Multiple criteria decision making based on bipolar picture fuzzy sets and extended TOPSIS

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Abstract

The notion of bipolar fuzzy sets (B_p FSs) has got much attention from the experts or decision-makers (DMs). B_p FSs have ample information in the form of two degrees called the positive belonging degree (P_v BD) and a negative belonging degree (N_v BD). In this article, we introduced the concept of bipolar picture fuzzy sets (B_Pc FSs) by connecting the concepts of B_p FSs and picture fuzzy sets (P_c FSs). Firstly, we presented the concept, operational rules, score, and accuracy functions of B_Pc FSs. Secondly, a distance measure is formulated for the B_Pc FSs and then implemented for the extension of TOPSIS. Thirdly, a multiple criteria decision making (MCDM) model is proposed to handle the uncertain MCDM problems. Lastly, a practical example related to the sum of money’s investment is exemplified to validate and effectiveness of the proposed model.

Keywords: Picture fuzzy sets, fuzzy sets, B_Pc FSs, linear programming model.

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1. Introduction

In 2013, Coung [4] introduced the generalization of fuzzy sets (FSs) [20] by presenting the idea of picture fuzzy sets (P_c FSs). P_c FSs consists of three well-known degrees, membership degree (MD), non-membership degree (NMD) and neutral degree (ND) so that 0 \leq MD + NMD + ND \leq 1. Cuong and Kreinovich [3] established various operational laws of P_c FSs to handle vague information perfectly. The notion of bipolar fuzzy sets (B_p FSs) [21, 22] have come to account as a superior device to portray the vagueness in the decision-making process. B_p FSs contain two elements called, the positive membership degree (P_MD) and the negative membership degree (N_MD) to represent the bipolar fuzzy (B_p F) information and the range both the degrees always lie in [-1, 1]. Currently, B_p FSs have been utilized in various fields of research [7, 9, 23–25]. Gul [5] presented several arithmetic and geometric operators for bipolar fuzzy information. Wei et al. [18] presented the concept of hesitant B_p FSs and its operational laws to deal with B_p F elements. Lu et al. [10] introduced the idea of bipolar 2-tuple linguistic fuzzy sets (B_p 2TLFSs). Further, Xu and Wei [19] suggested the dual B_p FSs and established many arithmetic laws to fuse the dual bipolar fuzzy data. Moreover, plenty of research work has been done on the B_p FSs for example, Hashim...
et al. [6] presented the idea of neutrosophic bipolar fuzzy sets and developed an algorithm to find the best medicine for some particular diseases, Riaz and Tehrim [12] gave the concept of cubic bipolar fuzzy sets (CBFSs), a generalization of BP FSs and implemented it in group decision making with the help of geometric aggregation operators.

The linear programming (LP) model introduced by Vanderbei [15], permits some target function to be minimized or maximized inside the system of given situational limitations. LP is a computational technique that enables DMs to solve the problems which they face in decision-making model. It encourages the DMs to deal with constrained ideal conditions that they need to make the best of their resources. Various experts utilized LP [1, 2, 8, 13, 16] in MCDM in different fields. Recently, Sindhu et al. [14] implemented the LP methodology with extended TOPSIS for picture fuzzy sets.

From the above discussion, it can be noticed that PB FSs and BP FSs are getting a lot of attention from the DMs and are playing an important role in the decision-making process. However, all these are concerned with discrete information due to which a chance of loss of information is present. In order to reduce the chance of loss of information, we presented the concept of BP FSs that consists of P,MD and N,MD in terms of fuzzy numbers. BP FSs also have a lot of information that helps the DMs to reach the best decisions in the MCDM problems. The weights of criteria appear to specify that the DMs identify the significance of people’s views and their influence on attaining the objective. Allocation of weights to the criteria epitomizes the importance of each decision criterion relative to each other. We apply the technique for order preference by similarity to ideal solution (TOPSIS) to get the objective function and then find out the weights of criteria under some constraints by using the LP model in this article.

The remaining part of the article is organized as follows. Section 2 briefly shows the basics like FSs and PB FSs to reach the notion BP FSs and the LP model that will be used to compute the weights of criteria. In Section 3, we introduced the concept of BP FSs, operational laws distance, and similarity measures of BP FSs. Based on the TOPSIS, an MCDM model is proposed in Section 4. In Section 5, the developed MCDM model is then applied to a practical example to select the best alternative. For the validity, effectiveness, and stability of the proposed MCDM model, we performed the sensitivity analysis in Section 6. Lastly, conclusions are drawn in Sections 7.

2. Preliminaries

A brief introduction of the notions FSs, PB FSs, BP FSs BP FSs and the LP model is presented in this section.

**Definition 2.1** ([20]). Let \( X = \{x_1, x_2, \ldots, x_n\} \) be a discourse set, a fuzzy set (FS) \( F \) on \( X \) is represented in terms of a functions \( m: X \rightarrow [0, 1] \) such as

\[
F = \{(x_i, m_F(x_i)) \mid x_i \in X\}.
\]

**Definition 2.2** ([4]). Let \( X = \{x_1, x_2, \ldots, x_n\} \) be a fixed set, a picture fuzzy set \( P_c \) on \( X \) is defined as:

\[
P_c = \{(x_i, \alpha_{P_c}(x_i), \gamma_{P_c}(x_i), \beta_{P_c}(x_i)) \mid x_i \in X, i = 1, 2, \ldots, n\},
\]

where \( \alpha_{P_c}(x_i), \beta_{P_c}(x_i), \gamma_{P_c}(x_i) \in [0, 1] \) are called the acceptance membership, neutral and rejection membership degrees of \( x_i \in X \) to the set \( P_c \), respectively and \( \alpha_{P_c}(x_i), \gamma_{P_c}(x_i) \) and \( \beta_{P_c}(x_i) \) fulfill the condition:

\[
0 \leq \alpha_{P_c}(x_i) + \gamma_{P_c}(x_i) + \beta_{P_c}(x_i) \leq 1, \text{ for all } x_i \in X.
\]

Also \( \zeta_{P_c}(x_i) = 1 - \alpha_{P_c}(x_i) - \gamma_{P_c}(x_i) - \beta_{P_c}(x_i) \), then \( \zeta_{P_c}(x_i) \) is said to be a degree of refusal membership of \( x_i \in X \) in \( P_c \). For our convenience, we can write \( P_k = (\alpha^k_{P_c}(x_i), \beta^k_{P_c}(x_i), \gamma^k_{P_c}(x_i)) \) as the picture fuzzy numbers (PB FSs) over a set \( P_c \), where \( k \) is positive integer.

**Definition 2.3** ([17]). Let \( P = (\alpha_{P_c}(x_i), \gamma_{P_c}(x_i), \beta_{P_c}(x_i)), P_1 = (\alpha^1_{P_c}(x_i), \gamma^1_{P_c}(x_i), \beta^1_{P_c}(x_i)), \text{ and } P_2 = (\alpha^2_{P_c}(x_i), \gamma^2_{P_c}(x_i), \beta^2_{P_c}(x_i)) \) be three PB FSs, then arithmetic operations are listed as follows:

1. \( P_1 \oplus P_2 = (\alpha^3_{P_c} + \alpha^2_{P_c}, \gamma^3_{P_c} + \gamma^2_{P_c}, \beta^3_{P_c} + \beta^2_{P_c}) \);
2. \( P_1 \otimes P_2 = (\alpha_{P_1}^+ \times \alpha_{P_2}^+, \gamma_{P_1}^+ \times \gamma_{P_2}^+, \beta_{P_1}^+ \times \beta_{P_2}^+); \)
3. \( \lambda P = (1 - (1 - \alpha_p^+)^\lambda, \gamma_{P_1}^+, \beta_{P_1}^+), \)
4. \( P_k^\lambda = (\alpha_{P_k}^+, 1 - (1 - \gamma_p^+)^\lambda, 1 - (1 - \beta_p^+)^\lambda), \)

**Definition 2.4** ([21, 22]). Suppose that \( X \) is a discourse set such that \( X = \{x_1, x_2, \ldots, x_n\} \), then a bipolar fuzzy set \( B_p \) on \( X \) is described as follows:

\[
B_p = \left\{ \left( x_i, (\alpha_{B_p(X_i)}, \beta_{B_p(X_i)}) \right) | x_i \in X, i = 1, 2, \ldots, n \right\}
\]

where \( \alpha_{B_p(X_i)} : X \rightarrow [0, 1] \), \( \beta_{B_p(X_i)} : X \rightarrow [-1, 0] \) are named as \( P_v \)BD and \( N_v \)BD of \( x_i \in X \) to \( B_p \), respectively.

**Definition 2.5** ([5]). Let \( B_p \), \( B_p^1 \) and \( B_p^2 \) be any three \( B_p \)FSs on \( X = \{x_1, x_2, \ldots, x_n\} \), then some aggregation operators are defined as:

1. \( B_p^1 \oplus B_p^2 = (\alpha_1^+ + \alpha_2^+ - \alpha_1^+ \times \alpha_2^+, \beta_1^+ \times \beta_2^+); \)
2. \( B_p^1 \odot B_p^2 = (\alpha_1^+ \times \alpha_2^+, \beta_1^+ - \beta_2^+ \times \beta_1^+); \)
3. \( \kappa B_p = (1 - (1 - \alpha_p^+)^\kappa, -\beta_p^+); \)
4. \( B_p^k = (\alpha^k)^k, \beta_p^k; \)
5. \( B_p^0 = (1 - \alpha_p^+, |\beta_p^+| - 1). \)

**Definition 2.6** ([15]). Vanderbei defined the LP model as follows:

Maximize: \( Z = c_1 y_1 + c_2 y_2 + c_3 y_3 + \cdots + c_n y_n, \)
Subject to:
\[
\begin{align*}
& a_{11} y_1 + a_{12} y_2 + \cdots + a_{1n} y_n \leq b_1, \\
& a_{21} y_1 + a_{22} y_2 + \cdots + a_{2n} y_n \leq b_2, \\
& \vdots \\
& a_{m1} y_1 + a_{m2} y_2 + \cdots + a_{mn} y_n \leq b_m, \\
& y_1, y_2, \ldots, y_n \geq 0,
\end{align*}
\]

where \( m, n \) represent the cardinalities of constraints and decision variables \( (y_1, y_2, \ldots, y_n) \), respectively. The solution \( (y_1, y_2, \ldots, y_n) \) is known as viable if it satisfies all the provided restrictions. The solution is used to compute the optimal solution of \( y_1, y_2, \ldots, y_n \) to maximize the linearly objective function \( Z \).

3. Bipolar picture fuzzy sets (BPCFSs)

In this section, we introduced the notion of BPCFSs by combining both \( B_p \)FSs and \( P_v \)FSs. Also, various operational laws are established and then a novel distance measure is proposed for bipolar picture fuzzy numbers (BPCFNs).

**Definition 3.1.** Suppose that \( X = \{x_1, x_2, \ldots, x_n\} \) is a discourse, then the BPCFSs \( B_{PC} \) on \( X \) is presented as:

\[
B_{PC} = \left\{ \left( x_i, (\alpha_{B_{PC}(x_i)}, \beta_{B_{PC}(x_i)}) \right) | x_i \in X, i = 1, 2, \ldots, n \right\}
\]

here \( \alpha_{B_{PC}(x_i)}^+ + \beta_{B_{PC}(x_i)}^+ \leq 1 \) and \(-1 \leq \alpha_{B_{PC}(x_i)}^- + \beta_{B_{PC}(x_i)}^- \leq 0 \) for all \( x_i \in X \).
Definition 3.2. Let $\bar{p} = (\alpha_{\bar{p}}^{1+}, \gamma_{\bar{p}}^{1+}, \beta_{\bar{p}}^{1+}, \alpha_{\bar{p}}^{1-}, \gamma_{\bar{p}}^{1-}, \beta_{\bar{p}}^{1-})$ and $\bar{p}_2 = (\alpha_{\bar{p}}^{2+}, \gamma_{\bar{p}}^{2+}, \beta_{\bar{p}}^{2+}, \alpha_{\bar{p}}^{2-}, \gamma_{\bar{p}}^{2-}, \beta_{\bar{p}}^{2-})$ be three $\text{BPa}_{\bar{p}}$FNs, then the operational rules are penned as:

1. $\bar{p}_1 \odot \bar{p}_2 = (\alpha_{\bar{p}}^{1+}, \gamma_{\bar{p}}^{1+}, \beta_{\bar{p}}^{1+}, \alpha_{\bar{p}}^{1-}, \gamma_{\bar{p}}^{1-}, \beta_{\bar{p}}^{1-})$
2. $\bar{p}_1 \odot \bar{p}_2 = (\alpha_{\bar{p}}^{2+}, \gamma_{\bar{p}}^{2+}, \beta_{\bar{p}}^{2+}, \alpha_{\bar{p}}^{2-}, \gamma_{\bar{p}}^{2-}, \beta_{\bar{p}}^{2-})$
3. $\lambda \bar{p} = ((1 - 1 - \alpha_{\bar{p}}^{+})^\lambda, (\gamma_{\bar{p}}^{+})^\lambda, (\beta_{\bar{p}}^{+})^\lambda)$
4. $\bar{p}^\lambda = ((\alpha_{\bar{p}}^{+})^\lambda, (1 - 1 - \gamma_{\bar{p}}^{+})^\lambda, (1 - 1 - \beta_{\bar{p}}^{+})^\lambda)$, where, $\lambda > 0$

Definition 3.3. Let $l$ and $q$ be two $\text{BPa}_{\bar{p}}$FNs of the $\text{BPa}_{\bar{p}}$FSSs $L$ and $Q$, respectively defined on a discourse set $X = \{x_1, x_2, \ldots, x_n\}$, then the distance $D_{\text{pc}}(L, Q)$ is defined as:

$$D_{\text{pc}}(L, Q) = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{1}{2} \| |(\alpha_{\bar{p}}^{+}(x_i) - \alpha_{\bar{p}}^{+}(x_i)) + |\gamma_{\bar{p}}^{+}(x_i) - \gamma_{\bar{p}}^{+}(x_i)| + |\beta_{\bar{p}}^{+}(x_i) - \beta_{\bar{p}}^{+}(x_i)| \|_{\bar{p}}^{+} \right)$$

Theorem 3.4. Suppose that, $D_{\text{pc}}$ is a mapping $D_{\text{pc}} : \text{BPa}_{\bar{p}}$FSSs($X$) × $\text{BPa}_{\bar{p}}$FSSs($X$) → $[0, 1]$, then $D_{\text{pc}}(L, Q)$ is a distance measure if the following four conditions hold:

1. $0 \leq D_{\text{pc}}(L, Q) \leq 1$;
2. $D_{\text{pc}}(L, Q) = 0$ iff $L = Q$;
3. $D_{\text{pc}}(L, Q) = D_{\text{pc}}(Q, L)$;
4. $D_{\text{pc}}(L, R) \geq D_{\text{pc}}(L, Q)$ and $D_{\text{pc}}(L, R) \geq D_{\text{pc}}(Q, R)$, for any $L, Q, R \in \text{BPa}_{\bar{p}}$FSSs($X$).

Proof. Since the proofs of 1 – 3 are obvious, thereby, we need to prove the last condition 4. For any $l = (\alpha_{\bar{p}}^{+}, \gamma_{\bar{p}}^{+}, \beta_{\bar{p}}^{+}, \alpha_{\bar{p}}^{-}, \gamma_{\bar{p}}^{-}, \beta_{\bar{p}}^{-}) \in L$, $q = (\alpha_{\bar{p}}^{+}, \gamma_{\bar{p}}^{+}, \beta_{\bar{p}}^{+}, \alpha_{\bar{p}}^{-}, \gamma_{\bar{p}}^{-}, \beta_{\bar{p}}^{-}) \in Q$, and $r = (\alpha_{\bar{p}}^{+}, \gamma_{\bar{p}}^{+}, \beta_{\bar{p}}^{+}, \alpha_{\bar{p}}^{-}, \gamma_{\bar{p}}^{-}, \beta_{\bar{p}}^{-}) \in R$ be three $\text{BPa}_{\bar{p}}$FNs defined on $X$ so that, $L \subseteq Q \subseteq R$, then based on Definition 3.3, we get:

$$|\alpha_{\bar{p}}^{+}(x_i) - \alpha_{\bar{p}}^{-}(x_i)| \geq |\alpha_{\bar{p}}^{+}(x_i) - \alpha_{\bar{p}}^{-}(x_i)|,$$  \hspace{1cm} (3.1)

$$|\alpha_{\bar{p}}^{+}(x_i) - \alpha_{\bar{p}}^{+}(x_i)| \geq |\alpha_{\bar{p}}^{+}(x_i) - \alpha_{\bar{p}}^{+}(x_i)|,$$  \hspace{1cm} (3.2)

$$|\alpha_{\bar{p}}^{+}(x_i) - \gamma_{\bar{p}}^{+}(x_i)| \geq |\alpha_{\bar{p}}^{+}(x_i) - \gamma_{\bar{p}}^{+}(x_i)|,$$  \hspace{1cm} (3.3)

$$|\alpha_{\bar{p}}^{+}(x_i) - \beta_{\bar{p}}^{+}(x_i)| \geq |\alpha_{\bar{p}}^{+}(x_i) - \beta_{\bar{p}}^{+}(x_i)|,$$  \hspace{1cm} (3.4)

$$|\alpha_{\bar{p}}^{+}(x_i) - \beta_{\bar{p}}^{+}(x_i)| \geq |\alpha_{\bar{p}}^{+}(x_i) - \beta_{\bar{p}}^{+}(x_i)|,$$  \hspace{1cm} (3.5)

$$|\alpha_{\bar{p}}^{+}(x_i) - \gamma_{\bar{p}}^{+}(x_i)| \geq |\alpha_{\bar{p}}^{+}(x_i) - \gamma_{\bar{p}}^{+}(x_i)|.$$  \hspace{1cm} (3.6)

By adding Eqs. (3.1)-(3.3) and (3.4)-(3.6), we get:

$$|\alpha_{\bar{p}}^{+}(x_i) - \alpha_{\bar{p}}^{+}(x_i)| + |\alpha_{\bar{p}}^{+}(x_i) - \beta_{\bar{p}}^{+}(x_i)| + |\alpha_{\bar{p}}^{+}(x_i) - \gamma_{\bar{p}}^{+}(x_i)|,$$

$$\geq |\alpha_{\bar{p}}^{+}(x_i) - \alpha_{\bar{p}}^{+}(x_i)| + |\alpha_{\bar{p}}^{+}(x_i) - \beta_{\bar{p}}^{+}(x_i)| |\alpha_{\bar{p}}^{+}(x_i) - \gamma_{\bar{p}}^{+}(x_i)|,$$  \hspace{1cm} (3.7)

$$|\alpha_{\bar{p}}^{+}(x_i) - \alpha_{\bar{p}}^{+}(x_i)| + |\alpha_{\bar{p}}^{+}(x_i) - \beta_{\bar{p}}^{+}(x_i)| + |\alpha_{\bar{p}}^{+}(x_i) - \gamma_{\bar{p}}^{+}(x_i)|,$$

$$\geq |\alpha_{\bar{p}}^{+}(x_i) - \alpha_{\bar{p}}^{+}(x_i)| + |\alpha_{\bar{p}}^{+}(x_i) - \beta_{\bar{p}}^{+}(x_i)| |\alpha_{\bar{p}}^{+}(x_i) - \gamma_{\bar{p}}^{+}(x_i)|.$$  \hspace{1cm} (3.7)

By adding Eqs. in (3.7), we have:

$$|\alpha_{\bar{p}}^{+}(x_i) - \alpha_{\bar{p}}^{+}(x_i)| + |\alpha_{\bar{p}}^{+}(x_i) - \beta_{\bar{p}}^{+}(x_i)| + |\alpha_{\bar{p}}^{+}(x_i) - \gamma_{\bar{p}}^{+}(x_i)|$$
Proof.
The proof of this Theorem can be completed on the same steps as Theorem 3.4.

1. Then the weighted distance measure $D_{pc}(L,R) \geq D_{pc}(L,Q)$,

and similarly we can prove that, $D_{pc}(L,R) \geq D_{pc}(Q,R)$. 

Generally, weights of the criteria have a great influence on the results of the decision making process, therefore, a weighted distance measure between two Bp$_c$FSs is developed on the basis of Definition 3.3 as following.

Definition 3.5. Let L and Q be two Bp$_c$FSs defined on a discourse set $X = \{x_1, x_2, \ldots, x_n\}$ and $w_j$ be the weights of the m criteria such that $\sum_{j=1}^{m} w_j = 1$. Then the weighted distance measure $D_{pc}^w(L,Q)$ is defined in the following way,

$$D_{pc}^w(L,Q) = \sum_{i=1}^{n} w_j \left( \begin{array}{c} |\alpha_{\text{Bp}_c}(x_1) - \alpha_{\text{Bp}_c}^+(x_1)| + |\alpha_{\text{Bp}_c}^-(x_1) - \beta_{\text{Bp}_c}^+(x_1)| + |\beta_{\text{Bp}_c}^+(x_1) - \beta_{\text{Bp}_c}^-(x_1)| \\ + |\alpha_{\text{Bp}_c}(x_1) - \alpha_{\text{Bp}_c}^+(x_1)| + |\alpha_{\text{Bp}_c}^-(x_1) - \beta_{\text{Bp}_c}^+(x_1)| + |\beta_{\text{Bp}_c}^+(x_1) - \beta_{\text{Bp}_c}^-(x_1)| \\ + |\alpha_{\text{Bp}_c}(x_1) - \alpha_{\text{Bp}_c}^+(x_1)| + |\alpha_{\text{Bp}_c}^-(x_1) - \beta_{\text{Bp}_c}^+(x_1)| + |\beta_{\text{Bp}_c}^+(x_1) - \beta_{\text{Bp}_c}^-(x_1)| \\ + \max \left[ |\alpha_{\text{Bp}_c}(x_1) - \alpha_{\text{Bp}_c}^+(x_1)|, |\alpha_{\text{Bp}_c}^-(x_1) - \beta_{\text{Bp}_c}^+(x_1)|, |\beta_{\text{Bp}_c}^+(x_1) - \beta_{\text{Bp}_c}^-(x_1)| \right] \end{array} \right)$$

Theorem 3.6. Suppose that $X = \{x_1, x_2, \ldots, x_n\}$ is discourse set, then the weighted distance measure $D_{pc}^w$ between the two Bp$_c$FSs satisfies the following properties:

1. $0 \leq D_{pc}^w(L,Q) \leq 1$;
2. $D_{pc}^w(L,Q) = 0$ iff $L = Q$;
3. $D_{pc}^w(L,Q) = D_{pc}^w(Q,L)$;
4. $D_{pc}^w(L,R) \geq D_{pc}^w(L,Q)$ and $D_{pc}^w(L,R) \geq D_{pc}^w(Q,R)$, for any $L,Q,R \in \text{Bp}_c\text{FSs}(X)$.

Proof. The proof of this Theorem can be completed on the same steps as Theorem 3.4.

Definition 3.7. Let L and Q be two Bp$_c$FSs defined on a discourse set $X = \{x_1, x_2, \ldots, x_n\}$. Then a similarity measure $\tilde{S}_{pc}(L,Q)$ based on Definition 3.5 is defined as:

$$\tilde{S}_{pc}(L,Q) = 1 - D_{pc}^w(L,Q).$$
Definition 3.8. Suppose that $X = \{x_1, x_2, \ldots, x_n\}$ is discourse set, then the weighted distance measure $D^W_{pF}$ between the two $B_{pF}$FSs satisfies the following properties:

1. $0 \leq S_{pF}(L, Q) \leq 1$;
2. $S_{pF}(L, Q) = 1$ iff $L = Q$;
3. $S_{pF}(L, Q) = S_{pF}(Q, L)$;
4. $S_{pF}(L, R) \geq S_{pF}(L, Q)$ and $S_{pF}(L, R) \geq S_{pF}(Q, R)$, for any $L, Q, R \in B_{pF}$FSs($X$).

4. Bipolar picture fuzzy TOPSIS ($B_{pF}$-TOPSIS) for MCDM

In this section, we proposed an MCDM model for $B_{pF}$ information based on TOPSIS, named $B_{pF}$-TOPSIS and LP technique is implemented to evaluate the weights of criteria, under various constraints. A linear objective function of weights is computed with the help of the first four steps of TOPSIS and then used the remaining steps to recognize the best alternative. Let $B = \{B_1, B_2, \ldots, B_n\}$ be a discrete set of alternatives, and $S = \{S_1, S_2, \ldots, S_m\}$ be the collection of criteria with $w = \{w_1, w_2, \ldots, w_m\}$, where $\sum_{j=1}^{m} w_j = 1$ is the weight vector of the criteria $S_j$ where $j = 1, 2, 3, \ldots, m$. A $B_{pF}$ decision matrix ($B_{pF}$DM) is represented by $B_{pF} = [\Delta_{ij}]_{n \times m}$ with $\alpha_{ij}$ as a degree of positive acceptance, $\gamma_{ij}$ degree of positive neutral and $\beta_{ij}$ degree of positive rejection while, $\alpha_{ij}$ as a degree of negative acceptance, $\gamma_{ij}$ degree of negative neutral and $\beta_{ij}$ degree of negative rejection of $B_i$ ($i = 1, 2, \ldots, n$), respectively. The proposed $B_{pF}$-TOPSIS consists of the following steps.

Step 1. Form a $B_{pF}$DM, $B_{pF} = [\Delta_{ij}]_{n \times m}$ based on the information provided by the DMs.

Step 2. Find out the bipolar picture fuzzy positive ideal solution ($B_{pF}$FPIS) denoted by $\Delta^+$ and bipolar picture fuzzy negative ideal solution ($B_{pF}$FNIS) denoted by $\Delta^-$, respectively for beneficial criteria,

$$\Delta^+ = \left( \left( \max_j (\alpha_{ij}^+), \max_j (\gamma_{ij}^+), \max_j (\beta_{ij}^+) \right) \right),$$

$$\Delta^- = \left( \left( \min_j (\alpha_{ij}^-), \min_j (\gamma_{ij}^-), \min_j (\beta_{ij}^-) \right) \right) \right).$$

Step 3. Based on Definition 3.7, calculate the degree of weighted similarity $\tilde{S}_{pi}^+$ between $B_{pF}$FPIS $\Delta^+$ and each alternative as well as the degree of weighted similarity $\tilde{S}_{pi}^-$ between $B_{pF}$FNIS $\Delta^-$ by using the Eqs. below, respectively:

$$\tilde{S}_{pci}^+(B_i, \Delta^+) = 1 - D^W_{pF}(B_i, \Delta^+), \quad (4.1)$$

$$\tilde{S}_{pci}^-(B_i, \Delta^-) = 1 - D^W_{pF}(B_i, \Delta^-), \quad (4.2)$$

where, $1 \leq i \leq n$.

Step 4. Based on Eqs. (4.1) and (4.2) established an model to find the objective function $Z$ to compute the weights of criteria under the given constraints,

$$Z = \sum_{i=1}^{n} (\tilde{S}_{pci}^+(B_i, \Delta^+) - \tilde{S}_{pci}^-(B_i, \Delta^-)). \quad (4.3)$$

Step 5. Based on LP model described in Section 2, compute the weights $w_j$ of the criteria $U_j$ where $j = 1, 2, 3, \ldots, m$ such that the objective function $Z$ is maximized.

Step 6. Calculate the degree of similarity $\tilde{S}_{pci}^+$ and $\tilde{S}_{pci}^-$ among each alternative and the elements obtained in $B_{pF}$FPIS $\Delta^+$ and $B_{pF}$FNIS $\Delta^-$, respectively.

Step 7. Compute the relative closeness $R_{ci}$ of alternative $B_i$ with respect to the $B_{pF}$FPIS $\Delta^+$ as:

$$R_{ci} = \frac{\tilde{S}_{pci}^+}{\tilde{S}_{pci}^+ + \tilde{S}_{pci}^-}. \quad (4.4)$$

The larger the value of the relative closeness $R_{ci}$ of the alternatives with regard to the $B_{pF}$FPIS $\tilde{S}_{pci}^+$ means that, we get the best alternative from different alternative $B_i$, where $1 \leq i \leq n$. 

M. S. Sindhu, M. Ahsan, A. Rafiq, I. A. Khan, J. Math. Computer Sci., 23 (2021), 49–57 54
5. Practical example

In this section, an example of the MCDM problem of alternatives is used as the illustration of the application of the proposed MCDM model. Consider an organization that needs to recruit the technical staff to manage the technical issues of the organization. In order to resolve the issue, DM arrange the interview of the five short-listed candidates (alternatives), $B = \{B_1, B_2, \ldots, B_5\}$ under the following four beneficial criteria $Q = \{Q_1, Q_2, Q_3, Q_4\}$ such that: $Q_1$ (advancement in technology), $Q_2$ (market potential), $Q_3$ (the ability of vendors) and $Q_4$ (formation of employment and the innovations in technology and of science). The five possible alternatives are to be evaluated by using the bipolar picture fuzzy decision matrix $B_pFDM$, $\hat{B}_p = [\Delta_{ij}]_{5 \times 4}$ presented in Table 1.

Step 1. Information provided by the DM is written as $BP_cFDM$, $B_{pc} = [b_{ij}]_{5 \times 4}$.

|       | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ |
|-------|-------|-------|-------|-------|
| $B_1$ | $[0.5,0.3,0.1,0,-0.2,-0.1,0.5]$ | $[0.8,0.1,0.1,-0.3,-0.4,-0.2]$ | $[0.5,0.3,0.1,-0.5,-0.3,-0.0]$ | $[0.9,0.0,0.1,-0.4,-0.3,-0.2]$ |
| $B_2$ | $[0.7,0.1,0.2,-0.3,-0.4,0.1]$ | $[0.1,0.6,0.2,-0.2,-0.1,0.3]$ | $[0.6,0.3,0.1,-0.4,-0.5,-0.1]$ | $[0.7,0.1,0.1,-0.4,-0.3,-0.1]$ |
| $B_3$ | $[0.5,0.0,0.2,-0.3,-0.4,-0.2]$ | $[0.8,0.1,0.0,-0.5,-0.3,-0.2]$ | $[0.1,0.8,0.1,-0.2,-0.1,0.6]$ | $[0.6,0.2,0.1,-0.4,-0.3,-0.2]$ |
| $B_4$ | $[0.5,0.0,0.2,-0.3,-0.4,-0.3]$ | $[0.1,0.1,0.4,-0.3,-0.4,-0.2]$ | $[0.3,0.5,0.2,-0.1,0.4,-0.3]$ | $[0.7,0.2,0.1,-0.4,-0.3,-0.1]$ |
| $B_5$ | $[0.5,0.4,0.0,-0.4,-0.5,-0.1]$ | $[0.6,0.2,0.1,-0.3,-0.7,0.0]$ | $[0.6,0.2,0.2,-0.3,-0.4,-0.3]$ | $[0.1,0.7,0.2,-0.3,-0.2,-0.1]$ |

Step 2. The $B_pFPIS$ denoted by $\Delta^+$ and $B_pFNIS$ represented by $\Delta^-$ are: $\Delta^+_{pc} = ([0.8,0.4,0.2,-0.4,-0.5], [0.8,0.6,0.2,-0.5,-0.4,-0.5], [0.6,0.8,0.2,-0.5,-0.4,-0.6], [0.9,0.2,0.4,-0.5,-0.5,-0.2]), \Delta^-_{pc} = ([0.5,0.1,0.0,-0.2,-0.1,0.1], [0.1,0.1,0.0,-0.2,-0.2,0.0], [0.1,0.2,0.1,-0.1,-0.1,0.0], [0.1,0.0,0.1,-0.3,-0.2,-0.1]).$

Step 3. Evaluate the degree of weighted similarity $\tilde{S}_p_{ij}$ between $B_pFPIS \Delta^+$ and each alternative as well as the degree of weighted similarity $\tilde{S}_p_{ij}$ between $B_pFNIS \Delta^-$ by using the Eqs. (4.1) and (4.2).

Step 4. Based on Eq. (4.3), we get the linear objective function $Z$ as:

$$Z = 0.8800w_1 + 1.6933w_2 + 1.6800w_3 + 0.8893w_4.$$

Step 5. Based on LP model as described in Section 2, compute the weights $w_j (j = 1, 2, 3, 4)$ of criteria with distinct limitation given below:

$$\max Z = 0.8800w_1 + 1.6933w_2 + 1.6800w_3 + 0.8893w_4,$$

subject to:

$$0.9000w_1 + 0.5000w_2 + 0.1000w_3 + 0.6000w_4 \geq 0.2000,$$

$$0.9000w_1 + 0.5000w_2 + 0.1000w_3 + 0.6000w_4 \leq 0.3500,$$

$$0.3000w_1 + 0.1100w_2 + 0.7000w_3 + 0.5000w_4 \geq 0.0500,$$

$$0.3000w_1 + 1.1000w_2 + 0.7000w_3 + 0.5000w_4 \leq 0.0550,$$

$$0.2000w_1 + 0.5000w_2 + 0.2000w_3 + 0.4000w_4 \geq 0.0300,$$

$$0.2000w_1 + 0.5000w_2 + 0.2000w_3 + 0.4000w_4 \leq 0.0350,$$

$$1.0000w_1 + 1.0000w_2 + 1.0000w_3 + 1.0000w_4 = 1,$$

$$0.1000 \leq w_1 \leq 0.20000,$$

$$0.2500 \leq w_2 \leq 0.3000,$$

$$0.3500 \leq w_3 \leq 0.4000,$$

$$0.1500 \leq w_4 \leq 0.2000,$$

we get, $w_1 = 0.1000; w_2 = 0.4000; w_3 = 0.3500$ and $w_4 = 0.1500$.

Step 6. On the basis of weights of criteria as obtained in Step 5, compute the degree of similarity $\tilde{S}_p^{\Delta^+}_{PC_i}$ and $\tilde{S}_p^{\Delta^-}_{PC_i}$ amongst each alternative and the elements obtained in $B_pFPIS \Delta^+$ and $B_pFNIS \Delta^-$, respectively,
we get: $\bar{S}_{PC1}^+ = 0.6230; \bar{S}_{PC2}^+ = 0.6230; \bar{S}_{PC3}^+ = 0.6500 \bar{S}_{PC4}^+ = 0.6660; \bar{S}_{PC5}^+ = 0.6410$, and $\bar{S}_{PC1}^- = 0.6610; \bar{S}_{PC2}^- = 0.6700; \bar{S}_{PC3}^- = 0.6420; \bar{S}_{PC4}^- = 0.6590; \bar{S}_{PC5}^- = 0.6520$.

**Step 7.** From Eq. (4.4), we obtain values of relative closeness $R_{C1}$ of each alternative $B_1$ with respect to the $B_PFPIS \Delta^+$ as:

$$R_{C1} = 0.4852; \quad R_{C2} = 0.4818; \quad R_{C3} = 0.5031; \quad R_{C4} = 0.5026; \quad R_{C5} = 0.4957.$$  

It reveals that, $R_{C3} \succ R_{C4} \succ R_{C5} \succ R_{C1} \succ R_{C2}$, $\Rightarrow B_3 \succ B_4 \succ B_5 \succ B_1 \succ B_2$ that is, $B_3$ is the best option or alternative.

6. Sensitivity analysis

In order to see the validity and stability of the proposed $B_PcF$-TOPSIS, a weighted sensitivity analysis is performed [11]. According to Mareschal [11], mostly MCDM techniques require the quantitative weights of the criteria which are sometimes difficult to get because we cannot be sure that the DMs have provided the precise weights to the criteria. Thereby, it is important to compute what changes occur by altering the weights of criteria. If there are minor or no changes happened then we are more confident about the results. In light of our performed sensitivity analysis, we examined the four criteria individually by increasing the weights from 2 to 10 percent randomly. We see that there is no minor change that happened in the arrangement of the criteria which represents that our $B_PcF$-TOPSIS MCDM model is effective and stronger.

| Relative closeness | Original Values | 2 percent increase | 5 percent increase | 10 percent increase |
|-------------------|----------------|-------------------|-------------------|-------------------|
| $R_{C1}$          | 0.4852         | 0.4847            | 0.4840            | 0.4828            |
| $R_{C2}$          | 0.4818         | 0.4813            | 0.4804            | 0.4799            |
| $R_{C3}$          | 0.5031         | 0.5032            | 0.5033            | 0.5036            |
| $R_{C4}$          | 0.5026         | 0.5027            | 0.5028            | 0.5031            |
| $R_{C5}$          | 0.4957         | 0.4956            | 0.4954            | 0.4951            |

7. Conclusions

We introduced the concept of bipolar picture fuzzy sets, operational rules, and extended the TOPSIS named $B_PcF$-TOPSIS in this article. On the basis of the novel distance measure, an MCDM model ($B_PcF$-TOPSIS) is developed to select the best alternative. A sensitivity analysis is performed to strengthen our MCDM approach. In the future, we shall establish aggregation operators like Bonferroni, and Hamy mean for $B_PcF$-FSs and implement these operators to solve the MCDM problems. Also, we shall present the concept of interval-valued bipolar picture fuzzy sets (IV$B_PcF$-FSs) and operational laws. Further, we shall apply the IV$B_PcF$-FSs in various group decision-making problems, like signature theory, signal processing, and operations management.

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