Precision Analysis Based on Complicated Error Simulation for the Orbit Determination with the Space Tracking Ship

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Abstract. The space tracking ship is a moving platform in the TT&C network. The orbit determination precision of the ship plays a key role in the TT&C mission. Based on the measuring data obtained by the ship-borne equipments, the paper presents the mathematic models of the complicated error from the space tracking ship, which can separate the random error and the correction residual error with secondary low frequency from the complicated error. An error simulation algorithm is proposed to analyze the orbit determination precision based on the two set of the different equipments. With this algorithm, a group of complicated error can be simulated from a measured sample. The simulated error groups can meet the requirements of sufficient complicated error for the equipment tests before the mission execution, which is helpful to the practical application.

1. Introduction
The space tracking ship is a very important part of the TT&C system in China, especially when the spacecraft is far away from our airspace. With the continuous improvement of the precision requirement of the orbit determination, the development of the measurement technology of the space tracking ship has also been greatly improved.

In the TT&C mission, both of the two set of the ship-borne equipments can track the target and accomplish the orbit determination. In order to choose the better result, it’s necessary to do the precision analysis before the mission execution. The key point is the simulation for the complicated error of the equipments.

The space tracking ship is a moving platform, which the tracking process is under dynamic conditions including swing, rotation, vibration etc. Therefore, the target tracking data needs to be corrected with the corresponding ship’s position and attitude. Except the system error and the random error, the tracking data includes the residual error generated by the correction process of ship-pose, hull’s deformation and wave refraction, which makes the main difference between the results measured by the ship and the ground stations.

Based on the analysis of the error character, the simulation of the complicated error is presented in this paper with spline function, wavelet technology and time series modeling technique. Furthermore, the comparison analysis between the two set of equipments is studied for the orbit determination precision, which can offer the reasonable reference for the result selection in the mission.

The remainder of this paper consists of three sections plus conclusions. Section 2 introduces the orbit determination method of the space tracking ship. The simulation method of the complicated error...
is presented in section 3. Section 4 discusses the precision comparison analysis of the two different equipments. Conclusions are resumed in section 5.

2. The Orbit Determination Method

Using the ship’s location \((\lambda, B, H)\) and the orbit elements \((a_0, e_0, i_0, \Omega_0, \omega_0, M_0)\) at one time \(t_f\), the orbit determination can be accomplished based on the following dynamic models.

The orbit elements described in 2000.0 geocentric celestial coordinate can be obtained by orbit propagation when the satellite state parameters are given. In order to get the initial simulated measured data of \((R_0, E_0, A_0)\), the results need to be transferred into the inertial system of the space tracking ship firstly. Then the measured data of the two sets of the equipment \((R_a, E_a, A_a)\) and \((R_b, E_b, A_b)\) can be obtained by adding the corresponding complicated error to the initial \((R_0, E_0, A_0)\). Using the measured data, the orbit elements \((a_a, e_a, i_a, \Omega_a, \omega_a, M_a)\) and \((a_b, e_b, i_b, \Omega_b, \omega_b, M_b)\) can be calculated with the Unit Vector Method (UVM). Hence it’s easily to do the comparison analysis of the orbit determination. The flow chart is shown in Figure 1.

The complicated error \(z(t)\) is given by

\[
\begin{align*}
2
\end{align*}
\]
where $s(t) + c(t)$ is the system error which needs processing with the spline function and wavelet frequency-division. $s(t)$ can be approximated by spline function with sparse equidistant nodes where $c(t)$ is approximated by spline function with dense equidistant nodes instead. $e(t)$ is the random error which can be approximated by the auto-regression model.

In this paper, the cubic B-spline is selected, which is given by

$$z(t) = s(t) + c(t) + e(t)$$

where $a_j$ are the model coefficients which can be estimated by the least square method, $M_i$ is the basis function, which has the form of

$$M_i(t) = \begin{cases} 0, & |t| \geq 2 \\ \frac{|t|}{2} - t^2 + \frac{2}{3}|t|^3 < 1 \\ \frac{|t|^3}{6} + t^2 - 2|t|^3 + \frac{4}{3}|t|, & 1 \leq |t| < 2 \end{cases}$$

The auto-regression model of $p$ th-order can be expressed by

$$e_t = \varphi_1 e_{t-1} + \ldots + \varphi_p e_{t-p} + \varepsilon_t$$

where $\varphi_1, \varphi_2, \ldots, \varphi_p$ are the auto-regression coefficients, $\varepsilon_t$ is the white noise.

### 3.2. The complicated error simulation

The complicated error samples are provided by the residual error from the intercomparison analysis between the precise orbit and the flight test or the star tracking result. The error fitting model is

$$\hat{z}(t) = \hat{s}(t) + \hat{c}(t) + \hat{e}(t) = \sum_{j=1}^{m+1} \hat{a}_j M_4 \left( \frac{t - T_j}{h} \right) + \sum_{j=1}^{n+1} \hat{b}_j M_4 \left( \frac{t - T_j}{h} \right) + \hat{\varphi}_1 e_{t-1} + \ldots + \hat{\varphi}_p e_{t-p} + \varepsilon_t$$

where $\hat{a}_1, \hat{a}_0, \ldots, \hat{a}_{m+1}$ can be considered as the sample data of the random variable $\omega$. Assuming $\omega \sim U(t_1, t_2)$ where $U$ means uniform distribution, $t_1$ and $t_2$ can be estimated with the maximum likelihood function, which are $t_1 = \min_j a_j$ and $t_2 = \max_j a_j$. In the simulation process of $s(t)$, $\hat{a}_{-1}, \hat{a}_0, \ldots, \hat{a}_{m+1}$ are given by the sampling from $\omega \sim U(t_1, t_2)$. Consequently, the complicated error simulation can be completed by the following steps:

**Step 1**: establish the fitting model $\hat{z}(t)$ after the spline fitting and wavelet frequency-division of the sampled error data.

**Step 2**: calculate the parameters $\hat{a}_{-1}, \hat{a}_0, \ldots, \hat{a}_{m+1}, \hat{b}_{-1}, \hat{b}_0, \ldots, \hat{b}_{n+1}$ and $\hat{\varphi}_1, \ldots, \hat{\varphi}_p$.

**Step 3**: calculate $\hat{t}_1$ and $\hat{t}_2$, and get $a_1, a_0, \ldots, a_{m+1}$ by sampling from $\omega \sim U(t_1, t_2)$.

**Step 4**: simulate $s(t)$ by using the equation $s(t) = \sum_{j=1}^{m+1} a_j M_4 \left( \frac{t - T_j}{h} \right)$.

**Step 5**: simulate $c(t)$ which the principle is the same as the simulation of $s(t)$.

**Step 6**: simulate $e(t)$ with $\hat{\varphi}_1, \ldots, \hat{\varphi}_p$ based on the auto-regression model of $P$ th-order.

**Step 7**: Finish the complicated error simulation with the equation (1).

### 4. The Simulation Method for Complicated Error
Taking the measured data from the equipment A in a mission as the test data, the real error and the simulated error is shown in figure 1.

![Figure 1. The real error and the simulated results](image1)

**Figure 2.** The real error and the simulated results

According the calculating process showed in Figure 1, the precision analysis is carried out with the residual error sample from the equipment A and B in a ship’s mission. The results are presented in Table 1.

| Table 1. The Precision Analysis Comparison Between Equipment A and B |
|---------------------------------------------------------------|
| **Equipment**       | **A**     | **B**     | **A**     | **B**     |
|---------------------|-----------|-----------|-----------|-----------|
| △a/(m)              | 363.33    | -330.31   | -271.99   | -292.56   |
| △e                  | 8.17e-5   | -8.29e-5  | -4.24e-5  | -4.27e-5  |
| △i/(°)              | 0.0443252 | 0.045014  | 0.0463275 | 0.0466115 |
| △ΩG/(°)             | -0.077497 | -0.078611 | -0.0703   | -0.070932 |
| △λ/(°)              | 0.093879  | 0.098608  | 0.057     | 0.056     |
| △R/(m)              | 2833.57   | 2886.68   | 2014.44   | 2036.25   |
| △V/(m/s)            | 7.9       | 7.8       | 6.4       | 6.5       |

5. **Conclusion**

The ship-swaying and hull’s deformation make the tracking error of the space tracking ship much more complicated than the one of the ground station. In the near future, the space tracking ship still has the irreplaceable advantages in our TT&C system. Hence it’s helpful to the engineering application with the complicated error simulation. In this paper, the errors of the different frequency are separated successfully with the spline function, wavelet frequency-division and auto-regression model. Based on this, the complicated error is simulated by modeling of the different error components. The orbit determination simulation shows the method’s validity, which could be used for equipments’ evaluation in the mission preparation phase.

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