CP violating asymmetries of $b$ quarks and leptons
in $e^+e^- \to t\bar{t}$ and supersymmetry

review

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ABSTRACT

The distributions of the single decay $b$-quarks and leptons from $e^+e^- \to t\bar{t}$ assuming CP violation are reviewed. Different asymmetries, sensitive independently to CP violation in the production and in the decay, and sensitive to the real and imaginary parts of $\delta^p$ and $\delta^Z$ are defined. The analytic expressions are general and independent on the model of CP violation. In most of them all phase space integrations are fulfilled analytically. Numerical results in the MSSM with complex couplings are presented.

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1 Introduction

The large mass of the top quark allows to probe high energies, where new physics might show up as well. In the last years a number of papers consider CP violating observables in processes with top quarks as tests of physics beyond the Standard Model (SM). There are several reasons for this:

1. Owing to its large mass ($m_t = 175 GeV$) [1], the top quark will decay before forming a hadronic bound state. Therefore its polarization will not be deluted by possible hadronization processes and it can be determined by the distributions of its decay products. CP violation is sensitive to the polarization of the top-quark.

2. Theoretical predictions are more reliable as they are free of the hadronization uncertainties.

3. Due to the GIM mechanism, the effects of CP violation in SM are very small. Thus, observation of CP violation in the top-quark physics would be an indication of physics beyond the SM. Supersymmetric models and models with more than one Higgs doublet are at present the most favoured candidates. They provide new sources of CP violation [2] that lead to CP non conservation at one loop level.

For testing of CP invariance one has to compare the decays of the top quark with those of the anti-top quark. It is important that in the future $e^+e^-$ colliders the top–antitop quark pairs will be copiously produced and their decay modes will be studied in the same experiment.

In general CP violation enters both the production and decay processes. In the distribution of the $t$-decay products it enters through the $t$-polarization. This means that if the $t$-quark would decay unpolarized (due to possible hadronization processes) the distribution of its decay products would not be sensitive to possible CP violation. As shown in [3] the effects of depolarization due to such hadronization processes for the top quark are very small.

We shall consider the $t$ ($\bar{t}$) - quarks produced in $e^+e^-$ annihilation [4]. They will be identified by their decay products. As the top quark actually does not mix with

\footnote{CP violation in $t\bar{t}$ production at hadron colliders has been discussed in [4].}
the quarks from the other generations, its only decay mode in SM is $t \rightarrow bW$. Consequently, information about the $t$-polarization will be carried both by the $b$-quarks and by the $W$’s. We shall consider the general expressions for CP violation comparing the angular [5] and energy [6] distributions of $b$ and $\bar{b}$ in the CP conjugate processes:

$$e^+ + e^- \rightarrow t\bar{t} \rightarrow b + X', \quad e^+ + e^- \rightarrow t\bar{t} \rightarrow \bar{b} + \bar{X}'.$$ (1)

$X'$ ($\bar{X}'$) stands for $\bar{t}W$ ($tW'$) irrespectively how the $W$’s are identified. Previously the effects of the dipole moment form factors in (1) were considered in [7]. The CP violating asymmetries organized from the $b$ and $\bar{b}$ quarks in processes (1) have the advantage that they have no background: If the $b$ and $\bar{b}$ quarks are produced directly in $e^+e^- \rightarrow b\bar{b}$ the final state is CP even and thus cannot induce any CP violating asymmetries. The $b$ and $\bar{b}$ quarks may come also from the decays of $W^\pm$: $e^+e^- \rightarrow W^+W^-$, $W^\pm \rightarrow bc$, that are Cabbibo supressed and CP violation can be of academic interest only. Thus, measuring CP violation through (1) does not require reconstruction of the processes event by event. However, a clear identification of the jets from $b$ and $\bar{b}$ is necessary. Different methods of $b$-tagging are considered in [5].

The $W$’s can be studied through the angular [8, 9, 10, 11] and energy [12, 13, 14] distributions of the leptons from the decays $W^\pm \rightarrow l^\pm \nu$:

$$e^+ + e^- \rightarrow t\bar{t} \rightarrow b l^+ X, \quad e^+ + e^- \rightarrow t\bar{t} \rightarrow \bar{b} l^- \bar{X}.$$ (2)

Here we shall study CP violation comparing the angular distributions of $b, l^+$ and $\bar{b}, l^-$ with special emphasis on triple product correlations [9].

The longitudinal polarizations of $e^+$ and $e^-$ are also taken into account. There are three vertices that can introduce CP violation:

- In the $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices CP violation is introduced by the electric $d^\gamma(s)$ and weak $d^Z(s)$ dipole moment form factors:

$$e \mathcal{V}_\mu^\gamma = e \left( \frac{2}{3} \gamma_\mu - i \frac{d^\gamma(s)}{m_t} \mathcal{P}_\mu \gamma_5 \right),$$

$$g_Z \mathcal{V}_\mu^Z = g_Z \left( \gamma_\mu (g_V + g_A \gamma_5) - i \frac{d^Z(s)}{m_t} \mathcal{P}_\mu \gamma_5 \right),$$ (3) (4)

where $\mathcal{P}_\mu = p_\mu - p_{\bar{t}} \mu$, $g_V = (1/2) - (4/3) \sin^2 \Theta_W$, $g_A = -(1/2)$, and $g_Z = e/\sin 2\Theta_W$ with $e$ the electro–magnetic coupling constant. The electoweak
dipole moment form factors $d^\gamma,Z(s)$ are functions of $s$, so that $d^\gamma(0)$ and $d^Z(m_Z^2)$ determine the electric and weak dipole moments of the top quark. $d^\gamma,Z(s)$ can be introduced only by an interaction in the production process $e^+e^- \rightarrow t\bar{t}$ that is both $P$ and $T$ violating, and through $CPT$ invariance, also $CP$ violating. Very recently a nice review on the dipole moment form factors appeared \cite{15}.

- The $tbW$ vertex, that determine the weak decays of the $t$ and $\bar{t}$ quarks, see eq. (83), is written in the form:

$$
V^t_\alpha = \frac{g}{2\sqrt{2}} \left( \gamma_\alpha(1-\gamma_5) + f^{t}_L \gamma_\alpha(1-\gamma_5) + \frac{g^{t}_R}{m_W} P^{t}_\alpha(1+\gamma_5) \right),
$$

$$
V^{\bar{t}}_\alpha = \frac{g}{2\sqrt{2}} \left( \gamma_\alpha(1+\gamma_5) + f^{\bar{t}}_L \gamma_\alpha(1+\gamma_5) + \frac{g^{\bar{t}}_R}{m_W} P^{\bar{t}}_\alpha(1-\gamma_5) \right)
$$

with $P^t = p_t + p_b$, $P^{\bar{t}} = p_{\bar{t}} + p_b$. In eqs. (3) and (4) we have kept only the terms that do not vanish in the approximation $m_b = 0$. Contrary to the electroweak dipole moment form factors $d^\gamma,Z(s)$, the form factors $f^{t,\bar{t}}_L$ and $g^{t,\bar{t}}_R$ have both $CP$–invariant and $CP$–violating contributions:

$$
f^{t,\bar{t}}_L = f^{SM}_{L} \pm if^{CP}_{L}, \quad g^{t,\bar{t}}_R = g^{SM}_{R} \pm ig^{CP}_{R}
$$

where the superscript $SM$ ($CP$) denotes the CP invariant (CP violating) contributions to the form factors. In analogy with the dipole moment form factors, we have explicitly written the $i$ in front of $f^{CP}_L$ and $g^{CP}_R$, that comes from the imaginary CP violating coupling. Both the CP invariant and CP violating parts of the formfactors $f^{SM(CP)}_L$ and $g^{SM(CP)}_R$ have real and imaginary parts. If neglecting absorptive parts of the amplitude, $\Im m f^{SM}_L = \Im m f^{CP}_L = 0$ and $\Im m g^{SM}_R = \Im m g^{CP}_R = 0$, then CP invariance implies that the form factors of the top and antitop quarks are real and equal:

$$
f^{t}_L = f^{\bar{t}}_L = \Re e f^{SM}_L, \quad g^{t}_R = g^{\bar{t}}_R = \Re e g^{SM}_R
$$

For understanding the mechanism of CP violation, it is important to distinguish CP violation in the production from CP violation in the decay processes. As we shall see, the distributions of the decay products are mainly sensitive to CP violation in the production, CP violation in the decay vertex being suppressed by the amount
of the SM $t$-polarization. In order to study CP violation in the decay more useful appears the difference between the partial decay rates of $t$ and $\bar{t}$ [16, 17]:

$$A_{CP} \equiv \frac{\Gamma (t \to bW^+) - \Gamma (\bar{t} \to \bar{b}W^-)}{\Gamma (t \to bW^+) + \Gamma (\bar{t} \to \bar{b}W^-)}.$$  \hspace{1cm} (9)

This difference propagates into the differences of the total number in events of the CP conjugate processes (1) and these of (2).

In general the CP violating pieces in the production and decay vertices have contributions from both real and absorptive parts of the amplitude. In accordance with this we have two types of observables: CP violation in the absorptive parts of the amplitude ($\Im m d^{\nu,Z}, \Im m f_L^{CP}, \Im m g_R^{CP}$) enter the energy and the angular distributions. Such an observable is also $A_{CP}$, which is proportional to the absorptive parts of the CP violating contributions of the $tbW$ vertex. In order to measure CP violation in the real parts of the amplitude one has to consider triple correlations of the type

$$\mathcal{T} = (q_1 q_2 q_3) \equiv (q_1 \times q_2 \cdot q_3)$$  \hspace{1cm} (10)

where $q_{1,2,3}$ can be any one of the 3–momenta in each of the processes (1) or (2). Triple product correlations of particle momenta and spin for a general study of $CP$ violation in $tt$ production in $e^+e^-$ annihilation and in $pp$ collisions have been proposed in [8, 18]. The correlations (10) are called T-odd as they change sign under the a flip of the 3-momenta involved. However, this does not imply $T$, or through $CPT$ also $CP$ violation. The time reversal operation $T$ implies not only reverse of the particle momenta and spin, but also interchange of the initial and final states. When loop corrections are included the correlations $\mathcal{T}$ can arise either from absorptive $CP$ invariant parts in the amplitude (so-called final state interactions [19]), or from $CP$ violation. The former effect is a consequence of the unitarity of the $S$–matrix and it is our background. It can be eliminated either by taking the difference between the process we are interested in and its $CP$ conjugate or by direct estimates. $T$–odd correlations in the SM due to gluon or Higgs boson exchange in the final states have been considered in [20, 21].

The contributions of the real and imaginary parts of the CP violating form factors to different pieces of the cross section can be understood as follows. The CP violating terms in the cross section come from the interference of the (real) tree level SM amplitude and the CP violating loop corrections. The i from the CP
violating coupling and the i from the absorptive part of the loop guarantee that the energy and angular distributions are real. The triple products \( i \varepsilon_{\alpha\beta\gamma\delta} p_1^\alpha p_2^\beta p_3^\gamma p_4^\delta \) \( (p_i \text{ is any of the 4-vectors of (1) and (2)}) \) when written in the laboratory frame. The i from the CP violating coupling and the i in front of \( \varepsilon_{\alpha\beta\gamma\delta} \) guarantee real contribution to the cross section only if real parts of the loop corrections are involved.

The presented formula for the distributions and the asymmetries are general and model independent. At the end we present numerical results in the Minimal Supersymmetric Standard Model (MSSM) with complex constants, where the CP violating form factors appear at one loop level. We have used the results of [22], where a complete analysis of \( d^r \) and \( d^s \) in the MSSM was performed.

## 2 The Formalism

In order to obtain analytic expressions for the distributions of the decay products in the sequential processes (1) and (2) we follow the formalism of [23]. This approach allows a clear physical interpretation of the different contributions in the cross section in terms of the polarization vectors of the decaying particle. For the cross sections of (1) in the narrow width approximation for the top quark \( (\Gamma_t \ll m_t) \) we write

\[
d\sigma^{b}_{\lambda\lambda'} = d\sigma^{t}_{\lambda\lambda'} \frac{d\Gamma_t}{\Gamma_t} \frac{E_t}{m_t}, \quad d\sigma^{b}_{\lambda\lambda'} = d\sigma^{t}_{\lambda\lambda'} \frac{d\Gamma_t}{\Gamma_t} \frac{E_t}{m_t}.
\]

For the cross sections of (2) in the narrow width approximation for \( t \) and \( W \) \( (\Gamma_t \ll m_t, \Gamma_W \ll m_W) \) we have

\[
d\sigma^{t^+}_{\lambda\lambda'} = d\sigma^{t}_{\lambda\lambda'} \frac{E_t}{m_t \Gamma_t} \frac{d\Gamma_t}{\Gamma_t} \frac{E_{W^+}}{m_W \Gamma_W}, \\

\[
d\sigma^{t^-}_{\lambda\lambda'} = d\sigma^{t}_{\lambda\lambda'} \frac{E_t}{m_t \Gamma_t} \frac{d\Gamma_t}{\Gamma_t} \frac{E_{W^-}}{m_W \Gamma_W}.
\]

Here \( d\sigma^{(t\bar{t})}_{\lambda\lambda'} \) is the differential cross section for \( t (\bar{t}) \) production in \( e^+e^- \) annihilation, \( \lambda, \lambda' \) are the degrees of longitudinal polarization of the \( e^- \), \( e^+ \) beams. \( d\Gamma_t (d\Gamma_{\bar{t}}) \) is the differential decay rate for \( t \rightarrow bW^+ \) (\( \bar{t} \rightarrow \bar{b}W^- \)) when the \( t \) (\( \bar{t} \)) quark is polarized, its polarization determined in the previous production process, and \( d\Gamma_{W^\pm} \) is the differential decay rate of \( W^\pm \rightarrow l^\pm \nu \), with the polarization states for \( W^\pm \) determined in the preceding \( t \) (\( \bar{t} \)) decay, \( E_{t(\bar{t})} \) and \( E_{W^\pm} \) are the energies of \( t \) (\( \bar{t} \)) and \( W^\pm \). All
quantities are in the c.m. system of $e^+e^-$. $\Gamma_t$ and $\Gamma_W$ are the total decay widths of $t$ and $W$. From (11) and (12) the cross sections of (1) and (2) are obtained in terms of the polarization 4-vector $\xi^t$ ($\xi^t_i$) of the $t$ ($i$) quarks.

For the differential cross sections of (1), assuming CP violation both in the production and decay vertices of the top-quark we obtain (3):

$$
\begin{align*}
    d\sigma^{b(b)}_{\lambda:\lambda'} &= \sigma^{b(b)}_0 \left\{ A^{b(b)}_{SM} + A^{b(b)}_d + A^{b(b)}_{g_R} \right\} d\cos\theta_{t(i)} \, d\Omega_{b(b)} \\
    A^{b(b)}_{SM} &= 1 \pm \alpha_lm_t \frac{(\xi^{t(i)}_{SM} p_{b(b)})}{(p_ip_b)} \\
    A^{b(b)}_d &= \pm \alpha_lm_t \frac{(\xi^{t(i)}_{CP} p_{b(b)})}{(p_ip_b)} \\
    A^{b(b)}_{g_R} &= \mp2 \left\{ 3m_f^{CP} + 3m_g^{CP} \frac{m_t^2 - m_W^2}{m_W(m_t^2 + 2m_W^2)} \right\} - 2\alpha_lm_t \left\{ 3m_f^{CP} + 3m_g^{CP} \frac{2m_t(m_t^2 - m_W^2)}{m_W(m_t^2 - 2m_W^2)} \right\} \frac{(\xi^{t(i)}_{SM} p_{b(b)})}{(p_ip_b)}
\end{align*}
$$

Here $\sigma^{b(b)}_0$ is the cross section of (1) for unpolarized decaying top quarks, $A^{b(b)}_{SM}$ is the contribution from the SM $t$-quark polarization, $A^{b(b)}_d$ describes the CP violating pieces due to $d^\gamma Z$, and $A^{b(b)}_{g_R}$ – those due to CP violation in the decay. We consider the SM at tree level, which implies that the SM form factors $f_L^{SM}$ and $g_R^{SM}$ are neglected. We use a reference frame in which the $z$-axis points the direction of $q_e$, $q_e$ and $p_{t(i)}$ determine the $xz$-plane, $\cos\theta_{t(i)}$ is the scattering angle of $t$ ($i$). The polarization vectors $\xi^t_i$ and $\xi^t_i$ get contributions from the SM and from CP violating interactions:

$$
\xi^{t(i)} = \xi^{t(i)}_{SM} + \xi^{t(i)}_{CP}.
$$

For the differential cross section of (2) we obtain:

$$
\begin{align*}
    d\sigma^{\pm} &= \sigma^{\pm}_0 \left\{ A^\pm_{SM} + A^\pm_d + A^\pm_{g_R} \right\} d\cos\theta_{t(i)} \, d\Omega_{b(b)} \\
    A^\pm_{SM} &= 1 \mp \alpha_lm_t \frac{(\xi^{t(i)}_{SM} p_{\pm})}{(p_{t(i)} p_{\pm})} \\
    A^\pm_d &= \mp \alpha_lm_t \frac{(\xi^{t(i)}_{CP} p_{\pm})}{(p_{t(i)} p_{\pm})} \\
    A^\pm_{g_R} &= \mp2\alpha_l \left\{ 3m_f^{CP} + 3m_g^{CP} \frac{m_t^2}{m_W} \frac{1 - \frac{m_W^2}{2(p_{t(i)} p_{\pm})}} \right\}
\end{align*}
$$
\[ -2\alpha_l \, 3 m \, g_R^{C_P} \frac{\xi_{\text{SM} p_6(\bar{b})}}{m_W} \]
\[ +2\alpha_l m_t \left[ 3 m \, f_L^{C_P} + \frac{(p_t p_6)}{m_t m_W} \, 3 m \, g_R^{C_P} \right] \frac{\xi_{\text{SM} p_7(\bar{i})}}{(p_t(p_6))} \]
\[ \pm 2\alpha_l \, \Re \, g_R^{C_P} \frac{\varepsilon_{\xi_{\text{SM} p_7(\bar{i})} p_2 + p_6(\bar{b})}}{m_W (p_t(p_6))}. \]

(21)

The indices ± correspond to \( l^+ \) and \( l^- \) production. \( \sigma_0^{\pm} \) is the tree level SM cross section of (2) for unpolarized decaying top quarks, \( A_{SM}^{\pm} \) is the contribution from the SM polarization, the terms \( A_{d}^{\pm} \) and \( A_{g_R}^{\pm} \) contain the \( CP \)-violating correlations due to \( d_{\gamma,Z} \) and due to CP violation in the decay, respectively. The quantity \( \varepsilon(p_1 p_2 p_3 p_4) \) is abbreviation of \( \varepsilon_{\alpha \beta \gamma \delta} p_\alpha p_{\beta} p_{\gamma} p_{\delta} \).

Eqs. (13) and (18) are our basic formula. They imply that CP violation enters the angular and energy distributions of the \( b \) quarks and the leptons only through the polarization of the top quarks. The coefficients \( \alpha_b \) and \( \alpha_l \) determine the sensitivity of the \( b \)-quarks and the leptons to the \( t \)-polarization. We have [8]:

\[ \alpha_b = \frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2}, \quad \alpha_l = 1. \]  

(22)

The sensitivity to CP violation in the production plane is determined by \( \alpha_b \) or \( \alpha_l \) and \( \xi_{CP} \). The sensitivity to CP violation in the decay is determined by \( \alpha_b \) or \( \alpha_l \), the form factors \( f_{L}^{C_P}, \) \( g_{R}^{C_P}, \) and the SM top quark polarization \( \xi_{SM} \). Thus, the sensitivity to \( f_{L}^{C_P} \) and \( g_{R}^{C_P} \) in the distribution of the decay products is suppressed by the amount of the SM \( t \)-polarization. In the next section we shall give the explicit expressions for \( \xi_{t,i} \). The first terms in \( A_{g_R}^{\pm} \) and \( A_{g_R}^{\pm} \) are independent on the top-polarization, which implies that CP violation in the decay will enter the total cross sections.

\( \sigma_0^{b} \) and \( \sigma_0^{l} \) determine the differential SM cross sections of (1) and (2) for totally unpolarized decaying top quarks with longitudinally polarized initial electron–positron beams. We have:

\[ \sigma_0^{b(\bar{b})} = \alpha_{em}^2 \frac{3\beta}{2s} \frac{\Gamma_{t \rightarrow bW}}{\Gamma_t} \frac{m_t^2 E_{b(\bar{b})}^2}{(m_t^2 - m_W^2)^2} N_{\lambda \lambda'}^{l(i)} \]  

(23)

\[ \sigma_0^{\pm} = \frac{\alpha_{em}^2}{\sin^4 \Theta_W} \frac{3\beta}{32\pi s} \frac{m_t^2 - 2(p_t(p_6) p_7(p_6))}{m_t \Gamma_t m_W \Gamma_W} \frac{E_{b(\bar{b})}^2}{m_t^2 - m_W^2} \frac{E_{l(\bar{l})}^2}{m_W^2} N_{\lambda \lambda'}^{l(i)}. \]  

(24)

Here \( E_{b(\bar{b})} \) and \( E_{l(\bar{l})} \) are the energies of the \( b \)-quarks and the final leptons in the
c.m.system:
\[ E_b = \frac{m_t^2 - m_W^2}{2E} \frac{1}{1 - \beta \cos \theta_{tb}}, \quad E_b = \frac{m_t^2 - m_W^2}{2E} \frac{1}{1 - \beta \cos \theta_{tb}} \] (25)

\[ E_{t^+} = \frac{m_W^2}{2 \left[ E(1 - \beta \cos \theta_{t^+}) - E_b(1 - \cos \theta_{tb}) \right]}, \] (26)

\[ E_{t^-} = \frac{m_W^2}{2 \left[ E(1 - \beta \cos \theta_{t^-}) - E_b(1 - \cos \theta_{tb}) \right]}, \] (27)

\( \sqrt{s} \) is the total c.m.energy, \( \beta = \sqrt{1 - 4m_t^2/s} \) is the velocity of the \( t \) quark,

\[ \cos \theta_{tb} = \frac{(p_t \cdot p_b)}{|p_t| |p_b|} = \sin \theta_t \sin \theta_b \cos \phi_b + \cos \theta_t \cos \theta_b, \quad \text{etc.} \] (28)

We take \( m_b = 0 \). \( \Gamma_{t \to bW} \) is the partial decay width of the top quark for the decay \( t \to bW \):

\[ \Gamma(t \to bW) = \frac{G_F m_t^3}{8\sqrt{2}\pi} \left( \frac{m_t^2 - m_W^2}{m_t^2} \right)^2 \frac{2m_t^2 + 2m_W^2}{m_t^2} |V_{tb}|^2, \] (29)

where \( V_{tb} \) is the corresponding element in the CKM mixing matrix. In (23) and (24) we take \( \Gamma_{t \to bW}/\Gamma_t = 1 \). We use the notation:

\[ N_{\lambda \lambda'}^{(t)} = (1 + \beta^2 \cos^2 \theta_{t(t)}) F_1 + (1 - \beta^2) F_2 \pm 2 \beta \cos \theta_{t(t)} F_3. \] (30)

The dependence on the beam polarizations comes through the functions \( F_i, i = 1, 2, 3 \), given by

\[ F_i = (1 - \lambda \lambda') F_i^0 + (\lambda - \lambda') G_i^0 \] (31)

where

\[
\begin{align*}
F_{1,2}^0 & = \frac{4}{3} - \frac{4}{3} c_V g_Y h_Z + (c_Y^2 + c_A^2) (g_Y^2 \pm g_A^2) h_Z^2 \\
G_{1,2}^0 & = \frac{4}{3} c_A g_Y h_Z + 2 c_V c_A (g_Y^2 \pm g_A^2) h_Z^2 \\
F_{3}^0 & = g_A h_Z + 4 c_V c_A g_Y g_A h_Z^2 \\
G_{3}^0 & = -\frac{4}{3} c_V g_A h_Z + 2 (c_Y^2 + c_A^2) g_Y g_A h_Z^2
\end{align*}
\] (32)

The quantities \( c_V = -(1/2) + 2 \sin^2 \Theta_W \), and \( c_A = (1/2) \) are the SM couplings of \( Z \) to the electron, \( h_Z = [s/(s - m_Z^2)] / \sin^2 2\Theta_W \).
3 The polarization vector

The amplitude for $e^+e^- \rightarrow t\bar{t}$, assuming CP violation, is

$$\mathcal{M} = i \frac{e^2}{s} \bar{v}(q_e) \gamma_\mu u(q_e) (\mathcal{V}^\gamma)_\mu - i \frac{g_2^2}{s - m_Z^2} \bar{v}(q_e) \gamma_\mu (c_V + c_A \gamma^5) u(q_e) (\mathcal{V}^Z)_\mu .$$  \hspace{1cm} (33)

Now we will give the expressions [5] for the polarization four–vectors $\xi^t_\mu$ and $\xi^l_\mu$ including the dependence on the electric and weak dipole moment form factors. $\xi^{t,\bar{t}}$ determine the spin density matrices of the decaying $t$ and $\bar{t}$ quarks:

$$\rho(p_t) = \frac{1}{2} (1 + \gamma^5) \Lambda(p_t), \quad \rho(-p_{\bar{t}}) = -\frac{1}{2} (1 + \gamma^5) \Lambda(-p_{\bar{t}}),$$

$$\Lambda(p_t) = \sum_r u_r(p_t) \bar{u}_r(p_t) = (\slashed{p}_t + m_t) .$$

As $(p_t \xi) = 0$, in general the polarization vector $\xi^t_\mu$ can be decomposed covariantly along three independent four–vectors orthogonal to $p_t$: two of them, $Q^t_\mu$ and $Q^\mu_\mu$ are in the production plane:

$$Q^t_\mu = q^t_\mu - \frac{(p_t q_e)}{m_t^2} p^\mu_t, \quad Q^\mu_\mu = q^t_\mu - \frac{(p_t q_e)}{m_t^2} p^\mu_t$$  \hspace{1cm} (34)

and the third one is normal to it: $\varepsilon_{\mu\alpha\beta\gamma} p^\alpha_t q^\beta_e q^\gamma_\mu$. Most generally, we can write:

$$\xi^t_\mu = P^t_{e}(Q_e)_\mu + P^t_{\bar{e}}(Q_{\bar{e}})_\mu + D^t \varepsilon_{\mu\alpha\beta\gamma} p^\alpha_t q^\beta_e q^\gamma_\mu .$$  \hspace{1cm} (35)

The components $P^t_{e(\bar{e})}$ get contributions from both SM and CP violating terms. The SM at tree level does not contribute to the normal component $D^t$. Thus we have:

$$P^t_{e(\bar{e})} = P^t_{e(\bar{e})}(SM) + P^t_{e(\bar{e})}(CR), \quad D^t = D^t(CP) .$$  \hspace{1cm} (36)

The polarization 4-vector is determined by the expression [23]

$$\xi^t_\mu = \left( g_{\mu\nu} - \frac{p_\mu p_\nu}{m_t^2} \right) \frac{\text{Tr}[\mathcal{M} \bar{\Lambda}(p_t) \mathcal{M} \Lambda(p_t) \gamma^{\nu} \gamma^{5}]}{\text{Tr}[\mathcal{M} \Lambda(p_t) \mathcal{M} \Lambda(p_t)]}$$  \hspace{1cm} (37)

where $\mathcal{M}$ is the amplitude [23]. The projection operator $(g_{\mu\nu} - m_t^{-2} p_\mu p_\nu)$ guarantees the condition $(\xi p_t) = 0$. In the c.m.system the SM contribution to $P^t_{e(\bar{e})}(SM)$ at tree-level is

$$P^t_e(SM) = \frac{2m_t}{s} \frac{1}{N_{\lambda\lambda'}^{t}} \left[(1 - \beta \cos \theta_t)(G_1 - G_3) + (1 + \beta \cos \theta_t)G_2 \right]$$  \hspace{1cm} (38)

$$P^t_{\bar{e}}(SM) = -\frac{2m_t}{s} \frac{1}{N_{\lambda\lambda'}^{t}} \left[(1 + \beta \cos \theta_t)(G_1 + G_3) + (1 - \beta \cos \theta_t)G_2 \right]$$  \hspace{1cm} (39)
are C–odd and CP–odd, while \( \xi \) are C–even and P–odd functions of the coupling constants in the production process \( e^+e^- \to \bar{t}t \). This implies that \( H_i \) are C–odd and CP–odd, while \( D_i \) are P–odd and CP–odd quantities.

The polarization four–vector \( \xi^t \) for the anti–top is obtained through C–conjugation. This leads to the following replacements in the expressions for \( \xi^t \), \( F_i \), \( G_i \), \( H_i \), and \( D_i \):

\[
p_t \to p_{\bar{t}}, \quad (2/3)e \to -(2/3)e, \quad g_V \to -g_V, \quad d^{\gamma,Z}(s) \to -d^{\gamma,Z}(s).
\]

\( H_i = (1 - \lambda \lambda') H_i^0 + (\lambda - \lambda') D_i^0 \)

where

\[
H_1^0 = \left( \frac{5}{3} - c_V g_V h_Z \right) d^\gamma(s) - \left( \frac{5}{3} c_V h_Z - (c_V^2 + c_A^2) g_V h_Z^2 \right) d^Z(s)
\]

\[
H_2^0 = h_Z d^\gamma(s) + 2 c_V c_A g_A h_Z^2 d^Z(s)
\]

\[
D_1^0 = -c_A g_V h_Z d^\gamma(s) - \left( \frac{5}{3} c_A h_Z - 2 c_V c_A g_V h_Z^2 \right) d^Z(s)
\]

\[
D_2^0 = -c_V g_A h_Z d^\gamma(s) + \left( c_V^2 + c_A^2 \right) g_A h_Z^2 d^Z(s).
\]
We have:

$$\xi^t_\mu = P^t_e(Qe)_\mu + P^t_e(Q\bar{e})_\mu + D^t_{\varepsilon\mu\beta\gamma} p^\alpha_\mu q^\beta_\alpha q^\gamma_\beta.$$  \hspace{1cm} (48)

where

$$\bar{Q}_e^\mu = q^\mu_\mu - \frac{(p\mu q_e)}{m_t^2} p^\mu_\mu, \quad \bar{Q}_{\bar{e}}^\mu = q^\mu_\mu - \frac{(p\mu q_{\bar{e}})}{m_t^2} p^\mu_\mu.$$  \hspace{1cm} (49)

In analogy to eq.(36) we define:

$$P^t_e = P^t_e(SM) + P^t_e(CP), \quad D^t = D^t(CP).$$  \hspace{1cm} (50)

and obtain:

$$P^t_e(SM) = \frac{2m_t}{s} \frac{1}{N^1_{\lambda\lambda'}}[(1 - \beta \cos \theta_t)(G_1 + G_3) + (1 + \beta \cos \theta_t)G_2]$$  \hspace{1cm} (51)

$$P^t_e(SM) = -\frac{2m_t}{s} \frac{1}{N^1_{\lambda\lambda'}}[(1 + \beta \cos \theta_t)(G_1 - G_3) + (1 - \beta \cos \theta_t)G_2]$$  \hspace{1cm} (52)

$$P^t_e(CP) = -\frac{1}{m_t N^1_{\lambda\lambda'}}[(1 + \beta \cos \theta_t - \beta^2 \sin^2 \theta_t) \Im H_1 + (\beta \cos \theta_t + \beta^2) \Im H_2]$$  \hspace{1cm} (53)

$$P^t_{\bar{e}}(CP) = -\frac{1}{m_t N^1_{\lambda\lambda'}}[(1 - \beta \cos \theta_t - \beta^2 \sin^2 \theta_t) \Im H_1 + (\beta \cos \theta_t - \beta^2) \Im H_2]$$  \hspace{1cm} (54)

$$D^t(CP) = \frac{8}{s} \frac{1}{m_t s N^1_{\lambda\lambda'}}[\Re D_1 - \beta \cos \theta_t \Re D_2]$$  \hspace{1cm} (55)

The expressions for the cross sections of (1) and (2) are naturally expressed in terms of the dimensionless combinations:

$$P^{t(\pm)}_e = \frac{s}{m_t} (P^t_e - P^t_{\bar{e}}) = \mathcal{P}^{t(\pm)}_e(SM) + \mathcal{P}^{t(\pm)}_e(CP),$$

$$D^{t(\pm)}_e = sm_t D^{t(\pm)}_e = \mathcal{D}^{t(\pm)}_e(CP)$$  \hspace{1cm} (56)

4 The process $e^+ e^- \rightarrow bX$

4.1 The differential cross section

In this section we summarize the results of [5] and [6]. Using the explicit expressions eqs.(35) and (48) for the top and the anti–top quark polarization four–vectors we obtain from (33) the analytic formula for the cross sections of (1) in the
where $\hat{q}$ and $\hat{p}$ are unit 3-vectors in the direction of the particles. $\sigma_{b(\bar{b})}^0$ is given in eq.(23). In (57) we have kept only the dependence on the electro weak dipole moment form factors and neglected CP violation in the decay of the top quark.

4.2 The angular distributions of $b$ and $\bar{b}$ quarks

Integrating (57) over $\cos \theta_{t(i)}$ and $\varphi_{b(\bar{b})}$ we obtain the $\cos \theta_{b(\bar{b})}$–distribution of the $b(\bar{b})$ quarks in the c.m.system:

$$d \sigma_{\lambda \lambda'}^{b(\bar{b})} = \sigma_{0}^{b(\bar{b})} (\lambda, \lambda') \left\{ 1 (\pm) \alpha_{b} \frac{m_{t}^{2}}{m_{t}^{2} - m_{W}^{2}} \frac{E_{b(\bar{b})}}{\sqrt{s}} \left[ \mathcal{P}_{+}^{t(i)} \left( 1 - \frac{1 - \beta \cos \theta_{t(i)}}{1 - \beta^{2}} \right) \\ - \mathcal{P}_{-}^{t(i)} \left( \cos \theta_{t(i)} - \beta \cos \theta_{b(\bar{b})} \right) \right] \\ + \frac{s \beta}{2m_{t}^{2}} \mathcal{P}_{t(i)}^{t(i)}(\hat{q}_{t(t)} \hat{p}_{b(\bar{b})}) \right\} \, d \cos \theta_{b(\bar{b})} d \Omega_{b(\bar{b})}, \quad (57)$$

The two independent combinations of the dipole moment form factors $H_{1}$ and $H_{2}$ enter (58), which implies that studying the angular distribution of the $b$ and $\bar{b}$ quarks one can obtain information about both $\Im m H_{1}$ and $\Im m H_{2}$.

These formulae coincide with the analogous SM expressions obtained in [21] for the unpolarized $e^{+}e^{-}$ and with [24] for polarized $e^{+}e^{-}$.
4.3 The energy distributions of $b$ and $\bar{b}$ quarks

Using (57) it is straightforward to obtain the energy distribution of the $b$ and $\bar{b}$ quarks if one moves to the frame where the $z$–axis points into the direction of the top quarks [6]:

$$
\frac{d\sigma_{\lambda\lambda'}^{b(\bar{b})}}{dx_{b(\bar{b})}} = \frac{\pi\alpha_{em}^2}{s} \frac{m_t^2}{m_t^2 - m_W^2} \left(c_0^{b(\bar{b})} + c_1^{b(\bar{b})} x_{b(\bar{b})}\right)
$$

(59)

where

$$
c_i^b = c_i^{SM} + c_i^{CP}, \quad c_i^\bar{b} = c_i^{SM} - c_i^{CP}
$$

$$
c_0^{SM} = N_{tot} + 4\alpha_b G_3, \quad c_0^{CP} = 8\alpha_b \Im m H_1,
$$

$$
c_1^{SM} = -8\alpha_b \frac{m_t^2}{m_t^2 - m_W^2}, \quad c_1^{CP} = -16\alpha_b \frac{m_t^2}{m_t^2 - m_W^2} \Im m H_1.
$$

We have used the conventional dimensionless energy variables $x_{b(\bar{b})} = \frac{2E_{b(\bar{b})}}{\sqrt{s}}$ and the notation

$$
N_{tot} = (3 + \beta^2)F_1 + 3(1 - \beta^2)F_2.
$$

(60)

Note that the linear behaviour of the spectra is introduced only by the top polarization – both $c_i^{SM}$ and $c_i^{CP}$ are proportional to $\alpha_b$. This may serve as a good analyser of the spin of the top quark in SM [24]. Studying the energy distributions one cannot obtain information about $d^\gamma$ and $d^Z$ independently – only one combination $H_1$ enters the CP violating terms $c_{0,1}^{CP}$ in (59).

4.4 CP violating asymmetries

The electroweak dipole moment form factors $d^{\gamma,Z}(s)$ have both real and imaginary parts. To obtain information about the dipole moment form factors from the differential cross section is a difficult task and it acquires very high precision of measurements. In the following we consider different integral observables, sensitive to $\Re d^{\gamma,Z}(s)$ and to $\Im d^{\gamma,Z}(s)$ separately.

i) $\Im m d^{\gamma,Z}(s)$ can be measured both by the angular-$\cos\theta_b$ and the energy asymmetries.

CP invariance for the angular distribution (58) implies:

$$
\frac{d\sigma^{b}_{\lambda\lambda'}(\cos\theta_b = \cos\theta)}{d \cos\theta_b} = \frac{d\sigma^{b}_{-\lambda'-\lambda}(\cos\theta_b = \pi - \cos\theta)}{d \cos\theta_b}.
$$

(61)
Note that in this equation and in the following ones, the first lower index of $d\sigma^b$ and $d\bar{\sigma}^b$ denotes the degree of longitudinal polarization of the electron beam and the second one that of the positron beam.

Let $\sigma_F^{(b)}(\lambda, \lambda')$ and $\sigma_F^{(\bar{b})}(\lambda, \lambda')$ denote the cross section of $b$ and $\bar{b}$ produced in the forward and backward hemispheres, respectively. Let $A_{FB}^{(b)}(\lambda, \lambda')$ is the standard forward–backward asymmetries for the $b$ and $\bar{b}$ quarks:

$$A_{FB}^{(b)}(\lambda, \lambda') = \frac{\sigma_F^{(b)}(\lambda, \lambda') - \sigma_B^{(b)}(\lambda, \lambda')}{\sigma_F^{(b)}(\lambda, \lambda') + \sigma_B^{(b)}(\lambda, \lambda')}.$$ 

(62)

Then we define the CP violating asymmetry $A_{FB}^E$:

$$A_{FB}^E = A_{FB}^{b}(\lambda, \lambda') + A_{FB}^{\bar{b}}(-\lambda', -\lambda) = -12\alpha_b \Im \frac{3mH_2}{N_{tot}}.$$ 

(63)

Other CP violating angular asymmetries, including also the general analytic expressions for their dependence on the experimental cuts are presented in [5].

CP invariance for the energy spectra of $b$ and $\bar{b}$ implies:

$$\frac{d\sigma_F^{b}(x_b = x)}{dx_b} = \frac{d\sigma_F^{\bar{b}}(x_b = x)}{dx_b}.$$ 

(64)

The corresponding integrated energy observable $A_{E}^E$, indicating CP violation is

$$A_{E}^{E} = R_{b}(\lambda, \lambda') - R_{\bar{b}}(-\lambda', -\lambda) = -4\alpha_b \beta \Im \frac{3mH_1}{N_{tot}},$$ 

(65)

where

$$R_{b}(\lambda, \lambda') = \frac{N^{b}(x > x_0, \lambda, \lambda') - N^{b}(x < x_0, \lambda, \lambda')}{N^{b}(\lambda, \lambda')} \equiv \frac{\Delta N^{b}(\lambda, \lambda')}{N^{b}(\lambda, \lambda')}$$

$$x_0 = \frac{x_{min} + x_{max}}{2}, \quad x_{min} = \frac{2(m_t^2 - m_W^2)}{s(1 + \beta)}, \quad x_{max} = \frac{2(m_t^2 - m_W^2)}{s(1 - \beta)},$$

(66)

$N^{b}(x > x_0, \lambda, \lambda')$ is the number of $b(\bar{b})$ quarks with $x > x_0$ for beam polarizations $\lambda$, $\lambda'$. $N^{b}(\lambda, \lambda')$ is the total number of $b$ ($\bar{b}$) quarks (the total cross section) of (60):

$$N^{b}_{tot} = N^{\bar{b}}_{tot} = \frac{\pi \alpha_{em}^2}{s} \beta \frac{\Gamma_{t \rightarrow MV}}{\Gamma_t} N_{tot},$$

(67)

$N_{tot}$ is given by (60).

The electroweak dipole moment form factors $d^\gamma Z(s)$ enter two combinations $H_1$ and $H_2$. The asymmetries $A_{FB}$ and $A_{E}$ provide two independent measurements
of their imaginary parts and thus of $\Im m d^\gamma$ and $\Im m d^Z$. Through $H_1$ and $H_2$ these asymmetries depend on the beam polarization that can strongly enhance (or decrease) the effects of CP violation we are interested in. Measurements performed with opposite beam polarizations can be used to disentangle $H_i^0$ from $D_i^0$. In analogy to the standard forward-backward asymmetries (62) we define the following polarization asymmetries:

$$P_{FB}^{b(\bar{b})} = \frac{(1 - \lambda \lambda') \cdot (\sigma_F^{b(\bar{b})} - \sigma_B^{b(\bar{b})})(\lambda, \lambda') - (\sigma_F^{b(\bar{b})} - \sigma_B^{b(\bar{b})})(-\lambda, -\lambda')}{(\sigma_F^{b(\bar{b})} + \sigma_B^{b(\bar{b})})(\lambda, \lambda') + (\sigma_F^{b(\bar{b})} + \sigma_B^{b(\bar{b})})(-\lambda, -\lambda')}.$$  (68)

Then the CP violating asymmetry is

$$P_{FB} = P_{FB}^b + P_{FB}^{\bar{b}} = -12 \alpha_b \beta \Im m D_0^0 \frac{3 \Im m D_1^0}{N_{tot}^0}.$$  (69)

where

$$N_{tot}^0 = N_{tot}(\lambda = \lambda' = 0) = (3 + \beta^2) F_1^0 + 3(1 - \beta^2) F_2^0.$$  (70)

Analogously we define the polarization CP violating asymmetry for the energy spectra:

$$P_E = R_P^b - R_P^{\bar{b}} = -4 \alpha_b \beta \Im m D_1^0 \frac{3 \Im m D_1^0}{N_{tot}^0},$$  (71)

where

$$R_P^{b(\bar{b})} = \frac{(1 - \lambda \lambda') \cdot \Delta N^{b(\bar{b})}(\lambda, \lambda') - \Delta N^{b(\bar{b})}(-\lambda, -\lambda')}{N_{tot}^{b(\bar{b})}(\lambda, \lambda') + N_{tot}^{b(\bar{b})}(-\lambda, -\lambda')}.$$  (72)

ii) The real parts of $d^\gamma, Z(s)$ can be singled out by measuring triple product correlations [8, 18]. A suitable asymmetry is given by [4]

$$O_{b(\bar{b})} = \frac{N[[q_e p_t \bar{p}_b] > 0] - N[[q_e p_t \bar{p}_b] < 0]}{N[[q_e p_t \bar{p}_b] > 0] + N[[q_e p_t \bar{p}_b] < 0]},$$  (73)

where $N[[q_e p_t \bar{p}_b] > 0(< 0)]$ are the number of $b$ quarks produced above/below the production–plane $\{q_e, p_t\}$ at a given polarization $\lambda, \lambda'$.

As in general $O_{b(\bar{b})}$ gets also CP invariant contributions from absorptive parts in the SM amplitude, the truly CP violating contribution will be singled out through the difference:

$$O_{b(\bar{b})}^{CP}(\lambda \lambda') = O_{b(\bar{b})} - O_{b(\bar{b})}^{\lambda \lambda'} = -\alpha_b \frac{3 \beta \pi \sqrt{s} \Im m D_1}{{2m_t^2} N_{tot}^0}.$$  (74)
In the above equation $O_i^\bar{b}$ refers to process (1) when the $\bar{t}$ decays. A non-zero value of (74) would imply CP violation in the $t\bar{t}\gamma$ and/or $t\bar{t}Z$ vertices.

For the above asymmetries we have obtained expressions in which all phase space integrations have been performed analytically.

5 The process $e^+e^- \rightarrow blX$

5.1 The differential cross section

From (18) and the expressions for $\xi^t$ and $\xi^\bar{t}$ we can obtain the differential cross section $d\sigma^\pm$ of processes (2) in the c.m. system. If we keep in (18) only the components of $\xi^t$, $\xi^\bar{t}$ CP that are normal to the production plane, i.e. the terms proportional to $D^t$ and $D^\bar{t}$ in eqs. (35) and (48), we shall obtain the dependence of $d\sigma^\pm$ on the triple product correlations of type (10). In the c.m. system we have [9]:

$$d\sigma^\pm = \sigma^\pm_{SM} \{ 1 + \frac{1}{1 - \beta(\hat{p}_t\hat{p}_l^\pm)} \left[ (\hat{q}_e\hat{p}_t\hat{p}_l^\pm) C_{1}^\pm + (\hat{q}_e\hat{p}_t\hat{p}_b) C_{2}^\pm + (\hat{p}_t\hat{p}_t^\pm) C_{3}^\pm + (\hat{q}_e\hat{p}_l^\pm) C_{4}^\pm \right] \} d\Omega_t d\Omega_b d\Omega_l$$

(75)

where $\hat{q}_e$, $\hat{p}_t$, etc. denote the corresponding unit 3-vectors. $\sigma^\pm_{SM}$ determines the expression for the SM cross sections of (2):

$$\sigma^\pm_{SM}(\lambda, \lambda') = \sigma^\pm_{0}(\lambda, \lambda') A^\pm_{SM}.$$  

(76)

The expressions for $\sigma^\pm_0$ and $A^\pm_{SM}$ are given by (24) and (19). For the functions $C_i$ we obtain [9]:

$$C_{1}^\pm = \mp \beta \left[ \frac{\mathcal{P}_t^{(t)}(CP)}{2} - \frac{m_t}{\sqrt{s}} E_{b(b)} \mathcal{P}_t^{(t)}(SM) \frac{\text{Reg}_{CP}}{m_W} \right]$$

(77)

$$C_{2}^\pm = \mp \beta \frac{m_t}{\sqrt{s}} E_{b(b)} \mathcal{P}_t^{(t)}(SM) \frac{\text{Reg}_{CP}}{m_W}$$

(78)

$$C_{3}^\pm = \mp \beta \frac{m_t}{\sqrt{s}} E_{b(b)} \mathcal{P}_t^{(t)}(SM) \frac{\text{Reg}_{CP}}{m_W}$$

(79)

$$C_{4}^\pm = \pm \frac{m_t}{\sqrt{s}} E_{b(b)} \mathcal{P}_t^{(t)}(SM) \frac{\text{Reg}_{CP}}{m_W}$$

(80)

2Note the i in the definitions of $d\gamma(s)$ and $dZ(s)$ in (3) and (4) that leads to the appearance of the real form factors in $C_i$ in stead of the imaginary ones in [9].
where $P_{\pm i}$ and $D_{\pm i}$, introduced previously in (56) are given by

$$P_{+ i}(SM) = -\frac{4}{N_{\lambda\lambda'}} \left[ \pm G_3 + \beta \cos \theta_{t(i)} (G_1 - G_2) \right]$$

$$P_{- i}(SM) = \frac{4}{N_{\lambda\lambda'}} \left[ G_1 + G_2 \pm \beta \cos \theta_{t(i)} G_3 \right]$$

$$D_{+ i}(CP) = \frac{8}{N_{\lambda\lambda'}} \left[ D_1 \pm \beta \cos \theta_{t(i)} D_2 \right] .$$

The result of (77) - (80) can be easily understood. The correlations $(\hat{q}_l \hat{p}_t \hat{p}_l)$ and $(\hat{q}_l \hat{p}_t \hat{p}_b)$ contain the production and $t$-decay planes, and thus CP violation from both the production and the decay may contribute to $C_1$ and $C_2$. The triple products $(\hat{p}_t \hat{p}_l \hat{p}_b)$ and $(\hat{q}_l \hat{p}_l \hat{p}_b)$ contain only the decay plane, and thus CP violation only in the decay vertex may enter $C_3$ and $C_4$. From the explicit expressions for $C_i$, one can see however that $C_2$ gets no contribution from the production process. This is a result of a cancelation due to the same $V-A$ form of the $tbW$ and $l\nu W$ vertices.

The terms proportional to $\Re q^R_{t\alpha}$ always enter multiplied by the SM-polarization and the kinematic factor $E_b m_t / \sqrt{s}$. Consequently $C_1$, the only term that contains $\Re \epsilon_\gamma$, $\Re \epsilon^Z$ is the dominant one.

The $\cos \theta_l$-distribution of the decaying leptons, that will depend on $\Im m_{\gamma,Z}$ was obtained in [11]. The analytic expression is the same as that for the $\cos \theta_b$-distribution, eq.(58), but for the replacement $\alpha_b \rightarrow \alpha_l$, $\Gamma_{t \rightarrow bW} \rightarrow \Gamma_{t \rightarrow b\nu}$. This can be understood having in mind the same $\gamma_\alpha(1 - \gamma_5)$ form of the $tbW$ and $l\nu W$ vertices.

Analytic expressions for the energy distribution of the secondary leptons $l^+$ and $l^-$ in case of CP violation in the production process were obtained in [12, 13] and later, including also CP violation in the decay in [14]. Observables sensitive to CP violation in the production and the decay are discussed in [14].

5.2 Triple product correlations

With the set of triple products $T_1 = (q_e p_t p_{l^+})$, $T_2 = (q_e p_t p_b)$, $T_3 = (p_t p_{l^+} p_b)$, and $T_4 = (q_e p_{l^+} p_b)$, we define the following observables for processes (2):

$$O'_i = \frac{N[T_i > 0] - N[T_i < 0]}{N[T_i > 0] + N[T_i < 0]}$$

where $N[T_i > 0(< 0)]$ is the number of events in which $T_i > 0(< 0)$. For example a nonzero value of $O'_1$ would mean that there is a difference between the number of
events in which \( l^+ \) are above and bellow the production plane \((q_e, p_t)\).

As \( O_t^i \) are T-odd asymmetries, they get contributions from final state interactions, too. The truly CP violating observables are the differences:

\[
O_{CP}^i = O_t^i - O_{\bar{t}}^i, \quad i = 1, 2, 4 \quad \text{and} \quad O_{CP}^3 = O_3^t + O_3^{\bar{t}},
\]

where \( O_t^i \) refer to process (2) when the \( t \)-quark decays, and \( O_{\bar{t}}^i \) refer to process (2) when \( \bar{t} \) decays.

As the considered triple products are not orthogonal, each observable \( O_{CP}^i \) will get in general contributions from all functions \( C_k, k = 1, 2, 3, 4 \). From the explicit expressions (75)-(80) one can show that \( C_1 \) enters the asymmetries \( O_{CP}^1, O_{CP}^2 \) and \( O_{CP}^4 \), and being the dominant contribution it determines their magnitude and sensitivity to CP violation in the production process. To \( O_{CP}^3 \) the terms \( C_3 \) and \( C_4 \) contribute, and this implies that it is sensitive to CP violation in the decay process only. Therefore, a nonzero value of \( O_{CP}^1, O_{CP}^2 \) and \( O_{CP}^4 \) will be an indication of CP violation in the production plane, and \( O_{CP}^3 \) will measure CP violation in the decay. However, because of the suppression factors (\( t \)-polarization and kinematics) \( O_3 \) will be too small to measure CP violation. This model independent analysis was confirmed by our numerical results performed in the MSSM \( \Box \).

### 6 CP violation in the \( t \) decay vertex

The matrix elements for \( t \to bW^+ \) and \( \bar{t} \to \bar{b}W^- \) in case of CP violation are:

\[
M_t = \bar{u}(p_b)V^t_\alpha u(p_t)\epsilon^\alpha(p_{W^+}) \quad \text{and} \quad M_{\bar{t}} = \bar{u}(p_b)V^{\bar{t}}_\alpha u(p_{\bar{t}})\epsilon^\alpha(p_{W^-}),
\]

where \( V^{t,\bar{t}} \) are given by (\( \Box \)) and (\( \Box \)). As shown in Sect. 2, eqs. (refsigmab) and (18) imply that the energy and angular distributions of the \( b \) quarks and the leptons, including the possible triple product correlations are actually sensitive to CP violation in the production process only. CP violation in the decay \( t \)-quark vertex leads to a nonzero value between the partial decays widths of \( t \to bW \) and \( \bar{t} \to \bar{b}W \) – the asymmetry \( A_{CP} \), eq.(9). From the interference of the tree level amplitude and the loop corrections containing the terms \( f_L \) and \( g_R \), we obtain (10):

\[
A_{CP} = 2 \left[ 3m f_L^{CP} + 3m g_R^{CP} \frac{m_t(m_t^2 - m_W^2)}{m_W(m_t^2 + 2m_W^2)} \right].
\]
In processes (1) and (2) this quantity can be measured by the asymmetries $\Delta^b$ and $\Delta^l$ respectively:

$$\Delta^b = \frac{N^b_{tot}(\lambda, \lambda') - N^\bar{b}_{tot}(-\lambda', -\lambda)}{N^b_{tot}(\lambda, \lambda') + N^\bar{b}_{tot}(-\lambda', -\lambda)} = A_{CP},$$

$$\Delta^l = \frac{N^+_{tot}(\lambda, \lambda') - N^-_{tot}(-\lambda', -\lambda)}{N^+_{tot}(\lambda, \lambda') + N^-_{tot}(-\lambda', -\lambda)} = A_{CP}. \tag{85}$$

Here $N^{b(\bar{b})}_{tot}$ and $N^{\pm}_{tot}$ are the total number of $b(\bar{b})$ quarks produced in (1), and of $l^\pm$ produced in (2). $\Delta^b$ and $\Delta^l$ measure CP violation in the $t$-decay vertex only, irrespectively of CP violation in the production process. Note that if $t \to bW$ is the only decay mode of the $t$-quarks, as it is actually in SM, $\Delta^b$ would be zero by the CPT theorem.

7 Numerical results in MSSM

Up to now our expressions for the asymmetries were general and model independent. Here we shall give numerical results for the CP violating asymmetries in the Minimal Standard Supersymmetric Model (MSSM).

In the SM CP violation appears through the phase of the CKM mixing matrix only if the three generations of quarks mix. This contribution is small, restricted by the unitarity condition on the mixing matrix. Further, again due to unitarity the dipole moment form factors $d^\gamma$ and $d^Z$ are at least two-loop effect and hence of academic interest only. The contribution of the self-energy loop in SM to the CP violating vertex was also shown to be extremely small [17].

In the Lagrangian of the MSSM additional complex couplings are introduced [4] that lead to CP violation within one generation only. This leads to CP violation at one loop, free of the unitary suppression. As the masses of the SUSY particles are not expected to be much heavier than the mass of the top quark, the radiative corrections through which the CP violating form factors are induced will not be strongly suppressed by the masses of the particles in the loop.

There are two physical complex couplings in the MSSM Lagrangian – the parameter $\mu$ in front of the Higgsino mass term, and the dimensionless parameter $A_f$ in the soft SUSY breaking piece of the Lagrangian. The magnitude of the CP violating form factors depend strongly on the phases of these parameters. The
parameter $A_f$ depends on flavour. Measurements of the electric dipole moment (EDM) of the neutron put constraints on some of these phases [25]. This is the so called supersymmetric fine-tuning CP problem – usually one concludes that either the phases involved in the EDM of the neutron are very small or the masses of the first generation of the squarks are in the TeV region. A complete analysis of the constraints on the SUSY parameters from measurements of the EDM’s of the neutron and the electron was done in [26]. Using supergravity with grand unification (GUT) there are attempts [27] to constrain also the phases of $A_t$ and $A_b$ that enter the CP violating form factors of the top quark. For our numerical analysis we shall not make any additional assumptions about GUT except unification of the gauge couplings, i.e. we do not assume unification of the scalar mass parameters and the parameters $A_f$. As the mechanism of SUSY breaking is not known, an unambiguous decision about CP violation in SUSY will be provided by experiment.

7.1 The asymmetries due to $d^\gamma$ and $d^Z$

In order to estimate the observables sensitive to CP violation in the production process we have used the results for the electro weak dipole moment form factors $d^\gamma$ and $d^Z$ as obtained in [22], where a complete analysis was performed with gluino, charginos and neutralinos exchanged in the loops.

There are two types of observables – sensitive to $\Re d^\gamma, d^Z$ and to $\Im d^\gamma, d^Z$. As an illustration, on Fig.1 and Fig.2 the values of $A^{FB}_{\lambda \lambda'}$, eq (53), and the values of $O^{CP}_b$, eq. (74), as functions of the c.m. energy $\sqrt{s}$ are shown. The asymmetry $A^{FB}_{\lambda \lambda'}$ is determined by the dependence on $\sqrt{s}$ of $\Im d^\gamma, d^Z$, and the asymmetry $O^{CP}_b$ – by $\Re d^\gamma, d^Z$. The figures are presented for the following values of the SUSY parameters: $M = 230$ GeV, $|\mu| = 250$ GeV, $m_{t_1} = 150$ GeV, $m_{t_2} = 400$ GeV, $m_{b_1} = 270$ GeV and $m_{b_2} = 280$ GeV. The following GUT relations between the gaugino mass parameters have been assumed: $m_{\tilde{g}} = (\alpha_s/\alpha_2) M \approx 3M$ and $M' = (5/3) \tan^2 \Theta_W M$.

Clearly seen are the spikes in the $\sqrt{s}$ behaviour of $A^{FB}_{\lambda \lambda'}$ and $O^{CP}_b$. They are due to the thresholds of the intermediate particles in the loop and are already present in the dipole moment form factors $\Im d^\gamma, d^Z$ and $\Re d^\gamma, d^Z$ as discussed in detail in [22]. Their position is determined by the spectra of the particles in the loop, their magnitude – by the strength of the couplings. For example, the spikes at $\sqrt{s} = 400$GeV and $\sqrt{s} = 590$GeV are due to the thresholds of $\tilde{\chi}^+_1 \tilde{\chi}^-_1$ production with $m_{\chi^+_1} = 200$GeV and of $\tilde{\chi}^+_2 \tilde{\chi}^-_2$ production with $m_{\chi^+_2} = 295$ GeV, respectively. The
asymmetries depend strongly on the polarization of the electron and positron beams \(\lambda, \lambda'\). The figures show the asymmetries for different beam polarizations, taking \(\lambda = -\lambda' = (-0.8, 0, 0.8)\). Notice the strong dependence on the polarization of the electrons. For \(\sqrt{s} < 700 \text{GeV}\) the asymmetries are much bigger if the electrons are left handed. This is more clearly pronounced for \(\mathcal{O}'_{CP}\). In general the asymmetries are of the order of \(10^{-3}\).

### 7.2 The asymmetries due to \(f^L_{CP}\) and \(g^R_{CP}\)

The asymmetry \(A_{CP}\) is determined by \(\Im m f^L_{CP}\) and \(\Im m g^R_{CP}\) - eq. (84). In order that \(\Im m f^L_{CP}\) and \(\Im m g^R_{CP}\) are different from zero, we need at the same time, CP violating complex couplings in the Lagrangian, provided here by the MSSM, and non vanishing absorptive parts in the loop integrals. Since only the absorptive parts of the loop SUSY amplitude enter \(\Im m f^L_{CP}\) and \(\Im m g^R_{CP}\), the main contribution would come from diagrams in which one of the on–shell loop particles is the lightest SUSY particle – the neutralino \(\tilde{\chi}^0_1\). There are two such diagrams: with \((\tilde{\chi}^0_1 - \tilde{t}_1 - \tilde{\chi}^+_i)\) and with \((\tilde{t}_1 - \tilde{\chi}^0_1 - \tilde{b}_L)\) in the loop (\(\tilde{t}_n\) are the massive scalar-top states, \(\tilde{\chi}^+_i\) are the chargino states, the mass of the \(b\)–quark has been neglected). The present experimental bounds on the gluino and the scalar top masses forbids kinematically the diagram with \((\tilde{t}_L - \tilde{g} - \tilde{b}_L)\) in the loop that could lead to a big contribution [17]. Full expression of the contribution from the different diagrams was obtained in [16], where also numerical integration was performed. Detail analysis of the dependence on the SUSY mass parameters was carried out in [28].

The full expression is rather combursome, however a rather simple expression, that gives the order of magnitude of the effect can be obtained if only the diagram with \((\tilde{t}_1 - \tilde{\chi}^0_1 - \tilde{b}_L)\) is considered. For a very light neutralino, if we neglect the mixing between the gaugino components \(\tilde{W}_3\) and \(\tilde{B}\) as compared to that between the gaugino and Higgsino components \(\tilde{W}_3(\tilde{B})\) and \(\tilde{H}^0\), as suggested by the minimal supergravity models [29] and parametrize the possible imaginary couplings by introducing a single CP violating phase \(\sin \delta_{CP}\) we obtain [16]:

\[
A_{CP} \simeq -\frac{\alpha_{em}}{\sin^2 \Theta_W} \frac{\sqrt{2}}{4 \sin \beta} \frac{m_i^2}{m_{\tilde{t}_1}^2 + 2M_{\tilde{W}_3}^2} \times \frac{m_{\tilde{\chi}^0_1}}{M_W} \times \\
\ln \left| 1 - \frac{(m_i^2 - M_{\tilde{W}_3}^2)(m_i^2 - m_{\tilde{t}_1})}{m_{\tilde{t}_1}^2 m_{\tilde{b}_L}^2} \right| \sin \delta_{CP}. \quad (86)
\]
For maximal $CP$ violation ($\sin \delta_{CP} = 1$), $m_{\tilde{t}_L} = 150$ GeV and $m_{\tilde{\chi}^0} = 20$ GeV (near the experimental bound), we have:

$$A_{CP} \simeq 0.059 \times \frac{\alpha_{em}}{\sin^2 \Theta_W}, \text{ for } m_{\tilde{b}_L} = 200\text{GeV}$$

(87)

$$A_{CP} \simeq 0.026 \times \frac{\alpha_{em}}{\sin^2 \alpha \Theta_W} \text{ for } m_{\tilde{b}_L} = 300\text{GeV}.$$  

(88)

which is an asymmetry of the order of $10^{-3}$. This value is obtained also taking into account the constraints from the EDM's of the neutron and the electron [28].

8 Conclusions

CP violation in the $\gamma t\bar{t}$ and $Z t\bar{t}$ vertices in the production process $e^+ e^- \rightarrow t\bar{t}$, and CP violation in the $tbW$ vertex in the $t$-decays $t \rightarrow bW$ or $t \rightarrow bW \rightarrow bl^+\nu$ have been assumed. Studying the single $b$-quark and lepton distributions we have defined asymmetries that can disentangle CP violation in the production from CP violation in the decay. The angular and energy asymmetries are actually sensitive to CP violation in the production process only, CP violation in the decay being suppressed by the amount of the SM top quark polarization, for the secondary leptons it is suppressed also kinematically. Appropriate angular and energy asymmetries that determine independently the real and imaginary parts of the dipole moment form factors $d^\gamma(s)$ and $d^Z(s)$ are defined. CP violation in the decay can be measured through the difference between the total number of $b$ and $\bar{b}$ quarks or $l^+$ and $l^-$ from the decay of the $t$ and $\bar{t}$ quarks. Particular attention is paid to the polarization of the top quark.

Analytic expressions for the considered distributions and asymmetries are obtained. These expressions are general and independent on the definite model of CP violation. In the formula that involve only the $b$ quarks all phase space integrations have been carried out analytically. A numerical analysis of the asymmetries have been performed in the Minimal Standard Supersymmetric Model with complex phases. In this model the effects turn out to be of the order of $10^{-3}$. With the planned luminosities for the $e^+ e^-$ linear collider this is on the borderline of detectability. However other models of CP violation, for example the two-Higgs doublet model, can give much higher asymmetries [30]. Observation of the asymmetries discussed above will be interesting as it would be a definite signal of physics beyond the SM.
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Figure Captions

1. FIG. 1 The asymmetry $A_{\lambda\lambda'}^{FB}$ that measures $\Im m d_{\gamma,Z}$, eq. (63) as a function of $\sqrt{s}$ (GeV) for different beam polarizations: $\lambda' = -\lambda$; $\lambda = -0.8$ (dashed line), 0 (full line) and 0.8 (dotted line).

2. FIG. 2 The asymmetry $O_{\lambda\lambda'}^{CP}$ that measures $\Re e d_{\gamma,Z}$, eq. (74) as a function of $\sqrt{s}$ (GeV) for different beam polarizations: $\lambda' = -\lambda$; $\lambda = -0.8$ (dashed line), 0 (full line) and 0.8 (dotted line).
Fig. 1: $A^{FB}_{\lambda\lambda'}$
Fig. 2: $\mathcal{O}_b^{CP}(\lambda \lambda')$