Magnetic field analysis of solenoid driven by alternating current

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Abstract. In order to elaborate the magneto-optical modulation process in detail and clarify the distribution of the magnetic field in the solenoid. In this paper, the magnetic field model of the solenoid with sine wave is established by Maxwell's equation. The boundary condition is determined by the law of Ampere loop and the law of electromagnetic induction. The exact function expression of the magnetic field is obtained and the relevant factors are analyzed in detail. The simulation results show that the magnetic field in the solenoid with sine wave has good sine characteristic and the magnetic field is closely related to the factors such as the frequency of the driving signal and the axial magnetic field at any point inside the solenoid is much larger than the circumference magnetic field at that point. So, when the magnetic field at a point inside the solenoid is analyzed, it is possible to ignore the circumferential magnetic field at that point. But outside the solenoid, both directions of magnetic field need to be considered. In this paper, the method of studying the magnetic field of the solenoid provides a reference for the detailed analysis of the magnetic field inside and outside the solenoid under different signal driving.

1. Introduction
The magneto-optical modulation technology based on Faraday effect has attracted more and more attention in the research field of military aerospace and high-precision technology because of its advantages of high angle measurement accuracy and strong anti-interference ability. The precise description of the magnetic field inside and outside the energized solenoid is one of the key technologies, the parameters such as the magnitude of the magnetic field are directly related to the Faraday rotation angle. Literatures 1 and 2 establish a sine wave driven solenoid internal and external electromagnetic field model [1-2], but there is no detailed analysis of the factors affecting the internal and external magnetic fields of the solenoid. The description of the sinusoidal-driven solenoid magnetic field is too simplified in literatures 3 and 4. Document 5 only simplified the analysis of the electromagnetic field distribution inside the solenoid, and did not analyze the influencing factors of the magnetic field [5]. Literatures 6 and 7 studied the magnetic field distribution of a solenoid driven by a DC signal according to Biot-Savar's law [6-7]. At present, research on the magnetic field of the solenoid is mostly concentrated on a constant magnetic field, and the precise description of the magnetic field of the solenoid driven by the alternating signal is less.

In this paper, Maxwell's equation is used to construct the internal and external magnetic field model of the solenoid driven by alternating signals. The boundary conditions are determined by combining the Ampere loop law and the electromagnetic induction law. The exact function expression of the magnetic field is obtained, and the related factors are analyzed in detail.
2. Magnetic field model driven by sinusoidal wave

In view of the cylindrical symmetrical structure of the solenoid, a suitable cylindrical coordinate system and a rectangular coordinate system are established in the solenoid space, as shown in Figure.1.

![Figure 1. The coordinate system of a solenoid.](image)

In Fig. 1, \( R \) is the solenoid radius, \( r \) is the radial to central axis distance, \( \phi \) is the distance from the radial direction to the central axis.

Inlet sinusoidal alternating current \( I = I_0 e^{(-i \omega t)} \) into the solenoid, among them, \( I_0 \) and \( \omega \) are the amplitude and frequency of the current signal. Due to the uniformity and symmetry of the solenoid, regardless of the components of the magnetic field \( B \) and the electric field \( E \) in the radial direction, the magnetic field \( B \) and electric field \( E \) of the solenoid can be expressed as:

\[
\vec{B}(r,t) = B_\phi(r,t)\hat{\phi} + B_z(r,t)\hat{z} \tag{1}
\]

\[
\vec{E}(r,t) = E_\phi(r,t)\hat{\phi} + E_z(r,t)\hat{z} \tag{2}
\]

In the formula, \( B_\phi \), \( B_z \) are the magnetic field strengths of the circumferential direction and the axial direction of the solenoid respectively; \( E_\phi \) and \( E_z \) are the electric field strengths of the circumferential direction and the axial direction of the solenoid respectively; \( r \) is the distance from the radial direction to the central axis; \( t \) is the line tube energization time; \( z \) is the radial direction of the solenoid.

Substituting equations (1) and (2) into Maxwell’s equations yields, got

\[
\frac{\partial E_\phi}{\partial r} = i\omega B_\phi \tag{3}
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} (r B_\phi) = i\mu\epsilon\omega E_\phi \tag{4}
\]

\[
\frac{\partial B_\phi}{\partial \phi} = +i\mu\omega E_\phi \tag{5}
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) = +i\omega B_z \tag{6}
\]

In above formula, \( \mu \) and \( \epsilon \) are the permeability and dielectric constant of the medium inside the solenoid. Equation (3-6) is a basic model for solving the electromagnetic field of the solenoid. It can be seen that the electric field and the magnetic field of the solenoid driven by the alternating signal are alternating signals and closely related. The magnetic fields in each direction are analyzed in detail below.

2.1. Axial magnetic field model

Can be obtained by subtracting \( E_\phi \) from equations (5) and (6)

\[
\frac{\partial^2 B_z}{\partial r^2} + \frac{1}{r} \frac{\partial B_z}{\partial r} + \mu\epsilon\omega^2 B_z = 0 \tag{7}
\]
Let \( k^2 = \mu \varepsilon \omega^2 \), then equation (7) can be expressed as

\[
\frac{\partial^2 B_z}{\partial r^2} + \frac{1}{r} \frac{\partial B_z}{\partial r} + k^2 B_z = 0
\]  

(8)

The general solution of the axial magnetic field of the solenoid \( B_m \) obtained by equation (8) is

\[
B_m(r,t) = C_1 H_1^0(kr) \exp[-i(\alpha t + \phi_m)]
\]  

(9)

Where: \( B_m(r,t) \) is the axial magnetic field strength of the solenoid at the moment \( t \) from the central axis \( r \); \( C_1 \) is the amplitude of the axial magnetic field of the solenoid; \( H_1^0(kr) \) is the Hankel function, \( H_1^0(kr) = J_0(kr) + iN_0(kr) \), and \( N_0(kr) \) is the second-order zero-order Bessel function; \( \phi_m \) is the phase shift of the solenoid magnetic field.

When using the nature of the alternating electromagnetic field[1], determining boundary conditions \( r = R \), the following formula can get

\[
B_m(k_r R) = \mu_0 \frac{N}{\Delta} l = \mu_0 n l
\]  

(10)

Where: \( N \) is the number of turns of the coil included in the integral path, and \( n \) is the number of turns of the coil.

The amplitude \( C_1 \) of the axial magnetic field of the solenoid is solved by equations (10) and (3)

\[
C_1 = \frac{\mu_0 n l_0 \sin \phi_0}{J_0(k R) \sin \delta_0 - N_0(k R) \cos \delta_0}
\]  

(11)

In the formula, \( J_0, N_0 \) are the first-order zero-order and second-order zero-order Bessel functions.

2.2. Circumferential magnetic field model

The same can be obtained by formulas (3) and (4):

\[
\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} + k^2 E_z = 0
\]  

(12)

The general solution of the axial electric field \( E_z \) of the solenoid obtained by solving equation (12) is:

\[
E_z(r,t) = C_2 H_1^0(k_r r) \exp[-i(\alpha t + \phi_z)]
\]  

(13)

Substituting equation (13) into equation (3), get the circumferential magnetic field size as

\[
B_{\phi_0} = (-i k_r C_2 / \omega) H_1^0(k_r r) \exp[-i(\alpha t + \Phi_{\phi_0})]
\]  

(14)

In the above formula, parameter \( C_2 \) size is

\[
C_2 = \frac{\mu_0 \omega l_0 \cos \phi_0}{2\pi(k_r R) J_1(k_r R) \sin \delta_2 - N_1(k_r R) \cos \delta_2}
\]  

(15)

3. Powered solenoid magnetic field simulation

Based on the model established above, the axial and circumferential magnetic fields of the solenoid are expressed as:

\[
B_m(r,t) = C_1 H_1^0(k_r r) \exp[-i(\alpha t + \phi_m)]
\]  

(16)

\[
B_{\phi_0} = (-i k_r C_2 / \omega) H_1^0(k_r r) \exp[-i(\alpha t + \Phi_{\phi_0})]
\]  

(17)
It can be seen from equations (16) and (17) that the main factors affecting the external magnetic field of the solenoid are the current signal frequency and the distance from the central axis. The matlab software is used to analyze the effects of the two parameters, frequency $\omega$ and distance $r$, on the magnetic field at various points in the solenoid.

### 3.1. Frequency characteristics of solenoid-changing magnetic field

Set the solenoid radius $R = 0.035m$, current signal amplitude $I_0 = 0.2A$. The outside of the solenoid is air, its dielectric constant $\varepsilon_0 = 8.85 \times 10^{-12}$, and permeability $\mu_0 = 4\pi \times 10^{-7}$. The internal medium of the solenoid is selected from TGG magneto-optical glass, and its dielectric constant is $3.8\varepsilon_0$, magnetic permeability is $\mu_0$. Take $r = 0.05m$, $f = (200MHz, 400MHz, 800MHz)$, through matlab simulation to obtain the external axial magnetic field $B_{zo}$ and circumferential magnetic field $B_{\Phi 0}$ of the solenoid at different frequencies, as shown in Figure 2:

![Figure 2](image-url)

Figure 2. Simulation of frequency characteristics of external alternating magnetic field of solenoid

It can be seen from Figure 2 that the solenoid magnetic field has good sinusoidal characteristics when excited by the high frequency current signal. As the frequency increases, the amplitudes of the axial and circumferential magnetic fields first increase and then decrease, and the period of change also decreases.

### 3.2. Radial distribution characteristics of the alternating magnetic field of a solenoid

Study the distribution law of the external magnetic field of the solenoid in the radial direction. At this time, the frequency is taken as $f = 200MHz$, and the solenoid structural parameters and the medium parameters are unchanged. The distances selected are 0.004m, 0.005m, 0.006m, and 0.007m.

The simulation shows the axial magnetic field of the solenoid and the circumferential magnetic field at different distances, as shown in Figure 3:

![Figure 3](image-url)

Figure 3. Radial distribution characteristics of alternating magnetic field outside a solenoid
As can be seen from Figure 3, the solenoid magnetic field has good sinusoidal characteristics. And as the distance increases, the smaller the amplitude of the axial and circumferential magnetic fields, which is consistent with the actual situation, the magnetic field can be considered zero when the distance is infinite. But the cycle of change between the two remains unchanged.

Further comparing the numerical results of the solenoid magnetic field simulation, it is found that the values of the axial magnetic field of the solenoid and the amplitude of the magnetic field in the circumferential direction are basically the same. Therefore, in the analysis of the solenoid magnetic field, both directions of the magnetic field need to be considered.

4. Conclusion
In this paper, Maxwell's equation is used to establish the internal and external magnetic field model of the solenoid driven by the sine wave current signal. The influence of relevant factors on the internal and external magnetic field of the solenoid is analyzed in detail, which better describes the internal and external magnetic field of the solenoid driven by the sine wave signal. The results show that the internal and external magnetic fields of the solenoid driven by sinusoidal current have good sinusoidal characteristics, and the magnetic field is closely related to the parameters of the driving signal frequency. The axial magnetic field at any point inside the solenoid is much larger than this point. Therefore, when approximating the magnetic field at a certain point inside the solenoid, the circumferential magnetic field at that point can be ignored, and only the axial magnetic field at that point can be considered. But outside the solenoid, both directions of the magnetic field need to be considered.

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