Hybrid simulations of parallel propagating large-amplitude Alfvénic fluctuations

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ABSTRACT

Large amplitude Alfvénic fluctuations tend to be unstable to parametric instabilities which result in a decay process of the initial wave into different daughter waves depending upon the amplitude of the fluctuations and the plasma beta. The propagation angle with respect to the mean magnetic field of the daughter waves plays an important role in determining the type of decay. We have revisited this problem by means of hybrid simulations. Starting from either monochromatic or non-monochromatic circularly polarized fluctuations, we have investigated the stability of Alfvénic fluctuations, the saturation mechanisms of the decay process and the final nonlinear state reached at different pump wave amplitudes and plasma beta values. As opposed to one-dimensional simulations where the instability is suppressed for increasing plasma beta values, we find that the decay process in multi-dimensions persists at large values of the plasma beta via the filamentation/magnetosonic decay instabilities. In general, the decay process acts as a trigger both to develop a perpendicular turbulent cascade and to enhance field-aligned wave-particle interactions. We find indeed that the saturated state is characterized by a turbulent plasma displaying a field-aligned beam at the Alfvén speed and increased temperatures that we ascribe to \( n = 0 \) resonances and pitch angle scattering in phase space. By addressing the stability of large amplitude Alfvénic fluctuations, and by providing a mechanism that may contribute to turbulence development as well as to collisionless plasma heating and particle acceleration, these results are relevant to the solar wind and to space and astrophysical plasmas in general.

Keywords: keywords

1. INTRODUCTION

Collisionless or weakly collisional turbulent plasmas are typically found in space and astrophysical environments. It is the case of the heliosphere and the solar wind, the outflow of plasma continually emitted by the sun and that permeates our solar system. Large amplitude fluctuations in the plasma velocity and magnetic field, known as Alfvénic fluctuations, are commonly observed in the solar wind. Such fluctuations are almost incompressible, and they display the typical velocity-magnetic field correlation that characterizes Alfvén waves propagating away from the sun (Coleman Jr 1967; Belcher & Davis Jr 1971). In spite of such a high degree of correlation, Alfvénic fluctuations in the solar wind are characterized by a well-developed power-law spectrum that dominates the low-frequency range of the solar wind fluctuations energy (Bavassano et al. 1982; Horbury et al. 2005; Bruno & Carbone 2013). It is thought that these Alfvénic fluctuations might be generated near the sun and that they may contribute to coronal heating and solar wind acceleration (Erdélyi & Fedun 2007; Verdini et al. 2009), problems that are still under debate in the community.

In the absence of collisions, plasma heating by Alfvén waves is believed to be due to resonant wave-particle interactions that contribute to the conversion of the wave energy into particle thermal energy (Marsch & Tu 2001; Hollweg & Isenberg 2002). On the other hand, \textit{in-situ} observations (Cranmer 2009; Vasquez et al. 2007; Hellinger et al. 2013; Sorriso-Valvo et al. 2018) and numerical simulations (MacBride et al. 2008; Karimabadi et al. 2013) support the idea that dissipation of turbulent fluctuations might contribute significantly to plasma
heating. Turbulent fluctuations involve magnetic and kinetic energy transport across scales and its conversion into heat at the small dissipative scales. However, internal energy generation involves different channels, such as kinetic instabilities, resonant wave-particle interaction (Buti et al. 2000; TenBarge et al. 2013; Cranmer 2014; Chen et al. 2019) and magnetic reconnection within coherent structures (Parashar et al. 2009; Drake et al. 2009), among others, and the very nature of the dissipation process(es) is still puzzling.

In this regard, the evolution of proton temperature in the solar wind shows a strong departure from the predictions of the double adiabatic theory. Preferential particle heating in the perpendicular direction to the local magnetic field is typically observed (e.g. Matteini et al. (2006) and references therein), and the proton distribution function displays many non-thermal features, such as temperature anisotropies and a secondary proton population with a drift velocity of the order of the local Alfvén speed (Marsch 2006). Interestingly, kinetic simulations have shown that a field aligned proton beam forms self-consistently through the decay of an initial large amplitude Alfvénic fluctuation (Araneda et al. 2008; Matteini et al. 2010), pointing to the fact that kinetic effects might play an important role in regulating the dynamics at the large, MHD scales.

Large amplitude Alfvén waves are known to be unstable and to decay into compressible and secondary Alfvénic modes through three or four-wave resonances that lead to a variety of decay instabilities depending on the plasma beta and dispersive effects. Such is the case of parametric decay (Galeev & Oraevskii 1973; Derby Jr 1978), modulational, and beat instabilities (Sakai & Sonnerup 1983; Wong & Goldstein 1986; Nariyuki & Hada 2007; Jayanti & Hollweg 1993). Parametric instabilities (in the broader sense) of Alfvén waves (or of a spectrum of Alfvén waves) have been widely studied over the years through theoretical approaches (Derby Jr 1978; Goldstein 1978; Malara & Velli 1996), and numerical simulations adopting both MHD (Ghosh et al. 1994a,b; Malara et al. 2000; Del Zanna et al. 2001) and kinetic models (Terasawa et al. 1986; Matteini et al. 2010; Verscharen et al. 2012; Nariyuki et al. 2012).

In particular, the traditional parametric decay instability has attracted much attention over the years both in the context of turbulence and plasma heating. This type of decay is most efficient at low values of the plasma beta and it essentially involves the decay of a pump Alfvén wave into a lower frequency reflected Alfvén wave and a forward sound wave. For this reason, parametric decay remains an appealing process because it provides a natural mechanism for the production reflected modes, which is essential for the triggering of a turbulent cascade. Indeed, recently it has been proposed as a viable mechanism to initiate the turbulent cascade in the solar wind acceleration region (Chandran 2018; Réville et al. 2018), while global MHD simulations of the solar wind have also shown that the parametric decay instability can contribute substantially to solar wind heating and acceleration, thanks to the generation of compressible modes that, in the absence of kinetic effects, naturally steepen into shocks (see, e.g., Shoda et al. (2019)). The traditional parametric decay has been also invoked as a possible source for the generation of inward modes and solar wind turbulence in the inner heliosphere, but expansion effects are known to inhibit its development, essentially because the parametric decay process is strongly suppressed as the plasma beta increases at larger heliocentric distances (Tenerani & Velli 2013, 2020; Del Zanna et al. 2015).

Despite much work on parametric instabilities, less attention has been devoted to kinetic effects in multi-dimensions. The multidimensional nature of parametric instabilities of a parallel propagating Alfvén wave was first investigated via two-fluid linear theory by Kuo et al. (1988) and later in the work by Viñas & Goldstein (1991, 1992) where it was shown that the oblique propagation of the daughter waves allows for additional parametric instabilities depending on the angle of the density perturbation with respect to the mean magnetic field. These studies showed that while oblique modes naturally emerge when the pump wave itself is in oblique propagation or in two-dimensional turbulence (as observed for example in Matteini et al. (2010); Primavera et al. (2019)), perpendicular and quasi perpendicular modes can grow as the result of a different decay process of an Alfvén wave in parallel propagation, known as the filamentation and the magneto-acoustic instability, respectively. Such highly oblique modes have been reported previously in two-dimensional numerical simulations (Ghosh et al. 1994b,a; Gao et al. 2013; Comișel et al. 2018, 2019).

The picture emerging from the aforementioned work in multi-dimensions seems to suggest that the growth rate of the decay instability becomes larger for larger plasma beta, the opposite trend that is found in the one-dimensional representation. Indeed, in the low beta plasma regime the traditional parametric decay is expected to be the dominant process, but the filamentation/magneto-acoustic instability, which depends on the plasma beta only weakly (Viñas & Goldstein 1992), is likely to become the dominant process as the plasma beta increases. This may have important implications, not only with regard to plasma heating, as we
shall see, but also for turbulence in the solar wind, where an increasing content of reflected waves (cross-helicity) and an evolving turbulent spectrum is observed with increasing heliocentric distance (Bavassano et al. 2000). In this paper we have therefore revisited the stability of Alfvén waves in parallel propagation using 1, 2 and 3D hybrid simulations to explore the combined effect of multi-dimensionality and kinetic proton physics at different values of the plasma beta and pump wave amplitude. We have considered both monochromatic and non-monochromatic left-handed circularly polarized Alfvénic fluctuations and we have investigated how the decay process and its saturation and nonlinear stages depend on the pump wave amplitude, plasma beta and dimensionality. We found that the overall decay process involves a superposition of modes due to parametric and filamentation instability that survives at values of the plasma beta well above unity, contrary to the one-dimensional case where the dynamics is restricted to the mean magnetic field-aligned direction. In the 2D and 3D simulations, we find that the instability triggers a wave energy dissipation process resulting in a turbulent energy cascade that develops preferentially in the perpendicular direction. At the same time, the decay of the initial fluctuation induces also the formation of a field-aligned proton beam drifting at the Alfvén speed regardless of the plasma beta, and which appears to be associated to a strong perpendicular particle heating. While perpendicular heating is affected by the pump wave amplitude, it is surprisingly observed whenever the decay occurs, regardless of the dimensionality and the plasma beta.

This paper is organized as follows: In section 2 we present the quasi-neutral hybrid model and the numerical setup that we have employed in this study. Section 3 is divided in three parts. In subsection I we describe the global dynamics of the instability for different plasma beta, wave amplitude and dimensionality of the problem. The spectral properties of the electromagnetic field are presented in subsection II where we focus on the turbulent properties at different scales. Finally, in subsection III we discuss the effect of the proton/electron beta and the wave amplitude on the proton heating and acceleration, and address the problem of wave-particle interactions by proposing a possible mechanism to explain the observed features. In section 4 we summarize our results.

2. MODEL AND SIMULATION SETUP

We have employed a hybrid model where electrons are treated as a massless and isothermal neutralizing fluid while the proton dynamics is described by the Vlasov-Maxwell equations (Eqs. 1). The coupling with the electromagnetic fields is given by the low-frequency and non-relativistic Maxwell’s equations, where quasi-neutrality \( n_i = n_e = n \) is assumed and the electric field is determined via the generalized Ohm’s law:

\[
\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla f_i + \frac{e}{m_i} (\mathbf{E} + \frac{\mathbf{v}_i}{c} \times \mathbf{B}) \cdot \nabla f_i = 0
\]  

(1a)

\[
\frac{\partial \mathbf{B}}{\partial t} = -c \nabla \times \mathbf{E}, \quad \mathbf{J} = \frac{c}{4\pi} \nabla \times \mathbf{B}
\]  

(1b)

\[
\mathbf{E} + \frac{\mathbf{u}_i}{c} \times \mathbf{B} = -\frac{k_B T_i}{e n} \nabla n + \frac{\mathbf{J}}{e n} + \eta \nabla \times \mathbf{B},
\]  

(1c)

with \( c \) the speed of light, \( e \) the electron charge, \( k_B \) the Boltzmann constant and \( T_e \) is the electron temperature. The proton number density \( n \) and the proton bulk velocity \( \mathbf{u}_i \) are computed from the moments of the distribution function \( \int n f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} \) and \( n \mathbf{u}_i = \int v f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} \) respectively. In this work we made use of the CAMELIA code (see e.g. Franci et al. (2018)), which is a hybrid particle-in-cell code that uses the current advance method (Matthews 1994) and Boris scheme for the particle pusher, with good stability and long term accuracy.

The numerical setup consists of a large amplitude, large scale Alfvénic fluctuation propagating along the mean magnetic field \( \mathbf{B}_0 \), that we take along the \( x \)-axis. Periodic boundary conditions are imposed in all directions of the computational box. Lengths are normalized to the proton inertial length \( d_i = c/\omega_p \) with \( \omega_p = (4\pi n e^2/m_i)^{1/2} \) the proton plasma frequency. Time is expressed in units of the inverse of proton gyrofrequency \( \Omega_{ci}^{-1} = (\mathbf{e} B_0/m_e c)^{-1} \) and velocities are normalized to the Alfvén speed \( v_A = B_0/(4\pi n m_i)^{1/2} \). The plasma beta for both ions and electrons is defined as \( \beta_{p,e} = 8\pi n k_B T_{p,e}/B_0^2 \). Since in almost all simulations we have \( \beta_p = \beta_e \), we will just use the symbol \( \beta \) to indicate the proton beta, unless otherwise specified. We have included the resistive term in the generalized Ohm’s law to improve energy conservation by avoiding energy accumulation at the grid scales. The resistive coefficient is defined in units of \( 4\pi \eta \omega_p^{-1} \) and the associated length scale is chosen to be greater than the grid size but smaller than any other scale of interest (i.e. smaller than the proton inertial length or proton gyroradius depending on the plasma beta).

We initialize the system using an isotropic homogeneous plasma with uniform particle density and proton velocities randomly distributed with a Maxwellian distribution function at given temperature \( T_p \) and a fixed number of particles per cell (npp). The initial pump
Alfvén wave is initialized with a wave number $n_0 = 4$ and wave vector $k_0 = 2\pi n_0/L$, $L$ being the box size in units of proton inertial length (we use a square or cube box with equal sides), that satisfies the condition $\delta u = - (\omega_0/k_0) \delta b$, with $|\delta b| = \delta b_0$ the amplitude of the pump wave normalized to the mean magnetic field magnitude $B_0$. The wave frequency is determined from the normalized dispersion relation $k_0^2 = \omega_0^2/(1 - \omega_0)$ for left-handed circularly polarized waves in parallel propagation. The pump wave is only weakly dispersive, with $(k_0 d_i)^2 \ll 1$ (see also Table 1).

The monochromatic and non-monochromatic initial Alfvénic fluctuation is given by the following general form for the magnetic field:

$$\delta b_z = \delta b_0 \cos(\phi(k_0, x))$$
$$\delta b_y = - \delta b_0 \sin(\phi(k_0, x)).$$

The phase of the pump wave is defined as:

$$\phi(k_0, x) = k_0 x + \epsilon \sum_{m=n_1, m\neq n_0}^{n_f} \frac{1}{m} \cos(k_m x + \phi_m).$$

By introducing the phase in this way, a superposition of multiple modes can be obtained with the same circular polarization and parallel propagating fluctuations with an imposed power spectrum that scales as $k^{-2}$ between the modes $k_i = 2\pi n_i/L$ and $k_f = 2\pi n_f/L$. $\phi_m$ is a random phase in the interval $[0, 2\pi]$ and $\epsilon$ controls the deviation from a monochromatic wave, which is recovered for $\epsilon = 0$ (Malara & Velli 1996; Del Zanna et al. 2015; Tenerani & Velli 2013; Réville et al. 2018). Note however that the expression in the phase is not monotonic, so if the amplitude of the epsilon term is large enough, the wave does not remain exactly left-hand, but might have short intervals of right handed arcs. A summary of the numerical and plasma parameters adopted in this work can be found in Table 1.

3. RESULTS

I. Global dynamics

In Fig. 1 we show the time evolution of the kinetic and magnetic energy of the fluctuations and thermal proton energy of two representative simulations that we use as a reference to summarize the main properties of the dynamical evolution of the system. In particular, we show results from a monochromatic case (A2D-IIh) and for a non-monochromatic case (C2D-I) with the same plasma parameters, $\beta = 0.5$ and $\delta b_0 = 1.0$. The bottom panel shows the time evolution of the parallel and perpendicular temperature profiles (black and red colors, respectively) and of the temperature anisotropy $T_\perp/T_\parallel$ (green color). We have introduced the parallel and perpendicular temperatures defined in terms of the decomposition of the pressure tensor according to the direction of the total magnetic field as: $p_\parallel = p : \hat{b}\hat{b}$ and $p_\perp = p : (I - \hat{b}\hat{b})/2$. The pressure tensor $p = \int (u - v_p)(u - v_p) f(r, v, t) dv$ is obtained from the particle velocity distribution and $\hat{b} = b/|b|$ is the direction of the total magnetic field.

For the monochromatic case, three different stages can be identified during the evolution: initially, the wave propagates without significant dispersion, the kinetic and magnetic energy oscillate around a mean value, while the proton temperature remains constant. After this initial stage, the wave energy is transformed into particle heating. Finally, saturation of the instability slows down the heating process and the system achieves a steady state condition with almost constant kinetic, magnetic and thermal energy. It is noted that the total energy of the system is not perfectly conserved during the simulations, in fact, there is a relative error of the order of 3% and some numerical heating is present in the simulations.

In the non-monochromatic case, on the other hand, the decay of the initial fluctuation occurs earlier than in the monochromatic simulation, in agreement with previous studies, owed to the fact that many modes are simul-
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Table 1. Initial conditions for the simulations presented in this paper.

| Run     | res | L(d_i) | Δx  | ppc | β_p | β_e | δb_0 | n_i | n_f | ϵ  | η  |
|---------|-----|--------|-----|-----|-----|-----|------|-----|-----|----|----|
| A1D-I   | 512 | 128    | 0.25| 10000 | 0.25 | 0.25 | 1.0  | 0   | 0   | 0.002 | 0.002 |
| A1D-II  | 512 | 128    | 0.25| 10000 | 0.5  | 0.5  | 1.0  | 0   | 0   | 0.002 | 0.002 |
| A1D-III | 512 | 128    | 0.25| 10000 | 2.0  | 2.0  | 1.0  | 0   | 0   | 0.002 | 0.002 |
| A2D-I   | 2048 | 128    | 0.0625 | 10000 | 0.25 | 0.25 | 1.0  | 0   | 0   | 0.0004 | 0.0004 |
| A2D-II  | 2048 | 128    | 0.0625 | 10000 | 0.5  | 0.5  | 1.0  | 0   | 0   | 0.0004 | 0.0004 |
| A2D-III | 512 | 128    | 0.25 | 1024  | 0.5  | 0.5  | 1.0  | 0   | 0   | 0.002 | 0.002 |
| A2D-IV  | 512 | 128    | 0.25 | 1024  | 2.0  | 2.0  | 1.0  | 0   | 0   | 0.002 | 0.002 |
| A3D-I   | 256 | 128    | 0.5  | 512    | 0.5  | 0.5  | 1.0  | 4   | 10  | 0.5  | 0.004 |
| B1D-I   | 512 | 128    | 0.25 | 10000 | 0.5  | 0.5  | 1.0  | 0   | 0   | 0.002 | 0.002 |
| B1D-II  | 512 | 128    | 0.25 | 10000 | 0.5  | 0.5  | 1.0  | 0   | 0   | 0.002 | 0.002 |
| B2D-I   | 2048 | 128   | 0.25 | 1024  | 0.75 | 0.75 | 1.0  | 4   | 10  | 0.5  | 0.002 |
| B2D-II  | 512 | 128    | 0.25 | 1024  | 2.0  | 2.0  | 1.0  | 4   | 10  | 0.5  | 0.002 |
| C3D-I   | 256 | 128    | 0.5  | 512    | 0.5  | 0.5  | 1.0  | 4   | 10  | 0.5  | 0.004 |
| C2D-I   | 512 | 128    | 0.25 | 1024  | 0.75 | 0.75 | 1.0  | 4   | 10  | 0.5  | 0.002 |
| C2D-II  | 512 | 128    | 0.25 | 1024  | 2.0  | 2.0  | 1.0  | 4   | 10  | 0.5  | 0.002 |
| C2D-III | 512 | 128    | 0.25 | 1024  | 4.0  | 4.0  | 1.0  | 4   | 10  | 0.5  | 0.002 |
| C2D-V   | 512 | 128    | 0.25 | 1024  | 8.0  | 8.0  | 1.0  | 4   | 10  | 0.5  | 0.002 |
| D2D-I   | 512 | 128    | 0.25 | 1024  | 0.5  | 0.5  | 1.0  | 4   | 10  | 0.5  | 0.002 |
| D2D-II  | 512 | 128    | 0.25 | 1024  | 0.5  | 0.5  | 1.0  | 4   | 10  | 0.5  | 0.002 |
| E2D-I   | 512 | 128    | 0.25 | 1024  | 0.5  | 0.5  | 1.0  | 4   | 10  | 0.5  | 0.002 |
| E2D-II  | 512 | 128    | 0.25 | 1024  | 0.5  | 0.5  | 1.0  | 4   | 10  | 0.5  | 0.002 |

Instantaneously present (Malara & Velli 1996; Tenerani & Velli 2013; Réville et al. 2018). However, the decay process in the non-monochromatic case is relatively slower and more gradual, and therefore the heating rate is smaller than in the monochromatic case, although the temperatures reached at the end of simulations, at $t = 10000\Omega_{ci}^{-1}$, are roughly the same. It is also noted that the proton heating observed in the simulations are pretty similar between 1D, 2D and 3D setup, with a slightly larger proton temperature in multi-dimensions than in 1D simulation. The dominant heating mechanism seems to be a one-dimensional process. We will address this problem in section 3.III.

The density fluctuations and the proton temperatures for monochromatic waves at different plasma beta and wave amplitudes are presented in Fig 2. Similar trends are observed for the non-monochromatic cases and the three dimensional simulation that, for the sake of brevity, we do not show here. Results for 1D simulations are plotted (dashed lines) as a reference for purely parallel dynamics. Here, the total temperature is defined as $T = \frac{1}{3}(T_∥ + 2T_⊥)$ and $\Delta T = T(t) - T(t = 0)$ represents the net change from the initial value of the total temperature. The left panels of Fig. 2 display results for different plasma beta and initial wave amplitude $δb_0 = 1$. As can be seen, the evolution of the decay process in the 1D simulations closely follows predictions from MHD linear theory. The initial nonlinear wave is subject to decay instability which becomes slower as the (electron) plasma beta increases. For the amplitudes considered here, the instability is suppressed at large plasma beta ($β = 2$), where the wave is more likely to decay via modulational or beat instabilities, with smaller growth rates and hence a slower decay process. The slight increase in density fluctuations that can be seen in the upper panel of Fig. 2 for the 1D case does not indeed correspond to the disruption/decay of the pump Alfvén wave. What is interesting to highlight is the different trend displayed by the 2D simulations with the plasma beta. In the low beta regime the 2D simulations are characterized by a growth rate similar to the 1D cases, although the decay occurs later than in
Figure 2. The standard deviation of the density fluctuations, mean parallel temperature, mean perpendicular temperature and temperature difference in time for simulations labeled with runs A. 1D simulations (dashed lines) and 2D simulations (solid lines).

the 1D, in agreement with previous MHD studies comparing parametric decay from one to three dimensions (Del Zanna et al. 2001). However, the 2D simulations display a rapid decay process even in the large beta case and, contrary to the 1D case, the growth rate tends to increase with the plasma beta. Accordingly, the temperature evolution and resulting proton heating is different between the 1D and 2D simulations. As we have mentioned above, there is no decay in the 1D simulations for $\beta = 2$, and no energy exchange with particles occurs in this case. On the contrary, in the 2D simulations a strong particle heating is observed both in the parallel and perpendicular directions, for small and large beta values. We ascribe such differences between the 2D and 1D set of simulations to the onset of filamentation/magnetosonic decay simultaneously to the main
parametric decay process, and to the resulting nonlinear dynamics.

The right panels of Fig. 2 show the same quantities on the left panel but for fixed beta ($\beta = 0.5$) at different initial wave amplitudes. The growth rate increases with the amplitude, as is known from linear theory. Strong proton heating is observed as the pump wave amplitude increases, a trend that we find also in the non-monochromatic cases. Interestingly enough, proton heating occurs in both parallel and perpendicular directions in both 1D and 2D simulations, pointing to the fact that most of the heating mechanism(s) must be ascribed to a 1D dynamics. We defer such a discussion to the subsection III.

By way of illustration, the decay process for run A2D-IIh and A2D-IIIh is presented in Fig. 1. The left panels show the superposition of the 2D FFT of the $y$-component of the magnetic field (black contours) and density (red contours) at the maximum of the the most unstable modes shown on the right panels.

For the sake of completeness we also display in Fig. 4 the density and parallel magnetic field waveform at different stages of their evolution for run A2D-II. The plots show slices along the parallel and perpendicular axis to the mean magnetic field for both quantities at different times. Before the complete decay of the pump wave, the parametric instability is already established, as can be seen from the generation of well defined, coherent density fluctuations (top panel). The anti-correlation between density and parallel magnetic field is a signature of the slow mode character of the growing fluctuations which is persistent even after the posterior distortion of the parent wave (bottom panel).

II. Spectral properties
The decay of the parent Alfvén wave into secondary modes triggers nonlinear interactions that ultimately lead to the establishment of a turbulent cascade. At saturation of the instability, an energy spectrum spanning scales all the way down to sub-proton scales develops preferentially in the perpendicular direction to the mean magnetic field. In Fig. 5 we show the resulting magnetic and electric field energy spectra for the same monochromatic cases shown in the left panels of Fig. 2.

In the top panels we plot the reduced 1D spectra in the perpendicular direction of the parallel ($E_{B||}$, $E_{E||}$) and perpendicular ($E_{B\perp}$, $E_{E\perp}$) components of the electromagnetic field fluctuations at saturation. We also plot the reduced spectrum of density fluctuations for the same period and also the initial density fluctuations as a reference to quantify the noise level in the simulations. We also plot the spectral ratio between $B||$ (gray) and $\rho$ (black) with $B\perp$ normalized with the KAW linear prediction for each simulation.

The reduced 1D perpendicular spectrum is computed as $E_{\delta A}(k_{\perp}) = \int dk_{||} A(k_{||}, k_{\perp})$, with $A(k_{||}, k_{\perp})$ the 2D spectral energy density. The vertical dashed line marks the location of $k_{\perp} \rho_i = 1$, with $\rho_i = \sqrt{\beta_i} d_i$ the proton gyroradius.

The power spectrum of magnetic field components in the parallel direction is less developed and the spectrum is dominated by the daughter waves and its harmonics (not shown), but in the transverse direction the magnetic field shows a broad inertial range with a Kolmogorov-like spectrum ($\sim k_{\perp}^{-5/3}$). A spectral break is also observed when approaching proton scales, marking the transition to another turbulent regime at sub-proton scales. The spectral break occurs at the larger of the proton scales depending on the plasma beta, in agreement with previous numerical simulations (Franci et al. 2016). At the transition, the magnetic field steepens, with a slope of $-7/3$, while the electric field flattens, with a slope of approximately $-1/3$, in agreement with in-situ measurements (Alexandrova et al. 2009; Sahraoui et al. 2013; Chen 2016), fluid and kinetic simulations (Dmitruk & Matthaeus 2006; Howes et al. 2011; Franci et al. 2015).

The change of turbulence regime from the large to the small scales is marked by the increase of plasma compressibility, an effect that we observe in all of our sets of simulations. In order to characterize small scale fluctuations, we consider spectral field ratios that are known to provide a useful tool to investigate the polarization properties of turbulent fluctuation (Gary & Smith 2009; Chen et al. 2013; Grošelj et al. 2017; Chen & Boldyrev 2017; Cerri et al. 2019). In the bottom panels of Fig. 5 we present the spectral ratio $R_1 \equiv C_1 E_{\delta B||} / E_{\delta B\perp}$ and $R_2 \equiv C_2 E_{\delta \rho} / E_{\delta B\perp}$ for each simulation. The ratios are normalized to the rms of each field and to the theoretical prediction from KAW at different
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parallel component of the Hall electric field and ∆T

In the top panel of Fig. 6, we show the rms value of the Hall term, but with smaller amplitude (not shown). In the parallel component of electron pressure to the parallel component of the Hall term involves the density and the parallel magnetic field fluctuations (fluctuations of \( B_z \)). In the bottom panel of Fig. 6 we present the parallel (black lines) and perpendicular (grey lines) spectrum of the ratio between these two terms. Solid, dashed and dotted lines refer to the same plasma beta cases reported in the top panel, with each curve being taken at the maximum of the parallel electric field and averaged over 50Ωci. At large scales, the ratio between electron pressure to Hall term at transverse scales \((k_L)\) appears to increase as the plasma beta increases. This trend does not come as a surprise since \( P_{LD}(E_{\parallel}p_e)/P_{LD}(EH) \propto \beta_e^2/4 \) and it is consistent with the formation of slow modes along the perpendicular direction (cfr Fig. 4). In the parallel direction \((k_\parallel)\), instead, the Hall term is larger than the electron pressure term at all beta values, as a consequence of the presence of strong currents produced by the decay of the pump wave. However, at sub-proton scales, the electron pressure dominates over the Hall term in both parallel and perpendicular directions. Again, particle noise may contribute to overestimate the level of density fluctuations observed at small scales.

As already mentioned, the generation of parallel electric field fluctuations is crucial for the particle heating and acceleration observed during the decay process of the pump wave and until the steady-state condition is reached at saturation stage, a problem that we address in the next section.

Figure 6. (Top). Temporal evolution of the rms of the parallel Hall electric field component (green lines) and the total temperature change for different plasma beta simulations. (Bottom). Spectral ratio of the parallel component of electron pressure to the parallel Hall term for the same simulations shown in the bottom panel.

plasma beta \( C_1 = \beta_t(1 + T_e/T_i)/2 + \beta_t(1 + T_e/T_i) \) and \( C_2 = 4 / ((1 + T_i/T_e)(2 + \beta_t(1 + T_e/T_i))) \) with \( \beta_t = \beta_p + \beta_e \). KAW theory predicts a value of unity for \( R_1 \) and \( R_2 \) at sub-proton scales. This would correspond to strong compressive magnetic fluctuations in nearly pressure balance in the kinetic range. In our set of simulations, where we considered \( T_e/T_i = 1 \), the values of \( R_{1,2} \) for the magnetic field are in rough agreement with the theory, but the level of density fluctuations at scales smaller than proton scales largely exceeds the linear prediction. This could be due to nonlinearities and/or the presence of different wave activity like slow modes, whistler or others plasma modes in the sub-proton range. It is also noted that particle noise beyond proton scales may also dominate the density spectrum and therefore the ratio overestimates the expected values.

The parallel electric field is developed during the collapsing stage of the pump wave and it plays a crucial role in the particle heating observed during the decay process until saturation. Interestingly, the gross contribution to the parallel electric field comes from the Hall effect rather than from the electron pressure term. Indeed, the electron pressure presents the same trend as the Hall term, but with smaller amplitude (not shown). In the top panel of Fig. 6, we show the rms value of the parallel component of the Hall electric field and \( \Delta T \) for different beta simulations. As can be seen, there is a clear correlation between the Hall parallel electric field and particle heating in this set of simulations.

One can find that the spectral ratio between the parallel component of electron pressure to the parallel component of the Hall term involves the density and the parallel magnetic field fluctuations (fluctuations of \( B_z \)). In the bottom panel of Fig. 6 we present the parallel (black lines) and perpendicular (grey lines) spectrum of the ratio between these two terms. Solid, dashed and dotted lines refer to the same plasma beta cases reported in the top panel, with each curve being taken at the maximum of the parallel electric field and averaged over 50Ωci. At large scales, the ratio between electron pressure to Hall term at transverse scales \((k_L)\) appears to increase as the plasma beta increases. This trend does not come as a surprise since \( P_{LD}(E_{\parallel}p_e)/P_{LD}(EH) \propto \beta_e^2/4 \) and it is consistent with the formation of slow modes along the perpendicular direction (cfr Fig. 4). In the parallel direction \((k_\parallel)\), instead, the Hall term is larger than the electron pressure term at all beta values, as a consequence of the presence of strong currents produced by the decay of the pump wave. However, at sub-proton scales, the electron pressure dominates over the Hall term in both parallel and perpendicular directions. Again, particle noise may contribute to overestimate the level of density fluctuations observed at small scales.

As already mentioned, the generation of parallel electric field fluctuations is crucial for the particle heating and acceleration observed during the decay process of the pump wave and until the steady-state condition is reached at saturation stage, a problem that we address in the next section.

III. Proton heating

The proton dynamics is illustrated in Fig. 7 for the monochromatic simulation with \( \beta = 0.5 \) and \( \delta b_0 = 1 \). The left panels show contours of the proton distribution function in phase space \((x - v_x)\) at three different times from top to bottom. The corresponding reduced distribution functions of parallel and perpendicular velocities \((v_x \text{ and } v_y)\) are shown on the right panels. We show the particle evolution at three different stages of the evolution. At \( t = 320\Omega_e^{-1} \), phase space vortexes form with the same wavelength of that of density fluctuations developed by the parametric instability, with not yet significant heating at that time of the evolution. After this initial stage, a “piston-like” mechanism mediated by the parallel electric field allows for the generation of a secondary proton population propagating parallel to the mean magnetic field. The beam travels at the Alfvén speed, with signatures of particle trapping...
clearly visible in the phase space. The proton beam is persistent and remains stable when a steady state condition is reached. Proton beam formation during parametric/modulational instabilities is a well known process that was found in 1D kinetic simulations (Araneda et al. 2008; Matteini et al. 2010). Here we investigate the origin and its possible relation to the strong perpendicular heating that we observe in our simulations.

To explore the plasma heating and the proton beam formation, we have performed the same numerical experiment that consists of a non-monochromatic initial perturbation with different plasma beta. Although the monochromatic case provides good insights into the physics of parametric instabilities, it remains an idealized problem. Previous numerical simulations using MHD and kinetic models (Malara et al. 2000; Tenerani & Velli 2013; Matteini et al. 2010; Primavera et al. 2019) have explored the parametric instability using non-monochromatic Alfvén wave packets in 1D and 2D configurations and it is known that it continues operating with a slight reduction of the growth rate, essentially because a broad-band spectrum tends to quench coherent resonances (Malara & Velli 1996).

Figure 7. (Left). Ion phase space $x - v_x$ at different stages of the evolution for the run A2D-IIh. (Right). Reduced distribution function for the $x$-component of the particle velocities (solid blue lines) and $y$-component of the particle velocities (dashed lines). The initial distribution function is plotted in red.

Figure 8. Comparison of different macroscopic quantities for simulations labeled with run C (Top). The normalized standard deviation of density fluctuations. (Middle). Parallel magnetic fluctuations and (Bottom) total temperature difference for different plasma beta condition.

We have run 1D, 2D and 3D simulations for the non-monochromatic wave using the same numerical parameters and we have obtained similar results for the three configurations (not shown).

The results from the 2D simulations with $\delta b_0 = 1$ and different plasma beta are summarized in Fig. 8. Density fluctuations are larger for low beta plasma and become progressively smaller as the beta increases. The amplitude of parallel magnetic field fluctuations seems not to be affected by the plasma beta, but the time for them to exponentially grow is affected by the plasma beta.

The proton heating shows different stages also in these non-monochromatic cases. There is an initial strong temperature increase and a beam formation, that corre-
lates with the enhancement of the parallel magnetic field fluctuations. A secular stage is followed where the heating rate is significantly reduced. The final temperature gained by the particles is similar to the monochromatic cases and so are the trends with the plasma beta and pump wave amplitude of $\Delta$.

Since the distribution function is averaged over several gyroperiods, the number of particles in the beam is affected by the finite amplitude effects and longer tails in the distribution are found with larger wave amplitude. Such trends are not affected by the plasma beta, rather, they are controlled by the amplitude of the pump wave.

The averaged particle distribution function (PDF) over $100\Omega_{ci}$ during the secular stage is presented in Fig. 9. We computed the average PDF right after the end of the instability is established and when the particle temperature is statistically constant. It can be noted that the beam forms around the Alfvén speed for all the beta cases, although it is not clearly visible at larger beta values. The distribution function in the perpendicular direction, instead, is a Maxwellian and, as mentioned above, the total change of perpendicular temperature is not affected by the plasma beta.

The effect of the wave amplitude on the final distribution function is presented in Fig. 10. Even if the heating of particles depends on the amplitude of the pump wave, the beam formation along the mean magnetic field is persistent. The core of the distribution is mostly affected by the finite amplitude effects and longer tails in the distribution are found with larger wave amplitude. Since the distribution function is averaged over several gyroperiods, the number of particles in the beam is affected by the averaging.

The understanding of the overall proton heating due to the unstable behavior of Alfvénic fluctuations and the corresponding energy transport toward smaller scales is fundamental to understand the implications on the plasma heating typically observed in solar and astrophysical context. The nature of parallel and perpendicular heating comes from different physical mechanisms and, therefore, we first focus on the parallel heating, which is in fact due mainly to the beam generation, and subsequently we discuss the perpendicular heating and the possible mechanism.

According to previous work, the electron beta plays a crucial role on the saturation process because the electron temperature contributes to the strength of the parallel electric field via the pressure gradient term in the Ohm’s law (see Eq.(1b)). It was suggested that the saturation of the instability was due to particle trapping by the field-aligned electric field generated by density fluctuations, which would lead to a field-aligned beam whose velocity appears to depend on the plasma beta. Hybrid simulations with $\beta_e = 0$ have shown that the beam formation is indeed inhibited and the saturation process results on the steepening of the ion acoustic wave, just like in the fluid description (Matteini et al. 2010). In the same spirit, we present in Fig. 11 the results for the non-monochromatic case with $\beta_p = 0.5$ and $\delta b_0 = 1.0$ for two different scenarios: a case with ($\beta_e = 0.5$) and a case with cold electrons ($\beta_e = 0$).

As can be seen, the two simulations do not display significant differences in the PDFs. Not only the perpendicular temperature achieved at the end of the process is the same for both simulations (right panel) but, importantly, the field-aligned beam (left panel) is persistent in a plasma with $\beta_e = 0$, even though a less populated beam is observed for cold electrons. Indeed, the gradient of the electron pressure, although not the dominant term in Ohm’s law, contributes to the trapping of the particles so that more particles can get trapped for the
$\beta_e = 0.5$ case. The comparison between these two setups therefore points to the fact that it is the Hall term that contributes the most to the field-aligned electric field and to the the beam formation.

According to Vlasov linear theory, proton heating by Alfvén waves is possible via resonant damping and particle can exchange energy with the wave only at discrete resonance interaction restricted by the condition $\omega - k_\parallel v_\parallel = n\Omega_{ci}$, with $\omega$ the frequency of the wave, $v_\parallel$ is the particle velocity and $k_\parallel$ is the wave number parallel to the magnetic field.

Linear theory implies that for $n = 0$, when the phase speed of the wave is of the order of the proton parallel velocity, particles can resonate with the wave and then they can gain or lose parallel velocity ($v_\parallel \leq \omega/k_\parallel$ or $v_\parallel \geq \omega/k_\parallel$ respectively). This resonant wave-particle interaction at $n = 0$ represents two different physical interaction, the Landau damping, driven by a parallel electric field, and the transit time damping (TTD) (Fisk 1976; Achterberg 1981), which is the magnetic analog of Landau damping. In TTD it is the mirror force $F_{mir} = \mu \nabla_\parallel B$, with $\mu = mv_\perp^2/2B$ the conserved particle magnetic moment, to play the analogous role of the parallel electric field in Landau damping and, as such, it can also contribute to the parallel heating by driving a process of particle diffusion in phase space. Interactions with $n \neq 0$ instead lead to cyclotron resonances in which the particles resonate with the oscillating electric and magnetic field (Hollweg & Isenberg 2002). This kind of wave-particle interaction results in the violation of the conservation of $\mu$ so that particles can experience strong perpendicular energization. In general these wave-particle resonances are important because they can lead to parallel/perpendicular heat-

Figure 11. Reduced distribution function averaged over the steady state stage for simulations labeled with runs F. (Left). $x$-component representing the parallel particle velocity. (Right). $y$-component representing one component of the perpendicular particle velocity.

Figure 12. Spatial profile different quantities for run A1D-I. Right axis show the values for proton parallel temperature (blue solid lines), perpendicular temperature (red solid lines), parallel (blue dashed dashed) and perpendicular components of the Hall electric field (red dashed lines). Left axis present the values of the gradients of $B$. These quantities are plotted for forth different stages of the evolution: (a) at $t = 100\Omega_{ci}^{-1}$, (b) at $t = 200\Omega_{ci}^{-1}$, (c) at $t = 400\Omega_{ci}^{-1}$ and (c) at $t = 700\Omega_{ci}^{-1}$.

ing produced by pitch-angle scattering resulting in an isotropization process of the particle distribution function by particle diffusion in velocity space (Kennel & Engelmann 1966; Lynn et al. 2012).

As we have shown in section I, the proton heating does not display major differences between 1D and 2D simulations whenever the wave is subject to parametric instabilities. In fact, the perpendicular heating is roughly the same, although a somewhat larger parallel temperature is obtained in the 2D simulations. In this sense, the main proton heating mechanism at play seems to be the same process regardless of the dimensionality of the system and of the proton/electron plasma beta. In
this way we can analyze the proton heating process for a 1D case without loss of generality. Moreover, the fact that in these numerical experiments the proton heating is essentially a one-dimensional mechanism, we can rule out contributions from stochastic heating possibly due to KAWs or in general to obliquely propagating modes (Chen et al. 2001; Chandran et al. 2010), which cannot develop in a 1D dynamics. However, the slight enhanced heating observed in the 2D systems could be due to the additional contribution to the parallel Hall electric field allowed by 2D effects.

In Figure 12, we present the spatial profiles of the parallel and perpendicular temperature and also the Hall term components \(E_{H_x} = E_{H_z} = b_zj_y - b_yj_z\) and \(E_{H_\perp} = \sqrt{E_{H_x}^2 + E_{H_z}^2} = B_0\sqrt{j_y^2 + j_z^2}\) and the gradients of the magnetic field \((\partial_y B = -B^{-1/2}E_{H_x})\) with the current component \(j_y = -\partial_x b_z\) and \(j_z = \partial_y b_y\), at different stages of the evolution for run A1D-I. Initially, the circularly polarized Alfvén wave is characterized by a constant-B state and zero parallel Hall electric field because of the initial condition (Fig. 12 (a)). Once the decay comes into play, the initial wave collapses and parallel density fluctuations are driven by ponderomotive force, because the constant-B condition breaks up. This is a one dimensional process that is reminiscent of the collapse of Alfvénic wave packets previously observed in other works (Buti et al. 2000). Here we find that the disruption of the wave together with dispersive effects generate gradients of \(B\) in the field-aligned direction which tend to steepen in some localized regions. There it follows an enhancement not only of density fluctuations but also of the Hall electric field through the current density components \(j_y\) and \(j_z\). Such steepened wavefronts with enhanced Hall electric field propagate at the Alfvén speed (not shown). The combined effect of particle trapping by the growing electric field fluctuations, and acceleration from the localized Hall electric field at the steepened fronts of the Alfvén wave contribute to the acceleration of particles into a field-aligned beam at the Alfvén speed, enhancing the number of resonating particles with the fluctuations themselves. We suggest that both type of \(n = 0\) resonances (Landau and TTD damping associated to the gradients of \(B\)) might be responsible for the parallel proton heating (Fig12 (b-c)). Once the resonating fluctuation energy is transformed into particle heating, the system achieves a steady state, with small gradients of \(B\), a persistent beam and a strong parallel and perpendicular heating (Fig12 (d)). As mentioned above, perpendicular heating might be produced by pitch-angle scattering processes due to the acceleration and redistribution of particles in the field-aligned direction of phase space.

We argue that pitch-angle scattering is efficiently mediated by the Alfvén wave, which indeed couples parallel and perpendicular particle dynamics. There is also a high correlation between large perpendicular temperature and strong perpendicular Hall electric field (Fig. 12 (b-c)). However, we notice that the increment of parallel and perpendicular temperatures occurs simultaneously and at the same rate, favoring the idea that perpendicular heating is probably due to pitch-angle scattering. We plan to develop a more detailed investigation of the beam formation and its possible relation to both wave steepening and perpendicular heating in future works.

4. SUMMARY AND CONCLUSION

We have performed hybrid simulations of large amplitude, parallel propagating Alfvénic fluctuations subject to parametric instabilities. We have considered both monochromatic and non-monochromatic Alfvénic fluctuations and we have investigated how their stability, nonlinear evolution and saturation is affected by the amplitude of the pump wave, the plasma beta and the dimensionality of the system (1D vs 2 and 3D).

In multi-dimensional systems the initial decay process can be described as a superposition of both the parametric decay instability and the filamentation/magnetosonic instability. The former leads to parallel propagating density fluctuations and Alfvénic modes, while the latter leads to the formation of perpendicular pressure-balanced fluctuations of density and magnetic field. Importantly, the filamentation instability becomes the dominant process at beta values larger than unity, contrary to one-dimensional systems where the decay appears to be strongly suppressed.

The decay process naturally results in a well developed turbulent cascade of electromagnetic (and internal) energy that develops preferentially in the transverse direction to the mean magnetic field, and that shows similar properties for all the plasma beta values considered. At saturation of the instability, the electromagnetic energy spectrum displays a Kolmogorov-like inertial range at large scales. At sub-proton scales, a steepened magnetic field and a flattened electric field spectrum is observed, displaying power-law scalings which appear to be consistent with kinetic Alfvén wave turbulence. We have also quantified the nature of the fluctuations at sub-proton scales by means of spectral ratio analysis. Using this diagnostic we found that the strong magnetic compressibility at sub-proton scales is consistent with KAW linear theory, though the level of density fluctuations largely exceeds the theoretical prediction. However, the number of particles per cell constraints the particle noise...
at small scales. This may be responsible for the observed discrepancy with the KAW’s prediction, although other effects not included in KAW linear theory might be present, from nonlinearities to the presence of different wave activity (like slow modes or whistlers).

While a developed turbulent state is not observed in the 1D systems, we found that, whenever the decay occurs, the saturated state is always characterized by a heated plasma displaying a persistent field-aligned beam localized at the Alfven speed. In our results, we have not found substantial differences of particle heating for different plasma beta and between 1D, 2D and 3D simulation systems, monochromatic or non-monochromatic pump waves. In fact, we found that the overall heating largely depends on the amplitude of the pump wave, thus the available reservoir of energy for particle heating, and the proton energization closely follows the decay process of the pump wave. A somewhat larger parallel temperature is observed in 2D than in 1D simulations, which could be explained in terms of the additional contribution to the parallel Hall electric field due to 2D effects. However, the overall proton heating seems to be a one-dimensional mechanism that leads to roughly the same final perpendicular heating for 1D and 2D. This rules out mechanisms such as stochastic heating due to KAWs or in general due to the presence of oblique modes, and favors field-aligned mechanisms involving wave-particle resonances which are enhanced by the beam formation.

Interestingly, the beam forms also when electrons are cold ($\beta_e = 0$), pointing to the fact that it is the field-aligned Hall electric field to play the dominant role in accelerating the beam of particles. Such electric field is enhanced at the steepened edges of the pump wave and we argue that it mediates wave-particle interactions via both Landau and transit time damping. Landau and transit time damping are expected to become effective once the beam is formed, so that there is an increased number of resonating particles with parallel electric field and magnetic field magnitude fluctuations, respectively. The perpendicular heating is attributed to a stochastic mechanism that works as an isotropization process of the distribution function via pitch-angle scattering of the accelerated particles.

In conclusion, the decay process acts as a trigger to develop a turbulent cascade and to enhance wave-particle interactions, the latter resulting in a field-aligned beam and efficient plasma heating, reproducing in this way some features which are observed in the solar wind. Moreover, we have shown that such a decay process remains efficient also at large values of the plasma ($\beta > 1$) which makes these results relevant not only to space plasmas, but also to astrophysical environments where the plasma beta can reach values well above unity.

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REFERENCES

Achterberg, A. 1981, Astronomy and Astrophysics, 97, 259
Alexandrova, O., Saur, J., Lacombe, C., et al. 2009, Physical review letters, 103, 165003
Araneda, J. A., Marsch, E., Adolfo, F., et al. 2008, Physical review letters, 100, 125003
Bavassano, B., Dobrowolny, M., Mariani, F., & Ness, N. F. 1982, Journal of Geophysical Research: Space Physics, 87, 3617, doi: 10.1029/JA087iA05p03617
Bavassano, B., Pietropaolo, E., & Bruno, R. 2000, Journal of Geophysical Research: Space Physics, 105, 15959
Belcher, J., & Davis Jr, L. 1971, Journal of Geophysical Research, 76, 3534
Bruno, R., & Carbone, V. 2013, The solar wind as a turbulence laboratory. Living Rev Sol Phys 10: 2
Buti, B., Velli, M., Liewer, P. C., Goldstein, B. E., & Hada, T. 2000, Physics of Plasmas, 7, 3998
Cerri, S. S., Groselj, D., & Franci, L. 2019, Frontiers in Astronomy and Space Sciences, 6, 64
Chandran, B. D. 2018, Journal of plasma physics, 84
Chandran, B. D. G., Li, B., Rogers, B. N., Quataert, E., & Germaschewski, K. 2010, The Astrophysical Journal, 720:503
Chen, C. 2016, Journal of Plasma Physics, 82
Chen, C., Boldyrev, S., Xia, Q., & Perez, J. 2013, Physical review letters, 110, 225002
Chen, C., Klein, K., & Howes, G. G. 2019, Nature communications, 10, 1
Chen, C. H., & Boldyrev, S. 2017, The Astrophysical Journal, 842, 122
Chen, L., Lin, Z., & White, R. 2001, Physics of Plasmas, 8, 4713
Coleman Jr, P. J. 1967, Planetary and Space Science, 15, 953
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Comišel, H., Narita, Y., & Motschmann, U. 2019 in , Copernicus GmbH, 835–842
Comišel, H., Nariyuki, Y., Narita, Y., & Motschmann, U. 2018 in , Copernicus GmbH, 1647–1655
Cranmer, S. R. 2009, Living Reviews in Solar Physics, 6, 3
—. 2014, The Astrophysical Journal Supplement Series, 213, 16
Del Zanna, L., Matteini, L., Landi, S., Verdini, A., & Velli, M. 2015, Journal of Plasma Physics, 81
Del Zanna, L., Velli, M., & Londrillo, P. 2001, Astronomy & Astrophysics, 367, 705
Derby Jr, N. 1978, The Astrophysical Journal, 224, 1013
Dmitruk, P., & Matthaeus, W. 2006, Physics of Plasmas, 13, 042307
Drake, J. F., Cassak, P., Shay, M., Swisdak, M., & Quataert, E. 2009, The Astrophysical Journal Letters, 700, L16
Erdélyi, R., & Fedun, V. 2007, Science, 318, 1572
Fisk, L. 1976, Journal of Geophysical Research, 81, 4633
Franci, L., Hellinger, P., Guarrasi, M., et al. 2018, Journal of Physics: Conference Series, 1031, 012002, doi: 10.1088/1742-6596/1031/1/012002
Franci, L., Landi, S., Matteini, L., Verdini, A., & Hellinger, P. 2016, The Astrophysical Journal, 833, 91, doi: 10.3847/1538-4357/833/1/91
Franci, L., Verdini, A., Matteini, L., Landi, S., & Hellinger, P. 2015, The Astrophysical Journal Letters, 804, L39
Galeev, A. A., & Oraevskii, V. N. 1973, Soviet Phys. Doklady, Engl. transl., 7
Gao, X., Lu, Q., Li, X., Shan, L., & Wang, S. 2013, Physics of Plasmas, 20, 072902
Gary, S. P., & Smith, C. W. 2009, Journal of Geophysical Research: Geospace Physics, 114
Goldstein, M. L. 1978, The Astrophysical Journal, 219, 700
Grošelj, D., Cerri, S. S., Navarro, A. B., et al. 2017, The Astrophysical Journal, 847, 28
Hellinger, P., Trávníček, P. M., Štverák, Š., Matteini, L., & Velli, M. 2013, Journal of Geophysical Research: Space Physics, 118, 1351
Hollweg, J. V., & Isenberg, P. A. 2002, Journal of Geophysical Research: Space Physics, 107, SSH
Horbury, T., Forman, M., & Oughton, S. 2005, Plasma physics and controlled fusion, 47, B703
Howes, G. G., TenBarge, J. M., Dorland, W., et al. 2011, Physical review letters, 107, 035004
Jayanti, V., & Hollweg, J. V. 1993, Journal of Geophysical Research: Space Physics, 98, 13247
Karimabadi, H., Roytershteyn, V., Wan, M., et al. 2013, Physics of Plasmas, 20, 012303
Kennel, C., & Engelmann, F. 1966, The Physics of Fluids, 9, 2377
Kuo, S., Whang, M., & Lee, M. 1988, Journal of Geophysical Research: Space Physics, 93, 9621
Lynn, J. W., Parrish, I. J., Quataert, E., & Chandran, B. D. 2012, The Astrophysical Journal, 758, 78
MacBride, B. T., Smith, C. W., & Forman, M. A. 2008, The Astrophysical Journal, 679, 1644
Malara, F., Primavera, L., & Veltri, P. 2000, Physics of Plasmas, 7, 2866
Malara, F., & Velli, M. 1996, Physics of Plasmas, 3, 4427
Marsch, E. 2006, Living Reviews in Solar Physics, 3, 1
Marsch, E., & Tu, C.-Y. 2001, Journal of Geophysical Research: Space Physics, 106, 8357
Matteini, L., Landi, S., Hellinger, P., & Velli, M. 2006, Journal of Geophysical Research: Space Physics, 111
Matteini, L., Landi, S., Velli, M., & Hellinger, P. 2010, Journal of Geophysical Research: Space Physics, 115
Matthews, A. P. 1994, Journal of Computational Physics, 112, 102
Nariyuki, Y., & Hada, T. 2007, Journal of Geophysical Research: Space Physics, 112
Nariyuki, Y., Hada, T., & Tsubouchi, K. 2012, Physics of Plasmas, 19, 082317
Parashar, T., Shay, M., Cassak, P., & Matthaeus, W. 2009, Physics of Plasmas, 16, 032310
Primavera, L., Malara, F., Servidio, S., Nigro, G., & Veltri, P. 2019, The Astrophysical Journal, 880, 156
Réville, V., Tenerani, A., & Velli, M. 2018, The Astrophysical Journal, 866, 38
Sahraoui, F., Huang, S., Belmont, G., et al. 2013, The Astrophysical Journal, 777, 15
Sakai, J.-I., & Sonnerup, B. Ö. 1983, Journal of Geophysical Research: Space Physics, 88, 9069
Shoda, M., Suzuki, T. K., Asgari-Targhi, M., & Yokoyama, T. 2019, The Astrophysical Journal Letters, 880, L2
Sorriso-Valvo, F, C., S, P., et al. 2018, Solar Physics, 293
TenBarge, J., Howes, G., & Dorland, W. 2013, The Astrophysical Journal, 774, 139
Tenerani, A., & Velli, M. 2013, Journal of Geophysical Research: Space Physics, 118, 7507
—. 2020, Plasma Physics and Controlled Fusion, 62, 014001
Terasawa, T., Hoshino, M., Sakai, J.-I., & Hada, T. 1986, Journal of Geophysical Research: Space Physics, 91, 4171
Vasquez, B. J., Smith, C. W., Hamilton, K., MacBride, B. T., & Leamon, R. J. 2007, Journal of Geophysical Research: Space Physics, 112

Verdini, A., Velli, M., & Buchlin, E. 2009, The Astrophysical Journal Letters, 700, L39

Viñas, A., & Goldstein, M. 1992, in Solar Wind Seven (Elsevier), 577–581

Viñas, A. F., & Goldstein, M. L. 1991, Journal of plasma physics, 46, 129

Wong, H., & Goldstein, M. 1986, Journal of Geophysical Research: Space Physics, 91, 5617

Verscharen, D., Marsch, E., Motschmann, U., & Müller, J. 2012, Physical Review E, 86, 027401