Determination of an acoustic reflection coefficient at the inlet of a model gas turbine combustor for power generation

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Abstract. A phenomenon that potentially influences the reliability of power generation systems is the presence of thermo-acoustic oscillations in the combustion chamber of a land-based gas turbine. To develop specific measures that prevent the instability, it is essential to predict and/or evaluate the underlying physics of the thermo-acoustics, which requires the acoustic boundary condition at the exit of the burner, that is, at the inlet of the combustor. Here we report a procedure for calculating acoustic reflection coefficients at the burner exit by utilizing two microphone method (TMM) for dynamic pressure signals. The procedure has been verified by comparing its results with reported ones and further successfully employed to determine the acoustic boundary condition of the burner of a partially-premixed model gas turbine combustor.

1. Introduction
Combustion instability (CI) in practical combustion systems including land-based gas turbines of the lean premixed combustion technique, which is employed for the reduction of the emission of pollutants such as nitrogen oxides, is one of the ongoing issues requiring proper solutions yet to be prepared for through further studies [1]. The phenomenon is generated from the excitation of certain resonance mode of the acoustic field in combustion chamber induced by the feedback originated from the perturbed heat release rate attributable to the fluctuation of equivalence ratio and/or of the velocity of the pre-mixture of oxidizer and fuel of natural gas in most cases.

Fuel flexibility has also become a very keen issue for the utility of the power generation [2]. Alternative natural gases such as coal-derived gas, synthetic natural gas (SNG), biomass and landfill gases, refinery off-gas, etc., whose chemical compositions are more likely to vary, may exacerbate CI, because most land-based gas turbine combustors are sensitive to change in operation conditions. When the systems experience the CI, thereby, the magnitude of dynamic pressure wave of the chamber may nonlinearly increase and end up with the limit cycle.

In general, the dominant frequency, the growth rate, and the amplitude of the limit cycle of the instability are of concern during from the design stage of gas turbine combustors to their commercial operations including tuning after regular overhauls. Some of the predictive approaches for this issue including the lumped network method [1, 3], finite difference method (FDM) [4], etc., deal with the Helmholtz equation, instead of the full Navier-Stokes equation [5]. The Helmholtz equation, time-independent version of the wave equation, mostly uses the flame transfer function which is determined
either analytically or experimentally, as a flame response model [6, 7]. Besides, the acoustic boundary conditions of the calculation domain which are also determined either analytically [3] or experimentally [8, 9], in terms of either reflection coefficient or impedance, are needed for the prediction of CI. Kim et al. [8] measured spatially distributed flame transfer functions and reflection coefficient at the single dominant frequency and then predicted combustion dynamics of lean-premixed combustor including CI. They measured the coefficient at the inlet of the lean-premixed combustor with the wave decomposition method. Recently, the result of the CI evaluated with the reflection coefficients measured as a function of frequency has been reported [9].

In this study, a method to determine the reflection coefficient on the boundary plane of a model gas turbine combustor as a function of frequency have been developed by using the two microphone method (TMM) to predict the CI in the gas turbine combustor for power generation. Theory associated with this approach has basically been borrowed from ISO standards [10] that were prepared for the determination of the absorption coefficients of sound absorbing materials. Factors associated with the characteristics of the model gas turbine combustors such as the mean flow and thermo-viscous dissipation effects were also taken into account. The validity of the developed procedure was verified by comparing the results with those of the benchmark experimental data. Further, it has been used to determine the reflection coefficient of a partially-premixed model gas turbine combustor burning SNG [11].

![Figure 1. Geometry of two microphones for measuring reflection coefficient at an impedance plane [10].](image)

2. Theory and development of analysis code
The acoustic reflection coefficient $R$ of the plane wave inside the duct of a model gas turbine combustor is determined by the pressure transfer function (PTF) $H_{12}$ that is measured with the two microphones installed in front of the impedance plane (see Figure 1). The theory associated with this method is as follows. The planar incident wave $p_i$ and planar reflected wave $p_r$ in a duct (or in an impedance tube) are expressed as in the following equations (1) and (2), respectively.

$$p_i = \hat{p}_i e^{jk_0 x}$$

$$p_r = \hat{p}_r e^{-jk_0 x}$$

where $\hat{p}_i$ and $\hat{p}_r$ are the magnitudes of $p_i$ and $p_r$ in the duct, and the constant $k_0$ is a complex wave number. The sound pressure of $p_1$ and $p_2$ at the two microphone positions are as expressed in the following equations (3) and (4), respectively.

$$p_1 = \hat{p}_1 e^{jk_0 x_1} + \hat{p}_r e^{-jk_0 x_1}$$

$$p_2 = \hat{p}_1 e^{jk_0 x_2} + \hat{p}_r e^{-jk_0 x_2}$$
The transfer function for the incident wave $H_I$ can be expressed as in the following equation (5).

$$H_I = \frac{P_{2I}}{P_{1I}} = e^{\imath k_s(x_1 - x_2)} = e^{\imath k_sS}$$  \hspace{1cm} (5)

where $s(x_1 - x_2)$ denotes the distance between the two microphones. Similarly, the transfer function for the reflected wave alone $H_R$ can be expressed as in the following equation (6).

$$H_R = \frac{P_{2R}}{P_{1R}} = e^{\imath k_s(x_1 - x_2)} = e^{\imath k_sS}$$  \hspace{1cm} (6)

The pressure transfer function $H_{12}$ for the total sound field in the duct can be derived from equations (3) and (4) and the relation of $\hat{P}_R = \hat{R} \hat{P}_I$ as follows:

$$H_{12} = \frac{P_2}{P_1} = e^{\imath k_s x_2} + \text{Re}e^{\imath k_s x_2}$$ \hspace{1cm} (7)

Substituting equations (3) and (4) into equation (7) yields the reflection coefficient $R$ at the impedance plane ($x = 0$) as follows:

$$R = \frac{H_{12} - H_I}{H_R - H_{12}} e^{\imath k_s x_1}$$  \hspace{1cm} (8)

Figure 2 illustrates the flowchart of the procedure for determining the PFT of model gas turbine combustor and thereof the reflection coefficient at the combustor inlet. Sound pressures $p_1$ and $p_2$ at the two microphone positions are time-domain data, and, to convert them into frequency domain, the certain data block of finite length is taken to perform fast Fourier Transform (FFT). The original dynamic pressure data were divided into blocks of the certain length, and the degree of overlapping between two consecutive blocks was taken as a parameter. For most of runs to be discussed, there were around 100 blocks. When FFT is performed, the time weighted function (that is, window function) is usually applied to reduce errors attributable to the aperiodicity and discontinuity of the data block. In MATLAB code developed in this study, one of three window functions (i.e., uniform (or boxcar), Hanning, and Hamming windows) can be selectively chosen during the transformation.

![Figure 2. Flowchart for a MATLAB code calculating PTF and thereof reflection coefficient $R$ at the inlet of a model gas turbine combustor.](image)
Figure 3. A typical $p'$ data as a time record of finite length: (a) the raw data, (b) the selected block of the data, (c) the modified with a boxcar window, the same as (b), (d) the modified with Hanning window and (e) the modified with Hamming window.

Figure 3 shows a typical $p'$ data as a time trace of finite length and its modifications with various windows. The PTF $H_{12}$ in equation (7) was calculated with the dynamic pressure data $p_1$ and $p_2$ taken from the two microphones by utilizing MATLAB built-in function $tfestimate$. The reflection coefficient $R$ was then determined by using equation (8).

Figure 4. Benchmark data used for verifying the reflection coefficient calculation code developed in this study: (a) the PTF and (b) the corresponding $R$. 
The developed MATLAB code, based on the flowchart in Figure 2, has been tested with a set of benchmark data shown in Figure 4 for its validation. Figure 4(a) depicts a calculated PTF of the TMM data, in terms of its magnitude and phase, taken from an impedance tube mimicking a lean premixed model gas turbine combustor, which is similar to the one in Figure 1. Figure 4(b) shows the calculated reflection coefficient $R$, in terms of the magnitude and phase. In order to verify the developed code in this study, the PTF in Figure 4(a) has been provided into the code, and its result of the calculated reflection coefficient is shown in Figure 5. The nearly identical reflection coefficients in Figure 5 and Figure 4(b) show that the developed code works well.

![Figure 5](image)

**Figure 5.** Calculated reflection coefficient in this study with the PTF in Figure 4(a).

3. Application to a model gas turbine combustor case

The combustion characteristics of a partially premixed model gas turbine combustor (see Figure 6) were investigated by varying both fuel composition of synthetic natural gas (see Figure 7) and other operation conditions [11]. Dynamic pressure transducers (DP01 - DP11; PCB® model 102M205) installed at 11 locations along the axis of both the partially premixed burner and the combustor, as shown in Figure 6, to identify the spatial distribution of dynamic pressure. The dynamic pressure signals from DP01 and DP02 were used for determining the reflection coefficient. The sampling frequency of the data is 10 kHz. Dynamic pressure signals from DP01 and DP02 for several test cases shown in Figure 7(a) have been input to the MATLAB code to calculate their reflection coefficients at the inlet of the combustor which is just upstream of flange shown in Figure 6. There is a flow straightener made out of the perforated metal disk at the inlet. Figures 8(a), 8(b), 8(c), and 8(d) show the time traces of dynamic pressure, their zoomed-in data, the corresponding pressure transfer function $H_2$, and the calculated reflection coefficient at the inlet of the combustor for the test case of 11 in Figure 7(a). Fuel composition of the test case is 12.5% H$_2$, 12.5% CO, and 75% CH$_4$ in volume. Since the fundamental frequency of the 1st longitudinal mode of the combustor would be less than 500 Hz, the result of the reflection coefficient is shown up to the frequency, as shown in Figure 8(d). The inlet of the combustor is frequently considered to be “choked” acoustically, at which the reflection...
coefficient is 1, if there is no experimental measurement available. As shown in Figure 8(d), the magnitude of the measured reflection coefficients at the inlet is generally in the vicinity of unity within the frequency range and thereby, the acoustic results obtained in this study could be concluded to be reasonable.

(a) Test cases with different fuel compositions of SNG and (b) their peak dynamic pressures measured at the dump plane of the combustor with DP03 [11].

Figure 7. (a) Test cases with different fuel compositions of SNG and (b) their peak dynamic pressures measured at the dump plane of the combustor with DP03 [11].

Figure 8. Reflection coefficient measurements at the inlet of the partially premixed model gas turbine combustor: (a) the time trace of TMM signals with DP01 and DP02, (b) the time trace of the TMM signals for the first 0.05 s, (c) the corresponding PTF $H_{12}$, and (d) the calculated reflection coefficient at the inlet of the combustor for the test case of 11.
Currently, the calculated reflection coefficient is being provided to the CI prediction codes including the finite element method (FEM)-based Helmholtz solver [4] and an open source OSCILOS (open source combustion instability low order simulator) [12] to predict the CI characteristics of the partially-premixed model gas turbine combustor [11]. The occurrence of CI and their dominant frequencies to be predicted are to be compared with those of the FFT analysis of the corresponding experimental dynamic pressure data measured at the dump plane of the combustor, DP03 in Figure 6.

4. Conclusion
A MATLAB code that can determine the acoustic reflection coefficient on the acoustic boundaries of model gas turbine as a function of frequency has been developed. The feasibility of the developed code was verified with the experimental benchmark data. Further, the reflection coefficient at the inlet of partially-premixed model gas turbine combustor has been calculated with dynamic pressure data measured synchronously at two locations of an experimental rig, resulting in quite reasonable values. A prediction of the CI for the model gas turbine combustor is under way with the calculated reflection coefficient at its inlet and other input parameters including the flame transfer functions, and will be reported in the near future.

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References
[1] Lieuwen T C and Yang V 2005 Combustion Instabilities in Gas Turbine Engines: Operational Experience, Fundamental Mechanisms and Modelling (Reston, VA: AIAA, Inc.)
[2] International Energy Agency 2012 World Energy Outlook 2012: Executive Summary. Available from: http://www.worldenergyoutlook.org/publications/weo-2012/
[3] Cha D J et al 2009 Journal of Mechanical Science and Technology 23 1602-1612
[4] Kim S K et al 2010 Journal of the Korean Society of Propulsion Engineers 38(5) 445-455
[5] Dowling A P and Stow S R 2003 Journal of Propulsion and Power 19(5) 751-764
[6] Balachandran R et al 2005 Combustion and Flame 143 37-55
[7] Palies P et al 2010 Combustion and Flame 157 1698-1717
[8] Kim K T et al 2010 Combustion and Flame 157 1718-1730
[9] Richecoeur F et al 2013 Comptes Rendus Mecanique 341 141-151
[10] ISO 1998 Acoustics–Determination of Sound Absorption Coefficient and Impedance in Impedance Tubes Reference No. ISO 10534-2
[11] Lee M C 2014 An Experimental Study on Combustion Instability and NOx Emission Characteristics of H2/CO/CH4 Syngas in a Gas Turbine Combustor Ph.D. thesis (Seoul: Seoul National University)
[12] http://www.oscilos.com