Implications of $\bar{B} \to D^0 h^0$ Decays on $\bar{B} \to D K$, $\bar{D} K$ Decays

Chun-Khiang Chua

Institute of Physics, Academia Sinica,
Taipei, Taiwan 115, Republic of China

Wei-Shu Hou

Physics Department, National Taiwan University,
Taipei, Taiwan 10764, Republic of China

Abstract

The recently observed color suppressed $\bar{B}^0 \to D^0 \pi^0$, $D^0 \eta(\prime)$, $D^+_s K^-$ and $D^0 \bar{K}^0$ decay modes all have rates larger than expected, hinting at the presence of final state interactions. We study rescattering effects in $\bar{B} \to D P$, $\bar{D} K$ and $\bar{D} K$ modes in the quasi-elastic approach, which is extended to accommodate $D^0 \eta'$ without using U(3) symmetry. The $D^0 \bar{K}$ modes are of interest in the determination of the unitarity angle $\phi_3/\gamma$. The updated $D P$ data are used to extract the effective Wilson coefficients $a_1^{\text{eff}} \simeq 0.92$, $a_2^{\text{eff}} \simeq 0.22$, three strong phases $\delta \simeq 62^\circ$, $\theta \simeq 24^\circ$, $\sigma \simeq 127^\circ$, and the mixing angle $\tau \simeq 2^\circ$. The values of $\delta$ and $\theta$ are close to our previous results. The smallness of $\tau$ implies small mixing of $D^0 \eta_1$ with other modes. Predictions for $D^0 K^-$, $D^+ K^-$ and $D^0 \bar{K}^0$ agree with data. The framework applies to $\bar{B} \to \bar{D} K$, and rates for $\bar{D}^0 K^-$, $D^- K^0$, $D_s^- \pi^0$, $D_s^- \eta$ and $D_s^- \eta'$ modes are predicted. From $B^- \to \bar{D}^0 K^-$ and $D^0 K^-$ rates, we find $r_B = 0.09 \pm 0.02$.

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I. INTRODUCTION

The color-suppressed decays $B^0 \rightarrow D^{(*)0}\pi^0$ [1, 2] and $D^0\eta, D^0\omega$ [1] were observed for the first time in 2001. Recently, improved measurements of $B^0 \rightarrow D^{(*)0}(\pi^0, \eta, \omega)$ [3, 4] and the first observation of the $D^0\eta'$ mode have been reported by BaBar [3] and confirmed by Belle [5]. Other color suppressed modes, such as $D_sK^-$ and $D^0K^0$, have also been observed [6, 7]. All these modes have branching ratios that are significantly larger than earlier theoretical expectations based on naive factorization, indicating the presence of non-vanishing strong phases, which has attracted much attention [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. Shortly after the first observation of the color suppressed modes became known, we proposed [11] a quasi-elastic final state rescattering (FSI) picture, where the enhancement of color suppressed $D^0h^0$ modes can be understood as rescattering from the color allowed $D^+\pi^-$ final state. The framework is applicable to $B \rightarrow D K, D K$ decays.

The color-allowed $B^- \rightarrow D^0K^-$ and color-suppressed $\overline{D}^0K^-$ decays are of interest for the determination of the unitary phase angle $\phi_3(\gamma) \equiv \arg V_{ub}^*$, where $V$ is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. To be more specific, the amplitude ratio $r_B$ and the strong phase difference $\delta_B$ for $\overline{D}^0K^-$ and $D^0K^-$ decay modes, which are governed by different CKM matrices as depicted in Fig. 1, are defined as

$$r_B = \left| \frac{A(B^- \rightarrow \overline{D}^0K^-)}{A(B^- \rightarrow D^0K^-)} \right|, \quad \delta_B = \arg \left[ \frac{e^{i\phi_3} A(B^- \rightarrow \overline{D}^0K^-)}{A(B^- \rightarrow D^0K^-)} \right]. \quad (1)$$

The weak phase $\phi_3$ is removed from $A(B^- \rightarrow \overline{D}^0K^-)$ in defining $\delta_B$. The $r_B$ and $\delta_B$ parameters are common to the $\phi_3$ determination methods of Gronau-London-Wyler (GLW) [19], Atwood-Dunietz-Soni (ADS) [20] and “$DK$ Dalitz plot” [21, 22], where one exploits the interference effects of $B^- \rightarrow D^0K^- \rightarrow f_{CP}K^-$ and $B^- \rightarrow \overline{D}^0K^- \rightarrow f_{CP}K^-$ amplitudes. Note that the $r_B$ parameter, which governs the strength of interference, is both color and CKM suppressed, hence hard to measure directly.

Through the $DK$ Dalitz plot method, the BaBar and Belle experiments already find $\gamma = 70^\circ \pm 44^\circ \pm 10^\circ \pm 10^\circ$ and $\phi_3 = 64^\circ \pm 19^\circ \pm 13^\circ \pm 11^\circ$, respectively [23, 24], where the last error comes from modelling of $D$ decay resonances across the Dalitz plot for, e.g. $D^0 \rightarrow K_S\pi^+\pi^-$. Although similar results on $\phi_3$ are obtained, the corresponding $r_B$ values are quite different for BaBar and Belle. Belle reports $r_B = 0.21 \pm 0.08 \pm 0.03 \pm 0.04$, while BaBar gives $r_B < 0.19$ at 90% confidence level. Note that an average $r_B = 0.10 \pm 0.04$
is found by the UT fit group, by combining analyses using all three methods [25]. As the strength of interference is governed by the size of $r_B$, the larger error in the $\gamma$ value of BaBar reflects the smallness of their $r_B$. Given the present experimental situation that Belle and BaBar have quite different $r_B$ values and the critical role it takes in $\phi_3/\gamma$ extraction, it is important to give a theoretical or phenomenological prediction of $r_B$ and $\delta_B$. Not much work has so far been done.  

The enhancement in rates of the color-suppressed $DP$ modes could imply [15] a larger $r_B$. It is thus of interest to study $DP$ and $\overline{D}K$ modes together. In fact, it was noted in Ref. [11]

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1 In the preparation of this paper, we note that a calculation in the pQCD approach has been reported [26].
that the quasi-elastic approach used in the \(DP\) system can be applied to the \(DK\) system. Rescattering parameters are basically non-perturbative and can only be fitted from data. We shall use the \(B \to D\pi, D\eta^0, D_s\bar{K}\) decay rates to extract the rescattering parameters, which are then used to predict \(DK\) and \(DK\) rates. In this way, we are able to deduce what value of \(r_B\) is preferred in the quasi-elastic rescattering scenario.

Our previous analysis on FSI effects in \(D^0h^0\) modes was based on early data. Recent experimental updates show some variations. For example, the \(D^0\pi^0\) rate has dropped while the \(D^0\eta\) rate is larger. Furthermore, the \(D^0\eta'\) mode has finally been measured. In our earlier study [11], because of the absence of the \(D^0\eta'\) mode, we ignored it and approximated \(D^0\eta\) by \(D^0\eta_8\), by argument of the \(UA(1)\) anomaly and small singlet–octet (or \(\eta–\eta'\)) mixing. The same approach was applied to the study of the charmless case [27]. Given the long standing problem of the \(B \to \eta'K\) rate, it is of interest to clarify the \(\eta_1\) issue in \(B\) decays [28]. With the emerging new data, it is time to update the quasi-elastic rescattering approach, and verify the approximations made.

In Sec. II we extend the quasi-elastic rescattering framework to include \(D^0\eta'\). Because of the \(UA(1)\) anomaly, we use SU(3) rather than U(3) symmetry. As a consequence, we need three phases and one mixing angle as rescattering parameters. These same parameters also enter the rescattering in the \(DK\) and \(DK\) systems. In Sec. III we carry out a numerical study. The effective Wilson coefficients and rescattering parameters are obtained by using current \(B \to DP\) data. The \(B \to DK\) rates are then predicted and compared with current data. We then proceed to study the \(B \to DK\) system and make predictions for \(r_B\) and \(\delta_B\). The conclusion is then offered in Sec. IV.

II. FINAL STATE RESCATTERING FRAMEWORK

A. Quasi-elastic Rescattering

Let \(H_W\) be the weak decay Hamiltonian. In the absence of weak phase (or if they are factored out), \(H_W\) is time-reversal invariant. By using time reversal invariance of \(H_W\) and the optical theorem, we have (see, for example, [11, 29])

\[
2 \text{Im} \langle i; \text{out}|H_W|B \rangle = \sum_j T_{ji}^* \langle j; \text{out}|H_W|B \rangle, \tag{2}
\]
where $T$ is the $T$-matrix of strong scattering, and the phase convention $T \mid \text{in} \rangle = \mid \text{out} \rangle$ under time-reversal operation is used. This is the master formula of FSI in $B$ decay. In particular, for $B$ decay to two body final state with momentum $(p_1, p_2)$, we have,

$$-2i \text{ Im } A(p_B \rightarrow p_1 p_2) = \sum_j \left( \prod_{k=1}^j \int \frac{d^3 q_k}{(2\pi)^3 2 E_k} \right) (2\pi)^4 \delta^4 \left(p_1 + p_2 - \sum_{k=1}^j q_k\right) \times M^* (p_1 p_2 \rightarrow \{q_k\}) A(p_B \rightarrow \{q_k\}),$$

where the optical theorem is used to all orders of the strong interaction, but only to first order of the weak interaction. Eq. (3) relates the imaginary part of the two body decay amplitude to the sum over all possible $B$ decay final states $\{q_k\}$, followed by $\{q_k\} \rightarrow p_1 p_2$ rescattering. The solution to the above equation is

$$A = S^{1/2} A^0,$$

where $A^0$ is real, and $S = 1 + i T$. The weak decay picks up strong scattering phases $[30]$. It has been pointed out that elastic rescattering effects may not be greatly suppressed at $m_B$ scale, while inelastic rescattering contributions may be important. But we would clearly lose control if the full structure shown in Eq. (3) is employed. Even if all possible $B$ decay rates can be measured, it would be impossible to know the phases of each amplitude. Furthermore, we know very little about the strong rescattering amplitudes. However, in Eq. (3) the subset of two body final states that may be reached via elastic rescatterings stand out compared to inelastic channels. It has been shown from duality arguments $[32]$, as well as a statistical approach $[29]$, that inelastic FSI amplitudes tend to cancel each other and lead to small FSI phases. We shall therefore separate $\{q_k\}$ into two body elastic channels plus the rest, and concentrate on the contribution of the former.

We will consider $DP$ final states, where $P$ stands for a pseudoscalar meson. We consider $D$ and $P$ within SU(3) multiplets, for example the $D$ anti-triplet of $D^0$, $D^+$ and $D_s^+$, the $\Pi$ octet that contains pions, kaons and the $\eta$ ($\eta_8$ component) meson, as well as the $\eta_1$ (mixing of physical $\eta$ and $\eta'$). Thus, we call this quasi-elastic rescattering.

**B. Rescattering Formalism including $D^0\eta_1$**

In Ref. [11] we treated $D\Pi \rightarrow D\Pi$ rescattering and considered only the $\eta_8$, since at that time $B^0 \rightarrow D^0\eta'$ was not yet reported. Here we wish to extend the formalism to include
function coefficients. For example, we have (in Eq. (6) can be reproduced easily using this pictorial approach by matching the flavor wave integrals as $r$, to charge exchange, annihilation, and flavor singlet exchange, respectively. The coefficients $M$ in Eq. (6) is obtained by separating the $D\Pi \rightarrow D\Pi$ scattering amplitude $M$ into three independent components, $M_{0,a,e}$, and defining

$$
\mathcal{T} = \begin{pmatrix}
\frac{r_0 + r_a}{\sqrt{2}} & \frac{r_a - r_0}{\sqrt{2}} & r_a & \frac{r_a + r_0}{\sqrt{6}} & \frac{r_a + r_0}{\sqrt{3}} \\
\frac{r_a - r_0}{\sqrt{2}} & r_0 + \frac{r_a + r_0}{2} & \frac{r_a + r_0}{2\sqrt{3}} & \frac{r_a + r_0}{\sqrt{6}} & \frac{r_a + r_0}{\sqrt{3}} \\
\frac{r_a}{\sqrt{2}} & \frac{r_a}{\sqrt{2}} & r_0 + r_a & \frac{r_a - 2r_0}{\sqrt{6}} & \frac{r_a - 2r_0}{\sqrt{3}} \\
\frac{r_a + r_0}{\sqrt{6}} & \frac{r_a + r_0}{2\sqrt{3}} & \frac{r_a - 2r_0}{\sqrt{6}} & r_0 + \frac{r_a + r_0}{6} & \frac{r_a + r_0}{3\sqrt{2}} \\
\frac{r_a + r_0}{\sqrt{6}} & \frac{r_a + r_0}{\sqrt{3}} & \frac{r_a + r_0}{\sqrt{3}} & \frac{r_a + r_0}{3\sqrt{2}} & r_0 + \frac{r_a + r_0}{3}
\end{pmatrix}, (6)
$$

and $\mathcal{T}'$ has the same structure as $\mathcal{T}$ by SU(3) symmetry, but with $r_i$ replaced by $r_i'$. In addition, we have $A_{D^0\pi^-} = (1 + i r_0' + i r_e')A_{D^0\pi^-}^0$. The $r_i'$’s are discussed below.

Eq. (6) is obtained by separating the $D\Pi \rightarrow D\Pi$ scattering amplitude $M$ into three independent components, $M_{0,a,e}$, and defining

$$
r_i = \int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} E_1 E_2 (2\pi)^4 \delta^4(p_1 + p_2 - q_1 - q_2) M_i(p_1 p_2 \rightarrow q_1 q_2). (7)
$$

Note that the $M_0, M_a$ and $M_e$ amplitudes correspond respectively to the three independent SU(3) combinations $\text{Tr}(D_{in}D_{out})\text{Tr}(\Pi_{in}\Pi_{out})$, $\text{Tr}(D_{in}\Pi_{in}D_{out})$ and $\text{Tr}(D_{in}\Pi_{out}\Pi_{in}D_{out})$ of $D_{in}\Pi_{in} \rightarrow D_{out}\Pi_{out}$ scattering. For $D\Pi \leftrightarrow D^0\eta_1$ and $D^0\eta_1 \leftrightarrow D^0\eta_1$ scattering, we denote the corresponding integrals as $\bar{r}_i$ and $\bar{r}_i$, respectively. Had U(3) rather than SU(3) symmetry held in the light pseudoscalar sector, $\bar{r}_i$ and $\bar{r}_i$ would have been identified with $r_i$.

We give a pictorial representation of $r_e, r_a, r_0$ in Fig. 2 which can be seen as corresponding to charge exchange, annihilation, and flavor singlet exchange, respectively. The coefficients $r_i$ in Eq. (6) can be reproduced easily using this pictorial approach by matching the flavor wave function coefficients. For example, we have $(r_a - r_e)/\sqrt{2}$ for $D^+\pi^- \rightarrow D^0\pi^0$ rescattering. Exchange rescattering ($r_e$), the first diagram of Fig. 2 projects out the $d\bar{d}$ component of $\pi^0$ on the right hand side. This give a $-1/\sqrt{2}$ factor from the $\pi^0$ wave function. Similarly, the second diagram of Fig. 2 projects out the $u\bar{u}$ component of $\pi^0$, hence gives $r_a/\sqrt{2}$. These
diagrams also provide further information. For example, it is easy to see from the second
diagram that annihilation rescattering ($r_a$) is responsible for $D^+\pi^- \rightarrow D_s^+K^-$, since there
is no $s$ quark before rescattering. The $M_i$s in Eq. (7) can be understood analogously. For
example, the $M_e$ operator $(D_{in})_i(\Pi_{out})_k(\Pi_{in})^j(D_{out})^j$ corresponds to $2$ the $D(c\bar{q}_i)\Pi(\bar{q}_j q^k) \rightarrow D(c\bar{q}^j)\Pi(\bar{q}^i q_k)$ process with the exchange of $i$th and $j$th antiquark, hence it is the exchange rescattering operator.

For Cabibbo suppressed $B \rightarrow D^+K^-$ and $D^0\bar{K}^0$ modes, we have

\[
\begin{pmatrix}
A_{D^+K^-} \\
A_{D^0\bar{K}^0}
\end{pmatrix}
= \begin{pmatrix}
1 + i r'_0 & i r'_0 \\
i r'_e & 1 + i r'_0
\end{pmatrix}
\begin{pmatrix}
A_{D^0\bar{K}^0} \\
A_{D^0\bar{K}^0}
\end{pmatrix},
\]

(8)

2 Superscripts and subscripts are assigned according to the field convention with $q^i$ and $\bar{q}_j$ as quark and antiquark fields, respectively.
which can be easily obtained by using the pictorial approach shown in Fig. 3. It is clear that “annihilation” rescattering is impossible in this case. For \( D^0 K^- \), we have

\[
A_{D^0 K^-} = (1 + ir'_0 + ir'_e)A_{D^0 K^-}^0.
\]

For the \( B^- \rightarrow D \bar{K} \) decays, we have

\[
\begin{pmatrix}
A_{D^0 K^-} \\
A_{D^- K^0} \\
A_{D^- \pi^0} \\
A_{D^- \eta_8} \\
A_{D^- \eta_1}
\end{pmatrix} = S^{1/2}
\begin{pmatrix}
A_{D^0 K^-}^0 \\
A_{D^- K^0}^0 \\
A_{D^- \pi^0}^0 \\
A_{D^- \eta_8}^0 \\
A_{D^- \eta_1}^0
\end{pmatrix},
\]

where \( S^{1/2} = (1 + iT)^{1/2} = 1 + iT' \), with

\[
T =
\begin{pmatrix}
r_0 + r_a & r_a & \frac{r_a - 2r_e}{\sqrt{2}} & \frac{r_a + 2r_e}{\sqrt{2}} & \frac{r_a + r_e}{\sqrt{2}} \\
r_a & r_0 + r_a & -\frac{r_e}{\sqrt{2}} & \frac{r_e - 2r_a}{\sqrt{2}} & \frac{r_e + 2r_a}{\sqrt{2}} \\
\frac{r_e}{\sqrt{2}} & -\frac{r_e}{\sqrt{2}} & r_0 & 0 & 0 \\
\frac{r_e - 2r_a}{\sqrt{2}} & \frac{r_e - 2r_a}{\sqrt{2}} & 0 & r_0 + \frac{2}{3}(r_a + r_e) & -\frac{\sqrt{2}}{3}(\tilde{r}_a + \tilde{r}_e) \\
\frac{r_e + 2r_a}{\sqrt{2}} & \frac{r_e + 2r_a}{\sqrt{2}} & 0 & -\frac{\sqrt{2}}{3}(\tilde{r}_a + \tilde{r}_e) & \tilde{r}_0 + \frac{\tilde{r}_a + \tilde{r}_e}{3}
\end{pmatrix},
\]

as one can easily verify using the pictorial approach shown in Fig. 4. The zeros are a consequence of assuming isospin symmetry. It is important to note that, due to charge conjugation invariance and \( SU(3) \) symmetry of the strong interactions, these \( r_i^{(t)} \), \( \tilde{r}_i^{(t)} \) and \( \check{r}_i^{(t)} \) coefficients are identical to those in Eqs. (6) and (8).

By solving \( S^\dagger S = 1 \), we obtain [11].
\[
(1 + i r_0) = \frac{1}{2}(1 + e^{2i\delta}),
\]
\[
i r_e = \frac{1}{2}(1 - e^{2i\delta}),
\]
\[
i r_a = \frac{1}{8}(3U_{11} - 2e^{2i\delta} - 1),
\]
\[
i(r_a + r_e) = \frac{3}{2\sqrt{2}}U_{12},
\]
\[
i(r_0 + \frac{r_a + r_e}{3}) = U_{22} - 1,
\]

where

\[
U = U^T = \begin{pmatrix}
\cos \tau & \sin \tau \\
-\sin \tau & \cos \tau
\end{pmatrix}
\begin{pmatrix}
e^{2i\theta} & 0 \\
0 & e^{2i\theta}
\end{pmatrix}
\begin{pmatrix}
\cos \tau & -\sin \tau \\
\sin \tau & \cos \tau
\end{pmatrix},
\]

where we have set the overall phase factor \((1 + i r_0 + i r_e)\) in \(S\) to unity. This phase convention
is equivalent to choosing the $A_{D^0\pi^-}$ amplitude to be real.

We stress that the above solution satisfies $S^\dagger S = 1$ in all three cases of $DP$ ($P$ now stands for strangeness 0 pseudoscalar), $D\bar{K}$ and $\bar{D}K$. The $r'_i$, $\bar{r}'_i$ and $\tilde{r}'_i$ in $S^{1/2}$ can be obtained by using the above formulas with phases ($\delta, \theta, \sigma$) reduced by half. We need three phases and one mixing angle to specify FSI effects since one does not have nonet symmetry in the light pseudoscalar sector. An extra phase as well as mixing angle arise from including $\eta_1$ in our analysis. We will resort to data to see how far the $DP$ system differs from the nonet symmetric case. At the same time, we will use $r'_i$, $\bar{r}'_i$ and $\tilde{r}'_i$ to predict $D\bar{K}$ and $\bar{D}K$ rates, and compare with data whenever possible.

C. SU(3) Decomposition

It is instructive to see the phases and angle given in Eq. (11) in light of SU(3) decomposition. Let us consider the $DP$ case first. $D$ is an anti-triplet ($D(\bar{3})$), while $P$ can be reduced to an octet [$\Pi(8)$] and a singlet ($\eta_1$). The $D(\bar{3}) \otimes \Pi(8)$ can be reduced into a $\bar{3}$, a $6$ and a $\bar{5}$ (see, for example, [33]), while $D(\bar{3})\eta_1$ is another anti-triplet. Denoting the latter as $\bar{3}'$, it can mix with the $\bar{3}$ from $D\Pi$ via a $2 \times 2$ symmetric (from time reversal invariance) unitary matrix $U$, which appears already in Eqs. (11) and (12). The invariance of the strong interaction under SU(3) transformation gives

$$S = |\bar{15}\rangle\langle \bar{15}| + e^{2i\delta}|6\rangle\langle 6| + (|\bar{3}\rangle\langle \bar{3}'|) \cdot U \cdot \left(\begin{array}{c} |\bar{3}\rangle \\ |\bar{3}'\rangle \end{array}\right).$$

(13)

It is now clear that, with the choice of vanishing phase in $S_{\bar{5}5\bar{15}\bar{15}}$, $2\delta$ is the phase of $S_{66}$ and $U$ is the mixing matrix in the anti-triplet sector. Note that in the master formula Eq. (11) one should use $S^{1/2}$. This can be easily obtained by reducing all phases in the right-hand-side of the above equation by half.

Three remarks are in order. It is important to emphasize that, by charge conjugation invariance of the strong interaction, the above $S$-matrix can also be applied to the $\bar{D}P$ case with $|\bar{15},\bar{6}\rangle$ and $|\bar{3}(\bar{0})\rangle$ replaced by $|15,\bar{6}\rangle$ and $|3(\bar{0})\rangle$, respectively. Second, the $D^0\eta_8$ and $D^0\eta_1$ are not physical final states. The physical $\eta, \eta'$ mesons are defined through

$$\left(\begin{array}{c} \eta \\ \eta' \end{array}\right) = \left(\begin{array}{cc} \cos \vartheta & -\sin \vartheta \\ \sin \vartheta & \cos \vartheta \end{array}\right) \left(\begin{array}{c} \eta_8 \\ \eta_1 \end{array}\right),$$

(14)
with the mixing angle \( \vartheta = -15.4^\circ \) [34]. In the previous analysis [11], \( D^0 \eta \) was approximated as \( D^0 \eta_8 \), while \( D^0 \eta_1 \) was ignored. This corresponds to vanishing mixing angle \( \tau = 0 \) in \( \mathcal{U} \), and dropping the \( \overline{\mathcal{B}} \).

Before we end this section, let us specify \( A^0 \) in the above formulas. We use factorization amplitudes for \( A^0 \), which are [9], for each physical final state \(^3\)

\[
\begin{align*}
A_{D^0 \pi^-}^f &= V_{cb} V_{ud}^* (T_f + C_f), & A_{D^+ \pi^-}^f &= V_{cb} V_{ud}^* (T_f + E_f), \\
A_{D^0 \pi^0}^f &= \frac{V_{cb} V_{us}^*}{\sqrt{2}} (-C_f + E_f), & A_{D^+ K^-}^f &= V_{cb} V_{us}^* E_f, \\
A_{D^0 \eta_8}^f &= \frac{V_{cb} V_{us}^*}{\sqrt{6}} (C_f + E_f), & A_{D^0 \eta_1}^f &= \frac{V_{cb} V_{us}^*}{\sqrt{3}} (C_f + E_f), \\
A_{D^0 K^-}^f &= V_{ub} V_{cs}^* (C_f + a_f), & A_{D^- \overline{\mathcal{B}}^0}^f &= V_{ub} V_{cs}^* a_f, \\
A_{D^0 \eta_8}^f &= \frac{V_{ub} V_{cs}^*}{\sqrt{6}} (t_f - 2a_f), & A_{D^0 \eta_1}^f &= \frac{V_{ub} V_{cs}^*}{\sqrt{3}} (t_f + a_f), \\
A_{D^+ \pi^0}^f &= \frac{V_{ub} V_{cs}^*}{\sqrt{2}} t_f, \\
\end{align*}
\]

where the super- and subscripts \( f \) indicate factorization amplitude, and

\[
\begin{align*}
T_f &= \frac{G_F}{\sqrt{2}} a_1^{\text{eff}} (m_B^2 - m_D^2) f_P F_0^{BD}(m_B^2), \\
C_f &= \frac{G_F}{\sqrt{2}} a_2^{\text{eff}} (m_B^2 - m_P^2) f_D F_0^{BP}(m_B^2), \\
E_f &= \frac{G_F}{\sqrt{2}} a_2^{\text{eff}} (m_D^2 - m_P^2) f_B F_0^{0 \rightarrow DP}(m_B^2), \\
t_f &= \frac{G_F}{\sqrt{2}} a_1^{\text{eff}} (m_B^2 - m_P^2) f_D F_0^{BP}(m_B^2), \\
c_f &= \frac{G_F}{\sqrt{2}} a_2^{\text{eff}} (m_B^2 - m_P^2) f_B F_0^{0 \rightarrow DP}(m_B^2), \\
a_f &= \frac{G_F}{\sqrt{2}} a_2^{\text{eff}} (m_D^2 - m_P^2) f_D F_0^{BP}(m_D^2),
\end{align*}
\]

\( F_0^{BD(BP)} \) is the \( \overline{\mathcal{B}} \rightarrow D(P) \) transition form factor and \( F_0^{0 \rightarrow DP} \) is the vacuum to \( DP \) (time-like) form factor. Some SU(3) breaking effects are included in the decay constants and various form factors. In subsequent numerical study we take \( E_f = a_f = 0 \). In this case, it is seen that in the \( \overline{\mathcal{B}} \rightarrow DP \) decays, the \( a_1^{\text{eff}} \) and \( a_2^{\text{eff}} \) parameters accompany \( F_0^{BD}(0) \) and

\(^3\) To match the phase convention \( T \mid \text{in} \rangle = \mid \text{out} \rangle \) (after factoring out the CKM factor) we absorb the factor \( i \) of these factorization amplitudes into the \( B \) meson state [18].
TABLE I: Summary of experimental results for $B^0 \rightarrow DP$ modes [2, 3, 4, 5, 6, 7, 40].

| $B$ ($\times 10^{-4}$) | CLEO | BaBar | Belle | Average |
|------------------------|------|-------|-------|---------|
| $B^0 \rightarrow D^0\pi^0$ | $2.74^{+0.36}_{-0.32} \pm 0.55$ | $2.9 \pm 0.2 \pm 0.3$ | $2.31 \pm 0.12 \pm 0.23$ | $2.53 \pm 0.20$ |
| $B^0 \rightarrow D^0\eta$ | – | $2.5 \pm 0.2 \pm 0.3$ | $1.83 \pm 0.15 \pm 0.27$ | $2.11 \pm 0.33^a$ |
| $B^0 \rightarrow D^0s^+K^-$ | – | $0.32 \pm 0.12 \pm 0.08$ | $0.45^{+0.14}_{-0.12} \pm 0.11$ | $0.38 \pm 0.13$ |
| $B^0 \rightarrow D^0\eta'$ | – | $1.7 \pm 0.4 \pm 0.2$ | $1.14 \pm 0.20^{+0.10}_{-0.13}$ | $1.26 \pm 0.23^b$ |
| $B^0 \rightarrow D^+K^-$ | – | – | $2.04 \pm 0.50 \pm 0.27$ | $2.0 \pm 0.6$ |
| $B^0 \rightarrow D^0\bar{K}^0$ | – | – | $0.50^{+0.13}_{-0.12} \pm 0.06$ | $0.50 \pm 0.14$ |

$^a$The error is scaled by a factor of $S = 1.4$.

$^b$The error is scaled by a factor of $S = 1.1$.

TABLE II: Form factors in covariant light-front models [41]. For $B \rightarrow \eta'$ form factors the mixing angle and Clebsh-Gordan coefficients are included.

| $F_B^{B\pi}(m^2_{D,D_s})$ | 0.28 | $F_B^{BD}(m^2_{\pi,K})$ | 0.67 |
| $F_B^{B\eta}(m^2_{D,D_s})$ | 0.15 | $F_B^{B\eta'}(m^2_{D,D_s})$ | 0.13 |
| $F_B^{BK}(m^2_{D,D_s})$ | 0.43 | |

$F_B^{BP}(m^2_D)$, respectively. It should be noted that in the quark diagram approach [or the SU(3) approach] the full amplitudes (i.e. without the super- and subscripts $f$) are expressed as in Eq. (15), and one treats $T$ (tree), $C$ (color-suppressed), $E$ (exchange) and $A$ (annihilation) as complex topological amplitudes [35, 36, 37, 38, 39].

III. RESULTS AND DISCUSSION

In our numerical study, masses and lifetimes are taken from the Particle Data Group (PDG) [40]. We use the color suppressed branching ratios as stated in Table II. For other modes, such as $B^0 \rightarrow D^+\pi^-$ and $B^- \rightarrow D^0\pi^-$, $D^0K^-$ decays, we use PDG values [40]. We fix $V_{ud} = 0.9738$, $V_{us} = 0.2200$, $V_{cb} = 0.0413$, $V_{cs} = 0.996$, $|V_{ub}| = 3.67 \times 10^{-3}$, and use the decay constants $f_\pi = 131$ MeV, $f_K = 156$ MeV [40], and $f_{D(s)} = 200$ (230) MeV. Form factors are taken from the covariant light-front quark model calculation [41], where we list the relevant values in Table II.
To describe the processes with rescattering from factorization amplitudes, we have six parameters: the two effective Wilson coefficients $a_1^{\text{eff}}$ and $a_2^{\text{eff}}$, and the three rescattering parameters $\delta$, $\theta$ and $\sigma$, and one mixing angle $\tau$ in $S^{1/2}$. In subsection A, these parameters are fitted with $C = 1, S = 0$ $DP$ data, i.e. the $\bar{B} \to D^+\pi^-, D^0\pi^-, D^0\eta, D^0\eta'$ and $D_s^+K^-$ rates. We then use the extracted parameters to predict $(C = 1, S = -1) \bar{B} \to D^0K^-, D^+K^- \text{ and } D^0\overline{K}^0$ rates and compare with measurement. Predictions for the $(C = -1, S = -1) \bar{B} \to D\overline{K}$ modes, and the value for $r_B$, are given in subsection B.

### A. FSI Effects on $\bar{B} \to DP$ and $D\overline{K}$ Rates

Taking the $\bar{B} \to D^+\pi^-, D^0\pi^-, D^0\eta, D^0\eta'$ and $D_s^+K^-$ rates as input, we fit for $a_{1,2}^{\text{eff}}$ and the FSI phases and mixing. The fitted parameters are given in Table III where errors are propagated from the experimental errors. The factorization rates and the predicted $D\overline{K}$ rates are compared to experimental results in Table IV. The factorization rates are determined by using the $a_{1,2}^{\text{eff}}$ from the fit, but setting all FSI phases and mixing angle to zero. Unitarity is then implied automatically, i.e. sum of rates within coupled modes are unchanged by FSI.

Table IV illustrates the effect of FSI. $\bar{B}^0 \to D^0h^0$ (where $h^0 = \pi^0, \eta, \eta'$) rates are fed

| parameter | solution | parameter | solution |
|-----------|----------|-----------|----------|
| $a_1^{\text{eff}}$ | $0.92^{+0.04}_{-0.06}$ | $a_2^{\text{eff}}$ | $0.22^{+0.12}_{-0.09}$ |
| $\delta$ | $\pm(62.4^{+5.4}_{-5.6})^\circ$ | $\theta$ | $\pm(23.8^{+2.0}_{-7.2})^\circ$ |
| $\sigma$ | $\pm(127^{+46.8}_{-77.0})^\circ$ | $\tau$ | $(1.7^{+21.3}_{-3.9})^\circ$ |
| $1 + ir_0'$ | $(0.73 \pm 0.04) \pm (0.44 \pm 0.02)i$ | $ir_c'$ | $(0.27 \pm 0.04) \mp (0.44 \pm 0.02)i$ |
| $ir_a'$ | $(0.10 \pm 0.02) \mp (0.07 \pm 0.02)i$ | $i(r_a' + r_c')$ | $(-0.05^{+0.11}_{-0.13}) \pm (0.01^{+0.16}_{-0.11})i$ |
| $1 + ir_0' + i(r_0' + r_a')/3$ | $(-0.61^{+1.29}_{-0.39}) \pm (0.79^{+0.21}_{-1.37})i$ | $i(r_a' + r_c')$ | $(0.37 \pm 0.06) \mp (0.51 \pm 0.03)i$ |
| $1 + ir_0' + i(r_0' + r_a')/3$ | $(0.86 \pm 0.02) \pm (0.27^{+0.01}_{-0.02})i$ |
TABLE IV: The branching ratios of various $B \to DP$ and $D\bar{K}$ modes in $10^{-4}$ units. The second and third columns compare experiment with factorization model, and the last column gives the FSI results. The factorization results are obtained by using the same set of parameters but with FSI phases set to zero.

| Mode       | $B^{\exp}$ ($10^{-4}$) | $B^{\text{fac}}$ ($10^{-4}$) | $B^{\text{FSI}}$ ($10^{-4}$) |
|------------|-------------------------|-------------------------------|-------------------------------|
| $D^0\pi^-$ | 49.8 ± 2.9              | input                         | input                         |
| $D^+\pi^-$ | 27.6 ± 2.5              | 33.0$^{+3.0}_{-4.3}$          | input                         |
| $D^0\pi^0$ | 2.53 ± 0.20             | 0.51$^{+0.72}_{-0.34}$       | input                         |
| $D_s^+K^-$ | 0.38 ± 0.13             | 0                             | input                         |
| $D^0\eta$  | 2.11 ± 0.33             | 0.29$^{+0.41}_{-0.20}$       | input                         |
| $D^0\eta'$ | 1.26 ± 0.26             | 0.18$^{+0.26}_{-0.12}$       | input                         |
| $D^0K^-$   | 3.7 ± 0.6               | 3.91$^{+0.37}_{-0.32}$       | 3.91$^{+0.37}_{-0.32}$       |
| $D^+K^-$   | 2.0 ± 0.6               | 2.38$^{+0.21}_{-0.31}$       | 1.78$^{+0.20}_{-0.17}$       |
| $D^0\bar{K}^0$ | 0.50 ± 0.14 | 0.12$^{+0.17}_{-0.08}$       | 0.73$^{+0.08}_{-0.10}$       |

mostly by $B^0 \to D^+\pi^-$. Since these rescattering parameters are extracted from CP even measurements, the overall sign of phases is undetermined.

The FSI contributions for the $D_s^+K^-$, $D^0\pi^0$ and $D^0\eta_8$ rates from $D^+\pi^-$ rescattering are governed by $r'_a$, $r'_a-r'_e$ and $r'_a+r'_e$, respectively. The strength of $B(B^0 \to D_s^+K^-)$ $\simeq 4 \times 10^{-5}$ implies that $r'_a$ cannot be too small, i.e. $|r'_a| \simeq \sqrt{B(D_s^+K^-)/B(D^+\pi^-)} \simeq 0.12$. On the other hand the FSI enhances $D^0\pi^0$ rate from $0.5 \times 10^{-4}$ to $2.5 \times 10^{-4}$. Comparing these two modes, we have $|r'_e-r'_a| > |r'_a|$. Consequently, through the analysis of the two above modes, the size of FSI contribution to $D^0\eta_8$ is roughly determined. In fact, the FSI contribution alone already gives $B(D^0\eta_8) \simeq 2 \times 10^{-4}$, and after interference with the short distance contribution, one gets $B(D^0\eta_8) \simeq 3 \times 10^{-4}$. To fit the $D^0\eta$ data, the $D^0\eta_8$ amplitude interferes destructively, through $\sigma > 90^\circ$, with the $D^0\eta_1$ amplitude, while at the same time the $D^0\eta'$ amplitude gets enhanced through constructive interference. Although the $D^0\eta_1$ amplitude is small and does not be enhanced in FSI (due to the smallness of $\hat{r}_{e,a}'$), it still affects $D^0\eta$ and $D^0\eta'$ rates through interferences.

Utilizing the SU(3) framework, we can predict the results for $C = 1$, $S = -1$ $D\bar{K}$ modes. The prediction for $B \to D^0K^-$, $D^+K^-$ and $D^0\bar{K}^0$ rates are given in Table IV.
where the experimental data are also listed. These rates were not used in the fit for $a_i$ and FSI parameters. We see that the $D^0 K^-$, $D^+ K^-$ rates are in good agreement with data. The color-suppressed $D^0 \overline{K}^0$ rate is a bit larger than data, but still in reasonable agreement.

From Table III we observe that $1 + ir'_0 \simeq 0.85 e^{\pm i31^\circ}$ is almost perpendicular to $ir'_e \simeq 0.52 e^{\mp i58^\circ}$. We do not know the reason for this orthogonality, but this implies that the FSI amplitude from $D^+ K^- \rightarrow D^0 \overline{K}^0$ rescattering is almost perpendicular to the one from $D^0 \overline{K}^0 \rightarrow D^0 \overline{K}^0$ rescattering [c.f. Eq. (8)]. Consequently, we have

$$B(D^0 \overline{K}^0) \simeq |1 + ir'_0|^2 B^{\text{fac}}(D^0 \overline{K}^0) + |ir'_e|^2 B^{\text{fac}}(D^+ K^-),$$

which gives a very good approximation of the result shown in Table IV. As we shall see, a relation similar to Eq. (17) holds for the $B^- \rightarrow D^0 K^-$ case. From Eq. (17) we see that a $\sim 15\%$ reduction of $a_1^{\text{eff}} r'_e$ from its central value can reproduce the current $B(D^0 \overline{K}^0)$ central value. In fact, a smaller $B(D^0 \overline{K}^0) \simeq 0.5 \times 10^{-4}$ was predicted \cite{11} with a smaller $\delta$ extracted from earlier $D^0 h^0$ data, so the measurements probably have yet to settle. Note that the rate of the color-allowed $D^0 K^-$ mode is not affected by the quasi-elastic rescattering, just like $D^0 \pi^-$. 

**B. $B \rightarrow D K$ Rates and Prediction of $r_B$**

In this subsection, the $B \rightarrow D K$ rates and $r_B$, $\delta_B$ are predicted and compared with data.

Table VI gives the current experimental results on $r_B$, $\delta_B$ and $\phi_3/\gamma$ from the $DK$ Dalitz method \cite{23, 24}. Our predictions for $B \rightarrow D^0 K^-$, $D^- \overline{K}^0$, $D^- \pi^0$, $D^- \eta$ and $D^- \eta'$ decay modes are summarized in Table V.

|       | Belle          | BaBar          |
|-------|----------------|----------------|
| $r_B$ | $0.21 \pm 0.08 \pm 0.03 \pm 0.04$ | $< 0.19$ (90\% CL) |
| $\delta_B$ | $-23^\circ \pm 19^\circ \pm 11^\circ \pm 21^\circ$ | $-66^\circ \pm 41^\circ \pm 8^\circ \pm 10^\circ$ |
| $\phi_3(\gamma)$ | $64^\circ \pm 19^\circ \pm 13^\circ \pm 11^\circ$ | $70^\circ \pm 44^\circ \pm 10^\circ \pm 10^\circ$ |

\textsuperscript{4} In place of $\delta$, a different notation $\delta'$ was used in \cite{11}.

\textsuperscript{5} Note that we use the phase convention $CP |D^0| = |\overline{D}^0|$ and, consequently, our $\delta_B$ is related to those in \cite{23, 24} by a $\delta_B - \pi$ transformation.
The last two modes are purely by isospin. Consequently, the orthogonal to the one from and themselves (as one may check that the sum of their rates are roughly conserved under FSI).

Furthermore, we see from Table III that 1 + ir modes have $I = 0, 1$ components, the third mode is purely $I = 1$ while the last two modes are purely $I = 0$. Rescattering between $D_s^-\pi^0$ and $D_s^-\eta^{(i)}$ is forbidden by isospin. Consequently, the $D_s^-\eta$ and $D_s^-\eta'$ rates do not receive any contribution form the $D_s^-\pi^0$ mode. These two modes also do not rescatter much from $D^-K^0$ and $D^0K^-$, as the rescattering parameters are either suppressed by Clebsch-Gordan coefficients, or by the smallness of $r_e' + r_a'$ (c.f. Table III). Thus, these two modes basically rescatter among themselves (as one may check that the sum of their rates is unchanged in the presence of FSI). The sum of rates is consequently, the FSI amplitude from $D_s^-\eta$ and $D_s^-\eta'$ is not ruled out.

The $D_s^-\eta$ and $D_s^-\eta'$ rates are reduced and enhanced, respectively, through FSI between themselves. This is due to the destructive and constructive interference effects of $D_s^-\eta_8$ and $D_s^-\eta_1$ in the $\sigma > 90^\circ$ case as required from the $D^0\eta^{(i)}$ data. Note that within error we can also have $\sigma < 90^\circ$, so $B(D_s^-\eta) > B(D_s^-\eta')$ is not ruled out.

For the first three modes, the dominant source of rescattering is $D_s^-\pi^0$. It feeds $\overline{D}^0K^-$ and $D^-K^0$ through $r_e'/\sqrt{2}$ and $-r_e'/\sqrt{2}$, respectively. Note that these $\mathcal{S}^{1/2}$ matrix elements are similar to the $D^+K^- \rightarrow D^0K^0$ rescattering matrix element except for the $1/\sqrt{2}$ factor. Furthermore, we see from Table III that $1 + ir_8' + ir_a' \simeq 0.97e^{i32^\circ}$ is perpendicular to $i\gamma e^{i58^\circ}$. Consequently, the FSI amplitude from $D_s^-\pi^0 \rightarrow \overline{D}^0K^-$ rescattering is orthogonal to the one from $\overline{D}^0K^- \rightarrow \overline{D}^0K^-$ rescattering [c.f. Eq. (13)], and the rate of $\overline{D}^0K^-$ is roughly given by

$$B(\overline{D}^0K^-) \simeq B^{\text{fac}}(\overline{D}^0K^-) + \frac{|i\gamma e^{i58^\circ}|^2}{\sqrt{2}} B^{\text{fac}}(D_s^-\pi^0) \simeq B^{\text{fac}}(\overline{D}^0K^-) + 0.13 B^{\text{fac}}(D_s^-\pi^0), \quad (18)$$

### Table VI: Predictions for $B^- \rightarrow D^0K^-$ rates

| Mode            | $B^{\text{exp}} (10^{-5})$ | $B^{\text{fac}} (10^{-5})$ | $B^{\text{FSI}} (10^{-5})$ |
|-----------------|-----------------------------|-----------------------------|-----------------------------|
| $\overline{D}^0K^-$ | $-$                         | $0.17_{-0.11}^{+0.23}$     | $0.28_{-0.15}^{+0.23}$     |
| $D^-K^0$        | $-$                         | $0$                         | $0.05_{-0.03}^{+0.06}$     |
| $D_s^-\pi^0$    | $< 20$                      | $0.77_{-0.10}^{+0.07}$     | $0.59_{-0.05}^{+0.06}$     |
| $D_s^-\eta$     | $< 50$                      | $0.46_{-0.06}^{+0.04}$     | $0.17_{-0.09}^{+0.30}$     |
| $D_s^-\eta'$    | $-$                         | $0.30 \pm 0.03$            | $0.58_{-0.26}^{+0.12}$     |

Experimental limits [40] are shown in the second column. The third and fourth columns are factorization and FSI results, respectively, using the same parameters as Table III.
which is analogous to Eq. (17) for the $D^0\bar{K}^0$ case. Since the $D^0\bar{K}^0$ and $D^0K^-$ modes have similar relations and the FSI contributions are governed by the same parameter $ir'_e$, the $D^0\bar{K}^0$ mode may provide some estimation of the FSI effect in the $D^0K^-$ case. For example, if the current central value of $5 \times 10^{-5}$ for $D^0\bar{K}^0$ holds, we would need roughly a 15% reduction in $a_1^{\text{eff}}r'_e$ as discussed in the previous section. This corresponds to a 10% reduction in the predicted $D^0K^-$ rate or, equivalently, a 5% reduction in $|A_{D^0K^-}|$. Such a variation is within the error shown in Table VI.

The $B^- \to D^-\bar{K}^0$ decay is a pure annihilation decay mode. Its decay rate can give us an idea of the size of the annihilation amplitude. We set the short distance annihilation contribution to zero. Its rate then comes mainly from $D^+_s\pi^0$ rescattering, which interferes destructively with the rescattering contribution from $D^0K^-$. In terms of topological amplitudes we have $(a/t)_{D^0K^-} = 0.21 \pm 0.08$ and arg $(a/t)_{D^0K^-} = \pm (104.8^{+14.3}_{-10.2})^\circ$, where $a$ and $t$ are the full annihilation and tree amplitudes, respectively [see Eq. (15) and subsequent discussion]. The $(a/t)_{D^0K^-}$ ratio is numerically close to $a_2^{\text{eff}}/a_1^{\text{eff}}$. The long distance annihilation amplitude cannot be neglected.

The $A(B^- \to D^0K^-)$ and $A(B^- \to D^0K^-)$ amplitude ratio gives $r_B$ and $\delta_B$, and the values are shown in Table VII and compared with experiment. The $r_B$ parameter [c.f. Eq. (11)] governs the strength of the interference effect that is essential for $\phi_3/\gamma$ determination in the GLW, ADS and $D\bar{K}$ Dalitz methods. We see that our $r_B = 0.09 \pm 0.02$ prefers the BaBar result over the Belle result, while $\delta_B$ is in agreement with both BaBar and Belle results. Our $r_B$ value is also in good agreement with the fit from UTfit group, obtained by using all three methods of GLW, ADS and $D\bar{K}$ Dalitz analysis. The smallness of $r_B$ implies that we need more $B$ data to determine $\phi_3$.

It is interesting to note that the ratios $\sqrt{\mathcal{B}(D^0\pi^0)/\mathcal{B}(D^+\pi^-)}$, $\sqrt{\mathcal{B}(D^0\bar{K}^0)/\mathcal{B}(D^+K^-)}$ and $\sqrt{\mathcal{B}(D^0K^-)/\mathcal{B}(D^+_s\pi^0)}$ are enhanced by roughly 2.44, 2.85 and 1.47, respectively, from their factorization values. These enhancements are sometimes interpreted as [9, 10] enhancement in $|a_2/a_1|$. However, the measured ratios are nonuniversal. In particular, we would have a larger $r_B$ if we straightforwardly apply the $|a_2/a_1|$ ratio from the $D\pi$ and $D\bar{K}$ systems [15]. So let us try to understand the smallness of $r_B$ or, equivalently, the smallness of the enhancement in $\sqrt{\mathcal{B}(D^0K^-)/\mathcal{B}(D^+_s\pi^0)}$. 

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TABLE VII: Factorization and FSI results on $r_B$, $\delta_B$ with $|V_{ub}| = 3.67 \times 10^{-3}$, and compared to the experimental results \[23, 24, 25\]. For the phase convention adopted see footnote 5.

|     | Expt | fac  | FSI                |
|-----|------|------|--------------------|
| $r_B$ | 0.21 $\pm$ 0.08 $\pm$ 0.03 $\pm$ 0.04 (Belle) | 0.07 $\pm$ 0.03 | 0.09 $\pm$ 0.02 |
|     | $<0.19$ (90\% CL) (BaBar) | | |
| $\delta_B$ | $-23^\circ \pm 19^\circ \pm 11^\circ \pm 21^\circ$ (Belle) | 0 | $\mp(19.9^{+25.1}_{-13.9})^\circ$ |
|     | $-66^\circ \pm 41^\circ \pm 8^\circ \pm 10^\circ$ (BaBar) | | |

In the factorization approach we have

$$r_B^{\text{fac}} \simeq \frac{|V_{ub}V_{cs}|}{V_{cs}V_{us}} \frac{a_2^{\text{eff}}}{a_1^{\text{eff}} X_{DK} + a_2^{\text{eff}}} \simeq 0.07$$

(19)

with $X_{DK} \equiv (m_B^2 - m_D^2) f_K F_0^{BD}(m_B^2)/(m_B^2 - m_K^2) f_D F_0^{BK}(m_D^2) \simeq 1.08$. This $r_B^{\text{fac}}$ agrees with the common estimation. In Table VII we see that $r_B$ is indeed enhanced from its factorization value but the enhancement is mild. Although the $\overline{D}^0 K^-$ rate is enhanced by 70\% from its factorization value, the amplitude is only enhanced by 30\%. The rate of the color-allowed $D^0 K^-$ mode is not affected by the quasi-elastic rescattering. Consequently, $r_B$ does not differ much from its factorization prediction. Note that although the predicted $\overline{D}^0 K^-$ rate has a large error, the error in $r_B$ is much reduced as it is a ratio and, furthermore, a ratio of amplitudes.

It is instructive to compare $B^- \to \overline{D}^0 K^-$ with the color suppressed modes $\overline{B}^0 \to D^0 \pi^0$ and $D^0 K^0$. Before rescattering, i.e. at factorization level, we have

$$\frac{A_{D^{0}\pi^{0}}^{f}}{A_{D^{+}\pi^{-}}^{f}} \simeq \frac{a_2^{\text{eff}}}{\sqrt{2} a_1^{\text{eff}} X_{D\pi}} \simeq 0.13,$$

$$\frac{A_{D^{0}K^{0}}^{f}}{A_{D^{+}K^{-}}^{f}} \simeq \frac{a_2^{\text{eff}}}{a_1^{\text{eff}} X_{DK}} \simeq 0.23,$$

$$\frac{A_{D^{0}K^{-}}^{f}}{A_{D^{+}\pi^{0}}^{f}} \simeq \sqrt{2} Y_{DK} a_2^{\text{eff}} a_1^{\text{eff}} \simeq 0.46$$

(20)

with $X_{DP} \equiv (m_B^2 - m_D^2) f_P F_0^{BD}(m_P^2)/(m_B^2 - m_P^2) f_D F_0^{BP}(m_D^2)$, which gives $X_{D\pi} \simeq 1.38$, and $Y_{DK} \equiv f_D F_0^{BK}(m_D^2)/(f_D F_0^{BK}(m_D^2)) \simeq 1.33$. The rates of the sources $D^+ \pi^-$ and $D^+ K^-$ are much larger than the corresponding color suppressed modes, hence the effects of FSI are
prominent. For $D^+\pi^-$, it can be traced to the source mode being relatively enhanced by $X_{D\pi}$, while the color suppressed $D^0\pi^0$ is further suppressed by $1/\sqrt{2}$ in the $\pi^0$ wavefunction. In the case of $B^- \to \overline{D}^0 K^-$, however, the situation is reversed. The color-suppressed mode $\overline{D}^0 K^-$ is relatively enhanced by $Y_{DK}$, while the $D_s^-\pi^0$ source mode receives the $1/\sqrt{2}$ wavefunction suppression. Thus, the factorization rate for $\overline{D}^0 K^-$ differs less from the $D_s^-\pi^0$ source rate, and the enhancement of $r_B$ through FSI is mild. Since the FSI effect is not simply multiplicative [see Eqs. (17) and (18)], we do not have a simplistic universal enhancement in $|a_2/a_1|$.

C. Discussion

Let us first offer some remarks on the fitted results of Tables III and IV. The effective Wilson coefficients $a_1^{\text{eff}} = 0.92^{+0.04}_{-0.06} [F_0^{BD}(0)/0.67]$ and $a_2^{\text{eff}} = 0.22^{+0.12}_{-0.09} [F_0^{B\pi}(m_D^2)/0.28]$ obtained in our fit to $DP$ data agree well with $|a_2| = 0.26 \pm 0.02$ from fit to $B \to J/\psi K$ data [42], and with the range of $a_1^{\text{eff}} \simeq 1$, $a_2^{\text{eff}} \simeq 0.2$–0.3 from various modes [42, 43]. The ratio $a_2^{\text{eff}}/a_1^{\text{eff}} \simeq 0.24$ is close to the one used in our previous analysis [11].

We find from data that a small mixing angle $|\tau| \ll 1$ is preferred. Thus, the approximation of treating $\eta$ as $\eta_8$ and ignoring $\eta_1$ taken in our previous analysis [11] is basically valid, and the FSI phases $\delta$ and $\theta$ correspond to those in [11]. In fact, the values for the phases $\delta \simeq \pm 62^\circ$ and $\theta \sim 24^\circ$ are consistent with the previous results of $\pm 48^\circ$ and $\pm 25^\circ$, respectively. We note that, as a consequence of the smallness of $\tau$, the phase $\sigma$ is less constrained from the present $D^0\eta^{(*)}$ data.

From the $r'_i$ values given in Table III we see that exchange rescattering is dominant over annihilation rescattering. Comparing $r'_i$ and $\tilde{r}'_i$ with $r'_i$, which should be identical in the U(3) limit, we see that U(3) is not a very useful limit for these modes.

In the present work the overall sign of FSI phases cannot be determined. We may obtain some information comparing to other work. For example, a pole model calculation [18] with some inputs, such as the Wilson coefficients $a_{1,2}^{\text{eff}}$ and cut-offs for form factors in strong interaction vertices, or the pQCD approach can give $D^0 h^0$ rates in good agreement with $D^0 h^0$ data and our results. For the $D\pi$ system, we have

$$
\left( \frac{C - E}{T + C} \right)_{D\pi} = -\sqrt{2} A_{D^0\pi^-}/A_{D^0\pi^-} \simeq 0.33 e^{\mp 116^i},
$$
$$
\left( \frac{C - E}{T + E} \right)_{D\pi} = -\sqrt{2} A_{D^0\pi^-}/A_{D^0\pi^-} \simeq 0.43 e^{\mp 116^i},
$$

(21)
where negative (positive) phase corresponds to the case of positive (negative) \( \delta, \sigma \) and \( \theta \).

Comparing to \( \frac{(C - E)}{(T + C)} = 0.33 \, e^{-50^\circ i} \) \[18\], \( 0.34 \, e^{-92^\circ i} \) \[14\] and \( \frac{(C - E)}{(T + E)} = 0.40 \, e^{-67^\circ i} \) \[18\], \( 0.32 \, e^{-111^\circ i} \) \[14\], the case of positive \( \delta, \sigma \) and \( \theta \) phases is preferred. It is interesting to note that, since the amplitudes for color suppressed modes in the \( D\pi \) system are fed dominantly from the same amplitude \( A_{D+\pi^-} \), by neglecting contribution from \( a_2 \), we have

\[
\left( \frac{C - E}{T + C} \right)_{D\pi} \simeq \frac{i r'_e - i r'_a}{1 + i r'_0 + i r'_e} \simeq 0.41 e^{+65^\circ i}, \quad \left( \frac{C - E}{T + E} \right)_{D\pi} \simeq \frac{i r'_a}{1 + i r'_0 + i r'_a} \simeq 0.49 e^{+89^\circ i},
\]

which give estimations within 30\% errors.

In the extraction of \( \phi_3 \) from \( B \to D K \), \( D K \) decays the sign of the strong phase \( \delta_B \) can be determined because one is making a \( CP \) violation study. The above preferable case of positive FSI phases \( \delta, \sigma \) and \( \theta \) leads to a negative \( \delta_B \) (c.f. Table VII), which is supported by the measured central values.

Let us turn to the question of \( r_B \) determination, and compare with other approaches.

We fitted six parameters, two effective Wilson coefficients, three FSI phases and one mixing angle, from rates of six modes, \( D^0\pi^- \), \( D^+\pi^- \), \( D^0\pi^0 \), \( D_s^+K^- \), \( D^0\eta \) and \( D^0\eta' \). These parameters are fully determined as the number of unknowns equals that of input and, consequently, the errors in parameters are propagated from data errors. To keep the above features we do not include more modes, such as the three \( D\bar{K} \) modes as input. In principle, we can also include them in the fit. However, as discussed already after Eq. \( \text{(18)} \), the \( r_B \) obtained in the new fit should be consistent with the one given here within errors.

One can extract the topological amplitudes from the \( D^0h^0 \) data as in Ref. \[12\]. However, it is not clear how to apply these amplitudes to the \( \overline{D}P \) system as the two topologies are not identical; for example, the form factor dependence in \( t_f \) and \( T_f \) in Eq. \( \text{(15)} \) are different. Furthermore, the annihilation amplitude, which turns out to be non-negligible in the \( \overline{D}P \) system, cannot be extracted in the \( D^0h^0 \) system.

In pQCD approach the \( D^0h^0 \) rates are explained through the enhancement in \( C \) from the incomplete cancellation in the non-factorization contribution \[14\]. Similar mechanism may lead to an enhanced \( r_B \). Recently, a calculation in the pQCD approach gives \( r_B = 0.093 \) \[26\], which is close to our result.

Color allowed \( \overline{B} \to D\pi \) modes can be calculated in the framework of QCD factoriza-
tion [44], but the color suppressed decay amplitudes with the emission of a heavy meson cannot. A process dependent $a_2$ approach is used [8, 10]. For a comparison to the present approach, see Ref. [11]. In this vein, soft colinear effective theory (SCET) have received some attention lately. Although SCET cannot predict the $D^0 h^0$ rate, the prediction on the similarity of $D^{*0} h^0$ and $D^0 h^0$ phase and the $B(D^0\eta')/B(D^0\eta)$ ratio agree with data within error [13]. It would be interesting to see the SCET prediction on $r_B$. We note that the strong phases extracted form $D^{*0} h^0$ in our FSI approach are similar to those in $D^0 h^0$ [11], in agreement with SCET. Consequently, $r_B$ in the $D^* K^-$ system could be similar to that presented here. Indeed, $r_B = 0.09 \pm 0.04$ [25] in the $D^* K^-$ system is given by the UTfit group and is close to our estimation for $B \rightarrow D K^-$. 

IV. CONCLUSION

We study quasi-elastic rescattering effects in $B \rightarrow D P$, $D \overline{K}$ and $\overline{D} K$ modes. The updated $\overline{B}^0 \rightarrow D^0 \pi^0, D^0 \eta, D^0 \eta'$ data, together with $D^+ \pi^-$ and $D^0 \pi^-$, are used to extract $a_1^{\text{eff}}$ and four rescattering parameters. We find the effective Wilson coefficients $a_1^{\text{eff}} \simeq 0.92$, $a_2^{\text{eff}} \simeq 0.22$, the strong phases $\delta \simeq 62^\circ$, $\theta = 24^\circ$, $\sigma \simeq 127^\circ$ and mixing angle $\tau \simeq 2^\circ$. The values of $\delta$ and $\theta$ are close to our previous results [11] ignoring $D \eta_1$. The smallness of $\tau$ implies small mixing of $D^0 \eta_1$ with other $DP$ modes, hence our previous approximation is valid. The predicted $B^- \rightarrow D^0 K^-$ and $\overline{B}^0 \rightarrow D^+ K^-$, $D^0 \overline{K}^0$ rates are in agreement with data. The formalism can be applied to $B \rightarrow \overline{D} K$ modes, and the rates for $\overline{D}^0 K^-$, $D^- \overline{K}^0$, $D^- \pi^0$, $D^- \eta$ and $D^- \eta'$ modes are predicted. In particular, we predict $r_B = 0.09 \pm 0.02$, which agrees with the UTfit extraction [25] and a recent pQCD result [26]. Our $r_B$ value prefers the lower value of the BaBar experiment and disfavors the Belle result, extracted from the $\phi_3/\gamma$ fit to $B^- \rightarrow \{D^0, \overline{D}^0\} K^-$ data using the $DK$ Dalitz method.

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