Investigate the complexity of the control system of the Norwegian traffic light using Petri net model

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Abstract. The Norwegian traffic light is more complicated than the traffic signals of the standard type that applies three colors of signals. The sequence of the standard type is green, yellow/amber, red, and return to the green signal as the initial state becomes a cycle of a traffic light. The Norwegian traffic light has one more stage, i.e., red-yellow that lights simultaneously. This fourth stage suggests the driver should be prepared to start driving. This state is at the end of the red signal. The implementation of the Norwegian traffic light at real-world can reduce the time delay of traveling although we apply it without using seven-segments as the countdown of timer stop remaining. The modification of the Norwegian traffic light changes the fourth stage into a light yellow color only. This paper aims to investigate the structural complexity of the control system using its Petri net model. We use simulation and invariants as the method of validation and verification of the traffic light model. The structural complexity of the Norwegian traffic light control system is the highest and the second is its modification. The standard system is the lowest. The algorithm for calculating the structural complexity uses the backward and forward incidence matrix of the Petri net model.

1. Introduction

There are many kinds of traffic lights variance in the world. The standard type applies the sequence of the three color of lights i.e., green, yellow/amber, and red. In mathematical modeling using Petri net, each color is represented as a state/event of a traffic light [1]. The sequence must be able to return to the initial state become a cycle of traffic light [2].

Proper implementation of traffic light promises travels safety and efficiency [3]. It also offers the advantages of better time sharing to the vehicles passing in different directions of the arms of an intersection [4]. Traffic light schedule proposes some benefits while it is properly installed. It also can improve the performance of an intersection capacity. However, traffic lights can cause excessive delays when it is poorly designed [5]. The traffic lights are the representative of the main control for measuring the traffic flow that is implemented in the urban networks.

The Norwegian traffic light has one more stage if it is compared to the standard type, i.e., red-yellow that lights simultaneously [6]. This fourth stage advises the drivers to get ready to start driving. Its stage is held when the red signal will end soon. The implementation of the Norwegian traffic light is able to reduce the travel time delay. Although, the implementation without using seven-segments as
the countdown timer of the stop remaining. Furthermore, the modification of the Norwegian traffic light changes the fourth stage into a light yellow color only.

It is quite evident that the reliability of traffic control directly depends on the relevance of the employed control methodologies [3]. These become a major topic of ITS (Intelligent Transportation System) research now. The simple control system offers a standard set of traffic lights, while the complexity of the control methodology promises some additional features in accordance with the wishes of the road users. However, the control methodology with high complexity always has a positive correlation with the level of construction complexity and expensive maintenance costs [7].

Referring to the layout in the physical domain, the simulation model complexity is classified into three categories. It is considered the static structural complexity and software complexity (computational & algorithmic) [8]. The structural complexity regards to the simulation models complexity of the physical layout of the modeled objects and the existing of the connection among the represented elements [9]. The software complexity includes the operational complexity and control logic by program codes of the model system [9].

It is based on the reasons above. This paper aims to investigate the structural complexity of the control system of the Norwegian traffic lights via its Petri net model. It is compared to the standard type and its modification. This paper contributes to help the authorities to choose the best system for implementation in the real world. The devices for measuring the complexity of various systems that can be used by everyone have not yet been developed. It can implement the Petri net model

This study has modeled the dynamic behavior of traffic lights. It uses basic Petri net like one of several mathematical modeling languages to express the Discrete Event System (DES) of a distributed network [10]. A basic Petri net is a directed and bipartite graph. The time interval of a state is represented using the of many number of tokens that exist in a place and it is displayed implicitly.

Many previous research has written the traffic lights control systems or about Petri net modeling. Adzkiya D [1] talked the modeling of the traffic light and its simulation. Huang et al. [11], has written modeling and analysis of traffic light control systems. Taiping et al., [12] told us about the complexity of performance analysis of a model of a manufacturing system. Murata T., [13] talked about Petri net properties, analysis, and its applications. Cassandras C. G. and Lafortune S., [14] told us the Petri net definitions. Soares M., [15] explained the architecture of the traffic lights model. Jamila O. and Sanja P., [16] made a survey of the dynamic schedule of the manufacturing system. Wang J. and Gan R., [17] investigated model Petri net of the information system. None of them reviewed the complexity of the traffic light control system.

This paper is organized as the following. Chapter 1 is the introduction. Chapter 2 reviews the research method and the basic Petri net, chapter 3 speaks of the Norwegian traffic light model, and chapter 4 presents the discussion of the results. Finally, in chapter 5 is written conclusions.

2. Research method
In the following is the research method.
   a. First, we built the Petri net model.
   b. For the model validation and verification, the simulation and place-invariants were used.
   c. It measured the complexity of the model.
   d. It made the structural complexity comparisons.

For the simulation, Petri net Simulator 2.0 is used. This chapter also explains the Petri net model, invariants, simulations, and the formulas for measuring structural complexity.

2.1. The formal definition and properties of Petri net
The traffic signals light up in a fixed sequence as a loop cycle [2]. The sequence of the standard traffic light signals are green, yellow, and red. The dynamics of the signal behavior is manifested as a DES. Its Petri net model consists of three places, namely green, yellow, and red. A place represents a state of traffic lights. The presence of a token marks an active state.

Petri net was introduced to the public for the first time by C.A. Petri in the early 1960s. An event of Petri net is represented using execution by a transition. The input place must satisfy the initial state
which implies a transition is to be enabled and ready to execute/ fire. The state will change after the execution by a transition and it is expressed using tokens deposited at the output place [1].

Definition 1. Petri net is 4-tuple \( PN = (P, T, F, W) \), [13].

\( P \) is a finite set of places, \( P = \{p_1, p_2, \ldots, p_m\}, m \in \mathbb{N}_+ / \) positive natural numbers. \( T \) is a finite set of transitions, \( T = \{t_1, t_2, \ldots, t_n\}, n \in \mathbb{N}_+ \). \( F \) is an arc set (flow of relations), \( F \subseteq (P \times T) \cup (T \times P) \). \( W \) is as weight function, \( W : F \rightarrow \{1, 2, 3, \ldots\} \).

Definition 2. A marking Petri net is \( N = (PN, Mo) \).

\( Mo : P \rightarrow \mathbb{N}_+ \), is the initial marking. The Petri net design must be correct. Consequently, the model must satisfy a set of desirable properties [1], [13],[14],[15]. In the following are the properties.

a. Able to reach a marking \( Mn \): A marking of \( Mn \) can be reached from the initial marking \( Mo \) if there exists a sequence of transition firing that transforms from \( Mo \) to \( Mn. Mn \in \mathbb{R}(N,Mo) \). In the following are the definitions \( N = (PN, Mo) \).

b. Reversible: The Petri Net state must be able to return to the initial marking.

c. Bounded: The marking of Petri Net is k-bounded if the number of tokens in each place does not exceed k for any reachable marking from \( Mo \), k is a non-negative integer.

d. Safe: A Petri Net is safe if it is 1-bounded [13].

e. Live: The live Petri net guarantees deadlock free of operation [13].

Definition 3. The pre-set and post-set of the transitions are the set of the input places and output places of the transition \( t \). In the following are the definitions \( \tau = \{p \in P \mid W(p, t) \geq 0\} \) and \( \tau' = \{p \in P \mid W(t, p) > 0\} \), respectively.

Definition 4. The pre-set and post-set of places are the set of the input transitions and output transitions of places \( p \). The definitions are \( \tau' = \{t \in T \mid W(t, p) > 0\} \) and \( \tau = \{t \in T \mid W(p, t) \geq 0\} \).

Definition 5. [1] The incidence matrix is the forward incidence minus the backward incidence, which is \( A = A_f - A_b \). The backward incidence matrix \( A_b \) is a matrix of the arc weights that connect place to transition. The weight is zero if there is no connection. The forward incidence matrix \( A_f \) is a matrix that states the connection of the transition to place. \( A(i, j) = w(t_j, p_i) - w(p_i, t_j) \).

2.2. Structure complexity analysis of the control system

The complexity of the control system relates to the complexity characteristics of the control system model. The elements and structure of the model reflect its static characteristics. Meanwhile, the characteristics of the state and the transformation process describe the dynamic behavior of the system [16]. The complexity of the elements is expressed by \( C_n \) and the complexity of the correlation is presented by \( C_r \). The complexity of the structure is presented with \( C_s \). The sum of \( C_n \) and \( C_r \) uses the correlation coefficient \( \lambda \) raises \( C_s \) [17].

The correlation coefficient of the system complexity is the ratio of the incoming arcs and the outgoing arcs of a place to the total number of connecting arcs.

\[
\lambda = \frac{1}{|\tau| + |\tau'|} \sum_{x \in X} \left( \frac{\left| \tau_x \right| + \left| \tau'_x \right|}{|X|} \right) \quad \text{which } x \in X
\]

where \( |X| \) is the number of all set elements \( X \) those have pre-set and post-set. Both are not allowed to be empty. The complexity of the elements of a net Petri model is a function of many number of places and transitions. In this case, all elements in the traffic light model always have pre-set and post-set. None of their pre-set or post-set is empty as in the Petri net management model.

\[
C_n = \frac{1}{2} \left( \frac{|\tau| - 1}{|\tau|} + \frac{|\tau'| - 1}{|\tau'|} \right)
\]

In the Eq. (3) is the correlation complexity of the places set [17].

\[
C_r = \frac{1}{|\tau| + |\tau'|} \left( \frac{|\tau|}{|\tau|} + \frac{|\tau'|}{|\tau'|} \right)
\]
In the Eq. (4) is the correlation complexity of the transitions set.

\[ C_r = \frac{1}{|P|} \sum_{j} \left( |T_j| + |P| \right) \]  

(4)

The average value of correlation complexity of the places set and the transitions set is \( Cr \). The complexity of the structure of the Petri net model is \( Cs \).

\[ Cr = \frac{C_p + C_r}{2} \]  

(5)

\[ Cs = \lambda Cn + (1 - \lambda)Cr \]  

(6)

Table 1. The structural complexity of Petri net model

| The structural complexity (Cs) | The meaning of structural complexity |
|-------------------------------|-------------------------------------|
| 0                             | Minimum                             |
| 0 < Cs < 1/2                  | Simple                              |
| 1/2                           | Ideal                               |
| 1/2 < Cs < 1                 | Complex                             |
| 1                             | Maximum                             |

2.3. The graphical model of the traffic light, and its incidence matrices

The Norwegian traffic light has one more stage while it compared to the standard type, i.e. red-yellow signals that turn on simultaneously [5]. The Modification of the Norwegian traffic light changes the fourth stage into a yellow signal only.

![Figure 1](image)

**Figure 1.** The Petri net model of a single traffic light.

The red rectangles are the enable transitions. The black dots are tokens

a. The standard traffic light type.

b. The Norwegian traffic light. It has an extra place as the control place than the standard type.

c. The Modification of Norwegian traffic light. It has two extra places as the control place.

In Eq. (7) is the backward incidence matrix and the forward incidence matrix of a single signal of the standard traffic light type. In Eq. (8) is the matrices of a single signal of the Norwegian type and in Eq. (9) is \( A_b \) and \( A_f \) of the modification of the Norwegian traffic light.
The simple model of the traffic light schedule consists of two phases only. It applies the fixed time strategy. It based on the empirical data. The traffic light phase is a travel scheduling model on the different vehicles flow in an intersection to avoid conflict. For example, phase 1 meets to vehicles approach come from the south and north arms, while phase 2 stands for the traffic coming from east and west arms. It assumes that the traffic lights schedule has solved the conflict of the vehicles flow. The marking of the signal model can be 1 or 0. It represents that the signals are turned on or off.

The marking of the model

\[
A_{b-1} = \begin{bmatrix}
T_{11} & T_{12} & T_{13} \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

\[
A_{f-1} = \begin{bmatrix}
T_{11} & T_{12} & T_{13} \\
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
\end{bmatrix}
\]

\[
A_{b-2} = \begin{bmatrix}
T_{21} & T_{22} & T_{23} & T_{24} \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 \\
\end{bmatrix}
\]

\[
A_{f-2} = \begin{bmatrix}
T_{21} & T_{22} & T_{23} & T_{24} \\
0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 \\
\end{bmatrix}
\]

\[
A_{b-3} = \begin{bmatrix}
T_{31} & T_{32} & T_{33} & T_{34} \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
A_{f-3} = \begin{bmatrix}
T_{31} & T_{32} & T_{33} & T_{34} \\
0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

\[
M(G_i) + M(Y_i) + M(R_i) = 1 \quad i = 1,3 \quad and \quad j = 1,2
\]

**Figure 2.** Petri net model of two phases of traffic lights (a) Standard traffic light Petri net model (b) Norwegian traffic light (c) Modification of Norwegian traffic light.

2.4. Invariants and the simulation result as the model validation and verification

Traffic light schedule does not allow conflicting of the traffic movement. It must be able to serve all signal phases and be able to return to the initial state [1]. It used place-invariants and simulations to analyze the model [11]. The invariants guarantee that the firing of the enable transitions will not vary.

\[
M(G_i) + M(Y_i) + M(R_i) = 1 \quad i = 1,3 \quad and \quad j = 1,2
\]
Invariant (1) mentions that there is one signal only that lights up. There are green (G), yellow (Y) or red (R). It also stands for modification of Norwegian traffic light model of an arm and its synchronization of phases north-south traffic to east-west traffic.

\[ 2M(G_{21}) + M(Y_{21}) + M(R_{21}) + M(C_{21}) = 2 \]  

Invariant (2) asserts that the token available in four places \( G_{21} \) (Green), \( Y_{21} \) (Yellow), \( R_{21} \) (Red), and \( C_{21} \) (Control) of Norwegian traffic light model in Figure 1.b.

\[ M(G_{31}) + M(C_{31}) + M(R_{31}) + M(C_{31}) = 1 \]  

Invariant (3) stands for modification of Norwegian traffic lights in Figure 1.c. It indicates that there is one token exists either in the place \( G_{31} \) (Green), \( C_{31} \) (Control_31), \( R_{31} \) (Red), or \( C_{32} \) (Control_32).

The synchronized traffic light must build the travel schedule and it can avoid conflict flow for both north-south traffic (phase 1) and east-west traffic (phase 2). It is in the Invariant (4). S replace the controls intermediation places to create a sequence of the north-south phase and the east-west phase.

\[ M(G_{31}) + M(G_{21}) + M(S_{12}) + M(S_{21}) + M(Y_{12}) + M(Y_{21}) = 1 \]  

Invariant (5), invariant (6), and invariant (7) stand for two phases of the Norwegian traffic light.

\[ M(R_{21}) + M(R_{22}) = 1 \]  

\[ M(G_{22}) + M(G_{22}) + M(S_{22}) + M(S_{22}) = 1 \]  

\[ 2M(G_{3}) + 2M(G_{22}) + M(Y_{21}) + M(Y_{22}) = 2 \]  

Invariant (8) up to invariant (12) stand for the synchronized of two phases of modification of Norwegian traffic light. The intermediation places are not included in the invariants.

![Figure 3. The simulation result of synchronized of two phases traffic light](image)

From Figure 3, the simulation result of synchronized of two phases traffic light:

a. Norwegian traffic light. The legend \( R_{21} \) stands for red north-south, \( Y_{21} \) is yellow north-south, \( G_{21} \) is green north-south, and \( S_{22} \) is the intermediation. The other for the east-west phase.

b. Modification of Norwegian traffic light. The figure illustrates the original Petri net model simulation results. It is displayed without applying a time interval of green and red signals. The figure presents four cycles of the traffic light.

\[ M(R_{ij}) + M(R_{ik}) + M(Y_{ij}) = 1, \quad j,k = 1,2 \text{ and } j \neq k \]  

Invariant (8)

\[ M(R_{11}) + M(R_{22}) + M(Y_{11}) + M(Y_{22}) = 1 \]  

Invariant (9)
The simulation result of the Norwegian traffic light applies 63 seconds for a time cycle. A step in Fig. 3. represents for 3 seconds. The green signal time interval is twenty-four seconds, the yellow signal is three seconds, and thirty-six seconds as the time interval of the red signal.

3. Results and Discussion

3.1. Algorithms for Calculating the Structural Complexity

{Algorithm for calculating the structural complexity of a system}
{It based on the backward incidence matrix and the forward incidence matrix}

Variables:

| \( P \) | = m the number of places  
| \( T \) | = n the number of transitions  
| \( F \) | = the number of arcs  
| \( |X| \) | = the number of all set elements \( X \) of the system that have pre-set and post-set.  
| \( \lambda \) | = the correlation coefficient  
| \( C_n \) | = the elements complexity  
| \( C_r \) | = the correlation complexity  
| \( C_s \) | = the structural complexity.

Begin

**Algorithm 1.** Count the number of places and transitions

Begin

\[ m := \text{the number of row of the backward or forward incidence matrix} \]
\[ n := \text{the number of column of the backward or forward incidence matrix} \]

End

**Algorithm 2.a.** Count the number of elements via the backward incidence matrix

Begin

For \( j = 1 \) to \( n \) do

\[ \text{Ab}_P = 0 \]

For \( i = 1 \) to \( m \) do

If the element of the backward incidence matrix \( \text{Ab}(i, j) \neq 0 \) then \( \text{Ab}_P = \text{Ab}(i, j) \)

End_do

if \( \text{Ab}_P = 0 \) then the place \( P(i) \) has no post-set and \( P(i) \notin X \) and \( X_p = m - 1 \)

{at least, one element from the row \( i^{th} \) is not zero. The place \( P(i) \) must have at least a post-set}

End_do

For \( i = 1 \) to \( m \) do

\[ \text{Ab}_T = 0 \]

For \( j = 1 \) to \( n \) do

If the element of the backward incidence matrix \( \text{Ab}(i, j) \neq 0 \) then \( \text{Ab}_T = \text{Ab}(i, j) \)

End_do

If \( \text{Ab}_T = 0 \) then the transition \( T(j) \) has no pre-set and \( T(j) \notin X \) and \( X_t = n - 1 \)

{at least, one element from the column \( j^{th} \) is not zero. The transition \( T(j) \) must have at least a pre-set}

End_do

\[ X = X_p + X_t \]

End

**Algorithm 2.b.** Count the number of elements via the forward incidence matrix

begin

{at least, one element from the row \( i^{th} \) is not zero. The place \( P(i) \) must have at least a pre-set}

{at least, one element from the column \( j^{th} \) is not zero. The transition \( T(j) \) must have at least a post-set}

End
Algorithm 3. check the number of places P(i) and transitions T(j)
Begin {P(i) and T(j) must have both pre-set and post-set }
1. Check P(i) and T(j) that have no pre-set and post-set via the backward incidence matrix
2. Check P(i) and T(j) that have no pre-set and post-set via the forward incidence matrix
For i = 1 to m do
    If P(i) of the backward incidence matrix = P(i) of the forward incidence matrix then X = X +1
End_do
For j = 1 to n do
    If T(j) of the backward incidence matrix = T(j) of the forward incidence matrix then X = X +1
End_do
End
Algorithm 4. Count the correlation coefficient \( \lambda \).
Algorithm 5. Count the complexity of the elements \( C_n = \frac{2m.n - m - n}{2.m.n} \)
Algorithm 6. Count the complexity of the correlation of the Places that have post-set
Begin
    Calculate the ratio of the output arc to all arcs connected to a place. Both output and input
    {the arcs number as the output of a place P(i) = the sum of all elements of row i\(^{th}\) in Ab}
    {the arcs number as the input of a place P(i) = the sum of all elements of row i\(^{th}\) in Af}
End
Algorithm 7. Count the complexity of the correlation of the Transitions that have pre-set
Begin
    Calculate the ratio of the input arc to all arcs connected to a transition. Both output and input.
    {the arcs number as the input of a transition T(j) = the sum of all elements of column j\(^{th}\) in Ab}
    {the arcs number as the output of a transition T(j) = the sum of all elements of column j\(^{th}\) in Af}
End
Algorithm 8. Count the complexity of the correlation \( C_r \)
Begin
    Count the average of \( C_p \) dan \( C_r \)
End.
Algorithm 9. Count the complexity of the structure of the system
End.

The maximum arc weight is one. The elements of the backward and the forward incidence matrices are 0 or 1. The weight is zero means there is no arc connecting between the two elements.

The duration of all signals is not written in the model. It is usually attached to each place. Duration is the time interval for which a signal must be active. Several models use Place Timed Petri net.

Algorithm 3 is not required for calculating the complexity of traffic light structures. Because all places and transitions in the model always have pre-set and post-set. The traffic light signal is a model of repetition of three states continuously and infinitely. It has a predetermined sequence.

3.2. Structure complexity of control systems
Based on the reason that there is no single phase of the traffic light, both about the structure complexity of the single phase and its multi-phases are separated in Table 2 and Table 3.
Table 2. Structure complexity of single phase of traffic light signal

| Traffic light type             | |P| |T| |F| |X| λ  | Cn  | Cp  | C_T | Cr  | Cs   |
|-------------------------------|---|---|---|---|---|---|---|---|---|---|---|---|
| Standard                      | 3 | 3 | 6 | 6 | 1/3 | 2/3 | ½  | ½  | ½  | 0.55556 |
| Norwegian                     | 4 | 4 | 14 | 8 | 7/8 | 3/4 | ½  | ½  | ½  | 0.71875 |
| Mod. of Norwegian             | 5 | 4 | 12 | 9 | 24/81 | 31/40 | ½  | ½  | ½  | 0.58148 |

Table 3. Structure complexity of multi-phases of traffic light signal

| Traffic light type             | Standard Number of phases | Norwegian Number of phases | Mod. of Norwegian Number of phases |
|--------------------------------|---------------------------|---------------------------|----------------------------------|
|                                | [P] = m | [T] = n | [F] | [X] | λ  | Cn  | Cp  | C_T | Cr  | Cs   |
|--------------------------------|-------|-------|-----|-----|---|----|----|-----|----|-----|
| Standard                       | 8     | 6     | 14  | 20  | 0.16327 | 0.08163 | 0.20408 | 0.85417 | 0.10204   |
| Norwegian                      | 12    | 9     | 24  | 30  | 0.10884 | 0.08163 | 0.20408 | 0.90278 | 0.10542   |
| Mod. of Norwegian              | 16    | 12    | 28  | 40  | 0.08163 | 0.08163 | 0.20408 | 0.92708 | 0.10204   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |
|                                | 0.5   | 0.5   | 0.5 | 0.5 | 0.5139 | 0.5139 | 0.5139 | 0.5139 | 0.5139   |

Figure 4. The structural complexity of the standard type, Norwegian, and Modification of Norwegian traffic light signals.

Table 2 and Table 3 are visualized into Figure 4. The structure of all types of traffic light signals are complex and they are not the simple systems. The structural complexity of a simple system is less than 0.5. Referring to Figure 4, the structural complexity of a single traffic light signal of the Norwegian traffic light and its modification are high. The complexity of the structure decreases while the number of phases increases. The value approximates and convergent to 0.5. It is the complexity value of the ideal system.

The system is not suitable to implement if the structural complexity value is divergent. It moves and approaches to 1. The complexity increases if the number of phases of the traffic lights is large. This system is not appropriate to be applied.

The standard types have the lowest complexity when compared to Norwegian traffic light and the modifications of Norwegian type. The Norwegian traffic light has the highest structure complexity. In
the middle is the structural complexity of the modification of the Norwegian traffic light. This ranking has not changed even though the number of traffic light phases increases.

The Norwegian traffic light is the most complicated systems while it is compared to the other competitors. The standard system is the simplest. The complex systems offer several additional features, but the system is usually easily damaged. It requires higher maintenance costs and intensive checking from the experts. This does not happen in a simple system.

Its implementation depends on the local situation. It will not be a problem if a complicated system is implemented while the budget is widely available and the number of experts is abundant. The available features will be very useful for road users. On the contrary, if the budget is limited and the number of experts is small, you shall use a simple system. The modification of the Norwegian traffic light is the solution when urgent and the situation does not meet the requirements. The modification of the Norwegian traffic light offers more features than the standard and it does not meet the demands as required by the Norwegian system.

4. Conclusion
We can calculate the structural complexity of the control system of the Norwegian traffic light via its Petri net model. The Norwegian traffic light is the most complex while it is compared to the standard type and its modification. The lowest order is a standard traffic light structure. In the intermediate order between them is the modification of the Norwegian traffic light. The Norwegian traffic lights control system and its modification are the appropriate signals for the implementation in urban road net. The standard type also promises travel safety.

Future research is about devices using Petri net to measure structural complexity. Everyone can use it to measure the complexity of a sophisticated system or social organizational system.

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