Explaining the anomalous $\Upsilon(5S) \to \chi_{bJ}(\omega)$ decays through the hadronic loop effect

Dian-Yong Chen$^{1,3}$, Xiang Liu$^{2,3,4}$ and Takayuki Matsuki$^{3,4}$

$^1$Theoretical Physics Division, Institute of Modern Physics of CAS, Lanzhou 730000, China
$^2$School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China
$^3$Research Center for Hadron and CSR Physics, Lanzhou University & Institute of Modern Physics of CAS, Lanzhou 730000, China
$^4$Tokyo Kasei University, 1-18-1 Kaga, Itabashi, Tokyo 173-8602, Japan

In this work, we carry out the study on $\Upsilon(5S) \to \chi_{bJ}(\omega)$ ($J = 0, 1, 2$) by considering the hadronic loop mechanism. Our results show that the Belle’s preliminary data of the branching ratios for $\Upsilon(5S) \to \chi_{bJ}(\omega)$ can be well reproduced in our calculation with a common parameter range, which reflects the similarity among these $\Upsilon(5S) \to \chi_{bJ}(\omega)$ decays of concern.

PACS numbers: 13.25.Gv, 12.38.Lg

In the past years, the Belle Collaboration has reported some novel phenomena relevant to the hidden bottom decays of $\Upsilon(5S)$. In Ref. [1], Belle indicated that the partial decay widths of $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-$ are $10^2$ times larger than those of $\Upsilon(nS) \to \Upsilon(mS)\pi^+\pi^-$, where $n, m = 1, 2, 3$ and $m > n$, which is the puzzle in the $\Upsilon(5S)$ hidden-bottom di-pion decays. There are two possible explanations for this puzzle. This investigation can, of course, provide a good test of the hadronic loop mechanism.

Theoretical Research Division, Nishina Center, RIKEN, Saitama 351-0198, Japan

Electronic address: chendy@impcas.ac.cn

Electronic address: matsuki@tokyo-kasei.ac.jp

In this work, we carry out the study on $\Upsilon(5S) \to \chi_{bJ}(\omega)$ ($J = 0, 1, 2$) by considering the hadronic loop mechanism. Our results show that the Belle’s preliminary data of the branching ratios for $\Upsilon(5S) \to \chi_{bJ}(\omega)$ can be well reproduced in our calculation with a common parameter range, which reflects the similarity among these $\Upsilon(5S) \to \chi_{bJ}(\omega)$ decays of concern.

PACS numbers: 13.25.Gv, 12.38.Lg

In the past years, the Belle Collaboration has reported some novel phenomena relevant to the hidden bottom decays of $\Upsilon(5S)$. In Ref. [1], Belle indicated that the partial decay widths of $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-$ are $10^2$ times larger than those of $\Upsilon(nS) \to \Upsilon(mS)\pi^+\pi^-$, where $n, m = 1, 2, 3$ and $m > n$, which is the puzzle in the $\Upsilon(5S)$ hidden-bottom di-pion decays. There are two possible explanations for this puzzle. Another possibility is that there is a tetraquark state $Y_b$ near $\Upsilon(5S)$. According to this assumption, Ali et al. also studied the $\pi^+\pi^-$ invariant mass spectrum and the $\cos \theta$ distribution of $Y_b \to \Upsilon(1S, 2S)\pi^+\pi^-$. They claimed that the experimental data can be well described under this explanation. However, as indicated in Ref. [4], the result of $Y_b \to \Upsilon(2S)\pi^+\pi^-$ is not consistent with the corresponding experimental data. That is, in their calculation of $Y_b \to \Upsilon(2S)\pi^+\pi^-$, they describe $\pi^+\pi^-$ data. If taking the same parameters to produce the $\cos \theta$ distribution, however, we found that the obtained $\cos \theta$ distribution cannot fit the experimental data. Furthermore, in Ref. [5], the authors also studied $\Upsilon(5S) \to \Upsilon(1S, 2S)\pi^+\pi^-$ by the rescattering mechanism, where the interference effect was considered. They met the same problem when fitting the experimental data of $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$. Thus, a new puzzle was proposed in Ref. [5]. Later, two charged bottomonium-like structures $Z_b(10610)$ and $Z_b(10650)$ were reported by Belle [6], which also stimulated the authors in Ref. [5] to find the relation between the observed $Z_b$ structures and the solution to this new puzzle. If introducing the intermediate $Z_b$ contributions in $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$, the new puzzle mentioned above can be nicely solved, which also results in the observation of the initial single pion emission mechanism in Ref. [7] to explain why there are two charged $Z_b$ structures near the $BB^*$ and $B^*B^*$ thresholds. More theoretical predictions of charged charmonium-like structures around the $D^*\bar{D}^*$ and $D^*\bar{D}^*$ threshold were, of course, given in Ref. [8].

The studies cited above show that the hadronic loop mechanism, as an important non-perturbative QCD effect, is indeed important to $\Upsilon(5S)$ decays. Before applying the hadronic loop mechanism to study the $\Upsilon(5S)$ decays, this mechanism was extensively applied to study the decays of the higher bottomonium and charmonium in Refs. [9–14] and achieved great successes.

Very recently, Belle announced their observation of $\Upsilon(5S) \to \chi_{bJ}(\omega)$ ($J = 0, 1, 2$), which indicates that the $\Upsilon(5S) \to \chi_{bJ}(\omega)$ decays also have large decay widths; i.e., the measured branch ratios of $\Upsilon(5S) \to \chi_{bJ}(\omega)$ are $< 3.4 \times 10^{-3}$, $(1.64 \pm 0.30 \pm 0.30) \times 10^{-3}$, and $(0.57 \pm 0.22 \pm 0.07) \times 10^{-3}$ with $J = 0, 1, 2$, respectively [15, 16]. It should be noticed that even though the tree-level contributions to $\Upsilon(5S) \to \chi_{bJ}(\omega)$ ($J = 0, 1, 2$) should be strongly suppressed due to the Okubo-Zweig-Iizuka rule, such large decay widths are observed, which again inspires our interest in understanding such quantities. In this work, we propose that the contribution from the hadronic loop should be considered in studying $\Upsilon(5S) \to \chi_{bJ}(\omega)$. To give a quantitative answer, we perform the concrete calculation, which is illustrated in the following. This investigation can, of course, provide a good test of the hadronic loop mechanism.

$\Upsilon(5S)$ as a higher bottomonium is above the threshold of a pair of bottom mesons, where $\Upsilon(5S)$ mainly decays into $B^{(*)}\bar{B}^{(*)}$, which means that there exists the strong coupling between $\Upsilon(5S)$ and a bottom meson pair. Thus, the hadronic loop effect can play an important role in the decay of $\Upsilon(5S)$, as just briefly reviewed above. Under the hadronic loop mechanism, these discussed $\Upsilon(5S) \to \chi_{bJ}(\omega)$ processes occur via the intermediate $B^{(*)}$ meson loops. In Fig. 1 the diagrams describing the $\Upsilon(5S) \to \chi_{bJ}(\omega)$ decays are given, where an intermediate bottom meson pair can transit into final states $\chi_{bJ}(\omega)$ by exchanging a proper bottom meson. Instead of the hadronic description for $\Upsilon(5S) \to \chi_{bJ}(\omega)$, we can give a quark level description of the hadronic loop contribution in Fig. 2. Here, a fermion line in red denotes bottom quark while a blue line corresponds to the light quark. $\Upsilon(5S)$ first dissolves into two virtual bottom mesons and then this bottom meson pair can turn into $\chi_{bJ}(\omega)$ via an exchange of an appropriate bottom meson. The matrix element of $\Upsilon(5S) \to \chi_{bJ}(\omega)$ via hadronic loop
The corresponding description at the hadron level is listed in Fig. 1.

**FIG. 1:** The necessary diagrams depicting \( \Upsilon(5S) \rightarrow \chi_{bJ} \omega \) decays under the hadronic loop effect.

![Diagram](image1)

**FIG. 2:** The quark level diagram depicting \( \Upsilon(5S) \rightarrow \chi_{bJ} \omega \) decay under the hadronic loop effect.

In the heavy quark limit, the wave function of a heavy-light meson is independent of the flavor and spin of the heavy quark; therefore, this wave function can be characterized by the angular momentum of the light degrees of freedom, which is \( \ell \), and \( |\ell\rangle \) corresponds to a degenerate doublet of states with the total angular momentum \( J = |\ell\rangle \pm 1/2 \). For the bottom meson with \( \ell = 0 \), the doublet formed by the bottom pseudoscalar and vector meson is represented in \([17, 20]\).

\[
H^0(\bar{Q}Q) = \frac{1 + \gamma}{2} \left[ B^\mu Y^\mu - B^0 Y^3 \right].
\]  

For the heavy quarkonium, the degeneracy is expected under the rotations of the two heavy quark spins, although the heavy quark flavor symmetry does not hold any more. This allows heavy quarkonium with the same angular momentum \( \ell \) to form a multiplet. For the bottomonia with \( \ell = 0, \eta_b \) and \( \Upsilon \) form a doublet in the form

\[
R^j(\bar{Q}Q) = \frac{1 + y}{2} \left[ \gamma^\nu Y^\mu - \eta_b Y^3 \right] \frac{1 - y}{2}.
\]  

In a similar way, a spin multiplet corresponding to the \( P \)-wave bottomonia is,

\[
p^j(\bar{Q}Q)^\mu = \frac{1 + y}{2} \left[ \gamma^\mu Y^\nu + \frac{1}{\sqrt{2}} \gamma^\rho v^\nu_{\rho \gamma} |\eta_b\rangle Y^\gamma \right] + \frac{1}{\sqrt{3}} (\gamma^\mu - Y^\mu) |\eta_b\rangle + \frac{R^j}{2} Y^3 |\eta_b\rangle \frac{1 - y}{2}.
\]

With these multiplets, we can construct the general form of the coupling between heavy quarkonium and heavy meson. The related effective Lagrangians involved in the present work are \([19]\)

\[
L_\Upsilon = i g_\Upsilon \text{Tr} \left[ R(\bar{Q}Q)^j \bar{B}(\bar{Q}Q)^j + H.c. \right],
\]

\[
L_\mu = i g_\mu \text{Tr} \left[ P(\bar{Q}Q)^j \bar{B}(\bar{Q}Q)^j + H.c. \right],
\]

where \( H^j(\bar{Q}Q) \) represents the heavy-light meson containing a heavy antiquark \( \bar{Q} \), which can be obtained by applying the charge conjugation operation to \( H^j(QQ) \). Expanding the above Lagrangians, we can obtain the following effective couplings:

\[
L_{\Upsilon(5S)B^0(\bar{B}^0)} = -i g_\Upsilon \text{Tr} \left[ \bar{B}B \gamma^\mu \gamma^\nu \frac{\partial}{\partial \nu} R^j(\bar{Q}Q)^j \right] + g_\Upsilon \text{Tr} \left[ \frac{\partial}{\partial \mu} R^j(\bar{Q}Q)^j \right] + i g_\Upsilon \text{Tr} \left[ \bar{B}B \gamma^\mu \gamma^\rho \gamma^\nu \frac{\partial}{\partial \nu} R^j(\bar{Q}Q)^j \right],
\]

\[
L_{\chi_b(5S)\omega(\bar{\omega})} = -g_{\chi_b(5S)\omega(\bar{\omega})} \chi_b^\mu \gamma^\rho \gamma^\nu \frac{\partial}{\partial \nu} \frac{\partial}{\partial \rho} \frac{\partial}{\partial \mu} R^j(\bar{Q}Q)^j + g_{\chi_b(5S)\omega(\bar{\omega})} \chi_b^\mu \gamma^\rho \gamma^\nu \frac{\partial}{\partial \nu} \frac{\partial}{\partial \rho} \frac{\partial}{\partial \mu} R^j(\bar{Q}Q)^j,
\]

\[
L_{\chi_b(5S)\rho(\bar{\rho})} = -g_{\chi_b(5S)\rho(\bar{\rho})} \chi_b^\mu \gamma^\rho \gamma^\nu \frac{\partial}{\partial \nu} \frac{\partial}{\partial \rho} \frac{\partial}{\partial \mu} R^j(\bar{Q}Q)^j + g_{\chi_b(5S)\rho(\bar{\rho})} \chi_b^\mu \gamma^\rho \gamma^\nu \frac{\partial}{\partial \nu} \frac{\partial}{\partial \rho} \frac{\partial}{\partial \mu} R^j(\bar{Q}Q)^j,
\]

\[
L_{\chi_b(5S)\rho(\bar{\rho})} = -i g_{\chi_b(5S)\rho(\bar{\rho})} \chi_b^\mu \gamma^\rho \gamma^\nu \frac{\partial}{\partial \nu} \frac{\partial}{\partial \rho} \frac{\partial}{\partial \mu} R^j(\bar{Q}Q)^j + i g_{\chi_b(5S)\rho(\bar{\rho})} \chi_b^\mu \gamma^\rho \gamma^\nu \frac{\partial}{\partial \nu} \frac{\partial}{\partial \rho} \frac{\partial}{\partial \mu} R^j(\bar{Q}Q)^j,
\]

where \( \mathcal{V} \) is the matrix of the vector octet, which is in the form

\[
\mathcal{V} = \left[ \begin{array}{cccc}
\mathcal{V}_{00} & \mathcal{V}_{0-} & \mathcal{V}_{0+} & \mathcal{V}_{05} \\
\mathcal{V}_{-0} & \mathcal{V}_{-+} & \mathcal{V}_{-5} & \mathcal{V}_{-1} \\
\mathcal{V}_{+0} & \mathcal{V}_{+-} & \mathcal{V}_{-+} & \mathcal{V}_{+5} \\
\mathcal{V}_{50} & \mathcal{V}_{5-} & \mathcal{V}_{5+} & \mathcal{V}_{55} \\
\end{array} \right].
\]
With the above effective Lagrangian, we can write out the amplitudes of the hadronic loop contributions to $\Upsilon(5S) \to \chi_{b1}\omega (J = 0, 1, 2)$. For $\Upsilon(5S) \to \chi_{b1}\omega$, the amplitudes corresponding to Fig. 1(a-1)-(a-4) are

$$M_{(a-1)} = \int \frac{d^4q}{(2\pi)^4} \left[ g T_B B e_{\mu\nu}^B (ip_{1\mu} - ip_{2\nu}) \right] - i g B B V e_{\mu}^B (ip_{1\nu}) - ig_{\chi_{b1}\omega} \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2 (\Lambda),$$

$$M_{(a-2)} = \int \frac{d^4q}{(2\pi)^4} \left[ g T_B B e_{\mu\nu}^B (ip_{1\mu} - ip_{2\nu}) \right] - i g_{\chi_{b1}\omega} \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2 (\Lambda),$$

$$M_{(a-3)} = \int \frac{d^4q}{(2\pi)^4} \left[ g T_B B e_{\mu\nu}^B (ip_{1\mu} - ip_{2\nu}) \right] - i g_{\chi_{b1}\omega} \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2 (\Lambda),$$

$$M_{(a-4)} = \int \frac{d^4q}{(2\pi)^4} \left[ g T_B B e_{\mu\nu}^B (ip_{1\mu} - ip_{2\nu}) \right] - i g_{\chi_{b1}\omega} \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2 (\Lambda),$$

respectively. Similarly, we can write out the amplitudes for $\Upsilon(5S) \to \chi_{b1}\omega$, corresponding to Fig. 1(b-1)-(b-4), which are

$$M_{(b-1)} = \int \frac{d^4q}{(2\pi)^4} \left[ g T_B B e_{\mu\nu}^B (ip_{1\mu} - ip_{2\nu}) \right] - i g B B V e_{\mu}^B (ip_{1\nu}) - i g_{\chi_{b1}\omega} \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2 (\Lambda),$$

$$M_{(b-2)} = \int \frac{d^4q}{(2\pi)^4} \left[ g T_B B e_{\mu\nu}^B (ip_{1\mu} - ip_{2\nu}) \right] - i g B B V e_{\mu}^B (ip_{1\nu}) - i g_{\chi_{b1}\omega} \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2 (\Lambda),$$

respectively. 

The hadronic loop contribution to $\Upsilon(5S) \to \chi_{b2}\omega$ is listed in Fig. 1(c-1)-(c-4). In these diagrams, the exchanged bottom meson can be $B$ meson or $B^*$ meson. The concrete amplitudes are collected in the Appendix.

Considering the isospin symmetry and charge symmetry, we obtain the total amplitude of $\Upsilon(5S) \to \chi_{b1}\omega$.

$$M_{\Upsilon(5S) \to \chi_{b1}\omega}^{\text{Tot}} = \sum_{j=1}^{4} M_{(i,j)},$$

where $i = a, b, c$ correspond to $\Upsilon(5S) \to \chi_{b1}\omega, \Upsilon(5S) \to \chi_{b1}\omega$, and $\Upsilon(5S) \to \chi_{b2}\omega$, respectively. The amplitudes of $M_{(a-j)}$ and $M_{(b-j)}$ have been presented in Eqs. (11)-(17). The amplitudes $M_{(c-j)}$ are defined as $M_{(c-j)} = M_{(b-j)} + M_{(b-j)^*}$. With above amplitudes, the partial decay width reads as

$$\Gamma_{\Upsilon(5S) \to \chi_{b1}\omega} = \frac{1}{24 \pi m_{\Upsilon(5S)^*}} \left| M_{\Upsilon(5S) \to \chi_{b1}\omega}^{\text{Tot}} \right|^2,$$

where the overline indicates the sum over the polarization vectors of $\Upsilon(5S)$ and $\omega$. In addition, we define $|\vec{p}_{\omega}| = \Lambda^{1/2}(m_{\Upsilon(5S)}^2, m_{\chi_{b1}}, m_{\omega})$ with the Källen invariant $\Lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$.

Adopting the similar approach, we can obtain the amplitudes of $\Upsilon(5S) \to \chi_{b1}\omega$ and $\Upsilon(5S) \to \chi_{b2}\omega$, which correspond to Fig. 1(b-1)-(b-4) and Fig. 1(c-1)-(c-4), respectively. In the amplitudes, we introduce a form factor in the monopole chagend boson mass. In the heavy quark limit, $B$ and $B^*$ are degenerate and the space wave functions of $B$ and $B^*$ are the same. Thus, in the present work, we parameterize the cutoff $\Lambda$ as $\Lambda = (m_B + m_{B^*})/2 + \alpha_\Lambda \Lambda_{QCD}$ with $\Lambda_{QCD} = 0.22$ GeV.
TABLE I: The coupling constants of $\Upsilon(5S)$ interacting with $B^{(*)}\bar{B}^{(*)}$. Here, we also list the corresponding branching ratios.

| Final state $\mathcal{B}(\%)$ | Coupling | Final state $\mathcal{B}(\%)$ | Coupling |
|-------------------------------|----------|-------------------------------|----------|
| $B\bar{B}$                   | 5.5      | $B\bar{B}^*$                  | 13.7     |
| $B^*\bar{B}^*$               | 38.1     |                               | 0.14 GeV$^{-1}$ |

Since $\Upsilon(5S)$ is above the threshold of $B^{(*)}\bar{B}^{(*)}$, the coupling constants between $\Upsilon(5S)$ and $B^{(*)}\bar{B}^{(*)}$ can be estimated by partial decay width of $\Upsilon(5S) \rightarrow B^{(*)}\bar{B}^{(*)}$. The partial decay width and the corresponding coupling constants are listed in Table I. If a vector boson multiplet is included, the effective Lagrangian both with pseudoscalars and vector bosons is constructed as in Refs. [17–20]. This Lagrangian includes only one gauge coupling $g_1$ in the heavy quark limit so that all of the coupling constants are related to this gauge coupling. In the heavy quark limit, the coupling constants of $\chi_{bJ}B^{(*)}\bar{B}^{(*)}$ are related to the gauge coupling $g_1$ by

$$g_{\chi_bBB} = 2 \sqrt{3} g_1 \sqrt{m_{\chi_b} m_B}, \quad g_{\chi_bBB^*} = \frac{2}{\sqrt{3}} g_1 \sqrt{m_{\chi_b} m_{B^*}},$$

$$g_{\chi_bBB^*} = 2 \sqrt{3} g_1 \sqrt{m_{\chi_b} m_{B^*}}, \quad g_{\chi_bBB} = 2 g_1 \sqrt{m_{\chi_b} m_B},$$

$$g_{\chi_bBB} = g_1 \sqrt{m_{\chi_b} m_B}, \quad g_{\chi_bBB^*} = 4 g_1 \sqrt{m_{\chi_b} m_{B^*}},$$

where we take the gauge coupling $g_1 = -\sqrt{\frac{3}{4\Lambda}}$ and $f_{\chi_b} = 175 \pm 55$ MeV is the decay constant of $\chi_{b0}$. The coupling constants between light vector mesons and bottom mesons are

$$g_{BBV} = g_{BB^*-V} = \frac{\beta g_V}{\sqrt{2}},$$

$$f_{BB^*-V} = \frac{f_{BB^*-V}}{m_{B^*}} = \frac{\lambda g_V}{\sqrt{2}}.$$

where the gauge coupling $\beta = 0.9$, $\lambda = 0.56$ GeV$^{-1}$, and $g_V = m_B/2 f_V$ with pion decay constant $f_V = 132$ MeV [22–23].

With above preparations, we can evaluate the hadronic loop contributions to $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decays. The $\alpha_A$ is introduced as a free parameter in the cutoff $\Lambda$ of the form factor. This parameter is usually dependent on particular process and taken to be of the order of unity. In Fig. 3 we present the $\alpha_A$ dependence of the branching ratio of $\Upsilon(5S) \rightarrow \chi_{b3}\omega$. The experimental data from the Belle Collaboration [15, 16] are also presented in comparison with our calculated results.

From Fig. 3 we notice that our theoretical estimate can reproduce the experimental data given by the Belle Collaboration [15, 16]. For $\Upsilon(5S) \rightarrow \chi_{b0}\omega$, only the upper limit was given by the experimental measurement, which is $\mathcal{B}(\Upsilon(5S) \rightarrow \chi_{b0}\omega) < 3.4 \times 10^{-3}$, where our result overlaps with the experimental data when taking the range $\alpha_A < 1.09$. As for the discussed $\Upsilon(5S) \rightarrow \chi_{b1}\omega$ and $\Upsilon(5S) \rightarrow \chi_{b2}\omega$ decays, our calculation can be fitted to the corresponding experimental values when taking $0.41 < \alpha_A < 0.48$ and $0.43 < \alpha_A < 0.54$, respectively. Moreover, we need to emphasize that there exists a common $\alpha_A$ range $0.43 < \alpha_A < 0.48$ for all $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decays, which reflects the similarity among these three decays. With this common $\alpha_A$ range, we can further restrict the branching ratio of $\Upsilon(5S) \rightarrow \chi_{b0}\omega$, which is $3.00 \times 10^{-4} < \mathcal{B}(\Upsilon(5S) \rightarrow \chi_{b0}\omega) < 4.05 \times 10^{-4}$, where this branching ratio is about 1 order smaller than the corresponding upper limit reported by Belle [15, 16], which can be tested in a future experiment.

In summary, being stimulated by the recent preliminary results of $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ released by Belle [15, 16], we have studied the $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decays through the hadronic loop mechanism. In the past years, there were some experimental [1, 6] and theoretical progresses [2, 3, 7, 8] on the $\Upsilon(5S)$ decays, which show that the hadronic loop mechanism can be an important effect on the $\Upsilon(5S)$ decays. The present investigation provides a further test of the hadronic loop effect. Our calculation indicates that the Belle data of $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ can be reproduced when the hadronic loop mechanism is considered in $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$. What is more important is that there exists a common $\alpha_A$ range for all $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decays, which is due to the similarity among $\Upsilon(5S)$ with $J = 0, 1, 2$. In addition, we further constrain the branching ratio of $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ by the obtained common parameter range, which can be tested in future experiments.

APPENDIX: THE DECAY AMPLITUDES OF $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$

We collected the $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decay amplitudes, i.e.,
\begin{align}
M_{c(-1)}^\theta &= \int \frac{d^4p}{(2\pi)^4} \left[ i g_{\gamma B B' E_{\mu \nu}} e^\mu_\nu (ip_{1\mu} - ip_{2\nu}) \right] - ig_{B B'} (-ip_{1\nu}) \\
&\quad - i q_{\nu} e^\nu_\mu \left[ -i g_{\gamma_{1\nu}} e^\mu_{\nu \sigma} (-ip_{2\alpha}) (-iq_{\beta}) \right] \times \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} q^2 - m_B^2 \mathcal{F}^2(\Lambda),
\end{align}

\begin{align}
M_{c(-1)}^\nu &= \int \frac{d^4p}{(2\pi)^4} \left[ i g_{\gamma B B' E_{\mu \nu}} e^\mu_\nu (ip_{1\mu} - ip_{2\nu}) \right] - 2 f_{B B' V} e_{\lambda \nu \theta \phi} \\
&\quad \times (ip^\nu_\alpha)e^{\alpha}_\mu (ip^\mu_\beta + iq^\beta_\phi) \left[ - g_{\gamma_{1\nu}} e^\mu_{\nu \sigma} (ip^\sigma_\delta) \right] e^{\alpha}_\beta \\
&\quad \times (-ip_{2\nu}) (-iq_{\nu}) \times \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} q^2 - m_B^2 \mathcal{F}^2(\Lambda),
\end{align}

\begin{align}
M_{c(-2)}^\theta &= \int \frac{d^4p}{(2\pi)^4} \left[ g_{\gamma B B' E_{\mu \nu}} (ip^\nu_0) e^\mu_\nu (-ip^\beta_2 + ip^\nu_1) \right] \\
&\quad \times \left[ -i g_{B B'} (-ip_{1\nu} - iq_{\nu}) e^{\nu}_\mu \right] \left[ -i g_{\gamma_{1\nu}} e^\mu_{\nu \sigma} (ip^\sigma_\delta) \right] e^{\alpha}_\beta \\
&\quad \times (iq_{\nu} (-ip^\alpha_\beta)) \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} q^2 - m_B^2 \mathcal{F}^2(\Lambda),
\end{align}

\begin{align}
M_{c(-2)}^\nu &= \int \frac{d^4p}{(2\pi)^4} \left[ g_{\gamma B B' E_{\mu \nu}} (ip^\nu_0) e^\mu_\nu (-ip^\beta_2 + ip^\nu_1) \right] \\
&\quad \times \left[ -2 f_{B B' V} e_{\lambda \nu \theta \phi} (ip^\nu_\alpha)e^{\alpha}_\mu (ip^\mu_\beta + iq^\beta_\phi) \right] \\
&\quad \times \left[ -g^\alpha_{\delta \beta} + p^\beta p^\alpha / m_B^2 \right] e^{\alpha \beta}_\nu \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} q^2 - m_B^2 \mathcal{F}^2(\Lambda),
\end{align}

\begin{align}
M_{c(-3)}^\theta &= \int \frac{d^4p}{(2\pi)^4} \left[ g_{\gamma B B' E_{\mu \nu}} (ip^\nu_0) e^\mu_\nu (ip^\beta_2 - ip^\nu_1) \right] \\
&\quad \times \left[ -2 f_{B B' V} e_{\lambda \nu \theta \phi} (ip^\nu_\alpha)e^{\alpha}_\mu (ip^\mu_\beta + iq^\beta_\phi) \right] \\
&\quad \times \left[ -g^\alpha_{\delta \beta} + p^\beta p^\alpha / m_B^2 \right] e^{\alpha \beta}_\nu \frac{1}{p_2^2 - m_B^2} q^2 - m_B^2 \mathcal{F}^2(\Lambda),
\end{align}

\begin{align}
M_{c(-3)}^\nu &= \int \frac{d^4p}{(2\pi)^4} \left[ g_{\gamma B B' E_{\mu \nu}} (ip^\nu_0) e^\mu_\nu (ip^\beta_2 - ip^\nu_1) \right] \\
&\quad \times \left[ -2 f_{B B' V} e_{\lambda \nu \theta \phi} (ip^\nu_\alpha)e^{\alpha}_\mu (ip^\mu_\beta + iq^\beta_\phi) \right] \\
&\quad \times \left[ -g^\alpha_{\delta \beta} + p^\beta p^\alpha / m_B^2 \right] e^{\alpha \beta}_\nu \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} q^2 - m_B^2 \mathcal{F}^2(\Lambda),
\end{align}

which correspond to Fig. [1](c-1)-(c-4), respectively.

Acknowledgments

This project is supported by the National Natural Science Foundation of China under Grants No. 11222547, No. 11175073, No. 11035006 and No. 11375240, the Ministry of Education of China (FANEDD under Grant No. 200924, SRDFP under Grant No. 20120211110002, NCET, the Fundamental Research Funds for the Central Universities); and the Fok Ying Tung Education Foundation (No. 131006).

[1] K. F. Chen et al. [Belle Collaboration], Phys. Rev. Lett. 100, 112001 (2008) [arXiv:0710.2577 [hep-ex]].
[2] C. Meng and K.-T. Chao, Phys. Rev. D 77, 074003 (2008) [arXiv:0712.3555 [hep-ph]].
[3] C. Meng and K.-T. Chao, Phys. Rev. D 78, 034022 (2008) [arXiv:0805.0143 [hep-ph]].
[4] A. Ali, C. Hambrock and M. I. Aslam, Phys. Rev. Lett. 104, 162001 (2010) [Erratum-ibid. 107, 049903 (2011)] [arXiv:0912.5016 [hep-ph]].
[5] D.-Y. Chen, J. He, X.-Q. Li and X. Liu, Phys. Rev. D 84, 094001 (2011) [arXiv:1104.2060 [hep-ph]].
074006 (2011) [arXiv:1105.1672 [hep-ph]].

[6] A. Bondar et al. [Belle Collaboration], Phys. Rev. Lett. 108, 122001 (2012) [arXiv:1110.2251 [hep-ex]].

[7] D. -Y. Chen and X. Liu, Phys. Rev. D 84, 094003 (2011) [arXiv:1106.3798 [hep-ph]].

[8] D. -Y. Chen and X. Liu, Phys. Rev. D 84, 034032 (2011) [arXiv:1106.5290 [hep-ph]].

[9] C. Meng and K. -T. Chao, Phys. Rev. D 78, 074001 (2008) [arXiv:0806.3259 [hep-ph]].

[10] D. -Y. Chen, J. He, X. -Q. Li and X. Liu, Phys. Rev. D 81, 074006 (2010) [arXiv:0912.4860 [hep-ph]].

[11] X. -H. Liu and Q. Zhao, Phys. Rev. D 81, 014017 (2010) [arXiv:0912.1508 [hep-ph]].

[12] D. -Y. Chen, X. Liu and T. Matsuki, Phys. Rev. D 87, no. 5, 054006 (2013) [arXiv:1209.0064 [hep-ph]].

[13] D. -Y. Chen, X. Liu and T. Matsuki, Phys. Rev. D 87, no. 9, 094010 (2013) [arXiv:1304.0372 [hep-ph]].

[14] Q. Zhao, G. Li and C. -H. Chang, Phys. Lett. B 645, 173 (2007) [hep-ph/0610223].

[15] M. Shapkin, Physics at Belle experiment, https://indico.cern.ch/event/269671/session/8/contribution/41/material/slides/0.pdf (2014), talk in the XXX-th International Workshop on High Energy Physics “Particle and Astroparticle Physics, Gravitation and Cosmology: Predictions, Observations and new Projects”, June 23–27, 2014, Protvino, Russia.

[16] X. H. He et al. [Belle Collaboration], [arXiv:1408.0504 [hep-ph]].

[17] O. Kaymakcalan, S. Rajeev, J. Schechter, Phys. Rev. D30, 594 (1984).

[18] Y. S. Oh, T. Song and S. H. Lee, Phys. Rev. C 63, 034901 (2001) [arXiv:nucl-th/0010064].

[19] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Rept. 281, 145 (1997) [arXiv:hep-ph/9605342].

[20] P. Colangelo, F. De Fazio and T. N. Pham, Phys. Lett. B 542, 71 (2002) [arXiv:hep-ph/0207061].

[21] E. V. Veliev, H. Sundu, K. Azizi and M. Bayar, Phys. Rev. D 82, 056012 (2010) [arXiv:1003.0119 [hep-ph]].

[22] H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, T. M. Yan and H. L. Yu, Phys. Rev. D 47, 1030 (1993) [arXiv:hep-ph/9209262].

[23] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin and H. L. Yu, Phys. Rev. D 46, 1148 (1992) [Erratum-ibid. D 55, 5851 (1997)].

[24] M. B. Wise, Phys. Rev. D 45, R2188 (1992).

[25] G. Burdman and J. F. Donoghue, Phys. Lett. B 280, 287 (1992).