Fundamentals of Size Separation

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Abstract

Screening and classification are used to separate particles based on differences in size. Industrially, these processes are done on a continuous basis to produce two (or more) product streams of varying degrees of fineness. Screening involves the separation of particles based on the probability of passage through a series of apertures of uniform size. Variables such as the number of presentations per second, size of the particle relative to aperture size, and retention time on the screen all are important in determining the probability of passage through the screen. In classification, the separations are determined by the movement of particles in a fluid, typically water or air. In this case, the probability of a particle reporting to the coarse or fine stream depends on the relative effect of gravity and fluid drag. This paper describes the factors affecting both screening and classification, the types of devices used in the separations, and the effects of staging to improve process performance.

Introduction

Size separation involves the partitioning of particles on the basis of size. Industrially, this process is done on a continuous basis, producing a coarse stream and a fine stream. However, in some applications, multiple streams of varying degrees of fineness are produced. The separations are performed using a wide range of devices, which generally can be grouped into one of two categories — screening or classification. Screening involves the separation of particles based on the probability of passage through a series of apertures of uniform size. Variables such as the number of presentations per second, size of the particle relative to aperture size, and retention time on the screen all play a major role in determining the probability of passage. In classification, the separations are determined by the movement of particles in a fluid, typically water or air. In this case, the probability of a particle reporting to the coarse or fine stream depends on the relative effect of gravity and fluid drag.

Screening

One method of size separation is on the basis of the probability of a particle passing through an aperture. Dry screening with a reciprocating motion is often used to separate particles at sizes greater than about 12 mm, while wet screening is generally employed for particle sizes down to about 0.5 mm. Dry, fine screening is usually performed with a gyratory motion. For finer size separations, classification devices are generally used. Wet classification is primarily carried out with hydrocyclones to separate particles down to about 10 μm, while solid-bowl centrifuges can be used for separations down to several micrometers. Dry classification is typically performed using one of the many types of commercially available mechanical classifiers for separating at sizes down to several micrometers.

In order to improve the performance of a given sizing operation, staging can be used. This involves reprocessing one of the product streams, with or without circulation. This not only improves separation efficiency, but it also can be used to decrease (or increase) the separation size of the circuit and to decrease the apparent bypass of fines to the coarse stream.

Screening

One method of size separation is on the basis of the probability of a particle passing through an aperture. For example, consider a flat screen plate having square openings of dimension s and centers of dimension c, hence a bar dimension of c−s. Ideally, the chance of a spherical particle, having a diameter d, passing through an opening would be 100% for all particles of relative size d/s<1, and 0% for all particles of relative size d/s>1 (see column I in Table 1). So obviously the relative size of the particle to the aperture is a factor affecting whether a particle passes through an aperture deck. However, the chance of such a particle...
This is because the screen deck only has 25% open area. Thus, the % open area is a factor in the chances of a particle being separated by the device. Therefore, there is a 75% chance of a particle not striking the bars.

As can be readily seen, these values are much different from those of a perfect separation. This is because the screen deck only has 25% open area. Therefore, there is a 75% chance of a particle whose dimension is d/s < 1 impacting a bar. If the bar dimension is 1/4 of the dimension of the square opening, i.e., c/s = 1.25, or 64% open area, then the chances increase as shown in column III because there is now only a 36% chance of such a particle impacting a bar, but still do not match the perfect separation values. Thus, the % open area is a factor in the chances of a particle of dimension d/s < 1 passing. Even if c/s → 1, i.e., approaching 100% open area, the probabilities of passing will not match the perfect separation values as shown in column IV. If particles become lodged in an aperture (blinding), this also reduces the % open area.

Note in column II that the chance of a particle of zero size passing is 25% or a 75% chance of not passing! This “not passing” value is given a unique symbol, a, because it represents the apparent bypassing of the separation process; that is, the process behaves as if it split or sent 75% of the feed directly to the coarse stream and only 25% had an opportunity to pass. Thus, there is a need to correct the chance of passing values for this bypassed material to determine how the device separated the apparent non-bypassed material. This is done by dividing the p(d) values by 1 - a/100. These corrected values, p'(d), are the same as those for 100% open area, shown in column IV.

The size modulus and shape modulus of the plot of these corrected values are used to characterize the separation. The size modulus, d_{50}, is defined as the particle size that has a 50% chance of passing (hence not passing). The shape modulus, k, is defined by the particle size that has a 75% chance of passing divided by the particle size that has a 25% chance of passing, and is called the sharpness index. The d_{50} value for column IV, actually d_{50}/c, is 0.3; the k value is 0.265. Note that for a perfect separation the d_{50} value is s and the k value is 1 (and a is zero). Apparent bypassing, i.e., a > 0, is commonly observed in industrial screening. In fact, the industry only guarantees “95% efficiency” which means, given the way they define efficiency, that they expect a 5% apparent bypass. It is interesting that if the values in column III are corrected, they are identical to the values in column IV, meaning that the chance of passing is the same for particles not striking the bars.

### Effects of Aperture Design on Separation

If the aperture is rectangularly shaped, say the length is 4 times the width, s, then the chance of a spherical particle approaching the deck normally without reflecting off the bar is [Gaudin, 1939]

\[
p(d) = 100 \left( \frac{1 - d/s}{c/s} \right)^2 \quad (1)
\]

Column II gives the values for the chance of reporting to the fine stream where c/s = 2; i.e., if the dimension of the bar equals the dimension of the square opening. As can be readily seen, these values are much different from those of a perfect separation. This is because the screen deck only has 25% open area. Therefore, there is a 75% chance of a particle whose dimension is d/s < 1 impacting a bar. If the bar dimension is 1/4 of the dimension of the square opening, i.e., c/s = 1.25, or 64% open area, then the chances increase as shown in column III because there is now only a 36% chance of such a particle impacting a bar, but still do not match the perfect separation values. Thus, the % open area is a factor in the chances of a particle of dimension d/s < 1 passing. Even if c/s → 1, i.e., approaching 100% open area, the probabilities of passing will not match the perfect separation values as shown in column IV. If particles become lodged in an aperture (blinding), this also reduces the % open area.

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\[
p(d) = 100 \left( \frac{1 - d/s}{c/s} \right) \left( \frac{4 - d/s}{3 + c/s} \right) \quad (2)
\]

As can be seen in column V, the chance of passing values for c/s = 2 (40% open area) are greater than for the square apertures (column II) with 25% open area, and, the corrected values are greater than the values in column IV, but still do not match the perfect values (column I). Thus the aperture geometry is also a factor affecting passage.

If the square openings are created by woven round wire rather than punched flat plate, and if the reflectance of the particle into the aperture after striking the wire is taken into account, then the chance of...
A spherical particle approaching normally a deck of square apertures is

\[ p(d) = 100 \left[ 1 - \frac{(c/s - 1 + d/s)(1 + \sqrt{1 + 8m^2})}{4(c/s)m} \right]^2 \]  

where \( m = \frac{c/s + 1 - d/s}{c/s - 1 + d/s} \) [Gaudin, 1939]. As can be seen in column VI, the chance of passing values for \( c/s = 2 \) are greater than for non-reflective, normal incidence (column II) and so are the corrected values. If the angle of incidence is less than 90\%, i.e., oblique incidence, then the chance of passing values for \( c/s = 2 \) are lower, as shown in column VII for a spherical particle striking the deck at 45\° [Gaudin, 1939], however the corrected values are greater. Thus the trajectory of approach to the aperture for a particle is a factor affecting passage.

**Effects of Operating Variables on Separation**

If a particle of relative size \( d/s < 1 \) that did not pass is given other opportunities, then the chance of that particle passing after \( n \) opportunities is

\[ 100(1 - \frac{q(d)}{100})^n \]

where \( q(d) = \frac{4(c/s)m}{(c/s - 1 + d/s)(1 + \sqrt{1 + 8m^2})} \) and the interval values, such as \( f_i = F(x_i) - F(x_{i-1}) \) are calculated for \( x_i/x_{i-1} = \sqrt{2} \), where \( F(x_i) \) is the cumulative fraction finer than size \( x_i \) in the feed stream, and \( n \) is the number of size intervals.

If this is done for a number of different operating conditions, then perhaps a systematic pattern in the changes in the chance of passing values, and consequently in \( a, d_{50} \), and \( \kappa \), may occur that can be used to predict the separation process. And since the size consist of the undersize stream does not depend upon the \( a \) value, hence, only on the corrected values, there is a unique \( d_{50} \) value for a particular \( \kappa \) value that will

**Screen Selection**

Obviously, there are other factors affecting the passage of the particles such as the amount of surface moisture, hence aggregation of the particles. Consequently, because of the multitude of factors acting individually and interactively, it is not possible to establish the chance of passage of each size particle based on first principles. Instead, manufacturers employ a design loading methodology whereby the feed rate/screen area that will produce acceptable chance of passing values is estimated. In addition, there are aperture systems that are not as easily analyzed as screen decks. For example, consider a “rotating probability” screen. The “screen deck” is created by fitting rods to a vertical rotating shaft. The rods, radiating from the central hub, create a horizontal circular deck. The “apertures” are the space between the radial rods, which progressively enlarge and have no supporting members. These “apertures” are typically larger than the feed. If the “deck” is not rotating, then all the feed particles will pass through the “apertures”. If the “deck” is rotated at very, very high speeds, then essentially none of the feed material will pass through the “apertures”. Thus, by controlling the speed of rotation, the “aperture size” is regulated.

Because the probability of collision between the particles and the rotating rods is not readily derivable, it is necessary to obtain the chances of passing via other means. For example, the values can be obtained by sampling around the device while it is operating at steady state. Then the chances of passing can be calculated from the size analysis of the feed stream, \( F \), the oversize stream, \( T \), and the undersize stream, \( Q \), as

\[ p_i = \frac{100q_i}{f_i(1+C)} \]

where \( C = \frac{\sum f_i - q_i}{\sum f_i - f_i} \) and the interval values, such as

\[ f_i = F(x_i) - F(x_{i-1}) \]
produce a fine stream analyzing 95% < a particular size.

For example, consider the results reported by Rogers and Brame [1985] for the high frequency screening of limestone slurries. They found, using a 0.35 m² pilot-plant Derrick machine that gave comparable performance to a 2.10 m² full size Derrick machine, that the slurry feed rate had no affect on the chance of passing values as long as the slurry feed rate did not exceed a maximum rate, which varied with the volumetric feed concentration, cvf, and aperture size. They found that $d_{50} = 1.08s_{50}^{0.00925}$, $0.2 < cvf < 0.5$, and $K = 0.93 d_{50}/s$. Using a functional form for the corrected values

$$p_i = \frac{100}{1 + (x_i/d_{50}) \exp [g(x_i/d_{50})^3 - 1]}$$

where $g = 0.08 \exp (4.25d_{50}/s)$, one can search for the $d_{50}$ value that will produce an undersize stream with the desired % passing a particular size by solving the algebraic relationship

$$Q_o = \sum_{i=1}^n p_i f_i$$

They also found that

$$a = 125 \frac{cvf}{1 - cvf} - 24.3, \quad 0.2 < cvf < 0.5$$

and that the apparent bypass essentially equaled the water split, the percentage of the water in the feed stream that reports to the coarse stream or $100T'/F'$, where the prime represents the mass flowrate of the water in the slurry (thus the feed slurry mass flowrate, $F''$, equals $F + F'$). Then, the mass flowrate of the solids in the undersize stream can be calculated as

$$Q = F(1 - a/100) \sum_{i=1}^n p_i f_i$$

and the solids concentration of the undersize stream can be calculated as

$$C_{vo} = \frac{1}{1 + \frac{1}{(1 - cvf) \sum_{i=1}^n p_i f_i}}$$

Obviously, the mass flowrate of the oversize stream is

$$T = F - Q$$

and the solids concentration of the oversize stream can be calculated as

$$C_vT = \frac{C_{vo}C_vT/(T/Q)}{C_{vo}(F/Q) - C_vf}$$

**Classification**

Size separations can also be carried out according to differences in the settling velocities of particles of various sizes, densities, and shapes. That is, the probability of a particle reporting to one of the product streams is a function of its settling velocity.

Consider the cylindrical device shown in Figure 1 containing a polydisperse particle system. If particles are moving within the cylinder as the result of settling (convection) and mixing (diffusion), it is possible to account for their movement into and out of an element (i.e., dz) within the device. Moreover, under conditions where the particles move independently of each other, then their settling velocity does not depend on solids concentration, i.e., free-settling conditions prevail. Since the separation occurs primarily in the vertical (z) direction, then the separating force (gravity) and the level of mixing must only be considered in this direction. If mixing is characterized by a single eddy-diffusion coefficient, then the rate of accumulation in an element $z$ to $z + dz$ can be given by

$$\frac{\partial c(x, p, z, t)}{\partial t} = D \frac{\partial^2 c(x, p, z, t)}{\partial z^2} - v(x, p) \frac{\partial c(x, p, z, t)}{\partial z}$$

where $c(x, p, z, t)$ = concentration of particles of size $x$ and density $p$ at depth $z$ and time $t$; $D$ = eddy diffusion coefficient; $v(x, p)$ = free settling velocity for particles of size $x$ and density $p$. The free-settling velocity can be calculated using Stokes equation for fine particles (i.e., particle Reynolds numbers less than ~0.1) or Newton's equation for coarser particles (i.e., particle Reynolds numbers greater than ~1000). However, it is often convenient to use Concha's equation to calculate the particle settling velocity, which can be used over the range of particle Reynolds number from about 0.1 to 10,000 [Concha and Almendra, 1979]. Equation 12 can then be used to describe particle movement under free settling conditions.

**Fig. 1** Cylindrical batch settling device.
Rather than calculate the particle concentration at various points in the separation device, it is more convenient to determine the fractional recovery of particles in a given product stream. The initial amount of material of a given size and density in a separation device of height, \( H \), is

\[
f(x,p)F = c_0(x,p) \int_0^H A(z) \, dz \tag{13}
\]

where \( f(x,p) \) = fraction of feed material of size \( x \) and density \( p \); \( F \) = mass of feed material; \( c_0(x,p) \) = initial concentration of particles of size \( x \) and density \( p \); \( A(z) \) = cross sectional area of the device at any vertical \( (z) \) direction.

If the particles are allowed to settle for some time, \( t \), and the cylinder is "cut" at some height \( (z') \), then the amount of material remaining in the upper compartment of the device is

\[
q(x,p,t)Q = \int_0^{z'} c(x,p,z,t)A(z) \, dz \tag{14}
\]

where \( q(x,p,t) \) = fraction of product material of size \( x \) and density \( p \); \( Q \) = mass of product material. The fraction of feed particles of size \( x \) and density \( p \) that remain in the upper compartment (i.e., probability of reporting to the fine stream) after time \( t \) is

\[
p(x,p,t) = \frac{q(x,p,t)Q}{f(x,p)F} = \frac{100}{HA_m} \int_0^H \frac{c(x,p,z,t)A(z) \, dz}{c_0(x,p)} \tag{15}
\]

where \( A_m \) is the mean value of the cross-sectional area. In the case of a cylindrical device, the cross-sectional area is constant and \( A(z) = A_m \). A solution for Equation 15 can be obtained by solving Equation 12 under the appropriate initial and boundary conditions [Klima and Luckie, 1989].

For size separations involving single density particles, the upper compartment would contain only fine particles, while the lower compartment would contain primarily coarse particles. However, since the feed was uniformly dispersed prior to separation, some fine particles will be found in the lower compartment. This fraction represents the apparent bypass of feed to the coarse stream and is typical of many wet size classification devices, e.g., hydrocyclones.

As is common in industry, the fraction of feed material of a given size reporting to the coarse stream (i.e., the size selectivity value) is generally desired. This is given by

\[
s(x;p,t) = \frac{t(x,p,t)T}{f(x,p)F} \tag{16}
\]

where \( t(x,p,t) \) = fraction of non-specification (i.e., coarse) material of size \( x \) and density \( p \) in the lower compartment after time \( t \); \( T \) = mass of coarse material. Obviously,

\[
s(x;p,t) = 1 - p(x;p,t)/100 \tag{17}
\]

The above treatment was derived for a batch settling system. However, Equation 15 can also be used to simulate continuous separations assuming that a lumped-parameter approach applies. For example, if it is assumed that no axial back-mixing occurs such that all particles remain in the device for the same amount of time and that the batch separation time is equivalent to the mean retention time, then the batch separations would be equivalent to continuous separations under plug flow conditions. In this case, after separation, the particles are split into two product streams, analogous to "cutting" of the batch device as shown in Figure 2. This situation is analogous to the wet classification pulp partition model discussed by Schubert and Neesse [1973]. However, in their case, a steady-state solution of the convection-diffusion equation (Equation 12) was used.

For a device as shown in Figure 2, separation occurs because particles of different sizes have different settling velocities, resulting in different trajectories. For example, consider a dispersed feed of particles of different sizes entering the device. Once in the device, large particles settle rapidly below the split point, ending up in the coarse product stream. On the other hand, finer particles that enter in the upper compartment may not have sufficient time to settle into the lower compartment and thus report to the fine product stream. Fine particles that enter into the lower compartment cannot reach the upper compartment and end up misplaced to the coarse product stream. These particles are often considered as bypass material, since they did not have the opportunity to make it to the fine product stream.

**Effect of Process Variables on Separation**

In order to examine the effects of the different fundamental parameters on size separation, a series of simulations can be performed using Equation 15 to generate size selectivity values. The particles are as-
Simulated to be settling in water, in a device 12 cm high. Simulations are performed for particle sizes in a \( \sqrt{2} \) series, i.e., 150, 105, 75, etc. down to 0.84 \( \mu m \). The baseline conditions include the following: cut height = 10.8 cm, settling time = 30 s, particle density = 2.65 g/cm\(^3\), diffusion coefficient = 1 cm\(^2\)/s, number of g’s = 1. Simulations are then performed by changing each variable, while holding the other variables constant, a process that cannot be performed in actual devices because of the confounding of variables.

In order to determine the characteristic parameters (cut size, sharpness index, and apparent bypass) for each simulation, the corrected size selectivity values are fitted to an appropriate mathematical function, i.e.,

\[
c(x;\rho,t) = \frac{s(x;\rho,t) - a/100}{1 - a/100}
\]

(18)

where \( a \) = apparent bypass of feed material to the coarse stream and

\[
c(x;\rho,t) = \frac{1}{1 + (x/d_{c0})^{2.1/2/ln(2)}k}
\]

(19)

where \( d_{c0} \) = cut size, i.e., size at which \( c(x;\rho,t) = 0.5 \); \( k \) = sharpness index, i.e., size at which \( c(x;\rho,t) = 0.25 \) divided by the size at which \( c(x;\rho,t) = 0.75 \). The simulation results are given in the following sections.

**Settling Time** — Figure 3 shows the variation of the size selectivity curves for retention times of 5, 10, 30, 60, 120, and 240 seconds. The corresponding parameters are given in Table 2. At very short times, the curve corresponds closely to the fraction remaining in the lower compartment, i.e., 10% in this case. This corresponds to a value of \( 1 - z'/H \). This is expected since at very short times, the particles will have settled only a short distance, giving a splitting action. Hence, the particle size distribution in both compartments would be expected to be similar to that of the feed, which would be represented by a horizontal line through 0.1 at time = 0.

As the separation time increases, the cut size and sharpness index decrease (Table 2). The decrease is significant up to 30 seconds, after which time very little change occurs. At long times, a limiting condition is reached in which there is a balance between the settling and mixing of the particles, i.e., steady-state condition. This condition will also result using the steady-state solution indicated previously [Schubert and Neesse, 1973]. The limiting curve will change, depending on the operating conditions and the level of mixing present.

**Level of Mixing** — Figure 4 shows the variation of the size selectivity curves with the level of mixing. As expected, increased mixing results in a worsening of the separation as shown by the flatter curves, which correspond to lower sharpness indices and much higher cut sizes (Table 3). In fact, at the highest level of mixing, the curve is very flat, indicating that very...

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**Table 2** Variation of size selectivity parameters with settling time.

| Settling Time, s | \( d_{c0}, \mu m \) | \( k \) | \( a \) |
|-----------------|-----------------|------|------|
| 5               | 177.1           | 0.88 | 0.10 |
| 10              | 111.3           | 0.63 | 0.10 |
| 30              | 91.9            | 0.49 | 0.10 |
| 60              | 90.0            | 0.46 | 0.10 |
| 120             | 89.9            | 0.46 | 0.10 |
| 240             | 89.9            | 0.46 | 0.10 |

**Fig. 3** Effect of settling time on the size selectivity curves.

**Fig. 4** Effect of mixing on the size selectivity curves.
little separation occurs. At even higher levels of mixing, no separation will occur and the final products will have the same composition as the feed, i.e., the feed stream is split into two product streams. Obviously, minimizing the level of mixing is critical to improving separator performance.

Cut Height—Figure 5 shows the variation of the size selectivity curves for cut heights of 1.2 (10%), 3.0 (25%), 6.0 (50%), 9.0 (75%), and 10.8 cm (90%). As can be seen, the curves shift downward and to the right as the cut height increases. This results in coarser cut sizes as more of the coarser material is split to the fine stream (Table 4). Moreover, at the fine sizes, each curve approaches the value of 1-\(z'/H\), which is equal to the apparent bypass for each curve. Interestingly, the sharpness indices are approximately constant at about 0.5 over the range of cut heights. This trend has been observed for industrial classifiers. The implication is that by changing the fraction of feed slurry that is split to the fine product stream (e.g., through changes in the geometry or operating conditions of the separator) the cut size can be varied without affecting the sharpness index. However, such changes will also produce an uncontrollable change in the apparent bypass, which may be detrimental to the overall separation.

Number of G’s—Figure 6 shows the variation in the size selectivity curves as a function of the number of g’s. It can be seen that the cut size decreases from 92 to 12 \(\mu\)m as the number of g’s increases from 1 to 50 (Table 5), while the sharpness indices remain constant at about 0.5. In all cases, the curves approach the limiting value of 0.1, i.e., the expected apparent bypass. Figure 7 is a log-log plot of cut size versus the number of g’s. It can be seen that it becomes increasingly difficult to achieve very fine cut sizes because of the large number of g’s required.
Particle Density — Figure 8 shows the variation in the size selectivity curves for different particle densities. As expected, the curves shift to the left with increasing particle density, indicating that denser particles separate at finer sizes (Table 6). The separation of the lightest particles is minimal. Since a constant cut height of 90% is used, the apparent bypass for all particles approaches 0.1, independent of density. As is the case for some of the previous runs, the sharpness indices are approximately constant at about 0.5.

Hindered-Settling Conditions
The above treatment was considered for free-settling conditions in which particle concentration did not impact the particle settling velocity. However, in many cases, separations are carried out under sufficiently high solid concentrations that hindered-settling conditions prevail. Such separations can be modeled in a manner similar to that for free-settling conditions. In this case, the rate of accumulation for particles of size $x$ to $x+\Delta x$ and density $\rho$ to $\rho+\Delta \rho$ in an element $z$ to $z+\Delta z$ is given by

$$\frac{\partial \phi(x,\rho,z,t)}{\partial t} = D \frac{\partial^2 \phi(x,\rho,z,t)}{\partial z^2} - \frac{\partial (V(x,\rho,z,t)\phi(x,\rho,z,t))}{\partial z}$$

where $\phi(x,\rho,z,t)$ = volume fraction of particles of size $x$ to $x+\Delta x$ and density $\rho$ to $\rho+\Delta \rho$ in element $z$ to $z+\Delta z$ at time $t$; $V(x,\rho,z,t)$ = velocity of particles of size $x$ to $x+\Delta x$ and density $\rho$ to $\rho+\Delta \rho$ in an element $z$ to $z+\Delta z$ at time $t$, with respect to the wall of the container; $\phi(z,t)$ = total volume fraction occupied by the solids in element $z$ to $z+\Delta z$ at time $t$.

As the particles settle, they displace liquid. For higher solids concentrations, the liquid displacement is significant. Thus the rate of accumulation for the liquid in element $z$ to $z+\Delta z$ is given by

$$\frac{\partial [1-\phi(z,t)]}{\partial t} = -\frac{\partial [U(z,t)(1-\phi(z,t))]}{\partial z}$$

where $1-\phi(z,t)$ = volume fraction occupied by the liquid in element $z$ to $z+\Delta z$ at time $t$; $U(z,t)$ = velocity of the liquid in element $z$ to $z+\Delta z$ at time $t$, with respect to the wall of the container.

Solving Equations 20 and 21 requires an equation to calculate the hindered-settling velocity for particles of various sizes and densities. One such equation is a modification of Concha's equation containing empirical factors to account for the effect of solids concentration on slurry viscosity. A description of this equation can be found elsewhere [Lee, 1989].

Equations 20 and 21 can then be solved using a finite difference technique to give the solids concentration at any point within the cylinder [Lee, 1989]. However, as with the free-settling equation, it is more convenient to calculate the size selectivity values. These values are given by

$$s(x;\rho,t) = \int_0^\infty \frac{\phi(x,\rho,z,t)dz}{\phi_0(x,\rho)}$$

### Particle Density

| Particle Density, g/cm³ | $d_{50}$, µm | $x$ | $a$ |
|------------------------|--------------|-----|-----|
| 1.30                   | 233.8        | 0.45| 0.10|
| 2.65                   | 91.9         | 0.49| 0.10|
| 3.30                   | 73.8         | 0.50| 0.10|
| 5.00                   | 57.7         | 0.51| 0.10|
| 7.50                   | 44.9         | 0.51| 0.10|

**Fig. 7** Variation of cut size with the number of g's.

**Fig. 8** Effect of particle density on the size selectivity curves.
where \( \phi_0(x,p) \) = initial volume fraction of particles of size \( x \) and density \( p \). Equation 22 has been used to evaluate the variation of size selectivity curves under hindered-settling conditions [Austin et al., 1992]. Very similar trends were obtained as given in the previous figures. This model has also been used to generate partition curves to evaluate density sorting separations [Cho and Klima, 1994; Klima and Cho, 1995].

**Wet Classification Devices**

Several types of devices are used to classify particles finer than about 100 \( \mu \)m. The wet devices include hydrocyclones and centrifuges and dry devices include mechanical air separators, counter-flow classifiers, and transverse-flow classifiers. The selection of the specific device depends on the desired cut size and production rate.

**Hydrocyclones** — Classifying hydrocyclones are used extensively in a wide range of industries including mineral, chemical, food, and pulp/paper to process materials in the size range from about 500 \( \mu \)m to 10 \( \mu \)m. A schematic showing the mode of operation of a hydrocyclone is given in Figure 9. Essentially, a hydrocyclone consists of hollow, cylindrical and conical sections, into which slurry is admitted, tangentially, at high velocity setting up a highly rotational flow, which imparts centrifugal forces to the particles. Slurry exits the device at the apex (the underflow) and through a central "vortex finder" at the hydrocyclone overflow. The larger and denser particles are concentrated close to the walls of the cone and exit primarily through the apex. Finer and lighter particles, together with the bulk of the carrier fluid, are forced to exit through the vortex finder.

![Fig. 9 Typical flow pattern of a hydrocyclone.](image)

**Centrifuges** — Solid-bowl (decanter) centrifuges can also be used for size classification of ultrafine particles (Figure 10). The slurry enters through the feed tube and is discharged radially onto the bowl. The (coarse) solids are collected on the bowl and are carried to the discharge end by a scroll, which rotates at a speed slightly slower than the bowl. The liquid and fines are carried hydraulically to the opposite end where they exit over an adjustable weir.

Unlike hydrocyclones, which are limited to separation sizes greater than about 10 \( \mu \)m, solid-bowl centrifuges can be used to separate particles finer than 5 \( \mu \)m. In a solid-bowl centrifuge, the gravity force can be changed by varying the rotational speed of the bowl, while keeping the flow rate constant. The result is the ability to classify at finer sizes, without a decrease in production capacity compared to a hydrocyclone, which requires smaller diameter (and lower capacity) units for finer separations. Moreover, the ability to regulate the centrifuge bowl speed (number of "g's"), flow rate (retention time), scroll speed, and weir height (water split), independently provides a high degree of control of the product size and separation efficiency. Solid-bowl centrifuges can produce more than 3000 g's and can treat a wide range of particulate slurries such as zirconia, alumina, calcium carbonate, and kaolin at sizes down to approximately 1 \( \mu \)m [Williamson and Bacon, 1977; Scheffler and Zahr, 1980]. They can be used either in stand-alone applications or as part of closed grinding circuits [Hennicke and Stein, 1989].

**Dry Classification Devices**

The vane classifier (Figure 11) is an example of a free-vortex counterflow classifier. The solids to be classified enter the outer cone dispersed and entrained in a gas stream. A cyclonic flow pattern is imparted on the feed stream before it passes through the adjustable vanes into the inner cone. As the vanes are closed down, an increase in the centrifugal motion causes more of the larger particles to strike the inner wall of the inner cone and drop out. The finer particles remain in the gas stream, exiting through the cen-
For a pilot-scale twin-cone classifier, the sharpness index was constant for a set vane position [NAS, 1981]. As the vanes were adjusted from 100% (an expansion classifier) to 50% to 25% (a pneumatic cyclone) open, the sharpness index increased from 0.3 to 0.5 but then decreased to 0.3. At the lowest air rate, the \(d_{50}\) value decreased with vane settling from 120 \(\mu m\) at the 100% open vane settling to 50 \(\mu m\) to 20 \(\mu m\). Increasing the air rate, thereby also increasing the feed rate, caused an increase in the cut size for each vane setting but a decrease in the apparent bypass at vane settings of 50% and 25% open area.

The free-vortex principle has been extended to fine classification. A built-in fan in the second chamber of the two-chamber Alpine Mikroplex Spiral classifier draws the classifying air into the classifier chamber while the feed is metered in separately. Adjustable vanes control the angle of air-flow approach to the center of the chamber where the fines flow out. The coarse particles are thrown to the outside and removed mechanically from the casing of the chamber. The sharpness index varies between 0.65 and 0.7, decreasing as the feed rate and hence solids loading increases. The cut sizes range between 2 \(\mu m\) and 30 \(\mu m\), depending on the rpm of the rotor. The \(d_{50}\) is inversely proportional to the rpm to the 1.4 power. Whereas, no apparent bypass has been reported by the manufacturer, values of approximately 5% have been measured. The forced vortex is less sensitive to solids loading than the free vortex.

The use of a rotor has been the cornerstone of the mechanical air classifier design. Rotor design has changed from blades to the multivane or post design (Figure 13). In this device, the feed material is dispersed in the airstream drawn through the rotor. Whether or not a particle exits in the central fine particle discharge depends on the force balance between the drag force of the particle being conveyed and the centrifugal force created by the rotor against it or the probability of collision, similar to the Ro-Pro screening device. The cut size is proportional to the rpm of the rotor and the square root of the relative density of the feed material. Values range between 5 \(\mu m\) and 150 \(\mu m\). Operating sharpness indices can reach 0.75.

Data for a pilot device using a blade rotor design gave a constant sharpness indices of 0.6 [Austin and Luckie, 1976]. There are several responses common to classifiers employing rotors. Increasing the feed rate without any other changes reduces the \(d_{50}\) value, the efficiency of the separation, and the yield. The
Data demonstrated a consistent pattern, in all cases, of the $d_{50}$ value decreasing to a minimum with increasing feed rate. Moreover, the efficiency of the classification is reduced with increasing feed rate because the apparent bypass value increases. The fine product yield also decreases with increasing feed rate, because the increased quantity of feed is merely bypassing to the coarse stream. Assuming the air flow rate to be proportional to fan speed, then the data for a rotor rpm of 1000 gave $d_{50}$ proportional to the air-flow rate to the 1.2 power. The 800 and 1400 rotor rpm data show that the $d_{50}$ value is inversely proportional to the rotor rpm. Industrial versions of this separator come in sizes up to 10 m in diameter.

A recirculation design (Figure 14) returns the gas to the classifier through the fan after the fine particles are removed from the gas stream. Such an arrangement requires an excellent solid/gas separator; otherwise the classification becomes less efficient. Interestingly, a perfect solid/gas separator would be a device having an apparent bypass of one. If the recirculated gas is entered through a secondary coarse stream classification section, then the classification is not less efficient unless the secondary classification is very inefficient.

It is quite common in the designs for fine classification to recontact the coarse stream transversely or in counterflow with air before discharging it (see Figure 13). This removes dry fine particles not removed in the primary classification. That is, these particles are swept back into the feed and given another chance to exit with the fine particles. Such an arrangement increases the overall sharpness index and reduces the overall apparent bypass. Another variation is to reenter the air from the solid/gas separation of the coarse stream.

The Matsuzaka Elbow-Jet classifier (Figure 15) is based on a transverse flow principle [Rumpf and Leschonski, 1967]. The stream of feed particles are accelerated to minimize the effect of gravity, and introduced into an air jet at right angles. The particles are fanned out in the classification zone with the trajectories for particles of the same hydrodynamic behavior, i.e., size and shape, being the same. Classification is achieved by mounting one or more cutters in the classification zone, thus dividing the feed into two or more fractions. A stream of fine particles of less than 5 μm can be produced in this manner.
The practice of recontacting a stream and the subsequent return to the feed is called stage classification. Based on the linear nature of the classification process, it is possible to calculate the overall chance of a particle reporting to a particular stream for any arrangement. For example, consider the arrangement where the coarse stream from the first stage is reclassified and the fine streams from both stages blended —

\[ s(x) = s_1(x) s_2(x) \]  

Thus the overall apparent bypass is

\[ a = \frac{a_1 a_2}{100} \]  

and hence will be reduced, thereby increasing the efficiency. In addition, the overall sharpness index will increase, but, unfortunately, the overall cut size will increase. The latter may not be critical since the 95% passing size in the fine stream is approximately equal to \( d_{50}/(\kappa + 0.16) \).

Consider instead the arrangement where the fine stream from the first stage is reclassified and the coarse streams from both stages blended —

\[ 1 - s(x) = (1-s_1(x))(1-s_2(x)) \]  

While the sharpness index may increase and the cut size decrease, the overall apparent bypass, given by

\[ a = \frac{a_1 a_2}{100-a_1 a_2/100} \]  

 decreases, resulting in a much more efficient separation.

Likewise, blending the coarse stream from the second stage which reclassified the fine stream from the first stage with the feed to the first stage, i.e.,—

\[ 1 - s(x) = \frac{1-s_1(x)}{1-s_2(x)(1-s_1(x))} \]  

The overall apparent bypass, given by

\[ a = \frac{a_1 a_2}{100-a_1 a_2/100} \]  

increases, reducing any overall efficiency increase, or perhaps making the separation overall efficiency less.

For example, consider the results for a 25 mm diameter hydrocyclone producing a 95% < 10 \( \mu \)m fine product. A vortex finder/apex diameter combination produces a cut size of 6 \( \mu \)m, sharpness index of 0.6, but...
an apparent bypass of 50%! Any attempt to lower the
apparent bypass in order to improve the efficiency
results in an increase in the cut size, coarsening the
product. However, by staging two of these hydrocyclones
such that the underflow from the first becomes
the feed to the second, after proper dilution, and the
overflow from the second is blended into the feed
to the first, it is possible by appropriate selection of
vortex finder/apex diameter combinations to produce
a $95\% < 10 \mu m$ product with an overall cut size of $7 \mu m$, an overall sharpness index of 0.67 and an overall
apparent bypass of 10%!

Conclusions

The separation of particles based on differences in
size are carried out using either screening or classifi-
cation. Screening relies on the probability of passage
of a particle through a series of apertures of uniform
size and depends on variables such as the number of
presentations per second, size of the particle relative
to aperture size, and retention time on the screen.
In classification, the probability of a particle reporting
to the coarse or fine stream depends on the relative
effect of gravity and fluid drag as the particles move
in a fluid. Staging can be used to improve process
performance through changes in cut size, sharpness
index, and apparent bypass.

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