Prediction of Moisture Distribution in Closed Ribbed Panel for Roof

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Abstract. Nowadays one of the possibilities to improve energy efficiency is the use of building elements with low air permeability, for example, sandwich panels with steel sheeting. However, these panels have one important disadvantage – a relatively small load-bearing capacity. This can be prevented by reinforcing the panel with antiseptized birch plywood ribs. For wood-based materials prediction of hygrothermal performance is important to avoid rot. Currently the methodology of ISO 13788:2012 is widely used assuming that moisture flux passes through the building envelope of any material. This assumption is not completely accurate with regard to a closed structure where no penetration of ambient humidity is possible. Therefore, in order to predict the distribution of moisture in such structure with the surfaces exposed to different temperatures and to assess the hazards of rot for plywood ribs, a methodology for closed building envelope is presented. To provide insight into expected results according to both methodologies, estimation for individual case with constant environmental conditions is given. According to the methodology for the closed building envelope no free water will occur. Therefore, it is believable that also no rot will be observed. This is contrary to the assessment according to the methodology of ISO 13788:2012, which predicts condensation.

1. Introduction
Nowadays, the when increase of energy resource costs is observed, more pronounced attention is paid to building envelope energy efficiency. To increase energy efficiency, elements with low air permeability are used more and more often. The second trend is the usage of prefabricated structural elements, thus reducing the labor-intensity at the building site. All these requirements are met by sandwich panels. However, these panels used as part of the combined roof structure may have insufficient load-bearing capacity required for covering the span resulting in additional purlins,
increased costs and labor intensity. One possibility to prevent that is reinforcing the panel by placing antiseptized birch plywood ribs into the insulation layer of the panel, which is a cost-effective solution. [1]

Rot is an important disadvantage of plywood and other wood-based materials; it can be caused by the appearance of free water. [2] Therefore, the analysis of moisture movement in the building envelope is required. Nowadays, the methodology of ISO 13788:2012 is widely used for this purpose. In accordance with this methodology, it is assumed that moisture flux is possible through the envelope of any material. This assumption does not reflect the actual situation in a closed structure such as a panel with steel sheeting. According to the methodology of this standard and also to the information presented in [3], the difference of water vapour pressure on both sides of the building envelope is the sole significant moisture transport potential for diffusion of water vapour. Whereas in accordance with [4] at least two moisture transport potentials should be taken into account where one of them is a temperature gradient since it can induce moisture flux directed contrary to the flux under water vapour pressure or moisture content gradients.

Therefore, the model for determination of moisture distribution in the closed building envelope has been developed, in which no moisture penetrates from the surrounding environment. Moisture flux under temperature gradient is also taken into account.

2. Prediction of condensation according to the methodology of ISO 13788:2012

According to the methodology of ISO 13788:2012, no moisture distribution across the depth of the panel is determined, only the probability of condensation is evaluated. The calculations are based on comparison of the partial water vapour pressure and the water vapour saturation pressure. At the interfaces of different materials where the former exceeds the latter, condensation occurs.

A constant one-dimensional moisture flux from the environment with higher partial vapour pressure to the environment with lower partial vapour pressure occurs due vapour diffusion:

\[
J = \delta_0 \left( \frac{p_i - p_c}{s_{d,i}} - \frac{p_c - p_e}{s_{d,c}} \right)
\]

where \(J\) – moisture flux (kg/(m²·s)); \(\delta_0 = 2 \times 10^{-10}\) (kg/(m·s·Pa)) – water vapour permeability of air with respect to partial vapour pressure; \(p_i = p_e + \Delta p\) – internal vapour pressure (Pa); \(p_c = \varphi_e \cdot p_{sat}\) – external vapour pressure (Pa); \(\Delta p\) – internal vapour pressure excess depending on the internal humidity class (Pa); \(\varphi_e\) – relative external air humidity (-); \(p_{sat}\) – water vapour saturation pressure (Pa); \(p_c = p_{sat}\) – partial water vapour pressure at condensation interface (Pa); \(s_{d,i}, s_{d,c}\) – respectively total water vapour diffusion-equivalent air layer thickness of the whole construction and of the layers from the external face to the condensation interface (m); \(s_d = \mu \Delta x\), where \(\mu\) – water vapour resistance factor (-); \(\Delta x\) – material layer thickness (m).

The water vapour saturation pressure \(p_{sat}\) at temperature \(T_c\) (°C) can be given by:

\[
p_{sat} = 610.5e^{\frac{17.266{T_c}}{T_c + 237.3}} \text{ if } T_c \geq 0 \degree C
\]

\[
p_{sat} = 610.5e^{\frac{2187.7{T_c}}{T_c + 237.3}} \text{ if } T_c < 0 \degree C \ [8]
\]
Accumulated moisture content per area at the condensation interface $M_c$ ($\text{kg/m}^2$) after time $\Delta t$ (s) can be determined by the following equation:

$$M_c = J \cdot \Delta t \quad (4)$$

3. Methodology for determination of moisture distribution in the closed building envelope

In the developed methodology for prediction of moisture distribution in the closed building envelope such as sandwich panel reinforced by plywood ribs the following assumptions are made:

- no moisture from environment air can penetrate inside the panel (the total moisture content is constant);
- built-in moisture flux under temperature and moisture gradients is one-dimensional;
- initially moisture is evenly distributed over the entire volume of the plywood rib.

Since the concentration of the built-in moisture due to the moisture movement across the depth of the plywood rib is time-varying, moisture distribution after time $\Delta t$ is calculated in several steps. At the start of calculations, the rib is divided into $n$ layers. Within each layer wood characteristics are determined depending on mean moisture $M_n$ (%) content and temperature $T_{k,n}$ (K) in the respective layer. Within one step it is assumed that characteristics of wood in each layer are constant.

Moisture distribution in plywood rib can be predicted by using the following algorithm:

1) Determine characteristics of wood in each layer by equations given below:

- density of wood $\rho_n$ ($\text{kg/m}^3$) if moisture content of wood $M_n \leq 30\%$:

$$\rho_n = 0.957 \rho_{12} \frac{100 + M_n}{100 + 0.6M_n} \quad (5)$$

where $\rho_{12}$ – wood density at moisture content $M = 12\%$ (kg/m$^3$) [5]

- specific gravity of wood $G_n$ (–):

$$G_n = \frac{\rho_n}{\rho_w \left(1 + \frac{M_n}{100}\right)} \quad [6]$$

- porosity of wood $v_{a,n}$ (–):

$$v_{a,n} = 1 - G_n (0.667 + 0.01M_n) \quad [7]$$

- thermal conductivity of wood $\lambda_n$ ($\text{W/(m} \cdot \text{K})$) if $M_n < 40\%$:

$$\lambda_n = 418 \left[G_n (5.18 + 0.096M_n) + 0.57v_{a,n}\right] \cdot 10^{-4} \quad [6]$$

- thermal resistance of layer $R_n$ (m$^2$-K)/W:

$$R_n = \frac{\Delta x}{\lambda_n} \quad (9)$$

where $\Delta x$ – layer thickness (m). [8]
2) Determine mean temperature $T_{K,n} (K)$ in each layer:

$$T_{K,n} = 273.15 + \left[ T_e + \sum_{i=1}^{n} \frac{R_n - 0.5R_n}{R_T} (T_i - T_e) \right]$$  \hspace{1cm} (10)

where $T_e$ – temperature of external air ($^\circ$C); $T_i$ – temperature of internal air ($^\circ$C); $\sum R_n$ – total thermal resistance of layers from the external face to the analyzed layer ((m$^2$·K)/W); $R_n$ – resistance of the analyzed layer ((m$^2$·K)/W); $R_T$ – total thermal resistance ((m$^2$·K)/W). [8]

3) Determine bound water diffusion coefficient transverse to the wood fiber direction $D_{T,n} (m^2/s)$ in each layer:

$$D_{T,n} = 0.7 \cdot 10^{-5} \cdot \exp \left( \frac{- \frac{9200 - 70M_n}{RT_{K,n}}}{(1 - \nu_{a,n})(1 - \sqrt{\nu_{a,n}})} \right)$$  \hspace{1cm} (11)

where $R = 2$ cal/mol – universal gas constant.

4) Determine the change of moisture content in each layer. Before that the moisture flux rate shall be known. One dimensional non-isothermal moisture flux in wood (in the hygroscopic range) may be written in the following form:

$$J = -G\rho_w \left[ D_T \left( \frac{M}{RT_K + 70M} \right) \frac{E_b}{T_K} \frac{\Delta T_K}{\Delta x} + D_T \frac{\Delta M}{\Delta x} \right]$$  \hspace{1cm} (12)

where $J$ – moisture flux (kg/(m$^2$·s)); $G$ – specific gravity of wood (–); $\rho_w = 1000$ kg/m$^3$ – normal density of water; $D_T$ – bound water diffusion coefficient transverse to the fiber direction (m$^2$/s); $M$ – moisture content of wood (%); $R = 2$ cal/mol – universal gas constant; $T_K$ – temperature (K); $E_b = 9200 - 70M$ – activation energy for bound water diffusion in the transverse direction (cal/mol); $x$ – distance in the direction of moisture flux (m). [9]

If the moisture flux rate is known, the concentration changes $\Delta c$ (kg/m$^3$) in time may be characterized by the Fick’s second law:

$$\frac{\Delta c}{\Delta t} = -\frac{\Delta J}{\Delta x}$$  \hspace{1cm} (13)

where $\Delta t$ – time (s); $\Delta J$ – difference between incoming and outgoing moisture flux rate in layer (kg/(m$^2$·s)); $\Delta x$ – layer thickness (m).

Equation (14) can be derived from equations (12) and (13) to determine the change of moisture concentration in each layer $\Delta c_n$ (kg/m$^3$) after time $\Delta t$ (s):
\[ \Delta c_n = \frac{\Delta t}{\Delta x} \Delta J = \Delta t \left( J_{n-1} - J_{n+1} \right) = \]

\[ = -\frac{G_n \rho_w \Delta t}{100(\Delta x)^2} \left[ D_{T,n-1} \left( \frac{M_{n-1}}{RT_{K,n-1} + 70M_{n-1}} \right) \left( \frac{T_{K,n-1} - T_{K,n}}{\Delta x} \right) + D_{T,n-1} \left( \frac{M_{n-1} - M_n}{\Delta x} \right) \right] \]

\[ - \left[ D_{T,n} \left( \frac{M_n}{RT_{K,n} + 70M_n} \right) \left( \frac{T_{K,n} - T_{K,n+1}}{\Delta x} \right) + D_{T,n} \left( \frac{M_n - M_{n+1}}{\Delta x} \right) \right] \]

\[ (14) \]

where indexes “\( n \)”, “\( n-1 \)” un “\( n+1 \)” – sequence number of the layer.

5) Determine the change of moisture content \( \Delta M_n \) (%) in each layer:

\[ \Delta M_n = \frac{\Delta c_n \cdot 100\%}{G_n \rho_w} \]

(15)

6) Determine total moisture content \( M_n \) (%) in each layer:

\[ M_{n,i} = M_{n,i-1} + \Delta M_n \]

(16)

where index “\( i \)” – the number of calculation step.

7) Repeat this algorithm in the next step by taking into account the changes of moisture content in each layer.

During calculations it is necessary to control that no naturally impossible situations occur when between two layers with increasing moisture content there is one with decreasing moisture content or vice versa. In such case it would be necessary to repeat the calculation by reduced step duration.

The method described above can be used for determination of moisture distribution if no water freezing can be observed, i.e. at temperatures below zero moisture content does not exceed relative amount of non-freezing moisture in the wood \( M^\ast \) (%). It can be characterized by the minimum temperature in layer \( T_{C,n} \)(°C):

\[ M^\ast = 12 + 19.5 \exp(0.055T_{C,n}) \]

(17)

This equation is valid for wood in the temperature interval from -50 °C up to 0 °C. The relative amount of non-freezing water at temperature -50 °C is equal to 13.2% and reaches its maximum value 31.5% at 0 °C. [10]

4. Characteristics of the research object

To compare the expected results obtained by both methods (ISO 13788:2012 and the method for closed building envelope), the sandwich panel with polystyrene insulation, steel sheeting and birch plywood ribs is analysed, see figure 1.
Figure 1. Cross section of ribbed sandwich panel: 1 – steel sheeting; 2 – polystyrene insulation; 3 – birch plywood ribs.

The calculation is carried out according to the methodologies given in sections 2 and 3. Both methodologies are applied for moisture analysis of 1 month (31 days = 2592000 seconds) long period. The assumed external temperature $T_e = -7.6\, ^\circ C$ is January mean temperature in Aluksne which leads to greatest amount of moisture on the condensate interface if compared with other months. [11] The interior temperature is assumed $T_i = +18\, ^\circ C$.

In case of calculation according to the methodology of ISO 13788:2012, the following input data are assumed: external air humidity $\varphi_e = 0.87$; internal vapour pressure excess $\Delta p = 270$ Pa for internal humidity class I and $\Delta p = 1080$ Pa for internal humidity class IV [8]; density of birch plywood $\rho = 700\, \text{kg/m}^3$; thermal conductivity $\lambda = 0.174\, \text{W/(m} \cdot \text{K)}$. Water vapour resistance factor $\mu = 40$ is taken as for solid wood rather than for plywood because the directions of adhesive layers are parallel to the flux of moisture. For steel sheeting of thickness 0.5 mm this factor is assumed $\mu = 10^6$. [12] During calculations it is assumed that cross section of the panel is divided into layers whose boundaries coincide with the boundaries of the material. Initially it is assumed that partial water vapour pressure increases linearly across the depth of the panel. Since the partial water vapour pressure $p_n$ cannot exceed the saturated water vapour pressure $p_{sat}$, the straight line $p_e - p_i$ is redrawn as a broken line $p_e - p_c - p_i$, see figure 2. The condensate will occur at point $p_c$.

In case of panel analysis as for closed building envelope, plywood ribs made of birchwood with built-in moisture $M = 12\%$ and density $\rho_{12} = 640\, \text{kg/m}^3$ are assumed. [5] Initially the calculations are done by dividing plywood rib into 10 layers. After repeating of calculations by dividing the panel into 20 layers it was concluded that the results differed by no more than 1.1% and no finer division of plywood rib is necessary. Therefore the thickness of each layer is assumed $\Delta x = 0.012\, \text{m}$. The calculation is carried out in 31 steps. Duration of each step is one day (86,400 seconds). To evaluate the influence of great temperature gradient on moisture distribution, additional case is analyzed when exterior temperature is $T_e = -41\, ^\circ C$. This temperature conforms to the absolute minimum of air temperature in Daugavpils that is possible once per 50 years and is the lowest temperature in Latvia. [11] These latter conditions may not be used for characterizing average conditions per month but only for demonstrating boundary values and distribution of moisture content in case of very low temperature.

5. Results

5.1. Results obtained according to the methodology of ISO 13788:2012

Results obtained according to the methodology of ISO 13788:2012 are shown in figure 2. The condensate interface is between insulation layer and exterior steel sheeting. After 31 days in case of moisture class I 0.19 g/m$^2$ of condensate and 1.12 g/m$^2$ in case of moisture class IV will be stored at condensate interface. In case of moisture class I condensate will occur in the layers of thickness 2.4
cm at the cold side of the panel and 12 cm in case of moisture class IV. As the condensation is predicted, appearance of rot is also not excluded.

5.2. Results obtained according to the methodology for the closed building envelope

Two cases are analyzed – with external temperature $T_e = -7.6 \, ^\circ C$ and $T_e = -41.0 \, ^\circ C$. According to equation (17) in both cases bound water will not freeze throughout all volume of the plywood rib and its flux will not be held up. During calculations it was found that moisture flux due the temperature gradient was $2 \times 10^3$ times greater than moisture flux caused by the moisture gradient at the end and at the start of the analyzed time period, respectively.

Moisture distribution in plywood rib is shown in figure 3. Numbering of the layers starts from the cold side of the panel. If $T_e = -7.6 \, ^\circ C$, maximum moisture content after 31 days will be 12.05% at layer No. 1. If $T_e = -41.0 \, ^\circ C$, maximum moisture content after the same period will be 12.03% at layer No. 17. Moisture flux due to temperature gradient increases with increasing temperature and vice versa. As a result, decreased wetting of the panel’s cold side and increased drying of the panel’s warm side can be observed if the case with exterior temperature $T_e = -41.0 \, ^\circ C$ is compared to the case with exterior temperature $T_e = -7.6 \, ^\circ C$. In both cases the change of moisture content in the middle layers is relatively small if compared with outer layers of the plywood rib. This can be explained by the fact that the middle layers receive and return the moisture to adjacent layers while the outer layers only receive or only return the moisture.

As the moisture content at moisture concentration areas is much less than moisture content at fiber saturation point ($M = 25 – 35\%$ [2]), no free water will be observed in plywood ribs. Moreover, it can be believed that moisture concentration will be much less because in natural conditions continuous changes of external conditions will induce the change of moisture flux direction. Therefore, it is reasonable to assume that no rot will occur.
Distribution of moisture content in plywood rib M, %

| No. | M, % | Te = -7.6 °C | Te = -41.0 °C |
|-----|------|--------------|---------------|
| 1   | 12.05* | 12.02        |
| 2   | 12.01  | 12.00        |
| 3   | 12.00  | 12.00        |
| 4   | 12.00  | 12.01        |
| 5   | 12.00  | 12.01        |
| 6   | 12.00  | 12.01        |
| 7   | 12.00  | 12.01        |
| 8   | 12.00  | 12.01        |
| 9   | 12.01  | 12.01        |
| 10  | 12.01  | 12.01        |
| 11  | 12.01  | 12.02        |
| 12  | 12.01  | 12.02        |
| 13  | 12.01  | 12.02        |
| 14  | 12.01  | 12.02        |
| 15  | 12.01  | 12.02        |
| 16  | 12.01  | 12.03        |
| 17  | 12.01  | 12.03*       |
| 18  | 12.00  | 12.03        |
| 19  | 11.98  | 11.98        |
| 20  | 11.87  | 11.74        |

Note.
* – maximum moisture content in plywood rib after 31 days or 2,592,000 seconds.

Figure 3. Moisture distribution in plywood rib divided into 20 layers along the depth of the panel.

6. Conclusions
Methodology for prediction of moisture distribution in the closed building envelope is presented. An insight into prospective results obtained according to this methodology is given by analysing a sandwich panel with steel sheeting, polystyrene insulation and birch plywood ribs at constant internal and external conditions. Comparison with the results according to ISO 13788:2012 methodology is also made.

During the calculations according to the methodology for the closed building envelope it was found that in case of sandwich panel temperature gradient may be the main moisture transport potential. For the analyzed cases, moisture flux under temperature gradient was even $10^5$ times greater than moisture...
flux due moisture gradient. Results show that at moisture concentration areas moisture content of a birch plywood rib changes from 12.00% to 12.05%. This is more than two times lower if compared with moisture content at the fiber saturation point. Therefore, no free water will be observed in plywood ribs. Consequently, theoretically it can be assumed that no rot will occur in this case.

Opposite conclusions can be drawn from the results obtained in accordance with the methodology of ISO 13788:2012. According to these results, condensation may occur in the panel. It may create favorable conditions for rot.

7. References
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