Gaia Cepheid parallaxes and ‘Local Hole’ relieve $H_0$ tension

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ABSTRACT

There is an $\approx 9 \pm 2.5$ per cent tension between the value of Hubble’s Constant, $H_0 = 67.4 \pm 0.5 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$, implied by the Planck microwave background power spectrum and that given by the distance scale of $H_0 = 73.4 \pm 1.7 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$. But with a plausible assumption about a Gaia DR2 parallax systematic offset, we find that Gaia parallax distances of Milky Way Cepheid calibrators are $\approx 12$–$15$ per cent longer than previously estimated. Similarly, Gaia also implies $\approx 4.7 \pm 1.7$ per cent longer distances for 46 Cepheids than previous distances on the scale of Riess et al. Then we show that the existence of an $\approx 150 \, h^{-1}\text{Mpc}$ ‘Local Hole’ in the galaxy distribution implies an outflow of $\approx 500 \, \text{km s}^{-1}$. Accounting for this in the recession velocities of SNIa standard candles out to $z \approx 0.15$ reduces $H_0$ by a further $\approx 1$ per cent. Combining the above two results would reduce the distance scale $H_0$ estimate by $\approx 7$ per cent from $H_0 \approx 73.4 \pm 1.7$ to $\approx 68.9 \pm 1.6 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$, in reasonable agreement with the Planck value. We conclude that the discrepancy between distance scale and Planck $H_0$ measurements remains unconfirmed due to uncertainties caused by Gaia systematics and an unexpectedly inhomogeneous local galaxy distribution.

Key words: distance scale.

1 INTRODUCTION

The history of measuring the Hubble Constant via the distance scale has been one of contention, with Hubble’s original value of $H_0 \approx 500 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$ gradually reducing to the present $H_0 \approx 73 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$. The problem has been that to estimate $H_0$ accurately we need to go to distances beyond the largest local inhomogeneities so that the recession velocity completely dominates any peculiar velocity and unfortunately this is beyond the reach of geometric distance indicators such as parallax and even primary distance indicators such as Cepheids.

Recently, there has been a tension noted between the $H_0$ estimated from models fitted to the acoustic peaks in the Planck CMB power spectrum which give $H_0 = 67.4 \pm 0.5 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$ (Planck Collaboration VI 2018) and the distance scale estimates which give $H_0 = 73.4 \pm 1.7 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$. This $\approx 9$ per cent discrepancy is now at the 3.5$\sigma$ level and is regarded as a serious tension in the Hubble parameter (Riess et al. 2016).

Although there remains the possibility that the uncertainties have been underestimated in both the distance scale and Planck $H_0$ results (see e.g. Feeney, Mortlock & Dalmasso 2018), here we focus on two developments that may act to reduce distance scale estimates of $H_0$. The first comprises new parallax distances to Milky Way Cepheids from Gaia DR2 (Gaia Collaboration 2018). We compare these to previous Cepheid parallax distances and also to main-sequence fitted distances for Cepheids in Galactic open clusters. We similarly compare directly to Cepheids on the distance scale of Riess et al. (2018a), calibrated by parallax and two other geometric methods.

Second, we review the evidence for the ‘Local Hole’ from the work of Whitbourn & Shanks (2014, 2016) and references therein. We then estimate the effect of the resulting outflow on scales of $\approx 150 \, h^{-1} \, \text{Mpc}$ on the redshifts of SNIa that are used in the Hubble diagram by fitting for $H_0$ and $\Omega_m$.

2 NEW CEPHEID PARALLAX DISTANCES FROM GAIA

Gaia DR2 has provided parallaxes of unprecedented statistical accuracy to many hundreds of Cepheid variables. Riess et al. (2018b) have analysed 46 of these where Hubble Space Telescope (HST) photometry exists and have concluded that their previous Cepheid P–L relation zero-points and distance scale have been confirmed, leaving the Hubble Constant at $H_0 = 73.4 \pm 1.7 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$. However, as Riess et al. (2018b) note, there are systematic uncertainties in the Gaia DR2 data particularly in the parallax zero-point and the effects of saturation that affect Cepheid distances. Lindegren et al. (2018) show that WISE quasars have an average Gaia parallax of $-29 \, \mu\text{as}$ when it should be zero. They warn that this offset may not be constant with sky position, star colour, or magnitude. They suggest that the offset could be fitted out in individual star samples. This is the approach used by Riess et al. (2018b) who found an average offset of $-46 \, \mu\text{as}$ against previous P–L based

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approach and compare distances from Gaia parallaxes directly with previous distance measurements.

We first consider three samples on which the P–L relation has traditionally been calibrated. Two of these comprise the parallax Cepheid samples discussed by Riess et al. (2018a), namely the 10 parallax stars from HST FGS measurements of Benedict et al. (2007) and the 7 HST WFC3 parallax stars of Riess et al. (2018a) (see Table 1). Similar to Riess et al. (2018b) we have excluded δ Cep and Y Sgr because they show negative parallaxes in Gaia DR2, possibly due to data corruption. To these we add the 14 Cepheids in 11 open clusters whose distances were obtained by main-sequence fitting by Laney & Stobie (1993) and Hoyle et al. (2003) who added NIR K band photometry to try and improve their reddening estimates and hence the distances (see Table 2).

The comparison of these previous distance moduli with the Gaia DR2 parallax distances with the 29 μas ‘quasar’ correction added can form the corrected distance moduli, \((m – M)_0\), are shown in Fig. 1. We see that there is a significant discrepancy that seems independent of distance. Conservatively, we compare with the Laney & Stobie (1993) distances to the open clusters – these are larger overall than the Hoyle et al. (2003) alternatives and may be considered the previous standard values. Note that the 29 μas correction only falls to <1 per cent of the parallax for \((m – M)_0 < 7.7\) mag i.e. only the Gaia parallaxes for FGS Cepheids of Benedict et al. (2007) are relatively immune to this systematic.

With the quasar offset and further excluding I Car, W Sgr, and RT Aur of Benedict et al. (2007) on the grounds they show the lowest parallax/error ratios as well as having saturated Gaia magnitudes with \(G < 6\) mag, the 23 remaining Gaia moduli are 0.30 ± 0.06 mag longer than the previous distance moduli or 14.8 ± 3 per cent greater in distance. Here and in what follows we have minimized \(\chi^2\) on the distance moduli differences with respect to the Gaia and the other sample’s errors (see Tables 1 and 2) combined in quadrature. Further excluding the five remaining FGS stars with \(G < 6\) in case their parallaxes are affected by saturation, the 18 Gaia distance moduli left are 0.32 ± 0.06 mag longer than the previous moduli or 15.9 ± 3 per cent greater in distance. Alternatively, excluding the 11 Cepheid open clusters of Laney & Stobie (1993) the remaining seven WFC3 and five FGS Cepheids with HST parallaxes (still excluding I Car, W Sgr, and RT Aur) the Gaia distance moduli are 0.25 ± 0.08 mag higher than previously or 12.2 ± 3.8 per cent longer in distance. For just the seven WFC3 stars the Gaia moduli are 0.28 ± 0.12 mag higher or 13.8 ± 5.7 per cent longer in distance. Note that for their distance scale, Riess et al. (2018a) report that their calibration route via Milky Way Cepheid parallaxes produces 4.8 ± 3.3 per cent longer distances than the NGC 4258 megamaser and eclipsing binaries routes combined. This partly explains why we further find that the final distance moduli reported for these seven WFC3 Cepheids in table 2 of Riess et al. (2018a) are 0.16 ± 0.08 mag or 7.6 ± 3.8 per cent longer than their HST parallax distances. Lastly, we note that each of the above differences remain significant if we assume the 46 ± 6 μas parallax correction of Riess et al. (2018b) rather than 29 μas e.g for all 23 previous calibrators the Gaia moduli are now 0.24 ± 0.07 mag longer c.f. 0.30 ± 0.06 mag.

The results of Fig. 1 as discussed so far are based on a comparison of Gaia parallaxes with HST parallax and main-sequence fitted distances to the Milky Way Cepheids previously used to calibrate the P–L relation. These give important contextual evidence that the Gaia distances may imply a longer Cepheid scale. But the most direct comparison with the scale of Riess et al. (2018b) comes from their sample of 46 Cepheids, also shown in Fig. 1. We follow these authors by excluding the Cepheid T Mon, bracketed in Fig. 1. We also note

**Figure 1.** Comparison between Cepheid distances based on Gaia parallaxes (assuming the 29 μas ‘quasar’ correction), compared to the HST parallaxes used by Riess et al. (2018a) to help zero-point their Cepheid scale. Also shown is the same Gaia comparison for Cepheids in open clusters with main-sequence fitted distances from Laney & Stobie (1993) and Hoyle, Shanks & Tanvir (2003). Finally, the distance moduli of the 46 Cepheids of Riess et al. (2018b) are also compared to those from Gaia. The Cepheids I Car, W Sgr, RT Aur, and T Mon are also plotted but with brackets because they were left out of numerical comparisons (see text).

distances to 46 Cepheids (see below). Similarly, Zinn et al. (2018) report an offset of \(-52.8 ± 2.4 ± 1(\text{syst.})\) μas on astroseismological/APOGEE spectroscopic distances to stars in the Kepler field. A range of offsets is also quoted in table 1 of Arenou et al. (2018).

Here, we shall assume as our baseline, the average \(-29\) μas offset from the quasars, estimated by Lindegren et al. (2018) for the Gaia collaboration, and add \(29\) μas to correct our Gaia parallaxes. Clearly the difference between this average value and, for example, the value of Zinn et al. (2018) emphasizes the possibility that the offset may vary with sky position or another parameter. But the reason we prefer the ‘quasar’ offset is that its ideal ‘model’ parallax of \(\pi = 0\) is indisputable unlike almost all other distance comparisons quoted by Arenou et al. (2018) and Zinn et al. (2018) (see also Stassun & Torres 2018). Unfortunately, the colour and magnitude ranges of quasars are too small to base reliable corresponding corrections for Cepheid parallaxes. But we have checked that adjusting the Gaia parallaxes for the possible dependence on ecliptic latitude given in fig. 7 (right) of Lindegren et al. (2018) changes our results insignificantly.

The non-linear \(r = 1/\pi\) relation between distance, \(r\), and parallax, \(\pi\), can also cause statistical and systematic bias in parallax distances. Luri et al. (2018) have made a thorough review of these effects for Gaia parallaxes, including approaches such as those of Lutz & Kelker (1973), Smith & Eichhorn (1996), and Bailer-Jones (2015). However, our fractional errors in Gaia parallax are generally \(\approx 10\) per cent (see Tables 1 and 2) and these authors agree that this makes them less susceptible to statistical bias. Indeed, tests on the samples used here assuming the average error/parallax ratio and cut at \(< 20\) per cent all gave \(< 0.5\) μas bias. Therefore we adopt a simple
Correcting by ±Cepheid ‘photometric’ distance moduli (see Fig.1) gives a smaller ± for their 46 Cepheids than this would imply a 4.7 increase in their distance scale. If we assumed a zero ± correction from zero to 0.029 mas gives an (m − M0) difference to 0.045 ± 0.03 mag. Thus the total difference is 0.08 ± 0.03 mag or 3.75 ± 1.4 per cent, similar to the 4.7 ± 1.7 per cent derived from parallaxes directly. Assuming zero correction, the Gaia (m − M0) are longer by 0.175 ± 0.03 mag or by 8.4 ± 1.4 per cent in distance. Thus the basic result remains that varying the assumed Gaia correction from zero to 0.029 mas gives larger Gaia distances by ≈2−8 per cent. So, at one end of this range, this difference might explain the discrepancy between the Cepheid parallax and megamaser/eclipsing binary Cepheid zero-points. At the other, the Cepheid scale calibrated by Gaia parallaxes would become more compatible with Planck. But until the systematic Gaia parallax offset becomes better determined it may be impossible to claim that Gaia has either contradicted or confirmed the current Cepheid scale.

Summarizing, assuming a 29 μas correction offset to Gaia parallaxes, we find 12−15 per cent longer distances to 23 Cepheids previously used as Cepheid P–L relation calibrators. Only about half of this increase might be expected based on difference between the previous parallax and the other geometric calibrations of

| Cepheid | Ref | HST π (mas) | HST (m − M0) | Gaia π (mas) | Gaia (m − M0) (29μas corr.) | Gaia G (mag) |
|---------|-----|-------------|-------------|-------------|--------------------------|-------------|
| SS CMA  | R18a| 0.389 ± 0.029| 12.05 ± 0.16| 0.201 ± 0.029| 13.19 ± 0.28              | 9.52        |
| XY Car  | R18a| 0.438 ± 0.047| 11.79 ± 0.23| 0.330 ± 0.027| 12.23 ± 0.16              | 8.94        |
| VX Per  | R18a| 0.420 ± 0.074| 11.88 ± 0.39| 0.330 ± 0.031| 12.23 ± 0.19              | 8.86        |
| VY Car  | R18a| 0.586 ± 0.044| 11.16 ± 0.16| 0.512 ± 0.041| 11.33 ± 0.17              | 7.33        |
| WZ Sgr  | R18a| 0.512 ± 0.037| 11.45 ± 0.16| 0.513 ± 0.077| 11.33 ± 0.31              | 7.65        |
| S Vul   | R18a| 0.322 ± 0.040| 12.46 ± 0.27| 0.305 ± 0.041| 12.38 ± 0.27              | 8.05        |
| X Pup   | R18a| 0.277 ± 0.047| 12.79 ± 0.37| 0.302 ± 0.043| 12.40 ± 0.28              | 8.30        |
| l Car   | B07 | 2.010 ± 0.20 | 8.48 ± 0.22 | 0.777 ± 0.256| 10.47 ± 0.72              | 3.79(DR1)   |
| ζ Gem   | B07 | 2.780 ± 0.18 | 7.78 ± 0.14 | 2.250 ± 0.30 | 8.21 ± 0.29               | 4.06        |
| β Dor   | B07 | 3.140 ± 0.16 | 7.52 ± 0.11 | 3.112 ± 0.284| 7.52 ± 0.20               | 4.10(DR1)   |
| W Sgr   | B07 | 2.280 ± 0.20 | 8.21 ± 0.19 | 1.180 ± 0.412| 9.59 ± 0.77               | 5.1(B07)    |
| X Sgr   | B07 | 3.000 ± 0.18 | 7.61 ± 0.13 | 3.431 ± 0.202| 7.30 ± 0.13               | 4.32        |
| FF Aql  | B07 | 2.810 ± 0.18 | 7.76 ± 0.14 | 1.810 ± 0.107| 8.68 ± 0.13               | 5.14        |
| T Vul   | B07 | 1.900 ± 0.23 | 8.61 ± 0.26 | 1.674 ± 0.089| 8.84 ± 0.11               | 5.44        |
| RT Aur  | B07 | 2.400 ± 0.19 | 8.10 ± 0.17 | 1.419 ± 0.203| 9.20 ± 0.31               | 5.9(B07)    |

Table 2. Cepheid distance moduli estimated from Gaia parallaxes compared to previous distance moduli estimated from main-sequence fitting for Cepheids in open clusters. We shall assume the Hoyle et al. (2003) uncertainties also apply to the Laney & Stobie (1993) distance moduli. Gaia parallaxes, π, are listed uncorrected for systematic offset whereas Gaia distance moduli are based on parallaxes corrected by adding 29 μas. G magnitudes are from Gaia DR2 except for l Car and β Dor where we prefer DR1 G magnitudes and W Sgr and RT Aur where we prefer the V magnitude of B07 converted to G using G ≈ V + 0.4. * denotes FGS stars rejected on grounds of Gaia saturation and high parallax fractional error.
Riess et al. (2018a). Gaia (+29 μas offset) parallaxes for the 46 Cepheids of Riess et al. (2018b) indicate a 4.7 ± 1.7 per cent increase in their Cepheid distances. This would imply their \( H_0 \approx 70.2 \pm 1.2 \text{ km s}^{-1}\text{Mpc}^{-1} \). However, this latter result depends on the assumed Gaia parallax offset. Assuming zero offset would give a bigger 13.9 ± 1.8 per cent reduction in the \( H_0 \) of Riess et al. (2018a).

### 3 \( H_0 \) AND THE ‘LOCAL OUTFLOW’

Next, we consider the peculiar velocity outflow caused by the ‘Local Hole’. Evidence for a local underdensity on the scale of \( \approx 150 \text{ h}^{-1}\text{Mpc} \) goes back to the galaxy count data of Shanks et al. (1984) and has been confirmed particularly in the Southern Galactic Cap in the 2dF Galaxy Redshift Survey by Busswell et al. (2004) and in 2MASS counts by Frith et al. (2003) and Frith, Metcalfe & Shanks (2006). More recently all sky redshift surveys including SDSS and 6dFGS have provided further confirmation as discussed by Keenan et al. (2012), Keenan, Barger & Cowie (2013), and Whitbourn & Shanks (2014, 2016). Here we follow Whitbourn & Shanks (2014) to estimate the effect of the ‘Local Outflow’.

We therefore use the three sky areas of Whitbourn et al. within each of which the \( \delta p_g(r)/\bar{\rho}_g \) was estimated as a function of redshift by dividing the observed \( n(z) \) limited at \( K < 12.5 \) to a homogeneous model. We then form:

\[
\frac{\delta p_g(< r)}{\bar{\rho}_g} = \frac{1}{V(r)} \sum_i \left( \frac{dn}{n} \right) \delta r \delta^2 V
\]

where \( \left( \frac{dn}{n} \right) \) are taken from averaging the data shown in figs. 3 (a, b, c) of Whitbourn & Shanks (2014), \( r \) are the corresponding comoving distances, \( \delta r \) is the comoving bin size, and \( V(r) \) is the spherical volume to radius, \( r \). We then apply linear theory to relate the fractional velocity change to the galaxy overdensity, \( \delta p_g(r)/\bar{\rho}_g \):

\[
\frac{\Delta v}{v} = -\frac{1}{3} \frac{\delta p_g(< r)}{\bar{\rho}_g} \frac{\Omega_m^{0.6}}{b}
\]

where \( b \) is the galaxy bias. Here we assume \( b = 1 \) as appropriate for \( K \) selected galaxies in the standard cosmological model (e.g. Whitbourn & Shanks 2014).

Fig. 2 shows the predicted outflow velocity from an area weighted average over the three regions of Whitbourn & Shanks (2014). This averaging implies a spherically symmetric underdensity but clearly this is only a rough approximation. In future work we shall explore the effect of relaxing this assumption. At \( z \approx 0.05 \) the ratio \( \Delta v/v \) peaks at 3.5 per cent while averaging \( \approx 1.8 \) per cent in the range 0.01 < \( z < 0.15 \) used by Riess et al. (2016). Here, we have assumed that the only contribution to \( \Delta v/v \) is from \( z < 0.1 \), leaving \( \Delta v/v \) to decline towards zero in the 0.1 < \( z < 0.15 \) range. So for a local determination of \( H_0 \) measured in this range, the effect of the ‘Local Hole’ will tend to lower \( H_0 \) by \( \approx 1.8 \) per cent. This effect will reduce the \( H_0 = 73.4 \pm 1.7 \text{ km s}^{-1}\text{Mpc}^{-1} \) distance scale measurement of Riess et al. (2018a) to \( H_0 \approx 72.1 \pm 1.6 \text{ km s}^{-1}\text{Mpc}^{-1} \).

We next check the effect of this Local Hole outflow on the Hubble Diagram to look for any inconsistency with the model standard fit. We therefore apply the correction \( \Delta v \) from Fig. 2 to obtain the corrected SNIa redshift, \( z_{\text{cor}} \):

\[
1 + z_{\text{cor}} = \frac{1 + z_{\text{SNIa}}}{1 + \Delta v/c}
\]

These corrected redshifts are then assumed to calculate the distance moduli of the Pantheon sample of 1048 SNIa of Scolnic et al.
with HST parallaxes could then more than reconcile their known 4.8 ± 3.3 per cent difference with the other geometric Cepheid calibrators based on eclipsing binaries and the NGC 4258 maser (Riess et al. 2018a). But much clearly depends on the value of the Gaia parallax systematic offset. We acknowledge arguments supporting the offset used by Riess et al. (2018b) from e.g. Zinn et al. (2018). Whether this current Gaia scale can compete with the alternative Cepheid geometric calibrations awaits an improved Gaia astrometric solution.

In terms of possible problems with the previous Cepheid calibrators, we note that Gaia parallaxes have the advantage over HST parallaxes that they are global, with no need of modelling background star distances. In the case of main-sequence distances, these fits assume a universal Galactic reddening law but a spatial dependence is increasingly discussed (Fitzpatrick & Massa 2007; Anderson et al. 2013). Then considering the effect of outflow due to the ‘Local Hole’ we have found that an ≈1.8 per cent decrease in average galaxy velocities out to z ≈ 0.15 is likely when the effect of local underdensities are taken into account. Here we have assumed linear theory in terms of relating underdensity to outflow velocity which is an approximation but others using more sophisticated models have come to similar conclusions (e.g. Hoscheit & Barger 2018). We have checked whether our linear outflow model leads to any inconsistency in the SNIa Hubble diagram using the data of Scolnic et al. (2018) but as long as a slight rise in $\Omega_m$ is allowed from $\Omega_m = 0.28$ to 0.33, c.f. $\Omega_m = 0.315 ± 0.0007$ from Planck Collaboration VI (2018), a few per cent drop in $H_0$ can be accommodated.

We have seen there is at least the possibility of a ≈4.7 per cent increase in the Cepheid distance scale implied by current Gaia parallaxes and a likely 1.8 per cent decrease in the average galaxy velocity out to z ≈ 0.15 after accounting for the ‘Local Outflow’. Together these effects would lead to an ≈7 per cent reduction in Hubble’s Constant, reducing from $H_0 = 73.4 ± 1.7$ km s$^{-1}$Mpc$^{-1}$ to $H_0 = 68.9 ± 1.6$ km s$^{-1}$Mpc$^{-1}$. Even without allowing for further systematic errors in Gaia parallaxes and our outflow analyses, we see that the tension with the Planck value of $H_0 = 67.4 ± 0.5$ km s$^{-1}$Mpc$^{-1}$ would be reduced to <1σ. It will be interesting to see whether improved Gaia parallaxes and better ‘Local Hole’ outflow models will confirm these current results.

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