**D_{s1}^*(2710) and D_{sJ}^*(2860) in the \( \tilde{U}(12) \times O(3,1) \)-scheme**

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Abstract. In order to classify the charmed-strange mesons, including puzzling \( D_{s0}^*(2317) \) and \( D_{s1}(2460) \), and recently observed \( D_{s1}^*(2710) \) and \( D_{sJ}^*(2860) \), we employ the \( \tilde{U}(12) \times O(3,1) \) level-classification scheme of hadrons proposed and developed by us in recent years. The scheme has a new degree of freedom, \( SU(2)_\rho \), which leads to a number of extra states out of conventional \( SU(6) \)-scheme, named chiralons. Applying this novel classification scheme, we investigate the strong decays of \( S \)- and \( P \)-wave \( c\bar{s} \) mesons with one pseudoscalar emission by using the covariant oscillator quark model. As a result it is shown that \( D_{s0}^*(2317) \) and \( D_{s1}(2460) \) are described as \( 1S^{\pi}_\rho \) and \( 3S^{\pi}_\rho \) chiralons, forming the \( SU(2)_\rho \) doublet with the ground-state \( D_s \) and \( D_{s1}^* \), respectively. Furthermore, the observed decay properties of \( D_{s1}^*(2710) \) is consistently explained as the vector chiralon \( 3P_1^{\rho} \). On the other hand, it is also found that the controversial narrow state, \( D_{sJ}^*(2860) \), does not fit as predicted properties of our \( P \)-wave vector chiralon.

Keywords: \( D_{s1}^*(2710) \), \( D_{sJ}^*(2860) \), Strong Decays, Covariant Oscillator Quark Model, the \( \tilde{U}(12) \times O(3,1) \)-scheme

PACS: 14.40.Lb,13.25.Ft,12.39.Ki

**INTRODUCTION**

In recent years we have proposed the \( \tilde{U}(12) \times O(3,1) \) level-classification scheme [1] of hadrons, which corresponds to a covariant extension of the non-relativistic \( SU(6) \times O(3) \)-scheme. In the new scheme, wave functions (WF) of composite hadrons are generally given by irreducible tensors of the \( \tilde{U}(12) \times O(3,1) \). It is to be noted that, at the rest frame of hadrons, the representations of the \( \tilde{U}(12) \) reduce to those of the \( U(12) \). Hence the hadronic states can be classified by representations of the \( U(12) \). The \( U(12) \) includes a new degree of freedom \( SU(2)_\rho \), called \( \rho \)-spin, in addition to conventional \( SU(6) \) (\( SU(2)_\sigma \times SU(3)_F \)), as \( U(12) \supset SU(6) \times SU(2)_\rho \). An important feature of the scheme is that we include, apart from the \( O(3,1) \) part, the '\( \rho \)-spin down' (\( \rho_3 = -1 \) component, which is treated as a fundamental building block to construct the WF, effectively realizing only inside hadrons. Accordingly, we expect to exist a number of extra states out of the \( SU(6) \)-framework, which is called chiralons. 3

To check the validity of our new classification scheme, we investigate systematically the strong decays of \( c\bar{s} \) mesons with one pseudoscalar emission by using the covariant oscillator quark model (COQM). Through the observed mass and results of decay study we present possible new assignments for observed charmed-strange mesons from the view point of the \( \tilde{U}(12) \times O(3,1) \)-scheme.

**FRAMEWORK OF THE COVARIANT OSCILLATOR QUARK MODEL**

We briefly recapitulate a framework of the COQM\(^4\) relevant to the present application. The WF of a composite \( s\bar{c} \) meson system is described by a bilocal field \( \Phi(x) \phi(x') \), and its Pauli conjugate defined by \( \Phi(x)\phi(x') = \Phi(x)\phi(x') = \phi(x)\Phi(x') \)

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2 Physical meaning of \( \rho_3 = -1 \) component is discussed in the Ref. [2].
3 More strictly, chiralons are defined as the states which includes at least one \( \rho_3 = -1 \) component in the spin WF.
4 The COQM have been successfully applied to various (static and non-static) problem of hadrons for many years. As relatively recent application, for example, see Ref. [3] and references therein.
\((\gamma_I)_{\alpha'}^{\alpha} (\Phi^I)_{\alpha'}^\beta (\gamma_I)_{\beta'}^\beta\), where \(\alpha\) and \(\beta\) represent Dirac spinor indices of \(s\)- and \(\bar{c}\)-quark, \(X\) and \(x\) denote the center of mass (CM) and relative coordinate four vectors, respectively.

We start from the Klein-Gordon type equation,

\[
\left( \frac{\partial^2}{\partial X^2} - \mathcal{M}(x)^2 \right) \Phi(X,x)_{\alpha}^{\beta} = 0. \tag{1}
\]

The squared-mass operator\(^5\) (in the pure-confining harmonic oscillator (HO) potential limit) is given by

\[
\mathcal{M}(x)^2 = d \left[ -\frac{1}{2\mu} \frac{\partial^2}{\partial x^2} + \frac{K}{2} x^2 \right] \left( d \equiv 2(m_s + m_{\bar{c}}), \quad \mu \equiv \frac{m_s m_{\bar{c}}}{m_s + m_{\bar{c}}} \right). \tag{2}
\]

We can define the plane wave expansion concerning the CM motion for mass (CM) and relative coordinate four vectors, respectively.

\[
\Phi(X,x)_{\alpha}^{\beta} = \int \frac{d^3P}{\sqrt{(2\pi)^3 2P_0}} (e^{iPX} \Phi(x,P)_{\alpha}^{(+)} + e^{-iPX} \Phi(x,P)_{\alpha}^{(-)}), \tag{3}
\]

with respect to each level \(M_n = \sqrt{P_0^2 - K}^2\) determined by the squared-mass eigen-equation. In the above the positive / negative frequency parts \(\Phi(x,P)_{\alpha}^{(+/-)}\) denotes the internal WF of relevant \((s\bar{c})\) mesons with a definite HO mass. The complete set of (boosted) LS-coupling basis, generally represented by

\[
\Phi(x,P)_{\alpha}^{(+/-)} \sim f_{\mu_1 \mu_2 \cdots}(v,x) \otimes (W(v)_{\alpha}^{(+/-)}), \tag{4}
\]

is used to expand the internal WF, where \(f(v,x)\) indicates the space-time part, while \(W(v)_{\alpha}^{(+/-)}\) does the spin part. Here \(v_\mu = P_\mu/M\) is four velocity of meson. The concrete expressions of former part \(f(v,x)\), being the eigen functions of \(\mathcal{M}^2\), are given by

\[
f_G(v,x) = \frac{\beta}{\pi} \exp \left( -\frac{\beta}{2} (x_\mu^2 + 2v_\mu x_\mu) \right) \rightarrow \frac{\beta}{\pi} \exp \left( -\frac{\beta}{2} (x^2 + x_\mu^2) \right) \quad (\beta = \sqrt{\mu K}) \tag{5}
\]

for \(S\)-wave ground states and

\[
f_v(v,x) = a_v^\alpha f_G(v,x,v) = \frac{1}{\sqrt{2\beta}} (\beta x_\nu - \partial \partial x_\nu) f_G(x,v) = \sqrt{2\beta} (x_\nu + v_\nu (x_\rho v_\rho)) f_G(x,v) \tag{6}
\]

for \(P\)-wave excited states, respectively. On the other hand, the later part \(W(v)_{\alpha}^{(+/-)}\) consists of the direct product of respective Dirac spinor bases, which simulates the transformation properties of relevant constituent quarks, as

\[
W_r^{(+)}(v)_{\alpha}^{\beta} = u_r^{(0)}(v)_{a}^{\alpha} v_r^{(\gamma)}(v)^{\beta}, \quad W_r^{(-)}(v)_{\alpha}^{\beta} = v_r^{(0)}(v)_{a}^{\alpha} u_r^{(\gamma)}(v)^{\beta}. \tag{7}
\]

Here the index \(r\) represents the eigenvalue of \(\rho_3\) in the rest frame \((v = 0)\). It should be noted that these spinors do not correspond to constituent quarks themselves. In fact, these contain only four velocity of hadron, hence the small component vanishes at the hadron rest frame.

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\(^5\) By imposing the definite metric-type subsidiary condition to freeze a redundant relative-time degree of freedom, we get the eigen-values of \(\mathcal{M}^2\) as \(M_0^2 = n\Omega + M_0^2\), where \(\Omega = d\sqrt{\frac{2K}{d}}\) and \(n = L + 2N\) \((N\) and \(L\) being the radial and orbital quantum numbers respectively), leading linear rising Regge trajectories.

\(^6\) For the anti-quark spinor \(v_r(v)\), it should be understood as \(\rho_3 = -\rho_3\).
S- AND P-WAVE s\̅c MESONS IN THE $\bar{U}(12) \times O(3,1)$-SCHEME

In this work we make the following assumptions for spin WF; only $\rho_3 = +1$ is allowed for c-quark, while both $\rho_3 = \pm 1$ can be realized for (rather) light s-quark.\textsuperscript{7} Resultant WF of S-wave states are given by

$$\Phi(x, P)^{(+)\alpha} = f_G(v, x) \left[ W^{(+)\alpha}_{\perp \nu}(v)|_{\langle S=0 \rangle} + W^{(+)\alpha}_{\perp \nu}(v)|_{\langle S=1 \rangle} + W^{(+)\alpha}_{\perp \nu}(v)|_{\langle S=0 \rangle} + W^{(+)\alpha}_{\perp \nu}(v)|_{\langle S=1 \rangle} \right] \beta$$

$$= f_G(v, x) \frac{1}{2 \sqrt{2}} \left[ (i \gamma_\nu \tilde{D}_{\nu\nu}(P) + i \gamma_\mu \tilde{D}_{\mu\nu}(P) + \tilde{D}_{\nu\nu}(P) + i \gamma_\nu \gamma_\mu \tilde{D}_{\mu\nu}(P) + 1 + \frac{i p_\mu}{M_1} \right] \beta,$$

(8)

where $\{D_0, D_{10}, D_{20}, D_{12}, D_{21}, \bar{D}_{12}, \bar{D}_{21}, \bar{D}_{20}, \bar{D}_{10}\}$ represent local s\̅c meson fields with $J^P = \{0^-, 1^-, 0^+, 1^+\}$, respectively. A superscript $\chi$ implies chiralon, s-quark being $\rho_3 = -1$. It should be noted that, in the relevant case, chiralons always have their ‘partners’ with opposite parity, same spin $J$, forming the $\rho$-spin doublet.\textsuperscript{8} Similarly, WF of P-wave states are given by

$$\Phi(x, P)^{(+)\alpha} = f_V(v, x) \left[ W^{(+)\alpha}_{\perp \nu}(v)|_{\langle S=0 \rangle} + W^{(+)\alpha}_{\perp \nu}(v)|_{\langle S=1 \rangle} + W^{(+)\alpha}_{\perp \nu}(v)|_{\langle S=0 \rangle} + W^{(+)\alpha}_{\perp \nu}(v)|_{\langle S=1 \rangle} \right] \beta$$

$$= \sqrt{2} \beta x_i f_G(v, x) \frac{1}{2 \sqrt{2}} \left[ (i \gamma_\nu \tilde{D}_{\nu\nu}(P) + i \gamma_\mu \tilde{D}_{\mu\nu}(P) + \tilde{D}_{\nu\nu}(P) + i \gamma_\nu \gamma_\mu \tilde{D}_{\mu\nu}(P) + 1 + \frac{i p_\mu}{M_1} \right] \beta,$$

(9)

where the local fields $\{D_{1\nu}, D_{2\mu\nu}, \bar{D}_{1\nu}, \bar{D}_{2\mu\nu}\}$ correspond to $J^P = \{1^+, \{J = 0, 1\}^+, 1^-, \{J = 0, 1\}^-\}$ states, respectively.\textsuperscript{9}

PIONIC / KAONIC DECAYS

Next we explain a procedure for calculating the pionic / kaonic decays of $D_s$ mesons, applying the COQM. It can be considered that decays proceed through a single quark transition via emission of a local pion / kaon. We introduce the decay interactions as follows:

$$S_{\text{int}} = \int d^4x_1 \int d^4x_2 \langle \Phi(-)(x_1, x_2) V(x_1) \Phi(+)(x_1, x_2) \rangle,$$

(10)

where $x_1$ and $x_2$ denote the space-time coordinates of s- and $\bar{c}$-quarks related to CM and relative coordinates as $X_{\mu} = (m_s x_{1\mu} + m_c x_{2\mu})/(m_s + m_c), x_{\mu} = x_{1\mu} - x_{2\mu}$, and $(\cdot \cdot \cdot)$ means taking trace concerning flavor and Dirac indices. Two types of vertex factors, $V(x_1) = V_{ND}(x_1) + V_{D}(x_1)$, denoting non-derivative and derivative couplings,\textsuperscript{10} are

$$\langle \Phi(-)(x_1, x_2) V_{ND}(x_1) \Phi(+)(x_1, x_2) \rangle = d g_{V_{ND}} \langle \bar{\Phi}(-)(x_1, x_2) \gamma_\mu \phi_{ps}(x_1) \Phi(+)(x_1, x_2) \rangle,$$

(11)

$$\langle \Phi(-)(x_1, x_2) V_{D}(x_1) \Phi(+)(x_1, x_2) \rangle = \frac{dg_{V_{D}}}{2m_s} \langle \bar{\Phi}(-)(x_1, x_2) (\gamma_{\nu}\gamma_{\mu}\phi_{ps}(x_1) - M_{\nu\mu}) \Phi(+)(x_1, x_2) \rangle.$$

(12)

Rewriting the above with CM and relative coordinates by

$$\Phi(+)(x_1, x_2) \sim \Phi(+)(x, P)e^{i p\cdot x}, \bar{\Phi}(-)(x_1, x_2) \sim \bar{\Phi}(-)(x, P)e^{-i p\cdot x}$$

(13)

\textsuperscript{7} In the Ref. [2], $\rho_3 = -1$ component for c-quark is taken into account.

\textsuperscript{8} Clearly degeneracy of mass (in the HO limit) for the $SU(2)_2$ doublets is badly broken. Thus the $SU(2)_2$ should be considered, in contrast to the $SU(2)_\rho$-symmetry, just to offer a tool which gives a new perspective on classifying hadronic states.

\textsuperscript{9} All Lorentz indices of local fields satisfy the subsidiary conditions; $v_{\mu} D_{\mu\nu} = v_{\mu} D_{\nu\mu} = v_{\mu} D_{\mu\nu} = v_{\mu} D_{\nu\mu} = v_{\mu} D_{\mu\nu} = v_{\mu} D_{\nu\mu} = 0, D_{\mu\nu} = D_{\nu\mu}, D_{\mu\nu} = D_{\nu\mu}, D_{\mu\nu} = D_{\nu\mu} = 0$.

\textsuperscript{10} In the non-relativistic limit, the first term contributes only the transitions between chiralons and non-chiralons, accompanied by $\rho_3$-change. On the other hand, the second term contributes only transitions among non-chiralons or chiralons themselves, which gives the well known $\sigma \cdot (q - \frac{1}{m_Q} p_Q)$ vertex.
TABLE 1. Possible assignments of $S$- and $P$- wave $D_s$ mesons in the $\bar{U}(12)_{SP} \times O(3,1)$-scheme

| $n$ | $L$ | $P$   | $V$   | $S\bar{L}$ | $A\bar{L}$ |
|-----|-----|-------|-------|------------|------------|
| 0   | 0   | $1^1S_0$ | $1^3S_1$ | $1^3S_0^+$ | $1^3S_1^+$ |
|     |     | $0^-$   | $1^-$   | $0^+$      | $1^+$      |
|     |     | $(2.11 \text{ GeV})$ | $(2.46 \text{ GeV})$ | |
| $D_{s1}(1968)$ | $D_{s1}^*(2112)$ | $D_{s0}^*(2317)$ | $D_{s1}(2460)$ |
| 1   | 1   | $1^1P_1$ | $1^3P_{J=0,1,2}$ | $1^1P_1^+$ | $1^1P_1^+$ |
|     |     | $1^+$   | $(0,1,2)^+$ | $1^+$       | $(0,1,2)^+$ |
|     |     | $(2.57 \text{ GeV})$ | $(2.87 \text{ GeV})$ | |
| $D_{s1}(2536)$ | $D_{s0}^*(2573)$, $D_{s1}^*(2573)$, $D_{s2}(2573)$ | $D_{s1}^*(2710)$ | $D_{s0}^*(2866)$, $D_{s1}^*(2860)$, $D_{s2}(2860)$ |

and

$$
\phi_{ps}(x_1) \sim \phi_{ps}(q) e^{-i\eta x_1} = \left( \begin{array}{ccc} 
\frac{\pi^0}{\sqrt{2}} & \frac{\eta(8)}{\sqrt{6}} & \frac{K^+}{\sqrt{3}} \\
\pi^- & \frac{\pi^0}{\sqrt{2}} & \frac{K^0}{\sqrt{3}} \\
K^- & \frac{K^0}{\sqrt{3}} & \eta(8) 
\end{array} \right) e^{-i(q_\mu - P_{\mu} - q_\mu') x}, \quad (q_\mu = P_{\mu} - P_{\mu}'),
$$

we obtain a formula to calculate the decay amplitudes as

$$
T = d_{\text{GND}} \int d^4x (\Phi^{(-)}(P', x)) i\gamma_\mu (P(x)) e^{-i\frac{2m_\mu}{2m} x}$$

$$
+ g_D \int d^4x (\Phi^{(-)}(P', x)) \gamma_\mu q_\mu \sigma_{\mu\nu} \left( P_\nu + P_{\nu'} - \frac{d}{2m_\nu} i(\vec{\partial}_{x,\nu} - \vec{\partial}_{x,\nu'}) \right) \phi_{ps}(q) \Phi^{(+)}(P, x) e^{-i\frac{2m_\mu}{2m} x}. \quad (16)
$$

**ASSIGNMENTS AND NUMERICAL RESULTS**

New charmed strange meson $D_{s1}^*(2860)$ was first observed by the BaBar collaboration[7] in the $DK$ channel of $e^+e^-$ inclusive measurement with $M = 2856.6 \pm 1.5 \pm 5.0\text{MeV}$ and $\Gamma = 48 \pm 7 \pm 10\text{MeV}$. Shortly after, a $J^P = 1^-$ state $D_{s1}^*(2710)$ was reported by the Belle collaboration[9] with $M = 2708 \pm 9\pm 1\text{MeV}$ and $\Gamma = 108 \pm 23\pm 36\text{MeV}$ in the $DK$ invariant mass distribution of $B$-decay. In the latest report from the BaBar collaboration[8], the $D_{s1}^*(2860)$ and $D_{s1}^*(2710)$ were seen in both $DK$ and $D^*K$ decay modes with the ratios of branching fraction

$$
\frac{\text{BR}(D_{s1}^*(2710) \rightarrow D^*K)}{\text{BR}(D_{s1}^*(2860) \rightarrow D^*K)} = 0.91 \pm 0.13 \pm 0.12, \quad \frac{\text{BR}(D_{s1}^*(2860) \rightarrow D^*K)}{\text{BR}(D_{s1}^*(2860) \rightarrow DK)} = 1.10 \pm 0.15 \pm 0.19. \quad (17)
$$
To understand the nature of these newly observed states, many theoretical efforts have been done, but it is still a subject of controversy.

Now we discuss a classification based on the $\bar{U}(12) \times O(3,1)$-scheme, shown in Table 1. Subsequent results for pionic / kaonic transition widths in comparison with experiments are also given in Table 2. In both tables, some predicted, but experimentally missing states are underlined. In consideration of unexpectedly lower mass, $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$ are plausible candidates for our $S$-wave chiralons. Thus we assign them to $D_{s0}^T$ and $D_{s0}^T$ chiralons in Eq. (8), respectively. A issue raised by relevant assignments is whether there are additional (conventional) $c\bar{s}$ $P$-wave states, $D_{s1}$ and $D_{s1}^*$, in Eq. (9). The mass of those $P$-wave $1^+$ non-chiralons is expected to be about $\sim 2.57$ GeV from that of typical $D_{s2}^*(2573)$, being much heavier than $S$-wave chiralons. The results of strong decays, assuming the existences of $D_{s0}^*(\sim 2.57)$ and $D_{s1}^{1/2}*(\sim 2.57)$ non-chiralons in Table 2 indicate that it is not contradict with present experiment, owing to predicted large widths. On the other hand, the experimental known state $D_{s1}(2536)$ is naturally explained as $D_{s1}^{3/2}$ non-chiralons, being mixing partner of $D_{s1}^{1/2}$. Next we make a rough estimate the mass of the $P$-wave chiralons, by using global HO mass relation, $m_s^2 = \Omega + m_0^2$, derived from Eq. (2). As a result, we predict two vector chiralons, $D_{s}^0$ and $D_{s}^0$, with the mass about $2.7 \sim 2.9$ GeV. These states are possible candidate of recently reported $D_{s1}^*(2710)$ and $D_{s1}^*(2860)$. We calculate the strong decays to check this possibility, and found that $D_{s1}^*(2710)$ meson is consistently explained as the vector chiralon $^1P^0$. On the other hand, $D_{s1}^*(2860)$, does not fit as predicted properties of our $P$-wave vector chiralon.

CONCLUDING REMARKS

In conclusion, the $D_{s0}^*(2317)$, $D_{s1}^*(2460)$, and $D_{s1}^*(2710)$ are good candidates for $c\bar{s}$ chiralons through their observed masses and decay properties. The existence of $P$-wave non-chiralon $D_{s0}^*$ and $D_{s1}$ with broad width ($\sim 180$ MeV) and higher mass ($\sim 2.57$ GeV) appeared in the $DK$ and $D'K$ spectrum, respectively, and that of $J^P = \{0, 1, 2\}$ $P$-wave chiralons with the mass $2.7 \sim 2.9$ GeV should be checked in future experiment.

ACKNOWLEDGMENTS

This work was supported in part by Nihon University Research Grant for 2008.

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11 For example, the preceding work[5, 6] predicts $D_{sJ}^*(2860)$ and $D_{sJ}^*(2710)$ as $2P$ and $2S$ (or $1D$) states of conventional $c\bar{s}$ states.
12 Though the decay studies we have used following parameters:
(i) coupling constants; $g_{0} = g_{1}/\sqrt{2}f_{\pi}$, $g_{0} = 0.55, g_{s} = 0.45$. (ii) $SU(2)_{P}$-symmetric) mass; $M_{0} = 2.26$ GeV, $M_{1} = 2.69$ GeV. (iii) Regge slope inverse; $\Omega = 2.160$ GeV$^{2}$, determined from $M(D_{sJ}^*(2573))^{2} - M(D_{sJ}^*(2112))^{2}$ (iv) quark mass ratio; $m_{c}/m_{s} = 0.65 \times 10^{-4}$. (v) $\eta^{(8)} - \eta_{c}$ mixing angle; $\sin^{2}\theta = 0.65 \times 10^{-4}$.
13 More properly, mixed states $|D_{s1}^{1/2}) = \sqrt{2/3}|D_{s1}^{1}) - \sqrt{1/3}|D_{s1}^{1})$ and $|D_{s1}^{3/2}) = \sqrt{1/3}|D_{s1}^{3}) + \sqrt{2/3}|D_{s1}^{3})$ are realized in the heavy quark limit.