A Robust GPS signal Acquisition Technique using Discrete Wavelet Transform

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Abstract

A novel Global Positioning System (GPS) signal acquisition scheme based on Discrete Wavelet Transform (DWT) is proposed to improve acquisition performance. Acquisition in GPS system is primary and important step to calculate the code phase in Pseudo Random Noise (PRN) code and Doppler shift in carrier frequency of received GPS signal, which is used for continuous tracking of the satellite. The performance of a GPS receiver system depends on the ability to precisely measure the code range and Doppler shift continuously in the presence of noise. Conventional GPS receivers use Fast Fourier Transform (FFT) to carry out convolution in the acquisition process. This method is unable to precisely acquire the GPS signal in the presence of noise. In this paper, a substitute algorithm using DWT is proposed to perform convolution in order to carry out robust, less complex GPS signal acquisition. The proposed algorithm is implemented on Intermediate Frequency (IF) GPS signal with different levels of noise and also compared with the conventional FFT search algorithm. The proposed algorithm allows implementation of the FFT with a reduced number of samples (2500 instead of 5000 samples), is implemented by collecting the one millisecond period data sampled at 5MHz. Performance of the algorithms are compared based on correlation ratio. Results show that the proposed algorithm using DWT outperforms the FFT search method of acquisition.

Keywords: GPS; acquisition; convolution; FFT; Discrete Wavelet Transform.

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1. Introduction

Global Positioning System (GPS) is a satellite-based navigation system\(^1\). In GPS system, Code Division Multiple Access (CDMA) with Direct-Sequence Spread Spectrum (DSSS) technique is used. It utilizes a PRN sequence with chip rate of 1.023MHz to spread the navigational data of 50Hz signal over a wider spectrum. This multiplexed signal is transmitted over a carrier frequency of 1575.42MHz using Binary Phase Shift Keying (BPSK) modulation. The length of the each GPS Coarse/ Acquisition (C/A) code is 1023 chips with 1ms period and used by different satellites, which are quasi-orthogonal\(^2\).

A GPS receiver receives RF signal through antenna and the signal is preprocessed in the RF frontend block to proper amplitude and gives it to down-converter. The carrier frequency of the GPS signal is converted to a desired output IF using a Local Oscillator (LO) in the RF frontend. An analog to digital converter (ADC) is used to digitize the IF GPS signal. After the signal is digitized, software is used to process the signal. Acquisition is the first signal processing operation performed on the IF GPS signal\(^3\). Acquisition identifies the satellites visible to the receiver and provides the estimation of the Doppler shift in carrier frequency and delay in C/A code of the satellite signals. These parameters are given as input to the tracking, is used to find the phase transition of the navigation data. After identifying the available satellites and acquiring their signal parameters, tracking is done through parallel channels\(^4\).

In tracking, C/A code and carrier are removed by measurement of the code phase and Doppler frequency precisely. So performance of acquisition processes directly influence the performance of tracking operation. The most affecting source of acquisition error is due to the noise on the transmission channel. The presence of the noise affects seriously the precision in the measurements of acquisition process.

On the other hand, there are some situations where GPS system will not work like in high multi-path environments, and in the presence of some intentional or unintentional jamming signals\(^5\). Jamming signals are easy to produce. Similar to transmission channel noise, it raises the level of background against which the GPS signal must be detected. Such jammer signal can be produced by just adding some white Gaussian noise \( N(10, 0.1) \) with some mean \( (10) \) and variance \( (0.01) \) to the received signal.

Hence, the objective of this paper is to develop a robust GPS signal acquisition system that improves the performance of the system over noisy transmission channel and in the presence of other sources of noise. An acquisition algorithm based on DWT is proposed for a robust GPS signal acquisition system and to facilitate its implementation. Correlation ratio is used as parameter to analyze the performance of the algorithm. Correlation ratio is defined as the ratio of difference between first two maximum peak magnitudes obtained in the search method to the second maximum of correlation peak i.e.

\[
\text{Correlation ratio} = \frac{\text{First max peak value} - \text{Second max peak value}}{\text{Second max peak value}} 
\]  

(1)

DWT is used in the proposed algorithm to compute correlation in the process of acquisition. In this algorithm DWT is mainly used to de-noise the GPS signal and to decrease the number of samples used there by complexity of acquisition process is simplified. To de-noise the GPS signal soft thresholding is applied to the wavelet coefficients obtained from the DWT. In this paper, one millisecond GPS signal is taken to implement the algorithms, which is sampled at 5MHz. So data contains 5000 samples for 1ms duration. DWT is used to carry out the FFT on 2500 points instead of 5000 point FFT without degrading the performances of the acquisition system. One of the very important characteristics of wavelet transform, concentrating the energy of the signal in some coefficients, allows to apply FFT on reduced number of points. Hence the complexity of the acquisition system is simplified.

2. Wavelet Transform

Wavelet transformation became an extremely useful tool for doing high performance signal processing of non-stationary and transient signals like GPS signals, because it provides an alternative to conventional Fourier transform. An important feature of wavelet transform is its high precision in time and frequency resolution. Wavelet
transform leads to relatively low complexity in computation. It uses a small wave called mother wavelet that has finite energy to analyze the signals. It allows localization in both the time domain and frequency domain using translations of the mother wavelet and dilation of mother wavelet respectively. It presents the signal in another representation called wavelet coefficients which is more useful form for some applications like data compression, de-noising etc. The wavelet coefficients are calculated by applying translation and dilation operations to the mother wavelet. This transformation of signal into wavelet coefficients is called wavelet transform. The wavelet transform is applied to many types of signal processing applications. Basically wavelet transform is of two types, continuous wavelet transform (CWT) and discrete wavelet transform (DWT). If the signal is continuous in time, the wavelet transform is called continuous wavelet transform. It contains lot of redundant information, so if it is uniformly sampled then it is called discrete wavelet transform.

2.1. Discrete Wavelet Transform

In the DWT, signal can be analyzed by passing it through an analysis filter bank. This analysis filter bank consists of a low pass and a high pass filter at each decomposition stage. When a signal passes though these filters, it splits into two bands at each stage. The low pass filter, which corresponds to averaging information, extracts the coarse information of the signal. The high pass filter, which corresponds to a differentiation operation, extracts the detail information of the signal. The output of the filtering operation is the decimated by two and these are called approximate and detailed coefficients respectively. Similarly, signal can be synthesized from wavelet coefficients by reverse operation. Synthesis filter bank consists of up sampling followed by a low pass filter and a high pass filter at each stage of synthesis. Analysis filter bank and synthesis filter banks of single stage DWT is shown in Fig. 1.

![Fig. 1(a) Analysis filter bank (b) Synthesis filter bank](image)

3. Signal acquisition

Acquisition is a synchronization process that gives estimates of the code offset and the carrier Doppler shift. These parameters are used to initialize the tracking loop. GPS signal acquisition is a two dimensional search process in which a locally generated C/A code and IF carrier are correlated with the received satellite signal. The correct correlation is identified by measurement of the output power of the correlation. In other words, when both the code and carrier Doppler matches with the received signal, the signal is de-spread and a carrier is recovered from the GPS signal. The result of the two dimensional search is an estimate of the code offset to within one sample of digitised C/A code and the Doppler shift to within half the Doppler search bin size (1000 Hz). The short length of the GPS signal required for acquisition is length of one C/A code (1 ms). In a conventional receiver, acquisition is performed using FFT search algorithm.

3.1. FFT search algorithm

Acquisition is a process of searching sequentially through two dimensions combining frequency and code phase. It is an exhaustive process to search the correct Doppler and code shifts. If one of the parameter could be eliminated from the search procedure, it will speed up the process significantly. Parallel search scheme based on FFT and Inverted FFT (IFFT) are being used traditionally for acquisition. FFT search algorithm converts the GPS signal from
time domain into frequency domain and thus it eliminates one parameter. Fig. 2 illustrates the block diagram of FFT search algorithm.

As shown in Fig. 2, the input GPS IF signal is expressed by Equation (2) and the locally generated carrier signals is expressed as equation (3).

$$ y_{IF}(t_k) = A d(t_k - \tau) c(t_k - \tau) \sin (2\pi (f_{IF} - f_d) t_k - \varphi) + n_k $$  

$$ y_c(t_k) = c(t_k - \tau) \exp (j2\pi (f_{IF} - f_d) t_k) $$

Where, $y_{IF}(t_k)$ is the input GPS IF signal at sample time $t_k$ and $y_c(t_k)$ is the locally generated carrier signal. $A$ is the signal amplitude which indicates the power of the signal, $d(t_k)$ is the satellite navigational data with a rate of 50bit/s, $c(t_k)$ stands for the C/A code of one GPS satellite, $\tau$ is the delay of the code chip. $f_d$ is the Doppler shift of carrier, $f_{IF}$ is the incoming digitized GPS IF signal frequency, $\varphi(t_k)$ represents the initial phase of the GPS satellite, $n_k$ is a white Gaussian noise ($N(0,0.01)$).

The correlation operation between digitised carrier signal replica and the digitised input GPS IF signal is defined as the following equation.

$$ z(n) = \sum_{m=0}^{N-1} y_c(m) y_{IF}(m+n) $$  

(4)

where, $m$ represents the index of the sampling time sequence and $N$ represents the number of samples. The circular correlation is defined in FFT as

$$ \text{FFT}[z(n)] = \text{FFT} \left[ \sum_{m=0}^{N-1} y_c(m) y_{IF}(m+n) \right] $$

$$ = \text{FFT} [ y_c(n) ] \text{FFT}^* [ y_{IF}(n) ] $$

(5)

where FFT is Fast Fourier Transform and FFT* is complex conjugate of FFT.

Equation (5) can be written in frequency domain as
\[ Z(K) = Y_c(K)Y_{\text{IF}}^*(K) \]  

Correlation function in time domain can be written as

\[ z(n) = \text{IFFT}(Z(K)) = \text{IFFT}(Y_c(K)Y_{\text{IF}}^*(K)) = \text{IFFT}([\text{FFT}[y_c(n)])\text{FFT}^*[y_{\text{IF}}(n)]]) \]  

The highest absolute value of correlation power obtained from equation (7) gives the required Doppler shift and code phase of GPS signal.

### 3.2. Proposed algorithm

In this section, GPS signal acquisition with the use of DWT algorithm is proposed. DWT is applied to carrier removed GPS signal and the locally generated PRN code. It de-noise the GPS signal and decreases the length of signals used to calculate the product of convolution by FFT. The proposed algorithm is summarized as follows and is depicted in Fig. 3

- Transform the signal to the wavelet approximate coefficients, \( a[n] \) and detailed coefficients, \( d[n] \)
- Apply thresholding function \( S_\tau \) with a threshold parameter \( \tau \neq 0 \), i.e. \( S_\tau (a[n]) \)
- Apply a non coherent correlation using FFT search algorithm (as in Fig. 2)
- Reconstruct the signal from the shrunk wavelet coefficients

One of the important feature of the wavelet transform is de-noising i.e. it separates the signal from noise by thresholding wavelets coefficients. In the proposed algorithm Donoho’s soft thresholding is used

\[ S_\tau(x) = \begin{cases} 
   x - \tau \text{sgn}(x) & \text{if } |x| > \tau \\
   0 & \text{if } |x| \leq \tau 
\end{cases} \]  

This shrinks the coefficients towards zero. Threshold parameter \( \tau \) is related to the variance of the Gaussian noise to the wavelet coefficients.

![Fig.3 Acquisition using Discrete Wavelet Transform](image-url)
4. Results and Discussions

GPS IF signal is used for the implementation of the proposed algorithm, consists of 1ms duration, carrier of 1.25MHz, sampled at 5MHz and signal to noise ratio is fixed at –20dB.

The acquisition of GPS signal is implemented using FFT search and DWT based search algorithms. In both the algorithms, Doppler search range observed is ±10KHz in steps of 500Hz, the search index for Doppler frequency is from 1 to 41 (1.24MHz – 1.26MHz) and the C/A code phase is from 1 to 5000 samples. It is observed that a maximum correlation peak is detected at 3126 C/A code phase sample and 27th frequency bin (i.e. 1.253MHz), which indicates that the beginning of C/A code is located at this sample. For both the algorithms the acquired Doppler shift and code phase are observed to be the same.

Figure 4 shows the correlation in 2D plot in the case of noisy signal with N (0, 0.01) as a function of code phase, frequency and correlation peak. From this figure it is found that the correlation peak obtained by proposed method (2505) is greater than by FFT search method (2428). Hence the proposed method (see Fig. 4(b)) gives more accurate peak than the one obtained when using FFT search algorithm (see Fig. 4(a)).

Figure 5, presents the correlation in 2D plot in the case of noisy signal with N (15, 0.1). Noise signal (i.e. N(15,0.1)) as an intentional interference which is easy to produce and can cause a perfectly functioning receiver to stop working completely. From this figure it is found that the proposed method still works (Fig. 5(b)) and the FFT method fails to detect the signal (Fig.5 (a)).

Acquisition processes with the proposed algorithm and FFT search method are implemented with different levels of noisy signals i.e. Gaussian noise of different mean values of 0, 0.1, 1, 10 and 15. Figure 6 shows 1D plot of obtained correlation peaks for different noisy signals. From this it is observed that the proposed method is robust for the noise. Table 1 summarizes the correlation peaks and correlation ratios obtained by the acquisition processes using FFT and proposed algorithm for different levels of noisy signals. The largest value of correlation ratio for FFT search method is 0.2356 which is smaller than the correlation ratio obtained by proposed method. Whereas the correlation ratio observed reaches to zero it means that there is second peak which is equal in magnitude to first peak. From Table1, the minimum correlation ratio obtained by FFT search method at N (15, 0.01) is 0.0292, which indicates that the FFT search method fails to detect the signal and acquire the parameters in the presence of noise. On the other hand correlation ration obtained by proposed method is 0.6578 at the same noise level.

![Fig. 4. Signal acquisition with noise N (0, 0.01) (a) using FFT search algorithm (b) using proposed algorithm](image-url)
Fig. 5. Signal acquisition with noise $N(15, 0.01)$ (a) using FFT search algorithm (b) using proposed algorithm

Fig. 6. Correlation peaks obtained (a) using FFT search algorithm (b) using proposed algorithm

Table 1 Comparison of Correlation ratios

| Noise      | With FFT first peak | With FFT second peak | With DWT first peak | With DWT second peak | Correlation ratio With FFT | Correlation ratio With DWT |
|------------|---------------------|----------------------|---------------------|----------------------|----------------------------|----------------------------|
| $N(0,0.01)$ | 2426.6              | 1964.7               | 2305.3              | 1332.3               | 0.2356                     | 0.8805                     |
| $N(0.1,0.01)$ | 2180.5              | 1905.7               | 2278.0              | 1316.9               | 0.1442                     | 0.7299                     |
| $N(1,0.01)$  | 2338.8              | 1926.8               | 2304.8              | 1343.7               | 0.2138                     | 0.8641                     |
| $N(10,0.01)$ | 2463.5              | 2368.3               | 2846.3              | 1551.9               | 0.0402                     | 0.8341                     |
| $N(15,0.01)$ | 2464.2              | 2536.3               | 2696.4              | 1626.5               | 0.0292                     | 0.6578                     |

4. Conclusions

GPS signal acquisition is implemented using FFT search algorithm and proposed algorithm using DWT. The GPS signal consist of 5000 samples of data is used for testing the algorithms. From the results it is found that the code offset and Doppler shift acquired from GPS signal at SNR of $-20$ dB and zero mean noise is same in both the methods. However, the correlation ratio is found to be high in the proposed algorithm (0.8805) compared to the FFT search method (0.2356). This low correlation ratio value represents possibility of false alarm in FFT search method.
From the results, it is also found that the correlation ratio obtained by FFT search method decreases drastically (0.0292) as noise level increase. On the other hand proposed method performed well (0.6578) for the same. Hence proposed method is very useful to implement GPS receivers in real time hostile environment.

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