Solutions of Stress Intensity Factors of Multiple Internal Axial Cracks in Hollow Cylinders

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Abstract. This paper solved numerically the stress intensity factors of multiple internal axial cracks in hollow cylinders under internal pressure. Semi-elliptical shaped cracks are modelled using ANSYS finite element program and the stress intensity factors are calculated using interaction domain integral which is based on the J-integral. There are two important parameters are considered such as crack aspect ratio, \( a/t \) is 0.2, 0.4 and 0.75 and crack spacing distance, \( d/c \) is 0.4, 0.7 and 1.1. For multiple internal axial cracks under internal pressure, stress intensity factors along the crack front decreased when crack aspect ratio, \( a/t \) is increased. It is also revealed that the role of crack spacing on the stress intensity factor is insignificant especially for the shallow cracks \( (a/t < 0.4) \). However, for the crack having \( a/t = 0.75 \), increasing the distance of \( d/c \) capable to increase the SIFs due to interaction effect.

1. Introduction

Mechanical reliability assessment considering fracture mechanics approach became an important tool in analyzing the role of flaws on the structural integrity especially on the fitness-for-service (FFS) procedures and life-extension programs of critically designed components \[1\]. Pipes are widely used to transport gas or liquid medium in the fastest way. Like other structures, pipe is also exposed to the external elements for examples harsh environment and other type of surface defects leading to form a premature cracks \[2\] and under certain circumstances, internal crack could be occurred frequently \[3\]. Most of the previous works focused on the single crack formed whether internally \[4\] or externally \[5\]. European flaw assessment procedure called SINTAP for an example \[6\] also recently covered the full scale pipe tests containing through wall and surface cracks under internal pressure and four pint bending moments. The goal is to predict the failure loads and the marginal safety.

Due to harsh environment, the formation of multiple cracks are unavoidable since surface corrosion capable to degrade such surface on the various locations. According to literature, there are tremendous amount of works dealing with the single crack \[7-10\]. Some of these works investigated the crack internally or externally especially on the surfaces of pipes or hollow cylinders. However, it is hard to find the behavior of multiple cracks occurred either on the external or internal surfaces of pipes under internal pressure.

Therefore, this paper presents a preliminary work to numerically solve the problems of multiple internal axial crack in pipes. There are three relative crack depth ratio, \( a/t \) is used such as 0.2, 0.4 and 0.75 with a normalized crack length, \( \rho = 0.5 \). It is also assumed that the pipe followed the rule of \( R/t = \)}
0.5. In order to study the effect of distance between two crack, the ratio of \( d/c = 0.4, 0.7 \) and 1.1 is used. ANSYS finite element program through the use of ANSYS Parametric Design Language (APDL) is used to facilitate the process and the interaction domain integral method is used to calculate the stress intensity factor. Then, the roles of stress intensity factors on the crack geometries are investigated and discussed.

2. Methodology

Axial internal crack is numerically modelled using ANSYS finite element software. Figure 1 shows the axial internal crack in a hollow cylinder subjected to internal pressure where \( t \) is a thickness, \( d \) is a distance between two cracks, \( a \) is a minor, \( c \) is a major radiuses of semi-elliptical shaped cracks, \( P \) is a point along the crack front and \( x \) is its distance measured from the central point of cracks. Since the crack geometries are symmetrical, only a quarter finite element model is constructed as in Figure 2.

Special attention is given to the crack tip where singular iso-parametric element is used and the other region of cylinder is modelled using regular element. Stress intensity factor is determined for ten points along the crack front based on the domain integral method. In this work, the ratio of the inner radius of the cylinder to the thickness, \( R/t = 0.5 \) is selected, the normalized crack length, \( \rho = 0.5 \) (as in equation 1) is considered, relative crack depth, \( a/t \) is 0.1, 0.4 and 0.75 and the distance between the cracks, \( d/c \) are varied such as 0.4, 0.7 and 1.1. The stress intensity factor, \( K \) is then normalized according to equation (2). In order to analyze the role between two cracks, an interaction factor, \( \psi \) is introduced and it is defined as a ratio between the SIFs obtained from two and single cracks. It is determined according to equation (3).

\[
\rho = \frac{c}{\sqrt{R_c t}} \\
F = \frac{K}{\sigma \sqrt{\pi a}} \\
\psi = \frac{F_{\text{multiple crack}}}{F_{\text{single crack}}} \\
\text{where, } \sigma = \frac{PR}{t}
\]

Before the model is further used, it is very important to validate with the previous model (Kim et al., 2004). Since multiple internal axial cracks are not available then single crack of similar geometry is used as in Figure 3. For validation purpose, only the stress intensity factors at the deepest point along the crack are considered (\( x/h = 0.0 \)). It is found that the present model is well similar with the existing model.

![Figure 1. Internal axial cracks in hollow cylinder](image)
3. Results and Discussion

In this work, there are two main crack fronts as in Figure 1. Then, it is divided into four sections A, B, C and D. According to the numerical analysis, crack fronts B and C interacted to each other while A and D remain intact. Therefore, for discussion crack fronts B and C are emphasized. On the other hand, since the crack is symmetrical either crack front B and C is presented. Figure 4 shows the effect of crack aspect ratios, $a/t$ on the normalized mode I stress intensity factor (SIF) when different values of crack spacing, $d/c$ is used. Figure 4 also reveals that the SIFs have an identical pattern where lower values of $a/t$ produced higher SIFs and higher $a/t$ produced lower SIFs. For the crack with $a/t = 0.2$, the SIFs are almost flattened along the crack front. This is due to since for $a/t = 0.2$, the crack front is almost flattened. This behavior of SIFs are almost similar with the results calculated by Kim et al. [10]. In the outer region of crack front, the SIFs showing some irregularities due to singularity problem since the SIFs are sharply increased.

Figure 5 shows the effect of crack distance, $d/c$ on the SIFs for a certain value of $a/t$. It is indicated that the role of $d/c$ only significant for the case of higher value of $a/t$ for an example $a/t = 0.75$. Figure 5(a) shows the effect of $d/c$ on the SIFs for $a/t = 0.2$. It is revealed that the SIFs along the crack fronts are almost identical meaning that varying $d/c$ insignificantly affected the SIFs. This is due to for $a/t = 0.2$, the integrity of the tube is strong enough to sustain under the action of internal pressure and the effect of such defects are minimized. However, when deeper cracks are used ($a/t > 0.4$), the effect of $d/c$ became obvious where the factor of $d/c$ capable to decrease the SIFs as in Figures 5(b) and 5(c). This phenomenon is called the crack shielding since the formation of multiple cracks are not contributed to the increment of SIFs.
Figure 4 Stress intensity factors along the crack fronts when crack aspect ratio, $a/t$ is varied for different crack spacing distance, $d/c$ (a) 0.4, (b) 0.7 and (c) 1.1.
Figure 5 Stress intensity factors along the crack fronts when crack spacing, $d$ is varied for different crack aspect ratio, $a/t$ (a) 0.2, (b) 0.4 and (c) 0.75.
Figure 6 Effect of crack spacing, d/c on the interaction factor, \( \psi \) for different crack aspect ratio, a/t (a) 0.2, (b) 0.4 and (c) 0.75.
Figure 6 reveals a strong evidence the role of $d/c$ on the SIFs obtained from multiple cracks. Figure 6(a) indicates the factor of crack interactions for $a/t = 0.2$ where different $d/c$ is used. The factor of $d/c$ is significant when higher $a/t$ is implement as in Figures 6(b) and 6(c). According to Figure 6(c), the effect of $d/c$ is tremendous especially on the deeper cracks. The existence of multiple internal cracks positioned axially are not contributed to increase the SIFs but it is capable to decrease which is indicated the shielding behavior.

4. Conclusion

In this work, ANSYS finite element program is used to model and analyze the roles of crack geometries ($a/t = 0.2, 0.4$ and $0.75$) and crack inter-distance ($d/c = 0.4, 0.7$ and $1.1$) on the SIFs along the crack front. According to the numerical analysis, it can be concluded that the parameter of $d/c$ played an important role in lowering the SIFs especially when deeper crack is used ($a/t = 0.75$). If shallow crack is used ($a/t = 0.2$), the effect of $d/c$ is insignificant where the SIFs along the crack fronts are almost identical. The decrement of SIFs for the case of cracks with $a/t = 0.75$ due to the phenomenon called a shielding effect.

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