1D Quantum ring: A Toy Model Describing Noninertial Effects on Electronic States, Persistent Current and Magnetization

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Abstract Inertial effects can affect several properties of physical systems. In particular, in the context of quantum mechanics, such effects has been studied in diverse contexts. In this paper, starting from the Schrödinger equation for a rotating frame, we propose a toy model to describe the influence of noninertial effects on physical properties of a one-dimensional ring in the presence of a uniform magnetic field. We first study how the electronic states are affected by rotation. Then, we investigate how the persistent current and the magnetization in the ring are influenced by temperature and rotating effects.

1 Introduction

Quantum Mechanics is one of most well-succeed scientific theories, with a wide range of fascinating phenomena and applications. In Condensed Matter Physics, it is an essential tool in describing the behavior of physical systems. In this context, low dimensional systems consist of a very fruitful subject for investigation because of their emergent physics. Such materials have been attracted much attention nowadays.

Since the experimental discovery of graphene [1], the interest in carbon structures based-materials is increasing [2]. Several types of structures in the nano and mesoscopic scales can be imagined and synthesized. In the physics of low dimensions materials, a relevant example of systems is the quantum rings [3]. Besides the small size, these structures are fascinating [4] and exhibit several possibilities for investigation, covering since fundamental aspects of the quantum mechanics to applications. For instance, the effects of confining quantum particles due to some types of potential can be investigated. Also, it is possible to study magnetic properties such as magnetization [5]. Another aspect that could be explored is the emergence of persistent currents in such systems when in the presence of magnetic flux. In all these cases, temperature plays a fundamental role, since the coherence phase length \( L_\phi \) increases significantly at low temperatures [6].

Several types of quantum rings have been proposed in the literature. A well-known model to study the electronic properties of such systems is due to Tan and Inkson [7], which considers a quantum particle constrained to a ring due to the presence of a radial confining potential \( V(r) \), which \( r \) being the radial coordinate in the \( x - y \) plane. Posteriorly, they extended the model by incorporating discussions about the existence of persistent currents and magnetization in such systems [8].

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After these contributions, other models dealing with two-dimensional rings also took place. Bulaev and collaborators, for instance, have studied the effect of surface curvature on the persistent currents [9]. Ref. [10] makes a theoretical investigation on persistent currents in distorted quantum rings, taking into account a geometrical potential [11]. The energy levels of quantum rings of arbitrary shape threaded by a magnetic field are studied in Ref. [12].

The study of the physical properties of quantum rings also has been examined for some specific materials, like graphene [13–15]. The study of two-dimensional quantum rings is not limited to understanding the electronic states and persistent currents. Optic properties and quantum information quantities also can be investigated [16–18].

One-dimensional (hereafter, 1D) rings also exhibit several impressive features. They are simple structures that allow us to investigate relevant physical properties and also applications to other physical systems. An important feature of these devices is that they are very appropriate to study the Aharonov-Bohm (AB) effect [19,20] for bound states. The AB effect establishes that the fundamental quantity in a description of the quantum system is the vector potential and not the magnetic field. It gives a physical significance to the potential vector in quantum mechanics. If we perform an interference experiment in a region where there is a magnetic flux without a correspondent magnetic field, then the wave function acquires a phase. It is also surprising the explanation to the AB effect for bound states [21]: We can imagine a particle confined in a 1D quantum ring, where there is no field but only a magnetic flux [22]. In this case, the influence of the flux it is manifested by modifying the energy levels. Such systems show a large number of possible applications and have been explored in different contexts. For example, a study about persistent currents in 1D rings at finite temperature can be accessed in Ref. [23]. The effects of both spin-orbit and Zeeman interactions on persistent currents in a 1D ring also it was analyzed [24]. Besides, geometrical phases and persistent currents in 1D rings can be induced by Lorentz-violating terms in the standard model extension [25]. Quantum rings also can be considered as a device to perform quantum computing [26].

Also, the study of 1D rings in the context of relativistic quantum mechanics has been considered. In [27], for instance, it was investigated the behavior of persistent current on a 1D ring with a stub. The presence of scattering potentials in 1D quantum rings also was considered [28]. Ref. [29] deals with the problem of obtaining relativistic persistent currents. In Ref. [30], the study of relativistic one-dimensional rings it is performed by employing a continuum model and a lattice one. Another aspect of studying 1D quantum rings is related to the description of thermodynamics properties. Examples can be accessed in [31,32].

The aspects mentioned above about the physical properties of low dimensional systems are related to electromagnetic interactions. On the other hand, another relevant issue in quantum mechanics refers to describing how a system in a non-inertial frame can affect the dynamics. [33,34]. Also, it is known that electromagnetic interactions and inertial effects can play similar influences in physical systems. To clarify it, let us make some remarks. First, we can compare the Lorentz force and inertial forces. When an electron moves with linear velocity \( \mathbf{v} \) in the presence of an electric field \( \mathbf{E} \) and a magnetic field \( \mathbf{B} \), it experiences the Lorentz force

\[
F_{\text{Lorentz}} = e\mathbf{E} + e(\mathbf{v} \times \mathbf{B}) = e\left[-\nabla V + \mathbf{v} \times (\nabla \times \mathbf{A})\right],
\]

where \( e \) is the electron charge and \( V \) and \( \mathbf{A} \) are the scalar and the vector electromagnetic potentials, respectively. Similarly, for a particle of mass \( m_e \) in a non-inertial frame the force is expressed as

\[
F_{\text{inertial}} = 2m_e(\mathbf{v} \times \mathbf{A}) - m_e\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}),
\]

where \( \boldsymbol{\Omega} \) is the angular frequency and \( \mathbf{r} \) is the radius of the orbit. The first term on the right side of the Eq. (2) refers to the Coriolis force and the second one corresponds to the Centrifugal force. If we consider \( \boldsymbol{\Omega} \) as a constant field, then \( \nabla \cdot \boldsymbol{\Omega} = 0 \). It means we can write that \( \boldsymbol{\Omega} = \nabla \times \mathbf{a} \). This way, the rotation can be written in terms of the curl of a vector field \( \mathbf{a} \), similarly to the magnetic field \( \mathbf{B} \), which can be written in terms of the vector potential \( \mathbf{A} \). Also, if we take into account that the centrifugal force is conservative, then it is possible to think in a corresponding scalar potential. After these considerations, we can write the relation

\[
F_{\text{inertial}} = m_e[-\nabla V_{\text{inertial}} + 2\mathbf{v} \times (\nabla \times \mathbf{a})].
\]

This expression looks like the right side of the Eq. (1), showing a similarity between electromagnetic fields and rotation.

We can look such analogies in the view of quantum mechanics. Since the work of Aharonov and Bohm, several analogs of that effect has been presented and investigated [35,36], including a rotational one. It is known as the Aharonov-Carmi (AC) effect [37,38]. The main idea of the AC effect is the following: Suppose a
particle in a rotating ring, subjected to the force given by Eq. (2). In principle, it is possible to apply an electric and a magnetic field in such a way that the inertial effects and electromagnetic effects cancel one each other. Then, the particle does not feel any forces. However, a quantum phase can arise in an interference experiment performed under these conditions [39], and electronic states can be affected by the AC effect [40].

Another similarity between noninertial effects and electromagnetic fields can be explored from the Barnett effect, where it is possible to obtain a magnetization generated by rotation [41–43]. Relations between the Hall effect and rotation also be established [44–46]. Noninertial effects also have been studied in the context of nanostructures, like carbon nanotubes [47,48] and fullerenes [49,50].

Motivated by these analogies between rotation and electromagnetic fields, in this manuscript we propose a toy model to study the influence of noninertial effects on electronic states, persistent current, and magnetization for a 1D quantum ring. We have chosen to deal with a one-dimensional system here because although it only demands a simplified description, at the same time, it is possible to discuss fundamental aspects involving rotation and magnetic fields in a quantum system.

2 The model

Let us construct the Schrödinger equation describing the dynamics of a spinless quantum particle of mass $m_e$ in a rotating frame in the presence of a uniform magnetic field. Following the Ref. [51], the nonrelativistic quantum description of a particle in a rotating frame can be done through a Galilei boost with velocity $v$, given by

$$U = e^{itv \cdot px - im_e v \cdot x},$$  \hspace{1cm} (4)

connecting two inertial frames $F_0$ and $F'_0$. More explicitly, we have

$$x' = x - vt, \quad t' = t.$$  \hspace{1cm} (5)

Here, the unprimed coordinates refers to the referential $F_0$ while the primed coordinates are related to the referential $F'_0$. The Schrödinger equation in the referential $F'_0$ is written as

$$\left( i \frac{\partial}{\partial t'} + \frac{1}{2} m_e v^2 \right) \psi (x', t') = \frac{1}{2m_e} \left( p' - m_e v \right)^2 \psi (x', t').$$  \hspace{1cm} (6)

It is worth to note that this equation can be obtained from the usual Schrödinger equation by using the minimal coupling ($\mu = 0, 1, 2, 3$)

$$p^\mu \rightarrow p^\mu - m_e A^\mu, \quad A^\mu = \left( -\frac{1}{2} v^2, v \right)$$  \hspace{1cm} (7)

with $A^\mu$ being the gauge field. Now, let us assume that the system described by the Eq. (6) it is in a region where there is a uniform magnetic field in the $z$ direction. An electromagnetic interaction can be included in the Schrödinger equation via a minimal coupling, $p^\mu \rightarrow p^\mu - eA^\mu$. In this way, Eq. (7) can be written as

$$p^\mu \rightarrow p^\mu - eA^\mu - m_e A^\mu,$$  \hspace{1cm} (8)

with the usual gauge given by

$$A^\mu = (A^0, A), \quad A = \frac{1}{2} Br \hat{\phi}$$  \hspace{1cm} (9)

where $A$ is the vector potential. Following Ref. [51], let us assume that $F'_0$ rotates with a constant angular velocity $\Omega$ with respect to the frame $F_0$. The Schrödinger equation describing the system is

$$\frac{1}{2m_e} \left[ \left( p - eA - m_e \Omega \times r \right)^2 - \frac{1}{2} m_e (\Omega \times r)^2 \right] \psi = E \psi.$$  \hspace{1cm} (10)
Considering that the particle is constrained to move in a circle of radius $r_0$, Eq. (10) takes the form

$$\frac{\hbar^2}{2m_e r_0^2} \left( \frac{\partial}{\partial \phi} - l' - \frac{m_e \Omega}{\hbar} r_0^2 \right)^2 \psi (\phi) - \frac{m_e e}{\hbar} \Omega^2 r_0^2 \psi (\phi) = E \psi (\phi),$$

(11)

with $l' = \Phi'/\Phi_0$, where $\Phi' = \pi r_0^2 B$ is the magnetic flux passing through the ring and $\Phi_0 = \hbar/e$ corresponds to the quantum flux. The solutions of Eq. (11) are of the form

$$\psi (\phi) = e^{im\phi}.$$  

(12)

The continuity of the wave function $\psi (\phi)$ in $\phi = 2\pi$ demands that $m$ must be an integer number. We can solve Eq. (11) using (12) to find the energy eigenvalues depending of the quantum number $m$ and the physical parameters of the system. Such energies are given explicitly by the expression

$$E_m = \frac{\hbar^2}{2m_e r_0^2} \left( m - l' - \frac{m_e \Omega r_0^2}{\hbar} \right)^2 - \frac{1}{2}m_e \Omega^2 r_0^2,$$

(13)

with $m = 0, \pm 1, \pm 2, \pm 3, \ldots$. Thus, it generalizes the model of Ref. [22], now incorporating noninertial effects. This result can be rewritten as

$$E_m = \frac{\hbar^2}{2m_e r_0^2} (m - l')^2 - \hbar \Omega (m - l').$$

(14)

3 Electronic properties

We know that a non-rotating one-dimensional ring in the presence of a magnetic field has its energy affected by the magnetic flux passing through the ring. Considering the energy as a function of the magnetic flux, we see each state $E_m$ describes one parabola with a minimum at $l' = m$, with energy equals to zero. In our toy model, besides these effects of electromagnetic origin, we have contributions to the energy spectrum coming from the rotation. The rotation changes the positions of the parabolas on the horizontal axis, corresponding to the minimum $l' = m - m_e \Omega r_0^2/\hbar$. Thus, rotation affects the relationship between the minimum values of energy and the states. Besides, the parabolas are shifted to the right (left) if the ring is rotating in the clockwise (anticlockwise) direction. The new energy minimum due to the rotation is given by $E_{\text{min}} = -m_e \Omega^2 r_0^2/2$ and it is independent of the direction of rotation. It is a well-known fact that the energy spectrum of a 1D ring is a periodic function of the magnetic flux, oscillating with a $\Phi_0 = \hbar/e$ period [52]. Rotation does not change it. Rotation produces a combined effect with the magnetic field: an energy shift, given by $\hbar \Omega l'$, which corresponds to the last term on Eq. (14). This energy shift affects all the energy states in the same way, since this term does not depends on the quantum number $m$.

To visualize the behavior of the system with respect to the magnetic field and rotation, we plot the energy $E_m$ as a function of $\Phi'/\Phi_0$ for several values of $m$ (Fig. 1), considering two different ring radius.

Fig. 1 shows the sketch of the energy as a function of $l'$ for rings with radius 100 nm (panel (a)) and 400 nm (panel (b)). We consider $\Omega = 0$ Hz and $\Omega = 1.0$ GHz. Since we are considering an anticlockwise rotation, the parabolas are shifted to the left. We can see that the effects of rotation are more prominent for the ring of 400 nm. Thus, the influence of rotation on the energy spectrum depends on the size of the system.

A relevant aspect concerning these energy states is it related to the degeneracy. We know that in the absence of magnetic flux and rotation, the energy of an electron confined in a 1D ring is just $\hbar^2 m^2/2m_e r_0^2$ and the energies are doubly degenerated, except for the $m = 0$ state. The introduction of a magnetic field breaks the degeneracy in $m$ (that is, for a given $l'$, the different allowed values of $m$ do not all carry the same energy anymore). This can be checked by analyzing Eqs. (13) and (14). It is worth noting, however, since the flux is quantized, different combinations of $(m - l')$ can provide the same values of energy. The magnetic flux controls the energy profile showed in Fig. 1. A particularly curious case happens when $l' = -m_e \Omega r_0^2/\hbar$, which corresponds to $\Omega = -w_c/2$ (being $w_c = eB/m_e$ the cyclotron frequency). In this case, the rotation cancels the effect of the uniform field on the kinetic term of the energy. In this case, the energy assumes the form

$$E_m = \frac{\hbar^2 m^2}{2m_e r_0^2} - \frac{1}{2}m_e r_0^2 \Omega^2 = \frac{\hbar^2}{2m_e r_0^2} (m^2 - l'^2),$$

(15)
(a) $r_0 = 100 \text{ nm}$

(b) $r_0 = 400 \text{ nm}$

Fig. 1 Energy (Eq. (13)) as a function of $\Phi'/\Phi_0$ for the case without and with rotation. We consider rings with radius $a r_0 = 100 \text{ nm}$ and $b r_0 = 400 \text{ nm}$. Note that for $\Phi'/\Phi_0 = 0$ the eigenvalues are doubly degenerate, except for $m = 0$. The continuous lines describe the behavior of a single electron.

Thus, for $\Omega = -\omega_c/2$, the influence of rotation on the energy levels corresponds only to an energy shift. This can be justified since both rotation and magnetic flux contributes to the total angular momentum of the particle. This way, it is possible to tune the field and rotation to modify (or not) the states. In particular, we can define an effective angular momentum given by $j_m = \pm m - (l' + \lambda \Omega)$, where $\lambda = m e r_0^2/\hbar$.

Another pertinent point consists in to study only the effect of rotation on the energies (making $l' = 0$). In this case, $E_m(l' = 0) = (\hbar^2 / 2m_e r_0^2)m^2 - \hbar \Omega m$. We can think the term $\hbar \Omega m$ as a rotation-momentum angular coupling. This term can increase or decrease the energy. For instance, if the ring is rotating in the anticlockwise direction, then a state $-m$ is affected differently than your opposite $m$. It tells us that, in the absence of magnetic flux, rotation can break the degeneracy between these states, in a similar way to the case in which there is no rotation but the magnetic flux is present.

In Fig. 2a, we sketch the energy as a function of $m$ and $\Omega$, considering a ring with radius $r_0 = 400 \text{ nm}$. We can notice the energy eigenvalues have a linear dependence on the rotation parameter. The slope of the function depends on the value of $m$. Also, the slope is positive (negative) if $m < 0$ ($m > 0$), as we can see in Fig. 2b, where we show a plot of the energy as a function of $\Omega$, considering the states with $m = \pm 1$.

We can compare the orders of magnitude between the terms in the Eq. 14. The first term corresponds to the energy levels of a particle confined in a quantum ring and includes the magnetic flux. The second one refers to the contribution due to rotation. Let us consider a ring of 200 nm, with $\Omega = 10^9 \text{ Hz}$, $B = 0.012 \text{ T}$ and $m = 1$. In this case, the first term gives 0.82 $\mu$eV, while the second one provides 0.61 $\mu$eV, showing that the inertial contribution can have the same order of magnitude of the first term, depending on the parameters of
Fig. 2 In panel a, the energy profile as a function of $\Omega$ and $m$. In panel b, the sketch of the energy as a function of $\Omega$, emphasizing only the states with $m = \pm 1$. We consider a ring with radius $r_0 = 400$ nm.

Fig. 3 Energy of a single electron as a function of $\Phi'/\Phi_0$ and $\Omega$ for a ring of 400 nm.

the system. Figure 3 shows the complete profile of a single electron energy as a function of the magnetic flux and rotation simultaneously in a ring of 400 nm. The states are shifted to the right or left depending on the direction of rotation, while the oscillations related to the magnetic flux are maintained.

4 Persistent Current

Persistent currents are frequently studied in the context of 1D and 2D quantum rings. Usually, the ring encloses a magnetic flux.

The persistent current in the ring can be calculated using different methods. Since the energy explicitly depends on magnetic flux, the easier way to obtain the persistent current is through the Byers-Yang relation [53]:

$$I_m = -\frac{\partial E_m}{\partial \Phi'} = -\frac{1}{\Phi_0} \frac{\partial E_m}{\partial l'},$$

(16)

where $E_m$ is given by Eq. (13). We obtain

$$I_m = \frac{e\hbar}{2\pi m_e r_0^2} \left( m - l' - \frac{m_e \Omega r_0^2}{\hbar} \right).$$

(17)

This expression provides the partial currents, relative to the states labeled by the quantum number $m$. The total current it is

$$I = \sum_m I_m f(E_m),$$

(18)

where $f(E_m)$ is the Fermi distribution function [54], which we have included at this point to accommodate the effects of temperature on the persistent current in the ring [9]. When $\Omega = 0$ Hz, it is known that the current...
Fig. 4  Persistent currents as a function $I'$ and $\Omega$, for zero temperature a, and $T = 3$ mK b. In c, persistent currents as a function of $I'$ for $\Omega = -1.0$ GHz (red color line), $\Omega = 0$ Hz (green color line) and $\Omega = 1.0$ GHz (blue color line). The continuous and dashed lines correspond to $T = 0$ K and $T = 1.0$ mK, respectively. We consider $r_0 = 400$ nm, and 10 electrons

presents a periodic behavior relative to the magnetic flux. If the system is rotating, a shift on the function $I_m(I')$ appears. This is intimately related to the shift of the parabolas describing the energy states discussed previously.

We can point some curious aspects for the partial currents described by Eq. (17). We can note this expression contains a usual term for the partial currents (when $\Omega = 0$), given by $e\hbar(m - l')/2\pi m r_0^2$. This term is a purely quantum contribution that shows up because of the magnetic flux and depends on the size of the system

Additionally, there is a term that appears due to the rotation, which can be rewritten as $e\Omega / 2\pi$. The letter does not depend on the ring size. In Fig. 4, we show the sketch of the persistent current. In Fig. 4a, we plot both the profile and the corresponding density plot of the current as a function of the magnetic flux $I'$ and rotation $\Omega$ at zero temperature. We can see that the rotation keeps the oscillations but introduces a linear contribution to the current. In Fig. 4b, we make a similar plot, but the ring is kept at a temperature of $T = 3.0$ mK. In Fig. 4c, we plot the current as a function of the magnetic flux at temperatures $T = 0$ K and $T = 1.0$ mK, considering $\Omega = 0$ Hz, $\Omega = 1.0$ GHz and $\Omega = -1.0$ GHz. We can see the temperature tends to smooth out the oscillations as well as the effect of decreasing the persistent current amplitude. Analogously to the case discussed in the previous section, we can also compare the orders of magnitude of the terms in the expression for the persistent current. In this case, we find that the inertial contribution can be comparable with the usual term which depends on $m$ and $I'$ when the temperature is zero. By considering a ring with a radius of 400 nm and $\Omega = 10^9$ Hz, for example, we can obtain the same order in both terms taking $B = 0.01$ T.

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1 See, for example, Eq. (16) of Ref. [30]
5 Magnetization

The magnetization $M$ can be evaluated by using the following expression [9]:

$$M = -\frac{\partial U}{\partial B} = -\sum_{m} M_{m} f(E_{m}), \quad (19)$$

where $M_{m} \equiv -\frac{\partial E_{m}}{\partial B}$ defines the magnetic momentum and $f(E_{m})$ is the Fermi distribution function. In the present case, we have

$$M_{m} = \frac{e\hbar}{2m_e} \left( m - l' - \frac{m_e \Omega r_0^2}{\hbar} \right). \quad (20)$$

If the system is not rotating, the persistent current and the magnetization are related by $M_{m} = \pi r_0^2 I_{m}$. This result remains valid when we include inertial effects. The magnetic flux is associated with the term $e\hbar (m - l')/2m_e$ on the Eq. (20), as expected. The other term on the expression for $M_{m}$ is related to the rotation. It is an analog of the Barnett effect for the present case.

In Fig. 5a, we show a 3D plot of the magnetization as function of $l'$ and $\Omega$ at zero temperature. This same magnetization profile at a temperature $T = 3.0 \text{ mK}$ is sketched in Fig. 5b.

The magnetization as a function of $l'$ for some fixed values of $\Omega$ and the temperature is sketched in Fig. 5c. We can note a similarity between the magnetization profile and the persistent current in Fig. 4, showing that
the rotation does not change the correspondence between current and magnetization. The modifications in the magnetization profile at zero temperature due to the rotating effects can have the same order of magnitude that the first term in Eq. (20), similarly what we have found in the previous sections for the energy and persistent current.

6 Conclusions

We proposed a toy model to study the problem of a quantum particle constrained to a rotating 1D ring in the presence of a magnetic field. We have investigated the influence of noninertial effects on the electronic states as well as persistent current and magnetization in this system. First, we have studied the energy spectrum of the system, focusing on the influence of magnetic flux and rotation parameters. The energy levels exhibit an oscillatory behavior as a function of the magnetic flux. Noninertial effects modify the energy eigenstates by introducing an energy shift, which occurs even when there is no magnetic field. Rotation is responsible for a new minimum value for the energies $E_m$. All these effects of rotation on the energy eigenstates depend on the size of the system.

With respect to the current, we saw it depends on both magnetic flux and rotation. We included temperature effects on the persistent current, by evaluating the sum of the partial currents using the Fermi distribution. Adopting a similar procedure, we have obtained the magnetization of the system. We verified that rotation does not destroy the usual relationship between current and magnetization. We saw the temperature affects both current and magnetization in a similar way, changing the form of the peaks of such quantities as viewed as functions of the magnetic flux. Concerning electron filling, we noted that the current and magnetization only suffer a change in their amplitudes when the number of electrons changes. Thus, we just focused on the effect of rotation, magnetic field, and temperature by considering a fixed number of particles.

We also saw that rotation affects the physical properties of the system in a similar way to the magnetic field. Depending on the system parameters, the effects generated by rotation and magnetic field can have the same order of magnitude. Thus, noninertial effects could be useful in studying relevant aspects of the physics of quantum rings.

With this letter, we hope to contribute to the understanding of inertial effects on quantum systems from a simple model. Although the low dimensionality of the system, it was possible to think about the aspects involving the interplay between electromagnetic fields and noninertial effects in the light of quantum mechanics of low dimensional systems. We believe that the toy model presented here can be useful for future studies dealing with more complicated structures. As a future perspective, the problem of rotating quantum rings in the presence of magnetic impurities could be addressed.

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