Logic, Individuals and Concepts

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Abstract

This extended abstract gives a brief outline of the connections between the descriptions and variable concepts. Thus, the notion of a concept is extended to include both the syntax and semantics features. The evaluation map in use is parameterized by a kind of computational environment, the index, giving rise to indexed concepts. The concepts are inhabited into language by the descriptions from the higher order logic. In general the idea of object-as-functor should assist the designer to outline a programming tool in conceptual shell style.

Introduction

The notion of an object arises for different purposes, especially in specific applied systems and often inspired by accidents. Issue where the objects come from usually is distinct from exact development and tends to mathematical consideration. The remarks here can be taken as a suggestion to group numerous aspects of ‘object’ to result in a general computational framework that gives a suitable scheme. This scheme can be useful as a primitive frame to put important ideas of data objects modeling in a certain order.

The individuals in a problem domain are briefly discussed in Section 1 and they are thought to be coupled into a single set D. Most of them are possible objects and can be converted into the actual objects. The difference is captured by the assignments that play a role of possible worlds, as shown in Section 3. The idea of possible worlds is more understood in a theoretical research and is not yet adopted as everyday tool of database designer and developer. For convenience imagine the actual object as an assigned possible object with respect to an index being specified. In a theory the set of possible worlds is represented by a suitable mathematical structure e.g. a category. To capture the dynamics of a problem domain this category represents the ‘evolving of events’, that is shortly covered in Section 3.

The descriptions of selfcontained couples of individuals generate concepts. Concepts are described and represented according to scheme of comprehension in a higher order logic.

1 Structuring a problem domain

The notion of a problem domain is a cross point for a lot of researches. A choice of the object brings the most difficulties – this is an atomic entity, the unit that generates the compound entities. Thus the objects induce an inductive class that contains all their representations.

Objects do not produce the homogeneous set. It is separated into at least three counterparts: sets of the actual A, possible D and virtual V objects. As usually they induce the natural inclusion A ⊆ D ⊆ V. In fact, for i ∈ I this inclusion transforms into Ai ⊆ D ⊆ V.

1.1 Objects as individuals

The individuals in a problem domain take part in all the constructions. Mainly the individuals are fixed by the individualizing functions. The discussions of their nature is excessively troublesome. But the most convenient reason is the possibility to express the individuals within the framework of some logical language. Therefore the properties of this language have the dominant importance. A selec-
tivity of the language has to enable capturing those minimal objects named individuals. Furthermore the individuals have to be comprehended to give rise to more vivid objects. Comprehension in a language takes the central place.

1.2 Comprehension

Let \( \Phi \) be an individualizing function that corresponds to the logical sentence or formula. The selective power of this formula permits the unique identification of a distinct object \( \Phi h \) in a language. This uniqueness is announced as a principle of comprehension (with the description \( I \)):

\[
\| I y \Phi(y) \| = h \iff \{ h \} = \{ h' \in D \mid \| \Phi(h') \| = 1 \} \\
T = \{ h : D \mid \Phi \} \equiv I y : [D] \forall h : D (\Phi \iff y(h))
\]

Second expression enables type \( T \) within a language, and the type indicates the set of individuals for a power sort \( [D] \).

2 Logic, individuals and concepts

Linkage of individuals and formulae is established in a logical language.

2.1 Language

The language tends to define and manipulate the objects of different kinds and gives a logical snapshot. This snapshot does not depend on any external parameters the same way in a database theory:

- Object ::= Atom | Complex
- Atom ::= Constant | Variable
- Complex ::= Constant_function(Object) |
| [Object, Object] |
| Object(Object) |
| Object \in Object |
- Logical_formula ::= Equation | Compound
- Equation ::= (Variable=Complex) |
| Logical_formula \land Logical_formula |
| Logical_formula \lor Logical_formula |
| \exists Variable.Logical_formula |
| \forall Variable.Logical_formula |

The objects with the proper computational aspects are, as usually, pairs \([\cdot, \cdot]\), applications \(\cdot(\cdot)\), and inclusions \(\cdot \in \cdot\) which generate a class of equations. The equations are counterparts in the compounds. The target of this is to support the object of a special kind, namely the concept.

2.2 Building the concepts

Concepts are mainly the basic building blocks. They are separated into generic concepts and indexed concepts. Note that new concepts generate the definition dimension and are introduced by the descriptions like:

\[
\begin{align*}
\text{New} \_ \text{concept} & = I y : \text{Power} \_ \text{sort} \\
& \quad \forall x : \text{Sort} \\
& \quad (y(x) \leftrightarrow \text{Logical} \_ \text{formula}(x))
\end{align*}
\]

Generic concepts are often used as a kind of the representation. They are intentional objects – and are correspondent to sort or type symbols – which are interpreted as sets. Initially the generic concepts are established to represent generic ideas of physical or abstract objects that are distinct and understood in a problem domain.

2.2.1 Indexed concept

Indexed concept results in a family of concepts each of them being dependent on the parameter (index, assignment etc.). Its counterparts are as follows:

\[
i \in I \quad \text{Index: Object } i \text{ is selected from a set } I. \text{ Index identifies the valid configuration of database.}\\
\]

\[
h(i) \in T \quad \text{Type: Object } h(i) \text{ is an indexed individual that is contained in the type.}\\
\]

\[
\| \Phi(\Phi h) \| i \quad \text{Substitution: Object } \Phi h \text{ with type } I \times T \text{ is a valid substitution of formula } \Phi \text{ and } h \text{ is isomorphic to pair } [i, h(i)] \text{ for projections } p, q: p([i, h(i)]) = i \text{ and } q([i, h(i)]) = h(i).\\
\]

\[
C'(\{i\}) \quad \text{Instantiation: Object } C'(\{i\}) = \{ h(i) \mid \| \Phi(\Phi h) \| i = 1 \} \text{ is an indexed concept and } C'(\{i\}) \in [T]. \text{ It is the intentional object with the extension that is generated by the substitutions of } \Phi.\\
\]

\[
C(I) \quad \text{Variable concept: Object } C(I) \in [I \times T] \text{ is a variable concept that generates the family of indexed concepts: } C(I) = \{ C(\{i\}) \}_{i \in I} = \{ h(i) \mid \| \Phi(\Phi h) \| i = 1 \}_{i \in I}.\\
\]

Therefore indexed concept indicates the elements of \([T]\): when \(i\) ranges \(I\) then \(C(\{i\})\) ranges \([T]\).

2.2.2 Evolvent

Let \(B\) and \(I\) be the sets of indexes. The mapping \(f : B \to I\) reaches elements \(B\) from \(I\) so that \(f(b) = i\) for some \(i \in I, b \in B\). If the elements of \(B\) and \(I\) are understood as events then \(f\) is therefore assumed to be evolvent of events. The reversed order of reading the mapping \(f\) is selected for technical convenience.

Any way, evolvents capture more dynamics in a problem domain. The steps for indexing concept as above are following.

\[
b \in B \quad \text{Index: Object } b \text{ is selected from a set } B. \text{ Index identifies the ‘new’ configuration of database.}\\
\]
\[ h(b) \in T \quad \text{Type: Object } h(b) \text{ is an indexed individual that type contains. For evolvent } f \text{ the composition gives } h(i) = (h \circ f)(b). \text{ Thus } b \text{ is the 'new' configuration of database and } i \text{ is an 'old' one. From this point of view } h \circ f \text{ is an } f\text{-shifted image of individual } h \text{ along the evolvent } f. \text{ Therefore the possibility to observe the } f\text{-shifted individuals } h \circ f \text{ in } b \text{ instead of } h\text{-individuals in } i \text{ is available. The remainder of steps repeats the steps above with slightly modified indexes.} \]

\[ \| \Phi(\overline{\tau}) \|_f b \quad \text{Substitution: Object } \overline{\tau} \circ f \text{ with type } B \times T \text{ is a valid substitution of formula } \Phi \text{ and } h \text{ is isomorphic to pair } [b, (h \circ f)(b)] \text{ for projections } p, q; \quad p([b, (h \circ f)(b)]) = b \text{ and } q([b, (h \circ f)(b)]) = h(i). \]

\[ C_f'(b) \quad \text{Instantiation: Object } C_f'(\{b\}) = \{(h \circ f)(b) \mid \| \Phi(\overline{\tau}) \|_f b = 1\} \text{ is an indexed concept and } C_f'(\{b\}) \in [T]. \text{ It is the intentional object with the extension that is generated by the substitutions of } \Phi. \]

\[ C_f(B) \quad \text{for the clear reason: set of configurations } B \text{ accepts only those events that are } f\text{-shifted from } I \text{ and } C_f(B) \subseteq C(B). \]

(Note that in combinatoric logic the substitution has a slightly modified form: \[ \| (\lambda x. \Phi) i \| = [h(i)/x] (\| \Phi \| i).] \)

### 2.3 Computational model

Language has to be enforced by the external parameters, or stages of knowledge, or assignments etc. Assignments enable language to capture a family of snapshots, or view. This view is partially analogous to the view in a database theory, but only partially. The computational model with views is the following.

\[
\begin{align*}
\text{Concept} & = \text{individual} \quad \text{assignment} \\
\text{individual} & = \text{state} \quad \text{assignment} \\
\text{individual} & = \text{Concept} \quad \text{assignment} \\
\text{state} & = \text{individual} \quad \text{assignment} \\
\text{Logic} & = \text{Logical formula} \quad \text{assignment} \\
\text{Logical formula} & = \text{Logic} \quad \text{assignment} \\
\text{Truth value} & = \{\text{true, false}\}
\end{align*}
\]

To explicate the advantages of the approach let us first establish the links between logic and concepts.

### 2.4 Logical revelation of the concepts

Logical formulae generate the concepts by the set-theoretic definitions:

\[ \text{Concept} = \{\text{individual} \mid \text{Logical formula}\} \]

The concepts are fixed in a language by the descriptions with the comprehension captured from a higher order logic. Think of objects as having being described.

To manipulate objects the computational tool is needed. The basic set of the objects is the following: \( \bot \) (the logical constant), \( g \) (the functional constants), \( (\cdot) \) (the application operator, or the functional variable), \( \in \) (the set constructor).

The model is to be constructed to reflect the resulting evolutions by the induction on the object complexity:

**Objects:**

\[
\begin{align*}
&\| I \| = \text{false} \\
&\| x = y \| I = \| x \| I = \| y \| I \\
&\| gx \| I = \| g \| \| x \| I \\
&\| x, y \| I = \| x \| I, \| y \| I \\
&\| x(y) \| I = \| x \| \| 1_A \| f \| y \| I \\
&\| y \in x \| I = \| y \| I, \| x \| 1_A I
\end{align*}
\]

**Logical objects:**

\[
\begin{align*}
&\| (\Phi \land \Psi) \| I = \| \Phi \| I \land \| \Psi \| I \\
&\| (\Phi \lor \Psi) \| I = \| \Phi \| I \lor \| \Psi \| I \\
&\| (\Phi \Rightarrow \Psi) \| I = f : B \rightarrow I \land \| \Phi \| f \| B \Rightarrow \| \Psi \| f \| B \\
&\| \forall x. \Phi \| I = f : B \rightarrow I \land b \in H_T(B) \Rightarrow \| b/x \| \| \Phi \| f \| B \\
&\| \exists x. \Phi \| I = \exists a \in H_T(I).a/x \| \| \Phi \| I
\end{align*}
\]

The computational model above gives rise to the syntax- and-semantic object \( \| \cdot \| \cdot \), the evaluation map. To understand its properties the idea of variable domain is needed. Note that the construction separates the system of concepts and the managing of them.

In the above the notation \( [a/x](\| \cdot \| \cdot) \) means the valuation \( \| \cdot \| \cdot \) at \( a \) fixed so that \( a \) matches \( x \). The notation \( \| \cdot \| f \cdot \) means the valuation that matches \( \| x \| f \) with each of the relevant variables \( f \)-shifted valuation.

### 3 Variable domains and variable concepts

#### 3.1 Variable domains

Suppose that all the individuals are gathered into a single general set \( D \) – a set of all possible individuals. They are possible relative to some predefined theory, and this assumption is not too restrictive but is fruitful to bring more accuracy into reasonings. Assume the events concerning the management of the living conditions for individuals. The additional assumption needs the law of ‘event evolving’, say \( f : B \rightarrow I \) when the events evolve from \( I \) to
The notion of possible world is more known in a database theory as the valid configuration of database.

Thus the individuals depend on $I$ and are mainly the functions like $h : I \rightarrow T$ where $T$ is a type symbol that indicates the distinct couple of individuals. The different way to understand the individuals leads to the pairs $h' = [i, h(i)]$ where $i \in I$ and $h \in T$.

**Definition 3.1 (variable domain)** The variable domain arises as the set $H_T(I) = \{h \mid h : I \rightarrow T\}$ giving rise to the mathematical object that is known as the functor $H_T(I)$ with the parameters $T$ and $I$. When $i$ ranges $I$ then $h(i)$ ranges $T$.

Indeed, the variable domain varies along the evolvent when the events evolve. For $f : B \rightarrow I$ the functor $H_T(f)$ represents the transition from the ‘old’ world $I$ to the ‘new’ world $B$: $H_T(f) : H_T(I) \rightarrow H_T(B)$. The reasoning in terms of elements gives the mapping $H_T(f) : h \in H_T(I) \mapsto h \circ f \in H_T(B)$ for evolvent $f : B \rightarrow I$.

Thus the individuals depend on $I$ and are mainly the functions like $h : I \rightarrow T$ where $T$ is a type symbol that indicates the distinct couple of individuals. The different way to understand the individuals leads to the pairs $h' = [i, h(i)]$ where $i \in I$ and $h \in T$.

**Note that this kind of concept depends on the world $I$**:

$$C(I) = \{h \mid \| \Phi(x) \| I = 1\} \subseteq H_T(I)$$

3.2 Variable concepts

The variable domain denotes the range of a variable. The logical formula $\Phi(x)$ with free variable $x \in H_T(I)$ reflects the idea. The logic and problem domain are separated by the evaluation $\| \cdot \| \cdot$ that depends both on the formula $\Phi(x)$ under evaluation and the world $I$ for individuals. As an object the valuation has the properties of functor:

$$\| \cdot \| f : B \rightarrow I \text{ is } \| \cdot \| f B$$

where $\| \cdot \| f$ is the $f$-shifted valuation of some formula. To build a subset of $H_T(I)$ the set $\{h \mid \| \Phi(x) \| I = 1\}$ should be fixed, denoted by $C(I)$ and named a concept.

**Note that this kind of concept depends on the world $I$**:

$$C(I) = \{h \mid \| \Phi(x) \| I = 1\} \subseteq H_T(I)$$

3.3 Diagrams

The sample diagram for all data objects in use is shown in Figure 3.

The entities in this figure conform to the following set of domain equations:

- `Type_of(assignment)=Assignment`
- `Type_of(individual)=Type`
- `Type_of(individual_concept)=Assignment \times Type`
- `Type_of(individual_concept)=Variable_concept`
- `Type_of(Variabie_concept)=Power_set(Assignment \times Type)`
- `Type_of(individual)=Concept`
- `Type_of(Concept)=Power_set(Type)`

**Conclusions**

The connection of the variable domains and variable concepts is left out of this extended abstract scope. The typeless computational model based on variable domains is left out of this extended abstract scope as well. As could be shown, this general model, free of any choice of specified evaluation map, does exist corresponding to an equational theory with products.

Some of the important data objects such as type, relation, function value and abstraction can be derived from the comprehension.

The objects in use inherit both syntax and semantic of the initial idea of object. This leads to and object-as-functor computations and generates a family of variable domains.

Variable concepts can be embedded into the computational model and inherit the logical properties of the objects. The higher order logic (with the descriptions) is in use.

Variable concept gives a natural representation of database view. Indexed concepts give a sound basis to create database snapshots. The parameter of variable concept ranges a category of evolvents.

**References**

[BLR97] C. Beeri, A.Y. Levy, and M.-Ch. Rousset. Rewriting queries using views in description logic. In Z. Meral Ösoyoglu, editor. Proceedings of the 16-th ACM SIGART-SIGMOD-SIGART Symposium on Principles of Database Systems (PODS 1997), Tucson, Arizona, May, 12–14, 1997. ACM, N.Y, 1997.

[Bor95] Alex Borgida. Description logics in data management. IEEE Transactions on Knowledge and Data Engineering, 7(5):671–682, 1995.

[Bor96] Alex Borgida. On the relative expressiveness of description logics and predicate logics. Artificial Intelligence, 82:353–367, 1996.
Figure 1: Diagram of variable concept.

[Bro95] M.L. Brodie. *Interoperable Information Systems: Motivations, Challenges, Approaches, and Status*. Russian Basic Research Foundation, Moscow, Russia, April, 6-7, 1995.

[Hul97] R. Hull. Managing semantic heterogeneity in databases: A theoretical perspective. In Z. Meral Özsoyoglu, editor, *Proceedings of the 16-th ACM SIGART-SIGMOD-SIGART Symposium on Principles of Database Systems (PODS 1997)*, Tucson, Arizona, May, 51–61, 1997. ACM, N.Y. 1997.

[Rou76] N. Roussopoulos. *A semantic network model of databases*. PhD dissertation, Toronto University, 1976. An early object-oriented approach via ‘frames’ was established. The frames are treated as intensions, and the associated relations as extensions.

[Sco80] D.S. Scott. Relating theories of the $\lambda$-calculus. In J. Hinhley and J. Seldin, editors, *To H.B. Curry: Essays on combinatory logic, lambda calculus and formalism*, pages 403–450. New York and London, Academic Press, 1980.

[Wol93] V.E. Wolfengagen. Computational aspects of data objects. In *Proceedings of the workshop on Advances in Database and Information Systems, ADBIS’93, May 11-14, Moscow*, 1993, pages 1–13, Moscow, May 1993.

[Wol99] V. Wolfengagen. Event driven objects. In V. Wolfengagen and Ch. Freytag, editors, *Proceedings of the 1-st International Workshop on Computer Science and Information Technologies (CSIT’99)*, Moscow, Russia, January, 18–22, 1999. JurInfoR-MSU Press, University Press of MEPhI.