Features in the primordial power spectrum: constraints from the CMB and the limitation of the 2dF and SDSS redshift surveys to detect them

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ABSTRACT

We allow a more general (step-function) form of the primordial power spectrum than the usual featureless power-law Harrison–Zeldovich (with spectral index $n = 1$) power spectrum, and fit it to the latest Cosmic Microwave Background data sets. Although the best-fitting initial power spectrum can differ significantly from the power-law shape, and contains a dip at scales $k \sim 0.003 \, h \, \text{Mpc}^{-1}$, we find that $\Omega_m \approx 0.24$, consistent with previous analyses that assume power-law initial fluctuations. We also explore the feasibility of the early releases of the 2dF and SDSS galaxy redshift surveys to see these features, and we find that even if features exist in the primordial power spectrum, they are washed out by the window functions of the redshift surveys on scales $k < 0.03 \, h \, \text{Mpc}^{-1}$.

Key words: cosmology:theory – cosmic microwave background – early Universe

1 INTRODUCTION

With the release of new Cosmic Microwave Background (CMB) data from DASI (Halverson et al. 2001), BOOMERANG (Netterfield et al. 2001) and MAXIMA (Lee et al. 2001), and the 2dF redshift survey (Percival et al. 2001) nearing its completion, our ability to constrain cosmological models has improved significantly. The new CMB data removed the ‘baryon crisis’ caused by the unexpectedly low amplitude of the second peak in the temperature power spectrum, and the standard model of the universe now seems to be a flat Friedmann–Robertson–Walker universe, with 30% matter ($\sim 5\%$ baryons + $\sim 25\%$ cold dark matter (CDM)), 70% is dark energy, commonly parameterized by a cosmological constant, and with the current value of the Hubble parameter $H_0$ being around 70 km sec$^{-1}$ Mpc$^{-1}$ (e.g. Efstathiou et al. 2001; Wang, Tegmark & Zaldarriaga 2001).

Some assumptions about the underlying cosmological model are necessary in order to extract these parameters from the data. One common assumption is that the initial power spectrum of the density fluctuations is a featureless power law $P_m(k) \propto k^n$, where $k$ is the comoving wavenumber. The spectral index $n$ is found empirically to be close to the Harrison–Zeldovich value $n = 1$. This scale invariant primordial power spectrum is what typically comes out of models for inflation. However, there is no definite model for inflation, and some models predict features in $P_m(k)$. For example, in supersymmetric double inflationary models, with two inflaton fields and two ‘trigger’ fields, the power spectrum of density fluctuations is found to have a bump with superimposed oscillations on intermediate scales (Lesgourgues 2000). Features in $P_m(k)$ can also be produced in double inflation (Polarski & Starobinsky 1992), and in one-field inflation with a feature in the inflaton potential (Starobinsky 1992). The fluctuation spectrum may be sensitive to physics at length scales below the Planck length (Brandenberger & Martin 2001; Martin & Brandenberger 2001), and attempts have been made at extracting mass fluctuation spectra from models inspired by string theory (Khouri et al. 2001; Kempf 2001; Kempf & Niemeyer 2001; Easther, Greene & Shiu 2001). In Kempf & Niemeyer (2001) and Easther et al. (2001) the primordial power spectrum was found to be of the Harrison–Zeldovich form, with a normalization depending on a short distance cutoff. More realistic models will probably give rise to a $k$-dependent imprint (Easther et al. 2001). It has been pointed out that models of this type may run into problems in the form of an excessive creation of Planck energy particles at the present era (Starobinsky 2001), but it should at any rate be clear that the theoretical motivation for investigating more general forms for the initial power spectrum of density fluctuations is substantial.

On the observational side, several claims of indications for features in $P_m(k)$ have been made (Broadhurst et al. 1990; Griffiths, Silk & Zaroubi 2001; Atrio-Barandela et al. 2001; Barriga et al. 2001; Hannestad, Hansen & Vil...
lante 2001; Einasto et al. 1999; Gramann & Hütsi 2001; Silberman et al. 2001). Griffiths et al. (2001) and Hannestad et al. (2003) found that the CMB data favor a bump-like feature in the power spectrum at a scale $k \sim 0.004 \ h \mpc^{-1}$ ($h$ is the dimensionless Hubble parameter; $H_0 = 100h \ \text{km sec}^{-1} \text{Mpc}^{-1}$). Barriga et al. (2001) introduced a step-like feature in the range $k \sim 0.06-0.6 \ h \mpc^{-1}$ and found that this spectral break gave a good fit to both the CMB data and the data from the AP01 survey (Maddox et al. 1990). Atrio-Barandela et al. (2001) investigated the temperature power spectrum in CDM models with a matter power spectrum $P_m(k)$ at redshifts $z \sim 10^3$ of the form $P_m(k) \sim k^{-1.9}$ for $k > 0.05 \ h \mpc^{-1}$. This form was derived by Einasto et al. (1999) by analyzing observed power spectra of galaxies and clusters of galaxies. Gramann & Hütsi (2001) studied the mass function of clusters of galaxies with this form of $P_m(k)$ and found that the predicted number density of clusters was smaller than the observed one. However, these authors found that they could get a good fit to the mass function with a $P_m(k)$ having a dip-like feature at $k \sim 0.1 \ h \mpc^{-1}$, and that this $P_m(k)$ also was consistent with data from other cosmological probes like peculiar velocities and CMB.

One of the reasons for considering alternatives to a scale-invariant $P_m(k)$ was that the CMB data before April 2001 indicated that the amplitude of the second acoustic peak in the temperature power spectrum was low, resulting in a baryon density $\Omega_b h^2 \sim 0.03$ outside the limits set by Big Bang Nucleosynthesis (BBN), which gives a 95% confidence interval $\Omega_b h^2 = 0.020 \pm 0.002$ (Burles, Nollett & Turner 2001). The new CMB data show a higher second peak, and the values for $\Omega_b h^2$ obtained with a power-law $P_m(k)$ are now consistent with standard BBN (Wang et al. 2001).

The motivation for the work presented in this paper is different: we want to check if the presently available CMB data allow for deviations from a scale-invariant $P_m(k)$, and if this is the case, whether these can be seen in the early releases of the 2dF and SDSS data. We will therefore consider more general shapes for $P_m(k)$ in our analysis.

For simplicity, we will in our analysis vary only $\Omega_m$ and $P_m(k)$. We assume a flat Universe, and consider parameters like $\Omega_b$ and $h$ to be well constrained by other cosmological probes. The reader should note that these rather restrictive assumptions mean that the error bars we obtain for the estimated quantities will be much smaller than obtained in an analysis with more free parameters, like that of Wang et al. (2001). Our aim in this paper is not to do the most general model fitting of the CMB+2dF, but to check two things: firstly, can the current CMB data allow features in $P_m(k)$? Secondly, can these features be seen in the early releases of the 2dF and SDSS data?

The outline of this paper is as follows: In Section 2 we introduce the models for $P_m(k)$ under consideration, and describe our procedure for fitting them to the CMB data. In Section 3 we present the results of this procedure, and in Section 4 we discuss possible constraints from the 2dF and SDSS galaxy redshift surveys. Finally, in Section 5 we summarize and discuss our results.

## 2 THE PRIMORDIAL POWER SPECTRUM AND CMB ANISOTROPIES

The power spectrum of fluctuations can be written as

$$P(k) = P_m(k)T^2(k),$$

where $T(k)$ is the transfer function (which modifies the initial power spectrum during the radiation dominated era). To investigate more general forms for $P_m(k)$, we let

$$P_m(k) = AkS(k),$$

where $A$ is a constant and $S(k)$ parameterizes the deviations from scale invariant initial fluctuations, and set up the models as follows. We modified the publicly available CMBFAST code (Seljak & Zaldarriaga 1996) to include two alternatives for $S(k)$:

- a ‘sawtooth’-shape, with ‘teeth’ equally spaced in $\ln(k)$.
- a set of ‘top-hat’ steps, equally spaced in $\ln(k)$ and with amplitudes $a_i, i = 1, \ldots, N$

To be specific, we defined the ‘sawtooth’ spectrum following Wang & Mathews (2000) as

$$S(k) = \begin{cases} a_1, & k \leq k_1 = k_{\text{min}} \\ \frac{k - k_{i-1}}{k_{i} - k_{i-1}} a_{i-1} + \frac{k_{i} - k}{k_{i} - k_{i-1}} a_i, & k_{i-1} < k < k_i \\ a_N, & k \geq k_N = k_{\text{max}} \end{cases}$$

where

$$k_i = k_1 \left( \frac{k_N}{k_1} \right)^{(i-1)/(N-1)}, \quad i = 2, \ldots, N - 1,$$

with $k_{\text{min}} = 0.001 \ h \mpc^{-1}$ and $k_{\text{max}} = 0.1 \ h \mpc^{-1}$. Since the connection between the harmonic $\ell$ in the CMB power spectrum $C_\ell$ and $k$ is roughly $\ell \approx kd_A$ where for a flat universe the angular-diameter distance to the last scattering surface is well approximated by (Vittorio & Silk 1991):

$$d_A = \frac{2c}{H_0 \Omega_m^{1/2}},$$

where $c$ is the speed of light, the wave-number range $0.001 < k < 0.1 \ h \mpc^{-1}$ corresponds for $\Omega_m = 0.3$ approximately to $10 < \ell < 1000$ which is nicely covered by the CMB data.

Alternatively, we let $P_m(k)$ be defined by a set of ‘top-hat’ steps,

$$S(k) = \begin{cases} a_1, & k \leq k_1 \\ a_i, & k_{i-1} < k < k_i \\ a_N, & k \geq k_{N-1}, \end{cases}$$

with $k_i$ given by (4). The comoving wavenumber $k$ is measured in units $h \mpc^{-1}$, where $h$ is the dimensionless Hubble parameter.

Both spectra are thus completely specified by $N$ and the values of $a_1, \ldots, a_N$. In our calculations we chose $N = 4$. We also let the matter density $\Omega_m$ be a free parameter, but the other parameters were kept fixed at $H_0 = 72 \ \text{km sec}^{-1} \text{Mpc}^{-1}$ (Freeland et al. 2001), $\Omega_b h^2 = 0.02$ (Burles et al. 2001), $N_e = 3.04$ (see e.g. Bowen et al. 2001), and we assumed a flat universe with no massive neutrinos and no reionization. We therefore left with a five-dimensional parameter space to search for the best-fitting model. This was done by minimizing the $\chi^2$ statistic given by

$$\chi^2(p) = \sum_{ij} [\Delta T_i^2 - \Delta T_i^2(p)](C^{-1})_{ij}[\Delta T_j^2 - \Delta T_j^2(p)],$$

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where \( p = (\Omega_m, a_1, a_2, a_3, a_4) \) is the parameter vector, \( \Delta T^2 \) are the measured CMB fluctuations,

\[
\Delta T^2(p) = \frac{T_0^2}{2\pi} \sum_k W_{kl} C_l(p),
\]

(8)

\( C_l(p) \) is the COBE-normalized output from CMBFAST, \( T_0 = 2.726 \text{ K} \), \( W_{kl} \) is the window function, and \( C_{ij} \) is the covariance matrix of the observations. The observations, \( \Delta T^2 \), \( W_{kl} \), and \( C_{ij} \) are taken from (Wang et al. 2001). (The absence of the usual 1/\( \ell \)-factor in Eq. (8) is a result of their definition of the window function). We computed the \( \chi^2 \) on a grid of \( 10^5 \) models, and located the region of parameter space containing the global minimum. In this region we performed a more accurate search for the optimal parameters using the downhill simplex method (Press et al. 1992).

3 RESULTS

In Fig. 1 we show the best models for the two types of \( P_\ell (k) \) we consider. For comparison, we have also plotted the power spectrum for a model with \( P_\ell (k) = Ak \) (i.e. \( S(k) \equiv 1 \) and \( \Omega_m = 0.24 \)). The best-fitting ‘top hat’ and ‘sawtooth’ models both have \( \chi^2 \approx 32 \) for the 24 data points. The high \( \chi^2 \) values are partly caused by the bandpowers centered at \( \ell = 2 \) and \( \ell = 50 \) in the compilation of Wang et al. Removing these points leads to lower values for the minimum \( \chi^2 \), but has no significant effect on our estimated values for \( \Omega_m \), and the parameters of \( P_\ell (k) \). Note that the CMB only constrains the quantity \( \Omega_m h^2 \), but since we have fixed \( h \) in our analysis, we obtain an estimate for \( \Omega_m \) directly. In Fig. 2 we show the \( S(k) \) which provide the best fit to the data, with \( a_1 \) scaled to 1 for clarity. The best-fitting parameters, mean values and confidence intervals can be found in Table 1. The confidence intervals for the parameters were obtained by the standard approach of constructing marginalized likelihoods for each parameter by integrating out the other parameters from the likelihood \( \mathcal{L} \propto \exp(-\chi^2/2) \). The tight confidence intervals, in particular on \( \Omega_m \), reflect the restrictive assumptions we have made about other cosmological parameters.

We see from Fig. 2 that both the best-fitting \( S(k) \) have a dip at \( k \sim 0.003 \text{ h Mpc}^{-1} \). In fact, for the ‘top hat’ \( P_\ell (k) \) the data favour \( a_1 \approx a_2 \) and \( a_3 \approx a_4 \). This means that there are effectively only two parameters in this spectrum: the position of the dip and the relative sizes of the amplitudes. We have checked this by repeating the analysis with more amplitudes in the spectrum, and found that the data in this case still favour a \( P_\ell (k) \) with a dip at \( k \sim 0.003 \text{ h Mpc}^{-1} \).

Thus, in this case we can reduce the parameter space to three dimensions: \( \Omega_m \), the ratio \( R \) between the two amplitudes defining \( P_\ell (k) \), and the position in \( k \)-space \( k^* \) of the dip. The best-fitting values and confidence limits are given in Table 2. For completeness, we also made a calculation for the ‘sawtooth’ spectrum with this reduced set of parameters (i.e. keeping just one ‘tooth’ in the spectrum, and using its amplitude and position in \( k \)-space as free parameters, see the tables). The marginalized likelihood distributions for \( \Omega_m \), \( R \), and \( k^* \) are shown in Fig. 3. We see that \( \Omega_m \) is well constrained to a narrow range around 0.24, and the size of the dip \( R \) similarly constrained to be around 1.6, consistent with the results for four steps. The scale \( k^* \) at which the break occurs has a broader distribution. In the calculation with four steps, this scale was at \( \sim 4.6 \cdot 10^{-3} \text{ h Mpc}^{-1} \), but from Fig. 2 and Table 2 we see that when we allow \( k^* \) to vary, we can only constrain it to be in the range \( \sim 0.001-0.005 \text{ h Mpc}^{-1} \).

4 COULD 2dF AND SDSS DETECT FEATURES IN THE PRIMORDIAL POWER SPECTRUM?

Combining data from different cosmological probes can in many cases lead to tighter constraints on the cosmological parameters, see e.g. Efstathiou et al. (2001) and Wang et al. (2001). Since we saw in the previous section that the CMB does not rule out deviations from a scale-invariant \( P_\ell (k) \), it is interesting to see if we can obtain further constraints
from the matter power spectrum as estimated from the 2dF and SDSS galaxy redshift surveys.

Assuming a simple scale-independent biasing model with a bias parameter $b$, the galaxy power spectrum (linear theory) is predicted to be

$$P_g(k) = b^2 P_m(k) = b^2 A(k) S(k) T^2(k)$$

(9)

where $T(k)$ is the transfer function. The recent analysis of 2dF+CMB data by Lahav et al. (2001) found $b = 1.0 \pm 0.1$ on comoving scales of $0.02 < a < 0.15$ h Mpc$^{-1}$. Using CMBFAST, we computed $P(k)$ for $S(k)$ and the best-fitting ‘top hat’ and ‘sawtooth’ $S(k)$, with the results shown in Fig. 4. However, to compare with the galaxy power spectrum from 2dF (Percival et al. 2001), we must convolve $P_g(k)$ with the 2dF window function:

$$P_{\text{conv}}(k) \propto \int P_g(|k - q|)/|W_q|^2 d^3q,$$

(10)

where $<|W_q|^2>$ is the spherical average of the the 2dF window function, approximately given by

$$<|W_q|^2> = \frac{1}{1 + (k/a)^2 + (k/b)^4},$$

(11)

with $a = 0.00342$, and $b = 0.00983$. As shown in Appendix A, the convolution integral can be rewritten as

$$P_{\text{conv}}(k) \propto \int_0^\infty K(k, k') P_g(k') dk',$$

(12)

where $K(k, k')$ is given by an integral over the 2dF window function [1] that can be evaluated analytically. As discussed in Appendix A, for $k < 0.1$ h Mpc$^{-1}$, $K(k, k')$ is almost a delta function $\delta(k' - k)$, so in this region $P_{\text{conv}}(k) = P(k)$. For low $k$, $K(k, k')$ is a broad distribution, and the main contribution to the convolution integral comes from values of $k'$ larger than $\sim 0.01$ h Mpc$^{-1}$, so that $P_{\text{conv}}(k)$ is nearly independent of $k$.

As a result, the features in $P(k)$ introduced by $S(k)$ are washed out by the convolution, as can be seen from Fig. 3. The results show that the present data from the 2dF survey cannot give us information about the power spectrum at wavenumbers smaller than about $0.03$ h Mpc$^{-1}$, everything on larger scales is washed out. There is no relation between the wiggles visible in 2dF power spectrum, which is the result of observing a single realization of the true power spectrum convolved with the window function of the survey, and the features we introduced in $P_g(k)$. The wiggles in the 2dF power spectrum may be signatures of baryon oscillations, but may also be a result of correlated noise. The scale of the observed wiggles is $\Delta k = 0.03$ Mpc$^{-1}$, which is the same scale over which power is correlated, so the wiggles could well be the result of correlated noise. This tentative conclusion is supported by the recent analysis of the published sampled carried out by Tegmark, Hamilton & Xu (2001).

We also compared our calculated $P_g(k)$ with the recent
estimate of the power spectrum from the Sloan Digital Sky Survey (SDSS), taken from Dodelson et al. (2001), where the SDSS data were analysed to obtain the deconvolved three-dimensional power spectrum. The SDSS results are given as a set of bandpowers in $k$-space, and thus the calculated power spectrum $P_g(k)$ must be transformed in a way analogous to Eq. (3); in the $i$th bin, the bandpower $P(k_i)$ is given by

$$P(k_i) = \sum_j W_{SDSS}(i,j) P_g(k_j).$$

where $k_i, k_j$ are the central values of the $k$ bins. The window functions $W_{SDSS}$ can be found in Dodelson et al. (2001). Our results are shown in Fig. 5. Only results for the magnitude bin $r^* = 21 - 22$ are shown, but the same conclusion is true for the other three magnitude bins given in Dodelson et al. (2001). We see that the same conclusion applies to the SDSS spectrum as to the one from 2dF: at present there is no information on the scale where we find features in $P_{in}(k)$. However, this conclusion only applies to the presently available data, as both 2dF and SDSS will, once completed, give information about larger scales than those probed by the data used in this paper.

5 CONCLUSIONS

We have analysed the new and updated CMB data, relaxing the usual assumption of power-law initial fluctuations. We found that the value of $\Omega_m$ we could extract was consistent with other analyses, e.g. $\Omega_m = 0.24^{+0.05}_{-0.04}$ for the ‘top hat’ $P_{in}(k)$ with varying position of the break. We should point out that the small error bar on $\Omega_m$ found in our analysis is mainly a result of the restrictive priors we put on other quantities. For example, allowing the Hubble constant to vary would have led to a significant increase in the uncertainty in $\Omega_m$ since the CMB power spectrum depends on $\Omega_m$ only through the matter density $\omega_m = \Omega_m h^2$.

We find that the present CMB data allow the initial power spectrum of the density fluctuations, $P_{in}(k)$ to have quite significant features, in particular a dip at a comoving wavenumber $k \sim 0.003 h$ Mpc$^{-1}$, which corresponds to $\ell \sim 40$. This dip can be understood as follows: Increasing $\Omega_m h^2$ decreases the amplitudes of the peaks in the CMB power spectrum, and also shifts their positions to lower multipoles $\ell$. The position of the first peak is well determined by the data, and the models pay a higher price in terms of the $\chi^2$ for not fitting it than they do for not fitting the amplitude. The position of the first peak in the compilation of CMB data we have used is well fitted by $\Omega_m h^2 = 0.12$ (Wang et al. 2001), which for $h = 0.72$ gives $\Omega_m = 0.24$. Fig. 5 shows that the CMB power spectrum of a model with this value of $\Omega_m h^2$, but a Harrison–Zeldovich $P_{in}(k)$, lies consistently above the data points for all but the lowest values of $\ell$. The freedom in the ‘sawtooth’ and ‘top hat’ $P_{in}(k)$ allows the amplitude to be fitted by reducing $P_{in}(k)$ at comoving wavenumbers above $k \sim 0.003 h$ Mpc$^{-1}$.

From the preceding discussion it also follows that the parameters of the ‘sawtooth’ and ‘top hat’ $P_{in}(k)$ are sensitive to calibration errors in the data points, and also that the choice of normalization for the models plays a role. We have normalized all our CMB power spectra to COBE, using the prescription given by Bunn & White (1997). Recent work by Lahav et al. (2001), Reiprich & Boehringer (2001), Seljak (2001), and Viana, Nichol & Liddle (2001), found that $\sigma_8$, the rms mass fluctuations in $8 h^{-1}$ Mpc spheres, is $\sim 20\%$.
lower than predicted by the COBE normalization. If we had used these results to normalize our CMB power spectra, the size of the dip we found in $S(k)$ would have been reduced. However, our goal here was to test the maximal features that can still be consistent with the CMB data. Therefore the analysis with the constraint of COBE normalization can be viewed as an upper limit on the amplitude of the features.

We also compared the early-release 2dF and SDSS galaxy power spectra to the theoretical predictions from our best-fitting $P_n(k)$, and we found that the observed power spectra are not sensitive to the features in $P_n(k)$ at comoving scales $k < 0.03$ h Mpc$^{-1}$. There is no relation between the features we find in $P_n(k)$ and the wiggles observed in the 2dF power spectrum.

Furthermore, we point out again that we have fixed all parameters except $\Omega_m$ and $P_n(k)$ in our analysis. Combining extra degrees of freedom in $P_n(k)$ with a full-scale analysis of the CMB data, would lead to larger error bars on the cosmological parameters. The more accurate measurements of the CMB fluctuations expected in the near future, analysed jointly with data sets from other cosmological probes, will hopefully allow us to put tighter constraints both on the primordial fluctuations and the parameters defining the geometry of the Universe.

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APPENDIX A: THE CONVOLUTION INTEGRAL

The convolution of the power spectrum $P(k)$ with the window function is given by

$$\hat{P}_{\text{conv}}(k) \propto \int P_g(\mathbf{k} - \mathbf{q}) |W_k(q)|^2 d^2q.$$  \hfill (A1)

Assuming $P$ is isotropic, and that only the spherical average of the final power spectrum is of interest, one can replace the window function with its spherical average, which for 2dF can be approximated by

$$\langle |W_k|^2 \rangle = \frac{1}{1 + \alpha k^2 + \beta k^4},$$  \hfill (A2)

where $\alpha = 5.55 \times 10^4$, $\beta = 1.071 \times 10^8$. Choosing $\mathbf{k}$ as the z-axis in the integration, the convolution integral can be written as

$$\hat{P}_{\text{conv}}(k) \propto \int_{0}^{\infty} P_g(\mathbf{k} - \mathbf{q}) |W_k|^2 d^2q$$

$$= 2\pi \int_{0}^{\infty} dq q^2 \langle |W_k|^2 \rangle$$

$$\times \int_{-1}^{1} d(\cos \theta) P_g(\sqrt{k^2 + q^2 - 2kq \cos \theta}),$$  \hfill (A3)

where $\theta$ is the angle between $\mathbf{k}$ and $\mathbf{q}$. (Here and in the following we ignore the normalization constant. In practical calculations it is taken care of by dividing by $\int |W_k|^2 d^2q$.)

On substituting $k'^2 = k^2 + q^2 - 2kq \cos \theta$, $d(\cos \theta) = -k' dk'/kq$, the integral becomes

$$\hat{P}_{\text{conv}}(k) \propto \frac{2\pi}{k} \int_{0}^{\infty} dq q |W_k|^2 \int_{|k-q|}^{k+q} dk' P_g(k').$$  \hfill (A4)

With the simple approximation Eq. (12) to the 2dF window function, the convolution integral can be simplified further by changing the order of integration in Eq. (14). We integrate over the region in the $(k', q)$-plane bounded by the lines $k' = k + q$, $k' = k - q$, $q \leq k$, and $k' = q - k$, $q > k$, so changing the order of integration is easy, and the result is

$$\hat{P}_{\text{conv}}(k) \propto \frac{2\pi}{k} \int_{0}^{\infty} dk' P_g(k') \int_{|k'-q|}^{k'+q} dq |W_q|^2$$

$$\approx \int_{0}^{\infty} K(k,k') P_g(k') dq.$$  \hfill (A5)

With the 2dF window function, one can obtain an analytical expression for $K(k,k')$:

$$K(k,k') = \frac{2\pi k'}{k} \int_{|k'-q|}^{k'+k} dq q \frac{1 + \alpha q^2 + \beta q^4}{1 + \alpha x + \beta x^2}$$

$$= \frac{\pi k'}{k} \left[ \text{arctanh}[\xi (k' - k) + \lambda] \right.$$

$$- \left. \text{arctanh}[\xi (k' + k) + \lambda] \right],$$  \hfill (A7)

where $\eta = 1.2055 \times 10^{-3}$, $\xi = 2.5822 \times 10^3$, and $\lambda = 1.0307$.

Numerical plots of $K(k,k')$ for various values of $k$ shows that it can be roughly approximated by a Gaussian,

$$K(k,k') \propto \exp \left(-\frac{(k' - \mu_k)^2}{2\sigma_k^2} \right).$$  \hfill (A8)

and that for $k > 0.1$ h Mpc$^{-1}$, $\sigma_k \ll k$, and $\mu_k \approx k$, so that the Gaussian approaches $\delta(k' - k)$. In this regime of $k$, we will therefore have

$$\hat{P}_{\text{conv}}(k) \propto \int_{0}^{\infty} \delta(k' - k) P_g(k') dk' = P_g(k).$$  \hfill (A9)

For $k \ll 0.1$ h Mpc$^{-1}$, the Gaussian is very broad, $\sigma_k \gg k$ and $\mu_k \gg \mu_k$. To illustrate what happens in this regime of $k$, we take $P_g(k) \propto k$ for $k < k_b$ and $P_g(k) \propto k^{-2}$ for $k > k_b$. Then

$$\hat{P}_{\text{conv}}(k) \propto \int_{0}^{k_b} \exp \left(-\frac{(k' - \mu_k)^2}{2\sigma_k^2} \right) k' dk'$$

$$+ \int_{k_b}^{\infty} \exp \left(-\frac{(k' - \mu_k)^2}{2\sigma_k^2} \right) \frac{dk'}{k'^2}. $$  \hfill (A10)

Both integrals can be evaluated analytically in terms of the error function erf$(x)$. Numerically, it turns out that it is a reasonable approximation in this regime to replace the
Gaussian by its peak value, which gives

\[
\hat{P}_{\text{conv}}(k) \propto \int_0^{k_b} k'dk' + \int_{k_b}^{\infty} \frac{dk'}{k'^2} \\
= \frac{k^2}{2} + \frac{1}{k_b} \approx \frac{1}{k_b}
\]

**(A11)**

Note that this is independent of \(k\), which agrees with the numerical results for the convolved \(P(k)\).

**REFERENCES**

Atrio-Barandela F., Einasto J., Müller V., Mücket J. P., Starobinsky A. A., 2001, ApJ, 559, 1

Barriga J., Gaztanaga E., Santos M., Sarkar S., 2001, MNRAS, 324, 977

Bowen R., Hansen S. H., Melchiorri A., Silk J., Trotta R., 2001, astro-ph/0110636

Brandenberger R. H., Martin J., 2001, Mod. Phys. Lett. A, 16, 999

Broadhurst, T. J. Ellis R. S., Koo D. C., Szalay A. S., 1990, Nat, 343, 726

Bunn E. F., White, M., 1997, ApJ, 480, 6

Burles S., Nollett K. M., Turner M. S., 2001, Phys. Rev. D, 63, 063512

Dodelson S. et al., 2001, astro-ph/0107421

Easther R., Greene B. R., Kinney W. H., Shiu G., 2001, Phys. Rev. D, 64, 103502

Efstathiou G. P. et al., 2001, astro-ph/0109152

Einasto J. et al., 1999, ApJ, 519, 441

Freedman W. L. et al., 2001, ApJ, 553, 47

Gramann M., Hütsi G., 2001, MNRAS, 327, 538

Griffiths L. M., Silk J., Zaroubi S., 2001, MNRAS, 324, 712

Halverson N. W. et al., astro-ph/0104485

Hannestad S., Hansen S. H., Villante F. L., 2001, Astropart. Phys., 16, 137

Kempf A., 2001, Phys. Rev. D, 63, 083514

Kempf A., Niemeyer J. C., 2001, Phys. Rev. D, 64, 103501

Khoury J., Ovrut B. A., Steinhardt P. J., Turok N., hep-th/0103239

Lahav O. et al., 2001, astro-ph/0112162

Lee A.T et al., 2001, ApJ, 561, L1

Lesgourges J., 2000, Nucl. Phys. B, 582, 593

Lidsey J. E., Liddle A. R., Kolb E. W., Copeland E. J., Barreiro T., Abney M., Rev. Mod. Phys., 69, 373

Maddox S. J., Efstathiou G. P., Sutherland W. J., Loveday J., 1990, MNRAS, 242, 43P

Martin J., Brandenberger R. H., 2001, Phys. Rev. D, 63, 123501

Netterfield C.B. et al., astro-ph/0104460

Percival W. J. et al., astro-ph/0105252

Polarski D., Starobinsky A. A., 1992, Nucl. Phys. B, 385, 623

Press W. W. et al., 1992, Numerical Recipes in Fortran, 2. ed. Cambridge University Press

Reiprich T. H., Böhringer H., 2001, astro-ph/0111285

Seljak U., 2001, astro-ph/0111360

Seljak U., Zaldarriaga M., 1996, ApJ, 554, 47

Silberman L., Dekel A., Eldar A., Zehavi I., 2001, ApJ, 557, 102

Soung-a-lee T., Bond J. R., Knox L., Efstathiou G. P., Turner M. S., 1998, in Roszkowski L., ed, Proc. COSMO-97. World Scientific Proceedings of COSMO-97, Ambleside, England, Ed. L. Roszkowski, World Scientific

Starobinsky A. A., 1992, JETP Lett., 55, 489

Starobinsky A. A., 2001, JETP Lett., 73, 371

Tegmark M., Hamilton A. J. S., 2001, astro-ph/0111575

Vittorio N., Silk J., 1991, ApJ, 385, L9

Viana P. T. P, Nichol R. C., Liddle A. R., 2001, astro-ph/0111394

Wang X, Tegmark M., Zaldarriaga M., astro-ph/0105091

Wang Y., Mathews G., 2000, astro-ph/0011351

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