$K \rightarrow \pi\nu\bar{\nu}$ decays and CKM fits

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After a brief introduction to the so-called flavour problem, we discuss the role of $K \rightarrow \pi\nu\bar{\nu}$ decays in shedding new light on this issue. In particular, we review the theoretical uncertainties in predicting $\Gamma(K \rightarrow \pi\nu\bar{\nu})$ within the SM, the sensitivity of these observables to New Physics scenarios, and the status of their experimental determination.

1 Introduction: the flavour problem

There is no doubt that the Standard Model (SM) provides a successful and economical description of particle physics up to energies of $O(100 \text{ GeV})$. However, it is very natural to consider this model only as the low-energy limit of a more general theory, or as the renormalizable part of an effective field theory valid up to some still undetermined cut-off scale $\Lambda$. We have no direct indications about the value of this cut-off, but theoretical arguments based on a natural solution of the hierarchy problem suggest that $\Lambda$ should not exceed a few TeV.

From this perspective, the goal of indirect New Physics (NP) searches can be viewed as the search for the effective non-renormalizable interactions, suppressed by inverse powers of $\Lambda$, which encode the presence of new degrees of freedom at high energies. These operators should naturally induce large effects in processes which are not mediated by tree-level SM amplitudes, such as $\Delta F = 1$ and $\Delta F = 2$ flavour-changing neutral current (FCNC) transitions. Up to now there is no evidence of these effects and this implies severe bounds on the effective scale of several dimension-six operators (more than 100 TeV for the effective scale of the $\Delta S = 2$ operators contributing to $K^0 - \bar{K}^0$ mixing). The apparent contradiction between these high bounds on $\Lambda$ and the expectation $\Lambda \sim \text{TeV}$, dictated by the electroweak hierarchy problem, is a manifestation of what in many specific NP frameworks goes under the name of flavour problem.

In the last few years the flavour problem has been considerably exacerbated by the new precise data in the $B$ system, which show an excellent consistency of the various observables used to (over-)constrain the CKM unitarity triangle \cite{11}. One could therefore doubt about the need for new precision measurements. However, the present consistency of CKM fits should not be over emphasized and there are various reasons why a deeper study of FCNCs and, particularly, of rare $K$ decays, would still be very useful.

First of all, in order to constrain the parameter space of possible SM extensions, we cannot simply test the consistency of the SM hypothesis, as is usually done in present CKM fits. In principle, all observables potentially sensitive to NP, namely all short-distance dominated FCNCs, should be left as free parameters. In other words, we should try to perform in the flavour sector something similar to what has been done in the electroweak sector with the model-independent fits of the oblique corrections (see e.g. Ref. \cite{2}). A completely model-independent approach to the flavour problem is very difficult, because of the larger number of couplings involved. Nonetheless, we can already try to constrain the parameter space of a series of rather general NP frameworks, such as

1. models with Minimal Flavour Violation \cite{3,4,5};
2. models with large NP effects in $b \rightarrow s$ FCNC transitions and not in $b \rightarrow d$ and $b \rightarrow s$ ones (or permutations) \cite{6};
3. models with large NP effects only in $\Delta F = 2$ FCNC amplitudes \cite{7,8,9};
4. models with large NP effects only in $Z$-penguins FCNC amplitudes \cite{10};

and a few other cases of well-defined effective field theories. As can by easily understood, in all these scenarios a substantial progress with respect to the present situation would be obtained by the inclusion of the precise $\Delta F = 1$ constraint from $K \rightarrow \pi\nu\bar{\nu}$ decays (see e.g. Refs. \cite{5,11}).

A second important argument in favour of precise measurements of $K \rightarrow \pi\nu\bar{\nu}$ widths, is the fact that most of the observables used in present CKM fits, such as $\epsilon_K$, $\Gamma(b \rightarrow u(\nu))$ or $\Delta M_{B_s}$, suffer from irreducible theoretical errors at the 10% level (or above). In the perspective of reaching a high degree of precision, it would be desirable to base these fits only on observables with theoretical errors at the percent level (or below), such as the CP asymmetry in $B \rightarrow J/\psi K_S$. As we shall review in the following section, the $K_L \rightarrow \pi^0\nu\bar{\nu}$ width belongs to this category.

2 Theoretical predictions of $\Gamma(K \rightarrow \pi\nu\bar{\nu})$

The $e \rightarrow d\nu\bar{\nu}$ transition is one of the rare examples of weak processes whose leading contribution starts at $O(G_F^2)$. At
the one-loop level it receives contributions only from Z-penguin and W-box diagrams, as shown in Fig. [1] or from pure quantum electroweak effects. Separating the contributions to the one-loop amplitude according to the intermediate up-type quark running inside the loop, we can write

\[ \mathcal{A}(s \to d\bar{v}) = \sum_{q=u,c,t} V_{qs}^* V_{qd} A_q, \]  

where \( V_{ij} \) denote the elements of the CKM matrix. The hierarchy of these elements would favour up- and charm-quark contributions; however, the hard GIM mechanism of the perturbative calculation implies \( A_q \sim m_q^2/M_W^2 \), leading to a completely different scenario. The top-quark contribution turns out to be the leading term both in the real and in the imaginary part of the amplitude. This structure implies several interesting consequences for \( \mathcal{A}(s \to d\bar{v}) \): it is dominated by short-distance dynamics, therefore its QCD corrections are small and calculable in perturbation theory; it is very sensitive to \( V_{td} \), which is one of the less constrained CKM matrix elements; it is likely to have a large CP-violating phase; it is very suppressed within the SM and thus very sensitive to possible new sources of quark-flavour mixing.

Short-distance contributions to \( \mathcal{A}(s \to d\bar{v}) \), are efficiently described, within the SM, by the following effective Hamiltonian \[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}}\frac{\alpha}{\sin^2 \theta_W} \sum_{i=\text{u,c,t}} \left[ \lambda_i X_{NL}^i + \lambda_i X_i \right] \times (\bar{s}d)_{\nu} A(\bar{v}d)_{\nu} A, \]  

where \( x_i = m_i^2/M_W^2 \) and, as usual, \( \lambda_q = V_{qs}^* V_{qd} \). The coefficients \( X_{NL}^i \) and \( X_i \), encoding top- and charm-quark loop contributions, are known at the NLO accuracy in QCD \[ [13,14] \] and can be found explicitly in \[ [12] \]. The theoretical uncertainty in the dominant top contribution is very small and it is essentially determined by the experimental error on \( m_t \). Fixing the \( \overline{\text{MS}} \) top-quark mass to \( m_t(m_t) = (166 \pm 5) \text{ GeV} \) we can write

\[ X_i = 1.51 \left( \frac{m_t(m_t)}{166 \text{ GeV}} \right)^{1.15} = 1.51 \pm 0.05. \]  

The simple structure of \( H_{\text{eff}} \) leads to two important properties of the physical \( K \to \pi^0\bar{\nu}\bar{\nu} \) transitions:

- The relation between partonic and hadronic amplitudes is exceptionally accurate, since hadronic matrix elements of the \( \bar{s}d\bar{v}d \) current between a kaon and a pion can be derived by isospin symmetry from the measured \( K_{\ell\alpha} \) rates.
- The lepton pair is produced in a state of definite CP and angular momentum, implying that the leading SM contribution to \( K_L \to \pi^0\bar{\nu}\bar{\nu} \) is CP-violating.

**Figure 1.** One-loop diagrams contributing to the \( s \to d\bar{v} \) transition.

The largest theoretical uncertainty in estimating \( \mathcal{B}(K^+ \to \pi^+\bar{\nu}\bar{\nu}) \) originates from the charm sector. Following the analysis of Ref. \[ [12] \], the perturbative charm contribution is conveniently described in terms of the parameter

\[ P_0(X) = \frac{1}{4^4} \left[ \frac{2}{3} X_{NL}^c + \frac{1}{3} X_{NL}^t \right] = 0.39 \pm 0.06, \]  

where we have used \( \lambda = 0.2240 \pm 0.0036 \) \[ [1] \]. The numerical error in the r.h.s. of Eq. (4) is obtained from a conservative estimate of NNLO corrections \[ [12] \]. Recently also non-perturbative effects introduced by the integration over charmed degrees of freedom have been discussed \[ [15] \]. Despite a precise estimate of these contributions is not possible at present (due to unknown hadronic matrix elements), these can be considered as included in the uncertainty quoted in Eq. (4).\(^1\) Finally, we recall that genuine long-distance effects associated to light-quark loops are well below the uncertainties from the charm sector \[ [16] \].

With these definitions the branching fraction of \( K^+ \to \pi^+\bar{\nu}\bar{\nu} \) can be written as

\[ \mathcal{B}(K^+ \to \pi^+\bar{\nu}\bar{\nu}) = \frac{\bar{k}_+}{\bar{r}_K} \left[ (\text{Im} \lambda_i)^2 {X^2_i} + \left( X_i^t \text{Re} \lambda_i P_0(X) + \text{Re} \lambda_i X_i \right)^2 \right], \]  

where \[ [12] \]

\[ \bar{k}_+ = r_K \frac{3 \sigma^2 \mathcal{B}(K^+ \to \pi^0e^+\nu)}{2 \pi^2 \sin^4 \theta_W} = 7.50 \times 10^{-6} \]  

(6)

and \( r_K = 0.901 \) takes into account the isospin breaking corrections necessary to extract the matrix element of the \( (\bar{s}d)_{\nu} \) current from \( \mathcal{B}(K^+ \to \pi^0e^+\nu) \) \[ [17] \].

The case of \( K_L \to \pi^0\bar{\nu}\bar{\nu} \) is even cleaner from the theoretical point of view \[ [13] \]. Because of the CP structure,
only the imaginary parts in \( \Delta M_B \) –where the charm contribution is absolutely negligible– contribute to \( \Delta A(M_2 \rightarrow \pi^0 \nu \bar{\nu}) \). Thus the dominant direct-CP-violating component of \( \Delta A(K_L \rightarrow \pi^0 \nu \bar{\nu}) \) is completely saturated by the top contribution, where QCD corrections are suppressed and rapidly convergent. Intermediate and long-distance effects in this process are confined only to the indirect-CP-violating contribution \([19]\) and to the CP-conserving one \([20]\), which are both extremely small. Taking into account the isospin-breaking corrections to the hadronic matrix element \([17]\), we can write an expression for the \( K_L \rightarrow \pi^0 \nu \bar{\nu} \) rate in terms of short-distance parameters, namely

\[
\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}} = \frac{\bar{k}_L}{\Lambda^2} (\text{Im} \lambda_t)^2 X^2(x_t)
\]

\[
= 1.48 \times 10^{-11} \times \left[ \frac{\text{m}_t(m_t)}{166 \text{ GeV}} \right]^{2.30} \left( \frac{\text{Im} \lambda_t}{10^{-4}} \right)^2 ,
\]

which has a theoretical error below 3%.

At present the SM predictions of the two \( K \rightarrow \pi \nu \bar{\nu} \) rates are not extremely precise owing to the limited knowledge of both real and imaginary parts of \( \lambda_t \). Taking into account the latest input values reported in Ref. \([11]\) and the corresponding global fit of the CKM unitarity triangle, we find \( \text{Re} \lambda_t = -(3.11 \pm 0.21) \times 10^{-4} \) and \( \text{Im} \lambda_t = (1.33 \pm 0.12) \times 10^{-4} \), which yield to

\[
\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = (0.77 \pm 0.11) \times 10^{-10} ,
\]

\[
\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}} = (0.26 \pm 0.05) \times 10^{-10} .
\]

These results are perfectly compatible with the previous recent estimates reported in Ref. \([11,21]\): however, it is interesting to note that the central value in the prediction of \( \mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \) has increased by \( \approx 7\% \). The main reason for this enhancement is the higher value of \( \text{Re} \lambda_t \), resulting from a new analysis of the constraints imposed by \( |V_{ub}| \) and \( \Delta M_B \) \([11]\). As pointed out in Ref. \([21]\), the errors in Eqs. \(9,10\) can be reduced if \( \text{Re} \lambda_t \) and \( \text{Im} \lambda_t \) are directly extracted from \( \lambda_{CP}(B \rightarrow J/\Psi K_S) \) and \( \epsilon_K \); however, this procedure introduces a stronger sensitivity to the probability distribution of the (theoretical) estimate of \( B_K \). Combining the two approaches (the extraction of \( \text{Re} \lambda_t \) and \( \text{Im} \lambda_t \)) via a global fit to the CKM matrix or a direct extraction of \( \text{Re} \lambda_t \) and \( \text{Im} \lambda_t \) via \( \lambda_{CP}(B \rightarrow J/\Psi K_S) \) and \( \epsilon_K \) leads to a solid upper bound of \( 1.0 \times 10^{-10} \) on \( \mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} \) \([21]\), which represent an interesting benchmark for NP searches.

The high accuracy of the theoretical predictions of \( \mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \) and \( \mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \) in terms of modulus and phase of \( \lambda_t = V_{ts}^* V_{td} \) clearly offers the possibility of very interesting tests of flavour dynamics. Within the SM, a measurement of both channels would provide two independent pieces of information on the unitary triangle, or a complete determination of \( \tilde{\rho} \) and \( \tilde{\eta} \) from \( \Delta S = 1 \) transitions. In particular, \( \mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \) defines an ellipse in the \( \tilde{\rho}-\tilde{\eta} \) plane and \( \mathcal{B}(K_L^0 \rightarrow \pi^0 \nu \bar{\nu}) \) an horizontal line (the height of the unitarity triangle). Note, in addition, that the determination of \( \sin 2 \beta \) which can be obtained by combining \( \mathcal{B}(K^0_L \rightarrow \pi^0 \nu \bar{\nu}) \) and \( \mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \) is extremely clean, being independent from uncertainties due to \( m_t \) and \( V_{ub} \) (contrary to the separate determinations of \( \tilde{\rho} \) and \( \tilde{\eta} \)) \([19]\).

As already mentioned, the short distance nature of the \( s \rightarrow d \nu \bar{\nu} \) transition implies a strong sensitivity of \( K \rightarrow \pi \nu \bar{\nu} \) decays to possible SM extensions \([22]\). Observable deviations from the SM predictions are expected in many specific frameworks, including low-energy supersymmetry \([10,23]\), models with extra chiral \([24]\) or vector-like quarks \([25]\), and models with large extra dimensions \([26]\), just to mention the specific frameworks which have received most of the attention in the last few years. Present experimental
data do not allow yet to fully explore the high-discovery potential of these modes. Nonetheless, it is worth to stress that the evidence of the $K^+ \to \pi^+ \nu \bar{\nu}$ transition obtained by BNL-E787 already provides highly non-trivial constraints on the realistic scenarios with large new sources of flavour mixing (see e.g. Ref. [10, 22, 23]). As illustrated in Fig. 1 even within the pessimistic framework of MFV, a precise measurement of the $K_L \to \pi^0 \nu \bar{\nu}$ rate would provide –in a long term perspective– one of the most significant constraint on possible new degrees of freedom.

3 Experimental status and present impact on CKM fits

The search for processes with missing energy and branching ratios below $10^{-10}$ is definitely a very difficult challenge, but has been proved not to be impossible: two $K^+ \to \pi^+ \nu \bar{\nu}$ candidate events have been observed by the BNL-E787 experiment [27]. The branching ratio inferred from this result,

$$\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu}) = \left(1.57 \pm 1.75_{-0.82}^{+0.6} \right) \times 10^{-10}, \quad (11)$$

has a central value substantially higher than the SM prediction in [29], but is compatible with the latter once the large errors are taken into account. In a few years this result should be substantially improved by the BNL-E949 experiment, whose goal is to collect about 10 events (at the SM rate). In the longer term, a high-precision result on this mode will arise from the CKM experiment at Fermilab, which aims at a measurement of $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})$ at the 10% level [28].

Unfortunately, the progress concerning the neutral mode is much slower. No dedicated experiment has started yet (contrary to the $K^-$ case) and the best direct limit is more than four orders of magnitude above the SM expectation [29]. An indirect model-independent upper bound on $\Gamma(K_L \to \pi^0 \nu \bar{\nu})$ can be obtained by the isospin relation [22]

$$\Gamma(K^+ \to \pi^+ \nu \bar{\nu}) = \Gamma(K_L \to \pi^0 \nu \bar{\nu}) + \Gamma(K_S \to \pi^0 \nu \bar{\nu}) \quad (12)$$

which is valid for any $s \to d\nu \bar{\nu}$ local operator of dimension $\leq 8$ (up to small isospin-breaking corrections). Using the BNL-E787 result (11), this implies $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu}) < 1.7 \times 10^{-9}$ (90% CL). Any experimental information below this figure can be translated into a non-trivial constraint on possible new-physics contributions to the $s \to d\nu \bar{\nu}$ amplitude. In a few years this goals should be reached by E931a at KEK: the first $K_L \to \pi^0 \nu \bar{\nu}$ dedicated experiment. This experiment should eventually be upgraded in order to reach a SES of $10^{-13}$ and collect up to $10^5 K_L \to \pi^0 \nu \bar{\nu}$ events at the future JPARC facility [30]. So far, the only approved experiment that could reach the SM sensitivity on $K_L \to \pi^0 \nu \bar{\nu}$ is KOPIO at BNL, whose goal is the observation of about 50 signal events (at the SM rate) with signal/background $\approx 2$ [29].

Although the experimental result in (11) is not very precise yet, it is already quite instructive trying to use it to constrain some of the general NP frameworks discussed in the introduction. As an example, in Fig. 3 we show the result of CKM unitarity-triangle fit allowing arbitrary NP contributions to $B_d-B_d$ mixing [11]. With this general frameworks, which includes one extra complex parameter with respect to the SM case, the standard CKM constrains from $A_{CP}(B \to J/\psi K_S)$, $\Delta M_{B_d}$ and $\Delta M_{B_s}/\Delta M_{B_d}$, cannot be used. In absence of the $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})$ information, we would find two preferred $\bar{\rho}-\bar{\eta}$ regions, corresponding to the overlap of $\epsilon_K$ and $|V_{ub}|$ constraints [51]. This degeneracy persist even if we include the preliminary $A_{CP}(B \to \pi^+ \pi^-)$ data from $B$ factories [9]. On the other hand, the $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})$ result in (11) breaks this degeneracy with a slight preference toward the non-standard solution in the left quadrant. As can be seen in Fig. 3 this indication is not statistically significant yet, but it provides a good illustration of the main points of this discussion: there is still a lot to learn about FCNC transitions and the measurements of $K \to \pi \nu \bar{\nu}$ rates provide a unique opportunity in this respect.

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