Quark Condensates in Non-supersymmetric MQCD

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Abstract

A set of non-supersymmetric minimal area embeddings of an M-theory 5-brane are considered. The field theories on the surface of the 5-brane have the field content of N=2 SQCD with fundamental representation matter fields. By suitable choice of curve parameters the N=2 and N=1 superpartners may be decoupled leaving a semi-classical approximation to QCD with massive quarks. As supersymmetry breaking is introduced a quark condensate grows breaking the low energy $Z_F$ flavour symmetry. At $\theta = (\text{odd}) \pi$ spontaneous CP violation is observed consistent with that of the QCD chiral lagrangian.

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1 Introduction

The massless states living on the surfaces of D-branes (for a review see [1]) in type IIA string theory correspond to fields in a D+1 dimensional gauge theory on the branes surface [2]. D-brane constructions can therefore be used to engineer field theories. In the string theory perturbative techniques can be used to identify the effective field theory, and gauge theories in 4 dimensions have been realized with N=2 and N=1 [3]-[9] and N =0 [4][6]-[11] supersymmetry. To understand the strong dynamics of these gauge theories one must study the IR properties of the brane constructions. Within the context of string theory this is a difficult problem since it involves the understanding of the cores of branes where the dilaton vev blows up. Perturbative type IIA string theory is though the zero radius limit of 11 dimensional M-theory. By considering related configurations in theories where the radius of the eleventh dimension is brought up from zero a continuous understanding of the interplay between D4 and NS5 branes may be obtained since they correspond to the same M5 brane in some places wrapped on $x^{10}$. A semi-classical approximation may then be obtained for the IR behaviour of the configurations by making a minimal area embedding of the M5 brane. These configurations have field theories on their surfaces with the particle content of the string theory field theories but with in addition extra Kaluza Klein modes with masses of order the scale at which strong coupling dynamics sets in [3]. One hopes that by moving to strong coupling in M-theory the low energy dynamics remains in the same universality class as the theory on the type IIA branes. For N=2 and N=1 theories in 4 dimensions this is born out by the recovery of the field theory solution from the brane configurations. For the one non-supersymmetric M5 brane minimal area embedding studied to date [3][9]-[11], which has a low energy field theory described by softly broken N=2 SQCD [10], confinement and the existence of QCD string solutions has been observed [3][9]. The calculation of the string tension in the brane configuration [1] and the field theory [10] do not agree but this is not surprising since the loss of supersymmetry relaxes the symmetry constraints on the semi-classical approximation. Nevertheless one may hope to deduce qualitative behaviour from the approximation. A recent analysis [11] of these configurations has demonstrated the non-trivial spectral flow in the theories with changing theta angle previously observed in field theory [12] with perturbing soft supersymmetry operators [12]-[15]. The novel property of the string theory construction is though that there is no restriction to small supersymmetry breaking operators and one may decouple all superpartners.

In this paper we extend these previous analyses by introducing a non-supersymmetric minimal area embedding of an M5 brane that corresponds to a IIA configuration with semi-infinite D4 branes that contribute quark flavours in the fundamental representation of the gauge group. As supersymmetry breaking is switched on it is consistent to interpret the
extra parameter with a quark condensate. In the limit of decoupling all superpartners one
observes that the $Z_F$ remnant of the quarks axial symmetry is broken by the IR dynamics
again demonstrating the existence of a quark condensate. Following [11] one observes non-
trivial spectral flow as the bare theta angle is changed in the theory. At $\theta = (\text{odd}) \pi$
there are phase transitions as two of $F$ discrete vacua interchange. This is precisely the behaviour
of the QCD chiral lagrangian [16].

2 Non-supersymmetric Type IIA Configurations

Let us begin from the standard type IIA brane construction that realizes a 4 dimensional
N=2 $SU(N) \times SU(F)$ gauge theory with matter fields in the $(N, \bar{F})$ representation. From
left to right in the $x^6$ direction the branes are

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
& \# & R^4 & x^4 & x^5 & x^6 & x^7 & x^8 & x^9 \\
\hline
\text{NS5} & 1 & - & - & - & \bullet & \bullet & \bullet & \bullet \\
\text{D4} & N' & - & - & \bullet & \bullet & [-] & \bullet & \bullet & \bullet \\
\text{NS5}' & 1 & - & - & - & \bullet & \bullet & \bullet & \bullet \\
\text{D4}' & F & - & \bullet & \bullet & [-] & \bullet & \bullet & \bullet \\
\text{NS5}'' & 1 & - & - & - & \bullet & \bullet & \bullet & \bullet \\
\hline
\end{array}
\]

(1)

$R^4$ is the space $x^0 - x^3$. A dash $-$ represents a direction along a brane’s world volume
while a dot $\bullet$ is transverse. For the special case of the D4-branes’ $x^6$ direction, where the
world volume is a finite interval corresponding to their suspension between two NS5 branes
at different values of $x^6$, we use the symbol $[-]$. The field theory exists on scales much
greater than the $L_6$ distance between the NS5 branes with the fourth space like direction of
the D4-branes generating the couplings of the gauge groups in the effective 4D theory.

The $U(1)_R$ and $SU(2)_R$ symmetries of the N=2 field theory are manifest in the brane
picture. They correspond to isometries of the configuration; an SO(2) in the $x^4, x^5$ directions
and an SO(3) in the $x^7, x^8, x^9$ directions.

The matter multiplets are provided by $4-4'$ strings. Alternative realizations of the matter
fields are possible. For example the NS5$''$ brane may be replaced with a D6 brane [3]. The
matter fields remain but the D4-D6 boundary conditions freeze the flavour gauge field and by
supersymmetry its superpartners (when the configuration is broken to N=1 supersymmetry
the adjoint matter field of the frozen N=2 gauge multiplet begins to propagate and is the
“meson” of dual SQCD). The flavour gauge multiplet may also be frozen by taking the NS5$''$
to infinity leaving a semi-infinite D4$'$ brane [4].
Supersymmetry may be broken in the configuration in a number of ways (see [4][6][9]-[11] for previous discussions). The NS5 branes may be rotated in the $x^4, x^5, x^7, x^8, x^9$ space (rotations from the N=2 configuration into the $x^6$ direction cause the NS5 branes to cross changing the topology of the configuration in such a way that it can no longer be easily identified with a field theory). Only the rotations of two of the three NS5 branes correspond to changing the parameters of the field theory since a rotation of the third can be reproduced by rotations of the other two and a rotation of the whole configuration. There are six parameters describing the rotations of each of the two NS5 branes (say the end two) (SO(5)/SO(2)/SO(3)).

Supersymmetry may also be broken by forcing the D4 and D4′ branes to lie at angles to each other. In the supersymmetric configuration for the D4′ to end on the NS5′ the $x^6 - x^9$ coordinates of these branes must be shared at the point where they meet. There is a choice for the $x^4 - x^5$ coordinates since the NS5′ lies in those directions. This choice determines the minimal length of the $4 - 4′$ strings and hence the mass of the matter multiplet (for the N=2 configurations a mass term and an adjoint vev are indistinguishable in both the field theory and the brane configuration corresponding to the D4s freedom to move in the $x^4 - x^5$ directions, the precise identification with a mass term is therefore valid for N=1 configurations). To preserve supersymmetry the $x^4 - x^5, x^7 - x^9$ coordinates of the point where the D4′ ends on the NS5′′ are then completely determined by the choice of mass. If we are willing to break supersymmetry this need not be the case. In general we may make arbitrary choices for the four point like coordinates of the NS5′′ so that the D4′ corresponds to an arbitrary vector in that four dimensional space. Its position relative to the D4 branes is then described by three angles.

The positions of the branes in these configurations break supersymmetry and hence we expect there to be supersymmetry breaking parameters introduced in the low energy field theory lagrangian. These parameters must be the supersymmetry breaking vevs of fields in the string theory since at tree level there are no supersymmetry breaking parameters. The vevs occur as parameters because the fluctuations of those fields are being neglected in the field theory; such fields are spurions. They have a natural interpretation in the brane configuration. The fields are those describing the positions of the branes and their fluctuations are neglected because the infinite branes are very massive. If though we choose to include these fields in the field theory description they occur subject to the stringent constraints of N=2 supersymmetry [14][15]. The spurions whose vevs correspond to the supersymmetry breaking parameters must be the auxiliary fields of N=2 multiplets. This constraint is sufficient to identify the spurions.

Field theoretically N=2 Yang Mills theory without matter is very restrictive on where
spurions may occur. The unique possibility for lowest dimension operators is that spurion fields occur as vector fields in the prepotential as $F = (S_1 + iS_2)A^2$. The scalar spurion vevs generate the gauge coupling $\tau$. These spurions are natural candidates to correspond to the supersymmetry breaking induced by rotations of the NS5 branes. When we allow the auxiliary fields of the spurions to be non-zero we obtain the tree level masses

$$-\frac{N_c}{8\pi^2}Im \left((F_1^* + iF_2^*)\psi_A^*\psi_A + (F_1 + iF_2)\lambda^\alpha\lambda^\alpha + i\sqrt{2}(D_1 + iD_2)\psi_A^*\lambda^\alpha\right)$$

$$-\frac{N_c}{4\pi^2Im(s_1 + is_2)}\left(|F_1|^2 + D_1^2/2)Im(a^\alpha)^2 + (|F_2|^2 + D_2^2/2)Re(a^\alpha)^2ight)$$

$$+ (F_1 F_1^* + F_1^* F_2 + D_1 D_2)Im(a^\alpha)Re(a^\alpha)$$

A number of consistency checks support the identification. Switching on any one of the six independent real supersymmetry breakings in the field theory leaves the same massless spectrum in the field theory as in the brane picture when any one of the six independent rotations of the NS5 brane is performed. The field theory and brane configurations possess the same sub-manifold of N=1 supersymmetric configurations.

With the introduction of matter fields in the field theory a single extra spurion field is introduced associated with the quark mass. The only possibility is to promote the mass to an N=2 vector multiplet associated with $U(1)_B$. Switching on its auxiliary field vevs induce the tree level supersymmetry breaking operators

$$2Re(F_M\tilde{q}q) + D_M\left(|q|^2 - |\tilde{q}|^2\right)$$

These breakings are therefore natural candidates to play the role of the breakings induced by the angles between the D4 and D4’ branes. Again a number of consistency checks support this identification. There are three independent real parameters in both the field theory and the brane picture. The scalar masses in the field theory break $SU(2)_R$ but leave two $U(1)_R$ symmetries of the supersymmetric theory intact. The scalar masses may always be brought to diagonal form by an $SU(2)_R$ transformation that mixes $q$ and $\tilde{q}^*$. In the resulting basis there is an unbroken $U(1)$ subgroup of $SU(2)_R$. In the brane picture the D4’ branes lie at an angle in the $x^6 - x^9$ directions breaking the $SU(2)_R$ symmetry but leaving two $U(1)_R$ symmetries unbroken.

The resulting field theory potential has an interesting dependence on the quark mass. The potential is of the form

$$V \simeq D(|q|^2 - |\tilde{q}|^2) + m^2(|q|^2 + |\tilde{q}|^2) + (|q|^2 - |\tilde{q}|^2)^2$$

For $m = 0$ the theory has a moduli space where $(|q|^2 - |\tilde{q}|^2) = -D/2$. For $m^2 < D$ the theory has a unique vacuum with $\langle q \rangle = 0$ and $\langle |\tilde{q}|^2 \rangle = (D - m^2)/2$. In both cases the colour
The gauge group is broken to an $SU(N - F)$ subgroup. For $m^2 > D$ there is a unique vacuum at the origin of moduli space and the gauge groups are unbroken. This same behaviour can be seen in the brane picture. Consider for example the N=1 configuration of Fig 1 in which we move the central NS5 brane in the $x^9$ direction to put the D4 branes at an angle. There are two possible results after switching on $D$. One is that the D4 branes remain connected as before. The other is that F of the D4 branes disconnect from the central NS5 brane breaking the surface field theory to an $SU(F) \times SU(N - F)$ gauge group. Which of these is energetically preferred depends on the relative lengths of the separation between the NS5s in the $v$ direction (the field theory mass) and the separation of the NS5s in the $x^9$ direction (corresponding in the field theory to the size of $D$). From Fig 1 the higgs branch is preferred when

$$m^2 < 2d^2 - 2LL' - 2\sqrt{L^2 + d^2} \sqrt{L'^2 + d^2}$$ \hspace{1cm} (5)

Fig. 1: An N=1 configuration with two NS5 branes in the $w$ direction and one in the $v$ direction. The same configuration after the central NS5 brane has been moved in the $x^9$ direction by displacement $d$, switching on scalar mass soft breakings.

Finally we note that the interpretation of the angles between D4 branes as the expectation values of the auxiliary fields of the $U(1)_B$ gauge multiplet is equivalent to the standard lore from N=1 theories that motion of NS5 branes in the $x^9$ direction corresponds to switching on a Fayet-Iliopoulos term for that field. Above we have simply considered the $U(1)_B$ field as a spurion rather than a propagating field (this is because, when we move to M-theory...
configurations below, the classical $U(N)$ gauge theory on the brane’s surface is known to be broken to $SU(N)$ with the dynamics freezing the $U(1)$ field [3].

The $SU(N) \times SU(F)$ brane configuration had six parameters associated with the rotations of each of the end two NS5 branes which are associated with the auxiliary field vevs of the two gauge coupling spurions. The three parameters describing the angles between the D4 and D4’ branes correspond to the auxiliary field vevs of the mass spurion. To study the dynamics of QCD with matter below we will take the NS5” brane to infinity removing the six parameters associated with its rotations and decoupling the flavour gauge multiplet. The resulting theory has nine supersymmetry breaking parameters.

3 IR Dynamics from M-theory

To obtain the IR dynamics of the theory it is neccessary to study the short distance behaviour of the brane configurations. In the string theory the strongly coupled core of the NS5 brane prevents detailed study of the NS5 D4 brane junction. Instead we move to M-theory [5]-[7] by increasing the $x^{10}$ dimension’s radius, $R$, from zero. The NS5 and D4 branes are then aspects of the same M5 brane, wrapped in places around the $x^{10}$ dimension. We may now perform a semi-classical minimal area embedding of the surface to study the IR dynamics of the theory on the brane’s surface.

Following Witten [6] we generalize the supersymmetric curves by the addition of extra parameters. To find as many configurations as possible we let the configuration wallow in the full six dimensional space $\vec{X} = (x^4, ..., x^9)$ and $x^{10}$ which is picked out by its compact nature. The ansatz is

$$\vec{X} = Re(\vec{p}z + \vec{q}/z + R\vec{r}\ln z + R\vec{s}\ln(z - m))$$

$$x^{10} = -NcIm(ln z) + NfIm ln(z - m)$$

(6)

$\vec{p}$ and $\vec{q}$ are complex 6 vectors, $\vec{r}$ and $\vec{s}$ real 6 vectors (this reality is required to keep $\vec{X}$ single valued allowing a physical interpretation of the configuration as a field theory [11]).

To enforce that the description is of a minimal area embedding we require the vanishing of the two dimensional energy-momentum tensor

$$T_{zz} = g_{ij}\partial_z X^i\partial_z X^j = 0$$

(7)

where $g_{ij}$ is the background metric
\[ ds^2 = \sum_{i,j=0}^{9} \eta_{ij} dx^i dx^j + R^2 (dx^{10})^2 \]  

(8)

Enforcing this condition leads to the constraints

\[ \vec{p}^2 = \vec{q}^2 = \vec{p} \cdot \vec{r} = \vec{p} \cdot \vec{s} = \vec{q} \cdot \vec{r} = \vec{q} \cdot \vec{s} = 0 \]

\[ -2\vec{p} \cdot \vec{q} + R^2 \vec{r}^2 = R^2 N_c^2, \quad \vec{r} \cdot \vec{s} = -N_c N_f, \quad \vec{s}^2 = N_f^2 \]

(9)

There are 36 real parameters in the ansatz that are reduced by these constraints and by using the background SO(6) isometries. We choose to use 11 of the SO(6) degrees of freedom to fix parameters; this leaves an \( SO(2) \otimes SO(3) \) symmetry unfixed corresponding to leaving the \( U(1)_R \otimes SU(2)_R \) of the field theory unfixed. A further 2 parameters may be removed by scalings of \( z \) (the parameter \( m \) must also be scaled). Finally the 7 complex constraints and 2 real constraints reduce the parameter count by a further 16. We are left with 7 real parameters in \( \vec{p}, \vec{q}, \vec{r} \) and \( \vec{s} \) and 2 in \( m \). This should be compared with the field theory in which we argued that there were 2 associated with the gauge coupling and theta angle, 2 with the mass parameter and 9 with soft breakings. The mismatch indicates that we do not have the most general set of field theories (equivalently when we take the \( R \to 0 \) limit in the M-theory curves we do not recover the entire set of type IIA brane configurations). This is not a surprise since the ansatz taken above is not the most general solution to (7). The configurations above though are a subset of the non-susy field theories with both gaugino and scalar masses and we proceed to analyse those since they contain configurations with non-supersymmetric QCD with matter in the IR.

A convenient parameterization of the curve is obtained as follows. We use the constraint \( \vec{p}^2 = 0 \), the rescaling of \( z \) (\( m \) must also be rescaled) and 8 of the SO(6) isometry degrees of freedom (leaving 7 degrees of isometry freedom which correctly corresponds to having fixed a single plane in 6-space leaving an \( SO(2) \times SO(4) \) isometry group) to set

\[ \vec{p} = (1, -i, 0, 0, 0, 0). \]

(10)

The constraints \( \vec{p} \cdot \vec{s} = 0, \vec{s}^2 = N_f^2 \) and 3 of the remaining isometries may be used to set

\[ \vec{s} = (0, 0, N_f, 0, 0, 0) \]

(11)

\( \vec{p} \cdot \vec{r} = 0 \), the \( \vec{r} \cdot \vec{s} \) constraints and a further 2 isometries may be used to set

\[ \vec{r} = (0, 0, -N_c, 0, 0, a/R) \]

(12)
Note that we have used 2 of the 3 SO(3) or SU(2) degrees of freedom here. In the field theory we have therefore used SU(2) \(_R\) to set two of the N=2 spurion components to zero. We are left with the isometries corresponding to the two \(U(1)_R\) symmetries of the N=1 configurations unfixed.

The remaining constraints on \(\vec{q}\) lead to a form

\[
\vec{q} = (\eta + \epsilon, -i\eta + i\epsilon, 0, \xi + \lambda, -i\xi + i\lambda, 0)
\]  

and the constraints

\[
\eta\epsilon + \xi\lambda = 0
\]

\[
\epsilon = a^2 / 4
\]

The curve may then be written in terms of the more familiar variables \(v = (x^4 + ix^5)\) and \(w = (x^7 + ix^8)\) as

\[
v = z + \frac{a}{z} + \frac{\xi}{z}
\]

\[
w = \frac{\xi}{z} + \frac{\lambda}{z}
\]

The supersymmetric configurations with semi-infinite D4s can be recovered by setting \(a = 0\). The constraints then set \(\epsilon = 0\) and \(\lambda = 0\). The curve is described in the remaining space by \(x^9 = 0\) and setting \(t = exp - (x^6/R + ix^{10})\)

\[
t = \kappa z^N/(z - m)^F
\]

The constant \(\kappa\) is undetermined by the minimal area embedding and will be fixed to ensure the curve has the correct flow as quark masses are taken to infinity and decoupled \[8\]. The curve has two \(U(1)\) symmetries associated with rotation in the \(v\) and \(w\) planes which are broken by the parameters of the curve. The symmetries may be restored by assigning the parameters spurious charges

\[
\begin{array}{cccccccccc}
& v & w & t & z & m & \eta & \epsilon & \xi & \lambda & \kappa \\
U(1)_v & 2 & 0 & 0 & 2 & 2 & 4 & 0 & 2 & 2 & 2(F - N) \\
U(1)_w & 0 & 2 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\
\end{array}
\]  

| v | w | t | z | m | \eta | \epsilon | \xi | \lambda | \kappa |
|---|---|---|---|---|-----|--------|---|-------|-------|
| 2 | 0 | 0 | 2 | 2 | 4 | 0 | 2 | 2 | 2(F - N) |
| 0 | 2 | 0 | 0 | 0 | 0 | 2 | -2 | 0 |       |
4 Supersymmetric Configurations

We first review the supersymmetric theories described by the curve when \( a = \epsilon = \lambda = 0 \)

\[
v = z + \frac{\eta}{z}, \quad w = \frac{\xi}{z}, \quad t = \kappa z^N / (z - m)^F
\]  

(18)

From the string theory limit we know that the curve describes \( N=2 \) SQCD broken to \( N=1 \) with an adjoint mass. If \( \xi \rightarrow 0 \) the curve describes \( N=2 \) SQCD at one point on moduli space and if \( \eta \rightarrow 0 \) the curve describes \( N=1 \) SQCD with the adjoint matter decoupled. Furthermore the parameter \( m \) plays the role of the field theory matter field mass term, \( m_Q \). It is natural therefore to associate the \( U(1)_v \) and \( U(1)_w \) symmetries with the field theory \( U(1)_R \) symmetries

\[
\begin{array}{c|cccccccc}
W & A & Q & \tilde{Q} & m_Q & m_A & \Lambda_{N=2} \\
\hline
U(1)_R & 1 & 2 & 0 & 0 & 2 & -2 & 2 \\
U(1)_{R'} & 1 & 0 & 1 & 0 & 2 & 0 \\
\end{array}
\]  

(19)

where \( m_Q \) is a common matter field mass, \( m_A \) the adjoint field mass, and \( \Lambda_{N=2} \approx \exp(2\pi i \tau / (2N - F)) \) the strong coupling scale of the \( N=2 \) theory. We can then make identifications between the field theory parameters and brane configuration parameters. In particular we may identify \( \xi \) with \( m_A \Lambda_{2b_0/N} \), \( \eta \) with \( m_Q^{F/N} \Lambda_{2b_0/N} \) and the curve parameter \( m \) with the field theory quark mass. The constant \( \kappa \) may now be identified with \( m^{F-N} \) in order that as a single quark mass is taken to infinity the curve correctly flows to the curve with \( F \rightarrow F - 1 \).

While the curve's parameters in general break the two \( U(1) \) symmetries, asymptotically discrete subgroups are preserved. Asymptotically as \( \Lambda_{N=2} \rightarrow 0 \) the curve is given by

\[
\begin{align*}
z \rightarrow \infty & \quad w = 0 & \quad t = v^{N-F} m_Q^{F-N} \\
z \rightarrow 0 & \quad v = 0 & \quad t = \left( \frac{m_A}{w} \right)^N \Lambda_{N=2} m_Q^{F-N}
\end{align*}
\]  

(20)

The asymptotic curve leaves \( U(1)_w \) unbroken (allowing \( m_A \) to transform spuriously) whilst only a \( Z_{2N-F} \) subgroup of the \( U(1)_v \) remains (allowing \( m_A \) and \( m_Q \) to transform). This is precisely the effect of the anomaly on the two \( U(1)_R \) symmetries of the field theory.

4.1 \( N=1 \) SQCD

The \( N=1 \) curve with the adjoint matter field completely decoupled may be obtained by taking \( \eta \rightarrow 0 \) at fixed \( \xi \) and is described by

\[
v = z, \quad w = \frac{\xi}{z}, \quad t = z^N m^{F-N} / (z - m)^F
\]  

(21)
The curve again has two $U(1)$ symmetries which correspond to the field theory symmetries

\[
\begin{array}{cccccc}
W & Q & \bar{Q} & m_Q & \Lambda_{b_0} \\
U(1)_R & 1 & 0 & 0 & 2 & 2(N - F) \\
U(1)'_R & 1 & 1 & 1 & 0 & 2N \\
\end{array}
\] (22)

where $\Lambda = \exp(2\pi i \tau/b_0)$ and $b_0 = 3N - F$. We may make the identifications $\xi = \Lambda_{b_0/N} m_Q^{F/N}$ and $m = m_Q$. The UV field theory displays $Z_N$ and $Z_{N-F}$ discrete subgroups of these symmetries. Viewing the curve asymptotically and as $\Lambda \to 0$

\[
z \to \infty \quad w = 0 \quad t = v^{N-F} m_Q^{F-N}
\]

\[
z \to 0 \quad v = 0 \quad t = \left(\frac{1}{w}\right)^N \Lambda_{b_0} m_Q^{F-N} 
\] (23)

The $U(1)_v$ and $U(1)_w$ symmetries (allowing $m_Q$ to transform spuriously but not $\Lambda$) are indeed asymptotically broken to $Z_N$ and $Z_{N-F}$ discrete subgroups. Other combinations of the two $U(1)_R$ symmetries may also be identified in the asymptotic curve. For example $U(1)_A$ symmetry is given by the rotations

\[
\begin{array}{cccccc}
v & w & t & z & \Lambda_{b_0} & m_Q \\
U(1)_A & -2 & 2 & 0 & -2 & 2F & -2 \\
\end{array}
\] (24)

The N=1 theory behaves like supersymmetric Yang Mills theory below the matter field mass scale and has $N$ degenerate vacua associated with the spontaneous breaking of the low energy $Z_N$ symmetry. In the curve this corresponds to the $N$ curves in which $\xi_n = \xi_0 \exp(2\pi in/N)$ (equivalently $\Lambda_{b_0} = \Lambda_{b_0} \exp(2\pi in)$). In the UV these curves can be made equivalent by a $Z_N$ transformation.

5 Non-Supersymmetric Solutions

Let us now consider switching on susy breaking in the configurations through $a \neq 0$ or equivalently switching on $\epsilon$.

5.1 Softly Broken N=1 SQCD

We begin by switching on $\epsilon$ in the N=1 configuration. The curve is now

\[
v = z + \frac{\bar{\epsilon}}{\bar{z}} \quad w = \frac{\Lambda_{b_0/N} m_Q^{F/N}}{z}, \quad t = z^N m_Q^{F-N} / (z - m_Q)^F, \quad x^9 = 4e^{1/2} R e \ln z \quad (25)
\]
The D4 branes lie in the $x^6$ and $x^9$ directions. This distortion does not break the $U(1)_v$ or $U(1)_w$ symmetries and so from the discussion above we deduce that in the $R \to 0$ limit we have introduced the supersymmetry breaking terms

$$D \left( |q|^2 - |\bar{q}|^2 \right)$$

(26)

The $w$ plane has also been rotated from its supersymmetric position as can be seen from the non-holomorphic nature of the first equation. The gaugino in these configurations will also therefore be massive (and break both $U(1)_R$ symmetries). We may identify the parameter $\epsilon$ with field theory parameters from its symmetry charges. Since it is chargeless under both $U(1)$s we may only identify it as a function of

$$m_\lambda^N \Lambda^{b_0} m_Q^F$$

(27)

To complete the identification we note that the field theory retains a $Z_F$ subgroup of $U(1)_A$ symmetry even after the inclusion of the soft breaking terms. Requiring this property of the curve asymptotically forces

$$\epsilon = \left( m_\lambda^N \Lambda^{b_0} m_Q^F \right)^{1/N}$$

(28)

Asymptotically the curve is then

$$z \to \infty \quad w = 0 \quad t = v^{N-F} m_Q^{F-N}$$

$$z \to 0 \quad v = \bar{m}_\lambda \bar{w} \quad t = \left( \frac{1}{v} \right)^N \bar{m}_\lambda^N \Lambda^{b_0} m_Q^{F-N}$$

(29)

which possesses a $Z_F$ subgroup of the $U(1)_A$

| $U(1)_A$ | $v$ | $w$ | $t$ | $z$ | $\Lambda^{b_0}$ | $m_Q$ | $m_\lambda$ | $D$ |
|-----------|-----|-----|-----|-----|-------------|------|--------|-----|
| $-2$      | $2$ | $0$ | $-2$| $2F$| $-2$        | $0$  | $0$    |     |

The curves with different $\Lambda^{b_0}_n$ are no longer equivalent on scales at which $\epsilon$ can not be neglected since the $Z_N$ symmetry is broken. This is the behaviour of the field theory described in [12] where one of the $N$ vacua of the N=1 model becomes the true vacuum of the N=0 model, the others become metastable vacua. The softly broken field theory solutions exhibit first order phase transitions at $\theta_{phys} = (\text{odd})\pi$ where two of the $N_c$ vacua of the SQCD theory become degenerate minima of the model. CP symmetry is spontaneously broken by the two minima. This behaviour has been identified in [13] for non-supersymmetric M-theory curves describing softly broken SQCD with $F = 0$. To do so the authors of [14] allowed the parameter equivalent to $a$ to take complex values which allows $x^9$ to take multiple values. It is not clear that the resulting brane configuration still corresponds to the field
theory. We note here that similar behaviour may be observed for these configurations too. For simplicity set the phases of \( m_Q \) and \( m_\lambda \) to zero (this may in general be ensured by making \( U(1)_R \) rotations; the theta angle is then the physical theta angle \( \theta_{phys} = \theta_0 + N\theta_\lambda + \text{arg}(\text{det} m_Q) \)). Shifting \( \theta \) from 0 to \( \pi \) has the effect of shifting \( \Lambda_{b_0/N} \rightarrow \Lambda_{b_0/N} \exp(i\pi/N) \) and \( \Lambda_{N-1} \rightarrow \Lambda_{b_0/N} \exp(-i\pi/N) \). These two curves are then identical up to complex conjugation (CP symmetry in the field theory). We deduce that as in the field theory the two vacua are degenerate and break CP symmetry.

From the softly broken field theory \[13\] we expect that after supersymmetry breaking a quark condensate will form. The quantity \( \Sigma = m_\lambda^{N/F} \Lambda_{b_0/F} \) has the correct \( U(1)_R \) charges to play this role. It’s dimension may be corrected by a function of the symmetry invariants \( |m|, |\Lambda| \) and \( |m_\lambda| \). It is \( \Sigma \neq 0 \) that breaks the \( Z_F \) symmetry of the curve in the IR. Note that a shift of the bare \( \theta_0 \) angle shifts \( \Sigma \rightarrow \Sigma \exp(i\theta_0/F) \).

### 5.2 Decoupling All Super-Partners

If we let \( \xi \rightarrow 0 \) for fixed \( \epsilon \) then we eliminate all R-chargeful parameters except \( m_Q \). The gaugino condensate has therefore switched off and there is no parameter which may play the role of the gaugino mass. We conclude that the gaugino has been decoupled from the field theory. The curve is now

\[
v = z + \frac{(m_Q \Sigma)^{F/N}}{z}, \quad t = z^N m_Q^{F-N}/(z - m_Q)^F, \quad x^9 = 4(m_Q \Sigma)^{F/2N} \text{Re } \ln z \quad (31)
\]

The configuration in the \( R \rightarrow 0 \) limit describes two NS5 planes in the \( v \) direction connected by \( N \) D4 branes with \( F \) semi-infinite D4 branes in the \( x^6, x^9 \) directions. We conclude from the absence of an adjoint matter field and gaugino that the configuration is in fact one where one NS5 has been reversed by a rotation relative to the usual supersymmetric set up and is an anti-NS5 brane.

We expect in the field theory that all superpartners will have been decoupled from the theory since without a massless gaugino there is nothing to stabilize the scalar masses and they will radiatively acquire masses of order the UV cut off. The curve provides support for this interpretation. In the \( m_Q \rightarrow 0 \) limit \( t \sim z^{N-F} \) and the gauge group has been broken to an \( SU(N - F) \) subgroup. In the field theory with non-zero \( D \) but \( m_Q = 0 \) there is a moduli space. The gauge group may be broken leaving a mass gap below the UV scale of the field theory. All particles decouple at the cut off except the \( SU(N - F) \) subgroup and there is therefore no renormalization of the scalar mass. For small non-zero \( m_Q^2 < D \), as discussed
above, the classical theory has a vacuum that higgs the gauge group at scales below $D$. The scalar fields exist in the field theory below the UV cut off and therefore their masses will be renormalized typically growing to of order the UV scale and hence greater than $D$. The quantum theory would therefore not be expected to have a higgsing vacuum for any $m_Q$. This is the behaviour of the curve for non-zero $m_Q$, leading some support to the hypothesis that scalar masses are being generated radiatively.

With the decoupling of the gaugino the $U(1)_v$ symmetry becomes $U(1)_A$ as may be seen in (22). Asymptotically the curve posseses a $Z_F$ subgroup of the axial symmetry of the quarks although explicitly broken by the quark masses. In the interior of the curve the symmetry is additionally broken by the quantity $\Sigma$. We conclude that a quark condensate has formed breaking the chiral symmetry.

There are $F$ distinct curves where $\Sigma_n = \Sigma_0 \exp(i2\pi n/F)$ which are identical asymptotically upto a spurious $Z_F$ transformation (since the mass explicitly breaks the $Z_F$ symmetry the curves are inequivalent). We can also observe a non-trivial spectral flow with changing theta if we again allow a complex values. Setting the phase of $m_Q$ zero (this can be arranged by a $U(1)_A$ transformation leaving the physical theta angle $\theta_{phys} = \theta_0 + \arg(detm_Q)$) and shifting $\theta \to \pi$ shifts $\Sigma_0 \to \Sigma_0 \exp(i\pi/F)$ and $\Sigma_{(F-1)} \to \Sigma_0 \exp(-i\pi/F)$. The two curves are again interchanged by complex conjugation. We deduce that for these values of $\theta$ there are two degenerate vacua which spontaneously break CP. This is precisely the behaviour observed in the QCD chiral lagrangian with changing $\theta$ angle [16].

We conclude that the M-theory strong coupling extension of the type IIA string configuration describing QCD in the IR behaves qualitatively as we would expect QCD to behave.

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