Mathematical modeling in cognitive process research

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Abstract. There was proposed a method for studying and mapping the cognitive activity of the brain. The experiment simulates a flight situation: the appearance of an image, recognition, a motor act (inner speech). Electroencephalogram was recorded with cognitive load. Two figures are presented on the laptop screen - a circle and a square. At the moment of presentation of the stimulus, the matching device sends a synchronizing pulse to one of the inputs of the electroencephalogram amplifier. The subject mentally names a figure with a larger area. A selective filter system suppresses background activity. The resonant frequencies of the filters are changed to reveal a single cognitive response to a stimulus. The parameters of the model are individual for each subject. There was developed a model of a diagnostic device. The results obtained for the "circle" and "square" figures coincide at time interval corresponding to the work of the sensory visual system, but they differ at the stage of internal speech, which corresponds to pronouncing a longer word by the syllables. These results demonstrate the effectiveness of modeling in studies on the localization of the cognitive process and the diagnosis of neurological disorders.

1. Introduction
Aviation and astronautics give us a large number of specific extreme situations, the outcome of which depends on the organization of connections in a person's cortex [1], the parameters of his response to incoming information (speed of sensory perception, time of awareness of the situation, time of motor act), and the ability to make the right decision [2].

An academician V.I. Vernadsky noted that the method of penetrating the unknown, which is characteristic of the human brain and which is now called the cognitive process (CP) and cannot be represented by a model [3]. The cognitive process, like awareness and comprehension, completes sensory perception and recourse to memory with the synthesis of new knowledge. Modeling the birth of a brilliant idea is also complicated by the fact that human memory, in contrast to the memory of a computer, has the properties of holographic and fractal [4]. Science so far only explores the temporal aspects of the thought process - changes in the local potentials of neural ensembles, manifested in an electroencephalogram (EEG). In this case, the electrical image of the CP is not recorded directly, since it is accompanied by numerous background processes of the brain, which have an order of magnitude greater amplitude.
The development of numerous methods for analyzing EEG (frequency, spectral, bispectral, correlation, coherent, soliton analyzes) makes it possible to carry out more and more in-depth studies, but there is still no single approach to the analysis of background (spontaneous) and evoked brain activity (evoked potentials for afferent and efferent stimuli).

According to the theory of functional systems (FS) academician P.K. Anokhin [5, 6], who assigns the brain the role of a control element in all functional systems, even those that arise randomly under the influence of a specific situation, each system has certain connections and interactions between different areas of the cerebral cortex. Physically, connections are expressed in electrical activity and the synchronization of this activity.

The FS is structurally a closed automatic control system with negative feedback: the current position is compared with the desired one (the acceptor of the result of the action), and the difference serves as a control signal for the actuator. The emerging specific situational FS may include others that have evolved at an earlier time. The memory included in any FS consists of a set of neural ensembles (NA). The property of the intersection of NA, predicted in his time by D. Hebb, when some neurons can enter several ensembles at once, determine its fractality, nesting, and multilevelness. Experiments with excision of sections of the cortex and electrophysiological studies show that the memory of each event is distributed over more or less extensive areas of the brain. Materially, information about various events is stored not as the excitation of a certain number of neurons, but as a configuration of the arisen connections in the NA. Two aspects of the cognitive process should be considered - topography (localization and connections of the neural ensembles included in the CP) and dynamics - the rate of the process.

The first aspect shows the individual characteristics of a person, neurological profile and possible neurological disorders. CP dynamics is a more flexible indicator and additionally shows the action of many factors. Evoked potentials (EP) of various modalities are one of the main tools for the study of CP. They have shown their sensitivity when exposed to pharmacological drugs [7], electrical pain stimulus [8], when assessing the results of transcranial magnetic stimulation [9-11], cognitive training [12], sound stimulation [13] and even when choosing a diet [14] and dance training [15].

Connections in CGM are formed in ontogenesis and can also change if the subject has neurological disorders [11]. The main method for studying connections in CMM is to assess the degree of synchronization of the background EEG in different leads - to determine the functions of coherence [12, 13] and (less common) correlation [14]. Obtaining such estimates is associated with significant processing time and, not having a unified methodological approach, does not provide sufficient reliability and repeatability of results on equipment from different developers [15, 16]. The coherence function is defined as the ratio of the Fourier transforms of the cross-correlation function and the autocorrelation functions of two signals [17]. The above-mentioned discrepancies [15] can be explained by the use of various calculation algorithms that the developers do not publish. In computer systems for EEG recording and processing, when calculating the Fast Fourier Transform (FFT), digital windows are used that zero out the signal value at the edges of the analyzed epoch. The correlation function of a periodic signal (EEG rhythm) in this case practically does not differ from the signal itself - in terms of the rhythmic component. Therefore, the first thought is to use the signal itself instead of the autocorrelation function when evaluating the synchronization. The second step is to abandon the spectral transformations of the FFT and replace them with the square of the transfer function of the equivalent selective filter.

Mapping of instantaneous EEG values, which was quite popular in the early 90s, is currently not widely used either in medical or research practice. The lack of publications on neuro-mapping techniques in recent years confirms this. EP mapping is of much greater interest, however, the traditional method of EP registration - accumulation and averaging of signals from one or two electrodes, does not allow obtaining any significant result.

The goal is to develop an adaptive mathematical model that makes it possible to separate the evoked bioelectrical activity of the brain from the background, to build algorithms for mapping the evoked
activity and quantitatively to assess the level of synchronization of the background activity according to the parameters of the resulting model.

2. Methodology
The paper proposes to use the EEG registration method as a basis for automatic construction (including in real time) of a mathematical model (MM) of the background EEG and determination of the CP graph as the difference between the EEG and the MM output.

In the course of the experiment, the EEG was recorded using the Neurokartograf-5 computer diagnostic system (MBN, Moscow), installed in the Clinical Diagnostic Center of the Tula Regional Clinical Hospital. Records of background activity and records with cognitive load were obtained. The subjects on the laptop screen were presented with two figures - a square and a circle. The task was to assess which of the figures occupies a large area and mentally name it. As a result of the assessment, the subject mentally pronounced the word "circle" or "square". The experiment involved student athletes.

At the moment of presentation of the stimulus, the pulse generator (made on the stm32f103 microcontroller), upon command from the laptop, issues a sync pulse, which is displayed on the EEG.

MM was based on the recurrent equation of a digital selective filter with a tunable resonant frequency. The filter parameters were automatically changed depending on the frequency of the dominant EEG rhythm.

Filtration was as follows: the EEG signal was fed to three series-connected selective Butterworth filters BF1, BF2, BF3. Each of the filters has an initial tuning frequency \( f_H \) corresponding to the middle of the analyzed frequency range, for example: theta, alpha, beta. Selective filters BF1, BF2, BF3 were automatically adjusted to the frequency of the dominant rhythm in the selected frequency range by the periodometric method: the counters measure the time intervals between the moments when the output signals of the filters of the maximums and minimums pass. The tuning frequency \( f_H \) is calculated as the reciprocal of several values of the measured period.

After the filters were adjusted, their parameters (their number will depend on the ratio of the amplitudes of the fundamental signal harmonics) with a given level of accuracy will represent the background EEG signal in the sense that the white noise passed through the constructed filter will have spectral characteristics that coincide with the characteristics of the background EEG. It was supposed to form a synchronization estimate based on the results of modeling selective filters that select different frequency components of the signal. The following assumptions were made for neuro-mapping:

1. The boundaries of the mapped zone passed through the electrodes Fp1, F7, T3, T5, O1, O2, T6, T4, F8, Fp2.
2. Through the electrodes F3, C3, P3, Oz; F4, C4, P4, Oz; F7, T3, T5, O1; O2, T6, T4, F8 passed through great circle arcs. The mapping process was carried out at each discrete time of the EEG recording (step \( \Delta t = 0.005 \) s) after digital filtering of the signal. On the mapping plane, the following coordinates of the electrodes in a circle of radius \( r \) and coordinates of the center \( X_C, Y_C \) were taken.
\[ y_{e7} = y_{e8} = Yc - r \cdot \cos(0.5\pi); \]
\[ x_{e7} = Xc - r \cdot \sin(0.3\pi); \quad x_{e8} = Xc + r \cdot \sin(0.3\pi); \]
\[ y_{e3} = y_{e4} = Yc - r \cdot \cos(0.3\pi); \]
\[ x_{e3} = Xc - 0.6r \cdot \sin(0.2\pi); \quad x_{e4} = Xc - 0.6r \cdot \sin(0.2\pi); \]
\[ y_{e5} = y_{e6} = Yc + r \cdot \sin(0.2\pi); \]
\[ x_{e5} = Xc - r \cdot \cos(0.2\pi); \quad x_{e6} = Xc + r \cdot \cos(0.2\pi); \]
\[ y_{e8} = y_{e9} = Yc + r \cdot \cos(0.2\pi); \]
\[ x_{e8} = Xc - 0.6r \cdot \sin(0.2\pi); \quad x_{e9} = Xc - 0.6r \cdot \sin(0.2\pi); \]
\[ y_{e1} = y_{e2} = Yc + r \cdot \cos(0.1\pi); \]
\[ x_{e1} = Xc - r \cdot \sin(0.1\pi); \quad x_{e2} = Xc + r \cdot \sin(0.1\pi); \]
\[ y_{e0} = Yc + r; \quad x_{e0} = Xc; \]
\[ \text{where } y_{ec3}, x_{ec3}, y_{ef7}, x_{ef7}, y_{ef3}, x_{ef3}, y_{et3}, x_{et3}, y_{et5}, x_{et5}, y_{e9}, x_{e9}, y_{e4}, x_{e4}, y_{et4}, x_{et4}, y_{et6}, x_{et6}, y_{e9}, x_{e9}, y_{e4}, x_{e4}, y_{e0}, x_{e0}, y_{e0}, x_{e0} - \text{are the coordinates of the corresponding electrodes.} \]

To determine the values of EEG signals in the interelectrode space, the method of spline interpolation at four points was used.

### 3. Results and discussion

To conduct the experiment, a matching device was developed that receives a signal from a laptop, on the screen of which a visual stimulus is displayed and sends a synchronizing pulse to one of the inputs of the EEG amplifier. The generator has two input interfaces - USB and RS232, which makes it possible to combine the equipment with other medical devices.

The EEG recording is processed by software (programming language VB6, VB2008Express), as a result, a map of the evoked activity for the corresponding moment is displayed on the screen.

Figure 1 shows a block diagram of one of three selective filters.

After the end of the process of tuning filters BF1 - BF3 and attenuation of transient processes in them, the dispersion of signals at their outputs is determined. The spectrum of signals in all ranges is defined as the square of the transfer function of the selective filters BF1-BF3.

An estimate of the signal spectrum in each EEG derivation is obtained by the formula:

\[ S(\omega) = A_1^2(\omega)D_1 + A_2^2(\omega)D_2 + A_3^2(\omega)D_3 \]  \hspace{1cm} (2)

where \( A_1(\omega), A_2(\omega), A_3(\omega) \) are the amplitude characteristics of the filters;
\( D_1, D_2, D_3 \) are the variances of the filter outputs.

For the obtained estimates of the spectra, the maximum values are determined, relative to which they are normalized. A quantitative assessment of the level of connectivity of the two processes is obtained by the formula:

\[ K(\omega) = S_{11}(\omega)S_{22}(\omega), \]  \hspace{1cm} (3)

where \( S_{11}(\omega), S_{22}(\omega) \) are the normalized estimates of the spectra of the two leads.

Figure 2 shows the filtering processes and the results of calculations of synchronization levels for two hypothetical EEG derivations.

The considered methodological approach can be applied to the analysis of the background EEG (without a clear cognitive load). As it was noted in [9], the level of synchronization of cortical connections changes during transcranial stimulation that can be quantitatively shown by the proposed method.
The authors are developing a diagnostic device designed for mass neurological screening of the child population, built according to the described method - prototyping an electronic amplifier unit, developing screening software and a service program for collecting the necessary database on patients with different types of disorders.

The hardware part of the device is assembled on the STM32F446 microcontroller and two specialized ADS1299 microcircuits, each of which is an 8-channel delta-sigma ADC with analog buffer amplifiers designed to work with human biopotentials and allowing EEG recording from the scalp without additional external active elements. The bit width of each ADC channel is 24 bits. The internal structure of the microcircuit allows you to select a sampling rate from 250 to 16 thousand samples per second and change the gain of analog buffers over a wide range.

The digital data stream undergoes internal primary filtering. Secondary filtering using the microcontroller software is carried out in order to cut off the entire frequency spectrum above 45 Hz. Further processing of the received data occurs in the microcontroller. The device is powered by a battery to reduce interference.
The second aspect of CP (dynamics in time) is also proposed to be investigated within the framework of the proposed approach. The traditional method involves the use of cognitive evoked potentials (CEP) based on multiple presentation of a stimulus.

![Figure 2](image)

**Figure 2.** The results of mathematical modeling. Graphs of signals and the level of synchronization of two EEG derivations (a) and (b)): signals x - input signal; y1 - output signal of the first filter; y11 is the difference between the input signal and the output signal of the first filter; y2, y3 - output signals of the second and third filters; c) graphs S1, S2 - spectra estimates; K is the level of synchronization.

The use of the previously proposed method makes it possible to register single realizations of the response to a stimulus and to trace the localization of the maximum of evoked activity at different points in time after the application of an external stimulus and to identify the areas involved in the observed process. It was previously noted that the CEP had several maxima observed at different times in different zones of the crust. According to academician P.K. Anokhin's EP maximum corresponds to the passage of a signal through a certain nervous structure (center) \[5\]. Having calculated the position of the instantaneous maximum of the evoked activity, we can establish in which center the information processing is currently taking place.

To determine the values of EEG signals in the interelectrode space, the method of spline interpolation at four points is sequentially applied.

1. The values of the EEG signals are determined along the "arcs of the great circle" (assumption of item 2) for the points of intersection of the arcs with the secant plane perpendicular to the plane of symmetry of the brain.

1.1. Point coordinates are calculated by the position of the cutting plane relative to the forehead - yb.

1.2. Four signal values are determined at the intersection of the cutting plane with the arcs of the great circle. Let us denote by the letter S the function of spline interpolation at four points.
where $x$ is the coordinate of the calculated point; $x_1, x_2, x_3, x_4$ are the coordinates of the control points for which the values of the interpolated functions are given; $y_1, y_2, y_3, y_4$ are the values of the functions at the reference points.

\begin{align}
\begin{align}
    u_1 &= S(y_b, yef7, yet3, yet5, yeo1, uf7, ut3, ut5, uo1) \\
    u_2 &= S(y_b, ye4, yec3, yep3, yez, uf3, uc3, up3, uoz) \\
    u_3 &= S(yb, ye4, yec4, yep4, yez, uf4, uc4, up4, uoz) \\
    u_4 &= S(yb, ye8, yet4, yet6, yeo2, uf8, ut4, ut6, uo2),
\end{align}
\end{align}

where $u_1, u_2, u_3, u_4$ – are the values of the EEG signals at the points of intersection of the secant plane with the arcs of the great circle; $uf3, uf4, uf7, uf8, ut3, ut4, ut5, ut6, uc3, uc4, up3, up4, uo1, uo2, uoz$ - are the values of the EEG signals at the corresponding electrodes, and $uo2 = S(yoz, yet5, yeo1, yeo2, yet6, ut5, uo1, uo2, ut6)$.

2. The EEG signal values at each point of the section are obtained by the following formula:

$$u_x = S(x, x_1, x_2, x_3, x_4, u1, u2, u3, u4),$$

where $x$ is the coordinate of the point in the section; $u_x$ is the value of the EEG signal at the point $x$.

Signal values in all successive sections are stored in a square matrix of 200 * 200 elements. After filling the matrix, the search for the global maximum and minimum is carried out. The interval between the maximum and minimum is divided into six equal intervals, according to which the corresponding color code is assigned to the element. Since the matrix is square, the edge elements outside the circular mapping area do not receive color features. The frontal zone of the map is not displayed due to the use of these amplifier channels for supplying the sync pulse.

A color map is displayed on the graphic field of the program corresponding to a discrete moment of time along with a color scale, boundary values of the maximum and minimum of the signal for a given moment of time, as well as the time relative to the moment of stimulus delivery.

Figure 3 shows six sequential maps (from left to right, from top to bottom), constructed for the moments of time, spaced from the moment of presentation of the stimulus with a step of $\Delta t = 0.05$ s. In the experiment, the area of the circle was larger than the area of the square, the subject mentally pronounced the word "circle".

Figure 4 shows similar maps for the case when the area of the square exceeded the area of the circle. It is noteworthy that the first two cards in figure 3 and figure 4 are very similar. These maps correspond to the signal processing time in the visual cortex, and the second maps (100 ms) correspond to the time of the positive maximum of visual EP in a healthy person. Despite the external differences in the presented figures, information reaches the visual area of the cerebral cortex almost simultaneously. It should be noted that there is a fairly wide area of information interaction in the right hemisphere, which is most likely due to the specifics of the task.

As it was seen from figure 3, the maximum cognitive EP (P300) is reached by the time indicated in the literature when a simple figure (circle) is recognized. In this case, the maximum area is noted not only in the parietal leads, but in a much larger area of the right hemisphere (responsible for the analysis of images). Awareness of the "square" takes more time, a similar picture is observed by the time of 0.45 s, that which does not contradict the data of other researchers obtained by the traditional method [13].

In the described experiment, the work of the FS has a linguistic output in the form of phonemes and words. Recently, such studies, which are at the junction of neurolinguistics and electrophysiology, are being carried out at an outstripping pace [17-19] due to their great importance for the development of communication methods for people with disabilities.
Figure 3. Maps of the evoked activity of the cerebral cortex with a time step of 0.05 s upon presentation of a large circle and a small square.

Figure 4. Maps of evoked activity of the cerebral cortex with a time step of 0.05 s upon presentation of a large square and a small circle.

The moments of activation of speech formation and recognition centers observed during the development of the CGM response, making it possible to localize neurolinguistic phenomena and evaluate the effectiveness of various teaching methods make a real interest.
The degree of EEG synchronization in all possible pairs of leads were calculated according to the parameters of the model by calculation (without spectral transformations, which occurs when determining the coherence functions), that allowed to diagnose the presence of neurological disorders.

4. Conclusion
There was developed a mathematical model for selective filters that transmit the background EEG activity without distortion, which makes it possible to isolate the evoked bioelectrical activity of the brain, to evaluate the parameters of the cognitive process - from sensory response to awareness and motor act. The proposed method not only reduces the recording time of evoked potentials by orders of magnitude (units of presented stimuli instead of tens and hundreds), but also provides a new quality - the ability to compare the brain's response to each stimulus.

On the basis of the considered method, it is possible to build a new type of simulator for pilots and astronauts, which makes it possible to reveal a person's ability to predict the behavior of a complex system and make the right decisions in an extreme situation. There was proposed an algorithm for mapping the evoked activity. A mathematical model has been developed for selective filters that transmit the background EEG activity without distortion, which makes it possible to isolate the evoked bioelectrical activity of the brain, to evaluate the parameters of the cognitive process - from sensory response to awareness and motor act.

This approach is fundamentally new, and opens up prospects for the development of the latest systems for diagnosing various disorders, assessing the degree of professional fitness, building a neurointerface and a lie detector.

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References
[1] Gorman J C, Demir M, Cooke N J and Grimm D AV 2019 Evaluating sociotechnical dynamics in a simulated remotely-piloted aircraft system: a layered dynamics approach. *Ergonomics.* 62(5) 629 doi: 10.1080/00140139.2018.1557750.
[2] Schneider J, Saenz-Otero A, Klerman E and Stirling L 2018 Strategy development pilot study of sleep-restricted operators using small satellites with displays. *Aerosp. Med. Hum. Perform.* 89(12) 1036 doi: 10.3357/AMHP.5024.2018.
[3] Vernadsky V I 1977 *Reflections of a Naturalist.* Book. 2. Scientific Thought as a Planetary Phenomenon (Moscow: Nauka) p191
[4] Pribram K 1975 *Brain Languages* (Moscow: "Progress") p 416
[5] Anokhin P K 1980 *Key Questions of the Theory of Functional Systems* (Moscow: Nauka) p196
[6] Sudakov K V 2011 *Functional Systems* (Moscow: RAMS Publishing House) 320
[7] Holečková I, Kletečka J, Štěpánek D, Žídek S, Bludovský D, Pouska J, Mautner P and Přibáň V 2018 Cognitive impairment measured by event-related potentials during early and late postoperative period following intravenous or inhalation anaesthesia. *Clin. Neurophysiol.* 129(1) 246 doi: 10.1016/j.clinph.2017.10.038
[8] Lier E J, Oosterman J M, Assmann R, de Vries M and van Goor H 2020 The effect of Virtual Reality on evoked potentials following painful electrical stimuli and subjective pain. *Sci. Rep.* 10(1) 9067. doi: 10.1038/s41598-020-66035-4
[9] Rogasch NC, Zipser C, Darmani G, Mutanen TP, Biabani M, Zrenner C, Desideri D, Belardinelli P, Müller-Dahlhaus F and Ziemann U 2020 The effects of NMDA receptor blockade on TMS-evoked EEG potentials from prefrontal and parietal cortex. *Sci. Rep.* 10(1) 3168 doi: 10.1038/s41598-020-59911-6.
[10] Baumer F M, Pfeifer K, Fogarty A, Pena-Solorzano D, Rolle C E, Wallace J L, Rotenberg A and Fisher R S 2020 Cortical excitability, synaptic plasticity, and cognition in benign epilepsy with
centrotemporal spikes: A pilot TMS-EMG-EEG study. *J. Clin. Neurophysiol.* 37(2) 170 doi: 10.1097/WNP.0000000000000662.

[11] Kuo I J, et al. 2020 Neurophysiological signatures of hand motor response to dual-transcranial direct current stimulation in subacute stroke: a TMS and MEG study *J. Neuroeng. Rehabil.* 17(1) 72 doi: 10.1186/s12984-020-00706-1.

[12] Schroder E, Dubuson M, Dousset C, Mortier E, Kornreich C and Campanella S 2020 Training inhibitory control induced robust neural changes when behavior is affected: a follow-up study using cognitive event-related potentials. *Clin. EEG Neurosci.* 51(5) 303 doi: 10.1177/1550059419895146.

[13] Ferraz E, Gonçalves TDS, Freire T, Mattar TLF, Lamônica DAC, Maximino LP and Abreu P C P (2018) Effects of a phonological reading and writing remediation program in students with dyslexia: intervention for specific learning disabilities. *Folia Phoniatr. Logop.* 70(2) 59 doi: 10.1159/000489091.

[14] Edwards C G, Walk A M, Thompson S V, Reeser G E, Erdman J W Jr, Burd N A, Holscher H D and Khan N A 2020 Effects of 12-week avocado consumption on cognitive function among adults with overweight and obesity. *Int. J. Psychophysiol.* 148 13 doi: 10.1016/j.ijpsycho.2019.12.006.

[15] Zhu Y, et al. 2018 Effects of a specially designed aerobic dance routine on mild cognitive impairment. *Clin. Interv.* 13 1691 doi: 10.2147/CIA.S163067. eCollection 2018.

[16] Talalay I V, Kurgansky A V and Machinskaya R I 2018 Alpha-band functional connectivity during cued versus implicit modality-specific anticipatory attention: EEG-source coherence analysis. *Psychophysiology* 55 (12) e13269 doi: 10.1111/psyp, 13269.

[17] Cooney C, Folli R and Coyle D 2018 Neurolinguistics research advancing development of a direct-speech brain-computer interface. *iScience* 8 103 doi: 10.1016/j.isci.2018.09.016.

[18] Bakhtin O M, Krivko E M and Kiroi V N 2019 Electromyographic components associated with internal speech. *Journal of Medical Biological Research* 8 (2) 111 doi: 10.37482 / 2542-1298-Z001

[19] Romero A C L, Frizzo A C F, Chagas E F B, de Lima Isaac M 2020 Cortical auditory evoked potential in babies and children listeners. *Braz. J. Otorhinolaryngol.* 86(4) 395. doi: 10.1016/j.bjorl.2019.01.007.