Impact of Anchoring Steel Mast Structure on the State of Stress and Strain in Existing Masonry Buildings

Anna Kucharczyk 1, Krystyna Urbanska 1
1 Faculty of Civil Engineering, Architecture and Environmental Engineering, University of Zielona Góra, Licealna Str. 9, 65-417 Zielona Góra, Poland
k.urbanska@ib.uz.zgora.pl

Abstract. The article presents analytical calculations of a connection between a mast structure and a masonry wall which take into account basic destruction mechanisms for both the connectors and the masonry element. Numerical simulations which illustrate the distribution of stresses and strains in the wall within the linear and nonlinear behaviour of the structure have been presented.

1. Introduction

The intensified development of mobile telephony makes the operators face the problem of providing their users with sufficient coverage. Antenna transmitters are usually mounted on detached structures such as masts and towers, made in steel or pre-stressed technologies, placed at considerable distances from cities, which may result in unsatisfactory coverage.

To solve this problem, mast structures with significantly lower height were started to be built on existing buildings or structures such as chimneys, towers, wind farms, silos, etc. Masts are most often mounted to reinforced concrete ceilings. If, for structural or technological reasons, it is not possible to mount the mast core to the floor, it is still possible to install the mast structure on a wall or on a coating of a chimney or a silo. The first solution requires the use of anchor bolts for a given type of material the wall is made of (brick, aerated concrete blocks, etc.), the second requires the use of perimeter beams. An example of a mast mounted to a brick wall may be a mast whose core is fixed to a wall of a church tower.

The article presents the results of analytical calculations taking into account the determination of the reaction values at the node where the mast structure is anchored to the brick wall. The authors also examined the load bearing capacity of this connection due to the shearing and the pulling – out of the connectors.

In addition, numerical simulations were presented to describe the behaviour of the masonry structure in the linear and nonlinear states. They illustrate the effect of the mast anchorage on the stress and strain state in individual components of the wall. The model used in the non-linear numerical analysis was based on the plastic-degradative destruction model for CDP (Concrete Damaged Plasticity) concrete, which is implemented in the ABAQUS system. It combines models based on the theory of plasticity and on the mechanics of destruction.
2. Analytical solution of anchoring a mast structure in a wall element

The connection of a mast structure with an existing brick wall should be considered as a connection between the mast core and the building. While examining the anchorage ultimate limit state, according to [1], various forms of destruction of both the connector and the wall should be considered. The mechanisms of destruction may be divided into two types – a mechanism in which the connector is pulled-out from the wall and the shear of the anchor. The following cases are regarded as destructions caused by the pulling-out of the anchor bolt: steel is destructed by breaking (figure 1a), destruction due to the pulling-out of the anchor (figure 1b), destruction of the wall due to a single anchor and a group of anchors (figure 1c) and destruction due to a brick or a block being pulled out (figure 1d). The destructions related to the shear of the connector include: the destruction of steel without taking into account bending (figure 1e) and a local destruction caused by a single anchor and by a group of connectors (figure 1f). If the anchorage is close to the corner of the wall, its damage should also be taken into account, together with the material of the wall (hollow or full blocks).

![Possible forms of destruction of steel anchors and brick walls](image)

A mast with a total height of 7.07m, made of round tubes performed with S235 steel anchored in a wall was analysed. Reactions in the node (table 1) were determined using the Autodesk ROBOT Structural Analysis program for the most unfavourable combination of interactions taking into account such loads of the mast structure as: own weight, equipment mounted on the mast, icing of the structure and the wind impact.

| Nsd  | Vsd,x | Vsd,y | Msd,x | Msd,y | MT,Sd |
|------|-------|-------|-------|-------|-------|
| [kN] | [kN]  | [kN]  | [kNm] | [kNm] | [kNm] |
| 59.232 | 1.681 | 9.016 | 3.039 | 0.076 | -0.085 |

The carried out calculations referred to a 2.80m high wall, made of solid bricks (compressive strength $f_c=35$MPa) and mortar (strength $f_z=12$MPa) (figure 2a), both determined from experimental studies [3]. The bottom mast plate, 250 x 250 x 20mm, was placed at a height of 0.3 m from the bottom edge of the wall. Anchor connectors were assumed as injection anchors for FIS A M16 wall structure with strength class 8.8., the anchorage depth was assumed $h = 100$mm. The spacing of the anchors in the anchor plate is shown in figure 2 b). A 15-cm-long circular pipe (88.9 x 5mm) was welded to the anchor plate to keep the mast core away from the wall.
Figure 2. a) View of fastening the anchor plate to the brick wall, b) spacing of the anchor connectors in the plate.

In order to examine the load bearing capacity of the connection in terms of the pulling-out and the shear of the anchor, the resultant forces acting on each connector were determined (table 2). The numbering of the anchors is shown in figure 2 b).

| Anchor No. | Pulling-out force [kN] | Shear force [kN] | Shear force x [kN] | Shear force y [kN] |
|------------|------------------------|-----------------|-------------------|-------------------|
| 1          | 23.47                  | 2.44            | 0.55              | 2.38              |
| 2          | 23.03                  | 2.20            | 0.55              | 2.13              |
| 3          | 6.59                   | 2.40            | 0.30              | 2.38              |
| 4          | 6.14                   | 2.15            | 0.30              | 2.13              |

2.1. Checking the load bearing capacity of the anchors and the wall for pulling-out

2.1.1. Checking the resistance in terms of destruction of the steel (the anchor is made of) by pulling out

\[ N_{sd} = 23,47 kN \leq N_{rd,s} = \frac{N_{rk,s}}{\gamma_{M,s}} = \frac{160}{1,5} = 106,67 kN \]  \hspace{1cm} (1)

\( N_{sd} \) – a design value of the force acting on the most stressed anchor,
\( N_{rd,s} \) – a design value of the resistance of the anchor for the pulling-out-caused steel destruction,
\( N_{rk,s} \) – a characteristic value of the anchor resistance for pulling-out-caused steel destruction,
\( \gamma_{M,s} \) – the partial tensile safety factor.

\[ N_{rk,s} = A_s \cdot f_{uk} = 200 \cdot 800 = 160000N = 160kN \] \hspace{1cm} (2)

\( A_s \) – the area of the connector active surface, \( f_{uk} \) – the characteristic strength of the bolt at tension.

\[ \gamma_{M,s} = \frac{1,2}{f_{yk} / f_{uk}} = \frac{1,2}{640 / 800} = 1,5 \] \hspace{1cm} (3)

\( f_{yk} \) – the characteristic plasticity border for the bolt.
2.1.2. Checking the resistance for destruction caused by pulling-out of the anchor

\[ N_{\text{sd}} = 23.47kN \leq N_{\text{rd},p} = \frac{N_{\text{rk},p}}{\gamma_{m,m}} = \frac{160}{2.5} = 64kN \]  

(4)

\( N_{\text{rd},p} \) – the design resistance of the anchor for the pulling-out-caused destruction,
\( N_{\text{rk},p} \) – the characteristic value of the resistance of the anchor for the pulling-out-caused destruction,
\( \gamma_{m,m} \) – the partial safety coefficient for destruction of the injection anchor (for masonry walls), amounting 2.5.

2.1.3. Checking the resistance for the destruction of the wall caused by a single anchor

\[ N_{\text{sd}} = 23.47kN \leq N_{\text{rd},b} = \frac{N_{\text{rk},b}}{\gamma_{m,m}} = \frac{160}{2.5} = 64kN \]  

(5)

\( N_{\text{rd},b} \) – the design value of the tensile resistance for of a single anchor in the event of the wall destruction,
\( N_{\text{rk},b} \) – the characteristic tensile resistance of a single anchor in the event of the wall destruction.

Bearing capacities \( N_{\text{rk},p} \) and \( N_{\text{rk},b} \) of the connectors are determined according to recommendations given in respective approvals. In calculations, load bearing capacity may be assumed as in formula (2).

2.1.4. Checking the resistance for the destruction of the wall caused by a group of anchors

\[ N_{\text{sd}} = 59.232kN \leq N^g_{\text{rd}} = \frac{N^g_{\text{rk}}}{\gamma_{m,m}} = \frac{640}{2.5} = 256kN \]  

(6)

\( N_{\text{sd}} \) – the design value of the pulling-out force,
\( N^g_{\text{rd}} \) – the design value of the resistance of a single anchor for destruction in the case when a group of anchors acts,
\( N^g_{\text{rk}} \) – the characteristic resistance of a single anchor for destruction in the case when a group of anchors acts.

\[ N^g_{\text{rk}} = N_{\text{rk},b} \cdot \alpha_N \cdot n = 160 \cdot 2 \cdot 2 = 640kN \]  

(7)

\( \alpha_N \) – ratio of the 5% tension force acting on a group of anchors (determined in experimental research) \( \leq 2.0 \), for two anchors situated along a line, \( n \) – is the number of connectors situated along a line.

2.1.5. Checking the load bearing capacity for the case when a brick is being pulled-out

\[ N_{\text{sd}} = N_{\text{sd}(1)} + N_{\text{sd}(2)} = 46.50kN \leq N_{\text{rd},pb} = \frac{N_{\text{rk},pb}}{\gamma_{m,m}} = \frac{138.875}{2.5} = 55.55kN \]  

(8)

\( N_{\text{sd}} \) – the sum of resultant forces acting on the anchors, placed in the same brick,
\( N_{\text{rd},pb} \) – the design value of load bearing capacity of a single / a group of anchors in the case when a single brick is being pulled-out,

\[ N_{\text{rk},pb} = 2 \cdot l_{\text{ceg}} \cdot b_{\text{ceg}} \cdot (0.5 \cdot f_{\text{v0}} + 0.4 \sigma_d) + l_{\text{ceg}} \cdot h_{\text{ceg}} \cdot f_{\text{v0}} = 138.875kN \]  

(9)

\( l_{\text{ceg}} \) – the brick length = 250mm, \( b_{\text{ceg}} \) – the brick width = 120mm, \( h_{\text{ceg}} \) – the brick height = 65mm, \( f_{\text{v0}} \) – the initial compressive strength of the wall, determined based on Tabl. 3.4 [2] = 0.3N/mm², \( \sigma_d \) – the design value of stresses resulting from loading the wall with the load of the ceiling = 5N/mm².
2.2. Checking the load bearing capacity of the anchors and the wall for shear

2.2.1. Checking the bearing capacity in the case of steel destruction regardless of bending

\[ V_{sd} = 2.44kN \leq V_{rd,s} = \frac{V_{rk,s}}{\gamma_{M,s}} = \frac{80}{1.5} = 53.33kN \]  

(Vsd – the design value of the shearing force acting on an anchor, 
Vrd,s – the design value of shear resistance for steel destruction regardless of bending 
Vrk,s – the characteristic load bearing capacity for steel destruction regardless of bending – assumed as 50% of the load bearing capacity NRk,s.

2.2.2. Local destruction of a brick due to a single anchor acting

\[ V_{sd} = 2.44kN \leq V_{rd,b} = \frac{V_{rk,b}}{\gamma_{M,m}} = \frac{80}{2.5} = 32kN \]  

(Vrd,b – the design value of the resistance of a single anchor in the case when a single brick is damaged, 
Vrk,b – the characteristic load bearing capacity for shear of a single anchor in the case when a single brick is damaged – assumed as 50% of the value of load bearing capacity NRk,s.

2.2.3. Local destruction of a brick due to acting of a group of anchors

\[ V_{sd} = 9.19kN \leq V_{rd} = \frac{V^{g}_{rk}}{\gamma_{M,m}} = \frac{320}{2.5} = 128kN \]  

(Vsd – the sum of resultant forces acting in the anchors, 
VRd – the design resistance for shear of the wall due to acting of a group of anchors, 
VRk – the characteristic resistance for shear of the wall due to acting of a group of anchors.

\[ V^{g}_{rk} = V_{rk,b} \cdot \alpha_{V} \cdot n = 80 \cdot 2 \cdot 2 = 320kN \]  

2.2.4. Pulling-out of a brick due to shear

\[ V_{sd} = V_{sd(1)} + V_{sd(2)} = 4.64kN \leq V_{rd,pb} = \frac{V_{rk,pb}}{\gamma_{M,m}} = \frac{134,375}{2.5} = 53.75kN \]  

(Vrk,pb – the characteristic load bearing capacity of an anchor / a group of anchors in the case when a single brick is being pulled-out due to shear.

\[ V_{rk,pb} = (2 \cdot l_{cegly} \cdot b_{cegly}) \cdot (0.5 \cdot f_{v,k0} + 0.4\sigma_d) = (2 \cdot 250 \cdot 125) \cdot (0.5 \cdot 0.3 + 0.4 \cdot 5) = 134,375kN \]  

(Vsd – the sum of resultant forces acting in anchors placed in one element of the wall.

The carried out analytical calculations indicate that the connection anchoring the mast to the wall satisfies all the load-bearing capacity conditions for pulling-out and shear. The largest strain occurs when the value of load bearing capacity for brick pull-out Nbd,pb amounts 84%. The remaining load bearing capacity conditions do not exceed 40%.
3. Modelling of a masonry structure

A part of a wall structure to which the steel mast is attached was modelled as a non-homogeneous material consisting of masonry elements (bricks) connected with mortar. Due to its two-component nature, consisting of regularly repeated components, the structure was modelled with the Finite Element Method. The micromodelling technique was used to analyse the masonry structure. Thus, the finite element is placed within one type of material. The condition of the brickwork structure after exceeding the stresses resulting in the formation of the first scratches is described with the use of a non-linear analysis based on the plastic-degradative model of concrete destruction proposed by [4] and developed by [5]. It combines elements of both the model based on the theory of plasticity and the model based on the mechanics of destruction.

It is based on two main mechanisms of destruction: cold cracking at tension and crushing at compression. Degradation of elastic properties of the material, in this case the reduction of stiffness, is described by two scalar destruction parameters: at tension – \( d_t \) and at compression – \( d_c \). The parameters take values from 0, which means no damage, to 1 - meaning total degradation.

Constitutive relations at tension and compression describing the plastic action of the material with simultaneous use of scalar measures of damage \( d \) are shown by the following relationship:

\[
\sigma_t = (1 - d_t)E_0(\varepsilon_t - \tilde{\varepsilon}_t^{pl}), \\
\sigma_c = (1 - d_c)E_0(\varepsilon_c - \tilde{\varepsilon}_c^{pl})
\]  

where \( \sigma_t, \varepsilon_t, \sigma_c, \varepsilon_c \) are the stresses, strains at tension and compression, \( \tilde{\varepsilon}_t^{pl} \) and \( \tilde{\varepsilon}_c^{pl} \) are equivalent plastic strains, whereas \( E_0 \) is the elasticity modulus of undamaged material. Expressing the constitutive equations by effective stresses \( \tilde{\sigma} \), allows joining the elastic-plastic model with the degradation model.

\[
\tilde{\sigma}_t = \frac{\sigma_t}{1 - d_t} = E_0(\varepsilon_t - \tilde{\varepsilon}_t^{pl}), \\
\tilde{\sigma}_c = \frac{\sigma_c}{1 - d_c} = E_0(\varepsilon_c - \tilde{\varepsilon}_c^{pl})
\]  

The characteristics of the model in the 1D state, i.e. the stress-strain relationship assumed in the calculations, is shown in figure 4.

![Figure 3. Relation between stress and strain under uniaxial loading a) in tension b) in compression](image-url)
In both cases, the reduction of wall load bearing capacity, and thus the assignment of a non-zero value to the destruction parameter, occurs after reaching a certain ultimate stress, i.e. for tension - after exceeding $\sigma_0$, which defines ultimate tensile strength and for compression - after exceeding $\sigma_{cu}$. The decrease in load bearing capacity is defined as tension softening (figure 3a) and compression softening, which occur, though after prior stiffening (figure 3b).

Figure 3 also shows equivalent plastic strains, which can be determined by the relationships presented below:

$$\tilde{\varepsilon}_t^{pl} = \varepsilon_t^{ck} - \frac{d_t}{1-d_t} \frac{\sigma_t}{E_0}, \quad \tilde{\varepsilon}_c^{pl} = \varepsilon_c^{in} - \frac{d_c}{1-d_c} \frac{\sigma_c}{E_0} \tag{18}$$

where $\varepsilon_t^{ck} = \varepsilon_t - \varepsilon_{0t}^{el}$ is a scratch - causing strain at tension, whereas $\varepsilon_c^{in} = \varepsilon_c - \varepsilon_{0c}^{el}$ is inelastic compression strain. $\varepsilon_{0t}^{el} = \sigma_t / E_0$ and $\varepsilon_{0c}^{el} = \sigma_c / E_0$ describe elastic deformations. The moment of scratch occurrence is determined by the plasticity surface.

The change in stiffness is characterized by the tensile $d_t$ and compression failure parameters $d_c$, which are non-decreasing functions of the strain state $d_t(\varepsilon_t^{ck})$ and $d_c(\varepsilon_c^{in})$, where $\varepsilon_t^{ck}$ is a scratch-causing strain at tension, whereas $\varepsilon_c^{in}$ is inelastic strain at compression. Parameters $d$ can take values from 0, for undamaged material, to 1 - for material with an open crack.

4. Numerical simulation of the connection between the mast and the masonry structure

The carried out analysis included a fragment of a brick wall with a steel mast fixed to it with an anchor plate. Numerical simulations were carried out for two cases of anchor bolts fastening. Case No. 1 presents a situation where all four bolts are placed in masonry elements (i.e. bricks) (figure 4a). Case No. 2 describes a situation where two upper bolts are placed in the wall joint (figure 4b).

![Figure 4](image-url)  
Figure 4. Masonry fragment in numerical analysis a) Case 1, b) Case 2 - description in the text

The purpose of numerical calculations was to analyse the strength of the selected fragment of the wall. Figure 5 presents the geometry of the wall fragment and the anchoring plate, MES digitisation (total number of elements: 185131- linear tetrahedral elements of type C3D4, total number of nodes: 35104) and the support conditions. The analysed wall fragment (dimensions 51 x 36.5 cm), was performed with full brick (25 x 12 x 6.5 cm). The masonry elements were joint with 1.0 cm thick mortar. The dimensions of the anchoring plate were: 25 x 25 cm.

The stress and strain state was induced in the wall by applying reactions to the plate anchoring the steel mast to the wall, calculated in point 2, and a load - to the upper surface of the wall with an intensity of 5 N/mm².

Calculations were performed with the non-linear analysis system Abaqus/Standard using the CDP model for the mortar material, which is based on the presented model of concrete destruction. The application of the model described in point 3 in the ABAQUS system requires: values of stresses in compression and tension describing the yield strength, uniaxial strength and biaxial compression strength. In addition, it is required to determine the area of plastic potential by giving the appropriate
parameters, i.e.: dilatation angle, eccentricity of hyperbola, parameter \( K_c \). In order to account for the destruction of material, it is necessary to give the rule of evolution of functions \( d_t \) and \( d_c \).

![Figure 5. Geometry of specimen, mesh and boundary conditions fragment of masonry](image)

However, in the implemented model, it is not possible to take into account anisotropic properties of the material, therefore in the presented example the wall is treated as an isotropic material. In addition, the perfect continuity between the brick elements and mortar was assumed. For the brick material, the elastic-plastic material characteristic was adopted, without taking into account the material softening mechanism. Table 3 provides information on the material parameters of the wall components. The data have been estimated based on our own experimental studies [3]. Poisson’s ratio for brick and mortar was based on literature [6, 7].

| Material parameters                  | Brick | Mortar |
|--------------------------------------|-------|--------|
| Young Modulus \( E \) [MPa]          | 2578  | 712    |
| Poisson Ratio \( \nu \)              | 0.2   | 0.25   |
| \( \sigma_{c0} \) [MPa]              | 28.0  | 9.6    |
| \( \sigma_{cu} \) [MPa]              | 35.0  | 12.0   |
| \( \sigma_{t0} \) [MPa]              | -     | 4.3    |
| Inelastic strain \( \varepsilon_{in} \) | -     | 0.02   |
| Scratch – causing strain \( \varepsilon_{t}^{ck} \) | -     | 0.002  |

In addition, the following parameters determining the shape of the plasticity surface were assumed for the mortar: \( \varepsilon = 0.1 \); \( \varphi \) (dilation angle) = 38°; \( \sigma_{b0} / \sigma_{c0} = 1.16 \); \( K_c = 0.667 \). It was also assumed that the steel anchor plate is made of a material with perfectly elastic-plastic characteristic. The following values were assumed for steel: Young Modulus \( E = 210 \) GPa, Poisson’s ration \( \nu = 0.3 \) and yield strength for the plate \( f = 225 \) N/mm\(^2\), for the anchors \( f = 640 \) N/mm\(^2\). The model assumes a full contact between the anchor bolts and the wall. In the applied computer program, this contact is modelled as "TIE". However, a "GENERAL" type contact was adopted between the plate and the wall, not taking into account the friction between the two elements.

The stress distribution maps, presented below, include both considered cases, i.e. the plate fixed into the bricks (Case 1) and the plate fixed into the mortar (Case 2). A slight increase, by 11.5%, in stress occurring in the wall area is observed for the bolts fixed into the mortar.
Figure 6. Distributions of stress $\sigma$ von Mises ($e+01=10$MPa) a) Case 1, b) Case 2.

The analyses of strain distribution reveal the concentration of stresses in the contact areas between the anchor bolts and the wall and in the mortar material, especially in the second case (figure 7).

Figure 7. Distribution of maximum principal elastic strains at final loading for Case 2

The figure presents the distribution of stress and strain in the brick wall, caused by loads which, according to calculations in point 2, should not cause damage to the wall components. However, it should be remembered that such mast structures are often fastened to existing objects. It is therefore necessary to assess the technical condition of the substrate and assess its strength. Lack of expertise may result in the formation of stresses exceeding or nearing the compressive or tensile strength of the wall components in the area of fastening.

The simulation results obtained with the assumption that the tensile strength of the mortar is only 10% of the compressive strength are presented below. The place of possible damage due to tension with the use of a scalar damage measure $d_t$ is presented on the basis of the DAMAGET parameter distribution map in figure 8.

Figure 8. Distribution of tensile damage variable $d_t$ at final loading

The damage is located in the mortar whose map of stress just before reaching the ultimate tensile strength is shown in figure 9a). When the tensile strength is reached, the process of destruction starts in the horizontal joints, followed by a decrease in stresses in the post – critical range, according to the assumed weakening function. Whereas figure 9b) presents the final picture of stress distribution $\sigma$ von Mises.
5. Conclusions
The analysis of the impact of a mast structure on the strength of a wall fragment at the anchorage place, presented in the article, has shown that even at small partition thicknesses, maximum strength of masonry elements does not exceed the allowable stresses. When this type of a structure is to be fastened to an existing building, the technical condition of the wall should always be assessed. It should be assessed on the basis of external observations and macroscopic examinations. Calculations based on data not compliant with the strength of masonry components may lead to an emergency condition. The numerical analyses performed, based on the obtained maps of stress and strain distributions as well as the on the maps of distributions of destruction parameters in individual incremental steps enabled observation of the most strenuous places. In addition, they have indicated the places where the structure starts to crack and the possible development of the crack depending on the place where the mast is fastened to the wall and on the adopted material parameters. It may be concluded that placing the anchor bolts in the wall joints does not have a significant impact on the load bearing capacity of the connection.

References
[1] ETAG 029 Guideline for European technical approval of metal injection anchors for use in masonry. Annex C: Design methods for anchorages, 2013.
[2] EN 1996-1-1. Eurocode 6 – Design of masonry structures – Part 1-1: General rules for reinforced and unreinforced masonry structures, 2009.
[3] K. Urbańska, “Analysis of masonry structures in the linear and nonlinear ranges”, PhD Thesis, Supervisor: M. Kuczma, Zielona Góra, December 2008. (in Polish).
[4] J. Lubliner, J. Oliver, S Oller., E. Onate, “A Plastic-Damage Model for Concrete”, Int. J. Solids Structures, vol. 25, pp. 299-326, 1989.
[5] J. Lee G. L. Fenves, “Plastic-damage model for cyclic loading of concrete structures”, Journal of Engineering Mechanics, pp. 892-900, 1998.
[6] S. Pietruszczak, X. Niu, “A mathematical description of macroscopic behaviour of brick masonry”, Int. J. Solids Structures, 29, No. 5, pp. 531-546, 1992.
[7] A. Anthoine, “Derivation of the in-plane elastic characteristics of masonry through homogenization theory”, Int. J. Solids Structures 32, No. 2, pp. 137-163, 1995.