Action for \textit{IIB} Supergravity in 10 dimensions

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Abstract

We review the construction of a manifestly covariant, supersymmetric and $SL(2R)$ invariant action for \textit{IIB} supergravity in $D=10$.

Talk given at the workshop \textit{Quantum Aspects of Gauge Theories, Supersymmetry and Unification}, Greece, September 1998.

PACS: 04.65.+e; Keywords: Supergravity, ten dimensions, duality

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1 Introduction

$D = 10$, $IIB$ Supergravity acquired new life, on one hand from the discovery of different dualities among string theories and among these and $D = 11$ supergravity and, on the other hand, from the related discovery of $D$-branes. Its bosonic sector consists of the graviton, two scalars, a complex rank two tensor and a real rank four tensor with selfdual field strength. The fermions are two gravitinos with the same chirality and two spin 1/2 fermions with opposite chirality w.r.t. the gravitinos. The presence of a self–dual tensor (chiral boson) is at the origin of the difficulty related with a covariant lagrangian formulation of this theory. A common statement is that such a formulation cannot exist. The statement is correct if with ”covariant action” one means a ”globally defined covariant action”, but sometimes it can be useful to deal with covariant actions even if they are not globally defined.

Recently a new method [1], [2] has been proposed to write covariant actions with manifest duality and/or in presence of chiral bosons and it is natural to apply this method to get a manifestly covariant action for $D = 10$, $IIB$ supergravity. This action has been constructed in [3], at the bosonic level, and in [4] for the complete theory, and will be presented in section 3. To write the complete action, one needs the trasformation rules under supersymmetry. The supersymmetry trasformations and the field equations of $IIB$ supergravity are well known at a non lagrangian level [5], [6] and can be conveniently derived in a superspace approach [4], [6], [7]. In section 2 we review and slighty improve this approach, the improvement being that we present the rheonomic parametrizations not only for the basic curvatures but also for their duals.

2 Superspace approach

In $D = 10$, $IIB$ supergravity the superspace coordinates are $Z^M \equiv (x^m, \theta^\mu, \bar{\theta}^\bar{\mu})$, where $x^m$ ($m = 0, ..., 9$) are the space–time coordinates and $\theta^\mu$ ($\mu = 1, ..., 16$) with their complex conjugates $\bar{\theta}^{\bar{\mu}}$ are Grassmann variables. Our normalization for a $p$-superform $Y_p$ is $Y_p = 1/p! dZ^{M_1}...dZ^{M_p} Y_{M_1...M_p}$ where the $dx^m$.

\footnote{For instance, in a recent paper [8] the covariant action of type $IIB$ supergravity obtained in [3], [4] has been used to obtain the quadratic action for the physical fields in an $AdS_5 \times S_5$ background.}

\footnote{For a different approach, see [9].}
are anticommuting and the $d\theta^\mu$ are commuting, so that we can suppress the wedge symbol in the products of (super)forms. $D = 10$, $IIB$ supergravity is invariant under the global S duality group $SL(2, R) \approx SU(1, 1)$, which acts only on the bosons. To keep this symmetry manifest, the scalars are described by two complex $SU(1, 1)$ doublets $V^I_\pm$, ($I = 1, 2$), which parametrize the coset $SU(1, 1)/U(1)$ and satisfy $V^J_+ V^I_+ = 1$. The indices $I, J$ are lowered and raised from the left with the metric $\epsilon^{IJ}$ and $\epsilon_{IJ}$ where $\epsilon_{12} = -\epsilon_{21} = 1 = \epsilon^{21} = -\epsilon^{12}$. $V^I_+$ and $V^I_-$ are superfields (0-superforms) with $U(1)$-charges $+1$ and $-1$ respectively and satisfy the reality condition $V^I_- = V^I_+ \bar{\psi}$. The $U(1)$ connection is the one-superform $Q = iV^I_+ dV_- I$ and the curvatures of the scalars are

$$R_1 = V^I_+ dV^I_-, \quad \bar{R}_1 = V^I_- dV^- I, \quad (1)$$

which are $SU(1, 1)$ invariant one-superforms with $U(1)$ charges $\pm 2$. The other geometrical objects are the supervielbeins $e^A$, the Lorentz valued superconnection $\omega^A_B$, a complex antichiral spinor superfield $\Lambda$ and the $p$–superform potentials $A_p$, where $p = 2, 4, 6, 8$. The supervielbeins $e^A \equiv (e^a, e^\alpha, \bar{e}^\alpha)$, with torsion $T^A = De^A$ ($a = 0, ..., 9; \alpha = 1, ..., 16$), describe the graviton and the gravitinos. The superconnection $\omega^A_B$, with Lorentz curvature $R^A_B$, can be expressed in terms of $e^A$ due to the torsion constraints. The gravitinos $e^\alpha \equiv \psi^\alpha$ and $\bar{e}^\alpha \equiv \bar{\psi}^\alpha$ are complex conjugate chiral spinor one-superforms. $\Lambda$ and $\bar{\Lambda}$ describe the spin 1/2 fermions. $D = d + \omega + iqQ$ denotes the $U(1)$ and $SO(1, 9)$ covariant differential for a superfield or a superform with $U(1)$ charge $q$. The two rank–two tensors are described by the $SU(1, 1)$ doublet two-superform $A^I_2 = -\bar{A}^I_2$, and the chiral boson by the real four-superform $A_4$. In addition to $V^I_\pm$ only $e^a$ and $\Lambda_\alpha$ are charged under $U(1)$ and have charges $1/2$ and $3/2$ respectively (and, of course $\bar{e}^\alpha$ and $\bar{\Lambda}_\alpha$ have charge $-1/2$ and $-3/2$). We shall also introduce the duals of $A_2$, that is, the doublet six-superform $A^6_I = \bar{A}^I_6$, and the duals of the scalars. Here we meet a little surprise: the duals of the scalars are a triplet of eight-superforms $A_8^I = \bar{A}^I_8$ which belong to the adjoint representation of $SU(1, 1)$. The space-time differentials $dA_8^I|_{\theta = 0}$ are the Hodge duals of the three $SU(1, 1)$ conserved currents; they are closed and, therefore, locally exact. However, they are not independent. There is a linear relation between them so that, after the gauge fixing of the local $U(1)$ simmetry, only two scalars and two

\[ \text{The complex conjugate } \phi^* \text{ of a complex field } \phi \text{ will be denoted by } \bar{\phi} \text{ but for an } SU(1, 1) \text{ doublet } \psi = (\psi_1 \psi_2) \text{ we define } \bar{\psi} = \tau_1 \psi^* \text{ where } \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \]
dual eigth–forms survive. The curvatures for the superform potentials \( A_p \) are

\[
R^I_3 = dA^I_2, \]

\[
R_5 = dA_4 + iA_{2I} dA^I_2, \]

\[
R^I_7 = dA^I_6 - i dA_4 A^I_2 + 1/3 A_{2J} dA^J_2 A^I_2, \]

\[
R_{9(IJ)} = dA_{9(IJ)} - 1/2 \left( (dA^I_6 + i/2 dA_4 A^I_2 - 1/12 A^I_2 A^I_2 A_2 K A^K_2) A^J_2 + I \leftrightarrow J \right),
\]

and the gauge transformations of the potentials are such that these curvatures are invariant.

Instead of \( R^I_3, R^I_7, R_{9(IJ)} \) it is convenient to use the \( SU(1, 1) \) invariant curvatures

\[
R_3 = -V^I_+ R_{3I}, \quad \bar{R}_3 = -V^I_- R_{3I}, \quad R_7 = -V^I_+ R_{7I}, \quad \bar{R}_7 = V^I_- R_{7I}, \]

\[
R_9 = V^I_+ R_{9I} V^J_+ V^J_-, \quad \bar{R}_9 = V^I_- R_{9I} V^J_+ V^J_-, \quad R^{(0)}_9 = -V^I_+ R_{9I} V^J_- V^J_-.
\]

Taking the differential (or covariant differential) of these curvatures and of the torsion \( T^A \) one can derive the associated Bianchi identities. Then, solving the Bianchi identities, under suitable constraints, one obtains the torsion and curvature rheonomic parametrizations (that include the constraints). For \( R_p, p = 1, 3, 5, 7, 9, 9^{(0)} \) one gets

\[
R_p = F_p - C_p,
\]

where \( F_p \) indicates the components of \( R_p \) along \( e^a \), i.e.

\[
F_p = \frac{1}{p!} e^{a_1} \cdots e^{a_p} F_{a_1 \cdots a_p},
\]

and the \( C_p \), which involve the gravitino superforms \( \psi^\alpha \), \( \bar{\psi}^\alpha \) and the spinor superfields \( \Lambda_{\alpha} \) and \( \bar{\Lambda}_{\alpha} \), are given by

\[
C_1 = 2\psi \Lambda,
\]

\[
C_3 = \frac{1}{2} e^a e^b (\bar{\psi} \Gamma_{ab} \Lambda) + \frac{i}{2} e^a (\psi \Gamma_a \bar{\psi}) = C^\Lambda_3 + C^\psi_3,
\]

\[
C_5 = \frac{1}{5!} e^{a_1} \cdots e^{a_5} (\bar{\Lambda} \Gamma_{a_1 \cdots a_5} \Lambda) - \frac{1}{3!} e^a e^b e^c (\bar{\psi} \Gamma_{abc} \bar{\psi}) = C^\Lambda_5 + C^\psi_5,
\]
\[ C_7 = \frac{1}{6!} \epsilon^{a_1 \ldots a_6} (\bar{\psi} \Gamma_{a_1 \ldots a_6} \Lambda) - \frac{i}{2 \cdot 5!} \epsilon^{a_1 \ldots a_5} (\psi \Gamma_{a_1 \ldots a_5} \psi) = C_7^\Lambda + C_7^{\psi}, \quad (13) \]

\[ C_9 = \frac{2}{8!} \epsilon^{a_1 \ldots a_8} (\psi \Gamma_{a_1 \ldots a_8} \Lambda), \quad (14) \]

\[ C_9^{(0)} = -i \left[ \frac{3}{9!} \epsilon^{a_1 \ldots a_9} (\bar{\Lambda} \Gamma_{a_1 \ldots a_9} \Lambda) + \frac{1}{2 \cdot 7!} \epsilon^{a_1 \ldots a_7} (\bar{\psi} \Gamma_{a_1 \ldots a_7} \psi) \right] \]

\[ = \left[ C_9^{(0)} \Lambda + C_9^{(0) \psi} \right]. \quad (15) \]

Here \( C_p^\Lambda \) (\( C_p^{\psi} \)) indicates the terms of \( C_p \) that depend on (are independent of) \( \Lambda \). For the torsion \( T^A \) and for \( D \Lambda \), the rheonomic parametrizations are

\[ T^a = De^a = i \bar{\psi} \Gamma^a \psi \]

\[ T = D \psi = \frac{1}{2} \epsilon^a \epsilon^b T_{ba} + \epsilon^a \left[ N_a \bar{\psi} + i \left( \frac{1}{192} F_{ab_1 \ldots b_4} \Gamma^{b_1 \ldots b_4} + L_a \right) \psi \right] \]

\[ - (\bar{\psi} \Lambda) \bar{\psi} - \frac{1}{2} (\bar{\psi} \Gamma^a \psi) \Gamma_a \Lambda \]

\[ DA = \epsilon^b D_b \Lambda + \frac{i}{2} F^a \Gamma_a \bar{\psi} + \frac{i}{24} F^{abc} \Gamma_{abc} \psi, \quad (18) \]

where

\[ L^a = -21 \Lambda^a + \frac{3}{2} \Lambda^b \Gamma_{ab} + \frac{5}{4} \Lambda_{abc} \Gamma^{bc} - \frac{1}{4} \Lambda^{bcd} \Gamma_{abcd} - \frac{1}{48} \Lambda_{ab_1 \ldots b_4} \Gamma^{b_1 \ldots b_4}, \quad (19) \]

\[ N^a = \frac{3}{16} \left( -F_{abc} \Gamma^{bc} + \frac{1}{9} F^{bcd} \Gamma_{abcd} \right), \quad (20) \]

\( \Lambda_{a_1 \ldots a_9} = 1/16 \bar{\Lambda} \Gamma_{a_1 \ldots a_9} \Lambda \), and \( F_{a_1 \ldots a_5}^{(+)} \) is the self–dual part of \( F_{a_1 \ldots a_5} \).

These parametrizations determine, on one hand, the supersymmetry transformations of the components fields, which are given by the covariant Lie derivatives of the associated superfields along the susy parameter \( \epsilon^A = (0, \epsilon^a, \bar{\epsilon}^\alpha) \), evaluated at \( \theta = d\theta = 0 \) (modulo \( \epsilon \)-dependent gauge transformations). Notice that \( F_{a_1 \ldots a_5}^{(+)} \) enters only in the susy transformation of the gravitinos.

On the other hand, they force the theory to be on shell and imply, therefore, the field equations. The field equation for the rank–four tensor is just the self–duality condition for \( F_5 \). The equations of motion for the rank-two tensors and for the scalars are equivalent to the duality conditions between \( F_3 \) and \( F_7 \) and between \( F_1 \) and \( F_9 \) respectively:

\[ \ast F_5 = F_5, \quad (21) \]

\[ \ast F_3 = F_7, \quad (22) \]
\[ *F_1 = F_9, \quad (23) \]
\[ 0 = F_9^{(0)}. \quad (24) \]

The hodge dual of \( F_p \) is the \((10 - p)\)-superform \((*F)_{10-p}\), whose intrinsic components are

\[ (*F)_{a_1...a_{10-p}} = \frac{1}{p!} \epsilon_{a_1...a_{10-p}}^{b_1...b_p} F_{b_1...b_p}. \quad (25) \]

Eq. (21) is, indeed, the self-duality condition for the chiral tensor, and the field equations for the rank two tensors and the scalars are obtained by taking the covariant differentials of eqs. (22) and (23) and using the Bianchi identities for \( R_7 \) and \( R_9 \) respectively. Eq. (24) is the linear relation between the three currents \( dA_{8I}^J \), mentioned above, since \( F_9^{(0)} \) is purely imaginary.

### 3 Covariant action

In this section we will apply the general method of [1], [2] to write a covariant, supersymmetric action for \( D = 10 \), IIB supergravity. Being a component action, we shall deal with fields and forms, not superfields and superforms. However, we shall use the same symbols of the previous section to indicate those objects evaluated at \( \theta = d\theta = 0 \).

As a first step, let us discuss the action for a free chiral boson in a ten dimensional bosonic curved background. The basic ingredients are:

i) a four-form potential \( A_4(x) \) with curvature \( F_5 = dA_4 \);

ii) a metric \( g_{mn}(x) \) (or the vielbeins \( e^a = dx^m e_m^a \));

iii) an auxiliary scalar field \( a(x) \) with its related one-form

\[ v = \frac{da}{\sqrt{-g^{mn}\partial_m a \partial_n a}} = e^a v_a, \quad (26) \]

normalized such that \( v^2 = v^a v_a = -1 \). Moreover, we associate to the anti-selfdual part of \( F_5 \), \( F_5^{(-)} \equiv F_5 - *F_5 \), the four form obtained via the interior product with \( v \): \( f_4 \equiv i_v F_5^{(-)} \).

The self duality condition, which has to be produced by the action, is \( F_5^{(-)} = 0 \); but, due to the identity

\[ F_5^{(-)} = -vf_4 + *(vf_4), \quad (27) \]
$F_5(\cdot) = 0$ is equivalent to $f_4 = 0$.

The covariant action which, after gauge–fixing of new bosonic symmetries (see below), leads to this equation is

$$S_0 = \frac{1}{4} \int (F_5 \ast F_5 + f_4 \ast f_4).$$

(28)

It yields the field equations

$$d(da \tilde{f}_4) = 0,$$

(29)

$$d(da \tilde{f}_4 \tilde{f}_4) = 0,$$

(30)

where $\tilde{f}_4 = \frac{f_4}{\sqrt{-\left(\partial a\right)^2}}$.

In addition to the standard gauge trasformation for $A_4$, i.e. $\delta A_4 = d\lambda_3$, $\delta a = 0$, the action (28) is invariant under the new local symmetries

$$\delta_1 A_4 = -\phi \tilde{f}_4, \quad \delta_1 a = \phi,$$

(31)

$$\delta_2 A_4 = \psi_3 da, \quad \delta_2 a = 0,$$

(32)

where $\phi(x)$ is an infinitesimal scalar but $\psi_3(x)$ is a finite 3-form. The symmetry (31) implies that $a(x)$ is pure gauge (but the gauge $a = 0$ is forbidden, since $S_0$ is non polinomial in $a$ and becomes singular in $a = 0$).

The general solution of the field eq. (29) is $vf_4 = d\tilde{\psi}_3 da$, and under the finite trasformation (32) $vf_4$ trasforms as $vf_4 \mapsto vf_4 + d\psi_3 da$, so that, with this trasformation, one can reach the selfduality condition $f_4 = 0$ i.e. $F_5(\cdot) = 0$.

This approach can be generalized to cover the case of a chiral boson $A_4$ interacting with matter fields, denoted collectively by $\chi$. The recipe is the following:

i) replace $F_5 = dA_4$ with $F_5 = dA_4 + \tilde{C}_5(x)$ where $\tilde{C}_5$ is a 5-form representing the coupling between $A_4$ and $\chi$;

ii) add the Wess-Zumino term

$$S_{WZ} = \frac{1}{2} \int \tilde{C}_5 dA_4 = \frac{1}{2} \int F_5 dA_4;$$

(33)

iii) add the action of $\chi$ in absence of $A_4$, $S_\chi = L_{10}(\chi)$, where $L_{10}$ is a local $\chi$-dependent ten-form.
One can verify that the action

$$S = \int \left( \frac{1}{4} (F_5 \ast F_5 + f_4 \ast f_4) + \frac{1}{2} F_5 dA_4 + L_{10}(\chi) \right)$$

(34)

is invariant under the symmetries (31), (32) and, after gauge fixing, yields the selfduality condition $F_5^{(-)} = 0$.

For $D = 10$, IIB SUGRA, the matter fields $\chi$ are the graviton, the gravitinos, the complex fermion $\Lambda$, the complex tensor $A_2$ and the scalars. According to (3) and (8), $\tilde{C}_5$ is

$$\tilde{C}_5 = i(A_{2I} dA_{2}^I) + C_5,$$

(35)

where $C_5$ is defined in eq. (12).

The problem is now to find $L_{10}$ such that the action $S$ becomes invariant under $SU(1,1)$, under local $U(1)$ and local supersymmetry. Invariance under $SU(1,1)$ and $U(1)$ is automatic if $L_{10}$ is neutral with respect to $U(1)$ and contains only covariant derivatives and fields and forms which are scalars under $SU(1,1)$. Supersymmetry is more delicate. The suSy transformations, as given by the superspace approach, close only on–shell, which in principle is not a problem. But in the superspace approach ”on shell” includes also the self–duality condition $F_5^{(-)} = 0$ which, in our lagrangian approach, is not a field equation (it arises only after gauge fixing of the new symmetry (32)). The problem concerns, actually, only the susy trasformations of the gravitinos which, as mentioned above, are the only ones which involve $F_5^{(+)}$, the self–dual part of $F_5$. This means that the susy transformation of the gravitino is determined only modulo terms which are proportional to the self–duality condition. Eventually these terms have to be fixed such that the complete action becomes susy invariant. At first sight it is by no means obvious that such terms exist, but a detailed analysis leads to a quite simple solution of this problem. One has to assume that the auxiliary field $a(x)$ is invariant under supersymmetry – it is a non propagating field and has no fermionic partner – and in the (on–shell) susy trasformation of the gravitinos, one has to replace $F_5^{(+)}$ with

$$K_5 = F_5 + v f_4.$$

(36)

The five–form $K_5$ is an interesting object. It is selfdual, $K_5 = *K_5$, it coincides with $F_5^{(+)}$ if the selfduality condition (21) holds, it is invariant under the transformations (31) and it is proportional to the field equation (29) under (32).
Now we can write the action for $D = 10$, IIB SUGRA:

\[
S = \int E_{ab} R^{ab} + \frac{1}{3} E_{abc} (i \bar{\psi} \Gamma^{abc} D \psi + \text{c.c.}) + 4E_a (i \bar{\Lambda} \Gamma^a D \Lambda + \text{c.c.}) + \\
+ \frac{1}{4} (F_5 * F_5 + f_4 * f_4) + \frac{1}{2} F_5 dA_4 - \frac{i}{2} (A_{2I} dA_{I}) C_5 + \\
+ 2 \left[ \bar{F}_3 * F_3 + \left( C_7 \bar{F}_3 - \frac{1}{2} C_7 C_3 + \text{c.c.} \right) \right] + \\
+ 2 \left[ \bar{F}_1 * F_1 + \left( C_9 \bar{F}_1 - \frac{1}{2} C_9 C_1 + \text{c.c.} \right) \right] + \\
+ \left( \frac{1}{2} \bar{C}^{-\psi} C_3^\Lambda - 2 \bar{C^\Lambda} C^{-\psi}_3 + \text{c.c.} \right) + \frac{1}{2} C^\Lambda_5 C^{-\psi}_5 - 3E (\bar{\Lambda} \Gamma^a \Lambda) (\bar{\Lambda} \Lambda a), \quad (37)
\]

where

\[
E_{a_1...a_p} = \frac{1}{(10-p)!} \epsilon_{a_1...a_p b_1...b_{10-p}} e^{b_1}...e^{b_{10-p}}, \quad (38)
\]

and the $C_p$ are defined in (10)–(14). The first line of eq. (37) contains the Einstein action and the kinetic terms for the fermions (gravitinos and $\Lambda$). The second line is the action for the self-dual tensor. The third and the fourth lines describe the kinetic actions for the rank-two tensors and for the scalars respectively. The four–fermions interactions are given in the last line.

The covariant action $S$ is manifestly invariant under $SL(2, \mathbb{R})$, the local $U(1)$, the gauge symmetries of the potentials $A_2$ and $A_4$ and under the new local symmetries (31), (32). It yields the correct field equations and, after gauge fixing, the self–duality constraint for the chiral boson. It is also invariant under local supersymmetry, as it can be checked with a long calculation. The easiest way to do this is to lift to superspace all the forms which appear in $S$ (except the term $f_4 * f_4$), compute their covariant Lie derivative along $\epsilon$ and use the rheonomic parametrizations for the torsion and the curvatures. The term $f_4 * f_4$ cannot be lifted to superspace since it contains the auxiliary field $a(x)$ which is not a superfield ($\delta \epsilon a(x) = 0$) and its variation must be calculated by hand. It is remarkable that under local supersymmetry transformations, the terms involving $A_4$ (the second line of (37)) transform in expressions containing $F_5$ and $\nu^a$ only in the combination (36). This variation can thus be cancelled by making the replacement $F_5^{(+)} \mapsto K_5$ in the SUSY trasformation of the gravitinos. Let us mention that, extending the method proposed in [1], one can also write actions [4], where the basic fields, rank-two tensors and scalars and their duals, six-forms and eight-forms, appear in a symmetric way, as already shown in [10] for the case of $D = 11$.
supergravity.

Acknowledgements. We are grateful to P. Pasti and D. Sorokin for their interest in this work and useful discussions. This work was supported by the European Commission TMR programme ERBFMPX-CT96-0045 to which K.L. and M.T. are associated.

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