The value distributions and uniqueness of q-shift difference polynomials of entire functions

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Abstract. We first consider the zeros of a big class generalized q-difference polynomials in entire function which come from the Hayman problem. Second, we research about uniqueness result of the class q-difference polynomials in entire function which share a value. Our results improved or generalize the previous known results.

1. Introduction
Suppose that the reader knows the notions of the theory (see [1]). For example, a polynomials \( P(z) \) (including the constant functions) is a small function of the exponential function \( e^z \). An exponential function \( e^z \) is a small function of \( 2 \exp(z) \).

We also suppose that the reader knew the notions of the theory of mermorphic functions. For example, how to say two meromorphic functions share the \( a \) value or a function CM or IM? One meromorphic function can be unique by sharing five values, one entire function can be unique by sharing four values. If two meromorphic functions share four or three values, what can one say? One entire function and its derivative can be unique by two different values, and so on. (see. [2-3])

In the last ten years, B. Halburd and Y. Chiang developed the Nevanlinna theory in the view of difference and q-difference, respectively. There are many authors began to considered the difference problems in the complex plane again (In fact, there exist a little results on complex difference equations which studied by Yanagihara, W. Bergweiler, K. Ishizaki, S. Bank at the early time), many results about the value distributions and uniqueness of meromorphic(entire) functions have been studied using the difference or q-difference to instead of the differential. The reader can find some system results in this direction in the book of Z.C. Chen or some papers. (2-11).

In 1959, W. K. Hayman [12] gave the following famous problem (Picard type theorem).

**Theorem A** Let \( n \geq 1 \), and \( f \) be a transcendental entire function. Then we have \( f^n f' \) can take each non-zero value \( a \in \mathbb{C} \) infinitely many times.

In the same paper, Hayman proved the problem holds for \( n \geq 2 \), Clunie [13] proved the problem holds for \( n = 1 \).

Laine and Yang [4] obtained the same result of Hayman problem by using the notation of difference. This is, \( f^n f(z+c) \) can take each finite non-zero value infinitely many times. Note if \( n = 1 \), the analog result does not holds. The counterexample has been given in [4].
M.L. Fang [14] proved that if \( f \) be a trans. entire function, then \( f^n(f - 1)f^- \) can take each non-zero value \( a \in \mathbb{C} \) infinitely many times. The corresponding result for difference polynomials has been obtained by J.L. Zhang in [5], this is, \( f^n(f - 1)f(z + c) - \alpha \) has infinitely many zeros, \( \alpha \) is a small function of \( f \) (In the following \( \alpha \) is the same notion). Recently, Chen and Chen [6] continue to extend the above result.

**Theorem B** Let \( n \geq 2 \), and \( f(z) \) be a finite order transcendental entire functions, \( n, l, t \), \( \mu_j(j = 1, \cdots, t) \) be positive integers and \( c_j(j = 1, \cdots, t) \) be distinct nonzero constants. Then

\[
f^n(f^{-1})\prod_{j=1}^{t}(f(z + c_j))^{\mu_j} - \alpha \text{ has infinitely many zeros.}
\]

Zhang and Korhonen [2] obtained two q-difference versions theorems. This is, \( f^n f(qz - a) \) and \( f^n(f - 1)f(qz - a) \) has infinitely many zeros for \( n \geq 2 \), where \( a \in \mathbb{C} \).

In this article, we will study investigate the zeros of the above generalized results.

**Theorem 1** Let \( f \) be a trans. entire function whose order is zero, and \( q_j(j = 1, \cdots, t) \) be nonzero finite constants, \( \sigma = \sum_{j=1}^{t} \mu_j \), \( \mu_j \in \mathbb{N} \). Let \( P(\omega) = a_1\omega^j + a_{j-1}\omega^{j-1} + \cdots + a_0 \) (\( \neq 0 \)) be a polynomial, where \( a_1, \cdots, a_0(\neq 0) \) are constants. If \( n \geq 2 \), then \( f^n P(f)\prod_{j=1}^{t}(f(q_j z))^{\mu_j} - \alpha(z) \) has infinitely many zeros. Especially, if \( P(\omega) \equiv c_0 \in \mathbb{C} \setminus \{0\} \), then the result holds for \( n \geq 1 \).

**Remark 1** If \( P(\omega) \equiv c_0 \in \mathbb{C} \setminus \{0\} \), \( s = 1 \) and \( \sigma = 1 \), from Theorem 1, we can get a better result than Zhang and Korhonen, where the condition \( n \geq 2 \) can be removed. If \( P(\omega) = \omega - 1 \), \( s = 1 \) and \( \sigma = 1 \), then from Theorem 1, we can get the result of Zhang and Korhonen.

Uniqueness problems for entire functions is a very important direction in Nevanlinna theory (see. [1]). Yang and Hua [15] first consider and proved the uniqueness theorem related to Hayman result. In [9], X. Qi, Liangzhong. Yang and Kai. Liu also got a uniqueness result related to difference polynomials corresponding to I. Laine and C. Yang. Chen and Chen [7] proved the uniqueness theorem related to difference polynomials corresponding to Theorem B. Zhang and Korhonen [2] proved the uniqueness theorems related to the above results obtained by themselves.

Naturally, we consider the uniqueness theorem corresponding to Theorem 1.

**Theorem 2** Let \( f \) and \( g \) be two trans. entire functions whose order are zero and let \( q_j(j = 1, \cdots, t) \) are nonzero constants, \( n \geq l + 8\sigma \), \( \sigma, \mu_j \), \( P(f) \) are defined in Theorem 1. If

\[
F_l = f^n P(f)\prod_{j=1}^{t}(f(q_j z))^{\mu_j} \quad \text{and} \quad G_l = g^n P(g)\prod_{j=1}^{t}(g(q_j z))^{\mu_j}
\]

share the small function \( \alpha(z) \) CM, then

(I) if \( P(\omega) = \omega^j - 1 \) and \( n \geq \sigma + 2t + 3 \), then we can obtain the result of \( f(z) \equiv kg(z) \), where \( k \) is a constant which satisfies the relation \( k^l = k^{n+\sigma} = 1 \);

(II) if \( P(\omega) \equiv c_0 \in \mathbb{C} \setminus \{0\} \) and \( n \geq \sigma + 3 \), then we can also obtain the result of \( f(z) \equiv kg(z) \), here \( k \) also is a constant which satisfies the relation \( k^{n+\sigma} = 1 \).

**Remark 2** In this theorem, we consider a unified uniqueness theorem of q-difference polynomials by using the different method with Zhang and Korhonen in [4]. But our result is not include the result of Zhang and Korhonen. Our method comes from the paper of Chen and Chen.
In the next, we talk about the organization of the article. Some very useful tools will be introduced in the section 2. Proofs of Theorems 1 and 2 are given in Sections 3 and 4. At last, we will give some concluding remarks in Section 5.

2. Preliminary

In the part, we will give some lemmas which play some important roles in the proofs of Theorems 1 and 2. First, we give the q-shift difference result related to the logarithmic derivative lemma. The lemma is very useful when one consider q-difference polynomials. (see [7])

Lemma 1 If \( \mathcal{f}(z) \) is a meromorphic function whose order is zero, and \( q \in \mathbb{C} \setminus \{0\} \), then we have

\[
m(r, \frac{\mathcal{f}(qz)}{\mathcal{f}(z)}) = S(r, \mathcal{f}).
\]

By the following lemma, one can estimate the characteristic function of \( \mathcal{f}(qz) \) and find the relation of \( \mathcal{f}(qz) \) with \( \mathcal{f}(z) \). (see. [2, Theorem 1.1])

Lemma 2 If \( q \in \mathbb{C} \setminus \{0\} \), \( \mathcal{f}(z) \) is a non-constant meromorphic function whose order is zero, then we have

\[
T(r, \mathcal{f}(qz)) = T(r, \mathcal{f}(z)).
\]

The next lemma is proved by C. C. Yang, which describes the characteristic function relation between a meromorphic function and its polynomials. (see. [1, Theorem 1.12])

Lemma 3 Let \( \mathcal{f} \) be a meromorphic function, and \( P(f) = b_p f^p + b_{p-1} f^{p-1} + L + b_1 f \), where \( p \) is a natural number and \( b_i (i = 1, \ldots, p) \) are small functions. Then

\[
T(r, P(f)) = p T(r, f) + S(r, f).
\]

The following lemma is obtained by Yi, which is a standard tool, when one consider two function share a value. Especially, the power of function is higher.

Lemma 4 ([1, Theorem 7.10]) If \( \mathcal{f} \) and \( \mathcal{g} \) are meromorphic, the two functions share 1 value counting multiplicities, then we can have the three results:

(i) \( T(r, \mathcal{f}) \leq N_2(r, \frac{1}{\mathcal{f}}) + N_2(r, \mathcal{f}) + S(r, \mathcal{f}) + N_2(r, \frac{1}{\mathcal{g}}) + N_2(r, \mathcal{g}) + S(r, \mathcal{g}) \)

and \( T(r, \mathcal{g}) \) also has the above inequalities; or

(ii) \( \mathcal{f}g \equiv 1; \) or

(iii) \( \mathcal{f} \equiv \mathcal{g}. \)

At last, we use the method of [11, Lemma 12], one can get the key lemma in the proof of Theorem 2.

Lemma 5 If \( \mathcal{f} \), \( \mathcal{g} \) are two transcendental entire functions whose order is zero, \( n(\geq 1), l(\geq 0) \), \( \mu_j(\geq 0)(j = 1, 2, \ldots, t) \) are some integers, \( \sigma = \mu_1 + \cdots + \mu_t \), \( q_j(j = 1, \cdots, t) \) are finite constants and if \( P(f) \) is a polynomial which is defined in Theorem 1. Let

\[
f^n P(f) \prod_{j=1}^{t} (f(q_j z))^{\mu_j} \equiv g^n P(f) \prod_{j=1}^{t} (g(q_j z))^{\mu_j}.
\]

Then (I) when \( P(\omega) = \omega l - 1 \) and \( n \geq \sigma + 2t + 3 \), then one have the result of \( \mathcal{f}(z) \equiv k \mathcal{g}(z) \), where \( k \) is a constant which satisfies the relation \( k^l = k^{\sigma} = 1; \)

(II) when \( P(\omega) \equiv c_0 \in \mathbb{C} \setminus \{0\} \), we have \( \mathcal{f}(z) \equiv k \mathcal{g}(z) \), here \( k \) also is a constant which satisfies the relation \( k^{n+\sigma} = 1. \)
Remark 3 If \( P(\omega) \) is a general polynomial, then we cannot find the relations between the function \( f \) and \( g \), directly. In the article, we just study Cases (I), (II). In the future, we will consider the case of \( P(\omega) = (\omega - 1)^l (l \geq 2) \).

3. Proof of Theorem 1

Proof. For the convenience, we denote

\[
F_i = f^P(f) \prod_{j=1}^{i} f(q_j z)^{\mu_j}. \tag{3.1}
\]

Using Lemma 3, one can get

\[
T(r, f^{n+\sigma} P(f)) = (n + \sigma + l) T(r, f) + S(r, f).
\]

Since \( f \) is entire, we have \( N(r, f) = 0 \), therefore,

\[
T(r, f^{n+\sigma} P(f)) = m(r, f^{n+\sigma} P(f)) + S(r, f).
\]

By Lemma 1, and \( q_j (j = 1, 2, ..., t) \) are not zeros, we know that

\[
m(r, \frac{f(z)}{f(q_j z)}) = S(r, f).
\]

Hence

\[
m(r, f^{n+\sigma} P(f)) \leq m(r, F_i) + m\left(r, \frac{f^{n+\sigma} P(f)}{F_i}\right) + S(r, f)
\]

\[
\leq m(r, F_i) + \sum_{j=1}^{i} m\left(r, \frac{f(z)^{\mu_j}}{f(q_j z)^{\mu_j}}\right) + S(r, f)
\]

\[
\leq m(r, F_i) + \sum_{j=1}^{i} \mu_j m\left(r, \frac{f(z)}{f(q_j z)}\right) + S(r, f)
\]

\[
\leq T(r, F_i) + S(r, f).
\]

From the above four inequalities or equalities, one can get

\[
(n + \sigma + l)T(r, f) \leq T(r, F_i) + S(r, f). \tag{3.2}
\]

Meanwhile, the following can be got by Lemmas 2 and 3

\[
T(r, F_i) = T(r, f^P(f) \prod_{j=1}^{i} f(q_j z)^{\mu_j})
\]

\[
\leq (n + \sigma + l + o(1)) T(r, f). \tag{3.3}
\]

Using (3.2) and (3.3), one can get

\[
T(r, F_i) = (n + \sigma + l + o(1)) (T(r, f)). \tag{3.4}
\]

Next, by the Nevanlinna SMT, one can deduce

\[
T(r, F_i) \leq \bar{N}(r, \frac{1}{F_i}) + N\left(r, \frac{1}{F_i - \alpha}\right) + S(r, F_i)
\]

\[
\leq \bar{N}(r, \frac{1}{f}) + N\left(r, \frac{1}{P(f)}\right) + \sum_{j=1}^{i} \bar{N}\left(r, \frac{1}{f(q_j z)}\right) + N\left(r, \frac{1}{F_i - \alpha}\right) + S(r, f)
\]
\[ \leq N\left(r, \frac{1}{f}\right) + N\left(r, \frac{1}{P(f)}\right) + \sum_{j=1}^{l} N\left(r, \frac{1}{f(q_j,z)}\right) + \overline{N}\left(r, \frac{1}{F_1 - \alpha}\right) + S(r, f) \] (3.5)

By the Nevanlinna FMT, Lemmas 2 and 3,
\[ N\left(r, \frac{1}{f}\right) \leq T\left(r, \frac{1}{f}\right) = T(r, f) + O(1), \] (3.6)
\[ N\left(r, \frac{1}{P(f)}\right) \leq T\left(r, \frac{1}{P(f)}\right) = T(r, P(f)) + O(1) \leq IT(r, f) + O(1), \] (3.7)
\[ \sum_{j=1}^{l} N\left(r, \frac{1}{f(q_j,z)}\right) \leq \sum_{j=1}^{l} T\left(r, \frac{1}{f(q_j,z)}\right) = \sum_{j=1}^{l} T(r, f(q_j,z)) + S(r, f) \]
\[ \leq tT(r, f) + S(r, f), \] (3.8)

By (3.5)-(3.8), we can get the following
\[ T(r, F_1) \leq (1 + l + t)T(r, f) + \overline{N}\left(r, \frac{1}{F_1 - \alpha}\right) + S(r, f). \] (3.9)

Using (3.2) and (3.9), we get
\[ \overline{N}\left(r, \frac{1}{F_1 - \alpha}\right) \geq (n + \sigma - t - 1)T(r, f) + S(r, f). \]

We get \( n + \sigma - t - 1 \geq 1 \) by \( n \geq 2 \) and \( \sigma \geq t \). Hence \( F_1 - \alpha \) has infinitely many zeros.

When \( P(\omega) = c_0 \in \mathbb{C} \setminus \{0\} \), this is, \( P(\omega) \) becomes a constant, then \( F_1 = c_0 f^n \prod_{j=1}^{l}(f(q_j,z))^\nu_j \) and \( F_1 - \alpha(z) \) has finitely many zeros. Notice \( f \) is entire, and its order is zero, there exists a polynomial \( h \), satisfying \( F_1 - \alpha(z) = h(z) \). Hence,
\[ (n + \sigma)T(r, f) = T(r, f^{n+\sigma}) = m(r, f^{n+\sigma}) + S(r, f) \]
\[ \leq m\left(r, F_1 \frac{f^{n+\sigma}}{F_1}\right) + S(r, f) \]
\[ \leq m(r, F_1) + \sum_{j=1}^{l} m\left(r, \frac{f(z)^{\nu_j}}{f(q_j,z)^{\nu_j}}\right) + S(r, f) \]
\[ \leq T(r, h) + S(r, f) \]
\[ \leq T(r, h) + T(r, \alpha) + S(r, f) = S(r, f). \]

It is a contradiction. Therefore, \( F_1 - \alpha \) has infinitely many zeros. Q.E.D.

### 4. Proof of Theorem 2

Proof. For the convenience, we denote
\[ F = \frac{F_1}{\alpha}, \quad G = \frac{G_1}{\alpha}. \]

Obviously, one can write \( F, G \) share 1 C-M. According to the above proof, one can get
\[ T(r, F_1) = (n + l + \sigma + o(1)) \cdot T(r, f), \] (4.1)
\[ T(r, G_1) = (n + l + \sigma + o(1)) \cdot T(r, g). \] (4.2)

By the definition of \( F_1 \), we have
$$N_2(r, \frac{1}{F_1}) \leq N_2(r, \frac{1}{f'}) + N_2(r, \frac{1}{P(f)}) + \sum_{j=1}^{r} N_2(r, \frac{1}{f(q_j z')}) + S(r, f)$$

$$\leq 2N(r, \frac{1}{f}) + lT(r, f) + 2 \sum_{j=1}^{r} N(r, \frac{1}{f(q_j z)}) + S(r, f)$$

$$\leq (l + 2 + 2t + o(1)) \cdot T(r, f). \quad (4.3)$$

By the definition of $F_1$, the poles of $F_1$ just come from the zeros of small function $\alpha$. Therefore $N_2(r, F_1)$ equals to $S(r, f)$.

By the same way, we have

$$N_2(r, G_1) + N_2(r, \frac{1}{G_1}) \leq (l + 2 + 2t + o(1)) \cdot T(r, g). \quad (4.4)$$

Note that $F, G$ share 1 C-M, from Lemma 4, we know there exist three relations between $F$ and $G$.

If the case (i) occurs, then by (4.1)-(4.4) one get

$$T(r, F_1) + T(r, G_1) \leq 2(l + 2 + 2t + o(1)) \cdot T(r, f)$$

$$+ 2(l + 2 + 2t + o(1)) \cdot T(r, g) \quad (4.5)$$

Using (4.1), (4.2) and (4.5) gives

$$[(n + l + \sigma) - 2(l + 2 + 2t)]T(r) \leq S(r).$$

Since $n \geq 8\sigma + l$, we have $n + \sigma + l > 2(2 + l + 2t)$. It is a contradiction.

If the case (ii) holds, then $F_1 G_1 = \alpha^2$. This is,

$$f^n P(f) \prod_{j=1}^{l} (f(q_j z))^{\mu_j} g^n P(g) \prod_{j=1}^{l} (g(q_j z))^{\mu_j} = \alpha^2.$$  

When $P(f)(z) = f(z)' - 1$, we have the zeros of $f$ come from the zeros of small function $\alpha$, the zeros of $f - 1$ also come from the zeros of small function $\alpha$. By Nevanlinna SMT again, one can get a contradiction.

When $P(f) \equiv c_0 \in \mathbb{C} \setminus \{0\}$, one can obtain

$$F_1(z) = c_0 f^n \prod_{j=1}^{l} (f(q_j z))^{\mu_j}, \quad G_1(z) = c_0 g^n \prod_{j=1}^{l} (g(q_j z))^{\mu_j}.$$  

Note that $F, G$ share 1 CM, there exists an entire function $\beta(z)$ such that

$$F_1 - \alpha \quad \frac{G_1 - \alpha}{\alpha} = e^{\beta}.$$  

Notice that the order of $f$ is zero, we can conclude that the order of $F_1$ is also zero. Similarly, the order of $g$ is zero, the order of $G_1$ is also zero. The order of $e^{\beta}$ is zero. Therefore $e^{\beta}$ is a constant $c$, where $c$ is not equal to zero.

Rewriting the above equation, gives

$$c G_1 = F_1 + c \alpha = \alpha.$$  

If $c \neq 1$, by the Nevanlinna SMT deduces
\[ T(r, F_i) \leq \mathcal{N}(r, \frac{1}{F_i}) + \mathcal{N}(r, \frac{1}{F_i+c\alpha - \alpha}) + S(r, f) \]
\[ \leq \sum_{j=1}^{i} \mathcal{N}(r, \frac{1}{f(q_j z)^{\nu_j}}) + \mathcal{N}(r, \frac{1}{f}) + \mathcal{N}(r, \frac{1}{G_i}) + S(r, f) \]
\[ \leq (1+t + o(1))(T(r, f) + T(r, g)). \] (4.6)

By (3.2), we have
\[ (n+\sigma)T(r, f) \leq T(r, F_i) + S(r, f). \]

Putting (4.6) into this expression, we have
\[ (n+\sigma-t-1-o(1))T(r, f) \leq (1+t + o(1))T(r, g). \]

In the same way,
\[ (n+\sigma-t-1-o(1))T(r, g) \leq (1+t + o(1))T(r, f). \]

Combining the two expression, one can obtain
\[ (n+\sigma-2t-2)T(r) \leq S(r). \]

Notice that \( n+\sigma-2t-2 > 0 \) by \( n \geq \sigma+3 \) and \( \sigma \geq t \), we obtain a contradiction.

When \( c=1 \), one have \( G_i = F_i \), this is,
\[ f^n \prod_{j=1}^{i} (f(q_j z))^{\nu_j} = g^n \prod_{j=1}^{i} (g(q_j z))^{\nu_j}. \]

If \( k = f/g \), by the above equality, we have
\[ k^n(z) = \frac{1}{\prod_{j=1}^{i} (k(q_j z))^{\nu_j}}. \] (4.7)

Thus from Lemmas 1 and 3, one have
\[ nT(r, k) = T(r, k^n) + S(r, k) = T\left(r, \frac{1}{\prod_{j=1}^{i} (k(q_j z))^{\nu_j}}\right) + S(r, k) \]
\[ \leq \sum_{j=1}^{i} m(r, \frac{1}{k(q_j z)}) + \sum_{j=1}^{i} N(r, \frac{1}{k(q_j z)}) + S(r, k) \]
\[ \leq \sigma m(r, \frac{1}{k}) + \sigma N(r, \frac{1}{k}) + S(r, k) \leq (\sigma + o(1))T(r, k), \]

it contradicts with the relation \( n > \sigma \). Therefore, one know \( k \) must be a constant.

By (4.7), we can obtain that \( k^{\sigma+\sigma} = 1 \), this is, \( f(z) = kg(z) \) and \( k^{\sigma+\sigma} = 1 \).

When the case (iii) occurs, we have \( F_i \equiv G_i \). Using Lemma 5, we also have the result of Theorem 2.

5. Conclusion
In the article, we first consider the zeros about a class q-difference polynomials form as
\[ f^n P(f) \prod_{j=1}^{i} (f(q_j z))^{\nu_j} - \alpha(z). \] In fact, we obtain a Picard theorem of the q-difference polynomials which generalizes some classic results. Especially, we prove that the result of Zhang and Korhonen ([2], Theorem 4.1) holds under the condition \( n \geq 1 \), which is a better result. On the other hand, we can
notice that the first two term of $f^n P(f) \prod_{j=1}^{x} (f(q_j z))^{\alpha_j} - \alpha(z)$ is $f^n P(f)$, $f^n P(f)$ is also a polynomial. By the same method in this article, one will get that $P(f) \prod_{j=1}^{x} (f(q_j z))^{\alpha_j} - \alpha$ has infinite many zero, where $P(f)$ is new polynomials whose zeros are multiple(at least one). The result also generalized a result of Na Li in 2013. Second, we consider a unified uniqueness theorem of q-difference polynomials by using the different method with Zhang and Korhonen in [4]. The method is better when one deal with the high power. If $P(f)$ is a general polynomials, we can not get a linear relation of two meromorphic functions. There exists a fact, we have to mention that the theorem 2 does not include the results of Zhang and Korhonen.

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