I. INTRODUCTION

The detection of primordial gravitational waves in the Cosmic Microwave Background (CMB) offers an exciting possibility for probing the physics around the GUT scale. In particular, the inflationary paradigm predicts the generation of nearly scale-invariant primordial tensor and scalar perturbations with the tensor-to-scalar ratio less than the order of 0.1. Since the spectral index of scalar perturbations was measured by the Planck satellite in high precision, the precise bounds on the tensor-to-scalar ratio allow us to test inflationary models from the future precise measurement of tensor-to-scalar ratio.

Many of the single-field inflationary models proposed so far belong to a class of Horndeski theories—the most general Lorentz-invariant scalar-tensor theories with second-order equations of motion. In fact, the leading-order power spectra of tensor and scalar perturbations were derived for inflationary models in the framework of Horndeski theories. These results were employed to place observational constraints on individual models (such as slow-roll inflation, k-inflation, Starobinsky inflation, Higgs inflation) from the WMAP and Planck data.

There exist more general modified gravitational theories beyond the Horndeski domain. Choosing the so-called unitary gauge in which the perturbation of a scalar field \(\phi\) on the flat Friedmann-Lemaître-Robertson-Walker (FLRW) background vanishes, the Horndeski Lagrangian can be expressed in terms of geometric scalar variables appearing in the 3+1 Arnowitt-Deser-Misner (ADM) decomposition of space-time. In Horndeski theories the coefficients in front of such geometric scalars have particular relations, but it is possible to consider extended theories with arbitrary coefficients. Gleyzes-Langlois-Piazza-Vernizzi (GLPV) theories can be incorporated in the EFT approach of Ref. by taking into account additional geometric scalar quantities (associated with spatial derivatives up to 6-th order) to the Lagrangian mentioned above. These quantities include the 6-th derivative \(\nabla_i R_{jk} \nabla^i R^{jk}\) and the scalars constructed...
from the acceleration vector \( a_i = \nabla_i \ln N \). In the non-projectable version of Hořava-Lifshitz gravity where the lapse \( N \) depends on both time and space \( [14] \), the contribution like \( a_ia^i \) in the Lagrangian can alleviate the instability problem \( [15, 46] \) present in the projectable version (in which \( N \) depends on time alone).

In Ref. \( [48] \) the second-order action for scalar perturbations was derived for the generic EFT Lagrangian encompassing Horndeski/GLPV theories and Hořava-Lifshitz gravity. This result can be useful for the computation of the primordial scalar power spectrum generated during inflation and for discussing conditions under which the ghosts and instabilities are absent (see Ref. \( [38] \) for a review).

In this paper we employ such a general EFT approach for the study of gravitational waves on the flat FLRW background. Our analysis is more generic than those of Refs. \( [33, 41, 47] \) in that higher-order spatial derivatives appearing in Hořava-Lifshitz gravity are explicitly taken into account for the computation of the second-order action of tensor perturbations. Unlike Ref. \( [48] \), we do not consider the terms associated with the broken spatial diffeomorphism. We provide a general formula for the inflationary power spectrum of gravitational waves by taking into account slow-roll corrections to the leading-order spectrum.

This paper is organized as follows. In Sec. \( \text{II} \) we present the action of our EFT approach and briefly review how several modified gravitational theories are incorporated in our general framework. In Sec. \( \text{III} \) we derive the second-order action and the equation of motion for tensor perturbations. In Sec. \( \text{IV} \) we obtain the spectrum of gravitational waves generated during inflation and in Sec. \( \text{V} \) we apply the results to concrete modified gravitational theories. Sec. \( \text{VI} \) is devoted to conclusions.

\section*{II. THE GENERAL EFT ACTION OF MODIFIED GRAVITY}

The EFT of cosmological perturbations is based upon the 3+1 decomposition of space-time described by the line element \( [40] \)

\[
ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt),
\]

where \( N \) is the lapse function, \( N^i \) is the shift vector, and \( h_{ij} \) is the three-dimensional spatial metric. Introducing a unit vector \( n_\mu = (-N, 0, 0, 0) \) orthogonal to the constant \( t \) hypersurfaces \( \Sigma_t \), the induced metric \( h_{\mu\nu} \) on \( \Sigma_t \) can be expressed of the form \( h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \).

The extrinsic curvature is defined by \( K_{\mu\nu} = h^\lambda_{\mu\nu\lambda\lambda} = n_\nu a_{\nu\rho} + n_\rho a_{\nu\rho} \), where a semicolon represents a covariant derivative and \( a_{\nu\rho} = n^\lambda n_{\nu\lambda} \) is the acceleration vector. The scalar quantities that can be constructed from the extrinsic curvature are the trace of \( K_{\mu\nu} \) and the square of \( K_{\mu\nu} \), i.e.,

\[
K = K^{\mu\mu}, \quad S = K_{\mu\nu}K^{\mu\nu}.
\]

The internal geometry of \( \Sigma_t \) is characterized by the three-dimensional Ricci tensor \( R_{\mu\nu} = \Box R_{\mu\nu} \), which is dubbed the intrinsic curvature. From \( R_{\mu\nu} \) we can construct the following scalar quantities:

\[
R = R_{\mu\mu}, \quad Z = R_{\mu\nu}R^{\mu\nu}, \quad U = R_{\mu\nu}K^{\mu\nu}.
\]

Since it is possible to express the Riemann tensor \( R_{\mu\nu\lambda\sigma} \) in terms of the Ricci tensor and scalar in three dimensions, we do not need to consider scalar combinations associated with \( R_{\mu\nu\lambda\sigma} \).

In Hořava-Lifshitz gravity there are other scalar quantities that generate spatial derivatives up to 6-th order:

\[
Z_1 = \nabla_i R^i, \quad Z_2 = \nabla_i R^j \nabla^i R^jk.
\]

We can also take into account the terms like \( R^i R^j R^k \) and \( R R^j R^k \), but they are irrelevant to the dynamics of linear scalar perturbations on the flat FLRW background. Hence we do not incorporate those terms in the following analysis.

In the original Hořava-Lifshitz gravity \( [24] \) the space-time foliation is preserved by the space-independent reparametrization \( t \rightarrow t'(t) \), so the lapse \( N \) is assumed to be a function of time \( t \) alone (which is called the projectability condition). This can be extended to a non-projectable version in which the lapse depends on both the spatial coordinate \( x^i \) (\( i = 1, 2, 3 \)) and \( t \) \( [14] \). Since the acceleration \( a_i = \nabla_i \ln N \) does not vanish in this case, we can also consider the following scalar combinations:

\[
\alpha_1 \equiv a_i a^i, \quad \alpha_2 \equiv a_i \Delta a^i, \quad \alpha_3 \equiv R \nabla_i a^i, \\
\alpha_4 \equiv a_i \Delta^2 a^i, \quad \alpha_5 \equiv \Delta R \nabla_i a^i,
\]

where \( \Delta \equiv \nabla_i \nabla^i \).

The action of general modified gravitational theories that depends on the above mentioned scalar quantities is given by

\[
S = \int d^4x \sqrt{-g} L (N, K, S, R, Z, U, Z_1, Z_2, \alpha_1, \ldots, \alpha_5; t),
\]

where \( g \) is a determinant of the metric \( g_{\mu\nu} \) and \( L \) is a Lagrangian. As we will briefly review in the following, the action \( (2.6) \) encompasses the Horndeski/GLPV theories and Hořava-Lifshitz gravity.

First of all, Horndeski theories are described by the Lagrangian

\[
L = G_2(\phi, X) + G_3(\phi, X) \Box \phi + G_4(\phi, X) R - 2G_4 X \phi - \phi^{\mu\nu} \phi_{\mu\nu} + G_5(\phi, X) G_{\mu\nu} \phi^{\mu\nu} + \frac{1}{3} G_5 X \phi \phi^{\mu\nu}(\Box \phi)^3 \]

\[
-3(\Box \phi) \phi^{\mu\nu} \phi_{\mu\nu} - 2 \phi^{\mu\nu} \phi_{\mu\nu} \phi^{\rho\sigma} \phi_{\rho\sigma},
\]

where \( \Box \phi \equiv (g^{\mu\nu} \phi_{\mu\nu})_\phi \), and \( G_j \) \( (j = 2, \ldots, 5) \) are functions in terms of a scalar field \( \phi \) and its kinetic energy \( X = g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \). \( R \) and \( G_{\mu\nu} \) are the Ricci scalar and the Einstein tensor in four dimensions, respectively. Here
and in the following, a lower index of $L$ denotes the partial derivatives with respect to the scalar quantities represented in the index, e.g., $G_{j,x} \equiv \partial G_j/\partial X$. In unitary gauge we have $\phi = \phi(t)$ and $X = -\phi(t)^2/N^2$, where $a$ dot represents a derivative with respect to $t$. Hence the dependence of $\phi$ and $X$ in the action (2.7) is interpreted as that of the lapse $N$ and the time $t$. In fact, we can express the Lagrangian (2.7) of the form  \[ L = A_2(N,t) + A_3(N,t)K + A_4(N,t)(K^2 - S) + B_4(N,t)R + A_5(N,t)K_3 + B_5(N,t)(U - K'R/2), \]  (2.8) where $K_3 = K^3 - 3KK_{\mu\nu}K^{\mu\nu} + 2K_{\mu\nu}K^{\mu \lambda}K^{\nu \lambda}$. Up to quadratic order in perturbations on the flat FLRW background we have that $K_3 = 3H(2H^2 - 2KH + K^2 - S)$, where $H = \dot{\phi}/(Na)$ is the Hubble parameter. Horndeski theories have the following correspondence  

\[ A_2 = G_2 - XF_{3,\phi}, \]  

(2.9)  

\[ A_3 = 2(-X)^{3/2}F_{3,X} - 2\sqrt{X}G_{4,\phi}, \]  

(2.10)  

\[ A_4 = -G_4 + 2XG_{4,X} + XG_{5,\phi}/2, \]  

(2.11)  

\[ B_4 = G_4 + X(G_{5,\phi} - F_{5,\phi})/2, \]  

(2.12)  

\[ A_5 = -(-X)^{3/2}G_{5,X}/3, \]  

(2.13)  

\[ B_5 = -\sqrt{-XF_5}, \]  

(2.14)  

where $F_3$ and $F_5$ are auxiliary functions satisfying $G_3 = F_3 + 2XF_{3,X}$ and $G_{5,X} = F_5/(2X) + F_{5,X}$. From Eqs. (2.11)-(2.14) the following two relations hold  

\[ A_4 = 2XB_{4,X} - B_4, \quad A_5 = -XB_{5,X}/3, \]  

(2.15)  

under which the number of 6 independent functions reduces to 4.

GLPV generalized Horndeski theories in such a way that the coefficients $A_1, B_1, A_4,$ and $B_5$ are not necessarily related to each other. The general action (2.6) can incorporate both Horndeski and GLPV theories described by the Lagrangian (2.23).

The action (2.6) also covers Hořava-Lifshitz gravity given by the Lagrangian  

\[ L = \frac{M_{pl}^2}{2}(S - \lambda K^2 + R + \eta_1\alpha_1) - \frac{1}{2}(g_2R^2 + g_3Z + g_2\alpha_2 + \eta_3\alpha_3) - \frac{1}{2M_{pl}^2}(g_4Z_1 + g_5Z_2 + \eta_4\alpha_4 + \eta_5\alpha_5), \]  

(2.16)  

where $M_{pl} = 2.435 \times 10^{18}$ GeV is the reduced Planck mass, and $\lambda, \eta_1, \ldots, \eta_4, g_2, \ldots, g_5$ are constants. The original Hořava-Lifshitz gravity [24] corresponds to the case $\alpha_1 = \cdots = \alpha_5 = 0$, whereas its healthy extension [44] involves the dependence of acceleration.

### III. THE SECOND-ORDER ACTION FOR GRAVITATIONAL WAVES

#### A. Cosmological perturbations

The perturbed line element involving the four scalar perturbations $\delta N, \psi, \zeta, E$, and tensor perturbations $\gamma_{ij}$ can be written of the form  

\[ ds^2 = -(1 + 2N)dt^2 + 2\partial_i\psi dx^i dt + a^2(t)\left[(1 + 2\zeta)\delta_{ij} + 2\partial_i\partial_j E\right]dx^i dx^j, \]  

(3.1)  

where $a(t)$ is the scale factor, and  

\[ \hat{h}_{ij} = \delta_{ij} + \gamma_{ij} + \frac{1}{2}\delta^{mk}\gamma_{im}\gamma_{kj}, \]  

(3.2)  

denotes the perturbation $\gamma_{ij}$ is traceless and divergence-free, i.e., $\gamma_{ii} = 0$. The last term on the r.h.s. of Eq. (3.2) was introduced for the simplification of calculations, but it does not affect the second-order action of gravitational waves [50].

Choosing the gauge $E = 0$, the spatial infinitesimal transformation vector is fixed. To fix the time transformation vector in Horndeski and GLPV theories we choose the unitary gauge in which the field perturbation $\delta \phi$ vanishes, so the dependence on the scalar field $\phi$ and its kinetic term $X$ does not explicitly appear in the action (2.23). In the projectable version of the Hořava-Lifshitz gravity [24], the lapse $N$ is a function of $t$ alone and hence $\delta N = 0$. In the non-projectable Hořava-Lifshitz gravity [44], there is no such restriction for the gauge choice.

On the flat FLRW background described by the line element $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$, the extrinsic curvature and the intrinsic curvature are given, respectively, by $K_{ij} = H\delta_{ij}$ and $\ddot{R}_{ij} = 0$, where a bar represents the background values. Then, the scalar quantities appearing in the Lagrangian $L$ of Eq. (2.6) are $N = 1$, $K = 3H$, $S = 3H^2$, $\ddot{R} = \ddot{U} = 0$, $Z_1 = Z_2 = 0$, and $\alpha_1 = \alpha_2 = \cdots = \alpha_5 = 0$.

Expanding the action (2.6) up to second order in scalar perturbations for the spatial gauge choice $E = 0$, we can obtain the equations of motion for the background and linear scalar perturbations without fixing the temporal gauge. Varying the first-order perturbed action with respect to $\delta N$ and $\delta \sqrt{F}$, respectively, the background equations are given by  

\[ \tilde{L} + L_N = 3HF = 0, \]  

(3.3)  

\[ \tilde{L} - \tilde{F} = 3HF = 0, \]  

(3.4)  

where  

\[ F \equiv L_K + 2HL_S. \]  

(3.5)  

The linear scalar perturbation equations derived by varying the second-order action in terms of $\delta N, \psi,$ and $\zeta$ are presented in Ref. [43].
B. The second-order action for tensor perturbations

Let us derive the second-order action of Eq. (2.10) for gravitational waves. Regarding the extrinsic curvature, tensor perturbations satisfy the relations $K = 3H$ and $\delta K_i^j = \delta^{ik}\dot{\gamma}_{kj}/2$. Up to first order, the three-dimensional Ricci tensor reads

$$R_{ij} = -\frac{1}{2}a^2 \Delta \gamma_{ij}. \quad (3.6)$$

The three-dimensional Ricci scalar from tensor perturbations is a second-order quantity, which is given by

$$R = \frac{1}{4} \delta^{ij} \delta^{kl} \gamma_{ij} \Delta \gamma_{kl}. \quad (3.7)$$

Then the quantity $Z_1$ is fourth-order in perturbations.

On using the above relations, the second-order action for tensor modes reduces to $S_h^{(2)} = \int d^3x \ a^4 L_h^{(2)}$, where $L_h^{(2)} = L_S \delta K_i^j \delta K_j^i + \mathcal{E} R + L_z \mathcal{R}_i^j \mathcal{R}_j^i + L_z \Delta Z_2$. More explicitly, it is given by

$$S_h^{(2)} = \int d^3x \ \frac{a^3}{4} \delta^{ij} \delta^{kl} (L_S \delta_{ij} \delta_{kl} + \gamma_{ij} \mathcal{O}(\gamma_{kl})), \quad (3.8)$$

where

$$\mathcal{E} = L_R + \frac{3}{2} H L_M, \quad \mathcal{O}_t = \mathcal{E} \Delta + L_z \Delta^2 - L_z \Delta^3. \quad (3.9)$$

Note that there are no contributions to $S_h^{(2)}$ from the scalars (2.22). The condition for avoiding the tensor ghost corresponds to $L_S > 0$.

Varying the action (3.8) with respect to $\dot{\gamma}_{ij}$, we obtain the equation of motion

$$\ddot{\gamma}_{ij} + \left(3H + \frac{L_S}{L_S} \right) \dot{\gamma}_{ij} - c_t^2 \Delta \gamma_{ij}$$

$$- \frac{L_z}{L_S} \Delta^2 \gamma_{ij} + \frac{L_z}{L_S} \Delta^3 \gamma_{ij} = 0, \quad (3.11)$$

where

$$c_t^2 \equiv \frac{\mathcal{E}}{L_S}. \quad (3.12)$$

In the absence of spatial derivatives higher than second order, $c_t$ exactly corresponds to the propagation speed of gravitational waves. In order to avoid the small-scale Laplacian instability in this case, we require that $c_t^2 > 0$.

General Relativity corresponds to $G_{A} = M_{pl}^2/2$ and $G_{T} = 0$ in the Horndeski Lagrangian (2.24), i.e., $-A_4 = B_4 = M_{pl}^2/2$ and $A_3 = B_5 = 0$ in Eq. (2.28). In this case we have $L_S = M_{pl}^2/2$, $\mathcal{E} = M_{pl}^2/2$, $c_t^2 = 1$, $L_z = 0$, and $L_z \Delta^2 = 0$, so Eq. (3.11) reduces to $\ddot{\gamma}_{ij} + 3H \gamma_{ij} - \Delta \gamma_{ij} = 0$.

IV. THE INFLATIONARY GRAVITATIONAL WAVES

In this section we derive the power spectrum of gravitational waves generated during inflation.

A. The power spectrum in Fourier space

We expand the tensor perturbation $\gamma_{ij}(x, \tau)$ into the Fourier series as $\gamma_{ij}(x, \tau) = \int \frac{d^3k}{(2\pi)^3} e^{i k \cdot x} \hat{\gamma}_{ij}(k, \tau)$, where

$$\hat{\gamma}_{ij}(k, \tau) = \sum_{\lambda=+, \times} \hat{h}_\lambda(k, \tau) e^{i(\lambda)}(k). \quad (4.1)$$

Here, $k$ is a comoving wavenumber, $\tau \equiv \int a^{-1} dt$ is the conformal time, and $e^{i(\lambda)}(\lambda = +, \times)$ are symmetric polarization tensors. The polarization tensors are transverse $\langle k_j e^{i(\lambda)} \rangle = 0$ and traceless $\langle e^{i(\lambda)} \rangle = 0$ with the normalization satisfying $e^{i(\lambda)}(k) e^{i(\lambda)}(k) = \delta_{\lambda\lambda'}$. We write the Fourier mode $\hat{h}_\lambda(k, \tau)$ of the form

$$\hat{h}_\lambda(k, \tau) = h_\lambda(k, \tau) a_\lambda(k) + h_\lambda^*(k, \tau) a_\lambda^*(k^-) - k, \quad (4.2)$$

where the annihilation and creation operators $a_\lambda(k)$ and $a_\lambda^*(k^-)$ obey the commutation relation $[a_\lambda(k), a_\lambda^*(k^-)] = \delta_{\lambda\lambda'}(k - k')$.

On the quasi de-Sitter background the conformal time is given by $\tau \simeq -1/(aH)$, so that the asymptotic past and future correspond to $\tau \to -\infty$ and $\tau \to 0$, respectively. The tensor power spectrum $P_h(k)$ is defined by the vacuum expectation value of $\gamma_{ij}$ in the $\tau \to 0$ limit, as $\langle 0 | \gamma_{ij}(k_1, \tau) \gamma_{ij}^*(k_2, \tau) | 0 \rangle = \left(2\pi^2/k_1^3 \right) \delta^{(3)}(k_1 + k_2) P_h(k_1)$. On using Eqs. (4.1) and (4.2), it follows that

$$P_h(k) = \frac{k^3}{2\pi^2} \left( |h_+ (k, 0)|^2 + |h_\times (k, 0)|^2 \right). \quad (4.3)$$

B. Equation of motion for a canonical field

A canonically normalized field $v_\lambda(k, \tau)$ is defined by

$$v_\lambda(k, \tau) \equiv z h_\lambda(k, \tau), \quad z \equiv a \sqrt{L_S/2}. \quad (4.4)$$

Then the kinetic term in the action (3.3) can be expressed as $S_K = \int d\tau d^3x \sum_{\lambda=+, \times} |v_\lambda'|^2/2$, where a prime represents a derivative with respect to $\tau$. From Eq. (3.3) each Fourier component $v_\lambda(k, \tau)$ obeys the equation of motion

$$v''_\lambda + \left[ K(k, \tau) - \frac{z''}{z} \right] v_\lambda = 0, \quad (4.5)$$

where the function $K(k, \tau)$ is defined as

$$K(k, \tau) \equiv c_t^2 k^2 \left( 1 + c_1 \frac{k^2}{a^2 M_{pl}^2} + c_2 \frac{k^4}{a^4 M_{pl}^4} \right). \quad (4.6)$$
and
\[ c_1 \equiv -\frac{L_z M_{\text{pl}}^2}{\mathcal{E}}, \quad c_2 \equiv -\frac{L_z^2 M_{\text{pl}}^4}{\mathcal{E}}. \] (4.7)

In the context of low-energy effective field theories, we will discuss the case where \( K(k, \tau) \approx c_I^2 k^2 \), such that the linear form of the dispersion relation, \( \omega = c_I k \), is not modified by the nonlinear terms in Eq. (4.6) well below the cut-off of the theories. Otherwise, we would need to know the UV-completion of the theories, or our treatment would break down. In fact, the non-linear terms are suppressed for the physical wavenumber \( k_{\text{max}} = k/\alpha \) much below the cut-off value \( k_{\text{phys}} = M_{\text{pl}}/|c_1|^{1/2} \) or \( M_{\text{pl}}/|c_2|^{1/4} \). In Hořava-Lifshitz gravity \( \text{[24]} \) and in the trans-Planckian physics studied in Refs. \( \text{[51–55]} \), \( k_{\text{phys}} \) is close to \( M_{\text{pl}} \), i.e., \( |c_1| \sim |c_2| \sim O(1) \). In the EFT approach of inflation advocated by Weinberg \( \text{[27]} \), the cut-off scale is slightly smaller than \( M_{\text{pl}} \), say \( \sqrt{c} M_{\text{pl}} \), where \( \epsilon = -H/H^2 \) is the slow-roll parameter typically of the order of 0.01.

In the following we shall focus on the situation in which the cut-off scale \( k_{\text{max}} \) is much larger than the Hubble parameter \( H \) during inflation. In this case the Hubble radius crossing occurs in the linear regime of the dispersion relation (i.e., \( K \approx c_I^2 k^2 \)), so that the second and third terms in the parenthesis of Eq. (4.6) are regarded as small corrections to the first term. In other words, the parameter defined by
\[ \sigma \equiv \frac{c_1 H_k^2}{M_{\text{pl}}} \] (4.8)
is much smaller than 1, where \( H_k \) is the Hubble parameter at \( c_I k = a H \).

According to the previous discussion, we will solve Eq. (4.6) iteratively, and write its solution in the form
\[ v_\lambda = v_\lambda^{(0)} + v_\lambda^{(1)}, \] (4.9)
where the leading-order perturbation \( v_\lambda^{(0)} \) obeys the equation of motion
\[ v_\lambda^{(0)}'' + \left( c_I^2 k^2 - \frac{z''}{z} \right) v_\lambda^{(0)} = 0. \] (4.10)

The field \( v_\lambda^{(1)} \) induced by the nonlinear corrections to Eq. (4.6) satisfies
\[ v_\lambda^{(1)}'' + \left( c_I^2 k^2 - \frac{z''}{z} \right) v_\lambda^{(1)} = -c_I^2 \frac{k^4}{a^2 M_{\text{pl}}^2} \left( c_1 + c_2 \frac{k^2}{a^2 M_{\text{pl}}^2} \right) v_\lambda^{(0)}. \] (4.11)

In order to solve Eq. (4.11), we take into account the slow-roll inflationary corrections to the leading-order solution on the de Sitter background \( \text{[50]} \). We then substitute the leading-order solution into Eq. (4.11) to obtain an iterative solution of \( v_\lambda^{(1)} \).

C. Solutions to the tensor equations of motion

In the following we assume that the parameters defined by
\[ \epsilon \equiv \frac{\dot{H}}{H^2}, \quad \epsilon_S \equiv \frac{L_z s}{H L_S}, \quad s \equiv \frac{c_I}{H c_t} \] (4.12)
are much smaller than unity during inflation. Then the quantity \( z''/z \), up to next to leading-order corrections, can be estimated as
\[ \frac{z''}{z} = 2(a H)^2 \left( 1 - \frac{1}{2} \epsilon + \frac{3}{4} \epsilon_S \right). \] (4.13)

Introducing a dimensionless variable
\[ y \equiv \frac{c_I k}{a H}, \] (4.14)
its time derivative obeys \( y'' = -a H y(1 - \epsilon - s) \). Then, Eq. (4.10) can be expressed as
\[ (1 - 2\epsilon - 2s)y'' \frac{d^2 v_\lambda^{(0)}}{dy^2} - s \frac{dv_\lambda^{(0)}}{dy} + \left( y^2 - 2 + \frac{3}{2} \epsilon_S \right) v_\lambda^{(0)} = 0. \] (4.15)

Here and in the following, we drop contributions of the slow-roll corrections of the order of \( c_I^2 \). In other words, we deal with the first-order slow-roll parameters as constants.

The solution to Eq. (4.15), after approximating by neglecting non-linear terms in the slow-roll parameters, is given by
\[ v_\lambda^{(0)}(y) = y^{(1+s)/2} [a_k H^{(1)}(y) \left( 1 + \epsilon + s \right) y] + \beta_k H^{(2)}(y) \left( 1 + \epsilon + s \right) y, \] (4.16)
where \( a_k \) and \( \beta_k \) are integration constants, \( H^{(1)}(x) \) and \( H^{(2)}(x) \) are Hankel functions of the first and second kinds respectively, and
\[ \nu = \frac{3}{2} + \epsilon + \frac{1}{2} \epsilon_S + \frac{3}{2} s. \] (4.17)

The Bunch-Davies vacuum corresponds to the choice \( \beta_k = 0 \). On using the property \( H^{(1)}(x \gg 1) \approx -\sqrt{2/(\pi x)} e^{i(x+(3-2\nu)\pi/4)} \), the solution in the asymptotic past reads
\[ v_\lambda^{(0)}(y \gg 1) \approx -a_k \sqrt{\frac{2}{\pi}} \frac{y^{s/2}}{\sqrt{1 + \epsilon + s}} e^{i(1+\epsilon+s)y+(3-2\nu)\pi/4}. \] (4.18)

The coefficient \( a_k \) is determined by the Wronskian condition \( v_\lambda^{(0)}(y) \epsilon'' - v_\lambda^{(0)*} \epsilon'(y) = i \), such that (up to second order in the slow-roll parameters)
\[ a_k = -\frac{1}{4} \sqrt{\frac{\pi}{c_I c_t k}} (2 + \epsilon + s), \] (4.19)
where $c_{ik}$ is the value of $c_i$ at $c_{ik} = aH$ (i.e., at $y = 1$).

For the derivation of Eq. (4.14) we used the property that
any time-dependent function $f(\tau)$ on the quasi de Sitter background can be expanded around $y = 1$ (denoted by the subscript $k$), as $f(\tau) = f(\tau_k) - (f(\tau_k) / h_k \log(\tau/\tau_k))$. For $\mu$ much smaller than 1 the quantity $y$ is also expanded as $y^u \simeq 1 + \mu \log(\tau/\tau_k)$, so the variation of $c_i$, $H$, and $L, S$ can be quantified as

$$c_i = c_{ik} y^{-s}, \quad H = H_k y^s, \quad L, S = L, S_k y^{-s}. \quad (4.20)$$

Substituting Eq. (4.19) and $\beta_k = 0$ into Eq. (4.14), we obtain

$$v_\lambda^{(0)}(y) = -\frac{\sqrt{\pi}}{2} \frac{aH}{(c_ik)^{3/2}} (1 + \frac{1}{2} \epsilon + \frac{1}{2} s) \times y^{3/2} H^{(1)}(v, (1 + \epsilon + s) y). \quad (4.21)$$

Using the property $H^{(1)}(x \to 0) = -(i/\pi) \Gamma(\nu)(x/2)^{-\nu}$ and the relations (4.20), the solution $v_\lambda^{(0)}(y) = v_\lambda^{(0)}(y)/z$ long after the Hubble radius crossing ($y \to 0$) reduces to

$$h_\lambda^{(0)}(0) = i \frac{H_k}{\sqrt{2\pi L, S_k}} \frac{2\Gamma(\nu)}{(c_ik)^{3/2}} (1 - \epsilon - s). \quad (4.22)$$

Expanding the function $2\Gamma(\nu)$ around $\nu = 3/2$, it follows that

$$h_\lambda^{(0)}(0) = i \frac{H_k}{\sqrt{2\pi L, S_k}} \frac{1}{(c_ik)^{3/2}} \left[ 1 + (1 - \gamma - \ln 2) \epsilon \right. \right.$$}

$$\left. + \frac{1}{2}(2 - \gamma - \ln 2) \epsilon S + \frac{1}{2} (4 - 3\gamma - 3\ln 2) s \right]. \quad (4.23)$$

where $\gamma = 0.5772...$ is the Euler-Mascheroni constant.

The next step is to derive the solution to Eq. (4.11) by using the leading-order solution of Eq. (4.21) on the de Sitter background (obtained by setting $\epsilon = \epsilon_S = s = 0$ and $a = -1/(H\tau)$ with $H$ = constant), i.e., $v_\lambda^{(0)}(\tau) = -i \frac{1}{(1 + i c_k \kappa \tau)} e^{-i c_k \kappa \tau}/\sqrt{2\pi (c_ik)^{3/2}}$. The speed of propagation for this mode, for large $k$'s, coincides, by construction, with $c_\tau$, such that this choice is consistent with the assumption that the corrections do not modify the standard propagation of gravitational waves. Integrating Eq. (4.11) after substitution of the leading-order solution of $v_\lambda^{(0)}$, the resulting particular solution is given by

$$v_\lambda^{(1)}(\tau) = \frac{e^{-i c_k \kappa \tau}}{H^2} \frac{\sqrt{2\pi \sqrt{c_ik}}}{5c_ik^2 / M_{pl}^2} \times \left[ 5 \left( 5c_ik^2 - 7c_k^2 H^2 / M_{pl}^2 \right) (3c_k \kappa \tau - 2c_k^2 k^4 \tau^4) \right. \right.$$}

$$- 10i \left. \left( 5c_ik^2 - 7c_k^2 H^2 / M_{pl}^2 \right) c_k^4 k^4 \tau^4 \right.$$}

$$\left. - 6c_k^2 H^2 / M_{pl}^2 (7 + 2i c_k \kappa \tau) c_k^4 k^5 \tau^5 \right]. \quad (4.24)$$

The correction $v_\lambda^{(1)}(\tau)$ has an oscillatory part $e^{-i c_k \kappa \tau}$, which by construction, follows the oscillations of the dominant contribution, $v_\lambda^{(0)}(\tau)$. Long after the Hubble radius crossing ($\tau \to 0$), the perturbation $h_\lambda^{(1)}(\tau) = v_\lambda^{(1)}(\tau)/z$ approaches

$$h_\lambda^{(1)}(0) = -i \frac{H}{8\sqrt{L, S_k} (c_ik)^{3/2}} \left( 5c_1 H^2 / c_{l1}^2 M_{pl}^2 - 7c_2 H^4 / c_{l2}^2 M_{pl}^4 \right). \quad (4.25)$$

Since we are not interested in the next-order solution to Eq. (4.25), we can replace $H, c_i, L, S$ for $H_k, c_{ik},$ and $L, S_k$, respectively.

D. The spectrum of inflationary gravitational waves

The tensor power spectrum is known by substituting $h_\lambda^{(0)}(0) = h_\lambda^{(0)}(0) + h_\lambda^{(1)}(0)$ into Eq. (4.22), as

$$P_h(k) = \frac{H^2}{\pi^2 L, S_k c_{ik}} \left[ 1 - 2(C + 1) \epsilon - C \epsilon_S - (3C + 2) s \right.$$}

$$\left. - 5 \sigma_{c_{ik}} + 7c_2 \sigma_{\sigma}^2 \right] \left( \frac{1}{4c_{ik}^2} \right), \quad (4.26)$$

where $C = \gamma - 2 + \ln 2 = -0.72963...$ and $\sigma$ is defined by Eq. (4.28). The leading-order power spectrum is given by $P_{h_{\text{lead}}}(k) = H^2 / (\pi^2 L, S_k c_{ik}^2)$. The last two terms in the square bracket of Eq. (4.26), which correspond to the corrections induced by spatial derivatives higher than second order, are suppressed by the factor $\sigma \approx H^2/(k_{\text{max}}^2)$. Provided that $\sigma / c_{ik}^2 \ll \epsilon$, these terms are smaller than the slow-roll corrections.

We introduce the tensor spectral index $n_t$, as

$$n_t = \left. \frac{d \ln P_h(k)}{d \ln k} \right|_{c_{ik}=aH}. \quad (4.27)$$

On using the property $d \ln k / d t_{\text{cik}} = aH$ and defining the following slow-roll parameters

$$\eta \equiv \frac{\dot{\epsilon}}{H \epsilon}, \quad \eta_S \equiv \frac{\dot{\epsilon}_S}{H \epsilon_S}, \quad \delta_s \equiv \frac{\dot{\sigma}}{H \sigma}, \quad (4.28)$$

it follows that

$$n_t = -2\epsilon - \epsilon_S - 3s - 2\epsilon^2 - 5\epsilon_s - \epsilon \epsilon_S - \epsilon_S s - 3s^2 \right.$$}

$$- 2(C + 1) \epsilon_S - C \epsilon_S \eta_S - (3C + 2) s \delta_s + \frac{5}{4c_{ik}^2} \sigma^2 \left( \epsilon + s \right), \quad (4.29)$$

which should be evaluated at $c_{ik} k = aH$. The leading-order spectral index is given by $n_t^{\text{lead}} = -2 \epsilon - \epsilon_S - 3s$.

V. APPLICATION TO CONCRETE THEORIES

We estimate the inflationary tensor power spectrum and its spectral index in concrete modified gravitational theories by using the general results derived in Sec. IV.
A. Theories with higher-order derivative potentials

Let us consider the theories described by the Lagrangian

\[ L = \frac{M_{pl}^2}{2}(\mathcal{S} - \lambda \mathcal{K}^2 + \mathcal{R}) + A_2(N,t) + A_3(N,t)K \]

\[ + \frac{M_{pl}^2}{2} \eta_1 \alpha_1 - \frac{1}{2} (g_2 \mathcal{R}^2 + g_3 \mathcal{Z} + \eta_2 \alpha_2 + \eta_3 \alpha_3) \]

\[ - \frac{1}{2M_{pl}^2} (g_4 \mathcal{Z}_1 + g_5 \mathcal{Z}_2 + \eta_1 \alpha_4 + \eta_5 \alpha_5). \]

For \( A_2 = A_3 = 0 \) this corresponds to the Lagrangian of Horava-Lifshitz gravity, including both the projectable (\( \alpha_i = 0 \)) and non-projectable (\( \alpha_i \neq 0 \)) versions.

We take into account the terms \( A_2(N,t) \) and \( A_3(N,t)K \) in Eq. (5.1) to realize inflation by a scalar degree of freedom. In fact, the Lagrangian \( L = (M_{pl}^2/2)R + \mathcal{G}_2(\phi, X) + \mathcal{G}_3(\phi, X) \) reduces to Eq. (5.1) for \( \lambda = 1, A_2 = G_2 - X \mathcal{F}_3, \phi, \eta_1 = \cdots = \eta_5 = 0 \) and \( g_2 = \cdots = g_5 = 0 \) in unitary gauge, where we used the fact that the four-dimensional Ricci scalar is expressed as \( R = \mathcal{S} - \mathcal{K}^2 + \mathcal{R} \) up to a boundary term. The field \( \phi \) is responsible for the cosmic acceleration as it happens for k-inflation (\( G_3 = 0 \)) and potential-driven slow-roll inflation (\( G_3 = 0 \) and \( G_2 = -X/2 - \mathcal{V}(\phi) \)).

Since \( L_S = E = M_{pl}^2/2, c_1 = g_3, \) and \( c_2 = g_5, \) Eqs. (4.2(c)) and (4.2(d)) read

\[ \mathcal{P}_h(k) = \frac{2H^2}{\pi^2M_{pl}^2} \left[ 1 - 2(C + 1)\epsilon - \frac{5}{4} \sigma + \frac{7g_5}{4g_3^2} \epsilon^2 \right], \]

\[ \eta_t = -2\epsilon - 2\epsilon^2 - 2(C + 1)\epsilon \eta + \frac{5}{2} \sigma - \frac{7g_5}{4g_3^2} \epsilon, \]

where \( \sigma = g_3 H^2/M_{pl}^2. \) If \( g_3 = g_5 = 0, \) then the last two terms in Eqs. (5.2) and (5.3) vanish. In this case, the above tensor power spectrum reduces to the one in standard slow-roll inflation.

The contributions from the terms \( A_2(N,t) \) and \( A_3(N,t)K \) do not directly appear in Eqs. (5.2)-(5.3), but they affect the tensor power spectrum indirectly through the background equations of motion.

Since the leading-order spectrum is \( \mathcal{P}_{h,\text{lead}}(k) = 2H^2/(\pi^2M_{pl}^2), \) the energy scale of inflation is directly known from the measurement of primordial gravitational waves. More concretely, we have \( H_k/M_{pl} \approx \pi \sqrt{r\mathcal{P}_s(k)/2}, \) where \( \mathcal{P}_s(k) \approx 2.2 \times 10^{-9} \) is the observed scalar power spectrum, and \( r = \mathcal{P}_{h,\text{lead}}(k)/\mathcal{P}_s(k) \) is the tensor-to-scalar ratio. On using the observational bound \( r \lesssim 0.2, \) we have that \( H_k/M_{pl} \lesssim 4 \times 10^{-5}. \) Hence, for \( |g_3|, |g_5| \lesssim 1, \) the corrections induced by spatial derivatives have a higher order than second order are suppressed compared to the slow-roll corrections (typically of the order of 0.01). Provided that \( H \) decreases during inflation, the spectrum of gravitational waves is red-tilted (\( n_s \approx -2 \epsilon < 0 \)). From the background Eqs. (5.3) and (5.4), we obtain \( M_{pl}^2(3\lambda - 1)\dot{H} = A_{2,N} + 3H A_{3,N}. \) If \( \lambda > 1/3, \) then the condition \( \dot{H} < 0 \) translates to \( A_{2,N} + 3H A_{3,N} < 0. \)

In unitary gauge the field kinetic energy is given by \( X = -N^{-2}\phi^2, \) so the Hubble parameter decreases for \( A_{2,N} + 3H A_{3,N} < 0. \)

B. Horndeski theories

In unitary gauge the Lagrangian \( \mathcal{L}_h \) of Horndeski theories is equivalent to Eq. (2.8) with the relations (2.7)-(2.14). In this case we have \( L_S = G_4(1 + \epsilon_1) \) and \( E = G_4(1 + \epsilon_2), \) where

\[ \epsilon_1 = -\frac{2XG_4X}{G_4} - \frac{XG_5 \phi}{2G_4} + \frac{H(-X)^{3/2}G_5X}{G_4}, \]

\[ \epsilon_2 = \frac{XG_5 \phi}{2G_4} - \frac{XG_5X \phi}{G_4}. \]

The terms \( \epsilon_1 \) and \( \epsilon_2, \) which involve \( X, \) work as the slow-roll corrections to the leading-order contribution \( G_4. \) The tensor propagation speed square is given by \( c_T^2 \approx 1 - \epsilon_1 + \epsilon_2 + O(\epsilon^2), \) and hence \( s = \epsilon_2 \eta_2/2 - \epsilon_1 \eta_1/2 + O(\epsilon^2), \) where \( \eta_j \equiv \dot{\epsilon}_j/(\epsilon \epsilon_f) \) with \( j = 1, 2. \) In the following we set \( G_4 = (M_{pl}^2/2)F(\phi, X), \) where \( F(\phi, X) \) is a dimensionless function with respect to \( \phi \) and \( X. \) Then the slow-roll parameter \( \epsilon_S \) can be expressed as \( \epsilon_S = \epsilon_F + \epsilon_1 \eta_1 + O(\epsilon^3), \)

where \( \epsilon_F \equiv F/(H F) \). The tensor power spectrum and its spectral index, up to next to leading-order terms, read

\[ \mathcal{P}_h(k) = \frac{2H^2}{\pi^2M_{pl}^2F} \left[ 1 - 2(C + 1)\epsilon - C \epsilon_F + \frac{\epsilon_1}{2} - \frac{3\epsilon_2}{2} \right], \]

\[ \eta_t = -2\epsilon - \epsilon_F - 2\epsilon^2 - \epsilon \eta_1 + \frac{1}{2} \epsilon \eta_2 - \frac{3}{2} \epsilon_2 \eta_2 \]

\[ -2(C + 1)\epsilon \eta + C \epsilon_F \eta_1, \]

where \( \eta_F \equiv \dot{\epsilon}_F/(H \epsilon_F). \)

Compared to Eq. (5.2), the leading-order power spectrum \( \mathcal{P}_{h,\text{lead}}(k) = 2H^2/(\pi^2M_{pl}^2F) \) of Eq. (5.6) is divided by the term \( F. \) This term is associated with the conformal factor \( \Omega^2 \) under the transformation \( \dot{g}_{\mu\nu} = \Omega^2 (\phi, X) g_{\mu\nu}. \) In the following we study the case in which the conformal factor depends on \( \phi \) alone, i.e., on \( t \) in unitary gauge. This assumption is justified provided that the \( X \) dependence in \( \Omega^2 \) works only as slow-roll corrections to the leading-order \( \phi \)-dependent term. Under the conformal transformation \( \dot{g}_{\mu\nu} = \Omega^2(t) g_{\mu\nu}, \) the coefficients \( A_4 \) and
respectively, where $A_4 = -G_4[1 + O(e)]$ and $B_4 = G_4[1 + O(e)]$ from Eqs. (2.11) and (2.12). Since the second terms in the parentheses of Eqs. (5.8) and (5.9) can be regarded as slow-roll corrections, we have $A_4 = -\Omega^{-2}G_4[1 + O(e)]$ and $B_4 = \Omega^{-2}G_4[1 + O(e)]$. Choosing the conformal factor $\Omega^2 = 2G_4/M^2_{pl} = F$, it follows that $A_4 = -(M^2_{pl}/2)[1 + O(e)]$ and $B_4 = (M^2_{pl}/2)[1 + O(e)]$.

Under the conformal transformation $\tilde{g}_{\mu\nu} = \Omega^2(t)g_{\mu\nu}$, the structure of the Lagrangian (2.8) is preserved with the modified leading-order coefficients $\tilde{A}_2 = \Omega^{-4}A_2$, $\tilde{A}_3 = \Omega^{-3}A_3$, $\tilde{A}_4 = \Omega^{-1}A_4$, and $\tilde{B}_5 = \Omega^{-1}B_5$ in the presence of slow-roll corrections (including the derivative $\tilde{\Omega}/(N\Omega)$) (22). This means that, for the choice $\Omega^2 = F$, the leading-order tensor spectrum in the transformed (Einstein) frame can be derived by setting $L_S = M^2_{pl}/2$ and $\epsilon_t = 1$ in Eq. (2.23), i.e., $\tilde{P}_h^{lead}(k) = 2\tilde{H}_k^2/(\pi^2 M^2_{pl})$. Since the Hubble parameters in two frames are related to each other as $H = [H + F/(2NF)]/\sqrt{F}$, the spectrum $\tilde{P}_h^{lead}(k)$ is equivalent to $P_h^{lead}(k) = 2H_k^2/(\pi^2 M^2_{pl} F)$ at leading order in slow-roll. Provided that the null energy condition is not violated in the Einstein frame the Hubble parameter $\tilde{H}$ decreases, in which case the tensor power spectrum is red-tilted.

The above properties can be notably seen in the Higgs inflationary scenario with the scalar-field potential $V(\phi) = (\lambda/4)(\phi^2 - v^2)^2$ and the function $F = 1 + \zeta\phi^2/M^2_{pl}$, where $\zeta$ is a non-minimal coupling (12) (see also Refs. [59]). In order to realize the self-coupling $\lambda$ of the order of 0.1, the non-minimal coupling is constrained to be $\zeta = O(10^4)$ from the CMB normalization. For $\zeta \gg 1$ the quantity $F$ is related to the number of e-foldsings $N_e$ from the end of inflation, as $F \approx 4N_e/3$ (60), which is much larger than $O(1)$ on scales relevant to the CMB anisotropies. The action in the Einstein frame is characterized by a canonically normalized field with the potential $\tilde{V} = V(\phi)/F^2$ (61), in which case the tensor spectrum $\tilde{P}_h^{lead}(k) = 2\tilde{H}_k^2/(\pi^2 M^2_{pl})$ is red-tilted due to the decrease of $\tilde{H}$.

C. GLPV theories

Let us proceed to the GLPV theories in unitary gauge, i.e., the Lagrangian (2.8). In this case the functions $L_S$ and $\mathcal{E}$ are given by $L_S = -A_4(1 + \epsilon_1)$ and $\mathcal{E} = B_4(1 + \epsilon_2)$ respectively, where

$$\epsilon_1 = 3HA_5/A_4, \quad \epsilon_2 = B_5/(2B_4).$$

Provided that $\epsilon_1$ and $\epsilon_2$ are regarded as slow-roll corrections to the leading-order terms of $L_S$ and $\mathcal{E}$, we have $c^2_{t,lead} = -(B_4/A_4)(1 - \epsilon_1 + \epsilon_2)$. The difference from Horndeski theories is that $A_4$ and $B_4$ are not related with each other, so $c^2_t$ generally differs from 1. Then the leading-order tensor spectrum is given by

$$P_h^{lead}(k) = \frac{H_k^2}{\pi^2|A_4|c^2_{t,lead}},$$

where $c^2_{t,lead} = -B_4/A_4$.

We perform the disformal transformation given by $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \Gamma(\phi, X)\partial_\mu\phi\partial_\nu\phi$, where $\Gamma(\phi, X)$ is a function in terms of $\phi$ and $X$ (62, 63). In Ref. (22) it was shown that the structure of the GLPV action is preserved under this transformation. The coefficients $A_4$ and $B_4$ in the Lagrangian (2.8) are transformed as

$$\tilde{A}_4 = \sqrt{1 + \Gamma X}A_4, \quad \tilde{B}_4 = B_4/\sqrt{1 + \Gamma X}.$$

In the new frame the tensor propagation speed square is given by $c^2_{t,lead} = -\tilde{B}_4/\tilde{A}_4 = c^2_{t,lead}/(1 + \Gamma X)$. If we choose the function

$$\Gamma = -\frac{1 - c^2_{t,lead}}{X},$$

then it follows that $c^2_{t,lead} = 1$. In this case, the coefficients in Eq. (2.8) are transformed as $\tilde{A}_2 = A_2/c_{t,lead}$, $\tilde{A}_3 = A_3$, $\tilde{A}_4 = c_{t,lead}A_4$, $\tilde{B}_4 = B_4/c_{t,lead}$, $\tilde{A}_5 = c^2_{t,lead}A_5$, and $\tilde{B}_5 = B_5$. Since $c^2_{t,lead} = 1$ in the new frame, the leading-order spectrum becomes $P_h^{lead}(k) = 2\tilde{H}_k^2/(\pi^2|\tilde{A}_4|)$. If we make the conformal transformation $\tilde{g}_{\mu\nu} = \Omega^2(t)g_{\mu\nu}$ further with $\Omega^2 = 2|\tilde{A}_4|/M^2_{pl}$, the resulting leading-order spectrum reduces to $P_h^{lead}(k) = 2\tilde{H}_k^2/(\pi^2 M^2_{pl})$.

The above discussion shows that the combination of the disformal and conformal transformations, $\tilde{g}_{\mu\nu} = \Omega^2(t)g_{\mu\nu} + \Gamma(\phi, X)\partial_\mu\phi\partial_\nu\phi$, can lead to a metric frame in which the leading-order tensor power spectrum is of the standard form that depends on the Hubble parameter $\tilde{H}_k$ alone. This conclusion is consistent with the recent results of Ref. (47) in which the authors took the EFT approach without having the direct connection to particular modified gravitational theories.

VI. CONCLUSIONS

We have studied gravitational waves on the flat FLRW background for the general action (2.6) that encompasses most of the modified gravitational theories proposed in the literature–including Horndeski theories, GLPV theories, and Hořava-Lifshitz gravity. The equation of motion (5.11) for tensor perturbations, which follows from the second-order action (5.8), involves the spatial derivatives higher than second order for the theories where the Lagrangian $L$ depends on $\dot{Z}$ or $\ddot{Z}$. 
We derived the inflationary power spectrum of gravitational waves under the condition that the cut-off scale $k_{\text{max}}^\text{phys}$ associated with the non-linear terms of Eq. (4.10) is much larger than the Hubble parameter $H_0$ at $c_t k = a H$ during inflation. On using the small parameter $\sigma$ of the order of $H_0^2/(k_{\text{max}}^\text{phys})^2$, the solution to Eq. (4.5) is obtained iteratively on the de Sitter background. Taking into account the slow-roll corrections to the leading-order solution as well, the resulting tensor power spectrum is given by Eq. (4.20) with the spectral index (4.29).

The corrections from the higher-order spatial derivatives to the leading-order power spectrum are suppressed by the factor $\sigma/c_t^2$. This conclusion is consistent with the effect of modified trans-Planckian dispersion relations on the inflationary power spectrum of gravitational waves. For $k_{\text{phys}}^\text{max}$ close to $M_{\text{pl}}$ and for $c_t k$ not very much smaller than 1, the corrections induced by the spatial derivatives higher than second order are smaller than the slow-roll corrections arising from the deviation from the de Sitter background.

We applied our general formula of the inflationary tensor power spectrum to a number of concrete modified gravitational theories. For the Lagrangian (5.1), which encompasses kinetic braiding models and Hořava-Lifshitz gravity, the leading-order spectrum is directly related to $H_k$, as $P_k^\text{lead}(k) = 2H_k^2/(\pi^2 M_{\text{pl}}^2)$.

In Horndeski theories, where the tensor propagation speed is 1 at leading-order in slow-roll, $P_k^\text{lead}(k)$ involves a dimensionless factor $F = 2G_A/M_{\text{pl}}^2$ in the denominator. Under the conformal transformation $\hat{g}_{\mu\nu} = F g_{\mu\nu}$, the spectrum in the Einstein frame simply reduces to $P_h^\text{lead}(k) = 2\hat{H}_k^2/(\pi^2 M_{\text{pl}}^2)$.

In GLPV theories the leading-order tensor spectrum (5.11) involves the terms $A_k$ and $c_t^2 A_{k,\text{lead}} = -B_4/A_4$. We showed that, under the disformal transformation $\hat{g}_{\mu\nu} = \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi, X)\partial_{\mu}\phi\partial_{\nu}\phi$, it is possible to find a frame in which $c_t^2 A_{k,\text{lead}} = 1$ and $A_4 = -M_{\text{pl}}^2/2$ up to slow-roll corrections. Thus the prediction of inflationary gravitational waves is robust in that there exists the metric frame in which the leading-order spectrum is simply proportional to $\hat{H}_k^2$ in a vast class of modified gravitational theories.

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