On three-body $B^0 \to D^{*-} D^{(*)0} K^+$ decays and couplings of heavy mesons to light pseudoscalar mesons

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Abstract

We analyze the decay modes $B^0 \to D^{*-} D^0 K^+$ and $B^0 \to D^{*-} D^{*0} K^+$ and, using the available experimental data, we find bounds for the constants $g$ and $h$ describing the strong coupling of heavy mesons to light pseudoscalar mesons. Both the decay channels are dominated by broad $L = 1$ charm resonances; the dominance is effective also in $B^0 \to D^- D^0 K^+$ and $B^0 \to D^- D^{*0} K^+$. 
1 Introduction

Recently, the BaBar Collaboration has observed the three-body $B^0$ decay modes

\begin{align*}
B^0 &\to D^*-D^0K^+ \quad (1) \\
B^0 &\to D^*-D^{*0}K^+ \quad (2)
\end{align*}

measuring the branching fractions [1]:

\begin{align*}
\mathcal{B}(B^0 \to D^*-D^0K^+) &= (2.8 \pm 0.7 \pm 0.5) \times 10^{-3} \quad (3) \\
\mathcal{B}(B^0 \to D^*-D^{*0}K^+) &= (6.8 \pm 1.7 \pm 1.7) \times 10^{-3} \quad (4)
\end{align*}

For the decay channel $B^0 \to D^*-D^0K^+$, a measurement has also been reported by the Belle Collaboration [2]:

\begin{align*}
\mathcal{B}(B^0 \to D^*-D^0K^+) &= (3.2 \pm 0.8 \pm 0.7) \times 10^{-3} \quad (5)
\end{align*}

The results (3), (4) and (5) (although preliminary) represent a significant improvement with respect to the previously available data, obtained by the CLEO Collaboration:

\begin{align*}
\mathcal{B}(B^0 \to D^*-D^0K^+) &= (0.45^{+0.25}_{-0.19} \pm 0.08) \times 10^{-2} \quad (6)
\end{align*}

The purpose of this note is to interpret the observations made so far, using the results (3)-(5) together with the existing datum on the two-body decay $B^0 \to D^*-D^{*+}$ [4]:

\begin{align*}
\mathcal{B}(B^0 \to D^*-D^{*+}) &= (19 \pm 6) \times 10^{-3} \quad (6)
\end{align*}

The interest for $B$ transitions into a pair of $D^{(*)}$ and a Kaon is manifold. It has been proposed to use the modes $B^0(\bar{B}^0) \to D^-D^+K_S$ and $B^0(\bar{B}^0) \to D^{*-}D^{*+}K_S$ (analogous to (2)) to investigate CP violation effects in neutral $B$ decays at the B factories [5]. Such processes are induced at the quark level by the transitions $b \to c\bar{c}s$ and $\bar{b} \to c\bar{c}\bar{s}$ and are Cabibbo-favoured as in the case of $B^0(\bar{B}^0) \to J/\psi K_S$, with a tiny penguin contribution. Studies of the time-dependent Dalitz plot would provide us with information about the weak mixing angles, namely the phase $\beta$ related to the $B^0 - \bar{B}^0$ mixing. In particular, since amplitudes with different strong phases corresponding to various intermediate states contribute to the three-body $B \to D^{(*)}D^{(*)}K_S$ decays, one envisages the possibility of measuring both $\sin(2\beta)$ and $\cos(2\beta)$ by suitable Dalitz plot analyses [4].
Another reason of interest concerns the possibility of carrying out tests of factorization for nonleptonic $B$ decays. It is reasonable to assume that the modes (1) and (2) mainly proceed through two-body intermediate states, such as

$$B^0 \to D^{*-} D_s^X,$$

followed by the strong transition

$$D_s^X \to D^{(*)0} K^+.$$  \hfill (7)  

$D_s^X$ are charmed strange mesons; a typical diagram is depicted in fig.1. In the factorization approximation the amplitude of the process in (7) is expressed as the product of the semileptonic $B^0 \to D^{*-}$ matrix element and the $D_s^X$ current-vacuum matrix element. In the infinite charm quark mass limit, the only contributions with non-vanishing $D_s^X$ current-vacuum matrix elements correspond to the states $D_s^X = D_s^*$ and $D_{s0}$ (with their radial excitations) for $B^0 \to D^{*-} D^0 K^+$, and $D_s^X = D_s^*$, $D_s$ and $D_{s1}^*$ (together with their radial excitations) for $B^0 \to D^{*-} D^{*0} K^+$. $D_{s0}$ and $D_{s1}^*$ are positive parity mesons belonging to the $s^P_\ell = \frac{1}{2}^+$ heavy meson $(\bar{s}c)$ doublet, $s^P_\ell$ being the spin-parity of the light degrees of freedom in the meson. Therefore, the number of independent amplitudes contributing to (1) and (2) is limited, and it is possible to study relations, e.g., with the mode $B^0 \to D^{*-} D_{s}^{*+}$ for which the experimental datum (6) is available. Moreover, one

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Diagram contributing to the decay $B^0 \to D^{*-} D^{(*)0} K^+$. The box represents a weak transition, the dot a strong vertex.}
\end{figure}
can exploit a possible dominance of the positive parity intermediate states to study the features of these (so far) unobserved states.

There is a further reason of interest in the processes (1) and (2). If the main mechanism for the three-body $D^*-D^{(*)0}K^+$ final states is the production of a pair of $D^*-D_s^X$ mesons followed by the strong transition (8), one can use such decay modes to access the couplings of heavy mesons to light pseudoscalar states. For these quantities little experimental information is currently available. The CLEO Collaboration has provided the first determination of the strong coupling constant $g_{D^*D\pi}$ governing the transition $D^{*+} \rightarrow D^0 \pi^+$, using the recent measurement of the total width of the $D^{*+}$ meson \[3\]

$$\Gamma(D^{*+}) = 96 \pm 4 \pm 22 \text{ KeV} \quad (9)$$

together with the experimental branching fraction $B(D^{*+} \rightarrow D^0 \pi^+) = (67.7 \pm 0.5) \times 10^{-2}$ \[4\]. The result for the coupling, defined by the matrix element

$$< D^0(k) \pi^+(q) | D^{*+}(p, \epsilon) >= g_{D^*D\pi} \epsilon \cdot q \quad (10)$$

($\epsilon$ is the $D^*$ polarization vector), is:

$$g_{D^*D\pi} = 17.9 \pm 0.3 \pm 1.9 \quad (11)$$

Rewriting $g_{D^*D\pi}$ in terms of an effective coupling $g_D$:

$$g_{D^*D\pi} = \frac{2\sqrt{m_D m_{D^*}}}{f_\pi} g_D = \frac{2\sqrt{m_D m_{D^*}}}{f_\pi} g \left( 1 + \mathcal{O}\left( \frac{1}{m_c} \right) \right) \quad (12)$$

one translates the result (11) into

$$g_D = 0.59 \pm 0.01 \pm 0.07 \quad (13)$$

In the heavy quark limit the parameter $g$ in (12) describes the strong coupling of charmed mesons as well as of beauty mesons to the members of the octet of light pseudoscalars; therefore, neglecting $SU(3)_F$ breaking effects, this parameter enters some matrix elements governing the transitions in (8). In addition, together with analogous couplings, $g$ represents a basic quantity in the heavy-quark chiral effective theory \[8, 9\], and therefore it is worth searching information about it from all available experimental data, and comparing the results with the predictions that vary in the rather wide range $0.2 < g < 0.7$ \[10\]. This is a purpose of the present note.

In the next section we analyze the decay modes (1) and (2) and discuss how to access the relevant strong couplings. Numerical results follow in section 3. The conclusions are drawn at the end of the note.

\[1\]This result updates the upper bound provided by the ACCMOR Collaboration: $\Gamma(D^{*+}) < 131 \text{ KeV}$ at 90\% c.l. \[3\].
2 Decay modes $B^0 \to D^{*-}D^0K^+$ and $B^0 \to D^{*-}D^{*0}K^+$

Let us consider the processes (1) and (2):

$$B^0(p) \to D^{*-}(p_-, \epsilon_-)D^0(p_D)K^+(q)$$
$$B^0(p) \to D^{*-}(p_-, \epsilon_-)D^{*0}(p_{D^*}, \epsilon)K^+(q)$$

with the momenta $p = m_B v$, $p_- = m_{D^*} v_-$ and $p_{D(*)} = m_{D(*)} w$ expressed in terms of the heavy-meson four-velocities $v$, $v_-$ and $w$. Neglecting penguin contributions, the processes are governed by the effective weak Hamiltonian:

$$H_W = \frac{G_F}{\sqrt{2}} V_{cs} V_{cb}^* a_1 \bar{b} \gamma_\mu (1 - \gamma_5) c \bar{c} \gamma_\mu (1 - \gamma_5) s$$  \hspace{1cm} (14)

where $G_F$ is the Fermi constant, $V_{ij}$ are CKM matrix elements and the parameter $a_1$ reads $a_1 = \left(c_1 + \frac{c_2}{N_c}\right)$, with $c_{1,2}$ short-distance Wilson coefficients and $N_c$ the number of colors. Dalitz plot variables of the decays can be defined:

$$s = (p_{D(*)} + q)^2$$
$$s_- = (p_- + q)^2$$  \hspace{1cm} (15)

and a set of invariant variables, in terms of the four-velocities $v, v_-$ and $w$, can be introduced:

$$v \cdot v_- = \frac{m_B^2 + m_{D^*}^2 - s}{2m_B m_{D^*}}$$
$$v \cdot w = \frac{m_B^2 + m_{D(*)}^2 - s}{2m_B m_{D(*)}}$$
$$v_- \cdot w = \frac{m_B^2 + m_K^2 - s - s_-}{2m_{D(*)}}$$
$$v \cdot q = \frac{s + s_- - m_{D^*}^2 - m_{D(*)}^2}{2m_B}$$
$$v_- \cdot q = \frac{s_- - m_{D^*}^2 - m_K^2}{2m_{D^*}}$$
$$w \cdot q = \frac{s - m_K^2 - m_{D(*)}^2}{2m_{D(*)}}$$  \hspace{1cm} (16)

In the plane $(s, s_-)$ the accessible kinematical region is defined by the conditions

$$\left(m_{D(*)} + m_K\right)^2 \leq s \leq \left(m_B - m_{D^*}\right)^2$$
$$\left(s_-\right)_- \leq s_- \leq \left(s_-\right)_+$$  \hspace{1cm} (17)
and all the couplings in (19) can be expressed in terms of two different coupling constants

$$\lambda^{1/2}(s, m^2_K, m^2_{D_{1s}}) \lambda^{1/2}(s, m^2_B, m^2_{D_{1s}})$$

\(\lambda\) being the triangular function.

We assume that the decays \(B^0 \to D^{*-}D^{(*)0}K^+\) proceed through polar diagrams such as the one depicted in fig. 1, computed adopting the factorization approximation for the weak transition. In the case of \(B^0 \to D^{*-}D^0K^+\), the pole can be either a vector \((J^P = 1^-)\) meson: \(D^+_s\), or a scalar \((J^P = 0^+)\) meson: \(D_{s0}\), with their radial excitations. For the decay \(B^0 \to D^{*-}D^{*0}K^+\), the possible poles are: \(D^{s+} (J^P = 1^-)\), \(D^+_s (J^P = 0^-)\) and \(D^{*+}_{1s} (J^P = 1^+)\) and their radial excitations. Therefore, the calculation of the amplitudes in fig. 1 requires the strong vertices

$$< \bar{D}^0(p_D)K^+(q)|D^+_s(p_{D^*_s}, \epsilon_s)> = g_{D^*_sDK}(\epsilon_s \cdot q)$$

$$< \bar{D}^0(p_D)K^+(q)|D^+_{s0}(p_{D_{s0}})> = g_{D_{s0}DK}$$

$$< \bar{D}^{*0}(p_{D^*}, \epsilon)K^+(q)|D^+_s(p_{D^*_s}, \epsilon_s)> = \frac{i g_{D^*_sDK}}{m_{D^*_s}} \epsilon_{r \theta \phi} \epsilon_{s}^{* \rho} \bar{p}^*_{D^*_s} q^\rho$$

$$< \bar{D}^{*0}(p_{D^*}, \epsilon)K^+(q)|D_{s0}(p_{D_{s0}})> = g_{D^*_{s0}DK}(\epsilon^* \cdot q)$$

$$< \bar{D}^{*0}(p_{D^*}, \epsilon)K^+(q)|D^+_{1s}(p_{D^*_{1s}}, \epsilon_s)> = \frac{g_{D^*_sD^*_sDK}}{m_{D^*_s}} (\epsilon^* \cdot \epsilon_s)(p_{D^*} \cdot q).$$

and analogous matrix elements involving radial \(D^X_s\) resonances. In the heavy quark limit, all the couplings in (19) can be expressed in terms of two different coupling constants \(g\) and \(h\), for negative and positive parity \(D^X_s\) states, respectively. This can be shown considering the effective lagrangian describing the interactions of heavy mesons with the light pseudoscalars. In the limit \(m_Q \to \infty\) the heavy quark in the heavy mesons only acts as a static colour source, and the gluons decouple from the heavy quark spin \(s_Q\), thus implying a \(SU(2N_f)\) spin-flavour symmetry [1, 2]. At the opposite energy scale, for vanishing masses of the up, down and strange quarks, the QCD \(SU(3)_L \times SU(3)_R\) chiral symmetry is spontaneously broken, the Goldstone bosons being the octet of the light pseudoscalar mesons. Both the heavy quark spin-flavour and the chiral symmetries can be realized in a QCD effective lagrangian [8], where the term describing the strong interactions of the heavy negative and positive parity mesons with the light pseudoscalars reads:

$$\mathcal{L}_I = i \ g \ Tr\{H_b \gamma_\mu \gamma_5 A^\mu_{ba} \bar{H}_a\} + [ i \ h \ Tr\{H_b \gamma_\mu \gamma_5 A^\mu_{ba} \bar{S}_a\} + h.c.].$$

(20)
The fields $H_a$ in (20) describe the negative parity $J^P = (0^-, 1^-) \bar{q}Q$ meson doublet, with $s^{P}_\ell = \frac{1}{2}^-$:

$$H_a = \frac{(1 + \gamma^5)}{2} \left[ P_{a\mu} \gamma^\mu - P_a \gamma_5 \right],$$

(21)

the operators $P_{a\mu}$ and $P_a$ respectively annihilating the $1^-$ and $0^-$ mesons of four-velocity $v$ ($a = u, d, s$ is a light flavour index). Analogously, the fields $S_a$ describe the positive parity states, with $s^{P}_\ell = \frac{1}{2}^+$:

$$S_a = \frac{(1 + \gamma^5)}{2} \left[ P_{a\mu} \gamma^\mu \gamma_5 - P_a' \right].$$

(22)

The octet of the light pseudoscalar mesons is included in (20) through the field $\xi = e^{iMf_{\pi}/2}$, with

$$M = \begin{pmatrix} \sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}} \rho^0 + \sqrt{\frac{1}{6}} \eta & K^0 \\ K^- & K_0^- & -\sqrt{\frac{2}{3}} \eta \end{pmatrix}$$

and $f_\pi = 131$ MeV. Finally, the operator $A$ in (20) reads

$$A_{\mu ba} = \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right)_{ba}. $$

(24)

From the definitions in (19) and from eq.(20) it is straightforward to derive the relations

$$g_{D^*DK} = 2\sqrt{m_{D^*}m_D} \frac{g}{f_K}$$

$$g_{D_oDK} = -\sqrt{m_{D_o}m_D} \frac{m_{D_o}^2 - m_D^2}{m_{D_o}} \frac{h}{f_K}$$

$$g_{D^*D_oK} = 2m_{D^*} \frac{g}{f_K}$$

$$g_{D^*D_sK} = 2\sqrt{m_{D^*}m_{D_s}} \frac{g}{f_K}$$

$$g_{D_{s1}D^*K} = -2\sqrt{m_{D_{s1}}m_{D^*}} \frac{h}{f_K},$$

(25)

where we have kept some $SU(3)$ flavor breaking terms in the masses of the $D_s^X$ mesons and in the leptonic constant $f_K$.

On the other hand, in the factorization approximation the calculation of the weak transition (3) requires the semileptonic $B^0 \to D^*-$ matrix element and the decays constant of the poles $D_s^X$. In the heavy quark limit, the former is given in terms of the Isgur-Wise function $\xi$:

$$< D^*(v_-, \epsilon_-)|V^\mu - A^\mu|B(v) > = \sqrt{m_Bm_{D^*}} \xi(v \cdot v_-) \epsilon_{-\alpha}^*$$

$$\left( -i\epsilon^{\alpha\lambda\mu} v_{\lambda}v_{-\mu} - (1 + v \cdot v_-)g^{\alpha\mu} + v^\alpha v^\mu \right).$$

(26)
while the decay constants are defined by

\[<D_s^+(p_{D_s})|\bar{c}\gamma^{\mu}(1-\gamma_5)s|0> = if_{D_s}p_{D_s}^\mu_s\]
\[<D_s^{*+}(p_{D_s^*},\epsilon_s)|\bar{c}\gamma^{\mu}(1-\gamma_5)s|0> = f_{D_s^*}m_{D_s^*}\epsilon_s^{\mu_s}\]
\[<D_s^{0+}(p_{D_s^0})|\bar{c}\gamma^{\mu}(1-\gamma_5)s|0> = if_{D_s^0}p_{D_s^0}^\mu_s\]
\[<D_{s1}^{*+}(p_{D_{s1}^*},\epsilon_s)|\bar{c}\gamma^{\mu}(1-\gamma_5)s|0> = f_{D_{s1}^*}m_{D_{s1}^*}\epsilon_s^{\mu_s} .\]

In the heavy quark limit, the leptonic constants \(f_{D_s}\) and \(f_{D_s^*}\), as well as \(f_{D_s^0}\) and \(f_{D_{s1}^*}\), are simply related.

It is now straightforward to work out the amplitude of \(B^0 \to D^{*-}D^0 K^+\) proceeding via the \(D_s^*\) intermediate state:

\[
\mathcal{A}_1 = \frac{i\mathcal{K} f_{D_s^*} m_{D_s^*} g_{D_s^*DK} v}{s - m_{D_s^*}^2 + im_{D_s^*} \Gamma_{D_s^*}} \xi (v \cdot v_{-}) \epsilon_{-}^{\mu} \epsilon_{-}^{\nu} - q^{\mu} + \frac{(m_B v - m_{D^*} v_{-}) \cdot q (m_B v - m_{D^*} v_{-})^{\mu}}{m_{D^*}} \left\{ i \epsilon_{\mu \nu \beta} v_{+}^{\alpha} v^{\beta} - g_{\mu \nu} (1 + v \cdot v_{-}) + v_{\nu} (v_{-})_{\mu} \right\} . \]  

(28)

with \(\mathcal{K} = \frac{G_F V_{cs} V_{cs}^* a_1}{\sqrt{2}} \sqrt{m_B m_{D^*}}\) and \(\Gamma_{D_s^*}\) the \(D_s^*\) decay width. Analogously, the amplitude \(\mathcal{A}_2\) relative to the \(D_{s0}\) contribution to \(B^0 \to D^{*-}D^0 K^+\) reads:

\[
\mathcal{A}_2 = -\frac{\mathcal{K} f_{D_s^0} m_{D_s^0} g_{D_s^0 DK} v}{s - m_{D_s^0}^2 + im_{D_s^0} \Gamma_{D_s^0}} \xi (v \cdot v_{-}) \epsilon_{-}^{\mu} \epsilon_{-}^{\nu} - \frac{(m_B v - m_{D^*} v_{-}) \nu (1 + v \cdot v_{-}) + v_{\nu} (m_B v - m_{D^*} v_{-}) \cdot v_{-}}{m_{D_s^0}} \right\} . \]

(29)

As for \(B^0 \to D^{*-}D^{*0} K^+\), the amplitudes \(\mathcal{A}_1^*, \mathcal{A}_2^*, \mathcal{A}_3^*, \mathcal{A}_4^*, \mathcal{A}_5^*\) corresponding to the \(D_s^*, D_s, D_{s1}^*\) intermediate states are given by:

\[
\mathcal{A}_1^* = \frac{\mathcal{K} f_{D_s^*} m_{D_s^*} g_{D_s^* DK^*} v}{s - m_{D_s^*}^2 + im_{D_s^*} \Gamma_{D_s^*}} \xi (v \cdot v_{-}) \epsilon_{-}^{\mu} \epsilon_{-}^{\nu} - \frac{m_{D_s^*} w_{\phi} q_{\psi}}{m_{D_s^*}} \left\{ i \epsilon_{\mu \nu \beta} v_{+}^{\alpha} v^{\beta} - g_{\mu \nu} (1 + v \cdot v_{-}) + v_{\nu} (v_{-})_{\mu} \right\} ; \]

(30)

\[
\mathcal{A}_2^* = \frac{\mathcal{K} f_{D_s^*} g_{D^* DK^*} v}{s - m_{D_s^*}^2 + im_{D_s^*} \Gamma_{D_s^*}} \xi (v \cdot v_{-}) (m_B + m_{D^*}) (\epsilon \cdot q) (\epsilon^{*} \cdot v) ; \]

(31)

\[
\mathcal{A}_3^* = \frac{i \mathcal{K} f_{D_{s1}^*} m_{D_{s1}^*} g_{D_{s1}^* DK^*} v}{s - m_{D_{s1}^*}^2 + im_{D_{s1}^*} \Gamma_{D_{s1}^*}} \xi (v \cdot v_{-}) (w \cdot q) \epsilon_{-}^{\mu} \epsilon_{-}^{\nu} - \frac{m_{D_{s1}^*} w_{\phi} q_{\psi}}{m_{D_{s1}^*}} \left\{ - i \epsilon_{\tau \nu \beta} v_{+}^{\alpha} v^{\beta} + g_{\tau \nu} (1 + v \cdot v_{-}) - v_{\nu} (v_{-})_{\tau} \right\} - \frac{(1 + v \cdot v_{-})(m_B v - m_{D^*} v_{-})\nu (m_B v - m_{D^*} v_{-}) \cdot v_{-}}{m_{D_{s1}^*}} \right\} . \]

8
Expressions analogous to eqs. (28)-(32), with appropriate masses, widths, leptonic constants and strong couplings, hold for the contributions of the radial excitations of negative and positive parity mesons. Such contributions are suppressed by the small numerical values of the leptonic constants and of the effective couplings. This can be shown, for example, using the relativistic constituent quark model in ref. [13], where one obtains: $f_{D'_s}/f_{D_s} \simeq 0.73$, $D'_s$ being the first radial excitation of $D_s$. In the same model, using the method described in [14], one obtains: $g_{D'_sD^*_{s}K}/g_{D_sD^*_{s}K} \simeq 0.32$. Analogous reductions occur for positive parity states. A further suppression is due to the large decay width of the excited states. Therefore, one can conclude that the first radial excitations contribute to the amplitudes of the processes (1) and (2) by less than 15\% with respect to the contribution of the corresponding low-lying states, an uncertainty that can be included in the error affecting the effective couplings we are studying in this note.

3 Numerical analysis and discussion

On the basis of the above considerations, we write down the widths of the decay modes $B^0 \rightarrow D^*-D^0 K^+$ and $B^0 \rightarrow D^{*-}D^{*0} K^+$ as follows:

$$\Gamma(B^0 \rightarrow D^*-D^{(*)0}K^+) = \int_{m_{D^*}/m_{D}}^{m_{D^*}/m_{D}} ds \int_{(s_0)^+}^{(s_0)^-} ds_- \frac{d\Gamma}{ds ds_-},$$

with $\frac{d\Gamma}{ds ds_-}(B^0 \rightarrow D^*-D^{(*)0}K^+) = \frac{1}{(2\pi)^3} \frac{1}{32m_B^2} |A|^2$ and

$$A(B^0 \rightarrow D^*-D^0 K^+) = \sum_{i=1,2} A_i,$$

$$A(B^0 \rightarrow D^{*-}D^{*0} K^+) = \sum_{i=1,2,3} A_i^*.$$

The decay widths depend on the effective couplings $g$ and $h$. They also depend on SM parameters, such as $\frac{G_F}{\sqrt{2}} V_{cs} V_{cb}^*$, on the leptonic constants $f_{D_s}, \ldots$, on the Wilson coefficients $c_{1,2}$ as well as on the Isgur-Wise form factor $\xi$. All such parameters appear in the same combination in the factorized amplitude of the two-body decay in (3):

$$A(B^0 \rightarrow D^*-D^*_s) = \mathcal{K} f_{D_s} m_{D_s} \xi (v \cdot v_-) \epsilon^\alpha \epsilon^{\mu}_{-} [i \epsilon_{\rho \lambda \mu} v^\lambda v^\rho + g_{\alpha \mu} (1 + v \cdot v_-) - v_\alpha v_\nu] .$$

Therefore, in the ratios

$$R_D = \frac{B(B^0 \rightarrow D^*-D^0 K^+)}{B(B^0 \rightarrow D^{*-}D^*_s)},$$
\[ R_{D^*} = \frac{\mathcal{B}(B^0 \to D^{*-} D^{*0} K^+)}{\mathcal{B}(B^0 \to D^{*-} D_s^+)} \]  

(36)

one gets rid of the dependences on \( V_{cb}, V_{cs} \) and \( a_1 \). As for the Isgur-Wise function, the linear form

\[ \xi(v \cdot v_\perp) = 1 - \hat{\rho}^2 (v \cdot v_\perp - 1) \]  

(37)

is well suited due to the narrow range of momentum transfer involved in the decays we are considering. A strong correlation has been observed between the measured values of \( V_{cb} \) and the slope parameter \( \hat{\rho}^2 \) in the analyses of the semileptonic \( B^0 \to D^{*-} \ell \nu \) decay spectrum and in the studies of two-body \( B \) transitions in the factorization approximation [13, 14]. However, in the ratios (36) such a correlation is essentially removed, and similar results are obtained varying \( \hat{\rho}^2 \) in the range \( \hat{\rho}^2 = 1.38 - 1.54 \).

Concerning the leptonic constants, we use \( f_{D_s} f_{D_s^*} = 1 \) and \( f_{D_s} f_{D_s^{*+}} = \frac{f_{D_s^{*+}}}{f_{D_s^*}} = 1 \). The former ratio exactly holds in the infinite charm quark limit. The latter one allows us to reduce the number of input parameters, since a deviation from unity can be reabsorbed in the numerical result for the parameter \( h \). For the masses of the excited charm mesons, we use \( m_{D_s^{*+}} = m_{D_{s1}} = m_{D_s} + \Delta \), with \( \Delta = 0.5 \text{GeV} \) [17].

The final set of input quantities involves the decay widths of the intermediate states. One can neglect the \( D_s \) width (\( \Gamma(D_s) = 1.33 \pm 0.03 \times 10^{-9} \text{MeV} \) [4]). As for \( D_s^* \), as well as for the positive parity charmed states, the widths depend on the effective couplings \( g \) and \( h \). Using the experimental branching fractions \( \mathcal{B}(D_s^{*+} \to D_s^{+} \pi^0) \) and \( \mathcal{B}(D_s^{*+} \to D_s^+ \gamma) \), together with the central value of \( g \) in [13], one obtains \( \Gamma(D_s^*) = 1.03 \text{MeV} \); consequently, we use the expression \( \Gamma(D_s^*) = 1.03 \left( \frac{g}{0.59} \right)^2 \text{MeV} \) in the analysis for constraining the strong coupling \( g \). Moreover, assuming that the decay widths of the positive parity states are saturated by two-body transitions, one gets \( \Gamma(D_{s0}) = 180 \left( \frac{h}{0.56} \right)^2 \text{MeV} \) and \( \Gamma(D_{s1}) = 165 \left( \frac{h}{0.56} \right)^2 \text{MeV} \) [17].

Solutions of the equations

\[
R_D = R_D |_{\text{exp}} = 0.15 \pm 0.07 \\
R_{D^*} = R_{D^*} |_{\text{exp}} = 0.36 \pm 0.17 \]  

(38)

in the variables \((g, h)\) are found, considering the central values in (38), for \((g, h) = (0.05, -0.59)\) and \((g, h) = (0.0, +0.60)\). The solutions for \( g \) are smaller than the result in [13], while the results for \( h \) are compatible with the theoretical expectations \( h = -0.52 \pm 0.17 \) and \( h = -0.56 \pm 0.28 \) [17]. However, before drawing conclusions from these results, it is worth analyzing the 1– and 2–σ regions in the plane \((g, h)\), obtained
considering the experimental errors in (38). Such regions are depicted in fig.2. They are rather tightly bounded along the $h$ direction, while the dependence on $g$ is mild and the range of the allowed values of $g$ extends over all the values between $g = 0$ and the CLEO result eq.(13). Along the $h$ axis, the allowed regions correspond to $|h| = 0.6 \pm 0.2$. The conclusion is that the main contributions to the processes (1) and (2) are related not to the $0^−$ and $1^−$, $D_s$ and $D_s^*$ intermediate states, but to the positive parity $0^+$ and $1^+$ states $D_{s0}$ and $D_{s1}^*$, since the amplitudes display a minor sensitivity to the coupling of the negative parity intermediate states. This is interesting from the phenomenological point of view, since it implies that three-body $B^0 \rightarrow D^{*−}D^0K^+$ and $B^0 \rightarrow D^{*−}D^{*0}K^+$ decay modes are well suited for separately studying the properties of the (so far unobserved) $D_{s0}$ and $D_{s1}^*$ resonances [5]. The analysis can be done by studying the Dalitz plot of the three-body decay. For the mode $B^0 \rightarrow D^{*−}D^0K^+$ the expected differential decay width is depicted in fig.3. It has been obtained for $g = 0.5$, $h = −0.6$, $a_1 = 1.1$, together with $V_{cb} = 0.04$ and $V_{cs} = 0.974$ [4]. As for $f_{D_s}$, we use the value $f_{D_s} = 240$ MeV obtained from the fit of (6); it is compatible, within the errors, with the value reported by [4]: $f_{D_s} = 280 \pm 19 \pm 28 \pm 34$ MeV. The distribution for $g = 0.3$ is completely similar.

One notices that the main variation of the differential decay distribution occurs along the direction of the invariant $D^0K^+$ mass, a feature related to the unique topology of

Figure 2: $1 − \sigma$ (continuous lines) and $2 − \sigma$ (dashed lines) region in the $(g, h)$ plane, as obtained from the ratios $R_D$ and $R_{D^*}$ in (38). The vertical lines represent the result (13); the shaded area corresponds to the region excluded by the upper bound $g < 0.76$ from ref.[7].
the (Cabibbo and color allowed) amplitudes governing the mode (4). The Dalitz plot for $B^0 \to D^*-D^{*0}K^+$, depicted in fig.4, shows similar features.

Figure 3: Differential decay width $\frac{d\Gamma}{ds\,ds_-}$ (left) and Dalitz plot (right) of the decay $B^0 \to D^*-D^0K^+$. Units of $s$ and $s_-$ are GeV$^2$.

Figure 4: Differential decay width (left) and Dalitz plot (right) of the decay $B^0 \to D^*-D^{*0}K^+$. Units are as in figure 3.

The prominent role of the intermediate states $D_{s0}$ and $D_{s1}^*$ makes the processes (4) and (3) particularly promising for the analysis of broad orbital excitations of the $c\bar{s}$ meson system. It is worth noticing, however, that also other three-body $B$ decays can be well
suited for such an investigation. Examples are $B^0 \to D^- D^0 K^+$ and $B^0 \to D^- D^{*0} K^+$, for which no experimental results are available. Since the matrix element of $B^0 \to D^-$ can be related to $B^0 \to D^{*-}$ in the heavy quark limit, it is possible to determine, in the scheme described in the previous section, the properties of the channels $B^0 \to D^- D^{(*)0} K^+$. One predicts:

\[
\frac{\mathcal{B}(B^0 \to D^- D^0 K^+)}{\mathcal{B}(B^0 \to D^{*-} D^0 K^+)} = 2.11
\]

\[
\frac{\mathcal{B}(B^0 \to D^- D^{*0} K^+)}{\mathcal{B}(B^0 \to D^{*-} D^{*0} K^+)} = 0.27
\]  

(39)

which imply, considering the experimental data in (3)-(4):

\[
\mathcal{B}(B^0 \to D^- D^0 K^+) = (6.3 \pm 1.8) \times 10^{-3}
\]

\[
\mathcal{B}(B^0 \to D^- D^{*0} K^+) = (1.8 \pm 0.7) \times 10^{-3}
\]  

(40)

in a range accessible to current experiments. The expected decay distributions and the Dalitz plots, depicted in fig.5, are similar to those of the modes (3) and (4), with features that it will be interesting to experimentally investigate.

We conclude this section with a comment on the two main theoretical uncertainties in our approach, the use of the heavy quark limit both for beauty and charm quarks, and the factorization assumed for the nonleptonic matrix elements. The two uncertainties are correlated, and a quantitative assessment of their role is not a trivial task. If we consider them separately, we can presume that several $\frac{1}{m_Q}$ corrections are compensated in the ratios used as the basis of our analysis. As for factorization, the matrix elements governing the decays considered in this note are different from the matrix elements for which factorization has been proved in the infinite $b$ mass limit [18]. Nevertheless, the study of various processes of the type considered here, i.e. color allowed B transitions to charm mesons, shows that factorization reproduces the available data within their current errors [10]. Our analysis can be considered as a further test of factorization; experimental measurements will be helpful in shedding light on the size and the type of possible deviations.

4 Conclusions

The analysis of the decays $B^0 \to D^{*-} D^0 K^+$ and $B^0 \to D^{*-} D^{*0} K^+$ shows that they mainly proceed through positive parity intermediate states, $D_{s0}$ and $D_{s1}^*$. Other contributions are
Figure 5: Differential decay width (left) and Dalitz plot (right) of the transitions $B^0 \to D^- D^0 K^+$ (up) and $B^0 \to D^- D^{*0} K^+$ (down). Units are as in fig.3.

less significant, so that such three-body $B^0$ transitions appear to be well suited for studying the features of the low-lying orbital excitations of the ($\bar{s}c$) meson system. Currently available experimental data allow us to constrain the strong coupling between such orbital excitations, the negative parity charmed mesons and the Kaon, in the region $|h| = 0.6 \pm 0.2$, close to the expected values. An improvement in the accuracy of the measurements would further constrain this parameter. On the other hand, the coupling $g$ is found in a range which extends from zero to the CLEO measurement \([13]\).

The Dalitz plots relative to $B^0 \to D^{*-} D^0 K^+$ and $B^0 \to D^{*-} D^{*0} K^+$ are expected to display peculiar features, namely the main dependence on the invariant $D^{(*)0} K^+$ mass, that can be experimentally tested. The investigations of other modes, $B^0 \to D^- D^0 K^+$ and $B^0 \to D^- D^{*0} K^+$, can also provide us with information about positive parity charm states; their expected decay rates are accessible to current experiments.
References

[1] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0107056.

[2] H. Tajima [BELLE Collaboration], Talk given at the Lepton-Photon 01 Conference, XX International Symposium on Lepton and Photon Interactions at High Energies, Rome, Italy, 23rd-28th July 2001.

[3] CLEO Collaboration report CLEO CONF 97-26, EPS97 337.

[4] D. E. Groom et al. [Particle Data Group Collab.], Eur. Phys. J. C 15 (2000) 1.

[5] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Lett. B 425 (1998) 375 [Erratum-ibid. B 433 (1998) 441]. P. Colangelo, F. De Fazio, G. Nardulli, N. Paver and Riazuddin, Phys. Rev. D 60 (1999) 033002. T. E. Browder, A. Datta, P. J. O’Donnell and S. Pakvasa, Phys. Rev. D 61 (2000) 054009.

[6] A. Anastassov et al. [CLEO Collaboration], Phys. Rev. D 65 (2002) 032003.

[7] S. Barlag et al. [ACCMOR Collaboration], Phys. Lett. B 278 (1992) 480.

[8] M. B. Wise, Phys. Rev. D 45 (1992) 2188. G. Burdman and J. F. Donoghue, Phys. Lett. B 280 (1992) 287. T. M. Yan et al., Phys. Rev. D 46 (1992) 1148 [Erratum-ibid. D 55 (1992) 5851]. A.F.Falk and M.Luke, Phys. Lett. B 292 (1992) 119.

[9] For a review see: R. Casalbuoni et al., Phys. Rept. 281 (1997) 145.

[10] Reviews of recent calculations of the coupling $g$ can be found in: P. Colangelo and A. Khodjamirian, in “At the frontier of Particle Physics - Handbook of QCD”, edited by M. A. Shifman, (World Scientific, 2001) page 1495 (arXiv:hep-ph/0010175), and in D. Becirevic and A. L. Yaouanc, JHEP 9903 (1999) 021.

[11] M.B. Voloshin and M.A. Shifman, Sov. J. Nucl. Phys. 45 (1987) 292; 47 (1988) 511. N. Isgur and M.B. Wise, Phys. Lett. B 232 (1989) 113; B 237 (1990) 527. J.D. Bjorken, in ”Results and Perspectives in Particle Physics”, Proceedings of the 4th Rencontres de Physique, La Thuile, Italy, 1990, edited by M.Greco, Ed. Frontieres, France (1990) page 583. H. Georgi, Phys. Lett. B 240 (1990) 447.

[12] For a recent review see: F. De Fazio, in ”At the Frontier of Particle Physics - Handbook of QCD”, edited by M. A. Shifman, (World Scientific, 2001), page 1671 (arXiv:hep-ph/0010007).
[13] P. Colangelo, G. Nardulli and M. Pietroni, Phys. Rev. D 43 (1991) 3002.

[14] P. Colangelo, F. De Fazio and G. Nardulli, Phys. Lett. B 334 (1994) 175.

[15] Updated results on semileptonic $b \to c$ transitions can be found at the LEP $V_{cb}$ Working Group site: [http://lepvcb.web.cern.ch/LEPVCB/](http://lepvcb.web.cern.ch/LEPVCB/).

[16] Z. Luo and J. L. Rosner, Phys. Rev. D 64 (2001) 094001.

[17] P. Colangelo, F. De Fazio, G. Nardulli, N. Di Bartolomeo and R. Gatto, Phys. Rev. D 52 (1995) 6422. P. Colangelo and F. De Fazio, Eur. Phys. J. C 4 (1998) 503.

[18] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 591 (2000) 313.