Nonequilibrium thermal Casimir–Polder forces

S Y Buhmann and S Scheel

Quantum Optics and Laser Science, Blackett Laboratory, Imperial College London, Prince Consort Road, London SW7 2BW, UK
E-mail: s.buhmann@imperial.ac.uk

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Abstract

We study the nonequilibrium Casimir–Polder force on an atom prepared in an incoherent superposition of internal energy eigenstates, which is placed in a magnetoelectric environment of nonuniform temperature. After solving the coupled atom–field dynamics within the framework of macroscopic quantum electrodynamics, we derive a general expression for the thermal Casimir–Polder force.

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1. Introduction

The thermal fluctuations of the electromagnetic field present at finite temperature may interact with a single atom or molecule, resulting in the thermal Casimir–Polder (CP) force. It has been studied theoretically via linear response theory [1], the Lifshitz theory [2], quantum electrodynamics (QED) [3] and heuristic generalizations of the zero-temperature result [4] and it has been studied experimentally by spectroscopic means [5].

Thermal CP forces in nonequilibrium scenarios have recently received increased attention, where two cases can be distinguished. A nonequilibrium environment can be realized when an atom interacts with a body whose temperature differs from the environment temperature [6]. Alternatively, situations have been considered where the environment is at global thermal equilibrium but with a uniform temperature, where or only two of the atoms present are not fully thermalized with their local environment [7, 8]. In this paper, we generalize these two special cases to the full nonequilibrium situation of an atom in a nonequilibrium state placed within an arbitrary magnetoelectric environment of nonuniform temperature. We use macroscopic QED (recalled in section 2) to calculate the CP force from the thermal average of the quantum Lorentz force (section 3) followed by a short summary (section 4).

2. Macroscopic QED at finite temperature

Consider an environment of dispersive and absorbing magnetoelectrics of (relative) electric permittivity \(\varepsilon(r, \omega)\) and magnetic permeability \(\mu(r, \omega)\), which are both satisfying the Kramers–Kronig relations. This environment together with its electromagnetic field can be characterized by a Hamiltonian

\[ H_F = \sum_{\lambda=e,m} \int d^3r \int_0^\infty d\omega \, \hbar \omega \hat{f}_\lambda^\dagger(r, \omega) \cdot \hat{f}_\lambda(r, \omega) \]

where the bosonic variables \(\hat{f}_\lambda^\dagger\) and \(\hat{f}_\lambda\) are associated with the noise polarization \((\lambda = e)\) and magnetization \((\lambda = m)\) of the media. When a single atom or molecule with Hamiltonian

\[ \hat{H}_A = \sum_n E_n |n\rangle \langle n| \] (where \(E_n\) are eigenenergies and \(|n\rangle\) are molecular eigenstates) is placed at position \(r_A\) in this environment, it interacts with the electromagnetic field via an electric-dipole Hamiltonian

\[ \hat{H}_{AF} = - \sum_{m,n} d_{mn} \cdot \hat{E}(r_A) \hat{A}_{mn} \] (where \(d_{mn} = \langle m | d | n \rangle\) and \(\hat{A}_{mn} = |m\rangle \langle n|\)), so that the total Hamiltonian of the system reads

\[ \hat{H} = \hat{H}_A + \hat{H}_F + \hat{H}_{AF}. \]

The electric field can be expressed in terms of the dynamical variables according to

\[ \hat{E}(r) = \int_0^\infty d\omega \, \hat{E}(r, \omega) + \text{H.c.} \]

\[ = \int_0^\infty d\omega \sum_{\lambda=e,m} \int d^3r' \mathbf{G}_\lambda(r, r', \omega) \cdot \hat{f}_\lambda(r', \omega) + \text{H.c.} \] (1)

with the coefficients \(\mathbf{G}_\lambda\) being related to the classical Green tensor, \(\mathbf{G}\), by

\[ \mathbf{G}_e(r, r', \omega) = \frac{i}{\epsilon c^3} \sqrt{\frac{\hbar}{\pi \varepsilon_0}} \text{Im} \varepsilon(r', \omega) \mathbf{G}(r, r', \omega), \] (2)

\[ \mathbf{G}_m(r, r', \omega) = \frac{i}{\mu c \sqrt{\pi \varepsilon_0}} \text{Im} \mu(r', \omega) \left| \frac{\mathbf{v} \times \mathbf{G}(r', r, \omega)}{\mu(r', \omega) c} \right|^2. \] (3)
The Green tensor is the unique solution to the Helmholtz equation

\[
\left[ \nabla \times \frac{1}{\mu(r, \omega)} \nabla \times -\frac{\omega^2}{c^2} \epsilon(r, \omega) \right] G(r, r', \omega) = \delta(r-r')
\]

(4)

with \( G(r, r', \omega) \to 0 \) for \( |r-r'| \to \infty \), where the above definitions imply

\[
\sum_{k,\omega,m} \int d^3{s} \mathbf{G}_k(r, s, \omega) \cdot \mathbf{G}_m^\dagger(r', s, \omega) = \frac{\hbar \mu_0}{\pi} \omega^2 \text{Im} G(r, r', \omega).
\]

(5)

For a nonuniform temperature \( T=T(r) \), the thermal state of the environment is described by \( \hat{\rho}_T=\text{Tr}[\hat{\mathcal{H}}_T(r)/[k_B T(r)]]/\text{Tr}(\exp[-\hat{\mathcal{H}}_T(r)/k_B T(r)]) \) [where \( \hat{\mathcal{H}}_T=\int d^3r \hat{\mathcal{H}}_F(r) \) and \( k_B \) is the Boltzmann constant]. The relevant nonvanishing thermal averages of the dynamical variables are thus given by

\[
\{ \hat{f}_k(r, \omega) \hat{f}^\dagger_k(r', \omega') \} = n_T(r, \omega) \delta(r-r') \delta(\omega-\omega'),
\]

(6)

\[
\{ \hat{f}_k(r, \omega) \hat{f}^\dagger_k(r', \omega') \} = [n_T(r, \omega)+1] \delta(r-r') \delta(\omega-\omega'),
\]

(7)

where \( n_T(r, \omega) = 1/\exp[\hbar \omega/k_B T(r)]-1 \) is the average thermal photon number.

3. The thermal CP force

The thermal CP force on an atom prepared in an incoherent superposition of internal energy-eigenstates can in electric-dipole approximation be found from the average Lorentz force [10, 11] \( \mathbf{F}(r, I) = \{ \nabla \times \hat{\mathbf{E}}(r) \}_{r=r_{\text{eq}}} \). To evaluate this expression, one needs to solve the coupled atom-field dynamics. Using the Hamiltonian given in section 2, one finds

\[
\hat{\mathcal{A}}_{mn} = i\omega_{mn} \hat{A}_{mn} + \frac{i}{\hbar} \sum_k (d_{nk} \hat{A}_{mk} - d_{km} \hat{A}_{nk}) \cdot \hat{\mathbf{E}}(r, \omega).
\]

(8)

\[
\hat{f}_k(r, \omega) = -i\omega \hat{f}_k(r, \omega) + \frac{i}{\hbar} \sum_{m,n} d_{mn} \mathbf{G}_k^\dagger(r, r, \omega) \hat{A}_{mn}.
\]

(9)

We eliminate the field by formally integrating the second of these equations and substituting the result back into the first one, which we arrange in normal ordering. After invoking the integral relation (5), one obtains

\[
\hat{\mathcal{A}}_{mn}(t) = i\omega_{mn} \hat{A}_{mn}(t) + \frac{i}{\hbar} \sum_k \int_0^\infty \text{d}\omega \left\{ e^{i\omega t} \hat{E}^\dagger(\mathbf{r}, \omega) \right\} \times [d_{nk} \hat{A}_{mk}(t) - d_{km} \hat{A}_{nk}(t)] + e^{-i\omega t} [\hat{A}_{mk}(t) d_{nk} - \hat{A}_{kn}(t) d_{mk}]
\]

\[
- \hat{A}_{kn}(t) d_{km} \cdot \hat{\mathbf{E}}(r, \omega) + \hat{Z}_{mn}(t),
\]

(10)

with

\[
\hat{Z}_{mn}(t) = \frac{\hbar \mu_0}{2\pi} \sum_{kj} \int_0^\infty \text{d}\omega \omega^2 \text{Im} G(r, r, \omega) \times [d_{kn} \hat{A}_{mk}(t) - d_{km} \hat{A}_{nk}(t)] + e^{i\omega t} \hat{A}_{kl}(t) [d_{jl} \hat{A}_{mk}(t) - d_{jm} \hat{A}_{nk}(t)] + e^{-i\omega t} \hat{A}_{kl}(t) [d_{jm} \hat{A}_{mk}(t) - d_{jm} \hat{A}_{nk}(t)]
\]

(11)

denoting the zero-point contribution to the internal atomic dynamics. The thermal contribution can be determined iteratively by substituting the self-consistent solution

\[
\hat{\mathcal{A}}_{mn}(t) = e^{i\omega_{mn} t} \hat{A}_{mn} + \frac{i}{\hbar} \sum_k \int_0^\infty \text{d}\omega \int_0^t \text{d}t' e^{i\omega(t-t')} [\hat{A}_{mk}(t) d_{nk} - \hat{A}_{kn}(t) d_{mk}]
\]

\[
\times \hat{\mathbf{E}}(r, \omega) e^{-i\omega t} + \text{H.c.}
\]

(12)

to the truncated equation (10) [without \( \hat{Z}_{mn}(t) \)] back into equation (10) and taking thermal expectation values with the aid of equations (6) and (7). This leads to a closed system of equations \( \hat{\mathcal{T}}(\hat{\mathcal{A}}_{mn}) = i\omega_{mn} \hat{\mathcal{A}}_{mn} + \{ \hat{Z}_{mn}(t) + \hat{T}_{mn}(t) \} \) with a thermal contribution

\[
\{ \hat{T}_{mn}(t) \} = -\frac{1}{\hbar} \sum_k \int_0^\infty \text{d}\omega \int_0^\infty \text{d}^3 r n_T(r, \omega) \times \int_0^t \text{d}t' [e^{-i\omega(t-t')} + e^{i\omega(t-t')}] \times [e^{i\omega_{mn}(t-t')} d_{nk} \cdot \mathbf{G}_k(\mathbf{r}, \omega) \cdot \mathbf{G}_m^\dagger(\mathbf{r}, \omega)]
\]

\[
\times \{ d_{kl} [\hat{A}_{mk}(t) - d_{jm} \hat{A}_{nk}(t)] - e^{i\omega_{mn}(t-t')} d_{kn} \cdot \mathbf{G}_k(\mathbf{r}, \omega) \cdot \mathbf{G}_m^\dagger(\mathbf{r}, \omega) \}
\]

(13)

Assuming the atom–field coupling to be sufficiently weak, we can apply the Markov approximation by writing \( \langle \hat{\mathcal{A}}_{mn}(t) \rangle \simeq e^{-i\omega_{mn}(t-t')} \langle \hat{\mathcal{A}}_{mn}(t) \rangle \) and allowing the lower limit of the time integrals to tend to minus infinity, so that \( \int_0^t \text{d}t' e^{i\omega(t-t')} \simeq \pi \delta(x) + i\pi/\omega \). For a nondegenerate system, the off-diagonal elements of the (internal) atomic density matrix \( \hat{\sigma} \) decouple from each other as well as from the diagonal ones, and one finds that internal atomic dynamics follow the rate equations

\[
\sigma_{mn}(t) = -\Gamma_m \sigma_{mn} + \sum_k \Gamma_{nk} \sigma_{kn}(t),
\]

(14)

\[
\sigma_{mn}(t) = -i \omega_{mn} + \frac{1}{2} (\Gamma_m + \Gamma_n)/2 \sigma_{mn}(t)
\]

for \( m \neq n \) (15)

(15)

where \( \sigma_{mn} = \langle m | \hat{\sigma} | n \rangle = \langle \hat{\mathcal{A}}_{mn} \rangle \). As follows from equations (11) and (13), the total loss rates read

\[
\Gamma_n = \sum_k \Gamma_{nk}
\]

(14)

\[
= \frac{2\mu_0}{\hbar} \sum_k \omega_{nk} \Theta(\omega_{nk}) d_{nk} \cdot \text{Im} G(r, r, |\omega_{nk}|)
\]

\[
= \frac{2\mu_0}{\hbar} \sum_k \omega_{nk} \Theta(\omega_{nk}) d_{nk} \cdot \text{Im} G(r, r, |\omega_{nk}|)
\]

\[
= \frac{2\mu_0}{\hbar} \sum_k \omega_{nk} \Theta(\omega_{nk}) d_{nk} \cdot \text{Im} G(r, r, |\omega_{nk}|)
\]
Using the Markov approximation in the form $\langle \hat{A}_{mn}(t) \hat{A}_{lm}(\tau) \rangle = \delta_{nk}(\delta_{ml}(t-\tau))$, where $\Omega_{mn} = \omega_{mn} + i(\Gamma_m + \Gamma_n)/2$, which follows from equation (15) via the quantum regression theorem [12], one obtains

$$F(r_A, t) = \frac{i}{\hbar} \sum_{n,k} \int_0^\infty d\omega \sum_{k,e,m} \int d^3 s \nabla d_{nk} \times \mathbf{G}_e(r, s, \omega) \cdot \mathbf{G}_m^T(r_A, \omega) \cdot d_{nk} f_{r=r_A}$$

where $\mathbf{G}_e(r, s, \omega) = \mathbf{D}(r, s, \omega) + i(\omega - \Omega_{nk})$.

Using the Markov approximation in the form $\int_0^\infty d\omega \sum_{k,e,m} \int d^3 s \nabla d_{nk} \times \mathbf{G}_e(r, s, \omega) \cdot \mathbf{G}_m^T(r_A, \omega) \cdot d_{nk} f_{r=r_A}$, the thermal CP force for an atom prepared in an incoherent superposition of internal energy-eigenstates is given by $F(r_A, t) = \sum_n \sigma_{ma}(t) F_n(r_A)$ with force components

$$F_n(r_A) = \frac{1}{\hbar} \sum_{k,e,m} \int_0^\infty d\omega \sum_{k,e,m} \int d^3 s \nabla d_{nk} \times \mathbf{G}_e(r, s, \omega) \cdot \mathbf{G}_m^T(r_A, \omega) \cdot d_{nk} f_{r=r_A} + c.c.$$
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