Permissible Region Extraction Strategies for XLCT: A Comparative Study

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Abstract. Permissible region (PR) strategy has been used successfully to alleviate the ill-posedness of the X-ray luminescence computed tomography (XLCT) reconstruction problem. In the previous researches on the permissible region strategy, it is obvious that permissible region strategy can solve the reconstruction problem efficiently. This paper aims to research the performances of four types of permissible region extraction strategies, including a permissible region manually extraction strategy, a permissible region extraction strategy with a priori information of the surface nanophosphors distribution, a permissible region extraction strategy based on the first-time reconstruction result and a precise permissible region extraction strategy. In addition, some heuristic conclusions are provided for the future study in this paper. Fast iterative shrinkage-thresholding algorithm (FISTA) is used to reconstruct in this paper. The numerical simulation experiments and physical phantom experiments are setup to evaluate and illustrate the performances of the four different types of permissible region strategies.

Keywords. X-ray luminescence computed tomography (XLCT), comparative research, permissible region extraction strategy, FISTA.

1. Introduction

X-ray luminescence computed tomography (XLCT) is a hybrid imaging modality with rapid development. X-ray excitable nanophosphors are emitted by the high energy X-ray photons and the emitted optical photons can be measured for optical tomographic imaging [1]. XLCT can improve the depth imaging in contrast to fluorescence molecular tomography (FMT) [2, 3] and bioluminescence tomography (BLT) [4, 5].

In XLCT, there are two main processes, containing forward simulation process and inverse reconstruction process. The inverse reconstruction process is a seriously ill-posed problem which causes the calculation extremely difficult [6-8]. To alleviate the ill-posedness of the reconstruction, there are lots of studies on the traditional algorithms, such as Lp-norm regularization algorithm, compressed sensing algorithm and so on [9-11]. Besides, researchers have found the performance of reconstruction in the extracted permissible region (PR) is efficient and accurate, because the unknown variables are reduced in the limited reconstruction region and the complexity of solving the inverse problem can be reduced. In this paper, we studied on the performances of four different types of permissible region strategies to give some heuristic conclusions for the future research. In this paper, the numerical simulation and physical phantom experiments investigated five permissible regions extracted by four different permissible region strategies which are briefly introduced, including a permissible region manually extraction strategy (PRES1), a permissible region extraction strategy with...
2. Methods

2.1. Mathematical Model of Photon Propagation

Based on previous work, the concentration of nanophosphors emitted by X-rays can be expressed as \( S(r) = \varepsilon X(r) \rho(r) \), where \( \varepsilon \) is the light yield. \( S(r) \) Represents the nanophosphors target, \( \rho(r) \) is the concentration and \( X(r) \) represents the X-rays intensity at position \( r \), given by the Lambert-Beers’ law [16, 17]. The diffusion equation (DE) model with Robin-type boundary condition is used to approximate the photon propagation for XLCT [18]. Based on the finite-element method, \( b = AX \) is obtainable, where \( b \) is the measurement, \( A \) is the system matrix, \( X \) is the nanophosphors distribution. More detailed expression can be found in [18]. Generally, \( b = AX \) is extremely difficult to be solved directly. Therefore, \( b = AX \) is often transformed to

\[
\min \| AX - b \|_2^2 + \tau \| X \|_p,
\]

where \( \tau \) is the regularization coefficient. \( \| X \|_p \) is the Lp-norm regularization, where \( 0 \leq p \leq 1 \) or \( p = 2 \).

2.2. Four Different Types of Permissible Region Extraction Strategies

In this section, we introduced four different types of permissible region extraction strategies. In addition, FISTA which has widely been used in optical molecular tomography [19], is used in each extracted permissible region to reconstruct.

2.2.1. A Permissible Region Manually Extraction Strategy

It is proved that the permissible region can reduce the complexity of solving the problem which can reconstruct more efficiently. One of the strategies is manually extracting a permissible region. In the experiments, we manually extracted a large-scale and a small-scale permissible region, the region was defined as follow:

a) The large-scale permissible region \( PR_1 = \{ (x_i, y_i, z_i) | z_{bot} \leq z_i \leq z_{ab} \} \),

b) The small-scale permissible region \( PR_2 = \{ (x_i, y_i, z_i) | z_{bot} \leq z_i \leq z_{ab} \} \).

where \( z_{bot} \) and \( z_{ab} \), \( i = 1,2 \) denote the bottom and above thresholds at \( z \) axis. Reconstruct by FISTA in the large-scale permissible region \( PR_1 \) and small-scale permissible region \( PR_2 \), respectively. In details, the procedure of the strategy is presented in [12].

In addition, in the numerical simulation experiment, the bottom and above thresholds of the large-scale permissible region \( PR_1 \) are at 5 mm and 25 mm, and the ones of the small-scale permissible region \( PR_2 \) are at 15 mm and 25 mm. Due to the size of phantom, whose side length is only 20 mm, the bottom and above thresholds of the large-scale permissible region \( PR_1 \) are at 3 mm and 13 mm, and the ones of the small-scale permissible region \( PR_2 \) are at 8 mm and 13 mm, in the physical phantom experiment.

2.2.2. A Permissible Region Extraction Strategy with a Priori Information of the Surface Nanophosphors Distribution

The permissible region extraction strategy described in section 2.2.1 is without any priori information, which causes that the extraction is highly empirical. In this section, the permissible region extraction strategy is based on the surface nanophosphors distribution information to locate the permissible region more accurately. Generally, the real target is located near the maximum energy node of the surface nanophosphors distribution. According to the maximum energy node of each projection \( z_i \), a sub-permissible region \( \Omega_i \) is extracted with the manually set thresholds.
\(\sigma_1\) and \(\sigma_2\), i.e. \(\Omega_3 = \{(x, y, z) | z_1 - \sigma_1 \leq z \leq z_1 + \sigma_2\}\). It is supposed that there are \(M\) sub-permissible regions. The union of all the sub-permissible regions is denoted as the final permissible region, i.e. \(PR_3 = \Omega_1 \cup \Omega_2 \cup \cdots \cup \Omega_M\). Finally, reconstruct by FISTA in the final permissible region \(PR_3\). The detailed process of the strategy can be found in [13]. In addition, the thresholds \(\sigma_1\) and \(\sigma_2\) are assigned to 1 mm and 4 mm.

2.2.3. A Permissible Region Extraction Strategy Based on the First-Time Reconstruction Result. Another way to obtain the priori information is the first reconstruction result. To further reduce the complexity of calculating, compress the system matrix \(A_{nzmA}\) to \(A_{csm}\), by PCA. The cumulative percent of the system matrix’s (\(A_{nzmA}\)) eigenvalues \(CPV_i\) is mathematically expressed as \(CPV_i = \frac{\sum_{i=1}^{t} \lambda_i}{\sum_{i=1}^{\infty} \lambda_i}\), where \(t\) is the number of remained principal components and it is determined when \(CPV_i\) is no less than the preset threshold \(CPV_{thres}\) that is valued 90% in this paper. Then, obtain the temporary result \(X_{tcm}\) by Tikhonov with the system matrix \(A_{csm}\). Based on the nonzero values of \(X_{tcm}\), compressed the system matrix \(A_{nzmA}\) to \(A_{nzmoc}\). Then, solve the above equation by FISTA with \(A_{nzmoc}\) in the final permissible region \(PR_3\). The detailed process of this permissible region extraction strategy can be found in [14].

2.2.4. An Extraction Strategy to Determine a Permissible Region. This permissible region extraction strategy concentrated on extracting a permissible region with the certain location and size through multiple iterations. This permissible region extraction strategy only need the measured surface nanophosphors distribution as the priori knowledge.

There are two main steps in this permissible region extraction strategy, including locating the permissible region and determining the size of permissible region. While locating the permissible region, a fixed-size and big-enough cube is selected randomly as a temporary permissible region in which reconstruct by FISTA, iteratively. Then, determine the size of permissible region and keep the central point. Reduce the length side of the cube by a constant \(\alpha\). Reduce the side length of the cube iteratively until it reaches to 0. In each iteration, the cube with current side length is regarded as a temporary target and calculate the root mean square error \(RMSE_i\) between its surface nanophosphors distribution and the measured nanophosphors distribution. If \(RMSE_i\) is less than the threshold \(\beta\) and the recorded optimal root mean square error (RMSE) is larger than \(RMSE_i\) by more than \(\beta\), then \(RMSE_i\) is the new recorded optimal RMSE. After iteration, a final optimal RMSE is obtainable. According to the optimal side length the size of the final permissible region \(PR_3\). Finally, reconstruct in the final permissible region \(PR_3\) by FISTA.

The procedure of the precise permissible region extraction strategy is described in [15], in details. The influence of the side length (SLIC) and central position (CPIC) of initial permissible region is little to the result, which has been proved in [15]. In this paper, an initial permissible region whose side length (SLIC) was 10 mm, the constant \(\alpha\) was 0.5 mm, the threshold \(\beta\) was 1e-10, and the maximum iterations were 30, was used to reconstruct. In the simulation experiment, the initial central point (CPIC) was located at (10, 12, 21) mm. Because of the limited size of the phantom, the central point of initial permissible region (CPIC) was located at (8, 10, 6) mm.

3. Experiments
In this section, all experiment codes were written in MATLAB and ran on a desktop computer with 3.3 GHz Intel Xeon CPU E3-1230 v3 and 12GB RAM. In the numerical simulation experiment, cost time (Time), relative error (RE), contrast-to-noise ratio (CNR), reconstructed density (Reco. Dens.) and the
number of elements in the extracted permissible region (Num. Ele.) are used to assess the results reconstructed in different extracted permissible region. In addition, the detailed formations of RE and CNR can be found in [14, 21], respectively.

3.1. Numerical Simulation Experiment

The single target was a cylinder with a diameter of 2 mm, a height of 4 mm and located at (12.0, 15.0, 24.5) mm. The investigated region was the torso section of a digital mouse with 35 mm height and 6 organs, including muscle, stomach, lungs, liver, heart and kidneys. The optical parameters of different organs can be found in [20]. A mesh used for the forward simulation contained 16769 nodes and 88834 tetrahedral elements, and a mesh used for the reconstruction contained 5957 nodes and 29577 tetrahedral elements. In this experiment, there were four excitation sources with 90 deg intervals. The concentration of the target was 0.0218 μg/mm$^3$. Figures 1(a1-a5) show the results in five permissible regions (PR1-PR5), respectively. In addition, figures 1(a1-a5) show the 2D-view results at the plane of Z=24.5 mm and figures 1(b1-b5) show the 3D-view results. In addition, the corresponding quantitative results were shown in table 1.

![Figure 1. The reconstructed results in numerical simulation experiments.](image)

**Table 1.** Quantitative results of the numerical simulation experiments.

| PR   | Time (s) | RE (%) | CNR  | Reco. Dens. (μg/mm$^3$) |
|------|----------|--------|------|------------------------|
| PR1  | 119.63   | 2.5    | 49.49| 0.0078                 |
| PR2  | 45.80    | 2.5    | 49.50| 0.0078                 |
| PR3  | 18.75    | 2.9    | 2.19 | 0.0027                 |
| PR4  | 139.39   | 1.2    | 45.76| 0.0210                 |
| PR5  | 92.71    | 0.5    | 24.90| 0.0194                 |

3.2. Physical Phantom Experiment

In this physical phantom experiment, a cylinder phantom whose radius was 10 mm and height was 20 mm is regarded as the investigated region. The central position of cylinder-shaped target is located at (11.8, 13.4, 10.5) mm. The detailed experimental data could be found in [21]. In this experiment, a mesh used to reconstruct has the elements at 12566. Figure 2 illustrated the five extracted permissible regions, and figures 2(a-e) show different extracted permissible regions (PR1-PR5), respectively. Table 2 listed the corresponding quantitative results in the five permissible regions.
Figure 2. The permissible regions extracted by different strategies in three-dimensional view.

Table 2. The corresponding quantitative results of the physical phantom experiments.

| PR | Time (s) | RE (%) | CNR | Num. Ele. |
|----|----------|--------|-----|-----------|
| PR1 | 0.41     | 13.0   | 9.07| 7512      |
| PR2 | 0.07     | 13.0   | 8.24| 4216      |
| PR3 | 0.08     | 15.0   | 9.39| 4249      |
| PR4 | 1.21     | 12.0   | 12.18| 2308      |
| PR5 | 1.51     | 11.0   | 8.77| 2203      |

4. Discussion and Conclusion
In this paper, we investigated the performances of the four different permissible region extraction strategies, including a permissible region manually extraction strategy, a permissible region extraction strategy with a priori information of the surface nanophosphors distribution, a permissible region extraction strategy based on the first-time reconstruction result and a precise permissible region extraction strategy. According to the results of the numerical simulation experiment, the permissible region strategies described in section 2.2.1 and section 2.2.2, need to rely on the empirical information to extract the permissible region, which leads to obtain a rougher permissible region and reconstruct with more error than the ones obtained by the strategies described in section 2.2.3 and section 2.2.4. However, the reconstructed result is more accurate in a permissible region which is more similar as the real target, according to the reconstructed results of the permissible region extraction strategies expressed in section 2.2.3 and section 2.2.4. The strategy shown in section 2.2.1 extracts a permissible region completely empirically, and one shown in section 2.2.2 does with the measured surface nanophosphors distribution and manually selected thresholds. The strategy represented in section 2.2.3 is based on the non-zero values of the reconstructed result by the principal components of the system matrix. In addition, the strategy described in section 2.2.4 concentrates on locating the permissible region and determining its size as much as possible. According to the results of the above experiments, it is verified that the results of reconstruction in the permissible region which is more similar to the real target, is more accurate.

However, it is still a challenge to automatically obtain a permissible region which is more similar to the target in terms of morphology and space. Therefore, we will focus on the study about automatically obtaining the permissible region which is more similar to the real target in the future work.

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