Determining the magnetic dipole moment of the rho meson

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Abstract. We elaborate on the procedure followed to determine the magnetic dipole moment of the rho meson, \( \mu_\rho = 2.1 \pm 0.5 [e/2m_\rho] \), from the \( e^+e^- \rightarrow \pi^+ \pi^- 2\pi^0 \) process. Using preliminary data from the BaBar Collaboration and a vector meson dominance approach.

1. Introduction
The magnetic dipole moment (MDM) of the elementary particles has been a key tool to explore underlying properties of the fundamental interactions. For states made up of quarks, the properties of the strong interaction in the low energy regime can be explored. For long lived states, like the proton and neutron, the measurement of their corresponding MDM has helped to understand its relationship as a consequence of their quark structure. However, to date, there is no measurement of the MDM of any vector meson\[1\], a state made up of quark an antiquark in a spin 1 configuration. This fact is related to their extremely short lifetimes (\( \approx 10^{-23} \) s), which prevents experimentalists to apply standard MDM measurement techniques. Alternatives to determine the MDM, by indirect means, invoke the fact that the radiation emitted from the vector meson carries information of its electromagnetic structure\[2\] and thus, provided the dominant electric radiation is known, the subleading MDM effect can be identified. This is the same approach used to determine the MDM of the \( W \) gauge boson, a particle with the same characteristics of a vector meson but of fundamental nature.

In this proceedings we elaborate on the procedure we followed to perform the first determination of the MDM of a vector meson, namely the MDM of the \( \rho \) meson [3], obtained from the \( e^+e^- \rightarrow \pi^+ \pi^- 2\pi^0 \) process, using preliminary data from the BaBar Collaboration[4]. The \( \gamma^* \rightarrow 4\pi \) vertex is modeled in the vector meson dominance (VMD) approach.

2. The electromagnetic vertex
The \( e^+e^- \rightarrow \pi^+ \pi^- 2\pi^0 \) process has been measured for several experiments in the low energy regime. The leading channel contribution to the total cross section was found to be that involving...
the $\omega - \pi$ mesons intermediate state. Measurements at increasing energies made clear that this process was also sensitive to the channel where the pions are produced in pairs through the $\rho - \rho$ meson resonant states. This particularity opens the door to explore the $\rho$ electromagnetic vertex and by consequence the corresponding electromagnetic multipoles. The study we have performed in [3] uses these facts to determine the MDM by fitting the experimental cross section data, modeled in the VMD approach including all the relevant intermediate states, while using other observables to fix all the remaining parameters.

Let us begin with a brief discussion of the general features of such electromagnetic vertex. For a vector particle ($V$), the vertex for the process $V(q_1, \epsilon) \rightarrow V(q_2, \eta) \gamma(q)$ is defined from the electromagnetic current

$$< V(q_2)|J_{EM}(0)|V(q_1) > \equiv \eta^\dagger \epsilon^\mu \Gamma_{\mu\nu\lambda}^{\nu\lambda},$$

where $q_i$ are the momenta and $\epsilon$ and $\eta$ are the corresponding polarization tensors. The CP conserving electromagnetic vertex $\Gamma_{\mu\nu\lambda}$ can be decomposed into the following Lorentz structures

$$\Gamma_{\mu\nu\lambda} = \alpha(q^2)g^{\nu\lambda}(q_1 + q_2)^\mu + \beta(q^2)(g^{\mu\nu}q^\lambda - g^{\mu\lambda}q^\nu) - \gamma(q^2)(q_1 + q_2)^\mu q^\nu q^\lambda.$$

The electromagnetic form factors $\alpha(q^2)$, $\beta(q^2)$ and $\gamma(q^2)$ dependence on $q^2$ is modeled in VMD by including the hadronic degrees of freedom of relevance in the corresponding energy regime. In the static limit, such form factors are related to the electromagnetic multipoles as follows:

$$Q = \alpha(0)$$

is the electric charge (in $e$ units),

$$\mu = \beta(0) \equiv 1 + \kappa + \lambda$$

is the magnetic dipole moment (in $e/2M_V$ units) and the electric quadrupole is $|X_E| = \kappa - \gamma(0)M^2_V \equiv \kappa - \lambda$ (in $e/M_V^2$ units),

where we have included the relation to the parameters $\kappa$ and $\lambda$ which are also of common use in the literature to refer to the electromagnetic multipoles of spin-1 particles [5, 6]. For instance, at tree level, the standard model gauge structure predicts for the $W$ gauge boson to have $\alpha(0) = 1$, $\beta(0) = 2$ and $\gamma(0) = 0$ ($\kappa = 1$ and $\lambda = 0$), corresponding to $|Q| = 1$, $|\mu| = 2$ and $|X_E| = 1$. These values are usually taken as a reference for vector mesons.

The predictions for the MDM $\rho$ meson computed using approaches to the strong interaction, have been found to lay in the region from 1.9 to 3 in $e/2M_V$ units.

The $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ process has been measured by several experiments in a direct way at low energies [7], and preliminary data is available from the BaBar collaboration [4] using initial radiation technique in a wider range.

3. Modeling the $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ process

This process involves a standard electromagnetic vertex for the electron-positron annihilation, which is properly accounted by a QED description. The description we followed for the hadronic part is based on VMD. The energy range to be described goes from threshold up to 2.2 GeV. Upon the pair annihilation, the photon is considered as becoming a $\rho$ and $\rho'$ neutral vector mesons, under the VMD rules, with a relative phase of $180^\circ$.

We have considered the channels contributing to the four pions production, including the exchange of the $\pi$, $\omega$, $a_1$, $\sigma$, $f(980)$, $\rho$ and $\rho'(1450)$ mesons. Therefore, we are left with seven generic channels, each one accounting for several specific diagrams, corresponding to the allowed permutations of the momenta due to Bose-Einstein symmetry and charge conjugation. Note that not all the diagrams are independent, gauge invariance condition requires some of them to behave in such a way that the conditions is fulfilled. Of particular in the description is the case when the $\rho'$ is included. So far there is a very poor information on this particle and its decay modes. The structure of the triple vector meson vertex is expected to follow the same form as the electromagnetic vertex previously discussed but for a set of different couplings. We have made the assumption that for such a vertex involving the $\rho$ and $\rho'$ it takes exactly the same value than for the vertex involving only the $\rho$ meson. This assumption has been found to
Figure 1. Fit to the BaBar data for the total cross section $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$. We use the MDM as the only free parameter, while the other involved parameters are fixed from different observables.

be appealing[8] in the description of this process.
As we mentioned in the introduction, vector mesons are extremely short lived particles. In order to account for this fact a proper scheme must be used to include its unstable nature without breaking the electromagnetic gauge invariance. For this purpose, the vector meson propagator is considered to take the complex mass form.
Provided the couplings involved in the included channels are determined from different observables, the ones involving the electromagnetic vertex $\Gamma^{\mu\nu\gamma}$, namely, the $\beta$ and $\gamma$ parameters can be determined by fitting the total cross section of the $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ process.

In Figure 1, we show the total cross section data with a 10% systematic error bars [4], and the fit corresponding to $\beta$ as a free parameter and $\gamma = 0$. The fit considering $\beta$ and $\gamma$ as free parameters favors the same $\beta$ and restricts $\gamma$ to be in the range $(-1.1, 0.1)$. That is, the $\beta$ parameter accounts for the global description, while the $\gamma$ contribution enters at the end region. To determine the cross section error bars, we have taken into account the combined uncertainties coming from the couplings of the different channels, assumed as no correlated. We also explored the role of the model assumption regarding the $\rho'$ triple boson vertex, the global combination of couplings and mass was found to be consistent with data for up to a 10% deviation from the combination for the $\rho$. We determine the $\beta$ parameter error bar considering it as the responsible of the total uncertainties. In addition, to account for the model dependence, we have added a 20% error (added in quadrature). Thus, corresponding to a MDM

$$\mu_\rho = 2.1 \pm 0.5 \left[ \frac{e}{2m_\rho} \right].$$

4. Conclusions
In summary, we have determined the magnetic dipole moment of the $\rho$ meson, by studying its effect in the $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ cross section, in the VMD approach. The channel that contains
the electromagnetic vector meson vertex becomes relevant for energies between 1.5 and 2.2 GeV, while the remaining channels are always subdominant. We have found that the best fit to the BaBar data implies a value for the MDM of the $\rho$ meson of $\mu = 2.1 \pm 0.5 \left(\frac{e}{2m_{\rho}}\right)$. The quoted error bar takes into account the uncertainties coming from the couplings of the different channels and model assumptions, definite data on this process and detailed information on the $\rho'$ meson will be very useful for a more refined analysis.

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