Study of Vainshtein mechanism in Galileon theory

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Abstract. In this project, we compute the nonlinear power spectrum for a generalized galileon model. This calculation is a first step to get access to the nonlinear effective gravitational constant and therefore to the scale where the model differs from general relativity.

1. Introduction
Actually, despite the fact there is a cosmological model that is accepted as the standard model, which fits perfectly the observations, there are some problems which are unresolved such as the cosmological constant problem. It can be seen as a fine-tuning problem or why the vacuum energy gravitates so little. Modified gravity models which are often presented as a possible solution to this problem do not actually try to resolve the problem but instead replace the cosmological constant problem by a similar problem. But they can be seen as good laboratories to study gravity models beyond General Relativity (GR). In fact, in various situations such as brane models, an effective theory on the brane provides additional corrections to Einstein’s theory.

In higher dimensional theories, the models have more than 2 degrees of freedom. For example, the DGP model has 5 degrees of freedom which can be split into a massless graviton, a massless vector field and a scalar field. But the main interesting parts of the model are contained in the additional scalar field (along with the massless graviton). In fact, integrating out the additional dimension [1] gives rise to an effective theory describing the brane bending mode, known as galileon theory [5]. Higher dimensional models contains often scalar fields which can be very relevant to cosmology. In fact, more generically many theories of high-energy physics, such as string theory and supergravity, predict light gravitationally coupled scalar fields [3, 4].

We see therefore that the path towards modified gravity can be very rich and interesting. But cosmology demands the new degree of freedom to be incredibly light in order to be relevant. Unfortunately light particles are highly constrained by fifth force experiments and solar system observations. A light scalar field is required for cosmology but not from local tests which put severe upper bound on its mass. Therefore this light degree of freedom should hide in local experiments, that is to say a screening mechanism.

One of the most relevant mechanism dubbed Vainshtein mechanism corresponds to a scalar field coupled to matter but becomes weakly coupled because of a large kinetic term. In this context, we would like to study this mechanism in realistic situations. The very interesting object in modified gravity is the evolution of the gravitational constant. If it is suppose to deviate from the Newtonian constant at large scales, it should recover this same constant at
small scales because of some screening mechanism. This effective constant gives then naturally access to the radius at which a deviation from GR appears and then to possible observations. In this direction, the Vainshtein mechanism has shown to be very interesting but it is not very well understood in a realistic framework of formation of structures. It is usually derived for an isolated object in a spherically symmetric situation. One of the challenging problem is to understand the elephant problem or how various vainshtein radii interact. In this context, we can try to answer this question, by studying the Vainshtein radius in an N-body problem. Also, the Vainshtein mechanism seems to be less effective in time dependent problem. Therefore, we will try to address also this problem by looking to the time evolution of the Vainshtein radius in this context. The strategy is to be able to extract the non-linear gravitational constant during matter evolution, we will have access to the Vainshtein radius (when it deviates sensibly from the Newtonian constant) and we will have access to its evolution during the cosmological evolution.

To be able to compute the effective gravitational constant in a non-linear regime, we need to have access for this model to the non-linear power-spectrum and bispectrum. In this first project, we will compute the nonlinear power spectrum of this model by using standard perturbation theory and compare to numerical simulations.

2. The model
In this paper, we consider a generalized form of the galileon model \[5\], for which the action takes the following form

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{c_2}{2} (\partial \phi)^2 - \frac{c_3}{\Lambda_0^2} \Box \phi (\partial \phi)^2 - \frac{c_4}{4\Lambda^4} (\partial \phi)^4 \right]
\]

where \(g\) is the metric determinant, \(R\) is the Ricci scalar, \(M_P\) is the reduced Planck’s mass, \(c_2, c_3\) and \(c_4\) are dimensionless constants, \(\phi\) is the galileon scalar field and \(\Lambda_v\) with \(\Lambda_s\) are characteristic scales of the model.

2.1. Background
From this action we obtain the Friedmann equations:

\[
3M_P^2 H^2 = \rho_m + \rho_\phi
\]
\[
-2\dot{H} M_P^2 = \rho_m + \rho_\phi + P_\phi
\]
\[
\dot{J} + 3H J = 0
\]

where

\[
\rho_\phi = \frac{c_2 \dot{\phi}^2}{2} - \frac{3c_4 \phi^4}{4\Lambda_s^4} - \frac{6c_3 H \dot{\phi}^3}{\Lambda_0^3}
\]
\[
P_\phi = \frac{c_2 \dot{\phi}^2}{2} - \frac{c_3 \dot{\phi}^4}{4\Lambda_s^4} + \frac{2c_3 \phi^2 \dot{\phi}}{\Lambda_0^3}
\]
\[
J = c_2 \dot{\phi} - \frac{6c_3 H \phi^2}{\Lambda_0^3} - \frac{c_4 \phi^3}{\Lambda_s^4}
\]

To analyze the background, we transform the equations into a dynamical system where we define the variables \((x, y)\) by

\[
H = H dS \sqrt{\frac{x}{y}}
\]
\[
\dot{\phi} = \phi dS \sqrt{xy}
\]
while \((H_{dS}, \dot{\phi}_{dS})\) are defined during the de Sitter era. We define the scales

\[

l_v = \frac{2c^2H^2_{dS}M_{Pl}}{\Lambda_3^3}, \quad l_* = \frac{c^4H^2_{dS}M_{Pl}^2}{4\Lambda_4^4}, \quad r_{dS} = \frac{\dot{\phi}_{dS}}{M_{Pl}H_{dS}}
\]

(10)

During the de Sitter phase, we have \(x = y = 1\) and \(\dot{H} = \ddot{\phi} = \rho_m = 0\) which implies from the equations of motion, the constraints

\[

\frac{c^2}{6}r_{dS}^2 - l_v r_{dS}^3 - l_* r_{dS}^4 = 1, \quad -\frac{c^2}{6}r_{dS}^2 + \frac{l_*}{3}r_{dS}^4 = 1
\]

(11)

We notice that these relations are invariant under the transformations \(c_i \rightarrow \gamma c_i\) and \(r_{dS} \rightarrow r_{dS}/\gamma\) along with \(l_v \rightarrow \gamma^3 l_v\) and \(l_* \rightarrow \gamma^4 l_*\). We see therefore that it is convenient to define an invariant parameter \(\alpha = l_v r_{dS}^3\) and express the others as

\[

l_* r_{dS}^4 = -3 - \frac{3}{2}\alpha
\]

(12)

\[

c^2r_{dS}^2 = -12 - 3\alpha
\]

(13)

We have therefore shown, that the dynamics of the model depend on only 1 parameter. Notice that \(\alpha = -2\) corresponds to galileon model (from eq.(10,12) we have \(c_4 = 0\)). The system (eq.(2,3,4)) reduces to a 2D dynamical system of the following form

\[

\frac{dx}{d\ln a} = f_\alpha(x,y)
\]

(14)

\[

\frac{dy}{d\ln a} = g_\alpha(x,y)
\]

(15)

where \(a\) is the scale factor, \((f_\alpha, g_\alpha)\) are functions of \((x,y)\) and the only parameter of the model is \(\alpha\).

![Figure 1](image-url)  

Figure 1. Phase space using the Poincaré sphere \((X,Y) > 0\). The dashed green line is an invariant submanifold which separates two phase spaces. \(P_{dS}\) represents the de Sitter phase while \(P^1_m\) and \(P^2_m\) represents 2 matter phases. For definitions of Poincaré sphere and invariant manifold along the way to study a dynamical system we refer to [8].

The stability has been studied by many authors and therefore we will not reproduce it here. We refer to [9] for a full analysis. We found that during the de Sitter phase which is an attractor,
the propagation of the perturbed scalar field doesn’t have Laplacian instability if $-2 < \alpha < 0$. We have also 2 matter points; the point $P^1_m = (0,0)$ which is an attractor and therefore can’t be followed by an accelerated universe while the point $P^2_m = \left(-1 - 4/\alpha, 0\right)$ is saddle with $c^2_a = 4/3$ (sound speed).

From now, we will consider the case $\alpha = -1$, for which we have

\[
\frac{dx}{d\ln a} = -3x \frac{18 - 6x - 20xy - 9y^2 + 9xy^2 - 2x^2y^2 + 21xy^3 - 5x^2y^3 - 6 + x^2y^4}{2(6 - 4x - 12xy - x^2y^2)} \quad (16)
\]

\[
\frac{dy}{d\ln a} = -3y \frac{6 + 2x + 4xy + 9y^2 - 15xy^2 + 2x^2y^2 - 21xy^3 + 7x^2y^3 + 6x^2y^4}{2(6 - 4x - 12xy - x^2y^2)} \quad (17)
\]

The phase space of this system is represented in Fig. (1). It is therefore easy to integrate the system, for which we get (see Fig. (2))

![Figure 2](image)

**Figure 2.** Left: Evolution of the density of matter and scalar field. Right: Evolution of $w_{eff} = -1 + 2\dot{H}/(3H^2)$ as function of $\ln(1+z)$.

### 2.2. Standard perturbation theory

Considering the metric

\[
ds^2 = -(1 + 2\Phi)dt^2 + a^2(1 - 2\Phi)dx^2
\]

along with $\delta \phi$ the perturbation of the scalar field and $\delta_m = \delta \rho_m/\rho_m$, we found

\[
\Delta \delta \phi + \frac{c_3}{a^2 \Lambda_v^2 \beta_1} \left[ (\Delta \delta \phi)^2 - (\partial_i \delta \phi)^2 \right] = \frac{c_3}{\Lambda_v^2 \beta_2 M_{Pl}^2} a^2 \rho_m \delta_m \quad (19)
\]

\[
\Delta \Phi = \frac{a^2}{2 M_{Pl}^2} \rho_m \delta_m + \frac{c_3}{\Lambda_v^2 M_{Pl}^2} \delta^2 \delta \phi \quad (20)
\]

Along with the Vlasov equation up to the second moment, which represents mass conservation and Euler equation, we have

\[
\dot{\delta}_m + \frac{1}{a} \partial_t \left[ (1 + \delta_m)u^i \right] = 0 \quad (21)
\]

\[
\dot{u}_i + Hu_i + \frac{1}{a} u^j \partial_j u_i = -\frac{1}{a} \partial_i \Phi \quad (22)
\]
where \( u^i \) is the mean density flow and we defined
\[
\beta_1 = \frac{c_2^2}{2} - \frac{2c_3^2}{\Lambda_3^6} (\dot{\phi} + 2H \dot{\phi}) - \frac{c_4^2}{\Lambda_4^6 M_{Pl}^2} \dot{\phi}^4 - \frac{c_4^4}{2\Lambda_4^6} \dot{\phi}^2
\]
\[
\beta_2 = \frac{2}{\dot{\phi}^2} \beta_1
\]
(23)

Transforming into Fourier space
\[
\delta(t, x) = \frac{1}{(2\pi)^3} \int d^3k \delta(t, k) e^{i k \cdot x},
\]
(25)

\[
u^j(t, x) = \frac{1}{(2\pi)^3} \int d^3k -i k_j k^2 aH \theta(t, k) e^{i k \cdot x},
\]
(26)

and defining \( \theta = \nabla_i u^i / (aH) \), we have
\[
\frac{\delta_m}{H} + \gamma + \int \frac{d^3k_1 d^3k_2}{(2\pi)^3} \delta(\vec{k} - \vec{k}_{12}) \alpha(k_1, k_2) \delta_m(k_1) \theta(k_2) = 0
\]
(28)

\[
\frac{\dot{\theta}}{H} + (2 + \frac{\dot{H}}{H^2}) \theta + \frac{3}{2} \Omega_m \delta_m - \frac{c_3 \dot{\phi}^2}{\Lambda_3^6 M_{Pl}^2 a^2 H^2 k^2} \delta \phi
\]
\[
+ \int \frac{d^3k_1 d^3k_2}{(2\pi)^3} \delta(\vec{k} - \vec{k}_{12}) \beta(k_1, k_2) \theta(k_1) \theta(k_2) = 0
\]
(29)

\[
k^2 \delta \phi + \frac{c_3}{\Lambda_3^6 \beta_2 M_{Pl}^2} a^2 \rho_m \delta_m - \frac{c_3}{a^2 \Lambda_4^6 \beta_1} \int \frac{d^3k_1 d^3k_2}{(2\pi)^3} \delta(\vec{k} - \vec{k}_{12}) \gamma(k_1, k_2) \delta \phi(k_1) \delta \phi(k_2) = 0
\]
(30)

where \( \vec{k}_{12} = \vec{k}_1 + \vec{k}_2 \) and
\[
\alpha(k_1, k_2) = 1 + \frac{\vec{k}_1 \cdot \vec{k}_2}{k_2^2}
\]
(31)

\[
\beta(k_1, k_2) = \frac{\vec{k}_1 \cdot \vec{k}_2 | \vec{k}_1 + \vec{k}_2 |}{2k_1^2 k_2^2}
\]
(32)

\[
\gamma(k_1, k_2) = k_1^2 k_2^2 - (\vec{k}_1 \cdot \vec{k}_2)^2
\]
(33)

and modified Poisson equation has been used
\[
-k^2 \Phi = \frac{a^2}{2 M_{Pl}^2} \rho_m \delta_m - \frac{c_3 \dot{\phi}^2}{\Lambda_3^6 M_{Pl}^2} k^2 \delta \phi
\]
(35)

These equations can be solved iteratively
\[
Y(t, p) = \sum_{n=1} Y_n(t, p),
\]
(36)

where \( Y \) represents \( \delta_m \), \( \theta \) or \( \delta \phi \), and \( Y_n \) is the n-order solution of the expansion.
2.3. First order perturbations

Using these equations to first order, we have

\[
\frac{\dot{\delta}_m(t, k)}{H} + \left(2 + \frac{\dot{H}}{H^2}\right)\theta(1)(t, k) + \frac{2}{3} \Omega_m \delta_m(1)(t, k) - \frac{c_3 \phi^2}{\Lambda^3 M^2_P \alpha^2 H^2} k^2 \delta(1)(t, k) = 0
\]

(37)

\[
\frac{\ddot{\delta}_m(t, k)}{H} + 2H \delta_m(t, k) - \frac{3}{2} H^2 \Omega_m \frac{G_{eff}(t)}{G} \delta_m(t, k) = 0
\]

(38)

Combining the different equations, we can reduce the system to a second order equation for \(\delta_m\)

\[
\ddot{\delta}_m(t, k) + 2H \dot{\delta}_m(t, k) - \frac{3}{2} H^2 \Omega_m \frac{G_{eff}(t)}{G} \delta_m(t, k) = 0
\]

(40)

where \(G_{eff}\) is the linear gravitational constant

\[
\frac{G_{eff}}{G} = 1 + \frac{2c_3^2 \phi^2}{\Lambda^6 M^2_P \beta_2}.
\]

(41)

Figure 3. Evolution of the effective gravitational constant for \(\alpha = -1\). We see that the model differs from GR only recently and therefore is perfect for a dark energy model.

The eq.(40) has solutions of the following form \(\delta_m(t, k) = D(t)A(k)\) and we can find that \(D\) has 2 solutions corresponding to a growing mode \((D_+)\) and decreasing mode \((D_-)\). For the linear solution, it is sufficient to consider only the growing mode, which gives

\[
\delta_m^{(1)}(t, k) = D_+(t)A(k), \quad \theta^{(1)}(t, k) = -D_+(t)A(k),
\]

(42)

(43)

\[
\delta\phi^{(1)}(t, k) = -\frac{a^2 \rho_m c_3}{k^2 \Lambda^3 \beta_2 M^2_P} D_+(t)A(k).
\]

(44)
2.4. Second order perturbations

Going to second order, we found

\[
\frac{\dot{\delta}_m^{(2)}(t, k)}{H} + \frac{\ddot{\delta}_m^{(2)}(t, k)}{H^2} = -\int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \delta(k - k_{12}) \alpha(k_1, k_2) \delta_m^{(1)}(t, k_1) \delta_m^{(1)}(t, k_2) \theta^{(1)}(t, k_1) (45)
\]

\[
\frac{\dot{\delta}_m^{(2)}(t, k)}{H} + \left(2 + H \frac{H}{H^2}\right) \theta^{(2)}(t, k) + \frac{2}{3} \Omega_m \delta_m^{(2)}(t, k) - \frac{c_3 \dot{\phi}^2}{\Lambda_\rho M_{Pl} a^2 H^2} k^2 \dot{\delta}_m^{(2)}(t, k) =
\]

\[-\int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \delta(k - k_{12}) \beta(k_1, k_2) \theta^{(1)}(t, k_1) \theta^{(1)}(t, k_2) (46)\]

\[
k^2 \delta \phi^{(2)}(k) + \frac{c_3}{\Lambda_\rho^2 M_{Pl}^2} \frac{a^2 \rho_m \delta_m^{(2)}(t, k)}{\eta^2} = \frac{c_3}{\Lambda_\rho^2} \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \delta(k - k_{12}) \gamma(k_1, k_2) \delta \phi^{(1)}(t, k_1) \delta \phi^{(1)}(t, k_2) (47)\]

The second order gives an equation similar to eq.(40), but with an inhomogeneous equation

\[
\ddot{\delta}_m^{(2)}(t, k) + 2H \dot{\delta}_m^{(2)}(t, k) - \frac{3}{2} H^2 \Omega_m \frac{G_{eff}}{G} \delta_m^{(2)}(t, k) = \frac{\dot{\delta}}{a^2} E(t, k) + \frac{c_3 \dot{\phi}^2}{M_{Pl}^2 H_0^2} F(t, k) + \frac{1}{a} E(t, k) + F(t, k) (48)\]

where

\[
E(t, k) = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \delta(k - k_{12}) \alpha(k_1, k_2) \delta_m^{(1)}(t, k_1) \theta^{(1)}(t, k_1)
\]

\[
F(t, k) = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \delta(k - k_{12}) \beta(k_1, k_2) \theta^{(1)}(t, k_1) \theta^{(1)}(t, k_2)
\]

\[
I(t, k) = \frac{c_3}{a^2 \Lambda_\rho^2} \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \delta(k - k_{12}) \gamma(k_1, k_2) \delta \phi^{(1)}(t, k_1) \delta \phi^{(1)}(t, k_2)
\]

The solution can be written in the following form

\[
\delta_m^{(2)}(t, k) = D^2(t) \left( \mathcal{W}_\alpha(k) - \frac{2}{\tau} \lambda(t) \mathcal{W}_\gamma(k) \right) (49)
\]

where

\[
\mathcal{W}_\alpha(k) = \frac{1}{(2\pi)^6} \int d^3k_1 d^3k_2 \delta(k_{12} - k) \alpha^{(s)}(k_1, k_2) A(k_1) A(k_2), (50)
\]

\[
\mathcal{W}_\gamma(k) = \frac{1}{(2\pi)^6} \int d^3k_1 d^3k_2 \delta(k_{12} - k) \gamma(k_1, k_2) A(k_1) A(k_2), (51)
\]

with

\[
\alpha^{(s)}(k_1, k_2) = 1 + \frac{k_1 \cdot k_2 (k_1^2 + k_2^2)}{2k_1^2 k_2^2}, \quad \gamma(k_1, k_2) = 1 - \frac{(k_1 \cdot k_2)^2}{k_1^2 k_2^2}. (52)
\]

and \(\lambda(t)\) has a complicated expression and for that will not be written but it’s evolution is represented below.

Using all these results, it is easy to find \(\theta^{(2)}(t, k)\) and \(\delta \phi^{(2)}(t, k)\) that we will not reproduce here.
where the function during matter era $\lambda = \mu \approx 1$.

2.5. Third order perturbations

Following the same procedure, we can write the third order solution in the following form

$$
\delta^{(3)}_{m}(t, k) = D_{\chi}^{3}(t) \left( W_{\alpha\alpha}(k) - \frac{2}{7} \lambda(t) W_{\alpha\gamma R}(k) - \frac{2}{7} \lambda(t) W_{\alpha\gamma L}(k) - \frac{2}{21} \mu(t) W_{\gamma\gamma}(k) \right),
$$

(53)

where the function $\mu$ is represented in Fig.4, and

$$
W_{\alpha\alpha}(k) = \frac{1}{(2\pi)^6} \int dk_{1}dk_{2}dk_{3}\delta^{(3)}(k_{1} + k_{2} + k_{3} - k)\alpha\alpha(k_{1}, k_{2}, k_{3})A(k_{1})A(k_{2})A(k_{3}),
$$

$$
W_{\alpha\gamma R}(k) = \frac{1}{(2\pi)^6} \int dk_{1}dk_{2}dk_{3}\delta^{(3)}(k_{1} + k_{2} + k_{3} - k)\alpha\gamma R(k_{1}, k_{2}, k_{3})A(k_{1})A(k_{2})A(k_{3}),
$$

$$
W_{\alpha\gamma L}(k) = \frac{1}{(2\pi)^6} \int dk_{1}dk_{2}dk_{3}\delta^{(3)}(k_{1} + k_{2} + k_{3} - k)\alpha\gamma L(k_{1}, k_{2}, k_{3})A(k_{1})A(k_{2})A(k_{3}),
$$

$$
W_{\gamma\gamma}(k) = \frac{1}{(2\pi)^6} \int dk_{1}dk_{2}dk_{3}\delta^{(3)}(k_{1} + k_{2} + k_{3} - k)\gamma\gamma(k_{1}, k_{2}, k_{3})A(k_{1})A(k_{2})A(k_{3}),
$$

with

$$
\alpha\alpha(k_{1}, k_{2}, k_{3}) = \frac{1}{3}(\alpha^{(s)}(k_{1}, k_{2} + k_{3})\alpha^{(s)}(k_{2}, k_{3}) + 2 \text{ cyclic terms}),
$$

$$
\alpha\gamma R(k_{1}, k_{2}, k_{3}) = \frac{1}{3}(\alpha(k_{1}, k_{2} + k_{3})\gamma(k_{2}, k_{3}) + 2 \text{ cyclic terms}),
$$

$$
\alpha\gamma L(k_{1}, k_{2}, k_{3}) = \frac{1}{3}(\alpha(k_{1} + k_{2}, k_{3})\gamma(k_{2}, k_{3}) + 2 \text{ cyclic terms}),
$$

$$
\gamma\gamma(k_{1}, k_{2}, k_{3}) = \frac{1}{3}(\gamma(k_{1}, k_{2} + k_{3})\gamma(k_{2}, k_{3}) + 2 \text{ cyclic terms}).
$$

3. Power Spectrum

With the third order solution we can compute the second order power spectrum (1-loop correction) of matter density contrast using

$$
\langle \delta(t, k_{1})\delta(t, k_{2}) \rangle = (2\pi)^3\delta_{m}^{(3)}(k_{1} + k_{2})P_{\delta\delta}(t, k),
$$

(54)
with $\delta(t, k) = \delta_m^{(1)} + \delta_m^{(2)} + \delta_m^{(3)}$, which gives after some simplifications

$$P_{\delta \delta}(t, k) = D_+^2(t)P_L(k) + D_+^4(t) \left( P_{\delta \delta}^{(22)}(t, k) + 2P_{\delta \delta}^{(13)}(t, k) \right) ,$$

(55)

where $P_L(k)$ is the linear matter power spectrum (correlation function of the amplitude $A(k)$) and $P_{\delta \delta}^{(22)}(t, k)$ and $P_{\delta \delta}^{(13)}(t, k)$ are defined by

$$P_{\delta \delta}^{(22)}(t, k) = \frac{k^3}{98(2\pi)^2} \int dr P_L(rk) \int_{-1}^{1} dx P_L(k(1 + r^2 - 2rx)^{1/2})(7 - 4\lambda)r + 7x + 2(2\lambda - 7)rx^2)^2$$

$$\frac{(1 + r^2 - 2rx)^2}{(1 + r^2 - 2rx)^2},$$

(56)

$$2P_{\delta \delta}^{(13)}(t, k) = \frac{k^3}{252(2\pi)^2} P_L(k) \int dr P_L(rk) \left[ 12\mu \frac{1}{r^2} - 2(21 + 36\lambda + 22\mu) + 4(84 - 48\lambda - 11\mu)r^2$$

$$- 6(21 - 12\lambda - 2\mu)r^4 + \frac{3}{r^3} (r^2 - 1)^2 ((21 - 12\lambda - 2\mu)r^2 + 2\mu) \ln \left( \frac{r + 1}{|r - 1|} \right) \right].$$

(57)

After integration, we found

$$\text{Figure 5.} \quad \text{Evolution of the power spectrum as a function of the scale } k. \quad \text{In blue, the linear power spectrum, in red the 1-loop correction and in dashed red the halofit model for the nonlinear matter power spectrum.}$$

From Fig. 5, we see that our 1-loop correction approximates very well the power spectrum at all scales while of course the linear spectrum fails for $k > 0.1 \text{ Mpc}^{-1}$ (small scales), because of the nonlinearities. This result constitute the first part before getting the nonlinear effective gravitational constant which will be presented in a future project.

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