TNT equivalent of the shock wave energy generated during the supersonic flight of an aircraft

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Abstract. To assess the impact on human health of the sonic boom that occurs when an aircraft is flying at supersonic speed, and, accordingly, to solve the problem of noise reduction by optimizing the aircraft design, it is proposed to evaluate the shock wave energy using the TNT equivalent of a cylindrical explosion. An example of calculating the shock wave energy during flights of F4 and F18 aircraft at different altitudes is considered. To calculate the evolution of an acoustic pulse during its propagation from the boundary of the shock wave transition to the acoustic one, the wave equation and its solution are used, taking into account the inhomogeneity of the atmosphere, nonlinear effects, absorption and expansion of the wave front, as well as the results of ground-based measurements of acoustic pulses. The results of calculations of the dependence of the explosion energy on the flight altitude, as well as on the type of aircraft are explained on the basis of the formula for the atmospheric resistance force.

1. Introduction

The big noise caused by sonic boom when the flight vehicle flying supersonically will not only has an influence on human lives but also bring a destroy to the constructions, especially for the infrasonic boom. The impact of sonic boom on human lives is estimated in decibels, and the impact of shock waves from various sources on the destruction of structures is usually estimated based on the calculation of the TNT equivalent of the shock wave energy. For example, the power of the explosion of the Chelyabinsk meteoroid was determined at 1-3 kt TNT [1]. The noise level on the earth's surface of a civil supersonic aircraft Concorde flying at the altitude of 15 km is 133 dB, while the noise level of a civil aircraft during takeoff and landing is only about 90 dB [2]. Thus, the Concord was forbidden to fly over the continent supersonically because of the high sonic boom level, which played down the economy of Concord. The noise level of next generation supersonic transport is demanded lower than 70 dB, which is comparative with the transonic civil aircraft. Thus mitigating the sonic boom is an exigent problem for next generation supersonic transports development.

Sonic boom assessment methods through numerical analysis have been ever-evolving since the development of the fundamental theory. Salah et al. [3] reviewed the theory for both near and far from the aircraft, as well as various approaches to solving the problem and their approximations. The pressure disturbances generated by the supersonic aircraft, which length L is settled as the reference dimension, propagate through the atmosphere over hundreds of body lengths before reaching the ground form what is called sonic boom. The dimensions of L can be different, and in addition, as the sound...
boom propagates to the earth’s surface, the dominant physical processes that form the sound boom change. Three regions have been identified by theory: the near-field, the mid-field, and the far-field. Most acknowledged accurate methods for sonic boom prediction are based on this three-layer decomposition. The near-field corresponds to a region within some body lengths, where the nonviscous aerodynamic small perturbation theory applies far from the boundary layer region. The energy absorption of acoustic waves is not taken into account. On the other hand, when acoustic waves propagate through the far zone (tens of kilometers to the ground), the predominant influence of nonlinear phenomena is taken into account. Zone 2 should provide a natural transition between the results of calculations using aerodynamic theory and nonlinear acoustics. The disadvantage of this approach is that it does not provide a description of the features of the evolution of acoustic pulses in the middle zone, and assumes that acoustic and aerodynamic effects between the near and far fields can be calculated separately and added to each other. In this case, when forming the input data for calculations in the far field, the pressure perturbation is determined on a cylindrical surface described around an aircraft aligned with the flow direction. The matching allows making the pressure signature compliant with the acoustical assumption of local axisymmetry field, which is necessary to nonlinear acoustic propagation theory and ensured with an adequate choice of the distance from the aircraft. The signal correction due to the matching has a significant impact on the resulting ground signature as it increases its length as well as the shock amplitude and all the shocks have coalesced.

The last step of the pressure disturbances propagation toward the ground is the acoustic propagation of the matched signal using the inviscid nonlinear acoustic theory. It uses a ray-tracing approach to account for the refraction phenomena occurring during the propagation through a stratified atmosphere with vertical temperature, cross wind, and density gradients, to evaluate the extent of the “primary carpet,” i.e., the width of the corridor underneath the aircraft trajectory directly affected by sonic boom. To calculate the evolution of the acoustic shock pressure form along each acoustic beam, a first-order correction to the linear supersonic theory developed by Whitham [4] is used. The pressure waveform remains unchanged along the beam, with the exception of changes in amplitude, governed by the Blokhintsev invariant [5].

Cikulin [6] was the first to show that shock waves formed near bodies moving in a gas at a high supersonic speed can also be considered as the result of a cylindrical explosion. In 2003, Drozbheva and Krasnov [7] derived a wave equation and found its solution for describing the propagation of acoustic waves in an inhomogeneous atmosphere, taking into account nonlinear phenomena, absorption and expansion of the wave front for the case of a point explosion. The solution is not limited to approximations of geometric acoustics, and allows us to calculate the evolution of the shape of an acoustic pulse as it propagates in the atmosphere. In 2003, Drozbheva, Krasnov and Sokolova [8] developed this theory for the case of a cylindrical explosion. The purpose of this work is to use this theory to estimate the TNT equivalent of shock waves that occur during supersonic flight of aircraft.

2. Materials and methods

To determine the initial pressure perturbation at the boundary of the shock wave transition to the acoustic one, the empirical formula is used [6]:

$$\frac{\Delta p_u}{p_0} = \frac{k_2 k_3 (\cos \theta)^{1/2}}{r_2^{1/2} r_1^{1/2} (r_2^{1/2} - 0.7)^{1/2}}$$

where \(\Delta p_u\) is the overpressure; \(r_2 = r/\Lambda\) is the dimensionless radius of the shock wave; \(\Lambda = (E/p_0)^{1/2} = M_a d_r\) is the scale of a cylindrical explosion for bodies with a hemispherical head in the air; \(d_r\) is the diameter of the cylinder; \(M_a = V_r/c_0\) is the Mach number; \(V_r\) is the speed of the aircraft; \(r\) is the radius of the wave front; \(p_0\) is the background atmospheric pressure; \(K_2 = 0.4\) is the asymptotic coefficient; \(c_0\) is the speed of sound.

The duration of the positive compression phase is calculated by the formula:

$$t_u = \frac{\Lambda (\gamma + 1) k_2 (r_2^{1/2} - 0.7)^{1/2}}{2 \gamma c_0 (\cos \theta)^{1/2}}$$

where \(\gamma = c_p/c_v\) - the ratio of specific heat capacities at constant pressure and volume. It is assumed that the shock wave becomes weak at a distance from the axis of the cylinder \(R_2 = 3\). In this case [9] \(c_0/D \approx 0.9\), where \(D\) is the velocity of the shock wave front, and this val-
ue $c_0/D$ corresponds to the excess pressure $\Delta p_u/p_0 = 0.27$. The time of propagation of the shock wave from the axis of the cylinder to $R_z$, is calculated by the formula $t_0 = \tau t^0$, where $t^0 = \Lambda \sqrt{\rho_0/p_0}$ - dynamic time; $\tau$ - dimensionless value; $\rho_0$ - the density of the surrounding atmosphere. For $R_z = 3$ the value $\tau = 1.9$.

To describe the time form of the initial pulse $P'(t)$, the expression was used [8]:

$$
\begin{cases}
P'(t) = \Delta p_u \left(1 - t/t_u\right) \left(1 - \left(t/t_g\right)^2\right) \quad 0 \leq t \leq 0.4 t_g, \\
P'(t) = kt + d \to 0.4 t_g, \quad t \leq \tau_s, \\
P'(t) = 0 \to t > \tau_s,
\end{cases}
$$

where $\tau_s$ is the time for which the area of the refraction wave becomes equil to the area of the compression, $d = P'(0.4 t_g) - 0.4 k t_g$ and $k = \frac{d \rho}{d t}$ at $t = 0.4 t_g$. The relationship between the pressure perturbation and the velocity $v(t)$ of the motion of hydrodynamic particles under the action of an acoustic wave is determined by the formula $P'(t) = \rho_0 v(t) c$.

The calculation of the characteristics of acoustic waves during their propagation from the initial wave front to the earth's surface is determined by the formula: $v(t) = \frac{b(z)}{c v(z, p_0, c_0)} \frac{d}{d t} (\ln U)$, where $\varepsilon = \frac{\eta + 1}{2}$; $b = \left(\zeta + \frac{4}{3} \eta\right) + \left(\frac{1}{c_p} - \frac{1}{c_v}\right)$; $\chi$; and $\eta$ are the bulk and shear viscosities, $\chi$ is the thermal conductivity; $U(z, t_p) = \frac{1}{2 q v/\eta} \int_0^\infty U_{in}(0, t') \exp\left[-\frac{(t-t')^2}{4 q^2}\right] dt'$; $q^2 = \int \frac{b}{2 \rho_0 c_0} d z$; the propagation time of an acoustic wave is determined by the expression $t_p = t + \int \frac{f(z)}{c_0} d z$, where

$$
f(z) = \left(1 - \frac{3 a + d}{2 \rho_0 \frac{d c_0}{d z}} - \frac{3 a}{2 \rho_0 \frac{d \rho_0}{d z}}\right), \quad a = \zeta + \frac{4}{3} \eta; \quad d = \left(\frac{1}{c_p} - \frac{1}{c_v}\right) \chi.
$$

The function $U_{in}$ describes the initial shape of the acoustic signal at the initial height $z_{in}$, which can be obtained from the initial velocity profile $v_{in}$ as $\ln U_{in} = \frac{\varepsilon c_0 \rho_0}{b} \int V_{in} d t$.

### 3. Results and discussions

Measurements of sonic boom during flight of supersonic aircraft were carried out in the United States from July 31 to August 7, 1987 and published in papers [10,11]. Each aircraft flew in a straight line at a constant supersonic speed at different altitudes and at different Mach numbers. To measure the sonic boom, rows of sensors were used, located both perpendicularly and parallel to the flight path of the aircraft. At the same time, radar control of each aircraft flight and atmospheric parameters was provided on the basis of radio-wind probes and ground measurements in the area of flight routes. For the calculations, we used the results of measurements of acoustic pulses recorded by sensors located directly under the flight path of the F-4 and F-18 supersonic aircraft. Aircraft flight data for selected experiments are presented in table 1.

| Type of aircraft | Date  | Registration Time, UTC | Mach number | Height, km | Latitude, ° | Longitude, ° |
|------------------|-------|------------------------|-------------|------------|-------------|--------------|
| F-4              | 3.08.87| 17:29:59               | 1.1         | 4.389      | 35.178      | 118.59       |
| F-4              | 31.07.87 | 15:11:20             | 1.2         | 4.876      | 35.156      | 118.60       |
| F-4              | 3.08.87 | 14:48:33               | 1.2         | 8.900      | 35.190      | 118.60       |
| F-18             | 8.08.87 | 15:10:36               | 1.1         | 4.267      | 35.163      | 118.60       |
| F-18             | 6.08.87 | 14:44:12               | 1.3         | 9.144      | 35.190      | 118.59       |
| F-18             | 8.08.87 | 14:57:05               | 1.4         | 13.716     | 35.188      | 118.59       |
The conditions in the atmosphere play an important role in the propagation of an acoustic wave. To obtain data at altitudes, radio-wind probes were launched three times a day, and ground measurements of atmospheric parameters were carried out after each recording of a sonic boom. For the experiments presented in Table 1, in [11], the profiles of atmospheric parameters are given only for 3 and 8 August. To carry out model calculations of the acoustic field, a special program was created in which the international model NRLMSIS00 [12] was used to calculate the atmospheric profiles. To determine the calculation errors for this model, a comparison was made between the calculation results and experimental data for August 3 and 8. For example, figure 1 shows the results of comparing the profiles for August 3.

![Figure 1](image1.png)

**Figure 1.** Results of comparison of model and experimental temperature and pressure profiles for August 3 1987, 1 – model, 2 – experimental data.

One can see a good agreement of the pressure profiles, and a slight difference in the temperature profiles. Similar results were obtained for three other cases.

In the model calculations of acoustic pulses, the inverse problem was solved, namely: an increasing value of the explosion energy of a cylindrical charge of a unit length was consistently set, and then an acoustic pulse on the earth's surface was calculated. When the model and experimental pulses coincided on the Earth's surface, a decision was made about the magnitude of the explosion energy. Figure 2 shows the results of calculations of pulses on the Earth's surface during supersonic flight of the F-4 aircraft and for the F-18 aircraft.

![Figure 2](image2.png)
Figure 2. Model acoustic pulses and pulses measured on the Earth's surface during supersonic flight of aircraft at different altitudes. 1 – model, 2 – experimental data.

The initial acoustic pulses at the boundary of the transition of a shock wave to an acoustic one during supersonic flights of F-4 and F-18 aircraft have the same N-shaped shape, and differ only in amplitude and duration; for example, figure 3a shows one of the pulses.

The pulse amplitude during the flight of the F-4 at altitudes of 4.389 km, 4.876 km and 8.9 km, respectively, was equal to: 37 Pa, 36.7 Pa and 34.8 Pa, and the duration was 0.05 s, 0.04 s and 0.05 s. The pulse amplitude during the flight of the F-18 at altitudes of 4.267 km, 9.144 km and 13.716 km, respectively, was 37 Pa, 34.5 Pa and 32.6 Pa, and the duration for all cases was about 0.04 s. The results of calculating the equivalent energy of a cylindrical charge of a unit length from the flight altitude of a supersonic aircraft are shown in figure 3b.

The shock wave is the main source of resistance experienced by a body moving at supersonic speed. Therefore, the observed regularities can be explained on the basis of the formula for the drag force of the atmosphere $F = \rho V^2 SC_D/2$, where $V$ is the velocity of the body relative to the atmosphere, $S$ is the cross-sectional area of the midsection, and $C_D$ is the aerodynamic drag coefficient. In particular, the decrease in the charge energy with an increase in the flight altitude of the aircraft is explained by a lower density value and, accordingly, less resistance of the atmosphere with an increase in altitude. The lower charge energy for the F-18 aircraft is also explained by the lower resistance of the atmosphere, which in this case is associated with a smaller mid-section value.
It can be assumed that most of the energy of the equivalent cylindrical charge \( E \) released per unit length \( dl \) is spent on work performed by the atmospheric drag force. In this case, you can write \( E \approx \frac{1}{2} \rho_0 V^2 S \cdot C_D dl \). Whence, knowing \( \rho_0 \) and \( V \), it is easy to determine the value of \( SC_D \). Obviously, at \( S=\text{const} \), it becomes possible to determine the dependence of the aerodynamic coefficient \( C_D \) on the speed \( V \).

4. Summary
It is proposed to use the TNT equivalent of a cylindrical explosion to estimate the energy of a shock wave that occurs during a supersonic flight of an aircraft. An example of calculating the shock wave energy for flights of F-4 and F-18 aircraft at different altitudes is considered. To calculate the evolution of an acoustic pulse during its propagation from the boundary of the shock wave transition to the acoustic one, the wave equation and its solution are used, taking into account the inhomogeneity of the atmosphere, nonlinear effects, absorption and expansion of the wave front, as well as the results of ground-based measurements of acoustic pulses. The solution is not limited to approximations of geometric acoustics. The results of calculations of the dependence of the explosion energy on the flight altitude, as well as on the type of aircraft, are explained on the basis of the formula for the atmospheric resistance force. The energy value of an equivalent cylindrical charge of unit length (1 m) varies in the range from 0.155 to 0.96 kg.

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