Method Study of Parameter Choice for a Circular Proton-Proton Collider *

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Abstract: In this paper we showed a systematic method of appropriate parameter choice for a circular pp collider by using analytical expression of beam-beam tune shift limit started from given design goal and technical limitations. A parameter space has been explored. Based on parameters scan and considerations from RF systems, a set of appropriate parameter designed for a 50Km and a 100Km circular proton-proton collider was proposed.

Key words: circular proton-proton collider, parameter choice, beam-beam tune shift limit

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1 Introduction

With the discovery of Higgs boson on LHC, the world high-energy physics community is investigating the feasibility of a Higgs Factory as a complement to the LHC for studying the Higgs and interested in the frontier of high energy. The CERN people are busy planning the LHC upgrade program, including HL-LHC and HE-LHC. They also plan a more inspiring program called FCC, including FCC-ee and FCC-lh. Both the HE-LHC and the FCC-lh are proton-proton colliders aiming to explore the high energy frontier and expecting to find new physics [1][2][3][4]. Chinese accelerator physicists also plan to design an ambitious machine called CEPC-SPPC(Circular Electron Positron Collider-Super Proton Proton Collider). The CEPC-SPPC program contains two stage. The first stage is an electron-positron collider with center-of-mass energy 240GeV to study Higgs properties carefully. The second stage is a proton-proton collider at center-of-mass energy more than 70TeV [5][6][7]. The SPPC design is just starting. We developed a systematic method of how to make an appropriate parameter choice for a circular pp collider by using analytical expression of beam-beam tune shift started from the required luminosity goal, beam energy, physical constraints at IP and some technical limitations.

2 Beam-Beam tune shift limit

In storage ring colliders, due to quantum excitation and synchrotron damping effects, the particles are confined inside a bunch. In $e^+e^-$ colliders, the quantum excitation is very strong and the position for each particle is random and the state of the particles can be regarded as a gas, where the positions of the particles follow statistic laws. Apparently, the synchrotron radiation is the main source of heating. Besides, when two bunches undergo collision at an interaction point (IP), every particle in each bunch will feel the deflected electromagnetic field of the opposite bunch and the particles will suffer from additional heatings. With the increase of the bunch particle population $N_c$, this kind of heating effect will get stronger. There is a limit condition beyond which the beam emittance will blow up. This emittance blow-up mechanism introduce a limit for beam-beam tune shift which was well discussed in reference [8]:

$$\xi_{y,max} \leq 2845 \sqrt{\frac{\tau_y}{6\pi R N_{IP}}} = \frac{2845}{2\pi} \sqrt{\frac{T_0}{\tau_y \gamma N_{IP}}} \quad (1)$$

In pp circular colliders, the synchrotron damping effect is very weak. The position for each particle is not like that for electron which is random and the state of the particles cannot be regarded as a gas. Due to the lack of strong synchrotron radiation, the particles inside a bunch are very cold and one can trace each particle without missing it. When the bunches suffer from the strong nonlinear beam-beam forces, some particles located in the outer part of the bunch undergo nonlinear force induced stochastically motions. The number of this heated particles, $N_{p,h}$, can be estimated by $N_{p,h} = f(x) N_p$ [9]. With

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\[ f(x) = 1 - \frac{2}{\sqrt{2\pi}} \int_0^x e^{-t^2} \, dt \]  

(2)

Where \( N_p \) is the particle number inside a bunch, \( x \) is the limit between the cold core and the heated region. On this condition, the limit for beam-beam tune shift can be expressed as [9]:

\[ \xi_{y,max} = \frac{2845\gamma}{f(x)} \sqrt{\frac{r_p}{6\pi RN_{IP}}} = \frac{2845}{2\pi f(x)} \sqrt{\frac{T_0}{\tau_y \gamma N_{IP}}} = \frac{\xi_1}{f(x)} \]  

(3)

\[ f(x) = 1 - \frac{2}{\sqrt{2\pi}} \int_0^x e^{-t^2} \, dt \]  

(4)

\[ x^2 = \frac{4f(x)}{\pi \xi_{y,max} N_{IP}} = \frac{4f(x)^2}{\pi \xi_1 N_{IP}} \]  

(5)

Where \( N_{IP} \) is the number of interaction point (When there are \( N_{IP} \) interaction points, the independent heating effects have to be added in a statistical way), \( R \) is the dipole radius, \( r_p \) is the classical radius of proton, \( \tau_y \) is the transverse damping time and \( T_0 \) is the revolution time.

3 Machine parameters choice

The design goal of energy of SPPC is about 70-100TeV using the same tunnel with CEPC which is about 50Km. A larger circumference like 100Km for SPPC is also being considered. We want to use the superconducting magnets which is about 20T [10]. We can develop a systematic way to calculate the parameter starting from the maximum beam beam tune shift limit and the design goal. Our design goal is: luminosity \( L_0 \), beam energy \( E_0 \), ring circumference \( C_0 \) and IP numbers \( N_{IP} \). Table 1 shows the goals, known quantities and constants.

Table 1. The design goal and known quantities.

| Parameter                     | Value                       |
|-------------------------------|-----------------------------|
| Circumference                 | \( C_0 = 54.7Km \)          |
| Beam Energy                   | \( E_0 = 35TeV \)           |
| IP numbers                    | \( N_{IP} = 2 \)            |
| Luminosity                    | \( L = 1.0 \times 10^{36} \) |
| Total straight section length | \( L_{SS} = 7595m \)        |
| Arc filling factor            | \( f_1 = 0.79 \)            |
| Bunch filling factor          | \( f_2 = 0.80 \)            |
| Energy gain(15~20)            | \( Gain = 16.67 \)          |
| Total/inelastic cross section | \( \sigma_{cross} = 140mbarn \) |
| Light speed                   | \( c = 3 \times 10^8 m/s \) |

The luminosity for pp collider can be written as [4]:

\[ \mathcal{L} = \frac{I_b \xi_u \gamma}{e \beta^* r_p} F_{ca} F_h \]  

(6)

\[ \mathcal{L}_0 = \frac{I_b \xi_u \gamma}{e \beta^* r_p} \]  

(7)

Where, \( F_{ca} \) is the luminosity reduction factor due to cross angle [11]:

\[ F_{ca} = \frac{1}{\sqrt{1 + (\frac{\xi_{y,max}}{2\sigma_y})^2}} \]  

(8)

\( F_h \) is the luminosity reduction factor due to hourglass effect [12]:

\[ F_h = \frac{\beta^*}{\sqrt{\pi \sigma_z}} \exp (\frac{\beta^*^2}{2\sigma_z^2}) K_0 (\frac{\beta^*^2}{2\sigma_z}) \]  

(9)

Put \( \xi_{y,max} \) into the luminosity formula, we can get:

\[ \mathcal{L}_0 = \frac{I_b \xi_u \gamma}{2\pi r_p \sigma_y^2 f(x)} \frac{2845}{1} \left[ \frac{I_b P_{SR}}{2E_0 N_{IP}} \right] \]  

(10)

And, then the beta function at IP can be written as:

\[ \beta_* = \frac{2845}{2\pi r_p \sigma_y^2 f(x)} \frac{1}{\mathcal{L}_0} \frac{I_b P_{SR}}{2E_0 N_{IP}} \]  

(11)

The RMS IP spot size:(\( \sigma_*^* = \sigma_x^* = \sigma_y^* \))

\[ \sigma_* = \sqrt{\beta^* \epsilon} = \sqrt{\beta^* \frac{\epsilon_n}{\gamma}} \]  

(12)

Beta at the 1st parasitic encounter with bunch separation \( \Delta t \):

\[ l_1 = c \times \Delta t \]  

(13)

\[ \beta_1 = \beta^* + (\frac{l_1}{2})^2 \]  

(14)

RMS spot size at the 1st parasitic encounter:

\[ \sigma_1 = \sqrt{\beta_1 \epsilon} = \sqrt{\beta_1 \frac{\epsilon_n}{\gamma}} \]  

(15)

The full cross angle [4]:

\[ \theta_c = 2 \times 6\sigma_1 \frac{l_1}{l_1/2} = 24\sigma_1 \frac{l_1}{l_1} \]  

(16)

We can rewrite \( F_{ca} \) as:

\[ F_{ca} = \frac{1}{\sqrt{1 + \Phi^2}} \]  

(17)

\[ \Phi = \frac{\sigma \beta_c}{2\sigma^*} = \frac{12\sigma_1 \sigma_1}{l_1 \sigma^*} = \frac{12\sigma_1 \sqrt{\beta^* \epsilon}}{l_1 \sqrt{\beta^* \epsilon_n \gamma}} = \frac{12\sigma_1 \sqrt{\beta_1 \epsilon}}{l_1 \sqrt{\beta_1 \epsilon_n \gamma}} = 12 \sqrt{\frac{\sigma^2}{(c \Delta t)^2} + \frac{1}{4(\beta^*/\sigma^*)^2}} \]  

(18)
Where $\Phi$ is Piwinski angle, $\beta^*$ is beta function at IP, $\sigma_z$ is bunch length and $\Delta t$ is the bunch separation.

When the luminosity reduce less than 10% due to the crossing angle effect, we have $F_{cw} \geq 0.9$. From equation(17) we get:

$$\Phi \leq 0.434822 \text{(rad)}$$  \hspace{1cm} (19)

Bunch numbers:

$$n_b = \frac{T_0 f_2}{\Delta t}$$  \hspace{1cm} (20)

Bunch population:

$$N_p = \frac{I_b}{n_b f_{rec} e}$$  \hspace{1cm} (21)

Combining equation(11)(18)(19)(20)(21), we can get reasonable values of $\beta^* I_b \Delta t n_b N_p$ and the ratio $\beta^*/\sigma_z$, where should also consider the instability influence and the constraints from technic.

From the definition of beam beam tune shift [11]:

$$\xi_y = \frac{N_p r_p}{4\pi \epsilon_n}$$  \hspace{1cm} (22)

We can get the normalized emittance:

$$\epsilon_n = \frac{N_p r_p}{4 \pi \xi_{y,\text{max}}}$$  \hspace{1cm} (23)

Then we can calculate $\sigma^* \beta_1 \sigma_1 \theta_c$ and $F_h$. Finally, we get the final value of the luminosity:

$$\mathcal{L} = \mathcal{L}_0 F_{cw} F_h$$  \hspace{1cm} (24)

We can also calculate the follow parameters easily.

Energy loss per turn [13]:

$$U_0 = 0.00778 [MeV] \left(\frac{E_0 [TeV]}{\rho [m]}\right)^4$$  \hspace{1cm} (25)

SR power per ring:

$$P_{SR} = U_0 I_b$$  \hspace{1cm} (26)

Critical photon energy $[E_c]$ [13][14]:

$$E_c [K eV] = 1.077 \times 10^{-4} (E_0 [TeV])^2 B [T]$$  \hspace{1cm} (27)

Accumulated particles per beam:

$$N_{ACC} = N_p n_b$$  \hspace{1cm} (28)

Stored energy per beam:

$$W = N_{ACC} E_0 e = N_p n_b E_0 e$$  \hspace{1cm} (29)

ARC SR heat load [15]:

$$\text{SR heat load} = \frac{P_{SR}}{L_{Dipole}}$$  \hspace{1cm} (30)

Transverse damping time $[\tau_x]$ [10]:

$$\tau_x = \frac{2 E_0 T_0}{J_k U_0}$$  \hspace{1cm} (31)

Longitudinal damping time $[\tau_c]$ [10]:

$$\tau_c = \frac{2 E_0 T_0}{J_k U_0}$$  \hspace{1cm} (32)

Beam life time due to burn-off [11]:

$$\tau_{\text{burn-off}} = \frac{N_p n_b}{\mathcal{L} N_{IP} \sigma_{cross}} = \frac{N_{ACC}}{\mathcal{L} N_{IP} \sigma_{cross}}$$  \hspace{1cm} (33)

The time required to reach $1/e$ of the initial luminosity [11]:

$$\tau_{1/e} = (\sqrt{e} - 1) \times \tau_{\text{burn-off}}$$  \hspace{1cm} (34)

Other contributions to luminosity decay come from Toucheck scattering and from particle losses due to a slow emittance blow-up. An emittance blow-up can be caused by the scattering of particles on the residual gas, the nonlinear force of the beam-beam interaction, RF noise and IBS scattering effects. The synchrotron radiation damping decreases the bunch dimensions and can partially compensate the beam size blow-up due to the above effects. Assuming that the radiation damping process just cancels the beam blow up due to the beam-beam interactions and RF noise, one can estimate the net luminosity lifetime by [11]:

$$\tau_L = \frac{1}{\frac{1}{\tau_{\text{IBS}}} + \frac{2}{\tau_{\text{rest-gas}}} + \frac{1}{\tau_{1/e}}}$$  \hspace{1cm} (35)

If the run time $\tau_{\text{run}}$ fulfills equation(36), the integrated luminosity has the maximum value and the run time will be the optimum run time [11]:

$$\log \left(\frac{\tau_{\text{turn-around}} + \tau_{\text{run}}}{\tau_L} + 1\right) = \frac{\tau_{\text{run}}}{\tau_L}$$  \hspace{1cm} (36)

$$\tau_{\text{optimum}} = \tau_{\text{run}}$$  \hspace{1cm} (37)

Integrating the luminosity over one luminosity run $[fb^{-1}]$:

$$L_{\text{int}} = \mathcal{L} \tau_L (1 - e^{-\frac{\tau_{\text{run}}}{\tau_L}}) \times \frac{3600}{10^{39}}$$  \hspace{1cm} (38)

where $\tau_{\text{run}}$ is the optimum total length of the luminosity run.

The overall collider efficiency depends on the ratio of the run length and the average turnaround time. So the optimum average integrated luminosity/day $[fb^{-1}]$ is [11]:

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\[ L_{\text{tot}} = \frac{24}{\tau_{\text{run}}[\hbar] + \tau_{\text{turn-around}}[\hbar]} L_{\text{int}} \]  

(39) \hspace{1cm} n_b = \frac{T_0 f_2}{\Delta t} \]  

(55) \hspace{1cm} N_p = \frac{I_b}{n_b f_{\text{rev}} e} \]  

(56) \hspace{1cm} \epsilon_a = \frac{N_p r_p}{4\pi \xi y_{\text{max}}} \]  

(57) \hspace{1cm} F_h = \frac{\beta^*}{\sqrt{\pi} \sigma_z} \exp\left(\frac{\beta^*}{2\sigma_z^2}\right) K_0\left(\frac{\beta^*}{2\sigma_z^2}\right) \]  

(58) \hspace{1cm} \mathcal{L} = \mathcal{L}_0 F_{\text{ca}} F_h \]  

(59) \hspace{1cm} N_{\text{ACC}} = N_p n_b \]  

(60) \hspace{1cm} W = N_{\text{ACC}} E_0 e = N_p n_b E_0 e \]  

(61) \hspace{1cm} \text{SR heat load} = \frac{P_{\text{SR}}}{L_{\text{Dispol}}} \]  

(62) \hspace{1cm} \tau_x = \frac{2E_0 T_0}{J_x U_0} \]  

(63) \hspace{1cm} \tau_z = \frac{2E_0 T_0}{J_z U_0} \]  

(64) \hspace{1cm} \tau_{\text{burn-off}} = \frac{N_{\text{ACC}}}{\mathcal{L} N_{\text{IP}} \sigma_{\text{cross}}} \]  

(65) \hspace{1cm} \tau_{1/e} = (\sqrt{e} - 1) \times \tau_{\text{burn-off}} \]  

(66) \hspace{1cm} \tau_L = \frac{1}{\tau_{\text{IBS}} + \tau_{\text{rest-gas}} + \tau_{1/e}} \]  

(67) \hspace{1cm} \log\left(\frac{\tau_{\text{turn-around}} + \tau_{\text{run}}}{\tau_L}\right) + 1 = \frac{\tau_{\text{run}}}{\tau_L} \]  

(68) \hspace{1cm} \tau_{\text{optimum}} = \tau_{\text{run}} \]  

(69) \hspace{1cm} L_{\text{int}} = \mathcal{L} \tau_L \left(1 - e^{-\frac{\tau_{\text{run}}}{\tau_L}}\right) \times \frac{3600}{10^{39}} \]  

(70) \hspace{1cm} L_{\text{tot}} = \frac{24}{\tau_{\text{run}}[\hbar] + \tau_{\text{turn-around}}[\hbar]} L_{\text{int}} \]  

(71)
4 Compare the LHC parameter list with the parameter obtained by our method

To check our method, we use it to chose and calculate the LHC parameters and compare them with the LHC parameter list. The second column in Table 2 is the parameter obtained using our systematical method, which is reasonable and nearly with the parameters in LHC parameter list. This indicates that our method is reasonable and more powerful. We can use this method to design and choose parameters for any proton proton circular colliders.

Table 2. Compare the LHC parameter list with the parameter obtained by our method.

| LHC-list | LHC-new |
|----------|---------|
| Value    | Unit    |
| Beam energy [E_0] | 7 | 7 | TeV |
| Circumference [C_0] | 26.7 | 26.7 | km |
| Lorentz gamma [γ] | 7463 | 7463 |
| Dipole field [B] | 8.33 | 8.26 | T |
| Dipole curvature radius [ρ] | 2801 | 2826 | m |
| Bunch filling factor [f_2] | 0.78 | 0.80 |
| Arc filling factor [H] | 0.79 | 0.79 |
| Total dipole magnet length [L_{Dipole}] | 17599 | 17756 | m |
| Arc length [L_{ARC}] | 22476 | 22476 | m |
| Total straight section length [L_{SS}] | 4224 | 4224 | m |
| Energy gain factor in collider rings | 15.6 | 15.6 |
| Injection energy [E_{inj}] | 0.45 | 0.45 | TeV |
| Number of IPs [N_{IP}] | 4 | 2 |

Physics performance and beam parameters

| Peak luminosity per IP [L] | 1.0E+34 | 1.0E+34 | /cm²·s |
| Optimum run time | 15.2 | 10.46 | hour |
| Optimum average integrated luminosity/day | 0.47 | 0.42 | fb⁻¹ |
| Assumed turnaround time | 6 | 5 | hour |
| Overall operation cycle | 21.2 | 16.9 | hour |
| Beam life time due to burn-off [τ] | 45 | 40.65 | hour |
| Total / inelastic cross section [σ] | 111/85 | 111/85 | mbarn |

Beam parameters

| Beta function at collision [β⁺] | 0.55 | 0.56 | m |
| Max beam-beam tune shift per IP [ξ] | 0.0033 | 0.0032 |
| Number of IPs contributing to ΔQ | 3 | 2 |
| Max total beam-beam tune shift | 0.01 | 0.0064 |
| Circulating beam current [I_b] | 0.584 | 0.589 | A |
| Bunch separation [Δt] | 25 | 5 | 25 | 5 | ns |
| Number of bunches [n_b] | 2808 | 2848 |
| Bunch population [Np] | 1.15 | 1.15 | 10¹¹ |

| Normalized emittance [ε] | 3.75 | 4.39 | μm |
| RMS IP spot size [σ⁺] | 16.7 | 16.09 | μm |
| Beta at the 1st parasitic encounter [β⁺] | 26.12 | 32.37 | m |
| RMS spot size at the 1st parasitic encounter [σ⁺] | 114.6 | 138 | μm |
| RMS bunch length [σz] | 75.5 | 75.7 | mm |
| Accumulated particles per beam | 0.32 | 0.33 | 10¹⁵ |
| Full crossing angle [θc] | 285 | 441.16 | μrad |
| Reduction factor according to cross angle [Fca] | 0.8391 | 0.7788 |
| Reduction factor according to hour glass effect [Fh] | 0.9954 | 0.9956 |

5 Parameter choice for SPPC

5.1 Parameter scan

Using the method above, we scan the goal luminosity L₀ with different bending radius ρ, IP numbers N_{IP} and different ratio of β⁺/σz. Table 3 shows the input parameters. We get some meaningful results which are shown from Fig.1 to Fig.8.

Table 3. Input parameters for machine design.

| Energy E₀ | Circumference C₀ | Goal luminosity L₀ |
|-----------|-------------------|--------------------|
| 35.07eV  | 54.7Km            | (1 ~ 4) × 10³⁸ cm⁻² s⁻¹ |
| IP numbers N_{IP} | Bending radius | ratio of β⁺/σz |
| 2 ~ 4    | 5.9 ~ 6.5Km       | 10 ~ 20          |

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Fig. 1. Vertical beta at IP as the function of goal luminosity.

Fig.1 shows that larger luminosity needs smaller vertical IP beta function. Larger bending radius and more interaction points require smaller $\beta^*$ at the same goal luminosity.

Fig. 2. Vertical beam beam tune shift as the function of peak luminosity.

Fig.2 shows smaller bending radius and less interaction points give larger vertical beam-beam tune shift.

Fig. 3. Bunch population as the function of peak luminosity.

Fig.3 and Fig.4 show that larger luminosity needs larger bunch population or larger bunch number. Larger bending radius and more interaction points indicate larger bunch population or larger bunch number at the same goal luminosity.

Fig. 4. Bunch number as the function of peak luminosity.

Fig. 5. $F_{ca}$ as the function of $\Delta t$.

Fig. 6. $F_{ca}$ as the function of the ratio of $\beta^*$ and $\sigma_z$. 
Fig. 5 and Fig. 6 tell us that the reduction factor according to cross angle has relationship with bunch separation (Δt) and the ratio of IP beta and RMS bunch length (β*/σz). The maximum value of this factor is 1, and larger β*/σz makes this value nearer to 1. If we want this effect reduce the luminosity less than 10%, we should have Fca ≥ 0.9. The dashed line in Fig. 5 and Fig. 6 is the value equal to 0.9, and we can easily get the important information from the figures. We should choose a larger β*/σz, which about 15 is much reasonable and now the bunch separation is 25 ns. If we want to choose a smaller bunch separation like 5 ns, the ratio of β* and σz should be more than 20. We should consider both of them and choose the eclectic values. Fig. 7 shows the 3D diagram of the relationship of Fca, Δt and β*/σz.

Overall speaking, we should decrease IP numbers and increase bending radius in order to achieve higher luminosity. NIP = 2 is the reasonable minimum value for IP number. Assuming the maximum dipole arc filling factor is 80%, 5.9 km bending radius will be a limit for the 54.7 km ring.

5.2 Constraints form RF system

As long as a set of beam parameters is determined, we need to check the RF system to see if the bunch length can be achieved. Firstly, considering the synchrotron radiation energy loss has to be compensated by the RF cavities, one finds [16]:

\[ U_0 = eV_{rf} \sin(\phi_s) \]  

where \( V_{rf} \) is the total voltage of the RF cavities and \( \phi_s \) is the synchrotron phase. According to eq. (72), one gets

\[ \phi_s = \pi - \arcsin\left(\frac{U_0}{eV_{rf}}\right) \]  

We can estimate the RF frequency from the "pill-box" model. As the following picture shows. We can find the \( f_{rf} \) via the Maxwell’s equation and the boundary conditions [16].

\[ J_0\left(\frac{\omega}{cR_0}\right) = 0 \]  

\[ \frac{\omega}{c}R_0 = 2.405 \]  

\[ \frac{2\pi f_{rf}}{c}R_0 = 2.405 \]  

\[ f_{rf} = \frac{2.405c}{2\pi R_0} \]  

When the cavity inner radius \( R_0 = 30 \text{cm}, f_{RF} = 400 \text{MHz} \) is a reasonable choose [16].

In a storage ring with an isomagnetic guide field (one which has a constant radius \( \rho \) in the magnets and is straight elsewhere) the relative energy spread \( \sigma_e/E_0 \) can be expressed as [17]:

\[ (\delta_e)^2 = \left(\frac{\sigma_e}{E_0}\right)^2 = \frac{C_q \gamma^2}{J_{r0}} \quad \text{(isomag)} \]  

Fig. 7. The 3D diagram of the relationship of Fca, Δt and β*/σz.

Fig. 8. \( F_h \) as the function of the ratio of β* and σz.

Fig. 9. Pill-box model.
so,
\[ \delta_s = \gamma \sqrt{\frac{C_q}{}\sqrt{J_0}}_0} \]
(78)
where \( C_q = 1.2817 \times 10^{-12} \) m is a constant.

The nature bunch length is expressed by [17]:
\[ \sigma_i = \frac{\alpha_p R \delta_s}{\nu_s} \]
(79)
where, \( \alpha_p \) is the momentum compaction factor, \( R \) is the average radius of the ring. \( \nu_s \) is the longitudinal oscillation tune which can be expressed as:
\[ \nu_s = \sqrt{-\frac{\eta_p hc V_{rf} \cos \phi_s}{2\pi E_0 \beta_2^2}} \]
(80)
Where \( \eta_p \) is the phase slippage factor, when \( v \approx c, \beta \approx 1, \gamma \rightarrow \infty, \eta_p = \alpha_p - \frac{1}{2} \approx \alpha_p, \) and \( h = f_{rf} \pi \nu_s = f_{rf} T_0, \) we can rewrite \( \nu_s \) as follow:
\[ \nu_s = -\frac{\eta_p hc V_{rf} \cos \phi_s}{2\pi E_0} \]
(81)
And then the nature bunch length can be expressed as [17][18]:
\[ \sigma_i = \sqrt{-\frac{2\pi E_0 \alpha_p}{f_{rf} T_0 e V_{rf} \cos \phi_s}} R \delta_s \]
(82)
The energy acceptance can be expressed as [17][18]:
\[ \eta_{\text{acceptance}} = \frac{2U_0}{\pi \alpha_p f_{rf} T_0 \sqrt{\left[\sqrt{q^2 - 1} - \arccos \left(\frac{1}{q}\right)\right]}} \]
(83)
where, \( q = e V_{rf}/U_0. \) Combining the eqs.(82) and eqs.(83), we can get the RF frequency \( f_{rf} \) and the momentum compaction \( \alpha_p \) for given RF voltage \( V_{rf} \) and energy acceptance \( \eta. \)

The synchrotron frequency [18]:
\[ f_{syn} = \frac{\nu_s}{T_0} = \nu_s f_{\text{rev}} \]
(84)

Bucket area [18]:
\[ \text{bucket area} = \frac{16\nu_s}{\hbar |\eta_p| \sqrt{\cos \phi_s}} \alpha(\phi_s) \]
(85)
where the dimensionless function \( \alpha(\phi_s) \) is the bucket area normalized to the case when \( \phi_s = 0. \) For the case \( \eta_p < 0, \) we have
\[ \alpha(\phi_s) = \frac{1}{4\sqrt{2}} \int_{0}^{\pi} \left[ \cos \phi + \cos \phi_s - (\pi - \pi) \sin \phi_s \right]^{1/2} d\phi \]
(86)
when \( \phi_s = 0, \alpha(\phi_s) = 1, \) then the bucket area is \( \frac{16\nu_s}{\hbar |\eta_p|}. \)

The bucket half height [18]:
\[ \text{bucket half height} = \sqrt{\frac{2e V_0 \cos \phi_s - \frac{\pi - 2\phi_s}{2} \sin \phi_s}{\pi \beta_2^2 E_0 h |\eta_p|}} \]
(87)
when \( \phi_s = 0, \) we have bucket half height \( \frac{2\nu_s}{\hbar |\eta_p|}. \)

5.3 Machine parameter choice for SPPC

Combining the discussions above, we get a set of new design for the 54.7 km SPPC. We also tried to give a set of parameters for larger circumference SPPC, like 78Km or 100Km. Table 4 is the parameter list for SPPC. As a comparison, we put the parameter for LHC HL-LHC HE-LHC and FCC-hh together in Table 4. The first plan for SPPC is using the same tunnel with CEPC. The circumference is 54.7Km which is determined by CEPC. We choose the dipole field as 20T and get center-of-mass energy 70TeV. If we want to explore the higher energy, we should make the circumference larger. When we want to explore center-of-mass energy 100TeV and keep the dipole field 20T, the circumference should be 78Km at least. At this condition, there is hardly space to upgrade. So a 100Km SPPC is much better because the dipole field is only 14.7T at this condition. If we make the dipole field 20T too, we can get the center-of-mass energy as high as 136TeV.

Table 4. Parameter lists for LHC HL-LHC HE-LHC FCC-hh and SPPC.

|                         | LHC | HL-LHC | HE-LHC | FCC-hh | SPPC-PrecDR | SPPC-54.7Km | SPPC-100Km | SPPC-78Km |
|-------------------------|-----|--------|--------|--------|-------------|-------------|------------|-----------|
| Beam energy [GeV]       | 7   | 7      | 16.5   | 50     | 35.6        | 35.0        | 50.0       | 50.0      |
| Circumference [km]      | 26.7| 26.7   | 26.7   | 100     | 54.7        | 54.7        | 100        | 100       |
| Lorentz gamma [\(\gamma\)] | 7463| 7463   | 14392  | 53305  | 37942       | 37313       | 53305      | 72495     |
| Dipole field [\(B\)]   | 8.33| 8.33   | 20     | 16      | 20          | 19.69       | 14.73      | 20.03     |
| Dipole curvature radius [\(\rho\)] | 2801| 2801   | 2250   | 10416   | 5928        | 5922.6      | 11315.9    | 11315.9   |

Table 4 is the parameter list for SPPC. As a comparison, we put the parameter for LHC HL-LHC HE-LHC and FCC-hh together in Table 4. The first plan for SPPC is using the same tunnel with CEPC. The circumference is 54.7Km which is determined by CEPC. We choose the dipole field as 20T and get center-of-mass energy 70TeV. If we want to explore the higher energy, we should make the circumference larger. When we want to explore center-of-mass energy 100TeV and keep the dipole field 20T, the circumference should be 78Km at least. At this condition, there is hardly space to upgrade. So a 100Km SPPC is much better because the dipole field is only 14.7T at this condition. If we make the dipole field 20T too, we can get the center-of-mass energy as high as 136TeV.
| Beam parameters | Value |
|-----------------|-------|
| Bunch filling factor \([f_z]\) | 0.78  |
| Arc filling factor \([f_t]\) | 0.79  |
| Total dipole magnet length \([L_{Dipole}]\) | 17599 |
| Arc length \([L_{ARC}]\) | 22476 |
| Total straight section length \([L_{SS}]\) | 4224  |
| Energy gain factor in collider rings | 15.6 |
| Injection energy \([E_{inj}]\) | 0.45 |
| Number of IPs \([N_{IP}]\) | 4 |

**Physics performance and beam parameters**

| Parameter | Value |
|-----------|-------|
| Peak luminosity per IP \([L]\) | \(1.0\times10^{34}\) |
| Optimum run time | 15.2 |
| Optimum average integrated luminosity/day | 0.47 |
| Assumed turnaround time | 6 |
| Overall operation cycle | 21.2 |
| Beam life time due to burn-off \([r]\) | 45 |
| Total / inelastic cross section \([\sigma]\) | 111/85 |

**Beam parameters**

| Parameter | Value |
|-----------|-------|
| Beta function at collision \([\beta^*]\) | 0.55 |
| Max beam-beam tune shift per IP \([\xi]\) | 0.0033 |
| Number of IPs contributing to \([\Delta Q]\) | 3 |
| Max total beam-beam tune shift | 0.01 |
| Circulating beam current \([I_0]\) | 0.584 |
| Bunch separation \([\Delta t]\) | 25 |
| Number of bunches \([n_b]\) | 2808 |
| Bunch population \([N_p]\) | 1.15 |
| Normalized RMS transverse emittance \([\epsilon]\) | 3.75 |
| RMS IP spot size \([\sigma^*]\) | 16.7 |
| Beta at the 1st parasitic encounter \([\beta_1]\) | 26.12 |
| RMS spot size at the 1st parasitic encounter \([\sigma_1]\) | 114.6 |
| RMS bunch length \([\sigma_z]\) | 75.5 |
| Accumulated particles per beam | 0.32 |
| Full crossing angle \([\theta_c]\) | 285 |
| Reduction factor according to cross angle \([F_{Ca}]\) | 0.8391 |
| Reduction factor according to hour glass effect \([F_{H}\)] | 0.9954 |

**Other beam and machine parameters**

| Parameter | Value |
|-----------|-------|
| Energy loss per turn \([U_0]\) | 0.0067 |
| Critical photon energy \([E_c]\) | 0.044 |
| SR power per ring \([P_0]\) | 0.0036 |
| Stored energy per beam \([W]\) | 0.362 |

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6 Comparing beam-beam tune shift of SPPC with LHC HL-LHC HE-LHC and FCC-hh

In the parameter design of LHC HL-LHC HE-LHC and FCC-hh, the CERN people assume the beam-beam tune shift limit as a constant number [4][11]. But we can find the beam-beam parameter has relationship with several parameters. A method to estimate the maximum beam-beam tune shift limit was developed in reference [9]. We compare the calculated numbers with the parameter list chosen numbers and find that these calculated numbers by analytical expression are much reasonable according to the real experimental numbers. We can easily get the ratio of the beam-beam tune shift of the list chosen number and the calculated number. The result was shown in Table 5, we can find that HL-LHC’s choice is much overlarge and the other machines’ choices are more reasonable.

| Parameters                      | LHC 7TeV | HL-LHC 7TeV | HE-LHC 16.5TeV | FCC-hh 50TeV | SPPC Pre-CDR 35.6TeV | SPPC-54.7Km 35TeV | SPPC-100Km 50TeV | SPPC-100Km 68TeV | SPPC-78Km 50TeV |
|---------------------------------|----------|-------------|----------------|-------------|----------------------|-------------------|------------------|------------------|------------------|
| Number of IPs contributing to ΔQ| 3        | 2           | 2              | 2           | 2                    | 2                 | 2                | 2                | 2                |
| Max total beam-beam tune shift  | 0.01     | 0.015       | 0.01           | 0.01        | 0.012                | 0.013             | 0.0134           | 0.016            | 0.0146           |
| Max beam-beam tune shift per IP| 0.0033   | 0.0075      | 0.005          | 0.005       | 0.006                | 0.0065            | 0.0067           | 0.008            | 0.0073           |
| (parameter list)               |          |             |                |             |                      |                   |                  |                  |                  |
| Max beam-beam tune shift per IP| 0.00321  | 0.00321     | 0.00499        | 0.00685     | 0.00662              | 0.006559          | 0.006688         | 0.00801          | 0.00731          |
| (calculated)                   |          |             |                |             |                      |                   |                  |                  |                  |
| [ξy](parameter list)/[ξy](calculated) | 1.0287   | 2.3379      | 1.002          | 0.7299      | 0.9063               | 0.9910            | 1.001            | 0.9986           | 0.9999           |

7 Conclusion

In this paper, a systematic method of how to make an appropriate parameter choice for a circular pp collider by using analytical expression of beam-beam tune shift limit started from given luminosity goal, beam energy and technical limitations was developed. By using this method, we reveal the relations of machine parameters with goal luminosity clearly and hence give a parameter choice in an efficient way. We also show the parameter chose for a 50Km SPPC and larger circumference SPPC, like a 78Km SPPC or a 100Km SPPC.
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