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An application of the continuous Steiner symmetrization to Blaschke-Santaló diagrams.

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Summary: In this paper we consider the so-called procedure of Continuous Steiner Symmetrization, introduced by F. Brock [Proc. Indian Acad. Sci., Math. Sci. 110, No. 2, 157–204 (2000; Zbl 0965.49002); Math. Nachr. 172, 25–48 (1995; Zbl 0886.49010)]. It transforms every open set \( \Omega \subset \subset \mathbb{R}^d \) into the ball keeping the volume fixed and letting the first eigenvalue and the torsional rigidity respectively decrease and increase. While this does not provide, in general, a \( \gamma \)-continuous map \( t \mapsto \Omega_t \), it can be slightly modified so to obtain the \( \gamma \)-continuity for a \( \gamma \)-dense class of domains \( \Omega \), namely, the class of polyhedral sets in \( \mathbb{R}^d \). This allows to obtain a sharp characterization of the Blaschke-Santaló diagram of torsion and eigenvalue.

MSC:

49Q10 Optimization of shapes other than minimal surfaces
49J45 Methods involving semicontinuity and convergence; relaxation
49R05 Variational methods for eigenvalues of operators
35P15 Estimates of eigenvalues in context of PDEs
35J25 Boundary value problems for second-order elliptic equations
26D10 Inequalities involving derivatives and differential and integral operators

Keywords:

Blaschke-Santaló diagrams; continuous Steiner symmetrization; torsional rigidity; principal eigenvalue; 0886.49010

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