Discriminating among theories of spiral structure using Gaia DR2

J. A. Sellwood,∗ Wilma H. Trick,† R. G. Carlberg,‡ Johanna Coronado,§ and Hans-Walter Rix,¶

1 Steward Observatory, University of Arizona, 933 N Cherry Ave, Tucson AZ 85722, USA
2 Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, D-85748 Garching b. München, Germany
3 Department of Astronomy and Astrophysics, University of Toronto, Toronto, ON M5S 3H4, Canada
4 Max-Planck-Institut für Astronomie, Königstuhl 17, D-69117 Heidelberg, Germany

1 INTRODUCTION

The second data release from the Gaia mission (Gaia collaboration: Brown et al. 2018) has revealed a much sharper image of the phase space distribution of nearby stars (Gaia collaboration: Katz et al. 2018), confirming the existence of multiple “stellar streams” passing through the solar neighbourhood. Follow-up studies (e.g. Famaey et al. 2007; Bensby et al. 2007; Bovy & Hogg 2010; Pompeia et al. 2011) of the smaller star samples in the Hipparcos data, as well as the GALAH survey (Quillen et al. 2018), have revealed that each stream contained stars of a range of abundances and ages, leading to the well-established conclusion that the phase-space structure was probably created by dynamical processes within the disc of the Milky Way. Dynamical models for some or all of these features have been presented in many papers (e.g. Dehnen 2000; Quillen 2003; De Simone et al. 2004; Quillen & Minchev 2005; Chakrabarty 2007; Antoja et al. 2009; Sellwood 2010; Grand et al. 2015; Hunt et al. 2018).

Trick, Coronado & Rix (2018) note one advantage of converting the distribution of stars in velocity space into the phase space is that they contain information about the entire orbit of the star and not just its instantaneous position. However, Sellwood (2010) and McMillan (2011) showed that the conjugate angles deduced from the Hipparcos data were not uniformly distributed, as would be expected if the local stellar distribution were well mixed, and these additional variables therefore also contain useful information. It should be noted that the transformation from position-velocity space to action-angle variables assumes first a local model for the radial variation the Galactic gravitational potential and second that the gravitational potential of the Galaxy is closely axisymmetric, although both these assumptions are needed only over the radial range explored by the orbit. We describe the evaluation of these variables in §2.

Our purpose in this paper is to use the Gaia data to discriminate as far as possible among theories for the origin of spiral patterns in the discs of galaxies. Inner Lindblad resonance (hereafter ILR) features are predicted to be strong in the theoretical picture presented by Sellwood & Carlberg...
(2014), and should be weak or absent in most other theories. In particular, the ILR should be protected by a “Q-barrier” in the theory of Bertin & Lin (1996), and the ILR does not feature prominently in the two other theories: the massive clumps invoked as drivers by Toomre & Kalnajs (1991) and whose non-linear evolution was studied in simulations by D’Onghia et al. (2013) or in theories of continuously shearing spiral arms (e.g. Grand et al. 2012a,b; Baba et al. 2013; Roca-Fàbrega et al. 2013; Michikoshı & Kokubo 2018).

We use the sample of nearby stars selected from Gaia DR2 by Trick, Coronado & Rix (2018), and employ the same local standard of rest and Galactic parameters, $R_0 = 8$ kpc and $V_0 = 220$ km s$^{-1}$. We select stars whose distance from the Sun, projected into the Galactic plane, is $< 200$ pc. We also discard stars having sufficient vertical energy to reach $z > 200$ pc, i.e. those for which $(z/v)^2 + v_z^2 > (0.2v)^2$, where $v = 70$ km s$^{-1}$ kpc$^{-1}$, $z$ is in kpc, $v_z$ is in km s$^{-1}$. These two cuts result in a final sample of 323 378 stars, all of which are sufficiently nearby for the inverse Gaia parallax to yield a reliable distance. Figure 1 shows that distance uncertainties are better than 1% for the vast majority of our stars.

Trick, Coronado & Rix (2018) highlighted a number of coherent features in the distribution of action-space that correspond to features seen in the more usual phase-space, $U$ and $V$. These features are widely believed to have been created by dynamical processes. Here we examine them further and, in particular, expand the study to include the additional information contained in the angles conjugate to the actions.

2 ACTION-ANGLE VARIABLES

For simplicity, we will consider motions of stars in the plane of the disc only and neglect the component of motion normal to the Galactic plane. Although some interesting features in the vertical motions have been found (e.g. Antoja et al. 2018; Bland-Hawthorn et al. 2018), this paper is focused on the in-plane part of the velocity distribution. We consider only stars whose vertical excursions are confined to $|z| < 200$ pc, the rapid vertical oscillations of such stars should be largely decoupled from their horizontal motion (Sellwood 2014). Thus we need consider just two actions, $J_R$ and $J_\theta$ and two conjugate angles $\theta_R$ and $\theta_\phi$.

The classical integrals, $E = \Phi(R) + \frac{1}{2}(v_R^2 + v_\theta^2)$ and $L_z = Rv_\phi$, can readily be estimated from the Gaia data, together with some adopted model for the axisymmetric gravitational potential, $\Phi(R)$, in the disc mid-plane. We compute the radial action from the approximate expression

$$J_R = \frac{1}{\pi} \int_{R_p}^{R_o} \dot{R}dR \quad \text{with} \quad \dot{R}^2 = 2[E - \Phi(R)] - \frac{L_z^2}{R^2}, \quad (1)$$

where $R_p$ and $R_o$ are respectively the Galactic peri- and apocentric distances of the star where its radial speed, $\dot{R} = 0$, and the integral is normalized by $\pi$ because the integral is over half a full radial oscillation. This expression yields very nearly the same value (McMillan 2011) for $J_R$ as obtained from the more-accurate “torus fitting” method that takes account of 3D motion. Physically, the radial action is an angular momentum-like variable that quantifies the magnitude of the radial motion of a star and, for disc stars, it is typically an order of magnitude smaller than the orbital angular momentum.

Note that expression (1) for $J_R$, and the evaluation of the angles variables described below, require multiple orbit integrations for each star. Each individual quadrature is fast, so that the computational cost of evaluating them for all 323 378 stars in our sample is not particularly burdensome. However, the implicit relations between the angles and the peculiar velocities that we will need to invert...
repeatedly require many millions more evaluations, making it computationally advantageous to choose the simple functional form, $\Phi(R) = V_0^2 \log(R/R_0)$ implying an exactly flat rotation curve, for the local radial variation of the Galactic gravitational potential. We have preferred this form over the Galaxy model proposed by Bovy (2015), even though the coordinate conversions can be effected quite efficiently using a local “Stäckel fudge” (Binney 2012; Bovy 2015; Trick, Coronado & Rix 2018). We have verified that the distribution of values of the actions and angles is insensitive to the chosen model.

Figure 3 illustrates the meaning of the angle $w_\phi$, which is the Galactic azimuth of the star’s guiding centre and we choose $w_\phi = 0$ for the line from the Galactic centre that passes through the Sun. The angle $w_R$ is the phase of the star around its epicycle, and we choose $w_R = 0$ at apocentre. These angles are computed from the positions reported by Gaia, but both increase with time at the uniform rates $w_R = \Omega_R t$ and $w_\phi = \Omega_\phi \equiv L_z/R_g^2$, where $R_g$ is the Galactic radius of the star’s guiding centre. The period of the radial oscillation defines the uniform rate

$$\Omega_R = \frac{2\pi}{\tau_R} \quad \text{where} \quad \tau_R = 2 \int_{R_p}^{R_g} R^{-1} dR,$$

which implies that $w_R$ does not correspond precisely to the angular position of the star around its epicycle, marked $\theta$ in the sketch, because the radial motion is generally not perfectly harmonic. To evaluate $w_R$ for a star at a general point, we must integrate its orbit to find the time $t$ required for it to reach its present radius from the moment it passed through apocentre. Then $w_R = \Omega_R t$. It is always true that $w_R = \pm \pi$ exactly at pericentre and, because the epicycle motion is slower in the outer part, $w_R \gtrsim \pm \pi/2$ as the star crosses the circle at $R_g$ in respectively the inward and outward directions.

This discussion is valid for orbits of arbitrary eccentricity, but the radial motion becomes more nearly harmonic as the orbit eccentricity decreases, and the angular frequencies tend to the familiar definitions introduced long ago by Lindblad (e.g. Binney & Tremaine 2008):

$$\Omega_\phi \rightarrow \Omega_c \quad \text{and} \quad \Omega_R \rightarrow \kappa,$$

where $R \Omega_c^2 = \Phi/R$ and $\kappa^2 = 4\Omega_c^2 + R \Phi R \Omega_c^2 / dR$.

The mapping from the observed 4D Galactic phase-space coordinates of a star at $(R, \phi, v_R, v_\phi)$ to $(J_R, L_z, w_R, w_\phi)$ is well-behaved, and (as was stressed by Trick, Coronado & Rix 2018) any features present in the one projection must necessarily appear in any other, although in a distorted form. In this paper we will show that action-angle coordinates have the additional advantage that they clarify the connection between the substructure and its probable dynamical origin.

3 FEATURES IN PHASE SPACE

Figure 3 presents the phase space density of our star sample in four different projections. The top panel reproduces the helio-centric velocity-space $(U, V)$ distribution...
Figure 4. Estimated uncertainties in the quantities of interest here, derived for 100 randomly chosen stars. Each smudge shows the spread in values, on a logarithmic scale, that results from 1000 Monte Carlo realizations, using the uncertainties in each observed quantity, as reported for each star in Gaia DR2.

Figure 5. Scattering in action space for a self-similar $V = \text{const}$ galaxy model. Both actions are normalized by $L_{z,c}$, which is the angular momentum of a circular orbit at corotation. The broken lines mark the separate loci of CR (black) and Lindblad resonances, colored red for $m = 2$, green for $m = 3$ and blue for $m = 4$, computed using eq. (5) for orbits of finite eccentricity. The vectors, having arbitrarily chosen lengths, indicate possible changes computed from eq. (4). Formula (6) indicates that they should have slopes $l/m$.

3.1 Features in the action distribution

The substructure in the second panel of Figure 3 may be indicative of resonance scattering. Here we explain why we formed this view.

A test particle moving in a non-axisymmetric potential that rotates at the uniform rate $\Omega_{p}$ about the z-axis conserves neither its energy $E$ nor its angular momentum $L_z$, but Jacobi’s integral $E_J \equiv E - \Omega_{p} L_z$ is conserved (Binney & Tremaine 2008). Hence, any changes to the energy and angular momentum of a star must be related as

$$\Delta E = \Omega_{p} \Delta L_z. \quad (4)$$

When the non-axisymmetric part of the potential is weak and has $m$-fold rotational symmetry, resonances between the motion of a star arise whenever

$$m(\Omega_{p} - \Omega_{l}) = \Omega_{l} R,$$  \quad (5)

with $l = 0$ for the corotation resonance (CR) and $l = \pm 1$ for the Lindblad resonances. At the ILR the star overrides the wave, while at the OLR it is overtaken by the wave, and in both cases it experiences forcing at its natural radial frequency $\Omega_R$. Lynden-Bell & Kalnajs (1972) showed that only those stars that are in one of these resonances experience lasting changes to $E$ and $L_z$. The broken curves in Figure 5 mark the loci of the resonances for $l = 0$, $\pm 1$ and $m = 2$ (red), $m = 3$ (green), and $m = 4$ (blue) perturbations in a simple $V = \text{const}$ galactic potential. These curves, which are calculated exactly for eccentric orbits using eq. (5), have similar negative slopes in any reasonable galaxy potential.

Sellwood & Binney (2002), in just a few lines of algebra, were able to show that scattering at any one of these resonances required not only that $\Delta E = \Omega_{l} \Delta L_z$, but that
changes to the actions were simply related as

$$\Delta J_R = \frac{l}{m} \Delta L_z.$$  

(6)

Formally, this relation is for near circular orbits, but the predicted slope holds for quite eccentric orbits, as illustrated by the vectors in Figure 5, with $\Delta J_R$ computed exactly from eq. (4) and not the approximation (6).

The horizontal vectors indicate that $\Delta J_R = 0$ for moderate $\Delta L_z$ changes at CR where $l = 0$, with the implication that radial migration at this resonance occurs without increasing random motion.

However, scattering at either Lindblad resonance should be along trajectories of almost constant slope $l/m$ in $L_z$-$J_R$ space, which therefore have positive slope at the OLR and negative at the ILR. The slopes of the vectors are shallower for disturbances of higher rotational symmetry, i.e. larger $m$. Lynden-Bell & Kalnajs (1972) showed that, on average, stars lose $L_z$ at the ILR and gain at the OLR, so formula (6) predicts heating at both resonances, because $J_R$ is increased, as shown in Figure 5. Notice that the scattering vectors for all initial $J_R$ values shift stars off the OLR as they gain $L_z$, but the scattering vectors at the ILR are almost perfectly aligned with the resonance line for $m = 2$ (red) and misalignment develops slowly as $m$ increases. This near coincidence is not just a peculiarity of this potential and appears to hold for most in other reasonable galactic potentials. Thus stars at the ILR remain close to the resonance as they lose $L_z$, which allows large changes in $J_R$ to be built up, as we will demonstrate.

In summary, the second panel of Figure 3 manifests multiple density excesses having predominantly negative slopes that are similar to the slopes $\approx -0.5$ of all the major resonances in Figure 5. These features therefore suggest, but do not prove, that the stellar distribution in the neighbourhood of the Sun contains many stars that have been scattered at Lindblad resonances of several different disturbances having various pattern speeds.

### 3.2 Features in the angle distribution

The third panel of Figure 3 shows the distribution of the Gaia stars as a function of $w_R$ and $J_R$. As for all other projections of the 4D distribution, we see a rich substructure.

The narrow distribution of $w_\phi$ values in the bottom panel, is a consequence of our selection of stars close to the Sun. We see that stars near apocentre ($w_R = 0$) and pericentre ($w_R = \pm \pi$) all have $w_\phi \approx 0$, as expected. The largest values of $w_\phi$ arise for stars having eccentric orbits that are close to their guiding centre radii ($w_R \gtrsim \pm \pi/2$), which is consistent with the sketch in Figure 2. Because the $w_\phi$ distribution is so dominated by selection effects, we have not found this last projection very informative.

By comparing the three other projections of the Gaia data with models in the next section, we will argue that the current stellar distribution has been sculpted by multiple resonances over time.

### 4 Predictions of Various Theories of Spiral Structure

Here we calculate how an originally smooth distribution of stars in an axisymmetric potential model is changed when perturbed by various time-dependent, non-axisymmetric perturbations. Our aim is not to create a single model that can match all the features in the Gaia data, but more modestly to show qualitatively how some of features might have arisen.

Each of our perturbations is motivated by one of the current theories for the origin of spirals in galaxies. Sellwood & Carlberg (2014) proposed that spirals result from transient modes, which we describe more fully and test in §4.2. A number of authors have proposed that spirals are material arms, and we examine the consequences of this hypothesis in §4.3. Toomre & Kalnajs (1991) and D’Onghia et al. (2013) imagine that spirals result from the responses to mass clumps within the disk, which we model in §4.4. Finally, Bertin & Lin (1996) propose that spirals are long-lived, quasi-steady density waves, and we discuss their picture in §4.5.

#### 4.1 Method

We employ the method pioneered by Dehnen (2000), which assumes some reasonable distribution function (DF) for the Galaxy before the perturbation was introduced and uses the fact that the DF is conserved along any orbit from the moment a perturbing potential is added to the present day. Bowy (2015) also implemented the technique in the python package galpy. Since we need to know the DF only where we wish to compare it with data, its current value at any point $(R, \phi, v_R, v_\phi)$ can be obtained by integrating an orbit from that point backwards in time over the history of the perturbation to when the adopted model was smooth and axisymmetric, where the value of the DF can be obtained from its coordinates $(R', \phi', v'_R, v'_\phi)$ at that earlier moment.

We adopt this method to model phase space at the location of the Sun. For each point in $(L_z, J_R)$ space, say, we must determine what the values of those variables imply for the instantaneous velocities of a star that also passes through the position of the Sun. That is we must find the $(R, \phi, v_R, v_\phi)$ coordinates of that orbit for fixed $R = R_0$ and $\phi = 0$. Trivially, $v_R = L_z/R_0$, and we map $(L_z, J_R) \rightarrow (E, L_z)$ and then use $v_R^2 = 2[E - \Phi(R_0)] = (L_z/R_0)^2$. We then integrate back in time to find where the orbit was before the perturbation, to find the value of the DF at the starting point in phase space. Note the two possible signs of $v_R$ imply that two separate locations $(R', \phi', v'_R, v'_\phi)$ before the perturbation was introduced map to the same point in $(L_z, J_R)$ space, and we therefore sum both DF values. The calculation when $w_R$ is pre-specified is more difficult, since we need a numerical search over $(E, L_z)$ for the unique orbit that passes through $(R, \phi) = (R_0, 0)$ with phase $w_R$ and radial action $J_R$; once found, the $v_\phi$ and $v_R$ values are determined as before, but the value of $w_R$ removes the sign ambiguity of $v_R$.

We calculate the perturbed DF at the point $(R, \phi) = (R_0, 0)$, and make no attempt to average over the small area of radius 200 pc that contains the stars that we compare with our models. Our selection of such nearby stars was
partly motivated by the need to be able to neglect changes in phase space across this area.

We assume a $\sech^2 \gamma t/2$ time dependence for every perturbation, which asymptotes to exponential growth as $\exp(-\gamma t)$ for $\gamma t \ll -1$ and begins to decay after peaking at $t = 0$. We begin the integration at $t = 1.36$ Gyr, which is six orbit periods around the Sun ($2\pi R_0/V_0$) after the perturbation peaked at $t = 0$, and integrate backwards in time to when the perturbation had negligible amplitude. The choice of six orbit periods after saturation allows the perturbation to decay somewhat, but is before some of the perturbed features become affected by a slow phase wrapping. We justify this on the grounds that continued evolution of a single perturbation would not be observable in the $Gaia$ data because all theories, bar one, predict successive perturbations on a time-scale of a few orbits. The exception is the quasi-steady mode theory of Bertin & Lin (1996) that we argue (§4.5) predicts no phase space changes at all.

Our unperturbed Milky Way model is locally approximated as a disc having a simple Gaussian velocity distribution, with radial velocity dispersion $\sigma_R = 0.13V_0$. The precise degree of random motion in the disc not very important, since we are here concerned with the qualitative relative changes produced by each perturbation, and do not attempt a quantitative comparison. The DF is

$$f(E, L_z) \propto \exp(-1.5R_g)e^{-E/\sigma^2},$$

where $R_g = L_z/V_0$ is the guiding center radius and $E = E - E_c(L_z)$, is the excess energy above that of a circular orbit at $R_g$; $E_c = \Phi(R_g) + 5/2 V_0^2$. The $\exp(-1.5R_g)$ factor was chosen to yield the right variation with $L_z$. We “observe” the perturbed DF that results from each adopted perturbation from the location of the Sun at $R_0 = 8$ kpc and $\phi = 0$, and in the following figures we normalize the phase space density by its maximum value in every projection.

4.2 A transient spiral mode

Sellwood & Carlberg (2014) proposed that spirals result from relatively short-lived unstable modes. Specifically, this description means that each perturbation is a standing wave oscillation, having a constant shape that rotates at a steady rate and grows exponentially until it saturates. Their simulations revealed that each coherent mode had a lifetime at moderate amplitude of some 10 rotation periods at its corotation radius, and the disk generally supported a few such modes simultaneously, with new instabilities developing in constant succession. The superposition of several coherent modes gave rise to the continuously changing patterns that are observed in most simulations. While the classical density wave theory (Bertin & Lin 1996) also invokes modes, those authors expect each mode to be very slowly growing, or even “quasi-steady”, and the disk would support rather few such modes; i.e. they have a very different picture from that proposed by Sellwood & Carlberg (2014).

In order to calculate what to expect from a single transient spiral of the kind described by Sellwood & Carlberg (2014), we chose a 3-armed spiral perturbation with a pattern speed $\Omega_p = 14.4$ km s$^{-1}$ kpc$^{-1}$, for which the ILR lies at 8.1 kpc. We adopted a logarithmic spiral potential that had a pitch angle of 26° and a radial variation that peaked at corotation and decreased as a quartic polynomial to zero at a radius that was 80% of that of the ILR – i.e. it was weak, but non-zero, a little interior to the ILR. As described above, it had a sech$^2$ time dependence, with the asymptotic growth rate $\gamma$ = 0.1Ω$_p$. We believe these choices to be reasonable. We have obtained very similar results for $m = 2$ and $m = 4$ patterns, and have experimented with wide variations in the pattern speed.

Although Sellwood & Carlberg (2014) identified growing modes, and showed that the frequency persisted for some time after saturation, the assumption we have made here that the shape remains the same as the disturbance decays is convenient, but probably too simple.

Figure 6 shows three projections of the phase space density of stars for this first case. The DF is largely undisturbed
in all three panels except for narrow features that have been caused by ILR scattering. The space of the two actions, $J_R$ and $L_z$ (middle panel), displays a scattering tongue of the kind seen in the Gaia data (Figure 3) and has also been reported in simulations (Sellwood 2012; Sellwood & Carlb erg 2014). Both axes are normalized by the $L_z$ of the LSR, and the requirement that the stars must pass through the solar position naturally creates the parabolic lower boundary.

The bottom panel of Figure 6 shows $J_R$ as a function of its conjugate phase $w_R$, which reveals that the density in this space is also no longer smooth. Again we see that the DF is undisturbed over most of this projection of phase space. The variation at high $J_R$ is because stars of different $w_R$ that are now at the Sun have different home radii, where the DF has different normalizations. Also the sharp features near $|w_R| \approx 2$ are for stars that were strongly scattered by the ILR and are now moving in their epicycles, both inward and outward, past the Sun.

In an earlier paper, Sellwood (2010) argued that all strongly scattered stars should have the same values of $nw_R + lw_R$. It is clear that this expectation was wrong, since we now see two sharp features near $|w_R| \equiv 2$. The fact that it is symmetrical about $w_R = 0$ (apocentre) is not a coincidence, since recalculating with a shift of the initial phase of the spiral did not alter the symmetry about $w_R = 0$ or the positions of the scattering features, and only minor details of the sharp features differed.

Small changes to the pattern speed of the spiral shift the feature in the top panel vertically and the scattering tongue in the middle panel horizontally, while the changes in the $(w_R, J_R)$ are more interesting. As the ILR was shifted to larger radii, the DF was undisturbed for small $J_R$ over all $w_R$ and the scattering features very gradually moved towards $w_R = \pm \pi$ (for stars at pericentre) and became weaker. Moving the ILR to a point interior to the Sun caused the stars near apocentre ($w_R = 0$) to be affected and again those with enough radial action to have been strongly perturbed by the resonance produced similar peaks, also near $|w_R| = \pi/2$, but this time curving towards $w_R = 0$ for lower values of $J_R$. All this behavior seems physically very reasonable.

The consequence of scattering at the OLR is shown in Figure 7, for which we adopted a pattern speed of $\Omega_p = 40$ km s$^{-1}$ kpc$^{-1}$. In this case, the observer is just interior to the resonance, so the features in the bottom panel curve towards $w_R = 0$, since stars of low $J_R$ that have been affected by the resonance are near their apocentres, which is the opposite of the situation shown in Figure 6. More interesting is the feature in the middle panel; again the overall slope of the feature is negative, but at this resonance the scattering vectors have positive slope (Figure 5) with the consequence that higher phase space density is shifted up and to the right. Furthermore, since stars are moved off resonance as they are scattered, we do not see the sharp peak that was created at the ILR (Figure 6).

It is noteworthy that adjusting the pattern speed to $\Omega_p = 27$ km s$^{-1}$ kpc$^{-1}$, so that the CR was at 8.13 kpc, i.e. just exterior to the Sun left phase space near the Sun almost unchanged by the perturbation, as shown in Figure 8. This result is consistent with the prediction of formula (6), where stars simply change places with no heating. Furthermore, Lynden-Bell & Kalnajs (1972) showed that stars away from the major resonances suffer no lasting changes to their integrals, and we find similarly mild changes for observers located in a broad swath of radii around CR where few stars are found that have been affected by either of the Lindblad resonances.

\subsection*{4.3 A material arm model}

A number of authors (e.g. Grand et al. 2012a,b; Baba et al. 2013; Roca-Fàbrega et al. 2013; Michikoshi & Kokubo 2018) have argued that the spirals in their simulations are swing-amplified features having pitch angles that change continuously with time at a rate that is consistent with the local shear rate in their adopted galaxy model. In other
words, the features are material arms, and not density waves. Hunt et al. (2018), using the galpy code (Bovy 2015), adopted a potential of such a disturbance, combined with a model of the MW bar, to calculate its effect on the distribution of stars in \((U,V)\)-space, and argued that some cases resembled the distribution of Gaia stars.

Although the disturbance they adopted indeed sheared, it did not seem to us to resemble the classic image of swing-amplification presented by Toomre (1981). We have therefore adopted the different potential perturbation that is illustrated in Figure 9. The radial variation has peak amplitude at some chosen \(R_c = V_0/\Omega_c\) and is again as a quartic function that drops to zero at \(R = R_c(1 \pm 0.76)\). The azimuthal behaviour is that of a logarithmic spiral at all times, with the angle to the radius vector \(\psi = \Omega_c(t-t_0)\), which therefore changes as the perturbation shears with the flow; it is radial at \(t = t_0\) and \(\psi > 0\) for a trailing spiral. Its overall amplitude also scales with time as \(\text{sech}^2 \beta\) where \(\beta = 0.1[\Omega_c(t-t_0) - m]\), and \(m\) is the angular periodicity of the pattern; this shift in the argument ensures that the peak amplitude occurs at a trailing pitch angle of 45°, as expected for a flat rotation curve. The factor 0.1 gives about the right variation in amplitude around the peak and asymptotic growth and decay rates \(\sim e^{-2\beta}\) for \(|\beta| \gg 1\). We choose the maximum potential amplitude = 0.02 at \(R_c = 9\) kpc at a trailing angle of \(\psi = 45°\).

Note this potential perturbation was chosen to mimic a swing-amplified spiral, but unlike that presented by Toomre (1981), it has no underlying dynamics. The potential function contains tightly-wrapped ripples for \(\Omega_c|t-t_0| \gg a\) few, which in reality should be washed out by random motion in a warm disc. Also, the disturbance in our model has peak amplitude at \(R_c\) at all times, which ignores the radial propagation of the wave packet at the group velocity, as occured in Toomre’s dynamically self-consistent calculation. However, both these effects occur only when the spiral is quite tightly wrapped, i.e. when our perturbed amplitude is weak. In summary, our model captures the material shear of the disturbance while the amplitude is large and the pattern open, which is the behaviour of the simulations as described in the above-cited papers, while omitting any dynamical details of the early and late evolution, when the amplitude is very low.

Figure 10 presents three separate projections of the phase space distribution of stars that results from this disturbance. This quite strong perturbation has caused significant changes to the DF; naturally, scaling down the ampli-
4.4 A dressed mass clump

Toomre & Kalnajs (1991) developed the idea that co-orbiting mass clumps within the disc create a “kaleidoscope” of transient spiral features, which are caused by the collective response of the underlying disc to density inhomogeneities. The local simulation by these authors, in which shot noise from particles themselves created the inhomogeneities, was extended to global simulations by D’Onghia et al. (2013). The latter authors found that a sprinkling of heavy particles produced evolving multi-arm spiral patterns in their rather low mass disc. They also reported that non-linear effects could cause activity to persist after a single driving clump was removed.

It should be noted that each heavy particle quickly becomes dressed by an extensive trailing wake, whose mass far exceeds that of the imposed mass, and that the wake orbits at the angular rate of the perturbing source mass. The spiral patterns that result are caused by the superposition of these wakes, each of which has its own pattern speed.

In order to calculate changes in phase space density that would be predicted by a simplified model of this type, we must use the perturbing potential of a dressed point mass. The response of a locally stable stellar disc to a point mass orbiting at the circular speed was originally calculated by Julian & Toomre (1966) in the well-known “sheared sheet” model that neglects curvature. The \((R, \phi)\) coordinates in the disc become

\[
x = R - R_c \quad \text{and} \quad y = R_c \left[ \phi - \Omega_c \left(t - t_0\right) \right] \tag{8}
\]

in the sheared sheet, where \(\Omega_c\) is the circular angular frequency at \(R_c\), the radius of the sheet centre, which is the location of the perturbing mass.

The top panel of Figure 11 presents contours of the perturbed potential computed using the mathematical apparatus devised by Julian & Toomre (1966). The spatial scale of the response density in the Figure can be converted to physical units by multiplying by \(\lambda_{\text{crit}} = 4\pi G \Sigma / \kappa^2\), where \(\Sigma\) is the disc surface density, and \(\kappa\) the epicyclic frequency, both reckoned at \(R = R_c\); in the self-similar Mestel disc \(\lambda_{\text{crit}} = 2\pi R f\) at any radius, where \(f\) is the fraction of active mass.

The Figure illustrates the potential of a wake for a \(V = \text{const.}\) disc with \(Q = 1.5\).

To use this perturbing potential in Dehnen’s method, we must convert \((R, \phi)\) to \((x, y)\) using equations (8) and scale them by an adopted value for \(\lambda_{\text{crit}}\). It is inconvenient to adopt this exact potential, because it is computed only over the rectangle shown; the perturbed potential on the boundaries is small, but non-zero, which would introduce mild discontinuities as stars crossed the boundaries that would be problematic when integrating orbits. We have therefore adopted the analytic approximation for the perturbing potential illustrated in the lower panel of Figure 11. Its functional form is

\[
\Phi_w(y, x) = -A e^{-a} \quad \text{where} \quad \quad a = y'^2 + x'^2 \left(1 + 100 \sqrt{x'^2 + y'^2}\right) \tag{9}
\]

\[
y' = y/\lambda_{\text{crit}} \quad \text{and} \quad x' = x/\lambda_{\text{crit}} - 0.6 \tanh(\pi y'/4),
\]

with \(A\) being a scaling constant. The line \(x = 0.6 \lambda_{\text{crit}} \tanh(0.25\pi y/\lambda_{\text{crit}})\), is the ridge line of the approx-
Figure 11. The upper panel contours the negative potential of the response density to a steady, coorbiting mass, computed by the method of Julian & Toomre (1966) in the sheared sheet, for a $V=\text{const}$ disc with $Q = 1.5$. The $(y, x)$ coordinates, defined in the text, are in units of $\lambda_{\text{crit}}$. The lower panel shows the analytic approximation adopted here; the ridge-line of the approximate potential is marked in red and reproduced in the upper panel.

Figure 12. Three phase space projections resulting from a dressed mass clump model, whose centre orbits at $R = 9$ kpc, observed at $R = 8$ kpc, as usual. The color scale has the same meaning as in Figure 6.
Their model attributes “grand design” spirals to slowly growing, bi-symmetric modes. Since their picture also suggests that the mode is “quasi-steady”, it should not have begun to decay, as we assumed for the transient mode model in §4.2. Predictions for a currently existing pattern could readily be calculated, indeed Dehnen (2000) devised his method to make predictions for the Milky Way bar. The most recent density-wave model for the Milky Way is that presented by Lin, Yuan & Shu (1969), which we could test. However, Shu (2016) argues that the situation in the Milky Way is now known to be more complex than the earlier paper assumed, with evidence for a 4-arm pattern, which he argues may possibly result from subharmonics or from superposed patterns, perhaps with one being a response to the bar.

In the absence of a specific, testable model from this group, we make a few general observations. Their picture of spiral wave generation invokes a $Q$-barrier to shield the mode from damping at the ILR, and therefore the principal cause of features in the phase space distribution in the transient spiral mode model would be absent in their picture. Also the CR has a neutral effect (Figure 8). Furthermore, since spirals in this model are long-lived and slowly evolving, these authors argue that most galaxies will have supported rather few such patterns. We would therefore expect that the extensive structure observed by Gaia in the local phase space distribution of stars in the Milky Way (Figure 3) would not be predicted by their quasi-steady spiral wave theory.

4.6 Discussion

The results presented for the three separate transient spiral arm models show that each gives rise to distinct predictions for the change in the density of stars in all three projections of phase space. Furthermore, all three scenarios expect that the Galaxy has supported many spiral episodes that have peaked at different radii, which in the case of the Sellwood & Carlberg (2014) mode theory would correspond to different pattern speeds.

For many reasons, we do not here attempt to make a quantitative comparison between the prediction of these models and the Gaia data presented in Figure 3. First, the data suggest that the features in phase space result from a succession of perturbations, but we have so far computed the consequences of only single disturbances. Second, it is likely that the local phase-space distribution of stars has also been sculpted by other perturbations, notably the bar in the Milky Way (Bland-Hawthorn & Gerhard 2016) but also perhaps tidally induced responses, which would need to be included in any comprehensive model to confront the data. Third, we have made no attempt to match DF of the Milky Way, so that our choice of the undisturbed DF yields a good match of the perturbed data to the observations. Fourth, features in phase space may be blurred over time as stars are scattered by giant molecular clouds. Building a comprehensive model that includes all these aspects would be a major undertaking, which we defer to later work.

Both the material arm model, Figure 10, and the dressed clump model, Figure 12, predict multiple features from a single disturbance of large enough amplitude, and therefore multiple such disturbances would probably produce quite complex structure in all phase-space projections. However, the salient aspect of the data in the second panel of Figure 3 are a few features of negative slope, consistent with resonance scattering by a few patterns of low $m$.

In Figure 13, we have highlighted stars from the Gaia data in several distinct groupings by colour. We selected each coloured group as those stars lying within $\Delta L_z = 0.01$ of a line of slope $-0.5$. The intercept on the $L_z$ axis of each line was chosen by eye so that each selected group of stars included one of the more prominent features in the second panel of Figure 3. Because the loci of all the major resonances have approximately this slope, we are unable to say whether any selected feature corresponds to an ILR or an OLR, neither can we identify the rotational symmetry of...
the perturbation that caused it, and therefore we cannot estimate a pattern speed for any of these possible disturbances. The other two panels in this Figure indicate where each group of selected stars lies in velocity space and in $w_H-J_R$ space.

The coloured stars in Figure 13 do not correspond to every feature in the distribution of Gaia stars, neither should they. The different scattering events, if that is what are highlighted, will have occurred successively, the Milky Way hosts a bar (Bland-Hawthorn & Gerhard 2016), and the disc has possibly been subject to other perturbations that may have produced additional features in these different projections. However, the distributions of the variously coloured stars in Figure 13 are strikingly similar to the features in Figure 6, and the resemblance seems closer than to those predicted by either of the other models. It should be cautioned that this superficially compelling comparison, particularly in the bottom panels, owes a lot to the fact that the appearance in each projection is determined by the mapping of the 4D distribution to each 2D surface, and the only really significant features are the resonant ridges in the middle panel. However, we conclude that the data offer little support for the dressed mass clump model, none at all for the quasi-steady density wave model and, while the material arm model may not be excluded, the sharper features in the Gaia data appear more consistent with the transient spiral mode model of Sellwood & Carlberg (2014).

4.7 Material arms vs. modes

The popular claim that the arms are material features that shear with the flow reflects the apparent behaviour in the simulations. However, Sellwood & Carlberg (2014) argued that an apparently shearing, swing-amplified spiral can result from the super-position of two or more rigidly rotating patterns. They also presented evidence for multiple coherent waves from spectral analysis of their simulations of a low-mass disc that appeared to manifest the material arms of the kind reported in by other authors. Note also that the “dust-to-ashes” figure presented in Toomre (1981) was calculated as the superposition of multiple steady responses to a set of perturbers having a range of pattern speeds (private communication, A. Toomre c1986). The swing-amplified interpretation of simulated spirals is very begging, and indeed Sellwood & Carlberg (1984) at first argued that was what they were observing, and did not present evidence for the underlying coherent waves in the same simulations for several years (e.g. Sellwood 1989).

But while material arms are consistent with Toomre’s picture, they beg the question of what is the origin of the leading signal that is swing-amplified? None of the papers that have argued for this interpretation have even addressed this question, let alone provided a satisfactory answer.

If the patterns were linear responses to shot noise, even of dressed particles (Toomre & Kalnajs 1991), their amplitude clearly should decrease as $N_1^{-1/2}$. This possibility was already disproved by Sellwood & Carlberg (2014) who compared the behaviour from simulations in which the particle number ranged over three orders of magnitude.

Other tests by Sellwood & Carlberg (2014) also ruled out that non-linear coupling was responsible for the spirals. After erasing any coherent mass clumps by azimuthal shuffling of the particles in a partly evolved simulation, they showed that the same coherent waves were present in parallel simulations both after shuffling and in the continued simulation without shuffling. Had density variations at the time of reshuffling been responsible for subsequent patterns, then shuffling would have been equivalent to a fresh start, and the amplitude of spirals would have grown as slowly, or more slowly because random motion had increased slightly. Instead they observed more rapid and coherent growth than in the first start. This result proved that the axisymmetric changes caused by the earlier evolution had created conditions for a vigorous, global, linear instability that was not present in the original smooth disc.

The prominent self-excited spirals that develop in every large-$N$ simulation of disc galaxy models, that exclude forcing by clumps, bars, or companions, require that they are true instabilities of the dynamical system that is represented by the particles. The swing-amplified material arm interpretation, while superficially attractive and a correct description of the apparent behavior, is simply not a viable theory for spiral arm formation. The interpretation that the apparent features result from superposition of a number of transient spiral modes was placed on a sound footing by Sellwood & Carlberg (2014) who also offered a mechanism that could produce exponentially growing, global modes in a dynamically modified disc. Our present finding that the distribution of local stars in action-angle space is more consistent with multiple spiral modes than with transient material disturbances provides further evidence in support of their picture.

It might be objected that the transient disturbance adopted in §4.3 was not dynamically self-consistent, and therefore not a fair test. However, a fair test would be to consider, as did Toomre (1981), a spiral that is the superposition of a number of steady waves, which we have already shown would be more consistent with the Gaia data.

5 CONCLUSIONS

The second data release from the Gaia mission has refined our view of the phase-space distribution of stars near the Sun. The components of their motion parallel to the Galactic plane have long been known to manifest detailed substructure, which was seen more sharply in the first view of the new data (Gaia collaboration: Katz et al. 2018). Adopting a simple axisymmetric model for the Milky Way that has a locally flat rotation curve, enabled us to compute action-angle variables for a sub-sample of likely thin-disk stars within a cylinder of radius 200 pc centered on the Sun, and Figure 3 shows the density of stars in four different projections of this 4D phase space. The well-known substructure in velocity space maps into rich substructure in these other variables, which we note has some of the characteristic features of Lindblad resonance scattering by multiple perturbations having a range of pattern speeds and low-order rotational symmetry.

---

1 An animation illustrating this behaviour may be found at http://www.physics.rutgers.edu/~sellwood/spiral.html
In order to determine whether some of these features might have been caused by past spiral activity in the disc of the Milky Way, we computed the changes to the distribution of stars in phase space that would be caused by a single spiral pattern of the kind favored by each of the following three current theories for spiral arm formation: the transient spiral mode model (Sellwood & Carlberg 2014), the swing-amplified model of material arms (e.g. Grand et al. 2012a,b; Baba et al. 2013; Roca-Fàbrega et al. 2013; Michikoshi & Kokubo 2018), and the dressed mass clump model (Toomre & Kalnajs 1991; D’Onghia et al. 2013). We also argued that the quasi-steady density wave model (Bertin & Lin 1996) should not create pronounced features in the phase-space distribution of stars near the Sun. For each of the first three cases, we adopted a simplified transient potential that approximates that of an idealized, isolated spiral. We found that the inner Lindblad resonances of a single pure transient spiral mode produced the narrow features in phase space shown in Figure 6. Multiple broad features, see Figure 10, were created by a single shearing spiral, or material arm, and multiple fine features, Figure 12, resulted from the wake of single dressed particle. For each model, we experimented with changes to the pattern speed or, in the case of material arms, the radius of the perturbed potential minimum, and found that the locations of the features in each projection varied in ways that made dynamical sense.

We argued that the Gaia data are inconsistent with the dressed particle model, and while they could be consistent with the material arm model, some of the narrower scattering features in Figure 3 more strongly resemble the prediction of spiral modes that are expected to have a variety of pattern speeds and rotational symmetries. Other considerations discussed in §4.7 strongly disfavour the swing-amplified material arm model, although we also explain that the superposition of a number of steady spiral modes can give the visual impression swing-amplified shearing evolution.

We do not here claim to have presented a model that can account for all the features in the 4D phase space distribution. The rich substructure in the distribution of Gaia stars deserves additional effort to fully understand its origin.

ACKNOWLEDGEMENTS

We thank Scott Tremaine and James Binney for helpful discussions. JAS acknowledges the hospitality of Steward Observatory where most of this work was done. This work has made use of data from the European Space Agency (ESA) mission Gaia (https://www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC https://www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement.

REFERENCES

Antoja, T., Valenzuela, O., Pichardo, B., et al. 2009, ApJ, 700, L78
Baba, J., Saitoh, T. R. & Wada, K. 2013, ApJ, 763, 46
Beny, T., Oey, M. S., Felting, S. & Gustafsson, B. 2007, ApJ, 655, L89
Bertin, G. & Lin, C. C. 1996, Spiral Structure in Galaxies (Cambridge: The MIT Press)
Binney, J. 2012, MNRAS, 426, 1324
Binney J. & Tremaine S. 2008, Galactic Dynamics (Princeton University Press, Princeton NJ) (BT08)
Bland-Hawthorn, J. & Gerhard, O. 2016, ARA&A, 54, 529
Bland-Hawthorn, J., Sharma, S., Tepper-Garcia, T., et al. 2018, arXiv:1809.02658
Bovy, J. 2015, ApJS, 216, 29
Bovy, J. & Hogg, D. W. 2010, ApJ, 717, 617
Chakrabarty, D. 2007, A&A, 467, 145
Dehnen, W. 2000, AJ, 119, 800
De Simone, R. S., Wu, X. & Tremaine, S. 2004, MNRAS, 350, 627
D’Onghia, E., Vogelsberger, M. & Hernquist, L. 2013, ApJ, 766, 34
Famaey, B., Pont, F., Luri, X., Udry, S., Mayor, M. & Jorissen, A. 2007, A&A, 461, 957
Gaia collaboration: Brown, A. G. A., Vallenary, A., Prusti, T., et al. 2018, A&A, 616A, 1
Gaia collaboration: Katz, D., Antoja, T., Romero-Góê, M., et al. 2018, A&A, 616A, 11
Grand, R. J. J., Kawata, D. & Cropper, M. 2012a, MNRAS, 421, 1529
Grand, R. J. J., Kawata, D. & Cropper, M. 2012b, MNRAS, 426, 167
Grand, R. J. J., Bovy, J., Kawata, D., et al. 2015, MNRAS, 453, 1867
Hunt, J. A. S., Hong, J., Bovy, J., Kawata, D. & Grand, R. J. J. 2018, eprint arXiv:1806.02832
Julian, W. H. & Toomre, A. 1966, ApJ, 146, 810
Lin, C. C., Yuan, C. & Shu, F. H. 1969, ApJ, 155, 721
McMillan, P. J. 2011, MNRAS, 418, 1565
Michikoshi, S. & Kokubo, E. 2018, arXiv:1808.08060
Pompéia, L., Masseron, T., Famaey, B., et al. 2011, MNRAS, 415, 1138
Quillen, A. C. 2003, AJ, 125, 785
Quillen, A. C. & Minchev, I. 2005, AJ, 130, 576
Quillen, A. C., De Silva, G., Sharma, S., et al. 2018, MNRAS, 478, 228
Roca-Fàbrega, S., Valenzuela, O., Figueras, F., et al. 2013, MNRAS, 432, 2678
Sellwood, J. A. 1989, in Dynamics of Astrophysical Discs, ed. J. A. Sellwood (Cambridge: Cambridge University Press) p. 155
Sellwood, J. A. 2010, MNRAS, 409, 145
Sellwood, J. A. 2011, MNRAS, 410, 1637
Sellwood, J. A. 2012, ApJ, 751, 44
Sellwood, J. A. 2014, Rev. Mod. Phys., 86, 1
Sellwood, J. A. & Binney, J. J. 2002, MNRAS, 336, 785
Sellwood, J. A. & Carlberg, R. G. 1984, ApJ, 282, 61
Sellwood, J. A. & Carlberg, R. G. 2014, ApJ, 785, 137
Shu, F. H. 2016, ARA&A, 54, 667
Trick, W. H., Coronado, J. & Rix, H-W. 2018: eprint arXiv:1805.03653
Toomre, A. 1977, ARA&A, 15, 437
Toomre, A. 1981, in "The Structure and Evolution of Normal Galaxies", eds. S. M. Fall & D. Lynden-Bell (Cambridge, Cambridge Univ. Press) p. 111
Toomre, A. & Kalnajs, A. J. 1991, in Dynamics of Disc Galaxies, ed. B. Sundelius (Gothenburg: Göteborgs University) p. 341
This paper has been typeset from a \textsc{TeX}/\LaTeX\ file prepared by the author.