Non-Linear Relativity in Position Space

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We propose two methods for obtaining the position space of non-linear relativity, i.e. the dual of its usual momentum space formulation. In the first approach we require that plane waves still be solutions to free field theory. This is equivalent to postulating the invariance of the linear contraction between position and momentum spaces, and dictates a set of energy-dependent spacetime Lorentz transformations. In turn this leads to an energy dependent metric. The second, more problematic approach allows for the position space to acquire a non-linear representation of the Lorentz group independently of the chosen representation in momentum space. This requires a non-linear contraction between momentum and position spaces. We discuss a variety of physical implications of these approaches and show how they point to two rather distinct formulations of theories of gravity with an invariant energy and/or length scale.

I. INTRODUCTION

A number of paradoxes in quantum gravity (e.g. [1]) suggest that it may be desirable to re-examine the nature of Lorentz invariance. We briefly review one example. We know that the combination of gravity ($G$), the quantum ($\hbar$) and relativity ($c$) gives rise to the Planck length, $l_P = \sqrt{\hbar G/c^3}$, the Planck time $t_P = l_P/c$, and the Planck energy $E_P = \hbar/t_P$. These scales mark thresholds beyond which the classical description of space-time breaks down and qualitatively new phenomena are expected to appear. However, whatever quantum gravity may turn out to be, it is expected to agree with relativity for all experiments probing the nature of space-time at, say, length scales much larger than $l_P$. So the question arises: in whose reference frame are $l_P$, $t_P$ and $E_P$ the thresholds for new phenomena? For suppose that there is a physical length scale which measures the size of spatial structures in quantum space-times, such as the discrete area and volume predicted by loop quantum gravity. Then, if this scale is $l_P$ in one inertial reference frame, special relativity suggests it may be different in another observer’s frame – a straightforward implication of Lorentz-Fitzgerald contraction.

Solutions to these problems have been proposed within the framework of what is variously known as non-linear, deformed, or doubly special relativity (DSR) [2,3,1]. These theories may be understood as non-linear realizations of the Lorentz group [4], in which modified Lorentz transformations reduce to the usual ones at low energies, but leave invariant $E_P$, the borderline between classical and quantum gravity. These theories can explain the recent claims for high-energy cosmic ray anomalies [5–7,4] (see also [8–10]). This is due to the introduction of deformed dispersion relations (i.e. departures from $E^2 - p^2 = m^2$). Even though considerable doubts still hang over these results (and considerable caution should therefore be exercised by theorists), they are intriguing, and if true, very far-reaching indeed.

Given that deformed dispersion relations provide for observational input into these theories, a momentum space formulation has been preferred in the past (see [11–13] for notable exceptions). Once this choice is made, however, recovering position space is highly non-trivial. With loss of linearity, duals no longer mimic one another, i.e. vectors no longer transform according to the inverse linear transformation of co-vectors. Defining the spacetime structure of DSR is therefore highly non-trivial.

This is not merely a formal annoyance, as the physical interpretation of many results obtained in momentum space can only be fully assessed when the relation to position space is clarified. For instance, as recently discussed [14–16], one may have trouble defining physical velocity in these theories. The primary definition of velocity should be the “space-time” velocity:

$$v = \frac{dx}{dt},$$

which is the relevant quantity, for example, in gamma-ray timing experiments [17]. It has been argued that the Hamiltonian expression for the velocity,

$$v = \frac{dE}{dp}$$

should be preserved, as this is the group velocity in normal circumstances. However, this has never been generally proved (with [14] even claiming that this leads to contradictions; see also [16]). Clearly this matter can only be settled when one rediscovers position space from the usual momentum space formulation, and the answer will depend on the particular construction of position space employed. (This matter only adds complexity to the issues raised in [19,20,18]).

In this paper we propose two possible ways to rediscover position space in non-linear relativity. The first
approach might seem the least obvious, but turns out to be more rewarding in terms of physical implications and internal consistency. It should be seen as the heart of this paper. The second approach mirrors techniques used for setting up DSR in momentum space, but runs into severe problems. We spell them out in the hope that they might be soluble.

In the first approach we start by requiring that free field theories have plane wave solutions, with momenta satisfying the set of deformed dispersion relations characterizing a given DSR theory. Even though in Section II we give explicit examples of such field theories, our paper does not depend on their details, but only on the existence of plane wave solutions. That such solutions exist is equivalent to requiring that the contraction between momenta and positions remain linear (a linear contraction provides the invariant phase for plane waves).

In Section III it is found that the invariance of such a linear contraction fully fixes the form of the Lorentz transformations in position space. Our main result is that the new Lorentz transformations are energy dependent, and so the space-time metric becomes energy dependent too. The physical meaning of $E$ in this dependence is fully spelled out in Section IV. Further physical implications are explored in Section V.

The second approach, developed in Section VI, is more unimaginative: we simply introduce a non-linear representation of the Lorentz group in position space. This provides the invariant phase for plane waves). The second approach mirrors techniques used for internal consistency. It should be seen as the heart of this literature.

We wish to propose an associated position space in such a way that the momentum space formulation uniquely fixes the transformation laws in position space. This can be achieved by introducing an appropriately strong physical requirement; for example by demanding that there be plane wave solutions to free field theories. By this we mean solutions of the form

$$\phi = Ae^{-ip_a x^a}$$

where $p_a$ satisfies deformed dispersion relations (3), and the contraction $p_a x^a$ remains linear ($p_a x^a = px^0 + px^1$).

Without a linear contraction such waves would not be "plane".

Even though this is the only postulate used in this approach, it is possible to exhibit field theories satisfying this requirement. Considering, say, scalar field theory, one should set up equations of motion via the replacement

$$p_a \to i\partial_a$$

applied to the dispersion relations (3). For example, if the momentum space formulation is based upon the invariant:

$$E^2 - p^2 = m^2$$

we should have a "Klein-Gordon" equation:

$$\left(\eta^{ab} \frac{\partial_a}{1 - il_p \partial_0} \frac{\partial_b}{1 + il_p \partial_0} + m^2\right) \phi = 0,$$

This equation is linear in $\phi$, and has plane wave solutions $\phi = Ae^{-ip_a x^a}$ with $p_a$ satisfying the dispersion relations (8). It can be obtained from the Lagrangian:

$$L = \frac{1}{2} \left[ \eta^{ab} \frac{\partial_a}{1 + il_p \partial_0} \phi \frac{\partial_b}{1 - il_p \partial_0} \phi + m^2 \phi^2 \right].$$

Another example is the $U$ chosen in [1], for which the deformed dispersion relations are:

$$\frac{\eta^{ab} p_a p_b}{(1 - l_p p_0)^2} = m^2.$$

In this case prescription (7) leads to

$$\left(\eta^{ab} \frac{\partial_a}{1 - il_p \partial_0} \frac{\partial_b}{1 + il_p \partial_0} + m^2\right) \phi = 0.$$
Again there are plane wave solutions with $p_a$ satisfying (11).

These examples are given as illustration, and the rest of our paper does not depend on the details of DSR field theory. However we believe that this framework for relating deformed dispersion relations and field theory is likely to be the field theory extension of DSR. DSR was initially set up on purely kinematical grounds, considering classical point particles. With prescription (7) the field equations are always linear and have plane wave solutions or their most general superposition. The plane waves carry a momentum satisfying deformed dispersion relations. Even though we only considered scalar fields it is possible to generalize our construction to arbitrary spins. The result should be compared with the frameworks of [18,21] and we shall comment on this matter at the end of the next section.

If the deformed dispersion relations have a maximum energy (as is the case with (11)), such field theories have a natural “Lorentz invariant” energy cut-off, and so are finite when interactions are considered. Hence the issue of their renormalizability should not arise, since they are naturally finite. A more concrete example of the effect of deformed dispersion relations in effective field theory may be found in [22].

Finally, despite their aspect, the theories we have displayed are “local” and “causal”; clearly not in the sense of the linear Lorentz group, but in the modified sense of non-linear Lorentz transformations, which we shall now spell out.

### III. THE DUAL SPACE-TIME AND ITS METRIC

It turns out that the criterium used in the previous section for setting up field theory is sufficient to fully fix the properties of position space. If there are plane wave solutions for scalar fields, then the linear contraction the properties of position space. If there are plane wave solutions for scalar fields, then the linear contraction

$$t' = \gamma \left( t - v_x \frac{f(E)}{g(E)} \right) \frac{f(E')}{f(E)}$$

$$x' = \gamma \left( x - v_y \frac{g(E)}{f(E)} \right) \frac{g(E')}{g(E)}$$

$$y' = y \frac{g(E')}{g(E)}$$

$$z' = z \frac{g(E')}{g(E)}$$

where $E'$ is the Lorentz boosted energy. Hence the space-time Lorentz transformations now depend upon the energy, and space-time has become intrinsically mixed with energy-momentum.

Transformations (15) have the invariant:

$$s^2 = \eta_{ab} U^a(x) U^b(x) = -\frac{t^2}{f^2} + \frac{(x^i)^2}{g^2}.$$  

Hence the empty space metric is now

$$g_{ab}(E) = \text{diag} \left[ -\frac{1}{f^2(E)}, \frac{1}{g^2(E)}, \frac{1}{g^2(E)}, \frac{1}{g^2(E)} \right]$$

In order to preserve $g_{ab} g^{bc} = \delta_a^c$ we should also have the inverse metric:

$$g^{ab}(E) = \text{diag} [-\frac{f^2(E)}{g^2(E)}, g^2(E), g^2(E), g^2(E)]$$

Notice that in the low energy limit ($Ep \ll 1$) we have $g_{\mu\nu} \approx \eta_{\mu\nu}$ and $g^{\mu\nu} \approx \eta^{\mu\nu}$.

This structure should elucidate the difference between the field theories proposed in the previous section, and those of [18,21]. It was suggested in [18] that the non-linearities of DSR could – and should – be undone when setting up field theory. This was shown for fermionic fields first, but also claimed for all spins (including, presumably, those of spin 0, i.e. scalar fields). The argument centered on the absence of a well defined position space dual to the non-linear momentum variables.

This issue was first solved by [21] with the introduction of non-commutative position space. In this paper we have introduced instead an energy-dependent position space. Field theories based on these two approaches are quite distinct, and also crucially different from those of [18]. But both stress the non-trivial nature of DSR once a suitably modified position space is introduced.

### Notes

1. The argument that the non-linearities in DSR can be removed by making a redefinition of energy and momenta has been refuted by two further developments: the explicit example of 2+1 gravity [23] and the realization that DSR’s phase space has an invariant curvature [24,25].
IV. AN EXAMPLE AND SOME PHYSICAL CONSIDERATIONS

As an example we may consider the form of $U$ chosen in [1], associated with dispersion relations (11). In this case it was found that energy and momentum boosts in the x-direction assume the form

$$E' = \frac{\gamma(E - vp_x)}{(1 + (\gamma - 1)p_x E - \gamma l_x p_x)}$$

$$p'_x = \frac{\gamma(p_x - vE)}{(1 + (\gamma - 1)p_x E - \gamma l_x p_x)}$$

$$p'_y = \frac{p_y}{(1 + (\gamma - 1)p_x E - \gamma l_x p_x)}$$

$$p'_z = \frac{p_z}{(1 + (\gamma - 1)p_x E - \gamma l_x p_x)}$$

(19)

The energy-momentum invariant is (11), from which one can read off the forms of the functions $f$ and $g$. Thus we find that

$$t' = \gamma(t - vx)[1 + (\gamma - 1)p_x E - \gamma l_x p_x]$$

$$x' = \gamma(x - vt)[1 + (\gamma - 1)p_x E - \gamma l_x p_x]$$

$$y' = y[1 + (\gamma - 1)p_x E - \gamma l_x p_x]$$

$$z' = z[1 + (\gamma - 1)p_x E - \gamma l_x p_x]$$

(20)

Transformations (15) have the invariant:

$$s^2 = \frac{-t^2 + (x')^2}{(1 - l_x E)^2}$$

(21)

Faced with energy dependent space-time transformations and metric a natural issue is the physical meaning of $E$ in these transformations. Whose $E$ is this? To answer this question we note that our construction follows from demanding that free field theories should accept plane wave solutions. In turn this requires that the linear contraction providing their phases should be a scalar or invariant. Thus the $E$ appearing in the Lorentz transformations is the $E$ of the plane wave used to probe space-time.

A similar argument can be made for wave packets and “particles”. The transformation laws for the position of a given particle depend on its energy as seen by a given observer. Particles with different energy, at the same position and time, must transform differently and feel a different metric. Different observers see a given particle with different energies, and so assign to it different Lorentz transformations and metric.

So not only different observers see a given particle being affected by different metrics, but the same observer will assign different metrics to different particles. The whole construction is clearly not invariant in the sense of linear Lorentz transformations – it is invariant (by construction) in the sense of non-linear momentum Lorentz transformations.

In our picture space-time and energy and momentum have become intertwined. Even ignoring gravitation, we cannot talk about time and position without a particle being there. The properties of this particle’s position and time depend on its energy. At low energies, the dependence is very weak, so we have the illusion that space-time exists independently of the (test) particles that might fill them.

V. FURTHER PHYSICAL IMPLICATIONS

Lorentz contraction and time dilation are quite different in this theory. Let us consider first the example of transformations (20). If we boost a stationary rod with one endpoint attached to the origin and the other at $x = l$, we find that its tip transforms into $x' = l' - vt'$, with

$$l' = \frac{l}{\gamma}[1 + (\gamma - 1)p_x E_0].$$

(22)

where

$$E_0 = \frac{m}{1 + l_p m}$$

(23)

is the particle’s rest energy (see [4]; also Eq. (11)). This provides the revised Lorentz contraction formula, and we notice that for $E_0 = E_p$ all lengths are invariant: $l = l'$. From $x' = l' - vt'$ we see that the particle’s velocity in the new frame is $v$. This shows that the boost parameter $v$ is the origin’s velocity, something far from obvious as discussed in the literature.

Boosting $x = v_0 t$ we may also show that the law of addition of velocities is the same as in special relativity:

$$v' = \frac{v - v_0}{1 - vv_0}$$

(24)

The time dilation formula, on the other hand, becomes:

$$\Delta t' = \gamma \Delta t[1 + (\gamma - 1)p_x E_0].$$

(25)

More generally for the extreme case of a Planck energy wave ($p_0 l_p = 1$), at rest we have $p_a = (E_p, 0, 0, 0)$, so that

$$t' = \gamma^2(t - vx)$$

(26)

$$x' = \gamma^2(x - vt)$$

(27)

$$y' = y\gamma$$

(28)

$$z' = z\gamma$$

(29)

As $v \to c$, $t'$ and $x'$ approach infinity faster than regular Lorentz boosts by a factor of $\gamma$. There is also a transverse effect, absent in the usual theory.

Performing a similar exercise with the more general transformation (15) we find the time dilation formula

$$\Delta t' = \gamma \Delta t \frac{f(E')}{f(E)}$$

(30)
and the Lorentz contraction formula:
\[ t' = \frac{t - \frac{vx}{c^2}}{\gamma} \frac{g(E')}{g(E)} \]
(31)

In general the boost parameter \( v \) is not the velocity, of the moving frame, \( v_f \), which is
\[ v_f = v \frac{g(E')}{f(E')} \]
(32)

By transforming \( x = vt \) we would also arrive at the general law of addition of velocities. However, the result is at first cumbersome – and shall not be reproduced here – until we consider the following.

Setting \( m = 0 \) in the deformed dispersion relations (3), we find that the coordinate speed of massless plane waves (with phase \( x^a p_a = Et - p \cdot x \)) is
\[ c(E) = \frac{dx}{dt} = E = \frac{g(E)}{f(E)}. \]
(33)

This is in contradiction with the usual Hamiltonian expression
\[ c = \frac{dE}{dp}. \]
(34)

Thus, in this formalism we have an energy-dependent speed of light, but its speed is given by \( E/p \), not \( dE/dp \). This is also the speed of null signals, obtained by setting \( ds^2 = 0 \) in Eq. (16). Thus massless waves move along null surfaces. As we shall see now, the speed of light is also an observer invariant speed in a suitably modified sense.

Given (33), the general case of the law of addition of velocities may be written in a more amenabel form:
\[ \frac{v'}{c(E')} = \frac{\frac{dx}{dt} - \frac{v}{c(E)} \frac{dv}{dE} f(E')} {1 - \frac{v}{c(E)} \frac{dv}{dE} f(E')} \]
(35)

This shows that the law \( c = c(E) \) is an invariant. If light rays in one frame propagate with \( c(E) \), then in another frame their energy will be \( E' \) and their transformed speed will be \( c(E') \).

We finally note that we have not considered here wave packets, but it’s conceivable that their “space-time” speed differs from the expression above. This has the implication that the space-time Lorentz transformations for wave packets may be different than those for plane waves.

VI. A LESS PROMISING APPROACH

It was shown in [4] how to introduce a general non-linear representation of the action of the Lorentz group in momentum space, corresponding to given deformed dispersion relations. A formally identical procedure can be applied in position space. The non-linearity that arises in doing so destroys translational invariance. Thus, all considerations in this section are to be seen as statements made in reference to small displacements about the origin (which can be a generic point), in the same sense that free falling frames in general relativity correspond to local neighbourhoods. The position space in this section is thus comprised of a set of coordinates \( dx^a \) rather than global coordinates \( x^a \). Because they are all differences with respect to the origin they should not be added or subtracted. Indeed the non-linearity of their transformation would preclude such an operation (one could, of course add and subtract these quantities non-linearly and thus achieve covariance, but that is not necessary here).

In analogy with [4] we construct non-linear realizations of the Lorentz group in position space by means of a transformation \( V(x) \) such that the new generators of the Lorentz group with respect to space-time coordinates are
\[ K^i = V^{-1} L^i_0 V, \]
where
\[ L_{ab} = x_a \partial \partial x^b - x_b \partial \partial x^a \]
(37)

are the standard Lorentz generators. Exponentiation of these generators reveals a non-linear realization of the group. Naturally this requires that the contraction between infinitesimal displacements (position) and momentum be non-linear, for it to remain invariant. The contraction should be defined as
\[ \langle p, x \rangle = U_a (p) V^a (x) \].
(38)

Sadly, this implies the absence even locally of plane wave solutions, and thus of a suitable Hilbert space.

The first non-linear representation of the Lorentz group [11–13] to have been proposed precedes DSR and is an example of this procedure. Taking
\[ V^a (x) = x^a \frac{1}{t + R/c_0}, \]
leads to:
\[ t' = \frac{\gamma (t - vx)}{1 - (\gamma - 1) \frac{v}{c_0} + \gamma \frac{v}{c_0} \frac{v}{c_0}} \]
(40)
\[ x' = \frac{\gamma (x - vt)}{1 - (\gamma - 1) \frac{v}{c_0} + \gamma \frac{v}{c_0} \frac{v}{c_0}} \]
(41)
\[ y' = y \]
(42)
\[ z' = z \]
(43)

the transformation first proposed by Fock. This transformation produces an invariant length \( R \), but given that \( R \) is large instead of small this transformation doesn’t seem to play a role in quantum gravity. We now discuss conditions upon \( V \) leading to more interesting transformations.
Whereas the pure momentum space formulation of non-linear relativity treats the Planck energy \(E_P\) as an invariant, we now require that the Planck time \(t_P\) be an invariant. The only invariant times in linear relativity occur at zero and infinity, which implies for the non-linear case that \(V(t_P) = 0\) or \(V(t_P) = \infty\) should hold. However, we want the range of the transformed time coordinates, \(V(t)\), to cover the domain of the boost transformations for all values of the boost parameter \(v\). Thus, the range of \(V(t)\) should be \([0, \infty]\). In addition we need \(V \approx 1\) in the \(t \gg t_P\) limit such that the ordinary Lorentz transformations are recovered for large \(t\). Combining these conditions implies that \(V(t_P) = 0\) is the correct condition. (Note that in momentum space, on the other hand, the appropriate condition is \(U(E_P) = \infty\); this is discussed in detail in [4]).

The simplest theory we found with these properties is generated by the transformation

\[
V^a(x) = x^a \left(1 - \frac{t_P}{t}\right),
\]

which can be written as the exponential operator

\[
V = e^{-\frac{t_P}{t} D},
\]

where \(D = x^a \partial_a\) is a dilatation. The boost generators then take the form

\[
K^i = L^i_0 - \frac{t_P x^i}{t^2} D,
\]

which, in turn, induce a non-linear representation of a Lorentz boost in the \(x\)-direction given by

\[
t' = \gamma(t - vx) \left(1 - \frac{t_P}{t}\right) + t_P
\]

\[
x' = \gamma(x - vt) \left(1 - \frac{t_P}{t} + \frac{t_P}{\gamma(t - vx)}\right)
\]

\[
y' = y \left(1 - \frac{t_P}{t} + \frac{t_P}{\gamma(t - vx)}\right)
\]

\[
z' = z \left(1 - \frac{t_P}{t} + \frac{t_P}{\gamma(t - vx)}\right)
\]

From (47) we see that the time dilation formula is now

\[
\Delta t' = \gamma(\Delta t - t_P) + t_P.
\]

It follows that the duration \(\Delta t = t_P\) is invariant, as expected. We may now invariantly impose as a physical condition \(\Delta t > t_P\). Hence we may remove sub-Planckian durations from the theory in an invariant form, solving the paradox that opened this paper. Observers will never mix sub and super Planckian regimes.

The transformation just proposed may be refined in a variety of ways. In the proposed transformation we have broken time reversal invariance. This may be restored by choosing

\[
V^a(x) = x^a \left(1 - \frac{t_P}{t}\right) \quad \text{for} \quad t > t_P
\]

\[
= x^a \left(1 + \frac{t_P}{t}\right) \quad \text{for} \quad t < -t_P
\]

\[
= 0 \quad \text{otherwise}
\]

Then the Lorentz transformations derived above are valid for \(t > t_P\) (which implies \(t' > t_P\)). For \(t < -t_P\) one should swap the sign of \(t_P\). Durations \(-t_P < t < t_P\) may now be removed invariantly. If we want to remove sub-Planckian durations without having to posit an (invariant) physical conditions we could choose:

\[
V = \log \left(1 - \frac{|t_P|}{t}\right).
\]

The range \(-t_P < t < t_P\) does not have an image inside the resulting representation of the Lorentz group, and so is removed without the need to impose any supplementary condition.

If time durations smaller than \(t_P\) become non-physical, the concept of exact simultaneity becomes meaningless. Concomitantly, lengths (which are spatial separations evaluated at the same time) depend on how we redefine simultaneity. Thus, instead of deriving the Lorentz contraction formula with respect to exact simultaneity, we let \(t\) represent a fixed course-grained duration \(T\), to which we refer length measures. The resulting Lorentz contraction formula, obtained from (47) and (48) with this prescription, does not have a simple analytic form, but is straightforward to implement numerically. Naturally \(T \ll l\) but \(T \geq t_P\), so that lengths \(l\) smaller than \(l_P\) also become unphysical in this theory.

Finally note that the finite space-time invariant, say for (44), is

\[
\Delta s^2 = (\Delta t - t_P)^2 - \Delta x^2 \left(1 - \frac{t_P}{\Delta t}\right)^2.
\]

Hence Planck durations become null. This conclusion applies to other choices of \(V\) that leave the Planck scale invariant.

**VII. WEIGHING THE OPTIONS**

We presented two distinct ways to define position space in non-linear special relativity (which are by no means exhaustive). In the first construction we let the non-linear realization of the Lorentz group in momentum space fully fix space-time by requiring that the contraction should remain linear, and that free field theories should have plane wave solutions. The result is an energy-dependent space-time. In the second we allow for any non-linear representation of the Lorentz group to be introduced independently in position and momentum space. A non-linear contraction then carries the burden of enforcing
invariance. This precludes plane wave solutions and simple field theories and for this reason we strongly favour the first approach.

Furthermore the first approach proposed in this paper simplifies enormously the definition of a theory of gravity based on non-linear special relativity. None of the “doubly special” theories have so far been formulated to incorporate space-time curvature, and thus gravitation. A related issue is the fate of the metric structure of space-time in these theories, and by extension, in quantum gravity.

It has been suggested [1] that non-linear realizations of the Lorentz group (in real space) have no quadratic invariant, and thus no metric. At high energy (or at small distances) the concept of metric simply disintegrates. However, given that the structure constants of the symmetry algebra are left unchanged (we are merely changing representation), it could be that much of the usual connection-based construction of general relativity could still be salvaged. This program fits the second of our constructions, but it has not, so far, been accomplished. It may even be claimed that such an enterprise is impossible.

Curiously, a much simpler program emerges if we adopt the first of our constructions. In that case, the space-time transformations remain linear, albeit energy-dependent, and so retain a quadratic invariant. An energy-dependent metric can still be defined, such that in some sense, the metric “runs” with the energy. This approach leads to a very simple extension of general relativity, capable of accommodating doubly special relativity, and is explored in [29]. It remains to be seen how strong gravity phenomena – such as black hole dynamics and Hawking’s radiation – behave in this theory.

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