The mathematical model of *Eunice Siciliensis* lifecycle: a study on Lombok Baunyale Festival Preserving Effort

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Abstract. Baunyale Festival or sea worms (nyales) catching event is one of the traditions or heritage culture of the Lombok Sasak tribe community. The appearance of the colorful nyales on the sea surface in large quantities is actually a mass spawning of nyales. The most number of appearing nyales founded during the Baunyale festival is *Eunice (palola) siciliensis*. It was reported an excessive catching of nyales happened at the event in 2016 that endanger its existence. This paper is a scientific study of the dynamics of *Eunice (palola) siciliensis* growth represented in an ordinary differential equations system with a harvesting function. The system has an existing critical point that determined by the parameter that represents the number of population to be maintained \((M)\). The stability of the system is evaluated locally at the critical point using the linearization method. The critical point is stable for a specific condition of \(m\) and \(\sigma\), where the second parameter represent the predation rate of phytoplankton as the prey of nyales. The simulation shows that the population of the solution converges to the some amount of nyales populations. It means that the proportional type of harvesting function prevents the number of existing nyales for the festival.

Keywords: Baunyale, Proportional Threshold Harvesting Function, Mathematical Model.

1. Background
Knowing as Baunyale festival, the massive sea worms (nyale) catching event is held annually in Lombok, one of Indonesian islands that also well known as Komodo Island. The event organized on February or March, precisely five days after the full moon. The festival is completed by cultural and colossal traditional drama namely entitled Putri Mandalika (Saharudin, 2016). The festival attacks many domestic and foreign tourists and becomes the main government income as it could generate local community income.

Many kind of species founded in the festival, categorized as *polychaeta* class, the most common one is *Eunice (palola) siciliensis* species. In 1982 and 1983, there were 5 species of nyale founded in the festival (Jeki, et all, 1993). The appearance of many colorful nyale massively is actually its spawning activity. At that time the body of the adult worms is split in two parts, namely epitoki and atoki. The first part is released to the sea surface, while the second one is shrunk in the coral reef as their habitat to hidden (Bachtiar, et all, 2016). Annually, there are thousands peoples comes to Lombok and the surrounding area to join the festival. It was reported that in 2016 there were
26,386 peoples came to attend the festival in Mandalika resort. Some of them, 1,041 peoples, cached as much as 1,796 kg of nyale (Bacchiari, et all, 2016). Recently the population number of nyale seems decreasing and needs a preserving effort to maintain the sustainability of the festival. Mathematically, the effort of it could be done by governing the mathematical model of *Eunice* (*palola*) *siciliensis* lifecycle to support the government policy and regulation.

Mathematical model is an effective instrument that could be proposed its reliability to solve many daily phenomena. This paper governed the dynamic of the nyale population growth in every stage of its life. The model is represented as a nonlinear differential equations system with a proportional harvesting function. The objective of the mentioned function is to find the proportional number of nyale that have to be preserved such that the existing population could be available as much as possible when the festival is hold. The model is analyzed its stability at the critical point locally using linearization method to draw the long term behavior of the system. The simulation is also done to verify the fitness of the model in representing the phenomena.

2. The Model

The mathematical model of *Eunice* (*palola*) *siciliensis* is constructed by consider the lifecycle of the worms and divide it into five stages, that are zygot (*U*), larva *trochopore* (*L*), juvenile worm(*C*), adult worm (*F*), epitoki (*E*) and atoki (*A*). The phytoplankton is also considered to the model in case of the prey of larva, juvenile and adult worm stage worm in form of logistic function. The rate of such subpopulation with respect to time represents tits growth that comes in form of linear and nonlinear form of terms. The linear term draws the transition stage, while the nonlinear term appears because of the epitoki interactions that occur in spawning process that performs new zygot. This paper assumed zygot as the starting point of the worm lifecycle. A transfer diagram of the worm lifecycle is figured based on the reproduction phases (see Fig. 1).

![Figure 1. A transfer Diagram *Eunice*(*Palola*) *siciliensis* lifecycle](image)

Based on the diagram of Fig. 1, a nonlinear differential equations system (NDES) is derived

\[
\frac{dE}{dt} = \delta E + \theta A - \Psi L \\
\frac{dU}{dt} = \theta \sigma U H - \Psi L \\
\frac{dL}{dt} = \Psi \sigma L H - \rho C \\
\frac{dF}{dt} = \rho \sigma C H + \beta A - \delta F \\
\frac{dC}{dt} = \frac{1}{2} \delta F - \beta A \\
\frac{dH}{dt} = r H (1 - \frac{H}{k}) \\
\]
The positive value of parameter description is stated in Table 1. It is also assumed that it has no failure incidence happened in the transition process because of the internal and externally factors. There is also an adult worm harvesting rate by the community that represented in form of \( f(E,t) \).

### Table 1. The Parameters

| Parameters | Description | Value | References | Dimension |
|------------|-------------|-------|------------|-----------|
| \( \epsilon \) | The rate of spawning process to produce zygote | \( \frac{1}{2} \) | - | hour |
| \( \theta \) | The transition rate of zygote to larva trochopore | \( \frac{1}{2} \) | - | hour |
| \( \psi \) | The transition rate of larva trochopore to be juvenile | \( \frac{1}{12} \) | - | hour |
| \( \rho \) | The transition rate of juvenile to be adult worm | \( \frac{4320}{3600} \) | - | hour |
| \( \delta \) | The transition rate of adult worm to release the epitoki from atoki | \( \frac{1}{3600} \) | - | hour |
| \( M \) | The proportional number of epitoki to be maintained | 4000 | Existence and stability requirement | individu |
| \( \mu \) | The death rate of epitoki (posterior) | \( \frac{1}{5} \) | - | hour |
| \( q \) | The harvesting rate | 23% | Bachtiar et al., 2016 | individu |
| \( k \) | The carrying Capacity of phytoplankton | 96.125 | The number of population/area | Individu/\text{m}^2 |
| \( \sigma_1 \) | The predacy rate of phytoplankton by larva trochopore | 0.003 | Generated from the stability requirement | individu |
| \( \sigma_2 \) | The predacy rate of phytoplankton by juvenile worm | 0.003 | Generated from the stability requirement | individu |
| \( \sigma_3 \) | The predacy rate of phytoplankton by adult worm | 0.003 | Generated from the stability requirement | individu |
| \( r \) | The intrinsic rate of phytoplankton | \( \frac{24}{24} - \frac{1}{360} \) | Jam | |
| \( \tau \) | Propotionality number | \(< 23\%\) | Bachtiar et al., 2016 | - |
| \( \beta \) | The growth of atoki to regenerate itself being adult worm | 0.0001 | Generated from the stability requirement | individu |

### 3. The harvesting function

In this paper the worms catching activity in the \textit{Baunyale} festival is considered as harvesting. The harvesting is designed such that the solution of the model gives such periodic solution that represents the annually of the festival. On the other side, the harvesting function also has to give a threshold represents the minimal number of nyale existing that could be used as the preserving policy and regulation to the government.

This paper proposes a threshold proportional function that expressed as follow

\[
f(E,t) = \tau q (E - M)(\alpha(t) - \beta(t) \frac{1}{E})
\]

(8)

where \( q \) represents the harvesting rate, \( \tau \) is the proportional constant and \( M \) is the minimal number of nyale that have be maintained. The death rate of nyale (\( \alpha(t) \)) is assumed equal to its natural death rate, while the birth rate of nyale is assumed as the birth rate of epitoki such that \( \beta(t) = \frac{\alpha}{2} \). As aconsequence the simplification of the harvesting function in equation (8) that being substituted to equation (5) could be stated

\[
\frac{dE}{dt} = (\frac{1}{2} gF - \epsilon \mu E^2 - \mu \tau q E - \mu \tau q M) \left( \frac{2E}{1 - \epsilon \tau q E - \mu \tau q M} \right)
\]

(9)
4. The critical point

The critical point of the NDES in equations (1) - (7) could not be expressed explicitly such that it one of the dependent variable is considered as a parameter. In this case the implicit solution of the system as a function of $E$ could be derived. The choosing of $E$ as the independent variable of the implicit solution is because epitoki is the stage where the spawning that produces zygote, as the new generation of nyale, happened.

$$ T = A(E^*), C(E^*), F(E^*), H(E^*), L(E^*), U(E^*) $$

where

$$ A(E^*) = \frac{\mu(E^2 \varepsilon + E\varepsilon t - M \sigma t)}{\beta} $$
$$ C(E^*) = \frac{\mu^2(E^2 \varepsilon + E\varepsilon t - M \sigma t)^2}{\delta} $$
$$ F(E^*) = \frac{2\mu(E^2 \varepsilon + E\varepsilon t - M \sigma t)}{\delta} $$
$$ H(E^*) = \frac{\mu(E^2 \varepsilon + E\varepsilon t - M \sigma t)}{k^2 \sigma \varepsilon + E\varepsilon t} $$
$$ L(E^*) = \frac{\mu(E^2 \varepsilon + E\varepsilon t - M \sigma t)}{\Psi k^2 \sigma \varepsilon t} $$
$$ U(E^*) = \frac{\varepsilon E^2}{\beta} $$

and $E$ is the root of the polynomial

$$ P(E) = a_4 E^4 + a_3 E^3 + a_2 E^2 + a_1 E + a_0 $$

with

$$ a_4 = \mu \varepsilon (Eh^2 \sigma_4 c_2 c_3 - \varepsilon \mu) $$
$$ a_3 = (\mu \varepsilon (Eh^2 \sigma_4 c_2 c_3 - \varepsilon \mu) - \mu^2 \varepsilon r M \sigma t) $$
$$ a_2 = (-\mu^2 M \varepsilon (Eh^2 \sigma_4 c_2 c_3 - \varepsilon \mu) - \mu^2 \sigma M \varepsilon t \sigma_3 M) $$
$$ a_1 = 2\mu^2 \sigma M \varepsilon t \sigma_3 M $$
$$ a_0 = -\mu^2 \sigma M \varepsilon t \sigma_3 M $$

The existence of the critical point could be justified when equation (11) has a positive root at least. By Descartes rule it could be derived that the requirement of the existence is

$$ M < E + \frac{E\sigma^2}{\sigma t} $$

5. The stability

The stability of the system at the critical point is analysed locally by linearization method that gives a characteristic polynomial in $\lambda$ as follow

$$ c((b_2 \lambda + b_3)(\alpha_1 \lambda^2 + \alpha_2 \lambda^3 + \alpha_3 \lambda^4 + \alpha_4 \lambda^2 + \alpha_5 \lambda + \alpha_6)) = 0 $$

Assuming that the predation rate of phytoplankton by larva trochopore, juvenile and adult worm are equal, the stability requirement of the first term of equation (13) is

$$ M < E + \frac{E\sigma^2}{\sigma t} $$

As a consequence the requirement of a stable critical point existence is the supreme of the both requirements in equation (12) and (14).

The second term of equation (13) could be stated in form of the third order function of quadratic expression

$$ (c_1 \lambda^2 + c_2 \lambda + c_3)^3 = c_1 s \lambda^6 + 3c_1 c_2 \lambda^5 + 3c_1 c_3 \lambda^4 + 3c_2 c_3 \lambda^4 + 5c_1 c_2 c_3 \lambda^3 + c_2 c_3 \lambda^3 + 3c_1 c_2 \lambda^2 + 3c_2 c_3 \lambda^2 + c_3 \lambda^2 $$

where

$$ c_1 = a_1 $$

(16)
5.1 Characteristic equation simplification

The simplification of second term equation (13) that given on equation (15) leads to the value of \( c_1 \), \( c_2 \), and \( c_3 \) by substituting the value of parameters \( M, \varepsilon, \beta, \rho, \sigma, q, k, r \) and \( \tau \) that gives the values of \( \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6 \) and \( \alpha_7 \) and comes to the value of \( c_1 = 2.367 \times 10^5 \). Substitute the value of \( c_1 \) to the equation (17) gives

\[
3c_1^2 c_2 + 3c_1c_2^2 = \alpha_2
\]  
(17)

\[
6c_1c_2c_3 + c_3^2 = \alpha_4
\]  
(18)

\[
3c_1c_2^2 + 3c_2^2c_3 = \alpha_5
\]  
(19)

\[
3c_2c_3^2 = \alpha_6
\]  
(20)

\[
c_3^3 = \alpha_7
\]  
(21)

\[
3c_1^2 c_2 + 3c_1c_2^2 = \alpha_2
\]  
(22)

\[
5.2 Parameter value generating
\]

For the given values of equations (16) – (22) that substitute to the equation (21) gives the value of \( \beta \) that gives the value of \( \alpha_2 \). Later on, the value of \( c_2 \) from equation (23.c) and the value of \( c_3 \) from equation (23.d) that substitute to the equation (21) leads to the polynom of \( \beta \) that gives the root of it \( \beta = 0.0001 \). This value makes the value of \( \sigma \) must satisfy \( c_3 > 0 \). It is because the value of \( c_1 \) derived from equation (23.a) and the value of \( c_2 \) derived from equation (23.b) are positive, such that the possibility of equation (13) having the value of negative eigen value is satisfied in case of \( c_1, c_2 \) and are \( c_3 \) positive.

Using equation (23.c), substitution the value of \( \beta = 0.0001 \) leads to the positive value of \( c_3 \), such that the value of \( \sigma \) in equation (23.g) gives

\[
0.002468914432 < \sigma
\]  
(24)

This result justify the negative value of \( \lambda \) because it already satisfy \( M < E + \frac{\sigma^2}{\epsilon^2} \). Substitution the value of \( \varepsilon, M, \epsilon, \mu, q, k, r \) and \( \tau \) to the equation (24) also gives the requirement

\[
\sigma < 0.040568052075
\]  
(25)

Equation (24) and (25) are give the valid interval of 0.001 < \( \sigma < 0.04 \). On that interval, the values of \( \sigma \) that satisfy the negative value of eigen value are \( \sigma = 0.003 \) and \( \sigma = 0.004 \). This result is very important because the measurement of this parameter value is difficult to be done.
6. The simulation

To simulate the numerical solution of the model, some parameter values are stated in Table 1, while the initial condition are given as the starting point of the dependent variable $E(0) = 300, U(0) = 0, L(0) = 0, C(0) = 0, F(0) = 0, A(0) = 0, H(0) = 1000$. The initial condition of $E(0) = 30t$ is the real data taken from Baunyale Festival of 2016.

The Baunyale Festival 2016 are visited by 26,386 people, 23% of them were nyale catcher. The number of catch were 0.3 kg/ person such that the whole nyale being catch was 23% x 26,386 x 0.3 kg = 1,796.692 (Bachtiar, et.al, 2016). Assumed that the average weight of nyale is 1 gram, it could be predicted that the number of nyale to be catched is 300/ person.

They conducted research in Kuta and Seger Beach, Central Lombok, for six month starting from February 13th until November 23rd. The spawning was predicted happen on February 28th, 2016. After conducted field measurement, the data are identified in Zoology Laboratory, P2OLIPI, Jakarta. The measurement stated about the parameters value in every stage starting from zygot and larva. Beside broken the coral reef, the data of nyale was measured at the station of Kuta, Seger and Seriting beach from the sediment core or sediment grab. The sample was measure five times, that are on March, May, July, September and November.

6.1 The simulation of the transition

In this session the transition curve of nyale is figured with the given initial condition and stated parameters value. In case of every visitors catch 300 epitoki, it could be seen that its number will tend to zero with respect to time as a consequence of lifetime limitation (Fig. 2.a). Before dying, epitoki faces spawning process and produces zygot that decrease in a short time interval being larva trochopore (Fig.2.b). This characteristic is also found in the characteristic of larva in Fig.2.c. The transition phenomena could be seen in Fig.2.d. The decreasing of zygot is followed by the appearance of new individu of the next phase of cycle. As the next phase of larva, the juvenile grows to be the adult worm that split into two part of body named epitoki and atoki. Epitoki is actually the head part of nyale that usually when the Baunyale Festival held, while the tail part named atoki is kept living in the coral. Later on atoki will regenerate its body to be the new individu of adult nyale.

![Image](a)
![Image](b)
![Image](c)

Figure 2 : a. The decreasing of Epitoki  
 b,c. The population growth of zygot and larvae  
 d. The transition of zygot being larvae  
 e. The population growth of zygot and larvae
6.2 The dynamic of juvenile, adult worm and atoki

The adult nyale, that in the short term seems growth reaching to some number, will decrease and converge to some number of atoki. The convergence number 4000 seen in Fig 3 is nothing but the number of adult worm that have to be maintain in every festival ($M$). The figure shows that each initial condition of various values of Epitoki gives a solution that converges to $M$. This result indicates the minimum number of adult worm that has to be exist to sustain the annual festival.

Figure 3. The convergence number of adult worm for various number of initial condition of Epitoki

$$E(0) = 300, E(0) = 900, E(0) = 2700, E(0) = 8100 \text{ and } E(0) = 24300$$

Figure 4. The dynamic of juvenile, adult worm and atoki for the various value of epitoki

a. $E(0) = 300$  b. $E(0) = 900$  c. $E(0) = 2700$  d. $E(0) = 8100$  e. $E(0) = 24,300$
The mathematical model of *Eunice siciliensis* has an endemic critical point that determines the stable *Eunice siciliensis* life cycle for some restriction of condition, that is:

\[
\mathcal{T} = A(E^*), C(E^*), F(E^*), H(E^*), L(E^*), U(E^*) = \left( \frac{\mu(E^2 + E^* - M^2)}{\mu E^2 + E^* - M^2} \right) \frac{\delta}{E^2 \sigma_1 \sigma_2} \quad \text{The requirement of its stability is } M < E + \frac{E^2 + E^* - M^2}{\mu E^2 + E^* - M^2}. \]

The requirements the number of adult worms that have to be maintained in order to prevent the sustainability of *Baunyale* festival.

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7. **The concluding remark**

The phenomena of the convergence of adult nyale to some number of atoki is also shown in Fig.4. Not only the adult nyale, the solution also shows the convergence of the number of juvenile to the number of atoki when the initial condition of epitoiki is varying. Qualitatively we could see that the more the initial condition of epitoiki, the more the decreasing of the juvenile occur. The effort to gain the number of existing atoki becomes urgent to hold the decreasing number of juvenile.