Radiative heavy quark energy loss in an expanding viscous QCD plasma

Sreemoyee Sarkar,1 Chandrodoy Chattopadhyay,2 and Subrata Pal2

1Centre for Excellence in Basic Sciences, University of Mumbai, Vidyanagari Campus, Mumbai 400098, India
2Department of Nuclear and Atomic Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India

We study viscous effects on heavy quark radiative energy loss in a dynamically screened medium with boost-invariant longitudinal expansion. We calculate, to first order in opacity, the energy loss by incorporating viscous corrections in the single-particle phase-space distribution function within relativistic dissipative hydrodynamics. We consider the Grad’s 14-moment and the Chapman-Enskog-like methods for the nonequilibrium distribution functions. Our numerical results for the charm quark radiative energy loss show that, as compared to static fluid, an expanding ideal (nonviscous) fluid causes a much smaller energy loss. Viscosity in the evolution lead to somewhat enhanced energy loss which is insensitive to the underlying viscous hydrodynamic models used. Further inclusion of viscous correction induces larger energy loss, the magnitude and pattern of this enhancement crucially depend on the form of viscous corrections used.

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I. INTRODUCTION

High-energy heavy-ion collision experiments at the Relativistic Heavy-Ion Collider (RHIC) [1, 2] and the Large Hadron Collider (LHC) [3–5] have firmly established the formation of strongly interacting matter composed of color deconfined system of quarks and gluons. Such conclusion was based from relativistic viscous hydrodynamic analysis of the large anisotropic flow that requires a remarkably small shear viscosity to entropy density ratio of η/s = 0.08 – 0.20 [6–9]. This not only suggests that the matter formed is close to local thermal equilibrium but also provides a window to the initial state of the fireball immediately after the collision. Hydrodynamic and transport model have been widely used to study the properties of the hot and dense medium by exploring the collective flow of the soft (bulk) hadrons.

On the other hand, the suppression of high transverse momentum of light and heavy quarks produced in hard processes provides an excellent tool that allows tomographic studies of the QCD plasma [10–15]. The suppression is caused by the attenuation (energy loss) of the energetic partons via inelastic and elastic collisions during their propagation in the medium. Heavy quarks, in particular, provide a promising probe as these are formed in the early stages via hard scatterings, and their production in the plasma at later stages is largely suppressed owing to their large mass [15]. These primordial heavy quarks could thus explore various stages of the space-time evolution. While the energy loss is dominated by medium-induced gluon radiation at moderate and large transverse momentum, collisional energy loss has sizable contribution especially at low transverse momentum p_T [16]. However, the massive heavy flavor gives a reduced radiative energy loss at low p_T (the “dead cone effect”) in contrast to light partons [17]. Consequently, RHIC measurements of the heavy flavor suppression data up to p_T ≈ 5 GeV/c [18, 19] could be well reproduced in various distinct theoretical energy loss formalisms. Heavy ion collisions at LHC enable charm meson measurement at p_T > 20 GeV/c and thus provide the ideal ground to study heavy flavor suppression [20, 21]. In this ultra-relativistic regime the radiative energy loss becomes dominant. In contrast to pQCD predictions, a surprisingly large suppression pattern of high-p_T D-mesons was observed at LHC, quantified by the nuclear modification factor, R_AA = (dN/dp_T)AA/N_{bin}(dN/dp_T)_{pp}, defined as the ratio of the yield in AA and pp collisions scaled by binary nucleon-nucleon collisions. The crucial ingredient for a reliable suppression prediction relies on a precise energy loss calculation taking due consideration of the expansion and viscosity of the QCD medium.

Early calculations of the medium-induced radiative energy loss were based on “static QCD medium” consisting of randomly distributed static scattering centers. In such a static medium, the collisional energy loss exactly vanishes. Subsequently, the radiative energy loss in a dynamically screened QCD medium was developed for an optically thick plasma [22] and finite-size optically thin plasma [23, 24], more relevant for rapidly expanding medium formed in relativistic heavy ion collisions. Radiative energy loss in a plasma was also shown to receive corrections because of modified dielectric effect of the medium known as Ter-Mikayelian effect [25]. Further, calculation of radiative energy loss suffers complication due to Landau-Pomeranchuck-Migdal (LPM) effect [26–28] which introduces a controlled reduction of emitted gluon formation time. As the increase of quark mass increases the phase-shifts and reduces the destructive nature of LPM, the effect was found prominent for light quarks and gluons [10, 13, 14, 29, 30].

The existing calculations on energy loss have been performed purely for an ideal fluid using the equilibrium phase-space distribution function, and ignoring viscous effects. Since the QGP formed in relativistic heavy-ion collisions behaves like a near prefect fluid with a small η/s, it is imperative to account for the viscous effects in computing the radiative energy loss. In fact, the impor-
tance of viscosity of the medium has already been realized in several quantities/observables relevant for RHIC and LHC, such as heavy quark damping rate [31], anisotropic flow [6–9], event-plane correlations [32, 33], dilepton spectra [41, 45] etc. While few calculations have incorporated radiative energy loss only for viscous medium evolution [13, 36], a realistic and consistent calculation where viscosity is explicitly included in the computation of energy loss as well as in the expanding viscous medium is crucial.

In this paper, we present the first calculation of radiative energy loss with viscosity, in first-order in opacity, of heavy (charm) quark in a dynamically screened viscous QCD medium that undergoes boost-invariant longitudinal expansion. We employ causal second-order viscous hydrodynamics for the underlying evolution of the medium based on the Müller-Israel-Stewart (MIS) framework [37–39] and the recently derived dissipative equations from Chapman-Enskog-like approach of iteratively solving the Boltzmann equation in the relaxation time approximation [40, 42]. Viscous effects are incorporated in the single-particle distribution $f(x, p) = f_0(x, p) + \delta f_{\text{vis}}(x, p)$, via the nonequilibrium distribution function $\delta f_{\text{vis}}$. The single-particle distribution would modify the scattering cross section of the energetic parton with the medium and thereby the radiative energy loss. For the nonequilibrium distribution, we use the commonly used form based on Grad’s 14-moment approximation [13] and that obtained from Chapman-Enskog method. We shall show that viscosity, in general, enhances the energy loss, the enhancement is significant in the Grad’s method. However, the magnitude of the total energy loss with viscosity included both in the dynamics and phase space distribution is smaller than that in the static limit. Further, we find nonlinearity in the time dependence of radiative energy loss for a viscous plasma, which mimics the energy loss behavior due to coherent gluon radiation.

The paper is organized as follows. In Sec. II we introduce the dissipative hydrodynamic formalisms used and then compute in these models, to first order in opacity, the radiative energy loss in a dynamical viscous QCD medium with boost-invariant longitudinal expansion. In Sec. III we compare the results for radiative energy loss in ideal and viscous fluids and with further inclusion of viscous corrections due to Grad and Chapman-Enskog methods, by using initial conditions relevant to that produced in heavy-ion collisions at LHC. In Sec. IV we summarize and conclude. The technical details of the computation of energy loss are presented in Appendices A–C.

II. RADIATIVE ENERGY LOSS IN AN EXPANDING VISCOUS MEDIUM

In this section we compute the medium induced heavy flavor radiate energy loss in the boost-invariant longitudinal expansion of matter within second-order viscous hydrodynamics. The hydrodynamic evolution is governed by the conservation of energy-momentum tensor, $\partial_\mu T^{\mu\nu} = 0$, where

$$T^{\mu\nu} = cu^{\mu}u^{\nu} - P\Delta^{\mu\nu} + \pi^{\mu\nu}. \quad (1)$$

We shall work in the Landau-Lifshitz frame and disregard particle flow due to very small values of net-baryon number formed at RHIC and LHC. In the above equation, $\epsilon$ and $P$ are respectively the energy density and pressure in the fluid’s local rest frame (LRF), and $\pi^{\mu\nu}$ is the shear pressure tensor. $\Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu}u^{\nu}$ is the projection operator on the three-space orthogonal to the hydrodynamic four-velocity $u^\mu$ defined by the Landau-matching condition $T^{\mu\nu}u_\nu = cu^\mu$.

For Bjorken longitudinal expansion, we work in the Mihotek coordinates $(\tau, x, y, \eta_8)$ where proper time $\tau = \sqrt{x^2 - \eta^2}$, space-time rapidity $\eta = \ln[(t + z)/(t - z)]/2$, and four-velocity $u^\mu = (1, 0, 0, 0)$. The conservation equation for the energy-momentum tensor gives the evolution of $\epsilon$:

$$\frac{d\epsilon}{d\tau} = -\frac{1}{\tau} (\epsilon + P - \Phi), \quad (2)$$

where $\Phi \equiv -\tau^2 \pi^{\eta_8\eta_8}$ is taken as the independent component of the shear pressure tensor. For the three independent variables, we need two more equations, namely, the viscous evolution equation and the equation of state (EoS). In this work, we have used a conformal QGP fluid EoS with thermodynamic pressure $P = \epsilon/3$. The simplest choice for the dissipative equation would be the relativistic Navier-Stokes theory, where the instantaneous constituent equation for the shear pressure in the Bjorken case gives

$$\Phi = \frac{4\eta}{3} \theta. \quad (3)$$

Here $\eta \geq 0$ is the shear viscosity coefficient and the local expansion rate $\theta = 1/\tau$. However, this first-order theory suffers from acausality and instability.

The most commonly used second-order dissipative hydrodynamic equation, derived from positivity of the divergence of entropy four-current, is based on the works of Müller-Israel-Stewart (MIS) [37–39]. In the boost-invariant scaling expansion, the MIS dissipative equation,

$$\frac{d\Phi}{d\tau} + \frac{\Phi}{\tau_\pi} = \frac{4\eta}{3\tau_\pi} \theta - \lambda_\pi \Phi, \quad (4)$$

restores causality by enforcing the shear pressure to relax to its first-order value via the relaxation time $\tau_\pi = 2\eta/\beta_2$, where $\beta_2$ is the second-order transport coefficient. In the present study we consider $\tau_\pi = 2\eta/\beta_2 = 5\eta/(sT)$ corresponding to that obtained in a weakly coupled QCD. Further, the coefficient of the second-order term (in the expansion of the velocity gradients) for EoS of an ultra-relativistic gas is $\lambda_\pi = 4/3$. In the derivation of Eq. (4), pertaining to a system that is out of equilibrium, the nonequilibrium effects have been quantified via the phase-space distribution, $f(x, p) = f_0(x, p) + \delta f_{\text{vis}}(x, p)$,
where the nonequilibrium part of the distribution function, \( \delta f_{\text{vis}}(x, p) \), is usually obtained by expanding \( f(x, p) \) about the equilibrium distribution function \( f_0(x, p) = \exp[(u \cdot p)/T - 1]^{-1} \). The Grad’s 14-moment approximation [13] is the common choice of viscous correction in hydrodynamics, where the expansion in powers of momenta is truncated at quadratic order. For a system of massless particles in the absence of bulk viscosity and charge diffusion current, the Grad’s method for viscous correction gives

\[
\delta f_{\text{vis}} = f_0(1 + f_0) \frac{\pi_\mu \pi_\nu p^\mu p^\nu}{2(e + p)T^2},
\]

\[
= f_0(1 + f_0) \frac{3\Phi}{4(e + p)T^2} \left( \frac{p^2}{3} - p_z^2 \right),
\]

where the second line is the equivalent representation for boost-invariant longitudinal expansion (assumed to be along \( z \)-direction) of the fluid in the LRF [14]. This equation has been exclusively used in deriving the second-order MIS dissipative equation.

Alternatively, dissipative evolution equations can be obtained from Chapman-Enskog-like (CE) method by perturbative expansion of the Boltzmann transport equation using Knudsen number as a small expansion parameter [10-12]. By expanding the nonequilibrium distribution function \( \delta f_{\text{vis}} \) about the local equilibrium value, and iteratively solving the Boltzmann equation in the relaxation-time approximation, the second-order dissipative equation for the shear tensor in the boost-invariant form gives

\[
\frac{d\Phi}{dt} + \frac{\Phi}{\tau_\pi} = \frac{4\eta}{3\tau_\pi} \theta - \lambda_\pi \theta \Phi.
\]

In the Chapman-Enskog-like approach, the relaxation time naturally comes out to be \( \tau_\pi = 2\eta \beta_2 = 5\eta / (sT) \) and \( \lambda_\pi = 3\beta / 21 \) [10]. The corresponding nonequilibrium distribution function has the form

\[
\delta f_{\text{vis}} = f_0(1 + f_0) \frac{5\pi_\mu \pi_\nu p^\mu p^\nu}{8PT(u \cdot p)}
\]

\[
= f_0(1 + f_0) \frac{15\Phi}{16PT(u \cdot p)} \left( \frac{p^2}{3} - p_z^2 \right)
\]

We shall present the calculation details of heavy quark radiative energy loss for the Müller-Israel-Stewart dissipative hydrodynamics with Grad’s form of \( \delta f_{\text{vis}}(x, p) \). The results within the Chapman-Enskog approach can be obtained in a similar fashion, which will be presented at the end of this section.

We shall compute the energy loss in a dynamical QCD medium for a thick expanding plasma in the opacity expansion. In a static plasma the energy loss is calculated by expansion over the number of parton scatterers in the medium times the transport cross section, integrated over the path length \( L \) traversed by the heavy quark. In an expanding medium, the total energy loss is obtained by summing the instantaneous energy loss over the time spent by the quark in the plasma before reaching vacuum or the survival time in the plasma.

In principle, boost invariance expansion induces anisotropy in the medium, hence the energy lost by a quark depends on its direction of propagation relative to the fluid flow [15]. In this paper, we consider the propagation of the heavy quark to be along the fluid direction and relegate to future work the calculation of complicated directional dependence of energy loss. As in Ref. [22, 24], we restrict ourselves to first order in opacity, where an on-shell heavy quark of mass \( M \) and spatial momentum \( p \gg M \) produced in the remote past traverses along fluid flow, i.e. \( z \)-direction. On scattering with a parton in the medium, it exchanges a virtual gluon of momentum \( q = (q_0, \vec{q}) = (q_0, q_z, \vec{q}) \) and radiates a gluon with momentum \( k = (\omega, \vec{k}) = (\omega, k_z, \vec{k}) \). The heavy quark then emerges along the \( z \)-direction with a momentum \( p' = (E', \vec{p}') = (E', p_z', \vec{p}') \). As the gluon momentum is spacelike \( (q_0 \leq |\vec{q}|) \) and the radiated gluon momentum is timelike \( (\omega \geq |\vec{k}|) \), these contribute accordingly in the gluon propagators \( D^{\mu \nu}(q) \) and \( D^{\mu \nu}(k) \), respectively. The validity of soft gluon, \( (\omega \ll E) \), and soft rescattering, \( (|\vec{q}| \sim |\vec{k}| \ll k_z) \), approximations at high temperature \( T \) at LHC together with the energy-momentum conservation, \( p = p' + k + q \), enables us to write

\[
k = \left( \omega \approx k_z + \frac{k^2 + m_g^2}{2k_z}, k_z, \vec{k} \right),
\]

\[
p' = \left( E' \approx p_z' + \frac{B^2 + M^2}{2p_z'}, p_z' - (k + q), \vec{p}' \right),
\]

\[
p = \left( E \approx p_z' + k_z + q_z + \frac{M^2}{2(p_z' + k_z + q_z)}, p_z' + k_z + q_z, 0 \right).
\]

Here \( m_g \approx \mu / \sqrt{2} = gT \sqrt{N_c/3 + N_f/6} \) is the effective gluon mass in a thermalized QGP at temperature \( T \) with Debye screening mass \( \mu \).

The heavy quark energy loss per unit proper time \( \tau \), to first order in opacity, can be obtained by folding the heavy quark interaction rate \( \Gamma(E) \) with the energy \( \omega + \gamma_0 \) and averaging over the initial color of the quarks [16, 17].

\[
\frac{dE_{\text{dyn}}}{dt} = \frac{1}{D_r} \int d\omega \omega \frac{d\Gamma(E)}{d\omega} \approx \frac{E}{D_r} \int dx \frac{d\Gamma(E)}{dx}.
\]

The soft rescattering approximation \( \omega + \gamma_0 \approx \omega \) has been used, and \( x \) is the longitudinal momentum fraction of the quark carried by the emitted gluon. \( D_r \) is defined as \( [t_a, t_c][t_c, t_a] = C_2(G)C_R D_R \) with \( C_2(G) = 3 \), \( D_R = 3 \) and \( [t_a, t_c] \) is a color commutator. The interaction rate is given by

\[
\Gamma(E) = \frac{1}{2E} 2 \text{Im} \mathcal{M}_{\text{tot}} = \frac{1}{2E} (2 \text{Im} \mathcal{M}_{1,0} + 2 \text{Im} \mathcal{M}_{1,1} + 2 \text{Im} \mathcal{M}_{1,2}).
\]

which can be obtained by computing all the Feynman diagrams (see Appendix A-C) that contributes at first order in opacity for the radiative energy loss. \( \mathcal{M}_{1,0}, \mathcal{M}_{1,1} \)
and $\mathcal{M}_{1,2}$ are the corresponding loop diagrams of the scattering amplitudes where, zero, one, two ends of the radiated gluon $k$ are attached to the exchanged gluon $q$ (see Figs. 6 and 7 for illustration).

In the high temperature plasma, the exchanged gluon receives correction from medium partons. This many-body effects gets encoded in the hard thermal loop (HTL) gluon propagators \cite{18}. The effective 1-HTL gluon propagator has the form \cite{19, 50}:

$$iD_{\mu\nu}(l) = \frac{P_{\mu\nu}(l)}{l^2 - \Pi_T(l)} + \frac{Q_{\mu\nu}(l)}{l^2 - \Pi_L(l)},$$

(11)

where the gluon-momentum $l = (l_0, \vec{l})$. The usual ideal part for the longitudinal and transverse self energies \cite{19, 50} are:

$$\Pi_T(l) = \mu^2 \left[ \frac{y^2}{2} + \frac{y(1 - y^2)}{4} \ln \left( \frac{y + 1}{y - 1} \right) \right],$$

$$\Pi_L(l) = \mu^2 \left[ 1 - y^2 - \frac{y(1 - y^2)}{4} \ln \left( \frac{y + 1}{y - 1} \right) \right],$$

(12)

where $y \equiv l_0/\bar{\mu}$. For typical values of $\eta/s \lesssim 3/(4\pi)$, required to explain flow data at RHIC and LHC, small viscous corrections in $\Pi_{L,T}$ are obtained, which can be safely ignored. The transverse $P_{\mu\nu}(l)$ and longitudinal $Q_{\mu\nu}(l)$ projectors in the Coulomb gauge, has the nonzero components $P_{0i}(l) = -\delta^{0i} + l^0 l^i/l^2 = 1 - y^2$. The imaginary part of the exchanged gluon propagator is

$$D_{\mu\nu}^{\gamma}(q) = (1 + f(q)) \frac{1}{2} \text{Im} \left[ \frac{P_{\mu\nu}(q)}{q^2 - \Pi_T(q)} + \frac{Q_{\mu\nu}(q)}{q^2 - \Pi_L(q)} \right] \times \theta \left( 1 - \frac{q_0^2}{q_s^2} \right).$$

(13)

The distribution function $f(q) = f_0(q) + \delta f_{\text{vis}}(q)$ receives strong viscous correction $\delta f_{\text{vis}}(q)$ due to Grad’s 14-moment approximation of Eq. (5); the equilibrium gluon momentum distribution function $f_0(q) = \exp(q_0/T) - 1)^{-1}$. For the radiated gluon, $\Pi_L(k) \approx 0$ and $\Pi_T(k) \approx m_g$. Using the soft scattering limit ($\omega \gg |q| \sim |k| \sim gT$), and noting that $f(k) \ll 1$ for energetic partons \cite{22}, the cut propagator for the imaginary part of the radiated gluon becomes

$$D_{\mu\nu}^{\gamma}(k) \approx -2\pi \frac{P_{\mu\nu}(k)}{2\omega} \delta(k_0 - \omega),$$

(14)

where $\omega \approx \sqrt{k^2 + m_g^2}$. The cut propagator for the heavy quark is

$$D^>(p') \approx 2\pi \frac{1}{2E'} \delta(p_0' - E').$$

(15)

With the help of these propagators one can calculate the matrix amplitude squared for the diagrams (see Appendix A-C). The phase space factor for the cut diagrams receives in-medium viscous corrections. On computing the diagrams, and in conjunction with Eqs. (9) and (10), one can obtain the heavy quark radiative energy loss.

The contribution to the energy loss from the first set of diagrams, $2\text{Im}M_{0,1}$, (see Eq. (A11)) is given by

$$\frac{1}{E} \frac{dE}{d\tau} \bigg|_{1,0} = \frac{12\alpha_s^2 C_R T}{\pi} \int dx \frac{d^2k}{k^2} \frac{k^2}{(k^2 + \chi)^2} \Lambda(\tau),$$

(16)

where $\chi = M^2 x^2 + m_g^2$, and the strong coupling constant $\alpha_s = g^2/(4\pi)$. The medium informations are encoded within the quantity

$$\Lambda(\tau) = \int d^2q \frac{d\eta}{(2\pi)^2} \left( \frac{1 + \Phi}{4\pi T^3} \right) \frac{q^2 (1 - 3y^2)}{1 - y^2} F_{LT}(q),$$

$$\equiv \Lambda_0(\tau) + \delta\Lambda_{\text{vis}}(\tau).$$

(17)

where $\Lambda_0(\tau)$ and $\delta\Lambda_{\text{vis}}(\tau)$ stem from ideal and viscous correction due to Grad’s 14-moment approximation \cite{5} for $P = \epsilon/3$ equation of state. We have used the shorthand notation, $F_{LT} = F_L - F_T$, for the difference of the polarization tensors $F_{LT} = 2\text{Im} \Pi_Z(y)[(q^2 + \text{Re}\Pi_Z(y))^2 + (\text{Im}\Pi_Z(y))^2]^{-1}$, with $z \equiv (L, T)$, in terms of the dimensionless variable of Eq. (A10), viz. $y = |q| \cos \theta_0(\tau_0/\tau) |q^2| \cos^2 \theta_0(\tau_0/\tau)^2 + q^4 - 1/2$. It is evident from the energy loss expression, that the nature of divergence gets modified from ideal to the viscous Bjorken case due to an extra $q^2$ factor stemming from $\delta\Lambda_{\text{vis}}$.

The diagram $M_{1,2}$, where emission of a gluon occurs from the exchanged gluon, has been computed in Appendix B. The corresponding radiative energy loss is given by (see Eq. (B4)):

$$\frac{1}{E} \frac{dE}{d\tau} \bigg|_{1,2} = \frac{12\alpha_s^2 C_R T}{\pi} \int dx \frac{d^2k}{k^2} \frac{\chi}{(k^2 + \chi)^2} \Lambda(\tau).$$

(18)

Finally, the diagrams for $M_{1,1}$ can be computed as the product of the previous two diagrams $M_{1,0}$ and $M_{1,2}$. The resulting radiative energy loss gives (Eq. (C6) in Appendix C):

$$\frac{1}{E} \frac{dE}{d\tau} \bigg|_{1,1} = \frac{12\alpha_s^2 C_R T}{\pi} \int dx \frac{d^2k}{k^2} \frac{-2k \cdot (k + q)}{|(k^2 + \chi)(k^2 + \chi)|} \Lambda(\tau).$$

(19)

The total energy loss is obtained by summing Eqs. (16), (18), (19) as

$$\frac{1}{E} \frac{dE}{d\tau} = \frac{4\alpha_s C_R}{\pi \lambda_{\text{dyn}}} \int dx \frac{d^2k}{k^2}$$

$$\times \left[ \frac{k}{k^2 + \chi} - \frac{k + q}{(k^2 + \chi)} \right] \Lambda(\tau),$$

$$\equiv \frac{4\alpha_s C_R}{\pi \lambda_{\text{dyn}}} \int dx \frac{d^2k}{k^2} P_g(x, k, q) \Lambda(\tau).$$

(20)

In the above equation, a dynamical mean free path has been defined as $\lambda_{\text{dyn}}^{-1} = C_2(G)\alpha_s T = 3a_s T$. In contrast to a time-independent QCD medium \cite{22, 24} where $\lambda_{\text{dyn}}$ is constant, in the present expanding medium $\lambda_{\text{dyn}}$ and thereby the density of scatterers has a time dependence via the temperature which modifies the energy loss.
Using the Chapman-Enskog results for the viscous evolution equation and the corresponding nonequilibrium distribution function in the Grad’s approximation. However, in the CE method, the quantity Λ of (17) is replaced by

\[ \Lambda(\tau) = \int \frac{d^3q}{(2\pi)^3} \left( 1 + \frac{5\Phi}{4sT} \right) \frac{q(1 - 3y^2)}{1 - y^2} \mathcal{F}_{LT}(\eta), \tag{21} \]

which involves the nonequilibrium form of the distribution function in the CE method.

### III. RESULTS AND DISCUSSIONS

In this section we estimate numerically the effects of expanding viscous medium on the radiative energy loss in first order in opacity for a dynamically screened QCD medium. We consider a plasma with an initial temperature \( T_0 = 400 \text{ MeV} \) and proper time \( \tau_0 = 0.4 \text{ fm/c} \) that corresponds to (averaged) values obtained in Pb+Pb collision at LHC. Charm quark of mass \( M = 1.2 \text{ GeV} \) is assumed to traverse in the plasma that has an effective number of degrees of freedom \( N_f = 2.5 \) with a constant strong coupling constant \( \alpha_s = g^2/4\pi = 0.3 \) and Debye screening mass \( \mu \sim gT \).

Considering the case of an non-expanding static fluid at a constant temperature \( T_0 = 400 \text{ MeV} \), the momentum dependence of fractional radiative energy loss \( E^{-1}dE/dL \) of the charm quark is shown in Fig. \( I \) (black solid line) \( 22 \). In boost invariant longitudinal expansion of an ideal fluid, the temperature decreases with time as \( T = T_0(\tau_0/\tau)^{1/3} \). This enforces a smaller fractional energy loss, \( E^{-1}dE/d\tau \), of the charm quark as seen at a later time \( \tau = 1.2 \text{ fm} \) in Fig. \( I \) (green solid line).

With the inclusion of dissipation in the dynamical evolution, the temperature decreases at a slower rate and the entropy increases as compared to an inviscid fluid. In Fig. \( I \) we present the fractional radiative energy loss of charm quark in an expanding viscous medium with \( \eta/s = 1/4\pi \) at \( \tau = 1.2 \text{ fm/c} \) in the MIS (red dashed lined) and CE (blue dashed line) in absence of nonequilibrium part of the distribution function (i.e. \( \delta f_{\text{vis}} = 0 \)). Dissipative effects is seen to cause \( \geq 5\% \) larger energy loss for charm quarks with momentum \( p \geq 10 \text{ GeV} \) as compared to that with ideal flow. Such an enhanced energy loss may be attributed to a relatively higher instantaneous temperature of the viscous plasma. Although the temperature in the CE approach falls slightly faster with time as compared to that in the MIS theory, the energy losses in these viscous evolution frameworks are found to be practically insensitive.

Figure \( I \) also shows the fractional energy loss obtained by inclusion of viscous corrections in the single-particle distribution function using the Grad (red solid line) and Chapman-Enskog (blue solid line) methods at \( \tau = 1.2 \text{ fm/c} \). We find that nonequilibrium correction induces a significant increase in the energy loss; albeit the magnitude of \( E^{-1}dE/d\tau \) is still smaller than in a static fluid. The enhancement is particularly large for Grad’s 14-moment approximation as compared to the Chapman-Enskog correction for heavy quark momentum \( p \gtrsim 10 \text{ GeV} \).
for charm quarks propagating in a boost-invariant expanding fluid over a total time of \( \tau_f = 5 \) fm/c with various values of \( \eta/s \) in the Müller-Israel-Stewart (MIS) (top panel) and Chapman-Enskog (CE) (bottom panel) frameworks. Also shown is the fractional energy loss in a static plasma integrated over a path length of \( L = 5 \) fm. The initial conditions are the same as in Fig. 3.

This can be understood by comparing the (positive) contribution from viscous correction \( \delta f_{\text{vis}} \) to the energy loss in Grad and Chapman-Enskog approaches, namely, Eqs. (17) and (21). An extra factor \( q/5T \) in the integrand of \( \delta A_{\text{vis}}(\tau) \) in Grad’s method gives a larger energy loss. Moreover, this energy loss is seen to rapidly increase with the momentum \( p = (\sqrt{E^2 - M^2}) \) of the charm quark as the limit of integration \( q_{\text{max}} = \sqrt{4ET} \). On the other hand, the energy loss obtained in the Chapman-Enskog viscous correction shows a similar saturation pattern as that seen in an ideal fluid and in viscous medium with \( \delta f_{\text{vis}} = 0 \). At \( p < 10 \) GeV/c the energy loss has identical behavior for the two viscous corrections used here. Large viscous corrections due to Grad’s 14-moment approximation have been also found in the spectra and elliptic flow of hadrons at kinetic freezeout [6, 52, 53], as well as in the longitudinal Hanbury-Brown-Twiss-Radii radii of pions [41].

Figure 2 displays the proper time dependence of fractional radiative energy loss, \( E^{-1}dE/d\tau \), for a charm quark of momentum \( p = 20 \) GeV/c. As expected, the energy loss at all times in the expanding medium is smaller than in a static fluid. With increasing time, the decrease in the energy loss is essentially due to falling temperature. As discussed above, we find viscous medium induces a somewhat larger energy loss as compared to an ideal fluid at all times. Although at early times \( \tau \lesssim 5 \) fm/c, the MIS dissipative hydrodynamics with viscous correction results in maximum energy loss, at later times all the viscous fluids give nearly identical energy losses mainly due to negligibly small shear pressure tensor in the dilute medium. Of course, for charm quarks with momentum \( p > 20 \) GeV/c, the differences in \( E^{-1}dE/d\tau \) will sustain at large times as evident from Fig. 3.

We show in Fig. 3 the charm quark momentum dependence of the (total) fractional energy loss \( \Delta E/E \) at various values of \( \eta/s \) in the MIS and CE theories. The total \( \Delta E \) is obtained by summing the energy loss during the entire time traversed by the quark. In the present calculation we set this time as \( \tau_f = 5 \) fm/c, as the typical lifetime of the QGP phase at RHIC and LHC. On the other hand, in a static fluid, \( \Delta E \) refer to energy loss integrated over a path length of \( L = 5 \) fm. In an expanding medium the scattering rates decrease resulting in smaller \( \Delta E/E \) as compared to the static case. However, large viscous corrections give positive contribution to the energy loss that grows with \( \eta/s \), especially when nonequilibrium part of distribution function in the Grad’s approximation is considered. In fact, the energy loss results obtained in various dissipative hydrodynamic medium fall between those in the static fluid and ideal hydrodynamics.

It is important to note that in a static medium, the total energy loss can be expressed as \( \Delta E \sim S(E) \int dL/(3\alpha_s T) \), where \( S(E) \) is a function of the energy of charm quark. Hence the total energy loss increases linearly with the path length \( L \) traversed by the quark which is QCD analogue of QED Bethe-Heitler limit [11]. On the other hand, the corresponding energy loss for a time-dependent viscous medium may be written as \( \Delta E \sim \int d\tau \mathcal{C}(E, \tau, \eta)/(3\alpha_s T(\tau)) \), where \( \mathcal{C}(E, \tau, \eta) \) is a complicated function encompassing medium effects.
IV. SUMMARY

In this work we have presented a theoretical formulation of the radiative energy loss of heavy quark traversing in a viscous medium that undergoes boost-invariant longitudinal expansion. The calculation was performed in first order in opacity for a dynamically screened QCD plasma at finite temperature. We have derived the radiative energy loss by including two forms of viscous correction in the nonequilibrium phase-space distribution, namely the Grad’s 14-moment approximation and Chapman-Enskog (CE) method. Viscous contributions from dynamics only, in absence of viscous corrections in the single-particle phase-space distribution, resulted in the enhancements of the fractional energy loss energy by about 5% depending on the shear viscosity to entropy density ratio of $\eta/s = 0.08 - 0.24$. This energy loss was found to be similar in the MIS and CE dissipative hydrodynamic models. At the early stages of evolution, we found that inclusion of Grad’s approximation of viscous correction in the distribution function resulted in appreciably large increase of fractional energy loss that increased monotonically with momentum $p$ of the charm quark. On the other hand, in the Chapman-Enskog viscous correction, the enhancement was found to be comparatively smaller, and the energy loss was seen to saturate for $p \geq 10$ GeV/c. At later proper times, the energy losses in all the scenarios were found comparable due to small temperature and nearly vanishing shear stress tensor. The time integrated fractional energy loss in the Grad’s approximation was found higher than in the Chapman-Enskog method. The heavy quark radiative energy loss results presented in this work is crucial for the interpretation of D-meson nuclear modification factor.

Appendix A: Computation of diagrams $M_{1,0}$ and associated radiative energy loss in MIS theory

We present detailed calculation of the first set of diagrams corresponding to $M_{1,0}$. In general, we denote all the loop diagrams as $M_{1,i,j}$, where $i$ refers to the number of the exchanged gluon $q$ that are attached to the radiated gluon $k$, and $j = a, b, \ldots$ denotes the particular diagram in that class, computed in first order in opacity denoted by 1. The Feynman diagrams for the first set, namely $M_{1,0,a}$, $M_{1,0,b}$, $M_{1,0,c}$ and $M_{1,0,d}$ are shown in Fig. 5. These scattering diagrams are associated with two-cut HTL loop diagrams. We first compute the cut diagram $M_{1,0,a}^\gamma = 2 \text{Im} \ M_{1,0,a}$ (see Fig. 6).

$$M_{1,0,a}^\gamma = g^4 t_a t_c t_a \int \frac{d^4p'}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4}$$

$$\times \frac{2^\delta (p - p' - k - q)}{\pi^4} \left[ D_{\rho\nu}^\gamma(q) D^{\rho\nu}_{\sigma\rho}(k) - D_{\rho\sigma}^\gamma(q) D^{\rho\nu}_{\nu\rho}(k) - D_{\rho\nu}^\gamma(q) D^{\rho\nu}_{\nu\rho}(k) + D_{\rho\sigma}^\gamma(q) D^{\rho\nu}_{\sigma\rho}(k) \right].$$

($A1$)

FIG. 5. Feynman diagrams $M_{1,0}$ (a)-(d) contributing to heavy quark radiative energy loss to first order in opacity.

FIG. 6. Left: Scattering amplitude of $M_{1,0,a}$ diagram where a heavy quark of momentum $p$ suffers collisional interaction with medium partons via screened gluon of momentum $q$, resulting in emission of a gluon of momentum $k$ from outgoing quark. The blob represents medium modified gluon propagator. Right: HTL loop diagram of first order in opacity corresponding to $M_{1,0,a}$. Heavy quark scatters from medium partons via cut gluon propagator of momentum $q$ (with $q_0 \leq |q|$) resulting in emission of a cut gluon propagator with momentum $k$ (with $\omega > |k|$). The imaginary part of the diagram corresponds to the squared amplitude of the left diagram and integrated over phase-space.
The above equation consists of two parts: the medium interaction and the phase space factor. The interaction history is encoded in the exchanged and radiated gluon propagators $D_{\mu\nu}(q)$ and $D_{\mu\rho}(k)$, respectively; $D(p'), D(p' + k)$ are the fermionic propagators. To proceed further we write the vector contraction as

$$(2p' + k)^\mu P_{\mu\rho}(k)(2p' + k)^\sigma \approx 2p'^\mu P_{\mu\rho}(k)2p'^\sigma \approx -4 \left( \frac{p'^2 - (\vec{p}' \cdot \vec{k})^2}{|k|^2} \right), \quad (A2)$$

where we have $k^\rho P_{\mu\rho}(k) = 0$. By choosing the coordinate axis $\vec{q} = (|q|) (\sin \theta_q \cos \phi_q, \sin \theta_q \sin \phi_q, \cos \theta_q)$, $\vec{k} = (|k|) (\sin \theta_k \cos \phi_k, \sin \theta_k \sin \phi_k, \cos \theta_k)$ and $\vec{p}'$ along $z$ direction, one can evaluate the terms within the braces of Eq. (A2) as

$$\frac{p'^2 - (\vec{p}' \cdot \vec{k})^2}{|k|^2} \approx \frac{p'^2k^2}{k^2 + p'^2z^2} \approx \frac{k^2}{x}, \quad (A3)$$

where $x = k_z/p'_z$. Similarly, for the vector contraction with the exchanged gluon term one can write

$$p'^\mu \text{Im} P_{\mu\rho}p'^\rho \approx \frac{E^2q^2}{q^2 + q_x^2} \approx -p'^\mu \text{Im} Q_{\mu\rho}p'^\rho. \quad (A4)$$

Other approximations which we use are $q_x \sim |q|, |k| \ll k_z, q_x \ll k_z$. The longitudinal component of the emitted and radiated gluons obeys the following approximations, $k_z + q_x = k_z, p'_z + k_z + q_z \approx p'_z + k_z \approx k_z$ and $p'_z + q_z \approx p'_z$. For the energy delta function we thus obtain,

$$\delta(E - E' - \omega - q_0) \approx \delta(q_x - q_0). \quad (A5)$$

While writing the above equation it has been assumed that $M^2/2p'_z \ll 1, (k^2 + m_y^2)/2k_z \ll 1, ((k + q)^2 + M^2)/2p'_z \ll 1$. Similarly, for the propagator one can write,

$$(p' + k)^2 - M^2 = 2 \left( p'_z + \frac{(k + q)^2 + M^2}{2p'_z} \right) \times \left( k_z + \frac{k^2 + m_y^2}{2k_z} \right) - 2[k_z p'_z + k \cdot (-k + q)] \approx \frac{k^2 + M^2x^2 + m_y^2}{x}. \quad (A6)$$

By using Eqs. (A2)-(A6), along with Eqs. (14)-(15), the Eq. (A1) reduces to

$$M_{1,0} = 16g^4 t_a t_c t_at_c ET \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{2\omega \left( k_0^2 + M^2x^2 + m_y^2 \right)^2} \times (1 + f_q) \left( \frac{E^2q^2}{q^2 + q_x^2} \right) \left\{ 2\text{Im} \left( \frac{1}{q^2 - \Pi_L(q)} \right) - 2\text{Im} \left( \frac{1}{q^2 - \Pi_T(q)} \right) \right\} \times 2\pi \delta(p'_0 - p_0 - k_0 - q_0) \theta \left( 1 - \frac{q_0^2}{q^2} \right). \quad (A7)$$

In presence of viscous correction due to Grad \[5\] and in-medium modifications \[14\], the bosonic distribution function becomes

$$f(q) = f_0(q) + \frac{3\Phi}{4sT^3} \left[ \frac{q^2 + q_x^2(\tau_0/\tau)^2}{\tau_0} - \frac{q_x^2}{\tau_0^2} f_0(q)(1 + f_0(q)) \right], \quad (A8)$$

where $q_z = |q| \cos \theta_q$. In the high temperature plasma and small $q_0$, the equilibrium part of the distribution function can be approximated as $f_0(q)(1 + f_0(q)) \approx f_0(q) \approx 1/(1 + q_0/T - 1) \approx T/q_0$. Since $q_z^2 = q_x^2 - q_z^2 \approx q_0^2$, and using the delta function, we can write $q_0 \approx q_x, q_z^2 \approx -q_0^2$. To proceed, we have introduced a dimensionless variable $y = q_0/q \approx q_x(q)/q_z$, which can be also written as

$$y = \frac{|q| \cos \theta_q (\tau_0/\tau)}{\sqrt{q^2 + q_x^2(\tau_0/\tau)^2 + q_z^2}}. \quad (A9)$$

Limits on $y$ are decided by $\cos \theta_q \approx y \in [y_{\min}, y_{\max}]$. On performing the $p_0, k_0$ and $q_0$ integrations, we finally get,

$$M_{1,0} = 8g^4 t_a t_c t_at_c ET \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{2\omega \left( k_0^2 + M^2x^2 + m_y^2 \right)^2} \times \int qdq \ dy \ do \left\{ \frac{3\Phi}{4sT^3} \left( 1 + y^2 \right) \right\} \times \left\{ 2\text{Im} \Pi_L(y) \left( q^2 + \text{Re} \Pi_L(y) \right) + \left( \text{Im} \Pi_L(y) \right)^2 \right\} - \left\{ 2\text{Im} \Pi_T(y) \left( q^2 + \text{Re} \Pi_T(y) \right) + \left( \text{Im} \Pi_T(y) \right)^2 \right\}. \quad (A10)$$

It can be shown that contribution from the other three diagrams, $M_{2,0}^\tau, M_{3,0}^\tau, M_{4,0}^\tau$, has the same result but for the color factor. On summing all the four diagrams and using Eqs. (9) and (10), the heavy quark radiative energy loss with the Grad's viscous correction for this set:

$$\frac{1}{E} \frac{dE}{dT} = 2g^4 \left( t_a, t_c \right) \left[ t_c, t_a \right] \int_0^1 dx \left( k_0 \right)^{1/2} \frac{1}{k_0} \int_0^{2\pi} d\phi_k \int_0^{\pi/4} \int_{y_{\min}}^{y_{\max}} dy \left\{ \left( 1 + \frac{\Phi}{4sT^3} \right) \left( q^2 + \text{Re} \Pi_L(y)^2 + \left( \text{Im} \Pi_L(y) \right)^2 \right) \right\} \times \left\{ 1 + \frac{\Phi}{4sT^3} \left( q^2 + \text{Re} \Pi_T(y)^2 + \left( \text{Im} \Pi_T(y) \right)^2 \right) \right\} \frac{k^2}{\left( k^2 + M^2x^2 + m_y^2 \right)^2} \times \int_0^{\pi} d\theta_k \left( q^2 + \text{Re} \Pi_L(y)^2 + \left( \text{Im} \Pi_L(y) \right)^2 \right) \frac{k^2}{\left( k^2 + M^2x^2 + m_y^2 \right)^2}. \quad (A11)$$

With the help of the commutator relation, $[t_a, t_c] [t_c, t_a] = 3C_R D_R$, and defining the strong coupling constant $\alpha_s = g^2/(4\pi)$, the coefficient in front of the integral can be written as $3\alpha_s^2 C_R T^3/\pi^3$. We use the notation $F_{LT} = F_L - F_T$ for the difference of the polarization tensors $F_Z = 2\text{Im} \Pi_Z(y)(q^2 + \text{Re} \Pi_Z(y)^2 + \left( \text{Im} \Pi_Z(y) \right)^2)^{-1}$, with $Z = (L, T)$. The upper limits of integration are set to $q_{max} = \sqrt{ET}$ and $k_{max} = 2E\sqrt{1 - x}$ [31].
We follow similar algebra for vector contraction as used if due to Grad (see Eq. (A8)), one can compute the diagonal where we have defined

\[ \mathcal{M}_{1,2} = 2 \text{Im} \mathcal{M}_{1,2}, \]

where we have defined

\[ H = (2p-k)^\mu (2p'-k')^\nu D_{\mu \nu}(k') D_{\sigma \tau}(q) D_{\sigma \tau}(k') \]

\[ \times \left( g^{\sigma \tau}(k'+q)^\lambda + g^{\lambda \tau}(k-q)^\sigma - g^{\lambda \sigma}(k'+k)^\tau \right) \]

\[ \times \left( g^{\sigma \beta}(k'+q)^\alpha + g^{\alpha \beta}(k-q)^\sigma - g^{\alpha \sigma}(k'+k)^\beta \right). \]

(B2)

We follow similar algebra for vector contraction as used in Eqs. (A2)-(A6), the fermionic propagator can be expressed as

\[ (k+q)^2 - m_g^2 = m_g^2 - q^2 + 2 \left( k_z^2 + m_g^2 \right) \]

\[ \times \left( q_z^2 - k_z^2 + M^2 x^2 + m_g^2 \right) \]

\[ -2k_z q_z - 2k_z m_g^2 \]

\[ \approx -[(k+q)^2 + M^2 x^2 + m_g^2]. \]

(B3)

Further, using \( i \epsilon \delta_{abc} t_a t_c = [t_a, t_c] \) and the viscous correction due to Grad (see Eq. (A8)), one can compute the diagram of Eq. (B1). The corresponding radiative energy loss in Grad's 14-moment approximation:

\[ \frac{1}{E} \frac{dE}{d\tau} = \frac{2g^4 T [t_a, t_c][t_c, t_d]}{(2\pi)^5 D_R} \int 0 dx \int 0 kdk \int 0 d\phi_k \int 0 d\phi_k \int 0 d\phi_k \]

\[ \times q dq \int 0 d\phi_q \int 0 d\int \right]

\[ \times \left( 1 + \frac{\Phi}{4sT^3} \frac{q^2 (1 - 3y^2)}{1 - y^2} \right) F_{L.T}. \]

(B4)

**Appendix C: Computation of diagrams \( M_{1,1} \) and corresponding radiative energy loss**

We present calculations of the diagrams \( M_{1,1} \) where one of the ends of the exchanged gluon \( q \) is attached to the radiated gluon. This can be evaluated as the product of the previous two diagrams. For the first diagram \( M_{1,1,a} \) one can express

\[ M_{1,1,a} \approx \frac{g^4}{2E} f c^a t_b t_c t_d \int d^4p' d^4q d^4k \]

\[ \times \frac{1}{(p'+k)^2 - M^2 - i\epsilon} \]

\[ \times (2\pi)^4 \delta^4(p-p'-k-q) D_{\sigma \tau} (p') G, \]

(C1)

where we denote

\[ G \approx \left[ (2p-k)^\mu (2p'+k')^\nu D_{\sigma \tau}(q) \right] \]

\[ \times D_{\mu \nu}(k') D_{\mu \nu}(k') \]

\[ \times \left( g^{\sigma \tau}(k'+q)^\lambda + g^{\lambda \tau}(k-q)^\sigma - g^{\lambda \sigma}(k'+k)^\tau \right), \]

\[ \equiv G_1 + G_2 - G_3. \]

(C2)

Here,

\[ G_1 = \left[ (2p-k)^\mu D_{\sigma \tau}(k) (2p-q)^\nu \right] \]

\[ \times \left[ (k'+q)^\lambda D_{\sigma \tau}(k') (2p'+k')^\nu \right], \]

(C3)

\[ G_2 = \left[ (2p-k)^\mu D_{\sigma \tau}(k') (k-q)^\nu \right] \]

\[ \times \left[ (2p'+k')^\nu D_{\sigma \nu}(k) D_{\sigma \tau}(q) \right], \]

(C4)

\[ G_3 \approx \left[ (2p-k)^\lambda D_{\sigma \nu}(k) (2p'+k')^\nu \right] \]

\[ \times \left[ (k'+q)^\mu D_{\sigma \tau}(q) (2p-q)^\nu \right]. \]

(C5)

We consider only \( G_3 \) as it gives a dominant contribution in the approximations involving the kinematics noted in Appendix A. With the help of the above equations and viscous correction Eq. (A8), one can compute the energy loss for the diagram \( M_{1,1,a} \). The energy loss for the other diagrams in this set, \( M_{1,1,b}, M_{1,1,c}, M_{1,1,d} \) can be calculated accordingly. On summing all the four diagrams we get the total energy loss for this set as

\[ \frac{1}{E} \frac{dE}{d\tau} \left|_{1,1} \right. = \frac{2g^4 T [t_a, t_c][t_c, t_d]}{(2\pi)^5 D_R} \int 0 dx \int 0 kdk \int 0 d\phi_k \int 0 d\phi_k \int 0 d\phi_k \]

\[ \times q dq \int 0 d\phi_q \int 0 d\int \right]

\[ \times \left( 1 + \frac{\Phi}{4sT^3} \frac{q^2 (1 - 3y^2)}{1 - y^2} \right) F_{L.T}. \]

(C6)
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