Overview of $\tan \beta$ Determination at a Linear $e^+e^-$ Collider

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Abstract

The ratio of the vacuum expectation value of the two Higgs doublets, $\tan \beta$, is an important parameter of the general 2-Higgs-Doublet Model (2HDM) and the Minimal Supersymmetric extension of the Standard Model (MSSM). The expected uncertainty on the determination of $\tan \beta$ at a Linear Collider (LC) of at least 500 GeV center-of-mass energy and high luminosity is reviewed based on studies of neutral and charged Higgs boson production.

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OVERVIEW OF \( \tan \beta \) DETERMINATION
AT A LINEAR \( e^+e^- \) COLLIDER

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The ratio of the vacuum expectation value of the two Higgs doublets, \( \tan \beta \), is an important parameter of the general 2-Higgs-Doublet Model (2HDM) and the Minimal Supersymmetric extension of the Standard Model (MSSM). The expected uncertainty on the determination of \( \tan \beta \) at a Linear Collider (LC) of at least 500 GeV center-of-mass energy and high luminosity is reviewed based on studies of neutral and charged Higgs boson production.

Introduction
Various methods to determine \( \tan \beta \) at a LC exist and they have in common that a physical observable depends on \( \tan \beta \):

- The pseudoscalar Higgs boson, \( A \), could be produced via radiation off a pair of b-quarks: \( e^+e^- \rightarrow b\bar{b} \rightarrow bbA \rightarrow bbb\bar{b} \). The \( bbA \) coupling is proportional to \( \tan \beta \) and thus the expected production rate is proportional to \( \tan^2 \beta \).

- The \( b\bar{b}bb \) rate from the pair-production of the heavier scalar, \( H \), in association with the pseudoscalar Higgs boson \( e^+e^- \rightarrow HA \rightarrow b\bar{b}bb \) can be exploited. While the \( HA \) production rate is almost independent of \( \tan \beta \) the sensitivity occurs via the variation of the decay branching ratios with \( \tan \beta \).

*speaker
• The value of $\tan \beta$ can also be determined from the $H$ and $A$ decay widths, which can be obtained from the previously described reaction.

• The $t\bar{t}b\bar{b}$ rate from charged Higgs boson production can contribute to the determination of $\tan \beta$ from the reaction $e^+e^- \rightarrow H^+H^- \rightarrow t\bar{t}b\bar{b}$ because of the charged Higgs boson branching ratio variation with $\tan \beta$.

• In addition, the charged Higgs boson total decay width depends on $\tan \beta$.

The $b\bar{b}A \rightarrow b\bar{b}b\bar{b}$ bremsstrahlung process

The experimental challenge of this study is the low expected production rate and the large irreducible background for a four-jet final state, as discussed in a previous simulation [1]. The expected background rate for a given $b\bar{b}A \rightarrow b\bar{b}b\bar{b}$ signal efficiency is shown in Fig. 1. Taking a working point of 10% efficiency, we estimate the statistical error in determining $\tan \beta$ by $\Delta \tan^2 \beta/\tan^2 \beta = \Delta S/S = \sqrt{S + B}/S = \sqrt{200/100} \approx 0.14$, resulting in an error on $\tan \beta = 50$ of 7%. In the MSSM, the $b\bar{b}h$ signal would essentially double the number of signal events and have exactly the same $\tan \beta$ dependence, yielding $\Delta \tan^2 \beta/\tan^2 \beta \approx \sqrt{300/200} \approx 0.085$ for $\tan \beta = 50$ and the $\tan \beta$ error would be about 4%. Systematic errors arising from interference with the $hA \rightarrow b\bar{b}b\bar{b}$ reaction can be controlled [2].

Figure 1: Left: Final background rate versus $b\bar{b}A$ signal efficiency for $m_A = 100$ GeV, $\sqrt{s} = 500$ GeV and $L = 500$ fb$^{-1}$. Right: Corresponding $\tan \beta$ statistical error for $L = 2000$ fb$^{-1}$ and $m_A = 100, 150, 200$ GeV. For both plots, we take a fixed value of $m_b = 4.62$ GeV.
HA production: branching ratios and decay widths

The branching ratios for H, A decay to various allowed modes vary rapidly with \( \tan \beta \) in the MSSM when \( \tan \beta \) is below 20. Consequently, if these branching ratios can be measured accurately, \( \tan \beta \) can be determined with good precision in this range. As the H and A decay rates depend on the MSSM parameters, two cases are considered. In scenario (I), SUSY decays of the H and A are kinematically forbidden. Scenario (II) is taken from [3] in which SUSY decays (mainly to \( \tilde{\chi}_1^0 \tilde{\chi}_1^0 \)) are allowed. We assume event selection criteria with an event selection efficiency of 10% and negligible background, based on the expected b-tagging performance and kinematic event selection. The expected HA \( \rightarrow b\bar{b}b\bar{b} \) event rates and 1\( \sigma \) statistical bounds are shown in Fig. 2 as a function of \( \tan \beta \) for \( \sqrt{s} = 500 \text{ GeV} \) and \( L = 2000 \text{ fb}^{-1} \). The resulting bounds for \( \tan \beta \) are plotted in Fig. 3 (right) for MSSM scenarios (I) and (II).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{Expected e^+e^- \( \rightarrow HA \rightarrow b\bar{b}b\bar{b} \) event rates for 10\% efficiency and \( \pm 1\sigma \) bounds in scenarios (I) and (II) in the MSSM for \( m_A = 200 \text{ GeV}, \sqrt{s} = 500 \text{ GeV} \) and \( L = 2000 \text{ fb}^{-1} \).
}
\end{figure}

A previous HA simulation [5] indicates that about 25\% of the time wrong jet-pairings are made, which are attributed to the wings of the mass distribution. The \( m_{b\bar{b}} \) values from H and A decays are binned in a single distribution, since the H and A mass splitting is typically substantially smaller than the detector resolution of \( \Gamma_{\text{res}} = 5 \text{ GeV} \) for the large \( \tan \beta \) values considered. Our effective observable is the resolved average width defined by

\[
\Gamma_{HA}^R = \frac{1}{2} \left[ \sqrt{[\Gamma_{\text{tot}}^H]^2 + [\Gamma_{\text{res}}]^2} + \sqrt{[\Gamma_{\text{tot}}^A]^2 + [\Gamma_{\text{res}}]^2} \right].
\]

Its dependence on \( \tan \beta \) is shown in Fig. 3 for \( m_H \approx m_A = 200 \text{ GeV} \) in MSSM scenario (I) and it is very similar for scenario (II).
Figure 3: Left: Expected resolved width $\Gamma_{R}^{H,A}$, Eq. (1), for scenario (I) and $1\sigma$ upper and lower bounds with 10% selection efficiency. The statistical bounds include an additional efficiency factor of 0.75 for keeping only events in the central mass peak and assume a detector resolution of $\Gamma_{res} = 5$ GeV with a 10% uncertainty. Right: Expected precision on $\tan\beta$ ($1\sigma$ bounds) for $m_{A} = 200$ GeV, $\sqrt{s} = 500$ GeV and $\mathcal{L} = 2000$ fb$^{-1}$ based on the $e^{+}e^{-} \rightarrow b\bar{b}A + b\bar{b}H \rightarrow b\bar{b}b\bar{b}$ rate, the $e^{+}e^{-} \rightarrow HA \rightarrow b\bar{b}b\bar{b}$ rate and $\Gamma_{R}^{H,A}$.

In order to extract the implied $\tan\beta$ bounds, we must account for the fact that the detector resolution will not be precisely determined. There will be a systematic uncertainty which we have estimated at 10% of $\Gamma_{res}$, i.e. 0.5 GeV. This systematic uncertainty considerably weakens our ability to determine $\tan\beta$ at the lower values of $\tan\beta$ for which $\Gamma_{tot}^{H}$ and $\Gamma_{tot}^{A}$ are smaller than $\Gamma_{res}$. This systematic uncertainty should be carefully studied as part of future experimental analyses. Figure 3 shows also the expected $\pm 1\sigma$ experimental errors based on the measurement of $\Gamma_{R}^{H,A}$. An excellent determination of $\tan\beta$ will be possible at high $\tan\beta$. The $bbH/A$ and $H/A$ width methods are nicely complementary in their $\tan\beta$ coverage to the $HA \rightarrow b\bar{b}b\bar{b}$ rate method at lower $\tan\beta$.

**H$^{+}$H$^{-}$ production: branching ratios and decay widths**

The reaction $e^{+}e^{-} \rightarrow H^{+}H^{-} \rightarrow t\bar{b}t\bar{b}$ can be observed at a LC [6] and recent high-luminosity simulations [7] show that precision measurements can be performed. As soon as the charged Higgs boson decay into $tb$ is allowed this decay mode is dominant. Nonetheless, $BR(H^{\pm} \rightarrow tb)$ varies significantly with $\tan\beta$, especially for small values of $\tan\beta$ where the $tb$ mode competes with the $\tau\nu$ mode. The $H^{\pm} \rightarrow tb$ branching ratio and width are sensitive to $\tan\beta$ in the form $\Gamma(H^{\pm} \rightarrow tb) \propto$
As in the previous section, we use HDECAY [4] (which incorporates the running of the b-quark mass) to evaluate the charged Higgs boson branching ratios and decay widths. The $t\bar{b}$ partial width and the corresponding branching ratio have a minimum in the vicinity of $\tan\beta \approx 6 - 8$. In contrast to the variation of the branching ratio, the cross section for $e^+e^- \rightarrow H^+H^-$ production is largely independent of $\tan\beta$.

Our procedures for estimating errors for the $t\bar{b}$ rate and for the total width are similar to those given earlier for HA production rate and width in the $b\bar{b}b\bar{b}$ channel. For $m_{H^\pm} = 300$ GeV at $\sqrt{s} = 800$ GeV, a $H^+H^-$ study [7] finds that the $t\bar{b}$ final state can be isolated with an efficiency of 2.2%. For $m_{H^\pm} = 200$ GeV and $\sqrt{s} = 500$ GeV, we have adopted the same 2.2% efficiency and negligible background. Figure 4 shows the resulting $t\bar{b}$ rates and 1σ bounds for MSSM scenarios (I) and (II). The corresponding bounds on $\tan\beta$ are shown in Fig. 5 (right).

For the total width determination, we assume that we keep only 75% of the events after cuts (i.e. a fraction $0.75 \times 0.022$ of the raw event number), corresponding to throwing away wings of the mass peaks, and each $t\bar{b}$ event is counted twice since we can look at both the $H^+$ and the $H^-$ decay. We define a resolved width which incorporates the detector resolution $\Gamma_{\text{res}} = 5$ GeV:

$$\Gamma_{H^\pm}^R = \sqrt{[\Gamma_{\text{tot}}^{H^\pm}]^2 + [\Gamma_{\text{res}}]^2}. \quad (2)$$
Figure 5: Left: Expected resolved width $\Gamma_{R,H^\pm}$, Eq. (2), for scenario (I) and 1σ upper and lower bounds with 2.2% selection efficiency. The statistical bounds include an additional efficiency factor of 0.75 for keeping only events in the central mass peak and assume $\Gamma_{\text{res}} = 5$ GeV with a 10% uncertainty. Right: Expected precision on $\tan\beta$ (1σ bounds) for $m_{H^\pm} \approx m_A = 200$ GeV, $\sqrt{s} = 500$ GeV and $\mathcal{L} = 2000$ fb$^{-1}$ based on the $e^+e^- \rightarrow H^+H^- \rightarrow t\bar{b}t\bar{b}$ rate and $\Gamma_{R,H^\pm}$.

Estimated errors are based on the width measurement for 10% systematic error in $\Gamma_{\text{res}} = 0.5$ GeV. The resolved width $\Gamma_{R,H^\pm}$ for scenario (I) is given in Fig. 5 and it is very similar for scenario (II). It also shows resulting $\tan\beta$ bounds. In comparison to the neutral Higgs boson methods (Fig. 3), we observe that for MSSM scenario (I) the $t\bar{b}t\bar{b}$ rate measurement gives a $\tan\beta$ determination that is quite competitive with that from HA production in the $b\bar{b}b\bar{b}$ final state. For MSSM scenario (II), the $t\bar{b}t\bar{b}$ rate gives an even better $\tan\beta$ determination than does the $b\bar{b}b\bar{b}$ rate. On the other hand, the width measurement from the $t\bar{b}t\bar{b}$ final state of $H^+H^-$ production is much poorer than that from the $b\bar{b}b\bar{b}$ final state of HA production.

By combining the $\tan\beta$ errors from all processes in quadrature we obtain the expected net errors on $\tan\beta$ shown in Fig. 6 for MSSM scenarios (I) and (II). The Higgs sector will provide an excellent determination of $\tan\beta$ at small and large $\tan\beta$ values. However, larger bounds are expected for moderate $\tan\beta$ in scenario (II) where SUSY decays of the $A, H, H^\pm$ are not significant. Further information on $\tan\beta$ could be obtained from the reaction $e^+e^- \rightarrow t\bar{t} \rightarrow t\bar{b}H^\pm \rightarrow t\bar{b}\tau\nu$, further Higgs decay branching ratios (e.g. $H \rightarrow WW, ZZ, hh$; $A \rightarrow Zh$; $H, A \rightarrow$ SUSY particles), the $H/A$ decay width from $bbH/A$ production, and the polarization of scalar taus.
Conclusions

A high-luminosity linear $e^+e^-$ collider will provide a precise measurement of the value of $\tan\beta$ throughout most of the large possible $\tan\beta$ range $1 < \tan\beta < 60$. In particular, we have demonstrated the complementarity of employing: a) the $b\bar{b}A + b\bar{b}H \rightarrow b\bar{b}b\bar{b}$ rate; b) the $HA \rightarrow b\bar{b}b\bar{b}$ rate; c) the average $H,A$ total width from $HA$ production; d) the $H^+H^- \rightarrow t\bar{b}t\bar{b}$ rate; and e) the $H^\pm$ total width from $H^+H^- \rightarrow t\bar{b}t\bar{b}$ production. Experimental challenges will be the required high total luminosity, an excellent b-tagging performance, and precision detector resolution and selection efficiency determinations.

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