Correction

Correction: Cabrera Martínez et al. On the Secure Total Domination Number of Graphs. Symmetry 2019, 11, 1165

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The authors wish to make the following corrections on paper [1]:

(1) Eliminate Lemma 1 because we have found that this lemma is not correct.
(2) Theorem 3 states that for any graph G with no isolated vertex,

$$\gamma(st(G)) \leq \alpha(G) + \gamma(G).$$

The result is correct, but the proof uses Lemma 1. For this reason, we propose the following alternative proof for Theorem 3.

Proof. Let D be a \(\gamma(G)\)-set. Let I be an \(\alpha(G)\)-set such that \(|D \cap I|\) is at its maximum among all \(\alpha(G)\)-sets. Notice that for any \(x \in D \cap I\),

$$\text{epn}(x, D \cup I) \cup \text{ipn}(x, D \cup I) \subseteq \text{epn}(x, I).$$

We next define a set \(S \subseteq V(G)\) of minimum cardinality among the sets satisfying the following properties.

(a) \(D \cup I \subseteq S\).
(b) For every vertex \(x \in D \cap I\),

\[
\begin{align*}
(b1) & \text{ if } \text{epn}(x, D \cup I) \neq \emptyset, \text{ then } S \cap \text{epn}(x, D \cup I) \neq \emptyset; \\
(b2) & \text{ if } \text{epn}(x, D \cup I) = \emptyset, \text{ ipn}(x, D \cup I) \neq \emptyset \text{ and } \text{epn}(x, D \cup I) \neq \emptyset, \text{ then either } \text{epn}(x, I) \setminus D = \emptyset \text{ or } S \cap \text{epn}(x, I) \setminus D = \emptyset; \\
(b3) & \text{ if } \text{epn}(x, D \cup I) = \emptyset \text{ and } \text{epn}(x, I) = \emptyset, \text{ then } S \cap N(\text{epn}(x, I)) \setminus \{x\} \neq \emptyset; \\
(b4) & \text{ if } \text{ipn}(x, D \cup I) = \emptyset, \text{ then } N(x) \setminus (D \cup I) = \emptyset \text{ or } S \cap N(x) \setminus (D \cup I) \neq \emptyset.
\end{align*}
\]

Since \(D\) and \(I\) are dominating sets, from (a) and (b) we conclude that \(S\) is a TDS. From now on, let \(v \in V(G) \setminus S\). Observe that there exists a vertex \(u \in N(v) \cap I \subseteq N(v) \cap S\), as \(I \subseteq S\) is an \(\alpha(G)\)-set. To conclude that \(S\) is a STDS, we only need to prove that \(S' = (S \setminus \{u\}) \cup \{v\}\) is a TDS of \(G\).

First, notice that every vertex in \(V(G) \setminus N(u)\) is dominated by some vertex in \(S'\), because \(S\) is a TDS of \(G\). Let \(w \in N(u)\). Now, we differentiate two cases with respect to vertex \(u\).

Case 1. \(u \in I \setminus D\). If \(w \notin D\), then there exists some vertex in \(D \subseteq S'\) which dominates \(w\), as \(D\) is a dominating set. Suppose that \(w \in D\). If \(w \in \text{ipn}(u, D \cup I)\), then \(I' = (I \cup \{w\}) \setminus \{u\}\) is an \(\alpha(G)\)-set such that \(|D \cap I'| > |D \cap I|\), which is a contradiction. Hence, \(w \notin \text{ipn}(u, D \cup I)\), which implies that there exists some vertex in \((D \cup I) \setminus \{u\}\) \(\subseteq S'\) which dominates \(w\).

Case 2. \(u \in I \cap D\). We first suppose that \(w \notin D\). If \(w \notin \text{epn}(u, D \cup I)\), then \(w\) is dominated by some vertex in \((D \cup I) \setminus \{u\}\) \(\subseteq S'\). If \(w \in \text{epn}(u, D \cup I)\), then by (b1) and the fact that in this case all vertices in \(\text{epn}(u, D \cup I)\) form a clique, \(w\) is dominated by some vertex in...
From now on, suppose that \( w \in D \). If \( w /\in ipn(u, D \cup I) \), then there exists some vertex in \( (D \cup I) \setminus \{u\} \subseteq S' \) which dominates \( w \). Finally, we consider the case in that \( w \in ipn(u, D \cup I) \).

We claim that \( ipn(u, D \cup I) = \{w\} \). In order to prove this claim, suppose that there exists \( w' \in ipn(u, D \cup I) \setminus \{w\} \). Notice that \( w' \in D \). By (1) and the fact that all vertices in \( epn(u, I) \) form a clique, we prove that \( uw' \in E(G) \), and so \( w /\in ipn(u, D \cup I) \), which is a contradiction. Therefore, \( ipn(u, D \cup I) = \{w\} \) and, as a result,

\[
epn(u, D \cup I) \cup \{w\} \subseteq epn(u, I).
\] (2)

In order to conclude the proof, we consider the following subcases.

Subcase 2.1. \( epn(u, D \cup I) \neq \emptyset \). By (2), (b1), and the fact that all vertices in \( epn(u, I) \) form a clique, we conclude that \( w \) is adjacent to some vertex in \( S \setminus \{u\} \subseteq S' \), as desired.

Subcase 2.2. \( epn(u, D \cup I) = \emptyset \) and \( epn(u, I) \setminus \{w\} \neq \emptyset \). By (2), (b2), and the fact that all vertices in \( epn(u, I) \) form a clique, we show that \( w \) is dominated by some vertex in \( S \setminus \{u\} \subseteq S' \), as desired.

Subcase 2.3. \( epn(u, D \cup I) = \emptyset \) and \( epn(u, I) = \{w\} \). In this case, by (b3) we deduce that \( w \) is dominated by some vertex in \( S \setminus \{u\} \subseteq S' \), as desired.

According to the two cases above, we can conclude that \( S' \) is a TDS of \( G \), and so \( S \) is a STDS of \( G \). Now, by the the minimality of \( |S| \), we show that \( |S| \leq |D \cup I| + |D \cap I| = |D| + |I| \). Therefore, \( \gamma_{st}(G) \leq |S| \leq |I| + |D| = \alpha(G) + \gamma(G) \), which completes the proof. \( \square \)

The authors would like to apologize for any inconvenience caused to the readers by these changes. The changes do not affect the scientific results.

Reference

1. Cabrera Martínez, A.; Montejano, L.P.; Rodríguez-Velázquez, J.A. On the secure total domination number of graphs. *Symmetry* 2019, 11, 1165. [CrossRef]

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