A New Method to Measure Hubble Parameter $H(z)$ Using Fast Radio Bursts

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Abstract

The Hubble parameter $H(z)$ is directly related to the expansion of our universe. It can be used to study dark energy and constrain cosmology models. In this paper, we propose that $H(z)$ can be measured using fast radio bursts (FRBs) with redshift measurements. We use dispersion measures contributed by the intergalactic medium, which is related to $H(z)$, to measure the Hubble parameter. We find that 500 mocked FRBs with dispersion measures and redshift information can accurately measure Hubble parameters using Monte Carlo simulation. The maximum deviation of $H(z)$ from the standard ΛCDM model is about 6% at redshift $z = 2.4$. We also test our method using Monte Carlo simulation. A Kolmogorov–Smirnov (K-S) test is used to check the simulation. The $p$-value of the K-S test is 0.23, which confirms internal consistency of the simulation. In the future, more localizations of FRBs make it an attractive cosmological probe.

Unified Astronomy Thesaurus concepts: Cosmological parameters (339); Radio bursts (1339)

1. Introduction

Since the creative work of Edwin Powell Hubble in 1929 (Hubble 1929), the fact that our universe is evolving and under expansion has been established. Seven decades later, the accelerating expansion of our universe was discovered by measurements of Type Ia supernovae (SNe Ia; Riess et al. 1998; Perlmutter et al. 1999), which changed our understanding of the universe again. The new findings have encouraged people to investigate the mysterious component, which is called dark energy, several different ways.

The cosmic expansion rate, expressed in terms of Hubble parameter $H(z) = \dot{a}(t)/a(t)$ with scale factor $a(t)$, is a powerful cosmological probe (for reviews, see Zhang et al. 2010). In flat ΛCDM cosmology, $H(z)$ can be expressed as

$$H(z) = H_0 \sqrt{\Omega_m + \Omega_\Lambda (1 + z)^3},$$  

where $H_0$ is the Hubble constant, $\Omega_\Lambda$ is the vacuum energy density fraction, and $\Omega_m$ is the matter density fraction.

The cosmic expansion rate $H(z)$ is a powerful tool for studying cosmological parameters (Samushia & Ratra 2006; Farooq et al. 2017; Tu et al. 2019), cosmological deceleration to acceleration transition (Farooq & Ratra 2013; Jesus et al. 2018; Yu et al. 2018), the Hubble constant (Busti et al. 2014; Chen et al. 2017; Wang & Meng 2017; Yu et al. 2018), and cosmic curvature (Clarkson et al. 2007, 2008; Yu & Wang 2016). Several methods have been proposed to measure $H(z)$. The first one is the differential age method, which was first put forward by Jimenez & Loeb (2002). Some efforts have been performed (Stern et al. 2010; Liu et al. 2012; Moresco et al. 2012; Zhang et al. 2014; Ratsimbazafy et al. 2017). However, it is difficult to select galaxies that can act as “cosmic chronometers” and determine the age of a galaxy. This method is relying on population synthesis simulations based on standard physics and cosmology. The second one is a radial baryon acoustic oscillation size method (Blake et al. 2012; Font-Ribera et al. 2014; Delubac et al. 2015; Alam et al. 2017). However, the Hubble parameter degenerates with the comoving distance and the derived $H(z)$ depends on the assuming cosmological model in this method. More recently, Amendola & Quartin (2019) proposed that $H(z)$ can be derived by measuring the power spectrum of density contrast and peculiar velocities of supernovae. In this paper, we propose that $H(z)$ can be measured using fast radio bursts (FRBs) with redshift measurements.

FRBs are very bright and short bursts with high brightness temperatures (Lorimer et al. 2007; Thornton et al. 2013; Champion et al. 2016; Katz 2018; Cordes & Chatterjee 2019; Platts et al. 2019), which are considered to have a cosmological origin. One of the significant characteristics is that FRBs have a large dispersion measure (DM), which is proportional to the integral of free electron density along the line of sight from the source to the observer. DM can be used as a cosmological probe. At present, more than 100 FRBs have been detected. Twenty of them are repeaters, most of which were discovered by the Canadian Hydrogen Intensity Mapping Experiment (CHIME; CHIME/FRB Collaboration 2019a, 2019b; Fonseca et al. 2020). So far, five FRBs have been localized and two of them are repeating bursts. A direct localization of FRB 121102 by the Very Large Array at redshift $z = 0.19$ (Chatterjee et al. 2017; Tendulkar et al. 2017) confirmed the cosmological origin of this source. Recently, the repeating FRB 180916.J0158+65 is localized to a nearby spiral galaxy ($z = 0.0337$; Marcote et al. 2020). For one-off FRBs, FRB 180924 was located in a position 4 kpc from the center of an early-type spiral galaxy at a redshift of 0.32 (Bannister et al. 2019). FRB 190523 was located in a few-arcsecond region containing a massive galaxy at redshift 0.66 (Ravi et al. 2019). FRB 181112 was localized in a galaxy at redshift 0.47 (Prochaska et al. 2019). More and more FRBs with measured redshifts will be detected in the future based on the high rate of FRBs, which reaches $10^6$ sk$^{-1}$ day$^{-1}$ (Thornton et al. 2013). Meanwhile, the CHIME telescope with an effective field of view of about 250 deg$^2$ can detect FRBs with an unexpected rate. It would provide a large data sample for measuring the Hubble parameter. FRBs with redshift and DM measurements will be useful cosmological and astrophysical probes, including for measuring baryon number density (Deng & Zhang 2014;
measuring cosmic proper distance (Yu & Wang 2017) and the cosmological parameters (Gao et al. 2014; Zhou et al. 2014; Li et al. 2018; Walters et al. 2018; Jaroszynski 2019), probing compact dark matter (Munoz et al. 2016; Wang & Wang 2018), and testing Einstein’s weak equivalence principle (Wei et al. 2015; Yu & Wang 2018; Xing et al. 2019). Kumar & Linder (2019) gave a quantitative estimation for the systematics control needed for using FRB dispersion measures as a distance probe.

In this paper, we propose a new method to measure the Hubble parameter using FRBs with redshift and DM measurements. This paper is organized as follows. In Section 2, we give an introduction of the method. In Section 3, our method is tested using a simulated FRB sample. Conclusions and discussion will be given in Section 4.

2. Method for Measuring $H(z)$

The observed $DM_{obs}$ includes contributions from the intergalactic medium (IGM), the Milky Way, and the local environment (including the host galaxy and the source). It can be expressed as

$$DM_{obs} = DM_{IGM} + DM_{MW} + \frac{DM_{loc}}{1+z}.$$  

(2)

$DM_{IGM}$ is the only parameter that contains the information of the Hubble parameter in this equation. The mean dispersion measure caused by the inhomogeneous IGM is given by

$$\langle DM_{IGM} \rangle = A \Omega_b H_0 \int_0^z \frac{F(z)}{E(z')} dz'.$$  

(3)

where $E(z) = H(z) / H_0$, $F(z) = (1 + z)f_{IGM}(z)f_e(z)$, and $A = 3c/8\pi Gm_p$. $\Omega_b$ is the cosmic baryon mass density fraction, $m_p$ is the mass of proton, and $f_{IGM}$ is the fraction of baryon mass in the IGM. $f_e = Y_H X_eH(z) + \frac{1}{2} Y_{He} X_eHe(z)$. $Y_H = 3/4$ and $Y_{He} = 1/4$ are the mass fractions of hydrogen and helium, respectively. $X_eH$ and $X_eHe$ are the ionization fractions of intergalactic hydrogen and helium, respectively. At $z < 3$, hydrogen and helium are fully ionized. So $X_eH = X_eHe = 1$. The values of $f_{IGM}$ are 0.82 and 0.9 at $z < 0.4$ and $z > 1.5$, respectively (Shull et al. 2012). In the redshift range $0.4 < z < 1.5$, $f_{IGM}$ may change linearly at 0.4 $< z < 1.5$ with a random deviation of 0.04 (Zhou et al. 2014).

We assume that the data are divided into several redshift bins and the averaged redshifts ($\bar{z}$) and dispersion measures ($\langle DM_{IGM} \rangle$) are known for each of them. Differentiating Equation (3), the Hubble parameter can be expressed as

$$H(z) = A \Omega_b H_0 F(z) \frac{\Delta \bar{z}}{\Delta DM_{IGM}},$$  

(4)

where $\Delta \bar{z}$ and $\Delta DM_{IGM}$ are the differences of $\bar{z}$ and $DM_{IGM}$ between two adjacent bins, respectively. The reciprocal of $\Delta \bar{z} / \Delta DM_{IGM}$ represents the first derivative of $DM_{IGM}$ with respect to redshift $z$. Only $DM_{IGM}$ depends on the Hubble parameter in Equation (2). Other components must be subtracted from $DM_{obs}$. For $DM_{loc}$, which contains the $DM_{host}$ and $DM_{source}$, one method has been proposed to determine it using low-$z$ FRBs (Yang & Zhang 2016). Zhang et al. (2019) studied the $DM_{host}$ using the IllustrisTNG simulation and found the value of $DM_{host}$ is almost independent of redshift for non-repeating FRBs at $z < 1.5$. $DM_{source}$ depends on the progenitors of FRBs. If FRBs are born in binary neutron star systems (Wang et al. 2016, 2020; Zhang 2020), the value of $DM_{source}$ is small. $DM_{loc}$ is divided by a $(1 + z)$ factor due to the time delay. Here $DM_{IGM}$ increases with redshift, so $DM_{loc}$ is not important at high redshifts. $\langle DM_{loc} \rangle = 200$ pc cm$^{-3}$ is assumed (Yu & Wang 2017). We subtract $DM_{loc}$ and leave its uncertainty into the total uncertainty $\sigma_{total}$. The total uncertainty $\sigma_{total}$ is

$$\sigma_{total}^2 = \sigma_{obs}^2 + \sigma_{MW}^2 + \sigma_{IGM}^2 + \left( \frac{\sigma_{loc}}{1+z} \right)^2.$$  

(5)

Since the measurements of DM are accurate, the uncertainties of $DM_{obs}$ and $DM_{WM}$ can be omitted compared with the much larger uncertainties of $DM_{loc}$ and $DM_{IGM}$. Following Thornton et al. (2013) and the numerical simulations of Quinn (2014), we choose $\sigma_{DM_{loc}} = 100$ pc cm$^{-3}$ in the analysis. Due to the inhomogeneity of IGM, the uncertainty of $DM_{IGM}$ is related to redshift, which can be expressed as (Kumar & Linder 2019)

$$\frac{\sigma_{DM_{IGM}}}{DM_{IGM}} = 20\% \sqrt{\frac{z}{1+z}}.$$  

(6)

Fortunately, if there are several FRBs from different sight lines but in a narrow redshift bin, uncertainty of the averaged $DM_{IGM}$ decreases by the square root of the number of FRBs in this bin.

The $DM_{MW}$ can be subtracted from pulsar observations (Taylor & Cordes 1993; Manchester et al. 2005). Some models have been proposed to describe the distribution $DM_{WM}$ (Cordes & Lazio 2002; Yao et al. 2017). Therefore, the value $DM_{E} = DM_{obs} - DM_{MW}$ is

$$DM_{E} = DM_{IGM} + \frac{DM_{loc}}{1+z} = A \Omega_b H_0 \int_0^z \frac{F(z)}{E(z')} dz' + \frac{DM_{loc}}{1+z}.$$  

(7)

According to the results of Zhang et al. (2019), we assume that the value of $DM_{loc}$ does not evolve with redshift significantly. The uncertainty of $DM_{E}$ is

$$\Delta DM_{E} = \Delta DM_{IGM} - \frac{DM_{loc} \Delta \bar{z}}{(1+z)^2}.$$  

(8)

Thus, we can calculate $\Delta \bar{z}$ from

$$\Delta \bar{z} = \frac{1}{\Delta DM_{IGM} + \frac{DM_{loc}}{(1+z)^2}}.$$  

(9)

The effect of $DM_{loc}$ is not important at high redshifts. From Equation (3), the uncertainty of $DM_{IGM}$ is

$$\Delta DM_{IGM} = \Delta DM_{E} + \frac{DM_{loc} \Delta \bar{z}}{(1+z)^2} = A \Omega_b H_0 F(z) \frac{\Delta \bar{z}}{E(z)}.$$  

(10)

Then we can derive the error of $H(z)$ as

$$\left( \frac{\sigma_{H(z)}}{H(z)} \right)^2 = \left( \frac{\sigma_{\Omega_b H_0}}{\Omega_b H_0} \right)^2 + \left( \frac{\sigma_{F(z)}}{F(z)} \right)^2 + \left( \frac{\sigma_{\Delta \bar{z}}}{\Delta DM_{IGM}} \right)^2.$$  

(11)

Here we assume that the uncertainty of $F(z)$ is 0.04, of $f_{IGM}$ is 0.04 (Zhou et al. 2014), and of $\sigma_{\Omega_b H_0}$ is 0.01.
Assuming that an FRB data set \((z, \text{DM}_{\text{obs}})\) is available, we use the following steps to derive the Hubble parameter \(H(z)\).

1. Deriving the data set \((z, \text{DM}_E)\) by subtracting \(\Delta \text{DM}_{\text{MW}}\).
2. Separating the data set \((z, \text{DM}_E)\) into five redshift bins, and then calculating the averaged redshifts, the averaged \(\text{DM}_E\) and the uncertainty of \(\text{DM}_E\). Then we obtain a data set \((z, \langle \text{DM}_E \rangle\), and \(\sigma_{\text{DM}_E}\).
3. Deriving \(\frac{\Delta z}{\Delta \text{DM}_E}\), then \(\frac{\Delta z}{\Delta \text{DM}_{\text{MW}}(z)}\) can be calculated using Equation (9). Equation (4) can be used to calculate the Hubble parameter \(H(z)\). The uncertainty of \(H(z)\) is derived from Equation (11).

### 3. Monte Carlo Simulations and Results

Monte Carlo simulation is used to test the efficiency of our method. Monte Carlo simulation is the easiest way to estimate the uncertainty of measured \(H(z)\) and its dependence on the number of FRBs. Flat ΛCDM cosmology with parameters \(\Omega_b = 0.0493\), \(\Omega_m = 0.308\), \(\Omega_{\Lambda} = 1 - \Omega_m\), and \(H_0 = 67.8\ \text{km s}^{-1}\text{Mpc}^{-1}\) is assumed (Planck Collaboration et al. 2016). The redshift distribution of FRBs is assumed as \(f(z) \propto z e^{-z}\) in the redshift range \(0 < z < 3\) (Yu & Wang 2017).

\(H(z)\) is derived as follows. (i) Simulating a data set \((z, \text{DM}_E, \sigma_{\text{DM}_E})\) using Equations (5) and (7). (ii) Then we calculate the Hubble parameter \(H(z)\) and \(\sigma_{H(z)}\) using the above method. (iii) Last, we test our method using the Kolmogorov–Smirnov (K-S) test. The left panel of Figure 1 shows the 500 mock DMs data using Monte Carlo simulation and the theoretical DM function. The right panel gives the five binned average \(\text{DM}_E\) with an error of about 20 pc cm\(^{-3}\). The average redshift \(z\) and \(\langle \text{DM}_E \rangle\) can be obtained. Then, the Hubble parameter \(H(z)\) is derived using Equations (4) and (9). In Figure 2, we give the confidence regions of \(H(z)\) for different redshifts. The best-fit values of \(H(z)\) with 1σ errors are \(H(0.65) = 97.89^{+5.87}_{-5.37}\ \text{km s}^{-1}\text{Mpc}^{-1}\), \(H(1.21) = 140.46^{+2.57}_{-2.37}\ \text{km s}^{-1}\text{Mpc}^{-1}\), \(H(1.79) = 183.14^{+5.90}_{-5.61}\ \text{km s}^{-1}\text{Mpc}^{-1}\), and \(H(2.37) = 225.88^{+12.45}_{-12.34}\ \text{km s}^{-1}\text{Mpc}^{-1}\). Figure 3 presents the derived Hubble parameter \(H(z)\) with 1σ errors and the theoretical \(H(z)\) function. The derived \(H(z)\) is consistent with the theoretical \(H(z)\). The maximum derivation is about 6% at redshift \(z = 2.4\). The maximum error of \(H(z)\) is \(\sigma_{H(z)} \approx 0.06\) using 500 FRBs, which means the measured Hubble parameter is reliable.

To check internal consistency of the simulation, the K-S test is used. The \(p\)-value of the K-S test is considered as the likelihood. In probability theory, if \(n\)-independent random variables all obey the standard normal distribution, the sum of the squares of the these variables obey the chi-squared \((\chi^2)\) distribution. In order to test the deviation between the derived \(H(z)\) and the theoretical \(H_0\), we construct a variable, i.e., \(\Delta H(z) = H(z) - H(z)_0\), which obeys normal distribution. Therefore, we compare \(\sum_{i=1}^{4} \Delta H_i^2(z)\) with the chi-squared distribution. First, we simulate 1000 times and derive a data set of \(H(z)\). Then, the probability density of \(\sum_{i=1}^{4} \Delta H_i^2(z)\) can be obtained from

\[
\Delta \chi^2 = \delta p C^{-1} \delta p^T,
\]

where \(C\) is the covariance matrix of the Hubble parameter \(H(z)\), and \(\delta p\) is the matrix of difference between the theoretical and simulated values. Last, we compare it with the chi-squared distribution, which has the same degree of freedom. The blue histogram of Figure 4 shows the probability density of \(\sum_{i=1}^{4} \Delta H_i^2(z)\) from 1000 simulations and the red line is the probability density of the chi-squared \((\chi^2)\) distribution with four degrees of freedom. The \(p\)-value of K-S test is 0.229, which is larger than 0.05. The two distributions are consistent and the simulated value and the theoretical value are drawn from the same distribution.

Below, the uncertainty of \(H(z)\) derived from our method is discussed. According to Equation (11), the parameter \(\Omega_b H_0^2\) will cause uncertainty of the derived \(H(z)\). The value of the Hubble constant \(H_0\) based on Cepheids and SNe Ia is different as compared to the result based on the cosmic microwave background (CMB) observations. Riess et al. (2019) derived the best estimation of \(H_0 = 74.03 \pm 1.42\ \text{km s}^{-1}\text{Mpc}^{-1}\) using Cepheid variables, masers in NGC 4258, and Milky Way parallaxes. In addition, Freedman et al. (2019) found \(H_0 = 69.8 \pm 2.5\ \text{km s}^{-1}\text{Mpc}^{-1}\) based on a calibration of the tip of the red giant branch applied to SNe Ia. However, \(H_0 = 67.8 \pm 0.9\ \text{km s}^{-1}\text{Mpc}^{-1}\) is derived from the CMB (Planck Collaboration et al. 2016). The difference between them is larger than 4σ. Here we use the best constraint on \(\Omega_b H_0^2\) from Planck CMB data.
Furthermore, when using FRB as a cosmological probe, systematic uncertainty cannot be ignored. The dispersion measure induced in the local vicinity directly by the source may depend the location of the FRB and the properties of its host galaxy. Here we assume that $s = -100 \text{ pc cm}^{-3}$ and $s = z \text{DM}_I \text{IGM}$ (Kumar & Linder 2019). Due to the selection effect of FRB observations, the DM values observed from different lines of sight may be different. Therefore, in our simulation, we average the DM$_{\text{IGM}}$ in a small redshift bin.

4. Conclusions and Discussion

Fast radio bursts are mysterious astrophysical phenomena that may provide a new distance probe. In this paper, we propose a new method to measure the Hubble parameter using DM$_{\text{IGM}}$ of FRBs. The Hubble parameter $H(z)$ is a key parameter for revealing the nature of cosmic expansion and dark energy. Through Monte Carlo simulations, we used 500 FRBs to measure $H(z)$. The K-S test confirms that Monte Carlo simulation is valid to estimate the uncertainty of measured $H(z)$. We find that the deviation between

Figure 2. The probability distributions of derived Hubble parameters at redshifts $z = 0.65, 1.21, 1.79,$ and $2.37$ from 500 simulated FRBs. Contours represent the $1\sigma$, $2\sigma$, and $3\sigma$ confidence levels.

Figure 3. $H(z)$ and $1\sigma$ errors derived from 500 mock FRBs are shown as blue dots. The red line represents the theoretical $H(z)$ function. The deviation between the simulated value of $H(z)$ and the theoretical one is about 6% at $z = 2.4$.

Figure 4. The distribution of the constructed $\sum_{i=1}^4 \Delta H_i^2(z)$ from 1000 simulations (blue histogram) and the probability density function of the chi-squared ($\chi^2$) distribution with four degrees of freedom (red line). The $p$-value of the K-S test is 0.229, which supports the two samples being drawn from the same distribution.
the simulated value of $H(z)$ and the theoretical one is small, and the error is only 0.06. However, the observed DM contains several contributions. The non-cosmological contributions to DM and their possible variations with direction and redshift must be extensively investigated.

Thanks to the high rate of FRBs (Thornton et al. 2013), a large FRB sample with redshifts can be built in the future. There is an increasing number of observation projects that conduct FRB observations, such as the CHIME (CHIME/FRB Collaboration 2019a), the Australian Square Kilometre Array Pathfinder (ASKAP; Shannon et al. 2018), and the Five-hundred-meter Aperture Spherical Telescope (FAST), which discovered the largest sample of FRB 121102 (Li et al. 2019). However, the determination of redshifts for FRBs is still a challenge due to the limitation of observation technology. In our method to measure $H(z)$ using FRBs, a large catalog of localized FRBs needs to be built up. The Square Kilometre Array (SKA) can detect FRBs at a rate of $10^3 \text{sky}^{-1} \text{day}^{-1}$ out to a redshift of about 3 (Fialkov & Loeb 2017). If 5% of the detected FRBs can be localized, the redshifts of about 10 FRB host galaxies can be measured per night for a mid-to-large-sized optical telescope (Walters et al. 2018). This indicates that a large catalog of localized FRBs could be built up. With a large number of observed FRBs, a reliable measurement of the Hubble parameter dependence on the redshift will become possible and will serve as a powerful cosmological probe.

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