Calculation of dephasing times in closed quantum dots

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Dephasing of one-particle states in closed quantum dots is analyzed within the framework of random matrix theory and Master equation. Combination of this analysis with recent experiments on the magnetococonductance allows for the first time to evaluate the dephasing times of closed quantum dots. These dephasing times turn out to depend on the mean level spacing and to be significantly enhanced as compared with the case of open dots. Moreover, the experimental data available are consistent with the prediction that the dephasing of one-particle states in finite closed systems disappears at low enough energies and temperatures.

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Quantum coherence of electrons in closed quantum dots has attracted much interest in recent years [1, 2, 3]. Electron-electron interactions are believed to be one of the main sources of dephasing in disordered systems at low temperatures. Compared to low-dimensional metals and semiconductors [4], substantial modifications of this dephasing mechanism are caused by the confinement of the quantum dot which leads to discrete energy levels. In particular, the dephasing rate was predicted [2] to disappear at low excitation energies, $\epsilon < \Delta \sqrt{g / \ln g}$ where $\Delta$ is the mean level spacing and $g$ is the dimensionless conductance of the dot.

Whereas there are a number of ways to measure the dephasing times in open quantum dots [1, 5], the situation is much more complicated in closed dots. Only a few experiments have attempted to study dephasing in closed quantum dots. Most of these have focused on the relaxation of highly excited states, verifying the continuous transition at $\epsilon \propto g \Delta$ [6]. Some signatures of dephasing in thermalized states have been studied by Patel et al. [7], who analyzed the statistical distribution of the conductance maxima $G_{\text{max}}$ (the height of the Coulomb blockade peaks). They found that the ratio of standard deviation to mean peak height $\sigma(G_{\text{max}}) / \langle G_{\text{max}} \rangle$ is smaller than what random matrix theory (RMT) predicts [8], and attributed this reduction to dephasing effects. More recently, Folk et al. [9] suggested to use the dependence of the conductance upon applying a magnetic field $B$,

$$\alpha = \frac{\langle G_{\text{max}} \rangle_{B \neq 0} - \langle G_{\text{max}} \rangle_{B = 0}}{\langle G_{\text{max}} \rangle_{B \neq 0}} \ ,$$

as a probe of dephasing times. This is the closed dot analog of the weak localization magnetoconductance which was analyzed earlier for open dots [10]. RMT predicts $\alpha = 1/4$ [11, 12], while Folk et al. measured considerably lower values of $\alpha$, down to $\alpha \approx 0$ for the largest quantum dot with the maximal ratio $k_B T / \Delta$ ($T$ is the temperature, $k_B$ the Boltzmann constant). interactions. Beenakker et al. [13] theoretically analyzed the situation in which the phase-breaking inelastic relaxation rate $\Gamma_{\text{in}}$ far exceeds the mean tunneling rate (inverse dwell time in the dot) $\Gamma$. It turns out that, in this limit, $\alpha$ is reduced much stronger than what found experimentally. Thus, they concluded that in the experiment [9] $\Gamma_{\text{in}} < \Gamma$. However, as noted in Refs. [10] and [13], the lack of a quantitative theory of the crossover regime $\Gamma_{\text{in}} \sim \Gamma$, prevents a full analysis of the experimental results.

In this Letter, we study theoretically the effect of arbitrary inelastic scattering on the conductance of a closed quantum dot. We develop an analytical approach that allows to evaluate $\alpha$ [Eq. (1)] and compare the results to the numerical solution. The approximate results are found to reasonably describe the behavior in the experimentally relevant temperature regime. Our calculations allow for the first time to extract dephasing times of low lying (thermally excited) states in closed quantum dots. We observe a clear enhancement of the dephasing times relative to earlier results for open quantum dots [7]. Moreover, contrary to the analysis of open quantum dots [5] which showed a dependence on temperature alone, we find a dependence on both $T$ and $\Delta$. From our analysis it follows that the measurements of Folk et al. [9] are not inconsistent with vanishing dephasing rate for low excitation energies [3].

In the experimentally relevant regime $h \Gamma_{\text{in}}, h \Gamma < k_B T, \Delta$, each state of the quantum dot is determined by a tuple $\{n_i\}$ of occupation numbers for the single particle eigenstates with energies $E_i$ and spins $S_i$. The electrons can tunnel between the dot and the two leads. The left (L) and right (R) leads differ due to the applied voltage $V$. The electrons in the leads are thermalized and distributed according to the Fermi function $f_{\text{FD}}(E) = [1 + e^{E/(k_B T)}]^{-1}$. The probability $P_N(\{n_i\})$ to find a given set of occupation numbers $\{n_i\}$ with a total of $N$ electrons (restricted to $N \in \{N, N + 1\}$ due to the Coulomb blockade) obeys the following Master equation [14]:

$$\frac{d}{dt} P_N(\{n_i\}) = \sum_{\{n_i\}'} \Gamma_{\text{in}}(\{n_i\} \rightarrow \{n_i\}') \frac{d}{dt} P_{N'}(\{n_i\}') - \sum_{\{n_i\}''} \Gamma_{\text{out}}(\{n_i\}'') \frac{d}{dt} P_{N''}(\{n_i\}'') .$$

In the experimentally relevant regime $\hbar \Gamma_{\text{in}}, \hbar \Gamma < k_B T, \Delta$, each state of the quantum dot is determined by a tuple $\{n_i\}$ of occupation numbers for the single particle eigenstates with energies $E_i$ and spins $S_i$. The electrons can tunnel between the dot and the two leads. The left (L) and right (R) leads differ due to the applied voltage $V$. The electrons in the leads are thermalized and distributed according to the Fermi function $f_{\text{FD}}(E) = [1 + e^{E/(k_B T)}]^{-1}$. The probability $P_N(\{n_i\})$ to find a given set of occupation numbers $\{n_i\}$ with a total of $N$ electrons (restricted to $N \in \{N, N + 1\}$ due to the Coulomb blockade) obeys the following Master equation [14]:
\[
\frac{dP_n(n_i)}{dt} = \sum_{j \lambda} \delta_{n_0,0} \Gamma_j^\lambda [(1-f_j^\lambda)P_{N+1}(\{n_i\}_{-j}) - f_j^\lambda P_N(\{n_i\})] + \sum_{jk} \delta_{n_0,0} \delta_{n_1} [\Gamma_{in}^j P_N(\{n_i\}_{-j}) - \Gamma_{in}^k P_N(\{n_i\})]
\]

Here, \(\{n_i\}_{-j} (\{n_i\}_{-j})\) are the tuples obtained from \(\{n_i\}\) by adding (removing) one electron in the one-particle eigenstate \(j\), and \(f_j^\lambda = \int_{E_j}^\lambda ((\delta_{jL} - 1/2)eV - \mu)\), where \(\lambda \in \{L,R\}\) and \(\mu\) is the effective chemical potential, including the charging energy.

The first terms in \(\frac{dP_n(n_i)}{dt}\) describe the tunneling of electrons between the dot and the leads. The additional terms \(\Gamma_{in}^j\) in \(\frac{dP_n(n_i)}{dt}\) describe inelastic scattering processes between the dot’s one-particle eigenstates \(j\) and \(k\). We assume that these are caused by thermal bosonic fluctuations at temperature \(T\) and neglect any back-coupling of the scattering to this bath state. Under the assumptions that the coupling strength is independent of the specific levels involved and there is no spin-scattering, one arrives at \((\omega_{jk} = E_j - E_k)\)

\[\Gamma_{in}^j = \Gamma_0 \frac{\text{sgn}(\omega_{jk}) D(\omega_{jk}) \exp(\omega_{jk}/(kBT))}{\exp(\omega_{jk}/(kBT)) - 1} \delta_{j\lambda},\]

where \(D(E)\) is the bosonic density of states. As will be shown below, the suppression of \(\alpha\) is quite robust to the specific model of interaction, and depends mainly on the total inelastic scattering rate \(\Gamma_{in}\). We consider \(\Gamma_{in}\) as a phenomenological parameter, to be determined experimentally. Since the experimental quantum dots \(\frac{d\Gamma_{in}}{d\Gamma}\) contain a large number of electrons, \(N \gg 1\), they can be described by RMT \(\frac{d\Gamma_{in}}{d\Gamma}\). In particular, the tunneling rates are Porter Thomas distributed \(\frac{d\Gamma_{in}}{d\Gamma}\) for the Gaussian orthonormal ensemble (with a magnetic field). The difference between \(P_1(r)\) and \(P_2(r)\) leads to the above mentioned of \(\alpha = 1/4\) in the absence of inelastic scattering \(\frac{d\Gamma_{in}}{d\Gamma}\).

The inelastic scattering model \(\frac{d\Gamma_{in}}{d\Gamma}\) is exponentially cut off to states outside an energy window of \(O(kBT)\) and, thus, \(\Gamma_{in}\) vanishes at low temperatures. At \(kBT \gg \Delta\) the other hand, there are many states \(M \propto T/\Delta\) connected by the inelastic scattering. Therefore, for \(T \to \infty\), the total inelastic scattering rate \(\Gamma_{in}/T \to \infty\) and the result of \(\frac{d\Gamma_{in}}{d\Gamma}\) is approached. In the leading order in \(\Delta/kBT\) the solution of the Master equation \(\frac{d\Gamma_{in}}{d\Gamma}\) is the equilibrium distribution

\[G = \frac{e^2}{2kB^T} \frac{k_BT}{\Delta} \Gamma_{in}^{R}\]

\(\frac{d\Gamma_{in}}{d\Gamma}\), i.e., \(\alpha = 0\) (here, \(\Gamma_{in}^{R}\) is the mean tunneling rate to lead \(\lambda\), \(\Gamma = \Gamma_{in}^{L} + \Gamma_{in}^{R}\)). Two corrections arise in the next order in \(\Delta/kBT\): (i) the effect of a finite total inelastic scattering rate and (ii) \(\Gamma_{in}^{R}\) is replaced by an average \(M\) levels around the Fermi energy, i.e., \(\Gamma_{in}^{R} \to \langle \Gamma_{in}^\lambda \rangle = \sum_f f_j(1-f_j)\Gamma_{in}^\lambda \). (Here and in the following \(\langle\rangle\) is the Fermi function at both leads for \(V = 0\).) This introduces corrections \(O(1/M)\) due to correlations between the nominator and denominator of Eq. \(\frac{d\Gamma_{in}}{d\Gamma}\). We calculate the former and take into account the latter by solving the Master equation using perturbation theory in \(\Gamma_{in}/\Gamma_{in}\) where \(\Gamma_{in}^{R} = \sum_{k \neq j} \Gamma_{in}^{jk}(1-f_k)\) as a result \(\frac{d\Gamma_{in}}{d\Gamma}\)

\[G = \frac{e^2}{k_BT} P_{eq}(N) \frac{(\langle \Gamma_{in}^{L}\rangle (\langle \Gamma_{in}^{R}\rangle) - (\langle \Gamma_{in}^{R}\rangle^2 + (\langle \Gamma_{in}^{L}\rangle^2 - 2\langle \Gamma_{in}^{L}\rangle \langle \Gamma_{in}^{R}\rangle)\langle \Gamma_{in}^\lambda \rangle^2)}{\Gamma_{in}^\lambda \langle \Gamma_{in}^{L} + \Gamma_{in}^{R}\rangle^2}\]

\(\alpha = 1 - \frac{\Delta}{12kB^T} + \frac{\Delta}{2kB^T}\).

where \(P_{eq}(N)\) is the equilibrium probability to have \(N\) electrons in the dot and \(\Gamma_{in}^{R} = \Gamma_{in}^\lambda/(1 - f_j)\). We neglect the weak \(j\)-dependence of \(\Gamma_{in}^\lambda\). The total inelastic scattering rate \(\Gamma_{in}\) is defined as value of \(\Gamma_{in}^\lambda\) at Fermi energy, i.e., \(\Gamma_{in} = \Gamma_{in}^\lambda\) for \(E_j = 0\).

In the following, we employ an approximation inspired by this high-\(T\) expansion. To first order in voltage, one can write \(P_N(\{n_i\}) = P_{eq}(\{n_i\}) (1 + \frac{1}{N^2} \sum_j \delta_{n_j,i} \Psi(j)\)\), where \(P_{eq}(\{n_i\})\) is the equilibrium value of \(P_N(\{n_i\})\). With this approximation of final states to scatter to, one can replace the sum over many individual terms \(P(\{n_i\}_{k-\lambda})\) in the Master equation \(\frac{d\Gamma_{in}}{d\Gamma}\) by an averaged quantity

\[\sum_k \Gamma_{in}^{jk} P_{N+1}(\{n_i\}_{k-\lambda}) - \Gamma_{in}^{jk} P_{N+1}(\{n_i\}) = \frac{eV}{k_BT} P_{eq}(\{n_i\}) \sum_k \Gamma_{in}^{jk} [\Psi(k) - \Psi(j)]\]

\[= \frac{eV}{k_BT} P_{eq}(\{n_i\}) [\sum_k \Gamma_{in}^{jk} [\Psi(k) - \Psi(j)]]\]

\[= \frac{eV}{k_BT} P_{eq}(\{n_i\}) \sum_k [\sum_j f_j(1-f_j)\Psi(j)] (\sum_j f_j(1-f_j))\]
Γ, which can be achieved by changing the contact setting. A first step in this direction was done in Eq. (8) which is assumed to be unaffected by the contact setting. The theoretical dependence of ΓL on T, as a function of ∆ and T, was not scattered at all. These happen with probabilities proportional to ΓL/R and are employed in the following. As we are interested in this regime we assumed a picket fence distribution with spacing ∆ between consecutive spin-degenerate levels (E2j = E2j−1 = jΔ; Γj2j = Γj/2; Γj = Γ/2).

We tested the range of validity of this high-temperature approximation against the numerical solution of the Master equations (2). The latter is obtained by solving the Master equation (2) by sparse matrix inversion (8). Figure 1 compares values of α, as calculated using the first three terms in the Δ/T expansion, with the numerical values. The agreement is very good for sufficiently high temperatures, and reasonable even for low T. In the whole temperature regime, the deviations are within current experimental accuracy.

It thus appears that our analytical approach provides a reliable way to determine Γin from the experimental measurements of α, in the whole temperature regime. For future experiments we provide Fig. 2, which presents α as a contour-plot in the space spanned by kBT/Δ and Γin/Γ.

A direct experimental test would be provided by measuring values of α in a given dot at fixed T, as a function of Γ (which can be achieved by changing the contact setting). The theoretical dependence of α on Γ involves a single fitting parameter, i.e., the unknown total scattering rate Γin which is assumed to be unaffected by the contact setting. A first step in this direction was done in Eq. (8), and in the inset of Fig. 3 we compare the prediction of our high temperature approximation with the measurements of α for three different values of Γ. An excellent agreement is obtained, though more data points are required for reliable conclusions.

We now use the above theory to extract dephasing rates from the data points (mean values and error bars).
of Folk et al. [4]. Figure 3 presents these estimates as symbols and error bars, respectively, and compares them with open dot values [7]. A clear enhancement of the dephasing times compared to open dots is observed. In addition, dephasing times strongly depend on $\Delta$ (as can be seen at $T = 45$ mK). This is in contrast to open dot results [4]. An additional suppression of $\alpha$ for $k_B T < \Delta$, resulting from level-spacing fluctuations [14, 17] was not included in our analysis, and therefore our results underestimate the dephasing times for $k_B T < \Delta$. In addition, the result for the $\Delta = 0.9\mu eV$ quantum dot which is consistent with $\tau_0 = 0$ should be interpreted carefully since the result implies $\hbar \Gamma_{\text{in}} > \Delta$ and the Master equation is not applicable anymore. Based on our analysis, the recent experiment [8], measuring dephasing in closed quantum dots is consistent with dephasing due to electron-electron interaction alone, including the prediction of the critical vanishing of dephasing rate. However, given the large error bars of the current experimental data, one cannot exclude an algebraic behavior or even a saturation of the dephasing rates for $T \to 0$. Nevertheless, the behavior is clearly different from that of open quantum dots [7] and is $\Delta$-dependent.

In conclusion, we provide a theoretical approach to extract the inelastic scattering rate in closed dots based on measurements of the weak-localization correction $\alpha$. Analyzing a recent experiment by Folk et al. [4], we see a clear enhancement of the dephasing time compared with open dots values. There is no inconsistency with theoretical predictions for electron-electron interaction, in particular, a vanishing dephasing rate at a critical $\Delta - \text{dependent}$ temperature. We note, however, that the available experimental data is limited and has considerably statistical uncertainties. Future experiments are necessary and we offer Figure 3 to extract the temperature and level-spacing dependence of the inelastic scattering rate and to thoroughly test the prediction of a diverging dephasing time.

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[18] For $k_B T < 1.6\Delta$, we took into account all configurations $\{n_i\}$ which involve levels in $[-4k_B T..-4k_B T]$ with $P^{eq}(n_i)/\min(n_i) > \exp(-5\Delta/k_B T)$; for $k_B T > 1.6\Delta$ the interval was $[-3k_B T..3k_B T]$ and the exponential cut-off $-4.5\Delta/k_B T$, see [14] for details.