An Effective Action for Finite Temperature QCD with Fermions

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Using lattice perturbation theory at finite temperature, we compute for staggered fermions the one-loop fermionic corrections to the spatial and temporal plaquette couplings as well as the leading $Z_N$ symmetry breaking coupling. Numerical and analytical considerations indicate that the finite temperature corrections to the zero-temperature calculation of A. Hasenfratz and T. DeGrand are small for small values of $\kappa$, but become significant for intermediate values of $\kappa$. The effect of these finite temperature corrections is to ruin the agreement of the Hasenfratz-DeGrand calculation with Monte Carlo data. We conjecture that the finite temperature corrections are suppressed nonperturbatively at low temperatures, resolving this apparent disagreement. The $Z_N$ symmetry breaking coupling is small; we argue that it may change the order of the transition while having little effect on the critical value of $\beta$.

1. INTRODUCTION

It has been known for some time that the effects of heavy dynamical fermions can be included in Monte Carlo simulations using a hopping parameter expansion of the fermion determinant. This is done routinely in perturbative QED where electron loops are included in an effective action. Recently Hasenfratz and DeGrand [1,2] have performed a zero-temperature calculation of the shift in the lattice gauge coupling constant induced by staggered dynamical fermions and applied the result to the finite temperature phase transition in QCD. Their result for the shift in the critical coupling, in the form $\beta_{F}^{\text{pure}} = \beta_{c} + \Delta \beta_{F}$, was found to hold rather well down to small values of the fermion mass, even for $N_{t} = 4$; this is particularly surprising since $\Delta \beta_{F}$ is calculated using lattice perturbation theory at zero-temperature. In order to understand the effects of finite temperature, we have calculated the one-loop fermionic corrections to the spatial and temporal plaquette couplings, as well as the leading $Z_N$ symmetry breaking coupling.

2. RENORMALIZATION OF $\beta$ AT FINITE TEMPERATURE

The one-loop finite temperature fermionic correction to the $O(A_{t}^{2})$ term in the gauge field lattice action is given by

$$
S_{\text{eff}} = -g^{2} \sum_{p_{\mu}} \frac{1}{N_{t}} \int_{-\pi}^{\pi} \frac{d^{3}p}{(2\pi)^{3}} \text{Tr} \{ \hat{A}_{\mu}(p) \} \times \hat{A}_{\nu}(p) \frac{\beta}{2N_{c}} \left[ D_{\mu\nu}^{(0)}(p) + D_{\mu\nu}^{(1)}(p) - D_{\mu\nu}^{(2)} \right]
$$

with

$$
D_{\mu\nu}^{(0)}(p) = 4 \left[ \delta_{\mu\nu} \sum_{\alpha} \sin^{2} \left( \frac{p_{\alpha}}{2} \right) - \sin \left( \frac{p_{\mu}}{2} \right) \sin \left( \frac{p_{\nu}}{2} \right) \right]
$$

$$
D_{\mu\nu}^{(1)}(p) = -\frac{1}{2} \sum_{k_{0}} \frac{1}{N_{t}} \int_{-\pi}^{\pi} \frac{d^{3}k}{(2\pi)^{3}} \text{Tr} \left[ R(k_{\mu}) \times S^{-1} (k - p/2) R(k_{\nu}) S^{-1} (k + p/2) \right]
$$

where the vertex functions are given by

$$
R(k_{\mu}) = i \gamma_{\mu} \cos(k_{\mu}) \quad Q(k_{\mu}) = -i \gamma_{\mu} \sin(k_{\mu})
$$
with no sum over $\mu$, and the inverse fermion propagator is given by
\[ S(k) = \frac{1}{2\kappa} + i\gamma_\mu \sin(k_\mu). \] (6)

At finite temperature there are two independent tensors of order $p^2$ which are four-dimensionally transverse[3]. This manifests itself as separate renormalizations of the spatial and temporal gauge couplings. We find that
\[ \Delta \beta_s = -N_c \sum_{k_0} \frac{1}{N_t} \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^d} \Phi(k; 1, 2) \] (7)
with
\[
\Phi(k; \mu, \nu) = 32B^{-2}(k) \cos^2(k_\mu) \cos^2(k_\nu) \\
-4096B^{-4}(k) \sin^2(k_\mu) \cos^2(k_\mu) \\
\times \sin^2(k_\nu) \cos^2(k_\nu)
\] (8)

and
\[ B(k) = \frac{1}{\kappa^2} + 4 \sum_{\alpha} \sin^2(k_\alpha). \] (9)

A different result is obtained for $\Delta \beta_t$ by replacing $\Phi(k; 1, 2)$ in (7) with $\Phi(k; 0, 1)$. Figure 1 compares $\Delta \beta$ per fermion (spatial and temporal) vs. $\kappa$ for $N_t = 4$ and $N_t = 6$ with the zero temperature result of Hasenfratz and DeGrand. The finite temperature values approach the zero temperature result from below as a consequence of the antiperiodic boundary conditions. Note that they ruin the agreement between the zero-temperature calculation and the Monte Carlo results for intermediate to large values of $\kappa$.

The connection between the the zero and finite temperature result can be understood more physically by transforming the sum over Matsubara frequencies into a sum over images using the Poisson summation formula for antiperiodic boundary conditions. The shift in the spatial plaquette coupling is then given by
\[
\Delta \beta_s = -2N_c \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n+1}} \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^d} \Phi(k; 1, 2) \\
\times \cos(nN_tk_0)
\] (10)
with a similar result for $\Delta \beta_t$. This form has a simple physical interpretation: the integer $n$ labels the net number of times the fermion wraps around the lattice in the temporal direction. Numerically, the dominant corrections to the zero-temperature result come from the first few values of $n$, with the $n = 1$ and $n = 2$ terms accounting for more than 90% of the finite temperature correction for $\kappa \leq 2.0$.

Figure 1. $\Delta \beta$ per fermion (spatial and temporal) vs. $\kappa$ for various $N_t$.

Figure 2. Diagrams contributing to finite temperature corrections for the renormalization of $\beta$.

The finite temperature corrections result from the $O(A_2^d)$ expansion of image diagrams such as
those depicted in Figure 2. The vertical segments of the image diagrams are powers of Polyakov loops. Given that $Z_3$ symmetry is maintained in the confined phase of QCD, the contributions of these terms is negligible below $\beta_c$. Thus, the finite temperature corrections to $\beta$ may be suppressed nonperturbatively in the confined regime. In particular, near $\beta_c$ the zero-temperature result will hold for $\Delta \beta$.

3. $Z_N$ SYMMETRY BREAKING IN THE EFFECTIVE ACTION

There is another set of terms induced by the fermion determinant at finite temperature [4]. Using the hopping parameter expansion and the Poisson summation formula, we find an additional contribution to the effective action:

$$S_{\text{eff}} = \sum_{\vec{x}} \sum_{n=1}^{\infty} (-1)^{n+1} h_n(\kappa) \text{Re} \left[ \text{Tr} P^n(\vec{x}) \right]$$

where $P(\vec{x})$ denotes a Polyakov loop and the couplings $h_n$ are given by

$$h_n(\kappa) = -4N_t \int_{-\pi}^{\pi} \frac{d^4q}{(2\pi)^4} \ln \left[ \frac{1}{4\kappa^2} + \sum_\mu \sin^2(q_\mu) \right] \cos(nN_tq_0)$$

The maximum values of the $h_n$ are obtained when $m_F = 0$. For $N_t = 4$ we find $h_1^{\text{max}} = 0.107$, $h_2^{\text{max}} = 0.00445$, and increasingly smaller values for higher order couplings. Including just the $h_1$ term in the action of an otherwise pure QCD Monte Carlo simulation, we observe that this additional source of $\Delta \beta$ is small compared to the renormalization of the plaquette couplings discussed in the preceding section. For example, even $h_1 = 0.1$ at $N_t = 4$ leads to a shift in $\beta$ per fermion of 0.00325. This value of $h_1$ corresponds to $m_F = 0.17$ which yields a zero-temperature predicted shift in $\beta$ per fermion of 0.104.

The effect of the $h_1$ term in the effective action, however, is not trivial. We find a first-order phase transition for values of $h_1$ smaller than approximately 0.08. Including zero-temperature plaquette coupling renormalization, the endpoint of this first-order phase transition line maps to the point (0.394, 4.68) in the $(m_F, \beta)$-plane for the case of sixteen degenerate staggered fermion species. This is near the endpoint of the first-order phase transition observed in simulations with dynamical staggered fermions [5].

A variety of workstations were used to obtain the results presented in this section. Typically 42000 sweeps were made on $10^3 \times 4$ lattices. Definitive numerics will require substantially greater computational resources.

4. CONCLUSIONS

Finite temperature corrections to the gauge coupling renormalization lift the degeneracy of the spatial and temporal couplings, but the results are in conflict with Monte Carlo data. An additional finite temperature shift in $\beta$, due to an induced coupling to Polyakov loops, is negligible. This coupling does, however, influence the order of the transition and may be the most important factor in determining the end point of the first-order deconfinement phase transition line. As we have shown, it is plausible that the success of the zero-temperature calculation in determining the critical value of $\beta$ is due to a nonperturbative suppression of finite temperature effects in the low temperature regime. Specifically, the small expectation value of the Polyakov loops at low temperatures indicates a suppression of those quark paths which account for finite temperature corrections.

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$\Delta \beta_{\text{fermion}}$ vs $\beta_c$ and $\beta_{\text{bare}}$ for zero temperature.