An Improved Nonlinear Cumulative Damage Model Considering the Influence of Load Sequence and Its Experimental Verification

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Abstract: According to the change characteristics in the toughness of the metal material during the fatigue damage process, the fatigue tests were carried out with the standard 18CrNiMo7-6 material. Scanning the fracture with an electron microscope explains the lack of linear cumulative damage in the mechanism. According to the obtained results, a nonlinear damage accumulation model which considered the loading sequence state under the toughness dissipation model was established. The recursive formula was devised under two-level. The fatigue test data verification of three metal materials showed that using this model to predict fatigue life is satisfactory and suitable for engineering applications.

Keywords: nonlinear cumulative damage; loading sequence; fatigue damage; toughness dissipation; two-level loading

1. Introduction

The failure of most engineering structures or mechanical parts is caused by the accumulation of fatigue damage caused by a series of cyclic loads. Factors affecting the accumulation of fatigue damage include load size, loading sequence, load history (number of actions), and load path. The cumulative effect of fatigue damage directly determines the life and reliability of mechanical parts. Scholars have done a lot of work in the field of fatigue cumulative damage and have proposed many fatigue cumulative damage theories and calculation models, which are mainly divided into linear fatigue cumulative damage theory, bilinear cumulative damage theory, and nonlinear fatigue cumulative damage theory [1]. The commonly used linear fatigue cumulative damage theory [2] (Miner’s rule) does not consider the influence of the load sequence, but the fact that Miner’s rule ignores the effects of load sequence and load interaction make lifetime estimations obtained by this rule unsatisfactory [3,4]. Although the bilinear cumulative damage theory considers the effect of load sequence on crack growth to a certain extent, its theoretical model cannot accurately simulate the actual damage process because it is difficult to determine the inflection point of crack growth. Therefore, the accuracy of life prediction is not high [5,6]. For the strain control of austenitic stainless steel, Taheri [7] proposed a conservative model of fatigue damage accumulation under variable amplitude load. This model does not require the constitutive law but considers plasticity through the cyclic strain stress curve.

In order to overcome the shortcomings of the linear damage accumulation Miner’s rule, a wide range of nonlinear damage accumulation models have been developed. According to the change characteristics of fatigue ductility, and based on the theory of continuum damage mechanics, Yuan [8] proposed a modified nonlinear uniaxial fatigue damage accumulation model. The model could be used to predict the failure of the specimen.
and explain the whole process of fatigue damage accumulation. Biezma [9] developed a practical and simple correction factor ensuring that the linear summation of damage was conservative, so as to take the sequence effect into account in random loading.

Nonlinear fatigue cumulative damage theory believes that the load sequence has a serious impact on fatigue cumulative damage [10–13]. Although these models are often capable of producing satisfying results for a specific set of experiments, Miner’s rule remains the most widely used for fatigue design under variable amplitude loading. However, some models have recently been developed, which do not require extensive testing. Many fatigue damage accumulation theories have been proposed to remedy the drawbacks of Miner’s rule, and a majority of these models are based on non-linear accumulation laws. Benkabouche [14] proposed a method for the prediction of the fatigue-life for different materials subjected to constant amplitude multiaxial proportional loading.

The non-linear fatigue damage accumulation models can be classified into the following categories: damage curve based models, continuum damage mechanics models, interaction between the various loadings considered models, energy-based damage methods, physical properties degradation-based models, ductility exhaustion-based methods, thermodynamic entropy-based damage theories. Detailed comments on some of these models can be found in [15].

Based on the fatigue test data of the high–low loading and low–high loading of 18CrNiMo7-6 steel, an improved nonlinear cumulative fatigue damage model is proposed based on the ductile dissipation model in the nonlinear cumulative damage theory. By analyzing the damage model, the load interaction parameters can be obtained and added to the ductile dissipation model, and the value of the parameter is determined through the experimental data.

This paper explains from the mechanism why Miner’s rule has different damages under two-level loading. This paper verifies the fatigue life of several commonly used metal materials such as 18CrNiMo7-6 steel, 45 steel, and aluminum alloy under two-level loading using the proposed improved model. A comparison is made among the results calculated by the test data, the Miner’s rule, the original model, and the modified model with little relative error, which proves the validity of the proposed model. The revised model is designed to facilitate the use of engineers. The coefficient selection is simpler than other nonlinear cumulative damage models, and the prediction results are more accurate than similar models.

2. Damage Accumulation Theory

The most widely used linear damage accumulation theory is Miner’s theory [16]. The theory defines the fatigue damage $D$ as the ratio of the number of cycles $n$ under a certain stress to the fatigue life $N_f$ of the material under the stress:

$$D = \frac{n}{N_f}$$  \hspace{1cm} (1)

Miner’s theory believes that under the action of multiple levels of different stress amplitudes, fatigue failure occurs

$$\sum \frac{n_i}{N_{f_i}} = 1$$  \hspace{1cm} (2)

where $n_i$ is the number of cycles under the $i$th stress level; $N_{f_i}$ is the fatigue life under the $i$th stress.

Taking into account the decrease in the material’s bearing capacity under cyclic loading, nonlinear fatigue damage accumulation theory introduces the concept of the material’s physical property degradation into damage accumulation [17], a typical tough dissipation model proposed by Ye Duyi [18]. According to the Griffith fracture criterion [19], San-
dor [20] established the empirical relationship between material fatigue toughness and static toughness, which was verified by a large number of experimental results [21,22]:

\[
\frac{U}{W_f} = \left(\frac{e_a}{e_f}\right)^4
\]  

(3)

\(U\) is the initial toughness without damage, \(W_f\) is fatigue toughness, \(e_a\) is the applied stress amplitude, \(e_f\) is the breaking strength of the material.

For metal materials with certain damage, Formula (3) can be rewritten as:

\[
\frac{U_N}{W_{fN}} = \left(\frac{e_a}{e_{fN}}\right)^4
\]  

(4)

\(U_N\) is the toughness after \(N\) cycles of loading, \(W_{fN}\) is the remaining fatigue toughness, \(e_{fN}\) is the residual breaking strength of the damaged material.

From the energy consumption process of fatigue damage, the fatigue damage variable is defined as [20]:

\[
D_N = \frac{\sum_{i=1}^{N} \Delta W_i}{W_f} = \frac{W_f - \sum_{i=N+1}^{N_f} \Delta W_i}{W_f} = 1 - \frac{W_{fN}}{W_f}
\]  

(5)

\(\sum_{i=1}^{N} \Delta W_i\) represents the plastic hysteresis energy accumulated and dissipated under a certain damage state, \(N\) is the load cycles experienced, \(N_f\) is the cycle of fatigue fracture.

Substitute Formulas (3) and (4) into Formula (5):

\[
D_N = 1 - \left(\frac{e_{fN}}{e_f}\right)^4 \frac{U_N}{U}
\]  

(6)

For materials with a power-hardening law, according to the experimental results [18], the tensile strength of the material does not show a sharp decline until it is close to fracture. The Formula (6) is further simplified as:

\[
D_N = 1 - \frac{U_N}{U}
\]  

(7)

This is the calculation formula of the damage variable defined by the material’s toughness dissipation. Its physical meaning is that the degree of fatigue damage of the material can be measured by the amount of change in the metal’s energy or the ability to absorb deformation and fracture during the fatigue process. In order to obtain the damage evolution law under the ductile dissipation model, the damage variable calculation, Formula (7), and the ductile dissipation model, Formula (8) [18], are combined together as

\[
U_N = U + \left(\frac{U - U_{N_{f-1}}}{\ln N_f}\right) \ln\left(1 - \frac{N}{N_f}\right)
\]  

(8)

where \(U_{N_{f-1}}\) is the residual toughness of the material after \(N_{f-1}\) cycles of loading, that is, the energy absorbed by the material under the tensile load before fatigue fracture. For most fatigue problems, because the macroscopic fracture presents brittle fracture characteristics,
there is no obvious necking phenomenon; therefore, $D_{N_f-1} \approx 1$ [18], the fatigue damage evolution law with toughness as a parameter is obtained as:

$$D_N = \left(1 - \frac{U_{N_f-1} + U}{U_{N_f} + U} \right) \ln\left(1 - \frac{N}{N_f} \right)$$

$$= - \frac{D_{N_f-1}}{\ln N_f} \ln(1 - \frac{N}{N_f}) \approx - \frac{\ln(1 - \frac{N}{N_f})}{\ln N_f}$$  \hspace{1cm} (9)

Deriving from Formula (9), the fatigue damage evolution formula can be obtained as:

$$\frac{dD_N}{dN} = \left[\left(\frac{N_f - N}{\ln N_f} \right) \ln N_f \right]^{-1}$$  \hspace{1cm} (10)

According to the principle of damage equivalence, the remaining life fraction ($N_2/N_f$) under the second-stage load and the occurrence of fatigue failure after the first-stage load is applied for a certain cycle of cycles $N_1$, and can be derived from Formula (9). Linear cumulative damage is expressed

$$\frac{N_2}{N_f} = \left(1 - \frac{N_1}{N_f} \right) \frac{\ln N_2}{\ln N_f}$$  \hspace{1cm} (11)

where $N_f$, and $N_f$, respectively, correspond to the number of fatigue fracture cycles at two different stress levels.

### 3. Stress Test and Results

#### 3.1. Test Conditions

According to the design requirements of a wind-turbine gearbox, the test material is 18CrNiMo7-6 forged steel, which is surface-hardened. The chemical composition is shown in Table 1. The test uses a small sample of a smooth cylindrical shape; the shape is shown in Figure 1. The test sample after fatigue is shown in Figure 2. The fatigue test was carried out on the PWS-E100 electro-hydraulic servo universal testing machine (shown in Figure 3) manufactured by Jinan Times Tester Co., Ltd, Jinan, China. with a load–stress ratio $R = -1$, a test frequency of 10 Hz, and the loading form is a sine wave. Material strength limit $\sigma_b$ and the yield strength $\sigma_y$ were determined by the static stretching of the test machine.

| Element | Content      |
|---------|--------------|
| C       | 0.15~0.21%   |
| Si      | 0.40%        |
| Cr      | 1.50~1.80%   |
| Mn      | 0.50~0.90%   |
| Ni      | 1.40~1.70%   |
| P       | <0.035%      |
| Mo      | 0.25~0.35%   |

#### 3.2. Test Methods

Five specimens are used for the static tensile test and the average maximum tensile strength of the material is $\sigma_b = 1100$ MPa. Then, every five test pieces are used together to carry out the fatigue test under different stress amplitudes. The test conditions are: symmetrical cycle $R = -1$, sine wave loading, and a frequency of 10 Hz. The average number of cycles for each group of experiments is shown in Table 2.

According to the data in Table 2, the fitting function of the material S-N curve is obtained, in Figure 4 ($S = (2.318 \times 10^4) \times N^{-0.002028} - 2.208 \times 10^4$). The S-N curve is used
to obtain the fatigue-fracture cycle $N_f$ at various stress levels $e_a (R = -1)$. According to the trend line, the fatigue limit of 18CrNiMo7-6 is taken as 350 MPa.

Figure 1. Test piece size.

Figure 2. Test sample after fatigue.

Figure 3. Test machine.

Table 2. Fatigue test results.

| Loading Stress Amplitude/MPa | N/Number of Cycles |
|-----------------------------|--------------------|
| 1100                        | 0.5                |
| 832                         | 471                |
| 728                         | 3132               |
| 624                         | 23,414             |
| 520                         | 246,809            |
| 416                         | 3,189,720          |
| 353                         | 9,230,000          |
According to the obtained S-N curve, the two-stage loading test plan is designed as follows: take the stress value at the inflection point of the curve 520 MPa, for high–low load application conditions; firstly, cycle \( n_1 \) times with 520 MPa stress \( (N_{f1} \approx 240,000) \); and then apply a 420 MPa stress \( (N_{f2} \approx 3,200,000) \) cycle until breaking (corresponding number of cycles \( n_2 \)). For low–high load conditions, after 420 MPa stress cycle \( n_1 \) times, apply a 520 MPa stress cycle until breaking (corresponding number of cycles \( n_2 \)). Every five test pieces are a group, and the average data from the test is shown in Table 3.

### Table 3. 18CrNiMo7-6 two-stage loading test data.

| Two-Stage Loading Stress Level/MPa | Test Value |
|-----------------------------------|------------|
|                                   | \( n_1 \) | \( n_1/N_{f1} \) | \( n_2 \) | \( n_2/N_{f2} \) | \( n_1/N_{f1}+n_2/N_{f2} \) |
| 520–420                           | 60,000     | 0.25              | 1,480,000 | 0.463            | 0.7125            |
|                                   | 120,000    | 0.50              | 850,000   | 0.266            | 0.766             |
|                                   | 156,000    | 0.65              | 324,800   | 0.1015           | 0.7515            |
| 420–520                           | 800,000    | 0.25              | 216,000   | 0.90             | 1.15              |
|                                   | 1,350,000  | 0.42              | 177,600   | 0.74             | 1.16              |
|                                   | 2,000,000  | 0.63              | 144,000   | 0.60             | 1.23              |

In the sequence of load application, the first high-stress load will have a greater impact on the overall damage as the data in Table 3. In high–low loading, the total damage value \( n_1/N_{f1} + n_2/N_{f2} \) is less than 1; in low–high loading, the total damage value \( n_1/N_{f1} + n_2/N_{f2} \) is greater than 1. Unlike in the linear damage accumulation model, the total damage value in the cumulative model is always equal to 1.

In order to further analyze the tension–compression fatigue fracture mechanism, the fatigue fracture specimen was sliced and then analyzed by the JSM-5610LV scanning electron microscope, as shown in Figures 5 and 6.

After binarization, the black pixels are in the form of dimples. According to statistics, the pixels in Figure 7 occupy the total selection area of S1 (0.921%), and the pixels in Figure 8 occupy the total selection area of S2 (9.79%). \( S2/S1 = 10.63 \). High–low loading significantly promotes the formation of dimples.

Binarize the middle part of Figures 5 and 6 to obtain Figures 7 and 8.
Regardless of the loading mode of high–low or low–high, the section shows the characteristics of multiple crack sources, and the interactive influence of different crack source propagation paths forming different ridge topographies. It can be clearly found that under low–high loading, the number of dimples in the cleavage zone formed on the fracture surface is less than the number of dimples formed on the fracture surface.
under high–low loading. The formation of dimples under the repeated action of normal stress accelerates the formation of microscopic voids caused by plastic deformation of the material in a small range during the high–low loading process. This difference ultimately leads to different total damage changes after low–high loading and high–low loading. That is, under the low–high loading form, the first-level low load forms the exercise effect. In the high–low loading mode, the first-level high load does not form an exercise effect and the overall fatigue life decreases rapidly with the increase in the first-level high-load cycle. Due to the presence of dislocations in the material, dislocation clusters are easily formed at grain boundaries, phase boundaries, and material defects during the tension–compression process, which leads to stress concentration and induces the initiation and growth of microvoids, eventually leading to fracture. High–low loading causes a significantly higher number of micro-holes to be generated than in the case of micro-holes generated by low–high load loading, which means that the number of original microcrack sources under high–low loading is large. These microcracks are easier to connect and propagate to form cracks during cyclic loading, resulting in a decrease in the fatigue life of the material compared to low–high load loading.

4. Improved Cumulative Damage Model

According to the Formula (11) mentioned above, the load ratio effect parameter $n$ is introduced, so the improved cumulative damage model is expressed

$$D'_2 = \left( \frac{\sigma_2}{\sigma_1} \right)^b D_2$$

(12)

Formula (12) reflects the effect of load loading sequence on damage, and the damage relationship between the improved model and the original model as:

$$D'_2 \begin{cases} 
\geq D_2 & \sigma_2 \geq \sigma_1 (Low - High) \\
\leq D_2 & \sigma_2 \leq \sigma_1 (High - Low)
\end{cases}$$

According to Formula (11), the number of cycles $n_1$ under the first-stage load amplitude $\sigma_1$ is equivalent to the equivalent number of cycles $n'_2$ under the second-stage load amplitude $\sigma_2$, expressed as:

$$D_1 = -\frac{\ln \left( 1 - \frac{n_1}{N_{f1}} \right)}{\ln N_{f1}} = -\frac{\left( \frac{\sigma_2}{\sigma_1} \right)^b \ln \left( 1 - \frac{n'_2}{N_{f2}} \right)}{\ln N_{f2}} = D'_2$$

(13)

$$\frac{n'_2}{N_{f2}} = 1 - \left( 1 - \frac{n_1}{N_{f1}} \right)^{\ln N_{f2}/\ln N_{f1}} \left( \frac{\sigma_1}{\sigma_2} \right)^b$$

(14)

$$D = -\left( \frac{\sigma_2}{\sigma_1} \right)^b \ln \left( 1 - \frac{n'_2 + n_2}{N_{f2}} \right) \frac{\ln N_{f2}}{\ln N_{f1}}$$

(15)

When the total damage degree is 1, the specimen is damaged. Formula (15) $D = 1$, represents the material under the action of two levels of load, after the number of cycles $n_1$ of the first level load amplitude $\sigma_1$, and the remaining life fraction of the second level load amplitude $\sigma_2$, expressed

$$\frac{n_2}{N_{f2}} = \left( 1 - \frac{n_1}{N_{f1}} \right)^{\ln N_{f2}/\ln N_{f1}} \times \left( \frac{\sigma_1}{\sigma_2} \right)^b - \left( 1 - \frac{N_{f2}}{N_{f2}} \right)^{\ln N_{f2}/\ln N_{f1}} \left( \frac{\sigma_1}{\sigma_2} \right)^b$$

(16)
By analogy, the total damage under the multi-stage load and the remaining life fraction $n_i/N_{f_i}$ of the last stage load $\sigma_i$ can be derived as:

$$D = \sum_{i=1}^{n} \left( \frac{-\sigma_i}{\sigma_{i-1}} \right)^n \ln \left( 1 - \frac{n_i + n_i}{N_{f_i}} \right)$$

(17)

$$n_i \frac{N_{f_i}}{N_{f_i}} = \left( 1 - \frac{n_{i-1}}{N_{f_i-1}} \right)^{\frac{\ln N_{f_i}}{m_{n_i} N_{f_i}}} \left( \frac{\sigma_i}{\sigma_{i-1}} \right)^b - \left( 1 \frac{N_{f_i}}{N_{f_i}} \right)^{b}$$

(18)

According to the data obtained from the test, for 18CrNiMo7-6 material, $b = 2.8$. One can substitute $b = 2.8$ into Formula (16) as:

$$n_i \frac{N_{f_i}}{N_{f_i}} = \left( 1 - \frac{n_{i-1}}{N_{f_i-1}} \right)^{\frac{\ln N_{f_i}}{m_{n_i} N_{f_i}}} \left( \frac{\sigma_i}{\sigma_{i-1}} \right)^2.8 - \left( 1 \frac{N_{f_i}}{N_{f_i}} \right)^{2.8}$$

(19)

5. Test Results and Analysis

In order to verify the effectiveness of the proposed improved model, based on the fatigue test data of the material 18CrNiMo7-6 listed in Table 3, the life prediction results of this model, the linear damage accumulation model, and the ductile dissipation model, are compared. The results are shown in Table 4. In addition, for 45# steel and Al-2024 aluminum alloy, commonly used in mechanical engineering, calculations and comparisons are also made based on the experimental data of the literature [23,24]. The results are listed in Tables 5 and 6. For each material in the corresponding loading mode, the relative error between the experimental value and the theoretical calculation is shown in Figures 9–14.

Table 4. 18CrNiMo7-6 two-stage loading test data and the remaining life prediction results of each model.

| Two-Stage Loading Stress Level/MPa | Test Value | Miner’s Rule (Formula (1)) | Ductile Dissipation Model (Formula (11)) | The Proposed Model (Formula (19)) |
|-----------------------------------|------------|---------------------------|------------------------------------------|----------------------------------|
|                                   | $n_1$ | $n_1/N_{f_1}$ | $n_2$ | $n_2/N_{f_2}$ | $n_2/N_{f_2}$ | Relative Error/% | $n_2/N_{f_2}$ | Relative Error/% | $n_2/N_{f_2}$ | Relative Error/% |
| 520–420                           | 60000  | 0.25 | 148000 | 0.583 | 0.75 | 28.64% | 0.7062 | 21.13% | 0.5168 | 11.35% |
|                                  | 120000 | 0.50 | 850000 | 0.266 | 0.50 | 87.97% | 0.4325 | 62.59% | 0.2038 | 23.38% |
|                                  | 156000 | 0.65 | 324800 | 0.1015 | 0.35 | 244.8% | 0.2810 | 176.8% | 0.0899 | 11.43% |
| 420–520                           | 800000 | 0.25 | 216000 | 0.9 | 0.75 | 16.67% | 0.7883 | 12.41% | 0.8818 | 2.02% |
|                                  | 1,350000 | 0.42 | 177600 | 0.74 | 0.55 | 25.67% | 0.6356 | 14.10% | 0.7872 | 6.38% |
|                                  | 2,000000 | 0.63 | 144000 | 0.60 | 0.35 | 41.67% | 0.4443 | 25.95% | 0.6518 | 8.63% |

Table 5. The 45# steel two-stage loading test data and remaining life prediction results of each model.

| Two-Stage Loading Stress Level/MPa | Test Value | Miner’s Rule (Formula (1)) | Ductile Dissipation Model (Formula (11)) | The Proposed Model (Formula (19)) |
|-----------------------------------|------------|---------------------------|------------------------------------------|----------------------------------|
|                                   | $n_1$ | $n_1/N_{f_1}$ | $n_2$ | $n_2/N_{f_2}$ | $n_2/N_{f_2}$ | Relative Error/% | $n_2/N_{f_2}$ | Relative Error/% | $n_2/N_{f_2}$ | Relative Error/% |
| 331.5–284.4                      | 12500  | 0.25 | 250400 | 0.5008 | 0.7500 | 49.76% | 0.7055 | 40.87% | 0.5755 | 14.91% |
|                                  | 25000  | 0.50 | 168300 | 0.3366 | 0.5000 | 48.54% | 0.4314 | 28.17% | 0.2671 | 20.64% |
|                                  | 37500  | 0.75 | 64500  | 0.1290 | 0.2500 | 93.80% | 0.1861 | 44.29% | 0.6970 | 10.41% |
| 284.4–331.5                      | 125000 | 0.25 | 37900  | 0.7580 | 0.7500 | 1.06% | 0.7888 | 4.07% | 0.8608 | 13.56% |
|                                  | 250000 | 0.50 | 38900  | 0.7780 | 0.5000 | 35.73% | 0.5646 | 27.42% | 0.6970 | 10.41% |
|                                  | 375000 | 0.75 | 43400  | 0.8680 | 0.2500 | 71.20% | 0.3188 | 63.27% | 0.4858 | 44.03% |
Table 6. Al-2024 aluminum alloy two-stage loading test data and remaining life prediction results of each model.

| Two-Stage Loading Stress Level/MPa | Test Value | Miner’s Rule (Formula (1)) | Ductile Dissipation Model (Formula (11)) | The Proposed Model (Formula (19)) |
|-----------------------------------|------------|-----------------------------|------------------------------------------|----------------------------------|
|                                   | n1         | n1/Nf1                      | n2/Nf2                                   | Relative Error/%                  |
| 200–150                           | 30,000     | 0.2000                      | 228,700                                  | 0.5319                           | 50.40%                          | 0.7844                          | 47.47%                          | 0.5623                          | 5.71%                           |
|                                   | 60,000     | 0.4000                      | 101,050                                  | 0.2350                           | 155.32%                         | 0.5735                          | 144.05%                         | 0.2677                          | 13.91%                          |
|                                   | 90,000     | 0.6000                      | 76,050                                   | 0.1769                           | 126.12%                         | 0.3689                          | 108.53%                         | 0.0941                          | 46.80%                          |
| 150–200                           | 86,000     | 0.2000                      | 144,500                                  | 0.9633                           | 16.95%                          | 0.8146                          | 15.43%                          | 0.9153                          | 4.98%                           |
|                                   | 172,000    | 0.4000                      | 133,500                                  | 0.8900                           | 32.58%                          | 0.6254                          | 29.73%                          | 0.8186                          | 8.02%                           |
|                                   | 258,000    | 0.6000                      | 81,700                                   | 0.5447                           | 26.57%                          | 0.4309                          | 20.89%                          | 0.6992                          | 28.36%                          |

Figure 9. (520–420 MPa) The relative error of each model under different n1/Nf1 (18CrNiMo7-6).

Figure 10. (420–520 MPa) The relative error of each model under different n1/Nf1 (18CrNiMo7-6).

Figure 11. (331.5–284.4 MPa) The relative error of each model under different n1/Nf1 (45# steel).
Table 6.

| n1/Nf1 | Formula 1 | Formula 11 | Formula 19 |
|--------|------------|------------|------------|
| 0.20   | 0.25       | 0.40       | 0.60       |
| 0.40   | 0.25       | 0.40       | 0.60       |
| 0.60   | 0.25       | 0.40       | 0.60       |

Figure 12. (284.4–331.5 MPa) The relative error of each model under different n1/Nf1 (45# steel).

Figure 13. (200–150 MPa) The relative error of each model under different n1/Nf1 (Al-2024).

Figure 14. (150–200 MPa) The relative error of each model under different n1/Nf1 (Al-2024).

It can be seen from Figures 9 and 10, for 18CrNiMo7-6 material, under high–low load loading, as the proportion of high load continues to expand, the relative error of Miner’s rule increases significantly. The relative error of the tough dissipation model is also gradually increasing, but the overall value is smaller than Miner’s rule. The relative error between the result obtained by the improved model and the actual value is the smallest among these three models. In the case of low–high loading, the error between the results obtained by the three models and the actual value is smaller than the error obtained in the case of high–low loading. This also shows that, from the side, different loading sequences have different effects on the fatigue life.

It can be seen from Figures 11 and 12, for 45# steel, under high–low load loading, as the proportion of high load continues to increases, the relative error of Miner’s rule increases significantly. The relative error fluctuation of the toughness dissipation model
is relatively stable, and the overall value is smaller than Miner’s rule. The relative error between the result obtained by the improved model and the actual value is the smallest among these three models. In the case of low–high loading, the error between the results obtained by the three models and the actual value is smaller than the error obtained in the case of high–low loading. Like the results of 18CrNiMo7-6, both reflect that under high–low loading, as the proportion of high load continues to increase, the relative error value is higher than under low–high loading, and the proportion of low load continues to increase.

It can be seen from Figures 13 and 14, for (Al-2024), under high–low loading, as the proportion of high load continues to increase, the relative error stability of the results obtained by the ductile dissipation model is better than that of the Miner’s rule. Under the same conditions, the accuracy of the results from the proposed model is higher than that of Miner’s rule and the ductile dissipation model. Under low–high loading, the results of each model are relatively stable. The accuracy of the improved model is the highest.

From the data in Tables 4–6, the linear damage accumulation model (Miner’s rule) assumes that damage is not related to the load state, damage accumulation is similarly not related to the load sequence, the interaction between loads cannot be considered, and the deviation from the test results is the largest. The relative error stability of the results obtained by the ductile dissipation model is better than that of the Miner’s rule. The improved model, proposed in this paper, increases the influence factors of the sequential stress sequence and magnitude and the error is smaller than the original ductile dissipation model. In this paper, the improved model is extended and applied to commonly used 45# steel and Al-2024 aluminum alloy. The error is larger than that of 18CrNiMo7-6, but it is still smaller than the original toughness dissipation model, indicating that the model in this paper has better material applicability.

6. Conclusions

(1) Based on the ductile dissipation theory, a nonlinear fatigue cumulative damage model considering the loading sequence is established, that is, an improved toughness dissipation model, which can consider the impact of loading sequence on damage with parameters which are simple and suitable for engineering applications;

(2) The fracture sections of the 18CrNiMo7-6 specimens, which were scanned by electron microscope, explain from the mechanism why Miner’s rule has different damages under two-level loading. The results of the electron microscope showed that the number of dimples formed on the fracture surface under low–high load was less than the number of dimples formed on the fracture surface under high–low load. This indicated that the number of micro-crack sources in the cross-section was relatively small, and the probability of micro-cracks connecting and expanding into cracks was relatively small, resulting in a small totally effective level of damage under low–high load cycles and a longer fatigue life;

(3) The improved model proposed in this paper is based on the test data of 18CrNiMo7-6 forged steel. This paper uses #45 steel, AL-2024 aluminum alloy and the nonlinear damage accumulation processes of other common materials, to predict that the life under two-stage load has smaller errors and better accuracy than the classic ductile dissipation model, indicating that the improved model has a good material applicability. Although this article uses three kinds of metal materials to test and verify the established model and has achieved good results, for other types of materials and test environments, much test verification and further research are needed. Further in-depth research can be done on issues such as the quantification of toughness dissipation, the improvement of model accuracy, and material verification.

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Nomenclature

- $b$: model parameter
- $D$: fatigue damage
- $D_i$: accumulated damage up to and including load step $i$
- $e_a$: applied stress amplitude
- $e_f$: breaking strength of the material
- $e_{f/N}$: residual breaking strength of the damaged material
- $i$: load step number
- $n$: number of cycles
- $n_i$: the number of cycles under the $i$th stress level
- $N$: fatigue life
- $N_f$: the cycle of fatigue fracture
- $N_{fi}$: the fatigue life under the $i$th stress level
- $U$: the initial toughness without damage
- $U_N$: toughness after $N$ cycles of loading
- $U_{Nf-1}$: the residual toughness of the material after $N_f - 1$ cycles of loading
- $W_f$: fatigue toughness
- $W_{fN}$: remaining fatigue toughness
- $\sigma_1$: the first-stage load amplitude
- $\sigma_2$: the second-stage load amplitude

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