Confirming the molecular nature of the $Z_b(10610)$ and the $Z_b(10650)$

Martin Cleven$^1$, Qian Wang$^2$, Feng-Kun Guo$^2$, Christoph Hanhart$^1,3$, Ulf-G. Meißner$^1,2,3$, and Qiang Zhao$^4$

$^1$Institut für Kernphysik and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D–52425 Jülich, Germany
$^2$Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D–53115 Bonn, Germany
$^3$Institute for Advanced Simulation, Forschungszentrum Jülich, D–52425 Jülich, Germany
$^4$Institute of High Energy Physics and Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, Beijing 100049, China

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Abstract

The decays of the $Z_b(10610)$ and the $Z_b(10650)$ to $\Upsilon(nS)\pi$, $h_b(mP)\pi$ and $\chi_{bJ}(mP)\gamma$ ($n = 1, 2, 3$, $m = 1, 2$ and $J = 0, 1, 2$) are investigated within a nonrelativistic effective field theory. It is argued that, while the decays to $\Upsilon(nS)\pi$ suffer from potentially large higher order corrections, the $P$-wave transitions of the $Z_b$ states are dominated by a single one loop diagram and therefore offer the best possibility to confirm the nature of the $Z_b$ states as molecular states and to further study their properties. We give nontrivial and parameter-free predictions for the ratios of various partial widths of the $Z_b$ and $Z'_b$ into final states with $h_b(mP)\pi$ and $\chi_{bJ}(mP)\gamma$. While such relations appear naturally in the molecular picture for the mentioned transitions, they are not expected to hold for any other scenario. In addition, the branching fractions for the neutral $Z_b$-states to $\chi_{bJ}\gamma$ are predicted to be of order $10^{-4}$–$10^{-3}$. This provides a fine test of the molecular nature in future high-luminosity experiments.

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1 Introduction

Recently the Belle Collaboration found two narrow structures, namely the $Z_b(10610) = Z_b^\pm$ and the $Z_b'(10650) = Z_b'^\pm$, in the $\Upsilon(nS)\pi^\pm$ ($n = 1, 2, 3$) and $h_b(mP)\pi^\pm$ ($m = 1, 2$) invariant masses of the $\Upsilon(5S) \rightarrow \Upsilon(nS)\pi^+\pi^-$ and $\Upsilon(5S) \rightarrow h_b(mP)\pi^+\pi^-$ decays processes [1]. In their latest data, also the open-bottom channels $B^*\bar{B}$ and $B^*\bar{B}^*$ are seen [2]. The fact that they lie in the bottomonium mass region and they are charged means that they cannot be the conventional bottomonium mesons, and their isospins are 1. The observation of the neutral state with a mass consistent with that of the $Z_b(10610)$ [3] presents a further confirmation that they are members of isotriplets. If they exist as observed by the Belle Collaboration, they must be exotic states with a pair of hidden $b\bar{b}$ and valence light quarks. Considering parity and charge parity, their quantum numbers should be $I^G(J^P) = 1^+(1^+)$, and the charge parity of the neutral state is negative.

Assuming that the total width of $Z_b^{(l)}$ is saturated by the seven channels already observed experimentally, i.e. $\Upsilon(nS)\pi$ ($n = 1, 2, 3$), $h_b(mP)\pi$ ($m = 1, 2$), $BB^* + B^*\bar{B}$ and $B^*\bar{B}^*$, Belle gives the branching ratio of each channel in $\Upsilon(5S)$ three–body decays [2]. The proximity of the states to the $B^{(*)}\bar{B}^*$ thresholds leads to the suggestion that they could be hadronic molecules of the corresponding states [4, 5, 6, 7, 8, 9, 10, 11, 12], to be distinguished from the compact $b\bar{q}b\bar{q}$ tetraquarks [11, 13, 14]. By hadronic molecules, we mean states composed of hadrons — they can be bound states (poles on the physical sheet with respect to the bottom-meson channel with a mass smaller than the threshold value), resonances or virtual states (both on the second sheet with respect to the relevant bottom-meson channel — the former above, the latter below the threshold). Based on a nonrelativistic effective field theory (NREFT) [15, 16], the authors of Ref. [5] show that the $h_b(1P, 2P)\pi^\pm$ data can be described within the bound state scenario. Therein, the $Z_b^{(l)}B^{(*)}\bar{B}^*$ coupling constants are related to the binding energies using a model-independent relation for $S$-wave shallow bound states [17, 18]. However, since now data on the decays of $Z_b^{(l)}$ to open bottom channels are available, we can chose a more general ansatz and take these couplings from data directly — in this way our results are valid for bound states, resonances as well as virtual states. Since we start from the assumption that the $Z_b$ states are purely molecular states, their decays into the $\Upsilon(nS)\pi$ and $h_b(mP)\pi$ can only happen via $BB^* + B^*\bar{B}$ (for simplicity, $BB^*$ will be used to represent $BB^* + B^*\bar{B}$ in the following) and $B^*\bar{B}^*$ loops. Since both $Z_b$'s are located very close to the corresponding open-bottom threshold, the system can in principle be examined by the NREFT approach.

As shown in Refs. [15, 16], a systematic power counting can be established for the $\pi$ or $\eta$ emissions between charmonium states. Because the $S$-wave and $P$-wave heavy quarkonia couple to the open-flavor heavy meson and anti-meson in a $P$-wave and an $S$-wave, respectively, the transitions studied in Ref. [16] can be classified into three groups, namely transitions between the $S$-wave heavy quarkonium states, $P$-wave states, and between the $P$- and $S$-wave states. The decay amplitudes of different groups have their own nonrelativistic velocity counting. Because the $Z_b$ states have positive parity, they couple to the bottom and anti-bottom mesons in an $S$-wave. Thus, the transitions $Z_b \rightarrow \Upsilon(nS)\pi$ are analogous to those between $P$- and $S$-wave quarkonia, while the $Z_b \rightarrow h_b(mP)\pi$ processes are similar to those between two $P$-wave quarkonia. The main difference is that the normal heavy quarkonium transitions with the emission of a pion studied in Ref. [16] break isospin symmetry while the $Z_b$ decays do not. Thus, by studying the $Z_b$ ($Z_b'$) $\rightarrow \Upsilon(nS)\pi$ and $h_b(mP)\pi$, we can examine the power
counting rules established in Ref. [16], and also better understand the properties of these two exotic states.

Further insight can be gained by studying radiative decays of the neutral $Z_b$ states into bottomonia. Because the $\chi_{bJ}$ ($J = 0, 1, 2$) states are the spin partners of the $h_b$ of the same principal quantum number, they couple to the bottom mesons with the same coupling constant in the heavy quark limit. Thus, since molecular states can decay via bottom meson loops only, the radiative decays $Z_b^{(\ell)0} \rightarrow \chi_{bJ}(nP)\gamma$ are related to the pionic decays $Z_b^{(\ell)\pm} \rightarrow h_b(nP)\pi^{\pm}$. Note that these transitions would be unrelated if the $Z_b$ states were of tetraquark nature.

Similar considerations were made in Ref. [19] for hindered M1 transitions of the $P$-wave charmonia. These decay channels have not been observed so far, but they could be potentially important in confirming the molecular nature of the $Z_b$ states, and thus are worthwhile to study experimentally.

In this paper, we will assume that the $Z_b^{(\ell)}$ are dynamically generated from the $B^{(*)}\bar{B}^{*}$ interactions, i.e. hadronic molecules of the $B^{(*)}\bar{B}^{*}$. We will try to identify the quantities which are sensitive to such a scenario. Section 2 contributes to the power counting in the NREFT framework of the relevant decays, which are the decays of the $Z_b$ states into the $\Upsilon\pi$, $h_b\pi$, $\chi_{bJ}\gamma$ — in particular we show that not all decays are accessible to the formalism. The numerical results are given in Sec. 3. In Sec. 4, we compare our results to previous calculations and make some comments. A brief summary is presented in Sec. 5. The loop function used in the calculations and the decay amplitudes in the NREFT are collected in Appendix A. As a cross check, we also calculate the same quantities using a Lorentz covariant formalism, and the formulas are summarized in Appendix B.

2 Power Counting

In Ref. [15], a NREFT method was introduced to study the meson loop effects in the heavy quarkonium transitions. The power counting scheme was analyzed in detail in Ref. [16]. The key quantities here are the velocities of the intermediate mesons. In this section, we briefly review the ideas of Refs. [15, 16] together with an improved discussion for the higher loop diagrams.

In general, the heavy meson velocities relevant for the decay of some particle $X$ may be estimated as $v_X \sim \sqrt{|M_X - 2m_B|/m_B}$, where the absolute value indicates that the formula can be used for both bound systems as well as resonances. The analogous formula holds when the two heavy mesons merge to a quarkonium in the final state. According to the rules of a nonrelativistic effective field theory [20] (for a review, see e.g. [21]), the momentum and non-relativistic energy count as $v_X$ and $v_X^2$, respectively. For the integral measure one finds $v_X^5/(4\pi)^2$. The heavy meson propagator counts as $1/v_X^2$. The leading order $S$-wave vertices do not have any velocity dependence, while the case for the $P$-wave vertices is more complicated: it scales either as $v_X$ when the momentum due to $P$-wave coupling contracts with another internal momentum, or as the external momentum $q$ when $q$ is contracted.

We start with the radiative transitions as shown in the upper row of Fig. 1. If the $Z_b$ states are molecular states, their spin wave functions contain both $s_{bb} = 0$ and 1 components [4], where $s_{bb}$ is the total spin of the $bb$ component. Thus, the radiative decays of the $Z_b$ states into the spin-triplet $\chi_{bJ}$ can occur without heavy quark spin flip and survive in the heavy quark limit. This is different from the M1 transitions between two $P$-wave heavy quarkonia which have been analyzed in Ref. [19]. Both the couplings of $Z_b$ and $\chi_{bJ}$ to a pair of heavy
mesons are in an $S$–wave, and the photon coupling to the bottom mesons is proportional to the photon energy $E_\gamma$. For the diagram of Fig. 1(a) the amplitude therefore scales as
\[
\bar{v}^5 \frac{1}{(4\pi)^2 \bar{v}^2} E_\gamma \sim \frac{E_\gamma}{(4\pi)^2 \bar{v}},
\]
where the velocity that appears is $\bar{v} = (v_Z + v_\chi)/2 \simeq v_\chi/2$ \textsuperscript{22}, since $v_Z \sim v_Z' \simeq 0.02$, if the central values of the measured $Z_b^{(l)}$ masses, 10607.2 MeV and 10652.2 MeV, are used, while $v_\chi$ ranges from 0.12 for the $\chi_bJ(3P)$ to 0.26 for the $\chi_bJ(2P)$ to 0.37 for the $\chi_bJ(1P)$. Here we used the mass of the $\chi_bJ(3P)$, 10.53 GeV, as reported by the ATLAS Collaboration \textsuperscript{23}. In the following, we will count $\bar{v}$ as $\mathcal{O}(v_\chi)$. The scaling ensures that the amplitude gets larger when the bottomonium in the final state is closer to the open-bottom threshold. Thus, we expect for the absolute value of the decay amplitude from this diagram
\[
\left| \frac{A_{\chi_bJ(1P)\gamma}}{E_\gamma} \right| : \left| \frac{A_{\chi_bJ(2P)\gamma}}{E_\gamma} \right| : \left| \frac{A_{\chi_bJ(3P)\gamma}}{E_\gamma} \right| \sim \frac{1}{v_1} : \frac{1}{v_2} : \frac{1}{v_3} = 1 : 1.4 : 3.1,
\]
if the $\chi_bJ(nP)B\bar{B}$ coupling constants take the same value. Diagram (a) can be controlled easily in theory. Thus, clear predictions can be made whenever diagram (a) dominates. In the following we will identify such dominant decays based on the power counting for the NREFT.

As for Fig. 1(b), the coupling $\chi_bB\bar{B}\gamma$ cannot be deduced by gauging the coupling of $\chi_bJ$ to a $B\bar{B}$-meson pair. Thus, it has to be gauge invariant by itself and proportional to the electromagnetic field strength tensor $F^{\mu\nu}$. This gives a factor of photon energy $E_\gamma$ and the amplitude of Fig. 1(b) scales as
\[
\frac{v_2^5}{(4\pi)^2 \frac{v_2^2}{v_2^2}} E_\gamma \sim \frac{E_\gamma v_Z}{(4\pi)^2},
\]
where we have assumed that the corresponding coupling is of natural size. Thus, diagram (b) is suppressed compared to diagram (a) at least by a factor of $v_Z v_\chi < 0.01$ for the decay to $\chi_bJ(1P)$ and even smaller for the excited states.

The situation is more complicated for the graph displayed in Fig. 1(c). Here we have a two-loop diagram, so that the velocities running in different loops are significantly different — the one in the loop connected to the $Z_b$ is $v_Z$, and the other is $v_\chi$. It is important to count them separately since $v_Z \ll v_\chi$ due to the very close proximity of the $Z_b$ to the threshold\textsuperscript{1}. The

\textsuperscript{1}The concept applied here is analogous to the scheme by now well established for the effective field theory for reactions of the type $NN \rightarrow NN\pi$ — see Ref. \textsuperscript{24} for a review.
internal pion momentum scales as the larger loop momentum, and thus the pion propagator should be \( \sim 1/(m_B^2 v^2_\pi) \). This leaves us with

\[
\frac{v_Z^5}{(4\pi)^2} \left( \frac{1}{(v_Z^2)^2} \frac{1}{(4\pi)^2} \frac{1}{(v_\chi^2)^2} \frac{E_\pi g g}{m_B^2 v^2_\pi} \right) \frac{m_B^4}{F_\pi^2 F_\pi} \sim \frac{v_Z E_\pi g^2 m_B^2}{v_\chi (4\pi)^2 \Lambda^4_\chi},
\]

where \( F_\pi \) is the pion decay constant in the chiral limit, the factor \( m_B^4 \) has been introduced to give the same dimension as the estimate for the first two diagrams, and the hadronic scale was introduced via the identification \( \Lambda_\chi = 4\pi F_\pi \). Thus the two-loop diagram is suppressed compared to the leading one, Eq. (3), by a factor \( v_Z g^2 m_B^2 / \Lambda^2_\chi \sim 0.1 \), where we used for the coupling \( B^* \to B\pi \) the value \( g = 0.5 \) (a recent lattice calculation gives \( 0.449 \pm 0.051 \) \cite{25}), and \( \Lambda_\chi \sim 1 \) GeV. It can easily be seen that Fig. 1 (d) gives the same contribution which also reflects the fact that they are both required at the same order to ensure gauge invariance. Thus, from our power counting it follows that the loop diagrams of Fig. 1 (b)-(d) provide a correction of at most 10%. We will therefore only calculate diagram (a) explicitly and introduce a 10% uncertainty for the amplitudes which corresponds to 20% for the branching ratios. Higher loop contributions are to be discussed later.

Next we consider the hadronic transition \( Z_b \to h_b \pi \). This decay has already been studied in Ref. 5 in the same formalism. Again, since the \( h_b \) has even parity, its coupling to the bottom mesons is in an \( S \)-wave. In addition, the final state must be in a \( P \)-wave to conserve parity, such that the amplitude must be linear in the momentum of the outgoing pion, \( q \). We therefore find for the one-loop contribution of Fig. 1 (e)

\[
\frac{v_Z^5}{(4\pi)^2} \left( \frac{1}{(v_Z^2)^2} \frac{1}{(4\pi)^2} \frac{1}{(v_\chi^2)^2} \frac{q q F_\pi}{m_B^2} \right) \sim g q F_\pi \frac{q F_\pi}{v_\chi v_\chi},
\]

while Fig. 1 (f) gives

\[
\frac{v_Z^5}{(4\pi)^2} \left( \frac{1}{(v_Z^2)^2} \frac{1}{(4\pi)^2} \frac{1}{(v_\chi^2)^2} \frac{q}{m_B^2} \frac{q}{m_B^2} \right) \sim v_Z q F_\pi \frac{q F_\pi}{\Lambda^2_\chi}.
\]

Notice that the pion has to be emitted after the loop if the \( Z_b \) is a pure hadronic molecule, so that the velocity in the counting should be \( v_Z \) instead of \( v_h \). This is suppressed by \( v_h v_Z / g \) which leads to a correction of the order of 2% noticing that \( v_h \simeq v_\chi \).

The two-loop diagram Fig. 1 (g) contributes as

\[
\frac{v_Z^5}{(4\pi)^2} \left( \frac{1}{(v_Z^2)^2} \frac{1}{(4\pi)^2} \frac{1}{(v_\chi^2)^2} \frac{1}{(4\pi)^2} \frac{1}{v_\chi v_\chi} \right) \frac{g g g g}{m_B^2 v^2_\pi} \frac{m_B^2}{F_\pi^2 F_\pi^2} \frac{m_B^2}{F_\pi^2 F_\pi^2} \frac{m_B^2}{F_\pi^2 F_\pi^2} \frac{m_B^2}{F_\pi^2 F_\pi^2} = \frac{v_Z g F_\pi g^2 m_B}{v_\chi v_\chi \Lambda^4_\chi},
\]

where we have used that the energy from the \( \pi B \to \pi B \) vertex can be identified with the energy of the outgoing pion \( 26 \), \( E_\pi \sim q \). The \( B^* B \pi \) vertex contributes a factor of the external momentum \( q \) since the \( Z_b \to h_b \pi \) is a \( P \)-wave decay, and this is the only \( P \)-wave vertex. Therefore, this diagram is suppressed compared to the leading loop, Eq. (5), by a factor \( v_Z m_B q / \Lambda^2_\chi \) which is smaller than 10%. Thus, also for the transitions \( Z_b^{(0)} h_b \pi \) we may only calculate the leading one-loop diagrams, Fig. 1 (e), and assign an uncertainty of 10% to the rates which gives an uncertainty of 20% for the branching ratios. Higher loop contributions are to be discussed later.

Finally, we consider at the decay channel \( Z_b \to \Upsilon \pi \). Here the final state is in an \( S \)-wave, but the coupling of the \( \Upsilon \) to \( BB \) is in a \( P \)-wave. For diagram (e) the momentum due to this
coupling has to scale as the external pion momentum. Together with the pionic coupling that is also linear in the pion momentum, the amplitude is thus proportional to $q^2$. The one-loop diagram for $Z_b \rightarrow \Upsilon \pi$ via Fig. 1 (e), is therefore estimated as

$$\frac{\bar{v}^5}{(4\pi)^2 (\bar{v}^2)^3 F_{\pi}} \approx \frac{g q^2 F_{\pi}}{v \Lambda^2_\chi}.$$  \hfill (8)

The diagram Fig. 1 (f) on the other hand gives

$$\frac{v_Z^5}{(4\pi)^2 (v_Z^2)^2 F_{\pi}} m_B \sim \frac{v_Z E_{\pi} F_{\pi} m_B}{\Lambda^2_\chi},$$  \hfill (9)

where $m_B$ is introduced in order to get the same dimension as Eq. (8). Compared to the one-loop diagram (e) this is a relative suppression of order $v_\Upsilon v_Z m_B / q$ which is less than 10% for all the $\Upsilon$ states, where the value of $v_\Upsilon$ is about 0.46, 0.33 and 0.22 for the $1S$, $2S$ and $3S$ states, respectively. The two-loop contribution with the exchange of a pion, Fig. 1 (g), is estimated as

$$\frac{v_Z^5}{(4\pi)^2 (v_Z^2)^2 (4\pi)^2 (v_Z^2)^2} \frac{1}{m_B^2 v_\Upsilon^2} \frac{1}{m_\Upsilon} \frac{1}{F_{\pi}} \frac{g E_{\pi} F_{\pi} m_B^5}{\Lambda^4_\chi} \sim \frac{v_Z v_\Upsilon g E_{\pi} F_{\pi}}{\Lambda^2_\chi} m_B^3.$$  \hfill (10)

Thus, the strength of two-loop diagram relative to leading one-loop diagram is estimated for the $\Upsilon \pi$ as $v_Z v_\Upsilon^2 m_B^3 / (\Lambda^2_\chi q)$. Numerically, this corresponds to a factor of around 0.6 for the $\Upsilon(1S, 2S)\pi$ and 0.7 for the $\Upsilon(3S)\pi$ amplitudes. As a consequence, the branching ratios for these transitions can only be calculated with rather large uncertainties up to 100%.

The heavy meson velocities relevant for the mentioned transitions range from 0.02 to 0.5 — in momenta this is a range from 0.1 to 2.5 GeV. While pion contributions are expected to be suppressed significantly and can be controlled within chiral perturbation theory for pion momenta of up to 500 MeV and smaller, higher pion loop contributions might get significant for momenta beyond 1 GeV. We will now study those higher pion loops within the power counting scheme outlined above. We will start with the three-loop diagrams, as shown in Fig. 2 considering first diagram (a). The results can be easily generalized to higher loops as shown below. Compared to the two-loop diagrams in Fig. 1 there are one more pion propagator and two more bottom meson propagators in the transition and none of them is connected to the external heavy quarkonia. In addition, there is no two-bottom-meson unitary cut present. As a consequence, we have to use a relativistic power counting — c.f. Ref. [20]. Then pion momentum and energy are of order $m_B v_{b\bar{b}}$, with $v_{b\bar{b}}$ the velocity of the $B$-meson connected to the $b\bar{b}$-meson in the final state. Both the energies and momenta of the additional bottom mesons are now of the same order, such that the bottom-meson propagator
is counted as $1/\nu_{\bar{b}b}$. The integral measure reads $v_{\bar{b}b}^4/(4\pi)^2$. Therefore, the additional factor as compared to the two-loop diagrams is

$$\frac{v_{\bar{b}b}^4}{(4\pi)^2} \frac{1}{v_{\bar{b}b}^4} \frac{(g v_{\bar{b}b})^2}{F^2} m_B^2 = \left( \frac{g m_B v_{\bar{b}b}}{\Lambda_\chi} \right)^2.$$ \hspace{1cm} (11)

If $v_{\bar{b}b} \sim 0.4$, then the three-loop diagram (a) is of similar size as that of the two-loop diagrams. This is the case for the processes with the $h_b(1P)$, $\chi_{bJ}(1P)$, and $\Upsilon(1S,2S)$. For smaller values of $v_{\bar{b}b}$, it is suppressed. In diagram (b) there is only one more bottom meson propagator and the additional $B\pi B\pi$ vertex is in an S-wave. We obtain

$$\frac{v_{\bar{b}b}^4}{(4\pi)^2} \frac{1}{v_{\bar{b}b}^4} \frac{1}{F^2} m_B^2 = \left( \frac{m_B v_{\bar{b}b}}{\Lambda_\chi} \right)^2.$$ \hspace{1cm} (12)

Four and higher loop diagrams that cannot be absorbed by using physical parameters may now be estimated by applying a proper number of factors of the kind of Eqs. (11) and (12). It is easy to see that also additional topologies provide analogous factors. Since $m_B/\Lambda_\chi \sim 5$, higher loops get increasingly important, if $v_{\bar{b}b} > 0.2$. For $v_{\bar{b}b} \sim 0.2$, the three and more loops are of the same order as the two-loop diagrams, which is the case for the 2P states, and thus suppressed in comparison with the one-loop contribution. For the 3P bottomonia in the final state, the value of $v_{\bar{b}b}$ is even smaller, and the multiple loops are even suppressed as compared to the two-loop contribution.

To summarize the findings of the power counting analysis, we conclude that the calculation of one-loop triangle diagrams as depicted in Fig. 1 (a) and (e) is a good approximation, with a controlled uncertainty, to the transitions of the $Z_b^{(1)}$ into the $\chi_{bJ}(2P,3P)\gamma$ and $h_b(2P)\pi$ — for the $h_b(3P)\pi$ the phase space is too limited. But similar calculations are not applicable to the decays into the 1P states as well as the $\Upsilon(nS)\pi$. For the decays $Z_b \to \Upsilon(nS)\pi$, the contribution from the two-loop diagrams is not suppressed or even enhanced compared to the one-loop contribution. In light of this discussion, it becomes clear why the pattern of branching fractions for these channels (see Tab. 1. $\mathcal{B}[\Upsilon(2S)\pi] > \mathcal{B}[\Upsilon(3S)\pi] \gg \mathcal{B}[\Upsilon(1S)\pi]$ cannot be reproduced by calculating the three-point diagrams in the NREFT formalism, which always favors the 1S to the 2S transition and the 2S compared to the 3S transition due to the factor $q^2$ in Eq. (5).
3 Results

In this section, we investigate quantitatively the decays of the \( Z_b \) states through heavy meson loops. The coupling of these states to the heavy meson fields \( H_a = \vec{V}_a \cdot \sigma + P_a \) and \( \bar{H}_a = -\vec{V}_a \cdot \bar{\sigma} + \bar{P}_a \) with \( V_a (\bar{V}_a) \) and \( P_a (\bar{P}_a) \) annihilating the vector and pseudoscalar (anti-)heavy mesons, respectively, is given by the Lagrangian

\[
L_{Z} = i z \frac{2}{2} \left( \bar{Z}_b^{\dagger} H_a \sigma^i \bar{H}_b \right) + \text{H.c.,}
\]

where the \( Z_b \) states are given by a \( 2 \times 2 \) matrix

\[
Z_{ba}^i = \begin{pmatrix} \frac{1}{\sqrt{2}} Z_{0i}^{0i} & \frac{Z_{0i}^i}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} Z_{0i}^{0i} \end{pmatrix}.
\]

We incorporate the experimental observation that the \( Z_b^{(i)} \) couples only to \( B^{(*)} \bar{B}^* \) via the Lagrangian,

\[
L_{Z,Z'} = z' \varepsilon^{ijk} \bar{V}^{ij} Z^{j} \bar{V}^{kl} + z \left[ \bar{V}^{ij} Z^{i} P^{j} - \bar{P}^{i} Z^{j} V^{ij} \right] + \text{H.c.}
\]

Fitting to the experimental data we find

\[
z = (0.79 \pm 0.05) \text{ GeV}^{-1/2}, \quad z' = (0.62 \pm 0.07) \text{ GeV}^{-1/2},
\]

and especially

\[
\frac{z}{z'} = 1.27 \pm 0.16
\]

which deviates from unity by \( 2\sigma \). This deviation indicates a significant amount of spin symmetry violation, which, however, is not unnatural for very-near-threshold states where small differences in masses may imply huge differences in binding energies resulting in significantly different effective couplings, as already discussed in Ref. [5]. The nonrelativistic Lagrangian for the \( \chi_{bJ} \) coupling to a pair of heavy mesons can be found in Ref. [29], and the one for the magnetic coupling of heavy mesons in Ref. [30].

With the amplitudes given in Appendix A it is straightforward to calculate the decay widths of the \( Z_b^{(i)} \to h_b(mP)\pi \), which are proportional to \( g_1^2 \), where \( g_1 \) is the \( P \)-wave bottomonium–bottom meson coupling constant. In the ratio defined as

\[
\xi_m := \frac{\Gamma[Z_b^{(i)} \to h_b(mP)\pi]}{\Gamma[Z_b \to h_b(mP)\pi]},
\]

\( g_1 \) is cancelled out. With the meson masses listed in Tab. I, we obtain

\[
\xi_1 = 1.21 \left| \frac{z'}{z} \right|^2 = 0.75, \quad \xi_2 = (1.53 \pm 0.43) \left| \frac{z'}{z} \right| = 0.95 \pm 0.36.
\]

where the first error in the second term is the theoretical uncertainty due to neglecting higher order contributions (see the discussion in the previous section), and the second one also includes the uncertainty of \( z'/z \) added in quadrature. Due to the theoretically uncontrollable higher order contributions for the decays into the \( 1P \) states, no uncertainty is given for \( \xi_1 \).

The predictions are consistent with their experimental counterparts (see Tab. I)

\[
\xi_1^{\text{Exp}} = 1.65 \pm 0.96, \quad \xi_2^{\text{Exp}} = 2.13 \pm 1.44.
\]
Table 2: The ratios $\xi$ of different decay modes in both NREFT and a relativistic framework as compared with the experimental data. The NREFT values quoted without uncertainties may be understood as order-of-magnitude estimates.

|        | NREFT | Rel. | Exp. |
|--------|-------|------|------|
| $\Upsilon(1S)\pi$ | 0.7   | 0.7  | 0.47 ± 0.22 |
| $\Upsilon(2S)\pi$ | 0.9   | 0.8  | 0.34 ± 0.15 |
| $\Upsilon(3S)\pi$ | 2 ± 2 | 1.6  | 0.48 ± 0.20 |
| $h_b(1P)\pi$      | 0.8   | 0.7  | 1.65 ± 0.96 |
| $h_b(2P)\pi$      | 1.0 ± 0.4 | 0.9 | 2.13 ± 1.44 |

Here, new measurements with significantly reduced uncertainties would be very desirable. A collection of ratios for the decays of the $Z_b$ states to $h_b(mP)\pi$ and $\Upsilon(nS)\pi$, respectively, is presented in Tab. 2. The uncertainties, whenever they are under control theoretically, are also included. The significant deviations for the $\Upsilon(nS)\pi$ results from the experimental numbers appear natural, given that for those transitions higher loops were argued to be at least as important as the one-loop diagram included here, as outlined in detail in the previous section. As a cross check of our nonrelativistic treatment, we also calculated the same quantities using a Lorentz covariant formalism with relativistic propagators for all the intermediate mesons (the formalism is summarized in App. B). The results are denoted as ‘Rel.’ in Tab. 2. They should be compared with the central values of the NREFT results. The difference reflects relativistic corrections to the nonrelativistic treatment of the bottom-meson propagators. In our case, the difference between relativistic and NREFT calculations here and in the following never exceeds 15% for the rates and thus is well below the uncertainty due to higher loops.

It is important to ask to what extent the above predictions can be used to probe the nature of the $Z_b$ states. The $Z_b$ and $Z_b'$ are away from the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds by similar distances, $m_{Z_b} - m_B - m_{B^*} \approx m_{Z_b'} - 2m_{B^*}$. Additionally, due to the heavy quark spin symmetry, one may expect that the loops contribute similarly to the decays of these two states into the same final state. This is indeed the case. We find that the ratios aside of $|z'/z|^2$ are basically the phase space ratios, which are

$$\frac{|q(Z_b' \to h_b(1P)\pi)|^3}{|q(Z_b \to h_b(1P)\pi)|^3} = 1.20, \quad \frac{|q(Z_b' \to h_b(2P)\pi)|^3}{|q(Z_b \to h_b(2P)\pi)|^3} = 1.53. \quad (18)$$

This implies that the ratios for the decays into the $h_b(mP)\pi$ are determined by the ratio of the partial decay widths for the open-bottom decay modes,

$$\xi_m \approx \frac{PS_m}{PS_m'} \frac{\Gamma(Z_b' \to B^{*+}\bar{B}^{*0})}{\Gamma(Z_b \to B^{+}B^{0} + B^{0}B^{*+})}. \quad (19)$$

where $PS_m'$ is the phase space for the decays $Z_b' \to h_b(mP)\pi$. Such a relation cannot be obtained were the $Z_b$ states of tetraquark structure because the decay of a $b\bar{q}b\bar{q}$ tetraquark into the $h_b\pi$ knows nothing about the decay into the open-bottom channels. Here, if we impose spin symmetry for $z$ and $z'$, one would get for $\xi_m$ simply the ratio of phase spaces. In reality the second factor which is the square of the ratio of effective couplings, c.f. Eq. (14), deviates from unity due to spin symmetry violations enhanced by the proximity of the $B^{(*)}\bar{B}^*$ thresholds as discussed above.
cause the πZ of in Tab. 3. The values quoted without uncertainties may be understood as order of magnitude estimates.

|, m, l, k, j, mP) are those into the \( bJ \) states. The results for the ratios \( \Omega \) are collected in Tab. 3.

Using the branching ratio \( B(Z_b^{(l)} \rightarrow h_b(1P,2P)\pi^+) \), we find that the branching ratios of \( Z_b^{(l)} \rightarrow \chi_{bJ}(1P,2P)\gamma \) are of order \( 10^{-4} \sim 10^{-3} \). The largest branching fractions of the \( \chi_{bJ}(mP) \) are those into the \( \gamma \Upsilon(nS) \), and the \( \Upsilon(nS) \) can be easily measured. Thus, the final states for measuring the \( Z_b^0 \rightarrow \chi_{bJ}\gamma \) would be the same as those of the \( Z_b^0 \rightarrow \Upsilon(nS)\pi^0 \) because the \( \pi^0 \) events are selected from photon pairs. This means that the detection efficiency and background of these two processes would be similar. In the preliminary experimental results [3], the \( Z_b(10610)^0 \) event number collected in the \( \Upsilon(1S,2S)\pi^0 \) channels is of order \( \mathcal{O}(100) \). Given that the luminosity of the future Super-KEKB could be two orders of magnitude higher than KEKB, such transitions will hopefully be measured. Furthermore, one may also expect to measure these radiative transitions at the LHCb. Note that the experimental confirmation of the ratios given in Tab. 3 would be a highly nontrivial evidence for the molecular nature of the \( Z_b \) states.

Lacking knowledge of the \( \chi_{bJ}(3P)B\bar{B} \) coupling constant, the transitions into the \( 3P \) states cannot be predicted parameter-free. However, they can be used to check the pattern in Eq. (2) predicted by the power counting analysis. The decay widths of the \( Z_b \rightarrow \chi_{bJ}(mP)\gamma \) are

| Table 3: The ratio \( \Omega \) and the corresponding branching fractions for all possible radiative decays. Uncertainties are given, whenever they can be controlled theoretically (see text). The values quoted without uncertainties may be understood as order of magnitude estimates. |
|---|---|---|---|
| \( \chi_{b0}(1P)\gamma \) | \( 5 \times 10^{-3} \) | \( 1 \times 10^{-4} \) | \( 4 \times 10^{-3} \) |
| \( \chi_{b1}(1P)\gamma \) | \( 1 \times 10^{-2} \) | \( 3 \times 10^{-4} \) | \( 1 \times 10^{-2} \) |
| \( \chi_{b2}(1P)\gamma \) | \( 2 \times 10^{-2} \) | \( 5 \times 10^{-4} \) | \( 2 \times 10^{-2} \) |
| \( \chi_{b0}(2P)\gamma \) | \( (6.3 \pm 1.8) \times 10^{-3} \) | \( (2.7 \pm 1.5) \times 10^{-4} \) | \( (4.2 \pm 1.2) \times 10^{-3} \) |
| \( \chi_{b1}(2P)\gamma \) | \( (1.3 \pm 0.4) \times 10^{-2} \) | \( (5.6 \pm 3.2) \times 10^{-4} \) | \( (1.3 \pm 0.4) \times 10^{-2} \) |
| \( \chi_{b2}(2P)\gamma \) | \( (1.9 \pm 0.5) \times 10^{-2} \) | \( (8.3 \pm 4.5) \times 10^{-4} \) | \( (1.8 \pm 0.5) \times 10^{-2} \) |

| | \( \Omega \) | Branching Fraction | \( \Omega \) | Branching Fraction |
|---|---|---|---|
| \( \chi_{b0}(1P)\gamma \) | \( 5 \times 10^{-3} \) | \( 1 \times 10^{-4} \) | \( 4 \times 10^{-3} \) |
| \( \chi_{b1}(1P)\gamma \) | \( 1 \times 10^{-2} \) | \( 3 \times 10^{-4} \) | \( 1 \times 10^{-2} \) |
| \( \chi_{b2}(1P)\gamma \) | \( 2 \times 10^{-2} \) | \( 5 \times 10^{-4} \) | \( 2 \times 10^{-2} \) |
| \( \chi_{b0}(2P)\gamma \) | \( (6.3 \pm 1.8) \times 10^{-3} \) | \( (2.7 \pm 1.5) \times 10^{-4} \) | \( (4.2 \pm 1.2) \times 10^{-3} \) |
| \( \chi_{b1}(2P)\gamma \) | \( (1.3 \pm 0.4) \times 10^{-2} \) | \( (5.6 \pm 3.2) \times 10^{-4} \) | \( (1.3 \pm 0.4) \times 10^{-2} \) |
| \( \chi_{b2}(2P)\gamma \) | \( (1.9 \pm 0.5) \times 10^{-2} \) | \( (8.3 \pm 4.5) \times 10^{-4} \) | \( (1.8 \pm 0.5) \times 10^{-2} \) |

Heavy quark spin symmetry allows one to gain more insight into the molecular structure. Because the \( \chi_{bJ}(mP) \) are the spin-multiplet partners of the \( h_b(mP) \), the radiative decays of the neutral \( Z_b^{(0)} \) into \( \chi_{bJ}(mP)\gamma \) can be related to the hadronic decays of the \( Z_b^{(5)} \), no matter whether they are neutral or charged, into the \( h_b(mP)\pi \). It is therefore useful to define the following ratios

\[
\Omega[Z_b^{(5)}, \chi_{bJ}(mP)] := \frac{\Gamma(Z_b^{(5)} \rightarrow \chi_{bJ}(mP)\gamma)}{\Gamma(Z_b^{(5)} \rightarrow h_b(mP)\pi^0)}.
\]
Table 4: The ratios defined in Eq. (21) for all channels. Uncertainties are given, whenever they can be controlled theoretically.

| Channel | Ratios |
|---------|--------|
| $Z_b \to \chi_{bJ}(1P)\gamma$ | $1 : 2.5 : 3.7$ |
| $Z_b' \to \chi_{bJ}(1P)\gamma$ | $1 : 2.9 : 4.4$ |
| $Z_b \to \chi_{bJ}(2P)\gamma$ | $1 : (2.1 \pm 0.6) : (2.9 \pm 0.8)$ |
| $Z_b' \to \chi_{bJ}(2P)\gamma$ | $1 : (3.0 \pm 0.9) : (4.2 \pm 1.2)$ |

proportional to $g_{1,mP}^2$. Taking the same value for $g_{1,mP}^2$, the explicit evaluation of the triangle loops gives $1 : 1.8 : 4.4$ for the ratios defined in Eq. (2) with $J = 1$. The values are close to the ones in Eq. (2), and thus confirm the $1/\bar{v}$ scaling in Eq. (1) of the amplitudes.

As observed in Ref. [16], the NREFT leading loop calculation preserves the heavy quark spin structure. Because the $Z_b$ contains both $s_{bb} = 0$ and 1 components, the leading contribution to its transitions into the normal bottomonia, which are eigenstates of $s_{bb}$, comply with the spin symmetry. This conclusion should be true no matter what nature the $Z_b$'s have as long as the spin structure does not change. Thus, one expects that the branching fraction ratios of the decays of the same $Z_b$ into a spin-multiplet bottomonia plus a pion or photon, such as

$$B(Z_b^{(t)} \to \chi_{b0}(mP)\gamma) : B(Z_b^{(t)} \to \chi_{b1}(mP)\gamma) : B(Z_b^{(t)} \to \chi_{b2}(mP)\gamma),$$

are insensitive to the structure of the $Z_b$. This statement may be confirmed by observing that our results, as shown in Tab. 4 agree with the ratios $1 : 2.6 : 4.1$ (for the $1P$ states) and $1 : 2.5 : 3.8$ (for the $2P$ states) which are obtained solely based on heavy quark spin symmetry in Ref. [31]. One may expect a derivation from the spin symmetry results, which is due to the mass difference between the $B$ and $B^*$ mesons, to be of order $O(2\Lambda_{QCD}/m_B) \sim 10\%$. The central values given in Tab. 4 deviate from the spin symmetry results by at most 20%, and they are fully consistent considering the uncertainties.

4 Comparison with other works on $Z_b$ decays

Since their discovery in an impressive number of theoretical works the molecular nature of the $Z_b$ states was investigated. In this section we compare in some detail our approach to the calculations in Refs. [32, 33, 34, 35] which deal with some of the decays considered in our paper. Common to most of these works is that, contrary to our approach, the second part of the one loop integral shown in Fig. 1(e) is either approximated [32, 33] or calculated differently [34]. Especially, by this the analytic structure of the loop gets changed converting it to a topology of type (f) in that figure, since the second $B^{(*)}\bar{B}^{(*)}$ cut was removed from the loop. However, our power counting gives that it is exactly this cut that drives the enhancement of the one loop diagrams compared to the two-loop diagrams.

Since the loop of type (f) gives the wave function at the origin in $r$-space, the formalism applied in Refs. [32, 33, 34] is basically identical to that used in the classic calculations for the decay of positronium into two photons. However, as discussed in detail in Ref. [34], it is applicable only if the range of the transition potential from the constituents to the final state is significantly shorter ranged than the potential that formed the molecule — a scale not to
be mixed up with the size of the molecule which can be very large for a shallow bound state. However, the range of the binding momentum of the $Z_b$ states is not known and might well be of the order of the range of the transition potential (at least as long as the final bottomonium is not a ground state). In such a situation in Ref. [36] it is proposed to calculate the full loop function for the transitions, as done here in our work, which in effect means to expand around the limit of a zero range potential that forms the bound state. In that paper it is also shown that the potentially most important corrections to the transition rate cancel, such that the uncertainty of the procedure is given by the binding momentum of the molecule in units of the range of forces and not of the order of the final momenta in units of the range of forces. This gives an additional justification for the approach we are using. We now discuss the formalisms of Refs. [32, 33, 34, 35] in some more detail.

In Ref. [32] an effective field theory called X-EFT is used. It is valid for hadronic molecules with small binding energies so that the pion mass and the heavy meson hyperfine splitting are hard scales. The decays of the $Z_b^{(*)}$ can be represented by a bubble with two vertices, one connecting the $B^{(*)} \bar{B}^*$ to the $Z_b^{(*)}$ states and the other is a local operator for the $B^{(*)} \bar{B}^* \pi (\bar{b}b)$ coupling. The coefficient of the local operator depends on the pion energy, and is obtained by matching to the tree-level diagrams in heavy hadron chiral perturbation theory. For more details, we refer to Refs. [37, 29]. The ratios of $\Gamma(Z' \rightarrow \Upsilon(3S)\pi)/\Gamma(Z \rightarrow \Upsilon(3S)\pi)$ and $\Gamma(Z' \rightarrow h_b(2P)\pi)/\Gamma(Z \rightarrow h_b(2P)\pi)$ were calculated in Ref. [32] assuming $z = z'$. As outlined in the discussion below Eq. (19), in this case these ratios are just the ratios of phase spaces and thus our results for them agree for $z = z'$. The method of Ref. [33] is a phenomenological variant of the approach outlined above.

Also in Ref. [34] the transition from the intermediate $B^{(*)} \bar{B}^*$-system to the final $\pi(\bar{b}b)$ system was assumed to be local, however, here the strength of this local operator was calculated differently: the authors estimate it via the overlap integral of the $\bar{b}b$ component in the $B^{(*)} \bar{B}^*$ wave function with the outgoing $\bar{b}b$ pair in the presence of a dipole operator. While this procedure is certainly justified when there are $1P$ states in the final state — here the relative momenta between the two $B$ mesons are beyond 2.5 GeV and indeed in this case our effective field theory does not converge anymore (c.f. Sec. 2) — we regard it as questionable for the $2P$ states. There is one more difference, namely the fact that in the formalism of Ref. [34], the transitions to the $\gamma \chi_b J(nP)$ final states are disconnected from those to the $\pi h_b(nP)$ states, while in our approach they are connected as discussed in detail above. Thus, an experimental observation of the decay of one of the $Z_b$ states to, say, $\gamma \chi_b J(2P)$ would allow one to decide on the applicability of our approach.

Similar to our work, in Ref. [35] the full heavy meson loop is evaluated, but regularized with a form factor. Absolute predictions are given for the transitions using a model to estimate the $B^{(*)} \bar{B}^{(*)} (\bar{b}b)$ coupling — it is difficult to judge the uncertainty induced by this. In a first step in that work a cut-off parameter was adjusted to reproduce each individual transition. It is found that the cut-off parameters needed for the $h_b \pi$ transitions are typically larger and closer together than those needed for the $\Upsilon \pi$ transitions. This hints at form factor effects being not very significant in the former decays. This interpretation is also supported by the observation that the ratios of decay rates — the same quantities as investigated here — are found to be basically independent of the form factor. In this sense the phenomenological studies of Ref. [35] provide additional support for the effective field theory calculation presented here, although a well-controlled error estimate cannot be expected from such a method.
5 Summary

In this paper, we assume that both $Z_b$ and $Z'_b$ are hadronic molecules predominantly coupling to $BB^*$ and $B^*B^*$, respectively, in line with the data by the Belle Collaboration. As a consequence of this assumption, the $Z_b$ states can only couple through $B(B^*)\bar{B}^*$-loops. Using NREFT power counting we argue that

- the decay channels $Z_b^{(i)} \rightarrow \Upsilon(nS)\pi$ as well as the transitions into the ground state $P$-wave bottomonia in the final state can not be controlled within the effective field theory, since higher loop contributions are expected to dominate the transitions;
- model-independent predictions can be provided for $Z_b^{(i)} \rightarrow h_b(2P)\pi$ and radiative decays $Z_b^{(i)} \rightarrow \chi_{bJ}(2P)\gamma$.

The ratios for $Z_b$ and $Z'_b$ decays into the same final states $h_b(mP)\pi$ are consistent with the experimental data. Our results reflect the fact that those ratios are essentially the ratio of the corresponding phase space factors times the ratio of the $Z_bB\bar{B}^*$ and $Z'_bB^*\bar{B}^*$ couplings squared. If further experimental analysis with higher statistics could underpin this fact, it would be a very strong evidence for the molecular interpretation since such a relation cannot be obtained from, e.g., a tetraquark structure.

Furthermore, we calculate branching fractions for the final states $\chi_{bJ}(mP)\gamma$. They are predicted to be of order $10^{-4} \sim 10^{-3}$. Although this is clearly a challenge to experimentalists, a confirmation of these rates would strongly support the molecular picture. It is noted that the ratios of a certain $Z_b$ into bottomonia in the same spin multiplet are insensitive to the structure of the $Z_b$, and may be obtained solely based on heavy quark spin symmetry.

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A Nonrelativistic approach

The basic three-point loop function worked out using dimensional regularization in $D = 4$ is

$$I(m_1, m_2, m_3, \bar{q}) = \frac{-i}{8} \int \frac{d^Dl}{(2\pi)^D} \left[ \frac{1}{l^0 - \frac{\bar{p}}{m_1} + i\epsilon} \frac{1}{l^0 + b_{12} + \frac{\bar{p}}{m_2} - i\epsilon} \frac{1}{l^0 + b_{12} - b_{23} - \frac{(\bar{q} - \bar{q})^2}{m_3} + i\epsilon} \right]$$

$$= \frac{\mu_{12}\mu_{23}}{16\pi} \sqrt{\frac{a}{\mu}} \left[ \tan^{-1} \left( \frac{c' - c}{2\sqrt{a(c - i\epsilon)}} \right) + \tan^{-1} \left( \frac{2a + c - c'}{2\sqrt{a(c' - a - i\epsilon)}} \right) \right],$$

(A.1)

where $m_i (i = 1, 2, 3)$ are the masses of the particles in the loop, $\mu_{ij} = m_i m_j / (m_i + m_j)$ are the reduced masses, $b_{12} = m_1 + m_2 - M$, $b_{23} = m_2 + m_3 + q^0 - M$, with $M$ the mass of the
are initial particle, and
\[ a = \left( \frac{\mu_{23}}{m_3} \right)^2 q^2, \quad c = 2\mu_{12}b_{12}, \quad c' = 2\mu_{23}b_{23} + \frac{\mu_{23}}{m_3} q^2. \]

In terms of the loop function given above, the amplitudes for \( Z_b^+ \) and \( Z_b^{+'} \) decays into \( h_b\pi^+ \) are
\[
A_{Z_b^+ h_b} = \frac{2\sqrt{2}g g_1 z_1}{F_\pi} \sqrt{M_{h_b}M_{Z_b}} \epsilon_{ijk} q_i^e Z_{b}^e h_b \ [I(M_B, M_{B'}, M_{B'}, \bar{q}) + I(M_{B'}, M_B, M_{B'}, \bar{q})], \quad (A.2)
\]
and
\[
A_{Z_b^{+'} h_b} = \frac{2\sqrt{2}g g_1 z_2}{F_\pi} \sqrt{M_{h_b}M_{Z_b'}} \epsilon_{ijk} q_i^e Z_{b'}^e h_b \ [I(M_{B'}, M_{B'}, M_{B'}, \bar{q}) + I(M_{B'}, M_B, M_{B'}, \bar{q})], \quad (A.3)
\]
respectively. In all these amplitudes, both the neutral and charged bottom and anti-bottom mesons have been taken into account. The amplitudes for \( Z_b^{+'+0} \) into \( \chi_{bJ}\gamma \) read
\[
A_{Z_b^{+'+0} \chi_{b0}\gamma} = - \frac{2}{3} i\beta g_1 z \sqrt{M_{\chi_b}M_{Z_b}} \epsilon_{ijk} q_i^e Z_{b}^e \gamma \ [I(M_B, M_{B'}, M_{B'}, \bar{q}) - 3I(M_{B'}, M_B, M_{B'}, \bar{q})], \quad (A.4)
\]
\[
A_{Z_b^{+'+0} \chi_{b1}\gamma} = - 2i \frac{\sqrt{2}}{3} \beta g_1 z' \sqrt{M_{\chi_b}M_{Z_b}} \epsilon_{ijk} q_i^e Z_{b}^e \gamma \ I(M_B, M_{B'}, M_{B'}, \bar{q}), \quad (A.5)
\]
\[
A_{Z_b^{+'+0} \chi_{b1}\gamma} = 2i \beta g_1 z \sqrt{M_{\chi_b}M_{Z_b}} (q^i g^{jk} - q^k g^{ij}) \epsilon_{Z_b}^e \epsilon_{Z_{b'}}^e \gamma \ I(M_{B'}, M_B, M_{B'}, \bar{q}), \quad (A.6)
\]
\[
A_{Z_b^{+'+0} \chi_{b1}\gamma} = - 2i \beta g_1 z \sqrt{M_{\chi_b}M_{Z_b}} (q^i g^{jk} - q^k g^{ij}) \epsilon_{Z_b}^e \epsilon_{Z_{b'}}^e \gamma \ I(M_{B'}, M_B, M_{B'}, \bar{q}), \quad (A.7)
\]
and
\[
A_{Z_b^{+'+0} \chi_{b2}\gamma} = \sqrt{2} i\beta g_1 z \sqrt{M_{\chi_b}M_{Z_b}} q^i (g^{im} \epsilon_{ikl} + g^{il} \epsilon_{ikm}) \epsilon_{Z_b}^e \epsilon_{Z_{b'}}^e \gamma \ I(M_B, M_{B'}, M_{B'}, \bar{q}), \quad (A.8)
\]
\[
A_{Z_b^{+'+0} \chi_{b2}\gamma} = \sqrt{2} i\beta g_1 z \sqrt{M_{\chi_b}M_{Z_b}} (q^i (g^{im} \epsilon_{ikl} + g^{il} \epsilon_{ikm}) + q^l \epsilon_{ijm} + q^m \epsilon_{ijl}) \times \epsilon_{Z_b}^e \epsilon_{Z_{b'}}^e \gamma \ I(M_B, M_{B'}, M_{B'}, \bar{q}), \quad (A.9)
\]
respectively.

**B Relativistic approach**

In this appendix, we formulate a Lorentz covariant framework for a parallel study of the \( Z_b^{(')} \) decays. In such a framework, more terms will appear as relativistic corrections that are generally neglected in the heavy quark limit. Therefore, a comparison between these two prescriptions will serve as a cross-check of the NREFT results, and more importantly as a confirmation of the validity of the NREFT power counting.
The bottom meson fields are defined as
\[ H_1 = \left( \frac{1 + \gamma^5}{2} \right) [P^*_\mu \gamma^\mu - \gamma_5 P], \quad H_1^\dagger = \gamma^0 \left( \frac{1 + \gamma^5}{2} \right) \gamma^0, \quad H_2 = \gamma^0 \left( \frac{1 + \gamma^5}{2} \right) P^*_\mu \gamma^\mu + \gamma_5 P, \quad \bar{H}_2 = \gamma^0 H_2^\dagger \gamma^0, \]
where \( P^* \) and \( P \) represent the \((B^{*+}, B^{*0}, B^{*s})\) and \((B^+, B^0, B^s)\) fields, respectively, which annihilate the corresponding particles, while \( \bar{P}^* \) and \( \bar{P} \) are the fields of their antiparticles.

The Lagrangians for the photon coupling to the bottom mesons are
\[ \mathcal{L}_{\gamma B B} = ig_2 \text{Tr} \left[ R_{bb} \bar{H}_2 a \gamma^\mu \partial_\mu \bar{H}_1 a \right] + \text{h.c.}, \quad (B.10) \]
\[ \mathcal{L}_{\gamma B B} = ig_1 \text{Tr} \left[ P^\mu_{bb} \bar{H}_2 a \gamma^\mu \bar{H}_1 a \right] + \text{h.c.}, \quad (B.11) \]
Under the similar convention, the \( Z_b \) field can be expressed as
\[ P_{Z}^\mu = \left( \frac{1 + \gamma^5}{2} \right) Z^\mu \gamma_5 \left( \frac{1 - \gamma^5}{2} \right), \quad (B.12) \]
and the effective interaction between the \( Z_b \) and a pair of bottomed meson reads
\[ \mathcal{L}_{ZBB} = i z^{(i)} \text{Tr} \left[ P_{Z,ab}^\mu \bar{H}_2 a \gamma^\mu \bar{H}_1 a \right] + \text{h.c.}, \quad (B.13) \]
The Lagrangian for the pion coupling to a pair of bottom mesons is
\[ \mathcal{L} = ig \text{Tr} \left[ H_b \gamma_5 A^\mu a_h \right], \quad (B.14) \]
where \( A_\mu = \left( \xi^i \partial_\mu \xi - \xi \partial_\mu \xi^i \right)/2, \) with \( \xi = \exp(i \sqrt{2} \phi/F_x). \) The relativistic form of the Lagrangian for the photon coupling to the bottom mesons is
\[ \mathcal{L}_\gamma = \frac{e \beta Q_{ab}}{2} F^{\mu\nu} \text{Tr} \left[ H_b^\dagger H_a \gamma^\mu H_\sigma \right] + \frac{e Q}{2m_Q} F^{\mu\nu} \text{Tr} \left[ H_\sigma H_a \gamma^\mu \right]. \quad (B.15) \]
With these effective Lagrangians, we obtain for the amplitudes of \( Z_b^{(i)} \to h_b \pi \)
\[ A_{Z_b h_b} = \frac{2 \sqrt{2} g g_{b1} z}{F_x} \epsilon_{q e h_b} \epsilon_{Z_b} \left( C_0 [MB, MB^*, MB^*] + C_0 [MB^*, MB, MB^*] \right), \quad (B.16) \]
\[ A_{Z_b h_b} = \frac{2 \sqrt{2} g g_{b1} z}{F_x} \epsilon_{q e h_b} \epsilon_{Z_b} \left( C_0 [MB^*, MB^*, MB^*] + C_0 [MB^*, MB^*, MB^*] \right). \quad (B.17) \]
and to \( \Upsilon \pi \)
\[ A_{Z,\Upsilon} = \frac{2 \sqrt{2} g g_{b1} z}{F_x} (-\epsilon_\Upsilon \cdot \epsilon_Z) \left( C_0 [MB, MB^*, MB^*] + 2C_2 [MB, MB^*, MB^*] \right) + C_0 [MB^*, MB, MB^*] + 2C_2 [MB, MB^*, MB^*] + q \cdot \epsilon_\Upsilon q \cdot \epsilon_Z \left( C_0 [MB^*, MB, MB^*] + 2C_2 [MB^*, MB^*, MB^*] \right) \quad (B.18) \]
\[ A_{Z'_b\tau} = \frac{2\sqrt{2} g g_2 z}{F_\pi} (\varepsilon_Z \cdot \varepsilon_\tau |q|^2 (C_0 [M_{B^*}, M_{B'}, M_B] + C_2 [M_{B'}, M_{B'}, M_B]) + C_0 [M_{B'}, M_{B'}, M_B] + C_2 [M_{B'}, M_{B'}, M_B]) + q \cdot \varepsilon_\tau q \cdot \varepsilon_Z (C_0 [M_{B^*}, M_{B'}, M_B]) + C_2 [M_{B'}, M_{B'}, M_B]) - C_0 [M_{B'}, M_{B'}, M_B] - C_2 [M_{B'}, M_{B'}, M_B]). \] (B.19)

The amplitudes for \( Z'_b^{(t)} \rightarrow \chi_{b0}\gamma \) are

\[ A_{Z'_b^{(t)}\chi_{b0}\gamma} = \frac{i e g_1 z \varepsilon_{\bar{q}v_{\gamma}}}{\sqrt{3}} C_0 [M_{B^*}, M_{B'}, M_B] - C_0 [M_{B^*}, M_{B'}, M_B]) \] (B.20)
\[ A_{Z'_b^{(t)}\chi_{b0}\gamma} = i2 e g_1 z \beta \varepsilon_{\bar{q}v_{\gamma}} C_0 [M_{B^*}, M_{B'}, M_B] \] (B.21)
\[ A_{Z'_b^{(t)}\chi_{b2}\gamma} = i2 \sqrt{2} e g_1 z \beta \varepsilon_{\bar{q}v_{\gamma}} \gamma_5 \chi_{b0} \sigma_\alpha C_0 [M_{B^*}, M_{B'}, M_B] \] (B.22)

\[ A_{Z'_b^{(t)}\chi_{b0}\gamma} = -i2 e g_1 z \beta \varepsilon_{\bar{q}v_{\gamma}} C_0 [M_{B^*}, M_{B'}, M_B] \] (B.23)
\[ A_{Z'_b^{(t)}\chi_{b0}\gamma} = -i2 e g_1 z \beta \varepsilon_{\bar{q}v_{\gamma}} C_0 [M_{B^*}, M_{B'}, M_B] \] (B.24)
\[ A_{Z'_b^{(t)}\chi_{b2}\gamma} = i2 \sqrt{2} e g_1 z \beta \varepsilon_{\bar{q}v_{\gamma}} \gamma_5 \chi_{b0} \sigma_\alpha C_0 [M_{B^*}, M_{B'}, M_B] \] (B.25)

respectively. Here, \( C_0 \) is the standard relativistic three-point scalar function which is similar to the definition of \( I \) in App. \( A \).

\[
\int \frac{(2\pi \mu)^{4-D}}{i\pi^2} \int \frac{d^D l}{\sqrt{(|l^2 - m_1^2|)(|p - l^2 - m_2^2|)(|l - q|^2 - m_3^2)}, \quad (B.26)
\]

where the incoming four-momentum of the \( Z_0 \) is \( p \), and the light outgoing particle four-momentum is \( q \). For simplicity, we do not explicitly include the first three arguments in \( C_0 \) in the above formulae. With the same convention, \( C_2 \) is a coefficient that arises from the tensor reduction of the three-point vector loop,

\[
C^\mu = \frac{(2\pi \mu)^{4-D}}{i\pi^2} \int \frac{l^\mu d^D l}{\sqrt{(|l^2 - m_1^2|)(|p - l^2 - m_2^2|)(|l - q|^2 - m_3^2)}}, \quad (B.27)
\]

which is similar to the integral \( I_1 \) defined in Ref. \( [16] \).

To compare with the NREFT method, we note the following treatments in the heavy quark limit:

- By expressing the heavy meson momentum \( M_Q v_\mu = m_Q v_\mu + k_\mu \) in the heavy quark limit, where \( v \) is the heavy meson velocity and \( k \) is the residual momentum of the order of \( \Lambda_{QCD} \), the derivative in Eq. \( (15.10) \) only gives the difference of the residual momentum between the intermediate bottomed mesons \( [11] \). Namely, the residual momentum \( k \) is the integral momentum in the meson loops instead of \( v \).

- The denominators \( (l^2 - m_1^2)(p - l^2 - m_2^2)(l - q)^2 - m_3^2) \) of the meson loops contain two independent external momenta \( p \) and \( q \). For the incoming momentum \( p \), we set...
p = M_Z v following the convention of Ref. [10], where v = (1, 0, 0, 0) defines the rest-frame of the initial particle. By doing this, the vector three-point function defined in the loop integrals becomes $C^\mu = M_Z v^\mu C_1 + (M_Z v^\mu - q^\mu) C_2$ which can be compared with the NREFT amplitudes term by term. It is interesting to note that the coefficient of $C_1$ vanishes in the decay amplitude because of cancellation.

- Another difference between this scheme and the NREFT comes from the contraction term $q \cdot \epsilon_T$, where q is the external light meson (e.g. pion) momentum. Note that the time component appears as an additional contribution compared to the NREFT formalism. It is proportional to $|q| E_\pi / m_\pi$, which is a relativistic correction and relatively suppressed with respect to the space component $|q| E_\pi / m_\pi$.

- The relativistic corrections also arise from the mass difference between $m_B$ and $m_{B^*}$. One notices that in Eqs. (13.18) and (13.19), the terms proportional to $q \cdot \epsilon_T q \cdot \epsilon_Z$ are given by the $D$-wave transition. In the heavy quark limit with $m_B = m_{B^*}$, exact cancelations occur within the integral functions $C_0$ and $C_2$, respectively. This means if the $Z_b$ and $Z'_b$ are indeed the $BB^*$ and $B^*B^*$ molecular states, respectively, their $D$-wave decays into the $\Upsilon \pi$ will be highly suppressed. This can be understood by noticing that the heavy quark spin decouples from the system in the heavy quark limit, and the total angular momentum of the light quarks $s_\bar{q}q$ is a good quantum number. If the $Z'_b$ states are $S$-wave $B^{(*)}B^*$ hadronic molecules as assumed here, there is no spatial angular momentum in the system. Thus, the light quark system has $s_\bar{q}q = 0$ or 1, and therefore cannot couple to a spinless pion in a $D$-wave. A similar statement was made very recently in Ref. [42]. Here we notice that the decay of a $bb\bar{q}q$ tetraquark state with $s_\bar{q}q = 2^-$ would decay into $b\bar{b}\pi$ dominantly in a $D$-wave.

- We also note the convention for the sums of the polarizations for the vector and tensor particles: $\epsilon_1^{\mu} \epsilon_2^{\nu} \gamma_{\mu,\chi_1}^{(h_b,\chi_2)} \gamma_{\nu,\chi_1}^{(h_b,\chi_2)} = -g^{\mu \nu} + u^{\mu} u^{\nu} \equiv \bar{g}^{\mu \nu}$ and $\epsilon_1^{\mu} \epsilon_2^{\nu} \epsilon_{\chi_1 \chi_2}^{\alpha \beta} = \frac{1}{2} (g^{\mu \alpha} g^{\nu \beta} + g^{\nu \alpha} g^{\mu \beta}) - \frac{1}{4} g^{\mu \nu} \bar{g}^{\alpha \beta}$. They allow us to separate out the relativistic contributions in the scalar loop diagrams which can then be compared with the NREFT formulae explicitly.

The relativistic formalism is advantageous for providing a cross check of the NREFT results and singling out the effects arising from the relativistic corrections. For the bottomonium and bottomed meson system discussed here, it shows that the relativistic corrections are indeed small and these two methods are consistent with each other. Alternatively, it should be cautioned that in the relativistic formalism, when the terms proportional to $|q| E_\pi / m_{\bar{Q}Q}$ are not obviously suppressed in comparison with the three-momentum $|q|$, i.e. $E_\pi / m_{\bar{Q}Q}$ is sizeable, a different scheme with a form factor [43, 44] might be used to control the large momentum contributions and non-local effects of each couplings.

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