THE USE OF SCHOONSCHIP AND FORM 
IN PERTURBATIVE LATTICE CALCULATIONS *

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Using the formal languages Schoonschip and Form, we have developed general codes that are able to carry out all the algebraic manipulations needed to perform analytic lattice calculations, starting from the elementary building blocks (propagators and vertices) of each Feynman diagram. The main difficulty resides in the fact that, although there are many built in instructions to deal with Dirac gamma-matrices, Schoonschip and Form have been conceived having in mind a continuum theory, which is invariant with respect to the Lorentz group. On the lattice, on the contrary, a field theory is only invariant with respect to the hypercubic group, contained in the (euclidean) Lorentz group and not every pair of equal indices should be summed over. Being impossible to directly use the ‘gammatrics’ of Schoonschip and Form as they are, special routines have been developed to correctly treat gamma matrices on the lattice, while using as much as possible of the built in Schoonschip and Form commands.

We have used our codes to compute, in 1-loop perturbation theory in lattice QCD, the renormalization constants and mixing coefficients of the operators that enter in the determination of the first two moments of deep inelastic scattering structure functions.

1. Introduction

In this contribution we present a short report on the use of some algebraic manipulation computer languages, namely Schoonschip and Form, to carry out the large and complex manipulations which occur in certain analytic perturbative lattice calculations. We have employed our codes to compute in lattice QCD matrix elements

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of operators related to the first two moments of the structure functions measured in Deep Inelastic Scattering (DIS), both using the standard Wilson action and the improved nearest-neighbor action. Up to now we have completed the calculations of the 1-loop renormalization constants and mixing coefficients of the fermion and gluon rank two operators ($O_{q}^{\mu\nu}$ and $O_{g}^{\mu\nu}$) that are related to the first moment of DIS structure functions, and of the rank three operator ($O_{q}^{\mu\nu\tau}$) related to the second moment.

Consequently to the use of improvement in lattice QCD, there is indeed a large number of diagrams to be computed and, in most of them, even in the simplest case of the rank two operator, algebraic manipulations are very complicated, each Feynman diagram giving rise to a huge number of terms. For this reason, we have found unavoidable to check independently, by means of suitable algebraic manipulation programs, all the computations made by hand. For $O_{q}^{\mu\nu\tau}$ we almost completely have to rely on computer results.

The first code we developed was written using the Schoonschip language, and was used in the case of $O_{q}^{\mu\nu}$ and $O_{g}^{\mu\nu}$. Afterwards, we upgraded this code to make it able to perform the more complex calculations concerning $O_{q}^{\mu\nu\tau}$. In this case it is an impossible task to calculate all diagrams by hand; the agreement for the few of them that we could really compute by hand was nevertheless complete. Further checks were later offered by an independent code developed by Beccarini using Form, and by another Form code constructed by us, that we have further upgraded to meet the requirements of the higher complexity of the operators $O_{q}^{\mu\nu\tau}$ that we are presently studying.

2. Moments of Structure Functions and Lattice QCD

In QCD the lowest twist operators appearing in the light-cone expansion of the product of two hadronic weak currents, relevant to DIS, are

$$O_{S}^{\mu_{1}\cdots\mu_{N}} = \frac{1}{2^{N}} \bar{\psi} \gamma_{\mu_{1}} D_{\mu_{2}} \cdots D_{\mu_{N}} (1 \pm \gamma_5) \psi,$$

$$O_{NS}^{\mu_{1}\cdots\mu_{N}} = \frac{1}{2^{N}} \bar{\psi} \gamma_{\mu_{1}} D_{\mu_{2}} \cdots D_{\mu_{N}} (1 \pm \gamma_5) \frac{\lambda_{f}}{2} \psi,$$

$$O_{S}^{NS} = \sum_{\rho} \text{Tr} \left[ F_{\rho\mu_{1}} D_{\mu_{2}} \cdots D_{\mu_{N-1}} F_{\rho\mu_{N}} \right],$$

where the $\lambda_{f}$'s are flavor matrices. The operators are gauge invariant and all have twist two; they are symmetrized with respect to all Lorentz indices. $S$ and $NS$ superscripts refer to Singlet and Non Singlet flavor structures.

The operators of rank two and three in (1) are related to the first two moments of the DIS structure functions via the Wilson operator expansion of the product of two electromagnetic currents (we do not consider in this talk the $\gamma_{5}$ contributions). The knowledge of the hadronic matrix elements of these operators is necessary for the theoretical evaluation of the moments of the $x$-distributions of quarks and gluons.
inside the hadrons (x is the momentum fraction carried by the struck parton inside the hadron).

Our work consists in the determination of the 1-loop renormalization constants and mixing coefficients of the lattice fermion and gluon operators of rank two and the renormalization constant of the fermion operator of rank three that correspond to the continuum expressions given in (1). The calculations were done using the nearest neighbor improved lattice QCD action and include the results for the renormalization constants and mixing coefficients pertinent to the standard Wilson action. All these constants are needed to renormalize the lattice operators and be able to extract physical hadronic matrix elements from numbers obtained in Monte Carlo QCD simulations.

Lattice QCD represents today the only viable way of evaluating from first principles the hadronic matrix elements needed for the computation of the moments of the structure functions. The nearest neighbor improved QCD action (also known as the “clover-leaf” action), is obtained by adding to the standard Wilson action for one flavor $f$

$$S_{LAT}^f = a^4 \sum_n \left[ -\frac{1}{2a} \sum_\mu \left[ \bar{\psi}_n (r - \gamma_\mu) U_{n,\mu} \psi_{n+\mu} + \bar{\psi}_{n+\mu} (r + \gamma_\mu) U_{n,\mu}^+ \psi_n \right] 
+ \bar{\psi}_n \left( m_f + \frac{4r}{a} \right) \psi_n \right] - \frac{1}{g_0^2} \sum_{n,\mu\nu} \left[ \text{Tr} \left[ U_{n,\mu} U_{n+\mu,\nu} U_{n+\nu,\mu} U_{n,\nu}^+ \right] - N_c \right],$$

(2)

the nearest-neighbor interaction term

$$\Delta S^f = -ig_0 a^4 \sum_{n,\mu\nu} \frac{r}{4a} \bar{\psi}_n \sigma_{\mu\nu} F_{n,\mu\nu} \psi_n.$$  

(3)

Here $F_{n,\mu\nu}$ is not the naive lattice “plaquette”

$$P_{n,\mu\nu} = \frac{1}{2ig_0 a^2} (U_{n,\mu\nu} - U_{n,\mu\nu}^+),$$

(4)

with $U_{n,\mu\nu} = U_{n,\mu} U_{n+\mu,\nu} U_{n+\nu,\mu} U_{n,\nu}^+$, but rather the average of the four plaquettes lying in the plane $\mu\nu$, stemming from the point $n$:

$$F_{n,\mu\nu} = \frac{1}{4} \sum_{\mu\nu=\pm} P_{n,\mu\nu} = \frac{1}{8ig_0 a^2} \sum_{\mu\nu=\pm} (U_{n,\mu\nu} - U_{n,\mu\nu}^+).$$

(5)

The use of this action has been proved to remove from on-shell hadronic matrix elements all terms that in the continuum limit are effectively of order $a$, $a$ being the lattice spacing, provided that in the calculation of a fermionic Green function, each fermion field undergoes the rotation

$$\psi \rightarrow \left( 1 - \frac{ar}{2} \vec{D} \right) \psi, \quad \bar{\psi} \rightarrow \bar{\psi} \left( 1 + \frac{ar}{2} \vec{D} \right).$$

(6)
The use of Schoonschip and Form

With these requirements, the difference between continuum and lattice is lowered from

$$\langle p | \hat{O} | p' \rangle_{\text{Monte Carlo}} = a^d \left[ \langle p | \hat{O} | p' \rangle_{\text{phys.}} + O(a) \right]$$

(7)

(where $\langle p | \hat{O} | p' \rangle_{\text{phys.}}$ is the physical matrix element we want to extract from Monte Carlo data and $d$ is its physical dimension) to

$$\langle p | \hat{O} | p' \rangle_{\text{IMPR. Monte Carlo}} = a^d \left[ \langle p | \hat{O} | p' \rangle_{\text{phys.}} + O(a/\log a) \right].$$

(8)

Using this recipe, the systematic error related to the lattice discretization drops in many cases from 20 – 30 percent down to 5 – 10 percent.

The numbers we want to compute are the renormalization constants that connect the bare lattice operators on the lattice, $O(a)$, to finite operators, $\hat{O}(\mu)$, renormalized at a scale $\mu$:

$$\hat{O}(\mu) = Z_{lk}(\mu a) O_k(a).$$

(9)

The constants $Z_{lk}$ are fixed in perturbation theory by the same renormalization conditions used in the continuum. In the flavor Singlet case there is a mixing between quark and gluon operators of rank two that have the same conserved quantum numbers. However, with the choice $\mu \neq \nu$ and the definition (5) of the gauge field strength we avoid mixing with operators of lower dimensions, and therefore the need for subtractions of power divergences. We thus write:

$$\hat{O}^q = Z_{qq} O^q + Z_{qg} O^g$$

$$\hat{O}^g = Z_{gg} O^g + Z_{gq} O^q,$$

(10)

where the $Z$'s are determined by imposing the renormalization conditions:

$$\langle q | \hat{O}^q(\mu) | q \rangle = \langle q | O^q(a) | q \rangle |_{p^2 = \mu^2}$$

$$\langle g, \sigma | \hat{O}^g(\mu) | g, \sigma \rangle = \langle g, \sigma | O^g(a) | g, \sigma \rangle |_{p^2 = \mu^2} = 0$$

$$\langle q | \hat{O}^g(\mu) | q \rangle = \langle q | O^g(a) | q \rangle |_{p^2 = \mu^2} = 0$$

$$\langle g, \sigma | \hat{O}^q(\mu) | g, \sigma \rangle = \langle g, \sigma | O^q(a) | g, \sigma \rangle |_{p^2 = \mu^2} = 0.$$

(11)

Similarly in the case of $O^q_{\mu\nu\tau}$ there is no mixing with lower dimensional operators, if one chooses $\mu \neq \nu \neq \tau$. In fact in the “quenched” approximation (no internal quark loops) $O^q_{\mu\nu}$ and $O^g_{\mu\nu\tau}$ are simply multiplicatively renormalizable (no mixing with other operators).

3. What our Programs do

We have developed general codes able to automatically carry out all the necessary algebraic manipulations, starting from the elementary building blocks of the calculation represented by the expressions of propagators, vertices and Fourier transforms.
of the operators. The final output of our codes can be cast either in the form of an analytic expression (to be possibly compared with the calculations made by hand) or in a format ready to be introduced in a suitable Fortran code for the final numerical loop integration.

The main difficulty in using Schoonschip and Form in lattice calculations resides in the fact that, although these languages have many built in instructions to deal with gamma matrices, they have been conceived having in mind a continuum theory, which is invariant with respect to the (euclidean) Lorentz group $O(4)$. On the lattice, on the contrary, where the theory is only invariant with respect to the hypercubic group $H(4)$, we cannot use the usual summation conventions and the standard rules to rearrange gamma matrices. To make an example, one simple and frequently occurring term, like $\sum_\lambda \gamma_\lambda p_\lambda \sin k_\lambda$, is not properly handled by these languages. Actually, this term is wrongly reduced by Schoonschip and Form to $\not{p} \sin k_\lambda$, because the first two equal indices that are encountered are by default assumed to be contracted. It turns out that most of the terms that arise on the lattice are improperly handled by a straight use of the Schoonschip and Form commands.

Being impossible to directly use the “gammatrics” of Schoonschip and Form as they are, it has been necessary to develop special routines to correctly treat gamma matrices on the lattice, while using as much as possible of the built in Schoonschip and Form commands. From our efforts at least three sets of routines have grown up, and we have checked for each of them that the rather complicated Dirac algebra (we have products of up to seven gamma matrices) is carried out correctly.

The CPU times needed to run our programs has greatly shrunk with the evolution of the various codes developed in the last three years. The first Schoonschip code we used in the case of the rank two operator was running on a Sun 3 workstation, with CPU times that varied, depending on the complexity of the diagrams, from 20 seconds up to 5 minutes for the most complicated cases. This was quite reasonable for our purposes, but when turning to the rank three operator the time required by the appropriately modified Schoonschip code was one order of magnitude higher. More recently we have developed new codes written using Form that run on VAX-VMS machines. These programs are considerably faster, and the typical CPU times are back in the minute range. There has been a final reduction of these values when we have turned to an HP-UX 9000/735 machine.

Another big problem we have encountered was the limitation on the working memory allowed by Schoonschip and Form on the different machines we have used. This is of primary relevance in the first stages of the computation, when all vertices, propagators etc. are expanded, up to the first and second order in the lattice spacing $a$, for the rank two and three operators respectively. In our case, when dealing with the rank three case, we have several products of up to ten trigonometric functions, each one to be expanded to second order in $a$. Their product thus contains tenths of thousands of monomials. A large part of them does not contribute to the final expression, to the order in $a$ we are interested in, and has to be killed at the earliest
possible stage of the calculation.

As a last remark, we want to say that, besides being more portable on different machines, Form appears to be superior to Schoonschip, at least for our kind of calculations (not involving $\gamma_5$ and limited to four dimensions). Form seems indeed to be faster and to allow an easier localization of the errors. We have also found very useful the larger set of wildcards that Form owns compared to Schoonschip.

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