Abstract

The $T$–odd top–quark chromoelectric dipole moment ($t$CEDM) is probed through top–quark–pair production via gluon fusion at the CERN Large Hadron Collider (LHC) by considering the possibility of having polarized protons. The complete analytic expressions for the tree–level helicity amplitudes of $gg \rightarrow t\bar{t}$ is also presented. For the derived analytic results we determine the 1–$\sigma$ statistical sensitivities to the $t$CEDM form factor for (i) typical $CP$–odd observables composed of lepton and anti-lepton momenta from $t$ and $\bar{t}$ semileptonic decays for unpolarized protons, and (ii) a $CP$-odd event asymmetry for polarized protons by using the so-called Berger-Qiu (BQ) parametrization of polarized gluon distribution functions. We find that at the CERN LHC, the $CP$-odd energy and angular correlations can put a limit of $10^{-18}$ to $10^{-17}$ $g_s cm$ on the real and imaginary parts of the $t$CEDM, while the simple $CP$-odd event asymmetry with polarized protons could put a very strong limit of $10^{-20}$ $g_s cm$ on the imaginary part of the $t$CEDM.
The top quark with a mass of \( m_t = 174 \pm 6 \) GeV has recently been observed at Tevatron\(^{[1]}\). This large mass compared to the other light quark masses implies that the top quark may be susceptible to new physics effects at TeV-scale, not readily observable in lighter quarks.

Among the properties of the heavy top quark which should be measured at the CERN LHC are its basic couplings to gauge bosons. In this Letter we investigate the possibility of extracting the \( CP \)-odd gluon-top-quark effective couplings through top-quark pair production by gluon fusion at the planned CERN LHC.

Production of \( t\bar{t} \) by gluon fusion followed by \( t \) and \( \bar{t} \) semileptonic decays has been studied to extract the real part of the \( T \)-odd \( tCEDM \) form factor with use of optimal observables\(^{[2]}\). We extend this work\(^{[2]}\) by considering the possibility of having a nonzero imaginary part of the \( tCEDM \) and extract the imaginary as well as real parts of the \( tCEDM \) through a few typical \( CP \)-odd lepton and antilepton correlations\(^{[3]}\) of the \( t \) and \( \bar{t} \) semileptonic decays. Gunion, Yuan and Grzadkowski in Ref. \(^{[4]}\) have demonstrated that \( CP \) violation in the Higgs sector can be directly probed using polarized gluon-gluon collisions at the CERN LHC with polarized protons. We apply the same method in extracting the imaginary part of the \( tCEDM \) and compare its attainable limits with those obtained through the \( CP \)-odd lepton and antilepton correlations from the \( t \) and \( \bar{t} \) semileptonic decays with unpolarized protons.

The general effective Lagrangian for gluon–top–quark interaction includes not only the SM terms of dimension-four but also the two terms of dimension-five:

\[
\mathcal{L}_M = \frac{1}{2} g_s \left( \frac{c_t}{2m_t} \right) \bar{t} \sigma^{\mu\nu} G_{\mu\nu}^a T_a t, \quad \mathcal{L}_E = \frac{i}{2} g_s \left( \frac{\tilde{c}_t}{2m_t} \right) \bar{t} \sigma^{\mu\nu} \gamma_5 G_{\mu\nu}^a T_a t, \tag{1}
\]

where \( \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu] \), \( G_{\mu\nu}^a \) is the gluon field strength, and \( T_a = \frac{1}{2} \lambda_a \) (\( a = 1 \) to \( 8 \)). The coefficient, \( \mu_t \equiv g_s(c_t/2m_t) \), is then called the top-quark chromomagnetic dipole moment (\( tCMDM \)) which, as in the case of QED, receives one-loop contributions in QCD so that its size is of the order \( g_s \alpha_s/\pi m_t \). This would very likely dilute any contributions to \( \mu_t \) due to new physics. On the other hand, the coefficient in the \( \mathcal{L}_E \), \( d_t \equiv g_s(\tilde{c}_t/2m_t) \), is called the \( tCEDM \), which violates \( T \) invariance. Within the Standard Model (SM), this \( tCEDM \) can arise only at three or more loops.
so that it is estimated to be extremely small (|\Delta t| \leq 10^{-30} g_s cm). The tCEDM, however, can be much larger in certain models of CP violation such as the models with CP-nonconservation through Higgs boson exchange, in which the form factor may be about $10^{-20} g_s cm$. In this light, a nonvanishing tCEDM should be a strong indication of new physics. Therefore, we will exclusively focus on the measurement of the tCEDM.

Generally the tCEDM (and likewise the tCMDM) is a function of momentum transfer. In most model calculations, the real part of the tCEDM is nonvanishing and constant to a good approximation, while its imaginary part can be nonvanishing only when the momentum transfer is larger than the $t\bar{t}$ production threshold. The productions of $t\bar{t}$ pairs in proton-proton collisions at the CERN LHC are predominantly through the gluon-gluon fusion diagrams shown in Figure 1(a)-1(c). When we include the tCEDM interaction, we must include the Feynman diagram of Figure 1(d) in order to preserve QCD gauge invariance. The imaginary part of the tCEDM form factor in Figure 1(a) and (b) is vanishing due to the zero momentum transfers of the on-shell gluons, but it might be nonvanishing in Figure 1(c) and (d) in which the momentum transfers are larger than the $t\bar{t}$ production threshold. With this feature in mind, we assume for simplicity that the real and imaginary parts of the tCEDM are constant and non-vanishing. Certainly, the momentum-transfer dependence of the tCEDM should be taken into account in more realistic model calculations and their comparisons with experimental data. This momentum-transfer dependence will be elaborated on in our future work.

The tCEDM term modifies the $g_{tt}$ vertex in momentum space as

$$\Gamma^\mu(k, p, p′) = -ig_s T_a \left[ \gamma^\mu + \left( \frac{\tilde{c}_t}{2m_t} \right) \sigma^{\mu\nu} \gamma_5 k_\nu \right],$$

(2)

where $k = p - p'$ is the gluon four-momentum, and $p$ and $p'$ are the four-momenta of incoming and outgoing top quarks, respectively. In addition, QCD gauge invariance yields a dimension-five $ggtt$ contact term, whose expression is given by

$$\Gamma^{\mu\nu} = g_s^2 f^{abc} T_c \left( \frac{\tilde{c}_t}{2m_t} \right) \sigma^{\mu\nu} \gamma_5,$$

(3)
Figure 1: Tree-level Feynman diagrams for $gg \rightarrow t\bar{t}$. The diagram of (d) is needed to preserve gauge invariance. The black circles denote the vertices modified by the tCEDM.

where $f^{abc}$ is the SU(3)$_C$ group structure constants. This additional term describes the Feynman diagram of Figure 1(d).

In evaluating the explicit form of helicity amplitudes we employ a set of covariant gluon polarization vectors ($\lambda = \pm$):

$$
\epsilon_1(\lambda) = \epsilon_2^*(\lambda) = -\frac{\lambda}{\sqrt{2}}(n_1 + i\lambda n_2),
$$

where $n_1$ and $n_2$ are defined as

$$
n_1^\mu = \frac{N}{2} \left[ (p_1 - p_2)^\mu + \frac{p_1 \cdot (k_1 - k_2)}{(k_1 \cdot k_2)}(k_1 - k_2)^\mu \right], \quad n_2^\mu = N \frac{\epsilon^{\mu\alpha\beta\nu} p_{1\alpha} k_{1\beta} k_{2\nu}}{(k_1 \cdot k_2)},
$$

with the normalization factor

$$
N = \left[ \frac{2(k_1 \cdot p_1)(k_1 \cdot p_2)}{(k_1 \cdot k_2)} - m_t^2 \right]^{-1/2}.
$$

These gluon polarization vectors greatly facilitate our analytic calculations in an arbitrary reference frame due to the useful relations ($i, j = 1, 2$):

$$
n_i \cdot n_j = -\delta_{ij}, \quad k_i \cdot n_j = 0, \quad p_1 \cdot n_1 = -p_2 \cdot n_1 = -\frac{1}{N},
$$

$$
p_i \cdot n_2 = 0, \quad k_i \cdot \epsilon_j = 0, \quad \epsilon_1(\lambda_1) \cdot \epsilon_2(\lambda_2) = -\delta_{\lambda_1\lambda_2}.
$$
It is now straightforward to calculate the tree-level helicity amplitudes for $gg \rightarrow t\bar{t}$. Noting $[T_a, T_b] = i f^{abc} T_c$, we decompose the helicity amplitudes into two parts - a symmetric part and an antisymmetric part with respect to color indices, and present their analytic expressions in the gluon-gluon c.m. frame as

$$M_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}} = \frac{2\pi\alpha_s}{1 - \hat{\beta}^2 \cos^2 \Theta} (\{T_a, T_b\} S_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}} + [T_a, T_b] A_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}}),$$

where $\Theta$ is the scattering angle between an incoming gluon and a top quark, and $\lambda_1, \lambda_2$ and $\sigma/2, \bar{\sigma}/2$ are the helicities of two gluons, $t$ and $\bar{t}$, respectively. For convenience we introduce a $t$CEDM parameter $\tilde{d}_t = \tilde{c}_t / 2m_t$, which is related with $d_t$ by $d_t = g_s \tilde{d}_t$, and expand the symmetric and asymmetric parts of the helicity amplitudes with respect to $\tilde{d}_t$ as

$$S_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}} = S_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}}^0 + (i\tilde{d}_t) S_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}}^1 + (i\tilde{d}_t)^2 S_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}}^2,$$

$$A_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}} = A_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}}^0 + (i\tilde{d}_t) A_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}}^1 + (i\tilde{d}_t)^2 A_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}}^2. \tag{9}$$

The SM contributions, $S_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}}^0$ and $A_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}}^0$, to the helicity amplitudes of $gg \rightarrow t\bar{t}$ are easily evaluated and they are given by

$$S_{\lambda, \sigma; \sigma}^0 = -\frac{4m_t}{\sqrt{s}} (\lambda + \sigma \hat{\beta}),$$

$$S_{\lambda, \sigma; -\sigma}^0 = 0,$$

$$S_{\lambda, -\sigma; \sigma}^0 = \frac{4m_t}{\sqrt{s}} \sigma \hat{\beta} \sin^2 \Theta,$$

$$S_{\lambda, -\sigma; -\sigma}^0 = 2\hat{\beta} (\lambda \sigma + \cos \Theta) \sin \Theta, \tag{10}$$

and

$$A_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}}^0 = \hat{\beta} \cos \Theta S_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}}^0. \tag{11}$$

Note that the asymmetric parts of the helicity amplitudes have the same structure as the symmetric parts except for the helicity-independent kinematic factor $\hat{\beta} \cos \Theta$. This remarkable factorization property is due to a general theorem that in any gauge theory a factorization of the internal-symmetry-(charge-) index dependence and the polarization (spin) dependence into
separate factors holds for any tree-level four-particle amplitude when one or more of the four particles are gauge bosons. This factorization property has been checked by an explicit calculation of both the symmetric and antisymmetric parts.

The explicit form of the terms linear in $\tilde{d}_t$ are given by

\begin{align}
S_{\lambda,\lambda;\sigma,\sigma}^1 &= 2\sqrt{s} \left[ \frac{8m_t^2}{s} + \hat{\beta}(\hat{\beta} - \lambda\sigma) \sin^2\Theta \right], \\
S_{\lambda,\lambda;\sigma,-\sigma}^1 &= -4m_t\lambda\hat{\beta}\sin\Theta \cos\Theta, \\
S_{\lambda,-\lambda;\sigma,\sigma}^1 &= 2\sqrt{s}\hat{\beta}^2 \sin^2\Theta, \\
S_{\lambda,-\lambda;\sigma,-\sigma}^1 &= 0; \\
A_{\lambda,\lambda;\sigma,\sigma}^1 &= \frac{8m_t^2}{\sqrt{s}}(\lambda\sigma + \hat{\beta}) \cos\Theta, \\
A_{\lambda,\lambda;\sigma,-\sigma}^1 &= -4m_t\lambda \sin\Theta, \\
A_{\lambda,-\lambda;\sigma,\sigma}^1 &= 2\sqrt{s}\hat{\beta}^3 \cos\Theta \sin^2\Theta, \\
A_{\lambda,-\lambda;\sigma,-\sigma}^1 &= 0,
\end{align}

and that of the terms quadratic in $\tilde{d}_t$ by

\begin{align}
S_{\lambda,\lambda;\sigma,\sigma}^2 &= -2m_t\sqrt{s}\lambda \left[ \frac{4m_t^2}{s} + \hat{\beta}(\hat{\beta} - \lambda\sigma) \sin^2\Theta \right], \\
S_{\lambda,\lambda;\sigma,-\sigma}^2 &= 4m_t^2\hat{\beta}\sin\Theta \cos\Theta, \\
S_{\lambda,-\lambda;\sigma,\sigma}^2 &= -2m_t\sqrt{s}\sigma\hat{\beta}\sin^2\Theta, \\
S_{\lambda,-\lambda;\sigma,-\sigma}^2 &= -\sigma\hat{\beta}\sin\Theta \left[ \frac{4m_t^2}{s} \cos\Theta + \lambda\sigma(1 - \hat{\beta}^2 \cos^2\Theta) \right]; \\
A_{\lambda,\lambda;\sigma,\sigma}^2 &= -4\sigma m_t^2 \cos\Theta \left( \frac{2m_t}{\sqrt{s}} \right), \\
A_{\lambda,\lambda;\sigma,-\sigma}^2 &= 4m_t^2 \sin\Theta, \\
A_{\lambda,-\lambda;\sigma,\sigma}^2 &= -2\sigma m_t\sqrt{s} \beta^2 \cos\Theta \sin^2\Theta, \\
A_{\lambda,-\lambda;\sigma,-\sigma}^2 &= -\sigma\hat{\beta}^2 \sin^3\Theta,
\end{align}

where $\lambda, \sigma = \pm, \sqrt{s}$ is the gluon-gluon c.m. energy, and $\hat{\beta} = \sqrt{1 - \frac{4m_t^2}{s}}$. We note in passing that the tree-level factorization, which is valid for the SM amplitudes, is spoiled in the full amplitudes by the $t$CDM.
\[ \mathcal{M}_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}} = -(-1)^{(\lambda_1 - \lambda_2) + (\sigma - \bar{\sigma})} \mathcal{M}_{-\lambda_2, -\lambda_1; -\sigma, -\bar{\sigma}}. \] (16)

From Eqs. (10)-(15), we can easily check that the terms linear in \( \tilde{d}_t \) violate CP, while the SM terms and the terms quadratic in \( \tilde{d}_t \) preserve CP.

In the present work, we investigate the CP-violating effects from the tCEDM in two distinct experimental situations at the CERN LHC - unpolarized and polarized proton-proton collisions.

In the unpolarized case, the initial gluon-gluon configurations with unpolarized gluons are invariant under CP so that a detection of the tCEDM requires information on the top quark polarization. Fortunately, the left-handed nature of the weak decay and the very short life time\(^3\) of the top quark due to its large mass allow us to determine the top quark polarization quite easily. We employ the semileptonic top quark decays \( t \rightarrow bW^+ \rightarrow b\ell^+\nu_\ell \) (\( l = e, \mu \)), which can be easily identified from the formidable hadronic backgrounds in the energetic proton-proton collisions and which provides the most efficient handle for top-quark polarization. The resolving power of the \( t \) and \( \bar{t} \) polarizations in the \( t \) and \( \bar{t} \) semileptonic decays in their rest frames is determined by the decay density matrices, which are given in the \( t \) and \( \bar{t} \) helicity bases by

\[
D^t_l = \frac{1}{2} \begin{pmatrix}
1 + \cos \theta & \sin \theta e^{i\phi} \\
\sin \theta e^{-i\phi} & 1 - \cos \theta
\end{pmatrix},
\]

\[
\bar{D}^\bar{t}_l = \frac{1}{2} \begin{pmatrix}
1 + \cos \bar{\theta} & \sin \bar{\theta} e^{i\bar{\phi}} \\
\sin \bar{\theta} e^{-i\bar{\phi}} & 1 - \cos \bar{\theta}
\end{pmatrix},
\]

respectively. Here, \( \theta(\bar{\theta}) \) and \( \phi(\bar{\phi}) \) are the polar and azimuthal angles of \( l^+(\bar{l}^-) \) from the \( t(\bar{t}) \) semileptonic decay with respect to the polarization vector of positively polarized \( t(\bar{t}) \), respectively. These decay density matrices enter the five-fold differential cross section of the sequential process \( gg \rightarrow t\bar{t} \) \( \rightarrow (\ell \nu_b)(\bar{\ell}\bar{\nu}_b) \)

\[
d\hat{\sigma} = \frac{\hat{\beta}}{8\pi s} B_1B_1\bar{\Sigma}(\Theta; \theta, \phi; \bar{\theta}, \bar{\phi})d\cos\Theta \frac{d\cos\phi d\phi d\cos\bar{\theta} d\bar{\phi}}{4\pi},
\]

through a multiplicative combination \( \bar{\Sigma} \) of the production amplitudes and the decay density matrices as

\[
\bar{\Sigma} = \frac{1}{256} \sum_{\sigma, \bar{\sigma}} \sum_{\sigma', \bar{\sigma}'} \sum_{\text{color}} \lambda_1, \lambda_2 = \pm \lambda \mathcal{M}_{\lambda_1, \lambda_2; \sigma, \bar{\sigma}} \mathcal{M}^*_\lambda \bar{M}_{\lambda_1, \lambda_2; \sigma', \bar{\sigma}'} \bar{D}^t_{\sigma, \bar{\sigma}} \bar{D}^\bar{t}_{\sigma', \bar{\sigma}'},
\]

(19)
where $B_l$ and $\bar{B}_l$ are the branching ratios of the $t$ and $\bar{t}$ semileptonic decays with $l = e, \mu$. This angular dependence, $\Sigma$, has sixteen independent lepton and antilepton angular correlations, which in principle allow us to fully reconstruct the information on the top-quark pair production by gluon fusion.

In the laboratory frame, however, the gluon-gluon c.m. frame is not fixed and there are two unidentifiable neutrinos so that the top and antitop momenta are not fully reconstructible. These experimental obstacles force us to employ $CP$-odd energy and angular correlations composed of lepton and antilepton momenta directly measurable in the laboratory frame. Generally, those $CP$-odd observables can be classified into two categories - even or odd under naive time reversal $\tilde{T}$, which changes the sign of the time variable, but does not interchange the initial and final states.

Following the classification of Ref. [3], we may employ two $CP$-odd and $CPT$-even correlations as

$$A_1 = \hat{p}_g \cdot (\vec{p}_l \times \vec{p}_\bar{l}), \quad T_{33} = 2(\vec{p}_l - \vec{p}_\bar{l})_3(\vec{p}_l \times \vec{p}_\bar{l})_3, \quad (20)$$

and three $CP$-odd and $CPT$-odd correlations as

$$A_E = E_l - E_\bar{l}, \quad A_2 = \hat{p}_g \cdot (\vec{p}_l + \vec{p}_\bar{l}), \quad Q_{33} = 2(\vec{p}_l + \vec{p}_\bar{l})_3(\vec{p}_l - \vec{p}_\bar{l})_3 - \frac{2}{3}(\vec{p}_l^2 - \vec{p}_{\bar{l}}^2). \quad (21)$$

However, the vector correlations $A_1$ and $A_2$ should vanish due to Bose symmetry of the initial $gg$ system so that these vector correlations are not useful in detecting the $tCEDM$. So, we have to use $T_{33}$ as a $CPT$-even correlation proportional to the real part of the $tCEDM$, and $A_E$ and $Q_{33}$ as $CPT$-odd correlations proportional to the imaginary part of the $tCEDM$.

The quantity which should be actually evaluated is the expectation value of a given $CP$-odd correlation $O_X$ over the $pp \to t\bar{t}$ production rates, whose expression is given by

$$\langle O_X \rangle = \frac{\int O_X g(x_1)g(x_2)d\hat{\sigma}dx_1dx_2}{\int g(x_1)g(x_2)d\hat{\sigma}dx_1dx_2} \quad (22)$$
where \( g(x) \) is the gluon distribution function, and \( x_1 \) and \( x_2 \) are momentum fractions of two gluons. In order to determine its statistical significance, \( \langle O_X \rangle \) should be compared with the statistical fluctuation \( \langle O_X^2 \rangle \). When only statistical uncertainties are taken into account, an observation different from the SM expectations at 1-\( \sigma \) level requires

\[
\langle O_X \rangle \geq \sqrt{\frac{\langle O_X^2 \rangle}{N_{tt}}},
\]

for the total number of events \( N_{tt} = \epsilon B_l B_\bar{l}\mathcal{L}_{pp}\sigma(pp(gg) \rightarrow t\bar{t}) \) with the acceptance efficiency \( \epsilon \) and the \( pp \) integrated luminosity \( \mathcal{L}_{pp} \).

We assume for the values of experimental parameters in our numerical analysis

\[
\epsilon = 10\%, \quad B_l = B_\bar{l} = 21\% \quad \text{for} \quad l = e, \mu, \quad \sqrt{s} = 14 \text{ TeV}, \quad \mathcal{L}_{pp} = 10 \text{ fb}^{-1}, \quad m_t = 175 \text{ GeV},
\]

and for an unpolarized gluon distribution function employ the GRVHO parametrization\(^{[12]}\), which includes higher order corrections.

Table 1: Attainable 1-\( \sigma \) limits on the real and imaginary parts of the \( t \)CEDM, \( d_t \), through the CP-odd correlations, \( T_{33} \), \( A_E \) and \( Q_{33} \) with the parameter set \(^{[24]}\).

| observable | Attainable 1-\( \sigma \) limits |
|------------|---------------------------------|
| \( T_{33} \) | \(|Re(d_t)| = 0.899 \times 10^{-17} g_s \text{cm} \) |
| \( A_E \) | \(|Im(d_t)| = 0.858 \times 10^{-18} g_s \text{cm} \) |
| \( Q_{33} \) | \(|Im(d_t)| = 0.205 \times 10^{-17} g_s \text{cm} \) |

Table 1 list the attainable 1-\( \sigma \) limits on the real and imaginary parts of the \( t \)CEDM, \( d_t \), through the CP-odd correlations, \( T_{33} \), \( A_E \) and \( Q_{33} \) with the parameter set \(^{[24]}\). Quantitatively, \( T_{33} \) and \( Q_{33} \) enable us to probe the real and imaginary parts of \( t \)CEDM up to the order of \( 10^{-17} g_s \text{cm} \), respectively, and \( A_E \) allows us to probe the imaginary part of the \( t \)CEDM down to \( 10^{-18} g_s \text{cm} \). The numerical results for the real parts are found to be consistent with those in Ref. \(^{[2]}\).
Let us now move to the other situation where proton beams are polarized. If gluons in a positively polarized proton are polarized, this polarization transmission can be utilized to have initial $CP$-odd gluon-gluon configurations, which allow us to probe $CP$-violating effects in gluon fusion without full reconstruction of the final states. More concretely, $CP$ violation is directly probed by the difference between its production rates through gluon-gluon fusion processes for colliding proton beams of opposite polarizations. An easily-constructed $CP$-odd asymmetry is the rate asymmetry $A \equiv \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$, which has been used as a probe of $CP$ violation in the Higgs sector\cite{[4]}. Here, $\sigma_\pm$ in the process $gg \to t\bar{t}$, is the cross section for $t\bar{t}$ production in collision of an unpolarized proton to a proton of helicity $\pm$.

Before folding the gluon distribution functions with the cross section of the subprocess $gg \to t\bar{t}$, we calculate the square of the matrix elements by summing over the final $t$ and $\bar{t}$ polarizations and, assuming that $\tilde{c}_t$ is small and keeping the terms up to linear in $\tilde{c}_t$, we then obtain

$$|M_{\lambda_1,\lambda_2}|^2 = \frac{8\pi^2 \alpha_s^2}{(1 - \beta^2 \cos^2 \Theta)^2} T_{\lambda_1,\lambda_2}$$

where

$$T_{\pm\pm} = \frac{1}{16} \left[ (1 + \beta^2) \pm 4Im(\tilde{c}_t) \right] \left( \frac{7}{3} + 3\beta^2 \cos^2 \Theta \right),$$

$$T_{\pm\mp} = \frac{1}{2} \beta^2 \sin^2 \Theta (2 - \beta^2 + \beta^2 \cos^2 \Theta) \left( \frac{7}{3} + 3\beta^2 \cos^2 \Theta \right).$$

With the above expressions we can see that the difference and sum of production cross sections of opposite proton helicities are given by

$$d\sigma_+ - d\sigma_- \sim g_1 \Delta g_2 (T_{++} - T_{--}) \sim g_1 \Delta g_2 Im(\tilde{c}_t), \quad d\sigma_+ + d\sigma_- \sim g_1 g_2 \sum_{\lambda_1,\lambda_2} T_{\lambda_1,\lambda_2},$$

where $g_1(g_2)$ is the gluon distribution function for the unpolarized (polarized) proton, $\Delta g_2 = g_{2+} - g_{2-}$, $g_{1,2} = g_{1,2+} + g_{1,2-}$, and the $\pm$ subscripts of $g_{1,2}$ and $T_{\lambda_1,\lambda_2}$ indicate gluons with $\pm$ helicity. So, the rate asymmetry $A$ contains the information only on the imaginary part of the $tCEDM$. 

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Crucial for our numerical analysis is the degree of polarization that can be achieved for gluons at the CERN LHC. The amount of gluon polarization in a positively polarized proton beam, defined by the structure function difference \( \Delta g(x) = g_+(x) - g_-(x) \), is not currently known precisely. The relative behavior of \( \Delta g(x) \) compared to the unpolarized gluon distribution \( g(x) \) is theoretically constrained in the \( x \to 1 \) and \( x \to 0 \) limits: \( \Delta g(x)/g(x) \to 1 \) for \( x \to 1 \) and \( \Delta g(x)/g(x) \propto x \) for \( x \to 0 \). Several simple models which satisfy these constraints suggest that a significant amount of the proton’s spin could be carried by the gluons. The European Muon Collaboration (EMC) data on the polarized structure function \( g_1^p(x) \) is also most easily interpreted if this is the case. Since the purpose of our work does not aim at the properties of the polarized gluon distribution function, we simply employ in our analysis the BQ parametrization among various parametrizations on \( \Delta g \). Furthermore, we do not consider the scale evolution of \( \Delta g \), but simply use the distribution at \( Q^2 = 100 \text{ GeV}^2 \).

Analytically, the BQ parametrization is defined to satisfy

\[
\Delta g(x) = \begin{cases} 
g(x) & (x > x_c) \\
(x/x_c)g(x) & (x < x_c) 
\end{cases},
\]

where \( x_c \approx 0.2 \) yields a value of \( \Delta g \approx 2.5 \) at \( Q^2 = 10 \text{ GeV}^2 \). For the unpolarized gluon distribution, we use the GRVHO parametrization as for the previous study.

To obtain a numerical indication of the observability of the asymmetry \( A \), we include all possible \( t \) decay modes so that the net branching ratio is assumed to be unity. The statistical significance of the asymmetry can then be computed as

\[
N_{SD} = \frac{N_+ - N_-}{\sqrt{N_+ + N_-}} \equiv \text{Im}(\tilde{c}_t) \frac{\Delta \hat{N}}{\sqrt{\hat{N}}},
\]

where \( N_+(N_-) \) is the number of \( t\bar{t} \) events predicted for positive(negative) proton, \( N = N_+ + N_- \), and \( \Delta \hat{N} \) is defined as \( N_+ - N_- = \text{Im}(\tilde{c}_t)\Delta \hat{N} \). For a realistic detection efficiency \( \epsilon \), we have only to rescale the number of events by this parameter, \( N \to \epsilon N \). Taking \( N_{SD} = 1 \), we obtain the 1-\( \sigma \) attainable limits on the imaginary part of the \( t \) CEDM, \( \text{Im}(d_t) \), as

\[
|\text{Im}(d_t)| = \frac{g_s}{2m_t} \sqrt{\frac{1}{N} \frac{N}{\Delta \hat{N}}},
\]
Table 2: The number of $t\bar{t}$ events $N$, the ratio $\Delta \hat{N}/N$, and the attainable 1-$\sigma$ limits $|\text{Im}(d_t)|$, for $p_T$-cuts with $\sqrt{s} = 14$ TeV, $m_t = 175$ GeV and $\mathcal{L} = 10$ fb$^{-1}$.

| $p_T$-cuts (GeV) | $N(\times 10^6)$ | $\Delta \hat{N}/N$ | $|\text{Im}(d_t)|(\times 10^{-20}g_s\text{cm})$ |
|------------------|-----------------|-------------------|---------------------------------|
| 0                | 2.62            | 1.44              | 0.766                           |
| 20               | 2.55            | 1.42              | 0.788                           |
| 40               | 2.36            | 1.37              | 0.847                           |
| 60               | 2.08            | 1.30              | 0.951                           |
| 80               | 1.74            | 1.22              | 1.107                           |
| 100              | 1.41            | 1.14              | 1.313                           |

Experimentally, a sizable transverse-momentum cut may be needed to reduce the formidable hadronic backgrounds and so we take into account the $p_T$-cut dependence in determining the number of $t\bar{t}$ events $N$, the ratio $\Delta \hat{N}/N$ and the 1-$\sigma$ limits of $|\text{Im}(d_t)|$ for an integrated luminosity of 10 fb$^{-1}$. There is, however, no drastic $p_T$-cut dependence as can be clearly seen in Table 2. A large $p_T$ can be, therefore, taken to reduce large amount of background effects without spoiling the attainable limits on the $t$CEDM. Remarkably, the attainable 1-$\sigma$ limits for the imaginary part, $\text{Im}(d_t)$, are much more stringent than those obtained through the lepton and antilepton correlation $A_E$ of $t$ and $\bar{t}$ decay products (See Table 1.), although the same value of acceptance efficiency is taken in the polarized case. Numerically, it is possible to put a limit on the imaginary part of the $t$CEDM, $\text{Im}(d_t)$, up to the order of $10^{-20}$ $g_s\text{cm}$ at 1-$\sigma$ level.

Certainly, more precise theoretical estimates and experimental determinations of polarized gluon distribution in polarized proton must be performed. If $\Delta g(x)$ is precisely known, then detection of $A$ is relatively straightforward. At any rate, taking into account the theoretical constraints on the $x \rightarrow 0$ and $x \rightarrow 1$ limits of $\Delta g(x)$, and various model parametrizations, we do not regard our choice of the simple BQ parametrization as unlikely. Consequently, the ability of polarizing one of the proton beams at the CERN LHC could provide a unique opportunity for detecting $CP$ violation in $gg \rightarrow t\bar{t}$ as well as in the Higgs sector.
To summarize, the real and imaginary part of the $t$CEDM can be measured through the top-quark pair production by gluon fusion to a precision of the order of $10^{-18} \, g_s \text{cm}$ by use of the lepton and antilepton correlations of the $t$ and $\bar{t}$ semileptonic decays in unpolarized proton-proton collisions, and the imaginary part of the $t$CEDM to a precision of the order of $10^{-20} \, g_s \text{cm}$ by use of the asymmetry between production rates for positively versus negatively polarized protons, if a reasonable amount of the proton polarization is transmitted to the gluon distributions. This large enhancement with proton polarization could be a reasonable motivation for having polarized LHC beams.

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