Impact of High Energy Mining-Induced Seismic Shocks from Different Mining Activity Regions on a Multiple-Support Road Viaduct

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Abstract: In this paper, the dynamic responses of a large-scale multiple-support road viaduct to mining-induced seismic events registered in two regions of mining activity were compared. The regions differ in geological structure, which results in discrepancies in the dominant frequency content. Spatial variation of ground motion causing the kinematic excitation non-uniformity was accounted for in the dynamic analyses of this large-scale structure. Non-uniform mining-induced kinematic excitation models were proposed, with respect to the specificity of mining origin quakes. The dynamic performance of the viaduct was determined using three different methods of calculation: the time history analysis, the response spectrum analysis, and the multiple support response spectrum analysis. Both the uniform and non-uniform kinematic excitation models were adopted for the dynamic performance assessment. The research revealed that the dynamic response of some members of the structure, determined using the non-uniform excitation model, was significantly greater than that obtained for the uniform one. Hence, in the dynamic analysis of multiple-support structures under mining-induced events, the effect of spatial variation of ground motion should be considered. The study pointed out that the commonly used response spectrum analysis may lead to the underestimation of the dynamic response of large-scale multiple-support structures. Instead, the multiple support response spectrum method, which takes into account the non-uniformity of ground motion, is recommended as a conservative approximation. This method provides a safe upper estimation of the full-dynamic analysis results of large-scale structures under mining-induced tremors. Finally, the research indicated that the dynamic performance of a structure strongly depends on the frequency range attributed to a specific mining region. The dynamic performance of identical engineering structures under tremors of similar maximal amplitudes may differ significantly due to discrepancies in frequency contents of shocks occurring in various mining regions.

Keywords: mining-induced seismicity; dynamic response; bridge dynamics; multiple-support structure; non-uniform kinematic excitation; multiple support response spectrum method

1. Introduction

The growing demand for fuel and mineral extraction has become a fundamental challenge to the sustainable development of mining clusters. In the next decades, scientists will have to find a balance between the economic benefits and the environmental damage caused by mining. Mining works have a significant impact not only on the environment but on civil surface infrastructure as well. The reduction of the environmental damage caused to land and air resources by mines [1–3] must go hand-in-hand with the development of infrastructure protection strategies [4]. The extraction of coal, natural gas, oil, or copper is the main generator of additional static (ground deformation) [5–7] and
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dynamic (shocks of mining origin) loads, which have a substantial impact on engineering structures [8].
In recent years, the evaluation of risk resulting from mining-induced tremors for surface infrastructure
became a task of intensive studies, since energies and amplitudes of vibration caused by high-energy
mining-induced events are comparable to those resulting from small earthquakes. So far the vast
majority of the studies on the influences of mining shocks on surface structures have been focused
on residential buildings [9–13]. The recognition of mining-related effects on untypical engineering
structures is still insufficient. In particular, the effects of mining shocks on large-scale structures are not
well researched.

To assess the impact of shocks triggered by mining activities on engineering objects, dynamic
analyses are carried out using various calculation methods. On account of similarities between energies
and amplitudes of vibrations caused by high-energy mining-related tremors and small earthquakes,
methods applied in seismic engineering can also be utilized for the evaluation of dynamic responses
of structures to mining tremors. Due to the relatively simple application and in accordance with
international standards and national codes recommendations [14–16], scientists and engineers often
use simplified dynamic analysis methods, in particular, response spectrum analysis (RSA) [17–19].
This approach allows for the estimation of the stress level in structure elements without advanced
numerical calculations and leads to correct results for typical infrastructure facilities, such as residential,
industrial, or warehouse buildings. However, mining activity areas are often highly urbanized,
with many atypical structures, the design process of which requires more advanced calculation tools.
The development of computational techniques opens new perspectives in the field of modeling for
example large-scale engineering structures.

In the case of the dynamic analysis of large-scale objects, the spatial variation of earthquake
ground motion (SVEGM) must be taken into consideration [20]. In such cases, the non-uniformity of
kinematic excitation, both in terms of amplitude and frequency, may play a key role in the dynamic
response level. Most of the simplified methods like the response spectrum analysis mentioned above,
do not take into account this excitation complexity. These methods are based on the assumption that the
ground motion under the structure’s supports is uniform. Hence, in the case of large-scale structures,
with dimensions comparable to the length of the seismic wave, such simplified analysis may lead to
significant errors, for example underestimation of the dynamic response to ground motion [21,22].

Typical large-scale objects, sensitive to the above-mentioned excitation non-uniformity, are called
multiple-support structures [23]. The dynamic analysis of such objects, taking into account the
non-uniform ground motion, is usually carried out using full-time dynamic analysis (time history
analysis—THA). The vast majority of such studies address objects localized in areas of seismic
activity. The non-uniform kinematic excitations of seismic origin are mostly considered in scientific
research analyzing long bridges and footbridges [24–27], footbridges [28], tunnels [29], dams [30,31],
pipelines [32], and large-dimensional cooling towers [33]. There are only a few works on the dynamic
response of multiple-support structures, subjected to non-uniform kinematic excitation triggered by
mining-induced shocks [34,35].

In addition to THA, there is another method meeting the requirements of the SVEGM effect
used for the assessment of the dynamic performance of large-dimensional structures, called the
multiple support response spectrum (MSRS) analysis [36]. The MSRS method is based on the RSA,
but additionally takes into account the non-uniformity of ground motion. So far MSRS has been
utilized mainly in the seismic analysis [37,38], but not in studies concerning the dynamic analysis
of structures exposed to non-uniform excitation of mining origin. The lack of such investigations
has become a genesis of the present study on the numerical analysis of the dynamic response of the
multiple-support structure subjected to mining-related seismic events. Both methods, the full-time
dynamic analysis (THA) and the approximate multiple support response spectrum (MSRS), require
the assumption of the non-uniform kinematic excitation models. Such models are widely accessible
in literature but, being appropriate for natural seismic cases, they do not address characteristics of
mining-induced excitations. This notion leads to an urgent need to create models of non-uniform
kinematic excitation dedicated strictly for mining-related tremors and suitable for utilizing in the above-mentioned methods. Mining-related seismicity is a significant problem in regions where numerous underground mines are located. For example, an urgent need to protect both existing and newly designed engineering structures against mining-induced shocks exists in two regions of strong mining activity in Poland: the Upper Silesian Coal Basin and the Legnica-Głogow Copper District. The regions differ in geological structure, which results in differences in parameters characterizing mining-triggered tremors, like energy, the frequency contents, and the lengths of shocks.

The main objective of this paper is to assess and compare the dynamic performance of a multiple-support road viaduct subjected to the mining-induced seismic events that occurred in two different regions of mining activity. The main criterion of the selection of these events was the difference in their frequency contents. The dynamic performance of similar structures located in these two particular mining regions may differ in the dynamic response level, due to the dissimilarities of the dominant frequency ranges of both shocks, even if their peak ground accelerations are equal.

To accomplish the above-mentioned objective the acceleration-time histories registered in both regions were applied as the input kinematic excitations. The stress levels in the structural members of the viaduct under the action of both mining tremors were compared. The dynamic performance of the viaduct subjected to the mining-triggered excitations was determined using three different methods of calculation: the time history analysis (THA), the response spectrum analysis (RSA), and the multiple support response spectrum (MSRS) analysis. Both the uniform and non-uniform kinematic excitation models were adopted for the dynamic performance assessment.

Comparison of results of these simultaneous computations allowed for evaluation of the impact of the non-uniformity of excitation on the structure’s dynamic response level, as well as assessing the applicability of both simplified approaches (RSA and MSRS) for proper, conservative estimation of stress levels for both excitation models. This was the second purpose of the research.

Investigations on the dynamic performance of large-dimensional structures subjected to spatially varying natural seismic excitation can be found in numerous recent studies. However, the problem of non-uniformity of kinematic excitation is, to the best of the authors’ knowledge, rarely if ever addressed in the context of mining-induced events. The novelty of this research lies in:

- Proposing the models of non-uniform kinematic excitation of mining origin;
- Adopting the multiple-support response spectrum method for assessing the dynamic performance of a multiple-support structure under mining-triggered shock; and
- Performing comprehensive research on the dynamic performance of a multiple-support structure exposed to non-uniform kinematic excitations of mining origin.

The above-mentioned aspects have not been highlighted in the available literature, and make this study innovative in the fields of civil surface infrastructure protection strategies and sustainable resources extraction development.

2. Materials and Methods

2.1. Non-Uniformity of Kinematic Excitation of Multiple-Support Structures

In the case of long multiple-support structures, the dimensions of which are comparable with a seismic wavelength, the problem of the spatial variation of earthquake ground motion (SVEGM) becomes an issue [20]. Generally, authors suggest that incorporating the effect of non-uniformity of seismic excitation decreases the dynamic response of a structure, due to a reduction of average amplitudes of ground motion. On the other hand, some authors mention that occurrence of pseudo-static effects, resulting from differences in excitation in particular points of foundations, may lead to an increased response in some structural members [20,28].

In practice, four non-uniformity reasons are most commonly taken into account during dynamic analyses: the wave passage effect, the incoherence effect, the attenuation effect, and the site effect [20,23].
The wave passage effect results from the difference in the arrival times of the wavefront at different locations, so the subsequent ground points localized in the direction of wave propagation repeat the same oscillations with a time delay. The time lag is directly related to the distances between the structure’s supports and the seismic shock wave velocity, which strongly depends on the ground stiffness. The effect of incoherence, arising from the scattering of waves in the heterogeneous ground, can be taken into account by means of the coherency function, based on the power and cross-power spectrum density. The function parameters should be defined based on experimental studies. The attenuation effect describes the amplitude decay with increasing distance from the vibration source. This effect strongly depends on the soil conditions in the object localization area and the energy of the earthquake. The relation between the amplitude reduction and the distance from the source point can take different forms for different seismic regions [10,39]. The site effect depends directly on local ground conditions. The complicated system of heterogeneous soil layers results in the modifications of the seismic wave velocity, amplitude, and frequency domain, and in consequence, in various ground motion which causes the complex dynamic response of the object.

In the paper, the problem of non-uniform kinematic excitation was solved using the classical equation of motion for a multi-degrees of freedom system. The classic formula describing the ground motion can be written as [20]:

\[
\begin{bmatrix}
M_{ss} & M_{sg} \\
M_{gs} & M_{gg}
\end{bmatrix}
\begin{bmatrix}
\ddot{X}_s \\
\ddot{X}_g
\end{bmatrix}
+ 
\begin{bmatrix}
C_{ss} & C_{sg} \\
C_{gs} & C_{gg}
\end{bmatrix}
\begin{bmatrix}
\dot{X}_s \\
\dot{X}_g
\end{bmatrix}
+ 
\begin{bmatrix}
K_{ss} & K_{sg} \\
K_{gs} & K_{gg}
\end{bmatrix}
\begin{bmatrix}
X_s \\
X_g
\end{bmatrix}
= 
P
\]  

(1)

In Equation (1) \( M \), \( C \), \( K \) mean the mass, damping, and stiffness matrices, respectively, with indices \( s \) referring to un-supported degrees of freedom, \( g \)—supported ones, whereas \( sg \) and \( gs \) representing the coupling matrices associated with both types of degrees of freedom. \( X_s \) is the total displacement vector of the structure points. \( X_g \) and \( P \) mean the ground displacement vector and the reaction vector at the supports, respectively.

The total displacement of the system is a sum of the quasi-static (\( X_s^{qs} \)) and the dynamic (\( X_s^d \)) components (Equation (2)). The quasi-static displacements of the non-supported degrees of freedom are obtained from the equation of motion based solely on the stiffness matrix and the supports’ motion, with the exclusion of the dynamic effects (Equation (3)):

\[
X_s = X_s^{qs} + X_s^d,
\]

(2)

\[
X_s^{qs} = -K_{ss}^{-1}K_{sg}X_g = R X_g.
\]

(3)

Substituting the quasi-static displacements (Equation (3)) into the equation of motion (Equation (1)) leads to Equation (4), from which the dynamic displacements \( X_s^d \) can be obtained:

\[
M_{ss}\ddot{X}_s^d + C_{ss}\dot{X}_s^d + K_{ss}X_s^d = -(M_{sg} + M_{ss}R)\ddot{X}_g - (C_{sg} + C_{ss}R)\dot{X}_g.
\]

(4)

In the case of kinematic excitation, the term comprising damping components (\( C_{sg} + C_{ss}R \)) is signficantly smaller than the mass damping, and as such can be omitted [15,20]. After the above-mentioned simplification, Equation (4) takes the form:

\[
M_{ss}\ddot{X}_s^d + C_{ss}\dot{X}_s^d + K_{ss}X_s^d = -(M_{sg} + M_{ss}R)\ddot{X}_g.
\]

(5)

Analyzing Equations (3) and (5) one can notice that the total displacements of a structure depend solely on the ground accelerations. It should be noted, that the vector \( \ddot{X}_g \) contains accelerations of all supports of a structure, and the time histories of ground accelerations are usually registered by a seismological station at one control point only. While analyzing multiple-support large-scale structures the variation of earthquake ground motion should be additionally accounted for. Hence, a model of
kinematic excitation has to be implemented. On the basis of accelerations registered at one control point and the adopted kinematic excitation model, accelerations of all supports of a structure can be determined. In the model of non-uniform mining-induced kinematic excitation adopted in the present study, three sources of non-uniformity, discussed earlier in this section, were taken into consideration: the wave passage effect, the incoherence effect, and the attenuation effect. The site effect was neglected in this case due to the high homogeneity of ground conditions at the analyzed location.

2.2. Data on the Mining-Induced Shocks Registered in the Selected Mining Activity Regions

2.2.1. The Mining-Induced Shock Registered in the Upper Silesian Coal Basin

The first shock analyzed in this study was registered by the seismic station located in the Upper Silesian Coal Basin (USCB). The shock duration was over 6 s, with the strongest intensity phase lasting the first 3 s. The energy of the shock was $1 \times 10^7$ J, which puts it in the category of high-energy events [10]. The maximum horizontal peak ground acceleration (PGA) of the event reached 0.35 m/s$^2$ (in the WE direction), whereas in the NS and vertical directions PGA equaled 0.28 m/s$^2$ and 0.12 m/s$^2$, respectively. For the purposes of this study, all registered data was scaled up proportionally, so that PGA in the horizontal direction WE was raised to 1.7 m/s$^2$. This value corresponds to the PGA of a mining-induced seismic shock registered in the USCB region in September 2015 [4], classified as a high energy event of $1 \times 10^9$ J. The acceleration-time histories of the shock in three directions are presented in Figure 1, whereas the frequency spectra are shown in Figure 2.

![Figure 1. The time history of the ground acceleration registered in the USCB region: (a) horizontal direction WE; (b) horizontal direction NS; (c) vertical direction Z.](image1.png)

![Figure 2. The frequency spectra of the acceleration up to 30 Hz registered in the USCB region: (a) horizontal direction WE; (b) horizontal direction NS; (c) vertical direction Z.](image2.png)

2.2.2. The Mining-Induced Shock Registered in the Legnica-Glogow Copper District

The second tremor taken into consideration in the research was registered by the seismic station in the Legnica-Glogow Copper District (LGCD). The total duration of this event was 5 s, with the phase of strong intensity over 2 s. The PGA in the WE and NS directions reached 0.8 m/s$^2$ and 0.37 m/s$^2$, respectively, whereas the vertical PGA equaled 0.45 m/s$^2$. The shock energy of $5 \times 10^7$ J, (classifying...
the tremor as a high-energy event [10]) was five times larger than that of the shock recorded in the USCB region. Again, for the purposes of this study, all registered data was scaled up proportionally, so that the PGA in the horizontal WE direction was raised to 1.7 m/s². This value corresponds to the maximum PGA of a real mining-induced seismic shock registered in the LGCD region in January 2003 [4]. The components of the shock acceleration and its frequency spectra are presented in Figures 3 and 4, respectively.

![Figure 3](image1.png)

**Figure 3.** The time history of the ground acceleration registered in the LGCD region: (a) horizontal direction WE; (b) horizontal direction NS; (c) vertical direction Z.

![Figure 4](image2.png)

**Figure 4.** The frequency spectra of the acceleration up to 30 Hz registered in the LGCD region: (a) horizontal direction WE; (b) horizontal direction NS; (c) vertical direction Z.

Scaling up the maximum values of PGA to the same value for both analyzed regions enabled focusing on the discrepancies in the frequency spectra only, and not taking into consideration differences in the shock intensities. It is also worth noticing that the value of 1.7 m/s² corresponds to maximal recorded PGA of mining tremors in both regions in the last two decades [4]. The dynamic responses of the bridge to the tremors with such a large value of maximum PGA may be considered as substantial and crucial results for the assessment of the mining tremors impact on the structure.

2.2.3. The Comparison of Frequency Spectra of the Selected Shocks

The shocks’ frequency contents differ for various mining regions in Poland. Typically mining-induced shocks are registered with sensors covering the frequency range up to 30 Hz. Dominant frequency ranges of mining tremors registered in the Legnica-Glogow Copper District generally consist of higher frequencies than those registered in the Upper Silesian Coal Basin [10].

The dominant frequencies of the selected shock from the USCB region in all directions are located in the range from 1.6 to 4.8 Hz, with a noticeable peak at 3.5 Hz (see Figure 2). The frequencies of the mining shock registered in the LGCD region show maxima at the dominant frequencies from 5 to 10 Hz, with a visible peak at 7 Hz. Additionally, in the direction WE the amplified acceleration occurs at the frequency range from 17 to 22 Hz with a clear peak at 20 Hz.

Hence, it is clearly noticeable that the tremors from both analyzed mining regions differ significantly, as far as the dominant frequency ranges are concerned.
2.2.4. Amplitude Decay with Distance for the Selected Mining Activity Regions

The attenuation effect describes the reduction in ground motion intensity with increasing distance from a vibration source. This phenomenon is more rapid in the case of mining shocks compared to seismic events [10]. Hence, the reduction of tremors’ amplitudes is non-negligible and may play an important role in the dynamic response of long structures. The attenuation function adopted in this study is based on the empirical dependence of the level of vibration amplitudes on distance from the epicenter, elaborated by seismologists for the analyzed mining regions [10,39]:

\[ a(r) = a_e H(r) \]  \hspace{1cm} (6)

where \( a(r) \) is the acceleration at a distance \( r \); \( a_e \) is the acceleration in the epicenter zone; \( r \) is the distance from the epicenter (km); and \( H(r) \) is the empirical function describing the decrease of amplitudes resulting from geometric damping outside the epicenter zone.

The empirical function \( H(r) \) takes the following forms:

for the Upper Silesian Coal Basin [39]:

\[ H(r) = 1.53r^{0.155}e^{-0.65r} + 0.014 \]  \hspace{1cm} (7)

and for the Legnica-Glogow Copper District [10]:

\[ H(r) = 0.8575r^{-1.0098} \]  \hspace{1cm} (8)

The graphs of \( H(r) \) functions for the Upper Silesian Coal Basin and the Legnica-Glogow Copper District are presented in Figure 5.

![Figure 5](image-url)  \hspace{1cm} Figure 5. The empirical functions \( H(r) \) adopted in the determination of the reduction of vibration amplitudes with the distance from the epicenter.

2.3. Calculation Methods of the Dynamic Response of a Structure to Kinematic Excitation

In the present study, the complex dynamic performance of the viaduct subjected to the mining-induced excitation was determined using three different methods of calculation of the dynamic response to kinematic excitation: the time history analysis (THA), the response spectrum analysis (RSA), and the multiple support response spectrum (MSRS) analysis.

2.3.1. Time History Analysis (THA)

The THA algorithm [40] is based on the direct integration of equations of motion at each time increment. Due to its high accuracy, the THA method is suggested in EC standards [14] for both linear and nonlinear dynamic analyses. For the THA algorithm kinematic excitation can be defined by the vector of the supports’ accelerations. Both the uniform and non-uniform kinematic excitation models...
can be adopted for the dynamic performance assessment of a structure using the THA. In this study, the mining shocks recorded in the USCB and LGCD regions (see Figures 1 and 3, respectively) were the sources of accelerations of the viaduct supports.

2.3.2. Response Spectrum Analysis (RSA) with Spectral Curves for the Selected Mining Regions

The RSA is a simplified method of determining the impact of vibration on a structure [40], based on the assumption that any structure movement can be represented by a superposition of its modal responses. Due to this assumption, the application of the superposition principle limits this method to linear problems. The RSA leads to the upper estimation of the structure response (displacements) only, without providing it as a function of time. The method allows for estimating the dynamic behavior of a structure under uniform kinematic excitation only.

The RSA requires adopting spectral curves (response spectrum functions) representing the peak response of a simple harmonic oscillator subjected to a base motion. In the EC standards [14], for seismic analysis purposes, the spectral functions are stochastically determined for typical seismic shocks, taking into account the local soil conditions at a structure’s location. However, for the purposes of mining-induced seismicity, response spectrum curves related to different mining regions have been developed on the basis of representative tremors and local soil conditions.

In the present study, the functions of the local response spectra have been adopted for the discussed regions of mining activity. In both cases, the damping ratio of 5% was assumed. The following response spectrum functions [10,41] were applied (Figure 6):

\[
D_a^{USCB}(f) = \begin{cases} 
0.25 & f \leq 1 \text{ Hz} \\
0.921 f - 0.671 & f \in (1 - 3.3) \text{ Hz} \\
1.8 + 2 f^{-1} & f \in (3.3 - 10) \text{ Hz} \\
1 + 10 f^{-1} & f > 10 \text{ Hz} 
\end{cases} \tag{9}
\]

\[
D_a^{LGCD}(f) = \begin{cases} 
0.25 & f \leq 1 \text{ Hz} \\
0.6 f - 0.4 & f \in (1 - 5) \text{ Hz} \\
2.6 & f \in (5 - 7.14) \text{ Hz} \\
1 + 11.43 f^{-1} & f > 7.14 \text{ Hz} 
\end{cases} \tag{10}
\]

![Figure 6. The response spectrum curves related to: (a) the USCB region; (b) the LGCD region.](image)

It is worth mentioning that the applied response spectrum curves were developed on the basis of many years of research on mining-induced seismicity, and they correlate with the representative tremors and local soil conditions occurring in the USCB and LGCD regions.

2.3.3. Multiple Support Response Spectrum (MSRS) Analysis

The novelty of the present study lies in the implementation of the multiple support response spectrum (MSRS) method for the estimation of the dynamic performance of the viaduct under
where the correlation matrices are given by: the correlation matrix between the modal displacements. The determination of these matrices plays a key role in the MSRS algorithm. The elements of the correlation matrices are given by:

\[ \sigma_{ki} = (\Phi^T \cdot M \cdot \Phi)^{-1} \]

The maximum response of the structure may be calculated using the Complete Quadratic Calculation (CQC) algorithm [36,40]. The total response of the structure can be estimated using Equation (12), or in the simpler version of Equation (13):

\[ u^2 = \sum_k \sum_l a_k \cdot d_l \cdot \rho_{uz} \cdot u_{gl} \cdot u_{gl}' + \sum_k \sum_l b_{kl} \cdot D_{kl} + \sum_k \sum_l \sum_j b_{kl} \cdot b_{lj} \cdot \rho_{uz} \cdot D_{kl} \cdot D_{lj} \]

\[ u = (B^T \cdot l_{uu} \cdot B + B^T \cdot l_{uz} \cdot \Phi_{BD} + \Phi_{BD}^T \cdot l_{zz} \cdot \Phi_{BD} + \Phi_{BD}^T \cdot l_{zu} \cdot B)^{1/2} \]

In the above formulas B = a_i \cdot u_i' and \( \Phi_{BD} = b_{ki} \cdot D_{kj} \), whereas \( l_{uu} = \rho_{u_i u_j} \), \( l_{uz} = \rho_{u_i s_j} \), \( l_{zz} = \rho_{s_i s_j} \) are the correlation matrices: \( l_{uu} \) is the correlation matrix between the displacements at the supports, \( l_{uz} \) is the correlation matrix between the supports displacements and modal displacements and \( l_{zz} \) is the correlation matrix between the modal displacements. The determination of these matrices plays a key role in the MSRS algorithm. The elements of the correlation matrices are given by:

\[ l_{uu} = \rho_{u_i u_j} = \frac{1}{\sigma_{u_i} \cdot \sigma_{u_j}} \int S_{u_i u_j}(\omega) d\omega, \]

\[ l_{uz} = \rho_{u_i s_j} = \frac{1}{\sigma_{u_i} \cdot \sigma_{s_j}} \int S_{u_i s_j}(\omega) d\omega, \]

\[ l_{zz} = \rho_{s_i s_j} = \frac{1}{\sigma_{s_i} \cdot \sigma_{s_j}} \int S_{s_i s_j}(\omega) d\omega. \]

In the above formulae \( \sigma_a \) is a mean square value of the ground displacements (Equation (17)) and \( \sigma_z \) is a mean square value of the normalized structure’s response (Equation (18)):

\[ \sigma_a = \int S_{u_i u_j}(\omega) d\omega, \]

\[ \sigma_z = \int h_i(\omega)^2 \cdot S_{u_i u_j}(\omega) d\omega. \]

The h function is the frequency response function of the i-th modal equation, while the \( S_{u_i u_j} \) and \( S_{s_i s_j} \) are the spectral density functions at the supports defined by the displacement (u) or acceleration (\( \ddot{u} \)). Both functions are based on the power spectral density of ground acceleration \( S_{u_g} \) and the coherency function \( \gamma \), and can be determined from the following formulae:

\[ S_{u_i u_j}(\omega) = \frac{\gamma_{ij}(\omega)}{\alpha^2} \cdot S_{u_g}(\omega), \]
\begin{align}
S_{u_i u_j}(\omega) &= \frac{\gamma_{ij}(\omega)}{\omega^2} S_{u_k}(\omega), \\
S_{u_i u_j}(\omega) &= \gamma_{ij}(\omega) S_{u_k}(\omega).
\end{align}

To conduct the dynamic analysis of a structure using the MSRS method, the algorithm parameters must be defined. The power spectral density function \( S_{u_k} \) can be determined based on the response spectral functions, using formula (22) taken from the literature [36]:

\[
S_{u_k}(\omega) = \frac{\alpha^2}{\omega^2 + \omega_{ff}^2} \left( \frac{2 \xi \omega}{\pi} + \frac{4}{\pi \tau} \right) \left( \frac{D(\omega, \xi)}{p_0(\omega)} \right)^2,
\]

where \( p_0 = \sqrt{2 \ln(2.8 \omega \tau / 2 \pi)} \), \( \tau \) is the duration time of excitation and \( \omega_{ff} \) and \( \Theta \) are stochastic parameters, most often adopted as: \( \omega_{ff} = 0.705 \) and \( \Theta = 3.0 \).

2.3.4. Comparison of the Introduced Calculational Methods of the Dynamic Behavior of Structures

The general comparison of the calculational methods presented in this section can be summarized as follows:

- The THA is more precise than methods based on the modal analysis (RSA and MSRS). It allows for finding the time-histories of structure response measures (stresses, strains, displacements) at any element. It also enables incorporating different problem nonlinearities like complex material behavior, large displacements or strains, or unilateral contact conditions.
- The methods based on system eigenmodes are much more cost-effective than direct time integration of all the degrees of freedom of the system in the dynamic analysis performed with THA.
- Taking into account the non-uniformity of ground motion, both in THA or by adopting MSRS method may play an important role in the analysis of the dynamic response of large-scale structures to seismic shocks. However, one should bear in mind that the correct definition of the non-uniform kinematic excitation model requires the knowledge of local soil conditions and in situ tests. Such a model for the current analysis is presented in Section 3.2.

2.4. The Description of the Analyzed Road Viaduct

2.4.1. Structural Layout and Material Data of the Viaduct

The analyzed object is an existing road viaduct designated for traffic and pedestrian communication, localized in Chrzanow, Southern Poland. The location is a coal-mining region, so the viaduct can be subjected to mining shocks.

The viaduct is created as an arch bridge (Figure 7). The load-bearing system contains a continuous girder and two arches. The girder consists of two five-span pre-stressed concrete beams integrated with a concrete slab (Figure 8c), additionally linked by crossbars in the middle span and over supports (Figure 8a). The span lengths are 18.5 m, 23 m, 80 m, 23 m, and 18.5 m. The longest span is suspended to the arches by two sets of 14 hangers located on both sides of the girder in equal spacings of 4 m (see Figure 8b). Each beam is pre-stressed by 15 tendons with an area of 150 mm$^2$. 
Three kinds of pot bearings are used: fixed, unidirectional, and multidirectional. Over the third support, a pair of fixed and unidirectional bearings is used. Over other supports pairs of unidirectional and multidirectional bearings are applied in a manner ensuring structure stability, yet providing freedom of some displacements to minimize unnecessary forces due to over-constraints.

Loads from the superstructure to the supports are transferred by a system of pot bearings. Due to the deck geometry (the longitudinal and the transverse decline), the height of the pillars differs from 4.2 m on the left side to 5.5 m on the right side. Of the span lengths are 18.5 m, 23 m, 80 m, 23 m, and 18.5 m. The longest span is suspended to the arches by two sets of 14 hangers located on both sides of the girder in equal spacings of 4 m (see Figure 8b). Each beam is pre-stressed by 15 tendons with an area of 150 mm².

Concrete classes used for the viaduct’s structural elements are C50/60 (arches), C40/50 (girder), and C30/37 (pillars, abutments). Prestressing tendons are made of steel grade Y1860, while steel grade S460 is used for the hangers.

The girder is supported on two concrete abutments and four double pillars acting as intermediate supports, and founded on pile foundations. Due to the deck geometry (the longitudinal and the transverse decline), the height of the pillars differs from 4.2 m on the left side to 5.5 m on the right side. Loads from the superstructure to the supports are transferred by a system of pot bearings. Three kinds of pot bearings are used: fixed, unidirectional, and multidirectional. Over the third support, a pair of fixed and unidirectional bearings is used. Over other supports pairs of unidirectional

**Figure 7.** The road viaduct in Chrzanow, Southern Poland.

**Figure 8.** The main dimensions of the viaduct (cm): (a) top view; (b) side view; (c) cross-section.
and multidirectional bearings are applied in a manner ensuring structure stability, yet providing freedom of some displacements to minimize unnecessary forces due to over-constraints.

Concrete classes used for the viaduct’s structural elements are C50/60 (arches), C40/50 (girder), and C30/37 (pillars, abutments). Prestressing tendons are made of steel grade Y1860, while steel grade S460 is used for the hangers.

2.4.2. The Numerical Model of the Viaduct

The finite element model of the viaduct (Figure 9) was created with the ANSYS Workbench software [42]. Numerous types of finite elements were used: eight-node brick elements (for modeling beams, crossbars, arches, and pillars), continuum shell elements (for the slab), and truss elements (for the hangers). The total number of elements in the model amounted to 530,000. The density of the mesh was determined based on convergence analysis, with the natural frequency values serving as the convergence criterion. In order to account for the frictional forces acting in sliding pot bearings, the horizontal springs of constant stiffnesses \( k = 4e7 \text{ N/m} \) were introduced [43]. Due to efficiency reasons rigid structural elements, like foundation footings or abutments, were neglected in the numerical model, since they have the negligible influence on the dynamic characteristics of the object. Moreover, the soil to structure interaction was also neglected in the model.

Figure 9. The numerical model, mesh detail, and finite elements used: 1—truss elements; 2—solid elements; 3—continuum shell elements.

The material parameters applied to the numerical model correspond to the type of planned dynamic analysis. The response spectrum analysis necessitates the linearization of both material and geometric behavior. Therefore linear elastic material characteristics are used in all kinds of dynamic analysis. The pre-compression of the girders is incorporated in the model by the modification of its Young modulus according to Equation (23) [44]:

\[
E_{cs} = \frac{E_{cm} \cdot A_c + E_s \cdot A_s}{A},
\]

where \( E_{cm}/E_s \) are Young moduli of concrete/steel and \( A_c, A_s, A \) are cross-section areas of concrete, steel, and the whole element. The material data used in the numerical model is summarized in Table 1.
Table 1. Material parameters applied in the numerical model of the viaduct.

| Element                  | \( P \) (kg/m\(^3\)) | \( E \) (GPa) | \( \nu \) (-) |
|--------------------------|------------------------|---------------|---------------|
| Concrete (supports)      | 2600                   | 32            | 0.15          |
| Concrete (arch)          | 2600                   | 37            | 0.15          |
| Concrete (girder)        | 2400                   | 42            | 0.15          |
| Structural steel (hangers)| 7800                   | 200           | 0.30          |

3. Results

3.1. The Natural Frequencies and Modes of Vibration of the Viaduct

During the first stage of the dynamic analysis, the natural frequencies and modes of vibration of the viaduct were calculated. The numerically evaluated natural frequencies as well as the modes of vibration were verified by the in situ experiment. In Figure 10 the mounting of accelerometers on the viaduct is illustrated.

Figure 10. The mounting of accelerometers in the measurement points of the viaduct [45].

The experimental modal assessments of the viaduct, presented in detail in [45], was based on the operational modal analysis (OMA) techniques. Stochastic subspace identification (SSI) [46] was implemented to obtain natural frequencies of the viaduct and the time domain decomposition (TDD) [47] was utilized for mode shapes evaluation. The comparison of the natural frequencies of the numerical model with those obtained based on the in situ experiment, presented in Table 2, shows high conformity, with discrepancies below 10%.

Table 2. Comparison of the numerical and experimental natural frequencies of the viaduct.

| Mode | Natural Frequency (Hz) | Differences [%] | MAC |
|------|------------------------|-----------------|-----|
| 1    | 1.62                   | 1.49            | 8.7 | 0.803 |
| 2    | 2.27                   | 2.48            | 8.4 | 0.998 |
| 3    | 3.55                   | 3.36            | 5.8 | 0.873 |
| 4    | 14.47                  | 14.26           | 1.5 | 0.883 |
| 5    | 16.1                   | 14.72           | 9.4 | 0.873 |

To verify the compliance of the numerical mode shapes of the viaduct (see Figure 11) with the experimental results, the modal assurance criterion (MAC) [48] was used. Over 95% of the obtained MAC matrix diagonal values, related to the same modes, were greater than 0.8 (see Table 2), whereas the off-diagonal values (related to different modes) were smaller than 0.2, which indicates a high level of compatibility of the proposed numerical model with the real structure [48].

Figure 10. The mounting of accelerometers in the measurement points of the viaduct [45].
Table 2. Comparison of the numerical and experimental natural frequencies of the viaduct.

| Mode | Natural Frequency (Hz) | Differences [%] | MAC
|------|----------------------|-----------------|-----|
|      | FE Analysis | OMA             |                  |     |
| 1    | 1.62     | 1.49             | 8.7              | 0.803|
| 2    | 2.27     | 2.48             | 8.4              | 0.998|
| 3    | 3.55     | 3.36             | 5.8              | 0.873|
| 4    | 14.47    | 14.26            | 1.5              | 0.883|
| 5    | 16.1     | 14.72            | 9.4              | 0.873|

To verify the compliance of the numerical mode shapes of the viaduct (see Figure 11) with the experimental results, the modal assurance criterion (MAC) [48] was used. Over 95% of the obtained MAC matrix diagonal values, related to the same modes ($MAC_{ii}$), were greater than 0.8 (see Table 2), whereas the off-diagonal values (related to different modes) were smaller than 0.2, which indicates a high level of compatibility of the proposed numerical model with the real structure [48].

![Figure 11. The calculated modes of vibration: (a) first; (b) second; (c) third; (d) fourth; (e) fifth.](image)

For the dynamic analysis, the Rayleigh model of mass and stiffness proportional damping was applied [40]. The damping coefficients $\alpha = 0.2815$ and $\beta = 0.0028$ were used. The parameters were calculated based on the damping ratios of 5% and the frequencies corresponding to the first and the second vertical modes of vibration.

3.2. The Proposed Model of Non-Uniform Mining-Induces Excitation Used for the Time History Analysis

In the present study, the viaduct dynamic performance was numerically assessed using the THA, under the action of both the uniform and non-uniform excitation models. In the case of the uniform excitation model, the time-acceleration histories of the tremor recorded in three directions in the USCB region (see Figure 1) or in the LGCD region (see Figure 3) were applied to all structure supports: WE shock component in the horizontal direction along the viaduct axis, NS component in the transverse horizontal direction, and Z component in the vertical direction. In the case of the model of non-uniform mining-induced excitations, the original records were assigned only to the first support. Signals applied to the consecutive supports were modified, to incorporate the phenomena discussed in Section 2.1 (i.e., the effect of incoherence, the wave passage effect, and the attenuation effect).
In order to account for the local soil conditions, the in situ experiment was carried out by the authors of this study. The ground accelerations under the viaduct were induced by a seismic vibrator (see Figure 12) and recorded at different points of the field. The obtained data was the basis for determining the apparent wave velocity and coefficients of the coherence function. The comprehensive description of the performed experiments, as well as the details of determination of the above-mentioned parameters, are presented in [45].

Figure 12. The Birdwagen MARK IV seismic vibrator during the in situ test [45].

3.2.1. The Effect of Incoherence

The signals applied to the consecutive supports of the viaduct were firstly modified to incorporate the phenomenon of incoherence, by means of the algorithm described in papers [49,50], in which the authors developed a method of conditional stochastic simulation of seismic wave propagation using the spatiotemporal correlation function in the time domain. The algorithm is used to generate acceleration-time histories at various points of ground motion random field. The process of signal generation is based on the reference event specified at one location, wave velocity, and the adopted coherence function:

$$\gamma = \sigma^2 e^{-\frac{d_i^2}{2\pi\alpha^2}}$$  \hspace{1cm} (24)

where $\sigma$ is the standard deviation of the field of the ground motion; $d_i$ is the distance between the first and the $i$-th support; $\alpha$ is the scale parameter depending on local geological and topographical conditions and specifying the degree of correlation between the points of the field; $v$ is the mean apparent seismic wave velocity. The parameters of the coherence function (24) $\sigma = 0.964$ and $\alpha = 10.64$, determined based on the above mentioned in situ test, coincide well with the values most commonly used by other authors [50,51]. The apparent wave velocity in the ground, $v = 243$ m/s, was determined based on the cross-correlation function of accelerations recorded at points located along the direction of wave propagation.

3.2.2. The Wave Passage Effect

In the model of non-uniform excitation, it is assumed that subsequent points of the ground in the direction of wave propagation repeat the same motions with a time delay dependent on wave velocity. Having the experimentally determined velocity $v = 243$ m/s and the distances between the supports of the viaduct the time delay was calculated and implemented to modify the acceleration-time histories at subsequent supports of the viaduct.
3.2.3. The Attenuation Effect

The attenuation effect represents the amplitude decay with increasing distance from the vibration source. As mentioned in Section 2.2.4, the attenuation function, Equation (6), was adopted in this study, with empirical formulae elaborated for the discussed mining regions: Equation (7) for USCB and Equation (8) for LGCD. According to these equations, the signals at the consecutive supports were scaled using the reduction factor, appropriate for the analyzed region (Table 3).

Table 3. Reduction factors of the amplitude accelerations for the consecutive supports of the viaduct in the USCB and the LGCB regions.

| Supports No | 1    | 2    | 3    | 4    | 5    | 6    |
|-------------|------|------|------|------|------|------|
| Amplitude reduction for USCB region | 1.00 | 0.991| 0.983| 0.942| 0.936| 0.928|
| Amplitude reduction for LGCB region | 1.00 | 0.997| 0.991| 0.972| 0.967| 0.963|

3.2.4. Resultant Data of the Non-Uniform Mining-Induced Excitation for the Selected Mining Regions

Applying the above-discussed modifications to the recorded tremors (presented in Figures 1 and 3) results in the acceleration-time histories of the subsequent viaduct supports in three directions. For example, the horizontal acceleration component WE for both analyzed mining regions is presented in Figure 13. It is worth pointing out that two analyzed mining activity areas, i.e., the USCB and the LGCD region, differ significantly in geological structure. This results in discrepancies in parameters characterizing mining-induced tremors, like energy, the lengths of shocks, and, especially, the frequency contents.

Figure 13. The horizontal (WE) acceleration-time histories of subsequent viaduct supports obtained based on the model of non-uniform mining-induced kinematic excitation for: (a) the USCB region; (b) the LGCD region.
3.3. Adoption of the Multiple-Support Response Spectrum (MSRS) Method for the Assessment of the Dynamic Performance of the Viaduct under Mining-Induced Excitations

Since the MSRS method takes into account the non-uniformity effects, the response spectrum functions have to be determined separately for each support of the viaduct. The original local spectral functions developed for both the USCB and the LGCD regions, presented in Section 2.3.2 (Equations (9) and (10)), were applied in three directions at the first support of the viaduct only. The spectral functions applied to the consecutive supports were modified to incorporate the phenomena of incoherence and attenuation. The original spectral functions have been modified using the coherency function with parameters presented in Section 3.2.1 and the relationship described by Equation (22). Additionally, the reduction factors determined for the USCB and the LGCB regions (see Table 3) were applied to the consecutive supports’ accelerations, to include the attenuation effect. The modified response spectrum functions applied to all supports in the MSRS method for both mining regions are presented in Figure 14.

![Figure 14](image)

**Figure 14.** The response spectrum functions used in MSRS analysis for: (a) the USCB; (b) the LGCD region.

4. Discussion

4.1. Comparative Analysis of the Dynamic Responses of the Viaduct to Mining-Induced Shocks

The dynamic performance of the viaduct under the mining-induced excitations was determined using different calculational methods: the time history analysis under both uniform and non-uniform excitations, the response spectrum analysis, and the multiple support response spectrum analysis.

Comparison of the results of these simultaneous computations allowed for the evaluation of the impact of the non-uniformity of excitation on the structure’s dynamic response level, as well as assessing the applicability of the simplified approaches (RSA and MSRS) for proper, conservative estimation of stress levels for both excitation models.

The dynamic response of the viaduct was determined in all structure members. The response histories were recorded and assessed in over 200 elements. Due to the different predicted behavior of the elements located in the span and the support zones, the detailed analysis is presented for six representative elements placed in those zones (Figure 15). Elements W1–W3 were situated in the span zones, whereas elements W4–W6 were located above the supports.
Figure 15. The location of the elements selected for discussing the results of the dynamic analysis.

The dynamic responses of the viaduct, in terms of extremal principal stresses (maximal or minimal), calculated at the elements W1-W6 for shocks from both mining activity regions are compared in Figures 16–19. The results of the THA with the uniform and non-uniform excitation models are marked by the black and red solid lines, respectively. The results of the RSA and MSRS analysis are presented by the dashed and solid grey lines, respectively.

Figure 16. Comparison of extremal principal stresses obtained from different calculational methods for the mining-induced shock in the USCB region at the span zone elements: (a) W1; (b) W2; (c) W3.
Figure 17. Comparison of extremal principal stresses obtained from different calculation methods for the mining-induced shock in the USCB region at the support zone elements: (a) W4; (b) W5; (c) W6.

Figure 18. Comparison of extremal principal stresses obtained from different calculation methods for the mining-induced shock in the LGCD region at the span zone elements: (a) W1; (b) W2; (c) W3.
Figure 19. Comparison of extremal principal stresses obtained from different calculation methods for the mining-induced shock in the LGCD region at the support zone elements: (a) W4; (b) W5; (c) W6.

The comparison of the principal stresses for the USCB tremor, determined by various calculational methods at the span zone elements W1–W3, is shown in Figure 16.

In the span zones, the stresses obtained by the THA under non-uniform excitation are smaller than those determined under uniform excitation. Both approximate methods, RSA and MSRS, lead in this case to a conservative upper estimation of the acceleration-time histories given by the THA. However, the stresses determined with the MSRS are smaller than those acquired using the RSA and closer to the results of the THA. Hence, the MSRS method provides a more accurate solution, better estimating the full-dynamic analysis results.

In Figure 17 the principal stresses at the support zone elements W4–W6 for the USCB tremor are compared. In the vicinity of the supports, the stresses obtained by the THA with non-uniform excitation are up to 15% greater than those obtained with uniform excitation. It can be noticed that under uniform excitation the RSA provides safe (upper) stress estimation of the THA. However, this method underestimates the results obtained by the THA under non-uniform excitation. Although the underestimation is not very significant (about 10%), the results given by the RSA cannot be considered a correct solution to the problem. The comparison of results from the RSA and MSRS shows that the incorporation of non-uniformity effects in the MSRS method results in an increased stress level in the support zones. The key observation is that the dynamic response obtained from the MSRS is greater than that from the THA. This makes the MSRS a conservative method, providing a safe estimation of the full-dynamic analysis results.

The comprehensive analysis of the results obtained for the LGCD region, presented in Figures 18 and 19 leads to quite similar observations.

In the span zones taking into account the tremor non-uniformity leads to a reduction of stress levels obtained from the THA (see Figure 18). The RSA gives a safe upper estimation of the THA results.
for both excitation models. The MSRS seems to give a conservative estimation of THA calculated stresses only when the excitation nonuniformity is accounted for. However, in some span zone elements, MSRS analysis generates stress levels lower than those obtained from the THA under uniform excitation.

Contrary to the span areas, in the support zones accounting for the shock nonuniformity in THA results in an increase of the stress level up to 30% (see Figure 19). What is important, for elements W4-W6, RSA underestimates the peak stress values obtained by THA under non-uniform excitation by at least 10%. Hence, the RSA method, being nonconservative, cannot be recommended as an approximate method for determining the dynamic response of structures that undergo non-uniform excitations.

4.2. Assessment of the Dynamic Performance of the Viaduct Subjected to the Mining-Induced Events from Different Mining Activity Regions

The level of the dynamic response of a structure to a kinematic excitation depends not only on the shock amplitude level, but on the frequencies of excitation as well. The dominant frequency contents are different for various mining regions. In particular, the dominant frequency ranges of mining-induced tremors registered in the Legnica-Glogow Copper District consist of higher frequencies than those registered in the Upper Silesian Coal Basin [10].

To assess the dynamic performance of the viaduct under the shocks registered in both considered mining regions (USCB and LGCD), the maximum stress levels obtained by THA in all analyzed elements for the non-uniform excitation model were collated in Table 4.

Table 4. Comparison of the stresses obtained for the shock from the different mining regions.

| Element | Maximum Stress Obtained for Region (MPa): | Stress Ratio (-) |
|---------|------------------------------------------|------------------|
|         | USCB | LGCD |                  |
| W1      | 1.00 | 0.27 | 3.7             |
| W2      | 0.80 | 0.17 | 5.9             |
| W3      | 1.27 | 0.32 | 4.0             |
| W4      | 3.85 | 0.97 | 4.0             |
| W5      | 3.80 | 0.65 | 5.5             |
| W6      | 2.09 | 0.46 | 4.5             |

The comparison of maximal principal stresses at point W4, obtained for the mining-induced non-uniform shocks from both regions, based on THA and MSRS methods is provided in Figure 20.

Figure 20. Comparison of the maximal principal stresses obtained for the mining-induced shocks from the USCB and the LGCD regions at point W4 based on the THA and the MSRS methods.

The acceleration-time histories registered in the USCB and the LGCD regions (see Figures 1 and 3, respectively) show the same PGA value, since they were both scaled up to 1.7 m/s². However, the frequency spectra of these tremors differ significantly. The dominant frequencies of the shock from
the USCB region were located in the range from 1.6 to 4.8 Hz (see Figure 2), whereas the amplitudes of
the shock registered in the LGCD region show maxima at frequencies from 5 to 10 Hz (see Figure 4).

It is clearly visible from Table 4 and Figure 20 that the dynamic response of the viaduct to the
mining-induced event is 4 to 5 times greater for the shock from the USCB than the LGCD region, despite
both having the same PGA. It indicates that the viaduct dynamic performance is strongly dependent
on the dominant frequency range of mining tremors attributed to each region. The dominant frequency
range of the shock from the USCB region included the first four natural frequencies of the viaduct (see
Table 2). This resulted in the resonance effect which tended to increase the dynamic response. One can
conclude that the dynamic performance of identical engineering structures located in various mining
regions under tremors of similar maximal amplitudes may differ significantly, due to dissimilar shock
frequency contents. This assessment was the main objective of this research.

5. Conclusions

In this study, the dynamic performance of a multiple-support road viaduct subjected to
mining-induced kinematic excitations registered in various mining activity regions was assessed.
Three methods of calculation were used: the time history analysis (THA), the response spectrum analysis
(RSA), and the multiple support response spectrum (MSRS) analysis. The uniform and non-uniform
kinematic excitation models were adopted for dynamic performance evaluation. The following
conclusions can be formulated based on the analyses performed:

• The dynamic response of some parts of the structure, like the support zones, obtained using the
  non-uniform excitation model was meaningfully greater than that calculated for the uniform
  excitation. The research proved that some members of a structure may be strongly affected by
  pseudo-static effects resulting from the non-uniformity of kinematic excitation. Hence, in the
  assessment of the dynamic response of large multiple-support structures to mining-induced events,
  the effect of spatial variation of ground motion seems to be vital and should be accounted for.

• The commonly used RSA method may lead to an underestimation of the dynamic response of
  a structure to non-uniform mining-induced kinematic excitation. Hence, as nonconservative,
  the RSA cannot be recommended as an appropriate method in determining the dynamic response
  of large-scale structures that undergo non-uniform mining-induced excitation.

• The performed investigation demonstrated that the dynamic response obtained from the multiple
  support response spectrum analysis was greater than the response received from the THA
  analyses. The MSRS method, which takes into account the non-uniformity of ground motion, is a
  conservative approximation and provides a safe upper estimation of the full-dynamic analysis
  results of large-scale structures under mining-induced tremors.

• In the dynamic analyses of multiple-support structures subjected to spatially varying ground
  motion, an adequate model of non-uniform mining-induced kinematic excitation should be
  applied, taking into account features characterizing events arising in mining zones. In the
  proposed models the main aspects of ground motion non-uniformity, specific for quakes of mining
  origin, were accounted for. The parameters of the models, like the velocity of seismic wave
  propagation, coherence, and attenuation functions, should be obtained experimentally for the
  local soil condition in mining activity zones.

• Mining activity regions usually differ in geological structure and topography, which results in
discrepancies in frequency contents of mining-induced shocks in these zones. The presented
research indicated that the dynamic performance of a structure strongly depends on a frequency
range attributed to the specific mining region. The dynamic performance of identical engineering
structures located in various mining regions under tremors of similar maximal amplitudes may
differ significantly, due to the dissimilarity of frequency contents.

The last conclusion is of great importance for large-scale infrastructure erected in mining activity
zones. If the lower natural frequencies of a structure fall into a frequency range typical for shocks
in a given mining region, the amplification of the structural dynamic response may appear, due to the resonance phenomenon. For this reason, the predictive modal analysis of such structures should be conducted. The analysis methods presented in this research may be useful at the design stage of large-scale multiple-support structures situated in mining activity zones, regarding infrastructure protection strategies and minimizing the risk of the resonance phenomenon appearance.

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**References**

1. Cai, Y.; Li, X.; Xiao, W.; Zhang, W. Simulation of mining-induced ground damage using orthogonal experiments to determine key parameters of super-large coalface: A case study in Shendong Coalfield in China. *Appl. Sci.* 2020, 10, 2258. [CrossRef]

2. Zhironkin, S.; Selyukov, A.; Gasanov, M. Parameters of transition from deepening longitudinal to continuous lateral surface mining methods to decrease environmental damage in coal clusters. *Energies* 2020, 13, 3305. [CrossRef]

3. Cehlář, M.; Janočko, J.; Šimková, Z.; Pavlík, T.; Tyulenev, M.; Zhironkin, S.; Gasanov, M. Mine sites after mine activity: The Brownfields Methodology and Kuzbass Coal Mining case. *Resources* 2019, 8, 21. [CrossRef]

4. Pachla, F. *Influence of Mining Shocks on Surface Structures*; Cracow University of Technology Publishing House: Cracow, Poland, 2019. (In Polish)

5. Cai, Y.; Verdel, T.; Deck, O. Using plane frame structural models to assess building damage at a large scale in a mining subsidence area. *Eur. J. Environ. Civ. Eng.* 2020, 24, 283–306. [CrossRef]

6. Franzu, A.; Deck, O.; DeJong, M.J. Charts for the mining-induced deflection of buildings. *Can. Geotech. J.* 2020, 1–7. [CrossRef]

7. Liu, Z.; Cui, B.; Liang, Y.; Guo, H.; Li, Y. Study on foundation deformation of buildings in mining subsidence area and surface subsidence prediction. *Geotech. Geol. Eng.* 2019, 37, 1755–1764. [CrossRef]

8. Foulger, G.; Wilson, M.; Gluyas, J.; Julian, B.; Davies, R. Global review of human-induced earthquakes. *Earth-Sci. Rev.* 2018, 178, 438–514. [CrossRef]

9. Kuzniar, K.; Stec, K.; Tatara, T. Comparison of approximate assessments of the harmfulness of mining shocks using ground and building foundation vibrations. *J. Meas. Eng.* 2018, 6, 218–225. [CrossRef]

10. Tatara, T. *An Influence of Surface Mining-Related Vibration on Low-Rise Buildings*; Scientific Notebooks of Cracow University of Technology: Cracow, Poland, 2002. (In Polish)

11. Salajka, V.; Kalab, Z.; Kala, J.; Hradil, P. Response of the residential buildings structure on load technical seismicity due to mining activities. *World Acad. Sci. Eng. Technol.* 2009, 50, 61–69.

12. Malinowska, A.; Hejmanowski, R. Building damage risk assessment on mining terrains in Poland with GIS application. *Int. J. Rock. Mech. Min.* 2010, 47, 238–245. [CrossRef]

13. Pachla, F.; Tatara, T. Dynamic resistance of residential masonry building with structural irregularities. In *Seismic Behaviour and Design of Irregular and Complex Civil Structures III*; Geotechnical, Geological and Earthquake Engineering; Springer: Berlin/Heidelberg, Germany, 2020; Volume 48.

14. EN 1998-1:2004. **Eurocode 8: Design of Structures for Earthquake Resistance Part 1: General Rules, Seismic Actions and Rules for Buildings**; CEN: Brussels, Belgium, 2005.

15. EN 1998-2. **Eurocode 8: Design of Structures for Earthquake Resistance—Part 2: Bridges**; CEN: Brussels, Belgium, 2005.

16. PN-B-02170:2016. Assessment of the Harmfulness of Vibrations Transmitted through Ground to Buildings; Polish Standard. (In Polish); Polish Committee for Standardization: Warsaw, Poland, 2016.

17. Pachla, F. The impact of predicted vibrations from mining shocks on the viaduct—case study. *Vib. Proced.* 2019, 23, 93–98. [CrossRef]

18. Ayuddin, A. Global structural analysis of high-rise hospital building using earthquake resistant design approach. *Sinergi* 2020, 24, 95–108. [CrossRef]
19. Świdziński, W.; Korzec, A.; Woźniaczyk, K. Stability analysis of Żelazny Most tailings dam loaded by mining-induced earthquakes. In *Seismic Behaviour and Design of Irregular and Complex Civil Structures II, Geotechnical, Geological and Earthquake Engineering*; Springer: Berlin/Heidelberg, Germany, 2016; Volume 40.

20. Zerva, A. *Spatial Variation of Seismic Ground Motions: Modeling and Engineering Applications*; CRC Press/Balkema—Taylor & Francis Group: Boca Raton, FL, USA, 2009.

21. Lupoi, G.; Franchin, P.; Lupoi, A.; Pinto, P. Seismic fragility analysis of structural system. In Proceedings of the 13th World Conference on Earthquake Engineering, Vancouver, BC, Canada, 1–6 August 2004; Paper No. 408.

22. Zong, Z.; Zhou, R.; Huang, X. Seismic response study on a multi-span cable-stayed bridge scale model under multi-support excitations. Part I: Shaking table tests. *J. Zhejiang Univ.-Sci.* 2014, 15, 351–363. [CrossRef]

23. Leger, P.; Idé, M.I.; Paultre, P. Multiple-support seismic analysis of large structures. *Comput. Struct.* 1990, 36, 1153–1158. [CrossRef]

24. Burdette, N.J.; Elnashai, A.S.; Lupoi, A.; Sextos, A.G. Effect of asynchronous earthquake motion on complex bridges. I: Methodology and input motion. *J. Bridge Eng.* 2008, 13, 158–165. [CrossRef]

25. Sextos, A.G.; Kappos, A.J. Evaluation of seismic response of bridges under asynchronous excitation and comparisons with Eurocode 8–2 provisions. *Bull. Earthq. Eng.* 2009, 7, 519–545. [CrossRef]

26. Zembaty, Z. Vibrations of bridge structure under kinematic wave excitations. *J. Struct. Eng.* 1997, 123, 479–487. [CrossRef]

27. Sextos, A.G.; Karakostas, C.; Lekidis, V.; Papadopoulos, S. Multiple support seismic excitation of the Evripos bridge based on free-field and on-structure recordings. *Struct. Infrastruct. Eng.* 2015, 11, 1510–1523. [CrossRef]

28. Drygala, I.J.; Dulinska, J.M.; Polak, M.A. Seismic assessment of footbridges under spatial variation of earthquake ground motion (SVEGM): Experimental testing and finite element analyses. *Sensors* 2020, 20, 1227. [CrossRef]

29. Miao, Y.; Yao, E.; Ruan, B.; Zhuang, H. Seismic response of shield tunnel subjected to spatially varying ground motion. *Tunn. Undergr. Space Technol.* 2018, 77, 216–226. [CrossRef]

30. Davoodii, M.; Jafari, M.K.A.; Sadrolddini, S.M. Effect of multi support excitation on seismic response of embankment dams. *Int. J. Civ. Eng.* 2013, 11, 19–28.

31. Akbari, M.; Hariri-Ardebil, M.; Mirzabozorg, H. Nonlinear response of high arch dams to nonuniform seismic excitation considering joint effects. *J. Eng.* 2013, 2013, 912830. [CrossRef]

32. Tsinidis, G.; Di Sarno, L.; Sextos, A.; Furtner, P. A critical review on the vulnerability assessment of natural gas pipelines subjected to seismic wave propagation. Part 2: Pipe analysis aspects. *Tunn. Undergr. Space Technol.* 2019, 92, 103056. [CrossRef]

33. Dulinska, J.M. Cooling tower shell under asynchronous kinematic excitation using concrete damaged plasticity model. *Key Eng. Mater.* 2013, 535–536, 469–472. [CrossRef]

34. Boroń, P.; Dulinska, J.M. Assessing the dynamic response of a steel pipeline to a strong vertical mining tremor using the multiple support response spectrum method. *Trans. Tech.* 2019, 2, 97–108. [CrossRef]

35. Burkacki, D.; Wójcik, M.; Jankowski, R. Numerical investigation on behaviour of cylindrical steel tanks during mining tremors and moderate earthquakes. *Earthq. Struct.* 2020, 18, 97–111.

36. Der Kiureghian, A.; Neuenhofer, A. Response spectrum method for multi-support seismic excitations. *Earthq. Eng. Struct. Dyn.* 1992, 8, 713–740. [CrossRef]

37. Savor Novak, M.; Lazarevic, D.; Atalic, J.; Uros, M. Influence of multiple-support excitation on seismic response of reinforced concrete arch bridges. *Appl. Sci.* 2020, 10, 17. [CrossRef]

38. Shen, J.; Li, R.; Shi, J.; Zhou, G. Modified Multi-Support Response Spectrum Analysis of structures with multiple supports under incoherent ground excitation. *Appl. Sci.* 2019, 9, 1744. [CrossRef]

39. Mutke, G.; Stec, K. Seismicity in the Upper Silesian Coal Basin, Poland: Strong regional seismic events. In *Rockbursts and Seismicity in the Mines*; A.A.Balkema: Rotterdam/Brookfield, The Netherlands, 1997; pp. 213–219.

40. Chopra, A.K. *Dynamics of Structures*, 4th ed.; University of California at Berkeley: Berkeley, CA, USA, 2012; Pearson.

41. Czerwionka, L.; Tatara, T. Standard response spectra from chosen mining regions at Upper Silesian Coalfield. *Trans. Tech.* 2007, 2–8, 11–18.

42. *Workbench User’s Guide. Release 18.2*; ANSYS, Inc: Canonsburg, PA, USA, 2017.
43. Sipple, J.; Sanayei, M. Full-Scale Bridge Finite-Element Model Calibration Using Measured Frequency-Response Functions. *J. Bridge Eng.* 2014, 20, 04014103. [CrossRef]

44. Bednarski, Ł.; Sienko, R.; Howiacki, T. Estimation of the value and the variability of elastic modulus of concrete in existing structure on the basis of continuous in situ measurements. *Cem. Wapno Beton* 2014, 6, 396–404.

45. Boron, P. Dynamic Response Analysis of the Multi-Support Structure to Mining Shocks Using Multiple Support Response Spectrum. Ph.D. Thesis, Cracow University of Technology, Cracow, Poland, June 2019. (In Polish)

46. Van Overshee, P.; de Moor, B. *Subspace Identification for Linear System–Theory–Implementation–Applications*; Kluwer Academic Publishers: Boston, MA, USA, 1996.

47. Daniotti, N.; Cheynet, E.; Jakobsen, J.B.; Snæbjörnsson, J. Damping estimation from full-scale traffic-induced vibrations of a suspension bridge. *J. Comput. Civ. Eng.* 2019, 2019, 171–179.

48. Ewins, D.J. *Modal Testing: Theory, Practice and Application*, 2nd ed.; Research Studies Press Ltd.: Philadelphia, PA, USA, 2000.

49. Jankowski, R. Numerical simulation of space-time conditional random fields of ground motions. *Comput. Sci.* 2006, 3993, 56–59.

50. Jankowski, R.; Wilde, K. A simple method of conditional random field simulation of ground motions for long structures. *Eng. Struct.* 2000, 22, 552–561. [CrossRef]

51. Vanmarcke, E.; Fenton, G. Conditioned simulation of local fields of earthquake ground motion. *Struct. Saf.* 1991, 10, 247–264. [CrossRef]

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