Model Independent Bounds on Magnetic Moments of Majorana Neutrinos

Nicole F. Bell, Mikhail Gorchtein, Michael J. Ramsey-Musolf, Petr Vogel, and Peng Wang

California Institute of Technology, Pasadena, CA 91125, USA

(Dated: September 23, 2006)

We analyze the implications of neutrino masses for the magnitude of neutrino magnetic moments. By considering electroweak radiative corrections to the neutrino mass, we derive model-independent naturalness upper bounds on neutrino magnetic moments, \( \mu_\nu \), generated by physics above the electroweak scale. For Dirac neutrinos, the bound is several orders of magnitude more stringent than present experimental limits. However, for Majorana neutrinos the magnetic moment contribution to the mass is Yukawa suppressed. The bounds we derive for magnetic moments of Majorana neutrinos are weaker than present experimental limits if \( \mu_\nu \) is generated by new physics at \( \sim 1 \) TeV, and surpass current experimental sensitivity only for new physics scales \( > 10 - 100 \) TeV. The discovery of a neutrino magnetic moment near present limits would thus signify that neutrinos are Majorana particles.

I. INTRODUCTION

In the Standard Model (minimally extended to include non-zero neutrino mass) the neutrino magnetic moment is given by [1]

\[ \mu_\nu \approx 3 \times 10^{-19} \left( \frac{m_\nu}{\text{TeV}} \right) \mu_B. \]  

(1)

An experimental observation of a magnetic moment larger than that given in Eq. (1) would be an unequivocal indication of physics beyond the minimally extended Standard Model. Current laboratory limits are determined via neutrino-electron scattering at low energies, with \( \mu_\nu < 1.5 \times 10^{-10} \mu_B \) [2] and \( \mu_\nu < 0.7 \times 10^{-10} \mu_B \) [3] obtained from solar and reactor experiments, respectively. Slightly stronger bounds are obtained from astrophysics. Constraints on energy loss from astrophysical objects via the decay of plasmons into \( \nu \pi \) pairs restricts the neutrino magnetic moment to be \( \mu_\nu < 3 \times 10^{-12} \mu_B \). Neutrino magnetic moments are reviewed in [2] [4] [7], and recent work can be found in [8] [9].

It is possible to write down a naïve relationship between the size of \( \mu_\nu \) and \( m_\nu \). If a magnetic moment is generated by physics beyond the Standard Model (SM) at an energy scale \( \Lambda \), as in Fig. 1a, we can generically express its value as

\[ \mu_\nu \sim \frac{eG}{\Lambda}. \]  

(2)

where \( e \) is the electric charge and \( G \) contains a combination of coupling constants and loop factors. Removing the photon from the same diagram (Fig. 1b) gives a contribution to the neutrino mass of order

\[ m_\nu \sim GA. \]  

(3)

We thus have the relationship

\[ m_\nu \sim \frac{\Lambda^2}{2m_\nu} \frac{\mu_\nu}{\mu_B}, \]

\[ \sim \frac{\mu_\nu}{10^{-18} \mu_B} [\Lambda(\text{TeV})]^2 \text{ eV}, \]  

(4)

which implies that it is difficult to simultaneously reconcile a small neutrino mass and a large magnetic moment.

\[ \text{FIG. 1: a) Generic contribution to the neutrino magnetic moment induced by physics beyond the standard model. b) Corresponding contribution to the neutrino mass. The solid and wavy lines correspond to neutrinos and photons respectively, while the shaded circle denotes physics beyond the SM.} \]

However, it is well known that the naïve restriction given in Eq. (1) can be overcome via a careful choice for the new physics. For example, we may impose a symmetry to enforce \( m_\nu = 0 \) while allowing a non-zero value for \( \mu_\nu \) [11] [12] [13], or employ a spin suppression mechanism to keep \( m_\nu \) small [14]. Note however, that these symmetries are typically broken by Standard Model interactions. The original version of the well-known Voloshin mechanism [10] involved imposing an \( SU(2)_\nu \) symmetry, under which the left-handed neutrino and antineutrino (\( \nu \) and \( \bar{\nu} \)) transform as a doublet. The Dirac mass term transforms as a triplet under this symmetry and is thus forbidden, while the magnetic moment term is allowed as it transforms as a singlet. However, the \( SU(2)_\nu \) symmetry is violated by SM gauge interactions. For Majorana neutrinos, the Voloshin mechanism may be implemented using flavor symmetries, such as those in [11] [12] [13]. These flavor symmetries are not broken by SM gauge interactions but are instead violated by SM Yukawa interactions, provided that the charged lepton masses are generated via the standard mechanism through Yukawa couplings to the SM Higgs boson.\(^1\) By calculating neutrino magnetic moment contributions to

\(^1\) If the charged lepton masses are generated via a non-standard
$m_\nu$, generated by SM radiative corrections, we may thus obtain general, “naturalness” upper limits on the size of neutrino magnetic moments.

In the case of Dirac neutrinos, a magnetic moment term will generically induce a radiative correction to the neutrino mass of order \[ m_\nu \sim \frac{\alpha \Lambda^2}{16\pi m_e \mu_B} \mu_\nu \sim \frac{\mu_\nu}{3 \times 10^{-15} \mu_B} [\Lambda(\text{TeV})]^2 \text{ eV}. \] (5)

If we take $\Lambda \approx 1 \text{ TeV}$ and $m_\nu \lesssim 0.3 \text{ eV}$, we obtain the limit $\mu_\nu \lesssim 10^{-15} \mu_B$, which is several orders of magnitude more stringent than current experimental constraints.

The case of Majorana neutrinos is more subtle, due to the relative flavor symmetries of $m_\nu$ and $\mu_\nu$ respectively. The transition magnetic moment $[\mu_\nu]_{\alpha\beta}$ is antisymmetric in the flavor indices $\{\alpha, \beta\}$, while the mass terms $[m_\nu]_{\alpha\beta}$ are symmetric. These different flavor symmetries play an important role in our limits, and are the origin of the difference between the magnetic moment constraints for Dirac and Majorana neutrinos.

It has been shown in [16] that the constraints on Majorana neutrinos are significantly weaker than those obtained in [15] for Dirac neutrinos, as the different flavor symmetries of $m_\nu$ and $\mu_\nu$ lead to a mass term which is suppressed by charged lepton masses. This conclusion was reached by considering one-loop mixing of the magnetic moment and mass operators generated by Standard Model interactions. The authors of Ref. [16] found that if a magnetic moment arises through a coupling of the neutrinos to the neutral component of the SU(2)$_L$ gauge boson, the constraints obtained in [16] for $\mu_{\tau e}$ and $\mu_{e\mu}$ are comparable to present experiment limits, while the constraint on $\mu_{e\tau}$ is significantly weaker. Furthermore, constraints on magnetic moments generated only via a coupling to the $U(1)_Y$ (hypercharge) gauge boson were found to be additionally suppressed. Thus, the analysis of [16] lead to a model independent bound that is less stringent than present experimental limits.

We shall show that two-loop matching of mass and magnetic moment operators implies stronger constraints than those obtained in [16] if the scale of the new physics $\Lambda \gtrsim 10 \text{ TeV}$. Moreover, these constraints apply to a magnetic moment generated by either the hypercharge or SU(2)$_L$ gauge boson. In arriving at these conclusions, we construct the most general set of operators that contribute at lowest order to the mass and magnetic moments of Majorana neutrinos, and derive model independent constraints which link the two. We thus obtain a completely model independent naturalness bound that – for $\Lambda \gtrsim 100 \text{ TeV}$ – is stronger than present experimental limits (even for the weakest constrained element $\mu_{e\mu}$).

Our key findings are summarized in Table II.

Our result implies that an experimental discovery of a magnetic moment near the present limits would signify (i) neutrinos are Majorana fermions and (ii) new lepton number violating physics responsible for the generation of $\mu_\nu$ arises at a scale $\Lambda$ which is well below the see-saw scale. Implications for neutrinoless double beta decay in theories with low scale lepton number violation have been discussed in [17].

II. FRAMEWORK

Following Refs. [15, 16] we assume that the magnetic moment is generated by physics beyond the SM at an energy scale $\Lambda$ above the electroweak scale. In order to be completely model independent, the new physics will be left unspecified and we shall work exclusively with dimension $D > 4$ operators involving only SM fields, obtained by integrating out the physics above the scale $\Lambda$. We thus consider an effective theory that is valid below the scale $\Lambda$, respects the SU(2)$_L \times U(1)_Y$ symmetry of the SM, and contains only SM fields charged under these gauge groups. We assume that all the usual SM interactions are present, including the Yukawa couplings of the charged leptons to the SM Higgs. We shall also work with the flavor states $\nu_\alpha$ (i.e. the basis where the charged lepton masses are diagonal) and discuss the mass eigenstate basis in Section V. Note that in either basis, Majorana neutrinos cannot have diagonal magnetic moments, but are permitted non-zero transition moments.

The lowest order contribution to the neutrino (Majorana) mass arises from the usual five dimensional operator containing Higgs and left-handed lepton doublet fields, $L$ and $H$, respectively:

\[ [O_M]_{\alpha\beta} = (L_\alpha^T e H) (H^T e L_\beta), \] (6)

where $\epsilon = -i\tau_2$, $L^c = L^TC$, $C$ denotes charge conjugation, and $\alpha, \beta$ are flavor indices. The neutrino magnetic moment operator must be generated by gauge invariant operators involving the SU(2)$_L$ and U(1)$_Y$ gauge fields, $W^a_\mu$, $B_\mu$ and $\tilde{B}_\mu$ respectively. The lowest order contribution arises at dimension seven, so we consider the following operators,

\[ [O_B]_{\alpha\beta} = g' (L_\alpha^T e H) \sigma^{\mu\nu} (H^T \epsilon L_\beta) B_{\mu\nu}, \] (7)

\[ [O_W]_{\alpha\beta} = g (L_\alpha^T e H) \sigma^{\mu\nu} (H^T \epsilon \sigma^a L_\beta) W^a_{\mu\nu}, \] (8)

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g_{\epsilon ab} W_b^\rho W^\rho_\nu$ are the U(1)$_Y$ and SU(2)$_L$ field strength tensors, respectively, and $g'$ and $g$ are the corresponding

\[2\] We do not consider models with new light particles, such as $[18]$. 

---

mechanism, SM Yukawa interactions do not necessarily violate flavor symmetries. However, such flavor symmetries must always be broken via some mechanism in order to obtain non-degenerate masses for the charged leptons.
couplings. We also define a 7D mass operator as

\[ [O_M^D]_{\alpha\beta} = (T_{\alpha}^{\nu} c H) (H^{T} \epsilon L_{\beta}) (H^{H}) \]  

(9)

We note that the three 7D operators given by Eqs. 7-10 do not form a closed set under renormalization. The full basis of 7D operators may be found in the appendix.

Operators \( O_M^D \) and \( O_D^7 \) are flavor symmetric, while \( O_B \) is antisymmetric. The operator \( O_W \) is the most general 7D operator involving \( W^\alpha_{\mu\nu} \) (see the appendix for details). However, it is neither flavor symmetric nor antisymmetric; it is useful to express it in terms of operators with explicit flavor symmetry, \( O_W^\nu \), which we define as

\[ [O_W^\nu]_{\alpha\beta} = \frac{1}{2} \{ [O_W]_{\alpha\beta} \pm [O_W]_{\beta\alpha} \} \]  

(10)

Our effective Lagrangian is therefore

\[ \mathcal{L} = \frac{C_{5D}^M}{\Lambda} O_M^5D + \frac{C_{5D}^D}{\Lambda^3} O_M^7D + \frac{C_B}{\Lambda^4} O_B + \frac{C_W^+}{\Lambda^2} O_W^+ + \frac{C_W^-}{\Lambda} O_W^- + \cdots \]  

(11)

where the “\( \cdots \)” denote other terms that are not relevant to the present analysis.

Due to the differences in the flavor structure of \( O_W^\nu \) and \( O_W \), the phenomenological manifestations of \( C_W^\nu \) and \( C_W \) are quite distinct. After spontaneous symmetry breaking, the flavor antisymmetric operators \( O_B \) and \( O_W^\nu \) contribute to the magnetic moment interaction

\[ \frac{1}{2} [\mu_B]_{\alpha\beta} \nu^\alpha \sigma^\mu \nu^\beta \mathcal{F}_{\mu\nu} \]  

(12)

where \( \mathcal{F}_{\mu\nu} \) is the electromagnetic field strength tensor,

\[ \frac{[\mu_B]_{\alpha\beta}}{\mu_B} = \frac{2 m_e e^2}{\Lambda^2} \left( [C_B(M_W)]_{\alpha\beta} + [C_W^-(M_W)]_{\alpha\beta} \right) \]  

(13)

and the Higgs vacuum expectation value is \( \langle H^T \rangle = (0, v/\sqrt{2}) \). The flavor symmetric operator \( O_W^\nu \) does not contribute to this interaction at tree-level. Similarly, the operators \( O_M^D \) and \( O_D^7 \) generate contributions to the Majorana neutrino mass terms, \( \frac{1}{2} [m_\nu]_{\alpha\beta} \nu^\alpha \nu^\beta \), given by

\[ \frac{1}{2} [m_\nu]_{\alpha\beta} = \frac{v^2}{2 \Lambda} \left[ C_{5D}^M (M_W) \right] + \frac{v^4}{4 \Lambda^2} \left[ C_{5D}^D (M_W) \right] \]  

(14)

In sections III and IV below, we calculate radiative corrections to the neutrino mass operators (\( O_M^D \) and \( O_D^7 \)) generated by the magnetic moment operators \( O_W^\nu \) and \( O_B \). This allows us to determine constraints on the size of the magnetic operator in terms of the neutrino mass, using Eqs. 13 and 14. Our results are summarized in Table II below, where the quantity \( R_{\alpha\beta} \) is defined as

\[ R_{\alpha\beta} = \left| \frac{m_\tau^2 - m_\nu^2}{m_\tau^2 - m_\mu^2} \right| \]  

(15)

with \( m_\alpha \) being the masses of charged lepton masses. Numerically, one has \( R_{\tau e} \approx R_{\tau \mu} \approx 1 \) and \( R_{\mu e} \approx 283 \).

\[ \begin{array}{lll}
    \text{i) 1-loop, 7D} & \mu_B^{W_D} \leq 1 \times 10^{-4} \mu_B \left( \frac{m_{\nu_D}}{1 \text{eV}} \right) \ln^{-1} \frac{A^2}{10^9} R_{\alpha\beta} \\
    \text{ii) 2-loop, 5D} & \mu_B^{W_D} \leq 1 \times 10^{-9} \mu_B \left( \frac{m_{\nu_D}}{1 \text{eV}} \right) \left( \frac{1 \text{TeV}}{\Lambda} \right)^2 R_{\alpha\beta} \\
    \text{iii) 2-loop, 7D} & \mu_B^{W_D} \leq 1 \times 10^{-7} \mu_B \left( \frac{m_{\nu_D}}{1 \text{eV}} \right) \ln^{-1} \frac{A^2}{10^9} R_{\alpha\beta} \\
    \text{iv) 2-loop, 5D} & \mu_B^{W_D} \leq 4 \times 10^{-3} \mu_B \left( \frac{m_{\nu_D}}{1 \text{eV}} \right) \left( \frac{1 \text{TeV}}{\Lambda} \right)^2 R_{\alpha\beta} \\
\end{array} \]

Table I: Summary of constraints on the magnitude of the magnetic moment of Majorana neutrinos. The upper two lines correspond to a magnetic moment generated by the \( O_W^\nu \) operator, while the lower two lines correspond to the \( O_B \) operator.

III. SU(2) GAUGE BOSON

We first consider contributions from \( O_W^\nu \) to the mass operators. Contributions to \( O_M^D \) occur through matching of the effective theory onto the full theory at the scale \( \Lambda \), illustrated at one-loop order with the diagrams of Fig. 2. If the operator \( O_W^\nu \) is inserted at the vertex, these two diagrams sum to zero. This is no surprise, as these two diagrams contain no non-trivial flavor dependence. We therefore cannot obtain a contribution to the flavor symmetric mass operator by inserting the flavor antisymmetric operator \( O_W^\nu \).

However, one-loop matching does yield a contribution to \( O_M^D \) associated with the flavor symmetric operator \( O_W^\nu \) of order

\[ C_{5D}^M (\Lambda) \approx \frac{\alpha}{4 \pi \sin^2 \theta_W} C_W^+(\Lambda) \]  

(16)

where we have estimated the one-loop matching contributions using Fig. 2 and naive dimensional analysis (NDA). We note that this contribution arises from loop momenta of order \( \Lambda \), and that the precise numerical coefficient cannot be obtained without knowing the structure of the theory above this scale. We therefore follow the arguments of Ref. 19 and employ NDA to estimate these high momentum contributions.

We see that the one-loop contribution to the 5D mass term provides a strong constraint on \( C_W^+ \) but no constraint on the parameter \( C_W^- \). In general, \( C_W^\pm \) are unrelated parameters in the theory. If the new physics were to have no specific flavor symmetry/antisymmetry
it might be natural for $C_W^+$ to be of similar magnitude. Alternatively, given the strong constraint on $C_W^+$ arising from Eq. (16), a sizable magnetic moment requires $|C_W^-| \gg |C_W^+|$. We identify two scenarios, to be discussed below:

- $|C_W^-| \sim |C_W^+|$
- Arbitrary $|C_W^-|

A. Special case: $C_W^- \sim C_W^+$

We have seen that the flavor antisymmetric operator $O_W^-$ does not contribute to the 5D neutrino mass term at 1-loop order; thus a direct constraint on the magnetic moment is not obtained from the diagrams in Fig. 2. However, suppose we had a theory in which the coefficients of $O_W^-$ and $O_W^+$ were of similar magnitude, $C_W^- \sim C_W^+$. Then, using Eqs. (13), (14) and (16) we have

$$m_{\nu} \sim \frac{\alpha}{8\pi \sin^2 \theta_W} \frac{\Lambda^2}{m_e \mu_B} \frac{\mu_{\nu}}{0.4 \times 10^{-13} \mu_B} [\Lambda (\text{TeV})]^2 \text{ eV,}$$

and thus obtain a stringent $\mu_{\nu}$ bound similar to that for Dirac neutrinos.

We emphasize that Eq. (17) is not a model-independent constraint, as in general $O_W^-$ and $O_W^+$ are unrelated. While it might seem natural for the the new physics to generate coefficients of similar size for both operators, we could obtain finite $C_W^-$ and vanishing $C_W^+$ (at tree-level) by imposing an appropriate flavor symmetry.

B. General case: Arbitrary $C_W^-$.

We now consider the more general case where $C_W^+$ and $C_W^-$ are unrelated, and directly derive constraints on the the coefficient of the flavor antisymmetric operator, $C_W^-$.

1. 7D mass term — $O_W^-$

As the operator $O_W^-$ is flavor antisymmetric, it must be multiplied by another flavor antisymmetric contribution in order to produce a flavor symmetric mass term. This can be accomplished through insertion of Yukawa couplings in the diagram shown in Fig. 3. This diagram provides a logarithmically divergent contribution to the 7D mass term, given by

$$[C_{M}^{\mu D}(MW)]_{\alpha \beta} \sim \frac{3g^2}{16\pi^2} \frac{m_\alpha^2 - m_\beta^2}{v^2} \ln \frac{\Lambda^2}{M_W^2} [C_W^-(\Lambda)]_{\alpha \beta}$$

where $m_\alpha$ are the charged lepton masses, and the exact coefficient has been computed using dimensional regularization and renormalized with modified minimal subtraction. We note that Eq. (18) gives the $O_W^-$ contribution to the neutrino mass from all scales between $\Lambda$ and the scale of electroweak symmetry breaking to leading log order. Moreover, while all contributions of the type $(g^2 \ln \Lambda / m_W)^n$ could be resummed using the renormalization group, a study of analogous operator mixing for Dirac neutrinos suggests that it is sufficient to retain only the leading log contribution. Using this result – as well as Eqs. (13) and (14) to relate $C_W^-$ and $C_M^{\mu D}$ to $\mu_{\nu}$ and $m_{\nu}$ respectively – leads to bound (i) in Table I.

Note that this provides a weaker constraint than that in Eq. (17), as it is suppressed by the charged lepton masses, and also only logarithmically dependent on the scale of new physics $\Lambda$.
of the operator coefficients from the scale \( \Lambda \) to \( M_W \) since the effects are higher order in the gauge couplings and have a negligible numerical impact on our analysis.

Compared to 1-loop (7D) case of Eq. (15), the 2-loop (5D) matching leads to a mass contribution that is suppressed by a factor of 1/16\( \pi^2 \) arising from the additional loop, but enhanced by a factor of \( \Lambda^2/v^2 \) arising from the lower operator dimension. Thus, as we increase the new physics scale, \( \Lambda \), this two-loop constraint rapidly becomes more restrictive and nominally provides a stronger constraint than the 1-loop result once \( \Lambda \gtrsim 4\pi v \sim 4 \text{ TeV} \). Inclusion of the logarithmic \( \Lambda \)-dependence of one-loop mixing implies that the “crossover” scale between the two effects is closer to \( \sim 10 \text{ TeV} \).

IV. HYPERCHARGE GAUGE BOSON

Unlike the case of the \( SU(2)_L \) gauge boson, where a flavor symmetric operator \( O_W^5 \) exists, the operator \( O_B \) is purely flavor antisymmetric. Therefore, it cannot contribute to the \( O_W^5 \) mass term at one loop. As was noticed in [16], the one-loop contribution of \( O_B \) to the \( O_W^5 \) mass term also vanishes.

3. 7D mass term — \( O_B \)

If we insert \( O_B \) in the diagram in Fig. 3 the contribution vanishes, due to the \( SU(2) \) structure of the graph. Therefore, to obtain a non-zero contribution to \( O_M^5 \) from \( O_B \) we require the presence of some non-trivial \( SU(2) \) structure. This can arise, for instance, from a virtual \( W \) boson loop as in Fig. 4 [16].

![FIG. 5: Representative contribution of \( O_B \) to the 7D neutrino mass operator at two loop order.](image)

This mechanism gives the leading contribution of the operator \( O_B \) to the 7D mass term. The \( O_B \) and \( O_W \) contributions to the 7D mass term are thus related by

\[
\frac{\delta m_{\nu}}{\delta m_{\nu}}^B \approx \frac{\alpha}{4\pi \cos^2 \theta_W}, \quad (20)
\]

where \( \theta_W \) is the weak mixing angle and where the factor on the RHS is due to the additional \( SU(2)_L \) boson loop. This additional loop suppression for the \( O_B \) contribution results in a significantly weaker neutrino magnetic moment constraint than that obtained above \( O_W^5 \). The corresponding limit is shown as bound (iii) in Table I.

4. 5D mass term — \( O_B \)

However, the leading contribution of \( O_B \) to the 5D mass term arises from the same 2-loop matching considerations (Fig. 4) that we discussed in connection with the \( O_W \) operator. Therefore, the contribution to the 5D mass term is the same as that for \( O_W \), except for a factor of \( (g'/g)^2 = \tan^2 \theta_W \). We thus obtain

\[
[C_B^M(\Lambda)]_{\alpha\beta} \approx \frac{g'^2}{16\pi^2} \frac{m_{\alpha}^2 - m_{\beta}^2}{v^2} [C_B(\Lambda)]_{\alpha\beta}, \quad (21)
\]

corresponding to bound (iv) in Table I. Importantly, this is the strongest constraint on the \( O_B \) contribution to the neutrino magnetic moment for any value of \( \Lambda \).

V. FLAVOR STRUCTURE

All the magnetic moment bounds presented in Table I contain the factor \( R_{\alpha\beta} = m_\nu^2/(m_\alpha^2 - m_\beta^2) \). This factor arises from the Yukawa insertions that are necessary to obtain flavor symmetric mass terms from the flavor antisymmetric magnetic moment operators. The strongest constraints are those for the \( \mu_\tau \) and \( \mu_\mu \) transition moments, as \( R_{\tau e} \approx R_{\tau \mu} \approx 1 \). However \( R_{\tau e} = 283 \), so the constraint on the \( e-\mu \) transition moment is much weaker.

In the foregoing analysis, it was convenient to work in the flavor basis (where the charged lepton masses are diagonal). However, given \( \mu_{\alpha\beta} \), we can use neutrino mixing matrix to determine \( \mu_{ij} \), where \( i, j \) denote neutrino mass eigenstates. Rotating the neutrino magnetic moments from the flavor basis to the mass basis gives

\[
\mu_{ij} = \sum_{\alpha\beta} \mu_{\alpha\beta} U^\ast_{i\alpha} U_{j\beta} \quad (22)
\]

Since most elements of \( U_{ij} \) are large, it would be natural to expect that all elements of \( \mu_{ij} \) would be of similar size. In particular, all elements of \( \mu_{ij} \) will receive a contribution from \( \mu_{\mu e} \) (the most weakly constrained element in the flavor basis.) Therefore the \( \mu_{\mu e} \) constraint will translate into similar limits for all elements of \( \mu_{ij} \).

VI. DISCUSSION AND CONCLUSIONS

We have discussed radiative corrections to the neutrino mass arising from a neutrino magnetic moment coupling. Expressing the magnetic moment in terms of effective operators in a model independent fashion required constructing operators containing the \( SU(2)_L \) and hypercharge gauge bosons, \( O_W \) and \( O_B \) respectively, rather
than working directly with the electromagnetic gauge boson. We then calculated \( \mu \), naturalness bounds arising from the leading order contributions to both the 5D and 7D Majorana mass terms. These bounds are summarized in Table I.

At the TeV scale, the strongest bound comes from a 1-loop contribution to the 7D mass term as previously discussed in [10] [limit (i) in Table I]. However, this bound applies only to a \( \mu_5^B \) contribution to the magnetic moment, and not to a \( \mu_7^W \) contribution. Even for the \( \mu_7^W \) contribution, the bound on the \( e - \mu \) transition moment is weaker than current experimental limits.

However, we have shown that 2-loop contributions to the 5D mass term always provide the strongest bound on \( \mu_5^B \) [limit (iv) in Table I] and also provides the strongest bound on \( \mu_5^W \) provided \( \Lambda \gtrsim 10 \text{ TeV} \) [limit (ii) in Table I]. We have estimated this contribution – associated with matching the effective theory onto the (unspecified) full theory at high scales – using NDA. The resulting expressions for \( \mu_5^B \) and \( \mu_5^W \) differ only by a factor of \( \tan^2 \theta_W \). Taking the \( \mu_5^B \) limit (the weaker of the two) we have

\[
\mu_{\alpha\beta} \leq 4 \times 10^{-9} \mu_B \left( \frac{m_\nu_{\alpha\beta}}{1 \text{ eV}} \right) \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \left| \frac{m_\nu^2}{m_\nu^2 - m_\nu^2} \right| . \tag{23}
\]

For any scale \( \Lambda \), Eq. (23) is the most general naturalness bound on the size of the Majorana neutrino magnetic moment. It can only be evaded in the presence of fine tuning of the couplings or model-dependent suppression of the matching conditions at the scale \( \Lambda \).

Turning now to the current experimental situation, the best laboratory limit obtained from scattering of low energy reactor neutrinos is \( \mu \lesssim 0.7 \times 10^{-10} \mu_B \). Note that this limit applies to both \( \mu_{\tau e} \) and \( \mu_{\mu e} \), as the flavor of the scattered neutrino is not detected in the experiment. Taking the neutrino mass to be \( m_\nu \approx 0.3 \text{ eV} \) (as implied by cosmological observations, e.g., [20]), Eq. (23) gives

\[
\mu_{\tau \mu}, \mu_{\tau e} \lesssim 10^{-9} \left[ \frac{\Lambda}{\text{TeV}} \right]^2 \mu_B, \mu_{\mu e} \lesssim 3 \times 10^{-7} \left[ \frac{\Lambda}{\text{TeV}} \right]^2 \mu_B. \tag{24}
\]

We thus conclude that if \( \mu_{\mu e} \) is dominant over the other flavor elements, an experimental discovery near the present limits (e.g., at \( \mu \approx 10^{-11} \mu_B \)) would imply that \( \Lambda \lesssim 100 \text{ TeV} \). On the other hand, any model that leads to all elements of \( \mu_{\alpha\beta} \) having similar size would only be consistent with the \( m_\nu \) naturalness bounds if \( \Lambda \lesssim 10 \text{ TeV} \).

Our conclusions can be summarized according to the scale of the new physics, \( \Lambda \):

I) \( \Lambda \lesssim 10 \text{ TeV} \)
- No conflict with experimental limits.
- Both \( O_W \) and \( O_B \) contributions to \( \mu_\nu \) are possible, though \( O_W \) contributions are more tightly constrained.

II) \( \Lambda \gtrsim 10 \text{ TeV} \)
- \( \mu_{\tau \mu}, \mu_{\tau e} \) bounds stronger than experimental limits
- \( \mu_{\mu e} \) bound weaker than experimental limits
- Same limit irrespective of whether \( \mu_\nu \) generated by \( O_W \) and \( O_B \)

III) \( \Lambda \gtrsim 100 \text{ TeV} \)
- The \( \mu_{\alpha\beta} \) bound becomes stronger than current experimental constraints, for all \( \alpha, \beta \).

The limit on the the magnetic moment of a Dirac neutrino is considerably more stringent than for Majorana neutrino. This is due to the different flavor symmetries involved. In the Dirac case, no insertion of Yukawa couplings is needed to convert a flavor antisymmetric operator into a flavor symmetric operator, and the stringent limit \( \mu \lesssim 10^{-15} \mu_B \) holds (in the absence of strong cancellations). A significant implication is that if a magnetic moment \( \mu \gtrsim 10^{-15} \mu_B \) were measured, it would indicate that neutrinos are Majorana fermions, rather than Dirac. Moreover, the scale of lepton number violation would be well below the conventional seesaw scale.

**Acknowledgments**

We thank Vincenzo Cirigliano, Concha Gonzalez-Garcia, and Mark Wise for illuminating conversations. This work was supported in part under U.S. DOE contracts DE-FG02-05ER41361 and DE-FG03-92ER40701 and NSF award PHY-0555674. N.F.B was supported by a Sherman Fairchild fellowship at Caltech.

**APPENDIX A: FULL 7 DIMENSIONAL OPERATOR BASIS FOR MAJORANA NEUTRINOS**

Here, we analyze the full set of gauge invariant operators of dimensions five through seven containing left-handed lepton doublet fields, one Higgs doublet, and \( SU(2)_L \) and \( U(1)_Y \) gauge bosons. It is convenient to classify the operators according to their dimension and field content.

**Mass operator (two fermion and four Higgs fields):**

\[
O_M^{5D} = (L^cH) \left( H^T cL \right) \left( H^1 H \right) \tag{A1}
\]

**Magnetic moment operators (two fermions, two Higgs and one gauge boson field):**

\[
O_B = \left( L^cH \right) \sigma_{\mu\nu} \left( H^T cL \right) B^{\mu\nu} \tag{A2}
\]

\[
O_W^a = \left( L^cH \right) \sigma_{\mu\nu} \left( H^T c^aL \right) W^{\mu\nu}_a
\]

\[
O_W^B = \left( L^cH \sigma \right) \sigma_{\mu\nu} \left( H^T cL \right) W^{\mu\nu}_a
\]

\[
O_W^C = \left( L^cH \sigma \muL \right) \left( H e^aH \right) W^{\mu\nu}_a
\]
However, it can be shown that the operators involving $W_\mu^{\nu}$ are related via

$$2O_{W}^{\mu} - O_{W}^{\nu} = -2O_{W}^{\nu} + O_{W}^{\mu} = O_{W}^{\mu} - O_{W}^{\nu}. \quad (A3)$$

Furthermore, we have

$$[O_{W}^{b}]_{\alpha\beta} = [O_{W}^{a}]_{\beta\alpha}. \quad (A4)$$

Therefore, there is only one independent operator involving $W_\mu^{\nu}$, which we may choose to be $O_{W}^{b} \equiv O_{W}$. We note that the operator defined in Ref. [18],

$$O_{W}^{\text{Davidson}} = (L^c \sigma^a \sigma_{\mu\nu} L) (H^\alpha \rho_b^\nu H) W_{\mu\nu}^{^c} \epsilon_{abcd}, \quad (A5)$$

corresponds to the flavor antisymmetric component of $O_{W}$. Explicitly, $O_{W}^{\text{Davidson}} = O_{W}^{a} - O_{W}^{b} = 2O_{W}^{c}$.

Two fermion fields, two Higgs fields, and two derivatives:

$$O_{3}^{a} = (L^c \epsilon H) (D_\mu H T^c D^\mu L)$$
$$O_{3}^{b} = (D_\mu L^c \epsilon D^\mu H) (H T^c \epsilon L)$$
$$O_{3}^{c} = (D_\mu L^c \epsilon H) (T^c H D^\mu L)$$
$$O_{3}^{d} = (L^c \epsilon D^\mu H) (D_\mu H T^c \epsilon L) \quad (A6)$$

It is also possible to establish relationships between some of the $O_{3}^{a}$ operators. Transposition yields:

$$(O_{3}^{a})^T = - (D_\mu L^c T^c D^\mu H) (H T^c \epsilon L, C^T L)$$
$$= O_{3}^{b}, \quad (A7)$$

where the minus sign arises from the interchange of two fermion fields, and we use $T^c = -\epsilon$, $C^T = -C$. Since the transpose of a number is the same number again, we conclude that there is only one independent operator out of these two, which we may choose to be $O_{3}^{a} \equiv O_{3}^{b}$.

The same trick applied to the remaining two operators does not lead to any new relations. However, another relation can be obtained via integration by parts. We consider

$$0 = D_\mu (D_\mu L^c e H H T^c \epsilon L) \equiv \{11\} + \{12\} + \{13\} + \{14\}, \quad (A8)$$

where the numbers in square brackets denote the field on which the first and the second derivative act, respectively. The squared derivative acting on Higgs is zero by the equation of motion, and that acting on fermion fields leads to operators of the form $O_{W,B}$. Repeating this trick for all possible combinations, we obtain a system of 4 equations:

$$\begin{align*}
\{11\} + \{12\} + \{13\} + \{14\} &= 0 \\
\{12\} + \{23\} + \{24\} &= 0 \\
\{13\} + \{23\} + \{34\} &= 0 \\
\{14\} + \{24\} + \{34\} + \{44\} &= 0
\end{align*} \quad (A9)$$

$$\frac{1}{2} \{\{11\} + \{44\}\} \quad (A10)$$

which implies that the operator $O_{3}^{a}$ can be expressed in terms of the operators $O_{3}^{b}$ and $O_{W,B}$. We are thus left with just two independent operators with two derivatives:

$$O_{3}^{1} = (\bar{L}^c \epsilon H) (D_\mu H T^c \epsilon D^\mu L)$$
$$O_{3}^{2} = (\bar{L}^c L^c D_\mu H) (D^\mu H T^c \epsilon L). \quad (A11)$$

Two fermion fields, three Higgs fields, and one derivative:

$$O_{4}^{a} = \bar{c}^c \gamma_\mu (H T^c \epsilon a L) (D^\mu H T^c \epsilon a H)$$
$$O_{4}^{b} = \bar{c}^c \gamma_\mu (D^\mu H T^c \epsilon a L) (H T^c \epsilon a H) \quad (A12)$$

After fierzing, both operators reduce to just one independent operator:

$$O_{4} = \bar{c}^c \gamma_\mu (H T^c \epsilon L) (D^\mu H T^c H) \quad (A13)$$

Four fermion fields and one Higgs:

$$O_{5}^{a} = (\bar{L}^c L^c) (L^c \epsilon T^c) \epsilon_{ij} H_j$$
$$O_{5}^{b} = (\bar{L}^c \epsilon \sigma_\mu L) (L^c \sigma_{\mu\nu} \epsilon) \epsilon_{ij} H_j \quad (A14)$$

Thus \( \{O_{M}^{DP}, O_{B}, O_{W}, O_{4}^{a}, O_{4}^{b}, O_{4}^{S}, O_{4}^{F}\} \) form the complete set of 7D operators (where flavor labels have been suppressed).

References:

[1] W. J. Marciano and A. I. Sanda, Phys. Lett. B 67, 303 (1977); B. W. Lee and R. E. Shrock, Phys. Rev. D 16, 1444 (1977); K. Fujikawa and R. Shrock, Phys. Rev. Lett. 45, 963 (1980).
[2] J. F. Beacom and P. Vogel, Phys. Rev. Lett. 83, 5222 (1999); D. W. Liu et al., Phys. Rev. Lett. 93, 021802 (2004).
[3] H. T. Wong [TEXONO Collaboration], hep-ex/0605006;
    B. Xin et al. [TEXONO Collaboration], Phys. Rev. D 72, 012006 (2005); Z. Daraktchieva et al. [MUNU Collaboration], Phys. Lett. B 615, 153 (2005).
[4] G.G. Raffelt, Phys. Rep. 320, 319 (1999).
[5] M. Fukugita and T. Yanagida, Physics of neutrinos and applications to astrophysics, Chapter 10, Springer, Berlin, (2003), and references therein.
[6] B. Kayser, F. Gibrat-Debu and F. Perrier, World Sci. Lect. Notes Phys. 25, 1 (1989).
[7] H. T. Wong and H. B. Li, Mod. Phys. Lett. A 20, 1103
(2005).
[8] G. C. McLaughlin and J. N. Ng, Phys. Lett. B 470, 157 (1999); R. N. Mohapatra, S. P. Ng and H. b. Yu, Phys. Rev. D 70, 057301 (2004).
[9] A. B. Balantekin, hep-ph/0601113; A. de Gouvea and J. Jenkins, hep-ph/0603036; A. Friedland, hep-ph/0505165
[10] M. B. Voloshin, Sov. J. Nucl. Phys. 48, 512 (1988). For a specific implementation, see R. Barbieri and R. N. Mohapatra, Phys. Lett. B 218, 225 (1989).
[11] H. Georgi and L. Randall, Phys. Lett. B 244, 196 (1990).
[12] W. Grimus and H. Neufeld, Nucl. Phys. B 351, 115 (1991).
[13] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. 64, 1705 (1990).
[14] S. M. Barr, E. M. Freire and A. Zee, Phys. Rev. Lett. 65, 2626 (1990).
[15] N. F. Bell, V. Cirigliano, M. J. Ramsey-Musolf, P. Vogel and M. B. Wise, Phys. Rev. Lett. 95, 151802 (2005).
[16] S. Davidson, M. Gorbahn and A. Santamaria, Phys. Lett. B 626, 151 (2005).
[17] V. Cirigliano, A. Kurylov, M. J. Ramsey-Musolf and P. Vogel, Phys. Rev. Lett. 93, 231802 (2004).
[18] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. 63, 228 (1989).
[19] C. P. Burgess and D. London, Phys. Rev. D 48, 4337 (1993).
[20] D. N. Spergel et al., astro-ph/0603449