Aberration Structure of Spot of Dispersion in the Image of Point at the Decentering Elements of the Optical System

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Abstract. Desentering is named displacement of elements of the optical system of transverse optical axis. At the decentering of elements of the optical system its axial symmetry is violated. The structure of spots of dispersion in the image of points of object is thus violated. It is comfortably to execute the analysis of structure of representing points in area of primary aberrations (aberrations of Zeydelya). It is shown that at the decentering of elements of the optical system in the diffraction image of points, which do not lie on an optical axis, new aberration with the meridional and sagyttal constituents appears along with the known primary aberrations (coma, astigmatism, dystorsia). The aberration spot described by the got expression has the appearance of circle, the radius of which is determined by not only the coordinate on the pupil of the optical system but also by the value of the decentering.

In general case the meridional and sagyttal constituents of a transversal aberration can be presented by expansion in the power row on to variables $l, m$ and $M$, where $l$ is the coordinate of point of object in a meridional plane, $m$ and $M$ are the coordinates of point in the plane of entrance pupil. The number of possible members of the third order is equal ten. They can contain the followings product of variables $l$, $m$ and $M$ [1]:

$$m^3, m^2l, m^2M, m^2M, m^2l, mMl, M^2m, M^2l, ml^2, Ml^2.$$  

We will notice that if in expansion of functions $\delta g'$ and $\delta G'$, determining the constituents of transversal aberration in meridional and in sagyttal planes accordingly, to change a sign at to the variable $M$, the value of meridional making aberration $\delta g'$ will not change, and, consequently, decomposition in the row of function $\delta g'$ not must contain a variable $M$ in an odd degree. As the sagyttal constituent of aberration at the change of sign of variable $M$ must also change a sign, saving an absolute value by unchanging, presentation of function $\delta G'$ power alongside can not contain members with the even degrees of variable $M$, including in a zeroing degree.

In 1856 the Munich astronomer L Zeidel got expressions for the coefficients of power row of the third order relatively variables $l, m$ and $M$ in decomposition in the row of functions $\delta g'$ and $\delta G'$, containing the structural parameters of the axis-symmetric optical system in general case. L Zeidel showed that the coefficients of ten possible members of the third order of power row are not
independent of each other, because of what the number of different coefficients is taken to five. Thus expressions determining aberrations $\delta g'$ and $\delta G'$ and members of the third order containing only in general case it is possible to present in a kind [2]:

$$\delta g' = Am\left(m^2 + M^2\right) + Bl\left(3m^2 + M^2\right) + Cl^2m + El^3,$$

$$\delta G' = AM\left(m^2 + M^2\right) + 2BmM + Di^2M.$$  

At decentering elements of the optical system (at displacement elements across an optical axis) its axial symmetry is violated. Therefore for the analysis of influencing of decentering elements of the optical system on aberration of image comfortably intersection ray with the plane of object to define by coordinates $l$ and $L$ in meridional and sagyttal planes accordingly. For this purpose we will turn coordinate axes in the planes of object, image and entrance pupil about optical axis on a corner $\varphi$ and we will designate new coordinates in the plane of object by letters $\bar{T}$ and $\bar{L}$, and new coordinates of intersection ray with the plane of entrance pupil – letters $\bar{m}$ and $\bar{M}$ . For the transition from old coordinates to new we have the followings formulas:

$$l = \bar{T} \cos \varphi + \bar{L} \sin \varphi,$$

$$m = \bar{m} \cos \varphi + \bar{M} \sin \varphi,$$

$$M = -\bar{m} \sin \varphi + \bar{M} \cos \varphi.$$  

In addition,

$$\bar{T} \sin \varphi = \bar{L} \cos \varphi.$$  

Constituents $\delta \bar{g}'$ and $\delta \bar{G}'$ to transversal aberration in the new system of coordinates related to the former constituent $\delta g'$ and $\delta G'$ by formulas:

$$\delta \bar{g}' = \delta g' \cos \varphi - \delta G' \sin \varphi,$$

$$\delta \bar{G}' = \delta g' \sin \varphi + \delta G' \cos \varphi.$$  

If we replace in (1) and (2) variables $l$, $m$ and $M$ on their values, determined by (3), (4) and (5), and using (6) here, we get:

$$\delta g' = A(m^2 + M^2)(m \cos \varphi + M \sin \varphi)$$

$$+ B\left(\bar{T} \cos \varphi + \bar{L} \sin \varphi\right)\left[3\bar{m}^2 + \bar{M}^2 \right] - 2\left(\bar{m}^2 \sin^2 \varphi - 2\bar{m}\bar{M} \sin \varphi \cos \varphi + \bar{M}^2 \cos^2 \varphi\right)$$

$$+ C\left(\bar{T}^2 + \bar{L}^2\right)(m \cos \varphi + M \sin \varphi) + E(\bar{T}^2 + \bar{L}^2)(T \cos \varphi + L \sin \varphi).$$

$$\delta G' = A\left(m^2 + M^2\right)(M \cos \varphi - m \sin \varphi)$$

$$+ 2B\left(\bar{T} \cos \varphi + \bar{L} \sin \varphi\right)\left(M \sin \varphi \cos \varphi + \bar{m} \bar{M} \cos^2 \varphi - \bar{m} \bar{M} \sin^2 \varphi\right)$$

$$- m^2 \sin \varphi \cos \varphi + D(\bar{T}^2 + \bar{L}^2)(M \cos \varphi - m \sin \varphi).$$

We will put the got correlations in (7) and (8). Using (6), we will transform the last expressions to the kind:

$$\delta \bar{g}' = A(m^2 + M^2) + B\left[3m^2 + M^2\right] + CT\left(mT + mL\right) + DL\left(mL - MT\right) + ET\left(T^2 + L^2\right),$$

$$\delta \bar{G}' = A\bar{m}\left(m^2 + M^2\right) + B\left[m^2 + 3M^2\right] + L\left(l + 2\bar{m}M\right),$$

$$+ CE\left(MT - mL\right) + DT\left(MT - mL\right) + ET\left(T^2 + L^2\right).$$

It is possible to cast aside hyphens unnecessary now above letters. Differentiating (11) and (12) and replacing differentials by eventual differences, taking into account this remark we get
$$\Delta \delta g' = A \left[ (3m^2 + M^2) \Delta m + 2mM \Delta M \right]$$
$$+ B \left[ (3m^2 + M^2) \Delta l + 2(3ml + ML) \Delta m + 2(Ml + mL) \Delta M + 2mM \Delta l \right]$$
$$+ C \left[ L^2 \Delta m + (2ml + ML) \Delta l + ml \Delta l + lL \Delta m \right]$$
$$+ D \left[ L^2 \Delta m + (2ml - ML) \Delta l - ml \Delta l - lL \Delta m \right]$$
$$+ E \left[ (3l^2 + L^2) \Delta l + 2lL \Delta l \right].$$

\(\Delta \delta G' = A \left[ (m^2 + 3M^2) \Delta m + 2mM \Delta m \right]
$$+ B \left[ (m^2 + 3M^2) \Delta l + 2(ml + ML) \Delta m + 2(3ML + ml) \Delta M + 2mM \Delta l \right]$$
$$+ C \left[ L^2 \Delta m + (2ML + ml) \Delta l + ml \Delta l + lL \Delta m \right]$$
$$+ D \left[ L^2 \Delta m + (2ML - ml) \Delta l - ml \Delta l - lL \Delta m \right]$$
$$+ E \left[ (l^2 + 3L^2) \Delta l + 2lL \Delta l \right].$$

For the analysis of the change of aberration structure of the spot of dispersion in the image of a point it is comfortably to pass to the arctic system of coordinates:

\[ m = a \rho \cos \alpha, \quad l = r \cos \beta, \]
\[ M = a \rho \sin \alpha, \quad L = r \sin \beta. \]

Let \( \delta \) be the displacement of the main point of the optical element (center of curvature of the spherical surface) in the direction perpendicular to the optical axis of the system. Thus

\[ \Delta m = \Delta l = -\delta \cos \gamma, \]
\[ \Delta M = \Delta L = -\delta \sin \gamma. \]

Then (13) and (14) it is possible to transform to the kind:

$$\Delta \delta g' = -a^2 \rho^2 \delta (A + B) \left[ 2 \cos \gamma + \cos(2\alpha - \gamma) \right]$$
$$- a \rho \delta \left[ (4B + C + D) \cos(\beta - \gamma) \cos \alpha + (2B + C - D) \cos(\alpha - \beta - \gamma) \right]$$
$$- r^2 \delta \left[ C \cos \beta \cos(\beta - \gamma) + D \sin \beta \sin(\beta - \gamma) + 2E \cos \gamma + E \cos(2\beta - \gamma) \right].$$

$$\Delta \delta G' = -a^2 \rho^2 \delta (A + B) \left[ 2 \sin \gamma + \sin(2\alpha - \gamma) \right]$$
$$- a \rho \delta \left[ (4B + C + D) \cos(\beta - \gamma) \sin \alpha - (2B + C - D) \sin(\alpha - \beta - \gamma) \right]$$
$$- r^2 \delta \left[ C \sin \beta \cos(\beta - \gamma) - D \cos \beta \sin(\beta - \gamma) + 2E \sin \gamma + E \sin(2\beta - \gamma) \right].$$

We will consider each of the aberrations, which bring into the image by transversal displacement of any element of the optical system separately.

Let aberrations \( \delta g' \) and \( \delta G' \) is determined by the first members of (15) and (16). We have thus:

$$\Delta \delta g' + 2(A + B) a^2 \rho^2 \delta \cos \gamma = -(A + B) a^2 \rho^2 \delta \cos(2\alpha - \gamma),$$
$$\Delta \delta G' + 2(A + B) a^2 \rho^2 \delta \sin \gamma = -(A + B) a^2 \rho^2 \delta \sin(2\alpha - \gamma).$$

Raising the left and right parts of these expressions in a second power and laying down them, we get

$$\left[ \Delta \delta g' + 2(A + B) a^2 \rho^2 \delta \cos \gamma \right]^2 + \left[ \Delta \delta G' + 2(A + B) a^2 \rho^2 \delta \sin \gamma \right]^2 = (A + B)a^4 \rho^4 \delta^2$$

(17)
We will notice that the size of corner $\gamma$ makes practical sense only at adding up of the aberrations, which bring into the image by arbitrary transversal displacement of row of elements of the optical system. Therefore in examined concrete case, not violating community of conclusion, it is fully possible to put a corner $\gamma = 0$. Thus (17) assumes a form:

$$\left[(\Delta \delta g')^2 + 2(A + B) a^2 \rho^2 \delta + (\Delta \delta G')^2 = (A + B)^2 a^4 \rho^4 \delta^2\right]. \tag{18}$$

From here it is necessary that at transversal displacement of any element of the optical system an equal and identically directed coma is bring into representing every point of object.

Let's aberrations $\Delta \delta g'$ and $\Delta \delta G'$ are determined by the second members of (15) and (16). We have thus:

$$\Delta \delta g' = -\left[(4B + C + D) \cos(\beta - \gamma) \cos \alpha + (2B + C - D) \cos(\alpha - \beta - \gamma)\right] a \rho r \delta, \tag{19}$$

$$\Delta \delta G' = -\left[(4B + C + D) \cos(\beta - \gamma) \sin \alpha + (2B + C - D) \sin(\alpha - \beta - \gamma)\right] a \rho r \delta. \tag{20}$$

But $\rho r / R = \sin \alpha = (n'/n) \sqrt{\sin \alpha' \sin \gamma}$, where $R$ is the distance from the axial point of element of the optical system to the axial point of object. From here we find that $m = R' \sigma'_i = R' \sigma' \cos \alpha$, and $m = R' \sigma'_s = R' \sigma' \sin \alpha$.

Lets the corner $\gamma = 0$. Then (19) and (20) assume an air:

$$\Delta \delta g' = -(4B + C + D) \cos(\beta - \gamma) + (2B + C - D) \cos(\beta + \gamma) m r \delta, \tag{21}$$

$$\Delta \delta G' = -(2B + C - D) \sin(\beta + \gamma) m r \delta. \tag{22}$$

And in this case from those considering it is comfortably to put a corner $\gamma = 0$. Then

$$\Delta \delta g' = -3(2B + C) m r \delta, \tag{23}$$

$$\Delta \delta G' = -(2B + C - D) L m r \delta. \tag{24}$$

In addition, at $\alpha = 0$:

$$\rho r = \frac{m}{R'}, \sigma'_i = \sigma', \text{ and } \sigma'_s = 0. \text{ Here the meridional constituent of curvature of surface of image }$$

$$z'_i = \frac{\Delta \delta g'}{\sigma'_i} = \frac{\Delta \delta G'}{\sigma'_s} = -3R'(2B + C) / \delta. \tag{25}$$

We will notice that type of transversal aberration, the sagyttal constituent of which is determined by (24), while not obvious.

Lets the corner $\alpha = \pi / 2$. Then (19) and (20) assume an air:

$$\Delta \delta g' = -(2B + C - D) \sin(\beta + \gamma) M r \delta, \tag{26}$$

$$\Delta \delta G' = \left[(4B + C + D) \cos(\beta - \gamma) - (2B + C - D) \cos(\beta + \gamma)\right] M r \delta. \tag{27}$$

Supposing a corner $\gamma = 0$ we get

$$\Delta \delta g' = -(2B + C - D) L M r \delta, \tag{26}$$

$$\Delta \delta G' = -2(B + D) L M \delta. \tag{27}$$

At the

$$\alpha = \frac{\pi}{2}, \sigma'_i = 0, \sigma'_s = \sigma' = \frac{M}{R'}.$$

Thus

$$z'_s = \frac{\Delta \delta G'}{\sigma'_s} = -2R'(B + D) / \delta. \tag{28}$$

The type of transversal aberration, the meridional constituent of which is determined by (26), while not obvious too.

It ensues from (25) and (28) that over transversal displacement of element of the optical system brings to inclination of surfaces of the images formed by the narrow cones of rays in meridional and sagyttal planes, on corners accordingly equal:

$$\varepsilon'_i = \frac{z'_i}{l'} = -2n' R(3B + C) \delta, \tag{29}$$
\[ \varepsilon' = \frac{z'}{l'} = -2 \frac{n'}{n} R (B + D) \delta . \]  
(30)

Let's aberrations \( \Delta \delta g' \) and \( \Delta \delta G' \) are determined by the third members of (15) and (16). Thus at \( \gamma = 0 \) we have:

\[ \Delta \delta g' = -\left[ C \cos^2 \beta + D \sin^2 \beta + 2E + E \cos 2\beta \right] r^2 \delta , \]  
(31)

\[ \Delta \delta G' = -\left[ C \sin \beta \cos \beta - D \sin \beta \cos \beta + E \sin 2\beta \right] r^2 \delta . \]  
(32)

Expressions (31) and (32) it is easily to transform to the kind:

\[ \Delta \delta g' = -\left[ (C + 3E) \cos^2 \beta + (D + E) \sin^2 \beta \right] r^2 \delta = \]  
(33)

\[ \Delta \delta G' = -(C - D + E) \sin \beta \cos \beta r^2 \delta = -(C - D + E) l L \delta . \]  
(34)

It is easily to see that (33) for any chosen row of values of segment \( l \) describes family of parabolas, the orientation of which does not depend on a sign \( l \); equation (34) for any chosen row of values describes family of lines, a sign and size of angle of slope of which is determined by a sign and size of segment \( L \).

At the analysis of the aberrations determined by the second members of (15) and (16), were got (24) and (26), which it ensues from, that the sagyttal constituent \( \Delta \delta g' \) of transversal aberration is determined by a meridional coordinate \( m \), and a meridional constituent \( \Delta \delta G' \) is determined by a sagyttal coordinate \( M \).

Raising these correlations in a second power and laying down, we get:

\[ (\Delta \delta g')^2 + (\Delta \delta G')^2 = (2B + C - D)^2 L^2 \left( m^2 + M^2 \right) \delta^2 = (2B + C - D)^2 \delta^2 L^2 a^2 \rho^2 . \]  
(35)

Does the aberration spot described by expression (35) have the appearance of circle, the radius of which is determined not only by a coordinate \( \alpha \) on the pupil of the optical system, but also by the coordinate \( L \) at the decentering \( \delta \) in the direction of \( l \).

It is known that wave aberration is determined by expression [3]:

\[ W = -\frac{1}{R_0} \left[ (\delta g'dm + \delta G'dM) \right] , \]  
(36)

where \( R_0 \) is the radius of sphere of comparison.

Putting in this expression correlations (24) and (26), we get:

\[ W = -\frac{1}{R_0} \left( 2B + C - D \right) L \delta \left[ (Mdm + mDM) \right] = KLmM , \]  
(37)

where

\[ K = -\frac{2}{R_0} \left( 2B + C - D \right) \delta . \]

The got expression for the fixed value of coordinate \( L \) determines deformation of wave front as the simultaneous wring about axes \( m \) and \( M \) on the same size, but in different directions at the different signs of coordinates \( m \) and \( M \), as shown on a Figure 1.

Thus, (24) and (26) determine aberration, which is not characteristic for the axis-symmetric system, and is not a member of the known primary aberrations.
Figure 1. Deformation of wave front.

References
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[3] Sljusarev G G 1969 Methods of calculations of optical systems. (Leningrad: Mashinostroenie) 672