I. INTRODUCTION

The study to search for decision laws on the decision table by assessing the measures of decision laws as well as incremental approaches, determining decision laws ... has been studied by many groups of authors, such as in [7], [8], ... On the other hand, when the decision table is expanded into a decision block, then the study, proposing a model and algorithm to detect decision laws on the decision block has been studied by the authors as in [4], [5], [6]. However, the proposed models and algorithms when smoothing and roughening the values of index attributes on the decision block have not been studied until now. The purpose of this paper is to study the some properties about smoothing, roughening the values of the condition index attribute or decision index attribute on the decision block and on the slice of the decision block. From the results found of the smoothing, roughening the condition equivalence class or decision equivalence class partial pullulate or pullulate on the slice then the incremental calculation of the support matrices on the slice will be simpler and therefore faster than recalculating these matrices when smoothing, roughening the values of the condition index attribute or decision index attribute.

II - THE BASIC CONCEPT

II.1 The block, slice of the block

Definition II.1 [1]

Let \( R = (id; A_1, A_2,..., A_n) \) is a finite set of elements, where id is non-empty finite index set, \( A_i \) (i=1..n) is the attribute. Each attribute \( A_i \) (i=1..n) there is a corresponding value domain \( \text{dom}(A_i) \). A block \( r \) on \( R \), denoted \( r(R) \) consists of a finite number of elements that each element is a family of mappings from the index set id to the value domain of the attributes \( A_i \) (i=1..n).

\[ t \in r(R) \Leftrightarrow t = \{ t_i : id \rightarrow \text{dom}(A_i) \} \]  

The block is denoted by \( r(R) \) or \( r(id; A_1, A_2,..., A_n) \), sometimes without fear of confusion we simply denoted \( r \).

Definition II.2 [2],[3]

Let \( R = (id; A_1, A_2,..., A_n) \), \( r(R) \) is a block over \( R \). For each \( x \in id \) we denoted \( r(R_x) \) is a block with \( R_x = \{ x \} ; A_1, A_2,..., A_n \) such that:

\[ t \in r(R_x) \Leftrightarrow t = \{ t_i = t_x ^i \} \]  

where \( t_i ^x (x_j) = t ^x (x_j) \), i =1..n.

Then \( r(R_x) \) is called a slice of the block \( r(R) \) at point \( x \), sometimes we denoted \( r_x \).

II.2 Information block

DefinitionII.3[4]: Let block scheme \( R = (id; A_1, A_2,..., A_n) \), \( r \) is a block over \( R \). Then, the information block is a tuples of four elements \( IB = (U, A, V, f) \) with Uis a set of objects of \( r \) called space objects, \( A = \bigcup_{i=1}^{n} \text{id}^{(i)} \) is the set of index attributes of the objects, \( V = \bigcup_{x \in A} V^{(x)} \), \( V^{(x)} \) is the set of values of the objects corresponding to the index attribute \( x^{(0)}, fis \) an information function \( UA \rightarrow V \) satisfy:

\[ \forall u \in U, \forall x^{(0)} \in A \ we \ have \ f(u, x^{(0)}) \in V^{(x^{(0)})}. \]

We call \( f(u, x^{(0)}) \) is the value of the object \( u \) at the index attribute \( x^{(0)} \).

If \( V \) contains missing values in at least one index attribute \( x^{(0)} \) then we call \( IB \) is inadequate information block. In contrast IB is a complete information block, or simply IB is an information block.

DefinitionII.4[4]: Let block scheme \( R = (id; A_1, A_2,..., A_n) \), \( r \) is a block over \( R \), \( r \) is the slice of the block \( r \) at the point \( x \in id \). Then the slice of the information block at \( x \) is a tuples of four elements \( IB_x = (U, A, V, f_x) \) with \( U \) is a set of objects of \( r \) called space objects, \( A_x = \bigcup_{x \in A} x^{(0)} \) is the set of the index attributes...
attributes of the object on the slice at \( x \), \( V_x = \bigcup_{i=1}^{k} V_{x(i)} \), \( V_{x(i)} \) is the set of values of the objects corresponding to the index attribute \( x^{(i)} \), \( f_x \) is an information function \( U \backslash A_i = V_x \), \( f_x \) satisfies: \( \forall u \in U \backslash A_i, \forall x^{(i)} \in A_i \), we have: \( f(u, x^{(i)}) \in V_{x(i)} \).

II.3 Relationships are indistinguishable

**Definition II.5 [5]**

Let information block \( IB = (U, A, V, f) \). Then for each index attribute set \( P \subseteq A \), we define an equivalence relation, \( \text{sign} \{ u \} \) called non-discriminatory relations:

\[
\text{IND}(P) = \left\{ v \in U \mid (u,v) \in P \right\}
\]

and called non-discriminatory relations:

From the definition we have:

\[
\text{IND}(P) = \bigcap_{x \in P} \text{IND}(x(i)).
\]

Relation \( \text{IND}(P) \) divides \( U \) into equivalence classes, constitutes a subdivision of \( U \) sign \( U/\text{IND}(P) \) or simply \( U/P \).

With each \( u \in U \), the equivalence class contains \( u \) in relation \( \text{IND}(P) \), sign \( [u] \) is defined as follows:

\[
[u]_P = \{ v \in U \mid (u,v) \in \text{IND}(P) \}.
\]

By this definition we see: two elements \( u, v \in U \) belonging to the same equivalence class if and only if they have the same value on every index attribute in \( P \).

**Definition II.6 [5]**

Let information block \( IB = (U, A, V, f) \), \( P \), \( Q \subseteq A \) is the set of index attributes, \( U/P = \{ P_1, P_2, \ldots, P_n \} \), \( U/Q = \{ Q_1, Q_2, \ldots, Q_m \} \) is the partition generated by \( P \), \( Q \) respectively. Then we say partition by \( Q \) is more coarse than partition by \( P \), or partition by \( Q \) is smoother than partition by \( P \) if and only if:

\[
\forall P_i \in U/P, \exists Q_j \in U/Q: P_i \subseteq Q_j, \quad i = 1..m, \quad j = 1..n.
\]

II.4 Decision block

**Definition II.7 [5]**

Let information block \( IB = (U, A, V, f) \) with \( U \) is the space of objects, \( A = \bigcup_{x \in \text{IND}(A)} A = \bigcup_{i=1}^{k} x^{(i)} \), then information block \( IB \) is called the decision block and denoted by \( DB = (U, C, A, D, V, f) \), with \( C \) is the set of conditional index attributes and \( D \) is the set of decision index attributes.

From the definition of the decision block, we see: \( C \cap D = A \), \( C \cap D = \emptyset \), we can denote the decision block simply by: \( DB = (U, C, D) \).

**Definition II.8 [5]**: Let decision block \( DB = (U, C, D, V, f) \), with \( C \) is the set of conditional index attributes and \( D \) is the set of decision index attributes. Then the slice of the block decides at \( x (x \in \text{IND}(D)) \) is a tuple of four elements \( DB_x = (U, C \cap D, V_x, f_x) \) with \( U \) is the set of objects of \( r \) called the space of objects

\[
C_x = \bigcup_{i=1}^{k} x^{(i)}, \quad A_x = \bigcup_{x \in \text{IND}(D)} A^{(i)},
\]

\[
V_x = \bigcup_{i=1}^{k} \{ V_x(i) \} \text{ of values of objects corresponding to the index attribute } x^{(i)}, \quad f_x \text{ is an information function } U \backslash A_x \to V_x \text{ satisfying } \forall u \in U, \forall x^{(i)} \in A_x \text{ we have: } f(u, x^{(i)}) \in V_x(i).
\]

**Comment:**

Let decision block \( DB = (U, C, D, V, f) \), then, if \( id = \{ x \} \), the decision block \( DB \) degenerate into the decision table as known.

When studying the decision block, people want to find the decisive laws from there. In these decision laws, the conditional part corresponds to the conditional index attribute, the conclusions will correspond to the decision index attributes.

The decision laws found in the decision block are divided into two categories:

i) The laws are correct on the block.

ii) The laws are correct on each particular slice of the block.

II.5 The decision laws

**Definition II.9 [5]**

Let decision block \( DB = (U, C, D) \) with \( U \) is the space of objects:

\[
C = D \cup \bigcup_{i=1}^{k} x^{(i)}, \quad D = \bigcup_{i=1}^{k} x^{(i)}, \quad D' = \bigcup_{i=k+1}^{l} x^{(i)}
\]

Then:

\[
U/C = \{ C_1, C_2, \ldots, C_{l_1} \}, \quad U/C' = \{ C_1, C_2, \ldots, C_{l_1} \},
\]

\[
U/D = \{ D_1, D_2, \ldots, D_{l_2} \}, \quad U/D' = \{ D_1, D_2, \ldots, D_{l_2} \},
\]

correspondingly, the partitions are generated by \( C, C', D \), \( D' \). A decision law on a block is denoted by:

\[
C_i \Rightarrow D_j, \quad i = 1..m, \quad j = 1..k,
\]

and on the slice at point \( x \) is denoted by:

\[
C_i \Rightarrow D_j, \quad i = 1..m, \quad j = 1..k.
\]

**Proposition II.1 [5]**

Let decision block \( DB = (U, C, D) \) with \( U \) is the space of objects:

\[
C = \bigcup_{i=1}^{k} x^{(i)}, \quad D = \bigcup_{i=1}^{k} x^{(i)}, \quad C' = \bigcup_{i=k+1}^{l} x^{(i)}, \quad D' = \bigcup_{i=k+1}^{l} x^{(i)}, \quad x \ni d.
\]

Then:

\[
U/C = \{ C_1, C_2, \ldots, C_{l_1} \}, \quad U/C' = \{ C_1, C_2, \ldots, C_{l_1} \},
\]

\[
U/D = \{ D_1, D_2, \ldots, D_{l_2} \}, \quad U/D' = \{ D_1, D_2, \ldots, D_{l_2} \},
\]

Then: \( \forall C_i \in U/C, \forall D_j \in U/D \) we have:

\[
C_i = \bigcup_{x \ni d} C_{x,p}, \quad D_j = \bigcup_{x \ni d} D_{x,q} \text{ with } p \ni \{ 1,2,\ldots,t_x \}, q \ni \{ 1,2,\ldots,t_x \}, x \ni d.
\]

**Definition II.10 [5]**

Let decision block \( DB = (U, C, D) \), \( C_i \in U/C, \quad D_j \in U/D, \quad C_{x,p} \in U/C, \quad D_{x,q} \in U/D, \quad x \ni d \text{ then, support, accuracy and coverage of decision law } C_i \Rightarrow D_j \text{ on the block are:}

- Support: \( \text{Sup}(C_i, D_j) = |C_i \cap D_j| \),
Let decision block $DB=(U, C, D)$ where $C, D \subseteq \mathcal{U}$ is the space of objects, $a \in \mathcal{C} \cap \mathcal{D}$, $V$ is the set of existing values of the index attribute $a$. Suppose $Z=\{x_{q}\in \mathcal{U} | f(x_{q}, a) = z\}$ is the set of objects whose $z$ value is on the index attribute $a$. If $Z$ is partitioned into two sets $W$ and $Y$ such that: $Z=W \cup Y$, $W \cap Y=\emptyset$ with $W=\{x_{q}\in \mathcal{U} | f(x_{q}, a) = w, w \in \mathcal{V}_{w}\}$, $Y=\{x_{q}\in \mathcal{U} | f(x_{q}, a) = y, y \in \mathcal{V}_{y}\}$; then we say the $z$ value of the index attribute $a$ is smoothed to two new values $w$ and $y$.

**Theorem II.2[6]**

Let decision block $DB=(U, C, D, V, f)$, with $U$ is the space of objects, $a \in \mathcal{C} \cap \mathcal{D}$, $V$ is the set of existing values of the index attribute $a$. Then, two equivalent classes $E_{p}$, $E_{q}$ ($E_{p}, E_{q} \subseteq \mathcal{U} | f(x_{a}) = f(x_{a})$) are made rough into new equivalent class $E$ and only if for all $a \neq a: f(E_{p}, a) = f(E_{q}, a)$.

**Theorem II.3[6]**

Let decision block $DB=(U, C, D, V, f)$, with $U$ is the space of objects, $a \in \mathcal{C} \cap \mathcal{D}$, $V$ is the set of existing values of the index attribute $a$. Then, equivalent class $E_{a}$ ($E_{a} \subseteq \mathcal{U} | E \subseteq \mathcal{E}$) smoothed into two new equivalent classes $E_{p}$, $E_{q}$ and only if we can put: $f(E_{p}, a) = f(E_{q}, a) = y$ and $E_{p} \cup E_{q} = E$.

**III. RESEARCH RESULTS**
III.1 Smoothing, roughening the conditional equivalence classes on the decision block and on the slice.

Proposition III.1
Let decision block \( DB = (U, C \cup D, V, f) \), \( a = x^i \in C \), \( V_j \) is the set of existing values of the conditional index attribute \( a \). The \( z \) value of \( a \) is smoothed to two new values \( w \) and \( y \).

\[ C = \bigcup_{i=1}^{k} x^{(i)} \]  
\[ D = \bigcup_{i=1}^{k} x^{(i)} \]  
\[ C' = \bigcup_{i=1}^{k} x^{(i)} \]  
\[ D' = \bigcup_{i=1}^{k} x^{(i)} \]

\( U/C = \{ C_1, C_2, \ldots, C_m \} \), \( U/D = \{ D_1, D_2, \ldots, D_n \} \), \( U/D' = \{ D_1, D_2, \ldots, D_n \} \).

Suppose that if the conditional equivalence class \( C_i \subseteq U/C \), \( f(C_i) = z \) smoothed into two new conditional equivalence classes \( C_{pi}, C_{qi} \) with \( w, y \) values of \( a \) are roughened to the new value \( z \).

Proposition III.2
Let decision block \( DB = (U, C \cup D, V, f) \), \( a = x^i \in C \), \( V_j \) is the set of existing values of the conditional index attribute \( a \). The \( z \) value of \( a \) is smoothed to two new values \( w \) and \( y \).

\( C = \bigcup_{i=1}^{k} x^{(i)} \)  
\( D = \bigcup_{i=1}^{k} x^{(i)} \)  
\( C' = \bigcup_{i=1}^{k} x^{(i)} \)  
\( D' = \bigcup_{i=1}^{k} x^{(i)} \]

\( U/C = \{ C_1, C_2, \ldots, C_m \} \), \( U/D = \{ D_1, D_2, \ldots, D_n \} \), \( U/D' = \{ D_1, D_2, \ldots, D_n \} \).

Suppose, if two conditional equivalence classes \( C_p \subseteq U/C \), \( f(C_p) = w \) is made rough into new conditional equivalence class \( C_{p}\) then the slice \( r \) exists two conditional equivalence classes \( C_{p}, C_{q} \).

Suppose, if two conditional equivalence classes \( C_p \subseteq U/C \), \( f(C_p) = w \) is made rough into new conditional equivalence class \( C_{p}\) then the slice \( r \) exists two conditional equivalence classes \( C_{p}, C_{q} \).

Proof

i) From the smoothing of the conditional equivalence class \( C_i \) we have: \( C_i = C_{pi} \cup C_{qi} \).

ii) Assuming we have: \( C_j \) is smoothed sympathetic partially into two new conditional equivalence classes \( C_{nj} \) and \( C_{nj} \).

Prove

i) From the smoothing of the conditional equivalence class \( C_i \) we have: \( C_i = C_{pi} \cup C_{qi} \).

ii) Assuming we have: \( C_j \) is smoothed sympathetic partially into two new conditional equivalence classes \( C_{nj} \) and \( C_{nj} \).
∀a ≠ a', a ∈ E: f(C_a) = f(C_a') ⇒ ∀a ≠ a', a ∈ E' (1)
In slices r, then we have:
C_a ∈ U/C ⇒ ∀a ≠ a', a ∈ E': f(C_a) = f(C_a') (2)

We also have:
C_a ∈ U/C ⇒ ∀a ≠ a', a ∈ E': f(C_a) = f(C_a') (3)
From (1), (2) and (3) we infer:

∀a ≠ a', a ∈ E: f(C_a) = f(C_a').

Therefore, apply the necessary and sufficient conditions in the statement of the theorem 1.1, we have two conditionalequivalents classes C_a, C_b is made rough sympathetic into C_a by the roughening of two conditionalequivalents classes C_a C_b to C_a.

From the nature of the rough work two conditionalequivalents classes C_a C_b to C_a we have:
C_a = (C_a C_b) ⊇ (C_a C_b) = C_a.
From that: C_a ⊆ C_a.

Proposition III.4
Let decision block DB=(U, C ∪ D), a=x^0 ∈ C, V_i is the set of existing values of the conditional index attribute a, the w and y values of a are rounded to the new value z

C^w = \bigcup_{i=1, ..., n_x} C^{x^{w_i}}, C^y = \bigcup_{i=1, ..., n_x} C^{x^{y_i}}

D^w = \bigcup_{i=1, ..., n_x} C^{x^{w_i}}, x \in W.

U/C = \{C_1, C_2, ..., C_m\}, U/C^w = \{C_1^w, C_2^w, ..., C_m^w\},
U/D = \{D_1, D_2, ..., D_n\}, U/D^w = \{D_1^w, D_2^w, ..., D_n^w\},
C_{pq} ∈ U/C \cup U/D, f(C_{pq}) = w, f(C_{pq}) = y, D_{ab} ∈ U/D^w.

\textbf{Suppose}, if C_a C_b is made rough into two new conditionalequivalents class C_a (f(C_a) = z) and on the slice r, two conditionalequivalents classes C_a, C_b, C_a ⊆ C_b is made rough sympathetic into C_a then:

i) C_a C_b = C_a

ii) ∀D_{ab} ∈ U/D^w: \text{Sup}(C_a D_{ab}) + \text{Sup}(C_b D_{ab}) = \text{Sup}(C_a D_{ab}), \forall i = 1, 2, ..., n_x

Prove

Suppose we have: x ∈ C_a ∪ C_b ⇒ x ∈ C_a or x ∈ C_b. If x ∈ C_a then from the two conditionalequivalents classes C_a, C_b made rough into conditionalequivalents class C_a = f(x, a) = f(C_a) = (C_a).

On the other hand, applying the results of theorem 2.1 we have
∀a ≠ a': f(C_a) = f(C_a') = f(C_a) = f(C_a') = f(C_a) = f(C_a') = x ∈ C_a. Totally similar, when x ∈ C_b we also prove that x ∈ C_b.

So inference: (C_a C_b) ⊆ C_a. (5)

On the contrary, suppose x ∈ C_a. Because C_a and C_b made rough into C_a applying the results of theorem 2.1 we have:
∀a ≠ a': f(C_a) = f(C_a') = f(C_a) = f(C_a') = f(C_a) = f(C_a') = f(C_a), (5)

On the other hand, because is C_a (f(x, a) = z) but z is made rough from w and y ⇒ f(x, a) = w or f(x, a) = y.

- If f(x, a) = w ⇒ f(x, a) = f(C_a) = w ⇒ x ∈ C_a.
- If f(x, a) = y ⇒ f(x, a) = f(C_b) = y ⇒ x ∈ C_b.

So x ∈ C_a or x ∈ C_b ⇒ x ∈ C_a ∪ C_b.
Therefore, from x ∈ C_a or x ∈ C_b we have:
C_a ∪ C_b = C_a. (6)

Combined (5) and (6) we have: C_a ∪ C_b = C_a.
decision equivalence classes $D_i$ and $D_j$: then:

1. $D_i = D_i^w \cup D_i^d$,
2. $\forall C_j \in U/C^i$: $\text{Sup}(C_{j,p} \cap D_i) = \text{Sup}(C_{j,p} \cap D_j) + \text{Sup}(C_{j,p} \cap D_i^d)$, with $j = 1, 2, \ldots, t_c$.

**Prove**

i. From the smoothing of the decision equivalent class $D_i$, we see that: $D_i = D_i^w \cup D_i^d$.

ii. Assuming we have: $D_i$ is smoothed sympathetic partially into two new decision equivalence classes $D_{i,1}$ and $D_{i,2}$.

$D_i = D_{i,1} \cup D_{i,2}$ and $D_{i,1} \cap D_{i,2} = \emptyset$.

Other way: $\forall C_j \in U/C^i$: $\text{Sup}(C_{j,p} \cap D_i) = \text{Sup}(C_{j,p} \cap D_{i,1}) + \text{Sup}(C_{j,p} \cap D_{i,2})$.

We have: $D_{i,1} \cap D_{i,2} = \emptyset$.

So we infer: $\forall C_j \in U/C^i$: $\text{Sup}(C_{j,p} \cap D_i) = \text{Sup}(C_{j,p} \cap D_{i,1}) + \text{Sup}(C_{j,p} \cap D_{i,2})$.

From this result we see: column corresponding to the decision equivalence class $D_i$ in the support matrix for slice $r$ will be split into two new columns corresponding to two new decision equivalence classes $D_{i,1}$ and $D_{i,2}$.

Therefore, to calculate the value of the elements of these two new columns in the support matrix with slice $r$, we first calculate the values $\text{Sup}(C_{j,p} \cap D_i)$ with $j = 1, 2, \ldots, t_c$. From there, we infer the values $\text{Sup}(C_{j,p} \cap D_{i,1})$ and $\text{Sup}(C_{j,p} \cap D_{i,2})$ with $j = 1, 2, \ldots, t_c$.

**Proposition III.7**

Let decision block $DB = (U, C \cup D, V, f)$, $a = x^i \in D$, $V$ is the set of existing values of the decision index attribute $a$, the $w$ and $y$ values of $a$ are roughened to the new value $z$.

$$C = \bigcup_{i=1}^{t_c} x^{(i)}, \quad D = \bigcup_{i=1}^{t_c} x^{(i)} + \bigcup_{i=1}^{t_c} x^{(i)} + \bigcup_{i=1}^{t_c} x^{(i)}, \quad C^i = \bigcup_{i=1}^{t_c} x^{(i)}$$

$$D^i = \bigcup_{i=1}^{t_c} x^{(i)} + \bigcup_{i=1}^{t_c} x^{(i)}$$

$U/C^i = \{C_1, C_2, \ldots, C_n\}$, $U/C^i = \{C_1, C_2, \ldots, C_n\}$

$U/D = \{D_1, D_2, \ldots, D_n\}$, $U/D^i = \{D_1, D_2, \ldots, D_n\}$

Suppose, if two decision equivalence classes $D_p, D_q$ ($f(D_p, a) = w, f(D_q, a) = y$) is made rough into new decision equivalence class $D_{p,q}$ ($f(D_{p,q}, a) = z$) then on the slice $r$, two decision equivalence classes $D_{p,q}$ are made rough sympathetic into $D_{p,q}$.

**Prove**

i. Suppose we have: $u \in D_{i,1}, D_{i,2} = u \in D_{i,1} \cup D_{i,2}$, If $u \in D_{i,1}$ then two decision equivalence classes $D_{i,1}, D_{i,2}$ is made rough into decision equivalent class $D_{i,1} \cup D_{i,2}$

On the other hand, apply the results of the theorem 2.1 we have $v \in a, \alpha = f(D_{i,1}, a) = f(D_{i,2}, a) \Rightarrow f(v, a) = f(D_{i,1}, a) = f(D_{i,2}, a) = w \Rightarrow u \in D_{i,1}$, $u \in D_{i,2}$. Completely similar, if $u \in D_{i,2}$ then we also proved $u \in D_{i,2}$.

So inference: $(D_{i,1}, D_{i,2}) \subseteq D_{i,1}$ or $(D_{i,1}, D_{i,2}) \subseteq D_{i,2}$.

On the contrary, suppose $u \in D_{i,2}$, because $D_{i,1}$ and $D_{i,2}$ is made rough into decision equivalent class $D_{i,1} \cup D_{i,2}$ should apply the results of the theorem 2.1 we have: $v \in a, \alpha = f(D_{i,1}, a) = f(D_{i,2}, a) \Rightarrow f(v, a) = f(D_{i,1}, a) = f(D_{i,2}, a) = w \Rightarrow u \in D_{i,1}$.

- If $f(u, a) = w \Rightarrow f(D_{i,1}, a) = w \Rightarrow u \in D_{i,1}$.
- If $f(u, a) = y \Rightarrow f(u, a) = f(D_{i,2}, a) \Rightarrow u \in D_{i,2}$.

Therefore, from $u \in D_{i,1}$ or $u \in D_{i,2}$.

So: $D_{i,1} \cup D_{i,2} \subseteq D_{i,1}$ or $D_{i,1} \cup D_{i,2} \subseteq D_{i,2}$.

Combined (7) and (8) we have: $D_{i,1} \cup D_{i,2} = D_{i,1}$ or $D_{i,1} \cup D_{i,2} = D_{i,2}$.

ii. Because $D_{i,1}$, $D_{i,2}$ are decision equivalence classes, so we have: $D_{i,1} \cup D_{i,2} = D_{i,2}$.

On the other hand: $\forall C_{j,p} \in U/C^i$: $\text{Sup}(C_{j,p} \cap D_{i,1} \cup D_{i,2}) = |C_{j,p} \cap D_{i,1} \cup D_{i,2}| = |C_{j,p} \cap D_{i,1}| + |C_{j,p} \cap D_{i,2}|$

We have: $D_{i,1} \cap D_{i,2} = D_{i,1} \cap D_{i,2} = D_{i,1} \cap D_{i,2} = \emptyset$.

Inferred: $\text{Sup}(C_{j,p} \cap D_{i,1}) = |C_{j,p} \cap D_{i,1}| = \text{Sup}(C_{j,p} \cap D_{i,2})$

Therefore: $\forall C_{j,p} \in U/C^i$: $\text{Sup}(C_{j,p} \cap D_{i,1}) = \text{Sup}(C_{j,p} \cap D_{i,2})$

Thus, we see two columns of the support matrix on the slice corresponds to two decision equivalence classes $D_{i,1}, D_{i,2}$ are made rough sympathetic into a new column corresponding to the decision equivalence class $D_{i,1}$. The value of each element of the new column corresponds to $D_{i,1}$ as the total value of two elements of two columns corresponding to two decision equivalence classes $D_{i,1}$ and $D_{i,2}$.

**IV. CONCLUSIONS**

From the initial results on the decision block, the paper proposes and demonstrates some of the results of the relationship between roughing, smoothing the values of
conditional attributes or decisions for conditionalequivalents
classes or decision equivalence classes on the decision blocks
and on the slices. The smoothing of conditionalequivalents
classes or decision equivalence classes on the decision block
have a sympathetic the smoothing of conditionalequivalents
classes or decision equivalence classes respectively on the slice.
The roughening of conditionalequivalents classes or decision equivalence
classes on the decision block have a sympathetic the
roughening of conditionalequivalents classes or decision equivalence
classes on the slice. From these results,
calculation of support matrix on the slices is define as
the calculation of the support matrix on the block when the
smoothing, roughening of conditionalequivalents classes or
decision equivalence classes.
In special cases, the index set id = {x}, the information
blocks degenerate into information systemsthen these results
coincide with the results reported by many authors for the
information system. On the basis of these results we can
study the reverse relationship between slices of information
block with that block itself, in case the objects of the
information block are changed, some other results may be
considered in individual cases of information blocks, it adds the theoretical results of
the exploitation of decision rules on information blocks.

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