Why is Universe so dark?

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In this presentation prepared for a general audience, we briefly mention the shortcomings of standard model of universe. We then focus on the late time inconsistency of the model dubbed age crisis whose resolution requires the presence of a repulsive effect that could be sourced either by dark energy or by a large scale modification of gravity. By and large, our description is based upon Newtonian cosmology which is simple and elegant despite of its limitations. On heuristic grounds, we explain how a tiny mass of graviton could account for late time cosmic acceleration. We also include a brief discussion on the underlying physics of Type Ia supernovae explosion and the direct confirmation of late time acceleration of Universe by the related observations.

I. INTRODUCTION

The standard model of universe dubbed hot big bang is remarkably a successful scenario[1]. Its predictions about the expansion of universe and the existence of relic radiation were confirmed by observations several years back. Another of its success includes the synthesis of light elements in the early universe. The standard model relies on cosmological principle which states that universe is homogeneous and isotropic at large scales which is confirmed by observations. However, a homogeneous isotropic universe evolves always again into a smooth universe. Since we have structure in the universe, it means there should have been deviations from the smoothness in the early universe. These tiny perturbations of primordial nature were found by COBE in 1992. In the hot big bang model, there is a mechanism, namely, the gravitational instability that allows these small perturbations to grow into the structure that we see in the universe today. However, in the standard model, there is no way to generated these inhomogeneities. And this is termed as one of the drawbacks of the model. In spite of the great successes of the scenario, it seems that it has several logical in built inconsistencies. The flatness problem: Today the observed universe is spatially flat to a great accuracy with nearly 30% dark matter content but the standard model dynamics is such that evolution from the early times to the present universe requires ugly fine tunings. Secondly, there are causality issues dubbed horizon problem. The present Universe evolved from the early phase where Universe consisted of a large number of causally disconnected patches. Since the Cosmic Micro wave Background(CMB) is smooth to the level of one part in $10^{-5}$, question arises, how it happened. Since different patches in the early universe did not talk to each other, our present universe would have shown large patchiness in the CMB which it does not. Let us put it in the simple language. Suppose that all of us gather here today for the first time and we come from different Icelands which never ever interacted by any means. Let us put it in the simple language. Suppose that all of us gather here today for the first time and we come from different Icelands which never ever interacted by any means. It will be surprising if we happen to speak a common language and our views on all issues turn out to be identical. There is a remarkable paradigm known as cosmological inflation which beautifully addresses the aforesaid problems. And what is most important, it provides a quantum mechanical mechanism to generate small perturbations that seed structure in the universe[2].

The hot big bang still faces one more grave problem: The age of universe in the model turns out to be smaller(8-10 billion years) than the age of some well known objects(globular clusters with age in the range, 12-15 billion years). Let us emphasize that most of the contribution to the age comes from matter dominated universe. For example universe was only $10^{5}$ years old at radiation matter equality which is negligible compared to the age of universe. Clearly, the age problem is related to late time expansion of universe. It turns out that the only way to address this problem in the standard framework is to add some repulsive effect needed to overcome the gravitational attraction at late times thereby giving rise to late time cosmic acceleration. The direct confirmation of this surprising phenomenon came from supernovae Ia observations in 1998[3] and was later supported by other indirect probes.

Late time acceleration of universe is termed as one of the most remarkable discoveries of our times. But what causes this phenomenon is the puzzle of modern cosmology and there is no convincing answer to this question at present. Normal matter (cold dark matter/radiation or baryonic matter) is gravitationally attractive. We need an exotic matter repulsive in character which can account for late time acceleration. The hypothetical matter with the said unusual property is known as dark energy[4–14]. Cosmological constant $\Lambda$, in a sense, is the simplest! candidate for dark energy which is also consistent with all the observations at present. However, there are difficult theoretical issues associated with it. Similar effect can also be mimicked by slowly scalar rolling fields.

It is quite possible that there is no real dark energy in nature but gravity is modified at large scales such that it reduces to Einstein theory of general relativity in solar system where Einstein gravity is in excellent agreement with observations and the large scale modification gives rise to late time cosmic acceleration. Thus in this case, the repulsive effect is provided by modification of gravity at large scales which mimics dark energy. It is a challenging task to build a consistent modified theory of gravity which passes local tests and can account for late time cosmic
acceleration\cite{15}.

In this presentation, we give a lucid description of late time expansion of universe using Newtonian description and try to emphasize that the resolution of age crisis in the hot big bang cries for late time cosmic repulsion which could be sourced by cosmological constant/dark energy or by large scale modification of gravity. We also describe the underlying physics of supernovae Ia explosions.

\section{II. THE HOMOGENEOUS AND ISOTROPIC UNIVERSE IN NEWTONIAN DESCRIPTION}

Cosmology is the study of structure and evolution of universe as a whole and general theory of relativity is the valid description at cosmological scales. In general, Einstein equations are very complicated non linear equations but get simplified thank to the underlying symmetries of space time. Friedmann-Robertson-Walker (FRW) model is based upon the assumption of spatial homogeneity and isotropy of universe known as cosmological principle. Homogeneity means that universe looks same at any location whereas isotropy implies indistinguishability with respect to the change of direction. Thus homogeneity and isotropy imply that there is no preferred location and no preferred direction in the universe. Observations confirm the validity of cosmological principle at large scales of the order of 100 Mpc ($1Mpc \simeq 10^{24} cm$). In fact this is a maximal symmetry which allows dramatic simplification in Einstein equations. However, we would avoid here general relativity due to conceptual and technical complications. In what follows we would opt for Newtonian description which is simple and elegant though has serious limitations not to be spelled out here\cite{13}. Using the heuristic description based upon Newtonian approach would allow us to obtain the correct evolution equations.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Figure shows a typical galaxy seen by two observers "O" and "O'" who try to check the validity of Hubble law in a homogeneous and isotropic universe.}
\end{figure}

\subsection{A. Kinematic implications of homogeneity and isotropy}

Let us consider universe filled with matter in a homogeneous and isotropic manner\cite{13} and imagine a coordinate system at an arbitrary point with the origin at "O" such that the matter is at rest there. A priori we do not assume static universe and allow motion of matter around the origin, the true nature of evolution should be implied by the dynamics. A material particle with location specified by radius vector $r(t)$ at time $t$ moves with certain velocity $v$, see Fig. 1. We then ask for the most general form of velocity distribution or velocity field consistent with homogeneity and isotropy. It is not difficult to guess that such a velocity distribution is given by,

$$v = H(t)r(t) \tag{1}$$

where $H$ is known as Hubble parameter. As for isotropy, the radius vector transforms into a radius vector under rotations thereby an observer looking into a different direction would see same velocity field. Since $H$ does not depend upon $r$, homogeneity also goes through. Indeed, let us imagine that the observer from "O" moves to new location "O'" and observes the velocity distribution from the new location, see Fig. 1. In that case,

$$r'(t) = r(t) - r_O(t) \rightarrow v' = v - v_O = H(r - r_O) = Hr' \tag{2}$$

which implies that observer located at $O'$ also sees the same velocity distribution as the observer at "O". Velocity distribution given by (1) is known as Hubble law which is nothing but the expression of homogeneity and isotropy.
Hubble law is confirmed by observations, see Fig. 2. We should, however, remember that our description is non-relativistic such that the velocities figuring in (1) are much smaller than the velocity of light. Hubble determined recession velocities of galaxies by observing spectral lines of a known element emitted by galaxies and interpreting the result using the Doppler effect. If \( \lambda_e \) and \( \lambda_0 \) are the emitted and observed wavelengths, then

\[
\frac{\lambda_0 - \lambda_e}{\lambda_e} = \frac{v}{c}, \ (v \ll c)
\]

Since all length scales in the universe scale with the scale factor,

\[
\frac{v}{c} = \frac{\lambda_0}{\lambda_e} - 1 = \frac{a_0}{a(t_e)} - 1 \rightarrow 1 + z \equiv \frac{a_0}{a(t_e)}
\]

where \( z \) is called redshift and \( a_0 \) designate the numerical value of the scale factor at the present epoch when \( \lambda_0 \) is observed.

Let us further explore the implications of the symmetry expressed by (1). Formally integrating (1), we have,

\[
r(t) = r_{in} e^{\int H(t)dt} \equiv a(t) r_{in} \quad a(t) \equiv e^{\int H(t)dt} \to H = \frac{\dot{a}}{a}
\]

where \( a(t) \) dubbed scale factor expresses how distances between two points in the universe scale with time. The distance expressed by \( r(t) \) is called physical or proper distance between two points in the expanding universe whereas \( r_{in} \equiv r(t = t_{in}) \) gives the corresponding comoving distance. A comoving frame is the one which expands with the expanding universe, matter filling the universe appears at rest in this frame. Whole information about dynamics of a homogeneous and isotropic universe is imbibed in the scale factor \( a(t) \) and we need evolution equation to determine it.

![FIG. 2: Hubble diagram for small redshifts showing recession velocities versus the redshift. Figure confirms the linear relation between the recession velocity of a galaxies versus their distances.](image)

**B. Dynamics of Homogeneous and isotropic universe**

We now apply Newtonian description to FRW universe dynamics. Let us imagine a sphere of radius \( r \) and mass \( M \) in a homogeneous and isotropic universe filled with non-relativistic background matter of density \( \rho_b \), see Fig. 3. Let us further imagine a unit mass \( m \) on the surface of the sphere and ask for the force it experiences due to matter inside the sphere (we assume that matter outside the sphere does not contribute to force on \( m \) which is under question in an infinite universe, see Ref. [13] for details).

\[
\frac{d^2 r}{dt^2} = -\frac{GM}{r^2} \equiv -\frac{4\pi G}{3} \rho_b(t)r(t) \to \ddot{a} = -\frac{4\pi G}{3} \rho_b(t)
\]

In order to integrate this equation we need to know matter density. Making use of continuity equation, we have
FIG. 3: Figure shows a point particle with mass $m$ on the surface of a sphere of mass $M$ and radius $r$ in a homogeneous and isotropic universe filled with non relativistic background matter of density $\rho_b$.

\[
\begin{align*}
\frac{\partial \rho_b}{\partial t} + (\nabla \cdot \mathbf{v}) &= 0. \quad (7)
\end{align*}
\]

Using then the Hubble law for velocity field, we find,

\[
\frac{\partial \rho_b}{\partial t} + 3H\rho_b = 0 \rightarrow \rho_b = \rho_0 \left( \frac{a_0}{a} \right)^3 \quad (8)
\]

where the subscript "0" designate the corresponding quantities at the present epoch. We can now integrate (6) by substituting $\rho_b$ from (8) and multiplying (6) left right by $\dot{a}$,

\[
H^2 = \frac{8\pi G}{3}\rho_b - \frac{K}{a^2}; \quad K \equiv a_0^2 \left( \frac{8\pi G \rho_0}{3} - H_0^2 \right) \quad (9)
\]

where $K$ is constant of integration. The redundant set of evolution equations (6),(8) & (9) should tell us whether universe is static($\dot{a}, \ddot{a} = 0$) or evolving. Indeed, equations (6) & (9) do not admit a static solution as in this case $\rho_b = 0$(Eq.(9) could be satisfied for $K > 0$). Thus Newtonian cosmology gives rise to an evolving universe (this was first observed in 1895-96, see Ref[13] and references therein). This certainly went against the common perception at that time which existed till the work of Friedmann.

Let us note that our description is valid for non relativistic matter at scales much smaller than the Hubble scale. The equation of continuity (8) is not applicable to relativistic matter which has non zero pressure. For instance, in case of thermal radiation , $p_{\text{rad}} = \rho_{\text{rad}}/3$. The number density of photon in the radiation redshifts as $a^{-3}$ and since any distance in FRW universe scales proportional to the scale factor $a$, energy of each photon ($hc/\lambda$) would scale as $a^{-1}$ which tells us that $\rho_{\text{rad}} \sim 1/a^4$. It is therefore clear that at early epochs, radiation dominated over the cold dark matter. Obviously, radiation density would not satisfy the continuity equation (8). Pressure is relativistic effect which corrects energy density in general theory of relativity. The correct continuity equation reads as

\[
\frac{\partial \rho}{\partial t} + 3H(\rho + p) = 0 \rightarrow \rho_{\text{rad}} \sim \frac{1}{a^4} \quad (10)
\]

which gives the right behavior for the energy density of radiation.

As mentioned earlier, our framework does not apply to relativistic fluid which has non zero pressure. Can we understand Eq.(10) despite our limitations? There is a way out using thermodynamic considerations. Thermodynamics is a great science which applies to any system, be it relativistic or non-relativistic, classical or quantum—*thermodynamic description is universal*. Let us consider universe field by matter with energy density $\rho$ and pressure $p$ and imagine a unit comoving volume(the corresponding physical volume or proper volume is $4\pi a^3/3$) in the expanding universe. Assuming the expansion to be adiabatic, the first law of thermodynamics tells that

\[
dE + pdV = 0 \quad (11)
\]

Expressing energy density of the fluid through its mass density, we have,

\[
E = \frac{4\pi}{3} a^3 \rho c^2 \quad (12)
\]
Substituting (12) into (11), we obtain the continuity equation in the expanding universe,

$$\frac{\partial \rho}{\partial t} + 3H(\rho + \frac{P}{c^2}) = 0$$

(13)

We thus get the pressure correction to energy density expressed by Eq.(10) where we used the system of units with \( c = 1 \) (we shall adhere to this system of units but would reinstate \( c \) whenever needed for clarity). It is not surprising that the modified continuity equation applies to any fluid, be it relativistic such as radiation or non relativistic like cold dark matter with zero pressure.

### C. Age problem in hot big bang

Since we are working in Newtonian approximation, the matter density in (9) refers to cold dark matter such that \( \rho \sim a^{-3} \). Bringing in radiation density does not change the estimate (see discussion in the subsection E). Secondly, for simplicity, we assume here, \( K = 0 \) (invoking cosmology with non zero \( K \) also does not change the estimate). In that case, Friedmann equation tells us that \( \dot{a}(t) \sim a^{-1/2} \) which readily integrates to,

$$a(t) \sim t^{2/3} \rightarrow t = \frac{2}{3} \frac{1}{H}$$

(14)

and specializing to the present epoch we get the age of universe \( t_0 \),

$$t_0 = \frac{2}{3} \frac{1}{H_0}$$

(15)

depending upon the observational uncertainties, \( H_0^{-1} \) varies from 12 to 15 billion years[12] which is a very comfortable number as there are objects in the universe which have age in this range. However, the factor 2/3 in (15) spoils the estimate. Let us note that we have matter density in our description. However, bringing in the radiation density does not change the estimate; as mentioned in the aforesaid, universe was only 10\(^5\) years old at radiation matter equality. It will become clear in the discussion to follow that there is negligible contribution to age from early times when radiation dominated such that early universe modifications do not affect the age of universe. The age crisis is generically related to late time evolution and therefore one needs to correct the late time dynamics to address this problem. Before we address the age issue in the standard model, we would like to mention the broad features of steady state cosmology which is free from the said problem.

### D. Steady state theory

The standard model of universe is based upon the imperfect cosmological principle. One should wonder why universe should look same at all locations and in any directions only and why should’nt it look same at all the times also? a la perfect cosmological principle. Why space and time should not be treated at the same footing at cosmological scales in adherence to the basic principles of relativity? The theory based upon the perfect cosmological principle dubbed steady state theory would have been aesthetically far superior than the standard model of universe. It is interesting to note that the nineteenth century materialist philosophy—view on the genesis of universe was based upon a similar ideology which can be found in the classic work by Frederick Engels, ”Dialectics of nature”. According to dialectical materialism, Universe is infinite, had no beginning, no end and always appears same thereby leaving no place for God in such a universe.

Let us examine the broad features of the steady state cosmology. In this theory, the global properties of universe such as Hubble parameter and average density of Universe should remain constant. In that case the Hubble law readily integrates to

$$r(t) \sim e^{H_0 t}$$

(16)

If we now imagine a spherical region in such a Universe, its volume would increase exponentially \( (V \sim e^{3H_0 t}) \) which would change the matter density in the universe. In order to keep it constant in adherence to the perfect cosmological principle, we would then need to continuously create matter. Let \( m(t) \) be the mass created at time \( t \). Then the mass of matter needed to be created per second per unit volume is given by,

$$\frac{\dot{m}(t)}{V} = 3H_0\rho_0 \approx 10^{-27} \text{ kg km}^{-3} \text{ year}^{-1}$$

(17)
which implies creation of one hydrogen atom per year per unit volume. Let us note that a steady state universe in infinitely old \((r(t) \to 0 \text{ for } t \to -\infty)\) thereby it has no age problem. Secondly Universe is naturally accelerating in the steady state theory by virtue of perfect cosmological principle. Unfortunately, the theory is plagued with a grave problems related to thermalization of the microwave background radiation. However, the generalized steady state theory known as "Quasi Steady State Cosmology" (QSSC) formulated by Hoyle, Burbidge and Narlikar claims to explain the microwave background radiation as well as derive its present temperature which the big bang cannot do[16].

E. Cosmological constant: Newton’s law of gravitation should be complemented by Hook’s law

Since Newtonian cosmology gives rise to an evolving universe, there was an effort to modify the Newton’s law of gravitation such that local physics is left intact and static universe is realized. Let us write down the force on a unit mass on the surface of the sphere[13],

\[
F = -\frac{4\pi G}{3} \rho_b r
\]  

which implies that acceleration can be made zero provided we add to (18) a term proportional to \(r\) vector with positive constant of proportionality(we denote by \(\Lambda\)) thereby complementing Newton’s law of gravitation by Hook’s law,

\[
F = -\frac{4\pi G}{3} \rho_b r + \frac{\Lambda}{3} r
\]

which in terms of the scale factor can be cast as,

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho_b + \frac{\Lambda}{3}
\]

Eq.(20) readily integrates to yield the Friedmann equation,

\[
H^2 = \frac{8\pi G}{3} \rho_b - \frac{K}{a^2} + \frac{\Lambda}{3}
\]

It should be noted that unlike the normal matter, the positive cosmological constant gives rise to a repulsive effect. Similar effect can also be mimicked by scalar fields. These systems can be thought as an ideal fluid, for instance, cosmological constant represents a fluid with energy density, \(\rho_\Lambda = \frac{\Lambda}{8\pi G}\). By definition, a fluid which gives rise to a repulsive effect is known as dark energy. Let us point out that \(\rho_\Lambda\) does not satisfy (8), it rather satisfy the modified continuity equation (10) which tells us that \(p_\Lambda = -\rho_\Lambda\). Cosmological constant represents an exotic fluid with large negative pressure which characterizes dark energy. As mentioned before slowly rolling scalar fields also exhibit same behavior. Such a fluid turn gravity into a repulsive force. Cosmological constant is a relativistic object which we somehow captured in the Newtonian framework. It is interesting to note that,

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho_b + \frac{\Lambda}{3} \rightarrow \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho_b - \frac{4\pi G}{3} (\rho_\Lambda + 3p_\Lambda)
\]

which is also true for any relativistic fluid with energy density \(\rho_r\) and \(p_r\) present in the universe(in that case, \(\rho_\Lambda + 3p_\Lambda\) is replaced by \(\rho_r + 3p_r\)). The second term in (22) then gives rise to acceleration provided that \(p_r/\rho_r < -1/3\), this is what we mean by large negative pressure). In general, in presence of a relativistic fluid, it is clear from Eq.(21) that no pressure correction should occur in the Friedmann equation. As for the acceleration equation, it can be obtained by using the Friedmann equation and the modified continuity equation (10). Once we incorporate pressure correction, the evolution equations become identical to those obtained in Friedmann-Robertson-Walker (FRW) cosmology in the frame work of general theory of relativity with \(K = 0, \pm 1\). Let us also note that the evolutions equations in case of \(K = 0\) exhibit a symmetry under \(a(t) \rightarrow a\alpha(t)\) where \(\alpha\) is constant, thank to which we can normalize the scale factor to a priori given value at a given time, for instance, \(a(t = t_0) = a_0 = 1\) at the present epoch; otherwise it would depend upon the matter content in the Universe.

Cosmological constant was first introduced in 1895–96 within the framework of Newtonian description with a hope to get a static universe[13]. Similar attempt, within the frame work of general theory of relativity, was made by Einstein much later. Unfortunately, the static universe turned out to be an unstable solution. Indeed, the static
solution \( (\ddot{a}, \dot{a} = 0) \) is possible for a particular value of \( \Lambda = \Lambda_c = 4\pi G \rho_0 \), see Eq.\((6)\). Let us now perturb around the static solution, \( a(t) \to a(t) + \delta a(t) \). It is not difficult to observe using Eq.\((20)\) that,

\[
\delta a(t) = C \delta a(t) \to \delta a \sim e^{Ct} \quad \text{(for large t),} \quad C = \frac{4\pi G \rho_0}{3}
\]

which means that the static solution is unstable. Einstein then suggested to drop the cosmological constant from his equations. But this is not easy, if we do it at classical level, it would come back to us through quantum corrections. Let us retain it; it could address the age problem of the hot big bang which seems to be the only known solution within the standard framework. In 1998, it would turn out to be a blessing for the standard model.

Let us note that the integration constant occurring in Eq.\((9)\) can be zero, positive or negative. However, the recent Wilkinson Microwave Anisotropy Probe in the cosmic microwave background made it clear that \( K = 0 \) to a very good accuracy which means that we live in a critical Universe with density, \( \rho_0 = 3H_0^2 = \rho_{cr} \) and we would adhere to same in the discussion to follow. The criticality of universe fixes the total energy budget of universe. The study of large scale structure reveals that nearly 30 percent of the total cosmic budget is contributed by dark matter. Then there is a deficit of almost 70 percent and the supernovae observations tell us that the missing component is an exotic form of energy which turn gravity repulsive. The idea that universe is in the state of acceleration at present is now considered to be established in modern cosmology.

F. Cosmological constant can make the universe older

Let us go back to the age problem. We said that the factor 2/3 in Eq.\((15)\) spoils the estimate. Let us show that problem could be circumvented by introducing a repulsive effect. In early times universe is hotter and its constituents run away with large velocities, the role of gravity is such that it decelerates this motion. Let us suppose for a while that we can ignore gravity. In that case, \( v = \text{const} \), Hubble law then implies that,

\[
t = \frac{1}{H} \to t_0 = H_0^{-1}
\]

which is the correct estimate for the age of universe. But we have matter in the universe which is attractive in nature and causes deceleration of expansion. More is the matter density, less time is required to reach a given expansion rate, in particular the present Hubble rate thereby giving rise to smaller age of universe. It is therefore clear that most of the contribution to age comes from late times. In the early universe, radiation dominated, its energy density was large thereby a negligible contribution to the age of universe. Indeed, the age of universe at radiation matter equality, was just 10^5 years. Thus it is sufficient to consider the matter dominant universe for the estimation of age. Further, in the matter dominated regime, more and more contribution comes from later and later stages of expansion\(^{13, 15}\).

At present, there is around 30\% matter in the universe and we can not ignore gravity. To put it in layman’s language, let us consider two trains which are running with a speed of 100 km/h. If we wish them to reduce their speed to 50km/h, the train with the superior breaks will take less time to achieve the said speed than the one with inferior breaks. The presence of matter plays the role of gravitational breaks with regard to the expansion of universe. How can we make the gravitational breaks inferior and improve the age of universe? The only known way out in the standard model to address this problem is provided by a repulsive effect which could encounter the gravitational attraction. The presence of cosmological constant is equivalent to a repulsive effect and it should increases the age of universe. Indeed let us rewrite the Friedmann equation in the following convenient form,

\[
H^2 = H_0^2 \left[ \Omega_m (\frac{a_0}{a})^3 + \Omega_\Lambda \right], \quad \Omega_m = \frac{\rho_m}{\rho_{cr}}, \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}
\]

which can be easily integrated to obtain the age of universe,

\[
t_0 = \frac{1}{H_0} \int_0^\infty \frac{dz}{(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} = \frac{2}{3} \frac{1}{\Omega_\Lambda^{1/2}} \ln \left( \frac{1 + \Omega_\Lambda^{1/2}}{\Omega_m^{1/2}} \right)
\]

where \( a_0/a = (1+z) \). The variable \( z \) known as redshift quantifies the effect of expansion. For observed values of density parameters (\( \Omega_m \simeq 0.3 \& \Omega_\Lambda \simeq 0.7 \)), we find that \( t_0H_0 \simeq 1 \) which is not surprising as we have dominant repulsive effect represented by the cosmological constant. As mentioned in the introduction, the early time inconsistencies of the standard model of universe can be taken care off by inflation. This is really amazing that resolution of late time problem of the model also requires accelerated expansion of universe. Thus, the hot big bang model should be sandwiched between two phases of acceleration: Inflation at early epochs and cosmic acceleration at late times.
III. REPULSIVE EFFECT FROM LARGE SCALE MODIFICATION OF GRAVITY

Since we are not using Einstein equations, our discussion is of heuristic nature. The main effort here is to convey the underlying ideas. In the preceding description, we tried to capture the late time cosmic acceleration by introducing a dark energy component which was for simplicity represented by cosmological constant. There is an alternative thinking in cosmology that gravity is modified at large scales which causes repulsion responsible for late time cosmic acceleration. Several schemes of large scale modification have been investigated in the literature in the recent years. Amongst them, massive gravity sounds more promising though it has not yet given a satisfactory result in cosmology. Let us try to understand the notion of mass in gravity. We better understand electromagnetic interactions where the force between two charged particles situated at a distance is due to the exchange of photons with zero rest mass thank to which the force is of long range character. The Newtonian force between two masses has similar behavior and the particles dubbed gravitons which are exchanged in gravitational interaction are also massless. Roughly speaking, if we assume that graviton has a tiny non zero mass, the effect of mass would be felt at large distances. The effect of large mass would be felt at small scales, say in the solar system which is untenable as Newton’s law provides an accurate description of physics there. The size of the observed universe is given $H_0^{-1}$ which corresponds to mass scale of the order of $10^{-33}$ eV. At the onset such a mass scale should be safe for the local physics where Newton’s law is in good agreement with observations. Let us write down the gravitational potential of a massive body with mass $M$ at a distance $r$ from the body in case the graviton has a tiny mass $m$,

$$\Phi_m = -\frac{GM}{r}e^{-mr}, \ m \sim H_0$$

which clearly reduces to Newton’s potential at small scales. However, at large scales($r \sim R_H$ such that $mr \sim 1$), the exponential factor or the Yukawa suppression becomes operational giving rise to the weakening of gravity. In the standard picture presented in the previous sections, the latter can happen only due to a repulsive effect a la cosmological constant. It is remarkable that cosmological constant gets linked to the mass of a fundamental particle, the graviton which is a novel perspective. This served as one of the motivations for the formulation of non linear massive gravity[15, 17, 18].

A. Relevant scales and number estimates

In order to have a feeling about the scales involved, let us remember that the size of our solar system is about $10^{14}cm$ and the size of our galaxy is around $10^{22}cm \equiv 10^4pc$ (Parsec (pc) is the convenient unit for distance used in cosmology). Galaxies form clusters containing a large number of galaxies, for instance, Coma cluster contains around 1000 galaxies. Clusters then for super clusters such that clusters in them are joined by filament like structures with voids in between them as large as 50 Mpc ($1Mpc \equiv 10^6pc$). Universe appears smooth beyond 100 Mpc. Universe really is clumsy at small scales and consists of very rich structure of galaxies, local group of galaxies, clusters of galaxies, super clusters and voids. These structures typically range from 1 kpc to 100 Mpc. The study of large scale structures in the universe shows no evidence of new structures at scales larger than 100 Mpc.

The Hubble parameter sets important scale(s) in the universe. As we know that the age of universe $t_0$ is given by, $t_0 \sim H_0^{-1}$. One can also ask for the time photon traveled since the big bang till the present epoch known as the visible size of the universe designated by $R_H$. Strictly speaking, photons can only travel to us from the so called last scattering surface. However, the universe was very young at that time and bringing in the last scattering surface would not change our estimate for $R_H$,

$$R_H = ct_0 \simeq \frac{c}{H_0} \simeq 10^{28} \text{ cm} \equiv 10^4 \text{ Mpc}$$

Next let us consider the mass scale associated with the cosmological constant. Using Friedmann equation (switching off matter density), we have,

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} \simeq M_p^2 H_0^2 \rightarrow M_\Lambda^2 \equiv \rho_\Lambda^{1/2} \simeq M_p H_0 \simeq (10^{-3}eV)^2$$

which should not be confused with the mass of graviton which is of the order of $H_0 \simeq 10^{-33}eV$ and in case dark energy is described by a slowly rolling scalar field, its mass should also be of this order. It is also interesting to find the corresponding distance scale,

$$c\Lambda^{-1/2} \simeq \frac{c}{H_0} = R_H$$
Thus the cosmological constant sets the largest distance scale or the smallest energy scale in the universe. It is interesting that the associated mass scale is of the order of the mass of neutrino. Can we probe it in the laboratory? Since, to the best of our knowledge, dark energy does not directly interact with matter, it can only be felt through its gravitational impact. In order to check for its local influence, let us estimate the Newtonian acceleration caused by the cosmological constant at the present epoch. Using Eq. (6), we have,

$$ \text{Newtonian acceleration} = \frac{4\pi G}{3} r (-\rho_m^0 + 2\rho_\Lambda) $$

Using Eq. (31), we can have crude idea about the local influence of cosmological constant (rigorously speaking, our framework should not apply for small scales). The acceleration caused by $\Lambda$ is negligible at small scales as compared to the deceleration due to matter, for instance, $\rho_m^0 \sim 10^{-24} g/cc$ in solar system much smaller than $\rho_\Lambda \simeq 10^{-29} g/cc$ and the contribution of $\Lambda$ can safely be ignored. However, situation changes crucially at large scales of the order of $R_H$ where $\rho_m^0$ and $\rho_\Lambda$ are of the same order of magnitude,

$$ \text{Newtonian acceleration} = \frac{H_0^2}{2} (-\Omega_m + 2\Omega_\Lambda) R_H $$

Indeed, given the present values of the dimensionless density parameters ($\Omega_m \simeq 0.3; \Omega_\Lambda \simeq 0.7$), the quantity in the parenthesis on the right hand side in Eq. (32) is of the order of one thereby the Newtonian acceleration becomes sizable ($R_H \simeq 10^{28} cm, H_0 \simeq 10^{-18}/s$); it is of the order of $10^{-9}m/s^2$. Hence, the influence of cosmological constant is important at cosmological scales which justifies the designation given to this constant.

Cosmological constant sets the lowest fundamental mass scale in the universe. Is there a fundamental mass scale that corresponds to highest energy scale? The mass scale below which we can trust classical description of gravity is known as Planck scale such that the Planck mass $M_p \simeq 10^{19}GeV$.

IV. COSMIC ACCELERATION AND ITS OBSERVATIONAL CONFIRMATION

In the preceding discussion, we argued that the known mechanism to resolve the age crisis in hot big bang is provided by the introduction of a repulsive effect which is naturally mimicked by cosmological constant. Let us note that in the framework of Einstein gravity, cosmological constant does not require ad hoc assumption for its introduction. It is always there, indeed, it is the other way around that we need a theoretical justification if we want to drop it out. At low energies there no known symmetry that would allow us to do that. So, all is well, we need cosmological constant to address a grave problem of the standard model of Universe and it is there in Einstein theory. However, the agreement related to age of Universe is an indirect one and we need the direct confirmation of cosmic acceleration which was provided by the supernovae Ia observations in 1998.

A. Luminosity distance and cosmic history

The distance to an object in the universe can be found out by knowing the absolute luminosity $L$ and the apparent or relative brightness $B$. In static universe, the amount of energy received per unit area from an object of Luminosity $L$ at a distance $d$ from the source is given by,

$$ B = \frac{L}{4\pi d^2} $$

Thus knowing $L$ and measuring $B$, one can find out the distance from the source and this is how astronomers determine the distance of luminous objects in the universe. Obviously, one should identify the objects whose intrinsic luminosity is known, which are called standard candles. Secondly, one should find out the generalization of (33) in the expanding universe. There are two effects which gives rise to the decrease in $B$, the redshift of each photon and the difference in observed rate as compared to the emission rate. It is easier to compute the desired quantity in comoving frame as universe looks static in this frame. Let us venture a bit into relativity. In Minkowski space time, the distance between two space time points is defined as,

$$ ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 $$

In an expanding universe, its obvious analog is,

$$ ds^2 = -c^2 dt^2 + a^2(dx^2 + dy^2 + dz^2) = -c^2 dt^2 + a^2(r^2 dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)) $$

$$ (35) $$
where \((x,y,z)\) or \((r,\theta,\phi)\) are the comoving coordinates and \(t\) is the cosmic time. We can define, \(d\eta = dt/a\) dubbed conformal time such that,

\[
ds^2 = a^2(-c^2d\eta^2 + r^2d\theta^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)\tag{36}
\]

which is the FRW metric for spatially flat geometry \((K=0)\). Computing luminosity distance is now easier than fishing in the bucket. We work in the comoving coordinates and conformal time, i.e., in Minkowski static space time and then translate back to FRW,

\[
L_c = \frac{hc dN}{\lambda_c d\eta} = \frac{hc dN}{\lambda dt} = La^2 \tag{37}
\]

where \(L_c\) is comoving Luminosity, \(N\) is the total number of photons radiated by the source at time \(t\) and \(\lambda_c\) is the comoving wavelength. For simplicity, we have assumed that all the photons are emitted with the same wave length. The apparent brightness is now given by,

\[
B = \frac{L_c}{4\pi r^2} = \frac{L}{4\pi d_L^2}; \quad d_L \equiv (1+z)r \tag{38}
\]

where \(d_L\) is the luminosity distance in the expanding universe and \(r\) is the comoving distance to the object from the observer which is constant for a given object but will be different for different objects. Since, for a photon propagating radially, \(c dt = adr\), the comoving distance can be expressed through the Hubble parameter

\[
d_L = c(1+z) \int_0^{t_0} \frac{1}{a(t)} dt = c(1+z) \int_0^z \frac{dz'}{H(z')} \cdot \left(\frac{dt}{a} = -\frac{dz}{H(z)}\right) \tag{39}
\]

which tells that the luminosity distance depends upon the history of universe. In general, it depends upon the cosmological models. However, we can extract important information from the above expression without invoking a particular model. Indeed, for small \(z\), we have,

\[
d_L \simeq \frac{c}{H_0} z \tag{40}
\]

that gives us Hubble law independent of the background cosmological model which is not surprising as the later is consequence of homogeneity and isotropy of universe for \(v \ll c\) or small redshift. The higher order corrections in \(z\) are model dependent. It is instructive to retain the first order correction to Hubble law(by invoking the series expansion, \(H(z) = H_0 + (\partial H/\partial z)|_{z=0}z + . . . \), in (39)) ,

\[
d_L \simeq \frac{c}{H_0} z \left(1 + \frac{1 - q_0}{2} z\right); \quad q_0 \equiv -\left(\frac{\ddot{a}}{aH^2}\right)_{t=t_0} \tag{41}
\]

where \(q_0\) is the deceleration parameter which is negative for accelerating universe as \(\ddot{a} > 0\) in this case. It is interesting to note from Eq.(41) that the first order correction to Hubble law distinguishes the accelerating and decelerating models irrespective of their details such that \(d_L\) is larger in case of accelerating cosmology. Hence, luminosity distance is an important indicator of the cosmic history.

Astronomers often quote there observations in terms of distance modulus defined as,

\[
\mu = m - M = 5 \log_{10} \left(\frac{d_L}{pc}\right) - 5 = 5 \log_{10} \left(\frac{d_L}{Mpc}\right) + 25 \tag{42}
\]

where \(m\) and \(M\) are the apparent and absolute magnitudes of Supernovae. Distance modulus is the observable which is quoted in the literature for the Type Ia supernovae observations.

Using Eq.(42) we can express the luminosity distance through distance modulus,

\[
d_L = 10^{0.2\mu+1} \text{ pc} = 10^{0.2\mu-5} \text{ Mpc} \tag{43}
\]

Using the data from Type Ia Supernovae (Union2.1 [19] of 580 data points) available for the observational values of \(\mu\), we can calculate the corresponding values of the luminosity distance from Eq.(43). We can also estimate the uncertainty in the luminosity distance data from the Union2.1 data [19]. Indeed, if we have a function \(f(x,y,...)\) and the errors in \(x,y,...\) are given as \(\sigma_x, \sigma_y,...\) then we can calculate the error in \(f\) as,

\[
\sigma_f^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + ... + 2 \left(\frac{\partial f}{\partial x}\right) \left(\frac{\partial f}{\partial y}\right) \sigma_{xy} + .... \tag{44}
\]
FIG. 4: Luminosity distance is plotted against redshift $z$ for flat $\Lambda$CDM model. $\Omega_{m0}$ and $\Omega_{\Lambda}$ in the plot are the present values of matter and dark energy density parameters. Black dots are observational values of $d_L$ and black bars are $1\sigma$ error bars of $d_L$ calculated from Union2.1 data set [19].

FIG. 5: Distance modulus is plotted against redshift $z$ for flat $\Lambda$CDM model. $\Omega_{m0}$ and $\Omega_{\Lambda}$ in the plot are the present values of matter and dark energy density parameters. Black dots are observational values of $\mu$ and black bars are $1\sigma$ error bars for $\mu$ taken from Union2.1 data set [19].

which then gives the uncertainty in $d_L$,

$$\sigma_{dL} = 0.2 \ln(10)d_L \sigma_{\mu},$$

where $\sigma_{\mu}$ is the error in the measurements of distance modulus $\mu$.

In Fig. 4 we have shown the variation of luminosity distance with redshift for three different cases of $\Lambda$CDM model and compared them with the observational data calculated from Union2.1 data set [19]. It should be noticed that error bars are large at large redshifts. It is clear, however, from Fig. 4 that the case $\Omega_{m0} \sim 0.3$ and $\Omega_{\Lambda} \sim 0.70$ is best fitted though the other cases cannot be clearly excluded because of the large error bars of $d_L$ at large redshifts. In Fig. 5, we plot distance modulus $\mu$ versus the redshift and compare with the Union2.1 data set [19]. As in case of distance modulus, error bars are not very large, it is better seen from Fig. 5 that the case $\Omega_{m0} \sim 0.3$ and $\Omega_{\Lambda} \sim 0.70$ is best fitted and others are less favored.

B. Problem with cosmological constant

It is clear from the aforesaid that the standard model of Universe to be consistent with observations should contain a small parameter, the cosmological constant. It should be noticed that constant energy plays here a decisive role
which is specific to general theory of relativity though we deliberately kept it out of discussion. For instance, in
mechanics, adding a constant to potential does not reflect in the underlying physics.

All seems to be well with cosmological constant at classical level. Problem arises if we bring in here the quantum
mechanics perceptions. Let us try to explain it in simple words. There are different types of fields which are present in
the universe, for instance the electromagnetic field that we are acquainted with. A field is a complicated object which
is a collection of infinite many harmonic oscillators each with energy levels given by, \( E_n = (n + 1/2)\hbar \omega; \ n = 0, 1, 2, ... \)
such that there is always a zero point energy, namely, \( \hbar \omega/2 \) and it is important in the present context context. When
we sum up the zero point energy of all the oscillators, we obtain the so called vacuum energy, \( \rho_{\text{vac}} \) which is formally
infinite. Since we know nothing beyond the Planck scale, we cut it off there adopting \( \rho_{\text{vac}} = M_p^4 \) as an expression of
our ignorance. This quantity should now be added to \( \rho_\Lambda \) present in equations of motion. The observed value then is
given by,

\[ \rho_{\Lambda}^{\text{obs}} = \rho_\Lambda + \rho_{\text{vac}} \]  

(46)

We computed \( \rho_{\text{vac}} \) and we know \( \rho_{\Lambda}^{\text{obs}} \) from observations but do not in general know \( \rho_\Lambda \) which is a parameter present
in the framework at classical level. But since \( \rho_{\text{vac}}/\rho_{\Lambda}^{\text{obs}} \sim 10^{120} \), one thing is sure and certain that the cancelation
between \( \rho_{\text{vac}} \) and \( \rho_\Lambda \) should be accurate to one part in \( 10^{-120} \). We do not know any mechanism which could give rise
to such a fantastic cancelation and explain as to why this quantity is so small or why the vacuum energy gravitates so
insignificantly. At low energies, there is no known symmetry which could give rise to the required cancelation. This
is the famous cosmological constant problem which needs to be addressed.

C. The standard candles and their underlying physics in brief

For the construction of the Hubble diagram, we need the luminous objects of known absolute luminosity or standard
candles such that their distance can be found from their apparent luminosity. Supernovae Ia are best suited as
cosmological distance indicators. Their absolute luminosity can be determined from the physical conditions responsible
for their formation. The type Ia supernovae form in binary star systems in case one of the companion stars is a white
dwarf with mass below the Chandrasekhar limit 1.4\( M_\odot \) and the other is a red giant.

Before we go ahead let us say few words about the life cycle of stars which crucially depends upon their mass. Stars
shine due to fusion of hydrogen into helium in their core such that the support against gravity is provided by the
thermal pressure. However, as the stars run out of the hydrogen fuel, their core begins collapsing under gravitational
pressure thereby getting hotter whereas hydrogen fusion still continue in outer shells. The core which gets hotter and
hotter due to contraction then pushes the outer layers of the star outward. As a result the ejecting layers of the star
expand and cool transforming the star finally into a red giant. What happens further crucially depends upon the mass
of the core. For stars with masses in the range of the sun, the ejection process continues leaving behind a hot core
dubbed white dwarf which eventually cool. The white dwarf is gravitationally a very compact object typically with
a mass of the order of \( M_\odot \) and radius around that of the earth such that the matter density in the star is given by,
\( \rho \sim 10^{12} \text{gm/cc} \). It is composed of nuclear waste Carbon and Oxygen supported by the electron degeneracy pressure
alone as its temperature is nearly zero.

In case the mass of the collapsing core of star is between \( M_{cs} \) and and \( 3M_\odot \), the system ends up with neutron star.
The core with mass more than three solar masses collapses into black hole.

Let us now discuss the mechanism of supernovae Ia explosion. In the binary system, the white dwarf begins
accreting matter from the companion star, the reg giant, and gets smaller further. But there is a limit to this process,
as the white dwarf reaches a mass equal to the Chandrasekhar limit \( M_{cs} \), the degeneracy pressure fails to support
the gravitational pressure. Its temperature during compression increases to the level sufficient for igniting the carbon
fusion which leads to a violent supernovae Ia explosion.

In what follows, we briefly outline the underlying physics of the explosion. The white dwarf consists of plasma
of carbon-oxygen nuclei and electrons with temperature around zero kelvin. As the density in white dwarf is high,
some of the electrons are forced to occupy higher momentum states due to Pauli exclusion principle. In this case, the
electron pressure dubbed degeneracy pressure is of quantum nature which supports the star against its gravitational
pressure. The electron degeneracy pressure is given by,

\[ P_{dg} = \frac{1}{3} \int_0^\infty v p n(p) dp \]  

(47)

where \((v, p)\) designate the velocity and momentum of the electron and \(n(p)\) is the number density of electrons per unit
momentum interval. The degeneracy pressure \( P_{dg} \) can be computed using quantum statistical mechanics. In a simpler
way, one can use the Heisenberg uncertainty relation to estimate it. Indeed, the minimum volume per electron in the
phase space is given by, \( \Delta x \Delta y \Delta z \Delta p_x \Delta p_y \Delta p_z \sim h^3 \). The three dimensional space breaks into small cells of volume \( \Delta V = \Delta x \Delta y \Delta z \). In accordance with Pauli exclusion principle, this elementary volume can accommodate at most two electrons. The number density of electrons inside the star is given by, \( 2/\Delta V \) and since at low temperature, electrons occupy all the quantum energy states up to a maximum value \( E_F \) dubbed Fermi energy with the corresponding momentum \( p_F \), we can estimate the total number of electrons in the white dwarf,

\[
N_e = \frac{8\pi V}{h^3} \int_0^{p_F} p^2 dp = \left( \frac{3h^3 N_e}{8\pi V} \right)^{1/3}
\]

where \( V \) is the volume of star. The number of electrons in the star can be expressed through its density \( \rho \),

\[
N_e = V \frac{\rho Z}{M_p A} \sim \frac{\rho}{M_p} \left( \frac{Z}{A} \approx \frac{1}{2} \right)
\]

where \( Z(A) \) is atomic number(mass number) and \( M_p \) is the mass of proton. In case the mass of white dwarf is much smaller than \( M_{\odot} \), electrons in the white dwarf can be treated non relativistically \( (v << c) \) such that \( v \approx p/m_e \),

\[
P_{dg} = \frac{8\pi}{3m_e h^3} \int_0^{p_F} p^4 dp = \frac{8\pi}{15 m_e h^3} p_F^5 = k_1 \rho^{5/3}; \quad k_1 = \left( \frac{3h^3}{8\pi} \right)^{2/3} \frac{1}{5m_e M_{p/3}^2}
\]

where \( \rho \) is matter density in the white dwarf. Let us note that the degeneracy pressure is inversely proportional to the mass of particles thereby the pressure due to nucleons is negligible as compared to the pressure caused by electrons. Next, we need to compare the degeneracy pressure with gravitational pressure of the white dwarf. The self gravitational energy or the binding energy of the white dwarf is given by,

\[
E_g = -\frac{3GM^2}{5R}
\]

where \( M \) is mass and \( R \) is radius of the star. The corresponding gravitational pressure is given by the thermodynamic expression,

\[
P_g = -\frac{\partial}{\partial V}E_g = - \left( \frac{4\pi}{375} \right)^{1/3} GM^{2/3} \rho^{4/3}
\]

The degeneracy pressure encounters the gravitational pressure of the white dwarf and equating the two we obtain,

\[
k_1 \rho^{5/3} = \left( \frac{4\pi}{375} \right)^{1/3} GM^{2/3} \rho^{4/3} \rightarrow R \sim \frac{1}{M^{1/3}}
\]

Thus the volume of white dwarf is inversely proportional to its mass. Consequently, as white dwarf gets heavier during accretion process, its gets smaller and thereby denser. The electrons then are forced to occupy still higher momentum states such that \( p_F \) is large. The non relativistic approximation then ceases to apply. In this case we should use the relativistic expression for velocity,

\[
v = \sqrt{\frac{p}{m_e}} \sqrt{1 + \left( \frac{p}{m_e c} \right)^2}
\]

while computing the degeneracy pressure. In this case, it is possible to compute the integral in \( (50) \) in the closed form. It would be instructive to quote the expression of \( P_{dg} \) in the limit \( p/mc \gg 1 \),

\[
P_{dg} \approx k_2 \rho^{4/3} - k_3 \rho^{2/3}; \quad k_2 = c \left( \frac{3h^3}{8\pi M_p^2} \right)^{1/3}; \quad k_3 = c^3 \left( \frac{\pi m_e^2}{24h^3 M_p^2} \right)^{1/3}
\]

The first leading term corresponds to the ultra-relativistic case \( (v \approx c) \) and the second term provides the first order correction. Equating \( (55) \) to the gravitational pressure, we arrive at an important relation,

\[
R \sim M^{1/3} \sqrt{1 - \left( \frac{M}{M_{\odot}} \right)^{2/3}}; \quad M_{\odot} = \left( \frac{5k_2}{G} \right)^{3/2} \left( \frac{3}{4\pi} \right)^{1/2} \approx 1.7M_{\odot}
\]
FIG. 6: Figure shows a binary system with a white dwarf and companion red giant. In the upper panel, the white dwarf accretes mass from the outer layer of the red giant and shrinks. As the mass of white dwarf reaches the Chandrasekhar limit, the Carbon fusion ignites leading to supernova Ia explosion shown in the lower panel of the figure. Image: NASA/CXC/M Weiss.

FIG. 7: Supernova Ia light curve shows the variation of relative brightness versus the days after explosion. The figure is taken from the site www-supernova.lbl.gov.

It is interesting to note that if we drop the correction to the leading term in Eq.(55), i.e., if we stick to the ultra-relativistic limit and then compare the degeneracy pressure to the gravitational pressure, the radius dependence drops out giving rise to,

$$M = \left( \frac{5k_2}{G} \right)^{3/2} \left( \frac{3}{4\pi} \right)^{1/2} \equiv M_{\text{ch}}$$

(57)

which tells that $M_{\text{ch}}$ is the limiting value of the mass and Eq.(55) tells us how radius of the star depends upon its mass as $M \to M_{\text{ch}}$. In order to get better feeling for the limiting mass of the white dwarf let us show that the said limit is related to the stability of the star. In the relativistic approximation, the kinetic energy of the degenerate electron gas in the star is given by,

$$E_K \simeq c p_F \simeq \frac{\hbar c N_p^{1/3}}{R} \simeq \frac{\hbar c N_p^{1/3}}{R}$$

(58)

where $N_p$ is the number of baryons in the star and $p_F$ is given by Eq.(48). Thus, the total energy of the white dwarf can be estimated as,

$$E_{\text{tot}} \simeq \frac{\hbar c N_p^{1/3}}{R} - \frac{G N_p M_p^2}{R}$$

(59)
Stability then demands that the maximum number of baryons the white dwarf can accommodate should correspond to $E_{\text{tot}} = 0$,

$$N_p^{\text{max}} \sim \left( \frac{h c}{G M_p^2} \right)^{3/2} \rightarrow M_{ch} \simeq M_p \left( \frac{h c}{G M_p^2} \right)^{3/2} \tag{60}$$

which is the same order of magnitude we obtained by comparing the degeneracy pressure with the gravitational pressure.

Our estimates depend upon the assumption we made for simplicity that matter density inside the star is constant. The realistic calculations which incorporate density variation inside the white dwarf, give, $M_{ch} \simeq 1.44 M_\odot$. Our estimate formally tells us that as $M \rightarrow M_{ch}$, the radius goes to zero.

Actually, as the mass of the white dwarf increases, temperature inside the star rises due to compression where as the volume is still controlled by the degeneracy pressure. The temperature in the core of the star increases to the level that Carbon fusion ignites which further heats it up. The white dwarf can not expand and cool like an ordinary star as its volume is controlled by the degeneracy pressure and the thermal pressure is negligible, contraction in this case continues till the thermal energy remains below Fermi energy. As a result, temperature rises to the extent that thermal conduction can no longer cope up with nuclear burning giving rise to run away nuclear reactions. What happens thereafter is a topic of an active debate at present. However, there is a broad consensus that as runaway nuclear reaction ensues, the matter in the star quickly fuses into iron-peak elements releasing enormous amount of energy giving rise to supernova explosion about 5 billion times brighter than the Sun. The luminous event can be identified by its light curve which shows the rapid increase of luminosity to a maximum value and then disappears in one to two months time. The last supernova in our galaxy was seen in 1604; around 300 supernovae are seen from other galaxies every year. Since these events are rare, their systematic search using their light curves needs to be carried out.

Based upon the aforesaid discussion, we draw the following important conclusion. As the underlying physics mechanism for supernovae Ia explosions is common or all the supernovae Ia form under the same physical conditions, their absolute luminosity is same. Hence, the supernovae which look dimmer are farther deeper in the universe. The observational investigations of their luminosity distances provides important information of the cosmic history.

### V. Discussion and Outlook

We described the kinematics and dynamics of homogeneous and isotropic universe in the framework of Newtonian cosmology. Since the material is prepared for a wider audience, we often resorted to heuristic arguments to convey the underlying physics ideas. We mentioned shortcomings of the standard model of Universe and focussed on the late time inconsistency dubbed age crisis in the hot big bang and argued that accelerated expansion plays an important role in the history of universe. In the standard framework, in particular, the resolution of age problem requires a repulsive effect or late time acceleration which dominates over deceleration due to normal matter. The repulsive effect can be caused by a positive cosmological constant in the standard lore or by the large scale modification of gravity. We tried to impress upon that similar to the early universe problems related to flatness, horizon and primordial fluctuations whose resolution requires an early phase of accelerated expansion, the solution of age problem also asks for the late time cosmic acceleration.

Cosmological constant, in a sense, is the simplest device that can turn gravity into a repulsive force at late times. It is remarkable that the effect was directly seen in supernovae Ia observations in 1998. The repulsive effect can also be caused by the slowly rolling scalar fields or by a large scale modification of gravity. We briefly explained how a tiny mass of graviton could mimic cosmological constant like behavior.

We also included a simple discussion on the underlying physics of supernovae Ia explosions and their role of distance indicators for the understanding of cosmic history. The luminosity distance is larger in case of accelerating models such that the related supernovae Ia data clearly speaks in favor of late time cosmic acceleration which is indirectly supported by other observations. The early phase of cosmic acceleration a la inflation fixes the total cosmic energy budget out of which nearly $30\%$ is contributed by cold dark matter as revealed by the study of large scale structures and the missing $70\%$ is fixed by the observed late time acceleration of Universe.

At present, observations are not in position to distinguish between various options that might give rise to late time acceleration. The future surveys might unveil the true cause of the phenomenon. It is quite likely that there is nothing but $\Lambda$ which is responsible for late time acceleration. Neither the scalar fields nor any known scheme of large scale modification perform better than the cosmological constant both on theoretical and observational grounds. However, non linear massive gravity, despite several tough theoretical challenges it faces, still deserves attention. It links cosmological constant to the mass of a fundamental particle, the graviton and it has an in built mechanism of
de gravitation which is a novel perspective for addressing the cosmological constant problem. Cosmological constant problem is the problem which needs to be finally addressed.

Whatever may be the underlying reason, we need a dominant (effective) dark energy component at late times to support the old universe we live in. If the Universe is to grow that old it is today, it has to be predominantly dark which sounds like an Anthropic argument!

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