RADIATION FEEDBACK IN ULIRGs: ARE PHOTONS MOVERS AND SHAKERS?

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ABSTRACT

We perform multidimensional radiation hydrodynamics simulations to study the impact of radiation forces on atmospheres composed of dust and gas. Our setup closely follows that of Krumholz & Thompson, assuming that dust and gas are well-coupled and that the radiation field is characterized by blackbodies with temperatures $\gtrsim 80$ K, as might be found in ultraluminous infrared galaxies (ULIRGs). In agreement with previous work, we find that Rayleigh–Taylor instabilities develop in radiation supported atmospheres, leading to inhomogeneities that limit momentum exchange between radiation and dusty gas, and eventually providing a near balance of the radiation and gravitational forces. However, the evolution of the velocity and spatial distributions of the gas differs significantly from previous work, which utilized a less accurate flux-limited diffusion (FLD) method. Our variable Eddington tensor simulations show continuous net acceleration of the gas and never reach a steady state. In contrast, our FLD results show little net acceleration of the gas and settle into a quasi-steady, turbulent state with low velocity dispersion. The discrepancies result primarily from the inability of FLD to properly model the variation of the radiation field around structures that are less than a few optical depths across. We consider the effect of varying the optical depth and study the differences between two-dimensional and three-dimensional runs. We conclude that radiation feedback remains a plausible mechanism for driving high-Mach number turbulence in ULIRGs with sufficiently high optical depths. We discuss implications for observed systems and galactic-scale numerical simulations of feedback.

Key words: galaxies: ISM – hydrodynamics – ISM: jets and outflows – methods: numerical – radiative transfer

Online-only material: color figures

1. INTRODUCTION

Observations of star-formation in the Milky Way and other galaxies provide consistent evidence that some feedback mechanism or mechanisms hamper the collapse of interstellar gas to form stars. For example, The Kennicutt–Schmidt law (Kennicutt 1998) implies that, on average, only a few percent of the available gas actually collapses to form stars per dynamical time. Observations of molecular gas in rapidly star forming ultraluminous infrared galaxies (ULIRGs) indicate that turbulent velocities of up to $\sim 100$ km s$^{-1}$ are present (e.g., Downes & Solomon 1998). Most dramatically, galaxy scale outflows of cold, neutral gas are inferred in galaxies ranging from nearby dwarf starbursts to ULIRGs and rapidly star forming galaxies at high redshift (e.g., Heckman et al. 1990; Pettini et al. 2001; Schwartz & Martin 2004; Rupke et al. 2005).

Although a number of promising mechanisms have been proposed to explain these observations, we restrict our attention to the potential role of radiation pressure on dust grains in driving turbulence, hampering gravitational collapse, and launching outflows in such environments (Scoville et al. 2001; Murray et al. 2005; Thompson et al. 2005). This possibility has been explored extensively in recent years, with a number of studies considering how (in)effective radiation driving may be in various environments (e.g., Krumholz & Matzner 2009; Andrews & Thompson 2011; Hopkins et al. 2011, 2012; Wise et al. 2012; Krumholz & Thompson 2012, 2013; Socrates & Sironi 2013).

A key question for evaluating the importance of radiation pressure feedback is how much momentum is imparted to the gas by radiation pressure for a given radiative luminosity $L$. In particular, can the rate of momentum exchange significantly exceed $L/c^2$? It has recently been argued by Krumholz & Thompson (2012, hereafter KT12) and Krumholz & Thompson (2013) that the Rayleigh–Taylor instability (hereafter RTI; see, e.g., Chandrasekhar 1961) plays a significant role in limiting the exchange of momentum between radiation and dusty gas to values $\sim L/c$. Since a general analytical calculation of the linear growth of the radiative RTI does not exist (see, e.g., Mathews & Blumenthal 1977; Koliik 1977; Jacquet & Krumholz 2011; Jiang et al. 2013), the KT12 results are based on results from numerical simulations. KT12 found that radiation pressure was ineffective at accelerating gas to observationally inferred velocities due to the RTI.

In this paper, we revisit the results of KT12, who numerically solve the equations of radiation hydrodynamics using an implementation of the flux-limited diffusion (FLD) algorithm in the ORION code (Krumholz et al. 2007). We utilize both a variable Eddington tensor (VET) method (Davis et al. 2012; Jiang et al. 2012) and our own version of the FLD method (Y.-F. Jiang et al., in preparation), each of which are implemented as part of the Athena astrophysical fluid dynamics code (Stone et al. 2008). These techniques have previously been used to study the radiative RTI, but focused on a very different opacity regime (Jiang et al. 2013). Although we reproduce some aspects of the KT12 results, we find significant discrepancies that arise from inaccuracies of the FLD treatment. Our results confirm that the RTI limits the exchange of momentum between radiation and...
gas. However, contrary to the previous results, we find that gas can still be accelerated efficiently by radiation pressure when the VET algorithm is used, as long as the systems are sufficiently optically thick and luminous enough to reach the Eddington limit at depth.

The plan of this work is as follows. We review the equations solved and summarize our numerical methods in Section 2. We describe the setup of our numerical simulations, including formulations for the opacities, boundary conditions, and initial conditions, in Section 3. We summarize our key results from our numerical simulation in Section 4 and discuss their implications in Section 5. We provide our conclusions in Section 6.

2. EQUATIONS AND NUMERICAL METHODS

In this work, we solve the equations of radiation hydrodynamics using the VET implementation (Sekora & Stone 2010; Davis et al. 2012; Jiang et al. 2012). The primary set of equations to be solved are the coupled systems of hydrodynamics and the radiation moment equations, which are obtained by integrating the radiation transfer equation over frequency and angle (Mihalas & Mihalas 1984). The standard fluid equations are solved using a finite volume formulation as discussed in Stone et al. (2008). We solve the radiation system in the mixed frame approach (see, e.g., Mihalas & Mihalas 1984), where the radiation moments are Eulerian frame quantities, while the emissivities and opacities correspond to the comoving frame quantities (see Mihalas & Klein 1982). We follow the derivation of Lowrie et al. (1999), which retains all terms of the order of \( v/c \) and some terms of the order of \( (v/c)^2 \). The equations correspond to mass conservation

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}
\]

momentum conservation

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v} + \mathbf{P}) = \rho \mathbf{g} - \mathbf{S}_r(\mathbf{P}), \tag{2}
\]

and energy conservation

\[
\frac{\partial E}{\partial t} + \nabla \cdot (E \mathbf{v} + \mathbf{P} \cdot \mathbf{v}) = \rho \mathbf{g} \cdot \mathbf{v} - c S_r(E), \tag{3}
\]

In the above, \( \rho \) is the gas density, \( \mathbf{v} \) is the fluid velocity, \( \mathbf{g} \) is the gravitational acceleration, and \( E \) is the total (fluid) energy density \( E = p/(\gamma - 1) + \rho v^2/2 \). The pressure tensor \( \mathbf{P} \) is defined as \( \mathbf{P} = \pi I \), where \( p \) is the gas pressure and \( I \) is the identity matrix.

The quantities \( \mathbf{S}_r(\mathbf{P}) \) and \( S_r(E) \) are the radiation momentum and energy source terms, respectively. They are given by

\[
\mathbf{S}_r(\mathbf{P}) = -\sigma_f \left[ \mathbf{F}_r - (v E_r + v \cdot \mathbf{P}_r) \right]/c + \mathbf{v} \sigma p a T^4 - \sigma_E E_r)/c, \tag{4}
\]

and

\[
S_r(E) = \left( \sigma p a T^4 \right) - \sigma E E_r + \frac{\mathbf{v} c^2}{c} \cdot \left[ \mathbf{F}_r - (v E_r + v \cdot \mathbf{P}_r) \right]. \tag{5}
\]

Here \( E_r \) is the radiation energy density, \( \mathbf{F}_r \) the radiation flux, \( \mathbf{P}_r \) is the radiation pressure tensor, \( a_r \) is the radiation constant, and \( T \) is the gas temperature. These equations include frequency-averaged weighted opacities, corresponding to the energy mean opacity \( \sigma_E \), the Planck mean opacity \( \sigma_p \), and the flux mean opacity \( \sigma_f \) (Mihalas & Mihalas 1984). In this work, we neglect any scattering contribution to the opacity. The radiation source terms on the right-hand side of Equations (2) and (3) are included via a modified Godunov method, as described in Jiang et al. (2012).

In order to compute these quantities, we solve the radiation moment equations representing conservation of radiation energy

\[
\frac{\partial E_r}{\partial t} + \nabla \cdot \mathbf{F}_r = c S_r(E), \tag{6}
\]

and conservation of radiation momentum

\[
\frac{1}{c^2} \frac{\partial F_r}{\partial t} + \nabla \cdot \mathbf{P}_r = \mathbf{S}_r(\mathbf{P}). \tag{7}
\]

This radiation system is solved using a backward-Euler formulation (Jiang et al. 2012).

2.1. The Short Characteristics Method

The above equations are incomplete because there is no evolution equation for the radiation pressure tensor. We address this by computing the eponymous VET \( f = \mathbf{P}_r/E_r \). We approximate \( f \) by solving the time-independent equation of radiation transfer of the form

\[
\hat{n} \cdot \nabla I = \sigma_f \left( \frac{\alpha c}{4 \pi} T^4 - I \right), \tag{8}
\]

where \( I \) is the specific intensity for an angle defined by the unit vector \( \hat{n} \). Hence, we have adopted a gray opacity treatment in which the characteristic opacity is assumed to correspond to the flux mean opacity. For the computation of the \( f \), we make no distinction between the comoving and Eulerian frames, since Equation (8) drops all order \( v/c \) terms or higher, including a \( c^{-1} \partial I/\partial t \) term that would otherwise be present. The neglect of the \( c^{-1} \partial I/\partial t \) term is typically a good approximation whenever \( v/c \ll 1 \), but the neglect of velocity-dependent terms generally requires \( \tau v/c \ll 1 \) if the characteristic optical depth for the system. In the results presented here \( v_{\text{max}}/c \sim 10^{-4} \) and \( \tau_{\text{max}} \sim 15 \), so the neglect of these terms is a good approximation.

On each timestep, this equation is solved using the method of short characteristics (Kunasz & Auer 1988), as described in Davis et al. (2012). As the radiation transfer calculation proceeds, the mean intensity \( J \), the first moment \( \mathbf{H} \), and the second moment \( \mathbf{K} \) are tabulated in each grid zone. These are then used to compute \( f = K/J \) for the subsequent timestep. For our two-dimensional (2D) simulations, we integrate the radiation field along 84 rays, distributed nearly uniformly over the half unit sphere, taking advantage of the reflection symmetry of 2D Cartesian domains. Our single three-dimensional (3D) run uses 168 rays. Further discussion of the choice of angular grid and impact of angular resolution is provided in the Appendix.

2.2. The Flux-limited Diffusion Method

We have also implemented an FLD algorithm in Athena. Here, we only briefly describe the equations and implementation. A more detailed description of the algorithm and its implementation will be provided in a forthcoming work (Y.-F. Jiang et al., in preparation).

In FLD, one drops the radiation momentum equation from the evolution equations, replacing it with a diffusion equation for the flux

\[
\mathbf{F}_r = -\frac{c^2}{\sigma_f} \nabla E_r, \tag{9}
\]
where \( \lambda \) is the flux-limiter. The radiation energy equation is then solved by substituting this relation for \( F_r \) in Equations (4)–(6). Following Levermore & Pomraning (1981), we assume a flux limiter of the form

\[
\lambda = \frac{1}{R} \left( \coth \frac{R - 1}{R} \right)
\]

\[
R = \frac{|\nabla E_r| + \beta}{\sigma F E_r}.
\]

To specify \( R \), we use the Eddington tensor

\[
f = \frac{1}{2}(1 - f) f + \frac{1}{2}(3 f - 1) \hat{f} \hat{f}
\]

with \( f = \lambda + \lambda^2 R^2 \) and \( \hat{f} = \nabla E_r/|\nabla E_r| \).

Following Shestakov & Offner (2008), we have added a \( \beta \) parameter to the definition of \( R \), which acts as an effective floor on \( R \) in optically thin regions. This addition was necessary to robustly obtain convergence in our backward-Euler scheme, in which one needs to solve a large matrix equation. We find poor convergence for our multigrid solver unless \( \beta \gtrsim 10^{-4} \).

These equations are implemented in a manner analogous to the VET method described above. The source terms are coupled to the hydro equations using our modified Godunov method and the radiation energy equation is solved using a backward-Euler algorithm. Finally, we note that \( F_r \) is the Eulerian frame flux in the above equations. Strictly speaking, it is the comoving frame flux for which the diffusion approximation applies in high optical depth media. However, the low vales of \( v/c \) in the simulations lead to very small differences between the Eulerian and comoving frame fluxes.

Due to the ad hoc aspects of this approximation, it is not as reliable as our VET method, which evolves the radiation momentum equation and uses a solution of the radiation transfer equation to estimate \( f \). Modulo the velocity-dependent terms that we neglect in computing \( f \) (and only in computing \( f \)), the accuracy of our approximation can always be improved by increasing the angular resolution. In contrast, FLD will always be inaccurate at some level in regions of the flow where the characteristic optical depths are near unity or less.\(^{6}\) For realistic problems, we can enhance the accuracy of our VET solution by increasing the angular and spatial resolution of our grid (Y.-F. Jiang et al., in preparation). There is no comparable mechanism for improving accuracy in FLD. Simply increasing the spatial resolution will not make FLD converge to the correct solution. Well known deficiencies of this method include its inability to cast shadows (see, e.g., Hayes & Norman 2003) because the radiation field diffuses around opaque barriers, even in optically thick environments. We have implemented FLD in Athena only to facilitate comparison of our VET calculation with previous FLD-based results, since it aids in determining which aspects of the calculation depend on the transfer method.

3. SIMULATION SETUP

Our simulation setup follows the computations performed in KT12. The goal is to simulate the evolution of molecular gas at temperatures where dust dominates the opacity and radiation forces can exceed gravity, so that RTI may have a significant impact on the dynamics. These conditions are most likely found in ULIRGs and high-redshift, star forming galaxies, but the results may generalize to other rapidly star forming environments. This setup does not attempt to directly replicate any specific star forming environment, but instead makes a number of simplifying assumptions to focus on assessing the impact of the RTI.

We assume that the gas and dust are strongly coupled, both dynamically and thermally. This assumption should be quite reasonable for ULIRGs, as discussed in Appendix of Krumholz & Thompson (2013). Hence, the gas and dust are assumed to share a common temperature (hereafter simply referenced as the gas temperature \( T_g \)), and any momentum exchanged between the dust and radiation field is rapidly shared via collisions with the gas.

Following KT12, the Planck \( \kappa_P \) and Rosseland \( \kappa_R \) mean opacities are given by

\[
\kappa_P = 0.1 \left( \frac{T}{10 \text{ K}} \right)^2 \text{cm}^{-2} \text{g}^{-1},
\]

\[
\kappa_R = 0.0316 \left( \frac{T}{10 \text{ K}} \right)^2 \text{cm}^{-2} \text{g}^{-1}.
\]

This \( \kappa \propto T^2 \) scaling approximately holds for dust in thermal equilibrium with a blackbody radiation field for \( T \lesssim 150 \text{ K} \) (Semenov et al. 2003). This explicitly assumes the characteristic temperature of the radiation and \( T \) are equal. Since we evolve the radiation energy density, we can define a characteristic radiation temperature \( T_r = (E_r/a_s)^{1/4} \) and check if \( T_r \approx T \). This is generally a good approximation, although modest differences are present in some regions, which motivated us to perform alternative simulations with \( \kappa_{R,P} \propto T^2 \). The two sets of simulations agreed very closely with each other so we report only the \( \kappa_{R,P} \propto T^2 \) here. These assumptions imply that the radiation field is characterized by blackbodies with \( T_r \propto T \approx 80–200 \text{ K} \). Since the main source of radiation is massive stars emitting primarily in the UV, this analysis assumes the direct stellar radiation has already been reprocessed by the dust and converted to infrared radiation.

The simulations are performed on a Cartesian grid, with a single 3D case and multiple 2D calculations. For simplicity, the gravitational acceleration \( g \) is assumed to be constant and the radiation field is sourced by a constant flux \( F_{\ast} \), incident at the lower boundary. This flux defines a characteristic temperature \( T_s = (E_s/a_s c)^{1/4} \), from which we can define a characteristic sound speed \( c_s^2 = k T_s/(\mu m_H) \), scale height \( h_s = c_s^2/g \), and sound crossing time \( t_s = h_s/c_s \). Following KT12, we assume \( \mu = 2.33 \) and choose \( T_s = 82 \text{ K} \). Corresponding to \( F_s = 2.54 \times 10^{13} \text{ L}_\odot \text{ kpc}^{-2} \) and \( \kappa_{R,s} = 2.13 \text{ cm}^2 \text{ g}^{-1} \). The remaining free parameters are \( g \) and the surface mass density \( \Sigma_s \), which can be specified in terms of the dimensionless Eddington ratio\(^{7}\)

\[
f_{E,s} = \frac{\kappa_{R,s} F_s}{g c}
\]

and optical depth

\[
\tau_s = \kappa_{R,s} \Sigma_s.
\]

\(^6\) Although FLD is often claimed to be accurate in the optically thin limit, it assumes \( |\nabla E_r| \rightarrow E_r \), which only applies to a radiation field with perfectly parallel rays. This is typically only obtained at large distances from a point source emitter and is not general.

\(^7\) Note that this constitutes only a local assessment of the force balance. Since \( g \) varies with height and radius in real systems, a local balance between radiation and gravitational forces simply means the disk is radiation pressure supported. The global Eddington ratio of the system may be much less than unity.
If summarized in Table 1.

For the lower boundary condition of the VET runs, we apply reflecting boundary conditions on $F_{rx}$ and require

$$F_{ry} = F_y + v_y E_y + (v \cdot P_y) y.$$  (16)

Hence, the comoving flux in the ghost zones is equal to $F_y$. We specify $E_y$ by integrating the time-independent vertical momentum equation, assuming the Eddington tensor at the lower boundary of the computational domain applies throughout the ghost zones. At the upper boundary, both components of $F_y$ are continued in to the ghost zones and we specify $E_y$ using

$$E_y = \frac{J}{H_y} F_{ry},$$  (17)

where $H$ and $J$ are the first moment and mean intensity (respectively) returned by the short characteristics module. In other words, the ratios of the vertical component of the first moment to the “zeroth” moment for the two calculations are forced to match identically in the ghost zones.

For the lower boundary condition of the FLD computations, we compute the flux limiter using Equation (10) and solve for $E_y$ using Equation (9), assuming $F_{ry} = F_y$ and no horizontal gradient in energy density. At the upper boundary, we assume a fixed $E_y = a T_y^4$ in the ghost zones for consistency with KT12.

For the lower boundary in the short characteristics module, we integrate the outgoing (downward) radiation intensity over angle $J^-$. We then assume isotropic incoming (upward) radiation. The intensity of the incoming radiation is normalized so that the corresponding angle integrated upward intensity $J_+$ obeys $H_y = J_+ - J_- = F_y$. At the upper boundary, we assume that the intensity of incoming radiation is zero. Periodic boundary conditions in the horizontal direction are imposed by iterating the short characteristics solution to convergence, as described in Davis et al. (2012).

### 4. RESULTS

KT12 computed the equilibrium profiles for one-dimensional (1D) atmospheres obeying the assumptions of Section 3, assuming Equation (9) holds. They find that for a given value of $\tau_\infty$ there is a maximum value for $f_{E,\infty}$ ($f_{E,\text{crit}}$) for which their iterative method converges. Above this value, no equilibrium solution exists because the radiation and gravitational forces cannot be forced to match exactly due to the temperature dependence of the opacities. Their results for $\kappa \propto T^2$ are summarized in their Figure 2. Although their results are derived using the FLD approximation, we expect that their boundary curve should approximately demarcate the locus of stable equilibrium solutions for our VET calculations, as well.

Here we consider a range of $\tau_\infty$ and $f_{E,\infty}$ as summarized in Table 1. The first ($\tau_\infty = 10$, $f_{E,\infty} = 0.02$) falls in the range where
Figure 1. Density distribution for four snapshots from the T10_F0.02 simulation, which is expected to be stable. Each panel shows the bottom quarter \((z/h) \leq 64\) of the domain. The times correspond to \(t/t_s = 10, 30, 50, \) and \(70\), as labeled. The atmosphere initially expands upward, but falls back and then oscillates, similar to the KT12 results for the same parameters.

(A color version of this figure is available in the online journal.)

an equilibrium exists, while the remaining runs \((\tau_s = 1, 3, \) and \(10; f_{E,\text{cr}} = 0.5\) fall in the unstable regime. We choose most of these parameters to match simulations performed by KT12.

4.1. Stable Case

We first consider the T10_F0.02 run, which we run using our VET module. The parameters for this run place it in the stable regime, according to the 1D equilibrium solutions and the 2D simulation results of KT12. We initialized the simulation with a sinusoidal perturbation only (i.e., no random fluctuations), as described in Section 3.1 and ran it for 80\(t_s\).

Figure 1 shows four snapshots of the density from this run. At the beginning of the run, the large optical depth and constant incident flux require \(E_r\) to increase. Since the radiation and dusty gas are well-coupled, the increase in \(T_{\text{rad}}\) drives a corresponding rise in \(T\). This leads to an increase in the opacity and a corresponding rise in the optical depth and radiation force. As a result, the atmosphere briefly expands upward, relaxes, and begins to oscillate around a quasi-equilibrium state.

This transient evolution and subsequent oscillatory behavior are most clearly seen in the mass-weighted mean velocity

\[
\langle v \rangle = \frac{1}{M} \int \rho v dV,
\]

and mass-weighted velocity dispersion

\[
\sigma^2 = \frac{1}{M} \int \rho (v - \langle v \rangle)^2 dV,
\]

where \(M = \int \rho dV\) is the total mass in the atmosphere. The top panel of Figure 2 shows \(\sigma_x\) and \(\sigma_y\), along with the total velocity dispersion \((\sigma = (\sigma_x^2 + \sigma_y^2)^{1/2})\), and the bottom panel shows \(\langle v_y \rangle\).

The initial transient acceleration lead to growth in both the vertical and horizontal velocity dispersion, although the vertical component initially grows faster. After \(\sim 3t_s\), both \(\langle v_y \rangle\) and \(\sigma_x\) have already reached their maximum for the simulation. After another \(\sim 10t_s\), both quantities settle down into oscillations, with \(\langle v_y \rangle\) varying about zero. The horizontal velocity dispersion is also oscillatory, albeit on a longer timescale with a period greater than \(50t_s\). Comparison with the top panel of Figure 7 in KT12 shows close agreement between the results of the two different codes.

At the end of the run, the simulation is still oscillating, albeit with a slow decay in \(\langle v_y \rangle\). Reductions in the kinetic energy due to “numerical viscosity” and diffusive damping of compressive motions by the radiation field will eventually damp the oscillatory behavior and the simulation will presumably settle into a steady state. However, this would seem to require a significantly longer run time and we are not interested in the detailed properties of equilibrium. We stop the run at \(80t_s\), which is long enough to confirm that we reproduce the KT12 results with our VET formalism for this stable regime.

4.2. Unstable Case

We now focus on the unstable regime, considering simulations with \(\tau_s = 3\) and \(f_{E,\text{cr}} = 0.5\). This run has the lowest ratio of \(f_{E,\text{cr}}/f_{E,\text{crit}}\) considered in KT12 for the unstable regime. Furthermore, it had the weakest transient acceleration at the beginning of the simulation, reached the lowest maximum vertical extent, and had the lowest maximum velocity dispersion. In this sense, the run represented the worst case for driving turbulence and outflows among the runs considered by KT12.

In our work, we consider two runs: one using our FLD module (T3_F0.5_FLD) and one using the VET module (T3_F0.5) in order to assess the impact of the radiation transfer method. In principle, we could simply compare our VET method directly against the ORION FLD results, but we perform Athena FLD runs to control for differences in the hydrodynamics algorithms between Athena and ORION.

The simulations were initialized with the goal of nearly reproducing the KT12 setup. One notable exception is that both runs have sinusoidal and random perturbations, as described in Equation (15). We made this choice after running a simulation with the VET module that only included sinusoidal perturbations. In that case, the development of non-linear structure (due
Figure 2. Top panel: mass-weighted mean velocity vs. time for the T10_F0.02 simulation. Bottom panel: mass-weighted velocity dispersion vs. time for the same run. The curves correspond to $\sigma$ (solid), $\sigma_x$ (dashed), and $\sigma_y$ (dotted). In both panels, the velocities are normalized to the initial isothermal velocity $c_*=0.54\text{ km s}^{-1}$. After a transient acceleration, the atmosphere settles into slowly damped oscillatory behavior.

to the RTI, as discussed below) was slower than seen in KT12 and most of the mass in the shell was able to reach the top of the domain before this structure produced a significant feedback on the radiative acceleration of the shell. Since the sinusoidal perturbation is an artificial construction and because we were interested in the evolution of an atmosphere with significant non-linear structure, we seeded the RTI with additional random perturbations. This allowed for faster development of the RTI in the VET runs. We also initialize the FLD run with random perturbations to facilitate direct comparison between our FLD and VET runs. As we shall see, the VET run shows stronger coupling between gas and radiation than found in the FLD run. The inferred coupling would have been even stronger if we had not added random perturbations to seed the RTI.

We first consider the results from T3_F0.5_FLD, which we run for $200t_\star$. This is run in a taller box than the T10_F0.02 simulation, with the initial condition and boundary conditions described in Section 3. Note that we began the simulations with $\beta = 10^{-4}$ in Equation (10) to place a floor on $R$ and avoid convergence problems in our backward Euler scheme. This was sufficient for the first $\sim 100t_\star$, but we had to increase $\beta = 4 \times 10^{-4}$ at $t = 100t_\star$ to ensure convergence thereafter.

As in the T10_F0.02 run, there is an initial increase in $T_{\text{rad}}$ at the base of the domain due to the finite optical depth, constant incoming flux, and close thermal coupling between gas, dust, and radiation. This increase in $T$ drives an increase in $\kappa_R$, resulting in an increase of the radiation force above the gravitational force. Hence, the lower part of the atmosphere quickly becomes super-Eddington, and the vast majority of the mass is driven upward in a thin shell.

Figure 3 shows snapshots of $\rho$ and $T_{\text{rad}}$ from four different times. By $25t_\star$, the shell has already begun to break up, with some material running behind the shell (or even falling back) in a number of plumes. This behavior is consistent with expectations that the shell should be subject to the RTI when accelerated against gravity by the radiation forces under these conditions (see Section 5.3 for further discussion). Our results are also qualitatively consistent with those of KT12, who also attribute the non-linear structure to RTI. The growth of non-linear structure in our run is somewhat faster and the structures are less uniform, consistent with our additional random perturbations, which seed the growth of smaller-scale structures.

Subsequent evolution is also consistent with KT12. The RTI plumes grow into high-density filaments interspersed with lower density channels. Since $F_\nu$ is largest in the low-density channels, there is an anti-correlation of $F_\nu$ and $\rho$ that reduces the momentum exchange between the radiation field and gas. The radiation forces become sub-Eddington in the optically thick, high-density filaments and the majority of the matter falls back by $50t_\star$. This behavior is reinforced by a drop in the volume averaged optical depth, which leads to a drop in $T_{\text{rad}}$ (and therefore $T$) at the base of the atmosphere, further reducing the radiation force. By $75t_\star$, almost all of the mass is contained in the bottom $\sim 50h_\star$ of the domain and remains there for the duration of the run, which we stop at $200t_\star$.

As expected from the lack of a 1D equilibrium, the matter at the base of the domain does not settle into a hydrostatic equilibrium, but instead remains in a turbulent state, with a moderate velocity dispersion and very little average vertical motion. Again, this quasi-steady state turbulent flow is qualitatively and quantitatively consistent with the results in KT12.

We now turn our attention to the VET run labeled T3_F0.5. This simulation began in a domain with the same dimensions, resolution, and initial condition as in the T3_F0.5_FLD run.
Figure 3. Top panels: density distribution for four snapshots from the T3_F0.5_FLD simulation. Each panel shows the full simulation at the labeled times. Bottom panels: radiation temperature $T_r$ distribution computed from the same snapshots as in the top row. The atmosphere heats up and is accelerated vertically as a shell. The shell becomes unstable, breaks up, and mass falls back to the bottom of the domain, reaching a quasi-steady state of turbulence at late times. (A color version of this figure is available in the online journal.)

Figures 4 and 5 show four snapshots of $\rho$ and $T_{\text{rad}}$ (respectively) from T3_F0.5. Initially, the dynamics of the simulation are qualitatively consistent with the T3_F0.5_FLD run. Again, $T_{\text{rad}}$ and $T$ rises rapidly at the base of the domain and the bulk of the mass is launched upward as a shell. The RTI seems to grow slightly slower than in the FLD run, but is still fairly rapid and significant growth of non-linear structure is apparent by $39t_*$. The difference between the FLD and VET runs become more apparent in the subsequent evolution. As in the T3_F0.5_FLD run, there is a flux-density anti-correlation that allows high-density filaments to become sub-Eddington even as the volume-averaged structure remains super-Eddington. Some of these high-density filaments do fall back to the base of the domain, where they are reflected by the boundaries. Others are disrupted and reaccelerated by the radiation field as they disperse. The overall evolution leads to a gradual filling of the volume with $\rho \gtrsim 10^{-4}\rho_*$ gas, although most of the mass remains in several dense filaments.

At $\sim 83t_*$, the dense gas reaches the upper boundary of the initial domain and the assumptions made for the radiation boundary condition (near vacuum with no incoming radiation) cease to be valid. This leads to an abrupt, unphysical increase in $T_{\text{rad}}$ at the upper boundary. Therefore, we restart the simulation at $t = 80t_*$ in a domain with the $L_y = 2048h_*$ and $N_y = 4096$ (i.e., we double the height at fixed resolution). All variables are copied into the bottom half of the new domain. Velocities in the upper half of the domain are set to zero. All other material variables, $F_r$ and $E_r$, are averaged on the upper most grid zones and the upper half of the domain is uniformly initialized with these values. The Eddington tensor is recomputed with the short characteristics module. Subsequent evolution matches the original evolution until $\sim 83t_*$ when the breakdown of the vertical boundary condition alters the evolution in the smaller domain. We continue our integration in the larger domain until $158t_*$, when the dense gas again reaches the upper boundary of the larger domain. Again, most of the mass is concentrated in a few of the densest filaments, but the remainder is nearly...
Figure 4. Density distribution $\rho$ for four snapshots from the T3_F0.5 simulation. Each panel shows the full simulation domain at the labeled times. Due to gas with high $\rho$ reaching the vertical boundary, the simulation was restarted at $t = 80t_*$ in a domain with double the vertical extent, as described in the text.

(A color version of this figure is available in the online journal.)

volume filling with $\rho \gtrsim 10^{-4} \rho_*$, except for transient periods when most of the volume near the bottom of the domain can have $\rho \lesssim 10^{-6} \rho_*$.

Figure 5 shows that there is relatively little horizontal variation in $T_{\text{rad}}$, consistent with T3_F0.5_FLD and KT12. In contrast to T3_F0.5_FLD, the vertical distribution spreads out vertically with time as matter fills the domain, but temperature at the base of the domain remains relatively constant, with $T \approx T_{\text{rad}} \sim 2.2T_*$. Modest but short-lived increases in $T_{\text{rad}}$ are seen and correspond to fall back and reflection of dense filaments at the lower boundary (see, e.g., the second panel at $78t_*$). These values are modestly higher than the mean values of $T_{\text{rad}}$ at the base in T3_F0.5_FLD, although there is greater fluctuation in $T_{\text{rad}}$ in the T3_F0.5_FLD run.

Figure 6 shows a comparison of volume-averaged quantities and their ratios from the T3_F0.5_FLD and T3_F0.5 simulations. The top panel compares the characteristic Eddington ratio

$$f_{\text{E,V}} = \frac{\langle \kappa \rho F_{\text{rv}} \rangle}{cg(\rho)},$$

where $\langle \rangle$ denote volume averages (e.g., $\langle \rho \rangle = L_-^{-1}L_-^{-1} \int \rho dxdy$). Both runs have an initial super-Eddington period that is followed by a gradual decline to sub-Eddington values, although this decline is more rapid and deeper in the FLD case. The Eddington ratio is initially slightly greater in the VET run than FLD, but some of this is due to issues with our VET boundary condition and deviation of the initial condition from thermal equilibrium. The value of $F_{\text{rv}}$ initially exceeds the target $F_*$ (up to $\sim 37\%$), but declines to with $1\%$ of $F_*$ by $5t_*$, accounting for some of the difference. Both simulations show a return to an Eddington ratio near unity, although T3_F0.5_FLD tends to have $f_{E,V}$ fluctuating near but typically just below unity, while T3_F0.5 tends to remain moderately super-Eddington for most of the run. Although the differences are rather modest, the upshot is that T3_F0.5 receives a nearly continual, modest upward acceleration while T3_F0.5_FLD settles into a quasi equilibrium state. This evolution is apparent in the evolution of $\langle v_y \rangle$ in Figure 7. The trend in $f_{E,V}$ suggests that T3_F0.5 might settle into a similar equilibrium, but with a much larger-scale height and higher velocity dispersion.
Figure 5. Radiation temperature distribution $T_r$ for the four snapshots from the T3_F0.5 simulation shown in Figure 4. Each panel shows the full simulation domain at the labeled times. Due to gas with high $\rho$ reaching the vertical boundary, the simulation was restarted at $t = 80\,t^*$ in a domain with double the vertical extent, as described in the text. (A color version of this figure is available in the online journal.)

In order to explore the evolution and impact of optical depth variations, it is useful to define two characteristic averages for the optical depth at the base of the domain. The first is the volume-weighted mean optical depth

$$\tau_V = L_y \langle \kappa R \rho \rangle,$$  

and the second is the flux-weighted mean optical depth

$$\tau_F = L_y \frac{\langle \kappa R \rho F_{ry} \rangle}{\langle F_{ry} \rangle}.$$  

The middle panel of Figure 6 shows that $\tau_V$ correlates closely with $f_{\text{E},V}$ as both evolve over the course of the simulations. Large values of $\tau_V$ correspond to large values of $\kappa R \propto T^2$ because $T \simeq T_{\text{rad}}$ and $T_{\text{rad}}$ is roughly proportional to $\tau_V^{1/4}$ throughout much of the domain. Note, however, that it is not quite true that $\tau_V$ determines the Eddington ratio as $f_{\text{E},V}$ can be larger in T3_F0.5, even when $\tau_V$ is greater for T3_F0.5_FLD.

This occurs primarily because the nature of anti-correlation between $F_{ry}$ and $\rho$ differs in the two runs.

The bottom panel of Figure 6 shows the ratio $\tau_F/\tau_V$. Note that $\tau_F$ multiplies $\langle F_{ry} \rangle = F_*$ to give the momentum per unit area transferred from the radiation to the gas while $\tau_V$ would be the characteristic value for the total infrared optical depth $\tau_{\text{IR}}$ in a uniform medium. Hence, this ratio gives an estimate of how much the flux–density anticorrelation reduces the momentum coupling between radiation and gas. Our $f_{\text{trap}} = f_{\text{trap}} + 1$, where $f_{\text{trap}}$ is trapping factor discussed in KT12. Our estimate of $\tau_F \simeq 6$ agrees with KT12's $f_{\text{trap}} = 5$ for $\tau_{\ast} = 3$, $f_{\text{E},\ast} = 0.5$.

Overall, Figure 6 leaves the impression that the FLD and VET simulations yield very similar results, particularly at late times, in contrast with the impression provided by comparison of Figure 3 to Figures 4 and 5. The key point is that even modest variations $F_r$ can lead to rather large differences in the outcome when systems are near an Eddington ratio of unity.

The differences are somewhat more apparent in Figure 7, which shows the evolution of $\langle v_y \rangle$ and $\sigma$. For both simulations,
Figure 6. Top panel: volume-averaged Eddington ratio vs. time for the T3_F0.5 (solid black) and T3_F0.5_FLD (dotted red) simulations. Middle panel: volume-averaged optical depth vs. time for the same simulations as in the top panel. Bottom panel: ratio of the flux-weighted mean optical depth to the volume-averaged optical depth vs. time for the same simulations as in the upper panels. The vertical dashed curves denote the times where the T3_F0.5 simulation was restarted in larger domains.

(A color version of this figure is available in the online journal.)

Figure 7. Top panel: mass-weighted mean velocity vs. time for the T3_F0.5 (solid black) and T3_F0.5_FLD (dashed red) simulations. Bottom panel: mass-weighted velocity dispersion vs. time for the same runs. In both panels, the velocities are normalized to the initial isothermal velocity $c_s = 0.54 \text{ km s}^{-1}$.

(A color version of this figure is available in the online journal.)
the evolution of $\langle v_y \rangle$ is largely determined by the value of $f_{E,V}$ in Figure 6. When $f_{E,V} > 1$, $\langle v_y \rangle$ increases and when $f_{E,V} < 1$, $\langle v_y \rangle$ decreases. In both simulations, there is an early period of transient growth, followed by a decline after the RTI sets in. For T3_F0.5_FLD, $\langle v_y \rangle$ becomes negative as most of the mass falls back, but eventually settles into a quasi-steady state, with $\langle v_y \rangle$ simply fluctuating near zero. For T3_F0.5, $\langle v_y \rangle$ grows for the majority of the run.

For both runs, the x and y components of $\sigma$ show a modest initial increase, followed by a faster rises in $\sigma_x$ as the RTI sets in and the shells start to break apart. The subsequent evolution for T3_F0.5_FLD involves a weak decline in $\sigma$, compensated by a slow increase in $\sigma_y$. These trends roughly cancel so that $\sigma$ fluctuates about $\sim 5c_s$ for the majority of the simulation, consistent with the results of KT12. The T3_F0.5 evolution displays a more continuous rise in $\sigma$, with declines at $\sim 75t_*$ and $170t_*$, when $\langle v_y \rangle$ increases quickly during periods of higher $f_{E,V}$. When we stop the run near $250t_*$, $\sigma$ is increasing, but still below the maximum, which was a factor of $\sim 3$ larger than in T3_F0.5_FLD.

### 4.3. Dependence on Optical Depth

Our initial focus on $t_*=3$ and $f_{E,V}=0.5$ is motivated by a desire to compare with KT12, but also by the fact that this gives parameters that are broadly consistent with observational constraints on luminous ULIRGs, like Arp 220, which has one of the highest inferred surface densities and star formation rates in the local universe. Many local star forming galaxies have lower surface densities and, therefore, lower dust optical depths. Higher optical depths may also be obtained in local regions of star forming galaxies and may be common in galaxies at high redshift.

These considerations motivate runs with $t_*=1$ (T1_F0.5) and $t_*=10$ (T10_F0.5), both assuming $f_{E,V}=0.5$. Figure 8 compares $f_{E,V}$, $\tau_V$, and $\tau_F/\tau_V$ for these runs, along with T3_F0.5. The T10_F0.5 (solid black) shows a similar profile for $f_{E,V}$ as in T3_F0.5 (red dotted), but with a notably larger initial transient. This follows from the larger $\tau_V \gtrsim 50$ in this run and the non-linear dependence of $\tau_V$ on $t_*$ results from the $T^2$ dependence of the opacity. However, we still have $f_{E,V} \sim 1$ due to the regulating effects of the RTI, which show up as a lower ratio of $\tau_F/\tau_V$. Figure 9 shows the evolution of $\langle v_y \rangle$ and $\sigma$. Due to the large initial transient, $\langle v_y \rangle$ grows quickly for T10_F0.5 and a significant fraction of the mass reaches the top of simulation ($L_\gamma = 2048r_*$) at $t = 65t_*$, considerably faster than the $t \sim 150t_*$ that it takes for the T3_F0.5 to reach the same height. Although $\sigma$ increases faster in the T10_F0.5 run, $\sigma \sim 15c_s$ at the end of both simulations.

The evolution of the T1_F0.5 (dashed blue) is qualitatively different. There is a very brief initial transient after which $f_{E,V} \lesssim 1$ for the remainder of the run. We find $\tau_V \sim 2$ and $\tau_F/\tau_V$ is always slightly below unity. As a result, there is never any significant net upward acceleration. The mass that is initially accelerated upward falls back and $\langle v_y \rangle$ oscillates about zero. The mass settles down into a turbulent state reminiscent of the T3_F0.5_FLD run, but with even smaller velocity dispersion.

Since the fiducial Eddington ratio $f_{E,V}=0.5$, the mass-weighted average opacity would have to increase by a factor of two in order to obtain $f_{E,V} \sim 1$, which requires the
mass-weighted average of $T^2$ to be $2T_e^2$. If the flow were homogeneous this would be possible, but only a very small reduction in the trapping factor will prevent this. As we will see in the discussion section (Equation (32)), we need $\tau_F \sim 0.94\tau_V$ to maintain $f_{E, V} \sim 1$. This value is approached in Figure 8, but not sustained.

4.4. Spatial Resolution Study

We have also considered the impact of spatial resolution on our simulations. Most of our simulations assume $L_{(x,y)}/N_{(x,y)} = 0.5h_*$, leading to a large number of grid zones in both spatial dimensions. These high resolutions are attainable in 2D simulations but would be very computationally demanding for 3D calculations. Therefore, we have performed simulations at a medium resolution $L_{(x,y)}/N_{(x,y)} = h_*$ (T3_F0.5_MR) and low resolution with $L_{(x,y)}/N_{(x,y)} = 2h_*$ (T3_F0.5_LR) to study the impact of resolution.

Figure 10 compares T3_F0.5 (black solid), T3_F0.5_MR (red dotted), and T3_F0.5_LR (blue dashed). The lower-resolution runs evolve with a time dependence similar to the high-resolution runs, but spend most of their time at lower Eddington ratios. At early times, the RTI develops more slowly as resolution decreases. However, the development of inhomogeneities in the non-linear phase has a larger affect. The Eddington ratio falls more quickly and takes longer to recover to $f_{E, V} > 1$. Both $\tau_V$ and $\tau_F/\tau_V$ are lower, consistent with a stronger flux-density anticorrelation in these lower-resolution runs. The mass is more concentrated in narrower and denser filaments in the lower-resolution runs. This can be seen by comparing the density snapshot from T3_F0.5_LR in the right-hand panel of Figure 11 with $t = 78t_*$ snapshot of T3_F0.5 in Figure 4.

In spite of these differences, the lower-resolution runs still have a significant amount of moderately dense ($\rho \sim 10^{-3}\rho_*$) material reaching the top of the box and we need to stop both the T3_F0.5_MR and T3_F0.5_LR runs at $t \approx 103t_*$, when enough mass has reached the upper boundary to invalidate the vacuum boundary condition on the incoming radiation. In comparison, the T3_F0.5 simulation reached the same point at $t = 83t_*$. Furthermore, the bulk of the mass has received much less acceleration in the low-resolution runs and has a lower $\langle v_y \rangle$ and $\sigma$. Most of the mass remains in the lower half of the box in both T3_F0.5_MR and T3_F0.5_LR, whereas $\rho$ is more uniformly distributed with height in the T3_F0.5 run.

The upshot is that higher resolution leads to a more diffuse density distribution and enhances the volume-averaged radiation force. There is a slight trend of increasing $f_{E, V}$ from T3_F0.5_LR to T3_F0.5_MR, but the two low-resolution runs are rather similar. This may imply that even higher resolution would further enhance or feedback or could simply indicate that one needs to resolve the $h_*$ scale in the simulation. We have not performed higher-resolution runs to test for convergence due to the computational expense, but this should be explored further in future applications.

4.5. Comparison of 2D and 3D Simulations

It is well known that the non-linear development of hydrodynamics RTI differs in 2D and 3D simulations with and without magnetic fields (e.g., Young et al. 2001; Stone & Gardiner 2007). However, we know of no published work that considers the development of radiative RTI in 3D. Since our results
Figure 10. Top panel: volume-averaged Eddington ratio vs. time for the T3_F0.5 (solid black), T3_F0.5_MR (dotted red), and T3_F0.5_LR (dashed blue) simulations. Middle panel: volume-averaged optical depth vs. time for the same simulations as in the top panel. Bottom panel: ratio of the flux-weighted mean optical depth to the volume-averaged optical depth vs. time for the same simulations as in the upper panels.

(A color version of this figure is available in the online journal.)

Figure 11. Density distribution $\rho$ for snapshots from the T3_F0.5_3D (left) and T3_F0.5_LR (right) simulations. Each panel shows the full simulation domain at $t = 100t_*$.

(A color version of this figure is available in the online journal.)
Figure 12. Top panel: volume-averaged Eddington ratio vs. time for the T3_F0.5_3D (solid black) and T3_F0.5_LR (dotted red) simulations. Middle panel: volume-averaged optical depth vs. time for the same simulations as in the top panel. Bottom panel: ratio of the flux-weighted mean optical depth to the volume-averaged optical depth vs. time for the same simulations as in the upper panels.

(A color version of this figure is available in the online journal.)

thus far have suggested that modest changes in the angular distribution of the radiation field can have a large affect on the dynamics, it is important to see how well our conclusions from 2D simulations carry over the 3D case. Due to the substantial increase in computational cost, we must perform a single 3D calculation (T3_F0.5_3D) at lower resolution, equivalent to the T3_F0.5_LR run in 2D.

Figure 12 compares $f_{E/V}$, $\tau_V$, and $\tau_F/\tau_V$ for T3_F0.5_3D (solid black) and T3_F0.5_LR (dotted red). The evolution of all three parameters overlaps for the first $t \sim 25 t_*$, when the flow is in the linear or early non-linear phases of RTI growth. Subsequent growth shows a significant enhancement for all three quantities in the 3D run. In the 3D run, $f_{E/V}$ remains above unity from $t = 48 t_*$ until the end of the run while the 2D run fluctuates about unity. This leads to a monotonic increase in $\langle v_y \rangle$. Both simulations must be stopped when $\rho \sim 10^{-3} \rho_*$ material reaches the upper boundary. Both simulations finish with $\sigma \sim 7 c_s$, but $\langle v_y \rangle = 8 c_s$ in T3_F0.5_3D, more than twice the final value of $3.5 c_s$ in T3_F0.5_LR.

These results suggest that the Eddington ratio is modestly enhanced in 3D. Figure 11, which shows a 2D slice through the $\rho$ distribution of T3_F0.5_3D, provides some indication of why this enhancement occurs. While the 2D density distribution is dominated by a number of narrow filaments, $\rho$ is more diffusely distributed in a larger number of lower density filaments. The overall density field is less structured inhibiting the escape of radiation through low-density channels and enhancing the coupling between matter and radiation. These results suggest that the dimensionality of the simulation will matter for properly assessing the role of radiative driving in dust environments.

5. DISCUSSION

5.1. The Dependence on the Radiation Transfer Method

In this work, we have revisited the calculations of KT12, but have been unable to reproduce all of their results with our more accurate VET radiation transfer algorithm. In contrast, our own implementation of the FLD algorithm seems to agree well with their calculations, indicating that discrepancies arise from the use of the FLD approximation, rather than implementation differences between the Athena and ORION codes. Although we reproduce several aspects of KT12’s results (see Section 4), in this section we focus primarily on the discrepancies and describe the deficiencies of the FLD algorithm that we believe are primarily responsible.

First, we note that our VET calculations do closely reproduce the KT12 results in the stable regime. Therefore, qualitative differences in the evolution arise from the development of non-linear structure, driven by the RTI. Although the development of the RTI is slightly slower in the VET case, some general aspects of the evolution are similar to the FLD simulations: a thin shell develops, becomes unstable to RTI, breaks up into high-density filaments interspersed with low-density channels, and eventually settles to state with $f_{E/V} \sim 1$ at late times.

In spite of these qualitative similarities, the differences in transfer methods have a fairly striking impact on the evolution of the velocity and spatial distributions of the gas. For the majority of the VET simulation, the radiation force exceeds gravity and the net vertical velocity is always positive. In contrast, the FLD results only see a transient acceleration phase followed by fallback of most of the gas, and settle into quasi-steady state
turbulence with a scale height and velocity dispersion which are much too low to explain observed systems.

We would like to understand what aspects of the FLD method lead to these discrepancies in evolution. We note that since both simulations reach Eddington ratios near unity, only modest differences are needed to produce divergent outcomes. The top panel of figure 13 shows a snapshot of $\rho$ for the T3_F0.5 run at $t = 25t_\ast$. We focus on a small fraction of the domain where the shell is being disrupted. The bottom panel compares $F_r$ with the FLD prediction for the flux using Equation (9) with the values $E_r$ in the VET run. The color map shows the flux ratio on a logarithmic scale while arrows denote the directions of the VET (white) and FLD (green) fluxes.

In the densest regions, our VET calculations are nearly in the diffusion limit where the FLD method is reliable, and the fluxes are in approximate agreement. In lower-density regions, where the diffusion approximation is poor, the magnitude of the FLD flux is much larger (up to factors $\sim 30$) than VET and the directions may be significantly different, even anti-parallel. For example, below and adjacent to the large plume in the center of the figure, the FLD fluxes point downward or horizontal while the VET fluxes point nearly upward almost everywhere in the domain. Since the radiation forces are proportional to $F_r$, the FLD fluxes would not be opposing the downward motion of the plume to the same extent as the VET flux and may even be reinforcing it. In contrast, the underdense regions near $x \sim 0$, $-115h_\ast$ and with $160 \leq y/h_\ast \leq 190$ have implied FLD fluxes that significantly exceed the VET fluxes. This means that (relative to VET) the FLD radiation forces would be more effective at reinforcing the development of the low-density channels that are forming. Overall, the tendency is for FLD to reinforce the development of non-linear structure to a greater extent than the VET algorithm, which is consistent with the faster non-linear development of the RTI in the FLD runs.

One objection to the comparison in Figure 13 is that the values of $E_r$ in the FLD run will not generally be the same as those in the VET run. Figure 14 avoids this issue by comparing $\rho$ and $F_{r,y}/E_r$ in the T3_F0.5 run to the same quantities in the T3_F0.5_FLD run. In both cases, we focus on a $256h_\ast \times 300h_\ast$ subsets of the simulation domains. The upper boundaries of these subsets are chosen to so that less than an optical depth of intervening matter lies between the top of the subset and the top boundary of the simulation domain, where vacuum boundary conditions are imposed.

Since the mean-free path of photons is proportional to $1/(\kappa\rho)$, $F_{r,y}$ will generally be larger when $\rho$ is smaller and vice versa. Such an anti-correlation is apparent in both runs and is particularly clear when we scale $F_{r,y}$ with $E_r$. Such...
anti-correlations are expected to be particularly strong if the gradient in $E_r$ is relatively uniform and the diffusion limit applies (Equation (9) with $\lambda \to 1/3$). However, the optical depths across most of the filaments seen in Figure 14 are of the order of unity or less, so the diffusion limit is not valid and the radiation flux is expected to be less sensitive to local values of $\rho$. Instead, it is determined by the geometry of sources of strong emission and the integrated optical depth along different lines of sight to these sources. The local value of the radiation field is determined by non-local properties of the flow. The VET algorithm accurately captures the variation of $F_{ry}/E_r c$ in this regime, because the Eddington tensor follows from the calculation of the angle-dependent radiation transfer equation. In contrast, the FLD radiation field is highly constrained by the ad hoc assumptions that underlie the approximation. There is a single preferred direction (parallel to $\nabla E_r$) and the ratio $F_{ry}/E_r c$ is determined by $R$ (Equation (10)). Since $|\nabla E_r|/E_r$ and $T$ are relatively uniform, the sharp variation of $\rho$ dominates the variation of $R$ and therefore $\lambda(R)$. This strong dependence of $\lambda(R)$ on $\rho$ leads to an unphysically high level of anti-correlation between $F_{ry}/E_r c$ and $\rho$ in low-to-moderate optical depth regions.

The upshot is that FLD tends to overestimate the contrast in $F_{ry}$ between the high-density filaments and the low-density channels, where most of the radiation escapes. In effect, the FLD radiation field is more effective at “punching holes” through the $\rho$ distribution and then reinforces the resulting channels to a greater degree than in the more accurate VET calculations. This allows more of the radiation to escape through the low-density channels, leading to a lower average radiation force for the majority of the gas.

Finally, we note that the FLD and VET methods agree well in regions where structures are moderately optically thick ($\tau \gtrsim 5$) and the diffusion approximation applies. This suggests that the FLD results may still be relevant for systems where the photospheric Eddington ratio is low. In these systems (i.e., systems with low $f_{E,\ast}$ and large $\tau_*$), the radiation force can only exceed the gravitational force at high optical depth, where $T$, and therefore $k_R$, is sufficiently large.

5.2. Comparison to Observed Systems

Although the current simulation setup is conducive to exploring the role of RTI, the choice to start with a hydrostatic initial condition limits the relevance to observations of real star forming galaxies. This is because the hydrostatic length scale in the problem is $h_s = 6.4 \times 10^{-4}$ pc (for $T_s = 82$ K and $f_{E,\ast} = 0.5$) and $L_s = 2048 h_s = 1.3$ pc. Hence the domain is nearly two orders of magnitude too small to contain a realistic ULIRG disk. Other simplifications, such as the assumption of a constant $g$, the approximate gray opacity, and the constant incident infrared flux, may also have a strong effect on the evolution.

Nevertheless, we can reach some tentative conclusions. One quantity of interest is the mass-weighted velocity dispersion $\sigma$. For T3_F0.3, $\sigma$ reaches a maximum $\simeq 14 c_s \simeq 7.5 \text{ km s}^{-1}$. 

\begin{figure}[h!]
\centering
\includegraphics[width=\textwidth]{figure14.png}
\caption{Top panels: maps of density $\rho$ for snapshots of the T3_F0.5 (left) and T3_F0.5_FLD (right) runs. Bottom panels: maps of the ratio of the vertical component of flux to energy density $F_{ry}/E_r$ for snapshots of the T3_F0.5 (left) and T3_F0.5_FLD (right) runs. The $256 h^* \times 300 h^*$ subsets of the domain are shown at $t = 50 t^*$ in the T3_F0.5 run and at $t = 42 t^*$ for the T3_F0.5_FLD. Note that the bottom of the T3_F0.5_FLD subset corresponds to the base of the domain while the bottom of the T3_F0.5 subset is $120 h^*$ above the base of the domain. There is a clearly correlation between $\rho$ and $F_{ry}/E_r$ in both calculations, but the degree of correlation is much stronger in the T3_F0.5_FLD run. (A color version of this figure is available in the online journal.)}
\end{figure}
However, since no steady-state is reached and the gas appears to be unbound in this simple setup, there is no evidence that this value of $\sigma$ would be characteristic of realistic self-gravitating system for either T3_F0.3 or T10_F0.3. It is difficult to assess what implications the trend toward $f_{E, V} \sim 1$ will have for the subsequent evolution of $\langle v_y \rangle$ and $\sigma$. The system is very sensitive to the precise value of $f_{E, V}$ when $g$ is constant. If, on average, $f_{E, V}$ continues to be slightly greater than one, then acceleration will continue and we would expect the scale height and velocities to grow with time indefinitely. In a real system, the vertical variation of $g$ would presumably play a significant role in determining an equilibrium scale height and velocity dispersion.

In contrast, the T1_F0.3 run with $\tau_c = 1$ seems to settle into a steady state with $\sigma \lesssim 3c_\infty$. This is perhaps not surprising because for $\tau_c \sim 1$ the trapping factor is not expected to be particularly large to begin with. Hence, for lower optical depth we conclude that radiative driving is likely ineffective due to the RTI, although this conclusion may be sensitive to the simplifying assumptions discussed above. This could mean that radiative feedback does not play a dominant role in less extreme starbursts and lower surface density ULIRGs (see, e.g., Krumholz & Thompson 2013) unless the radiation approaches the Eddington limit. Or, it may be that feedback in systems with lower mean optical depths might be dominated by regions with significantly higher than average optical depth. Assessing the role of radiative feedback in such systems will be an important goal for future work.

Another question of interest is whether these simulations are consistent with the radiation driving of large-scale outflows. Our results show that the simulations approach an Eddington ratio near unity for a constant value of $g$. However, if we assume that the gas in real ULIRGs is distributed in a disk-like geometry, the vertical component of gravity $g(\gamma)$ will increase from near zero at the midplane (where $\gamma = 0$) to a maximum value set by the overall potential of the galaxy. It seems plausible that an Eddington ratio near unity will also be reached in this case, but it is not clear what would select the specific value of $g_0$ where the radiation forces and gravity balance. However, if this hypothetical equilibrium behaves like our simulations, we would expect a fraction of the gas to be accelerated well beyond this characteristic $g_0$ because the opacity typically increases while $g(\gamma)$ decreases as we approach the midplane. Therefore, gas in low-density channels can be efficiently accelerated to high velocities, conceivably approaching the escape velocity from the galaxy. Such acceleration of low-density gas to high velocities is seen in our simulations. For example, when we stop the T3_F0.5 run, 4.1% and 1.2% of the gas had $v_y$ greater than $3\sigma_y$ and $4\sigma_y$, respectively. In typical ULIRGs, only a few percent of the gas is observed in the cold neutral outflows, so even a small tail of high-velocity gas could explain the observed outflow rates and velocities if $\sigma_y \sim 100 \text{ km s}^{-1}$ can be obtained in more realistic simulations.

In this respect, a prominent role of RTI in the dynamics could help solve a potential problem with radiation driving—that it cannot simultaneously explain both the presence of a radiation supported, quasi-equilibrium gas disk and the launching of outflows. If all the gas experienced the same radiative acceleration, it would all be in state of hydrostatic equilibrium or it would all be accelerated in an outflow. In principle, this could be resolved if outflows were driven primarily from localized regions that significantly exceed the local Eddington ratio. However, in an RTI-dominated picture, the bulk of the gas can be in a quasi-equilibrium turbulent state with $\langle v_y \rangle \sim 0$, but the presence of low-density channels with larger than average $F_\nu$ could still allow a modest fraction of the gas to be accelerated out the gas disk. In principle, this could allow radiation to launch outflows even in galaxies that are well below their global Eddington limit (see Socrates & Sironi 2013), although more realistic calculations are required to assess the effectiveness of this mechanism.

Another question is whether the inhomogeneities driven by the RTI have a notable impact on the spectrum emitted by the dusty gas. The simulation domain remains very optically thick to UV radiation, so unless the young stars are very near the photosphere, the inhomogeneities will not significantly alter the spectrum at these wavelengths. However, the inhomogeneities could, in principle, allow emission from hotter regions to escape, increasing the mid-infrared flux. We consider this possibility by using our short characteristics solver to generate a spectrum from a single snapshot of our T3_F0.5 run at $t = 75\tau_c$. For this exercise, we assume radiative equilibrium and only consider the dust absorption opacity. We model the frequency-dependent dust opacity using a model with $R_V = 5.5$ and assuming a gas to dust mass ratio of 105 (Weingartner & Draine 2001; Draine 2003). Although we do not have detailed knowledge of the dust opacity for ULIRGs and high-redshift galaxies, a higher value of $R_V$ is typical for giant molecular clouds in our galaxy and these models have previously been used provide satisfactory models of star forming galaxy SEDs (e.g., Chakrabarti & McKee 2005).

Figure 15 shows the resulting spectrum compared to a calculation with the same mass surface density, but, assuming uniform temperature model with $T = 97\text{ K}$, which approximately corresponds to the average photospheric temperature in the simulation. Some enhancement over the isothermal model is apparent at all wavelengths, because radiation coming from deeper, hotter regions can escape, but this enhancement is larger at shorter wavelengths. The spectrum peaks at shorter wavelengths than are typically observed in ULIRG spectral energy densities, but this follows from the simulation’s assumption of a fairly large photospheric temperature. The assumed temperature might be characteristic of hotter regions within a system like Arp 220, and the emission at $\lesssim 10\text{ m}$ would not be out of line with observed systems. Therefore, it does not seem to be the case that the inhomogeneities will allow too much short wavelength flux to emerge.
5.3. Assessing the Role of the RTI

We attribute the growth of structure in our simulations primarily to the RTI, but other instability mechanisms may play a role, in principle. Section 5.1 of KT12 provides a fairly comprehensive and persuasive discussion why the RTI is dominant and other instability mechanisms, such as radiative driving of unstable acoustic modes (Shaviv 2001; Blaes & Socrates 2003), are likely to be absent or unimportant. Here we simply summarize some of the most salient results and refer the reader to KT12 for further details.

Although a general solution for linear growth of the radiative RTI has never been formulated (see, e.g., Mathews & Blumenthal 1977; Krolik 1977; Jacquet & Krumholz 2011; Jiang et al. 2013), numerical experiments (Jiang et al. 2013) suggest that the instability will grow at a rate that can be approximated by \( t^{-1} \sim \sqrt{2\pi g/\lambda_x} \), if \( \lambda_x \) is a characteristic horizontal wavelength. This is the analytic prediction in the optically thick limit of an incompressible flow if the density contrast at the base of the shell is large (i.e., Atwood number of unity). Radiative diffusion effects can slow the rate of growth, but typically only by factors of the order of unity unless radiation pressure is much greater than gas pressure. For our simulations, this timescale evaluates to \( t_\eta \sim L/\sqrt{2\pi h_c} \) and we have \( t_\eta \sim 6t_\eta \) for \( \lambda_x \sim 256b_h \). The growth of the instability on small scales is even faster and there is ample time for growth of the RTI to explain the large-scale non-linear structure observed at \( t = 25t_\eta \) (and earlier).

Further evidence that the evolution is due to RTI is provided by calculations we performed with constant, temperature-independent opacities in the limits of pure scattering or pure absorption. The simulation setup for these runs were identical to the previous estimates of \( \tau_{IR} \) with \( \kappa/T_{mp}\Sigma_\ast \), where \( T_{mp} \) is the temperature at the base of the simulation domain. For T3_F0.5, this quantity is a factor of \( \sim 1.5 \) larger than \( \tau_x \), which we associate with \( \tau_{IR} \).

We can gain insight into the scalings of \( \tau_x, \tau_F \), and \( \eta \) as functions of \( \tau_\ast \) and \( f_{\eta,\ast} \), by assuming that all simulations will approach a characteristic horizontally averaged vertical profile. At late times our simulations can be fit approximately by

\[
\tau^4 = \tau_x^4 \left( \frac{2}{3} + \tau_F \right),
\]

where quantities with an overbar correspond to \( y \)-dependent (2D) or \( z \)-dependent (3D), horizontally averaged quantities. For example, \( \bar{\rho} = \rho \int \rho \, dx \) in 2D and \( \bar{\rho} = \rho \int \rho \, dx \int dy \) in 3D. The exceptions are

\[
\tau_F(y) = \int_y L_{\rho F} y \, dy',
\]

with \( y \) replaced by \( z \) in the 3D case.

To a reasonable approximation, we find that

\[
\tau_F \simeq \begin{cases} 
\tau_x, & \tau_x < 1, \\
\tau_x \tau_x & \tau_x \tau_x \end{cases},
\]

with a constant \( \eta \simeq \tau_F/\tau_x \). Using the fact that

\[
\frac{\partial \tau}{\partial \Sigma} = \kappa_{R,\ast} \left( \frac{\tau_x}{F_{\rho}} \right)^2,
\]

where \( \rho/dt \) is an estimate of the momentum transferred to the interstellar gas, \( \tau_{IR} \) is an estimate of the integrated optical depth through the dusty gas at infrared wavelengths, \( L \) is luminosity of a star (or star cluster), and \( \eta \) is an ad hoc reduction factor (\( \eta \simeq 1 \)) included to account for inhomogeneities in the surrounding gas.

Since \( \tau_{IR} \) corresponds to a volume-weighted average total optical depth, one can estimate \( \tau_{IR} \approx \tau_x \), while \( \tau_x \) represents the effective optical depth for momentum exchange. Therefore, we estimate the \( \eta \) in Equation (23) as the ratio \( \tau_{IR}/\tau_x \approx 0.87, 0.68, \) and 0.4 for runs T1_0.5, T3_0.5, and T10_F0.5, respectively. Here we have time-averaged the runs \( t = 30t_\eta \) to the end of the simulation. Values of \( \eta \simeq 0.4–0.87 \) are in the range considered

Note that Equation (24) implies \( E_{05} \approx 2F_{\rho}/c \), which is roughly the ratio returned by our short characteristics calculation at the top of the domain. For an approximately isotropically emitting, semi-infinite atmosphere, which is essentially what our periodic horizontal boundary conditions provide, this agrees well with standard gray treatments (Mihalas 1978). This is greater than the \( E_{05} \approx F_{\rho}/c \) assumed in FLD calculations, a limit that is only obtained at large distances from a point source.

\[\text{[Ref. to KT12 for further details.]}\]
with $d\Sigma = -\rho d y$, we find

$$\tau_V = \frac{3}{4} \eta (\kappa_{R,*}\Sigma)^2 + \sqrt{2} \kappa_{R,*}\Sigma.$$  \hfill (28)

By definition, $\tau_V(0) = \tau_V$ and $\Sigma(0) = \Sigma_* = \tau_*/\kappa_{R,*}$ so

$$\tau_V = \frac{3}{4} \eta \tau_*^2 + \sqrt{2} \tau_*.$$  \hfill (29)

If we assume that all simulations yield $f_{E,V} \approx 1$, we can infer that

$$\tau_F \approx \frac{\tau_*}{f_{E,*}} = \frac{cg}{F_*} \Sigma_*,$$  \hfill (30)

which is equivalent to KT12's Equation (43) with $(f_E) = 1$. Thus, for a given input flux $F_*$ and constant gravitational acceleration $g$, we can define a characteristic opacity

$$\kappa_E = cg/F_*,$$  \hfill (31)

which corresponds to the opacity where the Eddington ratio is unity. Then the optical depth $\tau_F = \kappa_E \Sigma_*$ can be used in place of $\eta \tau_F$ in Equation (23). In our calculations, $\kappa_E$ is only slightly larger than $\kappa_\nu$ near the photosphere for most of our simulations, which is in approximate agreement with conclusion of Krumholz & Thompson (2013) that the photospheric opacity is closer to the relevant opacity than the midplane value for evaluating momentum feedback. However, $\kappa_E$ could, in principle, significantly exceed the photospheric opacity for systems with lower $f_{E,*}$ and higher $\tau_*$.

We can estimate for $\eta = \tau_F/\tau_V$ by combining Equations (29) and (30) to obtain

$$\eta = \frac{2}{3\tau_*} \left(\sqrt{2 + 3\tau_*/f_{E,*}} - \sqrt{2}\right).$$  \hfill (32)

Finally, we can use Equation (32) in Equation (29) to obtain

$$\tau_V = \frac{\tau_*}{\sqrt{2}} \left[1 + \left(1 + \frac{3\tau_*}{2f_{E,*}}\right)^{1/2}\right].$$  \hfill (33)

Table 2 summarizes the time-averaged values for $\langle \tau_F \rangle$, $\langle \tau_V \rangle$, and $\langle \eta \rangle$ for runs T1_F0.5, T3_F0.5, and T10_F0.5. The quantities in parentheses are the estimates provided by Equations (30), (32), and (33). The estimates prove to be fairly accurate, particularly for the $\tau_* = 3, 10$ runs. For T1_F0.5, which fails to maintain $f_{E,V} \gtrsim 1$, we see that both $\eta$ and $\tau_F$ are slightly low, as anticipated. Therefore, we expect the estimates provided here to be useful predictions of how momentum feedback will scale with $\tau_*$ and $f_{E,*}$ (or, alternatively, $\Sigma_*$ and $F_*$, given $g$), as long as the system reaches an equilibrium with an Eddington ratio near unity.

The upshot of this analysis is that $\kappa_E$ would be the most suitable opacity to use in determining the trapping factor if the system being modeled has values of $T_*$ and $\Sigma_*$ that allow $\kappa_\nu$ to consistently reach or exceed $\kappa_{R,*}$ at the base of the domain. A seemingly necessary (but not sufficient) condition for this is that $\tau_V > \tau_F$. Combining Equations (30) and (33), this becomes

$$\tau_* > \frac{4}{3} \left(\frac{1}{f_{E,*}} - \sqrt{2}\right).$$  \hfill (34)

Note that all of the simulations except T10_F0.02 obey this constraint, but T1_F0.5 has $\tau_V$ barely exceeding $\tau_F$ and $\tau_F \lesssim \kappa_E \Sigma_*$ so it would seem this relation is only an approximate guide.

For our simulations with $f_{E,*} = 0.5$, $\kappa_E$ corresponds to $\sim 4 \text{ cm}^2 \text{ g}^{-1}$. More generally, one can estimate characteristic values for $\kappa_E$ using Equation (31) with $g = 2\pi G \Sigma_*/f_{\text{gas}}$ and $F_* = \epsilon c^2 \Sigma_*$ to obtain

$$\kappa_E \simeq \frac{2\pi G \Sigma_*}{\epsilon f_{\text{gas}} \epsilon \Sigma_*},$$  \hfill (35)

where $\epsilon$ is the fraction of rest mass energy converted into radiation by stars, $\Sigma_*$ is the star-formation rate per unit area, and $f_{\text{gas}}$ is the gas fraction of the total (stellar plus gas) mass. Scaling this relation in terms of characteristic values: $\epsilon_* = \epsilon/10^{-3}$, $\Sigma_0 = \Sigma_*/(1 \text{ g cm}^{-2})$, $\Sigma_* = \Sigma_*/(10^3 M_\odot \text{ yr}^{-1} \text{ kpc}^{-2})$, we find

$$\kappa_E \simeq 2.1 \text{ cm}^2 \text{ g}^{-1} \epsilon^{-1/3} \epsilon_*^{-1} \Sigma_0^{-1} f_{\text{gas}}^{-1}.$$  \hfill (36)

Given the uncertainties and approximations involved, this estimate is in rough agreement with the values of opacity assumed by previous authors (e.g., $5 \text{ cm}^2 \text{ g}^{-1}$; Hopkins et al. 2011).

Finally, we note that all of this analysis and discussion assumes that only radiation pressure and gravitational forces play a role. In real systems, mechanical feedback from stellar winds, supernovae, cosmic rays, magnetic fields, photoionization and photoelectric heating, etc. may be present. Even if they are not the dominant mechanisms of momentum transfer to the gas, they may still have a measurable impact on the gas velocity and density distributions. The implication that RTI will generically produce density inhomogeneities that drive $f_{E,V}$ toward a value $\sim 1$ may break down if any of these other mechanisms are strong enough to reorient the low-density channels and dense filaments. It seems much more likely that such reorientation will interfere with, rather than enhance, the escape of photons, leading to increased coupling and conceivably to generically super-Eddington configurations. This hypothesis can be tested with future calculations.

### 6. SUMMARY AND CONCLUSIONS

We have considered the role of the RTI in the interaction of infrared radiation fields, dust, and gas in rapidly star forming environments. We have focused on the regime of radiation-supported dense gas that may be present in some systems, such as ULIRGs. Our primary results stem from the numerical simulation of such environments, which are studied in a simplified problem setup with a constant gravitational acceleration, a constant incident infrared flux on the base of the domain, and initialized with a perturbed isothermal atmosphere to match previous calculations in KT12.

In the stable regime, we find that the atmosphere settles down into an equilibrium solution in agreement with the previous results. In the unstable regime, we confirm that the RTI develops...
and has a significant impact on subsequent evolution. However, we find that after the growth of the RTI, the evolution depends significantly on the choice of algorithm for modeling radiation transfer. Our VET simulations show a stronger coupling between radiation and dusty gas, leading to continuous net upward transfer. In contrast, our FLD calculations broadly reproduce the FLD results of KT12, finding weaker coupling between gas and radiation. This leads to a short initial burst of acceleration, followed by a period of fallback, finally settling into quasi-steady state turbulence with zero mean velocity and low-velocity dispersion. As a result, our VET calculations imply a much larger scale height and higher velocity dispersion than the FLD-based calculations.

We argue that these discrepancies result from limitations in the diffusion-based FLD algorithm, which lead to inaccuracies in modeling how the radiation field responds to structure in the gas distribution that is a few optical depths or smaller in size. These errors are related to the FLD radiation field’s tendency to diffuse around denser, optically thicker structures even when the diffusion limit does not apply. Relative to our VET calculations, this leads the FLD radiation forces to be more effective at opening and reinforcing low-density channels but less effective at disrupting high-density filaments, ultimately reducing the coupling between radiation and dusty gas.

Despite the discrepancies in the scale height and velocity distributions, it appears that both the VET and FLD simulations trend toward an approximate balance between the volume-averaged radiation and gravitational forces at late times. This confirms that the RTI limits momentum exchange, which is one of the key results of KT12 (see also Krumholz & Thompson 2013). Specifically, the rate of momentum transfer between radiation and dusty gas may scale approximately as \( \sim \tau_{E}L/c \), where \( L \) is the luminosity, \( \tau_{E} = \kappa_{E} \Sigma_{e} \), \( \Sigma_{e} \) is the mass surface density, and \( \kappa_{E} = \tau_{g}/F_{g} \), is the opacity for which the characteristic radiation and gravitational accelerations balance. However, this relation presupposes that the characteristic values of temperature and \( \Sigma \), allow the Rosseland mean opacity \( \kappa_{R} \) to reach \( \kappa_{E} \) in the deeper regions of the cloud or disk. Since \( \kappa_{R} \) is generally higher at larger optical depth, \( \tau_{g} \) will be lower than estimates of \( \tau_{g} \) that use opacities evaluated at the disk midplane or cloud center, where optical depth is maximum. Nevertheless, \( \kappa_{E} \) is generally larger (if only by factors of a few) than the opacity evaluated at the photosphere (see Krumholz & Thompson 2013). The values of \( \kappa_{E} \) found in our simulations do not differ significantly from values used in previous numerical simulations and modeling (e.g., 5 cm\(^2\) g\(^{-1}\); Hopkins et al. 2011).

Although the simulation setup considered here is useful for studying the development and saturation of the RTI, it is not optimally suited for making observational predictions. Future work with more realistic assumptions, such as a vertically varying gravitational acceleration, non-gray opacity, distributed radiation sources, and physically relevant simulation volumes, will be necessary to provide robust predictions and facilitate direct comparison with observations.

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**APPENDIX**

**IMPACT OF ANGULAR RESOLUTION**

The discussion in Section 5.1 assumes that the VET radiative transfer algorithm provides a more accurate realization of the radiation field than the FLD algorithm. Since we solve the momentum Equation (7), this should be true, as long as our method for computing the Eddington tensor is valid. As discussed in Section 2.1, the main approximation made in our computation of the Eddington tensor is the neglect of the order of \( v/c \) terms (which are retained in the radiation moment equations). We confirm after the fact that this is a very good approximation.

The only significant potential problem for our VET calculation would be if our angular grid of rays under-resolves the radiation field. Our experience has been that angular resolution used here is more than adequate for previous problems, but angular resolution requirements can vary from problem to problem. In the short characteristics method, the most apparent features in under-resolved systems are usually referred to as "ray-effects," which correspond to unphysical anisotropies in the radiation field correlated with the angular grid. Ray-effects can be particularly problematic when bright point sources dominate the radiation field and scattering opacity is negligible (see, e.g., Larsen & Wollaber 2008; Finlator et al. 2009). The intensity can be overestimated in grid zones that lie along ray directions that point directly back to the point source and underestimated in regions between rays, with a relative error that increases in magnitude with the number of grid zones traversed. This leads to the appearance of 'rays' or 'spokes' in radiation variables emanating radially outward from the point source.

Scattering introduces angular diffusion that can mitigate ray-effects (see, e.g., Larsen & Wollaber 2008), but is absent in the current calculations. Although bright point sources are also absent, dense filaments can lead to rather sharp variations in the emissivity. Ray-effects are most clearly apparent in the low-density regions well above the photosphere, reflecting variations in the emissivity near the photosphere that propagate along rays in the angular grid into the regions above. However, there is very little mass in this region and radiation is less important because the radiation forces are sub-Eddington here.

Ray-effects are less apparent below the photosphere, but must be present at some level. In order to assess their impact (if any), we reran the T3_F0.5 run with an alternative ray distribution that has effectively higher resolution in the \( x-y \) plane. Our default ray distribution is shown in the left panel of Figure 16. It attempts to cover the unit sphere uniformly, treating all three spatial directions, including the implied third dimension (\( z \)) on
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Figure 16. Ray distributions for a single octant of the unit sphere. The left panel shows our default distribution used for runs T10_F0.02 and T3_F0.5. Poloidal angles correspond to abscissas of Gauss–Legendre quadrature and azimuthal angles are distributed uniformly over the interval 0 to π/2 within each poloidal level. The right panel shows an alternate distribution with a single poloidal level (n_z = 1/√3), which we use to increase the effective angular resolution in the x–y plane. The axes correspond to the projection of the unit vector corresponding to the ray onto the Cartesian axes of the domain, e.g., n_x = ̂n ⋅ ̂x.

Figure 17. Top panel: volume-averaged Eddington ratio vs. time for the T3_F0.5 (solid black) and T3_F0.5_ALT (dotted red) simulations. Middle panel: volume-averaged optical depth vs. time for the same simulations as in the top panel. Bottom panel: ratio of the flux-weighted mean optical depth to the volume-averaged optical depth vs. time for the same simulations as in the upper panels. (A color version of this figure is available in the online journal.)

equal footing. This is formally required even though the domain is two-dimensional, because rays traveling at different angles relative to ̂z will travel different total path lengths for a given displacement in the x–y plane. This means that many of the rays nearly overlap when projected onto the x–y plane, and the effective angular resolution in the x–y plane is closer to ~24 rays per 2π radians, even though we have 84 rays total. The right panel shows our alternative distribution, in which all
rays have the same projection onto the $z$ axis ($n_z = 1/\sqrt{3}$ so that $f_{zz} = 1/3$). We distribute 80 total rays uniformly in azimuthal angle $\phi$ ($\tan \phi = n_y/n_x$). This amounts to a factor $\sim 3$ increase in the effective angular resolution in the $x$–$y$ plane even though the total number of angles is nearly identical. The trade off is that fixing $f_{zz} = 1/3$ can slightly modify the other diagonal components since the trace of the tensor must be unity ($f_{xx} + f_{yy} + f_{zz} = 1$).

Simulations using this alternative angular quadrature are denoted with $f_{zz} = 1/3$ in Table 1, while simulations using the original method are denoted as uniform. The impact of the angular quadrature on the evolution can be assessed by comparing simulations T3_F0.5 and T3_F0.5_AQ, which are identical except for the choice of angular quadrature. The higher effective azimuthal angular resolution significantly reduced ray-effects above the photosphere in T3_F0.5_AQ, but had relatively little effect on the Eddington tensor below the photosphere, suggesting ray effects were already minor in the original run. For comparison, we plot both the first 83 $t_*$ of T3_F0.5 (black solid) and T3_F0.5_ALT (red, dotted) in Figure 17. The global properties and evolution of the two simulation are nearly identical, indicating that our angular quadrature choice and resolution are not significantly impacting our results.

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