Top Quark Mixed Hybrid Meson and the Quark-Gluon Plasma

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Abstract

Using the method of QCD Sum Rules we estimate the energy of lowest energy top quark meson state with a hybrid admixture. This new estimate uses the previous estimates of hybrid charmonium and upsilon states, but with the mass of the top quark mass being much greater than the charm and bottom quark masses. We discuss production of mixed hybrid top quark mesons and possible detection of the creation of the Quark-Gluon Plasma via Relativistic Heavy Ion Collisions.

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1 Introduction

In previous estimates of mixed heavy quark hybrid mesons[1, 2] the charmonium meson state Ψ(2S) and upsilon meson state Υ(3S) state were shown to be approximately 50% standard meson and 50% hybrid meson, with the hybrid meson having an active gluon. QCD Sules were also used to estimate the mass of a scalar glueball[3].

The quarks associated with Ψ(2S), Υ(3S) are the charm and bottom quarks with masses $M_c \simeq 1.27$ GeV, $M_b \simeq 4.18$ GeV. Since the top quark mass, $M_t \simeq 173$ GeV[4] is much larger than the charm, bottom quark masses, $M_c, M_b$, top meson states have not been detected. Recently, however a signal for $t\bar{t}$ production in p-Pb collisions with $\sqrt{s_{NN}} = 8.16$ TeV has been observed[5]. Also, an important motivation for the present work is the recent study of the possible determination of the creation of the Quark-Gluon Plasma (QGP) via the production of $t\bar{t}$ events[6].

In order to estimate the energy of lowest energy hybrid top quark meson state we need the quark and gluon condensates. Note that FIG. 8 in Ref.[2] gave the dominant diagram so the charm quark condensate $<c\bar{c}>$ and bottom quark condensate $<b\bar{b}>$ were not included in the estimates of mixed charmonium and bottomonium hybrid mesons.

However, the top quark condensate, $<t\bar{t}>$, must be included in the present work. From Ref.[7] $<t\bar{t}> \simeq (126\text{GeV})^3$, which is used to estimate the mixed top quark meson correlator in section 3. Another important parameter to estimate the mass of a mixed top quark hybrid meson is the gluon condensate $<G^2>$, whose value is discussed and given below.

In the next section we give a brief review of the method of QCD Sum Rules, and in the following section we estimate the energy of lowest energy mixed hybrid top quark meson state using QCD Sum Rules. As with the charm and bottom $\Psi(nS), \Upsilon(nS)$states, we choose a top current so the $\Xi(nS)$ states are $1^{-+}$ states. Note that in Refs[1, 2] the $\Psi(2S), \Upsilon(3S)$ were found to be mixed hybrid states, while in the present research we find that the $\Xi(1S)$ is a mixed hybrid top meson state.

Since the top quark condensate, $<t\bar{t}>$ is large we must also include the top quark condensate correlator, $\Pi_{H}^{\mu\nu}(p)$, which was not necessary in estimates of the mixed hybrid $\Psi(nS), \Upsilon(nS)$states.

Also, after our derivation of the correlator of the mixed state, $\Pi_{H-H}^{\mu\nu}(p)$, we find there are terms needing Borel transforms that were not needed in Refs[1, 2].
2 Brief Review of the Method of QCD Sum Rules

The starting point of the method of QCD sum rules for finding $M_A$, the mass of a state $A$, is the correlator,

$$\Pi^A(x) = \langle | T[J_A(x)J_A(0)] | \rangle,$$

with $|$ the vacuum state and the current $J_A(x)$ creating the states with quantum numbers $A$:

$$J_A(x)| = c_A|A⟩ + \sum_n c_n|n; A⟩,$$

where $|A⟩$ is the lowest energy state with quantum numbers $A$. After a Fourier transform to momentum space,

$$\Pi^A(x) \Rightarrow \Pi^A(p),$$

where $p$ is the momentum. The Borel Transform, is defined by

$$B_{M_B^2} \Pi^A(p) = \Pi^A(M_B),$$

with $M_B$ the Borel mass. Defining the right-hand side (rhs) of the sum rule

$$\Pi(p)_{\text{rhs}}^A = \sum_k c_k(p)⟨0|O_k|0⟩,$$

where $c_k(p)$ are the Wilson coefficients and $⟨0|O_k|0⟩$ are gauge invariant operators constructed from quark and gluon fields, the final QCD sum rule is

$$\Pi^A(M_B) = B_{M_B^2} \Pi(p)_{\text{rhs}}^A.$$

The value of $M_A$ is found as the minimum in the plot of $M_A$ vs $M_B$.

3 Mixed Top Quark $1^{--}$ States Using QCD Sum Rules

Based on previous work, we assume that there is strong mixing between a top quark meson and a hybrid top quark meson with the same quantum numbers (as shown below).

3.1 Top Quark Meson Correlator

We now attempt to find the lowest $J^{PC} = 1^{--}$ state with a sizable admixture of a top meson and a hybrid top meson. The mixed vector ($J^{PC} = 1^{--}$) top, hybrid top current we use in QCD Sum Rules is

$$J^\mu = bJ_H^\mu + \sqrt{1-b^2}J_{HH}^\mu,$$

with

$$J_H^\mu = \bar{\Psi}_t \gamma^\mu \Psi_t$$

and

$$J_{HH}^\mu = \bar{\Psi}_t \Gamma_\nu G^{\mu\nu} \Psi_t,$$

where $\Psi_t$ is the top quark field, $\Gamma_\nu = C\gamma_\nu$, $\gamma_\nu$ is the usual Dirac matrix, $C$ is the charge conjugation operator, and the gluon color field is

$$G^{\mu\nu} = \sum_{a=1}^8 \frac{\lambda_a}{2} G^{\mu\nu}_a,$$

with $\lambda_a$ the SU(3) generator ($Tr[\lambda_a \lambda_b] = 2\delta_{ab}$).
Therefore the correlator for the mixed state:

$$\Pi_{H-H}^{\mu\nu}(x) = <0|T[J_\mu(x)J_\nu(0)]|0>$$  \hspace{1cm} (11)$$

is

$$\Pi_{H-H}^{\mu\nu}(x) = b^2 \Pi_{H}^{\mu\nu}(x) + (1 - b^2) \Pi_{HH}^{\mu\nu}(x)$$
$$+ 2b\sqrt{1 - b^2} \Pi_{HHH}^{\mu\nu}(x)$$  \hspace{1cm} (12)$$

$$\Pi_{H}^{\mu\nu}(x) = <0|T[J_\mu^H(x)J_\nu^H(0)]|0>$$
$$\Pi_{HH}^{\mu\nu}(x) = <0|T[J_\mu^{HH}(x)J_\nu^{HH}(0)]|0>$$
$$\Pi_{HHH}^{\mu\nu}(x) = <0|T[J_\mu^{H}(x)J_\nu^{HH}(0)]|0>$$ ,

where $\Pi_{H}^{\mu\nu}$, $\Pi_{HH}^{\mu\nu}$, $\Pi_{HHH}^{\mu\nu}$ are the correlators for the top quark meson, the hybrid top quark meson, the mixed meson and hybrid meson.

To estimate the mass of the mixed meson and hybrid meson we need the top quark condensate and the gluon condensate. The estimated value of the top quark condensate is\[7\]

$$<\bar{t}t> \simeq (126\text{GeV})^3$$  \hspace{1cm} (13)

In Refs\[9, 10\] $<\alpha_s G^2/\pi>$ were estimated, where $\alpha_s$ is the strong coupling constant and $\alpha_s/\pi \simeq 0.57$. Using this value of $\alpha_s/\pi$, from Refs\[9, 10\] the gluon condensate is

$$<G^2> \simeq 0.003\text{GeV}^4$$  \hspace{1cm} (14)

The basic diagrams for the top quark meson correlator are shown in Figure 1

![Figure 1: Heavy quark meson diagrams: (a) standard correlator; (b) Quark Condensate; (c) Gluon Condensate](image-url)
From Eqs.\(12\) the quark correlator in momentum space, corresponding to Figure 1 (a), is

\[
\Pi_{H}^{\mu\nu}(p) = g_{V}^{2} \int \frac{d^{4}k}{(2\pi)^{4}} Tr[S(k)\Gamma_{5}^{\mu}S(p-k)\Gamma_{5}^{\nu}] \]

\[
S(k) = \frac{k + M_{t}}{k^{2} - M_{t}^{2}} \]

\[
\Gamma_{5}^{\mu} = \gamma_{\mu}\gamma_{5} \]

\[
Tr[S(k)\Gamma_{5}^{\mu}S(p-k)\Gamma_{5}^{\nu}] = \frac{M_{t}^{2}Tr[\gamma^{\mu}\gamma^{\nu}] - k_{\alpha}(p-k)_{\beta}Tr[\gamma^{\alpha}\gamma^{\mu}\gamma^{\beta}\gamma^{\nu}]}{(k^{2} - M_{t}^{2})(p^{2} - M_{t}^{2})}.
\]  

(15)

The integrals needed to evaluate Eq.\(15\) are

\[
\int \frac{d^{4}k}{(2\pi)^{4}} \frac{1}{(k^{2} - M_{t}^{2})(p^{2} - M_{t}^{2})} = \frac{2M_{t}^{2} - p^{2}/2}{(4\pi)^{2}} I_{o}(p) \]  

(17)

\[
\int \frac{d^{4}k}{(2\pi)^{4}} \frac{k^{\mu}}{(k^{2} - M_{t}^{2})(p^{2} - M_{t}^{2})} = \frac{p^{\mu}}{(4\pi)^{2}} \left(\frac{1}{4}I_{o}(p) + 7/4\right) \]

\[
\int \frac{d^{4}k}{(2\pi)^{4}} \frac{k^{\mu}k^{\nu}}{(k^{2} - M_{t}^{2})(p^{2} - M_{t}^{2})} = \frac{g^{\mu\nu}p^{2}}{12(4\pi)^{2}} \left(\frac{5M_{t}^{2} - p^{2}}{2}I_{o}(p) + \frac{p^{\mu}p^{\nu}}{4\pi^{2}} \right) \]

with

\[
I_{o}(p) = \int_{0}^{1} \frac{d\alpha}{\alpha(1 - \alpha)p^{2} - M_{t}^{2}}.
\]  

(18)

Carrying out the traces and the \(k\) integral and dropping terms that vanish with a Borel transform

\[
\Pi_{H}^{\mu\nu}(p) = g^{\mu\nu} \frac{4g_{V}^{2}}{(4\pi)^{2}} p^{2}(M_{t}^{2} - \frac{p^{2}}{4})I_{o}(p).
\]  

(19)

With the scalar correlator \(\Pi^{S}\) defined by \(\Pi^{\mu\nu}(p) = (p_{\mu}p_{\nu}/p^{2} - g^{\mu\nu})\Pi^{V}(p) + (p_{\mu}p_{\nu}/p^{2})\Pi^{S}(p)\), the scalar correlator \(\Pi_{H}^{S}(p)\) is

\[
\Pi_{H}^{S}(p) = \frac{4g_{V}^{2}}{(4\pi)^{2}} p^{2}(M_{t}^{2} - \frac{p^{2}}{4})I_{o}(p).
\]  

(20)

The top quark condensate correlator corresponding to Figure 1 (b), \(\Pi_{Hc}^{\mu\nu}(p)\), with the condensate carrying no momentum so \(S_{c}(k) = -<\bar{t}t>M_{t}\delta(k-0)\) \(-M_{t}^{2}\), and

\[
\Pi_{Hc}^{\mu\nu}(p) = -\frac{g_{V}^{2}}{(2\pi)^{4}} <\bar{t}t>M_{t}g^{\mu\nu}\frac{M_{t}}{p^{2} - M_{t}^{2}} \]

\[
= -4g^{\mu\nu} \frac{g_{V}^{2}}{(2\pi)^{4}} <\bar{t}t>M_{t}\frac{M_{t}}{p^{2} - M_{t}^{2}}.
\]  

(21)

Therefore the scalar top quark condensate correlator is

\[
\Pi_{Hc}^{S}(p) = -\frac{4g_{V}^{2}}{(2\pi)^{4}} <\bar{t}t>M_{t}\frac{M_{t}}{p^{2} - M_{t}^{2}}.
\]  

(22)

Noting from Eq.\(14\), as in Ref.\(2\), we do not include the Gluon Condensate term, Figure 1 (c).

### 3.2 Top Quark Hybrid Meson Correlator

From Eqs.\(9\)\(10\) the top quark hybrid correlator\(1\)\(2\)

\[
\Pi_{HH}^{\mu\nu}(p) = 6ig_{V}^{2} \int \frac{d^{4}k}{(2\pi)^{4}} Tr[S(k)\Gamma_{5}^{\mu}S(p-k)\Gamma_{5}^{\nu}]Tr[G^{\alpha\beta}G^{\alpha\beta}](k).
\]  

(23)
with

\[ \text{Tr}(G^{\mu \alpha} G^{\nu \beta})(k) = -i4\pi^2 \left( g^{\alpha \beta} \frac{k^\mu k^\nu}{k^2} + g^{\mu \nu} \frac{k^\alpha k^\beta}{k^2} - g^{\alpha \nu} \frac{k^\mu k^\beta}{k^2} - g^{\mu \beta} \frac{k^\alpha k^\nu}{k^2} \right). \]  

(24)

In order to carry out the integrals for \( \Pi_{HH}^{\mu \nu}(p) \), in addition to the integrals in Eq(17) we need

\[ \int \frac{d^4 k}{(2\pi)^4 (k^2 - M_i^2)(p - k)^2 - M_i^2} = M_i^2 \left( \frac{2M_i^2 - p^2/2}{(4\pi)^2} I_o(p) + \text{ind of } p \right) \]  

(25)

\[ \int \frac{d^4 k}{(2\pi)^4 k^2(k^2 - M_i^2)(p - k)^2 - M_i^2} = \frac{1}{(4\pi)^2} \int_0^{\infty} d\rho \int_0^{\infty} d\lambda \int_0^{\infty} d\beta \int_0^{\infty} d\gamma \left( \frac{g^{\mu \nu}}{8} + \frac{p^\mu p^\nu \rho \gamma^2}{\lambda} \right) e^{-\rho^2 \rho (\gamma - \gamma^2)} \]

From Eqs(23 24 17 25)

\[ \Pi_{HH}^{\mu \nu}(p) = 6g_\alpha^2 |g^{\mu \nu} I_o(p)(16M_i^4 - 39M_i^2p^2 + p^4/2) + p^\mu p^\nu I_o(p)(63M_i^2/6 - 19p^2/2 - 10M_i^4/3p^2 - 28M_i^4/3p^2) \]

\[ -(24M_i^2 - 8p^2) \int_0^{\infty} d\rho \frac{\delta(1 - \lambda)}{\rho} \int_0^{\infty} d\alpha \int_0^{\infty} d\beta \int_0^{\infty} d\gamma \left( \frac{g^{\mu \nu}}{8} + \frac{p^\mu p^\nu \rho \gamma^2}{\lambda} \right) e^{-\rho^2 \rho (\gamma - \gamma^2)} \]  

(26)

Therefore, using \( \Pi^{\mu \nu}(p) = (p_\mu p_\nu/p^2 - g^{\mu \nu}) \Pi^V(p) + (p_\mu p_\nu/p^2) \Pi^S(p) \)

\[ \Pi_{HH}^S(p) = -ig_\alpha^2(6M_i^4 - 28.5M_i^2p^2 - 9p^4)I_o(p) \]  

(27)

\[ -(24M_i^2 - 8p^2) \int_0^{\infty} d\rho \frac{\delta(1 - \lambda)}{\rho} \int_0^{\infty} d\alpha \int_0^{\infty} d\beta \int_0^{\infty} d\gamma e^{-\rho^2 \rho (\gamma - \gamma^2)} \]

Note that from Ref[1] the Borel transform, \( B \), of the \( I_o(p) \) terms can be found, but the Borel transform of the terms \( p^{2n} e^{-\rho^2 \rho (\gamma - \gamma^2)} \) (with \( n=0,1,2 \)) must be derived.
The mixed top quark-top quark hybrid correlator, $\Pi_{HHH}^{HH}(p)$, is shown in figure 3 below.

![Figure 3: Mixed Top Quark, Hybrid Top Quark, Correlator](image)

From Ref[2]

$$\Pi_{HHH}^{S}(p) \simeq \pi^2 \Pi_{H}^{S}(p) .$$

(28)

Therefore from Eqs(20,22,27,28), with $b = -0.7[2]$, we obtain the scalar correlator $\Pi_{H-HH}^{S}(p)$ needed to obtain the mass of a mixed hybrid top quark meson

$$\Pi_{H-HH}^{S}(p) = (1 + \pi^2)\Pi_{H}^{S}(p) + \Pi_{H-H}^{S}(p) + \Pi_{H-H}^{S}(p) .$$

(29)

Next we must derive the Borel transform $B\Pi_{H-HH}^{S}(p)$ to obtain $\Pi_{H-HH}^{S}(M_B)$ from which we will estimate the mass of a top quark mixed hybrid meson.

### 4 Borel Transform of $\Pi_{H-HH}^{S}(p)$ and Estimate of the Mass of a Mixed Hybrid Top Quark Meson

With the Borel transform defined in Eq[1] the Borel transforms needed in this work are:

$$B_{M_B^2} I_0(p) = 2e^{\frac{2M_B^2}{M_B^2}} K_0(\frac{2M_B^2}{M_B^2})$$

$$B_{M_B^2} p^2 I_0(p) = 4M_t^2 e^{\frac{2M_B^2}{M_B^2}} [K_0(\frac{2M_B^2}{M_B^2}) + K_1(\frac{2M_B^2}{M_B^2})]$$

$$B_{M_B^2} p^4 I_0(p) = 4M_t^4 e^{\frac{2M_B^2}{M_B^2}} [3K_0(\frac{2M_B^2}{M_B^2}) + 4K_1(\frac{2M_B^2}{M_B^2}) + K_2(\frac{2M_B^2}{M_B^2})]$$

$$B_{M_B^2} \frac{1}{p^2 - M_B^2} = -e^{\frac{M_B^2}{M_B^2}}$$

$$B_{M_B^2} e^{-p^2 \rho(\gamma - \gamma^2)} = 0 .$$

(30)

Therefore from Eqs(20,22,27,29,31) $B\Pi_{H-HH}^{S}(p) = \Pi_{H-HH}^{S}(M_B)$, with $x = \frac{2M_B^2}{M_B^2}$, is[1]

$$\Pi_{H-HH}^{S}(x) \simeq g_0^2 M_t^4 e^{-x} [-1.260K_0(-x) - 1.548K_1(-x) - 210K_2(-x)]$$

$$+g_0^2 0.0026 \cdot \bar{t} \cdot M_t e^{\frac{M_B^2}{M_B^2}} .$$

(31)
The Modified Bessel functions of the second kind, $K_n(x)$ in Eq (31), are \[\]

\begin{align*}
K_0(x) &= \sqrt{\frac{\pi}{2x}} e^{-x} (1 - \frac{1}{8x} + \frac{9}{128x^2} - \frac{225}{3072x^3}) \\
K_1(x) &= \sqrt{\frac{\pi}{2x}} e^{-x} (1 + \frac{3}{8x} - \frac{15}{128x^2} + \frac{315}{3072x^3}) \\
K_2(x) &= \sqrt{\frac{\pi}{2x}} e^{-x} (1 + \frac{15}{8x} + \frac{105}{128x^2} - \frac{945}{3072x^3}) .
\end{align*}

(32)

Using $g_s^2 \simeq 1.49$ [4] and $M_t \simeq 173$ GeV, $C \equiv g_s^2 \sqrt{\frac{\pi}{2}} M_t^4 \simeq 1.67 \times 10^8$(GeV)$^4$. $D \equiv g_s^2 0.0026 < \bar{t} t > M_t \simeq 2.22 \times 10^5$(GeV)$^4$. Since $D \ll C$, we drop the last term in Eq (31). From Eqs (31,32), using $\sqrt{-x} = i \sqrt{x}$,

\[\Pi_{S-H-H}^H(x) \simeq -\frac{C}{i} \sqrt{\frac{1}{x}} e^x (3018 - \frac{816.75}{x} + 79.45 - \frac{1.85}{x^3}) .\]

(33)

As in Refs [1, 2], the mass of the top quark mixed hybrid meson, $M_{tHH}$ is obtained from the ratio of the derivative of $\Pi_{S-H-H}^H(M_B)$ with respect to $1/M_B^2$ to $\Pi_{S-H-H}^H(M_B)$:

\[M_{tHH}^2 = \frac{d}{d(1/M_B^2)} \Pi_{S-H-H}^H(M_B)/\Pi_{S-H-H}^H(M_B) .\]

(34)

Since $x = \frac{2M_t^2}{M_B^2}$, or $\frac{d}{d(1/M_B^2)} = 2M_t^2 \frac{d}{dx}$

\[M_{tHH}^2 = 2M_t^2 \frac{d}{dx} \Pi_{S-H-H}^H(x)/\Pi_{S-H-H}^H(x) .\]

(35)

From Eqs (35,33)

\[M_{tHH}^2 = 2M_t^2 \frac{3018 - 2325.8\frac{1}{x} + 1304.6\frac{1}{x^2} - 200.6\frac{1}{x^3} + 6.48\frac{1}{x^4}}{3018 - 816.75\frac{1}{x} + 79.45\frac{1}{x^2} - 1.85\frac{1}{x^3}} .\]

(36)

In Figure 4 below $M_{tHH}^2$ is plotted vs $M_B^2 = 2M_t^2/x$

Figure 4: Mixed top-hybrid top mass approximately 300 GeV
5 Results and Conclusions

As discussed in Ref[12] although the detection of the Quark-Gluon Plasma (QGP) produced in Relativistic Heavy Ion Collisions (RHIC) is difficult, the production of mixed heavy quark hybrid mesons via RHIC is a possible mechanism for detecting the QGP. In Ref[12] the mixed heavy quark hybrid charmonium meson state $\Psi(2S)$ and upsilon meson state $\Upsilon(3S)$ were considered. As discussed above $t\bar{t}$ production in p-Pb has been observed[5], and there as a possible determination of the creation of the QGP via the production of $t\bar{t}$ events[6]. The main result of our present work it that there is a mixed top quark hybrid meson state with a mass approximately 300 GeV, and with the increased energy at the LHC the production of this state could be used to detect the QGP.

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