Langevin equation in effective theory of interacting QCD pomerons in the limit of large $N_c$

S. Bondarenko *

*Email: sergey@fpaxp1.usc.es

University of Santiago de Compostela, Santiago de Compostela, Spain

October 23, 2018

Abstract

Effective field theory of interacting BFKL pomeron is investigated and Langevin equation for the theory, which arises after the introduction of additional auxiliary field, is obtained. The Langevin equations are considered for the case of interacting BFKL pomeron with both splitting and merging vertices and for the interaction which includes additional "toy" four pomeron interaction vertex. In the latest case an analogy with the Regge field theory in zero dimensions (RFT-0) was used in order to obtain this "toy" vertex, which coincided with the four point function of two-dimensional conformal field theory obtained in [31]. The comparison between the Langevin equations obtained in the frameworks of dipole and RFT approaches is performed, the interpretation of results is given and possible application of obtained equations is discussed.

1 Introduction

The scattering process of hadrons and nuclei in QCD with a large number of colors $N_c$ have been vigorously investigated in recent years along two main lines. The Color Glass Condensate (CGC) approach [5, 6], is formulated in the transverse position space and in large $N_c$ limit the evolution dynamics of the model is analyzed in the terms of color dipoles [7]. Another approach, the QCD Reggeon Field Theory (QCD-RFT), was written and investigated in the momentum space for the BFKL pomeron, [2, 3, 4], considering the picture of interaction of the pomeron in t-channel. The history of developing of such a picture begins from the pioneric paper [1] and it bases on the standard diagrammatic calculus developed for the interacting BFKL pomeron, having , as well, transverse position space formulation in large $N_c$ limit of the theory, see [8, 9, 10, 11, 12, 13]. Both approaches are expected to be valid also in the case of symmetrical treatment of target and projectile and may include not only a vertex of pomeron splitting but also a vertex of merging of two pomeron in one. It gives the possibility for attempts to formulate and calculate the pomeron loops contribution to the scattering amplitude and makes these approaches
formally similar. But, in spite of this formal similarity, these approaches are written and formulated in the different frameworks and, therefore, the identity of the approaches is not fully clear.

The QCD-RFT formulation of the high energy scattering uses a Lagrangian language describing the processes of scattering at high energy, \cite{10,11,12,13}, treating the processes of the high energy scattering in the terms of nonlocal effective field theory, \cite{14,15}, and keeps a lot of mutual features with this "older brother", see \cite{16}. The dipole CGC and so-called JIMLWK approaches are based on the consideration of the color dipoles as a main degrees of freedom in the high energy scattering in limit of large $N_c$, \cite{6,7}. It is proven, that on the level of the zero transverse dimensions both approaches describes the same physics, \cite{17}, in spite of the fact that these two approaches use very different pictures for the description of the scattering process. In QCD-RFT the calculations are based on the picture of t-channel propagating and interacting pomerons, whereas the CGC approach uses the picture of evolution of the dipoles of target and projectile in s-channel with interactions of dipole showers after the evolution. Therefore, considering the problem of pomeron’s loops contribution into the scattering amplitude, both theories support very different strategies for the accounting of the loops. In QCD-RFT, because of the Lagrangian formulation of the theory, a program of such calculation may be formulated as usual perturbative calculations and these calculations in general must be very difficult, as it is always happens for the loops calculations in a field theory. In CGC approach the problem of the calculations of the pomeron’s loops usually formulated as a problem of evolution equation with both, merging and splitting vertexes included and may be explicitly formulated in the terms of some effective Hamiltonian \cite{18}. A Hamiltonian formulation of the theory has been considered in QCD-RFT as well, \cite{13}, but the effective Hamiltonian formulation of the high energy scattering processes is out of the scope of the paper.

In this paper we will investigate another possible way to account the pomeron loops in the different high energy scattering approaches, namely a approach when loops contribution are compactly coded in the Langevin equation. On the level of CGC approach the different types of arising Langevin equations were considered in \cite{19,20,21}. The correct impact parameter treatment of the pomeron loops was obtained in \cite{20,21}, whereas the Langevin equation of \cite{19} may be considered only as a some effective model of a correct Langevin equation. Indeed, in \cite{19} the semiclassical treatment of large impact parameter behavior of the amplitude was used, when the sizes of the interacting dipoles are neglected in comparison to the impact parameter of the problem, that excludes a correct treatment of the pomeron’s loops. Therefore, talking about the comparison of the Langevin equations obtained in the framework of QCD-RFT approach with the Langevin equations of the dipole approach, we will have in mind mostly the comparison of the results of this paper with the results of \cite{20}. Nevertheless, we also will give the derivation of results of \cite{19} in the framework of QCD-RFT. It must be noticed here, that the equivalence of QCD-RFT and CGC approaches on the semiclassical level was considered as well in the framework of a generating functional approach and BFKL pomeron calculus of \cite{22,23}, where the problem of mutual description of QCD-RFT and CGC was formulated and resolved on the semiclassical level. But, as it was mentioned above, in our paper, in spite of the papers \cite{19,23}, the Langevin equation will be obtained with proper consideration of an whole impact parameter structure of the theory, where as a starting point the QCD-RFT approach will be used.

The paper is organized as follows. In the next section we will introduce and consider the main construction blocks of the QCD-RFT approach and clarify they relations with such physically relevant quantity as unintegrated gluon density. In the Section 3 we will obtain the Langevin equation for the theory with only triple pomeron vertexes of splitting and merging. In the Section 4 we will introduce a "toy" four pomeron interaction vertex and will consider the possibility to write the Langevin equation for the theory with such additional vertex. The Section 5 will contain the comparison between the Langevin equations obtained in s-channel dipole model and t-channel QCD RFT model. The Section 6 is a discussion of obtained results and conclusion of the paper.
2 Effective field theory of interacting pomerons

In this section we will formulate the main results concerning the pomeron effective field theory considered in [10] [11] [12] [13]. A first ingredient, which we will need in the further calculations, is a action of the pomeron effective theory:

\[ S = S_0 + S_I, \]  

(1)

where \( S_0 \) and \( S_I \) are the free and interacting parts of the action correspondingly:

\[ S_0 = \int dy dy' d^2 r_1 d^2 r_2 d^2 r'_1 d^2 r'_2 \Phi_1(y, r_1, r_2) G_{y-y'}^{-1}(r_1, r_2 | r'_1, r'_2) \Phi(y, r'_1, r'_2), \]  

(2)

and

\[ S_I = \frac{2 \alpha_s^2 N_c}{\pi} \int dy \int \frac{d^2 r_1 d^2 r_2 d^2 r_3}{r_{12}^2 r_{23}^2} \left( L_{13} \Phi(y, r_1, r_3) \right) \Phi(y, r_1, r_2) \Phi(y, r_2, r_3) + \]  

\[ + \frac{2 \alpha_s^2 N_c}{\pi} \int dy \int \frac{d^2 r_1 d^2 r_2 d^2 r_3}{r_{12}^2 r_{23}^2} \left( L_{13} \Phi(y, r_1, r_3) \right) \Phi(y, r_1, r_2) \Phi(y, r_2, r_3). \]  

(3)

In comparison with the action of [12] we omitted here the part of the action responsible for the interacting of pomerons with the target and projectile, the source terms, because for our following calculations these terms are not important. In the following derivations we also will not especially underline the fact that such quantities as \( r_i \) and/or \( k_i \) always denote the two dimensional vectors, it will be denoted as vectors only in the cases where precise definition of the vector structure of \( r_i \) and/or \( k_i \) will be needed. The part of the action, given by expression Eq. (3), reproduces the triple pomeron vertex in the large \( N_c \) limit with the use of an operator \( L_{13} \):

\[ L_{13} = r_{13}^4 p_{13}^2 = r_{13}^4 \nabla_1^2 \nabla_3^2. \]  

(4)

The propagator of the theory, \( G_{y-y'}^{-1}(r_1, r_2 | r'_1, r'_2) \), is defined throw the BFKL Hamiltonian [24] [25]:

\[ G_{y-y'}^{-1}(r_1, r_2 | r'_1, r'_2) = \left( \nabla_2^2 \nabla_1^2 \left( \frac{\partial}{\partial y} + H(r_1, r_2) \right) \right) \delta^2(r_1 - r'_1) \delta^2(r_2 - r'_2) \delta(y - y'). \]  

(5)

Now, let us consider only the triple pomeron interaction terms of the action and let us make a well known change of the variables in the action

\[ \phi(r_1, r_2) \rightarrow \frac{\Phi(r_1, r_2)}{r_{12}^2}, \phi^\dagger(r_1, r_2) \rightarrow \frac{\Phi^\dagger(r_1, r_2)}{r_{12}^2}, \]  

(6)

obtaining

\[ S_I = \frac{2 \alpha_s^2 N_c}{\pi} \int dy \int d^2 r_1 d^2 r_2 d^2 r_3 \left( \frac{L_{13}}{r_{31}^2} (r_{31}^2 \phi(y, r_1, r_3)) \right) \phi^\dagger(y, r_1, r_2) \phi^\dagger(y, r_2, r_3) + \]  

\[ + \frac{2 \alpha_s^2 N_c}{\pi} \int dy \int d^2 r_1 d^2 r_2 d^2 r_3 \left( \frac{L_{13}}{r_{31}^2} (r_{31}^2 \phi^\dagger(y, r_1, r_3)) \right) \phi(y, r_1, r_2) \phi(y, r_2, r_3). \]  

(7)

In order to relate the amplitude with the unintegrated gluon density, as a next step, we perform Fourier transform:

\[ \phi(y, r_1, r_2) = \int \frac{d^2 k_1 d^2 q_1}{(2 \pi)^2} e^{-i r_1 k_1 - i r_2 (q_1 - k_1)} \tilde{\phi}(y, k_1, q_1 - k_1) \]  

(8)

for the functions \( \phi(y, r_1, r_2) \) in Eq. (7). In the terms of the functions \( \phi(y, k_i, q_i - k_i) \) the triple pomeron term in the action will have the form:

\[ S_I = \frac{2 \alpha_s^2 N_c}{\pi} \int dy \int d^2 k_3 d^2 q_3 d^2 k_2 (\tilde{L}_3 \tilde{\phi}(y, k_3, q_3 - k_3)) \tilde{\phi}(y, q_3 - k_3, -k_2) \tilde{\phi}(y, k_2, k_3) + \]  

(9)
\[ + \frac{2\alpha_s^2 N_c}{\pi} \int dy \int d^2k_3 d^2q_3 d^2k_2 \left( \hat{L}_3 \tilde{\phi}(y, k_3, q_3 - k_3) \right) \tilde{\phi}(y, q_3 - k_3, -k_2) \tilde{\phi}(y, k_2, k_3) \]

where,

\[ \hat{L}_3 = \nabla^2_{k_3} k_3^2 (q_3 - k_3)^2 \nabla^2_{k_3}, \quad (10) \]

see Appendix A for the detailed derivation, and as usual, \( q \) and \( k \) are two dimensional vectors. Now we consider the case of interaction with zero momentum transfer, when:

\[ \int d^2 q_3 \tilde{\phi}(y, k_3, q_3 - k_3) = \int d^2 q_3 \varphi(y, k_3) \delta^2(q_3) = \varphi(y, k_3). \quad (11) \]

In this case in Eq. (11) we have two delta functions arisen from the substitutions:

\[ \tilde{\phi}(y, q_3 - k_3, -k_2) \rightarrow \varphi(y, q_3 - k_3) \delta^2(q_3 - k_3 - k_2) \quad (12) \]

\[ \tilde{\phi}(y, k_2, k_3) \rightarrow \varphi(y, k_3) \delta^2(k_3 + k_2) \quad (13) \]

that gives after the integration over \( k_2 \) and \( q_3 \) the triple pomeron vertex which is local in momentum space:

\[ S_I = \frac{2\alpha_s^2 N_c}{\pi} \int dy \int d^2k \left( \nabla^2_k k^4 \nabla^2_k \varphi(y, k) \right) \varphi^1(y, k) \varphi(y, k)^\dagger + \]

\[ + \frac{2\alpha_s^2 N_c}{\pi} \int dy \int d^2k \left( \nabla^2_k k^4 \nabla^2_k \varphi^1(y, k) \right) \varphi(y, k) \varphi(y, k). \quad (14) \]

It is clear, that the function \( \varphi(y, k) \) is a scattering amplitude for the case when we omit a momentum transfer, taking it equals zero, i.e. for the case of forward scattering. In the impact parameter representation this approximation corresponds to the semiclassical approximation of the large impact parameter limit, where the sizes of the interacting dipoles are neglected in comparison to the whole impact parameter of the problem. This \( \varphi(y, k) \) function we may connect with the unintegrated gluon (parton) density function \( f(k) \) with the help of the following expression [11, 26]:

\[ f(y, k) = \frac{N_c}{2\pi^2} k^4 \nabla^2_k \varphi(y, k), \quad (15) \]

and, therefore, the physical meaning of the \( \varphi(y, k) \) became to be clear: through Eq. (15) \( \varphi(y, k) \) defines the unintegrated gluon density function for the processes of forward scattering. Basing on the Eq. (9)-Eq. (10) and Eq. (14)-Eq. (15) it is easy to generalize the expression Eq. (15) for the case of non-zero momentum transfer:

\[ \hat{f}(y, k, q - k) = \frac{N_c}{2\pi^2} k^2 (q - k)^2 \nabla^2_k \tilde{\phi}(y, k, q - k), \quad (16) \]

here \( \tilde{\phi}(y, k, q - k) \) is the amplitude defined by Eq. (5) and \( \hat{f}(y, k, q - k) \) is a generalized (skewed) gluon (parton) distribution function. From Eq. (14) and Eq. (16) it is easy to see, that our initial amplitude \( \Phi(r_1, r_2) \) is simply Fourier transform of the \( \hat{f}(y, k, q - k) \) and vice versa:

\[ \hat{\tilde{f}}(y, k, q - k) = \frac{N_c}{2\pi^2} \int \frac{d^2r_1 d^2r_2}{(2\pi)^2} e^{i r_1 k + i r_2 (q - k)} \Phi(r_1, r_2) \quad \quad (17) \]

that determines very clear and transparency meaning for the amplitude \( \Phi(r_1, r_2) \). Considering the equations Eq. (15) and Eq. (16), which are the Poisson’s type equations, with the help of Green’s function for the two dimensional Poisson’s equation, we can write the inverse relation between \( \tilde{f}(y, k, q - k) \) and \( \tilde{\phi}(y, k, q - k) \):

\[ \tilde{\phi}(y, k, q - k, \theta_1) = \frac{2\pi^2}{N_c} \int d^2k' g(k, k', q) \hat{\tilde{f}}(y, k', q - k', \theta_2) \frac{1}{k'^2 (q - k')^2} = \quad (18) \]
and we rewrite this expression in the following form:

\[ \text{Eq. (18), it must be also possible to obtain the Bartels triple pomeron vertex written in the terms of the} \]

vertex in terms of unintegrated gluon density, see [26, 16] and Appendix B. In general, using Eq. (7) and
see also [26]. With the help of Eq. (14) and Eq. (19) we could obtain the expression for the triple pomeron
vertex in terms of unintegrated gluon density function \( \tilde{\rho}(y, k, q - k) \), see [8], but we do not consider this task in this paper.

So, in the further consideration, using the amplitudes \( \Phi(y, r, r_j) \) or \( \phi(y, r, r_j) \) defined in the transverse
position space we will always remember, that these quantities are related to the generalized (skewed)
 gluon density function \( \tilde{f}(y, k, q - k) \) in the momentum space.

### 3 Langevin equation in the theory with the triple pomeron vertex

#### 3.1 Langevin equation for the \( \Phi(y, r, r_j) \) field in the transverse position space

In order to introduce an auxiliary field in the theory we come back to the particular part of the action from the Eq. (3):

\[
S_{\Phi} = \frac{2\alpha_s^2 N_c}{\pi} \int dy \int \frac{d^2 r_1}{r_{12}^2} \frac{d^2 r_2}{r_{23}^2} \frac{d^2 r_3}{r_{31}^2} (L_{13} \Phi(y, r_1, r_3)) \Phi(y, r_1, r_2) \Phi(y, r_2, r_3) \tag{20}
\]

and we rewrite this expression in the following form:

\[
S_{\Phi} = \frac{1}{2} \int dy \frac{d^2 \rho_1}{\rho_{12}^2} \frac{d^2 \rho_2}{\rho_{12}^2} \Phi(y, \rho_1, \rho_2) \int dy' \frac{d^2 \rho_1'}{\rho_{12}^2} \frac{d^2 \rho_2'}{\rho_{12}^2} \Phi(y', \rho_1', \rho_2') \tag{21}
\]
Now we introduce on the scene an auxiliary field through the Gaussian integration over the auxiliary field \( \psi \):

\[
e^{S_{\Phi}} = N \int D[\psi]\exp \left\{ -\frac{1}{2} \int dy \frac{d^2 \rho_1 d^2 \rho_2}{\rho_{12}^4} \psi(y, \rho_1, \rho_2) \left( \frac{4 \alpha_s^2 N_c}{\pi} (L_{11'} \Phi(y, \rho_1, \rho_1')) \frac{\rho_{12}^4 \rho_{1'2'}^4}{\rho_{12}^4 \rho_{1'2'}^4} \right)^{-1} \delta(y - y') \delta^2(\rho_2 - \rho_2') \right\},
\]

where a distribution functional for \( \psi \) has the following form:

\[
W[\psi] = N \exp \left\{ -\frac{1}{2} \int dy \frac{d^2 \rho_1 d^2 \rho_2}{\rho_{12}^4} \psi(y, \rho_1, \rho_2) \left( \frac{4 \alpha_s^2 N_c}{\pi} (L_{11'} \Phi(y, \rho_1, \rho_1')) \frac{\rho_{12}^4 \rho_{1'2'}^4}{\rho_{12}^4 \rho_{1'2'}^4} \right)^{-1} \delta(y - y') \delta^2(\rho_2 - \rho_2') \right\},
\]

with

\[
N = \left( \int D[\psi] W[\psi] \right)^{-1}.
\]

Let us define a new part of the action of the theory:

\[
S_{\text{Aux}} = -\frac{1}{2} \int dy \frac{d^2 \rho_1 d^2 \rho_2}{\rho_{12}^4} \psi(y, \rho_1, \rho_2) \left( \frac{\alpha_s^2 N_c}{2\pi} (L_{11'} \Phi(y, \rho_1, \rho_1')) \frac{\rho_{12}^4 \rho_{1'2'}^4}{\rho_{12}^4 \rho_{1'2'}^4} \right)^{-1} \delta(y - y') \delta^2(\rho_2 - \rho_2') \int dy' \frac{d^2 \rho_1' d^2 \rho_2'}{\rho_{1'2'}^4} \psi(y', \rho_1', \rho_2'),
\]

and rewrite whole action in the following form:

\[
S = S_0 + S_{\text{Aux}} + S_{\Phi^4} - \int dy \frac{d^2 \rho_1 d^2 \rho_2}{\rho_{12}^4} \psi(y, \rho_1, \rho_2) \Phi^\dagger(y, \rho_1, \rho_2),
\]

where as a \( S_{\Phi^4} \) we denoted a second part of the action \( S_f \) from Eq. (23). Writing the equation of motion for the \( \Phi(y, \rho_1, \rho_3) \) field

\[
\delta S \left( \frac{\partial}{\partial y} + H(\rho_1, \rho_3) \right) \Phi(y, \rho_1, \rho_3) = 0
\]

we see, that we obtain the equation for the field \( \Phi \) which does not depend on the field \( \Phi^\dagger \) but rather on the field \( \psi \):

\[
G^{-1} \Phi(y, \rho_1, \rho_3) + \frac{2 \alpha_s^2 N_c}{\pi} \int \frac{d^2 \rho_2}{\rho_{12}^2 \rho_{23}^2} \Phi(y, \rho_1, \rho_2) \Phi(y, \rho_2, \rho_3) (L_{13}) - \frac{\psi(y, \rho_1, \rho_3)}{\rho_{13}} = 0,
\]

or

\[
\frac{2 \alpha_s^2 N_c}{\pi} \int \frac{d^2 \rho_2 \rho_{31}^2}{\rho_{12}^2 \rho_{23}^2} \Phi(y, \rho_1, \rho_2) \Phi(y, \rho_2, \rho_3) - (L_{13})^{-1} \psi(y, \rho_1, \rho_3) = 0.
\]
The field $\psi$ in Eq. (29) may be considered as a noise field with the following properties:

$$< \psi(y, \rho_1, \rho_2) > = 0 ;$$  \hspace{1cm} (30)

$$< \psi(y, \rho_1, \rho_2), \psi(y', \rho_2', \rho_1') > = \frac{4\alpha_s^2 N_c}{\pi} \rho_1^2 \rho_2^2 \rho_1' \rho_2' (L_{11'} \Phi(y, \rho_1, \rho_1')) \delta(y-y') \delta^2(\rho_2-\rho_2')$$  \hspace{1cm} (31)

that follows from the form of the distribution functional Eq. (23) for the $\psi$. The last term in Eq. (29) may be rewritten with the help of the following properties of initial Green’s function, i.e. Green’s function at zero rapidity. Indeed, let us consider this Green’s function $G_0(\rho_1, \rho_3|\rho_1', \rho_3')$, which has a form, see [25],

$$G_0(\rho_1, \rho_3|\rho_1', \rho_3') = \pi^2 \ln \frac{\rho_1^2 \rho_3^2}{\rho_1'^2 \rho_3'^2} \ln \frac{\rho_1'^2 \rho_3'^2}{\rho_1'' \rho_3''}$$  \hspace{1cm} (32)

and satisfies

$$(\rho_{13}^{-1} L_{13} (G_0(\rho_1, \rho_3|\rho_1', \rho_3'))) = (2\pi)^4 \delta^2(\rho_1-\rho_1') \delta^2(\rho_3-\rho_3').$$  \hspace{1cm} (33)

Using Eq. (33) we obtain an operator identity:

$$(\nabla_{\rho_1}^2)^{-1} (\nabla_{\rho_3}^2)^{-1} = \int \frac{d^2 \rho_1' d^2 \rho_3'}{(2\pi)^4} G_0(\rho_1, \rho_3|\rho_1', \rho_3').$$  \hspace{1cm} (34)

With the help of Eq. (34) we rewrite Eq. (29) in the following form:

$$\left( \frac{\partial}{\partial y} + H(\rho_1, \rho_3) \right) \Phi(y, \rho_1, \rho_3) + \frac{2 \alpha_s^2 N_c}{\pi} \int \frac{d^2 \rho_2 d^2 \rho_2'}{\rho_{12}^2 \rho_{23}^2} \Phi(y, \rho_1, \rho_2) \Phi(y, \rho_2, \rho_3) - \int \frac{d^2 \rho_1' d^2 \rho_3'}{(2\pi)^4} G_0(\rho_1, \rho_3|\rho_1', \rho_3') \frac{\psi(y, \rho_1, \rho_3)}{\rho_1^2} = 0.$$  \hspace{1cm} (35)

with the correlator for the function $\psi(y, \rho_1, \rho_3)$ given by Eq. (31). Both Eq. (29) and Eq. (35) with the auto correlator for the noise field Eq. (29) are the required Langevin equations of the QCD-RFT approach, valid for any value of the impact parameter of the problem. These equations are pretty complicated, therefore, let’s try to simplify them using the conformal basis representation for the pomeron fields and assuming special properties of the theory at high energy limit.

The following property of the field $\Phi(y, \rho_1, \rho_2)$ at high energy limit of the theory could help us to simplify the Langevin equation. Let us expand the $\Phi(y, \rho_1, \rho_2)$ field on the conformal basis formed by the functions $E_{\mu(n,\nu),\rho_0}(\rho_1, \rho_2)$,  \hspace{1cm} (27):

$$E_{\mu(n,\nu),\rho_0}(\rho_1, \rho_2) = \left( \frac{\rho_1}{\rho_{10} \rho_{20}} \right)^{\frac{1}{4}} \left( \frac{\rho_{12}^*}{\rho_{10} \rho_{20}} \right)^{\frac{1}{4}}$$  \hspace{1cm} (36)

and

$$L_{13} E_{\mu(n,\nu),\rho_0}(\rho_1, \rho_2) = \lambda_{\mu(n,\nu)}^{-1} E_{\mu(n,\nu),\rho_0}(\rho_1, \rho_2),$$  \hspace{1cm} (37)

with

$$\lambda_{\mu(n,\nu)} = \lambda_{\mu} = \frac{1}{((n+1)^2 + 4\nu^2) ((n-1)^2 + 4\nu^2)},$$  \hspace{1cm} (38)

where we used the same notations as in [12]. This expansion has the form

$$\Phi(y, \rho_1, \rho_2) = \sum_{\mu} E_{\mu(n,\nu),\rho_0}(\rho_1, \rho_2) \Phi_\mu(y) = \sum_{\mu} E_{\mu}(\rho_1, \rho_2) \Phi_\mu(y)$$  \hspace{1cm} (39)

where

$$\Phi_\mu(y) = \int \frac{d^2 \rho_1 d^2 \rho_2}{\rho_{12}^2} E_{\mu}(\rho_1, \rho_2) \Phi(y, \rho_1, \rho_2),$$  \hspace{1cm} (40)
see \[27\]. In Eq. (39) and in the following expressions the notation of the conformal summation always means
\[
\sum_{\mu} = \sum_{n=-\infty}^{\infty} \int d\nu \frac{\nu^2 + \frac{n^2}{4 \pi^2}}{\int d^2 \rho_0}.
\]  
(41)
At high energy limit, as it obtains for the BFKL equation at high energy limit, see for example \[29\], we can assume that the main contribution in the sum in Eq. (39) comes from the minimal conformal weight, namely when \(n = 0\) and \(\nu = 0\). In this case we have
\[
L_{12}\Phi(y, \rho_1, \rho_2) = L_{12} \sum_{\mu} E_{\mu}(\rho_1, \rho_2) \Phi_{\mu}(y) = \sum_{\mu} \lambda_{\mu}^{-1} E_{\mu}(\rho_1, \rho_2) \Phi_{\mu}(y) \simeq \sum_{\mu} E_{\mu}(\rho_1, \rho_2) \Phi_{\mu}(y) = \Phi(y, \rho_1, \rho_2).
\]  
(42)
Here we used the fact, that in the high energy limit when \(n = 0\) and \(\nu = 0\) we have \(\lambda_{\mu(n=0,\nu=0)}^{-1} = 1\). The same approximation was used, for example, in \[23\]. It must be clear, that this high energy approximation, which is based on the behavior of the BFKL amplitude, i.e. single pomeron, may be not correct in general in the theory of interacting pomerons. Now, supposing that the Eq. (42) is valid at high energy limit, we rewrite the Eq. (35) and Eq. (31) in the following form:
\[
\left( \frac{\partial}{\partial y} + H(\rho_1, \rho_3) \right) \Phi(y, \rho_1, \rho_3) + \frac{2\alpha_s^2 N_c}{\pi} \int \frac{d^2 \rho_2}{\rho_{12}^2 \rho_{23}^2} \Phi(y, \rho_1, \rho_2) \Phi(y, \rho_2, \rho_3) - \int \frac{d^2 \rho_1}{(2 \pi)^4} \frac{d^2 \rho_{1'}}{\rho_{11'}^2} G_0(\rho_1, \rho_3 | \rho_{1'}, \rho_{1'}) \psi(y, \rho_1, \rho_3) = 0.
\]  
(43)
\[
< \psi(y, \rho_1, \rho_2) > = 0; \quad < \psi(y, \rho_1, \rho_2), \psi(y', \rho_1', \rho_2') > = \frac{4\alpha_s^2 N_c}{\pi \left( \rho_{12}^2 \rho_{11'}^2 \rho_{23}^2 \right)^2} \Phi(y, \rho_1, \rho_2) \delta(y - y') \delta^2(\rho_2 - \rho_2'),
\]  
(44)
were we made the following substitution \(\psi(y, \rho_1, \rho_2) \rightarrow \psi(y, \rho_1, \rho_2)/\rho_{12}^4\). The equations Eq. (43)-Eq. (45) may be considered as a high energy approximation to the correct Langevin equation Eq. (29) with the auto correlator Eq. (31).

### 3.2 Langevin equation for the \(\varphi(y, k)\) field in the momentum space

Let us now consider the part of the action given by Eq. (14) for the case of the forward scattering and formulated in the momentum space:
\[
S_{\varphi} = \frac{2\alpha_s^2 N_c}{\pi} \int dy \int d^2 k \left( \nabla_k^2 \varphi(y, k) \right) \varphi(y, k) \varphi(y, k)^\dagger.
\]  
(46)
As it was mentioned previously, a dual expression of the \(S_{\varphi}\) part of the action in the coordinate space will represent a semiclassical approximation of the initial action Eq. (11)-Eq. (2), and, therefore, the results of this subsection are related to the Langevin equation of \[19\]. So, performing the same transformations as in the previous section, we rewrite the \(S_{\varphi}\) part of the action in the following form:
\[
S_{\varphi} = \frac{1}{2} \int dy \int d^2 k \varphi^\dagger(y, k) \left( \frac{4\alpha_s^2 N_c}{\pi} \left( \nabla_k^2 \varphi(y, k) \right) \delta(y - y') \delta^2(k - k') \right) \int dy' \int d^2 k' \varphi^\dagger(y', k').
\]  
(47)
Introducing an auxiliary field \(\psi(y, k)\) we write for the \(e^{S_{\varphi}}\):

\[
e^{S_{\varphi}} = N \int D[\psi] \exp \left\{ -\frac{1}{2} \int dy d^2k \psi(y, k) \frac{4\alpha_s^2 N_c}{\pi} (\nabla_k^2 k^4 \nabla_k^2 \psi(y, k)) \right\}
\]

\[
\delta(y - y') \delta^2(k - k') \int dy' d^2k' \psi(y', k') - \int dy d^2k \psi(y, k) \varphi^\dagger(y, k) \right\}. \tag{48}
\]

Reabsorbing the \(\left(\frac{4\alpha_s^2 N_c}{\pi} (\nabla_k^2 k^4 \nabla_k^2 \varphi(y, k))\right)^{-1/2}\) factor in the definition of the field \(\psi\), we obtain:

\[
e^{S_{\varphi}} = N \int D[\psi] \exp \left\{ -\frac{1}{2} \int dy d^2k \psi(y, k) \int dy' d^2k' \psi(y', k') \delta(y - y') \delta^2(k - k')
\]

\[
- \int dy d^2k \psi(y, k) \varphi^\dagger(y, k) \sqrt{\frac{4\alpha_s^2 N_c}{\pi} (\nabla_k^2 k^4 \nabla_k^2 \varphi(y, k))} \right\}. \tag{49}
\]

As in the previous case, \(N\) here is

\[
N = \left( \int D[\psi] \exp \left\{ -\frac{1}{2} \int dy d^2k \psi(y, k) \int dy' d^2k' \psi(y', k') \delta(y - y') \delta^2(k - k') \right\} \right)^{-1}. \tag{50}
\]

The whole action for the \(\varphi(y, k)\) and \(\varphi^\dagger(y, k)\) fields now takes the form:

\[
S = S_0 + S_{\text{Aux}} + S_{\varphi^\dagger} - \int dy d^2k \psi(y, k) \varphi^\dagger(y, k) \sqrt{\frac{4\alpha_s^2 N_c}{\pi} (\nabla_k^2 k^4 \nabla_k^2 \varphi(y, k))}. \tag{51}
\]

where

\[
S_{\text{Aux}} = -\frac{1}{2} \int dy \int d^2k \psi^2(y, k). \tag{52}
\]

Writing an equation of motion for the field \(\varphi(y, k)\)

\[
\frac{\delta S}{\delta \varphi^\dagger(y, k)} = 0 \tag{53}
\]

we obtain equation similar to Eq. \(\text{(28)}\):

\[
\hat{L}_k \left( \frac{\partial}{\partial y} + H(k) \right) \varphi(y, k) + \frac{2\alpha_s^2 N_c}{\pi} \varphi(y, k) \hat{L}_k - \psi(y, k) \sqrt{\frac{4\alpha_s^2 N_c}{\pi} (\hat{L}_k \varphi(y, k))} = 0, \tag{54}
\]

with the operator \(\hat{L}_k\) from the Eq. \(\text{(14)}\):

\[
\hat{L}_k = \nabla_k^2 k^4 \nabla_k^2. \tag{55}
\]

We rewrite Eq. \(\text{(54)}\):

\[
\left( \frac{\partial}{\partial y} + H(k) \right) \varphi(y, k) + \frac{2\alpha_s^2 N_c}{\pi} \varphi(y, k) - \left( \hat{L}_k \right)^{-1} \left( \psi(y, k) \sqrt{\frac{4\alpha_s^2 N_c}{\pi} (\hat{L}_k \varphi(y, k))} \right) = 0, \tag{56}
\]

where correlators for the auxiliary filed \(\psi(y, k)\) have the form:

\[
< \psi(y, k) > = 0; \tag{57}
\]

\[
< \psi(y, k), \psi(y_1, k_1) > = \delta(y - y_1) \delta^2(k - k_1). \tag{58}
\]
It is important to underline, that contrary to the simplifications obtained for the equations Eq. [43] and Eq. [15] for the \( \Phi(y,\rho_i,\rho_j) \) field at high energy limit, here we cannot write that \( \hat{L}_k \varphi(y, k) = \varphi(y, k) \). Indeed, such condition leads to the very non physical restriction on the unintegrated parton density function \( f(y, k) \) which follows from the Eq. [15] and Eq. [10]:

\[
\nabla_k^2 f(y, k) = \frac{1}{4} \int_{k^2}^{\infty} dk' \frac{f(y, k')}{k'^4} \log \left( \frac{k'^2}{k^2} \right),
\]

(59)

Of course, such identity can not be satisfied in general for arbitrary \( f(y, k) \) function. More of that, due the use of semiclassical approximation in derivation of this result, it is not clear at all, how such approximated Langevin equation may be used as a guideline for the pomeron loops calculations. Indeed, by definition, a loops calculation must correctly treat the impact parameter structure of the theory, whereas the Eq. (56) and Eq. (3.2) were obtained in the approximation of the large impact parameter.

4 Langevin equation for the theory with ”toy” four pomeron vertex

4.1 ”Toy” four pomeron vertex in the effective theory of the interacting pomerons

The Lagrangian of the RFT-0 model for the \( q \) and \( p \) pomeron fields, which includes also a four pomeron vertex, we could define in the following form:

\[
L = q \dot{p} + \mu qp - \lambda q (q + p)p + \lambda' q^2 p^2,
\]

(60)

see [14] [15] [17], where \( \mu \) is a bare pomeron intercept, \( \lambda \) is a vertex of triple pomeron interactions and \( \lambda' \) a four pomeron interaction vertex. For the case of ”fine tuning” of the vertexes, when \( \frac{\lambda}{\lambda'} = \frac{1}{2} \), that defines a ”magic” value of the four pomeron vertex \( \lambda' \), the Hamiltonian of the problem has a factorized form in the terms of \( q \) and \( p \) fields:

\[
-H = \mu (q - \frac{\lambda}{\mu} q^2)p - \lambda (q - \frac{\lambda'}{\lambda} q^3)p^2 = \mu (q - \frac{\lambda}{\mu} q^2)(p - \frac{\lambda'}{\lambda} p^2).
\]

(61)

We are not interesting in the further investigation of the RFT-0 model here, see more details in [17] [23] [30], but as a guideline for the derivation of our ”toy” four pomeron vertex, we will take the same as in Eq. (61) property of the factorizability of the Hamiltonian for the case of ”magic” value of the four pomeron vertex.

We will begin from the free part of the effective pomeron theory action, Eq. (2), written in the following form:

\[
S_0 = \frac{1}{2} \int dy \frac{d^2 r_1 d^2 r_2 d^2 r_3}{r_{13}^4} \left( \Phi(y, r_1, r_3) \frac{\partial (L_{13} \Phi(y, r_1, r_3))}{\partial y} - \frac{\partial (L_{13} \Phi(y, r_1, r_3))}{\partial y} \Phi(y, r_1, r_3) \right) +
\]

\[
+ \frac{1}{2} \int dy \frac{d^2 r_1 d^2 r_2 d^2 r_3}{r_{13}^4} \left( \Phi(y, r_1, r_3) (L_{13} H(r_1, r_3) \Phi(y, r_1, r_3)) + (L_{13} H(r_1, r_3) \Phi(y, r_1, r_3)) \Phi(y, r_1, r_3) \right).
\]

Here \( H(r_1, r_3) \) is a BFKL Hamiltonian. [21] [23] [27]. In order not to confuse this Hamiltonian with general Hamiltonian of the problem \( H \), the BFKL Hamiltonian and operators related with BFKL Hamiltonian will be always written with the arguments of the BFKL Hamiltonian, namely in the form \( H(r_i, r_j) \). Writing the Hamiltonian of the problem in the conformal basis, formed by functions \( E_\mu(r_1, r_3) \) of Eq. (30), we will omit in the further expressions the common integration factor

\[
\int dy \int \frac{d^2 r_1 d^2 r_2 d^2 r_3}{r_{13}^4}.
\]

(63)
The expression for the "free" part of the Hamiltonian in the conformal basis is the following:

\[
H_0 = \frac{1}{2} \sum_{\mu} E_{\mu}(r_1, r_3) \omega_{\mu} \lambda_{\mu}^{-1} \Phi_{\mu}(y) \sum_{\nu} E_{\nu}(r_1, r_3) \Phi_{\nu}(y) + \\
+ \frac{1}{2} \sum_{\mu} E_{\mu}(r_1, r_3) \Phi_{\mu}(y) \sum_{\nu} E_{\nu}(r_1, r_3) \omega_{\nu} \lambda_{\nu}^{-1} \Phi_{\nu}(y),
\]

(64)

where we used Eq. (37) and

\[
H(r_1, r_3) E_{\mu}(r_1, r_3) = -\omega_{\mu} E_{\mu}(r_1, r_3),
\]

(66)

with the eigenvalues \(\omega_{\mu}\) for the eigenfunctions \(E_{\mu}(r_1, r_2)\) of the BFKL Hamiltonian [28]. Before the definition of the "interacting" part of the Hamiltonian, let us introduce functions

\[
\Psi(y, r_1, r_3) = \int \frac{d^2r_2}{r_{12}^2 r_{23}^2} \Phi(y, r_1, r_2) \Phi(y, r_2, r_3) = \sum_{\mu} E_{\mu}(r_1, r_3) \Psi_{\mu}(y),
\]

(67)

and

\[
\Psi(y, r_1, r_3) = \int \frac{d^2r_2}{r_{12}^2 r_{23}^2} \Phi(y, r_1, r_2) \Phi(y, r_2, r_3) = \sum_{\mu} E_{\mu}(r_1, r_3) \Psi_{\mu}(y).
\]

(68)

With the use of the \(\Psi(y, r_1, r_3)\) and \(\Phi(y, r_1, r_3)\) functions the "interacting" part of the Hamiltonian, which corresponds to Eq. [39], obtains the following form:

\[
-H_I = \frac{2 \alpha_s^2 N_c}{\pi} \sum_{\mu} E_{\mu}(r_1, r_3) \lambda_{\mu}^{-1} \Phi_{\mu}(y) \sum_{\nu} E_{\nu}(r_1, r_3) \Phi_{\nu}(y) + \\
+ \frac{2 \alpha_s^2 N_c}{\pi} \sum_{\mu} E_{\mu}(r_1, r_3) \lambda_{\mu}^{-1} \Phi_{\mu}(y) \sum_{\nu} E_{\nu}(r_1, r_3) \Phi_{\nu}(y).
\]

(69)

(70)

Let us now assume the following anzats for the action with the "toy" four pomeron interaction vertex :

\[
S_{4P} = -C \int dy \int \frac{d^2r_1 d^2r_2}{r_{12}^4} \tilde{F}_{4P} \left( \sum_{\mu} E_{\mu}(r_1, r_3) \Psi_{\mu}(y) \sum_{\nu} E_{\nu}(r_1, r_3) \Psi_{\nu}(y) \right) =
\]

\[
= \int dy \int \frac{d^2r_1 d^2r_2}{r_{12}^4} \left( \sum_{\mu} (\tilde{F}_1 E_{\mu}(r_1, r_3)) \Psi_{\mu}(y) \sum_{\nu} (\tilde{F}_2 E_{\nu}(r_1, r_3)) \Psi_{\nu}(y) + \\
+ \sum_{\mu} (\tilde{F}_2 E_{\mu}(r_1, r_3)) \Psi_{\mu}(y) \sum_{\nu} (\tilde{F}_1 E_{\nu}(r_1, r_3)) \Psi_{\nu}(y) \right) =
\]

\[
= \int dy \int \frac{d^2r_1 d^2r_2}{r_{12}^4} \left( \sum_{\mu} f_{1\mu} E_{\mu}(r_1, r_3) \Psi_{\mu}(y) \sum_{\nu} f_{2\nu} E_{\nu}(r_1, r_3) \Psi_{\nu}(y) + \\
+ \sum_{\mu} f_{2\mu} E_{\mu}(r_1, r_3) \Psi_{\mu}(y) \sum_{\nu} f_{1\nu} E_{\nu}(r_1, r_3) \Psi_{\nu}(y) \right),
\]

(71)

where we introduced some operators \(\tilde{F}_1\) and \(\tilde{F}_2\) such that

\[
\tilde{F}_1 E_{\mu}(r_1, r_3) = f_{1\mu} E_{\mu}(r_1, r_3).
\]

(72)
Finally, again omitting the integration over $\int dy \int d^2 r_1 \, d^2 r_3 / r_{13}^4$, we write the Hamiltonian which is corresponding to this anzats:

$$H_{4P} \quad = \quad C \sum_\mu f_{1\mu} E_\mu(r_1, r_3) \Psi_\nu(y) \sum_\nu f_{2\nu} E^*_\nu(r_1, r_3) \Psi^*_\nu(y) + $$

$$+ \quad C \sum_\mu f_{2\mu} E_\mu(r_1, r_3) \Psi_\nu(y) \sum_\nu f_{1\nu} E^*_\nu(r_1, r_3) \Psi^*_\nu(y), \quad (73)$$

where the constant $C$ will be defined in the further derivation of the Hamiltonian. Now, collecting all terms Eq. (64), Eq. (69) and Eq. (73) together, we obtain:

$$H \quad = \quad \frac{1}{2} \sum_\mu E_\mu(r_1, r_3) \omega_\mu \lambda_\mu^{-1} \Phi_\mu(y) \sum_\nu E^*_\nu(r_1, r_3) \left( \Phi^*_\nu(y) - \frac{2\alpha^2 N_c}{\pi} \omega_\mu^{-1} \Psi^*_\nu(y) \right) + $$

$$+ \quad \frac{1}{2} \sum_\mu E^*_\mu(r_1, r_3) \omega_\mu \lambda_\mu^{-1} \Phi_\mu(y) \sum_\nu E^*_\nu(r_1, r_3) \left( \Phi^*_\nu(y) - \frac{2\alpha^2 N_c}{\pi} \omega_\mu^{-1} \Psi^*_\nu(y) \right) - $$

$$\quad - \quad \frac{\alpha^2 N_c}{\pi} \sum_\mu E_\mu(r_1, r_3) \Psi_\nu(y) \sum_\nu E^*_\nu(r_1, r_3) \lambda_\nu^{-1} \left( \Phi^*_\nu(y) - \frac{C\pi}{\alpha^2 N_c} \lambda_\nu f_{1\mu} f_{2\nu} \Psi^*_\nu(y) \right) - $$

$$\quad - \quad \frac{\alpha^2 N_c}{\pi} \sum_\mu E^*_\mu(r_1, r_3) \Psi^*_\nu(y) \sum_\nu E^*_\nu(r_1, r_3) \lambda_\nu^{-1} \left( \Phi^*_\nu(y) - \frac{C\pi}{\alpha^2 N_c} \lambda_\nu f_{1\mu} f_{2\nu} \Psi^*_\nu(y) \right). \quad (75)$$

Assuming for Eq. (75) the same factorization property as for the Hamiltonian of RFT-0, we find the following values for the $C, f_{1\mu}, f_{2\nu}$:

$$C \quad = \quad 2 \left( \frac{\alpha^2 N_c}{\pi} \right)^2, \quad f_{1\mu} = \omega_\mu^{-1}, \quad f_{2\nu} = \lambda_\nu^{-1}. \quad (79)$$

Continuing the derivation, we obtain for the Hamiltonian:

$$H \quad = \quad \sum_\mu E_\mu(r_1, r_3) \sum_\nu E^*_\nu(r_1, r_3) \left( \omega_\mu \lambda_\mu^{-1} \Phi_\mu(y) - \frac{\alpha^2 N_c}{\pi} \Psi_\mu(y) \lambda_\nu^{-1} \right) \left( \Phi^*_\nu(y) - \frac{2\alpha^2 N_c}{\pi} \omega_\mu^{-1} \Psi^*_\nu(y) \right) + $$

$$+ \quad \sum_\mu E^*_\mu(r_1, r_3) \sum_\nu E^*_\nu(r_1, r_3) \left( \omega_\mu \lambda_\mu^{-1} \Phi^*_\nu(y) - \frac{\alpha^2 N_c}{\pi} \Psi^*_\nu(y) \lambda_\nu^{-1} \right) \left( \Phi^*_\nu(y) - \frac{2\alpha^2 N_c}{\pi} \omega_\mu^{-1} \Psi^*_\nu(y) \right). \quad (77)$$

$$H \quad = \quad \sum_\mu \sum_{\nu, \lambda} \int \frac{d^2 r_1}{r_{12}^2} \frac{d^2 r_3}{r_{23}^2} E_\mu(r_1, r_2) E^*_\mu(r_1, r_3) \Phi_\nu(y) \Phi^*_\nu(y) = \sum_\mu E_\mu(r_1, r_3) \Psi_\mu(y), \quad (80)$$

and using the orthonormalization properties of $E_\mu, \Phi_\nu$, we obtain:

$$\Psi_\mu(y) = \sum_{w, \nu} \int \frac{d^2 r_1}{r_{12}^2} \frac{d^2 r_2}{r_{13}^2} \frac{d^2 r_3}{r_{23}^2} E_w(r_1, r_2) E^*_w(r_1, r_3) E^*_\mu(r_1, r_3) \Phi_w(y) \Phi^*_\nu(y) = V_{\mu, w, \nu} \Phi_w(y) \Phi^*_\nu(y), \quad (85)$$

12
where $V_{\mu,w,\nu}$ is the triple pomeron vertex in the conformal basis, see [31], and summation over repeating indexes $\mu, \nu$ is assumed in the form of Eq. (61). Another observation is concerning the omitted integration $\int \frac{d^2r_1 d^2r_2}{r_1^2 r_2^2}$. This integration over conformal functions $E_0(r_1, r_3)$ $E_0(r_1, r_3)$ in the Eq. (80) gives $\delta_{\mu\nu}$, that, excepting the integration over rapidity, determines the full Hamiltonian:

$$H = \sum_\mu \omega_\mu \lambda^{-1}_\mu \left( \Phi_\mu(y) - \frac{2\alpha_s^2 N_c}{\pi} \omega^{-1}_\mu V_{\mu,w,\nu} \Phi_w(y) \Phi_\nu(y) \right) \left( \Phi^\dagger_\mu(y) - \frac{2\alpha_s^2 N_c}{\pi} \omega^{-1}_\mu V_{\mu,w,\nu} \Phi^\dagger_w(y) \Phi^\dagger_\nu(y) \right),$$

in full analogy with the RFT-0 Hamiltonian Eq. (61). We see now, that the ”toy” four pomeron vertex in the conformal basis has the form

$$V_{4P} = \omega^{-1}_\mu \lambda^{-1}_\mu \left( \frac{2\alpha_s^2 N_c}{\pi} \right)^2 \sum_{w,\nu,w',\nu'} \Phi_w(y) \Phi_\nu(y) V_{\mu,w,\nu} V_{\mu,w',\nu'} \Phi^\dagger_w(y) \Phi^\dagger_\nu(y)$$

and the same vertex for the action in the usual field basis looks as follows:

$$S_{4P} = -2 \left( \frac{\alpha_s^2 N_c}{\pi} \right)^2 \int dy \int d^2r_1 d^2r_3 \int \frac{d^2r_2 d^2r_2'}{r_1^2 r_3^2 r_2^2 r_2'^2} \left\{ \left( L_{13} \Phi(y, r_1, r_2) \Phi(y, r_2, r_3) \right) \left( H^{-1}(r_1, r_3) \Phi^\dagger(y, r_1, r_2) \Phi^\dagger(y, r_2, r_3) \right) \right. + \left. \left( H^{-1}(r_1, r_3) \Phi(y, r_1, r_2) \Phi(y, r_2, r_3) \right) \left( L_{13} \Phi^\dagger(y, r_1, r_2) \Phi^\dagger(y, r_2, r_3) \right) \right\}. \tag{88}$$

From the form of this vertex it is clear, that at least for this ”toy” four pomeron vertex in the usual basis, the procedure described in the previous section gives very complicated auto correlator for the noise field. Indeed, now, due to the very complicated form of the kernel in the Eq. (88), this auto correlator will include a complicated expression with the $L$ and $H^{-1}$ operators with the square of the $\Phi(y, r_1, r_j)$ field. Therefore, we will write this expression only in Appendix C. Nevertheless, for the action in the conformal basis the possible expression for the auto correlator of the noise field looks much simpler and we will consider this derivation in the next section.

### 4.2 The Langevin equation for the ”toy” four pomeron vertex in the conformal basis

The action of the effective pomeron field theory in the conformal basis looks as follows:

$$S = \int dy \sum_\mu \left\{ \frac{1}{2} \Phi^\dagger_\mu(y) \lambda^{-1}_\mu \frac{\partial \Phi_\mu(y)}{\partial y} - \frac{1}{2} \Phi_\mu(y) \lambda^{-1}_\mu \frac{\partial \Phi^\dagger_\mu(y)}{\partial y} - \omega_\mu \lambda^{-1}_\mu \left( \Phi_\mu(y) - \frac{2\alpha_s^2 N_c}{\pi} \omega^{-1}_\mu V_{\mu,w,\nu} \Phi_w(y) \Phi_\nu(y) \right) \left( \Phi^\dagger_\mu(y) - \frac{2\alpha_s^2 N_c}{\pi} \omega^{-1}_\mu V_{\mu,w,\nu} \Phi^\dagger_w(y) \Phi^\dagger_\nu(y) \right) \right\}, \tag{89}$$

where we used the kinematic part of the action from the equation Eq. (62). The action describes the interaction of infinite number of the pomerons through infinite number of different triple pomeron and four pomeron vertexes in accordance with the results of [12], [13]. Let us now write the part of the action which contains the square of the $\Phi^\dagger$ field:

$$S_b = \int dy \sum_\mu \left( \frac{2\alpha_s^2 N_c}{\pi} \lambda^{-1}_\mu \Phi_\mu(y) V_{\mu,w,\nu} \Phi^\dagger_w(y) \Phi^\dagger_\nu(y) \right) - \int dy \sum_\mu \left( \frac{2\alpha_s^2 N_c}{\pi} \lambda^{-1}_\mu \Phi^\dagger_\mu(y) V_{\mu,w,\nu} \Phi^\dagger_w(y) \Phi^\dagger_\nu(y) \right). \tag{89}$$

13
Proceeding as before we obtain for the non-diagonal auto correlator for the noise field $\psi_{\mu}$.

We see, that we obtained the infinite number of Langevin equations for the fields $\Phi_{\mu}$.

Now, whole action Eq. (89) may be written as

$$S_{\Phi} = \frac{1}{2} \sum_{\mu} \int dy \Phi^\dagger_{\mu}(y) \left( \frac{4\alpha_s^2 N_c}{\pi} \lambda_{\mu}^{-1} \Phi_{\mu}(y) V_{\mu,\bar{\nu},\bar{\nu}} - 
- 2 \left( \frac{2\alpha_s^2 N_c}{\pi} \right)^2 \lambda_{\mu}^{-1} \omega^{-1}_{\mu} \Phi_{w'}(y) \Phi_{\nu'}(y) V_{\mu,w',\nu',w,\bar{\nu}} \right) \delta(y - y') \int dy' \Phi^\dagger_{\nu}(y') $$

(91)

and for the auxiliary field action we have:

$$S_{Aux} = -\frac{1}{2} \int dy \psi_{\nu}(y) \left\{ \sum_{\mu} \left( \frac{4\alpha_s^2 N_c}{\pi} \lambda_{\mu}^{-1} \Phi_{\mu}(y) V_{\mu,\bar{\nu},\bar{\nu}} - 
- 2 \left( \frac{2\alpha_s^2 N_c}{\pi} \right)^2 \lambda_{\mu}^{-1} \omega^{-1}_{\mu} \Phi_{w'}(y) \Phi_{\nu'}(y) V_{\mu,w',\nu',w,\bar{\nu}} \right) \right\}^{-1} \psi_{\nu}(y).$$

(92)

Now, whole action Eq. (89) may be written as

$$S = \int dy \sum_{\mu} \left\{ \frac{1}{2} \Phi^\dagger_{\mu}(y) \lambda_{\mu}^{-1} \frac{\partial \Phi_{\mu}(y)}{\partial y} - \frac{1}{2} \Phi_{\mu}(y) \lambda_{\mu}^{-1} \frac{\partial \Phi^\dagger_{\mu}(y)}{\partial y} - 
- \omega_{\mu} \lambda_{\mu}^{-1} \Phi_{\mu}(y) \Phi^\dagger_{\mu}(y) + \frac{2\alpha_s^2 N_c}{\pi} \lambda_{\mu}^{-1} \Phi_{w}(y) \Phi_{\nu}(y) V_{\mu,w',\nu',w,\bar{\nu}} \right\} + S_{Aux} - \sum_{\mu} \int dy \psi_{\nu}(y) \Phi^\dagger_{\mu}. $$

(93)

The equation of motion for each $\Phi_{\mu}$ field in this case has the form of Langevin equation:

$$\frac{\partial \Phi_{\mu}(y)}{\partial y} = \omega_{\mu} \Phi_{\mu}(y) - \sum_{w,\nu} \Phi_{w}(y) \Phi_{\nu}(y) V_{\mu,w,\nu} + \psi_{\mu},$$

(94)

where we are not summing up over the index $\mu$. The $\psi_{\mu}$ field in Eq. (94) we can consider as a noise field with the following correlators:

$$< \psi_{\mu}(y) > = 0;$$

(95)

$$< \psi_{w}(y), \psi_{\nu}(y_1) > = \frac{4\alpha_s^2 N_c}{\pi} \sum_{\mu} \lambda_{\mu}^{-1} \left( \Phi_{\mu}(y) V_{\mu,w',\bar{\nu}} - \frac{2\alpha_s^2 N_c}{\pi} \omega_{\mu}^{-1} \Phi_{w'}(y) \Phi_{\nu'}(y) V_{\mu,w',\nu'} V_{\mu,w,\bar{\nu}} \right) \delta(y - y_1). $$

(97)

We see, that we obtained the infinite number of Langevin equations for the fields $\Phi_{\mu}$ with very complicated non diagonal auto correlator for the noise field $\psi_{\mu}$. It must be underlined, that the evolution equations for the $\Phi_{\mu}$ fields are also very complicated, there we have summation over $w, \nu$ indexes in Eq. (94) that means integration and infinite summation due the definition of the summation procedure in Eq. (11).
5 The Langevin equation in the dipole approach

Now we will consider the results on Langevin equation obtained in [20] in the framework of the dipole CGC approach and will reproduce our Langevin equation basing on the equations of [20]. Let us consider the Langevin equation of [20] for the process of the scattering of an arbitrary numbers of dipoles off the target. The noise (fluctuation) term of the Langevin equation, corresponding to this process, was obtained in [20] and it has the following form:

\[
f(r_1, r_2, y) = C \int d^2 \rho_1 d^2 \rho_2 d^2 \rho_3 G_0(r_1, r_2|\rho_1, \rho_3) \frac{|\rho_2|}{\rho_{13}^2} \nabla_{\rho_1} \nabla_{\rho_2} \Phi(y, \rho_1, \rho_2) \nu(\rho_1, \rho_2, \rho_3, y) \tag{98}
\]

where we adapted the notations of [20] on the notations of the paper, here we introduced a \(\Phi(r_1, r_2)\) field as a RFT counterpart of the \(T_Y(r_1, r_2)\) function from [20]. The constant \(C\) in the expression Eq. (98) is related to the possible difference in normalization of the amplitudes in both papers and we do not fix it in the expression Eq. (98), for our following derivation it is not important. The noise field \(\nu(\rho_1, \rho_2, \rho_3, y)\) in Eq. (98) is a noise field from [20] with the following auto correlator:

\[
< \nu(\rho_1, \rho_2, \rho_3, y), \nu(\rho_1', \rho_2', \rho_3', y') > = \delta^2(\rho_1 - \rho_1') \delta^2(\rho_2 - \rho_2') \delta^2(\rho_3 - \rho_3') \delta(y - y') .
\]

Using Eq. (99), we can define the auto correlator for the field \(f(r_1, r_2, y)\):

\[
< f(r_1, r_2, y) , f(r_1', r_2', y') > = C^2 \int d^2 \rho_1 d^2 \rho_2 d^2 \rho_3 \frac{|\rho_1|}{\rho_{13}^2} G_0(r_1, r_2|\rho_1, \rho_3) \sqrt{\nabla_{\rho_1}^2 \nabla_{\rho_2}^2 \Phi(y, \rho_1, \rho_2)} \tag{100}
\]

\[
\int d^2 \rho_1' d^2 \rho_2' d^2 \rho_3' \frac{|\rho_1'|}{\rho_{13}^2} G_0(r_1', r_2'|\rho_1', \rho_3') \sqrt{\nabla_{\rho_1'}^2 \nabla_{\rho_2'}^2 \Phi(y, \rho_1', \rho_2')} < \nu(\rho_1, \rho_2, \rho_3, y) , \nu(\rho_1', \rho_2', \rho_3', y') > .
\]

The straightforward calculations gives with the help of Eq. (99):

\[
< f(r_1, r_2, y) , f(r_1', r_2', y') > = C^2 \int d^2 \rho_1 d^2 \rho_2 d^2 \rho_3 G_0(r_1, r_2|\rho_1, \rho_3) G_0(r_1', r_2'|\rho_1', \rho_3') \tag{101}
\]

\[
\sqrt{\nabla_{\rho_1}^2 \nabla_{\rho_2}^2 \Phi(y, \rho_1, \rho_2)} \sqrt{\nabla_{\rho_1'}^2 \nabla_{\rho_2'}^2 \Phi(y, \rho_1', \rho_2')} \frac{|\rho_{12}|}{\rho_{13}^2 \rho_{23}^2} \delta(y - y') .
\]

Now we will use Eq. (104) in order to rewrite Green’s functions \(G_0\) in the following form:

\[
G_0(r_1, r_2|\rho_1, \rho_3) = \nabla_{r_1}^{-2} \nabla_{r_2}^{-2} \delta^2(r_1 - \rho_1) \delta^2(r_2 - \rho_3) \tag{102}
\]

and

\[
G_0(r_1', r_2'|\rho_1, \rho_3) = \nabla_{r_1'}^{-2} \nabla_{r_2'}^{-2} \delta^2(r_1' - \rho_1) \delta^2(r_2' - \rho_3) . \tag{103}
\]

Inserting Eq. (102) and Eq. (103) into the Eq. (101) it is easy to see, that operators \(\nabla_{r_1}^{-2} \nabla_{r_2}^{-2}\) and \(\nabla_{r_1'}^{-2} \nabla_{r_2'}^{-2}\) now may be reabsorbed in the definition of our initial field \(f(r_1, r_2, y)\):

\[
f(r_1, r_2, y) \rightarrow \nabla_{r_1}^{-2} \nabla_{r_2}^{-2} \tilde{f}(r_1, r_2, y) . \tag{104}
\]

With this new noise field \(\tilde{f}(r_1, r_2, y)\), the equation of motion for the \(\Phi(r_1, r_2)\) field, or for the \(T_Y(r_1, r_2)\) amplitude from [20], obtains precisely the same additional noise term as in Eq. (103). Indeed, now the auto correlator for the \(\tilde{f}(r_1, r_2, y)\) has the form:

\[
< \tilde{f}(r_1, r_2, y) , \tilde{f}(r_1', r_2', y') > = C^2 \int d^2 \rho_1 d^2 \rho_2 d^2 \rho_3 \sqrt{\nabla_{\rho_1}^2 \nabla_{\rho_2}^2 \Phi(y, \rho_1, \rho_2)} \sqrt{\nabla_{\rho_1'}^2 \nabla_{\rho_2'}^2 \Phi(y, \rho_1', \rho_2')} \tag{105}
\]
Comparing this expression with the auto correlator given by Eq. (31), we see, that if we reabsorb $1/\rho r$
Performing integration and interchanging variables $r_1'$ and $r_2'$ in the $\tilde{f}(r_1', r_2', y')$, we obtain:

$$< \tilde{f}(r_1, r_2, y), \tilde{f}(r_2', r_1', y') > = C^2 \frac{x^2}{r_1^2 r_2'^2} \left( \nabla_{r_1}^2 \nabla_{r_1'}^2 \Phi(y, r_1, r_1') \right) \delta^2(r_2 - r_2') \delta(y - y'). \quad (106)$$

Comparing this expression with the auto correlator given by Eq. (31):

$$< \psi(y, \rho_1, \rho_2), \psi(y', \rho_2', \rho_1') > = \frac{4a^2 N_c}{\pi} \rho^3_1 \rho^3_2 \frac{x^2}{r_1' r_2'} (L_{11'} \Phi(y, \rho_1, \rho_1')) \delta(y - y') \delta^2(\rho_2 - \rho_2'), \quad (107)$$

we see, that if we reabsorb $1/\rho_i^4$ coefficient in definition of the noise field $\psi(y, \rho_i, \rho_j)$ in Eq. (107), then we obtain the same as Eq. (106) auto correlator for the noise field.

6 Discussion of results

As a first main result of the paper we consider a Langevin equations Eq. (29)-Eq. (31), Eq. (56)-Eq. (57) and Eq. (51)-Eq. (53) obtained in the framework of QCD-RFT. The meaning of these equations is simple. The derivation of them is achieved with the use of the classical action of the theory, i.e. action without pomeron loops. The introduction of the auxiliary field in the classical action instead the square of the $\Phi$ field leads to the Langevin equation for the field $\Phi$. Iterating calculations procedure for the $\Phi$ field together with the calculation of the auto correlator for the noise field $\psi$, see for example discussion in [19], will lead to the account of the pomeron loops contribution in the pomeron field $\Phi$. Proceeding, we will calculate all loops contribution to the pomeron field and will resolve whole quantum problem for this effective theory. Of course, practically, due the very complicated form of the auto correlators for the noise fields, this task looks at least not easy.

The interpretation of the Langevin equations in this paper and the interpretation of the Langevin equations from the papers [19, 20] are the same, in spite to the different forms of obtained equations. As in [19] and [20] we also obtained two different Langevin equations formulated in the different frameworks. Let us, therefore, discuss the similarities and differences in the forms of the Langevin equations, formulated in the momentum and coordinate spaces separately. First of all, we consider the obtained Langevin equation Eq. (50) for the $\varphi(k, y)$ field in the momentum space. The Eq. (50) looks very similar to the Langevin equations of [19]. There is only one move which reduces Eq. (50) to the corresponding equations in [19] and this move is an assumption about the action of the operators $L$ and $L^{-1}$ of Eq. (11), on the function $\varphi(k, y)$. If we assume, that the $\varphi(k, y)$ is a eigenfunction of $L$ and $L^{-1}$ with the unit eigenvalues then the corresponding equations will be the same. But, as it was argued in this paper, this move is impossible in the framework of the present approach because in this case a very unphysical restriction Eq. (59) on the form of the unintegrated gluon density function $f(y, k)$ is arising. Therefore, in general, without some approximation it is impossible to write Eq. (50) in the form obtained in [19]. More important, that the function $\varphi(k, y)$ is defined for the case of zero momentum transfer, i.e. there is no correct impact parameter dependence account of the amplitude is performed, and, therefore, it is in principal impossible describe loops contribution in the pomeron field with the use of the $\varphi(k, y)$ field. The Langevin equation for this function in the form of Eq. (50), therefore, may plays a role of some toy model which probably may describe some properties of real QCD. In general, this function also may be applied for the semiclassical solution of the problem, i.e. for the calculation of the "tree" pomeron structure, see [16]. Another application of this amplitude and semiclassical approach was developed in [22, 23], where the calculations of the amplitude were performed in the framework of a generating functional approach which
has a strong relations with the probabilistic interpretation of the BFKL pomeron interactions. It must be also noticed, that the approach developed in [23] has a mutual roots with the approach considered in this paper, focusing, nevertheless, mostly on the semiclassical solutions of the problem.

Concerning the correct Langevin equation for the $\Phi(y, r_i, r_j)$ field in the transverse position space we see, that the situation here is more obvious. The calculations in the previous section show the equivalence between the Langevin equation obtained in [20] in the framework of the s-channel dipole model and Langevin equation given by Eq. (29)-Eq. (31) which was obtained in the framework of the t-channel QCD-RFT. In spite to the differently written forms of the noise terms and auto correlators of the noise terms, the Langevin equations in both approaches are the same. The Langevin equations it is another description of the dynamics of the physical problem, and the same form of the Langevin equations in both approaches means a equivalent description of the quantum fields dynamics in QCD-RFT and dipole approaches. Using Langevin equation as a bridge we can pass from the QCD-RFT side to the dipole side of extended Balitsky-JIMWLK hierarchy and vice versa. We can conclude, therefore, that the $T_Y(r_i, r_j)$ function considered in [20] in the framework of the extended Balitsky-JIMWLK hierarchy of equations is equivalent to the pomeron field considered in the framework of t-channel QCD-RFT, where in both cases all pomeron loops contribution is included in. More of that, due the equivalence between the QCD-RFT approach and dipole model of [20] prooven in this paper and due the fact that in [20] was established the equivalence between approaches of [20] and [21], we can conclude, that our QCD RFT t-channel model is equivalent to the modified version of the JIMWLK equation of [21] as well. On the level of RFT in zero dimensions this fact was clarified in [17], and now we see, that the s-channel dipole and t-channel RFT approaches are indeed equivalent in the physical space of two transverse dimensions. This result we consider as a second main result of the paper.

The consideration of the theory with the “toy” four pomeron vertex in the conformal basis is similar, as it must be, to the derivations of [12, 13]. As in [12], we obtained the reformulated theory with infinite number of one dimensional pomerons, with only additional terms due the four pomeron vertex. In spite to the redefinition of the degrees of the freedom of the theory, the corresponding equations are not simpler then the equations in the usual basis. Indeed, the equation of motion of this theory contains infinite number of one dimension pomerons with an infinite number of different and very complicated interaction vertexes which include three integration and three infinite summation over repeating indexes. The equations in the form of Langevin dynamics also include very complicated correlators of Eq. (95), where the auto correlator for given conformal weights $\mu, \nu$ depends on all other conformal fields $\Phi$ of the theory. Nevertheless, as it was mentioned in [12], we can truncate the sums on some value of conformal weight $\mu$ and try to solve simplified truncated theory. Still, this task will be not easy and we leave it for the future studies. Concerning the form of the expression for this four pomeron vertex it is also interesting to note, that obtained in this paper vertex is the same as the four point function of two-dimensional conformal field theory derived in [31] from the conformal bootstrap point of view on the BFKL pomeron. In the given context it means, that conformal bootstrap in application to the interacting BFKL pomerons will lead to the factorized Hamiltonian of the problem with precisely zero ground state, see beginning of the Section 4. We do not know about the deep physical reasons for such relation between the conformal field theory in two dimensions and the Hamiltonian of the interacting BFKL pomerons, and, therefore, we do not consider here this subject, which potentially may be very interesting. Finishing the theme of four pomeron interaction vertex, we can also notice, that our vertex is not the same as the four pomeron vertex introduced in the [22] on the base of the physical reasons and arguments different from the considered in this paper.

Acknowledgments

I am especially grateful to Leszek Motyka and Mikhail Braun for the discussion on the subject of the paper. I thank also E.Levin, N.Armesto and C.Pajares for the interesting discussions and useful comments
I also gratefully acknowledge the support of the Ministerio de Educación y Ciencia of Spain under project FPA2005-01963, and by Xunta de Galicia (Consellería de Educación).
Appendix A:

Let us consider the following part of the action:

\[ \hat{S}_I = \int dy \int d^2 r_1 d^2 r_2 d^2 r_3 \left( \frac{L_{13}}{r_{31}^2} \phi^i(y, r_1, r_3) \right) \phi(y, r_1, r_2) \phi(y, r_2, r_3), \]  \hspace{1cm} (A.1)

where we omitted the coefficient in front of Eq. (18). Let us make the Fourier transform of the \( \phi \) functions:

\[ \phi(y, r_1, r_2) = \int \frac{d^2 k_1 d^2 q_1}{(2\pi)^2} e^{-i r_1 k_1 - i r_2 (q_1 - k_1)} \tilde{\phi}(y, k_1, q_1 - k_1), \]  \hspace{1cm} (A.2)

\[ \phi(y, r_2, r_3) = \int \frac{d^2 k_2 d^2 q_2}{(2\pi)^2} e^{-i r_2 k_2 - i r_3 (q_2 - k_2)} \tilde{\phi}(y, k_2, q_2 - k_2), \]  \hspace{1cm} (A.3)

\[ \phi^i(y, r_1, r_3) = \int \frac{d^2 k_3 d^2 q_3}{(2\pi)^2} e^{i r_3 k_3 + i r_1 (q_3 - k_3)} \tilde{\phi}^i(y, k_3, q_3 - k_3). \]  \hspace{1cm} (A.4)

Operator \( \frac{L_{13}}{r_{31}^2} \) throw the Fourier transform is changed to:

\[ \frac{L_{13}}{r_{31}^2} = r_{13}^2 \nabla_1^2 \nabla_3^2 \frac{FT}{r_{13}} \nabla_k^2 k_3 (q_3 - k_3)^2 \nabla_{k_3}^2 = \hat{L}_3. \]  \hspace{1cm} (A.5)

Inserting Eq. (A.2) and Eq. (A.3) back in Eq. (A.1) we obtain:

\[ \hat{S}_I = \int dy \int d^2 k_1 d^2 k_2 d^2 k_3 \int d^2 q_1 d^2 q_2 d^2 q_3 (\hat{L}_3 \tilde{\phi}^i(y, k_3, q_3 - k_3)) \tilde{\phi}(y, k_2, q_2 - k_2) \tilde{\phi}(y, k_1, q_1 - k_1) \]  \hspace{1cm} (A.6)

\[ \int \frac{d^2 r_1 d^2 r_2 d^2 r_3}{(2\pi)^2} \frac{d^2 r_3}{(2\pi)^2} e^{-i r_1 k_1 - i r_2 (q_1 - k_1)} e^{-i r_2 k_2 - i r_3 (q_2 - k_2)} e^{i r_3 k_3 + i r_1 (q_3 - k_3)}. \]  \hspace{1cm} (A.7)

Let us consider the last line of Eq. (A.4):

\[ \int \frac{d^2 r_1 d^2 r_2 d^2 r_3}{(2\pi)^2} \frac{d^2 r_3}{(2\pi)^2} e^{-i r_1 k_1 - i r_2 (q_1 - k_1)} e^{-i r_2 k_2 - i r_3 (q_2 - k_2)} e^{i r_3 k_3 + i r_1 (q_3 - k_3)}. \]  \hspace{1cm} (A.8)

and make there the following change of variables:

\[ \rho_1 = \frac{r_3 - r_1}{2} \]  \hspace{1cm} (A.9)

\[ \rho_2 = \frac{r_3 + r_1}{2} \]  \hspace{1cm} (A.10)

\[ R = \frac{r_2}{2} - \frac{r_1}{4} - \frac{r_3}{4} \]  \hspace{1cm} (A.11)

\[ r_2 = 2R + \rho_2. \]  \hspace{1cm} (A.12)

Changing variables in Eq. (A.9) we obtain:

\[ 4 \int \frac{d^2 R}{(2\pi)^2} \frac{d^2 \rho_1}{(2\pi)^2} \frac{d^2 \rho_2}{(2\pi)^2} e^{i R(k_1 - q_1 - k_2)} e^{i \rho_1 (k_1 + k_2 + 2k_3 - q_2 - q_3)} e^{i \rho_2 (q_3 - q_1 - q_2)}. \]  \hspace{1cm} (A.13)

Integration over \( R, \rho_1 \) and \( \rho_2 \) gives three delta function and after the integration over \( q_1, k_1 \) and \( q_2 \) such that

\[ q_1 = q_3 - k_2 - k_3 \]  \hspace{1cm} (A.14)

\[ k_1 = q_3 - k_3 \]  \hspace{1cm} (A.15)

\[ q_2 = k_2 + k_3 \]  \hspace{1cm} (A.16)

we obtain Eq. (9).
Appendix B:

Let us consider together Eq. (14) and Eq. (19):

\[ \hat{S}_I = \int d^2k \left( \nabla_k^2 \phi^\dagger(y, k) \right) \phi(y, k) \phi(y, k), \]  

(B.1)

and

\[ \phi(y, k) = \frac{\pi^2}{2N_c} \int_{k^2}^{\infty} dk' \frac{f(y, k')}{k'^4} \log\left( \frac{k'^2}{k^2} \right). \]  

(B.2)

\[ \phi^\dagger(y, k) = \frac{\pi^2}{2N_c} \int_{k^2}^{\infty} dk' \frac{f^\dagger(y, k')}{k'^4} \log\left( \frac{k'^2}{k^2} \right). \]  

(B.3)

The operator \( \nabla_k^2 \), because the rotational invariance of the problem (zero transfer momentum), can be rewritten in the following form:

\[ \nabla_k^2 = 4 \frac{\partial}{\partial k^2} \left( k^2 \frac{\partial}{\partial k^2} \right) = 4 \frac{\partial}{\partial x} \left( \frac{\partial}{\partial x} \right), \]  

(B.4)

here we changed the variables: \( k^2 \rightarrow x \). The whole expression Eq. (B.1) now may be rewritten in the terms of variable x:

\[ \hat{S}_I = 4\pi \int dx \frac{\partial}{\partial x} \left( x \frac{\partial}{\partial x} \right) (x^2 \nabla_k^2 \phi^\dagger(x)) \phi(x) \phi(x). \]  

(B.5)

Now we use, that

\[ f^\dagger(x) \propto x^2 \nabla_k^2 \phi^\dagger(x) \]  

(B.6)

\[ \phi(x) \propto \int_x^\infty dx' \frac{f(x')}{x'^2} \log\left( \frac{x'}{x} \right). \]  

(B.7)

Inserting these expressions in the Eq. (B.5) we obtain:

\[ \hat{S}_I \propto \int dx \frac{\partial}{\partial x} \left( x \frac{\partial}{\partial x} \right) (f^\dagger(x)) \int_x^\infty dx' \frac{f(x')}{x'^2} \log\left( \frac{x'}{x} \right) \int_x^\infty dx'' \frac{f(x'')}{x''^2} \log\left( \frac{x''}{x} \right). \]  

(B.8)

Integrating Eq. (B.8) by parts and omitting boundary terms of integration we obtain:

\[ \hat{S}_I \propto 2 \int dx \left( \frac{\partial}{\partial x} f^\dagger(x) \right) \left( x \int_x^\infty dx'' \frac{f(x'')}{x''^2} \log\left( \frac{x''}{x} \right) \int_x^\infty dx' \frac{f(x')}{x' x'^2} \right). \]  

(B.9)

The result of the second integration by parts is the following:

\[ \hat{S}_I \propto -2 \int dx f^\dagger(x) \left( \int_x^\infty dx'' \frac{f(x'')}{x''^2} \log\left( \frac{x''}{x} \right) \int_x^\infty dx' \frac{f(x')}{x' x'^2} \right) + \]  

(B.10)

\[ + 2 \int dx f^\dagger(x) \left( x \int_x^\infty dx'' \frac{f(x'')}{x''^2} \int_x^\infty dx' \frac{f(x')}{x' x'^2} \right) + \]  

\[ + 2 \int dx f^\dagger(x) \left( x \int_x^\infty dx'' \frac{f(x'')}{x'' x^3} \log\left( \frac{x''}{x} \right) \left( \frac{f(x)}{x^5} + \int_x^\infty dx' \frac{f(x')}{x' x^2 x'^2} \right) \right), \]

that gives finally:

\[ \hat{S}_I \propto 2 \int dx f^\dagger(x) \left( \int_x^\infty dx'' \frac{f(x'')}{x''^2} \int_x^\infty dx' \frac{f(x')}{x' x'^2} \right) + \]  

(B.11)

\[ + 2 \int \frac{dx}{x^2} f^\dagger(x) f(x) \int_x^\infty dx'' \frac{f(x'')}{x''^2} \log\left( \frac{x''}{x} \right). \]

We see that this vertex is the same as in [8], see also [16], excepting some not important for our consideration constant in front of the expression.
Appendix C:

Let us consider expression Eq. (88)
\[ S_{4P} = -2 \left( \frac{\alpha_s^2 N_c}{\pi} \right)^2 \int dy \int d^2 r_1 d^2 r_3 \int \frac{d^2 r_2 d^2 r'_2}{r_{12}^2 r_{23}^2 r_{12}'^2 r_{23}'^2} \]
\[
\left\{ \left( L_{13} \Phi(y, r_1, r_2) \Phi(y, r_2, r_3) \right) \left( H^{-1}(r_1, r_3) \Phi(y, r_1, r_2') \Phi(y, r_2', r_3) \right) + \\
+ \left( H^{-1}(r_1, r_3) \Phi(y, r_1, r_2) \Phi(y, r_2, r_3) \right) \left( L_{13} \Phi(y, r_1, r_2) \Phi(y, r_2', r_3) \right) \right\}. \quad (C.1)\]

and let us rewrite this expression in the following form:
\[ S_{4P} = -\frac{1}{2} \int dy \int dy' \int d^2 r_1 d^2 r_3 \int \frac{d^2 r_2 d^2 r''_2}{r_{12}^2} \Phi(y, r_1, r_2) \Phi(y, r_2, r_3) \]
\[
\cdot \left( \frac{2\alpha_s^2 N_c}{\pi} \right)^2 \int \frac{r_{12}^2 r_{23}^2}{r_{12}^2 r_{23}^2} d^2 r_2 \left\{ \left( L_{13} \Phi(y, r_1, r_2) \Phi(y, r_2, r_3) \right) H^{-1}(r_1, r_3) + \\
+ \left( H^{-1}(r_1, r_3) \Phi(y, r_1, r_2) \Phi(y, r_2, r_3) \right) L_{13} \right\} \delta(y - y') \delta^2(r_2' - r_2''). \quad (C.2)\]

From this expression for the $S_{4P}$ part of the action it is clear, that auto correlator Eq. (31) will be changed now to the following form:
\[
< \psi(y, r_1, r_2'), \psi(y', r_2, r_3) > = \frac{4\alpha_s^2 N_c}{\pi} \frac{r_{12}^2 r_{23}^2}{r_{12}^2} \left( L_{12} \Phi(y, r_1, r_2) \right) \delta(y - y') \delta^2(r_2' - r_2'') - \\
- \left( \frac{2\alpha_s^2 N_c}{\pi} \right)^2 \int \frac{r_{12}^2 r_{23}^2}{r_{12}^2 r_{23}^2} d^2 r_2 \left\{ \left( L_{13} \Phi(y, r_1, r_2) \Phi(y, r_2, r_3) \right) H^{-1}(r_1, r_3) + \\
+ \left( H^{-1}(r_1, r_3) \Phi(y, r_1, r_2) \Phi(y, r_2, r_3) \right) L_{13} \right\} \delta(y - y') \delta^2(r_2' - r_2''), \quad (C.3)\]

whereas the form of equation of motion Eq. (29) will stay unchanged.
References

[1] L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rept. 100, (1983) 1.

[2] L. N. Lipatov, Sov. J. Nucl. Phys. 23 (1976) 338 [Yad. Fiz. 23 (1976) 642]; E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP 45 (1977) 199 [Zh. Eksp. Teor. Fiz. 72 (1977) 377]; I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822 [Yad. Fiz. 28 (1978) 1597].

[3] L. N. Lipatov, Phys. Rept. 286 (1997) 131.

[4] V. S. Fadin and L. N. Lipatov, Phys. Lett. B 429 (1998) 127; M. Ciafaloni and G. Camici, Phys. Lett. B 430 (1998) 349; V. S. Fadin and R. Fiore, Phys. Lett. B 610 (2005) 61 [Erratum-ibid. B 621 (2005) 61]; V. S. Fadin and R. Fiore, Phys. Rev. D 72 (2005) 014018.

[5] I. Balitsky, Nucl. Phys. B 463 (1996) 99.

[6] J. Jalilian-Marian, A. Kovner and H. Weigert, Phys. Rev. D 59 (1999) 014015; J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, Phys. Rev. D 59 (1999) 014014; E. Iancu, A. Leonidov and L. D. McLerran, Nucl. Phys. A 692 (2001) 583; E. Iancu, A. Leonidov and L. D. McLerran, Phys. Lett. B 510 (2001) 133; E. Iancu and L. D. McLerran, Phys. Lett. B 510 (2001) 145; E. Ferreiro, E. Iancu, A. Leonidov and L. McLerran, Nucl. Phys. A 703 (2002) 489.

[7] A. H. Mueller, Nucl. Phys. B 415 (1994) 373.

[8] J. Bartels, Z. Phys. C 60 (1993) 471; J. Bartels and M. Wüsthoff, Z. Phys. C 66 (1995) 157; J. Bartels and C. Ewerz, JHEP 9909 (1999) 026.

[9] C. Ewerz, Phys. Lett. B 512 (2001) 239; C. Ewerz and V. Schatz, Nucl. Phys. A 736 (2004) 371; T. Bittig and C. Ewerz, Nucl. Phys. A 755 (2005) 616.

[10] M. A. Braun, Phys. Lett. B 483 (2000) 115.

[11] M. A. Braun, Eur. Phys. J. C 33 (2004) 113.

[12] M. A. Braun, arXiv:hep-ph/0504002

[13] M. A. Braun, Phys. Lett. B 632 (2006) 297.

[14] D. Amati, L. Caneschi and R. Jengo, Nucl. Phys. B 101 (1975) 397.

[15] R. Jengo, Nucl. Phys. B 108 (1976) 447;

[16] S. Bondarenko and L. Motyka, arXiv:hep-ph/0605185.

[17] S. Bondarenko, L. Motyka, A. H. Mueller, A. I. Shoshi and B. W. Xiao, arXiv:hep-ph/0609213.

[18] A. Kovner and M. Lublinsky, Phys. Rev. Lett. 94 (2005) 181603, Phys. Rev. D 72, 074023 (2005), Nucl. Phys. A 767, 171 (2006); Y. Hatta, E. Iancu, L. McLerran, A. Stafo and D. N. Triantafyllopoulos, Nucl. Phys. A 764 (2006) 423.

[19] E. Iancu and D. N. Triantafyllopoulos, Nucl. Phys. A 756 (2005) 419, Phys. Lett. B 610 (2005) 253; D. N. Triantafyllopoulos, Acta Phys. Polon. B 36 (2005) 3593; G. Sorey, Phys. Rev. D 72, 016007 (2005).

[20] E. Iancu and D. N. Triantafyllopoulos, Phys. Lett. B 610 (2005) 253.
[21] A. H. Mueller, A. I. Shoshi and S. M. H. Wong, Nucl. Phys. B 715 (2005) 440.

[22] E. Levin and M. Lublinsky, Nucl. Phys. A 763 (2005) 172.

[23] M. Kozlov, E. Levin and A. Prygarin, arXiv:hep-ph/0606260.

[24] L.N.Lipatov, Sov. Phys. JETP 63 (1986) 904, Nucl. Phys. B 715 (1991) 641, Phys. Rept. 286 (1997) 131;

[25] J. Bartels, L. N. Lipatov and G. P. Vacca, Nucl. Phys. B 706 (2005) 391;

[26] K. Kutak and J. Kwieciński, Eur. Phys. J. C 29 (2003) 521; K. Kutak and A. M. Staśto, Eur. Phys. J. C 41 (2005) 343; K. Kutak, DESY-THESIS-2006-034.

[27] L.N.Lipatov, in ”Perturbative QCD”, ed. A.H.Mueller, World. Sci. Singapore (1989).

[28] L.N.Lipatov, . Phys. Lett. B 251 (1990) 284.

[29] S. Bondarenko, M. Kozlov and E. Levin, Acta Phys. Polon. B 34, 3081 (2003).

[30] M. A. Braun and G. P. Vacca, arXiv:hep-ph/0612162; E. Levin and A. Prygarin, arXiv:hep-ph/0701178.

[31] G. P. Korchemsky, Nucl. Phys. B 550, (1999)397.