Dissipativity of Nonlinear Networked Control Systems Modeled by Markovian Jump System

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Abstract. Networked control system (NCS) is a control system in which the control loop is closed by a shared communication networked. There are some parameters that arise when using a network in an NCS such as packet dropout, time delay, limited bandwidth, and so forth. Due to the stochastic property of such parameters, the entire system under consideration realizes the Markovian jump system. This paper considers a dissipativity notion for a class of nonlinear networked control systems. Dissipativity refers to the existence of a supply rate function dealing with the system such that the system has a dissipative property. The main result in this paper is a solvability condition for achieving dissipativity of a class of nonlinear networked control systems modeled by Markovian jump system. The solvability condition is shown by the nonlinear matrix inequalities (NLMI).

1. Introduction

A networked control system (NCS) is a control system in which the control loop is closed via a shared communication network [1]. Compared to the conventional point-to-point system connection, the use of an NCS has advantages of reduced system wiring, ease of system diagnosis and maintenance, and increased system agility [2]. Since sensors, actuators, and controllers are connected via a shared band-limited communication network, there are some parameters which appear such as network-induced delays, packet dropout, bandwidth constraints, etc. The existence of these parameters will degrade performance of NCS or even cause instability [3]. Due to the widespread of the shared networks, NCS has attracted many researchers especially for large systems with distributed structure such as the distributed energy system, traffic control systems, water distribution in a city area, irrigation systems, etc. Though the theoretical foundation of NCS has been improved considerably during the last decade, many issues should be resolved before the all the advantages of NCS can be harvested.

A typical NCS is shown in Fig 1. In that illustration, a personal computer equipped with the data acquisition card enables the plant to send out the sensor data and receive the control data through the Ethernet LAN. A dedicated controller receives the sensor data, computes the control law for a certain purpose, and then sends out the control data through the same LAN. Because of the existence of the network parameter, a designed controller should be capable of overcoming that parameter. To cope with such situation, the network parameter has to be explicitly incorporated into a controller design.
using a certain model, such as Markov process, so that the system under consideration renders a Markovian jump system.

The Markovian jump systems (MJS) has attracted increasing attention in the recent literature, see e.g [4-9] for linear systems and [10-11] for nonlinear systems. Such systems are those having transition between models determined by a Markov chain. MJS is considered to be appropriate to model plants whose structures are subject to random abrupt changes due to component failures or repairs, sudden environmental changes, abrupt variations of the operating point of a nonlinear plant, changing subsystem interconnections, and so on [12]. There is a quite deep literature for a class of this system [5] and previous results relevant to this class of system in NCS modeling was reported in [4,6-8,10-13] for name a few. Whenever employing MJS in NCS modeling, the transition probabilities in the jumping process determine the overall system behavior [9-10].

In this paper, we address the dissipativity condition in terms of the nonlinear matrix inequalities (NLMI) in analysis of a class of nonlinear networked control system modeled by Markovian jump system. The dissipative concept used to analyze and design the control system was initially developed by Willems [14]. This concept concerns the analysis and design of control systems that use input-output properties based on the energy-related description [15]. Practically speaking, a system has a dissipative property if it always dissipates energy. When the dissipativity is assured, the stability problem can be solved. Meanwhile, the dissipative performance is a generalization of the performance measure, such as finite gain (H_{\infty}) and passivity [14]. Thus, the development of a dissipative framework will generalize the existing ones, including H_{\infty} control as well as passivity-based design.

Using a mathematical abstraction of the notions of physical power and energy, researchers have developed a stability analysis and designed a controller for various applications in dissipative system frameworks. Recent studies concerning the dissipative approach that are in line with our interest are reported in [16-20]. Aliyu has proposed a dissipativity analysis for the nonlinear Markovian jump system [16]. Zhang et al. have considered the dissipative problem of a class of stochastic hybrid systems and focused on the analysis of whether a stochastic hybrid system with time delay is stochastically asymptotically stable and strictly dissipative [17]. Mahmoud et al. have worked on a robust dissipative control problem applied to a class of hybrid multi-rate control models with time-delay and a switching controller [18]. Zhang et al. derived a linear state feedback controller in a reliable dissipative control problem for a class of stochastic hybrid systems [19]. Wang et al. addressed the dissipative problem for uncertain time-delay NCS [20]. The aforementioned papers consider the linear model of the networked control systems under consideration. So far, to the best of our knowledge, a dissipative control design for a nonlinear NCS modeled by the MJS has not been investigated yet. Therefore, the development of such a framework would extend the existing linear ones.

For convenience, we adopt the following notations in this paper. Capital letters denote matrices and small letters denote vectors. For real matrices or vectors, (\cdot)^T indicates their transpose. For a real square matrix P, we write P > 0 (resp., P < 0) if P = P^T is symmetric and positive (resp. negative).
Definition 1. Consider the following class of discrete system in the form of affine representation

\[ \Sigma_d: \begin{align*} x(k+1) &= f(x(k)) + g(x(k))u(k) \\ y(k) &= h(x(k)) + l(x(k))u(k) \end{align*} \]

where \( x(k) \) denote state variable of the system, \( u(k) \) is control input, and \( y(k) \) is an output. The functions \( f, g, h, \) and \( l \) are smooth mappings. The system \( \Sigma_d \) is said to be dissipative with respect to the supply rate \( W(u, y) \) if for all admissible \( u(k) \) there exists a storage function \( V(x) \geq 0 \) such that the following dissipation inequality holds

\[ V(x(k+1)) - V(x(k)) \leq W(y(k), u(k)); \forall u \in U, \forall y \in Y, \forall k \in \mathbb{Z} \]  \hspace{1cm} (2)

Inequality (2) could be said a step-wise dissipativity. By monotonicity assumption, we can get easily the following dissipation inequality

\[ V(x(k+1)) - V(x(0)) \leq \sum_{i=0}^{k} W(y(i), u(i)); \forall u \in U, \forall y \in Y, \forall i, k \in \mathbb{Z} \]  \hspace{1cm} (3)

Gupta in [22] classifies two notions of dissipativity, i.e dissipative and strictly dissipative based on the inequality sign in (2) or (3) respectively. A system is called dissipative if the inequality (2) or (3) satisfy. Meanwhile, the system is strictly dissipative if the mathematical operators in inequality (2) or (3) turns into the strict inequality.

Now, consider the Markovian jump nonlinear systems in the following discrete-time form

\[ \begin{align*} x(k+1) &= f(x(k), r(k)) + g(x(k), r(k))u(k) \\ y(k) &= h(x(k), r(k)) + l((x(k), r(k))u(k) \end{align*} \]

where \( x(k) \in X \subset \mathbb{R}^n \) is state vector with initial condition \( x_0 \), \( u(k) \in U \subset \mathbb{R}^m \) and \( y(k) \in Y \subset \mathbb{R}^p \) are system’s input and output respectively. The functions \( f, g, h, \) and \( l \) are the smooth and time-varying nonlinear mappings in the form of \( f: X \times Z \rightarrow \mathbb{R}^n, g: X \times Z \rightarrow \mathbb{R}^{m}, h: X \times Z \rightarrow \mathbb{R}^p \), and \( l: X \times Z \rightarrow \mathbb{R}^{p} \). All the mappings are functions of a discrete-time Markov chain taking values in a finite set \( Z = \{1, \ldots, N\} \). The Markov chain has transition probabilities \( p_{ij} = Pr(r(k+1) = j | r(k) = i) \) which satisfy \( p_{ii} > 0 \) and \( \sum_{j=1}^{N} p_{ij} = 1 \) for each \( i \in Z \). The initial conditions are given by specifying \( r(0) \) and \( x(0) \). Furthermore, dissipativity corresponds to the existence of a supply rate and a storage function as stated in the following definitions.
Definition 2. Consider a function \( W: \mathbb{R}^m \times \mathbb{R}^p \rightarrow \mathbb{R} \) associated with a system of the form (4). This function is called as the supply rate, if for any \( u \in U \subset \mathbb{R}^m \) and \( y \in Y \subset \mathbb{R}^p \), the system has the following property

\[
E\left[ \sum_{k=0}^{T} W(y(k),u(k)) \right] < \infty, \quad T = 0, 1, \ldots
\]  

(5)

Definition 3. A system of the form (4) with supply rate \( W(y(k),u(k)) \) is said to be locally dissipative in the stochastic sense, if there exists a nonnegative continuous function \( V(x(k),r(k)) : X \times Z \rightarrow \mathbb{R}^+ \), called the storage function, such that for all \( k \geq 0 \)

\[
E[V(x(k+1),r(k+1)) - V(x(k),r(k))] < E\left[ \sum_{k=0}^{T} W(y(k),u(k)) \right], \quad \forall x \in X, \forall u \in U
\]  

(6)

Concerning to the supply rate, if the form of a supply rate is as the followings

\[
W(y(k),u(k)) = \begin{pmatrix} y(k) \\ u(k) \end{pmatrix}^T \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \begin{pmatrix} y(k) \\ u(k) \end{pmatrix}
\]  

(7)

where \( Q, S, \) and \( R \) are suitable dimension with \( y(k) \) and \( u(k) \), then system (4) is said to be locally QSR-dissipative in the stochastic sense.

3. Main results

Proposition 1 below characterizes a necessary and sufficient condition for Markovian jump nonlinear system as in (4) to be locally stochastic QSR-dissipative.

Proposition 1. A system of the form (4) is said to be locally stochastic QSR-dissipative if and only if there exists a nonnegative storage function \( V(x(k),r(k)) : X \times Z \rightarrow \mathbb{R}^+ \) with \( V(0,r(k)) = 0 \), the supply rate (7), and two real functions \( \rho(x(k),r(k)) : X \times Z \rightarrow \mathbb{R}^+ \), \( \varepsilon(x(k),r(k)) : X \times Z \rightarrow \mathbb{R}^+ \), such that three conditions below hold for all \( i, j \in Z \)

\[
\sum_{j=1}^{N} p_{ij} [V(f(x(k),i),j) - V(x(k),i)] = h^T(x(k),i)R(x(k),i) - \rho(x(k),i)\varepsilon(x(k),i)
\]  

(8)

\[
\frac{1}{2} \sum_{j=1}^{N} p_{ij} \frac{\partial V(\zeta, j)}{\partial \zeta} \bigg|_{\zeta = f(x(k),i)} g(x(k),i) = h^T(x(k),i)Q(x(k),i) + h^T(x(k),i)S - \rho^T(x(k),i)\varepsilon(x(k),i)
\]  

(9)

\[
\frac{1}{2} \sum_{j=1}^{N} p_{ij} \frac{\partial^2 V(\zeta, j)}{\partial \zeta^2} \bigg|_{\zeta = f(x(k),i)} g(x(k),i) = l^T(x(k),i)Q(x(k),i) + 2l^T(x(k),i)S + R - e^T(x(k),i)\varepsilon(x(k),i)
\]  

(10)

Proof: See [23].

Proposition 1 states the necessary and sufficient conditions for the system represented in Equation (4) to be stochastically dissipative with respect to the supply rate (7). Based on that proposition, dissipativity is characterized by three elements, namely the existence of a positive definite function that can twice differentiable property, the real function \( \rho(x(k),r(k)) \), and the real function \( \varepsilon(x(k),r(k)) \). As reported by Wang et al., there is no specific method to determine the real function in order to conclude dissipativity of the underlying system [23]. However, if a class of the nonlinear form are affine, then the dependence on the two real functions can be eliminated.
To be precise, consider the following affine Markovian jump nonlinear system
\[
\begin{align*}
x(k+1) &= A(x(k), r(k))x(k) + B(x(k), r(k))u(k) \\
y(k) &= C(x(k), r(k))x(k) + D(x(k), r(k))u(k)
\end{align*}
\] (11)
and the quadratic storage function \(V(x(k), r(k)) = x(k)^T P(x(k), r(k)) x(k)\). Assume that matrix \(P(x(k), r(k))\) in the storage function only depends on the state of the Markov chain. Then, theorem 1 below characterizes the dissipativity in the stochastic sense for affine Markovian jump nonlinear system (11).

**Theorem 1.** Consider the affine Markovian jump nonlinear system (11). The system is locally stochastic QSR-dissipative if and only if there exists some positive definite matrices \(P_i\) such that the following state dependent LMI’s are satisfied for all \(i, j \in \mathbb{Z}, x \in X\)
\[
\begin{pmatrix}
    A_i^T(x)P_{ii}A_i(x) - P_i - C_i^T(x)QC_i(x) \\
    B_i^T(x)P_{ii}A_i(x) - (QD_i(x) + S)^T C_i(x) & B_i^T(x)P_{ii}B_i(x) - (R + S^T D_i(x) + D_i^T(x)S + D_i^T(x)QD_i(x)) \\
\end{pmatrix}
\neq 0
\] (12)
\[i,j \in \mathbb{Z}, x \in X\]
with \(A_i, B_i, C_i,\) and \(D_i\) denote system matrices in (11) when the plant mode is in mode \(i \in \mathbb{Z}\), i.e \(r(k) = i\). \(P_{ij}\) are matrices produced from \(P_i\) after multiplication by the transition probability of the associated Markov chain and given in the following form
\[
P_{ij} = \sum_{j=1}^{N} p_{ij} P_j, \forall i, j \in \mathbb{R}.
\] (13)

**Proof:** By using Proposition 1, substitution the storage function \(V(x(k), r(k)) = x(k)^T P(x(k), r(k)) x(k)\) and its derivative into (8)-(10) renders the following state dependent LMI
\[
\begin{pmatrix}
    A_i^T(x)P_{ii}A_i(x) - P_i - C_i^T(x)QC_i(x) \\
    B_i^T(x)P_{ii}A_i(x) - (QD_i(x) + S)^T C_i(x) & B_i^T(x)P_{ii}B_i(x) - (R + S^T D_i(x) + D_i^T(x)S + D_i^T(x)QD_i(x)) \\
\end{pmatrix}
= \begin{pmatrix}
    -\rho^T \rho \\
    -e^T e \\
\end{pmatrix}
\leq \begin{pmatrix}
    -\rho^T \epsilon \\
    -\epsilon^T e \\
\end{pmatrix}
< 0
\] (14)

Theorem 1 characterizes the dissipativity of affine Markovian jump nonlinear system based on matrices of its state space representation. With Theorem 1, QSR-dissipativity of the system (11) can be obtained by finding some positive definite matrices \(P_i\) for all mode in its Markov chain which satisfy a state dependent LMI (14). By Schur complement, the state dependent LMI (14) can be written in the 4 x 4 matrix blocks as follows
\[
\begin{pmatrix}
    P_i & * & * & * \\
    SC_i(x) & R + S^T D_i(x) + D_i^T(x) & * & * \\
    B_i(x) & * & P_{ii} & * \\
    -QC_i(x) & B_i(x) & 0 & -Q \\
\end{pmatrix}
> 0 \quad \forall i \in \mathbb{Z}, x \in X
\] (15)

**4. Concluding remarks**
The paper describes on dissipativity of affine nonlinear networked control systems via Markovian jump system approach. Dissipativity of such system is assured by the existence of positive definite matrices \(P_i\) which satisfy the solvability condition in terms of nonlinear matrix inequalities.

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