On the Non Unitary Neutrino Mixing Matrix in 331 Model with Three Higgs Triplets

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Abstract. The neutrino mixing phenomenon is studied within the non unitary Pontecorvo–Maki–Nakagawa–Sakata modified matrix (PMNS) in the context of the low energy limit of the minimal 331 model without right handed neutrinos as a deviation from the standard model. Moreover, comparison with the recent experimental data gives some stringent bounds on some physical parameters.

1. Introduction
During the last decade, many interest has been devoted to the study of the neutrino oscillation phenomenon [1]-[9], characterized by another mixing matrix similar to those of the quarks (Cabibbo-Kobayachi-Maskawa (CKM)) called Pontecorvo–Maki–Nakagawa–Sakata modified matrix (PMNS) [10]-[12]. It turns out that these mixing matrices are unitary within the standard interactions [13]-[16]. However, for non standard interactions (e.g. those represented by dimension five or six operators etc.) resulting as a low energy terms of a fundamental theory, the mixing matrix may become non unitary. Of course these non renormalizable operators are made of the standard model (SM) fields and it is renormalizable under the SM gauge group. It is very important to mention that the latest experimental data shows that it existsa possible window for small deviations towards non unitarity. The goal of this paper is to study such an effect within the general 331 model without right handed neutrinos[17] and the impact of the heavy mediators on neutrino oscillation phenomenon described by an effective non standard four fermions interaction (NSI) at low energies and its impact on the unitarity of the mixing angle matrix. In section 1, we present a brief review of the existing general theoretical approaches for this non unitary mixing matrix. In section 2, we consider the minimal 331 model with three triplets and sextet of higgs fields as a deviation from the standard model (NSI). In this case, the corresponding non unitary matrix diagonal elements are derived. Finally, at section 3, we discuss our results and draw conclusions.

2. The Model
The model is based on the gauge group $SU_L(3) \otimes U_X(1)$ and is anomaly free if the sum of all fermions (leptons and quarks) charges vanish and the number of families must be divisible by the number of color degrees of freedom. The matter (leptons and quarks) content in this model are with respect to the $SU_L(3)$ gauge group action:

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i) Left handed triplets denoted by $\psi_L$ consists of the two quarks generations:

$$\psi_L = \begin{pmatrix} u_L \\ d_L \\ s_L \\ c_L \end{pmatrix} \quad (1)$$

such that:

$$\hat{Q}_W \psi_L = \begin{pmatrix} \frac{1}{2} + \frac{\beta}{2\sqrt{3}} + X & 0 & 0 \\ 0 & -\frac{1}{2} + \frac{\beta}{2\sqrt{3}} + X & 0 \\ 0 & 0 & -\frac{\beta}{\sqrt{3}} + X \end{pmatrix} \psi_L \quad (2)$$

and

$$D_\mu \psi_L = \partial_\mu \psi_L - igW_\mu^a T^a \psi_L - ig_x X \mu T^9 \psi_L \quad (3)$$

Notice that the third components of the two triplets are new heavy quarks.

ii) Left handed antitriplets $\bar{\psi}_L$ consist of the 3 leptons and third quark generations:

$$\bar{\psi}_L = \begin{pmatrix} e_L \\ -\nu_e \\ \mu_L \\ -\nu_\mu \\ \tau_L \\ -\nu_\tau \\ b_L \\ d_L \\ s_L \end{pmatrix} \quad (4)$$

such that:

$$\hat{Q}_W \bar{\psi}_L = \begin{pmatrix} -\frac{1}{2} - \frac{\beta}{2\sqrt{3}} + X & 0 & 0 \\ 0 & \frac{1}{2} - \frac{\beta}{2\sqrt{3}} + X & 0 \\ 0 & 0 & \frac{\beta}{\sqrt{3}} + X \end{pmatrix} \bar{\psi}_L \quad (5)$$

and

$$D_\mu \bar{\psi}_L = \partial_\mu \bar{\psi}_L + igW_\mu^a T^a \bar{\psi}_L + ig_x X \mu T^9 \bar{\psi}_L \quad (6)$$

Notice that the 3rd member of the anti-triplet is chosen in a different manner depending on the considered variant of the model. In the minimal version of the 331 model where $\beta = \sqrt{3}$, one has: $E_i = l^c (l = e, \mu, \tau)$. In the case of $\beta = \frac{1}{\sqrt{3}}$, we have $E_i = \nu_i^c$ (c stands for conjugate).

iii) Right handed Singlets denoted by $\psi_R$ consist of all right handed quarks and leptons and such that:

$$\hat{Q}_W \psi_R = X \psi_R \quad (7)$$

and

$$D_\mu \psi_R = \partial_\mu \psi_R - ig_x X \mu T^9 \psi_R \quad (8)$$

where the $SU_L(3)$ generators are:

$$T^a = \frac{1}{2} a = \frac{1}{\sqrt{6}} T^9 = \frac{1}{\sqrt{6}}$$

such that:
\[ T \rho(T^a T^b) = \frac{\delta_{ab}}{2} \]  

(10)

And in the conjugate SU(3) representation the generators are:

\[ \hat{T}^a = -(T^a)^t \]  

(11)

\( \lambda^a \) are the Gell-man matrices and \( gX \) denote the couplings constants and \( X \) is the new additive quantum number corresponding to \( U_X(1) \) gauge group and it is the same for all members of a given (anti)triplet. The electric charge generator is related to the weak third isospin component \( \hat{T}^3 \) and hypercharge \( \hat{Y} \) through the Nishijima like relation:

\[ \hat{Q}_W = \hat{T}^3 + \frac{\hat{Y}}{2} = \beta \hat{T}^3 + X \]  

(12)

The gauge bosons matrix can be written as

\[
W_\mu = W_\mu^a T^a = 
\begin{pmatrix}
W^+_\mu & \sqrt{2} W^0_\mu & \sqrt{2} Y^Q_\mu \\
\sqrt{2} W^-_\mu & -W^3_\mu + \frac{1}{\sqrt{3}} W^8_\mu & \sqrt{2} Y^Q_\mu \\
\sqrt{2} Y^{-Q}_\mu & \sqrt{2} Y^{-Q}_\mu & \frac{-2}{\sqrt{3}} W^8_\mu
\end{pmatrix}
\]  

(13)

such that

\[
\hat{Q}_W W_\mu = \frac{1}{2}
\begin{pmatrix}
0 & 1 & \frac{1}{2} + \frac{\beta \sqrt{3}}{2} \\
-1 & 0 & -\frac{1}{2} + \frac{\beta \sqrt{3}}{2} \\
\frac{1}{2} - \frac{\beta \sqrt{3}}{2} & \frac{1}{2} - \frac{\beta \sqrt{3}}{2} & 0
\end{pmatrix} W_\mu
\]  

(14)

where

\[
W^\pm_\mu = \frac{1}{\sqrt{2}} (W^A_\mu \mp W^5_\mu) Y^Q_\mu = \frac{1}{\sqrt{2}} (W^A_\mu \mp W^5_\mu) Y^Q_\mu = \frac{1}{\sqrt{2}} (W^6_\mu \mp W^7_\mu)
\]  

(15)

It is worth to mention that the constant \( \beta \) plays a key role in the model according to its value various versions of the model have been proposed, which significantly differ from each other. In particular, only for some values of \( \beta \) the gauge bosons turn out to have an integer charges.

Choosing \( \beta = \frac{1}{\sqrt{3}} \), there are two charged bosons \( Y^\pm \) and two additional neutral gauge bosons \( V^0 \) and \( \tilde{V}^0 \). For comparison, in the minimal model \( (\beta = \sqrt{3}) \), one finds instead two charged bosons \( V^\pm \) and two doubly charged \( Y^{\pm \pm} \). In the Higgs sector, in order to have the conservation of the quantum number \( X \), the Higgs field in the invariant Yukawa terms involving a fermion triplet or a singlet has to be transformed either as triplets or a sextet. The triplets are denoted by as \( \chi, \rho \) and \( \eta \) such that:

\[
\chi = \begin{pmatrix}
\chi^A \\
\chi^B \\
\chi^0
\end{pmatrix}, \quad \rho = \begin{pmatrix}
\rho^+ \\
\rho^0 \\
\rho^-
\end{pmatrix}, \quad \eta = \begin{pmatrix}
\eta^0 \\
\eta^- \\
\eta^A
\end{pmatrix}
\]  

(16)

and the sextet by \( S \). The symmetry breaking is accomplished in two steps. At first, one has to recover the standard model as a low energy effective theory deriving from the 331 model after the spontaneous symmetry breaking \( SU_3(3) \otimes U_X(1) \rightarrow SU_2(2) \otimes U_X(1) \) at a given scale, that has to be properly identified with a vev of a Higgs field. The choice for this is
\[ \langle \chi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}, \quad \langle q \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \eta \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v' \end{pmatrix}, \quad \langle S \rangle = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & w & 0 \end{pmatrix} \]  

(17)

In this model we have six charged gauge bosons \( W^\pm, V^\pm q_v, Y^\pm q_v \) and two neutral gauge bosons \( Z \) and \( Z' \). After SSB and Higgs mechanism they acquire the following masses:

\[ M_W^2 = \frac{g^2}{4} v^2 Z^2, \quad M_Z^2 = \frac{g^2}{4} u^2 \left( 1 - \frac{v^2}{2u^2} + \frac{v'^2}{2u^2} \right), \]

(18)

\[ M_Y^2 = \frac{g^2}{4} u^2 \left( 1 + \frac{v^2}{2u^2} + \frac{v'^2}{2u^2} + 4w^2 \right), \]

(19)

\[ M_{Z'}^2 = \frac{g^2 u^2 c_w}{3(1 - \beta^2)} \left( 1 + \frac{v^2}{2u^2} \left( 1 + \beta^2 \frac{s_w^2}{c_w^2} \right) + \frac{v'^2}{2u^2} \left( \sqrt{3} \beta \frac{s_w^2}{c_w} \right) \right), \]

(20)

where

\[ v^2 = v^2 + v'^2 + w^2 v'^2 = v'^2 - v^2 - w^2. \]  

(22)

We remind that the following hierarchy among the vev's should hold \( u \gg v, v', w \). The lagrangian density of the charged and neutral currents in the leptonic sector is:

1)- Charged current mediated by the \( W^\pm \) bosons:

\[ \mathcal{L}^{CC}_W = \frac{g}{\sqrt{2}} \sum_{l=e, \mu, \tau} [\bar{v}_l Y^i_l l\gamma^2 W^+ \mu + \bar{v}_l Y^i_l l\gamma^2 W^- \mu] \]

(23)

2)- Neutral current mediated by \( Z \) boson:

\[ \mathcal{L}^{NC}_Z = \frac{g}{2c_w} \sum_{l=e, \mu, \tau} [\bar{v}_l Y^i_l l\gamma^2 l] - (1 - 2s_w^2) \bar{\tilde{t}}_l Y^i_l l + 2s_w^2 \bar{\tilde{t}}_R Y^i_l l + (1 - \sqrt{3} \beta) s_w^2 \bar{\tilde{e}}_l Y^i_l l + \bar{\tilde{e}}_l Y^i_l l] Z^\mu \]

(24)

3)- Current mediated by \( V^{\pm q_v} \):

\[ \mathcal{L}_V = \frac{g}{\sqrt{2}} \sum_{l=e, \mu, \tau} [\bar{v}_l Y^i_l l V^{-q_v} \mu + \bar{\tilde{e}}_l Y^i_l l V^{+q_v} \mu] \]

(25)

4)- Current mediated by \( Y^{\pm q_v} \):

\[ \mathcal{L}_Y = \frac{g}{\sqrt{2}} \sum_{l=e, \mu, \tau} [\bar{L}_l Y^i_l l Y^{-q_v} \mu + \bar{\tilde{e}}_l Y^i_l l Y^{+q_v} \mu] \]

(26)
5)- Neutral current mediated by $Z'$:

$$Q_{Z'}^{NC} = \frac{g}{2\sqrt{3}c_w(1-(1+\beta^2)s_w^2)} \sum_{l=e,\mu,\tau} [ \left( 1 + \sqrt{3} \beta \right) s_w^2 (\bar{v}_{l'i}' v_{iL} + \bar{l}_i y_{\mu} l_L) - 2\sqrt{3} \beta s_w^2 \bar{l}_{R'i} y_{\mu} l_{R'} - [2 - (2 - \sqrt{3} \beta (1 - \sqrt{3} \beta) s_w^2)] E_{l'i}' E_{lL} - \sqrt{3} \beta (1 - \sqrt{3} \beta) s_w^2 E_{l'i} y_{\mu} E_{lR} ] Z'^\mu ]$$

where $c_w = \cos \theta_w = \frac{M_w}{M_Z}, \ g^2 = 4\sqrt{2} M_Z^2 G_F$ and $s_w = \sin \theta_w$ ($G_F$ is the Fermi constant). It is very important to mention that the difference between this model and the standard model is the existence of the flavour changing neutral current related to the $Z'$ in the quark sector. This is due essentially to the different treatment of the third generation.

Now, one of the practical way to describe neutrino new interactions with matter is the so called non standard neutrino interactions (NSI) which consists of parametrizing the effects of new physics in neutrino oscillations. NSI for leptons operators are contained in the following lagrangian density:

$$\mathcal{L}_{eff}^{NSI} = -2\sqrt{2} G_F \sum_{\alpha, \beta} e_{\alpha \beta}^{LP} [\bar{v}_{\alpha} y_{\alpha} P] [\bar{v}_{\beta} y_{\beta} P v_{\beta}] \quad P = L, R$$

where $R = (1 + y^2)/2$ and $L = (1 - y^2)/2$. Here $l = e, \mu, \tau$ and $e_{\alpha \beta}^{LP}$ encodes the deviation from the standard model interactions between neutrino of flavor $\alpha$ with component $P$-handed of leptons, resulting in a neutrinos of flavor $\beta$. It is very important to mention that some of the parameters $e_{\alpha \beta}^{LP}$ can be extracted for example from the expressions of the differential of the cross sections for the elastic scattering processes $\nu_{\alpha} + e \rightarrow \nu_{\alpha} + e$. In fact, In the context of the $SU(3)_c \otimes SU(3)_L \otimes U(1)_N$ gauge group and after straightforward but tedious calculations, one can show that the differential cross section $\frac{d\sigma}{dy}$ has the following expression

$$\frac{d\sigma}{dy} = \frac{m_e E_e}{4\pi} [\xi_1^{ij} + \xi_2^{ij} (1 - y)^2 - \frac{m_e}{E_e} \xi_3^{ij} y]$$

Where $m_e$ is the electron rest mass, $E_e$ is the incident neutrino energy, $y = T_e/E_e$ is the rapidity ($T_e$ is the electron recoil energy at the final state) and

$$\xi_1^{ij} = (\Omega_1^{ij} \tilde{\Omega}_1^{ij} + \Omega_2^{ij} \tilde{\Omega}_2^{ij})/(M_{V_1}^2 M_{V_2}^2)$$

$$\xi_2^{ij} = (\Omega_1^{ij} \tilde{\Omega}_1^{ij} - \Omega_2^{ij} \tilde{\Omega}_2^{ij})/(M_{V_1}^2 M_{V_2}^2)$$

$$\xi_3^{ij} = (\Omega_1^{ij} \tilde{\Omega}_3^{ij} - 2\Omega_1^{ij} \tilde{\Omega}_2^{ij} + 2\Omega_2^{ij} \tilde{\Omega}_2^{ij})/(M_{V_1}^2 M_{V_2}^2)$$

with

$$\Omega_1^{ij} = A_i A_j + B_i B_j$$

$$\tilde{\Omega}_1^{ij} = \tilde{A}_i \tilde{A}_j + \tilde{B}_i \tilde{B}_j$$

$$\Omega_2^{ij} = B_i A_j - A_i B_j$$

$$\tilde{\Omega}_2^{ij} = B_i A_j - A_i B_j$$

$$\Omega_3^{ij} = A_i A_j + B_i B_j$$

$$\tilde{\Omega}_3^{ij} = \tilde{A}_i \tilde{A}_j + \tilde{B}_i \tilde{B}_j$$

$$A_1 = B_1 = \frac{4}{3}$$
\[ A_2 = B_2 = \frac{g}{4\sqrt{3}c_w \sqrt{1 - (1 + \sqrt{3}\beta) s_w^2}} [1 - (1 + \sqrt{3}\beta) s_w^2] \]

\[ A_3 = B_3 = A'_3 = B'_3 = \frac{g}{2\sqrt{2}} \]

\[ \tilde{A}_2 = \frac{g}{4\sqrt{3}c_w \sqrt{1 - (1 + \beta^2) s_w^2}} [1 - (1 + 3\sqrt{3}\beta) s_w^2] \]

\[ \tilde{B}_1 = \frac{g}{4c_w} (-\frac{1}{2}) \]

\[ \tilde{B}_2 = \frac{g}{4\sqrt{3}c_w \sqrt{1 - (1 - \sqrt{3}\beta) s_w^2}} [1 - (1 - \sqrt{3}\beta) s_w^2] \]

Now, it is easy to show that:

\[ |1 + \tilde{e}_{ee}^{L}| = \frac{4M_W^2}{g^2 (c_H^2 + c_A^2)} \sum_{L=0}^{1} \left| \frac{\rho_{11}^{ij} \rho_{11}^{ij} + \rho_{11}^{ij} \rho_{11}^{ij}}{M_W^2} \right| \]

\[ |1 + \tilde{e}_{ee}^{R}| = \frac{4M_W^2}{g^2 (c_A^2 - c_H^2)} \sum_{L=0}^{1} \left| \frac{\rho_{11}^{ij} \rho_{11}^{ij} - \rho_{11}^{ij} \rho_{11}^{ij}}{M_W^2} \right| \]

where \( c_H^2 = \frac{1}{2} + 2s_w \), \( c_A^2 = \frac{1}{2} \), \( s_w = \sin \theta_w \), \( M_{V_1}^2 = M_Z^2 \), \( M_{V_2} = M_{V_2'} \), and \( M_{\tilde{V}_3} = M_{W^+} \), and \( \tilde{e}_{ee}^{L} = \tilde{e}_{ee}^{L} / g_L \) = \( \tilde{e}_{ee}^{R} = \tilde{e}_{ee}^{R} / g_R \) are the relative left handed and right handed non unitary parameters (\( g_L \) and \( g_R \) are the total left and right handed couplings in the standard model charged and neutral currents).

3. Conclusions

Through this paper, we have derived some of the parameters encoding the deviation from the unitarity of the Pontecorvo–Maki–Nakagawa–Sakata modified neutrino mixing matrix (PMNS) in the context of the low energy limit of the minimal 331 model without right handed neutrinos. (more development of the subject and phenomenological study are under investigations).

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References

[1] Gonzalez-Garcia M Cand Maltoni M2008 Physics Reports460129.
[2] Cohen A G, Glashow S L and Ligeti Z 2009 Physics Letters B 678191.
[3] Pontecorvo B 1967 Sov. Phys. JETP 26 984.
[4] Davis Jr. R, Harmer D S and Hoffman K C 1968 Physical Review Letters 201205.
[5] Gribov V and Pontecorvo B 1969 Physics Letters B 28 493.
[6] Schechter J and Valle J W F 1980 Physical Review D 22 2227.
[7] Nakamura K et al. 2010 Journal of Physics G 37 1.
[8] Valle J W F 2006 Journal of Physics: Conference Series 53 473
[9] Boussahel M and Mebarki N 2011 Int. J. Mod. Phys. A 26 873.
[10] Maki Z, Nakagawa M and Sakata S 1962 Prog. Theor. Phys. 28 870.
[11] Bilenky S M, Hosek J and Petcov S T 1980 Phys. Lett. B 94 495.
[12] Schechter J and Valle J W F 1980, Phys. Rev. D 22 227.
[13] Doi M, Kotani T, Nishiura H, Okuda K and Takasugi E 1981 Phys. Lett. B 102 323.
[14] Beringer J et al. 2012 *Phys. Rev.* D86, 010001.
[15] Gonzalez-Garcia M C, Maltoni M and Schwetz T 2014 *JHEP* 1411 052.
[16] Altarelli G and Feruglio F 2010 *Rev. Mod. Phys.* 82 2701.
[17] Buras A J et al. 2013 *JHEP* 02 2023.