Consequences of a Pati-Salam unification of the electroweak-scale active $\nu_R$ model

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If right-handed neutrinos are not singlets under the electroweak gauge group as it was proposed in a recent model, they can acquire electroweak scale masses and are thus accessible experimentally in the near future. When this idea is combined with quark-lepton unification à la Pati-Salam, one is forced to introduce new neutral particles which are singlets under the Standard Model (SM). These “sterile neutrinos” which exist in both helicities and which are different in nature from the popular particles with the same name can have their own seesaw with masses in the keV range for the lighter of the two eigenstates. The keV sterile neutrinos have been discussed in the literature as warm dark matter candidates with wide ranging astrophysical consequences such as structure formation, supernova asymmetries, pulsar kicks, etc. In addition, the model contains W-like and Z-like heavy gauge bosons which might be accessible at the LHC or the ILC. An argument is presented on why, in this model, it is natural to have four families which can obey existing constraints.

I. INTRODUCTION

The deep question of whether or not the existence of neutrino masses has anything to do with parity violation in the weak interactions has been investigated in classic papers of Ref.[1] on the Left-Right symmetric model, linking the scale of “parity violation” (the mass of the gauge bosons $W_R$, $M_{W_R}$) to the neutrino masses $m_\nu$: $m_\nu \rightarrow 0$ as $M_{W_R} \rightarrow \infty$ (the V-A limit of the weak interactions). This interesting link stimulates further looks into the meaning of “parity violation” and its possible connection to neutrino masses.

The most popular mechanism, the so-called see-saw mechanism [2], for active neutrinos to have a very small mass is to enlarge the minimal SM by bringing in right-handed neutrinos and to have two widely separated masses: a Dirac mass $m_D$ coming from a lepton-number conserving term in the Lagrangian which should be much smaller than a Majorana mass $M_R$ coming from a lepton-number violating term. The nature of these two mass scales is very model-dependent. Nothing is known about either $m_D$ or $M_R$ but only about the ratio $m_D^2/M_R$ (the mass of the lighter neutrino) which is generally thought to be below an eV or so.

In a generic seesaw model, the right-handed neutrinos are sterile i.e. SM singlets. As a consequence, the Dirac mass $m_D$ is proportional to the electroweak breaking scale, although its Yukawa coupling is arbitrary. The right-handed neutrino Majorana mass $M_R$ on the other hand is generally thought to be close to a typical Grand Unified Theory (GUT) scale, an expectation which is generally based on the embedding of the SM into a larger gauge group such as e.g. $SO(10)$. (Light sterile neutrinos have been considered in models such as [2].)

If the right-handed neutrinos are not SM singlets, the situation can change dramatically. For example, if they are partners in doublets with right-handed charged leptons– the so-called mirror leptons which cannot be SM right-handed charged leptons because of neutral current constraints, $M_R$ is necessary related to $\Lambda_{EW}$ and furthermore they can contribute to the invisible Z width unless $M_R > M_Z/2$. This is a model presented in [4] where the right-handed neutrino masses are “confined” to a rather “narrow” range $M_Z/2 < M_R < \Lambda_{EW}$. (Another model where there exists SM non-singlet neutral leptons was discussed in [5].) Electroweak scale SM active right-handed neutrinos have an appealing aspect to them: They can be proved or disproved at colliders. One of the characteristic signals is like-sign dilepton events [4, 6, 7].

The model of [4] is based on the assumption of the existence of mirror fermions (both quarks and leptons) in the SM: For every $SU(2)_L$ SM left-handed doublet (e.g. $(\nu, e_L)$) there is a mirror right-handed doublet (e.g. $(\nu, e^\nu_R)$) and for every $SU(2)_R$ right-handed singlet (e.g. $e_R$), there is a mirror left-handed singlet (e.g. $e^\nu_R$). (The idea of mirror fermions have been entertained in e.g. [8].) In [4], contributions of the mirror charged leptons to Lepton-Flavor-Violating (LFV) processes such as $\mu \rightarrow e \gamma$ and $\tau \rightarrow \mu \gamma$ were discussed and constraints were put on couplings which have implications on both future LFV searches and like-sign dilepton events. (As mentioned in [10], the contribution to the S parameter from the mirror fermions (quarks and leptons) can be offset by the negative contribution to S from Higgs triplet and doublet and from the Majorana neutrinos [10].)

In [4], no attempt was made to unify quarks and leptons. One of the nice features of quark-lepton unification is the group-theoretical explanation of charge quantization. There are of course several ways to do so and that might involve a single Grand Unified (GUT) scale or several intermediate scales. One popular intermediate unification step is the Pati-Salam group $SU(4)_{RS}$ where quarks and leptons are grouped into the fundamen-
Mirror fermions and electroweak scale right-handed neutrinos: a review

Below we will review the essential elements of the model presented in [4]. We start with a change of notation concerning the weak group $SU(2)_L$. Since our model contains both left and right-handed fermions transforming in a similar way under $SU(2)_L$, it would be appropriate to change $SU(2)_L$ into $SU(2)_V$.

- **Gauge group**: $SU(3)_c \otimes SU(2)_V \otimes U(1)_Y$.
- **Fermion content**: (Mirror fermions will be accompanied with the superscript $M$ and the subscript $i$ below denotes a family index)
  
  \[
  l_L = \begin{pmatrix}
  \nu_L \\ e_L
  \end{pmatrix}_i = (1, 2, Y/2 = -1/2),
  \]

  \[
  l^M_R = \begin{pmatrix}
  \nu^M_L \\ e^M_L
  \end{pmatrix}_i = (1, 2, Y/2 = -1/2),
  \]

  \[
  q_L = \begin{pmatrix}
  u_L \\ d_L
  \end{pmatrix}_i = (3, 2, Y/2 = 1/6),
  \]

  \[
  q^M_R = \begin{pmatrix}
  u^M_L \\ d^M_L
  \end{pmatrix}_i = (3, 2, Y/2 = 1/6),
  \]

  \[
  e_{iR} = (1, 1, Y/2 = -1),
  \]

  \[
  e^M_{iL} = (1, 1, Y/2 = -1),
  \]

  \[
  u_{iR} = (3, 1, Y/2 = 2/3),
  \]

  \[
  u^M_{iL} = (3, 1, Y/2 = 2/3),
  \]

  \[
  d_{iR} = (3, 1, Y/2 = -1/3),
  \]

  \[
  d^M_{iL} = (3, 1, Y/2 = -1/3).
  \]

- **Higgs content**:  

  \[
  \Phi = \begin{pmatrix}
  \phi^+ \\ \phi^0
  \end{pmatrix} = (1, 2, Y/2 = 1/2)
  \]

  \[
  \tilde{\chi} = \frac{1}{\sqrt{2}} \tilde{\tau} \tilde{\chi} = \begin{pmatrix}
  \frac{1}{\sqrt{2}} \chi^+ \\ -\frac{1}{\sqrt{2}} \chi^+
  \end{pmatrix} = (1, 3, Y/2 = 1),
  \]

  \[
  \xi = \begin{pmatrix}
  \xi^+ \\ \xi^0 \\ \xi^-
  \end{pmatrix} = (1, 3, Y/2 = 0),
  \]

  \[
  \phi_s = (1, 1, Y/2 = 0).
  \]

Higgs triplets such as in (3) were considered earlier in various contexts [10].

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**Note**: The original text contains various mathematical expressions and notations that are typical in theoretical physics and particle physics literature. These are generally understood in the context of high-energy physics and may require specific knowledge of the notation and concepts used, such as SU(2) and SU(3) gauge groups, fermions, and Higgs doublets. The focus is on the extension of the SM gauge group to accommodate this fermion content leading to a unified model with interesting astrophysical consequences.
• Majorana mass

The interaction Lagrangian is listed in [4] and, in a more complete form, in [4]. Here, for the review purpose, we will just give the Yukawa interaction Lagrangian to illustrate the main point of [4]: The right-handed neutrino Majorana mass is of the order of the electroweak scale.

In what follows, we will assume as in [4] a global $U(1)_M$ symmetry under which the mirror fermions as well as $\tilde{\chi}$ and $\phi_S$ transform non-trivially while the SM fermions and Higgs doublet are singlets. This is first to prevent the appearance of a bare Dirac mass term for the neutrinos. Notice that the latter is known as Type II seesaw where, in a generic scenario, the Higgs triplet has a small VEV which is not the case here. However, it will be seen below that this is unnecessary when the SM gauge group is extended: Gauge invariance under the extended gauge group automatically forbids these terms. Furthermore, the same global $U(1)_M$ symmetry prevents a Majorana mass term for the left-handed neutrino while preventing a Majorana mass term for the mirror right-handed neutrino. Again, it will be seen below that the Higgs content of the extended gauge group insures the absence of this term.

The Majorana mass of the right-handed neutrinos comes from the following Lagrangian:

$$\mathcal{L}_M = i_i^M \sigma_2 g_M (\tau_2 \tilde{\chi}) l_i^M + H.c.$$  \hfill (6)

Here $l_i^M$ denotes a column vector with $n$ families to be general, and $g_M$ denotes a $n \times n$ matrix. When

$$\langle \chi^0 \rangle = v_M,$$  \hfill (7)

the Majorana mass matrix for the right-handed neutrinos is

$$M_R = g_M v_M.$$  \hfill (8)

• Dirac mass

The Dirac neutrino mass matrix comes from the following Lagrangian

$$\mathcal{L}_S = -\overline{\nu_L} g_{SI} \nu_R^{0,M} \phi_S + H.c.$$  
$$= - \overline{\nu_L} g_{SI} \nu_R^{0,M} + \overline{\nu_L} g_{SI} \nu_R^{0,M} \phi_S + H.c.,$$  \hfill (9)

where the superscript 0 is only relevant for the charged lepton part where, as shown in [4], the charged lepton mass eigenstates mainly come from the coupling to $\Phi$. Here $g_{SI}$ is an $n \times n$ matrix. With $\langle \phi_S \rangle = v_S$, the Dirac neutrino mass matrix is

$$m_D = g_{SI} v_S.$$  \hfill (10)

• See-saw

The see-saw mass matrix is

$$M_2 = \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}.$$  \hfill (11)

In [4], various implications of the model in which $M_R = O(100 \text{GeV})$ were discussed. In particular, in order for the light neutrino mass to be of $O(< 1 \text{eV})$, one should have $m_D \sim 10^5 \text{eV}$. We will come back to this point below.

• Custodial symmetry and $\rho = 1$

When the eigenvalues of $M_R$ are much larger than those of $m_D$, one obtains the usual seesaw formula for the lighter neutrinos

$$m_\nu = -m_D^T M_R^{-1} m_D,$$  \hfill (12)

where as the (almost) right-handed neutrino mass matrix is approximately $M_R$. As discussed in detail in [4], the VEV of $\chi^0, v_M$, can be of order of $\Lambda_{EW} \sim 246 \text{GeV}$ because of a remnant custodial $SU(2)$ that is guaranteed at tree level that comes from the breaking of a global $SU(2)_L \otimes SU(2)_R$ down to $SU(2)$ with $\chi$ and $\tilde{\chi}$ being grouped into its $(3, 3)$ representation as

$$\chi = \begin{pmatrix} \chi^0 & \xi^+ & \chi^+ \\ \chi^- & \xi^0 & \chi^- \\ \chi^- & \xi^- & \chi^{0, *}_s \end{pmatrix}.$$  \hfill (13)

Furthermore, the Higgs doublet can be written as

$$\Phi = \begin{pmatrix} \phi^0 & -\phi^+ \\ \phi^- & \phi^{0, *}_s \end{pmatrix},$$  \hfill (14)

which transforms as $(2, 2)$ under $SU(2)_L \otimes SU(2)_R$. $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)$ when

$$\langle \chi \rangle = \begin{pmatrix} v_M & 0 & 0 \\ 0 & v_M & 0 \\ 0 & 0 & v_M \end{pmatrix},$$  \hfill (15)

and

$$\langle \Phi \rangle = \begin{pmatrix} v_2 & 0 \\ 0 & v_2 \end{pmatrix}.$$  \hfill (16)

A detailed discussion of the minimization of the Higgs potential given in [17] indicates that the above VEVs take the form as shown in [15] in order to have a proper vacuum alignment.

This custodial symmetry guarantees that $\rho = 1$ at tree level. In consequence, $v_M$ can be of order of $\Lambda_{EW} \sim 246 \text{GeV}$. In fact, $\Lambda_{EW} = v = \sqrt{v_2^2 + 8 v_M^2} \approx 246 \text{GeV}$ with $M_W = g v/2$ and $M_Z = M_W/\cos \theta_W$. As discussed in [4], the active right-handed neutrinos have now electroweak scale masses and can be produced at colliders.
other issues are discussed in [4, 9]. Also, the presence of the mirror charged leptons give rise to LFV processes μ → eγ and τ → μγ [9] with implications concerning the search for like-sign dileptons.

• Mass scale issues

Several issues were discussed in [4] such as the one concerning the scale of the Dirac mass matrix. If we denote the largest eigenvalue of \( M_D \) by \( g^{\text{max}}_{\text{SI}} v_S \) and requiring that the light neutrino masses be less than \( O(1 \text{eV}) \), it can be seen that \( g^{\text{max}}_{\text{SI}} v_S \lesssim 10^5 \text{eV} \) for \( M_R \sim O(\Lambda_{\text{EW}}) \). For example, one has \( v_S \sim 10^5 \text{eV} \) for \( g^{\text{max}}_{\text{SI}} \sim O(1) \) or \( v_S \sim O(\Lambda_{\text{EW}}) \) for \( g^{\text{max}}_{\text{SI}} \sim O(10^{-6}) \). Some speculation on \( v_S \) was discussed in [4]. We will return to this issue below.

Another issue of interest is the usual question of why the mirror quarks and leptons are heavier than their SM counterparts because they have not yet been observed. This is a quintessential question that goes to the heart of particle physics: Why do quarks and leptons have the masses we think they have? Needless to say, there is at the present time no satisfactory answer to that question although there exists many models of quark and lepton masses. The same uncertainty applies to any extension of the SM, in this case the existence of mirror fermions in our model. However, we shall use, as one example, a particular model of quark masses and suggest how the disparity in masses in the two sectors may have something to do with the ratio of two breaking scales, or equivalently the ratio of the two masses: \( M_L \) and \( M_R \), where \( M_L \) is the scale of the breaking of \( SU(2)_L \otimes SU(2)_R \) to \( SU(2)_V \).

III. PARTIAL UNIFICATION OF SM AND MIRROR QUARKS AND LEPTONS:

\( SU(4)_P \otimes SU(2)_L \otimes SU(2)_R \otimes SU(2)_{l_L} \otimes SU(2)_{l_R} \)

The possibility of unifying quarks and leptons into a single representation of some larger gauge group which contains the SM is an old idea which still retains its appeal despite the present lack of direct experimental evidence. Among the many desirable features of the idea of unification is a rather natural explanation for charge quantization a.k.a. the relationship between quark and lepton charges, or that between quark and lepton hypercharge quantum numbers. In most cases, this relationship comes from the fact that the hypercharge generator is proportional to one of the generators of the unifying group.

In the SM, one may say that the requirement of anomaly cancellation automatically determines the hypercharge of, say, the leptons once that of the quarks is given, or vice versa. However, in our model with mirror fermions, anomaly cancellation can be realized between left and right-handed leptons separately, e.g., between \( l_L \) and \( l_R \), and similarly between left and right-handed quarks. As viewed from this angle, one might be tempted to conclude that charge quantization necessitates some form of unification, perhaps more so than in the SM.

The path to unification can be very diverse. It can go through several intermediate energy scales or it can go directly to the unification scale through a desert in between. In what follows, we shall choose the former path, namely one in which the SM gauge group is embedded into products of simple groups which could eventually merge into a simple unifying group.

A. Description of the model

In a similar fashion to [14, 15], we will assume that the gauge group \( SU(3)_c \otimes SU(2)_V \otimes U(1)_Y \) characterized by three different gauge couplings \( g_3, g_2 \) and \( g' \) is embedded in a group which is characterized by two different couplings: \( G_S(g_S) \otimes G_W(g_W) \). Here as in [14, 15], we will take \( G_S(g_S) \) to be the Pati-Salam gauge group \( SU(4)_P \) and \( G_W(g_W) \) to be \( SU(2)_L \otimes SU(2)_R \otimes SU(2)_{l_L} \otimes SU(2)_{l_R} \).

The pattern of symmetry breaking will be as follows

\[
G \rightarrow G_1 \rightarrow G_2 \rightarrow SU(3)_c \otimes SU(2)_V \otimes U(1)_Y,
\]

where

\[
G = SU(4)_P \otimes SU(2)_L \otimes SU(2)_R \otimes SU(2)_{l_L} \otimes SU(2)_{l_R},
\]

\[
G_1 = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes SU(2)_{l_L} \otimes SU(2)_{l_R} \otimes U(1)_Y,
\]

\[
G_2 = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_Y.
\]

Notice that, in the above, \( SU(2)_V \) is a diagonal subgroup of two of the \( SU(2)'s \) which we take to be

\[
SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V.
\]

In dealing with fermion assignments, it is useful to write down explicitly the charge operator as follows

\[
Q = T_{3V} + \frac{Y}{2},
\]

where

\[
T_{3V} = T_{3L} + T_{3R},
\]

\[
\frac{Y}{2} = T_{3L}' + T_{3R}' + \sqrt{\frac{2}{3}} T_{15},
\]

and

\[
T_{15} = \frac{1}{2\sqrt{6}} \begin{pmatrix} 1 & 1 & -3 \end{pmatrix}.
\]
The factor $\sqrt{\frac{2}{3}}$ denoted by $C_5$ in [14] is determined from the charge structure of fermions. For classification purposes shown below, we will use (22) and (23). (For the purpose of comparison, one can consult [14, 15].)

What we will show in the next section is the need for introducing electrically neutral, $SU(2)_V$ singlet left and right-handed fermions, the so-called “sterile neutrinos” $N_L$ and $N_R$, in order to be able to complete the fermionic assignment of our model.

**B. The need for the “sterile neutrinos” $N_L$ and $N_R$**

What are the fermions listed in the last section that can be grouped into a quartet of $SU(4)_P S$? From (22) and (23), one can easily obtain the following grouping under $SU(4)_P S \otimes SU(2)_L \otimes SU(2)_R \otimes SU(2)_L \otimes SU(2)_R$:

$$\Psi_L = (\left( \begin{array}{c} u_L \\ d_L \end{array} \right)_i , \left( \begin{array}{c} \nu_L \\ e_L \end{array} \right)_i ) = (4, 2, 1, 1, 1), \quad (26)$$

$$\Psi^M_R = (\left( \begin{array}{c} u_R \\ d_R \end{array} \right)_i , \left( \begin{array}{c} \nu_R \\ e_R \end{array} \right)_i ) = (4, 1, 2, 1, 1) \quad (27)$$

as can be seen from looking at (1a), (1c), (1b) and (1d). According to the symmetry breaking pattern (24), both SM left-handed doublets and the mirror right-handed counterparts transform as doublets under $SU(2)_V$ as laid out in the model of [4].

This leaves us with the $SU(2)_V$ singlets: (1e), (1f), (1g), (1h), (1i), and (1j). Here the charge operator takes the form:

$$Q_{\text{singlet}} = \frac{Y}{2} = T^3_{3L} + T^3_{3R} + \sqrt{\frac{2}{3}} T_{15}. \quad (28)$$

Noticing that

$$\sqrt{\frac{2}{3}} T_{15} = \left( \begin{array}{ccc} 0 & 1/6 & 1/6 \\ 1/6 & 0 & 1/6 \\ 1/6 & 1/6 & -1/2 \end{array} \right), \quad (29)$$

it is easy to see that one can group $e_{iR}$ and $d_{iR}$ into a quartet of $SU(4)_P S$ with $T^3_{3R} = -1/2$ and $T^3_{3L} = 0$. Similarly, one can group $e^M_{iL}$ and $d^M_{iL}$ into a quartet of $SU(4)_P S$ with $T^3_{3R} = 0$ and $T^3_{3L} = -1/2$. As it stands with the particle content of [4], it leaves us with the “orphaned” $u_{iR}$ and $u^M_{iL}$ which, in this new scheme, would have $T^3_{3R} = +1/2$ and $T^3_{3L} = +1/2$ respectively. In order to complete the fermion assignment, we propose to add the following $SU(2)_V$ electrically neutral singlets: $N_L$ and $N_R$. These singlets will be, in our scenario, the “sterile neutrinos” which are quite different from those with the same names. Here, these sterile neutrinos come with both chiralities whereas the common usage of the adjective “sterile” in the literature refers to mainly right-handed neutrinos. Let us remind ourselves that the right-handed neutrinos of the model of [4] are parts of doublets of $SU(2)_V$ and therefore are non-sterile. With these “sterile neutrinos”, $N_L$ and $N_R$, we can complete the fermionic assignment of the $SU(2)_V$-singlet sector as

$$\Psi_R = (\left( \begin{array}{c} u_R \\ d_R \end{array} \right)_i , \left( \begin{array}{c} N_R \\ e_R \end{array} \right)_i ) = (4, 1, 1, 1, 2), \quad (30)$$

$$\Psi^M_L = (\left( \begin{array}{c} u^M_{iL} \\ d^M_{iL} \end{array} \right)_i , \left( \begin{array}{c} N^M_{iL} \\ e^M_{iL} \end{array} \right)_i ) = (4, 1, 2, 1, 1). \quad (31)$$

The introduction of the “sterile neutrinos” $N_L$ and $N_R$, within the Petite Unification scheme, to complete the fermionic assignment is a rather surprising and interesting extension of the model of [4]. Let us notice that although this model contains the same gauge group and the same number of fermion degrees of freedom as the model of [14], the big difference lies in the interpretation of the meaning of mirror fermions and in the SM gauge group itself. Here, the weak $SU(2)$ gauge group comes from the breaking $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$ whereas it is the $SU(2)_L$ of $G_W$ in [14]. The new interpretation has a number of advantages over the old one such as the emergence of electroweak scale right-handed neutrinos with interesting implications [4, 9], a much larger Petite Unification scale, and several others that will be discussed below. In addition, this is an interesting bonus: there exists “sterile neutrinos” which could have important astrophysical implications to which we will come back in the following sections.

**IV. Masses of $N_L$ and $N_R$, and of Mirror Fermions**

Having introduced the sterile neutral leptons $N_L$ and $N_R$, one would like to have some hints on the possible mass ranges that one might expect for these leptons. Closer to home, the mirror fermions are assumed to be heavy since none has been detected so far. The question is why they are supposed to be heavier than their SM counterparts. In what follows, we will try to provide some rationale for this aspect of mirror fermions.

**A. Generalized “see-saw” involving $N_L$ and $N_R$**

In order to discuss neutrino masses, we need to study the Higgs sector of the model. In particular, we will concentrate in this section on the Higgs fields that couple to the fermions. In this section, we will leave out the intergenerational mass splitting and will focus mainly on the mass differences between quarks and leptons and, in particular, between the neutral leptons and the rest.

- **Dirac mass terms involving $N_L$ and $N_R$**

In this section, the Dirac masses for the neutral leptons involve mixing between $\nu_L$ and $N_R$, and
\( \nu_R^M \) and \( N_L \). This is different from the Dirac mixing terms between the \( SU(2)_Y \)-active \( \nu_L \) and \( \nu_R^M \).

For SM fermions, the Dirac mass term would be proportional to the product
\[
\Psi_L \times \Psi_R = (1 + 15, 2, 1, 1, 2). \tag{32}
\]

The Higgs field that can couple to this fermion bilinear can be of two types under \( SU(4)_{PS} \times SU(2)_L \times SU(2)_R \times SU(2)'_L \times SU(2)'_R \) (1, 2, 1, 1, 2) and/or (15, 2, 1, 1, 2). Note that the VEV of (1, 2, 1, 1, 2) would give equal masses to the SM quarks and leptons with the same \( G_W \) quantum numbers. If the (15, 2, 1, 1, 2) is also present, its VEV would split quark and lepton masses. We have the following notations
\[
\Phi_S = (1, 2, 1, 1, 2); \Phi_A = (15, 2, 1, 1, 2). \tag{33}
\]

Explicitly, one has
\[
\Phi_S = \left( \begin{array}{c} \phi_{S,u}^0 \\ -\phi_{S,d}^+ \\ \phi_{S,u}^0 \end{array} \right). \tag{34}
\]
and
\[
\Phi_A = \phi_A^\beta \frac{\lambda_\beta}{2}. \tag{35}
\]

where \( \frac{\lambda_\beta}{2} \) are the generators of \( SU(4)_{PS} \) with \( \beta = 1, ..., 15 \) and \( \phi_A^\beta \) is a 2 \times 2 matrix of the form shown in Eq. (34).

Similarly, for the mirror fermions we would have
\[
\Psi_L^M \times \Psi_R^M = (1 + 15, 2, 1, 2, 1). \tag{36}
\]

The related Higgs fields are
\[
\Phi_S^M = (1, 1, 2, 2, 1); \Phi_A^M = (15, 1, 2, 2, 1). \tag{37}
\]

Here, we also have
\[
\Phi_S^M = \left( \begin{array}{c} \phi_{S,u}^{0,M} \\ -\phi_{S,d}^{+,M} \\ \phi_{S,u}^{0,M} \end{array} \right). \tag{38}
\]
and
\[
\Phi_A^M = \phi_A^{\beta,M} \frac{\lambda_\beta}{2}. \tag{39}
\]

In the above the superscripts "S" and "A" refer to a singlet and an adjoint of \( SU(4)_{PS} \).

Let us define
\[
\tilde{\Phi}_S = \tau_2 \Phi_S^* \tau_2; \tilde{\Phi}_A = \tau_2 \Phi_A^* \tau_2, \tag{40}
\]
and
\[
\tilde{\Phi}_S^M = \tau_2 \Phi_S^{*,M} \tau_2; \tilde{\Phi}_A^M = \tau_2 \Phi_A^{*,M} \tau_2, \tag{41}
\]

One can now write down the Yukawa interactions which give rise to the Dirac masses as follows.
\[
\mathcal{L}_{Dirac} = y_1 \tilde{\Phi}_L \Phi_S \Psi_R + y_2 \tilde{\Phi}_L \Phi_A \Psi_R + y_3 \tilde{\Phi}_L \Phi_A \Psi_R + y_1^M \tilde{\Phi}_S^M \Phi_S^M \Psi_L^M + y_2^M \tilde{\Phi}_S^M \Phi_A^M \Psi_L^M + y_3^M \tilde{\Phi}_A^M \Phi_A^M \Psi_L^M + H.c. \tag{42}
\]

We assume the following VEVs:
\[
\langle \phi_{S,u}^0 \rangle = v_u ; \langle \phi_{S,d}^0 \rangle = v_d, \tag{43}
\]
\[
\langle \phi_{S,u}^{0,M} \rangle = v_u^M ; \langle \phi_{S,d}^{0,M} \rangle = v_d^M. \tag{44}
\]

Since 15 = 8 + 3 + 3 + 1 under the subgroup \( SU(3)_c \), one can only have
\[
\langle \Phi_A \rangle = \langle \phi_A^{15} \rangle \frac{\lambda_{15}}{2} ; \langle \Phi_A^M \rangle = \langle \phi_A^{M,15} \rangle \frac{\lambda_{15}}{2}. \tag{45}
\]

With
\[
\frac{\langle \phi_{A,u}^{15} \rangle}{2 \sqrt{6}} = v_{15,u} ; \frac{\langle \phi_{A,d}^{15} \rangle}{2 \sqrt{6}} = v_{15,d}, \tag{46a}
\]
\[
\frac{\langle \phi_{A,u}^{M,15} \rangle}{2 \sqrt{6}} = v_{15,u}^M ; \frac{\langle \phi_{A,d}^{M,15} \rangle}{2 \sqrt{6}} = v_{15,d}^M, \tag{46b}
\]

we obtain the following mass scales for the SM fermions
\[
m_U = y_1 v_u + y_2 v_d + y_3 v_{15,u} + y_4 v_{15,d}, \tag{47a}
\]
\[
m_{\nu_L,N_R} = y_1 v_u + y_2 v_d - 3(y_3 v_{15,u} + y_4 v_{15,d}), \tag{47b}
\]
\[
m_D = y_1 v_d + y_2 v_u + y_3 v_{15,d} + y_4 v_{15,u}, \tag{48a}
\]
\[
m_E = y_1 v_d + y_2 v_u - 3(y_3 v_{15,d} + y_4 v_{15,u}), \tag{48b}
\]

and, for the mirror fermions,
\[
m_{U,M} = y_1^M v_u + y_2^M v_d^M + y_3^M v_{15,u}^M + y_4^M v_{15,d}^M, \tag{49a}
\]
\[
m_{\nu_{R,L}} = y_1^M v_d + y_2^M v_u^M - 3(y_3^M v_{15,d}^M + y_4^M v_{15,u}^M), \tag{49b}
\]
\[
m_{D,M} = y_1^M v_d + y_2^M v_u^M + y_3^M v_{15,d}^M + y_4^M v_{15,u}^M, \tag{50a}
\]
\[
m_{E,M} = y_1^M v_d + y_2^M v_u^M - 3(y_3^M v_{15,d}^M + y_4^M v_{15,u}^M). \tag{50b}
\]

As we shall see below, the generalized “see-saw” involving \( \nu_L, \nu_R^M, N_L, \) and \( N_R \) will now also depend on \( m_{\nu_L,N_R} \) and \( m_{\nu_{R,L}} \). Furthermore, it will be seen that, with the existence of electroweak-scale right-handed neutrinos in our model, any additional “Dirac” term will be constrained to be small (how small this is will be the subject of the next section). There might be several ways to achieve this and we will show two of such possibilities.
\* \textbf{Majorana mass terms involving } \nu^M_R: \\

The electroweak-scale right-handed neutrino model of \[4\] invokes two Higgs triplets \[3\] and \[4\]. As mentioned above, the Majorana mass term involves \[3\] while \( \rho = 1 \) at tree-level requires the addition of the triplet \[4\]. In the context of \( SU(4)_P \otimes SU(2)_L \otimes SU(2)_R \otimes SU(2)'_L \otimes SU(2)'_R \), the fermion bilinear of interest would be

\[
\Psi^{M,T}_R \sigma_2 \Psi^M_R = (4 \times 4 = 6 + 10, 1, 1 + 3, 1, 1). \tag{51}
\]

Under the subgroup \( SU(3)_c \), one has the decompositions: \( 6 = 3 + 3 \) and \( 10 = 1 + 3 + 6 \). Since one is looking for a Higgs field that has a non-zero VEV, this Higgs field should contain a \textit{singlet} under \( SU(3)_c \) as well as being a SM triplet. In consequence, we shall take

\[
\Phi_{10} = (10 = 1 + 3 + 6, 1, 1, 1), \tag{52}
\]

where the \( SU(3)_c \) singlet part \((1,1,3,1,1)\) couples only to the leptons. The Lagrangian is

\[
\tilde{\mathcal{L}}_M = \Psi^{M,T}_R \sigma_2 g M (\tau_2 \Phi_{10}) \Psi^M_R. \tag{53}
\]

The VEV of \( \Phi_{10} \) is given by

\[
\langle \Phi_{10} \rangle = \langle(1,1,3,1,1)\rangle = v_M. \tag{54}
\]

Notice from \[8\], \[7\], \[62\] and \[63\] that \( 54 \) which involves an \( SU(3)_c \) singlet only gives a Majorana mass to the right-handed neutrinos as in \[8\], namely \( M_R = g M v_M \).

One important remark is in order at this point. In \[4\], in order to avoid a Majorana mass term of the type \( \nu^T_L \sigma_2 \nu_L \) which could, in principle, come from the coupling to the triplet \( \chi \), a global symmetry \( U(1)_M \) was imposed. In the present framework, one notices that \( \Psi^T_R \sigma_2 \Psi_L = (6 + 10, 1 + 3, 1, 1, 1) \) does not couple to \( \Phi_{10} \). In consequence, the Majorana mass term does not exist as long as \( \Phi_{10} \) is present and there is no need to invoke the \( U(1)_M \) symmetry.

The phenomenology of \( \Phi_{10} \) is quite interesting involving in particular its color-non-singlet components. This will be discussed in the phenomenology section below.

\* \textbf{Dirac mass terms involving } \nu_L and \nu^M_R: \\

What would be the equivalent of Eq. \[9\] for the neutrino Dirac masses and how does the singlet scalar field \( \phi_S \)?

Let us look at the following bilinear:

\[
\Psi_L \Psi^M_R = (1 + 15, 2, 2, 1, 1). \tag{55}
\]

From \[55\], it is clear that a bare Dirac mass term of the type \( \nu^T_L \Psi_L \Psi^M_R \) is \textit{not allowed} by gauge invariance. In \[4\], a global \( U(1)_M \) symmetry was imposed by hand to prevent such a term and we have just seen that it is not necessary to do so here.

Let us choose the following Higgs field with four real components

\[
\Phi_S = (1,2,2,1,1). \tag{56}
\]

How is \( \Phi_S \) a singlet like \( \phi_S \) under \( SU(2)_V \)? Obviously, it is not but one of its components is. To see this, let us recall that \( SU(2)_L \otimes SU(2)_R \sim SO(4) \) and a \( (2,2) \) of \( SU(2)_L \otimes SU(2)_R \) is just a quartet representation of \( SO(4) \). We can write

\[
\tilde{\Phi}_S = (\phi_S, \bar{\pi}_S) \in SO(4), \tag{57}
\]

where \( \bar{\pi}_S \) has three components. Under the diagonal subgroup \( SU(2)_V \) of \( SO(4) \), \( \bar{\pi}_S \) is a “vector” while \( \phi_S \) is a singlet. In summary, \textit{under} \( SU(2)_V \):

\[
\bar{\pi}_S \sim 3; \phi_S \sim 1. \tag{58}
\]

Explicitly, we have

\[
\Phi_S = \left( \begin{array}{cc}
\phi_S + i \pi_3^S & -\frac{1}{\sqrt{2}}(\pi_1^S + i \pi_2^S) \\
\frac{1}{\sqrt{2}}(\pi_1^S - i \pi_2^S) & \phi_S - i \pi_3^S
\end{array} \right). \tag{59}
\]

A few comments are in order at this point. A quick glance at \[59\] reveals a form that looks very much like how one would also present a complex Higgs doublet for the SM \( SU(2)_L \). The big difference lies however in the fact that \( \phi_S \) in \[59\] is now actually a \textit{singlet} of \( SU(2)_V \) unlike the case with the SM. Therefore, its VEV \textit{does not} break \( SU(2)_V \) as in \[4\].

The Lagrangian here is

\[
\tilde{\mathcal{L}}_D = \Psi_L g_{SI} \Phi_S \Psi^M_R + H.c. \tag{60}
\]

With \( \langle \phi_S \rangle = v_S \), we have

\[
\langle \Phi_S \rangle = \left( \begin{array}{cc}
v_S & 0 \\
0 & v_S
\end{array} \right). \tag{61}
\]

\[61\] gives rise to the Dirac neutrino mass matrix among \( \nu_L \) and \( \nu^M_R \) as \[10\], namely \( m_D = g_{SI} v_S \). Furthermore, \[63\] also gives rise to mass mixing between SM charged fermions and their mirror fermion counterparts. This has been thoroughly discussed in \[4\] and it was found that those mixings are negligible, for the changes to the eigenvalues of the charged fermions are of the form \( m_D^2 / (m_{fM} - m_{fSM}) \ll (m_{fM}, m_{fSM}) \) since, as we briefly review in \[11\], \( m_D \sim 10^2 \text{eV} \).

At this point, an important remark is in order here. Let us notice that \( \langle \Phi_S \rangle \neq 0 \) spontaneously break \( SU(2)_L \otimes SU(2)_R \) down to \( SU(2)_V \). As discussed below, the scale \( M_{LR} \) associated with \( SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V \) is of \( O(\leq 1 \text{ TeV}) \).
It is natural to ask whether or not one could identify $v_S$ with $M_{LR}$. If this is the case, the Yukawa coupling $g_{SI}$ would have to be $g_{SI} \sim 10^{-7}$ in order for $m_D \sim 10^5 eV$ in an electroweak scale see-saw scenario. There might not be anything unnatural about the smallness of this Yukawa coupling since the SM contains couplings of that order such as the electron Yukawa coupling. $g_{SI}$ needs not necessarily be of order unity. This might be the simplest scenario. Furthermore, as we shall see in the last section of the paper on families from spinors, it is quite natural in this framework to have a fourth family. There we argue that the fourth neutrino can be heavy ($> M_Z/2$) while the other three are light because $g_{SI}$ is generated at the one-loop level and can be $\sim 10^{-7}$. Another more complicated scenario with $g_{SI} \sim O(1)$ and $v_S \sim O(10^5 eV)$ is to add another similar Higgs field with a large VEV and forbids a coupling of the type \(\Phi^{67}\) by some global or discrete symmetry.

- **Majorana mass terms involving $N_R$**
  
  The appropriate fermion bilinear is
  \[
  \Psi_R^T \sigma_2 \Psi_R = (4 \times 4 = 6 + 10, 1, 1, 1, 1 + 3). \tag{62}
  \]
  
  In a similar fashion to \(\Phi \Phi^T\), we introduce a Higgs field
  \[
  \Phi_{10N} = (10 = 1 + 3 + 6, 1, 1, 1, 3), \tag{63}
  \]
  with a Lagrangian
  \[
  \mathcal{L}_{M,N} = \Psi_R^T \sigma_2 g_{M,N} (\tau_2 \Phi_{10N}) \Psi_R + H.c. \tag{64}
  \]
  
  Once more we notice that, as long as only $\Phi_{10N}$ is introduced, there is no coupling of the bilinear $\Psi_L^M \tau_2 \Psi_L^M$ to $\Phi_{10N}$ which, if present, would give rise to a Majorana mass term $N_R^T \sigma_2 N_L$.
  
  The VEV of $\Phi_{10N}$ will be
  \[
  \langle \Phi_{10N} \rangle = \langle (1, 1, 1, 1, 3) \rangle = v_{M,N}, \tag{65}
  \]
  giving rise to the Majorana mass for the right-handed sterile neutrino $N_R$:
  \[
  M_R^N = g_{M,N} v_{M,N}. \tag{66}
  \]

- **Dirac mass terms between $N_L$ and $N_R$**
  
  The Dirac mass term $N_R N_L$ is contained in
  \[
  \Psi_R \times \Psi_L^M = (1 + 15, 1, 1, 2, 2). \tag{67}
  \]
  
  Let us notice that because of \(\Phi \Phi^T\), there is no bare mass term $N_R N_L$ since it is forbidden by gauge invariance. \(\Phi \Phi^T\), in addition to $N_R N_L$, contains mixings between the right-handed SM charged fermions with the left-handed mirror fermions, all of which are $SU(2)_V$ singlets. One can choose for the Higgs field
  \[
  \Phi^N_S = (1, 1, 1, 2, 2). \tag{68}
  \]
  This Higgs field is of course $SU(2)_V$ singlet. This is similar to the case considered in \[4, 9\]. Its VEV is
  \[
  \langle \Phi_S^N \rangle = \left( \begin{array}{c} v_N^S \\ 0 \\ 0 \\ v_N^S \end{array} \right). \tag{69}
  \]
  
  The relevant Lagrangian is
  \[
  \mathcal{L}_D^N = \bar{\Psi}_R g_{S,N} \Phi_{S}^N \Psi_L^M + H.c., \tag{70}
  \]
  giving the following Dirac mass
  \[
  m_D^N = g_{S,N} v_N^S. \tag{71}
  \]
  
  Once again, a remark concerning the size of $v_N^S$ is in order here. If we assume that $v_N^S \sim v_S \sim O(\leq 1 TeV)$, it follows, from the discussion presented below, that $g_{S,N}^N \sim 10^{-5} - 10^{-7}$. The remarks made above concerning small values of the Yukawa couplings apply equally to this case. Let us also notice that the symmetry breaking pattern, at $M', SU(2)_L' \otimes SU(2)_R' \otimes U(1)_S \rightarrow U(1)_V$ requires a Higgs field with non-vanishing $U(1)_S$ quantum number which will not couple to $N_L N_R$ for the latter has a vanishing $U(1)_S$ quantum number. One will not have to worry about the scale $M$ (to be discussed below).

- **Generalized “see-saw”**
  
  One can now put the pieces obtained above to write a “see-saw” matrix which is now a $4 \times 4$ matrix instead of $2 \times 2$ one as follows.
  \[
  M_4 = \begin{pmatrix} 0 & m_D & m_{\nu_L N_R} & m_{\nu_L N_R}^M \\ m_D & M_R & m_{\nu_R N_L} & 0 \\ m_{\nu_L N_R} & m_{\nu_R N_L} & m_D^N & M_R^N \\ m_{\nu_L N_R} & 0 & m_D^N & M_R^N \end{pmatrix}. \tag{72}
  \]
  
  Let us notice that \(\Phi \Phi^T\) would decompose into two $2 \times 2$ blocs had $m_{\nu_L N_R} = m_{\nu_R N_L} = 0$, and its diagonalization becomes straightforward. For the upper $2 \times 2$ bloc, a previous discussion made in \[4\] implied that, for $M_R \sim O(100 GeV)$, $m_D \sim 10^5 eV$. The other elements of $M_4$, namely $m_{\nu_L N_R}, m_{\nu_R N_L}, m_{\nu_R N_L}, M_R^N$, are “unconstrained” at this stage. However, they are actually “constrained” in the sense that they might influence the active-sterile mixing angles as we shall see below.

- **Numerical examples**
  
  To gain some insight, let us vary $m_{\nu_L N_R}, m_{\nu_R N_L}, m_D^N, M_R^N$ and observe the pattern of mass eigenvalues. An exhaustive numerical
study is beyond the scope of this paper. However it will be useful to show a few numerical examples for illustration.

In the discussion that follows we will concentrate on the overall mass scales of various neutral lepton sectors and will ignore flavor differences for the time being. Several special cases will now be listed.

- \( m_{\nu LN_R} = m_{\nu RN_L} = 0 \):

There is, of course, no reason why this should be the case but, for the sake of clarity, it will be shown as a first step in our discussion. The mass eigenvalues are now straightforwardly given as

\[
M_R \pm \sqrt{M_R^2 + 4 m_D^2} \approx \left\{ \begin{array}{c} M_R \\
\frac{m_D}{M_R} \end{array} \right. , \quad (73)
\]

for the non-sterile sector, where the approximation comes from the case where \( m_D \ll M_R \), and

\[
M_R^N \approx \sqrt{(M_R^N)^2 + 4 (m_D^N)^2} , \quad (74)
\]

for the sterile sector, where Eq. (74) is left in its full form.

For (74), we have seen in [4] that \( m_D \sim 10^5 eV \) for \( M_R \sim O(100 GeV) \) in order for the light active neutrinos to have masses of \( O(1 eV) \). The same thing cannot be said about the sterile masses, although there appears to be interesting mass ranges in the keV region which might be of astrophysical interest. We shall come back to this aspect below.

- \( m_{\nu LN_R} = m_{\nu RN_L} \neq 0 \):

This is the simplest next step. However, since this involves couplings that mix the active neutrinos \( \nu_L \) and \( \nu_R^N \) with the sterile ones, \( N_L \) and \( N_R \), they will influence the mass eigenvalues of the active sector and also their mixtures in the mass eigenstates. The knowledge one has acquired in the determination of weak interaction couplings provides a strong constraint on these mixing. An exhaustive phenomenological study is beyond the scope of the present paper. However, some hints on what to expect will be presented here. Below, we will show two particular examples to see the correlations between these couplings and the mass eigenvalues and eigenstates of the active neutrino sector.

We will concentrate on scenarios in which the heavier of the sterile Majorana neutrinos has a mass ranging from a few MeV to a few hundreds of GeVs. This meant to be an example in which one can have both keV and MeV sterile neutrinos. Let us start with a few examples in which the heavier sterile neutrino has a mass of the order of \( M_R \). To be definite, we shall take as in [4] the following values for \( m_D \) and \( M_R \), namely \( M_R = 100 GeV \) and \( m_D = 10^{-6} M_R \), with the understanding that these values are mainly for illustration purposes.

\[
M_4 \approx \begin{pmatrix} 0 & 10^{-6} & 0 & 4 \times 10^{-9} \\
10^{-6} & 1 & 4 \times 10^{-9} & 0 \\
0 & 4 \times 10^{-9} & 0 & 1.8 \times 10^{-4} \\
4 \times 10^{-9} & 0 & 1.8 \times 10^{-4} & 1 \end{pmatrix} .
\]

The eigenvalues and eigenvectors are as follows (for \( M_R = 100 GeV \));

\[
m_1 \approx -0.1 eV ;
\]

\[
\tilde{\nu}_1 \approx -\nu_L + 10^{-6} \nu_R^M + 2.2 \times 10^{-5} N_L
-2.2 \times 10^{-11} N_R ,
\]

\[
m_2 \approx 100 GeV ;
\]

\[
\tilde{\nu}_2 \approx 10^{-6} \nu_L + \nu_R^M + 4 \times 10^{-9} N_L
-7.3 \times 10^{-13} N_R ,
\]

with the sterile ones, they will influence the mass eigenvalues of the active sector and also their mixtures in the mass eigenstates. The knowledge one has acquired in the determination of weak interaction couplings provides a strong constraint on these mixing. An exhaustive phenomenological study is beyond the scope of the present paper. However, some hints on what to expect will be presented here. Below, we will show two particular examples to see the correlations between these couplings and the mass eigenvalues and eigenstates of the active neutrino sector.

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\[
m_1 \approx -0.1 eV ;
\]

\[
\tilde{\nu}_1 \approx -\nu_L + 10^{-6} \nu_R^M + 2.2 \times 10^{-5} N_L
-2.2 \times 10^{-11} N_R ,
\]

\[
m_2 \approx 100 GeV ;
\]

\[
\tilde{\nu}_2 \approx 4 \times 10^{-9} \nu_L + 0 \nu_R^M + 1.8 \times 10^{-4} N_L
+N_R ,
\]

b) We now present another example in which the heavier sterile neutrino mass could be in the MeV range, in particular 10 MeV. For the sake of comparison, we will show an example in which the lighter of the sterile neutrinos has a mass around 3.25 keV and \( \sin \theta = 2.23 \times 10^{-5} \).

\[
M_4 \approx \begin{pmatrix} 0 & 10^{-6} & 0 & 4 \times 10^{-11} \\
10^{-6} & 1 & 4 \times 10^{-11} & 0 \\
0 & 4 \times 10^{-11} & 0 & 1.8 \times 10^{-6} \\
4 \times 10^{-11} & 0 & 1.8 \times 10^{-6} & 0.0001 \end{pmatrix} .
\]

with

\[
m_1 \approx -0.1 eV ;
\]

\[
\tilde{\nu}_1 \approx -\nu_L + 10^{-6} \nu_R^M + 2.2 \times 10^{-5} N_L
-3.6 \times 10^{-11} N_R ,
\]

\[
m_2 \approx 100 GeV ;
\]

\[
\tilde{\nu}_2 \approx -10^{-6} \nu_L - \nu_R^M - 4 \times 10^{-11} N_L
+2 \times 10^{-14} N_R ,
\]
\begin{equation}
  m_{S1} \approx -3.24 \text{ keV};
\end{equation}
\begin{equation}
  \tilde{v}_{S1} \approx -2.2 \times 10^{-5} \nu_L + 16.4 \times 10^{-11} \nu^M_R - N_L + 0.018 N_R,
\end{equation}
\begin{equation}
  m_{S2} \approx 10 \text{ MeV};
\end{equation}
\begin{equation}
  \tilde{v}_{S2} \approx -4 \times 10^{-7} \nu_L + 10^{-12} \nu^M_R - 0.018 N_L - N_R.
\end{equation}

We would like to mention in passing that, as the mass of the heavier sterile neutrino increases, the mixing among the sterile neutrinos also increases in the mass matrix.

- **Some comments on the numerics:**

We wish to make two remarks concerning the values chosen for the matrix elements above. The upper $2 \times 2$ bloc in (75) and (77) was taken from [4] and was chosen to give one electroweak scale mass eigenstate and one light ($\sim 0.1 \text{ eV}$) eigenstate for the active neutrino sector. The lower $2 \times 2$ bloc for the sterile sector was basically chosen phenomenologically to give a keV eigenstate and a heavier one with a mass ranging from 10 MeV to 100 GeV. Basically the elements $m_{\nu_L N_R}$ and $m_{\nu_R^M N_L}$ will then determine the mixing angles between the active and sterile sectors. They were chosen in such a way as to obey the various constraints imposed on sterile neutrinos [18, 19]. We shall come back to this aspect in the section on sterile neutrinos.

One might ask about the reasons why the mixing parameters between the active and sterile sectors in the mass matrices could be so small considering the fact that $m_{\nu_L N_R}$ (77b) and $m_{\nu_R^M N_L}$ (77c) are proportional to the same VEVs which give masses to the charged leptons and quarks. One first notices that even if we set $m_{\nu_L N_R} = 0$ and $m_{\nu_R^M N_L} = 0$ giving rise to two sets of relations, there would be no cancellations in the expressions for the charged lepton and quark masses. As a result, one might, from a phenomenological viewpoint set $m_{\nu_L N_R} \sim m_{\nu_R^M N_L} \sim O(10^{-11} M_R)$ for example. On the other hand the mixing $m_D$ (Eq. (10)) and $m_D^N$ (Eq. (71)) involve scalars which are $SU(2)_Y$ singlets which, as it has been argued in [4], could have vacuum expectation values smaller than the electroweak scale. There would be no need of cancellations there in contrast with $m_{\nu_L N_R}$ and $m_{\nu_R^M N_L}$.

A quick look at (75) and (77) reveals that, as the mass scales of the sterile sector decreases, the mixing between the active and sterile sectors also decreases within the context of our numerical examples. The astrophysical implications of these results will be discussed below. Needless to say that these examples are shown to illustrate some of the relationships between the sterile masses and their mixing with the active sector.

One last comment is in order here. The above numerical examples dealt with the overall mass scales and, in that sense, would look like a one generation case. A more “realistic” scenario would involve the usual three (or more) families. Nevertheless, one would expect that the above masses and mixing would not be changed much when three or more generations are involved. One might also expect that, if the heaviest among the light sterile neutrinos has a mass of a few keVs, some of the remaining eigenstates might be much lighter, even having eV masses.

### B. Masses of the charged mirror fermions

We have alluded above to the fact that mirror quarks and leptons should be heavier than their SM counterparts since they have not been observed so far. It is fair to ask the question: Why should they be heavier than the SM particles? First, let us remind ourselves that in this model the SM and mirror fermions are coupled to different Higgs scalars. In principle, there is no reason why the mass pattern of the two sectors should be similar.

It is without any doubt that the problem of fermion masses is one of the biggest mysteries of the SM and, although there are many models, no satisfactory answer has been found and widely accepted. However, to obtain some hint on why the mirror fermions are heavier than the SM particles, one might for instance take some ansatz that could “fit” the SM mass pattern and try to see how to adopt it to the mirror sector. For simplicity, let us use the ansatz of [20] (see also [21]). Also for simplicity we will focus on the quark sector in this discussion.

The hierarchical ansatz of [20] is simply the following matrix (ignoring the phase factors)

\begin{equation}
  M_H = m_3 \begin{pmatrix}
    0 & \epsilon^3 & 0 \\
    \epsilon^3 & \epsilon^2 & \epsilon^2 \\
    0 & \epsilon^2 & 1
  \end{pmatrix},
\end{equation}

with the mass eigenvalues to order $\epsilon^4$ being $-m_3 \epsilon^4$, $m_3 \epsilon^2$ and $m_3 (1 + \epsilon^4)$. [21] used $\epsilon_u = 0.07$ and $\epsilon_d = 0.21$ to reproduce the phenomenological mass hierarchies at the scale $M_Z$.

Let us assume a similar ansatz for the mirror quark sector

\begin{equation}
  M_M = m_M \begin{pmatrix}
    0 & \epsilon_M^3 & 0 \\
    \epsilon_M^3 & \epsilon_M^2 & \epsilon_M^2 \\
    0 & \epsilon_M^2 & 1
  \end{pmatrix}.
\end{equation}

Since $m_M$ cannot be too different from the electroweak scale, a value of $\epsilon_M$ similar to those of the SM quarks would engender light mirror quarks. To avoid this, $\epsilon_M$ would have to be significantly different from $\epsilon_u$ and $\epsilon_d$. Furthermore, in order to satisfy constraints coming from
the T parameter (to be discussed along with the S parameter in Section (VI)), we will assume, for the sake of discussion, that the up and down mirror quark sectors are “degenerate”, namely $\epsilon^u_M \sim \epsilon^d_M$. What if the mass difference between the SM and mirror sectors is due to the disparity between the scale $M_{LR}$ of the breakdown $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_{V}$ and $M_Z$? Let us remind ourselves that above $M_{LR}$, the SM and mirror fermions have separate gauge interactions, $SU(2)_L$ and $SU(2)_R$ respectively, while below that scale they interact with the same gauge bosons of $SU(2)_{V}$. We make the following ansatz:

$$\epsilon^u_M \sim \epsilon^d_M \sim \frac{M_{LR}}{M_Z} \epsilon_{SM} \ ,$$

(81)

where $\epsilon_{SM}$ is some value between $\epsilon_u$ and $\epsilon_d$. We take as an example $\frac{M_{LR}}{M_Z} = 10$, as discussed in the following section. The eigenvalues corresponding to $\epsilon^u_M \sim 0.8, 0.9$ are (1) $m_M(-0.37, 0.46, 1.55)$, (2) $m_M(-0.56, 0.51, 1.86)$ respectively. For $m_M \sim O(\geq 250\text{ GeV})$, all these mirror quarks are heavy. For example, with $m_M = 350\text{ GeV}$ and $\epsilon^u_M \sim 0.9$, one obtains the following three mass eigenvalues: $(-196, 179, 651)\text{ GeV}$. We will briefly discuss the constraints on such masses in the section on phenomenology. A similar consideration can be applied to the mirror lepton sector yielding heavy mirror leptons.

The above discussion is one of the probably many possibilities of rendering the mirror fermions heavy. We now turn our attention to phenomenological constraints and implications of the model.

V. CONSTRAINT FROM $\sin^2 \theta_W(M_Z)$

As we have discussed above, the SM is embedded into a PUT gauge group of the form $G = SU(4)_P \otimes SU(2)_L \otimes SU(2)_R \otimes SU(2)_{V} \otimes SU(2)_{H}$. The interpretation of the fermion content in the present model is however very different from that used for the same gauge group in [14,15]. As a result, the pattern of symmetry breaking and the computation of $\sin^2 \theta_W(M_Z)$ will be somewhat different here. The main purpose for computing $\sin^2 \theta_W(M_Z)$ is to constrain the Petit Unification mass scale.

The computation of $\sin^2 \theta_W(M_Z)$ in the breaking pattern [17] and, in particular, [23], is a little more complicated than a similar one in [14,15]. As a result, a certain caution is warranted. The difference with [14,15] lies with the fact that the weak $SU(2)$ group there was simply one of the $SU(2)$’s, namely $SU(2)_L$. This was referred to as an “unlocked” case. In the present model, the weak $SU(2)$ group comes from the breaking $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_{V}$. In the language of [14,15], this is a “locked” case and because of this, as we shall see below, the scales $\tilde{M}$ and $M$ turn out to be quite large. Our strategy for computing those scales will be as follows. First, we will derive an expression for $\sin^2 \theta_W(M_{LR})$ which depends on the three scales $M_{LR}$, $\tilde{M}$ and $M$. (Notice that $\tilde{\theta}_W$ refers to a slightly different angle than the usual one.) We then evolve the SM $\sin^2 \theta_W$ from its experimental value at $M_Z$ to a value at $M_{LR}$. Next, we derive a relation between $\sin^2 \theta_W(M_{LR})$ and $\sin^2 \theta_W(M_{LR})$. Using this relation, we then determine the possible values for the aforementioned three scales.

Unlike [14,15], our basic equations will start from the scale $M_{LR}$ instead of $M_Z$. This will be matched with the evolution of the $SU(3)c \otimes SU(2)_V \otimes U(1)_{Y}$ couplings up from $M_{LR}$ to $M_{LR}$. To be precise in our definitions, we first give a list of notations for the gauge couplings at various mass scales.

- The group $SU(3)c(g_3) \otimes SU(2)_V(g_2) \otimes U(1)_{Y}(g')$ at $M_Z$.
- The group $SU(3)c(g_3) \otimes SU(2)_L(gw) \otimes SU(2)_R(gw) \otimes U(1)_{Y}(g')$ at $M_{LR}$.

$$\frac{1}{g_w^2(M_{LR}^2)} = \frac{2}{g_w^2(M_Z^2)} + C_S^2 \left( \frac{M_{LR}}{M_Z} \right)^2 ,$$

(82)

where the factor of 2 in [22] comes from [23]. This will be used below to obtain a match at $M_{LR}$. Since we have $SU(2)_L^f(gw) \otimes SU(2)_R^f(gw) \otimes U(1)s(\tilde{g}_S) \rightarrow U(1)_{Y}(g')$ at $\tilde{M}$, one has

$$\frac{1}{(g')^2(M^2)} = \frac{2}{g_w^2(M^2)} + C_S^2 \left( \frac{M_{LR}}{M} \right)^2 ,$$

(83)

where we have used Eq. [24] and where $C_S^2 = 2/3$. Furthermore, at the scale $M$, one has

$$g_m(M^2) = \tilde{g}_S(M_Z^2) = g_S(M^2) .$$

(84)

One can now examine the evolution of the gauge couplings from $M_{LR}$ to $M$ in details.

Since the particle content of the $SU(2)$’s groups are symmetric and since it is assumed that the fermions and scalars have masses less than $M_{LR}$, one can use either $SU(2)_L$ or $SU(2)_R$ to study the evolution of the couplings from $M_{LR}$ to $M$. The basic equations used here are

$$\frac{1}{g_W^2(M_{LR}^2)} = \frac{(C_W')^2}{g_w^2(M^2)} + 2b_2 \ln \left( \frac{M}{M_{LR}} \right) ,$$

(85)
where \( C_S^2 = 2/3, (C_W')^2 = 1, C_W^2 = 2 \) and where
\[
b_1 = \frac{1}{48\pi^2} \left( \frac{20}{3} n_G + 7 \right),
\]
\[
b_2 = \frac{1}{48\pi^2} (2 n_G + 5 - 22),
\]
\[
b_3 = \frac{1}{48\pi^2} (4 n_G - 33),
\]
\[
\bar{b} = \frac{1}{48\pi^2} (4 n_G),
\]
with \( n_G \) (= left-handed plus right-handed) is the number of \( SU(2)^V \) doublets. (The number of families is \( n_G/2 \).) In addition we also list the coefficient related to \( SU(2)^V \) for the use in the evolution of the \( SU(2)^V \) coupling from \( M_Z \) to \( M_{LR} \):
\[
b_{2V} = \frac{1}{48\pi^2} (4 n_G + 7 - 22),
\]

A few words concerning the different factors in (89) (91) (92) (93) are in order. First, \( n_G \) refers to the number of \( SU(2)^V \) doublets and that includes both left-handed and right-handed fermions. Now, \( b_2 \) refers to \( SU(2)_L \) (or \( SU(2)_R \)) and, as a result, \( n_{LR} = n_G/2 \) resulting in the factor 2 \( n_G \) in (90) while it is 4\( n_G \) in (93). Second, the factors 7 in (89) and 5 in (90) comes from the counting of the number of scalar degrees of freedom as follows. From \( \Phi_S, \Phi_S^M \) and the \( SU(3)_c \)-singlet parts of \( \Phi^S \) and \( \Phi^M \), one obtains 8 Higgs doublets which contribute a factor of 4 to \( b_1 \). In addition, the two Higgs triplets contribute a factor 1 + 2 = 3 to \( b_1 \) giving a total of 7 from the scalar sector. On the other hand, above \( M_{LR} \), only 4 Higgs doublets contribute to \( b_2 \) since one now has e.g. \( SU(2)_L \) instead of \( SU(2)^V \) while below \( M_{LR} \) all 4 Higgs doublets contribute to \( b_{2V} \).

Eqs. (89) (90) (91) (92) are similar in forms to the ones used in (14) to derive \( \sin^2 \theta_W(M_Z^2) \) except that now we will use them to derive an expression for \( \sin^2 \theta_W(M_{LR}^2) \) and a relationship between these two quantities. Although the \( U(1)_{em} \) gauge coupling is transmogrified into the \( SU(2)^V \otimes U(1)_Y \) gauge couplings above the electroweak scale (or \( M_Z \)), we will keep the same notation above that scale, namely
\[
\frac{1}{e^2(M_{LR}^2)} = \frac{1}{g_W^2(M_{LR}^2)} + \frac{1}{(g')^2(M_{LR}^2)},
\]

Let us define a similar quantity involving \( g_W \), namely
\[
\frac{1}{\tilde{e}^2(M_{LR}^2)} = \frac{1}{g_W^2(M_{LR}^2)} + \frac{1}{(g')^2(M_{LR}^2)},
\]

and
\[
\tilde{a}(M_{LR}^2) = \frac{\tilde{e}^2(M_{LR}^2)}{4\pi}.
\]
Let us recall the definition of \( \sin^2 \theta_W(M_Z^2) \):
\[
\sin^2 \theta_W(M_Z^2) = \frac{c^2(M_Z^2)}{g_Z^2(M_Z^2)},
\]

with a similar expression evaluated at \( M_{LR} \). Let us now define \( \sin^2 \theta_W(M_{LR}^2) \) as
\[
\sin^2 \theta_W(M_{LR}^2) = \frac{\tilde{e}^2(M_{LR}^2)}{g_W^2(M_{LR}^2)},
\]

Obviously \( \sin^2 \theta_W(M_{LR}^2) \) is not the same as \( \sin^2 \theta_W(M_Z^2) \). One can easily derive a relation between the two as follows
\[
\sin^2 \theta_W(M_{LR}^2) = \frac{\sin^2 \theta_W(M_{LR}^2)/2}{1 - \sin^2 \theta_W(M_{LR}^2)/2},
\]

upon using Eqs. (94) (95) (97) (98). Since we will be comparing the predicted results with the usual \( \sin^2 \theta_W(M_Z^2) \), one can rewrite (99) as
\[
\sin^2 \theta_W(M_{LR}^2) = \frac{2 \sin^2 \theta_W(M_{LR}^2)}{1 + \sin^2 \theta_W(M_{LR}^2)}.
\]

From Eqs. (89) (90) (91) (92), one can readily derive the following formula for \( \sin^2 \theta_W(M_{LR}^2) \):
\[
\sin^2 \theta_W(M_{LR}^2) = \sin^2 \theta_W(1 - C_W^2 \frac{\tilde{a}(M_{LR}^2)}{\alpha_S(M_{LR}^2)})
\]
\[
-8\pi \tilde{a}(M_{LR}^2) K \ln \left( \frac{M_{LR}}{M} \right) + K' \ln \left( \frac{M_{LR}}{M} \right) \right),
\]

where, in the parlance of (14),
\[
\sin^2 \theta_W = \frac{(C_W')^2}{C_W^2 + (C_W')^2} = \frac{1}{2 + 1} = \frac{1}{3},
\]

\[
K = b_1 - 2 b_2 - \frac{2}{3} b_3 = \frac{125}{96\pi^2},
\]

\[
K' = C_S^2 \left( \bar{b} - b_3 \right) = \frac{22}{48\pi^2},
\]

and where we have used Eqs. (89) (90) (91) (92). Notice the interesting fact that the dependence on \( n_G \) drops out in both \( K \) and \( K' \).
To proceed with (101), we need to evaluate $\tilde{\alpha}(M_{LR}^2)$ and $\alpha_S(M_{LR}^2)$. From the above equations, one can relate $\tilde{\alpha}(M_{LR}^2)$ to the following measured quantities at $M_Z$ as follows

$$
\tilde{\alpha}^{-1}(M_{LR}^2) = \alpha^{-1}(M_Z^2)(1 - \frac{1}{2} \sin^2 \theta_W(M_Z^2)) - 8 \pi (b_1 + \frac{1}{2} b_{2V}) \ln \left( \frac{M_{LR}}{M_Z} \right).
$$

(105)

In (105), we will use $\alpha^{-1}(M_Z^2) = 127.934$, $\sin^2 \theta_W(M_Z^2) = 0.23113$. Furthermore, with the assumption of three or four (SM and mirror) families i.e. $n_G = 6, 8$, it can easily be seen that $\alpha_S(M_{LR}^2) \approx 0.117$. This last point is interesting on its own: In our model, QCD is nearly scale-invariant above $M_Z$!

The next step involves the extraction from $\sin^2 \tilde{\theta}_W(M_{LR}^2)$ of the values of $\sin^2 \theta_W(M_Z^2)$. Several steps are involved in this computation:

- From $\sin^2 \tilde{\theta}_W(M_{LR}^2)$, we can extract $\sin^2 \theta_W(M_{LR}^2)$ via Eq. (106).

- Next we calculate $\alpha^{-1}(M_{LR}^2)$ using

$$
\alpha^{-1}(M_{LR}^2) = \alpha^{-1}(M_Z^2) - 8 \pi (b_1 + b_{2V}) \ln \left( \frac{M_{LR}}{M_Z} \right)
$$

$$
= \alpha^{-1}(M_Z^2) - \frac{1}{6} \left( \frac{32}{3} n_G - 8 \right) \ln \left( \frac{M_{LR}}{M_Z} \right).
$$

(106)

- $\alpha^{-1}(M_{LR}^2)$ is obtained from $\alpha^{-1}(M_Z^2)$.

- Next we compute

$$
\alpha^{-1}(M_Z^2) = \alpha^{-1}(M_{LR}^2) + 8 \pi b_{2V} \ln \left( \frac{M_{LR}}{M_Z} \right)
$$

$$
= \alpha^{-1}(M_{LR}^2) + \frac{1}{6} \left( 4 n_G - 15 \right) \ln \left( \frac{M_{LR}}{M_Z} \right).
$$

(107)

- Finally, we obtain

$$
\sin^2 \theta_W(M_Z^2) = \alpha^{-1}(M_Z^2) \alpha(M_Z^2).
$$

(108)

As an example, we will show two cases corresponding to two values of the ratio $M_{LR}/M_Z$. These are shown in the tables below. The above two examples show the range of mass scales required to give values for $\sin^2 \theta_W(M_Z^2)$ consistent with experiment.

Although the above examples are far from being exhaustive, the mass scale pattern seems to be one in which $M_{LR} \lesssim 1 \text{ TeV}$ (masses of heavy $W_H$), $M \sim 10^7 \text{ GeV} \sim 10^8 \text{ GeV}$ (masses of $SU(2)_L \otimes SU(2)_R$ gauge bosons), and $M \sim 10^{15} \text{ GeV}$ (the Pati-Salam unification scale). Let us notice however that this is not a prediction for the mass scales. It is simply an illustration of the relationships between the various scales constrained by the experimental values of $\sin^2 \theta_W(M_Z^2)$. It goes without saying that $M_{LR}$ could be larger than $1 \text{ TeV}$ in which case the Pati-Salam unification scale could be higher than $10^{17} \text{ GeV}$. The phenomenological implications of these values will be discussed in the next section.

### VI. PHENOMENOLOGICAL IMPLICATIONS

In this section we will present a very brief discussion of the phenomenology, the details of which will be presented elsewhere. Three main issues among several others include the presence of mirror fermions [4], sterile neutrinos (in addition to the electroweak scale right-handed neutrinos), and heavy $W$-like gauge bosons.

#### Mirror fermions:

What are the effects of the mirror fermions or, in general, of extra chiral families on the $S$ and $T$ parameters? This question has been previously studied in the literature. For example, it was found that there are regions of parameter space in a two-Higgs doublet model that can accommodate even three additional chiral families [11, 12]. Our model contains eight Higgs doublets, one complex Higgs triplet and one real Higgs triplet where the counting include

| TABLE I: Average values of $\tilde{M}$ and $M$ for $\frac{M_{LR}}{M_Z} = 10$ and $n_G = 8$ subjected to the constraint $0.2308 \leq \sin^2 \theta_W(M_Z^2) \leq 0.2314$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\tilde{M}(\text{GeV})$ | $9.51 \times 10^6$ | $2.12 \times 10^7$ | $4.75 \times 10^7$ | $1.06 \times 10^8$ |
| $M(\text{GeV})$ | $9.51 \times 10^{17}$ | $2.12 \times 10^{16}$ | $4.75 \times 10^{15}$ | $1.06 \times 10^{15}$ |

| TABLE II: Average values of $\tilde{M}$ and $M$ for $\frac{M_{LR}}{M_Z} = 10$ and $n_G = 6$ subjected to the constraint $0.2308 \leq \sin^2 \theta_W(M_Z^2) \leq 0.2314$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\tilde{M}(\text{GeV})$ | $1.16 \times 10^7$ | $2.59 \times 10^7$ | $5.78 \times 10^7$ | $1.59 \times 10^8$ |
| $M(\text{GeV})$ | $1.16 \times 10^{17}$ | $2.59 \times 10^{16}$ | $5.78 \times 10^{15}$ | $1.59 \times 10^{15}$ |

| TABLE III: Average values of $\tilde{M}$ and $M$ for $\frac{M_{LR}}{M_Z} = 5$ and $n_G = 8$ subjected to the constraint $0.2308 \leq \sin^2 \theta_W(M_Z^2) \leq 0.2314$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\tilde{M}(\text{GeV})$ | $7.68 \times 10^6$ | $1.72 \times 10^7$ | $3.83 \times 10^7$ | $8.57 \times 10^7$ |
| $M(\text{GeV})$ | $7.68 \times 10^{16}$ | $1.72 \times 10^{15}$ | $3.83 \times 10^{15}$ | $8.57 \times 10^{14}$ |
TABLE IV: Average values of $\tilde{M}$ and $M$ for $\frac{M_{12}}{2} = 5$ and $n_G = 6$ subjected to the constraint $0.2308 \leq \sin^2 \theta_W (M_{\tilde{Z}}^2) \leq 0.2314$

\[
\begin{array}{cccc}
\tilde{M} (\text{GeV}) & 8.79 \times 10^6 & 1.96 \times 10^7 & 4.39 \times 10^7 & 9.83 \times 10^7 \\
M (\text{GeV}) & 8.79 \times 10^{16} & 1.96 \times 10^{17} & 4.39 \times 10^{18} & 9.83 \times 10^{19}
\end{array}
\]

only Higgs fields that develop VEVs (123, 124). Needless to say, we have the necessary ingredients to accommodate the extra mirror families. One can even work with a four family scenario (SM plus mirror) since one now has enough Higgs representations to offset any non-degeneracy of the extra families. In summary, one can have the correct S and T parameters in our model. We will now focus on various phenomenological aspects of mirror fermions.

The crux of the model presented in [4] is the existence of electroweak scale right-handed neutrinos which are non-sterile. Because of this fact, one could directly produce those right-handed neutrinos at colliders such as the upcoming LHC or the proposed ILC and check the validity of the seesaw mechanism. As mentioned in [4], one of most important signals of the model is the presence of like-sign dileptons at colliders. Such a signal would constitute a high-energy equivalent of neutrinoless double beta decay.

The like-sign dilepton events can come from the following subprocesses.

1) Production and subsequent decays of electroweak scale right-handed neutrinos:

\[
q + \bar{q} \rightarrow Z \rightarrow \nu_R^M + \nu_R^M \\
\rightarrow e_{R}^{M+,+} + e_{R}^{M,-} + W^\pm + W^\pm \\
\rightarrow e_{L}^{+} + e_{L}^{-} + W^\pm + W^\pm + \phi_S + \phi_S
\]

(110)

where $e_{R}^{M,+}$ could be real or virtual depending on the mass differences with $\nu_R$'s. The production cross section is estimated to be $\sigma \sim 400 \, fb$ for $M_R \sim 100 \, GeV$ at the LHC. One can also have e.g.

\[
u + \bar{\nu} \rightarrow W^+ \rightarrow \nu_R^M + e_R^{M,+} \\
\rightarrow e_{R}^{M,+} + e_{R}^{M,+} + W^- \\
\rightarrow e_{L}^{+} + e_{L}^{+} + W^- + \phi_S + \phi_S
\]

(111)

where $e$ and $e^M$ are generic notations for (SM and mirror) charged leptons. In the above processes, $\phi_S$ is the singlet scalar field which would be considered as missing energy. The W's could transform into jets or pairs of leptons. Depending on how heavy $\phi_S$ is, the signal could be quite interesting. If the mirror charged lepton is sufficiently long-lived (e.g. its decay could occur a few centimeters away from the beam pipe), the search for like-sign dileptons with displaced vertices would constitute perhaps a “clean” signal.

2) Direct production of like-sign mirror dileptons followed by like-sign SM leptons:

\[
W^+ + W^+ \rightarrow \chi^{++} \rightarrow e_R^{M,+} + e_R^{M,+} \\
\rightarrow e_{L}^{+} + e_{L}^{-} + \phi_S + \phi_S
\]

(112)

and similarly for the opposite sign process. Here one expects, at the LHC, a production cross section $\sigma \sim 3 \, pb$ for $M_{\chi^{++}} \sim 200 \, GeV$.

In a scenario in which $\nu_R^M$'s are all lighter than the mirror charged leptons, the direct production of lighter $\nu_R^M$'s is followed by the decay into $\nu_L$ plus $\phi_S$, all of which will become missing energy:

\[
q + \bar{q} \rightarrow Z \rightarrow \nu_R^M + \nu_R^M \\
\rightarrow \nu_L + \nu_L + + \phi_S + \phi_S
\]

(113)

All of the above processes can occur and it will be interesting to disentangle the two like-sign dilepton mechanisms. Beside the difference in cross sections, the process (110) has like-sign dileptons plus e.g. one or two jets and missing energy while the process (112) has a like-sign dilepton plus missing energy. This phenomenology will be presented elsewhere.

3) Last but not least, it would be interesting to study the phenomenology associated with mirror quarks. These quarks would be produced in colliders just like a typical SM heavy quarks e.g. the top quark. For the heavy mirror quarks to materialize into SM particles, one can look at the decay process $q^M \rightarrow q + \phi_S$. Here one should perhaps concentrate on displaced vertices depending on how long-lived the mirror quarks are [23].

• Sterile neutrinos $N_L$ and $N_R$:

As we have extensively discussed in Section IV various aspects of the sterile neutrinos $N_L$ and $N_R$, we would like to briefly discuss what these particles could do in astrophysics and cosmology, among others. A comprehensive study of various constraints on the sterile neutrino sector can be found in [18].

We have seen above that there are two types of sterile neutrinos in our model: $N_L$ and $N_R$. In the two numerical examples given in Section IV, one can have a situation in which the keV sterile neutrino is almost purely $N_L$. (Again, for simplicity, we discuss the one specific case although the keV
state could refer to the heavier among the light sterile neutrinos.) The first question to ask here is the following: What can a keV sterile neutrino do? The various possibilities have been extensively discussed (for a review and a list of references, see [12]) and we will just summarize some of the relevant points here. For definiteness, we will take as an example a 3.24 keV sterile neutrino (mostly $N_L$) with a mixing angle to the SM light neutrino sector of the order $2.2 \times 10^{-5}$ as shown in (75) and (77).

First, the possibility of keV sterile neutrinos being candidates for Warm Dark Matter (WDM) has been an exciting and active avenue of research [24]. For simplicity, we will just mention the two examples discussed in Section (IV). The first example has $m_{N_R} = 100 \text{GeV}$ and the mixing with with the SM $\nu_L$ being $4 \times 10^{-9}$. The second example has $m_{N_R} = 10 \text{MeV}$ and a mixing with the SM $\nu_L$ being $4 \times 10^{-7}$. For the first example, one has $\sin^2 \theta_S \approx 1.6 \times 10^{-17}$. From [18], one can see that this is well inside the allowed regions for $N_R \leftrightarrow \nu_e$, $N_R \leftrightarrow \nu_\mu$, and $N_R \leftrightarrow \nu_\tau$. For the second example with $m_{N_R} = 10 \text{MeV}$, one has $\sin^2 \theta_S \approx 1.6 \times 10^{-13}$. This is well inside the region forbidden by the CMB data [18] although it is allowed by accelerator and super-Kamiokande data [26]. For the kind of mixing angles considered in our examples, $m_{N_R}$ appears to be bounded from below by a few hundreds of MeVs. In low reheating cosmological scenarios, it is claimed that for $m_S > 30 \text{MeV}$, cosmological bounds no longer apply [30]. In our rather simple analysis here, the mixing of $N_R$ with the active sector is quite small and it is not clear how one could detect such an object.

- Heavy W-like gauge bosons:

Unlike extended models in which the electroweak SU(2) group is simply SU(2)$_L$ with other gauge groups being spontaneously broken at a larger scale than the electroweak one, our model is rather different in that the SM SU(2) group comes from SU(2)$_L \otimes SU(2)_R \rightarrow SU(2)_V$. At the scale $M_{LR}$, one has the following interaction Lagrangian

$$
\mathcal{L}_W = g_W \vec{J}_{LR}^\mu \cdot \vec{W}_{\mu\nu} + g_W \vec{J}_{LR}^\nu \cdot \vec{W}_{\mu\nu}
$$

$$
= \frac{g_W}{\sqrt{2}} \left( \vec{J}_{LR}^\mu \cdot \vec{W}_{\mu\nu} + \vec{J}_{LR}^\nu \cdot \vec{W}_{\mu\nu} \right)
$$

$$
+ \frac{g_W}{\sqrt{2}} \left( \vec{J}_{LR}^\mu - \vec{J}_{LR}^\nu \right) \cdot \frac{\vec{W}_{\mu\nu} - \vec{W}_{\nu\mu}}{\sqrt{2}},
$$

(114)

where

$$
\vec{J}_{LR}^\mu = \vec{f} \gamma^\mu \frac{(1 - \gamma_5)}{2} \vec{J}_{LR} f,
$$

(115)

and

$$
\vec{J}_{LR}^\nu = \vec{f} \gamma^\nu \frac{(1 + \gamma_5)}{2} \vec{J}_{LR} f^M.
$$

(116)

In (115) and (116), $f$ and $f^M$ refer to SM and mirror fermions respectively. From (114), one can identify

$$
\vec{W}_{\nu\mu} = \frac{\vec{W}_{\mu\nu} + \vec{W}_{\nu\mu}}{\sqrt{2}},
$$

(117)
As the electroweak gauge bosons while the orthogonal combination
\[
\tilde{W}_\mu' = \frac{\tilde{W}_{L\mu} - \tilde{W}_{R\mu}}{\sqrt{2}},
\]  
(118)
represents the heavy \(W'\) with mass \(\sim M_{LR}\). Notice that the electroweak gauge coupling denoted by the usual \(g_2\) is related to \(g_{W'}\) by
\[
g_2 = \frac{g_{W'}}{\sqrt{2}},
\]  
(119)
which is the same as Eq. \(32\). Below \(M_{LR}\), one can write the interaction Lagrangian involving \(W\) and \(W'\) as
\[
\mathcal{L}_W = g_2 (\bar{J}_L^+ \cdot \tilde{W}_{\nu\mu} + g_2 (\bar{J}_L^+ - J_R^+ \cdot \tilde{W}_{\mu}'.
\]  
(120)
The SM and mirror fermions interact with \(W'\) with the same strength but with a different sign. One can perhaps exploit this sign difference to isolate the contribution from \(W'\). From \(120\), one can see that, at low energies, the "fermi" constant involving \(W'\) is related to the usual Fermi constant by
\[
\frac{G'}{\sqrt{2}} = \frac{G_F}{\sqrt{2}} \left( \frac{M_W^2}{M_{W'}^2} \right)
\]  
(121)
Looking back at Section (V), one can deduce that \(G'/\sqrt{2} \sim 10^{-2} G_F\).

The present bounds on \(W'\) with standard couplings to the SM fermions are \(m > 800\,\text{GeV} \) (95% CL) for \(W \to e\nu, \mu\nu\) and \(m > 825\,\text{GeV} \) (95% CL) for \(Z'\) \(118\), although the latter’s bound is more model dependent. The phenomenology of these gauge bosons will be presented elsewhere.

**Other consequences:**

First, we notice that the previous consideration of the group \(SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \otimes SU(2)'_L \otimes SU(2)'_R\) where the PUTF scale was of the order of \(\text{ TeVs leading to severe a violation of the upper bound on } K_L \to \mu e \) by several orders of magnitude \(12\). This is no longer the case in the present scenario since the scale of the spontaneous breakdown of \(SU(4)_{PS} \to SU(3)_c \otimes U(1)_S\), \(M_s\) is of the order of a typical GUT scale \(11\). In fact flavor changing neutral current (FCNC) processes involving \(SU(4)_{PS}/SU(3)_c \otimes U(1)_S\) gauge bosons are totally suppressed in our present scenario.

Second, notice that the Higgs field which participates in the Majorana mass term for the (active) right-handed neutrino is \(\Phi_{10} = (10 = 1 + 3 \rightleftharpoons 6, 1, 3, 1, 1)\) \(12\). One expects the color non-singlet parts of \(\Phi_{10}\) (3 and 6) acquire a large mass of the order of \(M \sim 10^{15} - 10^{17}\) GeV. Although the \(SU(4)_{PS}/SU(3)_c \otimes U(1)_S\) gauge bosons do not induce proton decay, it can occur through the exchange of these color non-singlet scalars. This study will be presented elsewhere but one can briefly summarize the situation here by stating that modes such as \(p \to \pi^0 e^+\), \(p \to \pi^+ \bar{\nu}\) and \(p \to K^+ \bar{\nu}\) can in principle all occur and the decay rate can be under control.

**VII. FAMILIES FROM SPINORS**

- **Possibility of a fourth generation:** In Section (V), we have discussed the computation of \(\sin^2 \theta_W(M_Z)\) for three and four generations. One might be perhaps a little puzzled about the reason for even discussing the fourth generation case. And there is always the quintessential question of why the fourth active neutrino has to be much more massive than the other three (at least half the Z mass). Below we will present some phenomenological and theoretical reasons for why one might seriously consider the 4th family.

There is a quintessential question of why there exists three families of quarks and leptons. Whether or not there are more than three generations, in particular a fourth family, is another question that has been entertained over the years \(22\) but whose possibility was generally dismissed because of an apparent conflict with electroweak precision data, notwithstanding the fact that none has been observed so far. However, recent studies have revealed that not only was a fourth generation not ruled out by precision data but it might even have implications concerning the SM Higgs boson mass \(33\) and perhaps rare B decays \(34\). Its existence might even help bringing in coupling constant unification at the two-loop level \(32\). Furthermore, another recent analysis of experimental constraints on a fourth generation of quarks presented regions of allowed masses and mixing angles (between the fourth and the other three generations) which are more flexible than the widely quoted mass lower bounds \(30\). If the fourth family is not excluded experimentally and might even be detected in the future, one is again faced with the puzzle of family replication. Are there guiding principles which might help us to partially unravel this mystery?

Let us first notice that a spinor of \(SO(2n + 2m)\) decomposes into \(2^{m-1} \psi_+ + 2^{m-1} \psi_-\) of the subgroup \(SO(2n)_S\), where \(\psi_+, -\) are two distinct spinors of \(SO(2n)\). This fact has been exploited in a number of papers on family replication \(8, 37\). We will present a heuristic argument why, in our framework, it is desirable to have four generations.

One has \(SO(4) \approx SU(2) \otimes SU(2)\) at the Lie algebra level. (Group-theoretically, one actually has \(SO(4) \approx (SU(2) \otimes SU(2))/Z_2)\). Let \(\psi_+ = (2, 1)\) and \(\psi_+ = (1, 2)\) of \(SU(2) \otimes SU(2)\) and let this represent one family. In consequence, a spinor of
SO(2m + 4) decomposes into $2^{m-1}\psi_+ + 2^{m-1}\psi_-$ of SO(4) or $2^{m-1}$ families. If, in addition, one requires that SO(2m + 4), as a gauge theory, should be anomaly-free, one observes the following features: (1) $m = 1$ corresponds to one family and SO(4) which is not anomaly-free; (2) $m = 2$ corresponds to two families; (3) $m = 3$ corresponds to four families, etc...Case (1) is ruled out by the anomaly-freedom requirement; Case (2) is ruled out by observation. This leaves us with the simplest allowed case of $m = 3$ which corresponds to four families. This would correspond to the group SO(10). Notice that in this case one would have $\text{SO}(10) \rightarrow \text{SU}(4)_H \otimes \text{SU}(2)_L \otimes \text{SU}(2)_R$ where the subscript $H$ denotes “horizontal” or “family”. One might envision the following group:

$$\text{SU}(4)_P \otimes \text{SO}(10) \otimes \text{SO}(10)' \rightarrow$$

$$\text{SU}(4)_P \otimes \text{SU}(4)_H \otimes \text{SU}(2)_L \otimes \text{SU}(2)_R$$

$$\otimes \text{SU}(4)'_H \otimes \text{SU}(2)'_L \otimes \text{SU}(2)'_R.$$  \hspace{1cm} (122)

The unprimed and primed sectors have their own horizontal (family) gauge groups $\text{SU}(4)_H$ and $\text{SU}(4)'_H$ respectively.

It would be amusing if the above group comes from

$$\text{SO}(10)_P \otimes \text{SO}(10) \otimes \text{SO}(10)' \rightarrow$$

$$\text{SU}(4)_P \otimes \text{SO}(10) \otimes \text{SO}(10)'.$$  \hspace{1cm} (123)

The above presentation is in a nutshell the essence of the emergence of family replication from spinors.

- **What makes the fourth neutrino much heavier than the other three?**

The Z-boson width constrains the fourth neutrino to be heavier than $M_Z/2$. It is natural then to ask why this should be the case if a fourth generation exists. Although we do not have an answer to that question, we will give a sketch of one scenario where one could perhaps try as a first step toward finding that answer.

We will concentrate solely on the lepton sector in this section. The purpose is to obtain the ratio of the Dirac masses $m_D^{(3)}/m_D^{(4)} \sim 10^{-6}$ for the active sector and $m_D^{(3), N}/m_D^{(4), N} \sim 10^{-4}$ for the sterile sector according to the numerics discussed previously. Let us introduce under $\text{SU}(4)_P \otimes \text{SU}(4)_H \otimes \text{SU}(2)_L \otimes \text{SU}(2)_R \otimes \text{SU}(4)'_H \otimes \text{SU}(2)'_L \otimes \text{SU}(2)'_R$

$$\Phi_{HS} = (1, 1, 2, 1, 1, 1),$$  \hspace{1cm} (124)

$$\Phi_{HA} = (1, 15, 2, 2, 1, 1, 1).$$  \hspace{1cm} (125)

The Dirac mass term similar to $[9]$ can now be written, for the leptons, as

$$\mathcal{L}_H = \bar{\psi}_L (g_S \Phi_{HS} + g_S^\prime \Phi_{HA}) \psi + H.c.,$$  \hspace{1cm} (126)

where $\bar{l}_L$ and $l_R^M$ denotes a four-component (fundamental) representation of the family $\text{SU}(4)_H$. (A term with exactly the same couplings is present for the quarks.) With

$$\langle \Phi_{HS} \rangle = v_S,$$  \hspace{1cm} (127)

and

$$\langle \Phi_{HA} \rangle = \frac{v_A}{2\sqrt{6}} \begin{pmatrix} 1 & 1 \\ -3 & 1 \end{pmatrix},$$  \hspace{1cm} (128)

one readily obtain tree-level Dirac masses for the active neutrinos

$$m_D = g_S v_S + g_S^A \frac{v_A}{2\sqrt{6}},$$  \hspace{1cm} (129a)

$$m_D^{(4)} = g_S v_S - 3g_S^A \frac{v_A}{2\sqrt{6}},$$  \hspace{1cm} (129b)

for the common first three generations $[129a]$ and the fourth generation $[129b]$ respectively.

From $[129a]$ and $[129b]$, one first notices that $m_D \neq m_D^{(4)}$. Because of this difference, one might “fine-tune” the VEVs so that $g_S^A = -\frac{v_A}{2\sqrt{6}}$. If that can be done then one could have a situation in which $m_D \ll m_D^{(4)}$. A similar consideration can be considered for the sterile neutrino sector. These hints are under investigation.

It is beyond the scope of the paper to go deeper into this and related issues. This will be presented elsewhere. To summarize, we have presented an argument outlining the possibility of a heavy fourth neutrino, in addition to the three light ones. The main point of the argument is simply the fact that there is no reason to expect additional neutrinos to be as light as the SM neutrinos and that they violate the bound coming from the Z width.

**VIII. CONCLUSIONS**

A model with electroweak scale SM non-singlet right-handed neutrinos was presented in [4]. It contains a number of testable consequences such as lepton-number violating processes at colliders through the direct production of the right-handed neutrinos and their subsequent decays. In addition, there is a rich Higgs structure that can be probed at colliders such as the upcoming LHC and the proposed ILC such as the existence of doubly charged Higgs scalars contained in the model.

As shown in this paper, the attempt to unify quarks and leptons of the aforementioned model in the manner of Pati-Salam fails unless one introduces new neutral fermions which are SM singlets, the so-called sterile
neutrinos. However, unlike generic models of sterile neutrinos where they are usually thought of as SM-singlet right-handed particles, these new neutral fermions come in both helicities: $N_L$ and $N_R$. This Pati-Salam extension basically “completes” the fermionic assignments for the SM and mirror $SU(2)$ singlets: $(d_R, e_R)$, $(d^M_L, e^M_L)$, $(u_R, N_R)$ and $(u^M_L, N_L)$. The gauge extension of the SM in this case is the group $SU(4)_P \otimes SU(2)_L \otimes SU(2)_R \otimes SU(2)_L \otimes SU(2)_R$ with all the details given in Section III. This group is reminiscent of the Petite Unification model of [14] but differs from it in a major way, in terms of fermionic assignments and patterns of symmetry breaking. The computation of $\sin^2 \theta_W (M_Z)$ reveals the Pati-Salam scale to be of a typical GUT size. (In this sense, it has “grown up” and is no longer “Petite”.) It is shown in Section IX that it is reasonable to have keV sterile neutrinos which are only remotely constrained by the active sector. These keV sterile neutrinos could constitute a part (or all?) of warm dark matter and could be responsible for the so-called pulsar kicks.

The structure of the aforementioned gauge group and its fermionic representations is very suggestive of the way spinors of some orthogonal group decompose into spinors of its orthogonal subgroup. For instance, a spion of $SO(2m+4)$ decomposes into $2^{m-1} \psi_+ + 2^{m-1} \psi_-$ of $SO(4)$ or $2^{m-1}$ families of $SO(4)$. With $SO(4) \approx SU(2) \otimes SU(2)$, it is argued in Section VII why the simplest, anomaly-free case where $m = 3$ which corresponds to the group $SO(10)$ and to four families is an appealing scenario. The group that gives rise to this feature is argued to be $SU(4)_P \otimes SO(10) \otimes SO(10)'$ which breaks down to $SU(4)_P \otimes SU(4)_H \otimes SU(2)_L \otimes SU(2)_R \otimes SU(4)'_H \otimes SU(2)_L \otimes SU(2)_R$ with $SU(4)_H$ and $SU(4)'_H$ being family gauge groups. It is argued in Section VII how one might expect the fourth neutrino to be heavy.

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