HYPERONS AS SOLITONS IN CHIRAL QUARK MODEL

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ABSTRACT

In this talk we will discuss the phenomenology of the SU(3) Chiral Quark Model in which quarks interact via a self-consistent meson field, which takes a hedgehog soliton form. The classical part, i.e. the energy of the soliton, is exactly the same as in the two flavor case. The quantum corrections are calculated by an adiabatic rotation of the soliton resulting in a hamiltonian analogous to the one of the Skyrmion. A novelty is due to the mixed terms linear in the current quark mass and in the rotational velocity, connected to the anomalous part of the action, which get main contribution from the valence quarks. The resulting spectrum fits the data with a 10 % accuracy. At the same time the isospin splittings due to the $m_d - m_u$ mass difference are reproduced within experimental errors. Terms of the similar nature appear for some other observables like the axial coupling $g_A$ and it is argued that they cure the disease of too small $g_A$ inherent for all chiral models.

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1 INTRODUCTION: EFFECTIVE MODELS

It is still an outstanding problem to calculate the low energy properties of the hadronic spectrum. In fact evaluating some quantities in the high energy regime is in a sense much easier than to reproduce the old data on hadronic masses or magnetic moments. On the one hand the first principle calculations are plagued by enormous technical difficulties and on the other hand the calculations in the non-relativistic quark model (CQM), for instance, are theoretically unsound, although phenomenologically in many respects quite successful.

It is therefore tempting to try another approach, which would share some basic features with QCD and, in the same time, support the quark model ideas. The basic features of QCD with respect to the low energy limit are: chiral symmetry and confinement. The basic feature of the quark model is – not a surprise – the very existence of quarks. We will present here a model in which chiral symmetry breaking generates non-zero constituent mass $M$ for otherwise massless quarks coupled to Goldstone bosons$^{1,2}$, but we will almost completely ignore confinement. Not entirely though, since one has to assume that the physical degrees of freedom are color singlets.

Let us give some motivation. The generating functional of QCD involves quark and gluon fields. One can imagine the following scenario$^3$: first integrate out gluons. The resulting action would then describe the nonlinear and nonlocal many quark interactions. The next step would consist in linearizing this complicated action and expressing it in terms of local color singlet composite fields corresponding to pseudo-scalar mesons coupled in a chirally invariant way to quark fields. And finally we would have to integrate out quarks to end up with a pion (or $\pi$–$K$–$\eta$) effective lagrangian So we go through a chain of effective actions (see Refs.$^3$, $^4$, $^5$ for review and references):

\[ S_{QCD}[q, A] \rightarrow S_{\text{eff}}[q] \rightarrow S_{\text{eff}}[q, \pi] \rightarrow S_{\text{eff}}[\pi], \] (1)

It should be kept in mind that the arrows in Eq.(1) do not indicate a rigorous derivation of one action from another but rather educated guesses based mainly on symmetry principles and some physical input. As $S_{\text{eff}}[q, \pi]$ we will choose a semobosonized action of the Nambu–Jona-Lasinio (NJL) model, which is expected to follow from QCD in the instanton liquid model.
of the QCD vacuum\(^6,7\). Then we have:

\[
S_{\text{eff}}[U] = -\text{Sp} \log (i\partial - m - M U^\gamma_5).
\] (2)

where \(U^\gamma_5\) describes chiral fields \(\pi\) (or \(\pi-K-\eta\)) and \(m\) denotes here the current quark matrix. Eq.\((2)\) has to be regularized. The regularization procedure will introduce another implicit parameter: the cut-off \(\Lambda\).

From the gradient expansion of Eq.\((2)\) in meson sector one fixes \(m\) and additionally \(\Lambda = \Lambda(M)\). Therefore the model is very economical: there is in fact only one explicit parameter, namely \(M\). More complicated choices for \(S_{\text{eff}}\) have been also studied, and it seems that the results for the observables are in principle not changed\(^8\).

## 2 BARYONS AS SOLITONS

Now we will make crucial assumptions. We will assume that baryons can be described as solitons of the effective action of Eq.\((2)\). That means that the Goldstone fields described by matrix \(U\) are, in a sense, large and fulfill classical equations of motion. To this end we choose a hedgehog Ansatz for \(U\):

\[
U_0 = \left[ \begin{array}{cc} \overline{U}_0 & 0 \\ 0 & 1 \end{array} \right] \tag{3}
\]

where \(\overline{U}_0 = \cos P(r) + i\vec{n}\vec{\tau} \sin P(r),\) \(P(0) = \pi\) and \(P(\infty) = 0\) Then we rotate \(U_0\) adiabatically introducing a time-dependent SU(3) matrix \(A(t)\):

\[
U_0 \rightarrow A(t) U_0 A^\dagger(t), \quad A^\dagger \frac{dA}{dt} = i/2 \sum_{\alpha=1}^{8} \lambda_\alpha \Omega_\alpha.
\]

This procedure of introducing collective coordinates and the quantization of the system was developed in the context of the Skyrme model and was extensively discussed in the literature\(^9\).

In order to calculate baryon masses one usually considers the Euclidean correlation function for two baryonic currents \(J^0\):

\[
\langle J(T) J^+(0) \rangle \approx \int DU Dq Dq^+ J(T) J^+(0) e^{i \int d^4x q^\dagger (\partial + m + M U^\gamma_5)q} \\
\approx \Gamma^f \Gamma^g \int DU \prod_{i=1}^{N_c} G^{f_i g_i}_{U}(T,0) e^{-S_{\text{eff}}[U]} \\
\approx e^{-T(E_{\text{level}} + E_{\text{field}})} \approx e^{-T M_{\text{Baryon}}} \tag{4}
\]
where $\Gamma$ denotes schematically the projection of $N_c$ quarks on the color singlet baryonic state and $G_U$ is the quark propagator in the presence of the mean field $U$. From Eq. (4) one has to subtract the reference energy of the vacuum (no soliton), and apply the regularization procedure. Equation (4) states clearly that there are two different contribution to, in fact, any quantity, namely the valence part coming from the explicit quark propagator ($E_{\text{level}}$), and the sea contribution corresponding to the effective action ($E_{\text{field}}$).

Let us rewrite the effective action (2) in terms of the Euclidean spectral representation in a form ready for expansion in $\Omega$ and $m$:

$$S_{\text{eff}} = -N_c T \int \frac{d\omega}{2\pi} \text{Tr} \log (i\omega + H) \left[ 1 + \frac{1}{i\omega + H} (-i\gamma_4 A^\dagger m A + A^\dagger A) \right]$$

(5)

where $H$ is the hermitian static hamiltonian: $H = -i\gamma_4 (-i\gamma_i \partial_i + MU_0)$. The first term corresponds to the static soliton energy. Integrating by parts and subtracting the vacuum contribution we get:

$$S_{\text{eff}} = N_c T \int \frac{d\omega}{2\pi} \text{Tr} \left( \frac{i\omega}{i\omega + H} - \frac{i\omega}{i\omega + H_0} \right) = \frac{N_c T}{2} \sum_n (|E_n| - |E_n^0|).$$

(6)

So the static energy of the soliton is given by a sum of the differences of energies of the levels of the static hamiltonian $H$ with and without the soliton. This sum has to be of course regularized and the contribution of the valence level has to be added. Once the static energy of the soliton is found, selfconsitently or by variational methods, we can proceed further and expand Eq.(5) in powers of $\Omega$ and $m$.

Not all terms in this expansion should be regularized. Those which come from the imaginary part of the effective action need not any regularization. The imaginary part is connected to the anomalous term, or in other words, to the Wess-Zumino term. As an example let us consider a term linear in $\Omega_a$:

$$S_{\text{eff}} = -N_c T \int \frac{d\omega}{2\pi} \text{Tr} \left( \frac{1}{i\omega + H} \lambda_a - \frac{1}{i\omega + H_0} \lambda_a \right) \Omega_a$$

$$= -iN_c T \frac{1}{2\sqrt{3}} \Omega_8 \frac{1}{2} \text{Tr} (\text{sign}(H) - \text{sign}(H_0))$$

(7)

That only $\Omega_8$ contributes is due to the fact that $H$ is just the $SU(2)$ operator and what survives the trace is $\lambda_8$. The quantity $1/2 \text{Tr} (\text{sign}(H) - \text{sign}(H_0))$
counts the number of levels that crossed $E = 0$ line. The profound connection of Eq. (7) to the topology of our hedgehog Ansatz is probably clear to all readers familiar with the Skyrme model\textsuperscript{11,12}.

### 3 SEMICLASSICAL QUANTIZATION

Let us expand $S_{\text{eff}}$ up to the quadratic order in $\Omega$ (in Minkowski metric):

$$L_0 = -M_{\text{cl}}[P] + \frac{I_A[P]}{2} \sum_{i=1}^{3} \Omega_i^2 + \frac{I_B[P]}{2} \sum_{k=4}^{7} \Omega_k^2 - \frac{N_c}{2\sqrt{3}} \Omega_8$$

(8)

where:

$$I_{ab} = -\frac{N_c}{4} \int \frac{d\omega}{2\pi} \text{Tr} \left[ \frac{1}{i\omega + H} \lambda_a \frac{1}{i\omega + H} \lambda_b \right] = \begin{cases} I_A \delta_{ab} & \text{for} \ a, b = 1...3 \\ I_B \delta_{ab} & \text{for} \ a, b = 4...7 \\ 0 & \text{for} \ a, b = 8 \end{cases}$$

(9)

This lagrangian reminds the Skyrmion lagrangian. The quantization proceeds as in the Skyrme model case and the hamiltonian reads:

$$H_0 = M_{\text{cl}} + H_{\text{SU(2)}} + H_{\text{SU(3)}}$$

(10)

$$H_{\text{SU(2)}} = \frac{C_2(\text{SU(2)})}{2I_A}, \quad H_{\text{SU(3)}} = \frac{C_2(\text{SU(3)}) - C_2(\text{SU(2)}) - \frac{N_c^3}{12}}{2I_B}.$$

Here SU(3) is the flavor and SU(2) the rotational symmetry. It can be shown that the baryon wave functions are given in terms of SU(3) Wigner D functions:

$$\psi_{\text{baryon}}^{(\mu)}(A) = \sqrt{\text{dim}(\mu)} \langle Y, I, J_3 \mid D^{(\mu)}(A) \mid Y, I, J \rangle^*,$$

(11)

where $Y_R$ is in fact constrained due to the linear term in $\Omega_8$ to be 1. The lowest SU(3) representations which contain states with $Y = 1$ are\textsuperscript{13}: $\mu = 8$ and $\mu = 10$.

In order to fix $10 - 8$ splitting we need the constituent quark mass $M \approx 395$ MeV, certainly a reasonable number\textsuperscript{1}). What comes out too high is the absolute mass of the soliton $M_{\text{cl}} = 1.2$ GeV. There are however some negative corrections to it, which are of the order $O(1)$ whereas $M_{\text{cl}}$ is $O(N_c)$. Instead on insisting on the calculation of the absolute masses let us concentrate on the mass splittings.
4 MASS SPLITTINGS

To calculate mass splittings one has to expand Eq.(5) in powers of the current quark mass $m = \mu_0 \lambda_0 - \mu_s \lambda_s - \mu_3 \lambda_3$ ($\lambda_0 = \sqrt{2/3}$) where:

$$
\mu_0 = \frac{1}{\sqrt{6}} (m_u + m_d + m_s), \quad \mu_s = \frac{1}{\sqrt{12}} (2m_u - m_u - m_d), \quad \mu_3 = \frac{1}{2} (m_d - m_u).
$$

There will be terms of the order of $m$, $m^2$ and mixed terms: $m\Omega$. Note that: $A^\dagger \lambda_a A = D^{(8)}_{ab} (A) \lambda_b$. So we get lagrangian (in Minkowski space):

$$
L_m = -\sigma m_s + \sigma (m_s D^{(8)}_{88} + \frac{\sqrt{3}}{2} \Delta m D^{(8)}_{38}),
$$

$$
L_{m\Omega} = -\frac{2}{\sqrt{3}} m_s D^{(8)}_{8a} K_{ab} \Omega_b - \Delta m D^{(8)}_{3a} K_{ab} \Omega_b,
$$

$$
L_{m^2} = \frac{2}{9} m_s^2 (N_0(1 - D^{(8)}_{88})^2 + 3N_{ab} D^{(8)}_{8a} D^{(8)}_{8b})
+ \frac{2}{3\sqrt{3}} m_s \Delta m (-N_0(1 - D^{(8)}_{88}) D^{(8)}_{38} + 3N_{ab} D^{(8)}_{3a} D^{(8)}_{3b})
$$

where constant $\sigma$ is related to the sigma term $\Sigma = 3/2(m_u + m_d)\sigma$ ($\Sigma = 58$ MeV in this model). Similarly to tensor $I_{ab}$ of Eq.(9) $\sigma$, $N_{ab}$ and anomalous tensor $K_{ab}$ are given in terms of traces over certain Dirac and/or flavor matrices and denominators $1/(i\omega + H)$. Their explicit forms will be given elsewhere.

It is a matter of simple algebra to derive the pertinent hamiltonian and calculate the hyperon mass splittings up to the second order in $m_s$. Let us note that up to terms linear in current quark masses the Gell-Mann Okubo (GMO) mass formulae are reproduced:

$$
\Delta M_B^{(8)} = -\frac{F}{2} Y - \frac{D}{\sqrt{5}} (1 - \Gamma^2) + \frac{1}{4} Y^2), \quad \Delta M_B^{(10)} = -\frac{C}{2\sqrt{2}} Y.
$$

In the order $O(m_s^2)$ terms breaking GMO parametrization appear. They are however numerically negligible. In Fig.1 we plot GMO constants $F, D$ and $C$. Horizontal dashed lines represent experimentally allowed ranges. Long-dashed lines correspond to terms linear in $m_s$, whereas solid lines include $O(m_s^2)$ corrections. Since the $O(m_s^2)$ correction to $F$ is negligible, and since
Figure 1: Parameters $F$, $D$ and $C$ as functions of the strange quark mass

$F$ is given in terms of only one mass difference ($\Xi$–$N$) so that there are no experimental errors to it, we choose $m_s = 171$ MeV (vertical dashed line) to reproduce its experimental value. The arrows indicate the influence of the quadratic mass corrections on $D$ and $C$; they substantially improve agreement with experiment.

In Fig. 2. we plot the hadronic parts of isospin splittings for the octet$^{2,14}$. The long-dashed lines represent contribution linear in $\Delta m = m_d - m_u$, while solid lines include $m_s$ correction. In the order $O(m_s \Delta m)$ the experimental ranges for the isospin splittings are reproduced by $\Delta m \approx 3.5$ MeV (two vertical lines). The arrows indicate shift due to the $O(m_s \Delta m)$ terms. Here the common range shrinks to a value in a vicinity of $\Delta m \approx 4.3$ MeV. Taking into account the simplicity of the model this result seems to be surprisingly good.

5 FINAL REMARK: AXIAL COUPLINGS

We have developed a systematic approach to calculate nonperturbative quantities in the semibosonized NJL (or chiral quark) model. Since the mass splittings in the simplest version of the model (only one scalar coupling, no vector
mesons, no gluonic corrections, no boson loops) show surprisingly good agreement with experiment it is tempting to apply it to other observables. As an example let us consider the axial form-factors of hyperons. Let us first note that the very recent result on $g_A^{(3)}$ seems to solve the long lasting problem of underestimation of this quantity in NJL model\cite{15}. Similarly to the mass splittings $g_A^{(3)}$ is a series in $\Omega$ and $m$. For a long time it has been overlooked that there is a non-zero contribution to $g_A^{(3)}$ linear in $\Omega$. This contribution shifts the zeroth order value of approximately 0.8 by 0.4. Although this correction is large it acts in a right direction and puts the previously obtained result for a singlet axial current\cite{16} $g_A^{(0)} \approx 0.4$ (to which there are no $O(\Omega)$ corrections) on much safer ground. The above result should be compared with the EMC result of 0.13 ± 0.19\cite{17}. One should not forget though, that the experiment was done at the renormalization scale $< Q^2 > = 10 \text{ GeV}^2$. The evolution backwards in $Q^2$, although weak, tends to increase this value. So it is clear that both numbers are consistent with each other. The contribution to $g_A^{(0)}$ comes almost entirely from the valence part. This is consistent with the Skyrme model zero result for this quantity\cite{18}. On the other hand the model clearly shows that the matrix element of the singlet axial current is

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{Hadronic parts of isospin splittings as functions of the down-up mass difference for different values of the strange quark mass}
\end{figure}
substantially less than 1 in contrast to the naive quark model.

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