Dark Solitons with Majorana Fermions in Spin-Orbit-Coupled Fermi Gases

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Solitons, which maintain their solitary wavepacket shape while traveling, are crucially important in many physical branches. Recently, dark solitons have been experimentally observed in spin-balanced ultra-cold degenerate Fermi gases. Here we show that a single dark soliton can also exist in a spin-orbit-coupled Fermi gas with a high spin imbalance, where spin-orbit coupling favors uniform superfluids over non-uniform Fulde-Ferrell-Larkin-Ovchinnikov states, leading to dark soliton excitations in highly imbalanced gases. Above a critical spin imbalance, two topological Majorana fermions (MFs) without interactions can coexist inside a dark soliton, paving a way for manipulating MFs through controlling solitons. At the topological transition point, the atom density contrast across the soliton suddenly decreases to zero at the topological transition point, which may be used to experimentally detect topological solitons.

Recently MFs have attracted tremendous attention in various physical systems because of their fundamental importance as well as potential applications in fault-tolerant quantum computation. In this context, SO coupled fermionic superfluids have their intrinsic advantages for MFs because of their high controllability. Therefore another important question is whether topological Majorana excitations can exist inside dark solitons if such topological defects do exist in SO coupled DFGs with large spin imbalances.

In this Letter, we address these two important questions by studying dark solitons in DFGs trapped in one-dimensional (1D) harmonic potentials with the experimentally already realized SO coupling and spin imbalances. Here the spin imbalance is equivalent to a Zeeman field. In the absence of SO coupling, the FFLO state with an oscillating order parameter amplitude is the ground state with a large Zeeman field, which cannot support dark solitons. With SO coupling, we find

(i) SO coupling suppresses the FFLO state, leading to uniform BCS superfluids that support dark solitons. The parameter region for dark solitons and their spatial properties are obtained.

(ii) For substantially large spin imbalances, we find remarkably that two MFs can coexist inside a dark soliton without any interaction, beyond the general expectation that two MFs with overlapping wave-functions interact, leading to energy splitting that destroys MFs. Such solitons are topological solitons to be distinguished from solitons without MFs.

(iii) The experimental signature of MFs inside the dark soliton in the local density of states (LDOS) is characterized, which show an isolated zero energy peak at the center of the soliton. Moreover, the density contrast across the soliton suddenly decreases to zero at the topological transition point, which may be used to experimentally detect topological solitons.
System and Hamiltonian: Consider a SO coupled DFG confined in a 1D harmonic trap with the transversal confinement provided by a tightly focused optical dipole trap. The many-body Hamiltonian of the system can be written as

\[ H = \int dx \hat{\Psi}^\dagger(x) H_s \hat{\Psi}(x) - g \int dx \hat{\Psi}^\dagger(x) \hat{\Psi}^\dagger(x) \hat{\Psi}_\uparrow(x) \hat{\Psi}_\downarrow(x), \]

where the single particle grand-canonical Hamiltonian \( H_s = -\hbar^2 \partial_x^2/2m - \mu + V(x) + H_{SOC} + H_z \), the harmonic trapping potential \( V(x) = m \omega^2 x^2/2 \), \( \mu \) is the chemical potential, \( g \) is the attractive s-wave scattering interaction strength between atoms that can be tuned through Feshbach resonances, \( m \) is the atom mass, and \( \omega \) is the trapping frequency. \( \hat{\Psi}(x) = [\hat{\Psi}_\uparrow(x), \hat{\Psi}_\downarrow(x)]^T \) with the atom creation (annihilation) operator \( \hat{\Psi}_\uparrow(x) (\hat{\Psi}_\downarrow(x)) \) at spin \( \uparrow \) and position \( x \). We consider the exact Rashba and Dresselhaus SO coupling \( H_{SOC} = -i \hbar \alpha \partial_x \sigma_y \), where \( \sigma_i \) are Pauli matrices. The Zeeman field \( H_z = V_z \sigma_z \) generates the spin imbalance. This type of SO coupling and Zeeman field have been realized experimentally for cold atom Fermi gases using two counter-propagating Raman lasers that couple two atomic hyperfine ground states (i.e., the spin).

Within the standard mean-field approximation, the fermionic superfluids can be described by the Bogoliubov-de Gennes (BdG) equation

\[ \begin{pmatrix} H_s & \Delta(x) \\ \Delta(x)^* & -\sigma_y H_s^* \sigma_y \end{pmatrix} \begin{pmatrix} u_n \\ v_n \end{pmatrix} = E_n \begin{pmatrix} u_n \\ v_n \end{pmatrix}, \]

where \( u_n = [u_n \uparrow(x), u_n \downarrow(x)]^T \), \( v_n = [v_n \uparrow(x), -v_n \downarrow(x)] \) are the Nambu spinor wave-functions for the quasi-particle excitation energy \( E_n \), the order parameter \( \Delta(x) = -g \langle \hat{\Psi}_\downarrow(x) \hat{\Psi}_\uparrow(x) \rangle = -g/2 \sum_{E_n < E_F} u_n \uparrow \dagger u_n \downarrow f(E_n) + u_n \downarrow \dagger u_n \uparrow f(-E_n) \), and the atom density \( \rho_\uparrow(x) = 1/2 \sum_{E_n < E_F} |u_n \uparrow|^2 f(E_n) + |v_n \downarrow|^2 f(-E_n) \) with the energy cut-off \( E_c, f(E) = 1/(1 + e^{E/k_B T}) \) is the quasi-particle Fermi-Dirac distribution at the temperature \( T \). With the constraints of a fixed total number of atoms \( N = \int dx [\rho_\uparrow(x) + \rho_\downarrow(x)] \) and the definition of the order parameter, Eq. (2) can be solved self-consistently. To obtain a stationary soliton excitation in the superfluid, we choose \( \tanh(x) \) as the initial order parameter, and then solve the BdG equations self-consistently until the order parameter and density converge.

To solve the BdG equation, we expand \( u_n \) and \( v_n \) on the basis states of the harmonic oscillator to convert the equation to a diagonalization problem of a secular matrix. We consider \( N = 100 \) atoms with 300 harmonic oscillator states for the wave-function expansion. The energy cut-off \( E_c = 240 \hbar \omega \), which is large enough to ensure the accuracy of the calculation \( [23] \). We choose the single particle Fermi energy \( E_F = N \hbar \omega / 2 \) (neglecting zero point energy) in the absence of the SO coupling and Zeeman field and the harmonic oscillator length \( x_s = \sqrt{\hbar/m \omega} \) as the units of energy and length. For 1D Fermi gases, the interaction parameter \( g = -2\hbar^2/(ma_1D) \) with an effective 1D scattering length \( a_{1D} \). A dimensionless parameter \( \Gamma = -mg/n(0)\hbar^2 = \pi x_s/\sqrt{N a_{1D}} \), which is proportional to the ratio between the interaction and kinetic energy at the center, can be used to characterize the interaction strength \( [23] \). Here \( n(0) \) is the density at the trap center in Thomas-Fermi approximation. In experiments \( [23] \), this value can be as large as 8.0. We choose \( \Gamma = \pi \) in most of our calculations.

Dark solitons in spin imbalanced DFGs: In Fig. 1 we plot the order parameter and density profiles for an imbalanced SO coupled Fermi gas with different interaction strengths. With increasing interactions, both \( \Delta \) and the depth of the soliton increase while \( \mu \) (not shown here) decreases, signalling the crossover from BCS superfluids to BEC molecule bound states. The total size of the gas is also enlarged. Without SO coupling, the existence of a dark soliton leads to the depletion (enhancement) of spin \( \uparrow \) \( \downarrow \) component even with small Zeeman fields, thus the total atom density only has a small depletion at the soliton center. This is in sharp contrast to Fig. 1c,d with SO coupling, where a strong depletion of the total density (also for each spin compo-
nent) in the dark soliton is observed. In Fig. 3(b), we observe two length scales for the dark soliton. One is $K^{-1}_F$ defined using the local density approximation, corresponding to the steep slope and the oscillation wavelength. The other is the coherence length $\xi = \hbar v_F/\Delta$ with Fermi velocity $v_F = \hbar K_F/m$, corresponding to the smoother oscillation slope [17, 54], which is about $5K^{-1}_F, 2.2K^{-1}_F, 1.2K^{-1}_F, K^{-1}_F$ for $\Gamma = 0.65\pi, \pi, 1.5\pi, 2.5\pi$, respectively. These two scales are equivalent in the BEC limit, where the oscillation structure of the soliton vanishes.

The physical mechanism for the dark soliton in an imbalanced Fermi gas roots in the SO coupling. It is well known that without SO coupling, the ground state of an imbalanced Fermi gas is the FFLO phase [23] with a spatially oscillating order parameter amplitude. The FFLO state is dramatically suppressed with increasing SO coupling as shown in Fig. 2(a) because the BCS type of zero total momentum Cooper pairing can be formed in the same helicity band, which is energetically preferred than the FFLO state that is formed through the pairing between atoms in two different helicity bands [55, 56] with non-zero total momentum. In principle, a dark soliton can be created in all these phases. However, a soliton engineered in an oscillating FFLO state cannot manifest itself and only changes the phase of the FFLO state. In this sense, SO coupling makes it possible to generate a single dark soliton excitation with a high spin imbalance.

Two MFs inside a dark soliton: In a 1D spin imbalanced homogenous SO coupled superfluid, it is expected that two MFs appear at the edges when the system becomes topological with $V_z > \sqrt{\mu^2 + \Delta^2}$. For a Fermi gas in a harmonic trap, we replace $\mu$ with the effective chemical potential $\mu(x) = \mu - V(x)$ using the local density approximation. The corresponding phase diagram in a harmonic trap is plotted in Fig. 2(a). In the region $\Delta < V_z < \sqrt{\mu^2 + \Delta^2}$, the superfluid has a phase separation structure where the normal superfluid at the center is surrounded by a topological superfluid. Based on the phase separation structure, this region is called the partial topological superfluid (PTS) phase. The phase boundary between topological and normal superfluids located approximately at $|x| \approx \sqrt{2(\mu - \sqrt{\mu^2 + \Delta^2})/\hbar \omega^2}$. In this parameter region, there are four zero mode MFs, two located at the phase boundary between topological and normal superfluids, and two at the edges of the trap. When $V_z > \sqrt{\mu^2 + \Delta^2}$, the whole region becomes topological. This region belongs to the topological superfluid (TS) phase. Fig. 2(a) also shows that TS region is enhanced with increasing SO coupling because of the decrease of $\mu$.

The creation of a dark soliton at the trap center does not change the phase diagram shown in Fig. 2(a). However, the dark soliton induces two local gapped excitations (blue and green lines in Fig. 2(b)) that are similar to Andreev bound states in a vortex [17, 54]. When the Fermi gas enters the PTS region, two zero energy states (thus four MFs) appear and there are two minigap excitations inside the soliton. With sufficient large Zee- man fields, the Fermi gas enters the TS region and the maximum of the order parameter $\Delta_0$ decreases sharply. Without a dark soliton, there is only one zero energy mode (two MFs) located at the edge. In the presence of a dark soliton at the center, there are two zero energy states (four MFs), one around the edges and one inside the dark soliton.

To confirm that the zero energy excitations inside the dark soliton are MFs in the TS region, we consider a linear combination of the Bogoliubov quasi-particle operators $\gamma_0$ for states with $E_n \sim 0$ (here $E_2 > E_1$) in the numerical calculation to obtain spatially localized states. Due to the particle-hole symmetry, the Bog-
ubov quasi-particle operator should satisfy $\gamma_{0^n} = \gamma_{-n}$. With the choice $\gamma_L = (\gamma_{0^+} + \gamma_{0^+} + \gamma_{0^-} + \gamma_{0^-})/2$, $\gamma_R = (\gamma_{0^+} + \gamma_{0^+} + \gamma_{0^-} - \gamma_{0^-})/2$, $\gamma_{S1} = (\gamma_{0^+} + \gamma_{0^-})/2i$, $\gamma_{S2} = (\gamma_{0^+} + \gamma_{0^-})/2$, we obtain $\gamma^\dagger_L = \gamma_R$, $\gamma^\dagger_R = \gamma_L$, and $\gamma_{S}^\dagger = \gamma_{S}$ with $\sigma = 1, 2$, indicating that $\gamma_L$, $\gamma_R$ and $\gamma_{S}$ are all Hermitian Majorana operators. Their wave-functions, which respectively locate at the left and right edges, and inside the soliton, are plotted in the left panel of Fig. 3. Our numerical calculation results with $u_\uparrow = v_\uparrow$ and $u_\downarrow = v_\downarrow$ also reflect the characteristics of MFs. The wave-functions of zero energy excitations inside the soliton behave like $\sim \cos(\pi K_F x/2) \exp(-x^2/\xi_0^2)$ and $\sim \sin(\pi K_F x/2) \exp(-x^2/\xi_0^2)$, similar to Andreev bound states [17] but with $\xi_0$ larger than $\xi$. Clearly, two MFs can coexist inside the dark soliton without interactions, which are very different from widely known MFs in vortices or nanowire ends that interact due to wave-function overlapping, leading to energy splitting that destroys the zero energy states. The vanishing interactions between MFs inside the dark soliton can be understood through Kitaev’s toy model [57], where the interaction is proportional to $-\frac{i}{2} \cos(\delta \phi / 2) \gamma_{S1} \gamma_{S2}$ with the phase difference $\delta \phi$ between two sides of the soliton and the hopping parameter $t$. For the special case of a stationary soliton (dark soliton), $\delta \phi = \pi$ and there is no interaction.

**Experimental signature of topological solitons and MFs:** In the right panel of Fig. 3 we present the local density of states $25$ of the Fermi gas, $\rho_{\sigma}(x, E) = \sum_{E_n < E} \frac{[n_{\sigma n}^* \delta(E - E_n) + v_{\sigma} \delta(E + E_n)]}{2}$ with $\sigma = \uparrow, \downarrow$, which reflect the zero energy excitations at the places where MFs are locally accommodated. The LDOS could be experimentally measured using spatially resolved radio-frequency (rf) spectroscopy [58]. Clearly, in the TS region, the zero energy MF states appear at $x = 0$, where the dark soliton locates.

In experiments, the soliton structure can be measured by detecting the density contrast $P_\sigma = (n_{\sigma max} - n_{\sigma min})/n_{\sigma max}$ with the maximum $n_{\sigma max}$ and minimum $n_{\sigma min}$ of the density for spin $\sigma (P_\uparrow$ for the total density) in the soliton region. In Fig. 4 we plot the density contrast as a function of $V_z$. We see both $P_\uparrow$ and $P_\downarrow$ decrease, while $P_\uparrow$ is almost a constant with increasing $V_z$ before the appearance of MFs inside the soliton in the TS region. However, the soliton structure almost vanishes in the TS region with two MFs accommodating the soliton. The sudden decrease of the density contrast at the topological transition point is mainly due to the sharp decrease of the pairing order parameter, as shown by $\Delta_{\sigma}$ in Fig. 2(b), leading to the sudden decrease of the number of atoms participating in the pairing around the soliton. We note that this sharp change of $P_\sigma$ also occurs in Fermi gases without the trap, which we have confirmed in the lattice model. Moreover, the appearance of MFs inside the soliton further suppresses the density contrast for spin $\uparrow$, as the local MFs are capable of filling the notch. This suppression effect is notable only around the transition point. The sudden disappearance of the density contrast across the dark soliton provides another experimental signature for the appearance of MFs inside the dark soliton. From the insets, we see that the soliton density $n_1$ has a convex structure for $V_z > 0.53E_F$, where the density inside the soliton is larger than its surrounding. The transition to this convex structure leads to the kinks around $V_z = 0.53E_F$ observed in Fig. 2(b) and Fig. 3. This convex structure of soliton density is caused by the local quasiparticle excitations. In fact, without taking into account of such quasiparticle contributions to the density, $P_\uparrow$ is almost zero.

The 1D SO coupling and Zeeman fields considered here have already been achieved for $^{40}$K and $^{6}$Li fermionic atoms by coupling two hyperfine ground states using two Raman laser beams in experiments [50, 51, 53]. In experiment, the SO coupling and Zeeman field strength can be tuned through varying the laser intensity [59] or the setup of the laser beams. The SO coupling can be as large as $\alpha K_F \sim E_F$ and a Zeeman field can be readily tuned to $V_z \sim E_F$. The realization of 1D Fermi gases and the dark soliton can be similar as that in recent experiments [15], where an elongated 1D Fermi gas is confined in a harmonic trap with cylindrical symmetry (radial trapping frequency much larger than axial one) using a combination of weak magnetic trap (axial) and tightly focused optical trap (radial). Dark solitons can be experimentally created via phase imprinting [12, 13, 14], where a half of the cloud is shortly interacted with a laser beam to acquire the phase difference.

**Conclusion** To conclude, we showed that a single dark soliton excitation can exist in an imbalanced DFG with
SO coupling, in sharp contrast to the FFLO state without SO coupling. With a substantial spin imbalance, we found that two MFs can coexist inside one single dark soliton without interactions, which provides a new avenue for experimentally observing and manipulating MFs by controlling solitons as well as creating Majorana trains by engineering soliton trains [60].

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