The Redshift Dependence of the Alcock–Paczynsky Effect: Cosmological Constraints from the Current and Next Generation Observations

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Abstract

The tomographic Alcock–Paczynski (AP) test is a robust large-scale structure (LSS) measurement that receives little contamination from the redshift space distortion. It has placed tight cosmological constraints by using small and intermediate clustering scales of the LSS data. However, previous works have neglected the cross-correlation among different redshift bins, which could cause the statistical uncertainty being underestimated by ~20%. In this work, we further improve this method by including this multi-redshift’s full correlation. We apply it to the SDSS DR12 galaxies sample and find out that, for ΛCDM, the combination of AP with the Planck+BAO data set slightly reduces (within 1σ) Ω_m to 0.304 ± 0.007 (68.3% CL). This then leads to a larger H_0 and also mildly affects Ω_b h^2 and n_s as well as the derived parameters z_s, r_s, and z_0 but not τ, A_s, and σ_8. For the flat wCDM model, our measurement gives Ω_m = 0.301 ± 0.010 and w = −1.090 ± 0.047, where the additional AP measurement reduces the error budget by ~25%. When including more parameters into the analysis, the AP method also improves the constraints on Ω_c, Σ m, and Ω_m by 20%–30%. Early universe parameters such as d_n / d ln k and r, however, are unaffected. Assuming the dark energy equation of state w = w_0 + w_a z, the Planck+BAO+SNe Ia+H0+AP data sets prefer a dynamical dark energy at ≈1.5σ CL. Finally, we forecast the cosmological constraints expected from the DESI galaxy survey and find that combining AP with the CMB+BAO method would improve the w_0-w_a constraint by a factor of ~10.

Key words: cosmological parameters – dark energy – large-scale structure of universe

1. Introduction

The discovery of cosmic acceleration (Riess et al. 1998; Perlmutter et al. 1999) implies either the existence of a “dark energy” component in our universe or the breakdown of general relativity on cosmological scales (see Yoo & Watanabe 2012, for a recent review). The theoretical explanation and observational probes of cosmic acceleration have attracted tremendous attention, and are still far from being well understood or accurately measured (Weinberg 1989; Weinberg et al. 2013; Li et al. 2011; Miao & Huang 2018).

In an effort to probe the cosmic expansion history, large-scale structure (LSS) surveys are utilized to extract information about two key geometrical quantities; the angular diameter distance D_A and the Hubble factor H. If they were precisely measured as functions of redshift, then tight constraints can be placed on cosmological parameters, like the matter density Ω_m and the equation of state (EoS) of dark energy w.

The Alcock–Paczynski (AP) test (Alcock & Paczynski 1979) provides a geometric probe of D_A and H. Given a certain cosmological model, the radial and tangential sizes of distance objects or structures can be computed as Δr_A = r_A(z) Δz and Δr = (1 + z) D_A(z) Δθ, where Δz and Δθ are the observed redshift span and angular size, respectively. When incorrect cosmological models are assumed for transforming galaxy redshifts into comoving distances, the wrongly estimated Δr_A and Δr induces geometric distortion (see Figure 1). In galaxy redshift surveys, measuring the galaxy clustering in the radial and transverse directions enables us to probe the AP distortion, and thus place constraints on cosmological parameters (Ryden 1995; Ballinger et al. 1996; Matsubara & Suto 1996; Outram et al. 2004; Blake et al. 2011; Lavaux & Wandelt 2012; Alam et al. 2017; Mao et al. 2017).

The main difficulty of the AP test is that the radial distances of galaxies, inferred from their observed redshifts, are inevitably affected by the galaxy peculiar motions. This leads to apparent anisotropies in the clustering signal even if the adopted cosmology is correct. This effect, known as redshift space distortions (RSD), is usually much more significant than the AP distortion, and is notoriously difficult to be accurately modeled in the statistics of galaxy clustering (Ballinger et al. 1996; Jennings et al. 2011).

As a complementary method to apply the AP test, Marinoni & Buzzi (2010) proposed to statistically study a large number of galaxy pairs and search for the deviation from a symmetric distribution of direction; however, since the peculiar velocity distorts the observed redshifts and changes the apparent tilt angles of galaxy pairs. This method is also seriously limited by RSD (Jennings et al. 2011). In an effort to minimize RSD contamination, the shape of void regions (Ryden 1995; Lavaux & Wandelt 2012) has also been proposed as an AP probe. This approach has the advantage that the void regions are easier to model compared with dense regions, but has limitations in that it utilizes only low density regions of the LSS, and requires large samples to attain statistical significances and achieve competitive constraints (Mao et al. 2017).
Recently, a novel method of applying the AP test by investigating the redshift dependence of the distortion was proposed by Li et al. (2014, 2015). The method is motivated by Park & Kim (2010), where the authors found that adopting the wrong set of cosmological parameters would produce redshift-dependent distortion in the LSS. Li et al. (2014, 2015) applied this idea to the AP test analysis. The authors found that, on one hand, the anisotropies produced by the RSD effect are, although large, maintaining a nearly uniform magnitude over a large redshift range; on the other hand, the degree of anisotropies produced by AP varies much more significantly. So they developed a method searching for the AP distortion from the redshift evolution of the anisotropies in LSS.

A consequence of reducing the RSD effect is that, by avoiding the complex modeling of galaxy position and velocity distributions, it becomes possible to use galaxy clustering on scales as small as $6-40\ h^{-1}\ Mpc$. In this region, there exists a large amount of clustered structures (Zhang et al. 2019); thus enabling us to derive tight constraints on cosmological parameters. This large amount of information can hardly be utilized by other LSS statistical methods.

The first application of this AP method (hereafter the tomographic AP method) to observational data was performed in Li et al. (2016). The authors split the 1.13 million Sloan Digital Sky Survey (SDSS) Data Release 12 (DR12) galaxies into six redshift bins, measured their anisotropies 2PCFs, and quantified the redshift evolution of anisotropy. In combination with the data sets of cosmic microwave background (CMB), SNe Ia, baryon acoustic oscillations (BAO), and the $H_0$ measurements, the authors obtained $\Omega_m = 0.301 \pm 0.006$, $w = -1.054 \pm 0.025$ in a flat universe with cold dark matter and constant EoS dark energy components (hereafter wCDM). The error bars are reduced by as much as 40% by adding the method into the combination of CMB+SNe Ia+BAO+$H_0$.

As a follow-up study, Li et al. (2018) improved the method by developing a technique accurately approximating the 2PCFs in different cosmologies. This greatly reduces the computational expense of the 2PCFs, and thus enables the exploration of models with three or more parameters. Li et al. (2018) applied the method to constrain a model of dynamical dark energy $EoS\ w(z) = w_0 + \frac{w_a}{1+z}$ (hereafter $w_0$-$w_a$CDM), and improved the Planck+BAO+SNe Ia+$H_0$ constraint on $w_0$-$w_a$ by as much as 50%. Furthermore, in a very recent work, Zhang et al. (2019) combined the tomographic AP method with the BAO measurements, and obtained a Hubble constant $H_0 = 67.78^{+1.21}_{-1.86} \ km\ s^{-1} Mpc^{-1}$ (2.26% precision). The inclusion of AP reduces the error bar by 32%.

As a newly developed technique, the tomographic AP method shows promising potential in constraining cosmological parameters. However, it is still far from becoming as mature as the BAO method. To summarize, the method needs to be improved in three aspects:

1. We shall improve its methodology, by enhancing its statistical power, better understanding and estimating the systematical effects, and reducing the computational cost, etc.
2. So far the method has only been used to constrain a limited set of parameters, including $\Omega_m$, $w(z)$, and $H_0$. It is undoubtedly desirable to extend its application to more parameters and models.
3. Given that there are many ongoing and planned LSS surveys including DESI (Aghamousa 2016), EUCLID (Laureijs et al. 2011), and HETDEX (Hill et al. 2008), it is necessary to forecast the constraining power when the method is applied to these surveys.

In this work we explored all these three issues. In Section 2, we showed that the methodology used in Li et al. (2014, 2015, 2016, 2018) neglected the correlations among different redshift bins, which leads to an overestimation of statistical power and a large statistical fluctuation. We proposed a full covariance matrix approach to solve this problem and make the analysis statistically more perfect. In Section 3, we performed a comprehensive study on its constraints on a series of cosmological parameters. In Section 4, we forecast the cosmological constraints expected from the DESI survey. Conclusions of our work are given in Section 5.

2. Data and Methodology

The data and methodology used in this work are similar to those used in Li et al. (2014, 2015, 2016, 2018), except that we more completely evaluate the statistical uncertainties.
2.1. Data

2.1.1. SDSS DR12 Galaxies

The Sloan Digital Sky Survey (York et al. 2000), as the currently largest spectroscopic galaxy survey, has obtained spectra for more than three million astronomical objects. This created the most detailed three-dimensional maps of the universe ever made. BOSS (Baryon Oscillation Spectroscopic Survey; Dawson et al. 2012; Smee et al. 2013), as a part of the SDSS-III survey (Eisenstein et al. 2011), has obtained spectra and redshifts of 1.37 million galaxies selected from the SDSS imaging, covering a sky region of 9376 deg$^2$ and a redshift span of $0.1 < z < 0.75$. Its wide redshift coverage and large amount of galaxies makes it the best material for performing the tomographic analysis.

Following Li et al. (2016), we use the spectroscopic galaxy sample of SDSS-III BOSS DR12, containing the LOWZ catalog at $0.1 < z < 0.45$ and the CMASS catalog covering $0.4 < z < 0.7$ (Reid et al. 2016). For the purpose of a tomographic clustering analysis, we split the sample into six, nonoverlapping redshift bins of $0.15 < z_1 < 0.274 < z_2 < 0.351 < z_3 < 0.430 < z_4 < 0.511 < z_5 < 0.572 < z_6 < 0.693$. The total number of galaxies used in the analysis is 1,133,326.

2.1.2. Horizon Run 4 Mocks

We rely on the Horizon Run 4 (HR4; Kim et al. 2015) to estimate and correct the systematics. HR4 is a large $N$-body simulation with box size $L = 3150 h^{-1}$Mpc and number of particles 6300$^3$, produced under the WMAP5 (Komatsu et al. 2011) cosmological parameters ($\Omega_b$, $\Omega_m$, $\Omega_{\Lambda}$, $h$, $\sigma_8$, $n_s$) = (0.044, 0.26, 0.74, 0.72, 0.79, 0.96). Mock galaxy samples are then created using a modified version of the one-to-one correspondence scheme (Hong et al. 2016). Comparing the 2pCF of the mocks to the SDSS DR7 volume-limited galaxy sample (Zehavi et al. 2011), we found the simulated 2pCF shows a finger of god (FOG) feature (Jackson 1972) rather close to the observation. The projected 2pCF agrees with the observation within 1\sigma deviation on scales greater than $1 h^{-1}$Mpc Hong et al. (2016).\footnote{The boundaries are determined so that the number of galaxies are roughly the same in different redshift bins (for LOWZ and CMASS samples, respectively).}

2.1.3. MultiDark PATCHY Mocks

We utilize the set of 2000 MultiDark PATCHY mock catalogs (Kitaura et al. 2016) from the dark matter simulation to the covariance matrix. The MultiDark PATCHY mocks are produced using approximate gravity solvers and analytical-statistical biasing models, calibrated to the BigMultiDark $N$-body simulation (Klypin et al. 2016). The mock surveys can well reproduce the number density, selection function, survey geometry, and 2pCF measurement of the BOSS DR12 catalogs, and have been adopted in a series of works (see Alam et al. 2017, and references therein) to conduct clustering analysis of BOSS galaxies.

2.1.4. DESI Galaxies

The DESI Aghamousa (2016) observational program is a future project measuring the baryon acoustic feature of the LSS, as well as the distortion effects of redshift space. DESI provides high precision measurements of the universe’s expansion rate up to $z \sim 1.5$. The baseline assumption is that it runs over an approximately five year period covering 14,000 deg$^2$ in area. The DESI survey makes observations of four types of objects:

1. a magnitude-limited Bright Galaxy Survey (BGS; $0.05 < z < 0.45$) comprising approximately 10 million galaxies;
2. bright emission line galaxies (ELGs; up to $z = 1.65$) probing the universe out to even higher redshift;
3. luminous red galaxies (LRGs; up to $z = 1.15$), which extend the BOSS LRG survey in both redshift and survey area; and
4. quasi-stellar objects (QSOs) as direct tracers of dark matter in the redshift range of $0.65 < z < 1.85$.

The number density distribution of these galaxies is shown in Figure 2. Our forecast in Section 4 is based on these numbers.

2.2. Methodology

2.2.1. Quantifying the Anisotropy

Li et al. (2016; hereafter Li16) split the BOSS DR12 galaxies into six redshift bins, and computed the integrated 2pCF in

![Figure 2. Expected redshift distribution of the galaxy sample from the DESI survey. We plotted the number of galaxies $N_{gal}$ in 30 redshift bins. Among them, the 18 redshift bins of galaxies at $z \lesssim 1.9$ are used to forecast the performance of AP if applied to them. For comparison, the SDSS BOSS galaxies $N_{gal}$ are also plotted.](image-url)
each bin
\[ \xi_{\Delta_s}(\mu) \equiv \int_{s_{\text{min}}}^{s_{\text{max}}} \xi(s, \mu) \, ds, \]
where the correlation function \( \xi \) is measured as a function of \( s \), the distance separation of the galaxy pair, and \( \mu = \cos(\theta) \), with \( \theta \) being the angle between the line joining the pair of galaxies and the line-of-sight (LOS) direction to the target galaxy. The range of integration was chosen as \( s_{\text{min}} = 6 \, h^{-1} \text{Mpc} \) and \( s_{\text{max}} = 40 \, h^{-1} \text{Mpc} \). By focusing on the redshift dependence of anisotropy, the RSD effect is largely reduced, and it becomes possible to use the galaxy clustering down to \( 6 \, h^{-1} \text{Mpc} \).

To mitigate the systematic uncertainty from galaxy bias and clustering strength, Li16 further normalized the amplitude of \( \xi_{\Delta_s}(\mu) \) to focus on the shape, i.e.,
\[ \tilde{\xi}_{\Delta_s}(\mu) \equiv \frac{\xi_{\Delta_s}(\mu)}{\int_0^{s_{\text{max}}} \xi_{\Delta_s}(\mu) \, d\mu}. \]
A cut \( \mu < \mu_{\text{max}} \) is imposed to reduce the fiber collision and FOG effects.

### 2.2.2. The Redshift Evolution of Anisotropy

The redshift evolution of anisotropy, between the \( i \)-th and \( j \)-th redshift bins are quantified as
\[ \delta\tilde{\xi}_{\Delta_s}(z_i, z_j, \mu) \equiv \tilde{\xi}_{\Delta_s}(z_i, \mu) - \tilde{\xi}_{\Delta_s}(z_j, \mu), \]
where the systematics of \( \delta\tilde{\xi}_{\Delta_s} \) (hereafter \( \delta\tilde{\xi}_{\Delta_s, \text{sys}} \)), which mainly comes from the redshift evolution of the RSD effect, is measured from the Horizon Run 4 (Kim et al. 2015) N-body simulation and subtracted.

### 2.2.3. “Part-cov” Approach of Likelihood

To quantify the overall redshift evolution in the sample, Li16 chose the first redshift bin as the reference, then compared the measurements in higher redshift bins with it and summed up these differences. So we have the following \( \chi^2 \) function describing the total of evolution
\[ \chi^2_{\text{AP, part-cov}} = \sum_{i=2}^{6} \sum_{j=1}^{n_i} \sum_{k=1}^{n_j} p(z_i, \mu_{j_k}) \cdot (\text{Cov}^{-1})_{j_k,j_k} \cdot p(z_i, \mu_{j_k}), \]
where \( n_j \) denotes the binning number of \( \tilde{\xi}_{\Delta_s}(\mu) \), and \( p(z_i, \mu_{j_k}) \) is defined as
\[ p(z_i, \mu_{j_k}) \equiv \delta\tilde{\xi}_{\Delta_s}(z_i, z_{j_k}, \mu_{j_k}) - \delta\tilde{\xi}_{\Delta_s, \text{sys}}(z_i, z_{j_k}, \mu_{j_k}). \]
The covariance matrix \( \text{Cov} \), estimated from the 2000 MultiDark-Patchy mocks (Kitaura et al. 2016). In wrong cosmologies, the AP effect produces large evolution of clustering anisotropy, thus would be disfavored due to a large \( \chi^2 \) value.

In Equation (4) we labeled the \( \chi^2 \) function by the \( \chi^2 \) analysis by “part-cov,” to denote that this \( \chi^2 \) analysis does not include the correlations among different \( p(z_i) \) s. The different \( p(z_i) \) s are actually correlated with each other, in the sense that (1) since all the \( p(z_i) \) s takes the first redshift bin as the reference, the fluctuation in the first bin enters all \( p(z_i) \) s and makes them statistically correlate with each other; (2) the LSS at different redshift bins have correlations even if they are not overlapping with each other (galaxies lying near the boundary of a redshift bin have been affected by galaxies in the both nearby bins in the past structure formation era).

We tested and found that ignoring (2) does not lead to significant changes in the results, so it is a minor effect. But ignoring (1) leads to ~20% misestimation of the statistical error. The “part-cov” method relies on the first redshift bin as the reference. If this bin happened to have a large deviation (i.e., due to statistical fluctuation) from its statistical expectation, then all \( p \) s would be affected. This creates statistical error in the results, which have not been included in Li et al. (2016, 2018).

Appendix A shows the misestimation and large fluctuation of this part-cov approach. They would be overcome if we include the correlations among the \( p(z_i) \) s in the analysis.

#### 2.2.4. “Full-cov” Approach of Likelihood

We adopt the following formula to represent the \( \chi^2 \) function including all correlations,
\[ \chi^2_{\text{AP, full-cov}} = P \cdot \text{Cov} \cdot P, \]
where
\[ P = (\bar{p}(z_2, \mu_j), \ldots, \bar{p}(z_6, \mu_j)), \]
a vector containing \( (n_i - 1) \times (n_j) \) components, is built by joining all \( \bar{p}(z_i, \mu_{j_k}) \)s together. Here we redefine the \( P \) as the evolution between the nearby redshift bins, i.e.,
\[ \bar{p} \equiv \delta\tilde{\xi}_{\Delta_s}(z_i, z_{j_k}) - \delta\tilde{\xi}_{\Delta_s, \text{sys}}(z_i, z_{j_k}). \]
Actually, the results do not change if we still use the original \( P \) defined in Equation (5).\(^9\) We redefine \( P \) as \( \bar{p} \) so that the formula explicitly has no special redshift bin chosen as the reference.

#### 2.2.5. The Covariance Matrix

The covariance matrix \( \text{Cov} \), estimated from the MultiDark mocks, was shown in Figure 3. The upper panel shows the covariance matrix when we split \( \xi_{\Delta_s} \) into 20 bins in \( \mu \) space, while the lower panel shows the normalized covariance matrix (i.e., the correlation coefficient).

We find the following:
1. Since we have six redshift bins, in total we need \( 6 \times 1 = 5 \) \( \delta\xi_{\Delta_s, \text{sys}} \) to characterize the evolution among them. So the plot of total covariance matrix contains \( 5 \times 5 = 25 \) regions of “cells.”
2. The five “diagonal cells” describe the \( 20 \times 20 \) auto covariance matrix of \( \delta\xi_{\Delta_s}(\mu_j) \), \( \mu \)-bins close to each other have strong positive correlations, while \( \mu \)-bins very far away from each other have negative correlations imposed by the normalization condition.
3. The 20 “non-diagonal boxes” describe the cross-correlation among different \( \delta\xi_{\Delta_s} \) s. They have nonzero values.
4. Those \( \delta\xi_{\Delta_s, \text{sys}} \) s that have overlapping redshift bins are strongly correlated since they contain the same \( \xi_{\Delta_s}(\mu, z) \) (e.g., \( \xi_{\Delta_s}(\mu, z) \) and \( \xi_{\Delta_s, \text{sys}}(\mu, z) \) both depend on the \( \xi_{\Delta_s} \)).

\(^8\) The redshift evolution of anisotropy from RSD is, in general, smaller than those from AP (Li et al. 2014, 2015). But it still creates small redshift dependence in \( \xi_{\Delta_s}(\mu) \) (Li et al. 2014, 2015).

\(^9\) Once we include the full covariance matrix, the result does change no matter how we linearly transform the \( p \) s.
3. Cosmological Constraints

The cosmological constraints are derived from the likelihood method described in Section 2.2. We divide our discussion into two sections. In Section 3.1 we presented the constraints on the background parameters of \( w \), \( w_0 \), \( w_a \), \( \Omega_m \), and \( H_0 \), within the framework of \( \Lambda \)CDM, \( w \)CDM, and \( w_0w_a \)CDM models, respectively. These parameters are directly constrained by the AP method, which measures the geometry of the universe in the late time expansion era.\(^{10}\) In Section 3.2, we extend the scope and explore the other cosmological parameters, including the \( \Lambda \)CDM parameters and their derivations, and the 1-parameter extensions to \( \Lambda \)CDM and \( w \)CDM models.

We present the cosmological constraints when the AP likelihood is combined with several external data sets, including the full-mission Planck observations of CMB temperature and polarization anisotropies (Ade et al. 2016), the BAO distance priors measured from SDSS DR11 (Anderson et al. 2014), 6dFGS (Beutler et al. 2011), and SDSS MGS (Ross et al. 2015), the “JLA” SNe Ia sample (Betoule et al. 2014), and the Hubble Space Telescope measurement of \( H_0 = 70.6 \pm 3.3 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \) (Riess et al. 2011; Efstathiou 2014). They are exactly the same data sets as those used in Li et al. (2016, 2018).

### 3.1. Constraints on Background Parameters

Table 1 summarizes the Planck+BAO and Planck+BAO+AP constraints on the background parameters \( w \), \( w_0 \), \( w_a \), \( \Omega_m \), and \( H_0 \), within the framework of \( \Lambda \)CDM, \( w \)CDM, and \( w_0w_a \)CDM models. In what follows, we discuss them in detail.

#### 3.1.1. \( \Lambda \)CDM Parameters

The \( \Lambda \)CDM model with EoS \( w_\Lambda = -1 \) is the simplest candidate among a large number of dark energy models, with the Hubble parameter taking the form of

\[
H(z) = H_0 [\Omega_m (1 + z)^3 + \Omega_{\Lambda}]^{1/2},
\]

where \( \Omega_m + \Omega_\Lambda = 1 \) (we neglect curvature and radiation). Although the \( \Lambda \)CDM model seriously suffers from the theoretical fine tuning and coincidence problems (Weinberg 1989), it is in good agreement with most of the current observational data (Li et al. 2011).

Table 1 lists the constraints on \( \Omega_m \), \( H_0 \) derived from Planck+BAO and Planck+BAO+AP, respectively. Including AP in the analysis leads to a \( \lesssim 1 \sigma \) shift in the central values of \( \Omega_m \), \( H_0 \), i.e., from (0.310, 67.6) to (0.304, 68.1), respectively.

#### 3.1.2. \( w \)CDM Parameters

The simplest generalization to \( \Lambda \) is considering a constant dark energy EoS \( w \), and the Hubble parameter is given by

\[
H(z) = H_0 [\Omega_m (1 + z)^3 + \Omega_{de}(1 + z)^{3(1 + w)}]^{1/2},
\]

where \( \Omega_{de} \) is the current value of the dark energy density. If \( w = -1 \), then \( w \)CDM reduces to \( \Lambda \)CDM, with \( \Omega_{de} = \Omega_\Lambda \).

The upper panel of Figure 4 illustrates the constraint on \( w \) from the combination of Planck+BAO, Planck+BAO+AP, and Planck+BAO+SNe Ia+H0+AP. The mean values as well as the 68% and 95% limits are

\[
w = -1.031^{+0.067}_{-0.057} \quad \text{Planck + BAO};
\]

\[
w = -1.089^{+0.040}_{-0.054} \quad \text{Planck + BAO+AP};
\]

\[
w = -1.054^{+0.046}_{-0.052} \quad \text{SNeIa + H0 + AP}. \]

If we describe the decrement in the error bar (or equivalently, improvement in the precision) by \( \frac{\sigma_{w_{\Lambda}} - \sigma_{w_{\Lambda}+\text{AP}}}{\sigma_{w_{\Lambda}+\text{AP}}} \), then adding AP

\(^{10}\) There is one exception. The AP method alone cannot put constraints on \( H_0 \). The change in \( H_0 \) corresponds to a uniform rescaling of LSS, and produces no anisotropy. But the AP method can improve its constraint by breaking its degeneracy with other parameters (Zhang et al. 2019).
The uncertainties

Note.

contours + w panel: the constraint on
Mean Values and 68% Confidence Limits of Cosmological Parameters for the ΛCDM, wCDM, and w0wCDM CDM Models, from Combinations of Planck+BAO and Planck+BAO+AP

| Model       | Planck+BAO | +AP | Planck+BAO | +AP | Planck+BAO | +AP | Planck+BAO | +AP |
|-------------|------------|-----|------------|-----|------------|-----|------------|-----|
| Ω_m        | 0.3102(64) | 0.3041(67) | 0.306(13)  | 0.293(10) | 0.351(29)  | 0.300(17) | 0.351(29)  | 0.300(17) |
| H_0         | 67.63(48)  | 68.08(52)  | 68.3 ± 1.5 | 69.8 ± 1.2 | 64.0 ± 2.7 | 69.3 ± 1.7 | -0.51(30)  | -0.95(18)  |
| w_0         | -1.03(63)  | -1.090(47) | -1.031(63) | -1.090(47) | -0.51(30)  | -0.95(18)  | -1.47(83)  | -0.53(53)  |

Note. The uncertainties (of the last 2 digits of the numbers) are listed in the brackets.

Figure 4. Cosmological constraints within the framework of wCDM. Upper panel: the constraint on w from the combinations of Planck+BAO, Planck+BAO+AP, and Planck+BAO+SNe Ia+H_0+AP. Lower panel: likelihood contours (68.3% and 95.4% CL) in the Ω_m−w plane.

into the Planck+BAO combination reduces the errors by ~30%. The inclusion of AP also shifts the constraint toward negative EoS by ~1σ, making the results marginally consistent with w = -1 in 2σ. Further adding the SNe Ia and H_0 “pulled back” toward w = -1.

The lower panel of Figure 4 shows the marginalized constraint in the Ω_m−w plane. We see a positive degeneracy between the two parameters, and a shift of w toward negative values. Correspondingly, a smaller amount of Ω_m is preferred.

It is commonly believed that since the CMB data helps the BAO method to determine the absolute value of the sound horizon, tight constraints can be achieved if the two are combined. So we will use Planck+BAO as a “standard combination” and check how much the constraints improve after adding AP. In fact, because CMB and AP constrain different epochs of expansion history, combining them can also effectively reduce the uncertainties of parameters. This can be seen from Figure 10 of Li et al. (2016), where we find the Planck and AP contours have orthogonal directions of degeneracy in the Ω_m−w space. Actually, combining these two constrains give

\[ Ω_m = 0.295 ± 0.015, \quad H_0 = 69.7 ± 1.7, \quad w = -1.08 ± 0.05, \]

which are as tight as the Planck+BAO results.

The derived constrained from the full-covmat analysis is consistent with Li et al. (2016) except that here we obtained a larger error bar, mainly due to the inclusion of full covariance. A comparison between the two sets of results is presented in Appendix B.

To ensure the robustness of the results, we have conducted a series of checks about the systematics and the options of the AP analysis. We do not find any statistically significant effect on the derived results. These tests are briefly discussed in Appendix B.

3.1.3. w_0w_α CDM Parameters

We move on to a further generalization and consider a dynamical EoS dependence on z. As a simplest parameterization widely used in the literature, one can consider the first-order Taylor expansion of w_de with respect to (1 − a), i.e.,

\[ w_{de}(z) = w_0 + w_α(1 − a) = w_0 + w_α \frac{z}{1 + z}, \]

which is the well-known Chevallier–Polarski–Linder parameterization proposed by Chevallier & Polarski (2001) and Linder (2003). The Hubble parameter is

\[ H(z) = H_0[Ω_m(1 + z)^3 + Ω_{de}(1 + z)^{3(1+w_α+w_0)}e^{-3\frac{H_0}{H_0^2}}]^{1/2}. \]

Figure 5 shows the constraints on this dynamical dark energy model. The Planck+BAO combination cannot lead to effective constraints on the w_0+w_α parameters. We only obtain two weak bounds of w_0 > -1.2, w_α < 0.5 (95%), and the upper bound of w_0 and lower bound of w_α is left unconstrained. Adding AP
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Table 2
Mean Values and 68% Confidence Limits of Cosmological Parameters for the Base LCDM Model from Planck+BAO and Planck+BAO+AP Combinations

| Parameter    | Planck+BAO | +AP |
|--------------|------------|-----|
| \( \Omega_b h^2 \) | 0.02228(14) | 0.02255(14) |
| \( \Omega_c h^2 \) | 0.1189(10)  | 0.1179(12)  |
| \( 100\theta_{sc} \) | 1.0408(31)  | 1.0409(32)  |
| \( \tau \) | 0.081(17)   | 0.086(17)   |
| \( \ln(10^{10}A_s) \) | 3.094(33)   | 3.102(33)   |
| \( n_s \) | 0.9669(39)  | 0.9695(39)  |
| \( H_0 \) | 67.63(48)   | 68.08(52)   |
| \( \Omega_{m} \) | 0.6898(64)  | 0.6959(78)  |
| \( \Omega_{b} \) | 0.3102(64)  | 0.3041(68)  |
| \( \Omega_{b} h^2 \) | 0.14182(98) | 0.14092(10) |
| \( \Omega_{m} h^2 \) | 0.09591(30) | 0.09594(30) |
| \( \sigma_8 \) | 0.829(13)   | 0.829(14)   |
| \( \sigma_8 \Omega_{m}^{0.5} \) | 0.4615(88)  | 0.4572(89)  |
| \( \sigma_8 \Omega_{m}^{0.25} \) | 0.618(10)   | 0.616(11)   |
| \( \sigma_{8e} \) | 1.07\(^{+0.45}_{-0.33} \) | 1.05\(^{+0.46}_{-0.34} \) |
| \( 10^5A_s e^{-2\tau} \) | 2.208(72)   | 2.225(75)   |
| \( 10^5A_s e^{-2\tau} \) | 1.877(11)   | 1.873(11)   |
| \( \Delta E/Gyr \) | 13.806(22)  | 13.790(23)  |
| \( z_h \) | 1.04100(30) | 1.04112(32) |
| \( \Delta m_{avg} \) | 1059.64(30) | 1059.75(30) |
| \( m_{avg} \) | 147.49(25)  | 147.66(26)  |
| \( k_0 \) | 0.14038(29) | 0.14025(30) |
| \( \zeta_{eq} \) | 3374(24)    | 3352(25)    |
| \( \kappa_{eq} \) | 0.010297(72)| 0.010231(77)|
| \( 100\theta_{sc} \) | 0.4520(23)  | 0.4541(25)  |

Note. The uncertainties (of the last two digits of the numbers) are listed in the brackets.

\[ w_0 = -0.93^{+0.09}_{-0.20}, \quad w_A = -0.65^{+0.45}_{-0.40} \]

(Planck+BAO+SNeIa+H0+AP).

The main effect of adding AP is a \( \approx 0.7\sigma \) shift of \( w_A \) toward negative values (which can be seen evidently from the upper panel of Figure 5). As a result, a dynamical dark energy (i.e., \( w_A > -1 \)) is preferred at \( \approx 1.5\sigma \). The lower panel of the figure shows that the dark energy EoS is evolving from \( < -1 \) to \( > -1 \) from the high redshift epoch to the present. \( w = -1 \) is consistent with the constraint at \( 2\sigma \) CL. \(^{11}\)

3.2. Constraints on the Other Cosmological Parameters

3.2.1. LCDM Parameters

Table 2 summarizes the LCDM parameters (6 basic parameters, 21 derived; see Ade et al. (2016) for the explanation of their meanings) constrained by Planck+BAO and Planck+BAO+AP combinations.

In the LCDM framework, adding AP into the analysis only affects the constraint on \( \Omega_m \). The matter amount \( \Omega_m \) changes

\(^{11}\) In comparison, Li et al. (2018) found that adding AP into Planck+BAO+SNeIa+H0 leads to a dynamical dark energy at \( \approx 1\sigma \) CL together with 50% reduction of \( w_0-w_A \) parameter space.

Figure 5. Cosmological constraints for the w,CDM model. Upper panel: likelihood contours (68.3% and 95.4%) in the \( \Omega_m-w \) plane. Adding AP into Planck+BAO greatly improves the constraints on the \( w_0-w_A \) parameters. Low panel: evolution of \( w(z) \) in the 68.3% and 95.4% CLs. A dynamical dark energy crossing \( w = -1 \) is mildly favored \( \approx 1.5\sigma \) CL.

closes the constraints and yields to

\[ w_0 = -0.95^{+0.13}_{-0.18}, \quad w_A = -0.53^{+0.57}_{-0.43} \quad \text{(Planck + BAO + AP).} \]

The result is consistent with \( w(z) = -1 \) in 1\( \sigma \). This manifests the power of the AP method. Combining it with BAO significantly increases the amount of information extracted from the LSS data, and greatly tightens the dark energy constraint.

When further considering the SNe Ia and \( H_0 \) data sets, we find

\[ w_0 = -0.94^{+0.11}_{-0.11}, \quad w_A = -0.38^{+0.45}_{-0.39} \quad \text{(Planck + BAO + SNeIa + H0);} \]
from 0.310 to 0.304, which is a 1.0σ CL drop (hereafter we use the Planck+BAO error bar to quantify the CLs of the changes). This then affects the constraints on many other parameters via the degeneracy among the parameters. For the basic parameters, we find:

1. The cold dark matter density $\Omega_m h^2$ decreases by 1.0σ.
2. The baryon ratio $\Omega_b h^2$ is increased by $0.5\sigma$, which should come from the increase of $H_0$.
3. The scalar spectral index $n_s$, which has negative degeneracy between $\Omega_m h^2$, is decreased by $0.7\sigma$.
4. The Thomson scattering optical depth due to reionization, $\tau$, and the log power of the primordial curvature perturbations, $\ln(10^{10} A_s)$, have little degeneracy with the above parameters. So they are less affected (change $<0.3\sigma$).

This leads to a series of changes in the derived parameters:

1. Due to the negative degeneracy between $\Omega_m$ and $H_0$, the latter increased by $0.9\sigma$.
2. $\sigma_8$ is negatively correlated with both $\Omega_m$ and $H_0$; as a net effect, its value remains unchanged.
3. The acoustic scale $100\theta_s$, crucially determined by the CMB angular power spectrum measurement, remains less affected.
4. The $z_\mathrm{eq}$ and $r_\mathrm{eq}$, affected by the density of energy components, are decreased and increased by $0.8\sigma$; similar effects are found for $z_\mathrm{drag}$ and $r_\mathrm{drag}$.
5. The change in $\Omega_m$ and $H_0$ leads to corresponding change in the combinations of $\Omega_b h^2$, $\sigma_8\Omega_m^{0.5}$, and $\sigma_8\Omega_m^{0.25}$, but the only exception is $\Omega_m h^2$; its value is almost fully determined by the acoustic scale $\theta_s$, so it remains unchanged.
6. The age of the universe is rather sensitive to $\Omega_m$ and $\Omega_b h^2$; we find that it decreased by $0.7\sigma$.
7. The matter-radiation equality redshift $z_{\text{eq}}$ drops by $0.9\sigma$, i.e., it happens in the latter epoch. This is due to the drop in $\Omega_m$ and increment in radiation density $\Omega_r$ (because of larger $h$).
8. Affected by $z_{\text{eq}}$ and the fraction of energy components, $k_{\text{eq}} = a(z_{\text{eq}})H(z_{\text{eq}})$ and $\theta_{\text{eq}} = r_e(z_{\text{eq}})\Delta_{\theta}(z_{\text{eq}}, \text{tar})$ (the comoving wavenumber of perturbation mode that entered Hubble radius at $z_{\text{eq}}$, and the angular scale of the sound horizon at $z_{\text{eq}}$) are also changed by $0.9\sigma$.

The characteristic wavenumber for damping $k_{\text{eq}}$ (which determines the photon diffusion length), whose value is related to the fraction of energy components, is slightly changed by $0.4\sigma$.

Parameters directly determined by $\tau$ and $A_s$, including $10^9 A_s e^{-2\tau}$ describing small-scale damping of CMB due to Thomson scattering at reionization, and the reionization redshift $z_{\text{re}}$, all remain less affected.

### Table 3: Constraints on One-parameter Extensions to the $\Lambda$CDM and wCDM Models, for Combinations of Planck+BAO and Planck+BAO+AP

| Parameter | $\Lambda$CDM Extension | $w$CDM Extension |
|-----------|-------------------------|------------------|
| $\Omega_\Lambda$ | $0.0001\pm0.0042$ | $-0.015\pm0.0042$ |
| $\sum m_\text{nu}(\text{eV})$ | $<0.181$ | $<0.295$ |
| $N_{\text{eff}}$ | $2.97\pm0.34$ | $2.95\pm0.37$ |
| $dn_s/d\ln k$ | $-0.0025\pm0.0134$ | $-0.0025\pm0.0132$ |
| $r$ | $<0.113$ | $<0.111$ |

**Note:** Note that we quote 95% limits here.

9. Due to the degeneracy in their roles of governing the cosmic distances, $\Omega_\Lambda$ is negatively correlated with $\Omega_m$ and positively correlated with $w$. As shown in Table 3, after considering AP, the absolute value of $\Omega_\Lambda$ increases (0.28σ) and changes sign from minus to plus for the $\Lambda$CDM extension model. For the wCDM extension model, the absolute value of $\Omega_\Lambda$ becomes bigger (0.14σ), but the error is more tightly constrained (32% improvement).

10. Using the AP effect, the upper limit of the total neutrino mass is reduced (by 22% and 18%) for both $\Lambda$CDM extension and wCDM extension cases.

11. By comparing two scenarios of Planck+BAO and Planck+BAO+AP data sets, $N_{\text{eff}}$ is increased (0.6σ for $\Lambda$CDM extension, which is not obvious in the case of wCDM extension). The reduction in the error of $N_{\text{eff}}$ is small (from 11.4% to 10.7% for $\Lambda$CDM extension, and from 12.7% to 12.1% for wCDM extension).

12. The running of the spectral index $dn_s/d\ln k$ is typically small, while the error is not significantly changed in both cases.

5. Adding the AP test, it appears that the tensor-to-scalar ratio $r$ is widely constrained for $\Lambda$CDM extension, and has tighter constraints for wCDM extension; but...
considering the statistical significance (0.05σ and 0.02σ), the effect is really ignorable.

The contour plots of one-parameter extensions to the ΛCDM model for combinations of Planck+BAO and Planck+BAO+AP are illustrated in Figure 6. We see clearly that combining the AP method increases the mean values of $N_{\text{eff}}$, $\Omega_k h^2$, $n_s$, $H_0$, decreases $\Omega_c h^2$ and $\Omega_m$, and noticeably reduces the errors of $\Omega_m$.

The contour plots of one-parameter extensions to the $w_\Lambda$CDM model are illustrated in Figure 7. In this case adding AP helps sharpen the constraints $\Omega_k$, $\sum m_i$, $H_0$, $\sigma_8$, $\Omega_m$, and $w$ by 20%–30%.

4. Forecast

The number distribution of DESI galaxies is shown in Figure 2. Predicted constraints are made for the $w_\Lambda$CDM and $w_0$CDM.

Estimating the covariance matrix of our correlation-based estimator is a complicated job (Bernstein 1994; O’Connell et al. 2016). Among various terms that scale differently with $N_{\text{gal}}$, we found that

$$\text{Cov} \propto 1/N_{\text{gal}}, \quad (20)$$

where $N_{\text{gal}}$ is the number of galaxies, is already a good approximation. In Appendix B we tested it using SDSS galaxies and found that its error is $\lesssim 1.5\%–6\%$.

The DESI covariance matrices are then obtained simply using Equation (20). First, the covariance matrix of $\xi_{3D}(z_2, z_1, \mu)$ using SDSS galaxies is computed using the 2000 MD-PATCHY mocks. We then choose this matrix as the baseline, take a ratio of the $N_{\text{gal}}$ of SDSS and DESI, and multiply this matrix by a factor to get the DESI covariance matrix.

The contour plot of $\Omega_m$–$w$ in the $w_\Lambda$CDM model are illustrated in the top panel of Figure 8, using joint data sets of Planck+BAO, AP and Planck+BAO+AP, respectively. In the lower panels we show the contour plots of $w_0$–$w_a$ in the frame of the $w_0w_a$ CDM model. In all cases we find that the constraint greatly reduced after adding the AP method.

For the constraint on a single parameter, the performance of AP and Planck+BAO are comparable to each other. If considering the joint constraint on two or more parameters, then the different directions of degeneracy from the two sets of

13 We take the SDSS galaxies in redshift bins 1 and 2 as the baseline to infer the DESI covariance matrices in all redshift bins. The results are rather insensitive to the redshifts of the baseline galaxies.
results suggests that a greatly improved constraint can be achieved by combining them together.

In \( \Lambda \)CDM, adding AP reduces the constrained parameter space by 50\%, achieving a precision of

\[
\begin{align*}
\delta \Omega_m & \approx 0.003, \, \delta w & \approx 0.015 \\
\times (\text{Planck} + \text{DESI BAO} + \text{DESI AP}).
\end{align*}
\]

In \( w_0w_a \) CDM, the addition of AP greatly reduces the constrained region by a factor of 10, achieving a precision of

\[
\begin{align*}
\delta w_0 & \approx 0.035, \, \delta w_a & \approx 0.11 \\
\times (\text{Planck} + \text{DESI BAO} + \text{DESI AP}).
\end{align*}
\]

5. Concluding Remarks

We conduct a comprehensive study about the cosmological constraints derived from the tomographic AP method. Based on Li et al. (2014, 2015, 2016, 2018), we improve the methodology by including the full covariance among clustering in all redshift bins. We then apply it to current and future observational data.

When applying it to current observational data, we find the following:

1. The AP method noticeably improves the constraints on background evolution parameters \( \Omega_m, H_0, w, w_0, \) and \( w_a. \) When combining it with \textit{Planck}+BAO the parameters’ error bars are reduced by \( \sim 20\%\text{--}50\% \) (depending on the model and parameter).

2. Using \textit{Planck}+BAO+SNe Ia+\( H_0 \)+AP, a dynamical dark energy \( w_i \approx -1 \) is preferred at \( \approx 1.5\sigma \) CL.

3. In the framework of \( \Lambda \)CDM, adding AP into \textit{Planck} +BAO yields to a slightly smaller \( \Omega_m = 0.301 \pm 0.010, \) and a slightly larger \( H_0 = 68.9 \pm 1.2. \) This leads to \( \lesssim 1\sigma \) changes in \( \Omega_b h^2, n_s, z_*, r_*, \) and \( z_{re}, \tau, A_s, \) and \( \sigma_8 \) are less affected.

4. When considering one-parameter extensions to \( \Lambda \)CDM and \( w \)CDM models, we get improved constraints on \( \kappa, \sum m_p, \) and \( N_{\text{eff}} \) when combining AP with \textit{Planck}+BAO. Since AP only puts constraints on the late time expansion, early universe parameters \( d n_s / d \ln k \) and \( r \) are less affected.

We make a forecast of the \( w \)CDM and \( w_0w_a \) CDM constraints expected from \textit{Planck}+DESI. We find the AP’s constraints on \( \Omega_m, w, w_0, \) and \( w_a \) are as tight as the \textit{Planck}+BAO ones, while the directions of degeneracy from the two differ from each other. Thus, combining them significantly improves the power of constraint. Adding AP reduces the error bar of \( w \) by 50\%, and improves the \( w_0-w_a \) constraint by a factor of 10.

It should be pointed out that the many results presented in this work are not the optimistic ones. We expect the result to be further improved with improvement in the methodology, e.g., an optimistic binning scheme of the galaxies, more aggressive clustering scales, more precise estimation of the covariance matrix, and so on.

According to our tests, for current surveys the systematic effects cannot significantly affect the derived cosmological constraints. But it remains to be seen if this is true for future surveys.
galaxy surveys. In particular, the systematic effects are estimated using one set of simulations performed in a fiducial cosmology, so the cosmological dependence of the systematics remains to be investigated in future works. It could be solved by, e.g., interpolating among systematics estimated from several sets of simulations with different cosmologies, considering theoretical estimation of systematics, and so on.

The tomographic AP method is so far the best method in separating the AP signal from the RSD distortions and using it to deconvolve the w0wa constraint. The result has a special dependence on the choice of the redshift bin (here the first bin), and thus suffers from large statistical fluctuation. Here we conduct a simple test to see how large the above two effects are. We simply consider six independent variables obeying normal distribution, who share the same variance but have different mean values:

\[ X_i \sim \mathcal{N}(\mu = 10 i, \sigma^2 = 1), \quad i = 1, 2, 3, 4, 5, 6. \]  

We then use the ideas of Equations (4) and (6) to define a \( \chi^2 \) function characterizing the evolution among them:

\[ \chi_1^2 \equiv \sum_{i=2}^{6} (X_i - X_0)^2 / (\sigma^2 + \sigma^2), \]  

\[ \chi_2^2 \equiv \sum_{i=2}^{6} \sum_{j=2}^{6} \text{Cov}_{ij} (X_j - X_i). \]  

Figure 8. Expected constraints on \( w_{\text{CDM}} \) and \( w_{0w_a} \) CDM models from the DESI survey. Combining AP with Planck+BAO breaks the degeneracy between parameters, and thus reduces the uncertainty of \( w \) by 50% and improves the \( w_{0w_a} \) constraint by a factor of 10.

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Appendix A
The Full Covmat Method Compared with the Old Method

As described in Section 2, if one were ignoring the correlations between different \( \delta_i \)s and \( \delta_j \)s, and simply using Equation (4) to calculate the \( \chi^2 \), the result suffers from two problems:

1. The statistical power is overestimated because part of the correlations are not considered.
2. The result has a special dependence on the choice of the redshift bin (here the first bin), and thus suffers from large statistical fluctuation.

We then use the ideas of Equations (4) and (6) to define a \( \chi^2 \) function characterizing the evolution among them:

\[ \chi_1^2 \equiv \sum_{i=2}^{6} (X_i - X_0)^2 / (\sigma^2 + \sigma^2), \]  

\[ \chi_2^2 \equiv \sum_{i=2}^{6} \sum_{j=2}^{6} \text{Cov}_{ij} (X_j - X_i). \]
and here the covariance matrix simply takes the form of

\[ \text{Cov}_{ii} = \xi^2; \text{Cov}_{ij} = 1 \text{ for } i \neq j. \]  

(26)

We generate \(10^6\) sets of \(X_i\), compute their \(\xi^2\) values, and plot the result in Figure 9. The mean and root mean square are listed in the legend. We find that

1. in this case, the part-cov approach overestimates the \(\xi^2\) value by 57%, i.e., it overestimates the statistical significance of the evolution; and
2. the statistical fluctuation of the \(\xi^2\) derived from the part-cov approach is twice as large as the full-cov approach, which may lead to large bias when adopting this method to constrain cosmological parameters.

Figure 10 compares the Planck+BAO+AP constraint on \(w_{\text{CDM}}\) derived using the first-bin approach equation (yellow dashed line) and the full-covmat approach (blue line). We find

\[ w = -1.068 \pm 0.036 \pm 0.076 \text{ par-cov}, \]

\[ w = -1.089 \pm 0.040 \pm 0.104 \text{ full-cov}. \]

(27)

(28)

When using the full-covmat approach, the mean value was shifted toward the negative values by 0.02 (\(~0.4\sigma\)), and the upper/lower error bars are enlarged by 10%/26%, respectively.

The two sets of results are, still, in statistical consistency. The errors due to the defect of the first-bin approach are not serious.

The part-cov constraint is weaker than what was reported in Li et al. (2015; \(w = -1.054 \pm 0.025\)) because the difference in the choices of \(n_p\) (Li et al. 2015) adopted \(n_p = 6\)–40 and here we reduced it to 20–25. Different from Li et al. (2015), in this work, we adopt the technique developed in Li et al. (2018) to efficiently approximate the 2PCFs in different cosmologies, and increase the size of covariance matrix from \(n_p \times n_p\) to \(5n_p \times 5n_p\). Both changes make the analysis more sensitive to the noise in the 2PCFs. So we reduce the number of binning to reduce the noise in \(\xi_{\Delta p}\), which increases the reliability of the results, at the cost of scarifying some power of constraints.

Figure 9. Histogram of \(\chi^2\) values using six test variables as described in Appendix A. The full covariance matrix method results in reliable and stable estimation of \(\chi^2\)'s, while ignoring the covariance overestimates the \(\chi^2\)'s and suffers from significantly larger uncertainty.

Figure 10. \(w_{\text{CDM}}\). (1) The method we used in this paper (full covmat method) is different from Li et al. 2016 (where they do not use full covmat). The result is slightly different. (2) If we discard systematic correction (estimated from HR4 simulation), there is minor change in the results.

### Appendix B

Robustness Check

Li16 tested the robustness of the tomographic AP method in detail and found that the derived constraints on \(w_{\text{CDM}}\) are insensitive to the adopted options within the range of \(s_{\min} = 2\)–8 \(h^{-1}\) Mpc, \(s_{\max} = 30\)–50 \(h^{-1}\) Mpc, \(\mu_{\max} = 0.85\)–0.99, and number of binning \(n_p = 6\)–40. Li et al. (2018) reconducted the above tests in the \(w_{0\Lambda}\) CDM and obtained similar results. In what follows, we adopt the procedure of these two papers and test the robustness of the result. Our default set of options are \(s_{\min} = 6\) \(h^{-1}\) Mpc, \(s_{\max} = 40\) \(h^{-1}\) Mpc, \(\mu_{\max} = 0.97\), and \(n_p = 15\)–20.\(^{14}\)

In the two panels of Figure 10 we also plotted the results without conducting any systematics (blue dashed). We see that

\[^{14}\text{Our } n_p \text{ is smaller than the default choice of Li16 (where they more ambitiously chose } n_p = 20\text{–35); this weakens the constraints by a little bit, but increases the robustness of the results, and also reduces the noise in the likelihood.}\]
the peak value of $w$ remains almost unchanged; the only effect is a small (~10%) for the 95% contour) enlargement of the constrained region toward the larger value of $w$ and $\Omega_m$. This is similar to what we found in Li et al. (2018), i.e., for the analysis of current observations the systematics effect is not large enough to result in a statistically significant change of the results.

As a detailed test of the options adopted in the analysis, Figure 11 shows the mean values and 95% limits of the parameters, in cases of using a more binned binning scheme $n_y = 20–25$, a more conservative small-scale cut $s_7 = h^{-1}$ Mpc, and a more conservative cut of correlation angle $\mu_{\text{max}} = 0.85–0.95$. In all cases we find rather small change in the mean values ($< 0.2\sigma$) and the limits ($< 15\%$). So we conclude that the cosmological constraints obtained from the AP method do not sensitively depend on these options.

Finally, we test the precision of Equation (20) on the BOSS galaxies, and found that it works with satisfying precision (Figure 12). The difference between the covariance matrices estimated from Equation (20) and those directly computed using the mocks is $\lesssim 5\%$.

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Figure 11. $w$CDM. Plotted are 2σ regions of parameters. If we change the options in the analysis, changes in the results are minor.

Figure 12. Rough check of Equation (20) using six bins of SDSS DR12 galaxies (see Figure 2 of Li et al. 2016). The covariance matrices of $\xi_2 (z, z', \mu)$, $i = 1, 2, 3, 4, 5$ are computed using two methods: (1) measuring the scattering of $\delta_{i,1}$ from the 2000 MultiDark-Patchy mocks, as we did in Li et al. (2016). (2) Measuring $\delta_{i,1}$ from the mocks, and infer the other $\delta_{i,s}$ simply using Equation (20). The y-axis shows the ratio between the results of the two methods (method 2 over method 1). We find that the estimation (method 2) achieves < 2% precision for $\delta_{i,1}$, $\delta_{i,1}$, and $\delta_{i,1}$. In the related redshift bins (redshift bins 1–5), the galaxy number density $\delta_{i,1}$ varies in the range of $2 \sim 7 \times 10^{-3}$ h$^{-1}$ Mpc (as large as ~3 times). Given such a large fluctuation of $\delta_{i,1}$, Equation (20) still achieves good precision. In the sixth bin $\delta_{i,1}$ drops to significantly lower density ($1 \times 10^{-4}$ h$^{-1}$ Mpc$^{-3}$, ~5 times lower than the first and second bins), while the error of method 2 is still < 6%.