Abstract

It is pointed out that if the molecular interpretation of the recently observed resonance $X(3872)$ is valid, then nature may have prepared a good laboratory for us to examine the phenomenon of superradiance and subradiance of Dicke. The superradiance and supradiance factors are evaluated and the effects on the electromagnetic radiative decay of $X(3872)$ are discussed. Our results using coordinate space representation is similar to the momentum space results of Voloshin.
1 Introduction

The narrow resonance $X(3872)$ observed by the Belle Collaboration [1] in the decay channel $X \rightarrow \pi^+\pi^- J/\psi$ has attracted a great experimental and theoretical interest. It has been experimentally confirmed by CDF [2], DØ[3], and BabBar [4] Collaborations. Theoretically, different interpretations have been suggested to clarify the nature of the $X(3872)$ resonance. Charmonium options were considered for $X(3872)$ [5] while coupled channel effects were discussed in [6]. Exotic interpretations like a diquark-antidiquark (tetraquark) state $[cq][\bar{c}\bar{q}]$ [7, 8] and a $D^{*0} - \bar{D}^0$ molecule [9, 10, 11] bound by pion exchange were also considered.

In fact, $D - \bar{D}$ molecules were previously predicted [12, 13, 14, 15, 16]. Törnqvist [14, 16] showed that one-pion exchange potential is likely to bind a few states composed of two mesons and referred to such deuteronlike meson-meson bound states as deusons.

One of the motivations of the molecular interpretation of $X(3872)$ is the observation that $M(X)$ equals, within errors, $M(D^{*0}) + M(\bar{D}^0)$. Thus, the loosely bound molecule $D^{*0} - \bar{D}^0$ is expected to have a large $r_{r.m.s}$. Close and Page [9] estimated that $r_{r.m.s} \approx 7$ fm. They also pointed out that $D^0 - \bar{D}^0$ molecule does not exist because the $\pi D^0 \bar{D}^0$ vertex vanishes by parity conservation.

The electromagnetic radiative decay channel $X \rightarrow D^0 \bar{D}^0 \gamma$ can be very useful in providing an insight into the structure of the $X(3872)$. Voloshin [10] considered interference and binding effects in the radiative decays of $X(3872)$ and pointed out that the molecular component of $X(3872)$, $\frac{1}{\sqrt{2}} \left( |D^{*0} \bar{D}^0\rangle \pm |D^0 \bar{D}^{*0}\rangle \right)$, can be revealed by a distinct pattern of interference between the underlying decays of $D^{*0}$ and $\bar{D}^{*0}$.

It was Dicke [17] who first pointed out that the decay rate of an excited atom is affected if a second ground state atom is in its neighborhood. The decay rate can be enhanced up to double that of an isolated atom (superradiance) if the two atoms are in a symmetric state. If the two atoms are in an antisymmetric state, the decay rate is suppressed (subradiance). The superradiance and the subradiance factors are functions in the distance separating the two atoms and approaches one when the distance between the atoms is large compared to the wavelength of the emitted radiation. If the molecular interpretation of $X(3872)$ is confirmed, this resonance can be a good laboratory for investigating superradiance and subradiance.

In the present work, we evaluate the superradiance and the subradiance factors for the electromagnetic radiative decay of $X(3872)$ assuming that the molecular structure takes the form $\frac{1}{\sqrt{2}} \left( |D^{*0} \bar{D}^0\rangle \pm |D^0 \bar{D}^{*0}\rangle \right)$ using the coordinate space representation. Our results are similar to the results of Voloshin [10] evaluated in the momentum space representation. The coordinate space representation is more suitable for clarifying the interference in the radiative decay process.

In Sec. 2, we evaluate the electromagnetic radiative decay of the $D^*$ meson using the Golden Rule. In Sec. 3, we consider the electromagnetic radiative decay of $X(3872)$ and evaluate the superradiance and the subradiance factors. In Sec 4, we present the conclusions.

2 Electromagnetic radiative decay of the $D^*$ meson

The width of the electromagnetic radiative decay channel of an initial state $|i\rangle$ decaying into a final state $|f\rangle$ can be calculated using the Golden Rule

$$\Gamma = \sum_{\lambda} \int 2\pi |\langle f|H_{int}|i\rangle|^2 \frac{V_{\omega^2} d\Omega}{\hbar c^3 (2\pi)^3},$$

(1)
where $H_{\text{int}}$ is the electromagnetic interaction connecting the initial and final states. We sum over the photon polarization $\lambda$ and integrate over all photon directions.

The Hamiltonian of the $D$ meson, $D^*$ meson, and the electromagnetic fields can be written as

$$H = H_M + H_F + H_{\text{int}},$$

where $H_M$ is the meson Hamiltonian whose lowest eigenstates are $|D\rangle$ and $|D^*\rangle$, $H_F$ is the electromagnetic field Hamiltonian, and $H_{\text{int}}$ is the interaction Hamiltonian between the mesons and the electromagnetic fields. For the electromagnetic radiative decay of the $|D^*\rangle$ state, $H_{\text{int}}$ takes the form

$$H_{\text{int}} = -\vec{\mu} \cdot \vec{B},$$

where $\vec{\mu}$ is the magnetic moment and the $\vec{B}$ is the magnetic field. The magnetic field can be expanded in terms of the annihilation and the creation operators $a(\vec{k},\lambda)$ and $a^\dagger(\vec{k},\lambda)$ in the usual form

$$\vec{B}(\vec{x}) = i \sum_{\vec{k},\lambda} \sqrt{\frac{2\pi\hbar\omega}{V}} \hat{k} \times \vec{\varepsilon}(\vec{k},\lambda) \left[ a(\vec{k},\lambda)e^{i\vec{k} \cdot \vec{x}} - a^\dagger(\vec{k},\lambda)e^{-i\vec{k} \cdot \vec{x}} \right].$$

The matrix element of the transition is given by

$$\langle f | H_{\text{int}} | i \rangle = \langle D^\gamma | H_{\text{int}}(\vec{x}) | D^* \rangle = i \sqrt{\frac{2\pi\hbar\omega}{V}} \langle D | \vec{\mu} | D^* \rangle \cdot \hat{k} \times \vec{\varepsilon}(\vec{k},\lambda)e^{-i\vec{k} \cdot \vec{x}}.$$ (5)

From Eq. (1) and Eq. (5) and using the relation

$$\sum_{\lambda} \varepsilon_i(\vec{k},\lambda)\varepsilon_j(\vec{k},\lambda) = \delta_{ij} - \hat{k}_i\hat{k}_j,$$ (6)

we get

$$\Gamma_{D^*} = \int 2\pi \frac{2\pi\hbar\omega}{V} \langle D | \mu^i | D^* \rangle \langle D | \mu^j | D^* \rangle^* \left[ \delta_{ij} - \hat{k}_i\hat{k}_j \right] \frac{V\omega^2 d\Omega}{\hbar c^3 (2\pi)^3}. $$

Integrating over all angles of the emitted photon, we get

$$\Gamma_{D^*} = \frac{4\omega^3}{3c^3} |\langle D | \vec{\mu} | D^* \rangle|^2.$$ (7)

### 3 Electromagnetic radiative decay of $X(3872)$

Assuming the validity of the molecular interpretation of $X(3872)$, we can write the Hamiltonian of $X(3872)$ and the electromagnetic fields in the form

$$H = H_1 + H_2 + H_{12} + H_F + H_{\text{int}},$$

where $H_{1(2)}$ is the Hamiltonian of the first (second) meson, $H_{12}$ is the Hamiltonian describing the relative motion of the two mesons in the molecule, $H_F$ is Hamiltonian of the electromagnetic fields, and $H_{\text{int}}$ is the Hamiltonian of the interaction between the first and the second mesons with the electromagnetic fields. The lowest eigenstates of $H_1$ are $|D\rangle$ and $|D^*\rangle$, while the lowest eigenstates of $H_2$ are $|\bar{D}\rangle$ and $|\bar{D}^*\rangle$. 


The initial state can be written as \( |i⟩ = \frac{1}{\sqrt{2}} \left( |D^*\bar{D}⟩ \pm |D\bar{D}^*⟩ \right) \) while the final state is \( |f⟩ = |D\bar{D}\gamma⟩ \), and the interaction Hamiltonian, \( H_{\text{int}} = H_{\text{int}}(\vec{r}_1) + H_{\text{int}}(\vec{r}_2) \), is evaluated at the positions of the first and the second mesons \( \vec{r}_1, \vec{r}_2 \)

\[
\begin{align*}
\vec{r}_1 &= \vec{R} - \frac{M_2}{M_1 + M_2} \vec{r}, \\
\vec{r}_2 &= \vec{R} + \frac{M_1}{M_1 + M_2} \vec{r},
\end{align*}
\]  

(10)

where \( \vec{R} \) is the center of mass of the molecule and \( \vec{r} \) is the relative coordinate from the first to the second meson. In the center of mass system, \( \vec{R} = 0 \), and the matrix element of the transition \( \langle f | H_{\text{int}} | i⟩ \) is given by

\[
\langle f | H_{\text{int}} | i⟩ = \langle D\bar{D}\gamma | \{ H_{\text{int}}(\vec{r}_1) + H_{\text{int}}(\vec{r}_2) \} \frac{1}{\sqrt{2}} \left( |D^*\bar{D}⟩ \pm |D\bar{D}^*⟩ \right) 

= \frac{i}{\sqrt{2}} \sqrt{\frac{2\pi \hbar \omega}{V}} \left\{ |D|\bar{\mu}|D^*⟩ e^{+i\frac{M_1 M_2}{M_1 + M_2} \vec{k} \cdot \vec{r}} \pm \langle D|\bar{\mu}|D^*⟩ e^{-i\frac{M_1 M_2}{M_1 + M_2} \vec{k} \cdot \vec{r}} \right\} \cdot \vec{k} \times \vec{\varepsilon}(\vec{k}, \lambda).
\]

(11)

Since \( \langle D|\mu^i|D^*⟩ = \eta\langle D|\mu^i|D^*⟩ \), where \( \eta = -1 \), we get

\[
|\langle f | H_{\text{int}} | i⟩|^2 = \frac{12\pi \hbar \omega}{2} \frac{V}{V} \langle D|\mu^i|D^*⟩ \langle D|\mu^j|D^*⟩^* \delta_{ij} - \hat{k}_i \hat{k}_j \left\{ 1 \pm \eta \cos \vec{k} \cdot \vec{r} \right\}
\]

(12)

The electromagnetic radiative decay width of the molecular state \( X \) of the two mesons separated by a distance \( r \) is given by

\[
\Gamma^\pm_X(r) = \int 2\pi \frac{2\pi \hbar \omega}{V} \langle D|\mu^i|D^*⟩ \langle D|\mu^j|D^*⟩^* \left[ \delta_{ij} - \hat{k}_i \hat{k}_j \right] \left\{ 1 \pm \eta \cos \vec{k} \cdot \vec{r} \right\} \frac{V \omega^2 d\Omega}{\hbar c^3 (2\pi)^3}.
\]

(13)

To evaluate the angular integral, we write

\[
\int \left[ \delta_{ij} - \hat{k}_i \hat{k}_j \right] \left\{ 1 \pm \eta \cos \vec{k} \cdot \vec{r} \right\} d\Omega = 2\pi \left[ \delta_{ij} A_\pm + \hat{r}_i \hat{r}_j B_\pm \right].
\]

(14)

Multiplying both sides by \( \delta_{ij} \) and \( \hat{r}_i \hat{r}_j \) and summing over \( i \) and \( j \) we get two equation, which are solved for \( A_\pm \) and \( B_\pm \)

\[
A_\pm = \frac{4}{3} \pm 2\eta \left\{ \frac{\sin kr}{kr} + \frac{\cos kr}{(kr)^2} - \frac{\sin kr}{(kr)^3} \right\},
\]

\[
B_\pm = \pm 2\eta \left\{ -\frac{\sin kr}{kr} - 3\frac{\cos kr}{(kr)^2} + 3\frac{\sin kr}{(kr)^3} \right\}.
\]

(15)

From Eqs. (13,14,15), we get

\[
\Gamma^\pm_X(r) = \frac{\omega^3}{c^3} |\langle D|\mu^i|D^*⟩|^2 \left[ A_\pm + \cos^2 \theta B_\pm \right].
\]

(16)

where \( \theta \) is the angle between \( \bar{\mu} \) and \( \vec{r} \). For arbitrary directions of the magnetic moment, we take the average value of \( \cos^2 \theta = \frac{1}{3} \), and we get

\[
\Gamma^\pm_X(r) = \frac{\omega^3}{c^3} |\langle D|\mu^i|D^*⟩|^2 \frac{4}{3} \frac{1}{3} \left\{ 1 \pm \eta \frac{\sin kr}{kr} \right\}.
\]

(17)
Using Eq. (8) for the width of the $D^*$ meson, and noting that in our case $\eta = -1$, we can write Eq. (17) in the form

$$\Gamma_X^\pm (r) = \Gamma_{D^*} S^\pm (r),$$

(18)

where $S^\pm (r)$ is given by

$$S^\pm (r) = \left[ 1 \pm \frac{\sin kr}{kr} \right].$$

(19)

We can call $S^+(r)$ and $S^-(r)$ the superradiance and the subradiance factors when the two radiating mesons are separated by a distance $r$. These factors describe the effect of the interference between the two radiating sources $D^*$ and $\bar{D}^*$ on the decay width as a function of $kr$. We notice that since $\langle \bar{D} | \mu^i | D^* \rangle = \eta \langle D | \mu^i | D^* \rangle$, where $\eta = -1$, the symmetric state is subradiant while the antisymmetric state is superradiant, unlike the situation usually encountered in atomic physics where the symmetric state is superradiant and the antisymmetric state is subradiant.

Fig. 1 shows the dependence of the superradiance factor $S^+(r)$ and the subradiance factor $S^-(r)$ on $kr$. For $r = 0$ the the decay width is doubled in the superradiant state while the subradiant state is stable. As the distance between the two radiating mesons increases, both the superradiance and the subradiance factors approach one.

![Figure 1: The superradiance factor $S^+(r)$ (solid line) and the subradiance factor $S^-(r)$ (dashed line) against $kr$.](image)

The distance between the two mesons is not fixed but varies according to the molecular wavefunction which results from the solution of Schrödinger Equation for the Hamiltonian $H_{12}$ of Eq. (9). We follow Voloshin [10] and consider the molecular wavefunction as an $S$-state in the form

$$\psi(\vec{r}) = \sqrt{\frac{1}{2\pi r_0}} e^{-r/r_0},$$

(20)

where $r_0 = 1/\sqrt{2\mu w}$, $\mu = [M(D^*)M(\bar{D})]/[M(D^*) + M(\bar{D})]$ is the reduced mass, and $w = M(D^*) + M(\bar{D}) - M(X)$ is the binding energy of the molecule. Wavefunctions of nonzero values of orbital angular momentum can also be treated similarly.
The electromagnetic radiative decay width of the molecular state of $X(3872)$ can be evaluated by taking the average of $\Gamma_X^\pm(r)$ in the molecular state $\psi(\vec{r})$ considered in Eq. (20), thus we get

$$\Gamma_X^\pm = \int \psi(\vec{r})^* \Gamma_X^\pm(\vec{r}) \psi(\vec{r}) 4\pi r^2 dr,$$

which is easily evaluated, and we can write

$$\Gamma_X^\pm = \Gamma_D S^\mp,$$

where

$$S^\pm = \left[ 1 \pm \frac{2}{kr_0} \arctan \frac{kr_0}{2} \right].$$

This is our final result, and it is in agreement with the result obtained by Voloshin [10] using the momentum space representation in the calculation of the interference and the molecular wavefunction.

Fig. 2 shows the dependence of the superradiance factor $S^+$ and the subradiance factor $S^-$ on $kr_0$. For small $kr_0$ the superradiant limit of 2 and the subradiant limit of zero are reached, while for large $kr_0$ both factors approach one. It is interesting to note that while $S^\pm(r)$ oscillate about the value of 1 as shown in Fig. 1, $S^+$ is monotonically decreasing to 1 and $S^-$ is monotonically increasing to 1.

![Figure 2: The superradiance factor $S^+$ (solid line) and the subradiance factor $S^-$ (dashed line) against $kr_0$.](image)

For the decay of $X(3872)$ into $D^0\bar{D}^0\gamma$, $k = [M(D^0)^2 - M(D^0)^2]/2M(D^0) = 137$ MeV and $r_0 = 1/\sqrt{2\mu w} = 0.023$ MeV$^{-1}$ for $\mu = 966$ MeV and $w = 1$ MeV, so that $kr_0 = 3.1$. For these numerical values, $S^+ = 1.64$ and $S^- = 0.36$.

Measurements of the radiative decay width and comparison with Fig. 2 can provide valuable information about the structure and the dynamics of the state $X(3872)$. The molecular size, the binding energy, and the symmetry of $X(3872)$ can be studied in this way. This
state can be a good laboratory for investigating the superradiance and the subradiance phenomena. In addition to the neutral channel $D^0\bar{D}^0\gamma$, the charged channel $D^+\bar{D}^-\gamma$, although having a smaller width, will have larger binding energy $w$, smaller $r_0$ and consequently stronger interference effects.

4 Conclusions

The narrow resonance $X(3872)$ confirmed by many experiments may have a large component of its wave function as $D^*\bar{D}$ and $DD^*$ molecule. The electromagnetic decays of the $D^*$ and $\bar{D}^*$ mesons will reveal interference effects which depend on the distance between these two mesons and the wavelength of the emitted photon as pointed out by Dicke. Insight into the internal structure and the dynamics of $X(3872)$ can be gained by studying the electromagnetic radiative decay width into neural and charged $D$ and $\bar{D}$ mesons. Hadronic molecules are interesting in themselves and the phenomena of superradiance and subradiance may help reveal their features.

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