Quantum Vacuum Friction in highly magnetized neutron stars

A. Dupays\textsuperscript{1(a)}, C. Rizzo\textsuperscript{1}, D. Bakalov\textsuperscript{2} and G. F. Bignami\textsuperscript{3,4}

\textsuperscript{1} Laboratoire Collisions, Agrégats, Réactivité (UMR 5589, CNRS-Université de Toulouse, UPS), IRSAMC Toulouse, France, EU

\textsuperscript{2} INRNE, Bulgarian Academy of Sciences - Sofia, Bulgaria, EU

\textsuperscript{3} IUSS, Istituto Universitario di Studi Superiori - Pavia, Italy, EU

\textsuperscript{4} INFN, Sezione di Pavia - Via A. Bassi 6, I-27100 Pavia, Italy, EU

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Abstract – In this letter we calculate the energy loss of a highly magnetized neutron star due to Quantum Vacuum Friction (QVF). Taking into account one-loop corrections in the effective Heisenberg-Euler Lagrangian of the light-light interaction, we derive an analytic expression for QVF allowing us to take into account a magnetic field at the surface of the star as high as $10^{11}$ T. In the case of magnetars, with magnetic fields above the QED critical field, we show that the QVF is the dominating energy loss process. This has important consequences, in particular for the inferred value of the magnetic field. This also indicates the need for independent measurements of magnetic field, energy loss rate, and the braking index in order to fully characterize magnetars.

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Introduction. – Quantum Vacuum Friction (QVF) is a fundamental phenomenon \cite{1} related to the fact that the quantum vacuum can be regarded as a standard medium with its own energy density and electromagnetic properties. Well-known effects due to static vacuum properties are the Casimir effect \cite{2} and the vacuum magneto-electric optical properties \cite{3}. Recently, dynamical effects due to vacuum viscosity have been studied \cite{1}, in particular the photon radiation stimulated by moving mirrors \cite{4} (i.e. the dynamical Casimir effect) and the friction between perfectly smooth surfaces moving relative to each other \cite{5–7}.

Considering a classical magnetic dipole moment that is rotating in a standard medium with magneto-electric optical properties, relaxation effects (at the molecular level for example) produce retardation effects between the induced magnetization of the medium and the rotation of the magnetic dipole moment. In this case, the temporal delay between the medium’s response and the inductor gives rise to a classical dissipative frictional force.

In this letter, we investigate the friction effect between a non-stationary magnetic dipole moment and its induced quantum vacuum magnetic dipole moment. In this case the temporal delay between the vacuum response and the inductor is due to the finite velocity of light. We calculate the friction energy loss rate in the case of highly magnetized neutron stars. Pulsars are the most appropriate systems to look for such an effect. Pulsars are fast-rotating neutron stars, with a very high magnetic dipole moment tilted with respect to their rotational axis \cite{8}. Typically, their mass is of the order of the solar mass, and their radius of the order of 10 km. The magnetic field of neutron stars is of the order of $10^8$–$10^9$ T, while in the case of magnetars \cite{9} astrophysical observations seem to indicate that fields as high as $10^{11}$ T exist on their surfaces \cite{10–12}. Spinning periods of neutron stars range from tens of milliseconds for young stars to seconds, increasing with star age. Because of their fast rotation, retardation effects are significant. A rotating neutron star can thus be considered as a classical magnetic dipole moment weakly coupled to a bath, namely the quantum vacuum described by QED as electron-positron pairs due to quantum fluctuations. In a microscopic description, the energy is dissipated via the polarization process and through the annihilation of the polarized pairs which relax this excess of energy to the bath. This excess of energy corresponds to the frictional energy we calculate using a classical interaction between the star’s dipole moment and the bath. We show that, for a neutron star with a very high magnetic field, exceeding the Quantum ElectroDynamics (QED) critical field ($B_c = 4.4 \cdot 10^9$ T), the energy loss is essentially due to QVF, while the energy loss due to

\footnotesize\textsuperscript{\textsuperscript{(a)}E-mail: arnaud.dupays@irsamc.ups-tlse.fr}
the classical rotating dipole radiation becomes negligible. This applies in particular to magnetars, which associate a strong magnetic field and a spin period of a few seconds. To infer the magnetic field of such stars, one cannot use the classical dipole energy loss formula. Our results suggest that in order to describe magnetars one needs to measure the energy loss rate and the magnetic field independently. Taking into account QVF, we calculate the braking index [13] and show that its measurement provides a non-model-dependent determination of the magnetic field on the surface of a neutron star.

**Energy loss rate.** – To study the quantum vacuum magnetization in the presence of a magnetic field, we start with the effective Heisenberg-Euler Lagrangian $L$ magnetization in the presence of a magnetic field, we start given by [15]

$$F = e_0 E^2 - \frac{b^2}{\mu_0},$$  \hspace{1cm} (1)

$$G = \sqrt{\frac{c_0}{\mu_0}} \left( E \cdot B \right),$$  \hspace{1cm} (2)

where $\epsilon_0$ is the vacuum permittivity and $\mu_0$ the vacuum permeability. When one-loop corrections are included, $L_{HE}$ can be written as $L_{HE} = L_0 + L_1$, where $L_0 = F/2$ is the usual Maxwell’s term. Neglecting the electric field, $F = -B^2/\mu_0$ and $G = 0$. For $L_1$ we use the analytic expression derived by Heyl and Hernquist [16,17]

$$L_1 = \frac{e^2 B^2}{\epsilon_0} X_0 \left( \frac{B_c}{B} \right),$$  \hspace{1cm} (3)

where $B_c = m_e^2 e^2 / \hbar c \simeq 4.4 \cdot 10^9$ T is the critical field and

$$X_0(x) = 4 \int_0^{x/2} \ln(G(v+1)) dv + 1 \ln \left( \frac{1}{x} \right) + 2 \ln 4\pi$$

$$- \left( 4 \ln A + \frac{5}{3} \ln 2 \right) - \left( 4 \pi + 1 + \ln \left( \frac{1}{\gamma} \right) \right) x$$

$$+ \left[ \frac{3}{4} + \frac{1}{2} \ln \left( \frac{2}{\gamma} \right) \right] x^2,$$  \hspace{1cm} (4)

with $\ln A = 0.248754477$. Denoting the induced magnetization by $M_{qv}$, (i.e. the quantum vacuum magnetic dipole moment per volume element), we have [18]

$$H = -2 \frac{\partial L_{HE}}{\partial B} = \frac{B}{\mu_0} - M_{qv}. $$  \hspace{1cm} (5)

Using the analytic form (3) of the Lagrangian, we can calculate $M_{qv}$ to the first order of $a = e^2 / \hbar c \simeq 1/137$:

$$M_{qv} = \frac{a e^2 B^2}{2 \pi B_c^2 \mu_0} f_{qv}(B^2) B,$$  \hspace{1cm} (6)

with

$$f_{qv}(B^2) = \frac{1}{2} \left( \frac{B^4}{B^2} \right) X_0 \left( \frac{B_c}{B} \right) - \frac{B^3}{B^2} X_1 \left( \frac{B_c}{B} \right),$$  \hspace{1cm} (7)

where

$$X_{n} = \frac{1}{\Gamma(n+1)} \int_0^1 \left( \frac{1}{1-x} \right)^n dx,$$  \hspace{1cm} (8)

Let us now consider a neutron star rotating in vacuum (see fig. 1). We denote its magnetic dipole moment by $m$, $R$ its radius and $B_0$ the magnitude of the magnetic field at its surface ($B_0 \simeq \mu_0 m / 4\pi R^3$, where $m = \| m \|$). We define a fixed frame $(x, y, z)$ with the $z$-axis parallel to the rotation axis of this star. $(\theta, \varphi)$ stand for the spherical polar angles of $m$ in this fixed frame. If $p$ denotes the spinning period of the neutron star, $\omega = \omega t$, with $\omega = 2\pi/p$ and $m \cdot u_e = m \cos \theta$. At time $t$ the magnetic moment of the star produces a magnetic field $B(r, t)$. Let us denote by $r$ the position vector of a vacuum element of volume $dr = r^2 \sin \beta dr d\beta d\gamma$ where $(r, \beta, \gamma)$ stand for the spherical coordinates of $r$ in this fixed frame. Since $\omega r/c < 1$ inside the region where quantum vacuum magnetization is important, the leading term within the dipolar magnetic field approximation is [19]

$$B(r, t) \simeq \frac{(\mu_0)}{4\pi} \frac{3r(m(t-r/c) \cdot r)}{r^5} - \frac{m(t-r/c)}{r^3}.$$  \hspace{1cm} (9)

In this expression, retardation effects have been taken into account with the argument $t-r/c$ in $m$. According to eq. (6), the induced quantum vacuum magnetic moment at $r$ is given by

$$d m_{qv}(r, t) = \frac{a B^2(r, t) f_{qv}(B^2(r, t)) B(r, t)}{2\pi B_c^2 \mu_0} \int \frac{d\tau}{r^3}. $$  \hspace{1cm} (10)

At time $t + r/c$ the magnetic field $dB_{qv}$ produced by $d m_{qv}(r, t)$ at the center of the star is

$$dB_{qv}(0, t + r/c) \simeq \frac{(\mu_0)}{4\pi} \frac{3r(d m_{qv}(r, t) \cdot r)}{r^5} - \frac{d m_{qv}(r, t)}{r^3}.$$  \hspace{1cm} (11)
This field interacts with the magnetic dipole moment of the star. At this stage, quantum vacuum can be regarded as a standard medium. Therefore, the energy loss rate due to this friction is given by the classical formula [18]

$$\dot{E}_{qv} = - (mv(t + r/c) \times dB_qv(0, t + r/c)) \cdot \omega \cdot \mathbf{u}_z. \quad (12)$$

The total QVF energy loss rate is obtained by integrating eq. (12) over the space external to the star, and averaging it over time

$$\dot{E}_{qv} = \int_{r=R}^{+\infty} \int_{\beta=0}^{\pi} \int_{\gamma=0}^{2\pi} \langle \dot{E}_{qv} \rangle_z dt. \quad (13)$$

$\dot{E}_{qv}$ can thus be calculated numerically. Since retardation effects grow with $r$ while the star’s magnetic field decreases, the space region that mainly contributes to QVF is located around a few star’s radii and inside the light cylinder. Moreover, in the case of a dipolar magnetic field, using eq. (7), one can show that for $B_0 \lesssim 10^{10}$ T, $(B^2(r, t)/B^2_0)_{qv}(B^2(r, t)) \approx (4B^2(r, t)/45B^2_0)$ and we obtain the analytic expression

$$\dot{E}_{qv} \approx \alpha \left( \frac{3\pi^2}{4} \right) \frac{\sin^2 \theta \; B_0^3 R^4}{B^2_0 \mu_0 c^3 \; p^2}. \quad (14)$$

In fig. 2, we have plotted the function $\dot{E}_{qv}$ obtained numerically from eq. (13) vs. $B_0 \sin \theta$ (full line), for the Crab pulsar with a spinning period $p = 33.11$ ms. We have also plotted the star rotation radiation rate $\dot{E}_r$ vs. $B_0 \sin \theta$ given by the classical dipole model [20] (since in this case, QED corrections are not relevant [17]).

$$\dot{E}_r = \left( \frac{128\pi^5}{3} \right) \frac{\sin^2 \theta \; B_0^3 R^6}{\mu_0 c^3 \; p^2}. \quad (15)$$

Figure 2 clearly shows that for fields exceeding the QED critical field the star energy loss is essentially due to QVF. At this stage, it is important to stress that the spinning period dependence is not the same for the classical case and for the QVF. The ratio of classical-to-QVF losses decreases as $1/p^2$ for large $p$. QVF becomes more important for slowly rotating neutron stars.

This result has important consequences. At present, the value of the magnetic field on the surface of a neutron star is inferred by the energy loss of the star, derived by the measured value of the spin-down rate, assuming that the energy loss of the star is given by classical dipole model. In a case for which QVF should play an important role, the inferred value of the magnetic field should then be modified. Moreover, we see that the method used to obtain the value of the magnetic field gives $B_0 \sin \theta$ rather than $B_0$. One should thus assume the value of $\theta$ to get $B_0$ (typically $\theta$ is taken equal to $\pi/2$). In the following, taking advantage of the fact that this angular dependence is the same for $\dot{E}_{qv}$ and $\dot{E}_r$, we show that the inferred value of the magnetic field at the surface of a pulsar can be derived by the measurement of the star’s braking index.

**Braking index.** – The braking index $n$, is a fundamental parameter of pulsar electrodynamics describing the rate at which a magnetized neutron star loses rotational energy. This dimensionless quantity is given by

$$n = \frac{\nu \dot{\nu}}{\dot{\nu}^2}, \quad (16)$$

where $\nu$ is the spinning frequency of the pulsar and $\dot{\nu}$ (or $\ddot{\nu}$), denotes the first (or the second) derivative of $\nu$ with respect to time. From this definition one can see that the braking index can be determined from pulsar timing measurements without any assumption concerning the star’s structure and so provides crucial information for our understanding of the physics underlying pulsar spin-down. From a theoretical point of view, the pulsar’s rotational energy loss rate is given by

$$\dot{E} = 4\pi^2 I \nu \dot{\nu}, \quad (17)$$

where $I$ is the moment of inertia of the rotating star. Assuming a pure classical dipole energy loss mechanism, we obtain from eq. (15)

$$\dot{\nu} = - \frac{\dot{E}_r}{4\pi^2 I \nu} = - K_r \nu^3, \quad (18)$$

where $K_r = \dot{E}_r/4\pi^2 I \nu^4$. Taking the logarithmic derivative of this equation, we find $n = 3$. So far, braking indices of only six pulsars have been precisely measured, all of which are smaller than the value $n = 3$ expected for pure classical dipole radiation model (see table 1, second column). This result clearly shows that an additional energy loss mechanism should be included in the model to fit the data. Explanations of this discrepancy have already been suggested, as for example the effect of the relativistic pulsar wind [21] or the fact that the pulsar’s magnetism cannot be modelled with a simple dipolar...
Depending on the case, the inferred value becomes index measured so far. Taking into account QVF, eq. (18) provides a new coherent explanation for all the values of braking additionnal energy loss mechanism which can provide a very important test to confirm QVF predictions. Because of the gamma-ray emitting electron-positron cascades which occur for $\dot{E}$ above the threshold value of $\dot{E} \simeq 3 \times 10^{34}$ erg s$^{-1}$ [29,30], magnetars, i.e. pulsars with a magnetic field above the QED critical field (within the classical dipolar model) should not be gamma emitters. Detecting a gamma-ray emission from such a system, for example by the NASA GLAST mission due to launch in 2008, would provide evidence in favour of QVF which leads to smaller values of the magnetic field for magnetars compatible with gamma-ray emission.

**Conclusion.** – The energy loss of a strongly magnetized neutron star due to Quantum Vacuum Friction (QVF) has been studied. In neutron stars with magnetic fields of the order of $10^8-10^9$ T and spinning periods less than a second, this effect is negligible compared to classical energy loss. In the case of magnetars with high magnetic fields, and spinning periods of a few seconds, we have shown that the energy loss by QVF dominates the energy loss process. This has important consequences, in particular for the inferred value of the magnetic field. It also indicates the need for independent measurements of magnetic field, energy loss rate, and of the braking index full characterization of magnetars. QVF should also play an important role in the early stages of neutron star formation, and it should be taken into account in models of magnetars (see e.g. [31]), and its spindown history. On the other hand, macroscopic violation of the linearity of Maxwell’s equations predicted by QED [3] has not yet been proved. Experimental searches of vacuum birefringence are in progress (see [32] and references within). Evidence for QVF in the neighbourhood of a neutron star would thus be the first confirmation of a fundamental phenomenon.

**Table 1:** Spin and inferred magnetic field for pulsars with measured braking index $n$ taken from [22–27].

| Name           | $n$   | $\nu$ (s$^{-1}$) | $B_{0}^\text{inf}$ (10$^{12}$ G) | $B_{0}^\text{inf}=3$ (10$^{12}$ G) |
|----------------|-------|------------------|---------------------------------|-----------------------------------|
| J1846-0258     | 2.65  | 3.07             | 1.0                             | 49                                |
| B0531+21       | 2.51  | 30.2             | 12.4                            | 3.8                               |
| B1509-58       | 2.839 | 6.63             | 1.4                             | 15                                |
| J1119-6127     | 2.91  | 2.45             | 0.4                             | 42                                |
| B0540-69       | 2.140 | 19.8             | 12.4                            | 5.1                               |
| B0833-45       | 1.4   | 11.2             | 16.1                            | 3.4                               |

**Fig. 3:** Derivation of the inferred value of the magnetic field at the surface of the Crab pulsar B0531+21 using the analytical expression given by eq. (22).
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QED prediction, while a negative result of the observations would indicate the first limitation of the QED description of the vacuum.

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