Consequences of symmetries  
in renormalizing collinear effective theory

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Abstract

We consider effects of symmetries on renormalization properties of the collinear effective theory. We investigate which types of operators are possible in the effective theory satisfying gauge invariance, reparameterization invariance and residual energy invariance. Each symmetry puts a constraint on the possible structure of the theory, and there can appear only specific combinations of operators in the effective Lagrangian satisfying all the symmetry requirements. And the final effective Lagrangian is not renormalized to all orders in \(\alpha_s\) as long as no other nonlocal operators are induced at higher order. We explicitly prove this at one loop by renormalizing one-gluon vertices and discuss their features.

Strong interaction processes which involve energetic, massless particles can be described by the collinear effective theory [1–4]. It has been applied to sum Sudakov logarithms [1], to prove factorization [5,6] and study power corrections [7]. Symmetry properties such as reparameterization invariance, residual energy invariance, and gauge invariance have been investigated [8,9].

The collinear effective theory offers a systematic way to organize physical quantities in powers of a small parameter \(\lambda \sim p_\perp/E\), in which a massless energetic quark moves with energy \(E\), and transverse momentum \(p_\perp\). The momentum \(P^\mu\) of an energetic particle can be decomposed as

\[
P^\mu = \frac{\vec{p} \cdot \vec{n}}{2} n^\mu + p_\perp^\mu + k^\mu,
\]

where \(p^\mu = \frac{1}{2} (\vec{p} \cdot \vec{n}) n^\mu + p_\perp^\mu\) is the label momentum of order \(\lambda^0\) and \(\lambda\), respectively. The momentum \(k^\mu\) is the residual momentum of order \(\lambda^2\), which represents small fluctuation due to the strong interaction.

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In the collinear effective theory, we classify fields into three classes according to their momenta as collinear, soft and ultrasoft (usoft) fields. Their typical momenta scale as $E(\lambda^2, 1, \lambda)$, $E(\lambda, \lambda, \lambda)$, and $E(\lambda^2, \lambda^2, \lambda^2)$, respectively. In processes involving collinear quarks, the relevant fields are collinear quarks $\xi_n$, collinear gluons $A^\mu_n$ and usoft gluons $A^\mu_u$. The effective Lagrangian can be derived from the full QCD in terms of the collinear quark spinor $\xi_n$ which satisfies

$$\frac{\bar{\psi} \psi}{4} \xi_n = \xi_n, \ \bar{\psi} \xi_n = 0,$$  \hspace{1cm} (2)

where $n^2 = 0$, $\bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$. The effective Lagrangian is written as

$$L = \bar{\xi}_n \left[ n \cdot iD + i\bar{\psi} \cdot \frac{1}{n \cdot iD} i\bar{\psi} \right] \frac{\bar{n} \xi_n}{2},$$  \hspace{1cm} (3)

where $D^\mu = D^\mu_c + D^\mu_u$ is a covariant derivative under collinear and usoft gauge transformations. The covariant derivatives $D_c$ and $D_u$ are defined as

$$iD^\mu_c = \mathcal{P}^\mu - gA^\mu_n, \ iD^\mu_u = i\partial^\mu - gA^\mu_u.$$  \hspace{1cm} (4)

Here $\mathcal{P}^\mu$ is the operator which extracts label momenta from collinear fields. For example, if we apply $\mathcal{P}^\mu$ to a collinear spinor $\xi_n$ with label momentum $p^\mu$, we get

$$\mathcal{P}^\mu \xi_n = \left( \frac{\bar{n} \cdot p}{2} + p^\mu_\perp \right) \xi_n.$$  \hspace{1cm} (5)

Note that the derivative operator acting on a collinear field produces terms of order $\lambda^2$ since the label momenta are extracted.

At leading order in $\lambda$, Manohar et al. [8] have observed that, if we require only gauge invariance and reparameterization invariance, there can be operators in the effective Lagrangian, which are given by

$$O_2^{(0)} = \bar{\xi}_n i\mathcal{P} \frac{1}{n \cdot iD_c} i\mathcal{P} \bar{\xi}_n, \ O_3^{(0)} = \bar{\xi}_n iD_{\perp} \frac{1}{n \cdot iD_c} iD_{\perp} \bar{\xi}_n.$$  \hspace{1cm} (6)

Even though only $O_2^{(0)}$ appears at tree level, $O_3^{(0)}$ can be induced through radiative corrections. They have shown that the operator $O_3^{(0)}$ at leading order in $\lambda$ is not allowed due to the type-II reparameterization invariance, though the type-I reparameterization invariance does not exclude the operator. In this Letter, we extend the analysis to all orders in $\lambda$ by including all the possible operators in the collinear effective theory. We also present explicit
calculations for the renormalization of one-gluon vertices at one loop to show that the effective Lagrangian in a specific combination is not renormalized at this order.

We can obtain the operators similar to $O^{(0)}_2$ and $O^{(0)}_3$ to all orders. If we only require the collinear and usoft gauge invariance [9], the most general set of the operators in the effective Lagrangian is given by

\[
O_1 = \bar{\xi}_n n \cdot iD \frac{\bar{\Phi}}{2} \xi_n, \quad O_2 = \bar{\xi}_n iD \frac{\bar{\Phi}}{2} n \cdot iD \bar{\Phi} \frac{\bar{\Phi}}{2} \xi_n,
\]

\[
O_3 = \bar{\xi}_n iD \frac{\bar{\Phi}}{2} n \cdot iD \frac{\bar{\Phi}}{2} \xi_n, \quad O_4 = \bar{\xi}_n(-i\sigma_{\mu\nu})iD \frac{\bar{\Phi}}{2} \xi_n \sigma_{\mu\nu} G_{\mu\nu} \xi_n,
\]

(7)

where $\sigma_{\mu\nu} = i[\gamma_{\mu}, \gamma_{\nu}]/2$. Note that the operators $O_2$, $O_3$ and $O_4$ are not independent and the relation is given by $O_2 = O_3 + O_4$ due to the identity $\gamma_{\mu}\gamma_{\nu} = g_{\mu\nu} - i\sigma_{\mu\nu}$.

At tree level there only appears $O_1 + O_2$, but the operator $O_2$ can be decomposed into $O_3 + O_4$. And the question is whether the operators $O_3$ and $O_4$ receive different renormalization effects. To compare this situation with the heavy quark effective theory (HQET), the effective Lagrangian to order $1/m_Q$ in HQET is given by

\[
L_{\text{HQET}} = \bar{h}_v i v \cdot D h_v + \bar{h}_v \frac{(i\bar{\Phi})^2}{2m_Q} h_v
\]

\[
-\bar{h}_v i v \cdot D h_v + \bar{h}_v \frac{(iD)^2}{2m_Q} h_v - C_{\text{mag}}(\mu) \frac{g}{4m_Q} \bar{h}_v \sigma_{\mu\nu} G_{\mu\nu} v, \quad (8)
\]

where $iD^\mu = i\partial^\mu - gA^\mu$. At one loop, when we match the full theory with the HQET, the coefficient of the chromomagnetic operator $C_{\text{mag}}(\mu)$ is given by [10,11]

\[
C_{\text{mag}}(\mu) = 1 - \frac{3\alpha_s}{2\pi} \left( \ln \frac{m_Q}{\mu} - \frac{13}{9} \right). \quad (9)
\]

The last two operators in Eq. (8) are legitimate operators which are gauge invariant. However, when we include radiative corrections, the kinetic energy operator is not renormalized due to the reparameterization invariance, but the chromomagnetic operator receives nontrivial renormalization effects.

If we follow the same reasoning in the HQET, we could conclude that the kinetic energy operator $O_1 + O_3$ is not renormalized to all orders in $\alpha_s$. However, we can extend the reasoning further by including the residual energy invariance. The residual energy invariance along with the reparameterization
invariance guarantees that only the combination \( O_1 + O_3 + O_4 = O_1 + O_2 \) can appear in the Lagrangian, and is not renormalized to all orders.

In order to see how the argument goes, let us consider the transformation properties of each operator under the reparameterization transformation and the residual energy transformation. These transformations are classified in Ref. [8] as the type-I and the type-II transformations which are given by

\[
(\text{I}) \begin{cases} 
  n^\mu \to n^\mu + \Delta_\perp^\mu, \\
  \pi^\mu \to \pi^\mu,
\end{cases} \\
(\text{II}) \begin{cases} 
  n^\mu \to n^\mu, \\
  \pi^\mu \to \pi^\mu + \epsilon_\perp^\mu,
\end{cases}
\]

where the infinitesimal parameters \( \Delta_\perp^\mu \) and \( \epsilon_\perp^\mu \) satisfy \( n \cdot \Delta_\perp = \pi \cdot \Delta_\perp = n \cdot \epsilon_\perp = \pi \cdot \epsilon_\perp = 0 \). Under the type-I transformation, the transformation of each quantity in the Lagrangian is given by

\[
\begin{align*}
  n \cdot D &\to n \cdot D + \Delta_\perp \cdot D_\perp, \\
  D_\perp^\mu &\to D_\perp^\mu - \frac{\Delta_\perp^\mu}{2} \pi \cdot D - \frac{\pi^\mu}{2} \Delta_\perp \cdot D_\perp, \\
  \pi \cdot D &\to \pi \cdot D, \\
  \xi_n &\to (1 + \frac{1}{4} \Delta_\perp \pi) \xi_n.
\end{align*}
\]

Using Eq. (11), the infinitesimal change of the operators is given by

\[
\delta_{(I)} O_1 = \xi_n \left( i \Delta_\perp \cdot D_\perp \right) \overrightarrow{\pi} \xi_n = -\delta_{(I)} O_2 = -\delta_{(I)} O_3, \quad \delta_{(I)} O_4 = 0.
\]

Therefore the combinations \( O_1 + O_2, O_1 + O_3 \) and \( O_4 \) are invariant under the reparameterization (type-I) transformation.

Under the type-II transformation, each vector and the spinor \( \xi_n \) transform as

\[
\begin{align*}
  n \cdot D &\to n \cdot D, \\
  D_\perp^\mu &\to D_\perp^\mu - \frac{\epsilon_\perp^\mu}{2} n \cdot D - \frac{n^\mu}{2} \epsilon_\perp \cdot D_\perp, \\
  \pi \cdot D &\to \pi \cdot D + \epsilon_\perp \cdot D_\perp, \\
  \xi_n &\to (1 + \frac{1}{2} \epsilon_\perp \pi) \frac{1}{\pi} \xi_n.
\end{align*}
\]

From this, we obtain the change of \( O_1 \), which is given by

\[
\delta_{(II)} O_1 = \xi_n \left( i \Phi_\perp \frac{1}{\pi} \frac{1}{i D} \frac{1}{2} n \cdot D + n \cdot i D \frac{1}{2} \pi \cdot i D \right) \overrightarrow{\pi} \xi_n. 
\]

The variation of \( O_2 \) exactly cancels the change in \( O_1 \) and is given by

\[
\delta_{(II)} O_2 = -\delta_{(II)} O_1.
\]
The change of $O_3$ and $O_4$ is given by

\[
\delta_{\text{(II)}}O_3 = \xi_n \left[ i\bar{\Phi} \frac{1}{n \cdot iD} \frac{\epsilon_\perp}{2} iD_{\perp \mu} \frac{1}{\pi \cdot iD} iD_{\perp} - \frac{n \cdot iD}{\pi \cdot iD} \frac{1}{2} \epsilon_\perp \cdot iD_{\perp} \right]
\]

\[
- iD_{\perp \mu} \frac{1}{\pi \cdot iD} \epsilon_\perp \cdot iD_{\perp} \frac{1}{\pi \cdot iD} iD_{\perp} - \frac{\epsilon_\perp \cdot iD_{\perp}}{2} \frac{n \cdot iD}{\pi \cdot iD}
\]

\[
+ iD_{\perp \mu} \frac{1}{\pi \cdot iD} iD_{\perp} \frac{1}{\pi \cdot iD} i\bar{\Phi} \right] \frac{\bar{q}_n}{2} \xi_n,
\]

\[
\delta_{\text{(II)}}O_4 = \xi_n \left[ \bar{\Phi} \frac{1}{n \cdot iD} \frac{\epsilon_\perp}{2} \left( -i\sigma^{\mu \nu} \right) iD_{\perp \mu} \frac{1}{\pi \cdot iD} iD_{\perp \nu} \frac{\bar{q}_n}{2} \xi_n - n \cdot iD \frac{1}{\pi \cdot iD} \frac{\epsilon_{\perp \mu} \epsilon_{\perp \nu}}{2} \right]
\]

\[
- iD_{\perp \mu} \frac{1}{\pi \cdot iD} \epsilon_\perp \cdot iD_{\perp} \frac{1}{\pi \cdot iD} iD_{\perp \nu} - \frac{iD_{\perp \mu} \epsilon_{\perp \nu}}{2} \frac{n \cdot iD}{\pi \cdot iD}
\]

\[
+ iD_{\perp \mu} \frac{1}{\pi \cdot iD} iD_{\perp \nu} \frac{1}{\pi \cdot iD} i\bar{\Phi} \right] \frac{\bar{q}_n}{2} \xi_n.
\]

When we add the variations of $O_3$ and $O_4$, we can obtain the relation

\[
\delta_{\text{(II)}}(O_3 + O_4) = \delta_{\text{(II)}}O_2 = -\delta_{\text{(II)}}O_1.
\]

From this, we can conclude that the invariant combinations under the type-II transformation are $O_1 + O_2$ and $O_1 + O_3 + O_4$, which are the same.

To summarize the result, the reparameterization invariance (type-I) requires that $O_1$ should appear as a combination either of $O_1 + O_2$ or $O_1 + O_3$, and the operator $O_4$ itself is reparameterization invariant. There is no prescription between $O_1$ and $O_4$. And the residual energy invariance (type-II) requires that $O_1$ should appear as a combination either of $O_1 + O_2$ or $O_1 + O_3 + O_4$, which is the same as $O_1 + O_2$. Therefore the residual energy invariance puts a serious constraint on the structure of the effective Lagrangian, and, as a consequence, the combination $O_1 + O_2$ is not renormalized to all orders.

We can explicitly prove that the combination $O_1 + O_2$ is not renormalized at one loop. To be concrete, let us consider the renormalization of the effective Lagrangian at leading order in $\lambda$. In principle, we have to consider all the operators, but here we consider the renormalization of one-gluon vertices only to illustrate the point. The coefficients of the operators in the collinear effective theory are determined by matching amplitudes between the full theory and the effective theory. It is convenient to use the background field method [12] for an external gluon with two external collinear fermions and we employ the Feynman gauge in the calculation.
Fig. 1. Feynman diagrams of the full theory amplitudes for the one-gluon vertex to one loop with the background field method.

The Feynman diagrams for the one-gluon vertex to one loop in the full theory are given in Fig. 1. We use the on-shell renormalization and regulate both infrared and ultraviolet divergences using dimensional regularization with $D = 4 - 2\epsilon$. Because the effective theory has the same low energy behavior as in the full theory, the infrared divergences in the effective theory will exactly cancel the infrared divergences in the full theory in matching. Therefore one expects that the result will be independent of the choice of the infrared regulators.

We can write the amplitude for the Feynman diagrams in Fig. 1 as

$$-igA_{\mu a}\bar{\psi}(p')\Gamma^{\mu T^a}\psi(p).$$

The tree-level diagram which contributes to $\Gamma^\mu$ is simply $\gamma^\mu$, and the Feynman diagrams at one-loop in Fig. 1 yield

$$-ig\frac{\alpha_s}{4\pi}C_F\gamma^\mu\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}\right),$$

which is exactly cancelled by the wave function renormalization. This is because the current is conserved.

We can find the interaction in the collinear effective theory corresponding to Eq. (18) by noting that the relation between the spinor $\psi$ in the full theory and the spinor $\xi_n$ in the effective theory is given by

$$\psi(x) = \sum_p e^{-ip\cdot x}\left(1 + \frac{1}{\vec{n}\cdot i\vec{D}}\Phi_{\perp}^{n\perp}\right)\xi_n.$$  

The interaction can be written as

$$-g\bar{\psi}A\psi \rightarrow -g\bar{\xi}_n\left[n\cdot A_n + i\Phi_{\perp}^{n\perp}D\frac{1}{\vec{n}\cdot i\vec{D}}A_{n\perp} + A_{n\perp}D_{\perp}\Phi_{\perp}^{n\perp}\right]\eta_n^2\xi_n,$$

which gives the interaction term in the effective Lagrangian.
Fig. 2. Feynman rules for the propagator and the interaction vertices in the collinear effective theory. All the particles are collinear particles and the gluon momenta are incoming.

The effective Lagrangian at leading order in $\lambda$ is given by

$$L_0 = \xi_n \left[ n \cdot (iD - gA_n) + (\not{p} - g\not{A}) \frac{1}{n \cdot \not{p}} (\not{p} - g\not{A}) \right] \frac{\not{p}}{2} \xi_n. \quad (22)$$

And the Feynman rules for the interactions are given in Fig. 2. Here we omit vertices with an usoft gluon, which do not contribute to the following calculations. The relevant Feynman diagrams in the collinear effective theory for the renormalization of a single-gluon vertex are shown in Fig. 3. Here we also use the background field method for the external gluon field. There are also Feynman diagrams with usoft gluons in Fig. 3, but all of them vanish. When we add the first three diagrams in Fig. 3, the ultraviolet divergent part is given by

$$M_a + M_b + M_c = ig \frac{\lambda}{4\pi} \frac{T^a}{2N} \frac{1}{\epsilon} \left[ n_\mu + \frac{\gamma_\perp \not{p}_\perp}{n \cdot \not{p}} + \frac{\gamma_\perp \not{p}_\perp}{n \cdot \not{p}'} - \frac{\gamma_\perp \not{p}_\perp}{n \cdot \not{p}'} \frac{\not{p}_\perp \not{p}_\perp}{n \cdot \not{p}'} \right]. \quad (23)$$

If we regulate infrared and ultraviolet divergences using dimensional regularization, the $1/\epsilon$ pole in Eq. (23) is replaced by

$$\frac{1}{\epsilon} \to \frac{1}{\epsilon_{\text{UV}}} = \frac{1}{\epsilon_{\text{IR}}} \quad (24)$$

and the amplitude vanishes. In order to extract the ultraviolet divergence, we can regulate the infrared divergence by putting external particles slightly off the mass shell. Whatever regularization method we use, the infrared divergences in the full theory and the effective theory cancel in the matching.
Fig. 3. Feynman diagrams for the one-gluon vertex in the effective theory with the background field method. The wavy lines represent collinear gluons. Diagrams with usoft gluons, which do not contribute, are not shown.

The last two Feynman diagrams from a triple gluon vertex yield

\[ M_d + M_e = -ig \frac{\alpha_s}{4\pi} T^a \frac{1}{\epsilon} \left[ n_\mu + \frac{\gamma_\perp n_\perp}{n \cdot p'} + \frac{p'_\perp \gamma_\perp}{n \cdot p'} - \frac{p'_\perp n_\perp}{n \cdot p' n \cdot p} \right] \frac{\overline{p}}{n^2}. \]  

(25)

The contributions of all the Feynman diagrams in Fig.3 are given by

\[ M = -ig \frac{\alpha_s}{4\pi} C_F T^a \frac{1}{\epsilon} \left[ n_\mu + \frac{\gamma_\perp n_\perp}{n \cdot p} + \frac{p'_\perp \gamma_\perp}{n \cdot p'} - \frac{p'_\perp n_\perp}{n \cdot p' n \cdot p} \right] \frac{\overline{p}}{n^2}. \]  

(26)

The self-energy for a collinear quark is given by

\[ i\Sigma(p) = i \frac{\alpha_s}{4\pi} C_F \frac{1}{\epsilon} \frac{p^2}{n \cdot p^2} \frac{\overline{p}}{n^2}, \]  

(27)

and the ultraviolet divergence is the same as that in the full theory. When we add all these contributions, the ultraviolet divergent part vanishes and we have shown that the one-gluon vertex operator in \( O_1 + O_2 \) or \( O_1 + O_3 + O_4 \) is not renormalized at one loop.

This result is expected since there are only massless particles both in the full theory and in the effective theory. If we regulate both the infrared divergence and the ultraviolet divergence using dimensional regularization, any loop diagram either in the full theory or in the effective theory vanishes since there is no scale involved. Therefore matrix elements in each theory are given by their tree-level expressions. Furthermore, since the infrared divergence in the full theory is the same as the infrared divergence in the effective theory, the ultraviolet divergence in the effective theory should be the same as the ultraviolet divergence in the full theory, in which there is none in this case.
Fig. 4. Feynman diagrams at one loop in the collinear effective theory with the background field method in renormalizing the operator $O_4$. The square represents the vertex from the operator $O_4$. Wavy lines represent collinear gluons.

When we compare the renormalization behavior of the collinear effective theory with that of the HQET, there is a distinct difference. In HQET, when we calculate radiative corrections at one loop, the kinetic energy operator and the chromomagnetic operator do not mix. And the kinetic energy operator is not renormalized due to the reparameterization invariance. However, in the collinear effective theory, even though $O_1 + O_3$ and $O_4$ have reparameterization invariance, it is not guaranteed that radiative corrections of $O_1 + O_3$ and $O_4$ do not mix with each other.

In order to see this, let us consider the radiative corrections of $O_4$ at one loop. The relevant Feynman diagrams are shown in Fig. 4. It is not illuminating to show all the results, but if we calculate the radiative corrections of the operator $O_4$ in Fig. 4, there are terms proportional to $n \cdot A_n$, which are given by

$$g \frac{\alpha_s}{4\pi} \epsilon_n \left( \frac{1}{2N} + \frac{N \overline{n} \cdot (p + p')}{2 \overline{n} \cdot q} \ln \frac{\overline{n} \cdot p'}{\overline{n} \cdot p} \right) n \cdot A_n \frac{\ln \epsilon_n}{2 \epsilon_n}. \tag{28}$$

This is one definite example to show that the radiative corrections of $O_4$ mix with $O_1 + O_3$ at leading order in $\lambda$ and there appear additional nonlocal operators. However, if we consider the radiative corrections of $O_1 + O_2$, the logarithms that appear in Eq. (28) and the logarithms that appear in renormalizing the operator $O_3$ cancel each other and the result is given by Eq. (26). We can conclude that the reparameterization invariance and the residual energy invariance allow only the combination $O_1 + O_2$ at leading order in $\lambda$, and it is not renormalized at one loop. This fact has been used in calculating the form
factors for heavy-to-light currents to order $\lambda$ in Ref. [7].

We have shown explicitly that the single-gluon vertex obtained from $O_1 + O_2$ at leading order in $\lambda$ is not renormalized at one loop. We can extend this argument to the whole Lagrangian to all orders in $\alpha_s$. If we require gauge invariance, reparameterization invariance and residual energy invariance, the only possibility which includes the interaction we have considered, is $O_1 + O_2$. When we regularize both the ultraviolet and the infrared divergences with dimensional regularization, any loop diagrams in both theories vanish since there is no scale involved, and the divergence structure is the same in the full theory and in the effective theory to all orders. Therefore the whole Lagrangian is not renormalized to all orders in $\alpha_s$. However there is a caveat to claim this. The collinear effective theory is a nonlocal field theory in the coordinate $n \cdot x$ or momentum $\mathbf{n} \cdot \mathbf{p}$, but is local in other coordinates. It is possible to induce nonlocal operators which are not present in the tree-level Lagrangian through radiative corrections. With this in mind, we can say that the effective Lagrangian is not renormalized to all orders provided that no other nonlocal operators are induced by radiative corrections. We expect that the power counting method in Ref. [13] will offer a clue to see if nonlocal operators can exist, which are not present in the original Lagrangian.

Note that the situation is quite different in the HQET. For the renormalization of the chromomagnetic operator, the infrared divergences cancel in the matching, but the ultraviolet behavior in HQET and in the full theory is different because there is a heavy quark mass in the full theory, while there is no scale in the HQET. Furthermore, if we use the on-shell wave function renormalization, the wave function renormalization in the full QCD and in the HQET is different. Therefore there appears nontrivial matching condition for the chromomagnetic operator in the HQET. However, that is not the case in the collinear effective theory. The difference lies in the fact that we are dealing with massless quarks and gluons both in the full QCD and in the effective theory.

We have shown that the effective Lagrangian is not renormalized to all orders in $\alpha_s$ and to all orders in $\lambda$ due to the reparameterization invariance and the residual energy invariance combined with the collinear and the usoft gauge invariance. This is a very strong constraint imposed on the collinear effective theory, and it simplifies higher-order corrections in $\lambda$. The Wilson coefficients of various operators at higher order in $\lambda$ are the same as those operators at leading order in $\lambda$. Therefore we can save a lot of calculations in evaluating higher-order corrections to matrix elements of some operators, and the renormalization behavior of the Wilson coefficients of higher-dimensional operators is related to that of the Wilson coefficients of the operators at leading order.

We have considered the collinear sector which involves collinear quarks, collinear
gluons, and usoft gluons in the effective theory. What we have not considered here is the soft sector and the usoft sector, which have different symmetry structure. For example, there is no such symmetry as reparameterization invariance in the usoft sector. The usoft sector can be described rather well by the symmetries of the full QCD. Symmetry structure and power corrections for the currents composed of collinear fields, soft fields and usoft fields will be useful in exclusive and inclusive decays of heavy mesons and other high energy processes.

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References

[1] C. W. Bauer, S. Fleming and M. Luke, Phys. Rev. D 63 (2001) 014006.
[2] C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, Phys. Rev. D 63 (2001) 114020.
[3] C. W. Bauer and I. W. Stewart, Phys. Lett. B 516 (2001) 134.
[4] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D 65 (2002) 054022.
[5] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. Lett. 87 (2001) 201806.
[6] C. W. Bauer, S. Fleming, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, hep-ph/0202088.
[7] J. Chay and C. Kim, hep-ph/0201197.
[8] A. V. Manohar, T. Mehen, D. Pirjol, and I. W. Stewart, hep-ph/0204229.
[9] J. Chay and C. Kim, hep-ph/0205117.
[10] E. Eichten and B. Hill, Phys. Lett. B 243 (1990) 427.
[11] A. F. Falk, B. Grinstein and M. E. Luke, Nucl. Phys. B 357 (1991) 185.
[12] L. F. Abbott, Nucl. Phys. B 185 (1981) 189.
[13] C. W. Bauer, D. Pirjol, and I. W. Stewart, hep-ph/0205289.