Implicit-Discrete-Maximum-Principle-Based Production Optimization in Reservoir Development

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Abstract. Cost-effective development of proved oil resources is the goal of oil and gas field development workers. One of the ways to achieve this is to make the reservoir development in an optimal state based on the existing production conditions, which is also one of the core contents of intelligent oilfield. This paper proposes a new production optimization control method, which aims to maximize the net present value of reservoir development and production. By solving the mathematical model of development and production, the input and output control parameters of the reservoir are optimized in real time to obtain the optimal production plan. The conventional maximum principle method requires two time series solution calculations. In this paper, the coefficient matrix of the full implicit simulator is used to directly obtain the required adjoint equations. The efficient combination of simulation calculation and gradient solution of discrete maximum principle greatly improves the solution efficiency of the model and saves the calculation time. The reservoir examples are analyzed through theoretical research. The results show that the optimal production scheme is in line with the actual situation of the oilfield, which provides theoretical and technical support for the intelligent oilfield system.

1. Introduction

The active world economy has promoted the rapid growth of global disposable energy consumption. However, it is difficult to find new large-scale oil and gas fields. The exploration targets of oil companies have been turned to more complex areas. Therefore, how to maintain the balance of oil supply under the premise of such a rapid growth of disposable energy demand is a crucial problem that the oil industry is forced to solve [1]. Many of the large oil and gas fields were put into production in the 1960s and entered the stage of production decline in the 1990s. Under the impetus of this situation, some oil companies have been seeking solutions in order to reduce the rate of decline in production, so the concept of intelligent oil fields management has been proposed and widely concerned [2]. The reservoir development optimization theory method is a technical theory based on the above-mentioned concept. This theoretical method can not only timely analyze oil well production data and control reservoir occurrence, but also reduce uncertainties in reservoir development and improve reservoir recovery [3-5]. This research can provide scientific and effective technical support for realizing the informatization, digitalization and intelligent management of oil and gas exploitation engineering.
Generally speaking, the most representative optimization methods used for engineering problems are mainly divided into two categories: one is the global search algorithm represented by genetic algorithm and simulated annealing algorithm [6-9], and the other is the gradient algorithm represented by the steepest descent method and Newton iteration algorithm [10, 11]. The first category of global search algorithm needs to perform simulation estimation according to time series, which cannot guarantee the monotonous decrease or increase of the objective function, but they can find out the global optimal solution through large-scale calculation. The second category of gradient algorithm is more effective. It also needs to perform simulation estimation according to time series, but they can guarantee the single increase or single decrease of the objective function at each iteration to find a relatively better solution. However, the disadvantage of this method is that it can only ensure the local optimal solution for non-convex problems.

For reservoir simulation calculations, the number of simulated grids is in the hundreds of thousands, and it may take tens of hours to perform time series simulation calculations. This means that the gradient-based algorithm is more effective and feasible for this problem. In addition, for this kind of engineering problems, the goal is not to find the global optimal, as long as it can be determined that there is a better improvement to the current production situation, so the gradient algorithm is used for optimization of development control. This paper chooses the maximum principle method as a model of gradient algorithm.

2. Reservoir Production Optimization

Reservoir development optimization is to maximize the production efficiency by adjusting the injection and production of oil and water wells, which belongs to the optimal control problems [12]. The problem aims to improve the recovery rate and maximize the net present value of production by optimizing the production scheme of the reservoir [13, 14]. Compared with the conventional numerical simulation method, the biggest advantage of this method is that it is able to consider the long-term production period and automatically implement the optimal detailed development scheme for all wells of the reservoir.

The object of optimal control theory is the control system. The core problem is how to choose the control strategy for the given control system, and make the system optimal in a certain sense. We can describe it by a mathematical model. Combined with the general model of optimal control and the performance index of reservoir production optimization, the optimal control model for the production optimization is as follows

\[
\max J = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} R^{m,n}_u (x^{m,n+1}, u^m) + \theta(x^N)
\]

The constraints are described as:

\[
L^{m,n}_x (x^{m,n+1}, x^{m,n}, u^m) = 0
\]

\[
Au^m \leq b \quad \forall n \in (0, \cdots, N_m - 1), m \in (0, \cdots, N - 1)
\]

\[u_{mn}^m \leq u^m \leq u_{np}^m
\]

The specific parameters of the model are as follows:

1. \(J\) is the objective function, which is mainly composed of two parts: the performance of the optimal control \(R^{m,n}_u\) (net present value) and the extra cost expense \(\theta\) (such as depreciation or scrapped cost), where \(R^{m,n}_u\) is the increment of the performance index \(J\) in the nth sampling period. For the reservoir production optimization problem, it consists of variables such as the flow or saturation function (fluctuation efficiency) of the oil and water well, and its specific expression is

\[
R^{m,n}_u (u^m) = \frac{\sum_{j=1}^{N_o} (r_{o Q_{o,j}}^m - r_{w Q_{w,j}}^m) - \sum_{j=1}^{N_w} r_{w Q_{w,j}}^m}{(1+b)^{\phi_{mn}} / \Delta t^{m,n}}
\]
Where, $O_{o,j}$ is the oil production of a single well in the section $j$ of each time period, m$^3$; $Q_{w,j}$ is the water production of a single well in the section $j$ of each time period, m$^3$; $Q_{wi,j}$ is the water injection volume of a single well in the section $j$ of each time period, m$^3$; $r_o$ is the economic factor of oil production; $r_{wp}$ is the cost factor of water production; $r_{wi}$ is the cost factor of injection volume; $\Delta t$ is the time period, year; $b$ is the current interest rate; $N_p$ is the total number of production wells; $N_i$ is the total number of injection wells; $m$ is the control time step; $n$ is the simulated time step in each control time step;

(1) $L^{m,n}$ and the initial conditions of the reservoir constitute the reservoir dynamic system, which is composed of a fully implicit three-dimensional three-phase black oil model equation;

(2) $Au^m \leq b$ is a linear or nonlinear constraint condition; $u_{low}^m \leq u^m \leq u_{up}^m$ is the boundary constraint condition of the control variable;

(3) Others: $x^{m,n}$ is a dynamic variable (such as pressure, saturation and composition, etc.); $u^m$ is a control variable (such as oil well production flow pressure and the flow rate of oil and water wells, etc.); $N$ is the total number of control time steps; $N_m$ is the total number of simulation time steps in each control time step.

The significance of reservoir constraint control optimization problems is to optimize the injection and production control of the reservoir from the economic perspective, improve the development effect and recovery rate of the reservoir, and maximize the net present value of production$^{[13, 14]}$. The problem can be described as follows: while the control variables satisfy the linear constraint condition $u_{low}^m \leq u^m \leq u_{up}^m$ (single well production limit) and the nonlinear constraint condition $Au^m \leq b$ (such as the total injection quantity constraint), the optimal control $u^*(t)$ and corresponding optimal state for maximizing the performance index $J$ are solved.

3. The Solution of Optimal Control Model

3.1. Discrete Maximum Principle

Using discrete maximum principle to solving process is derived from the classical variational theory$^{[10, 12]}$, the essence of this theory is to combine the objective function equation with the existing equality constraints, convert the equality constraint optimization to unconstrained optimization, and convert the equations (1) to (4). So the augmented performance index for a production optimization problem is:

$$J_A = \sum_{m=0}^{N-1} \sum_{n=0}^{N_m-1} R_{m,n-1}^{m,n} \left(x^{m,n+1}, u^m\right) + \theta\left(x^N\right) + \sum_{m=0}^{N-1} \sum_{n=0}^{N_m-1} \left(\lambda^{m,n+1}\right)^T L_{m,n} \left(x^{m,n+1}, x^{m,n}, u^m\right)$$

$$= \sum_{m=0}^{N-1} \sum_{n=0}^{N_m-1} J_{m,n} \left(x^{m,n+1}, x^{m,n}, u^m, \lambda^{m,n+1}\right) + \theta\left(x^N, N_a\right)$$

In equation (6), only the constraints of the black oil model equation are considered. In the calculation, each constraint equation corresponds to a Lagrange multiplier vector, that is, the number of Lagrange multipliers is related to the number of calculation time steps and control variables. If the three-phase black oil model is divided into 2000 grids and 100 control time steps, the number of Lagrange multipliers to be solved should be $3 \times 2000 \times 100 = 6 \times 10^5$.

According to the discrete maximum principle, the first order variation of $J_A$ is:
\[
\delta J_A = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} \left( \frac{\partial J_A^{m,n}}{\partial x^{m,n}} \right) \delta x^{m,n} + \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} \left( \frac{\partial J_A^{m,n}}{\partial u^m} \right) \delta u^m + \frac{\partial \theta}{\partial x} \bigg|_{x^{N-L}N_a} \tag{7}
\]

Because of \( \delta x^{0,0} = 0 \), we can get that:

\[
\delta J_A = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} \left( \frac{\partial J_A^{m,n}}{\partial x^{m,n}} \right) \delta x^{m,n} + \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} \left( \frac{\partial J_A^{m,n}}{\partial u^m} \right) \delta u^m \tag{8}
\]

According to the definition the term \( \delta x^{m,n+1} \) is zero, and the necessary condition to obtain the functional extreme value is \( \delta J_A = 0 \), these terms are independent of each other, so the adjoint equation and the gradient solving equation are obtained, where the adjoint equation is:

\[
\begin{align*}
\frac{\partial J_A^{m,n}}{\partial x^{N-1,N_a}} + \frac{\partial J_A^{m,n-1}}{\partial x^{N-1,N_a}} = 0, & \quad m \leq N \\
\frac{\partial \theta}{\partial x^{N-1,N_a}} + \frac{\partial J_A^{N-1,N_a}}{\partial x^{N-1,N_a}} = 0, & \quad m = N
\end{align*}
\tag{9}
\]

The gradient solving equation is:

\[
\frac{\partial J_A^{m,n}}{\partial u^m} = \sum_{m=0}^{N-1} \left[ \frac{\partial R^{m,n}}{\partial u^m} + \left( \lambda^{m,n+1} \right)^T \frac{\partial L^{m,n}}{\partial u^m} \right] \forall m \in (0, \ldots, N-1) \tag{10}
\]

Since the Lagrange multiplier is used in the gradient solution, the equation (9) needs to be solved first. Substitute equation (6) into equation (9), we can get as follows:

\[
\begin{align*}
\frac{\partial R^{m,n-1}}{\partial x^{m,n}} + \left( \lambda^{m,n+1} \right)^T \frac{\partial L^{m,n}}{\partial x^{m,n}} = 0, & \quad m \leq N \\
\frac{\partial R^{m,n-1}}{\partial x^{N-1,N_a}} + \left( \lambda^{m,N_a} \right)^T \frac{\partial L^{m,n}}{\partial x^{N-1,N_a}} = 0, & \quad m \leq N \quad \text{(Boundary condition for each control step)} \\
\frac{\partial R^{N-1,N_a}}{\partial x^{N-1,N_a}} + \left( \lambda^{N_a} \right)^T \frac{\partial L^{N-1,N_a}}{\partial x^{N-1,N_a}} = 0, & \quad m = N \quad \text{(The final boundary condition)}
\end{align*}
\tag{11}
\]

In which, \( \lambda^{m,N_a} = \lambda^{m+1,0} \) and \( \lambda^{N_a} = \lambda^{N_a+1} \). In the above equation, there is a final boundary condition for each control time step, and this boundary condition is obtained from the calculation of the previous time step adjoint equation, and is inversely solved in the time series.

### 3.2 Optimization Procedures

a) Give the initial conditions and working system, and solve the reservoir model forward along the time scale, and save the state variable \( x \) (pressure and saturation of each grid) for each time step;

b) Using the principle of discrete maximum, the adjoint equation is solved inversely along the time scale. According to the reservoir simulation formula \( L \) and the net present value calculation function \( Y \), the partial matrices \( \frac{\partial R}{\partial x} \) and \( \frac{\partial L}{\partial x} \) in equations (9) and (10) are obtained by partial derivative of state variables and control variables, and then calculate the Lagrange multiplier for each time step;

c) After obtaining the Lagrange multiplier, use equation (10) to calculate the gradient \( \frac{\partial J_A}{\partial u^m} \) corresponding to all control variables;

d) According to the boundary conditions of the set control variables, the gradient is constrained by the logarithmic transformation method \([15]\) to obtain the gradient of the boundary constraints;
e) According to the obtained gradient value, a new control variable is obtained through linear search, and then performs inverse transformation of the logarithmic transformation to obtain the optimized control variable \( u \), and formulates a new production plan;
f) Repeat the optimization process until the gradient of all control variables is close to zero.

3.3. The Improvement of Optimization Algorithm

When using the maximum value principle to solve the gradient, according to the optimization calculation steps, it can be seen that the conventional algorithm needs to perform a time series forward reservoir simulation first, and then perform a sequence reverse gradient solution calculation, that is, it needs two large scale calculation. According to the characteristics of the adjoint equation and the gradient calculation equation, there are mainly four matrix items that need to be solved: \( \frac{\partial R}{\partial x}, \frac{\partial R}{\partial u}, \frac{\partial L}{\partial x}, \frac{\partial L}{\partial u} \). It can be found that \( x \) refers to the state variable (pressure and saturation of the reservoir), so \( \frac{\partial x}{\partial x} \) is actually the partial derivative of the state variable in Reservoir Flow Equation. According to the fully implicit numerical modeling solving, the Jacobian matrix is composed of such partial derivatives, so it is considered that the algorithm can be improved by combining the two solving processes. So the optimization calculation step can be changed as follow:

a) Give the initial conditions and working system, and solve the reservoir model forward along the time scale, and save the Jacobi matrix \( J_M \) of each time step, \( \frac{\partial G}{\partial x} \) and \( \frac{\partial G}{\partial u} \).

b) Using the principle of discrete maximum, the adjoint equation is solved inversely along the time scale. Get the required variables(\( \frac{\partial R}{\partial x}, \frac{\partial R}{\partial u}, \frac{\partial L}{\partial x}, \frac{\partial L}{\partial u} \)) directly based on the matrix items saved in each time step, and then calculate the Lagrange multiplier for each time step;
The remaining calculation steps c to f are unchanged.

4. Reservoir Case Studies

The model is a two-dimensional three-phase reservoir model with a grid of \( 11 \times 11 \times 1 \), grid size is \( \Delta x = \Delta y = 20 m, \Delta z = 10 m \). The permeability field is shown in Figure 1. The reservoir pressure is 14.1 MPa, and the bottom hole pressure of each oil well is initially set to 12.7 MPa, and the total injection volume is 200 m\(^3\)/d. The price of crude oil is 2,170 RMB/m\(^3\), the cost of processing produced water is 10RMB/m\(^3\), the cost of injecting water is 5RMB/m\(^3\), and the interest rate is 0.1. In order to compare the effects before and after optimization, only the production wells are optimized here. The initial bottom hole flow pressure of each well is 12.76 Mpa, and the total injection volume of the injection well remains unchanged. The lower boundary of the bottom hole pressure of the production well is set to 10.3 MPa and the upper boundary is 13.8 MPa. The simulated production time has a maximum step size of 30 days, the residual oil saturation is 0.2, and the irreducible water saturation is 0.3. The total production time is 365 days, and the optimization time step is divided into 4 steps.

The production of oil and water before and after optimization are shown in Figure 2. It can be seen from Figure 2 that on the 365th days, the cumulative oil production after the optimization is significantly increased, the cumulative water production is greatly reduced, and the final water cut is decreased by 7%. The distribution diagram of control step saturation at each time before and after optimization is shown as Figure 3.
According to figure 3(a) after the 90 days injection of water, fingering occurs along the hyperosmotic channel, the injected water breaks through to the production wells P2 and P3, resulting in water breakthrough in the early stage of the reservoir and low oil recovery. As shown in figure 3(b), the fingering phenomenon is obviously under better control. On the premise of injecting the same amount of water, the sweep efficiency is greatly improved. Figure 4 shows the results of the first set of iterative convergence calculations. Compared to the initial plan, the bottom hole flow pressure of the P2 and P3 wells has a certain increase at each time control step, that is, the production of these two wells is reduced. However, the bottom hole flow pressures of P1 and P4 decrease to a certain extent, and the production of the two wells is increased. Compared with the initial scheme, this scheme has achieved better development results, and it increases the net present value to 4.0×10^7 RMB.

5. Conclusions

(1) According to the actual situation of oilfield development, the optimization model of reservoir development control is established, and the model is solved by the principle of maximum value to achieve the purpose of optimizing reservoir development and production.

(2) In the gradient solution, based on the maximum value principle, the improved adjoint model is used to combine the reservoir simulation with the gradient solution, and the full implicit numerical simulation is used to replace the matrix calculation in the process of the maximal principle. The two computational processes are effectively combined, which simplifies the calculation process and greatly improves the computational efficiency. It also proves that the method is applicable not only to the oil-water two-phase model but also to the oil-water-gas three-phase multi-layer model;
Through the example analysis, the correctness and feasibility of the method are proved, which provides theoretical and technical support for the intelligent oilfield system.

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