Reliability Analysis of the Results of the Known Experiments on Measuring of the Sachs Form Factor Ratio Using the Rosenbluth Technique. 

Polarization of the Final Proton in the $e\bar{p} \rightarrow e\bar{p}$ Elastic Process

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A criterion for assessing the reliability of measurements of the Sachs form factor ratio using the Rosenbluth technique is proposed and applied to an analysis of three known experiments (Andivahis1994, Walker1994, Qattan2005) and a recent experiment on the CEBAF accelerator upgraded to 12 GeV at JLab (arXiv:2103.01842 [nucl-ex]). Based on the results of the JLab polarization experiments on measuring the ratio $\mu_p G_E/G_M$ in the $e\bar{p} \rightarrow e\bar{p}$ process, in the kinematics of the SANE Collaboration experiment (2020) on the measurement of double spin asymmetry in the $e\bar{p} \rightarrow ep$ process numerical calculations are performed for the $Q^2$ dependence of polarization transferred to the proton in the $e\bar{p} \rightarrow e\bar{p}$ process when the initial proton at rest is partially polarized along the direction of motion of the detected recoil proton.

INTRODUCTION

Experiments on studying the proton electric $G_E$ and magnetic $G_M$ form factors, the so-called Sachs form factors (SFFs), in the elastic scattering of unpolarized electrons by protons have been carried out since the mid-1950s [1]. All experimental data on the behavior of the SFFs were obtained using the Rosenbluth technique (RT), which is using the Rosenbluth cross section (in the one-photon exchange approximation) to the $ep \rightarrow ep$ process in the rest frame of the initial proton [2]

$$\sigma = \frac{d\sigma}{d\Omega_e} = \frac{\alpha^2 E_2 \cos^2(\theta_e/2)}{4 E_1^2 \sin^2(\theta_e/2)} \left(1 + \frac{\tau_p}{\varepsilon} G_E^2 + \frac{\tau_p}{\varepsilon} G_M^2 \right).$$

(1)

Here, $\tau_p = Q^2/4 M^2$, $Q^2 = -q^2 = 4 E_1 E_2 \sin^2(\theta_e/2)$ is the square of the momentum transferred to the proton; $M$ is the proton mass; $E_1$, $E_2$, and $\theta_e$ are the energies of the initial and final electrons and the electron scattering angle, respectively; $\varepsilon$ is the degree of linear polarization of the virtual photon [3–6] with the variability domain $0 \leq \varepsilon \leq 1$, $\varepsilon = [1 + 2(1 + p) \tan^2(\theta_e/2)]^{-1}$; and $\alpha = 1/137$ is the fine structure constant.

As follows from (1), at large $Q^2$, the main contribution to the $ep \rightarrow ep$ cross section comes from the term proportional to $G_M^2$, which already at $Q^2 > 1$ GeV$^2$ makes it rather difficult to extract the contribution of $G_E^2$. The RT was used to establish the experimental dependence of SFFs on $Q^2$ described up to $Q^2 = 6$ GeV$^2$ by the dipole approximation [7–11], and, for their ratio

$$R \equiv \frac{\mu_p G_E}{G_M}$$

the approximate equality $R \approx 1$ holds, where $\mu_p = 2.79$ is the proton magnetic moment.

In [4], Akhiezer and Rekalo proposed a method for measuring ratio $R$ based on polarization transfer from the initial electron to the final proton in the $e\bar{p} \rightarrow e\bar{p}$ process. Precision experiments conducted at JLab [12–14] using this method revealed a fast decrease in $R$ with increasing $Q^2$, which indicates SFFs scaling violation. This decrease in the region $0.4$ GeV$^2 \leq Q^2 \leq 5.6$ GeV$^2$ turned out to be linear. Repeated, more accurate measurements of ratio $R$ performed in [9, 15–17] in a wide $Q^2$ region up to 8.5 GeV$^2$ using both the Akhiezer–Rekalo method [4] and the RT confirmed the discrepancy of the results.

In [18], experimental values of $R$ are obtained by the SANE Collaboration using a third method, which is their extraction from measurements of double spin asymmetry in the process $e\bar{p} \rightarrow ep$ in the case where the electron-beam and the proton target are partially polarized. The degree of polarization of the proton target was $(70 \pm 5)$ %. The experiment was carried out at two electron-beam energies of 5.895 and 4.725 GeV and two values of $Q^2$, 2.06 and 5.66 GeV$^2$. The values of $R$ extracted in [18] agree with the results of the JLab experiments [12–17].

In [19, 20], a fourth method based on polarization transfer from the initial to the final proton was proposed, in which $G_E^2$ and $G_M^2$ can be extracted from direct measurements of the cross sections without and with proton spin flip in the elastic process

$$e(p_1) + \bar{p}(q_1, s_1) \rightarrow e(p_2) + \bar{p}(q_2, s_2)$$

(3)

when the initial proton (at rest) is completely polarized along the direction of motion of the final proton (detected recoil proton). This method also works in the two-photon exchange (TPE) approximation and allows squares of modules of generalized SFFs to be measured in a similar way [21].

To resolve the arising contradiction, it was assumed [22, 23] that the controversy in the experiments could result from the fact that the analysis ignored higher order radiative corrections, mainly TPE, the effects of which are
much more important in the Rosenbluth technique than in the method [4], because radiative corrections identically affect the observables of the longitudinal and transverse polarization of the recoil proton and thus are partially compensated in their ratio. The TPE contribution to the polarization transfer observables turned out to be small [24], as the calculations predicted [25, 26]. The idea proposed in [22, 23] stimulated a lot of theoretical studies of the TPE contribution [27–32] (see reviews [10, 11, 33–35] and references therein).

The two-photon exchange can be directly extracted by measuring the difference between the cross sections of the elastic $e^\pm p$ scattering processes. These experiments have recently been conducted at the VEPP-3 storage ring in Novosibirsk [36], at JLab (the EG5 CLAS experiment [37]), and at the DORIS accelerator at DESY (OLYMPUS experiment [38]) with the relevant data available for the region of $Q^2 < 2.1$ GeV$^2$. Their results have shown that allowances for the TPE may eliminate contradictions at $Q^2$ no larger than 2 GeV$^2$ [35].

The problem of highly accurately measuring the TPE contribution in the extended and so far largely unexamined region of $Q^2 > 2$ GeV$^2$ is supposed to be solved by the CLAS12 experiment [39] on measuring the $e^\pm p$ scattering cross section ratio using unpolarized $e^\pm$ beams from the CEBAF accelerator upgraded to 12 GeV at JLab. Its results will be decisive for unambiguously proving or disproving that the TPE is the main source of discrepancies and for verifying theoretical approaches based on the consideration of hadron and parton degrees of freedom that may compete in different parts of the $Q^2$ region under examination.

The polarized positron beam at JLab also provides the unique possibility of making the first measurement of the polarization transferred to the proton from the positrons in the elastic process $e^+ p \rightarrow e^+ p$ [40] and a comparison to the data [16, 17] on electron scattering may impose important limits on the hard TPE. The planned experiments [39, 40] will be an important supplement to the precision experiment conducted on the upgraded CEBAF accelerator at JLab [41] to measure ratio $R$ using the RT at beam energies of 2.2 to 11 GeV and much larger $Q^2$ of up to 15.75 GeV$^2$ obtained earlier. The results were analyzed in [41] using the improved procedures for calculating total radiative corrections (RCs) from [42]. Note that the RT-involving experiments [7, 8] were reanalyzed in [42], which made it possible to decrease the values of $R$ measured in those works.

In [43], three well-known experiments [7–9] were reanalyzed in the region of $Q^2 \leq 5$ GeV$^2$ using the RT, the improved RC calculation procedures from [42], and the TPE contribution calculated by the authors of [43]. Though they were the precision experiments, and one of them [9] even got a special name (Super-Rosenbluth), the discrepancies between the measurements by the RT and by the polarization method could be eliminated only for experiment [8] and only in the region of $Q^2 < 5$ GeV$^2$.

The goal of this work is to try to find out why the results of the experiments [7–9] failed to be reconciled in [43] with the results [17] and what can come out of a similar [43] reanalysis of the experimental results [41]. To this end, a criterion for assessing the reliability of RT measurements of the ratio $R$ is proposed and used to analyze the experimental measurements [7–9, 41]. Also, based on the results of the JLab polarization experiments on measuring ratio $R$ in the $e^+ p \rightarrow e^+ p$ process, a numerical analysis is given to the $Q^2$ dependence of the ratio of the cross sections without and with proton spin flip and to the polarization asymmetry in the $e^+ p \rightarrow e^+ p$ process when the initial (at rest) and final protons are completely polarized and have a common spin quantization axis coinciding with the direction of motion of the final proton (detected recoil proton). The longitudinal polarization transferred to the proton in the case of a partially polarized proton target is calculated in the kinematics of the SANE Collaboration experiment [18] on measuring double spin asymmetry in the $e^+ p \rightarrow e^+ p$ process.

I. CROSS SECTION FOR THE $e^+ p \rightarrow e^+ p$ PROCESS IN THE REST FRAME OF THE INITIAL PROTON

Let us consider spin four-vectors $s_1$ and $s_2$ of the initial and final protons with four-momenta $q_1$ and $q_2$ in process (3) in an arbitrary frame of reference. The conditions of orthogonality ($s_i q_i = 0$) and normalization ($s_i^2 = -1$) allow their time and space components $s_i = (s_{i0}, s_i)$ to be uniquely expressed in terms of their four-velocities $v_i = q_i/M$ ($i = 1, 2$)

\[ s_i = (s_{i0}, s_i), \quad s_{i0} = v_i, \quad s_i = c_i + \frac{(c_i v_i)}{1 + v_{i0}}, \]  

where the unit three-vectors $c_i$ ($c_i^2 = 1$) are the spin projection axes (spin quantization axes).

In the laboratory reference frame (LRF), where $q_1 = (M, 0)$ and $q_2 = (q_{20}, q_{2})$, we choose spin projection axes $c_1$ and $c_2$ such that they coincide with the direction of motion of the final proton

\[ c = c_1 = c_2 = n_2 = q_2/|q_2|. \]

Then spin four-vectors of the initial ($s_1$) and final protons ($s_2$) in the LF take the form

\[ s_1 = (0, n_2), \quad s_2 = (|v_2|, v_{20} n_2), \quad n_2 = q_2/|q_2|. \]

The method [19] is based on the expression for the differential cross section of process (3) in the LRF when the
initial and final protons are polarized and have a common spin projection axis $e$ (5)

$$\frac{d\sigma_{\delta_1, \delta_2}}{d\Omega_e} = \omega_+\sigma_{\uparrow\uparrow} + \omega_-\sigma_{\downarrow\uparrow},$$  \hspace{1cm} (7)

$$\sigma_{\uparrow\uparrow} = \sigma_M G_E^2, \sigma_{\downarrow\uparrow} = \sigma_M \frac{\tau_p}{\varepsilon} G_M^2,$$  \hspace{1cm} (8)

$$\sigma_M = \frac{\alpha^2 E_2 \cos^2(\theta_e/2)}{4E_e^3 \sin^2(\theta_e/2)} \frac{1}{1 + \tau_p}.$$  \hspace{1cm} (9)

Here, $\omega_\pm$ are the polarization factors

$$\omega_+ = (1 + \delta_1 \delta_2)/2, \quad \omega_- = (1 - \delta_1 \delta_2)/2,$$  \hspace{1cm} (10)

where $\delta_{1,2}$ are the doubled projections of the initial and final protons on the common axis of spin projections $e$ (5). Note that formula (7) is valid at $-1 \leq \delta_{1,2} \leq 1$.

The corresponding experiment on measuring squares of SFFs in the processes without and with proton spin flip can be implemented as follows. The initial proton at rest should be completely polarized along the direction of motion of the final proton (detected recoil proton). Measuring the $Q^2$ dependence of the differential cross sections $\sigma_{\uparrow\uparrow}$ and $\sigma_{\downarrow\uparrow}$ (8), one can also get information about the $Q^2$ dependence of $G_E^2$ and $G_M^2$ and thus measure them.

Note that formula (7), like (1), can be decomposed into a sum of two terms involving only $G_E^2$ and $G_M^2$. Averaging and summing (7) over polarizations of the initial and final protons, we obtain a different representation for cross section (1), designated as $\sigma_R$ [19]

$$\sigma_R = \sigma_{\uparrow\uparrow} + \sigma_{\downarrow\uparrow}.$$  \hspace{1cm} (11)

Consequently, the physical meaning of the decomposition of Rosenbluth formula (1) into a sum of two terms involving only $G_E^2$ and $G_M^2$ is that it is a sum of cross sections without and with proton spin flip in the case where the initial proton at rest is completely polarized along the direction of motion of the final proton.

Note that, in the literature, including textbooks on particle physics, it is often stated that the use of SFFs is merely convenient to make the Rosenbluth formula simple and compact. Since these formal considerations about their advantages also occur in the known monographs [44, 45] written many years ago, they have not been questioned and have been reproduced in the literature until now [46].

Cross section (7) can be written as

$$\frac{d\sigma_{\delta_1, \delta_2}}{d\Omega_e} = (1 + \delta_2 \delta_f)(\sigma_{\uparrow\uparrow} + \sigma_{\downarrow\uparrow}),$$  \hspace{1cm} (12)

$$\delta_f = \delta_f(R_\sigma - 1)/(R_\sigma + 1),$$  \hspace{1cm} (13)

$$R_\sigma = \sigma_{\uparrow\uparrow}/\sigma_{\downarrow\uparrow},$$  \hspace{1cm} (14)

where $\delta_f$ is the degree of longitudinal polarization of the final proton. In the case of a completely polarized initial proton ($\delta_1 = 1$), $\delta_f$ coincides with the ordinary definition of polarization asymmetry

$$A = (R_\sigma - 1)/(R_\sigma + 1).$$  \hspace{1cm} (15)

As follows from (8), the ratio of the cross sections without and with proton spin flip $R_\sigma$ (14) can be expressed in terms of the experimentally measured quantity $R = \mu_p G_E/G_M$

$$R_\sigma = \frac{\sigma_{\uparrow\uparrow}}{\sigma_{\downarrow\uparrow}} = \frac{\varepsilon G_E^2}{\tau_p G_M^2} = \frac{\varepsilon R^2}{\tau_p \mu_p}.$$  \hspace{1cm} (16)

The relative contribution $\Delta_\sigma$ of the term $\sigma_{\uparrow\uparrow}$ (8) involving $G_E^2$ to the cross section $\sigma_R$ (11) has the form

$$\Delta_\sigma = \frac{\sigma_{\uparrow\uparrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\downarrow\uparrow}} = \frac{R_\sigma}{1 + R_\sigma}.$$  \hspace{1cm} (17)

We recast formula (13) for the degree of polarization of the final proton in the standard notation, replacing $\delta_f$ with $P_r$ and $\delta_1$ with $P_t$

$$P_r = P_t(R_\sigma - 1)/(R_\sigma + 1).$$  \hspace{1cm} (18)

With constraint inversion in (18), we have the expression for $R^2$ as a function of $P_r/P_t$

$$R^2 = \mu_p \varepsilon \frac{1 + R_\sigma}{1 - R_\sigma}, \quad R_\sigma = \frac{P_r}{P_t},$$  \hspace{1cm} (19)

which can be used for extracting $R$ in the method of polarization transfer from the initial to the final proton [19, 20].
For numerical calculations of the $Q^2$ dependence of the polarization asymmetry $A$ (15), cross section ratio $R_{\sigma}$ (16), relative contribution $\Delta_{\sigma}$ (17), and polarization transferred to the proton $P_r$ (18) in the case of the dipole dependence ($R = R_d$) or its violation ($R = R_j$), we will use the parametrization

$$R_d = 1, \quad R_j = \frac{1}{1 + 0.1430 Q^2 - 0.0086 Q^4 + 0.0072 Q^6}.$$  

(20)  

(21)

The expression for $R_j$ is borrowed from [47], and the Kelly parametrization [48] can be used instead.

A. RESULTS OF NUMERICAL CALCULATIONS AND THEIR DISCUSSION

To clarify the general laws, the $Q^2$ dependence of cross sections ratio $R_{\sigma}$ (16) was numerically calculated for the electron-beam energies $E_1 = 1, 2, ..., 6$ GeV. The results are plotted in Fig. 1 for $R = R_d$ (20) (lines Rd1, Rd2, ..., Rd6) and $R = R_j$ (21) (lines Rj1, Rj2, ..., Rj6).

It follows from Fig. 1 that ratios of the cross sections without and with proton spin flip $R_{\sigma}$ (16) decrease with increasing $Q^2$ for all electron-beam energies. However, the decrease at $R = R_j$ is faster than at $R = R_d$ because of the denominator in the expression for $R_j$ (21). Note also that, at low electron-beam energies, the difference in the behavior of ratio $R_{\sigma}$ (16) at $R = R_d$ and $R = R_j$ is insignificant.

![Figure 1: Dependence of cross sections ratio $R_{\sigma}$ on $Q^2$ (GeV$^2$) for energies $E_1 = 1, 2, ..., 6$ GeV. Lines Rd1, Rd2, ..., Rd6 and Rj1, Rj2, ..., Rj6 correspond to ratios $R = R_d$ (20) and $R = R_j$ (21).](image)

It is clearly seen in Fig. 1 that the dependence of $R_{\sigma}$ on $Q^2$ for each electron-beam energy has a sharp boundary at $Q_{max}^2$, which is the maximum possible value of $Q^2$ corresponding to the backward (180°) electron scattering. Values $Q_{max}^2$ for the beam energies $E_1 = 1, 2, ..., 6$ GeV are presented in Table I, from which it follows that $Q_{max}^2$ is no larger than 10.45 GeV$^2$ for all energies considered.

Table I: Values $Q_{max}^2$ determining spectrum boundaries of the $R_{\sigma}$ dependence on $Q^2$ and values $(Q_0)^2_{(d,j)}$ at which $\sigma^{\uparrow\uparrow} = \sigma^{\downarrow\downarrow}$; polarization asymmetry $A$ (15) is zero in this case

| $E_1$ (GeV) | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 |
|-------------|-----|-----|-----|-----|-----|-----|
| $Q_{max}^2$ (GeV$^2$) | 1.277 | 3.040 | 4.868 | 6.718 | 8.578 | 10.443 |
| $(Q_0)^2_{(d)}$ (GeV$^2$) | 0.368 | 0.424 | 0.435 | 0.446 | 0.446 | 0.446 |
| $(Q_0)^2_{(j)}$ (GeV$^2$) | 0.436 | 0.380 | 0.391 | 0.402 | 0.402 | 0.402 |

Table I also presents the values $(Q_0)^2_{(d,j)}$ corresponding to the equality of the cross sections without and with proton spin flip $\sigma^{\uparrow\downarrow} = \sigma^{\downarrow\uparrow}$. In this case, their ratio is $R_{\sigma} = 1$ and the polarization asymmetry is zero. In the case of the dipole dependence, we have $(Q_0)^2_{(d)} \approx M^2/2$, where $M$ is the proton mass. If the dipole dependence is violated, we have $(Q_0)^2_{(j)} \approx 0.40$ GeV$^2$; i.e., equality of the cross sections $\sigma^{\uparrow\downarrow}$ and $\sigma^{\downarrow\uparrow}$ begins at approximately the same point where ratio $R$ begins linearly decreasing, thus, the points where $Q^2 = Q_0^2$ are, in a sense, distinguished.

The calculations depicted in Fig. 1 make it possible to understand why measurements of ratio $R$ using the RT are faced with difficulties at large $Q^2$. They should be conducted in the kinematics in which relative contribution $\Delta_{\sigma}$ (17) of the term $\sigma^{\uparrow\uparrow}$ to cross section $\sigma_R$ (11) is higher than Rosenbluth cross section measurement accuracy $\Delta_0$ in this experiment

$$\Delta_{\sigma} > \Delta_0.$$  

(22)

When $\Delta_0 \ll 1$, inequality (22) is reduced to

$$R_{\sigma} > \Delta_0.$$  

(23)
The requirements imposed by inequalities (22) and (23) can be considered the necessary conditions for performing reliable measurements. In the analysis of the experimental results, they can be used as a reliability assessment criterion for measurements.

The accuracy of Rosenbluth cross section measurements \( \Delta \sigma \) appearing in (22) and (23) is determined in the general case by statistical, systematic, and normalization uncertainties. Below in the reliability analysis of the experimental measurements [8], it will be established on the basis of the results [43] that \( \Delta \sigma \) is determined by the normalization uncertainty. Note that the limits on the kinematics of the experiment conducted using RT was not considered in the literature, including [42, 43, 49, 50]. Nevertheless, it seems to be an important aspect that deserves attention.

Tracing lines \( R d 1, Rd 2, ..., Rd 6 \) in Fig. 1, we make up Table II of values \( R \sigma \) (16) for \( E_1 = 1, 2, ..., 6 \text{ GeV} \) and \( Q^2 = 1, 2, ..., 9 \text{ GeV}^2 \). In this table, rows (columns) correspond to the same initial electron-beam energy \( E_1 \) (square of momentum transferred to proton \( Q^2 \)).

| \( E_1 \text{ (GeV)} \) | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 |
|------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( Q^2 \text{ (GeV}^2) \) |     |     |     |     |     |     |     |     |     |
| 6                      | 0.444 | 0.156 | 0.136 | 0.129 | 0.119 | 0.115 | 0.110 | 0.104 | 0.110 |
| 5                      | 0.440 | 0.209 | 0.179 | 0.160 | 0.150 | 0.145 | 0.140 | 0.134 | 0.139 |
| 4                      | 0.432 | 0.199 | 0.169 | 0.150 | 0.140 | 0.135 | 0.130 | 0.124 | 0.129 |
| 3                      | 0.415 | 0.175 | 0.145 | 0.126 | 0.116 | 0.112 | 0.107 | 0.101 | 0.106 |
| 2                      | 0.365 | 0.105 | 0.076 | 0.057 | 0.047 | 0.042 | 0.037 | 0.032 | 0.037 |
| 1                      | 0.114 |       |       |       |       |       |       |       |       |

For all Table II cells, except the one with \( R \sigma = 0.006 \), the relation \( R \sigma \geq 0.020 \) holds at \( Q^2 = 7.0 \) and \( 8.0 \text{ GeV}^2 \). Applying criterion (23), we come to the conclusion that, at \( Q^2 = 7.0 \text{ GeV}^2 \), measurements with the use of the RT should be performed with an accuracy no worse than 1.9 \%, and at \( Q^2 = 8.0 \text{ GeV}^2 \) the accuracy is required to be \( 0.3 \pm 0.5 \% \). Thus, the difficulties faced in the experimental measurements of ratio \( R \) at large \( Q^2 \) using the RT are the decrease in the relative contribution of the term \( \sigma^{1+} \) to the Rosenbluth cross section (11) and the necessity of increasing the accuracy of its measurement. Note that, in earlier experiments, measurements of the Rosenbluth cross sections with an accuracy higher than 2\% was an unsolvable problem for many reasons [51].

II. RELIABILITY ANALYSIS OF THE EXPERIMENTAL RESULTS [7–9, 41]

To analyze reliability of measurements of ratio \( R \) in experiments [7–9, 41], numerical calculations of relative contributions \( \Delta \sigma \) (17) were performed for all electron-beam energies \( E_1 \) and squares of momenta transferred to the proton \( Q^2 \) at which measurements were carried out in [7–9, 41]. The results are presented in Tables III, IV, V and VI respectively. Values \( E_1 \) (GeV) are given in the first columns, and \( Q^2 \) (GeV\(^2\)) are in the upper rows of the tables. Empty cells in the tables indicate that measurements were not performed at their corresponding values.

Reliability analysis of measurements in the experiment [8]. For this analysis, we refer to Fig. 15 (b) from [43] depicting results of a reanalysis of the experiments [7–9], which was performed using the improved procedures from [42] for the calculation of RCs and the added TPE contribution calculated in [43]. For the reader’s convenience, Figs. 15 (a) and 15 (b) from [43] are given below in Figs. 2(a), 2(b), respectively. It follows from Fig. 2(b) that measurements at \( Q^2 \leq 5.0 \text{ GeV}^2 \) in [8] with the added TPE contribution (Andivahis + TPE) agree well with the results [17], while at \( Q^2 = 5.0 \text{ GeV}^2 \) even the allowance for the TPE fails to eliminate discrepancies. That is why the measurement corresponding to the bottom cell of the column for \( Q^2 = 5.0 \text{ GeV}^2 \) in Table III is taken to be unreliable, i.e., insufficiently accurate.

From Table III and criterion (22), it follows that the accuracy of the measurements in [8] was at a level of \( 1.6 \pm 2.0 \)
% this interval to which there also belongs the normalization uncertainty of the Rosenbluth cross section measurement, which was 1.77 % (see [8, 42, 51]) for all $Q^2$ in the experiment [8]. Consequently, the measurement accuracy for $\Delta_0$ in criterion (22) should be identified with the normalization uncertainty. At this accuracy (1.77 %), reliability assessment criterion (22) does not hold for all cells in the diagonal of Table III at $Q^2 \geq 5.0$ GeV$^2$. Their corresponding measurements are regarded as unreliable; the values in Table III diagonal at $Q^2 \geq 5.0$ GeV$^2$ are in bold.

Table III: Values $\Delta_{\sigma}$ (17) at $R = R_{Q}$ (20) for $E_1$ (GeV) and $Q^2$ (GeV$^2$) used in the experiment [8]

| $E_1$ (GeV) \ $Q^2$ (GeV$^2$) | 1.75 | 2.50 | 3.25 | 4.00 | 5.00 | 6.00 | 7.00 | 8.83 |
|-----------------------------|------|------|------|------|------|------|------|------|
| 9.800                       | 0.097 | 0.083 | 0.067 | 0.055 | 0.006 |
| 5.507                       | 0.101 | 0.091 | 0.066 | 0.034 | 0.012 |
| 3.507                       | 0.129 | 0.091 | 0.066 | 0.034 | 0.012 |
| 3.400                       | 0.102 | 0.056 | 0.021 |
| 2.807                       | 0.080 | 0.028 |
| 1.968                       | 0.039 |
| 1.511                       | 0.061 |

The cell at $Q^2 = 8.83$ GeV$^2$ and $E_1 = 5.507$ GeV in Table III corresponds to $\Delta_{\sigma} = 0.006$, which requires measurement accuracy at a level of 0.3 ÷ 0.5 %. However, this level of accuracy was achieved only in experiment [52] in the region where $Q^2 < 1$ GeV$^2$. Note that at $Q^2 = 8.83$ GeV$^2$ the RT-based measurement procedure in [8] is violated, since in these experiments the measurements for each $Q^2$ should be performed at least at two, or better at three, electron-beam energies [53]. A similar conclusion about the unreliability of the measurements in [8] at $Q^2 \geq 5.0$ GeV$^2$ was drawn in [49].

Reliability analysis of measurements in the experiment [9]. Figure 2(b) also shows the results of the reanalysis of the experimental measurements [9] with the added TPE contribution (Qattan + TPE) presented as black diamonds. They are systematically higher than the green strip corresponding to the results of the polarization measurements in [17]. Calculations of relative contribution $\Delta_{\sigma}$ (17) in the kinematics of the experiment [9] are given in Table IV. Since the normalization error in [9] was 1.7 % [51], there is only one cell in the diagonal in Table IV with $E_1 = 2.842$ and $Q^2 = 4.10$ GeV$^2$ (with bold type) for which reliability assessment criterion (22) does not hold. The remaining discrepancies in [43] between Qattan + TPE and [17] are most probably caused by the underestimation of the normalization uncertainty in [9]. The values in Table IV make it possible to conclude that it was not 1.7 but 2.0 %. Note that, with approximate criterion (23), all measurements in [9] are classified as reliable [20].

Table IV: Values $\Delta_{\sigma}$ (17) at $R = R_{Q}$ (20) for $E_1$ (GeV) and $Q^2$ (GeV$^2$) used in the experiment [9]

| $E_1$ (GeV) \ $Q^2$ (GeV$^2$) | 2.64 | 3.20 | 4.10 |
|-----------------------------|------|------|------|
| 4.702                       | 0.129 | 0.103 | 0.072 |
| 3.772                       | 0.113 | 0.090 | 0.065 |
| 2.842                       | 0.093 | 0.059 | 0.017 |
| 2.262                       | 0.057 | 0.018 |
| 1.912                       | 0.020 |

Reliability analysis of measurements in the experiment [7]. Results of the calculations of relative contribution $\Delta_{\sigma}$ (17) for all $E_1$ and $Q^2$ used in the kinematics of experiment [7] are presented in Table V. Note that measurements [7] at each electron-beam energy $E_1$ were carried out in the region of small $Q^2$, located considerably far from $Q^2_{\text{max}}$. That is why the values in Table V are not small and satisfy the inequality $\Delta_{\sigma} \geq 0.086$. Since the normalization uncertainty of the measurement [7] was 1.9 % [7, 42], reliability assessment criterion (22) holds for all values in Table V. The remaining discrepancies can be due to the fact that either the reanalysis of the experiment [7] in [43] was not quite correct, which is hardly probable, or the normalization uncertainty in [7] is underestimated by about an order of magnitude (see Table V).

Table V: Values $\Delta_{\sigma}$ (17) at $R = R_{Q}$ (20) for $E_1$ (GeV) and $Q^2$ (GeV$^2$) used in the experiment [7]

| $E_1$ (GeV) \ $Q^2$ (GeV$^2$) | 1.000 | 2.000 | 2.500 | 3.000 |
|-----------------------------|------|------|------|------|
| 8.250                       | 0.149 | 0.125 |
| 7.500                       | 0.180 |
| 7.000                       | 0.179 | 0.147 | 0.124 |
| 6.250                       | 0.177 | 0.121 |
| 5.500                       | 0.175 |
| 5.000                       | 0.173 |
| 4.250                       | 0.172 |
| 4.008                       | 0.150 | 0.103 |
| 3.250                       | 0.296 | 0.155 | 0.116 | 0.086 |
| 2.800                       | 0.143 | 0.101 |
| 2.400                       | 0.282 | 0.126 |
| 1.594                       | 0.238 |
Reliability analysis of measurements in the experiment [41]. Results of the calculation of relative contribution $\Delta_{\sigma}$ (17) for all $E_1$ and $Q^2$ and used in the kinematics of the experiment [41] are presented in Table VI. Measurements in [41] were performed using the left and right high-resolution spectrometers, LHRS and RHRS. In Table VI, the values corresponding to the RHRS measurements are marked with an asterisk. The normalization uncertainty of the Rosenbluth cross section measurements by the LHRS and RHRS was 1.6 and 2.0 % respectively [41]. The values in bold in Table VI are those for which reliability assessment criterion (22) does not hold. Almost all of them, except for one, are related to the RHRS measurements and marked with an asterisk. The only unreliable LHRS measurement corresponds to the cell with the maximum values $E_1 = 10.587$ GeV and $Q^2 = 15.76$ GeV$^2$, where $\Delta_{\sigma} = 0.009$. For $E_1 = 10.587$ GeV, there are two rows in Table VI. The top row presents the single real, but unreliable, measurement, for which $\Delta_{\sigma} = 0.009$, and the bottom row presents the missed opportunities to perform reliable measurements, including those at $Q^2 > 2.0$ GeV$^2$. Thus, the kinematics used in [41] is not a good choice, and experiment [41] can hardly be considered a precision one, since about 40 % of its measurements do not meet the reliability assessment criterion.

Table VI: Values $\Delta_{\sigma}$ (17) at $R = R_d$ (20) for $E_1$ (GeV) and $Q^2$ (GeV$^2$) at which measurements were performed in the experiment [41]. Values with (without) an asterisk are for the RHRS (LHRS) measurements. The respective normalization uncertainties are 2.0 and 1.6 %

| $E_1$ (GeV) | $Q^2$ (GeV$^2$) |
|-----------|-----------------|
| 10.587    | 0.009           |
| 10.587    | 0.221           |
| 6.427     | 0.076           |
| 2.222     | 0.168           |

III. POLARIZATION TRANSFER FROM THE INITIAL TO THE FINAL PROTON IN THE ELASTIC $e\bar{p} \rightarrow e\bar{p}$ PROCESS

The method proposed in [19] for measuring squares of the SFFs in processes without and with proton spin flip requires a completely polarized proton target, which seems to be a matter for the quite distant future. As was noted above, in a wider sense it can be considered as a method based on polarization transfer from the initial to the final proton. The polarization transferred to the proton when the initial proton is completely polarized ($P_t = 1$) is determined by polarization asymmetry $A$ (15). To clarify generalities, the $Q^2$ dependence of polarization asymmetry $A$ (15) was numerically calculated for the electron-beam energies $E_1 = 1, 2, ..., 6$ GeV. The results are shown in Fig. 3 for $R = R_d$ (20) (lines $Ad_1$, $Ad_2$, ..., $Ad_6$) and $R = R_j$ (21) (lines $Aj_1$, $Aj_2$, ..., $Aj_6$).

Figure 3: Dependence of polarization asymmetry $A$ (15) on $Q^2$ (GeV$^2$) for electron-beam energies $E_1 = 1, 2, ..., 6$ GeV. Lines $Ad_1$, $Ad_2$, ..., $Ad_6$ and $Aj_1$, $Aj_2$, ..., $Aj_6$ correspond to ratios $R = R_d$ (20) and $R = R_j$ (21), respectively.

It is evident from the plots in Fig. 3 that, at $P_t = 1$, polarization asymmetry $A$ (15) changes, as it should, from $A = +1$ to $A = -1$, passing through 0 at $Q^2 = Q^2_0$. At $Q^2 > Q^2_0$, spin-flip cross section $\sigma^{\dagger}$ is larger than the non-spin-flip cross section $\sigma^t$, with their ratio being $R_{\sigma} < 1$. As a result, the helicity carried away by the recoil proton becomes negative. It reaches its maximum in absolute value $|A| = 1$ upon backward (180°) electron scattering. Note also that, at low electron-beam energies (e.g., at $E_1 = 1$ GeV), the difference in the behavior of asymmetry $A$ (15) at $R = R_d$ and $R = R_j$ is insignificant. At $E_1 > 1$ GeV and $Q^2 > 1$ GeV$^2$, the difference grows appreciable and the inequality of absolute values $|A_j| > |A_d|$ holds.
A. Proposed experiment on measuring the SFF ratio in the $e^p \to e^p$ process

In the general case, where the proton target is partially polarized ($P_t < 1$), the degree of longitudinal polarization transferred to the proton is defined by formula (18). At present, an experiment on its measurement appears to be quite feasible, since the target with a high degree of polarization $P_t = (70 \pm 5)\%$ has in principle been developed and was already used in [18]. It is for this reason that the proposed experiment should preferably be conducted at the facility used by the SANE collaboration [18] at the same $P_t = 0.70$, electron-beam energies $E_1 = 4.725$ and 5.895 GeV, and squares of momenta transferred to the proton $Q^2 = 2.06$ and 5.66 GeV$^2$. The difference between the proposed experiment and [18] is that the electron-beam should be unpolarized and the detected recoil proton should move strictly along the spin quantization axis of the proton target. Degrees of longitudinal and transverse polarization of the final proton were measured in [12–17]. In the proposed experiment, it is necessary to measure only the degree of longitudinal polarization of the recoil proton, which is an advantage when compared to the method [4] used in the JLab experiments.

The results of calculating the $Q^2$ dependence of polarization transferred to proton $P_j$ (18) in the kinematics of the experiment [18] are shown in Fig. 4, where lines $Pd5$, $Pd4$ (solid) and $Pj5$, $Pj4$ (dashed) are constructed for relations $R = R_d$ (20) and $R = R_j$ (21). Lines $Pd5$, $Pj5$, correspond to the electron-beam energy $E_1 = 5.895$ GeV, and lines $Pd4$, $Pj4$, correspond to $E_1 = 4.725$ GeV. The degree of proton target polarization is $P_t = 0.70$ for all lines in Fig. 4.

![Figure 4: Dependence of the degree of longitudinal polarization of the recoil proton $P_j$ (18) on the square of the momentum transferred to the proton $Q^2$ (GeV$^2$) for $E_1$ and $P_t$ used in [18]. Lines $Pd5$, $Pd4$ (solid) and $Pj5$, $Pj4$ (dashed) correspond to ratios $R = R_d$ (20) and $R = R_j$ (21).](image)

It follows from Fig. 4 that polarization transferred to the recoil proton depends appreciably on the form of the dependence of ratio $R$ on $Q^2$. In the case of SFFs scaling violation, i.e., when $R = R_j$, it noticeably increases in absolute value when compared to the case where $R = R_d$; i.e., inequalities $|Pj5| > |Pd5|$ and $|Pj4| > |Pd4|$ hold for all $Q^2$. A quantitative estimation of this difference is given in Table VII, which presents degrees of longitudinal polarization of the final proton $Pj5$, $Pd5$, $Pj4$, and $Pd4$ and their relative difference $\Delta_{dj5}$ and $\Delta_{dj4}$ (in percent) at two electron-beam energies of 5.895 and 4.725 GeV and two $Q^2$ of 2.06 and 5.66 GeV$^2$, where $\Delta_{dj5} = (Pj5 - Pd5)/Pj5$ and $\Delta_{dj4} = (Pj4 - Pd4)/Pj4$.

| $Q^2$ (GeV$^2$) | $Pd5$ | $Pj5$ | $Pd4$ | $Pj4$ | $\Delta_{dj5}$ % | $\Delta_{dj4}$ % |
|----------------|-------|-------|-------|-------|-----------------|-----------------|
| 2.06           | -0.36 | -0.55 | -0.47 | -0.56 | 16.6            | 16.1            |
| 5.66           | -0.63 | -0.69 | -0.63 | -0.69 | 9.1             | 6.4             |

It follows from Table VII that at $Q^2 = 2.06$ GeV$^2$ the relative difference between $Pj5$ and $Pd5$ is 16.6 %, and between $Pj4$ and $Pd4$ it is approximately the same, 16.1 %. At $Q^2 = 5.66$ GeV$^2$ this difference decreases to 9.1 and 6.4 % respectively.

Note that, after the measurement of the degree of longitudinal polarization of the recoil proton, $R$ is extracted using relation (19).

CONCLUSIONS

In this work, a reliability assessment criterion for measurements of the ratio $R$ using the RT is proposed, according to which the relative contribution of the $G_{E}^{2}$ involving term $\sigma^{\uparrow\uparrow}$ to the Rosenbluth cross section should be larger than the normalization uncertainty of the measurement of this cross section. Based on the results of the reanalysis [43], a reliability analysis was performed using this criterion for measurements in the known experiment [7–9] and for the recent experiment [41] at JLab’s CEBAF accelerator upgraded to 12 GeV. It follows from the analysis that, first, the measurements [8] at $Q^2 > 5$ GeV$^2$ are unreliable. Second, the remaining discrepancies between the measurements [9] (with added TPE contribution) and the results of the polarization experiments [17] observed
in [43] can result from the normalization uncertainty of the Rosenbluth cross section measurement in [9] being underestimated, being not 1.7% but 2.0%. Third, the remaining discrepancies found in [43] for the measurements in [7] can be due to the fact that either the reanalysis of the experiment [7] in [43] was not quite correct, which is hardly probable, or the uncertainties of the Rosenbluth cross section measurements in [7] are underestimated by about an order of magnitude. Fourth, the kinematics of the experiment [41] is not quite a good choice, because about 40% of the measurements performed in it are not reliable.

Based on the results of the JLab polarization experiments on measurement of the ratio $R$ in the $\vec{e}p \rightarrow \vec{e}p'$ process, a numerical analysis was performed for the dependence of the ratio of the cross sections without and with proton spin flip on the square of the momentum transferred to the proton and for the polarization asymmetry in the process when the initial (at rest) and final protons are completely polarized and have a common spin quantization axis coinciding with the direction of motion of the detected recoil proton. When the initial proton is partially polarized, the longitudinal polarization transferred to the proton is calculated in the kinematics used by the SANE collaboration [18] in the experiments on measuring double spin asymmetry in the $\vec{e}p \rightarrow ep'$ process. The polarization transferred to the proton is found to be noticeably sensitive to the form of the ratio $R$ dependence on $Q^2$, which can be useful for a new independent experiment on its measurement in the $\vec{e}p \rightarrow \vec{e}p'$ process.

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