Origin of abundance inhomogeneity in globular clusters

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ABSTRACT

We numerically investigate abundance properties of the Galactic globular clusters (GCs) by adopting a new ‘external pollution’ scenario. In this framework, GCs are assumed to originate in forming low-mass dwarfs embedded in dark matter subhaloes at very high redshifts ($z$) and thus be chemically influenced by field asymptotic giant branch (AGB) stars of the dwarfs during early GC formation processes. GCs within a dwarf galaxy therefore can be formed from the mixture of (i) gas ejected from the field AGB stars formed earlier in the dwarf and (ii) the interstellar gas infalling to the central region of the dwarf. In this external pollution scenario, the ratio of the total mass of infalling gas to that of AGB ejecta during GC formation in a dwarf ($s$) and the time-scale of gas infall ($\sigma_I$) are the most important key parameters that can determine abundance properties of GCs. We mainly investigate the abundance inhomogeneity among light elements (e.g. C, N, O, Na and Al) of stars in GCs by using the latest stellar yield models of metal-poor AGB stars with and without third dredge-up. Our principal results for the models with no third dredge-up, which are more consistent with observations, are as follows.

(i) Both $[\text{N/Fe}]$ and $[\text{C/Fe}]$ can be diverse among stars within a GC owing to chemical pollution from field AGB stars. $[\text{N/Fe}]$ distributions in some GCs can clearly show bimodality, whereas $[\text{C/Fe}]$ is monomodal in most models. $[\text{N/Fe}]$ distributions depend on $s$ such that models with smaller $s$ (i.e. larger mass fraction of AGB ejecta used for GC formation) show the $[\text{N/Fe}]$ bimodality more clearly.

(ii) N-rich, C-poor stars in GCs also have higher He abundances owing to pollution from massive AGB stars with He-rich ejecta. The number fraction of He-rich stars ($Y > 0.30$) is higher for the models with smaller $s$ and shorter $\sigma_I$ for $3 \leq s \leq 24$ and $10^5 \leq \sigma_I \leq 10^7$ yr. He abundances of stars correlate with $[\text{N/Fe}]$ and $[\text{Al/Fe}]$ and anticorrelate with $[\text{C/Fe}]$, $[\text{O/Fe}]$ and $[\text{Na/Fe}]$ within GCs in our models.

(iii) Although our model can much better explain the observed C–N and Mg–Al anticorrelations than previous theoretical models, it is in strong disagreement with the observed O–Na anticorrelation.

(iv) This model naturally provides an explanation for the large fraction of CN-strong stars without recourse to an implausible initial mass function.

Based on these results for the above external pollution scenario, we discuss the long-standing problem of the CN-bimodality prevalent in the Galactic GCs, the possible helium abundance inhomogeneity in these systems and their horizontal branch morphologies.

Key words: globular clusters: general – globular clusters: individual: $\omega$ Centauri – globular clusters: individual: NGC 6752 – galaxies: evolution – galaxies: star clusters – galaxies: stellar content.

1 INTRODUCTION

Since observational evidence of star-to-star abundance inhomogeneity among the light elements of stars in the Galactic globular clusters (GCs) was discovered (e.g. Cohen 1978; Peterson 1980; Norris et al.
The origin of the inhomogeneity has been extensively discussed both theoretically and observationally (e.g. Sneden et al. 1992; Norris & Da Costa 1995; Smith, Briley & Harbeck 2005; see Gratton, Sneden & Carretta 2004, for a recent review). So far the following two scenarios (or working hypotheses) have been proposed for the origin of the abundance inhomogeneity: (i) the primordial hypothesis and (ii) the mixing hypothesis (e.g. Sweigart & Mengel 1979; Cottrell & Da Costa 1981; Freeman & Norris 1981; Smith 1987; Suntzeff 1993; Kraft 1994; Denissenkov & Weiss 1996). The first hypothesis is that the observed inhomogeneity is due to the second generation of stars that formed from gas ejected from the first generation of evolved stars [e.g. asymptotic giant branch (AGB) stars] (‘primordial pollution’ or ‘self-pollution’ scenarios; e.g. Cottrell & Da Costa 1981).

Internal processes of stars, such as dredge-up of the CN-processing of envelope material in inner hydrogen-burning regions, are key for understanding the observed chemical inhomogeneity of GCs in the second mixing hypothesis (e.g. ‘deep mixing’; Smith 1987; Kraft 1994; Thoul et al. 2002). Although deep mixing is highly unlikely to occur in less evolved stars on the main sequence (MS) and subgiant-branch, star-to-star abundance inhomogeneity was found in these stars of some Galactic GCs (e.g. Cannon et al. 1998 for 47 Tuc). The self-pollution scenario is accordingly suggested to be more promising than the mixing (evolutionary) scenario (Da Costa et al. 2004; Gratton 2004, for a recent review).

Stellar ejecta from AGB stars and massive stars have been considered to play a key role in early chemical evolution of GCs for the self-pollution scenario (e.g. Fenner et al. 2004; Charbonnel & Prantzos 2006; Karakas et al. 2006; Prantzos & Charbonnel 2006; Bekki & Chiba 2007). Previous studies suggested that GCs are unlikely to retain AGB ejecta owing to ram pressure stripping by warm/cold gas of the Galactic halo and disc (e.g. Frank & Gisler 1976; Smith 1996; Gnedin et al. 2002). These imply that GCs are even more unlikely to retain AGB ejecta in protogalactic environments, where denser halo/disc gas can strip the AGB ejecta efficiently from GCs during hydrodynamical interaction between the gas and the AGB ejecta. These previous studies thus appear to suggest that self-pollution is not likely within GCs evolving in isolation, though numerical attempts have not yet been made to investigate the details of self-pollution processes within GCs.

Furthermore, the self-pollution scenario is suggested to have the following two problems in explaining physical properties of GCs. The first is that the observed large number fraction of CN-strong populations (∼0.5 for NGC 6752), which correspond to the second generation of stars formed from AGB ejecta, cannot be explained by the scenario without assuming very unusual and unrealistic initial mass functions (IMFs) (e.g. Smith & Norris 1982; D’Antona & Caloi 2004); the total mass of AGB ejecta from the first generation of stars in a GC is only 1–10 per cent of the total mass of the GC with a canonical IMF (Bekki & Norris 2006). The second is that the observed O–Na and Mg–Al anticorrelations and C-depleted stars in CN-strong populations cannot be reproduced quantitatively by chemical evolution models based on the self-pollution scenario (Fenner et al. 2004). This problem, however, could be due to the adopted AGB models, which have some uncertainties in predicting chemical yields (e.g. Denissenkov & Herwig 2003; Karakas & Lattanzio 2003; Ventura & D’Antona 2005a,b; Campbell et al., in preparation).

The most serious problem among these is the first one – the observed large fraction of stars that (in this scenario) formed from AGB ejecta. The very large mass fraction of ‘polluted’ second generation of stars in the self-pollution scenario requires that AGB ejecta used for the formation of this generation of stars originate from stellar systems that had total masses much larger than those of the present GCs and have already disappeared for some reason. There could be two possible scenarios that can satisfy the above requirement. In the first place, we propose: (i) the GCs were initially more massive than the present-day GCs by a factor of 10–100, (ii) AGB ejecta of the first generation of stars were consumed by the formation of the second generation with very high star formation efficiency and (iii) most (90–99 per cent) of the first generation of stars had been preferentially lost by some physical mechanisms such as tidal stripping due to the Galactic tidal field. This scenario can be viable if enough of the AGB ejecta can be converted into the second generation in less than 10⁷ yr after the completion of the first generation of Type II supernovae (SNe II) – otherwise, age differences between the first and the second generation would cause a large age spread inconsistent with the observed tightness of the colour–magnitude diagrams (CMDs) of GCs.

In the second scenario, proposed by Bekki (2006): (i) protogalactic clouds are located in the central regions of low-mass protogalaxies embedded in dark matter subhaloes, (ii) AGB ejecta of the host galaxy’s field stars, which formed earlier and surround the GCs, go into star formation within the clouds on a time-scale of ∼10⁷ yr (i.e. until the intracluster gas is expelled by SNe II) and (iii) the low-mass galaxies were destroyed by the Galactic tidal fields to become the Galactic halo components, whereas the GCs were tidally stripped to form the Galactic halo GCs. In this scenario, there can be very small age differences (∼10⁷ yr) between stars with different abundances in a single GC. This pollution process would be better called ‘external pollution’ rather than ‘self-pollution’, because most of AGB ejecta originate from outside the GCs. Although Bekki & Norris (2006) have recently discussed the origin of the possible non-solar abundance or ‘normal’ GCs have not been extensively studied using previous chemical evolution models based on the external pollution scenario.

The purpose of this paper is thus to discuss the origin of abundance inhomogeneity observed in the lighter elements (e.g. C, N, O, Na and Al) of GCs by using chemical evolution models based on the external pollution scenario. In particular, we discuss: (i) the [C/Fe]–[N/Fe] anticorrelation, (ii) CN bimodality, (iii) the O–Na and Mg–Al anticorrelations, (iv) the helium abundance and (v) observable predictions of the models. We adopt the AGB models recently developed by Campbell et al. (in preparation) in which the third dredge-up (3DUP) in low-mass AGB stars can, in effect, be switched off and on due to the use or non-use of the Ledoux criterion for convective boundaries. We mainly show the results of the models without the 3DUP, because the models can better explain the observed abundance inhomogeneities if they are included in the present chemical evolution models. We realize that these models are ad hoc, but they enable us to investigate the proposed formation mechanism.

2 THE MODEL

2.1 The external pollution scenario

We consider GC formation in the central regions of low-mass, protogalaxies embedded in dark matter subhaloes at very high z. The GC host galaxies can provide a deep gravitational potential that allows AGB ejecta to be retained and used for star formation in proto-GC clouds. In this scenario, both (i) AGB ejecta from field stellar populations
(i.e. the main components of the galaxies) and (ii) protogalactic interstellar gas (which falls into the central regions) can be mixed with each other and subsequently used for GC formation. Gas ejected from SNe II with velocities of $\sim 1000$ km s$^{-1}$ is assumed to be expelled from dwarfs during galaxy formation and thus irresponsible for the chemical evolution of forming GCs. After GC formation, the host galaxies merge with the proto-Galaxy and are completely destroyed by the strong Galactic tidal field. The field stellar components of the galaxies are dispersed into the Galactic halo region to become old, metal-poor halo stars, whereas GCs, which are not destroyed by the Galaxy owing to their initial compactness, become the Galactic halo GCs.

Star formation and chemical evolution are assumed to proceed until SNe II formed within proto-GCs prevent further star formation in this scenario. Therefore, star formation is assumed to continue for less than $\sim 10^7$ yr in the models in which stars with different masses form in a coeval way. Age differences of stars are typically less than 5 Myr for most models in the present study. Evolution of chemical abundances depends mainly on (i) how much AGB ejecta are mixed into fresh interstellar medium (ISM) and consequently converted into stars and (ii) the time-scale of gas infall into central regions of galaxies. Advantages and disadvantages of the external pollution scenario are discussed later in Section 4.1.

### 2.2 Star formation and gas mass evolution

The total gas mass $[M_g(t)]$ of a forming GC changes through the gas ejection of AGB stars, gas infall and star formation:

$$\frac{dM_g}{dt} = A(t) + I(t) - S(t), \quad (1)$$

where $A(t)$, $I(t)$ and $S(t)$ are the gas ejection rate of the field AGB stars, gas infall rate and star formation rate, respectively. In order to calculate $A(t)$ at each time-step for the field AGB stars (which have a total mass of $M_I$), we assume an IMF in number defined as $\psi(m) = A_0 m^{-\alpha}$, where $m_I$ is the initial mass of each individual star and the slope $q = 2.35$ corresponds to the Salpeter IMF. The normalization factor $A_0$ is a function of $M_I$, $m_I$ (lower mass cut-off) and $m_u$ (upper mass cut-off): $A_0 = \frac{M_I(2-q)}{m_u^{2-q} - m_I^{2-q}}$. \quad (2)

where $m_I$ and $m_u$ are set to be 0.1 and 120 M$_\odot$, respectively. We adopt $q = 2.35$ for all models in the present study. $A(t)$ between $t$ and $t + dt$ is accordingly described as

$$A(t) = \frac{1}{dt} \int_{m_1}^{m_2} f_\odot \psi(m) dm, \quad (3)$$

where $m_1$ and $m_2$ are masses of stars that turn off the MS (thus become an AGB star) at the time $t + dt$ and $t$, respectively (thus $m_1 < m_2$). $f_\odot$ describes the total gas mass ejected from an AGB star with initial mass $m_I$ and final mass ($m_f$). We derive an analytic form of $f_\odot$ ($= m_f - m_1$) from the observational data by Weidemann (2000) by using the least-square fitting method, and find

$$f_\odot = 0.916 M_I - 0.444. \quad (4)$$

In order to calculate the MS turn-off mass ($m_{TO} = m_1$ and $m_2$) at each time-step, we use the following formula (Renzini & Buzzoni 1986; Norman & Scoville 1988):

$$\log m_{TO}(t) = 0.0558 (\log t_\odot)^2 - 1.338 \log t_\odot + 7.764, \quad (5)$$

where $m_{TO}$ is in solar units and time $t_\odot$ is in years. We assume that the time $t = 0$ corresponds to the epoch ($T_0$) when the most massive AGB star (with $m_I = 8$ M$_\odot$) starts to eject gas in the present models. Therefore $t_\odot = t + T_0$ in the above equation (5).

The gas infall rate $I(t)$ is described as

$$I(t) = I_0 \exp \left[ \frac{-(t - T_I)^2}{2 \sigma_I^2} \right], \quad (6)$$

where $I_0$ is the normalization factor for the total mass of infalling gas, $T_I$ represents the epoch of the maximum infall rate and $\sigma_I$ describes the time-scale of gas infall. The adopted Gaussian distribution means that the gas infall rate increases, whether slowly or rapidly — depending on the adopted $\sigma_I$, for $0 < t < T_I$ and then decreases for $T_I < t$. The time-evolution of the gas infall rate and the gas ejection ratio of AGB stars for the model M1 is shown in Fig. 1.

The star formation rate $S(t)$ is described as

$$S(t) = S_0 M_g^\alpha, \quad (7)$$

where $S_0$ (a normalization constant) is determined for a given $\alpha$ so that most of the gas can be consumed within an order of $10^7$ yr. Although we investigate a range of $\alpha$, we show the results for the models with $\alpha = 1$ which can better explain observations.

Observations (e.g. tightness of CMDs of GCs) strongly suggest that the time-scale of GC formation should be quite short (an order of $10^7$ yr). We also consider that star formation is truncated owing to feedback effects of SNe II formed during GC formation: the present models with this truncation can keep abundance homogeneity in heavy elements (e.g. Fe) and thus can be consistent with physical properties of normal GCs in the Galaxy. Therefore we assume that gas can be converted into stars in GCs only for $0 \leq t \leq T_{end}$, where $T_{end}$ should be similar to or shorter than $10^7$ yr corresponding to the typical lifetimes of stars that explode as SNe II. We present the results of the representative models with $T_{end} = 10^7$ yr and $T_I/T_{end} = 0.5$ in the present study. The gas mass ($M_g$) is given in units of $M_I$ (the total mass of field stars). In the adopted IMF, $M_{AGB}$ is 0.01 ($\times M_I$) at $t = T_{end}$ for all models.

### 2.3 Abundance evolution

The time evolution of the abundance of the $i$th element is described as follows:

$$\frac{d(Z_i M_I)}{dt} = Z_{A,i}(t) A(t) + Z_{L,i}(t) I(t) - Z_i S(t), \quad (8)$$

where $Z_{A,i}(t)$ and $Z_{L,i}(t)$ are the $i$th abundance of AGB ejecta and infalling gas, respectively. $Z_{A,i}(t)$ is time-dependent, because of the varying time-scales of evolution for the different masses of AGB stars (see equation 5). We assume that $Z_{l,i}(t)$ is constant for each $i$th element and thus refer to it simply as $Z_{l,i}$ in the present study.

In order to calculate $Z_{A,i}(t)$, we use the latest chemical yield tables produced by Campbell et al. (in preparation) in which H, He, C, N, O, Na, 24Mg, 25Mg, 26Mg, Al, are listed for AGB stars with the masses ranging from 2.5 to 6.5 M$_\odot$ for a given [Fe/H] ($\sim -1.4$). These models follow normal AGB evolution with hot bottom burning (HBB), but we produce two sets of yields — one with the normal 3DUP and one without (see below for details). The chemical yield model without (with) the 3DUP is referred to as YA1 (YA2) in the present study. Table 1 shows the mass fraction of the above 11 species in AGB ejecta from Campbell et al. (in preparation).

We consider that $Z_{l,i}$ should be similar to typical values of old halo stars in the Galaxy (e.g. Goswami & Prantzos 2000; Venn et al. 2004) and the adopted values of $Z_{l,i}$ are summarized in Table 2 for four
different models. The details of the models without 3DUP are given in Campbell (2006). Throughout this study, we assume that [Fe/H] is the same between infalling gas and AGB ejecta. This assumption is reasonable, because we consider that gas from SNe II can be expelled from dwarf efficiently during galaxy formation. We do not include the effects of massive rotating stars on the chemical evolution of proto-GC clouds in the present study, because we intend to understand more clearly the roles of AGB stars in the chemical evolution: we need to derive separately the roles of AGB stars and those of massive rotating stars in the chemical evolution.

2.4 With and without 3DUP

We mainly show the results of the models that use the AGB yields with no 3DUP, though we investigate the models with and without 3DUP. It should be stressed here that the models with no 3DUP are purely hypothetical ones that have not yet been proved. Previous studies based on self-pollution models with 3DUP in AGB stars failed to explain many of the observations of abundance inhomogeneity in GCs (e.g. Fenner et al. 2004). If our models can successfully reproduce some observations that were not explained by previous ones, we can obtain some insights into the ‘culprits’ in the self-pollution or external pollution scenarios. The abundance inhomogeneity of GCs in the present models with no 3DUP originates from pure hydrogen burning, which is available in HBB AGB stars.

The present models with 3DUP cannot explain at all most of the observed correlations between different abundances (e.g. the [C/Fe]–[N/Fe] anticorrelation). For example, these models cannot produce stars with [C/Fe] < 0 and [N/Fe] > 0, because [N/Fe] increases with the increase of [C/Fe] during the chemical evolution owing to the C-rich ejecta of AGB stars. Furthermore, the observed CN bimodality of stars cannot be reproduced in these models, owing to the C-rich ejecta of AGB stars. In addition, the abundance inhomogeneity in GCs in the present models with no 3DUP originates from pure hydrogen burning, which is available in HBB AGB stars.

Notes. The mass fraction of AGB ejecta is given for each element. These yields are for AGB stars with masses of 6.5 M⊙ and [Fe/H] = −1.4. Stellar yields for the AGB stars with differences masses and metallicities are from Campbell et al. (in preparation).

2.5 Main points of analysis

One of the most important parameters that govern the chemical evolution of GCs in the external pollution scenario is the ratio (s) of the total mass of the infalling gas (Mg) to that of AGB ejecta (Magb). We investigate the models with s (Mg/Magb) ranging from 0.1 to 100 and thereby isolated the best values of s that can explain observations. The time-scale of gas infall (τg) is also an important parameter that can control final abundance distributions of GCs in the present study. We investigate models with τg ranging from 10^7 yr and show important results of the models.

Table 1. Stellar yields of AGB stars.

| Model | Y | Mg/Fe | Al/Fe | Fe/ | O/Fe | Na/Fe | Mg/Fe | C/Fe |
|-------|----|-------|-------|-----|------|-------|-------|------|
| Y1    | 0.23 | 0.25 | 0.3 | 0.3 | 0.5 | 1     |       |      |
| Y2    | 0.23 | 1.25 | 1.97 | 1.63 | 0.3 | 0.38 | 1.74  |      |
| Y3    | 0.23 | 0.25 | 0.3 | 0.3 | 0.5 | 0.3  |       |      |
| Y4    | 0.23 | −0.3 | −0.25 | 0.3 | −0.3 | 0.5  | 1     |      |

2.6 Notes on modeling

We mainly show the results of the models that use the AGB yields with no 3DUP and star formation histories of the external pollution scenario.

Further, at present there are no yields available for the so-called super-AGB stars, which are more massive AGB stars that experience thermal pulses after core carbon burning. The few calculations in existence (e.g. Ritossa, Garcia-Berro & Iben 1996; Gil-Pons et al. 2005; Siess 2006) show that these stars experience very hot HBB and very little dredge-up. Qualitatively, this is what is required to solve the GC problem, so our models without dredge-up may simulate to some extent the yields from these stars, while we await full evolution and nucleosynthesis calculations for the appropriate masses and compositions.

We close this section with some comments about the reliability of AGB star yields. We believe that the balance of evidence indicates that AGB stars with dredge-up are not the site of the abundance inhomogeneities seen in GC stars. However, there are still so many uncertainties that it may still be possible for these stars to produce the abundance profiles required. For example, the depth of dredge-up is still very uncertain, the temperature profile in the HBB envelope depends on the convection theory (e.g. Ventura & D’Antona 2005a,b,c as well as associated parameters, mass-loss and of course various reaction rates. Although previous studies (Ventura, D’Antona & Mazzetti 2002; Ventura & D’Antona 2005c) provided useful tables for stellar yields of AGB stars, the lack of tables for some abundances (e.g. the lack of a table for Al abundance in Ventura & D’Antona 2005c) currently does not allow us to investigate the O–Na and Mg–Al anticorrelations in a fully self-consistent manner. It is thus our future study to compare the present results with those based on chemical evolution models using tables from works other than Campbell (2006). More comments on AGB models with and without 3DUP are given in Appendix B.

Table 2. Initial abundances of infalling gas.

| Model | Y | C/Fe | N/Fe | O/Fe | Na/Fe | Mg/Fe | Al/Fe |
|-------|----|------|------|------|-------|-------|-------|
| Y1    | 0.23 | 0.0  | −0.25 | 0.3 | −0.3 | 0.5  | 0.1   |
| Y2    | 0.23 | 1.25 | −1.97 | 1.63 | −0.3 | 0.38 | 1.74  |
| Y3    | 0.23 | 0.25 | 0.3  | −0.3 | 0.5 | 0.3  |       |
| Y4    | 0.23 | −0.3 | −0.25 | 0.3 | −0.3 | 0.5  | 1     |

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Table 3. Model parameters.

| Model no. | Gas yield | AGB yield | s (M$_{IN}/M_{AGB}$) | $\sigma_I$ (in units of 10$^7$ yr) | $M_g(0)$ (in units of $M_f$) | Comments |
|-----------|-----------|-----------|----------------------|-------------------------------|----------------------------|----------|
| M1        | YI1       | YA1       | 3.0                  | 0.1                           | 0.01                       | The fiducial model |
| M2        | YI2       | YA1       | 3.0                  | 0.1                           | 0.01                       | With 3DUP   |
| M3        | YI3       | YA1       | 3.0                  | 0.1                           | 0.01                       | A model for NGC 6752 |
| M4        | YI1       | YA2       | 3.0                  | 0.1                           | 0.01                       | A model for $\omega$ Cen |
| M5        | YI1       | YA1       | 6.0                  | 0.1                           | 0.01                       | Smaller initial [C/Fe] |
| M6        | YI1       | YA1       | 12.0                 | 0.1                           | 0.01                       |                      |
| M7        | YI1       | YA1       | 24.0                 | 0.1                           | 0.01                       |                      |
| M8        | YI1       | YA1       | 3.0                  | 0.05                          | 0.01                       |                      |
| M9        | YI1       | YA1       | 3.0                  | 1.0                           | 0.01                       |                      |
| M10       | YI1       | YA1       | 3.0                  | 0.01                          | 0.001                      |                      |
| M11       | YI1       | YA1       | 3.0                  | 0.01                          | 0.005                      |                      |
| M12       | YI1       | YA1       | 3.0                  | 0.01                          | 0.029                      |                      |
| M13       | YI1       | YA1       | 3.0                  | 0.01                          | 0.01                       |                      |
| M14       | YI1       | YA1       | 0.17                 | 0.01                          | 0.001                      |                      |
| M15       | YI4       | YA1       | 3.0                  | 0.1                           | 0.01                       |                      |

Notes. Columns (2)–(3) yield types for infalling gas and AGB ejecta (shown in Tables 1 and 2), respectively. Columns (4)–(6) model parameters.

By changing $s$, $\sigma_I$, and $M_g(0)$, we can construct a model that can explain reasonably well the abundance distribution for a specific GC. For example, the model M11 shows a bimodal [N/Fe] distribution with the normalized number fraction of N-rich and C-depleted stars being equal to that of the N- and C-normal ones and thus is consistent with the observed CN distribution in NGC 6752 (e.g. Smith & Norris 1993). Since we are focusing on the general behaviours resulting from parameter dependences of the models, we do not discuss extensively the abundance distributions of individual GCs. We, however, just briefly discuss the origin of He-abundance distributions of different stellar populations in $\omega$ Cen in Appendix A, because this issue is quite topical and is currently being investigated extensively by different theoretical models (e.g. Bekki & Norris 2006; Maeder & Meynet 2006).

In order to compare the present models with previous ones (e.g. Fenner et al. 2004; Ventura & D’Antona 2005c), we compare the present results with the same observational ones (Yong et al. 2003 for NGC 6752) that have been already compared also with these previous studies in Section 3. We also compare the present results with other more recent observations on abundances of stars (in particular, less evolved stars) in GCs (e.g. Ramirez & Cohen 2002; Briley, Cohen & Stetson 2004a; Briley et al. 2004b; Carretta et al. 2005; Cohen, Briley & Stetson 2005) and assess the viability of the external pollution scenario later in Section 4.1.

3 RESULTS

3.1 The fiducial model

3.1.1 C and N abundances

Fig. 2 shows the time evolution of the fiducial model M1 in comparison with the model M2 on the [C/Fe]–[N/Fe] plane. There are differences in initial abundances of infalling gas between models M1 and M2 (e.g. [C/Fe] and [N/Fe]). Since these abundances are estimated for new stars formed at each time-step in a GC, each location of the line represents abundances of stars with given ages rather than the mean abundances of the GC. As the proto-GC cloud is polluted by ejecta from more massive field AGB stars ($m_I > 5M_\odot$), [N/Fe] and [C/Fe] in the cloud rapidly increases and decreases, respectively. Gas infall increases the total gas mass of the cloud (and thus the density) and consequently increases gradually star formation in the cloud. As a result of this, new stars that formed from the gas, which is a mix of infalling gas and AGB ejecta, can have higher [N/Fe] and lower [C/Fe]. As a larger amount of AGB ejecta is accumulated in the cloud at later times, new stars formed later can have higher [N/Fe] and lower [C/Fe] than those formed earlier. Thus stars with higher [N/Fe] can show lower [C/Fe] in the fiducial model.
model M1 and this anticorrelation between [N/Fe] and [C/Fe] can be seen in most models.

Although the fiducial model can qualitatively explain the observed anticorrelation in Fig. 2 (e.g. Norris et al. 1981; Smith & Norris 1993), it does not pass the observed location of the N-rich population in NGC 6752. This quantitative inconsistency between the model and the observation can be seen in the comparative model M2 with initially small [N/Fe] in the infalling gas that can significantly dilute [N/Fe] during star formation within GCs. In the present model, [N/Fe] always becomes significantly high (~1) when [C/Fe] becomes low irrespective of the infall rate of N-poor gas. Since only two data points (i.e. two stars) shown in Fig. 2 are obviously not enough to make any robust conclusions on the quantitative consistency of the present model with observations, we discuss the consistency of the model by using more data sets later in Section 4.1.

As a result of gradual increase (decrease) of [N/Fe] ([C/Fe]) due to AGB pollution, new stars formed at different times can have different [N/Fe] and [C/Fe] and thus show abundance inhomogeneity in [N/Fe] and [C/Fe]. Fig. 3, showing the normalized number distributions of stars with different [N/Fe] and [C/Fe] in M1, demonstrates that the [N/Fe] distribution has a clearer bimodality than the [C/Fe] distribution. Most N-rich stars with [N/Fe] > 1 show a lower [C/Fe] (< −0.3), as expected from chemical pollution from massive AGB stars. Owing to the adopted density-dependent star formation rate, the gas infall causes two peaks in the star formation rate in M1. New stars formed during the first peak of star formation can have lower [N/Fe], whereas those formed during the second peak can have higher [N/Fe] due to a higher degree of AGB pollution. The reason for the apparent absence of the bimodality in [C/Fe] is that [C/Fe] cannot dramatically change within a short time-scale (~10^7 yr) so that new stars can show a much narrower [C/Fe] distribution (i.e. the bimodality cannot be evident for the adopted binning). The final mean [C/Fe] and [N/Fe] at T = 10^7 yr are −0.09 and 0.85, respectively.

3.1.2 He abundance

As the gas mass of the proto-GC cloud increases owing to accumulation of AGB material in M1, the He abundance (Y) of the gas also increases in a monotonic fashion: the final mean Y at T = 10^7 yr is 0.26, whereas the final Y of stars formed at T = 10^7 yr is 0.33. As a result of this, new stars formed later will have higher Y than those formed earlier. Since AGB ejecta increases [N/Fe] rapidly, new stars that form later and thus have higher Y can show higher [N/Fe]. Fig. 4 clearly shows a positive yet non-linear correlation between Y and [N/Fe] in new stars of GCs for M1. This correlation can be seen in almost all models in the present study and thus is simply a result of H burning.

Owing to the time-evolution of Y in the gas cloud, new stars formed at different times show different Y, which results in a Y inhomogeneity in the GC. Fig. 5 clearly shows that as the proto-GC cloud is polluted by AGB ejecta to a larger extent, newly formed stars show a larger degree of Y inhomogeneity. Furthermore, the location of the peak in the Y distribution is also shifted to the higher values as the AGB pollution proceeds: the peak value changes from T = 1 Myr to 0.25 at T = 10 Myr. These results imply that the time-scale of GC formation is an important factor which can determine Y distributions of GCs.

Since abundances in the present model either increase or decrease in a monotonic fashion as AGB pollution proceeds, strong...
correlations or anticorrelations can be expected between different abundances of stars in a GC. Fig. 6 shows that Y can anticorrelate with [C/Fe], [O/Fe] and [Na/Fe], though the dependences are not necessarily linear. These correlations can be expected for any chemical evolution models of GCs using stellar yields of AGB stars with no 3DUP.

3.1.3 Mg–Al and Na–O anticorrelations

Fig. 7 shows the time evolution of M1 in the [Mg/Fe]–[Al/Fe] plane and the observations of bright red giant stars in NGC 6752 (Yong et al. 2003). As a natural result of ongoing chemical pollution of the proto-GC cloud by field AGB stars, the [Mg/Fe] and [Al/Fe] ratios of new stars within the GC decrease and increase, respectively, as AGB pollution proceeds. Although the observed [Mg/Fe]–[Al/Fe] anticorrelation can be well reproduced qualitatively, the observed slope of the anticorrelation and the ranges of [Mg/Fe] and [Al/Fe] are not consistent with the predicted values: a shallower slope of the anticorrelation, a wider range of [Mg/Fe] and a smaller range of [Al/Fe] are clearly seen in the model. None of the models shows slopes as steep as the observed one for NGC 6752.

Fig. 8 shows the time evolution of M1 in the [O/Fe]–[Na/Fe] plane as well as the observational plots of bright red giant stars in NGC 6752. The model predicts a positive correlation between [O/Fe] and [Na/Fe], which is obviously inconsistent with the observed [O/Fe]–[Na/Fe] anticorrelation. Furthermore, the predicted [Na/Fe] is too low to match the observed values, and this inconsistency can be seen in all models of the present study. This suggests that the derived inconsistency is due to the adopted AGB yield models in which no 3DUP is assumed. Also the present study with [O/Fe]–[Na/Fe] correlation is in contrast with Fenner et al. (2004) in which 3DUP is assumed and the derived [O/Fe]–[Na/Fe] anticorrelation is not quantitatively consistent with the observed one by Yong et al. (2003). The fact that both Fenner et al. (2004) and the present study fail to explain the observed [O/Fe]–[Na/Fe] anticorrelation quantitatively may well suggest that AGB pollution scenarios have a serious problem in terms of explaining the observed
3.2 Parameter dependences

3.2.1 The dependence on \( s \)

Fig. 9 shows the dependences of \([\text{C}/\text{Fe}]\) and \([\text{N}/\text{Fe}]\) number distributions normalized to their maximum values (denoted as \( F_x \)) on \( s \), i.e. the ratio of \( M_{\text{IN}} \) (the total mass of infalling gas) to \( M_{\text{AGB}} \) (the total mass of AGB ejecta) and thus controls the degree of AGB pollution. Fig. 9 clearly shows that both the shapes and the peak values of the distributions depend on \( s \), which suggests that \( s \) is an important parameter for \([\text{C}/\text{Fe}]\) and \([\text{N}/\text{Fe}]\) distributions. Bimodal distributions can be seen in \([\text{N}/\text{Fe}]\) for models with \( 3 \leq s \leq 24 \) and the fraction of N-rich stars with \([\text{N}/\text{Fe}] > 1\) is higher in the models with smaller \( s \) (i.e. larger mass fraction of AGB ejecta). As \( s \) becomes larger, both the difference between smaller and larger peaks in \([\text{N}/\text{Fe}]\) and \( F_x \) in the smaller peak become larger. The derived diverse distributions of \([\text{N}/\text{Fe}]\) can be compared with the observed diversity in the CN distributions of GCs (Norris 1988).

The \([\text{C}/\text{Fe}]\) distributions, on the other hand, do not depend so strongly on \( s \) and the four models show almost monomodal distributions. This reflects the fact that chemical pollution by AGB stars does not change \([\text{C}/\text{Fe}]\) as dramatically as \([\text{N}/\text{Fe}]\). The spread in \([\text{C}/\text{Fe}]\) is, however, dependent on \( s \) in the sense that the spread is larger for models with smaller \( s \) (i.e. a larger degree of AGB pollution). The predicted bimodal \([\text{N}/\text{Fe}]\) distribution and monomodal \([\text{C}/\text{Fe}]\) one are discussed later in Section 4.1 by comparing these predictions with the latest observations for less-evolved stars (e.g. MS stars).

Fig. 10 shows that \( Y \) distributions normalized to the maximum values \( (F_x) \) depend on \( s \) such that both the peak values and the spreads in the \( Y \) distributions are larger for the models with smaller \( s \) (a larger degree of AGB pollution). These results accordingly suggest that GCs with larger degrees of \( Y \) inhomogeneity will show higher \( Y \). The results shown in Figs 9 and 10 therefore suggest that GCs with higher fractions of N-rich stars with \([\text{N}/\text{Fe}] > 1\) can show higher \( Y \) and larger degrees of \( Y \) inhomogeneity. Since the fraction of N-rich stars in a GC can be observationally quantified by an \( 'r' \) parameter by counting the number fraction of CN-strong stars versus CN-weak stars (Norris 1988), the above predicted correlation can be tested by investigating correlations between the \( r \) parameters.
Abundance inhomogeneity

0.2 0.4 0.6 0.8 1
s=3
0.2 0.4 0.6 0.8 1
s=6
0.2 0.4 0.6 0.8 1
s=12
0.2 0.3 0.35

Figure 10. The dependences of \(Y\) distributions \((F_x)\) on \(s\).

and observable properties dependent on \(Y\) inhomogeneity, such as colour spreads between MS stars in the CMDs of GCs.

3.2.2 The dependence on \(\sigma_1\)

Fig. 11 shows that \([\text{N/Fe}]\) distributions depend strongly on \(\sigma_1\), which means that \([\text{N/Fe}]\) distributions can be controlled by the time-scale of gas infall. As seen in the \(s\) dependences of \([\text{N/Fe}]\), the models with higher peak values of \([\text{N/Fe}]\) show larger degrees of \([\text{N/Fe}]\) spreads in the distributions. It is significant that the model with \(\sigma_1 = 1.0\) shows an almost monomodal \([\text{N/Fe}]\) distribution with an apparently small dispersion of \([\text{N/Fe}]\) (i.e. no clear bimodality). \([\text{C/Fe}]\) distributions do not depend so strongly on \(\sigma_1\) as \([\text{N/Fe}]\) so that models show a relatively narrow range of \([\text{C/Fe}]\). It can be concluded from Figs 9 and 11 that (i) \([\text{C/Fe}]\) distributions are highly likely to be monomodal irrespective of \(s\) and \(\sigma_1\) and (ii)

\([\text{N/Fe}]\) distributions are likely to be bimodal and it depends on \(s\) and \(\sigma_1\) how clearly the distributions show the bimodality.

Fig. 12 shows that the models with smaller \(\sigma_1\) (i.e. more rapid gas infall) show larger degrees of \(Y\) inhomogeneity. However, a positive correlation between the peak \(Y\) and the degrees of \(Y\) inhomogeneity seen in models with different \(s\) (shown in Fig. 10) cannot be seen in these models with different \(\sigma_1\). Given the fact that mean \(Y\) should not be different between models with the same \(M_{\text{AGB}}\) (or \(s\)) yet different \(\sigma_1\), \(r\) parameters can be correlated only with the degrees of \(Y\) inhomogeneity (i.e. not with \(Y\)). The results shown in Figs 10 and 12 thus suggest that \(Y\) distributions are diverse depending on the two key parameters \(s\) and \(\sigma_1\). Observational implications of these results are discussed later in Section 4.4.

Thus the present study shows that \(Y\), \([\text{C/Fe}]\) and \([\text{N/Fe}]\) distributions in GCs are predicted to be diverse if GCs are formed from the mixture of infalling gas and AGB ejecta of field AGB stars in forming dwarfs at high \(z\). The results shown in Figs 10–12 can provide some physical basis not only for the observed diversity in CN distributions of GCs (e.g. Norris 1988) but also for the possible \(Y\) spreads in NGC 2808 (e.g. D’Antona et al. 2005) and \(\omega\) Cen (e.g. Bedin et al. 2004). It should be stressed here that the diverse distributions come from the combination of the external pollution scenario and the adopted AGB yield: neither the external pollution scenario assuming the 3DUP in AGB stars nor classical self-pollution scenarios assuming no 3DUP are able to predict diverse distributions similar to the observed ones.

4 DISCUSSION

4.1 Comparison with observations

It is more instructive for the present study to compare our model predictions on abundance inhomogeneity within GCs with observations for GC MS stars, rather than with the giant branch populations. Comparison with near-MS stars is to be preferred, because

\[\text{[C/Fe]}\]

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we do not have to consider abundance changes due to deep mixing in evolved stars. That said, there are relatively few studies that show observational data sets on abundances of MS populations in GCs (due to the difficulty in observing these faint objects by the current generation of telescopes). We thus use available near-MS data sets in order to understand how well the 'pure' AGB pollution scenario can explain the observations for Galactic GCs, and note that the data for evolved stars [i.e. red giant branch (RGB) and AGB stars] are not ideal for testing our models. We here mainly focus on relations between $[\text{C}/\text{Fe}]$ and $[\text{N}/\text{Fe}]$ using recent observational data sets from Briley et al. (2004a,b), Carretta et al. (2005), Cohen, Briley & Stetson (2002), Cohen et al. (2005) and Smith et al. (2005).

Fig. 13 shows the comparison between the predicted evolution of model in the $[\text{C}/\text{Fe}]-[\text{N}/\text{Fe}]$ plane and the observed $[\text{C}/\text{Fe}]$ and $[\text{N}/\text{Fe}]$ for less evolved stars such as MS populations, subgiants and dwarfs in NGC 6397 (Carretta et al. 2005), NGC 6752 (Carretta et al. 2005), 47 Tuc (Briley et al. 2004b; Carretta et al. 2005), M15 (Cohen et al. 2005), M13 (Briley et al. 2004a) and M5 (Cohen et al. 2002). Figs 13 and 14 clearly indicate that the models are much more consistent with observations for less evolved stars (Fig. 14) than for evolved ones (Fig. 13), though the models still have difficulty in explaining stars with low $[\text{C}/\text{Fe}]$ ($<-0.5$). Furthermore, the observed $[\text{C}/\text{Fe}]-[\text{N}/\text{Fe}]$ anticorrelation and the apparently non-linear dependence of $[\text{N}/\text{Fe}]$ on $[\text{C}/\text{Fe}]$ are both consistent with the theoretical predictions. These derived consistencies may well provide support for the adopted star formation histories of proto-GC clouds in the external pollution scenario.

Fig. 15 shows the comparison between the predicted $[\text{C}/\text{Fe}]$ and $[\text{N}/\text{Fe}]$ distributions and the observed ones for subgiants and stars at the base of RGB in M15 and for MS stars in 47 Tuc. It is clear from Fig. 15 that (i) both models and observations show almost monomodal distributions in $[\text{C}/\text{Fe}]$, (ii) the predicted peak (in normalized number fraction) in the $[\text{C}/\text{Fe}]$ distribution is significantly higher than those of the observations, (iii) the predicted bimodal distribution is more similar to the observed one in 47 Tuc than to the one in M15 and (iv) the locations of the two peaks in $[\text{N}/\text{Fe}]$ and $[\text{C}/\text{Fe}]$ can be qualitatively reproduced, (ii) the adopted 'pure' AGB-pollution scenario, however, cannot explain the observed smaller $[\text{C}/\text{Fe}]$ ($[\text{C}/\text{Fe}]<-0.5$) and (iii) it cannot explain stars with high $[\text{N}/\text{Fe}]$ (>1.5) and low $[\text{C}/\text{Fe}]$ (<−0.5). These results clearly suggest that a combination of deep mixing (e.g. see the new mechanism discussed by Eggleton, Dearborn & Lattanzio 2006) and primordial pollution from AGB stars in proto-GC clouds need to be considered for reasonable explanations of the observations.
are, however, not exactly the same as the observed ones. Thus the observed wider spread in [C/Fe] cannot be explained so well by the present models, though the monomodal [C/Fe] and bimodal [N/Fe] distributions are quite well reproduced by the models.

The present models also predict the mean values of [C/Fe] and [N/Fe] in GCs and thus can be compared with the ones derived from high-resolution spectroscopic studies of integrated spectra of GCs in nearby galaxies other than the Galaxy. Ongoing and future observational studies based on large ground-based telescopes with high-dispersion spectrographs enable us to derive [C/Fe] and [N/Fe] of GCs in other galaxies by model fitting (e.g. Bernstein et al. 2004a), for MS stars in 47 Tuc (green filled pentagons; Briley et al. 2004a), for MS stars in 47 Tuc (green filled pentagons; Briley et al. 2004b) and for subgiants and stars at the base of the RGB in M15 (magenta filled circles; Cohen et al. 2005) in the [C/Fe]–[N/Fe] plane. The time evolution of M1 and M15 is shown by thick solid and dotted lines, respectively. The typical error bars are 0.2 for [C/Fe] and [N/Fe] (see fig. 7 in Carretta et al. 2005).

4.2 Advantages and disadvantages of the external pollution scenario

4.2.1 Advantages

One of the serious problems in any self-pollution scenario is that the number fraction of N-rich, C-depleted stars in a GC for a canonical IMF cannot be large enough to explain the observed fraction (e.g. Smith & Norris 1982; D’Antona & Caloi 2004; Prantzos & Charbonnel 2006). This problem can be solved if we assume peculiar top-heavy IMFs (D’Antona & Caloi 2004), for which GCs can either be easily destroyed by the Galactic tidal field during their evolution (e.g. Chernoff & Weinberg 1990) or be disintegrated quickly due to loss of a large amount of mass through SNe II explosion (Bekki & Norris 2006). Since both N-rich and N-normal components can be formed from proto-GC clouds chemically polluted by field AGB stars (that do not become parts of GCs) in the external pollution scenario, there are no serious problems on number fraction of N-rich, C-depleted stars in the scenario with a canonical IMF. Recently, Pasquini et al. (2005) have suggested that the whole proto-GC cloud needs to be chemically enriched by a previous generation of stars based on the observational data of the Li abundance in nine turn-off stars in NGC 6752: this is consistent with the external pollution scenario.

GCs can be formed during or after the formation of the field stars in dwarfs at very high z in the external scenario. A consequence of this is that the field stars in dwarfs on average should be older and more metal-poor than GCs when GCs are formed in this scenario. Accordingly, if dwarfs with GCs can be destroyed by the tidal field of the proto-Galaxy to form the Galactic field halo stars and the halo GCs soon after GC formation, the field halo stars are highly likely to be more metal-poor than the halo GCs in this scenario. It is a well known observational fact that the metallicities of the most metal-poor haloes stars are lower than those of the most metal-poor GCs in the Galaxy (e.g. Freeman 1993). The earlier formation of field halo stars in comparison with GCs in the external pollution scenario can naturally explain this observation.
Since proto-GC clouds are chemically enriched by old field stars (not by the very first stars like Pop III stars) of low-mass dwarf galaxies in the present models, the clouds can have abundances similar to field stars (which can finally become halo stars) and be uniformly enriched by r- and s-process syntheses. Therefore, GCs formed from these clouds can have abundances similar to those of field stars except for light elements and accordingly explain recent observations that have reported little differences in [Fe/H] dependences of [Sr/Fe], [Y/Fe], [Ba/Fe] and [Eu/Fe] in MS turn-off stars and subgiants at the base of the RGB in some GCs (James et al. 2004). Stars in the models with no 3DUP keep initial abundances of s-process elements so that the models can be consistent with the observed possible evidence for homogeneous abundances of s-process elements in James et al. (2004). The external pollution from AGB stars with 3DUP would be inconsistent with the above observations by James et al. (2004): future observations on the degree of homogeneity in the abundances of s-process elements will give strong constraints on any AGB-pollution scenario.

Recent abundance studies of the Galactic open cluster have not yet revealed possible abundance spreads between stars of these clusters (e.g. Friel et al. 2003; De Silva et al. 2006). The external pollution scenario suggests that these clusters are highly unlikely to show abundance inhomogeneities, because they are formed from the Galactic gas clouds in which AGB pollution cannot proceed owing to very shallow gravitational potentials of the clouds. Hill et al. (2000) reported that Large Magellanic Cloud (LMC) clusters do not show Al–O anticorrelations, which implies that AGB pollution did not proceed in these systems. The external pollution scenario thus suggests that these LMC clusters, which shows discy kinematics (e.g. Freeman, Illingworth & Oemler 1983), were formed not within dwarfs that were the LMC’s building blocks but within the proto-LMC disc via a dissipative process at the formation epoch of the disc. The observed differences in some elemental abundances between the LMC clusters and the Galactic GCs (Johnson, Ivans & Stetson 2006) may well be understood in terms of clusters originating within the forming disc of the LMC.

4.2.2 Disadvantages

As pointed out by Fenner et al. (2004), chemical evolution models of GCs based on AGB-pollution have some difficulties in explaining quantitatively O–Na and Mg–Al anticorrelations and correlations between $^{24}\text{Mg}/^{26}\text{Mg}$ and light elements (e.g. [O/Fe] and [Na/Fe]). Although some improvement over previous models can be seen in the present models in which AGB stars experience HBB but no 3DUP, the observed O–Na anticorrelation cannot be explained by them. This incapability comes from the adopted stellar yield models of AGB stars rather than from improper models for early star formation histories within GCs. Given the fact that there are some uncertainties in model calculations of yields in AGB stars (Ventura & D’Antona 2005c; Campbell et al., in preparation), the above apparent failure does not necessarily mean that there is a serious problem with the adopted AGB-pollution scenario. Indeed, we remind the reader that no consistent solution currently exists. All proposals rely on tweaking of some inputs, be it over- or under-estimation of the adopted stellar yield model. One solution for the above problem would be to adopt AGB models with little 3DUP (i.e. not the models with no dredge-up) in order to avoid unreasonably small values of [Na/Fe] in the models. This contrived way of modelling, however, should not be regarded as a necessary solution for this problem. It may say something about the characteristics of low-metallicity AGB stars or perhaps higher mass super-AGB stars – maybe they do not suffer as much 3DUP as expected by our models.

4.3 The diversity in CN distributions

Previous observational studies showed that there exist variations in the strength of CN bands between the Galactic GCs (e.g. Norris & Smith 1981; Suntzeff 1981). Norris (1988) investigated the value of the ’r’ parameter, which is defined as the ratio of the number of CN-strong to CN-weak stars in a given system, for each of 12 GCs with $-1.9 \leq [\text{Fe/H}] \leq -1.2$ and thereby found that the values of the r parameter range from 0.22 (i.e. smaller fraction of CN-strong stars) to 3.29 (i.e. larger fraction). Norris, Freeman & Da Costa (1984) also suggested that the observed anticorrelation between the behaviour of the CN bands and that of the CH features in GCs (e.g. Norris & Cottrell 1979; Norris & Freeman 1982; Bell, Hesser & Cannon 1983; Norris, Freeman & Da Costa 1984) is difficult to explain in terms of the primordial pollution (or self-pollution) scenarios.

We have demonstrated that (i) the distributions of N and C are diverse depending on the two key parameters $s$ and $\sigma_t$, (ii) N distributions show clear bimodality in some models and (iii) C and N abundances anticorrelate with each other in the external pollution scenario. These results provide some physical basis for the origin of the observed diversity and for the observed C–N anticorrelation, [if we understand what determines the values of the above two key parameters in protogalaxies]. Owing to the lack of extensive numerical simulations on the dynamical fate of AGB ejecta in the central region of protogalaxies, it is unclear what physical processes in protogalaxies (e.g. the effectiveness of stellar feedback effects) can control $s$ and $\sigma_t$. We plan to derive $s$ and $\sigma_t$ and their dependences on physical properties of protogalaxies (e.g. masses and sizes) in our future high-resolution simulations and thereby discuss the origin of the observed diversity of CN distributions in GC in the context of galaxy formation.

4.4 Possible helium abundance inhomogeneity in GCs

Recent photometric observations of stars in $\omega$ Cen have discovered a double main sequence (DMS) in the CMDs of its stellar populations (Anderson 1997; Bedin et al. 2004). One of the most promising interpretation is that stars on the bluer main sequence (bMS) of the DMS represents a very helium-rich ($Y \gtrsim 0.3$) population (Norris 2004; Piotto et al. 2005). Furthermore, D’Antona et al. (2005) have reported helium abundance variation among MS stars of NGC 2808. Green (1980) first showed that there is a possible correlation between the helium abundance of GCs and the locations of GCs with respect to the Galactic Centre: inner GCs are more likely to show higher helium abundance. Recently Yong et al. (2005) have reported a possible trend between [Fe/H] and [Al/Fe] for giants stars in NGC 6752 and thus suggested that this trend can result from a possible correlation between the H–He ratio and [Al/Fe], which is a natural result of chemical pollution by H-burning ejecta. Although the total number of observations reporting the possible abundance spread in helium is currently very small, these observations contain valuable information not only on horizontal branch (HB) morphologies but also on helium pollution processes in proto-GCs (e.g. D’Antona et al. 2005, 2006).

Important predictions of the present external pollution scenario on helium abundance properties of GCs are (i) spread in helium abundance between stars, (ii) larger helium abundances in N-rich, C-depleted stars and (iii) smaller helium abundance in GCs formed...
from gas less polluted by field AGB stars. Furthermore, we can provide some prediction on colour distributions of stars for the MS populations of GCs by using the model developed by D’Antona et al. (2005). Differences in Y between stars with normal Y (=0.24) and those with higher/lower Y can be converted into colour difference \(\Delta(V-I)\) by using the following formula (D’Antona et al. 2005): 
\[
\frac{\Delta(V-I)}{\Delta Y} \approx -0.438.
\]

Fig. 16 clearly demonstrates that the colour distributions of stars in the fiducial model M1 shows significant dispersions in \(V-I\) of the MS. This dispersion can be observed as spreads in the CM relations of MS populations of GCs and thus as evidence for GC formation from AGB ejecta.

Several authors have pointed out that He abundance inhomogeneity of stars in GCs can be imprinted on photometric properties of stellar populations on the HB of GCs (e.g. Norris 2004; D’Antona et al. 2006; Lee et al. 2005). These authors suggested that a GC with a He-rich population can (e.g. NGC 2808) show a characteristic HB colour and luminosity distribution, a bimodal HB distribution and faint, very hot HB stars. Since a GC with a larger fraction of N-rich stars can have a higher degree of He abundance inhomogeneity in the external pollution scenario, the HB properties of GCs can be correlated with the fraction of N-rich stars (i.e. Y parameter). If the origin of the second parameter problem is related to the difference in helium abundance between GCs (see Catelan 2005, for a recent review), the external pollution scenario suggests that the origin of the problem can be closely associated with differences in AGB pollution processes (controlled mainly by the two parameters, \(s\) and \(\sigma\)) between different proto-GC clouds in the central regions of forming dwarfs at high \(z\).

### 4.5 Origin of the observed correlation between the number fraction of CN-strong populations and ellipticities of GCs

Norris (1987) investigated possible correlations between the Y parameter and other physical properties of GCs (e.g. metallicities and luminosities) and found that there exists a correlation between ellipticities (\(\epsilon\)) of GC shapes and the values of the Y parameters (see Smith 2002 for recent confirmation of this correlation). This result means that if the flattened shapes are due to rotational kinematics of GCs (rather than anisotropy of velocity dispersion), GCs with a larger amount of angular momentum are more likely to show larger \(r\). Norris (1987) suggested that if global rotation of a GC positively correlates with rotation of individual stars within the GC and if evolutionary mixing responsible for the CN abundance inhomogeneity is driven by stellar rotation, the observed correlation can be readily explained. Norris (1987), however, pointed out that this explanation has a difficulty in explaining the existence of CN-enhanced stars near the MS turnoff of 47 Tuc (Bell et al. 1983) unless the effects of rotation are efficient for MS stars. Furthermore, it is unclear why stars within GCs with a larger amount of intrinsic angular momentum would have a larger amount of stellar rotation in this scenario by Norris (1987).

In the present external pollution scenario, the values of the Y parameters depend strongly on the relative mass fraction of AGB ejecta with respect to the total gas mass (i.e. AGB ejecta + fresh infall gas) in proto-GCs: the higher the fraction is, the larger the \(r\) becomes. One of possible explanations for the above \(\epsilon-r\) correlation in the context of the external pollution is therefore that whenever a larger amount of AGB ejecta can be transferred into proto-GC regions (i.e. central regions of their host dwarfs), the gas is highly likely to have a larger amount of angular momentum, from which GCs with a larger amount of angular momentum and thus with larger \(\epsilon\) can be formed. We suggest that the above situation of the larger amount of AGB ejecta with higher (specific) angular momentum is likely if field stars in the central regions of dwarfs (thus, AGB polluters for proto-GCs) have rotational kinematics: AGB ejecta from such stars rotating dwarfs can also have orbital angular momentum (with respect to the centres of dwarfs) so that GCs formed from the ejecta in the centres can have rotational kinematics and thus large \(\epsilon\) after conversion of orbital angular momentum of gas into intrinsic spin angular momentum of GCs during GC formation.

It is thus our future study to investigate (i) whether orbital angular momentum of AGB ejecta from field stars rotating dwarfs can be really converted into intrinsic spin angular momentum of GCs and (ii) whether GCs have a larger amount of intrinsic spin angular momentum thus larger \(\epsilon\), if they are formed from gas with a larger fraction of AGB ejecta. High-resolution numerical simulations on stellar and gas dynamics in the central 0.1–100 pc of dwarfs at high \(z\) would be essential for this investigation. We plan to perform such chemodynamical simulations with AGB feedback effects (e.g. thermal heating and return of AGB ejecta to ISM) to understand the origin of the observed \(\epsilon-r\) correlation in the Galactic GCs. Nuclear dynamics in protogalaxies would be dependent on the depth of their gravitational potential wells thus on their masses and sizes. Our future simulations will therefore enable us to understand whether very flattened GCs (e.g. \(\omega\) Cen) can be formed in less or more massive dwarfs at high \(z\).

### 4.6 Origin of possible higher nitrogen abundances for some bright GCs in M31

Recently, Li & Burstein (2003) have found that the NH absorption line is far stronger in (three) metal-rich GCs of M31 than it is in the Galactic GCs at a given value of CH or [Fe/H]. Beasley et al. (2004) also have revealed that the near-ultraviolet cyanogen features of M31 GCs are strongly enhanced with respect to the Galactic GCs for \(-1.5 < [\text{Fe/H}] < -0.3\). These observations imply that [N/Fe] can be higher in some M31 GCs than in the Galactic ones.
for a same metallicity range. It has been also revealed that M31 GCs having very strong NH absorptions are more luminous with $-11 < M_V < -8.5$ (e.g. Burstein et al. 2004). This implies that the possibly higher [N/Fe] is due to a larger number of bright GCs in M31, which can have the higher values for some physical reasons. It remains, however, totally unclear why these bright M31 GCs have high [N/Fe].

The present models have shown that mean values of [N/Fe] of GCs, which should be compared with the [N/Fe] estimated from the integrated spectra of M31 GCs, can be as high as 0.9. This result, combined with recent theoretical results and observational ones on nucleus formation of galaxies (e.g. Bekki 2006; Bekki, Couch & Shioya 2006; Côte et al. 2006), can provide the following answer for the above question. Based on analytical calculations, Bekki (2006) pointed out that AGB ejecta can be more effectively trapped in nuclear region of more massive dwarfs embedded in dark matter haloes so that AGB pollution can proceeds to a greater extent in these dwarfs. Inward radial gas-transfer into nuclear regions, which is indispensable for the formation of compact stellar systems, is demonstrated to be more efficient in more massive dwarfs (Bekki et al. 2006). Photometric studies on nuclear properties of dwarfs based on the ACS Virgo Cluster Survey have recently revealed that stellar nuclei are more massive in more massive dwarfs (Côte et al. 2006). Therefore we can claim that if some bright GCs in M31 originate from nuclei of brighter nucleated dwarfs, the observed higher [N/Fe] can be naturally explained.

Thus the origin of the observed possible differences in [N/Fe] between M31 GCs and the Galactic ones could reflect the fact that the proto-M31 contained a larger number of brighter dwarfs where AGB pollution could proceed more effectively in their central regions to form GCs with higher [N/Fe]. These more massive dwarfs were destroyed to form the M31’s stellar halo during the hierarchical formation of M31 through merging of these dwarfs. Because of the mass–metallicity relation of field stellar populations of dwarfs (e.g. Durrell, Harris & Pritchet 2001), the developed stellar halo can show a higher metallicity and is consistent with observations (e.g. Reitzel, Guhathakurta & Gould 1998). If [N/Fe] of GCs can be controlled by nuclear dynamics of their host dwarfs at very high $z$, as suggested by the present study, more luminous galaxies would have both a larger number of GCs with high [N/Fe] and more metal-rich (thus redder) stellar haloes. We thus suggest that future high-resolution spectroscopic studies for GC abundances in galaxies beyond the Local Group, combined with wide-field imaging of stellar haloes in these galaxies, can give some constraints on the formation histories of GCs within dwarfs at very high $z$.

5 CONCLUSIONS

We have investigated the abundance inhomogeneity among the light elements (e.g. C, N, O, Na and Al) in stars in the Galactic GCs by adopting the external pollution scenario and using latest stellar yield models of metal-poor AGB stars with and without 3DUP. In the external pollution scenario, GCs within a forming low-mass dwarf embedded in a dark matter halo can be formed from the mixture of (i) gas ejected from the field AGB stars formed earlier in the dwarf and (ii) the interstellar gas infalling to the central region of the dwarf. We extensively investigated our models with no 3DUP over wide parameter ranges. We summarize our principal results of the models as follows.

(i) The ratio of the total mass of infalling gas to that of AGB ejecta during GC formation within a forming dwarf galaxy (described by the $s$ parameter) and the time-scale of gas infall ($\sigma_i$) are the most important parameters that can determine abundance properties of GCs.

(ii) Both [N/Fe] and [C/Fe] can be diverse among stars within a GC owing to chemical pollution from field AGB stars. [N/Fe] distributions in some GCs can clearly show bimodality, whereas [C/Fe] can be monomodal in most models. [N/Fe] distributions depend on $s$ such that models with smaller $s$ (i.e. larger mass fraction of AGB ejecta used for GC formation) show the [N/Fe] bimodality more clearly.

(iii) N-rich, C-poor stars in GCs also have higher He abundances owing to pollution from massive AGB stars with He-rich ejecta. The number fraction of He-rich stars (Y > 0.30) is higher for the models with smaller $s$ and shorter $\sigma_i$ for $3 \leq s \leq 24$ and $10^8 < \sigma_i < 10^7$ yr. He abundances of stars can correlate with [N/Fe] and [Al/Fe] and anticorrelate with [C/Fe], [O/Fe] and [Na/Fe] within GCs (and a simple consequence of H burning).

(iv) Although our models can much better explain the observed C–N and Mg–Al anticorrelations than previous theoretical models, the observed O–Na anticorrelation cannot by simply explained by any models with different parameters in this scenario. The inability of the present models to match the observations results from the adopted AGB yields and thus suggests that models using different AGB yields need to be explored to assess the viability of AGB pollution scenarios (for more details, see Appendix B).

(v) Extensive comparison between the models and the observations demonstrates that the predicted [C/Fe]–[N/Fe] anticorrelation can be more quantitatively consistent with observations for less evolved stars (MS populations, subgiants and dwarfs) than for evolved ones (e.g. red giants). The comparison also demonstrates that the observed [C/Fe] and [N/Fe] distributions for less evolved stars are at least qualitatively consistent with the model predictions.

(vi) One important prediction of our models is that GCs with higher degrees of abundance inhomogeneity should show a larger colour spread [e.g. $\Delta(V-I)$] in their MS populations as the result of bluer stars formed from He-rich gas. Accordingly, it would be worthwhile for future observational studies to investigate correlations between the $s$ parameters (i.e. the number ratio of CN-strong to CN-weak stars) and $\Delta(V-I)$ in GCs.

(vii) The values of the two key parameters ($s$ and $\sigma_i$) are suggested to be controlled by stellar and gas dynamics in the central regions of galaxies of GCs. Central stellar and gas dynamics (e.g. gas fuelling rates to the central 10 pc) have been suggested to be controlled by global properties (e.g. total masses and densities) of galaxies. Accordingly, abundance inhomogeneities of GCs may have fossil information on (i) formation and evolution histories of central regions of defunct dwarfs and (ii) physical properties of the dwarfs.

Thus we have shown that the CN bimodality and the number fraction of C-depleted, N-rich populations of a GC can give strong constants on the short-term (an order of $10^7$ yr) star formation history of the proto-GC cloud of the GC, if the cloud is chemically polluted by AGB stars. We accordingly suggest that any theoretical models for the origin of GC abundances need to explain not only the O–Na and Mg–Al anticorrelations but also the bimodal CN abundance distributions in GCs.

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APPENDIX A: A POSSIBLE MODEL FOR THE BMS IN ω CEN

Bedin et al. (2004) suggested that stars on the bMS of the DMS can be a very helium-rich (Y > 0.3) population. A number of recent investigations have suggested that a higher degree of He enhancement in the bMS is a viable explanation for the origin of the DMS (e.g. Norris 2004; Lee et al. 2005; Piotto et al. 2005) and recent analytical works by Bekki et al. (2006) have suggested that He enhancement due to self-pollution (or more correctly, external pollution) of AGB stars can explain the origin of the bMS. If, however, remains unclear whether such AGB pollution scenarios can quantitatively explain the observed abundance properties (e.g. [C/Fe], [N/Fe] and Y) in a self-consistent manner.

Fig. A1 shows Y, [C/Fe] and [N/Fe] distributions of the model M14 in which very small σi (i.e. a higher degree of AGB pollution), smaller σi (i.e. more rapid gas infall) and smaller Mj(0) (i.e. more rapid chemical enrichment) are adopted so that the model shows a large fraction of He-rich stars. Owing to the modelled very larger degree of AGB pollution, stars in this model shows a large fraction of He-rich (>0.3) and N-rich (>1) stars. Piotto et al. (2005) have reported that (i) the He-rich bMS shows a rather large nitrogen abundance ([N/M] ~ 1–1.5) and (ii) it is not very rich in C ([C/M] = 0). The result in Fig. A1 implies that these observations can be due to AGB pollution, though the C abundance of the bMS is not so consistent with the predicted one. An important result in Fig. A1 is that some fraction (50%) of stars show relatively normal Y (0.24–0.25). This suggests that if the bMS is formed from gas polluted by AGB stars, stars with relatively normal Y are also formed, though the mass fraction of the He-normal population is small. This may well provide a clue to the origin of the observed normal Y (=0.246) for the metal-intermediate RRL stars (that would have been formed with the bMS) in ω Cen (Sollima et al. 2006).

Very luminous GCs such as ω Cen in the Galaxy and G1 in M31 have long been suggested to originate from nuclei of ancient nucleated systems, where continuous gas infall and chemical enrichment were highly likely to occur (e.g. Hilker & Richtler 2000; Bekki & Freeman 2003; Bekki & Chiba 2004). As demonstrated above, the total mass of AGB ejecta used for the He-rich population in ω Cen needs to be ~6 times larger than that of infalling gas not enriched by AGB stars. It remains unclear whether this can really happen in the nuclei of dwarf galaxies that are considered to be progenitors of very luminous GCs like ω Cen and G1.

APPENDIX B: COMMENTS ON AGB MODELS WITH AND WITHOUT 3DUP

Current standard models of intermediate-mass AGB stars with 3DUP do not reproduce the observed C–N, O–Na or Mg–Al inverse-correlations observed in most GCs. We note, however, that the models of Ventura & D’Antona (2005b) do come close, due to their use and development of an alternative theory of convection, which alters the evolution of their models significantly (see e.g. Ventura & D’Antona 2005a). In the standard models, the combination of HBB and 3DUP during AGB evolution does predict enhancements of nitrogen, sodium and aluminium, and a reduction in oxygen – all of which are needed to explain the observations – however, they also predict an increase of carbon and magnesium, giving positive C–N and Mg–Al correlations which is in direct violation of the observations. In addition to this, the models do not quantitatively account for the large degree of O depletion and tend to overproduce Na,
Decoupling 3DUP from HBB exposes, the dependence of the yields on the interplay between the two processes and provides a possible solution to the mismatch between theory and observation. At the very least, by separating the effects it is a useful investigative tool. A salient result from our new stellar models, evident in Table 1, is that oxygen is now heavily depleted in the more massive stars. Also of importance is the fact that there is now an opposite trend for Mg (in the same stars) – it is depleted instead of produced. Furthermore, the C is depleted, whilst the N yield remains high. All of these features derive from the lack of 3DUP, as O is now not periodically replenished. C cannot increase and fresh fuel for the MgAl cycle is not available. The lack of 3DUP also implies that the sum of CNO nuclei will remain constant, as fresh CNO is not added to the envelope and the CNO burning cycles (occurring at the base of the convective envelope) conserve the number of CNO nuclei. All these features indicate that a better fit to the observations is expected (IMF withstanding). Also, as stated above, these models may simulate the effects of a previously ignored population of stars, the super-AGB stars. The main discrepancy we now have – in terms of comparing at a single mass – is with Na. At the high masses (i.e. temperatures) required to deplete O and Mg via the MgAl and ON cycles, Na is not produced enough, or rather it is destroyed too much by proton captures, resulting in low yields. Moving to lower masses (temperatures) improves the Na situation but prevents O and Mg from being depleted. Interestingly, this is the identical problem that Ventura & D’Antona (2005b) have with their best-fitting (for the Na–O inverse-correlation) model. Denissenkov & Herwig (2003) also report this problem. We note that the models of Ventura & D’Antona (2005b) are quite independent as they use a different treatment for overshoot, mass loss and the uncertainties in reaction rates). For example, the non-standard models of Ventura & D’Antona (2005b), which use a different convection model, can reproduce most of the abundance anomalies, although some overshooting and the adoption of a high mass-loss rate is required. In addition to the uncertainties in the input physics of all stellar models, numerical details can also significantly affect the results. For example, time-stepping during diffusive mixing can cause changes in the stellar structure, hence altering the temperature at the bottom of the convective envelope, and thus the resulting nucleosynthesis and yields (Siess, private communication). Attempts have been made to reconcile the (standard) models with the observations but have universally ended with a need to ‘tweak’ the relative effects of HBB and 3DUP to an unpalatable degree (e.g. Denissenkov & Herwig 2003), also suggested by Fenner et al. (2004). The models using a different convection formalism (Ventura & D’Antona 2005b) show a better agreement with observations but, as mentioned earlier, also need some overshooting and the adoption of a high mass-loss rate.

With the results of all these stellar modelling studies in mind, we have chosen to investigate the limiting case by utilizing yields from AGB models that do not experience any 3DUP, which we have calculated ourselves. Although ‘turning off’ 3DUP in our models is ad hoc, it was surprisingly easy to achieve. By adding an extra physical condition for the determination of convective boundaries (the molecular weight gradient) via implementing the Ledoux criterion rather than the Schwarzschild criterion, we found that no dredge-up at all occurred in our models. We note that we have not extended the convective boundaries past the formal ones given by the Ledoux criterion. We leave the detailed description of the models for a separate paper, but now discuss the nucleosynthetic results relevant for the current problem.

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