Dirac-Born-Infeld actions and Tachyon Monopoles

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ABSTRACT: We investigate magnetic monopole solutions of the non-abelian DBI action describing 2 coincident non-BPS D9-branes in flat space. Just as in the case of kink and vortex solitonic tachyon solutions of the full DBI non-BPS actions, as previously analyzed by Sen, these monopole configurations are singular in the first instance and require regularization. We discuss a suitable non-abelian ansatz and show it solves the equations of motion to leading order in the regularization parameter. Fluctuations are studied and shown to describe a codimension 3 BPS D6-brane. A formula is derived for its tension. We comment on the implication to our results from both the trace (Tr) and symmetrized trace (Str) prescriptions of the non-abelian DBI action of coincident non-BPS D9-branes.

KEYWORDS: Tachyon condensation, D-branes, Solitons Monopoles and Instantons.
1. Introduction

Tachyon condensation has been a subject of considerable investigation via the physics of non-BPS D-branes (for a comprehensive review see [1]). Such tachyons arise quite naturally in the open string spectrum when one considers non-BPS D-branes in type IIA or IIB string theories. A growing body of research has developed in open string field theory (for a review see [2] or [3, 4] for more recent works), boundary string field theory, (BSFT) [5, 6, 7, 8] and various effective actions around the tachyon vacuum [9, 10, 11, 12]1 to demonstrate Sen’s results [1] concerning the fate of the open string vacuum in the presence of tachyons.

In related developments, it was also shown that D-brane charges take values in appropriate K-theory groups of space-time. A major result is that all lower-dimensional D-branes can be considered in a unifying manner as non-trivial excitations on the appropriate configuration of higher-dimensional branes. In type IIB, it was demonstrated by Witten in [14] that all branes can be built from sufficiently many D9-anti-D9 pairs. In type IIA, Horava described how to construct BPS D(p − 2k − 1)-branes as bound states of unstable Dp-branes [15].

The mechanism of tachyon condensation into lower dimensional BPS D-branes has been verified in some cases at the level of tachyon effective action. In [16], Sen showed that tachyon kink solutions (that represent codimension one BPS D-branes) exist even when one considers the full non-linear DBI like action of a non-BPS D-brane in a flat background. Compared to their counterpart obtained in the truncated theories [7, 17, 18], these kinks are singular and require regularization. Remarkably, it was shown that in the limit where the regularization parameter is removed, the effective theory of fluctuations about the regularized tachyon kink profile, that depends only on a single spatial world-volume coordinate, are precisely those of a codimension 1 BPS D-brane and is described by a DBI action. Furthermore Sen also showed that in brane-antibrane systems, in which a single complex tachyon field is present, regularized vortex solutions to the equations of motion derived from the DBI non-BPS action exist, that naturally depend on two spatial worldvolume coordinates. Analysis of the fluctuations in this case again showed that to leading order, they are those of a codimension 2 BPS D-brane as described by the appropriate full non-linear DBI action.

In [19], we investigated the generalization of tachyon kink solutions to the case of the full non-linear non-abelian action of two coincident non-BPS D-branes. We showed that, in certain cases, starting with two non-BPS D9-branes, the fluctuations about the regularized non-abelian tachyon kink profile describe a coincident pair of BPS D8-branes.

1See [13] for a new proposal.
In this paper, we want to investigate codimension 3 magnetic monopole solutions, arising from the same DBI like action of two coincident non-BPS D9-branes, which correspond to one BPS D6-brane. Monopole solutions in certain truncations of tachyon models have already been studied in [17]. In this paper we wish to go beyond that analysis and study magnetic monopole solutions arising from the full non-linear non-abelian DBI like action, i.e., without assuming an action truncated in an expansion in derivatives of the tachyon field. From our understanding of the DBI tachyon kink and vortex solutions discussed above, we expect (and find) that such monopole solutions will again be singular in the first instance and require regularization.

Our starting point will be the effective description of two coincident non-BPS D9-branes proposed in [12]. This theory describes a non-abelian version of the DBI action in which the tachyon field transforms in the adjoint representation of the $U(2)$ gauge symmetry of the coincident non-BPS D9-brane world volume action. In the original construction of this action and its generalization to coincident non-BPS D$p$-branes, a standard trace prescription (which we denote as $Tr$) was taken over the gauge indices. Another prescription, motivated by string scattering calculations (at least to low orders in $\alpha'$ [20, 21]) is to take the symmetrized trace (which we denote by $Str$) over gauge indices. In both cases the expression being traced over is the same but the $Str$ prescription results in significantly more complicated terms in the action compared to $Tr$. We will discuss both prescriptions in this paper.

The structure of the paper is as follows. We begin in section 2 with a ’t Hooft-Polyakov monopole like ansatz for the $U(2)$ non-abelian DBI tachyon world volume theory and discuss constraints placed on it by requiring Dirac quantization of magnetic charge in section 3. In section 4, we show that with suitable regularization the magnetic monopole ansatz satisfies the equations of motion to leading order as the regularization parameter is switched off and a formula is derived for the D6 brane tension which depends implicitly on the non-BPS, non-abelian tachyon potential. By comparison to the vortex tachyon profiles, the function appearing in the $U(2)$ gauge field ansatz of the monopole appears not to have an analytic expression, though we derive a differential equation for it and together with its known asymptotic form, a numerical solution is expected to exist. In section 5 a study of the fluctuation spectrum about these monopoles shows them to be precisely described by a DBI action of a single BPS D6 brane in flat space. It is shown that to leading order in the regularization parameter this result can also be derived using the $Str$ prescription, the only difference being the appearance of $Str$ instead of $Tr$ in the expression for the D6 brane tension. We end with some conclusions and speculations.
2. The ’t Hooft-Polyakov Monopole and the DBI action

We begin by reviewing an effective DBI action for the coincident non-BPS D9-brane pair [12]. This system is unstable and it contains a tachyon in its spectrum, in particular, around the maximum of the tachyon potential, the theory contains a \( U(2) \) gauge field and four tachyon states represented by a \( 2 \times 2 \) hermitian matrix-valued scalar field transforming in the adjoint representation of the gauge group.

In this paper we are going to use the following DBI action for the two non-BPS D9-branes

\[
S_{\text{DBI}} = -Tr \int d^{10}x V(T) e^{-\phi} \sqrt{-\det (G_{\mu\nu})}
\]

where

\[
G_{\mu\nu} = g_{\mu\nu}\mathbb{1}_2 + B_{\mu\nu}\mathbb{1}_2 + \pi\alpha'(D_{\mu}TD_{\nu} + D_{\nu}TD_{\mu}) + 2\pi\alpha'F_{\mu\nu}
\]

In eq. (2.1), \( g_{\mu\nu}, B_{\mu\nu} \) and \( \phi \) are respectively the spacetime metric, the antisymmetric Kalb-Ramond tensor and dilaton fields whereas \( \mathbb{1}_2 \) is the \( 2 \times 2 \) unit matrix. The covariant derivative is defined to be \( D_\mu T = \partial_\mu T - i[A_\mu,T] \) and the field strength takes the usual form \( F_{\mu\nu} = \partial_\mu A_{\nu} - \partial_\nu A_{\mu} - i[A_{\mu},A_{\nu}] \). The tachyon kinetic term has been written in a symmetric form to make the integrand a Hermitian matrix [12]. Throughout the paper we will make use of conventions such that \( 2\pi\alpha' = 1 \).

For the potential, we shall only assume that

- \( V(T) \) is symmetric under \( T \rightarrow -T \),
- \( V(T) \) has a maximum at \( T = 0 \) and its minima are at \( T = \pm \infty \) where it vanishes.

Apart from a \( U(1) \) subgroup, the effective theory of two unstable \( D \)-branes, admits as a solution the ’t Hooft-Polyakov monopole, which is of the form

\[
T(x) = t(r) \frac{x_i}{r} \sigma_i,
\]

\[
A_i(x) = \frac{1}{2}(c - a(r))\epsilon_{ijk} \frac{x_j}{r^2} \sigma_k
\]

where \( r \) is the radial distance from the origin in the three transverse directions and \( c \) is a constant. The boundary conditions to be imposed at the origin are that \( t(0) = 0 \) and \( a(0) = c \), so as to avoid a singularity. The boundary conditions to be imposed at infinity are that both \( t(r) \) and \( a(r) \) go to a constant. Without loss of generality henceforth we will take \( c = 1 \).

It is actually more convenient to work in spherical coordinates to make use of the spherical symmetry of the solution. In these coordinates the tachyon and the gauge
fields take the form
\[ T = t(r) \left( \sin \theta \cos \phi \sigma_1 + \sin \theta \sin \phi \sigma_2 + \cos \theta \sigma_3 \right) \]
\[ A_r = 0 \]
\[ A_\theta = \frac{1}{2} \left( 1 - a(r) \right) \left( \sin \phi \sigma_1 - \cos \phi \sigma_2 \right) \]
\[ A_\phi = \frac{1}{2} \left( 1 - a(r) \right) \left( \sin \theta \cos \theta \sin \phi \sigma_2 + \sin \theta \cos \theta \cos \phi \sigma_1 - \sin^2 \theta \sigma_3 \right) \] (2.4)

The covariant derivatives of the tachyon are
\[ D_r T = t'(r) \left( \sin \theta \cos \phi \sigma_1 + \sin \theta \sin \phi \sigma_2 + \cos \theta \sigma_3 \right) \]
\[ D_\theta T = t(r) a'(r) \left( \cos \theta \sin \phi \sigma_2 + \cos \theta \cos \phi \sigma_1 - \sin \theta \sigma_3 \right) \]
\[ D_\phi T = t(r) a'(r) \sin \theta \left( \cos \phi \sin \theta \sigma_2 + \cos \phi \cos \theta \sigma_1 - \sin \theta \sigma_3 \right) \] (2.5)

and finally the tensor \( G_{\mu\nu} \) in (2.2) becomes
\[
G_{\mu\nu} =
\begin{pmatrix}
\eta_{\alpha\beta} \mathbf{1}_2 & \left( 1 + t'(r)^2 \right) \mathbf{1}_2 & F_{r\theta} & F_{r\phi} \\
-F_{r\theta} & A(r) \mathbf{1}_2 & F_{\theta\phi} \\
-F_{r\phi} & -F_{\theta\phi} & \sin^2 \theta A(r) \mathbf{1}_2
\end{pmatrix}
\] (2.7)

where we defined
\[ A(r) = r^2 + t(r)^2 a(r)^2. \] (2.8)

There is a potential ambiguity in how to take the determinant of the matrix (2.7), given that its elements are in general non-commuting. By choosing the standard definition for the determinant of \( G_{\mu\nu} \),
\[ \det G \equiv \frac{1}{3!} \epsilon^{\mu\nu\rho} \epsilon^{\mu'\nu'\rho'} G_{\mu\mu'} G_{\nu\nu'} G_{\rho\rho'} \] (2.9)

we obtain:
\[ -\det G = \sin^2 \theta \left[ (t'^2 + 1) \left( A(r)^2 + \frac{1}{4} (1 - a(r)^2)^2 \right) + \frac{1}{2} a'(r)^2 A(r) \right] \otimes \mathbf{1}_2. \] (2.10)

This definition has the nice feature that with our ansatz, \( \det G \) comes out to be proportional to the identity matrix in \( U(2) \) space. This will greatly simplify the analysis in what follows.
3. Dirac Quantization of Magnetic Charge

To evaluate the magnetic charge associated to the ansatz (2.3), we need to have a definition of the magnetic field. In a U(2) gauge theory, there is no unambiguous definition, but in a spontaneously broken theory, with unbroken group \(^2\) U(1), provided that the fields are close to the vacuum, a magnetic field can be defined:

\[ F_{\mu
u}^{EM} = \frac{1}{2} F_{\mu
u}^a \hat{T}^a \]  

(3.1)

where \( \hat{T}^a \) is a unit vector that points along the direction of the ‘Higgs’ field (in the present case the adjoint tachyon field \( T^a \)). In particular, \( \hat{T}^a = \frac{x^a}{r} \) and the physical magnetic field becomes:

\[ B_i = \frac{1}{2} \epsilon_{ijk} F_{jk}^{EM} = \frac{1}{4} \epsilon_{ijk} \frac{x^a}{r} F_{jk}^a. \]  

(3.2)

To find the total magnetic flux which is equal to the magnetic charge \( m \), we have to integrate the magnetic field over \( S^2_\infty \), the 2-sphere at infinity. The magnetic charge \( m \) enclosed in some Gaussian surface \( \Sigma \) enclosing the magnetic charge density is given by

\[ m = \int_{S^2_\infty} B_i dS_i = \lim_{r \to \infty} \frac{1}{4} \int_{S^2} \epsilon_{ijk} \frac{x^a}{r} F_{jk}^a dS_i \]  

(3.3)

Now \( dS_i = \epsilon_{ijk} dx^j \wedge dx^k \), so

\[ m = \lim_{r \to \infty} \frac{1}{2} \int_{S^2} F_{jk}^a \frac{x^a}{r} dx^j \wedge dx^k \]  

(3.4)

in polar coordinates, we can write

\[ dx^j \wedge dx^k = \partial_m x^j(r, \theta, \phi) \partial_n x^k(r, \theta, \phi) \, d\xi^m \wedge d\xi^n \]  

(3.5)

where \( \xi^n, n = 1, 2 \), correspond to the coordinates \( \theta \) and \( \phi \). We have

\[ m = \lim_{r \to \infty} \frac{1}{2} \int_{S^2} F_{jk}^a \frac{x^a}{r} \partial_m x^j(r, \theta, \phi) \partial_n x^k(r, \theta, \phi) d\xi^m \wedge d\xi^n \]  

= \lim_{r \to \infty} \int_{S^2} F_{\theta\phi}^a \frac{x^a(r, \theta, \phi)}{r} d\theta d\phi \]  

(3.6)

where the \( S^2 \) has radius \( r \). Using the definition of \( x^a(r, \theta, \phi) \) and the expressions derived before for \( F_{\theta\phi}^a \) we find:

\[ m = -\frac{1}{2} \lim_{r \to \infty} \int_{S^2} (1 - a(r)^2) \sin \theta \, d\theta d\phi \]  

\[ = -2\pi \lim_{r \to \infty} (1 - a(r)^2) \]  

(3.7)

\(^2\)Upon tachyon condensation the worldvolume of the D6-brane contains a U(1) gauge field.
Now this should be the magnetic charge and in the limit \( r \to \infty \) it does not depend on \( r \), the radius of the \( S^2 \) we enclose the magnetic monopole with. In the case of the tachyon monopole the core of the magnetic monopole is spread out over infinite volume, this is because the VEV of the tachyon is infinite (compared to a finite value in the 't Hooft-Polyakov case) and \( T^a \) approaches its VEV as \( r \to \infty \). Thus to capture all the enclosed magnetic charge we have to take the limit of \( r \to \infty \) for our surface. We can derive the necessary boundary condition at infinity on our function \( a(r) \) in order to satisfy Dirac quantization of magnetic charge:

\[
m = \frac{2\pi n}{e}
\]

for a charge \( n \) magnetic monopole where \( e \) is the electric charge. From the definition of the covariant derivative of the tachyon field \( T^a \) it is clear that \( e = -1 \). So for an \( n = +1 \) magnetic monopole, the magnetic charge is

\[
m = \frac{2\pi n}{e} = -2\pi \lim_{r \to \infty} \left( 1 - a(r)^2 \right)
\]

so that we have the boundary condition

\[
\lim_{r \to \infty} a(r)^2 \to 0
\]

Notice that in Cartesian coordinates we find that asymptotically \( B_i \sim \frac{1}{r^2} \frac{x^i}{r} \) where \( \frac{x^i}{r} \) is simply a unit radial vector, so the magnetic field is radial and its magnitude has the standard Coulomb form \( B = \frac{m}{4\pi r^2} \) with \( |m| = 2\pi \).

4. Energy-momentum tensor and \( D6 \)-brane tension

We now compute the energy-momentum tensor

\[
T^{\mu\nu} = -\text{Tr} \left( V(T) \sqrt{-\det G} (G^{-1})^{\mu\nu} \right)
\]

where

\[
G^{-1}_{\mu\nu} = \frac{1}{\det G} C^T_{\mu\nu},
\]

\( C_{\mu\nu} \) being the matrix of cofactors. In particular,

\[
(G^{-1})_{rr} = \frac{1}{\det G} \begin{vmatrix} G_{\theta\theta} & G_{\theta\phi} \\ G_{\phi\theta} & G_{\phi\phi} \end{vmatrix} = \frac{1}{\det G} \frac{1}{2} \epsilon^{ij} \epsilon^{ij'} G_{ii'} G_{jj'}
\]

\[
= \frac{1}{\det G} \frac{1}{2} \epsilon^{ij} \epsilon^{ij'} G_{ii'} G_{jj'}
\]
with \( i, j = \theta, \phi \). Therefore, we have that the energy-momentum tensor elements with one \( r \)-component are

\[
T_{rr} = -\text{Tr} \frac{V(T)}{\sqrt{-\text{det}G}} \sin^2 \theta \left[ \mathcal{A}(r)^2 + \frac{1}{4} (1 - a(r)^2)^2 \right] \otimes \mathbb{1}_2 ,
\]

\[
T_{r\theta} = -\frac{1}{2} \text{Tr} \frac{V(T)}{\sqrt{-\text{det}G}} \mathcal{A}(r) a'(r) \sin^2 \theta \left( -\sin \phi \sigma_1 + \cos \phi \sigma_2 \right),
\]

\[
T_{r\phi} = \frac{1}{2} \text{Tr} \frac{V(T)}{\sqrt{-\text{det}G}} \mathcal{A}(r) a'(r) \sin \theta \left( \sin \theta \sigma_3 - \cos \theta (\cos \phi \sigma_1 - \sin \phi \sigma_2) \right). \tag{4.4}
\]

However notice that for the tachyon ansatz \( T \sim x^a \sigma_a \) one has that \( T^2 \propto \mathbb{1}_2 \), therefore, for a potential of the form \( V(T^2) \) then also \( V(T) \propto \mathbb{1}_2 \), which means that we can directly act with the trace in the stress-energy tensor to eliminate some components. The only non-vanishing components of the stress-energy tensor are \( T_{rr} \), \( T_{r\theta} \) and \( T_{r\phi} \) which means that the overall conservation equation for the \( r \)-component reduces to \( \partial_r T_{rr} = 0 \). Evaluating the trace we obtain:

\[
T_{rr} = -\frac{2 \sin \theta V(T) (\mathcal{A}(r)^2 + \frac{1}{4} (1 - a(r)^2)^2)}{\sqrt{[(t(r)^2 + 1) (\mathcal{A}(r)^2 + \frac{1}{4} (1 - a(r)^2)^2) + \frac{1}{2} a'(r)^2 \mathcal{A}(r)]}} \tag{4.5}
\]

If we assume that the potential vanishes at infinity, then \( T_{rr} \) must vanish everywhere because it should not depend on \( r \), unless the function \( a(r)^2 \) in the numerator blows up fast enough. In the previous section we saw that in order to obtain the correct Dirac quantization for the magnetic charge, in the limit \( r \to \infty \), the function \( a(r) \) must approach a constant\(^3\). The conservation equation then tells us that \( T_{rr} \) should vanish for all \( r \). However, for \( r \) close to the origin, the potential is finite and \( T_{rr} \) doesn’t vanish and so at least for small \( r \) we require \( t'(r) \) or \( a'(r) \) to blow up. This forces us to consider a regularization of the form

\[
t(r) = \hat{t}(kr) , \quad a(r) = \hat{a}(kr) \tag{4.6}
\]

such that in the \( k \to \infty \) limit \( t'(r) \) and \( a'(r) \) go to infinity while keeping \( t(r) \) and \( a(r) \) fixed. In the large \( k \) limit:

\[
-\text{det}G = \sin^2 \theta k^2 \hat{t}^2 \left[ \hat{\mathcal{A}}^2(kr) + \frac{1}{4} (1 - \hat{a}(kr)^2)^2 + \frac{1}{2} \hat{a}'(kr)^2 \frac{\hat{\mathcal{A}}(kr)}{\hat{t}^2(kr)} \right] \otimes \mathbb{1}_2 \tag{4.7}
\]

where

\[
\hat{\mathcal{A}}(kr) \equiv r^2 + \hat{t}(kr)^2 \hat{a}(kr)^2 \tag{4.8}
\]

\(^3\)This constant is zero in our case having set \( c = 1 \), however, in general, the constant is \( c - 1 \).
The energy-momentum tensor becomes

\[
T_{rr} = -\frac{2\sin \theta V(T) \left( \dot{A}^2 + \frac{1}{4} (1 - \ddot{a})^2 \right)}{k \hat{t} \sqrt{\frac{1}{4} \left( \dot{A}^2 + \frac{1}{4} (1 - \ddot{a})^2 + \frac{1}{2} \dot{a}^2 \frac{A}{\hat{t}^2} \right)}}
\]  

(4.9)

and we see that \(T_{rr}\) vanishes everywhere in the large \(k\)-limit as required. This shows that the monopole solution is indeed a solution to the conservation equation and hence a consistent solution of the system e.o.m. Let us now calculate the tension associated with the D6-brane. We integrate the expression for \(T_{\alpha \beta}\) over the radial and angular coordinates to obtain:

\[
T_{\alpha \beta} = -4\pi \eta_{\alpha \beta} \text{Tr} \int_0^\infty dr V(\hat{t}(kr)) k \hat{t}'(kr) \sqrt{\frac{1}{4} \left( \dot{A}^2 + \frac{1}{4} (1 - \ddot{a})^2 + \frac{1}{2} \dot{a}^2 \frac{A}{\hat{t}^2} \right)} 1_2 \]  

(4.10)

Now we can perform coordinate transformations:

\[
y = \hat{t}(kr), \quad r \equiv \hat{r}(y) = k^{-1} \hat{t}^{-1}(y), \quad \ddot{a}(y) = \dot{\hat{a}}(kr) = \dddot{a}(k\hat{r}(y)),
\]

(4.11)

to obtain (in the large \(k\) limit)

\[
T_{\alpha \beta} = -8\pi \eta_{\alpha \beta} \int_0^\infty dy V(y) \sqrt{\frac{1}{4} \left[ (\dot{A}^2(y) + \frac{1}{4} (1 - \ddot{a}(y))^2 + \frac{1}{2} \ddot{a}(y) \dddot{a}(y)^2 \right]}}
\]

(4.12)

where

\[
\ddot{A}(y) = y^2 \ddot{a}(y)^2 + \frac{1}{k^2 \hat{t}^2(y)} \sim y^2 \ddot{a}(y)^2
\]

(4.13)
in the large \(k\)-limit. In a similar fashion to the kink and vortex calculations [16] most of the contribution to \(T_{\alpha \beta}\) comes from a small region in \(r\) space centered around \(\frac{1}{k}\). We can identify the tension of the D6-brane as:

\[
T_6 = 8\pi \int_0^\infty dy V(y) \sqrt{y^4 \ddot{a}(y)^4 + \frac{1}{4} (1 - \ddot{a}(y)^2)^2 + \frac{1}{2} y^2 \ddot{a}(y)^2 \dddot{a}(y)^2}
\]

(4.14)

The tension of the D6-brane is determined only by the tachyon potential since the function \(\ddot{a}(y)\) can be computed by minimizing the energy from the tensor component \(T_{00}\) and thus is determined implicitly in terms of \(V(y)\). This leads to the following differential equation for \(\ddot{a}(y)\):

\[
0 = \frac{\partial}{\partial y} \left( V(y) y^2 \ddot{a}(y) \dddot{a}(y) / \sqrt{y^4 \ddot{a}(y)^4 + \frac{1}{4} (1 - \ddot{a}(y)^2)^2 + \frac{1}{2} y^2 \ddot{a}(y)^2 \dddot{a}(y)^2} \right)
\]

\[
- V(y) \ddot{a}(y) \frac{4 y^4 \ddot{a}(y)^2 - (1 - \ddot{a}(y)^2) + y^2 \dddot{a}(y)^2}{\sqrt{y^4 \ddot{a}(y)^4 + \frac{1}{4} (1 - \ddot{a}(y)^2)^2 + \frac{1}{2} y^2 \ddot{a}(y)^2 \dddot{a}(y)^2}}.
\]
This equation is not easy to solve, and it seems there is no analytic solution. We notice that there is at least one trivial solution corresponding to \( \tilde{a}(y) \) being constant, more precisely, \( \tilde{a}(y) = 0 \). This solution is in agreement with the requisite boundary condition we found in (3.10) to obtain the correct Dirac quantization for the monopole magnetic charge.

Finally let us compare the tension \( T_{p-3} \) above, both to expression \( T_{p-1} \) for the codimension 1 BPS D-brane one finds from tachyon condensation on a non-BPS \( Dp \)-brane and to the expression \( T_{p-2} \) for the codimension 2 BPS D-brane on a \( Dp \bar{D}p \)-brane pair. There one obtains, respectively \[ \[16 \]

\[
T_{p-1} = \int_{-\infty}^{\infty} dy \, V(y) \\
T_{p-2} = 4\pi \int_{0}^{\infty} dy \, V(y) \sqrt{y^2 (1 - g(y))^2 + \frac{1}{4} g'(y)^2}
\]

By minimizing the tension \( T_{p-2} \) as a function of \( g(y) \), the following differential equation can be obtained

\[
\frac{\partial}{\partial y} \left( \frac{V(y)g'(y)}{\sqrt{y^2 (1 - g(y))^2 + \frac{1}{4} g'(y)^2}} \right) + \frac{4V(y)y^2(1 - g(y))}{\sqrt{y^2 (1 - g(y))^2 + \frac{1}{4} g'(y)^2}} = 0
\]  

(4.16)

Now although [16] does not discuss exact solutions of this equation one can derive one in certain cases. If we take a tachyon potential of the form \( V(y) = V_0 e^{-\beta y^2} \) given by boundary string field theory, this equation admits an exact analytic solution, namely

\[
g(y) = 1 - e^{-\frac{1}{\beta} y^2}.
\]

This solution gives the following tensions

\[
T_{p-1} = V_0 \sqrt{\frac{\pi}{\beta}}
\]

\[
T_{p-2} = 4\pi \sqrt{1 + \beta^2} V_0 \int_{0}^{\infty} dy \frac{y}{\beta} e^{-(\beta+1/\beta)y^2} = \frac{2\pi V_0}{\sqrt{1 + \beta^2}}
\]

(4.18)

in units where \( 2\pi \alpha' = 1 \), by choosing \( V_0 = \sqrt{2T_p} \) and \( \beta = 1 \) we get

\[
T_{p-1} = \sqrt{2\pi} T_p
\]

\[
T_{p-2} = 2\pi T_p
\]

(4.19)

which reproduce the correct descent relations.
5. World-volume action on the monopole

This section is devoted to analyze the world-volume calculation of the monopole. We plan to show that the world-volume theory of the monopole condensed on a Dp-brane results in a D(p-3)-brane, described by an action with a U(1) gauge theory. We begin by recasting the ansatz for the monopole in the following way:

\[ T(\vec{x}) = f(r)x_i\sigma_i \]
\[ A_i(\vec{x}) = g(r)\epsilon_{ijk}x_j\sigma_k \]  

(5.1)

then make the following ansatz for the fluctuating fields:

\[ \bar{T}(\vec{x}, \xi) = T(\vec{x} - \vec{t}(\xi)) = f(\hat{r})(x_i - \phi_i(\xi))\sigma_i \]
\[ \bar{A}_i(\vec{x}, \xi) = A_i(\vec{x} - \vec{t}(\xi)) = g(\hat{r})\epsilon_{ijk}(x_j - \phi_j(\xi))\sigma_k \]
\[ \bar{A}_\alpha(\vec{x}, \xi) = -\bar{A}_i(\vec{x}, \xi)\partial_\alpha \phi^i + a_\alpha(\xi) \otimes 1 \]  

(5.2)

In the previous expressions,

\[ \hat{r}^2 = (x_i - \phi_i(\xi))(x^i - \phi^i(\xi)) \]  

(5.3)

and the indices \( i, j = 1, 2, 3 \) run over the coordinates \( x_i \) transverse to the world volume whereas the indices \( \alpha, \beta = 0, 4, 5, \ldots, 9 \) run over the coordinates \( \xi_\alpha \) tangent to the world volume.

Using the fact that \( \partial_\alpha \bar{T} = -\partial_\alpha \phi^i \partial_i \bar{T} \) and that \( [\bar{A}_\alpha, \bar{T}] = -\partial_\alpha \phi^i [\bar{A}_i, \bar{T}] \) we obtain

\[ D_\alpha \bar{T} = -D_i \bar{T} \partial_\alpha \phi^i \]  

(5.4)

and similarly, using the fact that \( \partial_\alpha \bar{A}_j = -\partial_\alpha \phi^i \partial_i \bar{A}_j \) and defining \( f_{\alpha\beta} \equiv \partial_\alpha a_{\beta} - \partial_\beta a_\alpha \), we have

\[ F_{\alpha\beta} = -F_{ij}\partial_\alpha \phi^i \partial_\beta \phi^j + f_{\alpha\beta} 1, \]
\[ F_{\alpha j} = -\partial_\alpha \phi^i F_{ij} \]
\[ F_{i\alpha} = -F_{ij}\partial_\alpha \phi^j , \]
\[ \bar{F}_{ij} = \partial_i \bar{A}_j - \partial_j \bar{A}_i - i[\bar{A}_i, \bar{A}_j] \]  

(5.5)

From these we can proceed to compute the matrix elements of our determinant, by defining

\[ g_{ij} \equiv \frac{1}{2} \left( D_i \bar{T}D_j \bar{T} + D_j \bar{T}D_i \bar{T} \right) + F_{ij} \]  

(5.6)

we have

\[ G_{\mu\nu} = \left( \begin{array}{cc} G_{\alpha\beta} & G_{\alpha j} \\ G_{i\beta} & G_{ij} \end{array} \right) = \left( \begin{array}{cc} \eta_{\alpha\beta} + f_{\alpha\beta} + g_{ij}\partial_\alpha \phi^i \partial_\beta \phi^j & -\partial_\alpha \phi^i g_{ij} \\ -g_{ij}\partial_\beta \phi^j & \delta_{ij} + g_{ij} \end{array} \right) \]  

(5.7)
Next, we introduce a new matrix $\hat{G}_{\mu\nu}$ whose elements are $\hat{G}_{\alpha \nu} \equiv G_{\alpha \nu} + \partial_\alpha \phi^i G_{i \nu}$ and $\hat{G}_{i \nu} = G_{i \nu}$, namely

$$
\hat{G}_{\mu \nu} = \left( \begin{array}{c|c}
G_{\alpha \beta} & G_{\alpha j} \\
\hline
G_{i \beta} & G_{i j}
\end{array} \right) \equiv \left( \begin{array}{c|c}
G_{\alpha \beta} & G_{\alpha j} \\
\hline
\partial_\alpha \phi^i (G_{i \beta} G_{i j}) & \partial_\alpha \phi_i \end{array} \right) = \left( \begin{array}{c|c}
\eta_{\alpha \beta} + f_{\alpha \beta} \partial_\alpha \phi_j & \partial_\alpha \phi_j \end{array} \right) \right)
$$

(5.8)

If we were considering matrices whose elements were commuting, then clearly $\det G_{\mu\nu} = \det \hat{G}_{\mu\nu}$ because in that case the determinant would be invariant under the addition of a multiple of a row(column) to another row(column). This property follows from the fact that if each element in a row(column) is a sum of two terms, the determinant equals the sum of the two corresponding determinants. In our case the entries of the matrix $G_{\mu\nu}$ are $su(2)$ algebra-valued elements and therefore it is not clear $a \text{ priori}$ whether in this case that result should hold. However, notice that also in our case

$$
\det \hat{G}_{\mu \nu} \equiv \left| \begin{array}{c|c}
G_{\alpha \beta} + \partial_\alpha \phi^i G_{i \beta} & G_{\alpha j} + \partial_\alpha \phi^i G_{i j} \\
\hline
G_{i \beta} & G_{i j}
\end{array} \right| = \left| \begin{array}{c|c}
G_{\alpha \beta} & G_{\alpha j} \\
\hline
G_{i \beta} & G_{i j}
\end{array} \right| + \left| \begin{array}{c|c}
\partial_\alpha \phi^i G_{i \beta} & \partial_\alpha \phi^i G_{i j} \\
\hline
\partial_\alpha \phi_j & G_{i j}
\end{array} \right| (5.9)
$$

and the latter determinant is zero because $\partial_\alpha \phi^i$, being proportional to the identity in group space, commutes with all the other elements and, therefore, $\det G_{\mu\nu} = \det \hat{G}_{\mu\nu}$. Using the same arguments, we perform a final redefinition by introducing the matrix $\tilde{G}_{\mu \nu}$ whose elements are $\tilde{G}_{\mu \beta} = \hat{G}_{\mu \beta} + \hat{G}_{\mu j} \partial_\beta \phi^j$ and $\tilde{G}_{\mu j} = \hat{G}_{\mu j}$, namely

$$
\tilde{G}_{\mu \nu} = \left( \begin{array}{c|c}
\hat{G}_{\alpha \beta} & \hat{G}_{\alpha j} \\
\hline
\hat{G}_{i \beta} & \hat{G}_{i j}
\end{array} \right) \equiv \left( \begin{array}{c|c}
\hat{G}_{\alpha \beta} & \hat{G}_{\alpha j} \\
\hline
\frac{\partial_\alpha \phi^i}{\partial_\beta \phi_i} & \frac{\partial_\alpha \phi_i}{G_{i j}}
\end{array} \right) \right)
$$

(5.10)

Now, we take the determinant of the previous expression. Notice that the determinant of $G_{ij}$ is given by (2.10) upon the replacement of $\vec{x}$ by $(\vec{x} - t(\xi))$. This determinant has an explicit factor of $k^2$ which becomes dominant in the large $k$ limit, hence, we can ignore the off-diagonal contributions in computing $\det \hat{G}_{\mu\nu}$. We have

$$
-\det \hat{G}_{\mu \nu} \approx -\det G_{ij} \det \hat{G}_{\alpha \beta}
$$

(5.11)

So substituting this into the action gives:

$$
S = -8\pi \int d^7 \xi \int dr \ V(\hat{t}(kr)) k \hat{\dot{t}}(kr) \times \sqrt{\hat{A}^2 + \frac{1}{4} (1 - \hat{a}^2)^2 + \frac{1}{2} \hat{a}^2 \hat{A}^2 \hat{t}^2} \sqrt{-\det(\hat{G}_{\alpha \beta})}
$$

(5.12)
where we have redefined \( r = |\vec{x} - \phi(\vec{\xi})| \) and performed the coordinate transformation in (4.11). The integral over \( r \) is just the tension of the D6 found in (4.14) in the large \( k \)-limit, therefore, we obtain

\[
S = -T_6 \int d^5 \xi \sqrt{-\det \tilde{G}_{\alpha\beta}}
\]

(5.13)

where

\[
\tilde{G}_{\alpha\beta} = \eta_{\alpha\beta} + f_{\alpha\beta} + \partial_\alpha \phi^i \partial_\beta \phi_i
\]

(5.14)

This we recognize as the action of a BPS D6-brane, with the correct U(1) gauge theory.

### 6. Symmetrized trace

It has been shown that scattering amplitudes involving the tachyon can be obtained by an effective action with a symmetrized trace\(^4\). In this case, the effective action for a coincident non-BPS D9-brane pair is given by

\[
S = -\text{Str} \int d^{10} x V(T) e^{-\phi} \sqrt{-\det [g_{\mu\nu} \mathbb{1}_2 + B_{\mu\nu} \mathbb{1}_2 + 2\pi \alpha' (D_\mu T D_\nu T + F_{\mu\nu})]}
\]

(6.1)

In the above action the \( \text{Str} \) prescription means specifically that one has to first symmetrize over all orderings of terms like \( F_{\mu\nu}, D_\mu T \) and also individual \( T \) that appear in the potential \( V(T) \), therefore, it is not possible to plug our monopole ansatz into this action, but one has to, first, expand the square root, second, act with the symmetrized trace and finally use the ansatz.

In this paragraph we wish to shed some light, by doing some preliminary investigations, on tachyon condensation and brane descent relations in the non-abelian non-BPS DBI action with the \( \text{Str} \) prescription.

Again we are going to set \( \phi = B_{\mu\nu} = 0 \) and \( g_{\mu\nu} = \eta_{\mu\nu} \) and

\[
G_{\mu\nu} = \eta_{\mu\nu} + 2\pi \alpha' (D_\mu T D_\nu T + F_{\mu\nu})
\]

(6.2)

Before expanding the square root in the action (6.1), we rewrite \( G_{\mu\nu} \) above as in (5.10), namely

\[
\tilde{G}_{\mu\nu} = \left( \begin{array}{cc} \eta_{\alpha\beta} + 2\pi \alpha' f_{\alpha\beta} + \partial_\alpha \phi^i \partial_\beta \phi_i & \partial_\alpha \phi_i \\ \partial_\beta \phi_i & \delta_{ij} + 2\pi \alpha' (D_i T D_j T + F_{ij}) \end{array} \right)
\]

(6.3)

\(^4\text{Str}(M_1 \ldots M_n) \equiv Tr \sum_\sigma M_1 \ldots M_n \) where \( \sum_\sigma \) is a sum over all permutations of matrices in \( M_1 \ldots M_n \) divided by \( n! \).
Recall that the above expression can be obtained by adding appropriate multiple of rows and columns to other rows and columns. Now the question is: are we allowed to do such an operation in an action with the symmetrized trace? The answer is yes as long as the determinant is left invariant by this operation, that is to say as long as 
$$\det G = \det \tilde{G}.$$ Now recall that in the large $k$-limit, $D_i T, F_{ij} \sim k$ and, therefore, only the elements on the diagonal are the leading ones in this limit:

$$\det G = \det \tilde{G} \sim \det (\eta_{\alpha\beta} + 2\pi\alpha' f_{\alpha\beta} + \partial_\alpha \phi^i \partial_\beta \phi_i) \det (\delta_{ij} + 2\pi\alpha' (D_i T D_j T + F_{ij}))$$

(6.4)

In this limit, the action (6.1) factorizes out into two determinant terms and the symmetrized trace only acts on the first one as shown below, $f_{\alpha\beta}$ and $\partial_\alpha \phi^i$ commuting with $D_i T$ and $F_{ij}$. The result is

$$S = -S\text{Tr} \int d^3 x V(T) \sqrt{-\det [\delta_{ij} + 2\pi\alpha' (D_i T D_j T + F_{ij})]} \times$$

$$\times \int d^7 \xi \sqrt{-\det (\eta_{\alpha\beta} + 2\pi\alpha' f_{\alpha\beta} + \partial_\alpha \phi^i \partial_\beta \phi_i)}$$

(6.5)

from which we get that the tension of the $D6$-brane which lives transversally to the monopole has a tension given by the large $k$-limit of the following expression

$$T_6 = -S\text{Tr} \int d^3 x V(T) \sqrt{-\det (\delta_{ij} + 2\pi\alpha' (D_i T D_j T + F_{ij}))}$$

(6.6)

This tension reduces to the tension of a $D6$ that we found in (4.14) by replacing the $S\text{Tr}$ with the $\text{Tr}$ and by symmetrizing the tachyon kinetic term. It is interesting that in the case of the $S\text{Tr}$ the tension can only be obtained by expanding the square root order by order in $\alpha'$ and then take the large $k$-limit. For example, at lowest orders one would get

$$T_6 = -S\text{Tr} \int d^3 x V(T) \left(1 + \pi\alpha'(D_i T D^i T)ight.$$

$$+ (2\pi\alpha')^2 \left(-\frac{1}{4} D_i T D^i T D_j T D^j T + \frac{1}{8} (D_i T D_j T + F_{ij}) (D^j T D^i T + F^{ji}) \right)$$

$$+ \mathcal{O}(\alpha'^3) \right)$$

(6.7)

and similarly for higher orders.

7. Conclusions

In this paper, we have investigated codimension 3 magnetic monopole solutions, arising from the same DBI like action of two coincident non-BPS D9-branes. We have shown
the existence of singular monopoles that require regularization in a similar fashion to the kink and vortex soliton solutions of the DBI theory investigated by Sen [16]. An analysis of the fluctuations shows that in the limit where the regularization is removed, we recover the correct DBI action corresponding to a single BPS D6-brane. This extends the earlier results found by using truncated DBI like actions [17] and puts magnetic monopoles alongside kinks and vortices [16] as the possible products of tachyon condensation occurring in the full non-linear, non-BPS DBI actions and which yield fluctuation spectra that are described by the full DBI action corresponding to codimension 1, 2 and 3 BPS branes.

These results were obtained within the framework of the non-BPS action presented in [12]. Recently, [13], a modified version of this action (based on the results of [22, 23]) has been proposed. In this modified version, the tachyon field carries internal Pauli matrices \( \sigma_1 \) and \( \sigma_2 \) and was obtained by considering the disk level S-matrix element of one Ramond-Ramond field and three tachyon fields. In [13] the modified action was shown to be consistent with the S-matrix element of one gauge field and four tachyon fields. The modified action amounts to a multiplication of the tachyon potential \( V(T) \) in the symmetrized trace version of the non-BPS action [12] by a factor \( \sqrt{1 + \frac{1}{2}[T_i,T_j][T_i,T_j]} \) where \( T_i = T \sigma_i, \ i = 1,2 \). For large tachyon field values it was argued in [23] that one may compute the Str by expanding \( V(T_i) \) that such modifications resulted in effectively the potential \( V(T) \) being multiplied by a factor of \( T^4 \). The resulting modified potential still vanishes as \( T \to \infty \), so tachyon condensation is still expected to occur. Indeed one might argue that since the tachyon field configurations describing kinks, vortices and as we have shown, monopoles, are ‘large’ almost everywhere in the regularized theory (the tachyon field is infinite everywhere except at the maximum of \( V(T) \) where it is zero, in the unregularized theory) this large \( T \) approximation is justified. Nevertheless it would be interesting to see the details of tachyon condensation in such a modified DBI action, including an analysis of the fluctuation spectrum, and to see if they give the same results starting with the unmodified action in [12]. A first glance shows that at the very least, the formulae for the various tensions of the codimension 1, 2 and 3 BPS branes will change in that \( V(T) \) will be replaced by \( V(T)T^4 \).

Finally, we have only discussed tachyon condensation in flat space. When one considers curved backgrounds there are non-vanishing Ramond-Ramond forms and thus Wess-Zumino (WZ) terms appear in both the actions of BPS and non-BPS branes. Therefore it is natural to consider the origin of such Wess-Zumino terms when BPS D-branes emerge as a result of tachyon condensation. This has been studied some time ago in [24] in the case where a normal trace (as opposed to symmetrized trace) prescription is taken for the WZ term in the non-BPS D-brane action. More recently [25] and [26]
have studied higher order derivative corrections to the WZ terms in non-BPS D-brane actions via disk amplitude S-matrix calculations. It is certainly an interesting question to consider how such corrections modify the results of [24] when one considers tachyon condensation producing codimension 1, 2 and 3 BPS D-branes.

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