Fermion parity gap and topological degeneracy of one dimensional chain of fermions with spin-orbit coupling, Zeeman field, and intrinsic attractive interaction

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We examine the topological properties of a one-dimensional (1D) chain of fermions with spin-orbit coupling, Zeeman field, and attractive Hubbard interaction by numerically computing the pair binding energy, excitation gap, and susceptibility to local perturbations by density matrix renormalization group. Such a system can in principle be realized in a 1D optical lattice. We find that, in the presence of spatial interfaces introduced by a smooth parabolic potential, the variation of the pair binding energy and the excitation gap with the system size indicate an exponentially vanishing fermion parity gap and topological ground state degeneracy in the thermodynamic limit, consistent with recent works. However, the susceptibility of the ground state degeneracy to local perturbations indicate that the vanishing of the fermion parity gap in this number conserving system scales as a power-law in system size. We compare the present system with the more familiar system of an Ising antiferromagnet in the presence of a transverse field realized with Rydberg atoms, and argue that the degeneracy of this conventional symmetry-breaking system is identical to the 1D chain of fermions considered. Therefore, the degeneracy properties of the 1D chain of fermions cannot be attributed to topology.

Solid state semiconductor thin films and nanowires are theoretically predicted to support a topological superconducting (TS) phase in the presence of a proximity induced superconducting pair potential \(\Delta_{\text{ind}}\). Rashba spin-orbit coupling (SOC) \(\alpha\), and an externally applied Zeeman field \(h\), in the parameter space spanned by the weak coupling mean field equation \(h^2 > (\Delta_{\text{ind}}^2 + \mu^2)^{12}\). The TS phase is defined by the emergence of mid-gap non-Abelian topological quasiparticles known as Majorana zero modes (MZMs) localized at the topological defect. These systems have become leading candidates for the realization of topological quantum computation (TQC) owing to the tremendous experimental progress realized in recent years.

In parallel with the solid state systems, it has also been proposed that the system of ultracold fermions confined in optical lattice, in the presence of SOC, Zeeman field, and a mean-field \(k\)-wave superfluid pair potential supports a topological superfluid phase with MZMs as edge modes. Although effective SOC and Zeeman field can be generated in systems of ultracold atoms confined to one dimension (1D) by Feshbach resonance, the study of 1D fermions with intrinsic attractive interactions induced by Rashba SOC and Zeeman field in the framework of mean-field theory is problematic. This is because the reduced dimensionality results in strong pair phase fluctuations, and true superfluid long-range order in 1D is destroyed. This leads to the fundamental question: Can a 1D chain of fermionic atoms with SOC and Zeeman field support a TS phase with MZMs in the presence of intrinsic attractive interactions? The answer is important both for theory and experiments, since in addition to answering the related question of the existence of MZMs in a number conserving system (the 1D system is number conserving because the mean field superfluid order parameter \(\Delta_s = 0\)), the answer is also crucial for the ongoing experimental search for TS phase in ultracold atom systems.

It has been recently proposed that a 1D Fermi gas with SOC, Zeeman field, and intrinsic attractive interactions can support a TS phase in the presence of a smooth parabolic potential which is natural in ultracold atom systems confined by a harmonic trap potential. The parabolic potential introduces smooth interfaces between alternating “topological” and “trivial” regions created due to a spatially varying chemical potential. Here, the “topological” region is defined by the condition, \(|h| > \mu_{\text{eff}}\), where \(\mu_{\text{eff}} = \mu - V(r)\) represents the effective chemical potential (measured from \(k = 0\)) that includes the parabolic trap potential. The “crossing points” defined by \(|h| = \mu_{\text{eff}}\) mark the smooth interfaces between the “topological” and “trivial” regions of the 1D Fermi gas and have been proposed to host isolated MZMs for the appropriate values of the parameters. The ground state is doubly degenerate (up to exponentially small splitting for finite sized system) because of a topological degeneracy associated with fermion parity of the topological segments. Hence, an exponential decay of the excitation gap \(\Delta\) with increasing system size \(N\) and a similar decay in the pair binding energy indicating the absence of a fermion parity gap could confirm the existence of TS phase and MZMs in the 1D Fermi gas with attractive interactions.

In this work, we numerically study a 1D Fermi system, similar to the one proposed in Ref. (Fig. 1), to search for qualitative signatures of a phase transition in spectral properties such as the fermion parity gap and the excitation gap. We examine whether the excitation gap vanishes exponentially with size, considered to be an indication of the existence of a TS phase, or follows a power law as observed in number conserving systems. The question of exponential splitting of the ground state degeneracy is critical to the definition of a TS phase and is at the heart of topological protection of a putative Majorana qubit. As is clear from the Luttinger Liquid (LL) analysis of a spin-orbit coupled Fermi gas with attractive interactions, Bosonization of a clean system leads to an exponential degeneracy in pairs of SOC gases. Within this formalism, power law splitting can be generated by back-scattering induced phase slip terms shared between pairs of
wire. Microscopically, the back-scattering originates (at weak interactions) from scattering between different Fermi surfaces, and therefore, requires a breaking of momentum conservation by some impurity. In this work, we find that, in the presence of a smooth parabolic potential and for appropriate values of the parameters such as Zeeman field and interaction strength, the pair binding energy and the excitation gap of the clean 1D wire with attractive Hubbard interaction vary with the system size in a manner consistent with vanishing of the parity gap and a two fold ground state degeneracy in the thermodynamic limit, consistent with a recent work.

In the later part of this work we focus on a numerical study of the expectation values of local operators and the effect of impurity potentials on the ground state degeneracy. Consistent with expectations from previous work, we conclude that the ground state degeneracy of the spin-orbit coupled 1D Fermi gas with intrinsic attractive Hubbard interaction ceases to be exponential in the presence of local perturbations. We then compare the qualitative behavior discussed above with that of the degeneracy of quantum Ising anti-ferromagnets. We argue that the qualitative behavior of the degeneracy in the two systems is identical. Since the degeneracy of the quantum Ising anti-ferromagnet is associated with symmetry breaking, one cannot establish that the 1D fermi gas with attractive interactions is topological based on its degeneracy properties.

We consider a 1D Fermi gas with attractive on-site interaction $H_U$, uniform Zeeman field $H_Z$ and Rashba spin-orbit interaction $H_{SOC}$. A parabolic potential with tunable parameter $k$ controls the electron density profile ($H_{para}$). The Zeeman field ($h$) is applied in the $z$-direction and SOC interactions, with strength $\alpha$, act along a transverse direction ($x$-axis). The model Hamiltonian for the system can be written as,

$$H = H_t + H_U + H_{SOC} + H_Z + H_{para},$$

where,

$$H_t = -t \sum_{i,\sigma} \left( C_{i,\sigma}^\dagger C_{i+1,\sigma} + h.c. \right), \quad H_U = U \sum_i n_{i,\uparrow} n_{i,\downarrow},$$

$$H_{SOC} = +i \alpha \sum_i \left( C_{i,\uparrow}^\dagger C_{i+1,\downarrow} + C_{i,\downarrow}^\dagger C_{i+1,\uparrow} - h.c. \right),$$

$$H_Z = h \sum_i S_i^z, \quad H_{para} = \left( \frac{1}{2} k' r^2 \right) (n_{i,\uparrow} + n_{i,\downarrow}).$$

Here, the nearest neighbor hopping amplitude, $t = 1$, defines the scale for all other parameters in the calculations. $U$ is the on-site attractive Hubbard potential. The parabolic potential is of the form of $V(r) = \frac{1}{2} k' r^2$, where $k' = k/N^2$, $r = \frac{N+1}{2} - i$ with $i$ referring to the site index of the 1D lattice.

We study a low filling fraction of the electrons, $\nu = 0.10$, similar to where the electron filling fraction $\nu$ is defined as, $\nu = n/2N$, with $n$ representing the number of electrons in the $N$ orbital system. We focus on the attractive interaction regime $U \in [-1, -4]$.

We have used the density matrix renormalization group (DMRG) method, a state of the art numerical technique for calculating the eigenvalues and eigenvectors of low-dimensional systems, for the Hamiltonian in Eq. (1). This technique is based on the systematic truncation of the irrelevant degrees of freedom of the system, and the accuracy of the calculations depends on the number $(m)$ of most relevant degrees of freedom retained in successive steps. In the fermionic system under study, the spin degrees of freedom are not conserved; therefore, the Hamiltonian dimension is significantly large. The eigenvectors of the Hamiltonian, corresponding to $m \simeq 700$ largest eigenvalues of the density matrix have been retained to maintain a reliable accuracy. More than 10 finite DMRG sweeps have been performed for each calculation. The error in calculated energies is less than 1%.

We study two spectral characteristic of the system - the vanishing pair binding energy ($E_b$) or the parity gap and an exponential decay of the energy gap $\Delta$, defined as,

$$E_b(n, N) = \frac{1}{2} \left[ E_0(n + 1, N) + E_0(n - 1, N) - 2E_0(n, N) \right]$$

$$\Delta(n, N) = E_1(n, N) - E_0(n, N)$$

$E_0(n, N)$ and $E_1(n, N)$ are the ground state energy and the first excited state energy with $n$ electrons in the system of size $N$. In the absence of $U$ and $\alpha$, the spin up and spin down electronic bands split in the presence of $h$. But to create intra-band pairing correlations, an attractive $U$ is needed. Now the SOC interactions applied along $x$-direction generate a momentum dependent magnetic field along the $x$-axis (Insets: Fig. 1).

The non-topological phase is expected to be adiabatically connected to the conventional $s$-wave superconductor with Cooper pair as the only low energy degrees of freedom. The topological phase, on the other hand, is expected to harbor low-energy fermionic end modes, so that the fermion parity of the system is no longer gapped. We start by numeri-

![FIG. 1. (Color online) Schematic of the spatial profile of $\mu_{eff}$ in our model system. $r$ represents the 1D lattice index. For the “topological” region (shaded green), $h^2 > \mu_{eff}^2$, the rest represent “trivial” regions (shaded maroon). MZMs are predicted to occur at the crossing points, where $h^2 = \mu_{eff}^2$. Insets: Dispersion relations corresponding to each region. $\epsilon_F$ represents the Fermi energy.](image-url)
FIG. 2. (Color online) Variation of the pair binding energy $E_b$ with $1/N$, for various attractive Hubbard interaction strength $U$, for $\alpha = 0.20$, $h = 0.40$, $k = 3$, at $\nu = 0.10$. The dashed curves (green and red) represent power law fitting with finite intercept and the solid curve (black) represents a vanishing exponential. The fitting parameters, $(A, \alpha, \xi)$ for $U = -1.00$ are $(8.76, 1.39, 0.014)$. The power law fitting parameters $(C, B)$ for $U = -1.80$ and $U = -4.00$ are $(0.064, 1.23)$ and $(0.57, 0.58)$, respectively. The power law fitting for $U = -4.00$ and $U = -1.80$ and the non-zero intercept for $E_b$ in the thermodynamic limit indicate a non-topological phase for these parameters. The a vanishing intercept for $U = -1.00$ indicates possible existence of a topological phase for weakly attractive $U$, which is qualitatively consistent with the theoretical predictions in Ref. [39] for $\nu = 0.10$. The dashed curves represent power law fitting and the solid curve represents exponential fitting. The exponential fitting parameters, $(A, \alpha, \xi)$ for $U = -1.00$ are $(2.82, 0.88, 0.0018)$. The power law fitting parameters, $(G, \gamma)$ for $U = -1.80$ and $-4.00$ are $(1.45, 0.72)$ and $(1.78, 0.85)$, respectively.

FIG. 3. (Color online) Variation of $\Delta$ with $N$, for various Hubbard $U$, with fixed parameters, $\alpha = 0.20$, $h = 0.40$ and $k = 3$, at $\nu = 0.10$. The dashed curves represent power law fitting and the solid curve represents exponential fitting. The exponential fitting parameters, $(A, \alpha, \xi)$ for $U = -1.00$ are $(2.82, 0.88, 0.0018)$. The power law fitting parameters, $(G, \gamma)$ for $U = -1.80$ and $-4.00$ are $(1.45, 0.72)$ and $(1.78, 0.85)$, respectively.

physically searching for qualitative differences between the putative topological and non-topological phases by studying the size dependence of the parity gap $E_b$. We have shown the variation of $E_b$ with $N$ for different attractive Hubbard potentials $U$ in Fig. [2]. We find that the parity gap $E_b$, for the stronger binding energies $U = -4.00$ and $U = -1.80$ saturate to a finite intercept as $N$ increases. This is consistent with conventional Cooper pairing expected for the non-topological phase. In contrast, the parity gap $E_b$ is seen to vanish in the thermodynamic limit for a weakly attractive potential, $U = -1$, suggesting a possible topological phase in this limit. Here, parabolic potential with $k = 3$ has been kept fixed, and moderate Zeeman field ($h = 0.40$) and SOC strength ($\alpha = 0.20$) have been used.

Next, in Fig. [3] we show the variation of the excitation gap $\Delta$ for different attractive potentials, $U = -1.00, -1.80$ and $-4.00$. All the other parameters have been kept same as in Fig. [2]. First we note that the power-law decrease of the excitation energy with system size $N$ in the non-topological phase for $|U| = 4.00, 1.80$ is consistent with the excitations arising from phonon modes. By contrast, we find that the energy gap $\Delta$ for $U = -1$ fits better with an exponential dependence with the system size than a pure power-law. This, apart from possible finite size errors, is qualitatively consistent with the behavior of the pair binding energy $E_b$ shown in Fig. 2, demonstrating the possible existence of an exponential ground state degeneracy and a TS phase for $U = -1$. These results are also consistent with the theoretical predictions of the existence of a TS phase in spin-orbit-coupled Fermi gas in the presence of a parabolic trap potential[39]. The exponential suppression of $\Delta$ with $N$ is also consistent with numerics on a spinless analog of this system[39]. Below, we examine other indications of a topological phase - expectation values of a local operator and susceptibility to local perturbations - to carefully examine if the 1D spin-orbit coupled Fermi gas, even with exponential ground state degeneracy, can truly be considered a topological system.

Exponential splittings are only one indication of a topological phase. True topological degeneracy would require the exponentially small splitting to be robust to local perturbations. This is related to another indicator of a topological phase: the absence of an order parameter, or equivalently, local indistinguishability[50] of the pair of topological states. To determine whether the two exponentially degenerate topological states are locally distinguishable, we study a local operator defined as, $\Delta n_x = n_x - n_0 > 0$. $\Delta n_x$ represents the difference in local charge density between the lowest excited state, $|1\rangle$, and the ground state, $|0\rangle$, at a local position $(x)$ of the system. The averaged difference in the charge density $\overline{\Delta n_x}$ taken over the entire system - which is a global operator, vanishes for any arbitrary system size. This is because ours is a charge conserving system. However, we find that the local measurements at, say $x = 0.4N$ and $0.5N$, show a power law dependence of $\Delta n_x$ on $N$ (Fig. [4] for system sizes studied up to $N = 160$. Power law variation of $\Delta n_x$ with $N$ is observed for any other $x$ on the 1D wire too. This is in contrast to an exponential decay of $\Delta n_x$ as expected from a topological system. This observation suggests that the apparent exponential degeneracy of the number conserving spin-orbit coupled 1D Fermi gas is possibly different from what is expected in a topological phases.

While the tests based on exponential splitting of degener-
acy and local indistinguishability appear to produce conflicting evidence for topological degeneracy, local indistinguishability is intimately connected to the robustness of the topological degeneracy to local perturbations. To present this point more concretely, we consider impurity potentials at two sites in the bulk of the system, written as,

$$H_{\text{im}} = V_{\text{im}} \left(n_{N/2} + n_{N/2+1}\right)$$  \hspace{1cm} (3)

We take the values for $V_{\text{im}}$ in the range of $\Delta(N)$. In the absence of any impurity, $\Delta$ vanishes exponentially with $N$, at $U = 1.00$. On application of $V_{\text{im}} = 0.02$ and $0.10$, $\Delta$ now decays as a power law with $N$, as shown in Fig. 4. This indicates a definite departure from a topological phase, if any, that may have existed in the absence of the impurity potentials.

Let us now address the question of whether the ground state degeneracy, even if exponential, could truly be considered a topological degeneracy. The first order perturbation theory imply that the back-scattering potential, which is a local operator, has different expectation values in the two states of the putative topological qubit. The power law nature of the back-scattering potential is verified by our numerical results shown in Fig. 5. The local distinguishability of the two states involved in the ground state degeneracy as shown in Fig. 5 violates one of the key criteria for a topological qubit. To further illustrate the necessity of this criterion, let us consider a comparison to the degeneracy for a transverse field Ising antiferromagnetic chain realized with Rydberg atoms. Assuming the ordering direction to be along the $z$ direction, the ground state of the Ising anti-ferromagnet is two fold degenerate between states that have a non-zero on-site magnetization $<S_{z,j}> \neq 0$, where $S_{z,j}$ is the $z$ component of the magnetization at site $j$. The degeneracy in the Ising model is not topological but rather on associated with spontaneous breaking of the Ising symmetry ($S_z > -S_z$) generated by $S_{x,\text{tot}} = \sum_j S_{x,j}$. However, the symmetry-breaking is qualitatively different from a Ferromagnet in the sense that the magnetic order varies in space $<S_{z,j}> = (-1)^j M$, where $M$ is the amplitude of the order parameter. The two ground states of the Ising anti-ferromagnet are associated with opposite signs of $M$ and are split by an tunneling amplitude that goes to zero exponentially in the length of the system. Similar to the cold Fermi gas, this degeneracy is not split by a uniform symmetry breaking Zeeman field in the $z$ direction as long as the Zeeman field varies slowly in space as long as there are an even number of spins. This is because both states have vanishing total magnetization in the $z$-direction. However, this degeneracy can be seen to be non-topological from the fact that coupling to a magnetic impurity that creates a Zeeman field on a specific site would split the degeneracy by a finite amount. This is analogous to the back-scattering-induced splitting in the 1D Fermi gas. Technically, the symmetry breaking from local impurities is stronger in our example than in the 1D Fermi gas. This can be eliminated by considering a Wigner solid version where the position of the magnetic moments in the Ising anti-ferromagnet are considered to be fluctuating along the chain. This can be expected to introduce power-law fluctuations in $M$ that would reduce the magnetic impurity-induced splitting to powerlaw. This behavior of the degeneracy is qualitatively similar to the cold Fermi gas. Therefore, the exponential degeneracy in the presence of smooth potentials cannot be taken to be an indication of a topological protected degeneracy.

In summary, we discuss the possibility of the existence of a topological superfluid phase with the associated MZMs in
1D fermion systems in the presence of attractive interactions, spin-orbit coupling, and a Zeeman field, upon application of a confining parabolic potential as in a harmonic trap in the ultracold atom systems. We find that despite the exponential ground state degeneracy, shown in particular by the behavior of the pair binding energy, strictly speaking the spin-orbit coupled 1D Fermi gas is not in a topological phase by virtue of failing the crucial test of local indistinguishability. Because of the existence of a non-zero expectation value of a local operator that is able to distinguish the pair of phases involved in the ground state doublet, the ground state degeneracy is not topological, leading to its elimination by local perturbations such as impurity potentials.

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