Exotic nuclei viewed as clusters in the crust of neutron stars

E. Khan\textsuperscript{1}, M. Grasso\textsuperscript{1}, J. Margueron\textsuperscript{1}, Nguyen Van Giai\textsuperscript{1}, N. Sandulescu\textsuperscript{1,2}

\textsuperscript{1} Institut de Physique Nucléaire, Université Paris-Sud, IN2P3-CNRS
Orsay, F-91406, France
\textsuperscript{2} Institute for Physics and Nuclear Engineering, P.O. Box MG-6, 76900 Bucharest, Romania

E-mail: khan@ipno.in2p3.fr

Abstract.

In the inner crust of neutron stars there are clusters which have a similar structure with the neutron-rich nuclei. The clusters and the neutron gas in which they are immersed are commonly treated in the independent Wigner-Seitz cells approximation. Low-energy excitations are analyzed in these systems. The crust of neutron stars provides a unique framework for studying the evolution of low-lying modes from neutron-rich nuclei to a pure neutron gas. It is shown that the excitations with low multipolarities are concentrated almost entirely in one strongly collective mode which exhausts a very large fraction of the energy-weighted sum rule. Since these collective modes are located at very low energies compared to the giant resonances in standard nuclei, they may affect significantly the specific heat of baryonic inner crust matter of neutron stars. Investigations considering interaction between the clusters are called for.

1. Introduction

The crust of neutron stars provide a unique opportunity for investigating the hydrodynamic versus the microscopical picture of non-uniform nuclear matter. In the outer crust of these astrophysical objects, exotic nuclei form a Coulomb lattice in the presence of an electron gas [1]. When one goes deeply in the interior of the crust, density increases and nuclei are more and more neutron-rich up to the drip density $\rho \sim 4 \times 10^{11}$ g/cm$^3$ [2]. Beyond this limit neutrons drip out and very exotic structures of clusters surrounded by relativistic electrons and superfluid neutrons form a Coulomb lattice (inner crust). Low-lying collective excitations in the inner crust of neutron stars have been microscopically analyzed in Ref. [3]. By modeling the Coulomb lattice with spherical Wigner-Seitz (WS) cells [4], each cell has been studied as independent of all the others within the Hartree-Fock-Bogoliubov [5, 6] + quasiparticle random-phase approximation [7] (HFB + QRPA). The role of the low-lying collective modes, the supergiant resonances, has been for the first time underlined in Ref. [3] in connection with the evaluation of the specific heat and the cooling time [8] of a star. The WS cells which mostly contribute to the cooling time depend strongly on the pairing gap at low density [9], and the largest contribution comes from the intermediate and low density parts of the inner crust [9, 10] where the central cluster is most probably spherical.

Apart from the evolution from finite to infinite systems, inner crust WS cells provide also the opportunity to analyze strong asymmetries, i.e. very high excesses of neutrons with respect
to protons. The challenge of designing the nucleon-nucleon interaction for very neutron-rich systems is still an open question [11]. It is therefore necessary to develop a method involving experimental data on neutron-rich nuclei. A procedure to constrain the nucleon-nucleon interaction, aiming to accurately predict the properties of WS cells, is proposed. In this spirit these systems can be viewed as extreme extrapolations of very neutron-rich nuclei.

In the last years, several projects for a new generation of radioactive nuclear beam facilities have been started and are presently in progress [12]. These facilities will allow to explore more systematically regions closer to the drip lines. In the next decade more and more exotic nuclei will be produced and their properties will be experimentally accessible; the impact of strong neutron excess on low-lying excitations will be investigated.

In all the calculations mentioned above the specific heat was evaluated by considering only non-interacting quasiparticles states. However, the specific heat can be also strongly affected by the collective modes created by the residual interaction between the quasiparticles, especially if these modes appear at low-excitation energies.

2. Microscopic description of the crust

Free neutrons are not distinguished from those which are bound. One can show that the most favorable configuration in the cell is obtained when protons are clustered at its center. We can in this sense picture it as composed of a central nuclear cluster surrounded by an interacting gas of superfluid neutrons.

For both the nucleus and the WS cell we perform microscopic HFB + QRPA calculations at zero temperature and in spherical symmetry. We use the Skyrme interaction SLy4 [13] for the mean field and a zero-range density dependent interaction for pairing. As in Ref. [3] the parameters of the pairing interaction are chosen in order to have the same gap as obtained with a Gogny force [14].

We consider first ground state properties of the two systems. Within the HFB approach we have calculated the neutron and proton density profiles,

$$\rho_{q}(r) = \sum_{\nu l j} \frac{2j + 1}{4\pi} v_{\nu l j q}(r),$$

where $q$ stands for $n$ (neutrons) or $p$ (protons) and $\nu$, $l$, $j$ are the quantum numbers associated to the wave function $v$; $v$ indicates the lower component of the HFB quasiparticle wave function, solution of the equations,

$$H(r)u_{\nu l j q}(r) + \Delta(r)v_{\nu l j q}(r) = E_{\nu l j q}u_{\nu l j q}(r)$$

and

$$\Delta(r)u_{\nu l j q}(r) - H(r)v_{\nu l j q}(r) = E_{\nu l j q}v_{\nu l j q}(r).$$

In Eqs. (2,3), $H(r)$ contains the kinetic term and the Hartree-Fock mean field while $\Delta(r)$ represents the pairing field; $u$ and $v$ are the upper and lower components of the wave function and $E$ the corresponding quasiparticle energy. We have imposed Dirichlet (Von Neuman) boundary conditions for the even-(odd-)l wave functions.

Fig. 1 shows the density profile obtained for the $^{1800}$Sn and $^{982}$Ge cells. A cluster of nucleons is predicted around the center of the cell. At large radii, the density of the neutron gas depends on the cell, i.e. its location in the crust.

The QRPA equations, which correspond to the small-amplitude limit of the time-dependent HFB equations, are solved in coordinate representation. We integrate the Bethe-Salpeter equation for the QRPA Green function $G$,

$$G = (1 - G_0 V)^{-1} G_0,$$
Partial densities calculated with HFB for the WS cells $^{1800}$Sn (solid line) and $^{982}$Ge (dashed line).

where $G_0$ is the unperturbed Green function and $V$ the residual interaction, and we evaluate the response function $S$,

$$S(\omega) = -\frac{1}{\pi} Im \int d\vec{r} d\vec{r}' F^*(\vec{r}) G(\vec{r}, \vec{r}'; \omega) F(\vec{r}')$$

(5)

where $F$ is the excitation operator. Only particle-hole components of $G$ and $F$ appear in Eq. (5) [7].

3. Supergiant resonances in the WS cells

Within the HFB+QRPA formalism presented above we calculate the monopole neutron response in the cell $^{1800}$Sn. The unperturbed HFB response, built by non-interacting quasiparticle states, and the QRPA response are shown in Figure 2. As can be clearly seen, when the residual interaction is introduced among the quasiparticles the unperturbed spectrum, distributed over a large energy region, is gathered almost entirely in a peak located at about 3 MeV. This mode is extremely collective [3] and is denoted as supergiant resonance. This underlines the fact that this WS cell cannot be simply considered as a giant nucleus. As discussed below, the reason is that in this cell the collective dynamics of the neutron gas dominates over the cluster contribution. Apart from the monopole mode discussed above, we have also investigated the response of the WS cell to the dipole and quadrupole excitations: they display similar features, leading to the same qualitative conclusions.

This response can be compared with the one obtained in very neutron-rich nuclei. Let us take the example of $^{122}$Zr and the $^{1500}$Zr WS cell. In the neutron quadrupole spectrum of $^{122}$Zr we identify a low-lying $2^+$ state and a giant resonance region at higher energies. Due to the residual interaction, the low-lying $2^+$ state is split into two peaks at about 2 MeV and 3.2 MeV. In comparison the strength of the cell is concentrated around a very collective low-lying state of about 3.5 MeV (supergiant mode). It should be noted that the strength of the giant resonance is negligible with respect to the strength associated to the low-lying mode. This is due to the fact that the giant resonance is built with the contribution of particles bound to the
central cluster while a huge number of particles belonging to both the cluster and the free gas participate to the low-energy supergiant mode. To analyze the respective contributions to the mode, we have evaluated the transition density associated to the supergiant resonance in the cell, \( \delta \rho(r) = \rho(r) - \rho_0(r) \), where \( \rho_0 \) represents the equilibrium density and \( \rho \) the density of the excited system. The analysis of the radial profile of the transition density gives indications on the location of the particles which mainly participate to the excitation. Apart from a strong contribution in the external region where the free neutron gas is located, there is an important contribution at the cluster surface where the transition density is peaked. This means that the supergiant excitation is partly composed of two-quasiparticle configurations localized at the cluster surface. The internal part of neutron and proton transition densities are very similar to typical nuclei transition densities [7].

In order to trace the evolution of the low-lying excitations from neutron-rich nuclei to WS cells, QRPA calculations have been performed for systems located in between, that is beyond the drip-line. Fig. 3 shows the quadrupole response in \(^{300}\text{Sn}\). One observe the giant resonance, and already a strong low lying state. The magnitude of this state is due, as mentioned above, to the neutron of the continuum, which can have large spatial delocalisation. Hence supergiant resonances cannot be considered as the limit of low energy components of giant resonances but are rather generated by the neutron of the continuum. However their energy position depends on the single quasiparticle spectrum. As a conclusion, we have identified two contributions to the supergiant mode: the gas and the cluster surface.

On this basis, the following method should be observed in order to predict low-energy modes in the WS cells. The nucleon-nucleon interaction should be designed in order to reproduce the measured transition densities of the analogous neutron-rich nuclei (\(^{122}\text{Zr}\) in this case or lighter Zr isotopes). The combination between Coulomb excitation and proton scattering has proven to be an accurate tool to probe transition densities in neutron-rich nuclei, using the present HFB+QRPA approach ([15] and references therein). Once validated, this nucleon-nucleon interaction can be used to perform microscopic calculations in the WS cells. Note that this method is similar to that proposed to deduce the nuclear matter incompressibility from the

\[ S(E^*) (\text{fmMeV}^{-1}) \]

\[ E^* (\text{MeV}) \]

\( L=0 \)

**Figure 2.** Neutron monopole strength distributions for \(^{180}\text{Sn}\). The solid and dashed lines are the QRPA and unperturbed strengths, respectively.
measurements of the giant monopole resonances in nuclei [16].

The contribution due to the neutron gas in the external region suggests that a neutron gas filling the same volume of the cell should display a very similar quadrupole response. We have actually verified that decreasing the proton number by one order of magnitude the resulting neutron response has a peak shifted of only \(\sim 100 \text{ keV}\) with respect to the low-lying peak in the cell.

4. Conclusions
Nuclear clusters are predicted in the crust of neutron stars. Collective excitations in these systems have also been studied. The supergiant mode of the WS cells is built on two contributions, one due to neutrons in the cluster surface and the other due to neutrons in the external free gas. To predict the supergiant mode of the WS cell, we propose a method based on the experimental analysis of the low-lying \(2^+\) mode in neutron-rich Zr and Sn isotopes and on designing the nucleon-nucleon interaction for neutron-rich systems. Measurements of \(2^+\) modes for nuclei with extended neutron skins are therefore relevant for the prediction of neutron stars cooling properties. A theoretical description beyond the independent WS approximation is also of interest, and the clusters interaction in the crust should be taken into account in order to predict astrophysical quantities such as the specific heat of the crust.

[1] E.E. Salpeter, Astrophys. J. 134, 669 (1961).
[2] G. Baym, C. Pethick, and P. Sutherland, Astrophys. J. 170, 299 (1971).
[3] E. Khan, N. Sandulescu, and Nguyen Van Giai, Phys. Rev. C 71, 042801(R) (2005).
[4] J.W. Negele and D. Vautherin, Nucl. Phys. A 207, 298 (1973).
[5] P.G. de Gennes Superconductivity of Metals and Alloys (Addison-Wesley, Reading, MA, 1989).
[6] J. Dobaczewski, H. Flocard, and J. Treiner, Nucl. Phys. A 422, 103 (1984); M. Grasso, N. Sandulescu, N. Van Giai and R.J. Liotta, Phys. Rev. C 64, 064321 (2001); N. Sandulescu, Nguyen Van Giai, and R. J. Liotta, Phys. Rev. C69 045802 (2004); N. Sandulescu, Phys. Rev. C70 025801 (2004).
[7] E. Khan, N. Sandulescu, M. Grasso, and Nguyen Van Giai, Phys. Rev. C 66, 024309 (2002).
[8] J.M. Lattimer, K.A. Van Riper, M. Prakash and M. Prakash, Astro. J. 425, 802 (1994).
[9] C. Monroeau, J. Margueron, N. Sandulescu, Phys. Rev. C75 065807 (2007)
[10] J. Margueron, J. Navarro, and P. Blottiau, Phys. Rev. C 70, 028801 (2004).
[11] T. Otsuka, et al., Phys. Rev. Lett 95, 232502 (2005).
[12] Proceedings of the 17th International Conference on Cyclotrons and their Applications (October 2004), Tokyo.
[13] E. Chabanat, et al., Nucl. Phys. A 635, 231 (1998).
[14] G.F. Bertsch and H. Esbensen, Ann. Phys. (NY) 209, 327 (1991); E. Garrido, P. Sarriguren, E. Moya de Guerra, and P. Schuck, Phys. Rev. C 60, 064312 (1999).
[15] E. Becheva, et al., Phys. Rev. Lett. 96, 012501 (2006).
[16] J.P. Blaizot, Phys. Rep. 64, 171 (1980).