Analytical model for bearing capacity of two closely spaced foundations

A A Altawel\textsuperscript{1} and R R Shakir\textsuperscript{2}

\textsuperscript{1} Postgraduate Student of geotechnical engineering, Civil Engineering Department, College of Engineering, University of Thi-Qar, Iraq
\textsuperscript{2} Professor of geotechnical engineering, Civil Engineering Department, College of Engineering, University of Thi-Qar, Iraq

E-mail: akramagar@utq.edu.iq, akramagar@gmail.com, rrshakir@utq.edu.iq, rrshakir@yahoo.com.

Abstract. In this study, a new analysis has been developed to determine the bearing capacity of soil under interference of two asymmetric strip foundations conditions. The limit equilibrium method is used considering the soil failure theory under a rough based strip foundation. Three equations are developed to determine the values of the efficiency factors resulting from the effect of interference of foundations for the surcharge, cohesion, and weight. The effect of variable factors on the equations, which represent the angle of friction, the distance between the foundations and the width of the foundations has been studied, and the results have shown that when the interference occurs between the foundations, the soil bearing capacity increases with the increase in the angle of friction and decrease the distance between the foundations, while the change in the width of the foundations has just an effect on interference factor related to the soil weight, $\varphi$. Comparing the results of the present mathematical model with the results of available theoretical analyses and previous experimental studies shows acceptable and reliable agreement.

1. Introduction

In most constructional practices, the foundations are not constructed in isolation, but rather close to each other or bounded by adjacent obstruction construction. This type of situation has not been the subject of attention of the theories that have undertaken the analysis of bearing capacity of soil. There were a number of researchers discussed the individual foundation, some of them worth mentioning such as M. Y. Fattah et al. (2008) [1] and R. R. Shakir (2005, 2019) [2],[3]. This phenomenon leads to the development of the failure envelope under the foundations with a wider area than the case of the individual foundation[4]. In such cases, convergence often improves soil bearing properties[5]. Depending on the limit equilibrium method, theoretically, Stuart (1962) investigated the influence of interference shallow foundations by examined continuous shallow foundations rested on a layer of soil defined as non-cohesion, homogeneous dilated to an infinite depth, considering the failure mechanism that developed by Terzaghi (1943) for a single foundation [6].

Other studies considered the stress-characteristics solution method, which depended on the principle of that the isolated foundation failure mechanisms become deformed as a result of the existence of a neighboring foundation, thereby causing the consequent soil reaction on the base of foundation which become eccentric and inclined [7]. The probabilistic approach method was another try to analyses the problem of interference of multi-foundations which gave the possibility of examining bearing capacity.
considering an important parameter that could be the spatial engagement length of the soil in relation to the foundation width and spacing [8]. The upper bound limit analysis method also used to determine the bearing capacity of soil under interference foundations by assuming the geometry of the collapse mechanism for cohesion-less soil [9]. The lower bound limit analysis method is also used for analyzing the stability of soil supporting foundations under interference conditions [10].

To confirm the theoretical results, a series of laboratory tests have been carried out using model foundations which made a comparison with the theoretical results [8,11]. In spite of the considerable experimental and theoretical studies on the interference of two closely spaced foundations, clear understanding of the interference phenomena still more studies. Differences in both theoretical and experimental results were commonly found in the literature [10,11]. Therefore, a new theoretical model that may help in quantify the effect of interference is crucial.

In this study, a new analysis has been developed to determine the bearing capacity of soil under interference of two asymmetric strip foundations conditions. In this analysis, the limit equilibrium method is used considering the soil failure theory under a rough based strip foundation adopted by Terzaghi 1943. From this work, three equations were developed to determine the values of the efficiency factors resulting from the effect of interference of foundations for the surcharge, cohesion, and weight. After obtaining the results from these equations for different values of the angle of friction as well as different distances between the foundations, these results were compared with the results of available theoretical analyses and previous experimental studies, where the results were acceptable and reliable in the design works.

2. The theory

In this analysis, extracted for efficiency factor is done to find the effect of the interference of foundations on the soil's bearing capacity for the individual foundation, considering the soil failure theory under a rough based strip foundation adopted by Terzaghi 1943 [11] as shown in figure 1.

![Figure 1](image-url)

**Figure 1.** Failure surface in soil at ultimate load for a continuous rough rigid foundation as assumed by Terzaghi.

It illustrates the division of the failure envelope into three zones:
(I) a wedge-shaped zone located beneath the loaded strip, in which the major principal stresses are vertical.
(II) two zones of radial shear, ade and bde1, emanating from the outer edges of the loaded strip, whose boundaries intersect the horizontal at angles of $\theta$ and $45^\circ - \theta/2$.
(III) two passive Rankine zones.

The ultimate bearing capacity $q_u$ for a soil with cohesion, friction and weight may be expressed:

$$q_u = cN_c + qN_q + \frac{1}{2} \gamma BN_y$$  \hspace{1cm} (1)

Where: $c =$ cohesion, $q =$ surcharge, $\gamma =$ unit weight of soil, and $N_c, N_q$ and $N_y =$ bearing capacity coefficients
3. Statement of the problem and assumptions
For analytical purposes, to obtain the amount of efficiency factor as a result of the interference of foundations, the following assumptions are assumed:

1. Two asymmetric perfectly rough rigid strip foundations designated as left and right foundations, placed close to each other at a clear spacing \( S \), at the depth \( D_f \) from the surface of a semi-infinite homogeneous soil as shown in figure 2.

![Figure 2. interference problem assumption.](Image)

2. The foundations are considered to be nested when the interference distance exceeds the limits of the failure envelope of the foundation, in other words, when the distance \( S \) is less than the distance \( \frac{a}{b} \) or \( \frac{b_{1}}{b} \) as in the Figure (1), what is indicated is based on the Stuart 1962 hypothesis [6].

3. When interference occurs, the soil bearing capacity of one of the foundations considers as an active stress relative to the other foundation, and the effect value is equal to the interfering portion, which represents half of the soil bearing capacity for the second foundation.

4. Method of analyses
The theory of limit equilibrium is adopted to derive the governing equations to determine efficiency factor as a result of foundations' interference. The ultimate bearing capacity \( q_u \) will be equal to the passive force \( P_p \) on each face of the triangular wedge \( abd \) required to cause failure. The passive force will be a function of the surcharge, cohesion, unit weight, and angle of friction of the soil. In order to obtain \( P_p \), the method of superposition can be used to divide the force into three parts, \( P_{pq} \), \( P_{pc} \), and \( P_{py} \), which represent the passive force contributions of \( q \), \( c \), and \( \gamma \), respectively[11].

4.1. Relationship for \( P_{pq} \) \((\Phi \neq 0, \gamma = 0, q \neq 0, c = 0)\)
As shown in Figure (3), the forces due to surcharge are \( P_{pqL} \) (the passive force contribution of surcharge for the left side), \( P_{pqR} \) (the passive force contribution of surcharge for the right side), \( q \) (surcharge), \( P_{p(1)} \) (the Rankine passive force), \( F \) (the frictional resisting force along the arcs \( de \) and \( de_1 \)), and \( P_{qq2} \) (the force due to interference).

\[
P_{p(1)} = qK_p H_d = qH_d \tan^2 \left( \frac{45 + \gamma}{2} \right) \quad (2)
\]
Figure 3. passive force distribution of $P_{pq}$

$$H_d = \frac{B_1 \sin \alpha \theta \tan^0}{2 \cos \phi}$$  \hspace{1cm} (3)

where $\theta = 135 - \frac{\phi}{2}$ and $\alpha = 45 - \frac{\phi}{2}$.

$$P_{q,q,z} = \frac{q_{q,z}}{2} K_A H_q = \frac{q_{q,z}}{2} H_q \tan^2 \left(45 - \frac{\phi}{2}\right)$$  \hspace{1cm} (4)

$$H_q = \left\{ \frac{B_1 e^{\theta \tan^0}}{2 \cos(45 + \frac{\phi}{2})} - S \right\} \tan \alpha, \text{when } be_1' \leq S < bf_1'$$  \hspace{1cm} (5)

Taking the moment of all forces about point a for the left-side forces:

$$P_{pqL} \left( \frac{B_1}{4} \right) = q (ae') \left( \frac{ae'}{2} \right) + P_{p(1)} \frac{H_d}{2}$$  \hspace{1cm} (6)

or

$$P_{pqL} = \frac{qB_1 e^{2\theta \tan^0}}{4 \cos^2(45 + \frac{\phi}{2})}$$  \hspace{1cm} (7)

Also taking the moment of all forces about point b for the right-side forces:

$$P_{pqR} \left( \frac{B_1}{4} \right) = q (be') \left( \frac{be_1'}{2} \right) + P_{p(1)} \frac{H_d}{2} + P_{q,q,z} \frac{H_q}{2}$$  \hspace{1cm} (8)

or

$$P_{pqR} = \left\{ \frac{qB_1 e^{2\theta \tan^0}}{4 \cos^2(45 + \frac{\phi}{2})} + \frac{q_{q,z}}{B_1} \right\} \left\{ \frac{B_1 e^{\theta \tan^0}}{2 \cos(45 + \frac{\phi}{2})} - S \right\}^2 \tan^4 \left(45 - \frac{\phi}{2}\right), \text{when } be_1' \leq S < bf_1'$$  \hspace{1cm} (9)

Considering the stability of elastic wedge abd under the foundation (1):
\[ q_1 B_1 = P_{pqL} + P_{pqR} \ (10) \]

or

\[
q_1 = \begin{cases} 
\frac{q e^\theta \tan \phi}{2 \cos^2 (45 + \phi/2)} + q_2 \left[ \frac{\theta}{2 B_1 \cos (45 + \phi/2) - S/B_1} \right]^2 \tan^4 \left(45 - \frac{\phi}{2}\right), S \leq b e' \leq S < b f_1 \\
\frac{q e^\theta \tan \phi}{2 \cos^2 (45 + \phi/2)} + q_2 S^2 \tan^2 \beta \tan^2 \left(45 - \frac{\phi}{2}\right), S < b e' \end{cases} \ (11)
\]

where

\[
q_2 = \frac{q e^\theta \tan \phi}{2 \cos^2 (45 + \phi/2)} \ (12)
\]

then

\[
q_1 = q N_q \begin{cases} 
1 + \left[ \frac{\theta}{2 B_1 \cos (45 + \phi/2) - S/B_1} \right]^2 \tan^4 \left(45 - \frac{\phi}{2}\right), S \leq b e' \leq S < b f_1 \\
1 + \frac{S^2}{B_1^2} \tan^2 \beta \tan^2 \left(45 - \frac{\phi}{2}\right), S < b e' \end{cases} \ (13)
\]

and

\[
\xi_q = \begin{cases} 
1 + \left[ \frac{\theta}{2 B_1 \cos (45 + \phi/2) - S/B_1} \right]^2 \tan^4 \left(45 - \frac{\phi}{2}\right), S \leq b e' \leq S < b f_1 \\
1 + \frac{S^2}{B_1^2} \tan^2 \beta \tan^2 \left(45 - \frac{\phi}{2}\right), S < b e' \end{cases} \ (14)
\]

4.2. Relationship for \( P_{pc} (\phi \neq 0, \gamma = 0, q = 0, c \neq 0) \)

As shown in figure 4, the forces due to cohesion are \( P_{pcL} \) (the passive force contribution of cohesion for the left side), \( P_{pcR} \) (the passive force contribution of cohesion for the right side), \( P_{p(2)} \) (the Rankine passive force), \( C_\delta \) (the cohesive force along the lines \( \alpha d \) and \( \beta d = \frac{c R}{2 \cos \phi} \)), \( M_c \) moment due to cohesion \( c \) along arcs \( da \) and \( de_1 \), and \( P_{q(e)} \); the force due to interference.

\[ \text{Figure 4. passive force distribution of } P_{pc} \]
\[ P_{p(2)} = 2c\sqrt{K_p}H_d = 2cH_d\tan\left(\frac{45 + \frac{\theta}{2}}{2}\right) \]  \hspace{1cm} (15)

\[ M_c = \frac{c}{2\tan\frac{\theta}{2}}(r_1^2 - r_0^2) \]  \hspace{1cm} (16)

where: \( r_1 = r_0e^{\theta}\tan\frac{\theta}{2} \), and \( r_0 = \frac{b}{2\cos\frac{\theta}{2}} \)

\[ P_{qc2} = \frac{qc_2}{2}\sqrt{K_AH_q} = \frac{qc_2}{2}\sqrt{H_q}\tan \left(45 - \frac{\theta}{2}\right) \]  \hspace{1cm} (17)

Taking the moment of all forces about point a for the left-side forces:

\[ P_{pca} \left(\frac{B_1}{4}\right) = M_c + P_{p(2)}\frac{H_d}{2} \]  \hspace{1cm} (18)

or

\[ P_{pca} = \frac{cB_1}{2} \left[ \cot \theta e^{\theta \tan \frac{\theta}{2}} \frac{2}{2\cos^2\left(45 + \frac{\theta}{2}\right)} - \frac{1}{\cos \frac{\theta}{2} \sin \frac{\theta}{2}} \right] \]  \hspace{1cm} (19)

Also taking the moment of all forces about point b for the right-side forces:

\[ P_{pcR} \left(\frac{B_1}{4}\right) = M_c + P_{p(c)}\frac{H_d}{2} + P_{qc2}\frac{H_q}{2} \]  \hspace{1cm} (20)

or \( P_{pcR} = \) \[ \begin{cases} \frac{cB_1}{2} \left[ \cot \theta e^{\theta \tan \frac{\theta}{2}} \frac{2}{2\cos^2\left(45 + \frac{\theta}{2}\right)} - \frac{1}{\cos \frac{\theta}{2} \sin \frac{\theta}{2}} \right] + \frac{qc_2}{B_1} \left[ \frac{B_1 e^{\theta \tan \frac{\theta}{2}}}{2\cos\left(45 + \frac{\theta}{2}\right)} - S \right]^2 \tan^3 \left(45 - \frac{\theta}{2}\right), \overline{be}'_1 \leq S < \overline{bf}'_1 \\ \frac{cB_1}{2} \left[ \cot \theta e^{\theta \tan \frac{\theta}{2}} \frac{2}{2\cos^2\left(45 + \frac{\theta}{2}\right)} - \frac{1}{\cos \frac{\theta}{2} \sin \frac{\theta}{2}} \right] + \frac{qc_2}{B_1} s_2^2 \tan^2 \beta \tan \left(45 - \frac{\theta}{2}\right), S < \overline{be}'_1 \end{cases} \]  \hspace{1cm} (21)

Considering the stability of elastic wedge abd under the foundation (1):

\[ q_{c1}B_1 = 2C\sin\theta + P_{pca} + P_{pcR} \]  \hspace{1cm} (22)

\[ \begin{cases} c \cot \frac{\theta}{2} \left[ \frac{e^{\theta \tan \frac{\theta}{2}}}{2\cos^2\left(45 + \frac{\theta}{2}\right)} - 1 \right] + \frac{qc_2}{B_1} \left[ \frac{e^{\theta \tan \frac{\theta}{2}}}{2B_1\cos\left(45 + \frac{\theta}{2}\right)} - S \right]^2 \tan^3 \left(45 - \frac{\theta}{2}\right), \overline{be}'_1 \leq S < \overline{bf}'_1 \\ c \cot \frac{\theta}{2} \left[ \frac{e^{\theta \tan \frac{\theta}{2}}}{2\cos^2\left(45 + \frac{\theta}{2}\right)} - 1 \right] + \frac{qc_2}{B_1} s_2^2 \tan^2 \beta \tan \left(45 - \frac{\theta}{2}\right), S < \overline{be}'_1 \end{cases} \]  \hspace{1cm} (23)

where \( qc_2 = c \cot \frac{\theta}{2} \left[ \frac{e^{\theta \tan \frac{\theta}{2}}}{2\cos^2\left(45 + \frac{\theta}{2}\right)} - 1 \right] \) \hspace{1cm} (24)

then \( q_{c1} = cN_c \begin{cases} 1 + \left[ \frac{e^{\theta \tan \frac{\theta}{2}}}{2B_1(45 + \frac{\theta}{2})} - S \right]^2 \tan^3 \left(45 - \frac{\theta}{2}\right), \overline{be}'_1 \leq S < \overline{bf}'_1 \\ 1 + s_2^2 \tan^2 \beta \tan \left(45 - \frac{\theta}{2}\right), S < \overline{be}'_1 \end{cases} \]  \hspace{1cm} (25)
and \( \xi_c = \left\{ 1 + \left[ \frac{e^{\theta \tan \phi}}{2 B_1 \cos(45 + \frac{\phi}{2})} - \frac{S}{B_1} \right]^2 \tan^3 \left( 45 - \frac{\phi}{2} \right) , \frac{b e_1'}{f_1} \leq S < b f_1 \right. \\
1 + \frac{S^2}{B_1^2} \tan^2 \beta \tan \left( 45 - \frac{\phi}{2} \right) , S < \frac{b e_1'}{f_1} \right\} (26) \\
4.3. Relationship for \( P_{py} (\phi \neq 0, \gamma \neq 0, q = 0, c = 0) \)
As shown in figure 5, the forces due to weight are \( P_{pyL} \) (the passive force contribution of weight for the left side), \( P_{pyR} \) (the passive force contribution of weight for the right side), \( P_{p(3)} \) (the Rankine passive force), \( W_1 \) (the weight of spiral sector \( bde \) and \( bde_1 \)), \( W_2 \) (the weight of wedge \( be_1' \) and \( be_1e_1' \)), \( W_w \) (the weight of wedge \( abd' \)), and \( P_{q(2)} \) (the force due to interference).

\[
P_{p(3)} = \frac{1}{2} \gamma H_d^2 \tan^2 \left( 45 + \frac{\phi}{2} \right) (27)
\]

\[
P_{q(2)} = \frac{qz_2}{2} K_A H_q = \frac{qz_2}{2} H_q \tan^2 \left( 45 - \frac{\phi}{2} \right) (28)
\]

The area of spiral sector \( ade \) and \( bde_1 \) and the centroid can be found according to calculation adopted by Hijab 1956 [12] as shown in figure 6 and are expressed as:

\[
\frac{A}{r_0} = \frac{\left( \frac{r_1}{r_0} \right)^2 - 1}{4 \tan \phi} (29)
\]

\[
\frac{a'}{r_0} = \frac{4 \tan \phi \left( r_1^2 \right)^3 \left( 3 \tan \phi \sin \theta - \cos \theta \right) + 1}{3 \left( r_1 \right)^2 - 1 - \left( r_1 \right)^2 - 1} (30)
\]

\[
\frac{b'}{r_0} = \frac{4 \tan \phi \left( r_1 \right)^2 - 3 \tan \phi \sin \theta - \cos \theta}{3 \left( r_1 \right)^2 + 1 - \left( r_1 \right)^2 - 1} (31)
\]
where \( A \) is the area of spiral sector and \( a' \) and \( b' \) are the perpendicular distances from the centroid of spiral sector to \( r_0 \) and \( r_1 \) respectively.

\[
W_1 = \gamma A \tag{32}
\]

Taking the moment of all forces about point \( O \) for the left-side forces as shown in figure 7:

\[
P_{pyL} \left( \frac{R_1}{3} - x \right) = W_1 \sin \left( 45 + \frac{\phi}{2} \right) \left( a' + Y \cos \left( 45 - \frac{\phi}{2} \right) \right) - W_1 \cos \left( 45 + \frac{\phi}{2} \right) b' \\
+ W_2 \left( \frac{2}{3} ae' + x \right) + P_{p(3)} \left( \frac{2H_d}{3} + y \right) \tag{33}
\]

\[
P_{pyR} \left( \frac{R_1}{3} - x \right) = W_1 \sin \left( 45 + \frac{\phi}{2} \right) \left( a' + Y \cos \left( 45 - \frac{\phi}{2} \right) \right) - W_1 \cos \left( 45 + \frac{\phi}{2} \right) b' \\
+ W_2 \left( \frac{2}{3} be' + x \right) + P_{p(3)} \left( \frac{2H_d}{3} + y \right) + P_{qy2} \left( \frac{2H_q}{3} + y \right) \tag{34}
\]

However, the passive force \( P_{py} \) can be expressed in form:
Figure 8, passive force distribution of \( P_{pyr} \)

\[
P_{pyr} = \frac{1}{2} y h^2 K_{py} = \frac{1}{2} y \left( \frac{B_1 \tan \phi}{2} \right)^2 K_{py} = \frac{1}{8} y B_1^2 K_{py} \tan^2 \phi
\]

(35)

therefore

\[
P_{pyr} = \frac{1}{8} y B_1^2 K_{py} \tan^2 \phi
\]

(36)

and

\[
P_{pyr} = \frac{1}{8} y B_1^2 K_{py} \tan^2 \phi + \frac{p_{\gamma 2}}{(B_1/x)} \left( \frac{2H_a}{3} + y \right)
\]

(37)

or

\[
P_{pyr} = \frac{1}{8} y B_1^2 K_{py} \tan^2 \phi + \frac{q_{\gamma 2}}{(B_1/x)} H_q \left( \frac{2H_a}{3} + y \right) \tan^2 \left( 45 - \frac{\phi}{2} \right)
\]

(38)

Considering the stability of elastic wedge abd under the foundation (1):

\[
q_{\gamma 1} B_1 = P_{pyr} + P_{pyr} - W_w
\]

(39)

or

\[
q_{\gamma 1} = \frac{1}{2} y B_1 \left( \frac{1}{2} K_{py} \tan^2 \phi - \frac{\tan \phi}{2} \right) + \frac{q_{\gamma 2}}{(B_1/x)} H_q \left( \frac{2H_a}{3} + y \right) \tan^2 \left( 45 - \frac{\phi}{2} \right)
\]

(40)

where

\[
q_{\gamma 2} = \frac{1}{2} y B_2 \left( \frac{1}{2} K_{py} \tan^2 \phi - \frac{\tan \phi}{2} \right) = \frac{B_2}{B_1} \frac{1}{2} y B_1 \left( \frac{1}{2} K_{py} \tan^2 \phi - \frac{\tan \phi}{2} \right)
\]

(41)

then

\[
q_{\gamma 1} = \frac{1}{2} y B_1 N_{\gamma} \left[ 1 + \frac{B_2}{2B_1} H_q \left( \frac{2H_a}{3} + y \right) \tan^2 \left( 45 - \frac{\phi}{2} \right) \right]
\]

(42)

and

\[
\xi_{\gamma} = 1 + \frac{B_2}{2B_1} H_q \left( \frac{2H_a}{3} + y \right) \tan^2 \left( 45 - \frac{\phi}{2} \right)
\]

(43)

where

\[
H_q = \begin{cases} \frac{B_1 \tan \phi}{2 \cos (45 + \phi/2)} - S & \text{tan } \alpha, \text{when } \overline{be_1} \leq S \leq \overline{bf_1} \\ S \tan \beta, \alpha < \beta \leq 90 - \phi, \text{when } S < \overline{be_1} \end{cases}
\]

(44)

Note: the values of the coordinates \( x \) and \( y \) in equation (43) represent the location of the centroid \( O \) relative to the points \( a \) and \( b \) along the lines \( \overline{ae} \) and \( \overline{be_1} \) respectively, which produce the minimum value of \( P_{py} \) for isolated foundation.
Table 1. the values of \( x \) and \( y \) in calculation of \( \xi_y \)

| \( \phi \) | \( x \)  | \( y \)  | \( \phi \) | \( x \)  | \( y \)  |
|---|---|---|---|---|---|
| 1  | -0.09532 | -0.09367 | 23 | -0.12506 | -0.08277 |
| 2  | -0.09751 | -0.09417 | 24 | -0.12477 | -0.08103 |
| 3  | -0.10384 | -0.09854 | 25 | -0.12479 | -0.0795 |
| 4  | -0.10642 | -0.09924 | 26 | -0.12442 | -0.07774 |
| 5  | -0.11068 | -0.10142 | 27 | -0.12431 | -0.07618 |
| 6  | -0.11236 | -0.10117 | 28 | -0.12388 | -0.07443 |
| 7  | -0.11416 | -0.10117 | 29 | -0.12366 | -0.07284 |
| 8  | -0.11635 | -0.10114 | 30 | -0.12345 | -0.07127 |
| 9  | -0.1191  | -0.10173 | 31 | -0.12322 | -0.06971 |
| 10 | -0.11971 | -0.10044 | 32 | -0.1229 | -0.06812 |
| 11 | -0.12165 | -0.10028 | 33 | -0.12255 | -0.06654 |
| 12 | -0.12256 | -0.09925 | 34 | -0.12226 | -0.06501 |
| 13 | -0.1231  | -0.09792 | 35 | -0.12201 | -0.06351 |
| 14 | -0.12378 | -0.0967  | 36 | -0.12172 | -0.06202 |
| 15 | -0.12412 | -0.09524 | 37 | -0.12141 | -0.06053 |
| 16 | -0.12471 | -0.09398 | 38 | -0.12118 | -0.0591 |
| 17 | -0.12454 | -0.09216 | 39 | -0.12088 | -0.05766 |
| 18 | -0.12503 | -0.09084 | 40 | -0.12064 | -0.05625 |
| 19 | -0.12541 | -0.08946 | 41 | -0.12038 | -0.05486 |
| 20 | -0.1253  | -0.08773 | 42 | -0.12015 | -0.05349 |
| 21 | -0.12503 | -0.08593 | 43 | -0.11991 | -0.05214 |
| 22 | -0.12496 | -0.08429 | 44 | -0.11971 | -0.05081 |
| 23 | -0.12506 | -0.08277 | 45 | -0.11951 | -0.0495 |

5. Results and discussion

For two adjacent foundations, the analysis presented in this study showed that the bearing capacity of soil for an individual foundation is affected by a quantity called the efficiency factor, when a neighboring foundation is at a distance located within the failure zone of the first foundation, and therefore the impact factor will be reflected on both foundations.

In equations (14), (26) and (43), the efficiency factors’ values are found for surcharge, cohesion and weight respectively. Where these values have been affected by changing of width of foundations (B), angle of friction (\( \phi \)) and clear spacing between foundations (S). As shown in figures 9, 10 and 11, the interference of foundations produces an increase in the efficiency factors \( \xi_q, \xi_c \) and \( \xi_y \). This increase begins to rise concavely with the onset of the interfering, and then turns into a convex shape as the spacing (S) get decrease.

![Figure 9. Theoretical values of efficiency factor \( \xi_q \).](image-url)
5.1. Comparison of the efficiency factor with theoretical results
From the present analysis, the obtained variation of $\xi_y$ for the adjacent foundations with S/B was compared with the available theoretical solutions provided by Stuart 1962 on the basis of the limit equilibrium approach [6], Kumar and Ghosh 2007 by employing the method of stress characteristics [13], Kumar and Ghosh 2007 by using the upper bound finite elements limit analysis [9], and Kumar and Bhattacharya 2011 by using the lower bound finite elements limit analysis [10]. The comparison of the results has been made for $\phi = 35^\circ$ and provide in figure 11 for the rough base foundations.
Figure 12. A comparison of $\xi_y$ with the other available theoretical studies.

It is observed from the figure 11 that there is a variation in the efficiency factor values for the studies referred to, this variation can be attributed due the nature of the assumed collapse mechanism and the method of calculation in these studies. Also, it can be noted that as compared with all the available results in literature, for $S/B = 1.5$, the value of the efficiency factor $\xi_y$ was lower than the value produced by the Stuart analysis, while it was almost identical to the value produced by the Kumar and Ghosh analysis, mechanism1 (method of stress characteristics) [13], however it is higher than the values produced by the rest of the available theoretical analyses.

5.2. Comparison of the efficiency factor with experimental data

The present $\xi_y$ values for rough foundation are also compared with the experimental data reported by Stuart 1962 [6], Das and Larbi-Cherif 1983 [14], Khing et al. 1992 [15] and Ghosh et al. 2015 [16]. For all the chosen experimental data, the value of $\theta$ was found to vary between 35° and 40.3°. For making the comparison, from the present analysis, the values of $\xi_y$ corresponding to $\theta = 35^\circ$ and $\theta = 40^\circ$ were used. The comparison of all the results is provided in figure 12. It can be clearly observed, that the values provided by the Das and Larbi-Cherif experiment for a strip foundation with the angle of friction $\theta = 35^\circ$, represent the highest values obtained experimentally. The values provided by the Stuart experiment for a strip foundation with the same value of the friction angle that was used in the Das and Larbi-Cherif experiment, was less than its predecessor over the various values of $S/B$, however, it corresponds roughly to it at a value of $S/B$ equal to 1.5. The values provided by this analytical study for a strip foundation and friction angle $\theta = 35^\circ$ are less than the previous ones. With regard to other experiments, it is worth noting that the values of $\xi_y$ that were produced were close to the values of this study, except for the square foundations experiment of Ghosh et al., where the values of $\xi_y$ produced less than the values provided by this study. Based on the above, it can be noted that the present analysis provides generally the values of $\xi_y$ close to the lowest ones as compared with the reported experimental data. Thus, the results presented by this theoretical analysis can be considered practically acceptable, with desirability indicating its right, when was used for designing purposes.
6. Conclusion
The interference effect of two strip foundations placed close to each other at the depth $D_f$ from the surface of a semi-infinite homogeneous soil on the ultimate bearing capacity has been determined by using the limit equilibrium method, the following could be concluded according to this study:

1. The result of this analysis contributes to the production of three equations to find the values of the efficiency factors for the surcharge ($\xi_Q$), cohesion ($\xi_C$), and weight ($\xi_Y$).
2. As it appears in these equations that the variables that play a major role in influencing the values of the efficiency factors are the width of foundations, the spacing between foundations and the angle of friction.
3. This study showed an increase in efficiency factors with a decrease in the distance between the two foundations, in addition to that, there is an increase occurring in these factors due to an increase in the friction angle of the soil, on the other hand, especially in relatively close distances. The amount of increasing in efficiency factors will be less with increase the width of the foundation and this is due to the fading a part of interference effect as a result of increasing the failure envelope area with increasing the width of the foundation.
4. The variation in width of the two adjacent foundations shows an effect clearly on the value of $\xi_Y$, while it has no effect on the values of $\xi_Q$ and $\xi_C$. As increasing the width of one of the two adjacent foundations leads to an increase in the value of $\xi_Y$ for the other.
5. In comparison with the available theories and previous experiences of this problem, this analysis showed good results, and it can be adopted.

7. References
[1] M. Y. Fattah, M. F. Aswad, and M. M. Mahmood, “Evaluation Of The Method Of Stress Characteristics For Estimation Of The Soil Bearing Capacity,” Eng&Technology, vol. 26, no. 10, pp. 1184–1197, 2008.
[2] R. R. Shakir, “Undrained Bearing Capacity For Strip Footing On Two Layered Elastic-Plastic Clay By Using FEM,” Eng&Technology, vol. 24, no. 6, pp. 651–669, 2005.
[3] R. R. Shakir, “Probabilistic-based analysis of a shallow square footing using Monte Carlo simulation,” Eng. Sci. Technol. an Int. J., vol. 22, no. 1, pp. 313–333, 2019.
[4] R. R. Shakir and A. A. Altaweel, “Yield of Different Methods of Analytic Studies for Interfering of Shallow Foundations,” in AIP Conference Proceedings 2292, 2020.

Figure 13. A comparison of $\xi_Y$ with the other available experimental studies.
[5] M. Y. Fattah, K. T. Shlash, and H. A. Mohammed, “Bearing Capacity of Rectangular Footing on Sandy Soil Bounded by a Wall,” Arab. J. Sci. Eng., vol. 39, no. 11, pp. 7621–7633, 2014.
[6] J. G. Stuart, “Interference between foundations, with special reference to surface footings in sand,” Geotechnique, vol. 12, no. 1, pp. 15–22, 1962.
[7] J. Graham, G. P. Raymond, and A. Suppiah, “Bearing capacity of three closely-spaced footings on sand,” Geotechnique, vol. 34, no. 2, pp. 173–181, 1984.
[8] D. V Griffiths, G. A. Fenton, and N. Manoharan, “Undrained Bearing Capacity of Two-Strip Footings on Spatially Random Soil,” International Journal of Geomechanics, vol. 6, no. December. pp. 421–427, 2006.
[9] J. Kumar and P. Ghosh, “Upper bound limit analysis for finding interference effect of two nearby strip footings on sand,” Geotechnical and Geological Engineering, vol. 25, no. 5. pp. 499–507, 2007.
[10] P. Ghosh and S. Rukmini Kumar, “Interference effect of two nearby strip surface footings on cohesionless layered soil,” Int. J. Geotech. Eng., vol. 5, no. 1, pp. 87–94, 2011.
[11] K. Terzaghi, “Theoretical Soil Mechanics,” John Wiley and Son, New Yourk and London. 1943.
[12] W. A. Hijab, “A note on the centroid of a logarithmic spiral sector,” Geotechnique, vol. 6, no. 2, pp. 66–69, 1956.
[13] J. Kumar and P. Ghosh, “Ultimate bearing capacity of two interfering rough strip footings,” Int. J. Geomech., vol. 7, no. 1, pp. 53–62, 2007.
[14] B. Das and S. Larbi-Cherif, “Bearing capacity of two closely-spaced shallow foundations on sand,” Soils and Foundations, vol. 23, no. 1. pp. 1–7, 1983.
[15] K. H. Khing, B. M. Das, S. C. Yen, V. K. Puri, and E. E. Cook, “Interference effect of two closely-spaced shallow strip foundations on geogrid-reinforced sand,” Geotech. Geol. Eng., vol. 10, no. 4, pp. 257–271, 1992.
[16] P. Ghosh, P. K. Basudhar, V. Srinivasan, and K. Kunal, “Experimental studies on interference of two angular footings resting on surface of two-layer cohesionless soil deposit,” Int. J. Geotech. Eng., vol. 9, no. 4, pp. 422–433, 2015.