Modelling of Rolling of Strips with Longitudinal Ribs by 3-D Rigid Visco-plastic Finite Element Method

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A main feature of special shape strips is the local residual deformation on a normal flat. This paper considers a numerical simulation of the rolling of strips with longitudinal ribs as an example, to analyse the influence of friction variation on the convergence and results of simulation such as rolling force, rib height and forward slip by a 3-D rigid visco-plastic Finite Element Method (FEM). The effects of mesh division and the number of elements on the precision, stability and convergence of simulation are also discussed. This investigation shows that a frictional stress model with a variation in the deformation zone can provide satisfactory results. Suitable mesh division can improve the precision and convergence of the simulation. The simulation coupled with temperature field is also discussed, and the results are in good agreement with experimental values.

KEY WORDS: friction variation, mesh division, local residual deformation, simulation precision, rigid visco-plastic FEM and rolling temperature.

1. Introduction

The Finite Element Method (FEM) has been used widely in the analysis of metal forming processes. Metal rolling is a large deformation process where the plastic deformation is the main part and the elastic deformation is relatively small and it can be ignored. Therefore, the rigid visco-plastic FEM was introduced in the simulation of metal forming. In the past two decades, this method has been used widely in the analysis of metal rolling processes, such as flat rolling, shape rolling, slab rolling and special shape strip rolling. However, there are still some problems that need to be solved, including an improvement in the precision of calculations, stability and convergence of the simulations to promote their wider application in the metal forming industry. Especially for the simulation of the rolling of special shape strip with local residual deformation, an important problem is the effect of friction on the simulated results. It was found that friction varies in the deformation zone, and its variation and the effects of mesh division and number of elements have an influence on the precision and convergence of simulations for its practical application.

As an example, the strip with longitudinal ribs is a type of special shaped strip, on which there are multiple continuous ribs, as shown in Fig. 1. In general, this type of strip can be used in electrical equipment, the automobile industry and in ship structures. In particular, it is used in steel piles in civil construction, in which it is made into internal spiral-ribbed pipes, thus providing a stronger bond between the pipes and the concrete than in smooth-surfaced pipes. As this can lead to a potential saving of steel pipes and concrete, the application of this type of strip is expected to increase in civil construction.

In order to obtain detailed solutions regarding the influence of friction variation, the number of elements and mesh division on the simulation results and convergence, a full 3-D rigid visco-plastic FEM has been carried out for ribbed strip rolling process in this study. The results are examined, as well as the effect of rolling temperature is considered.

2. Formulation of Finite Element Method

The slightly compressible method in rigid visco-plastic FEM has been employed to solve many kinds of rolling problems. According to the variational principle, the real velocity field must minimise the following functional

\[ \Phi = \int_{V} \int_{V} k \sigma : \varepsilon_{d} \sigma + \int_{\partial V} \tau_{f} \Delta V' \sigma : \varepsilon_{d} + \int_{\partial V} T v ds \quad (1) \]

where the first term on the right-hand side is the work rate

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Fig. 1. Cross-section and plan view of the strip with ribs.
of plastic deformation, $\bar{\sigma}$ is an equivalent stress, and $K$ is a constant. In the calculation, the deformation zone was divided into two parts, the plastic zone and the rigid zone. When $\dot{\varepsilon} \leq \dot{\varepsilon}_0$, $K = 0.5$; and when $\dot{\varepsilon} > \dot{\varepsilon}_0$, $K = 1$. Here $\dot{\varepsilon}_0$ is a very small equivalent strain rate ($10^{-4}$). The second term on the right-hand side can be written as $\int_0^1 \tau_v (\sqrt{s^2 + k_1^2} - k_1) ds$. It represents the work rate of friction, where $\tau_v$ is the frictional shear stress, $V_s$ the relative slip velocity between surfaces of strip and the rolls, and $k_1$ a very small positive constant that can avoid the second singular point in the deformation zone. The third term on the right-hand side is the work rate of applied tension, $T$ is tension, $v$ is velocities at entry and exit sections, the “−” sign before the third term indicating the forward tension, and the “+” sign the backward tension.

The relative velocity of strip $V_g$ between the surfaces of the strip and the rolls is

$$V_g = [V_x - V_R \cos \alpha]^2 + [V_y - V_R \sin \alpha]^2]^{1/2}$$

......(2)

where $V_x$, $V_y$, $V_R$ are the velocity components at any point on the interface of the rolled material along $x$, $y$, $z$ directions, respectively. $V_R$ is the tangential velocity at the same point on the rolls, and $\alpha$ the angle between the line connecting the centre of the rolls and the radius containing the same point.

The functional in Eq. (1) can be written in matrix form as

$$\phi = \frac{1}{m+1} \int \int \int \bar{\sigma} \left[ \{v\}'^{T} \{B\}^{T} [Z] \{B\} \{v\} \right]'$$

$$+ \frac{1}{g} \{v\}'^{T} \{B\}^{T} \{C\}^{T} [\{B\}'^{T} \{v\}']^{1/2} dv$$

$$+ \int \tau_v \left[ \{v\}'^{T} [N]\{v\}' - 2V_R \{v\}'^{T} [N]^{T} \right]$$

$$\times \begin{bmatrix} \cos \alpha \\ 0 \\ \sin \alpha \end{bmatrix} + \begin{bmatrix} V_R \\ k_1 \\ V_R \\ k_1 \end{bmatrix} \right]^{1/2} ds = \int \int_{S_h} TVds$$

......(3)

where $[N]$ is the element shape function, and $[Z]$ is a constant matrix which is as follows:

$$[Z] = \begin{bmatrix} 4/9 & -2/9 & -2/9 & 0 & 0 & 0 \\ -2/9 & 4/9 & -2/9 & 0 & 0 & 0 \\ -2/9 & -2/9 & 4/9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/3 \end{bmatrix}$$

......(4)

and $m$ is the strain rate sensitivity index, $[B]$ is the element strain matrix, and $[c] = [1 1 1 0 0 0]^{T}$. When the functional $\phi$ is minimised by the rigid visco-plastic FEM using the formulation for slightly compressible materials, the stress tensor can be written as follows(14):

$$\sigma _{ij} = \frac{1}{g} \left( \frac{2}{3} \dot{\varepsilon}_j + \delta_{ij} (g - \frac{2}{9}) \dot{\varepsilon}_s \right)$$

......(5)

where $\dot{\varepsilon}_s$ is the strain rate tensor, $\delta_{ij}$ is the Kronecker delta, $g$ is a small positive constant, i.e. a factor conferring a small degree of compressibility, and $\dot{\varepsilon}_s = \dot{\varepsilon}_x + \dot{\varepsilon}_y + \dot{\varepsilon}_z$ is volumetric strain rate. The forward slip can be obtained from Eq. (6):

$$S_h = 1 - \frac{\sum_{i=1}^{m_i} \nu_i}{m_i V_R}$$

......(6)

where $\nu_i$ is the node velocity at the exit of rolled material; $m_i$ is the number of node at the exit of strip. The rolling force can be formulated by Eq. (7):

$$P = \int_{0}^{l} \sigma_{ij} dy$$

......(7)

where $l$ is the projected length of the deformation zone; $b_i$ is the width of strip in the deformation zone; $\sigma_{ij}$ is the stress along the thickness direction.

3. Simulation Conditions

3.1. FE Meshes for the Rolling of Strips with Ribs

Considering the symmetry of the deformation zone about $z$-$x$ plane, only half the strip was simulated. Eight-node isoparametric elements were used, and 108 elements and 266 nodes were used to discretize strip with 4 ribs, as shown in Fig. 2. In this figure, ABB’A’ represents the symmetrical surface, KLL’K’ the free surface, MNWX the exit section and M’N’W’X’ the entry section. A’M’ is the length of workpiece before the roll bite; MA is the length of workpiece after the roll bite; M’M is the length of workpiece in the roll bite.
3.2. Second Singular Point

In general, there is a neutral point at the roll-strip interface in rolling processes, and the direction of the relative slip velocity will change when the strip passes through the neutral point. In this case, it is prone to produce a divergent result, which is called the second singular point in this deformation zone. In order to avoid this phenomenon, a new function of relative slip velocity was used in the work described here. This function can be presented in $V'_g$ as

$$V'_g = V_g^2 + k_1^2$$

where $k_1$ is small positive constant. In this study, $k_1 = 10^{-3}$.

3.3. Frictional Boundary Conditions

The frictional boundary conditions at the roll-strip interface are very complex in metal forming processes. In this study, two different frictional stress models were used. One is the constant friction model, $i.e.$

$$\tau = \frac{m_1 \sigma_y}{\sqrt{3}}$$

where $m_1$ is friction factor and $\sigma_y$ is the yield stress. The second model is a modification of the Kobayashi frictional stress model\(^1\):

$$\tau = K_1 \frac{m_1 \sigma_y}{\sqrt{3}} \left( \frac{2}{\pi} \tan^{-1} \left( \frac{V'_g}{V_g} \right) \right)$$

where $K_1$ is a coefficient describing the changes of frictional shear stress in the deformation zone, which has two parameters $K_1$ and $K_2$. Here $K_1$ is for forward slip zone and $K_2$ is for backward slip zone; $V'_g$ is a small positive constant. The distribution of this frictional shear stress model is shown in Fig. 3. Based on $V'_g = 0$, the neutral point positions along the width of strip are determined in the program. For example, the neutral point on the RSS’R’ surface (see Fig. 2) is $\theta'$ (see Fig. 3), but on rib P’P section is $\theta''$, and on inclined contact surface OO’P’P varies between $\theta'$ and $\theta''$.

3.4. Velocity Boundary Conditions

The velocity boundary conditions for the rolling of ribbed strip, are listed in Table 1 (Fig. 2). $\beta$ is the angle between the tangential line which passes across a point in $y$-$z$ plane and $y$ direction.

| No. | Location | Velocity |
|-----|----------|----------|
| 1   | ABCDEFGHJKL | $u = u_f$ (inward), $v = 0$, $w = 0$ |
| 2   | ABKL     | $u = u_f$ (inward), $v = 0$, $w = 0$ |
| 3   | ABBB     | $v = 0$ |
| 4   | RSS’R’ etc | $w = u_f$ |
| 5   | AALL     | $w = u_f$ |
| 6   | Inclined contact surface, OOPP etc | $w = u_f$, $v = -\sigma_n g + V_g \sin g$ |

It should be noted that the relationship for the inclined surfaces is not valid when the metal does not touch the roll.

3.5. Convergence Criteria

There are mainly three kinds of convergence criteria employed in the simulation of strip with ribs.

1) Criterion of Relative Velocity Increment

$$\frac{\Delta V}{V} < \varepsilon_6$$

where $\varepsilon_6$ is a very small positive constant (10\(^{-6}\)). $\Delta V$ and $V$ are the norms of $\Delta V_i$ and $V_i$ receptively. 

2) Criterion of Total Energy Functional Increment

$$\Delta \phi_k = |\phi_k - \phi_{k-1}| < \varepsilon_7$$

where $\phi_k$ is the $k$th total functional, and $\phi_{k-1}$ the $(k-1)$th total functional.

$$\frac{\Delta \phi_k}{\phi_k} < \varepsilon_8$$

where $\varepsilon_8$ is a very small positive constant (10\(^{-4}\)).

3) Criterion of Stability of Free Surfaces

1) Width stability of the rolled materials

$$\Delta y = y_k - y_{k-1} < \varepsilon_y$$

where $k$ means the $k$th iteration, and $\varepsilon_y$ a very small positive constant (10\(^{-4}\)).

2) Rib height of the rolled material

The difference in the rib height, $\Delta Z$, between the $k$th and $(k-1)$th calculation is less than a very small positive constant $\varepsilon_z$ (10\(^{-4}\)).

$$\Delta Z = Z_k - Z_{k-1} < \varepsilon_z$$

If Eqs. (11), (13), (14) and (15) are satisfied, the convergent solutions are obtained.

4. Comparison of the Computational Results with the Experimental Ones

4.1. Experimental Conditions

Two experiments were performed to test the ribbed strips. The first one was carried out on an experimental 2-high hot rolling mill, the diameter of the upper roll (with grooves) was 306 mm, and that of the lower one 304 mm. The length of the roll barrels was 800 mm. The groove $E = 34$ mm (see Fig. 1). The second test was performed on 4-high hot tandem mills in industry, the strip with ribs is 6.0×1050 mm. The rolled material is carbon steel, and the constitutive behavior of which is described in Eq. (16)\(^1\).
where \(A_0, A_1, A_2\) are constants, \(T\) is the rolling temperature (°C), \(\varepsilon\) the strain, and \(\dot{\varepsilon}\) the strain rate.

### 4.2. Computational Results

#### 4.2.1. Effect of Reduction on the Results of Simulation

According to the first experimental conditions, a simulation was made in this study. Initial thickness \(H_0 = 6.0\) mm; width \(B_0 = 140\) mm; \(m_1 = 0.5\); rolling temperature is 950 °C. The friction variation \((K_1 = K_2 = 1.0)\) and \(V_a = 0.001\) are considered. Figs. 4 and 5 show the relationship between reduction and rolling force and rib height. It can be seen that the rolling force and rib height increase when the reduction increases, and calculated values are close to the measured ones\(^{14}\).

#### 4.2.2. Effect of Compressible Factor on the Results of Simulation

During the metal forming, the metal volume is approximately constant. However, the metal volume is considered slightly compressible according to the variation principle for rigid visco-plastic slightly compressible material. Figure 6 shows that the effect of compressible factor on rib height. It is seen that the height of the middle ribs decreases when \(g\) increases, but the height of side ribs increases. The difference between the height of middle ribs and sides becomes small when \(g\) increases. This is because the volumetric strain rate increases when \(g\) increases,\(^{29}\) and the middle ribs can be approximately considered as plane strain state.\(^{14}\) Therefore, the height of the middle ribs decreases. However, the spread decrease when the volumetric strain rate increases, this makes metal flow into the grooves of side ribs and increase the height of side ribs. In order to maintain the rate of volume strain less than 0.1% and convergence, the compressible factor \(g\) is chosen in the range 0.01–0.0001.\(^{41}\)

#### 4.2.3. Effect of Number of Elements and Mesh on the Results of Simulation

In the simulation, the following conditions were investigated. Initial thickness \(H_0 = 7.31\) mm; work roll diameter: \(D_e = 749.55\) mm; rolling temperature is 900 °C; reduction: \(\varepsilon = 19.4\%\); \(m_1 = 0.35\). The friction variation \((K_1 = K_2 = 1.0)\) and \(V_a = 0.001\) are considered, and there were 6 rows of elements in the \(x\) direction, 18 rows in the width \(y\) direction, and 1 to 4 arrays in the \(z\) direction. The partition of mesh and number of elements on simulation results is listed in Table 2. From this table, it is found that the number of elements increases from 108 to 432, the difference of the integrated variables (rolling force, rib height, forward slip and so on) is smaller with 6 rows of elements in the \(x\) direction. For example, the rolling force is 2.67%, height of middle ribs is 1.60%, and height of side ribs is 1.01%. When the number of elements is more than 200, the difference in the calculated results of the forward slip is smaller. The more the number of elements in \(x\) direction is, the smaller is the rolling force. Similar results were obtained for the case of the rib height variation.

When the number of elements is the same, the partition of mesh has an important influence on the results of simulation, which is shown in Table 3. The rolling force, height of middle ribs and side ribs obtained with mesh \(9 \times 18 \times 2\) are smaller than those with mesh \(6 \times 18 \times 3\), and the number of iterations minimizing the functional is smaller with mesh \(9 \times 18 \times 2\). This shows that suitable mesh can save the program running time and increase the rate of convergence.

\(\sigma = A_0 \exp(-3.5T) - A_1 \varepsilon^{0.244} - A_2 \dot{\varepsilon}^{0.13}\)

\((16)\)
4.2.4. Effect of the Friction Conditions on the Results of Simulation

The rolling forces obtained with the assumption of constant friction and friction variation is described in Table 4. The rolling force at constant friction is a little larger than that in the case of friction variation. However, the friction variation indicates the changes of friction direction at the neutral point in deformation zone. When the constant $V_a$ increases, the difference becomes large. The friction variation model can decrease the number of iterations (see Fig. 7). Therefore, the rate of convergence with friction variation model is increased. This shows that the friction variation model is more effective in the simulation of special shape strip.

When $K_1$ and $K_2$ change, for example, $K_1=1.0$, $K_2=1.1$, $V_t=0.001$, $m_1=0.35$ and $g=0.01$, the effect of the number of elements and the change of friction variation model on the results of simulation is shown in Table 5. It is seen that the rolling force and the number of iterations decrease as the number of element increases, and they almost do not change when the number of elements is more than 320. However, the forward slip increases as the number of element increases, and it also does not change when the number of element is more than 320. This shows that the simulation is more stable and convergent when the number of element is more than 320.

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higher the friction factor is, the higher the rolling force, rib height and forward slip become. The calculated rolling force and rib height are close to the measured values\textsuperscript{14) when $m_1=0.5$.

4.2.5. Effect of Coupled Rolling Temperature on Result of Simulation

For the second experimental conditions, the work roll diameter is $D_e=749.55$ mm; the industrial rolling conditions were considered.\textsuperscript{14) The friction variation ($K_1=K_2=1.0$) and $V_s=0.001$ are used in simulation. The rolling temperature changes during the rolling processes of the strip with ribs, and has an important influence on the results of simulation. In this study, when the mesh is $6 \times 18 \times 3$, $V_s=0.001$, $m_1=0.5$ and $g=0.001$, the temperature field is coupled with the deformation field, and the simulation results of rolling force, rib height and temperature field are shown in Figs. 11, 12 and 13 respectively. From Figures 11 and 12, it can be seen that the coupled computation results are in good agreement with measured values. Therefore, the coupled calculation increases the simulation accuracy.

From the cross-sectional temperature distribution Fig. 13 (reduction 25.7%), we can see the distribution of temperature field and the profile of the ribs. In the simulation, the slab initial temperature is 1200 °C at the furnace. The result shows that the section center has the highest temperature, reducing to lower values at the surfaces of ribs and the lowest temperature at both sides of the strip. The profile of ribs is in good agreement with the experimental one.\textsuperscript{14) Figure 14 shows that the relationship between reductions and number of iterations, it shows that the number of iterations for coupled calculation is less than that for non-coupled one. This is because the coupled calculation considers the temperature changes of strip, the calculation values of deformation and work rate for every iteration are closer to the real ones. So the computation with coupled temperature field needs less iteration, and it is more stable.

5. Conclusions

1) The process of rolling of strips with longitudinal ribs was solved successfully by rigid visco-plastic FEM with friction variation model, and the rolling process parameters such as rolling force, rib height and forward slip are obtained in this study.

2) The friction variation model indicates the changes of friction direction at the neutral point in the deformation zone, and decreases the number of iterations. Therefore, the rate of convergence can be improved. The rolling force, rib height and the number of iterations are small when $K_1=0.8$ and $K_2=1.0$.

3) The partition of mesh has an important influence on the results of simulation. The more the number of elements in $x$ direction is, the smaller the rolling force and rib height are. A suitable mesh can save the program running time and increase the rate of convergence.

4) Coupled rolling temperature field computations were performed and the results are close to the measured values than those without coupled calculation, and it has a smaller number of iterations. Therefore, the coupled calculation increases the simulation accuracy and
5) The rib height of middle ribs decreases when the compressible factor $g$ increases, but the rib height of side ribs increases. The difference between the height of middle ribs and sides becomes small when $g$ increases.

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