Experiments on the vortex wake of a swimming knifefish

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Abstract The knifefish species propels itself by generating a reverse Kármán street using an anal fin, and the propulsion of this species is known to be highly efficient (Blake in Can J Zool 61:1432–1441, 1983). Previous studies have suggested that there is an optimal swimming range for fish based on the amplitude and frequency of the reverse Kármán street. In the current study, experiments have been performed to measure the ratio between the amplitude and wavelength of vortices in the wake of a knifefish. It is suggested that the wave efficiency can be estimated by optimizing the thrust created by the reverse Kármán street for a given spacing ratio, and present observations have an average value of 0.89. The relationship established between spacing ratio and wave efficiency, in addition to the measured parameters, will be invaluable for bio-inspired designs based on the knifefish.

1 Introduction

In addition to understanding the ecological and evolutionary forces that shape animal diversity, the knowledge of fish swimming mechanisms is becoming increasingly important for the design and implementation of underwater vehicles. It is natural that designers look to fish for design inspiration since fish swimming has been exposed to evolutionary forces that could select for optimal swimming (e.g., Sfakiotakis et al. 1999). Fish inhabit a wide array of habitats, which place different performance demands. Thus, different fish species evolved swimming behaviors suitable for different habitats and tasks, and there exists a wide range of propulsive methods (Sfakiotakis et al. 1999). One of the swimming types recently suggested for use in underwater vehicles is that known as gymnotiform motion (Blake 1983). For their swimming robot, Shigaonkar et al. (2008) have taken inspiration from one of the gymnotiform swimmers: the knifefish. In the current study, the focus is on experiments of a live knifefish species (shown in Fig. 1).

The characteristic feature of gymnotiform motion is the anal fin (Fig. 1) that is used for thrust by creating a traveling sinusoidal wave along the length of its body (Blake, 1983). While this form of swimming does not produce the greatest thrust, it is known to have high hydromechanical efficiency ranging upward of 80 % (Blake 1983). In addition, the direction of the wave can be reversed implying that the fish can swim backward as easily as forward. Knifefish hunt for their prey by detecting disturbances in an electric field which suggests one of the reasons for this swimming behavior—the prey is detected by swimming past it and a quick change of direction ensures that it does not escape after being detected (Shigaonkar et al. 2008). According to Blake (1983), it is unclear whether the
electric sense preceded the gymnotiform motion in the evolution of the species. However, similar swimming behaviors are also observed in other weakly electric fish species, suggesting that this swimming mode co-evolved with electrical sense.

The vortex wake behind fish resembles a Kármán vortex street—only in reverse (Triantafyllou et al. 1991). To calculate thrust, Streitlien and Triantafyllou (1998) have reviewed different methods based on measurements of the wake and/or body of the fish. One of the three models they used was Kármán’s formulation for the drag induced by a vortex street with the suitable modification of reversing the sign of the circulation. Even though this model is relatively simplistic and does not account for three-dimensionality, it has been shown to be accurate to within 10% (Streitlien and Triantafyllou 1998). One of the challenges of using the Kármán model experimentally is the apparent need to measure the circulation; however, the thrust of the vortex street need not depend on the circulation directly since the speed of the vortex system, \( U_s \), and the circulation are linked (e.g., Weihs 1972). The vortex street thrust coefficient is defined as \( C_T_s = \frac{T_s}{\frac{1}{2} \rho U_f^2 a} \) where \( T_s \) is the thrust per unit depth (into the plane); \( \rho \) the density of water; \( U_f \) the speed of the fish; and \( a \) the vortex wavelength. Using the lateral vortex spacing, \( b \), the dependency of \( C_T_s \) on the vortex spacing ratio \( b/a \) can be written similarly to Bearman (1967)

\[
C_T_s = \frac{4}{\pi} \left( \frac{U_s}{U_f} \right)^2 \left[ \left( \frac{U_f}{U_s} - 2 \right) \frac{\pi b}{a} \coth \frac{\pi b}{a} + \coth^2 \frac{\pi b}{a} \right].
\]

The goal of the current study is to use experimental data to quantify the range of spacing ratios that could be expected in the wake of knifefish. These measurements allow the approximation of knifefish swimming efficiency and can help establish design criteria for those using the knifefish species in their bio-inspired designs (e.g., Curet et al. 2011).

2 Details of the experiments

In order to adhere to appropriate policies, regulations and laws for handling and care of vertebrate animals, the tests followed protocol 08-017 approved by Rutgers University Animal Care and Facilities Committee for studying the African Knifefish (Xenomystus nigri) swimming methods. Following the protocol, all tests were performed only by approved personnel at the Rutgers facility. Tests were performed in a recirculating acrylic tank comprising a test section with dimensions \( 8 \times 8 \times 38 \text{ cm}^3 \).

The height of the anal fin varies between 0.8 and 1 cm, and the data were taken underneath the fish in a horizontal plane (Fig. 1) located between 25 and 75% of the height of the anal fin using particle image velocimetry (PIV). The PIV system comprised a Nd:YAG laser and a digital PIV CCD Camera (TSI Powerview Plus 2 MP) at 14.5 Hz. In the current study, the focus is on six instances when the wake of the knifefish was in the field of view (11 \( \times \) 8 cm\(^2\)) of the camera: cases 1–3 while the fish was swimming forward and cases 4–6 while the fish was swimming backward. In cases 2 and 3, the flow is stationary and the fish swims freely, while in cases 1, 4, 5 and 6, a uniform flow of 8 cm/s is imposed.

3 Results and discussion

3.1 Vortex identification

To quantitatively measure the spacing ratio in the wake of the knifefish, it is necessary to identify the vortex centers unambiguously. The swirling strength, \( \lambda_{ci} \) is defined as the complex portion of the complex pair of eigenvalues of the two-dimensional velocity gradient tensor, and this method is often used on two-dimensional PIV data (e.g., Adrian et al. 2000). Once \( \lambda_{ci} \) is computed for each point, the locations with the highest swirling strength are identified as the vortex centers.

A sample velocity map has been plotted on top of a PIV image with vorticity contours in Fig. 2b and swirling strength contours in Fig. 2c. Also shown in Fig. 2c are the identified locations of high swirling strength highlighting the vortex centers.

3.2 Vortex spacing

One of the classical descriptions of vortex street wakes is the ratio of wavelength, \( a \), and the cross-stream distance between the two rows of vortices, \( b \) (Bearman 1967). These lengths have been computed according to Fig. 2a where the relationship between \( b \) and the amplitude of the anal fin is shown. The vortex wavelength is computed as the distance between vortices 1 and 3, and the cross-stream distance, \( b \), is defined as the shortest distance between the vortex at \( (x_2, y_2) \) and the line connecting vortices at \( (x_1, y_1) \) and \( (x_3, y_3) \). The number of vortex triplets used to estimate the spacing...
ratio ranged between 1 and 4 in each PIV frame, and each frame-average spacing ratio is presented in Table 1.

A wide range of spacing ratios is observed in Table 1 with the majority lying within the range of 0.2–0.4 found in bluff body wakes (e.g., Bearman 1967). Epps et al. (2009) report a wide range of spacing ratios for a robotic fish as well as four samples for a Giant Danio (Danio aequipinnatus). Analysis of variance (ANOVA) revealed that the spacing ratios for the knifefish ($\bar{m} \pm SD = 0.31 \pm 0.1$; Table 1) are statistically lower than the robot fish ($\bar{m} \pm SD = 0.56 \pm 0.13$; ANOVA $F_{2,25} = 20.4$ $p$ value < 0.001; Tukey Post Hoc $p$ value < 0.001). The knifefish spacing ratios were not significantly different than those of the Giant Danio ($\bar{m} \pm SD = 0.20 \pm 0.01$; Tukey Post Hoc $p$ value < 0.18; however, conclusions about the similarity in this case need to be made with caution due to the limited number of samples. One of the particular features of the knifefish is that the vortices are created by the sinusoidal shape and movement of the fin rather than the beating of its tail. Thus, rather than exclusively creating vortices behind its body, the knifefish creates vortices underneath its body (e.g., Fig. 2).

### 3.3 Efficiency

Bearman (1967) used Kármán’s drag equation for a vortex street and the $C_D/C_v(b/a) = 0$ stability criterion proposed by Kronauer to link the spacing ratio and the convection speed of vortices in the wake of bluff bodies. The schematic in Fig. 2a shows the reverse Kármán street generated in the wake of the knifefish and the corresponding motion in the opposite direction of a circular cylinder. Thus, by generating the same vortex arrangement as the cylinder while moving in the opposite direction, the drag induced by the vortex street is analogous to the thrust produced by the fish (Eq. 1). Thus, instead of creating a wake deficit, this vortex arrangement produces a jet-like profile in its wake indicating the generation of thrust as shown in Fig. 3. Near the end of case 2, the fish swims in the negative $x$-direction in the laboratory frame of reference; contours of the swirling strength $\omega$, points marking identified vortices, and lines showing the region over which the velocity profile of Fig. 3 was averaged. The axes in b and c are scaled by the largest lateral dimension of the fish, $d_f$, shown in a.

### Table 1 Wake parameters measured for different cases

| No. | $b/a$ | $U_f/U_w$ | $St$ | No. | $b/a$ | $U_f/U_w$ | $St$ |
|-----|-------|----------|------|-----|-------|----------|------|
| 1   | 0.37  | 0.79     | 0.29 | 4   | 0.26  | 0.90     | 0.23 |
|     | 0.29  | 0.87     | 0.25 |     | 0.19  | 0.97     | 0.18 |
|     | 0.27  | 0.89     | 0.24 |     | 0.04  | 1.00     | 0.04 |
| 2   | 0.35  | 0.81     | 0.28 | 0.13| 0.99  | 0.13     |      |
|     | 0.30  | 0.86     | 0.26 |     | 0.13  | 0.99     |      |
|     | 0.50  | 0.72     | 0.36 |     | 0.08  | 1.00     |      |
|     | 0.22  | 0.94     | 0.21 | 5   | 0.17  | 0.98     | 0.17 |
|     | 0.44  | 0.74     | 0.33 |     | 0.32  | 0.84     | 0.27 |
|     | 0.25  | 0.91     | 0.23 | 6   | 0.14  | 0.99     | 0.14 |
|     | 0.19  | 0.96     | 0.18 |     | 0.26  | 0.91     |      |
| 3   | 0.47  | 0.73     | 0.34 |     | 0.26  | 0.91     |      |
|     | 0.26  | 0.91     | 0.23 |     | 0.41  | 0.76     | 0.31 |
|     | 0.16  | 0.98     | 0.16 |     | 0.24  | 0.92     | 0.22 |

Each line is the average value for each PIV frame. The table is divided into forward (left side) and backward (right side) swimming.
ratio \( U_f/ U_w \) where \( U_w \) is the wave speed, or convection speed, of the vortex street (Sfakiotakis et al. 1999). In the assumptions behind Eq. 1, the wave speed of the vortices is defined as \( U_w = U_t + U_s \). Thus, the wave efficiency is calculated using Eq. 2 and

\[
\frac{U_t}{U_w} = \frac{1}{1 + U_s/U_t}. \tag{3}
\]

These data are added to Table 1 for each PIV measurement in all six cases, and the range of wave efficiencies found here is consistent with the range reported by Curet et al. (2011) of 0.6–0.9 for a bio-inspired robot knifefish. Thus, even though Eq. 3 is a two-dimensional approximation, it is observed to yield values which agree with previously measured wave efficiencies. Likewise, vortex streets are two-dimensional only at low Reynolds numbers, yet the prediction of convection speeds using a similar model (Bearman 1967) agree with published results for Reynolds numbers at which vortex streets are highly three-dimensional.

The efficiencies approach unity as the spacing ratio approaches zero; however, the thrust also approaches zero as \( b/a \to 0 \) (Eq. 1). Since the fish did not often appear in the image at the same time as its wake, it was not possible to estimate \( U_f \) very often from the current dataset. However, when it was possible, the wave slip was computed using the convection speed of the vortices as the wave speed (taken as the speed at the center of the vortex). The measured wave efficiency ranged between 0.85 and 0.89 which agrees with the predicted values based on the spacing ratios in Table 1.

In addition to the wave slip, there is an optimal Strouhal number \( (St = fb/Ut) \) range of 0.2 < \( St < 0.4 \) (Triantafyllou et al. 1991) for propulsive wakes. Since the frequency of the vortex street is \( f = U_w/a \), the Strouhal number can be written as

\[
St = \frac{U_w b}{U_t a}. \tag{4}
\]

The Strouhal numbers have been added to Table 1 using the measured spacing ratios and Eq. (3), and for most cases, the calculated Strouhal numbers fall inside the optimal range predicted by Triantafyllou et al. (1991). The cases falling outside of this optimal range correspond to cases when \( b/a \to 0 \).

4 Conclusions

Through PIV measurements on a swimming knifefish, the spacing ratio of the wake is quantified for several cases. The PIV measurements also show that the knifefish can create a thrust-generating vortex street wake while a portion of that wake remains underneath its body due to the unique anal fin. In addition, a link has been suggested between spacing ratio and efficiency (Eq. 3), and the efficiencies measured in this way agree with previous studies (Curet et al. 2011). While the two-dimensional approximations seem to match previously published wave efficiencies and optimal Strouhal numbers, Shirgaonkar et al. (2008) have shown the importance of the three-dimensional connections between the vortices, and further studies are required to address how the neglected three-dimensionality affects the swimming efficiency of knifefish.

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Fig. 3 Velocity profile in the wake of the fish for case 2, which is averaged between the lines shown in Fig. 2c. The vertical axis is scaled in the same way as Fig. 2.