Constraining unintegrated gluon distributions from inclusive photon production in proton-proton collisions at the LHC

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We compute the leading order (LO) $gg \to q\bar{q}\gamma$ and next-to-leading order (NLO) $gg \to q\bar{q}\gamma$ contributions to inclusive photon production in proton-proton collisions at the LHC. These channels provide the dominant contribution at LO and NLO for photon transverse momenta $k_{\perp}$ corresponding to momentum fractions of $x \leq 0.01$ in the colliding protons. Our computations, performed in the dilute-dense framework of the Color Glass Condensate effective field theory (CGC EFT), show that the NLO contribution dominates at small-$x$ because it is sensitive to $k_{\perp}$-dependent unintegrated gluon distributions in both of the protons. A $k_{\perp}$-factorization approximation to the full CGC EFT inclusive photon cross-section provides good agreement with data from CMS and ATLAS at center-of-mass energies of $2.76 \text{ TeV}$ and $7 \text{ TeV}$; $k_{\perp}$-factorization is broken in the CGC to $\sim 10\%$ due to coherent multiple scattering at small-$x$ for $k_{\perp} \leq 20 \text{ GeV}$. Such coherence effects are enhanced in nuclei to larger $k_{\gamma,\perp}$ and can be extracted from inclusive photon measurements in proton-nucleus collisions.

Photon production in high energy hadron-hadron collisions provides an excellent tool to probe the small-$x$ structure of hadron wavefunctions which are dominated by Fock states containing a large abundance of gluons. Their dynamics is described by the Color Glass Condensate (CGC) effective field theory (EFT) \textsuperscript{[1, 2]}. The dominant contribution to inclusive photon production at small-$x$, within the dilute-dense framework of CGC, is from the $qg \to q\gamma$ channel; it has been computed in several papers \textsuperscript{[3–6]}, with further applications to proton-proton (p+p) \textsuperscript{[7–9]} and proton-nucleus (p+A) collisions \textsuperscript{[10–15]}. Since the occupancy of gluons in the target is of order $1/\alpha_S$, our results in the limit of $\alpha_S$ are the QCD coupling, the quark from the proton scatters coherently off a gluon shockwave in the target. This channel is dominant in the fragmentation region where the hard photon is emitted off a large-$x$ valence quark scattering off the small-$x$ gluons in the target.

In \textsuperscript{[16]}, we computed the next-to-leading order (NLO) channel $gg \to q\bar{q}\gamma$ channel in the CGC EFT. For photon rapidities that are close to the central rapidity region of the collision, this process dominates over other contributions at this order \textsuperscript{[17, 18]}. It can be visualized as a fluctuation of a gluon from one of the protons into a quark-antiquark pair that scatters off the gluon shock wave of the other proton. Alternately, this gluon can first scatter off the shockwave before fluctuating into the quark-antiquark pair. In either case, the pair can emit a hard photon. If one probes small-$x$ values in either proton of $x \leq 0.01$, this NLO process will dominate over the stated LO contribution because the large gluon density in the proton overcompensates for the $\alpha_S$ suppression in the NLO cross-section arising from the splitting of the gluon into the quark-antiquark pair.

Our computation was performed within the dilute-dense approximation in the CGC EFT \textsuperscript{[19, 20]}, wherein one computes pair production (and subsequent photon emission) by solving the Dirac equation in the classical background field generated in the scattering process to lowest order in $\rho_p/k_{p,\perp}^2$ and to all orders in $\rho_t/k_{t,\perp}^2$. Here $\rho_p$ ($\rho_t$) are the color charge densities in the projectile (target) proton, and $k_{p,\perp}$ ($k_{t,\perp}$) are the associated transverse momenta. This approximation is strictly valid in the forward rapidity region where the momentum fraction $x_t$ of the parton from the “target” proton is much smaller than $x_p$, the momentum fraction of the parton from the “projectile” proton. Note that for these assumptions to be a priori robust, even the projectile parton should have $x_p \leq 0.01$. In our computations, we will cover kinematic regimes that will fall outside this preferred kinematic regime; the systematic uncertainties of the computation increase in that case due to the increased contributions of other channels and/or higher order effects. We note that the computation of heavy quark pairs $gg \to q\bar{q}$ in this framework (which, by Low’s theorem, is a limit of our results in the limit of $k_{\gamma,\perp} \to 0$) has been applied, with considerable success, to describe heavy quarkonium production in p+p collisions at RHIC and the LHC \textsuperscript{[21]}, in p+A collisions at both colliders \textsuperscript{[22–24]} and more recently, high multiplicity p+p and p+A collisions \textsuperscript{[25]}. In the latter case, the framework employed here also gives very good agreement with multiplicity distributions at the LHC \textsuperscript{[26]}.

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In this work, we will extend the comparison of the dilute-dense CGC EFT to LHC data to include single inclusive photon production in p+p collisions. There exist extensive published results from CMS and ATLAS on inclusive isolated photon production in p+p collisions at center-of-mass energies 2.76 TeV [27] and 7 TeV [28–30] for transverse momenta \( k_{\perp} > 20 \text{ GeV} \) at both mid- and forward rapidities. In the near future, 13 TeV results from the ATLAS and CMS will become available. It is further anticipated that the ALICE experiment will measure mid-rapidity photons at even lower \( k_{\perp} \). Especially promising too are forward LHC upgrades [31] such as LHCF [32] and the proposed ALICE FoCal [33]. The LHCf results for \( \sqrt{s} = 13 \text{ TeV} \) are available [34] for \( k_{\perp} > 200 - 300 \text{ GeV} \) which is outside small-x kinematics. This is also the case for the 13 TeV ATLAS data [35], where \( k_{\perp} > 125 \text{ GeV} \). We will here restrict our comparison to inclusive photon data from CMS and ATLAS for \( k_{\perp} < 50 \text{ GeV} \) where we expect the CGC computations to be reliable.

We begin by summarizing the CGC formulas for the LO and the NLO processes to explain our notations, approximations, and details of the numerical computation. The cross-section\(^1\) in the dilute-dense approximation of the LO process \( gg \rightarrow q\bar{q}(q_{\gamma}) \) in the dilute-dense collision is given by [3–6]

\[
\frac{d\sigma_{\text{LO}}}{d^2k_{\perp} \, d\eta_\gamma} = S_\perp \sum_f \frac{\alpha_s q_f^2}{16\pi^2} \int_{x_{p,\min}}^1 \! dx_p \, f_{q,f}^\dagger(x_p, Q^2) N_{l,Y_i}(q_{\perp} + k_{\gamma\perp}) \times \frac{1}{q^+ + t^+} \left\{ -4m_f^2 \left[ \frac{l^+ + (l \cdot k_{\gamma\perp})^2}{(q \cdot k_{\gamma\perp})^2} + \frac{q^2}{(l \cdot k_{\gamma\perp})(q \cdot k_{\gamma\perp})} \right] + 4 \left( l^2 + q^2 \right) \left[ \frac{l \cdot q}{(l \cdot k_{\gamma\perp})(q \cdot k_{\gamma\perp})} + \frac{1}{q \cdot k_{\gamma\perp} - 1} \right] \right\},
\]

(1)

where \( f_{q,f}^\dagger(x_p, Q^2) \) is the valence quark distribution function with \( Q^2 = (q_{\perp}^2, k_{\gamma\perp}^2) \). \( S_\perp \) is the transverse proton size, and \( m_f \) is the quark mass for flavor \( f \). The gluon shockwave in the dense target is represented by the dipole forward scattering amplitude,

\[
N_{l,Y_i}(k_{\perp}) = \frac{1}{N_c} \int_{x_\perp} e^{i(k_{\perp} \cdot x_\perp) \tau_c} \langle \bar{U}(x_\perp) U(0) \rangle_{Y_i}.
\]

(2)

In the above, the rapidity of the dense target is \( Y_i = \log(1/x_t) \), with \( x_t = \sqrt{2/s} (q^+ + k_{\gamma}^-) \) and \( \bar{U}(x_\perp) \) is a fundamental lightlike Wilson line. The light cone momenta of the incoming quark are \( l^+ = \sqrt{2/x_t} p^+ \) and \( l^- = m_f^2/(2l^+) \), those of the final state quark are: \( q^+ = l^+ - k_{\gamma}^+ \) and \( q^- = (q_{\perp}^2 + m_f^2)/(2q^+) \). Finally, those of the photon are \( k_{\gamma}^+ = k_{\gamma\perp} e^{\pm \eta_\gamma}/\sqrt{2} \).

We note that \( q^+ > 0 \) leads to \( x_p \geq x_{p,\min} \) with \( x_{p,\min} = \sqrt{2 k_{\gamma}^+}/\sqrt{s} \).

For inclusive photon production at NLO, as noted, there are three different channels in the gluon shockwave background of the target proton: \( gg \rightarrow qg \gamma \) [18], \( gg \rightarrow q^* q^* \rightarrow \gamma \) [17], and \( gg \rightarrow q\bar{q} \gamma \) [16]. The collinearly enhanced contributions in \( gg \rightarrow q\bar{q} \gamma \) are contained in the LO with evolved valence quark distributions, while the \( gg \rightarrow q^* q^* \rightarrow \gamma \) channel is suppressed by \( q^* \bar{q}^* \) phase space and flavor cancellation [17]. In the present work, we will consider the region close to mid-rapidity of \( 0 < Y_p < 2.5 \) where the \( gg \rightarrow q\bar{q} \gamma \) channel is the dominant contribution. The \( gg \rightarrow q\bar{q} \gamma \) channel, which may be expected to play an important role in the very forward region of the dilute projectile, will not be discussed in the following.

The inclusive cross section of the photon production from the \( gg \rightarrow q\bar{q} + \bar{q}(p) + \gamma(k_{\gamma}) \) channel can be expressed as [16],

\[
\frac{d\sigma_{\text{NLO}}}{d^2k_{\perp} \, d\eta_\gamma} = S_\perp \sum_f \frac{\alpha_s \alpha_s N_c^2 q_f^2}{64\pi^4 (N_c^2 - 1)} \int_{x_{p,\min}}^1 \! dx_p \int_{k_{\perp}}^1 \! k_{\perp} \frac{\varphi_p(Y_{p,t}, k_{1\perp}) N_{l,Y_i}(k_{\perp}) N_{l,Y_i} (P_{\perp} - k_{1\perp} - k_{\perp})}{k_{1\perp}^2} \times \left[ 2 \tau_{g,g}(k_{1\perp}; k_{1\perp}) + 4 \tau_{g,q\bar{q}}(k_{1\perp}; k_{1\perp}, k_{\perp}) + 2 \tau_{q\bar{q},q\bar{q}}(k_{1\perp}, k_{1\perp}; k_{\perp}) \right],
\]

(3)

where \( P_{\perp} = q_{\perp} + p_{\perp} + k_{\gamma\perp} \) and the rapidities are \( Y_{p,t} = \log(1/x_{p,t}) \) with

\[
x_p = \sqrt{2/s} (q^+ + p^+ + k_{\gamma}^+) \quad \text{and} \quad x_t = \sqrt{2/s} (q^- + p^- + k_{\gamma}^-)
\]

(4)

Here the light cone momenta of an on-shell particle with 4-momentum \( p \) are given by

\[
p^\pm = \frac{1}{\sqrt{2}} \sqrt{p_{\perp}^2 + m^2} \exp(\pm \eta_p).
\]

(5)

\(^1\) We use the following abbreviations: \( f_{q\perp} \equiv \int \frac{d^2q_{\perp}}{(2\pi)^2} \) and \( f_{x_{\perp}} \equiv \int d^2x_{\perp} \).
The unintegrated gluon distribution (UGD) in the dilute projectile $\varphi_p(Y_p, k_{1\perp})$ is defined as

$$\varphi_p(Y_p, k_{1\perp}) \equiv S_\perp \frac{N_c k_{1\perp}^2}{4\alpha_s N_p Y_p} N_p Y_p(k_{1\perp}),$$

where $N_p Y_p(k_{1\perp})$, the dipole amplitude is expressed in terms of the adjoint lightlike Wilson line $U(x_\perp)$ as

$$N_p Y_p(k_{1\perp}) = \frac{1}{N_c} \int_{x_\perp} e^{ik_{1\perp} x_\perp} \text{tr}_c(U(x_\perp) U^\dagger(0)) Y_p,$$

The product of fundamental dipoles in Eq. (3), to $O(1/N_c^2)$ in a large-$N_c$ expansion, represents general multigluon correlators describing the dense target; these too can be represented formally as UGDs [20].

The square brackets in Eq. (3) contain the hard factors for this process, where $\tau_{n,m}$ with $n,m \in \{g, q\bar{q}\}$ represents the Dirac trace,

$$\tau_{n,m} \equiv \text{tr}[(\gamma + m_f) T^{\mu}_n (m_f - \rho) \gamma_0 T^{\mu}_m \gamma_0],$$

with Dirac matrix products $T^{\mu}_n$ as specified in [16].

If $k_{1\perp}$ is much larger than the typical momenta exchanged from the dense target, namely $k_{1\perp}$ and $|P_{\perp} - k_{1\perp} - k_{\perp}|$, Eq. (3) simplifies to a $k_{\perp}$-factorized expression,

$$\begin{align*}
\frac{d^2 \sigma^{\text{NLO}}}{d^2 k_{\perp} \text{d} \eta_\gamma} &= S_\perp \sum_f \frac{\alpha_s N_c^2 q_f^2}{64 \pi^4 (N_f^2 - 1)} \int_{\eta_\gamma \eta_p} \int_{q_\perp p_\perp k_{1\perp}} \frac{\varphi_p(Y_p, k_{1\perp})}{k_{1\perp}^2} N_f, Y_f(P_{\perp} - k_{1\perp}) \\
&\times \left[ 2 \tau_{\gamma,g}(k_{1\perp}) + \tau_{q,q}(k_{1\perp}) + \tau_{\bar{q},\bar{q}}(k_{1\perp}) + 2 \tau_{q,\bar{q}}(k_{1\perp}) + 2 \tau_{\bar{q},q}(k_{1\perp}) \right],
\end{align*}$$

where $\tau_{n,m}$ takes the same form as in Eq. (8) for $n,m \in \{g, q, \bar{q}\}$ with the additional Dirac structures $T^{\mu}_g$ and $T^{\mu}_{\bar{q}}$ also specified as in [16].

It is crucial to note that we employ the valence quark distribution in Eq. (1). This explicitly defines our power counting of the dominant LO and NLO processes. If we use instead the quark distribution function including the sea quark contributions, this would mix up LO and NLO processes and therefore obscure the power counting. Therefore the flavor summation in Eq. (1) runs only over the valence $u$ and $d$ quarks, while the flavor summation in Eq. (3) and (9) runs over $u, d, s, c$ and $b$ quarks.

Prompt photon production includes both the direct photon component described by the above formulae as well as the contribution from fragmentation photons that we do not compute here. Experimentally, the two contributions can be separated by imposing an isolation cut along lines similar to that proposed in [36]; while this minimizes the fragmentation contribution, it does not eliminate it completely and this uncertainty is part of the quoted experimental systematic errors. We will adopt here the same isolation cut as used in the experiments to compare our results to the data. The above formulas must be convoluted with

$$\theta\left(\sqrt{(\eta_\gamma - \eta)^2 + (\phi_\gamma - \phi)^2} - R\right),$$

where $\theta(x)$ is the step function, $\eta, \phi$ are respectively the rapidity and the azimuthal angle of either $q$ or $\bar{q}$, while $\eta_\gamma$ and $\phi_\gamma$ denote the rapidity and the azimuthal angle of the photon. The CMS and the ATLAS experiments use $R = 0.4$, estimating the remaining fragmentation contribution to 10% of the total cross section [37]. We use $R = 0.4$ throughout this paper.

We will now present some of the numerical details in our computation of Eqs. (1), (3) and (9). For the valence quark distribution, we use the CTEQ6M set [38]. The small-$x$ evolution of the dipole distributions is obtained from the running coupling Balitsky-Kovchegov (rcBK) [39, 40], which is a good approximation to the general expression for the dipole forward scattering amplitude given by the Balitsky-JIMWLK hierarchy [39, 41–44]. In solving the rcBK equation numerically, the initial condition for the dipole amplitude at $x_0 = 0.01$ is given by the McLerran-Venugopalan (MV) model with anomalous dimension $\gamma = 1$, the saturation momentum at the initial $x_0$ of $Q_s^2 = 0.2$ GeV$^2$, and the IR cutoff for the running coupling $\Lambda_{\text{IR}} = 0.241$ GeV–see [45] for details of the rcBK initial conditions. With the initial condition fixed, the rcBK equation is solved to determine the dipole amplitude for $x < x_0$. For $x > x_0$, we use the extrapolation suggested in Ref. [21] wherein the UGD can be matched to the CTEQ6M gluon distribution.

\footnote{Hence, for the $gg \to q\bar{q}\gamma$ channel one needs to insert two step functions.}
The matching procedure fixes the proton radius $R_p$, to $R_p = 0.48$ fm, or equivalently $S_\perp = \pi R_p^2 = 7.24$ mb. Note that this value of $R_p$ is quite close to that extracted from saturation model fits to exclusive DIS data [46]. In our computations, we will take quark masses to be typically $m_u = m_d = 0.005$ GeV, $m_s = 0.095$ GeV, $m_c = 1.3$ GeV and $m_b = 4.5$ GeV. We will discuss later the effects of varying the parameters on model to data comparisons.

Evaluating the full CGC formula for the single inclusive photon cross-section as a function of photon transverse momenta $k_{\perp}$ and rapidity $\eta_\gamma$ in Eq. (3) involves performing 10-dimensional integrations while the simpler $k_{\perp}$-factorized approximation in Eq. (9) involves 8-dimensional integrations. Such multidimensional integrations are most efficiently performed by employing the VEGAS Monte Carlo (MC) algorithm. For the $k_{\perp}$-factorized integral, $10^8$ points were used to sample the approximate distribution of the integrand, until convergence with a significance of $\chi = 0.3$ was obtained. For the CGC calculations, we used the same algorithm but sampled the integrand with $10^9$ points. As a numerical check of our computation, we confirmed that in the small $k_{\perp}$ limit the NLO result reproduces the soft photon theorem—see Eqs. (B.7)-(B.11) in Ref. [16].

At low to moderate $k_{\perp}$, the full-CGC computation of the inclusive photon cross section based on (3) breaks $k_{\perp}$-factorization. This is also the case for inclusive quark production, as shown previously [47]. Our results for $k_{\perp}$-factorization breaking are shown in Fig. 1, where we plot the ratio of the full CGC inclusive photon cross-section to the $k_{\perp}$-factorized cross-section at $\sqrt{s} = 7$ GeV and $R = 0.4$. The results are plotted for central and forward photon rapidities, for individual flavor contributions, and for the net sum over flavors. The breaking of $k_{\perp}$-factorization is greater for forward rapidities and for decreasing quark mass, with negligible breaking of $k_{\perp}$-factorization observed for the heaviest flavor. Quantitatively, the breaking is maximally $\sim 10\%$ breaking at low $k_{\perp}$, approaching unity for $k_{\perp} \gtrsim 20$ GeV. As suggested by the discussion in [48], when $k_{\perp}$ is small, the quark-antiquark pair are more likely to both scatter off the gluon shockwave in the target; the $k_{\perp}$-factorized configuration, where multiple scattering of both the quark and antiquark does not occur, is therefore suppressed. As also suggested by Fig. 1, the reverse is true at large $k_{\perp}$.

Since the available data from the CMS and the ATLAS collaborations for p+p collisions at the LHC are for $k_{\perp} \gtrsim 20$ GeV, Fig. 1 clearly indicates that the $k_{\perp}$-factorized formula is a very good approximation for this momentum range. This is fortuitous since the computation involves fewer integrals than the full CGC expression. Therefore, for the rest of this paper, our numerical results are performed using the $k_{\perp}$-factorized formula in Eq. (9). Next, to illustrate the importance of the NLO ($gg \to q\bar{q}\gamma$) channel quantitatively relative to the LO ($gg \to q\gamma$) channel, we plot in Fig. 2 the NLO / (NLO+LO) fraction as a function of $k_{\perp}\gamma$. The left panel shows the collision energy dependence of the ratio for $\sqrt{s} = 0.2, 2.76, 7$ and 13 TeV with $\eta_\gamma = 1.0$. We observe that the NLO fraction of the inclusive photon cross-section at the highest RHIC energy of $\sqrt{s} = 0.2$ TeV is quite small, $\sim 10\%$. This is because, for the relevant $k_{\perp}\gamma$, quite large values of $x$ are probed in the proton where the gluon distribution does not dominate over that of valence quark distributions. However, already at $\sqrt{s} = 2.76$ TeV, the NLO contribution is more than 60\% even for the largest values of $k_{\perp}\gamma$ shown, and increasing the center-of-mass energy to $\sqrt{s} = 7$ TeV and 13 TeV enhances the NLO contribution to more than $\sim 90\%$. These results confirm that at LHC energies gluons dominate the proton wavefunction, even for photons with $k_{\perp} = 50$ GeV. The right panel shows the ratio for photon rapidities.
Before we discuss the comparison of our results to the LHC data, we will discuss the various sources of theoretical uncertainties. There is an overall degree of uncertainty in performing the Monte Carlo integrals, which is quantified by the error estimate of the VEGAS algorithm. This error estimate for the $k_\perp$-factorized inclusive cross-section is the range of 0 – 5% for all flavors. For the full CGC expression, which contains two more integrals, the errors are in the range of 5 – 15%. The inclusive isolated photon cross section (9) was calculated using the MC VEGAS algorithm as a function of $k_\perp$ and $\eta_\gamma$. The further integration over $\eta_\gamma$, required in the following, is calculated separately using the trapezoidal rule approximation, which gave a negligible error.

We should note, however, that there are other sources of uncertainty. We previously noted the $1/N_c^2$ corrections in using the BK truncation of the JIMWLK hierarchy. In practice, these have been observed to be significantly smaller, and may be especially so in the regime where $k_\perp$-factorization is applicable. Another source of systematic uncertainty are the values of the quark masses. Varying the quark masses in the ranges $m_u,d = 0.003 – 0.007$ GeV, $m_s = 0.095 – 0.15$ GeV, $m_c = 1.3 – 1.5$ GeV and $m_b = 4.2 – 4.5$ GeV, we observed that the cross section for $10$ GeV $< k_\perp < 50$ GeV varies by 5 – 10% for the light $u,d$, and $s$ quarks, while the heavier $c$ and $b$ quarks have small variations of order 0 – 5%. Based on these sources of uncertainty, we have included a systematic error band of 15% in comparisons to data. Further sources of uncertainty include higher order corrections to photon production as well as to the BK evolution. These are usually represented in the literature by a multiplicative constant “$K$-factor”. In our case, this is bundled together with the transverse area $S_\perp$. Though, as we noted, $S_\perp$ is constrained from the matching to parton distributions at large $x$, there can easily be 50% uncertainties in the overall cross-section that are absorbed by the extraction of the $K$-factor from comparison of the computed cross-sections to data. Until we can quantify the sources contributing to this $K$-factor separately, we should understand these sources of uncertainty as being “bundled” together in the value extracted.

After these preliminaries, we can now compare our LO+NLO results (employing the sum of Eqs. (1) and (9)) with the available LHC data. In Fig. 3, we show a comparison of our results to the $p+p$ 2.76 TeV CMS data on inclusive isolated photon production [27]. Likewise, the comparison to 7 TeV LHC data from CMS [29] and ATLAS [30] are shown in Fig. 4. For all the curves shown, a photon isolation cut $R = 0.4$ is applied. The CMS data for 7 TeV is presented in [29] as a rapidity bin averaged quantity. However, in Fig. 4, we multiplied this averaged quantity with the rapidity bin width $2 \Delta \eta$, to facilitate the comparison with ATLAS data. Note further that since the data sets integrate over symmetric intervals around the mid-rapidity regions, a factor of 2 was included in our comparison with data.

Our results describe both the 2.76 TeV and 7 TeV data quite well with a common $K$-factor of $K = 2.4$ corresponding to a best fit to the central values of the experimental data. We observe that this is very close to the $K$-factor of 2.5 extracted in computations of D-meson production in this dilute-dense CGC framework [25]. This is what one should expect since, as noted, the same channel as considered here, is the primary mechanism for charm pair production. In Fig. 5, using the same $K$-factor, we present our prediction for the inclusive photon cross-section at $\sqrt{s} = 13$ TeV with the same rapidity bins as shown in the previous plots.
FIG. 3. Comparison to the CMS p+p photon data at $\sqrt{s} = 2.76$ TeV [27]. The central line is obtained by multiplying our numerical results with a $K$-factor of $K = 2.4$. Here, and in subsequent plots, the band represents a 15% a systematic uncertainty of our calculation. See text for discussion.

FIG. 4. Comparison to the ATLAS and CMS p+p photon data at $\sqrt{s} = 7$ TeV [29, 30] across several rapidity bins. The central lines are obtained by multiplying our numerical results with a $K$-factor of $K = 2.4$.

We have presented in this work an important first step towards constraining the proton UGDs at small-$x$ from inclusive photon production at the LHC. We can summarize our results as follows. We have quantified the dominant contributions to inclusive photon production at LO and NLO. We found that the contribution of the NLO channel is significantly larger than the LO at central rapidities at the LHC. This is because for $k_{\perp} \leq 50$ GeV, the results are sensitive to small-$x$ values in the proton that have high gluon occupancy. We showed further that coherent rescattering contributions in the CGC that break $k_{\perp}$-factorization are of the order of 10%. The $k_{\perp}$-factorized framework gives good agreement with the CMS and ATLAS data, within the systematic uncertainties discussed above. Future publications will extend the analysis presented here to make predictions for p+A collisions and high multiplicity p+p and p+A collisions. Prior studies have only considered LO contributions to inclusive photon production. Another
FIG. 5. Predictions for the inclusive photon production at $\sqrt{s} = 13$ TeV across several rapidity bins. The central lines are obtained by multiplying our numerical results with a $K$-factor of $K = 2.4$.

important avenue where progress is required is in the computation of higher order effects which formally are NNLO in this approach but are essential to quantify running coupling corrections and for matching to results from collinear factorization computations at high $k_{\gamma \perp}$.

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