The theta-dependent Yang-Mills theory at finite temperature in a holographic description

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Abstract

The theta-dependent gauge theories can be studied by using holographic duality through string theory on certain spacetimes. Via this correspondence we consider a stack of $N_0$ dynamical D0-branes as D-instantons in the background sourced by $N_c$ coincident non-extreme black D4-branes. According to the gauge-gravity duality this D0-D4 brane system corresponds to Yang-Mills theory with a theta angle at finite temperature. We solve the IIA supergravity action by taking account into a sufficiently small backreaction of the D-instantons and obtain an analytical solution for our D0-D4-brane configuration. Then the dual theory in the large $N_c$ limit can be holographically investigated with the gravity solution. In the dual field theory, we find the coupling constant exhibits the property of asymptotic freedom as it is expected in QCD. The contribution of the theta-dependence to the free energy gets suppressed at high temperature which is basically consistent with the calculation by using the Yang-Mills instanton. The topological susceptibility in the large $N_c$ limit vanishes and this behavior remarkably agrees with the implications from the simulation results at finite temperature. Besides we finally find a geometrical interpretation of the theta-dependence in this holographic system.

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1 Introduction

The spontaneous CP violation in Quantum Chromodynamics (QCD) has been studied for very long time and such effects can usually be described by introducing a $\theta$ term to the four dimensional (4d) action for the gauge theories as \[ S = -\frac{1}{2g_{YM}^2} \text{Tr} \int F \wedge^* F + i \frac{\theta}{8\pi^2} \text{Tr} \int F \wedge F, \] where $g_{YM}$ is Yang-Mills coupling constant and the second term defines the topological charge density with a $\theta$ angle. While the experimental value of the theta angle is stringently small ($|\theta| \leq 10^{-10}$), the dependence of Yang-Mills theory and QCD on theta attracts great theoretical and phenomenological interests, e.g. the study of the large $N_c$ behavior \cite{2}, the glueball spectrum \cite{3}, the deconfinement phase transition \cite{4, 5} and the Schwinger effect \cite{6}. Particularly there is an open question in hadron physics, that is whether a theta vacua can be created in hot QCD. To figure out this issue, there have been some progresses achieved in \cite{7, 8, 9, 10, 11, 12} and one of the most famous proposal is to search for the chiral magnet effect (CME) in heavy-ion collisions \cite{13, 14, 15, 16} to confirm the theta dependence at high temperature.

On other hand the AdS/CFT correspondence, or more generally the gauge-gravity (string) duality, has rapidly become a powerful tool to investigate the strongly coupled quantum field theory (QFT) since 1997 \cite{17, 18, 19}. In the holographic approach to study QCD or Yang-Mills theory, a concrete model is proposed by Witten \cite{20} and Sakai and Sugimoto \cite{21, 22} (named as WSS model) based on the IIA string theory. This model is significantly successful since it almost includes all necessary ingredients of QCD or Yang-Mills theory in a very simple way, e.g. the fundamental quarks and mesons \cite{21, 22, 23}, baryon \cite{24, 25}, the phase diagram of hot QCD \cite{26, 27, 28, 29, 30}, glueball spectrum \cite{31, 32} and the interactions of hadrons \cite{33, 34, 35, 36, 37, 38}. Due to the non-perturbative properties of the theta dependence, it has
been recognized that the D-branes as D-instantons in the bulk geometry plays the role of the theta angle in the dual theory [39, 40, 41]. Via this viewpoint, the holographic correspondence of theta-dependence in QCD or Yang-Mills theory has been systematically studied by using the WSS model with D0-branes as D-instantons at zero temperature, or without temperature in [42, 43, 44, 45, 46, 47, 48, 49, 50].

Table 1: The configuration of \( N_0 \) smeared D0 and \( N_c \) black D4-branes with the compactified direction \( x^4 \). The “-” represents that the D-branes extend along this direction and “=” represents the direction where the D0-branes are smeared.

|        | 0 | 1 | 2 | 3 | 4 | 5(\( \rho \)) | 6 | 7 | 8 | 9 |
|--------|---|---|---|---|---|-------------|---|---|---|---|
| \( N_0 \) smeared D0-branes | = | = | = | = | - |             |   |   |   |   |
| \( N_c \) black D4-branes   | - | - | - | - | - |             |   |   |   |   |

To analyze the theta dependence at finite temperature, there have been various researches by using simulations and the results imply some large \( N_c \) behaviors are different from the situations of zero temperature or without temperature [1]. In the current status of the holographic approaches, the theta dependence at finite temperature is studied mostly in the \( \mathcal{N} = 4 \) super Yang-Mills theory by the D(-1)-D3 brane configuration e.g. [39, 51, 52]. In the opposite, few lectures discuss specifically QCD or Yang-Mills theory at finite temperature through the D0-D4 brane configuration. So we are motivated to fill this blank by exploring a way to combine the theta-dependent Yang-Mills at finite temperature with the IIA string theory. In our setup, we adopt the gravity background sourced by a stack of \( N_c \) black non-extreme D4-branes since the dual field theory in this background exhibits deconfinement at finite temperature [26]. Then we introduce \( N_0 \) coincident D0-branes as D-instantons into the D4-brane background by taking account into a very small backreaction to the bulk geometry. Hence the D-instantons are dynamical and this setup is coincident with the bubble D0-D4 configuration in [42, 43, 44, 45, 46, 47, 48, 49, 50]. In order to search for an analytical supergravity solution, we further assume that the D0-branes are homogeneously smeared in the worldvolume of the D4-branes and this D-brane configuration is illustrated in Table 1. Solving the effective 1d gravity action, we indeed obtain a particularly analytical solution, then we examine the coupling constants and renormalized ground-state energy by the gravity solution. The coupling constant indicates the property of asymptotic freedom and the free energy gets suppressed at high temperature. Besides the topological susceptibility in the large \( N_c \) limit vanishes. We find that all these results agree with the implications of simulation reviewed in [1], or the well-known properties of QCD and Yang-Mills theory, thus our work might offer a holographic way to study the issues proposed in [7, 8, 9, 10, 11, 12, 13, 14, 15].

Here is the outline of this manuscript. In Section 2, we first discuss the general formulas of

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\( ^2 \)Here we emphasize that the dual theory in the approach of the bubble D0-D4 configuration is defined at zero-temperature limit, or defined without a concrete temperature according to [26, 29, 30]. The dual theory includes a finite temperature is the distinguishing feature in our setup.
the black D0-D4 system based on the IIA supergravity. Then comparing them with the black
D4-brane solution, we obtain a particular solution by including some physical constraints. In
Section 3, we evaluate the coupling constant and free energy density by our gravity solution.
We also give an geometric interpretation of the theta-dependence in this D0-D4 system. The final
Section is the summary and discussion. Our gravity solution expressed in the $U$ coordinate is
summarized in the Appendix.

2 The supergravity description

2.1 General setup

In this section, let us explore the holographic description based on the
$N_0$ D0- and $N_c$ D4-branes with the configuration illustrated in Table 1. As the gauge-gravity duality is valid in the large $N_c$
limit, we can first define the 4d ‘t Hooft coupling as
$\lambda_4 = g_{YM}^2 N_c$ where $g_{YM}$ is the Yang-Mills
coupling and $\lambda_4$ is fixed when $N_c \to \infty$. Then in order to take into account a small backreaction
of the $N_0$ D0-branes, we further require $N_0 \to \infty$ while

$$\frac{N_0}{N_c} = C \text{ fixed, } C \ll 1.$$ (2.1)

Here $C$ is a fixed constant in the limitation of $N_c, N_0 \to \infty$ and we note that this limit is similar
as the Veneziano limit discussed in [29, 30]. Keeping this in mind, let us consider the dynamics
of the 10d bulk geometry which is described by the type IIA supergravity. In string frame the
action is given as,

$$S_{IIA} = \frac{1}{2\kappa_0^2} \int d^10 x \sqrt{-g} \left[ e^{-2\phi} \left( R + 4 (\partial\phi)^2 \right) - \frac{1}{2} |F_2|^2 - \frac{1}{2} |F_4|^2 \right],$$ (2.2)

where $2\kappa_0^2 = (2\pi)^7 l_s^8$ and $l_s = \sqrt{\alpha'}$ is the string length. $F_4 = dC_3, F_2 = dC_1$ is the Ramond-
Ramond four and two form sourced by the $N_c$ D4-branes and $N_0$ D0-branes. We have used $R$
and $\phi$ to denote the 10d scalar curvature and the dilaton field respectively. Since the D0-branes
as D-instantons are extended along $x^4$ direction and homogeneously smeared in the directions
of $\{x^0, x^i\}, i = 1, 2, 3$, we may search for a possible solution by using the metric ansatz written as [26, 29, 30],

$$ds^2 = -e^{2\lambda} dt^2 + e^{2\lambda} \delta_{ij} dx^i dx^j + e^{2\lambda} (dx^4)^2 + l_s^2 e^{-2\phi} dp^2 + l_s^2 e^{2\nu} d\Omega_4^2.$$ (2.3)

The the Ramond-Ramond $C_1$ form and its field strength $F_2$ is assumed to be,

$$C_1 = [h (\rho) + H] dx^4,$$
$$F_2 = dC_1 = \partial_\rho h dx^4 \wedge dp,$$ (2.4)
where $H$ is a constant and $h(\rho)$ is a function to be solved. To find a static and homogeneous solution by the ansatz (2.3), we further assume that the functions $\tilde{\lambda}, \lambda, \lambda_s, \varphi, \nu$ and the dilaton $\phi$ only depend on the holographic coordinate $\rho$. Hence the action (2.2) could be rewritten as an effective 1d action by inserting (2.3) (2.4) into (2.2) which leads to,

$$S_{IIA} = V \int d\rho \left[ -3\dot{\lambda}^2 - \dot{\lambda}_s^2 - \dot{\phi}^2 - 4\nu^2 + \frac{1}{2} e^{3\lambda + \tilde{\lambda} - \lambda_s + 4\nu + \varphi} \dot{h}^2 + V + \text{total derivative} \right].$$

(2.5)

We have used “.” to represent derivatives which are w.r.t. $\rho$ and,

$$V = 12e^{-2\nu-2\varphi} - Q_c^2 e^{3\lambda + \lambda_s + \tilde{\lambda} - 4\nu - \varphi}, \quad V = \frac{1}{2\kappa_0^2} V_3 V_{S^4} \beta_4 \beta_T l_3^3,$$

$$\varphi = 2\phi - 3\lambda - \tilde{\lambda} - \lambda_s - 4\nu, \quad Q_c = \frac{3\pi^2 l_s}{\sqrt{2}\kappa_0} \int_{S^4} F_4.$$

(2.6)

Here $\beta_4, \beta_T$ refers to the size of (time) $x^0$ and $x^4$ directions, $V_3$ represents the 3d spacial volume and $V_{S^4} = \frac{8\pi^2}{3}$ is the volume of a unit $S^4$. Then the solution for $C_1$ may immediately be obtained as,

$$\dot{h}(\rho) = -q_\theta e^{2\lambda_s - 2\phi},$$

(2.7)

where $q_\theta$ is an integration constant related to the $\theta$ angle and this would become more clear later. Note that the 1d action (2.5) has to be supported by the following zero-energy constraint

$$-3\dot{\lambda}^2 - \dot{\lambda}_s^2 - \dot{\phi}^2 - 4\nu^2 + \frac{1}{2} e^{3\lambda + \tilde{\lambda} - \lambda_s + 4\nu + \varphi} \dot{h}^2 - V = 0,$$

(2.8)

so that the equations of motion from the 1d effective action (2.5) would be coincident with those from the 10d action (2.2) if the homogeneous ansatz (2.3) is adopted.

Afterwards the full equations of motion could be obtained by varying the 1d action (2.5) which are given as,

\footnote{Since we would consider a 4d dual field theory at finite temperature, the $x^0$ and $x^4$ directions have to be compactified on $S^1$ as discussed in \cite{26, 27, 28, 29, 30} in this model. And $\beta_4, \beta_T \to \infty$ corresponds to the decompactified limit.}
\[\ddot{\lambda} - \frac{Q^2}{2} e^{6\lambda + 2\lambda_s + 2\tilde{\lambda} - 2\phi} = \frac{q_o^2}{4} e^{2\lambda_s - 2\phi},\]
\[\ddot{\lambda}_s - \frac{Q^2}{2} e^{6\lambda + 2\lambda_s + 2\tilde{\lambda} - 2\phi} = -\frac{q_o^2}{4} e^{2\lambda_s - 2\phi},\]
\[\ddot{\tilde{\lambda}} - \frac{Q^2}{2} e^{6\lambda + 2\lambda_s + 2\tilde{\lambda} - 2\phi} = \frac{q_o^2}{4} e^{2\lambda_s - 2\phi},\]
\[\ddot{\nu} + \frac{Q^2}{2} e^{6\lambda + 2\lambda_s + 2\tilde{\lambda} - 2\phi} - 3 e^{6\lambda + 2\lambda_s + 2\tilde{\lambda} - 4\phi + 6\nu} = \frac{q_o^2}{4} e^{2\lambda_s - 2\phi},\]
\[\ddot{\phi} - \frac{Q^2}{2} e^{6\lambda + 2\lambda_s + 2\tilde{\lambda} - 2\phi} = \frac{3q_o^2}{4} e^{2\lambda_s - 2\phi}.\] (2.9)

In order to find a solution for (2.9), let us introduce the new variables \(\gamma, p, \chi\) defined as,

\[\gamma = 6\lambda + 2\lambda_s + 2\tilde{\lambda} - 2\phi, \quad p = 6\lambda + 2\lambda_s + 2\tilde{\lambda} - 4\phi + 6\nu, \quad \chi = 2\lambda_s - 2\phi.\] (2.10)

So the (2.9) reduces to three simple equations,

\[\ddot{\gamma} - 4Q^2 e^{\gamma} = 0, \quad \ddot{p} - 18 e^{p} = 0, \quad \ddot{\chi} + 2q_o^2 e^{\chi} = 0.\] (2.11)

And the solution for the equations in (2.11) could be analytically obtained as,

\[\gamma = -2 \log \left[ a_1 - e^{-a_2 \rho} \right] - a_2 \rho + \log \left[ \frac{a_1 a_3^2}{2Q_o^2} \right],\]
\[p = -2 \log \left[ a_3 - e^{-a_4 \rho} \right] - a_4 \rho + \log \left[ \frac{a_3 a_4^2}{9} \right],\]
\[\chi = -2 \log \left[ a_5 + e^{-a_6 \rho} \right] - a_6 \rho + \log \left[ \frac{a_5 a_6^2}{4q_o^2} \right],\] (2.12)

where \(a_{1,2,3,4,5,6}\) are integration constants. According to (2.9), we on the other hand have,

\[\lambda - \tilde{\lambda} = b_1 \rho + b_2\]
\[\lambda - \lambda_s - \phi + \tilde{\lambda} = b_3 \rho + b_4,\] (2.13)

where \(b_{1,2,3,4}\) are additional integration constants. Altogether with (2.12) and (2.13), we could obtain the full solution for (2.3) as,
\[
\lambda = \frac{1}{8} (\gamma - \chi) + \frac{1}{4} (b_2 + b_1 \rho), \\
\lambda_s = \frac{1}{8} (\gamma + \chi) - \left( \frac{b_1}{4} + \frac{b_3}{2} \right) \rho - \frac{b_2}{4} - \frac{b_4}{2}, \\
\tilde{\lambda} = \frac{1}{8} (\gamma - \chi) - \frac{3b_2}{4} - \frac{3b_1}{4} \rho, \\
\phi = \frac{1}{8} (\gamma - 3\chi) - \left( \frac{b_1}{4} + \frac{b_3}{2} \right) \rho - \frac{b_2}{4} - \frac{b_4}{2}, \\
\nu = \frac{p}{6} - \frac{1}{8} (\gamma + \chi) - \left( \frac{b_1}{12} + \frac{b_3}{6} \right) \rho - \frac{b_2}{12} - \frac{b_4}{6}.
\]

(2.14)

Besides, the zero-energy constraint (2.8) reduces to the following relation,

\[-3a_2^2 + 8a_4^2 - 3a_6^2 - 20b_4^2 - 8b_1 b_3 - 8b_5^2 = 0.\]

(2.15)

While all the integration constants should be further determined by some additional physical conditions, we note that these integration constants could depend on \(q_\theta\) which is the only parameter in the solution.

2.2 A particular solution

In this section, let us discuss a particular solution to fix the integration constants in the supergravity solution obtained in the last section. Since \(|\theta|\) is usually very small in Yang-Mills theory, we consider a sufficiently small backreaction of the D-instantons (D0-branes) in the black D4 configuration. Therefore we require the solution (2.14) must be able to return to the pure black D4-brane solution if \(q_\theta \to 0\) i.e. no D0-branes. Hence the black D4-brane solution corresponds to the situation of \(C_1 = 0\) in (2.2) and in the near-horizon limit the solution is given as,

\[
ds^2 = \left( \frac{U}{R} \right)^{3/2} \left[ -f_T (U) dt^2 + \delta_{ij} dx^i dx^j + (dx^4)^2 \right] + \left( \frac{R}{U} \right)^{3/2} \left[ \frac{dU^2}{f_T (U)} + U^2 d\Omega^2_4 \right],
\]

\[
f_T (U) = 1 - \frac{U_3^3}{U^5}, \quad e^\phi = g_s \left( \frac{U}{R} \right)^{3/4}, \quad F_4 = 3R^3 g_s^{-1} \omega_4, \quad R^3 = \pi g_s N_c l_s^3,
\]

(2.16)

where \(g_s, \omega_4\) represents the string coupling constant and the volume form of \(S^4\). Accordingly we identify the solution (2.16) as the zero-th order solution of (2.12) and rewrite it in terms of \(\gamma, p, \chi\) defined as in (2.10),

\footnote{Strictly speaking, the black D4-brane solution corresponds to the situation that \(C_1\) is a constant because the IIA action (2.2) is invariant under the gauge transformation \(C_1 \to C_1 + dA\) where \(A\) is an arbitrary function. Thus we can choose a particular gauge condition so that \(C_1 = 0\) corresponds to the situation of the black D4-brane solution.}
\[ \gamma_0 = -2 \log \left[ 1 - e^{-3a_\rho} \right] - 3a_\rho + \log \left[ \frac{U_T^6}{g_s^2 R^6} \right], \]
\[ p_0 = -2 \log \left[ 1 - e^{-3a_\rho} \right] - 3a_\rho + \log \left[ \frac{U_T^6}{g_s^8 R^6} \right], \]
\[ \chi_0 = -2 \log [g_s]. \] (2.17)

This gives the relation of \( \rho \) and the usually used \( U \) coordinate in (2.16) as,
\[ \rho = -\frac{b_\theta}{3a} \log \left[ 1 - \frac{U_T^3}{U^3} \right], \quad a = \frac{\sqrt{3} Q_c U_T^3}{v_s^3 g_s}, \quad Q_c = \frac{3\pi N_c}{\sqrt{2}}. \] (2.18)

Here \( b_\theta \) is another constant depended on \( \theta \) which is required as \( b_\theta \to 1 \) if \( q_\theta \to 0 \). Comparing (2.17) with (2.12), it implies that in the limitation of \( q_\theta \to 0 \) there must be \( a_{1,3} \to 1, a_{2,4} \to 3a, a_5 a_6^2 \to \frac{g_s^2}{g_s^2}, a_6 \to q_\theta \) so that \( \gamma, p, \chi \) returns to \( \gamma_0, p_0, \chi_0 \) consistently. In this sense, we could in particular choose \( a_5 = 1, a_6 = 2 |q_\theta| g_s^{-1} \) so that \( a_1 = a_3 = 1, a_2 = 3a, b_2 = 0, b_4 = -\log [g_s] \) as the most simply solution. Moreover we require that \( g_{\nu 0} \sim \hat{\lambda}, g_{ij} \sim \lambda, g_{\Omega \Omega} \sim \nu \) has to behave as same as they are in the zero-th order solution (2.16) in the IR region (i.e. \( U \to U_T, \rho \to \infty \)) so that the holographic duality constructed on the \( N_c \) D4-branes basically remains in the low-energy theory. Therefore we have the following relations,
\[ b_1 = \frac{1}{2} (a_2 - a_6), \quad a_4 = \frac{a_2^2 + a_6^2}{a_2 + a_6}. \] (2.19)

On the other hand the zero-energy constraint (2.8) reduces to an extra relation to determine \( b_3 \) which is,
\[ b_3 = -\frac{1}{2} b_1 - \frac{\sqrt{2}}{4} \sqrt{-3a_2^2 + 8a_4^2 - 3a_6^2 - 18b_1^2}, \quad -3a_2^2 + 8a_4^2 - 3a_6^2 - 18b_1^2 \geq 0. \] (2.20)

The above constraints imply that our solution would be valid only if \( |q_\theta| \leq \frac{3}{2} a g_s \) and it is consistent with our assumption that the backreaction of D-instantons is sufficiently small. The constant \( b_\theta \) could be determined by additionally requiring that \( g_{U \nu} \sim \psi \) behaves as same as it in (2.16) at \( U = U_T \) and this gives
\[ b_\theta = \frac{9a_2^2 g_s^2 - 6ag_s q_\theta}{9a_2^2 g_s^2 + 4q_\theta^2}. \] (2.21)

For the reader’s convenience, we have summarized the current solution in the \( U \) coordinate in the Appendix and one could compare it with the zero-th order solution (2.16) directly. Notice that our solution also has the same behaves as (2.16) in the UV region (i.e. \( U \to \infty, \rho \to 0 \)).
3 The dual field theory

3.1 The running coupling

To start this section, let us examine the dual field theory interpretation of the above supergravity solution in Section 2.2 by taking account into a probe D4-brane moving in our D0-D4 background. The action for a non-supersymmetric D4-brane is given as,

\[ S_{D4} = -\mu_4 \int d^5x e^{-\phi} \text{STr} \sqrt{-\det (g_{(5)} + F)} + \frac{1}{2} \mu_4 \text{Tr} \int C_1 \wedge F \wedge F, \]  

(3.1)

where respectively \( \mu_4 = \frac{1}{(2\pi)^2 l_s^5} \), \( g_{(5)} \), \( F = 2\pi \alpha' F \) is the charge of the D4-brane, induced 5d metric and the gauge field strength exited on the D4-brane. We assume that the non-vanished components of \( F \) are \( F_{\mu\nu}(x) \delta_{1/2}(x^4 - \bar{x}) \). Then considering the \( x^4 \) direction is compacted on a circle \( S^1 \) with the period \( \beta_4 \), the action (3.1) can be expanded in powers of \( F \) as a 4d Yang-Mills theory with a \( \theta \) term,

\[ S_{D4} \approx -\frac{1}{2g_{YM}^2} \text{Tr} \int F \wedge* F + i \frac{\theta}{8\pi^2} \text{Tr} \int F \wedge F + \mathcal{O}(F^3), \]  

(3.2)

where the delta function is normalized as \( \beta_4 = \int dx^4 \delta(x^4 - \bar{x}) \) and the coupling constant \( g_{YM}, \theta \) are defined as,

\[ g_{YM}^2(U) = \left[ \mu_4 (2\pi \alpha')^2 \beta_4 e^{-\phi} \sqrt{g_{44}} \right]^{-1} = \frac{8\pi^2 g_s l_s}{\beta_4} \cosh \left[ \frac{q_\theta}{2g_s} \rho(U) \right], \]

\[ \theta(U) = \frac{i}{l_s} \int_{S^4_{D}} C_1 = \frac{i}{l_s} \int_{D} F_2 = \theta - \frac{\beta_4}{g_s l_s} \tanh \left[ \frac{q_\theta}{2g_s} \rho(U) \right], \]  

(3.3)

which are the running couplings. Since the asymptotic region of the bulk supergravity corresponds to the dual field theory, at the boundary \( \rho \to 0, U \to \infty \) the Eq.(3.3) defines the value of the Yang-Mills coupling constant and the \( \theta \) angle in the dual theory. In the large \( N_c \) limit, we should define the limitation \( \bar{\theta} = \theta/N_c [1, 2] \) and the t’Hooft coupling,

\[ \lambda_4(U) = \frac{8\pi^2 g_s l_s N_c}{\beta_4} \cosh \left[ \frac{q_\theta}{2g_s} \rho(U) \right]. \]  

(3.4)

According to the AdS/CFT dictionary, we remarkably find the Yang-Mills and t’Hooft coupling constant \( g_{YM}, \lambda_4 \) increase in the IR region \( \rho \to \infty, U \to U_T \) while they become small in the UV region \( \rho \to 0, U \to \infty \) with our D0-D4 solution. And this behavior is in qualitative agreement with the property of asymptotic freedom in QCD or Yang-Mills theory.

To close this subsection, let us simply evaluate the relation of \( q_\theta \) and \( \theta \). In the Dp-brane supergravity solution, the normalization of the Ramond-Ramond field \( F_{p+2} \) is given as \( 2k_0^2 \mu_p N_p = \int_{S_{8-p}} * F_{p+2} \) and this normalization with (2.7) would tell us that \( q_\theta \) is proportional to the number of D0-branes. Hence we have \( q_\theta \sim g_s N_0, N_0 = g_d D_0 V_4 \), where \( d_{D_0} \) is the number...
density of D0-branes and $V_4 \simeq (2\pi R)^3 \beta_T$ is the worldvolume of the D4-branes. In order to
include the influence of the D-instantons, we further assume that $d_{D_0}$ depends on $x^4$ because $x^4 = \theta R_4$ is periodic. This viewpoint implies that each slice in the D4-brane with a fixed $x^4$ corresponds to a theta vacuum in the dual field theory if we identify the coordinate $\theta$ to the theta angle in (3.2). So we could interpret that the 4d Yang-Mills action (3.2) is defined on a slice of the D4-brane with $x^4 = \bar{x}$, or namely with a theta angle $\theta = \bar{x}/R_4$ and it might offer a geometric interpretation of the theta-dependence in the dual field theory. Finally we can define the dimensionless density by using $\beta_4$ as
$I(\theta) = d_{D_0} \beta_4^{-4}$ which leads to $|q_\theta| \simeq 2g_s V_4 I(\theta) / \beta_4^4$.
Note that in the large $N_c$ limit $I(\theta)$ may be expected to be a function of $\theta/N_c$.

3.2 The thermodynamics

The thermodynamics in holography is based on the relation between the partition function of
the bulk supergravity $Z_{\text{SUGRA}}$ and the dual field theory (DFT) $Z_{\text{DFT}}$ as $Z_{\text{SUGRA}} = Z_{\text{DFT}}$ in the
large $N_c$ limit [19, 18, 17]. Hence the free energy density of the 4d theta-dependent Yang-Mills
theory $f(\theta)$ could be obtained by

$$Z = e^{-V_4 f(\theta)} = e^{-S_{\text{SUGRA}}^{\text{ren onshell}}},$$

(3.5)

where $V_4$ and $S_{\text{SUGRA}}^{\text{ren onshell}}$ respectively represents the 4d spacetime volume and the renormalized
onshell action of the bulk supergravity. For the duality to the thermal field theory, $V_4 = V_3 \beta_T$
and $S_{\text{SUGRA}}^{\text{ren onshell}}$ respectively refers to its Euclidean version. The temperature in the dual field
theory is defined by $T = 1/\beta_T$. So in order to avoid the conical singularities in the dual field
theory, it provides the relation with our D0-D4 solution.\footnote{It is not very obvious to find a relation as (3.6) just by requiring no singularities outside the horizon with our gravitational solution in Section 2.2. So we assume that our solution could return to (2.16) continuously if $q_\theta \to 0$ then we find the relation (3.6) is at least valid at order $O(q_\theta^4)$.}

$$2\pi T \simeq \left( \frac{3}{2} + \frac{q_\theta}{3g_s} \right) \frac{U_1^{1/2}}{R_3^{3/2}} + O(q_\theta^3),$$

(3.6)

Afterwards the renormalized Euclidean onshell action of the supergravity is given as,

$$S_{\text{SUGRA}}^{\text{ren onshell}} = S_{\text{IIA}}^E + S_{\text{GH}} + S_{\text{CT}},$$

(3.7)

where $S_{\text{IIA}}^E$ refers to the Euclidean version of IIA supergravity action (2.2) and $S_{\text{GH}}, S_{\text{CT}}$ refers
to the associated Gibbons-Hawking and the bulk counter-term which are respectively given as [29, 53].
\[ S_{IIA}^{E} = -\frac{1}{2k_{0}^{2}} \int d^{10}x \sqrt{g} \left[ e^{-2\phi} \left( R + 4(\partial \phi)^{2} \right) - \frac{1}{2} |F_{2}|^{2} - \frac{1}{2} |F_{4}|^{2} \right], \]
\[ S_{GH} = -\frac{1}{k_{0}^{2}} \int d^{9}x \sqrt{h} e^{-2\phi} K, \]
\[ S_{CT} = \frac{1}{k_{0}^{2}} \left( g_{s}^{1/3} \right) \int d^{9}x \sqrt{h} \frac{5}{2} e^{-7\phi/3}. \]  

(3.8)

Here \( h \) is the determinant of the boundary metric i.e. the slice of the bulk metric (2.3) at fixed \( \rho = \varepsilon \) with \( \varepsilon \to 0 \). And \( K \) is the trace of the extrinsic curvature at the boundary which is defined as,

\[ K = \frac{1}{\sqrt{g}} \partial_{\rho} \left( \frac{\sqrt{g}}{\sqrt{g_{\rho\rho}}} \right) \bigg|_{\rho=\varepsilon}. \]  

(3.9)

Then the actions in (3.8) can be evaluated by using the D0-D4 solution discussed in Section 2.2. After some straightforward but messy calculations, we finally obtain,

\[ S_{IIA}^{E} = \mathcal{V} \left[ 3 \frac{2}{2\varepsilon} - \frac{9}{4} \frac{a}{g_{s}} + \frac{7q_{\theta}}{2g_{s}} \right], \]
\[ S_{GH} = \mathcal{V} \left[ -\frac{19}{6\varepsilon} + \frac{7}{6} \frac{9a^{2}g_{s}^{2} - 36a_{s}q_{\theta} + 4q_{\theta}^{2}}{6a_{s}^{2} + 4g_{s}q_{\theta}} \right], \]
\[ S_{CT} = \mathcal{V} \frac{5}{3\varepsilon}, \]  

(3.10)

and the free energy density \( f(\theta) \) is therefore obtained by using (3.5) (3.10) with the relation of \( q_{\theta} \) and \( \theta \) which is calculated as,

\[ f(\theta, T) = -\frac{128N_{c}^{2} \pi^{4} T^{6} \lambda_{4}}{2187M_{KK}^{4} \lambda_{4}} + \frac{2M_{KK}^{2} \lambda_{4}}{3\pi^{2} T} I(\theta), \]  

(3.11)

where we have defined the Kaluza-Klein (KK) mass \( M_{KK} = 2\pi/\beta_{4} \) and rescaled \( I(\theta) \to (2\pi l_{s})^{3} M_{KK}^{2} I(\theta) \). Besides the function \( I(\theta) \) is turned out to be a periodic and even function of \( \theta \) i.e. \( I(\theta) = I(-\theta), I(\theta) = I(\theta + 2\pi k), k \in \mathbb{Z} \) and the energy of the true vacuum \( F(\theta) \) is obtained by minimizing the expression in (3.11) over \( k \),

\[ F(\theta, T) = \min_{k} f(\theta, T). \]  

(3.12)

While at finite temperature the exact theta-dependence of the ground-state free energy in Yang-Mills theory is less clear especially in the large \( N_{c} \) limit, the computation for one-loop contribution of instantons to the functional integral at sufficiently high temperature suggests that \( f(\theta) - f(0) \propto 1 - \cos \theta \) [11]. Although this theta-dependence is consistent with the gravitational constraints discussed in Section 2 i.e. \( q_{\theta} \to 0 \) if \( \theta \to 0 \), it does not have a definite limitation at
$N_c \to \infty$. Nonetheless if we assume the function $I(\theta)$ has a limit at $N_c \to \infty$, the topological susceptibility can be computed by expanding (3.11) in powers of $\bar{\theta}$ as,

$$f(\bar{\theta}) - f(0) = \frac{2M_5^5K\lambda_4}{3\pi^2T} \sum_{n=1}^{\infty} \frac{b_n}{2n!} \bar{\theta}^{2n}, \quad b_n = \frac{\partial^n f(\bar{\theta}, T)}{\partial \bar{\theta}^n} \bigg|_{\bar{\theta}=0}. \quad (3.13)$$

Thus the topological susceptibility reads

$$\chi(T) = \frac{\partial^2 f(\bar{\theta}, T)}{\partial \bar{\theta}^2} \bigg|_{\bar{\theta}=0} = \frac{2M_5^5K\lambda_4}{3\pi^2N_c^2T}b_2. \quad (3.14)$$

where $b_2$ should be a positive numerical number. The topological susceptibility (3.14) depends on temperature as expected while it becomes vanished in the large $N_c$ limit. We notice this large $N_c$ behavior remarkably agrees with the simulation results reviewed in [1] which indicates that the topological susceptibility has a vanishing large $N_c$ above the deconfinement temperature.

4 Summary and discussion

In this letter we holographically combine the IIA supergravity with the theta-dependent Yang-Mills theory at finite temperature. The bulk geometry is sourced by a stack of $N_c$ black D4-branes and $N_0$ D0-branes as D-instantons. As it is known in the pure black D4-brane solution, the dual field theory indicates deconfinement at finite temperature and adding D-instantons to the D4 background could describe the dynamics of the theta angle in the bulk. To keep this duality picture and include the dynamics of the D-instantons, we therefore consider a sufficiently small backreaction from the D-instantons to the bulk geometry, then a particular solution is found by solving the IIA supergravity action. Afterwards using our supergravity solution, we investigate the coupling constant and the ground-state energy as two most fundamental properties in the dual field theory. The behavior of the coupling constant exhibits the asymptotic freedom as in QCD or Yang-Mills theory and the theta contribution to the free energy density is suppressed at high temperature. The topological susceptibility is vanished in the large $N_c$ limit. Remarkably all these results are in qualitative agreement with various simulation results of the theta-dependent Yang-Mills theory at finite temperature. And we in addition propose a geometric interpretation of the theta-dependence in this system.

In our D0-D4 background, the dual theory should deconfine at the temperature $T \geq T_c$ where $T_c$ refers to the critical temperature of the deconfinement transition. Below $T_c$ the current supergravity solution would be invalid and the confinement in the dual theory should be described by the bubble D0-D4 background as discussed in [42, 43, 44, 45, 46, 47, 48, 49, 50]. Notice that the thermodynamical variables have different large $N_c$ limits in these two D0-D4
backgrounds. The $T_c$ could be obtained by comparing the free energy of our black \cite{3,11} and the bubble D0-D4 system \cite{42,43,44,45}, however the result will remain substantially unchanged as it is given in \cite{45} in the large $N_c$ limit. Another noteworthy point is that Eq.\((3.14)\) implies the instantons would be more unstable in the dual theory at high temperature due to the definition of the topological susceptibility in QFT \(\chi = -i \int d^4x \langle O(x)O(0)\rangle\), where \(O(x) = \text{Tr} F \wedge F\) is the glueball condensate operator. In other words, at extremely high temperature \(T \gg T_c\) the quantum fluctuations would destroy the glueball condensate in the dual theory in a very short time and the theta vacuum in the dual field theory decays soon to the true vacuum. This conclusion is basically consistent with e.g. \cite{7,8,9} and the D3-D(-1) approach in \cite{52}.

To finish this paper, let us give the final comments. Despite our holographic interpretation of the theta-dependence, the exact thermodynamics involving the theta angle is still challenging both in gauge-gravity duality and QFT, especially at finite temperature. In our theory, this is reflected in that the specific relation of \(q_\theta\) and \(\theta\) could not be determined naturally through the holographic duality, thus we have to further require the density of the D0-branes exactly controls the ground-state energy as the role of the theta parameter in the dual field theory. While this could consistently figure out the problem as we have done, the physical understanding of this constraint is not clear. And unfortunately, the analysis in QFT has not implied anything constructive yet, so we have to treat it as a particular constraint in this system and leave it to the future study.

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Appendix: The D0-D4 solution in the $U$ coordinate

We summarize the D0-D4 solution discussed in Section 2.2 here in the $U$ coordinate. The components of the metric are written as,

\[
d s^2 = g_{\mu\nu} dx^\mu dx^\nu + g_{44} (dx^4)^2 + g_{UU} dU^2 + g_{\Omega\Omega} d\Omega_4^2,
\]

(A-1)

where
\[ g_{00} = -\left( \frac{U_T}{R} \right)^{3/2} \frac{f_T(U)^{\frac{9+4Q^2}{9+4Q^2}}}{\sqrt{2}} g_1(U)^{1/2} g_2(U)^{-1/2}, \]

\[ g_{ij} = \frac{1}{\sqrt{2}} \left( \frac{U_T}{R} \right)^{3/2} \frac{g_1(U)^{1/2} g_2(U)^{-1/2} \delta_{ij}}{g_1(U)^{1/2} g_2(U)^{-1/2}}, \]

\[ g_{44} = \left( \frac{U_T}{R} \right)^{3/2} \sqrt{2} f_T(U)^{\frac{12Q}{9+4Q^2}} [g_1(U) g_2(U)]^{-1/2}, \]

\[ g_{UU} = \left( \frac{9+4Q^2}{9+6Q} \right)^{2/3} \left( \frac{R}{U_T} \right)^{3/2} \frac{g_1(U) g_2(U)^{1/2}}{\sqrt{2} f_T(U)}, \]

\[ g_{\Omega \Omega} = \left( \frac{9+4Q^2}{9+6Q} \right)^{2/3} \left( \frac{R}{U_T} \right)^{3/2} \frac{U^2 \left[ g_1(U) g_2(U) \right]^{1/2}}{\sqrt{2}} , \quad (A-2) \]

and the dilaton is

\[ e^\phi = g_s \left( \frac{U_T}{R} \right)^{3/4} f_T(U)^{\frac{Q(3-2Q)}{9+4Q^2}} g_1(U)^{3/4} g_2(U)^{-1/4}, \quad (A-3) \]

The parameter \( Q \) and functions \( g_{1,2} \) are defined as,

\[ g_1(U) = 1 + f_T(U)^{\frac{2Q(3+2Q)}{9+4Q^2}}, \quad g_2(U) = 1 - f_T(U)^{\frac{9+6Q}{9+4Q^2}}, \quad Q = \frac{|q_\theta|}{ag_s}. \quad (A-4) \]

Note that \( Q \) is a positive number and if it is sufficiently small we have \( f_T(U)^{\frac{12Q}{9+4Q^2}} \simeq 1 \), \( g_1(U) \simeq 2 \) in the region \( U \in (U_T + \varepsilon, \infty) \) where \( \varepsilon \to 0 \). The metric (A-2) and the dilaton (A-3) returns to the zero-th order solution (2.16) consistently if we set \( q_\theta, Q = 0 \).

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