Supergravity from a Massive Superparticle and the Simplest Super Black Hole

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ABSTRACT

We describe in superspace a theory of a massive superparticle coupled to a version of two dimensional $N = 1$ dilaton supergravity. The $(1 + 1)$ dimensional supergravity is generated by the stress-energy of the superparticle, and the evolution of the superparticle is reciprocally influenced by the supergravity. We obtain exact superspace solutions for both the superparticle worldline and the supergravity fields. We use the resultant non-trivial compensator superfield solution to construct a model of a two-dimensional supersymmetric black hole.

August 1997

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1 Introduction

Superparticles [1] have been useful in extracting some fundamental features of superstring theory [2] as well as some basic properties of topological defects of supersymmetric field theories [3]. A \( p \)-dimensional defect is referred to as a \( p \)-brane, the \( p = 0 \) case being the superparticle. Recently there has been a resurgence of interest in the study of superparticles in various dimensions, particularly for the massive case, because of the connection to D-branes [4].

There is, however, a general dearth in the literature of exact non-trivial solutions (let alone superspace ones) to supersymmetric problems, even for classical field theories. For classical supergravity, the solutions include wave-type solutions [5, 6], a supersymmetric generalization of an extreme Reissner-Nordstrom black hole [7], and a point supercharge on a flat background [8]. These solutions are non-trivial in the sense that they cannot be reduced by supersymmetry transformations to purely bosonic solutions. However they also have either vanishing torsion or vanishing gravitino stress-energy [8].

Motivated by the above, we consider in this paper the coupling of a massive superparticle to \( N = 1 \) supergravity in (1+1) dimensions. The comparative simplicity of 2D \( N = 1 \) supergravity makes it a natural theoretical laboratory for investigating how superparticles (and by extension, super \( p \)-branes) influence the behaviour of supergravity fields and spacetime, and vice versa. In this case, superparticle-supergravity coupling necessarily involves a dilatonic theory of supergravity, since the Einstein-Hilbert action is a topological invariant [9]. In general this would mean that both the supergravity fields and the dilaton influence the evolution of the superparticle. However, as mentioned below, there is a form of dilatonic supergravity in which the dilaton does not influence the classical evolution of the vielbein and superparticle.

This theory is a supersymmetric generalization of the (1+1) dimensional “R=T” theory, which has been of particular interest insofar as it has a consistent Newtonian limit [10] (a nontrivial issue for a generic dilaton gravity theory [11]). It also has an interesting set of solutions for many physical situations which closely resemble their (3 + 1) dimensional counterparts [12, 13, 14, 15]. The dilatonic part of the action is chosen so that the dilaton field decouples from the classical field equations. This ensures that the evolution of the gravitational field is determined only by the matter stress-energy (and reciprocally) [10, 12], thereby capturing the essence of general relativity (as opposed to classical scalar-tensor theories) in two spacetime dimensions. Indeed it is possible to interpret “R=T” theory as the \( D \rightarrow 2 \) limit of general relativity (as opposed to some particular solution(s)) [16].

In super “R=T” theory, the dilaton field classically decouples from the evolution
of the supergravity/matter system. Hence we have \((1+1)\) dimensional supergravity being generated by supersymmetric matter, and the evolution of the supermatter being influenced by the supergravity. For the case in which the supermatter is given by a single massive superparticle, we obtain exact non-trivial superspace solutions for both the superparticle worldline and the supergravity fields.

The outline of our paper is as follows. In section 2 we recapitulate the basic formalism for a massive superparticle in flat superspace, and in section 3 we couple the superparticle to supergravity. In section 4 we describe the form of dilaton \(\text{“} R = \Gamma \text{”}\) supergravity in superconformal gauge. In section 5 we examine the superparticle action in superconformal gauge and in sections 6 and 7 we solve for the supergravity compensator in the presence of a single superparticle. In section 8 we construct a model of a super black hole, based on the background generated by the compensator. The appendices contain the equations of motion, some component results, a discussion of the non-triviality of the solution, as well as the precise form of the super black hole vielbein. We close with some concluding remarks.

## 2 Massive Superparticle In Flat Superspace

The standard covariant action for the massive superparticle in two dimensions is

\[
I = -m \int d\tau \left[ \sqrt{-\pi^m \pi_n + \frac{i}{2} \bar{\theta} \Gamma^3 \dot{\theta}} \right]
\]  

(2.1)

with \(\pi^n = \dot{x}^n + \frac{i}{2} \bar{\theta} \Gamma^n \dot{\theta}\), where \((\Gamma^0)_{\alpha\beta} = \left( \begin{array}{cc} -1 & 0 \\ 0 & -1 \end{array} \right)\); \((\Gamma^1)_{\alpha\beta} = \left( \begin{array}{cc} -1 & 0 \\ 0 & 1 \end{array} \right)\) and

\((\Gamma^3)_{\alpha\beta} = \left( \begin{array}{cc} 0 & -1 \\ -1 & 0 \end{array} \right)\). The metric is \(\eta_{ab} = (-, +)\), for \(a = 0, 1\), and spinor indices are raised and lowered by \(\epsilon_{\alpha\beta} = \epsilon^{\alpha\beta} = \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right)\), where \(\bar{\theta}^\alpha = \theta^\alpha = \epsilon^{\alpha\beta} \theta_\beta\). The action is manifestly globally supersymmetric, reparametrization invariant and locally kappa-invariant.

For convenience we switch to light-cone coordinates, \((x^+, x^-) = \frac{1}{2}(x^0 \pm x^1)\), by defining also \((\pi^+, \pi^-) = \frac{1}{2} (\pi^0 \pm \pi^1)\) and identifying \(\theta^1 = \theta^+\) and \(\theta^2 = \theta^-\), so that in this notation the above action becomes

\[
I = -m \int d\tau \left[ \sqrt{-\pi^+ \pi^- - \frac{i}{2} (\theta^+ \dot{\theta}^- + \theta^- \dot{\theta}^+)} \right]
\]  

(2.2)

where \(\pi^+ = \dot{x}^+ + i \theta^+ \dot{\theta}^+, \pi^- = \dot{x}^- + i \theta^- \dot{\theta}^-\). We write the kappa-symmetry \((\kappa = \kappa(\tau))\) explicitly as

\[
\delta_\kappa x^+ = -i \theta^+ \delta_\kappa \theta^+, \quad \delta_\kappa x^- = -i \theta^- \delta_\kappa \theta^-
\]  

(2.3)
\[
\delta_{\kappa} \theta^+ = - \left( \kappa^+ + \kappa^- \frac{\pi^*}{\sqrt{\pi^2}} \right), \quad \delta_{\kappa} \theta^- = - \left( \kappa^- + \kappa^+ \frac{\pi^*}{\sqrt{\pi^2}} \right).
\]

(2.4)

The equations of motion are:

\[
\sqrt{\frac{\pi^*}{\pi^+}} = a, \quad \sqrt{\frac{\pi^*}{\pi^-}} = b
\]

\[
\dot{\theta}^- = a \dot{\theta}^+, \quad \dot{\theta}^+ = b \dot{\theta}^-
\]

(2.5)

where \(a, b\) are constants and \(ab = 1\). From (2.4), it is possible to choose a gauge in which one of the \(\theta\)'s is a constant, i.e., we take \(\dot{\theta}^- = 0\). This implies that \(\dot{\theta}^+ = 0\) likewise, so that both \(\theta^+\) and \(\theta^-\) are constants. Note that setting one of the thetas to zero is too strong a choice (and breaks manifest supersymmetry) [17]. For a free particle we have \(\pi^* = c\) and \(\pi^- = d\), both constants, so that \(\pi^2 = cd = a^2 c^2\) for a massive superparticle.

To make contact with the standard Green-Schwarz superstring formulation, we eliminate the square root from the action by introducing an einbein, \(g\), on the worldline of the superparticle, and obtain the action in the usual form

\[
I = -m \int d\tau \left[ g^{-1} \pi^* \pi^- - \frac{i}{2} (\theta^+ \dot{\theta}^- + \theta^- \dot{\theta}^+) + \frac{g}{4} \right].
\]

(2.6)

Varying with respect to \(g\) gives \(g = 2\sqrt{\pi^* \pi^-}\), and varying with respect to the particle coordinates gives back the previous equations of motion. The kappa-invariance is as before with the variation of the einbein given by

\[
\delta_{\kappa} g = 4i (\dot{\theta}^+ \kappa^- - \dot{\theta}^- \kappa^+) .
\]

(2.7)

En route to coupling the superparticle to supergravity, we introduce coordinates with world indices \(z^M = (x^\alpha, \theta^\mu)\), and the flat vielbein \(e_M^A\), defined so that \(\dot{e}^A \equiv \dot{z}^M e_M^A = (\dot{x} + i\dot{\theta}, \dot{\theta})\). We also introduce a gauge field \(\Gamma_A = (\Gamma_a, \Gamma_a)\) to describe the Wess-Zumino type term in the action. The action can then be rewritten as

\[
I = -m \int d\tau \left[ g^{-1} \dot{z}^M e_M^* \dot{z}^N e_N^* \eta_{\pi^* \pi^-} + \dot{z}^M e_M^A \Gamma_A + \frac{g}{4} \right]
\]

(2.8)

where \(\Gamma_+ = \frac{i}{2} \dot{\theta}^-, \Gamma_- = \frac{i}{2} \dot{\theta}^+, \Gamma_a = 0\) and \(\eta_{\pi^* \pi^-} = 1\).
3 Coupling to Supergravity

We now couple the superparticle to supergravity. Promoting the flat vielbein to the curved one, $E_M^A$, and $\Gamma$ to a general superfield, the action becomes

$$ I_P = -m \int d\tau \left[ g^{-1} \dot{z}^M E_M^A \dot{z}^N E_N^A \eta_{\#} + \dot{z}^M E_M^A \Gamma_A + \frac{g}{4} \right] $$

$$ = -m \int d\tau \left[ g^{-1} \dot{\hat{e}}^a \dot{\hat{e}}^a + \dot{\hat{e}}^4 \Gamma_A + \frac{g}{4} \right] \quad (3.1) $$

We define the $\kappa$-transformations of the coordinates in curved superspace as

$$ \delta_\kappa \mathcal{E}^a \equiv \delta_\kappa z^M E_M^a = 0 $$
$$ \delta_\kappa \mathcal{E}^a \equiv \delta_\kappa z^m E_M^a $$

where explicitly we have

$$ \delta_\kappa \mathcal{E}^+ = -(\kappa^+ + 2\frac{\kappa^-}{g} \dot{\hat{e}}^4) \quad , \quad \delta_\kappa \mathcal{E}^- = -(\kappa^- + 2\frac{\kappa^+}{g} \dot{\hat{e}}^4) $$

(3.3)

as well as

$$ \delta_\kappa g = 4i(\dot{\hat{e}}^+ \kappa^- + \dot{\hat{e}}^- \kappa^+) \quad . $$

(3.4)

The supergravity covariant derivative is given by $\nabla_A = \mathcal{E}^M_A \partial_M + \Omega_A$, where $\Omega_A = \omega_A M$ is the spin connection. We use ordinary derivatives $\partial_M$ for compatibility with the notation of forms. We define $\tilde{\nabla}_A = \nabla_A + \Gamma_A$, including now the gauge field, and we have

$$ \{ \tilde{\nabla}_A, \tilde{\nabla}_B \} = T_{AB}^C \tilde{\nabla}_C + R_{AB} M + F_{AB} $$

(3.5)

which defines the torsions, curvatures and gauge field strengths, respectively, where $\partial_{(M} \mathcal{E}_{N)}^A = T_{NM}^A + \omega_{(MN)}^A$. Also,

$$ F_{AB} = \nabla_{[A} \Gamma_{B]} - T_{AB}^C \Gamma_C $$

(3.6)

with $\{ \Gamma_A, \Gamma_B \} = 0$. The constraints on $\Gamma$ are

$$ \Gamma_{\#} = -i \nabla_+ \Gamma_+ \quad , \quad \Gamma_\# = -i \nabla_- \Gamma_- $$
$$ F_{+-} = F_{-+} = \nabla_+ \Gamma_- + \nabla_- \Gamma_+ = i $$

(3.7)

All other $F$’s are zero (as in the flat space case), consistent with the Bianchi identities [18].
The variation of the action under a kappa transformation is given by

$$
\delta I_P = -m \int d\tau \left\{ \delta g (-g^{-2} \hat{\epsilon}^* \hat{\epsilon} + \frac{1}{4}) + \hat{\epsilon}^B \delta \hat{\epsilon}^A [g^{-1}(T_{BA} \hat{\epsilon}^* + T_{AB} \hat{\epsilon}^* \plus F_{AB})] \right\}
$$  

(3.8)

where we have used the following expressions in the derivation

$$
\delta \hat{\epsilon}^A = \delta \hat{z}^M \epsilon^A_M + \hat{z}^M \delta \epsilon^A_M \\
= \partial_\tau (\delta \hat{z}^M \epsilon^A_M) - \delta \hat{z}^M \hat{z}^N \partial_N \epsilon^A_M + \hat{z}^M \delta \hat{z}^N \partial_N \epsilon^A_M \\
= D_\tau (\delta \hat{z}^M \epsilon^A_M) + \delta \hat{z}^M \hat{z}^N T^A_{NM} + \epsilon^B \delta \hat{z}^M \omega^A_{MB}
$$  

(3.9)

Substituting the explicit variations (3.3) and (3.4), we find \( \delta I_P = 0 \), provided the supergravity constraints (4.2) and those on \( \Gamma \), (3.7), are satisfied.

### 4 Dilaton Supergravity in Conformal Gauge

We now couple the superparticle to \( N = 1 \) dilaton supergravity, for which the action is

$$
I_D = \frac{1}{2\kappa} \int d^2x d^2\theta E^{-1}(\nabla_+ \Phi \nabla_- \Phi + \Phi R)
$$  

(4.1)

where \( \Phi \) is the dilaton superfield, \( R \) is the scalar supercurvature and \( E = s \text{det} E_A^M \).

We choose this action as opposed to that given in [19], for example, since the dilaton decouples from the evolution of the matter system in this case, and gives the supersymmetric analogue of the bosonic “R=T” system [10, 12].

The solution to the constraints is simplest in conformal gauge. The constraints are usually solved in terms of covariant derivatives \( \nabla_A = E_A^M D_M + \omega_A M \), that are expanded with respect to the standard flat supersymmetry covariant derivatives, \( D_A = (D_+, D_-) = (\partial_+ + i\theta^+ \partial_\psi, \partial_- + i\theta^- \partial_\bar{\psi}) \). However, the natural description for the superparticle is in terms of forms, and so we choose as a basis the ordinary derivatives \( \partial_M = (\partial_\mu, \partial_\nu) \) as mentioned earlier. In this basis we write \( \nabla_A = E_A^M \partial_M + \omega_A M \). We solve the constraints in conformal gauge in terms of the \( D \)’s and change to the other basis afterwards. The (1,1) supergravity constraints [18, 20] are

$$
\begin{align*}
\{ \nabla_+, \nabla_+ \} &= 2i \nabla_+ \\
\{ \nabla_+, \nabla_- \} &= \{ \nabla_-, \nabla_- \} = 2i \nabla_- \\
\nabla_+ &= R M \\
T^{+A} &= T^{-A} = 0
\end{align*}
$$  

(4.2)

where the constraints on the covariant derivatives are solved in conformal gauge in terms of the compensator superfield \( S \), as

$$
\begin{align*}
\nabla_+ &= e^S [D_+ + 2(D_+ S) M] \\
\nabla_- &= e^S [D_- - 2(D_- S) M] \\
\nabla_+ &= e^{2S} [\partial_+ - 2i(D_+ S) D_+ + 2(\partial_+ S) M] \\
\nabla_- &= e^{2S} [\partial_- - 2i(D_- S) D_- - 2(\partial_- S) M] \\
R &= 4e^{2S} D_- D_+ S
\end{align*}
$$  

(4.3)
From this we can read off the elements of $E_A^M$ and compute $E^{-1} = e^{-2S}$.

We now switch to the preferred basis and calculate the elements of $\mathcal{E}_A^M$. Inverting this matrix we obtain

$$\mathcal{E}_M^A = \begin{bmatrix} e^{-2S} & 0 & 2ie^{-S}D_+S & 0 \\ 0 & e^{-2S} & 0 & 2ie^{-S}D_-S \\ -ie^{-2S}\theta^+ & 0 & e^{-S}(1 - 2(D_+S)\theta^+) & 0 \\ 0 & -ie^{-2S}\theta^- & 0 & e^{-S}(1 - 2(D_-S)\theta^-) \end{bmatrix} \tag{4.4}$$

Therefore, in conformal gauge, the dilaton supergravity part of the action becomes

$$I_D = \frac{1}{2\kappa} \int d^2xd^2\theta (D_+\Phi D_-\Phi + 4\Phi D_-D_+S) \tag{4.5}$$

It is clear that the dilaton decouples from the supergravity-matter sector, provided the matter is independent of the dilaton. Indeed, the equations of motion for the full action, $I = I_D(\Phi, S) + I_M(\Psi, S)$, where $\Psi$ symbolically represents the supersymmetric matter sector, are

$$D_-D_+\Phi + 2D_-D_+S = 0 \tag{4.6}$$
$$\frac{2}{\kappa}D_-D_+\Phi + \frac{\delta I_M}{\delta S} = 0 \tag{4.7}$$

along with the matter field equations of motion. The solution of (4.6) is $\Phi = -2S$, and inserting (4.6) into (4.7) yields

$$-\frac{4}{\kappa}D_-D_+S + \frac{\delta I_M}{\delta S} = 0 \tag{4.8}$$

showing that the dilaton classically decouples from the supergravity-matter system.

## 5 Superparticle Action in Conformal Gauge

The action is

$$I_P = -m \int d^4z \int d\tau \left[ g^{-1}\dot{z}_0^N\mathcal{E}_M^N \dot{z}_0^M + \dot{z}_0^M\mathcal{E}_M^A \Gamma_A + \frac{\theta}{4} \right] \delta(z - z_0(\tau)) \tag{5.1}$$

where $z = (x, \theta)$ are the coordinates of the superspace, and $z_0(\tau) = (x_0(\tau), \theta_0(\tau))$ are the coordinates of the superparticle. We require the constraints on $\Gamma$ in conformal gauge. We define $\{\nabla_+, \nabla_\perp\} \equiv 2i\nabla_\mp$, and similarly for $\dot{\nabla}$, which implies that

$$\Gamma_+ = -ie^S(D_+\Gamma_+ + (D_+S)\Gamma_+)$$
$$\Gamma_- = -ie^S(D_-\Gamma_- + (D_-S)\Gamma_-) \tag{5.2}$$
Substituting in for $E$ and $\Gamma$ we obtain

\[
I_P = -m \int d^4z \int d\tau \left\{ g^{-1} e^{-4S} (\dot{x}_0^+ + i\theta_0^+ \dot{\theta}_0^-)(\dot{x}_0^- + i\theta_0^- \dot{\theta}_0^-) \right. \\
+ \left. ie^{-S}(\dot{x}_0^+ + i\theta_0^+ \dot{\theta}_0^-)[(D_+S)\Gamma_+ - \nabla D_+ \Gamma_+] \right. \\
+ \left. ie^{-S}(\dot{x}_0^- + i\theta_0^- \dot{\theta}_0^-)[(D_-S)\Gamma_- - \nabla D_- \Gamma_-] \right. \\
+ \left. e^{-S}(\dot{\theta}_0^+ \Gamma_+ + \dot{\theta}_0^- \Gamma_-) \right. \\
+ \left. \frac{g}{4} \delta(z - z_0(\tau)) \right\} (5.3)
\]

The complete action is the sum of (5.3) and (5.3).

It is convenient to define $G_\alpha = e^{S} \Gamma_\alpha$ and include (5.7) in the supergravity action by means of a Lagrange multiplier, $\lambda$. We obtain

\[
I_P = -m \int d^4z \int d\tau \left\{ g^{-1} e^{-4S} (\dot{x}_0^+ + i\theta_0^+ \dot{\theta}_0^-)(\dot{x}_0^- + i\theta_0^- \dot{\theta}_0^-) \right. \\
+ \left. i(x_0^+ + i\theta_0^+ \dot{\theta}_0^-)D_+ G_+ + i(x_0^- + i\theta_0^- \dot{\theta}_0^-)D_- G_- \\
+ \left. \dot{\theta}_0^+ G_+ + \dot{\theta}_0^- G_- + \frac{g}{4} \right\} \delta(z - z_0(\tau)) (5.4)
\]

and

\[
I_D = \frac{1}{2\kappa} \int d^2x d^2\theta [D_+ \Phi D_- \Phi + 4\Phi D_- D_+ S + \kappa \lambda e^{-2S}(D_+ G_- + D_- G_+ - ie^{-2S}S)] (5.5)
\]

We now perform a change of variables in the superparticle action, by first explicitly writing it in terms of $x_0^0$ and $x_0^1$, and then making the gauge choice $x_0^0 = \tau$ (static gauge) so that $\frac{dx_0^1}{dx_0^0} = -\frac{dz_0^1}{dz_0^0} \equiv \dot{x}_0$. We also rename $z^M = (x^0, x^1, \theta^\mu) = (t, x, \theta^\mu)$, so (5.4) becomes

\[
I_P = -m \int dt dx^0 d\theta \left\{ g^{-1} e^{-4S} \left[ \frac{1}{2}(1 + \dot{x}_0^0) + i\theta_0^+ \dot{\theta}_0^- \right] \left[ \frac{1}{2}(1 - \dot{x}_0^0) + i\theta_0^- \dot{\theta}_0^- \right] \right. \\
+ \left. i \left[ \frac{1}{2}(1 + \dot{x}_0^0) + i\theta_0^+ \dot{\theta}_0^- \right] D_+ G_+ + i \left[ \frac{1}{2}(1 - \dot{x}_0^0) + i\theta_0^- \dot{\theta}_0^- \right] D_- G_- \\
+ \left. \dot{\theta}_0^+ G_+ + \dot{\theta}_0^- G_- + \frac{g}{4} \right\} \delta(t - x_0^0) \delta(x - x_0^0) \delta(\theta^+ - \theta_0^+(x_0^0)) \delta(\theta^- - \theta_0^-(x_0^0)) (5.6)
\]

and doing the $x_0^0$ integration gives

\[
I_P = -m \int dt dx^1 d\theta \left\{ g^{-1} e^{-4S} \left[ \frac{1}{2}(1 + \dot{x}_0^0) + i\theta_0^+ \dot{\theta}_0^- \right] \left[ \frac{1}{2}(1 - \dot{x}_0^0) + i\theta_0^- \dot{\theta}_0^- \right] \right. \\
+ \left. i \left[ \frac{1}{2}(1 + \dot{x}_0^0) + i\theta_0^+ \dot{\theta}_0^- \right] D_+ G_+ + i \left[ \frac{1}{2}(1 - \dot{x}_0^0) + i\theta_0^- \dot{\theta}_0^- \right] D_- G_- \\
+ \left. \dot{\theta}_0^+ G_+ + \dot{\theta}_0^- G_- + \frac{g}{4} \right\} \delta(x - x_0(t)) \delta(\theta^+ - \theta_0^+(t)) \delta(\theta^- - \theta_0^-(t)) (5.7)
\]
The equations of motion are given in detail in Appendix A, and we just mention the general content here. Varying with respect to $\Phi$ shows that $\Phi$ decouples from the action; varying with respect to $\lambda$ gives back the constraint on $G$; and varying with respect to $g$ allows the elimination of the einbein from the action. The main equation of motion is the one for $S$, (A.2).

In section 2, we listed the equations of motion for the free superparticle in flat superspace. We reconsider those equations, written now in terms of the new variables. We have

\[
\begin{align*}
\dot{x}_0^+ + i\theta_0^+ \dot{\theta}_0^+ &= \frac{1}{2}(\dot{x}_0^0 + \dot{x}_0^1) + i\theta_0^+ \dot{\theta}_0^+ = c \\
\dot{x}_0^- + i\theta_0^- \dot{\theta}_0^- &= \frac{1}{2}(\dot{x}_0^0 - \dot{x}_0^1) + i\theta_0^- \dot{\theta}_0^- = d
\end{align*}
\]

which become

\[
\begin{align*}
\frac{1}{2}(1 + \dot{x}_0) + i\theta_0^+ \dot{\theta}_0^+ &= c \\
\frac{1}{2}(1 - \dot{x}_0) + i\theta_0^- \dot{\theta}_0^- &= d
\end{align*}
\]

(5.8)

When $\dot{\theta}_0 = 0$, the free superparticle moves with a constant velocity $\dot{x}_0 = 2c - 1 = 1 - 2d$.

## 6 Solution for Compensator and Motion of Superparticle

In solving this problem classically, we look for an explicit expression for the compensator $S$ in the dilaton supergravity, that is consistent with the motion of the superparticle. We solve (A.2) by analogy with the bosonic case. The observation that is crucial in the solution of the equations (A.1) to (A.6) is that if the solution for $S$ and its derivatives vanishes on the superparticle’s worldline, then the equations of motion reduce to those for a free superparticle in flat superspace. We look for such a solution.

Using this and noting that on the worldline, where $S = 0$, the constraint on the $G$’s becomes trivial, we obtain

\[
D_- D_+ S(z) = \frac{km}{2} \int dt \left\{ g^{-1} \left[ \frac{1}{2}(1 + \dot{x}_0) + i\theta_0^+ \dot{\theta}_0^+ \right] \left[ \frac{1}{2}(1 - \dot{x}_0) + i\theta_0^- \dot{\theta}_0^- \right] \right\} \delta^4(z - z_0(t))
\]

\[
= \frac{km}{2} \sqrt{\pi^2} \delta(x - x_0(t)) \delta(\theta^+ - \theta_0^+(t)) \delta(\theta^- - \theta_0^-(t))
\]

(6.1)
where $\sqrt{\pi^2} = \frac{i}{2}\sqrt{1 - \dot{x}_0^2}$ for a free particle.

We consider first the case of a superparticle at rest and assume that $S$ is time independent. Expanding it in a power series in $\theta$, we have

$$S(x, \theta) = -\frac{\kappa m}{4}[f(x) + \theta^+ g(x) + \theta^- h(x) + \theta^+ \theta^- k(x)] \quad (6.2)$$

Substituting (6.2) into (6.1) we find the following differential equations for the component fields

$$f'' = -\delta(x - x_0)$$
$$g' = i\theta_0^+ \delta(x - x_0)$$
$$h' = -i\theta_0^- \delta(x - x_0)$$
$$k = \theta_0^+ \theta_0^- \delta(x - x_0) \quad (6.3)$$

which are solved by

$$f = -\frac{1}{2}|x - x_0| = -\frac{1}{2}[(\Theta(x - x_0)(x_0 - x_0) + \Theta(x_0 - x)(x_0 - x)]$$
$$g = \frac{i}{2}\theta_0^+[\Theta(x - x_0) - \Theta(x_0 - x)$$
$$h = -\frac{i}{2}\theta_0^-[\Theta(x - x_0) - \Theta(x_0 - x)]$$
$$k = \theta_0^+ \theta_0^- \delta(x - x_0) \quad (6.4)$$

where $\Theta$ is the Heaviside function. Therefore we can write $S$ as

$$S = \frac{\kappa m}{4}\left\{\frac{1}{2}|x - x_0| + \frac{i}{2}(\theta^+ \theta_0^+ - \theta^- \theta_0^-)[\Theta(x - x_0) - \Theta(x_0 - x)] + \theta^+ \theta^- \theta_0^+ \theta_0^- \delta(x - x_0)\right\}$$
$$= -\frac{\kappa m}{8}\left\{|x - x_0 - i(\theta^+ \theta_0^+ - \theta^- \theta_0^-)|\right\} \quad (6.5)$$

where the second line is to be understood as a Taylor expansion. Note that $S$ and its derivatives vanish on the worldline of the superparticle, that is when $x = x_0$ and $\theta = \theta_0$.

The solution for a moving superparticle can be obtained by a Lorentz boost of $x, t$ and $\theta$, where we have

$$x' = \frac{x - \dot{x}_0 t}{\sqrt{1 - \dot{x}_0^2}}, \quad t' = \frac{t - \dot{x}_0 x}{\sqrt{1 - \dot{x}_0^2}},$$
$$x^\# = \zeta^2 x^\#, \quad x^\# = \zeta^{-2} x^\#,$$
$$\theta^+ = \zeta \theta^+, \quad \theta^- = \zeta^{-1} \theta^- \quad (6.6)$$

with $\zeta^2 = \sqrt{\frac{1 - \dot{x}_0}{1 + \dot{x}_0}}$. Applying this to (6.3) we obtain a new Lorentz transformed $S'$

$$S' = -\frac{\kappa m}{8}\frac{1}{\sqrt{1 - \dot{x}_0^2}}|x - x_0 - i(1 - \dot{x}_0)\theta_0^+ + i(1 + \dot{x}_0)\theta^- \theta_0^-| \quad (6.7)$$
which gives a solution in which $\dot{x}_0 = \text{constant}$ and $\dot{\theta}_0 = 0$. We have as the final solution for the compensator

$$S = -\frac{\kappa m}{8} \sqrt{1 - \dot{x}_0^2} |x - x_0(t) - i(1 - \dot{x}_0)\theta^+ \theta_0^+(t) + i(1 + \dot{x}_0)\theta^- \theta_0^-(t)|$$

(6.8)

It is straightforward to verify that (6.8) gives the full solution to (6.1).

We stress that this solution is non-trivial in that it cannot be obtained by an infinitesimal supersymmetry transformation from the bosonic solution for a particle moving in dilaton gravity. We discuss this issue in Appendix C, from both a superspace and a component viewpoint.

7 The Simplest Super Black Hole

The solution obtained in the preceding section is the supersymmetric version of that obtained for the gravitational field generated by a point particle in (1 + 1) dimensional R=T theory [10, 23]. In conformal gauge the metric for this latter solution is [23]

$$ds^2 = e^{2m|z|}(-dt^2 + dz^2)$$

(7.1)

where the parameter $m$ is the mass of the particle. This solution can be rewritten as

$$ds^2 = -(2m|w| + C)dt^2 + \frac{dw^2}{2m|w| + C}$$

(7.2)

under a straightforward transformation of coordinates, where $C = 1$.

Despite the fact that spacetime is flat everywhere outside of the particle, we can use it to construct a two dimensional black hole using the methods described in [11, 23]. Since the Ricci scalar is $r = -4m\delta(w)$, independent of the sign of $C$, the metric

$$ds^2 = -(2m|w| - |C|)dt^2 + \frac{dw^2}{2m|w| - |C|}$$

(7.3)

is also a solution of the field equation. This is a black hole whose event horizons are located at $w = \frac{|C|}{2m}$. This black hole spacetime may be constructed by taking two copies of Minkowski space, cutting each of them along the hyperboloids $T^2 - X^2 = m^2$, and gluing the spacetimes along their hyperboloids in a manner that does not generate closed timelike curves (i.e. by gluing the hyperboloids at positive $T$ together and the hyperboloids at negative $T$ together). Details of this construction are provided in refs. [12, 13]. A description of how this black hole can be understood to arise as the endpoint of gravitationally collapsing matter in (1 + 1) dimensions is given in refs. [14, 23].
We seek here the supersymmetric analogue of the spacetime described by (7.3), i.e. of a spacetime whose zweibein is
\[ e^a_m = \begin{pmatrix} \sqrt{\alpha} & 0 \\ 0 & \sqrt{\alpha^{-1}} \end{pmatrix} \] (7.4)
where \( \alpha \equiv 2m|w| + C \). For positive \( C \) the zweibein is that of a naked point particle of mass \( m \), whereas for \( C < 0 \) the zweibein is that of a black hole.

The first step, having found \( S \), is to determine the vielbein in other than superconformal coordinates - in particular, in superspace coordinates that are the analogue of those used in Schwarzschild gauge in the bosonic case (7.3). We construct a model of a supersymmetric black hole by examining the supercoordinate transformations that correspond to the ordinary \( x \)-space ones used to construct bosonic two-dimensional black holes in ref. [23]. Under a supercoordinate transformation \( z = (x, \theta) \rightarrow w = (u, \eta) \), the vielbein transforms as \( E^M_A(w) = \frac{dN}{dw} E^N_M(z) \), and we demand that in the transformed coordinates, the bosonic-bosonic corner of the vielbein matrix (corresponding to \( E^a_m \)) have the same form that the component zweibein \( e^a_m \) does in the purely bosonic case (7.4).

We define the transformations by analogy with the bosonic case, and we require \( (x = x_0, \theta = \theta_0) \) to correspond to \( (u = u_0, \eta = \eta_0) \) and also, \( u > 0(u < 0) \) when \( x > 0(x < 0) \). This ensures that \( \Theta(x - x_0) - \Theta(x_0 - x) \equiv \epsilon(x - x_0) = \epsilon(u - u_0) \). We expand each side of the supercoordinate transformation equation in powers of either \( \eta \) or \( \theta \), and determine the transformations of the \( \theta \)'s by matching both sides.

Consider first the case in which \( C > 0 \). We find that the supercoordinate transformation that relates the conformal coordinates to the Schwarzschild coordinates is:
\[ 2m|u - u_0 - i(\eta^+\eta_0^+ - \eta^-\eta_0^-)| + |C| = |C|e^{2m|x-x_0-i(\theta^+\theta_0^+ - \theta^-\theta_0^-)|} \] (7.5)
which implies that
\[
|x - x_0| = \frac{1}{2m} \ln \left( \frac{2m}{|C|} |u - u_0| + 1 \right)
\]
\[
\theta^\pm = \sqrt{|C|} \frac{\eta^\pm}{2m |u - u_0| + |C|/2m} \left[ 1 + \frac{i}{2}(\eta^+\eta_0^+ - \eta^-\eta_0^-) \frac{\epsilon(u - u_0)}{|u - u_0| + |C|/2m} \right]
\]
\[
\theta_0^\pm = \frac{\eta_0^\pm}{\sqrt{|C|}}
\]
This case corresponds to just a massive superparticle at rest, and no black hole. We find the scalar supercurvature in the new coordinate system to be
\[ R' = 8m\delta(u - u_0) \delta^{(2)}(\eta - \eta_0) \] (7.6)
and, as discussed in Appendix C, the transformed \( x \)-space component curvature is
\[
r' = (\nabla^2 R)' = -8m|C|\delta(u - u_0) \tag{7.7}
\]

For \( C < 0 \), we find that the supercoordinate transformations that model a super black hole can be split up into three regions:

**Region (i):** \( u - u_0 < \frac{-|C|}{2m} \)
\[
-2m[u - u_0 - i(\eta^+ \eta_0^+ - \eta^- \eta_0^-)] - |C| = |C|e^{-2m[x - x_0 - i(\theta^+ \theta_0^+ - \theta^- \theta_0^-)]} \tag{7.8}
\]
which implies that
\[
x - x_0 = \frac{-1}{2m} \ln\left(\frac{-2m}{|C|}(u - u_0) - 1\right)
\]
\[
\theta^\pm = \frac{i\sqrt{|C|}}{2m} \frac{\eta^\pm}{|u - u_0| - |C|/2m} \left[ 1 + \frac{i}{2}(\eta^+ \eta_0^+ - \eta^- \eta_0^-) \frac{1}{(u - u_0) - |C|/2m} \right]
\]
\[
\theta_0^\pm = \frac{-i\eta_0^\pm}{\sqrt{|C|}}
\]

**Region (ii):** \( |u - u_0| < \frac{|C|}{2m} \)
\[
2m[u - u_0 - i(\eta^+ \eta_0^+ - \eta^- \eta_0^-)] - |C| = -|C|e^{-2m[x - x_0 - i(\theta^+ \theta_0^+ - \theta^- \theta_0^-)]} \tag{7.9}
\]
which implies that
\[
x - x_0 = \frac{-1}{2m} \ln\left(1 - \frac{2m}{|C|}|u - u_0|\right)
\]
\[
\theta^\pm = \frac{\sqrt{|C|}}{2m} \frac{\eta^\pm}{|u - u_0| - |C|/2m} \left[ 1 + \frac{i}{2}(\eta^+ \eta_0^+ - \eta^- \eta_0^-) \frac{\epsilon(u - u_0)}{(u - u_0) - |C|/2m} \right]
\]
\[
\theta_0^\pm = \frac{-i\eta_0^\pm}{\sqrt{|C|}}
\]

**Region (iii):** \( u - u_0 > \frac{|C|}{2m} \)
\[
2m[u - u_0 - i(\eta^+ \eta_0^+ - \eta^- \eta_0^-)] - |C| = |C|e^{2m[x - x_0 - i(\theta^+ \theta_0^+ - \theta^- \theta_0^-)]} \tag{7.10}
\]
which implies that
\[
x - x_0 = \frac{1}{2m} \ln\left(\frac{2m}{|C|}(u - u_0) - 1\right)
\]
\[
\theta^\pm = \frac{i\sqrt{|C|}}{2m} \frac{\eta^\pm}{(u - u_0) - |C|/2m} \left[ 1 + \frac{i}{2}(\eta^+ \eta_0^+ - \eta^- \eta_0^-) \frac{1}{(u - u_0) - |C|/2m} \right]
\]
\[
\theta_0^\pm = \frac{-i\eta_0^\pm}{\sqrt{|C|}}
\]
We can compute $R'$ and $r'$ for these cases and we find

$$R' = 8m\delta(u - u_0)\delta^{(2)}(\eta - \eta_0)$$
$$r' = (\nabla^2 R)'| = -8m|C|\delta(u - u_0)$$

(7.11)

in region (ii), and $R' = r' = 0$ in regions (i) and (iii).

Equations (7.7) and (7.11) show that the supergravity solution we have obtained satisfies the field equations independently of the sign of $C$. For $C < 0$ we can perform the same construction as in ref. [10, 13], only now in superspace. One takes two copies of 2D flat superspace, cuts off the parts defined by spacelike bosonic hyperbolae, and then glues them together so that there are no closed timelike curves. This yields the solution given by regions (i)–(iii) above. Region (ii) corresponds to the region inside the super black hole, whereas regions (i) and (iii) correspond to the region outside the super black hole. The precise form of the vielbein corresponding to each region is given in Appendix D.

We close this section by commenting on the $C = 0$ case. For bosonic R=T theory the analogous construction would involve gluing two copies of $|T| < |X|$ Minkowski spacetime along their $|T| = |X|$ lightcones. The resultant manifold would be non-Hausdorff at $X = T = 0$. A possible resolution of this dilemma would be to glue only the right-hand Rindler wedges of each spacetime along their respective light cones, but it would be unclear how to avoid closed timelike lines in a manner that yielded a consistent gluing at $T = 0$. We shall not consider this case any further.

8 Conclusions

We have examined a $(1 + 1)$ dimensional dilaton supergravity theory in which the dilaton classically decouples from the supergravity-matter system. The stress-energy of the supermatter generates the supergravity, which in turn governs the evolution of all supermatter fields. We have found a non-trivial solution for the compensator superfield that describes the supergravity generated by a massive superparticle. We have also shown how to construct a two-dimensional super black hole from this supergravity solution.

A number of interesting questions arise from this work. Apart from being of interest in their own right, exact superspace solutions might permit us to make considerable headway in the interpretation of classical supergravity solutions. These interpretative problems have received only scant attention in the literature [3, 6] to date.

Recent progress in the $n$-body problem in $(1 + 1)$ dimensions [26] suggests the possibility of making progress in solving the super $n$-body problem in two dimensions
as well. As demonstrated in ref. [26], the motion in the bosonic case is quite complicated even for $n = 2$, and contains a variety of interesting features.

As a final comment, it would be of considerable interest to investigate the influence of quantum corrections on the gravitational and/or matter fields of such (classical) solutions.

Acknowledgments

We would like to thank Jim Gates and Marc Grisaru for discussions. This research was supported in part by a John Charles Polanyi Fellowship, NSERC of Canada, and an NSERC Postdoctoral Fellowship.

Appendices

A Equations of Motion

Varying the complete action with respect to $\Phi, S, x_0, \theta_0, G, \lambda$ and $g$ gives:

$$\Phi = -2S$$

$$\frac{2}{\kappa}D_-D_+\Phi(z) - \lambda e^{-2S}(D_+G_+ + D_-G_- - ie^{-2S})$$

$$+ 4m \int dt \left[ g^{-1}e^{-4S} \left( \frac{1}{2}(1 + \dot{x}_0) + i\theta_0^+\dot{\theta}_0^+ \right) \frac{1}{2}(1 - \dot{x}_0) + i\theta_0^-\dot{\theta}_0^- \right] \delta^4(z - z_0(t)) = 0$$

(A.2)

$$\left\{ -4g^{-1}e^{-4S} \frac{\partial S}{\partial x_0} \left[ \frac{1}{2}(1 + \dot{x}_0) + i\theta_0^+\dot{\theta}_0^+ \right] \frac{1}{2}(1 - \dot{x}_0) + i\theta_0^-\dot{\theta}_0^- \right\}$$

$$+ \frac{1}{2} \frac{d}{dt} \left[ g^{-1}e^{-4S}(-\dot{\theta}_0^- - i\theta_0^+\dot{\theta}_0^+) + iD_+G_+ - iD_-G_- \right] \delta(t - s) = 0$$

(A.3)
\[ + i \left[ \frac{1}{2} (1 - \dot{x}_0) + i \theta_0^{-} \dot{\theta}_0^{-} \right] \frac{\partial (D_- G_-)}{\partial \theta_0^{-}} + i \left[ \frac{1}{2} (1 + \dot{x}_0) + i \theta_0^{+} \dot{\theta}_0^{+} \right] \frac{\partial (D_+ G_+)}{\partial \theta_0^{+}} \]

\[ - \dot{\theta}_0^{-} + \frac{\partial G_+}{\partial \theta_0^{+}} - \dot{\theta}_0^{+} - \frac{\partial G_-}{\partial \theta_0^{-}} \]

\[ - \frac{d}{dt} \left[ g^{-1} e^{-4S} \left( \frac{1}{2} \dot{\theta}_0^{+} (1 - \dot{x}_0) - \dot{\theta}_0^{+} \theta_0^{-} \dot{\theta}_0^{-} \right) + \theta_0^{+} D_+ G_+ + G_+ \right] \varepsilon(t - s) = 0 \]

(A.4)

\[
\left\{ -4g^{-1} e^{-4S} \frac{\partial S}{\partial \theta_0^{-}} \left[ \frac{1}{2} (1 + \dot{x}_0) + i \theta_0^{+} \dot{\theta}_0^{+} \right] \left[ \frac{1}{2} (1 - \dot{x}_0) + i \theta_0^{-} \dot{\theta}_0^{-} \right] \\
+ g^{-1} e^{-4S} i \dot{\theta}_0^{-} \left[ \frac{1}{2} (1 + \dot{x}_0) + i \theta_0^{-} \dot{\theta}_0^{-} \right] - \dot{\theta}_0^{-} D_- G_- \\
+ i \left[ \frac{1}{2} (1 - \dot{x}_0) + i \theta_0^{-} \dot{\theta}_0^{-} \right] \frac{\partial (D_- G_-)}{\partial \theta_0^{-}} + i \left[ \frac{1}{2} (1 + \dot{x}_0) + i \theta_0^{+} \dot{\theta}_0^{+} \right] \frac{\partial (D_+ G_+)}{\partial \theta_0^{+}} \\
- \dot{\theta}_0^{+} + \frac{\partial G_+}{\partial \theta_0^{+}} - \dot{\theta}_0^{-} - \frac{\partial G_-}{\partial \theta_0^{-}} \\
- \frac{d}{dt} \left[ g^{-1} e^{-4S} \left( \frac{1}{2} \dot{\theta}_0^{-} (1 + \dot{x}_0) - \dot{\theta}_0^{+} \theta_0^{+} \dot{\theta}_0^{-} \right) + \theta_0^{-} D_- G_- + G_- \right] \varepsilon(t - s) = 0 \]

(A.5)

\[ m \int dt \left[ i \left( \frac{1}{2} (1 + \dot{x}_0) + i \theta_0^{+} \dot{\theta}_0^{+} \right) D_+ \delta^4 (z - z_0(t)) + \dot{\theta}_0^{+} \delta^4 (z - z_0(t)) \right] = \frac{\pi}{4} D_- (\lambda e^{-2S}) \]

\[ m \int dt \left[ i \left( \frac{1}{2} (1 - \dot{x}_0) + i \theta_0^{-} \dot{\theta}_0^{-} \right) D_- \delta^4 (z - z_0(t)) + \dot{\theta}_0^{-} \delta^4 (z - z_0(t)) \right] = \frac{\pi}{4} D_+ (\lambda e^{-2S}) \]

(A.6)

\[ D_+ G_- + D_- G_+ - i e^{-2S} = 0 \]

(A.7)

\[ g = 2 e^{-2S} \left( \frac{1}{2} (1 + \dot{x}_0) + i \theta_0^{+} \dot{\theta}_0^{+} \right) \left( \frac{1}{2} (1 - \dot{x}_0) + i \theta_0^{-} \dot{\theta}_0^{-} \right) \]

\[ = 2 e^{-2S} \sqrt{\frac{1}{2}} \sqrt{\frac{1}{2}} \]

(A.8)

as the equations of motion for the respective fields.

### B Component Action

We obtain the component superparticle action by following the method used in [27] for normal coordinate expansions of Green-Schwarz type \( \sigma \)-models. We modify the procedure slightly to bring the WZ-gauge choice in line with the superconformal

15
gauge choice, according to the discussion in the beginning of Appendix C. Following the notation of [27], superspace is parametrized by \( z^M = (x^m_0, 0) \) and \( y^M = (0, y^\mu) \) and we replace the WZ-gauge choice, \( \mathcal{E}_\mu^\alpha = \delta_\mu^\alpha \), with \( \mathcal{E}_\mu^\alpha = e^{-S} \delta_\mu^\alpha \). Then \( y^\alpha = y^\mu \mathcal{E}_\mu^\alpha = \theta_0^\alpha e^{-S} \) is the only change to the results in the above references. Defining \( V^A \equiv \bar{z}^M \mathcal{E}_M^A \), we find for the terms in the component superparticle action

\[
I^{(0)} = -m \int d\tau [g^{-1}V^*V^* + V^A \Gamma_A + \frac{g}{4}]
\]

\[
I^{(1)} = -m \int d\tau [-2ig^{-1}(V^\Xi y^+ + V^\Psi y^-) + iV^+ y^- + iV^- y^+]
\]

\[
I^{(2)} = -m \int d\tau [-2ig^{-1}(V^\Xi \mathcal{D} y^+ y^+ + V^\Psi \mathcal{D} y^- y^-) + i(\mathcal{D} y^+ y^- + \mathcal{D} y^- y^+)]
\]

\[
-2g^{-1} RV^\Xi y^+ y^- + 8g^{-1} V^\Xi y^+ y^-
\]

\[
I^{(3)} = -m \int d\tau [-8g^{-1}(V^- \mathcal{D} y^+ y^+ y^- + V^+ \mathcal{D} y^- y^- y^+)]
\]

\[
I^{(4)} = -m \int d\tau [-16g^{-1} \mathcal{D} y^+ y^+ \mathcal{D} y^- y^-]
\]

(B.1)

where all quantities are evaluated at \((x^m_0, 0)\) and \( \mathcal{D} \) is the world-line covariant derivative.

Specifically for the superparticle, we have:

\[
V^* = \frac{i}{2}(1 + \dot{x}_0) e^{-2S} \]

\[
V^\Xi = \frac{i}{2} (1 - \dot{x}_0) e^{-2S} \]

\[
V^+ = -i(1 + \dot{x}_0) \mathcal{D}^+ (e^{-S}) \]

\[
V^- = -i(1 - \dot{x}_0) \mathcal{D}^- (e^{-S}) \]

\[
\mathcal{D} y^\pm = (\mathcal{D} \theta^\pm) e^{-S} | + \theta^\pm (\mathcal{D} e^{-S}) | \]

(B.2)

where \( | = |_{\theta_0 = 0} \). We have also \( E_\mu^\alpha = e^{-S} \mathcal{D}^\mu \), and we introduce \( \tilde{\psi} \) with (lower world, upper tangent) indices, related to \( \psi \) by multiplication by the component zweibein. By analogy with Appendix C we can therefore write

\[
E_\pm = e^{-S} \mathcal{D}_\pm
\]

\[
= [e^{1/4} + \frac{i}{2} (\theta^+ \tilde{\psi}^+ - \theta^- \tilde{\psi}^-) + \frac{i}{2} \theta^+ \theta^- e^{-1/4} (i \bar{A} + \tilde{\psi}^- \tilde{\psi}^+) ] \mathcal{D}_\pm
\]

(B.3)

Therefore we have

\[
\mathcal{D}_+ e^{-S} | = \frac{i \tilde{\psi}^+}{2} \]

\[
\mathcal{D}_- e^{-S} | = -\frac{i}{2} \tilde{\psi}^- \]

\[
e^{-S} | = e^{1/4} \]

\[
y^\pm = \theta^\pm e^{1/4} \]

\[
\Gamma_A | = \gamma_A
\]

(B.4)
and substituting these expressions into (B.1), we obtain

\[
\begin{align*}
I^{(0)} &= -m \int d\tau \frac{g^{-1}}{4} (1 + \dot{x}_0)(1 - \dot{x}_0) e + \frac{1}{2} (1 + \dot{x}_0) e^{1/2} \gamma^\pm + \frac{i}{2} (1 - \dot{x}_0) e^{1/2} \gamma^- \\
&\quad + \frac{i}{2} (1 + \dot{x}_0) \psi^\pm \gamma_+ - \frac{i}{2} (1 - \dot{x}_0) \bar{\psi}^- \gamma + \frac{g}{4} \\
I^{(1)} &= -m \int d\tau \left[ -\frac{i}{2} g^{-1} (1 + \dot{x}_0)(1 - \dot{x}_0) e^{3/4} \{ \psi^{+\theta}_0 - \bar{\psi}^- \theta_0 \} \\
&\quad + \frac{i}{2} (1 + \dot{x}_0) \psi^{+\theta}_0 e^{1/4} - \frac{i}{2} (1 - \dot{x}_0) \bar{\psi}^- \theta_0 e^{1/4} \right] \\
I^{(2)} &= -m \int d\tau \left[ -ig^{-1} e \{(1 - \dot{x}_0)(\theta^{\pm\theta}_0) \theta_0^+ + (1 - \dot{x}_0)(\theta^{\pm\theta}_0) \theta_0^- \} + 2g^{-1} \frac{1}{2} e^{1/2} (1 + \dot{x}_0)(1 - \dot{x}_0) \psi^{+\theta}_0 \psi^{+\theta}_0 \bar{\psi}^- \theta_0 \right] \\
I^{(3)} &= -m \int d\tau \left[ 4g^{-1} e^{3/4} \{(1 - \dot{x}_0) \psi^{+\theta}_0 (\theta^{\pm\theta}_0) + (1 + \dot{x}_0) \bar{\psi}^{+\theta}_0 (\theta^{\pm\theta}_0) \} \theta_0^+ \theta_0^- \right] \\
I^{(4)} &= -m \int d\tau \left[ 16g^{-1} e (\theta^{\pm\theta}_0) (\theta^{\pm\theta}_0) \theta_0^+ \theta_0^- \right] 
\end{align*}
\]

It is simple to obtain the component form of the dilaton supergravity part of the action in conformal gauge from (4.5) using standard techniques [28].

C Component Results and Non-Triviality of Solution

Ordinarily, to go from superspace results to the usual \(x\)-space components, one uses the standard technique of fixing a Wess-Zumino gauge (\(\nabla_+ = \partial_+\)). However, as is discussed in [29], the usual procedure needs to be modified somewhat in order to find a Wess-Zumino gauge choice such that the superconformal gauge (\(E_\pm = \epsilon S D_\pm\)) is compatible with the ordinary \(x\)-space conformal gauge (\(e^m_a = \rho \delta^m_a\)). Using these results, we can write \(E_a\) in component conformal gauge as

\[
E_\pm = \epsilon^{S_\pm} D_\pm \\
= [e^{-1/4} + \frac{i}{2} (\theta^+ \psi^{+\theta}_0 + \theta^- \psi^{+\theta}_0) + \frac{i}{4} \theta^+ \theta^- e^{1/4} (iA + \psi^{+\theta}_0 \psi^{+\theta}_0)] D_\pm
\]  

(C.1)

Taylor expanding \(\epsilon^S\) allows us to identify the gravitino \((\psi^{+\theta}_0, \psi^{+\theta}_0)\) and the component auxiliary field of the supergravity multiplet, \(A\). Setting \(\kappa = 8\), we find

\[
\begin{align*}
e^{-1/4} &= e^{-M|x-x_0|} \\
\frac{i}{2} \psi^{+\theta}_0 &= e^{-M|x-x_0|} M \theta^+ \epsilon(x - x_0) \\
\frac{i}{2} \psi^{+\theta}_0 &= e^{-M|x-x_0|} M \theta^- \epsilon(x - x_0) \\
\frac{i}{4} e^{1/4} iA &= 2e^{-M|x-x_0|} M \theta^+ \theta^- \delta(x - x_0) \\
\frac{i}{4} e^{1/4} \psi^{+\theta}_0 &= -e^{-M|x-x_0|} M^2 \theta^+ \theta^- \psi^{+\theta}_0
\end{align*}
\]

(C.2)
We can now compute the scalar supercurvature, $R$, of the resulting background. From (4.3), we find

$$R = 8M\delta(x - x_0)\delta^{(2)}(\theta - \theta_0)$$  \hspace{1cm} (C.3)

which can be rewritten using (C.3) as

$$R = e^{1/4}[e^{1/4}iA + 2\theta^-\partial\psi^+ - 2\theta^+\partial\psi^- - 4\theta^+\theta^- (\partial^2 e^{-1/4} - e^{-1/4}M^2)]$$  \hspace{1cm} (C.4)

and compared with the general results in [21, 22]. The $x$-space Ricci curvature, $r$, is contained in $\nabla^2 R$. We find

$$\nabla^2 R| = 4e^{-1/4}(\partial^2 e^{-1/4} - e^{-1/4}M^2)$$

$$= -8M\delta(x - x_0)$$

$$= r$$  \hspace{1cm} (C.5)

since the other terms in $\nabla^2 R$, involving the gamma-trace of the gravitino, the curl of the gravitino and the auxiliary field, vanish in this simple case.

We turn now to the non-triviality of the solution – specifically, the fact that the solution for $S$ cannot be obtained by an infinitesimal supersymmetry transformation from the bosonic one. This can be seen in one of two ways: from a purely superspace argument or from components. In the former case, we note that the superspace described by $S$ has torsion, as expected in a bona fide supergravity solution. As the torsion $T_{AB}^C$ is a supercovariant quantity, its value remains unchanged under a suitable gauge transformation. Such a gauge transformation might set the gravitino to zero, but then simultaneously there must be a redefinition of the vielbein so that overall $T_{AB}^C$ is unchanged.

Alternatively, a supergravity solution can be seen to be trivial if one can find an infinitesimal spinor $\alpha(x)$ such that $\psi^\mu_a = D_a\alpha^\mu$ where $D_a = e^m_a\partial_m + \omega_a M$ is the component covariant gravitational derivative [24]. For example, the differential equation that $\alpha^+$ must satisfy, $\psi^+ = D^+\alpha^+$, becomes, using the above results

$$\partial[e^{-M|x-x_0|}\alpha^+] = 2M\theta^+_0 \epsilon(x - x_0)$$  \hspace{1cm} (C.6)

with the solution $\alpha^+ = 2M\theta^+_0 e^{M|x-x_0|} |x - x_0|$. We note that $\alpha^+$ is not well behaved at infinity, and so the solution for $S$ is not related to a trivial one by an infinitesimal supersymmetry transformation.
\[ \mathcal{E}'_M^A = \begin{pmatrix} (e^{-2S})' & 0 & -i(D_+ e^{-S})' & -i(D_+ e^{-S})' \\ 0 & \mathcal{E}'_u^t & \mathcal{E}'_u^t & \mathcal{E}'_u^t \\ \mathcal{E}'_+^t & \mathcal{E}'_+^t & \mathcal{E}'_+^t & \mathcal{E}'_+^t \\ \mathcal{E}'_-^t & \mathcal{E}'_-^t & \mathcal{E}'_-^t & \mathcal{E}'_-^t \end{pmatrix} \]  

(D.1)

where \((X)'\) means the object is to be evaluated in the transformed coordinates, and the rows and columns of the vielbein are labelled by \((t, u, +, -)\). We note that in obtaining \(\mathcal{E}'_M^A\) in its final form, we have performed two basis changes (one world, one tangent) in addition to the actual supercoordinate transformation. We reiterate that \(\epsilon(x - x_0) = \epsilon(u - u_0)\) by construction, and that \(\delta(x - x_0) = |C|\delta(u - u_0)\).

For \(C > 0\), we find

\begin{align*}
(e^{-2S})' &= \frac{2m}{|C|}|u - u_0 - i(\eta^+ \eta_0^+ - \eta^- \eta_0^-)| + 1 \\
(e^{-S})' &= \sqrt{\frac{2m}{|C|}}|u - u_0 - i(\eta^+ \eta_0^+ - \eta^- \eta_0^-)| + 1 \\
(D_+ e^{-S})' &= -(e^{-S})'(D_+ S)' \\
(D_- e^{-S})' &= -(e^{-S})'(D_- S)' \\
(D_+ S)' &= -2M(\theta^+ - \theta_0^+)[i\epsilon(u - u_0) - 2\theta^- \theta_0^- |C|\delta(u - u_0)] \\
(D_- S)' &= 2M(\theta^- - \theta_0^-)[i\epsilon(u - u_0) - 2\theta^+ \theta_0^+ |C|\delta(u - u_0)]
\end{align*}

\[(D.2)\]

where

\begin{align*}
|x - x_0| &= \frac{1}{2m}ln\left(\frac{2m}{|C|}|u - u_0| + 1\right) \\
\theta^\pm &= \frac{\sqrt{|C|}}{2m} \frac{\eta^\pm}{|u - u_0| + |C|/2m \pm \frac{i}{2}\eta^\mp \eta_0^\mp \epsilon(u - u_0)} \\
\theta_0^\pm &= \eta_0^\pm \sqrt{|C|}
\end{align*}

(D.3)

In addition, we also have

\begin{align*}
\mathcal{E}'_u^- &= \frac{1}{2m |u - u_0| + |C|/2m} (e^{-2S})' \\
\mathcal{E}'_u^+ &= \frac{1}{2m |u - u_0| + |C|/2m} (-iD_+ e^{-S})' - \frac{\sqrt{|C|}}{2m} \frac{\eta^+}{|X|^2}(\epsilon(u - u_0) + i\eta^- \eta_0^- \delta(u - u_0))(e^{-S})'
\end{align*}
\[\mathcal{E}_u' = \frac{1}{2m|u - u_0| + |C|/2m}\left(\frac{\epsilon(u - u_0)(iD_-e^{-S})'}{2m|Y|^2}(\epsilon(u - u_0) - i\eta^\dagger\eta_0^+\delta(u - u_0))(e^{-S})'\right)\]

\[\mathcal{E}_+^t = \sqrt{|C|} \frac{1}{2m|[X]}(-ie^{-2S}\theta^+)\]

\[\mathcal{E}_+^u = \sqrt{|C|} \frac{1}{2m|[X]}(-ie^{-2S}\theta^+)\]

\[\mathcal{E}_+^+ = \sqrt{|C|} \frac{1}{2m}[X](e^{-S})'(1 - 2(D_+S))\theta^+)\]

\[\mathcal{E}_+^- = \sqrt{|C|} \frac{1}{2m}[Y]^2(-\frac{i}{2}\eta^+\epsilon(u - u_0))(e^{-S})'\]

\[\mathcal{E}_-^t = \sqrt{|C|} \frac{1}{2m}[Y]^2(-ie^{-2S}\theta^-)\]

\[\mathcal{E}_-^u = -\sqrt{|C|} \frac{1}{2m}[Y]^2(-ie^{-2S}\theta^-)\]

\[\mathcal{E}_-^+ = \sqrt{|C|} \frac{1}{2m}[X]^2(\frac{i}{2}\eta^0\epsilon(u - u_0))(e^{-S})'\]

\[\mathcal{E}_-^- = \sqrt{|C|} \frac{1}{2m}[Y]^2(e^{-S})'(1 - 2(D_-S))\theta^-)\]

where we denote

\[X = |u - u_0| + |C|/2m + \frac{i}{2}\eta^0\eta^+\epsilon(u - u_0)\]

\[Y = |u - u_0| + |C|/2m - \frac{i}{2}\eta^+\eta^0\epsilon(u - u_0)\]

For \(C < 0\), we have the three different regions:

**Region (i):**

\[(e^{-2S})' = \frac{-2m}{|C|}[u - u_0 - i(\eta^+\eta_0^+ - \eta^-\eta_0^-)] - 1\]

\[(e^{-S})' = \sqrt{\frac{-2m}{|C|}[u - u_0 - i(\eta^+\eta_0^+ - \eta^-\eta_0^-)]} - 1\]

\[(D_+e^{-S})' = -\epsilon^{-S}'(D_+S)'\]

\[(D_-e^{-S})' = -\epsilon^{-S}'(D_-S)'\]

\[(D_+S)' = -2M(\theta^+ - \theta_0^+)[i\epsilon(u - u_0) - 2\theta\theta_0^+|C|\delta(u - u_0)]\]

\[(D_-S)' = 2M(\theta^- - \theta_0^-)[i\epsilon(u - u_0) - 2\theta^+\theta_0^+|C|\delta(u - u_0)]\]
where

\[ x - x_0 = \frac{-1}{2m} \ln \left( \frac{-2m}{|C|} (u - u_0) - 1 \right) \]

\[ \theta^\pm = \frac{\sqrt{|C|} \, i \eta^\pm}{2m \left[ (u - u_0) + |C|/2m \pm \frac{i}{2} \eta^\pm \eta_0^\mp \right]} \]

\[ \theta_0^\pm = \frac{i \eta_0^\pm}{\sqrt{|C|}} \quad \text{(D.7)} \]

In addition, we also have

\[ \mathcal{E}' u^- = \frac{-1}{2m (u - u_0) + |C|/2m} \left( e^{-2S} \right)' \]

\[ \mathcal{E}' u^+ = \frac{-1}{2m (u - u_0) + |C|/2m} \left( -i D_+ e^{-S} \right)' - \frac{\sqrt{|C|}}{2m} \left( \frac{\eta^+}{|X|^2} \right) (e^{-S})' \]

\[ \mathcal{E}' u^- = \frac{-1}{2m (u - u_0) + |C|/2m} \left( i D_- e^{-S} \right)' - \frac{\sqrt{|C|}}{2m} \left( \frac{\eta^-}{|Y|^2} \right) (e^{-S})' \]

\[ \mathcal{E}'^+ = \frac{\sqrt{|C|}}{2m} \left( \frac{i}{|X|} \right) (e^{-S})' (1 - 2(D_+ S) \theta^+) \]

\[ \mathcal{E}'^- = \frac{\sqrt{|C|}}{2m} \left( \frac{\eta^-}{|Y|^2} \right) (e^{-S})' \]

\[ \mathcal{E}'^- = \frac{\sqrt{|C|}}{2m} \left( \frac{i}{|Y|} \right) (e^{-S})' \]

\[ \mathcal{E}'^+ = \frac{\sqrt{|C|}}{2m} \left( \frac{i}{|X|^2} \right) (e^{-S})' \]

\[ \mathcal{E}'^- = \frac{\sqrt{|C|}}{2m} \left( \frac{i}{|Y|^2} \right) (e^{-S})' (1 - 2(D_- S) \theta^-) \]  \quad \text{(D.8)}

where we denote

\[ X = (u - u_0) + |C|/2m + \frac{i}{2} \eta^+ \eta_0^- \]

\[ Y = (u - u_0) + |C|/2m - \frac{i}{2} \eta^- \eta_0^+ \]  \quad \text{(D.9)}
Region (ii):

\[
(e^{-2S})' = \frac{-2m}{|C|} |u - u_0 - i(\eta \eta_0^+ - \eta_0^-)| + 1
\]

\[
(e^{-S})' = \sqrt{\frac{-2m}{|C|}} |u - u_0 - i(\eta \eta_0^+ - \eta_0^-)| + 1
\]

\[
(D_+e^{-S})' = -(e^{-S})'(D_+S)'
\]

\[
(D_-e^{-S})' = -(e^{-S})'(D_-S)'
\]

\[
(D_+S)' = -2M(\theta^+ - \theta_0^+)[i\epsilon(u - u_0) - 2\theta^- \theta_0^- |C| \delta(u - u_0)]
\]

\[
(D_-S)' = 2M(\theta^- - \theta_0^-)[i\epsilon(u - u_0) - 2\theta^+ \theta_0^+ |C| \delta(u - u_0)]
\]

where

\[
x - x_0 = \frac{-1}{2m} \ln(1 - \frac{2m}{|C|} |u - u_0|)
\]

\[
\theta^\pm = \sqrt{|C|} \frac{\eta^\pm}{2m} |u - u_0| - |C|/2m \pm \frac{i\eta^\pm \eta_0^\mp}{2m} \epsilon(u - u_0)
\]

\[
\theta_0^\pm = \frac{-\eta_0^\pm}{\sqrt{|C|}}
\]

In addition, we also have

\[
\mathcal{E}_u^t = -\frac{1}{2m \ |u - u_0| - |C|/2m} (e^{-2S})'
\]

\[
\mathcal{E}_u^+ = -\frac{1}{2m \ |u - u_0| - |C|/2m} (-iD_+e^{-S})' - \sqrt{|C|} \frac{\eta^+}{2m \ |X|} (e(u - u_0) + i\eta^- \eta_0^- \delta(u - u_0))(e^{-S})'
\]

\[
\mathcal{E}_u^- = -\frac{1}{2m \ |u - u_0| - |C|/2m} (iD_-e^{-S})' - \sqrt{|C|} \frac{\eta^-}{2m \ |Y|} (e(u - u_0) - i\eta^+ \eta_0^+ \delta(u - u_0))(e^{-S})'
\]

\[
\mathcal{E}_+^t = \sqrt{|C|} \frac{1}{2m \ |X|} (-ie^{-2S} \theta^+{})'
\]

\[
\mathcal{E}_+^u = \sqrt{|C|} \frac{1}{2m \ |X|} (-ie^{-2S} \theta^+{})'
\]

\[
\mathcal{E}_+^+ = \sqrt{|C|} \frac{1}{2m \ |X|} (e^{-S})' (1 - 2(D_+S) \theta^+{})'
\]

\[
\mathcal{E}_+^- = \sqrt{|C|} \frac{\eta^-}{2m \ |Y|} (e^{-S})' (1 - 2(D_+S) \theta^+{})'
\]

\[
\mathcal{E}_+^+ = \sqrt{|C|} \frac{\eta^-}{2m \ |Y|} (e^{-S})' (1 - 2(D_+S) \theta^+{})'
\]

\[
\mathcal{E}_+^- = \sqrt{|C|} \frac{\eta^-}{2m \ |Y|} (e^{-S})' (1 - 2(D_+S) \theta^+{})'
\]
\[
\mathcal{E}'_t = \frac{\sqrt{|C|}}{2m} \frac{1}{|Y|} (-ie^{-2S}\theta^-)',
\]
\[
\mathcal{E}'_u = -\frac{\sqrt{|C|}}{2m} \frac{1}{|Y|} (-ie^{-2S}\theta^-)',
\]
\[
\mathcal{E}'_+ = \frac{\sqrt{|C|}}{2m} \frac{\eta^+}{|X|^2} (\hat{\theta} \eta_0^- e(u-u_0)) (e^{-S})',
\]
\[
\mathcal{E}'_- = \frac{\sqrt{|C|}}{2m} \frac{1}{|Y|} (e^{-S})'(1 - 2(D_-S)\theta^-)',
\]
(D.12)

where we denote

\[
X = |u-u_0| - |C|/2m + \frac{1}{2} \eta^- \eta_0^- e(u-u_0)
\]
\[
Y = |u-u_0| - |C|/2m + \frac{1}{2} \eta^+ \eta_0^+ e(u-u_0)
\]
(D.13)

Region (iii):

\[
(e^{-2S})' = \frac{2m}{|C|} [u-u_0 - i(\eta^+ \eta_0^+ - \eta^- \eta_0^-)] - 1
\]
\[
(e^{-S})' = \frac{2m}{|C|} [u-u_0 - i(\eta^+ \eta_0^+ - \eta^- \eta_0^-)] - 1
\]
\[
(D_+ e^{-S})' = -(e^{-S})'(D_+ S)'
\]
\[
(D_- e^{-S})' = -(e^{-S})'(D_- S)'
\]
(D.14)

where

\[
x - x_0 = \frac{1}{2m} \ln\left(\frac{2m}{|C|} (u-u_0) - 1\right)
\]
\[
\theta^\pm = \frac{\sqrt{|C|}}{2m} \frac{i \eta^\pm}{[(u-u_0) - |C|/2m \pm \frac{1}{2} \eta^\pm \eta_0^\pm]}
\]
\[
\theta_0^\pm = -\frac{i \eta_0^\pm}{\sqrt{|C|}}
\]
(D.15)

In addition, we also have

\[
\mathcal{E}'_u^u = \frac{1}{2m (u-u_0) - |C|/2m} (e^{-2S})'
\]
\[
\mathcal{E}'_u^+ = \frac{1}{2m (u-u_0) - |C|/2m} (-i D_+ e^{-S})' - \frac{\sqrt{|C|}}{2m} \frac{i \eta^+}{|X|^2} (e^{-S})'
\]

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\begin{align}
\mathcal{E}_u^- &= \frac{1}{2m} \frac{1}{|C|/2m} (iD e^{-S})' - \frac{\sqrt{|C|} \ i \eta^-}{2m \ [|Y|]^2} (e^{-S})' \\
\mathcal{E}_+^+ &= \frac{\sqrt{|C|} \ i}{2m \ [X]} (-ie^{-2S} \theta^+)'
\end{align}

where we denote

\begin{align}
X &= (u - u_0) - |C|/2m + \frac{i}{2} \eta^+ \eta_0^- \\
Y &= (u - u_0) - |C|/2m - \frac{i}{2} \eta^+ \eta_0^+
\end{align}

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