Hidden Local Symmetry at One Loop

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Abstract
We show that one-loop corrections with the hidden gauge boson loop preserve in the low energy limit all the successful tree level predictions of the hidden local symmetry in the SU(2)_L × SU(2)_R/SU(2)_V chiral Lagrangian. Most amazingly, the ρ meson dominance of the pion electromagnetic form factor survives the one-loop corrections at any momentum, if and only if we take the parameter choice $a = 2$. For the choice $a = 1$ (“vector limit”), $a$ is not renormalized ($Z_a = 1$) and no deviation from $a = 1$ is induced by the loop effects of the hidden gauge bosons. Actually, $a = 1$ is a nontrivial ultraviolet fixed point.

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It is now a popular notion to identify the $\rho$ meson with the dynamical gauge boson of the hidden local symmetry in the $SU(2)_L \times SU(2)_R / SU(2)_V$ nonlinear chiral Lagrangian $[1]$. By setting a parameter choice $a = 2$ in this hidden local symmetry Lagrangian, we can successfully reproduce three phenomenological facts $[2]$:

1. The $\rho$-coupling universality, $g_{\rho\pi\pi} = g$, the KSRF relation (II), $m^2_\rho = 2 f^2_\pi g^2_{\rho\pi\pi}$, and the $\rho$ meson dominance of the electromagnetic form factor of the pion $[3]$.

2. Remarkably, we obtain the celebrated KSRF (I) relation, $g_\rho = 2 f^2_\pi g_{\rho\pi\pi}$, as an $a$-independent relation which is actually characteristic to the hidden local symmetry and hence may be regarded as a “low energy theorem” of the symmetry $[5]$. In fact it was proved to be a low energy theorem at tree level $[6]$.

Moreover, this hidden local symmetry Lagrangian has been applied to the strongly coupled Higgs model and/or the effective theory of the technicolor including the techni-$\rho$ meson (“BESS model”) $[7]$.

Recently, one-loop corrections of the pion loop have been extensively studied in the chiral Lagrangian (chiral perturbation theory $[8, 9]$), which succeeded in reproducing systematically the low energy hadron physics slightly away from the low energy limit dictated by the chiral symmetry. However, this approach appears to fail in reproducing the higher energy region, even considerably lower than the $\rho$ meson pole, and needs explicit degree of a new field, the $\rho$ meson. Amazing fact is that the finite part of the one-loop counter terms in the chiral perturbation theory is saturated by the tree level effects of the $\rho$ meson $[10]$. Thus the effective Lagrangian including the $\rho$ meson, if it yields successful tree level results, should be a good starting point towards constructing a true “effective field theory” including the quantum corrections.

As such we take the hidden local symmetry Lagrangian mentioned above. Actually, it was pointed out $[10, 11]$ that loop effects of the vector mesons are crucial to the $\pi^+ - \pi^0$ mass difference $[12]$. Our goal is thus to promote the hidden local symmetry Lagrangian into an “effective field theory” valid up to beyond the $\rho$ meson pole by including the full quantum effects.
In this paper we investigate one-loop effects of the hidden local gauge boson, and as a first step see whether or not the above successful tree level results survive the loop effects. First, we show that the “low energy theorem” remains intact even in the existence of the loop effects of the $\rho$ meson. Second, we show that the $a$-dependent results ($a = 2$), the $\rho$-coupling universality and the KSRF(II) relation, are satisfied in the low energy limit. Third, we show the $\rho$ meson dominance of the pion electromagnetic form factor still holds at one-loop level, if and only if we take $a = 2$. Finally, we show that for $a = 1$ there is no renormalization effect on $a$ ($Z_a = 1$) and the deviation from $a = 1$ (“vector limit” [13]) is not induced by the loop effects of the hidden local gauge bosons. This corresponds to the fact that $a = 1$ is a nontrivial ultraviolet fixed point of the $\beta$ function of $a$.

Let us start with the $[\text{SU}(2)_L \times \text{SU}(2)_R]_{\text{global}} \times [\text{SU}(2)_V]_{\text{local}}$ “linear” model [2]. We introduce two SU(2)-matrix valued variables, $\xi_L(x)$ and $\xi_R(x)$, which transform as

$$\xi_{L,R}(x) \rightarrow \xi'_{L,R}(x) = h(x)\xi_{L,R}(x)g^\dagger_{L,R},$$

(1)

where $h(x) \in [\text{SU}(2)_V]_{\text{local}}$ and $g_{L,R} \in [\text{SU}(2)_L,R]_{\text{global}}$. These variables are parameterized as

$$\xi_{L,R}(x) \equiv e^{i\sigma(x)/f_\sigma}e^{i\pi(x)/f_\pi} [\pi(x) \equiv \pi^a(x)\tau^a/2], \quad [\sigma(x) \equiv \sigma^a(x)\tau^a/2],$$

(2)

where $\pi$ and $\sigma$ are the pion and the “compensator” (would-be Nambu-Goldstone field) to be “absorbed” into the hidden gauge boson (the $\rho$ meson), respectively, and $f_\pi$ and $f_\sigma$ are the corresponding decay constants in the chiral symmetric limit. The covariant derivatives are defined by

$$D_\mu \xi_L(x) \equiv \partial_\mu \xi_L(x) - igV_\mu(x)\xi_L(x) + i\xi_L(x)eB_\mu(x)\tau_3/2,$$

$$D_\mu \xi_R(x) \equiv \partial_\mu \xi_R(x) - igV_\mu(x)\xi_R(x) + i\xi_R(x)eB_\mu(x)\tau_3/2,$$

(3)

where $g$ is the gauge coupling constant of the hidden local symmetry, $V_\mu (\equiv V^a_\mu \tau^a/2)$ the hidden gauge boson field (the $\rho$ meson), and $B_\mu$ denotes the photon field gauging the U(1)$_{em}$ part of the $[\text{SU}(2)_L \times \text{SU}(2)_R]_{\text{global}}$. 

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Thus we obtain the Lagrangian of the \([\text{SU}(2)_L \times \text{SU}(2)_R]_{\text{global}} \times [\text{SU}(2)_V]_{\text{local}}\) “linear” model, with the \([\text{SU}(2)_L \times \text{SU}(2)_R]_{\text{global}}\) being partly gauged by the photon field:

\[
\mathcal{L} = \mathcal{L}_A + a\mathcal{L}_V + \mathcal{L}_{\text{kin}}(V_\mu),
\]  

(4)

where \(a\) is a constant, \(\mathcal{L}_{\text{kin}}(V_\mu)\) denotes the (possibly induced) kinetic term of the hidden gauge boson, and \(\mathcal{L}_A\) and \(\mathcal{L}_V\) are given by

\[
\mathcal{L}_A = f_\pi^2 \text{tr} \left[ (\hat{\alpha}_\mu^\bot)^2 \right],
\]

\[
\mathcal{L}_V = f_\pi^2 \text{tr} \left[ (\hat{\alpha}_\mu^\|)^2 \right],
\]

(5)

with \(\hat{\alpha}_\mu^\bot\) and \(\hat{\alpha}_\mu^\|\) being the covariantized Maurer-Cartan 1-forms

\[
\hat{\alpha}_\mu^\bot(x) \equiv \frac{D_\mu \xi_L(x) \cdot \xi_L^\dagger(x) - D_\mu \xi_R(x) \cdot \xi_R^\dagger(x)}{2i},
\]

(6)

\[
\hat{\alpha}_\mu^\|(x) \equiv \frac{D_\mu \xi_L(x) \cdot \xi_L^\dagger(x) + D_\mu \xi_R(x) \cdot \xi_R^\dagger(x)}{2i}.
\]

(7)

Normalizing the kinetic term of \(\sigma\), we find

\[
f_\sigma^2 = af_\pi^2.
\]

(8)

Now the Lagrangian Eq. (4) gives in the unitary gauge, \(\xi_L^\dagger = \xi_R\ (\sigma = 0)\), the following tree level relations for the \(\rho\) meson mass \(m_\rho\), the \(\rho\)-\(\gamma\) transition strength \(g_\rho\), the \(\rho\pi\pi\) coupling constant \(g_{\rho\pi\pi}\) and the direct \(\gamma\pi\pi\) coupling constant \(g_{\gamma\pi\pi}\):

\[
m_\rho^2 = ag^2f_\pi^2,
\]

(9)

\[
g_\rho = agf_\pi^2,
\]

(10)

\[
g_{\rho\pi\pi} = \frac{1}{2}ag,
\]

(11)

\[
g_{\gamma\pi\pi} = \left(1 - \frac{a}{2}\right)e.
\]

(12)

For a parameter choice \(a = 2\), the above results reproduce the outstanding phenomenological facts:

(1) \(g_{\rho\pi\pi} = g\) (universality of the \(\rho\)-couplings),
(2) \( m_\rho^2 = 2g_\rho^{\pi\pi}f_\pi^2 \) (KSRF II) \[3\].

(3) \( g_{\gamma\pi\pi} = 0 \) (\( \rho \) meson dominance of the electromagnetic form factor of the pion) \[4\].

Moreover, Eqs.(10) and (11) lead to the KSRF relation \[3\] (version I)

\[ g_\rho = 2f_\pi g_{\rho\pi\pi}, \]

which is independent of the parameter \( a \) and hence is the decisive test of the hidden local symmetry \[3\]. Thus it was conjectured to be a “low energy theorem” of the hidden local symmetry \[3\] and was then proved at tree level \[3\].

Now, we consider the one-loop effects of the gauge boson of the hidden local symmetry. Let us introduce the gauge-fixing terms corresponding to the hidden gauge boson. We define for the hidden local symmetry an \( R_\xi \) gauge condition so as to cancel the quadratic vector-scalar mixing:

\[
\mathcal{L}_{GF}(V) \equiv -\frac{1}{\alpha} \text{tr} \left[ (\partial_\mu V_\mu)^2 \right] + \frac{i}{2} ag f_\pi^2 \text{tr} \left[ \partial_\mu V_\mu (\xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger) \right] \\
+ \frac{1}{16} \alpha a^2 g^2 f_\pi^4 \left\{ \text{tr} \left[ (\xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger)^2 \right] \\
- \frac{1}{2} \left( \text{tr} \left[ \xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger \right] \right)^2 \right\}.
\]

We also add the ghost Lagrangian corresponding to the gauge fixing:

\[
\mathcal{L}_{FP} \equiv i \text{tr} \left[ \bar{v} \left\{ 2\partial_\mu D_\mu v + \frac{1}{2} g^2 \alpha f_\pi^2 a (v\xi_L + \xi_L^\dagger v + v\xi_R + \xi_R^\dagger v) \right\} \right],
\]

where \( v \) denotes the ghost field. In the following calculation we choose the Landau gauge, \( \alpha = 0 \). In this gauge the would-be Nambu-Goldstone bosons \( \sigma \) are still massless, no other vector-scalar interactions are created and the ghost field couples only to the gauge fields. Since we are interested in the strong interaction effect, we consider the photon field as the external field and do not consider its loop effect.

\[^{\dagger}\text{Here we write the form in which the BRS transformation is transparent. Instead of the } \xi_L \text{ and } \xi_R, \text{ we can write this term using } \sigma \text{, which does not alter the results in this paper unchanged.}\]

\[^{\ddagger}\text{The tree level results Eqs.}\[1\]-(\[12\]) also hold in the Landau gauge.\]
For canceling the lowest derivative divergent part we redefine the normalization of the parameters and the fields such that

\[ a = Z_a a_r, \quad \epsilon = Z_e \epsilon_r, \quad g = Z_g g_r; \]
\[ V_\mu = Z_V^{1/2} V_{r\mu}, \quad \pi = Z_\pi^{1/2} \pi_r, \quad \sigma = Z_\sigma^{1/2} \sigma_r; \]
\[ f_\pi = Z^{1/2} f_{\pi r}, \quad f_\sigma = Z^{1/2} f_{\sigma r}. \]  
(16)

In the following calculations we define the $\rho$ meson mass parameter $m_\rho$ by

\[ m_\rho^2 \equiv a_r g_r^2 f_{\pi r}^2. \]  
(17)

Hereafter, we denote the pion momentum as $k_\mu$ and $q_\mu$, and the $\rho$ meson momentum as $p_\mu$. Throughout this paper we set the pion momentum on the mass-shell, $k^2 = q^2 = 0$.

The one-loop graphs contributing to the $\rho\pi\pi$ coupling are shown in Fig. 1. These contributions are given by

\[ \Gamma^{\rho\pi\pi}_{(a)} = \frac{g_r^2}{8 (4\pi)^2} F(a)(p^2), \]
\[ \Gamma^{\rho\pi\pi}_{(b)} = \frac{g_r^2}{4 (4\pi)^2} F(b)(p^2), \]
\[ \Gamma^{\rho\pi\pi}_{(c)} = \frac{g_r^2}{8 (4\pi)^2} \left[ \frac{1}{\epsilon} - \ln(-p^2) + \frac{8}{3} \right], \]
\[ \Gamma^{\rho\pi\pi}_{(d)} = \frac{g_r^2}{24 (4\pi f_{\pi r})^2} \left[ \frac{1}{\epsilon} - \ln(-p^2) + \frac{8}{3} \right], \]
\[ \Gamma^{\rho\pi\pi}_{(e)} = 0, \]
\[ \Gamma^{\rho\pi\pi}_{(f)} = \frac{g_r^2}{48 (4\pi f_{\pi r})^2} \left[ \frac{1}{\epsilon} - \ln(-p^2) + \frac{8}{3} \right], \]
\[ \Gamma^{\rho\pi\pi}_{(g)} = \Gamma^{\rho\pi\pi}_{(h)} = 0, \]  
(18)

where

\[ \frac{1}{\epsilon} \equiv \frac{2}{4 - n} - \gamma + \ln(4\pi), \]
\[ [\gamma : \text{Euler constant, } n : \text{the dimension of the integral}] \]  
(19)
and $F_{(a)}(p^2)$ and $F_{(b)}(p^2)$ denote certain complicated functions which have no divergent part and $F_{(a)}(p^2 = 0) = F_{(b)}(p^2 = 0) = 0$.

From Eq. (18) we can easily see that at $p^2 = 0$ there exist no contributions from the one-loop diagrams and hence no counter terms;

$$Z_v^{1/2} Z_a Z_g Z_{\pi} - 1 = 0. \tag{20}$$

Then we find that the $\rho\pi\pi$ coupling remains the same as the tree level in the low energy limit;

$$g_{\rho\pi\pi}(p^2 = 0, k^2 = 0, q^2 = 0) = \frac{a_r}{2} g_r. \tag{21}$$

Eq. (21) implies that for $a_r = 2$, the universality of the $\rho$-couplings remains intact in the low energy limit.

Similarly, one-loop graphs contributing to the $\rho$-$\gamma$ mixing are shown in Fig. 2. These are given by

$$\Gamma_{(a+b+c)}^{\rho\gamma} = \frac{1 + 2a_r - a_r^2}{12} (p_{\mu} p_{\nu} - p^2 g_{\mu\nu}) \frac{e_r g_r}{(4\pi)^2} \left[ \frac{1}{\epsilon} - \ln \left( -p^2 \right) + \frac{8}{3} \right], \tag{22}$$

which have no contributions at $p^2 = 0$ and hence no counter terms;

$$Z_e Z_v^{1/2} Z_a Z_g Z_{\pi} - 1 = 0. \tag{23}$$

Thus we find that the $\rho$-$\gamma$ mixing also remains the same as its tree level in the low energy limit;

$$g_{\rho}(p^2 = 0) = a_r g_r f_{\pi r}^2. \tag{24}$$

Comparing Eq. (23) with Eq. (20), we have

$$Z_e = 1, \tag{25}$$

which is in accord with the general theorem that the electromagnetic charge $e$ is not renormalized by the strong interaction.

From Eqs. (21) and (24), we obtain the desired “low energy theorem” (KSRF I);

$$g_{\rho}(p^2 = 0) = 2 f_{\pi r}^2 g_{\rho\pi}(p^2 = 0, p_{\pi}^2 = 0, p_{\pi}^2 = 0) \tag{26}$$
at one-loop level.

We next investigate loop effects on the KSRF(II) relation, \( m_\rho^2 = 2f_\pi^2 g_{\rho\pi\pi}^2 \), in
the low energy limit. To this end we calculate one-loop graphs for the \( \rho \) meson propagator, which are shown in Fig. 3. The contributions to the \( \rho \) meson propagator are given by

\[ \Gamma_\rho^{(a+b+c+d+e)} \rightarrow \frac{g_\rho^2}{2(4\pi)^2} \left( \frac{1}{\epsilon} - \ln m_\rho^2 + \frac{5}{6} \right). \]  

From Eq.(27) we can determine at \( p^2 = 0 \) the counter term

\[ Z_V Z_\sigma Z_\pi^2 \frac{2g_\pi^2}{4\pi^2} \left( \frac{1}{\epsilon} - \ln m_\rho^2 + \frac{5}{6} \right) \]  

in such a way as to obtain the \( \rho \) meson mass parameter \( M_\rho \) in the low energy limit;

\[ M_\rho^2(p^2 = 0) = m_\rho^2 = a_r g_\rho^2 f_\pi^2. \]  

Combined with Eq.(21), this yields the KSRF(II) relation for \( a_r = 2 \) in the low energy limit:

\[ M_\rho^2(p^2 = 0) = 2g_\rho^2 g_{\rho\pi\pi}(p^2 = 0, k^2 = 0, q^2 = 0) f_\pi^2. \]  

Finally, we come to the \( \rho \) meson dominance of the pion electromagnetic form factor. Let us first determine the counter term for the \( \gamma\pi\pi \) vertex;

\[ -ie \rho_{3\bar{c}}(k-q)\mu \left[ Z_\varepsilon Z_{\pi} \left( 1 - \frac{a_r}{2} Z_\sigma \right) - \left( 1 - \frac{a_r}{2} \right) \right]. \]  

We have already determined the renormalization constant \( Z_\varepsilon \) in Eq.(28). To obtain \( Z_\sigma \), we use the relation

\[ Z_\sigma = Z_\alpha Z_\pi \]  

which follows from Eq.(8).

\( Z_\pi \) and \( Z_\sigma \) are determined by renormalizing the wave functions of the \( \pi \) and \( \sigma \) fields at the on-shell point \( q^2 = 0 \) (remember that \( \sigma \) is massless in the Landau gauge). The one-loop graphs contributing to these propagators are shown in Figs.
\[ Z_{\pi} - 1 = -\frac{d\Gamma_{\pi}(q^2)}{dq^2} \bigg|_{q^2=0} = \frac{3a_r}{2} \frac{g_r^2}{(4\pi)^2} \left[ \frac{1}{\bar{\epsilon}} - \ln m_{\rho}^2 + \frac{5}{6} \right], \quad (33) \]
\[ Z_{\sigma} - 1 = -\frac{d\Gamma_{\sigma}(q^2)}{dq^2} \bigg|_{q^2=0} = \frac{3}{2} \frac{g_r^2}{(4\pi)^2} \left[ \frac{1}{\bar{\epsilon}} - \ln m_{\rho}^2 + \frac{5}{6} \right]. \quad (34) \]

From Eqs. (32), (33) and (34) we obtain
\[ Z_a - 1 = -\frac{3}{2} (a_r^2 - 1) \frac{g_r^2}{(4\pi)^2} \left[ \frac{1}{\bar{\epsilon}} - \ln m_{\rho}^2 + \frac{5}{6} \right]. \quad (35) \]

We note that for the parameter choice \( a_r = 1 \) the parameter \( a \) is not renormalized:
\[ Z_a = 1. \quad (36) \]

This implies that in the “vector limit” the loop effects of the \( \rho \) meson does not induce deviation from \( a = 1 \).

Combined with Eqs. (25), (33) and (35), the counter term for the \( \gamma\pi\pi \) vertex Eq. (31) now reads;
\[ -ie_\epsilon \bar{\epsilon}_{3bc} (k - q)_\mu \frac{3a_r (2a_r - 1)}{4} \frac{g_r^2}{(4\pi)^2} \left[ \frac{1}{\bar{\epsilon}} - \ln m_{\rho}^2 + \frac{5}{6} \right]. \quad (37) \]

Now, we investigate the one-loop effect on the \( \gamma\pi\pi \) vertex. The graphs which contribute to this vertex are shown in Figs. 6 and 7. The contributions from each graph in Fig. 6 are given by
\[ \Gamma_{(a)} = i e_\epsilon \bar{\epsilon}_{3bc} (k - q)_\mu \frac{a_r}{48} \frac{p^2}{(4\pi f_{\rho})^2} \left[ \frac{1}{\bar{\epsilon}} - \ln(-p^2) + \frac{8}{3} \right], \quad (38.a) \]
\[ \Gamma_{(b)} = -i e_\epsilon \bar{\epsilon}_{3bc} (k - q)_\mu \frac{1}{24} \frac{p^2}{(4\pi f_{\rho})^2} \left[ \frac{1}{\bar{\epsilon}} - \ln(-p^2) + \frac{8}{3} \right], \quad (38.b) \]
\[ \Gamma_{(c)} = 0, \quad (38.c) \]
\[ \Gamma_{(d)} = i e_\epsilon \bar{\epsilon}_{3bc} (k - q)_\mu \frac{9a_r}{8} \frac{g_r^2}{(4\pi)^2} \left[ \frac{1}{\bar{\epsilon}} - \ln m_{\rho}^2 + \frac{5}{6} \right], \quad (38.d) \]
\[ \Gamma_{(e)} = i e_\epsilon \bar{\epsilon}_{3bc} (k - q)_\mu \frac{3a_r}{8} \frac{g_r^2}{(4\pi)^2} \left[ \frac{1}{\bar{\epsilon}} - \ln m_{\rho}^2 + \frac{5}{6} + F(e)(p^2) \right], \quad (38.e) \]
\[ \Gamma_{(f)} = -i e_\epsilon \bar{\epsilon}_{3bc} (k - q)_\mu \frac{3a_r}{4} \frac{g_r^2}{(4\pi)^2} \left[ \frac{1}{\bar{\epsilon}} - \ln m_{\rho}^2 + \frac{5}{6} + F(e)(p^2) \right], \quad (38.f) \]
\[ \Gamma_{(g)} = 0, \quad (38.g) \]
where the function $F(e)(p^2)$ is defined by
\[ F(e)(p^2) \equiv -\frac{4}{3} \int_0^1 dx \ln \left(1 - \frac{x p^2}{m^2_\rho}\right) + \frac{2}{3} \int_0^1 dy \int_0^1 dx \ln \left(1 - \frac{xy(1-xy)p^2}{(1-y)m^2_\rho}\right). \]

Next we calculate the one-loop graphs through the tree-level direct $\gamma\pi\pi$ vertex (Fig. 7). These contributions are given by
\[
\begin{align*}
\Gamma_{(a)} & = -ie_r \epsilon_{3bc} (k-q) \frac{a_r^2 (2-a_r)}{8} \frac{g_r^2}{(4\pi)^2} F(h)(p^2), \\
\Gamma_{(i)} & = ie_r \epsilon_{3bc} (k-q) \frac{(2-a_r)(3a_r-4)}{48} \frac{p^2}{(4\pi f_\pi)^2} \left[\frac{1}{\epsilon} - \ln(-p^2) + \frac{8}{3}\right], \\
\Gamma_{(j)} & = 0,
\end{align*}
\]
where $F(h)(p^2)$ denotes a certain complicated function which has no divergent part and $F(h)(p^2 = 0) = 0$. In the zero photon momentum limit, $p^2 = 0$, the contribution of these graphs reduce to
\[
\begin{align*}
i e_r \epsilon_{3bc} (k-q) & \frac{3a_r (2a_r - 1)}{4} \frac{g_r^2}{(4\pi)^2} \left[\frac{1}{\epsilon} - \ln m^2_\rho + \frac{5}{6}\right].
\end{align*}
\]

Thus the counter term given in Eq.(37) precisely cancels the loop correction at $p^2 = 0$. No direct $\gamma\pi\pi$ interaction is induced by the one-loop effects of the hidden gauge bosons in the low energy limit for any value of $a_r$.

However, it is important to investigate the momentum dependence of the direct $\gamma\pi\pi$ vertex for really checking the $\rho$ meson dominance. Actually, away from the zero momentum $p^2 = 0$, the graphs, $\Gamma_{(a)}$, $\Gamma_{(b)}$, $\Gamma_{(e)}$, $\Gamma_{(f)}$, $\Gamma_{(h)}$ and $\Gamma_{(i)}$, make contributions to the higher order photon momentum. Generally, these contributions give rise to the direct $\gamma\pi\pi$ vertex, thus violating the $\rho$ meson dominance. (Of course for $a_r \neq 2$, tree level direct $\gamma\pi\pi$ vertex exist and hence obviously violates the $\rho$ meson dominance.) However, if we take the parameter choice $a_r = 2$, $\Gamma_{(a)}$ and $\Gamma_{(e)}$ are exactly canceled by $\Gamma_{(b)}$ and $\Gamma_{(f)}$, respectively;
\[
\begin{align*}
\Gamma_{(a)} + \Gamma_{(b)} & = 0, \\
\Gamma_{(e)} + \Gamma_{(f)} & = 0.
\end{align*}
\]
and $\Gamma^{\gamma\pi\pi}_{(h)}$ and $\Gamma^{\gamma\pi\pi}_{(i)}$ Eq. (10) vanish identically,

\[
\begin{align*}
\Gamma^{\gamma\pi\pi}_{(h)} &= 0, \\
\Gamma^{\gamma\pi\pi}_{(i)} &= 0.
\end{align*}
\] (43)

Therefore no direct $\gamma\pi\pi$ interaction is induced for all orders of photon momentum, if and only if we take the parameter choice, $a_r = 2$. Incidentally, for $a_r = 2$ there is no divergence in the momentum-dependent part of the $\gamma\pi\pi$ vertex, namely, for the electromagnetic form factor of the pion we need no higher derivative counter term like\[10\]
\[
L_9 \text{tr} \left[ F^L_{\mu\nu} \xi_L^{\dagger} \gamma^\mu \gamma^\nu \xi_L + F^R_{\mu\nu} \xi_R^{\dagger} \gamma^\mu \gamma^\nu \xi_R \right],
\] (44)

which is actually needed in the chiral perturbation theory without hidden gauge boson loop.

Finally, we make some comments on the renormalization-group equations for the parameters, $a_r$ and $g_r$, in the minimal subtraction scheme. In this scheme the $\beta$ functions for $a_r$ and $g_r$ are given by

\[
\begin{align*}
\beta_{a_r}(a_r) &\equiv \frac{d a_r}{d \mu} = -3a_r(a_r^2 - 1) \frac{g_r^2}{(4\pi)^2}, \\
\beta_{g_r}(g_r) &\equiv \frac{d g_r}{d \mu} = -\frac{87 - a_r^2}{12} \frac{g_r^3}{(4\pi)^2}.
\end{align*}
\] (45) (46)

The $\beta$ function for $a_r$, Eq.(45), has an ultraviolet fixed point at $a_r = 1$, which corresponds to the fact that the parameter $a$ is not renormalized if we set $a = 1$ from the beginning (see Eq.(36)). Eq.(46) implies that the hidden gauge coupling constant $g_r$ is asymptotically free for not so large value of $a_r$ ($a_r < \sqrt{87}$). These imply that for a reasonable value for $a_r$ in the low energy (for example $a_r = 2$), the parameter $a_r$ and the coupling constant $g_r$ go asymptotically to the value of “vector limit”\[13\] ($a_r = 1$ and $g_r = 0$), i.e., the “vector limit” is realized as the “idealized” high energy limit of the hidden local symmetry.

\[\text{For } a_r = 2, \text{ Eq.(43) as well Eq.(12) are correct not only in the Landau gauge but also in any } R_\xi \text{ gauge.}\]
In conclusion, we have shown that the successful tree level results of the hidden local symmetry hold at one-loop level. Thus the predictions of this symmetry has proved not accidental to the tree level. In particular, the celebrated KSRF(I) relation survives the loop effects and hence seems to be a true low energy theorem as was anticipated in Ref. [6]. If we further take the parameter choice $a = 2$, the $\rho$-coupling universality and the KSRF(II) relation also remain valid in the low energy limit. Most amazingly, the $\rho$ meson dominance still holds at one loop in the higher photon momentum not restricted to the low energy limit, if and only if we take $a = 2$. The “vector limit” $a = 1$ is an ultraviolet fixed point and is realized as the “idealized” high energy limit of the hidden local symmetry Lagrangian. It is highly desirable to study the full one-loop results of this Lagrangian at higher momentum $p^2 \simeq m_\rho^2$ and see whether or not this “effective field theory” survives up to, say, the $A_1$ meson mass region. Such results are then to be compared with the generalized hidden local symmetry Lagrangian having the $\rho$ and $A_1$ mesons on the same footing[3, 14]. It would also be worth applying the full quantum theory of the hidden local symmetry Lagrangian to the possible vector meson resonances in the dynamical electroweak symmetry breaking with large anomalous dimension, such as the walking technicolor[16], the strong ETC technicolor[17] and the top quark condensate models[18], etc.. Finally, our calculations were done in the Landau gauge for simplicity. It would be interesting to check our results in other gauges as well\(^\ast\).

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\(^\ast\)A formalism in which the gauge invariance is transparent is investigated by Tanabashi[19]

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Figure Captions

Fig. 1. 1-particle irreducible graphs contributing to the $\rho\pi\pi$ vertex.
Fig. 2. 1-particle irreducible graphs contributing to the $\rho$-$\gamma$ mixing.
Fig. 3. 1-particle irreducible graphs contributing to the $\rho$ propagator.
Fig. 4. The 1-loop contributions to the $\pi$ propagator.
Fig. 5. 1-loop contributions to the $\sigma$ propagator.
Fig. 6. 1-particle irreducible graphs contributing to the $\gamma\pi\pi$ vertex.
Fig. 7. 1-particle irreducible graphs contributing to the $\gamma\pi\pi$ vertex through the tree level direct $\gamma\pi\pi$ vertex.