On the value of $R = \Gamma_h/\Gamma_l$ at LEP

Maurizio Consoli $^1$ and Fernando Ferroni $^2$

Abstract

We show that the present experimental LEP average $R = \Gamma_h/\Gamma_l = 20.795 \pm 0.040$ is not unambiguous due to the presence of substantial systematic effects which cannot be interpreted within gaussian statistics. We find by Montecarlo simulation that the C.L. of the original LEP sample is only $3.8 \cdot 10^{-4}$. We suggest that a reliable estimate of the true $R$-value is $20.60 < R < 20.98$ which produces only a very poor determination of the strong coupling constant at the $Z$ mass scale, $0.10 < \alpha_s(M_Z) < 0.15$. 

1) Universit`a di Catania and INFN Catania
2) Universit`a di Roma ”La Sapienza” and INFN Roma I
The determination of the strong coupling constant at the Z-mass scale $\alpha_s(M_z)$ is of primary importance for a consistency check of perturbative QCD. In this context, the quantity $R$, defined as the ratio between the hadronic and the leptonic partial widths of the $Z$ boson, plays a fundamental role. Indeed, this particular observable, operatively defined through the ratio of the peak cross-sections in the corresponding hadronic and leptonic channels, can determine $\alpha_s(M_z)$ to a very high degree of accuracy thus allowing a direct comparison with the perturbative evolution of $\alpha_s$ from precise low-energy data for Deep Inelastic Scattering (DIS).

The theoretical prediction at one-loop in the electroweak theory and including $O(\alpha_s^3)$ perturbative QCD corrections, can be conveniently expressed by using the result of the recent analysis by Hebbeker, Martinez, Passarino and Quast [1] as

$$R^{\text{Th}} = R^{(o)} (1 + \delta_{\text{QCD}})$$

where $R^{(o)}$ is the purely electroweak value in the quark-parton model and $\delta_{\text{QCD}}$ is conveniently expressed as [1]

$$\delta_{\text{QCD}} = 1.06 \frac{\alpha_s}{\pi} + 0.9 \left( \frac{\alpha_s}{\pi} \right)^2 - 15 \left( \frac{\alpha_s}{\pi} \right)^3$$

By using the experimental LEP average presented at the Glasgow Conference [2]

$$R^{\text{LEP}} = 20.795 \pm 0.040$$

one deduces the value [2]

$$\alpha_s(M_z) = 0.126 \pm 0.006$$

or, by including all lineshape data, [3]

$$\alpha_s^{\text{LEP}}(M_z) = 0.127 \pm 0.005$$

Eqs.(4,5) should be compared with the prediction [4] from DIS (including a fair estimate of the theoretical error)

$$\alpha_s^{\text{DIS}}(M_z) = 0.113 \pm 0.005$$

As pointed out by Shifman [5], the discrepancy between Eqs.(4,5) and Eq.(6) is disturbing, implying a rather large difference in the values of the QCD scale parameter (in the $\overline{MS}$ scheme and with five flavours), namely $\Lambda_{\text{QCD}} \sim 500$ MeV rather than the value $\Lambda_{\text{QCD}} \sim 200$ MeV expected from the QCD sum rules based on the Operator Product Expansion approach.

The presence of a possible discrepancy with the low energy extrapolations provides a valid motivation to reconsider critically the meaning of the experimental LEP average presented in Eq.(3). Indeed, as we shall explicitly show in the following, the interpretation of the experimental data is not unambiguous and the average in Eq.(3) is faced with serious problems of statistical consistency.

The individual LEP measurements of $R$ in the various $\mu$, $\tau$ and electron channels, as presented by ALEPH, DELPHI, L3 and OPAL at the Glasgow Conference and summarized in ref. [2], are reported in Table 1.

These 12 individual measurements are not all statistically independent. However, in a first approximation, if one neglects the small correlation among measurements in the same experiment and treats all $R$-values as independent, one gets precisely the same average as obtained in ref. [2] by using the full covariance matrix. Thus, to good approximation, one may be tempted to consider the 12 individual measurements in Table 1 as belonging to a normal population governed by gaussian statistics.
To understand the possible presence of systematic effects, which can affect the global average in an uncontrolled way, we started reporting in a histogram the central values of the 12 individual measurements.

By inspection of fig.1 one discovers the following unexpected result: near the global average $R = 20.795 \pm 0.040$, where there should be a very large number of data, one finds a minimum of the probability since no experiment, in any individual channel, is reporting a central value lying in the interval $20.755 - 20.835$. The various measurements, instead, can be divided into two sets rather sharply peaked around $R \sim 20.92$ and $R \sim 20.66$.

In order to have a better qualitative understanding of the problem we have reported the 12 experimental data in sequence in fig.2 with their errors.

As one can see, all points lie at $\sim 1\sigma$ from the central value so that the $\chi^2$ is good indeed. However, a good value of the $\chi^2$ does not tell much on the gaussian nature of the data.

To obtain a quantitative description of this statement we decided to test the hypothesis of the common belonging of the measurements to a normal population having the observed mean value $R = 20.795$ and errors like those of each individual measurement. The variable chosen as a probe of non-normality is the kurtosis

$$
\gamma = \mu_4/\mu_2^2 - 3
$$

where $\mu_n = \int (x - \bar{x})^n f(x) dx$.

We have used a random number generator to produce a large number of equivalent copies starting from our original population of 12 measurements reported in Table 1. For any generated sample of 12 measurements, with their respective errors, we compute the mean $\bar{R}$, the standard deviation $\sigma$ and the kurtosis $\gamma$.

Table 1: $R_t$ values of the four LEP experiments.
Figure 2: The determination of $R_l$ for each experiment compared to the LEP average value (dashed band)

The distribution of $\bar{R}$ is shown in fig.3 for 10000 generated configurations of 12 measurements.

Figure 3: $\bar{R}$ from the MonteCarlo simulation of 10000 experiments in which the errors are assumed to be purely statistical

As one can see the value for $\bar{R}$ and its $\sigma$ are equal to those of LEP measurement (3) confirming the substantial statistical nature of each individual error. This provides a check of our approximation in neglecting the possible correlations among the errors in Table 1.

Fig. 4, on the other hand, shows that the probability of the initial LEP configuration in Table 1 is extremely small.

The MonteCarlo runs at high statistics ($10^6$ trials) show that the probability to have a result worse than the one observed in this experiment is $3.8 \cdot 10^{-4}$. The fact that the kurtosis distribution does not look what is expected for a gaussian population (null mean value and a symmetric distribution) depends on the fact that the estimator is biassed and only asymptotically gets to the expected value (12 samples are not close to infinity !). This circumstance is only aesthetical and does not affect the point we made.
Figure 4: Distribution of the kurtosis values obtained from the MonteCarlo simulation of 50000 experiments. The arrow shows where the actual LEP results fall.

In conclusion, our analysis indicates that the individual measurements in Table 1 can hardly be considered as belonging to a gaussian population since substantial systematic effects are needed to understand the kurtosis distribution in figure 4. As a consequence, the meaning of the global average in Eq.(3) (and therefore of Eqs.(4,5) ) is not entirely clear. This makes, at best, awkward a safe estimate of the true experimental $R$-value from the data reported in Table 1. The large probability contents for $R \sim 20.92$ and $R \sim 20.66$ (see fig.1 ) suggest that a reliable determination requires to define the error from the spread of the central values in fig.2, i.e. from a full region

$$20.60 \leq R \leq 20.98$$

This range, by itself, allows only a very poor determination of $\alpha_s$

$$0.10 \leq \alpha_s(M_z) \leq 0.15$$

of comparable precision to that attainable from the total and hadronic $Z$ widths and very far from the expected level of precision for LEP experiments.

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