Deuterium burning in Jupiter interior

M. Coraddu\textsuperscript{a,b}, M. Lissia\textsuperscript{b,a}, G. Mezzorani\textsuperscript{a,b}, P. Quarati\textsuperscript{b,c}

\textsuperscript{a}Physics Dept., Univ. Cagliari, I-09042 Monserrato, Italy
\textsuperscript{b}I.N.F.N. Cagliari, I-09042 Monserrato, Italy
\textsuperscript{c}Phys. Dept. and INFM, Politecnico Torino, I-10125 Torino, Italy

Abstract

We show that moderate deviations from the Maxwell-Boltzmann energy distribution can increase deuterium reaction rates enough to contribute to the heating of Jupiter. These deviations are compatible with the violation of extensivity expected from temperature and density conditions inside Jupiter.

1 Introduction

Jupiter emits more radiation than it receives from the sun: the origin of this excess heat is still uncertain and debated. Possible explanations are: release of gravitational potential energy due to the planet contraction and/or Helium sedimentation [1], decaying of radioactive isotopes in the core [2], or deuterium burning [3]. Each of these hypotheses has difficulties [4]; in particular, standard calculations of deuterium burning reaction rates predict negligible contribution to the planet thermal balance, in spite of the substantial enhancement due to electron and ion screening effects [3,5].

In a strongly coupled plasma, anomalous diffusion and time correlation effects originate non-Maxwellian two-body relative energy distribution that can be parameterized with and, in same cases, assume the same functional form that appears in contest of Tsallis non-extensive thermodynamics [6,7]. As demonstrated for the solar core, small changes of the tail of the energy distribution can strongly modify the fusion rates without affecting mechanical properties (hydrostatic equilibrium and the sound speed) that depend on the mean value of the distribution [8].

Since the internal conditions of Jupiter indicate the existence of a strongly coupled plasma, we investigate the effects of the consequent small deviations from the standard Maxwell-Boltzmann (MB) statistics on deuterium burning
rates and the possibility that deuterium burning could play, or have played in
the past, a role in Jupiter thermal balance.

2 Jupiter interior and standard deuterium burning

Jupiter interior is a mixture of liquid metallic hydrogen and helium, with
density and temperature within the ranges: \( \rho_J = 2 - 5 \text{ g cm}^{-3} \) and \( kT = 1 - 2 \text{ eV} \); during planet formation the central temperature should have been of the
order of \( 10 - 20 \text{ eV} \) [4]. If we assume the reference values \( \rho_J = 5 \text{ g cm}^{-3} \) and \( kT = 2 \text{ eV} \), the corresponding density of H, D, He and electrons are:
\( n_p = 2.4 \times 10^{24} \text{ cm}^{-3} \), \( n_D = 3 \times 10^{-5} n_p \), \( n_{He} = 6.25 \times 10^{-2} n_p \), and \( n_e = 1.125 n_p \).

Being the electron Fermi energy, \( E_F \approx 50 \text{ eV} \), much greater than the thermal
energy, the electron gas is fully degenerated. The electron and ion plasma
parameters, \( \Gamma_j = (Ze^2)/(\alpha^2 kT) \), where \( \alpha = (3Z/(4\pi n))^{1/3} \) is the Wigner-
Seitz radius, are: \( \Gamma_e \approx \Gamma_i \approx 16 \). Therefore, the interior of Jupiter is a Strong
Coupled Degenerated Plasma.

Jupiter excess energy flux \( \Phi_J = 5.4 \times 10^3 \text{ erg cm}^{-2} \text{ sec}^{-1} \) implies an excess
luminosity: \( L_J = 3.5 \times 10^{24} \text{ erg sec}^{-1} \). Each D(p, \( \gamma \))\(^3\)He reaction produces
an energy \( Q_{pD} = 5.493 \text{ MeV} \). Therefore, deuterium burning is relevant to
Jupiter thermal balance only if the rate is greater than a threshold \( r_t = (\rho_J L_J)/(M_J Q_{pD}) \approx 1 \text{ sec}^{-1} \text{ cm}^{-3} \), where we have used as Jupiter mass and
radius: \( M_J = 1.90 \times 10^{33} \text{ g} \) and \( R_J = 7.14 \times 10^9 \text{ cm} \).

Two-body reaction rates in a thermal plasma can be expressed as
\( (1+\delta_{ij})r_{i,j} = n_i n_j \langle \sigma v \rangle_M \), where \( \langle \sigma v \rangle_M \) is the thermal average of the reaction cross section
\( \sigma \) times the relative velocity \( v \), and \( n_i \) is the number density of species \( i \).

Cross sections between charged particles below threshold are dominated by
the penetration factor \( e^{-b/\sqrt{E}} \), where \( b = \pi \sqrt{2\mu Z_i Z_j \mu Z_j e^2/h} \) and \( \mu \) is the particle
reduced mass. In fact the astrophysical factor \( S(E) = \sigma(E) E \exp(b\sqrt{E}) \) has
a mild dependence on \( E \) in absence of resonances.

There exist no experimental determinations of \( S(E) \) for deuterium reactions
at energies in the eV range. Theoretically motivated low-energy extrapolations
can increase the deuterium [9] are: \( S(0) = 2.0 \times 10^{-7} \text{ MeV b} \) for D(p, \( \gamma \))\(^3\)He,
\( S(0) = 5.0 \times 10^{-2} \text{MeV b} \) for D(d, n)\(^3\)He and \( S(0) = 5.6 \times 10^{-2} \text{ MeV b} \) for
D(d, p)\(^7\)T.

If the thermal distribution is Maxwellian, \( \langle \sigma v \rangle_M \sim \int_0^\infty E \sigma(E) e^{-E/kT} dE \), ther-
monuclear rates are dominated by the product of the exponentials \( e^{-E/kT} \) (MB
distribution) and \( e^{-b/\sqrt{E}} \) (penetration factor), product that has its maximum
at the Gamow Peak Energy, $E_0$. In the weak screening regime $E_s < E_0$ we can use a saddle point expansion around $E_0$ and find:

$$r_{i,j} = f_e f_i \frac{n_i n_j}{1 + \delta_{ij}} \sqrt{\frac{2}{\mu (kT)^{3/2}}} I_{\text{max}} \Delta$$

(1)

where $E_0 = (bkT/2)^{2/3}$ is the Gamow Energy, $I_{\text{max}} = \exp(-3E_0/kT)$ is the integrand at $E = E_0$, and $\Delta = 4(E_0kT/3)^{1/2}$ measures the width of the Gamow Peak. The two correction factors, $f_e = (1 - E_s/E_0) \exp(E_s/kT)$ for electrons and $f_i = \exp(H(0)/kT)$ for ions, take care of screening.

In fact screening is very important in Jupiter interior. Electron screening lowers the coulomb barrier of $E_s = e^2/D_s$, where $D_s$ is the short-range screening distance, which is determined through the incipient Rudberg method [5]. Many body correlations modify also ion wave functions in plasmas: it is possible to define an equivalent mean field potential $H(r)$ which is related to the plasma parameter $\Gamma$ in the context of Ion Sphere Model: $H(0)/kT = 1.057\Gamma_s$ where $\Gamma_s$ is the screened ion plasma factor $\Gamma_s = \Gamma_i \exp(-a/D_s)$ and $\Gamma_i$ is the ion plasma factor.

Despite the large enhancement factors ($f_e \sim 10^{11}$, $f_i \sim 10^2$) deuterium burning remains negligible: $r_{pD} \sim 10^{-19}$ and $r_{DD} \sim 10^{-26}$ sec$^{-1}$ cm$^{-3}$. Rates are much lower than the threshold rate $r_t$ for any relevant temperature.

3 Non-Maxwellian deuterium burning

Non extensive Thermodynamics, introduced by Tsallis [10] has been applied to many different fields. In particular, it has been observed [6,7] that small deviations from Maxwellian distribution can be described by weakly non-extensive Tsallis distributions, and that such deviations can have dramatic consequences for nuclear reaction rates in the solar core [8].

We can calculate the effects of non-extensivity on nuclear reaction rates by substituting the Maxwell-Boltzmann distribution, $\exp(-E/kT)$, with the Tsallis one, $[1 - (1 - q)E/kT]^{q/(1-q)}$, inside the thermal average:

$$\langle \sigma v \rangle_M \sim \int_0^\infty e^{-b\sqrt{E}} \left(1 - (1 - q)\frac{E}{kT}\right)^{q/(1-q)} S(E + E_s) \, dE.$$  

(2)

The limit $q \to 1$ recovers the Maxwellian case. Analogously to the Maxwellian calculation, we find a $q$-dependent Gamow Peak Energy $E_{0q}$, and, in the weak
screening condition $E_s < E_{0q}$, we can compute again the reaction rates at first order in the saddle point expansion:

$$r_{i,j,q} = \frac{n_i n_j}{1 + \delta_{ij}} \sqrt{\frac{2}{\mu}} \left( 1 - \frac{E_s}{E_{0q}} \right) \frac{S(E_{0q})}{(kT)^{3/2}} I_{q,\text{max}} \Delta_q,$$

where now $E_{0q} = E_0 q^2(T,q)/q^{2/3}$ is the $q$-dependent Gamow Peak Energy,

$$I_{q,\text{max}} = \exp \left[ -\frac{b}{\sqrt{E_{0q} + E_s}} + \frac{q}{1-q} \ln \left( 1 - (1-q) \frac{E_{0q}}{kT} \right) \right]$$

is the integrand at $E = E_{0q}$, and

$$\Delta_q = \Delta \left( \xi(T,q)/q^{1/3} \right)^{5/2} \left( 1 + \frac{q(1-q)2E_0/(3kT)\xi^5(T,q)/q^{5/3}}{(1-(1-q)(E_{0q} - E_s)/kT)^2} \right)^{-1/2}$$

measures the peak width. The other terms are: $a = (1-q)b^{2/3}/(4q^2kT)^{1/3}$, $b = 1 + (1-q)E_s/kT$, and

$$3\xi(T,q) = -a + 2^{-1/3} \left[ (27b - 2a^3 - c)^{1/3} + (27b - 2a^3 + c)^{1/3} \right]$$

with $c = \sqrt{27b(27b - 4a^3)}$, while $E_0$ and $\Delta$ are the same as in the MB case.

Note that it is not possible to factorize an electron enhancement factor as for the MB distribution.

4 Discussion and conclusions

In Fig. 1 deuterium reaction rates are plotted for two different values of the $q$ parameter. We can observe that moderate deviations $(q - 1) \sim 0.1$ from the Maxwellian distribution increase deuterium burning rates above the threshold rate $r_t \approx 1\text{sec}^{-1}\text{cm}^{-3}$; therefore, these processes should contribute to heat Jupiter at the present epoch ($T \approx 1-2\text{ eV}$). If we read the graphs for $T \approx 10-20\text{ eV}$, which corresponds to temperatures of the planet during its formation, we realize that it was sufficient a smaller value, $(q - 1) \sim 0.03$, to effect the thermal balance in that period.

Reaction rates of the order of that required to heat Jupiter do not cause decrease significantly the deuterium density inside the planet; in fact a burning
rate equal to ten times the threshold rate would consume a significant fraction of deuterium only after a time of the order of $n_D/(10r_1) \sim 10^{11}$ years.

These considerations demonstrate that deuterium burning is a possible explanation for the Jupiter excess heat. Precise determinations of the conditions inside Jupiter and additional microscopic calculations are necessary for a better determination of the range of values of $q$ relevant to Jupiter interior.

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Fig. 1. Reaction rates for the reactions p+D (solid), D+D (dashed) and p+p (dot-dashed) as function of temperature for two values of the Tsallis parameter: $q = 1.03$ (upper frame) and $q = 1.1$ (lower frame).