Response Function of Asymmetric Nuclear Matter

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Abstract

The charge longitudinal response function is examined in the framework of the random-phase approximation in an isospin-asymmetric nuclear matter where proton and neutron densities are different. This asymmetry changes the response through both the particle-hole interaction and the free particle-hole polarization propagator. We discuss these two effects on the response function on the basis of our numerical results in detail.

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Since the development of experimental devices has enabled us to separate the charge longitudinal (Coulomb) response of nuclei from the spin transverse one in the electron scattering [1,2,3], these two response functions have been widely investigated both experimentally and theoretically [4]. Empirical response functions are now available for a variety of medium and heavy nuclei ranging from 40Ca to 208Pb, most of which have neutron excess. In order to examine the role of the neutron excess, several theoretical attempts have been made to calculate the response function of isospin-asymmetric nuclear matter taking into account the different phase space available for proton and neutron ph states [5,6,7]. This should not be, however, the whole story, because the asymmetry changes also the effective ph interaction [8]. In this letter, we examine how these two effects modify the charge longitudinal response function of the isospin-asymmetric nuclear matter, i.e. (i) the different ph phase space for protons and neutrons, and (ii) the change of the effective ph interaction.

We consider a nuclear matter of which the proton and the neutron densities are given by \( \rho_p, \rho_n \) respectively. Let the \( \varepsilon \) stand for the size of the asymmetry:

\[
\varepsilon = \frac{\rho_p - \rho_n}{\rho_p + \rho_n}.
\]

(1)

First, let us see how this asymmetry enters the expression for the response function \( R(\vec{q}, \omega) \) in the random-phase approximation (RPA) [6, 7], which is given by

\[
R(\vec{q}, \omega) = O^+ \frac{\Pi(\vec{q}, \omega)}{1 - V \Pi(\vec{q}, \omega)} O.
\]

(2)

where \( \vec{q} \) and \( \omega \) are the momentum and the energy transfers by the external operator \( O \). The symbols \( \Pi \) and \( V \) are the free ph propagator (Lindhard function) and the effective interaction (g-matrix), respectively. In the isospin-asymmetric nuclear matter, \( \Pi \) and \( V \) deviate from their symmetric nuclear matter values, and therefore can be expanded in powers of \( \varepsilon \) as

\[
V = V^0 + V^1 + V^2 + \cdots.
\]

(3)

\[
\Pi = \Pi^0 + \Pi^1 + \Pi^2 + \cdots.
\]

(4)

where the indices represent the order in powers of \( \varepsilon \). Substituting eqs. (3) and (4) into eq. (2), we see that in order to obtain the response exactly up to some order in \( \varepsilon \), we have to calculate both \( \Pi \) and \( V \) up to the same order.
We proceed in the following way. First, we explain the effective interaction, and then examine the response function.

The $g$-matrix in an isospin-asymmetric nuclear matter satisfies the Bethe-Goldstone equation formally identical to the one in the symmetric nuclear matter, i.e.

$$ g = v + v G Q g. $$

where $v$ is the bare two-nucleon interaction (Reid soft core potential), $G$, the two nucleon propagator including the self-energy correction, and $Q$, the Pauli exclusion operator. The difference of proton and neutron densities $\rho_p$ and $\rho_n$ generates the isovector component in $Q$ in addition to the isoscalar component present in the symmetric nuclear matter [8]. A detailed description of the resultant interaction will be discussed in a forthcoming paper [9]. Here we just show the part of the interaction which is relevant to the charge longitudinal response in the framework of RPA:

$$ g = g^0 + g^\tau \tau_1 \cdot \tau_2 + g^\alpha (\tau_1 + \tau_2) + \cdots. $$

where $\tau_1$ and $\tau_2$ are the isospin operators of two interacting nucleons. In the above expression, $g^0$ and $g^\tau$ already exist in the $g$-matrix in the symmetric nuclear matter, and $g^\alpha$ is the new component which appears due to the isospin asymmetry and is of first order in $\varepsilon$. The $g$-matrix of eq. (6) is invariant under the exchange of all protons and neutrons including the medium, because $\varepsilon$ changes its sign under such transformation. The ph interaction is derived from the above $g$-matrix as

$$ V_{pp} = g^0 + g^\tau + 2g^\alpha, $$

$$ V_{nn} = g^0 + g^\tau - 2g^\alpha, $$

$$ V_{pn} = V_{np} = g^0 - g^\tau $$

where $V_{pp}$, for example, is the interaction which describes the transition from a proton ph to another proton ph. We show the real and the imaginary parts of $g^0$, $g^\tau$ and $g^\alpha/\varepsilon$ in figures 1-a and 1-b, respectively, as functions of the energy. Note that the imaginary part of the effective interaction is the realization of the two-particle emission in the multiple scattering process described by the Bethe-Goldstone equation, eq. (5). We can see in the figure that the effect of the asymmetry appears mainly in the imaginary part of , and increases in the low
energy side. This is because the asymmetry is more important there than in the high energy region.

Let us now turn to the response function. It is straightforward to write down the expression for the Lindhard function in the isospin asymmetric nuclear matter. Let $\Pi^p$ and $\Pi^n$ be the Lindhard functions for proton and neutron, respectively. The total polarization propagator of eq. (4) can be written as

$$\Pi = \frac{1 + \tau_z}{2} \Pi^p + \frac{1 - \tau_z}{2} \Pi^n,$$


(8)

Actual calculations of the response $R$ of eq.(2) are carried on for the charge longitudinal operator

$$O = \frac{1 + \tau_z}{2} \exp(i \vec{q} \cdot \vec{r})$$

where we take the momentum transfer $q = 2.0$ fm$^{-1}$. Substituting equations (8) and (9) into eq. (2), we arrive at the following expression [6] for the response function $R(q, \omega)$:

$$R(q, \omega) = \frac{\Pi^p(q, \omega)}{1 - \tilde{V}_{pp} \Pi^p(q, \omega)}$$


(10)

where $\omega$ is the energy transfer from the external field, and $\tilde{V}_{pp}$ is a renormalized proton ph interaction which takes into account the neutron ph states, and is given by

$$\tilde{V}_{pp} = V_{pp} + V_{pn} \frac{\Pi^n}{1 - V_{nn} \Pi^n} V_{np}$$


(11)

The strength function, which is the observable in the quasielastic scattering, is then given by

$$-\frac{1}{\pi} Im R = \frac{1}{1 - \tilde{V}_{pp} \Pi^p} \left\{ Im \Pi^p + | \Pi^p |^2 Im \tilde{V}_{pp} \right\}$$


(12)

Now we present our numerical results. In the actual calculation, we have taken the proton and neutron fermi momenta as $k_F^p = 1.20$ fm$^{-1}$ and $k_F^n = 1.39$ fm$^{-1}$, which correspond to $\varepsilon = -0.213$ in eq. (1) (note that $\varepsilon = -0.2$ on average in $^{48}$Ca). Our results of the response function are shown in fig. 2. The dotted curve represents the free response ($-1/\pi Im \Pi^p$), and the dashed curve stands for the response of the symmetric nuclear matter, $R_0$, which is obtained by setting $\varepsilon = 0$ in eq. (12). The solid curve is the response function in the asymmetric nuclear
matter, eq. (12). The dotted-dashed curve shows the same response, but with the real part of the ph interaction alone. The difference between the dotted and the dashed curve shows the well known feature of the charge longitudinal response: the enhancement in the low energy side due to the attractive isoscalar interaction, and the quenching in the high energy side due to the isovector repulsive interaction [10]. A comparison of the dashed and the full curve immediately shows that the effect of the asymmetry slightly enhances the response in the quasielastic region, and quenches in the lower and higher energy side, as compared to the response of the symmetric nuclear matter. In the following, we look into our numerical results more closely.

First, we examine the effect of the imaginary part of our ph interaction, i.e., the difference between the dotted-dashed and the solid curve. We observe that the role of the imaginary part of the ph interaction is to remove the transition strength from the low energy side to the quasielastic and higher energy region where two-nucleon emission channel is more widely open. Second, in order to see separately how the asymmetry in \(V\) and in \(\Pi\) affect the response, we define two response functions \(R_V\) and \(R_{\Pi}\) which are respectively obtained by taking into account the change of the interaction and the ph propagator alone. In fig. 3, we show the differences of \(R_V\) and \(R_{\Pi}\) from the response of symmetric nuclear matter \((R_0)\) by the dotted and the dashed curve. We also show the difference of the full response, \(-1/\pi \text{Im}(R - R_0)\), by the solid curve. The figure shows that the asymmetry in the interaction lowers the response of the symmetric nuclear matter, while the asymmetry in the Lindhard function gives a sizable enhancement as a whole.

Next, we compare the whole strength integrated over all the energy transfer, i.e., the Coulomb sum rule [4, 11, 12]. We find that \(R_V\) and \(R_{\Pi}\) yield a 6 % reduction and a 3 % enhancement of the sum rule, and \(R\) a 3 % reduction, compared to the response of the symmetric nuclear matter.

We now focus on the energy dependence of the three curves in the figure. The behavior of \(-1/\pi \text{Im}(R_V - R_0)\), the dotted curve, can be understood as follows: first, the asymmetry here \((\rho_p < \rho_n)\) enlarges the two-proton emission channel compared to the symmetric case \((\rho_p = \rho_n)\), because the Pauli blocking is less operative for protons - thereby the imaginary part of \(V_{pp}\) of eq.(7) is negative and larger in magnitude than its symmetric limit \(g^0 + g^\tau\). Looking at this effect (enlargement of the two-proton emission channel) as a function of the ph energy \(\varepsilon\), we recognize that (i) for a large \(\omega\), this effect is fully operative because the whole decrement of the phase volume excluded by the Pauli blocking turns into
the increment of the phase space available for emitted protons, while (ii) for a
small w, this effect is less operative because only a part of the above decrement is
available. The combined effect on the response function is to remove the transition
strength from a low energy side to a high energy (several hundred MeV) region
where more two-proton emission channels are coupled to the response, therefore
the behavior of the dotted curve.

The asymmetry in the Lindhard function affects the response in a subtler way.
Here we just explain why this causes a larger deviation from the symmetric case
in the high energy region than the asymmetry in the ph interaction. In fig. 4, a
transition of a ph state to another is shown diagrammatically. The intermediate
states of the multiple scattering process (A in the figure) is accounted for by the
g-matrix, and does not appear explicitly in the actual calculation. On the other
hand, the ph phase space (B in the figure) is taken into account by the Lindhard
function explicitly. Now let us look at the asymmetric part of the proton and
neutron phase space in spaces A and B. Because of the short range nature of the
bare nucleon-nucleon interaction, the phase space A is located much higher in
energy (several hundred MeV) than B (around hundred MeV), which makes the
asymmetric part of the phase space is less important in A than in B. This is the
reason why the asymmetry plays a more important role in $R_\Pi$ than in $R_V$ in the
high energy (around one hundred MeV) region.

In summary, we have pointed out that in order to calculate a response func-
tion of asymmetric nuclear matter, it is necessary to account for the effect of the
asymmetry both in the effective interaction and in the ph propagator. We have
shown, on the basis of our numerical results, that these two effects are opposite in
sign and comparative in magnitude. We have also explained how and why these
two effects appear. It is now clear that further study of the response function of
$N \neq Z$ nuclei should simultaneously take these two effects into account.

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References

1. P. Barreau et al., Nucl. Phys. A402, 515 (1985).
2. Z. E. Meziani et al., Phys. Rev. Lett. 52, 2130 (1984); 54, 1233 (1977).
3. C. C. Blatchley et al., Phys. Rev. C34, 1243 (1986).
4. B. Frois and C. N. Papanicolas, Ann. Rev.Nucl. Part. Sci. 37, 133 (1987).
5. S. Stringari and E. Lipparini, Nucl. Phys. A473, 61 (1987).
6. W. M. Alberico, G. Chanfray, M. Ericson and A. Molinari, Nucl. Phys. A475, 233 (1987).
7. W. M. Alberico, A. Drago and C. Villavecchia, Nucl. Phys. A505, 309 (1989).
8. T. Cheon and K. Takayanagi, Phys. Rev. Lett. 68, 1292 (1992).
9. T. Cheon and K. Takayanagi, in preparation.
10. Shigehara, K. Shimizu and A. Arima, Nucl. Phys., A492, 388 (1989).
11. K. Takayanagi, Nucl. Phys. A516, 276 (1990); A522, 494 (1991); A522, 523 (1991).
12. G. Orlandini and M. Traini, Rep. Prog. Phys. 54, 257 (1991).
Figure Captions

Fig. [I]: The g-matrix of eq. (6) as a function of the starting energy $E_p$. (a) The real part, and (b) the imaginary part.

Fig. [II]: The charge longitudinal response of nuclear matter. The dotted and the dashed curve show the free response and the response of the symmetric nuclear matter, respectively. The solid curve shows the response of the asymmetric nuclear matter of eq. (12). The dotted-dashed curve is the same as the solid curve, but with the real part of the ph interaction alone.

Fig. [III]: The changes of the free response when the modification of the interaction alone (dotted line), of the ph propagator alone (dashed line) and both are taken into account (solid line).

Fig. [IV]: A transition process of a ph state into another. See the text.
Fig. 1

(a) Re $g$ (MeV fm$^3$)

(b) Im $g$ (MeV fm$^3$)
Fig. 2
\[-1 / \pi \, \text{Im} \, \Delta R(q, \omega) \, (x10^{-3} \, \text{MeV}^{-1})\]

Fig. 3
