Measurement of Wheel Radius in an Automated Guided Vehicle

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Abstract: In the case of automated guided vehicles using odometry, a very important issue is to know the actual rolling radius of the wheel used to calculate the position of the vehicle. This radius is not constant. Its changes depend on the elastic deformation of the band layer and wheel slip. The theoretical determination of the value of the radius and the nature of the change is very difficult. For this reason, it was decided to determine the rolling radii by the experimental method. For this purpose, an appropriate test stand was built and the proposed research method was checked. Within the tests, there was also obtained a number of interesting results characterizing the material used to build bands and the ranges of changes rolling radii for the tested material were specified.

Keywords: automated guided vehicle; wheel radius; wheel slip; odometry errors; measurement methods

1. Introduction

The use of an automated guided vehicle (AGV) is becoming more and more common. In many cases, the routes implemented by these vehicles are subject to constant changes and are adapted to the currently assigned task. The basic navigation system for most of these vehicles is odometry. The navigation process using odometry assumes the constancy of parameters such as rolling radii and wheelbases. Slips are also not included in this process. In real conditions, when working with different loads and on different surfaces, these parameters change their values and slides occur. Not taking these changes into account in the navigation algorithm leads to errors in determining the position.

An extensive review regarding odometry errors is included in papers [1,2]. There are several types of errors that affect the accuracy of the positioning process. These include systematic and unsystematic errors. The systematic errors that arise in determining the current position during the movement of the vehicle are aggregated [1,3]. Depending on the type of surface, the share of the systematic and the unsystematic errors in the error of position determination may be different. In addition, odometry errors can be caused by the same movement equations [4]. These equations describe any trajectory through a series of short sections. The accuracy of this approximation depends on the sampling frequency and vehicle speed.

Many methods are used to correct errors. The first works related to error correction concern systematic errors. Correction of these errors and calibration of the system takes place after traveling the preset route section and measuring the error of the final position [1,2,5–8].

Modern measurement techniques enable online measurements. A variety of techniques may be used for measurements, a brief review of which is contained in [9–11]. In most solutions using
additional measurement systems, extensive filtration methods are applied. These can be online methods such as at work [12–14] or offline as at work [15].

All the aforementioned works assume that the wheel and ground are rigid, cooperation between the wheel and the road takes place at one point and there is no slip. In recent years, there have been works including changes in rolling radius and occurrence of wheel slip [16–21].

Such changes in the rolling radius have been taken into account in the navigation process of the vehicle presented in Sec. 4.1 and intended for the carriage of loads. The shape of the route and the mass of the cargo carried during the operation of the vehicle is subject to change [22]. It depends on the currently assigned task. In the case of a change in the mass of the transported load or its position on the vehicle, the loads of individual wheels change. Along with these changes, the rolling radii of the vehicle wheels are subject to change. Failure to take this fact into account in the calculation algorithm will lead to errors. In order to reduce these errors, it is necessary to enter into the calculation algorithm appropriate values of rolling radii taking into account the current wheel load. This operation will be possible only after examining and determining the dependence of the rolling radius on the wheel load.

As part of the article, it was decided to describe the measurement methodology as well as the course of experimental research aimed at determining the dependence of the rolling radius on the wheel load in the vehicle under consideration.

The paper is organized as follows. Section 2 presents a research object and its characteristics and information about the wheel and its properties. Section 3 presents an overview of selected measurement methods. As part of the review, errors resulting from failure to maintain ideal measuring conditions were indicated. Taking into account the measurement capabilities and existing environmental conditions, the most adequate measurement method for the further stage of experimental research was indicated.

Section 4 presents the results obtained from experimental research. The wheel radii were determined in these tests. These tests were carried out for various loads. As part of the section, a statistical analysis of the measurement results was also carried out and characteristic values related to the actual wheel operating conditions were determined. Section 5 contains a discussion of the obtained results and their in-depth analysis. Finally, Section 6 presents the conclusions.

2. Determination of the Vehicle Position and the Role of the Rolling Radius

2.1. Movement of a Wheeled Vehicle

The basic method of determining the position in motion of an autonomous vehicle is odometry. Its rules are described in detail in the works [1–4,12]. There are many ways to drive and steer the vehicle.

Most autonomous transport vehicles use two modes of driving and steering. In the first one, independently powered rear wheels are used for driving and steering. In the second case, the front wheel is driven and steered. In both cases, the data necessary to control the vehicle, such as the rotational speed of the wheels or the steering angle of the front wheel, come from encoders installed at the respective wheels. Figure 1 shows the vehicle in the $X_0O_1Y_0$ reference system and the individual quantities characterizing the vehicle movement are marked.
The position of a selected point O in the base reference system X_0O_0Y_0 (Figure 1) is defined by the state vector X_j. The next position j + 1 of the vehicle point O is determined by the state vector X_{j+1}.

State vector X_{j+1} in the j + 1 iteration is expressed by:

$$X_{j+1} = X_j + V_{j+1}dt$$  \hspace{1cm} (1)

where:

$$X_{j+1} = \begin{bmatrix} x_{j+1} \\ y_{j+1} \\ \theta_{j+1} \end{bmatrix}, \quad X_j = \begin{bmatrix} x_j \\ y_j \\ \theta_j \end{bmatrix}, \quad V_{j+1} = \begin{bmatrix} v_{O,j+1} \cos(\theta_j + \omega_{j+1}dt) \\ v_{O,j+1} \sin(\theta_j + \omega_{j+1}dt) \\ \omega_{j+1} \end{bmatrix}. \hspace{1cm} (2)$$

Velocities \(v_{O,j+1}\) and \(\omega_{j+1}\) can be determined from the following relations, for a vehicle using measured data from the rear wheels:

$$v_{O,j+1} = (v_{R,j+1} + v_{L,j+1})/2, \hspace{1cm} (3)$$

$$\omega_{j+1} = (v_{R,j+1} - v_{L,j+1})/b, \hspace{1cm} (4)$$

where:

- \(v_{R,j+1}\) = speed of the right wheel in j + 1 step;
- \(v_{L,j+1}\) = speed of the left wheel in j + 1 step;
- \(\omega_{j+1}\) = angular velocity of the vehicle relative to the reference system X_0O_0Y_0;
- \(b\) = wheelbase of the driven wheels.

For a vehicle using measurement data from the front wheel:

$$v_{O,j+1} = v_{K,j+1} \cos(\alpha_{j+1}), \hspace{1cm} (5)$$

$$\omega_{j+1} = v_{K,j+1} \sin(\alpha_{j+1})/l, \hspace{1cm} (6)$$

where:

- \(v_{K,j+1}\) = speed of the front wheel in j + 1 step;
- \(\alpha_{K,j+1}\) = turning angle of the front wheel in j + 1 step.

To determine the speed of individual wheels: \(v_{R,j+1}, v_{L,j+1}, v_{K,j+1}\) the data from the encoders and the assumed values of the radius \(r\) of the vehicle wheel are used.

In the above consideration, it was assumed that the wheels are rigid and they roll without slipping, the contact between the wheel and the floor is a point contact and the radii \(r\) of the rear wheels are the same.
2.2. Wheel and Its Properties

The wheel radius used to determine the position from odometry calculations in real movement conditions is not a constant value. Its size is influenced by many factors. The vehicle wheel shown in Figure 2a has a flexible belt. Thanks to this, the wheel has the ability to damp vibrations in selected operating conditions. The physical model of the wheel is shown in Figure 2b. The wheel has longitudinal and circumferential elasticity, which was modeled by a spring. Additionally, the model features damping elements designed to dissipate vibrations.

![Figure 2](image-url)

**Figure 2.** Vehicle wheel (a) view; (b) model, g—deflection, rS—static radius.

In the case of a stationary wheel, the vertical force loading the wheel reduces the radius of the wheel by the value of deflection g (Figure 2b). In the case of a rolling wheel, circumferential deformations and slips occur in the wheel’s contact with the ground. The rolling radius of the wheel changes. These changes are additionally dependent on the type of horizontal forces and moments acting on the wheels. There may be two cases here. In one of them, the wheel is driven; the other is braked. The occurring phenomena have been comprehensively described in the works [23,24]. In real operating conditions of the wheel there is a slip and it is better than to use the concept of the rolling radius rt.

In the range of small slips up to 2–3%, the relationship between pressure forces and slip is linear in both cases of operation. This is shown schematically in Figure 3a.

![Figure 3](image-url)

**Figure 3.** Changes in wheel radius (a) as a function of static load; (b) during motion, under the driving force/braking force.

Changing the slip value affects the value of the rolling radius. The course of changes in the rolling radius is shown schematically in Figure 3b [25].
3. An Overview of Selected Measurement methods

3.1. Introduction

Rolling radius defined as the agreed size of the radius of such a rigid wheel, which at section \( L \) performs the same number of rotations \( n_k \). This describes Relationship (7):

\[
L = 2\pi r_t n_k. \tag{7}
\]

Thus, the rolling radius of the wheel \( r_t \) is determined by the formula:

\[
r_t = \frac{L}{2\pi n_k}. \tag{8}
\]

Wheel slip of the driven wheel \( s \) is:

\[
s = \frac{v_k}{v_x} = \frac{\omega r - \omega r_t}{\omega r} = 1 - \frac{r_t}{r}. \tag{9}
\]

Wheel slip of the braked wheel \( s \) is:

\[
s = \frac{v_k}{v_x} = \frac{\omega r_t - \omega r}{\omega r_t} = 1 - \frac{r}{r_t}. \tag{10}
\]

where:
- \( r_t \)—rolling radius;
- \( r \)—free radius;
- \( v_k \)—slip velocity;
- \( v_x \)—wheel center speed.

For the driven wheel in the slip range \( s \) from 0 to 1, the rolling radius \( r_t \) changes from \( r \) to 0. With the case of the rolling radius \( r_t = 0 \) we deal when the vehicle stands \( v = 0 \) and the driven wheels rotate at an angular speed \( \omega > 0 \). For a nondriven wheel in the slip range \( s \) from 0 to 1, the rolling radius \( r_t \) varies from \( r \) to \( \infty \). With the case of a rolling radius, \( r_t = \infty \) is when the vehicle moves at a speed \( v > 0 \) and the braked wheel is blocked \( \omega = 0 \). In the range of small slip to \( s = 0.02 \), the relationship between vertical wheel load and circumferential force is linear [22].

There are many advanced theoretical methods to determine the rolling radius as a function of the type of work and load. The accuracy of these methods depends on the knowledge of specific parameters. Sometimes, and in many cases, this requires additional experimental testing. For this reason, it was decided to determine the rolling radii directly on the basis of experimental research. According to Relationship (8), to determine the rolling radius, it is sufficient to measure the distance traveled by the road \( L \) and read the recorded data on the quantities made by the wheel revolutions \( n_k \). In this measurement method, it is very important to maintain the straight-line nature of the movement. In real traffic conditions, there will always be smaller or larger deviations from the theoretical route. These deviations are affected by the control system and incorrect initial position and values of the rolling radii entered into the navigation algorithm.

3.2. Methodology of Measurements in Curved Motion

With a large deviation of the actual track from the assumed theoretical course, the vehicle performs arc motion (Figures 4 and 5). Determination of rolling radii from the dependence (8) for such a course is subject to a significant error. The distance traveled along a curve differs from the length of the section connecting the beginning and the end of the curve.
3.3. Methodology of Measurements in Rectilinear Motion

From the encoders of individual wheels. Characteristic markers on the reference surface can be used to measure distance between the start and end position of a selected point of the vehicle. In the second method, characteristic markers on the reference surface can be used to measure distance $s_{w}$, e.g., door openings. In both methods, the distance $d$ from the reference surface is recorded and the counter reads the data from the encoders of individual wheels.

![Figure 4. An example of a curvilinear route.](image)

There are many ways to determine the path traveled in curvilinear motion. In the conducted preliminary tests, two methods were used, shown in Figure 6. The first method determines the distance $s_c$ between the start and end position of a selected point of the vehicle. In the second method, characteristic markers on the reference surface can be used to measure distance $s_{w}$, e.g., door openings. In both methods, the distance $d$ from the reference surface is recorded and the counter reads the data from the encoders of individual wheels.

![Figure 5. View of oscillations and interference on a selected fragment.](image)

Knowledge of the value of the radius of curvature $R$ and half arc angle $\phi$ makes it possible to determine the lengths of arcs traveled by the wheels and their rolling radii.

3.3. Methodology of Measurements in Rectilinear Motion

In order to meet the requirements for ensuring the straight-line movement of the vehicle, it is necessary to properly control and drive the vehicle. This can be obtained, for example, by moving along a given reference plane. As shown in Figure 7, as part of this movement, the fixed distance of the selected vehicle point and the assumed reference plane is maintained.

![Figure 6. The curve linear run and measurement data, $R$—radius of curvature, $d$, $d_a$—distance between vehicle and reference surface, $s_w$, $s_C$—measured path, $0$—middle of the arc, half arc angle $\phi$.](image)
Due to the imperfections of the reference surface and the measuring system, there may be three types of errors. The first of these is related to the nonlinearity of the reference surface. The second error is related to the structure of the reference surface. The third error is related to the accuracy of the measuring device. All three errors affect the measurement data which are used in the control system. This contributes to the formation of vehicle oscillations in relation to the assumed direction of movement. The course of such oscillations is shown in Figure 8a. Some of the errors in measurement data can be eliminated by applying real-time filtering. Excessive deviation from the set path can be partially eliminated by the use of real-time filtration. An example of using an online filter for measurement data entered into the control system is presented in [12]. The benefits of using such a filter are shown in Figure 8b.

The application of online filtration has reduced deviations from the set driving path and thus increased the accuracy of the rolling radius determination process. However, it is a big problem to find the appropriate reference surface in real measurement conditions. Some of the surfaces used in the research had discontinuities. The effect of these disturbances is visible peaks in Figures 4, 5 and 8.

3.4. Assessment of the Influence of Deviation from the Ideal Trajectory on the Value of the Rolling Radius

In the case of a vehicle with a propulsion system presented in Section 4.1 small errors in the values of the rolling radii and control system errors cause a slight oscillation \( A \) from the set driving path. For the angle of deviation from the given path \( \psi = 0 \), the vehicle moves along the assumed direction (Figure 9a). For an incorrectly defined initial position of the vehicle in which the deviation angle \( \psi \neq 0 \) the vehicle route deviates from the assumed direction of travel. The distance that can be covered is limited by the width of the corridor and the value of the angle \( \psi \) (Figure 9b).

Errors larger in the values of rolling radii cause a strong deviation \( C \) from the assumed trajectory. The actual vehicle movement is carried out in a curve (Figure 9c). In all cases, oscillations coming from the control system are visible on the course of the route.
In order to determine the effect of oscillation and deviation from the target track on measurement accuracy, a series of computer calculations were done.

The vehicle moves from the starting point to the endpoint. The direct distance \( L \) between these points is 40 m. Vehicle trajectory is not ideal and deviates from the line connecting the start and endpoints. Therefore, the actual \( L_r \) path traveled by the vehicle is greater than the distance \( L \). The first series of calculations concerned the course of motion shown in Figure 9a, while the second series of calculations concerned the course of motion shown in Figure 9c. In both cases, the distance traveled by the vehicle under the assumed traffic conditions, marked \( L_0 \), was determined. Using Dependence (11), the relative measurement error \( \Delta \) was determined.

\[
\Delta = \frac{L_r - L}{L} \tag{11}
\]

The results of the calculations are shown in Figure 10.

In both cases, it can be observed that the effect of the deviation \( C \) and oscillation \( A \) on the relative error is quite large. To ensure the accuracy the deviation had to be minimized. Hence, it was decided to use the modification of this method involving manual towing of the vehicle along a designated straight line. Several dozen attempts at various vehicle loads have been made with this method. The load values included future vehicle performance parameters.

The measurements were carried out in the corridors of a rigid surface. One of these is showing in Figure 11a. During the tests, the length of the distance \( L \) covered by the vehicle was measured. The value of the distance was measured by a laser rangefinder and ranged from 34 m to 42 m. The results of these tests and their analysis are presented in the next chapter. Basic research has been preceded by many trial tests.

**Figure 9.** Vehicle movement on the measuring section: (a) low values of rolling error \( \psi = 0 \); (b) low values of rolling error \( \psi \neq 0 \); (c) high values of rolling errors, \( L \)—direct distance between the start and endpoints of the route, \( A \)—amplitude of oscillation, \( C \)—deviation, \( \lambda \)—wavelength, \( \psi \)—angle deviation.

**Figure 10.** Relative error in the function: (a) of the amplitude \( A \) of oscillation at the \( \lambda = 5 \) m and \( L = 40 \) m; (b) of the deviation \( C \) at the \( A = 0 \) and \( L = 40 \) m.
The research object shown in Figure 11 was built at the Rzeszow University of Technology. It is intended for transporting goods. The vehicle has three wheels. The front wheel is a drive and steering wheel. This wheel is connected via gears with two DC electric motors. One of them built inside the wheel is used to drive the vehicle. The second engine is located on the vehicle frame and, through a worm gear, regulates the steering angle around the vertical axis of rotation. The rear wheels are support wheels. These wheels are connected with encoders (Figures 11b and 2a). Signals from encoders are used to control the movement of vehicles.

All vehicle wheels have an outer diameter of 200 mm and a width of 50 mm. There are polyurethane belts around the wheel circumferences. Compared to rubber, polyurethane is characterized by greater stiffness and therefore less elasticity. Wheels with such belting can be used for loads of 8500 N.

In order to ensure the possibility of autonomous operation, the vehicle is equipped with an onboard computer. All data necessary to control the movement of the vehicle was supplied to the onboard computer using the National Instruments NI USB-6343 measuring acquisition card. The scheme of the measuring system is shown in Figure 11b. Encoders were used to measure angular velocity and distance traveled data of individual wheels.

In the case of rear wheels, these were MHK40 encoders from Autonics Company. Encoders integrated with electric motors were used to measure the angular position, speed and traveled distance of the front wheel. The vehicle uses additional laser sensors to determine the position of the vehicle relative to the adopted reference surface. DT50-P2113 lasers were used to measure the distance perpendicular to the longitudinal axis of the vehicle, e.g., from the corridor wall. The DT500-A611 laser was used to measure the distance in the longitudinal direction.

Depending on the batteries used, the vehicle in working order had different weights. For low capacity batteries, the weight was 100 kg. With larger batteries enabling several hours of operation, the weight was 160 kg. The expected maximum speed of movement was 1 m/s. The expected maximum load is 2500 N.

In the vehicle from Figure 11, odometry was the primary navigation system. While the vehicle was moving, fixed rolling radius values were used in the calculation algorithm. These values were introduced into the calculation system at the beginning of the study. Additional measuring devices installed, such as laser rangefinders, allowed the current position of the vehicle to be determined in relation to the surroundings. Using the appropriate calculation systems [26] based on the measurements obtained, it is possible to correct errors [10,11].

4.2. Research Results and Their Analysis

As part of the basic research, 56 tests in four measuring series were carried out. In the individual series were the following loads: 790 N, 1093 N, 1376 N and 1800 N. On the basis of the obtained test results, the influence of the load on the rolling radii of the wheels was determined using the Statistica package. The average values and standard deviations in a given series were obtained in the individual
measurement series and were presented in Table 1. Coefficients of variation in individual measurement series were presented in Table 2. Apart from the measurements for the left wheel and the 1800 N load, the standard deviation values characterizing the dispersion of measurements are similar.

**Table 1.** Average values and standard deviations in the individual measurement series.

| Force $F_N$, N | Left Wheel | Right Wheel |
|---------------|------------|-------------|
| $r$, m        | $\sigma \times 10^{-6}$, m | $r$, m | $\sigma \times 10^{-6}$, m |
| 790           | 0.10046    | 25.632      | 0.10030 | 23.935 |
| 1093          | 0.10049    | 25.103      | 0.10034 | 24.587 |
| 1376          | 0.10054    | 20.679      | 0.10041 | 18.644 |
| 1800          | 0.10058    | 37.081      | 0.10046 | 20.101 |

**Table 2.** Coefficients of variation in individual measurement series.

| Force $F_N$ (N) | 790  | 1093 | 1376 | 1800 |
|----------------|------|------|------|------|
| Left wheel     | 0.000255 | 0.000250 | 0.000206 | 0.000369 |
| Right wheel    | 0.000239 | 0.000245 | 0.000186 | 0.000200 |

Figure 12a,b shows the determined regression curves together with a 95% confidence interval. For the left wheel, the correlation coefficient is 0.8617, and for the right wheel, is 0.9351 The correlation coefficient for the left wheel assumes smaller values due to the greater dispersion of the measurement results. The measure of this spread is the standard deviation (Table 1).

**Figure 12.** Linear regression with a 95% confidence interval estimated for (a) left wheel; (b) right wheel.

Figure 13 shows a comparison of the obtained rolling radius courses as a function of load for both wheels. These courses have a similar slope. Their nonparallelism is basically determined by the measurement for the left wheel loaded with 1800 N. This measurement is burdened with the largest error and the largest dispersion of the measuring points shown in Figure 12a.

**Figure 13.** Comparison of the obtained rolling radius courses.

Using the data from Table 1, slip coefficients for both wheels were determined. Figure 14 shows the relationship between the load of a given wheel and slip. The obtained relationships for the slip, $s = 0.1\%$ to $s = 0.3\%$ are linear.
As part of the study also determined the deflection characteristics of a flexible bend as a function of load. The graph presenting such characteristics is shown in Figure 15.

The graph presented in Figure 15 is a hysteresis loop, which is visible after the enlargement of the selected fragment. The dependence of the deflection on the pressure force does not differ much from the linear characteristic.

5. Discussion

The results obtained from experimental studies are characterized by small dispersion and thus high accuracy. A comparison of coefficients of variation with the values obtained from the calculation of relative errors (Figure 10) indicates that appropriate precision and diligence are maintained during the tests. During the experimental tests, it was tried to make the oscillation and deviation values smaller to \( A = 0.025 \) m and \( C = 0.3 \) m. The calculated values of the coefficient of variation Table 2 for both wheels, except for the load with the force \( F_N = 1800 \) N, are comparable. For a force load of \( F_N = 1800 \) N, the coefficient of variation for the left wheel is 84% higher. The values of the coefficient of variation are influenced by the impossible to completely eliminate deviations and oscillations as well as, for example, unexpected accidental slips in the contact of the wheel with the surface and measurement errors in data recording. The calculations of the slip coefficients \( s \) as a function of load carried out show their linear relationship (Figure 14). Equation (10) used for calculations takes into account the elastic deformation of the wheel and microslips in the contact between the wheel and the surface. According to the information contained in a series of studies and in Figure 3, for low values of slips \( s < 2\% \), the relationship between the force acting on the wheel and the slip is linear.

The diagram in Figure 15 clearly indicates the elastic properties of the belt material. This graph has the shape of a hysteresis loop. The field contained inside the hysteresis loop indicates the ability to damp vibrations.

The test results and their courses presented in Figures 14 and 15 are compatible with the examples shown in many works [23–25]. This proves the correctness of the adopted methodology and diligence in the course of research.

The results obtained and the relationships developed on their basis are very important for the automated guided vehicle. Especially in the vehicle shown in Figure 11. In the vehicle shown in
Figure 11, the rear wheels do not carry any driving or braking forces and are used to measure the distance traveled.

During the tests, the force loading the wheel did not exceed 21% of the maximum wheel load. By increasing the wheel load by 1010 N from 790 N to 1800 N, the relative increase in the rolling radius was 0.12%. Because with the slip below 2%, all changes are linear, a further increase in the load by the force of 1010 N and 2020 N will result in an increase of the rolling radius by 0.24% and 0.36%. The increments of the rolling radius are related to the value obtained under the load of 790 N. In the last case, the total force on the wheel will be 3820 N. This is less than half the allowable force.

Assuming that only one wheel is subject to load changes, the other wheel is loaded with a constant force $F = 790$ N, using the dependencies (1)–(4), it is possible to determine the deviation from the set route and the radius of the curve along which the vehicle will move. The results are presented in Table 3.

| Additional Loading Force $F_N$ Over the Initial Value $F_N = 790$ (N) | Radius of the Arc $R$ (m) | Deviation from the Track to the Road $L = 40$ m (m) |
|---|---|---|
| 1010 | 503 | 1.59 |
| 2020 | 251 | 3.20 |
| 3030 | 168 | 4.84 |

The analysis of the data in Table 3 shows that the differences in the load on the wheels cause significant changes in the trajectory carried out by the vehicle. Correction of the above errors can be made through additional navigation systems or by taking into account corrections made to the control system, coming from the load measurement system of a given wheel.

6. Conclusions

The conducted experimental tests showed the usefulness of the proposed measurement method. The main purpose of the research was to determine the relationship between the wheel’s rolling radius and its load. The main tests were preceded by a description of elastic wheel properties and presentation of possible measurement methods. After considering the preferred accuracy and availability of measuring equipment, the most favorable measurement methodology was selected. It is important to know the actual values of the wheel radii during vehicle operation. This is particularly true when odometry is the basic navigation system. This problem is even more important when the mass of loads carried by the vehicle changes within the permissible ranges for a given vehicle. The developed measurement methodology may also be useful for the initial determination of the value of wheel rolling radii. This is especially important for newly built vehicles. The wheel manufacturers do not provide the value of the rolling radii and, in addition, these wheels differ in dimensions. These differences probably fall within the tolerance of performance. However, from the point of view of odometry, they are important.

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