FORECASTING THE STRENGTH OF WOODEN STRUCTURES BY METHODS OF FRACTURE MECHANICS

Galina E. Okolnikova¹, Elena F. Shaleeva²

¹Department of Civil Engineering, Peoples' Friendship University of Russia (RUDN University), Moscow, Russia
²Foreign Languages Department, Peoples' Friendship University of Russia (RUDN University), Moscow, Russia

¹okolnikova_ge@rudn.university, ²shaleeva_ef@rudn.university

Corresponding Author: Galina Erikovna Okolnikova

https://doi.org/10.26782/jmcms.spl.9/2020.05.00009

Abstract

The article analyzes the operation of wood in the elements of real structures by the methods of the theory of elasticity and the methods of fracture mechanics; six types (systems) of crack development in the elements of wooden structures are considered, the anisotropy of wood is also taken into the account; The parameters of fracture mechanics for wooden beams with cuts were experimentally determined; values of stress intensity factors for pine wood were determined, graphical dependencies between stress intensity factors and notch depth for different crack opening models were also constructed, calculation results were compared using materials resistance methods (elasticity theory), fracture mechanics methods and experimental results; the conclusion is drawn about the applicability of linear fracture mechanics for predicting the strength of wooden structures.

Keywords: Fracture mechanics, stress intensity factor, crack resistance, wooden structures

I. Introduction

Since the late 60s of the last century, fracture mechanics has been used in the calculations of building structures, including plastics, both of natural (wood) and of artificial origin. Destruction mechanics is mostly widespread for such materials as steel, concrete [XXVIII] and reinforced concrete [I], [II].

This section briefly presents the most important, primarily experimental, studies on the crack resistance of wood and elements of wooden structures. In Sweden and Denmark, a group of scientists from different countries conduct systematic studies of the crack resistance of wood [XIII-III] within the framework of the CIB International...
Commission (International council for building research and documentation). On the basis of these works, a method for determining the critical energy release rate for wood has been proposed. L. Boström, proposed a method for determining the fracture energy on beam samples of variable cross section with a longitudinal notch; analyzed the crack resistance of pine, gave an assessment of various criteria for destruction [XII]. In France, at the University of Bordeaux under the direction of G. Valentin, along with extensive experimental tests, numerical methods for studying the crack resistance of various types of wood are widely used [V]. The works carried out in Finland by Fonselius A. are of particular interest, in which the crack resistance of wood under long-term loads is investigated [XXIV]. A lot of scientific work is devoted to the study of the viscosity of the destruction of wood and the definition of the parameters of fracture mechanics [XXV-XV]. The work [XXIII-XVIII] is devoted to the study of the operation of nailed connections by methods of fracture mechanics. Studies [VIII-IX] present the results of tests to determine the characteristics of the crack resistance of wood in wooden beams with notches (cut-outs).

Several techniques have been proposed for wood for the experimental determination of crack resistance. However, none of these methods has yet been officially approved in our country. The ability of wood to resist cracking when working under load is influenced by a large number of factors: the structure of the wood, the geometric dimensions of the structural elements, the duration of the impact of the load. Experimental data on the influence of these factors on the characteristics of the crack resistance of wood species grown in our country are not enough to form the normative basis for the calculation of wooden structures using the methods of fracture mechanics. Techniques for monitoring the crack resistance of wooden structures with defects in the form of cracks are still under development.

II. Research Purposes and Objectives

In this paper, the following tasks are set: to consider the crack resistance of wood in the elements of wooden structures from the standpoint of the methods of the theory of elasticity and the methods of fracture mechanics, to compare the theoretical crack resistance with the results of experimental studies. To assess the applicability of the methods of fracture mechanics according to the results of the comparison to describe the crack resistance of wood and predict the strength of wooden structures.

III. Some Features of Wood Structure

When determining the characteristics of wood, three natural axes of anisotropy of a tree trunk are considered (Fig. 1). The first one coincides with the direction of the longitudinal axis (direction of wood fibers) and is indicated by the letter L, the second axis is directed along the radius of the cross section of the tree trunk (radial direction), and the third coincides with the direction tangent to the annual layers (tangential direction T).

Fig. 1: Natural wood axis
Cylindrical anisotropy [VI, XVII, XVIII] is the most consistent with the wood structure, however, it is difficult to mathematically describe it. For small volumes of wood in which the curvature of the annual layers is neglected, one can adopt an orthogonal anisotropy (rectangular orthotropy) scheme, i.e. consider wood as a body that has three mutually perpendicular planes of elastic symmetry. The choice of an orthotropic model simplifies the solution of problems in the theory of elasticity in relation to elements of wooden structures that usually have a prismatic shape. Fig. 2 shows the natural axes and the corresponding designations for an arbitrary prism cut from wood (the main directions of the elastic symmetry of wood as an orthotropic body).

![Fig. 2: Coordinate axes tied to the natural axes of wood](image)

The greatest probability of cracking in wood occurs when it is stretched across the fibers. The resistance of wood in tension in the radial direction is higher than in tension in the tangential direction. This is mainly due to the location of prosenchymal cells, which form clear rows in the radial direction, and the presence of transverse reinforcing components of wood (cellulose microfibrils of parenchymal cells that make up the core rays) [XXVII, XVII]. The processes of deformation and destruction of the sample under tension in the tangential direction are adversely affected by internal (residual) stresses, which arise mainly due to shrinkage.

### IV. Description of the Elastic Work of Wood by the Methods of the Theory of Elasticity

The possibility of transferring the rhombic crystal structure to it is recognized to describe the elastic work of wood in the calculation of elements of wooden structures by the methods of the theory of elasticity. Due to this and taking into account the symmetry of the stress and strain tensor, the number of unknown coefficients can be reduced to 12 (in the case of a general consideration of an anisotropic body, the number of unknown coefficients is 81). Taking the location of the axes of orthotropy in accordance with Fig. 2, the generalized Hooke's law for wood (as for an orthotropic material) can be written in the following form (Table 1).

### Table 1: Hooke's generalized law

| Hooke's generalized coordinates | Relative deformations |
|---------------------------------|-----------------------|
| $\sigma_x = c_{11}\varepsilon_x + c_{12}\varepsilon_y + c_{13}\varepsilon_z$ | $\varepsilon_x = S_{11}\sigma_x + S_{12}\sigma_y + S_{13}\sigma_z$ |
| $\sigma_y = c_{21}\varepsilon_x + c_{22}\varepsilon_y + c_{23}\varepsilon_z$ | $\varepsilon_y = S_{21}\sigma_x + S_{22}\sigma_y + S_{23}\sigma_z$ |
| $\sigma_z = c_{31}\varepsilon_x + c_{32}\varepsilon_y + c_{33}\varepsilon_z$ | $\varepsilon_z = S_{31}\sigma_x + S_{32}\sigma_y + S_{33}\sigma_z$ |
| $\tau_{xx} = c_{44}\gamma_{xx}$ | $\gamma_{xx} = S_{44}\tau_{xx}$ |
| $\tau_{xy} = c_{55}\gamma_{xx}$ | $\gamma_{xy} = S_{54}\tau_{xy}$ |
| $\tau_{yx} = c_{66}\gamma_{xx}$ | $\gamma_{yx} = S_{64}\tau_{yx}$ |
Constant elasticity can be expressed through the elastic characteristics of the material as follows:

\[
\begin{align*}
S_{11} &= \frac{1}{E_x}; & S_{12} &= \frac{1}{E_y}; & S_{13} &= \frac{1}{E_z}; \\
S_{11} &= -\frac{\mu_{xy}}{E_x}; & S_{12} &= -\frac{\mu_{yx}}{E_y}; & S_{13} &= -\frac{\mu_{xz}}{E_z}; \\
S_{11} &= \frac{1}{G_{xy}}; & S_{12} &= \frac{1}{G_{xz}}; & S_{13} &= \frac{1}{G_{yz}};
\end{align*}
\]

(1)

Here \( E_x, E_y, E_z \) - elastic moduli; \( G_{xy}, G_{xz}, G_{yz} \) - shear moduli; \( \mu_{yx}, \mu_{xy}, \mu_{xz}, \mu_{xz} \) - Poisson's ratios.

For convenience, the matrix form of the Hooke's law (Table 2) is usually used with the designation of natural wood axes in accordance with Fig. 2.

**Table 2: The matrix form of the Hooke's generalized law**

| Hooke's generalized law | Elastic constant |
|-------------------------|------------------|
| \( \varepsilon_{kk} \) | \( \sigma_{kk} \) |
| \( \varepsilon_{ij} \) | \( a_{ij} \) | \( \sigma_{ij} \) |
| \( \tau_{AB} \) | \( a_{35} \) | \( \tau_{AB} \) |
| \( \tau_{AB} \) | \( a_{44} \) | \( \tau_{AB} \) |
| \( \varepsilon_{11} \) | \( a_{11} \) | \( \sigma_{11} \) |
| \( \varepsilon_{12} \) | \( a_{12} \) | \( \sigma_{12} \) |
| \( \varepsilon_{13} \) | \( a_{13} \) | \( \sigma_{13} \) |
| \( \varepsilon_{21} \) | \( a_{21} \) | \( \sigma_{21} \) |
| \( \varepsilon_{22} \) | \( a_{22} \) | \( \sigma_{22} \) |
| \( \varepsilon_{23} \) | \( a_{23} \) | \( \sigma_{23} \) |
| \( \varepsilon_{31} \) | \( a_{31} \) | \( \sigma_{31} \) |
| \( \varepsilon_{32} \) | \( a_{32} \) | \( \sigma_{32} \) |
| \( \varepsilon_{33} \) | \( a_{33} \) | \( \sigma_{33} \) |

(2)

V. Description of the Behavior of Wood from the Standpoint of Fracture Mechanics

In fracture mechanics, thin elliptical cavities (cracks) or perfectly thin cracks are usually considered as a model in the form of a mathematical section for which the stress and strain fields in the vicinity of the crack tip are searched for using linear fracture mechanics (linear elasticity theory) [XXVIII, XIX-XVIII].

**Stress-strain state in the vicinity of the crack tip in an orthotropic material.** When the mechanics of continuous deformable body are applied to wood, the hypothesis is accepted that the macrovolumes of “clean wood” (up to 1 cm³) are considered homogeneous, although in reality each element of the macrostructure has cavities and has different physical and mechanical characteristics.

In connection with the consideration of wood as an orthotropic material in the application of classical models of cracks (Fig. 3), it is necessary to accept some limiting prerequisites: 1- axis \( X \) coincides with one of the natural axes of wood; 2 - crack development occurs in the direction of the axis \( X \). Both assumptions will be made when choosing the direction of the axis parallel to the fibers, since, due to the anisotropy of the properties, the development of a crack along the fibers is most likely.
The starting point for determining the stress state in plates with cracks is a flat elastic problem with complex boundary conditions, the most effective method for which is recognized as the function of a complex variable.

Proceeding from the equation of the plane problem of the theory of elasticity in the absence of bulk forces, stresses can be expressed using the Erie functions:

\[
\begin{align*}
\sigma_x &= \frac{\partial^2 u}{\partial y^2}, \\
\sigma_y &= \frac{\partial^2 u}{\partial x^2}, \\
\tau_{xy} &= -\frac{\partial^2 u}{\partial x \partial y}
\end{align*}
\]

After substituting the values (3) into the generalized Hooke's law, taking into account the compatibility conditions for an orthotropic material, we obtain the following equation:

\[
S_{22} \frac{\partial^4 u}{\partial x^4} + (2S_{12} + S_{66}) \frac{\partial^4 u}{\partial x^2 \partial y^2} + S_{11} \frac{\partial^4 u}{\partial y^4} = 0,
\]

where \(S_{ij}\) - elastic constants (1).

Having taken in equation (4) that \(u - \) variable function \(Z_j\) as \(u(Z_j) = u(x + v_j y)\), we can get the characteristic equation:

\[
S_{11} v_j^4 + 2(S_{12} + S_{66}) v_j^2 + S_{22} = 0
\]

It is proved that equation (5) has no real roots. Using the functions of a complex variable, based on equations (4), (5), it is possible to determine the magnitudes of stresses and displacements in the immediate vicinity of the top of an ideal crack (if \(\frac{r}{l} < 1\) see fig.4):

---

**Fig. 3:** Three main models of crack opening: (I - crack of normal separation; II - crack of flat shear; III - crack of anti-flat shear)
where \( f \) - dimensionless functions. For orthotropic materials, they depend on both the crack geometry and the elastic characteristics of the material. Using the values of the stress intensity factors, equations (6) can be transformed and written in the following form (Table 3):

**Table 3: The stresses and displacements**

| For model I: | For model II: |
|-------------|--------------|
| \( \sigma_x = \frac{K}{2\pi R} \left[ \frac{\nu_1}{1 - \nu} \left( \frac{\nu_1}{1 - \nu} \right) \cos\theta \sin\theta - \frac{\nu_2}{1 - \nu} \left( \frac{\nu_2}{1 - \nu} \right) \cos\theta \sin\theta \right] \) | \( \sigma_x = \frac{K}{2\pi R} \left[ \frac{1}{1 - \nu} \left( \frac{\nu_1}{1 - \nu} \right) \cos\theta \sin\theta - \frac{1}{1 - \nu} \left( \frac{\nu_2}{1 - \nu} \right) \cos\theta \sin\theta \right] \) |
| \( \sigma_y = \frac{K}{2\pi R} \left[ \frac{\nu_1}{1 - \nu} \left( \frac{\nu_1}{1 - \nu} \right) \cos\theta \sin\theta - \frac{\nu_2}{1 - \nu} \left( \frac{\nu_2}{1 - \nu} \right) \cos\theta \sin\theta \right] \) | \( \sigma_y = \frac{K}{2\pi R} \left[ \frac{1}{1 - \nu} \left( \frac{\nu_1}{1 - \nu} \right) \cos\theta \sin\theta - \frac{1}{1 - \nu} \left( \frac{\nu_2}{1 - \nu} \right) \cos\theta \sin\theta \right] \) |
| \( \tau_{xy} = \frac{K}{\pi R} \left[ \frac{\nu_1}{1 - \nu} \left( \frac{\nu_1}{1 - \nu} \right) \cos\theta \sin\theta - \frac{\nu_2}{1 - \nu} \left( \frac{\nu_2}{1 - \nu} \right) \cos\theta \sin\theta \right] \) | \( \tau_{xy} = \frac{K}{\pi R} \left[ \frac{1}{1 - \nu} \left( \frac{\nu_1}{1 - \nu} \right) \cos\theta \sin\theta - \frac{1}{1 - \nu} \left( \frac{\nu_2}{1 - \nu} \right) \cos\theta \sin\theta \right] \) |

where \( R \) - real numbers; \( \nu_j \) - roots of the characteristic equation (5).

Designating \( a = 2S_{12} + S_{66}; b = \sqrt{a^2 - 4S_{11}S_{22}}; c = 2S_{11}, \)
we get:

\[
\nu_i = \sqrt{\frac{-a - b}{2c}}, \quad \nu_j = \sqrt{\frac{-a + b}{2c}}, \quad \nu_k = \sqrt{\frac{a - b}{2c}}.
\]

where \( i = \sqrt{-1} \) - imaginary number.

Using these equations, formulas are obtained for determining the rate of energy release for an orthotropic material:

\[
G_i = K_i^2 A_i, \\
G_{ij} = K_{ij}^2 A_{ij}
\]

where
Substituting the values of the elastic constants of wood, we can write:

\[
A_j = \sqrt{\frac{S_{11}S_{22}}{2} + \frac{2S_{12} + S_{66}}{2S_{11}}};
\]

\[
A_{ij} = \frac{S_{11}}{2S_{11}} \sqrt{\frac{S_{22}}{S_{11}} + \frac{2S_{12} + S_{66}}{2S_{11}}}
\]

When considering wood from the standpoint of fracture mechanics, it is important to know not only the model (type) of crack opening, but also its orientation relative to wood fibers, as well as the direction of its development. Considering the anisotropy of wood, there are six possible types or systems of cracks (Fig. 5). For their classification, a two-letter notation characterizing the orientation and direction of the crack development is used. In this case, the first letter indicates the direction of the normal to the crack plane (orientation of the crack), the second - the direction of the crack development. To evaluate the performance of wood in real construction, cracks of species RL and TL are of greatest interest. In experimental studies, specimens having initial incisions identical to cracks of the type considered are tested.

VI. Experimental Determination of Fracture Mechanics Parameters

![Fig. 5: System of designation of orientation and direction of crack development in wood](image)

![Fig. 6: Flexable elements with notches and with trimmings](image)
The greatest probability of cracking in wood occurs when it is stretched across the fibers and shear along the fibers, which is typical for wooden beams, including glued with different cuts and openings, frame nodes, locations of various mechanical connections in the body of wood (dowels, lamellar dowels, blocks, nails, etc.), connections of elements of wooden structures. Let us consider in more detail the bent elements of wooden structures, which have notches and undercuts (Fig. 6) [XXVIII, XVIII].

It is known that the stresses arising in a bendable element with a notch or frame can be calculated using the concept of net section. However, this method does not allow to take the stress concentration near the insert angles into account. Research results show that for beams with a notch, which is half the section height, the breaking load is reduced to 10% of the breaking load for integral section - gross, which is less than the breaking load for a beam with a notch according to the method of calculating the net section.

To analyze such elements from the standpoint of fracture mechanics, one can use the results obtained by the finite element method. For the inset depth equal to the \( \frac{1}{3} \) height of the beam \( \left( \frac{h}{3} \right) \) and the width \( \frac{1}{3} d \) and \( \frac{16}{3} d \), the stresses acting along the fiber direction were calculated \( (y < 0) \):

\[
\sigma_y(0,y) = \frac{6M}{bd^2} \left( \frac{0.133}{(2\pi r/d)^{0.5}} \right) = |y| \quad (11)
\]

\[
\tau_y(0,y) = \frac{6M}{bd^2} \left( \frac{0.2}{(2\pi r/d)^{0.5}} \right) = |y| \quad (12)
\]

It was assumed that wood is an orthotropic material, characterized by the following parameters:

\[ E_x / E_y = 16; \nu_{xy} = 0.25; E_y / G_{xy} = 16.5. \]

Stress values for a narrow crack-like notch are:

\[
\left\{ \begin{align*}
\sigma_y(0,y) &= \frac{6M}{bd^2} \left( \frac{0.138}{(2\pi r/d)^{0.5}} \right) = |y| \\
\tau_y(0,y) &= \frac{6M}{bd^2} \left( \frac{0.275}{(2\pi r/d)^{0.5}} \right) = |y|
\end{align*} \right. \quad (12)
\]

Comparison of expressions (11) and (12) shows that a narrow crack-like notch leads to a higher stress concentration. The results of experimental studies [XXVIII, XVIII], of bent wooden samples with a narrow notch and inset (Fig. 6), confirm this phenomenon — the breaking load for samples with a narrow notch was less than for samples with a conventional inset.

In case of crack development perpendicular to the direction of the fibers (Fig. 7), the stresses near the tip of the crack are:

- With angle \( \vartheta = 0^\circ \)
  \[ \sigma_y = K_y \sqrt{2\pi r}; \tau_y = K_y \sqrt{2\pi r} \quad (13) \]

- With angle \( \vartheta = \pm 90^\circ \)
  \[ \sigma_y = (0.12K_y \pm 0.82K_y) \sqrt{2\pi r}; \tau_y = (0.24K_y \pm 0.47K_y) \sqrt{2\pi r} \quad (14) \]
At the same time, the parameters of the orthotropy of the material are taken the same as above. Expressions in brackets in equations (14) can be considered as equivalent stress intensity factors in the direction along the fibers, perpendicular to the initial crack. Equations (14) can quantitatively explain the differences in the critical stress intensity coefficients for cracks developing according to the RL and LR schemes. For cracks of the RL type, the initial crack is located parallel to the direction of the fibers, and for LR cracks - perpendicular to the direction of the fibers. The values of the critical stress intensity factors, obtained experimentally for pine in the case of crack opening by model I are equal to:

\[
K_{\nu,LR} = 2.5 \text{MN} \cdot \text{m}^{-3/2} \quad \text{(for LR cracks)}
\]

\[
K_{\nu,RL} = 0.4 \text{MN} \cdot \text{m}^{-3/2} \quad \text{(for RL cracks)}
\]

where double line means equivalent stress intensity factor along the fibers. The value \(K_{\nu,LR}\) is less than \(K_{\nu,RL} (0.4 \text{MN} \cdot \text{m}^{-3/2})\) for the RL direction, however, has the same order of magnitude.

As a criterion for fracture under the joint action of normal and tangential stresses, the following equation can be used:

\[
K_{\nu,LR}/K_{\nu,RL} = 1,
\]

where the critical stress intensity factors \(K_{\nu,LR}\) and \(K_{\nu,RL}\) are calculated in the usual way (if the crack is parallel to the fibers) or as “equivalent” (if the crack is perpendicular to them).

Fig. 8 shows the dimensionless stress intensity factors \(K_{\nu}\) and \(K_{\nu,RL}\) for the two loading schemes as a function of the relative depth of the notch \((a/d)\).
Figure 9 shows the relationship between the limit moment $M_n$ in a notched beam and the maximum moment $M_o$ in the beam without a notch, as a function of the notch depth ($a/d$). The dotted line is the calculation results in accordance with the method of net sections, without taking into account the concentration of stresses. Solid lines correspond to the calculation by the methods of fracture mechanics. Experimental points were obtained on specimens with a narrow notch (bright) and on specimens with undercut (dark points). Figure 9 shows that for beams with cuts, calculation based on the methods of fracture mechanics gives much more accurate results than on the basis of the net section method.

VII. Conclusions

Real examples of the design of wooden structures show that the main cause of cracks in wooden elements are tensile stresses across the fibers and shear stresses along the wood fibers, in both cases the crack will develop along the fibers. As a result of these stresses, brittle or quasi-brittle fracture occurs. One example where structural solutions can cause brittle structural failure is wooden beams with various cuts and openings. For such practical cases, there arises the problem of calculating structures using methods that guarantee the safety of their operation and durability. The reliability of wooden structures is associated with the solution of the problem of developing a calculation method that takes into account the specificity of the structure of wood and the presence of defects of both natural and artificial origin in it. Since the methods of classical resistance of materials cannot determine the actual stress-strain state in the vicinity of the crack tip, the most suitable theory for describing the fragile behavior of wood is fracture mechanics.

Experimental studies confirm the applicability of linear fracture mechanics to describe the crack resistance of wood and to calculate the elements of wooden structures.

VIII. Acknowledgement

The publication was prepared with the support of the RUDN University Program “5-100”.

References

I. Bondarenko V.M., Bondarenko S.V. Engineering methods of the nonlinear theory of reinforced concrete. Moscow, stroiizdat, 1982, 288 p.

II. Danielsson H, Gustafsson PJ (2013) A three dimensional plasticity model for perpendicular to grain cohesive fracture in wood. EngFractMech 98. pp. 137–152.

III. Danielsson H., Gustafsson P.J., A probabilistic fracture mechanics method and strength (2011). pp. 407-419.

IV. Danielsson, H., Gustafsson, P. Fracture analysis of perpendicular to grain loaded dowel-type connections using a 3D cohesive zone model. Wood Material Science and Engineering Volume 11, Part 5, 2016, pp. 261-273.
V. G. Valentin, L. Boström, P.J. Gustafsson, A. Ranta-Mannus and S. Gowda, Application of Fracture Mechanics to Timber Structures: RILEM state-of-the-art report. VTT, ESPOO (1991).

VI. Gappoev M.M. Evaluation of the bearing capacity of wooden structures by the methods of fracture mechanics. Dis ... Dr. Tech. Science. - Moscow, 1996. 256p.

VII. Gustafsson, P.J., Danielsson, H. Perpendicular to grain stiffness of timber cross sections as affected by growth ring pattern, size and shape. European Journal of Wood and Wood Products. Volume 71, Issue 1, January 2013. pp. 111–119.

VIII. Hlaskova L, Orlowski KA, Kopecky´ Z, Jedinák M. Sawing processes as a way of determining fracture toughness and shear yield stresses of wood. BioRes 2015;10(3):5381–94.

IX. Jockwer R, Steiger R, Frangi A. State-of-the-art review on approaches for the design of timber beams with notches. J StructEng 2013; 140(3): 04013068-1–04013068-13.

X. Jockwer, R., Serrano, E., Gustafsson, P.-J., Steiger, R. Impact of knots on the fracture propagating along grain in timber beams. International Wood Products Journal 8 (1), 2017, pp. 39-44.

XI. Karpenko N.I. General models of reinforced concrete mechanics. M., stroiizdat, 1996, 416 p

XII. L. Boström, Method for Determination of the Softening Behavior of wood and the Applicability of a Nonlinear Fracture Mechanics Model, Doctoral thesis, Report TVBM-1012, Lund, Sweden, 1992.

XIII. Larsson, G., Gustafsson, P.J., Crocetti, R. Use of a resilient bond line to increase strength of long adhesive lap joints. European Journal of Wood and Wood Products, 2018, pp. 401-411.

XIV. Larsson, G., Gustafsson, P.J., Serrano, E., Crocetti, R. Bond line models of glued wood-to-steel plate joints. Engineering Structures, Volume 121. 2016, pp. 160-169.

XV. M. F. S. F. de Moura, M. A. L. Silva, J. J. L. Morais, N. Dourado. Mode II fracture characterization of wood using the Four-Point End-Notched Flexure (4ENF) test. Theoretical and Applied Fracture Mechanics, Volume 98, December 2018, Pages 23-29

XVI. Masalov A. Fracture resistance of bent glued wooden elements: Author's abstract. dis ... cand. tech. Sciences: 05.23.01 / Ing.-builds. in-t. - Voronezh, 1992. - 21 p.

XVII. NaychukA.Ya. Strength of elements of wooden structures in conditions of a complex inhomogeneous stress state. Moscow, 2006, p.
XVIII. Okolnikova G.E. Analysis of the work of nagelnyh joints of wooden structures from the standpoint of fracture mechanics // Makeyevka: "Bulletin of DonNACEA". 2011- 4 (90). - pp. 40-46.

XIX. Okolnikova G.E. Calculation of nagel compounds modified with pressed fiberglass bushings. - Moscow: MGOU, "Bulletin of MGOU", No. 1 (3), 2009, pp. 28 - 33.

XX. Okolnikova G.E. Investigation of the relationship between the fracture toughness of wood and the calculated resistance of wood to stretching along the fibers. Bulletin of MGOU. - Moscow: MGOU, No. 2, 2010, p. 23-26.

XXI. Orlovich R.B. Long-lasting strength and deformability of structures from modern wood materials under the main operational influences: Abstract. dis. ... Dr. techn. sciences. (05.23.01) / Leningr. Ing.-p., In-t. - L., 1991. - 51 p.

XXII. Pop O., Dubois F. Determination of timber material fracture parameters using mark tracking method. Construction and Building Materials, Volume 102, Part 2, 15 January 2016, pp. 977-984.

XXIII. R. Crocetti, P. J. Gustafsson, U. A. Girhammar, L. Costa, A. Asimakidis. Nailed Steel Plate Connections: Strength and Ductile Failure Modes. Structures, Volume 8, Part 1, November 2016, Pages 44-52.

XXIV. Ranta-Maunus, A. Fonselius, M., Kurkela, J., Toratti, T. Reliability analysis of timber structures. VTT Tiedotteita - ValtionTeknillinen Tutkimuskeskus.2001. pp

XXV. Shilang X.U., H.W. Reinhardt, M. Gappoev. Mode II fracture testing method for highly orthotropic materials like wood. International Journal of Fracture 75. September 1996, Volume 75, Issue 3, pp. 185–214.

XXVI. Sterley, M., Gustafsson, P.J. Shear fracture characterization of green-glued polyurethane wood adhesive bonds at various moisture and gluing conditions. Wood Material Science and Engineering. Volume 46, Issue 3, 2012. pp. 421-434.

XXVII. Tuturin SV, Shemyakin EI, Korotkina M.R. Destruction of wood during compression. // Bulletin of the Moscow State Forest University. – Moscow: 2005.-№3 (39). - From 56-71.

XXVIII. ZaitsevYu.V., Okolnikova G.E., Dorkin V.V. Fracture mechanics for builders: textbook.-2 nd ed., Rev. and additional. Moscow: INFRA-M, 2016. -216p.