Lateral radiative forces exerted by evanescent fields along a hyperbolic metamaterial slab

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Abstract. We show and investigate the optical forces acting on a particle in the vicinity of a planar waveguide which is filled with hyperbolic material and supports propagation across its plane (two-dimensional). The anisotropy axis of its medium lies in plane of the waveguide. In contrast to commonly considered pushing or pulling forces, acting in one-dimensional guiding structures, in the case of two-dimensional wave propagation, the angles between the momentum and the total energy flow may take any value around the circle. Accordingly, evanescent fields out of the slab exert lateral radiative forces on a nanoparticle oriented parallel to momentum being controllably different from the total energy flow direction. This provides a flexibility in manipulation by nanoparticles by employing suitably engineered hyperbolic structures.

1. Introduction

Laser beam manipulation of nanoparticles has found applications in bio-physics, micro-fluidics, and optomechanical devices. In literature, conservative (gradient) forces and non-conservative pushing and pulling forces exerted by electromagnetic fields have been mainly discussed. Especially the pulling forces have attracted particular interest due to its amazing property to act against the source of electromagnetic energy \cite{1}. In this context, hyperbolic metamaterials (HMMs) provide additional flexibility for controlling light propagation and enable construction of a platform supporting strong radiative pulling forces \cite{2}. However, in most of the papers devoted to such effects in HMMs, the forces are studied within a nanostructured metamaterial \cite{2, 3} hampering their experimental observation. Importantly, many works present the outcome of numerical simulations without explaining the reason for the appearance of the pulling forces in hyperbolic metamaterials.

In \cite{4}, the effect of the pulling force is explained in terms of partial energy flows propagating inside the HMM and outer space. Namely, the energy flow integrated over outer space, which is vacuum or isotropic dielectric, is always directed as the momentum. On the contrary, the energy flow integrated over the hyperbolic material cross-section, propagates in the opposite direction. Therefore, the total energy flow, i.e. integrated over full cross-section which includes HMM and external region, may be directed as the momentum or oppositely depending on the sign of waveguide dispersion. If one extends the aforementioned logic in the case of two-dimensional wave propagation, the angle between momentum and the total energy flow can take all possible values. Indeed, the energy flow in outer isotropic space is always parallel to the momentum, while the lateral radiative force on a neutral nanoparticle may be directed at an arbitrary angle with respect to the total energy flow or the group velocity.
2. Hyperbolic waveguide description and hybrid waves

A periodic metal-dielectric structure with the layers lying in the \((xz)\)-plane is considered as the hyperbolic metamaterial. Gold is assumed to be a metal with its relative permittivity defined by Drude formula in the optical range \([5]\). The components of the effective permittivity tensor are expressed as in \([6]\), with a fraction ratio of gold \(f = 0.05\) and under considered parameters: \(\varepsilon_{xx} < 0, \varepsilon_{zz} < 0, \varepsilon_{yy} > 0\). The external medium is vacuum, while the thickness of the HMM slab is taken equal to 250 nm.

In contrast to the case of one-dimensional wave propagation where the structure is isotropic in the \((xy)\)-plane \([4]\), the waves here are hybrid i.e. cannot be classified as TM and TE waves. Dispersion equation for hybrid waves in anisotropic slab, based on full-wave theory, has been derived using \(4 \times 4\) transfer matrix and solved numerically.

3. Waveguide dispersion and directions of momentum and energy flow

Fig. 1 illustrates waveguide dispersion in form of isofrequency contours on the map of the normalized wave vector components \((k_x/k_0, k_y/k_0)\), where \(k_0 = 2\pi/\lambda\) is the wavenumber in vacuum, calculated at the frequency \(\approx 231\) THz (\(\lambda = 1.3\) \(\mu\)m). In contrast to unbounded HMM there exists a countable number of modes with different field distribution along the the waveguide thickness, i.e. the \(z\)-axis. We show only three isofrequency loci to the lowest modes. High-order modes correspond to larger transverse components of the wave vector \(k_x, k_y\), see \([7]\). Thus,
several modes can be excited simultaneously by an external source, like a dipole that may make difficult their identification. However, the modes with large wavevectors have small propagation length and their influence disappears far from the source.

The vector of group velocity $v_g$ is perpendicular to the isofrequency curve at a point corresponding to the wavevector $k(k_x,k_y)$ and directed towards the increasing frequency. The group velocity has the same direction as the total energy flow or the Poynting vector integrated over whole cross-section. One can observe that in point 1 (corresponding to $k_y = 0$), the directions of the group velocity (the total energy flow) and momentum $k$ are opposite. It means that evanescent fields of these modes exert pulling forces on a particle placed nearby the HMM slab. At the same time, in point 2 the direction of the momentum (and the Poynting vector in outer space and the radiative force) forms an angle $\approx 90^\circ$ with the group velocity. Finally, at point 3, this angle is sharp tending to $180^\circ$ when $k_y \to 0$.

The radiative force, exerted on a dipolar particle, can be written as [8]:

$$\langle F \rangle = \frac{1}{4} \text{Re}\{\alpha \nabla |E|^2\} + \sigma \frac{1}{2} \text{Re}\{\varepsilon^2 \nabla \times \mathbf{H}^*\} + \sigma \frac{1}{2} \text{Re}\{i\varepsilon_0 k_0 (\mathbf{E} \cdot \nabla) \mathbf{E}^*\},$$

(1)

where $c$ is the speed of light and $\varepsilon_0$ is the permittivity of vacuum. The symbol $\alpha$ is used for the polarizability of a particle; in the case of a sphere with radius $r$ filled with material of relative permittivity $\varepsilon$ it is written as:

$$\alpha = \frac{\alpha_0}{1 - \alpha_0 k_0^2/(6 \pi \varepsilon_0)}, \quad \alpha_0 = 4 \pi \varepsilon_0 r^3 \frac{\varepsilon - 1}{\varepsilon + 2},$$

(2)

with $\sigma = k_0 \text{Im}\{\alpha\}/\varepsilon_0$. The first term in (1), related to the gradient forces, causes the attraction of the particle towards the slab interface due to the $z$-dependence of evanescent fields. In the second term, the lateral component of the Poynting vector exerts the force in $(xy)$-plane along the waveguide interface. The third term of (1) equals to zero for the evanescent field in a lossless isotropic medium as shown in [9]. Thus, the non-conservative dipolar force is proportional to the corresponding Poynting vector and it acts in the same direction.

In conclusion, we have demonstrated that not only pushing and pulling non-conservative forces can be observed in vacuum nearby anisotropic waveguide, but also forces directed at any angle to the group velocity (or, equivalently, the total energy flow).

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