Summary: In the chapter “Magic with a matrix” in [Hexaflexagons and other mathematical diversions. Chicago, IL: University of Chicago Press (1988)], M. Gardner describes a delightful “party trick” to fill the squares of a $d$-by-$d$ chessboard with nonnegative integers such that the sum of the numbers covered by any placement of $d$ nonthreatening rooks is a given number $N$. We consider such chessboards from a geometric perspective which gives rise to a family of lattice polytopes. The polyhedral structure of these Gardner polytopes explains the underlying trick and enables us to count such chessboards for given $N$ in three different ways. We also observe a curious duality that relates Gardner polytopes to Birkhoff polytopes.

MSC:
05A05 Permutations, words, matrices
52B12 Special polytopes (linear programming, centrally symmetric, etc.)
52B20 Lattice polytopes in convex geometry (including relations with commutative algebra and algebraic geometry)
52B25 Combinatorial properties of polytopes and polyhedra (number of faces, shortest paths, etc.)

Keywords:
Gardner polytopes; Birkhoff polytopes.

Full Text: DOI arXiv

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