Free convection heat transfer from two equal triangular cylinders confined in triangular enclosure

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Abstract

This paper studied the natural convection heat transfer from two equal-sized cylinders of triangular cross-section confined in triangular enclosure. The inner cylinders have hot surfaces and the outer enclosure has a cold surfaces. The annular space between inner cylinders and the outer one was filled with incompressible Newtonian fluid. The work examined the behavior of fluid flow in the process of transferring heat energy. The work was achieved numerically using the commercial code ANSYS-CFX. The adopted initial conditions were selected for these ranges: Prandtl number ($Pr = 1$ to $10^3$), Rayleigh number ($Ra = 10^3$ to $10^5$). It was found that the increment of Ra number increases the rate of heat transfer. However, the effect of Pr number on heat transfer is almost negligible.

Keyword: Triangular cylinders, buoyancy-driven flow, annulus, ANSYS-CFX, cold enclosure

1. Introduction

The buoyancy-driven flow in annular space has many industrial applications such as heat exchangers, nuclear reactors, solar collector and so on. Indeed many researchers have studied the heat transfer of free convection between two cylinders.

Laidoudi [1] determined the average Nusselt number of four cylinders located in circular enclosure. The obstacles are hot whereas, the enclosure is cold. The cylinders are arranged crossly. Yoo and Han [2] numerically studied the free convection in unsteady state. The studied space is horizontal and circular cross-section. The $Nu$ was determined as function of time. Sheremet and Pop [3] numerically examined the free convection between two circular obstacles of horizontal arrangement. The nanofluid was filled between those cylinders. The
values of Nu number of hot cylinder are computed and discussed. Also, the behavior of flow is discussed according to the pertinent parameters. Kuhnen and Goldstein [4] experimentally studied the heat transfer of natural convection between two spaces. The outer one is cold and the inner one is heated at constant temperature. The exact quantity of heat transfer was measured. Matin, and Khan [5] used the same geometry and the under the same conditions for the non-Newtonian power-law fluids. The Nu number was calculated and studied for different situations based on the nature of the fluid. Other recent studies are also reported for the same principle [6-10].

This work is reserved for exploring the natural convection from two identical triangular cylinders confined in triangular enclosure. We present here the roles of Rayleigh number and Prandtl number on the Nu number.

2. Description of studied geometry

Fig. 1 reflects the schematic presentation of studied geometry. It consists of two equal triangular cylinders of hot surfaces \( (T_h) \) confined within triangular enclosure of cold surfaces \( (T_c) \) all triangular forms are equilateral. The length of inner cylinder \( (d) \) to the outer one defines the blockage ratio \( (B = d/D = 0.2) \). The spacing \( S \) equals the distance between the base of the enclosure to the top dividing by 3. The temperature difference between the hot and cold surfaces generates a temperature gradient which is the source of heat and mass transfer.

![Diagram of studied geometry](image)

Fig.1 Diagram of studied geometry

3. Mathematical formulation and boundary conditions

The mathematical formulation of governing equations describing the thermal buoyancy in laminar flow and constant thermo-physical proprieties can be expressed in non-dimensions as:
\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}
\]
\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \text{Pr} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \tag{2}
\]
\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \text{Pr} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \text{Ra} \cdot \text{Pr} \cdot \theta \tag{3}
\]
\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \tag{4}
\]
where \( \text{Pr} \) and \( \text{Ra} \) indicates Prandtl number and Rayleigh number, respectively and they are given as:
\[
\text{Ra} = \frac{g \beta (T_h - T_c) d^3}{\nu \alpha}, \text{Pr} = \frac{\nu}{\alpha} \tag{5}
\]
The other parameters of governing equations are obtained after considering the following changes:
\[
X = \frac{x}{d}, \quad Y = \frac{y}{d}, \quad U = \frac{u d}{\alpha}, \quad V = \frac{v d}{\alpha}, \quad P = \frac{pd^2}{\rho \alpha^2}, \quad \theta = \frac{T - T_c}{T_h - T_c} \tag{6}
\]
The physical proprieties of working fluid are: kinematic viscosity \( \nu \), density \( \rho \), thermal diffusivity \( \alpha \) and the expansion coefficient \( \beta \).

The local Nusselt number around the surfaces of inner cylinder and the average value of each cylinder are calculated by following expressions:
\[
\text{Nu}_i = \frac{\partial \phi}{\partial n} \bigg|_{\text{wall}} \tag{7}
\]
\[
\text{Nu} = \frac{1}{l} \int_0^l \text{Nu}_i ds \tag{8}
\]
The appropriate boundary conditions used at the extremities of the computational domain are:

For hot cylinders: \( U = 0, V = 0, \theta = 1 \) \tag{9}

For the cold cylinder: \( U = 0, V = 0, \theta = 0 \) \tag{10}
4. **Numerical details**

The present simulations were accomplished using the package ANSYS-CFX. The code converts the partial form of governing equations into algebraic system then it solves them via finite volume method. This section presents the choice of adequate grid and the validation test. Fig. 2 shows the form of elements of the mesh. Indeed, a triangular form with non-uniform distribution of elements was generated for the mesh. The grid has concentration of elements around the inner cylinders. Table 1 shows the results of grid independency test; three grids of different size were generated. It is clear that when the number of mesh elements increases the variation of Nu number decreases and the variation becomes negligible between M2 and M3. Therefore, the grid M2 with 71900 elements can be adequate for this investigation.

![Fig. 2 structure of generated grid](image)

| Elements | Nu_1 | Nu_2 |
|----------|------|------|
| M1       | 46934| 3.93043 | 1.8045 |
| M2       | 71900| 3.92003 | 1.79854 |
| M3       | 107820| 3.90759 | 1.79239 |

5. **Results and discussions**

The free convection heat transfer in annular space of triangular cross-section is the main purpose of present study. The effects of geometrical form, thermo-proprieties of the fluid (Pr) and the strength of thermal buoyancy (Ra) on the fluid structure and temperature distribution are the goal of the work. For natural convection the heated parts of fluid particles is the source
of fluid motion. In the fact the hot fluid particles move up and the cold particles move down due to gravitational force.

Fig. 4 shows the variation of preventative contours of isotherms with $Ra$ and $Pr$. It is clear that the dissipation of hot temperature towards the upward direction increases with increasing the $Ra$ number. Also, the temperature gradient around the cylinders increases with $Ra$ number. On other hand, the effect of $Pr$ number is observed to be negligible. From these remarks, it can be expected that the fluid velocity in annular space increases with $Ra$, the heat transfer between the fluid and hot surfaces increases with $Ra$ number.

![Fig. 4 Isotherms around the triangular cylinders for different values of $Ra$ and $Pr$](image)

As it is observed from the contours of isotherms, the heated fluid particles move up and the cold parts move down. As result, recirculation zones appear in the annular space as it is shown in the Fig. 5. Two symmetrical vortices are observed in the annular space. The form and the size on the vortex depend on the value of $Ra$ number. On other hand, there is no effect
of Prandtl number on the flow structure. Furthermore, the center of the recirculation zone shifts up with increasing the Richardson number.

![Streamlines around the triangular cylinders for different values of Ra and Pr](image)

Fig. 5 Streamlines around the triangular cylinders for different values of Ra and Pr

Fig. 6 presents the variation of average Nusselt number of each inner cylinder with Ri and Pr numbers. The Nu characterizes the heat transfer rate, increasing Nu means increases of heat transfer rate. As it was expected from the analyzes of isotherms, increase in Ra number increases the heat transfer of both cylinders. Also, the variation of Pr number does not provoke any changes at the values of Nusselt number. Furthermore, the values of Nu number of lower cylinder (C1) are greater than the upper cylinder (C2) due to the cold source of fluid which is close to the lower cylinder (C1).
Conclusion

Numerical simulations were performed to study the free convection heat transfer in triangular enclosure containing two hot triangular cylinders. The effects of buoyancy strength (Ra) and the thermo-proprieties of the fluid on the flow structure and heat transfer rate were the main points of the study. At the end of the work some points were drawn:

- The heat transfer rate depends positively on Ra number;
- The thermo-physical properties of fluid have a limited effect on the performance of heat transfer;
- As usual, the structure of flow in annular space forms a two symmetrical bubbles and the center of each bubbles shifts upwardly with increasing the thermal buoyancy strength.
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