Dephasing via stochastic absorption: A case study in Aharonov-Bohm oscillations

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The Aharonov-Bohm ring has been the mainstay of mesoscopic physics research since its inception. In this paper we have dwelt on the problem of dephasing of AB oscillations using a phenomenological model based on stochastic absorption. To calculate the conductance in the presence of inelastic scattering we have used the method due to Brouwer and Beenakker. We have shown that conductance is symmetric under flux reversal and visibility of AB oscillations decay to zero as a function of the incoherence parameter thus signalling dephasing in the system. Some comments are made on the relative merits of stochastic absorption with respect to optical potential model, which have been used to mimic dephasing.

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Dephasing is defined as the process by which quantum mechanical interference is gradually destroyed. Dephasing of electronic phase coherence may be caused by it’s interaction with other quasi particles in the system. The notion of intrinsic decoherence and dephasing of a particle interacting with it’s environment is being investigated intensively. For, a comparison between theory and experiment, it is necessary to know how dephasing affects the various quantum phenomena related to transport. In our present work we will be interested in modeling dephasing phenomenologically. The Aharonov-Bohm (AB) interferometer is one of the best examples for analyzing how quantum interference effects are affected by dephasing. In this interferometer the phase of the electrons passing through the arms is modulated by the magnetic flux. It is not unlike the Young’s double slit experiment apart from the presence of the magnetic flux. In the Young’s double slit interferometer as we know the intensity is given by \( I = |\Psi|^2 = |\psi_1|^2 + |\psi_2|^2 + 2\Re(\psi_1^* \psi_2 e^{i\phi}) \), the part \( 2\Re(\psi_1^* \psi_2 e^{i\phi}) \) represents the interference term. Here \( \psi_1 \) and \( \psi_2 \) are the complex wave amplitudes across the upper and lower arms of the interferometer (this discussion is restricted to only two partial amplitudes for simplicity) and \( \phi \) is phase difference between the two wave amplitudes. If there is no phase relationship between the waves then the intensity will be \( I = |\psi_1|^2 + |\psi_2|^2 \). Dephasing thus suppresses the interference terms. In other words dephasing, leads to the vanishing of the off-diagonal elements of the density matrix.

Aharonov-Bohm effect in mesoscopic rings manifests itself as a periodic behavior of conductance as a function of magnetic flux piercing the loop. In an AB interferometer the electrons are treated quantum mechanically along the ring and they retain their phase memory across the entire sample and this gives rise to an operative definition of the phase coherence length \( L_\phi \) or incoherence length. On increasing the temperature \( L_\phi \) becomes less than the size of the ring and the phase coherence in the system gradually disappears and as a consequence the AB effect (the visibility of the AB oscillations) vanishes. Typically \( L_\phi \) scales with temperature \( T \) in a power law form, i.e., \( L_\phi = T^{-p} \) (\( p \) lies in the range 1 to 2).

There are different ways to model dephasing in such mesoscopic systems. A controlled way to introduce dephasing in the AB interferometer is to attach a voltage probe to the sample as in inset of Fig. 1 (Buttiker’s model). This voltage probe breaks the phase coherence by removing electrons from the phase coherent channel in the system and subsequently re-injecting them without any phase relationship. Another way to introduce dephasing is to add a spatially uniform imaginary potential \((-iV_1)\) (or optical potential) to the Hamiltonian which removes particles from the phase coherent motion in the system. In this case absorbed particles are identified as a spectral weight lost in the inelastic channels and they are re-injected back into the sample. In the method due to Zolla and Ezawa, the total transmission is defined after re-injection as the sum of two contributions one due to the coherent part and the other due to the incoherent part, i.e., \( T_{tot} = T_{coh} + T_{incoh} \). The incoherent part is calculated as \( T_{incoh} = \frac{T}{T_T} A \), herein \( T_T \) and \( T_I \) are the probabilities for right and left transmission from the region of inelastic scattering and \( A \) is the absorbed part which is given by \( A = 1 - T_{coh} - R_{coh} \).

This model has been used by several other authors as well to simulate inelastic scattering. But, this model has a problem, in the presence of magnetic flux it is shown to violate the Onsager’s two terminal symmetry relations, i.e., \( T_{tot}(\Phi) \neq T_{tot}(-\Phi) \), where \( \Phi \) is the magnetic flux, and hence is not suitable in modeling inelastic scattering whereas Buttiker’s model preserves the Onsager’s symmetry relations in the presence of magnetic flux. However, the Buttiker model suffers from a shortcoming, in that it describes only localized dephasing (at the point contact between system and third lead) instead of dephasing that occurs throughout the system in realistic situations.

Brouwer and Beenakker have removed the shortcomings of both the above stated models by mapping the three probe Buttiker’s method into a two terminal geometry, this is done by eliminating the transmission coefficients which explicitly depend on the third probe by means of unitarity of the S-matrix. They have considered a three terminal geometry in which one of the probes is
Application of the relations \(3,6,7\) of the entire system can be written as:

\[
\text{the ring to a reservoir at chemical potential to drain. In this model, a fictitious third lead connects the S - Matrix leads to which involve the voltage probe using the unitarity of I way that no current is drawn (}\alpha, \beta, \gamma\text{ used as a voltage probe in absence of magnetic flux (see inset of Fig. 1). A current } I = I_1 = -I_2 \text{ flows from source to drain. In this model, a fictitious third lead connects the ring to a reservoir at chemical potential } \mu_3 \text{ in such a way that no current is drawn (} I_3 = 0\text{). The 3X3 S-matrix of the entire system can be written as:

\[
S = \begin{pmatrix}
 r_{11} & t_{12} & t_{13} \\
 t_{21} & r_{22} & t_{23} \\
 t_{31} & t_{32} & r_{33}
\end{pmatrix}
\]

Application of the relations \(3,6,7\) \( I_p = \sum_q G_{pq}[\mu - \mu_q], p = 1, 2, 3 \text{ and } G_{pq} = (2e^2/h)T_{pq} \) yields the (dimensionless) two probe conductance \( G = \frac{h}{2e^2|\mu_1 - \mu_2|} \).

\[
G = T_{21} + \frac{T_{23}T_{31}}{T_{31} + T_{32}}
\]

and on elimination of the transmission coefficients which involve the voltage probe using the unitarity of the S - Matrix leads to:

\[
G = T_{21} + \frac{(1 - R_{11} - T_{21})(1 - R_{22} - T_{21})}{1 - R_{11} - T_{21} + 1 - R_{22} - T_{12}}.
\]

Now all the above coefficients are built from the 2X2 S matrix:

\[
S' = \begin{pmatrix}
 r_{11} & t_{12} \\
 t_{21} & r_{22}
\end{pmatrix}
\]

which represents the S Matrix of the absorbing system. The first term in Eq. 2 represents the conductance contribution from the phase coherent part. The second term accounts for electrons that are re-injected from the phase breaking reservoir, thereby ensuring particle conservation in the voltage reservoir model. For further calculation one can use the coefficients of the \( S' \) matrix of an absorbing system where absorption takes place uniformly.

There are different ways of introducing absorption in the system -(1) The well known imaginary potential (IP) model and (2) Stochastic absorption (SA). In the first case Hamiltonian becomes non-hermitian due to the presence of complex potential term \(-iV_t\), which leads to absorption (non conservation of particle number). This model suffers from some spurious features. In the scattering case, in the vicinity of the absorber, the particle experiences a mismatch in the potential (being complex) and therefore it tries to avoid this region by enhanced back reflection. Thus the IP model plays a dual role of an absorber as well as a reflector, i.e., absorption without reflection is not possible. Naively one expects the absorption to increase monotonically as a function of \( V_t \). However, the observed behavior is non-monotonic. At first absorption increases and after exhibiting a maximum decreases to zero as \( V_t \to \infty \). The absorber in this limit acts as a perfect reflector. During each scattering event an electron picks up an additional scattering phase shift due to \( V_t \) which along with multiple reflections leads to spurious scatterings (additional resonances) in the system. As a result of the above mentioned limitations there is need of a model wherein such spurious scatterings are absent and in the limit of large absorption does not correspond to a perfect reflector. Fortunately such a model exists, and this is the model of stochastic absorption. This model does not suffer from the drawbacks of the IP model as we will show towards the end of our work.

The SA model is not new it has earlier been dealt with in the context of ID localization. Stochastic absorption in the ring is inserted by the factor \( e^{-\alpha l} \) (or \( e^{-\alpha l} \)) in the complex free propagator amplitudes, every time we traverse the upper (or lower) arms of the ring (see Fig. 1). We have calculated the relevant transmission and reflection coefficients by using the S matrix method along with the quantum wave guide theory for a single channel case. In this model, average absorption per unit length is given by \( 2\alpha \). With this method we show that the calculated conductance \( (G) \) in Eq. 2 is symmetric under the flux reversal as required. The visibility of the AB oscillations rapidly decay as a function of \( \alpha \), indicating dephasing. Hence forth we refer to \( \alpha \) as an incoherence parameter. Increasing \( \alpha \) corresponds to increasing dephasing processes in the system or increase in temperature.

In Fig. 1, the length of the upper arm is \( l_1 \) and that of lower arm is \( l_2 \). The total circumference of the loop is \( L = l_1 + l_2 \). The loop is connected to two current leads. The couplers (triangles) in Fig. 1 which connects the leads and the loop are described by a scattering matrix \( S \). The S matrix for the left coupler yields the amplitudes \( O_1 = (\alpha', \beta_1', \gamma_1') \) emanating from the coupler in terms of the incident waves \( I_1 = (\alpha, \beta_1, \gamma_1) \), and for the right coupler yields the amplitudes \( O_2 = (\beta_2', \gamma_2') \) emanating from

![Aharonov - Bohm ring geometry. Inset shows the three probe model.](https://example.com/aharonov-bohm-ring.png)
the coupler in terms of the incident wave \( I_2 = (\delta, \beta_2, \gamma_2) \). The S-matrix for either of the couplers \( S \) is given by-

\[
S = \begin{pmatrix}
-\alpha & \beta \\
\beta & \gamma
\end{pmatrix}
\]

with \( \alpha = \frac{1}{2} \sqrt{(1 - 2\epsilon) - 1} \) and \( \beta = \frac{1}{2} \sqrt{(1 - 2\epsilon) + 1} \). \( \epsilon \) plays the role of a coupling parameter. The maximum coupling between reservoir and loop is \( \epsilon = \frac{1}{2} \), and for \( \epsilon = 0 \), the coupler completely disconnects the loop from the reservoir. As mentioned above incoherence is taken into account by introducing an attenuation constant per unit length as mentioned before.

The waves incident into the branches of the loop are related by the S Matrices for upper branch by-

\[
\begin{pmatrix}
\beta_1 \\
\beta_2
\end{pmatrix} = \begin{pmatrix} 0 & e^{ikl_1 e^{-\alpha l_1 \epsilon^{-\alpha l_1}}} \\
e^{ikl_1 e^{-\alpha l_1 \epsilon^{-\alpha l_1}}} & 0 \end{pmatrix} \begin{pmatrix} \beta'_1 \\
\beta'_2
\end{pmatrix}
\]

and for lower branch-

\[
\begin{pmatrix}
\gamma_1 \\
\gamma_2
\end{pmatrix} = \begin{pmatrix} 0 & e^{ikl_2 e^{-\alpha l_2 \epsilon^{-\alpha l_2}}} \\
e^{ikl_2 e^{-\alpha l_2 \epsilon^{-\alpha l_2}}} & 0 \end{pmatrix} \begin{pmatrix} \gamma'_1 \\
\gamma'_2
\end{pmatrix}
\]

These S matrices of course are not unitary \( S(\alpha)S(\alpha) = 1 \) but they obey the relation \( S(\alpha)S(-\alpha) = 1 \). The same relation is also obeyed by the S Matrix of the system in presence of imaginary potential. Here \( kl_1 \) and \( kl_2 \) are the phase increments of the wave function in absence of flux. \( \theta_{kl_1} \) and \( \theta_{kl_2} \) are the phase shifts due to flux in the upper and lower branches. Clearly, \( \theta_{kl_1} + \theta_{kl_2} = \frac{2\pi}{\Phi} \), where \( \Phi \) is the flux piercing the loop and \( \Phi_0 \) is the flux quantum \( \frac{\hbar e}{4\pi} \).

The transmission and reflection coefficients in Eq. 2 are given as follows- \( T_{21} = |\delta'|^2, R_{11} = |\alpha'|^2, R_{22} = |\gamma'|^2 \), \( T_{12} = |\delta|^2 \) wherein \( \delta', \delta, \alpha', \alpha \) are as depicted in Fig. 1.

After calculating the required reflection and transmission coefficients we graphically represent our results in the following figures. Throughout the discussion the physical parameters we give are in dimensionless units. We see that the coherent transmission \( T_{21} \) is not symmetric under the flux reversal however proper re-injection of carriers by Eq. 2 for the total conductance \( G \) plotted in Fig. 2 shows that the Onsager’s symmetry relations are restored, i.e., \( G \) is symmetric under flux reversal. In the inset of Fig. 2 we have plotted \( G \) versus flux for the same physical parameters except we have considered the weak coupling case (\( \epsilon = 0.10 \)). \( G \) is symmetric in flux and features are sharp as expected because of lifetime broadening of the energy levels is small in this case compared to the case for which \( \epsilon = 0.44 \).

In Fig. 3 we plot visibility \( (V) \) as a function of incoherence parameter \( \alpha \). Visibility is of course defined as-

\[ V = \frac{G_{max} - G_{min}}{G_{max} + G_{min}}. \]  

(3)

The plot shows that with increase in the value of the parameter \( \alpha \) the visibility exponentially falls off, reaching a point where it becomes zero corresponding to the disappearance of quantum interference effects. In the inset of Fig. 3 we have plotted the first few Fourier harmonics \( a_i \) (wherein \( i = 1 \) to 4) of \( G(\Phi) \) as a function of \( \alpha \). The harmonics exponentially fall off with increasing \( \alpha \) with exponent increasing as we go from 1 to 4. The \( nth \) order harmonic corresponds to the contribution from electronic
paths which encircle the flux n times. The harmonics can sometimes show non monotonic behavior depending on the physical parameters, however, the visibility is a monotonic function of the incoherence parameter $\alpha$.

In conclusion, we have shown that $G(\Phi)$ is symmetric under flux reversal in the presence of incoherent scattering represented by the incoherence parameter $\alpha$. For this we have used the procedure of Brouwer and Beenakker in the presence of magnetic flux and simulated absorption (dephasing) using the method of stochastic absorption. We have used this method of SA to study the behaviors of the various quantum phenomena, e.g., transport across resonant tunneling systems, current magnification effect in mesoscopic rings, delay and sojourn times in mesoscopic systems in the presence of incoherence. These results will be published elsewhere.

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