Coherently slowing light with a coupled optomechanical crystal array

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Abstract - We study the propagation of light in a resonator optical waveguide consisting in an evanescently coupled optomechanical crystal array. In the strong-driving limit, the Hamiltonian of the system can be linearized and diagonalized. In this case we obtain the polaritons, which are formed by the interaction of photons and the collective excitation of mechanical resonators. By analyzing the dispersion relations of polaritons, we find that the band structure can be controlled by changing the related parameters. It has been suggested that an engineerable band structure can be used to slow and stop light pulses.

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Introduction. – Owing to its important application in many fields, such as low-threshold lasing [1], pulse delay [2] and optical memories [3–5], slow light attracts a great deal of practical interest. A number of schemes to delay and store light have been suggested, such as electromagnetically induced transparency (EIT) in the atomic ensembles [6,7], photonic-crystal waveguide band edges [8,9], solid-state multilayer semiconductor structure [10], coupled resonator optical waveguide (CROW) [11–14], more complicated hybrid structure, e.g., coupled resonator optical waveguide doped with atoms [15–18].

For a static photonic structure, for example a bare CROW, due to the limitation of the delay-bandwidth product constraint, it is not suitable to stop light. To dynamically stop and release the light, a dynamically tunable system is required. Fan suggested that, if there are extra resonators side-coupling to the optical cavity cells of the CROW, the Fano interference can lead to a large change of the bandwidth of the system when a small refractive index modulation is employed [12]. The velocity of light can therefore be dynamically slowed down and even stopped. Unlike the case of EIT, the light is coherently stored in a static way in the resonance cavity array. Based on this idea, researchers have replaced optical resonators with atoms to couple to the resonators in the CROW and found that the light can be converted to collective excitations of atoms and then reversely converted and released [15,18].

Optomechanics opens a door to directly control the mechanical motion with light [19]. Many applications of optomechanics have been proposed, for example, using cooled nanomechanical oscillators to test quantum mechanics [20], ultra-sensitive detection of force and displacement [21,22], quantum optics and quantum information processing [23–25]. Meanwhile, as a new quantum system, optomechanics is also used to stop light. The EIT effect in cavity optomechanical system with a Bose-Einstein condensate (BEC) is suggested to slow the light [26]. Research groups led, respectively, by Painter [27] and Kippenberg [28] proposed the slowing-down of light based on EIT in optomechanics. The photons are mapped onto the phonon modes instead of onto internal atomic degrees of freedom in the case of EIT in atomic ensembles. In fact, as in the case of a CROW, an optical waveguide coupled to an optomechanical crystal array has been suggested to slow and stop the light pulse [29].

Motivated by the work mentioned above, this paper investigates the photon transmission in a homogeneous side-coupled optomechanical crystal array, in which

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Our scheme offers a patternable, compact and on-chip array of mechanical modes and then they can be stopped. The photons can be mapped onto the collective excitation of optical mode. By adjusting the refractive index of the photonic crystal, the band structure of CROW can be modulated, making mechanical mode and optical modes allowable for the photonic crystal. Each optomechanical crystal cell can be divided into common and differential modes of in-plane and out-plane motion of these nanomechanical resonators. For simplicity, we just consider the case where the gaps between the nanomechanical resonators time independent, i.e., the common mode case. Therefore, the coupling between neighboring optical cavities is constant. To excite the system, a probe optical signal is dropped in the optomechanical array in a side-coupled configuration, and the output signal is dropped out in a similar manner. With this consideration, the Hamiltonian of the system in the reference frame rotating with probe laser frequency \( \omega_p \) can be written as:

\[
H = H_0 + H_{int},
\]

\[
H_0 = \sum_i \hbar \delta \hat{a}_i^\dagger \hat{a}_i + \sum_j \hbar \omega_m \hat{b}_j^\dagger \hat{b}_j, \quad (1a)
\]

\[
H_{int} = \sum_j \hbar g \left( \hat{b}_j + \hat{b}_j^\dagger \right) \hat{a}_j^\dagger \hat{a}_j - \sum_j \hbar G \left( \hat{a}_j^\dagger \hat{a}_{j+1} + \hat{a}_{j+1}^\dagger \hat{a}_j \right). \quad (1b)
\]

Here \( \hat{a}_j^\dagger (\hat{a}_j) \) and \( \hat{b}_j (\hat{b}_j) \) are the creation (annihilation) operators of the optical cavity mode and the mechanical mode in the \( j \)-th optomechanical cell, respectively. \( \delta = \omega_c - \omega_p \) is the detuning between cavity field and probe laser, \( \omega_m \) is the mechanical resonator angular frequency, the constant \( g \) is the coupling strength between cavity field and mechanical resonator and \( G \) denotes the nearest-neighbor evanescent coupling of the intercavity.

When the intracavity fields have a large amplitude, i.e., in the strong-driving limit, we can linearize the Hamiltonian by setting \( f = (f) + \delta f \) \( (\hat{a}_j, \hat{a}_j^\dagger, \hat{b}_j, \hat{b}_j^\dagger) \), where \( (f) \) is the steady mean value and \( \delta f \) is the corresponding fluctuation around its steady value. With this ansatz, we then obtain the linearized Hamiltonian:

\[
H_0 = \sum_j \hbar \omega_m \delta \hat{b}_j^\dagger \hat{b}_j + \sum_i \hbar \delta \hat{a}_j^\dagger \hat{a}_j, \quad (2)
\]

\[
H_{int} = \sum_j \hbar g \left( \delta \hat{a}_j^\dagger \hat{b}_j + \hat{a}_j \delta \hat{b}_j^\dagger \right) - \sum_j \hbar G \left( \delta \hat{a}_j^\dagger \delta \hat{a}_{j+1} + \delta \hat{a}_{j+1}^\dagger \delta \hat{a}_j \right), \quad (3)
\]

where the effective detuning is \( \delta = \delta + \frac{\hbar}{2} \left( \langle \hat{b} \rangle + \langle \hat{b}^\dagger \rangle \right) \) and the effective coupling between light and mechanical vibration is \( \tilde{g} = g(a) \). In eq. (3) we have omitted the counter-rotational wave term in the interaction between the cavity field and the mechanical vibration.

**Dispersion of polariton.** – Let us study the Hamiltonian in the \( k \)-representation. Taking into account the periodic properties of the system, we can make Fourier transformations,

\[
A_k = \frac{1}{\sqrt{N}} \sum_j \delta \hat{a}_j e^{ikjL}, \quad (4a)
\]

\[
A_k^\dagger = \frac{1}{\sqrt{N}} \sum_j \delta \hat{a}_j^\dagger e^{-ikjL}, \quad (4b)
\]

\[
B_k = \frac{1}{\sqrt{N}} \sum_j \delta \hat{b}_j e^{ikjL}, \quad (4c)
\]

\[
B_k^\dagger = \frac{1}{\sqrt{N}} \sum_j \delta \hat{b}_j^\dagger e^{-ikjL}, \quad (4d)
\]

where \( A_k (A_k^\dagger) \) are the normal mode operators of the coupled optical cavity, \( B_k (B_k^\dagger) \) are the boson operators to
describe the collective excitation (phonon) of the mechanical resonators, \( k = 2\pi n/LN \) with \( n = 0, 1, \ldots, N - 1 \), and \( L \) is the distance of the intercavity. To satisfy the periodic boundary condition we used here, the number of photonic-crystal cavities \( N \) is required to be large. Inserting the above transformation relation into eqs. 1(a)–(c), we arrive at the new Hamiltonian

\[
H = \sum_k \hbar (\omega_m - i\gamma_m) B_k^\dagger B_k + \sum_k \hbar (\omega_{ph}(k) - i\gamma_c) A_k^\dagger A_k + \sum_k \hbar g \left( A_k^\dagger B_k + B_k^\dagger A_k \right).
\]

(5)

Here \( \omega_{ph}(k) = \tilde{\delta} - 2G \cos(kL) \) is the original dispersion property of the photon dependent on the quasi-momentum \( k \) in the side-coupled photonic-crystal cavity array. The Hamiltonian (5) describes the interaction of the photonic and phononic modes.

To decouple the Hamiltonian (5), we introduce the Bogoliubov transformation,

\[
A_k = u C_k + v D_k,
\]

(6a)

\[
A_k^\dagger = u C_k^\dagger + v D_k^\dagger,
\]

(6b)

\[
B_k = v C_k - u D_k,
\]

(6c)

\[
B_k^\dagger = v C_k^\dagger - u D_k^\dagger,
\]

(6d)

and the inverse transformation is

\[
C_k = u A_k + v B_k,
\]

(7a)

\[
C_k^\dagger = u A_k^\dagger + v B_k^\dagger,
\]

(7b)

\[
D_k = v A_k - u B_k,
\]

(7c)

\[
D_k^\dagger = v A_k^\dagger - u B_k^\dagger.
\]

(7d)

Since operators \( C_k \) and \( D_k \) must satisfy the Bosonic commutation relations

\[
\left[ C_k, C_k^\dagger \right] = 1,
\]

(8)

\[
\left[ D_k, D_k^\dagger \right] = 1,
\]

(9)

the transformation coefficients \( u \) and \( v \) have the relation \( u^2 + v^2 = 1 \). By substituting eqs. 6(a)–(d) into eq. (5), the Hamiltonian can be rewritten as

\[
H = (\omega_{ph}(k)u^2 + \omega_m u^2 - 2\tilde{g}uv)D_k^\dagger D_k + (\omega_{ph}(k)v^2 + \omega_m v^2 + 2\tilde{g}uv)C_k^\dagger C_k + ((\omega_{ph}(k) - \omega_m)uv + \tilde{g}(v^2 - u^2))D_k C_k^\dagger + ((\omega_{ph}(k) - \omega_m)uv + \tilde{g}(v^2 - u^2))D_k^\dagger C_k.
\]

(10)

Obviously, if

\[
(\omega_{ph}(k) - \omega_m)uv + \tilde{g}(v^2 - u^2) = 0,
\]

the Hamiltonian will have the diagonalized form

\[
H = \sum_k \hbar \omega_D(k)D_k^\dagger D_k + \sum_k \hbar \omega_C(k)C_k^\dagger C_k.
\]

(12)

From eqs. (7) and (12) one can note that the \( C_k \) and \( D_k \) operators represent two types of elementary excitations (phonon-photon polaritons) in the coupled optomechanical array system, which is the result of the coherent mixing of photons and phonons through coupling in each optomechanics cell. The dispersion relations of lower and upper branches polaritons are determined by

\[
\omega_C(k) = \frac{1}{2} \left( \omega_m + \omega_{ph}(k) - \sqrt{4\tilde{g}^2 + (\omega_{ph}(k) - \omega_m)^2} \right),
\]

(13)

\[
\omega_D(k) = \frac{1}{2} \left( \omega_m + \omega_{ph}(k) + \sqrt{4\tilde{g}^2 + (\omega_{ph}(k) - \omega_m)^2} \right).
\]

(14)

It is found that the original single optical band structure is split into two bands owing to the interaction between photons and phonons.

**Slowing light with tunable band structure.** – The bandwidths of lower and upper branches, respectively, are

\[
\Delta_{WC} = \omega_C(\pi) - \omega_C(0) = 2G + \Delta,
\]

(15)

\[
\Delta_{WD} = \omega_D(\pi) - \omega_D(0) = 2G - \Delta,
\]

(16)

where

\[
\Delta = \sqrt{\tilde{g}^2 + (\Delta_{OM}/2 - G + i\gamma_m)^2},
\]

(17)

\[
- \sqrt{\tilde{g}^2 + (\Delta_{OM}/2 + G + i\gamma_m)^2},
\]

(18)

with the detuning \( \Delta_{OM} = \tilde{\delta} - \omega_m \). Compared to the original optical band, the bandwidth of the lower-branch polariton is enlarged and the upper one is compressed. Moreover, the most important thing is that both of these bandwidths of lower and upper bands can be modulated by changing parameters, such as \( \tilde{g}, G \) and \( \Delta_{OM} \). Figure 2 shows a typical picture of the bandwidth of the lower-branch polariton dependent on \( \Delta_{OM} \), from which one can note that the bandwidth decreases with increasing \( \Delta_{OM} \).

In more detail, when \( \Delta_{OM}/2 \ll -\tilde{g} \), the bandwidth of the lower band \( \Delta_{WC} \approx 4G \), corresponding to the maximum bandwidth; when \( \Delta_{OM}/2 \gg \tilde{g} \), the bandwidth is approximately equal to zero.
Fig. 2: (Color online) The bandwidth of the lower-branch polariton as a function of $\Delta_0$. The insets are the corresponding band structures of the polariton when detuning $\Delta_{OM} = -100$ (a), $\Delta_{OM} = 0$ (b) and $\Delta_{OM} = 100$ (c). The other parameters are $G=1$, $g=5$, $\omega_m = 100$. Here $\Delta_{OM}$ is in units of $G$.

On the other hand, it is known that the group velocity of polaritons in a lattice is related to dispersion, 

$$v_{C,D} = \frac{\partial \omega_{C,D}(k)}{\partial k} = GL \sin(kL) \left( 1 + \frac{\omega_{ph}(k) - \omega_m}{\omega_{CD}} \right)$$

which is also dependent on the parameters $\tilde{g}$, $G$ and $\Delta_{OM}$ and therefore can be tuned. Such a tunable band structure leads to a tunable group velocity. Figure 3 shows the lower-branch polariton with momentum $kL = \pi/2$. We observe that its group velocity decreases rapidly from its maximum value $G$ and vanishes as $\Delta_{OM}$ increases. In fact, such tunable band structure can play an important role in optical communication and quantum memory; for example, Fan suggested using a tunable CROW to slow and stop the light pulse [31].

Here we briefly demonstrate the process, taking the lower-branch polariton as an example, to slow the light pulse in an optomechanical crystal array. To begin with, the optical cavity is adjusted to be resonant with laser frequency, so the detuning is $\Delta_{OM}/2 = -\omega_m/2 \ll \tilde{g}$. At this point, the bandwidth of the lower branch is largest and can accommodate the entire light pulse, and the lower-branch polariton is made up of photons, as shown in fig. 4. After the pulse enters completely into the optomechanical array, we then compress the bandwidth of the lower-branch polariton adiabatically by tuning the resonance frequency of the optical cavity until $\Delta_{OM}/2 = \omega_m/2 \gg \tilde{g}$. By further compressing the bandwidth, more and more photons are converted to mechanical modes in the lower branch; meanwhile, the velocity of polaritons slows down and approaches to zero.

Fig. 3: (Color online) Velocity of the lower-branch polariton. Other parameters are the same as in fig. 2.

From the point of conversion between photons and mechanical collective excitations, one can also understand the mechanism of light stopping. Because the total excitations number $N_k = B_k^\dagger B_k + A_k^\dagger A_k$ commutes with the Hamiltonian $H$, when adjusting some parameters, such as $\Delta_{OM}$, the total excitations number is conserved, while the numbers of photons and mechanical collective excitations $A_k^\dagger A_k$ and $B_k^\dagger B_k$ are not conserved since they do not commute with the Hamiltonian. Hence the photons and the mechanical collective excitations are mutually convertible, which results in mapping the light onto the mechanical vibration and vice versa. Figure 4 illustrates the conversion between the photons and the mechanical collective excitations. The number of mechanical collective excitation increases, from zero to unity, while the number of photons decreases from unity to zero, with increasing detuning $\Delta_{OM}$.

We keep in mind that the rate of tuning cavity frequency should be less than the band gap between the upper and
lower branches, which is given by

\[ \Delta_W(k) = \omega_D(k) - \omega_C(k) \]
\[ = \sqrt{4g^2 + (\Delta_{OM} - 2G \cos(kL))^2}. \]  

(20)

This limitation prevents the polaritons in the lower branch from jumping up to the upper branch. Obviously, when \( \Delta_{OM} \approx 2G \cos(kL) \), the band gap reaches a minimum value \( 2g \).

When studying the effect of the optical cavity and mechanical resonator decays, we should replace \( \omega_{cm} \) by \( \omega_{cm} - i\gamma_{cm} \). In this case the eigenfrequencies of lower and upper branches polaritons become

\[ \omega_C(k) = \frac{1}{2}(\omega_m - i\gamma_m + \omega_{ph}(k) - i\gamma_c - \omega_D), \]  

(21)

\[ \omega_D(k) = \frac{1}{2}(\omega_m - i\gamma_m + \omega_{ph}(k) - i\gamma_c + \omega_D), \]  

(22)

where

\[ \omega_{CD} = \sqrt{4g^2 + (\omega_{ph}(k) - i\gamma_c - \omega_m + i\gamma_m)^2}. \]  

(23)

The complex eigenfrequency implies that the lifetime of the polaritons \( \tau \) is finite, which is expressed as

\[ \tau^{-1}_{CD} = \Im(\omega'_{C,D}(k)), \]  

(24)

\[ = \frac{1}{2} (\gamma_m + \gamma_c). \]  

(25)

Equation (24) shows us that the storage time of the light pulse is determined by the decay of the optical cavity and mechanical resonator. Here we take the typical parameters \( \omega_c/2\pi = 200 \text{THz}, \omega_m/2\pi = 10 \text{GHz}, \omega_c/\gamma_c = 10^4, \) and \( \omega_m/\gamma_m = 10^3 \) as an example. In this situation, \( \gamma_m = 0.005 \gamma_c, \) thus \( \tau^{-1}_{CD} \approx \gamma_c = 0.2 \text{GHz}. \)

To practically stop the light in the coupled optomechanics array, we tune the detuning between optical cavity field and probe laser by adjusting the refractive index of the material, e.g., Si, which makes up the optomechanical crystal. In our scheme, the amplitude of the detuning modulation is of the same order of magnitude of the mechanical resonance frequency, so the refractive index shift should be

\[ \frac{\delta n}{n} \approx \frac{\delta \omega}{\omega_0} \sim 10^{-5}, \]  

(26)

which is feasible in practical optoelectronic devices [32]. Here we have taken the typical parameters \( \omega_c/2\pi = 200 \text{THz}, \omega_m/2\pi = 10 \text{GHz}, \omega_c/\gamma_c = 10^4, \) and \( \omega_m/\gamma_m = 10^3 \) [30].

Conclusion. – We have studied the model of light transmission in a spatial periodic optomechanical crystal array. The optical cavities of the array evanescently couple to each other one by one to form a CROW. In the strong-driving limit, we linearized the system and obtained the dispersion relations of lower- and upper-branch polaritons with the Bogoliubov transformation in the momentum space. Our results show that the modulation of detuning between optical cavity and laser light can vary the bandwidths of the polaritons, which has been demonstrated to be able to stop and release a light pulse.

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