Gravitational Waves from Phase Transition in Minimal SUSY $U(1)_{B-L}$ Model

Naoyuki Haba and Toshifumi Yamada

Institute of Science and Engineering, Shimane University, Matsue 690-8504, Japan

Abstract

Many extensions of the Standard Model include a new $U(1)$ gauge group that is broken spontaneously at a scale much above TeV. If a $U(1)$-breaking phase transition occurs at nucleation temperature of $O(100)$-$O(1000)$ TeV, it can generate stochastic gravitational waves in $O(10)$-$O(100)$ Hz range if $\beta_n/H_n = 1000$, which can be detected by ground-based detectors. Meanwhile, supersymmetry (SUSY) may play a crucial role in the dynamics of such high-scale $U(1)$ gauge symmetry breaking, because SUSY breaking scale is expected to be at TeV to solve the hierarchy problem. In this paper, we study the phase transition of $U(1)$ gauge symmetry breaking in a SUSY model in the SUSY limit. We consider a particular example, the minimal SUSY $U(1)_{B-L}$ model. We derive the finite temperature effective potential of the model in the SUSY limit, study a $U(1)_{B-L}$-breaking phase transition, and estimate gravitational waves generated from it.
1 Introduction

Many extensions of the Standard Model (SM) include a new $U(1)$ gauge group that is broken spontaneously, important examples being the minimal $U(1)_{B-L}$ model \cite{1,2,3}, the left-right symmetric model \cite{4,5} and Pati-Salam model \cite{6}. Usually, there is no theoretical reason to expect that the breaking scale of such $U(1)$ gauge group is at TeV scale. If the breaking scale is beyond the reach of new gauge boson searches at colliders, observation of stochastic gravitational waves generated from a $U(1)$-breaking phase transition is the key to testing such models \cite{7}. This is because the nucleation temperature of the phase transition is encoded by the peak position of gravitational wave spectrum, and Advanced LIGO \cite{8}, Advanced Virgo \cite{9} and KAGRA \cite{10} cover the region of $O(10) - O(100)$ Hz, which corresponds to nucleation temperature of $O(100) - O(1000)$ TeV if the speed of phase transition over the Hubble rate is 1000. Recent work on gravitational waves from the breaking of a new visible $U(1)$ gauge group that occurs separately from electroweak symmetry breaking includes \cite{11,12,13,14,15}.

If the breaking of a $U(1)$ gauge group occurs at a scale much above TeV, supersymmetry (SUSY) may play a crucial role in its dynamics, since we expect SUSY breaking scale to be at TeV to stabilize the electroweak scale with respect to Planck scale. In this paper, therefore, we study the phase transition of a $U(1)$ gauge symmetry breaking in a SUSY model and gravitational waves generated from it. We work in the SUSY limit, namely, we assume that the nucleation temperature is above the SUSY breaking scale so that soft SUSY breaking terms are negligible in the study of phase transition. For concreteness, we focus on the minimal SUSY $U(1)_{B-L}$ model\footnote{In a different context, gravitational waves in a SUSY $U(1)_{B-L}$ model has been discussed in Ref. \cite{16}.} which is by itself highly motivated because it can explain the origin of the seesaw scale, and if $B - L$ is broken by even charges, $R$-parity is derived and accounts for the stability of dark matter. To simplify our analysis on $U(1)_{B-L}$-breaking phase transition, we assume $R$-symmetry of the model. $R$-symmetry is well-motivated by itself because one can forbid $\mu H_u H_d$ term by $R$-symmetry thereby solving the $\mu$-problem.

Although we concentrate on the minimal SUSY $U(1)_{B-L}$ model, our study is applicable to a wide class of $U(1)$-gauge-extended SUSY models that contain superfields with $U(1)$ charge $+a$ and $-a$ and a gauge singlet $S$ to achieve the $U(1)$ breaking. Remarkably, this $U(1)$ need not be visible, i.e. the SM fields need not be charged under it, for the study of gravitational waves.

We comment in passing that SUSY models are more predictive than non-SUSY models about high-scale $U(1)$-breaking phase transitions. This is because in non-SUSY models where scalar field $\phi$ breaks an extra $U(1)$, no symmetry forbids the Higgs portal term ($H$ denotes the
SM Higgs field),

\[ \lambda_{\phi H} \phi^\dagger \phi H^\dagger H. \quad (1) \]

Suppose \( \phi \) develops a large vacuum expectation value (VEV) much above the electroweak scale. To achieve the correct electroweak symmetry breaking, one has two options; one fine-tunes the portal coupling \( \lambda_{\phi H} \) so that the emergent mass term \( \lambda_{\phi H} |\langle \phi \rangle|^2 H^\dagger H \) is negligible compared to genuine Higgs mass term \( m_H^2 H^\dagger H \); or one assumes that the genuine Higgs mass term nearly cancels the emergent mass term. In the latter case, the study of the \( U(1) \)-breaking phase transition involves the SM Higgs field and depends on unknown genuine Higgs mass term \( m_H^2 \), in addition to the \( U(1) \)-breaking scale. In SUSY models, such Higgs portal coupling is forbidden at the renormalizable level and hence is justifiably neglected.

This paper is organized as follows. In Section 2, we explain the minimal SUSY \( U(1)_{B-L} \) model, and derive the finite temperature effective potential for \( U(1)_{B-L} \)-breaking VEVs. In Section 3, we numerically compute the \( O(3) \)-symmetric Euclidean action for a high-temperature \( U(1)_{B-L} \)-breaking phase transition, calculate quantities that determine gravitational wave spectrum, and estimate stochastic gravitational waves generated from a \( U(1)_{B-L} \)-breaking phase transition. Section 4 summarizes the paper.

# 2 Finite Temperature Effective Potential in the Minimal SUSY \( U(1)_{B-L} \) Model

## 2.1 Minimal SUSY \( U(1)_{B-L} \) Model

The minimal SUSY \( U(1)_{B-L} \) model is defined as follows: The gauge group is \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \). The field content is that of the minimal SUSY Standard Model (MSSM) plus three isospin-singlet neutrinos \( N^c_i \) \( (i = 1, 2, 3) \) and \( \Phi, \overline{\Phi}, S \) with the following charge assignments.

\[ N^c_i : (1, 1, 0, 1), \quad \Phi : (1, 1, 0, -2), \quad \overline{\Phi} : (1, 1, 0, 2), \quad S : (1, 1, 0, 0) \quad (2) \]

As usual, for the MSSM fields, the lepton doublets \( L_i \) have \( B - L = -1 \), the lepton singlets \( E^c_i \) have \(+1\), the quark doublets \( Q_i \) have \( \frac{1}{3} \), the quark singlets \( U^c_i, D^c_i \) have \(-\frac{1}{3}\), and the Higgs fields \( H_u, H_d \) have \( 0 \). The most general superpotential reads

\[ W = W_{\text{MSSM}} + (Y_D)_{ij} H_u L_i N^c_j + Y_{M_i} \Phi N^c_i N^c_i \]
\[ + \lambda S(\overline{\Phi} \Phi - \frac{v^2}{2}) + \frac{m}{2} S^2 + \frac{k}{3} S^3. \quad (4) \]
Here, mass term $\mu \Phi \Phi$ is absorbed by a redefinition of $S$ and $m, \kappa$. $Y_D$ is the neutrino Dirac Yukawa coupling, and $Y_M$ is the coupling that generates Majorana mass for the right-handed neutrinos after $U(1)_{B-L}$ breaking. By a phase redefinition, we take $\lambda, v^2, Y_{Mi}$ to be real positive without loss of generality.

From now on, we assume $|m|^2 \ll v^2$ and $|\kappa| \ll 1$. This limit is obtained when the model has $R$-symmetry, under which superfield $S$ has $R = +2$ and $\Phi, \bar{\Phi}$ have $R = 0$, and the matter superfields have $R = +1$ and the Higgs superfields $H_u, H_d$ have $R = 0$. Assuming $R$-symmetry is advantageous for explaining the smallness of $\mu$ in $\mu H_u H_d$. In the rest of the paper, we neglect $|m|^2, \kappa$ and work with the $R$-symmetric superpotential,

$$W = W_{\text{MSSM}} \mid_{\text{without } \mu\text{-term}} + (Y_D)_{ij} H_u L_i N^c_j + (Y_M)_{ij} \Phi N^c_i N^c_j$$

As the mechanism for SUSY breaking (at zero temperature) is beyond the scope of this paper, we do not discuss soft SUSY breaking gaugino mass. The tree-level scalar potential involving $\Phi, \bar{\Phi}, S$ reads

$$V = \lambda S \Phi + Y_{Mi} N^c_i N^c_i + \lambda^2 |S\Phi|^2 + \lambda^2 \left|\Phi \Phi - \frac{v^2}{2}\right|^2$$

$$+ \frac{1}{2} g_{B-L}^2 \left(-2 |\Phi|^2 + 2 |\Phi|^2 - \sum_i (|N^c_i|^2 - |L_i|^2 + |E^c_i|^2 + \frac{1}{3} |Q_i|^2 - \frac{1}{3} |U^c_i|^2 - \frac{1}{3} |D^c_i|^2)^2 \right)$$

2.2 Finite Temperature Effective Potential

To compute the one-loop effective potential at zero and finite temperature, we need the field-dependent mass eigenvalues for bosonic and fermionic components. When SUSY is preserved, bosonic and fermionic components have the same set of mass eigenvalues. However, since SUSY is already broken at finite temperature, we must also consider SUSY-breaking configurations of VEVs, e.g., the case with $\langle \Phi \rangle \langle \bar{\Phi} \rangle \neq \frac{v^2}{2}$ giving $F$-term SUSY breaking, and the case with $\langle \Phi \rangle \neq \langle \bar{\Phi} \rangle$ giving $D$-term SUSY breaking. So, we study the mass eigenvalues of bosonic and fermionic components separately.

We use Landau gauge for $U(1)_{B-L}$ gauge theory.

Before deriving the field-dependent mass eigenvalues, we assume that the VEVs at any temperature satisfy

$$\langle \Phi \rangle \langle \bar{\Phi} \rangle = \text{(real positive)}, \quad \langle S \rangle = 0, \quad \langle N^c_i \rangle = 0.$$  

\footnote{By abuse of notation, we denote the scalar component by the same character as the superfield.}
Then, we take advantage of the $U(1)_{B-L}$ symmetry to set both $\langle \Phi \rangle, \langle \overline{\Phi} \rangle$ to be real positive, and rewrite these VEVs as

$$\langle \Phi \rangle \equiv \frac{1}{\sqrt{2}}h, \quad \langle \overline{\Phi} \rangle \equiv \frac{1}{\sqrt{2}}\bar{h} \quad (h > 0, \quad \bar{h} > 0).$$

(10)

The rest of the section is devoted to the study on the potential for $h, \bar{h}$.

The $(h, \bar{h})$-dependent mass eigenvalues for bosonic components are given as follows: We decompose the scalar components of $\Phi, \overline{\Phi}$ as $\Phi = \frac{1}{\sqrt{2}}(h + \phi + i\alpha), \overline{\Phi} = \frac{1}{\sqrt{2}}(\bar{h} + \tilde{\phi} + i\bar{\alpha})$ where $\phi, \tilde{\phi}$ represent CP-even components and $\alpha, \bar{\alpha}$ CP-odd components. The $(h, \bar{h})$-dependent mass matrix for $\phi, \tilde{\phi}$ is

$$\frac{1}{2} \begin{pmatrix} \phi & \tilde{\phi} \end{pmatrix} M_{\phi\phi} \begin{pmatrix} \phi \\ \tilde{\phi} \end{pmatrix} \quad \text{with} \quad M_{\phi\phi}^2 = \begin{pmatrix} 2g_{B-L}^2(3h^2 - \bar{h}^2) + \frac{1}{2}\lambda^2\bar{h}^2 & -4g_{B-L}^2 + \lambda^2h\bar{h} - \frac{1}{2}\lambda^2v^2 \\ -2g_{B-L}^2(h^2 - 3\bar{h}^2) + \frac{1}{2}\lambda^2h^2 \end{pmatrix},$$

(11)

and that for $a, \bar{a}$ is

$$\frac{1}{2} \begin{pmatrix} a & \bar{a} \end{pmatrix} M_{a\bar{a}}^2 \begin{pmatrix} a \\ \bar{a} \end{pmatrix} \quad \text{with} \quad M_{a\bar{a}}^2 = \begin{pmatrix} 2g_{B-L}^2(h^2 - \bar{h}^2) + \frac{1}{2}\lambda^2\bar{h}^2 & \frac{1}{2}\lambda^2v^2 \\ -2g_{B-L}^2(h^2 - \bar{h}^2) + \frac{1}{2}\lambda^2h^2 \end{pmatrix},$$

(12)

from which mass eigenvalues are obtained by diagonalization. The $(h, \bar{h})$-dependent masses for $S, N_i^c$ and the MSSM fields are

$$M^2_S|S|^2 + M^2_{N_i^c}|N_i^c|^2 + M^2_L|L_i|^2 + M^2_E|E_i^c|^2 + M^2_{Q_i}|Q_i|^2 + M^2_U|U_i^c|^2 + M^2_D|D_i^c|^2$$

(13)

with

$$M^2_S = \frac{1}{2}\lambda^2(h^2 + \bar{h}^2),$$

(14)

$$M^2_{N_i^c} = g_{B-L}^2(-h^2 + \bar{h}^2) + \frac{1}{2}Y_{M_i}^2h^2,$$

(15)

$$M^2_L = -g_{B-L}^2(-h^2 + \bar{h}^2), \quad M^2_E = g_{B-L}^2(-h^2 + \bar{h}^2), \quad M^2_Q = -\frac{1}{3}g_{B-L}^2(-h^2 + \bar{h}^2),$$

$$M^2_U = M^2_D = \frac{1}{3}g_{B-L}^2(-h^2 + \bar{h}^2).$$

(16)

Note that the mass of the MSSM fields solely comes from $D$-term SUSY breaking. The $(h, \bar{h})$-dependent mass term for the $U(1)_{B-L}$ gauge boson $X_\mu$ is

$$\frac{1}{2} M_X^2 X_\mu X^\mu \quad \text{with} \quad M_X^2 = 4g_{B-L}^2(h^2 + \bar{h}^2).$$

(17)
The $(h, \tilde{h})$-dependent mass eigenvalues of fermionic components are given as follows. Let $\psi_\Phi, \bar{\psi}_\Phi, \psi_S, \psi_{N^c_i}$ denote the fermionic part of $\Phi, \bar{\Phi}, S, N^c_i$, respectively, and let $\tilde{X}$ denote $U(1)_{B-L}$ gaugino. The $(h, \tilde{h})$-dependent Majorana mass matrix for fermionic components is given by

$$
\frac{1}{2} \begin{pmatrix} \psi_\Phi & \bar{\psi}_\Phi & \psi_S & \psi_{N^c_i} & \tilde{X} \end{pmatrix} M_F \begin{pmatrix} \psi_\Phi \\ \bar{\psi}_\Phi \\ \psi_S \\ \psi_{N^c_i} \\ \tilde{X} \end{pmatrix} \quad \text{with} \quad M_F = \begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{2}} \lambda \tilde{h} & 0 & 2g_{B-L} h \\ 0 & \frac{1}{\sqrt{2}} \lambda h & 0 & -2g_{B-L} \tilde{h} \\ 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} Y_{M_i} \tilde{h} & 0 & 0 & 0 \end{pmatrix}.\tag{18}
$$

The mass eigenvalues are obtained by diagonalizing $M_F^\dagger M_F$, and are given by $4g_{B-L}^2(h^2 + \tilde{h}^2)\), $4g_{B-L}^2(h^2 + \tilde{h}^2)\), $\frac{1}{2} Y_{M_i}^2 \tilde{h}^2\), $\frac{1}{2} \lambda^2(h^2 + \tilde{h}^2)\), $\frac{1}{2} \lambda^2(h^2 + \tilde{h}^2)\).

It is easy to verify that when $h = \tilde{h} = v$ so that SUSY is preserved, non-zero mass eigenvalues of bosonic components obtained from Eqs. (11)-(17) coincide with those of fermionic components (with the correct counting of degrees of freedom).

Finally, the finite temperature effective potential \[17\] for $h, \tilde{h}$ is obtained as

$$
V_{\text{eff}}(h, \tilde{h}; \mu, T) = \frac{1}{4} \lambda^2(h \tilde{h} - v^2)^2 + \frac{1}{2} g_{B-L}^2(h^2 - \tilde{h})^2 + \frac{1}{64\pi^2} \sum_j M_{Bj}^4 \left( \log \frac{M_{Bj}^2}{\mu^2} - \frac{3}{2} \right) - \frac{1}{64\pi^2} \sum j M_{Fj}^4 \left( \log \frac{M_{Fj}^2}{\mu^2} - \frac{3}{2} \right) + \frac{T^4}{2\pi^2} \sum_j J_B(M_{Bj}^2/T^2) - \frac{T^4}{2\pi^2} \sum_j J_F(M_{Fj}^2/T^2). \tag{19}
$$

$$
\text{Here, Eq. \[19\] represents the tree-level potential. Eq. \[20\] is the one-loop effective potential at zero-temperature, with } \mu \text{ being the renormalization scale in } \overline{\text{DR}} \text{ scheme. Eq. \[21\] is the temperature-dependent part of the potential, with } J_B(x^2) = \int_0^\infty dy y^2 \log(1 - \exp[y^2 + x^2]) \text{ and } J_F(x^2) = \int_0^\infty dy y^2 \log(1 + \exp[y^2 + x^2]). \text{ } M_{Bj}^2 \text{ denote the } (h, \tilde{h}) \text{-dependent mass eigenvalues for bosonic components, obtained by diagonalizing Eqs. \[11\], \[12\] and from Eqs. \[13\]-\[17\], with no duplication for real scalars, 2 duplications for complex scalars and 3 duplications for } X_\mu \text{ gauge boson. } M_{Fj}^2 \text{ denote the } (h, \tilde{h}) \text{-dependent mass eigenvalues for fermionic components, which are } 4g_{B-L}^2(h^2 + \tilde{h}^2)\), 4g_{B-L}^2(h^2 + \tilde{h}^2)\), \frac{1}{2} Y_{M_i}^2 \tilde{h}^2\), \frac{1}{2} \lambda^2(h^2 + \tilde{h}^2)\), \frac{1}{2} \lambda^2(h^2 + \tilde{h}^2)\), \text{ with 2 duplications for each.}

At temperature near or above the critical temperature, daisy diagrams cause breakdown of perturbation theory. This problem is remedied by replacing the tree-level masses of bosonic components $M_{Bj}^2$ in Eqs. \[11\]-\[17\] with loop corrected ones. \text{We follow Ref. \[18\] and only include } T^2\text{-proportional part of the one-loop correction}^{3} \text{, and make the following replacements}

\[3\] This recipe does not provide a good approximation at low temperature, since the decoupling of particles
for the mass of the scalar components of $\Phi, \overline{\Phi}, S, N_i^c$ and MSSM fields: 

\[
\mathcal{M}_{\phi\phi}^2 \rightarrow \mathcal{M}_{\phi\phi}^2 + \frac{3}{2} T^2 \left( \frac{8 g_{B-L} + 2 Y_i^2}{24} + 4 \lambda^2 \begin{pmatrix} 0 & 0 \\ 0 & 8 g_{B-L}^2 + 4 \lambda^2 \end{pmatrix} + \frac{3}{2} T^2 g_{B-L}^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) 
\]

\[
\mathcal{M}_{\alpha\alpha}^2 \rightarrow \mathcal{M}_{\alpha\alpha}^2 + \frac{3}{2} T^2 \left( \frac{8 g_{B-L} + 2 Y_i^2}{24} + 4 \lambda^2 \begin{pmatrix} 0 & 0 \\ 0 & 8 g_{B-L}^2 + 4 \lambda^2 \end{pmatrix} + \frac{3}{2} T^2 g_{B-L}^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) 
\]

\[
\mathcal{M}_S^2 \rightarrow \mathcal{M}_S^2 + \frac{3}{2} \frac{3}{2} T^2 \lambda^2 
\]

\[
\mathcal{M}_{N_i^c}^2 \rightarrow \mathcal{M}_{N_i^c}^2 + \frac{3}{2} T^2 \left( \frac{8 g_{B-L} + 2 Y_i^2}{24} + \frac{1}{2} T^2 g_{B-L} \right) 
\]

\[
\mathcal{M}_L^2 \rightarrow \mathcal{M}_L^2 + \frac{3}{2} T^2 \left( \frac{8 g_{B-L} + 2 Y_i^2}{24} + \frac{1}{2} T^2 g_{B-L} \right) 
\]

\[
\mathcal{M}_E^2 \rightarrow \mathcal{M}_E^2 + \frac{3}{2} T^2 \left( \frac{8 g_{B-L} + 2 Y_i^2}{24} + \frac{1}{2} T^2 g_{B-L} \right) 
\]

\[
\mathcal{M}_{Q_{1,2}}^2 \rightarrow \mathcal{M}_{Q_{1,2}}^2 + \frac{3}{2} T^2 \left( \frac{8 g_{B-L} + 2 Y_i^2}{24} + \frac{1}{2} T^2 g_{B-L} \right) 
\]

\[
\mathcal{M}_{Q_{3}}^2 \rightarrow \mathcal{M}_{Q_{3}}^2 + \frac{3}{2} T^2 \left( \frac{8 g_{B-L} + 2 Y_i^2}{24} + \frac{1}{2} T^2 g_{B-L} \right) 
\]

\[
\mathcal{M}_D^2 \rightarrow \mathcal{M}_D^2 + \frac{3}{2} T^2 \left( \frac{8 g_{B-L} + 2 Y_i^2}{24} + \frac{1}{2} T^2 g_{B-L} \right) 
\]

\[
\mathcal{M}_{U_{1,2}}^2 \rightarrow \mathcal{M}_{U_{1,2}}^2 + \frac{3}{2} T^2 \left( \frac{8 g_{B-L} + 2 Y_i^2}{24} + \frac{1}{2} T^2 g_{B-L} \right) 
\]

\[
\mathcal{M}_{U_{3}}^2 \rightarrow \mathcal{M}_{U_{3}}^2 + \frac{3}{2} T^2 \left( \frac{8 g_{B-L} + 2 Y_i^2}{24} + \frac{1}{2} T^2 g_{B-L} \right) 
\]

Here, the factor $\frac{3}{2}$ on the second and third terms on the right hand side reflects the fact that in SUSY theories, a bosonic loop correction is always accompanied by a fermionic loop correction with the same coupling constant, and that $T^2$-part of the fermionic one-loop correction to a boson mass is half the bosonic one-loop correction, and hence their sum is $\frac{3}{2}$ times the bosonic one. The bosonic part (i.e. part without factor $\frac{3}{2}$) of the second term comes from one-loop corrections via $D$-term and $F$-term quartic couplings, and that of the third term comes from one-loop corrections via gauge couplings. For the longitudinal component of the $U(1)_{B-L}$ gauge boson, we replace its mass, $(\mathcal{M}_X^2)^L$, as

\[
(\mathcal{M}_X^2)^L \rightarrow (\mathcal{M}_X^2)^L + \frac{3}{2} 8 g_{B-L}^2 T^2, 
\]
while the mass of the transverse component is unchanged.

In the rest of the paper, we use the finite temperature effective potential Eqs. (19)-(21) with replacements Eqs. (22)-(32), to study the $U(1)_{B-L}$-breaking phase transition in the minimal SUSY $U(1)_{B-L}$ model.

2.3 Behavior of the Finite Temperature Effective Potential

We numerically evaluate the finite temperature effective potential $V_{\text{eff}}(h, \bar{h}; \mu, T)$ Eqs. (19)-(21) (with replacements Eqs. (22)-(32)) for several benchmark parameter sets and study its behavior. The benchmarks we take are

$$(\lambda, g_{B-L}, Y_{M3}) = (0.01, 0.4, 1), (0.1, 0.4, 1), (0.01, 0.15, 1), (0.01, 0.4, 0.1)$$

(34)

and we fix $Y_{M1} = Y_{M2} = 0$ (only one right-handed neutrino has a large Majorana Yukawa coupling). We take $\mu = v$, which does not generate a large logarithm because $v$ is the only mass scale in the model. Then, the potential scales with $v^4$, and depends on $h, \bar{h}$ and temperature $T$ only through dimensionless quantities $h/v, \bar{h}/v, T/v$.

In Figs. 1, 2, 3, 4 we present $V_{\text{eff}}(h, \bar{h}; \mu, T) - V_{\text{eff}}(0, 0; \mu, T)$ on $(h, \bar{h})$ plane at the critical temperature $T = T_c$, at a high temperature slightly above $T_c$, and at the nucleation temperature $T_n$ (which we will evaluate in the next section).
Figure 1: $V_{\text{eff}}(h, \bar{h}; \mu, T) - V_{\text{eff}}(0, 0; \mu, T)$ on the plane of $(h, \bar{h})$ at the critical temperature $T = T_c = 0.098v$, at the nucleation temperature (which we will evaluate in Section 3) $T = T_n = 0.026v$, and at a higher temperature $T = 0.11v$, for the parameter set $(\lambda, g_{B-L}, Y_{M3}) = (0.01, 0.4, 1)$. The renormalization scale is set at $\mu = v$. The contours correspond, from outside to inside, to $V_{\text{eff}}(h, \bar{h}; \mu, T) - V_{\text{eff}}(0, 0; \mu, T) = 3 \cdot 10^{-3}v^4, 10^{-3}v^4, 3 \cdot 10^{-4}v^4, 10^{-4}v^4, 3 \cdot 10^{-5}v^4$. We caution that in the left panel, the barrier height is smaller than $3 \cdot 10^{-5}v^4$ and hence is not visible.

Figure 2: The same as Fig. 1 except that the parameter set is $(\lambda, g_{B-L}, Y_{M3}) = (0.1, 0.4, 1)$, namely, $\lambda$ is ten times larger. The temperatures are taken at the critical temperature $T = T_c = 0.232v$, the nucleation temperature $T = T_n = 0.222v$, and a higher temperature $T = 0.3v$. The contours correspond, from outside to inside, to $V_{\text{eff}}(h, \bar{h}; \mu, T) - V_{\text{eff}}(0, 0; \mu, T) = 3 \cdot 10^{-3}v^4, 10^{-3}v^4, 3 \cdot 10^{-4}v^4, 10^{-4}v^4, 3 \cdot 10^{-5}v^4$. 

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Figure 3: The same as Fig. 1 except that the parameter set is \((\lambda, g_{B-L}, Y_{M3}) = (0.01, 0.15, 1)\), namely, \(U(1)_{B-L}\) gauge coupling is smaller. The temperatures are taken at the critical temperature \(T = T_c = 0.074v\), the nucleation temperature \(T = T_n = 0.038v\), and a higher temperature \(T = 0.1v\). The contours correspond, from outside to inside, to \(V_{\text{eff}}(h, \bar{h}; \mu, T) - V_{\text{eff}}(0, 0; \mu, T) = 10^{-3}v^4, 3 \cdot 10^{-3}v^4, 10^{-3}v^4, 3 \cdot 10^{-4}v^4, 10^{-4}v^4\).

Figure 4: The same as Fig. 1 except that the parameter set is \((\lambda, g_{B-L}, Y_{M3}) = (0.01, 0.4, 0.1)\), namely, Majorana Yukawa coupling \(Y_{M3}\) is 1/10 smaller. The temperatures are taken at the critical temperature \(T = T_c = 0.105v\), the nucleation temperature \(T = T_n = 0.057v\), and a higher temperature \(T = 0.11v\). \(V_{\text{eff}}(h, \bar{h}; \mu, T) - V_{\text{eff}}(0, 0; \mu, T) = 10^{-3}v^4, 3 \cdot 10^{-3}v^4, 10^{-3}v^4, 3 \cdot 10^{-4}v^4, 10^{-4}v^4\).

Figs. 1, 2, 3, 4 show that the finite-temperature effective potential is nearly symmetric with respect to \(h\) and \(\bar{h}\) at temperature around or below the critical temperature \(T_c\). This indicates that even though only \(\Phi\), not \(\bar{\Phi}\), couples to the right-handed neutrino through Majorana Yukawa coupling \(Y_{M3}\), this asymmetry does not affect the potential.

Since the potential is nearly symmetric with respect to \(h\) and \(\bar{h}\), we can approximate the classical tunneling path from the metastable vacuum \((h, \bar{h}) = (0, 0)\) to an absolute vacuum.
by the line $h = \bar{h}$, because $\partial_h V_{\text{eff}} - \partial_{\bar{h}} V_{\text{eff}} \simeq 0$ and hence the equation of motion (with $-V_{\text{eff}}$) for $h - \bar{h}$ only admits a trivial solution $h - \bar{h} \simeq \text{(constant)}$. Under the above approximation, the phase transition is controlled by one-dimensional potential $V_{\text{eff}}(h, \bar{h}; \mu, T)$, which allows a qualitative discussion. The one-dimensional potential reads

$$V_{\text{eff}}(h, \bar{h}; \mu, T) = \frac{1}{4} \lambda^2 (h^2 - v^2)^2$$

(35)

$$+ \frac{1}{64\pi^2} \sum_j M_{Bj}^4 \left( \log \frac{M_{Bj}^2}{\mu^2} - \frac{3}{2} \right) - \frac{1}{64\pi^2} \sum_j M_{Fj}^4 \left( \log \frac{M_{Fj}^2}{\mu^2} - \frac{3}{2} \right)$$

(36)

$$+ \frac{T^4}{2\pi^2} \sum_j J_B \left( \frac{M_{Bj}^2}{T^2} \right) - \frac{T^4}{2\pi^2} \sum_j J_F \left( \frac{M_{Fj}^2}{T^2} \right)$$

(37)

where $M_{Bj}^2$ are now obtained by diagonalizing

$$\mathcal{M}_{\phi\phi} = \begin{pmatrix} (4g_{B-L}^2 + \frac{1}{2}\lambda^2)h^2 & (-4g_{B-L}^2 + \lambda^2)h^2 - \frac{1}{2}\lambda^2 v^2 \\ (4g_{B-L}^2 + \frac{1}{2}\lambda^2)h^2 & \frac{T^2}{16} \left( 32g_{B-L}^2 + 2Y_{M\bar{\mu}} + 4\lambda^2 \right) \end{pmatrix}$$

(38)

$$\mathcal{M}_{\phi\phi} = \begin{pmatrix} \frac{1}{2}\lambda^2 \left( h^2 \bar{v}^2 \right) + \frac{T^2}{16} \left( 32g_{B-L}^2 + 2Y_{M\bar{\mu}} + 4\lambda^2 \right) & 0 \\ 0 & 32g_{B-L}^2 + 4\lambda^2 \end{pmatrix}$$

(39)

and also from $\mathcal{M}_{\phi\phi} = \lambda^2 h^2 + \frac{1}{4}T^2 \lambda^2$, $\mathcal{M}_{N^c N^c} = \frac{1}{2}Y_{N^c\bar{\mu}}^2 h^2 + \frac{T^2}{8} (g_{B-L}^2 + 2Y_{M\bar{\mu}}^2) + \frac{3}{8} T^2 g_{B-L}^2$ (2 duplications for each), $(\mathcal{M}_{\phi\phi}^L)^L = 8g_{B-L}^2 h^2 + 12g_{B-L}^2 T^2$, and $(\mathcal{M}_{\phi\phi}^T)^T = 8g_{B-L}^2 h^2$ (2 duplications), while the MSSM particles become irrelevant. One might guess that increasing $g_{B-L}$ and decreasing $\lambda$ enhances the order of phase transition and hence the amount of latent heat, because the quartic coupling for $h$ is mostly $\lambda$, and the field-dependent mass for bosons $\phi, \bar{\phi}, X_\mu$ (which provides $h^3$ term in high-$T$ expansion) depends on $g_{B-L}^2 h^2$ times a big factor 4 or 8. However, increasing $g_{B-L}$ also enhances the thermal mass for these bosons (except for the transverse component of $X_\mu$), which diminishes their impact on the finite temperature effective potential. Therefore, we expect that the amount of latent heat (which is related to $\alpha_\theta(T_n)$ in the next section) is maximized for $\lambda \to 0$ and for some moderate value of $g_{B-L}$. On the other hand, $Y_{M\bar{\mu}}$ is expected to have a weaker impact on the latent heat because it only appears in the field-dependent mass for $N^c_i$ and is not accompanied by a big factor. All these expectations will be confirmed by the numerical study in the next section.
3 \( U(1)_{B-L} \)-breaking Phase Transition

3.1 \( O(3) \)-symmetric Euclidean Action

We calculate the \( O(3) \)-symmetric Euclidean action \([22, 23]\) for a high-temperature \( U(1)_{B-L} \)-breaking phase transition from the metastable vacuum \((h, \bar{h}) = (0, 0)\) to an absolute vacuum where \((h, \bar{h}) \neq (0, 0)\). Although we have seen in Section 2.3 that the potential is nearly symmetric with respect to \(h\) and \(\bar{h}\), we still consider a multi-field phase transition regarding \(h\) and \(\bar{h}\) as being independent. To compute the \( O(3) \)-symmetric Euclidean action for a multi-field phase transition, we use CosmoTransitions \([24]\). From the action computed, we derive the nucleation temperature, \(T_n\), the ratio of the trace anomaly divided by 4 over the radiation energy density of the symmetric phase at the nucleation temperature, \(\alpha_\theta(T_n)\), and the speed of the phase transition at the nucleation temperature, \(\beta_n\). They are defined as follows:

Let \( S_E(T) \) denote the Euclidean action. The tunneling rate per volume at temperature \(T\) is \(\Gamma/V = A(T)e^{-S_E(T)/T}\), where \(A(T)\) is a factor with milder \(T\)-dependence than \(e^{-S_E(T)/T}\). The nucleation temperature \(T_n\) satisfies

\[
H_n^4 = A(T_n)e^{-S_E(T_n)/T_n},
\]

where \(H_n\) denotes the Hubble rate at \(T = T_n\) in the symmetric phase. We estimate \(A(T_n)\) as \(A(T_n) \sim T_n^4\), and further approximate \(T_n\) by the \(U(1)_{B-L}\)-breaking VEV \(v\). Thus, we estimate \(T_n\) by the relation

\[
\frac{S_E(T_n)}{T_n} \sim -\log \left( \frac{g_* \pi^2}{30} T_n^4 \right),
\]

where \(M_*\) is the reduced Planck mass and \(g_* = 255\) is the effective relativistic degrees of freedom of the SUSY \(U(1)_{B-L}\) model (including \(\Phi, \bar{\Phi}, S\) fields). For example, when \(v = 100\) TeV, the right-hand side of Eq. (40) equals 117, and when \(v = 1000\) TeV, it equals 107. In the following analysis, we fix the right-hand side of Eq. (40) at about 117. \(\alpha_\theta(T_n)\) is given by

\[
\alpha_\theta(T_n) = \frac{1}{g_* \frac{\pi^2}{30} T_n^4} \left( -\frac{T}{4} \frac{\partial \Delta V}{\partial T} + \Delta V \right) \bigg|_{T=T_n}, \quad \Delta V = V|_{\text{symmetric phase}} - V|_{\text{broken phase}}.
\]

\(\beta_n\) satisfies

\[
\beta_n = -\frac{d}{dT} \left( \frac{S_E(T)}{T} \right) \bigg|_{T=T_n} = -H_n T \frac{d}{dT} \left( \frac{S_E(T)}{T} \right) \bigg|_{T=T_n}.
\]

As with Section 2.3, we take the renormalization scale at \(\mu = v\), which does not generate a large logarithm because \(v\) is the only mass scale in the model. Given \(\mu = v\), a quantity with mass dimension \(n\) scales with \(v^n\). In particular, \(T_n\) scales with \(v\), and so we present \(T_n/v\) in the plots.

In Fig. 5 we plot \(g_B-L\)-dependence of the nucleation temperature \(T_n\), for \(\lambda = 0.01, 0.1\) and \(Y_{M3} = 1, 0.1\), with \(Y_{M1} = Y_{M2} = 0\).
We find that $T_n/v$ has little dependence on $Y_{M3}$, and is much affected by $\lambda$.

In Fig. 5 we plot $g_{B-L}$-dependence of the trace anomaly divided by 4 over the radiation energy density $\alpha_\theta(T_n)$, for $\lambda = 0.01, 0.1$ and $Y_{M3} = 1, 0.1$, with $Y_{M1} = Y_{M2} = 0$. (We fix $Y_{M1} = Y_{M2} = 0$.)

$\alpha_\theta(T_n)$ is significantly enhanced for $\lambda = 0.01$ compared to the case with $\lambda = 0.1$. Interestingly, $\alpha_\theta(T_n)$ is maximized at $g_{B-L} \simeq 0.4$ when $\lambda = 0.01$, and at $g_{B-L} \simeq 0.6$ when $\lambda = 0.1$.

The dependence on $Y_{M3}$ is quite mild compared to those on $g_{B-L}$ and $\lambda$.

In Fig. 6 we plot $g_{B-L}$-dependence of the speed of phase transition in units of the Hubble rate at the nucleation temperature $\beta_n/H_n$ in logarithm, for $\lambda = 0.01, 0.1$ and $Y_{M3} = 1, 0.1$, with $Y_{M1} = Y_{M2} = 0$. (We fix $Y_{M1} = Y_{M2} = 0$.)
Figure 7: The speed of phase transition in units of the Hubble rate at the nucleation temperature $\beta_n/H_n$ Eq. (42), for various values of $U(1)_{B-L}$ gauge coupling $g_{B-L}$ and for $\lambda = 0.01, 0.1$ and $Y_{M3} = 1, 0.1$. (We fix $Y_{M1} = Y_{M2} = 0$.)

$\beta_n/H_n$ is exponentially enhanced for small values of $g_{B-L}$. In contrast, the dependence on $Y_{M3}$ is negligible.

Finally, we study how the above quantities vary with $\lambda$. We concentrate on an interesting case where $g_{B-L} = 0.4$ and $Y_{M3} = 1$, which has given the largest $\alpha_\theta(T_n)$ in the above plots when $\lambda = 0.01$. The dependence of $T_n, \alpha_\theta(T_n), \beta_n/H_n$ on $\lambda$ for $g_{B-L} = 0.4$ and $Y_{M3} = 1$ is found in Fig. 8.
Figure 8: The dependence of $T_n$, $\alpha_\theta(T_n)$, $\beta_n/H_n$ on $\lambda$ for $g_{B-L} = 0.4$ and $Y_{M3} = 1$.

It is observed that $T_n$ and $\beta_n/H_n$ increase linearly with $\lambda$, while $\alpha_\theta(T_n)$ decreases much rapidly.

### 3.2 Gravitational Waves

We estimate gravitational waves generated from a $U(1)_{B-L}$-breaking phase transition in the early Universe. In this subsection, we exclusively study the case with $\lambda = 0.1$, which gives $\alpha_\theta(T_n) \lesssim 0.1$ (see the right panel of Fig. 6). This selection is because the study on gravitational wave production in a strong phase transition $\alpha_\theta(T_n) > 0.1$ is currently under development (see, e.g., Refs. [25, 26]), while that in a weaker phase transition is relatively well established.

The sources of gravitational waves from a finite-temperature phase transition are (i) the energy momentum tensor of scalar field in colliding bubbles, (ii) that of sound waves of a surrounding plasma, and (iii) that of magnetohydrodynamic turbulence of a surrounding plasma. Source (i) is negligible unless the bubble wall runs away [28], and we expect non-run away behavior in a $U(1)_{B-L}$-breaking phase transition and hence neglect (i). For source (iii), the study
on the efficiency of converting latent heat to the energy of magnetohydrodynamic turbulence is at its early stage, and so we omit (iii) making a conservative estimate. Thus, we only consider source (ii).

It is claimed in Ref. [28] that the energy spectrum of gravitational waves generated by sound waves in a hot plasma in a phase transition with $\alpha_\theta(T_n) \lesssim 0.1$ can be expressed as (we rewrite the formula for gravitational wave energy over the critical density we observe today)

$$\frac{d\Omega_{gw}(k)}{d\log k} \bigg|_{\text{today}} = 3H_nL_{f,n} \frac{1}{2\pi^2} (kL_f)^3 (1 + \bar{p}/\bar{\tau})^2 \bar{U}_f \bar{P}_{gw}(kL_f) \times 2.6 \cdot 10^{-5} \left( \frac{255}{g_*} \right)^{1/3}, \quad (43)$$

where $L_{f,n}$ is a typical length scale of fluid motion at the nucleation temperature, $L_f$ is the redshifted value of $L_{f,n}$ today, and $\bar{P}_{gw}$ is a function only of the product $kL_f$. $\bar{U}_f$ is the enthalpy-weighted root mean square four-velocity of fluid at the nucleation temperature, and $1 + \bar{p}/\bar{\tau}$ is the ratio of enthalpy over energy. In this paper, we adopt Eq. (43). We further identify $L_{f,n}$ with the mean bubble separation $(8\pi)^{1/3}v_w/\beta_n$ [30] ($v_w$ denotes the bubble wall speed), and for $\bar{P}_{gw}$, we use a fitting of the simulation results in Ref. [29], which has improved on earlier works [27, 28]. For $(1 + \bar{p}/\bar{\tau})\bar{U}_f^2$, we use a fitting formula for the ratio of bulk kinetic energy over vacuum energy $\kappa(\alpha_\theta, v_w)$ derived in Ref. [31], and evaluate it as

$$(1 + \bar{p}/\bar{\tau}) \bar{U}_f^2 = \frac{\alpha_\theta(T_n)}{1 + \alpha_\theta(T_n)} \kappa(\alpha_\theta(T_n), v_w). \quad (44)$$

The calculation of the bubble wall speed $v_w$ is beyond the scope of the current paper, and we simply assume various values of $v_w$ that appear in the simulations of Ref. [29] and evaluate gravitational wave spectrum in each case.

Since we take the renormalization scale at $\mu = v$, $v$ is the only mass scale and a quantity with mass dimension $n$ scales with $v^n$. Accordingly, we present the gravitational wave spectrum in terms of frequency $k$ divided by the tree-level $U(1)_{B-L}$-breaking VEV $v$.

Our estimate on gravitational wave spectrum is presented in Fig. 9 for $\lambda = 0.1$ and for various values of $g_{B-L}$. Here, we fix $Y_{M3} = 1$ and $Y_{M1} = Y_{M2} = 0$. Lowering $Y_{M3}$ reduces the energy of the spectrum, but does not significantly change its shape and position. This is because $\alpha_\theta(T_n)$ decreases for $Y_{M3} = 0.1$, while $T_n$ and $\beta_n/H_n$ have little or no dependence on $Y_{M3}$ when $g_{B-L} \geq 0.25$, as seen in the right panels of Figs. [30] [7].
Figure 9: Energy spectrum of stochastic gravitational waves from a $U(1)_{B-L}$ breaking phase transition in the case where $\lambda = 0.1$. The horizontal line is frequency $k$ divided by the tree-level $U(1)_{B-L}$-breaking VEV $v$ in units of 100 TeV. From the upper-left to lower-right, each plot corresponds to different values of the $U(1)_{B-L}$ gauge coupling constant, $g_{B-L} = 0.25, 0.3, 0.55, 0.75$. In each plot, the black-solid, red-dashed and blue-dotted lines correspond to different assumptions on the bubble wall velocity with $v_w = 0.92, 0.72, 0.44$. We fix $Y_{M3} = 1$ and $Y_{M1} = Y_{M2} = 0$. Lowering $Y_{M3}$ reduces the energy of the spectrum, but does not significantly change its shape and position.

We find that stochastic gravitational waves are out of reach of Advanced LIGO and Virgo [32] for all values of $g_{B-L}$, provided $\lambda = 0.1$. However, if $\lambda$ is smaller than 0.1 and if the $U(1)_{B-L}$-breaking VEV is about 100 TeV or below, stochastic gravitational waves can be detected by these detectors. This is inferred from the fact that as seen in Fig. 8 $T_n$ and $\beta_n/H_n$ have linear, and thus mild dependence on $\lambda$, and so the shape and position of the spectrum does not change significantly with $\lambda$. In contrast, $\alpha_\theta(T_n)$ has violent dependence on $\lambda$, and so slight decrease in $\lambda$ can enhance the energy of the spectrum at the detectable level. To make a reliable prediction in the case with $\lambda < 0.1$, we need a further understanding of gravitation wave production in a phase transition with $\alpha_\theta(T_n) > 0.1$. 

4 Summary

We have studied the phase transition of a $U(1)$ gauge symmetry breaking in a SUSY model and the production of stochastic gravitational waves associated with it. We have concentrated on a particular model, which is the minimal SUSY $U(1)_{B-L}$ model with $R$-symmetric superpotential. We have worked in the SUSY limit by assuming that the nucleation temperature is above SUSY breaking scale so that soft SUSY breaking terms are negligible. We have derived the finite temperature effective potential for the $U(1)_{B-L}$ VEVs $h, \bar{h}$, and computed the $O(3)$-symmetric Euclidean action of a high-temperature $U(1)_{B-L}$-breaking multi-field phase transition. We have estimated stochastic gravitational waves generated from the phase transition, and discussed its detectability.

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