Study on dynamic response of slope under obliquely incident SV-wave

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Abstract. The change of the incident angle of the seismic wave will lead to the change of the mode of action of the earthquake, which will affect the dynamic response of the slope. In order to study the effect of oblique incidence of seismic SV wave on the dynamic response of rock slope, the problem of seismic fluctuation is transformed into wave source problem. The author uses the viscoelastic artificial boundary to simulate the scattering wave field and applies the equivalent loads to the boundary nodes, which realize the fluctuation input of obliquely incident seismic wave. The semi-infinite space numerical example verifies the validity and accuracy of the method. The results show that the angle of incidence and slope angle has a great impact on seismic dynamic response of slope surface. When the SV wave is incident obliquely, the peak ground acceleration of the rock slope gradually increases with the incident angle increasing, and the peak ground acceleration near the top of the slope increases with the increase of slope angle. The results of this paper can provide important reference for seismic stability analysis of slope.

1. Introduction

Earthquakes have occurred frequently in our country in recent years. The geological conditions of many mountains and many earthquakes in our country inevitably bring about a large number of earthquake-related slope problems. Especially in the construction and operation of water conservancy and hydropower projects, the instability of the slope near the slope of the dam in case of earthquake may lead to the loss of function of the dam and even lead to the risk of dam break. At present, great progress has been made in the study of the static stability of the slope of water resources and hydropower projects, and the relatively weak link is the dynamic stability of the slope under the action of strong earthquakes. The input mechanism of slope earthquakes and amplification effect of the earthquake along the slope need detailed analysis, and there are no comparatively mature and reliable evaluation methods and standards to adopt, which can be reflected in the three existing codes (Design specification for slope of hydropower and water conservancy project: DL/T 5353-2006; Design code for engineered slopes in water resources and hydropower projects: SL 386-2007; Code for seismic design of hydraulic structures of hydropower project: NB 35047-2015) related to the slope of hydroelectric projects. This leads to the slope of seismic design with greater randomness.
In recent years, several researchers have extensively investigated the impact of slope dynamic response under earthquake. In 1971, the United States Davis et al. measured the aftershocks of the San Fernando earthquake, and found that compared to the value of the foot, the value of Peak is several times larger [1]. The strong aftershock velocity observation records of two points on the Mount Cargill mountain Peak and the foot show that the duration of earthquake motions on the top of the Peak has significantly increased and the amplification effect is significant [2]. Xu et al. [3] found that the slopes have amplification effect on the input seismic wave by vertical and surface of the slopes with FLAC3D program. Li et al. [4] design and complete a large-scale shaking table model test of steep stratified rock slopes; Yang et al. [5] study dynamic response characteristics of anti-dip layered rock slope by large-scale shaking table model test, those testing results show that: seismic waves along the slope have a certain magnification effect in the rock slope. However, there are few studies on the dynamic response of rock slopes under oblique incident seismic waves. For the seismic design of structures, it is important to study the incident angle of the input wave, because the inclined input wave will cause severe vertical movement of the structure [6].

Based on a large number of previous researches, the authors adopted viscoelastic artificial boundary combined with the equivalent load input method to realize obliquely incident SV waves input. By analyzing and summarizing the numerical simulation results, the rock slope dynamic response law of the obliquely incident plane SV-wave under different slope angles is summarized.

2. The Finite Element Method of Obliquely Incident Seismic Wave
Reasonable ground motion input methods include two aspects: the selection of artificial boundaries and reasonable seismic wave field processing methods.

2.1. The viscous-spring artificial boundary
In the slope dynamic analysis, the seismic wave spreads to the infinite domain. Generally, it is necessary to cut out the limited-size calculation region from the semi-infinite continuum for analysis and calculation. However, the external wave is reflected on the truncated boundary of the limited foundation and artificially exaggerates the dynamic response of the calculation area. In order to eliminate this reflection effect, appropriate artificial boundary such as viscous boundaries [7] and viscoelastic boundaries [8] should be set on the truncated boundary to absorb the external waves and then simulate the radiation effect in the infinite foundation [9].

In this paper, the viscoelastic boundary is used to simulate, that is, a series of spring-damping elements in parallel with a linear spring and viscous damping are set at the artificial boundary. One end of the viscoelastic boundary is connected with the finite element of the inner domain, and the other end is connected with the far-field foundation. When the viscoelastic boundary completely absorbs the external scattering waves in the finite element region, the one end connected to the far-field foundation is subjected to seismic free-field motion. In this way, the input problem of earthquakes is transformed into the problem of free-field motion acting on springs and dampers on viscoelastic boundaries.

The spring stiffness coefficient and viscous damping coefficient at the two-dimensional artificial boundary are calculated according to the following expression:

\[
\begin{align*}
K_{BN} &= A_g \cdot \frac{E}{2 \rho_g} , \quad C_{BN} = \rho c_p \cdot A_g \\
K_{BT} &= A_g \cdot \frac{G}{2 \rho_g} , \quad C_{BT} = \rho c_s \cdot A_g
\end{align*}
\]

Where $K_{BN}$ and $K_{BT}$ are the normal and tangential spring constants, respectively; $C_{BN}$ and $C_{BT}$ are the normal and tangential damping coefficients, respectively; $\rho$ is the mass density; $G$ and $E$ are the
shear modulus and elastic modulus; \( c_p \) and \( c_s \) are the velocity of the P and SV wave, respectively. \( r_b \) is the distance between the scattering source and the artificial boundary [10].

2.2. The seismic wave input method

There are already many mature theories for the input of ground motion under viscoelastic artificial boundary conditions. A more reasonable theory that is currently accepted is the free field input theory of viscoelastic boundary. On the basis of finite element method, using the artificial boundary of viscous spring, the seismic motion of oblique incidence is converted into the equivalent nodal force on the truncated boundary of finite element model. The artificial boundary is set as viscoelastic boundary. The equivalent nodal force acting on the artificial boundary node can be expressed as:

\[
F_b = \left( K_b u_b^{ff} + C_b \dot{u}_b^{ff} + \sigma_b^{ff} n \right) A_b
\]

Where \( K_b \) is the stiffness matrix; \( C_b \) is the damping matrices; \( A_b \) is the area represented by the boundary node; \( n \) is the direction cosine of the outside normal vector. \( u_b^{ff}, \dot{u}_b^{ff} \) and \( \sigma_b^{ff} \) represent the free-field displacement vector, velocity vector, stress tensor at the boundary node, respectively. The following formulas are derived for calculating the equivalent load of the boundary nodes under the obliquely incident SV-wave.

According to the theory of wave [11], it can be seen that the waveform transform occurs when the seismic wave propagates to the free surface. When the SV wave is obliquely incident on the free surface at a certain angle, P waves and SV waves are derived in Figures 1 and 2. Set the displacement of the incident SV-wave as \( u_s(t) \), and the finite height of the foundation as \( H \) and the width as \( L \).

![Figure 1. Oblique incidence of plane SV-wave.](image1)

![Figure 2. Reflection of oblique incident SV waves on free surface.](image2)
According to the principle of wave superposition, the horizontal and vertical components of the arbitrary point in the half-space free field can be obtained. The horizontal and vertical displacements of free-field wave at the boundary can be expressed as:

\[
\begin{align*}
\quad u^x (x, y, t) &= u_x (x, y, t - \Delta t_j) \cos \alpha - B_2 u_y (x, y, t - \Delta t_j) \cos \alpha + B_2 u_y (x, y, t - \Delta t_j) \sin \beta \\
\quad v^y (x, y, t) &= -u_y (x, y, t - \Delta t_j) \sin \alpha - B_2 u_x (x, y, t - \Delta t_j) \sin \alpha - B_2 u_x (x, y, t - \Delta t_j) \cos \beta
\end{align*}
\]

(3)

In order to calculate the free-field stress wave propagation, the local-coordinate system \((\xi, \eta)\) is introduced. \(\xi\) is the propagation direction of the wave, \(\eta\) is the normal direction of the propagation direction. Assuming any point \((x, y)\) (where \(0 \leq x \leq L, 0 \leq y \leq H\)), the corresponding stress of the incident SV wave with the incident angle \(\alpha\) is:

\[
\begin{align*}
\sigma_x &= 2\tau_{\xi\eta} \sin \alpha \cos \alpha = -\frac{G \sin 2\alpha}{c_s} \hat{u}_s (x, y, t - \Delta t_j) \\
\tau_{xy} &= \tau_{\eta\xi} (\cos^2 \alpha - \sin^2 \alpha) = -\frac{G \cos 2\alpha}{c_s} \hat{u}_s (x, y, t - \Delta t_j) \\
\sigma_y &= -2\tau_{\eta\xi} \sin \alpha \cos \alpha = \frac{G \sin 2\alpha}{c_s} \hat{u}_s (x, y, t - \Delta t_j)
\end{align*}
\]

(4)

The corresponding stress of the reflected SV wave with the reflection angle \(\alpha\) is:

\[
\begin{align*}
\sigma_x &= -2\tau_{\xi\eta} \sin \alpha \cos \alpha = B_2 \frac{G \sin 2\alpha}{c_s} \hat{u}_s (x, y, t - \Delta t_j) \\
\tau_{xy} &= \tau_{\eta\xi} (\cos^2 \alpha - \sin^2 \alpha) = -B_2 \frac{G \cos 2\alpha}{c_s} \hat{u}_s (x, y, t - \Delta t_j) \\
\sigma_y &= 2\tau_{\eta\xi} \sin \alpha \cos \alpha = -B_2 \frac{G \sin 2\alpha}{c_s} \hat{u}_s (x, y, t - \Delta t_j)
\end{align*}
\]

(5)

The corresponding stress of the reflected P wave with the reflection angle \(\beta\) is:

\[
\begin{align*}
\sigma_x &= (\sigma_x \sin^2 \beta + \sigma_y \cos^2 \beta) = -B_2 \frac{\lambda + 2G \sin^2 \beta}{c_p} \hat{u}_s (x, y, t - \Delta t_j) \\
\tau_{xy} &= (\sigma_y - \sigma_x) \sin \beta \cos \beta = B_2 \frac{G \sin 2\beta}{c_p} \hat{u}_s (x, y, t - \Delta t_j) \\
\sigma_y &= (\sigma_x \cos^2 \beta + \sigma_y \sin^2 \beta) = -B_2 \frac{\lambda + 2G \cos^2 \beta}{c_p} \hat{u}_s (x, y, t - \Delta t_j)
\end{align*}
\]

(6)

3. Numerical Verification
In order to verify the accuracy of the above seismic wave input method, the authors select the mechanical model (Figure 3) to study the oblique incidence of uniform elastic half-space seismic waves, which has been studied by Liu and Lu [10] using other methods. The finite element calculation range is a \(762m \times 381m\) finite field and the far-field boundary is modeled using a viscoelastic boundary. The model medium has an elastic modulus of 13.23GPa, a mass density \(\rho\) of 2700kg/m\(^3\) and a
Poisson’s ratio of 0.25. Three monitoring points A, B and C are selected, and unit velocity pulses are used in incident waves:

$$u_0(t) = \frac{1}{2} \left[1 - \cos(2\pi ft)\right], \quad f = 4.0 \leq t \leq 0.25$$  \hspace{1cm} (7)

Figure 3. Computational model.  \hspace{1cm} Figure 4. Velocity time curve of input earthquake wave.

The incident wave uses a shear wave unit velocity pulse (Figure 4), the incident angles $\alpha = 0^\circ, 30^\circ$ are selected, respectively. When the plane SV wave is vertically incident, the amplitude of the incident wave and the reflected wave are superimposed on the free surface due to the reflection of the free surface. The displacement of the free surface is exactly twice that of the incident wave. While the incident wave is in 30 degree oblique incidence, both the value of wave and delay are different from the vertical incidence (Figure 5), and the calculation result is consistent with the studies of Liu and Lu [10]. It is proved that the oblique input method based on viscoelastic boundary can reflect the oblique incidence of Sv wave well and has high precision.

Figure 5. Time history of horizontal velocity at point A under different incident angles.

4. Analysis of Slope Dynamic Response
To simplify the study, the rock slope will be simplified into a uniform continuous elastic model. Figure 6 shows a two-dimensional simplified slope and Figure 7 shows a finite element model which is performed with finite element method combining viscoelastic artificial boundary with display dynamics calculation. The slope height $H$ is 100 m, the height of the point on slope surface is $h$, the relative height is $h/H$, slope angle $\theta = 45^\circ, 50^\circ, 60^\circ$, and $L$ is 300 m. The model medium has an elastic
modulus of 20GPa, a mass density $\rho$ of 2500 kg/m$^3$ and a Poisson’s ratio of 0.25. The authors use an artificial wave fitted in accordance with standard design response spectrum as an incident wave, of which the peak ground acceleration (PGA) was adjusted to 1 m/s$^2$ for facilitate the calculation. Figure 8 shows the time history of designed seismic motion as well as the corresponding response spectra, time span is 19.99 seconds.

For expression, $\varphi=A_a / C_\alpha$ is defined as the peak ground acceleration (PGA) amplification coefficient, where $A_a$ is the peak acceleration of seismic dynamic response at any point in the slope; $C_\alpha$ is the peak acceleration of the seismic dynamic response in the ground motion. According to related projects' experience, the authors have selected different slope angles ($\theta=45^\circ,50^\circ,60^\circ$) and different incident angles ($\alpha=0^\circ,15^\circ,30^\circ$), at the same time set the monitoring point along the slope surface. Based on the data recorded at each monitoring point, the acceleration distribution variation law of rock slope seismic response under different slope angles was discussed when the plane SV wave were obliquely incident.

4.1. Influence of Oblique Incidence on Acceleration of Slope Surface

Figure 9 shows the distribution law of PGA amplification coefficient of the slope surface by obliquely incident SV wave when the slope angles are 45$^\circ$, 50$^\circ$, and 60$^\circ$, respectively. The PGA amplification coefficient near the slope toe fluctuates locally when incident vertically. As the incidence angle increases, the PGA amplification coefficient increases. With the increase of elevation, the PGA
amplification coefficient of each point on the slope surface gradually increases, and the maximum value appears near the crest of the slope. It shows a non-linear growth trend and increases sharply at the top of the slope. The further away from the top of the slope, the smaller the PGA amplification effect.

(a) Slope angle (θ=45°)  

(b) Slope angle (θ=50°)

(c) Slope angle (θ=60°)

(d) X-acceleration time history at the crest of the slope under vertical incidence (θ=45°)

Figure 9. Distribution law of PGA amplification coefficient of slope surface under the obliquely incident SV-wave.

4.2. Influence of Slope Angle on Acceleration of Slope Surface

Figure 10 shows the relationship between the PGA amplification coefficient of the slope surface and slope relative height at different slope angles at vertical incidence and 30° oblique incidence. Near the foot of slope, the change of slope angle has no obvious effect on the dynamic response of rock slope. As the height h of the slope surface point is larger than 0.5H, the PGA amplification coefficient increases with the increase of the slope angle. The steeper the slope is, the greater the amplification effect at the crest will be.
Figure 10. Distribution law of PGA amplification coefficient of oblique incidence in different slope angle.

5. Conclusion
In this paper, the finite element input of oblique incident ground motion is realized by the viscoelastic boundary combined with the equivalent load wave input method. Taking the simplified slope model as the research object, the authors conducted sensitivity analysis on the seismic wave incident angle and the slope angle, and discussed the rock slope dynamic response law of obliquely incident plane SV-wave under different slope angles. The results show that the angle of incidence and slope angle has a great impact on seismic dynamic response of slope surface. When the SV wave is incident obliquely, the peak ground acceleration of the rock slope gradually increases with the incident angle increasing, and the peak ground acceleration near the top of the slope increases with the increase of slope angle. When the SV wave is oblique incident, the dynamic response of the slope surface is obviously different from the traditional vertical incident method. Therefore, it is necessary to consider the influence of oblique incidence of seismic SV waves in safety evaluation and design of slope stability analysis.

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