Determination of dynamic parameters of the elastically supported beam system model by optical and numerical methods

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Abstract. The article describes the design of a test facility that designed and manufactured by the authors for parametric studies of the features of free and forced vibrations of elastically supported dynamical systems. The results of a full-scale experiment are presented in the article, during which a frequency analysis of a beam with an intermediate elastic support is performed. Measurements are made with the use of the optical method of determining the movements and inductive sensors. A finite element model of the test facility was built in the Midas NFX environment with the aim to verify and evaluate the results. The results of the modal analysis performed for the finite element model of the test facility are compared with the natural frequency spectra obtained as a result of experiments. Conclusions are drawn about the prospects of using the test facility, the accuracy of the measuring equipment used, and about the way the finite element model built in the Midas NFX program environment corresponds to the full-scale model of the dynamic system in terms of its dynamic characteristics.

1. Introduction

Currently, when designing buildings and structures, a significant number of calculations related to the determination of the stress-strain state of structures are performed using software systems based on the finite element method (FEM). The accuracy of such calculations depends to varying degrees on the numerical approximation of the model, accepted assumptions and hypotheses. Often, errors caused by FEM inaccuracy can be estimated and minimized by selecting the optimal type of finite elements (FE) used in solving the problem, the size of the grid FE, the settings of the computational algorithm, etc. It is more difficult to estimate errors caused by a number of simplifications, such as idealization of construction materials, geometry, loads, etc.

In [1], notes a number of problems that arise during the transition from the physical model to its FE approximation. From the point of view of design reliability, the simplified representation of the design scheme is compensated by using a group of coefficients (reliability coefficient, operating conditions coefficient, etc.). However, for example, in nonlinear dynamical systems and in systems with feedback, it is quite difficult to provide a certain confidence interval for the results. There is a need to consider several calculation situations, which ultimately leads to less cost-effective solutions.

According to the recommendations given in [2] and [3] in dynamic calculations implemented in modern software systems, it is necessary to use mainly refined calculation schemes. In such schemes should be most considered features of the design (rigidity, pliability of the supports and the base, the spatial work of the construction, etc.). While in [2], the authors point out the possible error in the
determination of the natural frequencies of buildings and load-bearing elements. For approximate schemes, it varies in the range from 15 to 35 %, depending on the type of construction. In refined schemes, the error value can be 10-15%. Given the multidimensional nature of the practical problems being solved today, and as a result, the variety of forms of vibrations of structures, the results of calculations at this level of confidence in the dynamic parameters of the system become difficult to analyse.

One of the means of developing a verified FE model is an experiment. Experimental data can be obtained from a full-scale or scale model using various types of sensors. If necessary, the design as a whole or its separate element is considered. To determine the characteristics of linear design schemes, it is sufficient to consider the free vibrations of the structure.

In [4], a full-scale model of a girder bridge and a vehicle was created on a geometric scale of 1 to 10 in order to work out and test the proposed numerical models. The theoretical calculation scheme is performed in the Ansys program and consists of solid and shell finite elements. The monolithic reinforced concrete slab and diaphragms are modelled by the shell63 element, while the beams are modelled by the solsh190 element. The rigidity of the asphalt pavement of the bridge and the restrictive curb is ignored in the calculation. As a result, the difference between the measured and theoretical first frequency of natural vibrations of the superstructure was 4%.

The authors in [5] developed a finite element model of a three-axle truck EQ3166 in the LS-DYNA program for dynamic calculations of a road bridge with a mobile load. The car model consists of a total of 49 parts, 9 types of materials, 106 multi-node connections, 7008 nodes, 369 solid elements, 7959 shell elements and 237 beam elements. The nonlinear character of wheel deformation is modelled using the AIRBAG_SIMPLE_AIRBAG_MODEL option in LS-DYNA. The measured base frequency of the full-scale truck was 1.680 Hz, while the first frequency of the developed finite element model was 1.634 Hz. Thus, the error of the numerical model does not exceed 3 %.

The necessary experimental data about the investigated object can be obtained using mechanical, electromechanical, optical, electronic devices and their combinations. However, the use of mechanical devices is limited, especially in dynamic systems, where the recorded values from the sensors are functions of time. V. V. Volkov and S. Yu. Gridnev in [6] suggest using an optical method to analyse the movement of road bridge nodes from a mobile load. The authors use an algorithm that allows to determine changes in the coordinate of the investigated point by comparing several images of the superstructure during loading.

The issues of modelling dynamic mobile load on a span structure in modern finite element complexes are discussed in [7]. The authors investigate the possibility of using contact interaction mechanisms in the Midas NFX program to describe joint vibrations of a beam and a moving load.

The article [8] presents a FE model of the rubber-metal support part (RMSP), developed in order to clarify the dynamic behaviour of road bridges. The numerical modelling of the composite structure of the support material, the formation of interaction features of rubber and steel plates is of interest in this problem. In addition to the numerical implementation, in [9] describes an experimental analysis of the vibrations of the span structure of a steel-reinforced concrete bridge with the use of RMSP. Registration and processing of values of dynamic displacements of supports and deflections is performed by an optical device supplemented with special programs. The authors note the importance of taking into account the shear and vertical deformations of the RMSP in practical calculations.

Thanks to the spread of electronic gadgets with the ability to record photos and videos in high resolution, a wide range of researchers are becoming available to new opportunities in the experimental analysis of building structures. So in the article [10] to determine the movement of span structures or parts of them, it is proposed to use a smartphone with a pre-installed program D-Viewer. The program algorithm is based on tracking the movement of a light spot on the projection screen from a laser fixed to the investigated object. In order to test the proposed method, the authors measure the deflections of the beams of a reduced copy of the suspension bridge using iPhone 6 and iPhone 6 Plus smartphones and classic arrow indicators. The peak value of the oscillation amplitude measured by the two types of sensors differs by 1.71%.
The article describes the developed facility for studying the features of dynamic behaviour of elastically supported systems. Using the readings of inductive sensors and the results of optical measurements, the natural frequencies of vibrations of a beam with an intermediate elastic support are determined. To verify the results, the values of the frequencies and forms of natural vibrations of the investigated dynamic system are determined in the Midas NFX program environment. The results presented in the article complete the first stage of research in the field of experimental and theoretical analysis of the behaviour of elastically supported dynamical systems under the action of harmonic stationary and mobile loads.

2. Description of the design scheme of the investigated dynamic system
In mechanical engineering there is a whole class of dynamic machines and aggregates that have not rigid, but pliable junctions with the base or structural elements. In this case, the boundary conditions are represented by a system of elastic connections and rigid limiters, which are included in the operation when the set gap is selected. In construction practice, elastic connections are widely used in modelling pontoons of floating bridges.

Figure 1 shows the design scheme of the investigated dynamic system. The beam is pivotally supported at the ends and has one intermediate elastic connection in the middle.

![Design scheme of the investigated dynamic system](image)

Figure 1. Design scheme of the investigated dynamic system.

Horizontal longitudinal and transverse horizontal deformations of the beam are not taken into account due to their smallness. A linear deformation diagram was used for the material. The initial equilibrium state of the system was selected in such a way that after applying the gravitational load, the upper part of the supports was at the same level.

3. Development and production of a facility for obtaining experimental data
The experimental stand was made by the authors of the article in the framework of research on the analysis of the deformed state of elastically supported systems under the action of a moving load. Experimental data obtained on the test facility are more reliable than data taken from measuring equipment on a real structure, since in an enclosed space, undesirable effects of external factors are minimized, and the same loading can be implemented repeatedly. The General view of the manufactured installation and its main components are shown in figure 2.
Figure 2. General view of the facility (1-frame, 2-beam, 3-elastic support, 4-guides, 5-bogie with micrometer, 6-inductive sensors, 7-eccentric with adjustable speed).

The frame 1 is made of rolled paired channels that provide stability and rigidity of the structure. The frame is used for fixing the supports of the studied beam, measuring sensors and guides. The main parts and components are attached to the frame through split connections (bolts and screw clamps), which allows to change the parameters of the dynamic system if necessary.

Beam 2 at the ends has one fixed and one movable attachment with mating sections for the entrance and exit of the mobile vehicle model. The stiffness and weight of the beam may vary during experiments. To obtain the desired frequency range of the dynamic system, lead weights with a total mass of 168.6 g were suspended from the steel band.

At this stage, the beam has one intermediate elastic support 3 in the middle of the span, the rigidity of which was determined as a result of static tests. However, in General, the number and location of elastic supports can be changed if this is required during the experiment. The facility can also be supplemented with restrictive supports at the ends, which are included in the work when selecting the set gap, which makes it possible to study the structural nonlinear vibrations of the beam.

For checking the vertical marks of the beam before testing, as well as for determining static deflections, guides 4 and a bogie with micrometer 5 are used. The bogie can move freely along the beam, while in other directions its position is fixed by a system of roller bearings. The accuracy of the micrometer is 0.1 mm. The trolley can also be used to simulate the impact of a moving load.

The displacements of the studied dynamic system are recorded by means of optical control and subsequently processed using special software. The capabilities of optical measurement tools are limited by the frame rate when it comes to short-term processes or dynamic effects resulting from the influence of higher harmonics. To obtain a detailed spectrum of the system's natural frequencies, the test facility is equipped with inductive sensors 6. A real-time signal is recorded from inductive sensors, whose amplitude is proportional to the speed of the beam movement.

The article presents the results of the analysis of only free vibrations of the beam, but the versatility of the manufactured facility allows us to study forced vibrations when the load is moving. The authors performed a number of experiments aimed at studying the dynamic behaviour of elastically supported systems under the influence of harmonic stationary and moving loads. For this purpose, an eccentric motor 7 can optionally be installed on the beam, which exerts a harmonic force effect of a given amplitude. The load frequency can be changed using a single-channel pulse width modulator (PWM) power. The controller display shows the percentage of the output equivalent voltage to the rated voltage.
4. Optical method of measuring the movements of a dynamic system

In the optical method for determining spatial coordinates, which is used for analysing the deformed state of the installation, two stages can be distinguished.

At the first stage, a digital video camera is used to record beam vibrations at a certain time interval. In this case, the optical images projected on the light-sensitive matrix are converted to digital form and then recorded on an external storage medium.

At the second stage, the coordinates of the studied cross sections of the beam are calculated from the resulting video recording. To do this, we use the analysis of equidistant points in space and the correlation of their brightness images (figure 3). To determine the absolute movement of points, enter a scale factor that shows the number of pixels contained in a unit of length. It can be determined using a ruler or other template equidistant from the camera along with the system's calculated points (figure 4).

As you can see in the picture, the accuracy of the measurement results directly depends on the quality of the shooting. The higher the resolution of the recording device, the lower the error caused by the discretization of the image. So, for current research, the following value of the scale coefficient is accepted: \( \lambda = 8.8 \). Then the error in determination of displacements is 0.06 mm.

Measurement errors may also occur as a result of vibrations in the recording device itself. In order to avoid this effect, the camera tripod was installed at a sufficient distance from the sources of vibration on an isolated monolithic reinforced concrete structure.

A camera with a frequency of 30 frames per second was used in order to determine the dynamic movements of the beam. If we take into account Kotelnikov's theorem in the field of digital signal processing, we can say that the camera's capabilities allow us to analyse vibrations occurring with a frequency not exceeding 15 Hz. Often, the first natural frequencies of buildings and structures are below this value. However, in some cases, for example, if the building uses centrifuges, vibrating sieves or other dynamic equipment, the frequency of forced vibrations of structures may be higher and this will require the use of high-speed cameras for movement analysis. In this paper, inductive sensors are used to obtain the frequency spectrum of higher harmonics of an elastically supported beam.
5. Determination of free oscillation frequencies of a dynamic system using inductive sensors

The principle of operation of inductive sensors is based on changing the parameters of the magnetic field created by a permanent magnet when a metal object is introduced into the core. The inductor coil tends to keep its magnetic flux unchanged, so when the magnetized beam oscillates, an electric current occurs in the coil, resulting in a voltage at its ends. The value of the output signal is proportional to the product of the beam oscillation amplitude and frequency.

During the experiments, two inductive sensors were placed on the facility, as shown in figure 2. The sensor leads are connected to a 6.35 mm TRS jack for transmitting analog audio signal. Recording from the sensors was performed in WAV format, with two channels, with a sampling rate of 44.1 kHz.

In order to obtain the natural frequency spectrum, in the program Audacity performed a fast Fourier transform for the signal obtained when recording free vibrations of the beam. The best frequency resolution can be obtained when selecting Hanning window functions for fast conversion.

It should be noted that it is difficult to use inductive sensors to determine the speed and amplitude of the beam vibration, as the frequency of vibrations increases and the value of the voltage increases. In other words, the ratio of harmonics in the output signal will differ from the real one. However, inductive sensors can be successfully used to determine the frequencies of both free and forced vibrations of dynamic systems.

6. Description of the FE model of the experimental facility and results of modal analysis

The finite element model of the studied beam in the Midas NFX program environment is shown in figure 5. The beam in the calculation was modeled by 4-node rectangular finite elements of the shell, which have 5 degrees of freedom in the node. The shape and size of the finite element grid depended on the position of the supports and stability of the calculation. The concentrated masses from the additional weights and the elastic support were set using scalar elements that express the strain energy or kinetic energy when the selected node moves relative to the fixed support.

In order to create an accurate model of the dynamic system, the actual mass of the structure, the mass of additional weights, the geometric dimensions of the beam and other necessary characteristics of the system were determined using measuring instruments.

![Figure 5. Finite element model of an elastically supported beam.](image)

The stiffness of the elastic support was obtained as a result of a series of static tests, during which the deformations of the support were determined using the optical method.

The cross section of the beam is 37.5 × 1.8 mm. The beam material is steel with an elastic modulus $E = 2.14 \times 10^5$ MPa. Since the maximum stresses in the beam obtained during the experiments are significantly less than the proportionality limit, a linear diagram of steel deformation was used.
Figure 6. Forms of vibrations of a beam with an elastic support in the middle of the span.

The Midas NFX program contains functionality that allows performing modal and harmonic analysis of a FE model. The problem of determining the natural frequencies and forms of vibrations of the system is the first step to performing other, more complex types of dynamic calculations.

It should be noted that when determining the vibration frequencies of the beam in the Midas NFX FE complex, the influence of damping forces arising under the conditions of a full-scale experiment was not taken into account. This approach is due to the fact that the value of the damping coefficient $\delta$, established during the analysis of the oscillatory process, is significantly less than the first circular frequency of free vibrations of the dynamic system in the absence of friction $\omega_0$.

For the constructed FE model of an elastically supported beam, a modal analysis is performed and the first 8 natural frequencies and corresponding oscillation forms are obtained, which are shown in figure 6.

The figure shows that the elastic connection in the middle of the beam span affects the appearance of certain forms of free vibrations. Traditionally, the first form with a single half-wave is modified in
this case (figure 6 b), and its frequency grows with increasing elastic connection stiffness. As a result of modal analysis, it is established that the 6th form of beam oscillation occurs in a horizontal plane, and the 8th is a torsional form.

In General, the difference between the natural frequencies of the presented forms from the first to the eighth is significant and allows to analyze the capabilities of control and measuring equipment in a wide range when working with a full-scale model.

7. Analysis of the obtained experimental data
The purpose of the experiment was to determine the natural frequencies of vibrations of an elastically supported beam. Measurements were made by optical means of movement control, as well as inductive sensors.

At the first stage of the experiment, 10 seconds of background were recorded (the beam was at rest) in order to determine the measurement error caused by the vibration of the installation or the influence of external electromagnetic fields on the electrical circuit. Then, after deflecting the beam from the static equilibrium position, free damping vibrations occurred, which were recorded by inductive sensors and a video camera.

The positions of the cross-sections of the beam, which movements were analysed during the experiment, were selected taking into account the obtained forms of vibrations as a result of modal analysis. One of the inductive sensors was installed in the middle of the first span (section A), the second – in the first third of the second span (section B). Figure 7 shows the vertical movement of the cross section of the A beam with free vibrations.

As mentioned above, the capabilities of the video camera used theoretically allow you to record vibrations occurring at a frequency of no more than 15 Hz. For rice.7 you can see that the main frequency of the system is 11 Hz, while the frequency of the higher forms of the graph cannot be determined. It can also be noted that the amplitude of the beam vibrations in the section under consideration does not exceed 2 mm, so the displacements and deformations remain small, which allows us to use linear physical and geometric dependencies in the FEM equations.

Figure 8 shows the results of a fast Fourier transform performed using the Audacity program for the recorded signal with free vibrations of the beam.
Figure 8. Graph of the spectrum of free vibrations of the beam.

The spectrum graph shows several distinct peaks that correspond to the prevailing harmonics in the processed signal. The results obtained are in good agreement with the numerical calculation data. The natural oscillation frequencies of the investigated beam differ from the theoretical ones by no more than 2-3%. The comparison results are shown in table 1.

Table 1. The frequency of free oscillations.

| Form № | 1       | 2       | 3       | 4       | 5       | 6       | 7       | 8       |
|--------|---------|---------|---------|---------|---------|---------|---------|---------|
| FEM    | 10.8 Hz | 13.9 Hz | 33.2 Hz | 43.6 Hz | 72.2 Hz | 89.3 Hz | 98.2 Hz | 103.6 Hz|
| Experiment | 11 Hz   | 13 Hz   | 34 Hz   | 44 Hz   | 70 Hz   | -       | 99 Hz   | 102 Hz  |

Graph figure 8 lacks the 6th oscillation frequency, the theoretical value of which is 89.3 Hz. This suggests that the design and position of inductive sensors in this experiment does not allow us to analyse fluctuations occurring in the horizontal plane effectively enough. For instance, you can see pronounced peaks corresponding to the frequencies of 23 Hz and 28 Hz, which are not present in the results of modal analysis. The appearance of these harmonics on the graph can be explained by non-linearity in the electrical circuit, since they are multiples of the first two oscillation frequencies of the dynamical system.

8. Conclusions

The test facility was developed and manufactured in order to study the features of the dynamic behaviour of elastically supported beams. At the current stage, the research was limited to the study of free vibrations of the system. During the experiments, the natural forms and frequencies of vibrations were determined, and the results obtained were compared with the results of modal analysis performed in the Midas NFX program environment. On the basis of the conducted research we can draw the following conclusions and recommendations:
- The test facility made by the authors of the article, due to its universal design, can be used to solve a number of scientific and practical problems. By varying the number of elastic supports, beam stiffness, boundary conditions, and other system parameters, a large amount of empirical material can be collected to analyse the behaviour of elastically supported structures.

- Using the optical method of measuring displacements and electromagnetic sensors, it was found that the dynamic parameters of the FE model in the Midas NFX program correspond to its physical prototype. Based on these data, the next step is supposed to perform an analysis of an elastically supported dynamic system under the influence of harmonic stationary and mobile loads.

- The optical displacement measuring method can be used to analyse vibrations of a wide range of dynamic systems due to its accessibility and clarity of results. The accuracy of the data obtained depends largely on the resolution and frame rate of the video recording system. In addition, when conducting research, it is necessary to exclude the movement of the video camera itself during recording, or to control them with additional equipment.

- The use of inductive sensors and subsequent signal processing allowed us to obtain the natural frequency spectrum of the elastically supported beam, which is in good agreement with the results of modal analysis of the FE model. Inductive sensors do not allow to get information about the actual movement of the structure and forms of vibrations. Therefore, it is advisable to use this approach in conjunction with theoretical calculations or when there are already some ideas about the dynamic behaviour of the structure.

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