A unified leptoquark model confronted with lepton non-universality in \(B\)-meson decays

T. Faber\textsuperscript{*}, P. Meinzinger\textsuperscript{†} and W. Porod\textsuperscript{‡}

\textit{Institut für Theoretische Physik und Astrophysik, Uni Würzburg, Germany}

M. Hudec\textsuperscript{§} and M. Malinsky\textsuperscript{¶}

\textit{Institute for Particle and Nuclear Physics, Charles University, Czech Republic}

F. Staub\textsuperscript{∗∗}

\textit{Institute for Theoretical Physics (ITP), Karlsruhe Institute of Technology, Ensgesserstraße 7, D-76128 Karlsruhe, Germany and Institute for Nuclear Physics (IKP), Karlsruhe Institute of Technology, Hermann-von-Helmholtz-Platz 1, D-76344 Eggenstein-Leopoldshafen, Germany}

The anomalies in the \(B\)-meson sector, in particular \(R_{K^{(*)}}\) and \(R_{D^{(*)}}\), are often interpreted as hints for physics beyond the Standard Model. To this end, leptoquarks or a heavy \(Z'\) represent the most popular SM extensions which can explain the observations. However, adding these fields by hand is not very satisfactory as it does not address the big questions like a possible embedding into a unified gauge theory. On the other hand, light leptoquarks within a unified framework are challenging due to additional constraints such as lepton flavor violation. The existing accounts typically deal with this issue by providing estimates on the relevant couplings. In this letter we demonstrate that this setup has, in principle, all the potential to accommodate \(R_{K^{(*)}}\) and \(R_{D^{(*)}}\) while respecting bounds from other sectors usually checked in this context. However, it turns out that \(K_{L} \rightarrow \pi^{\pm}\pi^{\mp}\) severely constraints not only the vector but also the scalar leptoquarks and, consequently, also the room for any sizeable deviations of \(R_{K^{(*)}}\) from 1. We briefly comment on the options for extending the model in order to conform this constraint. Moreover, we present a simple criterion for all-orders proton stability within this class of models.

\section{I. INTRODUCTION}

In recent years a few anomalies in the \(B\)-meson sector have been observed by different experiments. The most striking one is a 3.5-\(\sigma\) deviation in the ratios

\[ R_{D^{(*)}} = \frac{\Gamma(\bar{B} \rightarrow D^{(*)}\tau\bar{\nu})}{\Gamma(B \rightarrow D^{(*)}\ell\bar{\nu})} \quad (l = e, \mu), \]

with

\[ R_{D}^{\text{exp}} = 0.388 \pm 0.047, \quad R_{D}^{\text{SM}} = 0.321 \pm 0.021, \]

from the Standard Model (SM) lepton universality expectations

\[ R_{D}^{\text{SM}} = 0.300 \pm 0.010, \quad R_{D}^{\text{SM}} = 0.252 \pm 0.005. \]

This was first reported by BaBar \cite{darmstadt} consistent with measurements by Belle \cite{belle1,belle2}. Recently this has been confirmed by LHCb in case of \(R_{D}\) \cite{lhcb1} at the 2.1-\(\sigma\) level. Additional deviations from lepton universality have recently been reported by LHCb in the ratio

\[ R_{K} = \frac{\Gamma(\bar{B} \rightarrow K\mu^{+}\mu^{-})}{\Gamma(B \rightarrow K e^{+}e^{-})} = 0.745^{+0.090}_{-0.074} \pm 0.036, \]

\[ R_{K^{*}} = \frac{\Gamma(\bar{B} \rightarrow K^{*}\mu^{+}\mu^{-})}{\Gamma(B \rightarrow K^{*}e^{+}e^{-})} = 0.69^{+0.11}_{-0.07} \pm 0.05 \]

in the dilepton invariant mass bin 1 GeV\(^2\) \(\leq q^{2} \leq 6\) GeV\(^2\) \cite{lhcb2,lhcb3}. These ratios are predicted to be 1 within the SM and are practically free from theoretical uncertainties. Equally intriguing is a discrepancy in the angular observables in the rare \(\bar{B} \rightarrow K\mu^{+}\mu^{-}\) decays measured by LHCb \cite{lhcb4} which, however, is subject to significant hadronic uncertainties \cite{lhcb5,lhcb6}. While the individual discrepancies are between 2 and 3 \(\sigma\), they all point in the same direction and amount to more than 4.5-\(\sigma\) deviations once combined in a fit \cite{lhcb7,lhcb8}.

In Refs. \cite{lhcb7,lhcb8} it has been shown that the deviations in \(R_{D}\) and \(R_{K}\) can be explained by an effective model adding one generation of scalar leptoquarks (LQs) with the quantum numbers of the right-handed \(d\)-quark and an additional scalar gauge singlet which couples to the LQs. However, it has been shown that this leads to a too large rate for \(b \rightarrow s\nu\nu\) \cite{lhcb9}. In Ref. \cite{lhcb10} another model with two different LQs, one with gauge quantum numbers of the
right-handed $d$-quark and one with charge $4/3$, has been presented which explains also neutrino masses at the 2-loop level. As has been shown in Refs. [18–25], another possibility to successfully accommodate the data is to use vector LQs. A somewhat more complete model containing two types of vector LQs to explain the two-photon excess, based on a Froggatt-Nielsen ansatz for the required coupling structures, has been presented in [26]. Beside the above mentioned violations of lepton-universality this model is also compatible with the neutrino data. Another possibility is that the required leptoquarks are bound states of strongly interacting fermions [27].

Most of these settings are effective models containing just the pieces required to account for the discussed experimental observations, which is clearly the first logical step to make when a new signal shows up. However, eventually one would like to understand the observations from a more fundamental perspective. Several attempts in this direction exist already in the literature [27,38]. Of course, the most attractive scenario would be a UV completion compatible with theoretical requirements like gauge-coupling unification with the potential to explain also the observed dark matter relic density.

From the GUT perspective the Pati-Salam (PS) model [39] emerges as the first and very natural candidate for a naturally light right-handed neutrino. The PS-breaking should occur above the (possibly heavy) right-handed fermions [27].

In what follows we consider the model proposed by Perez and Wise in Ref. [42]. For convenience, we briefly outline it here, focusing on the features related to flavor physics.

The model is based on the gauge group $G = SU(4)_C \otimes SU(2)_L \otimes U(1)_R$, where the first factor unifies the three colors of quarks with the lepton number. This group is spontaneously broken to $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ and further down to $G_{\text{vac}} = SU(3)_c \otimes U(1)_Q$, following the branching rules

$$Y = R + 1/2 \left[ B - L \right], \quad Q = T^3_L + Y,$$

where $[B - L] = \sqrt{8/3} T^3_C \begin{pmatrix} +\frac{1}{3} , & +\frac{1}{3} , & +\frac{1}{3} , & -1 \end{pmatrix}$.

The matching condition for the QCD coupling at the scale, where $SU(4)_C$ is broken, is simply $g_4 = g_1$.

The entire field content of the model is summarised in Table I. We also include information about other $U(1)$ charges which we need in Appendix A where the details of the baryon number conservation and lepton number violation are discussed.

The SM fermions together with the right-handed neutrinos are combined into three quadruplets under $SU(4)_C$ appearing in three copies representing different generations. On top of that, three fermionic gauge singlets $N$ necessary to generate the correct neutrino masses via inverse seesaw [43] are added.

The gauge field sector corresponding to $SU(4)_C$ consists of the gluons, $Z'$ and a vector leptoquark $X \sim (3,1,\pm \frac{4}{3})$ which mediates flavor violating processes such as $K_L \rightarrow e^\pm \mu^\mp$. This tight constraint implies that, for standard-size couplings, the mass of the vector leptoquark has to be at least 1600 TeV.

The scalar sector consists of three multiplets $\chi$, $\Phi$ and $H$, see Table I. The most general renormalizable scalar potential for these fields reads

$$V = m^2_2 |H|^2 + m^2_2 |\chi|^2 + m^2_{\Phi} \text{Tr}(|\Phi|^2) + \lambda_1 |H|^2 |\chi|^2 + \lambda_2 |H|^2 \text{Tr}(|\Phi|^2) + \lambda_3 |\chi|^2 \text{Tr}(|\Phi|^2) + \lambda_4 H^\dagger \chi^\dagger \Phi^i \chi + \text{h.c.} + \lambda_5 H^\dagger \text{Tr}(\Phi^i \Phi^j) H^j + \lambda_6 \chi^\dagger \Phi^i \chi + \lambda_7 |H|^4 + \lambda_8 |\chi|^4 + \lambda_9 \text{Tr}(|\Phi|^2)^2 + \lambda_{10} (\text{Tr}(|\Phi|^2))^2 + \left( \lambda_{11} H^\dagger_1 \text{Tr}(\Phi^i \Phi^j) H^j_1 + \lambda_{12} H^\dagger_1 \text{Tr}(\Phi^i \Phi^j_1) + \lambda_{13} H^\dagger_1 \text{Tr}(\Phi^i_1 \Phi^j) + \text{h.c.} \right) \right.$$

$$+ \lambda_{14} |\Phi^i|^2 \chi + \lambda_{15} \text{Tr}(\Phi^i_1 \Phi^j_1) + \lambda_{16} \text{Tr}(\Phi^i_1 \Phi^j) \text{Tr}(\Phi^i_1 \Phi^j) + \lambda_{17} \text{Tr}(\Phi^i_1 \Phi^j) \text{Tr}(\Phi^i_1 \Phi^j) + \lambda_{18} \text{Tr}(\Phi^i_1 \Phi^j_1) + \lambda_{19} \text{Tr}(\Phi^i_1 \Phi^j_1) \Phi^i \Phi^j$$

with $|H|^2 = H^\dagger_1 H^1$, $|\chi|^2 = \chi^\dagger_1 \chi$, $|\Phi|^2 = \Phi^i \Phi^i$ where $i,j$ are the $SU(2)$ indices. The trace is taken over the $SU(4)_C$ indices only. Notice that the terms proportional to $\lambda_{11}, \ldots, \lambda_{19}$ have been omitted in the original paper [42]: we include them here for completeness. However, for simplicity, in the numerical analysis in Sect. III we stick to the original setting [42] with the extra couplings deliberately set to zero.

The breaking of the $SU(4)_C$ group as well as the electroweak symmetry breaking is triggered by the corre-
\[ F_L = \begin{pmatrix} Q \\ L \end{pmatrix} \]

\[ f_{u} = \begin{pmatrix} u^c \\ \nu^c \end{pmatrix} \]

\[ f_{d} = \begin{pmatrix} d^c \\ e^c \end{pmatrix} \]

\[ N = \begin{pmatrix} (1,1,0) \end{pmatrix} \]

\[ \chi = \begin{pmatrix} (S_1^0) \\ (\chi^0) \end{pmatrix} \]

\[ \Phi = \begin{pmatrix} G \\ R_2 \\ 0 \end{pmatrix} + \sqrt{2} T^{13} H_2 \]

\[ A_\mu = \begin{pmatrix} (G_\mu \times \mu) \\ +T^{13}B_\mu \end{pmatrix} \]

\[ W_\mu = \begin{pmatrix} (1,3,0) \end{pmatrix} \]

\[ B_\mu = \begin{pmatrix} (1,1,0) \end{pmatrix} \]

**Gaugino Masses**

\[ M_{\tilde{G}} = \begin{pmatrix} (3,2,1/6) \end{pmatrix} \]

**Gravitino Masses**

\[ M_{\tilde{W}} = \begin{pmatrix} (3,1,2/3) \end{pmatrix} \]

\[ M_{\tilde{B}} = \begin{pmatrix} (1,0) \end{pmatrix} \]

\[ M_{\tilde{\chi}} = \begin{pmatrix} (1,1,0) \end{pmatrix} \]

\[ (\chi) = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \]

\[ (\Phi) = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \]

which are parametrised by \( v_1 = v_{ew} \sin \beta \) and \( v_2 = v_{ew} \cos \beta \), with \( v_{ew} \approx 246 \) GeV. The SM-like Higgs \( h \) is a superposition of the fields \( Re\{H_1^0, H_2^0, \chi^0\} \).

The fermion masses are generated by the following interactions between the scalars and fermions

\[ -\mathcal{L}_Y = f_{u}Y_{1}H_{1}F_{L} + f_{u}Y_{2}\Phi F_{L} + f_{d}Y_{3}H_{1}F_{L} + f_{d}Y_{4}\chi H_{1}F_{L} + f_{d}Y_{5}\chi F_{L} + \frac{1}{2} N \mu N + \text{h.c.} \]

In the broken phase, this leads to the following relations between the mass matrices for the SM fermions and the underlying Yukawa matrices:

\[ U_{u}^\dagger \tilde{m}_{u} U_{u} = \frac{v_{1}}{\sqrt{2}} Y_{1} + \frac{v_{2}}{2\sqrt{6}} Y_{2} \]

\[ U_{d}^\dagger \tilde{m}_{d} U_{d} = \frac{v_{1}}{\sqrt{2}} Y_{3} + \frac{v_{2}}{2\sqrt{6}} Y_{4} \]

Here \( \tilde{M} \) are diagonal and \( U \) and \( V \) are unitary matrices describing the relation between the gauge and mass eigenstates. We work in a basis where the lepton mass matrix is flavor-diagonal. The only constraints on \( U \)'s and \( V \)'s are that \( V_{CR} = V_{CR}^{\dagger} \) must be reproduced. In the current study, we shall assume that all the Yukawa matrices are symmetric in the flavor space and, hence, \( U_{u} = V_{u} \) and \( U_{d} = V_{d}^{\dagger} \). Besides simplicity, this is motivated by the idea that this model might eventually be embedded in a variant of the minimal \( SO(10) \) framework (such as proposed in Ref. [23]). Thus, we are left with just one mixing matrix \( V_{d} \), which we can choose freely. As we will see, this freedom is crucial for accommodating the \( B \) anomalies without violating other constraints from lepton flavor violating observables.
In the neutrino sector we have a $9 \times 9$ complex symmetric matrix in the basis $(\nu, \nu^c, N)$ which should yield the light-neutrino PMNS matrix as well as their measured mass differences.

The scalar leptoquarks in this model reside in $\Phi$ and $\chi$. In total, there are four physical scalars together with the would-be-Goldstone boson associated to the vector leptoquark. However, just the mixture of $R_2^{2/3}$, $R_2^{3/2}/\chi$ and $S_{1}$ is relevant to addressing the $B$-meson anomalies; the masses of the two physical eigenstates (to be called $R_{1,2}^{\prime}$) are approximately given by

$$m_{R_{1,2}^{\prime}}^2 = \left(\frac{\sqrt{3}}{4} \lambda_4 \tan \beta - \frac{\lambda_6}{8}\right) v_\chi^2 \pm \frac{\lambda_6}{4} v_\chi^2 + O(v_{cw})^2. \tag{13}$$

It is well known that for a proper explanation of $R_{K^(*)}$ the lighter one should have dominant $R_2^{2/3}$ component.

### III. LOW ENERGY OBSERVABLES

We turn now to a discussion of the relevant low-energy observables in the current model. First of all, we want to explain the observed deviation from the lepton universality in the $B$-meson decays. The Feynman diagram responsible for the tree-level contributions to $R_K$ via the scalar leptoquarks is depicted in Fig. 1. It is important to notice, however, that the same leptoquark which should explain the $B$-meson anomalies would also contribute to other observables. At the tree level, one can expect an impact on other meson observables like $B \rightarrow X_{s} \nu \nu$, $B_{s}^{0} \rightarrow l l$ or $K_{L} \rightarrow e \mu$. Moreover, there are important loop contributions to $B \rightarrow X_{s} \gamma$, $l \rightarrow l^' \gamma$ and $l \rightarrow 3l'$. An example of the responsible Feynman diagrams is shown in Fig. 2.

For our numerical study we used the Mathematica package SARAH\textsuperscript{10} [50] and extended\textsuperscript{3} it to support the model under consideration. In the first step, we used the model files to produce a spectrum generator based on SPheno [52] [53]. SPheno calculates the mass spectrum providing the option to include all one-loop and the important two-loop corrections to neutral scalar masses [54] [56] in the DR or the MS scheme. However, we are assuming here a full on-shell calculation of all masses, i.e., all shifts can be absorbed into counter-terms of the couplings leaving the mass spectrum unchanged.

In addition, SPheno provides an interface to HiggsBounds [57] [59] which we used to check the constraints on the neutral scalars. Moreover, SPheno calculates electroweak precision as well as flavor observables. The calculation of flavor observables is based on the FlavorKit functionality presented in Ref. [60]. We used this feature in order to calculate the values for all necessary lepton flavor violating observables including $K_{L} \rightarrow e \mu$. Moreover, the values of the Wilson coefficients relevant for the $B$-physics calculated by SPheno were passed to flavio [61] to obtain predictions for the $B$-physics observables.

In Table II we collect the input parameters for this study. The remaining parameters affect the heavy states which do not contribute to the observables discussed below. For the fermions we take as input the known quark and lepton masses, the CKM and the PMNS matrices using the best fit values reported in [62], the Yukawa couplings $Y_{2}$, $Y_{5}$ and $\tan \beta$. For an explanation of the $B$-physics observables we need an off-diagonal structure

\begin{table}
\centering
\begin{tabular}{|c|c|}
\hline
\textbf{Numerical input values} & \\
\hline
$Y_{2}$ & $\text{diag}(10^{-8}, 10^{-7}, 10^{-5})$. \\
$Y_{5}$ & $\text{diag}(10^{-2}, 5 \cdot 10^{-2}, 10^{-1})$. \\
$\theta_{12}$, $\theta_{13}$, $\theta_{23}$ & $\pi/2, 0, \pi/4$. \\
$v_\chi$ & $4 \cdot 10^{8}$ GeV. \\
m$_{A}$, $m_{R^{\prime}}$ & $2 \cdot 10^{5}$ GeV, 900 GeV. \\
$\tan \beta$ & 50. \\
\hline
\end{tabular}
\caption{Summary of the default input values used in this analysis except if stated otherwise. All other BSM scalars have masses of the order $O(m_{A})$.}
\end{table}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig1.pdf}
\caption{Tree-level contribution to $R_K$ via leptoquarks $R'$.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig2.pdf}
\caption{One-loop contribution to $\mu \rightarrow 3e$ via leptoquarks $R'$.}
\end{figure}

\textsuperscript{3} Details about the new feature to support unbroken subgroups in SARAH will be given in [51].
in $Y_d$. We therefore take $V_d$ as input, parametrizing it as

$$V_d = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix} \begin{pmatrix}
c_{13} & 0 & s_{13}e^{i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta} & 0 & c_{13}
\end{pmatrix} \begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix},$$

(14)
denoting $c_{ij} = \cos \phi_{ij}$ and $s_{ij} = \sin \phi_{ij}$, and vary all three angles in the range $[0, \pi]$ with $\delta = 0$ for simplicity. In the scalar sector we take the $SU(4)_C$-breaking VEV $v_\chi$ and the overall scale ($m_A$) of the heavy integer-charge Higgs bosons as input and set $\lambda_6$ such that the $R'_1 \simeq R_2$ leptoquark remains light. With $R'_0 \simeq R_2$ heavy, the mixing between the charge-2 components is negligibly small. Moreover, we fix the mass of the SM-like Higgs boson to 125.1 GeV by adjusting $\lambda_7$ accordingly. As we take the heavy Higgs bosons to be in the multi-TeV range, we are in the decoupling limit and fulfill automatically the experimental constraints on the observed Higgs boson.

As long as $R_D$ and $R_{D'}$ is concerned, there are in principle two ways to accommodate the data in the current scenario: (i) The model automatically contains a vector leptoquark of a suitable type [32]; however, its potential effects in $R_{D(\gamma)}$ are strongly suppressed by the need to be compatible with $K_L \to \mu e$ which pushes its allowed mass above 1600 TeV. (ii) A way larger effect than (i) is expected if the two scalar $SU(2)$-doublet leptoquarks in the spectrum remain light and, at the same time, their charge +2/3 components entertain a large mixing. This, however, is difficult to do in the current setting as it calls for a significant tuning of the scalar potential parameters; at the same time, BR$(\mu \to e\gamma)$ is driven way above the experimental limit. Hence, the simple model at stakes has serious issues with accommodating the existing $B \to D^{(*)}\ell\nu$ data.

By contrast, it is fairly easy to explain the currently observed values for $R_K$ and $R_{K^*}$ as demonstrated in Fig. 3. Here we have fixed the input parameters as given in Table II and varied one of the relevant angles. The new physics contributions to the Wilson coefficients are given by

$$C_{9,R_2}^{NP}ij = C_{10,R_2}^{NP}ij = -\frac{1}{4\,m_{R_1}^2} \left( Y_{4d}^\dagger \right)_{ij} \left( Y_{4d} \right)_{2j},$$

(15)
where $i = j = 1, 2$ for the contribution to $b \to see$ and $b \to s\mu\mu$, respectively. Using Eqs. (11) and (12) we calculate the Yukawa couplings $Y_3$ and $Y_4$ in terms of the lepton and quark masses as well as $V_d$. Exploiting the hierarchy in the fermion mass spectrum we get, to a good approximation,

$$C_{9,R_2}^{NPee} \simeq -\frac{3m_b m_e}{128 m_{R_1}^2 v_{ew}^2} (1 + \tan^2 \beta) f_-, $$

(16)

$$C_{9,R_2}^{NP\mu\mu} \simeq \frac{3m_b (m_\mu - m_e)}{128 m_{R_1}^2 v_{ew}^2} (1 + \tan^2 \beta) f_+,$$

(17)

with

$$f_\pm = 4 \cos 2\phi_{23} \sin 2\phi_{12} \sin \phi_{13} + \sin 2\phi_{23} (3 \cos 2\phi_{12} \pm 1 \pm 2 \cos 2\phi_{13} \sin \phi_{12}).$$

(18)

Combining this with

$$R_K \simeq 1 + \frac{1}{C_{9,R_2}^{NPee}^2} \left( |C_{9,R_2}^{NP\mu\mu}|^2 - |C_{9,R_2}^{NPee}|^2 \right),$$

(19)
from [63] one obtains an excellent analytic approximation to the numerical results shown in Fig. 3.

In Fig. 4 we display the contour lines $R_K$ and $R_{K^*}$ in the $m_{R_1}$-tan $\beta$ plane. The shaded regions indicate the 1-$\sigma$ regions consistent with present data. The increase of tan $\beta$ with increasing $m_{R_1}$ can easily be understood from Eqs. (16)–(17) as the new physics contributions scale as

$$\frac{Y_{4d}^2}{m_{R_1}^2} \propto \tan^2 \beta$$

(20)

We recall that the leptoquarks in general also contribute to lepton-flavor violating decays of the muon such as $\mu \to e\gamma$, see for instance [64]. There are two main
FIG. 4. Contour plots of $R_K$ and $R_{K^*}$ as a function of the leptoquark mass and $\tan \beta$. The shaded regions indicate the range preferred by current data. The remaining parameters are given in Table II.

FIG. 5. $\text{BR}(\mu \rightarrow e\gamma)$ as a function of $s$ which rescales the values of $Y_2$ in Table II. The other parameters are fixed as before.

Contributions to this observable in the current setting coming, namely, from heavy-neutrino and $W$-boson loops as well as leptoquark and quark loops. As an example, in Fig. 5 we show $\text{BR}(\mu \rightarrow e\gamma)$ as a function of an extra factor $s$ rescaling the $Y_2$ eigenvalues in Table II into $Y_2 \rightarrow sY_2$. For $s \gtrsim 5$ the leptoquark loops dominate whereas for $s \leq 1$ the neutrino loops are dominant. The narrow minimum is due to a negative interference between both contributions.\footnote{In principle, $s$ (and, hence, $Y_2$) may be even larger than that indicated in Fig. 5 if off-diagonal elements of $Y_5$ were invoked together with this negative interference. However, as $Y_2$ does not enter the calculation of $R_{K^*(\epsilon)}$ at the lowest order, we do not investigate this further.}

FIG. 6. Correlation between $\text{BR}(\mu \rightarrow e\gamma)$, $\text{BR}(\mu \rightarrow 3e)$ and $R_K$ obtained by varying $\phi_{12}, \phi_{13}, \phi_{23}$ in equal steps of $\pi/27$ in the range $[0, \pi]$ each. The other parameters are fixed as given in Table II. The color code indicates the value of $R_{K^*}$.

FIG. 7. Correlation between $\text{BR}(K_L \rightarrow e\mu)$ and $R_K$ obtained by varying $\phi_{ij}$ as in Fig. 6. The other parameters are fixed as given in Table II. The color code indicates the value of $R_{K^*}$ for each point.

Let us point out that in the current model the bounds from $\mu \rightarrow 3e$ are in general stronger than those from $\mu \rightarrow e\gamma$ in the range interesting for $R_K$ and $R_{K^*}$, see Fig. 6. The main reason for this is the negative interfer-
ence in $\mu \rightarrow e\gamma$ discussed above which does neither take place in the $Z$-penguins nor in the box-contributions to $\mu \rightarrow 3e$ (see Fig. [2]) for the same set of parameters.

In addition, we have checked that rare $\tau$ decays do not impose any constraints in the $R_{K^{(*)}}$-interesting regions. The same holds for rare $b$-decays, such as $B_{s,d} \rightarrow \mu^{+}\mu^{-}$, $B \rightarrow s\tau$, $B \rightarrow X_s\nu\nu$. Taking only the couplings of the scalar sector given in ref. [12] one would get a tight correlation between the masses of the scalar leptoquark of interest [13] and the scalar gluons leading to too large contributions to $\Delta M_K$ and $\Delta M_B$. However, this relationship gets broken by $\lambda_{14}$ in Eq. [7] implying that also these bounds can be avoided.

In any case, there is another stringent constraint to be considered, namely the bound on $K_L \rightarrow e\mu$. This mode is usually used as a limit on the mass of vector leptoquarks but it is typically not being taken into account for the scalar ones. As can be seen in Fig. [7] in the interesting region for $R_K$ the bound is violated by several orders of magnitude, thus ruling out this model even in those tuned parts of the parameter space where all other constraints can be satisfied.

It has been argued in [32] that additional fermions in vector-like representations of the gauge group can reduce the couplings of the SM-fermions to vector leptoquarks which, in turn, may be used for lowering the generic experimental limits for their masses. The same mechanism can in principle work also for the couplings of the scalar leptoquarks such that the constraints due to $K_L \rightarrow e\mu$ can be satisfied. However, a detailed exploration of this aspect is beyond the scope of this paper and will be elaborated on elsewhere [11].

We note for completeness that leptoquarks with masses of about 1 TeV are already constrained by the LHC searches. These, however, typically focus on the situation when the decays are dominated by one channel; in the scenarios where the LQs interact through multiple couplings the corresponding bounds must be re-evaluated.

We would also like to point out that the “LQ beta-decay modes”, i.e., the decays of a heavier LQ into its lighter $SU(2)_L$ companion and $W$ have not been considered so far within the collider searches. Nevertheless, depending on the exact mass splitting, they may be of significant interest, especially if the on-shell $W$ production is kinematically allowed[5].

IV. DISCUSSION AND CONCLUSIONS

Motivated by the successful attempts to explain the observed values of $R_{D^{(*)}}$ and $R_{K^{(*)}}$ by leptoquarks we study a unified $SU(4)_C \otimes SU(2)_L \otimes U(1)_{R}$ gauge model in order to demonstrate the challenges one faces when all additional relations inherent to a unified scenario are taken into account. Among these, the dominant role is typically played by the constraints on the Yukawa couplings from the quark and lepton masses and mixing data and/or the tight connection between the relevant gauge coupling and $\alpha_s$.

The model under consideration contains three different types of leptoquarks: a hypercharge-2/3 vector leptoquark and a pair of scalar leptoquarks with hypercharges 1/6 and 7/6. In its minimal version, with the SM fermion sector extended such that it supports the inverse-seesaw mechanism for neutrinos, one finds [42] that the kaon physics constrains the mass of the vector leptoquark to such an extent that it cannot significantly impact the $B$-physics observables.

We have shown that in the setting under consideration one can get the hypercharge-7/6 scalar leptoquark in the TeV range while, at the same time, have automatically the hypercharge-1/6 scalar leptoquark rather heavy. This feature implies that one can accommodate $R_{K^{(*)}}$ but not $R_{D^{(*)}}$. Furthermore, the allowed parameter space gets severely constrained by the bounds on rare monoo decays; in particular, $\mu \rightarrow 3e$ is more important than $\mu \rightarrow e\gamma$. Neither lepton flavor violating $\tau$-decays nor other $B$-physics observables lead to additional constraints. However, it turns out that no points in the available parameter space are compatible with $K_L \rightarrow e\mu$.

On the other hand, this does not imply that this kind of a leptoquark model is ruled out straight away as an explanation of $R_{K^{(*)}}$, because one can always enlarge the fermion sector by vector-like representations. In this way one may in principle reduce the couplings to the $d$-quark by mixing effects and, thus, avoid the bound due to $K_L \rightarrow e\mu$. This goes beyond the scope of this paper and will be elaborated on in a future study.

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Appendix A: Baryon number conservation

It has been noted in [42] that the model has an approximate extra $U(1)_F$ symmetry corresponding to the fermion number $F$, which is explicitly broken by the Majorana mass term for $N$. However, there is another independent accidental global symmetry $U(1)_M$, the charges

5 Let us note that in the model of our interest the LQ-doublet mass splitting would be below $m_W$ if we were to consider only the potential given in [42] but can be larger once the additional terms as in Eq. [8] are included.
of which are
\begin{align}
M(F_L^\alpha) &= M(\chi^\alpha) = +1, \\
M(f_{u_R}^c) &= M(f_{d_R}^c) = -1, \\
M(\Phi_\beta) &= M(H) = M(N) = 0, \\
M(A_{\mu}^\alpha) &= M(W_\mu) = M(B_\mu) = 0.
\end{align}
(A1)

Here \(\alpha, \beta\) denote \(SU(4)\) vector-like indices. Notice that we can obtain the \(M\)-charges of all the field multiplets by the prescription
\[ M = (\# \text{ upper } SU(4) \text{ indices}) - (\# \text{ lower } SU(4) \text{ indices}). \]
(A2)

The \(U(1)_M\) symmetry of each term in the Lagrangian is then guaranteed by the fact that every upper \(SU(4)\) index is contracted to a lower one, all carried by the dynamical fields. It is also clear that any hypothetical \(M\)-violating but \(SU(4)\)-preserving term necessarily contains the antisymmetric tensor \(\varepsilon_{\alpha \beta \gamma \delta}\). Hence, such type of a symmetry is realized in any \(SU(4)\) model whose field content does not allow for the \(SU(4)\) Levi-Civita symbol to occur in the interaction Lagrangian at the renormalizable level; for example, in \([12]\), the corresponding number is called \(B'\).

Having the \(M\)-charge at hand, we can combine it with the gauge charge \([\alpha]\) as
\[ B = \frac{1}{4}(M + [B-L]), \]
(A3)
which obviously yields the baryon number (see Table \([\alpha]\)).

As one can verify readily, in the model under consideration both \([B-L]\) and \(M\) are spontaneously broken by \(\langle \chi^0 \rangle\) whilst their sum \([A3]\) remains a good symmetry even in the asymmetric phase.

On a more general ground, one can rephrase the same argument as follows: If \(M\) defined as \([A2]\) is a good symmetry of the unbroken-phase theory, there is no \(SU(3)_c\) Levi-Civita tensor in its Lagrangian \(L_3\). This means that there is no \(SU(3)_c\) Levi-Civita tensor in the broken-phase Lagrangian \(L_3\) either. Consequently, a global charge defined as
\[ 3B = (\# \text{ upper } SU(3) \text{ indices}) - (\# \text{ lower } SU(3) \text{ indices}) \]
(A4)
generates a good symmetry of \(L_3\) and, hence, \(B\) – the usual SM baryon number – is perturbatively conserved.

Let us also note that there are two slightly different candidates for the lepton number, none of which is, however, related to a fully conserved quantity in our model. The first option is intuitive,
\[ L = B - [B-L]. \]
(A5)
Here, \(U(1)_L\) is a good symmetry of the classical action but it is spontaneously broken, together with \([B-L]\), by \(\langle \chi \rangle\). The alternative,
\[ L' = F - 3B, \]
(A6)
is, on the other hand, preserved by the vacuum but explicitly broken by the Majorana mass term because \(F\) is so.

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