THERMAL INSTABILITY AND MULTI-PHASE INTERSTELLAR MEDIUM IN THE FIRST GALAXIES

Tsuyoshi Inoue1 and Kazuyuki Omukai2

1 Division of Theoretical Astronomy, National Astronomical Observatory of Japan, Osawa 2-21-1, Mitaka, Tokyo 181-0015, Japan; tsuyoshi.inoue@nao.ac.jp
2 Astronomical Institute, Tohoku University, 6-3 Aramaki, Aoba, Sendai 980-8578, Japan; omukai@astr.tohoku.ac.jp

Received 2014 July 26; accepted 2015 March 23; published 2015 May 21

ABSTRACT

We examine the linear stability and nonlinear growth of the thermal instability in isobarically contracting background gas with various metallicities and far-UV (FUV) field strengths. Such a background medium can be expected for protogalactic clouds and shocked gas with metallicity \( Z/Z_\odot > 10^{-4} \). When the H\(_2\) cooling is suppressed by FUV fields (\( G_0 > 10^{-7} \)) or the metallicity is high enough (\( Z/Z_\odot > 10^{-3} \)), the interstellar medium (ISM) is thermally unstable in the temperature range 100–7000 K owing to the cooling by C \( \Pi \) and O \( \iota \) fine-structure lines. In this case, a bi-phase medium with a bimodal density probability distribution is formed as a consequence of the thermal instability. The characteristic scales of the thermal instability become smaller with increasing metallicity. Comparisons of the nonlinear simulations with different resolution indicates that the maximum scale of the thermal instability should be resolved with more than 60 cells to follow runaway cooling driven by the thermal instability. Under sufficiently weak FUV fields and with low metallicity, the density range of the thermal instability shrinks owing to the dominance of H\(_2\) cooling. As the FUV intensity is reduced, the bi-phase structure becomes less remarkable and eventually disappears. Our basic results suggest that, in early galaxies, (i) fragmentation by the nonlinear growth of thermal instability could determine the mass spectrum of star clusters for \( Z/Z_\odot < 0.04 \), and (ii) a thermally bistable turbulent ISM like our galaxy becomes ubiquitous for \( Z/Z_\odot \gtrsim 0.04 \), although the threshold metallicity depends on conditions such as thermal pressure, FUV strength, and redshift.

Key words: galaxies: formation – instabilities – shock waves – stars: formation

1. INTRODUCTION

The earliest generations of stars are believed to have played crucial roles in the cosmic evolution. The first stars that formed from the primordial pristine gas are considered to have been very massive with \( M \sim 100 M_\odot \) (Omukai & Nishi 1998; Bromm et al. 1999, 2002; Abel et al. 2002; Omukai & Palla 2003; McKee & Tan 2008; Yoshida et al. 2008; Hosokawa et al. 2011; Stacy et al. 2012; Hirano et al. 2014; Susa et al. 2014). Massive stars emit a copious amount of ultraviolet (UV) photons, which ionize the surrounding interstellar medium (ISM, e.g., Yorke 1986; Omukai & Inutsuka 2002; Hosokawa et al. 2011) and eventually reionize the entire intergalactic medium (Shapiro & Giurou 1987; Gnedin & Ostriker 1997). Metal-free galaxies exposed to strong radiation fields are also considered to be favorable sites of seed supermassive black hole formation (Bromm & Loeb 2003; Omukai et al. 2008; Agarwal et al. 2014; Dijkstra et al. 2014). The massive stars inject metal-enriched ashes as well as kinetic energy into their surroundings at the end of their lives (Madau et al. 2001; Mori et al. 2002). The accumulation of metals in the gas, raw material for star formation, elevates its cooling ability and thus causes a transition in the mass scale from very massive Population III stars to low-mass Population II stars (Omukai 2000; Bromm et al. 2001; Schneider et al. 2002; Omukai et al. 2005). The kinetic energy injection controls the star formation in a complex way: the shock compression by supernovae may promote star formation (Nagakura et al. 2009; Kumar & Johnson 2010), while evacuation of the gas from the halo would quench subsequent star formation (Bromm et al. 2003; Ritter 2012). However, the physical condition of the ISM in early galaxies (e.g., Norman & Spaans 1997; Aykutalp & Spaans 2011; Bialy & Sternberg 2014), which is the formation environment of second and later generations of stars, is still very uncertain in contrast to well-defined pristine environments where the first stars in the universe were formed.

One of the most conspicuous features of the Galactic ISM is its multi-phase nature, which is caused by thermal bistability (Field et al. 1969; Wolfire et al. 1995). The ISM in the Galaxy is known to be thermally unstable in the temperature range between about 100 and 7000 K owing to the metallic fine-structure line cooling (Field 1965). Development of a multi-phase structure as a consequence of the thermal instability has been studied intensively in the last decade by using (magneto-)hydrodynamic simulations. Hennebelle & Pérault (1999, 2000) and Koyama & Inutsuka (2000) carried out one-dimensional simulations of the thermally bistable ISM, and they found that the thermally stable warm neutral medium with a number density of \( n \approx 1 \text{ cm}^{-3} \) and temperature \( T \approx 10^4 \text{ K} \) can be destabilized by shock compression, and consequently, cold clumps are produced. By using two-dimensional simulations, Koyama & Inutsuka (2002) showed that the thermal instability creates a highly turbulent and clumpy medium in a shock compressed layer. Detailed statistical properties of such a clumpy medium due to the thermal instability are reported by Audit & Hennebelle (2005, 2010), Gazol et al. (2005), Hennebelle & Audit (2007), Hennebelle et al. (2007), and Gazol & Kim (2013), and the effect of magnetic fields on the thermal instability in multi-dimension was studied by Inoue & Inutsuka (2008, 2009), Heitsch et al. (2006, 2008), Vázquez-Semadeni et al. (2006, 2007), Hennebelle et al. (2008), and Banerjee et al. (2009) studied the formation of molecular clouds from shocked warm neutral medium via thermal instability including self-gravity and magnetic fields. Recently, Inoue & Inutsuka (2012) succeeded in reproducing the
observed physical state of molecular clouds, e.g., clump mass function, turbulent velocity law, etc., by carrying out a numerical simulation of molecular cloud formation from atomic H\textsuperscript{i} clouds by consistently taking into account chemical transitions in the presence of thermal instability and magnetic fields. The success of those numerical studies is quite encouraging as it implies that a similar numerical approach to the early galaxy ISM is also viable.

Thermal instability is also operative in hot primordial gases with $\sim 10^8$ – $10^9$ K owing to H and He\textsuperscript{*} bound–bound emission. Fall & Rees (1985) identified clumps formed by this instability as proto-globular cluster clouds. In smaller halos with virial temperatures $T_{\text{vir}} \lesssim 10^4$ K, H\textsubscript{2} formation and cooling can trigger thermal instability in the temperature range of 2000–7000 K (Sabano & Yoshii 1977; Yoshii & Sabano 1979, 1980; Murray & Lin 1989; Corbelli et al. 1997). In particular, Silk (1983) pointed out that enhanced cooling by rapid three-body H\textsubscript{2} formation makes a collapsing pre-stellar cloud thermally unstable and triggers its fragmentation around $10^5$–$10^6$ cm\textsuperscript{-3}. Although this instability appears too weak to cause fragmentation in reality (Abel et al. 2002; Omukai & Yoshii 2003; see also Greif et al. 2013), it might be relevant to the origin of the two-phase gas around the first protostars found in numerical simulations (Turk et al. 2010). In a Population II star-forming ISM with some metal enrichment, thermal instability is expected to be driven by the metallic fine-structure line cooling as in the Galactic ISM. For example, Suchkov et al. (1981) studied the thermal equilibrium state for gas with a metallicity $10^{-2}Z_\odot$ and discussed the fragmentation of proto-globular cluster clouds by thermal instability.

So far, however, development of a thermal instability in the low-metallicity medium has never been confirmed by multidimensional hydrodynamical simulations and its physical condition has not been studied comprehensively. In this paper, as the first attempt of this sort, we investigate the nonlinear growth of thermal stability in the ISM with a metallicity of $Z = 10^{-4} - 1Z_\odot$ by way of local three-dimensional simulations. As in the case of the Galactic ISM, post-shock regions can be thermally unstable in early galaxies. Shocks are considered to be prevalent in the formation of galaxies as a result of more frequent merging with other galaxies, inflow of the intergalactic gas, and compression by the supernova explosion after vigorous starbursts. Those post-shock layers are known to contract isobarically until the post-shock gas reaches thermal equilibrium (e.g., Yamada & Nishi 1998, Koyama & Inutsuka 2000). Even without shocks, with a metallicity of $Z \gtrsim 10^{-4}Z_\odot$, metal cooling causes the protogalactic clouds to contract nearly isobarically (Safranek-Shrader et al. 2014). Thus, here, we study thermal stability in isobarically contracting low-metallicity media.

The organization of this paper is as follows. In Section 2, we first examine when the condition for the thermal instability is satisfied in low-metallicity ISM and estimate the unstable scales for various metallicities and background UV field strengths. In Section 3, we describe the local three-dimensional simulation of the thermal-instability development in a low-metallicity medium. Finally, in Section 4, we summarize our findings and discuss their implications for low-metallicity star formation.

## 2. LINEAR STABILITY ANALYSIS

Before the full simulations, we explore the density/temperature ranges of thermal instability by applying the instability criterion derived from linear perturbation theory. Here, we take an isobarically contracting gas as the background medium against which we examine the thermal instability.

### 2.1. Isobarically Contracting Background

The evolution of the unperturbed background state is described by the energy equation,

$$\frac{de}{dt} = -\rho \frac{d}{dt}\left(\frac{1}{\rho}\right) + \Lambda,$$  

supplemented by the equation of state:

$$e = \frac{1}{\gamma - 1} \rho,$$  

where $e$ is the specific internal energy, $\rho$ is the mass density, $p$ is the thermal pressure, $\gamma$ is the adiabatic index, and $\Lambda$ is the net cooling rate per unit volume. Using the isobaricity $p = \text{const.}$, the above equations can be combined into the following single equation:

$$\frac{dp}{dt} = \frac{\gamma - 1}{\gamma} \rho \Lambda.$$  

Here we note again that the isobaric condition assumed for the background state is expected to hold both in the post-shock cooling layer and the protogalactic clouds, until the gas reaches thermal equilibrium, as long as the cooling timescale is shorter than the expansion timescale of an external force such as self-gravity. In Section 2.3, we show that the isobaricity would break down in the media with $Z \lesssim 10^{-4}Z_\odot$. We include a requisite minimum set of chemical reactions and cooling/heating processes, which are summarized in Tables 1 and 2, respectively. The whole chemical network and resulting cooling rate are solved self-consistently with the background isobaric contraction. We consider an optically thin medium, although our code is able to handle opaque cases as well (Inoue & Inutsuka 2012).

The background FUV intensity is expressed by the so-called the Habing parameter $G_\text{0}$, which indicates the field strength relative to that in the solar neighborhood and is defined by the radiation energy density in the energy range between 6 and 13.6 eV normalized by $5.29 \times 10^{-14}$ erg cm\textsuperscript{-3} Hz\textsuperscript{-1} (Habing 1968). The current most reliable estimate is by Mathis et al. (1983), who updated the solar neighborhood value to $G_\text{0} = 1.14$ (Draine 2011). The Habing parameter is related to another frequently used FUV parameter $J_{21}$, which is the specific intensity at the Lyman limit normalized by $10^{-21}$ erg s\textsuperscript{-1} cm\textsuperscript{-2} Hz\textsuperscript{-1} str\textsuperscript{-1}:

$$J_{21} = 20.9 \times G_0$$  

for the FUV spectral shape of Mathis et al. (1983).

We start the numerical integration of Equation (3) from a number density of $n_0 \equiv \sum n_i = 10$ cm\textsuperscript{-3} with abundances of $x_{\text{H}}(0) = 0.92$, $x_{\text{He}}(0) = 0.08$, $x_p(0) = 3 \times 10^{-3}$, $x_{\text{He}^+}(0) = 3 \times 10^{-3}$, $x_{\text{He}^2}(0)$, and $x_{\text{CO}}(0) = 1.4 \times 10^{-4}(Z/Z_\odot)$, where $x_i \equiv n_i/n_0$. Other abundances ($x_{\text{H}^+}$, $x_{\text{H}_2}$, $x_C$, and $x_{\text{CO}}$) are chosen initially to be zero. In the following, we indicate the
metallicity \( Z \) by using the parameter \([Z] \equiv \log(Z/Z_{\odot})\), where \([Z] = 0\) indicates the solar metallicity. The abundances of metals, dust, and polycyclic aromatic hydrocarbons (PAHs) are set to be proportional to the metallicity \( Z \). The fiducial value of the thermal pressure is chosen to be \( p/k_B = 10^{5} \text{ K cm}^{-3} \), consistent with the value for the collapsing cloud with \([Z] = -3\) in Safranek-Shrader et al. (2014)’s simulation.

### 2.2. Stability Criterion

According to the linear stability analysis for gas contracting isobarically with cooling (Balbus 1995; Koyama & Inutsuka 2000), thermal instability develops if the net cooling rate per unit volume \( \Lambda \) increases with decreasing temperature:

\[
\left( \frac{\partial \Lambda}{\partial T} \right)_p < 0, \tag{5}
\]

where the subscript denotes the invariant quantity under differentiation. Note that the same criterion holds for the thermal instability both in thermal equilibrium states (Field 1965) and in an isochorically cooling medium (Schwarz et al. 1972; Burkert & Lin 2000). Since our background solution ensures the isobaric condition and that the temperature decreases monotonically with time, criterion (5) is equivalent to the condition that the net cooling rate increases with time:

\[
\frac{d\Lambda}{dt} > 0. \tag{6}
\]

In a thermally unstable medium, density fluctuations with wavelength \( \lambda \) grow exponentially with time. The growth timescale is given by the cooling time \( t_{\text{cool}} \equiv \rho c_s/\Lambda \) for scales in

### Table 1

**Chemical Reactions**

| Reaction | Note | Reference |
|----------|------|-----------|
| H\(^+\) photo-dissociation | depends on FUV strength \( G_0 \) (visual extinction \( A_v = 0 \) is assumed) | 1 |
| H\(^+\) formation | collision with e | 2 |
| H\(_2\) formation on grains | dust temperature \( T_d = 10 \text{ K} \) is assumed | 4 |
| H\(_2\) formation with e | | 2 |
| H\(_2\) photo-dissociation | depends on \( G_0 \) (column density \( N_{H_2} = 0 \) and \( A_v = 0 \) is assumed) | 5 |
| H\(_2\) and CO dissociation | collisions with e, p, and H | 6 |
| H\(_2\) and CO dissociation | recombination of He\(^+\) | 1 |
| H, He, and C ionization | collisions with e, p, H, and H\(_2\) | 1, 6 |
| H\(^+\), He\(^+\), and C\(^+\) recombination | | 6, 7 |
| CO formation | depends on \( G_0 \) (\( A_v = 0 \) is assumed) | 8 |
| CO photo-dissociation | depends on \( G_0 \) (\( N_{H_2} = N_{CO} = A_v = 0 \) is assumed) | 8, 9 |
| C photo-ionization | \( G_0 \) (\( N_{C} = N_{H_2} = A_v = 0 \) is assumed) | 4 |

### Table 2

**Cooling and Heating Processes**

| Process | Note | Reference |
|---------|------|-----------|
| Photoelectric heating by PAHs | depends on \( G_0 \) (\( A_v = 0 \) is assumed) | 1, 2 |
| Cosmic ray heating | ... | 3 |
| H\(_2\) photo-dissociation heating | depends on \( G_0 \) (\( N_{H_2} = A_v = 0 \) is assumed) | 4 |
| Ly \( \alpha \) cooling | ... | 5 |
| C\(^+\) cooling (158 \( \mu \)m) | escape probability = 1 is assumed | 6 |
| O cooling (63 \( \mu \)m) | escape probability = 1 is assumed | 2, 6 |
| CO ro-vibrational cooling | escape probability = 1 is assumed | 7, 8, 9 |
| Cooling due to rec. of electrons with grains \& PAHs | ... | 1 |
| H\(_2\) cooling | collisions with H, H\(_2\), He, p, \& e | 10 |

References. (1) Bakes & Tielens (1994), (2) Wolfire et al. (2003), (3) Goldsmith & Langer (1978), (4) Black & Dalgarno (1977), (5) Spitzer (1978), (6) de Jong et al. (1980), (7) Hollenbach & McKee (1979), (8) Hosokawa & Inutsuka (2006), (9) Hollenbach & McKee (1989), (10) Glover & Abel (2008).

the range

\[ l_F \lesssim \lambda \lesssim l_{ac}, \tag{7} \]

where the lower and upper bounds are the Field length (Field 1965)

\[ l_F = \sqrt{\kappa T/\Lambda}, \tag{8} \]

and the acoustic length

\[ l_{ac} = c_s l_{cool}, \tag{9} \]

respectively. Here, \( c_s \) denotes the adiabatic sound speed, and \( \kappa = 2.5 \times 10^5 T^{1/2} \text{ erg cm}^{-1} \text{ s}^{-1} \text{ K}^{-1} \) (Parker 1953), the thermal conductivity in the neutral atomic gas. Fluctuations below the Field length (\( \lambda \lesssim l_F \)) are damped by thermal conduction, while those above the acoustic length (\( \lambda \gtrsim l_{ac} \)) cannot be causally connected within the cooling time and thus have a slower growth rate. Although the most unstable scale is given by \( l_F l_{ac} \), all the perturbations in the range of Equation (7) grow roughly at a similar rate of \( l_{cool}^{-1} \) (Field 1965).

### 2.3. Results of Linear Analysis

As described in Section 2.1, we numerically compute the isobaric contraction with \( p/k_B = 10^{5} \text{ K cm}^{-3} \) from \( (n, T) = (10 \text{ cm}^{-3}, 10, 000 \text{ K}) \) to \( (2000 \text{ cm}^{-3}, 50 \text{ K}) \) by integrating Equation (3) and the chemical reaction equations as the background model, and examine its thermal stability by using the criterion of Equation (6). In Figure 1, we show the acoustic (solid lines in the top panel) and Field lengths (solid lines in the bottom panel) as a function of background density for the case of \( G_0 = 1 \), i.e., the FUV strength is roughly the same as in the local ISM. Red, green, blue, magenta, and aqua represent the results of \([Z] = 0, -1, -2, -3, \) and \(-4\), respectively, and the solid line sections indicate the density range where the instability criterion is satisfied. The thin solid lines indicate the mass scale involved in the spherical region of the diameter \( l \).
The Astrophysical Journal, 805:73 (11pp), 2015 May 20

Inoue & Omukai

The local ISM is known to be thermally unstable in the temperature range $100 \, \text{K} \leq T \leq 5000 \, \text{K}$ owing to the fine-structure line cooling by [C II] $158 \, \mu\text{m}$ and [O I] $63 \, \mu\text{m}$ lines (Wolfire et al. 1995, 2003; Koyama & Inutsuka 2000). In the cases of $G_0 = 1$, since the cooling is always dominated by fine-structure lines, whose cooling rate can be fitted by (Koyama & Inutsuka 2002)

$$\Lambda \simeq 3 \times 10^{-25} n^2 T^{1/2} \exp\left(-T_{\text{line,2}}/T\right) \times \left(Z/Z_\odot\right) \text{erg cm}^{-3} \text{s}^{-1},$$

the cooling time, the acoustic scale, and the Field length can be written as:

$$t_{\text{cool}} \simeq 0.4 \, \text{Myr} \left(Z/Z_\odot\right)^{-1} n_{1/2}^{-3/2} \left(p/k_B\right)_5^{1/2} \times \exp\left[10^{-2} T_{\text{line,2}} n_{1/2} \left(p/k_B\right)_5^{-1}\right],$$

$$l_\Lambda \simeq 4 \, \text{pc} \left(Z/Z_\odot\right)^{-1} n_1^{-2} \left(p/k_B\right)_5 \times \exp\left[10^{-2} T_{\text{line,2}} n_1 \left(p/k_B\right)_5^{-1}\right],$$

$$l_F \simeq 0.013 \, \text{pc} \left(Z/Z_\odot\right)^{-1/2} n_{3/2}^{-3/2} \left(p/k_B\right)_5^{1/2} \times \exp\left[5 \times 10^{-3} T_{\text{line,2}} n_{3/2} \left(p/k_B\right)_5^{-1}\right],$$

where $n_1 = n/10 \, \text{cm}^{-3}$, $(p/k_B)_5 = (p/k_B)/10^5 \, \text{K} \, \text{cm}^{-3}$, and $T_{\text{line,2}} = T_{\text{line}}/100 \, \text{K}$. These analytic lines are also plotted as dashed lines in the top and bottom panels, which agree well with the numerical results.

In the top panel of Figure 1, we also plot the Jeans length $c_s \sqrt{\pi / G \rho}$ as a dotted line. Physically, when the acoustic scale is larger than the Jeans length, i.e., when the cooling timescale is longer than the freefall timescale, the gas contraction leads to pressure enhancement due to insufficient cooling, and we cannot assume isobaric background. According to the results of Safranek-Shrader et al. (2014), the gas with $[Z] \geq -3$ contracts nearly isobarically, while the $[Z] = -4$ run shows contraction with gradual increase of thermal pressure. Thus, our analysis assuming isobaric background contraction can be applied only for the case of $[Z] > -4$.

In numerical simulations, typically $1/10$ of the Jeans length is used as the refinement criterion for adaptive mesh refinement (AMR) in order to resolve the dynamics due to gravity (Truelove et al. 1997; Heitsch et al. 2001; Federrath et al. 2010). We see that the Jeans criterion manages to capture the acoustic scale (the largest scale of the thermal instability) only for the $[Z] \leq -3$ cases. Therefore, we should be careful about the use of AMR code when the Jeans criterion is applied. In the next section, through the convergence test of a nonlinear simulation, we show that we need to resolve the maximum scale of the thermal instability (the acoustic scale at $T \sim 5000 \, \text{K}$) with more than 60 cells in order to follow the fragmentation.

For lower FUV strength $G_0$, thermally stable H$_2$ cooling can dominate the cooling. Figure 2 shows the acoustic scales $l_{\text{ac}}$ for different metallicities and with $G_0 = 10^{-1}$ (panel (a)), $10^{-2}$ (panel (b)), $10^{-3}$ (panel (c)), and $10^{-4}$ (panel (d)). A thick line is used when the main coolant is metals, and a thin line is used when the main coolant is H$_2$. The general tendency is that a weaker FUV field reduces the unstable domain by metal cooling because of the domination of H$_2$ cooling. At low densities, unstable regimes from H$_2$ cooling appear. If the H$_2$ fraction is constant, the H$_2$ cooling would not cause thermally instability. In a dynamic medium, on the other hand, it can be unstable, because the H$_2$ fraction (or the amount of coolant) increases with density (i.e., the cooling rate can be enhanced as the temperature drops; $\partial \Lambda / \partial T|_p < 0$).

To indicate the strength of the thermal instability in a weak FUV environment, we calculate the e-folding number of the thermal instability during background contraction from $n = 10 \, \text{cm}^{-3}$ to $2000 \, \text{cm}^{-3}$:

$$N_t = \int_{d \Lambda / dt > 0} t_{\text{cool}}^{-1} dt,$$

where integration is done while the instability criterion Equation (6) is satisfied. Figure 3 shows the e-folding number as a function of the metallicity and the FUV strength. In the next section, based on nonlinear simulations, we show that
is necessary for the thermal instability to develop and create a bimodal density distribution, although the detailed threshold would also depend on the amplitude of the initial density fluctuation.

3. NUMERICAL SIMULATION OF POST-SHOCK FLOW

3.1. Method

In this section, we examine the nonlinear growth of the thermal instability in a post-shock medium by means of three-dimensional hydrodynamics simulations incorporating radiative cooling, heating, and chemical reactions. The basic equations to solve are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0,$$

$$\frac{\partial (\rho v)}{\partial t} + \nabla \cdot (\rho v \otimes v) = 0,$$

$$\frac{\partial (p e)}{\partial t} + \nabla \cdot \left\{ (p e + p) v \right\} = \nabla \cdot (\kappa \nabla T) - \Lambda,$$

along with the ideal gas equation of state

$$p = \frac{\rho k_B T}{\mu m_H}.$$  

The microphysics considered, i.e., chemical reactions, cooling/heating, and thermal conductivity, are the same as those used in the linear analysis in Section 2.1. We impose a temperature floor of 40 K, corresponding to the cosmic microwave background (CMB) temperature $T_{\text{CMB}}$ at redshift $z = 14$. We use a second-order Godunov method (van Leer 1979) for solving the hydrodynamical equations and a second-order piecewise-exact-solution method for the chemical reactions (Inoue & Inutsuka 2008, 2012). The cooling, heating, and thermal conduction terms are usually solved by using a 2nd order explicit scheme. When the local cooling time is shorter than the...
10% of the CFL timestep, the cooling and heating terms are solved implicitly using the bisection method.

3.2. Settings

We create a post-shock cooling medium by colliding two counter-streams at the $x = 0$ plane in a 3D box of extension $L_{\text{box}}/2 \geq x$ (also for $y$ and $z$) $\geq -L_{\text{box}}/2$. The average initial number density is set to be $n_{\text{i}1} = 2.5 \text{ cm}^{-3}$, with density fluctuations with a power-law spectrum with a Kolmogorov spectral index ($P_{\text{k3d}} \propto k^{-11/3}$) where the bracket means spatial averaging. The initial amplitude of the density fluctuations is chosen as $\Delta n_{\text{i}}/n_{\text{i}1} = 0.1$, where $\Delta n_{\text{i}}$ is the dispersion of the initial density field. The initial pressure is set to be constant at $p_1/k_B = 4000 \text{ K cm}^{-3}$ so that the density fluctuations are the entropy mode, which can be thermally unstable. The colliding counter-streams with velocity $v_1$ create a shocked slab with pressure $p_2 \approx (\rho_1 v_1^2 + p_1)$, which contracts isobarically by radiative cooling as long as the converging flows are maintained. The value of $v_1$ is chosen so that the average value of post-shock pressure becomes approximately the same as that used for the linear analysis in Section 2, i.e., $\langle p_2 \rangle/k_B = 10^4 \text{ K cm}^{-3}$ ($v_1 = 16 \text{ km s}^{-1}$). Since the acoustic scale, the maximum scale for the thermal instability, is inversely proportional to the metallicity, we scale the box size of the simulation as $L_{\text{box}} = 4(Z/Z_\odot)^{-1}\text{pc}$. We perform five runs with $[Z] = 0$, $-1$, $-2$, $-3$, and $-4$ under fixed FUV strength of $G_\text{f} = 1$, and three runs with $G_\text{f} = -2$, $-3$, and $-4$ under a fixed metallicity of $[Z] = -3$. We impose periodic boundary conditions at the $y, z = \pm L_{\text{box}}/2$ boundary planes. At the $x = \pm L_{\text{box}}/2$ planes, we set $v(t, \pm L_{\text{box}}/2) = \mp v_1$, $p(t, \pm L_{\text{box}}/2) = p_1$, and $\rho(t, x = \pm L_{\text{box}}/2, y, z) = \rho(t = 0, x = \pm L_{\text{box}}/2, y, z)$, indicating that there are continuous inflows with density fluctuations through these boundaries. Although our box scale $L_{\text{box}}$ can be extremely large in low-metallicity cases, e.g., $[Z] = -3$ and $-4$, this does not mean that a medium of such a large spatial extent is necessary for the thermal instability to grow. Recall that thermal instability is active at the smaller scale down to the Field length (see Equation (13)).

The numerical domain is divided into $1024^3$ cubic homogeneous cells, so the numerical resolution is $\Delta x = 3.9 \times 10^{-3}(Z/Z_\odot)^{-1}\text{pc}$. With this resolution, the acoustic scale is resolved in almost entire density range, while the Field length is resolved only in the low-density regime (see Figure 1). Thus, we have to bear in mind that, although structures below the resolution scale are not expressed in our simulation, clumps could be present even with smaller scales down to the Field length (Koyama & Inutsuka 2004; Aota et al. 2013). However, despite an insufficient resolution for the Field length, properties of the thermally bistable medium, e.g., the mass function of cold clumps, and the power spectra of velocity and density, are known to converge on large scales (Hennebelle & Audit 2007; see also Gazol et al. 2005; Vázquez-Semadeni et al. 2006). This is because most of the mass of the cold gas created by thermal instability is contained in large clumps that are formed

by the growth of large-scale fluctuations (the typical mass of the clumps will be discussed in Section 4.2).

3.3. Results

3.3.1. $G_0 = 1$ Cases

In Figure 4, we show snapshots of the density distribution on the $z = 0$ plane for four different metallicities $[Z] = 0$ (panel (a)), $-1$ (b), $-2$ (c), $-3$ (d), and $-4$ (e) with the same FUV strength $G_0 = 1$. The snapshots were taken at $t = 0.6(Z/Z_\odot)^{-1}\text{Myr}$, which corresponds to a few cooling times for the gas just behind the shock $(\langle n \rangle_{\text{ps}} \approx 10 \text{ cm}^{-3}$, see Equation (11)). In all the cases, clump formation by thermal instability is clearly visible in the shocked slabs. Note that the simulation run time above is required just because we start the simulations from such a low density as $n_{\text{ps}} \approx 10 \text{ cm}^{-3}$. In more realistic situations for the medium with $[Z] \leq -3$, an initial low-density gas would contract rapidly until the cooling time becomes comparable to the freefall time. This contraction makes the initial gas density for the growth of thermal instability much higher than the present setting of $n_{\text{sh}} \sim 10 \text{ cm}^{-3}$. Thus, the thermal instability can create clumps in a timescale shorter the present run-time because the cooling timescale decreases with the density as $0.6(Z/Z_\odot)^{-1}(n_{\text{ps}}/10 \text{ cm}^{-3})^{3/2}\text{Myr}$. Although the later beginning of the instability growth leads to a smaller e-folding number (Equation (14)), we expect a shorter dynamical timescale for the thermal instability due to the higher initial density.

A remarkable structural characteristic of the thermally unstable ISM is dense clumps embedded in a diffuse medium (Pikel’ner 1968; Field et al. 1969; Koyama & Inutsuka 2002; see also, Audit & Hennebelle 2005, 2010; Heitsch et al. 2005, 2006; Vázquez-Semadeni et al. 2006; Inoue & Inutsuka 2008, 2009, 2012; Hennebelle et al. 2008; Banerjee et al. 2009). Such a bi-phasic structure results from the fact that dense perturbations grow exponentially with time while the density in the surrounding diffuse medium increases more slowly by the overall contraction. Figure 4 demonstrates that, even in the lowest-metallicity case studied, $[Z] = -4$, the ISM has a bi-phasic structure consistent with a linear analysis (see Figure 1). Note that although the ambient diffuse gas itself is also cooling and will eventually be incorporated into the dense clumps, such a bi-phasic structure is maintained as long as thermally unstable gases are continuously supplied from the shock front (see, e.g., Inoue & Inutsuka 2012 for a recent simulation).

The bi-phasic nature can be observed more quantitatively in the probability distribution function (PDF) of the density shown in Figure 5. The red, green, blue, magenta, and aqua lines, respectively, represent the cases of $[Z] = 0$, $-1$, $-2$, $-3$, and $-4$ at $t = 0.6(Z/Z_\odot)^{-1}\text{Myr}$, and the black line shows the initial (preshock) state. The bimodal distribution of the post-shock gas ($n > 10 \text{ cm}^{-3}$) reflects the bi-phasic structure.

Because the preshock medium contains seed density inhomogeneities, the Richtmyer–Meshkov instability (Richtmyer 1960) is induced by interactions between the density fluctuations and the shocks, making the post-shock medium turbulent. Inoue et al. (2013) recently showed that the velocity dispersion of the post-shock turbulence agrees well with the growth velocity of the Richtmyer–Meshkov instability: $\Delta v \approx (\Delta \rho/\langle \rho \rangle)/(1 + \Delta \rho/\langle \rho \rangle)\langle v_{\text{sh}} \rangle$. 

---

3 If we fix the $v_1$ irrespective of the metallicity, one may think that the growth of thermal instability would be limited in low-$Z$ cases due to the small thickness of the shocked slab created by the converging flows compared to the $L_{\text{box}}$. However, because the thermal instability can always grow in the $y$- and $z$-directions, the fixed $v_1$ does not limit the growth rate of the thermal instability.
3.3.2. Lower $G_0$ Cases

To see the effect of enhanced H$_2$ cooling by $G_0$ reduction, we also perform simulations for $[Z] = -Z_0$ models with $G_0 = 10^{-2}$, $10^{-3}$, and $10^{-4}$. Their density snapshots and the density PDF are shown in Figures 6 and 7, respectively. We can see that the bimodal distribution becomes less remarkable as $G_0$ goes down and eventually disappears in the $G_0 = 10^{-4}$ case. This is because a weak FUV field suppresses the effect of the thermal instability as shown by the linear stability analysis (middle panel of Figure 2), which indicates that the thermally unstable density range shrinks when $G_0 < 10^{-2}$. Recalling the e-folding number shown in Figure 3, the vanishing of the bimodal distribution for $\log G_0 \lesssim -3$ cases indicates that $N_f \gtrsim 3$ is necessary for the thermal instability to develop fully, although the detailed threshold $N_f$ would also depend on the amplitude of the initial density fluctuation. It should be noted that the contrasting results of the $G_0 = 1$ run (effective thermal instability) and the $G_0 = 10^{-4}$ run (ineffective thermal instability) are reminiscent of the difference between the two converging-flow simulations, one for the thermally bistable.

Figure 4. Density snapshots at the $z = 0$ plane for the cases of $[Z] = 0$ (panel (a)), $-1$ (b), $-2$ (c), $-3$ (d), and $-4$ (e) with $G_0 = 1$. Data are sampled at $t = 0.6(Z/Z_0)^{-1}$ Myr, which corresponds to a few cooling time for the post-shock gas.
and \( \lesssim 10^3 \) and \( 10^24, 512 \) cell \( 3 \) \( \lesssim \) \( \lesssim \)

\[
\text{instability is suppressed due to numerical diffusion}
\]

The Astrophysical Journal, 805:73 (11pp), 2015 May 20

Inoue & Omukai

3.4. The Minimum Resolution Required

Here we consider how many numerical cells are necessary to capture the fastest growing mode of the thermal instability. For this purpose, we perform additional simulations for \([Z] = -3\) and \( G_0 = 1 \) with different cell numbers of \( N_{\text{cell}} = 1024^3, 512^3, 256^3, 128^3, \) and \( 64^3 \). In the top panels of Figure 8, we show the resulting density snapshots of these runs at \( t = 220 \) Myr. We also plot the mass weighted PDFs in the bottom panel. At this time, dense clumps with \( n \gg 10^3 \) \( \text{cm}^{-3} \) begin to appear for the case of the high-resolution runs with \( N_{\text{cell}} = 1024^3, 512^3, \) and \( 256^3 \). However, in the cases of \( N_{\text{cell}} \leq 128^3 \), the dense fragments have not yet formed, indicating that even the fastest growing mode of the thermal instability is suppressed due to numerical diffusion (Vázquez-Sepúlveda et al. 2003).

According to the results of the linear analysis in Section 2, the maximum scale of the thermal instability (the acoustic scale at \( T \approx 5000 \) K) is given as \( l_{\text{ac, max}} \approx 1 \times (Z/Z_\odot)^{-1} \text{ pc} \) (Equation (12)), which is resolved by \( N = l_{\text{ac, max}} / L_{\text{box}} = 0.25 N_{\text{cell}}^{1/3} \) cells. Thus, the result of the above comparison indicates that, at least we need to resolve \( l_{\text{ac, max}} \) with more than 60 cells to follow the fastest condensation mode. Note that, as we have mentioned in Section 3.2, the above minimum resolution does not ensure the convergence, since the convergence requires resolving the Field length for capturing the smallest fragments (Koyama & Inutsuka 2004). Even so, as we will discuss in Section 4.2, our criterion (\( \Delta x \lesssim l_{\text{ac, max}} / 60 \)) would be useful because large-scale clumps formed by the fastest growing mode have most of the mass of condensations for realistic seed fluctuations.

4. SUMMARY AND DISCUSSION

4.1. Findings

In this paper, we have examined the linear stability and nonlinear growth of the thermal instability in isobarically contracting gas with various metallicities and FUV field strengths. Our findings can be summarized as follows.

1. When the \( \text{H}_2 \) cooling is suppressed by FUV fields (log \( G_0 \gtrsim -3 \)), the ISM is thermally unstable in the temperature range \( 100 \text{ K} \leq T \lesssim 7000 \text{ K} \) owing to the cooling by \([\text{C II}] 158 \mu\text{m} \) and \([\text{O I}] 63 \mu\text{m} \) fine-structure lines regardless of metallicity. In this case, the ISM becomes bi-phasic with a bimodal density distribution (Figure 5).

2. The maximum (i.e., the acoustic scale \( l_{\text{ac}} \)) and minimum scales (i.e., the Field length \( l_F \)) of the thermal instability, which are well-fitted by Equations (12) and (13), respectively, become smaller with increasing metallicity. Comparisons of the nonlinear simulations with different resolution indicates that the maximum scale of the thermal instability should be resolved more than 60 cells to follow the fastest growing mode of the thermal instability (Section 3.4).

3. Under sufficiently weak FUV fields and with low metallicity, the density range of the thermal instability shrinks owing to the dominance of \( \text{H}_2 \) cooling (Figure 2). As the FUV intensity is reduced, bi-phasic structure becomes less remarkable and disappears eventually (Figure 7). These results indicate that the effect of the thermal instability is suppressed when the e-folding number is \( N_t \lesssim 3 \) in the case of our initial conditions, which corresponds to the conditions \([Z] \gtrsim -3 \) and log \( G_0 \lesssim -3 \) (see Figure 3).

4.2. Implications for the Formation of the First Galaxy

According to the recent numerical studies on the formation of the first galaxy (Safranek-Shrader et al. 2014), the gravitational fragmentation scale is set by the Jeans mass when the temperature of the isobarically contracting gas reaches the CMB temperature floor. The typical mass scale of such fragments is

\[
m_{1,\text{CMB}} \equiv \frac{4\pi}{3} \rho \left( \frac{l_1}{2} \right)^3 \approx 190M_\odot \left( \frac{\rho/k_B}{10^5 \text{ K cm}^{-3}} \right)^{-1/2} \left( \frac{1 + z}{10} \right)^2.
\]

where \( l_1 = c_s \sqrt{\pi/G\rho} \) is the Jeans length and the CMB temperature written in terms of the redshift \( T_{\text{CMB}} = 2.73(1 + z) \) is used. Safranek-Shrader et al. (2014) regarded this mass scale as the mass scale for stellar clusters in the first galaxy. In their simulation, however, clump formation by thermal instability may not be resolved properly because of their employment of the Jeans criterion. In the following, we speculate how the thermal-instability-induced fragmentation affects the fate of the ISM evolution.

The clump mass function for thermal instability is known to take a power-law form:

\[
N(m) \propto m^{(\delta - 3)/3 - 2} dm,
\]
\[ \delta \] is the power-law index of the 3D power spectrum of the seed density fluctuations (Hennebelle & Audit 2007; Hennebelle & Chabrier 2008), which gives \( \propto -N_\text{mm}(\delta) = 1.78 \) for realistic Kolmogorov-type fluctuations of \( \delta = 11/3 \). For this distribution, most of the mass is locked up in large fragments, with 50\% of the total mass being in the clumps of \( m \gtrsim 0.04 m_{\text{max}} \). Hence, we regard \( m_{\text{TI}} \equiv 0.04 m_{\text{max}} \) as the typical mass of the fragments due to thermal instability. Using the acoustic scale \( l_{\text{ac}} \) (Equation 12), the typical mass of the fragments can be written as
\[
m_{\text{TI}} \equiv 0.04 \times \frac{4\pi}{3} \rho \left( \frac{l_{\text{ac}}}{2} \right)^3
\approx 1.3 \times 10^4 M_\odot \left( \frac{Z}{0.01 Z_\odot} \right)^{-3/2} \left( \frac{T}{5000 \text{ K}} \right)^{5/2} \left( \frac{p/k_B}{10^5 \text{ K cm}^{-3}} \right)^{-2/3},
\]
where we have evaluated the mass scale at \( T = 5000 \text{ K} \) at which \( m_{\text{max}}(T) \) takes the maximum. Comparison of \( m_{\text{TI}} \) with \( m_{j,\text{CMB}} \) allows us to define the critical metallicity above which the typical clump mass falls below the gravitational fragmentation mass:
\[
Z_{\text{TI}} \equiv 0.042 Z_\odot \left( \frac{T}{5000 \text{ K}} \right)^{5/3} \left( \frac{p/k_B}{10^5 \text{ K cm}^{-3}} \right)^{-1/2} \left( \frac{1 + z}{10} \right)^{-2/3}.
\]

We here discuss the cases when the thermal instability fully develops although the thermal instability condition also depends on the FUV intensity as seen in Sections 2 and 3. If the metallicity is much lower than the critical value \( Z \ll Z_{\text{TI}} \),
most of the clumps created by thermal instability are above the gravitational fragmentation scale, suggesting that the thermal instability that happens prior to the Jeans instability could determine the mass scale of the stellar clusters in the first galaxy. Fragmentation by thermal instability would lead to a power-law distribution of the cluster mass spectrum as given by Equation (20) with a high-mass cutoff $m_{\text{max}}$. Note that our analysis based upon an isobarically contracting background can be reliable only for $[Z] > -4$ (see Section 2.3).

If $Z \gg Z_{\text{TJ}}$, on the other hand, most of clumps are much smaller than $m_{\text{J,CMB}}$, which indicates that even when the gravitationally bound objects are formed by dense gas that reaches the CMB temperature, they would be in the form of a complex of small-scale clumps, each of which is a gravitationally unbound object. In addition, the clumpy medium formed by the thermal instability is highly turbulent which would substantially affect the star formation activity (see, e.g., Inoue & Inutsuka 2012 for a recent simulation of such a molecular cloud in the present-day ISM). Thus, the star formation activity in the first galaxy with $Z \gg Z_{\text{TJ}}$ would be suppressed compared to those with $Z \ll Z_{\text{TJ}}$. There could be a transition in star formation from a starburst mode to a more gradual one around this metallicity.

The numerical computations were carried out on the XC30 system at the Center for Computational Astrophysics (CICA) of the National Astronomical Observatory of Japan and in part on the K computer at the RIKEN Advanced Institute for Computational Science (No. hp120087). This work is supported by Grant-in-aids from the Ministry of Education, Culture, Sports, Science, and Technology (MEXT) of Japan (23740154, 26287030 TI; 25287040 KO).

REFERENCES

Abel, T., Bryan, G. L., & Norman, M. L. 2002, Sci, 295, 93
Agarwal, B., Dalla Vecchia, C., Johnson, J. L., et al. 2014, MNRAS, 443, 648
Aota, T., Inoue, T., & Aikawa, Y. 2013, ApJ, 775, 26
Audit, E., & Hennebelle, P. 2005, A&A, 433, 1
Audit, E., & Hennebelle, P. 2010, A&A, 511, 76
Aykutalp, A., & Spaans, M. 2011, ApJ, 737, 63
Bakes, E. L. O., & Tielens, A. G. G. M. 1994, ApJ, 427, 822
Balbus, S. A. 1995, in ASP Conf. Proc., Vol. 80 (San Francisco, CA: ASP), 328
