Correlating new physics signals in $B \to D^{(*)}\tau\nu_\tau$ with $B \to \tau\nu_\tau$

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Abstract

Semileptonic and purely leptonic decays of B meson to $\tau$, such as $B \to D^{(*)}\tau\nu_\tau$ and $B \to \tau\nu_\tau$ are studied. Recognizing that there already were some weak hints of possible deviations from the SM in the measurements of $B(B \to \tau\nu_\tau)$ by BABAR and Belle and the fact that detection of the $\tau$ also occurs in the measurements of $B \to D^{(*)}\tau\nu_\tau$, we stress the importance of joint studies of these processes, whenever possible. For this purpose, as an illustration, we introduce the observable, $R(D^{(*)})/B(B \to \tau\nu_\tau)$ where, for one thing, the unknown systematics due to $\tau$ identification are expected to largely cancel. We show that all measurements of this observable are consistent with the existing data, within somewhat largish experimental errors, with the predictions of the SM. We stress that precise experimental measurement and comparison with theory of the branching ratio for $B \to \tau\nu_\tau$ is extremely important for a reliable search of new physics. Furthermore, in view of the anticipated improved precision in experiments in the next few years, in addition to $R(D^{(*)})$, host of other ratios analogous to $R(D^{(*)})/B(B \to \tau\nu_\tau)$ in the SM are suggested for lattice calculations as well, so that for more stringent tests of the SM, correlations in lattice calculations can be properly taken into account to enhance precision.

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1. Introduction

The observed excess in the branching fractions of the semitaunic decays, $B \to D\tau\nu_\tau$ and $B \to D^{(*)}\tau\nu_\tau$, has drawn a lot of attention in the recent years. The present experimental status is summarized in Fig. 1 [1].

Here, $R(D)$ and $R(D^{*})$ are defined as

\begin{align}
R(D) &= \frac{B(B \to D\tau^-\nu_\tau)}{B(B \to Dl^-\nu_l)}, \\
R(D^{*}) &= \frac{B(B \to D^{*}\tau^-\nu_\tau)}{B(B \to D^{*}l^-\nu_l)}. \quad (1)
\end{align}

As had been emphasized in several works [2, 3, 4, 5, 6, 7], the theory uncertainties in these observables are only a few percent, being independent of the CKM element $|V_{cb}|$ and also to a large extent, of the form-factors [1]. Interestingly, the $R(D^{(*)})$ values measured by BABAR [9] exceed SM expectations.

We note here the extremely small (SM) theory error quoted in $R(D^{(*)})$ (a lot smaller than that of $R(D)$) and emphasize that so far no lattice calculation of the ratio involving

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by 2.0σ and as much as 2.7σ respectively, and if taken together, disagree with the SM by about 3.4σ. 

On the other hand, older Belle results used to lie in between the SM expectation and the BaBar measurement and were consistent with both \(^{10}\) \(^{11}\), but $^{10}$LHCb announced the results of their first measurement of $\mathcal{R}(D^{(*)}))$, which uses both the leptonic and hadronic channels for the identification of $\tau$ reconstruction.

Moreover, since $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ are independent of $|V_{cb}|$, we emphasize the importance of analogous ratios for semileptonic decays $B \rightarrow \pi(p, \omega) f(\tau) \nu$ and similarly for $B \rightarrow D^{(*)} f(\tau) \nu$ (see eq.(14)).

2. SM vs. Experiments

2.1. $B(B^+ \rightarrow \tau^+ \nu)$

In Tables 1 and 2, the BaBar and Belle measured values of the $\text{Br}(B \rightarrow \tau \nu)$ are shown, using leptonic and hadronic decays of $\tau$ separately. Their combined results are also shown, and they are consistent with each other within errors.

| Decay Mode | $\alpha_b \times 10^{-4}$ | Signal yield | $\mathcal{B} \times 10^{-4}$ |
|------------|------------------------|-------------|-----------------|
| $\tau^+ \rightarrow e^+ \nu \nu$ | 2.47 ± 0.14 | 4.1 ± 9.1 | 0.35 ± 0084 |
| $\tau^+ \rightarrow \mu^+ \nu \nu$ | 2.45 ± 0.14 | 12.9 ± 9.7 | 1.13 ± 0056 |
| Leptonic | 4.92 ± 0.198 | 17 ± 13.3 | 0.739 ± 0.578 |
| $\tau^+ \rightarrow \pi^+ \nu$ | 0.98 ± 0.14 | 17.1 ± 6.2 | 3.69 ± 0022 |
| $\tau^+ \rightarrow \rho^+ \nu$ | 1.35 ± 0.11 | 24.0 ± 10.0 | 3.78 ± 0045 |
| Hadronic | 2.33 ± 0.178 | 41.1 ± 11.77 | 3.77 ± 1.12 |
| combined | 62.1 ± 17.3 | 1.83 ± 0.49 |

Table 1: The measured values of $\text{Br}(B^+ \rightarrow \tau \nu)$ by BaBar in various $\tau$ decay modes, and their combined value for $N_{B^\pm} = (467.8 \pm 5.1) \times 10^6$ \(^{14}\).

The expression for the branching fraction $\text{Br}(B \rightarrow \tau \nu)$ in the SM is given by

$E_{SM}(B^+ \rightarrow \tau^+ \nu) = \frac{G_F^2 m_B m_{\tau}^2}{8\pi} \left[ 1 - \frac{m_{\tau}^2}{m_B^2} \right] f_B^2 |V_{ub}|^2 \tau_{B^+}$, \((2)\)

where $G_F$ is the Fermi constant, $m_B$ and $m_{\tau}$, are the $B^+$ meson and $\tau$ lepton masses respectively,
\[
\tau_{B^+} = \text{the } B^+ \text{ meson lifetime. The branching fraction is sensitive to the } B \text{ meson decay constant } f_B \text{ and the Cabibbo-Kobayashi-Maskawa (CKM) matrix element } |V_{ub}|. \text{ With the numerical values of all the relevant parameters listed in table 3, we obtain}
\]
\[
B_{SM}(B^+ \rightarrow \tau^+ \nu_{\tau}) = (0.947 \pm 0.182) \times 10^{-4}. \quad (3)
\]

Numerical value of the CKM element |\(V_{ub}\)\| is obtained after fitting latest lattice calculation of \(B \rightarrow \pi l\nu\) form factors with the experimental measurements of the branching fraction from \(\text{BABAR}\) and \(\text{Belle}\), leaving the relative normalization as a free parameter, for details see Ref. \[16\] [17]. Here, we use |\(V_{ub}\)| i.e |\(V_{ub}^{\text{ME}}\)| with a more conservative error from \[16\] than that of HFAG or PDG (see table 3 or from \[17\]). Part of the reason we now feel more confident about this exclusive value of \(V_{ub}\)\| in that is found to be in excellent agreement with the value determined from exclusive baryonic B-decays \[18\].

In Fig. 2 the experimental measurements on \(Br(B \rightarrow \tau^+ \nu_{\tau})\) are compared with the SM predictions. In this figure, for the sake of completion, we also show the estimated branching fraction using the inclusive measurement of |\(V_{ub}\)|, i.e |\(V_{ub}^{\text{In}}\)| (table 3). The corresponding value of the branching fraction is given by.

\[
B_{SM}^{\text{In}}(B^+ \rightarrow \tau^+ \nu_{\tau}) = (1.413 \pm 0.175) \times 10^{-4}. \quad (4)
\]

From Fig. 2, we see that Belle measurement is roughly consistent with the SM irrespective of which \(V_{ub}\)\| (inclusive or exclusive) is used and \(\text{BABAR}\) measurement just mildly disfavors the SM when exclusive \(V_{ub}\)\| is used. On the other hand, if we consider only modes with leptonically reconstructed \(\tau\), \(\text{BABAR}\) is consistent with both \(V_{ub}\)\| values and Belle measurement slightly disfavors the inclusive \(V_{ub}\)\|.

### Table 2: The measured values of \(B(B^+ \rightarrow \tau^+ \nu_{\tau})\) by \(\text{Belle}\) in various \(\tau\) decay modes, and their combined value for \(N_{B^+ B^-} = 772 \times 10^3\) \[15\].

| Decay Mode     | \(\epsilon_k \times 10^{-4}\) | Signal yield | \(B \times 10^{-4}\) |
|----------------|------------------------------|--------------|---------------------|
| \(\tau^+ \rightarrow e^+ \nu_e \bar{\nu}_e\) | 6.8                          | 47 ± 25      | 0.90 ± 0.47         |
| \(\tau^+ \rightarrow \mu^+ \nu_{\mu} \bar{\nu}_{\mu}\) | 5.1                          | 13 ± 21      | 0.34 ± 0.55         |
| Leptonic       | 11.9                         | 60 ± 32.65   | 0.653 ± 0.355       |
| \(\tau^+ \rightarrow \pi^+ \nu_{\pi} \bar{\nu}_{\pi}\) | 4.0                          | 57 ± 21      | 1.82 ± 0.68         |
| \(\tau^+ \rightarrow \pi^0 \nu_{\pi} \bar{\nu}_{\pi}\) | 7.2                          | 119 ± 33     | 2.16 ± 0.66         |
| Hadronic       | 11.2                         | 176 ± 39.12  | 2.036 ± 0.452       |
| combined       | 23.1                         | 222 ± 50     | 1.25 ± 0.28         |

Figure 2: Graphical representation of the data shown in tables \[1\] and \[2\].

#### 2.2. \(\mathcal{R}(D^{(*)})\) and \(R_{\ell}(D^{(*)})\)

For the measurements of branching fractions \(Br(B \rightarrow D^{(*)} \tau \nu_{\tau})\), both \(\text{BABAR}\) \[9\] and \(\text{Belle}\) \[10\] use purely hadronic tagging of “the other B” and purely leptonic \(\tau\) decays coming from the \(B\) undergoing semi-tauonic decay. In one of their analyses (in 2016), Belle had used semi-leptonic tagging of “the other B” and measured \(\mathcal{R}(D^*)\) \[11\]. They are yet to publish their result on \(\mathcal{R}(D)\) using the same semileptonic tagging method for the other \(B\). Most recently, \(\text{Belle}\) has published another result on \(\mathcal{R}(D^*)\) along with their first measurement of \(R_{\ell}(D^*)\) \[12\] with the total available dataset of 772 million \(BB\) pairs. In that analysis, they have used hadronic \(\tau\) decays for \(\tau\) reconstruction and hadronic tagging for the other \(B\) (\(B_{tag}\)). In table 4 we list those measured values of \(\mathcal{R}(D)\) and \(\mathcal{R}(D^*)\) along with their SM predictions. The same data has been plotted in Fig 3. We note that like \(B(B \rightarrow \tau \nu_{\tau})\) the \(\text{Belle}\) 2015 data is consistent with the SM prediction, while the \(\text{BABAR}\) data exceeds the SM expectations by \(2.0\sigma\) and as much as \(2.7\sigma\) respectively for \(\mathcal{R}(D)\) and \(\mathcal{R}(D^*)\). On the other hand, while \(\text{Belle}\) measurement of \(\mathcal{R}(D^*)\)(2016) with leptonically tagged \(\tau\)s \[11\] is away from the corresponding SM prediction by \(1.6\sigma\), the most recent measurement with the full dataset is consistent with SM within \(0.6\sigma\) \[12\].

As mentioned in the introduction, to study the possibility of correlation in \(\tau\) decays affecting the analyses and also for other potentially useful pur-
Table 3: Input parameters used in obtaining theory predictions.

| Parameters | Values |
|------------|--------|
| $f_B$      | $0.191 \pm 0.007$ GeV $^{19,20}$ |
| $G_F$      | $1.1663787(6) \times 10^{-5}$ GeV$^{-2}$ $^{21}$ |
| $m_B$      | $5.27929 \pm 0.00015$ GeV $^{22}$ |
| $m_\tau$  | $1.77686 \pm 0.00012$ GeV $^{23}$ |
| $\tau_B^+$| $1.638(4)$ ps$^{-1}$ $^{24}$ |
| $\tau_{D^0}$ | $1.520(4)$ ps$^{-1}$ $^{24}$ |
| $|V_{ub}^{Exp}|$ | $(3.61 \pm 0.32) \times 10^{-3}$ $^{16}$ |
| $|V_{ub}|$   | $(4.41 \pm 0.15^{+0.15}_{-0.19}) \times 10^{-3}$ $^{25}$ |
| $|V_{cb}|$   | $(42.21 \pm 0.78) \times 10^{-3}$ $^{26}$ |
| $m_0(\mu = m_b)$ | $4.20 \pm 0.07$ GeV $^{27}$ |
| $m_c(\mu = m_b)$ | $0.901^{+0.113}_{-0.111}$ GeV $^{27}$ |
| $m_u$      | $0.00236 \pm 0.00024$ GeV $^{28}$ |
| $\lambda_1$ | $-0.15 \pm 0.15$ $^{29}$ |
| $\lambda_2$ | $0.12 \pm 0.01$ $^{29}$ |

Table 4: The SM predictions and the experimentally measured values of $R(D^*)$. For experimental results, the first uncertainty is statistical and the second one is systematic. The third row represents the Belle results published in 2015 $^{10}$. The fourth row represents Belle’s 2016 results of $R(D^*)$ $^{11}$, wherein the tagging method for “the other B” is different from the previous analysis. The fifth row represents Belle’s most recent results of $R(D^*)$ $^{12}$ with total available dataset.

| $R(D)$       | $R(D^*)$       |
|--------------|---------------|
| SM           | $0.300 \pm 0.008$ $^9$ |
| LHCb         | $0.276 \pm 0.034^{+0.039}_{-0.035}$ $^{12}$ |
| Belle(2016)  | $0.357 \pm 0.064 \pm 0.026$ $^{11}$ |
| Belle(2016, Tot. Data.) | $0.302 \pm 0.030 \pm 0.011$ $^{13}$ |
| BABAR        | $0.440 \pm 0.058 \pm 0.042$ $^{27}$ |
| Belle(2015)  | $0.299 \pm 0.003$ $^{10}$ |
| Belle(2016)  | $0.299 \pm 0.003$ $^{28}$ |
| LHCb         | $0.336 \pm 0.027 \pm 0.030$ $^{13}$ |

Figure 3: Graphical representation of the data given in Table 3. The Dark gray SM bands use the data from the first row of the table. The three vertical lines for Belle data of $R(D^*)$ are, respectively, (from right to left) Belle(2015)$^{10}$ (dark blue), Belle(2016)$^{11}$ (green) and latest Belle data(2016) with full dataset$^{12}$ (cyan).

poses, here we define a new observable $R_\tau(D^{(*)})$ as

$$R_\tau(D^{(*)}) = \frac{\mathcal{R}(D^{(*)})}{\mathcal{B}(B^+ \to \tau^+\nu_\tau)}$$

where $\mathcal{R}(D^{(*)})$ is normalized by $\mathcal{B}(B^+ \to \tau^+\nu_\tau)$.

In order to explicitly spell out the possible cancellation of $\tau$ systematics in this ratio, $R_\tau(D^{(*)})$ can be defined as explained below.

The definition of $\mathcal{R}(D^{(*)})$ as used in the experimental analyses is the average of all the $\mathcal{R}(D^{(*)})^i$, which are given as

$$\mathcal{R}(D^{(*)})^i = \frac{1}{N_\tau} \frac{N_{\text{sig(norm)}}}{N_{\text{Norm}} \epsilon_{\text{sig}}},$$

where $\mathcal{B}^i_\tau$ represents the branching fraction of the $i^{th}$ decay channel in which $\tau$ has been reconstructed. $N_{\text{sig(norm)}}$ and $\epsilon_{\text{sig(norm)}}$ represent the signal(normalization) events and the reconstruction efficiencies respectively.

On the other hand, the branching fraction in $B \to \tau\nu_\tau$ as defined in experimental analysis is given by

$$Br(B \to \tau\nu_\tau)^i = \frac{N_\tau}{2\epsilon^i_\tau N_{B^+}}.$$

where $\epsilon^i_\tau$ represents efficiency including the branching fraction of the $i^{th}$ decay mode of $\tau$, which is
defined as the average of all the reconstructed decay branching fractions to the number of fully surviving all the selection criteria including the determined by the ratio of the number of events in the last four rows are obtained using correlation.

Table 5: Our estimated values of $\tau$ using $R_D(D^{(*)})$. The values in the last four rows are obtained using corresponding results on $R(D^{(*)})$ listed in table 4. For the SM value, we use the first row of 4.

| SM (With $V_{ub}^{k}$) | $R_{s}(D^{(*)}) \times 10^3$ | $R_{d}(D^{(*)}) \times 10^3$ |
|------------------------|-----------------------------|-----------------------------|
| $3.17 \pm 0.01$        | $2.66 \pm 0.01$          |
| $2.12 \pm 0.07$        | $1.78 \pm 0.22$          |
| $5.96 \pm 2.26$        | $4.49 \pm 3.54$          |
| Belle(2015, Leptonic Tag) | $5.7 \pm 3.3$           | $4.49 \pm 3.54$           |
| Belle(2016, Leptonic Tag) | $-1.36 \pm 0.37$       |
| Belle(2016, Total Dataset, - Hadronic Tag) | $-4.62 \pm 2.56$       |

As the current measurements of $B \rightarrow \tau \nu$ have rather large errors, it may well be that for now the ratio is hiding NP beneath the errors; however, we are stressing its long term use as more data becomes available. Also, as alluded to before, we want to emphasize again that the consistency with the SM in Fig. 4 or table 5 does not necessarily mean that it rules out presence of new physics. It just means that the effects of new physics, if there, largely cancel in the ratios. Later we will illustrate this with a particular example of new physics, i.e., type-II 2HDM.

2.3. $\mathcal{R}(X_{c})$ and $\mathcal{R}_{c}(X_{c})$

We now consider the inclusive decay channel $B \rightarrow X_{c} \tau \nu$, along with the exclusive channels $B \rightarrow D^{(*)} \tau \nu$, discussed earlier. If there is NP in $b \rightarrow c \tau \nu_{c}$, it should show up in both the exclusive and inclusive channels. Inclusive semileptonic decays are theoretically clean compared to the respective exclusive decays, but experimentally challenging. The forthcoming experiments like Belle-II may allow a precise measurement of the branching fraction of $B \rightarrow X_{c} \tau \nu$. One potential advantage of the inclusive mode is that its branching ratio is expected to be larger than the exclusive modes. In a B-factory environment, as in Belle-II, the experimental detection may be facilitated by (partial) reconstruction of the “other B”. Unfortunately, at LHCb the inclusive measurements are always very challenging.

The SM expression for the differential decay rate of inclusive $\mathcal{B} \rightarrow \tau \nu X_{c}$ transitions including the power corrections at order $1/m_{b}^{2}$ in heavy quark effective theory (HQET) is [22, 33]
where \( q^2 = q^2/m_B^2 \), \( q = p_\tau + p_\nu \) is the dilepton momentum, \( x_\tau = m_\tau^2/q^2 = \rho_\tau/\tilde{q}^2 \), \( \rho = m_\tau^2/m_B^2 \), \( P = 1 - \tilde{q}^2 + \rho \), \( \lambda_1 \) and \( \lambda_2 \) parametrize the leading non-perturbative corrections of relative order \( 1/m_B^2 \).

Integrating this over the range \( \rho_\tau < \tilde{q}^2 < (1 - \sqrt{2})^2 \) gives us the total decay rate \( \Gamma_W \).

Like \( R(D) \), the ratio of inclusive decay rates is defined as

\[
R(X_c) = \frac{B(B \to X_c\tau\bar{\nu})}{B(B \to X_c\ell\bar{\nu})} \tag{9}
\]

and its SM value, considering the current world average \( B(B^- \to X_c\ell\bar{\nu}) = (10.92 \pm 0.16)\% \) [34, 35], is given by \( R(X_c)_{SM} = 0.225 \pm 0.006 \). In order to estimate \( R(X_c)_{exp} \), we take the ratio of the LEP average \( B(b \to X_c\tau^+\bar{\nu})_{lep} = (2.41 \pm 0.23)\% \) [36] and the world average for \( B(B^- \to X_c\ell\bar{\nu}) \), and we obtain \( R(X_c)_{exp} = 0.221 \pm 0.021 \).

Like \( R(D^{(*)}) \), we define \( R_c(X_c) \) by normalizing \( R(X_c) \) with \( B(B^+ \to \tau^+\nu_\tau) \). In Fig. 5 different values of \( R_c(X_c) \) are shown which are obtained for different values of the \( B(B^+ \to \tau^+\nu_\tau) \) taken from Babar and Belle measurements (for channels with leptonically tagged \( \tau \)s in Fig. 2). We note that the estimated values obtained using both the measurements are compatible with each other, also both of them are consistent with the SM. Again, as in the case of exclusive modes this does not necessarily mean that presence of all types of new physics are being ruled out.

2.4. \( R(\pi) \) and \( R_\pi^\mu \)

Since \( R(D^{(*)}) \) is independent of \( |V_{ub}| \), if the interpretation of new physics there is correct, then we should expect similar deviations in analogous semileptonic decays \( B \to \pi(\rho, \omega) \ell(\tau) \nu \) and similarly in \( B_s \) decays. Therefore, in addition to the above modes we also consider the decay \( B \to \pi\tau\nu_\tau \).

Earlier, in the literature this mode is considered for NP searches [37, 38, 39, 40], while \( B \to \pi(\mu, e) \), with \( \ell = \mu \) or \( e \), is used for the extraction of CKM element \( V_{ub} \) [41, 42]. The useful observable which is potentially sensitive to NP is defined as

\[
R(\pi) = \frac{B(B \to \pi\tau\bar{\nu}_\tau)}{B(B \to \pi\ell\bar{\nu}_\ell)} \tag{10}
\]

where the dependence and therefore the uncertainty due to \( V_{ub} \) cancels in the ratio; similarly, \( R(\rho, \omega) \) should also be studied.
Figure 5: SM (horizontal gray bands) estimation of $R_\tau(X_c)$ juxtaposed with experimental measurements (vertical bars).

In the SM, the differential decay rate for the decay $B \to \pi\tau\ell_\nu$ is given as [10]:

$$
\frac{d\Gamma(B \to \pi\tau\ell_\nu)}{dq^2} = \frac{8|\vec{p}_\pi|}{3} \frac{G_F^2 |V_{ub}|^2 q^2}{256\pi^3 m_B^2} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \left[H_0^2(q^2) \left(1 + \frac{m_\tau^2}{2q^2}\right) + \frac{3m_\tau^2}{2q^2} H_t^2(q^2)\right],
$$

where, $q$ is the four-momentum transfer between the B-meson and the final-state pion of the semileptonic decay, $|\vec{p}_\pi|$ is the absolute three-momentum of the final state pion,

$$
|\vec{p}_\pi| = \sqrt{\frac{m_B^2 + m_\pi^2 - q^2}{2m_B}}\left(m_\tau^2 - \frac{q^2}{2}\right)^2 - m_\tau^2
$$

and $H_{0/t}$ are helicity amplitudes defined as

$$
H_0 = \frac{2m_B|\vec{p}_\pi|}{\sqrt{q^2}} f_+(q^2)
$$

$$
H_t = \frac{m_\tau^2 - m_\pi^2}{\sqrt{q^2}} f_0(q^2).
$$

The form factors $f_{+/0}$ need to be calculated using non-perturbative methods, such as the lattice [16][17]. Setting $m_\tau$ to zero in eq.\ref{eq:rate} gives us the expression for $d\Gamma(B \to \pi\ell_\nu)/dq^2$ to an excellent precision. Taking the BCL coefficients and their correlations from ref.\cite{39}, we calculate $R(\tau)^{SM} = 0.598 \pm 0.024$. The error is around 4%, which is only slightly larger than the value quoted in that paper (0.641 ± 0.016) or essentially the same result of \cite{43}. Recent result from Belle \cite{44} gives us an upper limit on $B(B^0 \to \pi^-\ell^+\nu_\ell) < 2.5 \times 10^{-4}$. Dividing this with the present world average of $B(B^0 \to \pi^-\ell^+\nu_\ell) = (1.45 \pm 0.05) \times 10^{-4}$ \cite{45}, we get the upper limit of $R(\tau) < 1.784$.

We now introduce a different observable than our previous normalized ratio in eq.\ref{eq:ratio}:

$$
R_\tau = \frac{B(B \to \pi\tau\ell_\nu)}{B(B \to \tau\ell_\nu)},
$$

for which the SM prediction is $R_\tau^{SM} = 0.733 \pm 0.144$; the error is around 20%. We define $R_\tau$ in this way instead of $R(\tau)/B(B^+ \to \tau^+\nu_\tau)$ as in the former definition the dependence due to $V_{ub}$ cancels. In the latter definition the dependence on $V_{ub}$ will remain, though the error in the SM is still around 20%. Let us note in passing that a ratio analogous to eq.\ref{eq:ratio} in case of $B \to D(\pm)\tau\nu$ decays can also be useful.

Using the combined Belle result for $B(B \to \pi\nu_\ell)(table\ref{table:results})$, and the upper limit for $R(\tau)$ quoted above, we obtain the upper limit for $R_\tau < 2.62$.

3. Type II 2HDM Model

The 2HDMs with two complex Higgs doublets are amongst the simplest extensions of the SM which gives rich phenomenology due to the additional scalar bosons. The extended Higgs sectors have not yet been ruled out experimentally. The new features of the 2HDM includes three neutral scalar bosons. The extended Higgs sectors are amongst the simplest extensions of the SM.
In two Higgs doublet models, purely leptonic decays receive an additional contribution from charged Higgs, which can be factorized from the SM prediction [48 49 50 51 52 53].

$$B(B \rightarrow l \nu) = B(B \rightarrow l \nu)_{SM} (1 + r_H)^2.$$  \hspace{1cm} (15)

In 2HDM-II, the factor $r_H$ is given as,

$$r_H = \left( \frac{(m_u/m_b) - \tan^2 \beta}{1 + (m_u/m_b)} \right) \left( \frac{m_B}{m_{H^+}} \right)^2.$$  \hspace{1cm} (16)

Here, $\tan \beta$ is the ratio of the vacuum expectation values of the two Higgs doublets, $m_{H^+}$ is the mass of the charged Higgs and $m_u/m_b = (0.56 \pm 0.06) \times 10^{-3}$ [25] is the ratio of the $u$- and $b$-quark masses at a common mass scale.

The contributions of the charged Higgs to $B \rightarrow D^{(+)} \tau^\pm \bar{\nu}_\tau$ decays can be encapsulated in the scalar helicity amplitude in the following way [4, 37]:

$$H_s^{2HDM} \approx H_s^{SM} \times \left(1 - \frac{\tan^2 \beta}{m_{H^+}^2} \frac{q^2}{1 \mp m_c/m_b} \right).$$  \hspace{1cm} (17)

The denominator of the second term of the above equation contain $(1 \mp m_c/m_b)$, where the negative and positive signs are applied to $B \rightarrow D \tau^- \bar{\nu}_\tau$ and $\overline{B} \rightarrow D^* \tau^- \bar{\nu}_\tau$ decays, respectively. Here, $m_c/m_b = 0.215 \pm 0.027$ [27] is the ratio of the $c$- and $b$-quark masses at a common mass scale. Thus, the differential decay rate, integrated over angles, becomes [4, 2, 54],

$$\frac{d\Gamma \tau}{d q^2} = \frac{G_F^2 |V_{cb}|^2 |p_{D^{(*)}}|}{96\pi^3 m_B} \left( 1 - \frac{m_{D}^2}{q^2} \right)^2 \left( |H^+_\tau| \right)^2$$

$$\left. + |H^-|^2 + |H_0|^2 \right) \left( 1 + \frac{m_c^2}{2q^2} \right) \left( \frac{3m_b^2}{2q^2} \right) |H_s|^2.$$  \hspace{1cm} (18)

where $|p_{D^{(*)}}|$ is the three-momentum of the $D^{(*)}$ meson in the $B$ rest frame. Given that charged Higgs bosons are not expected to contribute significantly to $B \rightarrow D \tau^- \bar{\nu}_\tau$ decays, $R(D^{(*)})_{2HDM}$ can be described by a parabola:

$$R \left( D^{(*)} \right)_{2HDM} = R \left( D^{(*)} \right)_{SM} + A_{D^{(*)}} \frac{\tan^2 \beta}{m_{H^+}}$$

$$+ B_{D^{(*)}} \frac{\tan^4 \beta}{m_{H^+}^4},$$  \hspace{1cm} (19)

where,

$$A_D = -3.25 \pm 0.32, \quad A_{D^*} = -0.230 \pm 0.029$$

$$B_D = 16.9 \pm 2.0, \quad B_{D^*} = 0.643 \pm 0.085.$$
Figure 6: The allowed parameter spaces in 2HDM-II, which is obtained from $\mathcal{R}(D)$, $\mathcal{R}(D^*)$ and $\mathcal{B}(B \rightarrow \tau \nu)$ using the BaBar (left) and Belle (right) data. There is no common parameter space in 2HDM-II which satisfy simultaneously all the three excesses given by BaBar. However, there are common parameter spaces which are obtained as simultaneous solutions to all the three excess given by Belle data. The dotted vertical lines show $m_{H^+} = 540$ GeV.

Figure 7: Variations of $R_\tau(D)$ (left) and $R_\tau(D^*)$ (right) with the 2HDM-II parameter $r = \tan \beta / m_{H^+}$ for different values of $\tan \beta$. The 1σ experimental ranges are shown by the dotted (BaBar) and dashed (Belle) horizontal lines.
with BABAR

\[ R_\tau(D) \]

\[ R_\tau(D^*) \]

\[ m_H^+ \]

\[ \tan(\beta) \]

\[ \text{BABAR} \]

\[ \text{Belle} \]

\[ R_\tau(D) \]

\[ R_\tau(D^*) \]

\[ \text{Leptonic \tau Tag} \]

\[ \text{Hadronic \tau Tag} \]

\[ 0 \leq m_H^+ \leq 1000 \]

\[ \tan(\beta) = 5 \]

\[ \tan(\beta) = 20 \]

\[ \tan(\beta) = 35 \]

\[ \text{BABAR} \]

\[ \text{Belle} \]

\[ 0 \leq m_H^+ \leq 1000 \]

\[ 0 \leq r \leq 0.4 \]

\[ r > 0.4 \]

\[ 0.4 \leq m_H^+ < 1000 \]

\[ R_{\tau}(X_c) \]

\[ \text{2HDM-II} \]

\[ R_{\tau}(D) \]

\[ R_{\tau}(D^*) \]

\[ 0.2 \leq m_H^+ < 1000 \]

\[ 540 \leq m_H^+ \leq 1000 \]

\[ \text{LEP} \]

\[ \text{World Avg.} \]

\[ \text{HDM} \]

\[ \Gamma_W \]

\[ \hat{q}_2 \]

\[ \text{ref. [29]} \]

Figure 8: The allowed regions in \( \tan \beta - m_{H^+} \) parameter space, which are obtained as simultaneous solutions to \( R_\tau(D) \) and \( R_\tau(D^*) \) using the data given in table 5. The dotted vertical line shows \( m_{H^+} = 540 \text{ GeV} \).

Figure 9: Variation of \( R(X_c) \) with \( r = \tan \beta / m_{H^+} \) (blue region between dotted curves). Experimental range is shown by the orange region enclosed by dot-dashed horizontal lines. \( r = 0 \) corresponds to SM prediction for \( R(X_c) \).

Figure 10: Variation of \( R(X_c) \) with \( r = \tan \beta / m_{H^+} \) for different values of \( \tan \beta \) while \( 380 < m_{H^+} < 1000 \). Experimental ranges are shown by dotted (BABAR) and dashed (Belle) horizontal lines.

where, \( R = r^2 m_c m_b \), \( r = \tan \beta / m_{H^+} \), and the subindices \( W, H \), and \( I \) denote the \( W \) mediated(SM), Higgs mediated and interference contributions, respectively. \( \Gamma_W \) is given by the \( \hat{q}_2 \) integrated form of eq.(8). Other terms are listed in ref. [29].

Figure 11 represents the variation of \( \mathcal{R}(X_c) \) with \( r \). We note that the current data allows only the region \( r \leq 0.4 \), the region \( r > 0.4 \) is not allowed by the data.

The variations of \( \mathcal{R}_\tau(X_c) \) in 2HDM-II with the parameter \( r \) for various values of \( \tan \beta \) are shown in Fig. 10. Here too the \( m_{H^+} \) is varied in between \([540, 1000]\) as before. Also, in this case we note that
Figure 11: Allowed parameter space for $\tan \beta$ and $M_{H^+}$ obtained from the analysis of $R_\tau(X_c)$ in 2HDM-II. The dotted vertical line shows $m_{H^+} = 540$ GeV.

Figure 12: Variation of $R_\pi^\tau$ with $r = \tan \beta/m_{H^+}$ for different values of $\tan \beta$ while $540 < m_{H^+} < 1000$. The dotted horizontal line shows the experimental upper limit (section 2.4).

Figure 13: Allowed parameter space for $\tan \beta$ and $M_{H^+}$ obtained from the analysis of $R_\tau(\pi)$ in 2HDM-II.

the large values of $\tan \beta$ ($\gtrsim 30$) are not allowed by the current data. The experimental constraints in the $\tan \beta - m_{H^+}$ plane, obtained from the analysis of this observable is shown in Fig. 11.

3.3. $R_\tau^\pi$

The contributions of the charged Higgs Boson to $B \to \pi \tau \bar{\nu}_\tau$ decays can be incorporated into Eq. (11) by the replacement [37, 4]

$$H_t \to H_t^{SM} \times \left(1 - \frac{\tan^2 \beta}{m_{H^+}^2} \frac{q^2}{1 - m_u/m_t}\right).$$ (21)

So, just like eq. (19), $\mathcal{B}(B \to \pi \tau \bar{\nu}_\tau)$ can be described as a parabola,

$$\mathcal{B}(\pi)_{2HDM} = \mathcal{B}(\pi)_{SM} + A_\pi \frac{\tan^2 \beta}{m_{H^+}^2} + B_\pi \frac{\tan^4 \beta}{m_{H^+}^4},$$ (22)

where,

$$A_\pi = (-0.389 \pm 0.164) \times 10^{-3}$$

$$B_\pi = (0.418 \pm 0.258) \times 10^{-2}$$ (23)

Figures 12 and 13 show the Type-II 2HDM parameter space corresponding to the experimental upper limit given in section 2.4. We note that in this case $\tan \beta$ as large as 100 is allowed by the current data.

4. Summary & Outlook

Motivated by the reported indications of new physics signals in the experimental results from BABAR, Belle and LHCb in the ratio, $\mathcal{R}(D^{(*)})$, of semileptonic decays, in here we examine them along with $B \to \tau \nu_\tau$ decays. Since $\tau$ detection plays a central role in both categories, and because backgrounds in $B \to D^{(*)} \tau \nu$ are very different from those in $B \to \tau \nu$, it seems very useful to examine them both simultaneously whenever the data allows. Concretely, we define a new observable, $R_\tau(D^{(*)}) \equiv \mathcal{R}(D^{(*)})/\mathcal{B}(B \to \tau \nu_\tau)$. In this observable the (unknown) systematics, if any, due to the $\tau$ identification are expected to largely cancel. Our
analysis shows that this observable is remarkably consistent with SM with the data from both BaBar and Belle even though appreciable differences from the SM were reported in $R(D^{(*)})$ especially by BaBar. Since at present the errors in $B \rightarrow \tau \nu$ are rather large, it is certainly plausible that NP is for now hiding in the errors; our main purpose was to explore and suggest its use in the long run.

We emphasize that consistency of the experimental results on $R_{\tau}(D^{(*)})$ with the SM does not necessarily mean the absence of all new physics contribution in $R(D^{(*)})$ and/or $B \rightarrow \tau \nu$. For example, for a class of new physics models which affect both type of decay modes (as happens in 2HDM-II), NP contributions would also largely tend to cancel in $R_{\tau}(D^{(*)})$. In fact our analysis of $R_{\tau}(D^{(*)})$ explicitly shows that type II-2HDM in the region of $M_{H}$ larger than about 500 GeV with tanβ less than about 25 is allowed; in this important respect we reach at a different conclusion than the BaBar analysis. Indeed, the constraint obtained on the parameter space in $\tan \beta - m_{H^+}$ plane is very similar to the one obtained from $B(B \rightarrow \tau \nu)$; $\tan \beta \gtrsim 25$ is not allowed by the present data in the case when $m_{H^+} < 1 \text{ TeV}$.

This conclusion regarding the possible relevance of type-II 2HDM is of special significance to Supersymmetric theories as therein type II-2HDM is of special significance to Supersymmetric theories as therein type II-2HDM de-2

We cannot construct such a ratio for LHCb as so far at LHCb it has been difficult to measure the branching ratio for $B \rightarrow \tau \nu$.

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