A Classical Treatment of the Problems of Dark Energy, Dark Matter, and Accelerating Expansion

N K Spyrou
Astronomy Department, Aristoteleion University of Thessaloniki, 541.24 Thessaloniki, Macedonia, Hellas (Greece)
E-mail address: spyrou@helios.astro.auth.gr

Abstract. The dynamical equivalence with geodesic motions of the isentropic hydrodynamical flows, in the interior of a bounded gravitating perfect-fluid source, implies that the baryonic mass of a cosmological or astrophysical structure can be larger than its observationally determined mass. This can provide a classical explanation of the problems of the missing mass and of the flat rotation curves of disc galaxies.

The generalized mass density producing the above geodesic motions can be either positive, or negative, or even vanish, implying the possibility of a spatially increasing, or decreasing, or even vanishing acceleration, depending on the distance from the center of the source as compared to the inversion distance. The extra ingredient to the generalized mass density, stemming from the source’s internal physical characteristics, results in an extra, negative mass, beyond the baryonic mass of the source, which can be larger than the baryonic mass.

The above negative extra mass, specifically in the case of a supercluster of galaxies and in accordance with the WMAP observational data, can give a clear and precise physical meaning to the notions of dark energy, dark matter, and baryonic mass of a supercluster, which amount to, approximately, 77%, 18%, and 5%, respectively, of the supercluster’s total mass. This interpretation of the WMAP data is in accordance with a hot and extended supercluster of galaxies, as the manifestation of the cosmic-structure building and as a typical picture of the Universe, contributes to the clarification of the composition of the Cosmos, and excludes the existence of third (and higher)-order clusters of galaxies.

Treating the cosmological constant in the general-relativistic field equations as a source, in the form of a perfect fluid, the A-fluid, the cosmological constant does not prove to be really a constant, provided that the hydrodynamic flows in the A-fluid are not isentropic. This dictates the possibility of defining classically and for every structure its cosmological function. The cosmological function proves to share the numerical value and the basic properties of the cosmological constant, giving to the latter a clear physical meaning. Thus, the above unified scheme can also provide answers to problems, like repulsive gravity, accelerating/decelerating expansion of the Universe, stop-and-go Universe, Big Rip, and, finally, it predicts the existence of non-cosmological run-away motions at the inversion distance and beyond it.

Extended Summary
For a general-relativistic, bounded perfect-fluid source, the conservation of rest mass is a direct consequence of the adiabaticity of the hydrodynamical flows in the source and vice-versa, and the

---

1 This is an extended summary of the author’s contribution (key-note lecture) to the meeting Recent Developments in Gravity (2-6 June 2004, Mytilini, Lesvos, Hellas (Greece)).
isentropic flows in the source are dynamically equivalent to the geodesic motions in a conformal, fully defined virtual perfect-fluid source.

Specifically, in the Newtonian theory of gravity, the isentropic flows are dynamically equivalent to the geodesic motions in a generalized scalar potential, explicitly expressed in terms of the original source’s standard gravitational potential, rest-mass (baryonic-mass) density, isotropic pressure, and internal thermodynamic energy. This and the fact that usually the masses of the cosmological structures are determined observationally with the aid of geodesic motions, imply that the baryonic mass of a cosmological structure can largely exceed its observationally determined gravitationally active mass. The extra mass-density and mass, beyond the baryonic mass, involved depend on the structure’s internal physical characteristics, are both negative, and provide a natural classical (namely, non quantum) explanation of the problems of the missing mass, and of the flat-rotation curves of disk galaxies.

Treating the cosmological constant, \( \Lambda \), in the general-relativistic Einstein field equations as a source, in the form of an additional perfect-fluid source, the so-called \( \Lambda \)-fluid, \( \Lambda \) is practically this fluid’s energy density (equal to the opposite of the corresponding isotropic pressure), and the constancy of \( \Lambda \) is equivalent to the isentropicity of the hydrodynamical flows of the \( \Lambda \)-fluid. Hence, if the latter are not assumed adiabatic, then \( \Lambda \) is a function of the space-time coordinates rather than a constant.

If the \( \Lambda \)-fluid obeys an “isothermal” equation of state of the form obeyed by e.g. photons, neutrinos, gravitational waves or relativistic matter, then the \( \Lambda \)-fluid’s rest-mass density, energy density (and, hence, pressure, and \( \Lambda \)), and internal thermodynamic energy are all constant, properties, which, remarkably enough, characterize the quantum vacuum also.

In trying to describe semi-classically the cosmological constant, we use all the above as a guide and apply them in the context of the Newtonian theory of gravity. We suggest identifying the \( \Lambda \)-source’s rest-mass density with the so-called internal mass-density of the original fluid, which is negative. This enables us to define explicitly, as a function of the original fluid’s internal physical characteristics, the so-called cosmological function and the corresponding dark-energy density, for each of the Universe’s constituent-cosmological structures, and for the universe as a whole. Furthermore, in a unified way, it is proved that, for every cosmological structure, the cosmological function, namely, practically, the negative of the internal-mass density, can be used in defining the so-called inversion distance. This is the limiting distance from the center of the (spherically-symmetric) structure, beyond which the repulsive gravity dominates over the inwards attractive gravity and results in the outwards accelerated expansion, and inside which the inwards gravitational attraction prevails over the outwards accelerated expansion, and is, eventually, halted by a kind of pressure resulting in an equilibrium structure. The inversion distance, therefore, depending on the structure’s temperature and chemical composition, determines also the structure’s conventional (observable) linear dimensions.

Under quite general conditions, numerical applications show that, for all types of cosmological structures, the cosmological function and internal-mass (or dark-energy) density share the properties of the cosmological constant, and also describe the relative importance of the physical characteristics of the original fluid and of the \( \Lambda \)-source. Moreover the inversion distance (depending on the structure’s absolute temperature and chemical composition) approximates satisfactorily the conventional linear dimensions of realistic cosmological and astrophysical structures.

Furthermore, it is verified that, in the above unified framework and in the case of a supercluster of galaxies, the WMAP observational results for the composition of the Cosmos are consistently explained. To this end, the dark energy (which, according to the WMAP data, produces repulsive gravity) is identified with the internal mass in the region beyond the inversion distance (of dominance of the repulsive gravity). Moreover we identify the relative abundance of the above internal mass with the relative abundance of the dark energy, \( \sim 73\% \), as provided by the WMAP data. This and the assumption that the inversion distance is approximately equal to the conventional dimensions of a supercluster of galaxies (\( \sim 100 \text{ Mpc} \)) results in a hot and extended supercluster of matter temperature \( \sim 1.476 \times 10^4 \text{ K} \), and linear dimensions \( \sim 2.102 \times 10^{10} \text{ ly} \). So the cosmological importance of the hot fluid becomes evident. Then we evaluate the total mass of the supercluster \( \sim 7.137 \times 10^{14} \text{ m}_\odot \), consisting of
internal mass, namely, dark energy (~76.61%) and baryonic mass, namely, rest mass (~23.39%), in relative proportion ~3.3/1. Moreover, most (~82.52%) of the baryonic mass lies inside the inversion distance, and the rest (~17.48%) beyond it. Also, most (~98.45%) of the internal mass (dark energy) lies beyond the inversion distance, and the remaining part (~1.55%) inside it.

As it is known, the baryonic mass consists of the luminous baryonic matter and (the non-luminous baryonic matter, namely) the dark matter. The luminous baryonic matter (~5% of the total mass according to the WMAP data) is found to amount only ~25.91% of the rest (baryonic) mass up to the inversion distance. Also, interestingly, the dark matter (~18.38% of the total mass of the supercluster, in close agreement with the value ~22% provided by the WMAP data) consists of two parts, namely, on the one hand the remaining part of the rest (baryonic) mass lying inside the inversion distance, and on the other hand the baryonic mass lying beyond the inversion distance. The fact that only a minor part of the total baryonic mass contributes to the luminous baryonic matter, while the major part contributes to the dark matter, can explain the darkness (faintness) of the dark matter. Actually, the baryonic mass of the supercluster contributing to the dark matter occupies a three-dimensional volume ~10^5 times larger than the corresponding volume occupied by the baryonic mass (of the same temperature and chemical composition) contributing to the luminous baryonic matter. Similar arguments can explain the darkness of the dark energy (internal mass).

From all the above it is proved that the WMAP data are in accordance with a hot (as it is also indicated by studies of thermally virialized structures) and very extended supercluster of galaxies, resembling the typical picture of the whole cosmological Universe and thus, most probably, excluding the existence of third (and higher)-order clusters of galaxies in the Universe.

Knowing the values of the above physical parameters for a supercluster of galaxies, as described so far, we determine explicitly the properly defined mean value of the cosmological function, and conclude that its numerical value coincides with the generally accepted value of the cosmological constant for the Universe as a whole, namely, \(1.536 \times 10^{-55}\) cm\(^{-2}\). Moreover, using the notion of the inversion distance, we propose an explanation of the recent HST observations of distant Type 1a supernovae, implying that the expansion of the Universe was decelerating about 6.3 billion years ago, at \(z=0.46\), and then it switched to an accelerating expansion (stop-and-go Universe). Based on stellar and galactic-evolution considerations, as well as on black-hole-evaporation considerations, we verify that the cosmological function is a decreasing function of the time, and, based on this, we propose that the so-called Big-Rip phenomenon is not likely to happen. Furthermore, we prove that the parabolic (escape) velocity of the run-away motions at the inversion distance is a function of the structure’s gravitational field, absolute temperature and chemical composition, and, for a typical supercluster of galaxies, it is of the order of a few hundreds kilometers per second, approximately a few percent of the Hubble expansion velocity at the inversion distance. Interestingly, out of the three terms of the parabolic-velocity expression, the temperature-dependent term becomes the dominant one at distances larger than the inversion distance. Finally, we propose the observational verification of a new phenomenon, namely, to determine the parabolic (escape) velocity of the run-away motions external to the inversion distance of cosmological structure.

Acknowledgements
It is a pleasure to thank the organizers of the meeting and especially Dr. Yiannis Myritzis, for the warm hospitality and for his efforts towards a really interesting and successful meeting.

APPENDIX

1. Introduction and Outlook
In this Appendix and in the case of a general-relativistic bounded, gravitating perfect-fluid source, we describe the mutual dependence of the isentropicity of the flow motions and of the conservation of rest mass (§ 2), and also we examine the conditions, under with the so-called cosmological constant in the Einstein field equations can be treated as an independent, spatially and temporarily varying perfect fluid source (§ 3).
Further detailed calculations on the research results presented here will be published elsewhere (Spyrou, under preparation). Various aspects of the conformal dynamical equivalence, with applications, both classical and relativistic, at the astrophysical, galactic, and cosmological levels can be found in the References below.

2. Isentropicity versus Baryon Conservation in General Relativity

It is known that, in general relativity, the field equations for a gravitating perfect-fluid source

\[ \mathcal{G}_{ik} = -\frac{8\pi G}{c^4} T_{ik} \]  

(2.1)

are supplied by the equations of motion

\[ T^{ik}_{\ ,k} = 0 \]  

(2.2)

In Eqs (2.1) and (2.2), G is the gravitational constant and c the velocity of light in vacuum and, in the standard notation, \( \mathcal{G}_{ik} \) is the Einstein tensor,

\[ T_{ik} = (\varepsilon + p) u_i u_k - p g_{ik} \]  

(2.3)

is the fluid’s energy-momentum tensor, where \( g_{ik} \) are the components of the metric tensor, \( u_i \) the covariant components of the four velocity, a semi-colon denotes covariant derivative, and the mass-energy density \( \varepsilon \) is assumed to split as

\[ \varepsilon = \rho c^2 + \rho \Pi \]  

(2.4)

with the proper quantities \( \rho \), \( p \) and \( \rho \Pi \) being, respectively, the rest-mass density, proper isotropic pressure, and internal specific-energy density of the fluid source, the latter being the thermodynamic energy density that changes during the expansions or contractions of the fluid.

A direct consequence of Eqs (2.2) and (2.4) is

\[ \varepsilon u^i + (\varepsilon + p) u_j^i = 0 \]  

(2.5a)

or, equivalently,

\[
\left[ 1 + \frac{1}{c^2} \left( \Pi + \frac{p}{\rho} \right) \right] \left( \rho u^i \right)_{,i} + \\
\quad + \frac{1}{2} \left[ \Pi, j + p \left( \frac{1}{\rho} \right)_{,j} \right] \left( \rho u^i \right) = 0
\]  

(2.5b)

where a comma denotes usual partial derivative with respect to the spacetime coordinates.

Therefore, in general relativity, the conservation of rest mass (baryon number conservation) required by the equation

\[ (\rho u^i)_{,i} = 0 \]  

(2.6)

is compatible with the equations of motion if, and only if,

\[
\left[ \Pi, j + p \left( \frac{1}{\rho} \right)_{,j} \right] u^i = 0
\]  

(2.7)
This last equation is obviously satisfied under the assumption that the fluid’s hydrodynamical flows are adiabatic, in other words, the net change, \(dQ\), of the thermal content of a fluid’s volume element, defined through the first classical axiom of thermodynamics

\[
dQ = d\Pi + pd\left(\frac{1}{\rho}\right)
\]  

(2.8)

is always zero, namely,

\[
dQ = Q_u = d\Pi + pd\left(\frac{1}{\rho}\right) = 0
\]  

(2.9)

which is exactly Eq. (2.7)

Proceeding further to isentropic flows, it might be helpful to recall here that in reaching the general result (2.9) we can also the equilibrium hydrodynamics hypothesis, in the form of the constancy of the entropy, \(S\), along the flow lines namely, in view of the second classical thermodynamic axiom,

\[
dQ = TdS
\]  

(2.10)

\(T\) being the fluid distribution’s absolute temperature,

\[
0 = dS = S_u
\]

\[
= \frac{1}{T} dQ = \frac{1}{T} \left[ d\Pi + pd\left(\frac{1}{\rho}\right) \right]
\]  

(2.11)

\[
= \frac{1}{T} \left[ \Pi_u + p\left(\frac{1}{\rho}\right)_u \right] u^i
\]

In fact, in relativistic astrophysics and cosmology, we usually assume \(S\) to be a group invariant, whence

\[
S_u = 0, \text{ for every } u^i \text{ and } T
\]  

(2.12)

or, equivalently,

\[
\Pi_u + p\left(\frac{1}{\rho}\right)_u = 0
\]  

(2.13)

direct consequence of which is the adiabaticity condition (2.9) (or 2.7).

According to all the above, the conservation of rest mass (Eq. 2.6) and the conservation of entropy (Eq. 2.13) are not independent requirements in the framework of general relativity. The reason of their independence in the Newtonian limit \(\frac{1}{c^2} \to 0\) is that Eq. (2.5b) reduces simply to continuity equation

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0
\]  

(2.14)

We emphasize that isentropicity (and, hence, also adiabaticity) of the hydrodynamical flows could be physically necessary and useful. Actually, then, beyond the constancy of the matter content (the number of baryons) of a finite fluid volume element (Eqs. 2.6 or 2.14), also its thermodynamic content
remains unchanged along the flow line, in accordance with the usual assumption of an isolated physical system.

As emphasized long ago, the conservation of rest mass (2.6), under the assumption of the absence of any dissipative mechanisms in the fluid source, should be considered as a fundamental physical law supplementing the field equations (2.1). Assuming furthermore that the only baryons present are protons and neutrons of practically equally rest masses, the conservation law (2.6) is equivalent to the equally fundamental law of the conservation of the baryon number (practically Eq. (2.6) with \( \rho \) replaced by the baryon number density). Consequently, if we assume that the rest mass of a baryon remains constant, we conclude that the conservation of rest mass (2.6) is actually expressing the conservations of the baryonic mass. In what follows the motions rest-mass density and baryonic-mass density, as well as their conservation, and, finally, the motions rest mass and baryonic mass will not be distinguished from each other, respectively.

All these physical results concerning the adiabaticity or/and isentropicity of the hydrodynamic flows and their mutual dependence of the conservation of rest mass although emphasized long ago, since then, unfortunately, have been highly overlooked especially in astrophysical applications.

We note that, in the cosmological context of the Robertson-Walker line element in commoving coordinates, the two Eqs. (2.6) and (2.7) reduce to, respectively,

\[
\frac{d}{dt}(\rho R^3) = 0
\]

(2.15)

\[
\frac{d\varepsilon}{dt} + 3(\varepsilon + \rho) \frac{1}{R} \frac{dR}{dt} = 0
\]

(2.16)

where \( R(t) \) is the cosmological scale factor.

Direct consequence of Eqs (2.12) and (2.13) in this case is the relation

\[
\frac{d}{dt}(\varepsilon R^3) + \rho \frac{d}{dt}(R^3) = 0
\]

(2.17)

expressing the adiabaticity of the cosmic flows. Hence, also in the relativistic Robertson-Walker Cosmology, rest mass conservation implies adiabatic flows and vice versa.

3. A Cosmological Constant Versus a Cosmological Function

The inclusion of the cosmological constant \( \Lambda \) results in the following from of the field equations (2.1)

\[
G_{ij} + \Lambda g_{ij} = -\frac{8\pi G}{c^4} T_{ij}
\]

(3.1)

In the form (3.1), as distinguished from their form (2.1), the field equations are used for examining problems referring to the large scale structure of the Universe as a whole and, in particular, to the currently celebrated problem of the accelerating/ decelerating cosmological expansion, attributed to \( \Lambda \), as a property of the Universe as a whole. We shall propose here, that it is possible to use Eq. (3.1) to study any subsystem of the Universe in relation to the dark-matter, dark-energy, and accelerating expansion of the subsystem itself.

This role of \( \Lambda \) is usually studied by noticing that, it is possible to treat \( \Lambda \) as a source (the \( \Lambda \) source) writing Eqs. (3.1) as

\[
G_{ij} = -\frac{8\pi G}{c^4} T_{ij} - \Lambda g_{ij}
\]

(3.2)

Then, the energy-momentum tensor \( T_{ij}^\Lambda \) for the \( \Lambda \) source satisfies
\[ -\Lambda g_{ij} = -\frac{8\pi G}{c^2} T_{ij}^\Lambda \] (3.3)

If, moreover, the source \( \Lambda \) is assumed to be a fluid (the \( \Lambda \) fluid) characterized by the proper quantities \( \varepsilon_{\Lambda}, p_{\Lambda} \) (and \( \Pi_\Lambda \); see Eq. (3.11) below), then, in analogy to Eq. (2.3),

\[ T_{ij}^\Lambda = (\varepsilon_{\Lambda} + p_{\Lambda}) u_i u_j - p_{\Lambda} g_{ij} \] (3.4)

whence Eq. (3.3) reduces to

\[ -\Lambda g_{ij} = -\frac{8\pi G}{c^2} \left[ (\varepsilon_{\Lambda} + p_{\Lambda}) u_i u_j - p_{\Lambda} g_{ij} \right] \] (3.5)

Therefore, from the functional form of Eq. (3.5) we conclude that

\[ \varepsilon_{\Lambda} + p_{\Lambda} = 0 \] (3.6)

and

\[ \Lambda = -\frac{8\pi G}{c^2} p_{\Lambda} \left( = \frac{8\pi G}{c^2} \varepsilon_{\Lambda} \right) > 0 \] (3.7)

Eq. (3.6) means that, if \( \varepsilon_{\Lambda} \) and \( p_{\Lambda} \) are non-zero quantities of different signs, the \( \Lambda \) space-time is not empty. Eq. (3.7) expressing the cosmological constant in terms of \( p_{\Lambda} \), implies that, if \( \Lambda \) is positive, the energy density of the source \( \Lambda \) is positive \( (\varepsilon_{\Lambda} > 0) \) and its pressure is negative \( (p_{\Lambda} < 0) \), whence Eq. (3.6) can be satisfied.

Since, by its definition (3.7), \( \Lambda \) is practically the energy-density \( \varepsilon_{\Lambda} \), and since, in the cosmological context, \( \Lambda \) has been required to be constant, we may ask: What is the physical meaning of the constancy of \( \Lambda \)? Under which physical conditions can \( \Lambda \), considered as a source, be a constant?

In order to answer the above questions we shall use the fluid’s property of adiabaticity in the form of Eq. (2.20). If, additionally, we require the adiabaticity property to characterize the flow motions of the \( \Lambda \) perfect-fluid source, then we shall have

\[ \frac{d}{dt} (\varepsilon_{\Lambda} R^3) + p_{\Lambda} \frac{d}{dt} (R^3) = 0 \] (3.8)

Combining Eqs. (2.20), (3.6) and (3.8) we conclude, that, under the above conditions, \( \varepsilon_{\Lambda} \) is a constant,

\[ \frac{d\varepsilon_{\Lambda}}{dt} = 0 \] (3.9)

and so \( \Lambda \) is in fact a constant,

\[ \frac{d\Lambda}{dt} = 0 \] (3.10)

Now we remark that the constancy of \( \varepsilon_{\Lambda} \), namely, of \( \Lambda \), actually implies an equation of the form (2.9). This can be proved by assuming, in analogy to Eq. (2.4), the existence of the proper quantities \( p_{\Lambda} \) and \( \Pi_\Lambda \) such that

\[ \varepsilon_{\Lambda} = \rho_{\Lambda} c^2 + \rho_{\Lambda} \Pi_\Lambda \] (3.11)
where $\rho_{\Lambda}$ describes the “material content” and $\Pi_{\Lambda}$ the “energetic content” of the $\Lambda$ fluid. Inserting (3.11) into Eq. (3.9) and using Eq. (3.6) we obtain

$$
\frac{d\Pi_{\Lambda}}{dt} + p_{\Lambda} \frac{d}{dt}(\frac{1}{\rho_{\Lambda}}) = 0
$$

which, in analogy to Eq. (2.9), expresses the adiabaticity of the flows of the $\Lambda$ source.

In conclusion, the constancy of $\Lambda$, along with the condition (3.6), is equivalent to the adiabaticity of the flows of the $\Lambda$ source. This, of course, is true also inversely, namely, the adiabaticity of the flows of the $\Lambda$ source, along with the condition (3.6), implies the constancy of $\varepsilon_{\Lambda}$ and, hence, $\Lambda$.

Up to now we have considered the adiabaticity conditions for the original fluid and of the $\Lambda$ fluid holding independently of each other. This is natural to imply conservation of mass for the two fluids independently of each other, meaning that the density $\rho_{\Lambda}$ satisfies the conservation law

$$
\left(\rho_{\Lambda} u^i\right)_t = 0
$$

independently of the corresponding conservation law (2.6) for the original fluid. Consequently, the two fluids evolve independently of each other.

In examining the role of $\Lambda$ as a source, it appears that it is not always necessary to assume that mass conservation and adiabaticity are valid for the two energy-momentum tensors $T_{ij}$ and $T_{ij}^{\Lambda}$ independently of each other. Instead, these two assumptions can be considered to be valid for the total energy-momentum tensor

$$
T^{(1)}_{ij} = T_{ij} + T_{ij}^{\Lambda} = (\varepsilon + p + \varepsilon_{\Lambda} + p_{\Lambda}) u_i u_j - (p + p_{\Lambda}) g_{ij}
$$

In such a case we may demand that the generalized adiabaticity holds in the form

$$
dQ^{(1)} = d(Q + Q_{\Lambda}) = dQ + dQ_{\Lambda} = 0
$$

namely, there is always a mutual exchange of heat quantities between the original fluid and the $\Lambda$ fluid, with no losses from the otherwise isolated system of the two fluids. This means that the equations of motion (2.5a) hold for the system of the two fluids, for which $\rho$, $\varepsilon$ and $p$ are replaced by, respectively,

$$
\rho^{(1)} = \rho + \rho_{\Lambda}
$$

$$
\varepsilon^{(1)} = \varepsilon + \varepsilon_{\Lambda}
$$

$$
p^{(1)} = p + p_{\Lambda}
$$

Hence, specifically in the Robertson-Walker cosmological context, we shall have, in analogy to Eqs (2.15) and (2.16),

$$
\frac{d}{dt}\left[(\rho + \rho_{\Lambda}) R^3\right] = 0
$$

$$
d\left[(\varepsilon + \varepsilon_{\Lambda}) R^3 + (p + p_{\Lambda}) d(R^3)\right] = 0
$$

Direct consequence of Eqs (3.20) and (3.6) and (3.7) is
\[
\frac{d\varepsilon_\Delta}{dt} + \frac{c^4}{8\pi G} \frac{d\Lambda}{dt} = \frac{d\varepsilon}{dt} - 3(\varepsilon + p) \frac{1}{R} \frac{dR}{dt} \neq 0 \quad (3.21)
\]

Therefore, in the generalized case of the combined source of the two fluids (the original fluid and the $\Lambda$ fluid), we shall have more generally a cosmological function

\[
\Lambda = \Lambda(\vec{x}, t) \quad (3.22)
\]

(of either constant, \(\frac{d\Lambda}{dt} = 0\), or nonconstant, \(\frac{d\Lambda}{dt} \neq 0\), value $\Lambda$) rather than, necessarily, an absolute (cosmological) constant.

References
Chandrasekhar, S.1969, Ap. J. 158, 45-54.
Kleidis, K., and Spyrou, N.K. 2000, Class. Quantum Grav. 17, 2965-2982.
Spyrou, N.K.1997a, in The Physics of Ionized Gases, Proceedings of the 18th Summer School and International Symposium on the Physics of Ionized Gases (Kotor, Jugoslavia, 2-6 September 1996), eds. B. Vujicic, S. Djurovic and I. Puric, Institute of Physics, Novi Sad University, Yugoslavia, pp.417-446.
Spyrou, N.K. 1997b, in The Earth and the Universe, Volume in Honorem L. Mavrides, eds. G. Asteriadis, A. Bandellas, M. Contadakis, K. Katsambalos, A. Papademetriou and I. Tziavos, Thessaloniki, Greece, pp. 277-291.
Spyrou, N.K. 1997c, Facta Universitatis, 4, 7-14.
Spyrou, N.K. 1999, in Current Issues of Astronomical and Planetary Environmental Concern, Proceedings of International Seminar (Thessaloniki, 6-7 April 1999) ed. N.K.Spyrou, Astronomy Department, Aristoteleion University of Thessaloniki, Thessaloniki, Greece, pp.23-35.
Spyrou, N.K. 2001 “Conformal Invariance and the Nature of Cosmological Structures”, Invited Talk, Proceedings of The Conference on Applied Differential Geometry-General Relativity, and The Workshop on Global Analysis, Differential Geometry, Lie Groups, (Thessaloniki, 27 June-1July 2001) eds. G. Tsagas, C. Udriste, and D. Papadopoulos, pp. 101-107.
Spyrou, N.K 2002, On the Determination of the Masses of Cosmological Srtuctures in Modern Theoretical and Observational Cosmology, Proceedings of the 2th Hellenic Cosmology Meeting (Athens, 19-20 April 2001), eds. M. Plionis and S.Cotsakis, Kluwer Academic Publishers, Dordrecht, Vol. 276, pp.35-43.
Spyrou, N.K. 2003, Conformal Dynamical Equivalence and the Cosmological Expansion of a Realistic Universe, in Proceedings of the 10th Conference New Developments in Gravity, eds. K. Kokkotas and N. Stergioulas, Kluwer, Academic Publishers, Dordrecht, pp.90-96.
Spyrou, N.K. 2004a, A Classical Treatment of the Dark-Matter and Flat-Rotation-Curves Problems, in Proceedings of the 6th Hellenic Astronomical Conference (Penteli, Athens), September 2003 (in press).
Spyrou, N.K. 2004a, under preparation.
Spyrou, N.K. and Kleidis, K. 1999, “On the Nature of Nuclear Galactic Masses” Proceedings of JENAM 1999, Samos.
Spyrou, N.K., and Tsagas, C.G. 2004, Class. Quantum Grav. 21, 2435-2444 (gr-qc/0403116)