Performance of Media-based Modulation in Multi-user Networks
(Invited Paper)

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Abstract—High spectral efficiency and low power consumption are the most challenging requirements of 5G networks since the number of devices is increased drastically. Media-based modulation (MBM) is a promising scheme in order to achieve these requirements. In this paper, the spectral efficiency of MBM scheme in single- and multi-user networks is studied. To find the spectral efficiency, information theoretical methods are used and tight lower- and upper-bounds are derived for the achievable rate. The performance of the system is studied for both correlated and uncorrelated constellation diagrams. It is shown that the entries of the constellation diagram for the sum rate region of multi-user case are always correlated. The characteristic of covariance matrix is derived and its impact on performance assessed.

I. INTRODUCTION

In future wireless communication scenarios a huge number of devices will communicate tremendous amounts of data with low latencies and high energy efficiencies [1]. In order to achieve these goals, novel multi-antenna techniques as massive multiple-input multiple-output (MIMO), higher carrier frequencies as mmWave and novel modulation schemes as generalized frequency division multiplexing (GFDM) are considered. Often, one disadvantage of the technological approaches is that with increased hardware (e.g. number of radio frequency (RF) chains in massive MIMO) and complexity, the energy consumption is significantly increased. Therefore, we consider a recently proposed modulation scheme in [2] and study its applicability for multi-user scenarios.

The most common way in wireless communications to send information is to choose symbols from a complex modulation alphabet and transmit data over a fading channel. In other words, data is modulated before leaving the transmitter. In contrast to traditional methods, complex channel coefficients may be used to convey information. A method to modulate data after leaving the transmitter is introduced in [2] called MBM. The concept of MBM is generating a constellation diagram by perturbing the multipath fading channel. In this case, the constellation size is related to the number of perturbations as \(2^{N_P} = M\) where \(N_P\) is the number of the channel perturbations and \(M\) is the resulting constellation size. For each transmission, one of the channel realizations is chosen to send information and the receiver is able to decode received data due to the training period at the beginning of each coherence time of the channel.

Recently, the spectral efficiency of a Rayleigh fading single-input multiple-output (SIMO) channel is investigated and it is shown in [2] and [3] that by using MBM, the capacity of additive white Gaussian noise (AWGN) channel is achieved asymptotically. Since the coefficients of the fading channel are used to transmit data, training of the receiver is critical. This leads to the high complexity of the pilot training and afterward the decoding period in high data rates. To overcome these issues, so called layered multiple-input multiple-output MBM (LMIMO-MBM) is introduced and a recursive decoding algorithm for receiver side is presented [4]. Furthermore, the performance of LMIMO-MBM is investigated in [5] with secrecy constraint. In [6], bit error rate performance of MBM is examined when implemented to different physical layer techniques. A new technique to design reconfigurable antenna systems which promises low power and low cost that can be used to perturb the channel is introduced in [7]. Another approach to modulate information for MBM by using time-varying plasma is presented and experimental results are shown in [8]. Since the entries of the constellation diagram are generated by using channel state information (CSI), increasing the constellation size in MBM increase the complexity much more than the conventional modulation methods. In order to overcome this drawback, two channel state selection algorithms are proposed in [9]. In [10], polar coded MBM is introduced to achieve a better performance. Another scheme called differential MBM is given in [11] where a low-complexity maximum-likelihood detector is also proposed.

In this paper, we consider the single-user link with correlated constellation diagram entries and observe that correlation decreases the achievable spectral efficiency. We have observed that for the multiple-access channel (MAC), even if the entries of the constellation diagrams of each user are uncorrelated, the resulting constellation diagram of sum rate region is correlated.

The work of Merve Yüzgeçcioğlu has received partly funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 641985.
The covariance matrix for arbitrary number of users $K$ and modulation points $M$ is computed. Interestingly, the null space grows with increasing $K$ and $M$. The achievable rate region is compared to AWGN region and with the ergodic capacity region. It is shown that for asymptotic $M$, the impact of correlation vanishes.

The structure of the paper is as follows. In Section II the system models of the single- and multi-user network are introduced. Achievable rate performance of the system is studied in Section III and the numerical results are shown in IV. Finally, paper is concluded in V.

II. SYSTEM MODEL

A. Single user link

In MBM scheme, there are $M$ constellation points which are constructed from the channel coefficients by using different kind of hardware elements (RF mirrors, parasitic antennas, etc.). Transmitter chooses one of the constellation points to transmit the intended message. The resulting system model can be written as

$$y = \sqrt{P}x(m) + n,$$  \hspace{1cm} (1)

where $x \in \mathbb{C}^{M \times 1}$ contains the symbols from the constellation diagram with $M$ correlated entries, $x(m)$ is the $m$-th entry of the constellation diagram. Since we consider a single-input single-output (SISO) system, $x$ has Gaussian distribution with $\mathcal{N}(0, \Sigma)$ and the AWGN noise has $\mathcal{N}(0, \sigma^2)$. As a result, the probability density function of the output signal $y$ will be in form of a Gaussian mixture

$$f_Y(y) = \frac{1}{M} \sum_{m=1}^{M} \frac{1}{\sqrt{2\pi \sigma}} \exp \left\{ -\frac{[y - \sqrt{P}x(m)]^2}{2\sigma^2} \right\}. \hspace{1cm} (2)$$

B. Multiple access channel

In this section, a $K$-user MAC in Rayleigh fading environment is considered. Each user chooses one of the channel realizations to transmit the intended information and maps each message $m_k$ to the corresponding codeword $x_k(m_k)$ where $m = 1, \ldots, M$ and $k = 1, \ldots, K$. The input signals $x_k(m_k)$ are chosen from the constellation diagram with $M$ entries where $x_k$ has Gaussian distribution with $\mathcal{N}(0, \Sigma)$. The input-output relationship of a MAC is

$$y = \sqrt{P} \sum_{k=1}^{K} x_k(m_k) + n,$$  \hspace{1cm} (3)

where $m = 1, \ldots, M$ and $n$ is the AWGN with $\mathcal{N}(0, \sigma^2)$. Due to $M$ point constellation diagram with Gaussian components, the resulting distributions of the output $y$ is

$$f_Y(y) = \frac{1}{M^K} \sum_{m_1=1}^{M} \cdots \sum_{m_K=1}^{M} \frac{1}{\sqrt{2\pi \sigma}} \exp \left\{ -\frac{[y - \sqrt{P}x_1(m_1) + \cdots + x_K(m_K)]^2}{2\sigma^2} \right\}. \hspace{1cm} (4)$$

and the conditional pdf $y|x_k$ is

$$f_{Y|x_k}(y|x_k) = \frac{1}{M} \sum_{m=1}^{M} \frac{1}{\sqrt{2\pi \sigma}} \exp \left\{ -\frac{[y - \sqrt{P}x_k(m_k)]^2}{2\sigma^2} \right\}. \hspace{1cm} (5)$$

We observe from Eq. (4) that there are $M^K$ constellation points $x_1(m_1) + \cdots + x_K(m_K) = x(m_1, \ldots, m_K)$ for $1 \leq m_k \leq M$ and $1 \leq k \leq K$. These points are found by summation of each possibility among $K$ users with $M$ messages.

III. PERFORMANCE ANALYSIS

In this section, achievable rate region of the single-user link is studied for correlated constellation diagram entries. For the MAC case, both correlated and uncorrelated entries are taken into account in order to study the behavior of the system.

A. Single-user link

The achievable rate of the system can be calculated by using the information theoretical methods

$$I(x(m); y) = h(y) - h(n). \hspace{1cm} (6)$$

Here, the noise component has a Gaussian distribution with $\sigma^2$ variance, it is straightforward that the entropy can be found as $h(n) = \frac{1}{2} \log_2 (2\pi e \sigma^2)$. Since the received signal has Gaussian mixture distribution, calculation of the entropy of $y$ is not as easy as the noise component, as a matter of fact there exists no closed form solution for the entropy of a Gaussian mixture. A tight approximation is introduced to literature recently in [12]. By using this approximation method, entropy of the received signal can be bounded as

$$(\gamma + \alpha_{N,N'}) \log_2 e \leq h(y) \leq (\gamma + \beta_{N,N'}) \log_2 e. \hspace{1cm} (7)$$

The detailed explanation of the parameters can be found in [12].

1) Impact of the correlation: Depending on the hardware realization of the channel perturbations, correlation between the entries of $x(m)$ can occur. It is well known from the analysis of the multi-antenna links that the spatial correlation lowers the achievable rate, if the transmitter has no CSI and the receiver has perfect CSI, by applying majorization theory [13].

The entropy of a correlated Gaussian vector $x$ is a Schur-concave function of the eigenvalues of the correlation matrix $\Sigma$, i.e., if

$$\lambda(\Sigma_1) \geq \lambda(\Sigma_2) \quad \text{then} \quad h(\Sigma_1) \leq h(\Sigma_2), \hspace{1cm} (8)$$

with the majorization inequality $x \succeq y$ meaning $\sum_{m=1}^{l} x_{[m]} \geq \sum_{m=1}^{l} y_{[m]}$, $1 \leq l \leq M - 1$, and $\sum_{m=1}^{M} x_m = \sum_{m=1}^{M} y_m$, and entropy $h(\Sigma_1) = \log(2\pi e)^M \det(\Sigma_1)$.

The identity correlation matrix $\Sigma = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}$ is majorized by all other correlation matrices and provides the highest entropy. Since this result holds for all symmetric and concave functions
of the correlation matrix $\Sigma$, we expect a similar behavior for the MBM link and discuss the impact of correlation based on the numerical simulations in the next section.

B. Multiple access channel

The achievable rate region of a $K$ user multiple access channel is given by

$$\{R_S < I(x_S(m_S); y|x_{S^c}(m_{S^c})) \forall S \subseteq K = \{1, 2, \ldots, K\}\},$$

(9)

with $R_S = \sum_{k \in S} R_k$ and $x_S(m_S) = \{x_k(m_k)\}_{k \in S}$ and $S^c = K \setminus S$. The mutual information to define sum rate can be written as

$$I(x_1(m_1), \ldots, x_K(m_K); y) = h(y) - h(n).$$

(10)

Again the entropy of the noise $n$ can be written as

$$h(n) = \frac{1}{2} \log_2(2\pi e\sigma^2)$$

and for the remaining term $h(y)$ upper and lower bounds are found as in Eq. [7]. Similarly, the other mutual information expressions in [9], i.e., $I(x_S(m_S); y|x_{S^c}(m_{S^c}))$ can be derived.

1) Covariance matrix analysis: Interestingly, even if the original constellation diagram has uncorrelated entries the resulting signal set with $M^K$ entries will be correlated. The covariance matrix $\Sigma$ can be constructed by using the below description

$$\mathbb{E}\{x_i(m_i)x_j(m_l)\} = \begin{cases} \rho_{n,l}, & \text{if } i = j \text{ and } n \neq l \\ 1, & \text{if } i = j \text{ and } n = l \\ 0, & \text{if } i \neq j, \end{cases}$$

(11)

where $\rho_{n,l}$ is the correlation coefficient of the entries of the $k$-th user where $k = 1, \ldots, K$ and it is 0 for uncorrelated case. It can be seen from Eq. [11] that the joint constellation diagram of the users with uncorrelated constellation points will have a covariance matrix $\Sigma$ which is different than the identity matrix $I_{M^K \times M^K}$. Furthermore, the correlation matrix majorizes the identity matrix

$$\Sigma \succeq I_{M^K \times M^K} \succeq 0.$$  

Note that, a similar analysis can be performed for other multi-user communication scenarios, too. For both, the broadcast and the interference channel setup, similar superposition of the individual constellation points result in a correlation in the joint constellation diagram of the sum rate signal set.

The basic assumption in [2] is that entries of the constellation diagram are i.i.d. Gaussian zero-mean random variables. However, this is not true for the joint constellation diagram of a $K$-user MAC. Furthermore, the covariance matrix of the system can be defined as $\Sigma = \mathbb{E}\{x(m_1, \ldots, m_K)x(m_1, \ldots, m_K)^T\}$. To see this effect clearly, covariance matrix of a 2-user MAC with $M$ points constellation diagram is given

$$\Sigma = \begin{bmatrix} D_{M \times M} & I_{M \times M} & \cdots & I_{M \times M} \\ \vdots & \vdots & \ddots & \vdots \\ I_{M \times M} & I_{M \times M} & \cdots & D_{M \times M} \end{bmatrix}$$

(12)

The cases with more than 2 users and the correlated channel coefficients can be derived similarly by using Eq. [11]. However, it is tedious and results in a complicated structured covariance matrix. Nevertheless, it is possible to characterize the eigenvalues of these covariance matrices.

**Lemma 1.** The covariance matrix $\Sigma$ has $M^K$ eigenvalues. The largest eigenvalue is $KM^{K-1}$, there are $K(M - 1)$ eigenvalues which are $M^{K-1}$ and the remaining eigenvalues are zero.

**Remark 1.** There is a significant number of zero eigenvalues in the covariance matrix $\Sigma$, e.g. for $K = 3, M = 3$, there are 20 zeros out of 27 eigenvalues.

**Remark 2.** However, when $M$ goes infinity, the non-zero eigenvalues will be dominant and the effect of the correlation will vanish. This can be seen in Fig. [7] Here, the sum rate of a 2-user MAC is considered. In three different cases each user have 2, 4 and 16 uncaptured constellation diagrams, respectively. For the resulting constellation diagrams with 4, 16 and 256 entries, degradation causing from the correlation gets smaller with the increased size of constellation diagram.

IV. Numerical results

In this section, numerical results for both single- and multi-user system is presented. To have an insight into the behavior of the single-user system with MBM scheme under correlation, achievable rate results for the $N_P = 8, M = 256$ points constellation diagram with correlated and uncorrelated channel coefficients are shown. In Fig. [2] the numerical results for upper (UB) and lower bounds (LB) of the achievable rate are provided and compared with the Monte Carlo (MC)
is seen from this figure that the gap between AWGN capacity and the MBM scheme is a little bit wider for the sum rate than the individual rates of the users. This is the effect of inherent correlation of joint constellation diagram as stated in Eq. (11). Overall, the gap between the achievable rate region of the MBM scheme and the AWGN capacity region is small even with low number of channel perturbations. Numerical results show that with implementation of MBM scheme in a MAC system, communication in high data rates is possible without consuming additional power.

V. CONCLUSION

In this work, the achievable rate performance of MBM in both single- and multi-user network is studied. The effect of the correlated constellation diagram in a single user link is investigated. We have shown that the effect of the correlation on the spectral efficiency is considerably low. When the constellation diagram is expanded, degradation caused by the correlation vanishes, eventually capacity of the channel is achieved. Furthermore, the achievable rate region of the MAC is derived. Tight approximation of the achievable rate for both cases is derived and the tightness of the approximation is shown.

In future, this work may be extended to analyze the asymptotic behavior of MBM achievable rate for correlated constellation points when $M \to \infty$. Spectral efficiency of MIMO-MBM as described in [4] may be studied to investigate the impact of the spatial correlation on the achievable rate. And finally, model verification based on realistic RF perturbation approaches, e.g. RF mirrors [2]–[4] can be studied.

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