Is it possible to study neutrinoless $\beta\beta$ decay by measuring double Gamow-Teller transitions?

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Abstract. Searches of neutrinoless double-beta decay require information on the value of the nuclear matrix elements that rule the process to plan and interpret experiments. At present, however, even the matrix elements obtained with the most reliable many-body approaches do not agree to each other better than a factor two or three. A usual test of the many-body calculations is the comparison to several nuclear observables, but so far no nuclear structure property has been found to show a good correlation to neutrinoless double-beta decay. Here we propose that double charge-exchange experiments can offer a very valuable tool to provide insights on neutrinoless double-beta decay. Double charge-exchange reactions are being currently performed in various laboratories worldwide and aim to find the novel nuclear collectivity given by double Gamow-Teller excitations. Our results suggest a good linear correlation between double Gamow-Teller transitions to the ground state of the final nucleus and neutrinoless double-beta decay nuclear matrix elements. The correlation seems robust across $pf$-shell nuclei.

1. Introduction

Neutrinoless double-beta ($0\nu\beta\beta$) decay is possibly the best probe available to test whether neutrinos are its own antiparticle. While present $0\nu\beta\beta$ decay half-life limits are impressive, exceeding $10^{25}$ years for $^{136}$Xe, $^{76}$Ge and $^{130}$Te [1, 2, 3, 4], observing the decay rates predicted by the standard model of particle physics demands improving the half-life sensitivity of present experiments by two orders of magnitude, if the ordering of the neutrino masses is “inverted”, or by four orders of magnitude, if it is “normal” [5]. The estimated sensitivities, however, depend critically in the nuclear matrix elements (NMEs) that control the decay, $M_{0\nu\beta\beta}$ [6]

$$[T_{1/2}^{0\nu}]^{-1} = G_{0\nu} \left| M_{0\nu\beta\beta} \right|^2 m_{\beta\beta}^2,$$

(1)

with $G_{0\nu}$ a phase-space factor, and $m_{\beta\beta} = \sum U_{ei}m_i/m_e$ a combination of the neutrino masses and mixing matrix, normalized to the electron mass. Strategies to design future experiments exploring “inverted” and “normal” scenarios require a good knowledge of the NMEs.

Unfortunately nuclear theory is currently not in a position to provide very reliable NMEs. The best available calculations disagree between each other by a factor two or three when comparing results obtained with different nuclear many-body approaches [6]. These calculations are tested against experimental data on several nuclear structure properties: excitation spectra,
electromagnetic and $\beta$ decays [7], knockout and transfer reactions [8, 9], $\beta\beta$ decay with the emission of two neutrinos [10]... Various many-body approaches generally find good agreement with data when confronting these properties. By contrast, they differ significantly on their prediction of $0\nu\beta\beta$ decay NMEs. So far no nuclear observable has been found to be especially well correlated to $0\nu\beta\beta$ decay. The observation that $0\nu\beta\beta$ decay seems to be subtle on its own way has prevented to pin down the value of the NMEs.

2. Double Gamow-Teller transitions and $\beta\beta$ decay
Gamow-Teller (GT) strength distributions measured via charge-exchange experiments [11, 12, 13] are good tests of the many-body calculations. GT distributions have the advantage that they can probe transitions not accessed in $\beta$ decay due to the limited $Q_\beta$ value, and in addition they exhibit collective behavior best manifested in the GT giant resonance. Recently, there is a strong experimental interest in performing double charge-exchange experiments to access the novel nuclear collectivity indicated by double GT (DGT) transitions [14, 15, 16, 17]. Similarly to the GT case, this brings the opportunity to assess the validity of many-body calculations of $\beta\beta$ decays by testing predictions against measured DGT distributions.

Double charge-exchange reactions are a result of the strong interaction. On the other hand $\beta\beta$ decay is governed by the weak interaction—the reason why it is related to neutrinos. Nonetheless it is important to keep in mind that in both cases a pair of neutrons can become a pair of protons, via nucleon exchange in charge-exchange reactions, or via nucleon decay in $\beta\beta$ decay. In addition, the two processes are sensitive to the spin of the nucleons. We also note that $0\nu\beta\beta$ decay occurs at second-order in the weak interaction, and in principle it should depend explicitly on the states of the intermediate nucleus. However, it does not make much difference if one uses the so-called “closure approximation” that brings the NME only dependent on the initial ($i$) and final ($f$) nuclear states [18]. When doing so the $0\nu\beta\beta$ decay and DGT matrix elements read [6, 19]

$$M^{0\nu\beta\beta}(i \rightarrow f) = M_{GT}^{0\nu} + \frac{M_{F}^{0\nu}}{g_{A}^2} + M_{T}^{0\nu} = \sum_{X=GT,F,T}\langle f | \sum_{a,b} H_{X}(r_{ab}) S_{X} \tau^{+}_{a} \tau^{+}_{b} | i \rangle,$$

$$M^{DGT}(i \rightarrow f) = \langle f | \sum_{a,b} [\sigma_{a} \tau^{+}_{a} \times \sigma_{b} \tau^{+}_{b}] | i \rangle,$$

where the sums run over all nucleons, $\tau$ represents the isospin, and the DGT operator is coupled to $\lambda = 0, 2$. $S_{X}$ denote the GT, Fermi and tensor spin structures that contribute to $0\nu\beta\beta$ decay, and $H_{X}(r_{ab})$ are the so-called neutrino potentials corresponding to each spin structure—divided by $g_{A}^2$ in the Fermi case. The neutrino potentials depend on the distance between the decaying nucleons, and are given by [6]

$$H_{X}(r_{ab}) = \frac{2R}{\pi} \int_{0}^{\infty} q^2 dq J_{X}(q r_{ab}) h_{X}(q) q(q + \mu),$$

with $q$ the momentum transfer, $R = 1.2A^{1/3}$ fm the nuclear radius, $J_{F}(q r_{ab}) = j_{2}(q r_{ab}) = j_{0}(q r_{ab})$ and $j_{GT}(q r_{ab}) = j_{2}(q r_{ab})$ spherical Bessel functions, and $\mu$ the so-called closure energy [18]. The explicit form of the $h_{X}(q)$ functions can be found in Ref. [20].

The $M_{GT}^{0\nu}$ part with $S_{GT} = \sigma_{a} \cdot \sigma_{b}$ is by far dominant in $0\nu\beta\beta$ decay, accounting for almost 90% of the total NME. Therefore the operator structure that mainly drives $0\nu\beta\beta$ decay is that of DGT transitions with $\lambda = 0$ coupling. Overall, the main differences between the two processes are the presence of the neutrino potentials in $0\nu\beta\beta$ decay, and the capability of DGT transitions to connect states that differ in up to two units of angular momentum. In addition, in practice $0\nu\beta\beta$ NMEs only refer to decays to the ground state of the final nucleus. Other decays are very suppressed, or energy forbidden, due to the small—about a couple of MeV at most—$Q_{\beta\beta}$ values. In contrast, DGT transitions can populate final states excited up to several tens of MeV.
Figure 1. Correlation between the $0\nu\beta\beta$ decay NME $M^{0\nu\beta\beta}_{gs,i \rightarrow gs,f}$ and the DGT transition to the ground state of the final nucleus $M^{DGT}_{gs,i \rightarrow gs,f}$. Shell model results for initial calcium (Ca), titanium (Ti) and chromium (Cr) isotopes are shown in red, blue and orange, respectively. Calculations with the $pf$-shell GXPF1B [22], KB3G [23] and $sd−pf$ SDPFMU-DB [24] interactions are denoted by squares, circles and triangles, respectively. Open symbols indicate fully correlated nuclear states, while filled ones refer to nuclei restricted to seniority-zero configurations. The shell model results are compared to the non-relativistic energy-density functional (EDF) theory $^{48}$Ca result from Ref. [26] (light blue pentagon). The inset zooms in the results corresponding to fully correlated states.

3. Results for DGT and $0\nu\beta\beta$ decay matrix elements

The entire DGT distribution of the $\beta\beta$ emitter $^{48}$Ca was recently calculated in Ref. [19], using the shell model many-body framework. Ref. [19] considered a typical shell model space consisting of the $pf$-shell—this is, including the $0f_7/2$, $1p_{3/2}$, $1p_{1/2}$ and $0f_{5/2}$ single-particle orbitals—and an extended one comprising the $sd−pf$ configuration space—with the $0d_{5/2}$, $1s_{1/2}$ and $1d_{3/2}$ orbitals included as well. The results predicted a DGT distribution relatively independent of the size of the configuration space and the shell model interaction used.

An analysis of the sensitivity of the total DGT strength distribution to isovector—dominated by like-particle—and isoscalar—to which only proton-neutron contribute—pairing correlations showed that isovector pairing correlations tend to push the DGT giant resonance to higher energies [19]. Combining this sensitivity with the well-known dependence of the $0\nu\beta\beta$ decay NME to isovector pairing [21], Ref. [19] showed that, for moderate increases of the isovector pairing correlations, the energy of the $^{48}$Ca DGT giant resonance and the $0\nu\beta\beta$ decay NME value follow a simple linear correlation. The fact that a collective property such as the energy of the DGT giant resonance might be related to the value of an individual matrix element is very promising, and supports the similarity between DGT transitions and $0\nu\beta\beta$ decay. Further, the relation suggests that nuclear structure measurements of DGT transitions can be used to study, indirectly, $0\nu\beta\beta$ decay.

If there is a DGT transition that can be expected to be especially related to the $0\nu\beta\beta$ decay NME this is the DGT transition to the ground state of the final nucleus. In such case the final
states of the $0\nu\beta\beta$ decay and DGT transitions are the same. Moreover, if the initial state is a $0^+$ state—as the ground states of every even-even nucleus are—the DGT transition to the ground state can only be mediated by the $\lambda = 0$ channel, which implies that the spin structure of the DGT transition and the (dominant part of) the $0\nu\beta\beta$ decay are common as well. This implies that when only ground states are involved, and taking into account the validity of the closure approximation, the only difference between the $0\nu\beta\beta$ decay and the DGT transition matrix elements is caused by the neutrino potential $H(r_{ab})$. The potential signals the fundamental difference between the two processes: while $0\nu\beta\beta$ decay is driven by the weak interaction, DGT transitions, explored in charge-exchange experiments, are a result of the strong force.

Figure 1 shows the results of the shell model calculation of $0\nu\beta\beta$ decay NMEs compared to the DGT transitions to the ground state. The decays of calcium (shown in red), titanium (blue) and chromium (orange) isotopes are shown. Figure 1 considers a total of twenty six initial even-even nuclei, from the lightest $^{42}$Ca to the heaviest $^{60}$Cr. In addition, Fig. 1 explores two kind of nuclear states: the open symbols correspond to full shell model states, while the filled symbols indicate simplified states that entirely consist of pairs of neutrons and pairs of protons coupled to $0^+$—known as seniority-zero states. Finally, Fig. 1 shows results obtained using different configuration spaces and shell-model interactions: the $pf$-shell interactions GXPF1B [22] (squares) and KB3G [23] (circles), and the extended $sd-pf$ space SDPFMU-DB interaction [24] (triangles). The results were obtained with the shell model codes KSHELL [25] and NATHAN [7].

Figure 1 highlights a very good linear correlation between $0\nu\beta\beta$ decay NMEs and DGT transitions to the ground state. The correlation seems robust. It is valid across a very wide range of nuclei in this region—mass numbers $42 \leq A \leq 60$—and it is also common to the three shell model interactions used, in the usual and extended configuration spaces. Furthermore, the linear correlation remains valid regardless of whether we consider fully correlated shell model states or fairly simple seniority-zero wave functions. Because seniority-zero nuclear states overpredict the $0\nu\beta\beta$ decay NMEs, the last property implies that the correlation is valid for a very wide range of NME values $0 \lesssim M_{0\nu\beta\beta} \lesssim 10$. Such a broad range encompasses the factor-two-or-three range of NME values that covers the results of the different many-body calculations for any $\beta\beta$ emitter.

It is interesting to note that the $^{48}$Ca results obtained in the framework of energy density functional theory [26] (light blue pentagon in Fig. 1) are consistent with the linear correlation found in the shell model calculations. Reference [19] extended to heavier nuclei the comparison of $0\nu\beta\beta$ decay and DGT transitions to ground states, finding a linear correlation very similar to the one in Fig. 1 [19]. Again, for heavier nuclei the results from energy-density functional theory agree well with the correlation observed in the shell model calculations. In contrast, the results of the quasiparticle random phase approximation (known as QRPA) [27] behave very differently, with DGT transition values that tend to be persistently small [19].

How can we understand the existence of a rather universal linear correlation between $0\nu\beta\beta$ decay and DGT matrix elements? We can try to answer this question by studying the matrix element distributions as a function of the distance between decaying/transferred nucleons, and of the momentum transfer. For $0\nu\beta\beta$ decay the normalized distributions are defined as

$$C_{GT}(r) = \langle \tilde{f} \mid \sum_{ab} \delta(r-r_{ab}) H_{GT}(r_{ab}) \sigma_a \cdot \sigma_b \tau_a \tau_b|i\rangle/M_{GT}'$$

$$C_{GT}(p) = \langle \tilde{f} \mid \sum_{ab} \delta(p-q) \tilde{H}_{GT}(q) \sigma_a \cdot \sigma_b \tau_a \tau_b|i\rangle/M_{GT}'$$

An analogous definition can be used for DGT transitions, except that in that case there is no neutrino potential and the spin structure is defined with a different normalization $[\sigma_a \times \sigma_b]^{0} = -\frac{1}{\sqrt{3}} \sigma_a \cdot \sigma_b$ (the different definition does not affect the normalized distributions).
Figure 2. Normalized distributions of the Gamow-Teller part of the $0\nu\beta\beta$ decay NME $M_{0\nu}^{GT}$ (red) and and the DGT matrix element $M^{DGT}$ (orange), as a function of (a) the internucleon distance $C_{GT}(r)$, (b) the momentum transfer $C_{GT}(p)$. The results correspond to $^{48}\text{Ca}$ calculated with the $pf$-shell GXPF1B interaction.

Figure 2 shows the normalized radial and momentum distributions for $^{48}\text{Ca}$. The results show that the momentum distribution of the two processes is very different: while the DGT transitions in Eq. (3) do not entail any momentum transfer\(^1\), typical transferred momenta amount to $p \sim 100$ MeV in $0\nu\beta\beta$ decay. In contrast, the radial distributions of the two processes are much more similar: they are dominated by short-range internucleon distances of $r \lesssim 2$ fm. Moreover, the longer-range contributions that contribute to DGT transitions partially cancel, making the effective DGT range even more similar to that of $0\nu\beta\beta$ decay. We have observed that this partial cancellation is common to all the other DGT transitions in Fig. 1. We have to stress that the very short-range character of $0\nu\beta\beta$ decay and DGT transitions is a somewhat surprising outcome of the many-body calculations. Its validity should be carefully checked in more controlled ab initio calculations, such as those in Ref. [28].

The short-range character of $0\nu\beta\beta$ decay and DGT transitions to the ground state can explain the simple linear correlation observed in Fig. 1. References [29, 30] showed that when an operator probes only the short-range physics of low-energy states, the corresponding matrix elements factorize into a universal operator-dependent constant times a state-dependent number common to all short-range operators. Therefore, a linear relation is expected between two short-range operators, such as $0\nu\beta\beta$ decay and the corresponding DGT transition to the ground state. This explanation is also consistent with the quasiparticle random phase approximation results not being correlated: for this many-body method $0\nu\beta\beta$ decay NMEs are of short-range as in the shell model, but DGT transitions receive important contributions from much longer internucleon distances [27, 31]. To clarify this picture, further work should explore the physics of the long-range contributions in detail.

\(^1\) The identity that replaces the neutrino potential in Eq. (6) can be seen as the Fourier transform of a delta function in momentum space.
4. Conclusions
We have used the nuclear shell model to study the similarity between $0\nu\beta\beta$ decay NMEs and DGT transitions to the ground state, for a collection of twenty six transitions in the pf-shell. We have found that the $0\nu\beta\beta$ decay and DGT matrix elements are very well correlated. The correlation is common to all the transitions studied, and does not depend on the details of the nuclear interaction used, or the nuclear structure correlations permitted in the initial and final nuclei. Moreover, a result for $^{48}$Ca obtained with energy-density functional theory is consistent with the correlation found in the shell model calculations. Our findings suggest that future measurements of DGT transitions in double charge-exchange experiments can provide very valuable insights on $0\nu\beta\beta$ decay.

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