Dynamical symmetry breaking in Abelian geometrodynamics

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Abstract. A new tetrad is introduced within the framework of geometrodynamics for non-null electromagnetic fields. This tetrad diagonalizes the Einstein-Maxwell stress-energy tensor, any stress-energy tensor in a local and covariant way, and allows for maximum simplification of the expression of the electromagnetic field, in any curved four-dimensional Lorentzian spacetime, allowing for the identification of its degrees of freedom in two local scalars. New isomorphisms are proved. A new internal-spacetime mapping is established using these new tetrads. It is possible to map the local group of electromagnetic gauge transformations into the transformation groups of tetrad vectors on two local orthogonal planes. The planes that diagonalize the stress-energy tensor. We will discuss through a first order perturbative formulation the local loss of symmetry when a source of electromagnetic and gravitational field interacts with an agent that perturbs the original geometry associated to the source. The loss of symmetry will be manifested by the tilting of these planes under the influence of an external agent. In this strict sense the original local symmetry will be lost, however a new symmetry will arise. The purpose of this report is to show that the geometrical manifestation of local gauge symmetries is dynamic.

1. Introduction
A new tetrad is introduced within the framework of geometrodynamics for non-null electromagnetic fields. This tetrad diagonalizes the Einstein-Maxwell stress-energy tensor, any stress-energy tensor in a local and covariant way, and allows for maximum simplification of the expression of the electromagnetic field, in any curved four-dimensional Lorentzian spacetime, allowing for the identification of its degrees of freedom in two local scalars [1-4]. The Einstein-Maxwell equations will also be simplified. New isomorphisms are proved. The local group of electromagnetic gauge transformations is isomorphic to the new groups LB1 and LB2, independently. A new internal-spacetime mapping is established using these new tetrads [2, 3]. It is possible to map the local group of electromagnetic gauge transformations into the transformation groups of tetrad vectors on two local orthogonal planes. The planes that diagonalize the stress-energy tensor. LB1 is the group of local tetrad transformations comprised by SO(1,1) plus two different kinds of discrete transformations. One of these two discrete transformations is the full inversion or minus the identity two by two. The other discrete transformation is not Lorentzian because it is a reflection or flip with zeroes in the diagonal and ones off-diagonal also two by two. The local group of electromagnetic gauge transformations is isomorphic to the local group of tetrad transformations LB2 on the orthogonal local plane [1] Former Professor at Universidad de la República, Av. 18 de Julio 1824-1850, 11200 Montevideo, Uruguay.
as well, independently. LB2 is SO(2). These group results amount to proving that the no-go
theorems of the sixties like the S. Coleman and J. Mandula, the S. Weinberg or L. O’ Raifeartagh
versions are incorrect [5-7]. Not because of their internal logic, but because of the assumptions
made at the outset of all these versions. The explicit isomorphic link between the Abelian local
“internal” electromagnetic gauge transformations and the local tetrad transformations on special
orthogonal local planes is manifest evidence of these incorrect assumptions as has been proved.
Simply because the Lorentz transformations on a local plane in a four-dimensional curved
Lorentzian spacetime do not commute with Lorentz transformations on a different local plane in
general, element of contradiction with the no-go theorems assumptions. We will discuss through
a first order perturbative formulation the local loss of symmetry when a source of electromagnetic
and gravitational field interacts with an agent that perturbs the original geometry associated to
the source [8, 9]. It has already been proved that the local gauge groups are isomorphic to local
groups of transformation of special tetrads. These tetrads define at every point in spacetime
two orthogonal planes or blades such that every vector in these local planes is an eigenvector
of the Einstein-Maxwell stress-energy tensor. As the local gauge symmetry in Abelian or even
non-Abelian field structures in four-dimensional Lorentzian spacetimes is displayed through the
existence of local planes of symmetry that we will refer to as blades one and two, the loss of
symmetry will be manifested by the tilting of these planes under the influence of an external
agent. In this strict sense the original local symmetry will be lost. We will be able to prove in this
way that the new planes or blades at the same point will correspond “after the tilting generated
by perturbation” to a new symmetry. The purpose of this report is to show that the geometrical
manifestation of local gauge symmetries is dynamic. Despite the fact that the local original
symmetries will be lost, new symmetries will arise. A dynamic evolution of local symmetries
will be manifested. This result will produce a new theorem on dynamic symmetry evolution.
This new classical model will be useful in order to better understand anomalies in quantum
field theories. These new tetrads are useful in astrophysics spacetime evolution algorithms since
they introduce maximum simplification in all relevant objects, specially in stress-energy tensors
making the evolution relativistic differential equations simpler [10-12].

2. Diagonalization of the Einstein-Maxwell stress-energy tensor
Throughout the paper we use the conventions of reference [1]. In particular we use a metric
with sign conventions -+++. We will call our geometrized electromagnetic potential \( A_\mu \), where
\( f_{\mu \nu} = A_\nu \partial^\mu - A_\mu \partial^\nu \) is the geometrized electromagnetic field \( f_{\mu \nu} = (G^{1/2}/c^2) F_{\mu \nu} \). The stress-energy
tensor according to equation (14a) in reference [1], can be written as,

\[
T_{\mu \nu} = f_{\mu \lambda} f_{\nu}^{\lambda} + f^*_{\mu \lambda} * f_{\nu}^{\lambda},
\]

where \( * f_{\mu \nu} = \frac{1}{2} \epsilon_{\mu \nu \sigma \tau} f^{\sigma \tau} \) is the dual tensor of \( f_{\mu \nu} \), see section Appendix A for the object \( \epsilon_{\mu \nu \sigma \tau} \). The local duality rotation given by equation (59) in paper [1]
\( f_{\mu \nu} = \xi_{\mu \nu} \cos \alpha + * \xi^{\mu \nu} \sin \alpha \), allows to express the stress-energy tensor in terms of the extremal field \( T_{\mu \nu} = \xi_{\mu \lambda} \xi_{\nu}^{\lambda} + * \xi_{\mu \lambda} * \xi_{\nu}^{\lambda} \). We can write the extremal field as,

\[
\xi_{\mu \nu} = e^{-\nu \alpha} f_{\mu \nu} = \cos \alpha f_{\mu \nu} - \sin \alpha * f_{\mu \nu}.
\]

Extremal fields are local electromagnetic gauge invariants as it can be noticed from equation (2). Extremal fields satisfy the equation

\[
\xi_{\mu \nu} * \xi^{\mu \nu} = 0.
\]
gauge invariant, can be given when imposing condition (3) on equation (2), by the expression
\[
\tan(2\alpha) = -\frac{f_{\mu\nu} \ast f^{\mu\nu}}{f_{\lambda\rho} \ast f^{\lambda\rho}}.
\]
We have also available the general identity,
\[
A^{\mu\alpha} B^{\nu\alpha} - \ast B^{\mu\alpha} \ast A^{\nu\alpha} = \frac{1}{2} \delta_{\mu}^{\nu} A_{\alpha\beta} B^{\alpha\beta},
\]
which is valid for every pair of antisymmetric tensors in a four-dimensional Lorentzian spacetime \[1\]. When applied to the case \(A^{\mu\alpha} = \xi^{\mu\alpha}\) and \(B^{\nu\alpha} = \ast \xi^{\nu\alpha}\), it is straightforward to prove that condition (3) yields the equivalent condition,
\[
\xi_{\alpha\mu} \ast \xi^{\mu\nu} = 0.
\]
(5)

The extremal field \(\xi_{\mu\nu}\) and the scalar complexion \(\alpha\) have been previously defined through equations (22)-(25) in reference \[1\]. It is our purpose to find a tetrad that diagonalizes the Einstein-Maxwell stress-energy tensor. This tetrad will simplify the analysis of the geometrical properties of the electromagnetic field as we will see. There are four tetrad vectors that at every point in spacetime diagonalize the stress-energy tensor in geometrodynamics,

\[
\begin{align*}
V^{\alpha}_{(1)} &= \xi^{\alpha\lambda} \xi_{\rho\lambda} X^{\rho} & (6) \\
V^{\alpha}_{(2)} &= \sqrt{-\frac{Q}{2}} \xi^{\alpha\lambda} X_{\lambda} & (7) \\
V^{\alpha}_{(3)} &= \sqrt{-\frac{Q}{2}} \ast \xi^{\alpha\lambda} Y_{\lambda} & (8) \\
V^{\alpha}_{(4)} &= \ast \xi^{\alpha\lambda} \ast \xi_{\rho\lambda} Y^{\rho} & (9)
\end{align*}
\]

where \(Q = \xi_{\mu\nu} \xi^{\mu\nu} = -\sqrt{T_{\mu\nu} T^{\mu\nu}}\) according to equations (39) in manuscript \[1\]. \(Q\) is assumed not to be zero, because we are dealing with non-null electromagnetic fields. Non-null we clarify means basically that \(f_{\mu\nu} f^{\mu\nu} \neq 0\) and \(\ast f_{\mu\nu} f^{\mu\nu} \neq 0\). In turn and by definitions these last equations imply that \(\xi_{\mu\nu} \xi^{\mu\nu} \neq 0\). We are free to choose the vector fields \(X^{\alpha}\) and \(Y^{\alpha}\), as long as the four vector fields (6)-(9) do not become trivial. Two equations in the extremal field will be used extensively in this work, in particular, to prove that tetrad (6)-(9) diagonalizes the stress-energy tensor. The first equation is given by (64) in \[1\], also given in equation (5). When we replace \(A^{\mu\alpha} = \xi^{\mu\alpha}\) and \(B^{\nu\alpha} = \ast \xi^{\nu\alpha}\) in equation (4), the second identity is found,

\[
\xi_{\mu\alpha} \xi^{\nu\alpha} - \ast \xi_{\mu\alpha} \ast \xi^{\nu\alpha} = \frac{1}{2} \delta_{\mu}^{\nu} Q.
\]
(10)

When we make iterative use of (5) and (10) we find,

\[
\begin{align*}
V^{\alpha}_{(1)} T^{\beta}_{\alpha} &= \frac{Q}{2} V^{\beta}_{(1)} & (11) \\
V^{\alpha}_{(2)} T^{\beta}_{\alpha} &= \frac{Q}{2} V^{\beta}_{(2)} & (12) \\
V^{\alpha}_{(3)} T^{\beta}_{\alpha} &= -\frac{Q}{2} V^{\beta}_{(3)} & (13) \\
V^{\alpha}_{(4)} T^{\beta}_{\alpha} &= -\frac{Q}{2} V^{\beta}_{(4)} & (14)
\end{align*}
\]

In paper \[1\] the stress-energy tensor was diagonalized through the use of a Minkowskian frame in which the equation for this tensor was given in equations (34) and (38). In this work, we give the explicit expression for the tetrad in which the stress-energy tensor is diagonal. The freedom we have to choose the vector fields \(X^{\alpha}\) and \(Y^{\alpha}\), represents available freedom that we have to
choose the tetrad. If we make use of equations (5) and (10), it is straightforward to prove that (6)-(9) is a set of orthogonal vectors. Vectors (6)-(7) define or span the local plane or blade one. Vectors (8)-(9) define or span the local orthogonal plane or blade two. Let us introduce some names. The tetrad vectors have two essential components. For instance in vector $V^\alpha_{(1)}$, there are two main structures. First, the skeleton, in this case $\xi^{\alpha\lambda} \xi_\rho^\lambda$ which is a local electromagnetic gauge invariant as can be noticed from equation (2), and second, the gauge vector $X^\alpha$ which is gauge. The gauge vectors it was proved in manuscript [2, 4] could be anything that does not make the tetrad vectors trivial.

3. Electromagnetic potentials in Einstein-Maxwell geometrodynamics

Our goal is to simplify as much as we can the expression of the electromagnetic field through the use of an orthonormal tetrad, so its geometrical properties can be seen in an easier way. As it was mentioned above we would like to show this simplification through an explicit example by making a convenient and particular choice of the gauge vector fields $X^\alpha$ and $Y^\alpha$. In geometrodynamics, the Maxwell equations, $f^{\mu\nu} = 0$ and $*f^{\mu\nu} = 0$ say that two potential vector fields exist [13], $f_{\mu\nu} = A_{\nu\mu} - A_{\mu\nu}$ and $*f_{\mu\nu} = *A_{\nu\mu} - *A_{\mu\nu}$. For instance, in the Reissner-Nordström geometry the only non-zero electromagnetic tensor component is $f_{rr} = A_{t,t} - A_{t,r}$ and its dual $*f_{\theta\phi} = *A_{\phi\theta} - *A_{\theta\phi}$. The symbol “;” stands for covariant derivative with respect to the metric tensor $g_{\mu\nu}$ and the star in $*A_\nu$ is just a name, not the dual operator, meaning that $*A_{\nu\mu} = (\ast A_\nu)_\mu$. The vector fields $A^\alpha$ and $*A^\alpha$ represent a possible choice in geometrodynamics for the vectors $X^\alpha$ and $Y^\alpha$. It is not meant that the two vector fields have independence from each other, it is just a convenient choice for a particular example. A further justification for the choice $X^\alpha = A^\alpha$ and $Y^\alpha = *A^\alpha$ could be illustrated through the Reissner-Nordström geometry. In this particular geometry, $f_{rr} = \xi_{rr}$ and $*f_{\theta\phi} = *\xi_{\theta\phi}$, therefore, $A_\theta = 0$ and $A_\phi = 0$. Then, for the last two tetrad vectors (8)-(9), the choice $Y^\alpha = *A^\alpha$ becomes meaningful under the light of this particular extreme case, when basically there is no magnetic field. The choice $Y^\alpha = A^\alpha$ would make in the Reissner-Nordström geometry the vectors (8)-(9) to become zero and this would be meaningless.

4. Electromagnetic gauge transformations on blades one and two

Once we made the choice $X^\alpha = A^\alpha$ and $Y^\alpha = *A^\alpha$ the question about the geometrical implications of electromagnetic gauge transformations of the tetrad vectors (6)-(9) arises. We first notice that a local electromagnetic gauge transformation of the “gauge vectors” $X^\alpha = A^\alpha$ and $Y^\alpha = *A^\alpha$ can be just interpreted as a new choice for the gauge vectors $X_\alpha = A_\alpha + \Lambda_\alpha$ and $Y_\alpha = *A_\alpha + *\Lambda_\alpha$. When we make the transformation, $A_\alpha \rightarrow A_\alpha + \Lambda_\alpha$, $f_{\mu\nu}$ remains invariant, and the transformation, $*A_\alpha \rightarrow *A_\alpha + *\Lambda_\alpha$, leaves $*f_{\mu\nu}$ invariant, as long as the functions $\Lambda$ and $*\Lambda$ are local scalars. It is valid to ask how the tetrad vectors (6)-(7) will transform under $A_\alpha \rightarrow A_\alpha + \Lambda_\alpha$ and (8)-(9) under $*A_\alpha \rightarrow *A_\alpha + *\Lambda_\alpha$. Schouten defined what he called, a two-bladed structure in a spacetime [14]. These local blades or planes are the planes determined by the pairs $(V^\alpha_{(1)}, V^\beta_{(2)})$ and $(V^\alpha_{(3)}, V^\beta_{(4)})$.

Given the space constraint in these presentation we will limit ourselves to show a few illustrative results as far as tetrad transformations for gauge vector choice given by electromagnetic gauge transformations. The whole analysis is given in manuscript [2]. In order to simplify the notation we will write $\Lambda_\alpha = \Lambda$. First we study the change in (6)-(7) under $A_\alpha \rightarrow A_\alpha + \Lambda_\alpha$. Using the following notation, $C = (-Q/2) V^\alpha_{(1)} \Lambda^\sigma/( V^\beta_{(2)} V^\beta_{(2)} )$ and $D = (-Q/2) V^\alpha_{(2)} \Lambda^\sigma/( V^\beta_{(1)} V^\beta_{(1)} )$, several cases arise on blade one. We would like to calculate the norm of the transformed vectors $\tilde{V}^\alpha_{(1)}$ and $\tilde{V}^\alpha_{(2)}$.

$$\tilde{V}^\alpha_{(1)} \tilde{V}^\alpha_{(1)} = [(1 + C)^2 - D^2] V^\alpha_{(1)} V^\alpha_{(1)}$$

(15)
\begin{align}
\tilde{V}_\alpha^{(2)} \tilde{V}_\alpha^{(2)\alpha} &= \left[(1 + C)^2 - D^2\right] V_\alpha^{(2)\alpha} \, ,
\end{align}

where the relation \( V_\alpha^{(1)\alpha} V_\alpha^{(1)\alpha} = - V_\alpha^{(2)\alpha} V_\alpha^{(2)\alpha} \) has been used and \( V_\alpha^{(1)\alpha} \) assumed timelike for simplicity. In order for these transformations to keep the timelike or spacelike character of \( V_\alpha^{(1)\alpha} \) and \( V_\alpha^{(2)\alpha} \) the condition \( [(1 + C)^2 - D^2] > 0 \) must be satisfied. If this condition is fulfilled, then we can normalize the transformed vectors \( \tilde{V}_\alpha^{(1)\alpha} \) and \( \tilde{V}_\alpha^{(2)\alpha} \) as follows,

\begin{align}
\frac{\tilde{V}_\alpha^{(1)\alpha}}{\sqrt{-V^{\beta\beta}_{\alpha}(V^{(1)\beta}_{\alpha})}} &= \frac{(1 + C)}{\sqrt{(1 + C)^2 - D^2}} \frac{V_\alpha^{(1)\alpha}}{-V^{\beta\beta}_{\alpha}(V^{(1)\beta}_{\alpha})} + \frac{D}{\sqrt{(1 + C)^2 - D^2}} \frac{V_\alpha^{(2)\alpha}}{-V^{\beta\beta}_{\alpha}(V^{(2)\beta}_{\alpha})} \quad \text{(17)}
\frac{\tilde{V}_\alpha^{(2)\alpha}}{\sqrt{V^{\beta\beta}_{\alpha}(V^{(2)\beta}_{\alpha})}} &= \frac{D}{\sqrt{(1 + C)^2 - D^2}} \frac{V_\alpha^{(1)\alpha}}{-V^{\beta\beta}_{\alpha}(V^{(1)\beta}_{\alpha})} + \frac{(1 + C)}{\sqrt{(1 + C)^2 - D^2}} \frac{V_\alpha^{(2)\alpha}}{-V^{\beta\beta}_{\alpha}(V^{(2)\beta}_{\alpha})} \quad \text{(18)}
\end{align}

The condition \( [(1 + C)^2 - D^2] > 0 \) allows for two possible situations, \( 1+C > 0 \) or \( 1+C < 0 \), the proper transformations on blade one. For the particular case when \( 1+C > 0 \), the transformations (17)-(18) are telling us that an electromagnetic gauge transformation on the vector field \( A_\alpha \), that leaves invariant the electromagnetic field \( f_{\mu\nu} \), generates a boost transformation on the normalized tetrad vector fields \( \left( \frac{V_\alpha^{(1)\alpha}}{-V^{\beta\beta}_{\alpha}(V^{(1)\beta}_{\alpha})}, \frac{V_\alpha^{(2)\alpha}}{-V^{\beta\beta}_{\alpha}(V^{(2)\beta}_{\alpha})} \right) \). The case \( 1+C < 0 \), represents the composition of two transformations. An inversion of the normalized tetrad vector fields \( \left( \frac{V_\alpha^{(1)\alpha}}{-V^{\beta\beta}_{\alpha}(V^{(1)\beta}_{\alpha})}, \frac{V_\alpha^{(2)\alpha}}{-V^{\beta\beta}_{\alpha}(V^{(2)\beta}_{\alpha})} \right) \), and a boost. If the case is that \( [(1 + C)^2 - D^2] < 0 \), the vectors \( V_\alpha^{(1)\alpha} \) and \( V_\alpha^{(2)\alpha} \) will change their timelike or spacelike character,

\begin{align}
\tilde{V}_\alpha^{(1)\alpha} &= \left[-(1 + C)^2 + D^2\right](-V_\alpha^{(1)\alpha}) \quad \text{(19)}
\tilde{V}_\alpha^{(2)\alpha} &= \left[-(1 + C)^2 + D^2\right]V_\alpha^{(2)\alpha} \quad \text{(20)}
\end{align}

These are improper transformations on blade one. They have the property of being a composition of boosts and a discrete transformation given by \( \Lambda_\alpha^\mu = 0, \Lambda_\alpha^1 = 1, \Lambda_\alpha^2 = 1, \Lambda_\alpha^3 = 0 \). We notice that this discrete transformation is not a Lorentz transformation because it is a reflection. They might also be composed with an inversion, see reference [2] for the whole analysis. On blade or plane two, the choice \( Y_\alpha = *A_\alpha + *\Lambda_\alpha \) induces just local spatial rotation tetrad vector transformations. We reiterate that local tetrad electromagnetic gauge transformations can be interpreted as new or different gauge choices \( X_\alpha = A_\alpha + \Lambda_\alpha \) and \( Y_\alpha = *A_\alpha + *\Lambda_\alpha \).

5. Group Isomorphisms

We will just limit ourselves to state these new theorems proved in detail in reference [2].

**Theorem 1** The mapping between the local group of electromagnetic gauge transformations and the local group LB1 defined above is isomorphic.

**Theorem 2** The mapping between the local group of electromagnetic gauge transformations and the local group LB2 defined above is isomorphic.
6. Orthonormal tetrad
Then, at the points in spacetime where the set of four vectors (6)-(9) is not trivial, we can proceed to normalize,

\[ U^\alpha = \frac{\xi^\alpha \xi_{\rho \lambda} A^\rho}{\sqrt{Q/2} \sqrt{A_{\mu} \xi_{\mu \sigma} \xi_{\nu \sigma} A^\nu}} \]  
\[ V^\alpha = \frac{\xi^\alpha A_\lambda}{\sqrt{A_{\mu} \xi_{\mu \sigma} \xi_{\nu \sigma} A^\nu}} \]  
\[ Z^\alpha = *\xi^\alpha A_\lambda / (\sqrt{*A_{\mu} \xi_{\mu \sigma} \xi_{\nu \sigma} A^\nu}) \]  
\[ W^\alpha = *\xi^\alpha \xi_{\rho \lambda} A^\rho / (\sqrt{-Q/2} \sqrt{*A_{\mu} \xi_{\mu \sigma} \xi_{\nu \sigma} A^\nu}) \]  

In analogy with the electromagnetic case and without altering anything fundamental we assume for simplicity that \( A_{\mu} \xi_{\mu \sigma} \xi_{\nu \sigma} A^\nu > 0 \). \( *A_{\mu} \xi_{\mu \sigma} \xi_{\nu \sigma} A^\nu > 0 \) and \(-Q = \sqrt{T_{\mu \nu} T^{\mu \nu}} > 0\). See sections Appendix A and Appendix B for more details about this tetrad. The four vectors (21)-(24) have the following algebraic properties,

\[ -U^\alpha U_\alpha = V^\alpha V_\alpha = Z^\alpha Z_\alpha = W^\alpha W_\alpha = 1 \]  

Any other scalar product is zero. It is possible to find expressions for the metric tensor and the stress-energy tensor in the new tetrad (21)-(24). The new expression for the metric tensor is given by,

\[ g_{\alpha \beta} = -U_\alpha U_\beta + V_\alpha V_\beta + Z_\alpha Z_\beta + W_\alpha W_\beta \]  

The stress-energy tensor can be written in terms of the new tetrad (21)-(24) as,

\[ T_{\alpha \beta} = (Q/2) [-U_\alpha U_\beta + V_\alpha V_\beta - Z_\alpha Z_\beta - W_\alpha W_\beta] \]  

In order to find the expression for the electromagnetic field in terms of the tetrad (21)-(24), it is necessary to find some previous results. The extremal field tensor and its dual can be written, see reference [2],

\[ \xi_{\alpha \beta} = -2 \sqrt{-Q/2} U_{[\alpha} V_{\beta]} \]  
\[ *\xi_{\alpha \beta} = 2 \sqrt{-Q/2} Z_{[\alpha} W_{\beta]} \]  

Equations (28)-(29) are providing the necessary information to express the electromagnetic field \( f_{\mu \nu} = \xi_{\mu \nu} \cos \alpha + *\xi_{\mu \nu} \sin \alpha \) in terms of the new tetrad,

\[ f_{\alpha \beta} = -2 \sqrt{-Q/2} \cos \alpha \ U_{[\alpha} V_{\beta]} + 2 \sqrt{-Q/2} \sin \alpha \ Z_{[\alpha} W_{\beta]} \]  

We then proceed to apply all these geometrical elements to the Reissner-Nordström case with the choice \( X^\rho = A^\rho \) and \( Y^\rho = *A^\rho \), where the symbol * in this particular last case is not the Hodge operator but a particular nomenclature [8, 9, 15]. The line element for this spacetime is given by \( ds^2 = -(1 - \frac{2M}{r} + \frac{2q^2}{r^2}) dt^2 + (1 - \frac{2M}{r} + \frac{q^2}{r^2})^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \). In the standard spherical coordinates \( t, r, \theta, \phi \) the only non-zero components for the potentials will be \( A_t = -q/r \) and \( A_\phi = -q \cos \theta \). With these potentials we find that the only non-zero components for the electromagnetic tensor \( f_{\mu \nu} = A_{\nu \mu} - A_{\mu \nu} \) and its Hodge dual \( *f_{\mu \nu} = *A_{\nu \mu} - *A_{\mu \nu} \) are \( f_{tr} = -q/r^2 \) and \( *f_{t \phi} = q \sin \theta \). The symbol ; stands for covariant derivative with respect to the metric tensor \( g_{\mu \nu} \), in our case the Reissner-Nordström geometry. It is easy to check that the only non-zero components of the extremal field and its dual are \( \xi_{tr} = f_{tr} \) and \( *\xi_{t \phi} = *f_{t \phi} \). We
proceed again to write explicitly the only non-zero components of vectors (21)-(24) which will be useful when determining the geometric location of all conserved energy-momentum currents,

\[ U^\alpha = -\sqrt{q^2/q}/\sqrt{1 - \frac{2m}{r} + \frac{q^2}{r^2}} \] (31)

\[ V^r = \sqrt{1 - \frac{2m}{r} + \frac{q^2}{r^2}} \] (32)

\[ Z^\theta = -\sqrt{\cos^2 \theta/(r \sin \theta)} \] (33)

\[ W^\phi = -\sqrt{q^2 \cos^2 \theta/(q r \sin \theta \cos \theta)} \] (34)

In this particular coordinate system we would have to be careful because both vectors \( V^\alpha \) before normalizing would be zero at the coordinate value \( \theta = \pi/2 \). As the purpose of this section is not to find suitable coordinate coverings but to show that the conserved currents are vectors inside either blade one or two, we proceed to exhibit these currents and conserved charges taken from reference [16]. In reference [16] the energy-momentum currents are defined as \( T_{j}^{\alpha \beta \gamma} \xi_{\beta} \) for \( j : 1 \cdots 4 \), where the vectors \( \xi_{\beta} \) are Killing vector fields. These Killing vectors are defined by \( \xi_{1} = (1, 0, 0, 0) \), \( \xi_{2} = (0, \sin \theta, \cos \phi \cot \theta, 0) \), \( \xi_{3} = (0, \cos \phi, -\sin \phi \cot \theta, 0) \) and \( \xi_{4} = (0, 0, 0, 1) \). The corresponding conserved currents are then,

\[ J(\xi_1) = \frac{q^2}{r^4} \xi_1 \] (35)

\[ J(\xi_a) = \frac{q^2}{r^4} \xi_a \] (36)

Index \( a \in 2, 3, 4 \). The main objective of this section is to show that the conserved currents are located or belong to either blade one or blade two. We can now check comparing equations (35)-(36) with equations (31)-(34) that the conserved current \( J(\xi_1) \) belongs or is in the plane determined by the vectors \( (U^\alpha, V^\alpha) \), that is blade one, and the other three conserved currents \( J(\xi_a) \) for \( a : 2 \cdots 4 \) lie in the orthogonal plane or blade two determined by \( (Z^\alpha, W^\alpha) \). The conserved charges are calculated exactly as in reference [16]. The only one that is non-zero corresponds to the current vector inside blade one and it is given in the case where \( m^2 > q^2 \) and \( r_+ = m + \sqrt{m^2 - q^2} \) by the value \( Q_1 = 4 \pi q^2/r_+ \). Therefore we have proven our point which states that the four energy-momentum conserved currents belong to either blade one or two. From reference [16] we can easily see that the Bel currents [16-19] are also inside these blades. This finding should not be surprising, see section Appendix C. A vector inside blade one is invariant under local gauge transformations through the vector \( Y^\rho = \ast A^\rho \), and a vector inside blade two is invariant under local gauge transformations through the vector \( X^\rho = A^\rho \). This is because local Lorentz transformations on blade two do not affect the orthogonal blade one, and vice versa. All geometrical constructions presented in this section will help visualize the ideas that in a general setting we will present in the next section where we will study the dynamics of symmetries under a perturbative scheme.

7. First order perturbations in geometrodynamics

It is the purpose of this section to demonstrate that the whole geometric analysis in previous sections can still be reproduced step by step in a dynamic situation, see reference [8, 9]. We introduce to this end first order perturbations to the relevant objects where \( \varepsilon \) is an appropriate perturbative parameter,

\[ \tilde{g}_{\mu\nu} = g_{\mu\nu} + \varepsilon h_{\mu\nu} \] (37)

\[ \tilde{\xi}_{\mu\nu} = \xi_{\mu\nu} + \varepsilon \omega_{\mu\nu} \] (38)
The perturbation objects $h_{\mu\nu}$, $\omega_{\mu\nu}$ and the one we will introduce next for the electromagnetic tensor are of a physical nature caused by an external agent to the source of preexisting fields. It is worth stressing that they are not the result of a local first order coordinate transformation. We will raise indices with the perturbed metric $\tilde{g}^{\mu\nu} = g^{\mu\nu} - \varepsilon h^{\mu\nu}$. We can write the perturbed electromagnetic field through a new local duality transformation as,

$$\tilde{f}_{\mu\nu} = \cos \tilde{\alpha} \tilde{\xi}_{\mu\nu} + \sin \tilde{\alpha} \star \tilde{\xi}_{\mu\nu}.$$  \hspace{1cm} (39)

The perturbed local complexion $\tilde{\alpha}$ will not be explicitly involved in our analysis. As done in references [1, 2] we impose the new condition,

$$\tilde{\xi}_{\mu\nu} \star \tilde{\xi}_{\mu\nu} = 0.$$  \hspace{1cm} (40)

and through the use of the identity (4), which is valid for every pair of antisymmetric tensors in a four-dimensional Lorentzian spacetime [1], we will show that when applied to the case $A_{\mu\alpha} = \tilde{\xi}_{\mu\alpha}$ and $B^{\nu\alpha} = \star \tilde{\xi}^{\nu\alpha}$ it results in the equivalent condition,

$$\tilde{\xi}_{\mu\rho} \star \tilde{\xi}^{\mu\lambda} = 0.$$  \hspace{1cm} (41)

Even though we are developing a first order perturbative scheme, we avoid writing explicitly the first order approximations, specially in this section, in order to create a general framework that allows to understand the ideas with more clarity. The complexion, which is a local scalar, on account of imposing condition (40) can then be expressed as,

$$\tan(2\tilde{\alpha}) = -\tilde{f}_{\mu\nu} \tilde{f}^{\mu\nu} / \tilde{f}_{\gamma\delta} \tilde{f}^{\gamma\delta}.$$  \hspace{1cm} (42)

It will be useful for this matter to write explicitly the stress-energy tensor for the perturbed fields (37)-(38),

$$\tilde{T}_{\mu\nu} = \tilde{\xi}_{\mu\lambda} \tilde{\xi}^{\nu\lambda} + \star \tilde{\xi}_{\mu\lambda} \star \tilde{\xi}^{\nu\lambda}.$$  \hspace{1cm} (43)

We next proceed to write the four orthogonal vectors that will become an intermediate step in constructing the tetrad that diagonalizes the first order perturbed stress-energy tensor (43),

$$\tilde{V}_1^{\alpha} = \tilde{\xi}^{\alpha\lambda} \tilde{\xi}_{\rho\lambda} X^\rho,$$  \hspace{1cm} (44)  

$$\tilde{V}_2^{\alpha} = \sqrt{-\tilde{Q}/2} \tilde{\xi}^{\alpha\lambda} X^\lambda,$$  \hspace{1cm} (45)  

$$\tilde{V}_3^{\alpha} = \sqrt{-\tilde{Q}/2} \star \tilde{\xi}^{\alpha\lambda} Y^\lambda,$$  \hspace{1cm} (46)  

$$\tilde{V}_4^{\alpha} = \star \tilde{\xi}^{\alpha\lambda} \star \tilde{\xi}_{\rho\lambda} Y^\rho,$$  \hspace{1cm} (47)

In order to prove the orthogonality of the tetrad (44)-(47) it is necessary to use the identity (4) for the case $A_{\mu\alpha} = \tilde{\xi}_{\mu\alpha}$ and $B^{\nu\alpha} = \tilde{\xi}^{\nu\alpha}$, that is,

$$\tilde{\xi}_{\mu\alpha} \tilde{\xi}^{\nu\alpha} - \star \tilde{\xi}_{\mu\alpha} \star \tilde{\xi}^{\nu\alpha} = \frac{1}{2} \delta_{\mu}^\nu \tilde{Q},$$  \hspace{1cm} (48)

where $\tilde{Q} = \tilde{\xi}_{\mu\nu} \tilde{\xi}^{\mu\nu}$ is assumed not to be zero. We also need the condition (41). We are free to choose the vector fields $X^\alpha$ and $Y^\alpha$, as long as the four vector fields (44)-(47) do not become trivial. In this section we have essentially proved that we can build for the perturbed fields a replica of our previous formalisms and constructions used for the unperturbed fields. In particular, we are able to write our new local tetrad keeping the same local extremal skeleton structure as in the unperturbed case and define the new local planes of symmetry associated to the perturbed stress-energy tensor. The new local planes of symmetry will be tilted with respect to the unperturbed planes.
8. Dynamical symmetry breaking in geometrodynamics

In order to study the notion of symmetry breaking in geometrodynamics we will need the results from sections Appendix D and Appendix E. We proceed next to write the first order perturbed covariant derivative of a first order perturbed local contravariant current vector,

\[
\tilde{\nabla}_\mu \tilde{J}^\lambda = \frac{\partial \tilde{J}^\lambda}{\partial x^\mu} + \Gamma^\lambda_{\mu\nu} \tilde{J}^\nu + \varepsilon \tilde{\Gamma}^\lambda_{\mu\nu} J^\nu ,
\]

We can rewrite equation (49) as follows,

\[
\tilde{\nabla}_\mu \tilde{J}^\lambda = \nabla_\mu J^\lambda + \varepsilon \frac{\partial J^\lambda}{\partial x^\mu} + \varepsilon \Gamma^\lambda_{\mu\nu} J^\nu + \varepsilon \tilde{\Gamma}^\lambda_{\mu\nu} J^\nu ,
\]

where the first order perturbed local current has been written as \( \tilde{J}^\lambda = J^\lambda + \varepsilon J^\lambda (1) \). The objective of this section is to show that between an initial constant time hypersurface and an intermediate constant time hypersurface, just when the perturbation starts taking place, the unperturbed local currents are considered to be conserved, that is \( \nabla_\mu J^\mu = 0 \). This is because the unperturbed energy-momentum current for instance, lies inside the local blade one or blade two, that is to say, the local planes of gauge symmetry. Between the intermediate constant time hypersurface and a final hypersurface the original local current \( J^\mu \) will be conserved no longer. After the perturbation takes place, the ensuing conservation equation will be \( \tilde{\nabla}_\mu \tilde{J}^\mu = 0 \) for the perturbed local current. This is because the local planes of symmetry, both blade one and two, will be tilted by the perturbation with respect to the planes on the initial setting. There will be new local planes of symmetry at every point in spacetime. We can see through the new perturbed unnormalized vectors (44)-(47) that diagonalize the new perturbed stress-energy tensor (43), that the new local planes or blades of symmetry in spacetime after the perturbation took place, will no longer coincide with the old ones. This is the reason why after the perturbations already took place the equation \( \nabla_\mu J^\mu = 0 \) is no longer valid and according to equation (50) the following result will be correct,

\[
\nabla_\mu J^\lambda = -\varepsilon \frac{\partial J^\lambda (1)}{\partial x^\mu} - \varepsilon \Gamma^\lambda_{\mu\nu} J^\nu - \varepsilon \tilde{\Gamma}^\lambda_{\mu\nu} J^\nu .
\]

This is exactly what we might call dynamic symmetry breaking. The old currents \( J^\lambda \) will be no longer conserved, only the new ones \( \tilde{J}^\lambda \) will be. Using all the elements of analysis developed so far we will proceed to state the following theorem, see reference [8, 9],

**Theorem 3** The local orthogonal planes of symmetry or diagonalization of the Einstein-Maxwell stress-energy tensor and associated local groups of tetrad transformations LB1 and LB2 evolve as the continuous perturbation of an external agent takes place. Symmetries are continuously broken and transformed into new symmetries as the local planes of symmetry evolve.

9. Conclusions

A new tetrad that diagonalizes the Einstein-Maxwell stress-energy tensor for non-null electromagnetic fields was introduced. However, this tetrad has an inherent freedom in the choice of two vector fields. Geometrodynamics is an arena in which an explicit example or choice can be given for these two vector fields, because Maxwell’s equations are providing two vector potentials. The simplicity of the expression for the electromagnetic field in this new tetrad and the associated null tetrad becomes evident [2]. It was also proved that the local gauge group of electromagnetic gauge transformations is related to the group LB1, and also to the group LB2 through isomorphisms. It was also proved that when written in terms of the
new tetrad, the Einstein-Maxwell equations are substantially simplified [2]. We have been able to develop the concept of dynamic symmetry breaking for classical electromagnetic fields in a curved four-dimensional Lorentzian spacetime. The analogous notion developed under the realm of Quantum Field Theories for the Standard Model aimed at the creation of mass through a dynamical interacting mechanism [20-34]. In our work we have produced a dynamical breaking of symmetry through a change in spacetime curvature. The symmetries in the gauge theory of electromagnetic fields are understood through the isomorphisms proved in manuscript [2] as local Lorentz transformations on either plane one or two. These local groups have been named LB1 and LB2. New local tetrad vectors transform inside these blades under the action of these groups. Therefore, symmetry breaking is equivalent to a change in local planes or blades one and two through perturbations. When an external agent to the preexisting geometry perturbs the original system, the local planes of symmetry are tilted with respect to the original ones. Since there are conserved energy-momentum currents represented by vectors that are locally either inside blade one or blade two as we explicitly proved in the Reissner-Norström geometry case, and then in a general case in section Appendix D, symmetry breaking means that these currents will be inside the new local planes of symmetry after the perturbation takes place, which will be the perturbed ones. It is even possible to conceive and reproduce all these ideas in the quantum realm since quantum fluctuations are just perturbations, see reference [35]. We assumed that the energy-momentum currents conservation equations are locally invariant either under LB1 or LB2, evident in the Reissner-Norström geometry case and in a more general aspect for the energy-momentum currents introduced in sections Appendix C and Appendix D. The symmetries will correspond to new local orthogonal planes so that the new currents will also be inside the new planes. The old local conservation laws will no longer hold. There will be new ones associated to the new planes of symmetry. The vectors that locally diagonalize the old stress-energy tensor will no longer diagonalize the new perturbed stress-energy tensor. It is worth reminding ourselves that the Einstein-Maxwell stress-energy tensor is form covariant under perturbations because the perturbed version can be written exactly as the unperturbed version just using the perturbed metric tensor and the perturbed extremal fields. Even though the tetrad that diagonalizes the original stress-energy tensor is not the same as the new one for the perturbed stress-energy tensor, the tetrad vectors in both cases are locally structure invariant because the stress-energy tensors are structure invariant. In conclusion, in this work we have been able to prove that a change in curvature is associated with a local dynamic symmetry breaking process that we might reinterpret as an evolution of local symmetries into new local symmetries. There is a symmetry evolution, and we evaluate this evolution through the local plane symmetry evolution, or the evolution of blades one and two, which are the local planes of gauge symmetry. In the classical settings it is a continuous evolution while in the quantum setting it could be discrete. In other words, the local evolution of the LB1 and LB2 groups.

We can state the following conclusions as a summary.

- New orthonormal tetrad for non-null electromagnetic fields in four-dimensional curved Lorentzian spacetimes. This tetrad diagonalizes locally and covariantly the Einstein-Maxwell stress-energy tensor. Astrophysical applications in spacetime evolution [10-12].
- Isomorphisms between the local electromagnetic gauge group and the local groups of tetrad spacetime transformations LB1 and LB2. There is an isomorphism between kinematic states and gauge states of the gravitational fields, locally. These results can be extended to the non-Abelian cases [36-38].
- Maximum simplification of relevant tensors and field equations.
- New tetrads encode gravitational and electromagnetic gauge information.
- We are introducing an explicit “link” between the “internal” and the “spacetime”, so far detached from each other.
• Extension or generalization to non-Abelian theories with gauge groups $SU(2) \times U(1)$ and $SU(3) \times SU(2) \times U(1)$, see references [36-38].

• Hypotheses made at the outset of the no-go theorems [5-7] proved incorrect. Therefore, the no-go theorems are incorrect.

• Local gauge symmetries are dynamic. A change in curvature introduces through physical perturbations a tilt in the local orthogonal planes of symmetry. The local orthogonal planes of symmetry which are the local planes of diagonalization of the Einstein-Maxwell stress-energy tensor and associated local groups of tetrad transformations LB1 and LB2 evolve as the continuous perturbation of an external agent takes place. Symmetries are continuously broken and transformed into new symmetries as the local planes of symmetry evolve. There is a symmetry evolution.

• This new classical model will be useful in order to better understand anomalies in quantum field theories.

Appendix A.
The Levi-Civita pseudotensor can be transformed into a tensor through the use of factors $\sqrt{-g}$, where $g$ is the determinant of the metric tensor. We use the notation $e_{\alpha \beta \mu \nu} = [\alpha \beta \mu \nu]$ for the covariant components of the Levi-Civita pseudotensor in the Minkowskian frame given in [1],

$$e_{\alpha \beta \mu \nu} = \begin{cases} 1 & \text{if } \alpha \beta \mu \nu \text{ is an even permutation of } 0123 \\ -1 & \text{if } \alpha \beta \mu \nu \text{ is an odd permutation of } 0123 \\ 0 & \text{if } \alpha \beta \mu \nu \text{ are not all different} \end{cases}$$

It can be noticed that the signs in $e^{\alpha \beta \mu \nu}$ will be opposite to the standard notation [39]. The reason for this is that we want to keep the compatibility with [1] where the definition $e_{0123} = [0123] = 1$ was adopted. With these definitions we see that in a spacetime with a metric $g_{\alpha \beta}$,

$$e^{\alpha \beta \mu \nu} = e_{\alpha \beta \mu \nu} \sqrt{-g} = -\frac{[\alpha \beta \mu \nu]}{\sqrt{-g}}, \quad (A.1)$$

are the components of a contravariant tensor [39-41]. The covariant components of (A.1) are

$$e_{\alpha \beta \mu \nu} = e^{\alpha \beta \mu \nu} \sqrt{-g} = [\alpha \beta \mu \nu] \sqrt{-g}, \quad (A.2)$$

where

$$g_{\alpha \sigma} g_{\beta \rho} g_{\mu \kappa} g_{\nu \lambda} e^{\rho \kappa \lambda} = -g e_{\alpha \beta \mu \nu}, \quad (A.3)$$

is satisfied.

Appendix B.
The tetrad vectors $(U^\alpha, V^\alpha, Z^\alpha, W^\alpha)$ have the following expressions in the Minkowski reference frame given by equations (38)-(39) in [1],

$$U^0 = -A^0/\sqrt{-(A^0 A_0 + A^1 A_1)} \quad (B.1)$$

$$U^1 = -A^1/\sqrt{-(A^0 A_0 + A^1 A_1)} \quad (B.2)$$

$$V^0 = -\xi_{01} A^1/\left( |\xi_{01}| \sqrt{-(A^0 A_0 + A^1 A_1)} \right) \quad (B.3)$$

$$V^1 = -\xi_{01} A^0/\left( |\xi_{01}| \sqrt{-(A^0 A_0 + A^1 A_1)} \right) \quad (B.4)$$
\[ Z^2 = -\xi_{01} \ast A^3 / \left( |\xi_{01}| \sqrt{\ast A^2 \ast A_2 + \ast A^3 \ast A_3} \right) \quad (B.5) \]

\[ Z^3 = \xi_{01} \ast A^2 / \left| \xi_{01} \ast \ast A^2 \ast A_2 + \ast A^3 \ast A_3 \right| \quad (B.6) \]

\[ W^2 = \ast A^2 / \sqrt{\ast A^2 \ast A_2 + \ast A^3 \ast A_2} \quad (B.7) \]

\[ W^3 = \ast A^3 / \sqrt{\ast A^2 \ast A_2 + \ast A^3 \ast A_3} \quad (B.8) \]

where \( |\xi_{01}| = \sqrt{(\xi_{01})^2} \).

**Appendix C.**

In this section we will prove that if the locally conserved energy-momentum current \( T^{\mu\nu} \xi_{\nu} \) satisfies invariance either under the local groups LB1 or LB2, then the vector \( \xi_{\mu} \) has to lie either on blade two or blade one respectively, see reference [8, 9]. The stress-energy tensor can be written [2],

\[ T_{\mu\nu} = (Q/2) \left[ -U_\mu U_\nu + V_\mu V_\nu - Z_\mu Z_\nu - W_\mu W_\nu \right]. \quad (C.1) \]

We write the vector field \( \xi_{\mu} \) in a general way using the orthonormal tetrad (21)-(24),

\[ \xi_{\mu} = A [\cosh \phi U_\mu + \sinh \phi V_\mu] + B [\cos \varphi Z_\mu + \sin \varphi W_\mu]. \quad (C.2) \]

where \( A \) and \( B \) are local scalars as well as \( \phi \) and \( \varphi \). Equation (C.2) represents the superposition of a general vector on blade one and a general vector on blade two. The equation for conservation of the energy-momentum current will be,

\[ 0 = \left( T^{\mu\nu} \xi_{\nu} \right)_{;\mu} = (T^{\mu\nu} (A [\cosh \phi U_\nu + \sinh \phi V_\nu] + B [\cos \varphi Z_\nu + \sin \varphi W_\nu])_{;\mu}. \quad (C.3) \]

Using the orthonormal tetrad vectors (21)-(24) and equation (C.1) we can rewrite (C.3) as,

\[ 0 = ((Q/2) (A [\cosh \phi U_\nu + \sinh \phi V_\nu] + B [-\cos \varphi Z_\nu - \sin \varphi W_\nu])_{;\mu}. \quad (C.4) \]

From reference [2] we know that we can produce a full inversion on blade one in expression (C.4), \((U^\mu, V^\mu) \rightarrow (-U^\mu, -V^\mu)\).

\[ 0 = ((Q/2) (-A [\cosh \phi U_\nu + \sinh \phi V_\nu] + B [-\cos \varphi Z_\nu - \sin \varphi W_\nu])_{;\mu}. \quad (C.5) \]

Adding (C.4) and (C.5) we get,

\[ 0 = ((Q/2) (B [-\cos \varphi Z_\nu - \sin \varphi W_\nu])_{;\mu}. \quad (C.6) \]

Now substracting (C.4) and (C.5) we get,

\[ 0 = ((Q/2) (A [\cosh \phi U_\nu + \sinh \phi V_\nu])_{;\mu}. \quad (C.7) \]

If we now impose current conservation under boosts in expression (C.7) we necesarly get \( A = 0 \). Therefore, if we impose on conserved currents local gauge invariance under LB1, then the vector \( \xi_{\mu} \) must lie on blade two and equation (C.6) will be satisfied. Again in expression (C.4) we can produce a rotation on blade two \( \varphi \rightarrow \varphi + \pi \) and \((\cos \phi, \sin \phi) \rightarrow (-\cos \phi, -\sin \phi)\).

\[ 0 = ((Q/2) (A [\cosh \phi U_\nu + \sinh \phi V_\nu] - B [-\cos \varphi Z_\nu - \sin \varphi W_\nu])_{;\mu}. \quad (C.8) \]
Adding (C.4) and (C.8) we get,
\[ 0 = \left( (Q/2) \left( A \cosh \phi U^\mu + \sinh \phi V^\mu \right) \right);^\mu . \]  
\hspace{2cm} (C.9)

Now subtracting (C.4) and (C.8) we get,
\[ 0 = \left( (Q/2) \left( B \left[ \cos \varphi Z^\mu - \sin \varphi W^\mu \right] \right) \right);^\mu . \]  
\hspace{2cm} (C.10)

If we now impose current conservation under general spatial rotations in expression (C.10) we necessarily get \( B = 0 \). Therefore, if we impose local gauge invariance under LB2, then the vector \( \xi^\mu \) must lie on blade one and equation (C.9) will be satisfied. Summarizing the results in this section, from (C.6) and (C.9) we conclude that if we impose local gauge invariance either under LB1 or LB2 on the local energy-momentum current conservation equation (C.3), the vectors \( \xi^\mu \) have to lie either on blade two or blade one.

**Appendix D.**

The Reissner-Nordstr"om geometry is an exception in the sense that the complexion locally satisfies \( \tan(2\alpha) = 0 \). The Killing vector fields lie on either plane one or two. Locally, Killing vectors in principle would not have to be vectors lying on either of the two planes if we are talking about geometries other than the Reissner-Nordstr"om case. Then, the question arises about the existence of locally conserved current vectors lying on either planes in a more general dynamic geometry, for instance where non-null electromagnetic fields are present in a curved four-dimensional spacetime but without the spherical symmetry, see reference \[8, 9\]. That is to say, a spherically symmetric source under the dynamic perturbative action of an external agent as stated at the beginning of this work or specifically in section 7. Electromagnetic and gravitational fields would have to satisfy the Einstein-Maxwell equations even under perturbative interaction. The bottom line is that we are assuming that the perturbed fields \( \tilde{g}^\mu_\nu, \tilde{\xi}^\mu_\nu \) and \( \tilde{\alpha} \) will also satisfy the Einstein-Maxwell equations. In this section we will analyze conserved currents for the unperturbed case in general, not necessarily the Reissner-Nordstr"om geometry case. For the perturbed situation the analysis would be similar by replacing in the Einstein-Maxwell equations for the perturbed fields \( \tilde{g}^\mu_\nu, \tilde{\xi}^\mu_\nu \) and \( \tilde{\alpha} \). One simple way to see that there are always locally conserved currents lying on both planes or blades is the following. If we replace the electromagnetic field in terms of the complexion and the extremal field, see expression (62)-(63), we can see that the extremal field and the complexion must satisfy, in accordance to the Maxwell equations which are a subset of the Einstein-Maxwell equations,
\[ \xi^\mu_\nu;^\nu = - * \xi^\mu_\nu \alpha^\nu \]  
\hspace{2cm} (D.1)
\[ * \xi^\mu_\nu;^\nu = \xi^\mu_\nu \alpha^\nu, \]  
\hspace{2cm} (D.2)

where \( \alpha^\nu \) represents the derivative \( \partial \alpha / \partial x^\nu \). Therefore we can try and explore the vectors \( \xi^\mu_\nu \alpha^\nu \) and \( * \xi^\mu_\nu \alpha^\nu \), and see if they are conserved on one hand and if they belong to the planes one and two on the other hand. First we can see that due to the antisymmetry of the extremal field \( \xi^\mu_\nu \) and its dual \( * \xi^\mu_\nu \) and to the scalar nature of the complexion \( \alpha \) the following equations are satisfied,
\[ \xi^{\mu\nu};_\mu = -(\xi^{\mu\nu} \alpha^\nu)_;_\mu = 0 \]  
\hspace{2cm} (D.3)
\[ * \xi^{\mu\nu};_\mu = (\xi^{\mu\nu} \alpha^\nu)_;_\mu = 0. \]  
\hspace{2cm} (D.4)

An iterative use of equations (D.1)-(D.2) leads to equations (D.3)-(D.4). If the geometry is such that the complexion gradient \( \alpha^\nu \) is not trivial, then we have two conserved local vector
fields. Next we would like to see for instance if the vector $\xi^{\mu\nu} \alpha^\nu$ lies on plane one. Using property (10) and the normalized tetrad (21)-(24) it is evident to see that it lies on the plane generated by the vectors (21)-(22), that is blade one. A similar line of thinking for the vector $^*\xi^{\mu\nu} \alpha^\nu$ on blade two. We can summarize these results in the following table,

\[
U_\alpha \xi^{\alpha\beta} = \sqrt{-Q/2} V^\beta \quad (D.5)
\]

\[
V_\alpha \xi^{\alpha\beta} = \sqrt{-Q/2} U^\beta \quad (D.6)
\]

\[
Z_\alpha \ast \xi^{\alpha\beta} = \sqrt{-Q/2} W^\beta \quad (D.7)
\]

\[
W_\alpha \ast \xi^{\alpha\beta} = -\sqrt{-Q/2} Z^\beta \quad (D.8)
\]

Due to property (10) all other contractions are null. We can also observe that we can write the conserved currents as $T^{\mu\nu} \xi^\nu$. Using the property $T^{\mu\nu} T^{\gamma\nu} = (Q/2)^2 \delta^{\gamma\mu}$ we can find $\xi^\mu = \xi^{\mu\nu} \alpha^\nu/(Q/2)$ on blade one or $\xi^\mu = ^*\xi^{\mu\nu} \alpha^\nu/(Q/2)$ on blade two. The vector $\xi^\mu$ does not have to be necessarily a Killing vector field, nonetheless the energy-momentum current $T^{\mu\nu} \xi^\nu$ is conserved. Therefore, we proved our point. As long as the gradient of the complexion is not trivial or its contraction with the extremal field and its dual are not trivial, we always have a local conserved current on blade one and another one on blade two. By always we mean during the dynamical evolution. We would like to briefly remind that when a perturbative treatment is implemented, the vacuum Einstein-Maxwell equations can be written as,

\[
\hat{E}(g^{(0)}_{\alpha\beta}, \xi^{(0)}_{\alpha\beta}, \alpha^{(0)}) = 0 \quad (D.9)
\]

where $\hat{E}$ represents the set of nonlinear differential operators that generates the Einstein-Maxwell equations. The terms in (D.9) that are zero order in $\varepsilon$ will be satisfied automatically because $g^{(0)}_{\alpha\beta}$, the background metric, $\xi^{(0)}_{\alpha\beta}$ the background extremal field and $\alpha^{(0)}$ are a solution to the vacuum Einstein-Maxwell equations. To find the equations satisfied by the first order perturbation, we expand (D.9) in powers of $\varepsilon$, and write the set of first order equations as

\[
\hat{O}(g^{(1)}_{\alpha\beta}, \xi^{(1)}_{\alpha\beta}, \alpha^{(1)}) = 0 \quad (D.10)
\]

Since the perturbations $g^{(1)}_{\alpha\beta}$, $\xi^{(1)}_{\alpha\beta}$ and $\alpha^{(1)}$ can only appear linearly, $\hat{O}$ represents a set of linear differential operators. In the set of equations (D.9) we include all the Einstein-Maxwell equations. It is clear that all the analysis done through equations (D.1)-(D.2) and (D.3)-(D.4) can be reproduced analogously for the perturbed fields $\tilde{g}_{\mu\nu}$, $\tilde{\xi}_{\mu\nu}$ and $\tilde{\alpha}$. Perturbed fields will arise during the dynamical interaction process.

**Appendix E.**

In order to compare local current conservation laws we will need the first order perturbed covariant derivative of a vector. In this section we will display the main steps to obtain these calculations, see reference [8, 9]. We can start with the standard expression for the covariant derivative of a vector,

\[
V^\lambda_{\mu} = \frac{\partial V^\lambda}{\partial x^\mu} + \Gamma^\lambda_{\mu\nu} V^\nu \quad (E.1)
\]

where the expression for the affine connection is the usual,

\[
\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} \left( \frac{\partial g_{\mu\sigma}}{\partial x^\nu} + \frac{\partial g_{\nu\sigma}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma} \right) \quad (E.2)
\]
Following the literature in perturbative schemes, see [42, 43] and references therein as examples, we can write the first order perturbed affine connection as,

$$\tilde{\Gamma}^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} (h_{\mu\sigma;\nu} + h_{\nu\sigma;\mu} - h_{\mu\nu;\sigma}) \, ,$$  \hspace{1cm} (E.3)

where the covariant derivatives in (E.3) are calculated with the unperturbed (E.2) affine connection. We proceed then to write to first order the perturbed covariant derivative of a perturbed contravariant vector,

$$\tilde{\nabla}_\mu \tilde{V}^\lambda = \frac{\partial \tilde{V}^\lambda}{\partial x^\mu} + \Gamma^\lambda_{\mu\nu} \tilde{V}^\nu + \varepsilon \tilde{\Gamma}^\lambda_{\mu\nu} V^\nu \, ,$$  \hspace{1cm} (E.4)

where we have used now the operator $\nabla$ to indicate covariant derivative for notational convenience since we can write a tilde above it. The perturbed vector can be written $\tilde{V}^\lambda = V^\lambda + \varepsilon \psi^\lambda$, where $\psi^\lambda$ is a local vector field. When we think of $V^\lambda$ in a concrete example in this manuscript, we will be thinking of the local currents $J^\lambda$. It is important to stress that we are studying genuine physical perturbations to the gravitational and electromagnetic fields by external agents to the preexisting source. We are not introducing first order coordinate transformations of the kind $\tilde{x}^\alpha = x^\alpha + \varepsilon \zeta^\alpha$, where the local vector field $\zeta^\alpha(x^\sigma)$ is associated to a first order infinitesimal local coordinate transformation scheme [39].

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