Comparison of forces for the Crookes and Hettner radiometers

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Abstract
The Crookes radiometer (also known as a light mill) is a fascinating sunlight-powered device, in which a set of vanes is placed inside a glass bulb within which a partial vacuum has been pulled. The vanes then rotate when sunlight shines on the bulb. The reason for the turning of the vanes was subject to intense debate and many students still have an incorrect understanding of the device. We analyse the forces involved in a much less well-known radiometer (called the Hettner radiometer) and show that this can help us to understand the forces involved in the Crookes radiometer in a more unified way.

Keywords: radiometer, rarefied gases, thermophoresis

1. Introduction
The Crookes radiometer was invented by William Crookes in the nineteenth century (he originally observed the effect upon which the device is based during unrelated experimental work to determine the atomic mass of thallium) [1]. In the device (shown in the left-hand image of figure 1), one mounts a set of vanes inside a sealed glass bulb such that the vanes are able to rotate with negligible friction. A partial vacuum is then pulled inside the bulb. The vanes are black on one side and shiny on the other: the set of vanes then starts to rotate as soon as the radiometer is placed in light. Several intuitively appealing but incorrect explanations were provided for this rotation until the basis for a correct explanation was discovered by Maxwell [2]. The device attracted the further attention of Einstein several decades later who studied what is now known as the Einstein effect [3]. Needless to say, the device has a fascinating history on both the experimental and the theoretical side and is still used today for pedagogical purposes [4–6]. There have also been several numerical studies of the rotational motion of the vanes [7].

The Hettner radiometer is similar to the Crookes radiometer, but in this case the vanes are now horizontal such that the faces of the vanes are perpendicular to the ground. Since the vanes face downwards, the forces involved now occur across the surface of the vane, rather than at the
edges [8]. Unlike the Crookes radiometer where the two faces of the vanes are either black or glossy, each vane in the Hettner radiometer is split into a black half and a glossy half (shown in the right-hand image of figure 1). As well as describing an interesting device which is likely unfamiliar to most teachers, our discussion fits naturally into the usual basic physics curriculum, both in terms of kinetic theory and fluid dynamics. Students will have studied basic kinetic theory, but the usual qualitative discussions of random collisions of molecules and net motions of the gas can seem somewhat abstract and unmotivated.

In the radiometer, on the other hand, the reality of the collisions and the way that they on average influence the vanes to move in a certain direction are plain to see and very well-motivated once the correct physics has been explained in elementary terms. Similarly, discussions of fluid dynamics and low Reynolds number flows which some students will have studied might seem somewhat abstract and disconnected from the real world. The slow turning motion of the vanes through the gas in the radiometer gives an obvious, historically important example of such a flow and again, it should be easy to explain that there is a drag force on the vanes as they rotate. In this respect, it might help to purchase a few Crookes radiometers to use in class (these are available very cheaply). Finally, on a more philosophical point, both the Crookes and Hettner radiometer are good examples of how simple toy experiments can effectively act as testing laboratories for sophisticated theories and explore the limits of our theoretical knowledge.

2. Comparison of forces

We will begin by giving simplified summaries of the forces involved in the Crookes radiometer, and then explain how and why these differ from the forces which occur in the Hettner radiometer. One of the common incorrect explanations which will occur to a student is radiation pressure due to absorption of photons by the black side of the vanes, but this implies that the vanes should rotate in the opposite direction to the one which is observed in practice. However, in the device of Crookes, the main reason that a force occurs (roughly speaking) is that there is a significant temperature difference across the two sides of a porous surface (known as the thermal creep force) [2]. Clearly, a vane will heat up when exposed to light because of black body absorption of photons. A pressure difference at the surface of the vane drives a net drift of air molecules from the cold side to the warmer side, so the thermal creep force will push each vane in the direction of the net motion of the molecules [9]. Since this force occurs at the edges of the vanes, it can be changed by varying the vane thickness [10].

The thermal creep force is given by the following equation:

$$F_{TC} = -\frac{3}{4\sqrt{2\pi^2}} \frac{k}{\sigma^2} \frac{\Delta T}{L} S_v,$$

where $k$ is the Boltzmann constant, $\sigma$ is the average diameter of a molecule in the gas, $\Delta T$ is the temperature difference across the warm side of the vane to the cold side, $L$ is the thickness of the vane, and $S_v$ is the vertical area of the edge. Note that the mean free path of the gas $\lambda$ must be of the same order as the thickness of the vane for the above formula to make sense. The qualitative physics of the thermal creep force is still perhaps not obvious from the form of the equation above. The word ‘creep’ refers to the way that the flow of air molecules slowly ‘creeps’ along the edge of a vane towards the hot black surface.

This happens because gas molecules on average tend to flow from cold to hot close to the edges, so there is an overall net movement at the edges from the colder white side to the warmer...
black side, with the flow velocity being linearly dependent on the temperature gradient ∂θ/∂x. We have gas molecules moving to the edge from the hotter side and other molecules moving to the edge from the colder side, but on average the force exerted on the edge is greater when it is due to the molecules on the hotter side which have more kinetic energy. This means that there is a net force directed from the black side which is sufficient to cause the vanes to rotate in the direction of the white side. This resultant force is what is known as the thermal creep force.

There is, however, a second force involved which is known as the Einstein force. These forces act on the surface of the vane normal to the temperature gradient. In fact, it was believed for some time that the Einstein force was the main driving force of the radiometer until calculations showed that its magnitude was too small for it to be the dominant contributor (and we will see that it does not contribute at all in the Hettner radiometer) [11]. This force occurs due to the difference in energy absorbed by air molecules on either side of the vane. The equation for the Einstein force can be written as [12]:

\[ F_E = (2 - \alpha) \frac{15}{32\sqrt{2\pi}} \frac{k}{\sigma^2} \frac{\Delta T}{d} \lambda p, \]

where \( \alpha \) is the accommodation coefficient, \( d \) is the effective diameter of a molecule in the gas, and \( p \) is the gas pressure. Finally, one might also like to include the effects of air resistance, where the drag force is as usual:

\[ F_D = \frac{1}{2} C A p v^2, \]

where \( C \) is the drag coefficient, \( A \) is the cross-sectional area, \( p \) is the gas density, and \( v \) is the speed of the flow.

Again, it is perhaps not clear from the above equation what the Einstein force is in qualitative terms. The assumption of Einstein is that when a thin vane is placed in air along with the temperature gradient ∂θ/∂x, the pressure of the gas is constant in the air above the vane. At the edge of the vane, however, it is assumed that there is a very small region of transition in which the pressure is non-constant. In this region, the pressure is slightly higher on the warmer side compared to the colder side, which could theoretically contribute to causing the vane to rotate. Naively, one might guess that molecules which collide with the warmer side bounce away with increased momentum compared to molecules which collide with the cooler side, which would lead to a net pressure helping to push the vane in the direction of the white side. However, this does not happen because molecules which are moving faster collide more frequently with other molecules that are moving towards the warmer side from the rest of the gas, so that the net force would be the same if this were the only effect driving the rotation of the vanes. The larger temperature on one side decreases the local density of the gas molecules at the edge, so that the net force ends up cancelling out at both sides. The Einstein effect happens because the temperature difference at the edge of a vane means that the two pressures exerted on both sides of the vane in fact do not exactly cancel each other out.

The main interesting qualitative difference between the Crookes and Hettner radiometers is that for the Hettner radiometer the Einstein effect does not occur because of the alignment of the vanes, which implies that it is now the width rather than the thickness of the vane which is the characteristic length scale. One can still study the thermal creep shear force and the drag force on the vanes when they are horizontal as before but the analysis is more involved because the vanes move parallel to the flow and the problem is slightly different. For example, one can derive an expression for the net thermal creep force on the sides of a vane in the Hettner radiometer to be as follows [8]:

\[ F_{TC} = \frac{15x}{32\sqrt{2} \sigma_{CS}} \Delta T \alpha L' \min \left( \frac{W_v}{W_{Gas}}, 1 \right), \]

where \( x \) is a constant between zero and one, \( \sigma_{CS} \) is the hard sphere collision cross section, \( L' \) is the length of the vane with a slip length correction, \( W_v \) is the width of the vane, and \( W_{Gas} \) is the width of the temperature gradient in the gas. The minimum occurs because the ratio \( W_v/W_{Gas} \) takes values between zero and one. The above expression seems complicated but it is still the case that thermal creep ultimately explains why the vanes rotate.
The analysis of the drag force is also slightly different for the Hettner radiometer because the vane moves parallel to the flow and the dominant contribution to the drag force on a thin plate which moves parallel to a flow is skin friction. If we assume Stokes flow, the drag force is now

\[ F_D = C_D \rho v^2 LW, \]

where \( C_D \) is the drag coefficient for Stokes flow, \( L \) is the length of the vane, \( W \) is the width of the vane, \( \rho \) is the gas density, and \( v \) is the speed of the gas flow. However, note that the usual Stokes drag force equation does not take edge effects into account so one must add a certain correction term:

\[ F_{\text{edge}} = 1.6v\mu W, \]

where \( \mu \) is the dynamic viscosity of the gas [8].

### 3. Conclusion

We have seen that the essential difference between the Crookes and Hettner radiometers is primarily one of orientation: this difference in orientation then has a subtle effect on the physics involved. Since the Hettner radiometer removes the effect of the Einstein force, it is then possible to isolate and measure the effect of the thermal creep shear force much more easily in experiments and simulations. In summary, the Hettner radiometer has some very interesting differences from the Crookes radiometer whilst retaining the key qualitative phenomena. Despite this, it has been surprisingly little-studied and we have shown that it would merit some further study and comparison with the more familiar Crookes radiometer. The Crookes radiometer is itself quite frequently misunderstood and many incorrect explanations have been proposed for the physics involved in this device, so we have shown that understanding the Hettner radiometer is well-motivated simply because it can help to clear up some incorrect reasoning in the literature on the Crookes radiometer.

### Data availability statement

No new data were created or analysed in this study.