ON THE ORIGIN OF THE DEFLECTION OF LIGHT

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Abstract. Action at distance in Newtonian physics is replaced by finite propagation speeds in classical post–Newtonian physics. As a result, the differential equations of motion in Newtonian physics are replaced by functional differential equations, where the delay associated with the finite propagation speed is taken into account. Newtonian equations of motion, with post–Newtonian corrections, are often used to approximate the functional differential equations. In [16], a simple atomic model based on a functional differential equation which reproduces the quantized Bohr atomic model was presented. The unique assumption was that the electrodynamic interaction has a finite propagation speed. In [17], a simple gravitational model based on a functional differential equation which gives a gravitational quantification and an explanation of the modified Titius–Bode law is described. In [18], an explanation of the anomalous precession of Mercury’s perihelion is given in terms of a simple retarded potential, which, at first order, coincides with Gerber’s potential of 1898, and which agrees with the author’s previous works [16, 17]. In this paper, it is shown how the simple retarded potential presented in [18] also gives the correct value of the gravitational deflection of fast particles of General Relativity.

1. Introduction

The history of the deflection of light problem began in 1704, when Newton proposed in the conclusions of his treatise on Opticks [21] the following query:

Do not Bodies act upon Light at a distance, and by their action bend its Rays, and is not this action strongest at the least distance?

In fact, this suggestion is not so much radical, because on the basis of the corpuscular theory of light, and Newton’s laws of mechanics and

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gravitation, it is easy to conjecture that a ray of light could be deviated slightly while it passes near a big massive body, assuming that particles of light respond to gravitational acceleration similarly to particles of matter. Johann Georg von Soldner, in 1801, calculated the bending of light rays grazing the Sun’s disk, also referred to in [19], using classical mechanics and a hypothetical light velocity of around $3 \cdot 10^5$ km per second. A hundred years later, Einstein repeated the prediction (in the context of his theories) that starlight passing by the Sun would be deflected by the Sun’s gravity, so that the apparent distance between stars either side of the Sun when viewed during an eclipse would be smaller. The following computation of the deflection of the light, based in the Newtonian gravitation, is extracted from [20] and [26] where it is given a complete description of the historical development of the problem.

For any conical orbit of a small test particle of mass $m$ in a Newtonian gravitational field around a central mass $M$, the eccentricity of the unbounded hyperbolic orbit is given by

$$
\varepsilon = \sqrt{1 + \frac{2EL^2}{G^2m^3M^2}},
$$

where $E = \frac{mv^2}{2} - GmM/r$ is the total energy (kinetic plus potential), $L = mrv_t$ is the angular momentum, $v$ is the total speed, $v_t$ is the tangential component of the speed, and $r$ is the radial distance from the center of the mass. Since a beam of light travels at such a high speed, it will be in a shallow hyperbolic orbit around an ordinary massive object like the Sun.

Letting $r_p$ denote the closest approach (the perihelion) of the beam to the gravitating body, at which $v = v_t$, we have

$$
\varepsilon = \sqrt{1 + \left(\frac{r_pv_t^2}{GM}\right)^2 - \frac{2r_pv_t^2}{GM} = 1 - \frac{r_pv_t^2}{GM}},
$$

As expected, the test particle mass $m$ cancels out above. Now we set $v_t = c$ (the speed of light) at the perihelion, and from the geometry of the hyperbola we know that the asymptotes make an angle of $\beta$ with the axis of symmetry, where $\cos \beta = 1/\varepsilon$.

As the hyperbolic orbit shown in Figure 1, the total angular deflection of the beam of light is $\delta_N = \pi - 2\beta$, which for small angles $\beta$ and for $M$ much less than $r_p$, is given in Newtonian mechanics by

$$
\delta_N = \pi - 2 \arccos \left(\frac{1}{\varepsilon}\right) = \pi - 2 \arccos \left(\frac{GM}{GM - c^2r_p}\right) \approx \frac{2GM}{c^2r_p},
$$
Figure 1. The light ray from a star follows an unbound hyperbolic orbit about the Sun. For deflection on grazing incidence, the distance of closest approach is $r_p$.

The best natural opportunity to observe this deflection would be to look at the stars near the perimeter of the Sun during a solar eclipse. The mass of the Sun in gravitational units is about $M = 1475$ meters, and a beam of light just skimming past the Sun would have a closest distance equal to the Sun's radius, $r_p = 6.95 \cdot 10^8$ meters. Therefore, the Newtonian prediction would be $0.000004245$ radians, which equals $0.875$ seconds of arc.

In 1911, Einstein recaptures the idea of bending the light, see [10]. He used the equivalence principle and the equivalent mass-energy of a photon, together with Special Relativity, to predict that clocks run at different rates in a gravitational potential and the bending of light-rays in a gravitational field, even before he developed the concept of curved-space time. Oddly enough, the quantitative prediction given in this paper for the amount of deflection of light passing near a large mass was more or less identical to the old Newtonian prediction $0.83$ seconds of arc. It wasn’t until late in 1915, see [12], as he completed the General Relativity theory, that Einstein realized his earlier prediction was incorrect, and the angular deflection should actually be twice the size he predicted in 1911. Only in this second calculation, published in 1916, where he included the effect of space-time curvature, he obtained a value $\delta_{GR} = \frac{4GM}{c^2 r_p}$. 
2. DEFLECTION OF THE LIGHT FROM A RETARDED POTENTIAL

Action at distance in Newtonian physics is replaced by finite propagation speeds in classical post–Newtonian physics. As a result, the differential equations of motion in Newtonian physics are replaced by functional differential equations, where the delay associated with the finite propagation speed is taken into account. Newtonian equations of motion, with post–Newtonian corrections, are often used to approximate the functional differential equations, see, for instance, [5, 6, 7, 8, 15, 24, 25]. In [16] a simple atomic model based on a functional differential equation which reproduces the quantized Bohr atomic model was presented. The unique assumption was that the electrodynamic interaction has finite propagation speed, which is a consequence of the Relativity theory. A straightforward consequence of the theory developed in [16], and taking into account that gravitational interaction has also a finite propagation speed, is that the same model is applicable to the gravitational 2-body problem. In [17] a simple gravitational model based on a functional differential equation which gives a gravitational quantification and an explanation of the modified Titius–Bode law is described. In [18] an explanation of the anomalous precession of Mercury’s perihelion is given in terms of a simple retarded potential, which, at first order, coincides with Gerber’s potential of 1898, and which agrees with the author’s previous works [16, 17]. In the following, we show how the values of the anomalous precession of Mercury’s perihelion and the gravitational deflection of fast particles of General Relativity can be reproduced by a retarded potential.

First, we review the recent intends to solve the problem of the anomalous precession of Mercury’s perihelion and the gravitational deflection of fast particles of General Relativity in terms of Weber forces. In [11], Assis has proposed a theory of gravitation based on Mach’s principle, by postulating that the resultant force in any body is zero and with a Weber type force for gravitation. With a suitable coefficient for this force he was able to reproduce the advance of the perihelion of the planets as given by General Relativity, fixing in this way the only parameter of the theory. In [23] the gravitational deflection of fast particles was calculated and that the theory does not lead to the Einstein’s value for the deflection of light was shown, see [12]. In fact the valued obtained is twice as big. Moreover, in [23] it was shown how both results, the advance of perihelion and fast particle deflection, can be accommodated by using a modified Weber force. In 1848, Weber
presented a velocity dependent potential
\[ V = \frac{1}{r} \left[ 1 - \frac{\dot{r}^2}{2c^2} \right]. \]
from which the Weber force might be derived. This force form the basis of the classical Weber’s electrodynamics, see [2].

The basic equation of motion that it was analyzed in [23] is
\[ \frac{d^2\mathbf{r}}{dt^2} = -\frac{GM}{r^3} \left[ 1 + \frac{\xi}{c^2}(r \ddot{r} - \alpha \dot{r}^2) \right] \mathbf{r}, \]
where \( \mathbf{r} \) is the relative radius vector of the particle with respect to the Sun. The last term on the right-hand side of (1) is called the gravitational Weber force per unit of mass. With \( \alpha = 1/2 \), (1) is the expression adopted by Assis, see [1], who needed to fix \( \xi = 6 \) to obtain the right advance of the perihelion of the planets. A law of motion of type (1) was first proposed by Tisserand [27] with \( \alpha = 1/2 \) and \( \xi = 2 \), which corresponds to Weber’s law of electrodynamic action. In [23], it is considered the (small) deflection of a fast particle by the Sun with velocity tending to the speed of light, and with a distance of closest approach \( r_p \) much larger than \( GM/c^2 \). Moreover, the calculation in Cartesian coordinates, following the same reasoning of Corinaldesi and Papapetrou [9] for the calculation of the deflection of a spinning fast particle in a Schwarzschild field, was performed. Choosing the \( x \) axis perpendicular to the distance of closest approach with the \( y \) axis along it. Therefore, the deflection occurs in the plane \( xy \) with \( dy/dx = 0 \) at \( x = 0 \). If the deflection is small \( (r_p \gg GM/c^2) \), it is possible to write \( y = r_p \) and \( x = ct \), in the first approximation. We then have \( r^2 = c^2t^2 + r_p^2 \). Upon integration of \( d^2y/dt^2 \), with the condition \( dy/dt = 0 \) at \( t = 0 \), and writing \( x = ct \), the orbit equation becomes
\[ \frac{dy}{dt} = -\frac{GM}{r_p c^2} \left[ \frac{(1 + \xi)x}{\sqrt{x^2 + r_p^2}} - \frac{\xi(\alpha + 1)x^3}{3(x^2 + r_p^2)^{3/2}} \right]. \]

The angle of deflection of the path of the particle is \( \delta \approx (dy/dx)_{-\infty} - (dy/dx)_{\infty} \), or
\[ \delta = \frac{2GM}{r_p c^2} \left( 1 + \xi \right) \left( 1 - \frac{\xi(\alpha + 1)}{3} \right). \]
With the value \( \alpha = 1/2 \) adopted by Assis and the value \( \xi = 6 \) needed to reproduce the advance of perihelion, we would have \( \delta = 8GM/r_p c^2 \), which is twice as big as the value obtained by Corinaldesi and Papapetrou [9] for the deflection of a fast, spinless particle by a Schwartzchild field, which is also Einstein’s value \( \delta_{GR} \) for the deflection of light, see
Figure 2. The unbounded hyperbolic orbit of a fast particle around the Sun with mass $M$.

also [28]. To reproduce this value, we need to have from (2) the relation

\[ \xi (2 - \alpha) = 3. \]

Therefore, taking into account that the value of $\xi$ needed to reproduce the advance of perihelion is $\xi = 6$, if we want to construct a Weber force which also explains the deflection of the light, equation (3) gives for $\alpha$ the value of $\alpha = 3/2$, see [23].

In [14], the Weber-like forces are examined from the point of view of energy conservation and it is proved that they are conservative if and only if $\alpha = 1/2$. As a consequence, it is shown that gravitational theories employing Weber-like forces cannot be conservative and also yield both the precession of the perihelion of Mercury as well as the gravitational deflection of light.

In [18], the simple retarded potential was presented:

\[ V = -\frac{GMm}{r(t - \tau - \frac{r(t - r)}{c})}. \]

where $r(t - \tau - r(t - \tau)/c)$ is the distance between the masses when the potential was “emitted” to go from the emitting particle to the receiving particle and come back, and $\tau = r(t)/c$. This retarded potential coincides, at first orders, with Gerber’s potential of 1898 and gives an explanation of the anomalous precession of Mercury’s perihelion because if we develop the retarded potential [14] in powers of $\tau$ (up to second order in $\tau$), then we find that the gravitational force law
associated to this potential is given by

\begin{equation}
    f = -\frac{GMm}{r^2} \left(1 - \frac{3\dot{r}^2}{c^2} + \frac{6r\ddot{r}}{c^2}\right),
\end{equation}

This gravitational force is a Weber type force and it coincides with the force adopted by Assis [1] (which has the values \(\alpha = 1/2\) and \(\xi = 6\)).

In the following, we will see that the simple retarded potential \((4)\) also gives an explanation of the gravitational deflection of fast particles. Let \(t_0\) be the time when the test particle is in closest approach (the perihelion), i.e., \(r(t_0) = r_p\). To evaluate the bending we must take \(t \gg t_0\). If we consider a particle carrying out an unbounded hyperbolic orbit, at light velocity, and verifying the retarded potential \((4)\) we have Figure 2, where

\[d_1 = |r(t)| = \int_{t-t_0}^t \sqrt{(x'(s))^2 + (y'(s))^2} \, ds,\]

and

\[d_2 = |r(t - \tau)| = \int_{t-t_0}^{t-t_0-\tau(t-t_0)/c} \sqrt{(x'(s))^2 + (y'(s))^2} \, ds,\]

where \(r(t) = (x(t), y(t))\). This fact happens because the velocity of the particle in the orbit is the light velocity. Therefore, from Figure 2, we have that \(r(t - \tau) \ll r(t)\) and hence, in this case

\begin{equation}
    V = -\frac{GMm}{r(t - \tau - r(t-\tau)/c)} = -\frac{GMm}{r(t - r(t)/c + r(t-\tau)/c)} \approx -\frac{GMm}{r(t - \tau)}.
\end{equation}

If we develop the approximation of the retarded potential \((6)\) in powers of \(\tau\) (up to second order in \(\tau\)), we obtain

\begin{equation}
    V \approx -\frac{GMm}{r} \left[1 + \frac{\dot{r}}{r} \tau + \left(\frac{\ddot{r}}{r^2} - \frac{\dot{r}^2}{2r}\right) \tau^2\right],
\end{equation}

To develop some easier calculations we can reject, on the right hand side of expression \((7)\), the term with \(\ddot{r}\) (in fact this term is negligible and it only gives rise to terms of higher order). Hence, at this approximation, we obtain the velocity–dependent potential

\begin{equation}
    V \approx -\frac{GMm}{r} \left[1 + \frac{\dot{r}}{r} \tau + \left(\frac{\dot{r}^2}{r^2} - \frac{\dot{r}^2}{2r}\right) \tau^2\right],
\end{equation}

Introducing the expression of the delay \(\tau = r(t)/c\) in \((8)\) we have:

\begin{equation}
    V \approx -\frac{GMm}{r} \left[1 + \frac{\dot{r}}{c} + \frac{\dot{r}^2}{c^2}\right].
\end{equation}
On this basis, of this velocity-dependent potential function (9), the gravitational force law is given by substituting the potential function (9) into the equation:

\[
f = \frac{d}{dt} \left( \frac{\partial V}{\partial \dot{r}} \right) - \frac{\partial V}{\partial r} = -\frac{GMm}{r^2} \left( 1 - \frac{\dot{r}^2}{c^2} + \frac{2r\ddot{r}}{c^2} \right).
\]

Hence, we obtain a Weber type force with \( \alpha = \frac{1}{2} \) and \( \xi = 2 \). In fact, the force described in (10) coincides with the Weber type force proposed by Tisserand [27], which corresponds to Weber’s law of electrodynamic action. It is easy to see that these values verify equation (3) and hence the gravitational force law (10) gives the correct value of the gravitational deflection of fast particles of General Relativity.

3. Concluding remarks

We have seen how the values of the advance of the perihelion of the planets and the gravitational deflection of the fast particles of General Relativity can be reproduced with the simple retarded potential (4). In the first case, we directly develop the retarded potential in powers of \( \tau \). In the second case, first we impose that the velocity of the particle is close to the light velocity, which implies \( r(t - \tau) \ll r(t) \) and after we develop the approximation of the retarded potential in powers of \( \tau \). It should also mentioned that, although this model gives identical results as the one obtained by using General Relativity, these theories are based on different concepts and mathematical tools. In addition, the differences appear in the terms of higher order in \( \tau \), and the improvement of observations and experimental techniques will accomplish a good test for both theories in the future.

In [1], Assis proposes the postulate that the resultant force acting on any body is zero. With this postulate and the Weber force law (5) for gravitation, he obtains equations of motion and concludes that all inertial forces are due to gravitational interaction with other bodies in the universe, as it was suggested by Mach. All these arguments are accomplished in a strictly relational theory, see also [3]. Now, we have an important framework which explains the introduction in these models of the Weber’s force. These forces are, in fact, approximations of retarded forces, taking into account the finite propagation speed. A coherent theory of the inertia according with the Mach’s principle lacks to be given. We hope to give an answer in a future work.

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