Early Thermalization at RHIC

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Abstract

Triple-gluon elastic scatterings are briefly reviewed since the scatterings explain the early thermalization puzzle in Au-Au collisions at RHIC energies. A numerical solution of the transport equation with the triple-gluon elastic scatterings demonstrates gluon momentum isotropy achieved at a time of the order of 0.65 fm/c. Triple-gluon scatterings lead to a short thermalization time of gluon matter.

Keywords: Triple-gluon elastic scatterings; transport equation; thermalization

1. Introduction

Hadron spectra at low momentum can be fitted to thermal distributions. The disappearance of angular correlation of back-to-back jets means that one jet is dissolved into medium and thermalized. The thermalization of matter created in Au-Au collisions at RHIC is obvious. However, relevant importance is that the initially created matter thermalizes within a time of less than 1 fm/c \cite{1,2} at least in the regime near midrapidity \cite{3}. The early thermalization is concluded while the results of hydrodynamic calculations \cite{1,3-7} are in agreement with the elliptic flow data \cite{8,9} at $p_\perp < 2$ GeV/c. Triple-gluon elastic scatterings were proposed to explain the early thermalization puzzle in Ref. \cite{10}.

Why the gluon-gluon-gluon elastic scattering processes are needed at RHIC energies? The three-gluon to three-gluon scatterings get important while the gluon number density is high. To account for RHIC experimental data the gluon rapidity density $dN^g/dy \sim 1000$ is suggested for the initial gluon matter \cite{11,12}. Such a value of the rapidity density leads to a gluon number density of about $38$ fm$^{-3}$, which is high enough for triple-gluon scatterings to occur. When the gluon number density gets larger, three-gluon scatterings
become more important. The three-gluon scattering processes are anticipated to thoroughly overwhelm the two-gluon elastic scattering processes in heavy ion collisions at LHC energies. An estimate from minijet production gives initial gluon number densities of about 30 fm$^{-3}$ for RHIC and 140 fm$^{-3}$ for LHC [13]. This supports the importance of the three-gluon scattering processes.

Fig. 1: Scatterings of three gluons.

2. Triple-gluon scatterings

The two-gluon to two-gluon scattering processes were studied in perturbative QCD by Cutler and Sivers [14], Combridge, Kripfganz and Ranft [15]. The spin- and color-averaged squared amplitude $|M_{2\to2}|^2$ for the two-gluon elastic scatterings at order $\alpha_s^2$ was expressed in terms of the Mandelstam variables. For the triple-gluon elastic scatterings, many diagrams have
to be calculated by fortran codes. Some of the three-gluon to three-gluon scattering diagrams are presented in Figs. 1 and 2 and corresponding spin- and color-averaged squared amplitude $|M_{3 \to 3}|^2$ is calculated in perturbative QCD. A gluon four-momentum is labeled as $p_i = (E_i, \vec{p}_i)$ in the process $g(p_1) + g(p_2) + g(p_3) \to g(p_4) + g(p_5) + g(p_6)$. Then nine Lorentz-invariant variables are defined as

$$s_{12} = (p_1 + p_2)^2, \quad s_{23} = (p_2 + p_3)^2, \quad s_{31} = (p_3 + p_1)^2$$

$$u_{15} = (p_1 - p_5)^2, \quad u_{16} = (p_1 - p_6)^2$$
$$u_{24} = (p_2 - p_4)^2, \quad u_{26} = (p_2 - p_6)^2$$
$$u_{34} = (p_3 - p_4)^2, \quad u_{35} = (p_3 - p_5)^2$$

These independent variables are appropriate for expressing $|M_{3 \to 3}|^2$.

Fig. 2: Scatterings of three gluons.

Only the diagram $B_{\sim \sim}$ in Fig. 2 is calculable by hand. This process consists of two four-gluon couplings and one gluon propagator between the
two vertices. The spin- and color-averaged squared amplitude \(| \mathcal{M}_{B \sim \sim} |^2\) for the diagram \(B \sim \sim\) then depends on the three variables \(s_{12}, s_{23}\) and \(s_{31}\) in the simple form

\[
| \mathcal{M}_{B \sim \sim} |^2 = \frac{59049 g_s^8}{128 (s_{12} + s_{23} + s_{31})^2}
\]  

(1)

which is independent of the momenta of the three final gluons. This ensures that each of the final gluons runs away in any direction in momentum space with equal probability. In other words, local momentum isotropy must be attained through the scattering process. Here, \(g_s\) is the quark-gluon coupling constant, \(g_s^2 = 4\pi\alpha_s\) and \(\alpha_s = 0.3\) is used.

3. Thermalization of gluon matter

Since the triple-gluon elastic scatterings are important in the initial gluon matter, a transport equation of Boltzmann type must include the \(3 \rightarrow 3\) scattering term,

\[
\frac{\partial f_1}{\partial t} + \vec{v}_1 \cdot \vec{\nabla} f_1 = -\frac{g_G}{2E_1g_{22}} \int \frac{d^3p_2}{(2\pi)^32E_2} \frac{d^3p_3}{(2\pi)^32E_3} \frac{d^3p_4}{(2\pi)^32E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)
\times | \mathcal{M}_{2 \rightarrow 2} |^2 \left[ f_1f_2(1 + f_3)(1 + f_4) - f_3f_4(1 + f_1)(1 + f_2) \right] \\
- \frac{g_{G}^2}{2E_1g_{33}} \int \frac{d^3p_2}{(2\pi)^32E_2} \frac{d^3p_3}{(2\pi)^32E_3} \frac{d^3p_4}{(2\pi)^32E_4} \frac{d^3p_5}{(2\pi)^32E_5} \frac{d^3p_6}{(2\pi)^32E_6} \\
\times (2\pi)^4 \delta^4(p_1 + p_2 + p_3 - p_4 - p_5 - p_6) | \mathcal{M}_{3 \rightarrow 3} |^2 \\
\times [f_1f_2f_3(1 + f_4)(1 + f_5)(1 + f_6) - f_4f_5f_6(1 + f_1)(1 + f_2)(1 + f_3)]
\]  

(2)

where \(g_{22} = 2, g_{33} = 12\), the degeneracy factor \(g_G = 16\) and the velocity of a massless gluon \(v_1 = 1\). The gluon distribution function \(f_i\) depends on the position \(\vec{r}_i\), the momentum \(\vec{p}_i\) and the time \(t\). The first term on the right-hand-side of the equation exhibits the well-known \(2 \rightarrow 2\) scattering processes. The second term is a new term, which represents the \(3 \rightarrow 3\) scatterings. Six gluon distributions are involved in the new term. The \(3 \rightarrow 3\) scattering processes involve a larger phase space than the \(2 \rightarrow 2\) scattering processes.

The squared amplitude \(| \mathcal{M}_{3 \rightarrow 3} |^2\) has a very much long expression and substantially enhance the running time of computer in solving the transport equation. To gain a first sight at the physics determined by the transport
equation, initially produced gluons are assumed to uniformly distribute in a cylinder formed in a central heavy ion collision. Consequently, the gluon distribution function is only a function of momentum magnitude and time. We solve the equation starting at the time $t_{\text{ini}}$ when gluon matter is created and ending at the time $t_{\text{iso}}$ when gluon matter reaches local momentum isotropy. The transverse expansion of the initially produced gluon matter is neglected due to a short thermalization time.

Fig. 3: Gluon distribution functions versus momentum in different directions while gluon matter is just produced in the initial Au-Au collision. The dotted, dashed and dot-dashed curves correspond to the angles $\theta = 0^\circ, 45^\circ, 90^\circ$, respectively. The solid curve stands for the thermal distribution function.

For a central Au-Au collision at $\sqrt{s_{NN}} = 200$ GeV, an initial gluon distribution for the transport equation is obtained from Eq. (34) of Ref. [16],

$$f(\vec{p}, t_{\text{ini}}) = \frac{1.71 \times 10^7 (2\pi)^{1.5}}{g_G \pi R_A^2 Y(|\vec{p}| / \cosh(y) + 0.3)} e^{-|\vec{p}|/(0.9\cosh(y))-(|\vec{p}|\tanh(y))^2/8\bar{\theta}(Y^2-y^2)}$$

where $|\vec{p}|$ is in GeV, the nuclear radius $R_A = 6.4$ fm and $y$ is the rapidity. The time $t_{\text{ini}}$ is 0.2 fm/$c$ as estimated in HIJING Monte Carlo simulation [17]. Here, $Y$ is the maximum of rapidity and approximately equals 5. $\bar{\theta}(Y^2-y^2)$ is the step function which is zero for $Y < y$ or 1 for $Y \geq y$. Eq. (3) exhibits one form of initial gluon distribution in gluon matter which is not in thermal and chemical equilibrium. We draw in Fig. 3 the dotted, dashed and dot-dashed curves individually for the gluon distribution functions against the gluon momentum at the three angles $\theta = 0^\circ, 45^\circ, 90^\circ$ relative to the incoming beam direction. These curves can not coincide and do not exhibit local
momentum isotropy. Particularly, the gluon momentum distribution in the transverse direction differs considerably from one in the longitudinal direction.

![Gluon distribution functions versus momentum in different directions](image)

**Fig. 4:** Gluon distribution functions versus momentum in different directions while gluon matter just arrives at thermal equilibrium. The dotted, dashed and dot-dashed curves correspond to the angles $\theta = 0^\circ, 45^\circ, 90^\circ$, respectively. The solid curve represents the thermal distribution function.

The difference among the initial gluon distribution functions in different directions does not last for a long time since the transport equation alters them. Momentum isotropy can be established at the time $t_{iso} = 0.65$ fm/$c$ as shown by the dotted, dashed and dot-dashed curves in Fig. 4. The gluon distribution functions can be well fitted to the Jüttner distribution with nonequilibrium fugacity $\lambda$,

$$f(\vec{p}, t_{iso}) = \frac{\lambda}{e^{\beta |\vec{p}|/T} - \lambda}$$

where $T$ gives the temperature of gluon matter. The solid curve in Fig. 4 gives a fit of $\lambda = 0.065$ and $T = 0.75$ GeV. We get the thermalization time $t_{iso} - t_{ini} = 0.45$ fm/$c$. The fugacity and the temperature are higher than 0.05 and 0.55 GeV, and the thermalization time is less than 0.5 fm/$c$ obtained from the free streaming of partons in Ref. [16], respectively.

4. **Summary**

The three-gluon scattering processes proposed to study the early thermalization problem at high density have been reviewed. The gauge-invariant squared amplitude $| \mathcal{M}_{3\to3} |^2$ for all the triple-gluon scatterings is given at
the lowest order $\alpha_s^4$. $|\mathcal{M}_{3\to 3}|^2$ enters the transport equation to give a new contribution to the time dependence of the gluon distribution function. The evolution of gluon matter is dominated by the three-gluon scattering processes while the number density is high. With the initial gluon matter obtained from HIJING simulation, the transport equation gives the thermalization time of about 0.45 fm/$c$. The three-gluon scatterings considerably shorten the thermalization time of gluon matter at high density.

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References

[1] U. Heinz, P.F. Kolb, in: R. Bellwied, J. Harris, W. Bauer (Eds.), Proc. of the 18th Winter Workshop on Nuclear Dynamics, EP Systema, Debrecen, Hungary, 2002.
[2] E.V. Shuryak, Nucl. Phys. A715(2003)289c.
[3] T. Hirano, Phys. Rev. C65(2001)011901.
[4] P. Huovinen, Nucl. Phys. A715(2003)299c.
[5] K. Morita, S. Muroya, C. Nonaka, T. Hirano, Phys. Rev. C66(2002)054904.
[6] D. Teaney, J. Lauret, E.V. Shuryak, [nucl-th/0110037](http://arxiv.org/abs/nucl-th/0110037).
[7] K.J. Eskola, et al., Phys. Lett. B566(2003)187;
    K.J. Eskola, et al., Nucl. Phys. A715(2003)561c.
[8] K.H. Ackermann, et al., STAR Collaboration, Phys. Rev. Lett. 86(2001)402;
    R.J. Snellings, et al., for the STAR Collaboration, Nucl. Phys. A698(2002)193c;
    C. Adler, et al., STAR Collaboration, Phys. Rev. Lett. 87(2001)182301;
    C. Adler, et al., STAR Collaboration, Phys. Rev. C66(2002)034904;
    C. Adler, et al., STAR Collaboration, Phys. Rev. Lett. 90(2003)032301.
[9] R.A. Lacey, et al., for the PHENIX Collaboration, Nucl. Phys. A698(2002)559c;
    S.S. Adler, et al., PHENIX Collaboration, Phys. Rev. Lett. 91(2003)182301.
[10] X.-M. Xu, Y. Sun, A.-Q. Chen, L. Zheng, Nucl. Phys. A744(2004)347.
[11] K.J. Eskola, K. Kajantie, K. Tuominen, Phys. Lett. B497(2001)39.
[12] M. Gyulassy, P. Lévai, I. Vitev, Nucl. Phys. B594(2001)371;
M. Gyulassy, I. Vitev, X.-N. Wang, P. Huovinen, Phys. Lett. B526(2001)301.
[13] F. Cooper, E. Mottola, G.C. Nayak, Phys. Lett. B555(2003)181.
[14] R. Cutler, D. Sivers, Phys. Rev. D17(1978)196.
[15] B.L. Combridge, J. Kripfganz, J. Ranft, Phys. Lett. 70B(1977)234.
[16] P. Lévai, B. Müller, X.-N. Wang, Phys. Rev. C51(1995)3326.
[17] X.-N. Wang, M. Gyulassy, Phys. Rev. D44(1991)3501;
    X.-N. Wang, M. Gyulassy, Comput. Phys. Commun. 83(1994)307;
    X.-N. Wang, Phys. Rep. 280(1997)287.