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Detecting Majorana modes by readout of poisoning-induced parity flips

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Reading out the parity degree of freedom of Majorana bound states is key to demonstrating their non-Abelian exchange properties. Here, we present a low-energy model describing localized edge states in a two-arm device. We study parity-to-charge conversion based on coupling the superconductor bound states to a quantum dot whose charge is read out by a sensor. The dynamics of the system, including the readout device, is analyzed in full using a quantum-jump approach. We show how the resulting signal and signal-to-noise ratio differentiates between local Majorana and Andreev bound states.

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Topological superconductors host Majorana zero-energy modes [1,2] that store quantum information nonlocally, and are thereby in principle protected against local perturbations. Many protocols to detect this nonlocal storage have been theorized in fractional quantum Hall systems [3] and, more recently, in superconducting wires [4–12]. The latter are inspired by the idea of engineered topological superconductors, many of which have been realized experimentally. Transport spectroscopy and interference [13–19] indeed suggests the presence of Majoranas in such systems, but differentiating between topological and trivial states remains challenging [20–27]. Most importantly, however, transport via the states of interest themselves violates the conservation of fermionic parity essential to most topological qubit proposals.

The key question addressed in this Letter is thus how to test for Majoranas directly via the conserved parity of nonlocal Majorana pairs, using the same readout device as in the intended quantum information application. Typical parity measurement schemes are theorized to rely on a conversion to a charge or magnetic flux when the Majorana modes overlap [4,6,8–11,28–37]. Here, we consider parity-to-charge conversion with a quantum dot coupled to the subgap end states of superconducting wires, such that the dot charge measures the combined dot-subgap parity [6,10,11,35,36,38]. Integrating dots into semiconductor-superconductor structures is experimentally well developed [39–46]: Couplings are accurately tuned via gate voltages, and charge readout is performed via electromagnetic resonators [47–55], sensor dots [56–62], or quantum point contacts [63–67].

However, while the parity-to-charge conversion principle is well established, its implementation raises many fundamental questions which we answer in this Letter. First, can the charge detection, even in principle, differentiate Majorana modes from Andreev bound states? Second, do readout fluctuations provide additional information about these modes? And third, how do the coupling strengths affect the detection scheme?

We concretely study the system in Fig. 1(a): Two superconducting wires tunnel-coupled to a parity-to-charge converter dot (CD). The stable charge of this dot changes with subgap parity flips [6], which we assume to be simply due to rare but inevitable quasiparticle (QP) poisoning [68–73] on a timescale $t_{QP}$. As the occasional CD charge jumps affect a capacitively coupled sensor dot (SD), the parity flips are measurable via telegraph noise in the zero-bias SD conductance $G$ across two tunnel-coupled leads [Fig. 1(b)]. We calculate this conductance by explicitly mimicking the experimental lock-in charge readout technique [56,74]: Applying a small ac voltage

FIG. 1. (a) Subgap modes in two superconducting wires tunnel-coupled to a parity-to-charge converter dot [Eq. (1)] that capacitively couples to a charge sensor. (b) Sensor current $I_s(t)$ due to driven $\mu_L$, corresponding first harmonic $G_a$ [Eq. (2)], and dot-subgap parity $p_{d,SC}$. (c) Sample-averaged conductances $G_{\phi}$ of $G_{d,SC}=G_{\phi_{1,SC}}=1$ scaled by $G_T=\Gamma/(8T)$. (b) shares parameters with (c); other parameters are from Fig. 2.
at zero dc bias, the equal-frequency first harmonic current response provides a measure for $G$. The strength and unique feature of our approach is to quantify the conductance as directly sampled from the experimentally accessible readout signal $G(t)$, including SD backaction via a consistent quantum master equation [35,38,75,76] in a quantum-jump Monte Carlo simulation [77–80]. We thereby capture the effect of measurement-induced relaxation, thermal- and nonequilibrium noise, and of the lock-in drive as well as the readout integration time. This includes possible signal loss due to state projections away from the CD-subgap ground state [6,35,36,81].

The low-energy Hamiltonian for the single-level CD with energy $\epsilon$, $N$ superconducting subgap modes $i$ at energies $\epsilon_i$ in either wire, and the CD-subgap tunnel couplings $\lambda_i$ read ($|e|=\hbar=k_B=1$)

$$H_{\text{DC}} = en + \sum_{i=1}^{N} \epsilon_n i + \sum_{i=1}^{N} \lambda_i e^{\phi_i} d_i^\dagger [u_i |\alpha_i^\dagger| + |v_i| \alpha_i^\dagger] + \text{H.c.}$$

Here, $n = d^d d$ is the CD occupation with fermionic creation/annihilation operator $d_i^\dagger$, $d_i$; the subgap state occupations $n_i = \alpha_i^\dagger \alpha_i$ are likewise associated with the creation/annihilation operators $\alpha_i^\dagger$, $\alpha_i$. To justify the single-level dot picture, we assume both the CD single-particle level spacing and on-site Coulomb interaction to be large compared to the tunneling amplitudes $\lambda_i$ [38]. This implies sufficient CD-spin polarization, motivated by the large magnetic field needed for Majorana modes [82,83]. We, however, allow for subgap modes with $\epsilon_i \neq 0$, with spin-axis orientations different from the CD [84,85], and with (normalized, $|u_i|^2 + |v_i|^2 = 1$) couplings featuring unequal particle and hole components $|u_i| \neq |v_i|$ and mode-dependent phases $\phi_i \neq \phi_j$. Unlike previous works [6,10,35,36,38], which assumed a dot coupling to only one Majorana per wire, we thus represent $H_{\text{DC}}$ in terms of fermionic fields $\alpha_i^\dagger$, $\alpha_i$ instead of Majorana operators. These fermions are gauge to obtain $\lambda_i \geq 0$, so phases enter Eq. (1) exclusively via $e^{\phi_i}$. These depend on uncontrollable CD/subgap wave function details, but in the important case of one fermion per wire, the only relevant phase difference $\phi_2 - \phi_1 = \phi$ is flux ($\phi$) tunable.

The model (1) implies quantum fluctuations in all occupations $n, n_i$ and their sum $n_{\text{DC}} = n + \sum_{i=1}^{N} n_i$, but leaves the combined parity $n_{\text{DC}} = (-1)^{n_{\text{DC}}}$ a good quantum number. Our central question is how this parity $n_{\text{DC}}$—converted to the CD charge ($\langle n \rangle$) affecting the SD conductance [Figs. 1(a) and 1(b)]—distinguishes between finite- and zero-energy Andreev and Majorana wire modes. The capacitive coupling $H_{\text{cap}} = E_{\text{cap}} \rho_{ST}/\Delta T$ to the SD charge $n_S$ is quantified by $E_{\text{cap}}$. The SD tunnel-couplings to metallic noninteracting leads, assuming symmetric, energy-independent tunneling rates $\Gamma_{LR} = \Gamma$ [80,86]. The Supplemental Material [80] details how we obtain the time-resolved conductance from the quantum-jump method [77–79], with universal Lindblad operators [35,38,75,76] applicable even for the here relevant near-degenerate states. We assume weak coupling compared to the lead temperature, $\Gamma \ll T$, and lead-internal relaxation as the shortest timescale in the problem.

In brief, our method yields current-time series admitting sample averaging. After each numerical time step $\delta t \ll \Gamma^{-1}$ of each series, we record the accumulated number of electron jumps $J_{\text{e}}(t)$ to ($\eta = +$) and from ($\eta = -$) lead $r$, and calculate the current $I_{\text{e}}(t) = \sum_{\eta=\pm} \langle n_{\eta}(t) - n_{\eta}(t - \delta t) \rangle$ with the bandwidth ($1/\delta t < \Gamma$) reflecting detector-internal time averaging. Mimicking the experimental lock-in technique, the zero-bias conductance signal $G(t)$ is extracted from $I_{\text{e}}(t)$ by applying a low-amplitude voltage oscillation $\mu_{\text{r}}(t) = \mu_{\text{r}} + \delta \mu \sin(2\Omega t)$ with frequency $(1/\Omega_{\text{T}}) \ll \Omega \ll (1/\delta t)$, and by taking the first harmonic response divided by $\delta \mu \lesssim T$. Explicitly, each $G(t)$ sample averages over $N_{\text{osc}}$ voltage oscillations,

$$G(t) = \frac{\Omega \delta \mu / G_T}{N_{\text{osc}} \delta \mu} \sum_{n=0}^{\lfloor \delta \mu \Omega T \rfloor - 1} I_{\text{e}}(t + n \delta t) e^{-i n \delta \mu t},$$

where $G_T = \Gamma/(8T)$ is the spinless on-resonance conductance [86]. Figure 1(b) shows the difference between the raw conductance time series $I_{\text{e}}(t)/(\delta \mu G_T)$ and the first harmonic response. The sudden jumps causing the telegraph noise stem from randomly inserted $p_{\text{DC}}$ flips [68–73]. Just as in experiments, $G(t)$ filters out high frequencies via the Fourier transform, bounding the noise spectrum not by the inaccessible bandwidth $1/\delta t$, but by the well-controlled lock-in frequency $\Omega$.

The even/odd-parity conductances $G_{\text{e/o}} = \langle G \rangle_{M, p_{\text{DC}}=\pm 1}$ in Fig. 1(c) are averages over $M = N_{T} T_{\text{tra}}$ samples from $N_{T}$ trajectories $G(t)$ at fixed $p_{\text{DC}} = \pm 1$; each trajectory consists of $T_{\text{tra}}$ subsequent samples in a time $\Delta t = M_{T} N_{\text{osc}} \frac{2\pi}{\Omega_{\text{T}}} \ll 1_{\text{T}}$ expected to be within two QP poisonings [80], as exemplified by the typical $G$-plateau length in Fig. 1(b). The $G_{\text{e/o}}$ and their fluctuations $\delta G_{\text{e/o}} = \sqrt{\langle (G_{\text{e/o}} - G_{\text{e/o}}^2) \rangle}$ define the signal $S$ and signal-to-noise ratio (SNR) $D$:

$$S = |G_{\text{e/o}} - G_{\text{o}}|, \quad D = 2S/(\delta G_{\text{e/o}} + \delta G_{\text{o}}).$$

The ratio $D$ as a function of the tunable CD level $\epsilon$ and flux $\phi$ is our key observable to characterize the subgap states, as it sets the number of $G$ samples required for statistically significant parity distinguishability. We, however, also refer to $S$, mostly based on data in the Supplemental Material [80], to rule out or identify any nontrivial scaling between noise and signal. This furthermore allows us to estimate how the distinguishability would diminish with additional noise unaccounted for here.

The detector setup and SD level $\epsilon_S$ for optimal $D$ depends on both $E_{\text{cap}}$ and $\lambda_i$ [80], since $S, D > 0$ are due to $p_{\text{DC}}$-dependent CD-subgap hybridization inducing a $p_{\text{DC}}$-dependent conductance peak shift away from the classical resonances $\epsilon_S = 0$, $-E_{\text{cap}}$. We here focus on $E_{\text{cap}} > \lambda_i$, yielding nearly classical $G_{\text{o}}$ peaks and sizably deviating $G_{\text{e}}$ [Fig. 1(c)]. The largest $S$ then appears between Coulomb peaks, $\epsilon_S = -E_{\text{cap}}/2$, but this point is often susceptible to the here neglected higher-order-\(\Gamma\) effects such as Kondo resonances. We hence instead fix $\epsilon_S = 0$ and reduce lead-induced broadening by demanding $\Gamma, T \ll E_{\text{cap}}$. Larger $\Gamma, \delta \mu, T$ tend to improve $D$ [80], but we must keep $\Gamma, \delta \mu$, and especially $T$ smaller than $\lambda_i$; otherwise, capacitive backaction may drive
The system too far away from the near-ground state in Fig. 1(c) to obtain a signal [36,38]. Let us now first discuss the ideal limit in Fig. 2, with all subgap energies \( \epsilon_i/\lambda_i \to 0 \). The CD then effectively couples to only one subgap fermion [80], being a linear combination of all \( N \) subgap modes in Eq. (1),

\[
H_{\text{SC}} = \epsilon n + \lambda_{\text{eff}} (d^\dagger - d)\gamma,
\]

where \( \alpha \) is a fermionic annihilation operator and

\[
\lambda^\pm = \lambda_{\text{eff}} \sqrt{1 \mp \sqrt{1 - |Q|^2}}, \quad Q = \sum_{i=1}^N \frac{\lambda_i^2}{\gamma_{i\text{eff}}} e^{2\phi_i} |u_i||v_i|,
\]

with \( 0 \leq |Q| \leq 1 \) and effective coupling \( \lambda_{\text{eff}} = \sqrt{\sum_{i=1}^N \lambda_i^2/2} \). The capacitive readout likewise only senses the subgap parity of this one \( \alpha \) mode and the CD, \( \rho_{\alpha\alpha} = (-1)^{\epsilon_i+\epsilon_{i'}} \); the other \( N - 1 \) invisible subgap modes are inaccessible. For this single remaining mode, \( Q \) generalizes the Majorana quality factor of Refs. [87,88]: Interpolating between a pure particle- or

holelike (\( |Q| = 0 \)), and an equally particle- and holelike coupling (\( |Q| = 1 \)), the latter crucially maps to the single-Majorana case:

\[
H_{\text{SC}} \rightarrow \epsilon n + \lambda_{\text{eff}} (d^\dagger - d)\gamma,
\]

with \( \gamma = \alpha^\dagger + \alpha \). The sensor is then insensitive to \( \rho_{\alpha\alpha} \) [6], and yields \( S = D = 0 \) for any \( \epsilon, \lambda_i \) within the single-level CD approximation. We emphasize that for this \((\epsilon, \lambda_i)-\)independent parity insensitivity, \( \epsilon_i/\lambda_i \to 0 \) and \(|Q| = 1 \) are not only sufficient, but necessary [80]. Following Eq. (5), this means that all \( N \) subgap modes \( i \) in Eq. (1) must couple as zero-energy Majoranas to the CD, \(|u_i| = |v_i|\), and with equal \( \phi_i \) up to a \( \pi \) shift.

The key Majorana signature deriving from Eq. (6) is constant \( S, D = 0 \) with varying individual conductances \( G_{\alpha\alpha} \) in sweeps of the dot level \( \epsilon \) and, if possible, the coupling strengths \( \lambda_i \); Constant \( G_{\alpha\alpha} \) may merely indicate insufficient sensor coupling, and if \( S, D = 0 \) only for specific \( \epsilon, \lambda_i \), one can neither rule out fine tuning unrelated to Majoranas, nor the quasi-Majorana case [24] with a coincidently uncoupled zero mode. Importantly, such a sweep test is inherently robust to fluctuating \( \epsilon, \epsilon_i, \lambda_i \) due, e.g., \( 1/f \) noise [89–91], and towards unavoidable coupling asymmetries \( \lambda_i \neq \lambda_j \) [80].

Table I lists the subgap mode setups for which the suggested prescription can or cannot yield \((\epsilon, \lambda_i)-\)independent \( S, D = 0 \). The simplest case involves only one wire with only one \( (N = 1) \) coupled mode (see Refs. [87,88,92,93]). For this \( \phi \)-independent situation, Figs. 2(a) and 2(b) show that all \( \epsilon \) traces approach \( S, D = 0 \) close to the particle-hole symmetry point \( \epsilon = 0 \), but only a Majorana \(|u_i|^2 = 0.5 \rightarrow \text{Eq. (6)} \) yields \( S = D = 0 \) for all \( \epsilon \). This robustness is equivalent to the single-site protection in the minimal Kitaev chain [92,93], but using it to identify a Majorana in a single, actual wire is difficult. First, it relies on the SNR in the Andreev wire being \( \epsilon, \lambda_i \geq 0 \)-independent situation, Figs. 2(a) and 2(b) show that all \( \epsilon \) traces approach \( S, D = 0 \) close to the particle-hole symmetry point \( \epsilon = 0 \), but only a Majorana \(|u_i|^2 = 0.5 \rightarrow \text{Eq. (6)} \) yields \( S = D = 0 \) for all \( \epsilon \). This robustness is equivalent to the single-site protection in the minimal Kitaev chain [92,93], but using it to identify a Majorana in a single, actual wire is difficult. First, it relies on the SNR in the Andreev wire being \( \epsilon, \lambda_i \geq 0 \)-independent (see Refs. [87,88,92,93]). For this \( \phi \)-independent situation, Figs. 2(a) and 2(b) show that all \( \epsilon \) traces approach \( S, D = 0 \) close to the particle-hole symmetry point \( \epsilon = 0 \), but only a Majorana \(|u_i|^2 = 0.5 \rightarrow \text{Eq. (6)} \) yields \( S = D = 0 \) for all \( \epsilon \). This robustness is equivalent to the single-site protection in the minimal Kitaev chain [92,93], but using it to identify a Majorana in a single, actual wire is difficult. First, it relies on the SNR in the Andreev wire being \( \epsilon, \lambda_i \geq 0 \)-independent (see Refs. [87,88,92,93]).
and (b) one mode per wire, $\epsilon_{\text{subgap energies}}$, modes superpose to a fully electron/holelike effective mode in both wires giving rise to results in an $\epsilon,\lambda$ pair interference, any parity only, $D$ showing reduced noise and asymmetric wire couplings. A clear inference may, however, still be impeded by flux noise. Moreover, already small subgap mode energies can suppress the $\phi$ dependence after a perhaps challengingly short decay time $t_p$. On the flip side, extracting $t_p$ by varying measurement times may be an interferometric method to resolve deviations $\epsilon_i \neq 0$ more precisely than dc transport spectroscopy.

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