The Sketch of a Polymorphic Symphony

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Abstract
In previous work, we have introduced functional strategies, that is, first-class generic functions that can traverse into terms of any type while mixing uniform and type-specific behaviour. In the present paper, we give a detailed description of one particular Haskell-based model of functional strategies. This model is characterised as follows. Firstly, we employ first-class polymorphism as a form of second-order polymorphism as for the mere types of functional strategies. Secondly, we use an encoding scheme of run-time type case for mixing uniform and type-specific behaviour. Thirdly, we base all traversal on a fundamental combinator for folding over constructor applications.

Using this model, we capture common strategic traversal schemes in a highly parameterised style. We study two original forms of parameterisation. Firstly, we design parameters for the specific control-flow, data-flow and traversal characteristics of more concrete traversal schemes. Secondly, we use overloading to postpone commitment to a specific type scheme of traversal. The resulting portfolio of traversal schemes can be regarded as a challenging benchmark for setups for typed generic programming.

The way we develop the model and the suite of traversal schemes, it becomes clear that parameterised + typed strategic programming is best viewed as a potent combination of certain bits of parametric, intensional, polytypic, and ad-hoc polymorphism.

Arrangement

The running example Given is a tree, say a term. We want to operate on a certain subterm. We search for the subterm \( t \) in some traversal order, be it top-down and left-to-right. We want to further constrain \( t \) in the sense that it should occur on a certain path, namely below \( k \) nodes the fitness of which is determined by predicates. Once we found \( t \) below certain nodes \( n_1, \ldots, n_k \), we either want to select \( t \) as is, or we want to compute some value from \( t \), or we want to transform \( t \) in its context in the complete tree. This traversal scenario is illustrated in Fig. \( 1 \) for \( k = 2 \). This sort of problem is very common in programming over tree and graph structures, e.g., in adaptive object-oriented programming [16]. One might consider additional forms of conditions, e.g., certain kinds of nodes that should not be passed, or further forms of computation, e.g., cumulative computation along the path.

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Find the subtree \( t \) below \( n_2 \) in turn below \( n_1 \). We assume two predicates \( p_1 \) and \( p_2 \) to identify \( n_1 \) and \( n_2 \). We further assume a generic function \( f \) to identify and then to process \( t \).

**Sample code** In the present paper, we refrain from discussing real-world application snippets but we sketch an illustrative instance of the running example in Fig. 2. As one can see, we assume a combinator \( \text{belowlist} \) that captures the described scheme of path traversal. In the figure, we define a function \( \text{test42} \) to traverse into a sample term \( \text{term1} \) in terms of \( \text{belowlist} \). The traversal looks for a subterm of type \( \text{SortB} \) to extract its integer component if it occurs on a path with two \( \text{SortB} \) subterms with the integer components 1 and 3. There are a few blind spots in the figure (cf. “...”) which we will resolve in the course of the concert. As the following Hugs session shows, the traversal yields 42:

```
Main> test42
Just 42
Main>
```

A traversal scheme for the introductory problem

\[ f \ '\text{belowlist}' \ 'ps' = \ldots \]

A sample system of two datatypes

\[
\begin{align*}
\textbf{data} & \quad \text{SortA} = \text{SortA1} \ \text{SortB} \mid \text{SortA2} \\
\textbf{data} & \quad \text{SortB} = \text{SortB} \ \text{Int} \ \text{SortA}
\end{align*}
\]

A test term for traversal

\[
\text{term1} = \text{SortA1} \ (\text{SortB} \ 0 \ (\text{SortA1} \ (\text{SortB} \ 1 \ (\text{SortA1} \ (\text{SortB} \ 2 \ (\text{SortA1} \ (\text{SortB} \ 3 \ (\text{SortA1} \ (\text{SortB} \ 42 \ \text{SortA2}))))))))))
\]

Extract the integer from a \( \text{SortB} \) term; fail for other sorts

\[ \text{sortb2int} = \ldots \]

Insist on a \( \text{SortB} \) term with a specific integer

\[ \text{sortbEqInt} \ i = \ldots \]

An actual traversal

\[
\text{test42} :: \text{Maybe} \ \text{Int} \\
\text{test42} = (f \ '\text{belowlist}' \ [p1, p2]) \ \text{term1} \\
\textbf{where} f = \text{sortb2int} \\
p1 = \text{sortbEqInt} \ 1 \\
p2 = \text{sortbEqInt} \ 3
\]

Figure 2. Illustrative Haskell code for the running example
Side conditions The above traversal scenario should be complemented by a few useful side conditions. We assume that (i) we deal with many sorts, say user-supplied systems of named, mutually recursive datatypes such as syntaxes and formats. So a traversal will encounter terms of different types. We further require that (ii) traversal schemes are statically type-safe, that is, a traversal always delivers a type-correct result (if any) without even attempting the construction of any ill-typed term. Furthermore, we insist on (iii) a parameterised solution where the overall traversal scheme is strictly separated from problem-specific ingredients. Also, the kind of processor (recall selection vs. computation vs. transformation) should not be anticipated. Recall that the actual traversal in Fig. 2 is indeed synthesised by passing predicates and a processor to a presumably overloaded traversal scheme belowlist. Last but not least, we require that (iv) a proper combinator approach is adopted where one can easily compose traversals and schemes thereof. As for the assumed belowlist combinator, its definition should be a concise and suggestive one-liner based on more fundamental traversal combinators. With combinators, we can easily toggle variation points of traversal, e.g., top-down vs. bottom-up, left-to-right vs. right-to-left, first match vs. all matches, and so on.

Conducted by Strafunski If any of the above side conditions is given up, the treatment of the scenario becomes less satisfying. If you do not insist on static type safety, then you can resort to Prolog, or preferably to Stratego [23] which supports at least limited type checks. Types are however desirable to discipline the instantiation of traversal schemes. If you even want to do away with parameterisation and combinator style, then XSLT is your language of choice. If types appeal to you, then you might feel tempted to consider polytypic functional programming [2]. However, corresponding language designs do not provide support for generic programming in a combinator style because polytypic values are not first-class citizens. If you are as demanding and simplistic as we are, then you use the Strafunski-style of generic programming in Haskell as introduced by the present author and Joost Visser in [14].1 The style is centred around the notion of functional strategies — first-class generic functions that can traverse into terms of any type while mixing uniform and type-specific behaviour.

The movements No previous knowledge of strategic programming is required to enjoy this symphony. We develop a model of functional strategies based on Haskell 98 [19] extended with first-class polymorphism [11]. The symphony consists of four movements. In Sec. 1, the type Strategic of functional strategies is initiated and inhabited with parametrically polymorphic [21,24] combinators. In Sec. 2, we integrate a combinator adhoc to update functional strategies for a specific type. This combinator relies on run-time type case — a notion that was studied in the context of intensional polymorphism [28]. In Sec. 3, we integrate a fundamental combinator hfoldr to perform primitive portions of traversal by folding

1 URL http://www.cs.vu.nl/Strafunski where Stra refers to strategies as in strategic term rewriting [3,23], Fun refers to functional programming, and their harmonious composition is a homage to the music of Igor Stravinsky.
over the immediate subterms of a term without anticipation of recursion. This kind of folding can be regarded as a variation on polytypism [3]. In Sec. 4, we use overloading, say ad-hoc polymorphism [26,10], to treat different types of traversal (recall selection vs. computation vs. transformation) in a uniform manner. At this level of genericity, we succeed in defining a portfolio of highly parameterised traversal schemes, including the scheme `belowlist` that is needed in the running example. This portfolio demonstrates the expressiveness and conciseness of functional strategies.

1 Moderato

In this section, we initiate the type of generic functions that model functional strategies. For the sake of a systematic development, we will first focus on the most simple, i.e., the parametrically polymorphic [21,24] dimension of the corresponding type scheme. In the subsequent two movements, we go beyond this boundary by adding expressiveness for run-time type case and generic term traversal.

**First-class polymorphic functions** In Fig. 3, we use a type synonym `Parametric` with two parameters $\alpha$ and $\kappa$ to capture the general type scheme $\alpha \rightarrow \kappa \alpha$ of functional strategies. As one can see, the parameter $\alpha$ corresponds to the domain of the function type, and $\kappa$ is used to construct the co-domain from $\alpha$. Common options for the co-domain type constructor $\kappa$ are the identity type constructor $I$ and the constant type constructor $C$ as defined in the figure, too. Based on the type scheme `Parametric`, we define the ultimate datatype `Strategic` that captures the polymorphic function type for functional strategies. Polymorphism is expressed via the universal quantifier “$\forall$”, and the fact that we ultimately need to go beyond parametric polymorphism is anticipated via a class constraint “`Term $\alpha$`” for ‘strategic polymorphism’. Note that the “$\forall$” occurs in the scope of the component of the constructor $G$. This form of second-order polymorphism, where polymorphic entities are wrapped by datatype constructors, is called first-class polymorphism [11], and it is a common language extension of Haskell 98. (Implicit) top-level universal quantification would be eventually insufficient to construct strategy combinators.

**Strategic type schemes** In Fig. 3, we also define the identity type constructor $I$, the constant type constructor $C$, and the constructor $S$ for sequential composition of type constructors. Based on these, we derive three classes $MG$, $TP$, and $TU$ of generic functions from `Strategic`. The synonym $MG$ captures monadic strategies. Monadic style [25] is favoured here because that way we can deal with effects during traversal, e.g., success and failure, state, or nondeterminism. We could also consider extra variants for non-monadic strategies but we instead assume that the

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2 The code shown in the paper is fully operational in Haskell (tested with Hugs 98; Dec. 2001 version). A paper-specific Strafunski distribution with illustrative examples can be downloaded from the paper’s web-site [http://www.cwi.nl/~ralf/polymorphic-symphony/]. This distribution also comes with a strategic Haskell program that provides generative tool support for strategic programming against user-supplied datatypes.
The generic function type for functional strategies

\[
\text{type } \text{Parametric } \alpha \kappa = \alpha \rightarrow \kappa \alpha \\
\text{newtype } \text{Strategic } \kappa = G (\forall \alpha. \text{Term } \alpha \Rightarrow \text{Parametric } \alpha \kappa)
\]

\[\text{unG } (G s) = s\]

The identity type constructor

\[
\text{newtype } I \alpha = I \alpha \\
\text{unI } (I x) = x
\]

The constant type constructor

\[
\text{newtype } C u \alpha = C u \\
\text{unC } (C x) = x
\]

Sequential type constructor composition

\[
\text{newtype } S t t' \alpha = S (t' (t \alpha)) \\
\text{unS } (S x) = x
\]

Type-preserving and type-unifying strategies

\[
\text{type } MG \kappa m = \text{Strategic } (S \kappa m) \\
\text{type } TP m = MG I m \\
\text{type } TU u m = MG (C u) m
\]

\[-\text{ ‘Monadic Generic’ functions } \]

\[-\text{ Corresponds to } \forall \alpha. \alpha \rightarrow m \alpha \]

\[-\text{ Corresponds to } \forall \alpha. \alpha \rightarrow m u \]

Strategy application by unwrapping

\[
\text{apply } :: (\text{Term } \alpha, \text{Monad } m) \Rightarrow MG \kappa m \rightarrow \alpha \rightarrow m (\kappa \alpha)
\]

\[
\text{apply } s x = \text{unS } (\text{unG } s x)
\]

More convenient application for TP

\[
\text{applyTP } :: (\text{Monad } m, \text{Term } \alpha) \Rightarrow TP m \rightarrow \alpha \rightarrow m \alpha
\]

\[
\text{applyTP } s x = \text{apply } s x \gg\gg \text{return } \circ \text{unI}
\]

More convenient application for TU

\[
\text{applyTU } :: (\text{Monad } m, \text{Term } \alpha) \Rightarrow TU u m \rightarrow \alpha \rightarrow m u
\]

\[
\text{applyTU } s x = \text{apply } s x \gg\gg \text{return } \circ \text{unC}
\]

![Figure 3. Functional strategies as first-class polymorphic functions](image)

trivial identity monad is used for that purpose. The synonym TP instantiates MG via I so that Type-Preserving strategies are identified. The synonym TU instantiates MG via C so that Type-Unifying strategies are identified, that is, polymorphic functions with a fixed result type. At the bottom of Fig. 3, we provide trivial definitions of ‘application’ combinators. The combinator apply complements ordinary function application by some unwrapping that accounts for first-class polymorphism (cf. unG), and for type-constructor composition (cf. unS). The combinators applyTP and applyTU specialise apply for TP and TU to hide the employment of the datatypes I and C for co-domain construction. Their result types point out the basic type schemes for TP and TU without any noise.

\[^{3}\text{ We should revise our sample code from Fig. 2 to use applyTU in the synthesis of the traversal:}\]

\[
test42 :: \text{Maybe Int}
\]

\[
test42 = \text{applyTU } (f \text{ ‘belowlist’ } [p1, p2]) \text{ term1}
\]

\[\text{where...}\]
Parametrically polymorphic embedding

\[ \text{para} :: (\forall \alpha. \text{Parametric } \alpha \kappa \rightarrow \text{Strategic } \kappa) \]
\[ \text{para}\ s = G\ s \]

Identity strategy

\[ \text{idTP} :: \text{Monad } m \Rightarrow \text{TP } m \]
\[ \text{idTP} = \text{para} \ (S \circ \text{return} \circ I) \]

Constant strategy

\[ \text{constTU} :: \text{Monad } m \Rightarrow u \rightarrow \text{TU } u \kappa \]
\[ \text{constTU} \ u = \text{para} \ (S \circ \text{return} \circ C \circ \text{const } u) \]

Failure strategy

\[ \text{fail} :: \text{MonadPlus } m \Rightarrow \text{MG } \kappa \kappa \]
\[ \text{fail} = \text{para} \ (S \circ \text{const } mzero) \]

Figure 4. Parametrically polymorphic strategies

Left-to-right monadic sequential strategy composition

\[ \text{seq} :: \text{Monad } m \Rightarrow \text{TP } m \rightarrow \text{MG } \kappa \kappa \rightarrow \text{MG } \kappa \kappa \]
\[ \text{seq } f \ g = G \ (\lambda x \rightarrow S \ (\text{applyTP } f \ x \gg eq \text{apply } g)) \]

Value-passing with shared term argument

\[ \text{pass} :: \text{Monad } m \Rightarrow \text{TU } u \ k \rightarrow (u \rightarrow \text{MG } \kappa \kappa) \rightarrow \text{MG } \kappa \kappa \]
\[ \text{pass } f \ g = G \ (\lambda x \rightarrow S \ (\text{applyTU } f \ x \gg eq \lambda u \rightarrow \text{apply } (g \ u \ x)) \]

Composition of alternative strategies

\[ \text{choice} :: \text{MonadPlus } m \Rightarrow \text{MG } \kappa \kappa \rightarrow \text{MG } \kappa \kappa \rightarrow \text{MG } \kappa \kappa \]
\[ \text{choice } f \ g = G \ (\lambda x \rightarrow S \ (\text{apply } f \ x \ '\text{mplus}' \text{apply } g \ x)) \]

Figure 5. Composition combinators for functional strategies

**Nullary combinators** Let us start to inhabit the strategy types while restricting ourselves to parametrically polymorphic inhabitants. In Sec. 2 and Sec. 3, we will inhabit the type Strategic in other ways by employing designated combinators for ‘strategic polymorphism’. If we neglect the monadic facet of functional strategies for a second, then there are few parametrically polymorphic inhabitants: the identity function for the type-preserving scheme, and constant functions for the type-unifying scheme. This is generalised for the monadic setup in Fig. 4. For clarity, we define the function para that approves a parametrically polymorphic function as a member of Strategic. While its definition coincides with the constructor G, its type insists on a parametrically polymorphic argument (no mentioning of the Term class). Using para, we can define the identity strategy idTP and a combinator constTU for constant strategies. Relying on the extended monad class MonadPlus, we can also define the always failing strategy fail in terms of the member mzero denoting failure, e.g., Nothing in the case of the Maybe instance of the MonadPlus class. The inhabitant fail is meant to illustrate that functional strategies might exhibit success and failure behaviour, or they even might be non-deterministic depending on the actual choice of the MonadPlus instance.
Binary combinators In Fig. 5, we define three prime combinators to compose functional strategies. The combinator seq lifts monadic sequencing of function applications to the strategy level (cf. “⇒”). The combinator pass composes two strategies which share a term argument and where the result of the first strategy is passed as additional input to the second strategy. The combinator choice composes alternative function applications. The actual kind of choice depends on the (extended) monad which is employed in the definition (cf. class MonadPlus and its member mplus), e.g., the Maybe monad for partiality or the List monad for multiple results. In the definitions of the binary combinators, we use apply to deploy the polymorphic function arguments, and we use $G$ to re-wrap the composition as a polymorphic function of type Strategic. In composing functional strategies, we cannot use the disciplined para as a substitute for $G$ because, eventually, we want to compose functions that go beyond parametric polymorphism.

Rank-2 types A crucial observation is that the composition combinators seq, pass and choice really enforce us to use some form of second-order polymorphism [7,20]. Other function combinators like “◦” or “⇒” have rank-1 types, that is, they are quantified at the top-level. This implies that they (also) accept monomorphic function arguments. By contrast, the types of seq, pass and choice have rank-2 types, that is, they are quantified argument-wise. These types model insistence on polymorphic function arguments as required for generic programming in a combinator style. Rank-2 types are not just needed for the binary composition combinators, but also for the upcoming traversal primitive, and for non-recursive and recursive traversal combinators. Actually, generic traversal in a combinator style necessitates rank-2 types. This is because, in general, a traversal scheme takes universally quantified arguments. Nested quantification reflects that ingredients of traversals must be applicable to subterms of any type.

2 Larghetto

At this point in our polymorphic concert, we still lack the original expressiveness of functional strategies: the ability to perform traversal into terms while mixing uniform and type-specific behaviour. This section and the subsequent one are concerned with this ‘strategic polymorphism’. In the present section, we provide a combinator adhoc for type-specific customisation of functional strategies. In the next section, generic traversal into terms will be enabled.

Informal explanation The type of the adhoc combinator is given at the top of Fig. 6. The combinator allows us to update an existing strategy for a specific type via a monomorphic function. We call this idiom type-based function dispatch. We also use the term strategy update because of the affinity to point-wise modification of ordinary functions. In this sense, strategy update is about “type-wise” modification of a polymorphic function. Here are two samples of strategy update:

\[
\textit{negatebool} :: \text{TP Identity} \\
\textit{negatebool} = \text{adhocTP idTP (return } o \neg)\
\]
The adhoc combinator

\[
\text{adhoc} :: \text{Term } \alpha \Rightarrow \text{Strategic } \kappa \rightarrow \text{Parametric } \alpha \kappa \rightarrow \text{Strategic } \kappa
\]

\[
\text{adhoc poly mono = \ldots} \quad \text{-- Implemented via Term class; see below.}
\]

A convenient specialisation for \( TP \)

\[
\text{adhocTP} :: (\text{Monad } m, \text{Term } \alpha) \Rightarrow TP \ m \rightarrow (\alpha \rightarrow m \alpha) \rightarrow TP \ m
\]

\[
\text{adhocTP poly mono = adhoc poly } (\lambda x \rightarrow S \ (\text{mono } x \gg \text{return } \circ I))
\]

A convenient specialisation for \( TU \)

\[
\text{adhocTU} :: (\text{Monad } m, \text{Term } \alpha) \Rightarrow TU \ u \ m \rightarrow (\alpha \rightarrow m \ u) \rightarrow TU \ u \ m
\]

\[
\text{adhocTU poly mono = adhoc poly } (\lambda x \rightarrow S \ (\text{mono } x \gg \text{return } \circ C))
\]

Figure 6. Updating functional strategies by type-specific cases

Attempt to extract the integer from a \( \text{SortB} \) term

\[
\text{sortb2int} :: TU \ Int \ Maybe
\]

\[
\text{sortb2int} = \text{adhocTU fail } (\lambda (\text{SortB } i \bot) \rightarrow \text{Just } i)
\]

Insist on a \( \text{SortB} \) term with a specific integer

\[
\text{sortbEqInt} :: \text{Int} \rightarrow TU () \ Maybe
\]

\[
\text{sortbEqInt } i = \text{sortb2int ‘pass’ } (\lambda i’ \rightarrow \text{if } i' \equiv i \text{ then constTU } () \text{ else fail})
\]

Figure 7. Use of strategy update in the running example

\[
\text{return\_int} :: TU \ Int \ Maybe
\]

\[
\text{return\_int} = \text{adhocTU fail return}
\]

The strategy \text{negate\_bool} behaves like \text{idTP} most of the time but it performs negation when faced with a Boolean (cf. “\( \neg \)”). We use the Identity monad. The strategy \text{return\_int} is meant to recognise integers, that is, it simply fails if it does not find an integer. So we use the Maybe monad to deal with failure. Note that we use type-specialised variants \text{adhocTP} and \text{adhocTU} as defined in Fig. 6. They allow the strategic programmer to forget about wrapping and unwrapping \( I, C, \) and \( S \).

In Fig. 7, we use strategy update to resolve two blind spots in Fig. 2 — the sample code for our running example. The strategy \text{sortb2int} extracts an integer from a term of type \( \text{SortB} \). The strategy \text{sortbEqInt} insists on a specific extracted integer. This is a predicate-like strategy which either returns \text{Just } () \text{ or Nothing}.

Definition of strategy update

Given a strategy \text{poly} of type \( \text{Strategic } \kappa \), and a function \text{mono} of type \( \text{Parametric } \alpha \kappa \), the updated function \text{adhoc poly mono} branches according to the type \( \alpha' \) of an eventual input value \( v \): if \( \alpha = \alpha' \), then \text{mono} is applied to \( v \), otherwise \text{poly}. That is:

\[
\text{adhoc poly mono } v = \begin{cases} 
\text{mono } v, & \text{if type of } v = \text{domain of mono} \\
\text{poly } v, & \text{otherwise}
\end{cases}
\]

Strategy update can be considered as a simple and very disciplined form of type case at run-time. Such type case was studied in the context of intensional polymorphism [8,28]. One might feel tempted to compare strategy update to dynamic typ-
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but note that first-class polymorphic strategies, as designed in the present paper, operate on terms of algebraic datatypes as opposed to ‘dynamics’. Dynamic typing can be employed, however, in other models of functional strategies, e.g., in the way described in [14].

**Encoding scheme** A strategic programmer considers the *adhoc* combinator as a primitive. In Haskell, we cannot define the *adhoc* combinator once and for all, but it can be supported per datatype. This is precisely what the *Term* class is needed for in the definition of *Strategic* (recall Fig. 3). So we basically place the *adhoc* combinator in the *Term* class with the provision to add another member for generic traversal later. There are several ways to encode strategy update. We will explain here a scheme that is inspired by type-safe cast as of [27]. In fact, we revise this scheme to model a *single-branching type case* (as opposed to a plain type cast) while assuming *nominal type analysis* (as opposed to a structural one).

The scheme is illustrated in Fig. 8. In the *Term* class, we place a primed member *adhoc'* with an implicitly quantified type. This is necessary because we need to

| Initiation of the *Term* class for ‘strategic polymorphism’ |
|-------------------------------------------------------------|
| **class** *Term α where**  
| *adhoc* : *Term α* \(\Rightarrow\) *Strategic* \(κ\) \(\rightarrow\) *Parametric* \(α\) \(κ\) \(\rightarrow\) *Parametric* \(α'\) \(κ\) |

| Migration from an overloaded to a first-class polymorphic type |
|---------------------------------------------------------------|
| *adhoc* poly mono = \(G\) (*adhoc'* poly mono) |

| Helpers for ‘recording’ types of updates |
|-----------------------------------------|
| **class** *Term α where**  
| *int* : *Strategic* \(κ\) \(\rightarrow\) *Parametric* \(Int\) \(κ\) \(\rightarrow\) *Parametric* \(α\) \(κ\) |
| *sorta* : *Strategic* \(κ\) \(\rightarrow\) *Parametric* \(SortA\) \(κ\) \(\rightarrow\) *Parametric* \(α\) \(κ\) |
| *sortb* : *Strategic* \(κ\) \(\rightarrow\) *Parametric* \(SortB\) \(κ\) \(\rightarrow\) *Parametric* \(α\) \(κ\) |

| Initialisation of the ‘decision matrix’ via default declarations |
|---------------------------------------------------------------|
| **class** *Term α where**  
| *int* poly _ = \(\text{unG}\)* poly |
| *sorta* poly _ = \(\text{unG}\)* poly |
| *sortb* poly _ = \(\text{unG}\)* poly |

| The relevant slices of the *Term* instances |
|--------------------------------------------|
| **instance** *Term Int where**  
| *adhoc'* = *int* |
| *int* _ mono = mono |
| **instance** *Term SortA where**  
| *adhoc'* = *sorta* |
| *sorta* _ mono = mono |
| **instance** *Term SortB where**  
| *adhoc'* = *sortb* |
| *sortb* _ mono = mono |

Figure 8. Per-datatype support for strategy update
link the type parameter $\alpha$ of the class declaration to the type of $adhoc'$. In the type of $adhoc'$, the parameter $\alpha$ acts as a place-holder for the term type of the updating monomorphic function, and there is an extra parameter $\alpha'$ for the term type of the input term. The first-class polymorphic $adhoc$ is easily derived from $adhoc'$ via wrapping with $G$ as shown in the figure. The overall idea for the implementation of $adhoc'$ is to compare the term type $\alpha$ of the update with the term type $\alpha'$ of the ultimate input term. To this end, we need to encode a kind of decision matrix to check for $\alpha = \alpha'$ for all possible combinations of term types. The dimension for $\alpha$ is taken care of by overloading $adhoc'$ in $\alpha$. The dimension for $\alpha'$ is taken care of by extra helper members — one for each type. These helper members model strategy update for a specific type of update while they are overloaded in the input term type. This is exemplified in the figure with the helpers $\text{int}$, $\text{sorta}$, and $\text{sortb}$ for the sample term types in our running example (cf. Fig. 2). It makes sense to initialise the definitions of the helper members to resort to $\text{poly}$ by default. In any given instance of the Term class, we need to define $adhoc'$, and we override the default definition for the specific helper member that handles the instance’s type. The definition of $adhoc'$ simply dispatches to the helper member. The helper member is redefined to dispatch to $\text{mono}$ instead of $\text{poly}$. This is again illustrated for the sample term types in our running example. Note that we are faced with a closed-world assumption because we need one member per term type in our strategic program (see [14] for another model).

Mixture of specificity and genericity The contribution of strategy update is the following. It allows us to lift type-specific behaviour in a type-safe and transparent way to the strategy level. Note that parametric polymorphism only works the other way around: a polymorphic function can be applied to a value of a specific type, and in fact, the actual behaviour will be entirely uniform, independent of the specific type [21][24]. In generic programming with traversals, type-specific customisation is indispensable because a traversal is typically concerned with problem-specific datatypes and constructors. At a more general level, one can say that the $adhoc$ combinator implements the following admittedly very much related requirements regarding the tension between genericity and specificity:

(i) Separability of generic and type-specific behaviour
(ii) Composability of generic and type-specific behaviour
(iii) First-class status of updatable generic functions

In functional programming, the common approach to the definition of polymorphic functions with type-specific behaviour is to use ad-hoc polymorphism, say type and constructor classes [26][14] as supported in Haskell. This approach does not meet the above requirements because a class and its instances form a declaration unit. Any new combination of generic and type-specific behaviour has to be anticipated by a dedicated class. Hence, strategy update is complementary.
3 Allegretto

Ultimately, functional strategies are meant to perform term traversal. Hence, we need to add another bit of polymorphism. In fact, the kind of term traversal that we envisage can be regarded as a variation on polytypism [9]. That is, recursion into terms is performed on the basis of the structure of the underlying algebraic datatypes. First we will briefly categorise traversal approaches in previous work. Then we will generalise one of the approaches to contribute a very flexible means of traversal to the ongoing reconstruction of functional strategies.

Traversal approaches We identify the following categories:

(i) Recursive traversal schemes [18, 22, 15, 4]
(ii) One-layer traversal combinators [23, 12, 14]
(iii) Traversal by type induction [9]
(iv) Traversal at a representation-type level [3]

(i) Recursive traversal schemes, e.g., generalised folds or iterators over data structures, follow a fixed scheme of recursion into terms, and the style of composition of intermediate results is also intertwined with the recursion scheme. (ii) One-layer traversal combinators add flexibility because they do not recurse into terms by themselves but they rather capture single steps of traversals. Hence, different recursion schemes and different means to compose intermediate results can still be established by plain (recursive) function definition. The value of one-layer traversal combinators was identified in the context of strategic term rewriting [23] in the application domain of program transformation. (iii) Polytypic programming suggests that a traversal can also be viewed as a function that is defined by induction on the argument type. So one can capture term traversal schemes by polytypic definitions. However, these definitions could not be used in a combinator style because existing language designs do not cover ‘rank-2 polytypism’, that is, one cannot pass one polytypic function to another one. (iv) Finally, a programmer can resort to a universal representation type to perform traversal. Here we assume that the programmer is provided with implosion and explosion functionality to mediate between programmer-supplied term types and the representation type. While unrestricted access to a representation type corresponds to a less safe way of generic programming due to the potential of implosion problems, implementations of (i)–(iii) might very well be based on a representation type. To give an example, in [14], traversal combinators are implemented using a representation type without any exposure to the generic programmer. Hence, implosion safety can be certified.

Right-associative, heterogeneous fold In this symphony, we basically follow the one-layer traversal-combinator approach, but we generalise it in the following manner. Rather than making a somewhat arbitrary selection of traversal combinators as in previous work [23, 12, 14], we identify the fundamental principle underlying all one-layer traversal. To this end, we define a primitive combinator for folding over constructor applications. This kind of folding views the immediate subterms of a
Completion of the `Term` class for ‘strategic polymorphism’

```haskell
class Term α where
  hfoldr' :: HFoldrAlg κ → α → κ α
```

Migration from an overloaded to a first-class polymorphic type

```haskell
hfoldr :: HFoldrAlg κ → Strategic κ
hfoldr alg = G (hfoldr' alg)
```

Fold algebras

```haskell
data HFoldrAlg κ = A -- First-class polymorphic wrapper
  (∀ α β. Term α ⇒ α → κ (α → β) → κ β) -- Cons-like case
  (∀ γ. γ → κ γ) -- Nil-like case
```

Figure 9. Folding over constructor applications

```haskell
instance Term Int where
  hfoldr' (A _ z) i = z i

instance Term SortA where
  hfoldr' (A f z) (SortA1 b) = f b (z SortA1)
  hfoldr' (A _ z) SortA2 = z SortA2

instance Term SortB where
  hfoldr' (A f z) (SortB i α) = f α (f i (z SortB))
```

Figure 10. Per-datatype support for `hfoldr` — sample instances

term (say, its children) as a heterogeneous list of terms. Without loss of generality, we favour a right-associative fold in the sequel. We call the resulting fold operation `hfoldr` for right-associative, heterogeneous fold. Let us recall how the ordinary `foldr` folds over a homogeneous list. Given the fold ingredients `f` and `z` for the non-empty and the empty list form, the application of `foldr` to a list is defined as follows:

```
foldr f z [x₁, x₂, . . . , xₙ] = f x₁ (f x₂ (· · · (f xₙ z) · · ·))
```

The combinator `hfoldr` folds over a constructor application in nearly the same way:

```
hfoldr f z (C x₁ · · · xₙ) = f x₁ (f x₂ (· · · (f xₙ (z C)) · · ·))
```

Note the order of the children in a constructor application. Due to curried style, we have a kind of snoc-list, that is, the leftmost child is the head of the list, and so on. Also note that the empty constructor application `C` is passed to `z` so that the constructor can contribute to the result of folding.

**Encoding scheme** A strategic programmer considers `hfoldr` as a primitive. In Haskell, we cannot define the `hfoldr` combinator once and for all, but it can be supported per datatype — as in the case of strategy update. In Fig. 9, we complete the `Term` class accordingly. We add a primed helper member `hfoldr'` to the class, and we define the actual combinator `hfoldr` via wrapping with `G`. Due to the employment of first-class polymorphism, we need to pack the ingredients `f` and `z` for `hfoldr` in a datatype `HFoldrAlg`. This is a deviation from the curried style that is
used for the rank-1 $foldr$ for lists. The datatype constructor $A$ is used for packaging. The types of the two components deserve some explanation. The first component, i.e., the one that corresponds to $f$, is universally quantified in $\alpha$ and $\beta$ corresponding to the type of the heading child (i.e., the outermost argument in the constructor application), and the type of the constructor application at hand, respectively. The type of the recursively processed tail is $\kappa (\alpha \to \beta)$ because it ‘lacks’ a child of type $\alpha$. Given a constructor application $C \ x_n \cdots x_2 \ x_1$, the type variable $\alpha$ would be bound to the respective types of the $x_i$ in the several folding steps while the type $\beta$ variable would be bound to the corresponding remainders of $C$’s type. The second component of $A$, i.e., the one that corresponds to the base case $z$, is simply a parametrically polymorphic function for processing the empty constructor application. The per-datatype definition of $hfoldr$ is illustrated in Fig. 10 with the sample datatypes from our running example. We basically need one equation for $hfoldr'$ of the above form for each specific datatype constructor.

**One-layer traversal combinators** Based on $hfoldr$, one can define all kinds of one-layer traversal combinators. We indicate an open-ended list of candidates by reconstructing combinators that were presented elsewhere [23,13,14,12]:

- **allTP** Process all children; preserve the constructor.
- **oneTP** Try to process the children until one succeeds; preserve the constructor.
- **allTU** Process all children; reduce the intermediate results.
- **oneTU** Try to process the children until one succeeds; return the processed child.

One can think of further combinators, e.g., combinators dealing with different orders of processing children, monadic effects, and different constraints regarding success and failure behaviour.

The definition of the combinators is shown in Fig. 11. Let us explain the first one, that is, **allTP**. We construct a (type-preserving) fold algebra $A f z$ and pass it to $hfoldr$. The ingredient $z$ for the empty constructor application simply returns, i.e., preserves the constructor. Additional wrapping of $S$ and $I$ is needed to deal with our ‘datatypes-as-type-constructors’ encoding. The ingredient $f$ sequences three computations. Firstly, the tail $t$ is computed. Secondly, the head $h$ (i.e., the child at hand) is processed via $s$. Thirdly, the processed head $h'$ is passed to the computed tail $t'$ (modulo wrapping and unwrapping). The definition of the other combinators is similar. A challenging complication shows up when we want to define **oneTP** because it turns out that this fold over children is paramorphic in nature [17] while our combinator $hfoldr$ is catamorphic. We use a pairing technique adopted from [17] to encode the paramorphic fold with $hfoldr$ (cf. the new type constructor $D$). This complication also illustrates the potential for further generic function types, namely $MG (D Maybe) m$ in this case.
Process all children; preserve the outermost constructor

\[
\text{allTP} :: \text{Monad } m \Rightarrow TP m \to TP m
\]

\[
\text{allTP } s = hfoldr (A f z)
\]

where

\[
f h t = S \left( \text{do} \ t' \leftarrow \text{unS} \ t; h' \leftarrow \text{applyTP} \ s \ h \right.
\]

\[
\quad \text{return} (I \left( (\text{unI} \ t') \ h' \right))
\]

\[
z = S \circ \text{return} \circ I
\]

Try to process the children until one succeeds; preserve the outermost constructor

\[
\text{oneTP} :: \text{MonadPlus } m \Rightarrow TP m \to TP m
\]

\[
\text{oneTP } s = G \left( \lambda x \rightarrow S \left( \text{apply} \left( \text{poneTP} \ s \right) x \right) \right)
\]

\[
\text{maybe mzero} \circ \left( \text{return} \circ I \circ \text{fst} \circ \text{unD} \right)
\]

The Duplicate type constructor for encoding type-preserving paramorphisms

\[
\text{newtype } D t \alpha = D (t \alpha, t \alpha)
\]

\[
\text{unD} (D x) = x
\]

Paramorphic helper for \(\text{oneTP}\)

\[
\text{poneTP} :: \text{MonadPlus } m \Rightarrow TP m \to MG (D \text{Maybe}) m
\]

\[
\text{poneTP } s = hfoldr (A f z)
\]

where

\[
f h t = S \left( \text{do} \{ t' \leftarrow \text{unS} \ t; \text{tp} \leftarrow \text{maybe mzero} \left( \text{snd} \left( \text{unD} \ t' \right) \right); \text{do} \{ \text{tc} \leftarrow \text{maybe mzero} \left( \text{fst} \left( \text{unD} \ t' \right) \right); \right.
\]

\[
\quad \text{return} (D (\text{Just} (\text{tc} \ h), \text{Just} (\text{tp} \ h))) \right)
\]

\[
\text{mplus}' \left( \text{do} \ h' \leftarrow \text{applyTP} \ s \ h
\]

\[
\quad \text{return} (D (\text{Just} (\text{tp} \ h'), \text{Just} (\text{tp} \ h)))
\]

\[
\text{mplus}' \text{return} (D (\text{Nothing}, \text{Just} (\text{tp} \ h))) \right)
\]

\[
z x = S \left( \text{return} (D (\text{Nothing}, \text{Just} x)) \right)
\]

Process all children; reduce the intermediate results

\[
\text{allTU} :: \text{Monad } m \Rightarrow (u \to u \to u) \to u \to TU \ u \ m \to TU \ u \ m
\]

\[
\text{allTU } \text{op2} \ \text{unit} \ s = hfoldr (A f z)
\]

where

\[
f h t = S \left( \text{do} \ a \leftarrow \text{unS} \ t
\]

\[
\quad b \leftarrow \text{applyTU} \ s \ h
\]

\[
\quad \text{return} (C \left( \text{unC} \ a \ \text{op2} \ b \right))
\]

\[
z = S \circ \text{return} \circ C \circ \text{const} \ \text{unit}
\]

Try to process the children until one succeeds; return the processed child

\[
\text{oneTU} :: \text{MonadPlus } m \Rightarrow TU \ u \ m \to TU \ u \ m
\]

\[
\text{oneTU } s = hfoldr (A f z)
\]

where

\[
f h t = S \left( \text{cast} \left( \text{unS} \ t \right) \ \text{mplus} \ \text{cast} \left( \text{apply} \ s \ h \right) \right)
\]

\[
z = S \circ \text{const} \ \text{mzero}
\]

\[
\text{cast} \ c = c \gg \text{return} \circ C \circ \text{unC}
\]

Figure 11. Examples of one-layer traversal combinators
Sums of products vs. curried constructor applications  The combinator \textit{hfoldr} roughly covers the sum and product cases in a polytypic definition \cite{9} but without resorting to a constructor-free representation type. While a polytypic definition deals with the many children in a term via nested binary products, \textit{hfoldr} processes all children in a curried constructor application as is. While the different constructors of a datatype amount to nested binary sums in a polytypic definition, the base case of \textit{hfoldr} handles the empty constructor application. Also note that polytypic definitions employ a designated declaration form for type induction while functional strategies are defined as ordinary recursive functions. Finally, polytypic definitions are normally implemented by compile-time specialisation. Functional strategies only require a \textit{Term} interface for datatypes to enable \textit{hfoldr} and \textit{adhoc}.

4 Largo

We are now in the position to define all kinds of traversal schemes in terms of the strategy combinators and recursion. Before we present a portfolio of such schemes, we impose some extra structure on our combinator suite in order to allow for a uniform treatment of type-preserving and type-unifying traversal.

Overloading \textit{TP} and \textit{TU}  The strategy combinators are usually typed in a manner that they can be used to derive strategies of both type \textit{TP} and type \textit{TU}. Hence, one would expect that traversal schemes can be composed in a manner to work for both type schemes as well. There are two asymmetries which we need to address. Firstly, there is no obvious way to write fold algebras that cover both \textit{TP} and \textit{TU}. This is the reason why we have a \textit{oneTP} vs. a \textit{oneTU}, and an \textit{allTP} vs. an \textit{allTU}. These combinators should be overloaded. Secondly, there is no uniform way to \textit{combine} the execution of two strategies of the \textit{same} type. We can only compose \textit{alternative} branches via \textit{choice}. For the sake of sequential composition being generic, the \textit{seq} combinator insists on a type-preserving first argument. For the sake of value passing to deliver an extra input to the second operand, the \textit{pass} combinator insists on a type-unifying first argument. We suggest an overloaded composition combinator \textit{comb} as follows. In the \textit{TP} instance, sequential composition via \textit{seq} is appropriate: the term as returned by the first type-preserving application is passed to the second. In the \textit{TU} instance, both strategies are evaluated via \textit{pass}, and then, the two resulting values are combined via the binary operation of a monoid. Note that the \textit{Monoid} class is also appropriate to harmonise the types of the one-layer traversal combinators \textit{allTP} and \textit{allTU} for overloading. That is, the additional parameters for \textit{allTU} are assumed to correspond to the monoid operations.

In Fig. \ref{fig:overloaded}, the overloaded combinators for traversal (cf. \textit{one} and \textit{all}) and combination of strategies (cf. \textit{comb}) are defined. For completeness, we also overload \textit{idTP} and \textit{constTU} in a way that an always succeeding strategy is defined (cf. \textit{skip}). The overloaded combinators are placed in two Haskell type classes \textit{StrategicMonadPlus} and \textit{StrategicMonoid} in order to deal with the different class constraints for the overloaded combinators.
Overloading operator(s) involving `MonadPlus`

```haskell
class StrategicMonadPlus s where
  one :: s → s

instance MonadPlus m ⇒ StrategicMonadPlus (TP m) where
  one = oneTP

instance MonadPlus m ⇒ StrategicMonadPlus (TU u m) where
  one = oneTU
```

Overloading operators potentially involving `Monoid`

```haskell
class StrategicMonoid s where
  skip :: s
  all :: s → s
  comb :: s → s → s

instance Monad m ⇒ StrategicMonoid (TP m) where
  skip = idTP
  all = allTP
  comb = seq

instance (Monad m, Monoid u) ⇒ StrategicMonoid (TU u m) where
  skip = constTU mempty
  all = allTU mappend mempty
  comb = λf g → f `pass` λa → g `pass` λb → constTU (a `mappend` b)
```

Figure 12. Classes of generic function combinators

**Full traversal control** In Fig. 13, we define traversal schemes in three groups. The highly parameterised function `traverse` captures a rich class of traversal schemes as illustrated by the given instantiations. Some of the type-preserving instantiations of `traverse` were identified in [23]. In [13,12,14], some type-unifying instantiations were added. In the definition of `traverse`, we separate out the distinguishing parameters of such more specific traversal schemes. The argument `op` is the function combinator to compose term processing and recursive descent. The argument `t` defines how to descend into children. Finally, `f` is the strategy for term processing. Here is the normative type for this ‘mother of traversal’:

```haskell
traverse :: Monad m
  ⇒ (MG κ m → MG κ m → MG κ m) -- Composition
  → (MG κ m → MG κ m) -- Descent
  → MG κ m -- Processor
  → MG κ m
```

We call this a *normative* type because the inferable type is much more general, namely `traverse :: (a → b → c) → (c → b) → a → c`. Not even the normative type captures all our intentions, e.g., the one that `t` actually specifies descent into children. It is a topic for future work to capture such constraints in suitable types.

The scheme `traverse` is illustrated by several instances in the figure. The first two instances `all_rec` and `one_rec` refine `traverse` to opt either for descent with `all` or
Traversals with parameters for different kinds of traversal control

\[
\begin{align*}
\text{traverse } \text{op} & \quad \text{f} = f \circ \text{op} \circ (\text{traverse } \text{op} \quad \text{f}) \\
\text{all } \text{rec } \text{op} & = \text{traverse } \text{op} \quad \text{all} \\
\text{one } \text{rec } \text{op} & = \text{traverse } \text{op} \quad \text{one} \\
\text{full } \text{td} & = \text{all } \text{rec } \text{comb} \\
\text{full } \text{bu} & = \text{all } \text{rec } (\text{flip } \text{comb}) \\
\text{once } \text{td} & = \text{one } \text{rec } \text{choice} \\
\text{once } \text{bu} & = \text{one } \text{rec } (\text{flip } \text{choice}) \\
\text{stop } \text{td} & = \text{all } \text{rec } \text{choice} \\
\text{stop } \text{bu} & = \text{all } \text{rec } (\text{flip } \text{choice})
\end{align*}
\]

Traversals with propagation of an environment

\[
\begin{align*}
\text{propagate } \text{op} & \quad \text{f} \quad \text{u} \quad \text{e} = (f \quad e) \circ \text{op} \circ (u \quad \text{pass} \quad (\lambda e' \rightarrow t \quad (\text{propagate } \text{op} \quad \text{f} \quad u \quad e'))) \\
\text{full } \text{pe} & = \text{propagate } \text{comb } \text{all} \\
\text{once } \text{pe} & = \text{propagate } \text{choice } \text{one} \\
\text{stop } \text{pe} & = \text{propagate } \text{choice } \text{all}
\end{align*}
\]

Traversals based on a notion of path

\[
\begin{align*}
\text{f} \quad \text{beloweq } \quad \text{p} & = \text{once } \text{td } (p \quad \text{pass} \quad \lambda \rightarrow \text{once } \text{td} \quad \text{f}) \\
\text{f} \quad \text{below } \quad \text{p} & = (\text{one } f) \quad \text{beloweq } \quad \text{p} \\
\text{f} \quad \text{aboveeq } \quad \text{p} & = \text{once } \text{bu } (\text{once } \text{td } \quad \text{p} \quad \text{pass} \quad \lambda \rightarrow \text{f}) \\
\text{f} \quad \text{above } \quad \text{p} & = f \quad \text{aboveeq } \quad (\text{one } p) \\
\text{belowlist } \text{f} & = \text{foldl } \text{below } \quad \text{f} \circ \text{reverse} \\
\text{above } \text{f} & = \text{above } \quad \text{f} \circ \text{foldr } \text{above } (\text{constTU } \quad ()) \\
(p \quad \text{prepost } \quad \text{ps}' \quad \text{f}) & = (f \quad \text{above } \quad \text{f} \quad \text{ps}') \quad \text{below } \quad \text{ps}
\end{align*}
\]

Figure 13. Traversal schemes

one, respectively. The postfixes “...td” or “...bu” stand for top-down or bottom-up, respectively. The prefixes “full...”, “once...”, or “stop...” hint on the style of traversal control: full traversal where all nodes are processed vs. single hit traversal where the argument strategy has to succeed once vs. cut-off traversal where descent stops when a subterm was processed successfully. All the more concrete schemes are still overloaded as for the choice of TP vs. TU. To give an example, the scheme full td models full traversal of a term where all nodes are processed in top-down manner. In this example, the argument f for descent is instantiated with all (via all rec). The argument op for the composition of term processing and recursive descent is instantiated with comb.

Traversals with propagation The second group in Fig. 13 illustrates a class of traversal schemes that go beyond simple node-wise processing. In addition to traversal, propagation of an environment (abbreviated by the postfix “...pe”) is performed. The scheme propagate takes the same arguments op, t, and f as traverse. In addition, the scheme carries a generic function argument u to update the environment before descent into the children, and an argument e for the initial environment. Here is the normative type for propagate:
propagate :: Monad m
  ⇒ (MG κ m → MG κ m → MG κ m) -- Composition
  → (MG κ m → MG κ m) -- Descent
  → (e → MG κ m) -- Processor
  → (e → TU e m) -- Environment update
  → e -- Initial environment
  → MG κ m

The shown instances of *propagate* favour top-down traversal because bottom-up traversal seems to be less obvious when combined with propagation. It is interesting to notice that the scheme *propagate* provides an alternative to using the environment monad [25] in a strategic program.

**Path schemes** The last group in Fig. [13] deals with traversal constrained by the path leading to or starting from a node (say, a subterm). For this reason, we do not just use a processor strategy *f*, but we also use predicates *p* for constraints. The scheme beloweq attempts to process a subterm via *f* (not necessarily strictly) below a node for which *p* succeeds; dually for aboveeq. The below and above variants enforce the root of the processed subterm not to coincide with the constrained node. In the definition of the path schemes, we use the simpler traversal schemes *once bd* and *once bu* from above. The types of the path schemes are as follows:

\[
\text{beloweq}, \text{below}, \\
\text{aboveeq}, \text{above} :: (\text{MonadPlus } m, \text{StrategicMonadPlus } (MG \kappa m)) \\
  ⇒ MG \kappa m -- Processor \\
  → TU () m -- Predicate \\
  → MG \kappa m
\]

These types illustrate that predicates are encoded as type-unifying functions of type \(TU () m\) where \(m\) is normally the *Maybe* monad or another instance of the *MonadPlus* class. So success and failure behaviour encodes the truth value.

The final three schemes in the group elaborate the more basic path schemes to deal with *lists* of predicates. The schemes belowlist and aboveclist constrain the prefix or postfix of a node rooting a subtree of interest, respectively. The last scheme prepost combines belowlist and aboveclist. Note that the path scheme belowlist resolves the remaining blind spot in the sample code for our running example in Fig. [4].

[End Of Symphony]
**Appraisal**

The four movements of this symphony combine certain bits of parametric polymorphism, type case (i.e., intensional polymorphism), polytypism, and overloading to provide a concise and expressive model for functional strategies \[14\]. The development culminates in the last movement when typed highly parameterised traversal schemes are defined. This paper makes the following overall contributions when compared to previous work on strategic programming. Firstly, the developed model is more concise and expressive than in our previous work \[13\][12][14\]. Secondly, the defined traversal schemes reach new limits of parameterisation. We shall elaborate on these two claims accordingly.

The overall model “functional strategies = first-class polymorphic functions” was already sketched in \[14\]. In addition to working out this model, we improved it in two important ways. Firstly, the type schemes for type-preserving and type-unifying strategies are viewed as instances of one common type scheme Strategic. This allows us to define combinators like choice and adhoc without commitment to any specific type scheme. Secondly, we eliminated the need for an open-ended list of primitive one-layer traversal combinators. To this end, we designed a generic fold combinator for processing the immediate subterms of constructor applications. A related language construct is described in \[8\]: pattern matching is generalised in a way that generic access is granted to subterms of constructor applications. A limited form of folds over constructor applications was first proposed in \[12\] in a term-rewriting setting, but there it was by far less potent and it was difficult to type due to the limitations of the underlying type system.

The key idea to define traversal schemes in terms of one-layer traversal combinators carries over from the seminal work on term rewriting strategies \[23\]. The present paper reaches a new level of genericity for the following two reasons. Firstly, we effectively overload the two prime type schemes of generic traversal. This allows us to capture meaningful traversal schemes without anticipating their use for transformation vs. analysis. Secondly, we identify a few highly parameterised traversal schemes with parameters for various forms of control. These ‘skeleton’ or ‘meta’ schemes capture more concrete traversal schemes favoured in previous work.

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