Cyclic model based generalized predictive control of air-fuel ratio for gasoline engines

Madan KUMAR* and Tielong SHEN*
* Department of Engineering and Applied Science
Sophia University, 7-1 Kioi-cho, Chiyoda-ku, Tokyo,102-8554, Japan
E-mail: madanbhuit.10@gmail.com

Received 25 November 2015

Abstract
In four stroke internal combustion engines, optimization of engine performance with air-fuel ratio close to stoichiometric condition is still a challenging task specially in transient operation due to cycle-to-cycle coupling of combustion phenomena and gas dynamics in cylinder. In this paper, the cycle-to-cycle in-cylinder gas dynamics coupling model based air-fuel ratio control using the generalized predictive control law has been discussed and validated in which the input parameters of the discrete time model are updated on cyclic event based. With the discrete time model, a Kalman filter-based state variables such as total fuel mass, unreacted air and residual burnt gas are estimated and used to calculated the in-cylinder air-fuel ratio which reflect the cycle-to-cycle coupling effects of residual gas mass. Then based on model, a controller is designed to achieve the air-fuel control. Apart from this, the control performances of generalized predictive controller and PI controller have been compared. Finally, experimental validation results are demonstrated to show the effectiveness of proposed control scheme that is conducted on a full-scaled gasoline engine test bench.

Key words : Air-fuel ratio control, Cyclic discrete-time model, Kalman filter estimation, Generalized predictive control, Residual gas fraction, Combustion efficiency

1. Introduction

In the past decades, the modeling and control application for the optimization of engine performance such as fuel efficiency and emission have been focused by the researchers, and the target is recently aimed to reduce the burden of extra sensor and delay in feedback response signal which caused by the transient dynamics of engine and the sensors. Several modeling and control approaches are used for the optimization of fuel consumptions such as, stabilization of cyclic combustion instability (Shinmura et al. (2013)) which affects the cyclic combustion phenomena and hence air-fuel ratio, optimization of cooling and oil temperature based on frictional losses in engine (Kim et al. (2013)) and using the fast response of in-cylinder pressure sensors based feedback control of air-fuel ratio (AFR) in transient mode (Kumar and Shen, 2015, Pace and Zhu, 2014, Yang et al., 2013). It has been shown that the fuel efficiency specially in transient control can be improved when sufficient information of in-cylinder cyclic combustion phenomena, fluid dynamics variation and cycle-to-cycle coupling of engine systems are available. However, handling and control of the cyclic combustion phenomena is still a challenging task.

Recently, modeling and estimation methods for in-cylinder air-fuel ratio are investigated using cylinder pressure sensor signal (Pestana, 1989, Wibberley and Clark, 1989). In the literature (Arsie et al., 2014), the cylinder pressure based estimation of instantaneous AFR is proposed with considering the linear relation between AFR and 2nd or 3rd order moment of pressure cycle. The cyclic averaging AFR model and its identifications based on heat release profile is presented in (Tunestl and Hedrick, 2003). Moreover, the method of mean-value estimation of the air-fuel ratio based on the ionization current signal measurements has been discussed in (Lee et al., 2001), which shows the advantage of strong potential for real-time AFR estimation and combustion diagnostics of individual cylinders and engine cycles. However, still there are some drawbacks in considering the model for cyclic AFR, it need to consider the cyclic effect of
previous cycle combustion quality and quantity in AFR estimation due to the cyclic coupling of the residual gas fraction (Clerk, 1886, Daw et al., 1996, Daw et al., 1998).

To challenge the air-fuel ratio control problem, various research works have been reported in the past, such as PID control (Ebrahimi et al., 2012), sliding model control (Pace and Zhu, 2009), feed-forward controller with a purge event fueling system using a transport delay model (Zamanian et al., 2015), neural network feed-forward-feedback control (Zhai and Yu, 2008), linear quadratic (LQ) control (Pace and Zhu, 2014) and model based predictive control (MPC) (Wong et al., 2014). In recent, the generalized predictive controller (GPC) is widely in use due to the advantage covering the area of multi-variable, time-varying and delay in engine system with considering the assumptions of better accuracy in model prediction and optimizer efficiency. Comparison of control performances are also shown in paper (Wang et al., 2006), (Wang, L., 2009) and (Wong et al., 2014).

In this paper, a discrete-time model based generalized predictive control (GPC) is proposed for air-fuel ratio where the model represents the cyclic transient behavior of in-cylinder state variables under the assumptions of measurability of the cyclic total in-cylinder mass, residual gas fraction and combustion efficiency using the in-cylinder pressure signal. The cyclic fuel injection mass is chosen as control input and the model is represented as linear-time varying state equation with the state variables which denote total fuel mass ($m_f$), residual unreacted air mass ($m_{ra}$) and residual burnt gas mass ($m_b$), respectively. Then, Kalman filter algorithm is used for the estimation of these state variables and a generalized predictive control (GPC) law is applied to decide the fuel injection mass for the control of air-fuel ratio to desired value. Finally, validation of the proposed control scheme is illustrated on a full scaled gasoline engine test bench. The reminder of the paper is organized as follows; the experimental setup, procedure and problem descriptions are explained in next section. Controller design based on model is discussed following the modeling and estimation. Finally, result-discussion and conclusions are presented.

2. Experimental setup and problem descriptions

2.1. Experimental setup

A V6-type gasoline direct injection (GDI) engine is used for this research work. In the engine test bench (Fig. 1(a)), a low inertial dynamometer is coupled to engine to provide the desired facilitation of experimental environment. The engine specifications are given in Table 1. A prototype controller constructed by dSPACE (DS1106), dSPACE 2004 and on board ECU is used for the capturing of data and actuation signals such as the throttle position, fuel injection mass, spark advance and variable valve timing (VVT). The flow chart of instrumentation of engine setup is shown in Fig. 1(b).

![Fig. 1 (a) Experimental setup. (b) Experimental setup instrumentation flow chart.](image)

Table 1 Engine Specifications.

| Engine Type - 2 GR-FSE, V6, 3.5 L | Fuel system | Direct Injection |
|----------------------------------|-------------|-----------------|
| Compression ratio                | 11.8:1      |                 |
| Bore X Stroke (mm)               | 94 X 83     |                 |
| Displacement (cm³)               | 3456        |                 |
| Max. Power (kW)                  | 228 @ 6400 rpm |          |
| Max. Torque (Nm)                 | 375 @ 4800 rpm |          |

[DOI: 10.1299/jtst.2016jtst0009] © 2016 The Japan Society of Mechanical Engineers
In this experimental setup, the in-cylinder pressure sensor is equipped on the 5-th cylinder of V6-type engine which provides the measurement of each degree crank angle and then used for the measurement of total in-cylinder charge \( M_t(k) \), combustion efficiency \( C_f(k) \) and residual gas fraction \( r(k) \), where \( k \) denote the cycle index. Entire experiment for measurement and control have been conducted in speed constant mode. For measurement of \( M_t(k) \), two point pressure and volume data during compression stroke before start of combustion are used with assumptions of adiabatic compression process. \( C_f(k) \) is calculated by the total heat release calculation from pressure signal during the crank angle of start of combustion and exhaust valve opening and the heat supplied by the fuel energy. Similarly, \( r(k) \) is also calculated based on two point pressure data measurements during exhaust valve opening. The above measurements are then updated every cycle at the end of exhaust stroke. Detailed measurement procedures and model can be refer in (Kumar and Shen, 2015).

### 2.2. Problem descriptions

An investigation of cycle-to-cycle coupling in engine system is still a challenging task due to residual gas mass and energy transfer from a cycle to another which affects the engine performances such as, AFR and emissions. Considering the effects of residual gas mass such as residual fuel, residual air and burnt gas which retained in cylinder and mixed with the next cycle, will improve the engine dynamic performances. This challenge is motivated to develop a model which represents the effects of cyclic coupling residual gas mass in the engine in-cylinder behavior, and enable us to estimate AFR etc. A cyclic coupling of residual gas mass is shown in Fig. 2. As shown in Fig. 2, at the end of exhaust stroke, some residual gas trapped in-cylinder due to engine design and valve overlapping, and mixed with the next cycle fresh air and fuel mass which affects the next cycle combustion process too.

![Fig. 2 Cycle definition and cycle-to-cycle gas coupling phenomena.](image)

A discrete time time-varying cyclic transient model is proposed in which the state variables are chosen as in-cylinder total fuel \( m_f \), residual air \( m_{ra} \) and burnt gas \( m_b \). Then, based on the model, the states are estimated with Kalman filter. Furthermore, based on cycle definition in Fig. 2, in-cylinder cyclic air-fuel ratio \( \hat{\lambda}(k) \) is calculated from these estimated state variables as the ratio of total in-cylinder air \( m_{ind}(k)+m_{ra}(k) \) and total in-cylinder fuel \( m_f(k) \) is given in Eq. (1), which includes the coupling effects of in-cylinder residual gas fraction and hence this makes some offset from directly measured AFR by sensor at the exhaust manifold. The position of AFR estimation based on cyclic model and directly measured by sensor are represented in Fig. 3 (b). Where the notation \( m_{ind} \) denote the fresh inducted air during suction stroke and measured directly from air mass flow sensor and \( k \) is the cyclic index. Based on the cyclic model, a generalized predictive controller is designed. In Fig. 3 (a), it is shown that: cyclically decision of \( u_f \) such that the estimated air-fuel ratio, \( \hat{\lambda}(k) \) goes to desired air-fuel ratio, \( \lambda_d = 14.6 \), with considering the effects of cycle-to-cycle updated input parameters of transient model.
\[
\dot{\lambda}(k) = \frac{m_{ind}(k-1) + \hat{m}_r(k)}{m_f(k)} 
\]  
(1)

In Fig. 2, one cycle is defined from \(BDC(k)\) to \(BDC(k+1)\) in which cyclic process data available at \(BDC(k)\) is considered in \(k_{th}\) cycle. \(M_f(k)\) denote the cyclic residual gas mass. \(BDC\) denote the suction bottom dead center, \(TDC\) denote combustion top dead center, \(BDC_e\) and \(TDC_e\) denote the exhaust bottom dead center and top dead center, respectively. \(IVO\), \(IVC\), \(EVO\) and \(EVC\) denotes the intake valve opening, intake valve closer, exhaust valve opening and exhaust valve closer, respectively and \(\theta\) denote the crank angle. In Fig. 3 (a), \(\dot{\lambda}(k), u_f(k), m_{ind}(k-1)\) and \(\hat{\lambda}(k)\) denotes estimated air-fuel ratio, injected fuel mass, inducted fresh air and estimated state variables, respectively.

3. Modeling and estimation

The discrete time model (Kumar and Shen, 2015) which represents the cyclic behavior of in-cylinder state variables is given as follows,

\[
m_f(k+1) = (1 - C_f(k))r(k)m_f(k) + u_f(k) 
\]

\[
m_{ra}(k+1) = r(k)m_{ra}(k) - \lambda_d r(k)C_f(k)m_f(k) + r(k)m_{ind}(k-1) 
\]

\[
m_b(k+1) = r(k)m_b(k) + r(k)C_f(k)(1 + \lambda_d)m_f(k) 
\]

where \(\lambda_d\) represent the stoichiometric air-fuel ratio (14.6).

Under the assumptions,

1). \(m_{ind}(k-1) = G_m + \zeta(k)\)

2). \(y_{mes}(k) = M_f(k) - G_m\)

The model (Eq. (2)) can be represented as,

\[
x_m(k+1) = A_m x_m(k) + B_{1m} u_f(k) + B_{2m}(k)(G_m + \zeta(k)) 
\]

\[y(k) = C_m x_m(k) + \xi(k)\]

where \(G_m\) is constant and measured by the air mass flow sensor with filtering the noise, \(\zeta(k)\) is variance \((\zeta(k) \in N(0, \sigma^2))\), \(\xi(k)\) is the measurement noise, \(x_m(k)\) is the state variables and \(A_m(k), B_{1m}, B_{2m}(k)\) and \(C_m\) are input parameters of models which updates every cycle as given below,

\[
x_m(k) = \begin{bmatrix} m_f(k) & m_{ra}(k) & m_b(k) \end{bmatrix}^T, A_m(k) = \begin{bmatrix} (1 - C_f(k))r(k) & 0 & 0 \\ -\lambda_d r(k)C_f(k) & r(k) & 0 \\ r(k)C_f(k)(1 + \lambda_d) & 0 & r(k) \end{bmatrix}, B_{1m} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, B_{2m}(k) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

and \(C_m = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\).

Based on the above model, the in-cylinder air-fuel ratio is defined as follows:

\[
\hat{\lambda}(k) = \frac{m_{ind}(k-1) + \hat{m}_r(k)}{m_f(k)} 
\]

(4)
with the \( m_{af}(k-1), m_{rf}(k) \) and \( m_f(k) \) based on cycle definition in Fig. 2.

For the estimation of in-cylinder state variables such as \( m_f(k), m_{rf}(k) \) and \( m_{th}(k) \), a Kalman filter algorithm is used based on above model as,

\[
\dot{x}(k + 1) = A_m(k)\dot{x}(k) + B_{1m}u_f(k) + B_{2m}(k)(G_m + \zeta(k)) + L(y_{me}(k) - \hat{y}(k)) \tag{5}
\]

Kalman filter gain is calculated as,

\[
L(k) = P^*(k)C_m^T[C_mP^*(k)C_m^T + P]^{-1}
\]

\[
P^*(k) = A_m(k)P^*(k-1)A_m^T(k) + P_w
\]

\[
P^*(k) = [I - L(k)C_m]P^*(k)
\tag{6}
\]

where \( L(k) \) denotes the Kalman filter gain which can be calculated using Eq. (6). \( P_w \) and \( P_e \) are the covariance of process and sensor white noise, respectively, as Gaussian random process with zero mean. \( y_{me}(k) \) is system output measured from the experiment and \( \hat{y}(k) \) is estimated system output with Kalman filter algorithm.

4. Generalized predictive controller design

For implementing the generalized predictive controller (GPC) based on the state-space model (Eq. (3)), the model should be modify to adopt cyclic air-fuel ratio control objective. The objective is to control the in-cylinder cyclic air-fuel ratio (Eq. (4)) to desired value, hence Kalman filter-based estimation is used to perform the control due to the in-cylinder air-fuel ratio is not directly measurable. Hence, based on the control objective, we defined the cost function with a new system output \( y_f(k) \) as fresh fuel injected such that the air-fuel ratio will be controllable to desired value when difference between system output \( y_f(k) \) and its desired constant value \( u_d(k) \) approaches to zero. Where, \( y_f(k) = Cx(k) \) and \( C = \begin{bmatrix} 1 & -1/14.6 & 0 \end{bmatrix} \). The desired constant value \( u_d(k) \) is calculated from the ratio of fresh air mass inducted \( (G_a(k)) \) and stoichiometric air-fuel ratio \( (14.6) \) which is equivalent to the amount of fuel injected required for the keeping air-fuel ratio at stoichiometric level.

From optimization problem, the generalized cost function is:

\[
J = \sum_{i=1}^{N_p}(u_d(k) - y_f(k + i\hat{k}))^2 + \sum_{j=1}^{N_c}r_w\Delta u_f(k + j - 1|k)^2
\tag{7}
\]

where \( y_f(k + i\hat{k}) \) denote the future predicted output at \( k \) state, \( \Delta u_f(k) \) is the incremental in fuel injection in \( k \) state. \( N_p \) and \( N_c \) are the future predicted states and predicted control input signal at current cycle state \( k \) and considered 5 and 4, respectively. It should be noted that there is no analytic way to determine the parameter \( N_p \) and \( N_c \). In application, it might be used as tuning parameter based on try-and-error. In this paper, \( N_p \) and \( N_c \) are chosen as follows: In present model, the time delay for transient response is almost 5 cycle (Fig. 4). So, for covering the crucial dynamics response, \( N_p \) is considered as 5 because \( N_p \) is related to the step response of the system: the time interval \( (1,N_p) \) should contain the crucial dynamics of the process. The \( N_c \) value is usually taken equal or less than the \( N_p \) to get the robust control law in the case of modeling error. However, an increases of \( N_c \) may give a dramatic increases of computational burden and may also lead the instability in the case of modeling error. \( r_w \) is the weighting value for feedback signal which can be tuned to get the fast desired closed-loop control performance.

Using GPC controller law, the optimization problem defined as:

\[
\min_{\Delta u_f} J(\Delta u_f)
\tag{8}
\]

s.t.

\[
x(k + 1) = A(k)x(k) + B_1\Delta u_f(k) + B_2(k)\Delta G(k)
\]

\[
y_f(k) = Cx(k)
\]

Above discrete-time state space model (Eq. (8)) is in matrix formate which derivation details are given in Appendix A.

Equation (7) can be reformulated in matrix form to achieve the AFR control using GPC controller as,

\[
J = (R_x - Y)\hat{Y} + \Delta U^TR\Delta U
\tag{9}
\]

where \( R_x = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T \times u_d(k) \) is the desired signals with matrix dimension \( 1 \times N_p \) and \( R = r_w \times N_c \times N_c (r_w \geq 0) \) is the weighting value with matrix dimension \( N_c \times N_c \) for feedback signal which can be tuned to get the fast desired
closed-loop control performance. \( Y \) and \( \Delta U \) denoted the future predicted output and control input in vector at \( k_i \) state and represented as,

\[
Y = \begin{bmatrix}
y_1(k_i + 1|k_i) & y_1(k_i + 2|k_i) & \cdots & y_1(k_i + 5|k_i)
\end{bmatrix}^T \quad \text{and} \quad \Delta U = \begin{bmatrix}
\Delta u_f(k_i + 1|k_i) & \cdots & \Delta u_f(k_i + 4|k_i)
\end{bmatrix}^T.
\]

Taking the first derivative of the cost function \( J \) (Eq. (9)) and equate to zero \( \frac{\partial J}{\partial u_f} = 0 \), which gives the optimal solution for the control signal as,

\[
\Delta U = (\Phi^T \Phi + R)^{-1} \Phi^T \left( G_m(k)/14.6 - Fx(k_i) - \Psi \Delta G(k) \right) \tag{10}
\]

Based on the generalized predictive control principle, we consider the first element of the control vector at time \( k \) as the final incremental control variable. Then, the final control law is given by:

\[
u_f(k) = u_f(k - 1) + \Delta u_f(k) \tag{11}\]

where, \( \Delta u_f(k) = K \Delta U, \; K = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^{N_x \times 1} \).

\[
F = \begin{bmatrix}
CA(k) \\
CA^2(k) \\
CA^3(k) \\
CA^4(k)
\end{bmatrix}, \quad \Phi = \begin{bmatrix}
CB_1 & \cdots & 0 \\
CA(k)B_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
CA^4(k)B_1 & \cdots & CA^4(k)B_1
\end{bmatrix}, \quad \Psi = \begin{bmatrix}
CB_2(k) & \cdots & 0 \\
CA(k)B_2(k) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
CA^4(k)B_2(k) & \cdots & CA^4(k)B_2(k)
\end{bmatrix}
\]

The details derivation of parameters \( F, \Phi \) and \( \Psi \) are given in Appendix A.

It should be noted that the two system output have been defined for AFR control application based on above model. The model output \( y(k) \) based on constant vector \( C_m = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \) is used for state variables estimation with Kalman filter estimation algorithm, while model output \( y_f(k) \) based on \( C = \begin{bmatrix} 1 & -1/14.6 & 0 \end{bmatrix} \) is adopted for the model for AFR control objective using GPC controller. Then finally, based on the above control algorithm, the cyclic event based in-cylinder AFR is controlled to desired value (14.6) using the feedback incremental signal as \( \Delta u_f \). The experimental validation of cyclic event based control of AFR is discussed and validated in the following section.

### 5. Experimental validation

Based on the above model and GPC control law, the estimated cyclic air-fuel ratio control experiment has been performed and validated at various operating conditions. As it is mentioned in previous section, the future prediction of state variables and control input for the current states are chosen as 5 and 4, respectively during experiment. The fuel injection is selected as control input for the GPC controller during cyclic AFR control. A sample of experimental data (at 1000 rpm) for the control input future prediction as increment in control input (direct fuel injection (\( \Delta U \))) in vector form is presented below, which show the control performances effectiveness. The experiments are conducted with the step changes of throttle angle with the maintaining of transient environment of experiment in constant speed mode experiment to observe the control performances. It is shown that the \( \Delta U \) value is approaches towards zero with forward prediction.
steps and cycle. In perfect model system, the control input $\Delta U$ should be zero after some cycle passes during GPC control, but in real experiment, it is challenging task due to the stochastic behavior of engine dynamic system.

$$
\Delta U(k) = \begin{bmatrix}
-3.78 \\
-2.33 \\
-1.24 \\
-0.67 \\
\end{bmatrix}
$$

$$
\Delta U(k + 1) = \begin{bmatrix}
-2.88 \\
-1.78 \\
-1.02 \\
-0.57 \\
\end{bmatrix}
$$

$$
\Delta U(k + 2) = \begin{bmatrix}
-1.47 \\
-0.89 \\
-0.64 \\
-0.40 \\
\end{bmatrix}
$$

$$
\Delta U(k + 3) = \begin{bmatrix}
-0.26 \\
-0.12 \\
-0.17 \\
-0.16 \\
\end{bmatrix}
$$

For the validation of GPC controller based on cyclic transient model, several experiments are conducted at various constant speed operating condition, such as 1000, 1500 and 2000 rpm. During experiment, a step change in throttle is applied to simulate the transient environment and consequently the inducted air mass and air fuel ratio changes, but at the same time the controller predict the error based on changes in input parameters and provides the additional amount of fuel injection $\Delta u_f$ in the main fuel injection $u_f$ to maintain the AFR to desired level. In Fig. (5, 6, 7), it can be
Fig. 7 AFR control with GPC control algorithm at 2000 rpm.

Fig. 8 Comparison of control performance of GPC, PI and ECU at 1500 rpm in constant speed mode (Engine load changes (from 54 Nm ~ 120 Nm) with throttle changes).
clearly observed that, at the time of throttle changes, AFR is suddenly changes and adjusted to desired value gradually with time. In the same figures, $\Delta u_f$, $u_f$ are also plotted which show the amount of additional fuel injection increment by controller for AFR control. Generally, delay in AFR sensor signal occurs due to its position away from the combustion chamber (transportation delay) and the sensor response, but controlling of AFR with the in-cylinder pressure-based model will avoid such kinds of delay which evidents are shown in this results. At the same time, it can be also observed that there is some offset between $AFR_{model}$ and $AFR_{sensor}$ which mainly appears due to considering the effects of residual gas fraction (Residual air and fuel) in calculation of AFR in Eq. (1). With increasing the engine speed and the load, this offset decreases due to decreases of residual gas fraction. The amount of residual air and fuel available in cylinder also depends on the complex combustion phenomena (combustion efficiency) which proper modeling and measurement are still a challenging task. $AFR_{model}$ signal is used for the feedback control in GPC controller. When $AFR_{model}$ is forced to maintained the stoichiometric value (14.6) with GPC controller then sensor value will shift same offset during controlling the engine (Engine run slightly rich zone of AFR). In this research work, main objective is to validate the GPC controller performance based on calculated AFR with cycle-to-cycle transient model.

Apart from this, the GPC control performance based on model is also compared with the model based PI feedback control and commercial engine control unit (ECU) under the transient operating condition with constant speed and constant load which are shown in Fig. 8 and Fig. 9, respectively. In the case of constant speed mode result in Fig. 8, the model-based GPC and ECU shows the faster and stable response and the PI controller shows more fluctuation in transient state which depends upon the tuning of gain $K_p$ and $K_i$. By the decreases of $K_p$ gain, the fluctuation can be reduced but at the same time transient delay response will increases. However, ECU does not maintained the desired AFR which can be clearly observed in square dashed green line in Fig. 8. In the same figure, the fuel consumption and NOx emission are also plotted. The fuel consumptions by GPC controller show slightly higher than ECU value due to offset in estimated and measured AFR. The PI controller response shows the same fluctuation in fuel consumption signal also as in AFR signal. Similarly, same trained of response is also shown by NOx emission which revealed that GPC controller shows
better performance than PI and commercial engine control unit (ECU) and gives faster and stable feedback response in transient mode of experiment. However, at the same time due to the offset in $AFR_{model}$ and $AFR_{sensor}$, if GPC or PI controller implemented with $AFR_{model}$ feedback, the actual AFR ($AFR_{sensor}$) of engine will be slightly changes (Engine will run at slightly rich combustion at higher throttle) which makes the rich combustion and hence transient response and magnitude of NOx value in GPC and PI controller is lower compared to ECU control. Similarly, the validation of GPC controller and its comparison are also done with engine speed changes (constant load) and observed good agreement in control performance of model-based GPC which can be seen in Fig. 9. In Fig. 10, it can be also observed that the model based control performances is more stable (taking less time in stabilizing) compared to commercial ECU when a step changes in throttle is applied. The AFR controlled by ECU is not able to maintained at desired value but controlled by GPC satisfied the desired condition.

To verified the effectiveness of GPC control performance, the weighting factor $r_w$ is tunned to different value such as 5, 10 and 15 during the transient experiment (throttle angle changes) at 1000 rpm operating condition. In Fig. 11, the experimental results of control performance with tunning of weighting parameter $r_w$ are shown. It can be clearly distinguish from the signals (circled in Fig. 11) that the control performance is better and stable at $r_w$=10 compared to $r_w$=5. At $r_w$=15, the control performance is stable but takes longer time to approach desired value. Hence, based on above result, weighting parameter $r_w$ =10 is chosen for further analysis during the AFR control experiment.

6. Conclusions

A discrete-time cyclic event model based air-fuel ratio control is developed and validated. The generalized predicitive controller based on same model has been used in cyclic AFR control which considered the cycle-to-cycle gas dynamics coupling. The model parameters have been measured using the in-cylinder pressure signal on cycle basis and updated on every cycle which consists total in-cylinder charge, combustion efficiency and residual gas fraction. The control performances of GPC, ECU and PI have been compared which evident that the GPC controller shows better and stable performance than the other controller. It should be noted that the correct measurement of cyclic total in-cylinder charge, combustion efficiency and residual gas fraction are still a challenging task due to the cyclic imbalances and, if the noise does not satisfy the assumptions of stochastic properties (Gaussian distribution) and changes according to operating mode, then it may not be guarantee to get better control performances on different systems using the same model.

7. APPENDIX A

GPC controller derivation:

Based on the discussed cyclic state-space model in Eq. (3) for the control of cyclic AFR using the generalized predictive control, firstly we find the increment of state variables $\Delta x_m(k)$ and incremental in control input $\Delta u_f(k)$ by the taking of difference on both side in equation (Eq. (3)) and then based on this, the parameters for the control objective such as $F$, $\phi$, $\psi$ which minimized the cost function have been calculated which procedures are discussed below.

The incremental in state variables and control input are calculated as,
\[
\begin{align*}
\Delta x_m(k+1) &= x_m(k+1) - x_m(k) \\
\Delta u_f(k) &= u_f(k) - u_f(k-1) \\
\Delta G_m(k) &= G_m(k) - G_m(k-1).
\end{align*}
\]

From above increment of state space and control input, the state space model can be written as,

\[
\Delta x_m(k+1) = A_m(k)\Delta x_m(k) + B_{1m}\Delta u_f(k) + B_{2m}(k)\Delta G_m(k) \tag{12}
\]

Now, a new state variables vector is chosen to connect the \( \Delta x_m(k) \) to output \( y(k) \) as,

\[
\begin{bmatrix}
\Delta x_m^T(k) \\
y(k)
\end{bmatrix}
\]

and output increment is represented as,

\[
y(k+1) - y(k) = C_m(x_m(k+1) - x_m(k)) = C_m\Delta x_m(k+1) = C_mA_m(k)\Delta x_m(k) + C_mB_{1m}\Delta u_f(k) + C_mB_{2m}(k)\Delta G_m(k).
\]

Putting together, the following new state-space model appears:

\[
\begin{bmatrix}
\Delta x_m(k+1) \\
y(k+1)
\end{bmatrix} =
\begin{bmatrix}
A_m(k) & O_m^T \\
C_mA_m(k) & 1
\end{bmatrix}
\begin{bmatrix}
\Delta x_m(k) \\
y(k)
\end{bmatrix} +
\begin{bmatrix}
B_{1m} \\
C_mB_{1m}
\end{bmatrix}\Delta u(k) +
\begin{bmatrix}
B_{2m}(k) \\
C_mB_{2m}(k)
\end{bmatrix}\Delta G_m(k)
\]

\[
y_1(k) = \begin{bmatrix} O_m & 1 \end{bmatrix}\Delta x_m(k)
\]

If, we assume

\[
\begin{bmatrix}
\Delta x_m(k+1) \\
y(k+1)
\end{bmatrix}, A(k) =
\begin{bmatrix}
A_m(k) & O_m^T \\
C_mA_m(k) & 1
\end{bmatrix}, x(k) =
\begin{bmatrix}
\Delta x_m(k) \\
y(k)
\end{bmatrix}, B_1 =
\begin{bmatrix}
B_{1m} \\
C_mB_{1m}
\end{bmatrix}, B_2(k) =
\begin{bmatrix}
B_{2m}(k) \\
C_mB_{2m}(k)
\end{bmatrix}, and
\]

\[
C = \begin{bmatrix} O_m & 1 \end{bmatrix}
\]

Then, simplified final state-space model for GPC control can be represented as,

\[
\begin{bmatrix}
\Delta x_m(k+1) \\
y_1(k+1)
\end{bmatrix} = A(k)x(k) + B_1\Delta u_f(k) + B_2(k)\Delta G(k) \tag{13}
\]

\[
y_1(k) = Cx(k)
\]

where \( O_m = \begin{bmatrix} O & 0 & 0 \end{bmatrix} \), the triplet \( A(k), B_1, B_2(k) \) and \( C \) is called the augmented model, which is used in the design of predictive control.

Now we assume the predicted state variable optimization length \( (N_p) \) is 5 and length of predicted control input trajectory \( (N_c) \) is 4, which is predicted at \( k_i \) state. Based on the above state space model, the future predicted state variables at \( k_i \) state are calculated as,

\[
x(k_i + 1 | k_i) = A(k_i)x(k_i) + B_1\Delta u_f(k_i) + B_2(k_i)\Delta G(k_i)
\]

\[
x(k_i + 2 | k_i) = A^2(k_i)x(k_i) + A(k_i)B_1\Delta u_f(k_i) + \cdots + B_2(k_i)\Delta G(k_i + 1)
\]

\[
x(k_i + 3 | k_i) = A^3(k_i)x(k_i) + A^2(k_i)B_1\Delta u_f(k_i) + \cdots + B_2(k_i)\Delta G(k_i + 1)
\]

\[
x(k_i + 4 | k_i) = A^4(k_i)x(k_i) + A^3(k_i)B_1\Delta u_f(k_i) + \cdots + B_2(k_i)\Delta G(k_i + 1)
\]

\[
x(k_i + 5 | k_i) = A^5(k_i)x(k_i) + A^4(k_i)B_1\Delta u_f(k_i) + \cdots + B_2(k_i)\Delta G(k_i + 1)
\]

From the predicted state variables, the predicted outputs are calculated as substituting of state variables in output \( y_1 \) as,

\[
y_1(k_i + 1 | k_i) = CA(k_i)x(k_i) + CB_1\Delta u_f(k_i) + CB_2(k_i)\Delta G(k_i))
\]

\[
y_1(k_i + 2 | k_i) = CA^2(k_i)x(k_i) + CA(k_i)B_1\Delta u_f(k_i) + \cdots + CB_2(k_i)\Delta G(k_i+1)
\]

\[
y_1(k_i + 3 | k_i) = CA^3(k_i)x(k_i) + CA^2(k_i)B_1\Delta u_f(k_i) + \cdots + CB_2(k_i)\Delta G(k_i+1)
\]

\[
y_1(k_i + 4 | k_i) = CA^4(k_i)x(k_i) + CA^3(k_i)B_1\Delta u_f(k_i) + \cdots + CB_2(k_i)\Delta G(k_i+1)
\]

\[
y_1(k_i + 5 | k_i) = CA^5(k_i)x(k_i) + CA^4(k_i)B_1\Delta u_f(k_i) + \cdots + CB_2(k_i)\Delta G(k_i+1)
\]

Then the above future predicted output and control input at \( k_i \) step in vector form are represented as,

\[
Y = \begin{bmatrix}
y_1(k_1 + 1 | k_1) \\
y_1(k_1 + 2 | k_1) \\
\cdots \\
y_1(k_1 + 5 | k_1)
\end{bmatrix}^T \quad \text{and} \quad \Delta U = \begin{bmatrix}
\Delta u_f(k_1 + 1 | k_1) \\
\Delta u_f(k_1 + 2 | k_1) \\
\quad \cdots \\
\Delta u_f(k_1 + 4 | k_1)
\end{bmatrix}^T.
\]

Combining above both predicted output and control input in matrix form as,

\[
Y(k_i) = FX(k_i) + \Phi \Delta U(k_i) + \Psi \Delta G(k_i),
\]

where,

\[
F = \begin{bmatrix}
CA(k) \\
CA^2(k) \\
CA^3(k) \\
CA^4(k) \\
CA^5(k)
\end{bmatrix}, \quad \Phi = \begin{bmatrix}
CB_1 & \cdots & 0 \\
CA(k)B_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
CA^4(k)B_2 & \cdots & CA(k)B_2
\end{bmatrix}, \quad \Psi = \begin{bmatrix}
CB_2(k) & \cdots & 0 \\
CA(k)B_2(k) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
CA^4(k)B_2(k) & \cdots & CA(k)B_2(k)
\end{bmatrix}.
\]

Hence, after finding the cyclic event based control parameters \( F, \Phi, \Psi \) from above derivation, we uses these parameters in input signals for control input in Eq. (10).
Acknowledgment

The authors gratefully acknowledge to Toyota Motor Corporation for the supporting this research and valuable discussion and Dr. Mingxin Kang for helping in conducting the experiment:

References

Arsie, I., Di Leo, R., Pianese, C. and De Cesare, M., Estimation of in-cylinder mass and AFR by cylinder pressure measurement in automotive Diesel engines, In Proc. of 19th IFAC World Congress, Cape Town, South Africa, 2014, August, pp. 24-29.
Clerk, D., The gas engine, 1st ed., Longmans, Green and Co., 1886.
Daw, C. S., Finney, C. E. A., Green, J. B., Kennel, M. B., Thomas, J. F. and Connolly, F. T., A simple model for cyclic variations in a spark-ignition engine. SAE Technical Paper, No. 962086 (1996).
Daw, C. S., Kennel, M. B., Finney, C. E. A. and Connolly, F. T., Observing and modeling nonlinear dynamics in an internal combustion engine, Physical Review E, Vol. 57, No. 3 (1998), p. 2811.
Ebrahimi, B., Tafreshi, R., Masudi, H., Franchek, M., Mohammadpour, J. and Grigoriadis, K., A parameter-varying filtered PID strategy for airfuel ratio control of spark ignition engines, Control Engineering Practice, Vol. 20, No. 8 (2012), pp. 805-815.
Kim, H. I., Shon, J. and Lee, K., A Study of Fuel Economy and Exhaust Emission according to Engine Coolant and Oil Temperature, Journal of Thermal Science and Technology, Vol. 8, No. 1 (2013), pp. 255-268.
Kumar, M., and Shen, T., Estimation and feedback control of air-fuel ratio for gasoline engines, Control Theory and Technology, Vol. 13, No. 2 (2015), pp. 151-159.
Lee, B., Guezennec, Y. G. and Rizzoni, G., Estimation of cycle-resolved in-cylinder pressure and air-fuel ratio using spark plug ionization current sensing, International Journal of Engine Research, Vol.2 No. 4(2001), pp. 263-276.
Pace, S. and Zhu, G., Sliding Mode Control of a Dual-Fuel System Internal Combustion Engine, Proceedings of ASME Dynamic systems and Control Conference, 2009, pp. 881-887.
Pace, S. and Zhu, G. G., Transient Air-to-Fuel Ratio Control of an Spark Ignited Engine Using Linear Quadratic Tracking, Journal of Dynamic Systems, Measurement, and Control, Vol. 136, No. 2 (2014), p. 021008.
Pestana, G. W., Engine control methods using combustion pressure feedback, SAE Technical Paper, No. 890758 (1989).
Shinmura, N., Kubota, T. and Naitoh, K., Cycle-resolved computations of stratified-charge turbulent combustion in direct injection engine, Journal of Thermal Science and Technology, Vol. 8, No. 1 (2013), pp. 44-57.
Tunestål, P. and Hedrick, J. K., Cylinder air/fuel ratio estimation using net heat release data, Control engineering practice, Vol.11, No. 3 (2003), pp.311-318.
Wang, L., Model predictive control system design and implementation using MATLAB, Springer Science and Business Media, 2009.
Wang, S. W., Yu, D. L., Gomm, J. B., Page, G. F. and Douglas, S. S., Adaptive neural network model based predictive control for airfuel ratio of SI engines. Engineering Applications of Artificial Intelligence, Vol. 19, No.2 (2006), pp. 189-200.
Wibberley, P. and Clark, C. A., An investigation of cylinder pressure as feedback for control of internal combustion engines, SAE Technical Paper, No. 890396 (1989).
Wong, P. K., Wong, H. C., Vong, C. M., Xie, Z. and Huang, S., Model predictive engine air-ratio control using online sequential extreme learning machine, Neural Computing and Applications, 2004, pp. 1-14.
Yang, J., Shen, T. and Jiao, X., Model-Based Stochastic Optimal AirFuel Ratio Control With Residual Gas Fraction of Spark Ignition Engines, IEEE Transactions, Vol.22, No. 3 (2013), pp.896-910.
Zamanian, F., Franchek, M. A., Grigoriadis, K. M. and Makki, I., Feed-forward control of purge flow in internal combustion engines, International Journal of Engine Research, No. 1468087414565401(2015).
Zhai, Y. J., and Yu, D. L., Radial-basis-function-based feedforward-feedback control for airfuel ratio of spark ignition engines, Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, Vol. 222, No.3 (2008), pp. 415-428.