Possible approach to the analysis of nucleus-nucleus interactions at very high energies

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Abstract. Interactions of various nuclei in the frame of a simple geometrical approach are considered with the purpose to determine the number of nucleons participating in these interactions. It is shown that on average a part of interacting nucleons does not exceed one quarter of the total number of nucleons of both nuclei.

1. Introduction

Nucleus-nucleus interactions at very high energies (> $10^{15}$ eV in cosmic rays, > 1 TeV in colliders) are interesting and important from at least two points of view. Firstly, in these interactions the transition from nucleon-nucleon or/and quark-quark to interactions of many nucleons or/and quarks appears. Secondly, nucleus-nucleus interactions play an important role in cosmic ray investigations since cosmic rays consist mainly of nuclei (~60% at energies < $10^{15}$ eV), the fraction of which is increased at higher energies.

But namely at energies about $10^{16}$ eV, which correspond to several TeV in the center-of-mass system, various unusual events as in cosmic rays so in LHC experiments are observed [1, 2]. Numerous attempts of explanation of these results in the frame of existing models did not lead to success. And only a model of production of quark-gluon blobs with a large orbital momentum allows to explain in principle all observed events and phenomena from a single point of view [3, 4]. But their quantitative explanation requires more detailed information about nucleus-nucleus interactions.

In this paper a simple approach to nucleus-nucleus collisions is considered. In this approach, it is assumed that the nucleus is a hard sphere of radius $R$ with a constant density. Since in the considered model the interactions between nucleons are not taken into account and their concentration in nucleus volume is constant, collisions between nuclei can be considered in a pure geometrical approach. For transition to the number of nucleons in interacting parts of nuclei it is sufficient to multiply the volume by the density of nucleons.

The basic parameter of collisions of two nuclei is the impact parameter $b$: the distance between nuclei centers. Its value determines the volume of interacting parts of two nuclei. It can change between 0 and $(R_1+R_2)$, for equal nuclei up to $2R_0$. Correspondingly, the total number of interacting nucleons is equal to $\Delta A=\Delta A_1+\Delta A_2$. Below, two cases are discussed: collisions of two equal nuclei ($A_0$) and collisions of two different nuclei ($A_1$ and $A_2$).
2. Collisions of equal nuclei

The general scheme of interacting nuclei with the number of nucleons $A_0$ and radius $R_0$ moving along $z$-axis is shown in figure 1(a). Though colliding nuclei are spheres, interacting volume is described by two cylindrical surfaces of radius $R$. In figure 1(b) a nucleus without interacting part is shown. Therefore for the number of interacting nucleons we can obtain:

$$\Delta A = 2A_0 \frac{\Delta V}{V_1}$$

where $\Delta V(b) = 2 \int_{x_0}^{X_0} dx \int \frac{R_0^2 - x^2}{b - \sqrt{R_0^2 - x^2}} \sqrt{R_0^2 - x^2 - y^2} dy,$

and $x_0 = \frac{4R_0^2 - b^2}{2}$ is the point of intersection of the circles (see figure 1).

![Figure 1](image)

**Figure 1.** Geometry of the collision of equal nuclei.

The dependence $\Delta A(b)$ calculated by the formula (1) for $A_0 = 1$ ($1$ is a normalization value, but not one nucleon!), $R_0 = 1$ is presented in figure 2(a). It is interesting, that one half of colliding nucleons $\Delta A = A_0$ will interact when $b = 0.86R_0$, and $\Delta A = 0.8A_0$ when $b = R_0$.

![Figure 2](image)

**Figure 2.** Curves corresponding to collision of equal nuclei for $A_0 = 1, R_0 = 1$.

In order to calculate the average quantity of interacting nucleons we assume that the probability distribution law depends on the parameter $b$ only. In this case the effective cross-section is

$$d\sigma = 2\pi \cdot b \cdot db$$

and the total cross-section corresponds to the integral from 0 to $2R_0$; that is

$$\sigma_{tot} = \pi \cdot (2R_0)^2 = 4\pi \cdot R_0^2.$$ (3)

Then we can determine:

the probability $dP = \frac{d\sigma}{\sigma_{tot}} = \frac{b}{2R_0^2} db$ ,

the probability density $f(b) = \frac{b}{2R_0^2}$

as a function of the impact parameter $b$.

The dependence of the number of interacting nucleons $\Delta A f(b)$ on the impact parameter is shown in figure 2(b). According to figure 2(b), the most probable value of the impact parameter is
The value $\Delta A$ corresponding to it is $\Delta A(0.78R_0) = 1.12A_0$. Using the obtained dependence, one can calculate the average value $<\Delta A>$:

$$<\Delta A> = \int \Delta A dP = \int_0^{2R_0} \Delta A f(b) db.$$  

(6)

After calculations, we will get that

$$<\Delta A> = \frac{A_0}{2}.$$  

So, on average only one quarter of nucleons of two colliding nuclei participate in interactions.

3. Collisions of different nuclei

Now we consider collisions of nuclei with different radii $R_1$ and $R_2$, ($R_1 > R_2$ for definiteness) depending on the parameter $b$. There are three areas:

$$0 < b < R_1 - R_2 < b < \sqrt{R_1^2 - R_2^2} < b < R_1 + R_2$$  

(7)

3.1. The first area of interaction: $0 < b < R_1 - R_2$ (figure 3).

3.2. The second area of interaction: $R_1 - R_2 < b < \sqrt{R_1^2 - R_2^2}$ (figure 4).

In this case, the number of particles:

$$\Delta A = \frac{A_2}{2} + A_2 \frac{\Delta V_2}{V_2} + A_1 \frac{\Delta V_1}{V_1},$$

(10)

where

$$\Delta V_1 = 2 \int_{b}^{y_1} \int_{-x_1}^{x_1} \sqrt{R_1^2 - x^2 - y^2 - (y-b)^2} dy dx + 2 \int_{y_1}^{R_1} dy \int_{-x_2}^{x_2} \sqrt{R_1^2 - x^2 - y^2} dy,$$

$$\Delta V_2 = 2 \int_{b}^{y_1} \int_{-x_1}^{x_1} \sqrt{R_2^2 - x^2 - y^2 - (y-b)^2} dy dx + 2 \int_{y_1}^{R_1} dy \int_{-x_2}^{x_2} \sqrt{R_2^2 - x^2 - (y-b)^2} dy,$$

$$y_1 = \sqrt{\frac{R_1^2 - R_2^2 - b^2}{4b} + \frac{R_1^2 - R_2^2}{4b}}, \quad x_1 = \sqrt{R_2^2 - (y-b)^2}, \quad x_2 = \sqrt{R_1^2 - y^2}.$$  

(11)

(12)
3.3 The third area of interaction: $\sqrt{R_1^2 - R_2^2} < b < R_1 + R_2$ (figure 5).

Here the number of particles

$$\Delta A = A_1 \frac{\Delta V_1}{V_1} + A_2 \frac{\Delta V_2}{V_2}$$

(13)

where $\Delta V_1$ and $\Delta V_2$ are the volumes, corresponding to intersections of spheres of radii $R_1$ and $R_2$ with cylinders of radii $R_2$ and $R_1$, respectively:

$$\Delta V_1 = 2 \int_{-x_1}^{x_1} dx \int_{b - \frac{R_1^2 - x^2}{R_2^2 - x^2}}^{\sqrt{R_1^2 - x^2}} \sqrt{R_1^2 - x^2 - y^2} \ dy,$$

(14)

$$\Delta V_2 = 2 \int_{-x_1}^{x_1} dx \int_{b - \frac{R_2^2 - x^2}{R_2^2 - x^2}}^{\sqrt{R_2^2 - x^2 - (y - b)^2}} \sqrt{R_2^2 - x^2 - (y - b)^2} \ dy,$$

(15)

where $x_1$ is the point of intersection of the circles on the plane:

$$x_1 = \sqrt{R_2^2 - \frac{(R_2^2 - R_1^2 - b^2)^2}{4b^2}}.$$  

(16)

The curve of $\Delta A(b)$ for $R_1 = 1$, $R_2 = 0.5$, $A_1 = 1$, $A_2 = 0.5^3$ is shown in figure 6(a).
Figure 6. Curves corresponding to collision of nuclei with $R_1 = 1$, $R_2 = 0.5$, $A_1 = 1$, $A_2 = 0.5^3$.

Here we define $f(b)$ as:

$$f(b) = \frac{2b}{(R_1 + R_2)^2}.$$

From the curve of $\Delta A(b)f(b)$ (figure 6(b)) for such radii and numbers of particles we get the value: $<\Delta A> = 0.17$. Using the curve we can get that the most likely value of the impact parameter is $b = 0.65$ and a value of $\Delta A$, corresponding to $\Delta A(0.65) = 0.345$ (see figure 6(b)).

4. Conclusion
The average part of the number of interacting nucleons at collisions of nuclei depends on their sizes. The maximal average value corresponds to one quarter at collisions of two equal nuclei. At interactions of cosmic ray nuclei with atomic nuclei of the atmosphere ($A_2 = 14.5$) this value varies from 0.2 (helium and iron) to 0.25 (nitrogen and oxygen).

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