Relaxation Time and Relaxation Function of Quark-Gluon Plasma with Lattice QCD

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We propose a method which enables a QCD-based calculation of a relaxation time for a dissipative current in the causal and dissipative hydrodynamic equation derived by Israel and Stewart. We point out that the Israel-Stewart equation is not unique as a causal and dissipative hydrodynamic equation, and the form of the causal and dissipative hydrodynamic equation is determined by the shape of a spectral function reflecting the properties of elementary excitations in the system we consider.

Our method utilizes a relaxation function, which can be calculated from QCD using the linear response theory. We show that the relaxation function can be derived from a spectral function for a microscopic representation of the dissipative current. We also show that the Israel-Stewart equation is acceptable only as long as the calculated relaxation function is approximated well by an exponentially damping function, and the relaxation time can be obtained as its damping time constant. Taking a baryon-number dissipative current of a plasma consisting of charm quarks and gluons as a simple example, we present the first calculation of the relaxation function with use of the spectral function derived employing the quenched lattice QCD together with the maximum entropy method. The calculated relaxation function shows a strongly-oscillation damping behaviour due to the charmed vector hadron J/Ψ surviving above the deconfinement phase transition temperature in QCD. This result suggests that the applicability of the Israel-Stewart equation to the baryon-number dissipative current of the charm quark-gluon plasma is quite doubtful. We present an idea for the improvement of the Israel-Stewart equation by deriving the hydrodynamic equation consistent with the strongly-oscillation damping relaxation function.

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I. INTRODUCTION

The time evolution of the hot matter of quarks and gluons created in the Relativistic Heavy Ion Collider (RHIC) experiments at Brookhaven National Laboratory is well described by the relativistic hydrodynamic equation with almost no dissipative effects [1, 2, 3]. This fact means that the quark-gluon plasma (QGP) slightly above the deconfinement phase transition temperature T_C behaves as an almost ideal fluid, of which viscosity and heat conductivity, i.e., transport coefficients, are negligibly small. This interesting non-equilibrium property of QGP as an almost ideal fluid is considered to be one of the properties of a strongly interacting system, and consistent with hadronic excitations surviving in QGP shown by several groups using the lattice QCD [4, 5, 6, 7].

In order to reach a full understanding of this non-equilibrium property of QGP, we need to study the RHIC experimental data using the relativistic hydrodynamic equation with finite dissipative effects.

At present, a good candidate for the relativistic hydrodynamic equation with dissipative effects is considered to be the Israel-Steart equation [8, 9] instead of the standard equations by Eckart [10] and Landau and Lifshitz [11]. This is because the Israel-Stewart equation holds the causality in contrast with the Eckart and Landau-Lifshitz equations.

To assure the causality, the Israel-Stewart equation contains higher derivative terms with relaxation times, i.e., new parameters other than transport coefficients, which are dependent on materials we consider. An important matter is to determine these relaxation times from microscopic dynamics governing the materials.

In 1979, Israel and Stewart [9] calculated the relaxation times of a rarefied gas by using the relativistic Boltzmann equation with the Grad’s fourteen moment approximation [12]. These quantities have been used by several groups [13, 14, 15] to carry out (2+1)-dimensional hydrodynamic simulations based on the Israel-Stewart equation.

In 2008, Natsuume and Okamura [16] derived the relaxation times of a strongly interacting supersymmetric gauge-theory plasma by utilizing the dispersion relations of the hydrodynamic modes obtained with use of the AdS/CFT duality. However, a calculation of the relaxation times of QGP based on QCD is unfortunately not yet done.

The aim of this paper is (1) to construct a method which enables us to determine the relaxation times from the microscopic theory including QCD, and (2) to present the first lattice QCD calculation of the relaxation times of QGP on the basis of this method.

To construct the method, we utilize a relaxation function, which gives a linear but non-local (non-Markovian) relation between an external field applied to the system and the current induced in the system [17, 18]. The relaxation functions contain full information about non-equilibrium dissipative processes, e.g., transport coefficients and relaxation times. Moreover a microscopic calculation of the relaxation functions is straightforward using the linear response theory.
In this paper, we first express the relaxation function in terms of a spectral function for a conserved current, which can be calculated from the microscopic dynamics by a quantum-field theoretic method. This expression tells us that the relaxation function is determined by a whole behaviour of the spectral function in contrast to the transport coefficient defined as the low-frequency and long-wavelength limit of the spectral function. We show that, only as long as the calculated relaxation function shows an exponentially damping behaviour, the damping time constant can be identified as the relaxation time in the Israel-Stewart equation. Taking a baryon-number dissipative current of a plasma consisting of charm quarks and gluons (referred to as the charm quark-gluon plasma later on) as a simple example, we calculate the relaxation function from the spectral function obtained using the lattice QCD together with the maximum entropy method (MEM) \cite{14}. The obtained relaxation function shows a strongly-oscillation damping behaviour instead of an exponentially damping one, due to the charmed vector hadron $J/\Psi$ surviving above $T_C$. This disagreement suggests that the applicability of the Israel-Stewart equation to the baryon-number dissipative current of the charm quark-gluon plasma is doubtful. We show an idea for the improvement of the Israel-Stewart equation by deriving the hydrodynamic equation consistent with the strongly-oscillating damping relaxation function.

The conclusion of this paper is that the Israel-Stewart equation is not unique as the causal and dissipative hydrodynamic equation, and the form of the causal and dissipative hydrodynamic equation applicable to QGP is determined by the shape of the spectral function reflecting the properties of hadronic excitations in QGP. This indicates an important link between the non-equilibrium properties of QGP and the hadron spectroscopy at finite temperature.

This paper is organized as follows. In section II, we construct a method which enables a microscopic calculation of the relaxation times in the Israel-Stewart equation. For this purpose, the relaxation functions are utilized. In section III, we present the first calculation of the relaxation function for the baryon-number dissipative current in QGP, with use of the Monte Carlo simulation based on the lattice QCD. We discuss whether the exponentially damping relaxation function of the Israel-Stewart equation can be justified from a microscopic point of view. Section IV is devoted to the summary and the concluding remarks. In appendix A, taking a shear viscous dissipative process as a typical example, we show the detailed derivation of the microscopic representation of the relaxation function.

II. FORMULA FOR RELAXATION FUNCTION

In this section, we give a method which enables us to determine the relaxation time from the microscopic theory including QCD. Our discussion is based on the relaxation function established in the phenomenological relaxation theory \cite{17}. The microscopic representation of the relaxation function is given in the linear response theory \cite{17} \cite{18}.

A. Non-Markovian Constituent Equation Derived from Israel-Stewart Equation

To explain the physical meaning of the relaxation functions, let us start from the Israel-Stewart equation for a simple system, where the conserved quantities are the energy and the momentum, and the dissipative effect comes solely from the shear viscosity. We denote the energy-momentum tensor by

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - p \Delta^{\mu\nu} + \pi^{\mu\nu},$$

where $\epsilon = \epsilon(T)$ and $p = p(T)$ are the internal energy and the pressure at the temperature $T$, respectively, $u^\mu$ the flow velocity, $\pi^{\mu\nu}$ the shear viscous stress and $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$ the projection operator with $g^{\mu\nu} = \text{diag}(+1, -1, -1, -1)$. Here $u^\mu$ is normalized as $u^\mu u^\mu = 1$, and $\pi^{\mu\nu}$ is satisfied with $\pi^{\mu\nu} = \pi^{\nu\mu}$ and $u_\mu \pi^{\mu\nu} = \pi^{\nu\mu} = 0$, which are reproduced by $\pi^{\mu\nu} = \Delta^{\mu\nu\rho\sigma} \tilde{\pi}_{\rho\sigma}$ with the symmetric traceless projection operator $\Delta^{\mu\nu\rho\sigma} \equiv 1/2 (\Delta^{\mu\rho} \Delta^{\nu\sigma} + \Delta^{\mu\sigma} \Delta^{\nu\rho} - 2/3 \Delta^{\mu\nu} \Delta^{\rho\sigma})$ and the shear viscous stress before the projection $\tilde{\pi}_{\rho\sigma}$. It is noted that the Israel-Stewart equation is the system of the nine differential equations with respect to the nine independent variables, $T$, $u^\mu$ and $\pi^{\mu\nu}$. The equations consist of the continuity equation,

$$\partial_\mu T^{\mu\nu} = 0,$$

and the relaxation equation \cite{8} \cite{9},

$$\tau_\pi D \tilde{\pi}_{\rho\sigma} + \pi_{\rho\sigma} = 2 \eta \partial_\mu u_\sigma,$$

where $\eta$ and $\tau_\pi$ denote the shear viscosity and the relaxation time of the shear viscous dissipative process, respectively, and $D \equiv w^\nu \partial_\nu \equiv \partial / \partial \tau$ with the proper time $\tau$.

The solution of Eq. (3) for the shear viscous stress reads

$$\tilde{\pi}_{\rho\sigma}(\tau) = \int \text{d}\tau' \left[ \frac{\theta(\tau - \tau') \eta}{\tau_\pi} e^{-(\tau - \tau')/\tau_\pi} \right] 2 \partial_\rho u_\sigma(\tau').$$

Equation (4) tells us that $\pi^{\mu\nu} = \Delta^{\mu\nu\rho\sigma} \tilde{\pi}_{\rho\sigma}$ at the proper time $\tau$ depends on the history of the external field $2 \partial_\rho u_\sigma$ from past to present, and the weighting function is given by $\frac{\eta}{\tau_\pi} e^{-(\tau - \tau')/\tau_\pi}$. Notice that Eq. (4) is equivalent to that derived in the Stewart’s non-local thermodynamics \cite{20}, and has been recently rediscovered by Koide et al. \cite{21}. The meaning of Eq. (4) becomes clearer from the viewpoint of the phenomenological relaxation theory \cite{17}, where the linear but non-Markovian constituent equation,

$$\tilde{\pi}_{\rho\sigma}(\tau) = \int \text{d}\tau' R(\tau - \tau') 2 \partial_\rho u_\sigma(\tau'),$$
is well established and \( R(\tau) \) is identically the relaxation function. Therefore, we find that the Israel-Stewart causal hydrodynamics is equivalent to the special case of the phenomenological relaxation theory with

\[
R(\tau) = \theta(\tau) \frac{\eta}{\tau_\pi} \exp(-\tau/\tau_\pi),
\]

i.e., the exponentially damping relaxation function.

**B. Microscopic Representation of Relaxation Function Based on Spectral Function**

We first notice that the linear response theory\,\,[17, 18] enables us to calculate the relaxation function directly from the microscopic dynamics. We now consider the relaxation function of a generic dissipative process,

\[
J(t, x) = \int du \int dy R(t - u, x - y) X(u, y),
\]

where \( X(t, x) \) denotes the external field, \( J(t, x) \) the current induced by \( X(t, x) \), and \( R(t, x) \) the relaxation function. The linear response theory allows that the microscopic derivation of the relaxation function is straightforward. In fact, as shown in appendix A, we can derive the following relation,

\[
\dot{R}(t, x) = \theta(t) \int_{-\infty}^{0} ds \theta(s - t) i \langle [\hat{J}(t, x), \hat{J}(s, 0)] \rangle,
\]

where \( \hat{J}(t, x) \) is a Heisenberg operator and gives the microscopic representation for the induced macroscopic current \( J(t, x) \), and the bracket indicates the thermal average. Notice that the integrand of the right hand side of Eq.(5) is the retarded Green function, which has the following spectral representation,

\[
\dot{R}(t, x) = \theta(t) \int \frac{d^4 k}{(2\pi)^4} e^{-i(k \cdot x)} \int d\omega \frac{A(\omega, k)}{\omega - k^0 - i\varepsilon},
\]

where \( A(\omega, k) \) is the spectral function and \( \varepsilon \) the infinitesimal positive constant. Equations (8) and (9) lead us to the relation between \( R(t, x) \) and \( A(\omega, k) \) given by

\[
R(t, x) = 2 \theta(t) \int \frac{d\omega d^4 k}{(2\pi)^4} e^{-i(k \cdot x)} \frac{\tau_\pi}{\omega} A(\omega, k).
\]

This formula assures that the relaxation function can be obtained as the Fourier transformation of \( \pi A(\omega, k)/\omega \), i.e., the real part of the complex admittance\,\,[17], which is defined by

\[
\dot{R}(k^0, k) \equiv \int dt d^3 x R(t, x) e^{i(k^0 t - k \cdot x)} = \int d\omega \frac{-i}{\omega - k^0 - i\varepsilon} \frac{A(\omega, k)}{\omega}.
\]

Here we notice the two important properties concerning the complex admittance \( \dot{R}(k^0, k) \): (i) The low-frequency and long-wavelength limit of the complex admittance, \( \dot{R}(0, 0) = \pi A(\omega, 0)/\omega \) gives the transport coefficients. In fact, with use of this representation, many groups\,[22, 23, 24, 25] have calculated the transport coefficients of QGP employing the lattice QCD. (ii) Using \( \dot{R}(k^0, k) \), we can write down the generic relaxation equation equivalent to the non-Markovian constituent equation\,[7],

\[
\dot{R}^{-1}(k^0 = +i\partial_t, k = -i\nabla) J(t, x) = X(t, x).
\]

By combining Eqs.(11) and (12), we can construct the relaxation equation from a given spectral function.

The above observation tells us the following three points: (a) The relaxation function in QGP can be calculated from the spectral function for the microscopic current of QCD. (b) The Israel-Stewart equation can be valid for the description of QGP only as long as the relaxation function calculated in the microscopic way is an exponentially damping function, and the relaxation time of QGP can be obtained as its damping time constant. (c) If the relaxation function calculated from QCD is different from that derived from the Israel-Stewart equation, we should carry out the causal hydrodynamic calculation by incorporating the relaxation equation\,[12] or the non-Markovian constituent equation\,[7] into the continuity equation.

**C. Spectral Function Consistent with Israel-Stewart Equation**

It is interesting to investigate which spectral functions reproduce the exponentially damping relaxation function in the Israel-Stewart equation. Taking the shear viscous dissipative process at the rest frame as a typical example, we find it to be a Lorentzian function,

\[
\frac{\pi}{\omega} A(\omega, k) = \frac{\eta}{1 + \omega^2 \tau_\pi^2}.
\]

In fact, it can be checked that Eqs.(10) and (13) give the exponentially damping relaxation function,

\[
R(t, x) = \theta(t) \frac{\eta}{\tau_\pi} \exp(-t/\tau_\pi) \delta^3(x),
\]

and the correspondent complex admittance reads

\[
\dot{R}(k^0, k) = \frac{\eta}{1 - i k^0 \tau_\pi}.
\]

It is noteworthy that Eq.(12) with this complex admittance reproduces the Israel-Stewart-type relaxation equation,

\[
(1 + \tau_\pi \partial_t) J(t, x) = \eta X(t, x).
\]

Here we replace \( \partial_t \) with \( D = u^\mu \partial_{\mu} = \partial/\partial\tau \) in order to obtain the equation at an arbitrary frame from that at
the rest frame. Therefore it is found that the ansatz of the Lorentzian-type spectral function shown in Eq. (13) is an essential point of the Israel-Stewart equation. Notice that there is no reason to believe that the spectral function for the current of QCD is approximated well by the Lorentzian function in the low-frequency and long-wavelength region.

III. RESULTS FROM LATTICE QCD

In this section, we demonstrate the first lattice QCD calculation of the relaxation function. From the viewpoint of availability of lattice data, we choose the relaxation function for the baryon-number dissipative process. The continuity equation reads

\[ \partial_t N^\mu = \partial_\nu (n u^\nu + \nu^\mu) = 0, \]

and the non-Markovian constituent equation

\[ \nu^\mu(t, x) = D^\mu(t, x) \int \frac{d^3y}{(2\pi)^3} R(t-u, x-y) \times \partial_\nu \left( \frac{\mu B}{T}(u, y) \right), \]

where \( n \) and \( \mu B \) denote the baryon-number density and the baryon-number chemical potential, respectively. This relaxation function can be calculated from the spectral function of the spatial components of the baryon-number current, \( J^\mu = \sum_f \sum_{c=1}^{N_c} \tilde{\psi}_f c \gamma^\mu \psi_f \), where \( \tilde{\psi}_f \) denotes the flavor-c colour quark field, \( \gamma^\mu(\mu = 0, 1, 2, 3) \) the gamma matrix, \( \tilde{\psi}_f = \psi_f^\dagger \gamma^0 \), \( N_F \) the number of the flavour and \( N_C(= 3) \) the number of the colour, respectively. As a simple example, we consider the single-flavour deconfined system of which constituents are gluons and charm quarks \( (N_F = 1) \), i.e., the charm quark-gluon plasma. Furthermore, we treat a spatially integrated relaxation function,

\[ R(t) = \int d^3 x R(t, x) = 2 \theta(t) \int \frac{d\omega}{2\pi} e^{-i\omega t} \frac{\pi}{\omega} A(\omega), \]

where \( A(\omega) = A(\omega, 0) \) is a zero-momentum projected spectral function. This relation indicates that the calculation of \( R(t) \) requires an explicit form of \( A(\omega) \). Therefore we need to reconstruct \( A(\omega) \) from the temporal correlation function,

\[ D(\tau) = \int_0^\infty d\omega K(\tau, \omega) A(\omega), \]

obtained using Monte Carlo simulation based on the lattice QCD. To get the full information about \( A(\omega) \), we utilize MEM \( [19] \).

A. Outline of MEM Analysis

Here we shall give a brief account of MEM. The temporal correlation function \( D(\tau) \) and the associated \( A(\omega) \) are related by the Laplace transformation,

\[ D(\tau) = \int_0^\infty d\omega K(\tau, \omega) A(\omega), \]

where \( K(\tau, \omega) = \cosh[\omega(\tau-1/2T)]/\sinh(\omega/2T) \) is a kernel function. Since the number of data points in the temporal direction is finite, the direct inverse Laplace transformation from \( D(\tau) \) to \( A(\omega) \) is an ill-posed problem. MEM is a method to overcome this difficulty on the basis of the Bayes' theorem in the theory of statistical inference. Using MEM, we can infer \( A(\omega) \) as the most probable one which achieves a balance between reproducing the lattice data \( D(\tau) \) and being kept not so far from the default model \( m(\omega) \), defined by a plausible form of \( A(\omega) \). MEM has proved to be a powerful tool to investigate \( A(\omega) \) of mesons and baryons at both of \( T = 0 \) and \( T \neq 0 \) \( [19, 20, 21, 22] \).

Recently, the author \( [23] \) has improved MEM to avoid a numerical instability present in the study of the shear viscosity of the gluon plasma. An essential point of the improved MEM analysis is that the Laplace transformation combining \( D(\tau) \) and \( A(\omega) \) is converted as follows,

\[ D(\tau) = \int_0^\infty d\omega \left[ \frac{K(\tau, \omega)}{\pi} \right] \frac{\pi}{\omega} A(\omega). \]

Regarding \( \omega K(\tau, \omega)/\pi \) as a new kernel function, we can infer the real part of the complex admittance \( \pi A(\omega)/\omega \) directly from \( D(\tau) \). Further, in the study of the scalar tetraquark \( [24] \), the extension of this method, such as \( D(\tau) = \int_0^\infty d\omega [K(\tau, \omega)A(\omega)] [A(\omega)/A(\omega)] \) with \( A(\omega) \equiv A(\omega, \omega) \), has also been used to reduce the uncertainty of the inferred \( A(\omega) \). Aarts et al. \( [25] \) independently have prepared this method, and have applied this method to calculate the electrical conductivity of QGP.

In Fig. 11 we show the schematic overview which summarizes the way to obtain the relaxation function and the relaxation time of QGP on the basis of the lattice QCD and the improved MEM analysis.

Then we explain the default model used in the present analysis. The default model \( m(\omega) \) is usually given as the form predicted by the perturbative QCD calculation valid in the high-energy region, \( \omega \rightarrow \infty \). Due to the tree-level result, \( A(\omega) = m_0^2 \) with \( m_0 \equiv 1/4 \pi^2 \), the default model reads

\[ m(\omega) = m_{\text{mono}}(\omega) \equiv \pi m_0 \omega. \]

Such a monotonically increasing default model seems suitable to avoid the artificial peaks due to the choice of the default model, and using \( m_{\text{mono}}(\omega) \) we can identify any peak structure of the inferred \( A(\omega) \) as that derived from the lattice data not the default model. In this paper, we are interested in the applicability of the Israel-Stewart equation to QGP, i.e., the validity of the ansatz that the spectral function for the conserved currents is approximated by a Lorentzian function shown in Eq. (13). To test this ansatz, we carry out the MEM analysis based on the lattice QCD data.
on not only the monotonically increasing default model \( m_{\text{mono}}(\omega) \) but also the following Lorentzian-type one especially in the low-energy region,

\[
m(\omega) = m_{\text{lore}}(\omega) = \frac{1}{2} \left( 1 - \tanh \frac{\omega - \omega_0}{\Delta \omega} \right) \frac{\chi}{1 + \omega^2 \tau_j^2} + \frac{1}{2} \left( 1 + \tanh \frac{\omega - \omega_0}{\Delta \omega} \right) \pi m_0 \omega,
\]

where \( \omega_0 \) and \( \Delta \omega \) denote the center value and width of the border between the strongly interacting low-energy region and the asymptotically free high-energy one, respectively, and \( \chi \) and \( \tau_j \) denote the transport coefficient and relaxation time of the baryon-number dissipative process, respectively. If \( \pi A(\omega)/\omega \) inferred by the MEM analysis based on \( m(\omega) = m_{\text{lore}}(\omega) \) is quite different from the Lorentzian function in the low-frequency region, the applicability of the Israel-Stewart equation to QGP is doubtful.

### B. Numerical Result and Discussion

The system considered here is the charm quark-gluon plasma at \( T > T_c \). We use the quenched approximation in calculating \( D(\tau) \). A detailed set up of the lattice QCD numerical simulation is identical to that in Ref.\(^7\). We use the naive plaquette gauge action and the standard Wilson quark action with the gauge coupling constant \( \beta = 7.0 \), the bare anisotropy \( \zeta_0 = 3.5 \) and the fermion anisotropy \( \gamma_F = \kappa_T / \kappa_\sigma = 3.476 \) with the spatial (temporal) hopping parameter \( \kappa_T = \kappa_\sigma = 0.0825 \). These parameters reproduce the anisotropic lattice \( a_\sigma / a_\tau = 4 \) with the temporal (spatial) lattice spacing \( a_\sigma = 9.75 \times 10^{-3} \text{fm} \) \( (a_\tau) \) in the quenched-level simulation \(^4\). The adopted lattice size is \( 20^3 \times 46 \), where the spatial size \( L_\sigma = 0.78 \text{fm} \) and the temperature \( T = 1.62 T_c \). This spatial size seems small. By varying the spatial boundary condition, however, we have checked that \( L_\sigma \) is large enough to simulate the charm quark-gluon plasma in lattice \(^3\). The number of the gauge configurations is 100.

The improved MEM analysis is done using the lattice data \( D(\tau) \) \( (\tau = \tau_1, \cdots, \tau_f) \) with \( \tau_i / a_\tau = 4 \) and \( \tau_f / a_\tau = 42 \) and the default models \( m_{\text{mono}}(\omega) \) in Eq.\(^{20}\) and \( m_{\text{lore}}(\omega) \) in Eq.\(^{24}\). In Fig.\(2\) we show the inferred \( \pi A(\omega)/\omega \)'s, which have been checked to be stable against the change of \( (\tau_i, \tau_f) \) as \((5, 41), (6, 40)\) and \((7, 39)\). Figure \(2\)\(a\) indicates that \( \pi A(\omega)/\omega \) has no strength in the low-frequency region \( \omega < 3 \text{GeV} \) and the peak structure with large width around \( \omega \sim 4.5 \text{GeV} \). Notice that due to \( m_{\text{mono}}(\omega) \) having no strength in \( \omega < 3 \text{GeV} \) and taking a zero value at \( \omega = 0 \), we cannot get any information about the applicability of the Israel-Stewart equation to the...
baryon-number dissipative process of the charm quark-gluon plasma and the smallness of the transport coefficient of the baryon-number dissipative process \( \chi \) from this result. On the other hand, the peak structure with large width around \( \omega \sim 4.5 \text{ GeV} \) can be considered to be a signal of the charmed vector hadron \( J/\Psi \) surviving above \( T_c \). This result is in good agreement with those of the several groups \[4,5,6,7\]. Figure 2(b) shows that \( \pi A(\omega)/\omega \) is quite different from \( m_{\text{bare}}(\omega) \), the Lorentzian function in the low-frequency region. This result suggests that the applicability of the Israel-Stewart equation to the baryon-number dissipative process of the charm quark-gluon plasma is questionable. We have checked the stability of the inferred \( \pi A(\omega)/\omega \) against the change of \( \omega_0, \Delta \omega, \chi \) and \( \tau_f \).

We calculate the relaxation function expected to be quite different from the exponentially damping function. For this purpose, we carry out the Fourier transformation of the inferred \( \pi A(\omega)/\omega \) in Fig.2(a). Since \( \pi A(\omega)/\omega \) exhibits a linear divergence of \( \omega \) for \( \omega \to \infty \), the numerical Fourier transformation is unstable. To avoid this numerical instability, we convert Eq. \((19)\) into

\[
R(t) = \frac{1}{2\pi} \int_0^{\omega_{\text{cut}}} d\omega \cos \omega t \left( \frac{\pi}{\omega} A(\omega) - m_{\text{mono}}(\omega) \right),
\]

where we have used \( A(\omega) = -A(-\omega) \) and the following relation,

\[
\int_0^{\infty} d\omega e^{-i\omega t} \omega = -\frac{1}{t^2} \text{ for } t \neq 0.
\]

In Eq. \((24)\), \( \omega_{\text{cut}} \) denotes the cut-off frequency defined as \( \pi A(\omega)/\omega - m_{\text{mono}}(\omega) \sim 0 \) for \( \omega > \omega_{\text{cut}} \). Here we use \( \omega_{\text{cut}} = 100 \text{ GeV} > \pi/a_\tau = 63 \text{ GeV} \). In Fig.3, we show the calculated \( R(t) \) together with the error bars. The singularity of \( R(t) = 0 \) is inevitable because it is originated from the divergence of \( \pi A(\omega)/\omega \) at \( \omega \to \infty \) due to the composite operator \( J^\mu(-i\tau, x) = \sum_{f=1}^{N_f} \sum_{c=1}^{N_c} \bar{\psi}_f (-i\tau, x) \gamma^\mu \psi(-i\tau, x) f_c \) being ill-defined at the same coordinate. Therefore, we concentrate on \( R(t) \) for \( t > a_\tau = 9.75 \times 10^{-3} \text{ fm} \). As expected, \( R(t) \) is not a strongly exponential damping function but a strongly-oscillation damping function. This oscillation damping behaviour of \( R(t) \) is attributed to the peak structure with the finite width in \( \pi A(\omega)/\omega \) correspondent to the surviving \( J/\Psi \) at \( T = 1.62 T_c \). The relaxation time \( \tau_{\text{osc}} \) of the oscillation damping function is approximated by the inverse of the width of the low-lying peak structure. Accordingly, we can estimate \( \tau_{\text{osc}} \sim 0.1 \text{ fm} \) from Fig.3(a), which seems to be consistent with \( R(t) \) in Fig.3. The disappearance of \( J/\Psi \) at the higher temperature would drive the drastic change of \( R(t) \) and \( \tau_{\text{osc}} \).

**FIG. 3:** The solid line denotes the relaxation function \( R(t)/T^2 \) with \( T = 1.62 T_c \). The meaning of the horizontal position and length of the bars is explained in the caption of Fig.2. \( R(t)/T^2 \) exhibits a strongly-oscillation damping behaviour within the error bars, instead of the exponentially damping behaviour derived from the Israel-Stewart equation.

C. Relaxation Equation Applicable to Strongly Interacting System

Finally we discuss the form of a relaxation equation corresponding to the spectral function with the peak structures in the finite-frequency region. Here we consider the spectral function for the shear viscous dissipative process at the rest frame as a typical example,

\[
\frac{\pi}{\omega} A(\omega, k) = \frac{\eta (\omega_\pi^2 + \tau_\pi^{-2})/2}{(\omega - \omega_\pi)^2 + \tau_\pi^{-2}} + \frac{\eta (\omega_\pi^2)}{(\omega + \omega_\pi)^2 + \tau_\pi^{-2}},
\]

where \( \omega_\pi (\geq 0) \) denotes the peak position. Notice that Eq. \((27)\) agrees with Eq. \((19)\) by the setting of \( \omega_\pi = 0 \). Using Eqs. \((11) \) and \((27)\), we have the correspondent complex admittance,

\[
\tilde{R}(t^0, k) = \eta \frac{\omega_\pi^2 + \tau_\pi^{-2}}{\tau_\pi^{-1}} \frac{-ik^0 + \tau_\pi^{-1}}{\omega_\pi^2 + (-ik^0 + \tau_\pi^{-1})^2}.
\]

By combining Eqs. \((12) \) and \((28)\), we find the form of the relaxation equation corresponding to the spectral function with the two peak structures at \( \omega = \pm \omega_\pi \) to be

\[
\left[ \omega_\pi^2 \tau_\pi^{-1} + (1 + \tau_\pi \partial_t)^2 \right] J(t, x)
= \eta \left( \omega_\pi^2 \tau_\pi^{-1} + 1 \right) (1 + \tau_\pi \partial_t) X(t, x).
\]

It is found that the temporal derivative \( \partial_t \) appears also in the right hand side of Eq. \((29)\), in contrast with the Israel-Stewart-type relaxation equation \((10)\). We identify Eq. \((29)\) as a more probable form of the relaxation equation applicable to the strongly interacting system including QGP slightly above \( T_c \), of which spectral function has peak structures in the finite-frequency region.
As shown in subsection 11.13, the baryon-number dissipative process of the charm quark-gluon plasma cannot be described by the Israel-Stewart equation. We should carry out the causal hydrodynamic simulations by incorporating the relaxation equation based on Eq. (29) not Eq. (10) into the continuity equation (17).

IV. SUMMARY AND CONCLUDING REMARKS

First, in this paper, we have pointed out that the Israel-Stewart causal hydrodynamic equation is acceptable only as long as the relaxation function is an exponentially damping function, and the relaxation time can be obtained as its damping time constant. Using the linear response theory, we have derived the formula where the relaxation function is represented by the spectral function. This fact suggests that the applicability of the non-equilibrium statistical operator method proposed by Zubarev [18] together with the maximum entropy method is normally conserved. We have shown an idea for the improvement of the Israel-Stewart equation by deriving the relaxation function with use of the formula in Eq. (10), consistent with the strongly-oscillation damping relaxation function.

Then, taking the baryon-number dissipative process of the charm quark-gluon plasma for a simple example, we have calculated the relaxation function from the spectral function obtained using the quenched lattice QCD together with the maximum entropy method. The obtained relaxation function shows the strongly-oscillation damping behaviour within the error bars, not the exponential damping. The origin of the strongly-oscillation damping behaviour is the charmed vector hadron $J/\Psi$ surviving at $T = 1.62 T_C$. This fact suggests that the applicability of the Israel-Stewart equation to the baryon-number dissipative process of the charm quark-gluon plasma is questionable. We have shown an idea for the improvement of the Israel-Stewart equation by deriving the relaxation equation consistent with the strongly-oscillation damping relaxation function.

We conclude that the Israel-Stewart equation is not unique as the causal and dissipative hydrodynamic equation. By calculating the relaxation function from the spectral function with use of the formula in Eq. (10), we can investigate the applicability of the Israel-Stewart equation. It is noteworthy that the form of the causal hydrodynamic equation with dissipative effects applicable to QGP is determined by the shape of the spectral function reflecting the hadronic excitations in QGP. This indicates an important link between the non-equilibrium properties of QGP and the hadron spectroscopy at the finite temperature.

In the next paper [30], we will investigate the properties of the relaxation equation corresponding to the spectral function with the peak structures in the finite-frequency region. It seems an interesting subject to derive the relativistic hydrodynamic equation by combining the relaxation equation shown in Eq. (29) and the continuity equation based on the energy-flow equation by Landau [11] or the particle-flow equation by Tsumura et al. [31, 32], which have a stable equilibrium solution.

To discuss the more realistic system than the charm quark-gluon plasma, we must investigate the contribution of up, down and strange quarks to the spectral function for the baryon-number conserved current. The light vector hadrons composed of these quarks slightly above $T_C$ seem to play a more essential role for the slow and long-wavelength fluid behaviour of QGP than $J/\Psi$. The answer to this conjecture will be reported elsewhere. In addition, the calculation of the relaxation function and/or the spectral function for the shear and bulk viscous dissipative process of QGP is an interesting future study using the lattice QCD with the maximum entropy method or another non-perturbative method, such as the QCD sum rule or the AdS/CFT duality.

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APPENDIX A: MICROSCOPIC REPRESENTATION OF RELAXATION FUNCTION

In this section, we show the derivation of Eq. (8) based on the linear response theory together with the non-equilibrium statistical operator method proposed by Zubarev [18].

We consider a statistical operator $\hat{\rho}(t) = |\Psi(t)\rangle\langle\Psi(t)|$, where $|\Psi(t)\rangle$ denotes a state of the system. The time evolution of $\hat{\rho}(t)$ is governed by the Liouville equation,

$$\partial_t \hat{\rho}(t) + i [\hat{H}, \hat{\rho}(t)] = 0,$$  \hspace{1cm} (A1)

or the Neumann equation, i.e., the equation converted from Eq. (A1),

$$\partial_t \ln \hat{\rho}(t) + i [\hat{H}, \ln \hat{\rho}(t)] = 0,$$  \hspace{1cm} (A2)

both of which are equivalent to the Schrödinger equation, $i \partial_t |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle$ with $\hat{H}$ being the Hamiltonian. It is noted that Eqs. (A1) and (A2) hold the time-reversal symmetry. Therefore, the entropy of the system defined by $S(t) \equiv \text{Tr}\rho(t)\ln \rho(t)$ is normally conserved.

To obtain the non-equilibrium statistical operator $\hat{\rho}_{neq}(t)$, which exhibits the increase of the entropy, the breaking of the time-reversal symmetry is needed. In Ref. [18], Zubarev has proposed to construct $\hat{\rho}_{neq}(t)$ as the solution of the Neumann equation (A2) under the time-reversal asymmetric boundary condition $\hat{\rho}(t \to -\infty) = \rho_{neq}(t \to -\infty|0)$, where $\rho_{neq}(t|0)$ denotes the quasi-
equilibrium statistical operator defined by

$$\hat{\rho}_{\text{eq}}(t_1 | t_2) \propto \exp \left[ - \int \dd^3 x \, a_\mu(t_1, x) \hat{T}^{\mu 0}(t_2, x) \right], \quad (A3)$$

with $\text{Tr} \, \hat{\rho}_{\text{eq}}(t_1 | t_2) = 1$. Here, $\hat{T}^{\mu \nu}(t, x)$ denotes a energy-momentum tensor operator in the Heisenberg picture, which agrees with that of the Schrodinger picture at $t = 0$. $a_\mu(t, x)$ denotes a Lagrange multiplier characterizing the system. The physical meaning of $a_\mu(t, x)$ becomes clear from the setting of $a_\mu(t, x) = g_{\mu 0} / T$, which reproduces the equilibrium statistical operator $\hat{\rho}_{\text{eq}} \equiv e^{-\hat{H} / T} / \text{Tr} \, e^{-\hat{H} / T}$ with $\hat{H} = \int \dd^3 x \, \hat{T}^{00}(0, x)$.

It is noted that imposing the time-reversal asymmetric boundary condition $\hat{\rho}(t \to -\infty) = \hat{\rho}_{\text{eq}}(t \to -\infty | 0)$ to Eq. (A2) is equivalent to adding a time-reversal asymmetric source term to the right hand side of Eq. (A2),

$$\partial_t \ln \hat{\rho}(t) + i [\hat{H}, \ln \hat{\rho}(t)] = - \epsilon \ln \left[ \frac{\hat{\rho}(t)}{\hat{\rho}_{\text{eq}}(t)[0]} \right], \quad (A4)$$

where $\epsilon$ denotes an infinitesimal positive constant and the limit of $\epsilon \to +0$ follows the thermodynamical limit. $\hat{\rho}_{\text{eq}}(t)$ can be obtained as $\hat{\rho}_{\text{eq}}(t) = \lim_{\epsilon \to +0} \hat{\rho}_\epsilon(t)$, where $\hat{\rho}_\epsilon(t)$ denotes the solution of Eq. (A1) and $\lim_{\epsilon \to +0}$ is suppressed later on. The solution of Eq. (A4) reads

$$\ln \hat{\rho}_{\text{eq}}(t) = \epsilon \int_{-\infty}^t \dd s \, e^{\epsilon(s-t)} \ln \hat{\rho}_{\text{eq}}(s | s-t). \quad (A5)$$

A partial integration of the right hand side of Eq. (A5) leads us to

$$\hat{\rho}_{\text{eq}}(t) = e^{\hat{A} + \hat{B}} / \text{Tr} \, e^{\hat{A} + \hat{B}}, \quad (A6)$$

where

$$\hat{A} \equiv - \int_0^t \dd^3 x \, a_\mu(t, x) \hat{T}^{\mu 0}(0, x), \quad (A7)$$

$$\hat{B} \equiv \int_{-\infty}^t \dd s \, e^{\epsilon(s-t)} \int \dd^3 x \, \partial_\mu a_\nu(s, x) \hat{T}^{\mu \nu}(s-t, x). \quad (A8)$$

Here we have used $\partial_\mu \hat{T}^{\mu \nu} = 0$.

Notice that $\hat{B}$ contains $\partial_\mu a_\nu$, i.e., the gradient of the Lagrange multiplier characterizing the system. We shall consider the system not so far from the equilibrium state, where $\partial_\mu a_\nu$ is small. It is important that under this consideration we can regard $\hat{B}$ as a perturbation term to $\hat{A}$ and utilize the linear response theory, where $\partial_\mu a_\nu$ is identified as an external field. We expand $\hat{\rho}_{\text{eq}}(t)$ up to the first order of $\hat{B}$ as follows,

$$\hat{\rho}_{\text{eq}}(t) = \left( 1 + \int_0^t \dd \lambda \, e^{+\lambda \hat{A}} \left( \hat{B} - \langle \hat{B} \rangle_A \right) e^{-\lambda \hat{A}} \right) \hat{\rho}_A, \quad (A9)$$

where $\hat{\rho}_A \equiv e^{\hat{A}} / \text{Tr} \, e^{\hat{A}}$ and $\langle \hat{O} \rangle_A \equiv \text{Tr} \hat{\rho}_A \hat{O}$ with an arbitrary operator $\hat{O}$. Notice that $\hat{\rho}_A = \hat{\rho}_{\text{eq}}(t[0])$.

Using Eq. (A9), we calculate the energy-momentum tensor realized in the non-equilibrium state,

$$\hat{T}^{\mu \nu}(t, x) \equiv \text{Tr} \, \hat{\rho}_{\text{eq}}(t) \hat{T}^{\mu \nu}(0, x). \quad (A10)$$

We decompose $\hat{T}^{\mu \nu}(t, x)$ into the unperturbative term and the perturbative one as $\hat{T}^{\mu \nu}(t, x) = \hat{T}_0^{\mu \nu}(t, x) + \delta \hat{T}^{\mu \nu}(t, x)$, where

$$\hat{T}_0^{\mu \nu}(t, x) \equiv \langle \hat{T}^{\mu \nu}(0, x) \rangle_A, \quad (A11)$$

$$\delta \hat{T}^{\mu \nu}(t, x) = \int \dd u \, \dd^3 y \, R^{\mu \nu \rho \sigma}(t, x | u, y) \times \partial_\rho a_\sigma(u, y). \quad (A12)$$

Here $\hat{T}_0^{\mu \nu}$ represents the perfect-fluid part of the energy-momentum tensor, while $\delta \hat{T}^{\mu \nu}$ the dissipative part.

Using Eq. (A12), we make a relaxation function defined by

$$R^{\mu \nu \rho \sigma}(t, x | u, y) = \theta(t-u) e^{-\epsilon(t-u)} \times \left[ \int_0^t \dd \lambda \langle \hat{T}^{\mu \nu}(0, x) e^{+\lambda \hat{A}} \hat{T}^{\rho \sigma}(u-t, y) e^{-\lambda \hat{A}} \rangle_A - \langle \hat{T}^{\mu \nu}(0, x) \rangle_A \langle \hat{T}^{\rho \sigma}(u-t, y) \rangle_A \right]. \quad (A13)$$

Equation (A13) is one of the main results in this section. As the thermal average used in Eq. (A13) is defined with use of $\hat{\rho}_A = \hat{\rho}_{\text{eq}}(t[0])$ not $\hat{\rho}_{\text{eq}}$, Eq. (A13) seems inconvenient to calculate $R^{\mu \nu \rho \sigma}(t, x | u, y)$ employing a technique established in the finite-temperature field theory. To derive $R^{\mu \nu \rho \sigma}(t, x | u, y)$ written by $\hat{\rho}_{\text{eq}}$, we shall make an approximation for Eq. (A12). We consider a situation where a fluctuation of $a_\mu(t, x)$ is small. Notice that this approximation differs from that used in the linear response theory, and is implemented in a systematic way using the expansion,

$$a_\mu(t, x) = \bar{a}_\mu + \epsilon \delta a_\mu(t, x), \quad (A14)$$

where $\epsilon$ is an infinitesimal constant different from $\epsilon$. We evaluate a leading order of $\delta \hat{T}^{\mu \nu}(t, x)$ with respect to $\epsilon$. Due to $\partial_\mu a_\sigma(u, y)$ being the first order of $\epsilon$, the zeroth-order evaluation of $R^{\mu \nu \rho \sigma}(t, x | u, y)$ is sufficient.

First, using $a_\mu(t, x) = \bar{a}_\mu$, we have

$$\hat{A} = -\bar{a}_\mu \hat{P}^\mu, \quad (A15)$$

where the energy-momentum vector operator $\hat{P}^\mu \equiv \int \dd^3 x \, \hat{T}^{0 \mu}(0, x)$. With use of the unitary transformation for the temporal and spatial translation by $\hat{P}^\mu$,

$$e^{+\lambda \hat{A}} \hat{O}(t, x) e^{-\lambda \hat{A}} = \hat{O}(t + i \lambda \bar{a}, x + i \lambda \bar{a}), \quad (A16)$$

and the cluster property of the correlation function,

$$\langle \hat{O}(0, x) \hat{O}(t, y) \rangle_A = \langle \hat{O}(0, x) \rangle_A \langle \hat{O}(t, y) \rangle_A \quad \text{for} \quad |t| \to \infty, \quad (A17)$$
we can convert the right hand side of Eq. (A13) into
\[
\theta(t-u) e^{-\epsilon(t-u)} \int_0^1 d\lambda \int_{-\infty}^0 ds \langle \hat{T}^{\mu\nu}(0, x) \times \partial_s \hat{T}^{\rho\sigma}(u - t + (i\lambda + s)\bar{a}^0, y + (i\lambda + s)a) \rangle_A. \tag{A18}
\]
By replacing \( \partial_s \rightarrow -i \partial_\lambda \), integrating with respect to \( \lambda \) and using the translational invariance, we find Eq. (A18) to be
\[
\theta(t-u) e^{-\epsilon(t-u)} \int_{-\infty}^0 ds \times i \langle \hat{T}^{\mu\nu}(t-u, x-y), \hat{T}^{\rho\sigma}(s\bar{a}^0, s\bar{a}) \rangle_A. \tag{A19}
\]
Using Eq. (A19), we can rewrite Eq. (A12) as a Lorentz-covariant form,
\[
\delta T^{\mu\nu}(x) = \int d^4y R^{\mu\nu\rho\sigma}(x-y) \partial_\rho \rho_\sigma(y), \tag{A20}
\]
\[
R^{\mu\nu\rho\sigma}(x) \equiv \theta(x^0) e^{-\epsilon x^0} \int_{-\infty}^0 ds \times i \langle \hat{T}^{\mu\nu}(x), \hat{T}^{\rho\sigma}(s\bar{a}) \rangle_A. \tag{A21}
\]
Then, using \( \rho_\mu = u_\mu / T (u_\mu u^\mu = 1) \), we derive \( R^{\mu\nu\rho\sigma}(x) \) written by \( \rho_\rho_\sigma \). For this purpose, we utilize a decomposition of \( \delta T^{\mu\nu}(x) \) based on the irreducible tensors \( u^\mu, \Delta^{\mu\nu} = \delta^{\mu\nu} - u^\mu u^{\nu} \) and \( \Delta^{\mu\nu\rho\sigma} = 1/2 (\Delta^{\rho\nu} \Delta^{\mu\sigma} + \Delta^{\mu\rho} \Delta^{\nu\sigma} - 2/3 \Delta^{\mu\nu} \Delta^{\rho\sigma}) \), namely,
\[
\delta T^{\mu\nu}(x) = E(x) u^\mu u^{\nu} - \Pi(x) \Delta^{\mu\nu} + Q^\rho(x) u^{\rho} + Q^\nu(x) u^{\nu} + \pi^{\mu\nu}(x), \tag{A22}
\]
where \( E(x) \equiv \delta T^{ab}(x) u_a u_b \), \( \Pi(x) \equiv -\delta T^{ab}(x) \Delta_{ab} / 3 \), \( Q^\rho(x) \equiv \delta T^{ab}(x) u_a \Delta_{b\rho} \) and \( \pi^{\mu\nu}(x) \equiv \delta T^{ab}(x) \Delta_{ab\mu
u} \).

Here we focus the shear viscous stress \( \pi_{\mu\nu}(x) \). Using Curie’s principle \(^{29}\) and the spatial isotropy, we have
\[
\langle [\tilde{\pi}^{\mu\nu}(x), \tilde{T}^{\rho\sigma}(s u/T)] \rangle_A = \frac{\Delta^{\mu\nu\rho\sigma}}{10} \langle [\hat{\pi}^{ab}(x), \hat{\pi}_{ab}(s u/T)] \rangle_A. \tag{A23}
\]
where \( \tilde{\pi}^{\mu\nu} \equiv \Delta^{\mu\nu ab} \hat{T}_{ab} \). This relation leads us to
\[
\pi_{\mu\nu}(x) = \Delta^{\mu\nu\rho\sigma} \int d^4y R(x-y) 2 \partial^\rho u^\sigma(y), \tag{A24}
\]
\[
R(x) \equiv \theta(x^0) e^{-\epsilon x^0} \int_{-\infty}^0 ds \times i \frac{1}{10T} \langle [\tilde{\pi}^{\mu\nu}(x), \tilde{T}_{\mu\nu}(s u/T)] \rangle_A. \tag{A25}
\]
It is noted that, due to the integrand of \( R(x) \) in Eq. (A25) being a Lorentz-scalar function, the integrand takes the same value for an arbitrary \( u^\mu \). Therefore, we can set \( u^\mu = (1, 0, 0, 0) \) in Eq. (A25) without loss of generality, and using the spatial isotropy we arrive at
\[
R(x) = R(t, x) = \theta(t) e^{-\epsilon t} \int_{-\infty}^0 ds \times i \langle \hat{T}(t, x), \hat{T}(s, 0) \rangle. \tag{A26}
\]
Equation (A26) is the other of the main results in this section.
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