Tensor-polarized structure function $b_1$
by convolution picture for deuteron

W. Cosyn, Yu-Bing Dong, S. Kumano,  and M. Sargsian

1Department of Physics and Astronomy, Ghent University, Proeftuinstraat 86, B9000 Ghent, Belgium
2Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
3Theoretical Physics Center for Science Facilities (TPCSF), CAS, Beijing 100049, China
4KEK Theory Center, Institute of Particle and Nuclear Studies, KEK 1-1, Oho, Tsukuba, Ibaraki, 305-0801, Japan
5J-PARC Branch, KEK Theory Center, Institute of Particle and Nuclear Studies, KEK, and Theory Group, Particle and Nuclear Physics Division, J-PARC Center, 203-1, Shirakata, Tokai, Ibaraki, 319-1106, Japan
6Department of Physics, Florida International University, Miami, Florida 33199, USA

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There are polarized structure functions $b_{1-4}$ for the spin-1 deuteron. We calculated the leading-twist tensor structure function $b_1$ by using convolution description for the deuteron. We found large differences between our theoretical functions and HERMES experimental data on $b_1$. Although higher-twist effects should be considered in obtaining experimental $b_1$, it suggests a possible existence of new hadron physics mechanism for spin-1 hadrons. Furthermore, we found that there are significant distributions at large Bjorken $x$. In future, an experimental measurement is planned at JLab for $b_1$ and there is a possibility of a proton-deuteron Drell-Yan experiment at Fermilab with the tensor-polarized deuteron, so that further theoretical studies are needed for clarifying the physics origin of tensor structure in terms of quark and gluon degrees of freedom.

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I. INTRODUCTION

The discovery of nucleon spin puzzle created the field of high-energy spin physics. So far, the studies have been focused on spin-1/2 nucleon. It is known that there exist additional structure functions $b_{1-4}$ for a spin-1 hadron in charged-lepton deep inelastic scattering due to the existence of tensor structure [1, 2]. There are some theoretical studies on the tensor-polarized structure functions, and the first measurement of $b_1$ was reported by the HERMES collaboration [3]. A useful parametrization was proposed for the tensor-polarized parton distribution functions (PDFs) from the HERMES data [4] by using a constraint for the $b_1$ sum proposed for the tensor-polarized valence-quark distributions [5]. Although the $b_1$ data are not still accurate, the measurement indicates interesting features different from conventional theoretical calculations. As the deviation from the naive-quark-model prediction created the topic of nucleon spin puzzle, it is important to show a conventional theoretical estimate with the standard deuteron model since a possible experimental deviation may indicate a “tensor-polarization puzzle”.

As such a “standard” model of the deuteron for describing the twist-two structure function $b_1$, we use a convolution picture. Namely, the tensor-polarized structure function $b_1$ is given by unpolarized structure functions convoluted with tensor-polarized lightcone momentum distributions of the nucleon. There are such studies for $b_1$ in Refs. [2, 6]. We try to test their results by independent ways. We use two theoretical descriptions. One is a basic convolution model and another is the virtual-nucleon-approximation model. Such convolution models have been used also as a baseline calculation for the nuclear structure function $F_2^A$ at medium and large $x$ in which nuclear effects were taken into account through the binding and nucleon Fermi motion in a nucleus together with short-range correlations. These physics ingredients are contained in the spectral function, which is the nucleon’s four-momentum distribution in the nucleus. We directly apply such descriptions for calculating the structure function $b_1$. If the calculated $b_1$ distribution is much different from the HERMES measurement, a new hadron physics mechanism could be considered beyond the standard model of the deuteron in describing the tensor structure in terms of quark and gluon degrees of freedom [7].

The purpose of our research is to show the convolution result on $b_1$ as the standard theoretical estimate and to discuss its comparison with the HERMES data [8]. In particular, a new accurate measurement of $b_1$ will start in a few years at JLab [9] and a possible Drell-Yan experiment is considered at Fermilab with a tensor-polarized deuteron [10, 11]. Furthermore, other experiments are possible in principle for the tensor-polarized structure functions at BNL, EIC, J-PARC, GSI-FAIR, and IHEP@Russia.

II. THEORETICAL FORMALISMS FOR $b_1$

The cross section for deep inelastic scattering of charged lepton from a spin-1 hadron is given by a hadron tensor multiplied by a lepton tensor, and the hadron tensor is expressed in terms of eight structure functions as
The tensor-polarized structure functions are defined by the spin-1 polarization vector, hadron and virtual-photon momenta (\(P, q\)) to satisfy the current conservation \(q^\mu W_{\mu \nu} = q^\nu W_{\mu \nu} = 0\). The coefficients \(r_\mu, s_\mu, t_\mu, u_\mu\) are defined by the spin-1 polarization vector, hadron and virtual-photon momenta \((P, q)\), and initial and final spin states \((\lambda, \lambda')\), and their actual expressions should be found in Refs. \([2, 12]\). The structure functions \(b_3\) and \(b_4\) are twist-4, and \(b_1\) and \(b_2\) are leading-twist functions which are related to each other by the Callan-Gross type relation \(2\pi b_1 = b_2\) in the Bjorken scaling limit. We may first investigate the leading function \(b_1\) (or \(b_2\)).

The structure function \(b_1\) is expressed in terms of the tensor-polarized PDF's \(\delta_\tau f(x, Q^2)\) in the parton model as

\[
\begin{align*}
    b_1(x, Q^2) &= \frac{1}{2} \sum_i c_i^2 \left[ \delta_\tau q_i(x, Q^2) + \delta_\tau \bar{q}_i(x, Q^2) \right], \\
    \delta_\tau f(x, Q^2) &\equiv f_0(x, Q^2) - \frac{f^{+}(x, Q^2) + f^{-}(x, Q^2)}{2}.
\end{align*}
\]

(2)

Here, \(f^\lambda\) is an unpolarized parton distribution in the hadron spin state \(\lambda\) and \(c_i\) is the charge of the quark flavor \(i\). The Bjorken scaling variable \(x\) is defined as \(x = Q^2/(2M_N\nu)\) with the nucleon mass \(M_N\) and \(\nu = P \cdot q/M\), and its range is given by \(0 < x < 2\) for the deuteron.

There is a sum rule for \(b_1\) \([4, 5]\) in the similar way with the Gottfried sum rule \([13]\):

\[
\int dx b_1(x) = -\lim_{t \to 0} \frac{5}{24} t F_Q(t) + \frac{1}{9} \int dx \left[ 4 \delta_\tau \tilde{u}(x) + \delta_\tau \tilde{d}(x) + \delta_\tau \tilde{s}(x) \right],
\]

\[
\int \frac{dx}{x} \left[ F_2^p(x) - F_2^n(x) \right] = \frac{1}{3} + \frac{2}{3} \int dx \left[ \tilde{u}(x) - \tilde{d}(x) \right],
\]

(3)

where \(F_Q(t)\) is the electric quadrupole form factor for the spin-1 hadron and the first term, which comes from tensor-polarized valence-quark distributions, vanishes: \(\lim_{t \to 0} \frac{5}{24} t F_Q(t) = 0\). It could be used as a guideline in investigating \(b_1\). As the Gottfried-sum-rule violation initiated the studies of \(\tilde{u} - \tilde{d}\) \([13]\), a finite \(b_1\) sum could indicate tensor-polarized antiquark distributions. In fact, a finite \(b_1\) sum was suggested in the HERMES experiment, and it is interesting to measure the tensor-polarized antiquark distributions in the Fermilab-E1039 Drell-Yan experiment \([11]\).

For calculating \(b_1\) in the standard deuteron model with D-state admixture, we introduce two convolution models. (A) One is a basic convolution description and (B) another is a virtual nucleon approximation which includes higher-twist contributions.

### A. Theory 1: Basic convolution description

In the convolution description of nuclear structure functions \(W_{\mu \nu}^A\), the hadron tensor is given by the nucleonic one \(W_{\mu \nu}\) convoluted with the momentum distribution of the nucleon, so called spectral function \(S(p)\), as

\[
W_{\mu \nu}^A(P_A, q) = \int d^4 p S(p) W_{\mu \nu}(p, q),
\]

(4)

where \(p\) and \(P_A\) are momenta for the nucleon and nucleus, \(\phi_i(p')\) is the momentum-space wave function for the \(i\)-th nucleon. It explains major features of nucleon modifications at medium and large \(x\) \((x > 0.2)\) by the mechanisms of nuclear binding, Fermi motion, and short-range correlations contained in the spectral function.

The hadron tensor for the deuteron can be expressed by their helicity amplitudes of the virtual photon as \([2]\)

\[
A_{h,h,h}(x, Q^2) = \epsilon^{\mu}_h \epsilon^{\nu}_h W_{\mu \nu}(p, q),
\]

(5)

and the corresponding one \(\tilde{A}_{h,h,h}(x, Q^2)\) for the nucleon. Here, \(\epsilon^{\mu}_h\) is the photon polarization vector. The structure function \(b_1\) of the deuteron and \(F_1\) of the nucleon \((F_1^N)\) are expressed as \([2, 14]\)

\[
F_1^N = \frac{A_{++} + A_{--}}{2},
\]

(6)

in the Bjorken scaling limit. Using these equations, we obtain a convolution expression for \(b_1\) defined by the one per nucleon as

\[
\begin{align*}
    b_1(x, Q^2) &= \int \frac{dy}{y} \delta_\tau f(y) F_1^N(x/y, Q^2), \\
    \delta_\tau f(y) &\equiv f^{0}(y) - \frac{f^{+}(y) + f^{-}(y)}{2}.
\end{align*}
\]

(7)

The lightcone momentum distribution is expressed by the deuteron wave function \(\phi^H(p')\) as

\[
\phi^H(p') = \int d^3 p y |\phi^H(p')|^2 \delta \left( y - \frac{E - p_z}{M_N} \right).
\]

(8)

The variable \(y\) is the momentum fraction defined by \(y = M p \cdot q/(M_N P \cdot q) \approx 2 p^2/P^+\) where the lightcone momentum is defined by \(p^- \equiv (p_0^2 + p^2)/2\) by taking z-axis along the virtual-photon momentum direction. Using the deuteron wave function with the D-state
admixtures, we obtain
\[ \delta_{T_f}(y) = \int d^3p \, y \left[ -\frac{3}{4\sqrt{2}\pi} \phi_0(p)\phi_2(p) + \frac{3}{16\pi} |\phi_2(p)|^2 \right] \times (3\cos^2 \theta - 1 - 3) \left( y - \frac{p \cdot q}{M_N} \right), \] (9)

where \( \phi_0(p) \) and \( \phi_2(p) \) are S- and D-state wave functions. According to this basic convolution model, the structure function \( b_1 \) arises due to the D-state admixture. For calculating Eq. (7), we need the structure function \( F_1^N \). In our work, we use the leading-order (LO) expression with the longitudinal-transverse ratio \( R \) as
\[ F_1^N(x, Q^2) = \frac{1 + 4 M_N^2 x^2/Q^2}{2 x [1 + R(x, Q^2)]} F_2^N(x, Q^2), \]
\[ F_2^N(x, Q^2)_{\text{LO}} = \frac{3}{2} \sum_i e_i^2 [\tilde{q}_i(x, Q^2) + \tilde{q}_i(x, Q^2)]_{\text{LO}}. \] (10)

B. Theory 2: Virtual nucleon approximation

The cross section for the charged-lepton DIS from the polarized deuteron is expressed by the polarization factors and structure functions:
\[ \frac{d\sigma}{d\phi} = \pi^2 \alpha^2 \left[ \sum_{\text{LO}} F_{UU, T} + \epsilon F_{UU, L} \right] \]
\[ + T_{\parallel} \left( F_{UT, L/T} + \epsilon F_{UT, L/L} \right) \cos \phi_{T_i} \]
\[ + T_{\perp} \cos(2\phi_{T_i}) \epsilon F_{UT, T/T}, \] (11)

where \( \epsilon \) is the degree of the longitudinal polarization of the virtual photon, the details on the longitudinal and transverse polarization factors (\( T_{\parallel} \), \( T_{\perp} \)) and the angles (\( \phi_{T_i} \) ) should be found in Ref. [15]. Among the structure functions, \( F_{UT, L/T} \) and \( F_{UT, LT} \) are related to \( b_1 \) and they are expressed by the helicity amplitudes and tensor-polarized structure functions \( b_{1-3} \) as
\[ F_{UT, T_T} = \frac{2}{\sqrt{6}} \left( A_{++}+a - 2 A_{+0} + A_{+-} \right), \]
\[ = -\frac{1}{\sqrt{6}} \left[ 2(1+\gamma^2) x b_1 - \gamma^2 \left( \frac{1}{6} b_2 - \frac{1}{2} b_3 \right) \right], \]
\[ F_{UT, LT} = -\frac{\sqrt{2}}{3} x \frac{\sqrt{2}}{\gamma} \left( \frac{1}{6} b_2 - \frac{1}{2} b_3 \right), \] (12)

where \( \gamma = \sqrt{Q^2/\nu} \). From these equations, \( b_1 \) is expressed by these two structure functions as
\[ b_1 = -\frac{1}{1+\gamma^2} \sqrt{\frac{3}{8}} \left[ F_{UT, T_T} + F_{UT, LT} \right]. \] (13)

Now, we explain the virtual nucleon approximation (VNA) for calculating \( b_1 \). It considers the np component of the light-front deuteron wave function. The virtual photon interacts with one nucleon which is off the mass shell, while the second non-interacting “spectator” is assumed to be on its mass shell. Then, the structure functions are obtained by integrating over all possible spectator momenta \( p_N \):
\[ W_{\mu
u}^{\gamma N}(P, q) = 4(2\pi)^3 \int d\Gamma_N \frac{\alpha_s}{\alpha_i} W_{\mu
u}(p_i, q) \rho_D(x', \lambda), \] (14)

where \( d\Gamma_N \) is the phase space for the spectator nucleon. The factor \( 4(2\pi)^3 \) comes from the definition of deuteron lightcone wave function, and the factor \( \alpha_s/\alpha_i \) exists because the hadron tensor \( W_{\mu
u} \) is for the nucleon with momentum \( p_i \) instead of the nucleon at rest [15]. The lightcone momentum fractions are defined for the interacting (i) and spectator (N) nucleons as \( \alpha_i = 2 p_i^-/P^- \) and \( \alpha_N = 2 p_N^-/P^- = 2 - \alpha_i \). The deuteron density \( \rho_D(x', \lambda) \) is defined by the deuteron wave function \( \Psi(\vec{k}, \lambda', \lambda) \) as
\[ \rho_D(x', \lambda) = \sum_{\lambda, \lambda'} \frac{\left[ \Psi_D(\vec{k}, \lambda', \lambda) \right]}{\alpha_{\lambda'} \alpha_i}. \] (15)

The wave function \( \Psi_D \) is then expressed by the S- and D-wave components \( \phi_0 \) and \( \phi_2 \). Calculating the structure functions in Eq. (12) and using the relation of Eq. (13), we finally obtain \( b_1 \) in the VNA model:
\[ b_1(x, Q^2) = \frac{3}{4(1+\gamma^2)} \int \frac{k^2}{\alpha_i} dk \cos(\theta_k) \]
\[ \times \left[ F_1^N(x, Q^2) (6 \cos^2 \theta_k - 2) \right. \]
\[ - T^2 \frac{2 p_i \cdot q}{2} F_2^N(x, Q^2) (5 \cos^2 \theta_k - 1) \]
\[ \left. - \frac{\phi_0(k) \phi_2(k)}{\sqrt{2}} + \frac{\phi_2(k)^2}{4} \right], \] (16)

where \( \theta_k \) is the angle between \( k \) and \( q \), and \( T^2 \) is defined by \( T^2 = P_N^2 + q^2 + q^2 q^2 - L^2 P_N \cdot L / L^2 \) with \( L^2 = P^2 \mu + q^2 + q^2 / Q^2 \). As obvious from the above derivation, the \( b_1 \) of the VNA model includes higher-twist contributions, whereas the first basic model of Eq. (7) was obtained by using the relation (6) in the scaling limit.

III. RESULTS

We show results on \( b_1 \) by integrating the expressions in Eqs. (7) and (16). For this numerical evaluation, we choose the PDFs for calculating \( F_2^N \), the longitudinal-transverse ratio \( R \), and the deuteron wave function. They are taken as MSTW2008 (Martin-Stirling-Thorne-Watt, 2008) leading-order (LO) parametrization, the SLAC-R1998 parametrization, and the CD-Bonn wave function, respectively.

Since the average scale of the HERMES measurement is \( Q^2 = 2.5 \) GeV\(^2\), we show our result at this \( Q^2 \). In Eqs. (7) and (16), there are two components, \( \phi_0 \phi_2 \) and \( \phi_T^2 \), which are called SD and DD terms. These contributions are shown in Fig. 1 with total \( b_1 \) curves for the two theoretical descriptions. The order of magnitude is rather small and the distributions are less than \( 10^{-3} \).
We notice that these results are very different from previous convolution estimates of Refs. [2, 6] in the following points.

(1) Although theoretical formalisms are similar, our distributions, namely magnitude and $x$ dependence, are very different. Especially, the SD curves have opposite sign to the one in Ref. [6].

(2) The finite distributions exist at large $x$, even at $x > 1$ although there is no distribution in Ref. [6].

There are also relatively large differences between two theory results. We checked that both results become similar in the scaling limit, which indicates that higher-twist effects are the major sources for the differences. In addition, there are effects coming from slightly different normalizations for the lightcone wave functions [8].

Next, our results are compared with the HERMES data in Fig. 2. It is obvious that theoretical curves are much different from the data. In the measured $x$ range ($x < 0.5$), the experimental magnitude is one-order larger than both theoretical estimates. Furthermore, there are relatively large distributions even at large $x$ ($0.6 < x < 0.8$). Because the HERMES errors are large, we cannot draw a solid conclusion from this comparison. However, the large differences indicate that possibly a new hadron physics mechanism could be needed for their interpretation, although there are still some rooms to improve, for example, by considering higher-twist effects in extracting $b_1$ experimentally from the spin asymmetry $A_{zz}$ as pointed out in Ref. [8].

It is puzzling to find the large differences between the data and our standard convolution descriptions. In future, the JLab experiment will start in a few years to measure accurately $b_1$ at medium $x$ ($0.3 < x < 0.5$), and there is a possibly to measure the proton-deuteron Drell-Yan process in the Fermilab-E1039 Drell-Yan experiment [11]. Therefore, such a puzzle should be clarified by future studies; however, further theoretical studies are needed to clarify the situation and to consider a new mechanism to explain the HERMES data. Possibly, a new hadron spin field could be explored by such studies.

IV. SUMMARY

We calculated the tensor-polarized structure function $b_1$ by using the standard deuteron model with D-state admixture and the two convolution models. We found that our $b_1$ values are much smaller in magnitude than the HERMES data in the range $x < 0.5$. It could indicate possible existence of a new hadron mechanism for interpreting the large differences, although other contributions such as higher-twists should be investigated.

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[1] L. L. Frankfurt and M. I. Strikman, Nucl. Phys. A 405, 557 (1983).
[2] P. Hoodbhoy, R. L. Jaffe, and A. Manohar, Nucl. Phys. B 312, 571 (1989); R. L. Jaffe and A. Manohar, Nucl. Phys. B321, 343 (1989).
[3] A. Airapetian et al. (HERMES Collaboration), Phys. Rev. Lett. 95, 242001 (2005).
[4] S. Kumano, Phys. Rev. D 82, 017501 (2010).
[5] F. E. Close and S. Kumano, Phys. Rev. D 42, 2377 (1990).
[6] H. Khan and P. Hoodbhoy, Phys. Rev. C 44, 1219 (1991).
[7] G. A. Miller, Phys. Rev. C 89, 045203 (2014).
[8] W. Cosyn, Yu-Bing Dong, S. Kumano and M. Sargsian, arXiv:1702.05337.
[9] Proposal to Jefferson Lab PAC-38 (PR12-11-110), J.-P. Chen et al. (2011).
[10] S. Hino and S. Kumano, Phys. Rev. D 59, 094026 (1999); D 60, 054018 (1999).
[11] S. Kumano and Q. T. Song, Phys. Rev. D 94, 054022 (2016).
[12] S. Kumano, J. Phys. Conf. Ser. 543, 012001 (2014).
[13] S. Kumano, Phys. Rept. 303, 183 (1999); J.-C. Peng and J.-W. Qiu, Prog. Part. Nucl. Phys. 76, 43 (2014).
[14] T.-Y. Kimura and S. Kumano, Phys. Rev. D 78, 117505 (2008).
[15] W. Cosyn, M. Sargsian, and C. Weiss, to be submitted for publication.