Building a stress calculation model for dynamic deformation of metal rings under magnetic loads

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Abstract. This paper presents an analysis of dynamic deformation of metal rings under magnetic pulse loads. In addition to apparent advantages, such as low power consumption and high performance, the magnetic pulse method allows conducting series of controlled deformation experiments and determining strength properties of materials under uniform, rather than localized, deformation loads. The results of the conducted experiments were used to build a mathematical model for calculating forces that cause deformation of metal ring samples and circumferential stresses that occur in them.

1. Experimental technique

The approach that was used in the experiments is an evolution of the experimental technique for studying fracturing of metal samples under high-speed loads proposed by O. H. Zhang and K. Ravi-Chandar [1] and improved by researchers at Saint Petersburg State University [2, 3] to cover a wider range of deformation rates, as well as the method for recording sample fracture times.

The experiments used an inductance coil, which was placed coaxially on a metal ring sample. The repulsive force that occurred between the coil loops and the metal ring sample caused a deformation of the latter (see figure 1). The experiments showed that the optimum number of coil turns was five.

![Figure 1](image)

Figure 1. (a) Current flow diagram; (b) Ring sample loading diagram.

The experiments were conducted using a GKVI-300 narrow high-voltage pulse generator, which allowed generating voltages with amplitudes of 10 to 300 kV. The traditional loading circuit used for...
the study [1] is based on a concentrated capacitance (C) and a concentrated inductance (L). The authors of the studies [2, 3] demonstrated that such a circuit does not provide sufficiently fast load application, i.e. it is capable of applying loads only in a quasi-static mode. To address this shortcoming, improved loading circuits were introduced, which included a distributed capacitance and inductance and were capable of applying loads with conduction times of about 1 µs (see figure 2(a)) and pulse loads with durations of about 80 ns (see figure 2(b)).

Designations used in figure 2: Rch – charging resistor; C – capacitor; S – switch; IT – pulse transformer; FL – pulse forming line; OD – output device; HVW – high voltage electrode; L – coil (coreless solenoid); Sample – metal ring sample; RC – Rogowski coil; PD – photodiode; OSC – oscilloscope.

2. Mathematical model
In accordance with the diagram in figure 3, the Ampère’s force applied to the ring sample can be determined using the following equation:

\[ F(t) = \frac{\mu_0}{4\pi} \left( \frac{2I_1(t)I_2(t)}{a} + 2 \frac{\mu_0}{4\pi} \frac{2I_1(t)I_2(t)}{l_1} \cos \alpha_1 + 2 \frac{\mu_0}{4\pi} \frac{2I_1(t)I_2(t)}{l_2} \cos \alpha_2 \right), \]  

(1)

where \( I_1(t) \) is the current in the inductance coil, \( I_2(t) \) is the current in the metal sample, and \( \mu_0 \) is the magnetic constant.

Designations used in figure 3: \( F_0 \) – fracturing force; \( F_1 \) – force on the sample; \( l_1 \) – distance between the sample and the coil; \( l_2 \) – distance between the sample and the high voltage electrode; \( b_1 \) – distance between the sample and the Rogowski coil; \( b_2 \) – distance between the sample and the photodiode; \( \alpha_1 \) – angle between the sample and the coil; \( \alpha_2 \) – angle between the sample and the high voltage electrode; \( \alpha_3 \) – angle between the sample and the Rogowski coil; \( \alpha_4 \) – angle between the sample and the photodiode.
The amperages of the two currents are determined using the following system of equations, which describes electromagnetic oscillations in coupled circuits [2]:

\[ i_1 = i_{10}e^{pt}\sin\omega t, \]
\[ i_2 = Ae^{pt}\sin\omega t + Be^{pt}\sin\omega t, \]

where \( i_1 \) and \( i_2 \) are the currents in the circuit and the ring, respectively; \( i_{10} = U_c(L_1\omega)^{-1} \); \( U_c \) is the voltage of the charged capacitor; \( \omega = (\omega_0^2 - p^2)^{1/2} \), where \( \omega_0 = (L_4C)^{-1/2} \) and \( p = R_1(2L_4)^{-1} \);
\[ A = i_{10}L_{12}\left[-L_2 \left(p^2 + \omega^2\right) + R_2p\right]/\left[(R_2 - L_2p)^2 + \omega^2L_2^2\right]; \]
\[ B = -i_{10}L_{12}R_2\omega\sqrt{(R_2 - L_2p)^2 + \omega^2L_2^2}; \]
\( L_1 \) and \( L_2 \) are the inductances of the solenoid circuit and the ring sample; \( R_1 \) and \( R_2 \) are the resistances of the solenoid circuit and the ring sample; \( L_{12} = L_{21} \) is the mutual inductance.

Metal ring fracturing experiments allowed the authors to develop and implement a unique method for determining time of fracture, which is based on detecting sparks occurring between sample fragments at the moment of fracture. It also allowed to discover that sample fracture occurs closely to the yield point. Previous models for determining strength properties [2, 3], which described the deformation process in terms of brittle fracture, did not provide sufficiently detailed descriptions of the process. Therefore, it became necessary to build a mathematical model based on ductile fracture mechanisms.

The energy balance equation was used to derive the following equation describing the motion of a thin metal ring:

\[ \rho \left[ \frac{1}{2R} \left( \frac{dR}{dt} \right)^2 + \frac{d^2R}{dt^2} \right] + \frac{\sigma}{R} = \frac{q(t)}{h}, \]

where \( R \) is the radius of the ring, \( \rho \) is the density of the ring material, \( q(t) \) is the load applied on the inner surface of the ring (see figure 1(b)), \( \sigma \) is the stress within the sample, and \( h \) is the thickness of the sample.

The radial load on the metal ring sample was calculated using the following equation:

\[ q(t) = \frac{F(t)}{c}, \]

where \( c \) is the width of the ring sample.

In previous works [2, 3], Hooke’s law in a linear elastic approximation was used for the transition to the expression of the stress \( \sigma \). However, our experiments demonstrated that, as loading speed increases, oscillations occur in the pressure profile, which cannot be described by a linear model. In this paper, these experimental findings are explained in terms of dislocation dynamics using the Sokolovsky–Malvern model [4]:

\[ \frac{\partial \sigma}{\partial t} - \rho c^2 \frac{\partial \varepsilon}{\partial t} = g(\sigma, \varepsilon), \]

where \( \varepsilon \) is the deformation, \( c \) is the longitudinal speed of sound, \( t \) is the time, and \( g(\sigma, \varepsilon) \) is the relaxation function.

In [5], it was demonstrated that the function \( g(\sigma, \varepsilon) \) is proportional to the plastic shear deformation and can be expressed by the following equation:
\[ g(\sigma, \varepsilon) = -\frac{8}{3} \mu b N_m(\tau, \gamma) v_d(\tau, \gamma), \]  

(7)

where \( \mu \) is the shear modulus, \( b \) is the strength of dislocation, \( N_m(\tau, \gamma) \) is the mobile dislocation density, \( v_d(\tau, \gamma) \) is the mobile dislocation speed, \( \gamma \) is the plastic shear deformation, and \( \tau \) is the shear stress.

In the case of small deformations, the relationship between the mobile dislocation density, the initial dislocation density and the plastic deformation is linear:

\[ N_m = N_0 + \alpha \gamma, \]

(8)

where \( \alpha \) is the dislocation multiplication factor.

The dislocation speed can be expressed by the following equation:

\[ v_d = v_{dm} \exp\left(-\frac{\tau_0 + H\gamma}{\tau}\right), \]

(9)

where \( v_{dm} \) is the maximum possible dislocation speed, \( \tau_0 \) is the characteristic drag stress, and \( H \) is the strain hardening coefficient.

Thus, the equation (6) can be expressed in a uniaxial approximation as follows:

\[ \frac{\partial \sigma}{\partial t} - (\lambda + 2\mu) \frac{\partial \varepsilon}{\partial t} = \frac{8}{3} \mu b v_{dm} \left\{ N_0 + \frac{3\alpha}{8\mu} \left[ (\lambda + 2\mu) \varepsilon - \sigma \right] \right\} f(\sigma, \varepsilon), \]

(10)

where

\[ f(\sigma, \varepsilon) = \exp\left(-\frac{\tau_0 + \frac{3H}{8\mu} \left[ (\lambda + 2\mu) \varepsilon - \sigma \right]}{\frac{3}{4} \left[ \sigma - \frac{\lambda + 2}{3} \mu \varepsilon \right]} \right), \]

(11)

and \( H \) is the strain hardening coefficient.

3. Results

Using the relations (10) and (11), the motion equation (4) can now be used to determine the stress within the ring sample as a function of time.

On figure 4 shown the dependence of the circumferential stress on time, which was obtained using the above equations for a 2 mm wide aluminum ring sample.
As can be seen in figure 4, in contrast to the previously considered model for calculating circumferential stress [3], the mathematical calculation model based on dislocation motion provides a solution in form of a stress profile, which demonstrates a monotonic transition from the elastic precursor to the plastic front.

On figure 5 shown a comparison of the circumferential stress profile obtained in an experiment with a 3 mm wide aluminum ring sample, and the stress profile calculated for the same sample using the above model.

As can be seen in figure 5, the analytical stress function reflects the behavior of the experimentally obtained function.

This paper considers a method of magnetic pulse deformation of metal rings. It provides two dynamic loading diagrams that extend the range of strain rates in the samples under consideration as compared to the previously proposed ones. The paper describes a method for determining the electrodynamic force applied to ring samples and proposes a mathematical model for calculating circumferential stress that is based on the dislocation theory. The obtained results allow evaluating strength and stress-strain properties of materials over a wide range of energy parameters and strain rates.

This study was conducted under the financial support of the Russian Foundation for Basic Research as part of the Research Project No. 18-31-00199.
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