Examining a Round Vessel Relative to the Impact Posed By Porous Media and Constant Wall Permeability on the Creeping Flow: In Relation to Computational and Applied Mathematics Techniques

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Abstract--In this study, the main aim was to determine the impact of the parameters of porous media and constant wall permeability on the round vessel’s creeping flow. To determine the exact solution, the study employed the inverse technique. Also, the mathematical expression for leakage flux, flow rate, pressure distribution, wall shear stress, velocity components, and stream function were used to understand the flow behavior. The motivation was to determine how flow properties exist relative to porous media and wall permeability. From the findings, the study established that there is a reduction in fluid velocity when the permeability of the porous medium is low. The maximum permeation rate was recorded at 84% and was obtained following high osmotic pressure drop and increased wall permeability. Similarly, the porous media’s state of porosity did not affect the amount of permeable fluid, as well as leakage flow. Rather, these parameters dependent on the nature of constant wall permeability.

1. Introduction

In the recent past, material flows through porous media regimes have gained increasing scholarly attention. Some of the fields to which the regimes apply and have prompted the increasing attention include medical, industry, and engineering friends [1, 2]. Also, areas such as fluid flow involving obstructed renal tubules, lodging, well drilling, and the production of natural gases and petroleum call for several predictions that consider fluid flow outcomes as they interact with porous media [3]. It is also notable that many factors affect fluid motion. Boundary states that have also been documented to affect fluid motion include porous boundaries, oscillatory boundaries, moving boundaries, fluctuating boundaries, and stationery boundaries [4, 5]. The natural existence of cylindrical, circular, parabolic, rectangular, parallel plate, and duct channels or boundaries has also seen them gain increasing scholarly attention [6].

In vessels such as the intestine, capillaries, renal tubules, veins, and arteries, many flows (such as circulatory and gastric) allow for the deposition of waste material [7, 8]. The deposition causes diseased systems [9]. As such, normal vessels with porous media could be treated via frameworks responsible for evaluating supplementary drag forces that solid matrix numbers exert on the flow [10]. The matrix numbers could be represented by attributes such as fatty bunches, bacterial masses, and food items that have not been digested [4]. It is also notable that for these materials, disturbance to the flow could result from their viscosity in circular tubes [7], hence full or partial blockages [9, 10]. In this study, the central purpose was to examine material flow through a diseased or affected vessel and a healthy vessel. The motivation of the investigation was to determine the impact of parameters of constant wall permeability and porous media.
2. Methodology

The study considered an incompressible viscous fluid and its creeping flow through a round vessel. Also, porous media was used to fill the vessel. The radius and length of the vessel was set as $R$ and $L$ respectively. For the flow dynamics, the study considered radial and axial directions, translating into axisymmetric coordinates. Notably, the radial axis was perpendicular to the vessel’s axis while axial axis was in the direction or along the vessel’s axis. A central assumption that the study held was that the re-absorption was constant, rather than fluctuating. For the flow, the governing equations were derived as follows:

$$v = [v_r(r, z), 0, v_z(r, z)]$$

$$\frac{1}{\gamma} \frac{\partial}{\partial \gamma} (\rho v_r) + \frac{\partial v_r}{\partial z} = 0$$

$$\frac{\partial p}{\partial \gamma} = \mu \left( \frac{\partial^2 v_r}{\partial r^2} + \frac{1}{\gamma} \frac{\partial v_r}{\partial \gamma} - \frac{1}{\gamma^2} \frac{\partial^2 v_r}{\partial z^2} \right) - \frac{\mu v_r}{\gamma}$$

$$\frac{\partial p}{\partial \gamma} = \mu \left( \frac{\partial^2 v_z}{\partial r^2} + \frac{1}{\gamma} \frac{\partial v_z}{\partial \gamma} + \frac{\partial^2 v_z}{\partial z^2} \right) - \frac{\mu v_z}{\gamma}$$
3. Results and Discussion

Indeed, the analysis of the results was done in two ways. On the one hand, Mathematica aided in generating theoretical results in such a way that one flow parameter would be varied and others fixed. On the other hand, all physiological situation relevant values were used to generate physiological results. To ensure that the graphical representation was convenient, non-dimensionalization was introduced in the form:

$$\gamma^* = \frac{\gamma}{R}, z^* = \frac{z}{R}, U^* = \frac{U}{R}, P^* = \frac{p}{\mu U R}, k^* = \frac{k}{R^2}$$

In the equation above, the fluid entrance velocity was represented by \( U \). Also, the prediction of the results in the entirety was done in relation to the investigation along axial positions. These axial positions included the vessel exit where \( z=0.6 \), the middle of the vessel where \( z=0.3 \), and the entrance of the vessel where \( z=0.1 \). The figure below shows the initial results that were obtained. In the figure, it is evident that as the porous media’s parameter of permeability increased, there was an increase in radial velocity. However, the radial velocity was constantly symmetric at all positions as the analysis and material flow proceeded downstream. The factor that could explain this trend, as documented in the previous literature \([4, 9]\), is constant permeation.

**Figure 2: Correlation between the porous media’s permeability and radial velocity**

\[
\nu_\gamma = 0, \frac{\partial \nu_\gamma}{\partial \gamma} = 0 \text{ at } \gamma = 0 \\
\nu_\gamma = \nu_\rho, \nu_\rho = 0 \text{ at } \gamma = R \\
2\pi \int_0^R \lambda \nu_\gamma (\gamma, z) d\gamma = Q_0 \text{ at } z = 0, \\
p = p_0 \text{ at } z = 0 \text{ and } p = p_1 \text{ at } z = L \\
\frac{\partial}{\partial \gamma} \left[ \frac{1}{2} \frac{\partial \nu_\gamma}{\partial \gamma} \right] + \frac{\partial \nu_\gamma}{\partial z} \frac{\partial}{\partial \gamma} \left( \frac{1}{2} \frac{\partial \nu_\gamma}{\partial \gamma} \right) + \frac{\partial^2 \nu_\gamma}{\partial z^2} + \frac{1}{k} \left( \frac{\partial \nu_\gamma}{\partial z} \frac{\partial \nu_\gamma}{\partial \gamma} \right) = 0
\]

Hence the fractional permeation rate of:

$$FPR = \frac{Q(L) - Q(0)}{Q(0)} = \frac{2\pi R v_L^2 L}{Q}$$
Another trend that was investigated involved wall permeation velocity and radial velocity behavior. Specific findings demonstrated that with an increase in the wall permeability velocity rate, there was an increase in radial velocity. The figure below summarizes these findings.

![Figure 3: Correlation between the rate of wall permeability velocity and radial velocity](image)

When the axial velocity was investigated at the exit, the mid place, and the entrance of the vessel, an increase in the permeability k caused a gradual increase in axial velocity. However, an increase in wall permeation velocity was observed to cause a decrease in axial velocity. For increasing wall permeation velocity and the permeability, the study establish a linear increase in hydraulic pressure distribution in the vessel. Of significance to note is that most of these insights were gained from a rat kidney’s physiological data, which aided in explaining experimental outcomes and how they compared with theoretical outcomes in the previous literature.

Additional values that were analyzed included leakage flux, pressure drop, and wall permeability. From the results, the study found that in the wall experiencing constant permeation, maximum values for the leakage flow rate and pressure drop were obtained when wall permeability increased. Also, these results were compared with experimental data reported in the previous literature. From the comparison, an emerging theme or trend was that in the diseased vessel, there was a higher percentage of the difference in the rate of fractional permeation (between this study’s outcomes and the experimental data) than the case of the healthy vessel.

4. Conclusion and Future Directions

Given axial distance, this study focused on a round vessel. A central assumption was that wall permeability was a constant function. The experimental setup was arranged in such a way that some waste material was used to fill the vessel. The material acted as porous media. The interpretation of the outcomes considered parameters of constant wall permeation and the impact of porous media. To interpret the results, the study considered a rat kidney’s physiological data. From the findings, one of the themes that emerged was that an increase in media porosity increased the length of stay of the fluid within the vessel. In turn, there was an increase in FPR percentage (85%). The difference between the latter results of the study and the experimental data was found to be 11%. It is also notable that the study examined the behavior of a leaky diseased renal tubule, which represented a diseased permeable vessel. With constant wall permeability and the porous media’s permeability parameter approaching or tending to infinity, the homogenous fluid’s solution was obtained. Specific results demonstrated that with wall permeability approaching zero and the permeability parameter tending to infinity, the vessel would exhibit a homogenous fluid regime, a trend complemented by the presence of an impermeable boundary. These classical results in relation to the aforementioned experimental conditions about the permeability parameter and wall permeability reflected Poiseuille flow. With the permeability parameter set to free after wall permeability tends to zero, the walls of the vessel do not experience
permeation; hence, a homogenous wall ends up being retrieved due to the flow through the porous media regime. Given that the vessel’s assumed constant wall permeability was not an ideal scenario, the implication for the future of computation and applied mathematics is that the study’s results could be refined by considering relevant variable wall permeability of round vessels that provide room for partial obstruction.

5. References

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