Charmonium production in $e^+e^- \to \psi + X(\bar{c}c)$ and $e^+e^- \to D^*D_J$

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The dominance of $\chi_0$ in the data for $e^+e^- \to \psi + \chi_J(\bar{c}c)$ is shown to violate OZI factorization. Single gluon exchange gives a non-factorizing effective $S\cdot L$ interaction that generates a large scalar production amplitude. This also has observable effects near threshold in $e^+e^- \to D^{(*)}D_J$, where enhancements of $D^*D_0$ and $DD_1$ channels are predicted. Further tests and implications are discussed.

I. INTRODUCTION

Hadron decays occur dominantly when the colour sources (such as $Q\bar{Q}$) are separated by $O(\Lambda_{QCD})$ such that the energy in the fields is enough to enable the creation of a light $q\bar{q}$ pair. This has been embodied in the phenomenological OZI rule [1], incorporated in models [2] and more recently studied in lattice QCD [3]. It has been shown [4] that lattice QCD appears to confirm assumptions implicit in many models of strong decays, namely that there is a factorization of the constituent spin $S$ and total $J$ of the hadrons (in models this equates to a factorization of $S$ and $L$). Similar conclusions and results are being found in the AdS/QCD correspondence [5]. One consequence of factorization is that $e^+e^- \to V + [0^{++}]$ must be smaller than at least one of $e^+e^- \to V + [1^{++}]$ and $e^+e^- \to V + [2^{++}]$ [4]. This is in marked contrast to data on $e^+e^- \to \psi + X$ where the $\chi_0$ appears to dominate as reported in ref. [6].

Decays that involve the creation of heavy flavours where $2m_Q > \Lambda_{QCD}$ (e.g. $c\bar{c} \to c\bar{c} + \bar{c}c$) may differ radically from those involving creation of light flavours. The OZI process is suppressed because $2m_Q > \Lambda_{QCD}$ would require the colour fields of force to extend over excessive distances without having created light $q\bar{q}$. This is highly improbable. A way for such decays to be triggered is if the required energy is supplied by a hard process such as single gluon exchange. As the gluon will in general transmit information about $S$ and $L$ from the initial quarks to the created pair, we expect that factorization will not occur in such processes.

Braaten and Lee [7] present a calculation of $e^+e^- \to \psi + \chi_J$ which breaks the limits imposed by factorization, and which appears to be supported by the data – in particular the $\chi_0$ dominates the $\chi_1$ and $\chi_2$. This situation has drawn little comment either theoretically or experimentally despite its important ramifications for strong interaction phenomenology. The purpose of our paper is to: draw attention to the importance of Braaten and Lee’s results, expose the origin of their remarkable result given their assumptions, extend their result to other cases and develop stringent experimental tests of the OgE mechanism as opposed to factorized interactions.

In section 2 we review the predictions of factorization. In section 3 we study the general structure of the gluon-driven processes producing $\psi + \chi_J$ as a function of $J$ and expose the origin of the large $\chi_0$ amplitude in ref. [7]. We apply these results to the production of pairs of charmed mesons and find that they lead to an enhancement of $\bar{D}D_1$ and $\bar{D}^*D_0$ channels near threshold, but make no contribution to $\bar{D}^*D_1$ or $\bar{D}^*D_2$. Further, we predict that the helicity amplitude $\bar{D}^*(\pm 1)D_2(\mp 2) = 0$, whereas $\psi(\pm 1)\chi_2(\mp 2)$ is the largest of all the $\psi\chi_2$ amplitudes.
II. FACTORIZATION

A feature common to many models of hadron decays is that the constituent spins factorize from the total angular momentum of the hadrons. Ref \[8\] showed that this feature appears to be confirmed by lattice QCD \[8\], and refs \[4\] showed how this property explains relations among various amplitudes that are common to many many specific models. This factorization property (eq. 5 of ref \[4\]) underpins the empirical result that hadron loops (such as in $c\bar{c} \to D\bar{D}, D\bar{D}^*, D^*\bar{D}, D^*D^* \to c\bar{c}$) give universal mass shifts throughout an $L$ multiplet such as: $h_c, \chi_0, \chi_1, \chi_2 \[9\]$. It is also claimed to be consistent with AdS/QCD correspondence\[5\].

Ref \[4\] derived various consequences of factorization for strong decays of hadrons. In particular this work showed that factorization constrains the relative populations of final states in $e^+e^- \to V[3S_1] + (S,A,T)[3P_J]$. In the factorization scheme, the decay of a transversely polarized $^3S_1 \to ^3S_1 + ^3P_2$, with the tensor meson maximally polarized along the decay axis is predicted to vanish (see eq. 34 in \[4\]). This selection rule is a particular test of factorization. Also ref \[4\] found that for $^3S_1$ decay, the relative rates in $S$-wave, $S^2$, and $D$-wave, $D^2$ are:

$$^3S_1 + ^3P_0 : ^3S_1 + ^3P_1 : ^3S_1 + ^3P_2 : ^1S_0 + ^1P_1 = 3S^2 : 4S^2 + D^2 : 6D^2 : S^2 + D^2. \quad (1)$$

Among various consequences, of interest to the present discussion is that

$$\sigma(^3S_1 \to ^3S_1 + ^3P_1) = \frac{4}{3} \sigma(^3S_1 \to ^3S_1 + ^3P_0) + \frac{1}{6} \sigma(^3S_1 \to ^3S_1 + ^3P_2) \quad (2)$$

and hence

$$\sigma(e^+e^- \to ^3S_1 + ^3P_1) > \sigma(e^+e^- \to ^3S_1 + ^3P_0). \quad (3)$$

One of the central applications of the present paper will be to test these predictions against data on $e^+e^- \to \psi + \chi_J$ where preliminary indications are that the relation eq. 8 is violated \[6\].

The constraints of factorization become more powerful near threshold where $S$-wave dominates. For a $^3S_1$ initial state

$$\sigma(^3S_1 \to \psi\chi_2) \to 0$$

$$\sigma(^3S_1 \to \psi\chi_0) = \frac{3}{4} \sigma(^3S_1 \to \psi\chi_1). \quad (4)$$

Analogously, for a $^3D_1$ initial state

$$\sigma(^3D_1 \to \psi\chi_0) \to 0$$

$$\sigma(^3D_1 \to \psi\chi_1) = \frac{5}{3} \sigma(^3D_1 \to \psi\chi_2). \quad (5)$$

Finally one may allow for a coherent mixture of $^3S_1$ and $^3D_1$ initial state. Results become model dependent but $\sigma(\psi\chi_0)$ cannot be made larger than both $\sigma(\psi\chi_1)$ and $\sigma(\psi\chi_2)$. Thus in the region of threshold factorization forbids a dominant $\sigma(\psi\chi_0)$.

This is interesting in view of the data on $e^+e^- \to \psi + X$ at 10.6 GeV c.m. energy, which show three prominent enhancements $X$ in $e^+e^- \to \psi + X \[6\]$, consistent with being the $\eta_c, \eta'_c$ and $\chi_0$. The observed pattern of states appears radically different to what is seen for light flavous: the apparent prominence of $\chi_0$ with only a hint of $\chi_1$ and much suppressed $\chi_2$ contrasts with light flavous where $e^+e^- \to \omega f_2$ is clearly seen \[10\]. The charmonium data \[8\] are significantly above threshold and so our general restrictions against $e^+e^- \to \psi + \chi_0$ need not apply. However, we shall see that the large rate for this channel is a signal for factorization breakdown even away from threshold, and inspires the question: what is required to create a dominant $e^+e^- \to \psi + \chi_0$ amplitude?

III. GLUON EXCHANGE STRUCTURE

A. Non-relativistic reduction of the Feynman Amplitude

If we require in fig. 1 that the upper $c\bar{c}$ pair produce the $C = -$ meson then at leading order four diagrams contribute to $e^+e^- \to \psi\chi_J$. These consist of gluon emission from $c$ or $\bar{c}$ in either of two topologies: exchange within
the $\psi$ (figs. 1(a),1(c)) or within the $\chi$ (figs. 1(b),1(d)). To contrast with the factorization amplitudes most directly, we shall restrict our attention to the threshold region. As in ref. [1], we set the masses of the $\psi$ and $\chi$ each = $2m_c$.

We first make a non-relativistic reduction of the Feynman amplitudes into 2x2 block matrices sandwiched between two-component spinors. In so doing it is important to note the role of the (anti-)fermion propagator between the virtual photon and the gluon.

Matrix elements in the explicit non-relativistic limit are discussed in a consistent phase convention by Ackleh et al. [11], which we adopt here (see Appendix B of ref [11], especially eqs. B5-B7). Care is required to track phases and so we define here our choice of some convention dependent quantities. Dirac spinors are normalized to 1;Dirac spinors are normalized to 1;

\[ (1, 0) = \xi_+ = \eta_- \quad (0, 1) = \xi_- = -\eta_+ \] (6)

The momentum routing is defined in fig. 1.

The analysis of Ackleh et al. was for a $q\bar{q}$ wavefunction of well defined $^{2S+1}L_J$ with no constituent propagator effects considered. In $e^+e^-$ annihilation of the present paper, we explicitly consider the constituent propagator between the photon and exchanged gluon. This has the effect of introducing more than just a single $^{2S+1}L_J$ state for the photo-produced $c\bar{c}$, i.e. it is not simply $^3S_1$.

From the usual Feynman rules, the matrix element for fig. 1(a) is:

\[ \mathcal{M}^a = \bar{u}_1 i g_S \gamma^\nu \frac{i \left( \frac{1}{2} P_1 + \frac{1}{2} m_c \right)}{\left( \frac{1}{2} P_1 + k \right)^2 - m_c^2 + i\epsilon} \eta c \gamma^a v_4 - i d_{\nu\mu} \bar{u}_1 i g_S \gamma^\mu v_2. \] (7)

where $g_S$ is the strong coupling constant and $\epsilon_c$ is the ratio of the electron and $c$ quark charges and $\epsilon$ is the electron charge. Our primary interest is in the $J$-dependence of the production of $^3S_1 + ^3P_J$ near threshold.

In NRQCD the appropriate $L$ state is extracted by expanding the matrix element into a double Taylor series of the internal momenta $q_1$ and $q_2$ (about 0) and selecting the appropriate power of the internal momenta. The angular dependence of the appropriate term in the Taylor series is recombined with the spin of the hadron using covariant generalizations of Clebsh-Gordon coefficients and the radial dependence is absorbed into a model-dependent vacuum saturated matrix element. Finally the spin of the $^3P_J$ and its orbital angular momentum will be combined to obtain the appropriate $J$. This is the non-relativistic reduction of the covariant NRQCD technique for $^3S_1 + ^3P_J$.

The matrix element is effectively simplified to:

\[ \mathcal{M}^a = -i g_S e c \frac{8}{s} \sqrt{\frac{E + mc}{2mc}} \xi_1 \xi_2 \mathcal{N} \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \gamma^\nu \left( \frac{1}{2} P_1 + k + m \right) \gamma^a \left( \begin{array}{c} -\sigma \hat{q}_2 \\ 1 \end{array} \right) \eta_2 \gamma_2 \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \gamma_\nu \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \eta_1 \right] \] where $\hat{q}_2 \equiv q_2/2m_c$.

We generalise the above to allow the quarks of the flavour produced by the gluon to have mass $m_f$. Then we define $\delta = 2m_f/m_c$; in the particular case of $\psi + \chi$, $m_f = m_c$ and hence $\delta = 2$. For flavoured mesons with $m_f \to 0$ one has $\delta \to 0$. We shall retain the general form so that applications to other combinations of flavours may be made. The $\hat{q}_2 \equiv q_2/2m_c$ then generalises trivially to contain the relevant $m_f$ in the denominator: $\hat{q}_2 \equiv q_2/2m_f = \frac{1}{2} \hat{q}_2$. $N$ also has an implicit dependence on $\delta$ but the dependence does not matter for the relative rates we are interested in. Hence

\[ \mathcal{M}^a = N \xi_1 \xi_2 \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \gamma^\nu \left( m_c \gamma^0 - \frac{1}{2} P_1 \cdot \gamma + m_c \delta \gamma^0 - k \cdot \gamma + m_c \right) \left( \begin{array}{c} 0 \\ \sigma^a \end{array} \right) \left( -\sigma \hat{q}_2 \right) \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \eta_2 \right] \] (8)

\[ \left( \sigma \hat{q}_2 \right) \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \gamma^0 \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \eta_2 \]
We can split the equation into a time-like component \((\gamma^0 \gamma^0)\) and a space-like component \((\gamma\gamma)\). Taking the leading order terms for the time-like component,

\[
\mathcal{M}^{a \gamma^0 \gamma^0} = N \text{Tr} \left\{ \begin{pmatrix} 1^\dagger & 0 \\ 0 & m_c(2 + \delta) - \sigma \cdot q_2 \\ \sigma \cdot q_2 & -m_c \delta \end{pmatrix} \begin{pmatrix} 0 & \sigma^a \\ -\sigma^a & 1 \end{pmatrix} \begin{pmatrix} \eta_3 \xi_3 \end{pmatrix} \right\}
\]

Having written the matrix element in terms of block 2x2 matrices and particle, anti-particle Pauli spinors \((\xi, \eta)\), we can use

\[
\sum_{\lambda_1, \lambda_2} \langle 1, S | \frac{1}{2}, \lambda_1; \frac{1}{2}, \lambda_2 \rangle \eta_3 \xi_3 = -\frac{1}{\sqrt{2}} \sigma \cdot e^a (S)
\]

whereby the substitution, \(\eta_3 \xi_3 \rightarrow -\frac{1}{\sqrt{2}} \sigma \cdot e^a\) projects the matrix element into the spin triplet state. The terms of exactly one power of \(q_2\) are retained \((q_1 = 0\) ensures that only terms with zero powers of \(q_1\) were kept\). \(q_2\) is replaced by \(\varepsilon^*\) and its radial dependence absorbed into the vacuum saturated analogues of the NRQCD matrix elements. The result is the projection of the matrix element into the \(3S_1 + 3P\) state:

\[
\mathcal{M}^{a \gamma^0 \gamma^0} (\gamma^* \rightarrow 3S_1 + 3P) = \frac{1}{2} N \frac{2 + \delta}{2} \langle O_1 \rangle^{3S_1} \langle O_1 \rangle^{3P} \text{Tr} \left\{ \sigma \cdot a \sigma \cdot e^2 \sigma \cdot e^*_L \sigma \cdot e^*_1 \right\}
\]

and then using

\[
\frac{1}{2} \text{Tr} \left\{ \sigma^i \sigma^j \sigma^k \sigma^l \right\} = \delta^{ij} \delta^{kl} - \delta^{ik} \delta^{jl} + \delta^{il} \delta^{jk}
\]

we obtain

\[
\mathcal{M}^{a \gamma^0 \gamma^0} (\gamma^* \rightarrow 3S_1 + 3P) = \frac{1}{2} N \frac{2 + \delta}{2} \langle O_1 \rangle^{3S_1} \langle O_1 \rangle^{3P} \left( \varepsilon^*_2 \cdot a \varepsilon^*_L \cdot e^*_1 - \varepsilon^*_L \cdot a \varepsilon^*_2 \cdot e^*_1 + \varepsilon^*_1 \cdot a \varepsilon^*_2 \cdot e^*_L \right).
\]

where \(a\) is the polarisation of the photon, \(n\) is the polarisation of the spatial components of the gluon, \(\varepsilon^*_L\) is the orbital angular momentum of the \(3P\) meson and \(\varepsilon^*_1, \varepsilon^*_2\) are the spin polarisation tensors of the \(3S_1, 3P\) mesons respectively. The time-like components of the amplitudes cancel among the various graphs. Therefore we do not consider them further and focus on the space-like components.

For the space-like component, we have

\[
\mathcal{M}^{a \gamma \gamma} = -N \text{Tr} \left\{ \begin{pmatrix} 1^\dagger & 0 \\ 0 & \sigma^a \end{pmatrix} \begin{pmatrix} m_c(2 + \delta) - \sigma \cdot q_2 \\ \sigma \cdot q_2 \end{pmatrix} \begin{pmatrix} 0 & \sigma^a \\ -\sigma^a & 1 \end{pmatrix} \begin{pmatrix} \eta_3 \xi_3 \end{pmatrix} \right\}
\]

\[
= \mathcal{M}^{a \gamma \gamma} (\gamma^* \rightarrow 3S_1 + 3P) = \frac{1}{2} N \langle O_1 \rangle^{3S_1} \langle O_1 \rangle^{3P} \text{Tr} \left\{ (\sigma \cdot e^*_L \sigma \cdot a - \frac{\delta}{2} \sigma \cdot a \sigma \cdot e^*_L \sigma \cdot e^*_1) \sigma \cdot e^*_2 \sigma \cdot e^*_1 \right\}.
\]

Then as before, using eq.\(\text{[11]}\) this may be written

\[
\mathcal{M}^{a \gamma \gamma} (\gamma^* \rightarrow 3S_1 + 3P) = \frac{1}{2} N \langle O_1 \rangle^{3S_1} \langle O_1 \rangle^{3P} \left( \frac{\delta}{2} \varepsilon^*_1 \cdot a \varepsilon^*_L \cdot e^*_2 - \frac{\delta}{2} \varepsilon^*_L \cdot a \varepsilon^*_2 \cdot e^*_1 - \frac{\delta}{2} \varepsilon^*_2 \cdot a \varepsilon^*_1 \cdot e^*_L \right).
\]

The amplitude in fig. 1(b) is calculated in similar fashion and gives

\[
\mathcal{M}^{b \gamma \gamma} (\gamma^* \rightarrow 3S_1 + 3P) = \frac{1}{2} N \langle O_1 \rangle^{3S_1} \langle O_1 \rangle^{3P} \left\{ \sigma \cdot a \sigma^a \sigma \cdot e^*_L \sigma \cdot e^*_2 \sigma \cdot e^*_1 \right\}.
\]

Explicit calculation of the other diagrams confirms the symmetries:
\[ M^a \gamma \gamma^a (\gamma^* \rightarrow S_1 + P) = M^{b \gamma} \gamma (\gamma^* \rightarrow S_1 + P) = -M^{b \gamma} (\gamma^* \rightarrow S_1 + P) = -M^{d \gamma} (\gamma^* \rightarrow S_1 + P) \]

Thus the total amplitude at threshold becomes

\[ M(\gamma^* \rightarrow S_1 + P) = N(O_1)_{S_1} (O_1)_{3P} \left( (2\delta + 4)\epsilon_1^* \alpha \epsilon_L^* \epsilon_2^* - \delta \epsilon_L^* \alpha \epsilon_1^* \epsilon_2^* \right) \] (17)

For arbitrary final state meson momentum the amplitude is

\[ M(\gamma^* \rightarrow S_1 + P) \propto 4m_c^2 \epsilon_1 \cdot \epsilon_2 (6P_1^2 P_1 \cdot \epsilon_L - s \epsilon_2^2) + \epsilon_2 \cdot \epsilon_L s (s \epsilon_1^2 - 2P_1^2 \epsilon_1 \cdot P_2) + 24m_c^2 P_1 \cdot \epsilon_L (\epsilon_2^2 \epsilon_1 \cdot P_2 - \epsilon_1^2 \epsilon_2 \cdot P_1). \] (18)

\(\alpha\) is the photon’s helicity. We are grateful to Dr Jungil Lee for providing this expression. As in the threshold limit, this indeed reduces to our eq. 17 with \(\delta = 2\) for physical values of the photon helicity \(\alpha\).

Of the two contributions in eq. 17, only the \(\epsilon_L^* \alpha \epsilon_1^* \epsilon_2^*\) factorizes \(L\) and \(S\). The first term couples \(L_{3P}\) and \(S_{3P}\) and survives everywhere except the unphysical case of threshold for \(\delta = -2\). In the physical cases of interest, this term turns out to dominate.

### B. Amplitudes in NRQCD

In the particular limit of \(\psi\chi\) at threshold, \(\delta = 2\) the amplitude is:

\[ M(\gamma^* \rightarrow \psi + c\bar{c}(3P)) = 2N(O_1)_{S_1} (O_1)_{3P} (4\epsilon_1^* \alpha \epsilon_L^* \epsilon_2^* - \epsilon_L^* \alpha \epsilon_1^* \epsilon_2^* ) \] (19)

For charm pair production at threshold where \(\delta = 0\) the amplitude simplifies to:

\[ M(\gamma^* \rightarrow D^* + c\bar{c}(3P)) = N(O_1)_{S_1} (O_1)_{3P} (4\epsilon_1^* \alpha \epsilon_L^* \epsilon_2^* ) \] (20)

Note that \(N\) and the vacuum saturated matrix elements will be different in the charm pair case as opposed to the double charmonium case, but this will not matter for relative amplitudes in the \(D^* D_J\) channels of interest.

Eq. 18 is the starting point from which the amplitudes of ref 7 can be obtained. Our Pauli decomposition at threshold, eq. 17 reveals the origin of some particularly interesting results. In particular, for later reference, we draw attention to the NR reduction of the amplitude of the topology with O\(g\)E within the \(\psi\) (figs. 1(a) and 1(b)), equation 18, which involves the momentum \(q_2\) flowing through the photon-\(c\bar{c}\) vertex and along the virtual fermion line connecting photon to gluon. The photo-produced \(c\bar{c}\) pair is then clearly not in a simple \(S_1\) state. Physically, contributions other than \(S_1\) would vanish if the amplitude were proportional only to wavefunction at the origin; it is the spatial propagation away from the ‘origin’, associated with the propagator, that enables the non-zero amplitudes associated with these other configurations at threshold.

First consider the factorizing contribution in eq. 17 namely \(\epsilon_L^* \alpha \epsilon_1^* \epsilon_2^*\). The relative size of amplitudes arising from this term alone after combining with suitable Clebsch-Gordan coefficients in order to give helicity amplitudes for various \(\chi J\) production in association with \(\psi\) are as follows.

\[ \psi(-) \chi_2(++) = 1; \psi(0) \chi_2(+) = 1/\sqrt{2}; \psi(+) \chi_2(0) = 1/\sqrt{6} \] (21)

\[ \psi(0) \chi_1(+) = 1/\sqrt{2}; \psi(+) \chi_1(0) = 1/\sqrt{2} \] (22)

\[ \psi(+) \chi_0(0) = 1/\sqrt{3} \] (23)

Thus for the unphysical case \(\delta = -1/2\) where this term alone is present, one finds a result consistent with factorization as expected.
\[ \sigma(\psi\chi_2) : \sigma(\psi\chi_1) : \sigma(\psi\chi_0) = 5 : 3 : 1 \] (24)

In the particular limit of threshold and \( \delta = 0 \), only the “non-factorizing” contribution \( \varepsilon_2^* \cdot \varepsilon_L^* \equiv S_\chi \cdot L_\chi \) survives. This limit can be realised if the produced quarks have masses \( \to \) as in the production of charmed mesons, \( D^*, D J \).

The expectation values of this term, after combining with the Clebsch-Gordan coefficients for \( L \times S \to J \) appropriate for \( \psi \chi \) give, in the same normalisation as above

\[ \psi(+)\chi_J(0) = [(J0|11; 1 - 1) + (J0|1 - 1; 11) − (J0|10; 10)] \]

which vanishes for \( J = 1, 2 \) and is non-zero for \( J = 0 \) where the constructive interference of the three terms gives the large value of \( \sqrt{3} \).

For the case of \( e^+e^- \to \psi + \chi J \) the amplitude is given by eq. [19] The non-factorizing \( \varepsilon_2^* \cdot \varepsilon_L^* \equiv S_\chi \cdot L_\chi \) term dominates by a factor of 4 relative to \( \varepsilon_1^* \cdot \varepsilon_2^* \equiv S_\chi \cdot S_\psi \). While the factorizing term alone gave amplitude \( 1/\sqrt{3} \), the combination of the two terms now gives \( 1/\sqrt{3}(1 - 12) \). Thus in leading order of NRQCD at threshold, we find the following relative rates:

\[ \sigma(e^+e^- \to \psi[\chi_2; \chi_0]) = 5 : 3 : 121. \] (25)

This agrees with the result in ref [4] (to extract the OgE contribution, set \( Y = 0 \) in the amplitudes eqs. A3 in appendix A of ref [4]).

The same combination of Clebsch-Gordan coefficients arises in the longitudinal amplitude for \( e^+e^- \to \psi(0)\chi_J(0) \). Thus the origin of the large scalar amplitude at threshold is due to a \( S_\chi \cdot L_\chi \) transition operator between the initial state and the \( L = 1 \ \chi J \) state which vanishes for all but \( J = 0 \).

Note that the \( \psi\chi_1 \) and \( \psi\chi_2 \) amplitudes at threshold factorize. If the \( \psi\chi_2 \) amplitudes can be isolated, there is an interesting test concerning the non-zero \( \psi(-)\chi_2(++) \). In ref [4] (table III and eqs. 36 – 41), it was noted for a \( ^3S_1 \) initial state that factorization implies that the \( \psi(-)\chi_2(++) \) amplitude vanishes in S-wave, and that the D-wave amplitudes destructively cancel. In OgE however, there is more than simply \( ^3S_1 \) in the initial state with the result that the S-wave is non-zero and as noted above we find

\[ a[\psi(-)\chi_2(++)] : a[\psi(0)\chi_2(++)] : a[\psi(+)\chi_2(0)] = 1 : 1/\sqrt{2} : 1/\sqrt{6} \] (26)

in accord with S-wave dominance.

Thus the \( \psi(-)\chi_2(++) \) amplitude, far from vanishing, is predicted to be the dominant helicity state for \( \psi\chi_2 \). This provides an interesting test for the presence of non \( ^3S_1 \) contributions in \( e^+e^- \).

C. One gluon exchange amplitude for arbitrary momentum

Eq. [18] is the amplitude for arbitrary \( r \). From these amplitudes ref [7] obtains rates for \( \psi(\lambda_1) + \chi_J(\lambda_2) \), where \( \lambda_{1,2} \) are the helicities of the charmonium states, as a function of \( r^2 \equiv 16m_e^2/s \). At the 10.6 GeV c.m. energy of the data [6], where \( r^2 \equiv 16m_e^2/s = 0.28 \), ref [7] finds for the OgE contribution to the cross sections

\[ \sigma(\psi\chi_2 : \psi\chi_1 : \psi\chi_0) \sim 3 : 2 : 12. \] (27)

The \( L \) and \( S \) dependence factorizes in three of the terms; the only term coupling \( L \) and \( S \) is the one already identified. Compared to the results at threshold, the relative sizes of \( \sigma(\psi\chi_2) : \sigma(\psi\chi_1) \) have not changed much but there is an order of magnitude relative reduction of \( \sigma(\psi\chi_0) \) at the higher energy. Nonetheless, it is the non-factorizing term that continues to dominate. Relatively large scalar meson production is predicted as a robust phenomenon at all energies.

D. Charm pair production in NRQCD

The above analysis can be applied to charm pair production, \( e^+e^- \to D_J\bar{D}^{(*)} \) or \( \bar{D}_J D^{(*)} \). Near threshold, factorization predicts [4] that the ratios of cross-sections from initial \( ^3S_1, ^3D_1 \) or vector hybrid to the states \( D^*D_2 : D^*D_1(3P) : D^*D_0 \) are as shown in table [1]
As in the $c\bar{c} + c\bar{c}$ case we see here too that OZI factorization requires that the scalar production cannot be larger than both the axial and tensor channels.

The interesting feature for charm pair production is that the OZI, or “flux-tube breaking”, mechanism is dynamically allowed, but that OgE can also be anticipated to be present. The relative sizes of these OgE contributions to the channel is also non-zero.

It is interesting to note that the $\mathcal{e}_a^\dagger, a, \mathcal{e}_a^\dagger, \mathcal{e}^\dagger_2$ structure of eq. [20] is the same structure that, in eq. [19] was responsible for the large $\psi\chi_0$ amplitude in double charmonium production. Within the above approximation for $D^*D_J$ production, we see that the $D^*D_0$ channel is the only one driven by OgE, a maximal violation of factorization. The $DD_1(1^P_1)$ channel is also non-zero.

The double charmonium production is in practice the “worst case” for the scalar dominance. Yet it is clearly visible in the data, and our analysis at threshold shows why. For charmed meson production, where $g \to q\bar{q}$ with light flavours and $\delta \to 0$, the effect will necessarily be bigger, effectively infinite for reasonable parametrisation of $m_q/m_c$.

The effective absence of OgE contributions for $e^+e^- \to D^*D_1$ and $D^*D_2$, which becomes exact in the limit $\delta \to 0$, implies that the factorization selection rules should be particularly robust here. Thus we predict that at threshold, the helicity amplitude $D^*(\pm1)D_2(\mp2) = 0$, in contrast to the case of $\psi(\pm1)\chi_2(\mp2)$ where it is the largest of all the $\psi\chi_2$ amplitudes.

The OgE selection of $D^*D_0$ and $DD_1$ in the threshold region may play some role in generating the enhancement seen as $Y(4260)$ [12] (where charmed mesons in a relative S wave might rescatter to form $\psi\pi\pi$). We shall return to the $\gamma^* \to q\bar{q}$ process in more detail elsewhere.

### IV. CONCLUSIONS

The intrigue of $e^+e^- \to \psi + X$ can hardly be overstated. There is a clear spectrum of $C = +$ charmonium states, whose pattern is not yet explained and in addition an enigmatic structure around 3940 MeV. As this is in the mass region where $C = +$ charmonium hybrids could lurk [13], any theoretical modelling of this requires first understanding the population of the other bumps. The $\chi_J$ production in particular needs to be understood; it seems empirically dominated by $\chi_0$, at least when a mass fit is made to the cross section; no $J^{PC}$ analysis has been made. We have drawn attention to the fact that the dominance of $\chi_0$ contrasts radically with normal OZI expectation where $\chi_0$ would be relatively small. We demonstrated that OgE gives large scalar, confirming Braaten and Lee [7], and have exposed the origin and significance of this result which had not previously been recognised as a sharp test of dynamics.

Thus if dominance of $\psi\chi_0$ is confirmed over a range of $q^2$ away from threshold, this would support OgE as the dominant decay mechanism. Conversely, if data near threshold confirm $a[V(-)T(++)] \to 0$, this would signal factorization being dominant. In any event, we anticipate that the relative populations and helicity structures of $\psi\chi_J$ will vary with $q^2$. We recommend that this be investigated in $e^+e^-$ annihilation at super-B factories by means of ISR to access a range of energies. In particular experiment should attempt to measure the spin dependence of $e^+e^- \to \psi\chi_2$ as a function of $q^2$ and compare with the analogous amplitudes in $e^+e^- \to \omega f_2$ or $e^+e^- \to \rho f_2$.

When applied to charm pairs we find violation of OZI rules here too: OgE selects the $D^*D_0$ and $DD_1$ channels near threshold. The charm pair arena is interesting as it potentially enables us to test the relative role of OgE versus OZI dynamics. It also shows a possible mechanism for generating enhancements in these $S$-wave production channels which, by constituent rearrangement, may produce these structures, such as seen in $e^+e^- \to \psi\pi\pi$ at 4260 MeV [12].

Our approach has demonstrated the breakdown of factorization in OgE. Ackleh, et. al. implicitly noted this in ref [11] at least for on-shell constituents with a definite $2^{S+1}L_J$ initial state. Our work goes beyond this by exposing the important role propagator effects can play; in particular we have found this to be essential in matching to the

| $D^*D_2$ | $D^*D_1(3^P)$ | $D^*D_0$ |
|----------|---------------|----------|
| $^3S_1$  | 0             | 4        | 3        |
| $^3D_1$  | 3             | 5        | 0        |
| hybrid   | 6             | 3        | 4        |

TABLE I: Relative sizes of transversely polarized decay amplitudes from OZI in charm pair production, $\delta \to 0$. 


NRQCD work of ref [7], at least at threshold.

As a result, we have exposed the origin of the large amplitude for $\psi\chi_0$ relative to $\psi\chi_{1,2}$ which manifestly violates factorization and for which there are preliminary hints in data. We urge experiment to use spin analysis to confront this and other $J^P$ tests in both charm pair and light hadron production in order to sharpen understanding of the relative importance of OZI and non-factorizing dynamics.

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$$p_1 = \frac{1}{2} P_1 + 0$$
$$p_2 = \frac{1}{2} P_1 + 0$$
$$p_3 = \frac{1}{2} P_2 + q_2$$
$$p_4 = \frac{1}{2} P_2 - q_2$$

(a)

FIG. 1: The four topologies of the OgE model for $e^+e^- \rightarrow c\bar{c}c\bar{c}$. 

(b)

(c)

(d)