A Stimulating Explanation of the Extragalactic Radio Background

Andrea Caputo, Hongwan Liu, Siddharth Mishra-Sharma, Maxim Pospelov, and Joshua T. Ruderman

1 School of Physics and Astronomy, Tel-Aviv University, Tel-Aviv 69978, Israel
2 Department of Particle Physics and Astrophysics, Weizmann Institute of Science, Rehovot 7610001, Israel
3 Center for Cosmology and Particle Physics, Department of Physics, New York University, New York, NY 10003, USA
4 Department of Physics, Princeton University, Princeton, NJ 08544, USA
5 The NSF AI Institute for Artificial Intelligence and Fundamental Interactions
6 Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
7 Department of Physics, Harvard University, Cambridge, MA 02138, USA
8 School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA
9 William I. Fine Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA

(Dated: June 17, 2022)

Despite an intense theoretical and experimental effort over the past decade, observations of the extragalactic radio background at multiple frequencies below 10 GHz are not understood in terms of known radio sources, and may represent a sign of new physics. In this Letter we identify a new class of dark sector models with feebly interacting particles, where dark photons oscillate into ordinary photons that contribute to the radio background. Our scenario can explain both the magnitude and the spectral index of the radio background, while being consistent with other cosmological and astrophysical constraints. These models predict new relativistic degrees of freedom and spectral distortions of the cosmic microwave background, which could be detected in the next generation of experiments.

Introduction.—The cosmic microwave background (CMB) between 60–600 GHz is one of the most well-studied electromagnetic signals in science. In this frequency range, the CMB monopole dominates over astrophysical backgrounds, and is consistent with a blackbody distribution, with distortions limited to less than 1 part in 10^5 in a frequency range, the CMB monopole dominates over astrophysical backgrounds, and is consistent with a blackbody distribution, with distortions limited to less than 1 part in 10^5 [1]. Spatial fluctuations in the brightness of the CMB are on the level of 1 part in 10^5, and the power spectrum of these fluctuations forms a key pillar of modern cosmology [2]. These facts constitute strong evidence for the primordial origins of the CMB.

Much less is known about the extragalactic radio background (ERB) at frequencies \( \nu \lesssim 10 \) GHz. The ARCADE 2 collaboration combined their own observations [3] with measurements from other radio telescopes [4–7], spanning a frequency range of 22 MHz–90 GHz, and found that the ERB can be modeled as

\[
T(\nu) = T_0 + T_R \left( \frac{\nu}{310 \text{ MHz}} \right)^\beta,
\]

where \( T(\nu) \) denotes the brightness temperature. \( T \) is related to the photon spectrum via \( T(\omega) = \left( \frac{\pi^2}{\omega} \right) \frac{dN_{\gamma}}{d\omega} \), where \( dN_{\gamma}/d\omega \) is the number density of photons per unit energy (taking \( h = c = k_B = 1 \)), and \( \omega \equiv 2\pi\nu \). The power-law fit in Ref. [3] found \( T_0 = 2.725(1) \text{ K} \), \( T_R = 24.1(21) \text{ K} \) and \( \beta = -2.599(36) \). For \( \nu \gtrsim 100 \) GHz, this is consistent with the CMB temperature of \( T_{\text{FIRAS}} = 2.7255(85) \text{ K} \) measured by FIRAS [1, 8]. At lower frequencies, however, the power law is an excellent fit to \( T_{\text{exc}} \equiv T - T_0 \). A more recent re-analysis including all-sky maps from the LWA1 Low Frequency Sky Survey (LLFSS) reached a similar conclusion [9].

Explaining \( T_{\text{exc}} \) with known sources of radio emission has thus far been surprisingly difficult [10]. Almost all attempts to do so have relied on synchrotron emission, since synchrotron sources typically produce \( -3 \lesssim \beta \lesssim -2.5 \) [11, 12]. Extragalactic radio synchrotron sources such as active galactic nuclei and star-forming galaxies can produce \( \beta \approx -2.7 \) [13–17], and the source counts distribution is thought to be relatively well-understood. However, numerous studies have estimated the emission from extragalactic radio sources to be 3-10 times smaller than \( T_{\text{exc}} \) [16, 18–22] between 100 MHz–10 GHz. Electrons that are reaccelerated during cluster mergers have been proposed as a significant contribution to \( T_{\text{exc}} \) [23], albeit under optimistic assumptions; this mechanism also produces a softer power law than is required.

An alternative explanation of \( T_{\text{exc}} \) is contamination from unmodeled Galactic sources of radio emission, which needs to be subtracted from radio data to obtain the extragalactic component [24–26]. However, proposed additional sources of Galactic radio emission are strongly constrained [26–28]. More sophisticated modeling using cosmic ray propagation models disfavors the possibility of significant Galactic contamination [20].

A more exotic class of solutions involves radio emission...
from hypothetical early structures [29], including black holes [30–32] and star-forming galaxies [33] at high redshifts. These solutions typically require large, persistent magnetic fields [32], whose origin and impact on inverse Compton cooling are debated [34].

The ERB has also been studied in conjunction with 21-cm experiments, including EDGES [35], LEDA [36] and LOFAR [37]. These measurements are in tension with a $T_{\text{exc}}$ that is fully produced at high redshifts. In particular, the LEDA and LOFAR results constrain any cosmological radio background with $T_{\text{exc}} \propto \omega^{-2.6}$ at $\sim 10\%$ (13.2 < z < 27.4) [29] and $\sim 46\%$ (z = 9.1) [38] of the present-day radio excess respectively.

Another aspect of the ERB that makes astrophysical explanations difficult is the spatial smoothness of the emission [39], deduced from measurements of the anisotropy of the radio sky [39–44]. These measurements find that fluctuations in $T_{\text{exc}}$, $\Delta T/T_{\text{exc}}$, are $\sim 10^{-2}$ across a range of angular scales and frequencies; this is smoother than expected if radio emission is correlated with the present-day dark matter distribution [39].

New physics explanations proposed thus far have primarily focused on synchrotron emission from DM annihilation and decay [45–48]. However, these models also run into similar issues: they may result in nonsmooth emission; underproduce $T_{\text{exc}}$, unless the magnetic fields responsible for synchrotron production have unusual or unlikely properties [47, 48]; require a new, large population of faint sources [21], or require a large portion of the isotropic, extragalactic gamma-ray background to come from DM annihilation [46], which is disfavored [49, 50].

In this Letter, we show that a simple class of experimentally viable new-physics models can explain the amplitude, power-law dependence and smoothness of $T_{\text{exc}}$. These models rely on three basic ingredients: 1) a particle decaying into dark photons $A'$; 2) the presence of a thermal bath of $A'$ which stimulates this decay; and 3) $A'$ resonantly oscillating into radio photons. This class of models leads to $T_{\text{exc}} \propto \omega^{-5/2}$, close to the observed power-law dependence. Relatively low-$z$ resonant oscillations as well as $\text{d}n_{A'}/\text{d}\omega \propto \omega^{-1/2}$ prior to oscillations are both crucial elements of such models; we have not been able to identify viable alternatives. One possibility is the decay of DM particles to final states charged under $A'$, and $A'$ appearing as “dark bremsstrahlung”. While having a spectrum similarly enhanced in the infrared, the DM has to be light and millicharged, with an effective electromagnetic charge exceeding current experimental bounds [51]. Another alternative is DM decay into a dark photon $A''$, which then resonantly converts twice via $A'' \rightarrow A' \rightarrow \gamma$, with some dark states charged under a dark photon $A''$ facilitating the $A'' \rightarrow A'$ transition. However, two resonant conversions makes it difficult to produce a sufficiently large $T_{\text{exc}}$ while ensuring that $T_{\text{exc}} \propto \omega^{-5/2}$ over nearly three decades of frequency.

The remainder of this Letter is organized as follows. We begin by introducing the class of models and deriving the expected $T_{\text{exc}}$ from them. Next, we discuss our fit of a particular example model to the radio data, and relevant experimental constraints. We then move on to discuss the anisotropy of the ERB produced by the model. We conclude with future prospects for confirmation of this model. We adopt Planck 2018 cosmology throughout [2].

Model.—A particle physics model that has the following three features can generate an ERB with $T_{\text{exc}} \propto \omega^{-5/2}$:

1. a cold component of DM $a$ which is stable on cosmological timescales, but undergoes a two-body decay with lifetime $\tau_{\text{exc}}$ in vacuum (for simplicity, we take $a$ to be all of DM);

2. one of the daughter particles of the decay is a dark photon, $A'$, with an existing blackbody distribution, characterized by a temperature $T'(z) = T_0'(1+z)$, where $T_0'$ is its temperature today; and

3. the $A'$ has mass $10^{-15}$ eV $\lesssim m_{A'} \lesssim 10^{-9}$ eV and is emitted relativistically with energy $\omega_{A'}$. $A'$ mixes with the Standard Model photon $\gamma$ with kinetic mixing parameter $\epsilon$.

These three features are summarized in Fig. 1. The existence of the thermal population of $A'$ enhances the decay rate of $a$ due to Bose enhancement [52, 53], leading to a redshift-dependent effective decay lifetime $\tau(z)$, where

$$\tau(z) = \tau_{\text{vac}} \left[1 + n f_{A'}^{BB}(z)\right]^{-1},$$

with $f_{A'}^{BB} = (e^{\omega_{A'}/T'} - 1)^{-1}$ being the blackbody occupation number of $A'$ with energy $\omega_{A'}$.

There are several parameters that depend on the specifics of the model. The value of $n$ depends on the occupation number of the other daughter particle. In addition, $a$ can decay into $a = 1$ or 2 dark photons. Finally, $\omega_{A'}$ depends on the kinematics of the decay of $a$, with $\omega_{A'} = m_{a}/2$ if it decays into a pair of relativistic final states. In App. B, we describe a concrete particle physics model that realizes all three features. To guarantee $T_{\text{exc}} \propto \omega^{-5/2}$, processes such as inverse decays of $A'$ into $a$ cannot significantly distort either the power-law index of the $A'$ spectrum produced by the decay, or the blackbody distribution of $A'$ across all relevant frequencies. In accordance with this fiducial model, we fix $n = 2$, $\alpha = 1$, and $\omega_{A'} = m_{a}/2$ for all results that are specific to

![FIG. 1. A schematic representation of the key aspects of the proposed class of models.](image)
it, although we emphasize that this is only one example, out of potentially many, that we have studied in detail.

The small kinetic mixing between $A'$ and $\gamma$ enables resonant conversion between the two particles whenever $m_{A'}^2 = m_{\gamma}^2$ [54, 55], where $m_{\gamma}^2$ is the effective photon plasma mass squared. This quantity is proportional to the free electron number density, $n_e$. The converted photons ultimately form the present-day $T_{\text{exc}}$. We calculate the sky-averaged conversion probability per redshift $d\langle P_{A'\rightarrow \gamma}\rangle/dz$ taking into account inhomogeneities using the formalism developed in Refs. [55, 56] (see also Refs. [57–59]).

Radio background production.—The particle $a$ decays throughout cosmic history, producing a number density of $A'$ per unit energy given by (see App. A for a complete derivation, which follows from Ref. [60])

$$\frac{dn_{A'}}{d\omega}(\omega, z) = \frac{\alpha\rho_a(z)}{m_{a}\omega^2(z_\star)}H(z_\star)\Theta(z_\star - z),$$

where $1 + z_\star \equiv \omega_{A'}/(1+z)/\omega$ is the redshift at which a daughter $A'$ with frequency $\omega$ at redshift $z$ was produced, $H$ is the Hubble parameter, $\rho_a(z)$ is the DM energy density, and $\Theta$ is the Heaviside step function. The total spectrum of $A'$ is then a sum of this decay spectrum and the blackbody spectrum of the thermal $A'$ distribution, which is subdominant in the energy range of interest.

In comoving coordinates, the spectrum of photons produced is obtained by integrating the spectrum of dark photons times $d\langle P_{A'\rightarrow \gamma}\rangle/dz$, i.e.,

$$\frac{1}{(1+z)^3} \frac{dn_{\gamma}}{dx} = \int_{z}^{\infty} dz' \frac{d\langle P_{A'\rightarrow \gamma}\rangle}{dz'} (1+z')^3 \frac{dn_{A'}}{dx}(x, z'),$$

where $x \equiv \omega/[T_0(1+z)]$. For the range of $m_{A'}$ that is of interest, resonant conversions occur only after recombination. In the range $3 \times 10^{-4} \lesssim x \lesssim 0.2$, which are relevant for $T_{\text{exc}}$, these photons only evolve via redshifting, with the baryonic fluid being essentially transparent to them [53].

Substituting Eq. (3) into Eq. (4), we find

$$\frac{dn_{\gamma}}{dx}(x, z) = \frac{\rho_a(z)}{m_a} \frac{1}{x} \tau(z_\star) \frac{1}{x} \int_{x^{-1}}^{x^{-1}} dz' \frac{d\langle P_{A'\rightarrow \gamma}\rangle}{dz'},$$

This gives $dn_{\gamma}/dx \propto x^{-3/2}$, or $T_{\text{exc}} \propto \omega^{-5/2}$, the desired frequency dependence. Note that $z_\star$ must occur during matter domination in order for $H(z_\star) \propto x^{-3/2}$, while $d\langle P_{A'\rightarrow \gamma}\rangle/dz \propto x^{-1}$ is derived in Refs. [54, 55]. $\tau(z_\star) \propto x$ follows from Eq. (2) when $T' \gg \omega_{A'}$.

Eq. (5) is the main result of this paper. For our fiducial model, we find the following approximate parametric dependence of $T_{\text{exc}}$ at $z = 0$:

$$T_{\text{exc}}(\omega) \approx 10 K \left( \frac{\nu}{310 \text{ MHz}} \right)^{-5/2} \left( \frac{T_0}{0.2 T_0} \right) \left( \frac{10^{21} \text{s}}{\tau_{\text{vac}}} \right) \times \left( \frac{10^{-4} \text{ eV}}{m_{a}} \right)^{3/2} \left( \frac{2 \times 10^{-4} \text{ eV}}{10^{-4} \text{ eV}} \right) \left( \frac{P_{\text{GHz}}}{10^{-5}} \right),$$

where $P_{\text{GHz}}$ is $\langle P_{A'\rightarrow \gamma}\rangle$ for $\nu = 1$ GHz today, which the FIRAS measurement of the CMB energy spectrum limits to be $P_{\text{GHz}} \lesssim 10^{-2}$ [54, 56].

**Fit and constraints.**—We can now perform a fit of $T_{\text{exc}}$ predicted by our model to the measured data from Ref. [3]. There are six parameters in our fit: five parameters ($m_a, m_{A'}, \tau_{\text{vac}}, T_0', \epsilon$) from our new physics model, and $T_0'$. Fitting the radio data places some requirements on the model parameters. For $T_{\text{exc}}$ to be present across all data points, we require $m_a/2 \gtrsim 2\pi \times 10$ GHz. On the other hand, photons at 22 MHz must originate from decays of $a$ during matter domination to satisfy $T_{\text{exc}} \propto \omega^{-5/2}$, leading to the requirement that $m_a/2 \gtrsim 2\pi(1+z_{\text{eq}}) \times 22$ MHz, where $z_{\text{eq}}$ is the redshift of matter-radiation equality. Together, this means $8 \times 10^{-5} \text{ eV} \lesssim m_a \lesssim 6 \times 10^{-4} \text{ eV}$. Resonant conversion must predominantly occur after $z_{\gamma}$ for 10 GHz photons so that the full power law extends to at least that frequency; the requirements on $m_a$ show that $z_{\gamma} \lesssim 6$, which favors $10^{-14} \text{ eV} \lesssim m_a \lesssim 3 \times 10^{-13} \text{ eV}$. Finally, $T'$ must be sufficiently large for $f_{\text{BB}}' \gg 1$ and $\tau(z_{\gamma}) \propto x$ for the 10 GHz data points; this imposes $T_0'/T_0 \gtrsim 0.06$.

Several important experimental constraints exist on the class of models under consideration. First, $T_0' \lesssim 0.4 T_0$, in order to avoid introducing excessive effective relativistic degrees of freedom [2]. The kinetic mixing between $A'$ and $\gamma$ also leads to spectral distortions due to $\gamma \rightarrow A'$ resonant conversions; the FIRAS measurement of the CMB spectrum [1] leads to constraints on the order of $\epsilon < 10^{-7} - 10^{-5}$ in the $m_{A'}$ range of interest [55, 56].

There are also limits on the DM decay lifetime obtained from the CMB power spectrum, large scale structure and the Milky Way satellite population [63–68]; however, these results assume a constant decay width. Conservatively requiring the total energy density of $a$ that has decayed away with stimulated decay from the $A'$ thermal population by the present-day to be less than 2.2%, we find $\tau_{\text{vac}} \gtrsim 10^{19.8} \times \text{max} \{2.1, 6.3 (10^{-4} \text{ eV/} \omega_{A'}) (T_0'/0.2 T_0)\}$ (see App. C for details). We also note that the decaying particle $a$ may be a subcomponent $f_{\text{DM}}$ of DM, which evades the lifetime bound altogether if $f_{\text{DM}} \lesssim 2.2\%$ [67].

21-cm power spectrum measurements from both LEDA and LOFAR are both in tension with $T_{\text{exc}}$ being fully produced at high redshifts ($z \gtrsim 0.1$), placing strong constraints on models that produce $T_{\text{exc}}$ primordially. In our model, however, $A' \rightarrow \gamma$ resonant conversions occur predominantly at $z \lesssim 6$.

We explore the posterior on the model parameters using nested sampling [69–71] implemented in *dynesty* [72]. Our priors, described in detail in App. E, are constructed
such that they have zero probability density in parameter regions incompatible with 1) the FIRAS spectral distortion limits of Refs. [55, 56] and 2) the DM lifetime limit discussed above. Priors on $T_0$ and $\tau_0$ are also chosen to account for the $N_{\text{eff}}$ and FIRAS constraints on these parameters, respectively. The posterior on the excess temperature is shown on the left in Fig. 2. In computing $T_{\text{exc}}$ from our model, we include an irreducible contribution from unresolved extragalactic radio sources of $T_{\text{eg}} = 0.23 \, \text{K} \, (\nu / \text{GHz})^{-2.7}$ [16, 39], which is at least 3 times smaller than the measured brightness temperature. This parametrization of $T_{\text{eg}}$ is in excellent agreement with other independent estimates [73]. Although this power-law expression strictly applies in the range 100 MHz–10 GHz, we estimate the contribution outside of this frequency range by extrapolation. The marginal and joint posterior distributions are shown on the right in Fig. 2. The marginal posterior medians correspond to $m_{\chi} \simeq 2 \times 10^{-4} \, \text{eV}$, $m_{A'} \simeq 2.5 \times 10^{-14} \, \text{eV}$, $\tau \simeq 1.3 \times 10^{21} \, \text{s}$, $T_0' \simeq 0.22 \, T_0$ and $\epsilon \simeq 10^{-7}$. We find an excellent fit to the data over a wide range of allowed model parameters. In App. F, we show the extended corner plot for posterior distributions of all parameters of interest, along with other systematic variations; these do not qualitatively change our main result.

**Smoothness.**—Upper limits on the anisotropy of the ERB have been reported for $4 \times 10^3 \lesssim \ell \lesssim 6 \times 10^4$ in the 4–8 GHz range by VLA [40, 41] and ATCA [42], while actual measurements of the power spectrum have been made by LOFAR [44] and TGSS [43] at $\sim 140$ MHz for $10^2 \lesssim \ell \lesssim 10^4$. Such measurements can be challenging: observations at $\sim 140$ MHz disagree with each other by a factor of 3, and face issues such as galactic foreground contamination and calibration errors [44]. To assess the smoothness of $T_{\text{exc}}$ in our model, we take the 4–8 GHz upper limits and results from LOFAR—which reports a lower power—as approximate upper limits, noting that astrophysical sources can contribute more power [44], and that these observations are expected to improve.

There are two possible contributions to the ERB anisotropy produced by our model that are essentially independent: 1) decay anisotropy, due to DM density correlations from the point at which $a$ decays, and 2) conversion anisotropy, due to correlations in free electron density fluctuations $\delta_e$ during $A' \rightarrow \gamma$ conversions, since $m_e^2 \propto n_e$. The decay anisotropy was found to exceed the radio anisotropy power spectrum in the 4–8 GHz range, unless the decay that produces these photons happens at $z_\gamma \gtrsim 5$ [39, 47]. This is easily satisfied in the range of parameters providing a good fit.

We compute the conversion anisotropy power spectrum by first writing down the two-point correlation function

![Excess temperature graph](https://example.com/excess_temperature.png)
Anisotropy power spectrum, 140 MHz

**Fig. 3.** Predicted anisotropy power spectrum with normally (solid black line, below) and log-normally (solid black line, above) distributed baryon fluctuations. The model parameters are $m_{A'} = 3 \times 10^{-14}$ eV, and $\omega_{A'} = 2 \times 10^{-4}$ eV. Upper limits from VLA at 4.86 GHz [40] (purple triangles), 8.4 GHz [41] (orange triangles) and ATCA at 8.7 GHz [42] (green triangle) are shown, and have been rescaled to $T_{\text{exc}} = 255$ K, the expected value at 140 MHz. Representative data points of the power spectrum measured by LOFAR are shown in red [44].

of the conversion probability of $A' \rightarrow \gamma$ in two different directions in the sky. As such, it depends on the two-point function of $\delta_\nu$, which we model as either a Gaussian or a log-normal random field. The anisotropy power spectrum can then be obtained by performing a spherical harmonics decomposition, and using the Limber approximation [74–76] following the method outlined in Ref. [77]. Further details of our calculation can be found in App. D.

Fig. 3 shows the predicted anisotropy power spectrum in units of K$^2$, defined using the same conventions as in the CMB power spectrum [2], for normally and log-normally distributed $\delta_\nu$, with the result between these two choices shaded in blue. This can be taken as an estimate for the uncertainty associated with these distributions. This power should be compared to the LOFAR data, as well as the upper limits from VLA and ATCA at their respective frequencies $\nu$, which have been rescaled by $(140\text{ MHz}/\nu)^{2\beta}$, with $\beta = -2.6$; this assumes that $\Delta T/T_{\text{exc}}$ is independent of frequency. Our calculated anisotropy power spectrum for a fiducial choice of $m_{A'} = 3 \times 10^{-14}$ eV and $\omega_{A'} = 2 \times 10^{-4}$ eV lies below the 4–8 GHz measurements for both choices of the two-point PDF of $\delta_\nu$, while only the log-normal two-point PDF exceeds the low-$\ell$ measurements by LOFAR. However, significant scatter exists between adjacent frequency bands in the data for $\ell \lesssim 10^3$ [44]. Producing a sufficiently smooth radio background is thus highly plausible in our model.

**Conclusion.**—In this Letter, we have provided a new physics explanation for the ERB, which is brighter than expected from known astrophysical sources. Our model consists of a DM candidate which decays into dark photons in the presence of a thermal bath of the latter; the dark photons then resonantly convert to ordinary photons, producing the ERB. The anisotropy of the signal is expected to be relatively smooth, consistent with measurements of the anisotropy of the radio background [39, 44]. Future experiments may corroborate the predictions of our model. PIXIE [61] may be sensitive to spectral distortions expected from our model, and is expected to almost fully cover the 95% region of our posterior distribution in the $\epsilon - m_{A'}$ plane. The thermal population of $A'$ may also lead to a value of $N_{\text{eff}}$ that is detectable in future CMB experiments such as CMB-S4 [62]. The potential reach of both experiments are shown in Fig. 2.

**Acknowledgments.**—We thank J. Dowell for useful correspondence, and D. Aloni, A. Hook, J. Muñoz, N. Outmezguine, J. Pradler, M. Regis, M. Schmaltz, N. Sehgal and A. Urbano for helpful discussions. AC has made use of NASA’s Astrophysics Data System. This research was performed using the Princeton University Office of Information Technology’s Research Computing department. This work was also performed using the Princeton University Office of Information Technology’s Research Computing resources, services, and staff expertise. This work was also performed using the Princeton Research Computing resources at Princeton University which is a consortium of groups including the Princeton Institute for Computational Science and Engineering and the Princeton University Office of Information Technology’s Research Computing department. This research has made use of NASA’s Astrophysics Data System. This research was performed using the ArviZ [78], astropy [79, 80], CLASS [81], dynesty [72], HyRec [82], IPython [83], Jupyter [84], matplotlib [85], NumPy [86], pyftlog [87–89], seaborn [90], pandas [91], SciPy [92], and tqdm [93] software packages.
[1] D. J. Fixsen, E. S. Cheng, J. M. Gales, J. C. Mather, R. A. Shafer, and E. L. Wright, The Cosmic Microwave Background spectrum from the full COBE FIRAS data set, Astrophys. J. 473, 576 (1996), arXiv:astro-ph/9605054 [astro-ph].

[2] N. Aghanim et al. (Planck), Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. 641, A6 (2020), [Erratum: Astron.Astrophys. 652, C4 (2021)], arXiv:1807.06209 [astro-ph.CO].

[3] D. J. Fixsen et al., ARCADE 2 Measurement of the Extra-Galactic Sky Temperature at 3-90 GHz, Astrophys. J. 734, 5 (2011), arXiv:0901.0555 [astro-ph.CO].

[4] R. S. Roger, C. H. Costain, T. L. Landecker, and C. M. Swerdlyk, The radio emission from the galaxy at 22 mhz, Astron. Astrophys. Suppl. Ser. 137, 7 (1999), arXiv:astro-ph/9902213.

[5] K. Maeda, H. Alvarez, J. Aparici, J. May, and P. Reich, A 45-MHz continuum survey of the northern hemisphere, Astronomy and Astrophysics, Supplement 140, 145 (1999).

[6] C. G. T. Haslam, U. Klein, C. J. Salter, H. Stoffel, W. E. Wilson, M. N. Cleary, D. J. Cooke, and P. Thomasson, A 408 MHz all-sky continuum survey. I - Observations at southern declinations and for the North Polar region., Astronomy and Astrophysics 100, 209 (1981).

[7] P. Reich and W. Reich, A radio continuum survey of the northern sky at 1420 MHz, II, Astronomy and Astrophysics, Supplement 63, 205 (1986).

[8] D. J. Fixsen, The Temperature of the Cosmic Microwave Background, Astrophys. J. 707, 916 (2009), arXiv:0911.1955 [astro-ph.CO].

[9] J. Dowell and G. B. Taylor, The Radio Background Below 100 MHz, Astrophys. J. Lett. 858, L9 (2018), arXiv:1804.08581 [astro-ph.CO].

[10] J. Singal et al., The Radio Synchrotron Background: Conference Summary and Report, Publ. Astron. Soc. Pac. 130, 036001 (2018), arXiv:1711.09979 [astro-ph.HE].

[11] M. Wright, J. Dickel, B. Koralesky, and L. Rudnick, The Supernova Remnant Cassiopeia A at Millimeter Wave-lengths, Astrophys. J. 518, 284 (1999).

[12] P. A. G. Scheuer and P. J. S. Williams, Radio Spectra, The Annual Review of Astronomy and Astrophysics 6, 321 (1968).

[13] P. Platania, M. Bensadoun, M. Bersanelli, G. De Amici, A. Kogut, S. Levitin, D. Maino, and G. F. Smoot, A determination of the spectral index of galactic synchrotron emission in the 1-10 GHz range, Astrophys. J. 505, 473 (1998), arXiv:astro-ph/9707252.

[14] C. L. Carilli, R. A. Perley, J. W. Dreher, and J. P. Leahy, Multifrequency radio observations of Cygnus A - Spectral aging in powerful radio galaxies, Astrophys. J. 383, 554 (1991).

[15] R. J. Protheroe and P. L. Biermann, A New estimate of the extragalactic radio background and implications for ultrahigh-energy gamma-ray propagation, Astropart. Phys. 6, 45 (1996), [Erratum: Astropart.Phys. 7, 181 (1997)], arXiv:astro-ph/9605119.

[16] M. Gervasi, A. Tartari, M. Zannoni, G. Boella, and G. Sironi, The contribution of the Unresolved Extra-galactic Radio Sources to the Brightness Temperature of the sky, Astrophys. J. 682, 223 (2008), arXiv:0803.4138 [astro-ph].

[17] I. C. Niţu, H. T. J. Bevis, J. D. Bray, and A. M. M. Scala, An updated estimate of the cosmic radio background and implications for ultra-high-energy photon propagation, Astropart. Phys. 126, 102532 (2021), arXiv:2004.13596 [astro-ph.HE].

[18] T. Vernstrom, D. Scott, and J. V. Wall, Contribution to the diffuse radio background from extragalactic radio sources, Monthly Notices of the Royal Astronomical Society 415, 3641 (2011).

[19] J. J. Condon, W. D. Cotton, E. B. Fomalont, K. I. Kellermann, N. Miller, R. A. Perley, D. Scott, T. Vernstrom, and J. V. Wall, Resolving the radio source background: Deeper understanding through confusion, The Astrophysical Journal 758, 23 (2012).

[20] N. Fornengo, R. A. Lineros, M. Regis, and M. Taoso, The isotropic radio background revisited, JCAP 04, 008, arXiv:1402.2218 [astro-ph.CO].

[21] T. Vernstrom, R. P. Norris, D. Scott, and J. V. Wall, The deep diffuse extragalactic radio sky at 1.75 GHz, Monthly Notices of the Royal Astronomical Society 447, 2243 (2015), arXiv:1408.4160 [astro-ph.GA].

[22] M. J. Hardcastle, T. W. Shimwell, C. Tasse, P. N. Best, A. Drabent, M. J. Jarvis, I. Prandoni, H. J. A. Röttgering, J. Sabater, and D. J. Schwarz, The contribution of discrete sources to the sky temperature at 144 MHz, Astron. Astrophys. 648, A10 (2021), arXiv:2011.08294 [astro-ph.CO].

[23] K. Fang and T. Linden, Cluster Mergers and the Origin of the ARCADE-2 Excess, JCAP 10, 004, arXiv:1506.05807 [astro-ph.HE].

[24] R. Subrahmanyan and R. Cowish, Is there an Un-accounted for Excess in the Extragalactic Cosmic Radio Background?, Astrophys. J. 776, 42 (2013), arXiv:1305.7060 [astro-ph.CO].

[25] E. Orlando and A. Strong, Galactic synchrotron emission with cosmic ray propagation models, Mon. Not. Roy. Astron. Soc. 436, 2127 (2013), arXiv:1309.2947 [astro-ph.GA].

[26] M. G. H. Krause and M. J. Hardcastle, Can the Local Bubble explain the radio background?, Mon. Not. Roy. Astron. Soc. 502, 2807 (2021), arXiv:2101.05255 [astro-ph.HE].

[27] J. Singal, L. Stawarz, A. Lawrence, and V. Petrosian, Sources of the radio background considered, Monthly Notices of the Royal Astronomical Society 409, 1172 (2010).

[28] J. Singal, A. Kogut, E. Jones, and H. Dunlap, Axial Ratio of Edge-on Spiral Galaxies as a Test for Bright Radio Halos, Astrophys. J. Lett. 799, L10 (2015), arXiv:1501.00499 [astro-ph.GA].

[29] C. Feng and G. Holder, Enhanced global signal of neutral hydrogen due to excess radiation at cosmic dawn, Astrophys. J. 858, L17 (2018), arXiv:1802.07432 [astro-ph.CO].

[30] P. L. Biermann, B. B. Nath, L. I. Carsamet, B. C. Harms, T. Stanev, and J. Becker Tjus, Cosmic backgrounds due to the formation of the first generation of supermassive black holes, Mon. Not. Roy. Astron. Soc. 441, 1147 (2014), arXiv:1403.3904 [astro-ph.CO].
[31] A. Ewall-Wice, T. C. Chang, J. Lazio, O. Dore, M. Seiffert, and R. A. Monsalve, Modeling the Radio Background from the First Black Holes at Cosmic Dawn: Implications for the 21 cm Absorption Amplitude, Astrophys. J. 868, 63 (2018), arXiv:1803.01815 [astro-ph.CO].

[32] A. Ewall-Wice, T.-C. Chang, and T. J. W. Lazio, The Radio Scream from black holes at Cosmic Dawn: a semianalytic model for the impact of radio-loud black holes on the 21 cm global signal, Mon. Not. Roy. Astron. Soc. 492, 6086 (2020), arXiv:1903.06788 [astro-ph.GA].

[33] J. Mirocha and S. R. Furlanetto, What does the first highly-redshifted 21-cm detection tell us about early galaxies?, Mon. Not. Roy. Astron. Soc. 483, 1980 (2019), arXiv:1803.03272 [astro-ph.GA].

[34] P. Sharma, Astrophysical radio background cannot explain the EDGES 21-cm signal: constraints from cooling of non-thermal electrons, Mon. Not. Roy. Astron. Soc. 481, L6 (2018), arXiv:1804.05843 [astro-ph.HE].

[35] J. D. Bowman, A. E. E. Rogers, R. A. Monsalve, T. J. Mozdzen, and N. Mahesh, An absorption profile centred at 78 megahertz in the sky-averaged spectrum, Nature 555, 67 (2018), arXiv:1810.05912 [astro-ph.CO].

[36] G. Bernardi, J. T. L. Zwart, D. Price, L. J. Greenhill, A. Mesinger, J. Dowell, T. Eftekhari, S. W. Ellingson, J. Kocz, and F. Schinzel, Bayesian constraints on the global 21-cm signal from the Cosmic Dawn, Mon. Not. Roy. Astron. Soc. 461, 2847 (2016), arXiv:1606.06006 [astro-ph.CO].

[37] F. G. Mertens et al., Improved upper limits on the 21-cm signal power spectrum of neutral hydrogen at $z \approx 9.1$ from LOFAR, Mon. Not. Roy. Astron. Soc. 493, 1662 (2020), arXiv:2002.07196 [astro-ph.GA].

[38] A. Fialkov and R. Barkana, Signature of Excess Radio Background in the 21-cm Global Signal and Power Spectrum, Mon. Not. Roy. Astron. Soc. 486, 1763 (2019), arXiv:1902.02438 [astro-ph.CO].

[39] G. P. Holder, The unusual smoothness of the extragalactic unresolved radio background, The Astrophysical Journal 780, 112 (2013).

[40] E. B. Fomalont, K. I. Kellermann, M. C. Anderson, D. Weistrop, J. V. Wall, R. A. Windhorst, and J. A. Kristian, New Limits to Fluctuations in the Cosmic Background Radiation at 4.86 GHz Between 12 and 60 Arcsecond Resolution, The Astronomical Journal 96, 1187 (1988).

[41] R. B. Partridge, E. A. Richards, E. B. Fomalont, K. I. Kellerman, and R. A. Windhorst, Small-Scale Cosmic Microwave Background Observations at 8.4 GHz, Astrophys. J. 483, 38 (1997).

[42] R. Subrahmanyan, M. J. Kesteven, R. D. Ekers, M. Sinclair, and J. Silk, An Australia telescope survey for CMB anisotropies, Mon. Not. Roy. Astron. Soc. 315, 808 (2000), arXiv:astro-ph/0002467.

[43] S. Choudhuri, A. Ghosh, N. Roy, S. Bharadwaj, H. T. Intema, and S. S. Ali, All-sky angular power spectrum – I. Estimating brightness temperature fluctuations using the 150-MHz TGSS survey, Mon. Not. Roy. Astron. Soc. 494, 1936 (2020), arXiv:2003.07869 [astro-ph.CO].

[44] A. R. Offringa, J. Singal, S. Heston, S. Horiuchi, and D. M. Lueiro, Measurement of the anisotropy power spectrum of the radio synchrotron background, Mon. Not. Roy. Astron. Soc. 509, 114 (2021), arXiv:2110.00499 [astro-ph.CO].

[45] N. Fornengo, R. Lineros, M. Regis, and M. Taoso, Possibility of a Dark Matter Interpretation for the Excess in Isotropic Radio Emission Reported by ARCADE, Phys. Rev. Lett. 107, 271302 (2011), arXiv:1108.0569 [hep-ph].

[46] D. Hooper, A. V. Belikov, T. E. Jeltema, T. Linden, S. Profumo, and T. R. Slatyer, The Isotropic Radio Background and Annihilating Dark Matter, Phys. Rev. D 86, 103003 (2012), arXiv:1203.3547 [astro-ph.CO].

[47] J. M. Cline and A. C. Vincent, Cosmological origin of anomalous radio background, JCAP 02, 011, arXiv:1210.2717 [astro-ph.CO].

[48] K. Fang and T. Linden, Anisotropy of the extragalactic radio background from dark matter annihilation, Phys. Rev. D 91, 083501 (2015), arXiv:1412.7545 [astro-ph.HE].

[49] M. Ajello et al., The Origin of the Extragalactic Gamma-Ray Background and Implications for Dark-Matter Annihilation, Astrophys. J. Lett. 800, L27 (2015), arXiv:1501.05301 [astro-ph.HE].

[50] M. Lisanti, S. Mishra-Sharma, L. Necib, and B. R. Safdi, Deciphering Contributions to the Extragalactic Gamma-Ray Background from 2 GeV to 2 TeV, Astrophys. J. 832, 117 (2016), arXiv:1606.04101 [astro-ph.HE].

[51] H. Vogel and J. Redondo, Dark Radiation constraints on minicharged particles in models with a hidden photon, JCAP 02, 029, arXiv:1311.2600 [hep-ph].

[52] A. Caputo, M. Regis, M. Taoso, and S. J. Witte, Detecting the Stimulated Decay of Axions at RadioFrequencies, JCAP 1903 (03), 027, arXiv:1811.08436 [hep-ph].

[53] B. Bolliet, J. Chluba, and R. Battye, Spectral distortion constraints on photon injection from low-mass decaying particles, Mon. Not. Roy. Astron. Soc. 507, 3148 (2021), arXiv:2012.07292 [astro-ph.CO].

[54] A. Mirizzi, J. Redondo, and G. Sigl, Microwave Background Constraints on Mixing of Photons with Hidden Photons, JCAP 0903, 026, arXiv:0901.0014 [hep-ph].

[55] A. Caputo, H. Liu, S. Mishra-Sharma, and J. T. Ruderman, Modeling Dark Photon Oscillations in Our Inhomogeneous Universe, Phys. Rev. D 102, 103533 (2020), arXiv:2004.06733 [astro-ph.CO].

[56] A. Caputo, H. Liu, S. Mishra-Sharma, and J. T. Ruderman, Dark Photon Oscillations in Our Inhomogeneous Universe, Phys. Rev. Lett. 125, 221303 (2020), arXiv:2002.05165 [astro-ph.CO].

[57] K. Bondarenko, J. Pradler, and A. Sokolenko, Constraining dark photons and their connection to 21 cm cosmology with CMB data, Phys. Lett. B 805, 135420 (2020), arXiv:2002.08942 [astro-ph.CO].

[58] A. A. Garcia, K. Bondarenko, S. Ploekinger, J. Pradler, and A. Sokolenko, Effective photon mass and (dark) photon conversion in the inhomogeneous Universe, JCAP 10, 011, arXiv:2003.10465 [astro-ph.CO].

[59] S. J. Witte, S. Rosauro-Alcaraz, S. D. McDermott, and V. Poulin, Dark photon dark matter in the presence of inhomogeneous structure, JHEP 06, 132, arXiv:2003.13698 [astro-ph.CO].

[60] M. Pospelov, J. Pradler, J. T. Ruderman, and A. Urbano, Room for New Physics in the Rayleigh-Jeans Tail of the Cosmic Microwave Background, Phys. Rev. Lett. 121, 031103 (2018), arXiv:1803.07048 [hep-ph].

[61] A. Kogut et al., The Primordial Inflation Explorer (PIXIE): A Nulling Polarimeter for Cosmic Mi-
crowave Background Observations, JCAP 1107, 025, arXiv:1105.2044 [astro-ph.CO].

[62] K. Abazajian et al., CMB-S4 Science Case, Reference Design, and Project Plan (2019), arXiv:1907.04473 [astro-ph.IM].

[63] V. Poulin, P. D. Serpico, and J. Lesgourgues, A fresh look at linear cosmological constraints on a decaying dark matter component, JCAP 08, 036, arXiv:1606.02073 [astro-ph.CO].

[64] A. Nygaard, T. Tram, and S. Hannestad, Updated constraints on decaying cold dark matter, JCAP 05, 017, arXiv:2011.01632 [astro-ph.CO].

[65] A. Chen et al. (DES), Constraints on dark matter to dark radiation conversion in the late universe with DES-Y1 and external data, Phys. Rev. D 103, 123528 (2021), arXiv:2011.04606 [astro-ph.CO].

[66] S. Mau et al., Milky Way Satellite Census. IV. Constraints on Decaying Dark Matter from Observations of Milky Way Satellite Galaxies (2022), arXiv:2201.11740 [astro-ph.CO].

[67] T. Simon, G. F. Abellán, P. Du, V. Poulin, and Y. Tsai, Constraining decaying dark matter with BOSS data and the effective field theory of large-scale structures (2022), arXiv:2203.07440 [astro-ph.CO].

[68] S. Alvi, T. Brinckmann, M. Gerbino, M. Lattanzi, and L. Pagano, Do you smell something decaying? Updated linear constraints on decaying dark matter scenarios (2022), arXiv:2205.05636 [astro-ph.CO].

[69] J. Skilling, Nested sampling for general Bayesian computation, Bayesian Analysis 1, 833 (2006).

[70] F. Feroz, M. P. Hobson, and M. Bridges, MultiNest: an efficient and robust Bayesian inference tool for cosmology and particle physics, Mon. Not. Roy. Astron. Soc. 398, 1601 (2009), arXiv:0809.3437 [astro-ph].

[71] J. Skilling, Nested Sampling, in Bayesian Inference and Maximum Entropy Methods in Science and Engineering: 24th International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering, American Institute of Physics Conference Series, Vol. 735, edited by R. Fischer, R. Preuss, and U. V. Toussaint (2004) pp. 395–405.

[72] J. S. Speagle, dynasty: a dynamic nested sampling package for estimating bayesian posterior and evidences, Monthly Notices of the Royal Astronomical Society 493, 3132 (2020).

[73] M. Massardi, A. Bonaldi, M. Negrello, S. Riccicardi, A. Raccanelli, and G. de Zotti, A model for the cosmological evolution of low-frequency radio sources, Monthly Notices of the Royal Astronomical Society 404, 532 (2010), arXiv:1001.1069 [astro-ph.CO].

[74] D. N. Limber, The Analysis of Counts of the Extragalactic Nebulae in Terms of a Fluctuating Density Field, Astrophys. J. 117, 134 (1953).

[75] N. Kaiser, Weak gravitational lensing of distant galaxies, Astrophys. J. 388, 272 (1992).

[76] N. Kaiser, Weak lensing and cosmology, Astrophys. J. 498, 26 (1998), arXiv:astro-ph/9610120.

[77] N. Fornengo and M. Regis, Particle dark matter searches in the anisotropic sky, Front. Phys. 2, 6 (2014), arXiv:1312.4835 [astro-ph.CO].

[78] R. Kumar, C. Carroll, A. Hartikainen, and O. Martin, Arviz a unified library for exploratory analysis of bayesian models in python, Journal of Open Source Software 4, 1143 (2019).

[79] A. M. Price-Whelan et al., The Astropy Project: Building an Open-Science Project and Status of the v2.0 Core Package, Astron. J. 156, 123 (2018), arXiv:1801.02634.

[80] T. P. Robitaille et al. (Astropy), Astropy: A Community Python Package for Astronomy, Astron. Astrophys. 558, A33 (2013), arXiv:1307.6212 [astro-ph.IM].

[81] D. Blas, J. Lesgourgues, and T. Tram, The Cosmic Linear Anisotropy Solving System (CLASS) II: Approximation schemes, JCAP 1107, 034, arXiv:1104.2933 [astro-ph.CO].

[82] Y. Ali-Haimoud and C. M. Hirata, HyRec: A fast and highly accurate primordial hydrogen and helium recombination code, Phys. Rev. D83, 043513 (2011), arXiv:1011.3758 [astro-ph.CO].

[83] F. Perez and B. E. Granger, IPython: A System for Interactive Scientific Computing, Computing in Science and Engineering 9, 21 (2007).

[84] T. Kluyver et al., Jupyter notebooks - a publishing format for reproducible computational workflows, in ELPUB (2016).

[85] J. D. Hunter, Matplotlib: A 2d graphics environment, Computing In Science & Engineering 9, 90 (2007).

[86] C. R. Harris, K. J. Millman, S. J. van der Walt, R. Gommers, P. Virtanen, D. Cournapeau, E. Wieser, J. Taylor, S. Berg, N. J. Smith, R. Kern, M. Picus, S. Hoyer, M. H. van Kerkwijk, M. Brett, A. Haldane, J. P. del Río, M. Wiebe, P. Peterson, P. Gérard-Marchant, K. Sheppard, T. Reddy, W. Weckesser, H. Abbasi, C. Gohlke, and T. E. Oliphant, Array programming with NumPy, Nature 585, 357 (2020).

[87] J. D. Talman, Numerical fourier and bessel transforms in logarithmic variables, Journal of Computational Physics 29, 35 (1978).

[88] A. J. S. Hamilton, Uncorrelated modes of the nonlinear power spectrum, Mon. Not. Roy. Astron. Soc. 312, 257 (2000), arXiv:astro-ph/9905191.

[89] D. Werthmüller, prisae/pyfftlog: First packaged release (2020).

[90] M. Waskom et al., mwaskom/seaborn: v0.8.1 (september 2017) (2017).

[91] W. McKinney, Data structures for statistical computing in python, in Proceedings of the 9th Python in Science Conference, edited by S. van der Walt and J. Millman (2010) pp. 51 – 56.

[92] P. Virtanen et al., SciPy 1.0: Fundamental Algorithms for Scientific Computing in Python, Nature Methods https://doi.org/10.1038/s41592-019-0686-2 (2020).

[93] C. O. da Costa-Luis, tqdm: A fast, extensible progress meter for python and cli, JOSS 4, 1277 (2019).

[94] B. Grzadkowski, M. Iglicki, and S. Mróz, $\mathcal{T}$-channel singularities in cosmology and particle physics (2021), arXiv:2108.01757 [hep-ph].

[95] J. Redondo and M. Postma, Massive hidden photons as lukewarm dark matter, JCAP 0902, 005, arXiv:0811.0326 [hep-ph].

[96] M. LoVerde and N. Afshordi, Extended Limber Approximation, Phys. Rev. D 78, 123506 (2008), arXiv:0809.5112 [astro-ph].

[97] I. Kayo, A. Taruya, and Y. Suto, Probability distribution function of cosmological density fluctuations from Gaussian initial condition: comparison of one- and two-point log-normal model predictions with n-body simulations, Astrophys. J. 561, 22 (2001), arXiv:astro-ph/0105218 [astro-ph].
[98] J. Alsing and W. Handley, Nested sampling with any prior you like, arXiv e-prints, arXiv:2102.12478 (2021), arXiv:2102.12478 [astro-ph.IM].

[99] D. Rezende and S. Mohamed, Variational inference with normalizing flows, in Proceedings of the 32nd International Conference on Machine Learning, Proceedings of Machine Learning Research, Vol. 37, edited by F. Bach and D. Blei (PMLR, Lille, France, 2015) pp. 1530–1538.

[100] G. Papamakarios, E. Nalisnick, D. J. Rezende, S. Mohamed, and B. Lakshminarayanan, Normalizing flows for probabilistic modeling and inference, arXiv preprint arXiv:1912.02762 (2019).

[101] C. Durkan, A. Bekasov, I. Murray, and G. Papamakarios, Neural spline flows, in Advances in Neural Information Processing Systems, Vol. 32, edited by H. Wallach, H. Larochelle, A. Beygelzimer, F. d’Alché Buc, E. Fox, and R. Garnett (Curran Associates, Inc., 2019).

[102] M. Isi, W. M. Farr, and K. Chatziioannou, Comparing Bayes factors and hierarchical inference for testing general relativity with gravitational waves, arXiv e-prints (2022), arXiv:2204.10742 [gr-qc].

[103] G. B. Rybicki and A. P. Lightman, Radiative processes in astrophysics (1979).

[104] A. R. Bell, The acceleration of cosmic rays in shock fronts. II., Mon. Not. Roy. Astron. Soc. 182, 443 (1978).

[105] A. R. Bell, The acceleration of cosmic rays in shock fronts - I., Monthly Notices of the Royal Astronomical Society 182, 147 (1978).

[106] E. Fermi, On the origin of the cosmic radiation, Phys. Rev. 75, 1169 (1949).

[107] R. D. Blandford and J. P. Ostriker, Supernova Shock Acceleration of Cosmic Rays in the Galaxy, Astrophys. J. 237, 793 (1980).

[108] R. D. Blandford and J. P. Ostriker, Particle Acceleration by Astrophysical Shocks, Astrophys. J. Lett. 221, L29 (1978).

[109] M. Malkov and L. O. Drury, Nonlinear theory of diffusive acceleration of particles by shock waves, Rept. Prog. Phys. 64, 429 (2001).

[110] V. Domček, J. Vink, J. V. Hernández Santisteban, T. Delaney, and P. Zhou, Mapping the spectral index of Cassiopeia A: evidence for flattening from radio to infrared, Mon. Not. Roy. Astron. Soc. 502, 1026 (2021), arXiv:2005.12677 [astro-ph.HE].
Appendix

This appendix is organized as follows. In App. A we derive in detail the dark photon spectrum as a function of frequency and redshift in the class of models considered in the Letter. App. B gives an extended description of a particular model which presents all the ingredients necessary to explain the extragalactic radio background (ERB). App. C presents some of the important experimental limits on our model, including a derivation of the cosmological limits on the dark matter lifetime in the presence of stimulated decay, as well as limits on the dark sector temperature due to the presence of extra relativistic degrees of freedom. App. D presents the calculation of the anisotropy of the ERB due to photon-dark photon resonant conversions. In App. E we describe our data analysis procedure, before presenting in App. F our extended results, including a complete corner plot of our parameter space, and other systematics checks such as the use of different datasets and fits using the ARCADE 2 data points only. Finally, in App. G we give a brief summary of the spectrum of radiation expected from synchrotron sources.

Appendix A: Dark photon spectrum

In this section, we provide a detailed derivation of the dark photon spectrum as a function of frequency and redshift, in the presence of stimulated emission.

Consider a two-body decay of a heavy particle $a$ into two daughter particles, at least one of which is a dark photon $A'$, emitted relativistically with energy $\omega_{A'}$. In the presence of a blackbody distribution of $A'$ with temperature $T'$, $a$ undergoes stimulated decay, with a decay width $\Gamma(z)$ that is related to the usual vacuum decay rate $\Gamma_{\text{vac}}$ via

$$\Gamma(z) = (1 + n f_{A'}^{\text{BB}}) \Gamma_{\text{vac}},$$

where $f_{A'}^{\text{BB}} \equiv [\exp(\omega_{A'}/T') - 1]^{-1}$ is the blackbody occupation number at energy $\omega_{A'}$. $n$ is an integer that depends on the details of the model; $n = 2$ in the fiducial particle physics model that we describe below. Note that for $\omega_{A'} \ll T'$, we obtain $\Gamma(z) \simeq (n T' / \omega_{A'}) \Gamma_{\text{vac}}$, an approximation that we will use frequently to obtain parametric estimates.

Given $\Gamma(z')$, the number density of $A'$ produced per unit time at redshift $z'$ is

$$\frac{dn_{A'}}{dt} = \alpha \Gamma(z') \frac{\rho_a(z')}{m_a},$$

where $\alpha = 2$ if both daughter particles are $A'$ and $\alpha = 1$ otherwise, $\rho_a(z)$ is the energy density of $a$ at redshift $z$, and $m_a$ is the mass of $a$. With this, we can find the contribution to the dark photon number density at redshift $z$ per unit frequency $\omega$, due to a decay occurring at redshift $z'$:

$$\frac{dn_{A'}}{d\omega dz'} = \frac{dn_{A'}(z')}{dt} \frac{dt}{dz'} \frac{(1 + z')^3}{(1 + z')^3} \delta_D \left( \omega - \frac{1 + z'}{1 + z} - \omega_{A'} \right),$$

where the redshift factors rescale the number density of particles produced at redshift $z'$ down to redshift $z$, and the Dirac delta function enforces the fact that the decay produces dark photons with energy $\omega_{A'}$. Integrating over $z'$ gives

$$\frac{dn_{A'}}{d\omega} = \int_z^{\infty} dz' \frac{\rho_a(z')}{m_a} \frac{\alpha \Gamma(z') (1 + z')^3}{H(z)(1 + z)(1 + z')^3} \delta_D \left( \omega - \frac{1 + z'}{1 + z} - \omega_{A'} \right) = \frac{\alpha \rho_a(z)}{m_a} \frac{\Gamma(z_s)}{\omega H(z_s)} \Theta(z_s - z),$$

(A1)

where

$$1 + z_s \equiv \frac{\omega_{A'}}{\omega}(1 + z)$$

(A2)

is the redshift at which the decay of $a$ produced the dark photon at frequency $\omega$, with $\omega \leq \omega_{A'}$. We have also assumed that $\rho_a \propto (1 + z)^3$. This expression agrees with Ref. [60], if we simply set $\Gamma(z_s) \rightarrow \Gamma_{\text{vac}}$. In terms of comoving number density and $x \equiv \omega / T_0$ where $T_0$ is the CMB temperature today, we find

$$\frac{1}{(1 + z)^3} \frac{dn_{A'}}{dz} = \frac{\alpha \rho_{a,0}}{m_a} \frac{\Gamma(z_s)}{x H(z_s)} \Theta(z_s - z),$$

(A3)

which is almost constant in redshift, aside from the step function. $\rho_{a,0}$ is the dark matter energy density today. Eq. (A1) is the expression we use in the Letter.
Appendix B: Particle physics model

1. Model building considerations

As we explained in the Letter, the particle physics model we need to explain the radio background requires three features that we repeat here:

1. A cold component of DM a which is stable on cosmological timescales, but undergoes a two-body decay with lifetime $\tau_{\text{vac}}$ in vacuum. For simplicity, we take $a$ to be all of DM;

2. One of the daughter particles is a dark photon $A'$, which has an existing blackbody distribution of $A'$ characterized by a temperature $T'(z) = T'_0(1 + z)$, where $T'_0$ is its temperature today; and

3. $A'$ has a mass $10^{-15} \text{eV} \lesssim m_{A'} \lesssim 10^{-9} \text{eV}$, and is emitted relativistically with energy $\omega_{A'}$. $A'$ mixes with the Standard Model (SM) photon $\gamma$ with kinetic mixing parameter $\epsilon$.

These conditions appear simple to meet; for example, the model proposed in Ref. [60] where the DM is a dark axion decaying into two identical dark photons which oscillates into SM photons, could be supplemented by an additional thermal population of dark photons. However, there are several challenges to building a successful model. In order to produce the right power law in the ERB, the $A'$ power-law spectrum must maintain $dn_{A'}/d\omega \propto \omega^{-1/2}$ between $3 \times 10^{-4} \lesssim x \lesssim 0.2$, where $x \equiv \omega/T_{\text{CMB}}$, from the redshift of production $z_*(x)$ until resonant conversions $A' \to \gamma$ are mostly complete. Furthermore, the $A'$ blackbody spectrum also cannot be significantly distorted; specifically, $A'$ particles from the bath at each value of $x$ must be described by a blackbody spectrum at $z_*(x)$. In the model of Ref. [60], both of these requirements can be violated by inverse decays $A'A' \to a$ or other $A'$ scattering processes. Moreover, it is difficult to maintain thermal equilibrium in the blackbody spectrum through scattering with another particle in the dark sector bath, since such scattering processes likely distort the power-law spectrum significantly. For typical parameters required to produce the full $T_{\text{exc}}$, scattering between a low-energy $A'$ in the power-law spectrum and a blackbody $A'$ is too rapid to guarantee that the $A'$ blackbody spectrum remains undistorted at all relevant times. While it is possible that even with significant distortion to the blackbody spectrum, a reasonable fit to the ERB can still result, checking this possibility would require us to integrate the Boltzmann equation over a wide range of $A'$ frequencies. In this paper, we avoid this computational challenge by building a slightly different fiducial model without significant distortion.

Before discussing the details of our fiducial model, we will first discuss how to check the size of any $A'$ spectral distortion due to a single process. We begin by writing down the Boltzmann equation with the relevant process $A \to A' \to \gamma$:  

$$\frac{df_{A'}(x, t)}{dt} = C[f_{A'}] \omega, \quad (B1)$$

where $C[f_{A'}]$ is the collision integral, which we will define for particular processes below. Here, we adopt the convention that $f_{A'}$ is related to the number density via 

$$n_{A'} = 3 \int \frac{d^3 \vec{p}}{(2\pi)^3} f_{A'},$$

where the factor of 3 accounts for the degeneracy in the spin state of $A'$. To determine if any process causes a significant distortion to the spectrum of blackbody $A'$ or low-energy $A'$ from the decay of $a$, we divide Eq. (B1) by $f_{A'}$ and integrate with respect to time or, equivalently, redshift, to obtain the change in $\log f_{A'}$, which gives a measure of the fraction of $A'$ at that value of $x$ that undergoes a scattering process. Our criterion for a small distortion to the $A'$ spectra is therefore 

$$|\Delta \log f_{A'}(x)| = \int_{z_{\text{min}}}^{z_{\text{max}}} \frac{dz}{H(z)(1 + z)} \frac{|C[f_{A'}]|}{\omega f_{A'}(x, z)} \ll 1, \quad (B2)$$

where $z_{\text{min}}$ and $z_{\text{max}}$ are the lowest and highest redshifts respectively for which the collision term is important.

Since we are only interested in producing the radio excess over a finite frequency range, we only need to check that distortions are small over a limited range of $x$. The radio frequency data points with an excess temperature over the CMB temperature span the frequency range 22 MHz–10 GHz, which we can cover by considering $3 \times 10^{-4} < x < 0.2$. The power-law component at fixed $x$ is produced by decays of $a$ at a redshift $1 + z_* \equiv \omega_{A'}/(xT_0)$, where $\omega_{A'}$ is the energy of the emitted $A'$ at the point of decay (for $a$ decaying into two massless particles, this is simply $\omega_{A'} = m_a/2$).
and \( T_0 \) is the blackbody CMB temperature today. After it is produced, it must stay in this power law without undergoing significant distortions until the present day.\(^1\) For the blackbody component to effectively produce the stimulated emission that we need, on the other hand, the blackbody spectrum must not be significantly distorted prior to redshift \( z_\text{a}(x) \): distortions after this redshift are unimportant. This means that for the power-law spectrum, for each value of \( x \), we need to consider \( z_\text{min} = 0 \) and \( z_\text{max} = z_\text{a}(x) \), while for the blackbody spectrum, \( z_\text{min} = z_\ast(x) \).

### 2. Fiducial model

We will now describe our fiducial particle physics model, which has all the three properties required to produce the radio background laid out in the previous section. We will show that both the low-energy and blackbody distribution of the dark photons do not undergo any significant distortion.

Our fiducial model is a modified version of the model proposed in Ref. [60]. The important particles in this model are: \( i \) an axion-like dark matter \( a \) with mass \( m_a \) and decay constant \( f_a \), and \( ii \) two dark photons, which we label \( A_\text{osc} \) and \( A_\text{osc} \). The three requirements for producing the radio excess as discussed in the main body of the paper are satisfied as follows:

1. the DM decays through the process \( a \to A_\psi A_\text{osc} \);

2. a blackbody distribution of \( A_\text{osc} \) described by a temperature \( T' \) is also present, leading to stimulated decay of \( a \), and

3. the dark photon \( A_\text{osc} \) possesses a small kinetic mixing term with the SM photon, and has a mass \( 10^{-15} \text{ eV} \lesssim m_\text{osc} \lesssim 10^{-9} \text{ eV} \), so that it undergoes resonant conversions into the SM photon after recombination.

The terms in the dark sector Lagrangian that are relevant to us are therefore

\[
\mathcal{L} \supset \frac{1}{2} (\partial \mu a)^2 - \frac{1}{2} m_a^2 a^2 - \frac{a}{4 f_a} F_{\psi,\mu\nu} F_{\text{osc} \mu\nu} - \frac{\epsilon}{2} F_{\text{osc} \mu\nu} F_{\mu\nu} + \mathcal{L}_K, \tag{B3}
\]

where \( \mathcal{L}_K \) contains the other kinetic terms of the dark photons, including a mass of \( m_\text{osc} \) for \( A_\text{osc} \). Note that a discrete symmetry under which \( A_\psi \to -A_\psi \) and \( a \to -a \) explicitly forbids decays of \( a \) to a pair of dark photons of the same species, as well as mixing between \( A_\psi \) and the SM photon, naturally leading to the Lagrangian shown above. By allowing \( a \) to decay into two different dark photons, we can now introduce a fermion \( \psi \) that scatters rapidly with \( A_\psi \), to ensure that \( A_\psi \) always equilibrates into a thermal distribution with temperature \( T' \), without destroying the low-energy spectrum of \( A_\text{osc} \).\(^2\) With this modification, we overcome the main difficulty faced by the model in Ref. [60]: there are now no low-energy \( A_\text{osc} \) to interact with the blackbody distribution of \( A_\text{osc} \), which was the main source of distortion for the blackbody spectrum.\(^3\)

We are now ready to check that distortions to both the low-energy spectrum \( A_\text{osc} \) power-law spectrum and the \( A_\text{osc} \) blackbody are small. The most important process, which enters at order \( 1/f_a^2 \), is inverse decays, \( A_\psi A_\text{osc} \to a \), which leads to the largest distortion. The collision integral is given by

\[
C_{A_\text{osc} A_\psi \to a}[f_a^\text{osc}] = -\frac{f_a^\text{osc}}{6} \int \frac{d^3 k_{A_\psi}}{(2\pi)^3 2\omega_{A_\psi}} \int \frac{d^3 p_a}{(2\pi)^3 2E_a} (2\pi)^4 \delta_B^4(k_{\text{osc}} + k_{A_\psi} - p_a)|M_{A_\text{osc} A_\psi \to a f_a}(k_{A_\psi})|^2.
\]

We use \( k, \bar{k} \) and \( \omega \) for incoming four-momentum, three-momentum, and energy; likewise, we have \( p, \bar{p} \) and \( E \) for outgoing states. The subscript ‘osc’ represents quantities associated with \( A_\text{osc} \). There is no Bose enhancement in the outgoing state, since we can treat the entire population of \( a \) as having zero momentum. This also allows us to neglect the contribution from the backward reaction to the collision integral. The squared matrix element for this process (summed over initial and final states) is

\[
|M_{A_\text{osc} A_\psi \to a}|^2 = \frac{m_a^4}{2f_a^2},
\]

\(^1\) Technically, the dark photons can become significantly distorted after all resonant oscillations into photons have been completed, but for simplicity we set this more stringent requirement.

\(^2\) For simplicity, we assume that the entire dark sector is described by a common temperature \( T' \).

\(^3\) While this completes the basic description of our model, there are some requirements on \( \psi \) to keep this model tractable and avoid tracking the spectra of all of these particles as a function of time. First, Compton \( \psi A_\psi \to \psi A_\psi \) and double-Compton \( \psi A_\psi A_\psi \to \psi A_\psi \) scattering must be sufficiently rapid to ensure that \( A_\psi \) is always described by a simple, blackbody distribution. This is easily satisfied, as long as \( \psi \) has a large coupling to \( A_\psi \), and is sufficiently abundant. Second, the process \( a A_\text{osc} \leftrightarrow \psi A_\psi \) must not lead to significant distortion of \( A_\text{osc} \). This is hard to determine without tracking the full evolution of the spectrum of \( A_\text{osc} \), but it is possible to avoid this entirely by choosing a sufficiently massive \( \psi \) to kinematically forbid \( a A_\text{osc} \to \psi \psi \), and making \( \psi \) asymmetric. We find that \( m_\psi \sim 30 \text{ eV} \) with a coupling to \( A_\psi \) of \( \alpha_D = 1 \), with an asymmetric number density today of \( n_\psi,0 = 2.5 \times 10^{-14} \text{ cm}^{-3} \) can provide sufficiently efficient scattering with \( A_\psi \). This value of \( m_\psi \) is large enough to kinematically forbid \( a A_\text{osc} \to \psi \psi \). Introducing a similar fermion \( \chi \) with \( m_\chi \sim m_\psi \) that couples to \( A_\text{osc} \) instead guarantees that no distortion can occur when \( T' > m_\psi, m_\chi \), while allowing distortions to build up once \( T' < m_\psi, m_\chi \), and \( \psi, \chi \) has frozen out. This also prevents \( \psi \overleftrightarrow{\psi} \to a A_\text{osc} \) from being significant at \( T' < m_\psi, m_\chi \), since \( \psi \) annihilates away at \( T' \sim m_\psi \).
while the occupation number of $A_{\psi}$ is given by the blackbody occupation number, $f_{A_{\psi}} = [\exp(\omega_{A_{\psi}}/T') - 1]^{-1}$. These simple expressions allow us to perform the collision integral relatively easily, to obtain

\[ C_{A_{osc} A_{\psi} \rightarrow a}[f_{osc}] = - \frac{f_{osc} m_a^4 T'}{96 \pi \alpha_{osc} f_a^2} \log \left[ 1 - \exp \left( -\frac{1 + z_*}{1 + z} \right) \right], \tag{B4} \]

where $\omega_{osc} = x T_{CMB}$. With this expression, we can now assess the total distortion to the blackbody and power-law distributions of $A_{osc}$, and check that for typical model parameters, these distortions are small, based on the criterion set out in Eq. (B2). For the blackbody distribution, we want to perform the integral in Eq. (B2) starting from $z_{min} = z_*$, allowing us to approximate the collision term as

\[ C_{A_{osc} A_{\psi} \rightarrow a}[f_{osc}] = - \frac{f_{osc} m_a^4 T'}{96 \pi \alpha_{osc} f_a^2} \log \left( \frac{1 + z_*}{1 + z} \right). \tag{B5} \]

Integrating this gives

\[ |\Delta \log f_{osc}| \sim 2^{5/2} m_a 3/2 \gamma x \gamma x^{1/2} T_0^3/2 \sim 1.8 \times 10^{-4}\left(\frac{\gamma}{0.2}\right)^2 \left(\frac{2 \times 10^{-4} \text{eV}}{m_a}\right)^{3/2} \left(\frac{10^{21} \text{s}}{T_{vac}}\right)^{3/2} \left(\frac{x}{0.2}\right)^{1/2}, \tag{B6} \]

where $\gamma \equiv T_0^*/T_0$, a redshift invariant quantity. This indicates that the total distortion to the blackbody distribution is small.

For the low-energy spectrum, we integrate Eq. (B2) from $z_{min} = 0$ and $z_{max} = z_*$. The integral can be approximately done in two parts: i) $0 < 1 + z < (1 + z_*)/\sqrt{x}$, where we can expand the expression in Eq. (B4) using log$(1 - \alpha) \approx -\alpha$, and ii) $(1 + z_*)/\sqrt{x} < 1 + z < 1 + z_*$, where we can use the approximation in Eq. (B5). The second part of the integral dominates, but the total contribution to the distortion is given numerically by

\[ |\Delta \log f_{osc}| \sim 0.21 \left(\frac{\gamma}{0.2}\right)^2 \left(\frac{2 \times 10^{-4} \text{eV}}{m_a}\right)^{3/2} \left(\frac{10^{21} \text{s}}{T_{vac}}\right) \left(\frac{3 \times 10^{-4}}{x}\right)^{3/4}. \tag{B7} \]

This distortion appears to be somewhat large for the fiducial values shown here, but the relative uncertainty on the data point at $x = 3.8 \times 10^{-4}$ or 22 MHz is $\sim 25\%$, and so is enough to absorb the distortion obtained here. Moreover, the posterior distribution from our fits include larger values of $m_a$ and $T_{vac}$, which decreases the size of the distortion. We therefore conclude that inverse decays do not significantly distort either component of the $A_{osc}$ spectrum.  

Our proposed model therefore satisfies the requirement that all distortions to the $A_{osc}$ power law and blackbody spectrum are small, and is a viable model for explaining the ERB. We stress that many of the required conditions are invoked in order to simplify the analysis and to allow an unambiguous prediction of the ERB in this model. Some of the conditions on the distortions, for example, may be relaxed under a complete analysis, which would include numerically solving the Boltzmann equations for each mode.

Appendix C: Experimental constraints

1. Dark matter lifetime constraint with stimulated decay

Cosmological constraints on the decay lifetime of dark matter have been obtained for decays without stimulated emission [63–68]. These results show that if a subcomponent $f_{dcdm}$ decays between recombination and today, Planck 2018 and baryon acoustic oscillation (BAO) data limit $f_{dcdm} \leq 0.0216$ [67] at the 95% confidence level. This result is consistent with the uncertainty in the dark matter energy density reported by Planck 2018 [2]. From this, we can deduce a limit on the decay lifetime with stimulated emission, by adopting a limit of $f_{dcdm} = 0.0216$ for the proportion of dark matter that decays away by the present day. For a stimulated emission lifetime given by $\tau(z) = \tau_{vac} (1 + 2 f_{dcdm})^{-1}$, which is the expression we obtain with our fiducial model, we require\(^5\):

\[ \int_0^t \frac{dt}{\tau_{vac}} \left(1 + \frac{2}{e^{\omega_{dcdm}/T'} - 1}\right) < f_{dcdm}. \]

\(^4\) Although elastic scattering processes should be subleading at order $1/f_{dcdm}^4$, they can, in fact, be very rapid for two reasons. First, processes like $a A_{osc} \rightarrow a A_{osc}$ receive a large Bose enhancement from the existing population of $A_{osc}$, which significantly increases the scattering rate. Secondly, the matrix element of $a A_{osc} \rightarrow a A_{osc}$ contains a divergence. Such divergences are commonplace even in Standard Model processes, such as $e^- Z \rightarrow e^- Z$ [94]. In the context of cosmological fluids, these divergences are regulated by thermal corrections to the self-energy of $A_{osc}$, which impart an imaginary contribution to the mass of $A_{osc}$ [94]. We have checked that even including Bose enhancement and regulating this divergence with loop contributions from a thermal distribution of $\psi$ [56], all elastic scattering processes are indeed subdominant to the leading inverse decay process discussed here.

\(^5\) This corresponds to an approximation of the effect of stimulation, we leave a more complete treatment for future work.
Numerically, we find
\[
\int_0^t \frac{dt}{\tau_{\text{vac}}} \left( 1 + \frac{2}{\omega_{\text{A}}/T - 1} \right) \approx \frac{1}{\tau_{\text{vac}}} \times \begin{cases} 
1.5 \times 10^{18} s \left( \frac{2T_0'}{\omega_{\text{A}'}} \right), & \omega_{\text{A}'} < 3.2, \\
H_0^{-1}, & \omega_{\text{A}'} \geq 3.2,
\end{cases}
\]
which leads to a limit on the vacuum decay lifetime of
\[
\tau_{\text{vac}} > 10^{19} s \times \max \left[ 2.1, 6.3 \left( \frac{10^{-4} \text{eV}}{\omega_{\text{A}'}} \right) \left( \frac{\gamma}{0.2} \right) \right].
\] (C1)
This limit is roughly an order of magnitude stronger than the lifetime limits on DM decays without stimulated emission for our fiducial choice of parameters \[67\].

2. Constraints on relativistic degrees of freedom

We complete our discussion of the particle physics model by discussing its effect on \(N_{\text{eff}}\), and relevant constraints. \(A_{\text{osc}}, A_{\psi}\) contribute to \(N_{\text{eff}}\) as relativistic degrees of freedom. In addition, other fermions that couple to these bosons can contribute as well if they are sufficiently light. Assuming the existence of two such Dirac fermions on top of \(A_{\text{osc}}\) and \(A_{\psi}\) before recombination, the total energy density of these particles divided by the energy density of a neutrino is
\[
\Delta N_{\text{eff}} \approx \frac{2 \times 3 + 2 \times 4 \times \frac{7}{8}}{2 \times \frac{3}{8}} \frac{\gamma^4}{(\frac{4}{17})^{4/3}} = 0.04 \left( \frac{\gamma}{0.2} \right)^4.
\] (C2)
This is to be compared with the Planck measurement of \(N_{\text{eff}} = 2.99 \pm 0.17 \) \[2\], which limits \(\Delta N_{\text{eff}} < 0.34\) at the 95\% confidence level. Including the irreducible contribution from the thermal distribution of just one dark photon, which must be present in the class of models considered in this work, gives \(\Delta N_{\text{eff}} \approx 0.01(\gamma/0.2)^4\).

Appendix D: Anisotropy due to dark photon conversions

In this appendix, we obtain the conversion anisotropy power spectrum, which originates from variations in electron density fluctuations along two lines-of-sight separated in the sky by some angle. Our discussion follows Ref. \[77\] closely, but with some novel and peculiar features originating from the resonance conversion process; for clarity, we repeat many of the same calculations presented in that reference.

For dark photon conversions, the observed temperature in any direction in the sky \(\hat{n}\) at a fixed frequency is directly proportional to the total probability of conversion \(P(\hat{n})\) of dark photons into photons. We begin by defining the fluctuation of the conversion probability in some particular direction in the sky \(\hat{n}\), defined explicitly by \(\delta P(\hat{n}) \equiv P(\hat{n}) - \langle P \rangle\), where \(\langle P \rangle\) is the sky-averaged conversion probability. In any particular direction \(\hat{n}\), we have \[55, 56\]
\[
P(\hat{n}) = \frac{\pi \epsilon^2 m^4_{A'}}{\omega_0} \int_0^{z_*} dz \frac{\delta_D(m_{A'}^2(\hat{r}, z) - m_{A'}^2)}{H(z)(1 + z)^2},
\] (D1)
where \(m_{A'}^2(\hat{r}, z)\) is the effective plasma mass of photons along the particular line-of-sight, and \(\omega_0\) is the present-day, observed angular frequency of the photons. \(\hat{r} \equiv \hat{n} \chi(z)\), where \(\chi(z)\) is the comoving distance traveled by light between \(z\) and the present day. \(\omega_0(1 + z)\) is the energy of the daughter particle from the DM decay; photons that have energy \(\omega_0\) today were produced by decays at \(z_*\), which is the maximum redshift we integrate up to.

We can compute the sky-averaged conversion probability by integrating over the one-point probability density function (PDF) \(f_1(m_{\gamma}^2; z)\) of \(m_{\gamma}^2\), to obtain \[55, 56\]
\[
\langle P \rangle = \frac{\pi \epsilon^2 m^4_{A'}}{\omega_0} \int_0^{z_*} dz \int dm_{\gamma}^2 f_1(m_{\gamma}^2; z) \frac{\delta_D(m_{A'}^2 - m_{A'}^2)}{H(z)(1 + z)^2} = \frac{\pi \epsilon^2 m^4_{A'}}{\omega_0} \int_0^{z_*} dz \frac{f_1(m_{\gamma}^2 = m_{A'}^2; z)}{H(z)(1 + z)^2},
\] (D2)
so that
\[
\delta P(\hat{n}) = \frac{\pi \epsilon^2 m^4_{A'}}{\omega_0} \int_0^{z_*} \frac{dz}{H(z)(1 + z)^2} \left[ \delta_D(m_{\gamma}^2(\hat{r}, z) - m_{A'}^2) - f_1(m_{\gamma}^2 = m_{A'}^2; z) \right].
\] (D3)
We now decompose the observed conversion probability over the whole sky into spherical harmonics $Y_{\ell m}(\hat{n})$, with coefficients $a_{\ell m}$ given by

$$a_{\ell m} = \frac{1}{\langle P \rangle} \int d\hat{n} \delta P(\hat{n}) Y_{\ell m}^*(\hat{n}) .$$

The anisotropy power spectrum $C_\ell$ for the conversion probability, defined as $C_\ell \equiv \langle a_{\ell m}^* a_{\ell m} \rangle$, can then be computed as

$$C_\ell = \frac{1}{\langle P \rangle^2} \int d\hat{n} \int d\hat{n}' \langle \delta P(\hat{n}) \delta P(\hat{n}') \rangle Y_{\ell m}^*(\hat{n}) Y_{\ell m}(\hat{n}') ,$$

where $\langle \cdots \rangle$ in the integral should be interpreted as an all-sky average. Note that defined in the following manner, $C_\ell$ is dimensionless, since it describes fluctuations in conversion probability.

To make further progress, we define the quantity

$$Q(\hat{r}, z; \hat{r}', z') \equiv \left\langle \left( \delta_D(m^2_\gamma(\hat{r}, z) - m^2_{\alpha s}) - f_1(m^2_\gamma = m^2_{\alpha s}; z) \right) \left( \delta_D(m^2_\gamma(\hat{r}', z') - m^2_{\alpha s}) - f_1(m^2_\gamma = m^2_{\alpha s}; z') \right) \right\rangle ,$$

a two-point correlation function along two different lines-of-sight, described by comoving coordinates $\hat{r}$ and $\hat{r}'$, with photons passing through each point at redshifts $z$ and $z'$ respectively. In fact, homogeneity and isotropy guarantee that this function does not depend on $\hat{r}$ and $\hat{r}'$ separately, but only on $|\hat{r} - \hat{r}'|$. Inserting this into the expression above for $C_\ell$, we find

$$C_\ell = \frac{1}{\langle P \rangle^2} \left[ \frac{\pi c^2 m^4_{\alpha s}}{\omega_0} \right]^2 \int_0^{z_\gamma} dz W(z) \int_0^{z_\gamma} dz' W(z') \int d\hat{n} \int d\hat{n}' Q(|\hat{r} - \hat{r}'|, z, z') Y_{\ell m}^*(\hat{n}) Y_{\ell m}(\hat{n}') ,$$

where $W(z) \equiv \left[ H(z)(1 + z)^2 \right]^{-1}$. We can now write $Q$ in terms of its Fourier transform over $\hat{r} - \hat{r}'$, $\hat{Q}$, giving

$$Q(|\hat{r} - \hat{r}'|, z, z') = \int \frac{d^3 k}{(2\pi)^3} e^{ik \cdot (\hat{r} - \hat{r}')} \hat{Q}(k, z, z')$$

$$= (4\pi)^2 \sum_{p,s=0}^{\infty} \sum_{q=-p}^{p} \sum_{r=-s}^{s} \int \frac{d^3 \hat{k}}{(2\pi)^3} \hat{Q}(\hat{k}, z, z') j_p(kr) j_s(kr') Y_{p q}^* \hat{Y}_{p q} \hat{Y}_{s t}^* \hat{Y}_{s t}^*$$

$$= \frac{2}{\pi} \sum_{p=0}^{\infty} \sum_{q=-p}^{p} \int dk k^2 \hat{Q}(k, z, z') j_p(kr) j_p(kr') Y_{p q}^* \hat{Y}_{p q} \hat{Y}_{s t}^* \hat{Y}_{s t}^* .$$

In the second line, we have used the Rayleigh expansion for plane waves, and $j_p$ is the spherical Bessel function of order $p$. In the last line, we use the orthogonality of spherical harmonics to integrate over solid angle. Substituting this expression into Eq. (D5) and integrating over $\hat{n}$ and $\hat{n}'$, once again exploiting the orthogonality of spherical harmonics, gives

$$C_\ell = \frac{1}{\langle P \rangle^2} \left[ \frac{\pi c^2 m^4_{\alpha A}}{\omega_0} \right]^2 \frac{2}{\pi} \int_0^{z_\gamma} dz W(z) \int_0^{z_\gamma} dz' W(z') \int \frac{d^3 k}{r^2} \hat{Q}(k, z, z') j_\ell(kr) j_\ell(kr') .$$

Finally, we can simplify this integral further assuming the Limber approximation [74–76], which is a high-$\ell$ expansion that works particularly well for the multipoles in which we are interested [96] and allows us to approximate $k^2 j_\ell(kr) j_\ell(kr') \approx (\pi/2) \delta_D(k - \ell/r) \delta_D(r - r')/r^2$. This finally gives

$$C_\ell = \frac{1}{\langle P \rangle^2} \left[ \frac{\pi c^2 m^4_{\alpha A}}{\omega_0} \right]^2 \int_0^{z_\gamma} \frac{dz}{r^2(z)} W^2(z) H(z) \hat{Q}(k = \ell/r, z, z) .$$

This compact expression makes the calculation of the conversion anisotropy numerically tractable; in particular, the correlation function $Q$ defined in Eq. (D4) need only be evaluated at points along two different lines-of-sight which have equal redshifts.

We now need an expression for $\hat{Q}(k = \ell/r, z, z)$. First, we define $f_2(\rho, m^2_\gamma(\hat{r}), m^2_\gamma(\hat{r}'))$ as the two-point PDF for $m^2_\gamma$ at two different points, $\hat{r}$ and $\hat{r}'$, at redshift $z$, with $\rho \equiv |\hat{r} - \hat{r}'|$. Note that homogeneity and isotropy guarantee that $f_2$ only depends on $\rho$. With this, the averaging in Eq. (D4) is performed by integrating the quantity with respect to $m^2_\gamma(\hat{r})$ and $m^2_\gamma(\hat{r}')$, weighted by $f_2$, which gives

$$Q(|\hat{r} - \hat{r}'|, z, z) = f_2(|\hat{r} - \hat{r}'|, m^2_\gamma(\hat{r}) = m^2_{\alpha s}, m^2_\gamma(\hat{r}') = m^2_{\alpha s}; z) - |f_1(m^2_\gamma = m^2_{\alpha s}; z)|^2 ,$$

(D7)
with
\[ \dot{Q}(k, z) = 4\pi \int d\rho \, \rho^2 \, j_0(k \rho) Q(\rho, z, z) \]

Given both the one- and two-point PDF of \( m_g^2 \), we can now proceed to calculate the anisotropy power spectrum.

We demonstrate that the conversion of the free-electron number density can satisfy existing experimental bounds by assuming that 1) the free-electron number density is exactly equal to the baryon number density, so that the one- and two-point PDFs of \( m_g^2 \) are given by the one- and two-point PDFs of baryonic fluctuations \( \delta_b \), divided by one and two powers of \( m_g^2(z) \), the mean value of \( m_g^2 \) at redshift \( z \), respectively. This is a good assumption for \( z \lesssim 6 \), where resonant conversions generally take place, since reionization has been completed, and all baryons are ionized; and 2) the baryonic fluctuations follow analytic one- and two-point PDFs. We consider two analytic forms for the one- and two-point PDFs: normally distributed and log-normally distributed fluctuations, to give an indication of the dependence on the PDF. In both cases, the PDFs are fully specified by the correlation function \( \xi_b(\mathbf{r} - \mathbf{r}'; z) \), which is related to the power spectrum of baryons \( P_b \) via

\[ \xi_b(\rho; z) = \int \frac{d^3 k}{(2\pi)^3} j_0(k \rho) P_b(k; z), \]

with \( \sigma_b^2(z) = \xi_b(0; z) \) being the variance of baryonic fluctuations. For normally distributed fluctuations,

\[
\begin{align*}
  f_1(m_g^2 = m_A^2, z) &= \frac{1}{m_g^2(z) \sqrt{2\pi \sigma_b^2(z)}} \exp \left[ -\frac{g(m_A^2; z)^2}{2\sigma_b^2(z)} \right], \\
  f_2(\rho, m_g^2(\mathbf{r}) = m_A^2, m_g^2(\mathbf{r}') = m_A^2, z) &= \frac{1}{2\pi m_g^2(z) \sqrt{\sigma_b^2(z) - \xi_b(\rho; z)}} \exp \left( -\frac{g(m_A^2; z)^2}{\sigma_b^2(\xi_b(\rho; z) + \xi_b(0; z))} \right),
\end{align*}
\]

where \( g(m_A^2; z) = m_A^2(m_g^2(z) - 1) \), while for log-normally distributed fluctuations \([55, 97],\]

\[
\begin{align*}
  f_1(m_g^2 = m_A^2, z) &= \frac{1}{m_g^2(z) \sqrt{2\pi \Sigma^2(z)}} \exp \left[ -\frac{L^2(m_A^2; z)}{2\Sigma^2(z)} \right] \frac{1}{1 + g(m_A^2; z)}, \\
  f_2(\rho, m_g^2(\mathbf{r}) = m_A^2, m_g^2(\mathbf{r}') = m_A^2, z) &= \frac{1}{2\pi m_g^2(z) \sqrt{\Sigma^2(z) - X^2(\rho; z)}} \exp \left( -\frac{L^2(m_A^2; z)}{\Sigma^2(z) + X(\rho; z)} \right) \frac{1}{1 + g(m_A^2; z)^2},
\end{align*}
\]

where \( \Sigma^2(z) = \log[1 + \sigma_b^2(z)], X(\rho; z) = \log[1 + \xi_b(\rho; z)], \) and \( L(m_A^2; z) = \log[1 + g(m_A^2; z)] + \Sigma^2(z)/2. \)

In the limit that \( \rho \to \infty \), we obtain \( \xi_b \to 0 \) and \( f_2 \to f_2^* \) for both distributions, indicating that the two-point PDF factorizes into the product of two, independent one-point PDFs at sufficiently large separations, when correlations are unimportant.

Eqs. (D7) and (D6) give us the following general expression for \( C_\ell \), which is the main result of this section:

\[
C_\ell = \frac{1}{\langle P \rangle^2} \int_0^{z_{lim}} dz \frac{\langle \mathcal{P} \rangle^2}{r^2(z)} H(z) \left[ \frac{d\langle \mathcal{P} \rangle}{dz} \right]^2 \int d\rho \, 4\pi \rho^2 \, j_0(\ell \rho/r) \left[ f_2(\rho, m_g^2(\mathbf{r}) = m_A^2, m_g^2(\mathbf{r}') = m_A^2, z) - 1 \right],
\]

an expression that we can evaluate numerically for either choice of baryon fluctuation PDFs. Note that the integrand over \( \rho \) is finite as \( \rho \to 0 \). Following the convention used in CMB anisotropy power spectrum analyses as well as Ref. [44], the temperature anisotropy power spectrum is defined to be \( \ell(\ell + 1)C_\ell(T)^2 \), where \( T \) is the sky-averaged brightness temperature.

For small angular scales \((\ell \gtrsim 3000)\), upper limits on the anisotropy of the ERB have been obtained by the Very Large Array (VLA) at 4.86 GHz [40] and 8.4 GHz [41], as well as the Australia Telescope Compact Array (ATCA) at 8.7 GHz [42]. These measurements were made with the intention of looking for CMB anisotropies at the arcmminute scales, and all set an approximate upper limit of \( \Delta T/T_{sky} \lesssim 10^{-2} \) for the ERB [39]. More recently, LOFAR and TGSS observations have been used to measure the anisotropy power spectrum at \( \sim 140 \text{MHz} \) [43, 44]. This latest result confirms the fact that a currently unknown population of dim but numerous synchrotron sources is required to explain the ERB; they also find that these sources must exhibit some nontrivial clustering to produce the right power spectrum.

Fig. 4 shows the predicted anisotropy power spectrum, \( [\ell(\ell + 1)/2\pi]C_\ell(T)^2 \), as a function of \( \ell \), obtained by integrating Eq. (D10) numerically for both normally (lower line) and log-normally (upper line) distributed baryon
The range between the two lines, shaded in blue, gives some indication of the uncertainty of our prediction. Representative values of $m_{A'} = 3 \times 10^{-14} \text{eV}$ and $\omega_{A'} = 2 \times 10^{-4} \text{eV}$ have been chosen, but the results are qualitatively similar for other values of these parameters. This result should be compared with existing measurements of the anisotropy power, including 1) observed upper limits of the CMB anisotropy power spectrum at small scales between 4–9 GHz by the VLA [40, 41] and the ATCA, subsequently reinterpreted in Ref. [39] as an upper limit on the anisotropy power of the excess radio power above the CMB and known point sources, and 2) the LOFAR anisotropy power spectrum of the radio background at 140 MHz [44]. We take $\langle T \rangle = 255 \text{K}$, the expected radio temperature at 140 MHz based on the power-law fit in Eq. (1) of the Letter. The anisotropy power reported by the high frequency measurements at frequency $\nu_{\text{high}}$ has been rescaled by $(140 \text{MHz}/\nu_{\text{high}})^{2\beta}$, where $\beta = 2.6$; this assumes that the relative size of fluctuations $\delta T/T$ is independent of frequency, and can be rescaled by $\langle T \rangle^2$ at each frequency. Note that the observed upper limits in the 4–9 GHz range are not necessarily in tension with the $\ell \gtrsim 10^3$ results at 140 MHz, since the anisotropy can in fact be frequency dependent.

The experimental results for the anisotropy power spectrum shown in Fig. 4 cover $10^2 \lesssim \ell \lesssim 6 \times 10^4$, finding a power of $1$–$10^2 \text{K}^2$ across this range; this represents a temperature fluctuation of $0.004 \lesssim \Delta T/T \lesssim 0.04$, a result that seems unusually smooth if radio emission were correlated with large scale structure at low redshifts [39]. Our predicted conversion anisotropy power spectrum for the fiducial values shown here lies mostly below the measured anisotropy power, and is therefore consistent with these experimental results, except for $\ell \lesssim 800$ at 140 MHz. However, as mentioned in the Letter, lower-$\ell$ measurements of the anisotropy are difficult, and within the LOFAR dataset, significant scatter can be observed in the anisotropy power between adjacent frequency bands, which are subsequently combined to produce the final result shown here [44]. We have thus demonstrated that our model can in principle produce a sufficiently smooth conversion anisotropy power spectrum; more detailed studies involving more realistic one- and two-point PDFs, as well as potentially a more careful analysis of the 4–9 GHz radio data may be of interest in future work.

The theoretical prediction from our model only has a weak dependence on the observational frequency, coming from the integration limit $z_*=\star$; we have only included the largest fluctuations here.
Appendix E: Data analysis and statistical methodology

1. Likelihood and posterior sampling

We assume the likelihood of the observed brightness temperatures $T_{\text{obs},i}(\nu_i)$ given a model for the emission $T(\nu; \theta)$ to be Gaussian, where $i$ indexes individual data points and $\theta = \{m_a, m_{A'}, \tau_{\text{vac}}, \epsilon, T_0, T_0\}$ are the parameters of interest describing the fiducial new-physics model. Assuming uncorrelated measurements, the total likelihood for a dataset is given by $p(T_{\text{obs}} | \theta) = \prod_i \mathcal{N}(T_{\text{obs},i}(\nu_i) | T(\nu_i; \theta), \sigma_{T,i})$, where $\sigma_{T,i}$ are the corresponding standard deviation uncertainties describing the noise model.

Given a prior $p(\theta)$ as defined in Sec. E2 below, we infer an approximation for the posterior distribution $p(\theta | T_{\text{obs}}) = p(T_{\text{obs}} | \theta) p(\theta)/Z$, where $Z$ is the marginal evidence $Z = \int \mathcal{d}\theta p(T_{\text{obs}} | \theta) p(\theta)$. This is done through Monte Carlo nested sampling implemented in dynesty [72]. 1000 live points are used to model the posterior and sampling is performed until the estimated expected contribution to the log-evidence is less than $\Delta \log Z = 0.05$, with an otherwise default configuration of the static nested sampler.

2. Constrained prior definition

A baseline prior is assumed such that $\log_{10} m_a \sim \mathcal{U}(-7, -1)$, $\log_{10} m_{A'} \sim \mathcal{U}(-17, -10)$, $T_0/T_0' \sim \mathcal{U}(0, 0.4)$, $\log_{10} \tau_{\text{vac}} \sim \mathcal{U}(16, 25)$, $\log_{10} \epsilon \sim \mathcal{U}(-11, -4)$, and $T_0 \sim \mathcal{N}(2.7255, 0.00086)$, with the prior on $T_0$ motivated by an analysis of COBE/FIRAS CMB data in the 2.27–21.33 cm$^{-1}$ frequency range [1, 8], and the prior on $T_0'/T_0$ motivated by the constraints derived in Sec. C2. Furthermore, for our fiducial analysis we wish to incorporate external constraints on the considered parameter space as mentioned in the main body of this Letter. Practical implementations of nested sampling require that the prior distribution be provided as a transformation from the unit hypercube to the target prior density. Although such a transformation is readily available for commonly used prior specifications e.g., uniform- or Gaussian-distributed, transformations from an arbitrary prior such as that respecting the constraints laid out in the main Letter through combinations of multiple parameters are not always readily available or analytically tractable.

In order to construct an implicit prior for our purposes we follow a procedure similar to that outlined in Ref. [98] and use normalizing flows [99, 100], which use a series of bijections parameterized by neural networks with tractable Jacobians in order to define transformations from a simple base distribution to complex, expressive target distributions. Specifically, we generate a large number $10^6$ of samples from the baseline prior, remove the samples incompatible with the external constraints, and use a normalizing flow consisting of 8 neural spline flows [101] to learn a transformation from a standard Gaussian $\mathcal{N}(0, 1)$ into the constrained target prior density. Samples $u$ from the unit hypercube proposal can then be transformed to those corresponding to a standard Gaussian through an affine transformation $u \rightarrow 0 + 1 \cdot F^{-1}_N(u)$, where $F^{-1}_N$ is the inverse cumulative distribution function of the Gaussian distribution. These samples can then be further transformed to those on the constrained prior density by passing them through the learned normalizing flow.

3. Prior-predictive check

Generally in Bayesian analyses and in particular when non-trivial priors are utilized such as in this work, it is useful to conduct prior-predictive checks in order to instill confidence that the prior does not bias the analysis towards “favorable” regions of the parameter space. In that case, formally low-likelihood parameter points could still correspond to high posterior density regions.

Figure 5 shows the 68 and 95% contours of the prior induced on the excess temperature above the modeled extragalactic background, defined as $T_{\text{eq}} = 0.23 K(\nu/\text{GHz})^{-2.7}$ [16, 39]. It can be seen that the induced prior covers a large range of possible excess temperatures and does not bias the current analysis in favour of the observed data points (shown in black and red).
Prior-predictive check

FIG. 5. 68% (dark green) and 95% (light green) containment of the induced prior on the excess temperature above the modeled extragalactic background, $T_{\text{ex}} = 0.23 \left(\nu / \text{GHz}\right)^{-2.7}$ [16]. Measured data points from Ref. [3] are shown for comparison. The induced prior covers a large range of possible excess temperatures and does not bias the analysis in favour of the observed data points.

Appendix F: Extended results

1. Parameter posteriors

Figure 6 shows the individual and pair-wise marginal posteriors on all the modeled parameters obtained for the baseline analysis.

2. Systematic variations on the analysis

In this section we consider various systematic variations of our baseline analysis. In particular we consider:

1. results obtained using the data from Ref. [9] instead of those from Ref. [3]. Ref. [9] includes recent data points from the LWA1 Low Frequency Sky Survey (LLFSS), which spans a frequency range of 40–80 MHz. These new data points are consistent with the power-law fit in Ref. [3]. Ref. [9] also independently reanalyzed the data from ARCADE 2 and the other experiments used in Ref. [3], finding broad agreement, but with small changes to the central values and more significant changes to the error bars, despite using a similar data analysis method. Ref. [9] therefore provides a useful systematics check on how the ERB spectrum is extracted;

2. results obtained by fitting only the ARCADE 2 data $\gtrsim 3$ GHz and ignoring the lower-frequency radio measurements. In this case, we neglect stimulated emission, since this is not necessary to produce a good fit, simplifying the model and reducing to that first introduced in Ref. [60]. ARCADE 2 is the only experiment in our list that is designed primarily with absolute zero-level calibration in mind, in order to produce an accurate determination of the sky-brightness [10], which warrants a separate investigation; and

3. results obtained without the modeled contribution from unresolved extragalactic radio sources [16, 39]. This should concretely demonstrate that our model does not rely on unresolved extragalactic radio sources for a good fit, which can be anticipated from the fact that these sources are at least 3 times less bright than the measured $T_{\text{ex}}$. 
Figure 7 shows the excess temperature posterior for all these systematic variations, while Tab. I shows the posterior summaries for each case considered. The parameters ranges compatible with observations show minor variations from case to case, while providing a visually good fit to the data in all cases.

We summarize the inferred posteriors in Tab. I through the median and 68% containment of the individual marginal parameter posteriors (labeled “Marginal”) and the 68% highest posterior density intervals, describing the shortest interval in the higher-dimensional joint parameter space where 68% of the posterior mass is contained (labeled “HDPI”).
$\log_{10} m_A$ & $\log_{10} m_A$ & $T_0/T_0$ & $\log_{10} \tau_0$ & $\log_{10} \epsilon$ & $T_0$
\hline
Fixsen (Baseline) & Marginals & $-3.66^{+0.10}_{-0.26}$ & $-13.37^{+0.32}_{-0.68}$ & $0.22^{+0.05}_{-0.12}$ & $21.15^{+0.47}_{-1.10}$ & $-6.98^{+0.25}_{-0.60}$ & $2.7258^{+0.0003}_{-0.0007}$

No EG model & Marginals & $-3.96^{+0.11}_{-0.26}$ & $-13.22^{+0.28}_{-0.75}$ & $0.22^{+0.05}_{-0.11}$ & $21.48^{+0.58}_{-1.27}$ & $-7.23^{+0.27}_{-0.71}$ & $2.7259^{+0.0003}_{-0.0007}$

Dowell & Marginals & $-4.12^{+0.13}_{-0.33}$ & $-13.28^{+0.30}_{-0.74}$ & $0.22^{+0.05}_{-0.11}$ & $21.61^{+0.60}_{-1.26}$ & $-7.25^{+0.27}_{-0.78}$ & $2.7256^{+0.0003}_{-0.0007}$

Fixsen, no stim. ARCADE only & Marginals & $-3.38^{+0.13}_{-0.36}$ & $-13.59^{+0.32}_{-0.64}$ & - & $20.34^{+0.34}_{-0.65}$ & $-6.61^{+0.20}_{-0.45}$ & $2.7256^{+0.0002}_{-0.0007}$

HPDI & $-3.78^{+3.10}_{-3.10}$ & $-14.54^{+13.06}_{-13.06}$ & - & $19.36^{+20.80}_{-20.80}$ & $-7.15^{+6.08}_{-6.08}$ & $2.7249^{+2.7264}_{-2.7264}$

TABLE I. Posterior summaries for all the systematic variations as compared to our baseline analysis.

3. Power-law fit and model comparison

As a point of comparison, we also show results using a variant of the commonly-used power-law ansatz (equivalent to Eq. 1) for the excess temperature over $T_0$, 

$$T(\nu) = T_0 \left[ 1 + A_{\text{GHz}} \left( \frac{\nu}{1 \text{ GHz}} \right)^\beta \right]$$

(F1)

which is parameterized through the CMB black-body temperature $T_0$, excess temperature at 1 GHz $A_{\text{GHz}}$, and spectral index $\beta$. The results of these fits for the Fixsen et al and Dowell & Taylor datasets are shown in the left and right columns of Fig. 8, respectively. As previously reported in the literature, these parameterizations provide a formally good fit to the low-frequency radio data [9, 38]. Posterior summaries for these fits are given in Tab. II in the same format as Tab. I.

We show a quantitative comparison between the power law and new physics models as follows. For a given model $\mathcal{M}$ (in our case corresponding to either the new-physics model or the power-law ansatz) the model evidence, also known as the marginal likelihood, is defined for data $x$ and parameters $\theta$ as $\mathcal{Z}_x \equiv \int d\theta \, p_{\mathcal{M}}(x \mid \theta)p_{\mathcal{M}}(\theta)$ where $p_{\mathcal{M}}(x \mid \theta)$ and $p_{\mathcal{M}}(\theta)$ are the model likelihood and prior, respectively. The model evidence is a measure of compatibility between the model and data, and implicitly penalizes excessive model complexity—this is relevant in our case, since the new physics and power-law models have different number of parameters (3 and 6 respectively). The ratio of evidences between two models $\mathcal{M}_1$ and $\mathcal{M}_2$ is known as the Bayes factor, and can be formally used in a model comparison setting to quantify how much better of a fit $\mathcal{M}_1$ is compared to $\mathcal{M}_2$.

We show the log-Bayes factors in favor of the power-law model in the last column of Tab. II. The power-law model tends to provide a formally better fit using both the Fixsen et al and Dowell & Taylor datasets, corresponding to log-Bayes factors in favor of the power-law model of $\Delta \log Z = 1.89$ and $\Delta \log Z = 0.76$ respectively. This reflects the fact that a power law with $\beta \approx -2.6$ is a slightly better fit to the data points, rather than $\beta \approx -5/2$ as predicted by our model. We note however that given significant differences between models considered, a principled model comparison is difficult. Specifically, O(1) Bayes factors should not be taken at face value, since they can depend sensitively on chosen prior specifications and ranges; see Ref. [102] for a detailed discussion of this point. Finally, we note that even though synchrotron emission from unknown sources can plausibly produce a power law close to $\beta \approx -2.6$ (see App. G), this is highly dependent on the electron distribution within these individual sources as a function of source brightness and counts; it is therefore not immediately obvious that any future model ascribing $T_{\text{exc}}$ to currently unknown synchrotron sources would provide a better fit than our model does.
FIG. 7. Systematic variations on the analysis, same format as Fig. 2.

FIG. 8. Excess temperature for the power-law fit, same format as Fig. 2.
outside a supernova remnant, it can be shown that at linear level
physical shocks $[10^4–10^8]$. For example, if one considers shock waves which propagate in the interstellar medium
assumed above.

which generate the radiation. In particular, a power law is expected if the energy distribution is also a power law as
frequency given by the cyclotron frequency
charged particle accelerated in a magnetic field will radiate. Nonrelativistic particles emit cyclotron radiation, with a
synchrotron radiation from unresolved astrophysical sources. In this appendix, we briefly review synchrotron emission
favor of the power-law fit, with positive values corresponding to a preference for the power-law model.

As extensively discussed in the Letter, many astrophysical solutions to the ERB advocate for the presence of
synchrotron radiation from unresolved astrophysical sources. In this appendix, we briefly review synchrotron emission
with a particular focus on its spectral features. The basic physics behind synchrotron radiation is straightforward: a
charged particle accelerated in a magnetic field will radiate. Nonrelativistic particles emit cyclotron radiation, with a
frequency given by the cyclotron frequency $\omega_{\text{cyc}} = qB/m$, where $q$ is the particle charge, $B$ the magnetic field in which
it moves and $m$ its mass. Relativistic particles emit what we call synchrotron radiation: the frequency spectrum is
far richer, covering a range well away from the cyclotron frequency.

Taking into account strong relativistic effects (such as forward beaming), one finds that the characteristic frequency
of the synchrotron radiation of a single charged particle is $\omega_{\text{syn}} \sim \gamma^2 \omega_{\text{cyc}}$ [103]. The spectrum of the radiation is broad:
$\omega < \omega_{\text{syn}}$ the spectrum falls off like a power law $\propto \omega^{1/3}$, while for $\omega > \omega_{\text{syn}}$, the spectrum falls off exponentially.

This is, however, very different from what we observe in astrophysical systems. The reason is that we need to
consider what happens to the total synchrotron radiation emitted by a population of charged particles, with a given
energy distribution. Let us consider an ensemble of charged particles with number density distribution per unit energy
following a power law $dN/dE \propto E^\alpha$. The power per unit energy emitted by this ensemble of charges reads

$$\frac{dP}{dE} \propto \frac{dN}{dE} P_E \propto E^\alpha E^2 B^2,$$

where $P_E$ is the power emitted by a single particle, given by the Larmor formula $P_E \propto B^2 \gamma^2 \propto B^2 E^2$ (we drop here
all the irrelevant constants given that we are only interested in the final spectral shape of the synchrotron radiation power).
Let us now assume that the single-charge synchrotron spectrum is peaked enough to assume that all the power is emitted at the synchrotron frequency, $\omega_{\text{syn}}$. This implies a one-to-one relation between the observed frequency and the energy of the single particle $\nu \sim \omega_{\text{syn}}/(2\pi) \propto E^2 B$. It then follows

$$\frac{dP}{d\nu} = \frac{dP}{dE} \frac{dE}{d\nu} \propto E^{p+2} B \times \nu^{-1/2} B^{-1/2} \propto B \left( \frac{\nu}{B} \right)^{\frac{1+p}{2}},$$

and therefore the spectral index reads $\alpha = (1+p)/2$. In terms of the spectral index of brightness temperature, $\beta$, this
corresponds to $\beta = (p-3)/2$, since the brightness temperature is related to the intensity via $T_B \propto I_\nu/\nu^2$. Therefore,
the spectral index for synchrotron radiation is intrinsically related to the energy distribution of the charged particles
which generate the radiation. In particular, a power law is expected if the energy distribution is also a power law as
assumed above.

One well-known example for which a power-law distribution is expected comes from particle acceleration by astrophysical shocks [104–108]. For example, if one considers shock waves which propagate in the interstellar medium outside a supernova remnant, it can be shown that at linear level $p = -2$ [104, 105], which implies $\alpha = -0.5$ and $\beta = -2.5$. The spectral index predicted by the simplest scenario of shock acceleration coincides with the spectral index expected in our model, and is therefore slightly different than the power-law best-fit obtained in Ref. [3]. Nevertheless, steeper spectra are possible taking into account non-linear effects in the theory of diffusive shock acceleration [109]. As a matter of fact, these effects are crucial to match observations of the supernova remnant Cassiopeia A, which exhibits a steeper spectral index, $\alpha \simeq -0.77$ or $\beta \simeq -2.77$ [110].

| Config.       | Summary | $A_{\text{GHz}}$       | $\beta$        | $T_0$  | $\Delta \log Z$ |
|---------------|---------|------------------------|---------------|-------|-----------------|
| Fixsen        | Marginals | $0.43^{+0.02}_{-0.04}$ | $-2.58^{+0.02}_{-0.03}$ | 2.7258$^{+0.0003}_{-0.0003}$ | 1.89          |
| Power law     | HPDI    | [0.40, 0.47]           | $[-2.62, -2.55]$ | [2.7250, 2.7266] |                |
| Dowell        | Marginals | $0.54^{+0.01}_{-0.06}$ | $-2.58^{+0.02}_{-0.05}$ | 2.7255$^{+0.0003}_{-0.0003}$ | 0.76          |
| Power law     | HPDI    | [0.47, 0.61]           | $[-2.63, -2.53]$ | [2.7247, 2.7264] |                |

TABLE II. Posterior summaries for the power-law fits. The last column additionally shows the log-Bayes factor $\Delta \log Z$ in favor of the power-law fit, with positive values corresponding to a preference for the power-law model.