Research Article

On the Study of a Single-Period Principal-Agent Model with Taxation

Huan Wang and Wenyi Huang

School of Economic Mathematics, Southwestern University of Finance and Economics, 611130 Wenjiang, Chengdu, China

Correspondence should be addressed to Huan Wang; harrywang1213@smail.swufe.edu.cn

Received 27 November 2019; Accepted 24 February 2020; Published 26 March 2020

Academic Editor: Petr Hájek

Copyright © 2020 Huan Wang and Wenyi Huang. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper investigates a single-period principal-agent model with moral hazard. In the model, we implement bonus tax for the agent and analyze the effect of loss aversion by comparing with the results by Dietl et al. (2013). The existence and uniqueness of the optimal contracting problem are proved. Through an example, concrete illustrations of how loss aversion affects the compensation package are given. It is shown that although the agent’s efforts reduce, the fixed salary and marginal bonus paid by principal are increasing with the tax rate if the agent’s risk aversion and shocks in the economy are small. When the effect of loss aversion is sufficiently large, the curve of fixed salary is nonmonotonic, and the complementarity between fixed salary and marginal bonus disappears.

1. Introduction

We consider a single-period static contracting problem with moral hazard. To explore agent’s psychological effects, we introduce loss aversion into analysis of agent’s behaviors. The agent is supposed to be aware of his pretax and after-tax salaries. This income reduction incurred by taxation is variable if the agent is loss averse.

In our model, the principal needs to choose the compensation package, which contains the fixed salary and bonus rate. The amount of the bonus is the product of the bonus rate and output. To generate output, the agent should exert his efforts. Also, there is a shock in the economy, indicated by a random variable, fluctuating the output. In our setting, the principal is risk neutral and the agent has a mean-variance preference. The contract with initial time 0 only contains one period. To hire the agent, the principal must promise him an essential requirement, which is his reservation utility. As the contract terminates, the agent gets a lump-sum compensation. The salary package is assumed to be linear in our model.

Our single-period model develops from Dietl et al. [1]. They consider a linear compensation package to study how taxation affects the actions of the principal and agent. In the package, the taxation does not reduce the fixed salary and only shrinks the bonus with an unchanged proportion. In [1], Dietl et al. assume a risk neutral principal and a constant absolute risk averse (CARA) agent. When the agent is highly risk averse, the fixed salary and marginal bonus paid by principal show a trade-off relationship. Specifically, an increase in tax rate can boost the marginal bonus and cause a reduction of fixed salaries.

The differences between our work and Dietl et al. [1] are as follows. In the latter, the agent’s after-tax salary is used to compute his utility directly. In our setting, the distinction between the pretax and after-tax salaries distorts the agent’s feelings and in turn changes his utility. We consider the effect of wage reduction incurred by taxation. According to loss aversion in prospect theory, the reduction in compensation brings the agent an extra loss of utility. Therefore, we introduce disutility by taxation into the agent’s utility function. As an extended one, our model contains the benchmark model in Dietl et al. [1]. Considering effect of loss aversion in our model, the equilibrium becomes complex and the relationship between the optimal contract and parameters is more difficult to be analyzed. To make the
problem tractable, we adopt another expression method, which differs from that in [1]. In Dietl et al. [1], the parameter of tax rate emerges in the agent’s income function. Considering that the principal’s objective is linear (just the expectation of her profits), we put the taxation factor into the principal’s profit function. Through such layout, it becomes more convenient for us to formulate the equilibrium.

We prove the existence and uniqueness of the optimal contracting problem for a general form of cost function. Agent’s optimal efforts decrease with the taxation coefficient if the cost function has a quadratic form. Through a computational case, we give a concrete demonstration of how the optimal contract package is changed by tax rate. It is shown that there exists a complementarity between the fixed wage and marginal bonus when the effect of loss aversion is not huge. The fixed salary increases (or decreases) about the tax rate if the “risk parameter” is small (or big). The marginal bonus paid by principal changes, opposite with the fixed salary, increases with tax rate when the “risk parameter” is big. In addition, we illustrate the effect of loss aversion by comparing our results with those in Dietl et al. [1]. The fixed salary shows a “roller coaster” shape when both effect of loss aversion and risk parameter are big. The complementarity between the fixed wage and bonus rate disappears under this circumstance.

Taxation plays an important role in today’s compensation system. Many researchers focus on the influence of taxation when setting up their models. Smith and Watts [2] find that tax effect can explain the existence of some compensation plans. Ehrlrich and Radulescu [3] use the data in the UK to explore how a bonus tax changes the compensation structure. Radulescu [4] utilizes a principal-agent model to analyze the incidence of bonus tax. d’Andria [5] establishes a contracting multitasking model to study effects of various taxation schemes on innovations. Potential interactions between labor or payment taxation and feasibility of innovations are demonstrated. A complementary relationship between tax incentives and reduction in taxation rate is found. The complementarity shows a universality in various circumstances. Chang and Hu [6] consider a venture capitalist’s optimal contracting model of double-sided moral hazard. Their results show that fairness concerns change the optimal contract. Further simulated analysis is illustrated by adapting a constant elasticity of substitution (CES) function. A large amount of discrete or continuous-time models have been built up to investigate the optimal contracting problem (see Holmstrom and Milgrom [7], Sannikov [8], Cvitanić et al. [9], and so on).

A tendency of considering the effect of cognitive components on economic decisions of individuals emerges. A lot of researchers are devoted to studying people’s psychological or emotional factors, both empirically and theoretically. According to prospect theory, studied by Kahneman and Tversky [10], people’s behaviors are influenced by a psychology called loss aversion, a tendency to prefer avoiding loss to acquiring equivalent gains. As stated by Tversky and Kahneman [11], an identical amount of loss would cost people twice as much, psychologically as gains. To show that taxpayers are risk averse, Engström et al. [12] analyze data of 3.6 million Swedish taxpayers in the year 2006 by a regression kink and discontinuity approach. Rees-Jones [13] gives the evidence that taxpayers are loss averse when filing returns. To gather this phenomenon, Rees-Jones documents the 1979–1990 Internal Revenue Service (IRS) data. Using two complementary structural approaches, substantial potential policy impact of loss aversion is shown. Through all these evidences, we believe that the drop between agent’s pretax and after-tax salaries strongly distorts the agent’s feelings. Rather than verifying the evidence of whether taxpayers are loss averse, theoretically speaking, our goal in this paper is to analyze the incentive changes by such psychological effect.

The rest of the paper is organized as follows. Section 2 states a framework of our model. Section 3 establishes the derivation of the optimal contract. Section 4 gives a concrete example to analyze the effect of loss aversion caused by taxation. Conclusions are illustrated in Section 5.

2. Framework

2.1. Contract Relationship. The model in this paper, which follows the work of Dietl et al. [1], discusses a single-period principal-agent model with taxation. The economy is populated by a representative principal and a representative agent. To fix ideas, the principal is interpreted as the owner of a company and the agent as the manager. To run the company, the principal hires the agent by offering him a contract. The contract only contains one period. That is, the agent gets a lump-sum payment at the end of employment. To produce the output, the agent delivers his efforts. The output is described by

$$y = a + \varepsilon,$$

where $a$ is the level of agent’s efforts and $\varepsilon$ is a normally distributed random variable with mean 0 and variance $\sigma^2$. The random variable $\varepsilon$ indicates the aggregate shocks in the economy. Under the setting of moral hazard, agent’s efforts are unobservable for the principal. The output, however, is publicly known by both parties. The agent suffers a cost from delivering efforts, denoted by $c(a)$. The cost function $c(a)$ satisfies the next assumption.

Assumption 1. Let $c(a)$ be continuous on $\mathbb{R}^+$. Assume $c’(a) \geq 0$, $c''(a) \geq 0$, $c'''(a) \geq 0$, $c(0) = c’(0) = 0$, and $\lim_{a \to +\infty} c’(a) = +\infty$

From Holmstrom and Milgrom [7], we restrict our attention to the linear compensation scheme. (The reason why we consider such a scheme is from two aspects: (i) the linear compensation is widely adopted in literatures for its analytical convenience and (ii) the linear contract is found to be indeed optimal among all possible compensation schemes.) The agent gains $s(y) = \delta + \beta y$, where $\delta$ is the fixed salary and $\beta$ is the bonus rate. To introduce taxation, there exists bonus tax. More precisely, we assume that the taxation is also linear with bonus; i.e., $T = \mu \beta y$. Therefore, the total wages paid by principal are equal to $\delta + (1 + \mu) \beta y$

Usually, the wage obtained by agent is computed by $\delta + (1 - r) \beta y$ and the total salary the principal paid is $\delta + by$. In such setting, $r$ is the tax rate and $by$ is the bonus paid by principal. Technically, the two different schemes are of no
difference. The taxation factor $\mu$ in our notation is identical with $r/(1 - r)$ and the bonus rate $\beta$ equals $(1 - \tau)b$. The reason why we abandon the usual notation is for the computational convenience when solving the optimal problem. With slight abuse of appellation, we regard $\beta$ (or $(1 - \tau)b$) as the bonus rate and $(1 + \mu)\beta$ (or $b$) as the marginal bonus paid by principal hereafter.

The principal is assumed to be risk neutral and is concerned only about her expected profits. Assume a mean-variance expected utility for compensation for the risk averse manager. The agent’s compensation utility together with the cost of efforts is $E(s) - (r/2)\Var(s) - c(a)$. This mean-variance utility is equivalent to the exponential utility function through monotonic transformation. The factor $r$ is the severity of risk aversion in the constant absolute risk aversion (CARA) utility.

The problem of the principal is to choose the level of fixed salary $\delta$ and bonus rate $\beta$ to maximize her expected revenue, denoted by $\pi$. While designing the contract, the principal must follow two basic constraints: (i) the principal must ensure that the incentive-compatible condition is maintained and (ii) the contract must guarantee the agent an essential utility. Denoting this essential utility as $\hat{w}$, Dietl et al. [1] establish the principal’s problem as follows:

$$\max_{\delta, \beta} \quad \pi = E[y - s - T]$$

$$a \in \arg\max_{a \geq 0} E[U_A]$$

$$\begin{aligned} &\text{s.t.} \\
&\quad E[U_A] = E(s) - r\Var(s) - c(a) \geq \hat{w}. \end{aligned}$$

Since the contract only contains one period, there is no betray under the relationship of principal agent. The agent always works according to the incentive-compatible condition. The principal is capable of solving her optimal problems. In the next section, we give an introduction of loss aversion and consider such psychological effect when constructing our model.

2.2. Loss Aversion. As behavioral economics emerges, Kahneman and Tversky develop the prospect theory in [10]. One main argument of the prospect theory is loss aversion. Due to the psychological effect, an individual tends to regard losses as being more powerful as gains. A typical utility form to characterize loss aversion is given by

$$U(x) = \begin{cases} ax, & x \geq 0, \\ \lambda ax, & x < 0. \end{cases}$$

The parameter $\lambda$ is set to be larger than 1. From the function $U(x)$, the marginal value losses are larger than the marginal gain. As stated in [11], they find that an identical amount of loss would cost people twice as much, psychologically as gains. Such a psychological phenomenon suggests that the drop between the pretax and after-tax salary may strongly distort the agent’s feelings. In a principal-agent case, our confusion is that whether the agent only cares for his after-tax salary or he is obsessed with his loss caused by taxation. Some experiments on nonhuman animals show that different expectations given by the experimenters can significantly affect the emotions of the animals. If we follow the intuition inspired by loss aversion and consider taxation as a loss for the agent, this reduces his utility additively.

Following the idea stated above, to quantify the effect of loss aversion caused by taxation, we introduce disutility of taxation in the form of

$$\theta E(T) - \frac{r}{2} \theta^2 \Var(T).$$

The parameter $\theta$ is the loss-averse coefficient that signifies the level of loss aversion. Putting the disutility into the agent’s objective, his final utility function becomes

$$E(s) - \frac{r}{2} \Var(s) - \theta \left( E(T) - \frac{r}{2} \theta^2 \Var(T) \right) - c(a).$$

The final utility function contains three parts: (i) utility gained from compensation; (ii) disutility caused by taxation, and (iii) the effort cost. The corresponding optimal problem for principal is turned into

$$\max_{\delta, \beta} \quad \pi = (1 - (1 + \mu)\beta)\delta - \delta$$

$$a \in \arg\max_{a \geq 0} E[U_A]$$

$$\begin{aligned} &\text{s.t.} \\
&\quad E[U_A] = \delta + (1 - \theta\mu)\beta a - \frac{r\sigma^2}{2} \beta^2 (1 - \theta^2 \mu^2) - c(a) \geq \hat{w}. \end{aligned}$$

Problem (6) is different from problem (2) because we add the disutility term of taxation into the agent’s objective. Because of the factor $\beta$ in the disutility term, the change in agent’s objective influences both incentive-compatible constraints and participation constraints. Accordingly, it causes an alteration to the optimal compensation package. Considering the effect of loss aversion, the equilibrium in the employment relationship in our model is altered.

How the fixed salary $\delta$ and bonus rate $\beta$ are determined in the equilibrium? How does the optimal contract package $(\beta, \delta)$ change with the coefficient $\mu$? How does the equilibrium change if the agent is with loss aversion? In the next section, we derive the optimal contract based on the Lagrangian multiplier techniques.

3. Optimal Contracting

In this section, we derive the optimal contract for the principal by applying the Lagrangian multipliers. In Section 3.1, we deal with the incentive-compatible constraint. In the succeeding section, we give the necessity of the optimal contract and prove its sufficiency under proper conditions. In the last part of this section, we illustrate the economic meanings from the obtained result.
3.1. Incentive Compatibility. Before solving the principal’s problem, we give the agent’s incentive-compatible condition. Given the compensation scheme \((\delta, \beta)\), the agent chooses his best effort by maximizing his total utility. The problem for agent is

\[
\max_{a \geq 0} \left\{ \delta + (1 - \theta \mu) \beta a - \frac{r a^2}{2} \beta^2 \left( 2 - \beta^2 \theta^2 \mu^2 \right) - c(a) \right\}.
\]  

(7)

The corresponding first-order condition is

\[
(1 - \theta \mu) \beta = c'(a).
\]  

(8)

Since marginal costs of efforts and the bonus rate are both larger than zero, we need the following restriction to make (8) hold:

\[
\theta \leq \frac{1}{\mu}.
\]  

(9)

The first-order condition (8) indicates that the agent chooses his effort to achieve a balance. In the balance, his marginal cost of efforts equals the marginal revenue of efforts. The marginal revenue has two components: one is the direct income of exerting each unit of effort, which equals \(\beta\), and, for each unit of income, the agent pays \(\mu\) amount of taxation and this shrinks his utility by loss aversion factor \(\theta\). The final marginal revenue of efforts, which is \((1 - \theta \mu) \beta\), is the marginal income minus the disutility caused by loss aversion.

Since the cost function of effort is convex, the first-order condition (8) is also sufficient for the agent’s incentive-compatible problem.

3.2. Optimal Problem for the Principal. In this subsection, we proceed to solve the principal’s optimal contracting problem. Technically, the principal’s problem is a static optimal control problem with two constraints. The approach we use is the Lagrangian multipliers. The Lagrange \(\mathcal{L}\) with multiplier \(\lambda\) is defined as

\[
\mathcal{L}(a, \delta, \lambda) = a - \frac{1 + \mu}{1 - \theta \mu} \rho c'(a) - \delta
\]

\[
+ \lambda \left( \delta + \rho c'(a) - \frac{r a^2}{2} \frac{1 + \theta \mu}{1 - \theta \mu} (c'(a))^2 - c(a) - \omega \right).
\]  

(10)

We use the incentive-compatible condition to get rid of the argument \(\beta\) in (10). The Kuhn–Tucker conditions for \(a, \delta,\) and \(\lambda\) are

\[
\frac{\partial \mathcal{L}}{\partial a} = 1 - \frac{1 + \mu}{1 - \theta \mu} \rho c''(a) + \lambda a c''(a) = 0,
\]  

(11)

\[
\frac{\partial \mathcal{L}}{\partial \delta} = 1 + \lambda = 0,
\]  

(12)

\[
\frac{\partial \mathcal{L}}{\partial \lambda} = \delta + a c'(a) - \frac{r a^2}{2} \frac{1 + \theta \mu}{1 - \theta \mu} (c'(a))^2 - c(a) - \omega \geq 0.
\]  

(13)

Remark 1. Since the Lagrangian multiplier \(\lambda\) is positive in (12), the Kuhn–Tucker condition for \(\lambda\) must be binding. Namely, inequality (13) becomes equality. From (13), the fixed salary is set to maintain the agent’s participation constraint and does not take part in the decision of the optimal efforts.

Remark 2. The product \(r \sigma \rho\) has an economic meaning. It captures the agent’s attitudes towards risk. Therefore, we call \(r \sigma \rho\) as the “risk parameter” (The appellation “risk parameter” is noted in Dietl et al. [1].) and denote it by \(\rho\) hereafter.

The agent’s optimal effort \(a^*\) is determined by (11), which shows a clear economic significance of actions between two participants in the equilibrium. Rearranging parts of (11) and using \(c'(a) = (1 - \theta \mu) \beta\), we get

\[
1 - (1 + \mu) \beta = \left( \frac{1 + \theta \mu}{1 - \theta \mu} \rho c'(a) + \frac{1 + \theta \mu}{1 - \theta \mu} a \right) c''(a).
\]  

(14)

The left-hand side of above equation refers to the principal’s marginal benefit from hiring the agent. Since the principal is risk neutral, her marginal benefit is equal to the marginal output minus the marginal bonus for the agent. In the equilibrium, the principal must make her marginal benefit equal to the agent’s marginal utility, which is the right hand side of (14). If the principal’s marginal benefit is not identical with the agent’s marginal utility, a Pareto improvement exists. For instance, if we suppose that the sign “=” in (14) is a “>” sign, the principal can bring up the bonus rate to improve agent’s welfare. At the meantime, the agent can deliver more efforts to maintain the same profits for the principal. If agent’s marginal utility is larger than principal’s marginal benefit, the Pareto improvement is also implemented by applying the opposite strategy.

The following proposition presents the optimal condition for the principal’s problem and the uniqueness of the equilibrium is ensured.

Proposition 1. Suppose that the cost function \(c(a)\) satisfies Assumption 1. The unique equilibrium \((\delta^*, \beta^*, a^*)\) of problem (6) exists and is demonstrated by the following statements.

(i) The optimal linear compensation scheme is

\[
\beta^* = \frac{(1 - \theta \mu) - \mu (1 + \theta) a^* c''(a^*)}{(1 - \theta \mu) [(1 + \mu) + (1 + \theta) \rho c''(a^*)]},
\]  

(15)

\[
\delta^* = \omega - a^* c''(a^*) + c(a^*) + \frac{\rho}{2} \frac{1 + \theta \mu}{1 - \theta \mu} (c(a^*)^2).
\]  

(16)
(ii) The agent delivers his efforts according to the condition

\[ (1 - \theta \mu)\beta^* = c'(a^*). \]  

(17)

Proof. Since the function of \( c(a) \) is continuously differentiable, the Kuhn–Tucker condition for \( a \) contains all feasible optimal situations. Then, we check the second-order condition to ensure the maximum.

Let \( \varphi(a) = (\partial^2 L/\partial a^2) = 1 - ((1 + \mu)/(1 - \theta \mu))(c'(a) + ac''(a)) + (ac''(a) - \rho^2((1 + \mu)/(1 - \theta \mu))c'(a)c''(a)). \) With respect to \( a \), we have

\[
\varphi'(a) = -\frac{1}{1 - \theta \mu} \left\{ 1 + (2 + \theta)\mu + (1 + \theta \mu)\rho^2 c''(a) \right\} c''(a) + \left\{ (1 + \theta \mu)\alpha + (1 + \theta \mu)^2 c'(a) \right\} c''(a) \right\} (3). 
\]

(18)

Since \( c(3) (a) \geq 0 \), \( c''(a) \geq 0 \) and \( c'(a) \geq 0 \), we derive that \( \varphi(a) \leq 0 \). The result generated from the Kuhn–Tucker condition is the maximum point.

Next, we present the expression of the optimal bonus rate \( \beta^* \) and fixed salary \( \delta^* \). Rearranging the first-order condition of \( a \), we obtain

\[
\beta^* = \frac{(1 - \theta \mu) - \mu(1 + \theta)\alpha c''(a^*)}{(1 - \theta \mu)(1 + \mu) + (1 + \theta \mu)\rho c''(a^*)}. 
\]

(19)

Substituting the optimal efforts \( a^* \) into expression (13), we get

\[
\delta^* = \delta - a^* c'(a^*) + c(a^*) + \frac{\theta + \theta \mu}{2(1 - \theta \mu)} c'(a^*). \]

(20)

Finally, we check the uniqueness of the equilibrium. Since

\[
c'(a) = (1 - \theta \mu)\beta, \]

\[
\beta = \frac{(1 - \theta \mu) - \mu(1 + \theta)\alpha c''(a)}{(1 - \theta \mu)(1 + \mu) + (1 + \theta \mu)\rho c''(a)}, \]

we derive

\[
1 - \theta \mu = c'(a)[1 + \mu + (1 + \theta \mu)\rho c''(a)] + (1 + \theta \mu)\alpha c''(a). \]

(22)

Setting \( \psi(a) = c'(a)[1 + \mu + (1 + \theta \mu)\rho c''(a)] + (1 + \theta \mu)\alpha c''(a), \) we have

\[
\psi'(a) = c(3)(a)[(1 + \mu)\rho c'(a) + (1 + \theta \mu)\alpha] + c''(a)[1 + (2 + \theta)\mu + (1 + \theta \mu)\rho c''(a)] \geq 0. \]

(23)

\( \psi(a) \) is a monotone increasing function. For \( \psi(0) = 0 < 1 - \theta \mu \) and \( \lim_{a \to -\infty} \psi(a) = \infty \), the equation \( \psi(a) = 1 - \theta \mu \) has a unique solution \( a^* \). According to agent’s incentive-compatible condition (17), we derive that \( \beta^* \) is uniquely determined. From (13), \( \delta^* \) is unique.

\[ \square \]

3.3. Effects of Taxation. What we care about is how the taxation factor \( \mu \) affects the equilibrium. In this section, we want to find the relationship between the optimal equilibrium \( (\delta^*, a^*, \beta^*) \) and the taxation coefficient \( \mu \). The next proposition states how \( a^*, \delta^*, \) and \( \beta^* \) are changed by \( \mu \).

\[ \text{Proposition 2. Suppose that the cost function satisfies Assumption 1. In the equilibrium, the taxation factor } \mu \text{ affects the optimal effort } a^*, \text{ the bonus rate } \beta^*, \text{ and fixed salary } \delta^* \text{ according to} \]

\[
\frac{da^*}{d\mu} = \frac{c''(a^*)(a^* (1 + \theta) + (1 - \theta \mu)\beta^* \theta \rho) + (1 - \theta \mu)\beta^* + \theta}{\zeta}, \]

(24)

\[
\frac{d\delta^*}{d\mu} = \frac{\theta \beta^* c(3)(a^*) \left( \mu (1 + \theta) a^* - \beta^* \theta^2 \mu \rho \right) + \beta^* \rho}{(1 - \theta \mu) \zeta} \]

\[
+ \frac{c''(a^*) (c''(a^*) (2 \beta^* \theta^2 \mu \rho - (1 + \theta) a^*) + \beta^* ((\theta + 3) \theta \mu + \theta - 1 - \theta))}{(1 - \theta \mu) \zeta}, \]

(25)

\[
\frac{d\delta^*}{d\mu} = \rho \beta^* + ((1 + \theta \mu) \rho \beta^* - a^*) c''(a^*) \frac{da^*}{d\mu}, \]

(26)

where \( \zeta := c(3)(\mu(a^* \theta + a^* - \beta^* \theta^2 \mu \rho) + \beta^* \rho) + c''(a^*) (\theta \mu + 1) \rho + (\theta + 2) \mu + 1 \).

Proof. For notational convenience, we write \( c' = c'(a^*), \)

\( c'' = c''(a^*), \) and \( c(3) = c(3)(a^*). \) The optimal conditions are

\[
c'(a) = (1 - \theta \mu)\beta, \]

\[
\beta = \frac{(1 - \theta \mu) - \mu(1 + \theta)\alpha c''(a)}{(1 - \theta \mu)(1 + \mu) + (1 + \theta \mu)\rho c''(a)}. \]

(27)

Define

\[ \square \]
\[ g_1(a^*, \beta^*, \mu) = c'(1 - \theta\mu)\beta^*, \quad (28) \]

\[ g_2(a^*, \beta^*, \mu) = 1 - \theta\mu - (1 - \theta\mu)\beta^* \left[ 1 + \mu + (1 + \theta\mu)\rho c'' \right] - (1 + \theta\mu)a^* c''. \quad (29) \]

The partial derivatives are (We refer to \( g_{ia}, g_{ib}, \) and \( g_{ii} \) as the first-order partial derivatives of \( g_i(a^*, \beta^*, \mu), i = 1, 2. \)

\[ g_{ia} = c''(a^*), \quad g_{ib} = -(1 - \theta\mu), \quad g_{ii} = \theta\beta^*, \quad g_{ia} = -(1 + \theta)\mu c' \left[ (1 + \theta)\mu a^* + (1 - \theta^2\mu^2)\rho \beta^* \right], \]

\[ g_{ib} = -(1 - \theta\mu)(1 + \mu + (1 + \theta\mu)\rho c''), \quad g_{ii} = -((\theta + (1 + \theta)a^* c'' + \beta(1 - 2\theta\mu - 2\theta^2\mu\rho c''))) \]

\[ \rho \geq \rho_\beta := \frac{a^*(1 + \theta)c'' - \beta''((3 + \theta)\theta\mu + \theta - 1) + \theta}{2\beta^*\theta^2\mu c''}. \quad (35) \]

**Proof.** If \( \dot{c}^{(3)} = 0 \), expressions (24) and (25) become

\[ \frac{da^*}{d\mu} = -\frac{g_{ib}g_{ia} - g_{ia}g_{ib}}{g_{ia}g_{ib} - g_{ib}g_{ia}}, \quad \frac{d\beta^*}{d\mu} = \frac{g_{ib}g_{ia} - g_{ia}g_{ib}}{g_{ia}g_{ib} - g_{ib}g_{ia}} \]

\[ \frac{d\delta^*}{d\mu} = \rho \beta^* + ((1 + \theta\mu)\rho \beta^* - a^*)c'' \frac{da^*}{d\mu}. \quad (34) \]

Denoting \( \zeta := c^{(3)}(\mu(a^* \theta + a^* - \beta^*\theta^2\mu\rho) + \beta^* \rho) + c''(c''(\theta\mu + 1)\rho + (\theta + 2)\mu + 1) \), we get the results in the proposition. \( \square \)

**Corollary 1.** Assume that \( \dot{c}^{(3)}(a) \equiv 0. \) The optimal effort \( a^* \) decreases as the taxation factor \( \mu \) increases; i.e., \( (da^*/d\mu) < 0. \) The bonus rate \( \beta \) increases or decreases with \( \mu \) depending on the value of the parameter \( \rho \) like

\[ \frac{d\beta^*}{d\mu} = \frac{\beta^*((\theta + 3)\theta\mu + \theta - 1) - \theta}{(1 - \theta\mu)(c''(\theta\mu + 1)\rho + (\theta + 2)\mu + 1)}. \]
Noticing \( c'' \geq 0 \) and \( 1-\theta \mu \geq 0 \), all terms in \( da^*/d\mu \) are positive. Therefore, \( (da^*/d\mu) \leq 0 \). Calculating straightforward gives rise to

\[
\frac{d\beta^*}{d\mu} \geq 0 \implies \frac{a^* (1 + \theta) c'' - \beta^* ((3 + \theta) \theta \mu + \theta - 1) + \theta}{2 \beta^* \theta^2 \mu c''} \geq 0.
\]

(38)

According to Corollary 1, the agent always reduces his efforts when taxation factor \( \mu \) increases. However, the bonus rate \( \beta^* \) can either increase or decrease. Because the agent is loss averse, the effect of taxation factor \( \mu \) on the fixed salary \( \delta \) is still difficult to be vindicated. In the next section, we give a concrete analysis about the effect of taxation factor \( \mu \) and loss aversion coefficient \( \theta \) through an example.

4. Quadratic Effort Costs

In this section, we focus on a computational case in which the agent’s cost of efforts is quadratic. As a comparison, we use the results by Dietl et al. [1] as a benchmark. From now on, we refer to the model by Dietl et al. [1] as the benchmark model. The problem for the benchmark model is exactly problem (2) and the detailed analysis of the benchmark model is referred to as Propositions 1 and 2 in Dietl et al. [1]. Supposing that \( c'(a) = (1/2)a^2, \overline{w} = 0.1 \), and \( \mu \in [0, 1] \) (according to \( \tau = (\mu(1 + \mu)) \) as described in Section 2.1, the tax rate is fluctuated on the interval \([0, 0.5]\) when \( \mu = 0 \), it refers that there is no taxation; i.e., \( \tau = 0 \). If \( \mu = 1 \), it refers to the case where \( \tau = 0.5 \). The following proposition gives values of \((a^*, \delta^*, \beta^*)\) in the equilibrium.

Proposition 3. Suppose that the quadratic cost of efforts is given by \( c(a) = (1/2)a^2 \).

(i) The equilibrium is described by

\[
a^* = \frac{1-\theta \mu}{1+(2+\theta)\mu+(1+\theta)\rho},
\]

\[
\beta^* = \frac{a^*}{1-\theta \mu} = \frac{1}{1+(2+\theta)\mu+(1+\theta)\rho},
\]

\[
\delta^* = \overline{w} + \frac{1}{2} \left( \frac{1+\theta \mu}{1-\theta \mu} \right) a^2,
\]

\[
= \overline{w} + \left( \frac{1-\theta \mu}{2(1+(2+\theta)\mu+(1+\theta)\rho)} \right) \frac{1+(1+\theta)\rho-(1-\theta \mu)}{2(1+(2+\theta)\mu+(1+\theta)\rho)}.
\]

(ii) An increase of taxation rate \( \tau \) decreases the optimal efforts and bonus rate. That is,

\[
\frac{da^*}{d\tau} < 0 \text{ and } \frac{d\beta^*}{d\tau} < 0.
\]

The optimal fixed salary either increases or decreases depending on whether \( \rho \) is larger or lower than \( \rho_\theta \), i.e.,

\[
\frac{d\delta^*}{d\tau} \geq 0 \iff \rho \geq \rho_\theta = \frac{1-(1+\theta)\tau}{1-(1-\theta)\tau}.
\]

(iii) The marginal bonus paid by principal \((1+\mu)\beta^*\) increases (or decreases) by taxation rate \( \tau \) if the risk parameter \( \rho \) is larger (or lower) than \( \rho_\theta \), i.e.,

\[
\frac{d}{d\tau} (1+\mu)\beta^* \geq 0 \iff \rho \geq \rho_\theta = \frac{1-\tau}{1-(1+\theta)\tau}(1+\theta).
\]

Proof. For quadratic costs, it follows that \( c'(a) = a, c''(a) = 1, \) and \( c'''(a) = 0 \). Substituting these conditions into expressions (15) and (17), we have

\[
\begin{align*}
\alpha^* &= (1-\theta \mu)\beta^*, \\
\beta^* &= \frac{(1-\theta \mu) - \mu (1+\theta)a^*}{(1-\theta \mu)(1+\mu)(1+\theta)\rho),}
\end{align*}
\]

which results in

\[
\begin{align*}
a^* &= \frac{1-\theta \mu}{1+(2+\theta)\mu+(1+\theta)\rho),} \\
\beta^* &= \frac{1}{1+(2+\theta)\mu+(1+\theta)\rho).}
\end{align*}
\]

Through computations, we get

\[
\delta^* = \overline{w} + \frac{1-(1+\theta)\rho-(1-\theta \mu)}{2(1+(2+\theta)\mu+(1+\theta)\rho)^2}.
\]

For part (ii), it is straightforward to show that \((d\delta^*/d\tau) \geq 0 \iff (d\delta^*/d\mu) \geq 0\) and \((d\alpha^*/d\tau) \geq 0 \iff (d\alpha^*/d\mu) \geq 0\), because \((d\delta^*/d\mu) = (1-(1-\theta \mu))\rho\). Differentiating \( \mu \) in expressions of \( a^* \) and \( \beta^* \), we have

\[
\frac{da^*}{d\mu} = \frac{1}{(\theta+2)\mu + \theta \mu \rho + \rho + 1) < 0,
\]

\[
\frac{d\beta^*}{d\mu} = \frac{(\theta+2)(1-\theta \mu)}{(\theta+2)\mu + (1+\theta)\rho + 1) < 0.
\]

Namely, we obtain

\[
\frac{d}{d\mu} (1+\mu)\beta^* = \frac{1+\theta-(1+\theta)\mu}{(\theta+2)\mu + \theta \mu \rho + \rho + 1)^2}.
\]

Thus

\[
\frac{d}{d\mu} (1+\mu)\beta^* \geq 0 \iff \rho \geq \rho_\theta = \frac{1-\tau}{1-(1+\theta)\tau}(1+\theta).
\]

Namely, we obtain \((d\alpha^*/d\mu) \geq 0 \iff \rho \geq \rho_\theta = ((1-(1+\theta)\tau)/(1-(1-\theta)\tau)).\)

Remark 3. As stated in Proposition 3, the optimal efforts and bonus rate decrease with respect to \( \tau \) when the agent suffers a quadratic effort cost. As in the benchmark, the optimal effort is a decreasing function of taxation rate, which is the same in our model. In [1], Dietl et al. focus on how the marginal bonus \((1+\mu)\beta^*\) paid by principal changes by taxation rate. Therefore, the result established in Proposition 3 is similar with that in Dietl et al. [1]. The difference is that \( \rho_\theta \)
and $\rho_\beta$ are not constant in our model. The parameters $\rho_\beta$ and $\rho_\delta$ in Dietl et al. [1] equal 1 in their quadratic case. This concludes that the relation between $\delta^*$ or $(1-\mu)\beta^*$ and $\tau$ depends on the value of loss aversion factor $\theta$.

Giving an inspect in the expression of $\beta^*$ in (38), and bonus rate stay unchanged as taxation rate changes.

**Corollary 2.** Suppose that $\rho$ is larger than 1. If the loss aversion parameter $\theta$ and the risk parameter $\rho$ satisfy $\rho = ((1+\theta)/(1-\theta))$ or $\theta = ((\rho - 1)/(\rho + 1))$, the marginal bonus compensation $(1+\mu)\beta$ paid by principal in the optimal contract is constant and equals $1/(1+\rho)$.

**Proof.** The result is obtained through straightforward calculation after putting the condition $\rho = ((1+\theta)/(1-\theta))$ into (38). The detail is omitted.

For more visual analysis, we demonstrate our results by plotting concrete cases. For comparison, we exhibit the results in the benchmark models as well (notice that the relationship between $\mu$ and $\tau$ is $\mu = (\tau/(1-\tau))$. Moreover, $\mu$ and $\tau$ are positively dependent; i.e., $(d\mu/d\tau) = (1/(1-\tau)^2) > 0$). We first show the special case in Dietl et al. [1] when the parameter $\rho$ equals 1.

In Figure 1, it is shown that our model can be reduced to the benchmark model if $\theta = 0$. This is obvious because when $\theta$ is zero, loss aversion has no effect in our model. And the result is the same as that in Dietl et al. [1]. The fixed salary and bonus rate stay unchanged as taxation rate changes. The optimal effort $a^*$ is linearly decreased with taxation rate $\tau$.

In Figure 2, we show the static marginal bonus case stated in Corollary 2. The optimal effort and fixed salary in benchmark problem (2) are decreasing with $\tau$. The phenomenon stays the same in our model as well. In Dietl’s model, the marginal bonus is an increasing function of $\tau$ when the risk parameter $\rho$ is bigger than 1. When $\rho$ gets higher, the agent’s utility is more volatile. Hence, the agent is eager to search for a higher possible compensation to offset the utility loss caused by uncertainty. In our model, when $\rho = ((1+\theta)/(1-\theta))$, the marginal bonus paid by principal stays unchanged. On the one hand, the high risk parameter $\rho$ makes the agent ask for a high bonus. On the other hand, due to the existence of loss aversion, more taxation from the higher compensation brings about a potential loss for the agent in return. The two effects achieve a balance when $\rho = ((1+\theta)/(1-\theta))$. From this prospective, we can infer that the marginal bonus $(1+\mu)\beta^*$ is an increasing (decreasing) function of tax rate $\tau$ if the risk parameter $\rho$ is larger (lower) than $(1+\theta)/(1-\theta)$. Figure 3 actually exhibits these two circumstances. On the left side of Figure 3, the parameter $\rho$ equals 1.2 and $\theta$ is set to be a little larger than

![Figure 1: Our model versus benchmark model. Parameters set: $\theta=0$, $\rho=1$, $\overline{\rho} = 0.1$, and $\mu \in [0, 1]$.](image)
\( \frac{(\rho - 1)}{(1 + \rho)} \), while, on the left side of Figure 3, \( \theta \) is lower than \( \frac{(\rho - 1)}{(1 + \rho)} \).

Notice that when the risk parameter is less than 1, applying the results in Corollary 2, parameter \( \theta \) becomes negative. In reality, however, taxation could never bring people gains. It is meaningless to consider the case when \( \theta \) is less than 0. In the circumstance when \( \rho \) is less than 1, the marginal bonus paid by principal is always decreased by tax.
rate $\tau$ because $\theta$ is always larger than $(\rho - 1)/(\rho + 1)$. Figure 4 shows this situation. In benchmark problem (2), the optimal efforts and marginal bonus decrease as the tax rate goes high. The corresponding fixed salary in both benchmark model and our models increases. As shown in Figure 4, the fixed salary in our model (the cross-product line) is always higher than that in the benchmark model (the solid line). The marginal bonus paid by principal is lower in our model.

Among all cases above, the effect of loss aversion is set to be small; i.e., $\theta$ is small. As a consequence, the difference of agent’s compensations in this paper and Dietl et al. [1] is tiny. We proceed to investigate the situation when $\theta$ is relatively large.

In Figure 5, we study the circumstance when the effect of loss aversion is relatively huge. As shown, the optimal efforts always decrease by tax rate. In our model, $a^*$ is much smaller. As Corollary 2 states, when $\theta > ((\rho - 1)/(\rho + 1))$, $(1 + \mu \theta^2)$ is decreasing. In the benchmark model, as a contrast, the marginal bonus paid by principal is an increasing function of $\tau$. A finding in this case is that the fixed salary is not monotonic in our model. The curve of fixed salary has a “rollercoaster” shape. The optimal fixed salary reaches its maximum at point $\tau^*$ and then goes down afterwards. The complementarity between the fixed salary and marginal bonus disappears when the tax rate is larger than $\tau^*$. After reaching $\tilde{\tau}$, the fixed salary in our model is lower than that in benchmark problem (2). Since the marginal bonus in our model is always lower, both fixed salary and marginal bonus paid by principal in our model are lower than those in the benchmark model if $\tau$ is larger than $\tilde{\tau}$.

5. Conclusion

This paper considers a single-period principal-agent problem with moral hazard. Inspired by the research in behavioral economics, we consider the effect of loss aversion caused by taxation when building our model. Applying the Lagrangian multiplier method, we prove that the optimal contracting problem has a unique solution. Properties of the optimal contract are investigated when the agent’s cost function of efforts has a quadratic form. Moreover, we give an example to elaborate the effects of loss aversion.

As demonstrated by Dietl et al. [1], there exists a complementarity among the fixed salary and marginal bonus. Specifically, when the risk parameter is low, increasing taxation rate would reduce the marginal bonus and improve the fixed salary. The same phenomenon holds in our model when the coefficient of loss aversion is small. If the agent is strongly loss averse, however, the complementarity will disappear. Instead, the fixed salary...
has a “roller coaster” shape. As the tax rate increases, the fixed salary attaches its maximum and drops afterwards. Another result in our model is that the marginal bonus paid by principal always decreases with tax rate if the factor of loss aversion is huge, regardless of risk parameter. As a comparison, the marginal bonus increases about tax rate if the risk parameter is big enough as stated by Dietl et al. [1].

What interests us is whether the effects of risk aversion and loss aversion arrive at a balance in the model. As shown in the example, the balance does exist. When the risk parameter and coefficient of loss aversion are valued properly, the marginal bonus paid by principal stays the same whatever the tax rate is. Unfortunately, such property only holds when the risk parameter is big enough. In the situation when the risk parameter is low, technically, the marginal bonus can still stay unchanged if the factor of loss aversion is negative. However, it makes no sense in reality since the taxation could never bring individual gains.

The principal, the owner of a company, tends to seek for a compensation package with low fixed salary and high bonus rate. This is because when the agent’s efforts are hard to be monitored, such compensation package is supposed to promote incentives for the agent. As illustrated from our results, such contract scheme is achieved only when the effect of loss aversion is small and the risk parameter is big. The "big parameter" case usually shows up in new-economy firms because of uncertainty. In new economy, the owner of a start-up company is willing to motivate their agents by offering a high marginal bonus, usually through share allocations, equity incentives, or stock options. Oyer and Schaefer [14] take three economic justifications to investigate why many firms give stock options to employees. Ittner et al. [15] analyze different performance consequences of equity to three kinds of employees in new economy. As a policy guide from our results, implementing low taxes is a good way to support the new-economy industry since entrepreneurs can induce incentives for their employees when the loss aversion by taxation does not distort agents severely.

Our work mainly investigates how tax theoretically affects the behaviors of parties in an employment environment; any practical examples of using this principal-agent model are not found yet. An applicable research will be made to conduct a survey in different countries to verify the effect of loss aversion. Apart from the empirical studies, various works are to be done in the future. For instance, the single-period model is suitable to be extended into multiperiod model. Additionally, other psychological effects can be considered in suitable mathematical models and investigated empirically.
Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors’ Contributions

This article is a joint work of both the authors and they contributed equally to the final version of the paper. Both authors read and approved the final manuscript.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (no. 11471263).

References

[1] H. M. Dietl, M. Grossmann, M. Lang, and S. Wey, “Incentive effects of bonus taxes in a principal-agent model,” *Journal of Economic Behavior & Organization*, vol. 89, pp. 93–104, 2013.
[2] C. W. Smith Jr. and R. L. Watts, “Incentive and tax effects of executive compensation plans,” *Australian Journal of Management*, vol. 7, no. 2, pp. 139–157, 1982.
[3] M. von Ehrlich and D. Radulescu, “The taxation of bonuses and its effect on executive compensation and risk-taking: evidence from the UK experience,” *Journal of Economics & Management Strategy*, vol. 26, no. 3, pp. 712–731, 2017.
[4] D. Radulescu, “The effects of a bonus tax on manager compensation and welfare,” *FinanzArchiv*, vol. 68, no. 1, pp. 1–16, 2012.
[5] D. d’Andria, “Taxation and incentives to innovate: a principal-agent approach,” *Public Finance Analysis*, vol. 72, no. 1, pp. 96–123, 2016.
[6] J. Chang and Z. Hu, “Venture capital contracting with double-sided moral hazard and fairness concerns,” *Mathematical Problems in Engineering*, vol. 2018, Article ID 5296350, 13 pages, 2018.
[7] B. Holmstrom and P. Milgrom, “Aggregation and linearity in the provision of intertemporal incentives,” *Econometrica*, vol. 55, no. 2, pp. 303–328, 1987.
[8] Y. Sannikov, “A continuous-time version of the principal-agent problem,” *Review of Economic Studies*, vol. 75, no. 3, pp. 957–984, 2008.
[9] J. Civitanić, X. Wan, and J. Zhang, “Optimal compensation with hidden action and lump-sum payment in a continuous-time model,” *Applied Mathematics and Optimization*, vol. 59, no. 1, pp. 99–146, 2009.
[10] D. Kahneman and A. Tversky, “Prospect theory: an analysis of decision under risk,” *Econometrica*, vol. 47, no. 2, pp. 263–291, 1979.
[11] A. Tversky and D. Kahneman, “Advances in prospect theory: cumulative representation of uncertainty,” *Journal of Risk and Uncertainty*, vol. 5, no. 4, pp. 297–323, 1992.
[12] P. Engström, K. Nordblom, H. Ohlsson, and A. Persson, “Tax compliance and loss aversion,” *American Economic Journal: Economic Policy*, vol. 7, no. 4, pp. 132–164, 2015.
[13] A. Rees-Jones, “Loss aversion motivates tax sheltering: evidence from U.S. tax returns,” *SSRN Electronic Journal*, 2014.
[14] P. Oyer and S. Schaefer, “Why do some firms give stock options to all employees?: an empirical examination of alternative theories,” *Journal of Financial Economics*, vol. 76, no. 1, pp. 99–133, 2005.
[15] C. D. Ittner, R. A. Lambert, and D. F. Larcker, “The structure and performance consequences of equity grants to employees of new economy firms,” *Journal of Accounting and Economics*, vol. 34, no. 1–3, pp. 89–127, 2003.