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MODELS AND TECHNIQUES IN GEODETiC MONITORING
OF TECTONIC DEFORMATIONS IN GREECE

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Abstract
The assessment and interpretation of the geodetic results regarding the detection of
possible spatial displacements and the deformation parameters have to be combined
with a realistic geophysical model for the area. Usually, this study is carried out by
fitting the geodetic data to a polynomial function, which is considered sufficient to
describe adequately the deformation pattern. In terms of the computational steps
needed, this polynomial fitting can be accomplished simultaneously by the analysis
of the geodetic observations in a dynamic adjustment or non simultaneously in a
sequential approach. The main goal of this article is to give a short description of the
related methods and to present rigorous processing strategies for the analysis of
GNSS observations from continuously permanent stations in order to create a modern
and improved geodetic velocity field for Greece. Emphasis is given on the reference
frame definition problem.

Keywords: Deformation measurements, frame definition problem, GNSS networks,
Greek velocity field.

1. Introduction
Various geodetic methods for the extraction of displacements and deformation parameters are
recognized as useful techniques in many geophysical studies. Within the last fifty years a great
number of geodetic applications have been presented in the literature. They are based on repeated
observations from geodetic networks properly established in areas under investigation and the
analysis of the results between different epochs by means of appropriate models. All these methods
can be divided into three major categories according to the determination of: global scale
movements, regional movements and local deformations due to seismic or volcanic activity.

Local networks that cover relatively small areas include triangulation-trilateration in 2-d, geometric
leveling and GNSS networks. Furthermore, it is possible to process simultaneously in 3-d the previous
types of networks and moreover to have a common analysis of geodetic and photogrammetric
measurements or even a common analysis of geometrical geodetic measurements with measurements
related to the gravitational field. These networks can monitor the deformations of the earth’s crust due
to seismic activity, landslides, movements of road and mine slopes, etc. Networks established for the
volcano’s activity monitoring in order to predict its eruption, belong also in this class. Combining
measurements of GNSS systems, which record the geometric changes in the volcano area, with
measurements of the gravity field, which record the geometric changes and signals of the movements
and accumulations of magma beneath the volcano, provide important information about a possible
upcoming eruption a few dozen years before it happens.

Regional networks that cover a part of a country or even a continent consist mainly of permanent
GNSS stations with the aim to monitor the movements of plates in a continental scale and establish
national datum reference frames for geodetic activities in every country. The velocities determined by geodetic methods reflect the relative motions of tectonic plates - sections of earth’s crust bounded by faults - that extend to depths within the limits of the upper crust at least. These movements, where the first order effects of an earthquake deformation cycle has been removed, are smooth and usually described as rigid rotations of tectonic plates on the earth’s surface.

Global networks for monitoring the earth and its changes in time include GNSS, SLR (Satellite Laser Ranging), LLR (Lunar Laser Ranging) and VLBI (Very Long Baseline Interferometry) stations. The object of such networks is the creation of a global conventional geocentric reference frame characterized by high precision and homogeneity in space and time and the continuous determination of GNSS satellite orbits. Since the location of global stations are changing in time due to geophysical and tectonic processes and these changes are perceived by means of modern observations, the parameter of time must be considered in the analysis of the observations. Along with the coordinates of geodetic stations in a reference epoch, their changes over time are also resulted, e.g. the velocity of their movements. Ignoring these positional variations serious errors are introduced both in the satellite orbits and subsequently in any position on the earth’s surface but also in the assessment of deformation parameters and the velocity of the tectonic plates movement regardless of the type of a network.

The reference frame definition problem, known also as the datum problem or the zero order design problem, has received considerable attention since the pioneering work of Meissl (1965) and his famous “inner error theory”. Meissl’s method has been popularized by Blaha (1971) and Pope (1973) who developed a powerful method of evaluating the pseudoinverse matrix introducing the so called “solution space”. The relation of various solutions to Meissl’s inner constraints has been established by Baarda (1973) with the introduction of the so called S-transformation. Subsequently all methods have been described in full length by Grafarend and Schaffrin (1974), Felzer (1974), van Mierlo (1980), Koch (1982), Teunissen (1985). This problem dominated the geodetic literature in the 70s although it remains still opportune in GNSS applications and in the assessment of geodetic data for the detection of displacements and the estimation of deformation parameters.

The assessment and interpretation of the geodetic results for the detection of possible spatial displacements and the deformation parameters have to be combined with a realistic geophysical model for the area under consideration. Usually, fitting geodetic data to a polynomial function is considered sufficient to describe the deformation pattern what it is mainly described in the present paper. In terms of the computational strategy needed, polynomial fitting can be accomplished by one of the following (Rossiakopoulos, 2003; Dermanis, 2009): (i) simultaneously analysis of geodetic observations in a dynamic adjustment model, (ii) non simultaneously in a sequential approach of the dynamic adjustment, (iii) by a simple comparison of the results between any two epochs.

The main intention of the present paper is a short description of the methods used mainly in the analysis of satellite observations for studying the kinetic behavior of tectonic plates in Greece giving emphasis on the reference system definition problem, perhaps the most important problem as it concerns the estimability of unknown parameters. For example, a false choice of constraints in order to define the reference system could lead to a wrong picture of the movements of tectonic plates in a region. Depending on the kind and nature of the used geodetic data, e.g. original observations or derived coordinates from a network adjustment, the corresponding mathematical model may suffer from some specific problems, such as the inconsistency of the reference system or the existence of non-positive covariance matrices.

2. The reference system definition problem

The extended equations of observations are used in a dynamic treatment of temporal networks where the deformation of the network is considered and the measurements of all epochs take part in a simultaneously data process. Classical observations (angles, distances, height differences) or modern satellite observations (GNSS, VLBI, SLR, etc) can be analyzed. The linear observation equations for m epochs are written in matrix form as
The deformation networks can be treated as independent and be solved with the help of minimum constraints that define the reference system of coordinates without affecting the geometric characteristics of the network as defined by the observations. The inner constraints
\[ \mathbf{E} \mathbf{x} = \mathbf{0} , \quad \mathbf{E} \mathbf{u} = \mathbf{0} \] (2)
are considered the best choice in view of the connection of a time varying network, i.e. transforming the different network epoch solutions in a common reference system, at least for local networks, as it is the Volvi network (Fotiou et al., 2003), where the local deformations must be separated from the broader movements of tectonic plates. Representing as \( \mathbf{u} \) the vector of the temporal changes of the coordinates of network stations, each component is given by
\[
\begin{align*}
\Delta x_i &= x'_i - x_i = x_i(t') - x_i(t) \\
\Delta y_i &= y'_i - y_i = y_i(t') - y_i(t) \\
\Delta z_i &= z'_i - z_i = z_i(t') - z_i(t)
\end{align*}
\] (3)
The inner constraints \( \mathbf{E} \mathbf{u} = \mathbf{0} \) determine the center of the reference system keeping fixed the gravity center of the network, by defining as zero the sum of temporal coordinate variations
\[
\begin{align*}
\sum_{i=1}^{N} \Delta x_i &= \sum_{i=1}^{N} (x'_i - x_i) = 0 \\
\sum_{i=1}^{N} \Delta y_i &= \sum_{i=1}^{N} (y'_i - y_i) = 0 \\
\sum_{i=1}^{N} \Delta z_i &= \sum_{i=1}^{N} (z'_i - z_i) = 0
\end{align*}
\] (4)
The constraints for the network orientation are defined by the conditions that no rotation has happened around the axes through the gravity center of the network
\[
\begin{align*}
\sum_{i=1}^{N} [\Delta x_i, \Delta y_i, \Delta z_i] &= \sum_{i=1}^{N} [z_i(y'_i - y_i) - y_i(z'_i - z_i)] = 0 \\
\sum_{i=1}^{N} [\Delta x_i, \Delta y_i, \Delta z_i] &= \sum_{i=1}^{N} [x_i(z'_i - z_i) - z_i(x'_i - x_i)] = 0 \\
\sum_{i=1}^{N} [\Delta x_i, \Delta y_i, \Delta z_i] &= \sum_{i=1}^{N} [y_i(x'_i - x_i) - x_i(y'_i - y_i)] = 0
\end{align*}
\] (5)
They occur as a solution to discrete versus time-depended networks by applying the reset criterion of the relative angular momentum, known as Tisserand criterion in continuous networks. The scale of network in case it can’t be determined by the observations, is introduced through the constraint
\[
\sum_{i=1}^{N} [\Delta x_i, \Delta y_i, \Delta z_i, \Delta w_i] = \sum_{i=1}^{N} [x_i(x'_i - x_i) + y_i(y'_i - y_i) + z_i(z'_i - z_i)] = 0
\] (6)
keeping constant the mean square length of the network over time.
According to the above, the matrices \( \mathbf{E} \) and \( \mathbf{\dot{E}} \) have the analytical form
\[
\mathbf{E} = [\mathbf{E}_1 \ \mathbf{E}_2 \ \ldots \ \mathbf{E}_N]
\] (7)
where each sub-matrix \( \mathbf{E}_i \) corresponding to the point \( P_i \), is given by

\[
\mathbf{b} = \mathbf{A} \mathbf{x}_o + \mathbf{B} \mathbf{u} + \mathbf{v} \tag{1}
\]
where \( \mathbf{x}_o \) is the vector of coordinate corrections for the reference epoch, \( \mathbf{u} \) the vector of differences (displacement vector), \( \mathbf{A} \) and \( \mathbf{B} \) coefficient matrices and \( \mathbf{v} \) the vector of observational errors.
Consequently in a time dependent geodetic network a "common" reference system for all epochs is needed. The definition of the reference system in each epoch is based on analogous to the Meissl constraints, which are introduced for the coordinate differences in time. This procedure that has been proposed by Pelzer (1971) for geodetic applications of deformation measurements, is a discrete approximation to the definition of the reference system under time-continues data (Dermanis, 2002).

In the case of separate adjustments per epoch the constraints are incorporated as inner constraints on the unknown corrections of approximate coordinates, using common approximate coordinates for all epochs.

The inner constraints in a simultaneous adjustment of observations of all epochs are the corresponding process of the optimal fitting of the coordinates of any epoch to the corresponding coordinates at the reference epoch. Applying the well known 2-d or 3-d similarity transformation. Applying the similarity transformation between coordinates at different epochs the effects of different reference system definitions in the separate adjustments of observations are eliminated. Other effect are also reduced such as the systematic errors of GNSS data that affect the positions of the network stations as scale errors and rotational errors around the axes of the observation reference system. Furthermore a comprehensive, stable, for the wide area of the network, movement of the crust is reduced, usually described as a rigid transformation. While in the case of plane, rigid motion of a plate is represented by two translations and a rotation, studying larger areas, where the analysis should be carried out on the surface of the ellipsoid, the equivalent rigid motion is a rotation around a fixed pole with constant angular velocity. The corresponding parameters of the rigid motion are either the three cartesian components of the rotation vector, commonly referred to as "Euler vector", or its spherical components. Of course, a three-dimensional "rigid body" transformation may be applied instead of the Euler's pole technique as it is an equivalent solution. When initial information related to how the crust deforms is available, there are various ways to modify or extend the optimization criterion according to what was presented above, such as the generalization of inner constraints

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

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\[
W^2 W u = \min
\]

with "weight matrix" \( W \) determined according to the available information. The solution is given by means of the minimum constraints

\[
Hu = EWu = 0
\]

If all points of the network are involved in the inner constraints, the weight matrix \( W \) is the identity matrix \( W = I \). In the case of partial inner constraints, where only some points are selected for the reference system definition, for example points that are considered to be immovable, then the elements of the diagonal of \( W \) that correspond to these points are 1 while all other elements are 0. Generally matrix \( W \) introduces any information relevant to the parameters \( x \). By an appropriate choice of this matrix the various options, for solving the problem of the reference system definition in temporal networks, can be generalized and put in a common framework (Rossikopoulos, 2016), such as those given by Prescott (1981), Darby (1985), Segall and Matthews (1988) or by the various movement models such as those given by Solomon and Sleep (1974), Minster and Jordan (1978). For example, a choice for the reference system definition problem is to minimize the "movement" in a particular direction, through minimal constraints. This "outer coordinate solution" was given by Prescott (1981) in a specific application where the relative movement, and therefore the direction to minimize the temporal differences of the coordinates, should be parallel to the direction of the fault. Darby (1982) generalized the outer coordinate

\[
E_i = \begin{bmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

Consequently in a time dependent geodetic network a “common” reference system for all epochs is needed. The definition of the reference system in each epoch is based on analogous to the Meissl constraints, which are introduced for the coordinate differences in time. This procedure that has been proposed by Pelzer (1971) for geodetic applications of deformation measurements, is a discrete approximation to the definition of the reference system under time-continues data (Dermanis, 2002).

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\[
W^2 W u = \min
\]

with "weight matrix" \( W \) determined according to the available information. The solution is given by means of the minimum constraints

\[
Hu = EWu = 0
\]
solution noting that the preferred direction can be different at different stations and in the “model coordinate solution” of Segall and Matthews (1988) the displacement residuals are made as small as possible for a subset of stations only, a case of partial inner constraints.

3. The analytical approach in the estimation of deformations

Displacements may be treated as independent deterministic unknown parameters where their relation to an underlying function is ignored. This approach has the advantage that it is free from any doubtful assumptions about the structure of the underlying function and in addition the results of the simultaneous adjustment of observations are equivalent to those of separate adjustment per epoch.

On the other hand, the contrast between the discrete nature of the information, provided by the geodetic measurements, and the continuous information required to describe the deformation, makes clear that an appropriate approach to the analysis of geodetic data, related to the analysis of crustal motion, should not be limited to the classic techniques of adjustment of observations and parameter estimation but it has to somehow incorporate a solution to the problem of indirect interpolation. Applying interpolation techniques is not only a mathematical necessity for the estimation of deformation in the observation points but also for the expansion of the information at any point in the tectonically active area, a topic of obvious importance in geophysics. The dependence of displacements on underlying functions can be taken into account in different ways. One of these usually used is to introduce a more or less empirical model for the function, which involves unknown parameters to be estimated from the adjustment of the observations. Typical choices of an empirical model are linear combinations of known base functions with unknown coefficients. Based on the analytical functions

\[ u = Fa \] (11)

which are used to smooth-out the differential motions and to represent the deformation model, the observation equations are written as

\[ b = Ax + BFa + v \] (12)

where \( a \) are unknown parameters and \( F \) the matrix with elements depending on the used base functions. For the system definition problem in the network adjustment, the minimal constraints are introduced in a sequential form. Firstly the datum for the reference epoch is defined by applying the inner constrains

\[ E x_0 = 0 \] (13)

and afterwards the datum for any other epoch is defined by convenient minimal constraints. Although we use the extended model (12), we must minimize the norm

\[ a^T Fa = a^T Ra = \min \] (14)

of the displacements, instead of \( a^T a = \min \) that would lead to the inner constrains \( E a = 0 \). This solution (14) corresponds to the best fitting of the coordinates of the various epochs to a reference epoch. The minimal constraints, which correspond to the optimum criterion (14) and satisfy the condition

\[ a^T F^T Fa = a^T R a = \min \] (15)

have the form

\[ \tilde{E} a = 0 \] (16)

where \( \tilde{E} = \tilde{E} R = \tilde{E} F \) (17)

with \( \tilde{E} \) the inner constraints \( \tilde{E} a = 0 \) matrix

\[ \tilde{E} = \tilde{E} F (F^T F)^{-1} \] (18)

and \( \tilde{E} \) is the inner constraints matrix \( \tilde{E} u = 0 \) for the network of \( N \) points at epoch \( t_0 \). The deformation model (11) can be a space model, a time model or a space-time model. In the first case, where the displacements are considered as dependent on space only, we have for a point \( P_i \)

\[ u_j = F(x_i) a \] (19)
with the unknown parameters a common for all points at the same epoch. The observations equations for all \((m)\) epochs are
\[
\mathbf{b}_a = \mathbf{A}_a \mathbf{x}_a + \mathbf{v}_a
\]
\[
\mathbf{b}_a = \mathbf{A}_a \mathbf{x}_a + \mathbf{A}_a \mathbf{F}_a \mathbf{a}_a + \mathbf{v}_a
\]
\[
\mathbf{b}_a = \mathbf{A}_m \mathbf{x}_m + \mathbf{A}_m \mathbf{F}_m \mathbf{a}_m + \mathbf{v}_m
\]
(20)

Useful tools for creating analytic polynomial-type models of the form of the above relations result from the implementation methods of the theory of elasticity, as the deformation of the earth is associated with the classical case of continuous media mechanics. The deformation parameters in simultaneous adjustment of network observations of different epochs are usually treated as deterministic parameters, where the deformation of the entire network area or large part of it is considered homogenous or it is treated combined with analytical interpolation methods. The datum for the reference epoch is defined by applying the inner constrains
\[
\mathbf{E}_{xa} \mathbf{a}_a = \mathbf{0}
\]
(21)
and the datum for any other epoch is defined by the minimal constrains
\[
\mathbf{H}_{xa} \mathbf{a}_a = \mathbf{0}
\]
(22)
where
\[
\mathbf{H}_a = \mathbf{E}_a R_a = \mathbf{E}_a F_a = \sum_{i=1}^{N} \mathbf{E}_i F_i
\]
(23)
with
\[
\mathbf{R}_a = F_a^T F_a \quad \text{and} \quad \mathbf{E}_a = \mathbf{E}_a F_a (F_a^T F_a)^{-1}
\]
(24)
where \(\mathbf{E}_{xa} = [\mathbf{E}_1 \mathbf{E}_2 \ldots \mathbf{E}_N]\) is the inner constraints matrix \(\mathbf{E}_{xa} \mathbf{x}_a = \mathbf{0}\) for the network at epoch \(t_a\).

Based on the above, the selection of matrix \(\mathbf{R}_a\) depends on the choice of the minimum constraints. These constraints are selected on the basis to minimize the uncertainty of the elements of the vector \(\mathbf{u}_a\), due to the different definition of the reference system of the network in each epoch. This uncertainty is minimized for each epoch of measurements through the condition \(\mathbf{u}_a^T \mathbf{u}_a = \min\), which corresponds to the best fitting of the coordinates at the various epochs to the reference epoch. In equations (20) the deformation process was seen as a change from an initial to a final state of the continuous media without considering the time elapsed between the epochs. However, in many cases it is important to know the rate at which these changes occur, especially when we have a continuous data stream and the deformation is smooth in time. Examples of such applications were given by Morgan (1973), Bibby (1975, 1982) and Welsch (1986). In case the displacements are considered dependent on time only, the deformation model has the simple form
\[
\mathbf{u}_a = \mathbf{F}(t, t') \mathbf{u}_a
\]
(25)
If the movements are linear in time, we have the simple “velocity model”
\[
\mathbf{u} = \delta t_a \mathbf{u}
\]
(26)
where \(\mathbf{u}\) is the velocities vector and \(\delta t_a = t_a - t_c\) (or \(\delta t_a = t_a - t_{a-1}\)) the time difference. The first application of this velocity model was presented by Morgan (1973). Examples were given in Papo and Perelmuter (1983), Welsch (1986) and for vertical networks Vanicek et al. (1979), Mälzer et al. (1979). The observation equations for the \(m\) epochs are written as
\[
\mathbf{b}_a = \mathbf{A}_a \mathbf{x}_a + \mathbf{v}_a
\]
\[
\mathbf{b}_a = \mathbf{A}_a \mathbf{x}_a + \mathbf{A}_a \mathbf{F}_a \mathbf{a}_a + \mathbf{v}_a
\]
\[
\mathbf{b}_a = \mathbf{A}_m \mathbf{x}_m + \mathbf{A}_m \mathbf{F}_m \mathbf{a}_m + \mathbf{v}_m
\]
\[ b_m = A_m x_m + A_m F_m a + v_m \]  \hspace{1cm} (27)

where the unknown parameters \( a \) are common for all epochs at the same point. When discontinuities occur between epochs \( t_o \) and \( t_a \), the solution becomes significantly complicated. Movements are divided into long period, considered normal and occur between the epochs of discontinuities (e.g. epoch of a seismic event), and episodic occurring during the earthquakes activity. In such cases, the approaches can be grouped as follows:

- The simultaneous adjustment must be applied only for epochs between two "temporal discontinuities".
- The causes of the temporal discontinuities can be included in the analytical models. In this way we have analytical functions describing the long period movements between discontinuities and also functions for the episodic movements derived from the cause of discontinuity.

With reference to the epoch of measurements we should note that:

- The reference epoch is not necessarily a measuring epoch.
- There is no need for observations to refer to a "network measurement epoch" as every measurement is related to its own epoch.

The datum definition constraints are written as

\[ \bar{E} x_o = 0, \quad \bar{H} a = 0 \]  \hspace{1cm} (28)

where \( \bar{E} \) is the inner constraints matrix for the reference epoch and \( \bar{H} \) selected in order to minimize the norm

\[ u^T u = \sum_{a=1}^m u_a^T u_a = a^T \left( \sum_{a=1}^m F_a^T F_a \right) a = \min. \]  \hspace{1cm} (29)

given by

\[ \bar{H} = \bar{E} R = \bar{E} \sum_{a=1}^m F_a \]  \hspace{1cm} (30)

For the velocity model \( u = \delta t_o \) equation (28) becomes

\[ \bar{H} a = \left( \sum_{a=1}^m \delta t_o \right) \bar{E} u = 0 \quad \text{or} \quad \bar{E} u = 0 \]  \hspace{1cm} (31)

If the displacements are considered to be dependent on space and time, the analytical deformation model takes the form

\[ u_i = F(x_i, t, r, \tau) \]  \hspace{1cm} (32)

where the unknown parameters \( a \) are common for all epochs and all points and the observations equations for the \( m \) epochs have the same form as in (27). Examples are given in Bibby (1982), Snay et al. (1983, strain rate model) and Welsch (1986). The simultaneous adjustment of the observations of many epochs was presented by Whitten (1967), where movements are approached by functions of space and time of the above form. Subsequently relevant papers were given by Snay and Gergen (1978), Snay et al. (1983), Chrzanowski et al. (1986), Welsch (1986) and regarding the determination of the vertical movements by Holdahl (1978, 1980), Vanicek (1975), Vanicek et al. (1979), Holdahl and Hardy (1979) and Mälzer et al. (1979).

As in the previous case of dependent movements in time, the following should be noted:

- The reference epoch is not necessarily a measuring epoch.
- There is no need for observations to refer to a "network measurement epoch" as every measurement is related to its own epoch.

The minimal constrains which satisfy the condition \( u^T u \) for all epochs are written
\[ \hat{H}a = \left[ \sum_{i=1}^{N} E_i \left( \sum_{j=1}^{m} F_{ij} \right) \right] a = 0 \]  \hspace{1cm} (33)

where \( E \) is the sub- \( E \) corresponding to point \( P \) and \( F_{ij} \) the sub-matrix of \( F_{ik} \).

4. The sequential solution

The simplification of the algorithms implementing the above simultaneous adjustment of temporal observations lead to separate adjustment of observations, where specific algorithms are applied giving the same results as those of a simultaneous adjustment. These algorithms can be applied according to the type of available data. For example when the coordinates of the network stations and their covariance matrix are given for each epoch, they are considered as new (synthetic) observations. Generally, the analysis follows the steps:

a. Separate or single adjustment per epoch: This step includes the adjustment of the observations at each epoch, the statistical analysis and the final estimation of the coordinate set and its full covariance matrix at each epoch.

b. Best fitting of the coordinates at the various epochs to a reference epoch: The elimination of the difference between the coordinates at two distinct epochs, which is due to their different datum definition, is obtained by the optimal fitting of the coordinates of any epoch to the corresponding coordinates at the reference epoch, applying the well known 2-d or 3-d similarity transformation.

c. Adjustment with a deformation model: The coordinates at the reference epoch and the transformed ones as described in the second step are adjusted taking into account a deformation model. The mathematical model for all epochs is written in matrix form

\[
x_1 = x_o + F_1 a + v_1 \\
\vdots \\
x_n = x_o + F_n a + v_m
\]

or

\[
x = x_o + F a + v \hspace{1cm} \text{and} \hspace{1cm} \mathbf{v}^T \mathbf{W} \mathbf{v} = \sum_{i=1}^{m} v_i^2 W_{ii} = \min.
\]

where \( x_o \) is the vector of coordinates for the reference epoch and with the term \( F a \) to describes the displacement vector \( u \). The displacements can be considered as dependent in time or in space and time. For the time span of the analysis period (all epochs), the corresponding covariance matrices \( Q \) derived from the transformations, are semi-positive definite. In this case, any choice of the generalized inverse matrix \( W = Q^{-1} \) as weight matrix, and therefore the pseudo inverse matrix \( \mathbf{W}^* = Q_w^{1/2} \) leads to the best unbiased estimations for parameters \( u \).

The method is illustrated by Rossikopoulos et al. (1998), where GPS measurements are used associated to a 9-point geodetic network connecting the Greek and Italian coasts in the Ionian and Adriatic Sea. Other techniques for implementing the method of least squares for the adjustment of time-dependent networks are presented in the literature, such as the Kalman filters, where the equation \( u = \mathbf{F} a \) is the dynamic system that connects the epochs of measurements (Floyd et al. 2010), or other partitioned algorithms for the simultaneously adjustment of the temporal observations in a sequential mode (Rossikopoulos, 2016).

5. From discrete to continuous information

The fact that the analysis of the measurements provides only a discrete character, as a result of the discrete nature of the network stations, can be considered a disadvantage to the study of deformations using geodetic methods. In contrast the geophysical significant quantities have a continuous nature,
because they are dependent on what is happening in a very small area around each point of the under
studying area. For the extension of the information derived from the analysis of network
measurements at each point in the area, it is necessary to apply interpolation methods in order to
provide continuous information from discrete geodetic results. Of course, these disadvantages tend
to disappear today, as the technology of three-dimensional scanners develops rapidly, both in
terrestrial and space applications giving continuous information.

![Figure 1 - Horizontal velocities relative to ITRF2008 (Bitharis et al. 2015).](http://epublishing.ekt.gr)

Therefore the estimation of movements or the velocities or the geophysical parameters that can be derived
from these, at any point in the area covered by the control network, after the connection of its temporal
forms, interpolation methods are applied, where the area is divided into equal tectonic deformation
regions and an analytical function for each region, generally in polynomial form, is fitted at temporal
variations of the coordinates. A related method of analytical interpolation is that of piece-wise
polynomials or splines with smoothness conditions imposed at the boundaries of their domains. Spline
interpolation has been used by Bitharis et al. (2015), using bi-cubic splines for the interpolation of
velocities of 155 GNSS permanent stations in Greece (Fig. 1), at the nodes of an appropriately selected
grid. The continuous Greek geodetic velocity model relative to ITRF2008 is shown in figure 2.

The station coordinates and velocities depicted in figure 1, were estimated by a Kalman filtering
sequential approach. The connection of the temporal forms of the network was made through their
integration into the global reference system ITRF2008, which was conducted by means of the 7-
parameter similarity transformation for each epoch, using only IGS stations of the European part of
the studied Greek Continuous Satellite Network (GCSNet), excluding the Greek stations.

In addition to the “analytical approach” of movements, developed in this paper as a part of adjustment
algorithms, another way is to express the known limitations of the unknown functions of
the movements through a covariance function, which describes the statistical behavior of its values, by the
method of least squares collocation.
The least squares collocation is a generalized estimation method developed approximately in the 70s in order to analyze data of the gravitational field of the earth. Shortly after the method was applied to other scientific problems, for the analysis of observations related to quantities that depend on an unknown function. Collocation was first applied to the analysis of geodetic data for the calculation of strain parameters at the nodes of a grid in the area of Volvi in Greece (Dermanis et al., 1981) using the temporal variations of the network stations and in the area of Friuli in Italy (Bencini et al., 1982). Applications to the estimation of the vertical movements were given by Hein and Kistermann (1981), Kanngieser (1983) and El-Fiky et al. (1997). Papers on horizontal and vertical movements (El-Fiky and Kato, 1999; Wu et al., 2006; Kahle et al., 1995; Chatzinikos et al., 2013), were followed.

Besides the analytical interpolation methods and least squares collocation, other methods that could be applied to generalize any quantity resulting from the temporal variations of network stations are given in the literature, such as pure numerical method of moving average or the method of finite elements, which is the most popular in geodetic literature due to its simplicity in how it is applied.

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