Heavy \( qq \) interaction at finite temperature

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The first lattice QCD numerical study of heavy quark-quark potentials at finite temperature is reported. Using the quenched approximation, we evaluate the color anti-symmetric and symmetric potentials.

Recently, the diquark (a two-quark system) has attracted much attention with regard to high-energy phenomenology. Jaffe and Wilczek proposed that the recently discovered penta-quark state \((\Theta^+)^1\) is a bound state of \((ud)(ud)s\), where \((ud)\) represents for highly correlated \(u\) and \(d\) quark pairs.\(^2\) It is believed that at high baryon number density and low temperature there exists a family of color superconducting phases, due to the quark pairing driven by the BCS mechanism.\(^3\) (See Ref. 4) for a review of the history of diquarks and their role in high-energy reactions.)

A quark-quark system is a color anti-triplet (anti-symmetric) or sextet (symmetric) state:

\[
3 \times 3 = 3^* + 6,
\]

\[
\square \times \square = \square + \square.
\]

We believe that the quark-quark interaction is attractive and strong in the color anti-triplet channel, based on results obtained from the perturbation\(^5\) and instanton induced models.\(^6\) It is important to investigate the quark-quark potential using lattice QCD, which provides a non-perturbative and first principle basis for exploring the quark-quark interaction. To our knowledge, there has been only one such study, by Wetzorke and collaborators,\(^7\) in which diquark correlation functions were calculated.

In this work, we study the heavy quark-quark free energies at finite temperature that are obtained from the Polyakov line correlation (PLC). PLC was first investigated by McLerran and Svetitsky.\(^8\) The free energy \(F\) is given by

\[
e^{-\beta F} = \sum_{\phi} \langle \phi | e^{-\beta H} | \phi \rangle,
\]

where \( | \phi \rangle \) represents a state of gluons and heavy quarks. For the heavy quark-quark system of the color anti-triplet, we have

\[
| \phi \rangle = \epsilon_{abc} \psi^b(\vec{x}_1, t = 0) \psi^c(\vec{x}_2, t = 0) | \text{Gluons} \rangle.
\]

Here \(a, b\) and \(c\) are the color indices. Now, the summation in Eq. 2 is taken over \(a\) and all gluonic states. Using the relations\(^8\)

\[
\psi(\vec{x}, t) = L \psi(\vec{x}, 0),
\]

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\[
\{\psi^a(\vec{x}, t = 0), \psi^b(\vec{y}, t = 0)\} = \delta_{a,b}\delta_{\vec{x}, \vec{y}}, \quad (5)
\]

where

\[
L(\vec{x}) = \prod_{t=1}^{N_t} U_0(\vec{x}, t) \quad (6)
\]

and

\[
\epsilon_{abc}\epsilon_{ab'c'} = \delta_{b,b'}\delta_{c,c'} - \delta_{b,c'}\delta_{c,b'}, \quad (7)
\]

we obtain the free energies for the \(qq\) sector in the symmetric and anti-symmetric channels,

\[
\begin{align*}
\exp(-\beta F_s(R)) &= \frac{3}{4} \langle \text{Tr} L(\vec{x}_1)\text{Tr} L(\vec{x}_2) \rangle + \frac{3}{4} \langle \text{Tr} L(\vec{x}_1)L(\vec{x}_2) \rangle, \\
\exp(-\beta F_{as}(R)) &= \frac{3}{2} \langle \text{Tr} L(\vec{x}_1)\text{Tr} L(\vec{x}_2) \rangle - \frac{3}{2} \langle \text{Tr} L(\vec{x}_1)L(\vec{x}_2) \rangle, 
\end{align*}
\]

where \(F_s\) and \(F_{as}\) are the differences in the free energy for the cases with and without the static color symmetric (anti-symmetric) quarks located at \(\vec{x}_1\) and \(\vec{x}_2\), \(R = |\vec{x}_1 - \vec{x}_2|\) and \(\beta = 1/T\).

The formula (8) is gauge dependent, and therefore it requires the gauge fixing. We employ the stochastic gauge fixing quantization (SGFQ) with the Lorentz-type gauge fixing proposed by Zwanziger:

\[
dA^a_\mu d\tau = -\frac{\delta S}{\delta A^a_\mu} + \frac{1}{\alpha} D^{ab}_\mu(A)\partial_\nu A^b_\nu + \eta^a_\mu. \quad (9)
\]

Here, \(\alpha\), \(D^{ab}_\mu\) and \(\eta^a_\mu\) correspond to a gauge parameter, a covariant derivative and Gaussian random noise, respectively. The lattice formulation and a more detailed explanation of this algorithm can be found in Refs. 10–12).

In the quenched approximation, the system possesses \(Z_3\) symmetry. The gauge action is invariant under the transformation \(U_4(x) \rightarrow z U_4(x)\) or \(z^2 U_4(x)\) on a time-slice hyperplane, where \(z = \exp(2\pi i/3)\). In this transformation, \(\langle \text{Tr} L(\vec{x}_1)\text{Tr} L(\vec{x}_2) \rangle\) and \(\langle \text{Tr} L(\vec{x}_1)L(\vec{x}_2) \rangle\) obtain a factor of \(z^2\) or \(z^4\). After an infinite number of sweeps, the quantities in Eq. (8) vanish because \(1 + z^2 + z^4 = 0\). This is true also for the Polyakov line expectation value itself, and often \(|\text{Tr} L|\) or \((\text{Re}(\text{Tr} L)^3)^{1/3}\) is taken. The latter procedure brings the Polyakov lines around \(\pm 2\pi/3\) into regions around the positive real axis. This may be justified because this area is realized in full QCD, due to the dynamical quark effect. Therefore we restrict our simulations to the region satisfying \(-\pi/3 < \text{Phase } L < \pi/3\).

In order to calculate the color dependent Polyakov correlation functions at finite temperature, we performed the quenched \(SU(3)\) lattice simulation with the standard plaquette action. The values of the simulation parameters used are the same as in the previous study for the measurement of \(qq\) PLC functions in the QGP phase.\(^{13}\) The spatial lattice volume was \(24^3\), and the temporal lattice size was set to \(N_t = 6\), which determines the temperature as \(T = 1/N_t a\), where \(a\) is the lattice spacing. The corresponding critical temperature is estimated to be \(T_c \approx 256\) MeV in Ref. 14). The system temperature is changed when we vary the lattice cutoff, which
is determined by Monte Carlo renormalization group analysis.\textsuperscript{15) All of the PLCs are measured every ten Langevin steps and normalized by the value $\langle \text{Tr}L(0) \rangle^2$. We obtained between 3000 to 10000 data points after approximately 3000 steps were discarded as thermalization.

Typical behavior of the symmetric and anti-symmetric free energies at $T/T_c = 2.02, 3.04, 5.61$ is displayed in Fig. 1. The symmetric channel gives a repulsive force, while the anti-symmetric one gives an attractive force. As the system temperature is varied, all potentials change but their variations are observed to be small.

If there is a strong attractive force between quarks even at high temperature, diquark degrees of freedom should be taken into account for analyzing hadron production in high energy heavy ion collisions at RHIC and LHC. Therefore it is important to investigate the diquark force by the lattice QCD simulation. As a first step for such a study, we define the effective force

$$f(R,T)_{T^2} = -\frac{1}{T}(F(R+1) - F(R)).$$

The temperature dependence of the strength of this effective force is shown in Fig. 2. As the temperature increases, this strength decreases in both channels.

In the leading order perturbation (LOP), the coefficients of the color exchange terms in the symmetric and anti-symmetric channels are $C_{qq}[6] = +\frac{1}{2}$ and $C_{qq}[3^*] = -\frac{2}{3}$ respectively. We compare the force strengths in the symmetric and anti-symmetric channels and define their ratio $F_s/F_{as}$, which approaches the value $C_{qq}[6]/C_{qq}[3^*]$ =
Fig. 2. The temperature dependence of the effective force in the symmetric and anti-symmetric channels with Langevin step width $\Delta \tau = 0.03$.

$-0.5$ at short distances. Figure 3 displays values of this ratio evaluated at $T/T_c = 2.02, 3.04$ and $5.61$. We find that as the temperature increases, three values at short distances become comparable with the expected values: However, as the temperature decreases, it deviates from $-0.5$. There are several possible sources for this deviation at shorter distances for small $T$. One is numerical instability due to a long autocorrelation near $T_c$. If this is indeed the cause, then larger scale simulations in the vicinity of $T_c$ are required. Another possibility is the renormalization of PLC, i.e., $\langle TrL(R)L(0) \rangle = Z \exp(-F(R)/T)$. If $Z$ differs significantly from 1, we should take it into account when comparing $F$ with that obtained from the perturbation. Recently, the Bielefeld group has proposed that the proper normalization of PLC is determined by the comparison of the $T = 0$ $q\bar{q}$ potential data with the corresponding $T \neq 0$ $q\bar{q}$ singlet free energy at short distances. However, the determination for the normalization of the $qq$ channels using this method is not performed.

In the SGFQ algorithm with a Lorentz-type gauge fixing term, the $qq$ symmetric and anti-symmetric channels are studied by calculating the PLC functions. However, they are not gauge invariant. Figure 4 displays the gauge parameter $\alpha$ dependence of $F_s$ and $F_{as}$ for three values of $\alpha$ from 0.6 to 1.3. We find that the dependence on $\alpha$ in this range is small, and in particular, the basic features of the curves do not change.

In this paper we have reported the first non-perturbative lattice QCD simulation of the $qq$ symmetric and anti-symmetric free energies. Calculations were carried out
for both gauge dependent channels in the framework of the stochastic quantization with a Lorentz-type gauge fixing term. We find that the anti-symmetric free energy constitutes an attractive force, while the symmetric free energy constitutes an repulsive force.

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Fig. 4. Gauge parameter $\alpha$ dependence of the free energies in the symmetric and anti-symmetric channels at $T/T_c = 3.04$ ($\Delta \tau = 0.03$).

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