Scalar leptoquark effects on $B \to \mu \nu$ decay

Wei-Shu Hou, Tanmoy Modak and Gwo-Guang Wong
Department of Physics, National Taiwan University, Taipei 10617, Taiwan

Purely leptonic $B^-$ meson decays provide unique probes for physics beyond the Standard Model. We study the impact of a scalar leptoquark, $S_1$, on $B \to \mu \nu$ decay. We find that, for $m_{S_1} \sim 1$ TeV, the $S_1$ leptoquark can modify the $B \to \mu \nu$ rate significantly. Such a leptoquark can in principle also alter the $B \to \tau \bar{\nu}$ rate. However, current searches from LHC and low energy physics provide some constraints on the parameter space.

I. INTRODUCTION

Purely leptonic $B^-$ meson decays provide clean probes to new physics beyond the Standard Model (SM). The prime example is $B \to \tau \bar{\nu}$ [1], where the experimental observation [2] has provided one of the strongest constraints on parameters of a charged Higgs boson, especially in the so-called two Higgs doublet model (2HDM) type II [3] that automatically arises with supersymmetry. The $B \to \mu \nu$ decay is further helicity suppressed, and has not been observed so far. It was pointed out recently that, while the ratio $B(B \to \mu \nu)/B(B \to \tau \bar{\nu})$ is predicted to be the same for both SM and the popular 2HDM type II [1], the value could deviate [4, 5] from its central value, the significance moved up from 2 to 2.8σ [7]. Assuming the SM rate, the Belle II experiment should be able to observe $B \to \mu \nu$ decay with $\sim 5$ ab$^{-1}$ [8] in its early running. If the rate is actually larger than SM, observation would come sooner.

It is well known that leptoquarks (LQ) can also affect semileptonic and purely leptonic meson decays [3], which has been of some interest lately. The impact of a scalar LQ (SLQ) on $B \to \tau \bar{\nu}$ decays and the associated constraints have been discussed in Refs. [10,11]. In this paper we study the effect of the SLQ, $S_1$, on $B \to \mu \nu$ decay and discuss the possible constraints. For sake of comparison, the impact of $S_1$ on $B \to \tau \bar{\nu}$ decay is also considered. The experimental searches for SLQs at the LHC (see e.g. Refs. [12,13]) are based on the minimal Buchmuller-Rückl-Wyler model [14]. In our study, we allow $S_1$ to couple to different generations of quarks and leptons [9]. This work is therefore complementary to the previous study of Ref. [5] on $H^+$ effects in g2HDM, where the effective 4-Fermi operator approach was adopted to match the experimental presentation [7]. Our starting point would be from this New Physics (NP) Wilson coefficient language.

We find that deviations of the $B \to \mu \nu$, $\tau \bar{\nu}$ decay rates from their SM expectations are constrained in particular by direct searches at the LHC, as well as several low energy measurements. The direct search constraints arise primarily from $S_1$ pair production, followed by $S_1 \to q\mu\bar{\nu}$ decay [15,16], which cuts into the parameter space allowed by $B(B \to \mu \nu)$. Although $B(B \to \tau \bar{\nu})$ is less constrained, the case of complex $S_1$ Yukawa couplings is constrained by the electric dipole moment (EDM) of the neutron [11].

The paper is organized as follows. We give the formalism in Sec. II, then present our results on Wilson coefficients in Sec. III. We discuss possible constraints on SLQ Yukawa couplings in Sec. IV and summarize with some discussions in Sec. V.

II. FORMALISM

Let us consider the SLQ, $S_1$, which has quantum numbers $(3,1,1/3)$ under the SM gauge group. The relevant Lagrangian of $S_1$ interacting with SM quarks and leptons can be written as [4]

$$L = (y^Q_{ij} Q_i^c L_j S_1 + y^U_{ij} U_i^c U_j^r) S_1 + H.c.$$  \hspace{1cm} (1)

In the above, $Q_i^c$ ($L_j$) denotes the left-handed quark (lepton) doublet under $SU(2)_L$, while $U_i^c$ ($U_j$) denotes the right-handed up-type quark (charged lepton) singlet, and $i,j$ are generation indices. These fermion states are written in the down-type quark mass basis. After rotating to mass eigenbasis via the transformations $u^L_{Li} \to (V^\dagger)_{ij} u^L_{Lj}$, $d^L_{Li} \to d^L_{Li}$, $\ell^L_{Li} \to \ell^L_{Li}$, $\nu^L_{Li} \to U_{ij} u^L_{Lj}$, where $V$ and $U$ are the CKM and PMNS matrices, respectively, the Lagrangian of Eq. (1) can be expanded in the mass eigenbasis as

$$L = (V^* y^Q_{ij} \bar{u}^c_{Li} L_j S_1 - (y^U_{ij} U^c_{Lj} \bar{U}_i^c U_j^r) S_1 + y^R_{ij} \bar{\nu}^c_{Li} R \ell_j S_1 + H.c.$$  \hspace{1cm} (2)

where $L, R = (1 \mp \gamma_5)/2$.

\footnote{Concerning the stability of the proton, we turn off the coupling of SLQ to di-quarks by imposing appropriate symmetry. We also do not consider right-handed neutral leptons in this paper.}
For purely leptonic $B^-$ decays, the effective Hamiltonian is given by
\[ \mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{ub} \left( C_{VLL}^{ij} O_{VLL}^{ij} + C_{SLL}^{ij} O_{SLL}^{ij} \right), \] (3)
where
\[ C_{VLL}^{ij} = (\bar{u}_i \gamma^\mu L b_j) \left( \bar{\ell}_k \gamma_\mu L \nu_{\ell'} \right), \]
\[ C_{SLL}^{ij} = (\bar{u} L b) \left( \bar{\ell}_k L \nu_{\ell'} \right). \] (4)
The SM contributes only to the $V-A$ interaction via W-boson exchange, where the Wilson coefficients are written as $C_{VLL}^{ij} = C_{VLL}^{SM,ij} + C_{VLL}^{LQ,ij}$, with $C_{VLL}^{SM,ij} = \delta_{ij}$. The leptoquark contributions, at the $m_b$ scale, are given by
\[ C_{VLL}^{ij} \approx \frac{\sqrt{2}}{8G_F V_{ub}} \frac{y_{1i}^U \left( y^L U \right)_{3j}}{m_{S1}^2}, \]
\[ C_{SLL}^{ij} = -\frac{\sqrt{2}}{8G_F V_{ub}} \frac{y_{1i}^U \left( y^L U \right)_{3j}}{m_{S1}^2}, \] (5)
where we have approximated the factor $V_{ud} y_{ki}^L$, with $k$ summed over, as $y_{ki}^L$, i.e. we assume the other $y_{ki}^U$ factors do not overpower the CKM suppression of $V_{us}$ and $V_{ub}$, respectively.

The branching ratio for $B \to \ell \nu_{\ell'}$ in SM is well known
\[ B^{SM}_{B \to \ell \nu_{\ell'}} = |V_{ub}|^2 f_B^2 \frac{G_F^2 m_B m_{\ell'}^2}{8\pi \Gamma_B} \left( 1 - \frac{m_{\ell'}^2}{m_B^2} \right)^2. \] (6)
Adding the SLQ contributions, the branching ratio can be expressed as
\[ B(B \to \ell \nu_{\ell'}) = B^{SM}_{B \to \ell \nu_{\ell'}} \times \sum_{\ell' = e, \mu, \tau} \left| \delta_{\ell'\ell} - |C_{VLL}^{\ell'\ell'} e^{i\phi_{\ell'\ell}}|^2 \frac{m_B}{m_{\nu_{\ell'}}} \right| + |C_{SLL}^{\ell'\ell'} e^{i\phi_{\ell'\ell}}|^2, \] (7)
where the undetected anti-neutrino flavor $\ell'$ in the final state is summed over, and we have utilized the unitarity condition of PMNS matrix $U$. Eq. (7) is essentially the same form used by Belle [2], except that we allow for the phase(s) $\phi_{\ell'\ell}^{(V)}$ of $C_{S(V)L}^{\ell'\ell'}$, which is the phase difference between the product of Yukawa couplings and $V_{ub}$. Note that the Wilson coefficients corresponding to the scalar operators in Eq. (7) should be evolved to the $B$ meson scale via RGE [2]
\[ C_{SLL}^{\ell'\ell'} = -\frac{m_\ell(m_\ell)}{m_\ell(m_0)} \frac{\sqrt{2}}{8G_F V_{ub}} \frac{y_{1i}^U \left( y^L U \right)_{3j}}{m_{S1}^2}. \] (8)
where $m_\ell(m_\ell)$ and $m_\ell(m_0)$ are the $\overline{\text{MS}}$ running masses evaluated at $m_b$ and $m_{S1}$, respectively.

Defining the ratio
\[ R_B^{\mu/\tau} \equiv \frac{B(B \to \mu \bar{\nu})}{B(B \to \tau \bar{\nu})}, \] (9)
in SM one finds
\[ R_B^{\mu/\tau} |_{\text{SM}} = \frac{m_\mu^2 (m_B^2 - m_\mu^2)^2}{m_\tau^2 (m_B^2 - m_\tau^2)^2} \approx 0.0045, \] (10)
which is relatively precise (subject to mild QED corrections) since it involves only lepton masses. The prediction for 2HDM type II is the same \cite{11 4 5}.

The $B \to \ell \bar{\nu}$ ($\ell = \mu, \tau$) decay rates can deviate from SM predictions in the presence of the SLQ $S_1$. We shall ignore $y_{1i}^{L(R)}$ couplings and set them to zero for simplicity \cite{17}. This is similar to the treatment of Ref. \cite{18} where both $y_{1i}^{L(R)}$ and $y_{1i}^{L(R)}$ were assumed to be zero. Here we set $y_{1i}^{L(R)} = 0$, and note that these parameters enter in the electron EDM at one loop, hence receive severe constraints from the recent ACME result \cite{19}. But $y_{1i}^{L(R)}$ are less constrained. In the following, we will first present the Belle measurements of $B \to \mu \bar{\nu}$ and $B \to \tau \bar{\nu}$ decays in terms of $C_{S(V)L}^{\ell'\ell'}$ Wilson coefficients, then discuss in some detail the constraints on relevant $y_{1i}^{L(R)}$ Yukawa couplings, including other possible processes.

### III. CONSTRaining Wilson coefficients

From Eq. (7) one can see that both the Wilson coefficients $C_{SLL}^{\ell'\ell'}$ and $C_{VLL}^{\ell'\ell'}$ can alter $B \to \ell \nu_{\ell'}$ decay rates. However, the former is more efficient, as $C_{SLL}^{\ell'\ell'}$ receives the $m_B^2/m_\ell m_{\nu_{\ell'}}$ enhancement factor compared with the $C_{VLL}^{\ell'\ell'}$ term. This is especially true for $B \to \mu \bar{\nu}$ decay, where $m_B^2/m_\ell m_{\nu_{\ell'}} \sim 60$. For $B \to \tau \bar{\nu}$ decay, the $C_{SLL}^{\ell'\ell'}$ mechanism does not get large enhancement because $1/m_\tau$ is smaller than $1/m_\mu$, but there is still some advantage over $C_{VLL}^{\ell'\ell'}$ by $m_B^2/m_\ell m_{\nu_{\ell'}} \sim 4$. Hence, from here on we will primarily focus on $C_{SLL}^{\ell'\ell'}$, and touch only briefly on the $C_{VLL}^{\ell'\ell'}$ mechanism.

#### A. $B \to \mu \bar{\nu}$ decay

We first focus on $B \to \mu \bar{\nu}$ decay. Depending on the type of anti-neutrino flavor in the final state, there are different $C_{SLL}^{\ell'\ell'}$ Wilson coefficients that can modify the $B \to \mu \bar{\nu}$ rate. For muon anti-neutrino in final state, $C_{SLL}^{\mu\mu}$ interferes with the SM contribution, i.e. $\delta_{\ell'\ell} = 1$ in Eq. (7), while $C_{SLL}^{\ell\ell'}$ and $C_{VLL}^{\ell\ell'}$ effects add in quadrature for electron and tau anti-neutrino emission. But as already mentioned, $C_{SLL}^{\ell\ell'}$ receives stringent constraints, hence we ignore this Wilson coefficient for simplicity.

We plot $B(B \to \mu \bar{\nu})$ in the $[C_{SLL}^{\mu\mu}]$ vs $\phi_{\mu\mu}$ plane in the left panel of Fig. 1 setting all other Wilson coefficients to zero, while in the right panel we give the dependence of...
\( \mathcal{B}(B \to \mu \nu) \) on \( |C_{SL}^{\mu \nu}| \), where a \( \bar{\nu}_\tau \) is emitted. In generating Fig. 1 we used \( \mathcal{B}(B \to \mu \bar{\nu}_\mu)^{SM} \simeq 3.92 \times 10^{-7} \), which arises from utilizing \( f_B = 190 \) MeV from FLAG \cite{20}, and the exclusive value \( V_{ub}^{excl.} = 3.70 \times 10^{-3} \) \cite{2}.

Taking a closer look, Fig. 1(left) plots the contours for \( \mathcal{B}(B \to \mu \bar{\nu}) \) in the \( |C_{SL}^{\mu \nu}| \) vs \( \phi_{\mu \mu}^S \) plane. This is because the Wilson coefficients arising from extra Yukawa couplings of the SLQ are in general complex. The SM value for \( \mathcal{B}(B \to \mu \bar{\nu}) \) is given by the red solid lines, while the central value from Belle is illustrated by the black dotted line, with green dark (light) shaded regions illustrating the \( 1 \sigma \) (2\( \sigma \)) range \cite{7}. For \( \nu_\tau \) emission, which is not distinguished by experiment, the dependence of \( \mathcal{B}(B \to \mu \bar{\nu}) \) on \( |C_{SL}^{\mu \nu}| \) is given by blue dashed line in Fig. 1(right), which can be only constructive as it adds in quadrature. At this level of discussion, as one is using operator language with Wilson coefficients that follow the presentation by Belle, the plots bear similarity to those in Ref. \cite{5} that treat \( H^+ \) effects in q2HDM.

Before turning to \( B \to \tau \bar{\nu} \), let us compare the \( C_{SL}^{\mu \nu} \) and \( C_{VL}^{\tau \nu} \) mechanisms. We see from Fig. 1 that, to account for the Belle central value for \( \mathcal{B}(B \to \mu \bar{\nu}) \) \cite{7}, one needs

\[
|C_{SL}^{\mu \nu}| \lesssim 0.0344 \ (0.0026) \ \text{for} \ \phi_{\mu \mu}^S = 0 \ (\pi), \quad (11)
\]

while one finds from Eq. (7) that

\[
|C_{VL}^{\mu \nu}| \lesssim 0.1641 \ (2.1709) \ \text{for} \ \phi_{\mu \mu}^V = 0 \ (\pi), \quad (12)
\]

which are much larger in value. Similarly, \( |C_{SL}^{\mu \nu}| = 0.009 \) is sufficient to explain the Belle central value, but \( |C_{VL}^{\mu \nu}| \) would need to be 0.568 to produce the same effect. The required value of \( |C_{SL}^{\mu \nu}| \) for \( \phi_{\mu \mu}^S = 0 \ (\pi) \) (or 7) is about a factor of \( m_\bar{b}^2/m_\mu m_\nu \sim 60 \) smaller than that of \( |C_{VL}^{\mu \nu}| \) for \( \phi_{\mu \mu}^V = \pi \ (0) \), similarly for \( |C_{SL}^{\mu \nu}| \) versus \( |C_{VL}^{\mu \nu}| \). This illustrates that \( C_{SL}^{\mu \nu} \) provides a much more efficient mechanism to modify \( \mathcal{B}(B \to \mu \bar{\nu}) \) compared with \( C_{VL}^{\mu \nu} \).

**B. \( B \to \tau \bar{\nu} \) decay**

In Fig. 2(left) we give the contours of \( \mathcal{B}(B \to \tau \bar{\nu}) \) in the \( |C_{SL}^{\mu \nu}| \) vs \( \phi_{\tau \tau}^S \) plane, where \( \bar{\nu}_\tau \) is emitted, as well as
dependence on $|C_{32}^{SL}|$ in Fig. 2(right), for $\bar{\nu}_\mu$ emission. We have used the Belle average value of $B(B \to \tau \bar{\nu}) = (9.1 \pm 2.2) \times 10^{-5}$ from PDG, while the SM expectation is $8.73 \times 10^{-5}$ [3]. These are given by black dotted and red solid lines, respectively. The $1\sigma$ and $2\sigma$ allowed ranges are illustrated by the dark and light cyan shaded regions, while the blue dashed line depicts the dependence of $B(B \to \tau \bar{\nu})$ on $|C_{32}^{SL}|$ for Fig. 2(right). Unlike $B \to \mu \bar{\nu}$ decay, the $|C_{32}^{SL}|$ mechanisms do not have large enhancement factors over $|C_{33}^{SL}|$, as can be seen from Eq. (7). As a result, larger Wilson coefficient values are needed compared with the $B \to \mu \bar{\nu}$ case of Fig. 1.

IV. CONSTRAINTS ON LEPTOQUARK YUKAWA COUPLINGS

Our presentation so far is not so different from the $H^+$ study [5] in g2HDM with extra Yukawa couplings, as we follow the effective Hamiltonian approach used by Belle [7], except allowing the Wilson coefficients to be complex. The underlying physics is, however, quite different. To find the allowed parameter space for $S_1$ leptoquark, we turn to study $B(B \to \mu \bar{\nu})$ and $B(B \to \tau \bar{\nu})$ in terms of the $S_1$ Yukawa couplings.

A. Constraints from $B \to \mu \bar{\nu}$ and $B \to \tau \bar{\nu}$

For simplicity, in the left (right) panel of Fig. 3 we assume $y_{12}^R$ and $y_{32}^L$, i.e. equivalently $y_{\mu\mu}^R$ and $y_{\nu\mu}^R$, $y_{13}^R$ and $y_{33}^L$, i.e. equivalently $y_{\mu\tau}^R$, whereas we take $y_{13}^L$ and $y_{33}^R$, i.e. equivalently $y_{\nu\tau}^L$, as the only nonzero couplings for Fig. 4(right). As we focus in this subsection on the $|C_{32}^{SL}|$ mechanism, we drop the $S$ superscript from $\phi_{\nu\mu}$ from here on.

Let us understand Figs. 3 and 4 better. We have chosen $m_{S_1} = 1.2$ TeV for illustration. In both figures, the central value of Belle measurement, $+2\sigma$ and $-2\sigma$ ranges, are denoted by black dotted, dashed and dotted-dashed lines respectively. The red solid curves illustrate the SM expectation, but corresponds to rather sizable
\[ S_1 \text{ Yukawa couplings, which would be elucidated later.} \]

The left (right) panel of Fig. 3 corresponds essentially to Fig. 1 (right), where we set all \( C_{ij}^{SU} = 0 \) except \( C_{ij}^{SU} = c_{ij} \). However, we note that there is a negligibly small contribution (proportional to \( V_{ub} \)) in Fig. 3 (left) from \( C_{ij}^{SU} \) if \( y_{S2} \) is non-zero, which we neglect. A similar procedure is followed for Fig. 4.

For the detailed respective contours, in Fig. 3 (left) we have two sets of contours for Belle central values of \( B(B \to \mu^+\mu^-) \) for \( \phi_{\mu\mu} = 0 \), the other for \( \phi_{\mu\mu} = \pi \). The former requires larger values of \( |y_{S2}^L| \) and \( |y_{S2}^R| \) couplings to give the Belle \(-2\sigma\) range, and \( |y_{S2}^L| \) and \( |y_{S2}^R| \) couplings to give the Belle \(-2\sigma\) range. A similar explanation is needed for \( |y_{S2}^L| \) and \( |y_{S2}^R| \) couplings to give the Belle \(-2\sigma\) range. A similar explanation is needed for \( |y_{S2}^L| \) and \( |y_{S2}^R| \) couplings to give the Belle \(-2\sigma\) range.

The solid and dashed contours in Figs. 3 (left) and 4 (left) correspond to the SM expectations for \( B(B \to \mu^+\mu^-) \) and \( B(B \to \tau\tau) \), respectively, which require some explanation. One would obviously recover the SM value when Yukawa couplings vanish, as illustrated by the red solid straight lines. But for \( \phi_{\mu\mu} = \pi/2 \) in Fig. 3 (left), to compare with the constraint from neutron EDM discussed later. In this case, the SLQ contribution is purely imaginary, and adds in quadrature to the SM effect. Note that the Wilson coefficients are generally complex, as illustrated in Figs. 1 and 2. So, the actual interpretation in terms of SLQ Yukawa couplings are more complex than what is presented here.

### B. Muon anomalous magnetic moment

The muon anomalous magnetic moment \( a_\mu \) is defined via the coupling \( (\epsilon/4m_\mu)a_\mu \bar{\mu}\sigma_{\alpha\beta}\mu F^{\alpha\beta} \). The \( S_1 \) leptoquark can generate \( \Delta a_\mu \) radiatively \[ a_\mu \simeq -\frac{3}{8\pi^2} \frac{m_\mu^2}{m_S^2} \left\{ (|V_{tb}y_{S2}^L|^2 + |y_{S2}^R|^2) \left[ Q_{q'\mu} f_{q'}(x_t) + Q_S f_S(x_t) \right] + Q_{gS} g_S(x_t) \right\}, \]

where \( Q_{q'} = -2/3, Q_S = 1/3 \) for the leptoquark \( S_1 \), \( x_t = m_\tau^2/m_S^2 \), and the functions \( f_{q'}(x) \) and \( g_S(x) \) and \( Q_{gS}(x) \) can be found in Ref. 21. This will constrain \( y_{S2}^L \) and \( y_{S2}^R \), regardless of the value of \( y_{S2} \).

The current experimental world average 22 and the SM predicted 22 values show some deviation,

\[ a_\mu = (27.06 \pm 7.26) \times 10^{-10}, \]

corresponding to a long standing 3.7 \( \sigma \) discrepancy 22 that could be due to New Physics. However, for the plotted ranges in Figs. 3 and 4 the contributions from \( y_{S2}^L \) turn out to be negligible.

### C. \( \tau \to \mu\gamma \) decay

The branching ratio for \( \tau \to \mu\gamma \) is given by \[ B(\tau \to \mu\gamma) \simeq \frac{\alpha}{4\Gamma}\frac{(m_\tau^2 - m_\mu^2)\beta}{m_\tau^3} \bigg( |A_{\tau\mu}^L| + |A_{\tau\mu}^R| \bigg), \]

where \( \Gamma \) is the width, and

\[ A_{\tau\mu}^L = \frac{3}{16\pi^2 m_\tau^2} \times \left\{ (y_{S2}^{R}\gamma_{S2}^L m_\tau + |V_{tb}|^2 y_{S2}^{R}\gamma_{S2}^L m_\tau) [Q_{q'\mu} f_{q'}(x_t) + Q_S f_S(x_t)] - V_{tb} y_{S2}^{R}\gamma_{S2}^L m_\tau [Q_{q'\mu} g_{q'}(x_t) + Q_S g_S(x_t)] \right\}, \]

\[ A_{\tau\mu}^R = \frac{3}{16\pi^2 m_\tau^2} \times \left\{ (|V_{tb}|^2 y_{S2}^{R}\gamma_{S2}^L m_\tau + y_{S2}^{R}\gamma_{S2}^L m_\tau) [Q_{q'\mu} f_{q'}(x_t) + Q_S f_S(x_t)] - V_{tb} y_{S2}^{R}\gamma_{S2}^L m_\tau [Q_{q'\mu} g_{q'}(x_t) + Q_S g_S(x_t)] \right\}. \]

The current limits are \( B(\tau \to \mu\gamma) < 4.5 \times 10^{-8} \) from Belle 23 and 4.4 \times 10^{-8} from BABAR 24, both at 90\% C.L. Belle II may improve the limit by a factor of 100 8, which would provide some constraint on the parameter space via \( A_{\tau\mu}^L \), where the product \( y_{S2}^{R}\gamma_{S2}^L \) is proportional to \( m_\tau \). However, we find that the present constraints from Belle and Babar are again weaker than the range plotted in Figs. 3 and 4.

### D. EDM measurements

The ACME experiment has put stringent 19 constraints on electron EDM, \( d_e \), which prompted us to set
$y_{ij}^L/R$ to zero. The neutron EDM, $d_n$, imparts some constraint on the parameter space for $B(B \to \tau \bar{\nu})$ decays.

The effective Hamiltonian can be written as [11]

$$\mathcal{H}_{\text{eff}} = C_T O_T + C_\gamma O_\gamma + C_g O_g,$$

where the dimension-6 $O_T$ and dimension-5 $O_\gamma, g$ operators can be found in Ref. [11]. At one loop, the leptoquark $S_1$ will contribute to the neutron EDM with $\tau$ and $\mu$ running inside the loop. The contribution arising from the $\tau$ loop to the Wilson coefficients at the high scale can be written as

$$C_T \simeq -\frac{|V_{ub}|^2 |y_{ij}^L|^2 |y_{ij}^R|}{8m_S^2} \frac{|d_{ij}|}{|d_{ij}|^2} e^{-i\phi_{\tau\tau}},$$

$C_\gamma = -\frac{m_\tau |V_{ub}|^2 |y_{ij}^L|^2 |y_{ij}^R|}{96\pi^2 m_S^2} e^{-i\phi_{\tau\tau}} \left[ 4 + 3 \log(\mu^2/m_S^2) \right]$,

$C_g = -\frac{m_\tau |V_{ub}|^2 |y_{ij}^L|^2 |y_{ij}^R|}{64\pi^2 m_S^2} e^{-i\phi_{\tau\tau}},$

whereas the muon loop is suppressed by $m_\mu$. Note that $V_{ub}$ enters here through the first term of Eq. (2).

Neutron EDM depends on finite CPV phase. The contribution to neutron EDM can be expressed as [26]

$$d_n/e = -(0.44 \pm 0.06) \text{Im} C_\gamma - (1.10 \pm 0.56) \text{Im} C_g,$$

where $C_\gamma$ and $C_g$ are evaluated at 1 GeV, while $C_T$ does not contribute. We follow Ref. [11] for the RGE evolution of the Wilson coefficients from the $m_S$ scale.

The current 95% C.L. upper limit of neutron EDM, viz. $|d_n/e| < 3.6 \times 10^{-26}$ cm [27] (see also Ref. [28]) sets strong constraint on the parameter space in Fig. 4(left) for $\sin \phi_{\tau\tau} \neq 0$. As illustration, we use Eq. (22) and find the orange shaded excluded region for $\phi_{\tau\tau} = \pi/2$, i.e. purely imaginary Wilson coefficient in Fig. 2(left). Future measurements are expected to push the upper limit to $|d_n/e| < 10^{-28}$ cm [29], which is displayed as the thick orange line. This illustrates that future $d_n$ measurements can exclude the whole parameter space of $S_1$ that supports the current Belle central value for $B(B \to \tau \bar{\nu})$, if $\phi_{\tau\tau} = \pi/2$, i.e. the phase of $C_{\tau \tau}^{S \tau}$ is near maximal. Note that the constraint vanishes for $\phi_{\tau\tau} = 0$ or $\pi$, hence it should not be confused that the $\phi_{\tau\tau} = 0$ or $\pi$ contours for $B(B \to \tau \bar{\nu})$ are excluded. The parameter space of $B \to \mu \bar{\nu}$ decay is less constrained due to $m_{\mu}$ suppression.

We have mainly focused on the neutron EDM. Impact of other EDMs such as proton, deuteron and mercury can be found in more detail in Ref. [11] for $B \to \tau \bar{\nu}$.

E. Direct search

The scalar leptoquark $S_1$ can be singly or pair produced at the LHC in $pp$ collisions and subsequently decay into $u_i\ell_j$ and $d_i\nu_j$ final states (conjugate processes are always implied), depending on the values of $y_{ij}^R$ and $y_{ij}^L$. Several searches by ATLAS (e.g. Refs. [12, 16]) and CMS (e.g. Refs. [15, 30]) set strong limits on leptoquark mass and branching ratios. At the current collision energy of $\sqrt{s} = 13$ TeV, $S_1$ pair production via gluon fusion [16] is the dominant mechanism, while $qg$ initiated single leptoquark production is subdominant. For the range of $y_{ij}^R/L$ couplings in Figs. 3 and 4 we find that the most relevant constraints arise from Refs. [15, 16].

The strongest constraint comes from the ATLAS search [10] for SLQs at $\sqrt{s} = 13$ TeV with 36.1 fb$^{-1}$, with final states containing two or more jets, one muon or electron and missing energy, or two or more jets with two electrons or muons. The search gives 95% C.L. upper limits for branching ratios of leptoquark decaying into an electron and a quark, or a muon and a quark, for different values of leptoquark masses. As the final state jets are not tagged, the constraint on $S_1$ parameters will be modulated by $B(S_1 \to u\mu$) if $y_{ij}^L$ is nonzero. Using the 95% C.L. upper limit as $B(S_1 \to u\mu)$ [12] for leptoquark mass of 1.2 TeV, we find the purple excluded regions as displayed in Fig. 5.

The CMS search [15] sets limit on the mass vs branching ratio to $t\mu$ (and $t\tau$) to third generation leptoquarks. Although only $b\nu\ell$ type of SLQ couplings enter $B \to \ell\bar{\nu},$ there are corresponding $t\ell$ couplings. With our assumptions discussed above, for $B \to \mu\bar{\nu}$ decay of left (right) panel of Fig. 3, $S_1$ decays to $u\mu$ ($b\mu$) or $\bar{u}\ell$ ($b\ell$), while for $B \to \tau\bar{\nu}$ decay of left (right) panel of Fig. 4 $S_1$ decays to $u\tau$, $b\nu$ or $b\ell$ ($t\mu$), respectively. With the assumed couplings, one has e.g. $B(S_1 \to t\mu) \approx 0.3$ (0.5) for $y_{12}^{b\mu}$ (0.05) and $y_{12}^{b\ell}$ (0.2). For an SLQ with mass of 1.2 TeV, these branching ratios are below the observed [15] 95% C.L. upper limits at 0.56 and 0.64 for $t\mu$ and $t\tau$ decays, respectively. Similarly, constraints from CMS upper limits are also weaker than the parameter ranges given in Fig. 4. The sensitivity of the High Luminosity LHC (HL-LHC) in probing $S_1 \to t\mu$ is discussed in Ref. [31], where $y_{12}^{b\mu} \gtrsim 0.4$ is expected to be excluded at 95% C.L. Note that we have neglected CKM suppressed decays such as $S_1 \to c\mu$.

We remark that $y_{12}^R$ receives constraints from, for example, heavy resonance search in the dilepton final state via $t$ channel $S_1$ SLQ exchange. Refs. [32, 33] find $y_{12}^R \gtrsim 0.4$–0.5 are excluded for $m_{S_1} \sim 1$ TeV by such processes. However, the limit will weaken if $S_1$ decays to other final states such as $t\mu$ or $t\tau$, but the HL-LHC may be sensitive [31] to $B(S_1 \to t\ell) \sim 0.5$ (with $\ell = \mu, \tau$). The impact of direct searches with full HL-LHC dataset on the parameter space of $B \to \ell\bar{\nu}$ is worthy of further scrutiny, and will be studied elsewhere.

V. DISCUSSION AND SUMMARY

We offer a few brief remarks in passing. The decays of $Z$ and $W$ bosons can constrain the parameter space for $y_{ij}$. For example, $Z \to \tau\tau$ and $W \to \tau\nu$ exclude
$y^f_{3i} \gtrsim 1$ at $2\sigma$ for $m_S_1 \approx 1$ TeV \cite{11}. Such constraints are, however, in general weaker than the ranges plotted in Figs. 3 and 4. Note that the effect of $S_1$ on $B \to \tau \bar{\nu}$ has been previously discussed \cite{11}, and our discussion is only for comparison with $B \to \mu \bar{\nu}$. In principle, other SLQs such as $S_3$, $R_2$, $R_2$ (see Ref. \cite{9} for definition), as well as vector leptoquarks $U_1$, $U_3$, $V_2$, $V_2$ can all potentially affect $B \to \ell \bar{\nu}$ decays. We leave a detailed study of these parameters for the future.

In some sense, it is remarkable that rather large leptoquark Yukawa couplings as displayed in Figs. 3 and 4 remain unexplored. We have seen that, when light jets are involved, ATLAS data \cite{19} provide strong constraints. However, when only third generation quarks are involved, one does not easily access the rather strong constraints. However, when only third generation quarks are involved, one does not easily access the rather strong leptoquark Yukawa couplings that could enhance or suppress $B \to \mu \bar{\nu}$ or $B \to \tau \bar{\nu}$ decays. But this also illustrates the relative arbitrariness of the $S_1$ scalar leptoquark, where putting the mass above TeV scale on one hand escapes LHC detection, but on the other hand demands the rather large Yukawa couplings (the bottom Yukawa coupling is $\sim 0.02$ in SM) to have an effect on purely leptonic $B^-$ decays. In contrast, the $H^+$ effect of the general 2HDM that allows for extra Yukawa couplings is much more nuanced \cite{5}. The charged Higgs could be sub-TeV, with rather weak extra Yukawa couplings, but could still enhance $B \to \mu \bar{\nu}$ (less so for $B \to \tau \bar{\nu}$) within the Belle allowed range.

In summary, we have explored the constraints placed by current Belle results on the Wilson coefficients that can affect $B \to \mu \bar{\nu}$, $\tau \bar{\nu}$ decays, and then interpreted in terms of the Yukawa couplings of the $S_1$ scalar leptoquark. With $m_S_1$ set at 1.2 TeV, rather sizable Yukawa couplings are needed for enhancing the purely leptonic $B^-$ decays to the $2\sigma$ upper reach of Belle measurements. As one awaits eventual Belle II observation of $B \to \mu \bar{\nu}$ and improved measurements of $B \to \tau \bar{\nu}$, we find that neutron EDM can probe the CP violating phases of $S_1$ Yukawa coupling, while a large part of the rather large leptoquark Yukawa coupling range remains to be explored at hadron colliders.

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