Patterns Count-Based Labels for Datasets

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Abstract—Counts of attribute-value combinations are central to the profiling of a dataset, particularly in determining fitness for use and in eliminating bias and unfairness. While counts of individual attribute values may be stored in some dataset profiles, there are too many combinations of attributes for it to be practical to store counts for each combination. In this paper, we develop the notion of storing a “label” of limited size that can be used to obtain good estimates for these counts. A label, in this paper, contains information regarding the count of selected patterns–attributes values combinations–in the data. We define an estimation function, that uses this label to estimate the count of every pattern. We present the problem of finding the optimal label given a bound on its size and propose a heuristic algorithm for generating optimal labels. We experimentally show the accuracy of count estimates derived from the resulting labels and the efficiency of our algorithm.

I. INTRODUCTION

Data-driven decision systems are increasingly used today. The data on which these systems depend, as in much of data science, are often “found data”, namely, data that was not collected as part of the development of the analytics pipeline, but was rather acquired independently, possibly assembled by others for different purposes. When the decision is made by a machine-learned model, the correctness and quality of the decision depend centrally on the data used in the model training phase.

The use of improper, unrepresentative, or biased data may lead to unfair decisions, algorithmic discrimination (such as racism), and biased models [16]. Data-driven methods are increasingly being used in domains such as fraud and risk detection, where data-driven algorithmic decision making may affect human life. For instance, risk assessment tools, which predict the likelihood of a defendant to re-offend, are widely used in courtrooms across the US [6]. ProPublica, an independent, non-profit newsroom that produces investigative journalism in the public interest, conducted a study on the risk assessment scores output by a software developed by Northpointe, Inc. They found that the software discriminated based on race: blacks were scored at greater risk of re-offending than the actual, while whites were scores at lower risk than actual.

Further analysis [8] showed issues with other groups as well. For example, the error rate for Hispanic women is very high because there aren’t many Hispanic women in the data set. It is not only that there are fewer Hispanics than blacks and whites, and fewer women then men, but also fewer Hispanic women than one would expect if these attribute values were independently distributed. A judge sentencing a Hispanic woman presumably would like to be informed about this low count of Hispanic women in the data set and the consequent likelihood of greater error in the risk assessment.

When using “found data”, analysts typically perform data profiling, a process of extracting metadata or other informative summaries of the data [9]. Examples of information acquired in this process include statistics over the attributes’ values, their type, common patterns, and attributes correlations and dependencies. Such information may assist in mitigating the misuse of data and reduce algorithmic bias and racism. While informative and useful, data profiling is hard to do well, is usually not automated, and requires significant effort.

Even users of the data (or data analysis), and not just the analysts, may be interested in this sort of profiling information on the training data before they can trust the learned model. To help both the data analyst and the data user, the notion of a “nutrition label” has been suggested [15], [18], [21], [27], [28], [32]. The basic idea of a nutrition label is to capture, in a succinct label, data set properties of interest. Perhaps the single most important such property is a profile of the counts of various attribute value combinations. For instance, an analyst may wish to ensure a (close) to real-world distribution in the attribute’s values of the data, such as an equal number of males and females. Another concern may be the lack of adequate representation in the data for a particular group [8], such as divorced African-American females, or contrarily, a high percentage of data that represents the same group (data skew) [10]. The count information may also reveal potential dependent or correlated attributes. As a simple example, if all tuples representing individuals under 20 years old are also single, this may point out a possible connection between age and marital status.

Of course, interpretation of the count information depends on the intended use of the data set. Users performing different tasks may be interested in various parts of the data and their counts. Moreover, the thresholds set for skew or inadequate data may vary for different uses. Once the count information is available, it can be used to develop usecase-specific metadata warnings such as “dangerous intersected attribute combinations” or “inadequate representation of a protected group”.

In this paper, we propose to label datasets with information regarding the count of different patterns (attributes values combinations) in the data, which can be useful to determine fitness for use. Needless to say, there is a combinatorial number of such combinations possible. So, storing individual
counts for each is likely to be impossible. To this end, we focus on techniques to estimate these counts based on storing only a limited amount of information.

**Example 1.1:** COMPAS is a risk assessment commercial tool made by Northpointe, Inc. The COMPAS dataset was collected and published by ProPublica. as part of their investigation [1]. The full dataset contains 60,843 tuples with 29 attributes, including meaningful demographic groups such as gender, race, age, marital status, assessment reason, agency (e.g., pretrial, probation), language, legal stats, custody status, and supervision level. Four of these attributes are shown in a fragment of a simplified version of the dataset in Figure 1. This dataset description depicts the possible values of each attribute, and their count in the data, with the addition of counts for some attribute value combinations: gender and race in this example. Some immediate observations that can be made based on this information is that female and male are not equally represented in the data, and due to the low number of widows in the data, there is a high possibility that the number of Hispanic female widows is inadequate for the development of non-biased algorithm using this data.

![](image)

Fig. 1: Labels computed for (a simplified version of the) COMPAS dataset

Given a data set, if we do not know anything about value distributions in it, a common assumption to make is that of independence between attributes. One way we could control the size of stored information is to keep counts for only individual attribute values, and to estimate counts for attribute value combinations, assuming independence. However, this defeats the central purpose of profiling – we only get information about individual attributes (the “marginal distributions”) but nothing about any correlations. In the study of discrimination, there is a considerable examination of intersectionality, the whole point of which is to understand how the social consequence of being a member of a protected class on multiple axes is not simply the “sum” of each alone. For example, to understand the discrimination faced by black women it is not enough to understand independently the impact of race alone and gender alone. In other words, we have to ensure that our estimates for the count of any pattern in the database are at least approximately correct.

Histograms have long been used for similar purposes in relational databases, however, they do not do very well in high dimensions. Other prevalent techniques for selectivity estimation include sampling, and machine learning-based methods (see review in Section V). The former suffers from insufficient performance in the presence of skews and high selectivity queries, and the latter requires training and result in very complex models. Inspired by the concept of nutrition labels for datasets, a key requirement in our problem context is that the metadata annotation can be immediately comprehensible to a potential user of the dataset.

Our problem, intuitively, is to choose a small number of patterns (limited by a given space budget), among the exponential number, that can be used to estimate the count for any pattern with minimal error. We envisage this information being made available as meta-data with each data set. In deference to the idea of a nutrition label, we call our stored information a “label”. An important feature of our model that is missing in previously proposed models for data labeling is the ability to generate the labels in a fully automated manner.

We define our notion of data labels with respect to a subset of attributes $S$, as the count information of all possible values combination of attributes in $S$ appearing in the data. The size of the label is then determined by the space required for the count information. By making an independence assumption, individual attribute value counts can be used to estimate the joint distribution, but if we are additionally given selected intersection counts, how should we use these to estimate other intersection counts not provided? We present a model for this estimation in Section II. Given the estimation procedure, each label entails an error with respect to the real count of patterns in the data. The problem of finding an optimal label within a given bound on the label size is NP-hard.

A naive algorithm for the problem would traverse over all possible attributes subsets in increasing size order, compute the size of the corresponding label for each set, and choose the one that entails the minimal error within the space budget. We argue that in practice, the labels generated with a set of attributes $S$ is preferable over labels generated using any subset of $S$, and build upon this property an optimized heuristic for the problem of finding an optimal label (Section III).

We conduct an extensive experimental study (Section IV) to assess the quality of our proposed labels model and the labels generation algorithm’s performance using real-world datasets. Our experimental results demonstrate the high accuracy of the labels generated, even with a very limited space budget, and indicate the usefulness of our proposed optimized heuristic compared to the naive algorithm. They further show the scalability of the algorithm with respect to the generated label size, the data size, and the number of attributes.

We survey related work in Section V and conclude in Section VI.
In this section we present a novel model of label construction, based on counts. A summary of the notations used throughout the paper is shown in Table II. We assume the data is represented using a single relational database, and that the relation’s attributes values are categorical. Where attribute values are drawn from a continuous domain, we render them categorical by bucketizing them into ranges: very commonly done in practice to present aggregate results. In fact, we may even group categorical attributes into fewer buckets where the number of individual categories is very large.

### A. Patterns count information

We first define the notion of pattern which is the foundation for our label model.

**Definition 2.1 (Patterns):** Let $D$ be a database with attributes $A = \{A_1, \ldots, A_n\}$ and let $Dom(A_i)$ be the active domain of $A_i$ for $i \in [1..n]$. A pattern $p$ is a set $\{A_{i_1} = a_{i_1}, \ldots, A_{i_k} = a_{i_k}\}$ where $\{A_{i_1}, \ldots, A_{i_k}\} \subseteq A$ and $a_j \in Dom(A_{i_j})$ for each $A_{i_j}$ in $p$. We use $Attr(p)$ to denote the set of attributes in $p$.

**Example 2.2:** Consider the fragment of the simplified version of the COMPAS database given in Figure 2 $p = \{\text{age group= under 20, marital status = single}\}$ is a possible pattern and $Attr(p) = \{\text{age group, marital status}\}$.

**Definition 2.3:** We say that a tuple $t \in D$ satisfies a pattern $p$ if $t.A_i = a_i$ for each $A_i \in Attr(p)$. The count $c_D(p)$ of a pattern $p$ is the number of tuples in $D$ that satisfy $p$.

**Example 2.4:** Consider again the database given in Figure 2. The tuples 1, 3, 8, 10, and 14 satisfy the pattern $p = \{\text{age group= under 20, marital status = single}\}$ and thus the count of $p$ is $c_D(p) = 6$.

Information regarding the count of patterns appearing in the data can be useful to determine fitness for use. It may be used to ensure a (close to) real world distribution in the attribute’s values of the data to detect improper (underrepresented) or extremely high representation (data skew) of patterns, and potential dependent or correlated attributes. While full count of each pattern provides detailed and accurate description of the data, it can be extremely large. In fact it can have the same size as the data.

**Example 2.5:** As a simple example, consider a database $D$ with $n$ binary attributes $A_1, \ldots, A_n$, where each value combination $(b_1, \ldots, b_n)$, for $b_i \in \{0, 1\}$, appears exactly once. In this case the database, as well as the patterns count, includes $2^n$ tuples.

To this end, we propose an estimation function, which estimates a pattern count based on partial count information. Our basic intuition is that information regarding the count of individual attributes values is sufficient to provide a good estimate of any pattern count if there are no correlations within the attributes.

**Example 2.6:** Continuing with Example 2.5 given the counts $c_D(\{A_i = b_i\}) = \frac{2^n}{b_i}$, the count of the pattern $\{A_1 = 0, A_2 = 0, A_3 = 0\}$ may be estimated as

$$\frac{2^n}{\prod_{i=1}^{3} c_D(\{A_i = 0\})} = 2^n \cdot \left(\frac{1}{2}\right)^3 = 2^{n-3}$$

Intuitively, under the assumption that there are no correlations, the count of the pattern $\{A_1 = 0, A_2 = 0, A_3 = 0\}$ is the relative portion of the data (total number of $2^n$ tuples), that have the value 0 in the attribute $A_1$, $A_2$ and $A_3$, which is reflected in the sub-expressions $c_D(\{A_i = 0\})$ in the computation. In general, the count of the pattern $p = \{A_{i_1} = b_{i_1}, \ldots, A_{i_k} = b_{i_k}\}$ can be computed as

$$|D| \cdot \prod_{j=1}^{k} c_D(\{A_{i_j} = b_{i_j}\})$$

When we introduce correlations, the counts of individual attributes are no longer sufficient to provide a good estimation, as we next demonstrate.

**Example 2.7:** As a simple example, consider a database $D$ with $n$ binary attributes as described in Example 2.5 except that the values in the attributes $A_1$ are replaced such that the value of $A_1$ is equal to the value of $A_2$ for every tuple. The real count of the pattern $\{A_1 = 0, A_2 = 0, A_3 = 0\}$ is now $2^{n-2}$, where using only the individual count the pattern count estimation is $2^{n-3}$ with the same computation shown in Example 2.6.

We may remedy this problem by using additional count information. In the above example, the counts of the patterns $p = \{A_i = b_1, A_2 = b_2\}$ for $b_i \in \{0, 1\}$ is sufficient to provide an exact estimate for each pattern in the database.

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**TABLE I: Notation Table**

| $D$ | Dataset |
|-----|---------|
| $A$ | Attributes set in $D$ |
| $Dom(A_i)$ | Active domain of attribute $A_i$ |
| $\{\}$ | Pattern |
| $c_D(p)$ | The count of tuples in $D$ satisfying $p$ |
| $S$ | A subset of attributes ($S \subseteq A$) |
| $P_S$ | The set of all possible patterns over $S$ s.t. $c_D(p) > 0$ |
| $L_D(S)$ | A label of $D$ using $S$ |
| $\text{VC}$ | The value count of each value in $D$ |
| $\text{PC}$ | The pattern count of each tuples in $P_S$ |
| $\{\}$ | The pattern resulting when restringing $p$ to $S_i$ |
| $\text{Est}(p, l)$ | The estimation of a pattern $p$ using the label $l$ |
| $\text{Err}(l, p)$ | The error of $l$ with respect to $p$ |
| $\mathcal{P}$ | A set of patterns |
| $\text{Err}(l, \mathcal{P})$ | The maximal error of $l$ with respect to $p \in \mathcal{P}$ |

**Table II:** Sample data from a simplified version of the COMPAS dataset.

| Gender | Age group | Race | Marital status |
|--------|-----------|------|----------------|
| Female | under 20  | African-American | single |
| Male   | 20-39     | African-American | divorced |
| Male   | under 20  | Hispanic         | single  |
| Male   | 20-39     | Caucasian        | married |
| Female | 20-39     | African-American | divorced |
| Male   | under 20  | African-American | single  |
| Male   | 20-39     | Hispanic         | married |
| Male   | 20-39     | Hispanic         | married |
| Female | 20-39     | African-American | married |
Example 2.8: Given the patterns count \( c_D(\{A_1 = 0, A_2 = 0\}) = 2^{n-1} \) we can compute the count of \( \{A_1 = 0, A_2 = 0, A_3 = 0\} \) as
\[
2^{n-1}, \frac{c_D(\{A_3 = 0\})}{c_D(\{A_3 = 0\}) + c_D(\{A_3 = 1\})} = 2^{n-1}, \frac{1}{2} = 2^{n-2}
\]
In general, the count of any pattern \( p = \{A_{i_1} = b_{i_1}, \ldots, A_{i_k} = b_{i_k}\} \) (that contains \( \{A_1 = b_1, A_2 = b_2\} \) for \( b_i \in \{0, 1\} \)) can be computed as
\[
c_D(\{A_1 = b_1, A_2 = b_2\}) \prod_{j=3}^k \frac{c_D(\{A_{i_j} = b_{i_j}\})}{c_D(\{A_{i_j} = 0\}) + c_D(\{A_{i_j} = 1\})}
\]
Real world datasets are typically complex, and have correlations among attributes. One possible way to tackle this problem is to store more information about these (large) deviations from our initial independence assumption. The challenge is to spend wisely a limited space budget to capture exactly the deviations that induce greatest error in our estimates.

B. Patterns count based labels

We next define our notion of data label. A label is defined with respect to a subset \( S \) of the database attributes, and it contains the pattern count \( \text{(PC)} \) for each possible pattern over \( S \) and value count \( \text{(VC)} \) of each value appearing in \( D \). Given a subset of attributes \( S \subseteq \mathcal{A} \) we use \( \mathcal{P}_S \) to denote the set of all possible patterns over \( S \) (i.e., \( p \) with \( \text{Attr}(p) = S \)) such that \( c_D(p) > 0 \). The maximal number of patterns in \( \mathcal{P}_S \) is \( \prod_{A_i \in S} |\text{Dom}(A_i)| \).

Definition 2.9 (Label): Given a database \( D \) with attributes \( \mathcal{A} = \{A_1, \ldots, A_n\} \), and a subset of attributes \( S \subseteq \mathcal{A} \) a label \( L_S(D) \) of \( D \) using \( S \) contains the set \( \text{PC} = \{p_l, c_D(p_l)\} \) for each \( p_l \in \mathcal{P}_S \) and the set \( \text{VC} = \{\{A_i = a_j\}, c_D(\{A_i = a_j\})\} \) for each \( A_i \in \mathcal{A} \) and \( a_j \in \text{Dom}(A_i) \).

Example 2.10: Consider the database fragment given in Figure 2 the label resulting from use of the attributes set \( S = \{\text{age group}, \text{marital status}\} \) consists of the following:
\[
\text{PC} = \{(\text{age group} = \text{under 20}, \text{marital status} = \text{single}), 6\},
\{(\text{age group} = \text{20-39}, \text{marital status} = \text{married}), 6\},
\{(\text{age group} = \text{20-39}, \text{marital status} = \text{divorced}), 6\},
\{\text{gender} = \text{female}, 9\}, \{(\text{gender} = \text{male}), 9\},
\{(\text{race} = \text{Caucasian}), 6\},
\{(\text{race} = \text{Hispanic}), 6\},
\{(\text{marital status} = \text{single}), 6\},
\{(\text{marital status} = \text{married}), 6\},
\{(\text{marital status} = \text{married}) \}
\]
The label resulting from use of the attributes set \( S' = \{\text{gender}, \text{age group}\} \) consists of the same \( \text{VC} \) set and the following \( \text{PC} \) set:
\[
\text{PC} = \{(\text{gender} = \text{female}, \text{age group} = \text{under 20}), 3\},
\{(\text{gender} = \text{male}, \text{age group} = \text{under 20}), 3\},
\{(\text{gender} = \text{female}, \text{age group} = \text{20-39}), 6\},
\{(\text{gender} = \text{male}, \text{age group} = \text{20-39}), 6\}
\]
Note that for a given database \( D \), the \( \text{VC} \) set is determined and similar for every label of \( D \). This set may be large, for instance, the COMPAS dataset includes at least 10 meaningful demographic attributes as shown in Example 1.1 and the Credit Card dataset we used in our experiments has over 20 attributes, including demographic factors, credit data and history of payments (see Section IV for more details). As we show in the sequel this information is an integral part of the estimation method we propose. However, note that with a simple user interface, the label’s presentation may be manually refined and attributes can be filtered-out in order to adjust the information to the user’s interest.

Let \( D \) be a database with attributes \( \mathcal{A} \), and \( S_1 \) and \( S_2 \) be two subsets of attributes such that \( S_1 \subseteq S_2 \subseteq A \). Given a pattern \( p \in \mathcal{P}_{S_2} \), we use \( p|_{S_1} \) to denote the pattern that results when \( p \) is restricted to include only the attributes of \( S_1 \). Given a label \( l = L_{S_1}(D) \) the count estimate for a pattern \( p \in \mathcal{P}_{S_2} \) is
\[
\text{Est}(p, l) = c_D(p|_{S_1}) \prod_{A_i \in S_2 \setminus S_1} \sum_{a_j \in \text{Dom}(A_i)} c_D(\{A_i = a_j\})
\]
Example 2.12: Consider again the database given in Figure 2 and the label \( l = L_{S_1}(D) \) generated using \( S = \{\text{age group}, \text{marital status}\} \) shown in Example 2.10. The estimate of the pattern \( p = \{\text{gender} = \text{female}, \text{age group} = \text{20-39}, \text{marital status} = \text{married}\} \) using \( l \) is
\[
\text{Est}(p, l) = c_D(\text{age group} = \text{20-39}, \text{marital status} = \text{married}) \cdot \frac{c_D(\{\text{gender} = \text{female}\})}{\sum_{a_j \in \text{Dom}(\text{gender})} c_D(\{\text{gender} = a_j\})} = 6 \cdot \frac{9}{18} = 3
\]
Using the label \( l' = L_{S'}(D) \) generated from \( S' = \{\text{gender}, \text{age group}\} \), with a similar computation we obtain
\[
\text{Est}(p, l') = c_D(\text{gender} = \text{female}, \text{age group} = \text{20-39}) \cdot \frac{c_D(\{\text{marital status} = \text{married}\})}{\sum_{a_j \in \text{Dom}(\text{marital status})} c_D(\{\text{marital status} = a_j\})} = 6 \cdot \frac{6}{18} = 2
\]
We can then define the error of a label with respect to a pattern and a set of patterns.
Definition 2.13 (Estimation Error): The error of a label \( l = L_S(D) \) with respect to a pattern \( p \) is

\[
Err(l, p) = |c_D(p) - Est(p, l)|
\]

Example 2.14: Reconsider the estimates \( Est(p, l) \) and \( Est(p, l') \) of the pattern \( p = \{ \text{gender = female, age group = 20-39, marital status = married} \} \) shown in Example 2.12. The count of the pattern \( p \) in the database is 3, thus the error of \( l \) with respect to \( p \) is 0 and the error of \( l' \) is 1.

Abusing notation, we use \( Err(l, \mathcal{P}) \), for a set of patterns \( \mathcal{P} \), to denote the maximum error in the estimate for any individual pattern in \( \mathcal{P} \).

Error metric: There are multiple plausible error measures which can be classified into two groups: relative and absolute error measures. An example of relative error measure, commonly used in the field of selection estimation (see, e.g., [13], [23], [33]) is the proportion between the selectivity estimation and the true selectivity, called \( q \text{-error} \) [22].

\[
q\text{-error}(p) = \max \left( \frac{c_D(p)}{est(p)}, \frac{est(p)}{c_D(p)} \right)
\]

The \( q \text{-error} \) metric is relative, symmetric, and is usually preferred since it “fairly” penalize low selectivity estimations.

Selectivity estimation techniques are geared towards query optimizations, and relates to query plan quality [22], while our labels are designed to assist end users determine fitness for use. This difference plays a role when choosing the error measure. We choose to focus on the absolute maximum error (rather than mean for instance), as this definition of error is stiffer and gives us a sense of the error “bound” over a large number of patterns in the database. Our problem definition, its hardness and proposed solution holds also when using \( q \text{-error} \), and we report the resulting \( q \text{-error} \) of the generated labels in the out experiments (see Section [IV-B]).

C. Problem definition

We are now ready to define the optimal label problem.

Definition 2.15 (Optimal Label Problem): Given a database \( D \), with attributes \( \mathcal{A} \), a bound \( B_s \) over the label size, and a set of patterns \( \mathcal{P} \), the optimal label is

\[
\arg \min_{S \subseteq \mathcal{A}} Err(L_S(D), \mathcal{P}) \text{ such that } |P_S| \leq B_s
\]

Intuitively, the set of patterns \( \mathcal{P} \) may be defined as \( P_A \) (i.e., the set of all possible patterns that include all the attributes and every value for each attribute that appears in the data). In this case |\( \mathcal{P} \)| = |\( D \)| and an optimal label would be one that minimizes the error with respect to the count of tuples in the data. Our problem definition is more flexible, and allows the user to define a different pattern set, e.g., patterns that include only sensitive attributes.

To formally characterize the complexity of the optimization problem, we further need to define a corresponding decision problem. We define it as the problem of determining the existence of a label with size limited by the given bound and error which does not exceed a given error bound.

Definition 2.16 (Decision Problem): Given a database \( D \), with attributes \( \mathcal{A} \), a bound \( B_s \) over the label size, a set of patterns \( \mathcal{P} \), and an error bound \( B_e \), determine if there is a label \( L_S(D) \) with |\( P_S \)| \( \leq B_s \) and \( Err(L_S(D), \mathcal{P}) \leq B_e \).

We can show that (see proof in the appendix).

Theorem 2.17: The decision problem is NP-hard.

More complex approaches could consider overlapping combinations of patterns, derive best estimates from multiple labels, use partial patterns, and so on. Such complex approaches are left to future work.

III. OPTIMAL LABEL COMPUTATION

Given a database \( D \) with attributes \( \mathcal{A} = \{ A_1, \ldots, A_n \} \) and a bound \( B_s \), a naive algorithm for the optimal label computation would operate as follows: iterate over possible attributes sets, starting with set of size 2. At each iteration, compute the set of all possible labels with a fixed size, namely, at the \( i^{th} \) iteration the algorithm generate the labels \( \{ L_{S_1}(D), \ldots, L_{S_k}(D) \} \), where each \( S_j \) for \( j \in [1..k] \) is a subset of attributes of size \( i + 1 \). For each label generated, compute its size and error, and record the optimal label computed with size below the given bound. The algorithm terminates if the size of all the labels generated in the same iteration exceeds the bound (or when all possible subsets were generated). Intuitively, if every attribute subset of size \( i \) leads to a label with size greater than the given bound, then, every label generated using any attributes subset of size \( > i \) would also exceed the bound. The naive algorithm is unacceptably expensive. Therefore we developed a much faster heuristic solution for the optimal label problem.

A. Label estimation characterization

We start by characterizing the count estimation for a given pattern using a given label. Let \( D \) be a database with attributes \( \mathcal{A}, S \subseteq \mathcal{A} \) an attributes set and \( l = L_S(D) \) a label of \( D \) using \( S \).

Definition 3.1: Given a pattern \( p \), we say that the estimate of \( p \) using \( l \) is

- an exact estimation if \( Est(l, p) = c_D(p) \),
- an over estimation if \( Est(l, p) > c_D(p) \), and
- an under estimation if \( Est(l, p) < c_D(p) \).

Clearly, for every pattern \( p \) if \( Attr(p) \subseteq S \) then the estimate of \( p \) using \( l \) is an exact estimation. Moreover, we can show the following:

Proposition 3.2: Given two attribute sets \( S_1 \subseteq S_2 \subseteq \mathcal{A} \) and \( l_i = L_{S_i}(D) \) the labels of \( D \) using \( S_i \) for \( i = 1, 2 \) respectively, for every pattern \( p \) such that \( Attr(p) \not\subseteq S_2 \) let \( p' = p|_{Attr(p) \cap S_2} \) be the pattern resulting when restricting \( p \) to include only the attributes appearing in \( S_2 \). If the estimate of \( p' \) using \( l_1 \) is an over (under) estimation, and the estimate of \( p \) using \( l_2 \) is an over (resp., under) estimation then \( Err(l_2, p) \leq Err(l_1, p) \).

Example 3.3: Suppose we are interested in estimating the number of married Hispanic females under the age of 20 in the data. Proposition 3.2 states that if the estimation of a label \( l_1 \) consisting of the count for gender and age combinations
leads to an over (or resp. under) estimation of the pattern $p' = \{\text{gender} = \text{female}, \text{age} = \text{under 20}, \text{marital status} = \text{married}\}$, and the estimation using a label $l_2$ generated with the count of the gender, age and marital status leads to an over (or under) estimation of $p = \{\text{gender} = \text{female}, \text{age} = \text{under 20}, \text{race} = \text{Hispanic, marital status} = \text{married}\}$, then $Err(l_2, p) \leq Err(l_1, p)$.

Intuitively, for two attributes sets $S_1$ and $S_2$, if $S_1 \subseteq S_2$ the label generated using $S_2$ has more details than the one generated using $S_1$. In fact, based on Proposition 3.2 it is reasonable to assume that the pattern’s count estimation using $L_{S_2}(D)$ is more precise than the one using $L_{S_1}(D)$. We show that this assumption indeed holds in practice in our experiment (see Section V-E).

Our proposed solution is based on the above observation. Our algorithm is inspired by the Apriori algorithm [3] and the Set-Enumeration Tree for enumerating sets in a best-first fashion [26]. We start by defining a lattice over the possible labels, and then show how it can be used to compute the optimal label.

B. Labels lattice

We define a labels lattice as follows.

**Definition 3.4 (Labels lattice):** Given a database $D$ with attributes $A$, let $A^*$ be the set of all possible subset of $A$. The label lattice of $D$ is a graph $G = (V, E)$, where $V = A^*$ and $E = \{(S_1, S_2) \mid S_1 \subset S_2 \text{ and } \exists A_i \in A \text{ s.t. } S_1 \cup \{A_i\} = S_2\}$.

$S_1$ is a parent (child) of $S_2$ if there is an edge $(S_1, S_2)$ and $S_1 \subset S_2 \text{ (}S_2 \subset S_1\)$. Intuitively, $S_1$ is a parent of $S_2$ if $S_2$ can be obtained from $S_1$ by adding a single attribute $A \in A \setminus S_1$. Figure 3 depicts the label lattice of the database given in Figure 2. Figure 3 shows the label lattice exactly once in a top down scan as we next show. To this end we define the operator $gen(S)$ for a subset of attributes $S$ as follows.

**Definition 3.5:** Let $D$ be a database with attributes $A = \{A_1, \ldots, A_n\}$. We assume attributes are ordered, and for a given subset of attributes $S \subset A$ we use $idx(S)$ to denote the index of the attribute with maximal attribute index in $S$, namely $idx(S) = \max_j\{(A_i \mid A_i \in S)\}$, we define $gen(S) = \{S' \mid S' = S \cup \{A_j\} \text{ s.t. } idx(S') < j \leq n\}$.

For a given attributes set $S$, the set $gen(S)$ is the set of all children of $S$ in the label lattice of $D$.

**Example 3.6:** For the database $D$ given in Figure 2 and the attributes subset $S = \{\text{gender, race}\}$, $gen(S)$ is $\{\text{gender, race, marital status}\}$. Note that $\{\text{gender, age group, race}\}$ is a child of $S$ in the labels lattice, but is not included in $gen(S)$.

C. Top down algorithm

Algorithm 1 finds the optimal label using a top down traversal of the label lattice. The algorithm gets as input a database $D$, a set of patterns, and a bound $B_s$. It uses a queue $Q$ to generate a candidate list of attributes subset, $cands$, such that the size of the label generated using each candidate in the list does not exceed the bound $B_s$.

The algorithm first initializes the queue $Q$ with the set of attributes’ singletons using $gen(\{\}\) (line 1), and the candidates set $cands$ to an empty set (line 2). Then while the queue $Q$ is not empty (lines 3–9), the algorithm examines the first element in the queue $curr$ (line 4). It traverses over the elements in $gen(curr)$ (lines 5–9), and for each element $c$, checks if the size of the label generated by $c$ is not greater than $B_s$ (line 6). If so, the algorithm adds $c$ to the queue (line 7) and update the candidates list, by removing the parents of $c$ that are currently in $cands$ (line 8), and adding $c$ to the $cands$ list (line 9). Finally, the label that entails the minimal loss out of the set of all labels generated using the attributes sets in the $cands$ list is returned (line 10).

**Example 3.7:** Given the database $D$ shown in Figure 2 the pattern’s set $P$ that contains the set of all tuples in $D$, and the bound $B_s = 5$, the algorithm first initializes $Q$ to be $\{\{g\}, \{a\}, \{r\}, \{m\}\}$, and $cands$ to be an empty set. In the first iteration, $\{g\}$ is extracted from $Q$ and it’s children,
\[ \{g, a\}, \{g, r\}, \{g, m\} \], are generated using \( \text{gen}(\{g\}) \). Out of this set, \( \{g, a\} \) is the only subset that results in a label of size below 5, and therefore is added to \( Q \) and to \( \text{cands} \). In the next iteration, \( \{a\} \) is extracted from \( Q \), and the algorithm examines the elements in \( \text{gen}(\{a\}) = \{\{a, r\}, \{a, m\}\} \). The label generated with \( \{a, r\} \) is of size 3 and the label generated with \( \{a, m\} \) is of size 6, thus only \( \{a, r\} \) is added to \( Q \) and \( \text{cands} \). No other subset in the following iterations generates a label of adequate size, and the while loop terminates after all the elements in \( Q \) are extracted. Finally, \( \text{cands} \) contains \( \{g, a\} \) and \( \{a, m\} \), and the algorithm returns the label generated using \( \{a, m\} \) since it is the optimal in this case.

By traversing the lattice in a top-down fashion using the \( \text{gen} \) operator the algorithm generates each node in the lattice at most once. Furthermore, the nodes generated are only attribute sets that lead to labels with size below the given bound, and (in the worst case) their children.

**Proposition 3.8:** Given a database \( D \), a set of patterns \( \mathcal{P} \) and a bound \( B_s \), Algorithm \( \text{PCBL} \) generates each node in the label lattice at most once.

Algorithm \( \text{PCBL} \) avoids generating and exploring a large portion of the labels lattice, and in particular most of the labels that exceed the bound limit (which in practice are the majority, as shown by our experiments in Section [IV-D]).

### IV. Experimental Evaluation

We conducted experiments on real data to assess the quality of our proposed labels in estimating the data pattern’s count. The key concerns are the size of label and the error in estimation. We evaluated this trade-off and considered the impact of dataset parameters. We compared our label’s accuracy to the performance of a real DBMS estimator, and the conventional approach of sample based estimation using different error measures. A second issue we studied is the performance of the label generation algorithm. We examined scalability in terms of label generation time as a function of (i) label’s size bound, (ii) data size and (iii) number of data attributes. We also quantified the usefulness of the heuristic approach compared to the naive algorithm. Finally, we validated the assumption from Section [III-A] that more detailed labels lead to lower error. In this section, we report on all these experiments in turn. We begin with the setup we used.

#### A. Experimental setup

We used three real datasets with different numbers of tuples and attributes as follows.

**Blue Nile** Blue Nile is an online jewelry retailer. We used the dataset collected and used in [8] of diamonds catalog, containing 116,300 diamonds. The dataset has 7 categorical attributes for the diamonds: shape, cut, color, clarity, polish, symmetry, and florescence.

**COMPAS** The COMPAS dataset was collected and published by ProPublica [1]. It contains 60,843 records that includes demographics, recidivism scores, and criminal offense information. The total number of attributes in the original database was 29. We removed id attributes (person id, assessment id, case id), names (first, last and middle), dates and attributes with less than 2 values or over 100 values. We added the attribute age, with four age ranges, based on the date of birth attribute. The resulting dataset contains 17 attributes.

#### B. Label accuracy

We assessed the quality of the generated labels in estimating the data pattern’s count by examining the error induced by the labels of varying size with respect to the set of patterns appearing in the database. We varied the label’s size bound from 10 to 100 to generate labels with different size.

**Compared Baselines:** We have measured the accuracy of our proposed pattern count based label (PCBL, blue line in the graphs) to two baseline approaches.

**PostgreSQL** The PostgreSQL row estimation relies on 1D histograms. It stores the statistical data about the database in pg_statistic and random sampling while producing statistics.

**Sampling** Uniform random sample with growing size. The size of a sample that corresponds to the bound \( x = \lvert VC \rvert \). Given a sample \( S \) of size \( |S| \) for a dataset \( D \), and a pattern \( p \), we use \( c_S(p) \cdot \frac{|D|}{|S|} \) to estimate the count of \( p \) in \( D \), where \( c_S(p) \) is the count of \( p \) in \( S \).

**Error Measures:** We compared the quality of the estimation method with different error measures.

**Absolute error** Error was measured as the absolute value of difference in count between the actual and estimated count for each pattern. Recall that the absolute maximum error is our estimation error measure (as we defined it in Section [II]).

**Q-error** The factor by which an estimate differs from the actual count (see definition in Section [II]). This error measure is a standard accuracy metric in query estimation, where the accuracy is reported as mean q-error. To avoid division by zero, we set \( \text{est}(p) = 1 \) whenever the actual estimation was 0.

For all three datasets, we observed similar errors for the label generated by the optimal heuristic and the one generated by the naive algorithm (blue line in the graphs). In all cases, we observed a reduction in error as the size bound increased.

---

1 Recall that the bound \( B_s \) is over the pattern count set size \( |PC| \), see Section [III].
Mean q-error 1.00% 4 × 10

3.00%

100

80

60

50

40

20

10

48

18.4

1136

440

717

234

789

The maximal error of the sample based estimation was 575 (around 0.5%). The postgres maximal error was 1204 (1.04%) and the mean was about 7. In the sample based estimation we observed a small increase in the maximal error for a sample of 75 (corresponds to label with |PC| = 28, bound of 30). This is because the sample size is significantly smaller that the database size, thus \( \frac{|D|}{n} \) is larger that the count of all tuples in the data, which results in over estimation for all tuples in the sample, and estimation of 0 for the rest. In particular, if the count of a pattern is greater than 2 (as in one of the executions in this experiment) the overestimation is even higher. The mean error of the sample based method decreased from 18.44 for the smallest sample size \((\times 3 \text{ of the PCBL})\) to 17.04 in the largest sample (over \(\times 4\) of the PCBL).

For the COMPAS dataset, the size of the label generated when setting the bound to 10 was 9 and the maximum error induced by the generated labels was 494 (about 0.8%). For a label of size 87, generated with bound of 100, the maximum error was 378 (a little over 0.6%). In the postgres estimations the maximal error was 532 (0.87%) and mean error of 3.48. The maximal error of the sample based estimation was 1070 for the smallest sample size, and 782 for the largest.

The label obtained with bound of 10 contained 10 pattern-count pairs in the Credit Card dataset. The maximum error was 704 (2.3%). For a label with 92 pattern-count pairs (generated with the bound set to 100), we obtain maximum error of 607 (2.0%). The maximum observed error remains 607 when we increased the label size bound from 70 to 100 (generating labels of size 70 and 92 respectively). We note that the mean error decreased 2.978 to 2.974. To further demonstrate the trend, we examine the error of labels generated with bound set to 125 and 150, which generated labels of size 121 and 139 respectively. The postgres maximal error estimation was 717 (2.39%), with mean of 2.44. The maximal error of the average sample based estimation decreased from 789 to 453, which is slightly better than the results of the PCBL, however the mean error was higher, 6.25 to 5.59 (about \(\times 3\) of PCBL).

The mean q-error is shown in Figure 5. In all cases, PCBL outperformed the competitors, and we observed a decrease in the error as the label size grows. For the Blue Nile dataset the max q-error for the smallest label was 47 compared to average of 2039 using the corresponding samples. The mean was 2.4 and 10.2 respectively. The PCBL max q-error dropped down to 25 with mean of 1.8 using the largest label. The max error using the largest sample was 1335.4 and the mean was 9.4. The postgres maximal q-error in this case was 45 and the mean was 2.5. In the COMPAS dataset, the max q-error was 234 and 715 for the PCBL and sample methods respectively for the smallest bound, with mean of 3.4 and 5.2. For the largest bound the max q-error was 101 and 387, and the mean was 2.4 and 5.02 using PCBL and the sample respectively. In the postgres estimation we observed a max error of 234 and mean of 3.9. Finally, for the Credit Card dataset the observed max error was 47 in all label’s sizes using the PCBL, and the mean decreased from 1.8 to 1.7. Using the samples, the max error was 426.8 and 238.6, with mean of 4.1 and 3.7 for the smallest and largest samples respectively. The postgres max q-error was 47 and the mean was 1.8.
C. Label generation time

The next set of experiments aims at studying the scalability of the algorithms for label generation. We compared the performance of our proposed optimized heuristic algorithm (dark blue) to a baseline naive algorithm described in Section III (light blue). The reported times for the optimized heuristic are the total generation time, including both the candidates search time and finding the best label in the candidate set. Since the number of patterns is large (the same size of the database), the latter may be costly. However, as we use maximal error, we were able to optimize it as follows. We sort the patterns by count in a decreasing order. Then, when traversing the patterns, we compute the error for each one, while tracking the maximal error observed. Once we reach a pattern with lower count than the observed maximal error we terminate. On average, finding the optimal label out of the candidates set was 62.6% of the total running time in the BlueNile dataset, 18% in COMPAS and 44.4% in the Credit Card dataset.

Figure 6 depicts the running time as a function of the label’s size bound from bound 10 and up to 100. As the bound grows, the number of possible attributes subsets that may be used to generate an optimal label increases, which affect the generation time for both algorithms. The optimized heuristic outperform the naive algorithm since the number of subsets it consider is smaller (we give the actual number of subsets in Section IV.D). In the Credit Card dataset, the naive algorithm did not terminate within 30 minutes beyond bound of 50. For bound of 50 the naive algorithm running time was over 18 minutes. The optimal heuristics was able to compute the label for bound of 50 with about 3.5 minutes, and the label for the largest bound of 100 within 18 minutes.

The next experiments aim at assessing the effect of the data size (i.e., number of tuples) and the number of attributes, on the label generation time. We note that the number of attributes subsets examined by the algorithms to generate the optimal label depends (exponentially) on the number of attributes, whereas the number of tuples affects the examination time of each subset (i.e., measuring it’s size and error rate). Thus, we expect to see a moderate growth in the label generation time as a function of the database size, and a steep growth in the generation time as a function of the number of attributes.

To study the effect of the data size on the algorithm’s running time we gradually increased the data size by adding randomly generated tuples to the datasets. We increased the data size up to ×10 the original data size. We repeated each experiments 5 times and report the average running time of the label generation for the bound of 50 in Figure 7 (we observed similar trends for other bound setting). As expected, we observed a moderate growth with respect to the data size for all three datasets.

Interestingly, in the Credit Card dataset, the performance of both algorithms for the dataset with 60,000 tuples (45 and 24 seconds for the naive algorithm and the optimal heuristic respectively– first point in the rightmost graph in Figure 7) was better than their respective performance over the original 30,000 tuples (18 minute for the naive algorithm and 221 seconds for the optimal heuristic–3’rd point in the corresponding graph in Figure 9). The reason for that is that by adding new randomly generated tuples, we introduced new patterns that were missing in the original data. As a result, the number of attribute subsets examined by the algorithm, and in turn the overall running time of the algorithms, decreased. To illustrate, the number of attribute sets examined by the naive algorithm for the original dataset was 536,130 and 9,156 for the optimized heuristic. For the date (with randomly generated tuples) of 60,000 tuples the naive algorithm examined 12,926 attribute sets, and the optimized heuristic only 785 sets.

Figure 8 depicts the running time as a function of the number of attributes. We fixed the bound to 50 and varied the number of attributes in the datasets from 3 to |A| where A is the set of all attributes in the dataset. The effect on the running times was more notable in the COMPAS and the Credit Card datasets since they contain larger numbers of attributes. The results for these datasets are thus presented in log scale.

D. Effect of optimization

Recall that our heuristic optimizes the number of attribute sets examined during the search of the optimal label. To quantify the usefulness of our heuristic, we compared the number of attributes sets examined during the label generation by the optimized heuristic and the naive algorithm. We observed a gain of up to 99% in the number of subsets examined as shown in Figure 9.

For the BlueNile dataset we observed the lowest gain of 54%: 91 subsets examined by the naive algorithm compared with 42 by the optimized heuristic for a bound of 100. The largest gain in this dataset was 86% for a bound of 10 (56 for the naive algorithm and 8 for the optimized heuristic). For the largest bound, the naive algorithm generate 71% of all possible attributes subsets, while the optimized heuristic generate only 33%.

The gain in the COMPAS dataset varied from 96% (89,828 compared to 3,594 for a bound of 100), and up to 99% for a bound of 10 (9,384 by the naive algorithm compared to 106 by the optimized heuristic). In the worst case the naive algorithm examined 60% of of all possible attributes subsets, and the optimized heuristic examined only 3%.

For a bound of 50, the number of subsets generated for the Credit Card dataset by the naive algorithm was 536,130 whereas the optimal heuristic generated only 9,156 subsets, a gain of 98%. For a bound of 10 and 30 the gain was 99% (9,384 compared with 112, and 190,026 compared with 2,102 resp.). For a bound of 100 the heuristic algorithm generated 64,312 attributes subsets, only 0.4% of the total number of possible subsets (recall that the naive algorithm did not terminate within 30 minutes beyond bound of 50).

E. Sub-labels accuracy

The goal of our last experiment was to validate the assumption from Section III-A indeed takes place in practice. Namely, that the error entails from a label generated using a subset of...
attributes $S$ is at most the error entails by the label generated using any subset of $S$. To this end, we used the subset of attributes $S$ used to generate optimal label (for a bound of 100) for each dataset, and examine the error incur by the labels generated with each possible subset of $S$.

The dark bars in Figure 10 depict the maximum error for the optimal label for each dataset (orange for BlueNile, green for COMPAS and purple for the Credit Card). The light bars shows the maximum error of the labels generated from the attributes sets obtained by removing a single attribute from the set used to generate the optimal label.

For the BlueNile dataset, the optimal label was generated using the attributes cut, shape and symmetry. The maximum error for the label generated using this set of attributes was 0.49% (the dark orange bar). The light orange bars shows the maximum error rate observed for the labels generated using the sets \{cut, shape\}, \{cut, symmetry\}, \{shape, symmetry\}. The error in all cases was higher than the error of the optimal label (from 0.8% and up to 0.91%).

We observed similar results for the COMPAS dataset. The dark green bar shows the maximum error of the optimal label (0.62%). In this case the optimal label was generated using a set of six attributes: RecSupervisionLevel, RecSupervisionLevelText, DisplayText, Scale_ID, DecileScore and ScoreText. For each label generated from a set obtained by removing a single attribute from the set used to generate the optimal label we obtained a label with an higher error rate (shown in light green bars) from 0.72% and up to 0.81%.

Finally, the optimal label for the Credit Card dataset was generated using the attribute set containing the attributes education, marriage, age and PAY_AMT1, which describes the amount paid in September, 2005. The maximum error of the optimal label was 2.02% (the dark purple bar). In
Fig. 10: Optimal label vs. Sub labels error. For each data set, the dark bar indicates the performance with label bound set to 100. The light bars represent the error of the labels generated from the attributes sets obtained by removing a single attribute from the optimal set.

three out of the four attributes subsets (light purple bars), the maximum error was higher than the optimal label (from 2.2% to 2.34%). The error of the label generated using only education, marriage, age was similar to the optimal error.

To conclude, the result of this experiment supports our claim and indicates that the assumption (that a more specific pattern count leads to lower error in the count estimate) underlying our optimized heuristic indeed holds in practice.

V. RELATED WORK

With increasing interest in data equity in recent years, multiple lines of work have focused on labeling data and models in order to improve transparency, accountability and fairness in data science [15], [18], [21], [27], [28], [32].

Different data labeling models were studied in [15], [18], [28]. Data nutrition labels [18] are composed of modules, called widgets. Modules are stand-alone, and each provides a different flavor of information: metadata, provenance, variables, statistics pair, probabilistic model and ground truth correlations. The models vary in the manual effort required for their generation and their technical sophistication. Overall, the labels allow users to interrogate various aspects of the dataset. Our proposed label model may be assimilated as a widget or a module in the above models. An important feature of our model is the ability to automatically generate the labels.

Our proposed label model may be reminiscent of the minimum description length (MDL) principle [17], [25], an important concept in information theory and computational learning theory. The MDL principle addresses the problem of choosing the model that gives the shortest description of data. At a high level, the idea behind MDL is that the model that best captures or fits the important features of the data is the one that is able to compress the data most. The basic idea is then to use two parts to describe the data: the hypothesis (or model) and an encoding of the data using that model. In a way, our proposed approach is derived from a user perspective, and designed to allow for human visualization and interpretation. The typically complexity of ML models makes them ill-suited for such purpose.

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original data, the work of [5] uses multiple aggregate data to reconstruct the original data.

There is a wealth of work on lossy data compression [31], [34]. Various techniques were proposed for different application such as image compression [7] and text compression [24], [30]. While our proposed model of data labels may be considered as a new lossy data compression method, our intended usage of the labels is different and as a result we do not consider the decoding process of the entire compressed data in bulk.

VI. CONCLUSION

We have developed a “label” for a data set that can be used to determine the count for every pattern of attribute value combinations in the data set. Since these counts are typically central to determining fitness for use, and thus avoid generating biased models and data-driven algorithms, our work is in line with the many recent proposals for a data set label that allows users to determine fitness for use and build trust. Our labels can be fully automatically generated. Our label model is built upon an estimation function, that allows to estimate the count of every pattern, using partial count information in the label. We present an optimized heuristic for optimal label generation, and experimentally show the quality of our label and usefulness of our heuristic compared with a naive algorithm.

Given the label of a found dataset, in case we observe an undesirable property of the data, such as insufficient diversity or groups with inadequate representation, the next step for a data scientist would be to determine whether the data can be adjusted in a way that will fit their chosen tasks. For instance, the work of [8] proposed an approach for coverage enhancement for patterns with inadequate representation through data acquisition.

REFERENCES

[1] Compas recidivism risk score data and analysis. [https://www.propublica.org/datastore/dataset/compas-recidivism-risk-score-data-and-analysis](https://www.propublica.org/datastore/dataset/compas-recidivism-risk-score-data-and-analysis)
[2] Default of credit card clients data set. [https://archive.ics.uci.edu/ml/datasets/default+of+credit+card+clients](https://archive.ics.uci.edu/ml/datasets/default+of+credit+card+clients)
[3] Ziawasch Abedjan, Lukasz Golab, and Felix Naumann. Profiling relational data: a survey. VLDB J., 24(4), 2015.
[4] Rakesh Agrawal and Ramakrishnan Srikant. Fast algorithms for mining association rules in large databases. In Jorge B. Bocca, Matthias Jarke, and Carlo Zaniolo, editors, VLDB, Morgan Kaufmann, 1994.
[5] Faisal M. Almutairi, Charilaos I. Kanatsoulis, and Nicholas D. Sidiropoulos. PREMA: principled tensor data recovery from multiple aggregated views. CoRR, abs/1910.12001, 2019.
[6] Julia Angwin, Jeff Larson, Lauren Kirchner, and Surya Mattu. Machine bias, May 2016.
[7] Rashid Ansari, Nasir D. Memon, and Ersan Ceran. Near-lossless image compression techniques. J. Electronic Imaging, 7(3), 1998.
[8] Abolfazl Asudeh, Zhongjun Jin, and H. V. Jagadish. Assessing and remedying coverage for a given dataset. In ICDE, 2019.
[9] Nicolas Bruno, Surajit Chaudhuri, and Luis Gravano. Ssholes: A multidimensional workload-aware histogram. In Sharad Mehrotra and Timos K. Sellis, editors, SIGMOD. ACM, 2001.
[10] Irene Y. Chen, Fredrik D. Johansson, and David A. Sontag. Why is my classifier discriminatory? In Samy Bengio, Hanna M. Wallach, Hugo Larochelle, Kristen Grauman, Nicolò Cesa-Bianchi, and Roman Garnett, editors, NeurIPS, 2018.
[11] Graham Cormode, Minos N. Garofalakis, Peter J. Haas, and Chris Jermaine. Synopses for massive data: Samples, histograms, wavelets, sketches. Found. Trends Databases, 4(1-3), 2012.
[12] Amol Deshpande, Minos N. Garofalakis, and Rajeev Rastogi. Dependence is good: Dependency-based histogram synopses for high-dimensional data. In Sharad Mehrotra and Timos K. Sellis, editors, SIGMOD. ACM, 2001.
[13] Anshuman Dutt, Chi Wang, Azade Nazi, Srikanth Kundula, Vivek R. Narasayya, and Surajit Chaudhuri. Selectivity estimation for range predicates using lightweight models. Proc. VLDB Endow., 12(9), 2019.
[14] Michael R. Garey and David S. Johnson. Computers and Intractability: A Guide to the Theory of NP-Completeness. W. H. Freeman & Co., New York, NY, USA, 1990.
[15] Timnit Gebru, Jamie Morgenstern, Briana Vecchione, Jennifer Wortman Vaughan, Hanna M. Wallach, Hal Daumé III, and Kate Crawford. Datasheets for datasets. CoRR, abs/1803.09010, 2018.
[16] Venka Palaniappan and Shahram Latifi. Lossy text compression techniques. In Babak Akhgar, editor, IICS 2007, London, 2007. Springer London.
[17] Nicolas Bruno, Surajit Chaudhuri, and Luis Gravano. Stholes: A generative model for lossy text compression. VLDB J., 12(9), 2019.
[18] Mark H. Hansen and Bin Yu. Model selection and the principle of minimum description length. Journal of the American Statistical Association, 96, 1998.
[19] Faisal M. Almutairi, Charilaos I. Kanatsoulis, and Nicholas D. Sidiropoulos. Improved selectivity estimation by combining knowledge from sampling and synopses. Proc. VLDB Endow., 11(9), 2018.
[20] Margaret Mitchell, Simone Wu, Andrew Zaldivar, Parker Barnes, Lucy Vasserman, Ben Hutchinson, Elena Spitzer, Inioluwa Deborah Raji, and Timnit Gebru. Model cards for model reporting. In FAT*, pages 220–229. ACM, 2019.
[21] Guido Moerkotte, Thomas Neumann, and Gabriele Steidl. Preventing bad plans by bounding the impact of cardinality estimation errors. Proc. VLDB Endow., 2(1), 2009.
[22] Magnus Müller, Guido Moerkotte, and Oliver Kolb. Improved selectivity estimation by combining knowledge from sampling and synopses. Proc. VLDB Endow., 11(9), 2018.
[23] H. V. Jagadish, Nick Koudas, S. Muthukrishnan, Viswanath Poosala, Kenneth C. Sevcik, and Torsten Suel. Optimal histograms with quality guarantees. In VLDB. Morgan Kaufmann, 1998.
[24] Rashid Ansari, Nasir D. Memon, and Ersan Ceran. Near-lossless image compression schemes derived from the lempel-ziv algorithm. Comput. J., 42(1), 1999.
[25] Timnit Gebru, Jamie Morgenstern, Briana Vecchione, Jennifer Wortman Vaughan, Hanna M. Wallach, Hal Daumé III, and Kate Crawford. Datasheets for datasets. CoRR, abs/1803.09010, 2018.
[26] Ron Rymon. Search through systematic set enumeration. In Bernhard Nebel, Charles Rich, and William R. Swartout, editors, KR. Morgan Kaufmann, 1992.
[27] Anshuman Dutt, Chi Wang, Azade Nazi, Srikanth Kundula, Vivek R. Narasayya, and Surajit Chaudhuri. Selectivity estimation for range predicates using lightweight models. Proc. VLDB Endow., 12(9), 2019.
[28] Julia Stoyanovich and Bill Howe. Nutritional labels for data and models. IEEE Data Eng. Bull., 42(3), 2019.
[29] Jorma Rissanen. Modeling by shortest data description. IEEE Trans. Information Theory, 229. ACM, 2019.
[30] H. V. Jagadish, Nick Koudas, S. Muthukrishnan, Viswanath Poosala, Kenneth C. Sevcik, and Torsten Suel. Optimal histograms with quality guarantees. In VLDB. Morgan Kaufmann, 1998.
[31] Mark H. Hansen and Bin Yu. Model selection and the principle of minimum description length. Journal of the American Statistical Association, 96, 1998.
[32] Faisal M. Almutairi, Charilaos I. Kanatsoulis, and Nicholas D. Sidiropoulos. Improved selectivity estimation by combining knowledge from sampling and synopses. Proc. VLDB Endow., 11(9), 2018.
[33] Venka Palaniappan and Shahram Latifi. Lossy text compression techniques. In Babak Akhgar, editor, IICS 2007, London, 2007. Springer London.
[34] Jorma Rissanen. Modeling by shortest data description. Automatica, 14(5), 1978.
[35] Ron Rymon. Search through systematic set enumeration. In Bernhard Nebel, Charles Rich, and William R. Swartout, editors, KR. Morgan Kaufmann, 1992.
Fig. 11: Reduction input graph example

[34] Zhen Zhang and Victor K.-W. Wei. An on-line universal lossy data compression algorithm via continuous codebook refinement - part I: basic results. IEEE Trans. Information Theory, 42(3), 1996.

APPENDIX

A. Proof of Theorem 2.17

We prove Theorem 2.17 via a reduction from the vertex cover problem, a decision problem which we now define. For notational simplicity, and without loss of generality, we omit certain easy cases from the vertex cover problem. Namely, we require that the input graph contains at least two nodes and one edge and forbid self loops.

Definition A.1 (Vertex cover): Let \( G = (V, E) \) be an undirected graph. A set \( V' \subseteq V \) is a vertex cover of \( G \) if for every edge \( \{x, y\} \in E \) either \( x \in V' \) or \( y \in V' \).

Theorem A.2 ([14]): For each edge \( \{x, y\} \in E \) there are two possible values \( x \in V \) and \( y \in V \).

Proposition A.4: \( \sum_{P} \cdot |E| = 4 \cdot |E| \cdot |E| = 1 \)

Fig. 12: Reduction example’s output database

corresponding set of nodes in the graph are vertex cover using Lemma A.5 and (ii) prove the size bounds using Lemma A.8.

Lemma A.5: Let \( v_i = \{v_i, v_j\} \in E \), \( p = (A_E = x_i, A_i = x_i, A_i = x_j) \) be a pattern in \( P \) and \( S \subseteq A \) be an attributes subset. \( Err(L_S(D)) = 0 \iff A_i \in S \) and at least one of \( A_i \) or \( A_j \) in \( S \).

Proof A.6: Let \( e_r = \{v_i, v_j\} \in E \), \( p = \{A_E = x_r, A_i = x_i, A_i = x_j\} \) a pattern in \( P \) and \( S \subseteq A \) be an attributes subset.

Note that:

- \( cD(p) = |E| \)
- For each attribute \( A_i \):
  - \( cD(A_i = x_i) \)
  - \( cD(A_i = x_j) \)

We consider all possible cases as follows.

Without loss of generality assume that \( A_E \in S \) and \( A_i \in S \) then

\[ cD(p_s) = cD(\{A_E = x_r, A_i = x_i\}) = 2 \cdot |E| \]

and thus

\[ Est(p, L(D)) = 2 \cdot |E| \cdot \frac{1}{2} = |E| \]

Namely, the error in this case is 0.

If \( A_i \in S \) and \( A_j \in S \) but \( A_E \notin S \) and \( A_E \notin S \), we get \( cD(p_s) = cD(\{A_i = x_i, A_j = x_j\}) = |E| + 2 \cdot |E|^2 \) and thus

\[ Est(p, L(S)) = |E| + 2 \cdot |E|^2 \cdot \frac{1}{2} = 2|E| + 1 \]

Namely, the error in this case is \( |E| + 1 > 0 \).

Otherwise we get

\[ Est(p, L(S)) = |D| \cdot \frac{1}{|E|} \cdot \frac{1}{2} \]

\[ = \left(4 \cdot |E|^2 + 4 \cdot |E| \cdot |V|^2 + |V|^2 - |E| \right) \cdot \frac{1}{4 \cdot |E|} \]

Thus, the error in this case is greater than 0.

Corollary A.7: Let \( P \) be the patterns set generated by the reduction and let \( S \subseteq A \) be an attributes subset

\[ Err(L_S(D), P) = 0 \iff A_i \in S \] and

at least one of \( A_i \) or \( A_j \) in \( S \) for each \( \{v_i, v_j\} \in E \).

Lemma A.8: Let \( S \subseteq A \) be an attributes subset of size \( |S| = k + 1 \) for \( k \geq 1 \) such that \( A_E \in S \) then \( L_S(D) = 2 \cdot |E|^2 + 4 \cdot \sum_{i=1}^{k-1} i \cdot \) where \( E' = \{e_r = \{v_i, v_j\} \mid A_i \in S \text{ or } A_j \in S \} \text{ or } (S) \).
Proof A.9: The proof by induction on \(k\).

Base If \(k = 1\) then \(S = \{A_E, A_i\}\). Let \(E' = \{\{v_i, v_j\} \in E \mid \forall v_j \in V\}\), by the reduction construction \(D\) (and thus also \(L_S(D)\)) contains the patterns \(\{A_E = x_r, A_i = x_i\}\) and \(\{A_E = x_r, A_i = x_2\}\) for each \(e_r \in E'\), thus \(|L_S(D)| = 2 \cdot |E'|\) and the proposition holds.

Inductive step Assume the proposition holds for \(k > 1\). Let \(S = \{A_E, A_{i_1}, \ldots, A_{i_{k-1}}, A_i\}\), \(S' = \{A_E, A_{i_1}, \ldots, A_{i_{k-1}}\}\), \(E' = \{e_r = \{v_i, v_j\} \mid A_i \in S \text{ or } A_i \in S' \}\) (both) and \(E'' = \{e_r = \{v_i, v_j\} \mid A_i \in S' \}\) (or both). From the induction hypothesis \(|L_S(D)| = 2 \cdot |E'| + 4 \cdot \sum_{i=1}^{k-1} i\).

Adding the attribute \(A_i\) to the label increase the number of patterns by \(4\) for each \(A_i \in S'\):

- If \(e_r = \{v_i, v_{i_{k-1}}\} \in E\) then instead of 2 patterns \(\{A_E = x_r, A_{i_{k-1}} = x_{p}\}\) for each \(p \in \{1, 2\}\) we have 6 patterns in \(L_S(D)\): \(\{A_E = x_r, A_{i_{k-1}} = x_{p}\}, \{A_{i_{k-1}} = x_r, A_{i_{k-1}} = x_{m}\}\) for each \(p, m \in \{1, 2\}\) (4 patterns), \(\{A_{i_{k-1}} = x_1, A_{i_{k-1}} = x_1\}\) and \(\{A_1 = x_2, A_{i_{k-1}} = x_2\}\).
- If \(v_i, v_{i_{k-1}} \notin E\), the patterns \(\{A_{i_{k-1}} = x_p\}\) for each \(p \in \{1, 2\}\) are in \(L_S(D)\) (and not in \(L'_S(D)\)).

In addition, for every \(e_r \in E' \setminus E''\), \(L_S(D)\) contains 2 additional patterns: \(\{A_E = x_r, A_{i_{k-1}} = x_i\}\) and \(\{A_E = x_r, A_{i_{k-1}} = x_2\}\). Namely

\[
|L_S(D)| = |L_S(D)| + 2 \cdot |E'| + 2 \cdot |E''| + 4 \cdot k = 2 \cdot |E'| + 4 \cdot \sum_{i=1}^{k-1} i + 2 \cdot |E''| + 4 \cdot k = 2 \cdot |E'| + 4 \cdot \sum_{i=1}^{k} i
\]

and the estimate of \(p\) using \(l_2\) is

\[
Est(p, l_2) = \prod_{A_i \in \text{Attr}(p) \setminus S_2} \sum_{a_j \in \text{Dom}(A_i)} cD(\{A_i = a_j\})
\]

Without loss of generality, assume that \(Est(p', l_1) < cD(p')\), namely, the estimate of \(p'\) using \(l_1\) is an under estimation. Let further assume that the estimate of \(p\) using \(l_2\) is an under estimation, thus \(Est(p, l_2) < cD(p)\). Note that \(p'S_1 = p|S_1\) and \(p'S_2 = p|S_2\), thus \(cD(p'|S_1) = cD(p|S_1)\) and \(cD(p'|S_2) = cD(p|S_2)\). Moreover, since \(\text{Attr}(p') \subseteq S_2\), \(p'S_2 = p'\) and \(cD(p) = cD(p'|S_2)\). We have that

\[
Est(p, l_1) = \prod_{A_i \in \text{Attr}(p) \setminus S_1} \sum_{a_j \in \text{Dom}(A_i)} cD(\{A_i = a_j\})
\]

Since \(Est(p, l_2) < cD(p)\) and \(Est(p, l_1) < Est(p, l_2)\) we get \(Err(l_1, p) > Err(l_2, p)\).

Proof A.10: (Proposition A.4) Given an input for the vertex cover problem, a graph \(G = (V, E)\) and \(k\), let \(D\) be the database resulting from the reduction and \(\mathcal{P}\) the set of patterns. Assume that there exists a vertex cover of size \(k\) in \(G\), \(V' = \{v_{i_1}, \ldots, v_{i_m}\} \subseteq V\). Let \(S = \{A_E, A_{i_1}, \ldots, A_{i_m}\}\) be a subset of attributes. Since \(V'\) is a set cover, for every edge \((x, y) \in E\) either \(x \in V'\) or \(y \in V'\). Thus, from Corollary A.7 the error of \(Err(L_S(D), \mathcal{P})\) is 0. Moreover, from Lemma A.8 the size of \(L_S(D)\) is \(2 \cdot |E| + 4 \cdot \sum_{i=1}^{k-1} i\).

Assume that there exists a subset of attributes \(S = \{A_E, A_{i_1}, \ldots, A_{i_m}\}\) such that \(|L_S(D)| \leq 2 \cdot |E| + 4 \cdot \sum_{i=1}^{k-1} i\) and \(Err(L_S(D), \mathcal{P}) = 0\). From Corollary A.7 \(A_E \in S\). Let \(V' = \{v_{i_1}, \ldots, v_{i_m}\} \subseteq V\). We show that \(V'\) is a vertex cover of size at most \(k\). From Corollary A.7 \(V'\) is a vertex cover. Assume by contradiction that \(m > k\), then from Lemma A.8 the size of \(L_S(D)\) is \(2 \cdot |E| + 4 \cdot \sum_{i=1}^{m-1} i > 2 \cdot |E| + 4 \cdot \sum_{i=1}^{k-1} i\). Therefore \(V'\) is a vertex cover and \(|V'| \leq k\).

The proof of Theorem 2.17 follows immediately from Proposition A.4 and Theorem A.2.

B. Proof of Proposition 3.2

Proof A.11: Let \(D\) be a database with attributes \(A, S_1 \subseteq S_2 \subseteq A\) two attribute sets, \(l_i = L_S(D)\) the labels of \(D\) using \(S_i\) for \(i = 1, 2\) respectively and \(p\) a pattern such that \(\text{Attr}(p) \subseteq S_2\). Denoting \(p' = p|_{\text{Attr}(p) \cap S_2}\) the pattern resulting when restricting \(p\) to include only the attributes appearing in \(S_2\), the estimate of \(p'\) using \(l_1\) is

\[
Est(p', l_1) = \prod_{A_i \in \text{Attr}(p') \setminus S_1} \sum_{a_j \in \text{Dom}(A_i)} cD(\{A_i = p', A_i\})
\]