Can gravitational collapse and black-hole evaporation be a unitary process after all?

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Abstract

This paper shows a way of how to resolve the non-unitarity problem in black-hole physics without modifications of the basic principles of local quantum field theory.

Keywords: black holes, non-unitarity problem
I. INTRODUCTION

Right after Hawking’s discovery of the black-hole evaporation \[1\], it was realized by him \[2\] (see also \[3–6\] and \[7–9\]) that this effect leads to a violation of one of the basic principles of local quantum field theory (LQFT), namely the unitarity. This problem is also known as the incompatibility of the quantum theory principles with the general relativity ones or the breakdown of quantum predictability.\(^1\)

Nowadays it seems one awaits that the non-unitarity can be resolved at the level of quantum gravity. In this respect, the most promising role is expected to be played by the anti-de Sitter/conformal field theory (AdS/CFT) correspondence (for instance, see \[10\]).

One more possible resolution could be through rejecting some of the LQFT principles in favour of the unitarity. For example, this may be locality \[11\] (and references therein).

We will show in this paper how one may resolve the problem without modifications of the LQFT basic principles at the semi-classical approximation. In other words, we will show how classical theory of gravity, i.e. general relativity, and LQFT of matter fields could be still compatible with each other during gravitational collapse forming a black hole and its subsequent evaporation.

We will work within the algebraic framework of local quantum field theory. Specifically, we consider a scalar, non-interacting field \(\hat{\Phi}(x)\) in the spacetime geometry denoted as \(\mathcal{M}\) below. This field operator can by used to generate the so-called operator algebra \(\mathcal{A}(\mathcal{M})\). This algebra is composed of an identity operator \(\hat{1}\) and finite sums of finite products of the field \(\hat{\Phi}(x)\) smeared out over test functions \(\{f(x), x \in \mathcal{M}\}\), i.e. functions being smooth and compactly supported. States are defined as linear, positive and normalized functionals on \(\mathcal{A}(\mathcal{M})\). One can associate a certain Hilbert space representation \(\mathcal{H}\) of the algebra with a given state (through the Gelfand-Naimark-Segal construction). For more on this, see a basic reference \[12\] and recent reviews \[13–15\] devoted to LQFT in curved classical spacetimes.

Our main strategy is to extensively exploit the LQFT principles, properties and their subtle mathematical consequences, namely

(a) there exist unitarily inequivalent Fock space representations of the same canonical commutation relation in quantum field theory;

(b) a primary interpretation of quantum field theory is given in terms of local operations;

(c) a factorization of a field operator algebra \(\mathcal{A}(\mathcal{M})\) into a product of two commuting subalgebras \(\mathcal{A}_1(\mathcal{M}_1) \otimes \mathcal{A}_2(\mathcal{M}_2)\), where \(\mathcal{M}_1 \cup \mathcal{M}_2 \subset \mathcal{M}\) and the Cauchy surface \(\Sigma_{\mathcal{M}} = \Sigma_{\mathcal{M}_1} \cup \Sigma_{\mathcal{M}_2}\), does not lead to a Hilbert space factorization.

The explanation of these items in quantum field theory are in order: (a) The existence of unitarily inequivalent Fock space representations of \(\mathcal{A}(\mathcal{M})\) is related to the non-separability

\(^1\) The breakdown of classical predictability is related with the singularity inside the black hole, see, e.g. \[8\].
of its Hilbert space representation \cite{12,16,17}; (b) Since spacetime isometry plays a crucial role in defining field excitations, an interpretation of quantum field theory is generally impossible in terms of excitations \cite{18}. However, quantum field theory can be always interpreted in terms of local operations \cite{12}; (c) This is a subtle mathematical fact which will be exemplified below.

The outline of this paper is as follows. In Sec. I, we will show how the gravitational collapse can be still a unitary process. In Sec. III we will reinterpret the black-hole evaporation, which does not lead then to the information loss problem.

Throughout this paper the fundamental constants are set to unity, \( c = G = k_B = \hbar = 1 \).

II. THE PROPOSAL

We will follow the notations of reference \cite{2} in order to simplify our consideration and to avoid possible misunderstandings of the key point.

A. Operator algebra \( \mathcal{A}(\mathcal{M}) \) and its representations

One of the advantages of working within the algebraic framework is that one does not need at the outset to choose a concrete Hilbert space representation of the canonical commutation relation. Instead of the canonical commutation relation, one can consider a commutator between the quantum field \( \hat{\Phi}(x) \) at two spacetime points, namely

\[
[\hat{\Phi}(x), \hat{\Phi}(y)] = i\Delta(x, y),
\]

where \( \Delta(x, x') \) is the so-called causal propagator. It is given by a difference between the retarded and advanced Green function of the scalar field equation: \( \Box \Phi(x) = 0 \). One may call \( \Delta(x, x') \) as an algebraic structure in the set \( \mathcal{A}(\mathcal{M}) \). The spacetime metric of \( \mathcal{M} \) is now fixed and corresponds to the collapsing matter shell forming a spherical black hole which then evaporates in particular due to the backreaction produced by the field \( \hat{\Phi}(x) \).

We denote a set of the rest matter-field operators as \( \mathcal{A}_m(\mathcal{M}) \). The collapsing shell is supposed to be composed of these fields. We denote a physical Hilbert space representation\(^2\) of \( \mathcal{A}(\mathcal{M}) \) and \( \mathcal{A}_m(\mathcal{M}) \) as \( \mathcal{H} \). The representation \( \mathcal{H} \) is built on a physical state \( |\Omega\rangle \), such that \( \mathcal{A}(\mathcal{M}) \otimes \mathcal{A}_m(\mathcal{M}) |\Omega\rangle \) is dense in \( \mathcal{H} \). We also introduce \( \mathcal{H}_\Phi \subset \mathcal{H} \) which is a Hilbert space generated by operators from \( \mathcal{A}(\mathcal{M}) \) by acting on \( |\Omega\rangle \).

Shell state. If the quantum field under consideration does not contribute to an almost local operator\(^3\) \( \hat{O}_{\text{Shell}} \) composed of the matter fields building the collapsing shell, i.e.

\(^2\) It should be noted that this is actually the Fock space representation, i.e. a separable subspace in the whole non-separable representation of the field operators. It became common to call it as a Hilbert space.

\(^3\) For a definition of an almost local operator, see \cite{12}, Sec. II.4.1.
\( |\Omega_{\text{Shell}}\rangle = \hat{O}_{\text{Shell}} |\Omega\rangle \), then the quantum field \( \hat{\Phi}(x) \) is oblivious to the shell, but not to the spacetime geometry. Outside of the matter shell, \( \hat{O}_{\text{Shell}} \) vanishes, so that the shell state \( |\Omega_{\text{Shell}}\rangle \) becomes \( |\Omega\rangle \). Thus, the state \( |\Omega_{\text{Shell}}\rangle \) is a localized state which looks empty outside of the support of the operator \( \hat{O}_{\text{Shell}} \).

We actually make an assumption that one can associate to each single particle, say, a proton, state \( |p\rangle \) a localized operator \( \hat{O}_p \) from \( A_m(\mathcal{M}) \), which is in turn composed of the quark, gluon, electromagnetic field operators. This operator generates the state from the vacuum \( |\Omega\rangle \). This should be understood in the weak sense, i.e. \( \langle p|\hat{O}_p|\Omega\rangle \neq 0 \). The matter shell is then a complicated combinations of such operators of each particle of the shell. An association of a localized operator with a particle state is actually not a novel idea [12]. It is employed to describe composite particles (e.g. hadrons) to which one cannot ascribe any fundamental field operator.

The quantum field \( \hat{\Phi}(x) \) plays a role of a trial quantum field considered in the gravitational collapse. To make our set-up more realistic, we assume that \( \hat{O}_{\text{Shell}} \in A(\mathcal{M}) \otimes A_m(\mathcal{M}) \). Note that \( |\Omega_{\text{Shell}}\rangle \in \mathcal{H} \).

**Black-hole state.** It may correspond to ranges of Unruh vacua when the evaporating hole is present. We will denote this set of vacua as \( |\Omega_{\text{BH}}\rangle \), such that \( |\Omega_{\text{BH}}\rangle = \hat{O}_{\text{BH}} |\Omega\rangle \). The almost local operator \( \hat{O}_{\text{BH}} \) with a support of non-vanishing measure corresponds to \( \hat{O}_{\text{Shell}} \) after the black-hole formation. The evolution of the geometry leads to a change of the algebraic structure of \( A(\mathcal{M}) \) as well as \( A_m(\mathcal{M}) \). However, the composition of \( \hat{O}_{\text{BH}} \) and \( \hat{O}_{\text{Shell}} \) does not vary.

The choice of the Unruh vacua is based on the fact that the quantum field being in \( |\Omega_{\text{BH}}\rangle \) does not lead to a divergence of the field stress tensor on the future event horizon. This condition defines these states (see [7] and [19, 20] for a more rigorous definition). This choice of \( |\Omega_{\text{BH}}\rangle \) allows to have a self-consistent consideration of quantum gravity in the semi-classical approximation. The Unruh vacuum at a given moment of time will be also called as system’s vacuum, where the system is composed of the quantum matter and gravity fields. Note that \( |\Omega_{\text{BH}}\rangle \in \mathcal{H} \).

Up to now there does not exist a widely accepted theory of quantum gravity. Therefore, in this paper, we employ both general relativity (GR) and local quantum field theory up to regimes when one cannot trust anymore in their predictions. At the level of GR, the collapse of matter is described by a change of the spacetime structure. Its change leads to an appearance of a singularity at \( r = 0 \). In the vicinity of this point, GR is certainly unreliable. At the level of LQFT, the collapse of matter is described by a unitary evolution of the matter state from \( |\Omega_{\text{Shell}}\rangle \) to \( |\Omega_{\text{BH}}\rangle \). The black-hole state \( |\Omega_{\text{BH}}\rangle \) as noted above has a non-vanishing support in the vicinity of the singularity \( r = 0 \), e.g. due to the Heisenberg principle. In a sense, there should be a black-hole core of the original matter of the collapsing shell inside the horizon \( r = 2M \). The core is expected to be of a macroscopic size which
is dynamically generated. It is worth noting that it is legitimate to use GR only outside of the support of the black-hole core. Whenever one wants to "touch" the state $|\Omega_{BH}\rangle$ inside of its support, one generally should expect a discovery of quantum gravity degrees of freedom.\footnote{This is analogous to discovering gluons and quarks when one probes the structure of a proton in the high-energy collisions.} Quantum gravity is expected to resolve the problem of the breakdown of \textit{classical} predictability.

A similar (only to a certain extent) idea of representing a black hole was proposed in \cite{21} and further investigated in \cite{22, 23}.

One may also introduce three extra Hilbert spaces $H_-, H_H$ and $H_I$. These are associated with the states $|0_-, 0_{H}\rangle$ and $|0_I\rangle$, respectively, \cite{2}.

The state $|0_\pm\rangle$ is \textit{observer’s vacuum} (at past time infinity) or the initial vacuum state for scalar particles, such that

$$\hat{\Phi}(x) = \hat{a}(x) + \hat{a}^\dagger(x),$$

where

$$\hat{a}(x) = \sum_i f_i(x) \hat{a}_i,$$

$\hat{a}(x)|0_-\rangle = 0$ and $f_i(x)$ are mode functions. Following \cite{2}, we assume that at past time infinity observer’s vacuum coincides with system’s vacuum, i.e. $|0_-\rangle \cong |\Omega_{Shell}\rangle$ or, in terms of Hilbert spaces, $H_- \cong H_\Phi$.

The state $|0_I\rangle$ is \textit{observer’s vacuum} (at future time infinity) or the vacuum state for outgoing scalar particles. The state $|0_H\rangle$ is the vacuum state for particles falling into the black hole. One can equivalently rewrite (2) as an operator equality as follows

$$\hat{\Phi}(x) = \hat{b}(x) + \hat{b}^\dagger(x) + \hat{c}(x) + \hat{c}^\dagger(x),$$

where $\hat{b}(x)|0_H\rangle = 0$ and $\hat{c}(x)|0_H\rangle = 0$.

Following \cite{2}, one defines the final scalar particle vacuum state, $|0_+\rangle$, as $|0_H\rangle \otimes |0_I\rangle$. Hence, the Hilbert space representation of the algebraic structure \cite{1} corresponds to the factorized product $H_H \otimes H_I$.

\textbf{B. Splitting of operator algebra $A(\mathcal{M})$}

The field operators $\hat{O}_b(x)$ composed of $\{\hat{b}(x), \hat{b}^\dagger(x)\}$ and $\hat{O}_c(x)$ composed of $\{\hat{c}(x), \hat{c}^\dagger(x)\}$ commute with each other \cite{2}. This implies that one can express the total algebra of the field operators $A(\mathcal{M})$ as a factorized product of two commuting operator algebras i.e.
\( \mathcal{A}(H) \otimes \mathcal{A}(I) \), such that

\[
\hat{O}_a(x) \in \mathcal{A}(\mathcal{M}), \quad (5a)
\]
\[
\hat{O}_b(x) \in \mathcal{A}(I), \quad (5b)
\]
\[
\hat{O}_c(x) \in \mathcal{A}(H), \quad (5c)
\]

where field operators \( \hat{O}_a(x) \) are composed of \( \{ \hat{a}(x), \hat{a}^\dagger(x) \} \). This splitting of the total algebra can also be expressed as \( \hat{O}_a(x) = \hat{O}_c(x) \otimes \hat{O}_b(x) \). Note that operators of the form \( \hat{O}_b(x) \) have vanishing support inside of the hole, whereas operators of the form \( \hat{O}_c(x) \) outside of the hole.

C. **Unitary inequivalence of \( \mathcal{H}_\Phi \) and \( \mathcal{H}_H \otimes \mathcal{H}_I \)**

**Type III property of \( \mathcal{A}(\mathcal{N}) \).** There is a subtle mathematical result related to the operator factor subalgebras of \( \mathcal{A}(\mathcal{N}) \) in quantum field theory. This corresponds to their type III property. This essentially means that a splitting of \( \mathcal{A}(\mathcal{N}) \) into a factorized product of two its subalgebras \( \mathcal{A}(\mathcal{N}_1) \otimes \mathcal{A}(\mathcal{N}_2) \), where \( \mathcal{N}_1 \cup \mathcal{N}_2 \subset \mathcal{N} \) and the Cauchy surface of \( \mathcal{N} \) being \( \Sigma_{\mathcal{N}} = \Sigma_{\mathcal{N}_1} \cup \Sigma_{\mathcal{N}_2} \), does not lead to a Hilbert space factorization, i.e. \( \mathcal{H} \not\cong \mathcal{H}_1 \otimes \mathcal{H}_2 \).

Specifically, the factor subalgebra \( \mathcal{A}(\mathcal{N}_1) \) is said to be of the type III if for every projection \( \hat{E} \in \mathcal{A}(\mathcal{N}_1) \), there exists an element \( \hat{W} \in \mathcal{A}(\mathcal{N}_1) \) that maps \( \mathcal{H} \) isometrically onto \( \hat{E} \mathcal{H} \). In other words, \( \hat{W}^* \hat{W} = \mathbf{1} \) and \( \hat{W} \hat{W}^* = \hat{E} \). This implies that \( \hat{E} \) is equivalent to \( \mathbf{1} \), i.e. trivial. Thus, the subalgebra \( \mathcal{A}(\mathcal{N}_1) \) of local operators \( \mathcal{A}(\mathcal{N}) \) in \( \mathcal{N}_1 \) still acts irreducibly on \( \mathcal{H} \).

Note that this is not the case in quantum mechanics, because there one deals with systems with a finite number of degrees of freedom. One expresses this as operators in quantum mechanics are of the type I.

This fact was recently emphasized in [11] with applications to an entanglement. The difference between the type I and type III properties of operator algebras are contrasted in [24].

**Example 1: eternal Schwarzschild geometry.** Following [25], one may rewrite the Hartle-Hawking state \( |\Omega_{\text{HH}}\rangle \) as a thermal-field double state [16], i.e.

\[
|\Omega_{\text{HH}}\rangle = \frac{1}{Z} \prod_{\omega l m} \sum_{n=0}^{+\infty} e^{-\beta E_{\omega,n}/2} |n_L\rangle \otimes |n_R\rangle , \text{ where } E_{\omega,n} = \omega n \tag{6}
\]

and \( Z \) is a normalization factor. The frequency \( \omega > 0 \) is defined with respect to the vector \( \partial_t \) (\( t_s \) - the Schwarzschild time coordinate), and \( l, m \) are the orbital and magnetic numbers referring to a particular representation of the rotational symmetry of the black hole. The inverse temperature \( \beta \) is given by \( 1/T_H \), where \( T_H \) is the Hawking (H) temperature [1]. The states entering the right-hand side of (6) are defined as

\[
|n_L\rangle \equiv \frac{1}{\sqrt{n!}} (\hat{b}^L_{l,\omega m})^n |\Omega_{\text{LB}}\rangle , \tag{7}
\]

and

\[
|\Omega_{\text{HH}}\rangle = \frac{1}{Z} \prod_{\omega l m} \sum_{n=0}^{+\infty} e^{-\beta E_{\omega,n}/2} |n_L\rangle \otimes |n_R\rangle , \text{ where } E_{\omega,n} = \omega n \tag{6}
\]
and the same for $|n_R\rangle$ with $L \rightarrow R$ in the above formula. The states $|\Omega_{LB}\rangle$ and $|\Omega_{RB}\rangle$ are the “left” and “right” Boulware vacua. One can associate two Hilbert spaces, $H_{LB}$ and $H_{RB}$, to both these states. The state on the left-hand side of (6) is the only non-singular state on the black hole horizons.

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The normalization factor $Z$ in (6) is, rigorously speaking, infinite:

$$Z = \exp \left( + \frac{\pi}{96M} \delta(0) \sum_{l=0}^{+\infty} (2l + 1) \right),$$

where $M$ is a mass of the black hole. This can be stressed out by saying that the equality (6) is merely formal. It is well-discussed in [11] (and see also [16, 26]). It is worth emphasizing that it is thus illegitimate to interpret (6) as $|\Omega_{HH}\rangle \in H_{HH}$ in quantum field theory. It means $H_{HH}$ corresponding to the Hartle-Hawking state $|\Omega_{HH}\rangle$ is not unitarily equivalent to the factorized product $H_{LB} \otimes H_{RB}$.

Example 2: gravitational collapse. Following [2], one may rewrite the initial vacuum state $|0_-\rangle$ or $|\Omega_{Shell}\rangle$ ($\equiv |\Omega_{BH}\rangle$) as a thermal-field double state

$$|\Omega_{Shell}\rangle = \frac{1}{Z^{\frac{1}{2}}} \sum_A \sum_B \lambda_{AB} |A_I\rangle \otimes |B_H\rangle,$$

where $|A_I\rangle$ is the outgoing state with $n_{j_b}$ particles in the $j$th outgoing mode and $|B_H\rangle$ is the horizon state with $n_{k_c}$ in the $k$th mode going into the hole. For more details, see [2, 27] and [8, 28].

The normalization factor $Z$ allowing to have $\langle \Omega_{Shell}|\Omega_{Shell}\rangle = 1$, is infinite. Indeed, following, for instance, [12, 19], one can show that

$$Z = \text{Tr}(\hat{\rho}_{TH}) = \sum_A \sum_B |\lambda_{AB}|^2 = \infty,$$

i.e. $\hat{\rho}_{TH}$ is not a trace class operator. This means that the equality (9) is merely formal. In other words, one cannot interpret (9) as the initially pure state (the left-hand side of (9)) is transformed into an entangled state (the right-hand side of (9)) during the Hawking process. These states are actually orthogonal. If one insists on the equality (9), then one violates the unitarity. It is worth emphasizing that this unitarity violation is an origin of the ordinary non-unitarity in black-hole physics which was pointed out in [2] even if the final state would be still pure.

This result can be treated as a check of the above statement related to the type III property of the field operator algebra $A(M)$. Thus, one is forced to conclude that

$$H_{\Phi} \not\cong H_{H} \otimes H_{I} \quad \text{or} \quad H_{\Phi} \perp H_{H} \otimes H_{I}.$$
D. Proposal

Our proposal is as follows. One should not consider the final scalar particle vacuum state $|0_+\rangle$ as realizable physically. The reason is that this choice of the final state leads to the unitarity violation by hand. As discussed above, the physically realizable state is the Unruh vacuum $|\Omega_{BH}\rangle$ up to the moment of time when, in particular, the backreaction of the quantum field $\hat{\Phi}(x)$ can no longer be neglected. Thus, there is no evidence of non-unitarity, at least at the level of local quantum field theory.

The black hole singularity potentially invalidates the use of local quantum field theory at the end of the black-hole evolution. In other words, the subsequent evolution of $|\Omega_{BH}\rangle$ or even $|\Omega\rangle^6$ when $\langle \Omega_{BH}|\hat{T}_\phi(x)|\Omega_{BH}\rangle \sim O(1)$ and/or $\langle \Omega_{BH}|\hat{T}_m(x)|\Omega_{BH}\rangle \sim O(1)$ requires either quantum gravity or a certain semi-classical prescription allowing to describe the complete black-hole disappearance still in the semi-classical language. This should be understood in the same spirit of a treatment of the bounce in cosmology by semi-classical models (see, e.g. a recent review [29]). Therefore, black-hole solutions without curvature singularity [30] as well as non-singular “phenomenological” black-hole geometries [31–33] are of great interest.

However, as pointed out above, one should not trust in GR in the vicinity of the singularity. Nevertheless, a decrease of the horizon size is still a reliable prediction. Consequently, the fig. 5 in the second paper of [1] should not be taken for granted. At the level of LQFT, the black-hole evaporation might then lead to a disappearance of the horizon inside the support of the black-hole core through properly taking into account the backreaction.

III. CONCLUDING REMARKS

The non-unitarity problem appears, in particular, because the equivalence $\mathcal{H}_\Phi \cong \mathcal{H}_H \otimes \mathcal{H}_I$ as well as the equality in [9] were implicitly assumed. We have shown above that this isomorphism and the equality in [9] are actually impossible in quantum field theory.

It is also worth emphasizing this as follows. A representation of the Cauchy surface $\Sigma_M$ as a union $\Sigma_H \cup \Sigma_I$ does not lead to the Hilbert space factorization, i.e. $\mathcal{H}_\Phi$ and $\mathcal{H}_H \otimes \mathcal{H}_I$ are unitarily inequivalent in local quantum field theory. However, the contrary is usually employed to formulate the information loss problem or the breakdown of quantum predictability in black-hole physics.

Observer’s particle states being elements of $\mathcal{H}_I$ do not belong to the physical Hilbert space $\mathcal{H}$ of the system. That is a thermal gas of the excitations being elements of $\mathcal{H}_I$ cannot be represented in the Hilbert space $\mathcal{H}$. The field observables employed by an experimentalist at future time infinity are $\hat{O}_b(x) \in \mathcal{A}(I)$. The non-trivial thermal response of the pure state $|\Omega_{BH}\rangle$ at future time infinity is due to the fact these operators have vanishing support inside

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6 This would correspond to a phase transition.
of the black hole. Modelling this by a thermal density matrix $\hat{\rho}_T$ seems to be misleading, because one then needs to average the field operators over the states being not represented in the physical Hilbert space, i.e. $\mathcal{H}$. The effect should instead be understood in terms of local probes of system’s vacuum $|\Omega_{\text{BH}}\rangle$ by non-trivial field observables, i.e. $\hat{O}_b(x)$, by the experimentalist.

Schematically, an available set of the field observables for the stationary experimentalist evolves as follows

$$\hat{O}_a(x) \rightarrow \hat{O}_b(x),$$

such that system’s vacuum $|\Omega_{\text{BH}}\rangle$ is a Kubo-Martin-Schwinger (KMS)\(^7\) or thermal state with respect to $\partial_t s$ for operators of the type $\hat{O}_b(x)$ at the Hawking KMS parameter $\beta_\text{H} = 1/T_\text{H}$, and for operators of the type $\hat{O}_a(x)$ at the infinite KMS parameter $\beta$. For instance, a field operator modelling a detector considered in \([34]\) (see also \([12]\)) belongs to $\mathcal{A}(I)$ and non-trivially responses when it probes system’s vacuum. This is in complete agreement with the principles of quantum field theory and does not lead to the unitarity violation, because system’s vacuum $|\Omega_{\text{BH}}\rangle$ is pure all the time during the black hole formation and its subsequent gradual evaporation. The latter is due to the non-trivial vacuum polarization effect. A representation of the black-hole evaporation as due to the vacuum polarization effect has been suggested in \([35]\) (see also \([36]\)).

It seems the main problem is to take the backreaction of the quantum fields into account which should lead to deviations from the perfect thermality of system’s vacuum when it is probed by local observables available to the stationary experimentalist. In other words, the backreaction should lead to

$$\begin{align*}
\hat{O}_a(x) &\rightarrow \hat{O}_b(x) \rightarrow \hat{O}_b(x) + \delta\hat{O}_b(x) \notin \mathcal{A}(I) \\
\mathcal{A}(\mathcal{M}) &\rightarrow \mathcal{A}(I) & \mathcal{A}(\mathcal{M})
\end{align*}$$

such that $|\Omega_{\text{BH}}\rangle$ is only approximately a KMS state for modified operators $\hat{O}_b(x) + \delta\hat{O}_b(x)$.

It is worth emphasizing that although this would be a sign of the unitarity restoration, we do not need it, because the process is unitary all the time during the black-hole evolution as pointed out above.

From the argument based on the type III property of the factor operator subalgebras and its various representations, we have concluded that a density matrix usually introduced to describe future time experience of an observer is of no physical meaning. One may accept

\(^{7}\) The KMS state $|\Omega_\beta\rangle$ is a state which satisfies the KMS condition:

$$\langle \Omega_\beta|a_K^\dagger(\hat{A})\hat{B}|\Omega_\beta\rangle = \langle \Omega_\beta|\hat{B}a_K^{\dagger+i\beta}(\hat{A})|\Omega_\beta\rangle,$$

where both sides are analytic in the strip $0 < \text{Im}(t) < \beta$, continuous on its boundary, and $a_K(\hat{A}) \equiv \exp(+i\hat{K}t)\hat{A}\exp(-i\hat{K}t)$. $\hat{K}$ is a Hermitian operator corresponding to the Killing vector $K$. More details can be found, for instance, in \([12]\).
the contrary, but then we have a change of the representation of the operator algebra or a phase transition. Consequently, the (Hawking) particles detected at infinity would be physically of a different sort in comparison with those from which the collapsed matter shell was composed. For example, this is analogous to the well-known phase transition in QCD, where the change of the Fock space representation leads to a change of the notion of a particle: hadrons at low energies and quarks and gluons at energies above the QCD energy scale $\Lambda_{\text{QCD}} \approx 0.3$ GeV.\(^8\) Thus, there are no evidences in favour of the loss of the quantum coherence working in the framework of the algebraic QFT whenever one does not allow a phase transition during the black-hole formation.\(^9\)

Another face of the problem is the non-conservation of baryon number $n_b$. Initially, one has $\hat{B}|\Omega_{\text{Shell}}\rangle = n_b|\Omega_{\text{Shell}}\rangle$, where $\hat{B}$ is the baryon number operator. If the assumption of the association of a single particle state with an almost local quantum operator is right, then $\hat{B}$ counts the number of these operators building $\hat{O}_{\text{Shell}}$. Since the number of these operators is the same in $\hat{O}_{\text{BH}}$, the baryon number should be conserved, at least till one can no longer neglect the backreaction, because, as pointed out above, the horizon can hide under the black-hole core and the matter can be thrown away to spatial infinity.

It would be interesting to investigate our proposal further in detail for a presence of any unphysical or unacceptable consequences. It is worth noting that we are in agreement with a general argument in favour of the unitary evaporation of a black hole made in the context of the AdS/CFT conjecture.

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\(^8\) In this sense, we have an inequivalence of the Heisenberg and Schrödinger pictures in QFT. In certain situations, a quantum system "prefers" to change the representation (e.g. various phase transitions), in others, the algebraic structure (e.g. the Casimir and Hawking effect).

\(^9\) There is also no loss of the quantum coherence in flat space when the final Cauchy hypersurface fails to be a Cauchy hypersurface for Minkowski spacetime. This can be seen if one employs the Reeh-Schlieder theorem \(^{12}\) to the observable algebra of operators with a support over the final Cauchy hypersurface. Indeed, even with a subalgebra of the total algebra of all possible field operators, one can probe all elements of the Minkowski Hilbert space with an arbitrary precision, whereas the Minkwoski Hilbert space is generated by applying all possible field operators from the total algebra. This is not the case if one allows a factorization of the Minkowski Hilbert space when the final Cauchy hypersurface is merely a subset of the Cauchy hypersurface for Minkowski space.
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