Is there an imprint of Planck scale physics on inflationary cosmology?

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We study the effects of the trans-Planckian dispersion relation on the spectrum of the primordial density perturbations during inflation. In contrast to the earlier analyses, we do not assume any specific form of the dispersion relation and allow the initial state of the field to be arbitrary. We obtain the spectrum of vacuum fluctuations of the quantum field by considering a scalar field satisfying the linear wave equation with higher spatial derivative terms propagating in the de Sitter space-time. We show that the power spectrum does not strongly depend on the dispersion relation and that the form of the dispersion relation does not play a significant role in obtaining the corrections to the scale invariant spectrum. We also show that the signatures of the deviations from the flat scale-invariant spectrum from the CMBR observations due to quantum gravitational effects cannot be differentiated from the standard inflationary scenario with an arbitrary initial state.

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I. INTRODUCTION

The present upper limits on the temperature fluctuations of the microwave background radiation, corresponding to the density perturbations at the time of recombination, are less than 1 part in $10^5$ on scales larger than the Hubble radius. On these scales, the observations of the CMBR give the power spectral index $n$ to be $1.1 \pm 0.1$. These observations provide an upper bound on the amplitude of density fluctuations at that epoch. Inflationary paradigm provides a physical mechanism for the generation of seed density perturbations (see, for example, Ref. [1]). Many models of inflation require a period much longer than 70 e-foldings to solve the horizon and flatness problems of standard cosmology. In order to obtain the observed density inhomogeneities on the galactic scales, these models of inflation require the modes of vacuum fluctuations at the initial epoch of inflation to be smaller than Planck length $L_P \equiv (G\hbar/c^3)^{1/2}$ [2].

It is well known that the space-time structure at Planck scales is affected by the quantum gravitational effects and it is generally believed that $L_P$ acts as a physical cutoff for space-time intervals. The existence of a fundamental length implies that processes involving energies higher than Planck energies will be suppressed, and the ultraviolet behavior of the theory will be improved. All existing models of quantum gravity provide a mechanism for good ultra-violet behavior, essentially through the existence of the fundamental length scale. Even though the models of quantum gravity have some success, none of these have given a complete theory that works at the Planck scales.

If the ultimate theory of quantum gravity has a fundamental length scale, then the low-energy effective quantum field theories should have an imprint of the Planck scale. However, the standard formulation of quantum field theory does not take into account the existence of any fundamental length in the space-time. Attempts have been made to incorporate quantum gravitational effects with the help of two different approaches: (i) by introducing the fundamental length scale in a Lorentz-invariant manner in the Feynman Green function and (ii) by modifying the dispersion relation of the linear field equation.

In this work, our focus is on approach (ii). (For method (i) and its implications in conventional quantum field theory, see Ref. [3, 4].) In the context of inflationary cosmology, trans-Planckian effects have gained interest after the detailed work of Martin and Brandenberger [7, 8] who introduced modified dispersion relations in an ad hoc manner. The authors dealt with high-frequency dispersion relations, used earlier by Unruh [5] and Corley/Jacobson [6] for the analysis of deviations from the thermality of the Hawking spectrum, to the inflaton field and obtained the spectrum of the vacuum fluctuations after it crosses the Hubble radius (during the inflation). They found that while some hypothesized dispersion relation(s) to quantum field theory yield no change in the power spectrum of the fluctuations, some indicate the cosmological sensitivity to Planck scale physics. However, there have been arguments that the spectrum of vacuum fluctuations should not be sensitive to the Planck scale physics [10, 11].

All previous analyses of trans-Planckian effects on inflationary perturbations [7]-[13] have concentrated on specific form of the dispersion relations for a particular choice of initial state of the field. [Two sets of initial conditions which are used in the literature are Minimum energy and Instantaneous Minkowski vacuum.] In our analysis, we do not assume any specific form of the dispersion relation and also allow initial state of the field to be arbitrary. The only condition on the dispersion relation being that the non-linear dispersion relations become linear for $\lambda >> L_P$. From the earlier analyses, we would expect that the spectrum of fluctuations will strongly depend on the nature of the dispersion relation and the initial state of the field.

Using this strategy, we address the following questions:

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Is there an imprint of Planck scale physics on inflationary cosmology? and Can the deviations from the flat scale-invariant spectrum due to non-linear dispersion relations be differentiated from that of deviations from the flat spectrum in the standard inflationary scenario? We try to provide an answer by considering a quantum field satisfying the linear wave equation with higher spatial derivative terms propagating in the de Sitter space-time. We perform a general analysis for a class of dispersion relations whose field modes can be expanded as WKB modes over a range of initial states (conditions) for the quantum field. In this paper we are primarily interested in frequency evolution of modes and we ignore the actual creation of the modes. There are claims in the literature that the energy density of fluctuations of the inflation field is necessarily high and its back-reaction on the metric cannot be ignored [10, 12] (see also, Ref. [14]).

We show that the signatures of the deviations from the flat scale-invariant spectrum due to non-linear dispersion relations cannot be differentiated from the standard inflationary scenario with an arbitrary initial state. We supplement this claim by studying four dispersion relations for an arbitrary initial state of a quantum field Φ, in standard inflationary scenario and in dispersive field theory models, propagating in the de Sitter space-time.

\[ \Omega^2 = \frac{E_p^2}{2\pi} \log \left(1 + \frac{2\pi k'^2}{E_p^2}\right) \]

(Padmanabhan’s dispersion relation)

\[ \Omega^2 = k^2 \log^2(1 + k/k), \]

(κ-Poincare dispersion relation)

\[ \Omega^2 = k_0^2 \text{tanh}(k/k_0)^{n/(2^n)}, \]

(Unruh’s dispersion relation)

\[ \Omega^2 = k_0^2 \tan^{-2}[k/k_0], \]

(Ahlwalia’s dispersion relation)

where \( E_p \) is the Planck energy and \( k_0 \) is the inverse Planck length (or of the order of Planck scale). The dispersion relation (1) reproduces the exact density of states required for the black-hole by treating black-hole as a one-particle state of a non-local field theory [15], relation (2) is motivated by κ-Poincare algebra [9] while the last relation is the gravitationally modified expression for the wave-particle duality [16].

The organization of the paper is as follows: In Sec. (II), we calculate the power spectrum of the vacuum fluctuations for an arbitrary initial condition in (i) standard inflationary scenario (ii) dispersive field theory models, and interpret the results. Finally, in Sec. (III) we summarize the results.

## II. POWER SPECTRUM OF QUANTUM FLUCTUATIONS

In this section, we calculate the power spectrum of fluctuations for an arbitrary initial state of a quantum field Φ, in standard inflationary scenario and in dispersive field theory models, propagating in the spatially flat FRW background (specifically de Sitter). We show that using the observations of CMBR the signatures of the deviations from the flat scale-invariant spectrum due to non-linear dispersion relations cannot be differentiated from the standard inflationary scenario with an arbitrary initial state.

### A. Standard inflation

Let us consider scalar fields propagating in the flat FRW metric of the form

\[ ds^2 = dt^2 - a^2(t) dx^2 = a^2(\eta) \left[ d\eta^2 - d\mathbf{x}^2 \right], \]

where \( \mathbf{x} \) is the 3-space, \( t \) is the conforming time, \( \eta \) is the conformal time and \( t = \int a(\eta) d\eta \). For flat de Sitter \( a(\eta) = 1/(H\eta) \) \( [a(t) = \exp(HT)] \) and \( H \) is the Hubble constant during inflation. [Typical values of the inflationary expansion are \( E \approx 10^{15} \text{GeV} \), \( H \approx 5 \times 10^{-24} \text{ cm/s} \), start of the inflation \( \eta_i \approx -10^{-36} \text{ sec} \), and the end of inflation \( \eta_f \approx 70H^{-1} \).] The free massless, minimally coupled scalar field satisfies the Klein-Gordon equation,

\[ \Box \Phi \equiv \frac{d^2 \Phi}{d\eta^2} + 2 - \frac{1}{a^2(\eta)} \frac{da(\eta)}{d\eta} \Phi = 0. \]

The symmetry of the Robertson-Walker metric allows for separating variables in Eq. (6) and the scalar field can be decomposed as

\[ \Phi(\mathbf{x}, \eta) = \frac{1}{(2\pi)^{3/2}} \int_k \left[ a_k u_k(\mathbf{x}, \eta) + a_k^* u_k^*(\mathbf{x}, \eta) \right] dk, \]

where

\[ u_k(\mathbf{x}, \eta) = a(\eta)^{-1} \chi_k(\eta) \exp(ik \cdot \mathbf{x}). \]

[\( k \) is the comoving wave vector and \( k^2 = | \mathbf{k} |^2 \).] Note that the creation and annihilation operators \( a_k, a_k^* \) obey the usual commutation relations. The time-dependent part of the mode function satisfies the oscillatory equation

\[ \frac{d^2 \chi_k}{d\eta^2} + \omega_k^2(\eta) \chi_k(\eta) = 0, \]

where \( \omega_k^2(\eta) = k^2 - [a''(\eta)/a(\eta)] \). [The solution to the above equation reduces to the usual harmonic oscillator solution, \( \chi_k \sim \exp(\pm i \omega_k \eta) \), with a linear dispersion relation, \( \omega^2 = k_{ph}^2 = k^2/a^2(\eta) \), on scales much smaller than the Hubble length. \( k_{ph} \) is the physical momentum of the perturbation.] A complete set of mode solutions to Eq. (9) is obtained by imposing initial conditions \( \chi_k(\eta_i) \), \( \chi_k'(\eta) \) on a Cauchy surface \( \eta = \eta_i \) corresponding to a homogeneous initial state.

The solution to the above equation in the short-wavelength limit \( \lambda_{ph} << H^{-1} \) corresponds to the Minkowski modes, i.e.,

\[ \chi^I = \frac{a_k}{\sqrt{2k}} \exp(ik \eta) + \frac{b_k}{\sqrt{2k}} \exp(-ik \eta) \]
TABLE I: The dominant contribution and solution to the Eq. (16) in three different regimes. In this $\eta_i$ is the epoch at which inflation starts, $\eta_{II}$ is the epoch at which the non-linear dispersion relation can be approximated to be linear and $\eta_H$ is the epoch at which the perturbation leaves the Hubble radius.

| Regime | Epoch | Dominant contribution of $\tilde{\omega}_k^{\text{mod}}(\eta)$ | Solution |
|--------|-------|------------------------------------------------|-----------|
| I      | $\eta_{II} >> \eta \geq \eta_i$ ($\lambda_{ph} << H^{-1}$) | $W_k^I(\eta) = a(\eta)\Omega[k_{ph}(\eta)]$ | $\chi_k^I \sim [2W_k^I(\eta)]^{-1/2} \exp \left( \pm i \int W_k^I(\eta)d\eta \right)$ (WKB approximate solution) |
| II     | $\eta_H >> \eta \geq \eta_{II}$ ($\lambda_{ph} << H^{-1}$) | $W_k^{II}(\eta) = k$ | $\chi_k^{II} \sim \exp(\pm ik\eta)$ (Plane wave solution) |
| III    | $\eta \geq \eta_H$ ($\lambda_{ph} >> H^{-1}$) | $[W_k^{III}(\eta)]^2 = -[a''(\eta)/a(\eta)]$ | $\chi_k^{III} \sim C_k a(\eta)$ (frozen classical perturbation) |

where, $a_k$ and $b_k$ are determined by the choice of the initial conditions. Orthonormality of the modes gives,

$$|a_k|^2 - |b_k|^2 = 1 \quad (11)$$

Thus, $a_k$ and $b_k$ can be any arbitrary functions of $k$ satisfying the above condition. In the long wave-wavelength limit ($\lambda_{ph} >> H^{-1}$), the modes are frozen and the solution is given by

$$\chi_k^{II} = C_k a(\eta) = C_k \frac{H}{\eta}. \quad (12)$$

Demanding that $\chi^I$ and $\chi^{II}$ are continuous at $\eta_H$, $C_k$ can be determined in-terms of $a_k$ and $b_k$. [The junction condition at $\eta_H$ between the two modes correspond to the relation $\{2\pi a(\eta)/k = H^{-1}$, i.e., $\eta_H = 2\pi/k\}$. Thus, for the $k$ mode leaving the Hubble radius at $\eta_H$, we get

$$k^3|C_k|^2 = \frac{H^2}{2} + H^2 \left[ |b_k|^2 + |b_k|(|b_k|^2 + 1)^{1/2} \right]. \quad (13)$$

where we have used condition the (11) to write $|a_k|^2$ in-terms of $|b_k|^2$. The above result shows that the spectrum of fluctuations has a scale-invariant term plus a non scale-invariant term which strongly depends on the choice of the initial state ($b_k$). If we assume that on cosmologically relevant scales the inflaton field is in the vacuum state, corresponding to no inflaton particles corresponding to the initial condition,

$$\chi(\eta_i) = \frac{1}{\sqrt{2k}}; \quad \chi'(\eta_i) = \pm i \frac{\sqrt{k}}{2}, \quad (14)$$

we get $a_k = 1, b_k = 0$; which results in a flat scale-invariant spectrum.

B. Dispersive field theory model

As mentioned in the introduction, in many models of inflation, the physical length of the perturbations is smaller than the Planck length at the initial epoch implying we need to consider quantum gravitational corrections. One way of introducing quantum gravitational corrections is to modify the linear dispersion relation in an ad-hoc manner by breaking the Lorentz invariance i.e., by rewriting $\omega^2 = \Omega^2[k_{ph}(\eta)]$. Replacing $k^2$ in Eq. (9) as

$$k_{\text{mod}}^2 = a^2(\eta)\Omega^2[k_{ph}(\eta)] = a^2(\eta)\Omega^2 \left( \frac{k}{a(\eta)} \right), \quad (15)$$

we get,

$$\frac{d^2\chi_k}{d\eta^2} + \tilde{\omega}_k^{\text{mod}}(\eta)\chi_k = 0, \quad (16)$$

where,

$$\tilde{\omega}_k^{\text{mod}}(\eta) = a^2(\eta)\Omega^2 \left( \frac{k}{a(\eta)} \right) - \frac{a''(\eta)}{a(\eta)}. \quad (17)$$

The modified dispersion relations must satisfy the property $k_{\text{mod}} \sim k$ for $k << k_{ph}$. We have tabulated the three regimes of interest and the dominant contribution of $\tilde{\omega}_k^{\text{mod}}$ in these regimes in Table I. Using the notation in Table I, the solution to Eq. (16) in regime I, in the WKB limit is

$$\chi_k^I(\eta) = \frac{A_k}{\sqrt{2W_k^I(\eta)}} \exp \left( -i \int W_k^I(\eta)d\eta \right)$$

$$+ \frac{B_k}{\sqrt{2W_k^I(\eta)}} \exp \left( i \int W_k^I(\eta)d\eta \right), \quad (18)$$

where, $A_k$ and $B_k$ are constants and are determined by the choice of the initial conditions. Orthonormality of the modes gives,

$$|A_k|^2 - |B_k|^2 = 1. \quad (19)$$
Here again, we assume $A_k$ and $B_k$ to be arbitrary functions of $k$ satisfying the above condition. In regime II, solution to Eq. (16) is (Minkowski) plane wave solution, i. e.,

$$\chi^{II}_k(\eta) = \alpha_k \exp(-i k \eta) + \beta_k \exp(i k \eta).$$

(20)

The junction conditions of the wave modes $\chi^I_k/\chi^{II}_k$ and its derivatives at $\eta = \eta_1$ gives

$$\alpha_k = A_k \frac{\partial}{\partial \eta} \exp[-i I(\eta_1) + i k \eta_1],$$  

$$\beta_k = B_k \frac{\partial}{\partial \eta} \exp[i I(\eta_1) - i k \eta_1],$$

(21)

(22)

where,

$$\frac{\partial}{\partial \eta} = \sqrt{\frac{W^I_k(\eta_1)}{8 k^2}} + \frac{1}{\sqrt{8 W^I_k(\eta_1)}}.$$  

(23)

$$I(\eta) = \int_{\eta_1}^{\eta} W^I_k(\eta') d\eta' = \int_{\eta_1}^{\eta} a(\eta') \Omega[k_{ph}(\eta')] d\eta'.$$

(24)

In regime III, the solution is

$$\chi^{II}_k(\eta) = \frac{1}{a(\eta_1)} \chi^I_k(a(\eta_1)) = C_k^{mod} a(\eta),$$

(25)

where $C_k^{mod}$ is a constant whose modulus square gives the power spectrum of the density perturbations. Using the junction conditions at $\eta = \eta_H$, we get

$$C_k^{mod} = \frac{1}{a(\eta_H)} [\alpha_k \exp(-i k \eta_H) + \beta_k \exp(i k \eta_H)].$$

(26)

Rewriting $\alpha_k/\beta_k$ in-terms of $A_k$ and $B_k$, we get

$$|C_k^{mod}|^2 = a^{-2} \frac{\partial^2}{\partial \eta} \left[ |A_k|^2 + |B_k|^2 + \mathcal{R}(A_k B^*_k \exp[-2 i I(\eta_1) + 2 i k(\eta_1 - \eta_H)]) \right].$$

(27)

Using the condition (19), we obtain

$$|C_k^{mod}|^2 = \frac{H^2 k^{-2}}{4 k_{mod}(\eta_H)} \left( 1 + \frac{k_{mod}(\eta_H)}{k} \right)^2 \left[ \frac{1}{2} + |B_k|^2 + \sqrt{B_k^2 + |B_k|^2 \cos [-2 I(\eta_1) + 2 k(\eta_1 - \eta_H)]} \right].$$

(28)

Using the property that $k_{mod} \simeq k$ for $k << k_{pl}$, we get

$$k^3 |C_k^{mod}|^2 = \frac{H^2}{2} + H^2 |B_k|^2 + H^2 |B_k| \sqrt{|B_k|^2 + 1} \times \cos [-2 I(\eta_1) + 2 k(\eta_1 - \eta_H)].$$

(29)

The above result shows that the spectrum of fluctuations has a scale-invariant term plus a non scale-invariant term which strongly depends on the initial state ($B_k$) and the form of the dispersion relation ($\Omega$). If we assume the initial state of the field (Minimum energy state) to be

$$\chi_k(\eta_1) = \sqrt{\frac{1}{2a(\eta_1)}} \Omega^{-1/2} \left( \frac{k}{a(\eta_1)} \right),$$

(30)

$$k \chi'_k(\eta_1) = \pm \sqrt{\frac{a(\eta_1)}{2}} \Omega^{1/2} \left( \frac{k}{a(\eta_1)} \right),$$

(31)

we get $A_k = \exp[I(\eta_1)], B_k = 0$; which results in a flat scale-invariant spectrum.

C. Analysis and Interpretation of results

In subsections (II A) and (II B), we obtained the power spectrum of fluctuations for an arbitrary initial state in standard inflationary scenario and dispersive field theory models. In this subsection, we compare and interpret the results obtained. We first show that the third term in the right hand side of (29) cancels out. We then show that the signatures of the deviations from the flat scale-invariant spectrum due to non-linear dispersion relations cannot be differentiated from the standard inflationary scenario with an arbitrary initial state.

Using the property that the non-linear dispersion relations becomes linear for $k_{pl} >> k_{ph}$, we can write them in the general form

$$\Omega = k_{pl} F \left( \frac{k_{ph}}{k_{pl}} \right),$$

(32)

where $F[k_{ph}/k_{pl}] \rightarrow k_{ph}/k_{pl}$ in the large wavelength limit. Substituting the above form in (24), we get

$$I(\eta_1) = \frac{k_{pl}}{\eta_H} \times Q$$

(33)

where

$$Q = \int_{H\eta_1}^{H\eta_1} \frac{d(H\eta_1)}{(H\eta_1)} F \left( \frac{k(\eta_1)}{k_{pl}} \right).$$

(34)

Using the typical energy scale at the start of inflation to be GUT scale [$E \approx 10^{15} GeV$, $\eta_i \approx -10^{-36}$sec, $H \approx 10^{13} GeV^{-1}$], the prefactor $k_{pl}/H$ in the expression (33) is of the order $10^{10}$. This implies that unless the integral $Q$ is a small number (of the order $10^{-9}$), the argument of the $\cos$ term in (29) rapidly oscillates and can be set to zero. [This is because, in the finding $C_i$’s (spherical harmonics of the temperature anisotropy) we need to integrate over $k$ and the $\cos$ term vanishes.]

Assuming the amount of inflation to be $80 - \epsilon$ foldings, we get $\eta_{pl} \approx 10^{-38}$sec. To obtain a rough estimate of the integral $Q$, we consider four dispersions relations ($1 \sim 4$) - Padmanabhan’s, $\kappa$-Poincare, Unruh’s and Ahluwalia’s. The numerical value of the integral $Q$ in all these cases ranges between 0.5 to 0.7. [For numerical work, we use MATHEMATICA owing to its convenience]
and flexibility it offers.] Since, the argument of cosine in the expression (29) is large (of the order $10^9$) for all practical purposes, it can be set to zero. Thus, Eq. (29) reduces to

$$k^3|c_k|^2 = \frac{H^2}{2} + H^2|B_k|^2. \tag{35}$$

The following features are noteworthy regarding this result:

1. If the field modes are expanded as WKB modes and the initial state of the field is $A_k = 1; B_k = 0$, then the spectrum of fluctuations are flat scale-invariant for any general dispersion relation. [In the case of Corley/Jacobson dispersion relation, $\Omega$ becomes imaginary for $k > k_B$ and hence the expansion of modes as WKB modes is not valid.] In the standard inflationary scenario, the fluctuations are flat if the initial state of the field is $a_k = 1; b_k = 0$.

2. For an arbitrary initial state, the spectrum of fluctuations in the dispersive field theory models have a scale-invariant term plus a non-scale invariant term. The non-scale invariant term strongly depends on the initial state ($B_k$). However, the power spectrum does not depend strongly on the dispersion relation. Thus, the form of the dispersion relation does not play significant role in obtaining the corrections to the scale-invariant spectrum.

3. In our discussion, we have assumed $|b_k|$ and $|B_k|$ to be arbitrary functions of $k$. The form of $|b_k|$ and $|B_k|$ contribute to the non scale-invariant part of the power spectrum. Setting,

$$|B_k|^2 = |b_k|^2 + |b_k|(|b_k|^2 + 1)^{1/2}, \tag{36}$$

we observe that the spectrum of fluctuations in the standard inflationary scenario and in dispersive field theory model are same. Hence, the signatures of the deviations from the flat scale-invariant spectrum from the CMBR observations due to quantum gravitational effects cannot be differentiated from the standard inflationary scenario with an arbitrary initial state. In other words, the signatures of the deviation from the flat scale-invariant spectrum is not necessarily due to the trans-Planckian effects. Even in the standard inflation, the deviations from the flat spectrum can be obtained for a different choice of vacuum state – other than the Bunch-Davies vacuum.

### III. CONCLUSIONS

We have studied the quantum gravitational effects on the spectrum of primordial density perturbation in the inflationary epoch. We have incorporated the quantum gravitational corrections using non-linear dispersion relations to the quantum scalar field i.e., the scalar field satisfying the linear wave equation with higher spatial derivative terms propagating in the de Sitter space-time [and ignored the back-reaction of the particles on the metric]. We keep the form of the dispersion relation to be arbitrary and have not assumed any specific dispersive field theory model.

We have performed a general analysis of the power spectrum for a class of dispersion relations whose field modes can be expanded as WKB modes over a range of initial conditions for the quantum field. We have shown that the spectrum of fluctuations due to the quantum gravitational corrections has a scale-invariant term plus a non-scale-invariant term which strongly depends on $B_k$ (the initial state) and the form of the dispersion relation $\Omega$. However, the strong dependence on the dispersion relation cancels out due to the rapidly varying phase term. We have thus shown that the form of the dispersion relation do not play significant role in obtaining the corrections to the scale invariant spectrum. We have also shown that the deviations from the flat spectrum due to quantum gravitational effects (29) cannot be differentiated as opposed to the deviations from the flat spectrum for an arbitrary initial state in the standard inflationary scenario (13).

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