Total-derivative supersymmetry breaking

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Abstract

On an interval compactification in supersymmetric theory, boundary conditions for bulk fields must be treated carefully. If they are taken arbitrarily following the requirement that a theory is supersymmetric, the conditions could give redundant constraints on the theory. We construct a supersymmetric action integral on an interval by introducing brane interactions with which total derivative terms under the supersymmetry transformation become zero due to a cancellation. The variational principle leads equations of motion and also boundary conditions for bulk fields, which determine boundary values of bulk fields. By estimating mass spectrum, spontaneous supersymmetry breaking in this simple setup can be realized in a new framework. This supersymmetry breaking does not induce massless R-axion, which is favorable for phenomenology. It is worth noting that fermions in hyper-multiplet, gauge bosons, and the fifth-dimensional component of gauge bosons can have zero-modes (while the other components are all massive as Kaluza-Klein modes), which fits the gauge-Higgs unification scenarios.
1 Introduction

Supersymmetry has been motivated from various viewpoints including the gauge hierarchy problem and the gauge coupling unification as well as string theory. Combining it with higher-dimensional theories also attract a lot of attention from not only string theory but also phenomenologies. It is well known that a supersymmetric Lagrangian is invariant under the supersymmetry transformation up to total derivative terms. In four dimensions, the action is supersymmetrically invariant as fields fall to zero at spatial infinity.

In higher-dimensional theories with compactified extra-dimensions, such total derivative terms need to vanish for finite spaces with respect to extra-dimensions. Suppose that a total derivative term for a five-dimensional example is denoted as $\partial_y \Delta(x, y)$. Integrating it out over $y$ ($y_i \leq y \leq y_f$) leads to $[\Delta(x, y_f) - \Delta(x, y_i)]$. In a usual orbifold setup, bulk fields have their parity eigenvalues associated with orbifolding, which derive $\Delta(x, y_f) = \Delta(x, y_i) = 0$, and the action integral is supersymmetric. So, how about an interval compactification? An interval compactification has been intriguing for phenomenological model building, such as Higgsless models [1], gauge-Higgs unification models [2], and so on, since it has more varieties of boundary values for bulk fields. In models on intervals, it also seems to become supersymmetric if bulk fields are assigned to have the same boundary conditions as the orbifolding. In constructing the action integral on intervals, however, there are no inevitable conditions to fix boundary conditions for bulk fields unlike an orbifold compactification. If $y$-dependence of fields is specified beforehand so as to make an action integral supersymmetric, it corresponds to a setup of a constrained system. This constraint can be redundant in an interval compactification, while in an orbifold compactification it is the very orbifold condition that constrains the system. The assignment of boundary conditions for bulk fields should be treated carefully. The $y$-dependent profile for fields need to satisfy equations of motion and boundary conditions, that are derived from the variational principle for a supersymmetrically-invariant action integral. When the values of $\Delta(x, y_f)$ and $\Delta(x, y_i)$ are arbitrary in an interval compactification, it is nontrivial whether supersymmetry is preserved or not. In order that total derivative terms vanish in an interval compactification, another valid way may be to employ a cancellation. Since boundary contributions which cancel the variation $[\Delta(x, y_f) - \Delta(x, y_i)]$ make the whole theory supersymmetric, the boundary action integral, whose variation is $[-\Delta(x, y_f) - \Delta(x, y_i)]$, would cancel the total derivative terms.

In this paper, we construct a supersymmetric action integral on an interval by introducing brane interactions with which total derivative terms under the supersymmetry transformation become zero due to a cancellation. The variational principle leads equations of motion and also boundary conditions of bulk fields, which determine boundary values of bulk fields. By estimating mass spectrum, spontaneous supersymmetry breaking can be realized in this simple setup. This supersymmetry breaking does not induce massless R-axion, which is favorable for phenomenology. It is worth noting that fermions in hyper-multiplet, gauge bosons, and the fifth dimensional component of gauge bosons can have zero-modes (while the other components are all massive as Kaluza-Klein (KK) modes), which fits the gauge-Higgs unification scenarios.

Our point is that the action integral with boundary terms is invariant under supersymmetry transformation and that supersymmetry breaking occurs without twist for bound-

* Also in a grand unified theory, a five-dimensional interval compactification can realize a direct reduction of SO(10) to the standard model gauge group, while an orbifold compactification induces additional U(1) [3].
ary conditions. Such a supersymmetry breaking without twist is supported by the results in models given in Ref. [4], where the action integral is constructed with introduction of the indices SU(2)$_H$ as well as SU(2)$_R$. Treatment of symmetric models with these indices has been developed in Ref. [5]. By comparison, we will construct a new supersymmetric action integral with boundary terms without introducing the index SU(2)$_H$. Our framework has three critical advantages. Firstly, its consistency with earlier works is clear. Our boundary terms are composed of the hyper-multiplet scalar field and auxiliary field. The form is expected from four-dimensional couplings in Ref. [6]. In the formulation with the index SU(2)$_H$, the fermion seems to have boundary terms [4]. To check consistency with the case without employing SU(2)$_H$, several steps might be needed.$^\dagger$ The second advantage is simpleness of phenomenological application. A hyper-multiplet in our framework is described by an SU(2)$_R$ complex scalar, a Dirac fermion and an SU(2)$_R$ complex auxiliary field. Because our fermion is a usual Dirac fermion, a simple introduction of quarks and leptons is possible directly in a realistic application. Thirdly there is a clear indication for theoretical research. In our supersymmetric formulation, boundary terms for a hyper-multiplet are formed by only SU(2)$_R$-charged fields. This provides a transparent framework for developing further research on relation between R symmetry and supersymmetry. With these points and the new action integral, we can derive the observations also including the subjects associated with R-axion and the gauge-Higgs unification scenarios.

The paper is organized as follows. In Sec. 2, we construct a supersymmetric action integral with a hyper-multiplet. In Sec. 3, equations of motion and boundary conditions are derived based on the variational principle, and solutions for the equations are represented. We show that it gives rise to spontaneous supersymmetry breaking in a new framework. We conclude in Sec. 4 with some remarks.

## 2 Supersymmetric action

In five dimensions, the number of minimal supercharges is eight. Associated with the supersymmetry, R-symmetry is SU(2)$_R$ in the bulk. Simplectic Majorana fermions satisfy $\psi^i = \epsilon^{ij}C \psi_j^T$, whose component fields are written as\textsuperscript{4}

$$
\psi^1 = \left( \begin{array}{c} (\psi_L)_\alpha \\ (\bar{\psi}_R)^{\dot{\alpha}} \end{array} \right), \quad \psi^2 = \left( \begin{array}{c} (\psi_R)_\alpha \\ -(\bar{\psi}_L)^{\dot{\alpha}} \end{array} \right).
$$

(2.1)

Here $\epsilon^{12} = \epsilon_{21} = 1$, $C_{21} = -C_{12} = C_{34} = -C_{43} = 1$ and $C_{AB} = 0$ for the other matrix elements. The gamma matrices are given by

$$
\gamma^M = \left( \begin{array}{cc} 0 & \sigma^m \\ \sigma^m & 0 \end{array} \right), \quad \left( \begin{array}{cc} -i & 0 \\ 0 & i \end{array} \right),
$$

(2.2)

where $\{\gamma^M, \gamma^N\} = -2\eta^{MN}$. Here the capital letters $M$ run over 0, 1, 2, 3, 5 and $\sigma^m = (1, \bar{\sigma})$, $\sigma^m = (1, -\bar{\sigma})$. The five-dimensional spacetime is flat, and the extra-dimensional coordinate is denoted also as $y$. The fundamental region is taken as $0 \leq y \leq L$.

$^\dagger$In Ref. [4], standard orbifold boundary conditions are obtained as $H^\pm(x, y_b + y) = \pm s_b t_b H^\pm(x, y_b - y)$ and $\psi_{L,R}(x, y_b + y) = \mp s_b \psi_{L,R}(x, y_b - y)$ with $b = i, f$. Here the twist parameters for SU(2)$_H$ and SU(2)$_R$ are denoted as $s_b$ and $t_b$, respectively. When the SU(2)$_H$ twist is trivial $s_b = \pm 1$, fermions have zero mode and scalar masses depend on an SU(2)$_R$ twist which are consistent with our result.

$^\ddagger$ We follow the notation in Ref. [7].
For a hyper-multiplet, bulk fields are composed of an SU(2)$_R$ complex scalar $H^i$, a Dirac fermion $\psi$ and an SU(2)$_R$ complex auxiliary field $F_i$. The kinetic Lagrangian for bulk fields are given by

$$\mathcal{L} = -\partial_M H^i \cdot \partial^M H^i - \frac{i}{2} \left( \bar{\psi} \gamma^M \partial_M \psi - \partial_M \bar{\psi} \cdot \gamma^M \psi \right) + F_{i\bar{i}} F_{i\bar{i}}.$$  \hspace{1cm} (2.3)

In order that the Lagrangian is invariant under supersymmetry in the bulk, supersymmetry transformation is given by

$$\delta_\xi H^i = -\sqrt{2} \epsilon^{ij} \xi_j \psi, \hspace{1cm} (2.4)$$
$$\delta_\xi \psi = i \sqrt{2} \gamma^M \partial_M H^i \cdot \epsilon_{ij} \xi^j + \sqrt{2} F_i \xi^i, \hspace{1cm} (2.5)$$
$$\delta_\xi F_{i\bar{i}} = i \sqrt{2} \xi^i \gamma^M \partial_M \psi. \hspace{1cm} (2.6)$$

Here the transformation parameter $\xi^i$ is constant. The conjugate transformation is given by

$$\delta_\xi H^i \dagger = \sqrt{2} \epsilon_{ij} \bar{\psi} \xi^j, \hspace{1cm} (2.7)$$
$$\delta_\xi \bar{\psi} = i \sqrt{2} \partial_M H^i \cdot \epsilon^{ij} \xi^j \gamma^M + \sqrt{2} F_{i\bar{i}} \xi^i, \hspace{1cm} (2.8)$$
$$\delta_\xi F_{i\bar{i}} = -i \sqrt{2} \partial_M \bar{\psi} \cdot \gamma^M \xi^i. \hspace{1cm} (2.9)$$

The supersymmetry transformation of the Lagrangian in Eq. (2.6) is

$$\delta_\xi \mathcal{L} = -\partial_M \left( \sqrt{2} \epsilon_{ij} \bar{\psi} \left( \eta^{mN} + \frac{1}{2} \gamma^m \gamma^N \right) \xi^j \partial_N H^i + \frac{i \sqrt{2}}{2} \bar{\psi} \gamma^m \xi^i F_i \right) + \text{H.c.}, \hspace{1cm} (2.10)$$

which is decomposed in the four-dimensional derivative part and the extra-dimensional derivative part as $\delta_\xi \mathcal{L} = \partial_m \Delta^m + \partial_y \Delta_y$ with

$$\Delta^m(x, y) = - \left( \sqrt{2} \epsilon_{ij} \bar{\psi} \left( \eta^{mn} + \frac{1}{2} \gamma^m \gamma^n \right) \xi^j \partial_n H^i + \frac{i \sqrt{2}}{2} \bar{\psi} \gamma^m \xi^i F_i \right) + \text{H.c.}, \hspace{1cm} (2.11)$$
$$\Delta_y(x, y) = - \frac{1}{\sqrt{2}} \left( \bar{\psi} \left( \epsilon_{ij} \partial_y H^i + i \gamma^5 F_j \right) \xi^j + \epsilon_{ij} \bar{\psi} \left( \gamma^5 \gamma^m \partial_m H^i \right) \xi^j \right) + \text{H.c.}. \hspace{1cm} (2.12)$$

If extra-dimensional space were infinitely extended and all the fields fell to zero at spatial infinity, the action integral is supersymmetrically invariant. However, it is quite non-trivial when the extra-dimensional space is compactified in a finite space, where the action integral is transformed into

$$\int_0^L dy \ \delta_\xi \mathcal{L} = \Delta_y(x, L) - \Delta_y(x, 0). \hspace{1cm} (2.12)$$

Thus, it should be checked whether the total derivative terms with respect to $y$ vanish in the finite space setups.

In an orbifold compactification, taking $S^1/\mathbb{Z}_2$ for example, the spatial points of $y = 0$ and $y = L$ are fixed points with respect to the identification $y \sim -y$ and $L + y \sim L - y$, respectively. Under the orbifold parities $P_0$ and $P_1$ at $y = 0$ and $y = L$, bulk fields are expected to have their parity eigenvalues in a usual setup. For example, let us take orbifold parities as $P_0 = P_1 = +1$ for $(\xi^i_L, H^1, \psi_L, F_1)$ and $P_0 = P_1 = -1$ for $(\xi^i_R, H^2, \psi_R, F_2)$,
Then, from Eq. (2.11), the odd parity of \((\xi^L, H^2, \psi_R, F_2)\) = 0 and the even parity of \(\partial_y H^1\) = 0 realize \(\Delta_y = 0\). So the action integral is invariant under the supersymmetric transformation. Here we do not say orbifolding setup always has a supersymmetric action integral. In actual, a setup of Ref. [8] is not the case, where twists at boundaries can make a situation complicated. Anyhow, we should remind again that a boundary condition of \(\partial_y H^1\) = 0 for a bulk field which has a finite boundary value is needed to make a supersymmetric action integral in the above case. Then, how about the situation in an interval (where this condition is not obvious)?

In an interval compactification, there are more boundary conditions which can be taken based on the variational principle [9, 10]. Notice that the \(y\)-dependence profiles of bulk fields must be solutions to equations of motion with the boundary conditions. Since the equations of motion are derived from an action integral with supersymmetry, it is natural that the action integral is constructed without fixing boundary values of fields as initial conditions. Namely, instead of fixing boundary values of fields, we should construct a supersymmetric action integral at first, and obtain bulk mode equations of motion, then determine boundary values of bulk fields.

For the supersymmetric action integral, the simplest case is to take a vanishing net variation for each boundary, which realizes a supersymmetric action integral as shown in Eq. (2.12)\(^5\). Here we focus on a hyper-multiplet, while a construction of a supersymmetric action integral of a vector multiplet was discussed in Ref. [8]. Let us construct a supersymmetric action integral with a vanishing net variation for each boundary in a general interval setup. From the dimensional counting, candidates of Lagrangian terms on boundaries for the cancellation are

\[
\partial_y H^i \cdot H^i, \quad H^i \partial_y H^i, \quad \bar{\psi} \psi, \quad \bar{\psi} \gamma^5 \psi, \quad H^i F_i, \quad H^i \gamma^i F_i,
\]

and there are no other terms. We can show that their supersymmetry transformations and \(\Delta_y\) need a combination among Eq. (2.13) in order to cancel between supersymmetric transformations of bulk and boundary terms as

\[
\Delta_y \left[ \frac{1}{2} A \delta \xi (H^i \partial_y H^i) + \frac{1}{2} B \delta \xi (\epsilon^{ij} H^i F_j) + \text{H.c.} \right], \tag{2.14}
\]

where \(A\) and \(B\) are numbers. At \(y = L\), Eq. (2.14) is explicitly written as

\[
-\frac{1}{\sqrt{2}} \bar{\psi} \left[ (1 - A) \epsilon_{ij} \partial_y H^i + (i \gamma^5 + B) F_j \right] \xi^j
\]

\[
+ \frac{1}{\sqrt{2}} \epsilon_{ij} \partial_m \bar{\psi} \cdot (\gamma^5 + iB) \gamma^m H^i \xi^j + \frac{1}{\sqrt{2}} \epsilon_{ij} \partial_y \bar{\psi} \cdot (A + iB \gamma^5) H^i \xi^j. \tag{2.15}
\]

Note that brane interactions are only for scalar and auxiliary (SU(2)\(_R\) non-singlet) fields, and there are no interactions for fermion (SU(2)\(_R\) singlet) fields. At the other boundary \(y = 0\), similar terms are found in a parallel way. In order to vanish Eq. (2.15), \(A\) must satisfy \(A = 1\), and \(B\) must satisfy

\[
(i \gamma^5 + B) \xi^j = 0. \tag{2.16}
\]

\(^5\) Of course there is a more complicated possibility of \(\Delta_y (x, L) - \Delta_y (x, 0) = 0\) with \(\Delta_y (x, L) \neq 0\) and \(\Delta_y (x, 0) \neq 0\), which will be done in a future work.
Since the value of $\gamma^5$ depends on the chirality of $\xi^j$ and $B$ is just a number, there is no solution of $B$ for general $\xi^j$. Therefore, a whole four-dimensional $\mathcal{N} = 2$ supersymmetry cannot be preserved on boundaries, which is a well-known result as an origin of a four-dimensional chiral theory. Taking an eigenvalue of $\gamma^5$ for $\xi^i$, $B$ has a solution, and then a four-dimensional $\mathcal{N} = 1$ supersymmetric theory is obtained. Namely, if $\xi^i_L = 0$ and $\xi^i_R \neq 0$ at $y = L$, $-i\gamma^5\xi^i = \xi^i$ (for the notation $-i\gamma^5\xi^i = \xi^i_R$), which means that $B$ must satisfy $(-1 + B) = 0$, and $\mathcal{N} = 1$ supersymmetry is preserved with $B = 1$. If $\xi^i_R = 0$ and $\xi^i_L \neq 0$ at $y = L$, $-i\gamma^5\xi^i = -\xi^i$, which means $(1 + B) = 0$ and the $B = -1$ solution preserves $\mathcal{N} = 1$ supersymmetry. Hence, the supersymmetric action is given as

$$S = \int d^4x dy \left( -\partial_M H^i \cdot \partial^M H^i - \frac{i}{2} (\bar{\psi} \gamma^M \partial_M \psi - \partial_M \bar{\psi} \cdot \gamma^M \psi) + F^{ij} F_{ij} + \left[ \frac{1}{2} H^i_1 (\partial_y H^i + B_1 \epsilon^{ij} F_j) \delta(y - L) - \frac{1}{2} H^i_1 (\partial_y H^i + B_0 \epsilon^{ij} F_j) \delta(y + H.c.) \right] \right) \tag{2.17}$$

Here $B_s (s = 0, 1)$ should satisfy $(i\gamma^5 + B_s)\xi^j = 0$, which means $\mathcal{N} = 1$ supersymmetry is preserved as $\xi^j_R \neq 0$ and $\xi^j_L = 0 (B_s = 1)$ or $\xi^j_L \neq 0$ and $\xi^j_R = 0 (B_s = -1)$. It is worth noting that the added boundary action in the action integral Eq. (2.17) is only formed by the fields with charges of $\text{SU}(2)_R$.

3 Solving equations of motion

We have found the bulk and boundary action integrals invariant under supersymmetry in the previous section. Since we do not know the boundary values for bulk fields on intervals, it is natural to construct a supersymmetric action integral at first, and determine boundary values of bulk fields by the variational principle instead of fixing the values as the initial condition.\footnote{Fixing boundary values as the initial condition should correspond to fixing boundary conditions of the model on interval.} This is our standing point in this paper. Let us introduce solutions for equations of motion and boundary conditions from the variational principle, and estimate boundary values of bulk fields.

In the action integral Eq. (2.17), the direction of supersymmetry transformation can differ at each boundary depending on the values of $B_s$. A possibility to keep global $\mathcal{N} = 1$ supersymmetry would be to take the same direction $B_0 = B_1$. For example, we choose $\xi^j_L \neq 0$ and $\xi^j_R = 0$ at $y = 0, L$ where $B_0 = B_1 = -1$. Then the action integral Eq. (2.17) becomes

$$S = \int d^4x dy \left( -\partial_M H^i \cdot \partial^M H^i - \frac{i}{2} (\bar{\psi} \gamma^M \partial_M \psi - \partial_M \bar{\psi} \cdot \gamma^M \psi) + F^{ij} F_{ij} + \frac{1}{2} H^i_1 (\partial_y H^i - \epsilon^{ij} F_j) + H.c. \right), \tag{3.1}$$

where the boundary action is denoted as a total derivative term. The Dirac fermion in hyper-multiplet is $\text{SU}(2)_R$ singlet and does not have boundary interactions as shown in Eq. (3.1), so that the existence of zero-mode in the above boundary conditions is obvious. On the other hand, for the $\text{SU}(2)_R$ non-singlet fields, whether they have zero-modes or not is nontrivial since they have their boundary action integral. Under the variations of
SU(2)\(_R\) non-singlet fields as \(H^i \to H^i + \delta H^i\) and \(F_i \to F_i + \delta F_i\), the variation of the action integral is given by

\[
\delta S = \frac{1}{2} \int d^4 x dy \left( 2 \delta H^i \partial_M \partial^M H^i + 2 \partial^M \partial_M H^i \cdot \delta H^i + 2 \delta F^{ij} \cdot F_i + 2 F^{ij} \delta F_i \right) + \left[ \delta H^i \left( - \epsilon^{ij} F_j \right) + H^i \left( - \epsilon^{ij} \delta F_j \right) \right] \delta(y - L) + \partial_y (H^i \delta H^i (y - L)) - 2 \partial_y H^i \cdot \delta H^i \delta(y - L) - H^i \delta H^i \partial_y \delta(y - L) + \partial_y (\delta H^i \cdot H^i \delta(y - L)) - \delta H^i \cdot \partial_y H^i \delta(y - L) - \delta H^i \cdot H^i \partial_y \delta(y - L) + \left( - \epsilon_{ij} \delta F^{ij} \right) H^j \delta(y - L) + (\partial_y H^i - \epsilon_{ij} F^{ij}) \delta H^i \delta(y - L) + (\delta(y) \text{ terms}) \right].
\]

(3.2)

From this equation, \(\delta H^i\) terms mean

\[
2 \partial_{\delta M} \partial^M H^i - \epsilon^{ij} F_j \delta(y - L) + \epsilon^{ij} F_j \delta(y) + H^i \delta(0) \delta(y - L) - \partial_y H^i \cdot \delta(y - L) - H^i \partial_y \delta(y - L) + H^i \delta(0) \delta(y) + \partial_y H^i \cdot \delta(y) + H^i \partial_y \delta(y) = 0,
\]

(3.3)

and \(\delta F^{ij}\) terms mean \(2 F_i + \epsilon_{ij} H^j \delta(y - L) - \epsilon_{ij} H^j \delta(y) = 0\). Combining these equations induces three boundary conditions,

\[
2 F_i + \epsilon_{ij} H^j \delta(y - L) - \epsilon_{ij} H^j \delta(y) = 0,
\]

(3.4)

\[
\left[ 4 \partial_y H^i + \frac{3}{2} H^i \delta(0) \right]_{y=0} = 0,
\]

(3.5)

and a bulk equation of motion,

\[
\partial_M \partial^M H^i = 0.
\]

(3.6)

Here the behavior of fields near boundaries are treated as \(\partial_y H^i \big|_{y=0} = \lim_{\epsilon \to 0} \partial_y H^i \big|_{y=\epsilon}\) and \(\partial_y H^i \big|_{y=L} = \lim_{\epsilon \to 0} \partial_y H^i \big|_{y=L-\epsilon}\). With the mode expansion of \(H^i(x, y) = \sum_n \phi^i_n(x) H^i_n(y)\), the bulk mode equation means \(\partial_y^2 H^i_n = -m_n^2 H^i_n\).

Now we solve the equation of motion Eq. (3.6) under the boundary conditions Eqs. (3.4) and (3.5). From the bulk equation, a general solution apart from boundaries is

\[
H^i_n(y) = \sin(m_n y + \alpha),
\]

(3.7)

up to the normalization. Here \(m_n\) and \(\alpha\) are constants determined by boundary conditions in the following. Dependences on \(i = 1, 2\) are omitted when no confusion arises. At \(y = 0\), substituting Eq. (3.7) into the first equation in Eq. (3.5) gives

\[
4 m_n \cos \alpha + \frac{3}{2} \sin \alpha \cdot \delta(0) = 0.
\]

(3.8)

If \(\sin \alpha = 0\), this equation means \(m_n = 0\), so that trivially \(H^i(x, y) = 0\). Non-vanishing \(H^i\) requires \(\sin \alpha \neq 0\), which means \(\delta(0) = - (8/3) \cot \alpha \cdot m_n\). At \(y = L\), the second equation in Eq. (3.5) with the above equation leads to

\[
\frac{\tan m_n L}{m_n L} \approx \frac{16}{3} \frac{1}{L \delta(0)}.
\]

(3.9)
Since $\delta(0) \gg 1/L$, the mass eigenvalue is obtained as

$$m_n = \frac{n\pi}{L} \left( 1 + \frac{16}{3L\delta(0)} \right) \approx \frac{n\pi}{L}. \quad (3.10)$$

Thus we find the solution

$$H^i(x, y) = \sum_{n=1}^{\infty} \phi^i_n(x) \sin \left( m_n \left[ y - \frac{8}{3\delta(0)} \right] \right) \approx \sum_{n=1}^{\infty} \phi^i_n(x) \sin(m_n y), \quad (3.11)$$

which means the field $H^i(x, y)$ does not have zero-mode as $m_n \neq 0$. The absence of zero-mode is seen directly from $\delta(0)$-term in Eq. (3.5) and the bulk equation. The zero-mode of the bulk equation means a constant solution, and a constant $H^i$ does not fulfill Eq. (3.5) due to nonzero $\delta(0)$-terms which correspond to brane localized interactions of making supersymmetric action. The mass splitting between $\psi$ and $H^i$ is $\pi/L$, which is characterized by the dimensional quantity $L$. (Here, $L$ is only one dimensionful parameter in this model.)

We emphasize that the equations of motion with boundary conditions remove zero-mode for hyper-multiplet scalars. This means that the action integral is supersymmetric but the vacuum is not, that is, supersymmetry is \textit{spontaneously} broken. This mechanism of supersymmetry breaking has been realized in a very simple formulation. Relations between R-symmetry and supersymmetry breaking have been generally discussed [11]. Usually, in four dimensions, R-symmetry is needed for the spontaneous supersymmetry breaking, but there appears massless R-axion which induces phenomenological difficulties. How about our setup in five dimensions? In our present context with R-symmetry which corresponds to $\mathcal{N} = 1$ supersymmetry of $B_0 = B_1$, this supersymmetry is \textit{spontaneously} broken. Here massless R-axion is absent, since there is no scalar source of R-axion in our setup. It does not contradict the potential arguments of spontaneous supersymmetry breaking in four dimensions [11]. Notice also that gauginos have Dirac-type KK mass terms although there is R-symmetry. It is because KK mass is not a (simplectic) Majorana mass, so that this situation do not contradict with the arguments in Ref.[11]. Thus, gauginos can be massive without massless R-axion in this setup. This mechanism sheds light on to phenomenology. In a short summary, the boundary interactions, which are introduced to construct the supersymmetric action integral in a five-dimensional theory, realize the total derivative terms in the Lagrangian, and $\mathcal{N} = 2$ supersymmetry is completely broken to $\mathcal{N} = 0$ through the equations of motion.

In vector multiplets, gauginos and auxiliary fields are SU(2)$_R$ non-singlet fields. Gauginos, auxiliary fields, and real scalar which is not the extra-dimensional component of gauge bosons, have boundary terms for the supersymmetric action. Then, zero-modes exist only in gauge bosons and the fifth dimensional component of gauge bosons, if exist. This situation can be read from Ref. [8], where Scherk-Schwarz twist [12] and boundary actions are taken into account. It can be shown that a nonzero twist with a specific boundary condition (not taking the same values of $B_0$ and $B_1$ as above) under a certain setup (Scherk-Schwarz twist), SU(2)$_R$ non-singlet fields can also have zero-mode. This means $\mathcal{N} = 1$ supersymmetry is preserved after the compactification. Although our setup induces supersymmetry breaking, it is consistent with Ref.[8]. It is because we do not take the twists at the boundary conditions. As for the vacuum energy, our setup (\textit{spontaneous} supersymmetry breaking vacuum) might have higher magnitude than the setup of Ref.[8] (supersymmetry preserving vacuum). However, a dynamical mechanism
of compactification is still a mystery so that both setups are worth analyzing carefully. Anyhow, our setup is simple and supersymmetry breaking occurs without such a source as Scherk-Schwarz twist. And, hyper-multiplets and vector multiplets have zero-mode for hyper-multiplet fermions and five-dimensional gauge bosons. Since scalars of hyper-multiplets become heavy (nonzero-mode), the standard model Higgs field can not be regarded as a field in hyper-multiplets. However, things go well if the Higgs field can be identified as a part of the extra-dimensional component of five-dimensional gauge bosons, because it has zero-modes as mentioned above. This is a so-called gauge-Higgs unification model, and the gauge hierarchy problem for quadratic divergence for Higgs boson mass is solved not by supersymmetry but by the gauge-Higgs unification. Therefore, our spontaneous supersymmetry breaking gives a compelling way to extract just viable fields in the non-supersymmetric gauge-Higgs unification from a supersymmetric setup.

4 Summary and discussions

We have constructed a five-dimensional supersymmetric action integral where total derivative terms play a role of canceling the supersymmetry transformation of a boundary action integral. After deriving equations of motion and boundary conditions, we have found solutions for these equations. The solutions remove zero-mode for hyper-multiplet scalars. This means that the action is supersymmetric but the vacuum is not, that is, supersymmetry is spontaneously broken. This mechanism of supersymmetry breaking has been realized in a new formulation.

We have shown that zero-mode exists only for fermions in hyper-multiplets. Combining our result with the case of vector multiplets in Ref. [8], hyper-multiplets in the gauge theory have zero-modes only in fermions of the hyper-multiplets and bosons (four-dimensional gauge bosons and scalars corresponding to the fifth-dimensional component of gauge bosons). The mass of the Higgs boson depends on the compactification scale and a Wilson-line phase.

For a model building, it would be also possible to change the setup to produce a smaller mass splitting than that of the order of $1/L$. This may occur if additional twist contributions (suitable twists between $B_0$ and $B_1$, and Scherk-Schwarz) are included. In other words, supersymmetry may be broken at lower scales. A mixing of supersymmetry breaking from boundary action integrals and Scherk-Schwarz twists can reduce a magnitude of mass splitting. It has been shown that such a mixing can relax the lower bound to the lightest Higgs boson mass in a supersymmetric orbifold model [13], even if $1/L$ is of the orders of magnitude larger than $O(1)$ TeV.

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