Chameleon gravity and satellite geodesy

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ABSTRACT

We consider the possibility of the detection of a chameleon effect by an earth orbiting satellite such as LAGEOS, and possible constraints that might be placed on chameleon model parameters. Approximate constraints presented here result from using a simple monopole approximation for the gravitational field of the earth, along with results from the Khoury-Weltman chameleon model, solar system constraints obtained from the Cassini mission, and parameter bounds obtained from the LAGEOS satellite. It is furthermore suggested that a comparison of ground-based and space-based multipole moments of the geopotential could reveal a possible chameleon effect.

1. Introduction

Newtonian gravity, a Newtonian limit (weak fields, nonrelativistic motions) of general relativity, has been successful in describing gravitational phenomena in laboratory settings. However, models of modified gravity have been proposed which may give rise to small corrections to Newtonian gravity, perhaps manifesting themselves on different distance scales or under certain environmental influences. One such interesting modification is the “chameleon gravity” model [Khoury and Weltman 2004a,b], wherein the effective coupling of gravitation to matter can depend upon the environment. In particular, in regions of high mass density, the scalar component of the interaction (the scalar chameleon field, \( \phi \)) develops a large mass and a short range, so that its effect is suppressed, whereas in regions of low mass density the field can become effectively massless and long ranged, and consequently have a much more pronounced effect, reflected in a different value of the effective gravitational constant. Therefore, according to the chameleon model, earth-based gravity is different from space-based gravity.

Attention is focused here on the motion of a satellite, mimicking a “test particle”. Assuming the chameleon gravity hypothesis, a chameleonic acceleration of the satellite results, due to a departure from pure geodesic motion. Therefore, sensitive measurements of any such
departure can serve to set bounds on chameleon model parameters. Here, we use the reported bound on a fifth force coupling constant $\alpha$, obtained from a data analysis of the LAGEOS satellite, in conjunction with results from the Khoury-Weltman chameleon model, along with solar system constraints from the Cassini mission to establish possible bounds on the chameleon model. Numerical estimates are made, using a simple monopole model, as in Khoury and Weltman (2004a,b). These estimates lead us to establish approximate constraints on the chameleonic acceleration $a_\phi$, and constraints on the chameleon coupling parameter $\beta$ may be approximated, which describes the strength with which the chameleon field $\phi$ couples to matter. Furthermore, we suggest that a comparison of ground-based and space-based multipole moments of the geopotential could provide evidence for the existence or nonexistence of a measurable chameleon effect.

2. The chameleon model

The chameleon model can be considered to arise from a scalar-tensor theory where the scalar field $\phi$ couples to the Ricci scalar $\bar{R}[\bar{g}_{\mu\nu}]$ in the Jordan frame representation with metric $\bar{g}_{\mu\nu}$. A conformal transformation to the Einstein frame representation, with metric $g_{\mu\nu}$, removes the coupling of $\phi$ from the curvature, but a coupling to the matter sector emerges. The action is given by

$$
S = \int d^4x \sqrt{\bar{g}} \left\{ \frac{1}{2\kappa^2} \bar{R}[\bar{g}_{\mu\nu}] + \frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right\} + S_m \left[ A^2(\phi) g_{\mu\nu}, \psi \right] 
$$

where the Jordan frame metric $\bar{g}_{\mu\nu}$ and Einstein frame metric $g_{\mu\nu}$ are related by the conformal transformation $\bar{g}_{\mu\nu} = A^2 g_{\mu\nu}$ and $g = |\det g_{\mu\nu}|$. Matter fields are represented collectively by $\psi$ and $\kappa^2 = 8\pi G$. The matter part of the action is

$$
S_m = \int d^4x \sqrt{\bar{g}} \bar{\mathcal{L}}_m(\bar{g}_{\mu\nu}, \psi) = \int d^4x \sqrt{g} \mathcal{L}_m[A^2(\phi) g_{\mu\nu}, \psi] 
$$

The function $A(\phi) = \exp(\beta\kappa\phi)$ is an increasing function of $\phi$ with the constant $\beta$ representing the strength of the coupling of $\phi$ to matter. The potential $V(\phi)$ is taken to be a decreasing function of $\phi$, and a variation of the action $S$ with respect to $\phi$ leads to an effective potential in the Newtonian limit given by

$$
V_{eff}(\phi) = V(\phi) + \bar{\rho} A(\phi) = V(\phi) + \bar{\rho} e^{\beta\kappa\phi} 
$$

where $\bar{\rho}$ is a $\phi$ independent conserved energy density of nonrelativistic matter in the Einstein
frame. The action $S$ yields the equations of motion

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa^2 T_{\mu\nu} = -\kappa^2 \left[ T^{(\phi)}_{\mu\nu} + T^{(m)}_{\mu\nu} \right]$$

$$\Box \phi + \frac{\partial V}{\partial \phi} - \frac{\partial L_m}{\partial \phi} = 0$$

$$\frac{du^\nu}{ds} + \Gamma^\nu_{\alpha\beta} u^\alpha u^\beta - \left( \partial_{\mu} \ln A(\phi) \right) \left[ g^{\mu\nu} - u^\mu u^\nu \right] = 0$$

We use a metric with negative signature, $(+,-,-,-)$. The line element is $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$, the velocity of a test particle is $u^\alpha = dx^\alpha / ds$, $\Box \phi = \nabla^\mu \partial_{\mu} \phi$ with $\nabla^\mu$ the covariant derivative, and $T_{\mu\nu}$ is built from a chameleon part $T^{(\phi)}_{\mu\nu}$ and a matter part $T^{(m)}_{\mu\nu}$, with

$$T_{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\partial (\sqrt{g} L)}{\partial g^{\mu\nu}}$$

$$L = \frac{1}{2} (\partial \phi)^2 - V(\phi) + L_m$$

We will be interested in the Newtonian limit of the above model, in which case the equations of motion reduce to

$$\nabla^2 \Phi = \kappa^2 \left( T^{\phi}_{00} - \frac{1}{2} T^{\lambda}_{\lambda} \right) = \kappa^2 \left[ \frac{1}{2} \beta A(\phi) - V(\phi) \right]$$

$$\nabla^2 \phi = \frac{\partial V}{\partial \phi} + \beta \kappa \bar{\rho} A(\phi)$$

$$\ddot{a} = \frac{d^2 \vec{x}}{dt^2} = -\nabla \Phi - \nabla (\ln A)$$

where $\Phi = \frac{1}{2} h_{00}$ is the Newtonian gravitational potential, and we will assume no back reaction on $\Phi$ due to $\phi$. [The terms on the right hand side of (6a) are from the stress-energy tensor, which is generated by both matter and the chameleon field. Outside of a mass distribution the matter density $\bar{\rho} \approx 0$ and the contribution to $(T^{\phi}_{00} - \frac{1}{2} T^{\lambda}_{\lambda})$ is due to the chameleon, and reduces to $(T^{(\phi)}_{00} - \frac{1}{2} T^{(\phi)}_{\lambda\lambda}) = -V(\phi)$ for $r \gg r_S$, where $r_S$ is the Schwarzschild radius of the source. This term $V(\phi)$ is expected to be negligible in a low density region far from the source, where $\phi$ can become much larger than in a high density region (with $\kappa \phi \lesssim 1$).]

Let us note here that in the Newtonian limit the equation of motion for the scalar field can be written as $\Box \phi + \partial V_{eff}/\partial \phi = 0$ where $V_{eff}$ takes the form given in (3). [Refs. [Khoury and Weltman 2004a,b] can be consulted for further details.] Now consider the potential $V(\phi)$ to be given by $V = M^5 / \phi$, or perhaps more generally, by $V = M^{4+n} / \phi^n$. The functions $V(\phi)$ and $A(\phi)$ in the effective potential $V_{eff}$ have competing behaviors, so that $V_{eff}$ exhibits a minimum at some value $\phi_{min}$ which will depend upon the local matter density $\bar{\rho}$, as seen from (3). The mass of the scalar field $\phi$ is computed from $V_{eff}$ where the curvature $V''_{eff}(\phi)$ is evaluated at $\phi_{min}$, i.e., $m^2_{\phi} = V''_{eff}(\phi)|_{\phi_{min}}$. (For a potential of the form $V = M^{4+n} / \phi^n$ we have
\[ m_\phi^2 = [n(n+1)M^{4+n}\phi^{-(n+2)} + (\beta\kappa)^2 \bar{\rho} e^{\beta\kappa\phi}]_{\phi_{\min}}. \] This leads to a large mass \( m_\phi \) in regions of high density and a small mass in regions of low density. Therefore the range \( m_\phi^{-1} \) of the chameleon scalar is very small in regions of high density, but is much larger in regions of low density. It is for this reason that, according to the chameleon model, earth-based gravity (short ranged \( \phi \)) differs from space-based gravity (long ranged \( \phi \)), and why chameleon effects are hidden on earth. [For example, Eq.(12) in Ref.(Khoury and Weltman 2004b) gives values for the range of \( \phi \) in the atmosphere and above the atmosphere (and on solar system scales) of \( m_{\text{atm}}^{-1} \lesssim 1 \text{mm} - 1 \text{cm} \) and \( m_{\text{ss}}^{-1} \lesssim 10 - 10^4 \text{AU} \), respectively.]

Since the chameleon’s effects are hidden on earth, laboratory experiments reveal no deviations from Newtonian gravity. [See(Khoury and Weltman 2004a,b) for estimates and arguments that laboratory tests of gravity are satisfied.] Furthermore, a large body having a so-called “thin shell” will have a substantial chameleonic suppression in comparison to that of a small body having a “thick shell” (Khoury and Weltman 2004a,b). For example, a large (thin shelled) planet may behave differently from a small (thick shelled) satellite, with the satellite showing an extra chameleonic acceleration not exhibited by a planet. Consequently, the chameleon model successfully passes all existing solar system constraints (Khoury and Weltman 2004a, see Sec.VII).

### 2.1. Remarks

1. There are now two contributions to the acceleration \( \vec{a} \) of a test mass, the Newtonian part, \( \vec{a}_N = -\nabla \Phi \), and the scalar chameleonic acceleration, \( \vec{a}_\phi = -\nabla (\ln A) = -\beta\kappa \nabla \phi \). So from (6c) (“geodesic” equation), \( \vec{a} = \vec{a}_N + \vec{a}_\phi \).

2. The effective potential \( V_{\text{eff}}(\phi) \) depends upon the local matter density \( \bar{\rho} \), and the result is that in regions of high density, \( \phi \) is small, \( m_\phi \) is large, and the chameleon effect is weak or undetectable, with the effects of the scalar being of very short range. Then for a small \( \phi \), \( A(\phi) \approx 1 \) and in this case the chameleonic acceleration \( \vec{a}_\phi = -\nabla (\ln A) \approx 0 \) and \( \vec{a} \approx \vec{a}_N = -\nabla \Phi \).

3. However, in regions of low density, \( m_\phi \) is very small, the effects of the scalar are of long range, and the chameleon effect becomes stronger. Therefore, for a satellite in earth orbit outside the earth and the bulk of its atmosphere [taken to be roughly 10 km thick (Khoury and Weltman 2004b)], the chameleon effect can become stronger, with the possibility of a detectable deviation from geodesic motion.
3. Acceleration and Potential Fields

3.1. Effective gravitational potential and acceleration

The chameleonic “anomaly” would not be apparent near the surface of the earth, but outside the earth’s atmosphere it could become detectable. This leads to a question of whether there is any detectable discrepancy between gravitational fields and potentials determined by earth-based measurements and space-based measurements from satellite geodesy, both of which can possess high degrees of sensitivity. Presently, we consider possible deviations from geodesic motion for earth satellites, such as the LAGEOS satellite. If any measurable deviation from geodesic motion is associated with the chameleon effect, it becomes possible to establish approximate constraints on the chameleon model parameters.

From Eq. (6c) we have

$$\nabla \cdot \vec{a} = -\nabla^2 \Phi - \nabla^2 (\ln A) = -\nabla^2 \Phi - \beta \kappa \nabla^2 \phi \equiv -\nabla^2 \Psi$$

(7)

where the effective gravitational potential \(\Psi(\vec{r})\) is

$$\Psi = \Phi + \ln A(\phi) = \Phi + \beta \kappa \phi$$

(8)

and the effective gravitational acceleration is

$$\vec{a} = -\nabla \Psi = -\nabla (\Phi + \delta \Phi)$$

(9)

with \(\delta \Phi = \ln A = \beta \kappa \phi\). The 1st term \(\nabla^2 \Phi\) is given by the 1st equation in (6a). Using this with (6b) we can write

$$\nabla \cdot \vec{a} = -4\pi G \left\{ \bar{\rho}A - 2V + \frac{2\beta}{\kappa} \nabla^2 \phi \right\} = -4\pi G \rho_{eff}$$

(10)

where

$$\rho_{eff} \equiv \bar{\rho}A - 2V + \frac{2\beta}{\kappa} \nabla^2 \phi$$

(11)

We can therefore write a Poisson equation for \(\Psi\),

$$\nabla^2 \Psi = 4\pi G \rho_{eff} = 4\pi G \left\{ \bar{\rho}A - 2V + \frac{2\beta}{\kappa} \nabla^2 \phi \right\}$$

(12)

3.2. Multipole expansion

Let us apply this to a satellite orbiting earth outside the atmosphere, where \(r \gg r_S\) and we approximate \(\bar{\rho} = 0\). (The mass density is taken to be the ambient density in our neighborhood.
of the galaxy, \( \bar{\rho}_G \approx 10^{-24} \text{ g/cm}^3 \) (Khoury and Weltman 2004b), so that we take \( \bar{\rho} \approx 0 \). Also, from (Hees and Fuzfa 2012), (Anderson and Morris 2012), we have \( \ln A = \beta \kappa \phi \lesssim 2 \times 10^{-12} \ll 1 \) so that \( \ln A \ll 1 \). See, e.g., (Anderson and Morris 2012) for a transcription of the results of (Hees and Fuzfa 2012) into our notation and conventions.) Then the vacuum value of \( \phi \) becomes large (with \( \beta \phi \ll 1 / \kappa \)), \( m_\phi \) becomes small (low curvature, an almost flat chameleon potential \( V(\phi) \)), and \( V(\phi) \) becomes very small.

Let us obtain solutions for \( \Psi \) in terms of multipole moments which can be obtained from measurements at the earth’s surface, and multipole moments which can be obtained from satellite measurements. Any difference between the two sets of multipole measurements could indicate the presence of some type of screened gravity effect.

We start with the formal solution to the Poisson equation (12),

\[
\Psi(\vec{x}) = -G \int_V \frac{\rho_{\text{eff}}(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3x'
\]

along with

\[
\frac{1}{|\vec{x} - \vec{x}'|} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{1}{2l+1} \frac{r_<^l}{r_>^{l+1}} Y_m^*(\theta', \varphi') Y_m^l(\theta, \varphi)
\]

where \( \vec{x}' \) is the source point, \( \vec{x} \) is the field point, and \( r_< \) (\( r_> \)) is the smaller (larger) of \( |\vec{x}| \) and \( |\vec{x}'| \). We consider our system to be the earth, with volume \( V_E \), surrounded by the atmosphere forming a thin shell of volume \( V_{\text{atm}} \), along with any chameleon field \( \phi \) which may be present, and possibly nonnegligible above the atmosphere. The chameleon field levels off away from the earth, approaching an asymptotic value of \( \phi_\infty \) (Khoury and Weltman 2004b), with \( \nabla^2 \phi \to 0 \).

**At earth’s surface:** On the surface of the earth the chameleon effect is absent, and the density interior to the earth’s surface is \( \rho_E(\vec{x}') \), i.e., just the ordinary earth density. The density exterior to the earth’s surface is the atmospheric density \( \rho_{\text{atm}} \), along with a chameleon contribution to the density, given by (11), with \( \bar{\rho} = 0 \) (the chameleon effect is absent within the atmosphere, as well). We note that \( \rho_{\text{eff}} \to 0 \) as \( |\vec{x}| \to \infty \). Using (13) and (14) we have

\[
\Psi^m_l(\vec{x}) = -4\pi G \int_{V_E} d^3x' \left\{ \frac{\rho_E(\vec{x}')}{2l+1} Y_l^{m*}(\theta', \varphi') Y_l^m(\theta, \varphi) \right\}
\]

\[
- 4\pi G \int_{V_{\infty}} d^3x' \left\{ \frac{\rho_{\text{eff}}(\vec{x}')}{2l+1} \frac{r_<^l}{r_>^{l+1}} Y_l^{m*}(\theta', \varphi') Y_l^m(\theta, \varphi) \right\}
\]

\[
= -4\pi G \left\{ \frac{A^m_l Y_l^m(\theta, \varphi)}{2l+1} + \frac{I_l^m}{2l+1} r_<^l Y_l^m(\theta, \varphi) \right\}, \quad (r \approx r_E)
\]
where \( r_E \) is the average radius of the earth, radii \( r \) lie between the earth’s surface and the upper boundary of the atmosphere, \( V_\infty \) includes all of space outside of the earth and atmosphere, the \( \Psi_m^l \) are the multipole fields of \( \Psi = \sum_l m \Psi_m^l \), and the exterior and interior multipole moments are given by

\[
A_l^m = \int_{V_E} d^3x' \rho_E(x') r^{l+1} Y_l^m(\theta', \varphi')
\]

and

\[
I_l^m = \int_{V_\infty} d^3x' \frac{\rho_{eff}(x')}{r^{l+1}} Y_l^m(\theta', \varphi')
\]

respectively. Furthermore, we neglect the contribution due to the interior multipole moments \( I_l^m \) by virtue of the comparative smallness of \( \rho_{eff}/r^{l+1} \), or more specifically \( (r'|r_E)I_l^m \ll A_l^m/(r^{l+1}|r_E) \), with \( \rho_{eff} \to 0 \) at points distant from earth. We therefore have, approximately,

\[
\Psi(x) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \Psi_m^l(x) = -4\pi G \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_l^m \frac{Y_l^m(\theta, \varphi)}{2l + 1} \frac{1}{r^{l+1}}, \quad (r \approx r_E)
\]

where the \( A_l^m \) coefficients are determined by earth-based measurements. This earth-based potential is just the Newtonian potential \( \Phi \), i.e., \( \Psi = \Phi \) near earth’s surface.

**Above the atmosphere:** Above the atmosphere, but not at distances too far from the earth, we again ignore interior multipole terms due to the chameleon contribution to \( \rho_{eff} \), which is very small compared to \( \rho_E \) and vanishes well away from earth, (or more specifically we assume \( (r'|r_E)I_l^m \ll A_l^m/(r^{l+1}|r_E) \), for radial positions \( r \) of the satellite) but at the position of a satellite in orbit above the atmosphere we have the exterior moments

\[
B_l^m = \int_{V_{sat}} d^3x' \rho_{eff}(x') r^{l+1} Y_l^m(\theta', \varphi')
\]

where \( V_{sat} \) is the volume of space interior to a surface on which the satellite’s orbit lies, where on this surface we assume that \( \rho_{eff} = -2V + \frac{2\beta}{k} \nabla^2 \phi \) may be small, but is not assumed to vanish identically. We have that \( \rho_{eff} \geq \rho_E \) within the volume \( V_{sat} \). From (11), along with the expectation that \( |\nabla^2 \phi| \) will maximize at some finite distance above the atmosphere (see, for example, analytical and numerical solutions presented in (Khoury and Weltman 2004b)), we anticipate that \( B_l^m - A_l^m \neq 0 \), and may be measurable, if there does exist a chameleon effect. At points above the atmosphere we therefore have

\[
\Psi(x) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \Psi_m^l(x) = -4\pi G \sum_{l=0}^{\infty} \sum_{m=-l}^{l} B_l^m \frac{Y_l^m(\theta, \varphi)}{2l + 1} \frac{1}{r^{l+1}}, \quad (r > r_E)
\]
where the $B_{l}^{m}$ are determined by space-based satellite measurements.

**Acceleration:** The effective gravitational acceleration $\vec{a} = \vec{a}_{N} + \vec{\alpha}$ is given in terms of the gravitational potential $\Psi$ by $\vec{a} = -\nabla \Psi$. We can therefore write the acceleration, as measured by a satellite, as

$$\vec{a} = -\nabla \Psi = - \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \nabla \Psi_{l}^{m}(r, \theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \vec{a}_{l}^{m}; \quad \vec{\alpha}_{l}^{m} = -\nabla \Psi_{l}^{m}$$ (21)

where the $a_{i,l}^{m}$ are the multipole terms of the acceleration field components $a_{i}$, $(i = r, \theta, \varphi),$

$$a_{r,l}^{m} = -\partial_{r} \Psi_{l}^{m} = -4\pi G \left[ B_{l}^{m}(l+1)Y_{l}^{m}(\theta, \varphi) \right]$$ (22a)

$$a_{\theta,l}^{m} = -\frac{1}{r} \partial_{\theta} \Psi_{l}^{m} = -4\pi G \left[ -B_{l}^{m} \frac{1}{r^{l+2}} \partial_{\theta} Y_{l}^{m}(\theta, \varphi) \right]$$ (22b)

$$a_{\varphi,l}^{m} = -\frac{1}{r \sin \theta} \partial_{\varphi} \Psi_{l}^{m} = -4\pi G \left[ -B_{l}^{m} \frac{1}{r^{l+2}} \frac{\partial_{\varphi} Y_{l}^{m}(\theta, \varphi)}{\sin \theta} \right]$$ (22c)

For pure Newtonian gravity, characterized by a potential $\Phi$, i.e., for points near the earth’s surface where $\Psi \rightarrow \Phi$, or for points in space for the case that there is no chameleon field and $\Psi = \Phi$, we would have the same expressions for multipole terms with replacements $B_{l}^{m} \rightarrow A_{l}^{m}$.

4. Comparison

The potential $\Phi(r, \theta, \varphi)$ for pure Newtonian gravity is given by a multipole expansion with multipole moments $A_{l}^{m}$, and the potential $\Psi(r, \theta, \varphi)$ for chameleon gravity is given by (20). The difference between each multipole term is

$$\delta \Phi_{l}^{m} \equiv \Psi_{l}^{m} - \Phi_{l}^{m} = -4\pi G(B_{l}^{m} - A_{l}^{m}) \left[ \frac{1}{r^{l+1}} Y_{l}^{m}(\theta, \varphi) \right]$$ (23)

which would vanish identically if there were no chameleon field or chameleon effect. However, if a detectable chameleon field does exist, it would give rise to an “anomalous” acceleration $\vec{\alpha} = \vec{a} - \vec{a}_{N} \equiv \delta \vec{a}$ with multipole terms

$$\delta \vec{a}_{l}^{m}(r, \theta, \varphi) = -\nabla \delta \Phi_{l}^{m} = -(B_{l}^{m} - A_{l}^{m})(-4\pi G) \nabla \left[ \frac{1}{r^{l+1}} Y_{l}^{m}(\theta, \varphi) \right]$$ (24)

However, the acceleration field $\vec{a}$ measured near the earth would not exhibit an anomalous acceleration, i.e., $\delta \vec{a} = 0$, while the anomalous acceleration predicted by the chameleon effect
that could be detected by a satellite in orbit would be nonzero. From (23) and (24) we have a relative correction to the Newtonian fields, due to the chameleon effect,

$$\frac{\delta \Phi^m_l(\vec{r})}{\Phi^m_l(\vec{r})} = \left( \frac{B^m_l A^m_l - 1}{A^m_l} \right), \quad \frac{|\delta \vec{a}^m_l(\vec{r})|}{a^{m,N}_l(\vec{r})} = \left( \frac{B^m_l}{A^m_l} - 1 \right) \quad (25)$$

The $B^m_l$ coefficients are determined from satellite measurements of $\vec{a}$, and the $A^m_l$ coefficients are determined from the ground-based measurements of $\vec{a}$, where the chameleon effect disappears.

The acceleration $\vec{a}$ and its “anomalous” part $\delta \vec{a}$ are dominated by the monopole and quadrupole terms. (The $l = 1$ dipole term vanishes, as we take the coordinate origin to coincide with the center of mass.) The monopole radial acceleration “anomaly” is

$$\frac{|(\delta a^r_0)|}{(a^r_N)_0} = \left( \frac{B^0_0}{A^0_0} - 1 \right) \quad (26)$$

For a uniform sphere of mass having only a monopole Newtonian field and no chameleon anomaly, i.e., $A^m_l = B^m_l$, the gravitational field is $a^{N,r} = -GM/r^2 = -4\pi GA^0_0 Y^0_0 / r^2$ (see Eq. (22a), for example). In this case, the “geoid” would be a sphere at the sphere’s surface, and we would identify $A^0_0 = M/\sqrt{4\pi}$. If there is a chameleonic anomaly, with $B^0_0 \neq A^0_0$, then the “anomaly” may take an appearance of a slightly modified gravitational parameter, $(GM)_{eff} = GM + \delta(GM)$, with $\delta(GM)/(GM) \propto (B^0_0/A^0_0 - 1)$.

Although a gravitational anomaly resulting from differences in space-based and ground-based measurements of the gravitational field may be rather small, its existence could give some credence to the idea of some theory of modified gravity with a screening mechanism, such as the chameleon model of gravitation.

5. Numerical Estimates: Monopole Approximation

For a simple example, we now make a monopole approximation for the earth’s gravitational field, treating the earth as a uniform sphere of radius $R$ and mass $M = \int \rho_E d^3x$. The magnitude of the Newtonian gravitational field at a distance $r$ from the center of the earth is

$$a^r_N = \frac{4\pi GA^0_0 Y^0_0}{r^2} = \frac{GM}{r^2} \quad (27)$$

where $A^0_0 = M/\sqrt{4\pi}$ is the monopole ($l = 0, m = 0$) moment contribution to the gravitational field. Both the Newtonian acceleration $\vec{a}_N$ and the chameleonic acceleration $\vec{a}_\phi$ are directed radially inward (Anderson and Morris 2012). To get a numerical estimate for $\delta a(r)/a^r_N(r)$,
we use results that were reported in Ref. ([Anderson and Morris 2012]), where the anomalous acceleration $\delta a$ is identified with the chameleonic acceleration $a_\phi$. For the present case of an earth satellite we have the result

$$\frac{\left| \delta a_r \right|}{a_N} = \left( \frac{B_0^0}{A_0^0} - 1 \right) = \frac{|a_\phi|}{a_N} = 6\beta^2 \Delta_E \tag{28}$$

where $\beta$ is a constant denoting the strength of the coupling of the chameleon field to matter, and $\Delta_E$ is the “thin shell” factor for the earth (the gravitational “source”). Here, we are taking $\delta a_r$ to be an inward radial acceleration, which should appear to be a small radial acceleration of the satellite, in excess of the Newtonian acceleration. On the other hand, if it is determined that there is no inward radial component of $\delta \vec{a}$, then we must conclude that there is no chameleonic acceleration, i.e., $\beta = 0$. Also note that the ratio $B_0^0/A_0^0 = (GM)_{sat}/(GM)_E$ where $(GM)_{sat}$ is measured by satellite from space and $(GM)_E$ is measured near the earth’s surface.

Several estimates or bounds can be placed on the “anomalous” acceleration $\delta a_r$.

### 5.1. Estimated Bounds

**Khoury-Weltman bound:** In Refs. ([Khoury and Weltman 2004a] and [Khoury and Weltman 2004b]), the value of $\beta$ is taken to be on the order of unity, $\beta \approx 1$, and the thin shell factor for the earth is estimated to be $\Delta_E < 10^{-7}$. With these parameters adopted by KW, we have a rough estimate for an upper bound of $|\delta a_r|/a_N$ given by

$$\left( \frac{|\delta a_r|}{a_N} \right)_{KW} \lesssim 6\beta^2 \times 10^{-7} \approx 6 \times 10^{-7} \quad (\beta \approx 1) \tag{29}$$

**Cassini bound:** On the other hand, we can appeal to an upper limit based upon solar system constraints obtained by the Cassini mission, where limits were established ([Bertotti, Iess, and Tortora 2003]) on the parameterized Post-Newtonian (PPN) parameter $\gamma$,

$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5} \tag{30}$$

Hees and Fuzfa ([Hees and Fuzfa 2012]) have used this solar system-based constraint to obtain an upper bound on chameleon parameters, without making assumptions regarding the values of $\beta$ or the thin shell factors. This analysis was incorporated in Ref. ([Anderson and Morris 2012]) to obtain (see Ref. ([Anderson and Morris 2012]) for details)

$$\beta^2 \Delta_S \lesssim 3.3 \times 10^{-7} \tag{31}$$
where $\Delta_S$ is the thin shell factor for the sun. The relation between $\Delta_S$ and $\Delta_E$ is given by (Anderson and Morris 2012)

$$\Delta_E \approx \left( \frac{1}{3} \times 10^4 \right) \Delta_S$$

(32)

giving an upper bound on $\beta^2 \Delta_E$ based upon the Cassini constraint

$$(\beta^2 \Delta_E)_C \lesssim 10^{-3}$$

(33)

From (28) we then have a Cassini-based estimate

$$\left( \frac{|\delta a_r|}{a_N} \right)_C = 6\beta^2 \Delta_E \lesssim 6 \times 10^{-3}$$

(34)

**LAGEOS bound:** Consider now a fifth force Yukawa addition to the geopotential,

$$\Psi(r) = \Phi(r) + \delta \Phi(r) = -\frac{GM}{r} \left( 1 + \alpha e^{-r/\lambda} \right) = \Phi(1 + \alpha e^{-r/\lambda})$$

(35)

where $\alpha$ represents the coupling of the fifth force, which we will here assume to be universal, and $\lambda$ represents the range of the Yukawa potential. We are interested in the effect of $\delta \Phi = -\alpha \Phi e^{-r/\lambda}$ on a satellite where the ambient mass density is negligible, and $m_\phi \approx 0$ and $\lambda \to \infty$. In this case we have $\delta \Phi = \alpha \Phi$ and $\delta \Phi/\Phi = \alpha$. Identifying $\delta \Phi/\Phi = |\delta a_r|/a_N$ [see Eq.(25)] and using (28), we have

$$\frac{\delta \Phi}{\Phi} = \frac{|\delta a_r|}{a_N} = \left( \frac{B_0}{A_0} - 1 \right) = 6\beta^2 \Delta_E = |\alpha|$$

(36)

The upper bound on $\alpha$, as determined by the LAGEOS satellite, is quoted to be (Iorio 2003; Ciufolini and Wheeler 1995)

$$|\alpha| = \left( \frac{|\delta a_r|}{a_N} \right)_L = 6\beta^2 \Delta_E < 10^{-5} - 10^{-8}$$

(37)

Of course, for this fifth force to be associated with the chameleon effect, $\alpha$ should be positive.

To summarize, the deviation from Newtonian acceleration, as well as the fractional change in the monopole moment is, from (26), (28), (29), (34), and (37), estimated as

$$\frac{|\delta a_r|}{a_N} = \left( \frac{B_0}{A_0} - 1 \right) = 6\beta^2 \Delta_E \lesssim \begin{cases} 6 \times 10^{-7}, & (KW, \beta \approx 1) \\ 6 \times 10^{-3}, & (\text{Cassini}) \\ 10^{-5} - 10^{-8}, & (\text{LAGEOS}) \end{cases}$$

(38)
where use has been made of (29), using the original fiducial KW parameters with $\beta \approx 1$, along with (33), and (36). The discrepancy between earth-based and space-based monopole moments may be quite small, but the measurement of a deviation from geodesic motion for the LAGEOS satellite is fairly sensitive, and does not assume any particular values for $\beta$ or $\Delta_E$. The Cassini-based constraint allows much more freedom, but again, is based upon fewer assumptions than the KW estimate. The LAGEOS bound is seen to provide more restriction than the Cassini bound, and does not assume values for $\beta$ or $\Delta_E$.

**Note:** A cautionary note is in order here. The formalism presented here, and the subsequent constraints obtained, have been admittedly oversimplified. Effects on satellite acceleration beyond a simple Newtonian acceleration, along with a possible chameleonic correction, have been ignored. Such effects can include gravitational forces from the sun, moon, and other planets, solar radiation pressure, and general relativistic effects (which for a scalar-tensor theory might be accommodated by appropriate Eddington parameters). (See, for example, \textit{Combrinck 2011} and \textit{Damour and Esposito-Farese 1994} and references therein.) A meaningful comparison of theoretical predictions and satellite data is therefore expected to require delicate expertise, and is well beyond the scope and intent of the basic ideas presented here.

6. **Summary**

A theoretical framework has been proposed by which a gravitational anomaly due to a scalar “chameleon” field might be detected by comparing measurements of the gravitational field strength made at the earth’s surface and from a satellite in earth orbit. The chameleon model allows such an “anomalous” acceleration to appear at points far from a matter source (satellite in orbit), but the anomaly does not become manifest at points near a matter source (earth’s surface).

Gravitational field measurements made via satellite can be compared with earth-based measurements where the chameleon effect is absent. The presence of a difference in the two sets of measurements could indicate the existence of a chameleonic gravitational correction. On the other hand, clear evidence of an absence of anomaly could severely constrain the chameleon model of gravitation.

Estimates based upon a simple monopole approximation indicate that the relative difference between earth-based multipole moments $A^m_l$ and space-based multipole moments $B^m_l$ is expected to be very small, with an estimate for a difference in monopole moments given by (38) if the difference is due solely to the chameleon effect. If, for instance, the acceleration and coefficients $A^m_l$ are obtained on earth’s surface, near the geoid, at some position $(R, \theta, \varphi)$ and the corresponding acceleration and coefficients $B^m_l$ are determined by a satellite in orbit at position $(r, \theta, \varphi)$ (at approximately the same time to eliminate temporal differences), they can
be compared. Since the multipole moments are obtained from acceleration measurements, and not computed from the formal expressions involving $\rho_{\text{eff}}$, one need not be concerned with any dependence upon spatial uncertainties in the density $\bar{\rho}$. However, given the fact that the higher multipole moments are typically smaller than the lower ones, it is hopeful that sufficiently sensitive measurements of the difference $(B_{m}^{l} - A_{l}^{m})$ could indicate whether space-based gravity is indeed different from earth-based gravity.

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