MODELS OF REALISTIC REGGEONS FOR $t=0$ HIGH ENERGY SCATTERING

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Abstract

Several Regge-phenomenological models of forward scattering at high energies are considered. It was found that one of them corresponds to the form of finite series in $\log s$ with additional terms that represent the exchanges of non-degenerate Regge-trajectories. These trajectories are unique for both the scattering and resonance regions and satisfy the most realistic ideas on them.

1 Introduction

Since the middle of the last century Pomeron plays a fundamental role in describing the scattering at high energies. In spite of this, our understanding of the Pomeron nature is far from completeness and requires further specifications. At the same time the secondary Reggeons were undeservedly moved to the role reflected in their name.

In this paper, we will consider the Regge-phenomenological models of forward scattering at high energies, where both Pomeron and secondary Reggeons (further on Reggeons) satisfy the most realistic ideas on them.

From the Regge-theory point of view the basic characteristic of Pomeron and Reggeons is the intercept [1]. This concept has the most effective and economic form in the Donnachie-Landshoff (DL) model [2], which we considered as a standard reference for the models of total, elastic and diffractive cross sections. For the $pp$ and $\bar{p}p$ scattering the total cross-section in this model is:

$$\sigma_{tot}(s) = Xs^\varepsilon + Y_s^{-\eta},$$

where $Y_s$ corresponds to $pp$ and $\bar{p}p$ scattering with exchange degenerate Reggeon representing both $C = \pm 1(\rho, \omega, a_2, f)$ exchanges besides the Pomeron with intercepts given by $\alpha_P = 1 + \varepsilon$ and $\alpha_R = 1 - \eta$. The DL model fares reasonably well when fitting to the $pp$ and $\bar{p}p$ total cross sections and demands a generalization when fitting both the total cross sections and the real parts of the scattering amplitudes [3, 4]. Then the $pp$ and $\bar{p}p$ scattering of the total cross sections in this model will be:

$$\sigma_{tot}(s) = Xs^\varepsilon + Y_s^{-\eta_+} \mp Y_s^{-\eta_-},$$

where the last two terms represent the exchanges of non-degenerate $C = +1(a_2, f)$ and $C = -1(\rho, \omega)$ trajectories with intercepts $\alpha_{\pm} = 1 - \eta_{\pm}$, respectively. The sign of $Y_-$ term flips when fitting $pp$ data are compared to the $\bar{p}p$ data.

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It is well known that models (1) and (2) violate the unitarity. As it was pointed out in [3], the unitarity violation occurs at the energies only slightly above the Tevatron energy of 1.8 TeV, and therefore it is a problem of the present and not of the future. Not less successful models are those based on the more complex analytical properties of the scattering amplitude, not violating however the unitarity conditions [4, 5]:

\[
\sigma_{\text{tot}}(s) = A + B \log s + Y_+ s^{-\eta_+} \mp Y_- s^{-\eta_-},
\]

(3)

\[
\sigma_{\text{tot}}(s) = A + B \log^2 s + Y_+ s^{-\eta_+} \mp Y_- s^{-\eta_-}.
\]

(4)

An alternative way of effective account of the complex structure of the singularities is to try to mimic them by a two-component Pomeron built from two Regge singularities [5, 7, 8]. Note that the perturbative Pomeron has also a complex form. Recently detailed calculations in QCD indicated an existence of the two-component Pomeron [7].

The two-component Pomeron model for the total cross sections has the following form:

\[
\sigma_{\text{tot}}(s) = Z + X s^\varepsilon + Y_+ s^{-\eta_+} \mp Y_- s^{-\eta_-},
\]

(5)

where the second component corresponds to the intercept larger than 1 ($\varepsilon > 0$) and the first component corresponds to an intercept exactly localized at 1.

A new development of this model supposes that the X-component is fully universal, i.e. its coupling is the same in all hadron-hadron reactions [8, 9], while the first one is a non-universal Pomeron.

Another generalization of the multicomponent Pomeron is based on the assumption that QCD Pomeron corresponds to the infinite sum of gluon ladders with Reggeized gluons on the vertical lines [10]. Essentially at finite energies only a finite number of diagrams contributes, giving rise to a finite series in \(\log s\) [12] like:

\[
\sigma_{\text{tot}}(s) = A + B \log s + C \log^2 s + Y_+ s^{-\eta_+} \mp Y_- s^{-\eta_-}.
\]

(6)

Because of the fact that most of mentioned models have been tested with different data sets, it is very difficult to compare between them. Fortunately there exists a complete data set of total cross sections and ratios of the real part of scattering amplitude to the imaginary part for the \(pp, \bar{p}p, \pi^\pm p, K^\pm p\) scattering and for total cross sections in the case of the \(\gamma p, \gamma\gamma\) and \(\Sigma p\) scattering [12]. It was found in [3] with the data set [12] that the data cannot discriminate between a simple-pole fit (2) and asymptotic \(\log s\) (3) and \(\log^2 s\) (4) fits. The models examined in [3] satisfy the condition \(\chi^2/dof \approx 1\) with 16 fitted parameters at \(\sqrt{s}_{\text{min}} \geq 9\text{GeV}\). According to the program initiated in [3] by scrutinizing two earlier unexplored models for the Pomeron (5) and (6) with the weak degeneration as in (1) for \(\sqrt{s}_{\text{min}} \geq 10\text{GeV}\) it was found in [13] that the condition \(\chi^2/dof \sim 1\) satisfies with the same number of fitted parameters.

The analysis of all mentioned models shows that the values of fitted intercepts of Reggeons sufficiently differ (see Fig. 1). Moreover, the fitted values of intercepts are far from the available data for the corresponding mesonic resonances drawn on the Chew-Frautschi plot. Therefore a fairly conclusive analysis can be performed using, on the one hand, the data in the resonance region, and, on the other hand, those for forward scattering [14].
Below we suggest a Regge-phenomenological model (6) with both Pomeron and Reggeons satisfying the most realistic notions about them. It will be demonstrated that Pomeron as a finite sum of log $s$ terms (6) is quite enough, the coefficients of this series decrease with respect to each other as $\sim 1/5$ and $\sim 1/15$.

Nevertheless, at simultaneous fit of Pomeron and Reggeons parameters the values of the Reggeons intercepts coincide with the values obtained from Chew-Frautschi plot. In addition note that the $\chi^2/dof \sim 1$ criterion is reached at $\sqrt{s}_{min} = 8GeV$ energy.

Furthermore we study how compatible are the above mentioned models with the realistic interpretation of the unique Regge trajectory for both the scattering and resonance regions as well. Finally on the basis of this criterion we define the realistic intercept values of the supercritical Pomeron.

## 2 Finite log $s$ series for the Pomeron

Let us write the real and imaginary parts of the forward elastic scattering amplitude as

$$\frac{1}{s} ImA_{h_1h_2}(s) = ImP(s) + ImR(s),$$  \hspace{1cm} (7)
\[
\frac{1}{s} ReA_{h_1h_2}(s) = ReP(s) + ReR(s),
\]
where
\[
ImP(s) = \lambda_{h_1h_2} \left[ A + B \log \left( \frac{s}{s_0} \right) + C \log^2 \left( \frac{s}{s_0} \right) \right],
\]
\[
ReP(s) = \frac{\pi}{2} \lambda_{h_1h_2} B + \pi \lambda_{h_1h_2} C \log \left( \frac{s}{s_0} \right),
\]
come from the Pomeron contribution and
\[
ImR(s) = Y_{h_1h_2} s^{-\eta_1} \mp Y_{h_1h_2} s^{-\eta_2},
\]
\[
ReR(s) = \left[ Y_{h_1h_2} s^{-m} \cot \left( \frac{1 - \eta_1}{2} \pi \right) \mp Y_{h_1h_2} s^{-\eta_2} \tan \left( \frac{1 - \eta_2}{2} \pi \right) \right],
\]
correspond to the contribution of two non-degenerate Regge trajectories.

We have fitted the above model to the data on the total cross sections and the ratio of the real to imaginary part of \(pp, \overline{pp}, \pi^\pm p, K^\pm p, \gamma p\) and \(\gamma \gamma\) as well as the \(\Sigma p\)-scattering, starting from \(\sqrt{s_{\text{min}}} = 3 GeV\) up to the highest available energies. Similarly to ref. [6], we have studied the stability of our fit by varying the lower limit of the fit \(\sqrt{s_{\text{min}}}\) between 3 and 13 \(GeV\). The resulting fit (value of \(\chi^2/dof\)) as well as the dependence of one of the fitted parameters on the lower bound \(\sqrt{s_{\text{min}}}\) are presented in Fig. 2 and Fig. 3.

![Figure 2: Values of the f-Reggeon intercept as a function of the minimum energy considered in the fit. The lines are the same as in Fig. 1.](image)

The result of our fits is good as those of ref. [6]. The limiting value \(\chi^2/dof \approx 1\) is reached at \(\sqrt{s_{\text{min}}} = 8 GeV\).
An important result of our fit is that the coefficients of the finite series of Pomeron \( \mathcal{P} \) (see Table 1) decrease in their absolute values (as \( \sim 1/5 \) and \( \sim 1/15 \)) providing a fast convergence of the series and ensuring the applicability of this approximation at still much higher energies. The physical motivation of such a finite series representation of the Pomeron was discussed earlier - both in the context of Regge multipoles and QCD \([10, 11]\).

TABLE 1: Values of the fitted parameters in the finite series Pomeron model \((6)-(12)\), with a cut-off \( \sqrt{s_{\text{min}}} = 8 \text{GeV} \) and \( s_0 = 1 \text{GeV}^2 \) fixed.

| \(Y_1\) (mb) | \(Y_2\) (mb) | \( \eta_1 \) | \( \eta_2 \) | \( \chi^2/\text{d.o.f.} \) | \( \text{statistics} \) |
|-------------|-------------|-------------|-------------|----------------|----------------|
| \( pp \)    | \( \bar{p}p \) | \( Kp \)    | \( \Sigma p \) | \( \gamma p \times 10^{-4} \) | \( \gamma \gamma \times 10^{-4} \) |
| \( 10.5 \pm 0.7 \) | \( 2.201 \pm 0.076 \) | \( 0.138 \pm 0.011 \) | \( 0.300 \pm 0.015 \) | \( 0.563 \pm 0.013 \) | \( 1.0 \) |

However, this is not a single advantage of our model. Note that in the process of the fit the obtained intercepts of Reggeons well coincide with those calculated from the Chew-Frautschi plot \([14]\). As is seen in Fig. 1, our model belongs to the very few models that give realistic values of the intercept for the \( f \)-Reggeon. Regarding its quantum numbers the \( f \)-Reggeon is inseparable from Pomeron and thus the definition of its parameters is being model dependent.

The intercept value of \( \omega \)-Reggeon is in a quite good agreement with the values taken from the Chew-Frautschi plot \([14]\), despite of the simplicity of the calculations based on the degeneration of \( f/a \) and \( \omega/p \). Taking into consideration these properties of the model \((6)\) concerning the Reggeons parameters, one can estimate the realistic parameters of the Pomeron in a form of model \((6)\), if the parameters of Reggeons are fixed, in accordance with the values obtained from the fit of Chew-Frautschi plot \([14]\), still requiring the stability conditions to hold \( \chi^2/\text{dof} \sim 1 \). To do this we have chosen the following Reggeons intercept values \([14]\):

\[
\alpha_f = 0.6971, \quad \alpha_\omega = 0.4359, \quad \alpha_\rho = 0.4783, \tag{13}
\]

and here two Reggeons still contribute to the scattering amplitude:

- \( f \) and \( \omega \) for \( pp, \bar{p}p, K^\pm p, \Sigma p \)-scattering
- \( f \) and \( \rho \) for \( \pi^\pm p \)-scattering
- \( f \) for \( \gamma p, \gamma \gamma \)-scattering

Our motivation is as follows. As shown in \([13]\), the contribution of \( a_2 \)-meson \( \approx 0 \) and the contribution of \( \rho \)-meson to the total cross sections of \( pp, \bar{p}p \) and \( K^\pm p \)-scattering is comparable with the error of the \( \omega \)-Reggeon contribution. Moreover, we have not included to the fit the scattering data for neutral particles, for which the contribution of these Reggeons is very important. The result of the fit is shown in Fig. 3. As is seen from these data, the model gives a slightly better result than the previous ones with free parameter’s fit, due to the fact that the number of parameters is reduced by 2 and, that is essential, our suggestions about the realistic model \((6)\) are valid.
Figure 3: $\chi^2$/dof as the function of the minimum energy considered in the fit: 3P+2R - model (6) with both Pomeron and Reggeons parameters fitted; 3P+2R (fixed) - model (6) with Reggeons intercept values (13); 3P+2R Universal - model (14) with both Pomeron and Reggeons parameters fitted; 3P+2R (fixed) Universal - model (14) with Reggeons intercept values (13).

Recently it has been shown in [15] that the inclusion of G-universality improves sufficiently some models describing the scattering at t=0. We have included into our model the assumption about the G-universality that now has a form:

$\sigma_{pp} = A_{pp} + P(s) + R_{pp}^p(s)$,
$\sigma_{K^\pm p} = A_{K^p} + P(s) + R_{K^\pm p}(s)$,
$\sigma_{\pi^\pm p} = A_{\pi p} + P(s) + R_{\pi^\pm p}(s)$,
$\sigma_{\gamma p} = \delta A_{pp} + \delta P(s) + f_{\gamma p}(s)$,
$\sigma_{\gamma\gamma} = \delta^2 A_{pp} + \delta^2 P(s) + f_{\gamma\gamma}(s)$,
$P(s) = B \log s + C \log^2 s$,

$R_{pp}^p(s) = f_{pp} \mp \omega_{pp}$,
$R_{K^\pm p}(s) = f_{K^p} \mp \omega_{K^p}$,
$R_{\pi^\pm p}(s) = f_{\pi p} \mp \rho_{\pi p}$.

As is seen from the comparison of the fitting results, the latter model demonstrates a stable improvement at all energies. Unfortunately it is not valid for the $f$-Reggeon intercepts.

As one may expect, the version of our model with the assumption of the G-universality
Figure 4: $\chi^2/dof$ as the function of the minimum energy considered in the fit: The first four models correspond to the (2)-(5) models with fixed Reggeons parameters from (13); The last three models correspond to the model (14) with the assumption of the G-universality and fixed Reggeons intercept values (13).

and fixed Reggeons values gives the absolutely best description of the whole experimental data set used here (see Fig. 3).

If one can accept the reasonable criterion $\chi^2/dof \leq 1.08$ then as is seen from Fig. 3 all the four versions of our model satisfy this criterion yet at the lower limit of the fit $\sqrt{s_{\text{min}}} = 5$GeV.

3 Scattering models at $t=0$ with realistic values of Reggeons intercept

It is seen from Fig. 1 that the $f$-Reggeon intercept values are distinctly model-dependent and are far from the agreement with those obtained from the Chew-Frautschi plot. Here we shall check the criteria of applicability of the models under discussion if one uses the fixed Reggeons intercept values from the Chew-Frautschi plot, similarly to the previous section. The results of such check are shown in Fig. 4. It is interesting to note that most of the models obey the reasonable requirement $\chi^2/dof \leq 1.08$ for $\sqrt{s_{\text{min}}} = 8$GeV, while for the four versions of our model (14) this condition is valid at $\sqrt{s_{\text{min}}} = 5$GeV (see...
The severe requirement \( \chi^2/dof \leq 1 \) is satisfied by the Pomeron models for the boundary energy value of \( \sqrt{s_{\text{min}}} \geq 10\text{GeV} \).

\[
P(s) = A s^\varepsilon \quad (1) \quad P(s) = A + B \log^2 s \quad (3) \\
P(s) = A + B \log s \quad (2) \\
P(s) = A + B s^\varepsilon \quad (4)
\]

Among the model versions with the G-universality all the above models except for (2) obey this criterion.

![Figure 5: \( \varepsilon \) parameter of the supercritical Pomeron as a function of the minimum energy considered in the fit.](image)

4 Realistic intercept values for the Pomeron

Since Donnachie-Landshoff determined the Pomeron intercept value equal to 1.08, debates are still around it. One of the reason is in the unusual behavior of the Pomeron intercept in models of DIS, where at \( Q^2 \to 0 \) the intercept value of the Pomeron is close to the value obtained from the scattering region, and at \( Q^2 \to \infty \) it drastically increases up to 1.3–1.4. The situation became dramatic, because the intercept value of the BFKL Pomeron is also large. Scrupulous estimations for the Pomeron intercept values were carried out in [3, 4]. It was found that within the framework of the model (2) the Pomeron intercept equals to \( 1.104 \pm 0.002 \) and \( 1.096^{+0.012}_{-0.009} \). Herein we refitted the Pomeron intercept values in the spirit of [3] at fixed values of Reggeons (3) for boundary energies \( 3 - 13\text{GeV} \) utilizing the same data set [12]. At \( \chi^2/dof = 1.05 (\sqrt{s_{\text{min}}} = 9\text{GeV}) \) the intercept value is equal to \( 1.1020 \pm 0.0014 \), while at \( \chi^2/dof = 1 (\sqrt{s_{\text{min}}} = 11\text{GeV}) \) it is \( 1.0989 \pm 0.0016 \). Calculations are shown in Fig. 5. As is seen our result well coincides with the previous calculations [3, 4].
5 Conclusions

Summarizing the results and checking the Regge-phenomenological models it is possible to formulate reasonably criteria for its application:

1. The check of models always should be done by using the same data set (for example [2]).

2. The criterion of $\chi^2/dof \leq 1$ should be fulfilled.

3. The compatibility of parameters with the secondary Reggeons from the resonance region.

As the result of our analysis we have concluded that the most realistic model includes the Pomeron in the form of finite series in log $s$, satisfying the Froissart bound and two Reggeons for the best description of the forward scattering. In this model the criterion $\chi^2/dof \leq 1$ was reached yet at the energy of $\sqrt{s}_{\text{min}} = 8\text{GeV}$. Note that only this model gives the $f$-Reggeon intercept value that agrees with those extracted from the resonance region. The two-component Pomeron model (3) and the Froissaron model (4) with its G-universality versions satisfy the above criteria beginning from $\sqrt{s}_{\text{min}} = 9\text{GeV}$, the DL model (2) beginning from $\sqrt{s}_{\text{min}} = 11\text{GeV}$ and the dipole Pomeron model just from $\sqrt{s}_{\text{min}} = 12\text{GeV}$. If one will change the criterion 2. to the reasonable criterion $\chi^2/dof \approx 1.08$ then practically all the models are fulfilled.

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