EFFECTIVE STRING AMPLITUDES FOR HADRONIC PHYSICS

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ABSTRACT

We propose using the general structure and properties of conformal field theory amplitudes, in particular those defined on surfaces with boundaries, to explore effective string theory amplitudes for some hadronic processes. Two examples are considered to illustrate the approach. In one a natural mechanism for chiral symmetry breaking within the string picture is proposed. One consequence is that the vertex operator for pion emission (at zero momentum) behaves like a world sheet current evaluated on the string boundary. This fact is used to rederive, in a more general setting, hadronic mass relations found in the early days of string theory by Lovelace, and Ademollo, Veneziano and Weinberg. In the second example, we derive the general structure of the form factor for the emission of a pomeron (interpreted as a closed string) from a meson or baryon. The result reconciles the interpretation of the pomeron as a closed string, emitted from the interior of the meson or baryon world sheet, with the additive quark rules for total hadronic cross sections. We also review the difficulties involved in constructing complete effective string theories for hadrons, and comment on the relation between the intercepts of trajectories and the short distance behavior of the underlying theory.
1. Introduction

Originally string theory was a phenomenological theory of hadronic physics, developed to describe some striking features of hadrons and their interactions. Today, given the status of fundamental superstrings as that branch of theoretical physics perhaps least constrained by experiment, this history is often viewed as an ironic curiosity. The fact remains, however, that the experimental evidence for the string behavior of hadrons in some processes is as compelling today as it was 20 years ago (if not more so).† To a reasonable approximation the known mesons and baryons do lie on linear Regge trajectories. A wealth of high energy, modest momentum transfer, elastic scattering data is modelled well by single Reggeon (open string) exchange, with the exchanged trajectories coinciding with those found from the hadronic spectra. Ad–hoc phenomenological models incorporating Regge behavior and duality (e.g., the Koba–Nielsen amplitudes) often provide a reasonable fit with few free parameters to the experimental data for many scattering processes. Moreover there is considerable theoretical prejudice (encouraged by the flux tube picture, and the area confinement law found in the strong coupling expansion) that some limit of QCD (perhaps \( N_c \to \infty \)) should be exactly equivalent to some string theory.\(^4\)

There are, however, fundamental differences between the behavior of QCD and that of the complete, mathematically consistent string theories which we know how to construct. QCD displays distinctly un–string like behavior at short distances, where the interactions of small numbers of point–like quarks and gluons dominate (e.g., in deep inelastic, or high energy fixed angle, scattering). In all of the string models which have been fully constructed to date, the space–time string coordinates are represented by free massless bosons on the string world sheet, and no qualitatively new behavior appears even for arbitrarily short distances in space–time. If we consider mesons as valence quarks and anti–quarks linked by a QCD

\(^{†}\) Relevant data can be found in several reviews [1]. For Regge phenomenology see [2], and for a summary of some phenomenological applications of dual string models, with references, see [3].
flux tube then it is clear that the naive string picture must be modified (or break
down altogether) for processes in which the thickness of the flux tube becomes
important.

It is likely that this mistreatment of the short distance structure in the known
string models is ultimately responsible for the most persistent and troublesome
pathology of these theories from the point of view of hadronic physics: the ap-
pearance of undesirable states in the spectrum, i.e., the closed string trajectory
with intercept two which contains the graviton and the open string trajectory
with intercept one which includes a massless gauge boson. We would argue that
the intercepts of these trajectories are artifacts of considering unphysically short
strings, and that the leading effect (on the long distance physics) of altering the
short distance behavior of the string is, in fact, a shift in the intercepts of the
trajectories.

The present work is motivated by the following questions: To what ex-
tent can some string theory serve as an effective theory for QCD within the kinematical
regimes for which string behavior is observed in the data and expected on general
grounds? That is, can some consistent and tractible string theory usefully describe
hadronic string behavior if we agree to exclude processes where short distance
parton–parton interactions are known to dominate? Or, is it the case that some
corner of a complete string model (e.g., some trajectories, the closed string sector,
some highly excited states, or loop amplitudes) cannot be neatly divorced from the
short distance physics and will necessarily lead to undesirable features even in the
long distance predictions of the model? In this case can we still extract sensible
phenomenology from pieces of string models (at the level of individual amplitudes)
without the existence of a completely consistent theory? Is it possible with modern
string technology to systematize and justify the considerable empirical successes of
the (often ad–hoc) Regge and dual model phenomenology of 20 years ago?

We will not give definitive answers to these questions here. In the next sec-
tion we discuss some of the general issues and difficulties involved in formulating
theories of hadron strings, and provide some background and motivation for the approach employed in subsequent sections. In sections 3 and 4 we explore specific features of hadron physics within the string picture but in the absence of any completely satisfactory string model. Our chief tool is the understanding of the basic structure and properties of conformal field theory amplitudes,[5] especially those defined on surfaces with boundaries. The philosophy advocated is the usual one for effective theories (for example chiral Lagrangians[8]). We assume that many important features of the physics are direct consequences of the underlying symmetries involved. Any toy effective theory with these symmetry properties built in will necessarily correctly describe these physical features. We build in some physics assumptions, use our knowledge of the general structure of string amplitudes to isolate properties following from “string behavior”, and postulate these as correct physical features, notwithstanding the many problems which exist in the toy models considered. Apriori there is much guess work involved in deciding what constitutes “string behavior” and in deciding how certain physical features should be incorporated into the string picture; fortunately there exists a wealth of data from low energy hadronic physics which can be consulted for guidance, along with the knowledge that the underlying theory is QCD.

In section 3 we postulate a mechanism for chiral symmetry breaking within the string picture based on the generic behavior of conformal field theories defined on surfaces with boundaries. Some of the symmetry present in a bulk conformal field theory is necessarily broken by the choice of boundary conditions when world sheet boundaries are included. In this picture chiral symmetry breaking necessarily follows from confinement and the existence of mesons. Incorporating chiral symmetry is in itself sufficient to guarantee the usual current algebra results for low energy pion interactions. The incorporation of string behavior leads to additional results. The vertex operator for pion emission behaves like a world sheet current evaluated on the string boundary, and from this behavior follow mass relations between hadron states (call them $A$ and $A^*$) which are related by $S$-wave pion emission, $\alpha'(M_{A^*}^2 - M_A^2) = 1/2 \pmod 1$. These are reasonably well satisfied
experimentally.

In section 4 we consider closed strings, the pomeron, and total hadronic cross sections. We concentrate on a feature of the data which potentially poses a serious difficulty for the string picture. To an accuracy of about 5% total hadronic cross sections at high energy satisfy an additive quark rule, as if the pomeron coupled locally to the valence quarks in each hadron. In the string picture the pomeron is a closed string, which is emitted from anywhere in the interior of the string world sheet, and there seems to be no origin for any simple additive behavior. In section 4 we consider the form factor for the coupling of closed to open strings and show how an approximate additive behavior in fact generically arises. This gives some insight into why the naive quark model is so successful even for some processes where it seems inappropriate.

As one would expect, given the attention accorded to string models of hadrons in their first incarnation, there are few topics in this field without direct ancestors in the early literature. Some of the background provided in section 2 would have been considered common knowledge 15-20 years ago, but is included here because that is no longer the case today. The general picture of chiral symmetry breaking in string theory considered in section 3 can be viewed as a natural extension of a picture originally advocated by Susskind and collaborators in the parton string framework.\(^\star\) The possibility that chiral symmetry incorporated into dual models leads to mass relations between hadrons dates back to a classic paper of Lovelace\(^9\), and a generalization due to Ademollo, Veneziano, and Weinberg\(^{10}\). Their discovery within the context of Veneziano type models is placed on a more general footing within string theory here. In addition, there were extensive studies of the pomeron–meson form factor within various string models in the early literature\(^{13,14,15}\) which serve as a starting point for the discussion of section 4.

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\(^\star\) The incorporation of current algebra results into string amplitudes for pions was also considered from a somewhat different point of view in [12].
2. Effective hadronic strings

Consider pion–nucleon elastic scattering. If the momentum transfer is very large, perturbative QCD is valid, and we can view the event as the hard scattering of a quark or anti–quark in the pion off of a single quark in the nucleon, with additional hard gluons exchanged within the individual hadrons to prevent their fragmentation. The fixed angle scattering amplitude falls off as a power of the center of mass energy, $A \sim s^{-3}$, by the dimensional counting rules. On the other hand, when all of the momenta involved are small, pions and nucleons behave like point particles. An effective Lagrangian which incorporates an approximate (spontaneously broken) chiral symmetry gives a good description of the scattering event.

An effective string theory description for pion–nucleon scattering is useful if there exists some intermediate energy regime in which the scattering process is dominated by the exchange of extended, string–like, configurations. In this picture $\pi N$ scattering is dominated by the exchange of the $\rho$–trajectory (the $\rho$ meson together with the infinite tower of excited “$\rho$–strings" with ever increasing angular momentum), and the amplitude exhibits the corresponding Regge behavior, $A \sim s^{\alpha_{\rho}(t)}$, for large $s$. The principle evidence for this string behavior is the linearity of the $\rho$–trajectory function, $\alpha_{\rho}(t)$, as determined from meson spectroscopy (for $t > 0$) and from charge exchange $\pi N$ scattering (for $t < 0$) at energies up to tens of GeV and momentum transfers of a few GeV or less. It is data of this sort which an effective string theory for hadrons should address most directly.

Within what regime should the effective string picture be valid? More specifically, when does string perturbation theory begin to break down (i.e., loops grow in importance) and when does the finite thickness of QCD flux tubes become significant?† At least within the Regge limit (large $s$, fixed $t$) we have some indication of

† These issues are not unrelated in that the string world sheet can develop some effective thickness via a large number of small holes and handles. In an effective theory, however, we should include the short distance effects (to the extent possible) into the tree–level theory, and include only loop amplitudes defined on a larger distance scale.
when multiple string exchange becomes important. If the contribution from single trajectory exchange behaves for large \( s \) like \( s^{a't + \alpha_0} \), then the contribution from exchanging two such trajectories behaves like \( s^{a't/2 + 2\alpha_0 - 1} \). Thus the loop contributions become more important as \( s \) and \(|t|\) grow. This effect is apparent in the \( \pi N \) scattering data. For large \( s \) (\( \sim 200 \text{ GeV}^2 \)) the measured effective trajectory remains linear for small \(|t|\) but flattens out for \( t \) around \(-1 \text{ GeV}^2\).

It should be kept in mind that the string coupling constant is not obviously small. Comparing the current algebra result for \( \pi \pi \) scattering with the Lovelace–Shapiro–Veneziano four–point open string amplitude\(^{[9,17]}\), leads one to conclude that the string coupling is of the order \( g \approx (\sqrt{\alpha'} 4\pi f_{\pi})^{-1} \sim 1 \). It is only by virtue of kinematic factors (and factors of \( 2\pi \)) that the loop amplitudes are suppressed for some processes. This is analogous to the situation in chiral perturbation theory, which the string should match on to at low energies.

The effects of finite string thickness are more difficult to address. The fundamental dimensionful parameter in the theory is the string tension \( T \), or the related Regge slope, \( \alpha' = (2\pi T)^{-1} \). The latter, as measured from the rho trajectory, is \( .88 \text{ (GeV)}^{-2} \), corresponding to a string tension of \( .91 \text{ GeV/fm} \). In point particle theories large momenta are naturally associated with small distance scales. In a string theory, on the other hand, large momenta are naturally associated with large distances, via the string tension. Put simply, large momenta are typically exchanged via long strings, small momenta by small ones. This basic fact leads to a fundamental difficulty: in momentum space there is no simple way to identify or isolate processes involving short distance scales where the string picture breaks down. This scale, essentially the flux tube thickness, we expect (from lattice simulations and the size of heavy \( q \bar{q} \) bound states\(^{[18]}\)) to be of the order of \(.3 \text{ fm} \). It is inappropriate to eliminate such scales with a momentum cutoff of \( p \ll 1/\cdot3 \text{ fm} \approx .7 \text{ GeV} \) as one would do in an effective point particle theory. Not only would this eliminate precisely the regime which we are most interested in (where stringlike behavior is observed in the data), but it would not insure that short distance scales have been removed, since small momenta can signal the appearance of unphysically
short strings (shorter than .3 fm).

Directly imposing a short distance cutoff in space–time is no less problematic. The utility of an effective theory depends on its calculability; with current technology this restricts us to string theories constructed out of conformal field theories. By virtue of the conformal invariance, however, there is no direct relation between a short distance cutoff on the world sheet and one in space–time. Choosing a parametrization of the world sheet which behaves otherwise and then imposing a cutoff necessarily breaks the conformal invariance, rendering the sum over string world sheets virtually intractable.

These short distance problems are fundamentally connected with the most serious of the traditional problems plaguing string theories applied to hadronic physics. The consistent theories known 15 years ago, the bosonic and Ramond–Neveu–Schwarz (RNS) strings, suffer from a number of problems. The intercepts of the leading trajectories in the closed and open string sectors of the theory are 2 and 1, respectively, instead of \( \sim 1 \) and \( \sim 1/2 \) as required for the observed pomeron and \( \rho - \omega - A_2 - f \) trajectories; the simple bosonic (respectively RNS) string is consistent only in 26 (10) space–time dimensions; finally, the string amplitudes do not exhibit any parton–like behavior at short distances (e.g., power law fall off of the elastic scattering amplitude at high energy and fixed angle).

The last of these difficulties we have chosen to set aside in considering the string only as an effective theory. The first, the intercept problem, is critically important for the physics we wish to study. The “critical dimension” problem is, in comparison, only of secondary importance, and the name is a misnomer. What is constrained is not the number of space–time dimensions, but a measure of the number of degrees of freedom living on the string world sheet, the total central charge. It is easy to construct consistent strings in four dimensions provided we include internal degrees of freedom in the form of a conformal field theory with the appropriate central charge; the issue is then which sort of conformal field theory is most appropriate for hadronic applications (e.g., bosonic, fermionic, Liouville–like
as in so-called “non-critical” strings, etc.). While the intercept problem appears at tree-level, a complete specification of the degrees of freedom in the string can be postponed until loop-level. This will be our philosophy here, motivated by the observed string behavior in hadronic interactions. Because the u and d quark masses are small, hadron strings are easily broken, and the observed string behavior is dominated by “short” strings like the lowest few resonances on the $\rho$ or $K^*$ trajectories. On the other hand, in a world with all quark masses $\gg \Lambda_{\text{QCD}}$, we could concentrate on long strings (length$\gg$3 fm) for which an intercept shift of 1 or 1/2 is completely negligible. In this case (which is the one discussed in [19]) the intercept problem would become secondary and the specification of the string degrees of freedom would take precedence.

The origin of the intercept problem is the following. In the known consistent string theories the space–time coordinates are realized in terms of free bosons on the string world sheet. All open string vertex operators carrying momentum $k$ are of the form,

$$V_\phi(\zeta, k, x) = \zeta^{\mu_1\ldots\mu_J} \partial X_{\mu_1} \ldots \partial X_{\mu_J} e^{ik \cdot X(x)} \phi(x)$$  \hspace{1cm} (2.1)

where $\phi(x)$ is an operator in the internal conformal field theory. Like $\psi_\mu$ in the RNS model, $\phi$ need not be a Lorentz scalar, but for notational simplicity we assume that is the case in the following equations. The mass shell condition is that $V$ have conformal dimension one, $J + \Delta_\phi - \alpha' M^2 = 1$. The intercept of the leading trajectory of particles of this type is then,

$$\alpha_0 = 1 - \Delta_\phi.$$  \hspace{1cm} (2.2)

Closed string vertex operators have the form $V(k, z, \bar{z}) = V(k/2, z)\bar{V}(k/2, \bar{z})$ and have conformal dimension (1,1) on shell so that,

$$\alpha_0 = 2 - \Delta_\phi - \bar{\Delta}_{\bar{\phi}}.$$  \hspace{1cm} (2.3)

It is simple enough to choose internal conformal field theories with desirable operators $\phi$ to obtain trajectories with practically any intercept we might wish.
The point is that regardless of which conformal field theory is chosen, it will always contain the identity operator, which has dimension (0,0). Hence there will always be leading open and closed string trajectories with intercepts 1 and 2. In string theories with world sheet supersymmetry, the lowest states on these trajectories (both tachyons) in fact decouple from physical states, but finding a symmetry such that an entire trajectory decouples is an entirely more formidable task.

How is this problem related to that of cutting off or modifying the string at short distances? The following heuristic argument gives some idea of how the parameters in an effective theory are altered when the short distance behavior (in space–time) is modified. To implement the modification, let us allow the string tension to change with momentum scale, but constrained so that \( T(p) \) is analytic in \( p \), Lorentz invariant, and asymptotically constant for large \( p^2 \):

\[
\frac{1}{2\pi T(p)} \approx \alpha' + \frac{a}{p^2} + \frac{b}{(p^2)^2} + \ldots
\]  

(2.4)

As noted previously, it is appropriate for most string applications to hadron physics to associate large \( p^2 \) with large distances and hence with asymptotically constant string tension. This expression is valid only for sufficiently large \( p^2 \) (long distances) where the string picture is still appropriate. Given (2.4) the conformal dimension of that piece of the vertex operator carrying momentum is altered, and with it the behavior of the leading Regge trajectory,

\[
\alpha_{\text{leading}}(-p^2) = 1 - \Delta(e^{ip\cdot X}) = 1 - \frac{p^2}{2\pi T(p)} = 1 - a - \alpha' p^2 - \frac{b}{p^2} + \ldots
\]  

(2.5)

In other words, there is an effect of modifying the small \( p^2 \) (short distance) piece of the theory, even for arbitrarily large \( p^2 \) (long distances) which is precisely to shift the intercepts of all of the trajectories by a constant \( a \). The higher order corrections curve the trajectories for smaller \( p^2 \).

This argument links two basic problems but solves neither. Three possible approaches come to mind: 1) find an alternative to free bosons for incorporating
string coordinates which differs at short distances but remains conformal on the world sheet; 2) find a tractable way to sum over world sheets for non–conformally invariant theories; 3) restrict attention to pieces of string models (built from conformal field theories) in which the undesirable trajectories do not appear.

The first possibility would be ideal, allowing a systematic treatment even at the loop level; however, such a theory is tightly constrained and its existence can largely be ruled out, at least for theories which share the usual ghost structure of the bosonic or RNS strings.\textsuperscript{[20]} The second possibility requires new technology and may prove to be solvable only numerically. In the remainder of this work we will explore the third possibility, which should be appropriate for some processes whether or not either of the first two possibilities are realizable. Even if the complete string theory is not conformally invariant, we expect that for some processes the string amplitude should be dominated by the long distance degrees of freedom so that an effective conformal theory should be appropriate for computing that particular amplitude.

In an effective point particle theory one typically specifies a Lagrangian with some number of interaction terms whose form is constrained by a set of imposed symmetries. The various coupling constants are fixed by comparing the predictions of the theory with enough measured processes for which the effective description is valid. This done, predictions can be made for other processes and tested against experiment. Not much is learned if we are restricted to consider only a single amplitude or set of amplitudes, because there is considerable freedom in adding interaction terms and tuning coupling constants. An effective string theory in a sense suffers from the opposite problem. The symmetries imposed (world sheet conformal invariance, duality, etc.) are so constraining that we have not been able to formulate a complete theory in which we can tune the parameters to their physical values. To some extent, however, we can turn this fact to our advantage: precisely because the string symmetries are so constraining we have a chance of obtaining significant predictions from the effective theory even at the level of individual amplitudes.
The principle tool in this approach is the known general structure and properties of conformal field theory amplitudes. In the following sections of this paper we will demonstrate this approach with two examples; we will address only a few general points here. In a given amplitude we must specify the conformal dimensions of the states appearing and their fusion rules. Conformal invariance, duality and possibly other symmetries for each given case, then typically fix the form of the amplitude completely up to some coupling constants. The complexity of the amplitude and number of undetermined coefficients is governed primarily by the number of distinct primary fields which appear as intermediate states. The lowest order approximation to a four-point function, for example, assumes that only a single trajectory (and its daughters) is exchanged in each \((s, t\) and \(u\)) channel; this gives amplitudes which are finite sums of Veneziano terms and often leaves only the overall normalization undetermined. These were the amplitudes available for phenomenological study 20 years ago. If two distinct trajectories appear in each channel then the basic amplitudes are integrals of hypergeometric functions; more than two leads to integrals of generalized hypergeometric functions, etc..

Unless otherwise noted, we take the momentum dependence of the conformal dimensions of vertex operators to be as in the bosonic string, \(\Delta = -\alpha' p^2 + \ldots\). This is required to obtain the usual Klein–Gordon propagator. This, together with momentum conservation, fixes the momentum dependence of the conformal field theory amplitude to be as in the bosonic string, except for the possible momentum dependence residing in coupling constants (OPE coefficients). The conformal field theory amplitude must still be integrated over the appropriate moduli space of vertex operator positions to obtain the final string amplitude, so the final momentum dependence can be much different than found in bosonic string amplitudes.

When the conformal field theory amplitude is defined on a surface with boundaries we need (in addition to conformal dimensions and fusion rules) some information on the boundary conditions. The structure of the amplitude is correspondingly

\* The operators may be organized under some chiral algebra extended beyond the Virasoro algebra, but for present purposes we don’t need to know explicitly what that algebra is.
richer than in the bulk case. The necessary technology and notations employed here are collected in [7]. It is through the boundary conditions that the valence quark properties of mesons should be incorporated.

Without a complete string theory we have at best only some general guidelines for determining when an effective string amplitude should be appropriate. Processes which are obviously dominated by short distance physics, for example large momentum transfers via electroweak currents, or heavy q\bar{q} bound states, should be avoided. On the other hand the interactions of hadrons with soft electroweak probes, where vector meson dominance is appropriate, should be amenable to a string description, and light–heavy q\bar{q} states are a particularly interesting forum for these methods. As a practical matter the upper limit on momentum transfers which can be considered in soft hadronic processes should be determined by the expected onset of string loop corrections, rather than by the appearance of short distance physics. The possible need for a low momentum cutoff to avoid unphysically short strings is avoided in choosing by hand the intercepts of the exchanged trajectories.

Finally we come to hadronic states which are easily treatable within the string framework for some purposes but not for others. Consider again πN scattering. The exchanged ρ trajectory is modelled adequately by an open string. The observed string–like behavior for this amplitude says little, however, about the validity of interpreting the pion and nucleon as one dimensional strings. In this process they appear just as external point sources on the string world sheet. We expect some difficulties with the string interpretation in both cases. The pion is the smallest of hadrons built from light quarks, and so one might expect short distance effects (in particular spin–spin interactions) to alter the simple string behavior on the π trajectory. To the extent one can judge from only three observed states (π, b_1 and π_2) and modest t channel exchange data, this appears to be the case. The slope of the trajectory for t near 0 is shallower than that of the ρ trajectory, but curves upward so that between the b_1 and π_2 the two trajectories are almost parallel. To model an amplitude involving the pion trajectory with an effective string amplitude
we must decide how best to approximate this curved trajectory. In section 2 we will concentrate on the pion as a Goldstone boson, and accordingly treat the $\pi$ trajectory as linear with the universal Regge slope, and with its intercept near zero. This allows for the correct $\pi$ mass, but mistreats the higher lying states on the trajectory.

Baryons are problematic within the string picture. Generalizing the single flux tube picture for mesons, it is natural to picture flux tubes emanating from each of the three quarks in a baryon and joining at a point. The baryon is then “Y” shaped and its world sheet in space–time consists of three subsheets sewn together along a joining curve, with appropriate quark boundary conditions on the other edges (fig.1). In principle to compute an amplitude involving baryons we must compute the corresponding conformal field theory amplitude defined on this world sheet and then integrate over some moduli space of joining curves as well as over the positions of vertex operators. In the case of a meson tree amplitude we could (by the Riemann mapping theorem) always conformally map the world sheet into, for example, the unit disk or upper half–plane. For the three sheeted baryon world sheet there is no such simple result, and so the sum over world sheets remains formidable even after the nontrivial computation of the relevant amplitude on this surface has been performed.

To make computations involving baryons tractable we must reduce baryon world sheets to the form of meson ones. In the examples considered in this paper we will use two different approaches. If, in the amplitude being considered, there are no vertex operator insertions in the interior or on the boundary of one of the three subsheets of the baryon world sheet, then we can imagine first summing over all of the possibilities for that subsheet. What remains will be the computation of a correlation function in a (probably modified) conformal field theory on a meson–like world sheet. The second possibility is to consider the baryon as an open string with a boundary condition appropriate to a quark on one end, and some different boundary condition mocking up a “diquark” at the other end. This approach is particularly suited to processes, such as meson emission, which only
directly involve one of the three baryon subsheets; the details of how the other two subsheets are treated should be of secondary importance. Obviously we have no systematic handle on the validity of either of these approaches to baryons. Both are suspect if heavy quarks are involved. The best we can do is to compare the results of both approaches to see when they are compatible, and to determine how sensitively they depend on the details of the boundary conditions or conformal field theories considered.

3. Chiral symmetry breaking and the nature of the pion

The incorporation of chiral symmetry into a low energy effective Lagrangian is sufficient by itself to guarantee all of the current algebra results for soft pion amplitudes. If we assume in addition that in some intermediate energy regime pion amplitudes exhibit string behavior, do any other general predictions follow? At this stage we cannot construct a completely satisfactory string theory which reduces to the non–linear sigma model in the low energy limit. We can, however, consider string models with spontaneously broken symmetries, explore the general features of the resulting Goldstone bosons, and hope that we are getting some of the general features of the physics correct.

Consider then a simple toy string model: an orientable bosonic string with string coordinates giving rise to 4–dimensional space–time together with some internal conformal field theory (with $c = 22$) which includes an SU(2) WZW model as one piece. The remainder of the internal degrees of freedom we leave unspecified as they will play no role in the discussion. This model contains conserved SU(2)$\otimes$SU(2) Kac–Moody currents, $J^a(z)$ and $\bar{J}^a(\bar{z})$, on the string world sheet. These are naturally associated with chiral SU(2)$^L \otimes$SU(2)$^R$ currents in space–time (in momentum space),

\begin{align}
J^a_{\mu,L}(k) &= \int d^2 z J^a(z) \bar{\partial} X^\mu e^{ik \cdot X/2} \\
J^a_{\mu,R}(k) &= \int d^2 z \bar{J}^a(\bar{z}) \partial X^\mu e^{ik \cdot X/2} \tag{3.1}
\end{align}
For closed strings on shell (i.e., $k^2 = 0$ so that the above operators are con-
formally invariant on the string world sheet) these are recognized as the vertex
operators for the emission of massless SU(2)$_L \otimes$SU(2)$_R$ gauge bosons coupling to
the conserved currents in space–time. Ultimately, closed string gauge bosons are
undesirable for hadronic physics (and are in part a manifestation of the omnipresent
leading trajectory problem), but for our modest purposes at present it is easy to
sidestep this problem. At tree level we can isolate the closed string sector of the
model from the open strings which are of primary interest; in fact this model can
be adjusted to eliminate these closed string gauge bosons from the spectrum al-
together without altering the open string sector of the theory. For the moment,
though, the presence of this local symmetry will actually prove helpful in diagnos-
ing the nature of the symmetry breaking in this model.

In the closed string sector of this model both of the space–time currents are
exactly conserved,

$$
k \cdot J^a_L = \int d^2z J^a(z) k \cdot \bar{\partial} X e^{ik \cdot X/2} = \int d^2z \bar{\partial}[J^a(z)e^{ik \cdot X/2}] = 0
$$

$$
k \cdot J^a_R = \int d^2z J^a(\bar{z}) k \cdot \bar{\partial} X e^{ik \cdot X/2} = \int d^2z \bar{\partial}[J^a(\bar{z})e^{ik \cdot X/2}] = 0
$$

The integrals of the total derivatives identically vanish upon integration by parts
since closed string world sheets have no boundaries.

Once we consider world sheets with boundaries, that is open string “meson”
amplitudes, we find that at least half of this symmetry must be broken, depend-
ing on the boundary conditions chosen. We can always conformally map an open
string tree amplitude to the upper half plane with the real axis as boundary. If
we choose boundary conditions $J^a(x) = \bar{J}^a(x)$ on the real axis then $\bar{J}^a(\bar{z})$ is the
analytic continuation of $J^a(z)$ into the lower half plane. The familiar derivation
of the Ward identities following from a world sheet symmetry (in terms of contour
integrals of $J^a(z)$) can be applied as usual but only for a single SU(2) symmetry.$^6$
The surviving symmetry is the diagonnal SU(2) within SU(2)$_L \otimes$SU(2)$_R$, and all
boundary operators (i.e., the vertex operators for open strings) fall into representations of this single SU(2), not the full symmetry present in the bulk. More generally we could choose boundary conditions preserving some other SU(2) (i.e., $J^a(x) = M^{ab}\bar{J}^b(x)$ with $M^{ab}$ an SO(3) rotation matrix), or even break the symmetry completely, but no more than half of the original symmetry can be preserved once boundaries are included.

This is a generic feature of conformal field theories defined on surfaces with boundaries. If there are local world sheet symmetries present in the bulk theory, only a subset can be preserved in the presence of boundaries. The canonical example is the local conformal symmetry itself; the bulk (closed string) theory includes two independent Virasoro algebras, while the theory with boundaries (open string) includes only one.

What is the nature of this symmetry breaking in space–time? The simple string model we are considering can be constructed free from any world sheet sickness (i.e., violations of modular invariance) and so should have a consistent space–time interpretation. Since half of the local gauge symmetry present at closed string tree level is broken when we couple in open strings, the corresponding gauge bosons will acquire a mass. The only candidates for the extra degrees of freedom required to make these massive are the massless open string states with vertex operators,

$$V^a_G(k) = \int dx J^a(x)e^{ik\cdot X(x)} \quad . \quad (3.3)$$

This is the direct analogue (for gauge bosons in lower dimensional open string models) of the Cremmer–Scherk version of the Higgs mechanism.\textsuperscript{[21]} The lost symmetry is effectively spontaneously broken, with the open string states (3.3) playing the role of the would–be Goldstone bosons. These states mix with one combination of the closed string vector boson vertex operators,

$$A^a_\mu(k) \equiv \int d^2z [J^a(z)\bar{\partial}X_\mu - J^a(\bar{z})\partial X_\mu]e^{ik\cdot X/2} \quad . \quad (3.4)$$
The other combination of vector bosons remains massless,

\[ V^a_{\mu}(k) \equiv \int d^2 z [J^a(z) \bar{\partial} X_\mu + \bar{J}^a(\bar{z}) \partial X_\mu] e^{ik \cdot X/2} \, . \quad (3.5) \]

If we include the interchange of left and right moving degrees of freedom on the world sheet (i.e., holomorphic and anti–holomorphic), in defining the action of the space–time parity operation on the world sheet fields, then \( A^a_{\mu}(k) \) and \( V^a_{\mu}(k) \) transform as axial–vector and vector currents respectively.

If we just compute at open string tree level and don’t consider the closed string sectors of this model, then the states in (3.3) forestall being eaten and behave exactly like true Goldstone bosons arising from a spontaneously broken SU(2) chiral symmetry. In particular the current algebra/soft pion theorems will be satisfied. From (3.3) and (3.4) the coupling of the (off shell) axial vector current to the pion can be computed,

\[ \langle 0 | g A^a_{\mu}(k) | \pi^b(p) \rangle = ig \delta^4(p - k) \delta^{ab} p_{\mu} \, . \quad (3.6) \]

g is the string coupling constant, which we see is inversely proportional to \( f_\pi \) (as we would have found as well by comparing the amplitude for \( \pi \pi \) scattering with the current algebra result). Contracting (3.6) with \( p^\mu \) we would find the usual PCAC relation between the pion field and the divergence of the axial current were our pion not exactly massless.

The basic soft pion theorem (the Adler consistency condition\(^{[22]}\)) which states that amplitudes involving pions should vanish as the pion momentum is set to zero and any pole terms are removed, follows simply as well. The vertex operator (3.3) which inserts a pion can be written at zero momentum as \( \oint J^a(z) dz \) with the contour running around the world sheet boundary. Any tree level open string amplitude including this vertex operator will be an analytic function of \( z \) everywhere inside of this contour (and hence will vanish upon integration) except for possible poles when \( z \) sits atop one of the other vertex operators on the boundary.
But the contributions from these points are precisely pole terms in the space–time amplitude.

The fact that world sheet axial currents which are conserved in the interior of the world sheet but not on its boundaries imply the usual current algebra results was discussed in considerable detail by Susskind and collaborators within the parton string picture long ago.\cite{11} The explicit toy model considered here adds one new feature to the general scenario considered there: the vector and axial vector currents are intimately related on the world sheet, being different combinations of the same holomorphic and anti–holomorphic currents; as a consequence, the presence of a boundary necessarily breaks half of the symmetry (this needn’t be put in by hand). In other words, in this picture confinement (assumed \textit{ab initio} in the string picture) together with flux tubes with ends (mesons) necessarily imply the breaking of chiral symmetry.

In its details our toy model is inadequate to describe physical pions (there is, for example, a massless, spin one, isoscaler which couples to $\pi\pi$); nonetheless, we expect some features to survive in any realistic hadronic string model. In particular we expect the vertex operators for zero momentum pions to behave like currents evaluated on the world sheet boundary. In conformal field theory language this implies (among other things) that the zero momentum pion operator has simple fusion rules with other operators in the theory and as a consequence its correlation functions have a simple analytic structure. We will show momentarily that this leads to mass relations between hadronic states linked by S–wave pion emission, but first we must consider what sort of modifications of our toy model we should expect and allow for.

So far we have considered isospin and chiral–isospin currents living in the interior of the string world sheet. As we have seen, the chiral symmetry breaking is particularly clear (and automatic) in this case; however, the existence of these currents is problematic. Most glaring is the presence of gauge boson vertex operators in the closed string sector which promote the global symmetry into a local one
in space–time. In addition we have closed string states carrying nonzero isospin and exotic states with isospins greater than one. Finally, the intuition from the large N limit of QCD (which we expect to be string–like) is that world sheets with flavor currents in the interior should be suppressed, since flavor separations in the interior of the world sheet imply color separations as well.

To what extent should flavor degrees of freedom be represented in the interior of the string world sheet, or restricted to its boundaries? It is reasonable to expect that at long enough distances an effective string picture should be valid for a gauge theory with any values of $N_c$, $N_f$, and quark masses consistent with confinement. This needn’t be the same string theory for different values of these parameters, and presumably some limits lead to much simpler theories than others.

Very heavy quarks ($M^2 \gg \Lambda_{QCD}^2$) can only appear via (non–trivial) boundary conditions on the edge of the world sheet. Light quarks, on the other hand, at least renormalize the string tension. For $N_c$ very large this is probably their only effect on the interior of the world sheet, and the individual flavor degrees of freedom should be present only on the boundaries (e.g., via Chan–Paton factors). At the other extreme, if $N_c$ is small, quark flavor currents in the interior are probably not suppressed (especially if $N_f$ is relatively large, but consistent with confinement). In between these limits there should exist conserved flavor currents, which effectively have some limited extent of penetration into the interior of the world sheet.

While these different limits represent quite different physics in the bulk of the world sheet, we expect much of the boundary physics to be the same. That is, regardless of how free they are to penetrate in the interior, near the world sheet boundary we expect conserved isospin and chiral–isospin currents, with the latter broken as before by the boundary conditions. The pion vertex operator should again be a world sheet current operator evaluated at the string boundary, even if these currents do not actually propagate freely in the bulk.

Since they are ultimately only evaluated on the world sheet boundary, the nature of these currents can be somewhat more general than in our toy example.
We must, in particular, allow for half-integer spin currents as well as integer ones. We need only require that correlation functions involving these currents are well defined and single valued, and this can be achieved with fermionic boundary currents. There is no analogue for open strings of the modular invariance requirement under $\tau \to \tau + 1$ which forbids operators with half-integer spin in the bulk theory.

As an aside, we can use the partial independence of bulk and boundary theories to modify our toy model to remove the closed string gauge bosons. The simplest example which illustrates the possibilities are the allowed combinations of two Ising models (i.e., two Majorana fermions and their associated spin operators). There are two consistent, modular invariant theories which can be defined in the bulk theory: the naive tensor product of two Ising models, and a correlated tensor product which includes holomorphic and anti-holomorphic fermion bilinears which generate a $U(1) \otimes U(1)$ symmetry. When we include boundaries into the latter theory, at most the diagonal $U(1)$ survives; the other combination of fermion bilinears will mix with the “would-be Goldstone” boundary operator given by the fermion bilinear on the boundary. This same operator is present in the simple Ising$\otimes$Ising theory, even though the bulk $U(1)$ currents are not. In the simple Ising$\otimes$Ising theory there also exist operators in the boundary theory which explicitly break the $U(1)$ symmetry (the boundary spin operator of a single one of the Ising models). We can, however, exclude these operators from the theory if we so choose, by excluding some of the possible boundary conditions. There is no analogue of modular invariance for open strings which forces us to keep all of the allowed boundary conditions and operators.$^{[7]}$ In this truncated system of boundary operators we have a $U(1)$ symmetry. We could consider $SU(N) \otimes SU(N)$ Kac–Moody currents constructed from free majorana fermions in the same fashion. Or $U(1)$ currents can be extended to non-abelian ones on the boundary with the addition of Chan–Paton factors.

We are now in a position to place one of the more intriguing results from the early days of string theory on a more general footing. In 1968 Lovelace$^{[9]}$ (and independently Shapiro$^{[17]}$) considered a simple model of the Veneziano type for
\[ \pi^+\pi^- \text{ scattering}, \]

\[ A(s,t) = \beta \frac{\Gamma(1 - \alpha's - \alpha\rho)\Gamma(1 - \alpha't - \alpha\rho)}{\Gamma(1 - \alpha's - \alpha't - 2\alpha\rho)}. \] (3.7)

Lovelace noticed that the Adler consistency condition (vanishing of \( A \) at \( s = t = u = M^2_\pi \)) is satisfied given the physical value of \( \alpha\rho \), the intercept of the rho trajectory. Or, conversely, imposing the Adler condition forces \( \alpha'(M^2_\rho - M^2_\pi) \) to be an integer or half-integer. Subsequently, Ademollo, Veneziano and Weinberg\textsuperscript{[10]} (AVW) generalized the argument to amplitudes of the form \( \pi A \to BC \), assuming each amplitude to be a sum of Veneziano terms with each term vanishing at the Adler point. For an S–wave coupling of the pion to \( A \) to form an intermediate resonance \( A^* \) (and similarly \( B \) to \( B^* \), etc.) the Adler condition is nontrivial and leads to a quantization condition of the form \( \alpha'(M^2_{A^*} - M^2_A + M^2_{B^*} - M^2_B) = \text{integer} \). Comparing various different amplitudes and using the observed \( \pi - \rho \) splitting (\( \alpha'(M^2_\rho - M^2_\pi) = 1/2 \)) then leads to the relation,\textsuperscript{[10]}

\[ \alpha'(M^2_{A^*} - M^2_A) = 1/2 \pmod{1}. \] (3.8)

Comparing the measured mass squared differences in units of the string tension for pairs of hadrons linked by the emission of an S–wave pion we find:

| \( A^* - A \) | \( \alpha'(M^2_{A^*} - M^2_A) \) |
|-----------------|-----------------|
| \( \rho - \pi \) | .50 |
| \( K^* - K \) | .49 |
| \( D^* - D \) | .48 |
| \( B^* - B \) | .49 |
| \( \Delta - N \) | .56 |
| \( \Sigma(1385) - \Lambda \) | .59 |
| \( \Sigma(1385) - \Sigma \) | .44 |
| \( \Xi(1530) - \Xi \) | .54 |
| \( \Sigma_c - \Lambda_c \) | .70 |
The Regge slope is taken from the $\rho$-trajectory to be, $\alpha' = 0.88 \text{ (GeV)}^{-2}$. Only those states which are lowest lying on the respective trajectories are included; the values for higher lying pairs of states differ only by non-linearities in the trajectories. To the accuracy given in (3.9), isospin splittings may be ignored. Of the pairs listed, five were known to the authors of [10].

The mass squared differences are all (except for the charmed baryons) surprisingly close to $1/2$. This success is particularly puzzling, given that \textit{apriori} the four point amplitudes involved needn’t have anything like the simple Veneziano form. After all, given the possibilities for conformal field theories, there are correspondingly rich possibilities for consistent string amplitudes, all consistent with Regge behavior and duality. The Adler condition could arise from a simple multiplicative momentum factor and it is not difficult to construct amplitudes with the correct masses which don’t satisfy the Adler condition. As we will now show, however, given the picture for chiral symmetry breaking we have proposed, the mass relations of Lovelace and AVW follow even without assuming a Veneziano form for the four–point amplitudes.

Consider a three–point function coupling a pion to hadrons $A$ and $A^*$. This could be included as a factor in an $N$–point function in some limit, with either $A$ or $A^*$ appearing as an intermediate state, so we needn’t restrict them to be on the mass shell. In the limit in which the pion carries zero momentum the three–point function will vanish because of an explicit factor of momentum unless the pion has an S–wave coupling to the other hadrons, so we restrict our attention to this case. The relevant conformal field theory amplitude, evaluated in the upper–half complex plane with all vertex operators inserted on the real axis is fixed up to an overall constant by the conformal symmetry,

$$\langle V_\pi(z, k = 0)V_A(x_1, p)V_{A^*}(x_2, -p)\rangle =$$
$$\text{const.}(z - x_1)^{A^* - A - \Delta_\pi}(z - x_2)^{A - A^* - \Delta_\pi}(x_1 - x_2)^{\Delta_\pi - \Delta_A - \Delta_{A^*} + \alpha' p^2}$$

(3.10)

We have argued that the zero momentum pion vertex operator should behave
like a holomorphic current in the bulk theory evaluated on the world sheet boundary. Under this assumption the amplitude (3.10) should be well defined if we analytically continue in $z$ away from the real axis. In particular, (3.10) should be single valued under the operation of analytically continuing in $z$ around any closed contour. Assuming boundary conditions on the real axis which preserve isospin symmetry ($J^a(z) = \bar{J}^a(\bar{z})$ for $z$ real), we can smoothly continue the pion operator from the upper half–plane into the lower half–plane without difficulty. Choosing a contour surrounding both $x_1$ and $x_2$, (3.10) is single valued under the continuation provided that $2\Delta_\pi$ is an integer. This is just the restriction that the world sheet current have integer or half–integer spin. Imposing single–valuedness for the contour surrounding $x_1$ alone gives the condition,

$$\Delta_{A^*} - \Delta_A - \Delta_\pi = 0 \pmod{1} \quad (3.11)$$

The mass relations found by AVW then immediately follow, with $\alpha'(M^2_{A^*} - M^2_A)$ an integer or half–integer depending on the world sheet spin of the current associated with the pion. The half–integer spin current (such as occurs in the RNS model) is clearly chosen by the data. Reconciling this half–integer spin with the fact that the pion is massless, should ultimately provide a significant clue as to how the intercepts of the leading trajectories should be shifted in a complete string theory.

We can also justify, to some extent, the starting point of the Lovelace and AVW derivations. In a bulk conformal field theory correlation functions involving holomorphic operators, or more generally any operators which have simple fusion rules with all other operators, necessarily have a simple analytic structure. Four–point functions with one or more such operators can be written as the product of a single holomorphic function of the world sheet coordinates times an anti–holomorphic function, with the product single valued on the complex plane. Open string four–point tree amplitudes which are constructed from the corresponding boundary operators are integrals of the holomorphic half of the bulk amplitude; this gives rise to a finite sum of Veneziano type terms, the form assumed by Lovelace and
AVW. This form is natural, then, for any four-point function involving pions, if the other boundary operators have counterparts in some bulk theory. This is not required by the picture for chiral symmetry breaking we have been considering. The boundary operators in a conformal field theory need not have analogs in any consistent bulk theory. The consistency conditions (duality, modular invariance, etc.) which correlation functions of bulk or boundary operators must satisfy, while related, are not identical. If the boundary theory cannot be extended into a consistent bulk theory (which we believe will be the case at least for the operators appropriate for describing mesons involving one heavy quark) then the four-point functions including pions (with non-zero momenta) needn’t consist of a finite sum of Veneziano terms. The four pion amplitude itself is a special case. It should consist of Veneziano terms, and its form is constrained up to a few free parameters, with the original Lovelace–Shapiro amplitude the most natural possibility. It is encouraging that the Lovelace–Shapiro amplitude, with no free parameters, fits the $\pi\pi$ scattering data reasonably well.[20]

A few further comments on the entries of (3.9) and the validity of the AVW mass relations are in order. First, the derivation we have given is appropriate for baryons only to the extent that a baryon behaves like a meson string, for example using either of the approximations for baryons discussed in section 2. It is entirely reasonable, then, that the observed agreement of the baryons with the mass relation is systematically worse than that for the mesons. Second, we have been using an approximation in which the pion trajectory is treated as linear. It is measurably non-linear, and this fact is reflected in the failure of the AVW mass relation when any of the higher spin states on the pion trajectory are involved. Finally, there are likely problems with the higher lying states on the heavy meson trajectories as well, since these are not expected to be linear, but there is, at present, little data involving these states to consider.

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* Which may explain why linear “zero trajectories” are observed in pion–nucleon scattering.[23]
4. Closed strings, the pomeron, and the additive quark rules for total hadronic cross sections

In any given string model one can analyze the closed string spectrum and scattering amplitudes as easily as those for open strings. Were there compilations of glueball properties and scattering data one could compare directly with the string results to learn about this sector of the theory, which is free from complications due to world sheet boundary effects. Unfortunately this is not the case. Only a handful of glueball candidates exist (none positively identified) and, given that the closed string Regge slope is half that for open strings, it is too much to expect to identify several states on a single trajectory. Indeed there is no evidence that glueballs are narrow resonances at all.

The evidence for closed string behavior comes from exchanged t–channel trajectories in elastic and inclusive processes. At high energies \( (s > 100 \text{ GeV}^2) \) all total hadronic cross sections become approximately constant in energy (rising only logarithimically, or as a small power of \( s \)). Relating the total cross–section to the forward scattering amplitude via the optical theorem and assuming Regge behavior, this indicates the exchange of a trajectory with intercept near one with vacuum quantum numbers, the pomeron. An approximately linear trajectory with intercept one and slope consistent with half of the open string value (as predicted for a closed string) can be isolated in pp elastic scattering at moderate energies \( (P_{\text{Lab}} \sim 30 \text{ GeV}). \)\footnote{At higher energies the measured slope falls to only 1/3 to 1/4 that for the meson trajectories, consistent with the flattening of the slope expected from multiple pomeron exchange.} The fact that the total cross sections do not rise linearly in \( s \) is the most direct evidence that there is no hadronic string trajectory with intercept two.\footnote{In actuality such a linear rise would soon violate the Froissart bound, so one would expect to observe a near saturation of the Froissart bound at modest \( s \) if there were in fact a trajectory with intercept two.} In particular any mechanism for eliminating or decoupling the spin two “graviton” is,
by itself, insufficient to reconcile a string theory with hadronic physics if the rest of the trajectory remains.†

Leaving aside the problem of the pomeron intercept, let us consider the nature of its couplings to hadrons and determine whether the observed behavior is consistent with the interpretation of the pomeron as a closed string. Long ago it was observed that total hadronic cross sections at high energies seem to obey an additive quark rule, i.e., the total cross-section behaves like a sum of individual valence quark-quark cross sections. To illustrate, for $s$ around 200 GeV$^2$ the measured total cross sections are,$^{[25]}$

$$\sigma_{\pi N}, \sigma_{KN}, \sigma_{pp}, \sigma_{\Sigma N}, \sigma_{\Xi N} = 24, 20, 39, 33, 29 \text{ mb} \quad (4.1)$$

Using the pion–nucleon and kaon–nucleon total cross sections to extract cross sections for $u$ or $d$ quarks on $u$ or $d$ quarks and $u$ or $d$ quarks on $s$ quarks, respectively, the additivity assumption leads to the predictions,

$$\sigma_{pp}, \sigma_{\Sigma N}, \sigma_{\Xi N} = 36, 32, 28 \text{ mb} \quad , \quad (4.2)$$

in reasonable agreement with (4.1).‡ The agreement for cross sections on deuteron and helium targets is similar, though some nuclear shadowing is apparent. Donnachie and Landshoff, motivated by this feature of the data, have argued that the pomeron is small by hadronic standards, and couples locally to the valence quarks in a hadron rather like a $C=+1$ photon.$^{[26]}$

Within the string picture, the single pomeron exchange contribution to the forward scattering amplitude of two mesons is given by the open string non-planar, one-loop (“cylinder”) diagram. Computing a loop diagram requires more detailed

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† If enough states were absent from the trajectory, however, its coupling to hadrons at present energies might be sufficiently weak to avoid conflict with the data.
‡ We have made no attempt here to remove the small part of the total cross-section at these energies which is due to non-pomeron exchange although this should be done for a more definitive test of the additive quark rules.
knowledge of a string theory than a tree amplitude; problems which we could largely sidestep at tree level, such as undesirable trajectories or the additional degrees of freedom required for the cancelation of the conformal anomaly, cannot be avoided. Fortunately, in the Regge limit (large $s$, fixed $t$), the non–planar amplitude factorizes into a closed string propagator and form factors for the coupling of the closed string to each of the open strings.$^{[13,14,15]}$ The form factors can be computed at tree–level, so that we can avoid the need for specifying a complete string model. We will treat the pomeron coupling to mesons first, before turning to baryons.

Is the pomeron–meson–meson coupling in the string picture compatible with the observed additive quark rule? The valence quark content of a meson is determined by the choice of boundary conditions on the edge of the string world sheet. The pomeron, on the other hand, is emitted from the interior of the meson world sheet, not from the boundaries as an open string would be, so there seems to be no origin for any simple additive behavior. By duality we can think of the pomeron as coupling to the meson via an open string emitted from one of the two meson boundaries which then closes upon itself (fig.2), but this is not the origin of additivity. Duality equates the three pictures in fig.2, not the first with the sum of the last two. Since we would expect the pomeron exchange contribution to the forward scattering amplitude to involve distance scales for which the string picture is appropriate, this is a potentially serious problem for the effective string picture.

To address this issue, let us consider the pomeron–meson form factor in some detail. It is a three–point function for two open and one closed string. In the underlying conformal field theory this can be written in terms of a correlation function of two boundary operators and one bulk operator in the upper half–plane, $\langle \psi_{m}^{ab}(x_{1})\psi_{m}^{ba}(x_{2})\phi_{P}(z, \bar{z}) \rangle$. The superscripts $a$ and $b$ label boundary conditions along the real axis; the boundary operator $\psi_{m}^{ab}$ connects the two. We will use different boundary conditions to incorporate different valence quarks. The bulk–boundary–boundary three–point function has the same form as the holomorphic half of a bulk four–point function in the full complex plane,$^{[6]} \langle \psi_{m}^{ab}(x_{1})\psi_{m}^{ba}(x_{2})\phi_{P}(z)\bar{\phi}_{P}(\bar{z}) \rangle$. Using the conformal invariance of the amplitude we can fix three real coordinates.
The precise prescription requires a little care because the closed string vertex operator is off mass shell (i.e., not dimension (1,1)). In factoring the complete non-planar four-point function the usual closed string propagator (which appears in closed string tree amplitudes) is obtained if we fix one meson vertex operator at the origin, the other at infinity, and integrate the pomeron vertex over the unit circle,\(^{13,14,15}\) giving the form factor,

\[
g_{mmP}^{ab} = \pi \int_0^\pi d\theta \langle \psi_m^{ab} | \bar{\phi}_P(e^{-i\theta})\phi_P(e^{i\theta}) | \psi_m^{ba} \rangle . \tag{4.3}
\]

\(g_{mmP}^{ab}\) is constrained by conformal invariance to have the general form,

\[
g_{mmP}^{ab} = \pi \int_0^\pi d\theta \sum_q C_{Pq}^a C_{qmm}^{ab} G_q^q \left(1 - e^{-2i\theta}\right) \tag{4.4}
\]

\[= \pi \int_0^\pi d\theta \sum_r C_{Pr}^b C_{rmm}^{ba} \tilde{G}_r^r \left(1 - e^{-2i\theta}\right) .
\]

The second equality in (4.4) is a type of duality (fig.2).\(^7\) \(C_{Pq}^a\) is the operator product coefficient appearing in the short distance expansion for the pomeron vertex operator near the boundary with boundary condition \(a,\)

\[
\phi_P(z, \bar{z}) \sim (2\text{Im}z)^{\Delta_q - \Delta_P - \Delta_P} C_{Pq}^a \psi_q^{aa}(\text{Re}z) + \ldots , \tag{4.5}
\]

which governs the amplitude for the closed string \(\phi_P\) to transform into the open string \(\psi_q^{aa}.\) \(C_{qmm}^{ab}\) is the boundary operator OPE coefficient giving the open string three-point amplitude for the emission of \(\psi_q^{aa}\) from the boundary with boundary condition \(a\) of the open string \(\psi_m^{ba}.\) The couplings \(C_{Pr}^b\) and \(C_{rmm}^{ba}\) are the analogous quantities involving the boundary with boundary condition \(b.\) \(G_q^q\) and \(\tilde{G}_r^r\) are analytic functions of the variable \(\eta \equiv 1 - e^{-2i\theta}\) except at the possible branch points \(\eta = 0, 1,\) and \(\infty.\) For positive \(\theta \sim 0 (\eta \sim i\epsilon),\) \(G_q^q\) behaves like \(e^{\Delta_q - 2\Delta_P}.\)
For $\theta \sim \pi$ ($\eta \sim -i\epsilon$), $\tilde{G}^r$ behaves like $\epsilon^{\Delta_r - 2\Delta_P}$. We assume throughout that the pomeron vertex operator is diagonal ($\Delta_P = \bar{\Delta}_P$).

To explore the additivity of total cross sections within the string picture we will impose the following conditions on $g_{mmP}^{ab}$. For each open string boundary condition (i.e., quark flavor) we assume that the pomeron mixes directly only with a single, linear, open string trajectory. For $u$ or $d$ quarks this will be the $f$, for $s$ quarks the $f'$. For $c$ or $b$ quarks the trajectories will not be linear, and consequently our results will be suspect for this case. Further, we assume that the open string couplings are $SU(3)$ symmetric, so, for example we take $C_{KKf'}^{sus} = C_{KKf}^{usu} = C_{\pi\pi f}^{usu} \equiv C_{mmf}$. This is supported by Regge fits to various measured two to two scattering processes; the observed $SU(3)$ symmetry breaking is consistent with the assumption that it arises from shifts of the trajectory intercepts alone. Finally, we assume that $g_{mmP}^{ab}$ is a smooth function of $\Delta_q$ and $\Delta_r$ (i.e., of the underlying valence quark masses).

With these assumptions only a single $q$ and $r$ contribute to (4.4), and $C_{Pq}^a G^q = C_{Pr}^b \tilde{G}^r \equiv G_{qr}$. In order for $G_{qr} (1 - e^{-2i\theta})$ to have the correct behavior for $\theta$ near 0 and $\pi$ it must be part of a two-dimensional representation of the monodromy group. That is, the result of analytically continuing $G_{qr} (\eta)$ about any of its branch points is a linear combination of two functions, one behaving like $\eta^{\Delta_q - 2\Delta_P}$ as $\eta \to 0$ the other like $\eta^{\Delta_r - 2\Delta_P}$. This restricts $G_{qr} (\eta)$ to be a linear combination of terms involving hypergeometric functions, of the form,

$$
\eta^{\Delta_q - 2\Delta_P} (1 - \eta)^B [1 + \sum_{j=1}^M a_j \eta^j] F(\alpha, \beta; \Delta_q - \Delta_r + 1; \eta) .
$$

(4.6)

This form is further restricted by the following requirements:

1) symmetry under the interchange of $z$ and $\bar{z}$ in the original amplitude implies $G_{qr} (\eta) = e^{i\pi (\Delta_q - 2\Delta_P)} G_{qr} (\eta/(\eta - 1))$;

2) after analytically continuing $G_{qr} (1 - e^{-2i\theta})$ from $\theta = 0$ to $\theta = \pi$, only the terms behaving as $\eta^{\Delta_r - 2\Delta_P}$ should appear;
3) the limit in which \( q = r \) should be well defined and the result, \( G^{qr} \), symmetric under the above analytic continuation (i.e., symmetric under the interchange of \( \theta \) and \( \pi - \theta \)).

The final result (employing transformation and recursion formulas for the hypergeometric functions as needed and the identity \( F(a, 1/2 + a; 1 + 2a; \eta) = 2^{2a}[1 + (1 - \eta)^{1/2}]^{-2a} \)) is,

\[
G^{qr}(\eta) = C_{Pq}^{a} 2^{\Delta_q - \Delta_r} (-i\eta)^{\Delta_q - 2\Delta_r} (1 - \eta)^{\Delta_r - \frac{1}{4}(\Delta_q + \Delta_r)} [1 + \sum_{n=1}^{N} \alpha_n (\cos 2n\theta - 1)][1 + (1 - \eta)^{1/2}]^{\Delta_r - \Delta_q} .
\]  

(4.7)

Here \( \alpha_n \) are some linear combinations of the \( a_j \) in (4.6) and \( N = M/2 \) for \( M \) even; odd powers are incompatible with condition 3).

Analytically continuing (4.7) from \( \theta = 0 \) to \( \theta = \pi \) and imposing the duality symmetry we can relate the two bulk–boundary OPE coefficients and solve for their dependence on the boundary condition,

\[
4^{\Delta_q} C_{Pq}^{a} = 4^{\Delta_r} C_{Pr}^{a} \equiv 4^{-\alpha' t} \kappa .
\]  

(4.8)

\( \kappa \) is the bulk–boundary OPE coefficient in the simple bosonic string, and \( t \) is minus the pomeron momentum squared. Together with (4.7) this gives the general form for the pomeron–meson–meson form factor (4.3) consistent with world sheet conformal invariance, duality, and the assumption of pomeron mixing through a single meson trajectory for each boundary condition,

\[
g_{mmP}^{ab} = \int_{0}^{\pi} d\theta 2^{-\Delta_q - 2\Delta_r - 2\alpha' t} \kappa C_{mmf}^{ab} [1 + \sum_{n=1}^{N} \alpha_n (\cos 2n\theta - 1)](\sin \theta)^{\Delta_q - 2\Delta_r} (\cos \theta / 2)^{\Delta_r - \Delta_q} .
\]  

(4.9)

With a change of variables, \( x = \sin^2 \theta / 2 \), we can bring this expression into a more
symmetric form and integrate it,

\[
g_{mmP}^{ab} = \int_0^1 dx \kappa C_{mmf} 2^{-4\Delta_P - 2\alpha' t} x^{1/2} (\Delta_q - 2\Delta_P - 1) (1 - x)^{1/2} (\Delta_r - 2\Delta_P - 1) \left[ 1 + \sum_{n=1}^N a_n ((1 - 2x)^{2n} - 1) \right]
\]

\[
= \kappa C_{mmf} 4^{\alpha_P(t)} 2^{-\alpha' t} B\left( \frac{1}{2} (\alpha_P(t) - \alpha_q(t)), \frac{1}{2} (\alpha_P(t) - \alpha_r(t)) \right) \cdot \left[ 1 + \sum_{n=1}^N a_n \left( F(-2n, \frac{1}{2} (\alpha_P(t) - \alpha_r(t)); \frac{1}{2} (2\alpha_P(t) - \alpha_q(t) - \alpha_r(t)); 2) - 1 \right) \right].
\]

B is the Euler beta function, the coefficients \(a_n\) are linear combinations of the \(\alpha_n\) in (4.9), and in the last expression we have replaced the conformal dimensions by the related Regge trajectory functions,

\[
1 - \Delta_q = \alpha_q(t) = \alpha' t + \alpha_q
\]

\[
1 - \Delta_r = \alpha_r(t) = \alpha' t + \alpha_r
\]

\[
2 - 2\Delta_P = \alpha_P(t) = \frac{\alpha' t}{2} + \alpha_P.
\]

The hypergeometric function represents a finite sum of ratios of polynomials of the trajectory functions since the first argument is a negative integer.

In the limit \(q = r\) with \(N = 0\), (4.10) reproduces the pomeron form factor first found by Lovelace for the Veneziano model.\(^{[13]}\) The same expression is valid for the Neveu–Schwarz model as well.\(^{[15]}\) Equation (4.10) with \(N = 0\) is the direct generalization of this result, consistent with conformal invariance and duality, for pomerons coupling to meson strings with different “quarks” on either end. As one would expect, a change in one of the string boundaries has two effects: the conformal dimension \(\Delta_q\) of the boundary state appearing in the limit that the pomeron vertex operator approaches the boundary is altered, as is the relevant coupling, \(C_{Pq}^a\). Because of the duality constraint the two effects are completely correlated (c.f.,(4.8)).

We can finally test whether the additive quark rules for total cross sections arise within the string picture. Approximate additivity would mean that for vanishing
pomeron momentum,

\[ g_{mmP}^{ab} \approx \frac{1}{2} (g_{mmP}^{aa} + g_{mmP}^{bb}) \]  

(4.12)

We first consider the minimal case ((4.10) with \( N = 0 \)) for which (4.12) becomes,

\[ B(\frac{1}{2}(\alpha_P - \alpha_q), \frac{1}{2}(\alpha_P - \alpha_r)) \approx \frac{1}{2} [B(\frac{1}{2}(\alpha_P - \alpha_q), \frac{1}{2}(\alpha_P - \alpha_q)) + B(\frac{1}{2}(\alpha_P - \alpha_r), \frac{1}{2}(\alpha_P - \alpha_r))] \]

(4.13)

The case of physical interest (to explain (4.2)) is \( q \) and \( r \) taken as the \( f \) and \( f' \) mesons with intercepts near .5 and .1 respectively, together with a pomeron intercept of 1. For these values (4.13) holds to better than \( \frac{1}{2} \)%.

In fact for any values of these intercepts within the range from \( 0 < \alpha_q, \alpha_r < 1 \), (4.13) holds to better than 1%. For a range between \(-.9 \) and \( 1 \) the agreement is still better than 5%, and better than 10% even for very large negative intercepts (where our approximation of linear trajectories is in fact no longer trustworthy) provided \( |\alpha_r - \alpha_q| < 1 \). As an aside, for \( f \) and \( f' \) intercepts of .5 and .1 respectively, the minimal form factor predicts \( \sigma_{KN}/\sigma_{\pi N} = .74 \), in rough agreement with (4.1).

Numerically checking (4.12) for subsequent terms in (4.10) \( (n=1,2,3,\ldots) \), one finds that as \( n \) increases the additivity approximation for the individual terms improves and the contribution to the form factor slowly drops. Thus if the coefficients \( a_n \) are independent of \( \Delta_q \) and \( \Delta_r \) (or even slowly varying functions), and very large cancelations among the coefficients are avoided, then the general form factor \( g_{mmP}^{ab} \) is remarkably additive, in the sense of (4.12), over a broad range of the parameter space which includes the physically relevant values for the intercepts.

Why does (4.12) hold so well for (4.10)? It is not simply that the dominant contributions to the form factor come from pomeron emission near the world sheet boundaries. This is true for \( \alpha_q \) near 1 (where \( g_{mmP}^{ab} \) becomes infinite), but not for smaller \( \alpha_q \) where approximate additivity still holds. The success of (4.12) follows also from the structure of the underlying bulk–boundary–boundary three–point amplitude. This is a smooth analytic function of \( \theta \) between 0 and \( \pi \) except at the end points, where the holomorphic half of the pomeron vertex operator collides
with its anti–holomorphic image at the world sheet boundary. Thus the endpoints play a special role; for $\theta$ away from 0 and $\pi$ the amplitude is essentially determined by the behavior at the nearest branch point, with a smooth interpolation in the middle around $\theta = \pi/2$. As we consider terms in (4.10) with increasing $n$, the contribution of pomeron emission from the middle of the world sheet becomes less and less dependent on the behavior at the boundaries because of the factor $\cos^{2n}\theta$, further improving the additivity approximation for these terms.

We turn now to the form factor for pomeron emission from baryons. Ideally we would compute the conformal field theory amplitude for the pomeron vertex operator to be emitted from the interior of a world sheet consisting of three surfaces glued together along a curve (fig.1). We would then have to integrate this amplitude over some moduli space of joining curves as well as over the position of the pomeron operator. We would expect additivity to follow for the same heuristic reason given above for the meson case: the behavior of the amplitude should be governed by the singularities arising when the pomeron vertex operator approaches the world sheet boundary (which consists here of three lines joined at a point). Elsewhere the amplitude should be a smooth function of the position of the pomeron vertex operator, even on the joining curve. Unfortunately we lack the necessary technology for such a complete computation. We will consider the form factor only within the two approximations for baryons discussed in section 2.

Consider, then, $g_{BBP}^{abc}$, the form factor for pomeron emission from a baryon world sheet with boundary conditions $a, b, c$ on the three distinct edges. For that part of the amplitude in which the pomeron vertex operator is located on the subsheets of the world sheet bounded by the $a$ and $b$ boundaries, we can integrate over the possibilities for the third subsheet with boundary $c$ in order to reduce the problem to the same form as the meson case. Because we have integrated out this third subsheet, the conformal field theory on the new (meson–like) world–surface could be different from that in the pure meson case; however, the leading behavior of the amplitude for the pomeron near the boundary will be the same since $\Delta_p$, $\Delta_q$, and $C_P^{a}$ remain unchanged. Thus this contribution to the form factor is of the
same form as in the meson case, \( g_{mmP}^{ab} \).

To include the contribution to the form factor coming from pomeron emission from the subsheet bounded by \( c \), we can integrate out either the \( a \) or \( b \) subsheets. Clearly the complete form factor for baryons within this approximation is,

\[
g_{BBP}^{abc} = \frac{1}{2} (g_{mmP}^{ab} + g_{mmP}^{bc} + g_{mmP}^{ac})
\]  

(4.14)

The additivity of this form factor follows from the additivity of the meson form factor provided that the coefficients \( a_{n} \) are constant or change only slightly as we move from the meson case to the baryon case.

In the second approximation to baryons within the string picture, we consider an open string world sheet with a “quark” on one boundary and a “diquark” on the other, represented by boundary conditions \( a \) and \( bb \) respectively. For simplicity we will take the two quarks within the diquark to be degenerate, which we can always do if we restrict our attention to \( u, d, \) and \( s \) quarks and assume exact isospin symmetry. In order to incorporate three distinct quark masses we would have to modify the form factor to include functions transforming as a three dimensional representation of the monodromy group.

The computation of the form factor follows that for the meson case except for two features. In the short distance expansion for the pomeron vertex operator near the \( bb \) diquark boundary the coefficient of the boundary operator \( \psi_{r}^{bb} \) should be twice what it was for the single \( b \) boundary, \( C_{Pr}^{(bb)} = 2C_{Pr}^{b} \). Second, the form factor is not symmetric under the interchange \( \theta \leftrightarrow \pi - \theta \) even in the limit \( \Delta q = \Delta r \), since the two boundaries are inherently different. This means we must allow for \( \cos j \theta \) terms in (4.7) with \( j \) odd as well as even, and in fact such terms are required in order to achieve \( C_{Pr}^{(bb)} = 2C_{Pr}^{b} \). The final structure of the the form factor for
baryons within this approximation is,

\[ g_{BBP}^{ab} = \int_0^1 dx \kappa C_{mmf} 2^{-4\Delta_P-2\alpha' t_x} x^{1/2} (\Delta_r-2\Delta_P-1)(1-x)^{1/2}(\Delta_r-2\Delta_P-1) \]

\[ \cdot \left[ 1 + x + \sum_{n=1}^N [b_n + c_n(1-2x)]((1-2x)^{2n}-1) \right] \]

\[ = \kappa C_{mmf} 4^{\alpha_P(t)-2-\alpha' t} B\left(1/2 (\alpha_P(t) - \alpha_q(t)), 1/2 (\alpha_P(t) - \alpha_r(t))\right) \left[ 1 + \frac{\alpha_P(t)-\alpha_q(t)}{2\alpha_P(t)-\alpha_q(t)-\alpha_r(t)} \right] \]

\[ + \sum_{n=1}^N a_n(F(-2n, 1/2 (\alpha_P(t) - \alpha_q(t)); 1/2 (2\alpha_P(t) - \alpha_q(t) - \alpha_r(t)); 2)) - 1 \]

\[ + b_n(F(-2n-1, 1/2 (\alpha_P(t) - \alpha_r(t)); 1/2 (2\alpha_P(t) - \alpha_q(t) - \alpha_r(t)); 2)) \]

\[ - 1 + 2\frac{\alpha_P(t)-\alpha_q(t)}{2\alpha_P(t)-\alpha_q(t)-\alpha_r(t)} \] .

(4.15)

For the minimal form factor \((N=0)\) at zero pomeron momentum this reduces to,

\[ g_{BBP}^{ab} = g_{mmP}^{ab} \left[ 1 + \frac{\alpha_P - \alpha_q}{2\alpha_P - \alpha_q - \alpha_r} \right] . \]

(4.16)

For \(q = r\) this satisfies the additivity condition exactly (as does (4.14)), \(g_{BBP}^{aa} = \frac{3}{2} g_{mmP}^{aa}\). For \(\alpha_q\) and \(\alpha_r\) within the physically interesting range \(-0.5 < \alpha_q, \alpha_r < 1\) the additivity property and the agreement between the two approximations (4.16) and (4.14) for the baryon form factor, hold to 5% or better. There is no unambiguous choice for the higher order coefficients \(b_n\) and \(c_n\) in terms of the coefficients \(a_n\) appearing in the meson form factor, but it should be such that the behavior of the integrand near \(x = 0\) and \(x = 1\) approximates the behavior of the integrand in the meson form factor near \(x = 0\) and twice the integrand near \(x = 1\), respectively. Approximate additivity again results.
5. Conclusions

In this work we have advocated the use of effective string amplitudes for hadron phenomenology. This subject is an old one — the earliest days of Dual models included some surprisingly successful attempts at phenomenology using the Veneziano and Koba–Nielsen amplitudes — but today a more complete and systematic approach is possible using the technology of conformal field theories. The general structure and properties of amplitudes consistent with world sheet conformal invariance, duality and Regge behavior are well understood; examples are no longer restricted to the rather special case of Veneziano type models. The current challenge is finding the best means for incorporating various properties of hadrons within this framework. We believe in particular that the (under utilized) technology of conformal field theory amplitudes on surfaces with boundaries together with appropriately chosen boundary conditions, will make it possible to incorporate the properties of the valence quark content of hadrons.

The natural subjects for this approach are soft hadronic processes for which perturbative QCD is inapplicable and lattice gauge theory is often impractical. This should include soft hadronic contributions to weak processes (e.g., heavy meson decay constants, or the $\Delta I=1/2$ rule) as well as physics in the Regge regime, diffractive (pomeron) processes and hadron spectroscopy. The imposition of string behavior (linear trajectories, world sheet conformal invariance, duality) is highly constraining and reduces the number of free parameters available in traditional Regge phenomenology dramatically. Perhaps more important than these applications are the possibilities for improved qualitative understanding of inherently non–perturbative gauge theory phenomena (such as chiral symmetry breaking or the nature of the pomeron) which may have applications ranging from lattice gauge theory to technicolor.

We have considered two applications of these ideas to begin to illustrate the possibilities. In the first we proposed a natural mechanism for chiral symmetry breaking within the string picture based on a generic feature of conformal field
theories: that some of the symmetries present in the bulk theory are necessarily broken by the choice of boundary conditions when boundaries are included. In this picture, chiral symmetry breaking necessarily follows from confinement and the existence of mesons. One consequence of this picture is that the vertex operators for pion emission are particularly special, behaving (at zero momentum) like world sheet current operators evaluated on the string boundary. We used this fact, in turn, to rederive, under more general conditions, the hadronic mass relations first found by Lovelace\cite{9} and Ademollo, Veneziano and Weinberg\cite{10}. Even though the present derivation still includes a number of assumptions and the original suggestion is more than 20 years old, this remains a remarkable result: string behavior together with chiral symmetry essentially predicts the correct $\rho$ mass, as well as other mass relations.

In the second example we derived the general structure of the form factor for emission of a pomeron (interpreted as a closed string) from a meson or baryon, under modest assumptions. The result generalizes that found for the bosonic and RNS strings\cite{13,14,15}. Remarkably, within a broad parameter range including the physically most relevant values for the trajectory intercepts, these form factors display an additive behavior, as required to reproduce the additive quark rules for total hadronic cross sections which are observed experimentally. To the extent that QCD hadrons behave like flux tubes, this represents, to our knowledge, the first explanation in the completely nonperturbative regime of QCD for the success of the additive quark rules.

This result is not necessarily in conflict with Donnachie and Landshoff’s phenomenological explanation of the additive quark rule in which the pomeron couples locally to the valence quarks. In the string picture pomeron emission from the interior of the string world sheet away from the boundaries represents a significant contribution to the form factor. Because of conformal invariance, however, it is not clear which configurations of the string couplings in space–time provide the dominant contributions. While our results provide an alternative origin for the additive quark rule, it is possible that even in the string picture the pomeron effectively
couples locally to the string boundaries as measured in space–time.

The emergence of the additive quark rule from the string picture provides some insight into why the naive quark model is often successful even for processes in which sea quarks and gluons are expected to be important. In the string picture the boundary conditions on the edge of the world sheet, together with the behavior of the bulk operators as they approach the boundary, to a large extent govern the behavior of simple string amplitudes, even those, such as pomeron emission, where the interior of the world sheet (presumably built up from the contributions of gluons and sea quarks) is involved in a fundamental way.

Admittedly, settling for effective string amplitudes instead of complete string theories is less than completely satisfactory. Indeed one of the chief motivations for this approach is to gain insight into what sort of string behavior is most relevant for hadron physics as a step towards constructing a complete theory. At this stage we are restricted to tree amplitudes and therefore processes for which the narrow resonance approximation is appropriate. Further, the approximation of string–like behavior for hadrons as embodied in conformal field theory amplitudes, is completely uncontrolled. Judging from the measured linearity of hadron trajectories and the degree of success of earlier Regge and Dual model phenomenology, we expect results valid to 5–20% accuracy for hadronic processes dominated by string–like behavior, but there is no understanding of how to compute corrections to these results or determine precisely when the approximations break down. This will require new technology for summing over world sheets in the absence of conformal invariance. The present situation is in many ways analogous to the early days of current algebra prior to the systematic exploitation of effective chiral Lagrangians, only in this case we do not yet know how to handle the analog of the field theory with explicitly broken chiral symmetry.

Ultimately we would like to derive an effective string theory directly from QCD. This is particularly difficult because it involves comparing a second quantized theory (QCD) in which we can’t compute hadron amplitudes, with a first quantized
theory (string theory) in which basically all we can do is compute S–matrix elements perturbatively. A different formulation of one or both theories is required to even compare the two on the same footing. We have avoided addressing the question of whether QCD is in some limit exactly equivalent to some string theory, and if so how hard parton–like behavior arises in the string picture. We feel it is likely that the effective theory which best describes the observed string–like behavior of hadrons is not the the same as the exact string equivalent to some limit of QCD (assuming that one exists). Nonetheless, the former would provide great insight into the latter as well as providing a valuable tool for phenomenology.

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FIGURE CAPTIONS

1) A baryon world sheet.

2) The emission of a closed string from an open string can, by duality, be computed as a sum over open string states (emitted from one of the two boundaries) which then transform into the closed string.