Role of pion pole in hard exclusive meson leptonroduction

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Abstract. We consider the pion pole contribution and transversity effects determined by the $H_T$ and $\vec{E}_T$ Generalized Parton Distributions (GPDs) which are essential in hard pseudoscalar and vector meson leptonroduction. We investigate spin effects in the $\omega$ and $\rho^0$ reactions. It is shown that the pion pole contribution is very important in the $\omega$ production. Such effects in the $\rho^0$ channel are much smaller. Our results on spin asymmetries and spin density matrix elements in these reactions were found to be in good agreement with HERMES data.

1. Introduction
In this report, we discuss pion pole effects [1] and transversity twist-3 contributions from the $H_T$, $\vec{E}_T$ GPDs [2] which are important in explanation of spin effects in the light meson production at moderate $Q^2$. Our model of hard electroproduction of light mesons [3] is based on the handbag factorization at high photon virtuality $Q^2$ of the process amplitude into the hard meson electroproduction off partons, and GPDs [4].

The amplitudes of the pseudoscalar meson (PM) leptonroduction in the leading twist approximation are sensitive to GPDs $\vec{H}$ and $\vec{E}$. These contributions were found to be not sufficient to describe spin effects in the PM production at sufficiently low $Q^2$ [2]. To be consistent with experiment, one needs the essential contributions from the transversity GPDs $H_T$, $\vec{E}_T$ which go together with a twist-3 pion wave function [5]. We discuss the pion pole and transversity effects in the PM leptonroduction at HERMES and CLAS energies in section 2. Our results are in good agreement with experimental data.

The HERMES data on SDMEs for the $\omega$ production indicated essential contributions from unnatural parity exchanges [6]. It was found that the pion pole (PP) contribution [1] is significant in explanation of the large unnatural-parity effects observed by HERMES. The PP contribution to the $\rho^0$ production is much smaller with respect to the $\omega$ case. We discuss the PP effects in $\omega$ and $\rho$ production in section 3.

2. Handbag approach. Pseudoscalar meson leptonroduction.
Within the handbag approach the leading contributions to the meson production amplitude off proton at sufficiently high photon virtuality $Q^2$ can be described in factorized form [4] as a convolution of a hard meson subprocess amplitude off partons with the same helicities $\mathcal{H}^{a}_{\mu',\mu}$. 

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and GPDs as
\[
M_{\mu',\mu+} \propto \int_{-1}^{1} dx \mathcal{H}^{(0)}_{\mu',\mu+}(x,\xi,\ell) \quad \text{and} \quad M_{\mu',-\mu+} \propto \frac{1}{2m} \int_{-1}^{1} dx \mathcal{H}^{(0)}_{\mu',-\mu+}(x,\xi,\ell) \cdot E^a(\vec{q},\xi,\ell). \quad (1)
\]

Here \( a \) is a flavor factor and \( \mu \) and \( \mu' \) are helicities of the photon and produced meson.

GPDs contain information on the hadron structure. With the help of sum rules they are connected with hadron form factors, and information on the parton angular momenta can be extracted. In the forward limit \( t = 0 \) and zero skewness \( \xi = 0 \) GPDs are equivalent to ordinary Parton Distribution Functions (PDFs). We estimate GPDs by using the double distribution representation [7] which connects GPDs with PDFs. The PDFs parameterizations are obtained, e.g., from the analysis [8], or from the nucleon form factor study [9].

The handbag approach was successfully applied to light meson leptoproduction [3]. The cross sections and spin observables of light vector meson (VM) leptoproduction were found to be in good agreement with HERMES, COMPASS and HERA data.

The leading twist contributions are not sufficient to describe the experimental results on PM electroproduction at low \( Q^2 \). This can be found from analysis of \( A_{UT}^{\sin(\phi_5)} \) asymmetry which in the leading twist handbag approximation is small. However, HERMES found [10] that this asymmetry is large, about \( 0.5 \). This effect can be explained by the large contributions to the \( A_{UT}^{\sin(\phi_5)} \) asymmetry from the amplitude \( M_{0\perp,+,+} \). At low \( Q^2 \) the amplitudes \( M_{0\perp,+,+} \) are determined by the transversity GPDs \( H_T \) and \( \bar{H}_T \) contributions which have the twist-3 nature. Within the handbag approach the transversity GPDs are accompanied by a twist-3 meson wave function in the hard subprocess amplitude \( H \) [5] which is the same for both the \( M_{0\perp,+,+} \) amplitudes

\[
M_{0\perp,+,+}^{\text{tw}-3} \propto \int_{-1}^{1} dx \mathcal{H}_{0\perp,+,+}(x,\ldots) H_T; \quad M_{0\perp,+,+}^{\text{tw}-3} \propto \frac{\sqrt{-t}}{4m} \int_{-1}^{1} dx \mathcal{H}_{0\perp,+,+}(x,\ldots) \bar{H}_T. \quad (2)
\]

The \( H_T \) GPDs in the forward limit and \( \xi = 0 \) are equal to transversity PDFs \( \delta \) and are parameterized by using the model [11]. The double distribution is used to calculate GPDs as before. For details, see [5].

Information on \( \bar{H}_T \) is obtained now only from the lattice QCD [12]. The lower moments of \( E_T^2 \) and \( \bar{E}_T^2 \) were found to be quite large, have the same sign and a similar size. At the same time \( H_T^2 \) and \( \bar{H}_T^2 \) GPDs have a different sign. These properties of GPDs provide an essential compensation of the \( \bar{E}_T \) contribution in the \( \pi^+ \) amplitude, but \( H_T \) effects are not small there. For the \( \pi^0 \) production we have large \( \bar{E}_T \) contributions and smaller \( H_T \) effects.

We present here our results on the PM leptoproduction based on the handbag approach. In calculation, we use the leading contributions together with the transversity effects (2) which are essential at low \( Q^2 \).

In Fig.1, we show our results for the \( \pi^+ \) production which are in agreement with HERMES [13]. The dashed line indicates the model results for \( H_T = 0 \). It is closed to the PP contribution to the \( \pi^+ \) cross section. So in this channel the PP effects are very essential. The pion induced Drell-Yan process \( \pi^- p \to \gamma^* n \to l^+ l^- n \) which is a time-like analog of \( \pi^+ \) leptoproduction was investigated in [15]. It was found that the pion pole gives the predominant contribution in the longitudinal cross section of this process. Other effects are rather small in \( \sigma_L \).

In Fig. 2, we present the model results for the \( \pi^0 \) production cross section [5]. Here the PP contribution is absent but transversity effects are essential. At small momentum transfer the \( H_T \) contribution is visible and provides a nonzero cross section. At \( -t' \sim 0.2 \text{GeV}^2 \) the \( E_T \) contribution becomes predominant and gives a maximum in the cross section. A similar contribution from \( \bar{E}_T \) is observed in the interference cross section \( \sigma_{TT} \). Note that the \( \bar{E}_T \) effects in the unseparated cross section \( \sigma \) which is saturated by \( \sigma_T \) is strongly correlated with similar
effects in $\sigma_{TT}$. The fact that we describe well both the unseparated $\sigma$ and $\sigma_{TT}$ cross sections can indicate that transversity effects were probably observed in CLAS [14].

![Graph](image1)

**Figure 1.** Cross section of $\pi^+$ production at HERMES energies with the data [13]. Full line- unseparated cross section, Dashed line- model results for $H_T = 0$.

![Graph](image2)

**Figure 2.** $\pi^0$ production in the CLAS energy range together with the data [14]. Dashed-dot-dotted line- $\sigma = \sigma_T + \epsilon \sigma_L$, dashed line-$\sigma_{LT}$, dashed-dotted line- $\sigma_{TT}$.

3. **Large unnatural parity effects in $\omega$ production. Pion pole contribution.**

In most reactions the unnatural parity (UP) contributions are small with respect to the natural ones. The HERMES data on the spin density matrix elements (SDMEs) for the $\omega$ production indicate the strong contributions from UP effects [6]. It was found that the ratio of the unnatural and natural parity cross section, which was expected to be small, is larger than unity. This can be caused by the large pion pole contribution to this process.

![Diagram](image3)

**Figure 3.** Pion pole contribution to VM production. The UP helicity amplitudes determined by pion pole looks as follows:

$$M_{++,...}^{pole} \sim \frac{\rho_{\pi V}}{t-m_{\pi}^2} \frac{m \xi Q^2}{\sqrt{1-\xi^2}}$$

$$M_{+-,...}^{pole} \sim -\frac{\rho_{\pi V}}{t-m_{\pi}^2} \frac{\sqrt{\xi} Q^2}{2}$$

with

$$\rho_{\pi V} \sim g_{\pi V}(Q^2) g_{\pi NN} F_{\pi NN}(t).$$

In Fig 3, we show the PP contribution to the VM production together with the helicity amplitudes generated by the pion pole. These amplitudes have the UP nature and are controlled by the $\pi V$ transition form factor. The transition form factor $g_{\pi V}(0)$ can be determined from the VM radiative decay

$$\Gamma(V \rightarrow \pi \gamma) \sim \frac{\alpha_{em}}{24} |g_{\pi V}(0)|^2 M_V^3.$$  

We find

$$|g_{\pi \omega}(0)| = 2.3 GeV^{-1}; \quad |g_{\pi \rho}(0)| = .85 GeV^{-1}. \quad (4)$$
This means that $|g_{\pi\omega}(0)|$ is about 3 times larger with respect to $|g_{\pi\rho}(0)|$ and we should observe large PP effects in $\omega$ and small in $\rho$ production. The $Q^2$ dependence of $g_{\pi V}(Q^2)$ was extracted [1] from the HERMES data [6] on the ratio of the unnatural to the natural parity cross section $U_1$ at $Q^2 < 4\text{GeV}^2$.

We will discuss our results on PP effects in the $\omega$ and $\rho$ production [1] and give a comparison with HERMES data [6]. In calculations we use GPDs from our analysis of hard meson lepton production.

The natural and unnatural parity asymmetry $P$ is determined as follows:

$$P = \frac{d\sigma\, (\gamma_T^+ \rightarrow V_T) - d\sigma\, (\gamma_T^- \rightarrow V_T)}{d\sigma\, (\gamma_T^+ \rightarrow V_T) + d\sigma\, (\gamma_T^- \rightarrow V_T)}.$$  \hspace{1cm} (5)

If the UP contribution is small, we find $P \sim 1$. If it is large, the value of the $P$ asymmetry will be far from unity. The model results for this asymmetry for $\omega$ production are shown in Fig.4. We find that the PP contribution to the $\omega$ asymmetry give $P \sim -0.5$ (full line) in agreement with experiment. While neglecting the PP contribution we obtain $P \sim 0.5$ (dashed line). In this figure, we show for comparison the model results for CLAS energy $W = 3.5\text{GeV}$ by the dotted line and for COMPASS energy $W = 8\text{GeV}$ by the dashed-dotted curve. It can be seen that at COMPASS energies PP effects are rather small for $\omega$. For the $\rho$ production PP effects in $P$ asymmetry are shown in Fig. 5. They are small and the asymmetry is close to unity.

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{figure4}
\caption{$P(\omega)$ at HERMES. Solid line-with PP contribution, dashed line - without PP. Dotted line-for $W = 3.5\text{GeV}$ (CLAS), dashed-dotted line for $W = 8\text{GeV}$ (COMPASS).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{figure5}
\caption{$P(\rho^0)$ at HERMES. Solid line with PP, dashed line - without PP.}
\end{figure}

The SDME $r_{1-1} = \text{Im} r_{2-1}$ shows the difference of the natural and unnatural parity contributions

$$r_{1-1} = \frac{d\sigma\, (\gamma_T^+ \rightarrow V_T) - d\sigma\, (\gamma_T^- \rightarrow V_T)}{2d\sigma}.$$  \hspace{1cm} (6)

The results for this SDMEs for the $\omega$ production are shown in Fig. 6. We see that the PP effects are very strong for $\omega$. With PP contribution we obtain $r_{1-1} \sim -0.2$ and if we omit PP, $r_{1-1} \sim 0.2$. For the $\rho$ meson production the PP contribution is small and the results are close to the $\omega$ case without PP (see Fig.7). We find that HERMES experimental results can not be explained without consideration of the PP contributions.
Pion pole effects can be observed in spin asymmetries. We find two not small interference terms with PP

\[ \text{Im} M_{1++}^{N^*} M_{1++}^{pole} \quad \text{and} \quad \text{Im} M_{1++}^{U^*} M_{1++}^{pole} \]  

(7)

which contribute to spin asymmetries for longitudinal and transverse beam and target polarizations. Note that the \( M_{1++}^{U^*} \) amplitude in (7) is determined by \( \hat{H} \) effects. The PP contributions to asymmetries change the sign with the transition \( \pi V \) form factor.

Some examples for asymmetries in Figs. 8, 9 show substantially different results for the positive and negative sign of the \( \pi \omega \) transition form factor. This means that generally, the sign of this form factor might be determined experimentally. New HERMES data where \( \delta \) modulations of \( A_{UT} \) asymmetry was analysed [16] are probably consistent with the positive transition \( \pi \omega \) form factor. Unfortunately, the large experimental uncertainties prevent a definite conclusion on the sign of this transition form factor.

**Figure 6.** SDMEs for \( \omega \) production at HERMES.

**Figure 7.** SDMEs for \( \rho \) production at HERMES.

Solid line- with PP, dashed -without PP.

**Figure 8.** \( \sin(\phi - \phi_s) \) modulation of \( A_{UT} \) asymmetry.

Full line - positive, dashed- dotted line - negative \( \pi \omega \) transition form factor.

**Figure 9.** \( \sin(\phi_s) \) modulation of \( A_{UT} \) asymmetry.
4. Conclusion
We calculated the light meson lepton production reactions within the handbag approach, where the amplitudes factorize into hard subprocesses and GPDs [4]. The results on the cross sections and various spin observables based on this approach are in good agreement with data at HERMES, COMPASS and HERA energies at high $Q^2$ [3].

We considered pion pole contribution to $\pi^+$ lepton production. The PP effect in this case is determined by photon interaction with a charged pion. It was shown that PP gave an essential contribution to the leading twist longitudinal cross section [2]. Similar effects were found in the pion-induced Drell-Yan process [15]. The transversity $H_T$ and $\tilde{E}_T$ GPDs effects in the PM lepton production were investigated in [2, 5]. It was found that the transversity contributions are essential in the PM lepton production where they lead to large transverse cross sections which for $\pi^0$ production at low $Q^2$ exceed essentially the leading twist longitudinal cross section.

The pion pole effects in vector meson production have a different nature. They are determined by the $\pi V$ transition form factor. It is found that the PP contributions are essential in $\omega$ and rather small in $\rho^0$ production. Based on our approach we found that the model results for $\omega$ SDMEs [1] are in good agreement with HERMES experimental results [6]. The large unnatural-parity effects observed by HERMES can be explained by the substantial PP contributions there. The PP effects in $\omega$ spin asymmetries are sensitive to the sign of the $\pi\omega$ transition form factor [1]. The experimental test of these asymmetries by HERMES [16] does not give a definite conclusion on the sign of the $\pi\omega$ transition form factor because of the large experimental uncertainties.

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