An Ising Machine-Based Solver for Visiting-Route Recommendation Problems in Amusement Parks

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SUMMARY In an amusement park, an attraction-visiting route considering the waiting time and traveling time improves visitors’ satisfaction and experience. We focus on Ising machines to solve the problem, which are recently expected to solve combinatorial optimization problems at high speed by mapping the problems to Ising models or quadratic unconstrained binary optimization (QUBO) models. We propose a mapping of the visiting-route recommendation problem in amusement parks to a QUBO model solving it using Ising machines. By using an actual Ising machine, we could obtain feasible solutions one order of magnitude faster with almost the same accuracy as the simulated annealing method for the visiting-route recommendation problem.

key words: Ising machine, amusement park, combinatorial optimization problem, Ising model, quadratic unconstrained binary optimization

1. Introduction

1.1 Visiting-Route Recommendation Problem in Amusement Parks

In amusement parks, the waiting time and traveling time decrease visitors’ satisfaction, and hence finding an attraction-visiting route considering them improves visitors’ experience. For example, a method to find the shortest route considering the waiting time in a zoo have been proposed [1]. Minimizing the total required time of the route considering the waiting time and the reservation system in an amusement park has been also proposed [2], [3]. These studies solve the problem that extends a traveling salesman problem (TSP). TSP is one of the combinatorial optimization problems to find the shortest route to visit each location only once and returning to the starting location. In amusement parks, however, one attraction may be visited more than once depending on the visitor’s preference. Furthermore, the visitor does not want to visit the same attraction consecutively. There are many additional constraints as well as preferences in a visiting-route recommendation problem in an amusement park.

1.2 Ising Machines

Ising machines, which are recently expected to solve combinatorial optimization problems at high speed, have been developed [4]–[10] and utilized in many fields. A combinatorial optimization problem is to search for the optimal combination of decision variables so as to maximize or minimize the objective function under satisfying the given constraints. Ising machines search for the ground states of a model in statistical mechanics called an Ising model [11], where the ground states mean the lowest-energy states. Ising machines are expected to obtain quasi-optimal solutions at high speed by mapping these problems to Ising models or quadratic unconstrained binary optimization (QUBO) models. Mapping methods from combinatorial optimization problems into Ising models or QUBO models and the solutions obtained by using actual Ising machines have been discussed in [12]–[23].

1.3 Our Proposal

In this paper, we propose a mapping of the visiting-route recommendation problem in amusement parks to an Ising model. Firstly, we define the visiting-route recommendation problem in amusement parks as a combinatorial optimization problem. The purpose of the problem is to find the visiting route that maximizes the visitor’s satisfaction within the scheduled stay time. In the problem, the satisfaction and required time of each attraction, the traveling time between every two attractions, and the scheduled stay time are given. Secondly, we propose an objective function which maximizes the total satisfaction and constraint terms which minimize the energy of the Ising model when satisfying the constraints of the problem. Finally, we solve the visiting-route recommendation problem for two real amusement parks by an actual Ising machine and demonstrate the experimental results to evaluate our proposed method.

1.4 Contributions

Our contributions of this paper are summarized as follows:

1. We define the visiting-route recommendation problem
in amusement parks as one of the combinatorial optimization problems, and we propose a mapping method of the problem to a QUBO model.  

2. Problem Definition

As described in Sect. 1, in amusement parks, the shortest route is not always optimal for the visitors due to their preference, which differs from the setting of traveling salesman problems. Therefore, we introduce a satisfaction based on the visitor’s preference for each attraction and find the visiting route which maximizes the total satisfaction.

Assume that a set \( I \) of \((m + 1)\) attractions as \( I = \{a_0, a_1, \ldots, a_m\} \), the number \( n \) of attractions in a visiting route, and the scheduled stay time \( t_{\text{max}} \) are given. For simplicity of the notation, we assume that there is only one gate in the amusement park, and we regard the gate as a kind of attractions and \( a_0 \in I \) is the gate. Let \( s_i \) (\( 1 \leq s_i \leq 10 \)) and \( c_i \) be the satisfaction and required time (sum of attraction-waiting time and attraction-using time) of an attraction \( a_i \in I \) \((1 \leq i \leq m)\), respectively. Note that we assume that the required time of every attraction is a constant. Let \( d_{i,j} \) be the traveling time from an attraction \( a_i \in I \) to an attraction \( a_j \in I \). Then, the objective is to maximize the total satisfaction of a visiting route.

In our problem, we adopt the attraction-satisfaction model as follows: \(^1\) The visitor can visit an attraction more than once, where the satisfaction decreases at every visit after the first time. Let \( b \) be this decreasing value. Let \( v_i (1 \leq i \leq n) \) be the \( i \)-th attraction in the visiting route. Then the total satisfaction is expressed by

\[
S = \sum_{i=1}^{n} s_i v_i .
\]  

Note that the satisfaction \( s_i \) of an attraction \( a_i \) is decreased by \( b \) every time it appears in the visiting route after its first visit.

Then, the visiting-route recommendation problem in amusement parks is defined as follows:

**Definition 1:** The visiting-route recommendation problem in amusement parks consists of a set of attractions with their satisfaction and required time, the traveling time between every two attractions, the number of attractions in a visiting route, and the scheduled stay time. The goal is to find the visiting route which maximizes the visitor’s total satisfaction under the constraints as follows:

**Gate Constraint**
Both of the start point and the end point of the route must be the gate in the amusement park.

**Total Time Constraint**
The time to follow the route must be within the scheduled stay time.

**Simultaneous Attraction Visit Prohibiting Constraint**
The visitor must not visit multiple attractions at the same time.

**Continuous Attraction Visit Prohibiting Constraint**
The visitor must not visit the same attraction consecutively.

where the satisfaction decreases by \( b \) at every visit after the first time.

3. Ising Model Mapping of Visiting-Route Recommendation Problem in Amusement Parks

3.1 Ising Model and QUBO

An Ising model is a statistical mechanics model which represents macroscopic phenomena in systems with interacting many microscopic elements \([11], [15]\). For example, the Ising model is used to investigate the nature of magnetic materials. In a magnetic material, many microscopic elements called electron spins are gathered. The interaction is applied between the electron spins, and the local magnetic field is applied to every electron spin.

As shown in Fig. 1, the Ising model is defined on an undirected graph \( G = (V, E) \), where \( V \) is a set of vertices and \( E \) is a set of edges. A spin \( \sigma_i \) exists on each vertex \( i \in V \) and has a value of either +1 or −1. Then, the energy function \( H_{\text{Ising}} \) of the Ising model is defined by

\[
H_{\text{Ising}} = - \sum_{(i,j) \in E} J_{ij} \sigma_i \sigma_j - \sum_{i \in V} h_i \sigma_i ,
\]  

![Fig. 1](image_url)  

**Fig. 1** An example of Ising model.  

\(^{1}\)One of the extensions of the attraction-satisfaction model is that, we set the parameter \( b \) to be a variable for each attraction. However, since \( b \) depends on every visitor and it is too much burden for him/her to determine the value of \( b \) for every attraction beforehand. Another extension of the attraction-satisfaction model is that, we use the attraction-satisfaction model using division instead of subtraction, i.e., the satisfaction \( s_i \) is decreased to \( s_i/b \) every time the attraction \( a_i \) is visited after the first time. As discussed in Sect. 3.1, the QUBO model must be the “quadratic” polynomial of the binary variables. Introducing the attraction-satisfaction model using division leads to the “exponential” function of the binary variables, which cannot be acceptable in the QUBO model.

Based on the discussion above, the attraction-satisfaction model described here must be reasonable enough from the viewpoints of both practical use and QUBO mapping.
where \( J_{ij} \) is the interaction coefficient between the two spins \( \sigma_i \) and \( \sigma_j \), and \( h_i \) is the external local magnetic field acting on the spin \( \sigma_i \). Here, \( J_{ij} \) and \( h_i \) take real values. According to the principle in physics, lower energy states are more stable. The lowest energy state is called the ground state. Thus, the larger the interaction coefficient \( J_{ij} \) is, the more likely the two spins \( \sigma_i \) and \( \sigma_j \) tend to take the same value. The larger the external local magnetic field \( h_i \) is, the more likely \( \sigma_i \) takes +1.

Depending on the combinatorial optimization problem, it is more convenient to introduce a binary variable taking either 1 or 0 instead of a spin. The energy function in the form of using binary variables is called Quadratic Unconstrained Binary Optimization (QUBO). Let \( n_i \) be a binary variable, the energy function \( H_{\text{QUBO}} \) in the QUBO model is defined by

\[
H_{\text{QUBO}} = -\sum_{i,j \in E} v_{ij} n_i n_j - \sum_{i \in V} w_i n_i - \text{const},
\]

where \( v_{ij} \) is the interaction coefficient between the two binary variables \( n_i \) and \( n_j \), \( w_i \) is the coefficient of the linear term of the binary variable \( n_i \), and the third term of Eq. (3) is a constant value which does not depend on the binary variables. Here, \( v_{ij} \) and \( w_i \) take real values.

The binary variable \( n_i \) and the spin \( \sigma_i \) are transformed by the following equation:

\[
n_i = \frac{\sigma_i + 1}{2}.
\]

Therefore, the energy function of Ising model and the QUBO model are equivalent except for the constant value.

### 3.2 Ising Model Mapping of Visiting-Route Recommendation Problem in Amusement Parks

We propose an Ising model mapping of the visiting-route recommendation problem in amusement parks using QUBO. Let \( x_{t,i} \) (1 ≤ \( t \) ≤ \( n \), 1 ≤ \( i \) ≤ \( m \)) be binary variables as follows:

\[
x_{t,i} = \begin{cases} 1 & \text{(if} a_i \text{ is visited at } t\text{-th time slot)} \\ 0 & \text{(otherwise)} \end{cases}
\]

By using Eq. (5), the energy function \( H \) of visiting-route recommendation problem in amusement parks is formulated as follows:

\[
H = H_{\text{obj}} + \lambda_1 H_1 + \lambda_2 H_2 + \lambda_3 H_3
\]

where \( H_{\text{obj}} \) and \( H_k \) (1 ≤ \( k \) ≤ 3) correspond to the objective function and constraints of the visiting-route recommendation problem in amusement parks. The coefficients in RHS of Eq. (6), \( \lambda_k \) (1 ≤ \( k \) ≤ 3) are the positive hyperparameters. The definitions of \( H_{\text{obj}} \) and \( H_k \) are shown below.

#### 3.2.1 Objective Function \( H_{\text{obj}} \)

The objective function \( H_{\text{obj}} \) is an energy function corresponding to the total satisfaction of the route. In our problem, the visitor can visit an attraction more than once, where the satisfaction decreases by \( b \) at every visit after the first visit. The satisfaction obtained to visit the attraction \( a_i \) for the first time is \( s_i \). Let \( k_i \) be the number of visiting counts in the visiting route of the attraction \( a_i \), which can be expressed by:

\[
k_i = \sum_{t=1}^{n} x_{t,i}.
\]

The partial satisfaction \( S_i \) obtained to visit \( a_i \) can be expressed as the sum of the arithmetic sequence where the first term is \( s_i \), the common difference is \((-b)\) and the number of terms is \( k_i \). Note that we set \( b = 2 \) in our problem instance (See Sect. 4.2.3 in detail). Then we have

\[
S_i = s_i + (s_i - b) + (s_i - 2b) + \cdots + [s_i - (k_i - 1)b] = k_i (s_i - k_i + 1).
\]

The total satisfaction \( S \) of the visiting route is the sum of Eq. (8) for all the attractions \( a_i \in I \) and can be expressed by:

\[
S = \sum_{i=1}^{m} S_i = \sum_{i=1}^{m} k_i (s_i - k_i + 1).
\]

By using Eq. (7) and Eq. (9), \( H_{\text{obj}} \), which is an energy function corresponding to this objective, is given by

\[
H_{\text{obj}} = -S = -\sum_{i=1}^{m} \left( \sum_{t=1}^{n} x_{t,i} \left( s_i - \sum_{u=1}^{n} x_{u,j} + 1 \right) \right).
\]

where the sign is reversed so that the energy is minimized only when the total satisfaction is maximized. The QUBO mapping of this objective function is depicted as in Fig. 2 (a).

#### 3.2.2 Total Time Constraint \( H_1 \)

Total time constraint is that the time to follow the route must be within the scheduled stay time. In amusement parks, the stay time is roughly classified into two types: required time to use the attraction and traveling time between every two attractions. The total required time to use the attraction can be expressed as follows:

\[
T_{\text{required}} = \sum_{i=1}^{m} \sum_{t=1}^{n} c_{t,i} x_{t,i},
\]

where \( c_{t,i} \) shows the required time to use the attraction \( a_i \). The total traveling time can be expressed as follows:

\[
\sum_{i=1}^{n} \sum_{j=1}^{m} d_{i,j} x_{t,i} x_{t+1,j},
\]

where \( d_{i,j} \) shows the traveling time from the attraction \( a_i \) to the attraction \( a_j \).

In addition, we must consider the traveling time from
the gate to the first attraction and from the last attraction to the gate because of the gate constraint. As stated before, we regard the gate as a kind of the attractions and let \( g_0 \in I \) be the gate in the amusement park. Then, \( d_{0,i} \) is the traveling time from the gate to the attraction \( a_i \) and thus the total traveling time considering the gate constraint is given by

\[
T_{\text{traveling}} = \sum_{i=1}^{n-1} \sum_{j=1}^{m} d_{i,j} x_{i,j} x_{i+1,j} + \sum_{i=1}^{m} d_{0,i} x_{0,i} + \sum_{j=1}^{m} d_{1,i} x_{1,i}.
\] (13)

There can be two possible naive methods to express the total time constraint. The first naive method is to express the constraint as an inequality constraint and map it to the Ising model directly. Although there has already established the idea for mapping the inequality constraints, too many auxiliary binary variables must be introduced in this method, which definitely leads to difficulties in solving the problem in actual Ising machine [12]. The second naive method is to express the constraint as a soft constraint where the total time should be equal to the scheduled stay time. This method is derived from the intuition that making full use of the scheduled stay time is appropriate since the initial value method is derived from the intuition that making full use of the time should be equal to the scheduled stay time. This idea for mapping the inequality constraints, too many auxiliary binary variables must be introduced in this method, which definitely leads to difficulties in solving the problem.

3.2.3 Simultaneous Attraction Visit Prohibiting Constraint \( H_2 \)

Simultaneous attraction visit prohibiting constraint is that the visitor must not visit multiple attractions at the same time. One attraction must be assigned to one time slot, which is expressed as

\[
\sum_{i=1}^{m} x_{t,i} = 1 \quad \text{for } 1 \leq t \leq n.
\] (15)

Then we introduce Eq. (16) below. Equation (16) is minimized if and only if Eq. (15) is satisfied.

\[
H_2 = \sum_{i=1}^{n} \left( 1 - \sum_{j=1}^{m} x_{t,i} \right)^2.
\] (16)

\( H_2 \) takes the minimum value of 0 only when Eq. (15) is satisfied. The QUBO mapping of this constraint term is depicted as in Fig. 2 (c).

3.2.4 Continuous Attraction Visit Prohibiting Constraint \( H_3 \)

Continuous attraction visit prohibiting constraint is that the visitor must not visit the same attraction consecutively.

\[
\sum_{i=1}^{m} x_{t,i} x_{t+1,i} = 0 \quad \text{for } 1 \leq i \leq m.
\] (17)

Then we introduce Eq. (18) below. Equation (18) is minimized if and only if Eq. (17) is satisfied.

\[
H_3 = \sum_{i=1}^{m} \sum_{j=1}^{n-1} x_{t,i} x_{t+1,i}.
\] (18)

\( H_3 \) takes the minimum value of 0 only when Eq. (17) is satisfied. The QUBO mapping of this constraint term is depicted as in Fig. 2 (d).

3.2.5 The Number of Spins

The total energy function Eq. (6) is a quadratic polynomial
of $x_{i,j}$ which is given by the weighted sum of $H_{\text{obj}}$, $H_1$, $H_2$, and $H_3$. Since each binary variable $x_{i,j}$ corresponds to every spin in the Ising model, $m \times n$ spins are required for the visiting-route recommendation problem in amusement parks.

4. Experiments and Discussion

4.1 Purpose of Experiments

In this section, we demonstrate several experiments and discuss the results. The main purpose of our experiments is that we can obtain feasible solutions by using an actual Ising machine where a feasible solution means the solution satisfying all the constraints of the gate constraint, the total time constraint, the simultaneous attraction visit prohibiting constraint, and the continuous attraction visit prohibiting constraint shown in Sect. 2. In order to realize that, we construct the objective function and constraint terms so that they include only 2-body interactions and linear terms. Specifically, we express the total time constraint as the simple subtraction and construct the attraction-satisfaction model for the multiple visits to the same attraction using subtraction by a constant value.

In addition, we compare our proposed method with the software-implemented simulated annealing just for reference. Note that, we use the actual Ising machine, which can be accessible through the cloud services [6]. When running software-implemented simulated annealing, the software environment of macOS 10.15.1 on a 2.7GHz Intel Core i7 CPU with 16GB of memory is used. The programming language used is C++. Note that it is not completely a fair comparison, since the experimental setups are not completely the same.

4.2 Experimental Environment

4.2.1 Input Data

In our experiment, we pick up two real amusement parks as the sample A [26] and sample B [27]. The required time to visit the attraction $c_{i}$ and traveling time between every two attraction $d_{i,j}$ are set based on these real amusement parks. In general, the attraction with higher satisfaction tends to have longer waiting time. The satisfaction correlates positively with the waiting time, and hence we set $s_i$ according to Table 1, which ranges from 1 to 10. The attraction waiting time and the attraction-using time for the sample A and sample B are derived from [28] and [29], respectively. The scheduled stay time to be $t_{\text{max}} = 720$. Table 2 shows the satisfaction $s_i$, waiting time and required time $c_i$ of every attraction of the sample A and sample B. The statistics of the satisfaction $s_i$, waiting time, required time $c_i$ and traveling time $d_{i,j}$ are summarized in Table 3.

4.2.2 Hyperparameters

We need to decide the hyperparameters of $\lambda_1$, $\lambda_2$, and $\lambda_3$ in Eq. (6). Estimating based on the input data of the required time $c_i$ and the traveling time $d_{i,j}$, $H_1$ takes the values that is approximately 10–100 times larger than the other constraint terms. Therefore, setting the hyperparameters so that $\lambda_1 < \lambda_2, \lambda_3$ must be effective [19]. We experimentally evaluate the relationship between the hyperparameters and the quality of the solution in the case of $n = 10$. Figure 3 shows this evaluation result on 10 solutions obtained for each set of hyperparameters, where $\lambda_1 = i \times 10^{-3}$ ($1 \leq i \leq 100$, $i \in \mathbb{Z}$),
\[ \lambda_2 = \lambda_3 = j \times 10^{-1} \quad (1 \leq j \leq 100, \ j \in \mathbb{Z}). \]

In Fig. 3, the uncolored area indicates that no feasible solutions can be obtained, where a feasible solution means the solution satisfying all the constraints described in Sect. 2. Meanwhile, more than 90% of solutions obtained in the blue area satisfy all the constraints. The blue dots indicate the average total satisfaction, which is obtained by averaging the total satisfaction in feasible solutions. In the blue area which can obtain feasible solutions, the smaller the \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) are, the higher the average total satisfaction tends to be. Based on the results of the similar investigation for \( n > 10 \), we set \( \lambda_1 = 0.01n - 0.09 \) and \( \lambda_2 = \lambda_3 = 0.8n \) (\( n \geq 10 \)).

4.2.3 Parameter \( b \) of Attraction-Satisfaction Model

It is necessary to determine the parameter \( b \) which is the decreasing value for multiple visits to the same attraction defined in Sect. 2. We have investigated the effect of \( b \) for each sample where \( n = 10 \) and \( n = 15 \). Figure 4 plots the \( b \)-dependences of the maximum visiting counts \( V_{\text{max}} \) to the same attraction and the total satisfaction \( S \). Every plot is the average of 100 trials. \( V_{\text{max}} \) decreases as \( b \) becomes large and the results that an attraction is visited more than once cannot be obtained rarely where \( b \geq 3 \). In terms of the total satisfaction, the larger \( b \) (the less \( V_{\text{max}} \)) becomes, the higher \( S \) tends to be.

In this paper, since the model allows that multiple visits to the same attraction are assumed, we fix \( b = 2 \) based on these results. Note that we can also set \( b = 3 \) or larger depending on the visitors’ preferences in our proposed method. In our experiment, in order to see the effect of multiple visits to the same attraction, we set \( b = 2 \).

4.2.4 Ising Machine

In our experiment, we use a simulated-annealing-based (SA-based) Ising machine [6]. The SA-based Ising machine has 8192 spins and can set the interaction coefficient between every two binary variables \( x_i \) and \( x_j \) (\( i \neq j \)). Moreover, the SA-based Ising machine has 64 bits of precision on the quadratic term and 76 bits of precision on the linear term. Hence, we can input Eq. (6) which is an energy function of the fully-connected Ising model. The SA-based Ising machine can set the number of annealings \( N_{\text{annealing}} \). It performs the annealings in parallel and returns \( N_{\text{annealing}} \) results. Out of \( N_{\text{annealing}} \) states obtained by a single execution, we pick up the state that satisfies all the constraints and has the lowest energy as the solution. In this paper, we consider this process to be a single trial.

We experimentally evaluate the relationship between the parameters of the Ising machine and the quality of the solution. Figure 5 shows the relationship between the initial/final temperature and the average total satisfaction of 100 trials where we set \( n = 15 \) of the sample A. In Fig. 5(a), the dependence of the initial temperature \( T_{\text{initial}} \)
is shown, where \( T_{\text{initial}} = 10^0, 10^1, 10^2, 10^3 \) and the final temperature is fixed to \( T_{\text{final}} = 10^{-1} \). In Fig. 5 (b), the dependence of the final temperature \( T_{\text{final}} \) is shown, where \( T_{\text{final}} = 10^0, 10^{-1}, 10^{-2}, 10^{-3} \) and the initial temperature is fixed to \( T_{\text{initial}} = 10^1 \). The total satisfaction is higher on average when \( T_{\text{initial}} = 10 \) and \( T_{\text{final}} = 1 \). Regardless of the temperature parameters, the more the number of iterations \(^5\) is, the higher the total satisfaction tends to become (see Fig. 5). However, the best total satisfaction of 100 trials does not improve even if the number of iterations is larger than \( 1 \times 10^6 \). In addition, the annealing time increases with the number of iterations. Thus, we set the number of iterations to \( 1 \times 10^5 \). In the SA-based Ising machine, the temperature is updated by \( T \leftarrow T \times T_{\text{decay}} \) every \( I_{\text{update}} \) iterations. Hence, we set \( T_{\text{decay}} \) based on Eq. (19) so that the final temperature is \( T_{\text{final}} \).

\[
T_{\text{decay}} = \left( \frac{T_{\text{final}}}{T_{\text{initial}}} \right)^{I_{\text{update}}/I_{\text{iteration}}}. \tag{19}
\]

Finally, the parameters used in the SA-based Ising machine are summarized in Table 4. The temperature parameters should be changed according to the data scale of the problem. However, since the data scale of two samples used in our experiment are equivalent as shown in Table 3, we used the same parameters for the sample A and sample B.

### 4.2.5 Software-Implemented Simulated Annealing

For comparison, we implement a simulated annealing method (SA method) to solve the visiting-route recommendation problem in amusement parks. The SA method is performed as follows:

1. Set an initial visiting route \( r \) randomly. Let \( r_t (1 \leq t \leq n) \) be the \( t \)-th attraction in the visiting route.
2. Choose randomly \( t \) and change randomly \( r_t \) to another attraction \( r_t' \) so that \( r_t' \neq r_{t-1} \) and \( r_t' \neq r_{t+1} \). The change satisfies the continuous attraction visit prohibiting constraint. Let \( \Delta E \) be the energy difference in Eq. (6) between the current state and the new state. If \( \Delta E < 0 \), accept the new state. Otherwise, accept the new state with the probability \( p = \exp(-\Delta E/T_k) \), where \( T_k \) is the temperature at \( k \)-th outer iteration. The update rule is called the Metropolis method [15].
3. Repeat (2), “Inner loop” times. Reduce the temperature \( T_{k+1} = T_k \times T_{\text{decay}} \), where \( k \) is the current number of outer iterations and \( T_{\text{decay}} \) is the cooling rate.
4. Repeat (2) and (3), “Outer loop” times.

The SA method above can search in the solution space satisfying the gate constraint, simultaneous attraction visit prohibiting constraint and continuous attraction visit prohibiting constraint, i.e., \( H_2 = H_3 = 0 \) is always satisfied. Based on [30], the parameters in the SA method are summarized in Table 5.

### 4.3 Experimental Results

Figure 6 shows the examples of obtained results of the visiting-route recommendation problem in amusement parks by the proposed method. The blue nodes and the yellow node show the attractions and the gate in the amusement park, respectively. The darker the blue node is, the higher the satisfaction of the attraction is. \( S \) is the total satisfaction of the visiting route and \( t_{\text{total}} \) is the total time to follow the visiting route. In Fig. 6 (a), the leftmost attraction is visited twice where \( n = 10 \). In Fig. 6 (b), there also exists an attraction in the right side area, which is visited twice where \( n = 15 \). These results show the correct implementation of the attraction-satisfaction model defined in Sect. 2.

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**Table 4 Parameters in the SA-based Ising machine.**

| Parameter            | Value      |
|----------------------|------------|
| Number of annealings | \( N_{\text{annealing}} = 128 \) |
| Number of iterations | \( N_{\text{iteration}} = 1 \times 10^6 \) |
| Temperature update interval | \( I_{\text{update}} = 1 \times 10^3 \) |
| Initial temperature | \( T_{\text{initial}} = 10 \) |
| Temperature decay    | \( T_{\text{decay}} = 0.997 \) |

**Table 5 Parameters in the SA method.**

| SA method parameter | Value |
|---------------------|-------|
| Initial state       | Random |
| \( T_0 \)           | 10    |
| Cooling rate        | 0.99  |
| Inner loop          | 100   |
| Outer loop          | 1000  |

---

\(^{5}\)“Iteration” means the single update process in the QUBO model in the actual Ising machine [6]. Hence, “the number of iterations” shows the number of update processes in the QUBO model.
Since the scheduled stay time $t_{\text{max}}$ is fixed in our experiment, the smaller $n$ is, the more time the visitor can spend on one attraction. As a result, the attraction with more required time (i.e., higher satisfaction) tends to be selected. However, since the visitor can spend less time for traveling, the route tends to be short. The results satisfy all the constraints and show that the proposed method can successfully solve the visiting-route recommendation problem in the amusement park by mapping it to the Ising model.

Table 6 summarizes the experimental results of the proposed method using the Ising machine [6] and the SA method, where the number $n$ of attractions in a visiting route is 10–15. “$S$ (best)” and “$S$ (avg.)” are the highest and average total satisfaction among the feasible solutions obtained by 100 trials, respectively. “Time [ms]” is average execution time over 100 trials of each method. “Prob. [%]" shows the probability that the proposed method and the SA method can obtain a feasible solution. As mentioned earlier, the solutions obtained by the SA method always satisfy the gate constraint, simultaneous attraction visit prohibiting constraint and continuous attraction visit prohibiting constraint. However it should be noted that the SA method considered here sometimes obtains a solution that violates the total time constraint. Thus, the probability that the SA method can obtain a feasible solution is less than 100% in almost all cases.

The results suggest that our hyperparameters are reasonable enough to obtain feasible solutions satisfying the other three constraints. The total satisfaction which is the objective of the visiting-route recommendation problem in an amusement park is roughly equal between the proposed method and the SA method in the sample A and sample B. Focusing on the execution time, the proposed method achieves almost the same satisfaction as that of the SA method with one order of magnitude faster.

### Acknowledgments

This work was supported in part by JST CREST Grant Number JPMJCR19K4, Japan.

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