Reliability Analysis for the Gear of Power Assisting Cycle with Multi-Extremum Response Surface Method

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Abstract. To more accurately calculate the dynamic reliability for the center gear of power assisting cycle with various failure modes, the multi-extremum response surface method is adopted. The first, the gear’s torque, material density, elastic modulus and poisson's ratio are sampled in small batches as the random input samples, the dynamic extremum of deformation, stress and strain are obtained as output response by finite element model. Then, the multi-extremum response surface equation is established. Finally, multitudinous sample points are obtained by using Monte Carlo method and linkage sampling to multi-extremum response surface equation, which can used to calculate the reliability of the deformation, stress and strain of the gear in the comprehensive failure mode. The results show that the comprehensive reliability degree of gear is 0.9949 when the allowable deformation, stress and strain are 0.47 mm, 540 Mpa and 0.003 , respectively.

1. Introduction
The gears support complex loads during the operation of the power assisting cycle operation. Once the gears fail, the energy stored in the spring can’t be released and the whole system does not run. So, the structural reliability analysis of gear is very necessary.
In recent years, a structural analysis method considering the tooth contact of internal gear system is introduced through the comparison with the simplified gear system model and applied to the structural analysis of a 2-stage differential-type gearbox for wind turbine by Cho et al [1]. The key meshing states of the gear pair for the contact fatigue and the bending fatigue was investigated based on the cumulative fatigue criterion and the stress-life equation by Deng et al [2]. Although the problem of gear contact was studied in different degrees, they were based on deterministic analysis method, and it is not discussed further from the angle of probability.
Currently, scientists have presented a lot of research on the gear reliability analysis method, response surface method is the most widely used [3-6]. The reliability analysis method based on response surface and the Markov chain Monte Carlo (MCMC) were proposed to improve the precision of the response function, and the dependability sensitivity analysis was conducted to quantify the effects of assembly errors, machining errors and stochastic external loads on gear transmission reliability by Tong et al [7], then the reliability was calculated using the founded mathematical model by Monte Carlo method (MCM). However, these methods only consider the reliability of single failure mode analysis without considering the failure correlation of dynamic reliability problems.
In order to solve the problem of multiple failure mode correlation, the multi-extremum response surface method was introduced in the reliability analysis of gear. The gear's torque, material density,
elastic modulus and poisson's ratio were sampled as random input variables to calculate the overall maximum deformation and maximum stress and maximum strain as output response. Then, these dates were used to fit out multi-extremum response surface mathematical model, and to calculation reliability with the Monte Carlo method and sampling linkage technology [8]. Finally, Monte Carlo method [9], [10] was used to verify the accuracy and effectiveness of this method.

2. Multi-extremum response surface method

In order to solve the problem of structural reliability analysis under multiple failure modes, the multi-extremum response surface method (MERSM) is adopted. The principle of MERSM is as follows: Firstly, Monte Carlo method is used to sample random of input parameters, the dynamic extremum of deformation, stress and strain were obtained as output response by finite element model. The extremum response surface function was established to reflect the relationship between the input parameters and the extremum output. Then, a multi-extremum response surface mathematical model is established on the basis of the extremum response surface model, and the multi-extremum response surface model is used to replace the finite element model (FEM) for structural reliability analysis.

Assume that $m$ as output objects in the structural reliability analysis($m \in \mathbb{Z}$). If $X^{(i)}$ is an input at the $i$-th output objects, and term "i" varies from 1 to $m$, then the corresponding output response is $y^{(i)}$, the functions is shown as:

$$y^{(i)}(X) = a^{(i)} + \sum_{j=1}^{n} b_{j}^{(i)} X_{j}^{(i)} + \sum_{j=1}^{n} c_{j}^{(i)} (X_{j}^{(i)})^2 \quad (j = 1, 2, \ldots, n)$$

(1)

Where $x_{j}^{(i)}$ is the $j$-th component of the input variable $x$ in the $i$-th output response (object) and $a^{(i)}, b_{j}^{(i)}, \text{ and } c_{j}^{(i)}$ are the undetermined coefficients of constant term, linear term, and quadratic term, respectively. The number of undetermined coefficients is $m(2n+1)$.

The undetermined coefficients are gained based on least square method when the number of samples is enough, because the vector $D^{(i)}$ is formed by

$$D^{(i)} = [a^{(i)} \ b_{1}^{(i)} \ b_{2}^{(i)} \ldots b_{k}^{(i)} \ c_{11}^{(i)} \ c_{22}^{(i)} \ldots c_{kk}^{(i)}]$$

(2)

From (2), we can gain the undetermined coefficients of (1) and further the mathematical model of MERSM.

3. Certainty analysis of bicycle gear

The static analysis of the gear is carried out under the finite element simulation platform. The material of gear is 40Cr, and the boundary condition gear’s torque is 9152N / mm. 47768 nodes and 27651 elements are generated by using tetrahedral meshes, and the finite element mesh model of gears is shown in Figure 1.

The distribution of stress, deformation and strain of the center gear are shown in Figure 2, Figure 3 and Figure 4, respectively. It can be seen that the maximum stress and maximum strain of the gear occur at the tooth root line of the contact gear. The maximum displacement is tooth top position. The maximum stress, maximum displacement and maximum strain of the center gear are 506.44Mpa, 0.44673 mm, 0.0028407, respectively.
4. Reliability analysis of bicycle gear

4.1. Reliability analysis

In the light of the uncertainties of material parameters mentioned in the handbook, the torque, material density, Young's modulus, and Poisson's ratio are chosen as input random variables, which were assumed to follow normal distribution and be independent mutually. The distributions of input random variables are listed in Table 1.

| Input random variables | Young's modulus, $E$/ (Mpa) | Material density, $\rho$/ (kg/m$^3$) | Torque, $T$/ (N * mm) | Poisson's ratio, $\mu$ |
|------------------------|-------------------------------|---------------------------------------|------------------------|-------------------------|
| Mean                   | 211000                        | 7870                                  | 9152                   | 0.277                   |
| Standard deviation     | 6330                          | 236.1                                 | 274.56                 | 0.00831                 |

Using the MC method and Latin Hypercube Sampling technology simultaneously extract random input variables and output response at the location of the maximum deformation, maximum stress and maximum strain. Then, we can establish extreme response surface equation with these sample points by least square, finally the Multi-extremum response surface equation is shown as:
4.2. The results of reliability analysis

Through 10000 simulations on three extremum response surfaces by MC simulation method, and based on material handbook and some related material tests, it can be found that the allowable deformation, allowable stress and allowable strain are $u = 0.47 \text{ mm}$, $\sigma = 540 \text{ Mpa}$ and $\varepsilon = 0.003$, respectively. Referencing these material parameters, the results of gear reliability analysis are listed in Figure 5.

\[
\begin{align*}
\hat{y}_1 &= -67.8892 - 4.3026 \times 10^{-3} \rho + 1.1124 \times 10^{-10} E - 6.5839 \mu + 0.0865 T \\
&+ 3.1464 \times 10^{-8} \rho^2 - 3.1481 \times 10^{-22} E^2 + 18.8054 \mu^2 - 2.6825 \times 10^{-6} T^2 \\
\hat{y}_2 &= -2.4011 \times 10^{-4} - 2.7785 \times 10^{-9} \rho + 6.5854 \times 10^{-16} E - 1.7401 \times 10^{-6} \mu + \\
&4.3186 \times 10^{-7} T + 1.9956 \times 10^{-13} \rho^2 - 1.8523 \times 10^{-27} E^2 + 3.9624 \times 10^{-5} \mu^2 - 1.0927 \times 10^{-11} T^2 \\
\hat{y}_3 &= -0.0839 - 3.0803 \times 10^{-6} \rho + 1.3390 \times 10^{-13} E + 0.0798 \mu + 8.3817 \times 10^{-5} T + \\
&2.0600 \times 10^{-10} \rho^2 - 3.8401 \times 10^{-25} E^2 - 0.1269 \mu^2 - 2.9531 \times 10^{-9} T^2
\end{align*}
\]

5. Methods validation

In order to verify the validity of the multi-extremum response surface method based on the input variables in table 1, the reliability of MCM, MRSM and MERSM were compared at the same condition. Under different times of simulation, the method of calculating time and reliability are shown in table 2. The results show that MERSM is a kind of high precision and high efficiency in reliability analysis.

| Samples $10^2$ | MCM | MRSM | MERSM | Computational time/s | MCM | MRSM | MERSM | Computational Precision |
|---------------|-----|------|-------|----------------------|-----|------|-------|------------------------|
| $10^3$        | 6300| 0.97 | 0.62  | 0.99                 | 0.98| 0.99  | 0.99  |                       |
| $10^4$        | -   | 5.36 | 1.44  | -                    | 0.992| 0.988| 0.991 |                       |
| $10^5$        | -   | 15.28| 2.53  | -                    | 0.99573| 0.99642|            |                       |
6. Conclusions
(1) The location and value of maximum deformation, maximum stress and maximum strain are obtained by the deterministic analysis of gear contact. The comprehensive reliability of gear is 0.9949 when the allowable maximum of deformation, stress and strain are 0.47 mm, 540 Mpa and 0.003, respectively.
(2) The comparison shows that the calculation precision used MERSM is consistent with MCM, and superior than MRSM, the advantage of its high computational efficiency is more obvious with the increase of the number of simulation.

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References
[1] Cho J R, Jung K Y, Park M H, et al. Finite element structural analysis of wind turbine gearbox considering tooth contact of internal gear system. J. Journal of Mechanical Science and Technology, 2013, 27(7):2053-2059.
[2] Song D, Lin H, X H H, et al. Finite element analysis of contact fatigue and bending fatigue of a theoretical assembling straight bevel gear pair. J. Journal of Central South University, 2013, 20(2):279-292.
[3] Guan T M, Li J B, Lei L. Tooth Surface Contact Fatigue Reliability Analysis of Cycloidal Gear Based on Monte-Carlo. J. Advanced Materials Research, 2013, 605-607:811-814.
[4] Li G, Mi W, Lu K, et al. Reliability analysis on gear contact fatigue strength considering the effect of tolerance. J. Open Mechanical Engineering Journal, 2014, 8(1):630-635.
[5] Liu Z L, Zhao M M, Peng-Peng M A, et al. Large Helical Gears Contact Analysis and Contact Strength Reliability Analysis Based on ANSYS. J. Journal of Zhengzhou University, 2015.
[6] Zhao H, Ru Z, Chang X, et al. Reliability Analysis Using Chaotic Particle Swarm Optimization. J. Quality & Reliability Engineering International, 2014, 31(8):1537 - 1552.
[7] Tong C, Sun Z L, Chai X D, et al. Gear Contact Fatigue Reliability Based on Response Surface and MCMC. J. Journal of Northeastern University, 2016.
[8] Zhang C Y, Lu Cheng, Fei C W, et al. Linked Analysis of reliability of Aeroengine Blisk with dual extremum response surface method. J. Propulsion Technology, 2016, 37(6):1158-1164.
[9] Seila A F. Simulation and the Monte Carlo Method. J. Technometrics, 2012, 24(2):167-168.
[10] Cheng Q, Zhao H, Zhao Y, et al. Machining accuracy reliability analysis of multi-axis machine tool based on Monte Carlo simulation. J. Journal of Intelligent Manufacturing, 2015:1-19.