Effect of stress triaxiality on the damage of polyethylene

L M Han¹, S F Xue¹,² and Y Zhang¹
¹College of Pipeline and Civil Engineering, China University of Petroleum (East China), Qingdao, Shandong, 266580, China
E-mail: sfeng@upc.edu.cn

Abstract. In this study, the effect of the triaxial stress state on the stress/strain behavior and damage of polyethylene (PE) was investigated through a combination of experimental testing and finite-element (FE) simulation. Notched round bars with four different notch radii (0.5, 2, 5, and 20 mm) were stretched at a crosshead speed of 1mm/min until fracture. A phenomenological constitutive model was proposed to describe the mechanical behavior of PE specimens with different radii. Parameters of the constitutive model were adjusted until the curve of engineering stress-displacement from the FE simulation matched the experimental results. The FE model was further applied to obtain the data on stress triaxiality and true stress-strain curves of PE specimens. The results show that stress triaxiality and yield stress increase with the reduction of notch radii. However, the true stress decreases when the notch radii are reduced, suggesting that higher triaxiality aggravates the damage accumulation.

1. Introduction
Polyethylene (PE) has been widely used in the manufacture of water pipes, agricultural drainage, irrigation pipes, and gas pipes due to its excellent resistance to impact, and low-temperature nontoxic features [1]. However, in practical engineering applications, the damage of micro-voids, micro-cracks, and micro-pores in PE materials will degrade the material properties, reduce the bearing capacity of materials, and shorten the life of materials. Therefore, it is essential to study the main causes of PE damage.

The damage mechanism of PE depends on the stress states [2]. Stress triaxiality, a stress state parameter that can reflect the triaxial stress state in the stress field and the constraint degree of material deformation, directly affect plastic deformation and fracture of materials [3]. To the best of the authors’ knowledge, much attention has been devoted to the stress triaxiality study of metal materials, but less to PE. Bridgman [4] established a formula for calculating the initial stress triaxiality of cylindrical specimens with notches. The value of stress triaxiality changes during the deformation process. Hence, some modifications were made to the formula by the researchers. Based on experiments, the change law of stress triaxiality in the process of specimen deformation was investigated, and the corresponding stress triaxiality of fracture strain was modified by Borvik [5] using finite element (FE) method. The combination of the experiment with FE simulation was used by Xie Fan [6] to obtain the true stress-strain curve of the material and modified the relationship between the fracture strain and the stress triaxiality by obtaining the parameters of the crack initiation time utilizing FE. With the application of PE materials in engineering, the study on stress triaxiality of PE has gradually attracted the attention of scholars. A charge-coupled device (CCD) was used by Castagnet [7] to shoot the variation of the perimeter of the black line around the smallest cross-section during deformation, and image analysis...
software was used to process the evolution of the minimum cross-section diameter. A conclusion that stress triaxiality increases with the decrease of radial strain was drawn. The micro-notch surface was observed by Boisot [8] by scanning electron microscopy (SEM) in the flow-breaking test. He also revealed by combining experiments with numerical simulations that the stress triaxiality decreased with the radial strain and the change of volume at the stress-softening stage could reduce the stress triaxiality ratio. Ognedal [9] conducted tensile experiments and numerical simulation on mineral-filled polyvinyl chloride (PVC) and high-density polyethylene (HDPE) with and without preprocessing notch, which proved that phenomenological constitutive model could be used for high triaxial stress state. It is concluded that PVC material is sensitive to hydrostatic pressure, and plastic expansion increases with the increase of stress triaxiality. But, HDPE is a non-hydrostatic pressure-sensitive material which volume increases significantly when the triaxial stress is high.

On the basis of experimental testing and FE simulation, effects of triaxial stress states on the stress/strain behavior and damage in PE were researched in the present study. Besides, the constitution relation is established by applying the experimental data to the FE simulation. The obtained FE model was used as a tool to obtain information about stress triaxiality and true stress-strain curves of PE specimens.

2. Experiment

The triaxial stress state can be generated using notched round bar specimens [2]. The experiment was carried out using notched round bars with four different notch radii (0.5, 2, 5, and 20 mm), respectively, as shown in figure 1. The specimens were processed from HDPE material. All specimens were stretched to fracture at a crosshead speed of 1mm/min on a universal testing machine. It is noticeable that the diameter on the minimum section of all specimens is 6mm.

3. Numerical simulation

3.1. Finite element model

The two-dimensional axisymmetric FE model of the notched specimens was established and realized in ABAQUS/CAE (2016), as shown in figure 2. CAX8R axisymmetric 8-node elements are chosen to model the notched specimens. Note that due to the geometric symmetry, the FE model was for half of the specimen length and quarter of the cross-section. Boundary conditions were the same as those used in the experimental testing. That is, one end of the specimen was fixed, and the other end moved at the crosshead speed of 1mm/min.
3.2. Constitutive model

The constitutive relation attributing to obtain the true stress-strain curve of PE materials is divided into four stages [10-13]: (a) linear elastic stage, (b) non-linear elastic stage, (c) necking stage, and (d) hardening stage.

\[
\sigma(\varepsilon) = \begin{cases} 
\frac{E}{2(1+\nu)} \varepsilon & \varepsilon \leq \varepsilon_y \\
d \left[ a(\varepsilon + b)^{(-c)} \right]^{-1} - \left[ a(\varepsilon + b)^{(-c)} \right] & \varepsilon_y \leq \varepsilon \leq \varepsilon_n \\
\alpha k \varepsilon^N & \varepsilon_n \leq \varepsilon \leq \varepsilon_i \\
ke^{M\varepsilon^\beta} & \varepsilon \geq \varepsilon_i
\end{cases} (1)
\]

where \( \sigma \) is the equivalent stress (in MPa), \( \varepsilon \) is the equivalent strain; \( \varepsilon_y \) is the critical strain change from linear deformation to nonlinear deformation; \( \varepsilon_n \) is the critical strain for the one-set necking; \( \varepsilon_i \) is the strain at the beginning of exponential hardening. Other parameters \( (a, b, c, d, e, \alpha, k, N, M, \beta) \) are user-defined constant values. These constant values and strain intervals are modified until the engineering stress-displacement curves obtained from the experiment can be reproduced by FE simulation.

4. Results

4.1. Comparison of experiment and simulation

Figure 3 illustrates the engineering stress-displacement curves of experiment testing for notched specimens with radii of 0.5, 2, 5, and 20 mm. It can be noted that the yield stress (i.e., peak engineering stress) gradually increases, and the displacement at the fracture significantly decreases with the reduction of notch radii. The stress drop after the peak is more significant with the decrease of notch curvature radius.
Figure 3. The engineering stress-displacement curves of experiment.

Figure 4. Engineering stress-displacement curves of (a) R20, (b) R5, (c) R2 and (d) R0.5.

The engineering stress-displacement curves determined from experiment testing and FE simulation for notched specimens with radii of 0.5, 2, 5, and 20 mm are shown in figure 4(a)-(d), respectively. It
can be seen from figure 4 that experimental results were successfully reproduced by FE simulation before fracture. The last part of the FE simulation curve could not mimic the change of the experiment because the fracture was not considered. The specific parameters in the Equation (1) are listed in tables 1 and 2.

| FE model | Types of specimens | R20 | R5 | R2 | R0.5 |
|----------|-------------------|-----|----|----|------|
| Equation (1a) | E | 1600 | 1400 | 1400 | 1400 |
| | ν | 0.35 | 0.35 | 0.35 | 0.35 |
| Equation (1b) | a₁ | 31.952 | 34.817 | 34.692 | 35.601 |
| | b₁ | 0.02 | 0.018 | 0.018 | 0.018 |
| | c₁ | 0.1 | 0.1 | 0.1 | 0.1 |
| | d₁ | -31.2 | -30 | -29.7 | -31 |
| | e₁ | 14 | 14 | 14 | 13.51 |
| Section2 | a₂ | 26.19 | 26.345 | 24.91 | 22.65 |
| | b₂ | 0.02 | 0.02 | 0.02 | 0.02 |
| | c₂ | 0.1 | 0.1 | 0.1 | 0.1 |
| | d₂ | -19 | -14 | -16 | -13 |
| | e₂ | 15 | 14 | 14.654 | 14.652 |
| Equation (1c) | αk | 37.732 | 39.72 | 40.65 | 41.81 |
| | N | 0.175 | 0.225 | 0.235 | 0.275 |
| Equation (1d) | k₁ | 29.196 | 29.01 | 30.08 | 29.012 |
| | M₁ | 0.4 | 0.38 | 0.16 | 0.3 |
| | β₁ | 1.8 | 1.8 | 1.8 | 1.8 |
| Section2 | k₂ | 30.19 | 28.82 | 25.24 | 29.012 |
| | M₂ | 0.35 | 0.39 | 0.6 | 0.3 |
| | β₂ | 1.8 | 1.8 | 1.8 | 1.8 |
| Section3 | k₃ | 33.497 | 29.923 | 31.45 | 27.97 |
| | M₃ | 0.31 | 0.37 | 0.48 | 0.32 |
| | β₃ | 1.8 | 1.8 | 1.8 | 1.8 |
| Section4 | k₄ | 16.588 | 35.301 | 37.39 | 24.22 |
| | M₄ | 0.48 | 0.33 | 0.42 | 0.37 |
| | β₄ | 1.8 | 1.8 | 1.8 | 1.8 |
| Section5 | k₅ | 9.692 | 51.976 | 51.976 | 51.976 |
| | M₅ | 0.6 | 0.25 | 0.25 | 0.25 |
| | β₅ | 1.8 | 1.8 | 1.8 | 1.8 |
| Section6 | k₆ | 7.616 | 7.616 | 7.616 | 7.616 |
| | M₆ | 0.65 | 0.65 | 0.65 | 0.65 |
| | β₆ | 1.8 | 1.8 | 1.8 | 1.8 |

4.2. Stress triaxiality

4.2.1. The distribution of stress triaxiality. Stress triaxiality is defined as the ratio of the average stress to the equivalent stress.

\[ \eta = \frac{\sigma_m}{\sigma_{eq}} \] (2)
where, \( \sigma_m \) is the average stress, 
\[
\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}
\]
; \( \sigma_{eq} \) is the Mises equivalent stress, 
\[
\sigma_{eq} = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}}
\]
; \( \sigma_1, \sigma_2, \sigma_3 \) are the principal stresses in three directions.

In this paper, the stress triaxiality on the minimum section of four notched specimens under uniaxial tensile condition is obtained from initial elastic deformation in FE simulation. The distribution of stress triaxiality defined in Equation (2) is shown in figure 5, in which X represents the distance from the center to edge. The curves in figure 5 show that stress triaxiality increases with the decrease of notch radii. A smoother distribution curve of stress triaxiality is created by a bigger notch radius.

**Table 2.** Strain ranges for Equation (1) used in FE simulation

| FE model | Notched specimens |
|----------|-------------------|
|          | R20 | R5 | R2 | R0.5 |
| Equation (1a) | 0.00005 | 0.00005 | 0.00005 | 0.00007 |
| Equation (1b) | Section1 | 0.0005-0.003 | 0.0005-0.003 | 0.0005-0.003 | 0.0007-0.003 |
| Equation (1c) | Section2 | 0.003-0.075 | 0.003-0.06 | 0.003-0.06 | 0.003-0.06 |
| Equation (1d) | Section3 | 0.075-0.3 | 0.06-0.3 | 0.06-0.3 | 0.06-0.3 |
|          | Section2 | 0.3-0.8 | 0.3-0.8 | 0.3-0.6 | 0.3-0.8 |
|          | Section3 | 0.8-1.7 | 0.8-1.4 | 0.6-1.4 | 0.8-1.4 |
|          | Section4 | 1.7-2.2 | 1.4-2.2 | 1.4-1.8 | 1.4-1.8 |
|          | Section5 | 2.2-2.3 | 2.2-2.4 | 1.8-2.5 | 1.8-2.5 |
|          | Section6 | 2.3-2.4 | 2.4-3.0 |
|          | Section5 | 2.4-2.7 |

It can be noted that the position of highest stress triaxiality of four notch specimens is different. The highest stress triaxiality occurs at the center (X = 0) for specimens of R20, R5, and R2. However, this does not happen in the specimen with R0.5, where the maximum stress triaxiality occurs at X = 2.25 mm. Owing to the distribution of stress triaxiality, most researchers choose the stress triaxiality at the center of the minimum cross-section. However, this choice is not suitable for the specimen with small notch radii. The reason is that position with the highest stress triaxiality is somewhere between the center and the edge rather than the center point. In this paper, the highest stress triaxiality on the minimum cross-section of each specimen is taken as the modified equivalent stress triaxiality. The modified R20, R5, R2 and R0.5 was 0.3868, 0.5638, 0.9346, and 1.2182, respectively.

![Figure 5. Curves of stress triaxiality change with the change of X for the four specimens.](image-url)
4.2.2. Effect of stress triaxiality on the true stress-strain curve. Figure 6 presents the true stress-strain curves of four notch specimens determined from the FE simulation. It can be observed in figure 4 that the curve of engineering stress-displacement from FE simulation can match experimental results before the peak. Therefore, the curves in figure 6 were all taken from the beginning to the peak engineering stress. The true stress with smaller stress triaxiality is higher. This indicates that larger damage is generated under higher stress triaxiality. In addition, the true stress tends to be the same when the true strain value is larger than 0.3.

![Figure 6. True stress/strain curve of different stress triaxiality](image)

5. Conclusion
A method combining experimental testing and FE simulation has been used to investigate the effect of stress triaxiality of notched specimens on the damage. The experimental results show that the decrease of notch radii has caused the increase of the yield stress and significant reduction of the displacement. Using Kwon's constitutive model, the engineering stress-displacement curve obtained from the experiment can be reproduced by FE simulation. Stress triaxiality and true stress/strain curves of PE specimens have obtained from the FE simulation. Results from the FE simulation indicate that the distribution of stress triaxiality is not uniform in the minimum cross-section: it varies from the center to the edge. More importantly, stress triaxiality increase with the reduction of notch radii. However, the true stress decreases when the notch radii are reduced, suggesting larger damage is generated under higher triaxial stress state.

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