Active Control of Regenerative Chatter in Turning by Compensating the Variable Cutting Force

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This work was supported by the European Union FP7 DynXperts project (260073/FP7-2010-NMP-ICT-FoF), SPRI Elkartek COACVIRE (KK-2018/00084) project, and Spanish Ministry of Science and Innovation (CDTI-CERVERA framework) under MIRAGED (CER-20191001) project.

ABSTRACT During metal cutting operations performed by conventional machine tools (turning, milling ...) often appear vibrations due to the emergence of a variable force generated by the so-called regenerative effect. Such vibrations, known as regenerative chatter, may not be prevented at the machine design stage and often limit productivity severely. Therefore, a lot of effort has been put into developing solutions for this problem in the past. To compare the performance of such solutions, it is interesting to realistically reproduce in the laboratory the mentioned regenerative effect in a reliable, repeatable way and covering general machining conditions. With this objective, the paper presents an improvement of a Hardware-in-the-loop chatter simulator that creates this type of vibration for turning machines on a specifically designed mechanical structure. This simulator completely corrects the effects that the delay, introduced by the used equipment (by actuator and controller, mainly), has on the vibration and, moreover, it is capable of imposing general behaviors for any machine with a certain damping factor. Later, by installing inertial actuators on the mentioned structure, the operation of various active chatter control systems may be compared. In this work, the effort made to accurately create the variable machining force is harnessed to try its compensation, by generating with the inertial actuator an approximately inverse control action. In this context, the performance of the novel controller, based on the cancelation of the whole variable cutting force, is analyzed.

INDEX TERMS Hardware-in-the-loop simulation, regenerative chatter, active control, practical implementation, parameter uncertainty, real-time control, turning machines.

I. INTRODUCTION

The study of regenerative vibrations in machine tools is a classic subject of study, as it has serious practical implications. On the one hand, it reduces the life of the cutting tool and the mechanical components of the machine. On the other hand, it significantly impairs the quality of the workpiece surface finish. It is therefore important to prevent this effect from appearing, which involves being aware of it in as much detail as possible. The regenerative effect is a complex physical phenomenon, which not only depends on the dynamic characteristics of the machine (damping, inertia, stiffness, ...), but also on the cutting conditions (depth, spindle speed, workpiece material, ...). The first description of the effect is due to Tobias [1] and Tlusty [2]. Later, a closed-loop model that describes the chatter as a feedback interaction between the cutting process and the structural dynamics of the machine was proposed by Merrit [3]. Using such a model, the limit values that can take the cutting parameters (spindle speed and depth of cut) to maintain the stability of the process are described graphically by the so-called stability lobes. Thus, these diagrams are a very valuable tool to know, in advance, the parameters that may be used in each case in order to maintain the stability of the machining process.

It is obvious that if the stability zones could be extended, by increasing the stable depth of cut, this would allow increasing the metal removal speed and, therefore, the productivity of the machine. In fact, numerous techniques have been proposed in the literature for the reduction of these undesirable vibrations through many different ways [4]. Some authors
proposed special tool geometries to distort the regenerative effect in processes with tools of multiple flutes [5]–[7]. The spindle speed variation is another method that demonstrated effective chatter reduction. On the one hand, the discrete spindle speed tuning based on stability lobes can conduct the process conditions onto a high stability zone [8], [9]. On the other hand, a continuous spindle speed variation around the nominal speed can distort the periodical regenerative effect [10]–[12].

Nevertheless, the addition of external damping represents one of the most promising methods for chatter suppression without changing the original process conditions. For that purpose, passive tuned mass dampers are usually employed due to their simplicity and relatively low cost [13]. However, the employment of passive absorbers is not feasible in cases where dynamics of the system change during machining process. Active solutions can overcome these limitations and it is why they have been extensively studied. These active systems are roughly based on measuring the vibration and generating a force response that attempts to reduce the effect of the chatter, by means of a sensor-actuator pair placed near the tool [15], [16], [18], [20], [21], [23] or on the machine structure [14], [17], [19], [22], which guarantees adaptability to changing conditions.

Several active chatter control algorithms have been successfully proposed so far, such as direct position feedback (DPF), direct velocity feedback (DVF), and direct acceleration feedback (DAF) –see e.g. [24]. DVF is undoubtedly the most widely used, as it adds damping to the process, delaying the appearance of the chatter for any tool speed, mainly in low stability zones [25]. Also, compared to DPF and DAF, DVF is less affected by delays in the feedback loop [26] and is less likely to destabilize high-frequency modes [27], [28]. Finally, an active control technique that attacks the regenerative term using feedback has been proposed. This is the Delayed Position Feedback (DelPF), initially proposed in [25] and [29], and later studied in more detail in [30]. In such motivating work, the practical option of feedback of the acceleration measured in the previous tool pass, instead of the position (DelAF), is also validated. It has been observed that one of the main advantages of the DelPF (and DelAF) is based on the possibility of eliminating the control loop delay (computational, power electronics, actuator,...), by taking advantage of the revolution period. The present work aims to explore the possibility of compensating, by active control, not only the regenerative term of the chatter, as the DelPF (DelAF) does, but the whole variable force suffered by the tool during machining.

However, comparative studies on the performance that can be achieved using active chatter control techniques face an important practical problem. Such studies are normally carried out by pushing the system to its stability limit many times while applying the techniques under study with different machining parameters. This inevitably leads to rapid deterioration of the tool which, in turn, impairs the repeatability of active control comparison tests. Therefore, this work is dedicated firstly to the optimization of a mechatronic Hardware-in-the-loop (HiL) chatter simulator for orthogonal cutting. Thus, it is possible to reproduce in the laboratory, on a mechanical structure designed specifically for this purpose, the regenerative effect. This is achieved in a reliable and exact way, eliminating by software the effect of the delays that, inevitably, introduces the simulator hardware. Besides, this simulator allows to reproduce the general dynamic behavior of any machine with the same damping factor facing the regenerative chatter, by using dimensionless variables. Finally, an inertial actuator is coupled to the structure of the simulator to perform, by using rapid control prototyping, reliable real-time chatter active control tests. On such tests, we base the experimental results that are presented and serve as a basis for the conclusions drawn.

Section II is dedicated to explain the chatter model and the basic design of the hardware-in-the-loop simulator. Section III focuses on describing the improvements applied to the HiL simulator in order to avoid the effects of the delays of the equipment and, also, to open the possibility of imposing different damping factors in a general way to the machines-tool under study. Then, in section IV, it is discussed how to obtain experimental results independent of the machining parameters. The operation of the HiL system for the study of the active control of chatter in turning is detailed in section V. Finally, the new control algorithm, devoted to compensate the whole variable machining force, is analyzed in section VI and the conclusions are presented in section VII.

II. REGENERATIVE CHATTER HARDWARE-IN-THE-LOOP SIMULATOR

A. MODEL OF REGENERATIVE CHATTER IN TURNING

According to Altintas [31], machine tool chatter vibrations result from a self-excitation mechanism in the generation of chip thickness during machining operations. Cutting forces excite one of the structural modes of the machine-tool-workpiece system and a wavy surface is left behind.

As shown in Figure 1, after a full rotation, the tool confront the mentioned waves and the resultant chip thickness $h(t)$ turn out to be time-variant and dependant on the relative phase of

![Figure 1. Scheme of regenerative chatter vibrations in turning.](image-url)
the present \( y(t) \) and previous \( y(t - \tau) \) vibration displacements of the tool [31]

\[
h(t) = h_0 - [y(t) - y(t - \tau)] \tag{1}
\]

where \( h_0 \) is the constant feed of the tool into the workpiece and \( \tau \) is the time of one full rotation. It is well known that the variable cutting force \( F(t) \) may be considered proportional to the frontal area of the chip, that is, the product of the chip thickness \( h(t) \) and the depth of the cut \( b_p \)

\[
F(t) = K_f \cdot b_p \cdot h(t) \tag{2}
\]

where \( K_f \) is the cutting coefficient of the process. Now, the dynamic equation of motion is

\[
m \cdot \ddot{y}(t) + c \cdot \dot{y}(t) + K \cdot y(t) = F(t) \tag{3}
\]

If the machine-tool transfer function between the force and the displacement is defined as \( \Phi(s) \) in Laplace domain and making \( y(t - \tau) = y(s)e^{-\tau s} \), we have from (1)-(3):

\[
\frac{h(s)}{h_0(s)} = \frac{1}{1 + K_f \cdot b_p \cdot \Phi(s) \cdot (1 - e^{-\tau s})} \tag{4}
\]

being

\[
\Phi(s) = \frac{y(s)}{F(s)} = \frac{1}{ms^2 + cs + K} = \frac{-2\xi r}{1 - r^2} \left[ \frac{2(\xi r)^2}{k} \right] + j \frac{-2\xi r}{1 - r^2} \left[ \frac{(1 - r^2)^2 + (2\xi r)^2}{2k} \right] = G + jH \tag{5}
\]

where \( m \) is the mass, \( c \) is the damping coefficient, \( K \) is the stiffness \( (F/y) \), \( r \) is the ratio of chatter frequency to natural frequency \( (\omega_c/\omega_n) \), \( \xi \) is the ratio of the damping coefficient to the critical damping coefficient \( (c/c_c) \) and the critical damping coefficient is \( c_c = 2\sqrt{Km} \).

From (1)-(4), a closed-loop feedback diagram -Figure 2- for regenerative chatter has been introduced by Merrit [3]. Such a diagram may be used for analyzing the stability of the regenerative effect, from a control engineering perspective.

Operating with the characteristic equation of the closed-loop described in Figure 2, it follows

\[
b_p \lim = -\frac{1}{2 \cdot K_f \cdot G(\omega_c)} \tag{6}
\]

Expression (6) gives –see [3]– the limiting depth of cut as a function of the chatter frequency \( \omega_c \). Given that the depth of cut should always be positive, from expressions (5) and (6), the minimum depth of cut occurs at the maximum negative value of \( G \), \((G_{\min})\), when \( r = \sqrt{1 + 2\xi} \):

\[
b_p \text{ min} = \frac{2k(1 + \xi)}{K_f} \tag{7}
\]

Such a minimum depth of cut represents a limit in the machining process under study. Since it is independent of the chatter frequency, it is a fixed value, determined by the materials and geometries of the tool and the workpiece, limiting the unconditionally stable metal cutting.

A dimensionless limiting depth of cut may be obtained by the ratio

\[
\frac{r_b}{b_p \text{ min}} = \frac{G_{\min}(\omega_c)}{G(\omega_c)} = \frac{(1 - r^2)^2 + (2\xi r)^2}{-4\xi(1 + \xi)(1 - r^2)} \tag{8}
\]

In this way, the expression (6) represents the relationship between the limiting depth of cut and the chatter frequency, while the equation (8) gives the dimensionless limiting depth of cut ratio and chatter frequency ratio.

Now, we have to describe the relationship between the chatter frequency and the spindle speed \( N \). On the one hand, we define

\[
\tan \psi = \frac{H}{G} \tag{9}
\]

where \( \psi \) is the phase of the system transfer function. Operating as in, e.g. [3], we have the relation between the chatter frequency and the spindle speed \( N \), being \( \epsilon \) the phase difference between successive undulations on the workpiece surface and \( k \) the number of complete waves during one period of revolution \( \tau \):

\[
N = \frac{60}{\tau} = \frac{120\pi \cdot f_c}{2\psi + \pi} = \frac{120\pi \cdot f_c}{2\psi + \pi} = \frac{60 \cdot \omega_c}{2\psi + \pi} \tag{10}
\]

In order to obtain the dimensionless version, \( r_t \) is defined as the ratio between the tooth frequency \( f_t \) and the natural frequency \( f_n \) of the system, where

\[
\frac{f_t}{f_n} = \frac{N}{60} = \frac{1}{2\psi + \pi} \tag{11}
\]

being \( n_t \), the number of teeth of the tool \((n_t = 1 \text{ in the case of turning})\). Then, by using (10) and (11)

\[
r_t = \frac{N}{60f_n} = \frac{2\pi \cdot r}{2\psi + \pi} = \frac{2\pi \cdot r}{2\psi + \pi} \tag{12}
\]

where the phase could be obtained by

\[
\epsilon = 2 \cdot \tan^{-1} \left( \frac{-2\xi \cdot r}{1 - r^2} \right) + \pi \tag{13}
\]

From equations (6) and (10), a stability diagram between the limiting depth of cut \( b_{\text{lim}} \) and the spindle speed \( N \) may be created, by giving proper values to the chatter frequency \( \omega_c \). Besides, we may also create a dimensionless stability chart between \( r_b \) and \( r_t \) from expressions (8) and (12)-(13), using now \( r = \omega_c/(\omega_n) \) as variable. The dimensionless stability diagram has the advantage of allowing comparisons among different machining conditions, for systems showing the same
damping ratio $\zeta$. In this way, the stability study turns out to be more general and independent from machining circumstances.

B. CHATTER HIL SIMULATOR SETUP AND EXPERIMENTAL PRELIMINARY RESULTS

The regenerative feedback model of chatter presented in Figure 2 yields the conceptual idea of the HiL simulator. A steel cantilever beam represents the dynamics for a turning machine $\Phi(s)$ and an actuator (“shaker”) applies the cutting force $F(t)$ -expression (2)- to it. A sensor installed on the beam provides the vibration displacement $\gamma(t)$ and a fast processor, included on a programmable automation controller (PAC), calculates $F(t)$ in real-time. It is inspired by the interesting work by Ganguli [32]. Anyway, Ganguli did not correct the effect of the delays introduced by the chatter simulator hardware. Such delays have a great practical importance in the process stability, as will be shown later. Besides, he did not use the dimensionless environment to reproduce the behavior of any machine-tool with a given damping ratio. In this way, more general and confident experimental results are obtained in the present work.

The details about the mechanical design and the equipment may be consulted in [34]. The described design has been carried out in the laboratory and may be observed in Figure 3. In addition, the dynamical parameters are presented in Table 1. Once the beam vibrates as the tool of a turning machine suffering regenerative chatter, an inertial actuator may be attached to it to test the performance of different active control laws -see e.g. [25]. In fact, an actuator from Micromega Inc. -not yet included in Figure 3- has been attached to the beam, before the study on the dynamics of the system.

The vibration modes and damping of the cantilever beam are consistent with respect to real turning tools. On the other hand, the actual tools usually have more stiffness –see Table 1-. However, since dimensionless approach shall be deployed, the stiffness of the cantilever beam do not need to be consistent with respect to real turning tools. Indeed, as far as the damping is consistent, experimental results may be extrapolated to any mechanical system with similar damping ratio. Besides, we also offer the possibility of modifying such damping ratio to extend the study to any other machining scenario, observing some restrictions, described later, in section IV.A.

The constant feed of the tool into the workpiece and the cutting coefficient of the metal have been considered constants ($h_0 = 1e^{-6}$ m. and $K_f = 2500$ N/mm², respectively) in the regenerative effect model. Then, for different values of the spindle speed $N$, the depth of cut $b_p$ has been increased until the vibration becomes unstable (the modeled breaking of the chip saturates the vibration within some limits). In this way, the experimental stability lobes have been obtained, formed by discrete points, marked as red crosses, and are presented in Figure 4. By measuring the frequency of the vibration in such limiting points, Figure 5 is configured.

Then, to compare experimental and theoretical results, the stability lobes –see section II. A.- for this structure have been obtained from expressions (6) and (10), by giving values to $N$ and $k$. Such stability lobes are also depicted in blue in
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FIGURE 6. Closed-loop feedback model for regenerative chatter with the actuator delay included.

Figures 4 and 5. Only the first vibration mode is considered, given that the damping of the second mode is more than double of the first one -Table 1-, so the lobes have double of $b_{p\text{lim}}$. Anyway, as commented in section IV. A., the second mode acquires later some unexpected practical importance by causing certain usage limitations for the chatter HiL simulator, when using the dimensionless environment. In fact, it results important, at a practical level, that the used structure does not have modes with a similar dynamic stiffness, as is the case in this work.

A noticeable deviation is observed between experimental and theoretical results –see also [33], [34]-, partly preventing the desired behavior of the HiL simulator from occurring. In addition, to obtain more general results, the dimensionless stability diagrams –see section II. A.- should be obtained and the system damping ratio $\zeta$ selected at will. Thus, the main goal of this experimental set-up is to allow the confident comparison, within a perfectly repetitive machine-working scenario, of different anti-chatter active control strategies. In this way, we may infer the best choice for each machining scenario, understanding that each scenario may be defined, in a general way, by the damping ratio provided by the machining conditions (dimensionless environment). By doing so, the stability study turns out to be more general and, indeed, independent from machining circumstances, including the properties of the cutting tools.

III. OPTIMIZATION OF THE HARDWARE-IN-LOOP CHATTER SIMULATOR

As it could be extracted from the preliminary results presented in section II. B, two challenges have arisen to obtain a reliable and versatile chatter HiL test bench. On the one hand, the deviation observed between preliminary experimental and theoretical results should be clarified and, if possible, corrected. On the other hand, dimensionless stability diagrams for different values of system damping ratios should be extracted. Sections III and IV are devoted to describing the efforts dedicated to attain such goals.

A. ACTUATOR DELAY MODELING

At this point, the delay of the actuator, described in the phase diagram -see Figure 6 in reference [34]-, is put forward as the main source of the discrepancies observed in Figures 4 and 5 –see [33], [34]-. In order to confirm such a hypothesis, the actuator delay $\tau_a$ has been got into the regenerative chatter model -see Figure 6-.

Thus, the transfer function describing such a system is

$$h(s) = \frac{1}{1 + K_f \cdot b_p \cdot \Phi(s) \cdot e^{-s\tau_a} \cdot (1 - e^{-s\tau_a})}$$

And operating from (14) in the same fashion as in section II. A. from expression (4), we have

$$b_{p\text{lim}} = \frac{-1}{2 \cdot K_f \cdot [G(\omega_c) \cos(\omega_c\tau_a) + H(\omega_c) \sin(\omega_c\tau_a)]}$$

that describes the limiting values for the depth of cut to maintain the stability of the system when the delay of the shaker is considered. Now, the stability lobes are again obtained and presented in red curves -see Figures 7 and 8-.

As may be observed in Figure 7, now the correspondence with the experimental results about the critical depth of cut has been greatly enhanced. However, in figure 8 you may see notable discrepancies for high values of the chatter frequency. This is probably due to the imperfect linearity of the shaker phase -see again Figure 6 in reference [34]-. It must be taken into account that the phase function has been considered linear to obtain the $\tau_a$ delay in expressions (14)-(15) for the whole chatter frequency range. As we will see in the next section III. B. 2., this effect will have important consequences. Anyway, the importance of the $\tau_a$ delay has been confirmed.
Step 1: Procurement of the feasible chatter frequencies for each \( N \) (rpm): First, from any given value of the spindle speed, a set of feasible values for the chatter frequency \( f_c \) is obtained from the blue \( f_c/N \) curves of Figure 9 - the lobes we want to reproduce, related with the number of full waves between cuts \( k \).

Step 2: Election of the actual chatter frequency for each \( N \) (rpm): By using the \( b_{\text{lim}}/N \) functions that we want to implement - blue curves of Figure 9-, the factual value of \( k \) is obtained (corresponding to the lowest stability lobe) and, by using such \( k \), the exact \( f_c \) (accuracy is important) is elected from the set obtained in step 1.

Step 3: The compensating delay is obtained: By using the values of \( f_c \) and \( k \) obtained in the previous step, the compensating delay between successive passes of the tool is

\[
\tau' = \frac{2\pi k + \epsilon'(f_c)}{2\pi f_c} \tag{16}
\]

where, evidently, now \( \tau' \) do not match \( 60/N \), but will be the value introduced in the HiL simulator, in order to compensate the hardware delay. The compensating phase between subsequent undulations \( \epsilon'(f_c) \) is calculated, considering the delay of the shaker \( \tau_0 \), by

\[
\epsilon'(f_c) = 2\Psi'(f_c) + \pi \tag{17}
\]

given that

\[
\Psi' = \arctan \frac{H'}{G} \wedge \Phi'(s) = \Phi(s)e^{-s\tau_0} = G'(\omega_c) + jH'(\omega_c) \tag{18}
\]

Step 4: The compensating force “constant” of the process is obtained: Finally, by using the value of \( f_c \) obtained in step 2, the compensating force “constant” \( K_f' \) for the studied process is calculated by equalizing equations (15) and (6) to implement the stability lobes “without” the deviation due to the delay of the shaker, resulting in:

\[
K_f' = K_f \left[ \frac{G(\omega_c)}{(G(\omega_c) \cos(\omega_c \tau_a)) + H(\omega_c) \sin(\omega_c \tau_a)} \right] \tag{19}
\]

Remarks:

- The “compensating” items (16) and (19) must be recalculated for each value of the spindle speed \( N \). In this way, they turn out to be factual variables in the HiL software. Until now, the delay between passes and the force constant for the process have been considered as fixed parameters on the HiL chatter simulator.

- Evidently, when the delay of the shaker \( \tau_a \) tends to 0, \( \epsilon'(f_c) \) tends to \( \epsilon(f_c) \) and \( \tau' \) to \( \tau = 60/N \). In the same fashion, \( K_f' \) will converge to \( K_f \).

2) EXPERIMENTAL TESTS AND ERRORS DESCRIPTION

By implementing such a procedure on the processor of the cRIO, the experimental results are obtained and presented as blue crosses in Figures 9 and 10. In this case, the delay compensation is again achieved, but the deviation is significant and its causes deserve explanation.

B. ACTUATOR DELAY COMPENSATION

As it has been proved in the previous subsection III. A., the delay of the actuator means an important drawback to reproduce exactly the behavior of the assumed feedback model of regenerative chatter. At this point, it is useful to recall that the main goal is to reproduce, as exactly as possible, on the mechanical structure located at our lab, the behavior described by some stability diagrams - for example, the blue curves in Figures 4 and 5 -.

Bearing in mind the previous facts, a procedure –different from that proposed in [33]– for the compensation of the actuator delay is considered. The flexibility of the chatter HiL simulator, provided by the system software, will be used to compensate for the delay imposed by the hardware. Such a procedure, which takes advantage of the knowledge on the regenerative effect model, was formally proposed in [34]. Now, it has been experimentally tested, corrected, and, finally, validated here.

1) FICTITIOUS VARIABLES METHOD

Specifically, two parameters - the cutting coefficient of the process \( K_f \) and the period of revolution \( (\tau) \), considered \textit{a priori} as prefixed (for each workpiece-tool pair and spindle speed), will be used here as fictitious variables to compensate for the delay of the actuator (and other components, if necessary). The mentioned procedure is divided into four sequential steps - see [34] for further details -:
Evidently, the deviation observed in the experimental points obtained without correction (red crosses) in relation to the delayed mathematical model (14) (red solid line), especially in the chatter frequency -see Figure 10- is the origin of the errors occurred in the correction process. The proposed procedure uses the mathematical model, but the real behavior of the structure is described by the red crosses and, mainly the chatter frequency for high values of $N$, both are quite different. The problem is that, as it was remarked when step 2 was described, the accuracy is rather important. Let’s see how the error occurs when the procedure is applied for, e.g., $N = 4800$ rpm. In steps 1 and 2, the “desired” chatter frequency ($f_c = 125.7$ Hz.) is obtained from the blue solid line in Figure 10 (Point A. $N = 4800$ rpm). Step 3, by applying the mathematical model -expressions (16)-(18)-, provides the $\tau'$ relating to the $N' = 60/\tau'$ that gives such chatter frequency (Point B. $N' = 5361$ rpm). Nonetheless, the exact real behavior should be expected from the experimental characterization of the mechanical structure given by the red crosses. In this way, a $5^{th}$ order polynomial interpolation is extracted from such points, named as $f(N)$ in Figure 10 (dashed red line), and then we should impose $N'' = 5481$ rpm (Point C) to really obtain the desired $f_c$. Otherwise, the obtained frequency is $f'_c = f(N'') = 124.9$ Hz. (Point D), marked as a blue cross in Figure 10 for $N = 4800$ rpm. Finally, step 4 obtains the compensating force “constant” $K_f'$ but, if we use the model (Point B in Figure 9 for $N' = 5361$ rpm), the denominator of expression (19) will be $b''_p = 0.5098$ instead of the real needed value (Point C for $N'' = 5481$ rpm) $b''_p = 0.6753$. Such difference makes $K_f'$ to be too big and the chatter occurs too soon. In this way, the experimental corrected point (blue cross) in Figure 9 is located rather below the desired lobe (solid line in blue).

3) PROCEDURE CORRECTION AND EXPERIMENTAL VALIDATION

In order to avoid such errors, the interpolated experimental characterization $f(N)$ is used instead of the mathematical model to apply the compensating procedure. The obtained results acting that way are presented in Figures 11 and 12. The accuracy of the delay compensation now is very high, due to the use of the experimental characterization of the mechanical structure, previously obtained.

Note 1: In addition to the high accuracy, there is another major advantage related to the presented method. It may be easily deduced, from expressions (16)-(19) and the explanations presented above, that the “desired” stability lobes to reproduce in our HiL may be almost freely -with some limitations, commented later- imposed. In this way, the chatter HiL simulator may provide a suitable test-bench for different general turning machines and machining conditions.

IV. DIMENSIONLESS LOBES WITH DIFFERENT DAMPING RATIOS

Until now, the obtained stability lobes have been created by shaping diagrams between the chip width $b_y$ and the spindle speed $N$ (and the chatter frequency versus $N$). But, we may also create -see section II. A. dimensionless stability charts between $r_b$ and $r_t$ from expressions (8) and (12)-(13), using $r = \omega_c/\omega_n$ as a variable. It is well known that the dimensionless stability diagram has the advantage of allowing comparisons among different machining conditions, for systems showing the same damping ratio $\zeta$ -see section II. A.-. Thus, the stability study turns out to be more general and independent from machining circumstances. Anyway, to extend the study, even more, we have to develop a new procedure to impose different values on the damping ratio of the system under study. Another method to do that, by using a DVF extra loop, may be consulted in reference [33].

Nevertheless, exploiting the property, commented in note 1, of the method described in section III.B. and conceived to compensate for the delay of the used equipment, different dynamic behaviors could be also imposed on the simulator. In this way, different damping ratios may be reproduced in the test-bench, simply by previously calculating the stability lobes corresponding to different damping ratios and introducing them as the “desired” lobes to the mentioned method.

For verification purposes, the obtained experimental results in this way are presented next for a damping ratio of 2 %. As we can see in Figures 13 and 14, the obtained experimental results (blue crosses) are very tightly matched with the stability diagrams for a turning system with $\zeta = 0.02$.
stability diagrams related to systems with a larger damping ratio bring about higher chatter frequencies. To obtain such frequencies, we are forced to use higher values for \( N'' \) -see related explanations in subsection III. B. 2.-. The higher the values of \( N'' \), the sooner (smaller values of \( N \)) that the second mode arises on the beam.

**Note 3:** See, for example, the three experimental points circled in a black ellipse in Figure 9 (also appeared in Figure 11). Such points are related to chatter in the second mode, near 375 Hz. (that is why the corresponding chatter frequencies are not depicted in Figures 10 and 12) and the compensating methods can’t be applied without an extension study devoted to include the second mode.

As a direct consequence of such effect, the damping ratio to implement on the HiL system is limited to values slightly above 4 %, given that, in such cases, the second mode chatter appears before the reaching of the experimental minimum value of \( b_{\text{plim}} \). Evidently, a mechanical redesign of the HiL structure may be carried out to lift the second mode and extend the available chatter frequency range and, consequently, the range for the feasible damping ratios –see reference [33].

**V. CHATTER HARDWARE-IN-THE-LOOP TEST BENCH OPERATION**

The HiL test bench software is used intensively during the experimentation, given that a big number of critical points should be obtained to characterize the stability of the different processes. In this way, a practical Graphical User Interface (GUI) will be welcomed. Once that such GUI is developed, the experimental tests on active control of chatter may be performed more conveniently. A video file, illustrating the operation of the chatter HiL simulator and the active control tests performed on it, has been added to this publication.

**A. GRAPHICAL USER INTERFACE**

Let’s recall the experimental procedure. First, the spindle speed \( N \) and the depth of cut \( b_{\text{c}} \) are selected. Then, an external perturbation is introduced in the mechanical structure (it may be programmed and carried out by the shaker, or manually provoked with an impact hammer) and the temporal evolution of the vibrations are observed, in order to detect critically stable behavior due to the regenerative effect. The initial value of the depth of cut is low and, then, it is progressively increased until the critical value \( (b_{\text{plim}}) \) for this spindle speed \( (N) \) is reached (maintained vibration). At this moment, the chatter frequency \( (f_{\text{c}}) \) is obtained, with sufficient resolution, by a Fast Fourier Transform (FFT). Such values \( (N, b_{\text{plim}}; f_{\text{c}}) \) are stored and graphically represented on the screen. Thus, point by point, the stability lobes of interest are obtained by scanning the necessary values of \( N \).

The software is programmed by using LabView® from National Instruments and is composed of three Virtual Instruments (VIs): one running on the Host-windows, one on the cRIO Host-RT, and the last one on the cRIO FPGA. Anyway, the user must execute just the VI installed on...
the Host-windows, given that such VI executes afterward the other two, in proper sequence. The basic configuration (select or not the adimensional mode, introduce the damping ratio or the dynamic parameters of the system under study, select the interpolation method for the f(N), the representation options, etc,...) and the required parameters for each experiment (the N and $b_p$) are introduced by the user in this VI -see Figure 15-. Once the experiment is finished, the obtained experimental results are displayed in the corresponding graphics over the reference (theoretical or experimental) lobes, obtained by the VI, or introduced externally.

Each experiment requires a detailed visualization of the actual vibration in order to detect the critical points. This visualization and the calculation of the chatter frequency are processed on the cRIO Host-RT, which panel may be observed in Figure 16. In such a figure, a case of critical stability is presented: the initial perturbation occurs about the 30000 sampling instant (3 seconds from the beginning of the experiment) and the resulting vibration maintains its amplitude (upper left screen) since then. In such a case, the FFT of the collected data during this vibration (upper right screen) must be implemented and the result is presented numerical and graphically (lower screens). Then, the results are sent to the Host-windows VI. The volume of the collected data for the procurement of $f_c$ may be also selected by the user to balance the FFT resolution and the duration of the experiment and the processing of the data.

Finally, the VI running on the cRIO FPGA is in charge of the RT execution of the chatter model to create the regenerative effect. To do so, the vibration -position- is acquired, the variable force is calculated, considering the actual machining parameters, and then converted to a voltage signal, input to the shaker actuator. In addition, all the acquired and generated data are stored by using several FIFO registers and then transferred to the Host-RT, by using the DMA channels of the cRIO. In the Host-RT, such data is accumulated on TDMS files for possible (if the point under study turns out to be critical -see explanations above about Host-RT VI-) subsequent analysis. The start and stop of this VI are implemented by the Host-RT VI, after receiving the necessary values for each experiment. Such values and other information -the VI includes some additional filters for the signals process-about the VI execution is presented on the VI screen -see Figure 17-. Anyway, the execution of the VI is completely transparent for the user and, therefore, the visualization of this panel is optional.

### B. ACTIVE CONTROL TESTS

Now, we have the HiL system prepared to accurately reproduce the regenerative chatter suffered by different turning machines. The next step to study the performance of different algorithms of active control is based on the usage of an inertial actuator, attached to the mechanical structure of the HiL system, as can be seen now in Figures 18 and 19. In this case, the used inertial actuator is an IA-01-S from Micromega Dynamics, whose specifications can be seen in Table 2. Such an actuator is initially placed on the same vibration injection point used by the Shaker –see Figure 18- for the preliminary study on the potential of the new control algorithm -see section VI. A.-. Later, in order to create a more realistic scenario, it is non-collocated, 4 cms. away from the variable
force injection point (stinger location) –see Figure 19 and section VI. B.-. Actuators for active chatter control are often, for practical reasons, mounted farther away from the cutting tool in real applications. However, our only intention now is to demonstrate that mounting the actuator in a more realistic fashion does not imply a drastic change in the results obtained by comparing the different active control algorithms.

The inertial actuator is connected to a preconfigured Rack 02-01N control unit also from Micromega dynamics –see specifications in Table 2-. This control unit allows for the fast setup of a complete active damping solution with one or two 1N inertial actuators. Thanks to a robust embedded classical [31] Direct Velocity Feedback (DVF) controller included in the control unit, high damping ratios can be achieved on almost any structure or machine in minutes. Users may also eventually by-pass/disable such an internal controller and connect their own in order to investigate alternative active control laws, in an additive or independent way –Figure 20-.

Real-time deterministic implementation of the new controllers (Variable Force Compensators) is performed by using a rapid-prototyping system completely independent of that used in the HiL simulator. Such a system is based on the Simulink Real-Time tool from MathWorks. The execution takes place on an industrial computer with a processing power very similar to that offered by actual machine tool control platforms. The diagram describing the complete real-time test-bench may be consulted in Figure 20. Now, different active control laws may be tested in the HiL test bench.

VI. VARIABLE CUTTING FORCE COMPENSATION

At this point, a new type of active chatter control is considered. Actually, such a controller is tangentially inspired by the work done in the HiL simulator to reproduce the regenerative effect as realistically as possible. The idea behind it is: we have been able to generate the variable cutting force realistically, may we also be able to compensate for it by acting inversely in the same way?.

Obviously, this will never be entirely true, due, mainly, to the impact of the delays of the electronic equipment, the regenerative effect model mismatch, and the actuator limitations (delay, saturation, bandwidth, non-colocated position, ...). As seen in the previous sections, we may correct the effects of such delays, but it has been thanks to a lot of information that the active controller will not actually have. Therefore, the interest of the following experimental study is centered on verifying if the uncorrected effect of the delay in the operation of the anti-chatter controller is capable of completely disrupting the control action or if, on the contrary, it is still possible to substantially delay the appearance of the instability in the cut. In addition, the effect of the model
parameter uncertainty and the actuator limits may also be investigated.

To implement the new controller, we have to first calculate the tool position \( y(t) \) from the accelerometer signal, taking into account the sensor sensitivity \( K_s \) —see Figure 21—. Then, the variable force, expected from the regenerative effect, is obtained by using the known machining parameters \((K_f, b_p, N)\) in the theoretical model and, finally, such a force is implemented, in counter-phase, considering the actuator gain \( K_f \). Since, in practice, some machining parameters are known with little accuracy and/or have some variability, we have introduced an error of -10% in the \( K_f \times b_p \) term with respect to the one used in the chatter HiL simulator. A detailed scheme of the implementation may be seen in Figure 21.

Obtaining the position of the tool \( y(t) \) from the acceleration is quite laborious due, above all, to the sensitivity of the integrators to the offset that can appear in the input signals. This forces us to eliminate such offsets employing high-pass filters before each integration. On the other hand, the \( K_f \) gain in Figure 21 is used to keep the tool speed values \( \dot{y}(t) \) within the optimal range of values for its computation. Finally, we use the \( K_s \) gain to calibrate the tool position measurement \( y(t) \) obtained, compensating the gain errors that the filters may have created. This calibration is carried out by comparing this measurement with the related position laser measurement.

Additionally, the position measurement is cleaned, using high and low-pass filters in series, before entering the chatter force model. Thus, the relatively high number of required digital filters causes the delay (phase) in obtaining \( y(t) \) to increase. Alternatively, the tool position may be obtained by performing the double integration by means of hardware, as it is done in the recent work [36], which reports good results.

The algorithm described in figure 21, called Variable Force Compensator (VFC), is discretized with the same sampling period that uses the HiL chatter simulator (\( T=1 \times 10^{-4} \text{ s.} \)) and then executed deterministically (in hard real-time) on the Target of the test-bench described in section V.B. —see also Figure 20—. Initially, to preliminarily explore the potential -maximum- performance of the new VFC controller —see section VI.A.—, the real-time control tests are performed using a colocated sensor and actuator configuration —Figure 19—.

Besides, in such a preliminary study, the remedies described in section III to optimize the operation of the HiL simulator are not yet used. In this way, the effect of the HiL simulator enhancements on the performance of the studied control algorithms will be evident later on —see Notes 4 and 5 below-. Then, in section VI.B., to provide a more realistic scenario and obtain general results, independent of the specific machining parameters, the non-colocated configuration of sensors and actuators —Figure 19— is used, as well as the optimization options of the HiL simulator described in section III.

A. PRELIMINARY EXPERIMENTAL STUDY: POTENTIAL PERFORMANCE RESULTS

As stated before, this preliminary study aims to explore the potential -maximum- performance that may be reached by the proposed VFC controller, without yet trying to generalize the results, nor to accurately associate the dynamic behavior of the tool with a specific machining process. Thus, now our interest is limited to comparing the stability limits achieved with such a VFC controller with respect to the ones to be obtained without control and through a reference active control method (DVF). In this way, the real-time control tests are implemented initially without correcting the effects of the delays and without altering the damping of the mechanical structure of the chatter HiL simulator. Moreover, note that not correcting the mentioned delays makes the dynamic behavior of the chatter HiL simulator not able to be associated with a specific machining process accurately, but in principle, it would say that does not invalidate the comparison between several active chatter controllers, as it was done in previous works, e.g., [32]. Anyway, the possible impact of these corrections on controller’s performance must be carefully monitored and analyzed —see Note 4—.

First, the \( K_s \) gain —see Figure 20— is adjusted by following the Micromega manufacturer’s instructions and the built-in DVF control loop (control ON in Figure 20) is activated. Then, the \( r_s \) values that mark the stability limit —see a detailed description of the procedure in section V— are again obtained. Such values are represented by magenta asterisks, plotted over the uncontrolled experimental results in red crosses, for the stability lobes using the values \( k = 2 \) to 6 in equations \((8)\) and \((12)\), to obtain Figure 22. A substantial improvement, close to 90 %, in the minimum stability, is attained.
machines (circles = stable; cross = critical): Without active control (blue), DVF (magenta), VFC (black) and DVF+VFC (green).

FIGURE 23. Experimental general results for 2% damped turning processes. Stability lobes without control. However, in the case of chatter, the DVF control is deactivated (control OFF in Figure 20) and new stability tests, with the VFC control loop activated, are implemented. The $r_p$ values obtained in this way are plotted also in Figure 22 by using black asterisks. In this case, an improvement in the minimum process stability slightly higher, close to 100%, compared to the uncontrolled case, is achieved.

It may be seen that, with the VFC controller, an apparent improvement somewhat superior to that obtained with the standard built-in DVF is achieved. However, the DVF algorithm maintains an important practical advantage with respect to the newly proposed VFC algorithm, given that DVF does not need any prior knowledge about the machining process parameters. Meanwhile, the VFC needs to know approximately the value of $K_f$, $N$, and $b_p$ for each turning machining process. Remember that a −10% error in $K_f$ or $b_p$ is used here to reproduce approximately such effect, but it may be greater in practice under some circumstances. Therefore, it is not clear which of the algorithms is more convenient.

However, at this point, we realize that it is not necessary to make both control strategies contend. Both take action on different facets of the system dynamics: the DVF on the damping of the closed-loop system and the VFC on the regenerative effect itself. From this reasoning, it is presumed that both controllers may be applied simultaneously while sharing the same inertial actuator. Thus, we proceed to activate simultaneously (control ON again in Figure 20) the same controllers that we have previously studied separately, using the same settings. The stability lobes represented in Figure 22 with green asterisks are now obtained. There is a 240% improvement in the low stability level, higher than the sum of the improvements obtained previously with the DVF and VFC controllers separately, which is what we would expect a priori -in the best case-. This may be due to the combined effect of acting prematurely on the regenerative effect while having also increased the damping of the system.

Note 4: As said before, the experimental results obtained without correcting the chatter simulator hardware delay should be observed in order to detect other effects. In that way, looking carefully at figure 22, it may be seen that the effect of the DVF consists exclusively, as expected, in raising the stability lobes without control. However, in the case of the VFC, besides the remarkable increase in the stability, also the elimination of the delay effect with respect to the uncontrolled case may be noticed. In particular, the VFC lobes correct the displacement to the right, coinciding in shape with the theoretical without-delay lobes. In the DVF+VFC experimental lobes, this effect is partial. This observation induces to think that the VFC control action is being applied in an approximately "synchronized" way with the variable machining force produced by the chatter in the HiL simulator, what would be impossible in a real machining environment.

B. GENERAL EXPERIMENTAL STUDY: REALISTIC PERFORMANCE RESULTS

As stated previously -see section IV-, the dimensionless stability diagrams have the advantage of allowing comparisons among different machining conditions, for systems showing the same damping ratio $\zeta$ -see also section II.A-. Thus, the stability study turns out to be more general and independent from machining circumstances. Therefore, the experimental procedure, described in section VI.A., is applied again, but now correcting the delay of the chatter HiL simulator and generating dimensionless diagrams for all turning processes with a 2% damping ratio. Further stability tests have been carried out for the spindle speed values corresponding to the minimal stable depth of cut. For such values of $r$, the circles represent stable and the crosses unstable tests. These experimental results are added to those obtained without control -Figure 13- using again blue color for the uncontrolled delay-free case, black for VFC, magenta for DVF control, and green for DVF+VFC, resulting in Figure 23. The procedure for obtaining such results may be seen in the video file attached to this work. In particular, the obtaining of the critical stability point $r = 0.58$, $r_{\text{lim}} = 1.33$, (V point in Figure 23), with the DVF+VFC control active, is shown there.

As we can see, an improvement near 22% on the stability of the turning process has been experimentally demonstrated for this type of system and the DVF active control law. It may be also seen there that the appearance of the instability has been held back in a modest way (−10%) by the VFC. Finally, DVF and VFC have been also implemented now in an additive way. The result has proved to be good, delaying the appearance of instability in a significant way (−32%), bringing about as it may be seen as an additive improvement in this case.

Note 5: Now, the VFC results clearly less effective than the DVF, despite using the same inertial actuator, possibly due to the known insensitivity of the DVF to the delay on the control loop. Furthermore, probably the VFC was favored, during the preliminary study in section VI.A., by the uncorrected delay that appeared in the HiL simulator. Such delay is similar to the one that appears in the real-time execution of the VCF control prototype, so that both may be “balanced” to some extent -see Note 4 above-. But that does not happen in practice. Thus, the practical importance of the compensation of the delay in the chatter simulator is demonstrated, given that the results obtained otherwise -section VI.A.- are not totally trustworthy, since the VFC performance seems to have been overestimated.
Anyway, it has been experimentally shown that, when important aspects of the practical implementation are taken into account (delays, non-collocated sensors and actuators, errors in machining parameters, . . . ), the performance that can be achieved by the VFC algorithm is impaired with respect to its potential results. In fact, its use is only justified in additive combination with the classical DVF method, which is already widely used in the industry -see e.g. [37]-. Therefore, to extract a higher performance from such an algorithm, closer to its potential, more efforts must be implemented in order to improve the application of the corrective force, in the right magnitude, and at the right time.

VII. CONCLUSION

Active control of regenerative chatter is of high industrial interest, due to the remarkable productivity increases that may be obtained. Obviously, to study and compare the performance of such active chatter control systems, it is very interesting to be able to perform intensive real-time control tests in a reliable and repetitive way. However, it is difficult to do this on the real machine, given the rapid alteration of the properties of the cutting tool and the large number of machining parameters to be considered. In this work, the regenerative effect has been accurately recreated in the laboratory over an optimized Hardware-in-the-Loop simulator, by carefully eliminating the effects of the delays introduced by the equipment. Besides, the behavior of machines with different damping ratios are emulated. Then, by using dimensionless notation, general experimental results, independent of the machining parameters, are obtained. By coupling an inertial actuator to the mechanical structure of the chatter simulator, different active chatter control algorithms are reliably tested.

Finally, a new active chatter control algorithm, focused on compensating the variable force created by the regenerative effect, has been proposed and tested in a realistic scenario. The results obtained in combination with a classical active control algorithm show that the degree of stability of orthogonal cutting processes may be substantially improved. On the other hand, it is confirmed that it results mandatory to compensate for the delay that appears in the chatter HiL simulator in order to obtain reliable control performance experimental results.

As future work, the results obtained in the chatter simulator should be validated with some verification tests on the real machine, since real industrial problems of active systems, such as the actuator force saturation must be considered in control strategies’ comparison. Then, it is proposed to try to reduce the impact of certain aspects of the practical implementation on the performance obtained by the new control method. In particular, it is suggested to reduce the effect of the delays caused by the real-time execution of the control algorithm and that introduced by the inertial actuator. Such delays may be accurately measured and, therefore, it is possible to try to compensate them, in full or in part. In addition, it is also feasible to reduce the important effects that restrictions and uncertainty in machining parameters have on the controller performance to be achieved, by using e.g. model-based predictive control.

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