Super-D0-branes
at the endpoints of fundamental superstring:
an example of interacting brane system

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Abstract

We present a supersymmetric action functional for the coupled system of an open fundamental superstring and super–D0–branes attached to (identified with) the string endpoints. As a preliminary step the geometrical actions for a free super-D0-brane and a free type IIA superstring have been built. The pure bosonic limits of the action for the coupled system and of the equations of motion are discussed in some detail.

Introduction

Recently a way to obtain a supersymmetric action functional for interacting branes (intersecting branes and branes ending on branes) has been proposed. The systems involving open fundamental superstrings ending on super–Dp–branes are quite generic and, on the other hand, especially interesting. The case of superstring—super-D3-brane system has been discussed briefly in (see [2] for details).

Two types of such system are special and require separate consideration. One consists of the open superstring and a super–D9–brane (space–time filling brane). It has been elaborated in [3]. In this contribution we present a supersymmetric action functional for the system of the open superstring ending on

\footnote{Contribution to the Proceedings of the International Seminar ”Supersymmetries and Quantum Symmetries” (SQS'99, 27-31 July, 1999).}
(the dynamical) super-D0-branes or D-particles. In distinction to the general case \[1, 2\] neither Lagrange multipliers no auxiliary space–time filling brane are necessary in this case.

Note that this dynamical system provides a supersymmetric generalization of the 'string with masses at the endpoints' which has been considered in the early years of 'QCD string' \[4\].

1 Geometric action for free super-D0-brane

The geometric action \[5\] and the generalized action principle \[6\] for super–Dp–branes with \(0 < p < 9\) and \(p = 9\) has been constructed in \[7\], \[8\] respectively. However the super–D0–brane has not been considered in this framework.

The geometric action for the super–D0–brane has the form

\[
S_{D0} = m \int_{\mathcal{M}^1} \tilde{L}_1 = \int_{\mathcal{M}^1} \left( \tilde{\Pi}^m u_0^m(\tau) + i \left( d\tilde{\Theta}^1 \Theta^2 - \Theta^1 d\tilde{\Theta}^2 \right) \right),
\]

where \(m\) is the super-D0-brane mass parameter,

\[
\tilde{\Pi}^m = dX^m - id\bar{\Theta}^1 \sigma^m \Theta^1 - id\bar{\Theta}^2 \sigma^m \Theta^2
\]

is the basic covariant 1–form of the flat type IIA superspace,

\[
\tilde{\Pi}^m = dX^m - id\bar{\Theta}^1 \sigma^m \Theta^1 - id\bar{\Theta}^2 \sigma^m \Theta^2 = d\tau \tilde{\Pi}_\tau^m
\]

is its pull–back on the super–D0–brane world–line \(\mathcal{M}^1\)

\[
X^m = \tilde{X}^m(\tau), \quad \Theta^1 \bar{\Theta}^1 = \tilde{\Theta}^1 \bar{\Theta}^1(\tau), \quad \Theta^2 \bar{\Theta}^2 = \tilde{\Theta}^2 \bar{\Theta}^2(\tau) : \mathcal{M}^1 \rightarrow \mathcal{M}^{(10|32)},
\]

and \(u_0^m\) is a time–like unit length vector field

\[
u_0^m u_0^m = 1.
\]

It is convenient to consider \(u_0^m\) as a column of the Lorentz group valued matrix

\[
u_a^m = \begin{pmatrix} u_0^m, u_i^m \end{pmatrix} \in SO(1,9) \quad \Leftrightarrow \quad \nu_a^m \eta^{mn} \nu_b^m = \eta^{ab}.
\]

The conditions \(5\) include the normalization \(5\) as well as the orthogonality conditions for the vectors \(u_0^m, u_i^m\) (Lorentz harmonics \(3\))

\[
u_0^m u^i_m = 0, \quad u_i^m u^i_m = -\delta^{ij}.
\]
A doubly covered element for the $SO(1,9)$-valued matrix (8)

$$ v^A \mu \in Spin(1,9) $$

is related with (8) by the conditions of $\sigma$-matrix conservation

$$ u^{\alpha \nu}_{\mu} \sigma_{\nu}^{\alpha} = v^A \mu \sigma_{AB} v^B \mu, \quad u^{\bar{\alpha} \bar{\nu}}_{\bar{\mu}} = v^A \bar{\mu} \bar{\sigma}_{AB} v^B \bar{\mu}. $$

$A = 1, \ldots, 16$ can be treated as $SO(9)$ spinor index. Then the requirement of $SO(9)$ gauge symmetry makes natural an identification of the harmonics with homogeneous coordinates of the coset $SO(1,9)/SO(9)$ (cf. with [10]). Note that $SO(9)$ group possesses a symmetric charge conjugation matrix. When it is identified with unity matrix, the difference between upper and lower $SO(9)$ spinor indices disappears.

Substituting the $SO(9)$ invariant representation for $SO(1,9)$ sigma-matrices

$$ \sigma_{AB}^0 = \delta_{AB}, \quad \sigma_{AB}^i = \Gamma_{AB}^i, \quad \bar{\sigma}_{AB}^0 = \delta_{AB}, \quad \bar{\sigma}_{AB}^i = -\Gamma_{AB}^i, $$

one can decompose Eq. (8) into

$$ u^{(0)}_{\mu} \sigma_{\nu}^{\mu} \mu = v^A \mu \sigma_{AB} v^B \mu, \quad u^{i}_{\mu} \sigma_{\nu}^{i} \mu = v^A \mu \Gamma_{AB} v^B \mu. $$

$$ u^{(0)}_{\mu} \delta_{AB} = v^A \mu \delta_{AB} v^B \mu, \quad u^{i}_{\mu} \Gamma_{AB} = v^A \mu \bar{\sigma}_{AB} \bar{\mu} v^B \mu. $$

Similar relations can be obtained for the inverse $SO(1,9)/SO(9)$ harmonics

$$ v^A \mu \sigma_{\nu}^{\mu} \mu = \delta_{BA}^A, $$

$$ u^{(0)}_{\mu} \sigma_{\nu}^{\mu} \mu = v^A \mu \sigma_{AB} v^B \mu, \quad u^{i}_{\mu} \sigma_{\nu}^{i} \mu = -v^A \mu \Gamma_{AB} v^B \mu. $$

$$ u^{(0)}_{\mu} \delta_{AB} = v^A \mu \delta_{AB} v^B \mu, \quad u^{i}_{\mu} \delta_{AB} = v^A \mu \bar{\sigma}_{AB} \bar{\mu} v^B \mu. $$

The harmonics can be used to define a general supervielbein of the flat type $IIA$ superspace $E^A$ which possesses the $SO(9)$ invariant decomposition

$$ E^A = (E^{(0)}, E^i; E^{A1}, E^{A2}) $$

$$ E^{(0)} \equiv \Pi^{(0)}_{\mu} v^0_{\mu}, \quad E^i \equiv \Pi^{i}_{\mu} v^i_{\mu}, $$

$$ E^{A1} \equiv d\Theta^2 v^A_{\mu}, \quad E^{A2} \equiv d\Theta^2 v^A_{\mu}. $$

A new important property of this supervielbein (in comparison with the ”coordinate” one $(\Pi^{i}_{\mu}, d\Theta^2 v^A_{\mu})$) is that it permits covariant linear combinations of the different fermionic supervielbein forms, e.g. $E^{A1} \pm E^{A2}$. 
The structure equations of the flat type IIA superspace can be written as

\[ dE^{(0)} = -iE^{A1} \wedge E^{A1} - iE^{A2} \wedge E^{A2} + E^i \wedge f^i, \]  
\[ \mathcal{D}E^i \equiv dE^i + E^i \wedge A^j = -iE^{A1} \wedge E^{B2} \Gamma_{AB}^i + E^{(0)} \wedge f^i, \]  
\[ \mathcal{D}E^{A1} \equiv dE^{A1} + E^{B1} \wedge \frac{1}{4} A^{ij} \Gamma_{ij}^{BA} = \frac{1}{2} E^{B1} \wedge f^i \Gamma_{BA}^i, \]  
\[ \mathcal{D}E^{A2} \equiv dE^{A2} + E^{B2} \wedge \frac{1}{4} A^{ij} \Gamma_{ij}^{BA} = -\frac{1}{2} E^{B2} \wedge f^i \Gamma_{BA}^i. \]  

Here the 'admissible derivatives' of the harmonics \[5\] (i.e. the derivatives which preserve the conditions (6), (8))

\[ du_a^m = u_b^m \Omega^b_a \quad \iff \quad \begin{cases} du^{(0)}_a = u_b^m f^i, \\ du^i_a = -u^j_m A^{ji} + u^{(0)}_m f^i, \end{cases} \]  
\[ dv_{\mu}^A = \frac{1}{4} \Omega_{ba}^b v_{\mu}^B (\sigma_{ba})^A_B = \frac{1}{2} v_{\mu}^B f^i \Gamma_{BA}^i - \frac{1}{4} A^{ij} v_{\mu}^B \Gamma_{ij}^{BA}, \]  
\[ dv_{\mu}^A = -\frac{1}{4} \Omega_{ba}^b v_{\mu}^B (\sigma_{ba})^B_A = \frac{1}{2} f^i \Gamma_{AB}^i v_{\mu}^B + \frac{1}{4} A^{ij} \Gamma_{AB}^{ij} v_{\mu}^B \]  

have been used. In (22), (23), (24)

\[ \Omega^{ab}(d) = -\Omega^{ba}(d) \equiv u_a^m du_b^m = \begin{pmatrix} 0 & f^j \\ -f^i & A^{ij} \end{pmatrix} \]  

are so(1,9)–valued Cartan 1–forms. The forms

\[ f^i \equiv u^{(0)}_m du^m \]  
\[ A^{ij} \equiv u^j_m du^m \]  

are covariant with respect to local SO(9) transformations and provide a basis for the coset SO(1,9)/SO(9) while

\[ d\Omega_a^{\frac{b}{c}} - \Omega_a^{\frac{b}{c}} \wedge \Omega^b_{\frac{c}{d}} = 0 \quad \iff \quad \begin{cases} \mathcal{D}f^i = df^i + f^j \wedge A^{ji} = 0, \\ F^{ij} = dA^{ij} + A^k \wedge A^{kj} = -f^i \wedge f^j, \end{cases} \]
2 Gauge symmetries and equations of motion for free super–D0–brane

The simplest way to vary the geometric action is to calculate an external derivative of the Lagrangian 1–form \( \mathcal{L}_1 \)

\[
\mathcal{L}_1 = m \left[ E(0) + i \left( d\Theta^1 \mu \Theta^2 - \Theta^1 \mu d\Theta^2 \right) \right],
\]

(29)
cf. (1), (16), (17) and use the seminal formula

\[
\delta \mathcal{L}_1 = i_{\delta}(d\mathcal{L}_1) + di_{\delta}\mathcal{L}_1.
\]

(30)

Here \( i_{\delta} \) can be regarded as formal contraction of differential form with variation symbol, e.g.

\[
i_{\delta}d\Theta^1 = \delta\Theta^1, \quad i_{\delta}d\Theta^2 = \delta\Theta^2, \quad i_{\delta}\Pi^m = \delta X^m - i\delta\Theta^1 \sigma^m \Theta^1 - i\delta\Theta^2 \tilde{\sigma}^m \Theta^2.
\]

(31)

The basis (31) in the space of variations is more convenient than the original one \( (\delta X^m, \delta\Theta^1, \delta\Theta^2) \). The contractions \( i_{\delta}f^i, i_{\delta}A^{ij} \) shall be considered as parameters of independent transformations of the harmonic variables which preserve the conditions (6), (8) (admissible variations [5])

\[
\delta u^{(0)}_m = u^i_m i_{\delta}f^i, \quad \delta u^i_m = -u^j_m i_{\delta}A^{ij} + u^{(0)}_m i_{\delta}f^i,
\]

(32)

\[
\delta v^A_{\mu} = \frac{1}{2} v^B_{\mu} \Gamma^i_{BA} i_{\delta}f^i - \frac{1}{4} v^B_{\mu} \Gamma^{ij}_{BA} i_{\delta}A^{ij},
\]

\[
\delta v^\mu_A = \frac{1}{2} \Gamma^i_{AB} v^\mu_B i_{\delta}f^i + \frac{1}{4} \Gamma^{ij}_{AB} v^\mu_B i_{\delta}A^{ij}.
\]

(33)

External derivative of the Lagrangian 1–form (29) can be written as

\[
d\mathcal{L}_1 = m \left[ E \wedge f^i - i(E^{A1} - E^{A2}) \wedge (E^{A1} - E^{A2}) \right].
\]

(34)

Thus the variation of the action (1) is

\[
S_{\text{D0}} = m \int_{\mathcal{M}^1} \left( E^i i_{\delta}f^i - f^i i_{\delta}E^i - 2i(E^{A1} - E^{A2})i_{\delta}(E^{A1} - E^{A2}) \right),
\]

(35)

where we skipped the complete derivative term \( \int_{\mathcal{M}^1} di_{\delta}\mathcal{L}_1 \). The latter means that the D0-brane worldline is considered as a surface without boundary \( \partial\mathcal{M}^1 = 0 \) and, hence, there are no rejections for its identification with a boundary of some surface \( \mathcal{M}^1 = \partial\mathcal{M}^{1+1} \) (see below).
Only 16 of 32 fermionic variations

\[ i_\delta (E^A_1 - E^A_2) \equiv \delta \Theta^1_{\mu} v^A_{\mu} - \delta \Theta^2_{\mu} v^A_{\mu} \]  

are involved effectively in (35). Thus the remaining 16 variations

\[ \kappa^A \equiv i_\delta (E^A_1 + E^A_2) \equiv \delta \Theta^1_{\mu} v^A_{\mu} + \delta \Theta^2_{\mu} v^A_{\mu} \]  

can be regarded as parameters of a fermionic gauge symmetry of the model. This is the famous \( \kappa \)-symmetry \[11\]. Other gauge symmetries can be found by searching for the variations whose parameters are absent in (35). They are \( SO(9) \) symmetry \( (i_\delta A^{ij}) \) and the reparametrization \( (i_\delta E^{(0)} = \delta X_m u_m^{(0)}, \delta \Theta^{1,2} = 0) \).

Equations of motion for the super–D0-brane appear as a result of variations with respect to \( i_\delta f^i, i_\delta E^i = \delta X m u^i m \) and \( i_\delta (E^A_1 - E^A_2) \) respectively

\[ \tilde{E}^i \equiv \tilde{\Pi}^m_{\hat{u}^i m} = 0, \]  

\[ \tilde{f}^i \equiv \tilde{u}^{(0) m} d \tilde{u}^i m = 0, \]  

\[ \tilde{E}^A_1 - \tilde{E}^A_2 \equiv d \tilde{\Theta}^1_{\mu} v^A_{\mu} - d \tilde{\Theta}^2_{\mu} v^A_{\mu} = 0. \]  

It can be proved that these equations are equivalent to the standard equations of motion for the super–D0–brane \[13\]. In the gauge \( \tilde{X} m \tilde{u}_m^{(0)} = \tau, \tilde{\Theta}^{1,2} = 0 \), Eqs. (38), (39), (40) are equivalent to the set

\[ d \tilde{X} m = d \tau \tilde{p} m / m, \quad d p_m = 0, \quad p^2 m = m^2, \quad d \tilde{\Theta}^2_{\mu} = 0, \]  

which describes a massive superparticle.

3 Geometric action for type IIA superstring

The geometric action, superembedding approach and generalized action principle for type IIB superstring has been constructed in \[4, 14, 6\] respectively. Type IIA superstring has not been considered in this framework before.

The geometric action for type IIA superstring is

\[ S_{IIA} = \int_{M^{1+1}} \mathcal{L}_2 = \int_{M^{1+1}} \left( \frac{1}{2} \hat{E}^{++} \wedge \hat{E}^{--} - \hat{B}_2 \right), \]  

\[ ^2 \text{See} [12] \text{for the geometrical meaning of the} \kappa \text{-symmetry.} \]
where

\[ E^{\pm\pm} \equiv \Pi^m U^{\pm\pm}_m, \quad E^I \equiv \Pi^m U^I_m, \]

\[ B_2 = i\Pi^m \wedge \left( d\Theta^1 \sigma_m \Theta^1 - id\Theta^2 \sigma_m \Theta^2 \right) + d\Theta^1 \sigma_m \Theta^1 \wedge d\Theta^2 \sigma_m \Theta^2, \]

\[ \hat{\Pi}_m = d\hat{X}^m - i\hat{\Theta}^m \sigma_m \hat{\Theta}^1 - i\hat{\Theta}^2 \sigma_m \hat{\Theta}^2 = d\xi^m \hat{\Pi}_m (\xi) \]

is the pull-back of the 1–form (2) on the superstring worldsheet \( \mathcal{M}^{1+1} \) whose embedding into the type IIA superspace \( \mathcal{M}^{1+1} \rightarrow \mathcal{M}^{(10|32)} \) is defined by

\[ X^m = \hat{X}^m (\xi) \equiv \hat{X}^m (\tau, \sigma), \quad \Theta^1_\mu = \hat{\Theta}^1_\mu (\xi), \quad \Theta^2_\mu = \hat{\Theta}^2_\mu (\xi). \]

\( \hat{U}^{\pm\pm}_m (\xi) \equiv \hat{U}^0_m (\xi) \pm \hat{U}^9_m (\xi) \) are light–like Lorentz harmonic vectors [9], i.e. the components of the \( SO(1, 9) \) valued matrix

\[ U_m^a = \left( U_0^m, U^I_m, U_9^m \right) = \left( \frac{1}{2} (U^{++}_m + U^{--}_m), U^I_m, \frac{1}{2} (U^{++}_m - U^{--}_m) \right) \in SO(1, 9) \]

The spinor harmonics

\[ V_\mu^a = \left( V^{+}_\mu, V^{-}_\mu \right)^T \in Spin(1, 9) \]

\[ V_\alpha^\mu = \left( V^{-\mu}_q, V^{+\mu}_q \right) \in Spin(1, 9) \]

\[ V_\mu^a V_\mu^b = \delta^a_\beta : \quad V_q^{-\mu} V_{\mu p}^+ = \delta_q^p, \quad V_\mu^+ V_\mu^- = \delta_\mu^p, \quad V_q^{-\mu} V_{\mu p}^- = V_\mu^+ V_\mu^- = 0 \]

are related with (17) by Eqs. (9) which include, in particular,

\[ U^{++}_m \sigma^m_{\mu\nu} = 2 V_{\mu\nu}^+ V_{\mu\nu}^+; \quad U^{--}_m \sigma^m_{\mu\nu} = 2 V_{\mu\nu}^- V_{\mu\nu}^-, \]

\[ U^{--}_m \sigma^m_{\mu\nu} = 2 V_{\mu\nu}^- V_{\mu\nu}^-; \quad U^{++}_m \sigma^m_{\mu\nu} = 2 V_{\mu\nu}^+ V_{\mu\nu}^+. \]

The details about the stringy harmonics and Cartan forms (cf. (23))

\[ \Omega^{ab} \equiv U^a_m dU^b_m = \left( \begin{array}{ccc} 0 & \frac{f^{++} f^{--}}{2} & -\frac{1}{2} \omega \\ -\frac{f^{++} f^{--}}{2} & \frac{A^{IJ}}{2} & -\frac{f^{++} f^{--}}{2} \\ \frac{1}{2} \omega & -\frac{f^{++} f^{--}}{2} & 0 \end{array} \right) \]

can be found in Refs. [3, 14, 15, 16].

The external derivative of the Lagrangian 2–form

\[ \mathcal{L}_2 = \frac{1}{2} E^{++} \wedge E^{--} - i\Pi^m \wedge \left( d\Theta^1 \sigma_m \Theta^1 - id\Theta^2 \sigma_m \Theta^2 \right) + d\Theta^1 \sigma_m \Theta^1 \wedge d\Theta^2 \sigma_m \Theta^2 \]
can be calculated with the use of (51), (52), (53) and stringy counterparts of Eqs. (22)–(28)

\begin{equation}
\begin{aligned}
dL_2 &= -2iE^{+} \wedge E^{-q_1} \wedge E^{-q_1} + 2iE^{-} \wedge E^{q_2} \wedge E^{q_2} + \\
E^I \wedge \left( \frac{1}{2}E^{-} \wedge f^{+I} - \frac{1}{2}E^{+} \wedge f^{-I} + 2i \left( E^{q_1} \wedge E^{-q_1} + E^{q_2} \wedge E^{-q_2} \right) \gamma^I_{q\bar{q}} \right).
\end{aligned}
\end{equation}

The parameters of the stringy \( \kappa \)-symmetry can be identified with the contractions of those fermionic forms which are absent in the first line of Eq. (55)

\begin{equation}
\begin{aligned}
\kappa^{+q} &\equiv i_\delta E^{+q_1} = \delta \Theta^1_{\mu} u_{\mu}^+, \\
\kappa^{-q} &\equiv i_\delta E^{-q_2} = \delta \Theta^2_{\mu} v_{-q}^\mu.
\end{aligned}
\end{equation}

The second line of (55) determines, in particular, the transformations of the harmonics with respect to the \( \kappa \)-symmetry. Other gauge symmetries are \( SO(1,1) \times SO(8) \) \( (i_\delta \omega, i_\delta A^{IJ}) \) and the reparametrization \( (i_\delta E^{\pm \pm} = \delta X^m U^{\pm \pm}_m) \).

The equations of motion for the type IIA superstring can be obtained from (55). They are

\begin{equation}
\begin{aligned}
\hat{E}^I &\equiv \hat{\Pi}^m u^I_m = 0, \\
M_2^I &\equiv E^{-} \wedge f^{+I} - E^{+} \wedge f^{-I} + 4i \left( E^{q_1} \wedge E^{-q_1} + E^{q_2} \wedge E^{-q_2} \right) \gamma^I_{q\bar{q}} = 0,
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
E^{++} \wedge E^{-q_1} &\equiv \hat{\Pi}^m \wedge d\Theta^1_{\mu} u_{\mu}^- u_{m}^+ = 0, \\
E^{--} \wedge E^{+q_2} &\equiv \hat{\Pi}^m \wedge d\Theta^2_{\mu} v_{q}^+ u_{m}^- = 0.
\end{aligned}
\end{equation}

It can be proved that this set is equivalent to the standard equations of motion for the type IIA superstring (see [5] for the type IIB case).

### 4 Supersymmetric action functional for type IIA superstring with super–D0–branes at the endpoints

The main problem which should be solved to write down the action of interacting branes is: how to take into account an identification of the bosonic and fermionic superembedding functions on the intersection [1, 3]. However, this problem has the natural solution just for the system under consideration. Here the super–D0–brane worldline \( \mathcal{M}^4 \) should be considered as the boundary of the
superstring worldsheet $M^1 = \partial M^{1+1}$. Thus one can define an embedding of $M^1$ into $M^{1+1}$

$$\xi^m = \tilde{\xi}^m(\tau) : M^1 = \partial M^{1+1} \to M^{1+1}$$

and identify the super–D0–brane coordinate functions $\tilde{X}(\tau), \tilde{\Theta}^{1,2}(\tau)$ with the images of the superstring coordinate functions $\hat{X}(\xi), \hat{\Theta}^{1,2}(\xi)$ on the boundary

$$\tilde{X}(\tau) = \hat{X}(\tilde{\xi}(\tau)), \quad \tilde{\Theta}^{1,2}(\tau) = \hat{\Theta}^{1,2}(\tilde{\xi}(\tau)).$$

With this identification the action for the coupled system of an open fundamental superstring and super–D0–branes at the ends of the superstring is the direct sum of the actions (1) and (42)

$$S_{str} + D_0 = \int_{M^{1+1}} \hat{L}_2 + m \int_{\partial M^{1+1}} \tilde{L}_1.$$  

The variation of the action can be calculated as

$$\delta S_{str+D0} = \int_{M^{1+1}} i_\delta (dL_2) + m \int_{\partial M^{1+1}} (i_\delta L_2 + i_\delta dL_1).$$

A possibility is to require the vanishing of the bulk and the boundary variations in (64) separately. On the other hand, following \[1, 2, 3\], one can introduce the following current density distribution

$$j_1 = d\xi^m \varepsilon_{mn} \int_{\partial M^{1+1}} d\tilde{\xi}^n(\tau) \delta^2 \left( \xi - \tilde{\xi}(\tau) \right) = \varepsilon_{mn} d\xi^m j^m$$

with the property

$$\int_{M^{1+1}} j_1 \wedge \hat{A}_1 = \int_{\partial M^{1+1}} \tilde{A}_1.$$  

In (66) $\hat{A}_1$ is an arbitrary 1-form defined on the worldsheet $M^{1+1}$ and $\tilde{A}_1$ is its pull–back onto $M^1 = \partial M^{1+1}$. As it is easy to define the extension of the super–D0–brane Lagrangian form to the whole worldsheet (29), we can use (66) to lift the action (63) or its variation (64) to the integral over the whole worldsheet

$$S_{str+D0} = \int_{M^{1+1}} \hat{L}_2 + m j_1 \wedge \hat{L}_1,$$

$$\delta S_{str+D0} = \int_{M^{1+1}} i_\delta (dL_2) + m j_1 \wedge (i_\delta L_2 + i_\delta dL_1).$$

Here $\hat{L}_2$ is defined by Eq. (42), (54), $\hat{L}_1$ is the pull–back of the 1–form $L_1$ (29) on the worldsheet (16). In (68) the contractions of the forms (i.e. $i_\delta B_2 = i_\delta (1/2dZ^M \wedge dZ^N B_{NM}) = dZ^M \delta Z^N B_{NM}$) should be pulled back to the worldsheet and the contractions of the coordinate differentials in the second term should include the variations of the functions $\tilde{\xi}^m(\tau)$ (61), e.g. $i_\delta dX^m = \delta X^m + \delta \tilde{\xi}^m(\tau) \partial_m \tilde{X}^m |_{\xi=\tilde{\xi}(\tau)}$ (though the variations $\delta \tilde{\xi}^m(\tau)$ shall not produce independent equations).
5 D0-branes at the endpoints of bosonic string

In the pure bosonic limit our dynamical system describes a 'string with masses at the endpoints'. Such system has been studied in early years of QCD strings and partial solutions have been found [4]. However the geometric or first order formulation as well as 'extended variational problem' approach [2] (67), (68) for this system are new and, in our opinion, instructive.

The geometric action for the coupled system has the form

\[ S = \int_{M^{1+1}} \frac{1}{2} \hat{E}^{++} \wedge \hat{E}^{--} + mj_1 \wedge \hat{E}^{(0)}. \]  

(69)

Its variation with respect to harmonic variables

\[ \delta_h S = \int_{M^{1+1}} \left( \frac{1}{2} \hat{E}^I \wedge \hat{E}^{--} - \frac{1}{2} \hat{E}^I \wedge \hat{E}^{++} + m j_1 \wedge \hat{E}^i \delta f^i \right) \]  

(70)

produces the same algebraic embedding equations as in the case of a free string and free D0-brane(s)

\[ \hat{E}^I \equiv \hat{X}_m(\xi)U^I_m(\xi) = 0, \]  

(71)

\[ \hat{E}^i \equiv \hat{X}_m(\xi(\tau))u^i_m(\tau) = 0. \]  

(72)

This provides the possibility to simplify the variation with respect to the co-
ordinate functions considering it modulo Eqs. (71), (72)

\[ \delta S|_{\hat{E}^I = 0} = \frac{1}{2} \int_{M^{1+1}} \left( M^I_2 U^I_m + j_1 \wedge (\hat{E}^{++}U^{--}_m - \hat{E}^{--}U^{++}_m - 2mf^iu^i_m) \right) \delta \hat{X}_m \]  

(73)

Here

\[ M^I_2 \equiv E^{--} \wedge f^{++} - E^{++} \wedge f^{--} \]  

(74)

is the pure bosonic limit of the l.h.p. of the free superstring equation (58).

When considered together, Eqs. (71) and (72) relate the images of the stringy harmonics on the worldsheet boundary with the harmonics of the D0-

broanes. Indeed, Eq. (72) implies \( d\hat{X}_m = E^{(0)}u^{(0)}_m \). Substituting this relation into the pull-back of Eq. (71) on the boundary one arrives at

\[ u^{(0)}_m(\tau) U^I_m(\xi(\tau)) = 0. \]  

This implies

\[ u^{(0)}_m(\tau) = w^{--}(\tau)U^{++}m(\xi(\tau)) + U^{++}m(\xi(\tau))/4w^{--}(\tau) \]  

(75)

with some indefinite function \( w^{--}(\tau) \) (compensator for \( SO(1,1) \) symmetry). The relative coefficient in (73) is fixed by normalization conditions. Now, using
the $SO(9)$ gauge symmetry, we can chose the super–D0–brane harmonics to be expressed through the images of stringy ones by

$$u^m(\tau) = \left( U^{++}m(\xi(\tau)), u^{(9)m}(\tau) \right),$$  \hspace{1cm} (76)

$$u^{(9)m}(\tau) = w^{--}(\tau)U^{++}m(\xi(\tau)) - U^{++}m(\xi(\tau))/4w^{--}(\tau).$$

Then the set of Cartan forms (26) splits as $f^i = (f^I, f^{(9)})$ and, after some algebraic manipulations, the equations of motion can be written in the form

$$M^I_2 \equiv E^{--} \wedge f^{++I} - E^{++} \wedge f^{--I} = mj_1 \wedge f^I,$$  \hspace{1cm} (77)

$$j_1 \wedge \left( E^{(0)} - mf^{(9)} \right) = 0.$$  \hspace{1cm} (78)

When $m \neq 0$ the latter equation evidently implies

$$f^{(9)} \equiv u^{(9)m} du^{(9)}_m = \frac{1}{m}E^{(0)}.$$  \hspace{1cm} (79)

For $m \to \infty$ we can neglect the left hand side of Eq. (77) and the right hand side of Eq. (79). Thus we arrive at the free equations of motion (33) for the D0-branes. This means that D0–branes (or ’quarks’ [4]) with infinite mass(es) do not feel the influence of the open string. When $m \to 0$ Eq. (77) becomes the free string equation, while (78) implies that $j_1 = 0$, i.e. that the worldsheet has no boundary and, thus, the string is closed.

Concluding Remarks

The analyzes of the gauge symmetries of the action (33), (34) and the supersymmetric equations which follow from it will be the subject of a forthcoming article. We expect that they shall provide an important insights for future study of the generic system of interacting superbranes.

Another direction of the development of the present results is to elaborate the generalized action principle [3] and the superembedding approach [4, 14, 15] the super–D0–brane (see [4] for the super-Dp-branes with $0 < p < 9$ and [8] for $p = 9$). The basis for such study is provided by the geometric action (1).

Acknowledgments

The author is grateful to D. Sorokin, M. Tonin, B. Julia for useful conversations and for the hospitality at the Padova Section of INFN (Padova) and Laboratoire de Physique Theorique de l’Ecole Normale Superieure (Paris), where a
part of this work has been done. A partial support from the INTAS Grant 96-308 and the Ukrainian GKNT Grant 2.5.1/52 is acknowledged.

References

[1] I. Bandos and W. Kummer, *Phys.Lett.* **B462**, 254 (1999) (*hep-th/9905144*).

[2] Igor Bandos and Wolfgang Kummer, *papers in preparation*.

[3] Igor Bandos and Wolfgang Kummer, *Superstring 'ending' on super-D9-brane: a supersymmetric action functional for the coupled brane system, Preprint, TUW/99-11, *hep-th/9906041*, Nucl.Phys. **B** (1999) (in press).

[4] B.M. Barbashov and V.V. Nesterenko, *Introduction to Relativistic String Theory*. World Scientific 1990.

[5] I. A. Bandos and A. A. Zheltukhin, *JETP. Lett.* **54**, 421 (1991); *Phys. Lett. B288*, 77 (1992); *Phys.Part.Nucl. 25*, 453 (1994); *Class.Quantum Grav. 12*, 609 (1995).

[6] I. Bandos, D. Sorokin and D. Volkov, *Phys.Lett. B352* 269 (1995) (*hep-th/9502141*).

[7] I. Bandos, D. Sorokin and M. Tonin, *Nucl.Phys.B497*, 275 (1997).

[8] V. Akulov, I. Bandos, W. Kummer and V. Zima, *Nucl.Phys. B527*, 61 (1998) (*hep-th/9802032*).

[9] E. Sokatchev, *Phys. Lett. B169*, 209 (1987); *Class.Quantum Grav. 4*, 237 (1987).

[10] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky and E. Sokatchev, *Class.Quantum Grav. 1*, 498 (1984); 2, 155 (1985).

[11] J.A. De Azcarraga and J. Lukierski, *Phys.Lett. B113*, 170 (1982).

[12] D. Sorokin, V. Tkach and D.V. Volkov, *Mod.Phys.Lett. A4*, 901 (1989).

[13] E. Bergshoeff and P.K. Townsend, *Nucl.Phys. B490* (1997) 145.

[14] I. Bandos, P. Pasti, D. Sorokin, M. Tonin and D.V. Volkov, *Nucl.Phys. B446*, 79–119 (1995) (*hep-th/9501113*).

[15] P.S. Howe and E. Sezgin, *Phys.Lett B390*, 133 (1997); *B394*, 62 (1997); P.S. Howe, E. Sezgin and P.C. West, *Phys.Lett. B399*, 49 (1997); D. Sorokin, *Superbranes and superembeddings, hep-th/9906142*, Phys.Repts. (in press).