Novel Inter-file Coded Placement and D2D Delivery for a Cache-aided Fog-RAN Architecture

Kai Wan*, Daniela Tuninetti†, Mingyue Ji‡, Giuseppe Care*,
*Technische Universität Berlin, Berlin, Germany, {kai.wan, caire}@tu-berlin.de
†University of Illinois at Chicago, Chicago, USA, danielat@uic.edu
‡University of Utah, Salt Lake City, USA, mingyue.ji@utah.edu

Abstract—Maddah-Ali and Niesen (MAN) in 2014 showed that it is possible to serve an arbitrarily large number of cache-equipped users with a constant number of transmissions by using coded caching in shared-link broadcast networks. This paper considers a novel cache-aided “fog” Radio Access Network (Fog-RAN) architecture including a Macro-cell Base Station (MBS) and several Small-cell Base Stations (SBSs), serving users without caches. Some users, not in the reach of SBSs, are directly served by the MBS. Other users, in the reach of SBSs, are “offloaded” and receive information only from the SBSs through high throughput links. The SBSs have their own local caches. The MBS broadcasts packets to the SBSs and the directly served users. Subsequently, the SBSs can communicate among each other such that each SBS can obtain enough information to decode multicast files by its connected users. For the proposed Fog-RAN we study the memory-loads tradeoff for the worst-case demands. The main contribution of this paper is the design of a novel inter-file coded cache placement and a novel D2D caching scheme with shared-caches to handle the inter-SBS communication phase. The proposed scheme is proved to be order optimal within a constant factor when each SBS is connected to the same number of users. As by-product, the proposed inter-file coded placement can also be used in coded caching systems with heterogeneous cache sizes, where some users have no cache. The proposed coded placement depends on the network topology only by the number of SBSs and the number of users directly served by the MBS. Then, by relaxing the requirement of topology independence, we improve the proposed scheme by designing a cache placement with a subfile division dependent on network structure, which is exactly optimal in some memory size regimes. As another by-product, we also improve the existing shared-link caching scheme with shared-caches by this novel topology-dependent subfile division and placement.

I. INTRODUCTION

In content distribution network, traffic can be smoothed out by placing content in local caches “closer” to the end users during off-peak hours (placement phase), with the hope that the pre-fetched content will be requested during peak hours, in which case the number of broadcast transmissions from the server to the users (delivery phase) will be reduced. Coded caching was originally considered in [1] by Maddah-Ali and Niesen (MAN) for shared-link broadcast networks, where a server (equipped with N files) communicates to K users (with a cache able to store M files) through a shared noiseless channel. In the original MAN scheme, each of the N files is partitioned into a number of pieces, and each piece is placed uncoded into a number of user caches that depends on the cache size M; after this symmetric uncoded cache placement, MAN generates multicast coded messages (by a binary linear network code) that are simultaneously useful to many users; these coded multicast message delivery drastically reduces the download time, or network load, compared to traditional caching strategies. The MAN scheme is known to be optimal for shared-link broadcast networks under the constrain of uncoded cache placement [2], [3], and optimal to within a factor of 2 otherwise [4]. When no central server exists and the cache-aided users communicate among each other in the delivery phase, in [5] Ji et al. proposed a device-to-device (D2D) caching scheme which was proved to be order optimal within a factor of 8.

As claimed in [6]–[8], to bring the caching idea into the reality of next generation (5G) cellular networks, a great interest is given on hybrid architectures where the operator/Internet provider caches popular content files at the Fog Radio Access Networks (F-RAN). In this paper, we consider a novel architecture for F-RAN with caching as illustrated in Fig. 1. A Macro-cell Base Station (MBS) with a library of N files, is connected to H Small-cell Base Stations (SBSs) and K0 users. Each SBS has a memory to cache M files. There are also K1 users each of which is connected to one SBS. Users have no cache memory. The two populations of K0 and K1 reflect the very practical scenario where the SBSs offer a very high throughput over a limited coverage area (hotspots) and therefore there are some users outside the reach of the SBSs. In contrast, the users inside the hotspots can receive from the SBSs at much higher rate, which here for simplicity is considered [1]. The delivery phase contains three steps. In the first step, the MBS broadcasts packets to the connected SBSs and the K0 users. In the second step, we have a D2D caching scenario. More precisely, each SBS exchange coded packets among each other in function of their cached content and received packets from the MBS. In the third step, each SBS delivers packets to its connected users to let them recover the desired files. The goal is to find the tradeoff among the memory size, the MBS load (i.e., number of broadcasted bits by the MBS in the first step), and the load of the SBS inter-communication (i.e., number of total exchanged bits in the second step). We focus on the peak

1As a matter of fact, the same results are obtained by letting the capacity of these links finite, but sufficiently large with respect to the capacity of the broadcast links between the MBS and the SBSs and the K0 directly served users.
load, i.e., we consider the load under worst-case user demands. It is reasonable and practical to consider the MBS load and D2D load separately, because the transmissions over these two subnetworks typically take place in different frequency bands with very different signal bandwidths and operational costs (e.g., the MBS uses prime cellular frequencies, while the SBSs use a widely available mmWave band).

**Related Works:** The users in our model could be divided into two hierarchies, \( K_1 \) and \( K_0 \) users with and without access to SBS, respectively. Thus the proposed model is related to the coded caching systems with heterogeneous cache sizes. In [9]–[13], the authors let each user directly cache a subset of all bits in the library (i.e., uncoded cache placement) and designed different delivery schemes based on linear coding and random coding. MDS intra-file coded placement phase and a linear-programming based delivery was proposed in [14] for heterogeneous small cell networks. The authors in [15] proposed an optimal centralized caching scheme with two users and heterogeneous cache sizes. Specifically, they proposed an inter-file coded placement for the memory size pair \((M_1, M_2) = (N-1, 0)\) to improve the state-of-the-arts. Very recently, based on the strategy that the signals intended to serve users with small cache sizes can be used in decoding the cache contents of users with larger cache size, the authors in [16] proposed an MDS inter-file coded placement for the shared-link caching systems with three users and heterogeneous cache sizes. It was also extended to small memory size regime (total cache sizes of all users are less than \( N \)).

Since the throughput from each SBS to its connected users is infinite, the proposed model is also related to the shared-link caching systems with shared caches (or the shared-link caching systems with multiple requests), where there are \( H \) caches with size \( M \) and each user is connected to one cache. In [17], the authors used the MAN uncoded placement and proposed a multi-round delivery phase, which was proved to be optimal under the constraint that the placement is uncoded and without knowledge of network structure. A caching scheme based on coded cache placement was proposed in [18] for \( M \in [1/H, N/H] \).

**Contributions:** In this paper, we study the memory-loads tradeoff for the novel F-RAN architecture in Fig. 1. The main contributions are:

1) We propose a converse bound on this problem based on the submodularity of entropy, which tightens the converse bound if we directly use cut-set strategy in [1].

2) Observing that there are \( K_0 \) users without connection to any SBS and thus the MBS should broadcast the whole demanded files, we propose an inter-file coded placement with the MAN symmetric subfile division, such that the SBSs can leverage these broadcasted files.

To the best of our knowledge, it is the first inter-file coded placement for D2D caching problem. Opposite to the existing coded cache placement for the standard MAN coded caching problem, that can only lead a small gain in a limited memory size regime compared to the uncoded cache placement, in our model, the proposed inter-file coded cache placement can lead a significant gain in the whole memory size regime compared to the uncoded cache placement for the problem at hand. As a by-product, we can also use the proposed inter-file coded placement for shared-link caching systems with heterogeneous cache sizes, where there exist users without cache. It can be seen as the generalization of the inter-file coded placement in [15]. [16] for any number of users and any memory sizes. We emphasize that the results in this paper are independent works to the very recent paper [16].

3) In the first step of delivery, we use the shared-link caching scheme with shared-caches proposed in [17]. In the second step of delivery, instead of directly extending the shared-link caching scheme with shared-caches in [17] to D2D scenario, we propose a novel group-based D2D delivery scheme with shared-caches. We also prove that the proposed caching scheme is order optimal within a factor of 22 if each SBS is connected to the same number of users.

4) Observe that the MAN symmetric subfile division is independent on the network structure, we then propose a cache placement by combining the proposed inter-file coded placement and a novel subfile division method based on the network structure. We show that the proposed scheme is exactly optimal for some memory size regimes. As another by-product, for the shared-link caching problem with shared-caches, we also improve the scheme in [17] by this novel cache placement dependent on the network.

**II. System Model**

We use the following notation convention. Calligraphic symbols denote sets, bold symbols denote vectors, and sans-serif symbols denote system parameters. We use \(|\cdot|\) to
Definition 1. be successfully satisfied with delivery load pair be achievable if for the memory constraint the memory-sharing over these points. In the first class, we mainly consider two classes of corner points and take the objective is to determine \( R \). In this paper, we consider the first class; and if the shared-link broadcast transmission is ‘expensive’ we can focus on the second class.

We can also notice that when \( K_0 = 0 \) and there is no D2D link among SBSs, our model is equivalent to the shared-link caching model with shared-cache in [17] (or the shared-link caching model with multi-requests in [19]).

III. MAIN RESULTS

In this section, we first introduce the achieved region of the proposed caching scheme and its order optimality results. We then describe in details our proposed caching scheme.

Theorem 1 (Achievable Region of Proposed Scheme). For the proposed cache-aided F-RAN problem, the lower convex envelop of the following corner points is achievable,

\[
(M, R_b, R_d) = \left( \frac{t(N - K_0)}{H}, K_0 + \frac{\sum_{i=1}^{H-1} L_i}{t!}, 0 \right),
\]

\[
(M', R_b', R_d') = \left( \frac{t'(N - K_0)}{H}, K_0, \frac{1}{t'!} \sum_{S \subseteq [H]: |S| = t'+1} (g_1(S) + \frac{1}{t'} \max \{g_{t'+1}(S) - \max \{g_1(S), g_{t'}(S)\}\}) \right),
\]

for \( t \in [0 : H] \) and \( t' \in [H] \).

We also propose a converse bound based on submodularity of entropy, which will be proved in Appendix A.

Theorem 2. For the proposed cache-aided F-RAN problem, if a memory-loads couple \((M, R_b^*, R_d^*)\) is achievable, it must satisfy that

\[
R_b^* + R_d^* \geq K_0 + \max \{L_1 \left(1 - M/(N - K_0)\right), 0\}. \tag{3}
\]

We should notice that it is easy to derive \( R_b^* + R_d^* \geq K_0 + \max \{L_1 \left(1 - M/(N - K_0)\right), 0\} \) by the cut-set idea proposed in [1], which is looser than (3).

We then provide some order optimality results of the proposed scheme in Theorem 3.

Theorem 3 (Order Optimality). For the proposed cache-aided F-RAN problem where each SBS is connected to the same number of users, the caching scheme in Theorem 1 is order optimal within a factor of 22. More precisely, for any achievable couple \((M, R_b^*, R_d^*)\), the caching scheme in Theorem 1 can achieve the couple \((M, 11R_b^*, 2R_d^*)\).

Proof: In this case, we assume that \( L = L_1 = \cdots = L_H \).

Converse: We divide the library into two non-overlapping sets, \( N_1 = [N - K_0] \) and \( N_0 = [N - K_0 + 1, N] \). We consider the demand vectors where each user in \([K_1]\) demands a different file in \( N_1 \) and each user in \( U_0 \) demands a different file in \( N_0 \). Since \((M, R_b^*, R_d^*)\) is achievable, we have

\[
R_b^* + R_d^* \geq H(X_0, \{X_h : h \in [H]\})/B \\
\geq H(X_0, \{X_h : h \in [H]\}, \{F_i : i \in N_0\})/B + \epsilon \\
= H(X_0, \{X_h : h \in [H]\})/(F_i : i \in N_0)/B + K_0 + \epsilon. \tag{4}
\]

It was proved in [19] that for shared-link caching problem including N files and H users, where each user demands L files, the number of broadcasted bits is lower bounded by the lower convex envelop of \(11L_1^{H-1}/H+1\) for \( t = HM/N \in [0 : H] \). Hence,
we can also prove that $H(X_0, \{X_h : h \in [H]\})\{F_i : i \in N_0\}$ in (1) is also lower bounded by the lower convex envelop of $\frac{\mathcal{C}(h-t)}{H+1}$ for $t \in [0 : H]$.

Achievability: It can be seen that for the corner point in (1), we have $R_0 = K_0 + \frac{H(t+1-t')}{H+1}$. For the corner point in (2), we have $R_0' = \frac{L(t+1-t')^2}{(H+1)}$. Notice that $R_0'/(R_0 - K_0) = (t+1)/t$ when $t' = t \in [H]$. By memory-sharing of the above achieved corner points and comparing to the converse bound, we can prove Theorem 3.

A. Novel Inter-file Coded Cache Placement

To achieve the bound in Theorem 1, we propose a novel inter-file coded cache placement for $M = t(N - K_0)/H$ where $t \in [0 : N]$. We divide each file $F_i$ where $i \in [N]$ into $H$ non-overlapping and equal-length subfiles, $F_i = \{F_{i,W} : \forall W \subseteq [H], |W| = t\}$. Thus each subfile has $B/(h^t)$ bits. We define $F_W := \{F_{i,W} : i \in [N]\}$, for each $W \subseteq [H]$ where $|W| = t$. It can be seen that $F_W$ contains BN/($h^t$) bits. Then we let each SBS $h \in [H]$ cache $|F_W|/(N - K_0)/N$ random linear combinations of all bits in $F_W$ for each $W \subseteq [H]$ where $|W| = t$ and $h \in W$. Hence, it can be seen that each SBS totally caches $(H-1)B(N-K_0)/H = Bt(N-K_0)/H$ bits, satisfying the cache size constraint.

In the following, we introduce the delivery schemes to achieve the corner points in (1) and (2), respectively.

B. Delivery Scheme to Achieve (1)

In the delivery phase, we first satisfy the demands of the $K_0$ users and let each SBS $h \in [H]$ recover all bits in $F_W$ where $W \subseteq [H], |W| = t$ and $h \in W$. More precisely, we let the MBS broadcast $F_i$ where $i \in \bigcup\{d_k\}$ such that each user in $U_0$ can recover its desired file. If $\bigcup\{d_k\} < K_0$, we then let the MBS broadcast $\bigcup\{d_k\} - K_0$ files demanded by some users in $K_1$ and let SBSs forward the demanded files to these users. We denote the set of users in $K_1$ demanding these $\bigcup\{d_k\} - K_0$ files by $U_0$. Since we have satisfied the demands of the users in $U_0$, for each $h \in [H]$, we let $U'_0 = U_0 \setminus U$ and $U'_0 = |U'_0|$. In addition, each SBS $h \in [H]$ receives $K_0$ files (assuming that the union of all bits in these $K_0$ files is $F$). For each $W \subseteq [H]$ where $|W| = t$ and $h \in W$, SBS $h$ can recover $B(N - K_0)/H$ random linear combinations of $F_W \setminus \{F\}$ where $|F_W \setminus \{F\}| = B(N - K_0)/H$. Hence, SBS $h$ can recover all bits in this $F_W$.

In the following, we use the shared-link caching scheme with shared-caches in (17) to let each user in $K_1 \setminus U_0$ recover its desired file. Delivery contains $\bigcup\{d_k\} - K_0$ rounds. In each round $j$, we pick one non-picked user connected to each SBS $h \in [H]$ if it still remains non-picked users connected to SBS $h$. We assume that the set of picked users is $K_j$ and that the user in $K_j$ connected to SBS $h$ is $K_j(h)$. For each set $S \subseteq [H]$, we let the MBS broadcast

$$\bigoplus_{h \in S} F_{d_{K_j(h)}, S \setminus \{K_j(h)\}}.$$ 

It can be seen that in round $j$, each SBS $h$ can recover $F_{d_{K_j(h)}}$ and then forwards it to user $K_j(h)$.

The worst-case is that each user requests a different file and the worst-case load broadcasted by the MBS is $K_0 + \sum_{i=1}^{H-t} L_i^{(H-t')}$ as shown in (1).

Remark 1. We can extend the inter-file coded cache placement and the above delivery scheme to shared-link caching problem with heterogeneous cache sizes where there exist some users without cache. Assuming the library has $N$ files and there are totally $K$ users, of which do not have cache. For any caching scheme with uncoded cache placement which is symmetric across the $N$ files (e.g., the caching schemes in (2)-(13)), we divide each file $F_i$ into non-overlapping and equal-length subfiles, $F_i = \{F_{i,W} : \forall W \subseteq [K]\}$, where $F_{i,W}$ represents the bits exclusively cached by users in $W$. Due to the symmetry, we have $|F_{i,W}| = |F_{j,W}|$ for any $i, j \in [N]$. Due to the cache size constraint. We need $N \sum_{i=0}^{H-t} L_i^{(H-t')}$ M bits for each user $k \in [K]$. However, there are $K_0$ users without caches and thus the server should transmit the entire file demanded by them. So we can use the proposed inter-file coded cache placement. We let user $k$ store $M_k B(N - K_0)/N$ random linear combinations of all bits in $\{F_{i,W} : i \in [N], |W| \subseteq [K], k \in W\}$. In the delivery phase, the server broadcasts $K_0$ files which contains all the demanded files of the $K_0$ users. Hence, each user $k$ can recover all bits in $\{F_{i,W} : i \in [N], |W| \subseteq [K], k \in W\}$. We then use the corresponding delivery phase of the original caching scheme to let the remaining users recover their desired files. In conclusion, by this inter-file coded cache placement, we can achieve the same worst-cost load but with a lower memory size $M_k = M_k (N - K_0)/N$ compared to the need memory size $M_k$ of the original caching scheme with uncoded cache placement, for any user $k \in [K]$.

C. Delivery Scheme to Achieve (2)

As Section III-B, we first use the same way to satisfy the demands of the $K_0$ users and let each SBS $h \in [H]$ recover all bits in $F_W$ where $W \subseteq [H], |W| = t$ and $h \in W$. So it remains to satisfy the demand of each user in $U'_0$ for each $h \in [H]$. We propose a novel D2D caching scheme with shared-caches and use the following example to illustrate the main idea.

Example 1. We focus on the network structure illustrated in Fig. 1 with $H = 3$, $K_1 = 4$, $K_0 = 2$, $N = 6$ and $M = 8/3$. Hence, we have $t = HM/(N - K_0) = 2$. In the placement phase, we divide each file $F_i$ into $L_i^{(H-t)} = 3$ non-overlapping and equal-length subfiles, $F_i = \{F_{i,1,2}, F_{i,1,3}, F_{i,2,3}\}$ and each subfile has $B/3$ bits. We let each SBS $h \in [H]$ cache $B(N - K_0)/H = 4B/3$ random linear combinations of all bits in $F_{(h,h_1)}$, where $h_1 \in ([H] \setminus \{h\})$. It can be seen that each SBS caches $8B/3$ bits, satisfying the cache size constraint.

In the first step of delivery phase, we assume $d = (1 : 6)$. We let the MBS broadcast $F_{3}$ and $F_{6}$ such that users 5 and 6 can recover their desired files. Simultaneously, each SBS $h \in [H]$ can recover $F_{(h,h_1)}$, where $h_1 \in ([H] \setminus \{h\})$. 

In the second step, we have a D2D caching problem with shared-caches. We can directly extend the shared-link caching scheme with shared-caches in [17] to the D2D scenario. More precisely, the shared-link caching scheme in [17] have two rounds. In the first round, we can let the server/MBS serve the group of users 2, 3, and 4 by broadcasting \( F_{2,\{2,3\}} \oplus F_{3,\{3\}} \oplus F_{4,\{1,2\}} \). We can directly extend this sum in D2D scenario by splitting each subfile in the sum into two non-overlapping and equal-length pieces, \( F_{2,\{2,3\}} = \{ F_{2,\{2,3\}}, F_{2,\{2,3\}} \}, F_{3,\{3\}} = \{ F_{3,\{3\}}, F_{3,\{3\}} \} \) and \( F_{4,\{1,2\}} = \{ F_{4,\{1,2\}}, F_{4,\{1,2\}} \} \). We then let SBS 1 transmit \( F_{3,\{3\}} \oplus F_{4,\{1,2\}} \), SBS 2 transmit \( F_{2,\{2,3\}} \oplus F_{4,\{1,2\}} \), SBS 3 transmit \( F_{2,\{2,3\}} \oplus F_{3,\{3\}} \). In the second round, we let SBS 2 transmit \( F_{1,\{3\}} \) to satisfy the demand of user 1 in the second group. Hence, the total broadcast load in the second step is \( \frac{3}{6} + \frac{1}{3} = \frac{5}{6} \).

However, instead of directly extending the shared-link caching scheme with shared-caches, we can use another way to group users. We can move user 4 from the first group to the second one. To serve the new first group containing users 2 and 3, we let SBS 3 transmit \( F_{2,\{2,3\}} \oplus F_{3,\{3\}} \). To serve the new second group containing users 1 and 4, we let SBS 2 transmit \( F_{1,\{3\}} \oplus F_{4,\{1,2\}} \). Hence, the total broadcast load in the second step achieved by the new group method is \( \frac{1}{3} + \frac{1}{3} = \frac{2}{3} \), which is strictly less than the one achieved by direct extension.

Hence, in this example we can achieve the worst-case load \( (M, R_0, R_d) = (8/3, 2, 2/3) \). By the proposed converse bound in Theorem 2 we have when \( M = 8/3 \),

\[
R_0^* + R_d^* \geq 8/3 = R_0 + R_d. \tag{5}
\]

Hence, we prove the optimality of the proposed scheme, e.g., there does not exist a caching scheme achieving \( (8/3, R_0^*, R_d^*) \), where \( R_0^* < 2 \) and \( R_d^* = 2/3 \) nor \( R_0^* = 2 \) and \( R_d^* < 2/3 \). □

We are now ready to generalize the D2D caching scheme with shared-caches in the above example. To avoid heavy notation, with abuse of notation we assume \( L_i' \geq L_j' \) for any \( 1 \leq i < j \leq |H| \).

We focus on one set \( S \subseteq [H] \) where \(|S| = t + 1\), and we would like to let each user \( k \) in \( k \in U_h \) recover \( F_{d_k,S \setminus \{h\}} \), for each \( h \in [H] \). We divide all the users connected to SBSs in \( S \) into \( L_i' \) groups where \( i = \min_{j \in S} j \). In group 1, we pick one user connected to each SBS in \( S \). In group 2, we pick one of the non-picked user connected to each SBS in \( S \). We repeat it until group \( L_i' \). It can be seen that each of the first \( L_i' \) groups contains \( t + 1 \) users, where \( \frac{i + 1}{\max_{j \in S} j} \). It can also be seen that from group \( L_i' + 1 \) to group \( L_i' \) each group contains \( t \) users, where \( \frac{i}{\max_{j \in S \setminus \{i+1\}}} \). In addition, from group \( L_i' + 1 \) to group \( L_i' \), each group contains less than \( t \) users. If \( L_i' + 1 \leq L_i' + 1 \), from each group \( g \in [L_i' + 1] \), we move the user connected to SBS \( i_{t+1} \) to group \( L_i' + g \); otherwise, from each group \( g \in [L_i' - L_i'] \), we move the user connected to SBS \( i_{t+1} \) to group \( L_i' + g \). Hence, we finish the grouping procedure for the set \( S \). In the following, we generate transmitted packets for each group. Recall that the connected SBS of user \( k \) is \( h_k \). For group \( g \) of set \( S \), if it contains \( t + 1 \) users, for each user \( k \) in group \( g \), we divide each subfile \( F_{d_k,S \setminus \{h_k\}} \) into \( t \) non-overlapping and equal-length pieces,

\[
F_{d_k,S \setminus \{h_k\}} = \{ F_{d_k,S \setminus \{h_k\}, h} : h \in (S \setminus \{h_k\}) \}.
\]

We let each SBS \( h \in S \) transmit

\[
k \text{ is in group } g \quad \quad \oplus \quad \quad F_{d_k,S \setminus \{h_k\}, h}.
\]

If group \( g \) contains less than \( t + 1 \) users, we can choose one SBS \( h \in S \) which does not connected to any users in \( g \), and let SBS \( h \) transmit

\[
k \text{ is in group } g \quad \quad \oplus \quad \quad F_{d_k,S \setminus \{h_k\}}.
\]

After considering all the subsets of \( [H] \) with cardinality \( t + 1 \), each SBS can decode the files which its connected users demand and then forward the demanded files to its connected users. It can be seen that the worst case is when each user demands a different file and the achieved worse-case loads are in [2].

IV. FURTHER IMPROVEMENT

In the proposed inter-file coded cache placement, we first use the symmetry subfile division proposed by MAN in [1] such that for each \( t \)-subset of users, there exists a subfile of each file. We then encode subfiles by random linear combinations. In other words, this subfile division is without consideration of the network structure. In the following example, we will show that when one can design the placement as a function of the network topology, such symmetric placement can be outperformed by an asymmetric one where each file is divided in a topology-dependent way.

Example 2. We focus on the network illustrated in Fig. 1 with \( H = 3 \), \( K_1 = 4 \), \( K_0 = 2 \), \( N = 6 \) and \( M = 2 \). With \( R_d = 0 \), the proposed achievable scheme in Theorem 1 yields the memory-load triple \( (M, R_0, R_d) = (2,19/6,0) \). We improve it as follows.

**Placement Phase:** For each file \( F_i \), \( i \in [N] \), we divide it into two non-overlapping and equal-length subfiles, \( F_i = \{ F_{i,\{1\}}, F_{i,\{2,3\}} \} \). Each subfile has \( B/2 \) bits. Let SBS 1 cache MB = 2B random linear combinations of all bits in \( \{ F_{i,\{1\}} : i \in [N] \} \). We also let each of SBSs 2 and 3 cache MB = 2B random linear combinations of all bits in \( \{ F_{i,\{2,3\}} : i \in [N] \} \).

**Delivery Phase:** Assume the demand vector is \( d = (1 : 6) \). We first let the MBS broadcast \( F_1 \) and \( F_6 \) to satisfy the demands of users 5 and 6. Simultaneously, SBS 1 can recover all bits in \( \{ F_{i,\{1\}} : i \in [N] \} \). SBSs 2, 3 can recover all bits in \( \{ F_{i,\{2,3\}} : i \in [N] \} \). We then let the MBS broadcast \( F_{1,\{2,3\}} \oplus F_{3,\{1\}} \) to satisfy the demands of users 1 and 3, and broadcast \( F_{2,\{2,3\}} \oplus F_{4,\{1\}} \) to satisfy the demands of users 2 and 4. So we can achieve the memory-loads triple \( (2,3,0) \). By the converse bound in Theorem 2 we can also prove the optimality of this memory-load point.
We generalize the above example in the following theorem based on a novel subfile division with the consideration of the network structure. For each $G \in [H]$, we define a $G$-way partition $\Phi = \{G^b_1, \ldots, G^b_t\}$ (by the definition of partition we have $G^b_i \neq \emptyset$, $\cup_{i \in [G]} G^b_i = [H]$ and $G^b_i \cap G^b_j = \emptyset$ for all $i \neq j$), and such that groups are ordered such as $\sum_{h \in G^b_i} L_h \geq \sum_{h \in G^b_j} L_h \quad \text{for all } i \leq j.$ We then denote the set of all the $G$-way partitions by $Q_G$, for each $G \in [H]$.

**Theorem 4** (Improvement). For the proposed cache-aided F-RAN problem, the following memory-load points are achievable:

\[
(M, R_b, R_d) = \left(\frac{t(N - K_0)}{G}, \frac{\min_{\Phi \in Q_G} \sum_{r = t}^{G-t} \sum_{j \in G^b_r} \frac{L_j (G-r)}{G}}{t}, 0\right),
\]

for all $t \in [0 : G]$ and all $G \in [H].$

**Proof:**

**Placement Phase:** Consider $G$-way partition $\Phi$. For $M = \frac{t(N - K_0)}{G}$ where $t \in [0 : G]$, we divide each file $F_i$ where $i \in [N]$ into $\binom{G}{i}$ non-overlapping and equal-length subfiles, $F_i = \{F_{i,W} : W \subseteq [G], |W| = t\}$. Thus each subfile has $B/(\binom{G}{i})$ bits. Recall that $\mathcal{F}_W := \{F_{i,W} : i \in [N]\}$, for each $W \subseteq [G]$ where $|W| = t$. It can be seen that $\mathcal{F}_W$ contains $\binom{N}{W} \binom{G}{i}$ bits. We then let each SBS $h \in G^b_i$ cache the subfile $\mathcal{F}_{W}(\mathcal{N} - K_0)$ for random linear combinations of all bits in $\mathcal{F}_{W}$ for each $W \subseteq [G]$ where $|W| = t$ and $i \in W$. Hence, it can be seen that each SBS totally caches $\binom{N-K_0}{t} \binom{N}{W}$ bits, satisfying the cache size constraint.

**Delivery Phase:** It can be also seen that the SBSs in the same group have the same cache. Hence, it is equivalent to say that all users connected to the SBSs in one group share the same cache. Hence, we use the delivery phase described in Section II-B to achieve (6).

From the achievable scheme in Theorem 4 we can derive the following optimality result.

**Theorem 5** (Optimality). For the proposed cache-aided F-RAN problem, if there exists one $G$-way partition $\Phi$ such that $G^b_i = \{1\}$, the achievable couple $(M, R_b, R_d)$ is exactly optimal, where $M \geq (1 - 1/G)(N - K_0)$, $R_b = K_0 + L_1(1 - M/(N - K_0))$ and $R_d = 0$.

**Proof:**

Converse: The converse part follows directly from Theorem 2.

Achievability: When $G^b_i = \{1\}$ and $t = G - 1$, from (6), it can be proved that the following memory-loads couple is achievable

\[
\left(\frac{(G-1)(N - K_0)}{G}, \frac{\sum_{r = t}^{G-t} \sum_{j \in G^b_r} \frac{L_j (G-r)}{G}}{t}, 0\right) = ((1 - 1/G)(N - K_0), K_0 + L_1/(N - K_0))
\]

From memory-sharing between $M = \frac{(G-1)(N - K_0)}{G}$ and $M = N - K_0$, we can prove Theorem 5.

For any network, we can partition the SBSs into $G = H$ groups. In this case, the caching scheme is the same as the one achieved in (1). Hence, the corner points in Theorem 4 generally cover all the corner points in (1).

If in Theorem 4 we let $K_0 = 0$, it is equivalent to the shared-link caching problem with shared-caches. Since $K_0 = 0$, the proposed cache placement (as in Example 2) becomes uncoded. Furthermore, in some memory size regime, the caching scheme in Theorem 4 can be proved to be exactly optimal and strictly better than the shared-link caching scheme with shared-caches in (17), which was proved to be optimal under the constraint of uncoded cache placement and that the placement is independent on the network structure. Hence, by Example 2 and Theorem 4 we also show the sub-optimality of designing placement independent on the network for the shared-link caching problem with shared-caches.

Dividing users into groups and letting the users in the same group have the same cache was originally proposed in [20] for shared-link caching problem in order to reduce the sub-packetization level. But the the achieved load was increased compared to the MAN caching scheme. However, in this paper, we can see that for this asymmetric cache-aided network, this grouping strategy can reduce the sub-packetization level and also reduce strictly the achieved load.

We can also directly combine the proposed placement in Theorem 4 and the proposed D2D caching scheme with shared-caches in Section III-C To simplify this paper, we do not go into details.

V. NUMERICAL EVALUATIONS AND DISCUSSIONS

A. Numerical Evaluations

We give some numerical evaluations on the proposed results. In Fig. 2 we consider the network with $H = 4$, $K_1 = 16$, $K_0 = 4$, $N = 20$, $L_1 = 6$, $L_0 = 4$, $L_3 = 3$ and $L_4 = 3$. We fix $R_b = K_0 = 4$ and $R_d = 0$. In Fig. 3, we consider the network with $H = 4$, $K_1 = 22$, $K_0 = 8$, $N = 30$, $L_1 = 10$, $L_0 = 9$, $L_3 = 2$ and $L_4 = 1$. We fix $R_b = 0$ and plot the memory-load tradeoff $(M, R_d)$. To compare the proposed caching scheme, we also plot other caching schemes with “MAN Placement” (without inter-file coded placement) or with “Trivial D2D Delivery” (the D2D delivery scheme with shared-caches by trivially extending the shared-link delivery scheme with shared-caches in (17)). It can be seen the proposed caching scheme in Theorem 1 outperforms the other schemes.

In Fig. 3 we consider the network with $H = 4$, $K_1 = 22$, $K_0 = 8$, $N = 30$, $L_1 = 10$, $L_0 = 9$, $L_3 = 2$ and $L_4 = 1$. We fix $R_b = 0$ and plot the memory-load tradeoff $(M, R_d)$. We can see that the proposed inter-file coded placement with the novel subfile division in Theorem 4 outperforms the others. In addition, it can be also seen that when $M \geq 44/3$, the proposed scheme in Theorem 4 is optimal.

B. Conclusions and Further Discussions

In this paper, we studied a novel cache-aided F-RAN network and proposed a novel inter-file coded cache placement and a D2D caching scheme with shared-caches. Order
optimal within constant factor could be characterized when each SBS is connected to the same number of users. We also proposed a novel cache placement based on a novel subtree division dependent on the network, which is exactly optimal in some memory size regimes. In the extended version of this paper, we have further improvement and extensions:

1) Decentralized scenario The proposed inter-file coded placement is designed with the knowledge of $K_0$. However, in the decentralized systems, users may move and thus we may not know $K_0$ and $K_1$ during placement. In the extended version of this paper, we will see that the proposed inter-file coded placement is extensible to decentralized scenario if we know the distribution of $L_i$ where $i \in [N]$ and $K_0$. Similarly, the improved cache placement proposed in Section IV also depends on the network structure. In the decentralized scenario, if we know some SBSs would be connected to larger number of users and other SBSs would be connected to small number of users, we can also use the proposed cache placement in Section IV to group the SBSs.

2) $N < K$. In the extended version, we also consider the case where $N < K$, i.e., one file may be demanded by several users. We can combine the shared-link caching scheme with more users than files in [3] and the proposed caching schemes in this paper to leverage the multicast opportunities.

### Appendix A

**Proof of Theorem 2**

We divide the library into two non-overlapping sets, $\mathcal{N}_1 = [N - K_0]$ and $\mathcal{N}_0 = [N - K_0 + 1, N]$. In this proof, we consider the demand vectors where each user in $[K_1]$ demands a different file in $\mathcal{N}_1$ and each user in $\mathcal{N}_0$ demands a different file in $\mathcal{N}_0$. From [4], if $\left(M, R_0^*, R_1^*\right)$ is achievable, we have

$$R_0^* + R_1^* \geq H(X_0, \{X_h : h \in [H]\})/B + K_0 + \epsilon.$$  

(7)

We then focus on $H(X_0, \{X_h : h \in [H]\})/B$ for the simplicity, we denote $R = H(X_0, \{X_h : h \in [H]\})/B$ where $S = \{F_i : i \in N_0\}$.

To satisfy the demands of the users in $U_1$, we have

$$H(F_{d_1}, \ldots, F_{d_{k_1}} | Z_1, X_0, \{X_h : h \in [H]\}, S) \leq \epsilon_1$$  

(8a)

$$I(F_{d_1}, \ldots, F_{d_{k_1}}; X_0, \{X_h : h \in [H]\}|Z_1, S) + \epsilon_1$$  

(8b)

$$H(F_{d_1}, \ldots, F_{d_{k_1}} | Z_1, S) \leq \epsilon_1$$  

(8c)

$$H(F_{d_1}, \ldots, F_{d_{k_1}} | Z_1, S) \leq RB + \epsilon_1.$$  

(8d)

From (8d), we have

$$RB + \epsilon_1 \geq H(F_{d_1}, \ldots, F_{d_{k_1}} | Z_1, S)$$  

(9a)

$$= H(F_{d_1}, \ldots, F_{d_{k_1}} | S) - I(F_{d_1}, \ldots, F_{d_{k_1}}; Z_1 | S)$$  

(9b)

$$= L_1 B - I(F_{d_1}, \ldots, F_{d_{k_1}}; Z_1 | S).$$  

(9c)

Consider $\binom{N-K_0}{L_1}$ demand vectors where in each demand vector, the union of demanded files by the users in $U_1$ is different. Hence, among these $\binom{N-K_0}{L_1}$ demand vectors, each file in $[N - K_0]$ is demanded by users in $U_1$ for $\binom{N-K_0}{L_1-1}$ times. For each demand, we can write an inequality as (9c) and we sum all of these inequalities to obtain

$$(N-K_0) \binom{N-K_0}{L_1} (RB + \epsilon_1)$$  

$$\geq L_1 B \binom{N-K_0}{L_1} - \sum_{V \in [N-K_0]:|V|=L_1} I(\cup_{i \in V} F_i; Z_1 | S)$$  

(10a)

$$= L_1 B \binom{N-K_0}{L_1} - \sum_{V \in [N-K_0]:|V|=L_1} (H(Z_1 | S) - H(Z_1 | \cup_{i \in V} F_i, S))$$  

(10b)

$$= L_1 B \binom{N-K_0}{L_1} - \binom{N-K_0}{L_1} H(Z_1 | S) + \sum_{V \in [N-K_0]:|V|=L_1} H(Z_1 | \cup_{i \in V} F_i, S).$$  

(10c)

We then use the submodularity of entropy to lower bound

$$\sum_{V \in [N-K_0]:|V|=L_1} H(Z_1 | \cup_{i \in V} F_i, S).$$

More precisely, for two sets of independent random variables, $\mathcal{V}_1$ and $\mathcal{V}_2$, we have

$$H(X | \mathcal{V}_1) + H(X | \mathcal{V}_2) \geq H(X | \mathcal{V}_1 \cup \mathcal{V}_2) + H(X | \mathcal{V}_1 \cap \mathcal{V}_2).$$  

(11)
For \( \sum_{V \in [N-K_0]:|V|=L_1} H(Z_1 | \cup_{i \in V} F_i, S) \), we use the submodularity in \([11]\) multi-rounds to derive

\[
\sum_{V \in [N-K_0]:|V|=L_1} H(Z_1 | \cup_{i \in V} F_i, S) \geq \sum_{V \in [N-K_0]:|V|=L_1} aH(Z_1 | S) + bH(Z_1 | \cup_{i \in [N-K_0]} F_i, S). \tag{12}
\]

Notice that among all the sets \( V \subseteq [N-K_0] \) where \( |V| = L_1 \), each element \( j \in [N-K_0] \) appears totally \((N-K_0-1)\) times. Hence, in (12), we have \( b = \binom{N-K_0-1}{L_1-1} \) and \( a = \binom{N-K_0}{L_1} - b = \binom{N-K_0-1}{L_1-1} \). Furthermore, we have \( H(Z_1 | \cup_{i \in [N-K_0]} F_i, S) = 0 \). Hence, from (12), we have

\[
\sum_{V \in [N-K_0]:|V|=L_1} H(Z_1 | \cup_{i \in V} F_i, S) \geq \binom{N-K_0-1}{L_1} H(Z_1 | S). \tag{13}
\]

We take (13) into (10c) to obtain

\[
(L_1 + \varepsilon_1) \geq L_1 B \binom{N-K_0}{L_1} - L_1 H(Z_1 | S) - \binom{N-K_0-1}{L_1} H(Z_1 | S) \tag{14a}
\]

\[
= L_1 B \binom{N-K_0}{L_1} - \binom{N-K_0-1}{L_1-1} H(Z_1 | S) - \binom{N-K_0-1}{L_1} H(Z_1) \tag{14b}
\]

\[
\geq L_1 B \binom{N-K_0}{L_1} - \binom{N-K_0-1}{L_1-1} H(Z_1) \tag{14c}
\]

\[
= L_1 B \binom{N-K_0}{L_1} - \binom{N-K_0-1}{L_1} L_1 MB \frac{N-K_0}{N-K_0}. \tag{14d}
\]

From (14d), we get

\[
(L_1 + \varepsilon_1) \geq L_1 B - \frac{L_1 MB}{N-K_0}. \tag{15}
\]

We take (15) into (7) to obtain

\[
R_1^* + R_3^* \geq K_0 + \max \left\{ L_1 (1 - M/(N - K_0)), 0 \right\}, \tag{16}
\]

coinciding with Theorem 2.

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