Optimization of the number of pumping stations when tracing distribution lines of the external fire-fighting water supply system

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Abstract. New graphical algorithms of planning, optimal deployment of the pumps for the external firefighting are proposed and mathematical models of optimization when constructing external various water-supply pipeline networks for fire hydrants in a separate community that does not involve highways or special roads are also presented.

1. Introduction
As a rule, in the process of fire-fighting large amounts of fire-extinguishing agents are required and the main one among them is water. The main source of water in this situation proves to be the systems of external fire-fighting water supply pipelines (EFFWP). Simulation mathematical models and techniques for the assessment of water yield in the water supply circular and dead-end mains of external firefighting water supply networks are known and well-studied, they can be found in the extensive reference works [1-3].

2. Optimization of circular design for water supply pipeline with the use of traveling salesman problem (TSP) for different metrics
For the realization of the optimal water pipelines with a less length we present a standard formula from the work [1]. Coefficient of hydraulic resistance takes into account line and local hydraulic losses at the segment of pipeline between the taps and inside a tap and it can be determined from the expression:

\[
A = \frac{\rho}{2} \sum_{k=1}^{n} \frac{\xi_k(l) + \xi_k(m)}{F_k^2}
\]  

(1)

Here \(n\) is a number of the pipeline segments forming a concerned part of EFFWP; \(\xi_k(l)\) – coefficient of linear resistance at the \(i\)-th section of the pipeline; \(\xi_k(m)\) is a sum of coefficients of the local resistance values at the \(i\)-th section of the pipeline; \(F_k^2\) is cross-section area of the pipeline at the \(i\)-th section of the pipeline.

This formula just as many others demonstrates the actuality of the problem of minimization for the total length of the pipes fragments. Design of the minimal in length circular pipe connection for all of the fire-fighting pipelines (FPL) in the form of a closed graph, without circuits, where one edge of the
graph enters one of its node and one exits from the node is very similar to the known traveling salesman problem (TSP) [4]; or, in other words, obtaining of the Hamilton graph with the optimal length (Figure 1).

This problem can be solved by a simple computer search technique or with the use of special known algorithms [4]. However, in practice this problem is complicated since the size, shape and location of the contemporary buildings do not permit simulation of this optimization problem in Euclidian metrics (or simply in direct lines approach) (Figure 1).

In the real scenario the travelling salesman problem should be solved not in Euclidian metrics, but in the space metrics of $\ell^1_n$. Therefore, the program for computer solution of this problem should be written with the use of this metrics. The distance in this program is calculated with the use of expression (2).

$$\rho_m(M_1,M_2) = |x_1 - x_2| + |y_1 - y_2|$$

(2)

Computer program can operate with any type of metrics. Pipe arrangement of the broken line kind can be changed (see Figure 2).

Authors of this work could not find similar description of design of the circuit connection in the known literature as well as optimization of its total length with the use of the modified TSP.

One should note that in the modern concept of council estate development it is possible to distinguish several main kinds of the development: line building, cluster housing, regular housing, free housing and so on. Hence, for each kind of the development an individual water supply scheme to the hydrants of external water supply should be planned in details.
Thus, in the author’s opinion, circuit design of water supply is better fitted to the line building and cluster housing variants of the community development (Figure 3).

**Figure 3.** Kinds of the housing development: a) along a perimeter; b) line building; c) cluster housing.

### 3. Design of the dead-end mains of special type water supply networks

Assume that we are allowed of constructing the new type of pump station. This station of special operation after alert signaling can supply water for firefighting to the hydrants located at the immediate vicinity from the hotbed of fire and not supply water to the other points thus saving all of the possible expenses. So, there appears the problem – where this station should be located in order the total length of the pipes fragments would be the least one (Figure 4). The problem of searching coordinates for such station is reduced to the task of determination coordinates for Fermat-Torricelli–Steiner point. This task is solved analytically for a triangle, $n = 3$. For the total number of points with $n \geq 4$ the problem of determination of the point coordinated with the optimal sum of distances to the fixed $n$ points is not solved yet even for the case of points arrangement on the plane. Some results on this topic are considered in [5–8].

Let us formulate the problem in general: it is necessary to determine coordinates of $M$ point in Banach space so that the sum of distances from this point to the fixed $n$ points $(A_1,A_2,...,A_n)$ in the coordinate Banach space is minimal one.

In [6, 7] computer algorithms are presented as well as the programs for computer (approximate) solution of this task are described using Java2 and C++ languages, for any number of the points in the graph and with any preset accuracy. Accuracy can be set in the program basing on the practical appropriateness.
Figure 4. Schematic presentation of the location point for the pump station: circle designates position of the pump station (in Euclidian metrics), hatched rectangle designates position of the pump station in $l_1$ metrics.

Dashed line in Fig. 4 shows variant of external fire-fighting pipeline laying-out along direct route, while solid line shows laying-out route in $l_1$ metrics. So, will a position of the pump station be different one in case if the measurements are made in different metrics? This case is presented in Figure 4. To elucidate this effect two analytical issues are considered below.

4. On the mismatch of Ferma-Torricelli – Steiner (F-St) points in different metrics

In the finite-dimensional space all of the norms are equivalent. Constants of equivalence depend on the dimensionality and the increase with the growth of dimensionality - $N$.

For example, inequality with the constants of equivalence that are precise by the order for the spaces $l_1 \subset l_2 \subset l_\infty$, $\| \vec{C} \|_{l_1} \leq \sqrt{N} \| \vec{C} \|_{l_2} \leq N \| \vec{C} \|_{l_\infty}$. So, there appears a question: can coordinates of F-St. points be different when measuring distances in different metrics? Or if these points can be matched? In [9] it was proved that even if the “customer points” are arranged on the circle coordinates of F-St do not coincide and they have different coordinates while measurements are made in different metrics. First, this phenomenon was revealed (see [6-9]) with the use of computer graphics (Figure 5). Analytical proof of this fact is presented in [9].

5. Computer graphics

Mapping of F.-St. point is performed with the use of iteration programs where the search space is getting narrow and the values of the objective functions of the following type are calculated for each step of the algorithm utilized in these programs:

$$R_{l_1}(x, y) = \sum_{k=1}^{n} |x - x_k| + |y - y_k| \quad (3)$$

$$F_{l_2}(x, y) = \sum_{k=1}^{n} \sqrt{(x - x_k)^2 + (y - y_k)^2} \quad (4)$$

For the case of space $l_1$ (right part of the figure 5) and of space $l_2$ (left part of the figure 5) of metrics is presented.
Figure 5. Left part is a determination of F-St point and layout in Euclidian metrics, right part of the figure is made in the metrics of \(l_1\).

For the case of the problem concerned with the pipe lining in a case when complexity of the pipeline layout depends on the direction of lining, it is possible to consider the objective function of optimization with the complexity coefficients (weighing coefficients) of the following kind:

\[
F_s(x, y) = \sum_{k=1}^{n} \mu_k \sqrt{(x - x_k)^2 + (y - y_k)^2}
\]

Computer graphic is presented in Figure 6. In order to plot this graphic formula (5) was applied (5). Complexity vector was involved the calculation program: \(\bar{\mu} = (1; 1.5; 1.2; 2; 2; 1.4)\).

Figure 6. Computer graphics for the pumping stations system.

The reason for arising of these coefficients is well-known, for example, it is a tortuosity of the fire line – 1.2.

Note that in authors opinion water dead-end mains is best fitted to the ribbon and sporadic building of micro-district (figure 3).

Thus, some distinctions of design for various special water dead-end mains in the external firefighting systems of zone operation are considered in the article. Selected water supply is possible just into the district where the fire occurred. This leads to a considerable economy in the different problems as when building-up of the water-supply networks as in their maintenance. Basing on the developed current methods and the ways of the information transfer the authors justify advantages of these networks. When designing these networks different metrics for measuring the lengths of pipelines across the plane in a dependence on complexity of the pipeline layout, geometry of its arrangement, as
well as on the dimensions and shape of the buildings. For the layouts the problem of finding coordinates of Ferma-Steiner point is utilized for the different metrics. This problem is solved with the use of computer programs for numerical determination of the coordinates together with the iteration check of the objective function. Type of the function depends on the way (metrics) of the distance measurement.

One should note that construction of Steiner networks (associated with F-St points) in different metrics is applied for a long time and rather successively for a lot of applied problems, for example, in case of laying-out of the optimal oil pipelines networks. Associating of the more appropriate way of water supply in a dependence of their types should account for the way of micro-district building-up.

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