DLCQ of M-theory as the light-like limit

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ABSTRACT

We investigate whether DLCQ of M-theory can be defined as a limit of M-theory compactified on an almost light-like circle. This is of particular interest since the proofs of the matrix description of M-theory by Seiberg and Sen rely on this assumption. By the standard relation between M-theory on $S^1$ and IIA string theory, we translate this question into the corresponding one about the existence of the light-like limit of IIA superstring theory for any string coupling $g_s$.

We argue that perturbative string loop amplitudes should have a finite and well-defined light-like limit provided the external momenta are chosen to correspond to a well-defined DLCQ set-up. On the non-perturbative side we consider states and amplitudes. We show that an appropriate class of non-perturbative states (wrapped D-branes) precisely have the right light-like limit. We give some indications that non-perturbative corrections to string amplitudes, too, may behave as required in the light-like limit. Having perturbative and non-perturbative evidence, this suggests that type IIA superstring theory as a whole has a well-defined light-like limit (for any string coupling $g_s$) and hence that the same is true for M-theory.

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1. Introduction and summary

The notion of eleven-dimensional M-theory emerged [1] as the strong-coupling limit of IIA superstring theory. More precisely, M-theory compactified on a (space-like) circle of radius $R_{11}$ is identical to IIA superstring theory with coupling $g_s = R_{11}/\sqrt{\alpha'}$. While this sounds like a mere definition, the non-trivial conjecture is that in the $g_s \to \infty$ limit, the resulting theory has eleven-dimensional Lorenz invariance. Once one accepts this conjecture, a statement about M-theory can be translated into a statement about IIA superstring theory at generic coupling. More precisely, any question one might ask in M-theory compactified on a manifold $S^1 \times K$ can be translated into, and in principle be answered within IIA theory on $K$, provided we keep the string coupling $g_s$ \emph{generic} and do not restrict ourselves to perturbative physics (in $g_s$) only. By taking the appropriate $g_s \to \infty$ limit, IIA on $K$ of course corresponds to M on $K$.

The question we want to ask here is the following: Does M-theory compactified on a space-like circle of radius $R_s$ has a well-defined limit as $R_s \to 0$. This is of quite some interest since it is the basic assumption in Seiberg’s [2] and Sen’s [3] proof that the DLCQ of M-theory is given by the finite $N$ matrix model (for a pedagogical review, see [4]). Let me recall that Susskind conjectured [5] that the quantization of M-theory compactified on a light-like circle of radius $R$ (discrete light-cone quantization = DLCQ) in a sector of fixed $p_- = N/R$ is given by a U($N$) matrix quantum mechanics as obtained by reduction from ten-dimensional super Yang-Mills. The advantage of this DLCQ of M-theory with respect to the infinite momentum frame of Banks et al [6] is that various dualities are already manifest at finite $N$. Seiberg and Sen consider this DLCQ of M-theory as the limit of a compactification on an almost light-like circle, which in turn is Lorenz equivalent to compactification on a space-like circle of radius $R_s$. Since $2\pi R_s$ is the proper (Lorenz invariant) length of all the circles, the light-like limit is recovered for $R_s \to 0$. Under the assumption that this $R_s \to 0$ limit of M-theory makes sense, Seiberg and Sen argued, using various dualities, that the DLCQ of M-theory is indeed equivalent to IIA theory in a particular limit where $g_s \to 0$ and the string mass diverges, so that all that is left in a sector of non-zero D0 brane charge $\sim N$ is the corresponding U($N$) matrix model describing the dynamics of the open string ground states.

The key question remaining open is whether this $R_s \to 0$ limit exists. In case it does, it should actually define what is meant by DLCQ of M-theory. To elucidate this issue, Hellerman and Polchinski [7] considered quantum field theories compactified on a circle, in the $R_s \to 0$
limit. They found that, typically, one-loop diagrams with vanishing $p_-$ transfer are plagued with divergences $\sim \frac{1}{R_s}$, thus casting serious doubt on the existence of the required limit in M-theory. However, M-theory is certainly very different from ordinary quantum field theory, in particular due to the existence of extended objects: membranes and five-branes. In this respect it much more resembles string theory where the existence of the winding modes of the string plays a crucial role. The present author has investigated this $R_s \to 0$ limit for a four-point one-loop amplitude in type II superstring theory [8] and found that this limit is perfectly well-defined, even for vanishing $p_-$ transfer (the potentially troublesome case). It was also shown [8] that this limit reproduces the result of a direct DLCQ computation of the string amplitude.

While the point of view in [8] was to consider superstring theory as a possible analogy with M-theory, here we take a different attitude. We consider M-theory on a circle $S^1$ of radius $R_{11}$ as IIA theory with $g_s = R_{11}/\sqrt{\alpha'}$. The statement we would like to prove is:

**Statement A**: “The IIA superstring theory with coupling $g_s$, compactified on a space-like circle $S^1$ of radius $R_s$ has a well-defined limit as $R_s \to 0$.”

As discussed above, this then is equivalent to the

**Statement B**: “M-theory compactified on (space-like) $S^1 \times S^1$ with radii $R_{11} = \sqrt{\alpha'} g_s$ and $R_s$ has a well-defined limit as $R_s \to 0$.”

If statement A can be proved (as uniform convergence) for any string coupling $g_s$, then we have shown the

**Statement C**: “M-theory compactified on a space-like circle of radius $R_s$ has a well-defined limit as $R_s \to 0$.”

This then would fill the gap in Seiberg’s and Sen’s proofs.

Although a complete and rigorous proof of statement A for any $g_s$ certainly is beyond reach, we nevertheless now have much perturbative and non-perturbative knowledge about the IIA theory at our disposal in order to check this statement to quite some extent. This is the purpose of the present note.

On the perturbative side one has to consider arbitrary $N$-point genus-$g$ amplitudes in type II superstring theory compactified on a circle of radius $R_s$ and check that the $R_s \to 0$ limit is
well-behaved. As explained in [8], one has to consider amplitudes for external states with non-vanishing momenta in the compact direction (see [9] for a discussion of non-vanishing winding numbers also). In [8] the simplest case, \( N = 4, g = 1 \), was studied in detail. Already there it was clear that the same argument should apply to any \( N \)-point genus-1 amplitude. Here we will argue that similarly the \( R_s \to 0 \) limit should also exist for any genus-\( g \) amplitude.

On the non-perturbative side, the first think to do is to simply look at the spectrum of BPS states. We will argue that precisely those D-brane configurations that had a finite mass before compactifying the space-like direction do scale appropriately to survive and make sense in the light-like \( (R_s \to 0) \) limit. More ambitiously, one can look at non-perturbative corrections to \( N \)-point scattering amplitudes. Some important information can already be extracted from the D-instanton corrections to the \( R^4 \) coupling of the low-energy effective action [10]. We will see that these corrections do depend on \( R_s \) in exactly the way needed so that a well-defined \( R_s \to 0 \) limit of the full amplitude might exist!

Thus although not a proof, I believe that the perturbative and non-perturbative evidence presented in this note is rather encouragingly pointing towards the existence of a well-defined light-like limit \( (R_s \to 0) \) of M-theory.

2. Kinematics

Let me begin by briefly describing the limit we are interested in. We want to study type II superstring theory compactified on a space-like circle of radius \( R_s \) in the limit \( R_s \to 0 \). This is Lorenz equivalent to a compactification on an almost light-like circle, with \( R_s \to 0 \) corresponding to the light-like limit [2,7,8]. We will take the space-like compactified direction to be \( x^9 (R_s \equiv R_9) \), so that the corresponding momenta \( p_9 \) are quantized:

\[
x^9 \simeq x^9 + 2\pi R_s \quad , \quad p_9 = \frac{n}{R_s} .
\]  

Since we are interested in the \( R_s \to 0 \) limit it is convenient to write \( R_s = \epsilon R_0 \) where \( R_0 \) is kept fixed. Using a boost with parameter \( \beta = (1 - \epsilon^2/2)/(1 + \epsilon^2/2) \) we get a Lorenz equivalent coordinate system (denoted \( \tilde{x}^\mu \) with \( \tilde{x}^\pm = (\tilde{x}^0 \pm \tilde{x}^9)/\sqrt{2} \)) where

\[
\tilde{x}^+ \simeq \tilde{x}^+ + \pi \epsilon^2 R_0 \quad , \quad \tilde{x}^- \simeq \tilde{x}^- - 2\pi R_0 .
\]

For \( \epsilon \to 0 \) this gives a light-like compactification. Let’s make this more precise. Introduce yet
another coordinate system by \( \hat{x}^- = \tilde{x}^- \), \( \hat{t} = \tilde{x}^+ + \epsilon^2 \tilde{x}^- / 2 \). Then \( \hat{t} \) is a standard non-compact coordinate, while \( \tilde{x}^- \) still has period \( 2\pi R_0 \). The metric is \( ds^2 = -d\hat{t} d\hat{x}^- + \epsilon^2 (d\tilde{x}^-)^2 \) so that the light-like limit is indeed \( \epsilon \to 0 \). The momentum \( \hat{p}_- \) is quantized as \( \hat{p}_- = \frac{n}{R_0} \) while \( \hat{p}_t \) takes continuous values. The relation between the momenta in the different frames is easily seen to be

\[
\hat{p}_+ = \hat{p}_t , \quad \hat{p}_- = \hat{p}_- + \frac{\epsilon^2}{2} \hat{p}_t ,
\]

\[
p_9 = \frac{1}{\epsilon} \hat{p}_- , \quad p_0 = \frac{1}{\epsilon} \hat{p}_- + \epsilon \hat{p}_t .
\] (2.3)

Now \( \hat{p}_t = \hat{p}_+ \) is the DLCQ hamiltonian and should have a finite limit as \( \epsilon \to 0 \). Then, since \( \hat{p}_- = \frac{n}{R_0} \), one has \( p_9 = \frac{n}{\epsilon R_0} \) so that the space-like momentum blows up. Also, \( p_0 = \frac{n}{\epsilon R_0} + \mathcal{O}(\epsilon) \) so that the energies in this frame also scale as \( 1/\epsilon \).

3. Perturbative evidence

We now want to study string scattering amplitudes compactified on a space-like circle of radius \( R_s \equiv R_0 = \epsilon R_0 \). We have just seen that the external momenta we are interested in have fixed, generically non-zero \( \hat{p}_- \), i.e. fixed non-zero \( n \). This means that we want to look at spacelike momenta in the compact direction of the form \( p_9 = \frac{n}{R_s} = \frac{n}{\epsilon R_0} \) that blow up as \( \epsilon \to 0 \).

3.1. Genus one

Let me first briefly recall the proof of [8] that the four-point one-loop amplitude of type II superstring theory compactified on a space-like circle has a well-defined limit as \( R_s \to 0 \). This amplitude for massless (in the ten-dimensional sense) external states with fixed momentum quantum numbers \( n_r, \ r = 1, \ldots, 4 \) was given in [8] to be

\[
A_{(4)}^{\text{cl}} = \frac{(\pi \kappa)^4}{\alpha'^5} K_{\text{cl}} \int \frac{d^2 \tau}{(\text{Im}\tau)^2} \prod_{r=1}^3 \frac{d^2 \nu_r}{\text{Im}\tau} \prod_{s > r} \chi(\nu_{sr}, \tau)^{\alpha' k_r \cdot k_s} \times \sum_{n,m} \alpha' \frac{1}{R_s^2} \exp \left\{ -\pi \alpha' \frac{1}{R_s^2} \frac{1}{\text{Im}\tau} \left\| m + n \tau + \sum_{s=1}^4 n_s \nu_s \right\|^2 \right\}
\] (3.1)

where \( \chi(\nu, \tau) = 2\pi \exp[-\pi(\text{Im}\nu)^2/\text{Im}\tau] \theta_1(\nu, \tau)/\theta'_1(\nu, \tau) \), and \( k_r \cdot k_s \) denotes the full ten-dimensional scalar product of the external momenta (we write \( k_r \) rather than \( p_r \), as customary),
while $K_{cl}$ is the standard kinematic factor already present in the closed string tree amplitude [12]. The important point that was noticed in [8] is that although for vanishing $n_r$ the amplitude (3.1) diverges as $R_s \to 0$, for at least one non-vanishing $n_r$ it has a finite limit. Indeed, let $R_s = \epsilon R_0$ so that the relevant $\epsilon$-dependent factor is

$$
\frac{1}{\epsilon^2} \left( \frac{\alpha'}{R_0^2} \right) \exp \left\{ -\frac{\pi}{\epsilon^2} \left( \frac{\alpha'}{R_0^2} \right) \frac{1}{\text{Im} \tau} \left| m + n \tau + \sum_{s=1}^4 n_s \nu_s \right|^2 \right\} \tag{3.2}
$$

(in ref. [8] we took $R_0^2 = \alpha'$). As $\epsilon \to 0$ this yields a complex delta function:

$$
(3.2) \quad \to \quad \text{Im} \tau \ \delta^{(2)} \left( m + n \tau + \sum_{s=1}^4 n_s \nu_s \right). \tag{3.3}
$$

The net effect of the sum over $n$ and $m$ and the integration over the moduli $\nu_s$ then is to replace one of the $\nu_s$-integrations, say the $\nu_3$-integration, by a discrete sum over a lattice of $n_3^2$ values on the world-sheet torus.

It was shown in [8] that this amplitude exactly has the singularities required by unitarity, and no more. In particular the case $n_1 = -n_2, n_3 = -n_4$ of vanishing momentum transfer in the compact direction (which was the dangerous case [7]) is perfectly finite, except for poles corresponding to on-shell intermediate states, as it should.

It was already clear in [8] that the restriction to only four external states (amplitudes with less than four external states vanish) was of not much relevance for our argument. The basic point was the exponential factor, coming from the zero-modes (momenta and winding modes) after a partial Poisson resummation, and the $\frac{1}{R^2}$ factor from the measure of the momentum modes and again the partial Poisson resummation. It is pretty clear that for $N > 4$ external states, the expression (3.2) would simply be replaced by

$$
\frac{1}{\epsilon^2} \left( \frac{\alpha'}{R_0^2} \right) \exp \left\{ -\frac{\pi}{\epsilon^2} \left( \frac{\alpha'}{R_0^2} \right) \frac{1}{\text{Im} \tau} \left| m + n \tau + \sum_{s=1}^N n_s \nu_s \right|^2 \right\}. \tag{3.4}
$$

In the $\epsilon \to 0$ limit this again leads to the complex delta function (with the sum over $s$ now running from 1 to $N$) with the same net effect of discretizing one of the moduli $\nu_s$. One obtains a finite amplitude having only the singularities required by unitarity.
3.2. Higher genus

For the one-loop amplitude, the factor of $1/\epsilon^2 \sim 1/R_s^2$ was to be expected from T-duality. Indeed, as compared to the tree-level amplitude, the one-loop amplitude carries an extra factor of $g_s^2 = e^{2\phi}$. Under T-duality, $e^{2\phi}/R_s^2$ converts to $e^{2\tilde{\phi}}$ with no explicit $R_s$ dependence as the dual radius $\tilde{R}_s = \alpha'/R_s$ goes to $\infty$. The same type of argument shows that for a genus-$g$ amplitude there must be a factor $R_s^{-2g}$. Indeed, we get a factor of $R_s^{-2}$ for each handle from the momentum measures and partial Poisson resumming of the winding modes. This factor $R_s^{-2g} \sim \epsilon^{-2g}$ must combine with appropriate exponentials $\prod_{i=1}^g \exp(-\pi/\epsilon^2)\ldots |^2)$ to give a product of $g$ delta functions. These delta functions then e.g. fix one of the insertion points of external states and $g - 1$ of the Teichmüller parameters describing the genus-$g$ Riemann surface.

Rather than working this out in general, let me only consider a genus-2 surface in the limit where it looks like two tori joined by a long and narrow tube. An $N = N_1 + N_2$-point amplitude then looks like the product of an $N_1 + 1$-point one-loop amplitude with an $N_2 + 1$-point one-loop amplitude integrated over the modular parameter describing the long narrow tube. In the $R_s \to 0$ limit, each one-loop amplitude then indeed gives a complex delta function as discussed in the previous subsection, and the net effect is to discretize e.g. the insertion point of the long narrow tube on one of the tori, as well as the insertion points of one of the external states. According to our discussion for $g = 1$, this amplitude must be finite and only have those singularities that are required by unitarity. Although we have only discussed this very special geometry of the two-loop amplitude, it is already quite encouraging and shows how a similar result may well hold for the general case.

4. Non-perturbative evidence

4.1. D-branes

In section 2 we have argued that we should be interested in states with energies $p_0$ scaling as $1/\epsilon$ because only such states can correspond to a fixed non-vanishing $p_-$ and finite DLCQ energy. We will now show that this is indeed satisfied for all D-branes that had finite energy before compactification of $x^9$. 
First consider D0 branes. Before compactifying $x^9$, a D0 brane has a finite mass $T_0$. Upon compactification of $x^9$ on a circle of radius $R_9 = \epsilon R_0$, the D0 becomes a D1 wrapped around the dual circle of radius $\alpha' / R_0$ [13]. It thus has an energy $T_1 2\pi \alpha' / R_0 = 2\pi T_1 \alpha' / \epsilon R_0$ which indeed scales as $1 / \epsilon$ as required.

Next look at a D2 brane. To get a finite mass/energy in the theory with non-compact $x^9$ we can compactify two other (transverse) directions, say $x^7$ and $x^8$ with radii $R_7$ and $R_8$ and wrap the D2 around this $T^2$. If we now compactify $x^9$ with radius $R_9 = \epsilon R_0$, the D2 becomes a D3 wrapped around a $T^3$ with radii $R_7, R_8$ and $\alpha' / R_0$, so that its energy is $T_3 (2\pi)^3 R_7 R_8 \alpha' / R_0$, again $\sim 1 / \epsilon$, as claimed. However, one may also start with a D2 in the 8-9 direction (wrapped or not in the 8 direction). If $x^9$ is not compact this D2 has infinite extent, hence infinite energy. Upon compactifying $x^9$ it then becomes a D1 in the 8 direction. There is no way it’s energy can scale as $1 / \epsilon$. The argument is the same for higher branes.

Thus we see that D-branes that had finite energy before compactifying $x^9$, have an energy scaling as $1 / \epsilon$ once $x^9$ is compactified with radius $R_9 = \epsilon R_0$, showing they have finite DLCQ energy. Hence the non-perturbative states made up from D-branes of finite energy behave in just the right way to survive and make sense in the light-like limit.

4.2. Non-perturbative amplitudes

Of course, it is a formidable task to work out non-perturbative corrections to an arbitrary superstring amplitude in general. However, there are some limiting cases of such amplitudes where the full series of non-pertubative corrections is known. We may look at the low-energy limit where one can extract various terms of the effective action from the full string amplitudes. Particularly interesting are the $R^4$-couplings [10] because they are BPS protected and can only receive contributions from string tree-level, one-loop and non-perturbative effects. Let us first discuss in general what one might learn about the light-like limit from these couplings and then consider the explicit results of [10,11].

A priori, one might expect that one cannot extract any useful information for our purpose from a low-energy effective action, because, as we saw above, we are interested in scattering amplitudes with momenta in the compact dimension and energies diverging as $1 / \epsilon$ in the light-like limit. Nevertheless, the low-energy effective couplings or amplitudes being the low-energy
limits of some corresponding expressions valid at any energy scale, the former contain valuable information about the possible forms of the latter.

To illustrate this point, consider again the four-point one-loop amplitude (3.1) of section 3 before taking the \( R_9 \equiv R_9 \to 0 \) limit. Its low-energy limit (i.e. for \( p_9^\alpha = 0 \)) gives, among others, the one-loop contribution to the \( R^4 \)-coupling of the low-energy effective action. In particular, the dependence on the compactification radius is \( \frac{\alpha'}{R_9} \). This is the well-known reason why in the low-energy limit one must combine \( R_9 \to 0 \) with \( \alpha' \to 0 \). But we can turn the argument around. As we have seen in section 3, this \( \frac{1}{R_9^2} \)-dependence of the (one-loop) low energy effective \( R^4 \)-coupling is a necessary condition for the \( R_9 \to 0 \) limit of the high-energy (\( p_9^\alpha \sim \frac{1}{4} \)) amplitude to exist. The key point was to combine the \( \frac{1}{R_9^2} \) with the exponential zero-mode factor into (3.2) which gave the delta function (3.3).

Let us now turn to non-perturbative (D-instanton) corrections to such a string scattering amplitude. Because we are looking at the corrections to the same process as in section 3, we would expect a similar exponential zero-mode factor to be present again. Rather than depending on the torus modular parameter \( \tau \) as in (3.2) it might depend on the moduli of the D-instantons. We do not know these zero-mode factors and, of course, the present discussion is highly speculative. It nevertheless seems a fair guess that again one has a product of \( \frac{1}{R_9^2} \) and some other factor and that both combine into some delta function as \( R_9 \to 0 \) so that this limit turns out to be finite. At low energies however, this other factor should become trivial and only the \( \frac{1}{R_9^2} \) should remain. Although this is not the only possible scenario,\(^*\) it is the simplest one and it seems reasonable to expect that all the non-perturbative corrections to the \( R^4 \)-coupling in the effective action of the IIA string compactified on \( S^1 \) with radius \( R_9 \) should behave as \( \frac{1}{R_9^2} \).

The full series of these corrections is given in [11] and reads (including also tree-level and one-loop contributions)

\[
\frac{1}{R_9} j_{D=9}^{\text{IIA}} = 2 \zeta(3) e^{-2\phi} + \frac{2\pi^2 \alpha'}{3 R_9^2} e^{-\phi} \sum_{m,n \neq 0} \left| \frac{m}{n} \right| K_1(2\pi |mn| R_9 e^{-\phi}/\sqrt{\alpha'}) e^{2\pi i mn A} \]

(4.1)

where \( K_1 \) is the Bessel function and \( A \) the Wilson line of the RR one-form on the circle. The

\(^*\) Obviously, one could also expect other scenarios leading e.g. to multiple delta functions as we have seen for the perturbative genus \( g > 1 \) amplitudes in section 3.2. Then one could e.g. have a factor \( R_9^{-2N} \) for D-instantons of charge \( N \).
contribution of a D-instanton of charge $N = nm$ can be read off to be

$$\frac{4\pi\sqrt{\alpha'}}{R_9} e^{-\phi} \sum_{n|N} \frac{|N|}{n^2} K_1(2\pi|mn|R_9 e^{-\phi}/\sqrt{\alpha'}) e^{2\pi i N A}.$$  \hspace{1cm} (4.2)

For $R_9 \to 0$, the argument of this Bessel function becomes vanishingly small and, using $K_1(z) \sim z^{-1} + O(z, z \log z)$ as $z \to 0$, one gets for the D-instanton of charge $N$

$$\frac{2}{R_9^2} \left( \sum_{n|N} \frac{1}{n^2} \right) e^{2\pi i N A} \sim \frac{1}{R_9^2}.$$ \hspace{1cm} (4.3)

This is exactly as expected from our discussion! Although this is only rather indirect evidence that the non-perturbative contributions to the full string scattering amplitudes with non-vanishing $p_9^0 = \frac{n'}{R_9}$ have a finite $R_9 \to 0$ limit, it nevertheless points in the right direction. Note that (4.3) is valid as long as $NR_9 \ll e^{\phi}/\sqrt{\alpha'} = g_s$. Thus the limit $R_9 \to 0$ cannot be uniform for all $N$ in the full expression (4.1). Also, for $R_9 \to 0$ we used the small $z$ asymptotics of $K_1(z)$, while to make explicit the instanton order one would have to use the large $z$ asymptotics $K_1(z) \sim \sqrt{\pi/(2z)} e^{-z}$ to obtain an expansion in powers of $\exp \left[ -2\pi((R_9/\sqrt{\alpha'})e^{-\phi} + iA) \right]$. Clearly, small $R_9$ and small $e^{\phi}$ are very different regimes. In particular, (4.3) does not depend on $g_s = e^{\phi}$ at all, exactly as the one-loop contribution.

5. Conclusion

We have argued that perturbative string loop amplitudes should have a finite and well-defined light-like limit provided the external momenta are chosen to correspond to a well-defined DLCQ set-up. On the non-perturbative side we considered states and amplitudes. We showed that the appropriate class of non-perturbative states (D-branes with finite energy for non-compact $x^9$) have precisely the right light-like limit. We had less precise things to say about non-perturbative corrections to string amplitudes, but still displayed some indications that they, too, might behave as required in the light-like limit. Having perturbative and non-perturbative evidence, this suggests that type IIA superstring theory as a whole has a well-defined light-like limit (for any string coupling $g_s$) and hence that the same is true for M-theory.
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