QCD sum rules and thermal properties of Charmonium in the vector channel

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Abstract

The thermal evolution of the hadronic parameters of charmonium in the vector channel, i.e. the $J/\psi$ resonance mass, coupling (leptonic decay constant), total width, and continuum threshold is analyzed in the framework of thermal Hilbert moment QCD sum rules. The continuum threshold $s_0$, as in other hadronic channels, decreases with increasing temperature until the PQCD threshold $s_0 = 4m_Q^2$ is reached at $T \approx 1.22T_c$ ($m_Q$ is the charm quark mass) and the $J/\psi$ mass is essentially constant in a wide range of temperatures. The other hadronic parameters behave in a very different way from those of light-light and heavy-light quark systems. The total width grows with temperature up to $T \approx 1.04T_c$ beyond which it decreases sharply with increasing $T$. The resonance coupling is also initially constant beginning to increase monotonically around $T \approx T_c$. This behavior strongly suggests that the $J/\psi$ resonance might survive beyond the critical temperature for deconfinement, in agreement with lattice QCD results.

Keywords: Finite temperature field theory, hadron physics.

1. Introduction

We discuss here the thermal evolution of the hadronic parameters of $J/\psi$ in the vector channel, using thermal QCD Sum Rules \cite{1}. We refer the reader to the original article \cite{2} for details. This technique has been used previously in the light-light and in the heavy-light quark sector \cite{3}- \cite{5}, with the following emerging picture: (i) For increasing temperature, hadronically stable particles develop a non-zero width, and resonances become broader, diverging at a critical temperature interpreted as the deconfinement temperature ($T_c$). The thermal resonance broadening was first proposed in \cite{6}. ii) Above the resonance region, the continuum threshold in hadronic spectral functions, i.e. the onset of perturbative QCD (PQCD), decreases monotonically with increasing temperature. When $T \to T_c$, hadrons disappear from the spectrum. (iii) This scenario is also supported by the behavior of hadronic couplings, or leptonic decay constants, which approach zero as $T \to T_c$. Masses, on the other hand, do not to provide information about deconfinement.

The thermal behavior of the heavy-heavy quark correlator should be different from that involving at least one light quark since: a) In the light-light and heavy-light quark sector, the PQCD contribution is dominated by the time-like spectral function (annihilation term), which is relatively unimportant in relation to the light quark condensate contribution, being the scattering PQCD spectral function highly suppressed. Instead, for heavy-heavy quark systems this term becomes increasingly important with increasing temperature while the annihilation term only contributes near threshold; b) The non-perturbative QCD sector in the operator product expansion (OPE) of light-light and heavy-light quark correlators is driven by the light quark condensate, responsible for the behavior of the continuum threshold since $s_0(T)/s_0(0) \approx \langle \bar{q}q \rangle / \langle \bar{q}q \rangle$ \cite{4-7}. The light quark condensate is the order parameter for chiral symmetry restoration. In contrast, for heavy-heavy quark correlators the leading power correction in the OPE is that of the gluon condensate, which has a very different thermal behavior. In this approach, the criti-
cal temperature for deconfinement is a phenomenological parameter which does not need to coincide with e.g. the critical temperature obtained in lattice QCD [8]. In fact, results from QCD sum rules lead to values of \( T_c \) somewhat lower than those from lattice QCD. In order to compare with other approaches, we express our results in terms of the ratio \( T/T_c \).

We find for charmonium in the vector channel that the continuum threshold, \( s_0(T) \), decreases with increasing \( T \), being driven by the gluon condensate and the PQCD spectral function in the space-like region, until it reaches the PQCD threshold \( s_0 = 4 m_Q^2 \) at \( T \approx 1.22 T_c \) (\( m_Q \) is the charm quark mass). Below this value of \( s_0 \) the sum rules cease to be valid. The \( J/\psi \) mass remains basically constant as in the light-light or heavy-light systems. We have, however, a very different thermal evolution of the width and the coupling. Both are almost independent of \( T \) up to \( T \approx 0.8 T_c \), where the width begins to increase substantially, but then above \( T \approx 1.04 T_c \) it starts to decrease sharply, and the coupling increases also sharply. This suggests the survival of the \( J/\psi \) resonance above the deconfinement temperature.

The PQCD spectral function in the space-like region plays here a very important role. Non-relativistic approaches to charmonium at finite temperature would normally miss this contribution. In fact, the complex energy plane in the non-relativistic case would only have one cut along the positive real axis, which would correspond to the time-like (annihilation) region of PQCD. The space-like contribution \( \langle \omega^2 - Q^2 \rangle \leq 0 \) in the form of a cut in the energy plane centered at the origin for \( -|q| \leq \omega \leq |q| \), would not be present in the non-relativistic case.

### 2. Hilbert Moment QCD Sum Rules

We consider the correlator of the heavy-heavy quark vector current at finite temperature

\[
P_{\mu
u}(q^2, T) = \langle g_{\mu
u} q^2 - q_{\mu} q_{\nu} \rangle \Pi(q^2, T)
\]

\[
= i \int d^4x \ e^{iqx} \theta(x_0) \langle [V_{\mu}(x), V_{\nu}^+(0)] \rangle > > , \quad (1)
\]

where \( V_{\mu}(x) = : \bar{Q}(x) \gamma_{\mu} Q(x) : \) and \( Q(x) \) is the heavy (charm) quark field. The matrix element above is understood to be the Gibbs average in the quark-gluon basis. The imaginary part of the vector correlator in PQCD at finite temperature involves two pieces, one in the time-like region \( (q^2 \geq 4m_Q^2) \), \( \text{Im} \Pi_{\mu\nu}(q^2, T) \), which survives at \( T=0 \), and one in the space-like region \( (q^2 \leq 0) \), \( \text{Im} \Pi_{\mu\nu}(q^2, T) \), which vanishes at \( T=0 \). To leading order in PQCD we find

\[
\frac{1}{\pi} \text{Im} \Pi_{\mu\nu}(q^2, T) = \frac{3}{16\pi^2} \int_{-T}^{\infty} dx \left( 1 - x^2 \right)
\]

\[
\left[ 1 - n_F \left( \frac{|q| x + \omega}{2T} \right) - n_F \left( \frac{|q| x - \omega}{2T} \right) \right] , \quad (2)
\]

where \( \omega^2 = 1 - 4m_Q^2/q^2 \), \( m_Q \) is the heavy quark mass, \( q^2 = \omega^2 - q^2 \geq 4m_Q^2 \), and \( n_F(z) = (1 + e^{-z})^{-1} \) is the Fermi thermal function. In the rest frame of the thermal bath, \( |q| \to 0 \), the above result reduces to

\[
\frac{1}{\pi} \text{Im} \Pi_{\mu\nu}(\omega, T) = \frac{3}{8\pi^2} \frac{1}{m_Q^2} \left[ 1 - 2n_F(\omega/2T) \right] \times \theta(\omega - 2m_Q) . \quad (3)
\]

The quark mass is assumed independent of \( T \), a good approximation for \( T < 200 \text{ MeV} \) [9]. Only the leading order in the strong coupling will be considered here.

The PQCD piece in the space-like region demands a careful analysis. In the complex energy plane, and in the space-like region, the correlator \( \Pi(q^2) \), Eq.(1), has a cut centered at the origin and extending between \( \omega = -|q| \) and \( \omega = |q| \). In the rest frame this cut produces a delta function \( \delta(\omega^2) \) in the imaginary part of \( \Pi(q^2) \). The result is

\[
\frac{1}{\pi} \text{Im} \Pi_{\mu\nu}(\omega, T) = \frac{1}{2\pi} m_Q^2 \delta(\omega^2) \times
\]

\[
\left[ n_F \left( \frac{m_Q}{T} \right) \right] + \frac{2T^2}{m_Q^2} \int_{-m_Q(T)}^{m_Q(T)} y n_F(y) dy . \quad (4)
\]

The corresponding hadronic representation is parametrized in terms of the ground state resonance, the \( J/\psi \), followed by a continuum given by PQCD after a threshold \( s_0 > M_J^2 \). In the zero width approximation, the hadronic spectral function is

\[
\frac{1}{\pi} \text{Im} \Pi(s, T)_{\text{HAD}} = \frac{1}{\pi} \text{Im} \Pi(s, T)_{\text{RES}} \theta(s_0 - s) + \frac{1}{\pi} \text{Im} \Pi(s, T)_{\text{PQCD}} \theta(s - s_0)
\]

\[
= 2 f_J^2(T) \delta(s - M_J^2(T)) + \frac{1}{\pi} \text{Im} \Pi(s, T)_{\text{HAD}} \theta(s - s_0) , \quad (5)
\]

where \( s \equiv q^2 = \omega^2 - q^2 \). The leptonic decay constant is defined as \( < 0|V_{\mu}(0)|V(k) >= \sqrt{2} M_J f_J \epsilon_{\mu} \).

When considering a finite (total) width the following replacement will be understood

\[
\pi \delta(s - M_J^2(T)) \to \frac{M_J V_J(T) \Gamma_J(T)}{(s - M_J^2(T))^2 + M_J^2(T) \Gamma_J^2(T)} , \quad (6)
\]
The hadronic scattering term, due to current scattering off D-mesons, is negligible [2]. The correlation function \( \Pi(Q^2, T) \), Eq.(1), satisfies a once subtracted dispersion relation. To eliminate the subtraction one can use Hilbert moments, i.e.

\[
\varphi_N(Q^2, T) \equiv \frac{(-1)^N}{(N)!} \left( \frac{d}{dQ^2} \right)^N \Pi(Q^2, T)
\]

\[
= \frac{1}{\pi} \int_0^\infty \frac{ds}{(s + Q^2)^{N+1}} \text{Im} \Pi(s, T),
\]

where \( N = 1, 2, \ldots \), and \( Q^2 \geq 0 \) is an external four-momentum squared, to be considered as a free parameter. Using Cauchy's theorem in the complex s-plane, the Hilbert moments become Finite Energy QCD sum rules (FESR), i.e.

\[
\varphi_N(Q^2, T)|_{\text{RES}} = \varphi_N(Q^2, T)|_{\text{QCD}},
\]

where

\[
\varphi_N(Q^2, T)|_{\text{RES}} \equiv \frac{1}{\pi} \int_0^{\alpha(T)} \frac{ds}{(s + Q^2)^{N+1}} \text{Im} \Pi(s, T)|_{\text{RES}},
\]

\[
\varphi_N(Q^2, T)|_{\text{QCD}} \equiv \frac{1}{\pi} \int_0^{\alpha(T)} \frac{ds}{(s + Q^2)^{N+1}} \text{Im} \Pi(s, T)|_{\text{RES}} + \frac{1}{\pi} \int_0^{\infty} \frac{ds}{(s + Q^2)^{N+1}} \text{Im} \Pi(s, T) + \varphi_N(Q^2, T)|_{NP},
\]

and \( \text{Im} \Pi(s, T)|_{\text{RES}} \) is given by the first term in Eq.(5) modified in finite-width according to Eq.(6), and the PQCD spectral functions are given by Eqs.(3) and (4).

The dimension d=4 non perturbative term in the OPE is well known in the literature, see [2] for details. The dependence on \( N \) is quite cumbersome and it is proportional to the gluon condensate \( \langle \frac{\alpha_s}{\pi} G^2 \rangle \). At low temperatures, this condensate has been calculated in chiral perturbation theory [10]. In this framework the condensate remains essentially constant up \( T \sim T_c = 100 \text{ MeV} \), after which it decreases sharply. In order to go beyond the low temperature regime of chiral perturbation theory, lattice QCD provides the right tool. A good approximation [11] is given by the expression

\[
\langle \frac{\alpha_s}{\pi} G^2 \rangle = \langle \frac{\alpha_s}{\pi} G^2 \rangle \left[ \theta(T^* - T) + \frac{1 - \frac{T}{T^*}}{1 - \frac{T}{T_c}} \theta(T - T^*) \right]
\]

where \( T^* \approx 150 \text{ MeV} \) is the breakpoint temperature where the condensate begins to decrease appreciably, and \( T_c = 250 \text{ MeV} \) is the temperature at which \( \langle \frac{\alpha_s}{\pi} G^2 \rangle |_{T_c} = 0 \).

Returning to the \( Q^2 \) dependence of the Hilbert moments, Eq.(7), we shall fix \( Q^2 \) and \( s_0(0) \) from the experimental values of the mass, the coupling, and the width at \( T=0 \). At finite temperature there are non-diagonal (Lorentz non-invariant) condensates that might contribute to the OPE. Non-gluonic operators are highly suppressed [5], [12] so that they can be safely ignored. We have considered also a gluonic twist-two term in the OPE introduced in [13], and computed on the lattice in [14]. Its impact is small,(2-6)\%, and plays no appreciable role in the results.

3. Results

We begin by determining \( s_0 \) and \( Q^2 \) at \( T=0 \) from the moments, Eq.(8), and using as input the experimental values \[ 15 \] \( M_V = 3.097 \text{ GeV}, f_V = 196 \text{ MeV}, \) and \( T_V = 93.2 \text{ keV} \), as well as \( m_Q = 1.3 \text{ GeV} \), and \[ 16 \] \( \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle \approx 5 \times 10^{-3} \text{GeV}^4 \). In the zero-width approximation one finds from Eq.(9) that

\[
\frac{\varphi_1(Q^2)|_{\text{RES}}}{\varphi_2(Q^2)|_{\text{RES}}} = \frac{\varphi_1(Q^2)|_{\text{QCD}}}{\varphi_2(Q^2)|_{\text{QCD}}}. \tag{12}
\]

Given the extremely small total width of the \( J/\psi \) it turns out that the above relation also holds with extreme accuracy in finite width. Using Eq.(8) this leads to

\[
\frac{\varphi_1(Q^2)|_{\text{QCD}}}{\varphi_2(Q^2)|_{\text{QCD}}} = \frac{\varphi_1(Q^2)|_{\text{QCD}}}{\varphi_2(Q^2)|_{\text{QCD}}}, \tag{13}
\]

which depends only on the two unknowns \( s_0 \) and \( Q^2 \), and provides the first equation to determine this pair of parameters. The second equation can be e.g. Eq.(8) with \( N = 1 \). In this way we find that \( s_0 = 11.64 \text{ GeV}^2 \), and \( Q^2 = 10 \text{ GeV}^2 \) reproduce the experimental values of the mass, coupling, and width of \( J/\psi \) within less than 1\%. This whole set of hadronic parameters will then be used to normalize the corresponding parameters at finite temperature. In this way, see [2] for details, we were able to find the thermal evolution of \( s_0 \), the \( J/\psi \) mass, its width and its coupling. We show here only the behavior of the width and the coupling (Figs. 1 and 2) since these are the most important results of this analysis.

Both the width and the coupling can only be determined up to \( T_f \approx 1.1 T_c \), beyond which \( s_0(T) < M_V^2(T) \) and the FESR integrals have no longer a support. The temperature behavior of the width and the coupling shown in Figs. 1 and 2 strongly suggests the survival
of the $J/\psi$ above the critical temperature for deconfinement. This conclusion agrees with results from lattice QCD [8], but disagrees with non-relativistic determinations. As pointed out earlier, the reason for this disagreement might very well be the absence of the central cut (QCD scattering term) in the energy plane in non-relativistic frameworks.

Figure 1: The ratio $\Gamma_V(T)/\Gamma_V(0)$ as a function of $T/T_c$.

Figure 2: The ratio $f_V(T)/f_V(0)$ as a function of $T/T_c$.

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