Dynamics of F/D networks: the role of bound states

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Abstract. In a field theory model, we study, via numerical experiments, the role of bound states in the evolution of cosmic superstring networks, being composed by $p$ F strings, $q$ D strings and $(p, q)$ bound states. We find robust evidence for scaling of all three components of the network, independently of initial conditions. The novelty of our numerical approach consists of having control over the initial abundance of bound states. This indeed allows us to identify the effect of bound states on the evolution of the network. We also find an additional energy loss mechanism, resulting in a lower overall string network energy, and thus scaling of the network. This new mechanism consists of the formation of bound states with an increasing length.

Keywords: cosmology with extra dimensions, physics of the early universe
1. Introduction

Cosmic superstrings can be formed as a result of brane annihilations in the context of the brane-world scenario [1]–[4]. In the brane-world models, brane inflation takes place [5] while two branes move towards each other, and their annihilation releases the brane tension energy that heats up the universe to start the hot big bang era. Typically, strings of all sizes and types may be produced during the collision. Considering a IIB string theory in a (9 + 1)-dimensional spacetime, interactions of Dirichlet (D) branes lead to the unwinding and subsequent evaporation of higher-dimensionality branes, with the survival of three-dimensional (D3) branes embedded in a (9 + 1)-dimensional bulk—one of which could play the role of our universe—and D1 branes (D strings) [6]. Large fundamental (F) strings and D strings that survive the cosmological evolution become cosmic superstrings. They are of cosmological size and could play the role of cosmic strings [7,8], false vacuum remnants formed generically at the end of hybrid inflation within grand unified theories [9,10]. Cosmic superstrings have gained a lot of interest, particularly since it is believed that they may be observed in the sky, providing both a means of testing string theory and a hint for a physically motivated inflationary model (for a recent review, see, e.g., [11]).

Brane collisions lead also to the formation of bound states, \((p, q)\) strings, which are composites of \(p\) F strings and \(q\) D strings [12,13]. The presence of stable bound states implies the existence of junctions, where two different types of string meet at a point and form a bound state leading away from that point. Thus, when cosmic superstrings of different types collide, they cannot intercommute; instead they exchange partners and form a Y junction, as a consequence of charge conservation at the junction of colliding \((p, q)\) strings. The evolution of F and D strings and their bound states is a rather complicated problem, which necessitates both numerical as well as analytical investigations. Junctions may prevent the network from achieving a scaling solution, invalidating the cosmological model leading to their formation. In a number of studies, cosmic superstring evolution has been addressed via numerical experiments [14]–[20]. The formation of three-string junctions and kinematic constraints for their collisions have also been investigated analytically [21]–[23].
In what follows, we use a field theory model to address the question of the effect of junctions in the evolution of a cosmic superstring network, being composed of three components: $p$ F strings, $q$ D strings and their $(p, q)$ bound states. The results of our numerical investigations can be summarized as follows. Firstly, there is clear evidence for scaling of all three components of the network, independently of the chosen initial configurations. Secondly, the existence of bound states affects the evolution of the network. Thirdly, for $(p, q)$ strings there is a supplementary energy loss mechanism, in addition to the chopping off of loops, and it is this new mechanism that allows the network to scale. More precisely, the additional energy loss mechanism is the formation of bound states, whose length increases, lowering the overall energy of the network. We note that in our simulations we can have control over the initial population of bound states, and this renders our novel results particularly important.

2. The model

We adopt the model of [18]. This is a simple field theory model of $(p, q)$ bound states, in analogy to the Abelian Higgs model, which incorporates the main features of string theory. To represent the two different species of strings, the model includes two complex scalar fields, $\phi$ and $\chi$, of the Abelian Higgs model, coupled via a potential, so that bound states can be formed. The presence of stable bound states implies the existence of junctions, where two different types of string meet at a point and form a bound state leading away from that point.

We distinguish two different cases of cosmic superstring networks. The nature of cosmic superstrings depends on the particular brane inflationary model leading to their formation. In the case that both species of cosmic strings are BPS, the model is described by the action [18]:

$$\mathcal{S} = \int d^3x \, dt \left[ -\frac{1}{4} F^2 - \frac{1}{2} (D_\mu \phi)^* (D^\mu \phi) - \frac{\lambda_1}{4} \left( \phi \phi^* - \eta_1^2 \right)^2 
- \frac{1}{4} H^2 - \frac{1}{2} (D_\mu \chi)^* (D^\mu \chi)^* - \frac{\lambda_2}{4} \phi \phi^* \left( \chi \chi^* - \eta_2^2 \right)^2 \right],$$

(1)

where the covariant derivative $D_\mu$ is defined by

$$D_\mu \phi = \partial_\mu \phi - ie_1 A_\mu \phi,$$

$$D_\mu \chi = \partial_\mu \chi - ie_2 C_\mu \chi.$$  

(2)

For clarity in our discussion, we label the $\phi$ field as ‘Higgs’ and the $\chi$ field as ‘axion’, even though both fields are Higgs-like. The scalars are coupled to the $U(1)$ gauge fields $A_\mu$ and $C_\mu$, with coupling constants $e_1$ and $e_2$ and field strength tensors $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $H_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$, respectively. The scalar potentials are parametrized by the positive constants $\lambda_1, \eta_1$ and $\lambda_2, \eta_2$, respectively.

In the case that one species of string is non-BPS, we remove the second gauge field by setting $e_2 = 0$. In this way, this species of string is represented by the topological defect of a complex scalar field with a global $U(1)$ symmetry. Such defects are characterized by the existence of long-range interactions [24]—as opposed to local strings in which all energy density is confined within the string, so that local strings have only gravitational interactions—implying different consequences for the evolution of the network.
In flat space the classical equations of motion for the $\phi$ and $\chi$ fields, obtained from the above action, equation (1), are [18]

\[
\begin{align*}
\partial_\mu F^{\mu\nu} &= 2e_1 \text{Im} \phi^* D^\nu \phi, \\
\partial_\mu H^{\mu\nu} &= 2e_2 \text{Im} \chi^* D^\nu \chi, \\
D_\mu D^\mu \phi &= -2\lambda_1 \left( \phi^* \phi - \eta_1^2 \right) \phi - \lambda_2 \left( \chi^* \chi - \eta_2^2 \right) \phi, \\
D_\mu D^\mu \chi &= -2\lambda_2 \phi^* \phi \left( \chi^* \chi - \eta_2^2 \right) \chi.
\end{align*}
\]

By calculating numerically the tension of the $(p,q)$ strings, for a range of values of the charges, and then fitting the data to the square-root expression

\[
\mu_{(p,q)} = \mu_F \sqrt{p^2 + q^2 / g_s^2},
\]

where $\mu_F$ denotes the effective fundamental string tension after compactification and $g_s$ stands for the string coupling, we check [18] the validity of our model. We remind the reader that equation (4) is exact only in the BPS limit.

The potential is chosen such that bound states are energetically preferred over single strings. This can be seen from the form of the potential for the axion strings. At the core of the Higgs string the field vanishes, $\phi = 0$, and therefore the potential of the axion vanishes as well. Therefore it is energetically favourable for the axion string to be located at the core of the Higgs string. In the true vacuum of the Higgs, we have $|\phi| = \eta_1$, so the axion string behaves just like the Higgs string.

We employ this model to study, via numerical simulations, overall properties of the cosmic superstring network. In particular, we are interested in investigating the effect of junctions and their role in achieving scaling of the network.

In the actual simulations we include the expansion of the universe by considering strings of fixed co-moving thickness, according to the model of [25]–[28]. In a Friedmann–Lemaître–Robertson–Walker universe and considering the temporal gauge, $A_0 = 0$, the equations of motion for the scalar fields take the form

\[
\begin{align*}
\partial_0 F_{0i} + 2(1 - s) \frac{a'}{a} F_{0i} &= 2a^2 e_1 \text{Im} \phi^* D^0 \phi + \partial_j F_{ji}, \\
\partial_0 C_{0i} + 2(1 - s) \frac{a'}{a} C_{0i} &= 2a^2 e_2 \text{Im} \chi^* D^0 \chi + \partial_j C_{ji}, \\
\phi'' + 2\frac{a'}{a} \phi' + D_i D_i \phi &= -a^{2s} \left[ 2\lambda_1 \left( \phi^* \phi - \eta_1^2 \right) + \lambda_2 \left( \chi^* \chi - \eta_2^2 \right) \right] \phi, \\
\chi'' + 2\frac{a'}{a} \chi' + D_i D_i \chi &= -2\lambda_2 a^{2s} \phi^* \phi \left( \chi^* \chi - \eta_2^2 \right) \chi,
\end{align*}
\]

where the time variable is the conformal time $\tau$:

\[
ds^2 = a^2(\tau) \left( -dt^2 + dx^2 \right).
\]
terms for the scalar fields $\phi(x)$ as well as the kinetic term for the gauge fields have the discretized form:

$$\int d^3x\, dt D_i \phi D^i \phi^* \to a^3 a_t \sum_{x \in M_4,i} \frac{|e^{-iA^0_i \phi(x+a^\hat{i}) - \phi(x)}|^2}{a^2},$$  \hspace{1cm} (10)$$

$$\int d^3x\, dt D_0 \phi D_0 \phi^* \to a^3 a_t \sum_{x \in M_4} \frac{|e^{-iA^0_0 \phi(x+a^\hat{t}) - \phi(x)}|^2}{a^2},$$  \hspace{1cm} (11)$$

$$\int d^3x\, dt D_i F_{ij} F^{ij} \to a^3 a_t \sum_{x \in M_4,ij} \frac{[A^i_t(x) + A^i_0(x+a^\hat{i}) - A^j_t(x+a^\hat{j}) - A^j_0(x)]^2}{a^4}. \hspace{1cm} (12)$$

The damping terms multiply the velocities of the scalar fields, and the plaquettes are oriented along the time direction. The time evolution algorithm (Euler integration) is imposed by the lattice action involving links oriented along the time direction. In our simulations we set $s = 0$, instead of the $s = 1$ required by the continuum limit of the equations of motion (otherwise the defects will fall through the lattice); we also take, for simplicity, $a'/a = 1/\tau$.

We calculate the correlation length using the expression

$$\xi = \sqrt{V/L},$$  \hspace{1cm} (13)$$

where $V$ is the volume of the simulation box and $L$ is the total length of the string network inside the box (following the same approach as, for example, in [19]). We identify the location of the string by the value of the scalar fields. In the vacuum, the value of the scalar field is equal to the v.e.v. $|\phi| = \eta_1$, $|\chi| = \eta_2$, while at the core of the Higgs of the axion string $\phi = 0$, or $\chi = 0$. After a number of attempts we found that a threshold of $0.6 \times \eta_1$ is the best value to identify if a string is present at that particular lattice point. A smaller value results in the strings being perceived as much thinner, making it possible to even completely miss parts of the string. A larger value, closer to the v.e.v. of the Higgs for example, would make the strings thicker, with string loops being perceived as ‘blobs’. A shrinking loop would be seen as a collapsing blob, and such a blob will rebound, giving false violations of scaling. Also, at the core of the axion string the Higgs has a value $|\phi| \simeq 0.9\eta_1$ which is different from the v.e.v. of $\eta_1$ even if no string is present there [29]. This is simply due to the particular form of the potential which makes it energetically favourable for the Higgs to lower its value at the core of the axion string (and therefore favour the formation of bound states). If the threshold is very close to the v.e.v. of the Higgs, the Higgs strings will be very thick and we would mistakenly identify a Higgs string at the core of each axion string.

We therefore set an appropriate threshold for each field and count the number of lattice points where the absolute value of the field is below the threshold. If at a given point both fields are below their respective thresholds, we have identified a bound state.

The correct values for the correlation lengths take into account the cross-sectional area of the strings at the threshold we defined and the fact that we coarse-grain the lattice by writing out only every second lattice point; this effectively doubles the lattice
spacing. In the following, $a_C$ is the spacing of the coarse lattice:

$$\xi = \sqrt{\frac{V}{L}} = \frac{N^3 a_C^3}{N_B a_C^2 (N_{\text{cross}} a_C^2)} = a_C \sqrt{N_{\text{cross}} \frac{N^3}{N_B}},$$

(14)

where $N$ is the number of points on the side of the (coarse) lattice (128 in our case), $N_B$ is the number of lattice points where we identify a bound state and $N_{\text{cross}}$ is the number of lattice points contained inside the cross-sectional area of the string at the given threshold.

For our choice of threshold, the strings are roughly four points across in the coarse lattice and $a_C = 0.9$. Since the strings have slightly different thickness, depending if they are singly charged or in bound states, and the axion strings have a slightly different thickness than the Higgs ones, we take

$$a_C \sqrt{N_{\text{cross}}} \simeq 4.$$  

(15)

3. Results

We present our main results for the scaling of $(p, q)$ string networks, using simulations of the classical time evolution on lattices with three spatial dimensions. We consider two types of networks, namely local–global and local–local networks. The axion field has a global $U(1)$ symmetry, and therefore the axion strings display long-range interactions.

The lattice discretization of the equations of motion is discussed in [18]. We choose all parameters of the Lagrangian to have natural values:

$$\eta_1 = 1, \quad \lambda_1 = 1, \quad e_1 = 1, \quad \eta_2 = 1, \quad \lambda_2 = 1, \quad e_2 = 1.$$  

(16)

The lattice spacing is $\delta x = 0.45$, roughly half the characteristic length scale set by the masses of the scalar and gauge fields in the broken phase. The time step is $\delta t = 10^{-2}$. The simulations are carried out with boxes of volume $(256)^3$.

For each of the two (local–global and local–local) networks we consider two types of initial conditions: one in which a large percentage of strings ($\sim 50\%$ of the string length) is in bound states and the other one with a small percentage of strings ($\sim 2\%$ of the string length) is in bound states. To obtain an initial configuration with a large percentage of bound states we first run a simulation of a local–global network starting with the same configuration for the Higgs and axion strings, that is, all the strings are initially in bound states. The long-range interaction between the global strings causes the bound states to split [18] and we can pick an intermediate field configuration with the desired amount of bound states and use it as the starting point for the other simulations. We are able to achieve the small bound state percentage configuration by starting with a large percentage configuration and then shifting one of the fields by roughly half the initial correlation length.

The type of behaviour we observe is that, in most cases, the string network does have a scaling behaviour, but the slope, $\gamma$, of the correlation length, $\xi$, versus time, defined by

$$\xi (\tau) = \gamma \tau,$$

(17)

undergoes a discrete change as the network evolves. This change is most clearly visible in the case of a local–global string network, for both large and small amounts of bound
states. The most dramatic change is that for the correlation length of the bound states themselves, provided the amount of bound states is small.

For local–local networks, the change in correlation length is not detectable if there is a large amount of bound states, while if there is a small amount of bound states we find a case where the bound states themselves do not exhibit scaling behaviour. This violation of scaling by the bound states can be explained by taking into account that the formation of bound states gives an additional possibility for a network of \((p, q)\) strings to decrease its energy. Therefore, the length of the bound states increases as a function of time, violating the scaling behaviour which requires the total length to decrease. Once the length of the bound states has reached the maximum value, scaling resumes. This violation of scaling by the bound states does not lead to violation of scaling for the entire network, as can be observed from the evolution of the Higgs and axion strings.

In figure 1 we show the string correlation length for the Higgs and axion fields, as well as for their bound states, as a function of time. The initial configuration is a local–global network with a large amount of bound states. Clearly, there is convincing evidence for scaling of the three components of the network. This scaling is characterized with a distinct change of the correlation length slope during the network evolution.

In figure 2 we show the string correlation length for the Higgs and axion fields, as well as for their bound states, as a function of time. The initial configuration is a local–global network with a small amount of bound states. Again, scaling behaviour is apparent for all three components of the local–global network. The difference between the case plotted in figure 2 and the one plotted in figure 1, is the percentage of initial bound states. As one can easily observe from figure 2, if the initial state of a local–global network has a small amount of bound states, then the change of the correlation length slope during the evolution of the network is more acute.

We then draw the corresponding plots for local–local networks, with a large and a small amount of bound states in figures 3 and 4, respectively. We see, from figure 3, that for a local–local network with a large amount of bound states, all three components scale (as for the local–global case), whereas the change in correlation lengths during evolution is not really apparent (as opposed to the local–global case).

In figure 4, we plot the total (bounded + unbounded) correlation length of the Higgs, the total (bounded + unbounded) correlation length of the axion, as well as the correlation
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Figure 2. The Higgs (left), axion (middle) and bound state (right) string correlation length as a function of time. The network is a local–global one with a small amount of bound states. The data and linear fits for the two regimes are shown.

Figure 3. The Higgs (left), axion (middle) and bound state (right) string correlation length as a function of time. The network is a local–local one with a large amount of bound states. The data and linear fits for the two regimes are shown.

Figure 4. The Higgs (left), axion (middle) and bound state (right) string correlation length as a function of time. The network is a local–local one with a small amount of bound states. The data and linear fits for the two regimes are shown.

length of (just) the bound states, for a local–local network with a small amount of bound states. Here we explicitly see, in one example, that there is a violation of the scaling behaviour for the bound states, as a result of the increase of their length with time. However, this violation does not affect the entire network, which indeed scales. More precisely, even though the overall string length in bound states increases, the overall network does not freeze (which would mean that the network does not scale), because
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Figure 5. The correlation length for a single species of cosmic string. There is no change in the slope of the correlation length as a function of time.

Table 1. Slopes of the correlation length as a function of time, $\xi(\tau) = \gamma \tau$. We separate data fits for the first and second halves of the simulations, as shown in figures 1–5. The slopes given in the first four columns are taken from figures 1–4, while the slopes in the last column are taken from figure 5.

| Component     | L–G, large | L–G, small | L–L, large | L–L, small | Single Higgs |
|---------------|------------|------------|------------|------------|--------------|
| Higgs, 1st half | 0.26       | 0.24       | 0.21       | 0.15       | 0.22         |
| Axion, 1st half | 0.24       | 0.31       | 0.20       | 0.17       | —            |
| Bound, 1st half | 0.38       | 1.53       | 0.35       | -0.91      | —            |
| Higgs, 2nd half | 0.33       | 0.21       | 0.18       | 0.21       | 0.21         |
| Axion, 2nd half | 0.32       | 0.36       | 0.18       | 0.20       | —            |
| Bound, 2nd half | 0.68       | 0.21       | 0.32       | 0.69       | —            |

the string length of the unbound states decreases faster. Thus, by increasing the length of bound states we obtain a twofold energy loss mechanism leading to scaling: firstly, the bound states lose energy into radiation, and secondly, the length of unbound states decreases.

In figure 5 we show, for comparison, the correlation length of a single species of string, using the same initial data as the one used for the (first) Higgs of the $(p, q)$ strings. Clearly, there is no change in the slope of the correlation length as a function of time. This is an important check to ensure that finite size effects do not influence the results of the simulations.

We give explicitly the slopes, $\gamma$, of the correlation length, $\xi$, as a function of time, $t$, in table 1. We separate data fits for the first and second halves of the simulations, as shown explicitly in the figures above. The last column contains the slopes for the case of a single (Higgs) species of string, as obtained from the fits in figure 5.

4. Conclusions

In the context of brane-world models, brane collisions lead to the formation of large Fundamental (F) strings and/or D1 branes (D strings). Those who survive the cosmological evolution become cosmic superstrings, playing the role of their solitonic analogues. By observing strings in the sky, we may be able to test string theory for
the first (and maybe only) time. To know the observational consequences of cosmic superstrings, it is essential to master the properties and evolution of such networks.

Performing numerical experiments, we have studied the dynamics and overall properties of superstring networks. More precisely, we have performed field theory simulations of a model of \( p \) F and \( q \) D strings, and their \((p,q)\) bound states. We have investigated the effect of bound states and in particular the approach to scaling, which is crucial for the cosmological consequences of cosmic superstring networks. Defects which do not scale are cosmologically undesired, since they may over-close the universe, leading to the old monopole problem.

Our studies have clearly shown that the three components of the network scale, independently of the chosen initial conditions. In addition, by having control over the initial abundance of bound states, we were able to identify the effect of bound states on the overall network evolution. Finally, we have found that there is an additional energy loss mechanism, beyond the chopping-off loops. This new mechanism consists of the formation of bound states, resulting in a lower overall energy of the network, thus leading to scaling.

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References

[1] Sarangi S and Tye S H H, 2002 *Phys. Lett.* B 536 185 [SPIRES] [hep-th/0204074]
[2] Majumdar M and Davis A C, 2002 *J. High Energy Phys.* JHEP03(2002)056 [SPIRES] [hep-th/0202148]
[3] Jones N T, Stoica H and Tye S H H, 2003 *Phys. Lett.* B 563 6 [SPIRES] [hep-th/0303269]
[4] Dvali G and Vilenkin A, 2004 *J. Cosmol. Astropart. Phys.* JCAP03(2004)010 [SPIRES] [hep-th/0312007]
[5] Dvali G and Tye S H, 1999 *Phys. Lett.* B 450 72 [SPIRES] [hep-ph/9812483]
[6] Durrer R, Kunz M and Sakellariadou M, 2005 *Phys. Lett.* B 614 125 [SPIRES] [hep-th/0501163]
[7] Vilenkin A and Shellard E P S, 1994 *Cosmic Strings and Other Topological Defects* (Cambridge: Cambridge University Press)
[8] Sakellariadou M, 2007 *Springer Lect. Notes Phys.* 718 247 [hep-th/0602276]
[9] Jeannerot R, Rocher J and Sakellariadou M, 2003 *Phys. Rev.* D 68 103514 [SPIRES] [hep-ph/0308134]
[10] Sakellariadou M, 2008 *Springer Lect. Notes Phys.* 738 359 [hep-th/0702009]
[11] Sakellariadou M, 2008 *Phil. Trans. R. Soc. A* 366 2881 [0802.3379] [hep-th]
[12] Copeland E, Myers R C and Polchinski L, 2004 *J. High Energy Phys.* JHEP06(2004)013 [SPIRES]
[13] Leblond L and Tye S H, 2005 *J. High Energy Phys.* JHEP03(2005)055 [SPIRES]
[14] Sakellariadou M, 2005 *J. Cosmol. Astropart. Phys.* JCAP04(2005)003 [SPIRES] [hep-th/0410234]
[15] Avgoustidis A and Shelkovich E P S, 2005 *Phys. Rev.* D 71 123513 [SPIRES] [hep-ph/0410349]
[16] Copeland E J and Saffin P M, 2005 *J. High Energy Phys.* JHEP11(2005)023 [SPIRES] [hep-th/0505110]
[17] Hindmarsh M and Saffin P M, 2006 *J. High Energy Phys.* JHEP08(2006)066 [SPIRES] [hep-th/0605014]
[18] Rajantie A, Sakellariadou M and Stoica H, 2007 *J. Cosmol. Astropart. Phys.* JCAP11(2007)021 [SPIRES] [0706.3662] [hep-th]
[19] Urrestilla J and Vilenkin A, 2008 *J. High Energy Phys.* JHEP02(2008)037 [SPIRES] [0712.1146] [hep-th]
[20] Tye S H H, Wasserman I and Wyman M, 2005 *Phys. Rev.* D 71 103508 [SPIRES] [astro-ph/0503506]
[21] Tye S H H, Wasserman I and Wyman M, 2005 *Phys. Rev.* D 71 129906 (erratum) [astro-ph/0503506]
[22] Copeland E J, Kibble T W B and Steer D A, 2006 *Phys. Rev. Lett.* 97 021602 [SPIRES]
[23] Copeland E J, Kibble T W B and Steer D A, 2007 *Phys. Rev.* D 75 065024 [SPIRES] [hep-th/0611243]
[24] Moore G D and Stoica H, 2006 *Phys. Rev.* D 74 065003 [SPIRES] [hep-th/0605070]
[25] Ryden B S, Press W H and Spergel D N, *The evolution of networks of domain walls and cosmic strings*, 1989 *CFA-3011*
Dynamics of F/D networks: the role of bound states

[26] Press W H, Ryden B S and Spergel D N, *Dynamical evolution of domain walls in an expanding universe*, 1989 NSF-ITP-89-51, CFA-1870

[27] Moore J N, Shellard E P S and Martins C J A, 2002 *Phys. Rev. D* 65 023503 [SPIRES] [hep-ph/0107171]

[28] Bevis N, Hindmarsh M, Kunz M and Urrestilla J, 2007 *Phys. Rev. D* 75 065015 [SPIRES] [astro-ph/0605018]

[29] Hartmann B and Urrestilla J, 2008 Preprint 0805.4729 [hep-th]