Numerical simulations of impact Taylor tests

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Abstract. The finite element method is used for simulation of productivity in dynamic Taylor tests. Two different approaches for prediction of plastic deformation of finite element method is employed for numerical simulation of yielding under dynamic impact Taylor tests. Obtained results of modeling are compared with experimental ones. These are Johnson–Cook model and von Mises yielding criterion enhanced by incubation time approach. The simulation results have shown that the simplest method based on von Mises plasticity model provides good coincidence with experimental profiles of specimen shape in the course of deformation. The shortcoming is that the correct value of yield stress is depending on the loading rate and should be known beforehand. Thus, if there was a method to predict the value of dynamic yield stress to be used within von Mises criterion then this simple approach could be the optimal choice for simulation of dynamic plasticity in conditions of Taylor test.

1. Introduction

The behavior of the material is determined not only by its mechanical properties, but also by the intensity and speed of the load. It is well known that for many quasistatic problems plastic deformation of metals can be successfully predicted utilizing the classical von Mises model. There is a critical stress value (usually denoted as $\sigma_y$), so that for equivalent stress values exceeding this limit, a developed plastic deformation is expected. However, if the load is applied at a high-rate and intensity, the available experimental data testifies that the material can withstand stresses exceeding for a short period of time and yielding is initiated at stress level higher then. This limiting stress is usually called the dynamic yield stress $\sigma_y^d$.

The Taylor test is one of the simplest approaches to study yielding in materials undergoing intensive dynamic loading [1]. Originally the approach was suggested to determine critical yield stress $\sigma_y^d$ in dynamic conditions for soft metals, for example, aluminum or copper [2]. Typically, this experimental scheme provides a possibility to measure dynamic yield stress for very high level of strain and strain rate $\sim 10^5 \text{ s}^{-1}$.

In these experiments, cylindrical rod-shaped projectile impacts to a rigid anvil at a high velocity. The summoned shortening of the specimen provides a possibility to estimate dynamic yield limit of the material tested. There also exists a more complicated experimental scheme of symmetrical Taylor test where the specimen impacts exactly the same specimen, ensuring
contact rigidness due to the problem symmetry. However, thought the implementation of this experimental scheme leads to difficulties securing normal impact and alignment of specimen axes, this scheme has a number of advantages. The first advantage consists in absolute rigidity of contact, requested by the subsequent theoretical analysis of experimental results. Moreover, the symmetrical Taylor test removes the problems associated with frictional constraints at the interface between the specimen and the target. Another benefit of symmetrical Taylor test is a possibility of direct measurement of temporal dependency of specimen strain by the gauge fixed on the target sample. Methods of high-speed photography and optical interferometry provide data about intermediate deformation stages for both of the possible experimental schemes. Thus, Taylor tests and symmetrical Taylor test can provide extensive data about high-strain rate material deformation to be used for validation of constitutive material models.

A more detailed analysis of the experimental results, for example, measuring the mushroom-shaped profile of a deformed sample or studying the evolution of deformation using high-speed photography, initiated the development of new approaches and models for describing the various phenomena observed during dynamic plasticity of materials. Among them one can mention dislocation and twining approaches [3], Johnson–Cook (J–C) flow stress model and its modifications [4], Zerilli–Armstrong model [5], Mechanical Threshold Stress model [6]. The usual disadvantage of these approaches is a big number of model parameters and values that in most cases have no clear physical meaning and should be evaluated by fitting the theoretical to the experimental measurements. The big advantage of the traditional von Mises approach is the simplicity of the analysis and rather accurate prediction of the observed phenomena, should there be a proper way to calculate dynamic yield stress as a function of the loading rate. This paper is an attempt to propose an approach for calculation of dynamic yield stress based on the incubation time criterion originally proposed by N Morozov and Y Petrov for prediction of dynamic brittle fracture (e.g., [7]).

2. Numerical modeling

The finite element method (FEM) is a powerful tool for studying the problems of solid mechanics with complex geometry and the possibility of introducing complex rheological properties of the material. FEM was applied to simulate Taylor test utilizing commercially available ANSYS software [8]. To initiate the work, the two widely used material models of plasticity were chosen: the traditional von Mises approach and the Johnson–Cook flow stress model. The following equation describes the yielding process according to the Johnson–Cook model to be used within numerical simulation

$$\sigma_{eq} = [A + B(\bar{\varepsilon}^p)^n][1 + C \ln(\dot{\varepsilon}^*)][1 + (T^*)^m],$$

where $\sigma_{eq}$ is the equivalent stress; $A$, $B$, $C$, $n$ and $m$ are material constants; $\bar{\varepsilon}^p$ is the accumulated plastic strain; $\dot{\varepsilon}^*$ and $T^*$ are the dimensionless strain rate and temperature.

2.1. Problem statement

The axial symmetry of the sample allows us to consider a two-dimensional planar problem. Quad node plane elements are utilized. The model is presented in figure 1. The anvil is simulated as hardened maraging steel circular bar with sizes chosen to be large enough to avoid influence of the reflected waves on the specimen during the contact time. Thus, the anvil thickness is approximately 2.5 times larger as comparing to the specimen length and can vary from test to test. An initial gap between the specimen and the anvil is equal to the linear size of a finite element within the sample. Symmetrical boundary condition is satisfied by restricting the horizontal displacement for central specimen axis and vertical displacement for the rear anvil surface. Mechanical properties of the specimen material and the initial velocity are taken
Figure 1. Typical (a) initial and (b) deformed shapes of the body is meshed using four-node plane linear elements.

according to data presented in [2, 9]. Dynamic yield stress of von Mises model is determined by fitting the final specimen length between the simulation and experiment.

2.2. Verification
A comparative numerical analysis of various approaches was carried out to determine the most applicable method for predicting the dynamic fluidity process when testing metal samples by Taylor. Modeling results are compared to experimental data of Taylor tests for oxygen-free electronic (OFE) copper [9] where specimens were 75 mm length rods with length to diameter ratio of 4 : 1. Numerical simulations using J–C model are compared to the results received utilizing the traditional von Mises approach.

Experimental results [9] are giving the specimen side profile for three different time instances (figure 2). The utilized model parameters (as suggested in [9]) are given in table 1. As can be seeing in figure 2, the Johnson–Cook model does not provide good correspondence to the real sample profiles while the predictions utilizing the traditional von Mises approach provide a much better coincidence with experimental data in terms of the final sample length and the final diameter of the specimen contact surface. Thus, if the dynamic yield stress is given then the von Mises approach is applicable for predictions of dimensions of the final specimen shape.
Table 1. Values of parameters for simulation of tests [9] using the Johnson–Cook \((A, B, C, m, n)\) and Von Mises \((\sigma_y^d)\) models.

| \(A\) (MPa) | \(B\) (MPa) | \(C\) (MPa) | \(m\)  | \(n\)  | \(\sigma_y^d\) (MPa) |
|-------------|-------------|-------------|------|------|-------------------|
| 195         | 291.64      | 0.025       | 1.09 | 0.31 | 348               |

\[ V = 83 \text{ m/s} \]

Figure 2. Instrumental impact tests on the Taylor anvil to confirm the applicability of plasticity models under conditions of transitional deformation with experimental data [9].

The conditions of tests conducted by House [2] are also simulated to verify the applicability of von Mises approach for simulations of Taylor experimental scheme. They performed Taylor test on Oxygen Free High Conductivity (OFHC) copper rods of constant 7.57 mm diameter having several different lengths. Numerical simulation is performed the diameter/length ratio equal to 2.5. Thereof dynamic yield stress as a function of impact velocity was fitted from condition of correspondence of the final projectile length between the real experiments and simulations. The obtained data is presented in table 2.

As can be seeing in table 2, the coincidence between experimental data and simulated results employing the von Mises approach with fitted yield stress is rather accurate. In addition, it should be mentioned that the dynamic yield stress expectedly grows with the increase of the projectile velocity. High rates of impact stipulate a linear increase of sample deformation (figure 3), hence it is possible to calculate the average constant strain rate of load for each test. Typical projectile length as a function of time received in simulations is shown in figure 3. The straight segment (approximated with the dashed line) corresponds to deformation with constant strain rate and can be easily approximated within the simulation analysis.
Table 2. Experimental data [2] and simulated values of length alternation and dynamic yield stress.

| Impact velocity (m/s) | $\Delta t_{\exp}$ (mm) | $\Delta t_{\text{sim}}$ (mm) | $\sigma_y^d$ (MPa) |
|-----------------------|-------------------------|------------------------------|-------------------|
| 79.43                 | 0.9843                  | 0.984                        | 234               |
| 136.49                | 2.4067                  | 2.407                        | 263               |
| 168.25                | 2.2289                  | 2.23                         | 421               |
| 190.84                | 2.4829                  | 2.484                        | 475               |
| 255.94                | 3.2449                  | 3.245                        | 613               |

Figure 3. Typical temporal dependence of a projectile length.

2.3. Incubation time approach

Thus, utilizing the experimental data from [2] and numerical simulations, it is possible to obtain strain rate dependency of the dynamic yield stress. The received data points can be analyzed according to the incubation time approach [7]. The general form of the incubations time criterion for yield is

$$\frac{1}{\tau} \int_{t-\tau}^{t} \sigma(s) ds \leq \sigma_y,$$

where $\sigma_y$ is the quasi static yield stress and $\tau$ is the incubation time of plastic flow initiation. The observed linear law of loading can be represented by $\sigma(t) = H(t)E\dot{\varepsilon}t$, where $E$ is the Young modulus, $H(t)$ is the Heaviside step function and is the average strain rate of the impact load. Substitution of load pulse into the criterion gives an analytical expression for dynamic yield stress as a function of strain rate

$$\sigma(\dot{\varepsilon}) = \begin{cases} \sigma_y + E\tau\dot{\varepsilon}/2, & \text{if } \dot{\varepsilon} \leq 2\sigma_y/(E\tau); \\ \sqrt{2\sigma_y\tau E\dot{\varepsilon}}, & \text{if } \dot{\varepsilon} > 2\sigma_y/(E\tau). \end{cases}$$

Figure 4 demonstrates analytically received dependencies of dynamic yield stress versus load strain rate in comparison to experimental points discussed above. As follows from the performed
Figure 4. Strain rate dependency of dynamic yield stress calculated according to incubation time approach $\tau \in [160; 290] \, \mu s$. Black squares are experimental data [2].

analysis (see figure 4), for the values of incubation time close to $\tau = 220 \, \mu s$ it is possible to receive dynamic yield stress as a function of strain rate that will give predictions of the deformed sample geometries similar to the ones observed experimentally.

3. Conclusions
Considered plasticity models of von Mises and Johnson–Cook provide good correspondence to experimental measurements. The simple von Mises approach can be successfully utilized to predict plasticity within numerical computational schemes for simulation of dynamic Taylor impact tests, should there be a method to determine strain rate dependency of dynamic yield stress. The incubation time approach can be used for calculation of the correct strain rate dependency of the yield stress.

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References
[1] Taylor G I 1948 Proc. R. Soc. Lond. A 194 289–99
[2] House J W 1989 Taylor impact testing Preprint (Kentucky University Lexington)
[3] Borodin E N and Mayer A E 2015 Int. J. Plast. 74 141–57
[4] Johnson G R 1983 Proc. 7th Int. Symp. on the Ballistics pp 541–7
[5] Zerilli F J and Armstrong R W 1994 AIP Conf. Proc. 309 989–92
[6] Follansbee P S and Kocks U F 1988 Acta Metall. 36 81–93
[7] Gruzdkov A A, Sitnikova E V, Morozov N F and Petrov Yu V 2009 Math. Mech. Solids 14 72–87
[8] Kohnke P (ed) 2013 ANSYS Mechanical APDL Theory Reference (Canonsburg, PA: ANSYS Inc)
[9] Eakins D E and Thadhani N N 2006 J. Appl. Phys. 100 073503