Research Article

Finite-Time $H_\infty$ Control of Affine Nonlinear Singular Systems Subject to Actuator Saturation

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This paper discusses the finite-time stable and finite-time $H_\infty$ control problems of affine nonlinear singular systems subject to actuator saturation. Some sufficient conditions, to guarantee the system is finite-time stable, are established for the affine nonlinear singular systems subject to actuator saturation. First, the finite-time stable problem is investigated by the state undecomposed method, and then the finite-time robust $H_\infty$ control law is presented for the system. Finally, the effectiveness of the designed controllers is shown by an example of a nonlinear singular circuit system in this paper.

1. Introduction

Singular system, also called descriptor system, is applied to many areas, such as engineering, economic, and biological systems [1]. In the past decades, the singular system has attracted interest of more and more researchers, and a lot of results are proposed for the linear singular system [1–10]. However, due to the complexity of the structure, few explore the nonlinear singular systems [11–15]. With the linear matrix inequality (LMI) method, the author in [11] has studied the robust control problem in connection with a set of stochastically nonlinear singular jump systems, while the guaranteed cost control and stabilization problems have been investigated for a set of time-delay nonlinear singular systems in [12, 13], respectively. Based on the approach of state undecomposed, scholars have considered the asymptotic stabilization of the nonlinear singular system in [14], including systems subject to actuator saturation in [15]. In [16], the control design of singular discrete-time systems has been studied based on simulation relations and behavioral theory.

It is well known that actuator saturation can compromise the functions of the closed loop system and cause instability of the system. Hence, scholars have been studying the stabilization problem of systems subject to actuator saturation for two decades [2–5, 17, 18]. Scholars have studied the stability problem in connection with the singular system with input saturation via the Lyapunov method in [2]. For the linear singular system with actuator saturation, the studies are proposed based on the stabilization conditions of the closed loop system, and the domain of attraction has been estimated by LMI technique in [3], while the authors in [4] proposed the estimate of the domain of attraction by the saturated state feedback method. In [5], the stochastic stability has been investigated for singular discrete-time Markov jump systems subject to input saturation by LMI approach. The stabilization controller has been designed based on the adaptive dynamic programming algorithm on nonlinear systems with input saturation in [17]. Fridman et al. [18] have studied local stabilization and $H_\infty$ control for the system with time delay and input saturation via the LMI and Lyapunov–Krasovskii functional method. The robust output regulation problem has been studied for discrete-time singular systems subject to actuator saturation with an additional control term to the nonlinear feedback in [19]. Zong et al. [20] investigated the decentralized adaptive output feedback saturated control problem for interconnected nonlinear systems with strong interconnections.
For nonlinear systems with actuator fault and saturation, the authors in [21] applied the surface control technique to address the finite-time adaptive output feedback control and the authors in [22] used the sliding mode and neutral networks methods to design the adaptive fault-tolerant controller.

Unlike the asymptotical stability [23–26], the finite-time stable (FTS) is that the state (weighted) norm does not exceed a certain boundary within a fixed time $T$. In many practical applications, FTS plays an important role, such as analyzing the transient behavior of the controlled system within a finite interval. Due to its extensive engineering application background, the FTS problem has attracted much scholarly interest [6, 7, 27–42]. In [27, 28], the FTS problem of the linear system has been investigated. To guarantee FTS of switched linear systems subject to actuator saturation, the authors in [29] designed the FTS controllers with the time domain approach. The authors in [30, 31] gave the sufficient and necessary conditions of FTS for the impulsive linear system by using the LMI method. Ma et al. and Wang and Feng [6, 32] considered the FTS for singular discrete-time Markov jump systems subject to input saturation by using the Lyapunov–Krasovskii functional method and using the mode-dependent parameter approach, respectively. Feng et al. [7] investigated the FTS for the linear singular system with the LMI method. Based on the sliding mode control design, FTS and input-output FTS problems are, respectively, dealt with in [33–35] for a class of nonlinear systems. In [36–38], the finite-time asynchronous dissipative filtering, finite-time asynchronous $L_2$-gain control, and finite region asynchronous $H_{\infty}$ control have been, respectively, studied for nonlinear Markov jump systems while an annular finite-time $H_{\infty}$ filter has been considered for networked switched systems in [39]. The finite-time $H_{\infty}$ controller has been given for the nonlinear singular discrete-time system in [40, 41] and for the nonlinear singular continuous time system in [42], respectively. It is worth pointing out that there is another definition of finite-time stability, where all states of the system reach the equilibrium point within a fixed time $T$ and stay at the equilibrium point permanently [21, 43–47]. Based on the Hamiltonian function method, the authors in [43] studied the observer design problem of general nonlinear time-delay systems and gave the finite-time robust stabilization results; the authors in [44] investigated the finite-time stabilizability problem for a class of singular systems by the constructed new Lyapunov functional while the finite-time robust simultaneous stabilization and adaptive robust simultaneous stabilization have been investigated for nonlinear systems with time delay in [45, 46], respectively. In [47], the finite-time stabilization problem has been considered for a class of high-order stochastic nonlinear systems by using the backstepping method.

Because the control input is limited by the saturation nonlinear function, it is more difficult to design a control for nonlinear singular systems with input saturation compared with the case without actuator saturation. To the best of the knowledge, the authors are only aware of few results related to the FTS of the nonlinear singular systems with input saturation. Compared with the mentioned results, the main contributions of the paper are highlighted as follows:

1. The state feedback controllers designed in the paper have simple form, so it has low computational complexity
2. The design method proposed in the paper has low conservative criteria, and the singular matrix $E$ does not need to satisfy any restriction conditions
3. The nonlinear function $\phi(x)$ does not need to satisfy Lipschitz conditions

This paper, in Section 2, introduces the definition of FTS and provides the design method of FTS controller for affine nonlinear singular systems subject to actuator saturation (ANSSAS). With the state undecomposed method, Section 3 discusses the finite-time $H_{\infty}$ control problem and designs a corresponding controller for the ANSSAS with external disturbance. In Section 4, an example of a circuit system is given to illustrate the effectiveness of the proposed controllers, and the simulation curves are presented. Section 5 provides a brief conclusion.

Notations. In the paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $A \in \mathbb{R}_{\text{sym}}^{n \times n}$ implies that $A$ is an $n \times n$-matrix in real number field. $A^T$ is the transpose of matrix $A$. $\lambda_{\text{max}}(Q_1)$ and $\lambda_{\text{min}}(Q_1)$ are the maximum and minimum eigenvalues of square matrix $Q_1$, respectively. $Q_1 > 0$ ($Q_1 \geq 0$) implies that square matrix $Q_1$ is positive definite (positive semidefinite). The Euclidean norm of vectors $z$ is denoted by $\|z\|$.

2. FTS of ANSSAS

This section discusses the FTS problem for ANSSAS. Consider the ANSSAS as follows:

$$
\begin{align*}
E \dot{x}(t) &= \phi(x(t)) + B(x(t))\text{sat}(u(t)), \\
E \dot{x}(0) &= E\xi_0, \\
\phi(0) &= 0,
\end{align*}
$$

(1)

where $x(t) \in \mathbb{R}^n$ is the state, $E \in \mathbb{R}^{m \times n}$, $0 \leq \text{rank } E = r < n$; $B(x(t)) \in \mathbb{R}^{m \times n}$, $\phi(x(t)) \in \mathbb{R}^n$ is a sufficiently smooth vector field; $\text{sat}(u(t)) \in \mathbb{R}^m$ is the saturation nonlinearity control input, and

$$
\text{sat}(u_i(t)) = \begin{cases} 
 u_i(t), & u_i(t) > t_i, \\
 -t_i \leq u_i(t) \leq t_i, & i = 1, 2, \ldots, m,
\end{cases}
$$

(2)

To study system (1), we present the following definition and lemmas.

Definition 1 (see [48]). For any initial condition $E\xi_0$, if the resulted closed loop singular system is impulsive free, then the control law $u(x(t))$ is said to be admissible, and the original system is said to be impulse controllable.
Lemma 1 (see [49]). If a vector function $S(x(t))$ with $S(0) = 0$ (for $x(t) \in \mathbb{R}^n$) has continuous $n$ th-order partial derivatives, then $S(x(t))$ can be rewritten as

$$S(x(t)) = a_1(x(t))x_1(t) + \cdots + a_n(x(t))x_n(t) = A(x(t))x(t),$$

where $A(x(t)) = [a_1(x(t)) a_2(x(t)) \cdots a_n(x(t))] \in \mathbb{R}^{m \times n}$.

Lemma 2 (see [15]). Denote

$$\text{sat}(u(t)) = u(t) - \delta(t).$$

Then, there exists a positive real number $\zeta$ such that

$$\delta^T(t)\delta(t) \leq \zeta u^T(t)u(t),$$

where $0 < \zeta < 1$, $\delta(t) = [\delta_1(t), \delta_2(t), \ldots, \delta_n(t)]^T \in \mathbb{R}^n$, and $\delta_i(t)$ is the dead-zone nonlinearity function, $i = 1, 2, \ldots, m$.

According to [7, 27, 41], we introduce the definition as follows.

Definition 2. ANSSAS (1) is called FTS with respect to $(c_1, c_2, T, R)$, with $0 < c_1 < c_2$ and $R > 0$ if ANSSAS (1) is impulse controllable and $x^T(0)E^TREx(0) \leq c_1$ such that

$$x^T(t)E^TREx(t) < c_2, \forall t \in [0, T].$$

According to Lemma 1, system (1) can be transformed into

$$Ex(t) = A(x(t))x(t) + B(x(t))\text{sat}(u(t)).$$

To facilitate the analysis of system (6), we provide an assumption and a lemma.

Assumption 1. Rank $\begin{bmatrix} 0 & E & 0 \\ E & A(x(t)) & B(x(t)) \end{bmatrix} = n + \text{rank}E, \forall x(t) \in \mathbb{R}^n$.

Lemma 3 (see [14]). Assume Assumption 1 holds, then system (6) is impulse controllable.

Under Assumption 1, the following result is given.

Theorem 1. Consider ANSSAS (1) and its equivalent system (6). If Assumption 1 holds, there exist two positive real numbers $\beta$ and $\zeta$ and three matrices $K(x(t)) \in \mathbb{R}^{m \times n}$, $Q_1 \in \mathbb{R}^{n \times n}$, and $P \in \mathbb{R}^{n \times n}$ such that

$$(A(x(t)) - B(x(t))K(x(t)))^TPE + E^T(P(A(x(t)) - B(x(t))K(x(t))) + E^TB^TPBPE + \zeta K^T \beta E^TPE \leq 0,$$

$$\lambda_{\max}(Q_1)c_1e^{\beta t} < c_2\lambda_{\min}(Q_1),$$

then the FTS controller of system (1) can be given as follows:

$$u = -K(x(t))x(t),$$

where $c_2 > c_1 > 0$, $0 < \zeta < 1$, $R > 0$, $Q_1 > 0$, $P > 0$, and $P = R^{1/2}Q_1R^{1/2}$.

Proof. Applying (4) and (9) to system (6), it has

$$\dot{V}(x(t)) - \beta V(x(t))$$

$$= (Ex(t))^TPEx(t) + x^T(t)E^TPE\dot{x}(t) - \beta x^T(t)E^TPEx(t)$$

$$= x^T(t)(A(x(t)) - B(x(t))K(x(t)))^TPEx(t) + x^T(t)E^T(P(A(x(t)) - B(x(t))K(x(t)))x(t)$$

$$- 2x^T(t)E^TPB(x(t))\delta - \beta x^T(t)E^TPEx(t)$$

$$\leq x^T(t)(A(x(t)) - B(x(t))K(x(t)))^TPE + E^T(P(A(x(t)) - B(x(t))K(x(t)))x(t)$$

$$+ x^T(t)E^TPB(x(t))B^T(x(t))PEx(t) + \delta^T(t) - \beta x^T(t)E^TPEx(t)$$

$$\leq x^T(t)E^T(P(A(x(t)) - B(x(t))K(x(t)))^TPE + E^T(P(A(x(t)) - B(x(t))K(x(t))))x(t)$$

$$+ E^T(PB(x(t))B^T(x(t))PE + \zeta K^T(x(t))K(x(t)) - \beta E^TPE)x(t)$$

$$\leq 0,$$
which is
\[ V(x(t)) \leq \beta V(x(0)) \quad \forall t \in [0, T]. \] (12)

Next, we prove that system (10) is FTS. By integrating inequality (12) between 0 and T with \( t \in [0, T] \), it follows that
\[ \ln \frac{V(x(t))}{V(x(0))} \leq \beta t. \] (13)

It is clear that
\[ V(x(t)) \leq e^{\beta t} V(x(0)). \] (14)

Given the chain of inequalities as follows:
\[ V(x(t)) = x^T(t)E^T R^{1/2} Q_1 R^{1/2} E x(t) \]
\[ \geq \lambda_{\min}(Q_1) x^T(t) E^T R E x(t), \] (15)
\[ V(x(0)) e^{\beta t} = x^T(0) E^T R^{1/2} Q_1 R^{1/2} E x(0) e^{\beta t} \leq \lambda_{\max}(Q_1) x^T(0) E^T R E x(0) e^{\beta t}. \] (16)

According to \( x^T(0) E^T R E x(0) \leq c_1 \), putting together (14)–(16), we have
\[ x^T(t) E^T R E x(t) \leq \lambda_{\max}(Q_1)c_1 e^{\beta t}. \] (17)

From (8) and (17), it can be obtained that \( x^T(t) E^T R E x(t) < c_2, \forall t \in [0, T] \). So, system (1) is FTS with respect to \((c_1, c_2, T, R)\).

3. Finite Time \( H_{\infty} \) Control of ANSSAS

Based on Section 2, this section studies the finite-time \( H_{\infty} \) control law for the ANSSAS.

Consider ANSSAS as follows:
\[
\begin{aligned}
E x(t) &= \phi(x(t)) + B(x(t)) s(t) + E d(x(t)) w(t), \\
E x(0) &= E x_0, \phi(0) = 0, \\
y(t) &= h_2(x(t)), \\
z(t) &= h_1(x(t)),
\end{aligned}
\] (18)

where \( y(t) \in \mathbb{R}^r \) is the output, \( z(t) \in \mathbb{R}^q \) is the penalty signal, \( w(t) \in \mathbb{R}^s \) is the external disturbance, and \( d(x(t)) \in \mathbb{R}^{mxs}, E, x(t), \phi(x(t)), s(t), \) and \( B(x(t)) \) are the same as those in ANSSAS (1).

Choose \( h_1(x(t)) = L x(t) B^T x(t) \), \( h_2(x(t)) = d^T x(t) x(t) \), where \( L(x(t)) \) is full column rank. From Section 2, we know that we can design an admissible finite-time \( H_{\infty} \) control law \( u(t) \) for system (18) under Assumption 1. The design steps of the finite-time \( H_{\infty} \) controller are as follows. First, we design an admissible control law \( u \) such that the \( L_2 \) gain of the closed loop system is not greater than \( \gamma \), where \( \gamma > 0 \) is a given disturbance attenuation level. Next, we demonstrate that the resulted closed loop system is FTS when \( w(t) = 0 \). To design the finite-time \( H_{\infty} \) controller for the ANSSAS, we recall the following lemma:

**Lemma 4** (see [50]). Consider an affine nonlinear system:
\[
\begin{aligned}
\dot{x} &= f(x) + g(x) w, \quad f(x_0) = 0, \\
z &= h(x),
\end{aligned}
\] (19)

where \( x \in \mathbb{R}^n, w \in \mathbb{R}^s, \) and \( z \in \mathbb{R}^q \) are the state, disturbance, and penalty signal of the system, respectively. If there exists a function \( V(x) \geq 0 \) for which the Hamilton–Jacobi inequality
\[
\frac{\partial V}{\partial x} f(x) + \frac{1}{2} \frac{\partial^2 V}{\partial x^2} g^T g \frac{\partial V}{\partial x} + \frac{1}{2} \gamma^2 h \leq 0,
\] (20)
holds, then the \( L_2 \) gain of system (19) (from \( w \) to \( z \)) is bounded by \( \gamma \), i.e.,
\[
\int_0^T \|z(t)\|^2 dt \leq \gamma^2 \int_0^T \|w(t)\|^2 dt, \quad \forall w \in L_2[0, T],
\] (21)
where \( \gamma \) is a positive number.

Based on with, we give the following theorem.

**Theorem 2.** Consider ANSSAS (18). Suppose that Assumption 1 holds. Let
\[
u(t) = \left( \begin{array}{c}
K(x(t)) + \frac{1}{2} \left( L^T(x(t)) L(x(t)) + \frac{1}{1+T_{\infty}} \right) B^T(x(t))
\end{array} \right)
\]
\[x_{m} = K_1(x(t)) x(t).
\] (22)

If
\[
\begin{aligned}
(A(x(t)) - B(x(t)) K_1(x(t)))^T P E + E^T P (A(x(t)) - B(x(t)) K_1(x(t))) + \frac{1}{2} B(x(t)) L^T(x(t)) L(x(t)) B^T(x(t)) + \zeta \kappa_1^T(x(t)) K_1(x(t)) + E^T P B B^T P E \leq 0,
\end{aligned}
\] (23)
\[
\lambda_{\max}(Q_1)c_1 e^{\beta t} < c_2 \lambda_{\min}(Q_1),
\] (24)

then controller (22) is the finite-time \( H_{\infty} \) control law of ANSSAS (18), where \( P, Q_1, \beta, \zeta, c_1, \) and \( c_2 \) are the same as those in Theorem 1.
\[
\begin{aligned}
\dot{x}(t) &= (A(x(t)) - B(x(t))K_1(x(t)))x(t) - B(x(t))\delta(t) + E d(x(t))w(t), \\
y(t) &= d^T(x(t))E^T x(t), \\
z(t) &= L(x(t))B^T(x(t))x(t).
\end{aligned}
\] (25)

Choose a proper Lyapunov function
\[
V(x(t)) = x^T(t)E^TPEx(t);
\] according to (23), we have

\[
\dot{V}(x(t)) - \beta V(x(t)) = \frac{\partial^T V(x(t))}{\partial x(t)} \cdot x(t) - \beta x^T(t)E^TPEx(t)
\]

\[
= x^T(t)\left( (A(x(t)) - B(x(t))K_1(x(t)))^TPE + E^TP(A(x(t)) - B(x(t))K_1(x(t))) \right)x(t)
\]

\[
-2x^T(t)E^TPB(x(t))\delta(t) + w^T(t)d^T(x(t))E^TPEx(t) + x^T(t)E^TPEx(t)w(t) - 2x^T(t)E^TPEx(t)w(t)
\]

\[
+2\frac{\beta}{\gamma} x^T(t)E^TP d(x(t))d^T(x(t))E^TPEx(t) + \frac{1}{2} x^T(t)B(x(t))L^T(x(t))L(x(t))B^T(x(t))x(t)
\]

\[
\leq x^T(t)\left( (A(x(t)) - B(x(t))K_1(x(t)))^TPE + E^TP(A(x(t)) - B(x(t))K_1(x(t))) \right)x(t)
\]

\[
+2\frac{\beta}{\gamma} x^T(t)E^TP d(x(t))d^T(x(t))E^TPEx(t) + \frac{1}{2} x^T(t)B(x(t))L^T(x(t))L(x(t))B^T(x(t))x(t)
\]

\[
+\zeta K_1^T(x(t))K_1(x(t)) + E^TPB(x(t))B^T(x(t))PE \right)x(t) \leq 0.
\] (26)

By Lemma 4, the $L_2$ gain of system (25) is not more than $\gamma$.

Next, we prove that the system is FTS if $w(t) = 0$.
The following proof is the same as that in Theorem 1. So, the closed loop system (25) is FTS if \( w(t) = 0 \).

4. A Circuit Example

This section proposes a circuit example to show the effectiveness of the finite-time \( H_{\infty} \) controller designed in Theorem 2.

**Example 1.** Consider the circuit system as Figure 1, where \( i_w \) is a disturbance signal, \( u_1 = \varphi_1(q_1), u_2 = \varphi_2(q_2) \).

From Kirchhoff’s current law and voltage law, the circuit system can be expressed as

\[
\begin{bmatrix}
1 & 1 \\
0 & 0
\end{bmatrix} \dot{x}(t) = \begin{bmatrix}
-3x_1(t) - x_1(t) x_2^2(t) \\
x_1(t) - 6x_2(t) - 2x_2^3(t)
\end{bmatrix} + \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix} \text{sat}(v) + \begin{bmatrix}
-1 \\
0
\end{bmatrix} w(t).
\]  

Choose the penalty signal \( z = (1/2)[q_2, q_1]^T \). Thus, system (29) and \( z \) can be combined to

\[
\begin{bmatrix}
1 & 1 \\
0 & 0
\end{bmatrix} \dot{x}(t) = \begin{bmatrix}
-3x_1(t) - x_1(t) x_2^2(t) \\
x_1(t) - 6x_2(t) - 2x_2^3(t)
\end{bmatrix} + \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix} \text{sat}(v) + \begin{bmatrix}
-1 \\
0
\end{bmatrix} w(t),
\]  

where \( L(x(t)) = \begin{bmatrix}
1/2 & 0 \\
0 & 1/2
\end{bmatrix} \). According to \( \phi(0) = 0 \), we have

\[
A(x(t)) = \begin{bmatrix}
-3 - x_2^2(t) & 0 \\
1 & -6 - 2x_2^3(t)
\end{bmatrix}.
\]

It is not difficult to verify that Assumption 1 holds.

Let

\[
P = \begin{bmatrix}
2 & 1 \\
1 & 2
\end{bmatrix},
\]

\[
Q_1 = \begin{bmatrix}
2 & 1 \\
1 & 2
\end{bmatrix},
\]

\[
R = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix},
\]

\[
K(x(t)) = \begin{bmatrix}
5/2 & 0 \\
0 & 5/2
\end{bmatrix},
\]

\[
q_1 + q_2 = \text{sat}(I_x) = \frac{\varphi_1(q_1)}{R_3} - i_w,
\]

\[
0 = \text{sat}(U_x) + \varphi_1(q_1) - \varphi_2(q_2).
\]  

We introduce \( \varphi_1(q_1) = q_1, \varphi_2(q_2) = q_2^3 \). \( R_3 = (1/2) \Omega \), and \( |U_x| \leq 8, |I_x| \leq 6 \). Let \( v = [v_1, v_2]^T = [U_x, I_x]^T \), \( x(t) = [x_1(t), x_2(t)]^T = [q_1, q_2]^T \), and \( w(t) = i_w \). Then, system (28) can be rewritten as

\[
\text{for given disturbance attenuation } \gamma = 1; \text{choose some parameter values } c_1 = 0.5, c_2 = 3, T = 6, \beta = 0.1, \text{ and } \zeta = 0.1, \text{ then}
\]

\[
\begin{aligned}
(A(x(t)) - B(x(t))K_1(x(t)))^T P E + E^T P (A(x(t)) - B(x(t))K_1(x(t))) \\
+ \frac{2}{\gamma} E^T P E d(x(t))d^T(x(t))E^T P E \\
+ \frac{1}{2} B(x(t))L^T(x(t))L(x(t))B^T(x(t)) \\
+ \frac{1}{2} \zeta K_1^T(x(t))K_1(x(t)) + E^T PBB^T P E
\end{aligned}
\]  

\[
\begin{bmatrix}
-3.711 - 4x_2^2(t) & -5.319 - 4x_2^2(t) \\
-5.319 - 4x_2^2(t) & -6.230 - 4x_2^2(t)
\end{bmatrix} \leq 0,
\]

\[
\frac{\lambda_{\max}(Q_1)c_1}{\lambda_{\min}(Q_1)} = 3c_1e^{0.6} < c_2
\]

hold.

Obviously, it is illustrated that all conditions of Theorem 2 can be satisfied.
Figure 1: Nonlinear singular circuit system.

Figure 2: Response of $\omega = x^T(t)E^TREx(t)$ for the open loop system.

Figure 3: Response of $\omega = x^T(t)E^TREx(t)$ for the closed loop system.
Thus, we can give the following finite-time $H_{\infty}$ controller of system (29):

$$
\nu = \begin{bmatrix} \frac{5}{2} & \frac{5}{8} \\ \frac{5}{8} & \frac{5}{5} \end{bmatrix} x(t). \quad (33)
$$

To check the effectiveness of the control law (33), give $Ex(0) = [0.5, 0]^T$ and input a square-wave disturbance of amplitude $[0, 2]^T$ in the time duration $[1 \text{s} - 2 \text{s}]$ for the system. The response of $\bar{\omega} = x^T(t)E^T R E x(t)$ is presented in Figures 2 and 3 for the open-loop system and the closed loop system, respectively. It is clear that $x^T(t)E^T R E x(t) > 3$ in the open loop system $\forall t \in [3.5, 6]$, whereas $x^T(t)E^T R E x(t) < 3$ in the closed loop system $\forall t \in [0, 6]$. The responses of state $x$ and saturation input $\text{sat}(\nu)$ are given in Figures 4 and 5, respectively. According to Figures 2–5, it is clear that the circuit system (29) is FTS with respect to $(0.5, 3, 6, I)$ under the admissible $H_{\infty}$ control law (33).

5. Conclusion

This paper investigates the finite-time control problem for affine nonlinear singular systems subject to actuator saturation by using the state undecomposed method. First, saturation input is represented by control input and dead-zone nonlinear compensation. Then, the finite-time control law has been designed under sufficient condition of the system impulsive controllable. Based on with, the finite-time
$H_\infty$ control problem is solved via the suitable state feedback. New results on the finite-time control and finite-time $H_\infty$ control problems have been presented for affine nonlinear singular systems subject to actuator saturation. In the future, the input-output finite-time control problems can be studied for affine nonlinear singular systems subject to actuator saturation.

Data Availability
The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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References
[1] L. Y. Dai, Singular Control Systems, Springer-Verlag, Berlin, Germany, 1989.
[2] J. R. Liang, H. L. Choi, and J. T. Lim, "On stability of singular systems with saturating actuators," IEICE Transactions on Fundamentals of Electronics Communications and Computer Sciences, vol. 86, no. 10, pp. 2700–2703, 2003.
[3] Z. Lin and L. Lv, "Set invariance conditions for singular linear systems subject to actuator saturation," IEEE Transactions on Automatic Control, vol. 52, no. 12, pp. 2351–2355, 2007.
[4] Y. L. Li and Z. L. Liu, "Improved set invariance conditions for singular linear systems subject to actuator saturation," Control Theory and Applications, vol. 31, no. 7, pp. 955–961, 2014.
[5] S. P. Ma, C. Zhang, and S. Q. Zhu, "Robust stability for discrete-time uncertain singular Markov jump systems with actuator saturation," IET Control Theory and Applications, vol. 5, no. 2, pp. 255–262, 2011.
[6] Y. Ma, X. Jia, and D. Liu, "Finite-time dissipative control for singular discrete-time Markovian jump systems with actuator saturation and partly unknown transition rates," Applied Mathematical Modelling, vol. 53, pp. 49–70, 2018.
[7] J. E. Feng, Z. Wu, J. B. Sun, and Z. Cheng, "Finite-time control of linear singular systems subject to parametric uncertain and disturbances," in Proceedings of the 5th World Congress on Intelligence Control and Automation, Hangzhou, China, June 2004.
[8] J. Tian and S. Ma, "Existence of nonimpulsive unique solution and stability for discrete-time linear rectangular descriptor Markov jump systems," IEEE Transactions on Automatic Control, vol. 64, no. 10, pp. 4245–4251, 2019.
[9] J. Wang, Z. Huang, Z. Wu, J. Cao, and H. Shen, "Extended dissipative control for singularly perturbed PDT switched systems and its application," IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 67, no. 12, pp. 5281–5289, 2020.
[10] J. Wang, Z. G. Huang, Z. G. Wu et al., "Extended dissipative control for singularly perturbed PDT switched systems and its application," IEEE Transactions on Fuzzy Systems, vol. 67, no. 12, pp. 5281–5289, 2020.
[11] Q. Zhu, "Stabilization of stochastically singular nonlinear jump systems with unknown parameters and continuously distributed delays," International Journal of Control, Automation and Systems, vol. 11, no. 4, pp. 683–691, 2013.
[12] R. Q. Lu, H. Y. Su, J. Z. Wang, A. Xue, and T. Shi, "Robust optimal control for a class of nonlinear uncertain singular systems with time-delay," in Proceedings of the 2006 American Control Conference, pp. 5020–5024, Minneapolis, MN, USA, June 2006.
[13] J. C. Wu, S. L. Wo, and G. P. Lu, "Asymptotic stability and stabilization for a class of nonlinear descriptor systems with delay," Asian Journal of Control, vol. 13, no. 2, pp. 361–367, 2011.
[14] L. Sun and Y. Wang, "An undecomposed approach to control design for a class of nonlinear descriptor systems," International Journal of Robust and Nonlinear Control, vol. 23, no. 6, pp. 695–708, 2013.
[15] L. Sun, Y. Wang, and G. Feng, "Control design for a class of affine nonlinear descriptor systems with actuator saturation," IEEE Transactions on Automatic Control, vol. 60, no. 8, pp. 2195–2200, 2015.
[16] S. Haesaert, F. Chen, A. Abate, and S. Weiland, "Formal control synthesis via simulation relations and behavioral theory for discrete-time descriptor systems," IEEE Transactions on Automatic Control, vol. 66, no. 3, pp. 1024–1039, 2021.
[17] B. Zhao, L. H. Jia, H. B. Xia, and Y. Li, "Adaptive dynamic programming-based stabilization of nonlinear systems with unknown actuator saturation," Nonlinear Dynamics, vol. 93, pp. 2089–2103, 2018.
[18] E. Fridman, A. Pila, and U. Shaked, "Regional stabilization and Hoo control of time-delay systems with saturating actuators," International Journal of Robust and Nonlinear Control, vol. 13, no. 9, pp. 885–907, 2003.
[19] E. Jafari and T. Binazadeh, "Robust output regulation in discrete-time singular systems with actuator saturation and uncertainties," IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 67, no. 2, pp. 340–344, 2020.
[20] G. Zong, H. Sun, and S. K. Nguang, "Decentralized adaptive neuro-output feedback saturated control for INS and its application to AUV," IEEE Transactions on Neural Networks and Learning Systems, pp. 1–10, 2021.
[21] R. Ji, J. Ma, D. Li, and S. S. Ge, "Finite-time adaptive output feedback control for MIMO nonlinear systems with actuator faults and saturations," IEEE Transactions on Fuzzy Systems, pp. 1–15, 2020.
[22] M. Qian, Z. Zheng, and P. Cheng, "Adaptive NFTSM-based fault tolerant control for a class of nonlinear system with actuator fault and saturation," IEEE Access, vol. 7, pp. 107083–107095, 2019.
[23] X. Yi, R. Guo, and Y. Qi, "Stabilization of chaotic systems with both uncertainty and disturbance by the UDE-based control method," IEEE Access, vol. 8, no. 1, pp. 62471–62477, 2020.
[24] L. Liu, B. Li, and R. Guo, "Consensus control for networked manipulators with switched parameters and topologies," IEEE Access, vol. 9, pp. 9209–9217, 2021.
[25] T. Hou, Y. Liu, and F. Deng, "Stability for discrete-time uncertain systems with infinite Markov jump and time-delay," Science China: Information Sciences, vol. 64, p. 1C11, 2021.
[26] R. Peng, C. Jiang, and R. Guo, “Stabilization of a class of fractional order systems with both uncertainty and disturbance,” IEEE Access, vol. 9, pp. 42697–42706, 2021.

[27] F. Amato, M. Ariola, and P. Dorato, “Finite-time control of linear systems subject to parametric uncertainties and disturbances,” Automatica, vol. 37, no. 9, pp. 1459–1463, 2001.

[28] F. Amato, M. Ariola, and C. Cosentino, “Finite-time stabilization via dynamic output feedback,” Automatica, vol. 42, no. 2, pp. 337–342, 2006.

[29] X. Lin, X. Li, Y. Zou, and S. Li, “Finite-time stabilization of switched linear systems with nonlinear saturating actuators,” Journal of the Franklin Institute, vol. 351, no. 3, pp. 1464–1482, 2014.

[30] R. Ambrosino, F. Calabrese, C. Cosentino, and G. De Tommasi, “Sufficient conditions for finite-time stability of impulsive dynamical systems,” IEEE Transactions on Automatic Control, vol. 54, no. 4, pp. 861–865, 2009.

[31] F. Amato, G. De Tommasi, and A. Pironti, “Input-output finite-time stabilization of impulsive linear systems: necessary and sufficient conditions,” Nonlinear Analysis: Hybrid Systems, vol. 19, pp. 93–106, 2016.

[32] G. Wang and B. Feng, “Finite-time stabilization for discrete-time delayed Markovian jump systems with partially delayed actuator saturation,” Discrete Dynamics in Nature and Society, vol. 2016, Article ID 1304379, 12 pages, 2016.

[33] J. Song, Y. Niu, and Y. Zou, “Finite-time stabilization via sliding mode control,” IEEE Transactions on Automatic Control, vol. 62, no. 3, pp. 1478–1483, 2017.

[34] X. Lv, Y. Niu, and J. Song, “Finite-time boundedness of uncertain Hamiltonian systems via sliding mode control approach,” Nonlinear Dynamics, vol. 104, no. 1, pp. 497–507, 2021.

[35] J. Song, Y. Niu, and Y. Zou, “Finite-time sliding mode control synthesis under explicit output constraint,” Automatica, vol. 65, pp. 111–114, 2016.

[36] X. Zhang, S. P. He, V. Stojanovic et al., “Finite-time asynchronous dissipative filtering of conic-type nonlinear Markov jump systems,” Science China: Information Sciences, vol. 64, pp. 152206:1–152206:12, 2021.

[37] C. Ren, S. He, X. Luan, F. Liu, and H. R. Karimi, “Finite-time L2-gain asynchronous control for continuous-time positive hidden Markov jump systems via T-S fuzzy model approach,” IEEE Transactions on Cybernetics, vol. 51, no. 1, pp. 77–87, 2021.

[38] P. Cheng, S. He, X. Luan, and F. Liu, “Finite-region asynchronous H∞ control for 2D Markov jump systems,” Automatica, vol. 129, no. 2021, Article ID 109590, 2021.

[39] G. Zong, H. Ren, and H. R. Karimi, “Event-triggered communication and annular finite-time H∞ filtering for networked switched systems,” IEEE Transactions on Cybernetics, vol. 51, no. 1, pp. 309–317, 2021.

[40] X. Lu, X. Zhang, and L. Sun, “Finite-time H∞ control for nonlinear discrete Hamiltonian descriptor systems,” Journal of the Franklin Institute, vol. 354, no. 14, pp. 6138–6151, 2017.

[41] M. Li, L. Sun, and R. Yang, “Finite-time H∞ control for a class of discrete-time nonlinear singular systems,” Journal of the Franklin Institute, vol. 355, no. 13, pp. 5384–5393, 2018.

[42] M. Li and L. Sun, “Finite-time stabilisation for a class of nonlinear descriptor systems,” IET Control Theory & Applications, vol. 12, no. 17, pp. 2399–2406, 2018.

[43] R. Yang, G. Zhang, and L. Sun, “Observer-based finite-time robust control of nonlinear time-delay systems via Hamiltonian function method,” International Journal of Control, vol. 4, pp. 1–18, 2020.