Modelling of squeeze film between parallel rectangular plates of finite length considering inertia effects

Xueping Li, Xinhao Luo and Wei Jiang

State Key Laboratory of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan, China

E-mail: jiangw@hust.edu.cn

Abstract. Squeeze film of finite length rectangular plate is widely used in MEMS devices, immersion lithography machines and other engineering fields. The damping force of squeeze film of finite length rectangular plates can be obtained by multiplying the solution of infinite length rectangular plates with a correction factor depending on the aspect ratio. Whether the correction factor is the same for the inertia force is attractive to be studied. The distributions of velocity and pressure are obtained by solving the 3D Navier–Stokes (NS) equations with free and half sealed outlets, which shows the leakage of flow in the length direction causes the drop of maximum pressure comparing with the infinite case, which is about 56.6% for square plates. The squeeze film force ratios for 3D to 2D models with different aspect ratios have been obtained under the conditions of different film thickness, viscosity and rectangular width. It is concluded that the correction factor of damping force can be used to modify the inertia force. The damping and inertia coefficients can be easily extracted from the modified analytical solution of parallel rectangular squeeze film with finite length, which can be applied to the modelling of system dynamics.

1. Introduction

Squeeze film effect is a common phenomenon that involves hydrodynamic pressure developed from the relative motion of two surfaces. This squeeze film action plays a positive role in areas of squeeze film damper [1], MEMS harvester [2], squeeze film type rheometers [3]. However, the drag force due to this squeeze film effect will restrict the mechanical behaviour in fields of micro actuators [4, 5], atomic force microscopy [6, 7], and microcantilever biosensors [8]. For the optimized design of such devices in practical applications, the efficient and accurate predictions of hydrodynamic of squeeze film are the most concerned issues.

The geometry of the squeeze surface is the main factor affecting the pressure distribution of the film. For simple shapes such as circles and strips, approximate analytical solutions can be derived. The approximate analytical solution for parallel circle squeeze liquid film [9] can be derived from the Navier–Stokes (NS) equation, including the damping term, unsteady inertia term and nonlinear convective inertia terms. The strip can be equivalent to infinite long rectangular plate by neglecting the pressure variation across the length because the length of the strip is usually much larger than its width, which is similar to the assumption of long bearing approximations [10-12]. The approximate analytical solution for parallel strip squeeze liquid film considering inertia effects can also be derived [13]. For simplicity, the hydrodynamic influences on squeeze surface are equivalent to viscous damping and added mass by ignoring the nonlinear convective terms. The geometries of circle and strip for squeeze surface are instructive for study of the physical mechanisms and mathematics of squeeze film
lubrication but are of limited practical value. The geometry of rectangular plates is widely used in MEMS devices. Especially for those squeeze surface that are nearly square, the infinitely long rectangular plate model is no longer valid, which tends to overestimate the hydrodynamic mass. The deviations between the test and the theoretical results are ascribed to the non-parallel squeeze surface [14]. However, the square squeeze surface used in the experimental setup is perhaps another reason resulting in the deviations.

The squeeze film force of finite rectangular plate considering 3D geometry was obtained by using perturbation method [15] for Reynold’s equation, which is limited to compressible gas film and small squeeze numbers less than 1. The correction factor for finite rectangular plate without considering inertia effects has been deduced through method of separation of variables [16]. However, the inertia effects can be equivalent to an added mass to the plate, which plays a significant effect on the frequency change of system in some special conditions [7, 17]. The research on the correction of inertial force is rarely mentioned.

Although the 3D rectangular squeeze film with considering inertia effects can be simulated through finite element simulation, it is rather time consuming and lack of equivalent added mass and damping parameters, which is difficult to be applied in system modelling. In this paper, we have put forward the idea of using the damping force correction factor [13] to modify the inertia force. This new idea is simple but innovative and has been finally verified through simulations. The proposed analytical solution of parallel rectangular plate considering inertia effects provides an effective model for engineering applications.

The linear analytical solution of infinitely long rectangular squeeze film [13] has been adopted to be modified. The 3D to 2D NS equations are solved through the COMSOL Multiphysics. Comparisons of squeeze film force between 2D NS equations and the linear analytical solution with or without inertia effects have been done to prove that the linear analytical solution is accurate enough for small Reynold’s number and the correction of inertia force is urgent. The distributions of velocity and pressure are obtained by solving the 3D Navier–Stokes(NS) equations with free and half sealed outlets, which shows the leakage of flow in the length direction causes the drop of maximum pressure comparing with the infinite case, which is about 56.6% for square plate. The force ratios of squeeze film force for 3D to 2D models have been calculated to compare with the results derived from Reference [16].

2. Modelling of finite rectangular squeeze film

In this paper, the low viscosity liquid film with a thickness of hundreds microns under micro vibration conditions is considered. Figure 1 shows a finite rectangular plate with length \( L \) and width \( b \) that oscillates in close proximity to a fixed parallel surface with sinusoidal excitation \( \Delta h = \varepsilon \sin(\omega t) \), where \( \varepsilon \) is amplitude of squeeze and \( \omega \) is angular frequency of squeeze\((f=2\pi\omega)\). The initial film thickness is defined as \( h \).

![Figure 1](image)

**Figure 1.** Schematic of squeeze film between two parallel finite rectangular plates.

When assuming a constant pressure across the gap for \( h \ll b \) and \( h \ll L \), the Navier–Stokes equations can be reduced to [18]
Mesh independence tests have been performed such that further refinement of the mesh results in insignificant changes in the numerical results. The first term of Equations (1) and (2) refers to unsteady inertia and accounts for the acceleration of the liquid. The second and third terms characterize the convective inertia arising due to the confinement of the liquid. For small gaps with \( h/b \ll 1 \) and \( \varepsilon/h \ll 1 \), the nonlinear convective inertia terms are always much less than unsteady inertia terms [19]. For the infinitely long rectangular plate assumption, the velocities \( v \) is considered to be zero. Thus the NS equations will be reduced further to

\[
\rho \frac{\partial u}{\partial t} + \rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} = -\frac{\partial P}{\partial x} + \eta \frac{\partial^2 u}{\partial z^2} \quad \text{(1)}
\]

\[
\rho \frac{\partial v}{\partial t} + \rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \eta \frac{\partial^2 v}{\partial z^2} \quad \text{(2)}
\]

where \( u \) and \( v \) are the flow velocities in the \( x \)- and \( y \)-directions, respectively, \( \eta \) is the viscosity coefficient, \( \rho \) is the density and \( P \) is the pressure of the film. The first term of Equations (1) and (2) is the squeeze film force with the infinitely long rectangular plate assumption, defined as \( F_{\text{inf}} \) here, has been determined via the perturbation method for small Reynolds numbers, which can be expressed as [13]

\[
F_{\text{inf}} = \frac{\eta WL}{h^3} + \frac{\rho WL^3}{10h} \quad \text{(4)}
\]

where the first term of the right side represents the damping force defined as \( F_c \) and the last term represents the inertia force defined as \( F_i \).

For the squeeze film between parallel rectangular plates of finite length in practical applications, the infinitely long rectangular plate assumption is invalid. The correction factor is defined as the squeeze force of infinite to infinite case of parallel rectangular plate. The correction factor for the damping force of finite rectangular plate has been deduced through method of separation of variables [16] as

\[
\beta_c = \frac{192}{\pi^4} \sum_{n=1,3,5,...}^{\infty} \left( \frac{1}{2n^4} - \frac{\lambda}{\pi n^5} \tanh \left( \frac{n\pi \lambda}{2} \right) \right)
\quad \text{(5)}
\]

where the aspect ratio defined as \( \lambda = L/h \).

Figure 2 shows the boundary conditions of 2D and 3D finite element models. The atmospheric pressure \( P_0 \) has been prescribed for the free outlet boundary. The wall boundary means the velocity of the boundary layer near the wall is equal to zero. The four sides are set as free outlet boundary conditions for 3D model of free outlet, while a pair of sides for 3D model of half sealed outlet are set as wall boundary conditions. All the top surfaces are set with moving boundary condition with displacement of \( \Delta h \). All the lower surfaces are set with wall boundary condition. The approximate infinitely long rectangular plate 2D model is equivalent to the 3D model of half sealed outlet, which has also been calculated to explain the principle of the difference between 2D and 3D models for the analysis below. The squeeze film force of 3D model of free outlet, defined as \( F_{3D} \), can be obtained by integrating the pressure through the whole upper surface. The squeeze film force of 2D model, defined as \( F_{2D} \), can also be obtained by multiplying the length \( L \) after integrating the pressure through the upper surface.

In our simulations, the grids are gradually refined towards the solid wall boundaries, as shown in Figure 3. Mesh independence tests have been performed such that further refinement of the mesh results in insignificant changes in the numerical results. The simulations were performed for water with a constant temperature of 21.5°C. The viscosity and density of water are 0.972 mPa.s and 997.9 kg/ m³ respectively. For the purpose of comparing the effects of different viscosities, motor oil was also used in the simulations with viscosity of 169 mPa.s and 873.2 kg/ m³. All the simulations have been done with micro vibration of \( \varepsilon=1 \) μm and \( f=50 \) Hz.
3. Analysis of simulation results

Figure 4 shows the pressure and velocity distributions of 3D models with free and half sealed outlet with $\lambda=1$, $b=30$ mm, $h=180$ μm. The maximum pressure of 3D model with free outlets of Figure 4(a) is less than the 3D model with half sealed outlets of Figure 4(b), which is equivalent to the infinitely long rectangular plate hypothesis of 2D model. The leakage of flow in the $x$ direction causes this pressure drop in the maximum pressure, which can be reflected in Figure 4(c) and 4(d). There is almost no leakage flow velocity in the sealing ($x$) direction of Figure 4(d) while the leakage flow velocity of $x$ direction is significant of Figure 4(c) for $\lambda=1$. Figure 5 shows the maximum pressure with different $\lambda$ for 3D model with free outlets and 2D model with $b=30$ mm, $h=100$ μm. The maximum pressure of 3D model with free outlets drops sharply comparing with 2D model with decreasing $\lambda$, which is about 56.6% for $\lambda=1$. This shows that there is an unacceptable error by adopting the uncorrected two-dimensional analytical model for finite length rectangular plates.

Figure 2. Boundary conditions of 2D and 3D finite element models.

Figure 3. Mesh of 2D and 3D finite element models.

Figure 4. The pressure and velocity distributions of 3D models with different boundary conditions.
The 2D-NS equations are solved to compare with the linear analytical solutions $F_{2D}$ with and without inertia effects. All these three model have been calculated with the squeeze amplitude of 1 μm and squeeze frequency of 50Hz with film thickness from 80 μm to 280 μm. Figure 6 shows that the squeeze film forces of the linear analytical solution $F_{inf}$ with inertia effects are consistent with the simulation results of $F_{2D}$, which means the linear analytical solution is accurate enough for small Reynold’s number. The deviation of $F_c$ and $F_{2D}$ increases gradually as the film thickness getting thicker, which means inertial effects are decisive to accurately predict the squeeze film force for the moderate film thicknesses. In this case, effect of aspect ratio on the inertia force is more urgent to be studied.

$$F_{3D} = \beta_{F} F_{2D} = \beta_{F} F_{c} + \beta_{i} F_{i}$$  \hspace{1cm} (6)

where $\beta_{F}$ and $\beta_{i}$ are correction factors of $F_{2D}$ and $F_{i}$, respectively. If we can obtain the analysis results of $\beta_{F}$ from the 2D and 3D models, the $\beta_{i}$ can be proved from Equation (6) indirectly.

The force ratio is defined as amplitude ratio for the squeeze film force of 3D to 2D models, which is actually the same with $\beta_{F}$. The influence of film thickness, film viscosity and feature size $b$ of rectangular plate have been studied with different aspect ratio $\lambda$ from 1 to 25. The squeeze amplitude is 1 μm and the squeeze frequency is 50Hz, which ensures the linearity of the model with small Reynold’s number. Figure 7 shows that the force ratios of three different film thickness coincide well with $\beta_{c}$ with all the test range of $\lambda$ from 1 to 25. The same results can be obtained for different film viscosity in Figure 8. The force ratio with different feature size $b$ of rectangular plate for $\lambda=1$ and $\lambda=3$ in Figure 9 also show good consistency for $\beta_{E}$ and $\beta_{c}$. Form Figure 7 to Figure 9 we can obtain that the
correction factors of inertia force can be regarded as consistent with the correction factors of damping force. The corrected squeeze film force of finite rectangular plate can be expressed as

\[ F_{\text{finite}} = \beta_c \left( \frac{\eta WL^3}{h^3} - h + \frac{\rho WL^3}{10h} \right). \]  

Figure 9. The force ratio with different feature size \( b \) of rectangular plate.

Figure 10. The time depended squeeze film force for different models with \( h=180 \) μm.

With the corrected model mentioned above, the time depended squeeze film force for different models have been calculated with \( h=180 \) μm and \( \lambda=1 \). Figure 10 shows that the amplitude of inertia force is almost the same with damping force. The corrected model for finite rectangular plate can predict the squeeze film force well with considering inertial effects, which proved to be accurate and efficient. The modified inertia and coefficient can also be easily extracted.

4. Conclusions
Squeeze film of finite length rectangular plate is widely used in engineering fields. The corrected analytical solution of squeeze film force for infinite length rectangular plates is easy to be derived and proved to be effective without considering inertia effects. However, for the liquid film with Reynolds number around 1, the inertia effects must be considered. There is a lack of research on the correction of inertial force. In this paper, the 2D and 3D simulations of parallel rectangular squeeze film with finite length considering inertia have been calculated. The velocity and pressure distributions and the squeeze film forces are analysed with the following conclusions:

1) There is an unacceptable error in the use of the uncorrected two-dimensional analytical model for finite-length rectangular plates. The leakage flow in the length direction causes the drop of pressure.

2) The deviation of the linear analytical solution with and without inertia effects increases gradually as the film thickness getting thicker, which means inertial effects are decisive to accurately predict the squeeze film force for moderate film thicknesses. In this case, effect of aspect ratio on the additional mass is more urgent to be studied.

3) By comparing the force ratio with the damping force correction factor, we can infer that the correction factor of inertia force can be regarded as consistent with the correction factor of damping force. The damping and inertia coefficients can be easily extracted from the modified analytical solution of parallel rectangular squeeze film with finite length, which can be applied to the modelling of system dynamics.

For the squeeze film with large Reynolds numbers or large squeeze amplitude, the nonlinear convection inertia terms could be dominant. Thus the nonlinear squeeze film model of parallel rectangular squeeze film with finite length will be the focus of the future research.
Acknowledgements
The work is supported by the National Natural Science Foundation of China (No. 51675195, No. 51721092) and the National Major Science and Technology Projects of China (No. 2017ZX02101007-002).

References
[1] Chen W T, Chen S Y, Hu Z H, Tang J Y and Li H N 2020 Dynamic analysis of a bevel gear system equipped with finite length squeeze film dampers for passive vibration control Mech Mach Theory 147 18
[2] Chen S Y and Feng Z C 2017 Damping and stiffening forces of a squeeze film between two plates Nonlinear Dyn 87(3) 1763-72
[3] Esmonde H, See H and Swain M V 2008 Dynamic squeeze film rheometry for flat and spherical geometries using nonlinear spectral analysis Meas Sci Technol 19(11) 115404
[4] Burugupally S P and Perera W R 2019 Dynamics of a parallel-plate electrostatic actuator in viscous dielectric media Sensors and Actuators A: Physical 295 366-73
[5] Burugupally S P and Hoelzle D 2018 Experimental investigation of curved electrode actuator dynamics in viscous dielectric media Appl Phys Lett 113(7) 074102
[6] Lin S M 2010 Effective dampings and frequency shifts of several modes of an inclined cantilever vibrating in viscous fluid Precis Eng 34(2) 320-6
[7] Basak S, Raman A and Garimella S V 2006 Hydrodynamic loading of microcantilevers vibrating in viscous fluids J Appl Phys 99(11) 114906
[8] Singh P and Yadava R D S 2019 Effect of viscous axial loading on vibrating microcantilever sensors J Phys D: Appl Phys 52(34) 345301
[9] Esmonde H, Fitzpatrick J A, Rice H J and Axisa F 1992 Modelling and identification of nonlinear squeeze film dynamics Journal of Fluids and Structures 6(2) 223-48
[10] Sangeetha S and Kesavan S 2018 Pressure distribution with surface roughness for effect between porous infinitely long rectangular plates with MHD couple stress squeeze film lubrication J Phys: Conference Series 1000(1) 012006
[11] Irannejad M and Ohadi A 2015 Vibration analysis of a rotor supported on magnetorheological squeeze film damper with short bearing approximation: A contrast between short and long bearing approximations Journal of Vibration & Control 23(11) 1792-1808
[12] Antunes J, Moreira M and Piteau P 2003 Finite Length Bearings And Squeeze-film Dampers: A Two-dimensionalDynamical Solution WIT Transactions on Modelling and Simulation 33
[13] Huang S J, Borca-Tasciuc D A and Tichy J A 2011 A simple expression for fluid inertia force acting on micro-plates undergoing squeeze film damping P Roy Soc a-Math Phy 467(2126) 522-36
[14] Huang S, Borca-Tasciuc D A and Tichy J A 2015 Tilt effects on experimental measurement of squeeze film damping in microsystems Microfluidics and Nanofluidics 19(4) 891-7
[15] Sadd M H and Stiffler A K 1975 Squeeze film dampers: amplitude effects at low squeeze numbers J Eng Ind 97(4) 1366-70
[16] Khonsari M M and Booser E R 2017 Applied tribology: bearing design and lubrication (New York: John Wiley & Sons)
[17] Marrero V, Borca-Tasciuc D A and Tichy J 2010 On Squeeze Film Damping in Microsystems Journal of Tribology-Transactions of the Asme 132(3) 031701
[18] Veijola T 2004 Compact models for squeezed-film dampers with inertial and rarefied gas effects Journal of Micromechanics and Microengineering 14(7) 1109-18
[19] Lang J, Nathan R and Wu Q 2019 Experimental Study of Transient Squeezing Film Flow Journal of Fluids Engineering 141(8) 081110