Study the Relationship between Beta Decay Stability of Nuclide and its Shape for Some even-even Isobars

Naz T Jarallah 1, Murtadha S Nayyef 1
1 Collage of Education for Pure Science -Ibn Al-Haitham/University of Baghdad, Baghdad, Iraq
E-mail: naztalab2016@yahoo.com

Abstract. The aim of this work is to learn the relationship of the stability of (β) emitter isobars with their shape for some isobaric elements with even mass number (A=152 - 162). To reach this goal firstly the most stable isobar have been determined by plotting mass parabola (plotting the binding energy (B.E) as a function of the atomic number (Z)) for each isobaric family. Then three-dimensional representation graphics for each nucleus in these isobaric families have been plotted to illustrate the deformation in the shape of a nucleus. These three-dimensional representation graphics prepared by calculating the values of semi-axis minor (a), major (b) and (c) ellipsoid axis's. Our results show that the shape of nuclides which is represented the most stable isobar in mass parabola are not spherical but they have some deformation in their shape, and need to emit a gamma ray to a chive more stable status and get spherical shape. The stable nuclides, which are determined from mass parabola, are different from the nuclides, which have less value of deformation.

Keywords: Nuclear Binding Energy, Semi Empirical Mass Formula (SEMF), Mass Parabolas, the intrinsic quadrupole moment.

1. Introduction
The minimum part of the elements that keeps the basic features is the atom. An atom is made of a minor massive central called the nucleus; this nucleus is enclosed by orbiting electrons [1]. Nucleus consists of two kinds of separate nucleons (neutrons and the protons) these nucleons having nearly the same mass. Therefore, the binding energy of a nucleus is a dis-continuous function of its mass [2]. Cables of nuclei having similar mass number (A) but unlike in there number of protons (Z) and neutrons number (N) are called isobars. For example, the three nuclei 14C, 14O and 14N are isobars with mass number (A) = 14 [2]. Instability of nuclide causes the radioactivity. Which depends on, the symmetry between the neutrons and atomic number [3].

All nuclei which are unstable decay naturally in several ways. Isobars with a high extra of neutrons profit energy by translating a neutron into a proton. In the case of a surplus of protons, the reverse reaction may occur: i.e., the change of a proton into a neutron. These changes are called β-decays [4]. Beta particles (β) are electrons, which carry a negative or positive charge (e−&e+). In the state of (β−) decay atomic number (Z) increased by one unit. But in the case of (β+) decay the atomic number (Z) will be reduced by one unit. Some nuclei go through a radioactive conversion by taking an atomic electron, generally from K shell, releasing a neutrino in addition to decreases atomic number (Z) by one unit [5].

The deformation in the shape of nuclides arises from the method that valence nucleons arrangement themselves in an unfilled shell, in other words the deformation happens only when both proton and
neutron shells are slightly full. In the group, decay procedure the significant character is shell nuclear deformations. [3]

2. Theoretical Part

2.1. Semi Empirical Mass Formula (SEMF)

Semi Empirical Mass Formula (SEMF) is used to calculate mass, binding energy of nuclei, which is built on Liquid Drop Model (LDM) and originally expressed by C. F. V. Weizsäcker [2]. Then adapted by Bethe and other physician. It is built partly on model and partially on empirical measurement, thus this formula is named Semi Empirical mass formula (SEMF). This formula plays an important part in the advance of nuclear physics and it consists of five forms of energy terms that is, the volume, surface, coulomb, asymmetry and pairing energy along with corresponding five kinds of constants, while modifications have been done to the coefficients over the years, but the arrangement of the formula remains the similar till today. (SEMF) has been prolonged by addition unlike additional terms or been adapted by varying the dependence of several terms on A and Z slightly in an energy to expect the binding energies as nearer to experimental values [6].

The whole containing terms of the energy of (SEMF) takes the formula as below:

\[ B.E(A,Z) = a_vA - a_sA^{2/3} - a_cZ(Z-1)A^{-1/3} - a_d(A-2Z)^2A^{-1} \pm \delta A^{-1/2} + \eta \]

Where: \( a_v, a_s, a_c, a_d, \delta \) and \( \eta \) are the energy coefficients that denote to the volume, surface, Coulomb, asymmetry, pairing and shell energy terms, respectively [7].

One set of this factors is [8]: \( a_v = 15.8 \text{MeV}, \ a_s = 18.3 \text{MeV}, \ a_c = 0.72 \text{MeV}, \ a_d = 23.2 \text{MeV}. \)

And \( \delta = \begin{cases} +11.2 \text{MeV for (even N, even Z).} \\ 0 \text{ for (even N or odd Z, or even Z, odd N).} \\ -11.2 \text{ MeV for ( odd N,odd Z).} \\ 3 \text{ MeV (N and Z = magic number ).} \end{cases} \)

And \( \eta = \begin{cases} 2 \text{ MeV (N or Z = magic number and other is odd).} \\ 1 \text{ MeV (N or Z = magic number and other is even).} \\ 0 \text{ (N or Z = no magic number).} \end{cases} \)

2.2. Mass Parabola

One of the maximum uses of semi empirical mass formula (SEMF) is to determine the most stable isobar of a given mass number (A) from beta emission by plotting the binding energy as a function to the atomic number (Z) [9]. The isobars located on the sides of the parabola are not stable, thus to be more stable these nuclides will be decay. Nuclides with low atomic number (Z) side of the parabolic, decay by (\( \beta^- \)) emission toward to the increased of the atomic number (Z\(_A\)), but the nuclides with high atomic number(Z\(_A\)) decay in the opposite direction toward to the decreased of the atomic number(Z\(_A\)), in this case the decay being either by(\( \beta^+ \)) emission or electron capture (EC) [9].

2.3. Minor and major ellipsoid axis

In classical electrodynamics, the simplest model for a non-spherical homogeneous charge supply is a rotating ellipsoid with charge \( z \), major axis \( b \), and minor axis \( a \) [10]. To determine small (minor) \( a \) and large (major) \( b \) ellipsoid axises from following equations [3].

\[
\begin{align*}
\frac{<r^2>}{3} &= 5 - \frac{2\delta}{\delta^3} \quad (2) \\
\frac{<r^2>}{2a^2} &= 1.2A^{1/3} \quad (3)
\end{align*}
\]

Where:

\(< r^2 >$: The mean-squared charge distribution radius average which calculated by:

\[
< r^2 > = 0.61.2A^{1/3} \quad (A > 100) \quad (4)
\]

\( \delta \): quadruple deformation parameter denotes to the quantity of a deviation of shape of nucleus from spherical shape, which depends on the value of intrinsic quadruple moment (\( Q_0 \)) as described in this equation [11].
δ = \frac{0.75Q_0}{<r^2>^Z} \quad (5)

Where:

\(Q_0\): The intrinsic quadrupole moment.

The electric quadrupole moment \(Q_0\) determines the deviation of the nuclear charge distribution from spherical balance. So electric quadrupole moment \(Q_0\) represented one of the important amounts to fix the nuclide form, which is in go associated to the reduced transition probability \(B(E2)\):[10]

\[Q_0 = \left[ \frac{16\pi}{5} \frac{B(E2)\uparrow}{e^2} \right]^{\frac{1}{2}} \quad (6)\]

Depending on the Global Best Fit (GLOBAL) a known of the energy \(E\) (keV) of the \((2^+)\) state is wholly that required to make a calculation for \(B(E2)\uparrow (e^2b^2)\). [12]

\[B(E2)\uparrow = \frac{2.6 Z^2}{E_{\gamma_0} A^2} \quad (7)\]

Where \(E_{\gamma_0}\) is the Gamma ray energy transition in KeV, \(Z\) denotes the atomic number, \(A\) represents the mass number of a nucleus.

3. Results and Discussion

The study of the relationship of the beta decay stability of isobars with their shape for some isobaric elements with even mass number \((A=152 - 162)\) have been done by calculating the binding energy \((B.E)\) and plotting three-dimensional representation graphics for each nucleus in these isobaric families. The binding energy \((B.E)\) for nuclides under this study calculated by using Semi Empirical Mass Formula (SEMF) eq. (1). The results of binding energy \((B.E)\) were plotted as a function to the atomic number \((Z)\) for each isobar, so we get mass parabolas for different isobars as shown in the fig.1. From this figure the most stable isobars represented the lowest point of the mass parabola (the nuclide with the highest value of binding energy) for each isobar in this study.

Also we can show from Fig. 1 that the value of binding energy is increased by increasing the atomic number (isobars decay by emitting \(\beta^-\) particles) until reaching to the most stable isobar, and then decreased due to distance from the most stable isobar (isobars decay by emitting \(\beta^+\) particles).
Fig.1. Mass parabolas for isobars with mass number (A=152,154,156,158,160&162).

To plot mass parabolas for isobars with mass number (A=152-162), the values of binding energy (B.E) plotted as a function to the atomic number (Z) for every isobaric family, thus we grow mass parabolas for isobars under this study as shown in Fig.1.

From this figure we can see clearly that the mass parabola in this range of isobars (A=152-162) un equivalent i.e. the number of isobars which decay by (β+) emission or electron capture (e.c.) greater than isobars which decay by (β-) emission. The reason of that the weighty nuclides have an extra number of protons.

To determine the effect of beta decay stability on the shape of the isobar, plotting the shape of nuclides in (3-D) for each isobar in isobaric families with even mass number (A=152 - 162) have been done, from calculated the values of semi-axis minor (a), major (b) and (c) ellipsoid axis's, i.e. change the degree of deformation of nuclei as a result of the beta emission, the shapes of all nuclides were displayed in Figs. (2-7) respectively.

The semi-axis minor (a), major (b) and (c) ellipsoid axis's are calculated by using equations (2&3) with help of mean-squared charge distribution radius average< r^2 >, the quadruple deformation parameter(δ), intrinsic quadruple moment(Q₀), and reduced transition probability B(E2) ↑ where the values of these parameters are calculated from equations (4-7) respectively. The values of these parameters presented in Table 1, while the values of the semi-axis minor (a), major (b) and (c) ellipsoid axis's represented in Table 2.
Table 1. Nuclei symbols-mass number A, atomic number Z, neutron number N, decay mode, mean square radii \(< r^2 >\), root mean square radii \(< r^2 >^{1/2}\), reduced transition probabilities \(B(E2) \uparrow e^b\), quadrupole moment \((Q_0)\) and deformation parameter \((\delta)\) for even-even isobaric elements with even mass number \((A=152 - 162)\).

| Nucleus   | Z  | \(E_m\) Kev | decay mode | \(< r^2 >^{1/2}\) | \(\delta\) |
|-----------|----|-------------|------------|----------------|--------|
| Ce-152    | 58 | 81.71 β^-  | Most stable| 3.7          | 6.4     |
| Nd-152    | 60 | 72.51 β^-  | Most stable| 4.58          | 6.49    |
| Sm-152    | 62 | 121.7       | Most stable| 5.0819        | 4.9606  |
| Gd-152    | 64 | 344.2 β^+  | Most stable| 5.0774        | 4.9606  |
| Dy-152    | 66 | 613.8 β^+  | Most stable| 5.0950        | 4.9606  |
| Er-152    | 68 | 808.2 β^+  | Most stable| 5.0843        | 4.9606  |
| Yb-152    | 70 | 1531. β^-   | Most stable| 5.0423        | 4.9606  |
| Nd-154    | 60 | 70.81 β^-   | Most stable| 4.58          | 4.9823  |
| Sm-154    | 62 | 81.97 β^-   | Most stable| 5.1053        | 4.9823  |
| Gd-154    | 64 | 123.0       | Most stable| 5.1223        | 4.9823  |
| Dy-154    | 66 | 334.5 β^+  | Most stable| 5.1241        | 4.9823  |
| Er-154    | 68 | 560 β^+    | Most stable| 5.1129        | 4.9823  |
| Yb-154    | 70 | 821.3 β^+  | Most stable| 5.0875        | 4.9823  |
| Hf-154    | 72 | 1513 β^-    | Most stable| 5.031         | 4.9823  |
| Nd-156    | 60 | 66.91 β^-   | Most stable| 4.88          | 25.0378 |
| Sm-156    | 62 | 75.89 β^-   | Most stable| 4.58          | 5.0037  |
| Gd-156    | 64 | 88.96       | Most stable| 5.1420        | 5.0037  |
| Dy-156    | 66 | 137.8 β^+  | Most stable| 5.1622        | 5.0037  |
| Er-156    | 68 | 344.5 β^+  | Most stable| 5.1429        | 5.0037  |
| Yb-156    | 70 | 536.4 β^+  | Most stable| 5.1219        | 5.0037  |
| Hf-156    | 72 | 858 β^-    | Most stable| 0.53          | 5.0037  |
| Sm-158    | 62 | 72.81 β^-   | Most stable| 4.68          | 5.0037  |
| Gd-158    | 64 | 79.51       | Most stable| 5.1569        | 5.0037  |
| Dy-158    | 66 | 98.91 β^+  | Most stable| 5.1815        | 5.0037  |
| Er-158    | 68 | 192.1 β^+  | Most stable| 5.1761        | 5.0037  |
| Yb-158    | 70 | 358.2 β^-  | Most stable| 5.1498        | 5.0037  |
| Hf-158    | 72 | 476.3 β^-  | Most stable| 0.96          | 5.0037  |
| Sm-160    | 62 | 70.61 β^-   | Most stable| 4.78          | 5.0037  |
| Gd-160    | 64 | 75.26 β^-   | Most stable| 5.1734        | 5.0037  |
| Dy-160    | 66 | 86.78       | Most stable| 5.1951        | 5.0037  |
| Er-160    | 68 | 125.8 β^+  | Most stable| 5.2045        | 5.0037  |
| Yb-160    | 70 | 243.1 β^-  | Most stable| 5.1781        | 5.0037  |
| Hf-160    | 72 | 389.6 β^-  | Most stable| 1.16          | 5.0037  |
| Gd-162    | 64 | 71.7 β^-   | Most stable| 5.01          | 5.0037  |
| Dy-162    | 66 | 80.66       | Most stable| 5.2074        | 5.0037  |
Table 2. Nuclide symbols- mass number A, atomic number Z, neutron number N, semi-axes minor (a), major (b) and (c) ellipsoid axis’s for even-even isobaric elements with even mass number (A=152 - 162).

| Nuclide-A | Z  | N  | Present work | a fm | b fm | c fm |
|-----------|----|----|--------------|-----|-----|-----|
| Ce-152    | 58 | 94 |              | 4.832671 | 8.736734 | 4.832671 |
| Nd-152    | 60 | 92 |              | 4.718888 | 8.860257 | 4.718888 |
| Sm-152    | 62 | 90 |              | 5.152556 | 8.363151 | 5.152556 |
| Gd-152    | 64 | 88 |              | 5.6693025 | 7.63013 | 5.639025 |
| Dy-152    | 66 | 86 |              | 5.879667 | 7.341593 | 5.879667 |
| Er-152    | 68 | 84 |              | 5.949685 | 7.227893 | 5.949685 |
| Yb-152    | 70 | 82 |              | 6.077364 | 7.012221 | 6.077364 |
| Nd-154    | 60 | 94 |              | 4.741906 | 8.896378 | 4.741906 |
| Sm-154    | 62 | 92 |              | 4.880475 | 8.745217 | 4.880475 |
| Gd-154    | 64 | 90 |              | 5.200611 | 8.368045 | 5.200611 |
| Dy-154    | 66 | 88 |              | 5.716962 | 7.664827 | 5.716962 |
| Er-154    | 68 | 86 |              | 5.869696 | 7.402986 | 5.869696 |
| Yb-154    | 70 | 84 |              | 5.985538 | 7.243173 | 5.985538 |
| Hf-154    | 72 | 82 |              | 6.106263 | 7.038749 | 6.106263 |
| Nd-156    | 60 | 96 |              | 4.731159 | 8.967801 | 4.731159 |
| Sm-156    | 62 | 94 |              | 4.854387 | 8.835102 | 4.854387 |
| Gd-156    | 64 | 92 |              | 4.995248 | 8.676646 | 4.995248 |
| Dy-156    | 66 | 90 |              | 5.315129 | 8.287822 | 5.315129 |
| Er-156    | 68 | 88 |              | 5.762289 | 7.666892 | 5.762289 |
| Yb-156    | 70 | 86 |              | 5.907375 | 7.442784 | 5.907375 |
| Hf-156    | 72 | 84 |              | 6.027212 | 7.248076 | 6.027212 |
| Sm-158    | 62 | 96 |              | 4.859911 | 8.889288 | 4.859911 |
| Gd-158    | 64 | 94 |              | 4.941045 | 8.799379 | 4.941045 |
| Dy-158    | 66 | 92 |              | 5.122705 | 8.589103 | 5.122705 |
| Er-158    | 68 | 90 |              | 5.542384 | 8.051141 | 5.542384 |
| Yb-158    | 70 | 88 |              | 5.81032 | 7.664024 | 5.81032 |
| Hf-158    | 72 | 86 |              | 5.904734 | 7.518319 | 5.904734 |
| Sm-160    | 62 | 98 |              | 4.875051 | 8.932406 | 4.875051 |
| Gd-160    | 64 | 96 |              | 4.934808 | 8.866536 | 4.934808 |
| Dy-160    | 66 | 94 |              | 5.05928 | 8.725107 | 5.05928 |
| Er-160    | 68 | 92 |              | 5.333826 | 8.391706 | 5.333826 |
| Yb-160    | 70 | 90 |              | 5.689984 | 7.910012 | 5.689984 |
| Hf-160    | 72 | 88 |              | 5.872835 | 7.630846 | 5.872835 |
| Gd-162    | 64 | 98 |              | 4.933368 | 8.927636 | 4.933368 |
| Dy-162    | 66 | 96 |              | 5.039177 | 8.808651 | 5.039177 |
| Er-162    | 68 | 94 |              | 5.227157 | 8.586709 | 5.227157 |
| Yb-162    | 70 | 92 |              | 5.540321 | 8.184657 | 5.540321 |
| Hf-162    | 72 | 90 |              | 5.791067 | 7.829816 | 5.791067 |
| W-162     | 74 | 88 |              | 5.952148 | 7.584379 | 5.952148 |

Figs. (2-7) are a three-dimensional representation graphics for the isobar in isobaric family with even mass number (A=152 -162), these figures illustrate the change of the shape for a nucleus with emission of ($\beta^-, \beta^+$).
Fig. 2. Shape of even-even isobars $A=152$ and $(Z=58, 60, 62, 64, 66, 68 & 70)$.

Fig. 3. Shape of even-even isobars $A=154$ and $(Z=60, 62, 64, 66, 68, 70 & 72)$.

Fig. 4. Shape of even-even isobars $A=156$ and $(Z=60, 62, 64, 66, 68, 70 & 72)$.
From Figs. (2-7) which represent a three-dimensional shapes for the isobaric elements with even mass number (A=152 - 162) respectively, we distinguished that the shape of nuclides which is represented the most stable isobar in mass parabola (isobaric nuclide with highest value of binding energy) as a result of its decay by emitting ($\beta^-$, $\beta^+$) particle and electron capture, are not spherical but they have some deformation in its shape. This is due to the fact that the nuclei are not stable by emitting beta particles only (particle loss happens first from radioactive nuclides, maximum of the nucleus excitation energy was a loss if one particle emission, whereas only a minor quantity of angular momentum have been lost in this particle emission. So, gamma-ray release happens to discharge this angular momentum and the residual energy. Only (8–10) MeV are passed out from the excitation energy by every particle produced. So a band of gamma rays are observed until the nucleus be in ground state (stable)).[14] So the most stable isobar needs to emit a gamma ray to move the nucleus from the excited state to the ground state and change its shape to spherical shape.
While depending on Liquid Drop Model (LDM), isobars nuclei under study, which are considered the most stable isobar among the other isobars in same isobaric family because they have the largest binding energy, are not completely stable and therefore do not have a spherical shape. Also we can note from these figures that the deformation of isobar nuclei decreases with increasing atomic number, as well as for nuclei that have a magic number such as \((^{152}\text{Yb},^{154}\text{Hf})\) neutron magic number equal to \(\mathrm{N}=82\).

Finally, our results show that the stable nuclide which are determined from mass parabola \((^{70}\text{Yb},^{72}\text{Hf},^{76}\text{Hf},^{76}\text{Hf},^{72}\text{Hf},^{72}\text{Hf})\) for isobaric elements with even mass number \(\mathrm{A}=152 - 162\) respectively.

### 4. Conclusions

The results show that the shape of nuclides which is represented the most stable isobar in mass parabola are not spherical but they have some deformation in their shape, and need to emit a gamma ray to a chive more stable status and have the perfect spherical shape.

The stable nuclides which are determined from mass parabola are different from the nuclides which have less value of deformation.

### References

1. Nageshwari M. 2014 *Indian J. Sci. Technol.* 7(s5) 974
2. Nayyef M. S., Jarallah N. T. 2019 *AIP Conference Proceedings* 2190
3. Marid H. A., Jarallah N. T. 2019 *Energy Procedia* 157 270
4. Povh B., Rith K., Scholz C. Zetsche M. F. 2008 *Particles and Nuclei An Introduction to the Physical Concepts Sixth Edition Springer-Verlag Berlin Heidelberg*
5. Turner J. E. 2007 *Atoms radiation and radiation protection WILEY-VCH Verlag GmbH &Co.KGaA, Weinheim*
6. Ankita & Suthar B. 2016 *AIP Conf. Pro* 1728, 020024 ; doi: 10.1063/1.4946074.
7. Ghahramany N., Sarafraza H., Yazdankish E. 2013 *Universal Journal of Physics and Application* 7(1)8
8. Cook N.D. 2010 Models of the atomic nucleus Unification through a lattice of Nucleons *second Edition Springer-Verlag Berlin Heidelberg* ; DOI 10.1007/978-3-642-14737-1
9. Nayyef M. S., Jarallah N. T. 2020 *Ibn Al-Haitham J. for Pure & Appl. Sci.* 33 (4) 18
10. Buchmann J.A., Henley M. E. 2000 *Intrinsic Quadrupole Moment of the Nucleon Phys. Rev. C 63, 015202*
11. Boboshin I., Ishkhanov B., Komarov S., Orlin1 V., Peskov N., V. VarlamovV. 2007 *Investigation of Quadrupole Deformation of Nucleus and its Surface Dynamic Vibrations International Conference on Nuclear Data for Science and Technology*, Moscow, Russia : 65, doi:10.1051/ndata:07013
12. S. Raman, C. W. Nestor, JR., & P. Tikkanen. 2001 Transition probability From The Ground to The First -Excited 2+ state of Even- Even Nuclides, Atomic Data and Nuclear Data Tables 78 (1) : doi:10.1006/adnt.2001.0858
13. Angeli I., Marinova K.P. 2013 *Table of Experimental Nuclear Ground State Charge Radii, Atomic Data and Nuclear Data Tables* 99 69,doi:10.1016/j.adt.2011.12.006
14. Agular A. 2008 high spin nuclear structure of \(^{168,170}\text{Ta}\) and triaxial strongly deformed structure in \(^{160}\text{Yb}\) *Ph.D. Thesis Florida State University, College of Arts and Science*