(0) = 0$.

### 2. Main Results

#### 2.1. Notation.

Let $J = (0, T)$, where $T$ may be finite or infinite. For $p \in (1, \infty)$ we introduce the following function spaces:

- $W^{1,0}_p(\Omega) := C_0^\infty(\Omega)^{W^1_p}$.
- $X_0 := [L_p(\Omega)]^3$, $X_1 := [W^2_p(\Omega) \cap W^1_{p,0}(\Omega)]^3$.
- $L_p(J, X_0) = [L_p(J, L_p(\Omega))]^3$, $L_p(J, X_1) = [L_p(J, W^2_p(\Omega) \cap W^1_{p,0}(\Omega))]^3$, $W^1_p(J, X_0) = [W^1_p(J, L_p(\Omega))]^3$.
- $X_p = (X_0, X_1)_{1-\frac{1}{p},p} = \begin{cases} [W^{2(1-1/p)}_p(\Omega)]^3, & \text{if } 1 < p < 3/2, \\
\{u \in [W^{2(1-1/p)}_p(\Omega)]^3 : u|_{\partial \Omega} = 0\}, & \text{if } p > 3/2.
\end{cases}$

For $\mu \in (1/p, 1]$ we set

- $L_{p,\mu}(J, X_0) := \{u : J \to X_0 : [t \mapsto t^{1-\mu} u(t)] \in L_p(J; X_0)\}$,
- $E_{0,\mu}(J) := L_{p,\mu}(J; X_0)$,
- $E_{1,\mu}(J) := W^1_{p,\mu}(J; X_0) \cap L_{p,\mu}(J; X_1)$.
Taking origin and provide trade-off between singularity and additional fractional regularity.

1. one obtains 'classical' $L^p$ of the framework introduced in [42, 44].

Remark 1. The result obtained in Theorem 2.1 uses weighted norms $X_{p,\mu}$. For $\mu = 1$ one obtains 'classical' $L_p$ estimates. These norms account for singularity at the origin and provide trade-off between singularity and additional fractional regularity. Taking $p \to \infty$ allows to obtain “almost” $L_\infty$-estimates. This is reminiscent to some of the framework introduced in [43, 44].

2.2. Formulation of the result.

Theorem 2.1. Let $n \in \mathbb{N}$, $p > 1 + \frac{n}{2}$ and $\mu \in (\frac{2+p}{2p}, 1]$. Assume that $\phi$ satisfies Assumption 1. With reference to the problem (3)-(4) let

$$x(0) := (\Delta W(0), W_t(0), \Theta(0)) \in X_{p,\mu}.$$ 

Then the following assertions hold.

1. There exists $\rho > 0$ such that for all $|x(0)|_{X_{p,\mu}} \leq \rho$ and for every $T > 0$ there is a unique solution $x(t) = (\Delta W(t), W_t(t), \Theta(t))$ of (3)-(4) with maximal parabolic regularity

$$(\Delta W, W_t, \Theta) \in [L_{p,\mu}(J; W^2_p(\Omega))]^3 \cap [W^1_{p,\mu}(J, L_p(\Omega))]^3 \cap [BUC(J, W^{2p-2/2p}_p(\Omega))]^3.$$ 

2. If in addition $p > (n+4)/2$, $\mu \in (\frac{2+p}{4p} + \frac{1}{2}, 1]$ and $\phi'(s) \geq 0$ for all $s \in \mathbb{R}$, then the same conclusion holds with no restriction on the size of initial data, provided $T > 0$ is sufficiently small.

3. There exists $\omega > 0$ and a constant $C > 0$ such that for $|x(0)|_{X_{p,\mu}} \leq \rho$ the following exponential estimate holds:

$$|x(t)|_{X_{p,\mu}} \leq Ce^{-\omega t}|x(0)|_{X_{p,\mu}}, \quad t \geq 0.$$ 

4. For all $\sigma > 0$ there exists $\omega > 0$ (independent of $\sigma$) and a constant $C(\sigma) > 0$ such that for $|x(0)|_{X_{p,\mu}} \leq \rho$ the following exponential decay rate holds:

$$|x(t)|_{X_p} \leq C(\sigma)e^{-\omega t}|x(0)|_{X_{p,\mu}}, \quad t \geq \sigma,$$

where $C(\sigma) \to \infty$ as $\sigma \to 0$.

By specializing $\phi$ to $\phi(s) = s^3$ we obtain at once

Corollary 1. The result stated in Theorem 2.1 applies to the original model (1).
2.3. Comments.

(1) It is interesting to contrast the result of Theorem 2.1 with the one of Theorem 1.3 and Theorem 1.5 of [32] obtained for the original model (1)-(3) within the framework of $L^2$-theory. More specifically, in [32] global existence and exponential decay rates are shown in the so called finite energy which is $[L^2(\Omega)]^3$ for the variable $x(t)$. There is no uniqueness result obtained within this framework. This, of course, raises a familiar dilemma of discrepancy between uniqueness and globality of solutions. It is an interesting problem that is still open to the best knowledge of the authors.

(2) Unique and ”small” solutions for equations (1)-(3) have also been obtained in [32] within the framework of maximal regularity with the spaces $C^1(\bar{\Omega})$. However, the above framework leads to the ”loss” of incremental differentiability with respect to the initial data. This drawback is no longer present in Theorem 2.1 where the space $X_{p,\mu}$ is invariant under the flow.

(3) One can consider more general structure of linear matrix operator in (1) as long as it is associated with an exponentially stable semigroup. This is to say that the coefficients of matrix $M$ introduced in (8) may be arbitrary as long as all eigenvalues of $M$ have positive real parts.

We shall next address the issue of higher regularity of solutions given by Theorem 2.1. Among other things it will be shown below that under the additional assumption that $\phi \in C^\infty$, the solution $x(t)$ is infinitely many times differentiable in time away from $t = 0$.

**Theorem 2.2.** Under the Assumptions of Theorem 2.1 and with $\phi \in C^\infty(\mathbb{R})$ we obtain for all $k \in \mathbb{N}$ that $x^{(k)} \in e^{-\omega t}C^\infty_0(J_\sigma, X_p)$, for each $\sigma > 0$, where $J_\sigma = [\sigma, \infty)$. In addition, if $[s \mapsto \phi(s)]$ is real analytic, then $[0, \infty) \ni t \mapsto x(t)$ is real analytic with values in $X_p$.

3. Proof of Theorem 2.1

The proof employs techniques developed in the context of abstract parabolic problems and related maximal regularity.

3.1. Abstract parabolic problems and maximal regularity. Let $X$ be a given Banach space and $J = [0, T]$ or $J = [0, \infty)$ and let $A : D(A) \subset X \to X$ be a closed operator that is also densely defined. Consider an abstract Cauchy problem

$$u_t = Au(t) + f(t), \ t \in J, \ u(0) = u_0.$$  

**Definition 3.1.** We say that $A$ admits maximal $L_p$-regularity on $J$ with some $p \in (1, \infty)$ iff for each $f \in L_p(J; X)$ and $u_0 = 0$, problem (5) admits a unique solution $u \in \mathcal{E}(J) := W^1_p(J; X) \cap L_p(J; D_A)$, where $D_A := (D(A), | \cdot |_A)$.

The space $\mathcal{E}(J)$ is continuously embedded into $BUC(J; \text{tr} \mathcal{E})$ where the trace space $\text{tr} \mathcal{E}$ is defined as

$$\text{tr} \mathcal{E} = D_A(1 - 1/p, p) = (X, D_A)_{1-1/p, p}$$

and $(\cdot, \cdot)_{0,p}$ denotes the real interpolation method.

**Definition 3.2.** We say that the abstract inhomogeneous Cauchy problem admits maximal $L_p$-regularity, if the solution map

$$(f, u_0) \mapsto u$$
is a topological isomorphism
\[ L^p(J; X) \times \text{tr} \mathbb{E} \to \mathbb{E}(J) \subset BUC(J; \text{tr} \mathbb{E}) \]

In particular, the following estimate holds for operators \( A \) with maximal \( L^p \) regularity:
\[ |u|_{\mathbb{E}(J)} \leq M(J)(|f|_{L^p(J; X)} + |u_0|_{\text{tr} \mathbb{E}}). \]

3.2. Setting up (2)-(4) as an abstract parabolic problem. We define \([41, 37]\)
\[ U := W_t, \ Z := \Delta W \]
and set \( x := (Z, U, \Theta) \).

The differential operator \( \Delta \), equipped with zero Dirichlet boundary conditions,
generates an analytic semigroup on \( L^p(\Omega) \). With the above notation, the original
system can be written in the following operator form:
\[ x_t = \Delta \left[ \begin{array}{ccc}
0 & 1 & 0 \\
-1 & 0 & 1 \\
0 & -1 & 1
\end{array} \right] x - a \Delta \left[ \begin{array}{c}
0 \\
\phi(Z) \\
0
\end{array} \right]. \]

Denoting
\[ A := \Delta \left[ \begin{array}{ccc}
0 & 1 & 0 \\
-1 & 0 & 1 \\
0 & -1 & 1
\end{array} \right] = \Delta M \]
where \( M \) is a \( 3 \times 3 \) nonsingular matrix with eigenvalues having positive real parts. It
is easily seen that \( A \) is the generator of an exponentially stable analytic semigroup
\( e^{At} \) on \( X_0 := L^p(\Omega) \times L^p(\Omega) \times L^p(\Omega) \) and (7) can be rewritten as
\[ x_t = Ax + AF(x) \]
where
\[ F(x) := a \left[ \begin{array}{c}
\phi(Z) \\
0 \\
0
\end{array} \right]^\top. \]

Equation (11) is a nonlinear abstract parabolic system defined on \( X_0 \). The nonlinearity enters via
the generator \( A \), and so solvability of the system must depend on “maximal regularity” properties [12, 42, 47]. Since maximal regularity does not hold within the context of the \( L^\infty([0, T]; X_0) \)-topology [42], one should consider the
problem within the framework of \( L^p \)-spaces.

3.3. Representation as a quasilinear abstract parabolic system. Rewriting
\[ \Delta \phi(u) = \phi'(u)\Delta u + \phi''(u)|\nabla u|^2, \]
we obtain from (9) that
\[ x_t = Ax - a \left[ \begin{array}{c}
0, \phi'(Z)\Delta Z + \phi''(Z)|\nabla Z|^2, 0
\end{array} \right]^\top. \]

Denoting
\[ A(x) := A - a \left[ \begin{array}{ccc}
0 & 0 & 0 \\
\phi'(Z)\Delta & 0 & 0 \\
0 & 0 & 0
\end{array} \right] = A + B(Z), \]
leads to the consideration of a quasilinear system of the form:
\[ x_t = A(x)x + f(x), \]
where
\[ f(x) \equiv -a \left[ \begin{array}{c}
0, \phi''(Z)|\nabla Z|^2, 0
\end{array} \right]^\top. \]
Equation (11) is a quasilinear abstract parabolic system. Since $A = M\Delta$ where $M$ is a real valued $3 \times 3$ matrix with eigenvalues possessing positive real parts, the operator $A$ has maximal parabolic regularity when considered on the space $L_p(J; X_0)$ (see e.g. [14]). The interval $J$ can be extended to the positive real axis due to exponential stability of $e^{At}$. This of course implies that $A(0) = A$ enjoys maximal parabolic regularity on $J = (0, \infty) = \mathbb{R}_+$. By [47] the operator $A$ has the property of maximal parabolic regularity in the weighted $L_p$-spaces

$$L_{p,\mu}(J; X_0) := \{ u : J \to X_0 : [t \mapsto t^{1-\mu}u(t)] \in L_p(J; X_0) \},$$

where $\mu \in (1/p, 1]$. In particular, in [47] the authors have shown that the problem

$$v_t = Av + f, \quad v(0) = v_0$$

has a unique solution

$$v \in W^{1,1}_{p,\mu}(J; X_0) \cap L_{p,\mu}(J; X_0) =: \mathcal{E}_{1,\mu}(J)$$

if and only if $f \in L_{p,\mu}(J; X_0) =: \mathcal{E}_{0,\mu}(J)$ and

$$v_0 \in X_{p,\mu} := (X_0, D(A))_{\mu-1/p, p}$$

$$\begin{cases} 
[\omega_p^{2(\mu-1/p)}(\Omega)]^3, & \text{if } 1 < \mu p < 3/2, \\
\{ u \in [\omega_p^{2(\mu-1/p)}(\Omega)]^3 : u|_{\partial\Omega} = 0 \}, & \text{if } \mu p > 3/2.
\end{cases}$$

Moreover the estimate

$$|v|_{\mathcal{E}_{1,\mu}(J)} \leq C(|f|_{\mathcal{E}_{0,\mu}(J)} + |v_0|_{X_{p,\mu}})$$

holds for some constant $C > 0$.

Let $s(A) < 0$ be the spectral bound of $A$ and let $f \in e^{-\omega}L_{p,\mu}(J; X_0)$ as well as $v_0 \in X_{p,\mu}$ be given. Consider the problem

(12) $$v_t = Av + f, \quad v(0) = v_0$$

in $e^{-\omega}L_{p,\mu}(J; X_0)$. The scaled function $u(t) = e^{\omega t}v(t)$ then solves the problem

(13) $$u_t = (A + \omega)u + e^{\omega t}f, \quad u(0) = v_0.$$ 

Note that $s(A + \omega) = s(A) + \omega < 0$ if $\omega \in [0, -s(A))$. Since by assumption $e^{\omega t}f \in L_{p,\mu}(J; X_0)$ and $v_0 \in X_{p,\mu}$ it follows that there exists a unique solution $u \in \mathcal{E}_{1,\mu}(J)$ of (13). But this in turn implies that there exists a unique solution $v \in e^{-\omega}\mathcal{E}_{1,\mu}(J)$ of problem (12) satisfying the estimate

$$|v|_{e^{-\omega}\mathcal{E}_{1,\mu}(J)} \leq C(|f|_{e^{-\omega}\mathcal{E}_{0,\mu}(J)} + |v_0|_{X_{p,\mu}}).$$

In other words we have shown that the operator $A$ has maximal parabolic regularity in the weighted spaces $e^{-\omega}L_{p,\mu}(J; X_0)$ as long as $\omega \in [0, -s(A))$ and $\mu \in (1/p, 1]$.

The above allows to consider system (11) within this maximal regularity framework. In order to be able to use maximal regularity theory we need to verify several assumptions regarding the operator $A(x)$ and the forcing term $f(x)$. This is done below.
3.4. Supporting estimates. We shall present several estimates which will be used later for the proof of main theorems.

Lemma 3.3. Let \( p > \frac{n+2}{2p}, \mu \in (\frac{n+2}{2p}, 1] \) and \( \omega \geq 0 \). Then

1. The map \((V, x) \mapsto \phi'(V)\Delta x\) takes
\[
e^{-\omega}BUC(J, W_p^{2(\mu-1)/p}(\Omega)) \times e^{-\omega}L_{p,\mu}(J, X_1) \rightarrow e^{-\omega}L_{p,\mu}(J, X_0)).
\]

2. The map \((V, Z) \mapsto \phi''(V)(\nabla V \cdot \nabla Z)\) takes
\[
e^{-\omega}BUC(J, W_p^{2(\mu-1)/p}(\Omega)) \times e^{-\omega}L_{p,\mu}(J, W_p^2(\Omega)) \rightarrow e^{-\omega}L_{p,\mu}(J, L_p(\Omega))
\]

Proof. (1) For \( p > \frac{n+2}{2p} \) and \( \mu \in (\frac{n+2}{2p}, 1] \) one has \( 2(\mu - 1/p) - \frac{n}{p} > 0 \), hence
\[
W_p^{2(\mu-1)/p}(\Omega) \rightarrow L_\infty(\Omega).
\]
Therefore \( \phi'(V) \) is a multiplier on \( e^{-\omega}L_{p,\mu}(J, X_0) \). This along with the boundedness of \( \Delta : X_1 \rightarrow X_0 \) proves the claim.

(2) Since we already know that \( \phi''(V) \) is in \( e^{-\omega}BUC(J, L_\infty(\Omega)) \) it suffices to analyze the mapping \((V, Z) \mapsto \nabla V \cdot \nabla Z\). Our aim is to show that
\[
\nabla V \cdot \nabla Z \in e^{-\omega}L_{p,\mu}(J, L_p(\Omega))
\]
for \( V \in e^{-\omega}BUC(J, W_p^{2(\mu-1)/p}(\Omega)) \) and \( Z \in e^{-\omega}L_{p,\mu}(J, W_p^2(\Omega)) \), or alternatively
\[
\nabla V \in e^{-\omega}BUC(J, W_p^{2(\mu-1)/p-1}(\Omega)), \text{ and } \nabla Z \in e^{-\omega}L_{p,\mu}(J, W_p^1(\Omega))
\]

Applying Hölder’s inequality with \( r, \tilde{r} \) exponents yields
\[
\int_0^T \int_{\Omega} e^{\omega pt} |\nabla Z|^p |\nabla V|^{p(1-\mu)p} dx \, dt \leq
\]
\[
\leq |\nabla V|_{L_\infty(J, L_{p,\mu}(\Omega))}^p \int_0^T e^{\omega pt} |\nabla Z|_{L_{p,\mu}(\Omega)}^{p(1-\mu)p} \, dt.
\]
The choice of Hölder’s exponent will depend on the relation between \( p \) and \( n \). Since \( \mu > \frac{n+2}{2p} \), Sobolev’s embeddings imply
\[
W_p^{2\mu-1-2/p}(\Omega) \hookrightarrow L_n(\Omega)
\]
Moreover
\[
W_p^1(\Omega) \hookrightarrow \begin{cases} L_{np/(n-p)}(\Omega) & p < n \\ L_{\infty}(\Omega) & p > n \\ L_q(\Omega), q \in [1, \infty) & p = n \end{cases}
\]
If \( p < n \), we set \( r = n/(n-p) \) and \( \tilde{r} = n/p \). Conversely, if \( p > n \) then we choose \( r = \infty \) and \( \tilde{r} = 1 \).

In case \( p = n \) we use (17), (18) and the strict inequality for \( \mu \). This yields \( W_n^{2\mu-1-2/n}(\Omega) \hookrightarrow L_{n+\varepsilon}(\Omega) \) for a sufficiently small \( \varepsilon > 0 \). Defining \( \tilde{r} := (n+\varepsilon)/n > 1 \) and \( q = pr = p\tilde{r}/(\tilde{r} - 1) \) we finally obtain the desired estimate
\[
\int_0^T \int_{\Omega} e^{\omega pt} |\nabla Z|^p |\nabla V|^{p(1-\mu)p} dx \, dt \leq |V|_{L_{np/(n-p)}(J, W_p^{2(\mu-1)/p}(\Omega))}^p \int_0^T e^{\omega pt} |Z|_{W_p^2(\Omega)}^{p(1-\mu)p} \, dt,
\]
valid for all \( p > 1 + n/2 \) and \( \mu \in (\frac{n+2}{2p}, 1] \).

3.5. Solvability of a linear non-autonomous auxiliary problem. We begin with an auxiliary lemma which provides solvability for the linear equation with variable time and space coefficients. The coefficients are assumed to be sufficiently smooth (in line with maximal parabolic regularity) and also of sufficiently small variation. The corresponding result is given below.

Lemma 3.4. Let \( p > \frac{n+2}{2} \), \( \mu \in (\frac{n+2}{2p}, 1] \), \( \omega \in [0, -s(A)) \) and \( V \in e^{-\omega}BUC(J; W^{2, 1}_p(\Omega)) \) such that \( \|V\|_{e^{-\omega}BUC(J; W^{2, 1}_p)} \leq \rho \). Then there exists \( \rho_0 > 0 \) such that for all \( \rho \in (0, \rho_0) \) the linear problem

\[
x_t = A(V)x + f(V, x), \quad x(0) = x_0 \in X_{p, \mu}
\]

with

\[
A(V) = A + B(V), \quad B(V) = -a \begin{bmatrix} 0 & 0 & 0 \\ \phi'(V) \Delta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]

\[
f(V, x) = -a[0, \phi''(V)\nabla V \cdot \nabla Z, 0]^T
\]

has a unique solution \( x = (Z, U, \theta) \in e^{-\omega}E_{1, \mu}(J) \) which satisfies the estimate

\[
|z|_{e^{-\omega}E_{1, \mu}} \leq |C(\rho_0) + c||x_0||_{X_{p, \mu}}.
\]

where \( C(\rho_0) \to 0 \) when \( \rho_0 \to 0 \).

Proof. We first solve the problem

\[
v_t = Av, \quad v(0) = x_0
\]

in \( e^{-\omega}E_{0, \mu}(J) \). This yields a solution \( v = e^{At}x_0 \in e^{-\omega}E_{1, \mu}(J) \) satisfying the estimate

\[
|v|_{e^{-\omega}E_{1, \mu}} \leq C|x_0||_{X_{p, \mu}}
\]

Our next step is to homogenize the equation with respect to the initial data. For this we introduce change of variable \( w := x - v \), so that \( w|_{t=0} = 0 \). Then the sought after solution \( x \) can be expressed as \( x := w + v \) where \( w \) solves

\[
w_t = A(V)w + f(V, w) + g, \quad w(0) = 0,
\]

with \( g := -B(V)v - f(V, v) \in e^{-\omega}E_{0, \mu}(J) \) being a given function. The regularity \( g \in e^{-\omega}E_{0, \mu}(J) \) follows directly from Lemma 3.3. Thus, our goal is reduced to establishing well-posedness of (21). Writing \( A(V) = A + B(V) \), where \( A \) has maximal parabolic regularity in \( e^{-\omega}E_{0, \mu}(J) \), we may rewrite linear equation in \( w \) given in (21) as

\[
w = (\partial_t - A)^{-1}[B(V)w + f(V, w) + g]
\]

in the space \( e^{-\omega}E_{1, \mu}(J) \). By Lemma 3.3 and maximal parabolic regularity of \( A \), which then implies invertibility of \( (\partial_t - A)^{-1} \) from \( e^{-\omega}E_{0, \mu} \) into \( e^{-\omega}E_{1, \mu} \), we obtain
where $s > 0$ by the assumption imposed on $\phi$. Therefore, if $\rho \in (0, \rho_0)$ and $\rho_0 > 0$ is sufficiently small, a Neumann series argument yields the statement. Recall that equation for $w$ is linear ($w \mapsto f(V, w)$ is also linear). The above and maximal regularity imply the estimate for $w$

$$|w|_{e^{-\omega} E_{1, \mu}} \leq C|g|_{e^{-\omega} E_{0, \mu}} \leq C\rho^s|v|_{e^{-\omega} E_{1, \mu}} \leq C\rho^s|x_0|_{X_{p, \mu}}$$

The above estimate along with the estimate in (20) leads to the final conclusion in the Lemma.

### 3.6. Analysis of nonlinear equation and completion of the proof.

We shall apply Banach’s fixed point theorem. Let $\rho_0 > 0$ from the preceding lemma and

$$\mathcal{W} := \{W \in e^{-\omega} BUC(J; X_{p, \mu}) : W(0) = x_0 \text{ and } |W|_{e^{-\omega} BUC(J; X_{p, \mu})} \leq \rho \},$$

$\rho \in (0, \rho_0)$. For $W = (W_1, W_2, W_3) \in \mathcal{W}$ define $T(W) = x$ to be the unique solution of (19), where $V = W_1 \in e^{-\omega} BUC(J; \bar{X}_{p, \mu})$ and $\bar{X}_{p, \mu} := (L_p(\Omega), D(\Delta_D))_{1-1/p, \mu}$.

Note that $T$ is well-defined by Lemma 3.4 and we have the estimate

$$|T(W)|_{e^{-\omega} E_{1, \mu}} = |x|_{e^{-\omega} E_{1, \mu}} \leq |C(\rho_0) + c|x_0|_{X_{p, \mu}}.$$ 

From now on we assume that $|x_0|_{X_{p, \mu}} \leq \delta$. This yields

$$|T(W)|_{e^{-\omega} BUC(J; X_{p, \mu})} \leq M|T(W)|_{e^{-\omega} E_{1, \mu}} \leq M(C(\rho_0) + c)\delta.$$ 

Here $M \geq 1$ denotes the embedding constant from $e^{-\omega} E_{1, \mu} \hookrightarrow e^{-\omega} BUC(J; X_{p, \mu})$. Therefore, if $\delta = \rho/(M(C(\rho_0) + c))$, it follows that $T(\mathcal{W}) \subset \mathcal{W}$.

We shall now show that $T$ is a contraction on $\mathcal{W}$. Let $W, \bar{W} \in \mathcal{W}$ and $x = T(W)$, $\bar{x} = T(\bar{W})$. By the proof of Lemma 3.4 we have

$$x - \bar{x} = (\partial_t - A)^{-1}[B(V)x - B(\bar{V})\bar{x} + f(V, x) - f(\bar{V}, \bar{x})],$$

since $(x - \bar{x})(0) = 0$. It follows that

$$|x - \bar{x}|_{e^{-\omega} E_{1, \mu}} \leq C(|B(V)x - B(\bar{V})\bar{x}|_{e^{-\omega} E_{0, \mu}} + |f(V, x) - f(\bar{V}, \bar{x})|_{e^{-\omega} E_{0, \mu}}).$$

For the first term on the right side we estimate as follows.

$$|B(V)x - B(\bar{V})\bar{x}|_{e^{-\omega} E_{0, \mu}} \leq |B(V)(x - \bar{x})|_{e^{-\omega} E_{0, \mu}} + |(B(V) - B(\bar{V}))\bar{x}|_{e^{-\omega} E_{0, \mu}}$$

$$\leq C\rho^s|x - \bar{x}|_{e^{-\omega} E_{1, \mu}} + \rho|V - \bar{V}|_{e^{-\omega} BUC(J; \bar{X}_{p, \mu})}|\Delta\bar{x}|_{e^{-\omega} E_{0, \mu}}$$

$$\leq C\rho^s|x - \bar{x}|_{e^{-\omega} E_{1, \mu}} + \rho|V - \bar{V}|_{e^{-\omega} BUC(J; \bar{X}_{p, \mu})}|\bar{x}|_{e^{-\omega} E_{1, \mu}}$$

$$\leq C\rho^s|x - \bar{x}|_{e^{-\omega} E_{1, \mu}} + \rho^2|V - \bar{V}|_{e^{-\omega} BUC(J; \bar{X}_{p, \mu})}$$

$$\leq C\rho^s|x - \bar{x}|_{e^{-\omega} E_{1, \mu}} + \rho^2|W - \bar{W}|_{e^{-\omega} BUC(J; \bar{X}_{p, \mu})}.$$
for some $s > 0$, since $|V|_{e^{-\omega}BUC(J;\tilde{X}_{p,\mu})}, |\tilde{V}|_{e^{-\omega}BUC(J;\tilde{X}_{p,\mu})}, |\tilde{x}|_{e^{-\omega}E_{1,\mu}} \leq \rho$ and $\tilde{X}_{p,\mu} \hookrightarrow L_\infty(\Omega)$. In a similar way we obtain

$$|f(V, x) - f(\tilde{V}, \tilde{x})|_{e^{-\omega}E_{0,\mu}} \leq C\rho^s (|x - \tilde{x}|_{e^{-\omega}E_{1,\mu}} + |W - \tilde{W}|_{e^{-\omega}BUC(J;X_{p,\mu})}),$$

for some $s > 0$, by Assumption 1. Since

$$|T(W) - T(\tilde{W})|_{e^{-\omega}BUC(J;X_{p,\mu})} \leq M|T(W) - T(\tilde{W})|_{e^{-\omega}E_{1,\mu}} = M|x - \tilde{x}|_{e^{-\omega}E_{1,\mu}},$$

it follows that $T$ is a strict contraction on $W$ provided that $\rho > 0$ is sufficiently small. The contraction mapping principle yields a unique fixed point $x_\ast \in W$ of $T$, i.e. $T(x_\ast) = x_\ast$. By construction of $T$, the fixed point $x_\ast$ is the unique solution of (11) in $e^{-\omega}E_{1,\mu}$. Moreover, $x_\ast$ satisfies

$$|x_\ast|_{e^{-\omega}C_0(J;X_{p,\mu})} \leq M_1|x_\ast|_{e^{-\omega}E_{1,\mu}(J)} \leq M_1[C(\rho_0) + c]|x_0|_{X_{p,\mu}},$$

as well as

$$|x_\ast|_{e^{-\omega}C_0(J;X_{p,\mu})} \leq M_2|x_\ast|_{e^{-\omega}E_{1,\mu}(J)} \leq M_2\sigma \rightarrow, x_\ast|_{e^{-\omega}E_{1,\mu}(J)} \leq M_2\sigma^{-1}|C(\rho_0) + c|^{|x_0|_{X_{p,\mu}}},$$

where $J_\sigma = [\sigma, \infty)$ for some $\sigma > 0$ and $J = J_0$. Here the constant $M_1 > 0$ comes from the embedding (see (17))

$$e^{-\omega}E_{1,\mu}(J) \hookrightarrow e^{-\omega}C_0(J;X_{p,\mu})$$

and the constant $M_2 > 0$ does not depend on $\sigma > 0$. This can be seen as follows.

$$|x_\ast|_{e^{-\omega}E_{1,\mu}(J)} = \int_0^\infty \rho(t)|x_\ast(t)|_{X_1}' dt + \int_0^\infty \rho(t)|\dot{x}_\ast(t)|_{X_0}' dt$$

$$= e^{\omega\rho\sigma} \left( \int_0^\infty \rho(t)|x_\ast(t + \sigma)|_{X_1}' dt + \int_0^\infty \rho(t)|\dot{x}_\ast(t + \sigma)|_{X_0}' dt \right)$$

$$= e^{\omega\rho\sigma}|T(\sigma)x_\ast|_{e^{-\omega}E_{1,\mu}(J)} \geq \frac{1}{M_2} \sup_{r \geq 0} e^{\omega\rho(r + \sigma)}|x_\ast(r + \sigma)|_{X_p}'$$

$$= \frac{1}{M_2} \sup_{r \geq \sigma} e^{\omega\rho|t|} |x_\ast|_{X_p}$$

where $|T(\sigma)f|(|t|) := f(|t + \sigma|), \sigma \geq 0$, is the semigroup of left-translations and $M_2 > 0$ denotes the embedding constant associated to

$$e^{-\omega}E_{1}(J) \hookrightarrow e^{-\omega}C_0(J;X_{p,\mu}).$$

This yields the estimates

$$|x_\ast(t)|_{X_{p,\mu}} \leq M_1[C(\rho_0) + c]|x_0|_{X_{p,\mu}}, \quad t \geq 0,$$

and

$$|x_\ast(t)|_{X_{p,\mu}} \leq M_2 \sigma^{-1}|C(\rho_0) + c|^{|x_0|_{X_{p,\mu}}}, \quad t \geq \sigma > 0,$$

valid for all $|x_0|_{X_{p,\mu}} \leq \delta$. It follows that $x_\ast(t) \rightarrow 0$ in $X_{p,\mu}$ as $t \rightarrow \infty$ at an exponential rate and the trivial equilibrium of (11) is exponentially stable in $X_{p,\mu}$ for $\mu \in \left( \frac{n+2}{2p} + \frac{1}{2}, 1 \right]$. This proves assertion (1), (3) & (4).

If in addition $p > (n + 4)/2$, $\mu \in \left( \frac{n+4}{2p} + \frac{1}{2}, 1 \right]$ and $\phi'(s) \geq 0$ for all $s \in \mathbb{R}$ and $x_0$ is not necessarily small in $X_{p,\mu}$, then one can show that there exists a possibly small $T = T(x_0) > 0$ such that (22)-(23) has a unique solution

$$(\Delta W, W_t, \Theta) \in [L_{p,\mu}(J;W_2^2(\Omega))]^3 \cap [W_{p,\mu}(J,L_p(\Omega))]^3 \cap [BUC(J,W_2^{2\mu-2/p}(\Omega))]^3.$$

$J = [0, T]$. This follows from the lines of the proof of [26, Theorem 2.1], hence assertion (2) follows.

4. PROOF OF THEOREM 2.2

The proof of Theorem 2.2 follows from Theorem 2.1 and a suitable application of the implicit function theorem (see [13]), which gives both differentiability and analyticity of the nonlinear flow. The parameter trick which will be applied below goes back to [3] and in the context of maximal regularity it has been applied e.g.

in [46].

4.1. Differentiability of solutions. We will show that

$$[x \mapsto (A(x), f(x))] \in C^1(e^{-\omega}E_{1,\mu}(J); e^{-\omega}BUC(J; L(X_1, X_0)) \times e^{-\omega}E_{0,\mu}(J)).$$

where

$$A(x) = A + B(Z) \text{ and } f(x) = -a[0, \phi''(Z)|\Delta Z|^2, 0]^T$$

To this end, let $\phi$ satisfy Assumption 1 and, in addition, assume that $\phi \in C^3(\mathbb{R})$.

The natural candidate for $f'(x_s)x$ is

$$f'(x_s)x = -a \begin{pmatrix} \phi''(Z_s)|\nabla Z_s|^2 + 2\phi''(Z_s)(\nabla Z_s, \nabla Z) \\ 0 \\ 0 \end{pmatrix}.$$ 

We have

$$f(x_s + x) - f(x_s) - f'(x_s)x = -a[0, f_1(Z, Z_s) + f_2(Z, Z_s) + f_3(Z, Z_s), 0]^T,$$

where

$$f_1(Z, Z_s) := (\phi''(Z + Z) - \phi''(Z_s) - \phi''(Z_s)|\Delta Z_s|^2$$

$$f_2(Z, Z_s) := 2(\nabla Z_s, \nabla Z)(\phi''(Z_s + Z) - \phi''(Z_s))$$

and

$$f_3(Z, Z_s) := \phi''(Z + Z)|\nabla Z|^2$$

Since $\phi \in C^3(\mathbb{R})$, it is easy to check the desired $C^1$-property for $[x \mapsto f(x)]$ with the help of Lemma 3.3. In the same way (which is actually easier) one can show that $[x \mapsto A(x)]$ with $A(x) := A + B(x)$ is $C^1$ with

$$[B'(x_s)x]x_s = -a \begin{pmatrix} 0 \\ \phi''(Z_s)|\Delta Z_s|^2 + 2\phi''(Z_s)(\nabla Z_s, \nabla Z) \\ 0 \end{pmatrix}.$$ 

Let $x_s(t)$ be the solution according to Theorem 2.1. We introduce a new function $x_\lambda(t) := x_s(\lambda t)$ for $\lambda \in (1 - \epsilon, 1 + \epsilon)$ and $t \in J$. It follows that $\partial_t x_\lambda = \lambda(\partial_t x_s)(\lambda t)$, hence

$$\partial_t x_\lambda(t) + \lambda A(x_\lambda(t))x_\lambda(t) = \lambda f(x_\lambda(t)), \quad t \in J, \quad x_\lambda(0) = x_0(0).$$

Define a mapping $H : (1 - \epsilon, 1 + \epsilon) \times e^{-\omega}E_{1,\mu}(J) \rightarrow e^{-\omega}E_{0,\mu}(J) \times X_{p,\mu}$ by

$$H(\lambda, x) = (\partial_t x + \lambda A(x)x - \lambda f(x), x(0) - x_s(0))$$

Note that $H(1, x_s) = 0, H \in C^1((1 - \epsilon, 1 + \epsilon) \times e^{-\omega}E_{1,\mu}(J))$ and

$$D_xH(\lambda, x_s) = (\partial_t x + \lambda[A'(x_s)x]x_s + \lambda A(x_s)x - \lambda f'(x_s)x, x(0)),$$

by the differentiability properties of $A$ and $f$. This yields

$$D_xH(1, x_s)x = (\partial_t x + [B'(x_s)x]x_s + A(x_s)x - f'(x_s)x, x(0)),$$
where as before $A(x) = A + B(x)$. We already know that

$$|x_s| e^{-\omega E_{1,\mu}(t)} \leq C|x_s(0)|_{X_{p,\mu}}$$

by (22). This yields

$$||B'(x_s)x_s||_{e^{-\omega E_{0,\mu}(t)}} \leq a|\phi''(Z_s)|Z_s e^{-\omega E_{0,\mu}(t)} \leq a|\phi''(Z_s)| e^{-\omega E_{0,\mu}(t)} |Z| e^{-\omega E_{1,\mu}(t)} |Z_s| e^{-\omega E_{0,\mu}(t)} \leq C|\phi''(Z_s)| e^{-\omega E_{1,\mu}(t)} |x_s| e^{-\omega E_{1,\mu}(t)} \leq C\delta|\phi''(Z_s)| e^{-\omega E_{1,\mu}(t)},$$

if $|x_s(0)|_{X_{p,\mu}} < \delta$. Similarly one can show that

$$|B(x_s)x - f'(x_s)x_s + B(x_s)x - f'(x_s)x|$$

is a small perturbation of $|x \mapsto Ax|$. A Neumann series argument as in Lemma 5.3 yields that the operator $D_x H(1, x_s) : e^{-\omega E_{1,\mu}(t)} \rightarrow e^{-\omega E_{0,\mu}(t)} \times X_{p,\mu}$ is an isomorphism. Therefore, by the implicit function theorem (see e.g. Theorem 15.1 in [13]), we obtain a $C^1$-mapping $\Phi : (1 - \eta, 1 + \eta) \rightarrow e^{-\omega E_{1,\mu}(t)}$ such that $H(\lambda, \Phi(\lambda)) = 0$ for all $\lambda \in (1 - \eta, 1 + \eta)$ and $\Phi(1) = x_s$. By uniqueness it follows that $x(\lambda)(t) = x_s(\lambda t)$ for all $\lambda \in (1 - \eta, 1 + \eta)$ and $\Phi(\lambda) \in e^{-\omega E_{1,\mu}(t)}$. Therefore, by the implicit function theorem (see e.g. Corollary 15.1 in [13]), we obtain a $C^1$-mapping $\Phi : (1 - \eta, 1 + \eta) \rightarrow e^{-\omega E_{1,\mu}(t)}$. Evaluating at $\lambda = 1$ yields $t \mapsto t_0 e^{\omega E_{1,\mu}(t)}$ such that $\Phi(t_0) = x_s$. Consequently, we obtain $x(\lambda)(t) = x_s(\lambda t)$ for all $\lambda \in (1 - \eta, 1 + \eta)$. By uniqueness it follows that $x(\lambda)(t) = x_s(\lambda t)$ for all $\lambda \in (1 - \eta, 1 + \eta)$ and $\Phi(\lambda) \in e^{-\omega E_{1,\mu}(t)}$. Therefore, by the implicit function theorem (see e.g. Theorem 15.3) yields that $\Phi$ is real analytic, hence $\lambda \mapsto x_s(\lambda)$ is real analytic. Let $t_0 > 0$ be fixed and define $c(x) := x(t_0)$. It is easy to see that $c \in L(e^{-\omega E_{1,\mu}(t_0)}; X_p)$, hence $\lambda \mapsto x_s(\lambda t_0) = c(\lambda)$ is real analytic. But since this is true for any $t_0 > 0$, we obtain that $x_s(\lambda)$ is real analytic for all $\lambda > 0$ with values in $X_p$.

**References**

[1] S.A. Ambartsumian, M.V. Belubekyan and M.M. Minasyan. On the problem of vibrations of nonlinear elastic electroconductive plates in transverse and longitudinal magnetic fields. *International Journal of Nonlinear Mechanics*, 19:141-149, 1983.

[2] P. Acquistapace and B. Terreni. Some existence and regularity results for abstract non-autonomous parabolic equations. *Journal of Mathematical Analysis and Applications*, 99:9-64, 1984.
[3] S. Angenent. Nonlinear analytic semiflows. *Proc. Roy. Soc. Edinburgh Sect. A*, 115:91-107, 1990.
[4] G. Avalos and I. Lasiecka. Exponential stability of a thermoelastic system with free boundary conditions without mechanical dissipation. *SIAM Journal of Mathematical Analysis*, 29:155-182, 1998.
[5] G. Avalos and I. Lasiecka. Exponential stability of a thermoelastic system without dissipation. *Rend. Istit. Mat. Univ. Trieste*, Special Volume dedicated to memory of P. Grisvard, XXVIII:1-28, 1997.
[6] G. Avalos and I. Lasiecka. On the null-controllability of thermoelastic plates and singularity of the associated minimal energy function. Journal of Mathematical Analysis and its Applications, 10:34-61, 2004.
[7] G. Avalos and I. Lasiecka. Uniform decays in nonlinear thermoelasticity. In *Optimal Control, Theory, Methods and Applications*, Kluwer, 15:1-22, 1998.
[8] A. Benabdallah and M.G. Naso. Nullcontrolabilty of thermoelastic plates. *Abstract and Applied Analysis*, 7:585-599, 2002.
[9] G.Y. Bagdasaryan. Vibrations and Stability of Magnetoelastic Systems (in Russian). Yerevan, 1999.
[10] C. Dafermos. On the existence and asymptotic stability of solutions to the equations of nonlinear thermoelasticity. *Arch. Rat. Mechanics. Anal.*, 29:241-271, 1968.
[11] C. Dafermos and L. Hsiao. Development of singularities in solutions of the equations of nonlinear thermoelasticity. *Quart. Appl. Math.*, 44:463-474, 1986.
[12] G. Da Prato and P. Grisvard. Maximal regularity for evolution equations by interpolation and extrapolation. *Journal of Functional Analysis*, 58:107-124, 1984.
[13] K. Deimling *Nonlinear Functional Analysis* Springer, New York, 1985.
[14] R. Denk, M. Hieber and J. Prüss R$^p$-boundedness, Fourier Multipliers and Problems of elliptic and parabolic type. *Memoirs of the AMS*, no. 788, 2003.
[15] R. Denk and R. Racke. L$^p$-resolvent estimates and time decay for generalized thermoelastic plate equations. *Electronic Journal of Differential Equations*, no. 48, 2006.
[16] R. Denk, Y. Shibata and R. Racke. L$^p$ theory for the linear thermoelastic plate equations in bounded and exterior domains. *Konstanzer Schriften in Mathematik und Informatik*, 240, February, 2008.
[17] M. Eller, I. Lasiecka and R. Triggiani. Simultaneous exact-approximate boundary controllability of thermo-elastic plates with variable thermal coefficients and moment control. *Journal of Mathematical Analysis and its Applications*, 251:452-478, 2000.
[18] M. Eller, I. Lasiecka and R. Triggiani. Unique continuation result for thermoelastic plates. *Inverse and Ill-Posed Problems*, 9:109-148, 2001.
[19] D. Hasanyan, N. Hovakimyan, A.J. Sasane and V. Stepanyan. Analysis of nonlinear thermoelastic plate equations. In *Proceedings of the 43rd IEEE Conference on Decision and Control*, 2:1514-1519, 2004.
[20] S. Hansen and B. Zhang. Boundary control of a linear thermoelastic beam. *Journal of Mathematical Analysis and its Applications*, 210:182-205, 1997.
[21] S. Hansen. Exponential decay in a linear thermoelastic rod. *J. Math. Anal. Appl.*, 187:428-442, 1992.
[22] A.A. Ilyushin. *Plasticity. Part One. Elasticity-Plastic Deformations* (Russian). OGIZ, Moscow-Leningrad, 1948.
[23] S. Jiang and R. Racke. *Evolution Equations in Thermoelasticity*. Chapman and Hall, Boca Raton, FL, 2000.
[24] J.U. Kim. On the energy decay of a linear thermoelastic bar and plate. *SIAM Journal of Mathematical Analysis*, 23:889-899, 1992.
[25] H. Koch and I.Lasiecka. Backward uniqueness in linear thermo-elasticity with variable coefficients. *Functional Analysis and Evolution Equations*, special volume dedicated to G. Lumer, Birkhauser, 2007.
[26] M. Köhne, J. Prüss and M. Wilke On quasilinear parabolic evolution equations in weighted L$^p$-spaces. *J. Evol. Equ.*, 10, 443-463, 2010.
[27] J.Lagnese. The reachability problem for thermoelastic plates. *Archive for Rational Mechanics and Analysis*, 112:223-267, 1990.
[28] J. Lagnese. *Boundary Stabilization of Thin Plates*. SIAM, 1989.
[29] I. Lasiecka. Uniform decay rates for full von Karman system of dynamic thermoelasticity with free boundary conditions and partial boundary dissipation. *Communications in Partial Differential Equations*, 24:1801-1847, 1999.

[30] I. Lasiecka and C. Lebiedzik. Asymptotic behavior of nonlinear structural acoustic interactions with thermal effects on the interface. *Nonlinear Analysis*, 49:703-735, 2002.

[31] I. Lasiecka and C. Lebiedzik. Boundary stabilizability of nonlinear structural acoustic models with thermal effects on the interface. *C.R. Acad. Sci. Paris*, t 1. 329:187-192, 2000.

[32] I. Lasiecka, S. Maad, A. Sasane. Existence and exponential decay of solutions to a quasilinear thermoelastic plate system. *NODEA*, vol 15, pp 689-715, 2008.

[33] I. Lasiecka and T. Seidman. Blowup estimates for observability of a thermoelastic system. *Asymptotic Analysis*, 50:93-120, 2006.

[34] I. Lasiecka and R. Triggiani. Structural decomposition of thermoelastic semigroups with rotational forces. *Semigroup Forum*, 60:16-60, 2000.

[35] I. Lasiecka and R. Triggiani. *Control Theory for PDEs*, volume 1. Cambridge University Press, 2000.

[36] I. Lasiecka, M. Renardy and R. Triggiani. Backward uniqueness of thermoelastic plates with rotational forces. *Semigroup Forum*, 62:217-242, 2001.

[37] L. Librescu. *Elastostatics and Kinetics of Anisotropic and Heterogeneous Shell-type Structures*. Noordhoff, Leiden, 1975.

[38] L. Librescu, D. Hasanyan, Z. Qin and D. Ambur. Nonlinear magnetothermoelasticity of anisotropic plates in a magnetic field. *Journal of Thermal Stresses*, 26:1277-1304, 2003.

[39] J. Prüss. Maximal regularity for evolution equations in $L_p$-spaces. *Conf. Semin. Mat. Univ. Bari*, no. 285, 1-39, 2002.

[40] J. Prüss, G. Simonett. Maximal regularity for evolution equations in weighted $L_p$-spaces. *Arch. Math.,* 82, no. 5, 415-431, 2004.

[41] A. Lunardi. *Analytic Semigroups and Optimal Regularity in Parabolic Problems*. Birkhäuser, 1995.

[42] A. Lunardi. Abstract quasilinear parabolic equations. *Math. Ann.*, 267:395-415, 1984.

[43] A. Lunardi, Global solutions of abstract quasilinear parabolic equations. *Journal Differential Equations*, vol. 58, 228-242, 1985.

[44] A. Pazy. *Semigroups of Linear Operators and Applications to Partial Differential Equations*. Applied Mathematical Sciences, 44, Springer-Verlag, New York, 1983.

[45] J. Prüss. Maximal regularity for evolution equations in $L_p$-spaces. *Conf. Semin. Mat. Univ. Bari*, no. 285, 1-39, 2002.

[46] J. Prüss, G. Simonett. Maximal regularity for evolution equations in weighted $L_p$-spaces. *Arch. Math.,* 82, no. 5, 415-431, 2004.

[47] J.E. Muñoz Rivera, R. Racke. Smoothing properties, decay, and global existence of solutions to nonlinear coupled systems of thermoelastic type. *SIAM Journal on Mathematical Analysis*, no. 6, 26:1547-1563, 1995.

[48] J.E. Muñoz Rivera, R. Racke. Large solutions and smoothing properties for nonlinear thermoelastic systems. *Journal of Differential Equations*, no. 2, 127:454-483, 1996.

[49] H. Triebel. *Interpolation Theory, Function Spaces, Differential Operators*. North-Holland, 1978.