A Splitting Set Theorem for Epistemic Specifications

Richard Watson
Texas Tech University
Department of Computer Science
richard.watson@coe.ttu.edu

Abstract
Over the past decade a considerable amount of research has been done to expand logic programming languages to handle incomplete information. One such language is the language of epistemic specifications. As is usual with logic programming languages, the problem of answering queries is intractable in the general case. For extended disjunctive logic programs, an idea that has proven useful in simplifying the investigation of answer sets is the use of splitting sets. In this paper we will present an extended definition of splitting sets that will be applicable to epistemic specifications. Furthermore, an extension of the splitting set theorem will be presented. Also, a characterization of stratified epistemic specifications will be given in terms of splitting sets. This characterization leads us to an algorithmic method of computing world views of a subclass of epistemic logic programs.

Introduction
One of the most important areas in artificial intelligence is knowledge representation. Traditional logic programming has proven itself to be a powerful tool for knowledge representation. There are, however, limitations to the expressibility of traditional logic programming. In an attempt to overcome some of these limitations, new logic programming formalisms were introduced. These new formalisms expand the traditional formalism by including disjunction [Min82], classical negation [GL91], or both (in the case of extended disjunctive logic programs) [GL91]. These formalisms work well for certain classes of programs. Unfortunately, these formalisms do not always allow for the correct representation of incomplete information in the presence of multiple belief sets. As an attempt at solving this problem, the language of epistemic specifications was introduced [Ge91]. A good overview of each of the formalisms mentioned above can be found in [BG94].

As is usual with logic programming languages, the problem of answering queries is intractable in the general case. It is often useful, however, to find methods which simplify the query answering task for certain subclasses of programs. In [LT93], the usefulness of splitting sets for the investigation of answer sets was shown.

In this paper we will present an extended definition of splitting sets that will be applicable to epistemic specifications. This in turn leads to an extension of the splitting set theorem from [LT93]. As with EDLPs, there is a strong relationship between stratification and splitting sets. Using these ideas, we can develop an algorithmic method for computing world views of a subclass of epistemic logic programs.

An overview of the syntax and semantics of epistemic specifications is covered in Section 2. In Section 3 we present splitting sets for epistemic specifications and the main theorem of the paper, the splitting set theorem. Finally, Section 4 contains a discussion of stratification, how it is related to splitting sets, and an algorithm for computing world views of stratified programs which is based on the splitting set theorem.

Epistemic Specifications
The language of epistemic specifications is an extension of the language of extended disjunctive logic programs (EDLPs). In addition to the normal operators in EDLPs, the language of epistemic specifications also contains unary modal operators $K$ and $M$. $K$ should be read as “is known to be true” while $M$ is read as “may be believed to be true”. Atoms are defined in the usual way. Literals in the language of epistemic specifications are split into two types, objective literals and subjective literals. An objective literal is either an atom or an atom preceded by $
eg$ (classical negation). A subjective literal is an objective literal preceded by $K$, $M$, $\neg K$, or $\neg M$. Given an objective literal, $L$, we will refer to the set of four subjective literals that can be built from $L$ as $SubLit(L)$. Given a set of objective literals, $U$, $SubLit(U) = \{ X : L \in U \text{ and } X \in SubLit(L) \}$.

Syntax
The general form for rules in epistemic specifications is given in [Ge91]. In this paper we will restrict rules to the form:

$F_1 \text{ or } \ldots \text{ or } F_n \leftarrow G_1, \ldots, G_k, \not\exists G_{k+1}, \ldots, \not\exists G_m$

where $F_1, \ldots, F_n$ are objective literals, $G_1, \ldots, G_k$ are either objective or subjective literals, and $G_{k+1}, \ldots, G_m$ are objective literals. This form differs from the original
only in the fact that in [Gel94], \( G_{k+1}, \ldots, G_m \) were also allowed to be subjective literals. Notice however that for any subjective literal, \( G_i \), the value of \( G_i \) can never be unknown and hence not \( G_i \) is always equivalent to \( \neg G_i \). It can therefore easily be seen that the restricted form of rules above can be used without any loss of expressibility.

A collection of such rules will be referred to as an epistemic logic program or an epistemic specification. Given a rule, \( r \),

- head(\( r \)) refers to the the set of literals, \( \{ F_1, \ldots, F_n \} \) which occur in the head of the rule.
- pos(\( r \)) refers to the set of all objective literals, \( L \), such that either
  - \( L = G_i \) for some \( 1 \leq i \leq k \), or
  - \( G_i \in \text{SubLit}(L) \) for some \( 1 \leq i \leq k \).
- neg(\( r \)) refers to the set of literals, \( \{ G_{k+1}, \ldots, G_m \} \).
- lit(\( r \)) = head(\( r \)) \cup pos(\( r \)) \cup neg(\( r \)).

Given an epistemic logic program, \( \Pi \), Lit(\( \Pi \)) will denote the union of the sets lit(\( r \)) for all \( r \in \Pi \).

**Semantics**

We now move from the syntax of the language to the semantics. A rule with variables is considered to be a shorthand for the set of all ground instances of the rule. The truth or falsity of a literal in an epistemic logic program is determined by the *world views* of that program. A *world view* is a collection of sets of ground objective literals which satisfy certain properties. An objective literal, \( L \), is true with respect to a collection of sets of literals, \( W \), if it is true in each set in that collection (i.e. for each set \( A \in W, L \in A \)). If \( W \) is a collection of sets of objective literals and \( L \) is an objective literal then

- \( KL \) is true with respect to \( W \) (denoted \( W \models KL \)) if for each set \( A \in W, L \in A \),
- \( W \models ML \) iff there exists an \( A \in W \) such that \( L \in A \),
- \( W \models \neg KL \) iff \( W \not\models KL \), and
- \( W \models \neg ML \) iff \( W \not\models ML \).

A literal is true with respect to an epistemic logic program if it is true in every *world view* of that program. We will define the concept of a *world view* of an epistemic logic program in several steps.

First let us consider the case when \( \Pi \) is an epistemic logic program which does not contain and does not contain any subjective literals. A set of literals, \( A \), is called a *belief set* of \( \Pi \) iff \( A \) is a minimal set satisfying the following two conditions:

- For every rule \( F_1 \lor \ldots \lor F_n \leftarrow G_1, \ldots, G_k \in \Pi \) if \( G_1, \ldots, G_k \in A \) then \( \exists i, 1 \leq i \leq n \) such that \( F_i \in A \),
- If \( A \) contains a pair of contrary literals then \( A = \text{Lit.} \) (This belief set is called *inconsistent.*)

Next we consider an epistemic logic program, \( \Pi \), which contains *not* but does not contain subjective literals (such programs are extended disjunctive logic programs). For any such \( \Pi \) and any set \( A \subseteq \text{Lit}(\Pi) \), let \( \Pi^A \) be the program obtained from \( \Pi \) by deleting

- each rule that contains *not* \( L \) in its body where \( L \in A \), and
- all formulas of the form *not* \( L \) in the bodies of the remaining rules.

The resultant program \( \Pi^A \) does not contain *not* or subjective literals and therefore its belief sets are as defined above. We will say a set, \( A \), of literals is a belief set of \( \Pi \) if \( A \) is a belief set of \( \Pi^A \).

Finally, let \( \Pi \) be an arbitrary epistemic logic program. Let, \( W \), be any collection of sets of literals from \( \text{Lit}(\Pi) \) and let \( \Pi^W \) be the program obtained by

- removing each rule which contains a subjective literal, \( L \), where \( W \not\models L \), and
- removing all subjective literals from the bodies of the remaining rules.

Notice that \( \Pi^W \) does not contain subjective literals, therefore we can compute its belief sets as previously described. If \( W \) is the set of all of the belief sets of \( \Pi^W \) then \( W \) is a world view of \( \Pi \).

We will say that a world view of an epistemic logic program is *consistent* if it does not contain a belief set consisting of all literals. We will say an epistemic logic program is *consistent* if it has at least one consistent non-empty world view.

Intuitively, a belief set is a set of literals that a rational agent may believe to be true. A world view is a set of belief sets that a rational agent may believe to be true with respect to that “world”.

The following give examples of epistemic logic programs and their world views.

**Example 1** Let \( \Pi_1 \) be the program which consists of the rules:

1. \( p(a) \) or \( p(b) \leftarrow \)
2. \( p(c) \leftarrow \)
3. \( q(d) \leftarrow \)
4. \( \neg p(X) \leftarrow \neg Mp(X) \)

The set \( W = \{ \{ q(d), p(a), p(c), \neg p(d) \}, \{ q(d), p(b), p(c), \neg p(d) \} \} \)

consisting of two belief sets, can be shown to be the only world view of \( \Pi_1 \).

**Example 2** For the next example, consider the program, \( \Pi_2 \), consisting of the following two rules:

1. \( p(a) \leftarrow \neg Mp(a) \)
2. \( q(a) \leftarrow \neg Mp(a) \)

It can be seen that \( \Pi_2 \) has two world views: \( W_1 = \{ \{ q(a) \} \} \) and \( W_2 = \{ \{ p(a) \} \} \).
Example 3 As a final example, consider the program, \(\Pi_3\) consisting of only one rule,

\[
p(a) \leftarrow \neg K p(a).
\]

It can be shown that this program does not have a world view.

In general, to find the world view of an epistemic logic program one must either try all possible collections of sets of literals or guess. It is infeasible to try all combinations since, even for the case where the number of ground literals, \(n\), is finite, there are \(2^n\) possibilities. A guess-and-check method could possibly be used to find world views but the problem is how to create an algorithm which would make good “educated” guesses and would know when and if it has found all the of the world views.

In this paper, we are primarily interested in presenting a means of computing world views. As a first step in achieving this goal, we will limit ourselves to programs which have at most a finite number of world views. For the remainder of this paper we will only consider epistemic logic programs which do not contain function symbols and have a finite number of constants and predicate symbols.

### Splitting Sets

In this section we will present a definition of splitting sets of epistemic logic programs. The definition is an extension of the definition in (LT93). We will also present a version of the splitting set theorem that is applicable to epistemic logic programs.

**Definition 1 (Splitting Set)** A set, \(U\), of objective literals is a splitting set of an epistemic logic program, \(\Pi\), iff

- for every rule \(r \in \Pi\), if head\((r) \cap U \neq 0\) then lit\((r) \subset U\), and,
- if \(\Pi\) contains \(K\) or \(M\), then for any objective literal, \(p \in \text{lit}(\Pi)\), if \(p \in U\) then \(\neg p \in U\).

If \(U\) is a splitting set of \(\Pi\), we also say that \(U\) splits \(\Pi\). The set of all rules \(r \in \Pi\) such that \(\text{lit}(r) \subset U\) is denoted by \(b_U(\Pi)\) and is called the bottom of \(\Pi\) with respect to \(U\). The set \(\Pi \setminus b_U(\Pi)\) is called the top of \(\Pi\) with respect to \(U\).

Using a splitting set, one can break the computation of a world view of an epistemic specification into two parts, a bottom and a top. The basic idea is to first compute the world view of the bottom of the program. The world view of the top can then be computed, taking into consideration the what was already computed for the bottom. Finally the two parts are merged together to get the world view of the complete program.

The world view of the bottom can be computed without regard to the top since no literal which occurs in the head of a rule of the top can occur anywhere in the bottom. When computing the world view for the top however, one needs to take the world view of the bottom into consideration. The world view of the bottom of the program can be used to “reduce” the top of the program. We can remove from the top those rules which cannot be satisfied because the value of a literal computed in the bottom makes their bodies false. From the remaining rules one can remove the portions of the bodies of the rules that were determined to be true. The reduction is performed in two steps; one for subjective literals and one for objective ones.

To remove subjective literals we will introduce the idea of a restricted reduct.

**Definition 2 (Restricted Reduct)** Let \(\Pi\) be an epistemic logic program, \(W\) be a collection of sets of literals, and \(U\) be a set of literals. The restricted reduct is the program obtained from \(\Pi\) by:

1. removing from \(\Pi\) all rules containing subjective formulae \(G\) where \(G \in \text{SubLit}(U)\) and \(W \not\models G\).
2. removing all other occurrences of subjective formula \(G\) where \(G \in \text{SubLit}(U)\).

The resultant program will be denoted by \(\Pi^{(U,W)}\) and be referred to as the reduct of \(\Pi\) with respect to \(W\), restricted by \(U\).

In our intended use, \(\Pi\), would be the top of a program, \(U\), would be the set used to split the program, and \(W\) would be the world view of the bottom.

The following is an example of a restricted reduct.

**Example 4**

Let \[
\begin{align*}
W &= \{\{a, \neg b, d\}, \{a, \neg d\}\}, \\
U &= \{a, \neg a, b, \neg b, c, \neg c\}, \text{ and} \\
\Pi &= e \leftarrow a, \neg b, f, \\
g &\leftarrow Ka, h, \\
i &\leftarrow Mc, \\
j &\leftarrow Kd, k
\end{align*}
\]

then \(\Pi^{(U,W)} = e \leftarrow a, f, g \leftarrow h\).

Next we consider objective literals. Recall that the world view of the bottom of a program is in essence a set of belief sets, all of which are different. Because of this, the truth or falsity of the objective literals in the bodies of rules of the top may vary with respect to each belief set. Due to this fact, after performing the reduction described below, rather than being left with a single program, we have, in general, a different partially evaluated top for each belief set of the bottom.

**Definition 3 (Partial Evaluation)** Given two sets of objective literals, \(U\) and \(X\), and an epistemic specification, \(\Pi\), for which none of the literals from \(U\) or \(X\) occur subjectively in its rules, then \(e_{U}(\Pi, X) = \{r'; \exists \text{ rule } r \in \Pi \text{ such that } \text{pos}(r) \cap U \subset X \text{ and } \neg \text{lit}(r) \cap U \text{ is disjoint from } X, r' \text{ is the rule which results from removing each sub-formula of the form } L \text{ or not } L \text{ from } r, \text{ where } L \in U\} \). We refer to \(e_{U}(\Pi, X)\) as the partial evaluation of \(\Pi\) with respect to \(X\).

Here again, in our intended use \(\Pi\) would be the top of the program, \(U\) would be the splitting set used, and \(X\)
would be one of belief sets from the world view of the bottom.

As was mentioned above, after taking the restricted reduct of the top and then finding the partial evaluation of the result with respect to each of the belief sets of the bottom, we are often left with multiple “tops”. We cannot simply take the world view of each “top” and merge them together. The reason for this is that it does not guarantee that the truth of subjective literals in the merged world view are the same as they were in each “top”. To handle this problem we introduce the idea of a multi-view.

**Definition 4 (Multi-view)** Given epistemic logic programs $\Pi_1, \ldots, \Pi_n$, then a collection of sets of objective literals, $W$, is a multi-view of $\Pi_1, \ldots, \Pi_n$ iff

1. $W = \bigcup_{i=1}^n \text{ans}(\Pi_i^W) \setminus \{\text{Lit}\}$ (if $\exists i$ s.t. $\text{ans}(\Pi_i^W)$ is consistent)
2. $W = \{\{\text{Lit}\}\}$ (otherwise)

A multi-view, $W$, is consistent iff $W \neq \{\{\text{Lit}\}\}$. For each $\Pi_i$, the set of all belief sets of $\Pi_i^W$ is called the restricted view of $\Pi_i$ with respect to $W$.

Here a simple example of a multi-view.

**Example 5**

If $\Pi_1 = a \leftarrow b$ and $\Pi_2 = a \leftarrow c \leftarrow Kb$

then $\Pi_1, \Pi_2$ has only one multi-view, $\{\{a, b\}, \{a\}\}$.

Before we present the main theorem of the paper we must first present a new notation and a definition. Given a collection of sets of objective literals, $W$, and a set of literals, $U$, then

$W|_U = \{X : \exists W_i \in W, X = W_i \cap U\}$.

**Definition 5 (Safe)** Given an Epistemic Specification $\Pi$ with splitting set $U$ such that $\Pi_U = b_U(\Pi)$ and $\Pi_U = \Pi\setminus\Pi_U$, $\Pi$ is said to be safe with respect to $U$ iff $\forall W = \{W_1, \ldots, W_n\}$ if $\{W_1|_U, \ldots, W_n|_U\} \subseteq \text{ans}(\Pi_U^W)$ then $\forall A \in \text{ans}(\Pi_U^W) : (\pi_U(\Pi_U^W, A))$ is consistent.

**Theorem 1** Let $\Pi$ be an epistemic specification, $U$ be a splitting set of $\Pi$ such that $\Pi$ is safe with respect to $U$. If we denote $b_U(\Pi)$ as $\Pi_U$, and $\Pi\setminus\Pi_U$ as $\Pi_U^W$ then:

1. If $X = \{X_1, \ldots, X_n\}$ is a consistent world view of $\Pi_U$ and $Y$ is a consistent multi-view of

$$(e_U(\Pi_U^W(U, X), X_1), \ldots, e_U(\Pi_U^W(U, X), X_n))$$

then if $W = \{W_i : W_i = X_j \cup Y_k, \text{ where } X_j \in X, Y_k \in \text{ans}((e_U(\Pi_U^W(U, X), X_j))^Y) \text{ and } X_j \cup Y_k \text{ is consistent }\} \neq \{\}$ then $W$ is a consistent world view of $\Pi$.

2. If $W$ is a consistent world view of $\Pi$ then $\exists X, Y$ such that $X$ is a world view of $\Pi_U$, $Y$ is a multi-view of

$$(e_U(\Pi_U^W(U, X), X_1), \ldots, e_U(\Pi_U^W(U, X), X_n))$$

and

$$\forall W_i \in W(W_i|_U \subseteq X)$$

and

$$W_i|_U \in \text{ans}((e_U(\Pi_U^W(U, X), W_i|_U))^Y)$$

In the above theorem we require that the splitting set be safe with respect to the program. As we will show, this restriction is important. If one or more or the belief sets of the bottom does not have a consistent extension to the top, the value of subjective literals defined in the bottom may change. In this case, the above method may not compute a correct world view.

**Example 6** Consider the program, $\Pi_4$, with the following rules:

1. $p(a) \text{ or } p(b) \leftarrow$
2. $p(c) \leftarrow M(p(b)$
3. $p(d) \leftarrow p(b)$
4. $\neg p(d) \leftarrow p(b)$

If we split the program using

$$U = \{p(a), \neg p(a), p(b), \neg p(b), p(c), \neg p(c)\}$$

as a splitting set, then $b_U(\Pi_4)$, which consists of rules 1 and 2, has one world view which contains 2 belief sets, $\{p(a), p(c)\}$ and $\{p(b), p(c)\}$. With respect to the belief set $\{p(b), p(c)\}$, however, the top of the program is inconsistent. Using the method from the theorem above, not requiring the program be safe, we get one “world view”: $\{\{p(a), p(c)\}\}$. It can easily be seen however, that this is not a world view of $\Pi_4$. The only world view of the program is $\{\{p(a)\}\}$.

The error occurred because, since $p(b)$ was “possible” in the world view the bottom we concluded $p(c)$ was therefore true even though we later find that $p(b)$ is no longer “possible” after the computation of the top.

As can be seen from the definition, determining if a splitting set of a program is safe may be as difficult as finding the world views. We will give a property which more intuitive and easier to check. While it is less general, it is reasonable and encompasses a large number of interesting programs. Before we present the condition, we must first define satisfies.

**Definition 6 (Satisfies)** Given a program $\Pi$ and a collection of sets of literals from $\text{Lit}(\Pi)$, denoted $W$, then we will say $W$ satisfies the body of a rule, $r \in \Pi$, if each literal in the body is true with respect to $W$. We say $W$ satisfies $r$ if either $W$ does not satisfy the body of $r$ or at least one literal in the head of $r$ is true with respect to $W$.

We now present the property.

**Definition 7 (Guarded)** We will say that a program $\Pi$ is guarded with respect to a splitting set $U$ if
• II does not contain subjective literals, or
• for every pair of rules $R_1, R_2 \in II \setminus b_0(II)$ and for every collection of sets of literals from $\text{Lit}(II)$, denoted as $W$, if head($R_1$) and head($R_2$) contain contrary literals and $W$ satisfies all of the rules in $b_0(II)$ then either $W$ does not satisfy the body of $R_1$ or $W$ does not satisfy the body of $R_2$. Note that if a rule with an empty head can be rewritten as a rule which has the predicate ¬true as the head and by adding the rule

true ←

to the program. A program containing rules with empty heads is guarded with respect to $U$ if the program rewritten without such rules is.

It can be shown that, given any program II with splitting set $U$, if II is guarded with respect to $U$ then $U$ is safe with respect to II.

Splitting and Stratification

In this section we will give a definition of stratification for epistemic logic programs, show how it relates to splitting sets, and illustrate how the splitting set theorem can be used to simplify the computation of the world view of a stratified epistemic logic program. We will start out with the definition of stratification.

Definition 8 (Stratification) A partitioning $\pi_0, \ldots, \pi_z$

of the set of all literals of an epistemic logic program, II, is a stratification of II, if for any literal, $L_1 \in \pi_i$, then

$\neg L_1 \in \pi_i$

and for any other literal $L_2$ in $\text{Lit}(II)$ and any rule $r \in II$:

• if $L_1, L_2 \in \text{head}(r)$ then $L_2 \in \pi_i$,
• if $L_1 \in \text{head}(r)$ and $L_2$ occurs objectively in $\text{pos}(r)$ then there exists an $j \leq i$ such that $L_2 \in \pi_j$.
• if $L_1 \in \text{head}(r)$ and $L_2 \in \text{neg}(r)$ or $L_2$ occurs subjectively in $r$, then there exists $j < i$ such that $L_2 \in \pi_j$.

This stratification of the literals defines a stratification of the rules of II to strata $\Pi_0, \ldots, \Pi_k$ where a strata $\Pi_i$ contains all of the rules of II whose heads consists of literal from $\pi_i$. A program is called stratified if it has a stratification.

It can easily be seen that, given a stratified epistemic logic program, II, with stratification $\pi_0, \ldots, \pi_z$, the set of literal $U_i$ such that

$U_i = \bigcup_{j=1}^{i} \pi_j$

is a splitting set of II. With each stratified epistemic logic program we will then associate a sequence $U_0, \ldots, U_z$ of splitting sets formed as described.

This leads us to an algorithm for computing the world view a safe, stratified epistemic specification. Given an epistemic specification, II, with stratification $\pi_0, \ldots, \pi_n$ and associated splitting sets $U_0, \ldots, U_n$, such that II is safe with respect to $\{ \}$ and each $U_i$, we can compute the world view of II as follows:

1. Using the splitting set theorem, compute the world view, $W_1$, of $\Pi_0 \cup \Pi_1$ with splitting set $U_0$. Note that $b_{U_0} = \Pi_0$ and, by the definition of stratification, it does not contain not or any subjective literals. $\Pi_0$ is also safe with respect to $\{ \}$. From these two facts, it can be seen that $\Pi_0$ has a unique, consistent, world view which consists of all the belief sets of the EDLP $\Pi_0$.

2. Given the world view, $W_{i-1}$, of $\Pi_0 \cup \ldots \cup \Pi_{i-1}$, the world view, $W_i$, of $\Pi_0 \cup \ldots \cup \Pi_i$ can be computed using the splitting set theorem with the splitting set $U_{i-1}$.

Notice that $W_n$ is the world view of II. It can be seen from the definition of stratification that, in each step of the algorithm above, when we take the restricted reduct of the top of program we are left with a program which does not contain subjective literals. The multi-view is therefore simply the union of the world views obtained by taking the restricted reduct of the top and partially evaluating with respect to one of the belief sets of the world view of the bottom. To compute the world view of a safe, stratified, epistemic logic program therefore, one only needs to be able to compute the belief sets of extended disjunctive logic programs.

The following theorem, which is a slightly modified version of a theorem from [Vai94], also follows from the results above.

Theorem 2 Given any stratified, epistemic logic program, II, which is safe with respect to $\{ \}$ as well as each of the splitting sets associated with its stratification, the program II has a unique, consistent, world view.

Conclusion

In this paper, we expanded the results from [LT93] to include epistemic logic programs. We also presented definitions of what it means for a epistemic logic program to be safe, guarded, and stratified. This led to an algorithmic method for computing world views of a subclass of epistemic logic programs.

It should be noted that the belief sets of an extended disjunctive logic program are simply the answer sets (GL93) of that program. Recently, there have been considerable advances in the computation of such answer sets. One such system which shows great promise is DLV (Leo97). Using their system and the results in this paper, it should be a reasonable task to create a inference engine for the subclass of epistemic logic programs mentioned here.

As this paper is meant to form a basis for the computation of world views, we restricted ourselves to epistemic logic programs with a finite number of finite world views. We believe that the theorem presented here can
be expanded to cover programs with an infinite number
of infinite world views.

Acknowledgements
The author would like to thank Michael Gelfond and
the anonymous reviewers for their helpful comments.

References
[BG94] Chitta Baral and Michael Gelfond. Logic Pro-
gramming and Knowledge Representation. In Journal
of Logic Programming, 1994
[Leo97] Nicola Leone et al. The DLV system: Model
generator and application frontends. In Proceedings of
the 12th Workshop on Logic Programming, pages 128-
137, 1997.
[Gel91] Michael Gelfond. Strong introspection. In Pro-
cedings of AAAI-91, pages 386-391, 1991.
[Gel94] Michael Gelfond. Logic programming and rea-
soning with incomplete information. In Annals of
Mathematics and Artificial Intelligence, vol. 12, pages
89-116, 1994.
[GL90] Michael Gelfond and Vladimir Lifschitz. Logic
programs with classical negation. In Logic Program-
ming: Proceedings of the 7th International Conference,
pages 579-597, 1990.
[GL91] Michael Gelfond and Vladimir Lifschitz. Clas-
sical negation in logic programs and disjunctive
databases. In New World Computing, pages 365-387,
1991.
[GP91] Michael Gelfond and Halina Pryzmusinska.
Definitions in epistemic specifications. In Logic Pro-
gramming and Non-monotonic Reasoning, Proceedings
of the First International Workshop, pages 245-259,
1991.
[LT93] Vladimir Lifschitz and Hudson Turner. Splitting
a logic program. In Proceedings of the Eleventh Inter-
national Conference on Logic Programming, pages 23-
37, 1994.
[Min82] Jack Minker. On indefinite databases and the
closed world assumption. In Proceedings of CADE-82,
pages 292-308, 1982.
[Wat94] Richard Watson. An Inference Engine for Epis-
temic Specifications, 1994. M.S. Thesis, Department of
Computer Science, University of Texas at El Paso.