Lens candidates in the Capodimonte Deep Field in vicinity of the CSL1 object

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ABSTRACT

CSL1 is a peculiar object discovered in the OACDF. Photometric and spectroscopic investigation strongly suggest that it may be the first case of gravitational lensing by cosmic string. In this paper we derive and discuss a statistical excess of a gravitational lens candidates present in OACDF region surrounding CSL1. This excess cannot be explained on the basis of conventional gravitational lens statistic alone, but is compatible with the proposed cosmic string scenario.

Key words: cosmic string; galaxies; general; cosmology: gravitational lensing.

1 INTRODUCTION

Cosmic strings in cosmology were thoroughly discussed over the past decades (cf. Zeldovich (1980), Vilenkin (1981)). For instance, several authors proposed to explain the quick variability of QSO by the gravitational lensing by a cosmic string: Sazhin, Khlopov (1988), Schild, Masnyak, Hnatyk, Zhdanov (2004).

Accordingly to Allen & Shellard (1990) their expected number should be large enough to make them observable but, so far, most attempts to detect their signatures seem to have failed. In a previous paper Sazhin et al. (2003) (hereafter Paper I) we discussed the strange
properties of the gravitational lens CSL1 which, by photometric and spectroscopic investigation was shown to consist of two identical images of a giant elliptical galaxy at \( z = 0.46 \). The most relevant feature of these images is that their isophotes appear to be undistorted, thus leaving only two possible explanation: the first one is that we are dealing with the chance alignment of two galaxies and the second one is that CSL1 is produced by a gravitational lens which does not distort extended images. It has in fact to be stressed that usual gravitational lenses, i.e. created by a bound clump of matter, produce inhomogeneous gravitational fields which always distort background extended images (cf. Schneider, Ehlers, Falco (1992), Keeton (2003)). The only possible interpretation of CSL1 in the framework of the gravitational lensing theory is therefore that of lensing by a cosmic string. In the same paper we also pointed out that the safest way to disentangle between these two possible scenarios would be to obtain milliarcsecond resolution images of CSL1. While waiting for this type of data to become available, we explore here the possibilities offered by another method which has been discussed in the past by Vilenkin, Shellard (1994), Hindmarsh (1990), Bernardeau & Uzan (2000), Laix & Vachaspati (1996), and recently in Huterer & Vachaspati (2003).

The alignment of the background object (a galaxy) inside the deficit angle of the string is a stochastic process determined by the area of the lensing strip and by the surface density distribution of the extragalactic objects which are laying behind the string. The larger field of search we choose, the larger the number of lensed objects that we should find. All lensed objects will fall inside a narrow strip defined by the deficit angle computed along the string pattern. With an effective metaphor, we can say that this method calls for the creation of a ”new milky way” of galaxies, consisting of double and triple images, all located along the string pattern.

The exact number of expected lensed object will be determined by the deficit angle and geometry of the string as well as by the density of the extragalactic objects. In what follows we shall consider only galaxies since the QSO contribution may be shown to be negligible of optical wavelengths.

In the case of a straight string one can easily estimate the expected number of lensed galaxies as

\[
\langle N \rangle = n_g 2\delta l
\]

(1)

here \( n_g \) is density of galaxies per unit solid angle, \( \delta \) is deficit angle of the string, and \( l \) is length of the string in the chosen field. Both \( \delta \) and \( l \) are expressed as angular measures.
A more sophisticated case emerges if the string is assumed to be curved \cite{Huterer2003}. A simple estimation can be derived as follows. The length of a curved string is larger in comparison to a straight one. Therefore strip of the string will cover a larger area on the sky in the same patch. The length can be written as  

\[ l = R \left( \frac{R}{l_c} \right)^a \]

Here  \( R = |\vec{r} - \vec{r}_1| \) is distance from the point  \( \vec{r} \) to the point  \( \vec{r}_1 \),  \( l_c \) is a correlation interval. The parameter  \( a \) varies inside 0 (straight string) and 1 (in the case of random walk of the string); the last value corresponding to purely brownian motion (\( R \sim \sqrt{l} \)). In the case (\( a=1 \)), the expected number of lenses can be estimated as follows

\[ \langle N \rangle = 2 \frac{\delta}{l_c} n_g \Omega \]

where the product of the angular area  \( \Omega \) of the patch, by the surface density of galaxies  \( n_g \) gives the multiplication factor  \( N_g \) which is number of galaxies in the patch.

In Paper I and in the present work we take the R OACDF frame as reference one. We therefore need to estimate the expected number of galaxies in the R band down to the OACDF limiting magnitude (\( \sim 24 \, m_R \)).

Deep counts in the R are practically absent in the literature. A moderately deep survey was obtained in the R band \cite{Kummel2001}. Other surveys were obtained (cf. \cite{Gardner1996}) of different wavelength and therefore need to be interpolated in order to adjust to our case.

Another source of information comes from very deep counts such as those derived by \cite{Thomson1999, Gardner1998} from the Hubble Deep Field. Also in this case however, there are no R band.

By taking into account all the above data it becomes apparent that galaxy counts have large uncertainty in the interval of magnitudes  \( 20 < m < 24 \) (with a possible multiplication factor of 10). If we use the \cite{Kummel2001} and extrapolate them to  \( m = 24 \) in the R band we obtain our estimated total number of galaxies in the interval  \( 20 < m_R < 24 \) are in our \( (16' \times 16') \) field of 2200. It roughly corresponds to number of extended sources in analyzed patch of the OACDF. We would stress that this estimate is in good agreement with the number of galaxies actually observed in the OACDF in the same magnitude range.

Therefore by using the above estimation in the case of straight string one can expect 9 lenses along the string, and in the case of a random string one can expect a much larger number: up to 200 lenses depending of the value of  \( a \).
In this patch one can expect also lenses produced by galaxies or conventional lenses. The mean density of lenses and its application to cosmology was discussed in Fukugita et al. (1992), Kochanek (1993), Chiba & Yoshi (1999), Ofek et al. (2003).

One can estimate the mean number of conventional lenses as the product of the optical depth due lensing per the number of galaxies in the field. These estimation (within the uncertainty interval) provide us with a mean number of \( \lesssim 2 \) conventional lenses in our patch.

\section{Criteria for Candidates Identification}

\subsection{Simulations}

We performed extensive simulations of the gravitational effects of a cosmic string on a realistic background galaxy distribution. The simulation was performed as follows. The field was chosen to be 1000 \times 1000 pixels in size with a pixel size of 0\textquoteleft\textquoteright 238 in order to match the scale of the OACDF. We stress, however, that the adopted scale has no relevant effects on the results of the simulation. The simulated field therefore covers a region of ca. 4\textquoteleft\times 4\textquoteleft and the simulated number of galaxies is slightly more than 100 (as expected from Thomson et al. (1999) and Gardner (1998) and confirmed by investigation of the OACDF).

Each galaxy was modeled assuming a de Vaucouleurs \( r^{1/4} \) surface brightness profile with an artificial threshold at 10 effective radii. The galaxies were then distributed in the field varying randomly both their positions and their total apparent magnitudes. Then we smoothed the simulated image with the measured OACDF point spread function, and finally we added gaussian noise with zero mean and the same variance as in the R band of OACDF.

The string was assumed to be straight and to cross the simulated field in a nearly vertical direction.

When the angular distance of a galaxy image from the string is smaller than the deficit angle of the string, a lensing event occurs and, as it was shown in Paper I, when the string happens to cross a galactic image, a triple source appears.

By comparing different simulated field we found on average 3 to 5 lensing events per field, which in some cases were triple ones. This result is consistent with the theoretical expectations from Eq. [1].
2.2 The case of OACDF

Then we visually inspected a 4000 x 4000 pixels subsection of the OACDF field centered on CSL1. Accordingly to the above estimates we expect to find at least 7-9 lens candidates (an higher number is expected for curved string geometry).

The OACDF is a deep field observed in four broad bands (U, B, V, and R) plus 6 narrow bands (effective wavelengths at 753 nm, 770 nm, 791 nm, 815 nm, 837 nm, 914 nm). The description of the OACDF and frames one can find in Alcalá et al. (2003).

In order to select gravitational lens candidates we assumed the criteria listed in Schneider, Ehlers, Falco (1992): i) two or more images with small angular separation and ii) the same flux ratio in different bands. We therefore searched for objects having angular separation inside the interval 1"- 4"5. The low boundary being determined by the resolution on the field, and the upper one resulting from the above described simulation.

Finally, we introduced the following estimator

\[ e_{RV} = 1 - \frac{I_R}{I_r} \cdot \frac{I_V}{I_v} \]

where \( I_R \) is flux in R band from the brightest object and \( I_r \) is flux in R band from the weakest object, \( I_V \) is flux in V band from the brightest object and \( I_v \) is flux in V band from the weakest object. In fact, we used 8 estimators \( e_{RB}, e_{R753}, e_{R770} \) etc in order to exploit the information contained in all available bands.

In the case of images produced by a gravitational lens each estimator should be equal to zero. But, due to the presence of experimental errors, in the real data, it may assume slightly different values. Therefore, in order to have a reliable estimate of the possible range of variation, we derived the estimator for CSL1, which we assume (from spectroscopy) to be a confirmed case of lens.

The estimator may be written as follows:

\[-2.5 \log(1 - e_{RV}) = m_{1R} - m_{2R} + m_{2V} - m_{1V}\]

Here \( m_{1R}, m_{2R}, m_{2V}, m_{1V} \) are the brightest and the weakest component in the R and V band, respectively. It is very easy to find a rough estimate of the errors on the estimator. As far as the estimator is the sum of 4 independent random values, the total error of the estimator is approximately twice the error on individual magnitudes, because of the fact that squares are additive values. Therefore the expected error on the estimators is approximately 20% of the original value.
The visual inspection of the OACDF produced a final list of 11 very likely candidates. This number is in good agreement with our cosmic string scenario.

Most of them are weak; magnitudes in R band ranging from 19 to 24 mag (as expected by Huterer & Vachaspati (2003)). We also wish to stress that all candidates having the brightest component brighter than 21 are also extended objects.

In Fig. 1 we give, as an example, the spectral energy distributions derived from OACDF photometry the N1 candidate. The shapes of the spectral distribution of all other candidates are very similar to the one shown in Fig. 1 within the experimental errors.

It has to be stressed, however, that while in the case of microlensing, the similarity of photometric colors is the main criterium, in extragalactic lensing, spectroscopic confirmation is needed.

3 THE ANGULAR SEPARATION VS MAGNITUDE DIFFERENCE TEST

Theory predicts that, in the case of lensing of a point source by a cosmic string, the angular distance between images should be roughly equal to the deficit angle, while in the case of lensing of a background extended object we expect a weak correlation between the distance of the photometric centroids and the magnitude difference of the two images. More in detail, in the case when a small part of the background extended object falls inside the lensing strip, the weak secondary image forms on the other side of the string. In this case the distance between the centroids of the images is the sum of the deficit angle and of the size of the object itself. Therefore, a correlation between the ratio of the fluxes and the angular distances between the centers should arise; the actual shape depending on the exact brightness distribution within the lensed object.
In Fig. 2 we plot the central peaks distance against the magnitude difference in the $R$ band for all our lens candidates. The solid curve gives the trend derived from our simulations, while diamonds refer to our candidates. The existence of a weak correlation for the candidates can be considered as an argument in favor of the cosmic string model.

**CONCLUSION**

In Paper I we discussed the strange properties of CSL1: a peculiar object discovered in the OACDF which spectroscopic investigations proved to be the double undistorted image of an elliptical galaxy. Always in Paper I we showed that CSL1 could be interpreted as the first case of lensing by a cosmic string.

In the present work, starting from consideration that a cosmic string is an elongated structure which produces non local effects we investigated the statistics of lens candidates around the CSL1 position.

The result of extensive simulations led us to expect a relatively high number of lenses: 7-9. A prediction which seems to be confirmed by the investigation of a 16′ × 16′ area centered on CSL1. By applying the objective estimator described in the text we found, in fact, 11 candidates which show the lens properties listed in Schneider, Ehlers, Falco (1992). This figure is much higher than what has to be expected from normal gravitational lens statistics.

It has to be stressed however that firm conclusions, will be shown only when spectroscopic confirmations of these candidates will become available.
ACKNOWLEDGMENTS

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ABSTRACT
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Key words: cosmic string; galaxies; general; cosmology: gravitational lensing.

1 INTRODUCTION
Cosmic strings in cosmology were thoroughly discussed over the past decades (cf. Zeldovich (1980), Vilenkin (1981)). Accordingly to Allen & Shellard (1990) their expected number should be large enough to make them observable but, so far, most attempts to detect their signatures seem to have failed. In a previous paper Sazhin et al. (2003) (hereafter Paper I) we discussed the strange properties of the gravitational lens CSL1 which, by photometric and spectroscopic investigation was shown to consist of two identical images of a giant elliptical galaxy at $z = 0.46$. The most relevant feature of these images is that their isophotes appear to
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The align of the background object (a galaxy) inside the deficit angle of the string is a stochastic process determined by the area of the lensing strip and by the surface density distribution of the extragalactic objects which are laying behind the string. The larger field of search we choose, the larger the number of lensed objects that we should find. All lensed objects will fall inside a narrow strip defined by the deficit angle computed along the string pattern. With an effective metaphor, we can say that this method calls for the creation of a ”new milky way” of galaxies, consisting of double and triple images, all located along the string pattern.

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In the case of a straight string one can easily estimate the expected number of lensed galaxies as

\[ \langle N \rangle = n_g 2\delta l \]  \hspace{1cm} (1)

here \( n_g \) is density of galaxies per unit solid angle, \( \delta \) is deficit angle of the string, and \( l \) is length of the string in the chosen field. Both \( \delta \) and \( l \) are expressed as angular measures.

A more sophisticated case emerges if the string is assumed to be curved Huterer &
Vachaspati (2003). A simple estimation can be derived as follows. The length of a curved string is larger in comparison to a straight one. Therefore strip of the string will cover a larger area on the sky in the same patch. The length can be written as Huterer & Vachaspati (2003):

$$l = R \left( \frac{R}{l_c} \right)^a$$

Here $R = |\vec{r} - \vec{r}_1|$ is distance from the point $\vec{r}$ to the point $\vec{r}_1$. The parameter $a$ varies inside 0 (straight string) and 1 (in the case of random walk of the string); the last value corresponding to purely brownian motion ($R \sim \sqrt{l}$). In the case ($a=1$), the expected number of lenses can be estimated as follows

$$\langle N \rangle = 2\frac{\delta}{l_{ch}} n_g \Omega$$

where the product of the angular area $\Omega$ of the patch, by the surface density of galaxies $n_g$ gives the multiplication factor $N_g$ which is number of galaxies in the patch.

In Paper I and in the present work we take the R OACDF frame as reference one. We therefore need to estimate the expected number of galaxies in the R band down to the OACDF limiting magnitude ($\sim 24 m_R$).

Deep counts in the R are practically absent in the literature. A moderately deep survey was obtained in the R band Kummel & Wagner (2001). Other surveys were obtained (cf. Gardner et al. (1996)) of different wavelength and therefore need to be interpolated in order to adjust to our case.

Another source of information comes from very deep counts such as those derived by Thomson et al. (1999), Gardner (1998) from the Hubble Deep Field. Also in this case however, there are no R band.

By taking into account all the above data it becomes apparent that galaxy counts have large uncertainty in the interval of magnitudes $20 < m < 24$ (with a possible multiplication factor of 10). If we use the Kummel & Wagner (2001) and extrapolate them to $m = 24$ in the R band we obtain our estimated total number of galaxies in the interval $20 < m_R < 24$ are in our ($16' \times 16'$) field of 2200. It roughly corresponds to number of extended sources in analyzed patch of the OACDF. We would stress that this estimate is in good agreement with the number of galaxies actually observed in the OACDF in the same magnitude range.

Therefore by using the above estimation in the case of straight string one can expect 9 lenses along the string, and in the case of a random string one can expect a much larger number: up to 200 lenses depending of the value of $a$. 

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One can estimate the mean number of conventional lenses as the product of the optical depth due lensing per the number of galaxies in the field. These estimation (within the uncertainty interval) provide us with a mean number of $\lesssim 2$ conventional lenses in our patch.

2 CRITERIA FOR CANDIDATES IDENTIFICATION

2.1 Simulations

We performed extensive simulations of the gravitational effects of a cosmic string on a realistic background galaxy distribution. The simulation was performed as follows. The field was chosen to be $1000 \times 1000$ pixels in size with a pixel size of $0^\prime.238$ in order to match the scale of the OACDF. We stress, however, that the adopted scale has no relevant effects on the results of the simulation. The simulated field therefore covers a region of ca. $4' \times 4'$ and the simulated number of galaxies is slightly more than 100 (as expected from Thomson et al. (1999) and Gardner (1998) and confirmed by investigation of the OACDF).

Each galaxy was modeled assuming a de Vaucouleurs $r^{1/4}$ surface brightness profile with an artificial threshold at 10 effective radii. The galaxies were then distributed in the field varying randomly both their positions and their total apparent magnitudes. Then we smoothed the simulated image with the measured OACDF point spread function, and finally we added gaussian noise with zero mean and the same variance as in the $R$ band of OACDF.

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By comparing different simulated field we found on average 3 to 5 lensing events per field, which in some cases were triple ones. This result is consistent with the theoretical expectations from Eq. 1.
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Then we visually inspected a 4000 \times 4000 pixels subsection of the OACDF field centered on CSL1. Accordingly to the above estimates we expect to find at least 7-9 lens candidates (an higher number is expected for curved string geometry).

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