Detection of Fatigue Crack in Basalt FRP Laminate Composite Pipe using Electrical Potential Change Method

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Abstract. Novel modulation electrical potential change (EPC) method for fatigue crack detection in a basalt fibre reinforced polymer (FRP) laminate composite pipe is carried out in this paper. The technique is applied to a laminate pipe with an embedded crack in three layers [0°/90°/0°]. EPC is applied for evaluating the dielectric properties of basalt FRP pipe by using an electrical capacitance sensor (ECS) to discern damages in the pipe. Twelve electrodes are mounted on the outer surface of the pipe and the changes in the modulation dielectric properties of the piping system are analyzed to detect damages in the pipe. An embedded crack is created by a fatigue internal pressure test. The capacitance values, capacitance change and node potential distribution of ECS electrodes are calculated before and after crack initiates using a finite element method (FEM) by ANSYS and MATLAB, which are combined to simulate sensor characteristics and fatigue behaviour. The crack lengths of the basalt FRP are investigated for various number of cycles to failure for determining crack growth rate. Response surfaces are adopted as a tool for solving inverse problems to estimate crack lengths from the measured electric potential differences of all segments between electrodes to validate the FEM results. The results show that, the good convergence between the FEM and estimated results. Also the results of this study show that the electrical potential difference of the basalt FRP laminate increases during cyclic loading, caused by matrix cracking. The results indicate that the proposed method successfully provides fatigue crack detection for basalt FRP laminate composite pipes.

1. Introduction

A review of recent literature indicates that there is an increasing interest in the aircraft industry in monitoring the actual health condition of a structure in real time and while the structure is in service. It is anticipated that a real-time Structural Health Monitoring (SHM) could make some of the regular inspections unnecessary and allow maintenance only when required. This is particularly important for basalt FRP structures as a new composite material in which aircraft manufacturers are increasingly interested. For these new composite structures where the incidence of fatigue is low, SHM technique could potentially be economical and improve the safety of the structures. While there might be numerous benefits, there are some challenges and difficulties in operational conditions especially in early stage
that need to be dealt with. This challenges may require large computational efforts in the SHM and potential maintenance. The challenges feed by SHM system has been discussed by many researchers [1-7].

Non-destructive testing (NDT) methods have been found to be useful for in-situ evaluation of composite structures, where the structural integrity of laminate composite structures can be assessed effectively. There is a diverse range of NDE techniques for detecting fatigue crack in composite structures, the capabilities and limitations of each method are different. Each technique has its specific field of applicability although there is a level of overlap based on the size of defect and accuracy of detection and the ability to detect certain types of damage. For instance, it may be necessary to combine information obtained from acoustic emission and X-ray radiography to achieve a three-dimensional map of the complex array of fatigue cracks in a composite, however, no single method is capable of easily detecting, or identifying, all fatigue cracks with high level of accuracy, and at a low-cost [8].

ECS is one of the most mature and promising methods, which measures the capacitance change of multi-electrode sensors due to the change in dielectric permittivity being imaged. It can then reconstruct the cross-section images using the measured raw data utilizing a suitable algorithm. It has the characteristics such as being a low cost, fast response, non-intrusive method with a broad range of applications and with a high level of safety [9-13]. The need for a more accurate measurement of ECS has led to the study of the factors which influence and affect ECS sensitivity and the sensitive domain of ECS electrodes. There are three factors that have been studied and found that they affect ECS measurements, e.g. pipe’s material, inner dielectric permittivity [14-20] and the ratio of pipeline thickness and diameter [21-22]. Altabey [23] found that the environmental temperature also affects ECS sensitivity and sensitive domain of ECS electrodes with high percentage. Therefore, it was concluded that the environmental temperature should be considered as the fourth factor which influences the ECS measurement sensitivity.

In the present study, detection of fatigue cracks in a basalt FRP laminate composite pipe is attempted numerically using a new non-destructive evaluation (NDE) technique via utilizing ECS to detect the changes in the dielectric properties. An embedded crack due to fatigue internal pressure is created across the pipe thickness on which multiple electrodes are mounted. A finite element model (FEM) is developed to simulate the fatigue life behaviour before and after matrix cracking. Through measuring the capacitance values, capacitance change and electric potential difference of ECS electrodes are analyzed in order to obtain relationships between the electric potential differences and the fatigue life before and after cracking. The crack lengths in the basalt FRP pipe are investigated for various number of cycles to failure to determine crack growth rate. Applicability of the method for the detection of fatigue cracking is numerically investigated in detail using ANSYS and MATLAB software. The Response surfaces are adopted as a tool for solving inverse problems to estimate crack lengths from the measured electric potential differences of all segments between electrodes to validate the FEM results.

2. Principle of Electrical Capacitance sensor (ECS)

ECS was first introduced in the 1980s by a group of researchers from the US Department of Energy, at Morgantown Energy Technology Centre (METC), to measure fluidized bed systems [24-26]. The technique further developed and advanced rapidly during the past 10 years. It has gained attention and found important applications in monitoring industrial processes, due to its low cost and its operability under harsh environmental conditions.

ECS consists of an insulating pipe, measurement electrode, radial screen and earthed screen [27]. The measurement electrode is mounted symmetrically around the circumference of a pipeline. Radial screen is fitted between the electrodes to cut the electro-line external to the sensor pipeline and reduce the inter-electrode capacitance. The earthed screen surrounds the measurement electrodes to shield external electromagnetic noise. In most applications, ECS electrode is mounted outside the pipeline which is called external electrode ECS [28]. Electrical capacitance system includes a sensor.

ECS converts the permittivity of the piping system to inter-electrode capacitance, which is the ECS forward problem. Capacitance measuring circuit takes the capacitance data and transfers that to imaging
computer. Imaging computer reconstructs the distribution image with a suitable algorithm, which is called ECS inverse problem.

2.1. ECS System geometrical model
The model section comprises of an ECS column with 0.1 m inner diameter and 0.3 m height. The ECS is made up of the basalt FRP laminate composite pipe structure and having a ring of 12 electrodes (which are separated from each other by a small gap) on its outer periphery. Figure 1 shows the cross section of a 12-electrode ECS system, in which $R_1$ is the inner pipe radius; $R_2$ is the outer pipe radius; $R_3$ is the earthed screen radius and the radial screen is connected to the outer composite pipe. The ECS also includes radial guard electrodes to constrain the field lines from the excited electrode and to reduce the dependence of spacing between the electrodes and the screen as shown in the Figure. The function of the sensor includes measuring the capacitance between all possible combination pairs of the electrodes and converting the measured capacitance values in the voltage signals. The sensors physical specifications and the permittivity values of GFRE composite pipe are shown in Table 1.

| ECS system                | Specification          |
|---------------------------|------------------------|
| No. of electrodes         | 12                     |
| Space between electrodes  | 2 mm                   |
| Pipe diameter (d,)        | 94 mm                  |
| Pipe thickness (h)        | 6 mm                   |
| Earth Screen diameter     | 110 mm                 |
| Thickness of electrodes   | 1mm                    |
| Permittivity Basalt fibre/Polymer | $\varepsilon_b = 2.2 \text{ Fm}^{-1}$ |
| Permittivity of Water     | $\varepsilon_w = 80 \text{ Fm}^{-1}$ |
| Permittivity of Air       | $\varepsilon_a = 1.0 \text{ Fm}^{-1}$ |
| Excitation voltage        | $\varphi = 15$ Volts   |

2.2. The ECS composition and working principle
For the 12-electrode system the electrodes are numbered as shown in Figure 1. These electrodes are excited with an electric potential, one at a time, in increasing order. When one electrode is excited, the other electrodes are kept at ground potential, as shown in Figure 1, and act as detector electrodes. When electrode No. 1 is excited with a potential, the change $Q_{1,j}$ induced on the electrodes, $j = 2, \cdots, N$, can be measured. Next, electrode No. 2 is excited whereas, rest of the electrodes are kept at ground potential, and the induced charges $Q_{23}, Q_{24}, \cdots, Q_{2N}$ ($N = 12$) are measured. The measurement protocol continues until electrode $N-1$ is excited. Using these charge measurements, the inter electrode capacitance $C_{ij}$ can be computed using the definition of capacitance (Eq. 1) i.e.:

$$C_{ij} = \frac{Q_{ij}}{\Delta V_{ij}}$$

(1)

Where $Q_{ij}$ is the charge induced on electrode $j$ when electrode $i$ is excited with a known potential, $V_{ij}$ is the potential difference between electrodes $i$ and $j$ ($\Delta V_{ij} = V_i - V_j$). So the number of independent capacitance measurements $M = 66$ using Eq. 2 is:

$$M = \frac{N(N - 1)}{2}$$

(2)
It is important to note that these capacitances are dependent on the geometry of electrodes and are determined once the size and the location of the electrodes and the permittivity distribution $\varepsilon(x,y)$ are known. A change in the permittivity distribution $\varepsilon(x,y)$ is naturally reflected in the capacitance measurements. The actual capacitance changes measured will be very small, in the order of Pico or Femto Farad ($10^{-12}$ F or $10^{-15}$ F). Sequential electrodes are referred to as adjacent electrodes; have the largest standing capacitance, while diagonally or opposing electrodes will have the smallest capacitances.

**Figure 1.** Schematic representation of the measurement principle of 12-electrode ECS.

3. **Finite Element Simulation Model**

3.1. **Materials and fatigue testing Model**

The fatigue model for carrying out the analysis on a basalt FRP laminate composite pipe was manufactured using three layers of basalt fibre with orientation $[0º/90º/0º]$, and a polymer resin matrix with inner diameter $R_1=94$mm and outer diameter $R_2=100$mm. The thickness of all plies were 2mm. The corresponding elastic modulus values were $E_1=96.74$GPa, $E_2=E_3=22.55$GPa, and the Shear modulus values were $G_1=G_3=10.64$GPa, $G_2=8.73$GPa. Poisson coefficients were $\nu_1=\nu_2=0.3$, $\nu_2=0.6$ and the density was 2700 kg/m$^3$.

The basalt FRP pipe was modelled at room temperature and was subjected to internal fatigue pressure (P), using a sinusoidal wave load at a frequency of 10 Hz. One series of fatigue model was performed using the pressure ratio ($P_0$) between the applied (P) to the burst pressure ($P_{\text{max}}$) equal to 0.5, and the fatigue stress $S_1$ and the number of cycles to failure (N) were measured. Utilization of the power formula $S_f = aN_f^{-b}$ proved its suitability by giving acceptable values for the correlation factor (C.F) equal to 0.9677. The value of fatigue constants a and b for present pressure ratio were 222.6 MPa, -0.0731 respectively, and the results were plotted in Figure 2 in terms of fatigue stress versus the number of cycles to failure.
3.2. Dielectric Properties Change of ECS Model

In terms of Electrical Capacitance sensor (ECS), the forward problem is the problem of calculating the capacitance matrix C from a given set of sensor design parameters and a given cross-sectional permittivity distribution (\( \varepsilon \)). The forward model proposed for ECS [29] was based on finite element simulations. It was assumed that both the flow distribution and the electrical field during the measurement set were 2D and static. Changes in axial direction were neglected within the axial electrode length. Furthermore, free charges in the flow were also neglected. Thus, the system was governed by the following Poisson equation:

\[
\nabla \varepsilon(x,y) \nabla \varphi(x,y) = 0
\]

The 2D models considered in this study were constructed using commercially available finite element software, ANSYS (The Electrostatic Module in the Electromagnetic subsection of ANSYS (2014)) [30, 31]. The problem space was divided into triangular elements. In a region of ideal dielectrics and space charges, the potential distribution \( \varphi(x,y) \) inside the ECS was determined by solving the Poisson’s equation. For the boundary condition imposed on the ECS head by the measurement system, the potential distribution \( \varphi(x,y) \) could be found. The electric field vector \( E(x,y) \), the electric flux density \( D(x,y) \) and the potential function \( \varphi(x,y) \) are related as follows:

\[
E(x,y) = -\nabla \varphi(x,y)
\]

\[
D = \varepsilon(x,y)E(x,y)
\]

The change on the electrodes, and hence the inter electrode capacitances could be found using the definition of the capacitance and Gauss’s law based on the following surface integral:
\[ Q_{ij} = \int_{S_j} (\varepsilon(x,y) \nabla \varphi(x,y). \hat{n}) \, ds \]  \hspace{1cm} (6)

Where: \( \varepsilon(x,y) \) is the permittivity distribution, \( \nabla \varepsilon(x,y) \) is the divergence of permittivity distribution, \( \nabla \varphi(x,y) \) is the gradient of potential distribution, \( S_j \) is a surface enclosing electrode \( j \), \( ds \) is an infinitesimal area on electrode \( j \), \( \hat{n} \) is the unit vector normal to \( S_j \) and \( ds \) is an infinitesimal area on that.

4. Response surface for the electric potential change method

The response surface is a widely adopted tool for quality engineering fields [32]. The response surface methodology comprises curve fitting with regression to obtain approximate responses, design of numerical model to obtain minimum variances of the responses and optimizations using the approximated responses. In the present study, the response surface methodology is adopted as a solver for inverse problems. For the present study, predictions of fatigue crack length (\( L_c \)) from measured electric potential differences are one of the inverse problems. The response surface methodology brings two advantages; the inverse problems can be approximately solved without consideration of modeling, and the approximated response surfaces can be evaluated using powerful statistical tools. The response surface is described as follow:

\[ L_c = \beta_0 + \sum_{j=1}^{k} \beta_j E_j + \sum_{j=1}^{k} \beta_{jj} E_j^2 + \sum_{l=1}^{k-1} \sum_{j=l+1}^{k} \beta_{lj} E_l E_j \]  \hspace{1cm} (7)

Where: \( k \) is the number of variables. In the case of 12 electrode type specimens, there are 11 electric potential difference variables; \( E_{1-2}, E_{1-3}, \ldots, E_{1-12} \).

To improve the estimation performance of the crack length, the normalizations of the measured electric potential differences (\( E \)) are performed by means of the norm of the electric potential differences vector. Each element is divided by the square root sum, and replacement the vector \((E_{1-2}, E_{1-3}, \ldots, E_{1-12})\) in Eq. 7 with norm vector \((e_{1-2}, e_{1-3}, \ldots, e_{1-12})\). Each element is divided by the square root sum of all results as follows:

\[ (e_{1-2}, \ldots, e_{1-12}) = \left( \frac{E_{1-2}}{\sqrt{E_{1-2}^2 + \cdots + E_{1-12}^2}}, \ldots, \frac{E_{1-12}}{\sqrt{E_{1-2}^2 + \cdots + E_{1-12}^2}} \right) \]  \hspace{1cm} (8)

In the case that varies crack length (\( L_c \)), the total number of location is \( n \), the response surface can be expressed as follows using matrix expression.

\[ L_c = E \beta + \lambda \]  \hspace{1cm} (9)

Where: \( L_c = \begin{bmatrix} L_{c_1} \\ L_{c_2} \\ \vdots \\ L_{c_n} \end{bmatrix} \), \( E = \begin{bmatrix} 1 & E_{11} & E_{12} & \cdots & E_{1k} \\ 1 & E_{21} & E_{22} & \cdots & E_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & E_{n1} & E_{n2} & \cdots & E_{nk} \end{bmatrix} \), \( \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix} \), \( \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix} \)

Where: \( \lambda \) is an error vector, the unbiased estimator \( b \) of the coefficient vector \( \beta \) is obtained using the well-known least square error method as follows:

\[ b = (E^T E)^{-1} E^T L_c \]  \hspace{1cm} (10)

The variance-covariance matrix of the \( b \) is obtained as follows:

\[ \text{Cov}(b_i, b_j) = \sigma^2 (E^T E)^{-1} \]  \hspace{1cm} (11)

Where the \( \sigma \) is the error of \( L_c \). The estimated value of \( \sigma \) is obtained as follows:
\[ \sigma^2 = \frac{SS_E}{n - k - 1} \]  

\( SS_E \) is a square sum of errors, and expressed as follows:

\[ SS_E = L_c^T L_c - b^T E^T L_c \]  

The lack of fit is evaluated with the adjusted coefficient of the multiple determination \( R^2_{adj} \) [33, 34]; \( R^2_{adj} \) is defined as:

\[ R^2_{adj} = 1 - \frac{SS_E/(n - k - 1)}{S_{yy}/(n - 1)} \]  

Where: \( S_{yy} \) is the total sum of squares:

\[ S_{yy} = L_c^T L_c - \left( \sum_{i=1}^{n} L_{ci} \right)^2 / n \]  

The value of \( R^2_{adj} \) is equal to or lower than 1.0. A higher value of \( R^2_{adj} \) implies a better fit. When the response surface shows a very good fit, \( R^2_{adj} \) approaches 1.0. A good fit of the response surface means that the response surface gives good estimations for the DPC method used for the regression. Lower \( R^2_{adj} \) values means poorer estimations and the error band of the estimated result is wider.

5. Results and Discussions

5.1. Electrical potential change method for fatigue crack detection

To investigate the effect of the matrix crack on the electrical potential changes, FEM analysis of the electric field intensity of basalt FRP piping system were performed, using commercially available FEM code ANSYS ver. 14. 2D FEM software.

The approach taken by ANSYS 2D is to divide the different materials and geometries into triangular elements as shown in Figure 3, because many pipes are round under the circumstances of a smaller number of pixels, we can achieve higher accuracy to use the triangular mesh instead of rectangular grids, and then to represent the electric field (see Eq. (4)) within each element with a separate polynomial at six integration points location. For FEM simulating of pipe, electrostatic module (PLANES121), triangular 6-node, and the element has one degree of freedom, voltage, at each node. The 6-node elements have compatible voltage shapes and are well suited to model curved boundaries. The total number of elements used for the analyses was 4845. To improve the accuracy of mesh, we divided the meshed region again into 9825 elements. The applied electric potential (\( V = V_0 \)) was 15V (RMS) and another electrode was kept at ground (\( V = 0 \)) potential.

The software only computes the potential (\( \phi \)) and the electric field (\( E \)) values during each cycle (\( N_t \)) under fatigue test at the element nodes by using looping method in ANSYS through creating an analysis file for use during looping and then interpolates between these nodes to obtain the values for other points within the elements. The Simulations, and node potential distribution of basalt FRP laminated composite pipe under fatigue internal pressure before and after matrix crack initiates, due to fatigue stress for the ANSYS 2D simulation when electrode (1) is excited, are illustrated in Figure 4 right and left respectively. The blue area represents the region of the pipe system without potential i.e. \( \phi = 0 \) and the colored areas represent the region of the pipe that have the different potential (different node potential), i.e. the domain of electrode can be sensitive or detection domain.

From the Simulation Figure 4, we can make a conclusion that there is a significant difference before and after matrix crack initiates in the node potential and electric field intensity.
Using the scripting capabilities in ANSYS we can simulate the electrical potential change during cyclic loading. Figure 5 shows the finite element results obtained for the electric potential change, during cyclic loading of the basalt FRP laminate pipe, when electrode (1) is excited. The electric potential is almost constant with the increase of the number of cycles, before 25,000 cycles. After 25,000 cycles, the electric potential starts to increase.
The main objective of the FE modelling was to investigate the crack lengths of the basalt FRP for various number of cycles to failure. Figure 6 shows the relation between crack length and the number of cycles. Crack length curve shows a sharply-increasing relationship between the crack lengths versus the number of cycles once initial cracking was detected. At first, the results show no signs of crack for an initial large set of cycles, 3000–25,000 cycles, at 31% of fatigue stress level degradation.

Using the power formula $L_c = cN_f^d + e$ has proved its suitability by giving acceptable values for the correlation factor (C.F) equal to 0.986, in order to fit these plots to determine crack growth rate. The value of crack length constants c, d and e are -60.04, -0.2065 and 8.077 respectively, and the results are plotted in Figure 6 in terms of crack lengths versus the number of cycles to failure.

The crack length ($L_c$) as a function of cycles ($N_f$) and applied fatigue stress level ($S_f$) can be determined by combining Eqs. ($L_c = cN_f^d + e$) and ($S_f = aN_f^b$) and integrating from a set of starting crack length and cycle values ($L_0$ and $N_0$) to final values ($L$ and $N$):

$$\int_{L_0}^{L_f} \frac{cS_f}{a(L_c - e)} dL_c = \int_{N_0}^{N_f} N_f^{b-d} dN_f$$

(16)

Where a, b, c, d and e are the same as those determined in Eqs. ($L_c = cN_f^d + e$) and ($S_f = aN_f^b$).

**Figure 5.** Change of the contact electrode potential during cyclic loading of basalt FRP laminate.
5.2. Estimation of fatigue crack length

Table 2 show the comparison between the FEM and estimated results of the crack length ($L_c$) for basalt FRP laminate composite pipe of response surface method. The $R^2_{adj}$ of this result of crack length is 0.9728. The error band is defined as the maximum error of the crack length. The error band is less than 0.0477 mm. As a result, the response surfaces gave good estimations for FE data even for extrapolations crack length for basalt FRP laminate composite pipe.

Table 2. Estimations and Errors of response surfaces method Data (Unit mm)

| $N_f$  | $L_c$ (FEM) | Estimated $L_c$ | Error   |
|-------|-------------|-----------------|---------|
| 15132 | 0           | 0               | 0       |
| 23810 | 0.615       | 0.651           | 0.036   |
| 39998 | 0.9203      | 0.968           | 0.0477  |
| 51254 | 1.5111      | 1.5497          | 0.0386  |
| 66825 | 2.236       | 2.253           | 0.017   |
| 99549 | 2.8011      | 2.8462          | 0.0451  |
| 280670| 3.6137      | 3.6431          | 0.0294  |
6. Conclusions
In the present work, detection of the fatigue crack in a Basalt fibre reinforced polymer (FRP) laminated composite pipe is performed using the electric potential change (EPC) method by evaluating the dielectric properties of basalt FRP pipe by using electric capacitance sensor (ECS). The following conclusions can be drawn:

1. For the Basalt fiber reinforced polymer (FRP) laminated composite pipe, matrix cracking caused a significant increase in the electric potential. The electric potential is almost constant with the increase of the number of cycles, before 25,000 cycles. After 25,000 cycles, the electric potential starts to increase.

2. The crack length is sharply increases with the number of cycles increasing, once initial cracking was detected at 25,000. No signs of crack for an initially large set of cycles, 3000~25,000 cycles, at 31% of fatigue stress level degradation.

3. The crack lengths versus the number of cycle to failure curve was fitted using the power formula \( L_c = cN_f^d + e \), in order to determine the crack growth rate formula by combining this formula with the fatigue life formula \( S_f = aN_f^b \).

4. These response surfaces data gave good convergence for FEM data with \( R^2_{adj} \) of this result of crack length is 0.9728 within the error band of less than 0.0477 mm.

5. Finally, the present method successfully provides fatigue crack detection for a basalt FRP laminate composite pipe including the crack lengths investigation during cyclic loading and crack growth rate estimation.

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