Mathematical modeling of hydrodynamics and heat exchange in liquid channels of the thermoelectric cooling module

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Abstract. The article presents the results of the synthesis of the mathematical model of hydrodynamics of nonisothermic flow of heat-transfer fluid in channels bordering on the heat-absorbing surface of the thermoelectric cooler (TEC), based on the ANSYS CFX toolkit. The lengths of the unstabilized flow regions at the inlet and outlet of the channels were determined. The parameters of the non-uniform distribution of the coolant through the individual fluid channels were determined. A physical experiment was planned on the model of a local thermal stabilization system to determine the integral parameters of convective heat emission on the heat-absorbing and heat-generating surfaces of the TEC of a liquid-air circuit. The regression dependence of the heat-transfer coefficient on the heat-absorbing surface of the TEC on the determining parameters was found.

1. Introduction
Modern approaches to material science made it possible to produce thermoelectric coolers (TEC). Such devices have their unique advantages over other types of cooling systems. Their exclusive qualities include high reliability, durability, compact size, ecological efficiency, and the absence of vibrations, as well as operating in a wide range of temperatures. A significant amount of the research aimed at the development and improvement of above TEC have been conducted, however, thermoelectric technologies still have a long development path to a wide practical application [1-9].

The numerical method or the finite element analysis is one of the means that is widely used in the design and optimization of TEC in the early stages of the production process [10-14]. Several studies have described the experience of a successful appliance of finite element analysis for designing prototypes of thermoelectric devices [15, 16].

When modeling processes in a thermoelectric cooler, high accuracy of determining boundary conditions on its surfaces is necessary. In a number of works [17-29], to determine the heat-transfer coefficient on the "liquid" heat-absorbing surface of a TEC, the criterion relationships are used, which were obtained when summarizing the results of a thermal engineering experiment, which was conducted on "ordinary" materials without the influence of thermoelectric effects. In addition, at low Reynolds numbers, it is risky to use the dependencies for calculating the heat transfer coefficient obtained for laminar flows to calculate the heat exchange parameters in the initial portions of the hydrodynamic and thermal stabilization of the “liquid” channels of a TEC. In this regard, it is advisable to carry out a mathematical analysis that accompanies a thermal engineering experiment for
a specific version of the TEC design, in order to obtain generalized "working" dependencies on the definition of integral performance criteria of the TEC, such as cooling capacity, which can be used to optimize the TEC operation modes for energy saving of this TEC, as well as for designs similar to the TEC.

2. Materials and methods

2.1. Technical device under research

For mathematical and physical modeling, a cooling unit (CU) (figure 1) with thermoelectric coolers is adopted [30]. There are 24 thermoelectric coolers in total, which are connected in current in such a way that the cold junctions of the TEC are in thermal contact with sections for the movement of the cooled liquid, and the hot junctions are with the bases of air-cooled radiators.

![Figure 1. Cooling unit.](image)

The design provides 4 sections for the movement of the cooled heat-transfer agent. Each section has inside 11 mini-channels for the movement of fluid with a finned inner surface (figure 2).

![Figure 2. Layout of mini-channels of the fluid path.](image)

Figure 3 shows the design model of the simulation problem.

![Figure 3. Design model.](image)

The inlet fitting supplies fluid to the heat exchange path, after which the flow is divided into two parts, moving in the extreme and middle sections in the "forward" direction. After passing through the sections, the fluid mixes in the intermediate manifold and then is again divided into two sections and moves in the "opposite" direction to the outlet fitting. Figure 3 also shows 24 TECs using a number-letter designation, for example, 1x, 2x, or 3z.

2.2. Using the ANSYS Toolkit

Grid model of a flowing part consists of 1124 surfaces, formed by 3152 faces. The construction of the full grid model is limited by the resources of the computer used, so it was decided to divide the geometry into 47 separate bodies (44 channels, the inlet, intermediate and outlet manifolds). Figures 4-5 show examples of forming grid models of the flowing part elements of the device’s liquid path.
**Figure 3.** Design model of the heat exchange unit.

**Figure 4.** Grid model of the inlet manifold.
The use of the laminar flow mode option (taking into account that the Reynolds number in a separate mini channel is no more than Re=100) in the settings of the algorithm for numerical solution of the equations of momentum led to the fact that the convergence of the iteration process was not achieved [31]. This is due to the fact that the vortex flow in the inlet and outlet manifolds and the transition to the channels (or from them) of the fluid path cannot be solved under the assumption of a laminar flow regime (figure 6).

Further, the simulation was carried out under the assumption of a turbulent flow regime, despite the fact that in some mini-channels the movement became apparently laminar at some distance from the inlet. On the graphs of the calculated heat transfer coefficient in the channels, one can notice outlined zones of unstabilized flow in liquid channels (figure 7), which can take up to a quarter of the length of the heat exchange sections.

The layout of the channels (figure 2) was made for the development of the heat exchange surface. Based on the simulation in ANSYS CFX, the efficiency ratio of the finning of this design model was evaluated. When simulating heat transfer in the channels specifying adiabatic conditions on the edges (figure 8), the temperature of the liquid at the outlet, and, consequently, the amount of heat transferred to the wall, does not differ from the temperature of the liquid obtained under the condition of convective heat exchange of the first kind on the inner surfaces of the channels. Thus, the efficiency ratio of the finning was estimated by the formula $\eta_p = F_p / F_T$, where $F_p$ is the heat exchange surface area in figure 10, $F_T$ is the total area of the “cold” surfaces of the TEC, and it is equal to $\eta_p = 2.14$.

The value of flow rate non-uniformity in individual channels according to the results of the modeling in ANSYS can reach ten percent. Table 1 shows the data of calculating the distribution of the flow rate in the channels with the total flow rate in the liquid path of 0.05 kg/s. It can be noted that the difference between channel 5 and channel 2 in group 2 is 9.1%.
**Figure 6.** Contours of velocity stream function.

**Figure 7.** Distribution of the heat transfer coefficient values over the heat exchange surface.
Thus, according to the results of the modeling in ANSYS CFX, it can be concluded that it is inexpedient to direct the main efforts to search for or refine the criterion relations of convective heat exchange in order to find the local values of the heat transfer coefficient. A large number of epistemic uncertainties of the boundary conditions of heat exchange in fluid channels, among which we highlight the uneven distribution of flow through individual mini-channels, as well as the presence of extended hydrodynamically unstabilized sections in the fluid channels, leads to further efforts to find dependencies for the average heat transfer coefficient of the fluid path of the TEC with the preferred method of obtaining such a relationship by processing the experimental data.

Table 1. Mass flow rates through the channels of the cooling system.

| Channel  | Group 1     | Group 2     | Group 3     | Group 4     |
|----------|-------------|-------------|-------------|-------------|
| 1        | 0.00213065  | 0.00244662  | 0.00244924  | 0.00258665  |
| 2        | 0.0021868   | 0.00214863  | 0.00210964  | 0.00239107  |
| 3        | 0.0022488   | 0.002343    | 0.00222213  | 0.00231053  |
| 4        | 0.00230609  | 0.0023683   | 0.00209698  | 0.00232133  |
| 5        | 0.00229444  | 0.0023638   | 0.0021599   | 0.00223677  |
| 6        | 0.00238714  | 0.00232106  | 0.0020902   | 0.00226684  |
| 7        | 0.00237814  | 0.00235871  | 0.00214441  | 0.00222635  |
| 8        | 0.00223912  | 0.00223831  | 0.00208693  | 0.00230068  |
| 9        | 0.00220614  | 0.0023373   | 0.00221509  | 0.00228311  |
| 10       | 0.00221197  | 0.00215018  | 0.00210824  | 0.00236301  |
| 11       | 0.00216728  | 0.00246368  | 0.00248143  | 0.00255219  |

2.3. Experiment

Planning and implementation of the experiment to determine the dependence of the heat transfer coefficient on the heat-absorbing surface of the TEC and the processing of the results of the physical experiment were carried out using the commercial module ANSYS DesignXplore and the academic version of the IOSO NS program.

Figure 9 shows a scheme for testing a cooling unit as part of an electronic equipment thermal stabilization system. Figure 10 shows a photograph of an operative embodiment of the system.

In the planned experiment, the factors (variable parameters) take on the following values:

- $\Delta T$ - the temperature difference between the fluid at the inlet to the CU $T_{fix}$ and the environment $T_{vtx}$, i.e. $\Delta T = T_{fix} - T_{vtx}$;
- $N_{tmo}$ - the power consumption of the TEC CU, i.e. $N_{tmo} = U_{tmo} \cdot I_{tmo}$, where $U_{tmo}$ is the voltage applied to the TEC CU, and $I_{tmo}$ - the current strength;
- $N_{vent}$ - the electrical power consumption of the fans, i.e. $N_{vent} = U_{vent} \cdot I_{vent}$, where $U_{vent}$ and $I_{vent}$ are, respectively, the voltage and current supplied to the fans;
The coolant flow rate at the inlet to the CU.

**Figure 9.** Scheme of the test (1 - a container of liquid coolant, 2 - heater, 3 - pump, 4 - cooling block of the liquid air circuit, 5 - flowmeter, 6 - radiator with a fan, 7 - thermocouples for measuring temperatures at the inlet and outlet of the CU.

**Figure 10.** Operative embodiment of the local thermal stabilization system.

The experimental model was also provided with a thermocouple to determine the temperature of a hot surface of TEC. The cooling capacity of the CU, which was the outlet parameter (criterion), was determined according to the temperature difference of the coolant at the inlet and outlet of the installation. The physical experiment was based on the Optimal Space-Filling Design algorithm, it consisted of 25 points for 4 factors. Table 2 presents the matrix of the range of experiment plan with the measured and calculated values of the factors and criteria.

**Table 2.** Plan and results of the physical experiment.

| Plan point number | Uто, Jтмо, U vent, J vent, Tтмо, T vent, ΔT, V | ΔT, K | Vтмо, W | Nтмо, W | Qx, W |
|-------------------|---------------------------------|-------|--------|--------|-------|
| 1                 | 21.9 30.1 17.8 9.1 15 12.3 -14.059999 | 1.917 | 659.190002 | 161.979996 | 216.299271 |
| 2                 | 22.6 31.3 19.8 10.5 17.57 15.99 -10.530001 | 0.778 | 707.380005 | 207.899994 | 203.38884 |
| 3                 | 14.4 20 18.6 9.8 18.82 15.99 -11.73 | 1.778 | 288 | 182.280014 | 209.725693 |
| 4                 | 15.8 22.1 23 13 21.3 16.48 -8.210001 | 1.222 | 349.180023 | 161.979996 | 216.299271 |
| 5                 | 13.1 18 14.6 6.9 23.14 18.95 -5.950001 | 0.528 | 254.800003 | 100.740008 | 209.267029 |
| 6                 | 16.5 22.8 16.2 8 24.97 15.44 -4.33 | 1.306 | 235.800032 | 100.740008 | 209.267029 |
| 7                 | 11.7 16.4 20.2 11 26.89 19.11 -1.980001 | 0.611 | 191.87999 | 222.280014 | 209.725693 |
| 8                 | 19.2 27.5 22.6 12.9 28.61 24 -0.229995 | 2.25 | 294 | 222.280014 | 209.725693 |
| 9                 | 26.7 36.5 19 10.1 30.63 23.74 1.109999 | 1.556 | 974.550049 | 191.900009 | 443.924988 |
| 10                | 21.2 28.9 14.2 6.6 32 25.47 1.200001 | 1.472 | 612.679993 | 93.719994 | 397.815186 |
| 11                | 23.3 32.7 23.4 12.6 33.54 23.97 3.360001 | 1.162 | 761.909973 | 294.839999 | 462.006744 |
| 12                | 24.6 33.4 16.6 8.3 35.6 21.5 4.689999 | 0.974 | 821.640076 | 137.779999 | 405.228363 |
| 13                | 11.02 16.2 18.2 9.6 37.6 33.2 7.299998 | 2.167 | 178.524017 | 174.720016 | 392.982849 |
| 14                | 20.54 28.7 17 8.5 38.9 32.8 8.120001 | 2.333 | 589.498047 | 144.5 | 586.58252 |
| 15                | 10.34 15.6 22.2 12.4 40.86 33.82 10.380001 | 1.389 | 161.304001 | 275.279999 | 402.637695 |
| 16                | 18.5 26.1 21.8 12.2 42.7 21.07 11.440001 | 0.444 | 482.850006 | 265.959991 | 397.065094 |
| 17                | 12.41 17.2 17.4 8.8 44.68 31.92 14.130001 | 0.833 | 213.452011 | 153.119995 | 437.646667 |
2.4. Results and discussion. Processing the physical experiment results

The Response Surface tool of the ANSYS DesignXplore module was used to obtain the regression equations. The quality of the resulting model based on statistical criteria is estimated as high for the approximation method based on the full quadratic polynomial. The coefficient of determination of the model based on the full quadratic polynomial ($R^2$) is 0.99656, i.e. the proportion of the deviation variance of the dependent variable from its average value, explained by the approximation model under consideration (selected factors), is practically equal to 1. The maximum calculation error is 6.22%.

To obtain higher accuracy, a metamodel equation based on a cubic polynomial was obtained (the dimension of flow in the equation is $m^3$/hour).

$$Q_x = -934.07754 + 8.75298422 \cdot \Delta T + 29793.4812 \cdot \dot{V}_f +$$

$$+ 1.69455648 \cdot N_{mo} + 6.98283484 \cdot N_{vent} + 0.93180537 \cdot \Delta T^2 -$$

$$- 438970.47 \cdot \dot{V}_f^2 - 0.00157519 \cdot N_{mo}^2 - 0.02754901 \cdot N_{vent}^2 -$$

$$- 0.00294631 \cdot \Delta T^3 + 2447855.36 \cdot \dot{V}_f^3 + 0.638640E - 6 \cdot N_{mo}^3 +$$

$$+ 0.425975E - 4 \cdot N_{vent}^3 + 52.4775919 \cdot \Delta T \cdot \dot{V}_f - 0.01654813 \cdot \Delta T \cdot N_{mo} -$$

$$- 0.00502026 \cdot \Delta T \cdot N_{vent} - 9.6976619 \cdot \dot{V}_f \cdot N_{mo} - 28.439111 \cdot \dot{V}_f \cdot N_{vent} -$$

$$- 0.00295621 \cdot N_{mo} \cdot N_{vent} + 0.332329116 \cdot \Delta T \cdot \dot{V}_f \cdot N_{mo} +$$

$$+ 0.152462498 \cdot \Delta T \cdot \dot{V}_f \cdot N_{vent} + 0.112969E - 3 \cdot \Delta T \cdot N_{mo} \cdot N_{vent} +$$

$$+ 0.062229253 \cdot \dot{V}_f \cdot N_{mo} \cdot N_{vent} - 0.00174352 \cdot \Delta T \cdot \dot{V}_f \cdot N_{mo} \cdot N_{vent}$$

The maximum calculation error at point 7 of the plan is 0.5%, which, taking into account the presence of experimental errors in determining the parameters, can be considered an almost accurate result.

The design features of the channels for the movement of the liquid coolant and the flow-through part of air-cooled radiators lead to the fact that it is impossible to confidently select a set of criterion dependencies of heat exchange for the purpose of using them in boundary relations derived from the similarity theory. The simulation results showed that, firstly, a significant part of the liquid channels worked in the area of unstabilized fluid flow both at the inlet to the channels and at the outlet. Secondly, the developed (finned) surface of heat transfer from the liquid side leaves open the question of the prevailing mechanism (molar or molecular) of heat transfer from the coolant to the TEC wall. Thirdly, the heat transfer is carried out into a thermoelectrical material at various values of electrical energy supplied to the TEC. In this case, the heat is transferred from the cold to the hot junction, that is, the thermoelectric mechanism of heat transfer is activated with a significant influence of thermal processes taking place in accordance with the Fourier and Joule laws. The experimental studies showed a certain effect of the thermoelectric nature of the heat absorption in the cold junction on the intensity of heat transfer between the coolant and the TEC walls. Determining the average value of the heat transfer coefficient from the coolant to the heat-absorbing wall of the TEC by the formula
\[ \alpha_{se} = \frac{q_c}{(T_f - T_s)S_0}, \]  
where \( T_f \) is the average temperature of the coolant in the liquid channels of the TEC, \( S_0 \) is the surface area of the liquid channels involved in heat exchange for the points of the experiment plan, and after analyzing the sensitivity of the heat transfer coefficient depending on various factors, it was found that the available power of the thermoelectric cooler has a significant influence on the heat transfer coefficient. Thus, the generally accepted criterion relations (usually of the form \( Nu = f(Re, Pr) \)), obtained in experiments with materials without the influence of thermoelectric effects, should be identified not only taking into account the presence of sections of unstabilized coolant flow in transient modes but also taking into account the influence of thermoelectric effects on heat transfer intensity.

3. Conclusion

In the process of mathematical modeling using mathematical models of various hierarchical levels, the problem of the influence of the uncertainties of the external conditions of functioning of various TECs on a CU on the integral criteria for the functioning of the system was discovered. It is not only that different modules work at different values of the coolant temperature, which changes as it moves in the model’s channels, but a single TEC also works under non-constant boundary conditions on surfaces. This is due to the uneven distribution of the coolant through the individual channels of the liquid path of the unit and, as a consequence, the uneven heat supply on the cold side. In addition, external conditions, especially the ambient temperature, significantly affect the efficiency of a thermoelectric unit.

On the basis of the Response Surface Optimization technology, a metamodel for calculating the cooling power of the CU based on the polynomial approximation was obtained. The metamodel based on a full quadratic polynomial ensures the accuracy of determining \( Q_c \) of at least 6.22%, and the metamodel based on a cubic polynomial ensures an error in calculating the objective function of no more than 0.5%.

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