A Learning-Theoretic Framework for Certified Auditing of Machine Learning Models

Chhavi Yadav  
University of California San Diego  
cyadav@ucsd.edu

Michal Moshkovitz  
Tel Aviv University  
michal.moshkovitz@mail.huji.ac.il

Kamalika Chaudhuri  
University of California San Diego  
kamalika@eng.ucsd.edu

Abstract

Responsible use of machine learning requires that models be audited for undesirable properties. However, how to do principled auditing in a general setting has remained ill-understood. In this paper, we propose a formal learning-theoretic framework for auditing. We propose algorithms for auditing linear classifiers for feature sensitivity using label queries as well as different kinds of explanations, and provide performance guarantees. Our results illustrate that while counterfactual explanations can be extremely helpful for auditing, anchor explanations may not be as beneficial in the worst case.

1 Introduction

The recent success of machine learning (ML) has opened up new avenues for potential use in many high-stakes applications. For example, [5] uses ML based detectors for self-driving; [1] discusses ML models to help judges, and [7] discusses ML image systems to help doctors make decisions. However, this kind of wide-spread use requires us to be able to audit them extensively and verify that they possess certain desirable properties.

However, due to complexities in the pipeline and the black box nature of most machine learning models, auditing properties of machine learning models has been mostly performed in a fairly ad-hoc manner. In this work, we seek to address this limitation by proposing a formal learning-theoretic framework for auditing properties of machine learning models. Our framework formalizes auditing as a conversation between two entities – a data scientist, who holds a model, and an auditor who seeks to audit a specific property of the model. Due to confidentiality reasons, the auditor does not have direct access to the model, and can only access it through querying specific data points to the data scientist. The auditor’s goal is to determine if the model has some property using as few queries as possible, where the responses to a query may involve both labels and explanations.

To formalize our framework, we first propose a general formal definition for a good auditor that characterizes success in section 2. We then consider a general auditing property, and provide a fallback auditing strategy through drawing a connection between auditing in our framework and membership query active learning. This provides a baseline on the number of queries for auditing.

Next, in section 3 we exemplify our framework by considering a particular use-case: auditing if a model is sensitive to a specific feature. We consider two hypothesis classes – linear classifiers and extended thresholds – and two kinds of explanations – counterfactual and anchors – to be returned by the data scientist. For anchor explanations, we propose a novel algorithm called Active Augmentation that incorporates these explanations into the active learning process for faster learning and provide
performance guarantees. For counterfactual explanations, we provide algorithms for auditing that use a single query (for linear classifiers) and two queries (for extended thresholds). Our results illustrate that the two explanation methods differ greatly in their usefulness in auditing – while worst-case anchors do not provide any more information than labels, counterfactuals greatly bring down the number of queries to one or two.

In section 4 we validate our algorithms through experiments on two real datasets. We demonstrate a simple way of auditing neural networks by modeling them as extended thresholds. Other experiments show that while worst-case anchors do not provide any more information than labels, typical anchors do, thereby reducing the queries needed to audit. Our results indicate that our proposed active augmentation technique does help in auditing.

Lastly, in section 5 we discuss ways of dealing with an untruthful data scientist. We propose various types of lies the data scientist can tell, propose simple ways to verify explanations and strategies for the auditor when it catches the lying data scientist.

1.1 Related Work

Auditing has been used to uncover some undesirable behavior in ML models in the past. For instance, different kinds of biases were discovered by [6] in commercial facial recognition models, by [16] in automated speech recognition tools and by [24, 23] in automatic captions on Youtube. However, the aforementioned papers were based on ad-hoc dataset creation. Recently, there has been some effort towards streamlining and developing a structured process for auditing. For instance, [12] and [19] introduce datasheets for datasets and model cards and [20] introduce an end-to-end internal auditing process. However, these papers are on the engineering end. In contrast, we propose a formal theoretical framework which allows us to determine what kind of properties can be audited and under what conditions.

In the literature with performance guarantees, [13] considers auditing the accuracy of ML models. It also uses an interactive protocol between the verifier and the prover like our work and is related to a similar notion of property testing introduced by [4]. However, our paper is more general from these in the following sense – it formalizes auditing for general auditing tasks (not just accuracy) and also demonstrates the utility of explanations in auditing.

Specific statistical methods have been proposed by [15] to audit privacy and by [17, 14] to audit data deletion. These are properties of the learning algorithm rather than the model. In our paper we focus on the latter and hence our algorithms are not applicable to auditing privacy and data deletion. Extending our framework to broader settings is an important direction of future work.

Our work is also related to the literature on explanations, which provide a reason why for a specific prediction was made by the model for an input. Specifically, local explanations describe the behavior of the model around the input locally. Multiple local explanation methods exist in the literature like anchors [22], counterfactuals [25] and LIME [21] among others. In our proposed framework, data scientist responds with local explanations along with labels. [9] propose a method for learning a class of functions in an online fashion using explanations provided by humans. Even though we too use explanations, our algorithm for using explanations and our use-case is different from theirs, as explained in sections 2.4 and 5.

Another related notion is Active Learning which is discussed in detail in section 2.4.

2 Framework

Informally, an audit is an examination of an algorithm or model to check for consistency with specified norms or policies. To ground auditing for ML, we introduce the main components involved in the process, specifically: 1) the model/hypothesis to be audited \( h \), 2) the auditing property (AP) aka the norm/policy, 3) the Data Scientist (DS), aka the creator of the model \( h \) and has complete access to it and 4) the Auditor, aka the one who audits the model \( h \) for AP and has query access to \( h \) through the DS. Next we introduce notation and define these components mathematically.
2.1 Setup

Let $\mathcal{X} \subseteq \mathbb{R}^d$ be the set of instances and $\mathcal{Y}$ be the set of labels. Let $\mathcal{H}$ be the hypothesis class, which consists of hypotheses $h : \mathcal{X} \rightarrow \mathcal{Y}$. $\mathcal{H}$ is known to both the DS and auditor. We assume the model to be audited belongs to the hypothesis class, i.e. $h \in \mathcal{H}$. The DS has white box access to $h$, while auditor only has query access to it due to confidentiality reasons. Let $Z \subset \mathcal{X} \times \mathcal{Y}$ be the set of labeled examples such that $\forall z \in Z, z = (x, h(x))$.

When a query $x \in \mathcal{X}$ is asked to the DS, it returns the corresponding label and the local explanation. We refer to local explanation as explanation henceforth. Explanation provides information about behavior of the model locally as discussed in section 1.1. We consider two types of explanation methods in our paper: Counterfactuals \cite{25} and Anchors \cite{22}, defined formally as follows.

**Definition 1 (Counterfactual Explanations).** Given an input $x$, counterfactual explanations return the closest instance $x'$ that has a different label than $x$, i.e. $x' = \arg\min_{x': h(x') \neq h(x)} d(x, x')$ where $d(\cdot, \cdot)$ is a distance function and hypothesis $h \in \mathcal{H}$. In our case, the distance function is the l$_2$-norm, $d(x, x') = \|x - x'\|_2$.

**Definition 2 (Anchor Explanations).** Given an instance $x$ from a distribution $\mathcal{D}$, anchor explanation returns a hyperrectangle $A_x$ such that it contains $x$ and the points in the hyperrectangle have the same label as $x$ most of the times \cite{22,10}. Precision parameter $\tau$ measures quality of the anchor explanation and is given by the probability that a point sampled from $A_x$ according to $\mathcal{D}$ has the same label as $x$, $\tau = \Pr_{x' \sim \mathcal{D}}(h(x) = h(x'))$ for hypothesis $h \in \mathcal{H}$. Coverage parameter $c$ corresponds to the probability that a point sampled according to $\mathcal{D}$ lies in the $A_x$, $c = \Pr_{x' \sim \mathcal{D}}(x' \in A_x)$.

Since there exist multiple types of explanation methods, both DS and auditor agree upon one before the auditing process begins. Let $e_h : \mathcal{X} \rightarrow \mathcal{E}$ denote this explanation method where $\mathcal{E}$ represents the codomain of $e_h$ and $h \in \mathcal{H}$. For example, for counterfactuals, $\mathcal{E} = \mathcal{X}$.

We define auditing property (AP) using a continuous score function, $s : \mathcal{H} \rightarrow [0, 1]$. $s(\cdot)$ captures the extent to which the model “follows” the AP. $s(\cdot) = 0$ when AP is not followed by the model. The exact form of $s(\cdot)$ changes according to the AP. Auditor knows the score function but does not know its value for $h$. For example, consider “unfairness” to be the AP, $s(\cdot)$ is the l$_2$-norm, $d(x, x') = \|x - x'\|_2$. Auditor knows the score function but does not know its value for $h$.

We assume that the DS is truthful, meaning the label and explanation returned by the DS for $x$ are $y = h(x)$ and $e_h(x)$ respectively. In section 5 we discuss how to deal with an untruthful DS.

2.2 Auditing Framework

In this section, we conceptualize the auditing process as an interaction between the DS and auditor. Our protocol is as follows.

At each time step $t = 1, 2, \ldots$

- Auditor picks a new query $x_t \in \mathcal{X}$ and supplies it to the DS.
- DS returns a label $y_t \in \mathcal{Y}$ and an explanation $e_t \in \mathcal{E}$ to the auditor.
- Auditor decides whether or not to stop. If auditor decides to stop, it returns a decision $Y_a \in \{\text{Yes, No}\}$, otherwise it continues to the next time step.

Note that while in the proposed framework DS returns a label and an explanation, depending upon the auditing property, these can be replaced with suitable entities.

At the end of the auditing process, auditor responds with an answer denoted by $Y_a$, which takes values $\{\text{Yes, No}\}$. It is a random variable since both the DS and auditor can be random. Next we formally define a good auditor.

**Definition 3 (($\epsilon, \delta$)-auditor).** Auditor is an ($\epsilon, \delta$)-auditor for $\epsilon, \delta \in [0, 1]$, hypothesis class $\mathcal{H}$ and a score function $s(\cdot)$ if $\forall h \in \mathcal{H}$ the following conditions hold: 1) if $s(h) > \epsilon$, $\Pr(Y_a = \text{Yes}) \geq 1 - \delta$ and 2) if $s(h) = 0$, $\Pr(Y_a = \text{No}) = 1$. 3
According to this definition, a successful auditor should return a Yes with high probability when AP is followed by the model to a large extent and it should only say No when AP is not followed at all. When $0 < s(\bar{h}) \leq \epsilon$, we cannot place a guarantee on the auditor’s decision with a finite query budget.

Through the mathematical formulation of a good auditor and the definition of the score function, we can only audit properties that depend on the hypothesis like accuracy, fairness and so on. We plan to explore properties related to the learning algorithm in future work.

Query complexity $T$ of the framework is the total number of queries auditor asks DS before stopping. $T$ should be small for efficiency. Ideally, auditor should be efficient in both time and space along with being an $(\epsilon, \delta)$-auditor. In this paper we limit our focus to designing $(\epsilon, \delta)$-auditors. Next we discuss one such general auditor.

2.3 A General Auditor

Auditor\footnote{We use “auditor” and “an algorithm for the auditor” interchangeably to mean the auditor.} consists of two parts – strategy to pick next query and the stopping condition. Amongst the many, we describe some general ways to do both in this section.

Picking next query. At time $t$, auditor maintains a candidate search space $S_t$ over hypotheses\footnote{Search space does not have to be over hypotheses. As an example, for feature sensitivity auditing discussed in \sref{search-space}, search space can be over responsive pairs (defined later) or both the hypotheses and responsive pairs.} $\mathcal{H}$. Starting with $S_0 = \mathcal{H}$, the search space is narrowed down through queries made to the DS at each subsequent time step. Auditor chooses such a point to query that reduces the search space the most, no matter what explanation and label are returned by the DS. Formally, $x_t = \arg\max_{x \in \mathcal{X}} (\min_{x \in \mathcal{X}} |S_t - |S_{t+1}|| = \arg\max_{x \in \mathcal{X}} \text{val}_t$ where $|S_t|$ and $|S_{t+1}|$ denote the volume of the search spaces $S_t$ and $S_{t+1}$ and value$_t$ corresponds to the search space reduction by worst case label and explanation for a point $x$. Note that this is an inefficient algorithm, since the next query is searched over all the remaining points in $\mathcal{X}$ at each time step.

Let $Z_{t-1} \subseteq Z$ and $E_{t-1} \subseteq \mathcal{X} \times \mathcal{E}$ be the set of labeled examples and explanations acquired from the DS till $t-1$. We define the search space at $t$ to be the subset of hypotheses $\mathcal{H} \subseteq H$ that are consistent with all the labels and explanations in $Z_{t-1}$ and $E_{t-1}$ respectively. As an example for counterfactual explanations, the hypothesis $\bar{h}$ is consistent if $\forall (x, h(x)) \in Z_{t-1}$ and their counterfactuals $(x, x' = e_{\bar{h}}(x)) \in E_{t-1}$, $h(x) = \bar{h}(x)$ and $h(x') = \bar{h}(x')$.

Stopping. A simple stopping condition is when the auditor exhausts the query budget assigned to it. Query budget can be set to the number of queries enough to satisfy $3$ using the aforementioned query picking strategy for a fixed $\epsilon$, $\delta$ and hypothesis class $\mathcal{H}$. Another stopping condition is when for all hypotheses $\bar{h}$ in the search space, $s(\bar{h}) = 0$ or for all hypotheses $\bar{h}$ in the search space, $s(\bar{h}) > \epsilon$ or for all hypotheses $\bar{h}$ in the search space, $0 < s(\bar{h}) \leq \epsilon$. The auditor can immediately return a decision in this case.

Outline for the general auditor can be found in appendix alg\ref{alg:general-auditor}.

2.4 Similarities and Differences from Active Learning

Our framework with the auditor in section\ref{search-space},\ref{general-auditor} seems very closely connected to active learning.

The aim of active learning\footnote{Start of active learning where the learner synthesizes queries rather than sampling from the data distribution or selecting from a pool. Our framework is essentially partial learning in the MQAL setting where the oracle returns explanations in addition to labels. Here, the DS, assumed to be truthful, is the oracle and auditor is the learner.} is to learn a hypothesis in an interactive manner by selecting highly informative unlabeled data points for label queries. A variant of active learning is Membership Query Active Learning (MQAL)\footnote{Neither this nor any of the remaining footnotes are referenced in the text.} where the learner synthesizes queries rather than sampling from the data distribution or selecting from a pool. Our framework is essentially partial learning in the MQAL setting where the oracle returns explanations in addition to labels. Here, the DS, assumed to be truthful, is the oracle and auditor is the learner.

To formalize the connection, we observe that learning is a harder task than auditing and if there exists an algorithm that can learn a hypothesis, it can also audit it. Hence, an active learning algorithm can be used as a fallback auditing algorithm.

**Theorem 2.1.** If there exists a membership query active learner that can learn $\bar{h}$ exactly in $T$ queries, then the active learner can also audit in $T$ queries.
Proof can be found in the appendix section A.2. Note that this connection assumes that the score function can be calculated efficiently.

The key differences between active learning and our auditing framework are – 1) unlike active learning, partial learning of the hypothesis can be sufficient for auditing and 2) usage of explanations. For example, if all hypotheses \( h \) in the search space have \( s(h) = 0 \), the auditor can return a decision without learning \( h \) exactly; or learning only some parameters of \( h \) may be enough to audit as shown in section 3.2.1. Since partial learning can be sufficient for auditing, it also means that an auditing algorithm is not necessarily an active learner.

3 Auditing Feature Sensitivity

We introduce a specific instantiation of our framework for auditing ‘feature sensitivity’. If flipping a feature of an input leads to different prediction, we call it a sensitive feature for the model. Identifying such features leads to insights into the model’s working, helps in debugging, uncovers spurious correlations and harmful biases that it may have learnt.

3.1 Setup and Preliminaries

Let us denote the model to be audited by \( \hat{h} \). Input is \( x \in \mathcal{X}^d \) and output is \( y \in \{-1, 1\} \). \( x_i \) denotes the \( i^{th} \) feature of input \( x \). Next we introduce some definitions to help with the rest of the discussion.

**Feature of Interest** (FoI) is one of the input features that the auditor is auditing for. Two instances \( x^1, x^2 \in \mathcal{X} \) form a pair \( p_{ij} \) if for every feature \( k \) besides FoI \( s \), it holds that \( x^1_k = x^2_k \) and for the FoI, \( x^1_i \neq x^2_i \). For example, consider \( d = 2 \), we are auditing for our FoI \( x_2 \). Then \( (0.5, 1) \) and \( (0.5, 0) \) form a pair as they are equal in the first dimension but differ in the FoI. We call pair \( p_{ij} \) a responsive pair for \( h \in \mathcal{H} \), if \( h(x_i) \neq h(x_j) \). Unless mentioned, responsive pairs are with respect to \( \hat{h} \). For instance, if \( \hat{h}((0.5, 1)) = 1 \) and \( \hat{h}((0.5, 0)) = 0 \), then \( (0.5, 1) \) and \( (0.5, 0) \) form a responsive pair. Lastly, the FoI is a sensitive feature for hypothesis \( h \) if a responsive pair exists with respect to the FoI. For example, if the above responsive pairs existed, we would say that \( x_2 \) is a sensitive feature for \( \hat{h} \).

The Auditing Property (AP) is defined as the sensitivity of \( \hat{h} \) to the FoI, or equivalently, existence of responsive pairs with respect to the FoI. Let the set of all pairs be denoted by \( \mathcal{P} \). Then score function is the probability that a randomly drawn pair from \( \mathcal{P} \) is a responsive pair according to \( h \), that is \( s(h) = \Pr_{p_{ij} \sim \mathcal{P}}(p_{ij} \text{ is a responsive pair}) \). This score function can be interpreted as the fraction of responsive pairs. To be an \((\epsilon, \delta)\)-auditor, auditor should detect the existence of more than \( \epsilon \) fraction of responsive pairs with high probability and when the fraction is zero with probability one.

**A Simple Baseline: Random Testing.** A simple way to detect feature sensitivity is to query a large number of pairs at random from the DS. If any of them is a responsive pair, auditor knows that the feature is sensitive. The following theorem states a bound on the query complexity for this algorithm.

**Theorem 3.1.** For \( \epsilon, \delta \in (0, 1) \), auditor is an \((\epsilon, \delta)\)-auditor if it queries \( m \geq \frac{1}{\epsilon} \log \frac{1}{\delta} \) pairs.

Proof can be found in the appendix section A.3. Note that this baseline is independent of the hypothesis class. It does not learn \( \hat{h} \) (even partially) in order to audit or use explanations either. However, a lot of pairs have to be queried for a small \( \epsilon \) since the structure of the class is not exploited. Next we discuss cases where using explanations and exploiting the structure of hypothesis class achieves faster auditing.

Table 1: Query Complexity of auditing algorithms for linear classifiers

| Algorithm | Query Complexity |
|-----------|------------------|
| Baseline | \( O(\frac{1}{\epsilon} \log \frac{1}{\delta}) \) |
| AlgLC_{c} | 1 |
| AlgLC_{n} | \( O(d\log(\frac{2e}{\delta})) \) |

\(^1\)For some hypothesis classes learning the hypothesis class might be hard. In such cases auditing using baseline might be a better option than auditing through learning \( h \).
3.2 Linear Classifiers

In this section, we audit *Linear Classifiers* for feature sensitivity using different explanation methods. *Linear Classifiers* are defined as follows.

\[
\mathcal{H}_{LC} = \{ h_{w,b} \}_{w \in \mathbb{R}^d, b \in \mathbb{R}}
\]

\[
h_{w,b}(x) = \text{sign}(\langle w, x \rangle + b)
\]  

(1)

Note that responsive pairs exist for \( h \in \mathcal{H}_{LC} \) with respect to a FoI \( x_i \), if and only if \( w_i \) is non-zero.

3.2.1 Counterfactual Explanations

Recall that the counterfactual explanation \( x' \) for an input \( x \) is the closest point to \( x \) that has a different label. For linear classifiers, we observe that the difference \( x - x' \) is parallel to \( w \). The auditor tests if the FoI in \( x - x' \) is zero. This observation can be used to design an algorithm that uses a single query denoted by \( \text{AlgLC}_c \).  

**Theorem 3.2.** For any \( \epsilon \in [0, 1] \), auditor \( \text{AlgLC}_c \) is an \((\epsilon, 0)\)-auditor for \( \mathcal{H}_{LC} \) and score function \( s(\cdot) \) with \( T = 1 \) query.

Proof for the theorem can be found in the appendix section [A.5](#).

3.2.2 Anchor Explanations

We consider homogeneous linear classifiers \( (b = 0) \) in this section; however our techniques can be easily extended to non-homogeneous linear classifiers by considering \( d + 1 \) dimensions and concatenating \( 1 \) to \( x \) and \( b \) to \( w \). For simplicity, we assume our anchor explanations have perfect precision, \( \tau = 1 \) and coverage \( c = c_0 \) where \( c_0 \) is a scalar.

[2](#) proposed a spectral MQAL algorithm for actively learning a linear classifier; however, this algorithm is not capable of using explanations. We propose a way to incorporate anchor explanations into this algorithm through a procedure that we call *Active Augmentation*. The main idea is that anchor explanations give us a region of space around a point \( x \) where the labels are the same; we use this fact to generate more synthetic labeled examples (without actually querying the DS) that can be fed into the active learning algorithm. Through these observations we can come up with an auditing algorithm for linear classifiers using anchors, denoted by \( \text{AlgLC}_a \).

In the worst-case, anchor explanations for \( x \) are degenerate rectangles of the form \( \{ \lambda x \}_{\lambda \in \mathbb{R}^+} \) and do not provide any extra information than the label itself. Then the query complexity of auditing is the same as that of active-learning-without-explanations, as given by [2](#) and presented next.

**Theorem 3.3.** For every dimension \( d \), there exists \( c > 0 \) such that for any \( \epsilon \in (0, 1) \), auditor \( \text{AlgLC}_a \) is an \((\epsilon, 0)\)-auditor for \( \mathcal{H}_{LC} \) and score function \( s(\cdot) \) with \( T = O(d \log \frac{2d}{\epsilon}) \) queries.

Proof follows from the fact that the score function can be approximated by the weight of the FoI. If \( h \) is well approximated, then as a result, the score function can be estimated well too. More details can be found in appendix section [A.6](#). Typically, anchors are not worst-case and can lead to faster auditing, as demonstrated empirically in sec. [4.1](#).

3.3 Extended Thresholds

In this section, we audit *Extended Thresholds* for feature sensitivity. Intuitively, these are multi-threshold models, where the threshold to apply can change with one of the input features. Consider for example a biased loan granting model that grants loans based on different salary thresholds for male vs. females.

*Extended Thresholds* are defined mathematically as follows. Let input \( x = (x', g) \) where \( x' \in \mathbb{R}^d \), \( g \in \{0, 1\} \) and let \( l \leq u \) be two scalars.

\[
\mathcal{H}_{ET} = \{ h_{f,\theta_1,\theta_2} \}_{\theta_1,\theta_2 \in [l,u], f: \mathbb{R}^d \to \mathbb{R}}
\]

\[
h_{\theta_1,\theta_2,f}(x, g) = \begin{cases} 
+1 & \text{if } (1-g)\theta_1 + g\theta_2 \leq f(x') \\
-1 & \text{otherwise.}
\end{cases}
\]  

(2)
Here \( g \) is the feature to be audited for. If \( \theta_1 \neq \theta_2 \), then \( g \) is used to make a prediction, qualifying as a sensitive feature. Therefore, responsive pairs exist only when \( \theta_1 \neq \theta_2 \), in between \( \theta_1 \) and \( \theta_2 \). The auditing task boils down to finding if \( \theta_1 = \theta_2 \) and if \( \theta_1 \neq \theta_2 \), then what is \( |\theta_1 - \theta_2| \).

Note that \( f \) can be a neural network as well.

In appendix section A.7 we show that extended thresholds are a special case of 2D linear classifiers. Due to this reduction, auditors proposed for linear classifiers should apply to extended thresholds as well. However, the explanations considered in this case can be unrealistic as \( g \) is not limited to \{0, 1\}. Therefore we propose auditors AlgET\(_c\)\(^6\) and AlgET\(_a\)\(^7\) for counterfactuals and anchors respectively where the explanations are realistic. AlgET\(_c\) has a query complexity of two. AlgET\(_a\) is basically binary search and has a query complexity of \( \lceil 2 \log(\frac{1}{\epsilon}) \rceil \). As before, worst-case anchors do not help. A detailed treatment of this section can be found in the appendix A.8 and A.9.

### 4 Experiments

In this section we conduct experiments on real-world datasets to test some aspects of feature sensitivity auditing. Specifically, we answer the following questions:

1. Does active augmentation of typical anchors reduce query complexity for linear classifiers?
2. What is the average query complexity for auditing extended thresholds?

We use two datasets for our experiments - Adult Income\(^4\) and Covertype\(^5\). The FoI are gender and wilderness area type for Adult Income and Covertype respectively. Details about the datasets can be found in the appendix section A.10.1.

#### 4.1 Active Augmentation of Typical Anchors

As discussed in section 3.2.2, active augmentation of worst-case anchors does not help in reducing the query complexity of auditing. A natural question then is - do typical anchors help? Since auditing with anchors means learning the DS’s model for our algorithm, reduction in query complexity of learning implies reduction in that of auditing. Hence, we check experimentally if faster learning is achieved through active augmentation of typical anchor points.

**Methodology** We learn a linear classifier with weights \( w \) for each dataset; these correspond to the DS’s model. Then the weights are estimated during auditing using alg. \( XL \). We consider two different augmentations 1) worst-case anchor points which is equivalent to not using anchors 2) typical anchor points. We set the augmentation size (number of anchor points augmented) to a maximum of 30. Our anchors are hyperrectangles of a fixed volume surrounding the query point. We sample points with the same label as the query point from this hyperrectangle and augment to the set of queries in the typical case. Note that we have relaxed the \( \tau = 1 \) assumption here.

**Results** The results are shown in Figure 1. We see that the active augmentation of typical anchors drastically reduces the query complexity to achieve the same estimation error between weights as compared to not using anchors (or equivalently using worst-case anchors). This saving directly translates to efficiency in auditing. For example, in the adults dataset, less than 50% of the queries are needed to achieve a lower error with typical anchors than without them. This illustrates that anchor explanations can be helpful for auditing, suggesting an application for these explanations.

#### 4.2 Average Query Complexity of Extended Thresholds

The main question we investigate in this section is - how many queries are needed on average for auditing extended thresholds? The number of queries changes based on the type of explanation method, difference between thresholds, and numeric value of the thresholds. Value of thresholds affects query complexity for anchor explanations since alg. \( XL \) is a binary search algorithm.

\(^4\)https://archive.ics.uci.edu/ml/datasets/adult
\(^5\)https://archive.ics.uci.edu/ml/datasets/covertype
Methodology

We first propose a way to use NNs in the extended thresholds formalism. Next we utilize one of the learned NNs to demonstrate how the average query complexity changes with difference in thresholds.

Given input $x = (x', g)$, we learn a feedforward NN to predict one of the features in $x'$ using the remainder features of $x'$. This NN corresponds to the function $f$ in eq. 2. The idea of predicting one feature from the rest is inspired from Continuous-bag-of-words [18] in NLP, wherein the target word is predicted through its context. Next we learn different thresholds corresponding to $g$ on the aforementioned predicted feature. $g$ is set to the FoI. Fig. 3 in the appendix illustrates our methodology for learning extended thresholds with NNs.

For any NN $f$ learnt above, we randomly pick thresholds $\theta_1, \theta_2$ in the range of $f$, to be treated as the DS’s model. Next we use alg. 7 and 6 to audit and estimate $\theta_1, \theta_2$. We keep track of the number of steps needed to audit and average them over multiple runs. Note that here we are considering two dimensional features, one being $f(\cdot)$ and the other being the feature of interest.

Results

Appendix tables 2 and 3 display the learned thresholds and test accuracy for various predicted features. Fig. 2 displays average query complexity (AQC) results for the NN predicting education. For counterfactual explanations, AQC of auditing is fixed and equals two. For anchors, AQC decreases logarithmically with increasing difference in thresholds. This experiment confirms that the auditing problem is easier when the difference between the two thresholds is larger, i.e. a lot of responsive pairs exist.
5 Untruthful Data Scientists

A natural question to ask is what happens when a DS is not entirely truthful in the auditing process, as we assumed in Section 2.1. What kind of auditing is possible in this case?

Suppose the DS returns all labels and explanations from an entirely different model than \( \hat{h} \). In this case, there is no way for an auditor to detect it; but perhaps this can be dealt with in a procedural manner – like the DS hands its model to a trusted third party who answers the auditor’s queries.

What if the DS returns the correct labels but incorrect explanations? DS might be forced to return correct labels when the auditor has some labeled samples already and hence can catch the DS if it lies with labels. This scenario motivates verification of explanations. Next, we discuss schemes to verify anchors and counterfactuals for this scenario.

**Anchors.** For anchors, verification is possible if we have samples from the underlying data distribution \( D \). For a query \( x \), DS returns an unverified anchor \( A_x \) with precision \( \tau_x \) and coverage \( c_x \). The correctness of \( \tau_x \) can be detected as follows. First, get an estimate for the true value of the precision parameter by sampling \( n \) points from anchor \( A_x \) according to \( D \). Let the true and estimated values of precision parameter be \( \tau \) and \( \hat{\tau} \) respectively. \( \hat{\tau} \) approaches \( \tau \) with sufficiently large number of points as mentioned in lemma 1. Second, compare \( \tau_x \) with \( \hat{\tau} \). A large difference between \( \tau_x \) and \( \hat{\tau} \) implies false anchors.

**Lemma 1.** For any \( \Delta > 0 \) and integer \( n \), \( \Pr(|\tau - \hat{\tau}| \geq \Delta) \leq 2 \exp^{-2\Delta^2 n} \).

Lemma 1 is immediate from Hoeffding’s Inequality. Notice that the number of samples \( n \) required to verify precision changes with \( 1/\Delta^2 \). If the fraction of responsive pairs \( \epsilon \) equals \( \Delta \), our baseline in section 3.1 can audit with \( O(1/\Delta) \) samples without using explanations. Hence, in adversarial conditions where the probability of a lying DS is high, it is better to audit with our explanations-free baseline than auditing with anchors and verifying them.

Since coverage is the probability that a point sampled from \( D \) belongs to \( A_x \), it can be easily checked 1) by calculating the volume of \( A_x \) when all dimensions of \( x \) are bounded or 2) by sampling points from \( D \) when the features are unbounded.

**Counterfactuals.** Given \( x \), let \( x' \) be the unverified counterfactual explanation returned by the DS. There are two aspects to a counterfactual explanation – its label which should be different from \( x \) and it should be the closest such point to \( x \). The first aspect can be easily verified by querying \( x' \) from the DS. For the second aspect, we observe that finding the counterfactual is equivalent to finding the closest adversarial point. Deriving from adversarial robustness literature, verifying the closeness aspect can be a computationally hard problem for some hypothesis classes like discussed in [26]. However, we propose a sampling based algorithm to estimate the true counterfactual, assuming that the DS is lying. Firstly, sample points from the ball \( B(x, d(x, x')) \). For \( x' \) to be the true counterfactual, all points within the ball \( B(x, d(x, x')) \) should have the same label as \( x \). If a point \( x'' \) with a different label is sampled, select this point as an estimate of the true counterfactual and repeat the scheme with the new ball \( B(x, d(x, x'')) \). By following this procedure iteratively, we get closer to the correct counterfactual explanation as the radius of the ball reduces at each iteration. There are cases where our algorithm may not work well. We leave designing better algorithms for verifying closeness to future work.

Upon verification, if auditor finds that the DS is untruthful, it can choose to 1) stop auditing and declare that the DS is lying, 2) audit with estimated explanations or 3) audit with explanations-free baseline algorithm (section 3.1) since option 2 is computationally intensive.

6 Conclusions and Future Work

To summarize, we propose a general learning-theoretic framework for certified auditing, which is a major requirement for responsible ML. We instantiate our framework through auditing feature sensitivity for linear classifiers and provide two auditing algorithms based on different explanations. We show that while counterfactual explanations greatly bring down the query complexity of auditing, anchor explanations help auditing in the typical case, but not in the worst case. Our results illustrate that auditing can be a use-case for some existing explanation methods. We also discuss how to deal with an untruthful data scientist.
We believe that this work is a first step towards understanding certified auditing and much needs to be done before we can design useful certified auditors for complex machine learning models and properties. Some fruitful directions include designing certified auditors for other auditing properties and hypothesis classes, as well as incorporating other kinds of explanations or different information into the auditing process.

Acknowledgements This work was supported by NSF under CNS 1804829 and ARO MURI W911NF2110317. MM has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation program (grant agreement No. 882396), by the Israel Science Foundation (grant number 993/17), Tel Aviv University Center for AI and Data Science (TAD), and the Yandex Initiative for Machine Learning at Tel Aviv University. CY thanks Geelon So for providing valuable feedback on an early draft of the paper.

References
[1] G. Aini et al. A summary of the research on the judicial application of artificial intelligence. Chinese Studies, 9(01):14, 2020.
[2] I. Alabdulmohsin, X. Gao, and X. Zhang. Efficient active learning of halfspaces via query synthesis. In Twenty-Ninth AAAI Conference on Artificial Intelligence, 2015.
[3] D. Angluin. Queries and concept learning. Machine learning, 2(4):319–342, 1988.
[4] M.-F. Balcan, E. Blais, A. Blum, and L. Yang. Active property testing. arXiv preprint arXiv:1111.0897, 2011.
[5] M. Bojarski, D. Del Testa, D. Dworakowski, B. Firner, B. Flepp, P. Goyal, L. D. Jackel, M. Monfort, U. Muller, J. Zhang, et al. End to end learning for self-driving cars. arXiv preprint arXiv:1604.07316, 2016.
[6] J. Buolamwini and T. Gebru. Gender shades: Intersectional accuracy disparities in commercial gender classification. In Conference on fairness, accountability and transparency, pages 77–91. PMLR, 2018.
[7] I. Castiglioni, L. Rundo, M. Codari, G. Di Leo, C. Salvatore, M. Interlenghi, F. Gallivanone, A. Cozzi, N. C. D’Amico, and F. Sardanelli. Ai applications to medical images: From machine learning to deep learning. Physica Medica, 83:9–24, 2021.
[8] S. Dasgupta. Coarse sample complexity bounds for active learning. Advances in neural information processing systems, 18, 2005.
[9] S. Dasgupta, A. Dey, N. Roberts, and S. Sabato. Learning from discriminative feature feedback. Advances in Neural Information Processing Systems, 31, 2018.
[10] S. Dasgupta, N. Frost, and M. Moshkovitz. Framework for evaluating faithfulness of local explanations. arXiv preprint arXiv:2202.00734, 2022.
[11] V. Feldman. On the power of membership queries in agnostic learning. The Journal of Machine Learning Research, 10:163–182, 2009.
[12] T. Gebru, J. Morgenstern, B. Vecchione, J. W. Vaughan, H. Wallach, H. Daumé III, and K. Crawford. Datasheets for datasets. arXiv preprint arXiv:1803.09010, 2018.
[13] S. Goldwasser, G. N. Rothblum, J. Shafer, and A. Yehudayoff. Interactive proofs for verifying machine learning. In 12th Innovations in Theoretical Computer Science Conference (ITCS 2021). Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2021.
[14] Y. Huang, X. Li, and K. Li. Ema: Auditing data removal from trained models. In International Conference on Medical Image Computing and Computer-Assisted Intervention, pages 793–803. Springer, 2021.
[15] M. Jagielski, J. Ullman, and A. Oprea. Auditing differentially private machine learning: How private is private sgd? Advances in Neural Information Processing Systems, 33:22205–22216, 2020.
[16] A. Koenecke, A. Nam, E. Lake, J. Nudell, M. Quartey, Z. Mengesha, C. Toups, J. R. Rickford, D. Jurafsky, and S. Goel. Racial disparities in automated speech recognition. *Proceedings of the National Academy of Sciences*, 117(14):7684–7689, 2020.

[17] X. Liu and S. A. Tsaftaris. Have you forgotten? a method to assess if machine learning models have forgotten data. In *International Conference on Medical Image Computing and Computer-Assisted Intervention*, pages 95–105. Springer, 2020.

[18] T. Mikolov, K. Chen, G. Corrado, and J. Dean. Efficient estimation of word representations in vector space. *arXiv preprint arXiv:1301.3781*, 2013.

[19] M. Mitchell, S. Wu, A. Zaldivar, P. Barnes, L. Vasserman, B. Hutchinson, E. Spitzer, I. D. Raji, and T. Gebru. Model cards for model reporting. In *Proceedings of the conference on fairness, accountability, and transparency*, pages 220–229, 2019.

[20] I. D. Raji, A. Smart, R. N. White, M. Mitchell, T. Gebru, B. Hutchinson, J. Smith-Loud, D. Theron, and P. Barnes. Closing the ai accountability gap: Defining an end-to-end framework for internal algorithmic auditing. In *Proceedings of the 2020 conference on fairness, accountability, and transparency*, pages 33–44, 2020.

[21] M. T. Ribeiro, S. Singh, and C. Guestrin. " why should i trust you?" explaining the predictions of any classifier. In *Proceedings of the 22nd ACM SIGKDD international conference on knowledge discovery and data mining*, pages 1135–1144, 2016.

[22] M. T. Ribeiro, S. Singh, and C. Guestrin. Anchors: High-precision model-agnostic explanations. In *Proceedings of the AAAI conference on artificial intelligence*, volume 32, 2018.

[23] R. Tatman. Gender and dialect bias in youtube’s automatic captions. In *Proceedings of the First ACL Workshop on Ethics in Natural Language Processing*, pages 53–59, 2017.

[24] R. Tatman and C. Kasten. Effects of talker dialect, gender & race on accuracy of bing speech and youtube automatic captions. In *Interspeech*, pages 934–938, 2017.

[25] S. Wachter, B. Mittelstadt, and C. Russell. Counterfactual explanations without opening the black box: Automated decisions and the gdpr. *Harv. JL & Tech.*, 31:841, 2017.

[26] L. Weng, H. Zhang, H. Chen, Z. Song, C.-J. Hsieh, L. Daniel, D. Boning, and I. Dhillon. Towards fast computation of certified robustness for relu networks. In *International Conference on Machine Learning*, pages 5276–5285. PMLR, 2018.

[27] J. Ye, A. Maddi, S. K. Murakonda, and R. Shokri. Privacy auditing of machine learning using membership inference attacks. 2021.
A Appendix

A.1 General Auditor

In section 2.3, we discussed how to construct a general auditor. Next, we outline the same in alg. 1.

Algorithm 1 A General Auditor

1: \(S_0 := \hat{H}; \) stop_flag := False; \(t := 1\)
2: while \(!\text{stop_flag}\) do
3: \(x_t := \text{picking_next_query}(S_{t-1})\)
4: Auditor receives label \(y_t\) and explanation \(e_t\) from the DS
5: \(S_{t+1} := \text{update_search_space}(S_t, x_t, y_t, e_t)\)
6: stop_flag = \text{check_stopping_condition}(); //Auditor specified stopping condition, examples in sec. 2.3
7: end while
8: \(Y_a = \text{check_audit_result}()\) //Some computation to check auditing result, done by the auditor
9: return \(Y_a\)

Algorithm 2 picking_next_query()

1: Input: \(S_t\)
2: for \(x \in \mathcal{X}\) do
3: for \((e, y) \in \mathcal{E} \times \mathcal{Y}\) do
4: \(S_{t+1} := \text{update_search_space}(S_t, x, y, e)\)
5: end for
6: \(\text{value}_x = \min_{(e, y) \in \mathcal{E} \times \mathcal{Y}} |S_t|/|S_{t+1}|\)
7: end for
8: return \(\arg\max_{x \in \mathcal{X}} \text{value}_x\)

Algorithm 3 update_search_space()

1: Input: \(S, x, y, e\)
2: \(S_{\text{new}} := \{h \in S| h(x) = y, \text{check_consistent_explanation}(x, e_h(x), e)\}\) //Explanation method dependent consistency check, examples in sec. 2.3
3: return \(S_{\text{new}}\)

A.2 Similarities and Differences from Active Learning

To formalize the connection between active learning and auditing, we observe that learning is a harder task than auditing and if there exists an algorithm that can learn a hypothesis, it can also audit it. The following theorem states this connection.

**Theorem 2.1.** If there exists a membership query active learner that can learn \(\hat{h}\) exactly in \(T\) queries, then the active learner can also audit in \(T\) queries.

**Proof.** Once \(\hat{h}\) is known, \(s(\hat{h})\) returns the auditing decision. If \(s(\hat{h}) = 0\), the decision is No and if \(s(\hat{h}) > \epsilon\), the decision is Yes. \(\square\)

A.3 A Simple Baseline: Random Testing

A simple algorithm to detect feature sensitivity is to query a large number of pairs at random from the DS. If any of them is a responsive pair, auditor knows that the feature is sensitive. The following theorem states a bound on the query complexity for this algorithm.

**Theorem 3.1.** For \(\epsilon, \delta \in (0, 1)\), auditor is an \((\epsilon, \delta)\)-auditor if it queries \(m \geq \frac{1}{\epsilon} \log \frac{1}{\delta}\) pairs.

**Proof.** Let the set of all pairs be denoted by \(P\). Assume the fraction of responsive pairs is at least \(\epsilon\). Then the probability that a randomly drawn pair is not a responsive pair is given by,
\[ \Pr_{p_{ij} \sim P}(p_{ij} \text{ is not a responsive pair}) \leq 1 - \epsilon. \] If \( m \) pairs are drawn randomly, then the probability that all of the pairs are not responsive is bounded by \((1 - \epsilon)^m \leq \exp(-\epsilon m) \leq \delta,\) by the choice of \( m.\)

\section*{A.4 Connection between Model Parameters and Score Function}

\textbf{Notation} For vector \( v, \) the \( i^{th} \) feature is denoted \( v_i.\) Let hypothesis \( h_{w,b} \in \mathcal{H}_{\text{LC}}. \) When \( w, b \) are clear from the context, we simply write \( h. \) Let \( w' \) be the \((d - 1)\)-dimensional vector \([w_1 \ldots w_{d-1}]^{T}. \) Hence \( w \) is a concatenation of \( w' \) and \( w_d, \) denoted by the shorthand \( w = [w', w_d]. \) We assume that \( \|w'\|_2 = 1. \) Let \( x \) be a \( d \)-dimensional input to this hypothesis. Let \( x' \) be the \((d - 1)\)-dimensional vector \([x_1 \ldots x_{d-1}]^{T}. \) Let \( \bar{x} = [x_1 \ldots x_{d-1}, 1]^{T}. \) Without loss of generality, let the \( d^{th} \) feature be the feature of interest and let \( x_d \in \{0, 1\}. \)

The score function for a hypothesis \( h \) is given as, \( s(h) = \Pr\left(\left(x^i, x^j\right) \text{ forms a responsive pair}\right)\) where \((x^i, x^j)\) is sampled uniformly from the set of all pairs and labeled by \( h. \) Henceforth we use this score function.

In the following theorem, we bound the fraction of responsive pairs, also our score function, using the weight of our feature of interest, \( w_d. \) The score function is bounded by \( c \cdot |w_d| \) where \( c \) depends on the dimension of the input. This implies that if \( |w_d| \) is small, there are not a lot of responsive pairs (low score function value).

\begin{theorem}
Assume \( \forall x \in \mathcal{X}, \|\bar{x}\|_2 \leq 1. \) Let \( h[w', w_d], b \in \mathcal{H}_{\text{LC}}. \) Then
\[ s(h) \leq c \cdot |w_d| \]
where \( c = \frac{2^{d-2}}{\pi^{(d-1)/2} \Gamma\left(\frac{d+1}{2}\right)} \) is a constant for finite dimension \( d \) and \( \Gamma \) is Euler's Gamma function.
\end{theorem}

\textbf{Proof}. Let \( P \) be the set of all pairs of points. Let \( x^i \) and \( x^j \) denote two inputs forming a pair. Let the pair \((x^i, x^j)\) drawn uniformly from \( P \) form a responsive pair.

From the definition of a pair, \( x^i \) and \( x^j \) only differ in the \( d^{th} \) feature. Hence,
\[ w^T x^j + b = w^T x'^j + b + w_d x^j_d \]
\[ = w^T x'^i + b + w_d (1 - x^j_d) \]
\[ = w^T x'^i + b + w_d - w_d x^j_d. \]

Without loss of generality, let \( x^j_d = 0; \) Hence \( x^j_d = 1. \)
Therefore,
\[ w^T x^j + b = w^T x'^i + b + w_d \]
Next, writing the definition of a responsive pair for \( \mathcal{H}_{\text{LC}} \) we get,
\[ \text{sign}(w^T x^i + b) \neq \text{sign}(w^T x^j + b) \]
Substituting eq.\[\ref{eq:sign}\] into the RHS of eq.\[\ref{eq:sign}\] we get,
\[ \text{sign}(w^T x^j + b) \neq \text{sign}(w^T x'^i + b + w_d) \]
Expanding the LHS of eq.\[\ref{eq:sign}\] and substituting \( x^j_d = 0, \) we get,
\[ \text{sign}(w^T x'^i + b) \neq \text{sign}(w^T x'^i + b + w_d) \]
Eq.\[\ref{eq:sign}\] implies the following,
\[ 0 \leq w^T x'^i + b \Rightarrow w^T x'^i + b + w_d < 0 \]
\[ 0 > w^T x'^i + b \Rightarrow w^T x'^i + b + w_d \geq 0 \]
Combining the two equations in eq. 9 we get,

\[ 0 \leq w^T x + b < -w_d \]
\[ 0 < -(w^T x + b) \leq w_d \]

(10)

Note that the model (defined by \( w, b \)) is fixed. Hence the variables in the above conditions are the inputs \( x \).

Note that only one of the conditions in eq. 10 can be satisfied at any time, based on whether \( w_d \geq 0 \) or \( w_d < 0 \). The fraction of the inputs which satisfy one of the above conditions correspond to the fraction of responsive pairs and hence is the value of the score function.

Conditions in eq. 10 correspond to intersecting halfspaces formed by parallel hyperplanes. If \( \| x \|_2 \leq r \), the region of intersection can be upper bounded by a hypercuboid of length \( 2r \) in \( d-2 \) dimensions and perpendicular length between the two hyperplanes \( l = \frac{|w_d|}{\|w^\prime\|_2} \) in the \( (d-1) \)-th dimension.

Hence, we can upper bound score function \( s(h) \) as,

\[ s(h) \leq \left( \frac{2r}{{V_{d-1}(r)}} \right) \frac{d-2}{d-1} \cdot l \]

(11)

where \( l = \frac{|w_d|}{\|w^\prime\|_2} \) and \( V_{d-1}(r) \) is the volume of the \( (d-1) \)-dimensional ball given by \( \pi^{(d-1)/2} r^{d-1} \Gamma \left( \frac{d+1}{2} \right) \) and \( \Gamma \) is Euler’s Gamma function.

Upon simplification we get,

\[ s(h) \leq \frac{2^{d-2}}{\pi^{\left( \frac{d+1}{2} \right)}} \frac{l}{\Gamma \left( \frac{d+1}{2} \right)} \]

(12)

Assuming \( r = 1 \) and \( \|w^\prime\|_2 = 1 \), we can write eq. 12 as,

\[ s(h) \leq c \cdot |w_d| \]

(13)

where \( c = \frac{2^{d-2}}{\pi^{\left( \frac{d+1}{2} \right)}} \Gamma \left( \frac{d+1}{2} \right) \) is a constant for small dimensions.

A.5 Auditing Linear Classifiers with Counterfactual Explanations

In this section, we will prove that auditing linear classifiers using counterfactual explanations requires only one query. We denote our auditor by \( \text{AlgLC}_c \), as outlined in alg. 4 The proof goes by noting that the counterfactual explanation \( x^\prime \) returned by the DS is very close to the projection of input \( x \) and that \( x - x^\prime \) is parallel to \( w \). We consider the \( d \)-th feature to be our FoI without loss of generality.

Algorithm 4 \( \text{AlgLC}_c \) : Auditing Linear Classifiers using Counterfactuals

1: Query any point \( x \) from the DS
2: Auditor receives label \( y \) and explanation \( x^\prime \) from the DS
3: \( \hat{w} \leftarrow x - x^\prime \)
4: if \( \hat{w}_d = 0 \) then
5: return No
6: else
7: return Yes
8: end if

Lemma 2. Given hyperplane \( w^T x + b = 0 \), point \( x \) and its projection on the hyperplane \( x'' \), \( x - x'' = \lambda w \) where \( \lambda = \frac{w^T x + b}{\|w\|^2} \).

Proof. The projection, \( x'' \), of \( x \) on the hyperplane \( w^T x + b = 0 \), is found by solving the following optimization problem.
\[
\min_{x''} \|x'' - x\|^2 \\
\text{s.t.} \quad w^T x'' + b = 0
\]

Let \( L(x'', \lambda) \) be the lagrangian for the above optimization problem.

\[
L(x'', \lambda) = \|x'' - x\|^2 + 2\lambda(w^T x'' + b) \\
= \|x''\|^2 + \|x\|^2 - 2x^T x'' + 2\lambda w^T x'' + 2\lambda b \tag{15}
\]

Taking derivative of the lagrangian with respect to \( x'' \) and equating with zero we get,

\[
\frac{\partial L}{\partial x''} = 2x'' - 2(x - \lambda w) = 0 \\
x'' = \lambda w \tag{16}
\]

By substituting above equation in the constraint for the optimization problem \( w^T x'' + b = 0 \), we get

\[
\lambda = \frac{w^T x + b}{\|w\|^2}.
\]

Theorem 3.2. For any \( \epsilon \in [0, 1] \), auditor \( \text{AlgLC}_c \) is an \((\epsilon, 0)\)-auditor for \( \mathcal{H}_{LC} \) and score function \( s(\cdot) \) with \( T = 1 \) query.

Proof. Recall that the counterfactual explanation returned by the DS for input \( x \) is given as \( x' = \arg\min_{x': h(x') \neq h(x)} d(x, x') \) where \( d(x, x') = \|x - x'\|_2 \).

The projection of \( x \) on the hyperplane, \( x'' \) is the closest point to \( x \) on the hyperplane. Therefore \( x' = x'' + \Delta \) where \( \Delta \) is a vector in the direction of \( w \), \( \Delta = \gamma w \), \( \gamma \) is a very small non-zero constant.

Therefore, \( x - x' = x - (x'' + \Delta) = (x - x'') + \Delta \).

Using lemma\textsuperscript{[2]}

\[
\hat{w} := x - x' = \lambda w + \gamma w = c_0 w, \tag{17}
\]

where \( c_0 \) is a non-zero constant. \((x-x')\) is non-zero due to the definition of counterfactuals, specifically that they have different labels.

If \( w_d = 0 \), then it implies that \( \hat{w}_d = 0 \) and the feature has no effect on the prediction. Thus the score function is zero. Since \( \text{AlgLC}_c \) returns a \text{No} when \( \hat{w}_d = 0 \), it is always correct in this case. For all the other cases when \( w_d \neq 0 \), it implies that \( \hat{w}_d \neq 0 \) and therefore, the feature has an effect on the prediction. Since \( \text{AlgLC}_c \) returns a \text{Yes} when \( \hat{w}_d \neq 0 \), it is always correct.

Also \( \delta = 0 \) since our auditor and DS are deterministic.

Note that this is partial learning since 1) we do not need to learn \( w \) exactly and 2) we do not need to learn the bias term \( b \).

A.6 Auditing Linear Classifiers with Anchor Explanations

\textsuperscript{[2]} proposed a query synthesis spectral algorithm to learn homogeneous linear classifiers in \( O(d \log \frac{1}{\Delta}) \) steps where \( \Delta \) corresponds to a bound on the error between estimated and true classifier. They maintain a version space of consistent hypotheses approximated using the largest ellipsoid \( \varepsilon^* = (\mu^*, \Sigma^*) \) where \( \mu^* \) is the center and \( \Sigma^* \) is the covariance matrix of the ellipsoid. They prove that the optimal query which halves the version space is orthogonal to \( \mu^* \) and maximizes the projection in the direction of the eigenvectors of \( \Sigma^* \).

We propose an auditor \( \text{AlgLC}_c \) as depicted in alg.\textsuperscript{[5]} using their algorithm. The anchor explanations are incorporated through active augmentation. But, in the worst-case anchors are not helpful and hence the algorithm reduces essentially to that of \textsuperscript{[2]} (without anchors). In this section we find the query complexity of this auditor.
Notation In $A_{1gLC_{a}}$, $\varepsilon_{t}\cdot y = (\mu_{t}, \Sigma_{t})$ denotes the largest ellipsoid that approximates the version space (corresponds to search space in our case) at time $t$ where $\mu_{t}$ is the center and $\Sigma_{t}$ is the covariance matrix of the ellipsoid at time $t$. $N_{t}$ is the orthonormal basis of the orthogonal complement of $\mu_{t}$. $\alpha_{t}\cdot y$ is the top eigenvector of the matrix $N_{t}\Sigma_{t}N_{t}$. In the implementation by [2], some warm-up labeled points are supplied by the user, we denote this set as $W$. Let $W(t)$ denote the $t$-th element of this set. Let the $d_{t}\cdot y$ element of this set. Let the $H_{t}\cdot y$ of all classifiers consistent with the label be $H_{t}$. Let the set of all classifiers consistent with the label be $H_{t}$. From the definition of anchors and $\tau_{t}$, the precision parameter $\tau_{t}$ qualifies as a worst-case anchor. Therefore $A_{x}$ qualifies as a worst-case anchor.

**Lemma 3.** Given input $x$, a worst-case anchor for $A_{1gLC_{a}}$ is of the form $A_{x} = \{\lambda x | \lambda \in R^{+}\}$ with precision parameter $\tau_{t} = 1$.

**Proof.** Consider that given $x$, DS returns label $y$ and anchor explanation $A_{x}$ from the DS.

Let the set of all classifiers consistent with the label be $H = \{w : y(w, x) > 0\}$. From the definition of anchors and $\tau_{t} = 1$, the label of all points in anchor $A_{x}$ is also $y$. Then, the set of all classifiers consistent with $A_{x}$ is, 

$$H' = \{w : y(w, \lambda x) > 0\}$$
$$= \{w : \lambda y(w, x) > 0\}$$
$$= H$$

(18)

Hence the set of consistent classifiers remains the same despite anchors. Therefore $A_{x}$ qualifies as a worst-case anchor.

Next we give a bound on the number of queries required to audit using $A_{1gLC_{a}}$. With worst-case anchors, it means that we are just using the algorithm of [2], essentially without explanations and active augmentation. The auditor has a fixed $\varepsilon$ that it decides beforehand. $A_{1gLC_{a}}$ decides how many times it must run the algorithm of [3] such that for the fixed $\varepsilon$, it satisfies def. [5].

---

**Algorithm 5 $A_{1gLC_{a}}$: Auditing Linear Classifiers using Anchors**

1: **Input:** $T$, augmentation size $s$, set of warm-up labeled points $W$
2: set of queried points $Q := \emptyset$, $t := \text{size}(W)$
3: for $t = 1, 2, 3, \ldots, T + l$ do
4:   if $t <= l$ then
5:     $(x, y) := W(t)$
6:     $Q := Q \cup (x, y)$
7:     goto step 15
8:   else
9:     Query point $x^{t} := N^{t}\alpha^{t}\cdot y$ from the DS
10:    Auditor receives label $y$ and explanation $A_{x}$ from the DS
11:    $Q := Q \cup (x^{t}, y)$
12:   Sample randomly $s$ points $x^{1}, \ldots, x^{s}$ from $A_{x}$
13:   $Q := Q \cup \{(x^{1}, y), \ldots, (x^{s}, y)\}$
14:   end if
15: $\varepsilon^{(t+1)}\cdot y = (\mu^{(t+1)}, \Sigma^{(t+1)}) := \text{estimate_ellipsoid}(Q)$
16: $N^{(t+1)} := \text{update_N}(N^{(t+1)}\mu^{(t+1)}\Sigma^{(t+1)})$
17: $\alpha^{(t+1)} := \text{update_\_alpha}(N^{(t+1)}\Sigma^{(t+1)})$
18: //Exact formulae for steps 15, 16 and 17 can be found in [2]
19: end for
20: $\hat{\mu} = \mu^{T+1}$
21: if $|\hat{\mu}| \leq \Delta$ then
22:   return no
23: else
24:   return Yes
25: end if
Theorem 3.3. For every dimension $d$, there exists $c > 0$ such that for any $\epsilon \in (0, 1)$, auditor $A1gL_{a}$ is an $(\epsilon, 0)$-auditor for $H_{LC}$ and score function $s(\cdot)$ with $T = O\left(\frac{d \log \frac{2d}{\epsilon}}{\epsilon}\right)$ queries.

Proof. Let $w$ be the true classifier and $\hat{w}$ be the estimated classifier learnt by $A1gL_{a}$. Let the difference between $w$ and $\hat{w}$ be bounded by $\Delta$ as follows,
\begin{equation}
\|w - \hat{w}\|_2 \leq \Delta.
\end{equation}

The value of $\Delta$ will be set later on. Since $A1gL_{a}$ uses $|\hat{w}_d|$ to make its decision, the worst case is when the entire error in estimation is on the $d^{th}$ dimension. Hence, we consider $|w_d - \hat{w}_d| \leq \Delta$.

To guarantee that the auditor is an $(\epsilon, 0)$-auditor we need to verify for every hypothesis in the class that if $s(\cdot) = 0$, then the answer is No and if $s(\cdot) > \epsilon$, then the answer is Yes, see def. 3.

Importantly, $s(w)$ is zero only if $w_d = 0$ from theorem 1. If $w_d = 0 \Rightarrow |\hat{w}_d| \leq \Delta$, by eq. 19. Since $A1gL_{a}$ returns a No for $|\hat{w}_d| \leq \Delta$, $A1gL_{a}$ satisfies def. 3 when $s(w) = 0$ with $\delta = 0$.

Next, we have the case $s(w) > \epsilon$ when auditor should return a Yes with high probability. $A1gL_{a}$ returns a Yes when $|\hat{w}_d| > \Delta$. Hence for $A1gL_{a}$ to be correct, we need that $s(w) > \epsilon$ imply that $|\hat{w}_d| > \Delta$.

We can upper bound $s(w)$ using eq. 12 as,
\begin{equation}
s(w) \leq c \cdot |w_d|
\end{equation}

Since $\epsilon < s(w)$,
\begin{equation}
\epsilon < c \cdot |w_d|
\end{equation}

Since $|w_d - \hat{w}_d| \leq \Delta$,
\begin{equation}
\epsilon \leq c (|\hat{w}_d| + \Delta)
\end{equation}

On rearranging,
\begin{equation}
\frac{\epsilon}{c} \leq |\hat{w}_d| - \Delta
\end{equation}

Eq. 23 connects $\epsilon$ with $\Delta$ and $\hat{w}_d$. For $s(w) > \epsilon$, $|\hat{w}_d|$ should be greater than $\Delta$ for $A1gL_{a}$ to be correct. Hence, lower bounding the LHS of eq. 23 we get,
\begin{equation}
\Delta \leq \frac{\epsilon}{c} - \Delta
\end{equation}
\begin{equation}
\Delta \leq \frac{\epsilon}{c} - \Delta
\end{equation}
\begin{equation}
\Delta \leq \frac{\epsilon}{2c}
\end{equation}

We set $\Delta = \frac{\epsilon}{2c}$.

From Lemma 3, $A1gL_{a}$ reduces to [2]'s spectral algorithm in the worst-case. This algorithm has a bound of $O(d \log \frac{d}{\epsilon})$. Hence, the query complexity of $A1gL_{a}$ is $O(d \log \frac{2d}{\epsilon})$.

\section*{A.7 Extended Thresholds as Linear Classifiers}

According to the definition of linear classifiers, output is 1 if $w^T x + b \geq 0$ and -1 otherwise. Writing the if condition of extended thresholds in terms of linear classifiers, we get,
\begin{equation}
(g - 1)\theta_1 - g\theta_2 + f(x') \geq 0
\end{equation}
\begin{equation}
\Rightarrow (\theta_1 - \theta_2)g + f(x') + -\theta_1 \geq 0 \Rightarrow w = [1, \theta_1 - \theta_2]^T, b = -\theta_1
\end{equation}

Thus, extended thresholds are 2D linear classifiers with weights and bias as given above.
A.8 Auditing Extended Thresholds with Counterfactual Explanations

We propose AlgET to audit extended thresholds with counterfactual explanations. Auditor queries (1,0) and (u,1) from the DS. DS responds with threshold parameters of h inadvertently as (θ1,0) and (θ2,1) are the counterfactual explanations for the asked queries.

Algorithm 6 AlgETc: Auditing Extended Thresholds using Counterfactuals

1: Query points (l,0) and (u,1) from the DS
2: Auditor receives labels y0, y1 and explanations (θ1,0), (θ2,1) from the DS
3: if θ1 = θ2 then
4:    return No
5: else
6:    return Yes
7: end if

Theorem 2. For any ε ∈ [0,1], auditor AlgETc is an (ε,0)-auditor for HET and score function s(·) with T = 2 query.

Proof. Without loss of generality, let l = -1 and u = 1. Then (-1,0) and (1,1) are the two queries. We also consider computers with finite precision. Hence for example, if the point 0.999999... is saved as 1 by the computer.

Let θ1, θ2 correspond to thresholds for h. Next we prove that the counterfactual explanations returned by the DS for the queried points have to be (θ1, 0), (θ2, 1).

If θ1 ≤ θ2, (θ1, 0) is the closest point to (-1,0) with a different label and is therefore the counterfactual explanation. Note that (θ1, 1) cannot be the counterfactual since changing the second feature would add a cost of 1 to that of (θ1, 0). Similarly, (θ2, 1) is the closest point to (1,1). Hence (θ1, 0) and (θ2, 1) are returned as explanations by the DS.

If θ1 > θ2 and |θ1 - θ2| > 1, (θ1, 0) is the counterfactual for (-1,0) and (θ2, 1) is the counterfactual for (1,1).

If θ1 > θ2 and |θ1 - θ2| ≥ 1, (θ2, 1) is the counterfactual for (-1,0) and (θ1, 0) is the counterfactual for (1,1).

In all the cases, DS returns (θ1, 0) and (θ2, 1) as the explanations. This allows the auditor to exactly know h after querying 2 points. Responsive pairs exist between the thresholds. Hence if the thresholds are equal, it is implied that no responsive pairs exist and therefore FoI is not a sensitive feature for the model. The opposite case holds when the thresholds are not equal. Then, ε = |θ1 - θ2| and δ = 0 since the auditor and DS are deterministic in this case.

A.9 Auditing Extended Thresholds with Anchor Explanations

In this section, we propose AlgETa with guarantees to audit extended thresholds with anchor explanations.

Note that anchor explanations can only be provided in terms of f(x′), otherwise it is clear that the model is unfair. In our case, hyperrectangles correspond to intervals in [l, u]. Let us assume l = -1, u = 1, τ = 1 and fix coverage length c for simplicity.

Auditor AlgETa is as follows. Let search space at time t correspond to the interval [θt, m, θt, m]. Auditor queries the mid points (θt, m + θt, m)/2 from the DS. These queries halve the size of the search space, which is the maximum amount in the worst case. The worst case explanation is the interval between (θt, m + θt, m)/2 ± c and (θt, m + θt, m)/2 where c is for label -1 and +c for label 1. Note that these explanations do not provide extra information as they do not reduce the search space by any more amount than labels. Based on the output of the DS, auditor updates θt, m + 1 and θt, m + 1. Auditing stops when a responsive pair is detected or if the auditing budget is over. If a responsive pair is detected, the auditor returns Ya = Yes otherwise it returns Ya = No. This algorithm is demonstrated in algorithm 7.

Next we will prove that the query complexity of AlgETa is ⌈2 log(1/τ)⌉ as stated in theorem 3 using lemmas 4 and 5.
Algorithm 7 AlgET_a : Auditing Extended Thresholds using Anchors

1: **Input:** \( T \)
2: \( bp\_detected := \text{No}, \theta^1_{\text{min}} := -1, \theta^1_{\text{max}} := 1 \)
3: for \( t = 1, 2, 3, \ldots T \) do
4: Query points \( (\theta^t_{\text{min}} + \theta^t_{\text{max}} / 2, 0) \) and \( (\theta^t_{\text{min}} + \theta^t_{\text{max}} / 2, 1) \)
5: Auditor receives labels \( y_0, y_1 \) and explanations \( e_0, e_1 \) from the DS
6: if \( y_0 \neq y_1 \) then
7: \( bp\_detected = \text{Yes} \)
8: break
9: end if
10: if \( y_0 = -1 \) then
11: \( \theta^t_{\text{min}} := \theta^t_{\text{min}} \)
12: \( \theta^t_{\text{max}} := \theta^t_{\text{min}} + \theta^t_{\text{max}} / 2 \)
13: else
14: \( \theta^t_{\text{min}} := \theta^t_{\text{min}} + \theta^t_{\text{max}} / 2 \)
15: \( \theta^t_{\text{max}} := \theta^t_{\text{max}} \)
16: end if
17: end for
18: return \( bp\_detected \)

Lemma 4. At time \( t, \hat{h} \in S_t \).

**Proof.** This is true from the definition of the search space and \( \hat{h} \in \mathcal{H} \) assumption.

\( \theta_{1h}, \theta_{2h} \) are the thresholds \( \theta_1, \theta_2 \) for hypothesis \( h \). At each time step, the search space is reduced by half. \( \forall h \in S_t, \theta_1 \) and \( \theta_2 \) lie within an interval which narrows down at each time step. Responsive pairs exist within this interval. Hence, the probability of responsive pairs also reduces subsequently. This intuition is formalized below.

Lemma 5. At time \( t \), \( \exists \theta^t_{\text{min}}, \theta^t_{\text{max}} \in [-1, 1] \) such that, \( \forall h \in S_t \)
1. \( \theta_{1h}, \theta_{2h} \in [\theta^t_{\text{min}}, \theta^t_{\text{max}}] \)
2. \( |\theta_{1h} - \theta_{2h}| \leq 2^{-t+1} \)

**Proof.** At \( t = 0, S_0 = \mathcal{H} \).

\( \forall h \in S_0, \theta_{1h}, \theta_{2h} \in [-1, 1] \) and \( \theta_{1h} - \theta_{2h} \leq 2 \). Hence, \( \theta^0_{\text{min}} = -1 \) and \( \theta^0_{\text{max}} = 1 \).

Inductive Step: Consider at time \( t \), the lemma holds. At time \( t \), the auditor picks the queries \( (\theta^t_{\text{min}} + \theta^t_{\text{max}} / 2, \cdot) \).

Without loss of generality, using worst-case label \( = -1, \theta_{1h}^{t+1} = \theta_{\text{min}}^t, \theta_{2h}^{t+1} = (\theta^t_{\text{min}} + \theta^t_{\text{max}}) / 2 \).

\( \forall h \in S_{t+1}, \theta_{1h}, \theta_{2h} \in [\theta^t_{\text{min}} + (3\theta^t_{\text{min}} + \theta^t_{\text{max}}) / 4] \) or \( \theta_{1h}, \theta_{2h} \in [(3\theta^t_{\text{min}} + \theta^t_{\text{max}}) / 4, \theta^t_{\text{max}}] \).

Hence \( \theta_{1h}, \theta_{2h} \in [\theta^t_{\text{min}} + \theta^t_{\text{max}} / 2] \).

\( |\theta_{1h} - \theta_{2h}| \leq \max(\theta_{1h} - \theta_{2h}) = |\theta^t_{\text{min}} + \theta^t_{\text{max}}| = 2^{-t} \)

**Theorem 3.** For any \( \epsilon \in [0, 1] \), auditor \( \text{AlgET}_a \) is an \( (\epsilon, 0) \)-auditor for \( \mathcal{H}_{ET} \) and score function \( s(\cdot) \) with \( T = \lceil 2 \log(\frac{1}{\epsilon}) \rceil \) queries.

**Proof.** At time \( t \), responsive pairs exist between \( \theta^t_{\text{min}} \) and \( \theta^t_{\text{max}} \). Since each pair corresponds to 2 queries and using lemma 4 and 5, we get theorem 3.

A.10 Experiments

A.10.1 Datasets

Both Adults and Covertype have a mix of categorical and continuous features. Categorical features are processed such that each category corresponds to a binary feature in itself. For Adult dataset, the
output variable is whether Income exceeds $50K/yr. For Covertype, the output variable is whether forest covertype is category 1 or not.

A.10.2 Auditing Neural Networks using Extended Thresholds

Figure 3: Neural Networks as Extended Thresholds

Table 2: Thresholds learnt on Adults dataset

| Feature predicted by NN | \((\theta_1, \theta_2)\) | Test Accuracy % |
|-------------------------|-----------------|-----------------|
| Education               | (0.5, 1.5)      | 78.8            |
| Age                     | (2.6, 2.7)      | 75.79           |
| Hours/week              | (0.4, 2.2)      | 78.6            |
| Capital-gains           | (0, 0.2)        | 79.86           |
| Capital-losses          | (0.2, 0.4)      | 78.15           |

Table 3: Thresholds learnt on Covertype dataset

| Feature predicted by NN | \((\theta_1, \theta_2, \theta_3, \theta_4)\) | Test Accuracy % |
|-------------------------|---------------------------------|-----------------|
| Elevation               | (1, -1, 3, 0)                   | 66.65           |
| Aspect                  | (2, -1, 2, 2)                   | 64.79           |
| Scope                   | (2, -1, 5, 3.5)                 | 64.6            |