Anomalous viscosity of vortex hall states in graphene

M Rabiu1,‡, S Y Mensah2, S Y Ibrahim3 and S S Abukari2
1University for Development Studies, Faculty of Applied Science, Department of Applied Physics, Navrongo, GHANA
2University of Cape Coast, Laser and Fibre Optics Center, Cape Coast, GHANA
3University for Development Studies, Faculty of Mathematical Science, Department of Mathematics, Navrongo, GHANA
E-mail: mrabiu@uds.edu.gh

Abstract. In this work, we studied anomalous viscosity, $\eta$ of Fractional Quantum Hall States (FQHSs) in Graphene within quantum many-vortex hydrodynamics picture of Euler hydrodynamics. We emphasized on the lowest landau levels at $\beta = 3$ vortex-Laughlin-Helprin series, focusing on commonly observed fractions within $0 < \nu_G < 2$. At finite temperatures, we obtained an important geometric parameter, $\gamma$ which appears in addition to the anomalous term, $\xi$. Both effects greatly modify transport properties. The system’s property critically depend on this geometrical parameter; Hall expansion coefficient. It arises as a result of strained induced magnetic field which is modulated by temperature. We noticed that the coefficient unexpectedly enhances observation of weak fractions and may even expose new FQHSs. By fixing the quantity $\gamma T_0 \lesssim 10$, we observed both flatness and infinities in $\eta$ at different filling fractions. The flat regions are typically ideal for device applications. Whilst the infinities identified as gaps suggest temperatures at which fractions are observed. The singularities are not mono-points but show up many points within certain temperature ranges, which further confirmed multicomponent nature of Graphene.

Keywords: Euler hydrodynamics, Quantum hall fluid, Vortex dynamics, Fractional quantum hall state, Anomalous viscosity

‡ Corresponding author: mrabiu@uds.edu.gh
1. Introduction

Graphene is a lately discovered truly two-dimensional material that continue to inspire research interest, mainly because it is tipped as a potential candidate to replace Silicon in the electronic industry. In normal graphene, relativistic quasi-particles experience some kind of resistance to flow or normal viscosity. Under certain conditions the particles can form correlated state of matter or electronic fluid (EF) which flows just like liquid. However, when interactions are weak, in the regime of moderate doping and at low temperatures a shearing force, accompanied by shear viscosity, is produced. It establishes transverse velocity gradients that obstructs electron motion. The viscosity-to-entropy ratio takes on a universal minimal value (Mueller, 2009). The situation is completely different in Quantum Fluid (QF) which is formed in 2D electron gas with strong particle-particle interactions at low-temperatures when the gas is submitted to high magnetic fields. Individual particles loose their properties and only depend on the collective behavior (of the QF). The viscosity that is measured here is termed Lorentz shear viscosity after Avron (1995), Hall viscosity after Read (2009) and Anomalous viscosity after Wiegmann (2013, 2014). Quantum fluids are interesting and have occupied many academic minds especially due to their remarkable phenomenon. Including; superconductivity, superfluidity, ultracold atoms and Fractional Quantum Hall States (FQHSs). In a QF, an applied magnetic field perpendicular to a plane containing electric field creates transverse voltage. Flows associated with the Hall voltage produces no work and dissipationless. The anomalous viscosity can be revealed in the QF when time reversal symmetry is broken. It measures the response of stress tensor to time-varying strain, stress preserving deformations or lattice scale dilatations.

Recently, there has been a great deal of interest and renewed focus on the anomalous viscosity of QFs. In particular, Read obtained a universal relation $\eta = \left( \frac{\hbar \rho}{4} \right) S$ for (FQHS) of generic systems by studying the response to metric deformations. Siavah et al. (2014) obtained $\eta = \left( \frac{B}{8\pi} \right) \kappa$ for FQHS of relativistic particles in graphene by electromagnetic and gravitational response. The same general result, including anomalous term, has been obtained by Abanov (2013) within hydrodynamic picture of effective field theory and (Wiegmann, 2014) within the Euler hydrodynamic vortex picture. However, all these results lack the presence of temperature gradient which can have remarkable effect on $\eta$ and other transport coefficients.

1.1. Fractional quantum hall states

Our main focus in this work is the study of anomalous viscosity of FQHS in graphene within Euler hydrodynamics picture. The hydrodynamic fluid picture has the advantage of fewer parameters, lesser symmetries and only basic principles are employed. It leads naturally to incompressible FQHS and thus captures basic essential physics. A plethora of reasons motivated this studies. Specifically, it is due to the relativistic quasi-particles and multicomponent nature of the system. In graphene, there is an infinitely rich physics and many hierarchies of FQHS exist that makes its studies unlimited. For
instance, conductivity is anomalous and shows precisely Hall plateau at $\nu(e^2/h)$. Each fractional value, $\nu = 1/3, 2/5, 3/5, 2/3, 4/3, 5/3, 8/3, \ldots$, corresponds uniquely to an incompressible and dissipationless ground state. The ground states are holomorphic in nature and gapped from excited states. The states are characterized by a universal anomalous viscosity which we so much desire to compute in this work. An extra pseudo-spin internal degree, in addition to intrinsic electron spin further endow FQHS in graphene with an approximate SU(4) symmetry.

1.2. Quantum hall vortices and anomalous terms

A two-dimensional incompressible gas subject to perpendicular magnetic field at low-temperatures is prone to generation of extended states; vortices. In particular, the honeycomb lattice of graphene coupled with the magnetic field can lead to formation of vortices. Instead of the normal microscopic theory of quantum Hall fluid of electrons, we consider flows of quantized Hall vortices in the fluid as elementary objects forming themselves a highly correlated quantum fluid. If we further assume vortices are point-like and fermions with chiral flow, then vorticity becomes a fundamental object and its physical observation must be pursued. In the regime of long-wave slow motion, an accurate hydrodynamic description is allowed and does, in its own validity, not depend on the microscopic behavior of the electronic fluid. Vortex-vortex interactions will then be responsible for appearance of FQHS. The hydrodynamics of vortex matter presented here differs from Euler hydrodynamics (Stone, 1984) by an anomalous term. Within this many-vortex approach, the anomalous term has been derived for bulk single component non-degenerate systems in the recent series of papers by Wiegmann (2013a, 2013b) where the name anomalous viscosity reappeared. In the context of normal fluid flow, Mack (1948) had employed this jargon when he studied anomalous viscosity due to order-disorder transition. The term is also routinely used in plasma physics to describe different phenomenon. Here, the origin of the anomalous term is due to vortex velocity diverging at microscopic scale which locally deforms a metric locally and causes dilatation of particle coordinates. The divergence due to individual particles collectively manifest itself at macroscopic scale and give rise to the anomalous term. The systems may respond to the local deformations by producing strain. In the strained region, the anomalous term pushes vortices away from these highly strained areas to more relaxed places and accumulate charges there. If the system has either globally or locally curved edges, the anomalous term creates dipole moment which reconstruct the edge states.

Another important term arising from lattice scale dilatations at finite temperature is the Hall expansion coefficient. It now established that strained graphene induces giant synthetic magnetic field. At finite temperature, this field can get modulated allowing vortices to feel an effective magnetic field. Its is the Hall expansion that captures this contribution.

The remaining of the paper is organized as follows. In Section 2, we derived vorticity equation from Euler hydrodynamic equation through Boltzmann transport equation for
graphene. We obtained and quantized Helmholtz-Kirchoff vortex solutions. A FQHS constraint is derived from which we computed density-velocity pair. Laughlin-like many-vortex states are determined. Vortex flux and momentum conservation are subsequently derived. The stress tensor is deduced from which the anomalous viscosity is read off. In Section 3 we analyzed behavior of viscosity under density and temperature profiles for different filling fractions. We conclude in Section 4 highlighting possible applications of our results.

2. Theoretical Approach

2.1. Euler hydrodynamics and point vortices

Two dimensional Euler hydrodynamics can be straightforwardly derived following Boltzmann transport equation at local equilibrium, where particle distribution reduces to hydrodynamic equations for density and momentum;

\[ \frac{D_t \rho}{\alpha} = 0 \quad \text{and} \quad \frac{D_t \mathbf{p}}{\alpha} + \nabla p^\alpha = 0. \]  

(1)

Where the material derivative \( D_t \equiv \partial_t + \mathbf{u} \cdot \nabla \), \( \mathbf{p} = m_u \mathbf{u} \) is the momentum and \( \mathbf{u} \) is fluid velocity. \( m^* \) is the effective mass. \( p \) is the partial pressure and \( \alpha \) is the fluid component index. For graphene, \( \alpha \equiv (K' \uparrow, K \uparrow, K' \downarrow, K \downarrow) \). \( K, K' \) and \( \uparrow, \downarrow \) are the valley and spin indexes, respectively. Now, we defined the quantity \( \omega = \nabla \times \mathbf{u} \) as the vorticity which is non-zero for rotational incompressible flows. \( \omega \) is considered as frozen into the fluid and constitute its own fluid. It also assumes the single geometrical meaning as the electronic fluid in (1), i.e.,

\[ \frac{D_t \rho}{\nu} = 0 \quad \text{and} \quad \frac{D_t \omega}{\alpha} = 0, \]  

(2)

for a scaler pressure or an isentropic flow. The equation above means vorticity is simply advected with the fluid. This suggest two types of flow. The slow motion of the vortex fluid and the fast motion of the fluid. Helmholtz realized that a solution for (2) exists and consists of point-like vortices,

\[ \mathbf{u}^{\alpha} = -i \Omega z^{\alpha} + i \sum \frac{\Gamma^{\alpha}_i}{z^{\alpha}_i(t) - z^{\alpha}_j(t)} + \text{Boundary terms}, \]  

(3)

where \( z = x + iy \), \( u = u_x - iu_y \), \( \Gamma \) is the circulation and \( \Omega \) is the rotation/cyclotron frequency. Assuming flow in which the circulation, \( \Gamma^{\alpha}_i (= \Gamma^{\alpha}) \) is both minimal and chiral such that in the thermodynamic limit, \( \rho \rightarrow \infty \) rotation is compensated by large number of vortices. In magnetic field, the vortices of all components are smoothly distributed with fixed mean density, \( \rho_0 = (1/\pi)(\Omega/\Gamma) \). Borhn-Summarfelt phase-space quantization leads to quantization of the circulation \( m_\nu \Gamma^{\alpha} = 2\pi \beta^{\alpha}\hbar \). Where \( m_\nu \) is vortex mass and \( \beta^{\alpha} \) is an integer. In this case, vortices move such that

\[ v^{\alpha}_i(t) = -i \Omega z^{\alpha}_i(t) + i \sum \frac{\Gamma^{\alpha}}{z^{\alpha}_i(t) - z^{\alpha}_j(t)} + \text{Boundary terms}. \]  

(4)

Equation (4) is the Helmholtz-Kirchoff equation.
2.2. Temperature effect on dynamics of hall vortices and viscosity anomaly

In the following, we neglect contributions from the boundary terms and instead consider effects of temperature gradient. In the quantum Hall regime of dissipationless flow, fluid particles do not carry heat flux, but vortices do. Vortices move in response to temperature gradient, $\nabla T$. This is captured in the vortex density defined through momentum flux as

$$P_\alpha(r,T) = \sum_i \frac{m_v}{2} \left\{ \delta(r - r_i, T), v_i^\alpha \right\}. \quad (5)$$

Using the identity $\pi \delta(r) \equiv \bar{\partial} \frac{1}{z}$ and the ward identity $\sum_{i \neq j} \left[ \frac{1}{(z - z_i)z_j} \right] = \left[ \sum \frac{1}{(z - z_i)} \right] - \sum \left[ \frac{1}{(z - z_i)} \right]^2$, we arrived at (Wiegmann, 2013)

$$P_\alpha(r,T) = m_v \rho_v(r,T) \left[ u^\alpha + \frac{\Gamma^a}{4} \nabla^* \log \rho_v(r,T) \right]. \quad (6)$$

The vortex momentum flux equation, (6) is very crucial in this studies. In particular, our results is based on the second term which dictates discussions that follows. The term is responsible for anomalous behavior of the fluid when approaching a vortex. It is a quantum or micro-scale phenomenon which manifest itself at classical regime due to possible broken translation symmetry associated with lattice scale deformations. Its presents removes the poles seen in (4). It also produces stresses perpendicular to flow with no work nor dissipation. The momentum conservation following from broken translation invariance and in the presence of external forces yields

$$\dot{P}_a^\alpha + \nabla_b \Pi_{ab}^\alpha = \rho_v F_a^\alpha. \quad (7)$$

Where the stress tensor is $\Pi = m_v \rho_v u \cdot u + \eta(\nabla^* u + \nabla^* u) x^* (\cdot) \equiv x \times (\cdot)$. and $F$ is the force. The kinetic coefficient $\eta$ is anomalous viscosity. Except the temperature dependence, it has the same structure as the universal relation obtained by Avron (1995) for IQHS, Read (2009) for FQHS and recently by Abanov (2013), Wiegmann (2013, 2014) and recently by Siavah (2014) for graphene FQHS. Its is expressed as

$$\eta(r,T) = \frac{m_v \Gamma}{4} \rho(r,T). \quad (8)$$

2.3. Fractional quantum vortex hall states

A physical interpretation of $\Gamma$ in (8) in the context of known microscopic theory is in order. A potential candidate is the theory of FQHS. Achieving this purpose requires quantizing our hydrodynamic equations. This is done by following the lines of Wiegman (2013b). We start by embedding our system in Bargmann space. The following three steps help realized full quantization of the hydrodynamic Kirchoff equations.

(i) Quantizations of holomorphic coordinates: $\{ \bar{z}_i, z_j \} = i (2\ell^2 / h) \delta_{ij}$ with conjugate coordinates, $\bar{z} = -2\ell^2 \partial_z$, (ii) specification of choice of states: $\psi(z_1, z_2, \ldots, z_N)$ and (iii) definition of inner product: $\langle \psi' | \psi \rangle = \prod_i \int d^2z \exp[-(z_i \bar{z}_i / 2\ell^2)] \bar{\psi}' \psi$, where the
equilibrium inter-vortex separation is $\ell = \sqrt{1/(2\pi \rho_0 \beta)}$. These quantization rules allow us to write Kirchoff’s equation as

$$P^\alpha = -2\pi i \hbar \sum_i \delta(r - r_i, T) \left( \partial z^\alpha + \sum_{j \neq i} \beta^\alpha_{iz} - z_j^\alpha \right).$$

(9)

In the limit, $m_v \to 0$, such as in graphene, and at finite temperature vortices do fluctuate but confined to the lowest Landau levels (LLL). Under this condition the vortices can be regarded as being quasi-stationary so that the momentum flux annihilates ground states, $\sum_\alpha P^\alpha \psi_0 = 0$. Thus, it follows that, the multicomponent nature of our system leads directly to the Laughlin-Helprin-like Fractional Quantum Hall vortex states

$$\psi_0(z_1, z_2, \ldots, z_N) = \prod_{\alpha=1}^{4} \prod_{i<\beta} \left( z^\alpha_{ia} - z^\alpha_{ja} \right)^{n_{\alpha \beta}}, \quad \beta^\alpha \to n_{\alpha \beta}. \quad (10)$$

However, an additional term accounting for inter-component vortex interactions is necessary (Goerbig, 2012). Though, the resulting state disqualifies the $\psi_0$ from being the true ground state of $P$. The new state becomes

$$\psi_0(z_1, z_2, \ldots, z_N) = \prod_{\alpha=1}^{4} \prod_{i<\beta} \left( z^\alpha_{ia} - z^\alpha_{ja} \right)^{n_{\alpha \beta}} \prod_{\alpha<\beta} \prod_{i,j} \left( z^\alpha_{ia} - z^\beta_{ja} \right)^{n_{\alpha \beta}}. \quad (11)$$

The maximal exponent for an $i^\alpha$ quasi-particle is $N_\phi - 1 = m_\alpha (N_{\alpha} - 1) + n_{\alpha \beta} N_{\beta \neq \alpha}$. The shift and fractional filling sum over all components yields

$$S = \frac{1}{4} \sum_\alpha \left( m_\alpha - 1 \right) \quad \text{and} \quad \nu = N \left[ \sum_{\alpha, \beta} \left( m_\alpha N_{\alpha} + \sum_{\alpha \neq \beta} n_{\alpha \beta} N_{\beta} \right) \right]^{-1} \quad (12)$$

respectively. If the vortex system inherits the honeycomb lattice approximate SU(4) symmetry, the filling in the LLL becomes $\nu_G = \pm 2 + \nu$. The results obtained in (12) is crucial as far as seeking physical microscopic interpretation to the circulation, $\Gamma$ concerns us. Direct comparison with Read’s Hall viscosity relation allows a connection between the circulation and the shift. That is, $\Gamma = (\hbar/m_v)S$.

2.4. Effective magnetic field

As we have pointed out in the introduction, geometric deformations can lead to induction of giant magnetic fields. Vortices feel this contribution in addition to external magnetic field as an effective field, $B_v$. The anomalous viscosity for the graphene-vortex system takes the shape, after restoring all variables, $\eta(r, t, T) = (e\kappa/8\pi)B(r, t, T)$ with $\kappa = \nu_G S$ and $\rho = \nu_G(eB/h)$. If the field is frequency, $\omega$ dependent, we have

$$\eta(r, \omega, T) = \frac{e\kappa}{8\pi}B_v(r, \omega, T). \quad (13)$$

Where $r = |z_i - z_j|$ and $B_v(r, \omega, T) = B_v(\omega)B_v(r, T)/B_0$ which we now compute. Notice $\omega$ is different from the scalar form of vorticity ($\omega = \omega \hat{z}$) which uses the same Greek letter.
Applying the commutation relation \( \sum_{\alpha} [u^\alpha(r), \rho(r')] = -(\hbar/2m_v) \nabla^* \rho(r - r') \) it follows from (6) that
\[
v^\alpha(r, T) = u^\alpha(r, T) + \frac{\Gamma^\alpha}{2} \left( \frac{1}{2} - v^\alpha_G \right) \nabla^* \log \rho_v(r, T),
\]
(14)
and using \( \nabla \times u = 2\pi \Gamma(\rho_v - \rho_0) \), we have
\[
\nabla \times v^\alpha(r, T) = 2\pi \Gamma^\alpha \left[ \rho_v(r, T) - \rho_0 + \frac{1}{4\pi} \left( \frac{1}{2} - v^\alpha_G \right) \Delta^* \log \rho_v(r, T) \right].
\]
(15)
The fact that left hand side of this equation annihilates the ground sate means the magnetic field intensity can take the form
\[
B_v(r, T) = B_0 - \frac{\hbar}{4e} \left\{ \Delta^* \log B_v(r) + \Delta^* \log \left[ 1 - \gamma T(r) \right] \right\},
\]
(16)
where \( \xi = 1/2\nu_G - 1 \) is the anomalous term. The Hall expansion coefficient \( \gamma = -(1/B_v)(\partial B_v/\partial T) \) can be likened to the expansion coefficient and the minus sign is inferred from recent experimental results on graphene at low-temperatures (Vibhor, 2010; Duhee, 2011; Linas, 2014). Because strain in graphene induce giant synthetic magnetic fields, at finite temperatures \( \gamma \) can have great consequences on the system’s electronic transport properties. Thus, the coefficient can characterize stress and knowing it can be very critical in strain engineering.

To compute \( B_v(\omega) \), we recall that the force acting on vortices is Magnus force which is defined as
\[
\rho_v F = m_v \Omega \hat{z} \times (\rho_0 v - \rho_v u).
\]
(17)
The term in brackets is written in such a way that the force always stay constant in order that fluctuations in the LLL states are bounded. Replacing \( v \) from (15) in (17) and using \( \rho_v F = \dot{P} = -i\omega m_v \rho_v v \) up to leading order in gradients, we get
\[
B_v(\omega) = \frac{B_0 [1 + (\xi/2)(kl)^2]}{\omega/\Omega)^2 - (1 + (\xi/2)(kl)^2]^2}.
\]
(18)

3. Results and discussions

In this section, we analyzed the results obtained in the previous section. In particular, the anomalous viscosity obtained when (16) and (18) are put into (13).

In Figure 1 (Left), anomalous viscosity, \( \eta \) is studied over the filling fractions range \( 0 < v_G < 1 \). The separation of \( \eta \) from the normal \( \eta(T = 0) \) measures the strength of the fraction \( v_G \). The anomalous term, \( \xi \) makes observation of the \( v_G = 1/3 \) much more easier compared to the \( v_G = 2/3 \) which shows no significant change (indicated by the open squares curve). Surprisingly, at finite temperatures where \( \gamma T_0 \sim 7.2 \), the weak fractions, together with the already robust \( v_G = 1/3 \), become strong. This may be explain by the fact that \( \lambda \) increases the effective magnetic field by capturing strained induced field effects at finite temperature. In Figure 1 (Right) where \( \eta \) is observed for the series \( 0 < v_G < 1 \), the situation is complicated. The viscosity takes on negative
values for the reference $\xi = 0, T = 0$ case. It suggests why fractions in this series are weakly observed. Here, the rule of separation of $\eta$ from $\eta = 0$ applied only to the finite temperature curve. $\nu_G = 2/3$ may become more probable to observe. Effect of $\xi$ as $T \to 0$ produces only indistinguishable flows.

In Figure 1 (top), we showed the behavior of $\eta$ over low- and high-temperatures for the filling $0 < \nu < 1$. At extremely low-temperatures viscosity is flat which is ideal for physical device applications. It however, diverges to $+\infty$ for $\nu_G > 3/7$ and to $-\infty$ for $\nu_G < 4/7$ at critical temperatures, $T_C$. The opposite is observed at high-temperatures where $\eta$ fall to constant value. The significance of the divergences in the intermediate region is critical in determining which fractions are more likely observed. Figure 2 (bottom) shows $\eta$’s behavior in the series $1 < \nu < 2$. Even though viscosities are relatively small at high-temperatures, the system responds very quickly (becomes very sensitive) to rate of stress preserving deformations at low-temperature values.

In Figure 3, $\eta$ exhibits ranges of singularities as indicated by the white spaces. One can draw two conclusions from this phenomenon. (i) The wider width of the space, instead of a line, is indicative of multicomponent nature of our system. (ii) They may still be more unobserved fractions which are hidden in this spaces. The transition regions mean probing difficult in this areas are much more difficult.

4. Conclusions

In conclusion, we have computed anomalous viscosity of FQHS in graphene within hydrodynamics using quantum many-vortex picture of Euler hydrodynamics. The hydrodynamics formalism allowed a great deal of simplifications as the microscopic theory is completely unnecessary and only few variables, $\rho$ and $v$ are employed.

An important aspect of the anomalous viscosity is the anomalous $\xi$ and Hall $\gamma$ terms. Their effect come in two folds. (i) it corrects $\rho$ and introduces non-linearities
Figure 2. Normalized anomalous viscosity plots versus temperature at different fractions showing singularities at some critical temperatures.

in the fluid dynamics making flows at different FQHS distinguishable. (ii) $\xi$ captures temperature variations through $\gamma$ as soon as vortices are formed. The combined effect greatly enhanced observation of weaker fractions. Thus, we suggested ways of exposing new fractions and making weak ones stronger by controlling the $\gamma$ parameter. Knowing it is critical for strained-engineered devices in particular. Since device sensitivity to stress-preserving deformation depends on the factor.

At some specific temperature ranges, $\eta$ contains infinities. These divergences may be regarded as temperatures at which fractions show up and the range of temperatures yielding additional infinities confirms multicomponent nature of graphene.

Finally, our studies can guide future experiments towards observing new fractions. In particular, the temperature windows obtained may be probed, though away from the transition zones, for new fractions by controlling the Hall expansion coefficient parameter. Moreover, our work may resolve conflicts of different reported temperatures at which fractions, specifically $T_{\nu=1/3}$, are observed.
Figure 3. Anomalous viscosity behavior on temperature-filling fraction. Full squares indicate critical temperatures obtained in this study. The empty spaces may contain additional different critical values confirming multicomponent nature of the graphene-vortex hall fluid.

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