Nucleon Properties and Restoration of Chiral Symmetry at Finite Density and Temperature in an Effective Theory

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ABSTRACT

Modifications of baryon properties due to the restoration of the chiral symmetry in an external hot and dense baryon medium are investigated in an effective chiral quark-meson theory. The nucleon arises as a soliton of the Gell-Mann - Lévi \( \sigma \)-model, the parameters of which are chosen to be the medium-modified meson values evaluated within the Nambu - Jona-Lasinio model. The nucleon properties are obtained by means of variational projection techniques. The nucleon form factors as well as the nucleon delta transition form factors are evaluated for various densities and temperatures of the medium. Similar to the chiral phase transition line the critical curve in the \( T - \rho \) plane for delocalization of the nucleon is non-monotonic and this feature is reflected in all nucleon properties. At medium densities of about \( (2 - 3) \rho_{nm} \) the baryonic phase exists only at intermediate temperatures. For finite temperature and densities the nucleon form factors get strongly reduced at finite transfer momenta.

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1. Introduction

The QCD as a theory of the strong interaction is assumed to incorporate the spontaneous chiral symmetry breaking. The latter is believed to be a dominant mechanism in the low-energy sector of QCD and in particular, for the structure of the low-lying baryons. As it is suggested by the lattice QCD calculations\(^1\), at some finite temperature or/and at some finite density one generally should expect a restoration of the chiral symmetry. Hence, it is a common idea to consider the (partial) restoration of the chiral symmetry in a hot and dense medium as a relevant mechanism for a scale change which modifies the structure of the baryons immersed in it.

Since on one hand chiral models like Nambu – Jona-Lasinio\(^2\) and Gell-Mann – Lévy \(\sigma\)-model\(^3\) account for the chiral symmetry breaking and on the other they allow for the (partial) restoration of the symmetry at finite density and/or temperature those models seem to provide a suitable working scheme to study modifications of the meson and baryon properties in medium. It should be noted also that the critical value (of about 200 MeV) for the temperature is much lower than the typical cutoff used to regularize the NJL model. Indeed, the models show a significant success in description of both the static and dynamic properties of the nucleon arising as a non-topological soliton\(^4-6\)). Quite encouragingly, assuming that the nuclear medium can approximately be replaced by a uniform quark medium, the investigations\(^7-12\)) based on these models give a restoration at both finite temperature and density in a quantitative agreement with the Monte-Carlo lattice calculations as well as with the chiral perturbation theory\(^13\)). Medium\(^8,14-16\)) and temperature\(^17,18\)) effects in the properties of the mesons and of the nucleon have been successfully studied as well. Despite of that the meson and nucleon properties seem to be affected by the density and by the temperature in a similar way, the physical mechanism, however, driving the chiral symmetry restoration in each case, is different. At finite density the attractive interaction of the medium with the Dirac sea polarize it in a way that the quark condensate \(< \bar{q}q >\) gets reduced whereas at finite temperature the thermal fluctuations simply disorder the system. Therefore one might expect non-trivial effects to meson and baryon properties if both temperature and density are assumed finite. Indeed, the me-
son properties\textsuperscript{12,19} show a non-trivial temperature dependence at finite density. It is illustrated in Fig. 1 a) and b) where the constituent quark mass and the meson masses calculated\textsuperscript{19} within the NJL model are shown for four different baryon densities as a function of temperature. We also studied\textsuperscript{11} the modification of some static nucleon properties, namely the mass and the charge radius, due to the chiral symmetry restoration in hot and dense baryon medium. The nucleon appears as a soliton in the Gell-Mann – Lévy $\sigma$-model which is uniquely defined by medium-modified meson values coming from the NJL model. We found a non-monotonic critical curve for delocalization of the nucleon in the medium. The nucleon mass and the charge radius show also a non-monotonic temperature dependence at finite density.

In the present work we extend the study of ref.\textsuperscript{11} to nucleon form factors. Thus, the main task is to study in a systematic way the modification of the nucleon properties due to the restoration of the chiral symmetry in dense and hot medium. This is done by solving the medium-modified Gell-Mann – Lévy $\sigma$-model using angular and isospin projection combined with variational techniques. Various form factor and observables of the nucleon and the delta-isobar are evaluated.

### 2. The projected chiral soliton model

In our approach we assume a physical picture in which the nucleon, being off-shell by the presence of the medium, is simulated by an on-shell nucleon facing meson fields modified by the medium. In fact it means that the influence of the medium is expressed in terms of modified values of the pion decay constant, and the pion and sigma masses. To that end we use the NJL model in an approximation which consists of treating the baryon medium as an uniform quark matter neglecting the nucleon sub-structure in it. The modified values of $f_\pi$, $m_\pi$ and $m_\sigma$ are then used to define a Gell-Mann – Lévy $\sigma$-model for the nucleon (in medium) which is later solved by standard self-consistent methods.

We determine the properties of the nucleon employing the projected chiral soliton model\textsuperscript{5}). Here we briefly sketch its main points. It is given by the Gell-Mann–Lévy lagrangean\textsuperscript{3}) with valence quarks solved by a variational procedure with a spin and isospin projection.
The Gell-Mann - Lévy lagrangean reads:

\[
\mathcal{L} = \overline{\Psi} [i \gamma^\mu \partial_\mu - g(\sigma + i \gamma_5 \vec{\tau} \cdot \vec{\pi})] \Psi + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma + \frac{1}{2} \partial^\mu \vec{\pi} \partial_\mu \vec{\pi} - U(\sigma, \vec{\pi}).
\]  

(2.1)

with the meson self-interaction potential

\[
U(\sigma, \vec{\pi}) = \frac{\eta^2}{2} (\sigma^2 + \vec{\pi}^2 - \nu^2)^2 + f_\pi m_\pi^2 \sigma.
\]  

(2.2)

Besides the coupling constant \(g\) the others can be expressed by means of \(m_\pi, m_\sigma\) and \(f_\pi\):

\[
\eta^2 = \frac{(m_\sigma^2 - m_\pi^2)}{2 f_\pi^2}, \quad \text{and} \quad \nu^2 = \frac{(f_\pi^2 - m_\pi^2)}{\eta^2}.
\]  

(2.3)

In the present approach at finite density and temperature the lagrangean is simply modified by inserting their medium values. Actually the NJL model can be related to the Gell-Mann–Lévy sigma model via a gradient expansion and hence to combine these two models, as it is done here, seems to be reasonable.

In order to get a solitonic solution of lagrangean (2.1) we employ a variational procedure based on mean-field states with a generalized hedgehog structure and spin and isospin projection (described in detail in ref.\(^5\)). The trial functions for the nucleon is assumed to be product of coherent Fock states representing the sigma and pion fields and a quark state taken to be an antisymmetrized product in colour space of three valence quarks in the same 1s-orbit. The states of a good angular momentum and isospin are obtained from the classical solitonic solution by means of projection operators of the type:

\[
P_{MK(M_T K_T)}^{J(T)} \sim \int d\Omega D_{MK(M_T K_T)}^{(T)\ast} \hat{R}(\Omega),
\]  

(2.4)

where \(JM (TM_T)\) are spin (isospin) numbers and \(\alpha\) stands for the additional quantum numbers. \(R(\Omega)\) is the rotation operator in spin (or isospin) space and \(\Omega\) denotes the set of Euler angles in this space.

The obtained projected nucleonic solution with good spin and isospin numbers is used to evaluate the static nucleon properties as well as the nucleon and the nucleon-delta transition form factors. The form factors are evaluated in the Breit frame neglecting the recoil effects and assuming that the projected state of the nucleonic soliton is a good zero momentum state. In the case of the nucleon-delta transition form factors we work with the Rarita-Schwinger formalism in the.
limit of nucleon-delta mass degeneracy. One can find the details in refs.\textsuperscript{5}). For completeness we present the final expression for the form factors in terms of the projected state:

\[ G_E(q^2) = \int d^3r \, j_0(qr) < JTM M_T | j_{\mu}^0(r) | JTM M_T >, \]  

(2.5)

\[ \frac{G_M(q^2)}{2M_N} = \frac{3}{2} \int d^3r \, \frac{j_1(qr)}{qr} |r \times < JTM M_T | j_{\mu}^0(r) | JTM M_T >, \]  

(2.6)

\[ G_A(q^2) = \frac{2M_N}{\sqrt{q^2/4 + M_N^2}} \int d^3r \, \{ j_0(qr) < JTM M_T | A_i^\mu(r) | JTM M_T > - \sqrt{2}\pi j_2(qr) Y_2^\lambda < JTM M_T | A_i^\mu(r) | JTM M_T > |_{10} \}, \]  

(2.7)

\[ G_{\pi NN}(q^2) = -6M_N \int d^3r \, \frac{j_1(qr)}{qr} z < JTM M_T | \overline{\bar{J}_\pi}^3(r) | JTM M_T >. \]  

(2.8)

\[ \frac{G_{\pi NN}^M}{2M_N} = \sqrt{6} \int d^3r \, \frac{j_1(qr)}{qr} < \Delta^+_i | \bar{\mu}_0 | N^+_i >, \]  

(2.9)

\[ G_{\pi N\Delta} = 6M_N \frac{M_N^2}{q^2/4 + M_N^2} \cdot \frac{3}{2} \int d^3r \, \frac{j_1(qr)}{qr} z < \Delta^+_i | \bar{f}_\pi^0 | N^+_i >. \]  

(2.10)

Eqs.(2.5) and (2.6) are the electromagnetic Sachs form factors as the \( j_{\mu}^0 \) is the \( \mu \)-component of the electromagnetic current operator. The axial form factor is given by eq.(2.7) where the \( A_i^\mu \) is the \( i \)-space and \( \mu \)-isovector component of the axial current operator. Eq.(2.8) concerns the pion nucleon form factor evaluated by the pion source current \( \overline{\bar{J}_\pi} \). For the \( N - \Delta \) transition magnetic form factor (2.9) we use the standard decomposition for the matrix element of the electromagnetic current given by Jones and Scadron\textsuperscript{20}). For the \( \pi N \Delta \) form factor \( G_{\pi N\Delta} \) (2.10) the simplest coupling for the \( \pi N \Delta \) vertex\textsuperscript{21}) is assumed.
3. Nucleon as a soliton at finite density and temperature

In the present approach the nucleon appears as a self-consistent localized stationary solution (soliton) in a suitable modified Gell-Mann - Lévy sigma model solved in a variational procedure with a spin and isospin projection. The first problem we would like to look into is the limits which the restoration of the chiral symmetry in a hot and dense medium imposes on the existence of the nucleon as a soliton. In this model picture the lack of solitonic solution is taken as an indication for a delocalization of the baryonic phase. It is rather tempting to identify this with deconfinement but a such conclusion seems to be out of the scope of the model itself. Fig. 2 shows the resulting critical line for existing of the nucleon in the $T - \rho$ plane together with the chiral phase diagram expected from the meson sector. One can see that both critical curves have a nearly identical behaviour where the critical values for the baryonic phase are smaller than ones for the chiral phase transition. It means that the delocalization of the nucleon always happens before the restoration of the chiral symmetry takes place. Apparently the change of the chiral symmetry properties drives the delocalization of the nucleon. It should also be noted that for densities $(2-3)\rho_{nm}$ which are relevant for the high energy heavy-ion reactions the baryonic phase exists only at intermediate temperatures.

3.1. Nucleon static properties at finite density and temperature

Using the projected nucleonic solution with good spin and isospin quantum numbers we have also calculated at finite density and temperature all nucleon properties including the nucleon and nucleon-delta transition form factors as well. In this section we present the results concerning the nucleon static properties. The calculated nucleon mass as a function of the temperature at different densities is shown in Fig. 3. At zero density the nucleon mass shows no change at increasing temperatures up to $1/2T_c$ and decreases then monotonically at higher temperatures. Near the critical value $T_c$ the nucleon mass is noticeably reduced. Similar behaviour is concluded by Bernard and Meissner within the Skyrme model with vector mesons. It is interesting also to mention that Ellis et al. predict that the reduction of the nucleon mass will lead to an enhancement of the antibaryon production in heavy-ion collisions. At finite densities, however, the temperature dependence of the nucleon mass is quite different. For temperature up to about
the mass increases which means that the nucleon becomes more stable. At density above the critical value $\rho_c$ the nucleon exists only at intermediate temperatures which agrees with the critical line shown in Fig. 2. The delta mass as well as the nucleon-delta mass splitting have exactly the same behaviour as those of the nucleon mass shown in Fig. 3.

Fig. 4 and Fig. 5 illustrate the density and temperature dependence of the proton and neutron charge radii. Apart from the values at the origin they have similar behaviour. At the critical temperature value all curves show a clear trend to grow to infinity which indicates the delocalization of the nucleon. At vanishing medium density both quantities increase monotonically. At finite density values it changes as at intermediate temperatures the radii get reduced. The latter is a clear signal for a stabilization of the nucleon. This is very well seen at high densities where the nucleon has a finite radius only at intermediate temperatures. We found similar behaviour in the magnetic and axial r.m.s.radii at finite density and temperature.

It should be emphasized that the vanishing of the nucleon mass and the unlimited increase of the radius happen always along the critical line for the nucleonic soliton solution. It means that at least in present model picture the lack of the solitonic solution should be identified with a delocalization of the nucleon.

The proton magnetic moment calculated at finite density and temperature is depicted in Fig. 6. The same picture is valid for the neutron magnetic moment as well as for $N-\Delta$ transition magnetic moment. As can be seen the magnetic moment shows a behaviour similar to those of the charge radius but the medium effect is much less pronounced.

In contrast to the other nucleon properties the axial vector coupling constant and the pion nucleon coupling constant as well as the pion nucleon delta coupling constant show a very slight medium modification. Apart from the critical density and temperature values they stay almost constant. The $g_\Lambda$ and $g_{\pi N \Delta}$, however, are a bit more affected: the first quantity shows a slight decrease whereas the second one a slight increase. For instance, at temperature close to $T_c$ both show a common change of about 10% relative to the free value. At the critical temperature and density values all coupling constants vanish rapidly. Dey et al. concluded
a bit larger temperature effect in $g_{\pi NN}$. In their approach the $g_{\pi NN}$ is bound to decrease with the temperature. Their considerations, however, are based on a particular assumptions about the nucleon current and are limited to relatively low temperatures because of the approximations used.

3.2. Nucleon and nucleon-delta transition form factors at finite density and temperature

Using the projected nucleonic solution with good spin and isospin numbers in eqs.(2.5–2.2) we evaluate the nucleon and the nucleon-delta transition form factors. The results for the proton and neutron electric form factors are plotted in Fig. 7 and Fig. 8 for a zero and two times nuclear matter density and three different temperatures. As can be seen at zero density and relatively low temperatures (< 100 MeV) the effects are negligible. At finite density and temperature the form factor gets reduced at finite transfer momenta in comparison with the free one. One realizes, however, that similar to other nucleon properties at intermediate temperatures the density effects are partially suppressed by the temperature and the form factor are less reduced than in the case of vanishing temperature and finite density. The slope of the origin also changes which is reflected in the corresponding r.m.s. radii. Close to the critical values the temperature effects are dominant and strong enough to make the form factors practically vanishing at large transfer momenta.

One can find the same effects in the other form factors. In Fig. 9 the proton magnetic form factor is depicted. As in the previous case there is no change at zero density and low temperatures up to 100 MeV. The crossing known from the finite density study\textsuperscript{16}) is still presented. The magnetic $N – \Delta$ transition form factor has an identical behaviour. The last two form factors presented (see Fig. 10 and Fig. 11) are the axial and the pion nucleon form factors. The pion nucleon delta form factor is similar to the pion nucleon one. Both seem to be more affected by the medium at finite momentum transfers than the other form factors.
4. Summary

We have investigated the modification of the nucleon properties due to the restoration of the chiral symmetry in a hot and dense baryon medium within an effective chiral quark-meson theory. The nucleon appears as a soliton in the Gell-Mann – Lévy $\sigma$-model which is defined in terms of the modified values of the pion decay constant and the pion and sigma masses in the medium. These values are obtained from the NJL model. The critical curve in $T - \rho$ plane for delocalization of the nucleon is non-monotonic. The delocalization occurs before the chiral phase transition. At medium densities of about $(2-3)\rho_{nm}$ the baryonic phase exists only at intermediate temperatures. The nucleon properties evaluated by means of projection techniques also show non-monotonic dependence on temperature at finite density. At critical density and temperature the nucleon mass vanishes whereas the radii grow to infinity. At finite density and intermediate temperatures both quantities indicate a stabilization of the nucleon. At finite density and temperature values all form factors get reduced at finite transfer momenta. The reduction, however, is smaller than in the case of vanishing temperature.

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FIGURE CAPTIONS

1. The constituent quark mass $M$ (upper part) and the sigma and pion masses $m_\sigma$ and $m_\pi$ (lower part) as functions of temperature at different medium densities.

2. $T$-$\rho$ baryonic phase diagram. The solid curve is the critical line for the chiral symmetry restoration. The dashed one shows the critical line for the baryonic phase.

3. The nucleon mass as a function of the temperature at different medium densities.

4. The proton charge r.m.s.radius as a function of the temperature at different medium densities.

5. The neutron charge r.m.s.radius as a function of the temperature at different medium densities.

6. The proton magnetic moment as a function of the temperature at different medium densities.

7. The neutron electric form factor as a function of the temperature at different medium densities. The curves are given at zero and two times $\varrho_{nm}$ and the temperatures $T = 0, 100$ and $180$ MeV.

8. The proton electric form factor as a function of the temperature at different medium densities. The curves are given at zero and two times $\varrho_{nm}$ and the temperatures $T = 0, 100$ and $180$ MeV.

9. The proton magnetic form factor as a function of the temperature at different medium densities. The curves are given at zero and two times $\varrho_{nm}$ and the temperatures $T = 0, 100$ and $180$ MeV.

10. The axial form factor as a function of the temperature at different medium densities. The curves are given at zero and two times $\varrho_{nm}$ and the temperatures $T = 0, 100$ and $180$ MeV.
11. The pion nucleon form factor as a function of the temperature at different medium densities.