Effects of $A_t$ and $\mu$ phases on $B$ and $K$ physics
in the effective supersymmetric models

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In the minimal supersymmetric standard model (MSSM), the $\mu$ parameter and the trilinear coupling $A_t$ may be generically complex and can affect various observables at B factories. Working in the effective SUSY models and imposing the edm constraints from Chang-Keung-Pilaftsis (CKP) mechanism, we find that (i) there is no new large phase shift in the $B^0 - \bar{B}^0$ mixing from the $A_t$ and $\mu$ phases, (ii) CP violating dilepton asymmetry is smaller than 0.1%, (iii) the direct CP violation in $B \to X_s \gamma$ can be as large as $\sim \pm 16\%$, (iv) the $\text{Br}(B \to X_s l^+ l^-)$ can be enhanced by up to $\sim 85\%$ compared to the standard model (SM) prediction, and its correlation with $\text{Br}(B \to X_s \gamma)$ is distinctly different from the minimal supergravity scenario. Also, we find $1 \leq \epsilon_K/\epsilon_{K}^{SM} \leq 1.4$, and $\epsilon_K$ cannot be saturated by the $A_t$ and $\mu$ phases alone: namely, $|\epsilon_K^{SUSY}| \leq O(10^{-5})$ if the phases of $\mu$ and $A_t$ are the sole origin of CP violation.

1 Introduction

1.1 Motivations

In the minimal supersymmetric standard model (MSSM), there can be many new CP violating (CPV) phases beyond the KM phase in the standard model (SM). These SUSY CPV phases are constrained by electron/neutron electric dipole moment (EDM) and have been considered very small ($\delta \leq 10^{-2}$ for $M_{SUSY} \sim O(100) \text{ GeV}$). However there is a logical possibility that various contributions to electron/neutron EDM cancel with each other in substantial part of the MSSM parameter space even if SUSY CPV phases are $\sim O(1)$. Or one can consider effective SUSY models where decouplings of the 1st/2nd generation sfermions are invoked to evade the EDM constraints and also SUSY FCNC/CP problems. In such cases, these new SUSY phases may affect $B$ and $K$ physics in various manners. Closely related with this is the electroweak baryogenesis (EWBGEN) scenario in the MSSM. One of the fundamental problems in particle physics is to understand the baryon number asymmetry, $n_B/s = 4 \times 10^{-12}$, and currently popular scenario is EWBGEN in the MSSM. The EWBGEN is in fact possible in a certain region of the MSSM parameter space, especially for light stop ($120 \text{ GeV} \leq m_{\tilde{t}_1} \leq 175 \text{ GeV}$, dominantly $\tilde{t}_1 \simeq \tilde{t}_R$) with CP violating phases in $\mu$ and $A_t$ parameters. Then
one would expect this light stop and new CP violating phases may lead to observable consequences to $B$ physics.

In this talk, we report our two recent works related with this subject. We considered a possibility of observing effects of these new flavor conserving and CPV phases ($\phi_\mu$ and $\phi_{A_t}$) at $B$ factories in the MSSM (including EWBGEN scenario therein). More specifically, we consider the following observables: SUSY contributions to the $B^0 - \overline{B^0}$ mixing, the dilepton CP asymmetry in the $B^0\overline{B^0}$ decays, the direct CP asymmetry in $B \rightarrow X_s\gamma$, the branching ratio for $B \rightarrow X_s l^+ l^-$ and its correlation with the branching ratio for $B \rightarrow X_s \gamma$. The $B^0 - \overline{B^0}$ mixing is important for determination of three angles of the unitarity triangle. Also, last two observables are vanishingly small in the standard model (SM), and any appreciable amounts of these asymmetries would herald the existence of new CP violating phases beyond the KM phase in the SM. The questions addressed in our papers were how much these observables can be deviated from their SM values when $\mu$ and $A_t$ parameters in the MSSM have new CPV phases.

1.2 Model

In order to study $B$ physics in the MSSM, we make the following assumptions. First of all, the 1st and the 2nd family squarks are assumed to be degenerate and very heavy in order to solve the SUSY FCNC/CP problems. Only the third family squarks can be light enough to affect $B \rightarrow X_s \gamma$ and $B^0 - \overline{B^0}$ mixing. We also ignore possible flavor changing squark mass matrix elements that could generate gluino-mediated flavor changing neutral current (FCNC) process in addition to those effects we consider below. Recently, such effects were studied in the $B^0 - \overline{B^0}$ mixing, $B \rightarrow X_s \gamma$ and CP violations therein, and $B \rightarrow X_s l^+ l^-$, respectively. Ignoring such contributions, the only source of the FCNC in our model is the CKM matrix, whereas there are new CPV phases coming from the phases of $\mu$ and $A_t$ parameters in the flavor preserving sector in addition to the KM phase $\delta_{KM}$ in the flavor changing sector. In this sense, this paper is complementary to the earlier works.

1.3 Chang-Keung-Pilaftsis (CKP) EDM Constraints

Even if the 1st/2nd generation squarks are very heavy and degenerate, there is another important edm constraints considered by Chang, Keung and Pilaftsis (CKP) for large $\tan \beta$. This constraint comes from the two loop diagrams involving stop/sbottom loops, and is independent of the masses of the 1st/2nd
generation squarks.

\[
\frac{(d_f/e)_{\text{CKP}}}{M^2_A} = Q_f \frac{3 \alpha_{em} R_f m_f}{64 \pi^2 M^2_A} \sum_{q=t,b} \xi_q Q^2_q F \left( \frac{M^2_{3q}}{M^2_A}, \frac{M^2_{\tilde{q}}}{M^2_A} \right)
\]

(1)

where \( R_f = \cot \beta (\tan \beta) \) for \( I_3f = 1/2 \) \((-1/2)\), and

\[
\xi_t = \frac{\sin 2 \theta_{t\mu} \text{Im}(\mu e^{-i\delta})}{\sin^2 \beta v^2}, \quad \xi_b = \frac{\sin 2 \theta_{b\mu} \text{Im}(A_b e^{-i\delta})}{\sin \beta \cos \beta v^2},
\]

(2)

with \( \delta_q = \text{Arg}(A_q + R_q \mu^*) \), and \( F(x,y) \) is a two-loop function given in Ref. 13. Therefore, this CKP edm constraints can not be simply evaded by making the 1st/2nd generation squarks very heavy, and it turns out that this puts a very strong constraint on the possible new phase shift in the \( B^0 - \bar{B}^0 \) mixing.

1.4 Parameter Space

In the MSSM, the chargino mass matrix is given by

\[
M_{\chi^\pm} = \left( \begin{array}{cc} M_2 & \sqrt{2} m_W \sin \beta \\ \sqrt{2} m_W \cos \beta & \mu \end{array} \right).
\]

(3)

In principle, both \( M_2 \) and \( \mu \) may be complex, but one can perform a phase redefinition in order to render the \( M_2 \) is real. In such a basis, there appears one new phase \( \text{Arg}(\mu) \) as a new source of CPV. The stop mass matrix is given by

\[
M^2_{\tilde{t}} = \left( \begin{array}{cc} m^2_{\tilde{t}} & m_t (A_t - \mu / \tan \beta) \\ m_t (A_t - \mu / \tan \beta) & m^2_{\tilde{t}} + D_R \end{array} \right),
\]

(4)

where \( D_L = (1/2 - 2/3 \sin^2 \theta_W) \cos 2 \beta m^2_Z \) and \( D_R = 2/3 \sin^2 \theta_W \cos 2 \beta m^2_Z \). There are two new phases in this matrix, \( \text{Arg}(\mu) \) and \( \text{Arg}(A_t) \) in the basis where \( M_2 \) is real.

We scan over the MSSM parameter space as indicated below (including that relevant to the EWBGEN scenario in the MSSM) : 80 GeV < |\( \mu \}| < 1 TeV, 80 GeV < \( M_2 \) < 1 TeV, 60 GeV < \( M_A \) < 1 TeV, \( 2 < \tan \beta < 70 \), \( (130 \text{ GeV})^2 < M^2_{\tilde{q}} < (1 \text{ TeV})^2 \), \(- (80 \text{ GeV})^2 < M^2_{\tilde{q}} < (500 \text{ GeV})^2 \), \( 0 < \phi_{\mu} < 2 \pi \), \( 0 < |A_t| < 1.5 \text{ TeV} \). We have imposed the following experimental constraints : \( M_{\tilde{t}_1} > 80 \text{ GeV} \) independent of the mixing angle \( \theta_{\tilde{t}} \), \( M_{\chi^\pm} > 83 \text{ GeV} \), \( \text{Br}(B \to X_{s\gamma}) < 6.8 \% \), and \( 0.77 \leq R_\gamma \leq 1.15 \).

where \( R_\gamma \) is defined as \( R_\gamma = \frac{\text{BR}(B \to X_{s\gamma})^{\text{exp}}}{\text{BR}(B \to X_{s\gamma})^{\text{SM}}} \) and \( \text{BR}(B \to X_{s\gamma})^{\text{SM}} = (3.29 \pm 0.44) \times 10^{-4} \). It has to be emphasized that this
parameter space is larger than that in the constrained MSSM (CMSSM) where the universality of soft terms at the GUT scale is assumed. Especially, we will allow \( m_{L}^{2} \) to be negative as well as positive, which is preferred in the EWBGEN scenario. Since we do not impose any further requirement on the soft terms (such as radiative electroweak symmetry breaking, absence of color charge breaking minima, etc.), our results of the maximal deviations of \( B^{0} - \bar{B}^{0} \) mixing and \( A_{\text{CP}}^{b \rightarrow s \gamma} \) from the SM predictions are conservative upper bounds within the MSSM. If more theoretical conditions are imposed, the maximal deviations will be smaller. In the numerical analysis, we used the following numbers for the input parameters: \( m_{c} (m_{c}(\text{pole})) = 1.25 \text{ GeV}, \) \( m_{b}(m_{b}(\text{pole})) = 4.3 \text{ GeV}, \) \( m_{t}(m_{t}(\text{pole})) = 165 \text{ GeV} \) (these are running masses in the \( \overline{MS} \) scheme), and \(|V_{cb}| = 0.0410, |V_{tb}| = 1, |V_{ts}| = 0.0400 \) and \( \delta_{K} = \gamma(\phi_{3}) = 90^\circ \) for the CKM matrix elements.

2  \( B^{0} - \bar{B}^{0} \) Mixing

2.1 Phase Shift in the \( B^{0} - \bar{B}^{0} \) Mixing

The \( B^{0} - \bar{B}^{0} \) mixing is generated by the box diagrams with \( u_{i} - W^{\pm}(H^{\pm}) \) and \( \tilde{u}_{i} - \chi^{\pm} \) running around the loops in addition to the SM contribution. The resulting effective Hamiltonian is given by

\[
H_{\Delta B}^{\Delta B=2} = -\frac{G_{F}^{2}M_{W}^{2}}{(2\pi)^{2}} \sum_{i=1}^{3} C_{i} O_{i},
\]

where \( O_{1} = \bar{d}_{L}^{3} \gamma_{\mu} b_{L} \bar{d}_{L}^{3} \gamma_{\mu} b_{L}^{\beta}, O_{2} = \bar{d}_{L}^{3} b_{R}^{\mu} b_{R}^{\mu}, \) and \( O_{3} = \bar{d}_{L}^{3} b_{R}^{\alpha} \bar{d}_{L}^{3} b_{R}^{\alpha}. \) The Wilson coefficients \( C_{i}'s \) at the electroweak scale (\( \mu_{0} \sim M_{W} \sim M_{t} \)) can be written schematically as

\[
C_{1}(\mu_{0}) = (V_{td}^{*} V_{tb})^{2} \left[ F_{V}^{W}(3; 3) + F_{V}^{H}(3; 3) + A_{V}^{C} \right]
\]

\[
C_{2}(\mu_{0}) = (V_{td}^{*} V_{tb})^{2} F_{V}^{H}(3; 3)
\]

\[
C_{3}(\mu_{0}) = (V_{td}^{*} V_{tb})^{2} A_{S}^{C},
\]

where the superscripts \( W, H, C \) denote the \( W^{\pm}, H^{\pm} \) and chargino contributions respectively, and

\[
A_{V}^{C} = \sum_{i,j,k,l} \frac{1}{4} G_{i}^{(3,k)} G_{j}^{(3,k)} G_{i}^{(3,l)} G_{j}^{(3,l)} Y_{1}(r_{k}, r_{l}, s_{i}, s_{j}),
\]

\[
A_{S}^{C} = \sum_{i,j,k,l} \frac{1}{2} H_{i}^{(3,k)} G_{i}^{(3,l)} H_{j}^{(3,l)} Y_{2}(r_{k}, r_{l}, s_{i}, s_{j}),
\]
Here $G^i_{(3,k)}$ and $H^i_{(3,k)}$ are the couplings of $k$–th stop and $i$–th chargino with left-handed and right-handed quarks, respectively:

$$
G^i_{(3,k)} = \sqrt{2}C^*_{R1i}S_{ik1} - \frac{C^*_{R2i}S_{ik2}}{\sin \beta} \frac{m_t}{M_W},
$$

$$
H^i_{(3,k)} = \frac{C^*_{L2i}S_{ik1}}{\cos \beta} \frac{m_b}{M_W},
$$

and $C_{L,R}$ and $S_1$ are unitary matrices that diagonalize the chargino and stop mass matrices: $C_{L}^\dagger M_{\chi}^\dagger C_{L} = \text{diag}(M_{\tilde{\chi}_1^\pm}, M_{\tilde{\chi}_2^\mp})$ and $S_1 M_\tilde{t}^2 S_1^\dagger = \text{diag}(M_\tilde{t}_1^2, M_\tilde{t}_2^2)$. Explicit forms for functions $Y_1, Y_2$ and $F$'s can be found in Ref. $^{16}$, and $r_k = M_\tilde{t}_k^2/M_W^2$ and $s_i = M_{\tilde{\chi}_i}^2/M_W^2$. It should be noted that $C_2(\mu_0)$ was misidentified as $C_3^H(\mu_0)$ in Ref. $^{17}$. The gluino and neutralino contributions are negligible in our model. The Wilson coefficients at the $m_b$ scale are obtained by renomalization group running. The relevant formulae with NLO QCD corrections at $\mu = 2$ GeV are given in Ref. $^{18}$.

In our model, $C_1(\mu_0)$ and $C_2(\mu_0)$ are real relative to the SM contribution. On the other hand, the chargino exchange contributions to $C_3(\mu_0)$ (namely $A^C_3$) are generically complex relative to the SM contributions, and can generate a new phase shift in the $B^0 - \bar{B}^0$ mixing relative to the SM value. This effect can be in fact significant for large $\tan \beta (\approx 1/\cos \beta)$, since $C_3(\mu_0)$ is proportional to $(m_b/M_W \cos \beta)^2$. However, the CKP edm constraint puts a strong constraint for large $\tan \beta$ case, which was not properly included in Ref. $^{17}$. In Fig. $\ref{fig:2}$ (a), we plot $2\theta_d \equiv \text{Arg}(M_{12}^{\text{FULL}}/M_{12}^{\text{SM}})$ as a function of $\tan \beta$. The open squares (the crosses) denote those which (don’t) satisfy the CKP edm constraints. It is clear that the CKP edm constraint on $2\theta_d$ is in fact very important for large $\tan \beta$, and we have $|2\theta_d| \leq 1^\circ$. If we ignored the CKP edm constraint at all, then $|2\theta_d|$ could be as large as $\sim 4^\circ$. This observation is important for the CKM phenomenology, since time-dependent CP asymmetries in neutral $B$ decays into $J/\psi K_S, \pi\pi$ etc. would still measure directly three angles of the unitarity triangle even in the presence of new CP violating phases, $\phi_A$, and $\phi_\mu$. Our result is at variance with that obtained in Ref. $\cite{17}$ where CKP edm constraint was not properly included.

2.2 Dilepton Asymmetry

If we parametrize the relative ratio of $M_{\text{SM}}$ and $M_{\text{SUSY}}$ as $M_{\text{SUSY}}/M_{\text{SM}} = h e^{-i\theta}$, the dilepton asymmetry is given by

$$
A_{h\ell} = \left( \frac{\Delta \Gamma}{\Delta M} \right)_{\text{SM}} f(h, \theta) \equiv 4 \text{ Re}(\epsilon_B),
$$

(8)
where \( f(h, \theta) = h \sin \theta / (1 + 2h \cos \theta + h^2) \) and \( (\Delta \Gamma/\Delta M)_{SM} = (1.3 \pm 0.2) \times 10^{-2} \). We have neglected the small SM contribution. It is about \( \sim 10^{-3} \) in the quark level calculation, but may be as large as \( \sim 1\% \) if the delicate cancellation between the \( u \) and \( c \) quark contribution is not achieved. The result of scanning over the available MSSM parameter space is that \( |f(h, \theta)| \leq 0.1 \) so that \( |A_H| \leq 0.1\% \), which is well below the current data, \( A_H = (0.8 \pm 2.8 \pm 1.2)\% \). On the other hand, if any appreciable amount of the dilepton asymmetry is observed, it would indicate some new CPV phases in the off-diagonal down-squark mass matrix elements, assuming the MSSM is realized in nature.

2.3 \( \Delta M_B \)

On the contrary to the \( \theta_d \) and \( A_H \) discussed in the previous paragraphs, the magnitude of \( M_{12} \) is related with the mass difference of the mass eigenstates of the neutral \( B \) mesons: \( \Delta m_B = 2|M_{12}| = (3.05 \pm 0.12) \times 10^{-13} \) GeV, and thus it will affect the determination of \( V_{td} \) from the \( B^0 - \bar{B}^0 \) mixing. We have considered \( |M_{12}^{FULL}/M_{12}^{SM}| \) and its correlation with \( Br(B \to X_s \gamma) \) are shown in Fig. 1 (b). The deviation from the SM can be as large as \( \sim 60\% \) and the correlation behaves differently from the minimal supergravity case.
repeated the same analyses for $B^0_s - \overline{B^0_s}$ mixing. There is no large new phase shift ($2|\theta_s|$) in this case either, but the modulus of $M_{12}(B_s)$ can be enhanced by up to 60% compared to the SM value.

3 Direct Asymmetry in $B \to X_s\gamma$

The radiative decay of $B$ mesons, $B \to X_s\gamma$, is described by the effective Hamiltonian including (chromo)magnetic dipole operators. Interference between $b \to s\gamma$ and $b \to sg$ (where the strong phase is generated by the charm loop via $b \to c\bar{c}s$ vertex) can induce direct CP violation in $B \to X_s\gamma$, which is given by

$$A_{\text{CP}}^{b \to s\gamma} \equiv \frac{\Gamma(B \to X_s + \gamma) - \Gamma(B \to \bar{X}_s + \gamma)}{\Gamma(B \to X_s + \gamma) + \Gamma(B \to \bar{X}_s + \gamma)}$$

$$\simeq \frac{1}{|C_7|^2} \left\{ 1.23\text{Im} \left[ C_2 C_7^* \right] - 9.52\text{Im} \left[ C_8 C_7^* \right] \right. $$

$$\left. + 0.10\text{Im} \left[ C_2 C_8^* \right] \right\} \text{ (in %)}.$$

adopting the notations in Ref. [23]. We have ignored the small contribution from the SM, and assumed that the minimal photon energy cut is given by $E_{\gamma} \geq m_B(1 - \delta)/2$ ($\approx 1.8$ GeV with $\delta = 0.3$). $A_{\text{CP}}^{b \to s\gamma}$ is not sensitive to possible long distance contributions and constitute a sensitive probe of new physics that appears in the short distance Wilson coefficients $C_7, C_8$. [23]

The Wilson coefficients $C_7, C_8$ in the MSSM have been calculated by many groups, including the PQCD corrections in certain MSSM parameter space. [27] In this letter, we use the leading order expressions for $C_i$’s which is sufficient for $A_{\text{CP}}^{b \to s\gamma}$. After scanning over the MSSM parameter space described in Eq. (3), we find that $A_{\text{CP}}^{b \to s\gamma}$ can be as large as $\simeq \pm 16\%$ if chargino is light enough, even if we impose the edm constraints. Its correlation with $Br(B \to X_s\gamma)$ and chargino mass are shown in Figs. (2 (a) and (b) respectively. Our results are quantitatively different from other recent works, mainly due to the different treatments of soft terms. In the minimal supergravity scenario, this asymmetry is very small, because the $A_t$ phase effect is very small in the electroweak scale. [23] If the universality assumption is relaxed, one can accomodate larger direct asymmetry without conflicting with the edm constraints.

4 The Branching Ratio for $B \to X_s l^+ l^-$

Let us first consider the branching ratio for $B \to X_s l^+ l^-$. The SM and the MSSM contributions to this decay were considered by several groups. [28, 29] We use the standard notation for the effective Hamiltonian for this decay as
The new CPV phases in $C_{7,9,10}$ can affect the branching ratio and other observables in $B \rightarrow X_s l^+ l^-$ as discussed in the first half of Ref.\cite{12}. In the second half of Ref.\cite{12}, specific supersymmetric models were presented where new CPV phases reside in flavor changing squark mass matrices. In the present work, new CPV phases lie in flavor conserving sector, namely in $A_t$ and $\mu$ parameters. Although these new phases are flavor conserving, they affect the branching ratio of $B \rightarrow X_s l^+ l^-$ and its correlation with $Br(B \rightarrow X_s \gamma)$, as discussed in the first half of Ref.\cite{12}. Note that $C_{9,10}$ depend on the sneutrino mass, and we have scanned over 60 GeV $< m_{\tilde{\nu}} < 200$ GeV. In the numerical evaluation for $R_{ll} \equiv Br(B \rightarrow X_s l^+ l^-)/Br(B \rightarrow X_s l^+ l^-)_{SM}$, we considered the nonresonant contributions only for simplicity, neglecting the contributions from $J/\psi, \psi', \text{etc.}$. It would be straightforward to incorporate these resonance effects. In Figs. 3 (a) and (b), we plot the correlations of $R_{\mu\mu}$ with $Br(B \rightarrow X_s \gamma)$ and $\tan \beta$, respectively. Those points that (do not) satisfy the CKP edm constraints are denoted by the squares (crosses). Some points are denoted by both the square and the cross. This means that there are two classes of points in the MSSM parameter space, and for one class the CKP edm constraints are satisfied but for another
class the CKP edm constraints are not satisfied, and these two classes happen to lead to the same branching ratios for $B \to X_s \gamma$ and $R_{\mu\mu}$. In the presence of the new phases $\phi_\mu$ and $\phi_{A_0}$, $R_{\mu\mu}$ can be as large as 1.85, and the deviations from the SM prediction can be large, if $\tan \beta > 8$. As noticed in Ref. 12, the correlation between the $\text{Br}(B \to X_s \gamma)$ and $R_{\mu\mu}$ is distinctly different from that in the minimal supergravity case. In the latter case, only the envelop of Fig. 3 (a) is allowed, whereas everywhere in between is allowed in the presence of new CPV phases in the MSSM. Even if one introduces the phases of $\mu$ and $A_0$ at GUT scale in the minimal supergravity scenario, this correlation does not change very much from the case of the minimal supergravity scenario with real $\mu$ and $A_0$, since the $A_0$ phase becomes very small at the electroweak scale because of the renormalization effects. Only $\mu$ phase can affect the electroweak scale physics, but this phase is strongly constrained by the usual edm constraints so that $\mu$ should be essentially real parameter. Therefore the correlation between $B \to X_s \gamma$ and $R_{\mu\mu}$ can be a clean distinction between the minimal supergravity scenario and our model (or some other models with new CPV phases in the flavor changing case).

5 \( \epsilon_K \) in Our Model
5.1 $K^0 - \bar{K}^0$ Mixing

The new complex phases in $\mu$ and $A_t$ will also affect the $K^0 - \bar{K}^0$ mixing. The relevant $\Delta S = 2$ effective Hamiltonian is given by

$$H_{\Delta S=2}^{\text{eff}} = \frac{G_F^2 M_W^2}{(2\pi)^2} \sum_{i=1}^{3} C_i Q_i,$$

(10)

where

$$C_1(\mu_0) = (V_{td}^* V_{ts})^2 [F_V^W (3; 3) + F_V^H (3; 3) + A_C^V]$$
$$+ (V_{cd}^* V_{cs})^2 [F_V^W (2; 2) + F_V^H (2; 2)]$$
$$+ 2 (V_{td}^* V_{ts}) (V_{cd}^* V_{cs}) [F_V^W (3; 2) + F_V^H (3; 2)],$$
$$C_2(\mu_0) = (V_{td}^* V_{ts})^2 F_S^H (3; 3) + (V_{cd}^* V_{cs})^2 F_S^H (2; 2)$$
$$+ 2 (V_{td}^* V_{ts}) (V_{cd}^* V_{cs}) F_S^H (3; 2),$$
$$C_3(\mu_0) = (V_{td}^* V_{ts})^2 A_S^C,$$

(11)

where the charm quark contributions have been kept. $G^{(3, k)ii}$ and $H^{(3, k)ii}$ are the same as Eqs. (7) except that $m_b$ should be replaced by $m_s$ in the $K^0 - \bar{K}^0$ mixing. Note that $C_2(\mu_0)$ was misidentified as $C_{ij}^H(\mu_0)$ in Ref.\textsuperscript{17}. The gluino and neutralino contributions are negligible in our model. The Wilson coefficients at lower scales are obtained by renormalization group running. The relevant formulae with the NLO QCD corrections at $\mu = 2$ GeV are given in Ref.\textsuperscript{18}. As in the $B^0 - \bar{B}^0$ mixing before, $C_1(\mu_0)$ and $C_2(\mu_0)$ are real relative to the SM contribution in our model. On the other hand, the chargino exchange contributions to $C_3(\mu_0)$ (namely $A_S^C$) are generically complex relative to the SM contributions, and can generate a new phase shift in the $K^0 - \bar{K}^0$ mixing relative to the SM value. This effect is in fact significant for large $\tan \beta (\simeq 1/\cos \beta)$,\textsuperscript{17} since $C_3(\mu_0)$ is proportional to $(m_s/M_W \cos \beta)^2$.

The CP violating parameter $\epsilon_K$ can be calculated from

$$\epsilon_K \simeq \frac{e^{i\pi/4} \mathrm{Im} M_{12}}{\sqrt{2} \Delta M_K},$$

(12)

where $M_{12}$ can be obtained from the $\Delta S = 2$ effective Hamiltonian through $2M_K M_{12} = \langle K^0|H_{\Delta S=2}^{\text{eff}}|\bar{K}^0 \rangle$. For $\Delta M_K$, we use the experimental value $\Delta M_K = (3.489 \pm 0.009) \times 10^{-12}$ MeV, instead of theoretical relation $\Delta M_K = 2 \text{Re} M_{12}$, since the long distance contributions to $M_{12}$ is hard to calculate reliably unlike the $\Delta S = 2$ box diagrams. For the strange quark mass, we use the $\overline{\text{MS}}$ mass at $\mu = 2$ GeV scale: $m_s(\mu = 2\text{GeV}) = 125$ MeV. In Figs.\textsuperscript{4}
Figure 4: The correlations between $\frac{\epsilon_K}{\epsilon_K^{SM}}$ and the lighter chargino mass $M_{\tilde{\chi}_1^\pm}$ for (a) $2 < \tan\beta < 35$ and (b) $35 < \tan\beta < 70$, respectively. The squares (the crosses) denote those which (do not) satisfy the CKP edm constraints.

(a) and (b), we plot the results of scanning the MSSM parameter space: the correlations between $\frac{\epsilon_K}{\epsilon_K^{SM}}$ and (a) $\tan\beta$ and (b) the lighter stop mass. We note that $\frac{\epsilon_K}{\epsilon_K^{SM}}$ can be as large as 1.4 for $\delta_K = 90^\circ$ if $\tan\beta$ is small. This is a factor 2 larger deviation from the SM compared to the minimal supergravity case. The dependence on the lighter stop is close to the case of the minimal supergravity case, but we can have a larger deviations. Such deviation is reasonably close to the experimental value, and will affect the CKM phenomenology at a certain level.

5.2 Can $\epsilon_K$ Come Entirely from $A_t$ and $\mu$ Phases?

In the MSSM with new CPV phases, there is an intriguing possibility that the observed CP violation in $K_L \rightarrow \pi\pi$ is fully due to the complex parameters $\mu$ and $A_t$ in the soft SUSY breaking terms which also break CP softly. This possibility was recently considered by Demir et al. Their claim was that it was possible to generate $\epsilon_K$ entirely from SUSY CPV phases for large $\tan\beta \approx 60$ with certain choice of soft parameters. (Their choice of parameters leads to $M_{\chi^\pm} = 80$ GeV and $M_{\tilde{t}} = 85$ GeV, which are very close to the recent lower limits set by LEP2 experiments.) In such a scenario, only $\text{Im}(A_C^S)$ in Eq. (11) can contribute to $\epsilon_K$, if we ignore a possible mixing between $C_2$ and $C_3$ under QCD renormalization. In actual numerical analysis we have included
this effect using the results in Ref. We repeated their calculations using the same set of parameters, but could not confirm their claim. For $\delta_{KM} = 0^\circ$, we found that the supersymmetric $\epsilon_K$ is less than $\sim 2 \times 10^{-5}$, which is too small compared to the observed value: $|\epsilon_K| = (2.280 \pm 0.019) \times 10^{-3}$ determined from $K_{L,S} \to \pi^+\pi^-$.  

Let us give a simple estimate for supersymmetric $\epsilon_K$ with real CKM matrix elements, in which case only $C_3(\mu_0)$ develops imaginary part and can contribute to $\epsilon_K$. For $m_{\tilde{t}_1} \sim m_{\tilde{t}_2} \sim M_W$, we would get $Y_2 \sim Y_2(1,1,1,1) = 1/6$, and

\[
|G^{(3,k)i}| \leq O(1), \quad \text{and} \quad |H^{(3,k)i}| \sim \frac{m_s \tan \beta}{M_W},
\]

because any components of unitary matrices $C_R$ and $S_t$ are $\leq O(1)$. Therefore $\text{Im}(A^e_S) \leq O(10^{-3})$. Now using

\[
\text{Im}(M_{12}) = -\frac{G_F^2 M_W^2}{(2\pi)^2} f_K M_K \left( \frac{M_K}{m_s} \right)^2 \frac{1}{24} B_3(\mu) \text{Im}(C_3(\mu)), \quad (13)
\]

and Eq. (9), we get $|\epsilon_K| \leq 2 \times 10^{-5}$.

### 6 Conclusion

In conclusion, we assumed that the MSSM has new CPV phases beyond the CKM phase, and considered its observable consequences at $B$ factories and on $\epsilon_K$ without making universality assumption on the soft terms at the GUT scale. Our study includes the EW baryogenesis scenario in the MSSM. The main results can be summarized as follows.

- There is no appreciable new phase in the $B^0 - \overline{B^0}$ mixing ($|2\theta_d| \leq 1^\circ$), so that time-dependent CP asymmetries in neutral $B$ decays (into $J/\psi K_S, \pi\pi$ etc.) still measure essentially three angles of the unitarity triangle even if there are new complex phases in $\mu$ and $A_t$ parameters.

- The size of the $B^0 - \overline{B^0}$ mixing can be enhanced up to $\sim 60\%$ compared to the SM contribution, which will affect determination of $V_{td}$ from $\Delta m_B$.

- There is no large shift in $\text{Re}(\epsilon_B)$, and dilepton CP asymmetry is rather small ($|A_{tl}| \leq 0.1\%$).

- Direct CP asymmetry in $B \to X_s \gamma$ can be as large as $\sim \pm 16\%$ if chargino is light enough. This would encompass the interesting EWBGEN scenario in the MSSM.
• The branching ratio for $B \to X_s l^+ l^-$ can be enhanced up to $\sim 85\%$ compared to the SM prediction, and the correlation between $\text{Br}(B \to X_s \gamma)$ and $\text{Br}(B \to X_s l^+ l^-)$ is distinctly different from the minimal supergravity scenario (CMSSM) (even with new CP violating phases) in the presence of new CP violating phases in $C_{7,8,9}$ as demonstrated in model-independent analysis by Kim, Ko and Lee.

• $\epsilon_K/\epsilon_K^{SM}$ can be as large as 1.4 for $\delta_{KM} = 90^\circ$. This is the extent to which the new phases in $\mu$ and $A_t$ can affect the construction of the unitarity triangle through $\epsilon_K$.

• Fully supersymmetric CP violation is not possible even for large $\tan \beta \sim 60$ and light enough chargino and stop, contrary to the claim made in Ref. 17. With real CKM matrix elements, we get very small $|\epsilon_K| \leq O(10^{-5})$, which is two orders of magnitude smaller than the experimental value.

These results would set the level of experimental sensitivity that one has to achieve in order to probe the SUSY-induced CP violations at $B$ factories through $B^0 - \bar{B}^0$ and $A_{CP}^{b \to s\gamma}$ mixing. Our results are conservative in a sense that we did not impose any conditions on the soft SUSY breaking terms except that the resulting mass spectra for chargino, stop and other sparticles satisfy the current lower bounds from LEP and Tevatron. Therefore, one would be able to find the effects of the phases of $\mu$ and $A_t$ parameters by observing $A_{CP}^{b \to s\gamma}$ at B factories. Within our assumption, the results presented here are conservative since we did not impose any conditions on the soft SUSY breaking terms except that the resulting mass spectra for chargino, stop and other sparticles satisfy the current lower bounds from LEP and Tevatron.

Before closing this paper, we’d like to emphasize that all of our results are based on the assumption that there are no new CPV phases in the flavor changing sector. Once this assumption is relaxed, then gluino-mediated FCNC with additional new CPV phases may play important roles, and many of our results may change. For example, recently we have shown that both $\epsilon_K$ and $\epsilon'/\epsilon_K$ can be saturated by a single complex parameter $(\delta_{12}^d)_LL \sim O(10^{-3})$ with $O(1)$ phase in the mass insertion approximation in supersymmetric models, if $|\mu \tan \beta| \sim O(10 - 20)$ TeV. In this case, $\epsilon_K$ is saturated by $(\delta_{12}^d)_LL$, whereas $\epsilon'/\epsilon_K$ is given by the induced $(\delta_{12}^d)_LL \approx (\delta_{12}^d)_LL \times (m_s(A_s - \mu \tan \beta)/\tilde{m}^2) \sim 10^{-5}$. Remarkably, this can be achieved without any contradiction to various FCNC or EDM constraints.
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