SOME APPLICATIONS OF GROTHENDIECK DUALITY THEOREM

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Abstract. In this paper, we systematically apply Grothendieck duality theorem to simplify and reprove several theorems in different papers: Including a vanishing theorem in KMM\[18\], a theorem of Kollár’s paper \[10\], a vanishing theorem due to Kovács in \[17\] and a theorem of Fujino in \[5\]. We remark that all of the above are achieved by the same trick.

Introduction

We first briefly introduce the trick that will be used in this paper. Given a resolution $f : Y \to X$, on $Y$ there are effective divisor $E$ and some integral divisor $B$ we are interested in. Consider the following diagram of complex,

$$
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
f_* B \xrightarrow{\alpha} Rf_* B \\
\downarrow\beta \\
Rf_*(B + E)
\end{array}
\end{array}
\end{array}
$$

We show that $\beta$ is a quasi-isomorphism by Kawamata-Viehweg vanishing theorem. Dualize this diagram

$$
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
R\text{Hom}(f_* B, \omega_X) \xrightarrow{\phi^*} R\text{Hom}(Rf_* B, \omega_X) \xrightarrow{\psi^*} Rf_* \omega_Y(-B)[n]
\end{array} \\
\downarrow \beta^* \\
R\text{Hom}(Rf_*(B + E), \omega_X) \xrightarrow{\phi^*} Rf_* \omega_Y(-B - E)[n]
\end{array}
\end{array}
\end{array}
$$

where $\phi^*$ and $\psi^*$ are quasi-isomorphism by duality theorem. Then we use some vanishing theorem to get information of the map $\gamma$. For instance, if $B = 0$, the by Grauert vanishing $\gamma$ is a zero map except at the $-n$ degree. Then by the commutativity of the diagram and the fact that $\beta$ and $\psi^*$ are isomorphism, we conclude that $R\text{Hom}(f_* B, \omega_X)$ and the complex in the second row are zero except at the $-n$ degree.

For the reader’s convenience we recall Grothendieck duality theorem and fix the notations in this paper. Let $f : Y \to X$ be a proper morphism between two projective schemes. Then for any $\mathcal{F} \in D^-_{qcoh}(Y)$, we have

$$Rf_* R\text{Hom}_Y(\mathcal{F}, \omega_Y) \cong R\text{Hom}_X(Rf_* \mathcal{F}, \omega_X)$$

For a Cohen-Macaulay scheme $Y$ of dimension $n$, we have $\omega_Y \cong \omega_Y[n]$. In the whole paper, we let $n$ denote the dimension of all the varieties. Since we only deal

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with birational morphisms, this will not cause any confusion. If \( Y \) is normal, then 
\[ h^{-n}(\omega_Y[n]) = \mathcal{O}_Y(K_Y), \]
the extension of regular \( d \)-forms on smooth locus.

The organization of this paper goes as follows. In section one we prove a vanish-
ing theorem in KMM, and in section two we consider the log J terminal singularities. 
In the last section, we first prove a vanishing theorem of dlt pair and then apply it 
to the proofs of the theorems mentioned at the beginning. In the whole paper we 
work over complex numbers.

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questions via email.

1. A vanishing theorem in KMM

In this section we reprove Theorem 1-3-1 in [18], which is a vanishing theorem 
due to Elkik [3] and Fujita [9] in a generalized form. Besides Grothendieck duality 
theorem, the original proof also used spectral sequence and local duality.

**Theorem 1.1.** Let \( f : Y \to X \) be a proper birational morphism from a smooth 
variety \( Y \) to a variety \( X \) with divisor \( L, \tilde{L} \) on \( Y \). Assume there exist \( \mathbb{Q} \) divisors 
\( D, \tilde{D} \) and an effective divisor \( E \) on \( Y \) such that the following conditions are satisfied:

1. \( \text{supp } D \) and \( \text{supp } \tilde{D} \) are simple normal crossing and \( \lfloor D \rfloor = \lfloor \tilde{D} \rfloor = 0 \);
2. both \( -L - D \) and \( -\tilde{L} - \tilde{D} \) are \( f \)-nef;
3. \( K_Y = L + \tilde{L} + E \);
4. \( E \) is exceptional for \( f \).

Then, \( R^i f_* L = 0 \) for \( i > 0 \).

**Proof.** Consider the following diagram of complexes,

\[
\begin{array}{ccc}
  f_* \mathcal{O}_Y(L) & \xrightarrow{\alpha} & Rf_* \mathcal{O}_Y(L) \\
  \beta & & \downarrow \\
  & & Rf_* \mathcal{O}_Y(L + E)
\end{array}
\]

where \( f_* \mathcal{O}_Y(L) \) denotes the complex with the sheaf \( f_* \mathcal{O}_Y(L) \) at degree zero and 
zero elsewhere.

Here \( \beta \) is a quasi-isomorphism, since

\[
L + E = K_Y - \tilde{L} \\
= K_Y - \tilde{L} - \tilde{D} + \tilde{D} \\
= K_Y + (f\text{-nef and big})+ \text{fractional part.}
\]

By Kawamata-Viehweg vanishing we have \( R^i f_* \mathcal{O}_Y(L + E) = 0 \) for all \( i > 0 \). Also \( R^0 f_* \mathcal{O}_Y(L + E) = f_* \mathcal{O}_Y(L) \) by Lemma [12].

Apply duality to the diagram above we have

\[
\begin{array}{ccc}
  R\text{Hom}(f_* \mathcal{O}_Y(L), \omega_X^*) & \xleftarrow{\alpha^*} & R\text{Hom}(Rf_* \mathcal{O}_Y(L), \omega_X^*) \\
  \beta^* & & \downarrow \\
  R\text{Hom}(Rf_* \mathcal{O}_Y(L + E), \omega_X^*) & \xrightarrow{\psi^*} & Rf_* \omega_Y(-L)[n]
\end{array}
\]
where $\phi^*$ and $\psi^*$ are isomorphism by duality theorem. By the commutativity of this diagram we have

$$\beta^* \circ \psi^* = \alpha^* \circ \phi^* \circ \gamma$$

Note that $K_Y - L = K_Y - L - D + D$, so by Kawamata-Viehweg vanishing theorem we have $R^i f_* K_Y (-L) = 0$ if $i > 0$. In other words, the right hand side map is zero at the positive degree part. Since the left hand side is a quasi-isomorphism, we conclude that $R^i f_* K_Y (-L - E) = 0$ if $i > 0$. This implies $R^i f_* L = 0$ by condition (3), and then by symmetry we also have $R^i f_* L = 0$.

**Lemma 1.2.** (Lemma 1-3-2 in [18]) Let $f : Y \rightarrow X$ be a proper birational morphism from a nonsingular variety $Y$ onto $X$, $L$ be a line bund on $Y$ and $D$ a $\mathbb{Q}$-divisor. Assume that $\text{supp } D$ is simple normal crossing, $\lfloor D \rfloor = 0$, and $-L - D$ is $f$-nef. Then for any exceptional effective divisor $E$ on $Y$, we have

$$f_* \mathcal{O}_E (L + E) = 0$$

2. A vanishing theorem of dlt pair

In this section we prove a vanishing theorem of a dlt pair $(X, \Delta)$, where $X$ is a normal variety. We first prove a special case (Theorem 2.1) to illustrate the idea and then consider the more general case (Theorem 2.4). Theorem 2.1 is in fact a special case of Kovács's vanishing theorem, but use Fujino's idea we can also prove it (Corollary 2.10). After finished this paper, the author learned that Kollár and Kovács had also proved Theorem 2.4 by the notion of rational pair, which will appear in [11]. Base on Theorem 2.4 we prove several corollaries mentioned in the introduction.

One of the equivalent definitions of dlt singularities is that there is a log resolution (Szabó resolution [21]) $f : Y \rightarrow X$ such that the discrepancy $a(E; X, \Delta) > -1$ for any exceptional divisor $E$ on $Y$ (Theorem 2.44 in [13]).

**Theorem 2.1.** Let $(X, \Delta_X)$ be a dlt pair and let $f : Y \rightarrow X$ be a Szabó resolution. Then we can write

$$K_Y + \Delta_Y = f^*(K_X + \Delta_X) + P - Q,$$

where $P, Q$ are effective exceptional divisors, $\lfloor Q \rfloor = 0$ and $\Delta_Y$ is the strict transform of $\Delta_X$. Then we have

$$R^i f_* \mathcal{O}_Y (-\lfloor \Delta_Y \rfloor) = 0$$

for every $i > 0$.

**Proof.** Write

$$K_Y - f^*(K_X + \Delta_X) + \Delta_Y = P - Q,$$

Define

$$B = [P] = K_Y - f^*(K_X + \Delta_X) + \Delta_Y + Q + [P] - P,$$

then $B$ is $f$-exceptional and effective. Consider the following diagram,

$$\begin{array}{ccc}
  f_* \mathcal{O}_Y (-\lfloor \Delta_Y \rfloor) & \xrightarrow{\alpha} & Rf_* \mathcal{O}_Y (-\lfloor \Delta_Y \rfloor) \\
  \downarrow \beta & & \downarrow \\
  Rf_* \mathcal{O}_Y (B - \lfloor \Delta_Y \rfloor) & & \\
\end{array}$$

Note that

$$B - \lfloor \Delta_Y \rfloor = K_Y - f^*(K_X + \Delta_X) + \delta,$$
where $\delta$ is some effective simple normal crossing divisors such that $|\delta| = 0$. So by Kawamata vanishing $R^i f_*\mathcal{O}_Y(B - |\Delta_Y|) = 0$ for $i > 0$. On the other hand, we know that $X$ is normal so $f_*\mathcal{O}_Y(B - |\Delta_Y|) = f_*\mathcal{O}_Y(-|\Delta_Y|)$. So $\beta$ is in fact isomorphism.

Dualize this diagram and by relative duality we have
\[
\begin{array}{c}
\text{RHom}(f_*\mathcal{O}_Y(-|\Delta_Y|), \omega_X) \\
\downarrow \text{RHom}(Rf_*\mathcal{O}_Y(-|\Delta_Y|), \omega_X) \\
\text{RHom}(Rf_*\mathcal{O}_Y(B - |\Delta_Y|), \omega_X) \\
\end{array}
\]
where $\phi^*$ and $\psi^*$ are isomorphism by duality. In the future discussion, we call it diagram $A$.

By lemma 2.2, $R^i f_*\mathcal{O}_Y(|\Delta_Y|) = 0$ for $i > 0$, so $\gamma$ is a zero map except at degree $-n$. Since the isomorphism $\beta^* \circ \psi^*$ factors through $\gamma$, so take the $-n$th cohomology we then have the following diagram
\[
\begin{array}{c}
\text{Ext}^{-n}(f_*\mathcal{O}_Y(-|\Delta_Y|), \omega_X) \\
\downarrow \text{Ext}^{-n}(f_*\mathcal{O}_Y(-|\Delta_Y|), \omega_X) \\
\text{Ext}^{-n}(f_*\mathcal{O}_Y(|\Delta_Y| - B), \omega_X)
\end{array}
\]
Since every sheaf in this diagram is torsion free with rank one, $\gamma$ and $\alpha^* \circ \phi^*$ are both isomorphisms.

Dualize diagram $A$, since $\gamma$ is a quasi-isomorphism we have
\[
\begin{array}{c}
f_*\mathcal{O}_Y(-|\Delta_Y|) \\
\downarrow \text{RHom}(Rf_*\mathcal{O}_Y(-|\Delta_Y|), \omega_X) \\
\text{RHom}(Rf_*\mathcal{O}_Y(|\Delta_Y| - B), \omega_X)
\end{array}
\]
By this diagram, we conclude that $\alpha$ is an isomorphism, or equivalently,
\[
R^i f_*\mathcal{O}_Y(-|\Delta_Y|) = 0
\]
for every $i > 0$.

**Lemma 2.2.** Let $f : Y \to X$ be a proper morphism from a smooth variety $Y$, with a reduced simple normal crossing boundary $B$. Given a $f$-nef and big line bundle $L$ on $Y$ such that $L$ is also $f$-big at all the lc (log canonical) centers of $(Y, B)$. Then $R^i f_*K_Y(B + L) = 0$ for all $i > 0$. Particularly in our case, $R^i f_*K_Y(|\Delta_Y|) = 0$ for all $i > 0$.

**Proof.** Take an irreducible component $S \subseteq B$, then consider
\[
0 \to K_Y((B - S) + L) \to K_Y(B + L) \to K_S(B - S + L) |_S \to 0.
\]
Since $S$ is a lc center of $(Y, B)$, $L |_S$ is $f$-nef and big on $S$ by assumption. The lc center of $(S, (B - S) |_S)$ is the restriction from $(Y, B)$, so $L |_S$ is $f$-big on all of the lc centers of $(S, (B - S) |_S)$. As a result, by induction on dimension we conclude that $R^i f_*(K_S(B - S + L) |_S) = 0$.

On the other hand, if there is no irreducible component in $B$ then the statement is the classical Kawamata- Viehweg vanishing theorem. So by induction on the number of irreducible components of $B$, we conclude that $R^i f_*K_Y((B - S) + L) = 0$. Then after pushing forward the exact sequence, we see that $R^i f_*K_Y(B + L) = 0$. 

\[
\square
\]
In the case of Theorem 2.1 we consider the pair \((Y, |\Delta|)\) and the line bundle \(L\) is \(\mathcal{O}_Y\). Since \(f\) is birational, \(\mathcal{O}_Y\) is automatically \(f\)-nef and big. Since \(f\) is isomorphism at the generic point of \(|\Delta|\), \(\mathcal{O}_Y\) is \(f\)-big on the lc center. So \(R^if_*K_Y(|\Delta_Y|) = 0\) for all \(i > 0\).

\[\square\]

Remark 2.3. Lemma 2.2 is in fact a vanishing lemma of Reid- Fukuda type, see for example [5] Lemma 4.10. We include the proof here for the reader’s convenience.

More generally, Theorem 2.1 is true for any subset of \(\Delta_Y\) by the following theorem.

Theorem 2.4. Let \((X, \Delta_X)\) be a dlt pair and let \(f : Y \to X\) be a Szabó resolution. Notation as Theorem 2.1, then for any subset \(\Delta_Y^1 \subset \Delta_Y\), we have

\[R^if_*\mathcal{O}_Y(-|\Delta_Y^1|) = 0\]

for every \(i > 0\).

Proof. Write

\[K_Y + \Delta_Y = f^*(K_X + \Delta_X) + P - Q,\]

and let \(\Delta_Y^2 = \Delta_Y - |\Delta_Y^1|\). Then following the proof of Theorem 2.1 we consider

\[[P] - |\Delta_Y^1| = K_Y - f^*(K_X + \Delta_X) + [P] - P + Q + \Delta_Y^2\]

Note that in this case, \(\Delta_Y^2\) has some reduced component so we are not able to apply Kawamata vanishing theorem. However, we claim that \(R^if_*\mathcal{O}_Y([P] - |\Delta_Y^1|) = 0\) for \(i > 0\). Since \(f\) is a Szabó resolution, it is isomorphic at the generic points of log canonical centers of \((X, \Delta_X)\). As a result, \(R^if_*\mathcal{O}_Y([P] - |\Delta_Y^1|) = 0\) at those centers. One the other hand, by torsion-free theorem of Ambro and Fujino (Theorem 1.1 in [4] and Theorem 3.2 in [1]), the associate prime of \(R^if_*\mathcal{O}_Y([P] - |\Delta_Y^1|)\) are among the log canonical centers of \((X, \Delta_X)\). So the claim is proved.

Since \([P]\) is \(f\)-exceptional, we have \(f_*\mathcal{O}_Y([P] - |\Delta_Y^1|) = f_*\mathcal{O}_Y(-|\Delta_Y^1|)\). In conclusion, we have the following diagram such that \(\beta\) is a quasi isomorphism,

\[\begin{array}{c}
\alpha \quad Rf_*\mathcal{O}_Y(-|\Delta_Y^1|) \\
\downarrow \beta \\
Rf_*\mathcal{O}_Y([P] - |\Delta_Y^1|)
\end{array}\]

Then we dualize this diagram and follow the same argument in Theorem 2.1 to get the conclusion. \[\square\]

Corollary 2.5. (Theorem 5.22 in [13]) If \((X, \Delta_X)\) is a dlt pair, then \(X\) has rational singularities.

Proof. By Proposition 2.43 in [13], we can perturb \(\Delta\) a little bit so that \((X, \Delta)\) is a klt pair. In particular, \(|\Delta| = 0\).

Take the Szabó resolution \(f : Y \to X\), by Theorem 2.1 \(R^if_*\mathcal{O}_Y(-|\Delta_Y|) = R^if_*\mathcal{O}_Y = 0\), for \(i > 0\). By definition we assume that \(X\) is normal so \(f_*\mathcal{O}_Y = \mathcal{O}_X\).

As a result, \(f\) is a rational resolution. \[\square\]

Corollary 2.6. (Theorem 4.14 in [5], Proposition 2.4 in [6]) Given a dlt pair \((X, \Delta)\), let \(S = S_1 + \cdots + S_k\) be the reduced part of \(\Delta\) and \(T = S_1 + \cdots + S_j, j \leq k\). Then \(\mathcal{O}_X(-T)\) is Cohen-Macaulay. Moreover, as a reduced scheme \(T\) is Cohen-Macaulay and has Du Bois singularities.
Proof. We take the Szabó resolution and write
\[ K_Y + \Delta_Y = f'(K_X + \Delta_X) + P - Q, \]
Let \( \Delta_Y^1 \subseteq \Delta_Y \) be the strict transform of \( T \). By diagram A and the argument in the proof of Theorem 2.1, we have
\[ (2.1) \quad \text{Ext}^{-i}(f_*\mathcal{O}_Y(-\Delta_Y^1),\omega_X^i) = 0, \forall i \neq n. \]
By Corollary 2.5 \( X \) is Cohen-Macaulay so \( \omega_X^i \cong \omega_X[n] \). Then equation 3.1 implies that \( f_*\mathcal{O}_Y(-\Delta_Y^1) \) is Cohen-Macaulay, see for example Corollary 3.5.11 in [2]. Note that since \( \Delta_Y^1 \) is the strict transform of \( T \) on \( Y \), \( f_*\mathcal{O}_Y(-\Delta_Y^1) = \mathcal{O}_X(-T) \). This proves the first statement.

For the second statement, consider
\[ 0 \to \mathcal{O}_X(-T) \to \mathcal{O}_X \to \mathcal{O}_T \to 0. \]
Apply \( R\text{Hom}(\cdot,\omega_X^i) \) to this exact sequence we get
\[ \omega_T \to \omega_X \to R\text{Hom}(\mathcal{O}_X(-T),\omega_X^i) \to +1 \]
Since both \( \omega_X^i \) and \( R\text{Hom}(\mathcal{O}_X(-T),\omega_X^i) \) are nonzero only at the \(-n\) degree, \( \omega_T \) is nonzero only at the \(-n+1\) degree. So \( T \) is Cohen-Macaulay.

Push forward the following sequence
\[ 0 \to \mathcal{O}_Y(-\Delta_Y^1) \to \mathcal{O}_Y \to \mathcal{O}_{\Delta_Y^1} \to 0. \]
By Theorem 2.4 \( R^1f_*\mathcal{O}_Y(-\Delta_Y^1) = 0 \) so \( f_*\mathcal{O}_{\Delta_Y^1} = \mathcal{O}_T \). Using Theorem 2.4 and Corollary 2.5, we conclude that \( R^if_*\mathcal{O}_{\Delta_Y^1} = 0, \forall i > 0 \). In other words, \( \mathcal{O}_T \cong Rf_*\mathcal{O}_{\Delta_Y^1} \), which implies \( T \) is Du Bois by Corollary 2.4 in [11].

Remark 2.7. See similar property of semi divisorial log terminal pair in [8], Theorem 4.2.

The first result in Corollary 2.6 is generalized in the next Corollary. We say \( D \sim_{Q,\text{loc}} \Delta' \) if locally \( D - \Delta' \) is \( Q \)-Cartier.

Corollary 2.8. (Theorem 2 in [10]) Let \((X, \Delta)\) be dlt, \( D \) a \( Z \)-divisor and \( \Delta' \leq \Delta \) an effective \( Q \)-divisor on \( X \) such that \( D \sim_{Q,\text{loc}} \Delta' \). Then \( \mathcal{O}_X(-D) \) is Cohen-Macaulay.

Proof. We follow the set up in [10]. First by perturbing \( \Delta \), we can assume \((X, \Delta)\) is klt. Take a log resolution \( f : Y \to X \). Since \( D - \Delta' \) is \( Q \)-Cartier, we can write
\[ f^*(D - \Delta') = f_*^{-1}D - f_*^{-1}\Delta' - F, \]
where \( F \) is \( f \)-exceptional. Define
\[ D_Y = f_*^{-1}\Delta' + \{ F \} + f^*(D - \Delta'), \]
then \( f_*\mathcal{O}_Y(-D_Y) = \mathcal{O}_X(-D) \). We can also find effective and \( f \)-exceptional divisor \( B \) on \( Y \) such that
\[ B - D_Y = K_Y - \{ f \text{-trivial} \} + \{ \text{strict transforms} \} + \delta, \]
where \([ \delta ] = 0 \). Apply Lemma 2.2 we see that \( R^if_*\mathcal{O}_Y(B - D_Y) = 0, \forall i > 0 \). In particular, \( Rf_*\mathcal{O}_Y(B - D_Y) \) is quasi isomorphic to \( f_*\mathcal{O}_Y(-D_Y) \) as complexes. So far everything is the same as the set up in section 9 of [10].

Now we apply the trick in this paper, replace \([ \Delta_Y ] \) in diagram A by \( D_Y \), we have
Remark 2.9. See [10] for more systematic way to approach this problem by rational pairs.

Now we prove a vanishing theorem due to Kovács, which is a generalization of Theorem 2.1.

Corollary 2.10. (Theorem 1.2 in [17]) Let $(X, \Delta)$ be a log canonical pair and let $f : Y \to X$ be a proper birational morphism from a smooth variety $Y$ such that $\text{Exc}(f) \cup \text{Supp} f_*^{-1}\Delta$ is a simple normal crossing divisor on $Y$. If we write

$K_Y = f^*(K_X + \Delta) + \sum a_i E_i$

and put $E = \sum_{a_i = -1} E_i$, then

$R^i f_* O_Y(-E) = 0$

for every $i > 0$.

Proof. (The argument here essentially follows [7], we refer the readers to there for more details.) By Theorem 3.1 in [12] we can take dlt modification $g : Z \to (X, \Delta)$, so that $K_Z + \Delta_Z = g^*(K_X + \Delta)$ and $(Z, \Delta_Z)$ is a dlt pair. Take the Szabó resolution $h : Y \to Z$ and let $f = g \circ h$, then we have

$K_Y + E = f^*(K_X + \Delta) + P - Q$,

where $E = |h^{-1}\Delta_Z|$ and $|Q| = 0$.

By Theorem 2.1 $R^i h_* O_Y(-E) = 0$ for $i > 0$ and the proof of Corollary 2.6 implies that $h_* O_Y(-E) = I(Z, \Delta_Z)$. Then relative Kawamata-Viehweg-Nadel vanishing theorem implies that $R^i g_* h_* O_Y(-E) = R^i g_* I(Z, \Delta_Z) = 0$ for all $i > 0$. Then the Leray spectral sequence implies $R^i f_* O_Y(-E) = 0$ for every $i > 0$. Then we blow up some particular locus to make sure that $\text{Exc}(f) \cup \text{Supp} f_*^{-1}\Delta$ is simple normal crossing and $R^i f_* O_Y(-E)$ is not affected. For the detail, see the original argument in [7].

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