Electroweak Measurements on the Z Resonance

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Almost all precision electroweak measurements from the Z resonance made at the LEP storage ring by the ALEPH, DELPHI, L3 and OPAL experiments and those made using a polarized electron beam at the SLC by the SLD experiment are now final and have been published. Changes in the measurements since the last RADCOR meeting are discussed. The internal consistency of the measurements is considered. The impact of remaining theoretical uncertainties in the QCD sector are examined as well as the impact of experimental and theoretical uncertainties on the value of $\alpha_{\text{QED}}(m_Z)$.

Presented at the

5th International Symposium on Radiative Corrections (RADCOR–2000)
Carmel CA, USA, 11–15 September, 2000

*Work supported by the US Department of Energy grant DE-FG03-96ER40969.
1 Introduction

Between 1989 and 1995 the LEP collaborations collected more than 200 pb$^{-1}$ per experiment, which resulted in more than 17 million recorded Z decays. The LEP data included almost 50 pb$^{-1}$ of off-peak data, which is needed to determine the mass and width of the Z. The SLD at the SLC recorded collisions with a polarized electron beam and an unpolarized positron beam from 1992 until 1998, achieving electron polarizations as high 80% and a total data sample of more than 500,000 recorded Z decays. These large data samples provide the basis for well-known tests of the Standard Model.

Almost all of the precision measurements made at the Z resonance have now been published and the LEP and SLD electroweak working groups have almost completed a comprehensive review of these measurements which will shortly appear in Physics Reports[1]. Furthermore, several very complete reviews of the theory necessary for the interpretation of these measurements already exist, see, for example, References[2,3,4]. Rather than attempting to summarize this entire work in a few pages, I will briefly review changes in theory and measurement since the last RADCOR meeting and then consider three areas where some controversy exists: the determination of strong coupling constant, $\alpha_s$, the possible discrepancy between the Standard Model and measurements of forward-backward asymmetries in $b\bar{b}$ final states, and finally the impact on Higgs mass limits of the contribution of hadronic vacuum polarization to the running value of the electromagnetic coupling constant at the Z resonance, $\alpha(m_Z^2)$.

2 Changes since RADCOR 1998

The LEP and SLD collaborations present their measurements in terms of pseudo-observables which are closely related to the actual measurements, but include corrections for effects such as electromagnetic radiation and interference between photon mediated and Z mediated processes. These variables, together with other variables commonly used in electroweak fits are summarized in Table 1. The values of the Z mass, $m_Z$, the Z width, $\Gamma_Z$ and the peak hadronic cross section $\sigma_{\text{pole had}}$ require quite large corrections as illustrated in Figure 1 taken from Reference[5]. Large corrections are also needed to the forward-backward asymmetries of leptons and quarks. The largest corrections of all are for electron final states, where t-channel effects dominate in many regions of phase space. Radiative corrections for the left-right asymmetries measured by SLD and for polarized forward-backward asymmetries are much smaller, but nevertheless important. For example, the largest change between the preliminary SLD measurement of the left-right asymmetry ($A_{\text{LR}}$) and the final published value of $A_{\text{LR}}$ came from a correction to the beam energy which was based
Table 1: The summary of measurements included in the combined analysis of Standard Model parameters used by the LEP and SLD electroweak groups reproduced from References [5] and [6]. The electroweak measurements from pp colliders, $\nu N$ scattering, and LEP2 $m_W$ are described elsewhere in the RADCOR 2000 proceedings [7,8].

|                         | 2000 Result                  | 1998 Result                  | Standard Model fit | 2000 Pull |
|-------------------------|------------------------------|------------------------------|-------------------|-----------|
| $\Delta \alpha_{\text{had}}^{(5)}(m_Z^2)$ | 0.02804 ± 0.00065           | 0.02804 ± 0.00065           | 0.02804          | 0.0       |
| **LEP**                 |                              |                              |                   |           |
| $m_Z$ [GeV]             | 91.1875 ± 0.0021             | 91.1867 ± 0.0021             | 91.1874          | 0.0       |
| $\Gamma_Z$ [GeV]       | 2.4952 ± 0.0023              | 2.4939 ± 0.0024              | 2.4962           | -0.4      |
| $\sigma_{\text{pole}}^{\text{had}}$ [nb] | 41.540 ± 0.037               | 41.491 ± 0.058               | 41.480           | 1.6       |
| $R_\ell$                | 20.767 ± 0.025               | 20.765 ± 0.026               | 20.740           | 1.1       |
| $A^0_{\text{FB}}$       | 0.0171 ± 0.0010              | 0.0169 ± 0.0010              | 0.0164           | 0.8       |
| $\tau$ polarization:    |                              |                              |                   |           |
| $A_\tau$                | 0.1439 ± 0.0042              | 0.1431 ± 0.0045              | 0.1480           | -1.0      |
| $A_e$                   | 0.1498 ± 0.0048              | 0.1479 ± 0.0051              | 0.1480           | 0.4       |
| q̅$q$ charge asym.:     |                              |                              |                   |           |
| $\sin^2 \theta_{\text{eff}}$ | 0.2321 ± 0.0010          | 0.2321 ± 0.0010           | 0.23140          | 0.7       |
| $m_W$ [GeV]             | 80.427 ± 0.046               | 80.37 ± 0.09                | 80.402           | 0.5       |
| **SLD**                 |                              |                              |                   |           |
| $\sin^2 \theta_{\text{eff}}^{\text{lept}}(A_\ell)$ | 0.23098 ± 0.00026            | 0.23109 ± 0.00029          | 0.23140         | -1.6      |
| **Heavy Flavor**        |                              |                              |                   |           |
| $R_b$                   | 0.21653 ± 0.00069            | 0.21656 ± 0.00074           | 0.21578          | 1.1       |
| $R_c$                   | 0.1709 ± 0.0034              | 0.1735 ± 0.0044             | 0.1723           | -0.4      |
| $A^{0,b}_{\text{FB}}$   | 0.0990 ± 0.0020              | 0.0990 ± 0.0021             | 0.1038           | -2.4      |
| $A^{0,c}_{\text{FB}}$   | 0.0689 ± 0.0035              | 0.0709 ± 0.0044             | 0.0742           | -1.5      |
| $A_b$                   | 0.922 ± 0.023                | 0.867 ± 0.035               | 0.935            | -0.6      |
| $A_c$                   | 0.631 ± 0.026                | 0.647 ± 0.040               | 0.668            | -1.4      |
| pp and $\nu N$          |                              |                              |                   |           |
| $m_W$ [GeV]             | 80.452 ± 0.062               | 80.41 ± 0.09                | 80.402           | 0.8       |
| $\sin^2 \theta_W$      | 0.2255 ± 0.0021              | 0.2254 ± 0.0021             | 0.2226           | 1.2       |
| $m_t$ [GeV]             | 174.3 ± 5.1                  | 173.8 ± 5.0                 | 174.3            | 0.0       |

on a scan of the Z resonance [10].
Figure 1: Illustration of LEP line-shape parameter definitions[9].

For the five Z line-shape parameters (assuming lepton universality) as measured at LEP[11,12,13,14], the total error and the theoretical error on the determination of the pseudo-observables is shown in Table 3. In terms of the effective vector and axial and vector couplings of a given fermion to the Z, $g_{Vf}$ and $g_{Af}$:

$$\Gamma_f = \frac{G_FN_cm_Z^3}{6\pi \sqrt{2}} \left( R^f_V g_{Vf}^2 + R^f_A g_{Af}^2 \right) + \Delta_{QCD}$$  \hspace{1cm} (1)$$

$$A_f = 2 \frac{g_{Vf} g_{Af}}{g_{Vf}^2 + g_{Af}^2}.$$  \hspace{1cm} (2)

Here, $R^f_V$ and $R^f_A$ give corrections for final-state QED and QCD effects as well as quark masses, $\Delta_{QCD}$ for non-factorizable QCD effects. Note that $A_f$ depends only on the ratio of couplings.

In almost all cases the theory error on the extraction of the pseudo-observables is at least five times smaller than the experimental error. The exceptions occur for theory corrections involving electron final states where t-channel photon mediated processes are important. For example, the theoretical error on luminosity determined with small angle Bhabha scattering drives the error on $\sigma_{\text{had}}^\text{pole}$[13,14] and the effects of interference corrections on $R_e \equiv \frac{\Gamma_{\text{had}}}{\Gamma_e}$, and on $A_{\text{FB}}^{0,e} \equiv \frac{3}{4} A_e^2$ give a theoretical error which is of the same order as the total error on these quantities when lepton universality is assumed.
It should be stressed that because of the complexity of the fitting procedure used to extract the pseudo-observables from the several hundred cross section and asymmetries of each of the LEP experiments, it will be extremely difficult to incorporate any future improvements in the theory needed to determine the pseudo-observables. It is encouraging that, in general, these theory errors are quite small. The theory used to extract the pseudo-observables from the raw measurements has been very stable since RADCOR 1998 (also shown in Table 1) with two exceptions. The inclusion of third order initial state radiation correction shifted $\sigma_{\text{had}}^0$ by 0.023 nb or 70% of its present total error\cite{17}, and also led to a $\sim 0.5$ MeV shift on $m_Z$. Inclusion of initial-state radiation of $e^+e^-$ pairs gave rise to a $\sim 0.5$ MeV shift on $m_Z$ and $\sim 0.8$ MeV shift on $\Gamma_Z$\cite{18}.

The largest change in the experimental handling of the data was due to improved treatment of the errors on the determination of the beam energy. A lower energy systematic error was obtained for the 1995 LEP run than for the 1993 LEP run\cite{19}. To properly take this into account, the four experiments combined should give more weight to the 1995 data than each do individually. To test the effects of this reweighting, separate values of $m_Z$ were determined for each year as shown in Figure 2. Because of the consistency of $m_Z$ for the different periods, the numerical effect of this new procedure was small\cite{9}.

| Quantity                      | Total Error | Theory Error |
|-------------------------------|-------------|--------------|
| $m_Z$                         | 2.1 MeV     | 0.3 MeV      |
|                               | ($0.2 \times 10^{-4}$) | ($0.03 \times 10^{-4}$) |
| $\Gamma_Z$                    | 2.3 MeV     | 0.2 MeV      |
|                               | ($9.2 \times 10^{-4}$) | ($0.8 \times 10^{-4}$) |
| $\sigma_{\text{had}}^0$      | $\equiv \frac{12\pi \Gamma_{e^+e^-} - \Gamma_{\text{had}}}{m_Z^2 \Gamma_Z^2}$ | 0.037nb | 0.022nb |
|                               |             | ($8.9 \times 10^{-4}$) | ($5.3 \times 10^{-4}$) |
| $R_\ell$                      | $\equiv \frac{\Gamma_{\text{had}}}{\Gamma_\ell}$ | 0.025 | 0.004 |
|                               |             | ($12 \times 10^{-4}$) | ($1.9 \times 10^{-4}$) |
| $A_{\text{FB}}^0$            | $\equiv \frac{3}{4} A_e A_\ell$ | 0.0010 | 0.0001 |
|                               |             | (5.6%) | (0.6%) |
In terms of the parameters derived from the LEP line shape, the largest change between RADCOR 1998 and these results is that the ratio of the invisible width to the width for a single generation of charged leptons, \( \frac{\Gamma_{\text{inv}}}{\Gamma_{\ell}} \), is now slightly less than the Standard Model prediction giving a value for the number of neutrinos,

\[
N_\nu = 2.984 \pm 0.008,
\]

approximately 2 \( \sigma \) smaller than expected. In 1998, the \( N_\nu \) value was almost exactly 3. The change in the central value is largely due to an improved treatment of initial-state radiation that changed the value of \( \sigma_{\text{pole}} \). The error has also been significantly reduced because of a reduction in the luminosity theoretical error [15] since RADCOR 1998.

As is apparent from Table [1] there have been big improvements in the heavy quark measurements made by SLD. These are discussed in detail in Section [4].

3 Theoretical errors in the determination of \( \alpha_s \)

The LEP line-shape data allows a precise determination of \( \alpha_s \) from the effect of QCD final state corrections to the hadronic width. To lowest order, the hadronic width scales as

\[
\Gamma_{\text{had}} \propto 1 + \frac{\alpha_s}{\pi}
\]

which leads to the following dependences of LEP

![Figure 2: The value of \( m_Z \) determined in separate running periods of the LEP accelerator.](image)

m\_Z [GEV]

1990-1992
91.1904 \pm 0.0065

1993-1994
91.1882 \pm 0.0033

1995
91.1866 \pm 0.0024

average
91.1874 \pm 0.0021
pseudo-observables on $\alpha_s$:

\[
\Gamma_Z = \Gamma_{\text{had}} + 3\Gamma_\ell + 3\Gamma_\nu \propto 1 + 0.7\frac{\alpha_s}{\pi}
\]

\[
R_\ell = \frac{\Gamma_{\text{had}}}{\Gamma_\ell} \propto 1 + \frac{\alpha_s}{\pi}
\]

\[
\sigma^0_{\text{had}} = \frac{12\pi}{m_Z^2} \frac{\Gamma_{\text{had}}}{\Gamma_Z^2} \propto 1 - 0.4\frac{\alpha_s}{\pi}
\]

\[
\sigma^0_\ell = \frac{12\pi}{m_Z^2} \frac{\Gamma_\ell^2}{\Gamma_Z^2} \propto 1 - 1.4\frac{\alpha_s}{\pi}
\]

Here charged lepton universality has been assumed and the invisible width of the Z has been assumed to consist of the contribution from the three generations of neutrinos $(3\Gamma_\nu)$. Theoretically, the dependence of these quantities on $\alpha_s$ comes from the same contribution to $\Gamma_{\text{had}}$. (There are additional small top mass ($m_t$) contributions to the partial widths from $\alpha_s m_t^2$ and $\alpha_s^2 m_t^2$ corrections to the couplings $g_A t$ and $g_{Vt}$. The uncertainty on these corrections correspond to less than a 1.0 GeV uncertainty in the top mass and can therefore be ignored. This is discussed in more detail below.)

The cleanest measurements of $\alpha_s$ come from quantities which depend on the ratio of partial widths such as $R_\ell$, $\sigma^0_{\text{had}}$ and $\sigma^0_\ell$ where additional uncertainties from $m_t$ and from the Higgs mass ($m_h$) cancel.

The parameters $R_\ell$, $\sigma^0_{\text{had}}$, and $\sigma^0_\ell$ are not independent since $R_\ell = \frac{\sigma^0_{\text{had}}}{\sigma^0_\ell}$. The LEP parameter set includes $R_\ell$ and $\sigma^0_{\text{had}}$, hiding the constraint imposed by $\sigma^0_\ell$ in the correlation matrix. The value of $\alpha_s$ determined using $R_\ell$ alone is (for $m_h = 100$ GeV)

\[
\alpha_s(m_Z) = 0.1228 \pm 0.0038 \text{ (for } m_h = 900 \text{ GeV})
\]

which can be compared to that obtained from $\sigma^0_\ell$ alone

\[
\alpha_s(m_Z) = 0.1183 \pm 0.0030 \text{ (for } m_h = 900 \text{ GeV})
\]

The discrepancy between these two values is another manifestation of the small value of $N_\nu$ determined from these data. The result of the grand electroweak fit that uses other electroweak data to constrain the unknown Higgs mass and includes information from $\Gamma_Z$ is

\[
\alpha_s(m_Z) = 0.1183 \pm 0.0027.
\]

The error does not include any theoretical error from the QCD calculation of $\Gamma_{\text{had}}$.

The line-shape value of $\alpha_s(m_Z)$ is comparable to the recent world averages, such as the PDG average\[20\], $\alpha_s(m_Z) = 0.1181 \pm 0.002$ and an average of measurements based on NNLO calculations\[21\] $\alpha_s(m_Z) = 0.1178 \pm 0.0034$. There is some controversy concerning the theoretical error for the line-shape $\alpha_s(m_Z)$. Values in the literature differ by nearly an order of magnitude ranging from 0.0005 \[22\] to 0.003 \[21\]. Given the statistical precision of the line-shape $\alpha_s(m_Z)$ measurement it is worth examining the errors in some detail.
The most complete analysis of the error on $\alpha_s(m_Z)$ is given in Reference [23] which gives a detailed examination of the QCD calculations presently implemented in the commonly used programs TOPAZ0 [24] and ZFITTER [25]. The treatment here closely follows that of Reference [23], however, calculations which were not available at the time that this work was completed are also considered here. The effects of QCD on $\Gamma_{\text{had}}$ can be divided into 4 different categories: the dominant non-singlet terms which have the same effect on axial and vector neutral currents; corrections due to quark masses, dominated by uncertainties in the b-quark mass; singlet contributions and finally propagator corrections associated with the top mass $m_t$.

3.1 Uncertainties in massless non-singlet terms

The non-singlet axial and vector QCD corrections for massless quarks in $R_V^f$ and $R_A^f$ are equal (see Equation 1). This correction is known to third order and is given [22] by:

$$1 + \frac{\alpha_s}{\pi} + 1.40932\left(\frac{\alpha_s}{\pi}\right)^2 - 12.76706\left(\frac{\alpha_s}{\pi}\right)^3.$$  \hspace{1cm} (3)

One way to assess the errors due to missing higher order terms is by changing the QCD renormalization scale, which is explained in some detail in Reference [21]. In Reference [22] the renormalization scale $\mu$ is varied in the interval $e^{-2}(0.14) < \mu/m_Z < e^{2}(7.4)$. The total fractional variation of $\Gamma_{\text{had}}$ for this range of renormalization scales corresponds to a variation in $\alpha_s$ of 2.6%, giving errors of approximately $\pm 1.3%$. A similar study has been done in Reference [23] and a total variation in $\alpha_s$ of 1.0% is obtained when $\mu$ is varied in the smaller interval $0.5 < \mu/m_Z < 2.0$.

An alternative method, employed in Reference [23], is simply to estimate the possible error due to missing higher orders as equal to the last evaluated term. When this is applied to Equation 3, the cubic term corresponds to an estimated error on $\alpha_s$ of $\pm 1.8%$.

These uncertainties can be reduced by summing a large class of "$\pi^2$-terms", as is done in Reference [22]. For the measurement of $\alpha_s$ from the Z line shape, considering scale variations ($e^{-2}(0.14) < \mu/m_Z < e^{2}(7.4)$) and scheme dependence, Reference [22] suggests an error $\pm 0.4\%$ from QCD theory and that the value of $\alpha_s$ be scaled by

$$\alpha_s^{\text{improved}} = 1.006\alpha_s^{\text{ZFITTER}}.$$  \hspace{1cm} (4)

In Reference [26] a similar technique is applied to $e^+e^- \to \text{hadrons}$ at $\sqrt{s} = 31.6\text{GeV}$. In this case the improved value of $\alpha_s$ was 0.8% greater than the standard one, in agreement with Equation 4. Reference [24] does criticize Reference [22] for not having varied the scheme dependence sufficiently, but this would appear to be a technical objection as opposed to a practical one, as the variation in the scheme dependence made only a small contribution to the error.

The main controversy surrounding the $\alpha_s$ error centers on the validity of the summation of the higher order terms such as done in Reference [22]. This summation
has also been applied to the determination of $\alpha_s$ using information from hadronic decays of the tau lepton, $R_\tau$. Given the much larger value of $\alpha_s$ at this scale, it might be expected that any problems in the summation procedure would be amplified. Using a method called Contour Improved Fixed Order Perturbation Theory (CIPT) \[27\], which is similar to the $\pi^2$ summation of Reference \[22\], OPAL\[33\] obtains $\alpha_s(m_\tau^2) = 0.348 \pm 0.010(\text{exp}) \pm 0.019(\text{theory})$. However, it has also been found that when “renormalon” effects referred to as “renormalon chain resummation” (RCPT) \[29\] are included, $\alpha_s(m_\tau^2)0.306 \pm 0.005(\text{exp}) \pm 0.011(\text{theory})$ is obtained.

Since these two methods of determining $\alpha_s$ with $R_\tau$ marginally disagree by more than the theoretical error estimates, it is important that an error due to renormalon effects be included in any analysis of $\alpha_s$ from the Z line shape. Fortunately, the effects of renormalons were also included in the calculations of Reference \[22\] for $\Gamma_{\text{had}}$ and were found to have almost no effect.

Since these renormalon effects are small at the Z resonance, I conclude that the studies of $R_\tau$ give no evidence for additional QCD uncertainties in the non-singlet term and the 0.4% error estimate is appropriate. Of course, it cannot be excluded that there are other unknown effects, but this is true for all of the theoretical errors in the Z resonance studies as well.

3.2 Mass correction

The only significant contribution from the uncertainty in the mass corrections is from the b-quark mass. The uncertainties associated with these corrections can be significantly reduced if the running b-quark mass, $\overline{m}_b(m_Z) \simeq 2.77$ GeV is used.

These corrections are known to $\mathcal{O}(\alpha_s^3)$ for $R_V$ (vector current), but only to $\mathcal{O}(\alpha_s^2)$ for $R_A$ (axial-vector current). In Reference \[23\] uncertainties from missing orders are evaluated using scale variations ($0.5 < \mu/m_Z < 2.0$) and from the size of the $\mathcal{O}(\alpha_s^2)$ terms. The scale variation gave a total variation in $\alpha_s$ of 0.04%. The size of the axial-vector $\mathcal{O}(\alpha_s^2)$ term dominates the uncertainty, and corresponds to 0.05%. I adopt $\pm0.05\%$ as the error estimate from unknown higher orders in the mass corrections.

Propagating the error of the pole mass of the b-quark, $M_b = 4.7 \pm 0.2$ GeV, an error on $\alpha_s(m_Z)$ of $\pm0.31\%$ is obtained.

3.3 Singlet contributions

The error on the singlet contribution is due to uncertainties from the top quark mass and from possible missing higher orders. The error due to the top-quark mass is evaluated using ZFITTER or TOPAZ0 and is not included in the QCD error estimate. The QCD singlet contribution (including top mass dependent contributions with $m_t = 174$ GeV) scales $\Gamma_{\text{had}}$ by

$$1 - 0.63(\alpha_s/\pi)^2 - 2.69(\alpha_s/\pi)^3. \quad (5)$$
Varying the renormalization scale in the range \((0.5 < \mu/m_Z < 2.0)\) in this expression gives a total variation of \(\alpha_s\) of 0.26%, whereas dropping the third term changes \(\alpha_s\) by 0.38%. The larger value is taken as the error.

### 3.4 Propagator corrections

The behavior of the widths themselves, such as \(\Gamma_Z\) or \(\Gamma_{\text{had}}\), will differ from observables that depend on the ratio of widths, \(R_\ell\), \(\sigma^0_{\text{had}}\) and \(\sigma^0_\ell\). Since the \(\alpha_s\) correction to the propagator affects all partial widths equally, its effects will cancel in these observables. This correction is parameterized\(\text{[30]}\) by

\[
\delta \rho_{m_t} = \frac{3\sqrt{2} G_F m_t^2}{16\pi^2} (1 - 2.8599 \frac{\alpha_s}{\pi} - 14.594(\frac{\alpha_s}{\pi})^2)
\]

where \(m_t\) is the top pole mass and \(\alpha_s = \alpha_s(m_t) \simeq 0.11\). For \(m_t = 175\) GeV the partial widths of leptonic and neutrino final states scale with \(\alpha_s\) as

\[
1 + \delta \rho_{m_t}^{\text{QCD}} = 1 - 0.027 \frac{\alpha_s}{\pi} - 0.140(\frac{\alpha_s}{\pi})^2.
\]

Although this propagator correction is only a few percent of the QCD hadronic final-state correction, its theoretical uncertainty can be disproportionately large. Measurements of \(\alpha_s\) through quantities in which the propagator effects cancel, such as \(R_\ell\), \(\sigma^0_{\text{had}}\) and \(\sigma^0_\ell\) are therefore favored. Such observables also benefit from the cancelation of other effects in the propagator, such as \(m_h\) and \(m_t\) dependence, which are in fact much larger than the QCD effects.

The scheme and renormalization dependence of the QCD propagator correction has been evaluated in Reference \(\text{[30]}\) and is less than \(5 \times 10^{-5}\). Taking into account the Z branching ratio to hadrons (\(\sim 70\%\)), the corresponding additional error on \(\alpha_s\) determined from \(\Gamma_Z\) is 0.21%.

This renormalization scale uncertainty could also be viewed as an error on \(m_t\). For \(m_t = 175\) GeV, this corresponds to an uncertainty of 0.4 GeV, which is much smaller than the corresponding experimental uncertainty of 5 GeV.

In the extreme alternative approach of taking the last calculated term as the error estimate, a fractional error on \(\Gamma_Z\) of \(17 \times 10^{-5}\) is obtained, corresponding to an additional error on \(\alpha_s\) determined from \(\Gamma_Z\) of 0.7%.

Effects from the uncertainties of the QCD corrections (also known to second order) on the ratio of couplings for different fermions, \(g_W/g_A\), or equivalently \(\sin^2 \theta_W^{\text{eff}}\), have been justifiably neglected\(\text{[31]}\) in these error estimates. These corrections give rise to slight differences in the \(\alpha_s\) dependence of the Standard Model predictions for \(\Gamma_\nu\) and \(\Gamma_\ell\).
Table 3: Summary of the QCD error on $\alpha_s$ derived from various line shape observables based on the ratio of partial widths, $R_\ell$, $\sigma^0_{\text{had}}$ and $\sigma^0_\ell$ and from the total width of the Z, $\Gamma_Z$.

| effect                        | $\alpha_s$ from ratio of widths | $\alpha_s$ from $\Gamma_Z$ |
|-------------------------------|---------------------------------|---------------------------|
| missing orders, massless, non-singlet | 0.40%                           | 0.40%                     |
| missing orders, singlet       | 0.38%                           | 0.38%                     |
| missing orders, mass          | 0.05%                           | 0.05%                     |
| b-quark mass                  | 0.31%                           | 0.31%                     |
| propagator effects            | -                               | 0.21%                     |
| Total                         | 0.84%                           | 0.87%                     |

3.5 $\alpha_s$ summary

The effects discussed above are summarized in Table 3. I conservatively assume that the singlet and non-singlet contributions could be 100% correlated and sum their errors linearly. Another approach to possible correlation between theoretical uncertainties in different parts of the QCD calculation is taken by Reference [21]. This approach is based on an attempt to extract the overall dependence of $R_\ell$ on $\alpha_s$, including all contributions to the running quark masses and any residual propagator effects from a third order fit in $\alpha_s$ to the ZFITTER output as a function of $\alpha_s$ [32]. Note that because of the running quark masses and the propagator corrections the expansion contains terms beyond the third-order. The effects of these terms are included by the fit in the effective coefficients of the lower-order terms. The renormalization group equations are then applied to the resulting expansion. An error from renormalization scale uncertainties of +2.4%, -0.3% is obtained which is compatible to the result one obtains from adding the errors of the singlet and non-singlet contributions without the correction of Equation 4. Note that neither ZFITTER nor TOPAZ0 presently include this correction.

The other effects in Table 3 are added in quadrature. The contribution of the QCD uncertainty in the measurement of $\alpha_s$ from $R_\ell$, $\sigma^0_{\text{had}}$ and $\sigma^0_\ell$ is slightly smaller than that from $\Gamma_Z$ (or the derived quantity $\Gamma_{\text{had}}$) because the propagator corrections are smaller. Since the constraints on $m_t$ and $m_h$ are presently much looser than the uncertainty on the QCD effect in the propagator, this additional source of error could be ignored in the grand electroweak fit. However, at present this is numerically unimportant. Rounding the QCD theoretical uncertainty to 0.9% and applying the correction of Equation 4 gives an improved value of the strong coupling constant

$$\alpha_s(m_Z) = 0.1190 \pm 0.0027(\text{Exp.} + \text{EW}) \pm 0.0011(\text{QCD})$$

where the first error includes statistical, systematic and electroweak errors and the second error is due to QCD effects. This result is nearly as precise as the 2000 PDG [20] world average of $\alpha_s(m_Z) = 0.1181 \pm 0.002$. 

10
Figure 3: Comparison of $\sin^2 \theta_{\text{eff}}$ measured at SLC and LEP.

### 4 Measurements of $\sin^2 \theta_{\text{eff}}$ and $A_b$

The effective value of the weak mixing angle is given by ratio of effective vector and axial couplings

$$\sin^2 \theta_{\text{eff}} \equiv \frac{1}{4} \left( 1 - \frac{g_{V\ell}}{g_{A\ell}} \right)$$

and is closely related to $A_{\ell}$ as given by Equation 2. The most accurate value of $\sin^2 \theta_{\text{eff}}$ comes from the left-right asymmetry $A_{\text{LR}} = A_{\ell}$ as measured by SLD\textsuperscript{[10]}. Additional constraints come from polarized forward-backward asymmetries of leptons measured at SLD and from forward-backward asymmetries measured at LEP (see Table 2). LEP can also probe $A_e$ and $A_\tau$ directly using the tau polarization. The values presented here include an improved preliminary measurement from OPAL\textsuperscript{[34]} and a recent final result from DELPHI\textsuperscript{[35]}. The resulting values of $\sin^2 \theta_{\text{eff}}$ are given in Figure 3.

The forward-backward asymmetries of $b\bar{b}$ and $c\bar{c}$ events can also be used to determine $\sin^2 \theta_{\text{eff}}$ through the relations $A_{\text{FB}}^{b,c} \equiv \frac{3}{4} A_e A_c$ and $A_{\text{FB}}^{a,b} \equiv \frac{3}{4} A_e A_b$ as long as the Standard Model is used to calculate $A_e$ and $A_b$. The figure shows that there is a significant discrepancy between the quark based forward-backward measurements
Table 4: Comparison of measured and Standard Model values of $A_c$ and $A_b$. The LEP values are extracted using the LEP/SLD average of $A_\ell = 0.1500 \pm 0.0016$

|                | $A_b$                     | $A_c$                     |
|----------------|---------------------------|---------------------------|
| SLD            | $0.922 \pm 0.023 (-0.6 \sigma)$ | $0.631 \pm 0.026 (-1.4 \sigma)$ |
| LEP/SLD Average| $0.898 \pm 0.015 (-2.5 \sigma)$ | $0.623 \pm 0.020 (-2.2 \sigma)$ |
| Standard Model  | 0.935                     | 0.668                     |

and those made in the leptonic sector alone.

Since the leptonic measurements agree well, a possible explanation for this effect would be that the values of either or both $A_c$ and $A_b$ deviate from the Standard Model prediction. Using the polarized forward-backward asymmetry for $b\bar{b}$ and $c\bar{c}$ events $A_c$ and $A_b$ can be obtained directly [37]. The comparison of the SLD result and Standard Model is given in Table 4. The precision of the preliminary SLD result has been considerably improved since the 1998 RADCOR [37], (see Table 1 and Reference [5]) but the SLD data are not statistically precise enough to indict the Standard Model by themselves. It is unfortunate that SLD was prevented from running long enough to settle this issue.

Given that the SLD data cannot confirm a deviation in the values of $A_c$ and $A_b$, the remaining possibilities are a large statistical fluctuation or an unstated systematic error in some of the measurements. The most economical solution would be a systematic error affecting the LEP heavy quark measurements. However, there is no obvious source of such an error. On the experimental side, the total systematic error for $A_{0,b}^{FB}$ would have to be inflated by more than a factor of 5 to account for the discrepancy. The theoretical systematics are dominated by the correction to the observed asymmetry for gluon radiation. For a completely inclusive selection, the total QCD correction applied to the data is approximately 4%, of order the discrepancy between the LEP $A_{0,b}^{FB}$ average and the expected value from the Standard Model. However, most experimental analyses tend to reject events strongly affected by gluon radiation so the actual corrections are much smaller [38]. Furthermore, the experimental techniques used in jet-charged measurements uses data driven correction which attenuate the QCD effects still further. The residual QCD error on the LEP $A_{0,b}^{FB}$ measurements, largely from missing higher orders in the QCD calculation and from hadronization, is estimated to be 0.2%.

The LEP heavy quark results are not all published or final and it is expected that some of the techniques developed for $b$-mixing studies will result in an increase in the precision of some of the LEP results [39,40].
Table 5: Limits and values for Higgs mass determined from fits with the “usual” experiment
driven value\(^{[41]}\) of \(\Delta \alpha^5_{\text{had}}(m_Z^2)\) traditionally used by the LEP electroweak group and the
value of \(\Delta \alpha^5_{\text{had}}(m_Z^2)\) presented at ICHEP2000 including BES data \(^{[50]}\).

| \(\Delta \alpha^5_{\text{had}}\) | 0.02804 ± 0.00065 | 0.02755 ± 0.00046 |
|----------------|------------------|------------------|
| \(m_h\) | 60^{+52}_{-29}\) GeV | \(m_{\text{Higgs}} < 170\) GeV |
| \(m_h\) 95% C.L. upper limit | \(m_{\text{Higgs}} < 210\) GeV |

5 Impact of uncertainties in hadronic vacuum polarization

The constraint given by the LEP and SLD asymmetry data on the Higgs mass is
strongly dependent on the running value of \(\alpha\) parameterized by

\[
\alpha(m_Z^2) = \frac{\alpha(0)}{1 - \Delta \alpha_{\ell}(m_Z^2) - \Delta \alpha^5_{\text{had}}(m_Z^2) - \Delta \alpha_{\text{top}}(m_Z^2)}
\]

where \(\Delta \alpha_{\ell}(m_Z^2)\) is the contribution to vacuum polarization from leptons, \(\Delta \alpha_{\text{top}}(m_Z^2)\)
the contribution from top quarks and \(\Delta \alpha^5_{\text{had}}(m_Z^2)\) the contribution from the five
lightest quarks. The value of \(\Delta \alpha^5_{\text{had}}(m_Z^2)\) is derived from the measured cross section
for the process \(e^+e^- \to \text{hadrons}\) at low energies and currently limits the precision
with which \(\alpha(m_Z^2)\) can be determined.

The correlation between \(\Delta \alpha^5_{\text{had}}(m_Z^2)\) and the determination of the Higgs mass
from the electroweak data is shown in Figure 4. The value of the Higgs mass
determined from the fit is strongly correlated with the \(\Delta \alpha^5_{\text{had}}(m_Z^2)\) input. In Figure 5
various determinations \(^{[11][12][13][14][15][16][17][18][19][20]}\) of the \(\Delta \alpha^5_{\text{had}}(m_Z^2)\) are
shown. The LEP Electroweak group has generally used the value from Eidelmann
and Jegerlehner \(^{[11]}\) which is primarily data driven. It is interesting that the new
determination from Pietrzyk \(^{[10]}\), based on new data from BES \(^{[51]}\) presented at ICHEP
2000 agrees well with the result of theory driven results \(^{[12][13][14][15][16][17][18][19][20]}\) which
makes use of perturbative QCD. In any case the result of using the Pietrzyk result is
to move the Higgs mass prediction of the grand electroweak fit towards higher values
(see Table 3). We can expect that the error on \(\Delta \alpha^5_{\text{had}}(m_Z^2)\) will continue to decline
in the future as more data is collected by BES and other low energy electron-positron
storage rings.
Figure 4: The contours show the 1 $\sigma$ (47% C.L.), 2 $\sigma$ (91% C.L.) and 3$\sigma$ (99.5% C.L.) limits in the $\Delta\alpha_{\text{had}}^{5}(m_{Z}^{2})$-$m_{h}$ plane, for a data similar, but not identical to that of Table 1[36]. The upper bands show the value from $\Delta\alpha_{\text{had}}^{5}(m_{Z}^{2})$-$m_{h}$ from Reference [41] and the lower band shows preliminary results using the new preliminary BES data from Reference [50].
Figure 5: Compilation of values of $\Delta \alpha_{\text{had}}^5(m_Z^2)$ as a function of time. (See References [41, 42, 43, 44, 45, 46, 47, 48, 49], for older values see citation in Reference [42]). The last two results incorporate the new BES data [51].
6 Conclusions

The LEP results for the Z line shape and lepton asymmetries have been stable for some time and are now final and published\cite{9}. The theoretical error assigned to $\alpha_s$ determined from these data remains controversial. If the correction of Reference \cite{22} is applied,

$$\alpha_s(m_Z) = 0.1190 \pm 0.0027 (\text{Exp.} + \text{EW}) \pm 0.0011 (\text{QCD})$$

is obtained. This is competitive with the 2000 PDG\cite{20} world average of $\alpha_s(m_Z) = 0.1181 \pm 0.002$.

The SLD measurement of $\sin^2\theta_{\text{eff}}^{\text{lept}}$, based primarily on the left-right polarized asymmetry, is also now final and published\cite{10}. Its value agrees with that obtained from lepton asymmetries and $\tau$ polarization at LEP. However, the average from these lepton based results is in disagreement with the LEP heavy-quark measurements of $\sin^2\theta_{\text{eff}}^{\text{lept}}$. It is possible that the discrepancy could be explained by anomalous values of $A_b$ or $A_c$, but the direct measurements of these quantities by SLD are in agreement with both the Standard Model and the LEP values, assuming a Standard Model value for $A_e$.

The interpretation of these electroweak results in terms of limits on the Higgs boson mass depends on the value of $\Delta\alpha_{\text{Had}}^5(m_Z^2)$. New $e^+e^-$ cross section measurements from BES gives a data driven value $\Delta\alpha_{\text{had}}^5(m_Z^2)$ which agrees with previous theory driven determinations, resulting in a higher prediction for the Higgs boson mass.

Acknowledgments

I would like to thank the ALEPH, DELPHI, L3, OPAL and SLD collaborations and the LEP and SLD Electroweak Working Groups for their assistance in preparing this report. I am especially grateful to Martin Gr"unewald and G"unter Quast for providing the electroweak fits used in this report. I would also like to thank Dave Soper and Sigi Bethke for useful discussions about the application of QCD to the LEP line-shape results. Finally I wish to thank Dick Kellogg for his suggestions and encouragement.

References

[1] ALEPH, DELPHI, L3, OPAL and SLD Collaborations, the LEP Electroweak Working Group and the SLD Electroweak and Heavy Flavor Groups, “Precision Electroweak Measurements on the Z Resonance”, in preparation.

[2] G. Montagna, O. Nicrosini and F. Piccinini, Riv. Nuovo Cim. 21N9 (1998) 1.
[3] D. Bardin, M. Grunewald and G. Passarino, “Precision calculation project report,” hep-ph/9902452.

[4] D. Bardin and G. Passarino, “The Standard Model in the making: Precision study of the electroweak interactions,” Oxford, UK: Clarendon (1999) 685 p.

[5] ALEPH, DELPHI, L3, OPAL and SLD Collaborations, the LEP Electroweak Working Group and the SLD Electroweak and Heavy Flavor Groups, “A combination of preliminary electroweak measurements and constraints on the standard model,” hep-ex/0103048.

[6] ALEPH, DELPHI, L3, OPAL and SLD Collaborations, the LEP Electroweak Working Group and the SLD Electroweak and Heavy Flavor Groups, “A combination of preliminary electroweak measurements and constraints on the standard model,” CERN-EP-99-015.

[7] K. McFarland, “Electroweak Physics at the Tevatron”, RADCOR 2000.

[8] S. Wynhoff, “Standard model physics results from LEP2,” hep-ex/0101016.

[9] LEP Collaborations, “Combination procedure for the precise determination of Z boson parameters from results of the LEP experiments,” hep-ex/0101027.

[10] K. Abe et al., SLD Collaboration, Phys. Rev. Lett. 84 (2000) 5945.

[11] R. Barate et al., ALEPH Collaboration, Eur. Phys. J. C 14 (2000) 1.

[12] P. Abreu et al., DELPHI Collaboration, Eur. Phys. J. C 16 (2000) 371.

[13] M. Acciarri et al., L3 Collaboration, Eur. Phys. J. C 16 (2000) 1.

[14] G. Abbiendi et al., OPAL Collaboration, Eur. Phys. J. C 19 (2001) 587.

[15] B. F. Ward, S. Jadach, M. Melles and S. A. Yost, Phys. Lett. B 450 (1999) 262.

[16] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini and F. Piccinini, Nucl. Phys. B 547 (1999) 39.

[17] G. Montagna, O. Nicrosini and F. Piccinini, Phys. Lett. B 406 (1997) 243.

[18] A. B. Arbuzov, “Light pair corrections to electron positron annihilation at LEP/SLC,” hep-ph/9900750.

[19] L. Arnaudon et al., Z. Phys. C 66 (1995) 45.
[20] L. Montanet et al., Phys. Rev. D 50 (1994) 1173.
[21] S. Bethke, J. Phys. G G26 (2000) R27.
[22] D. E. Soper and L. R. Surguladze, Phys. Rev. D 54 (1996) 4566.
[23] K. G. Chetyrkin, J. H. Kuhn and A. Kwiatkowski, Phys. Rept. 277 (1996) 189.
[24] G. Montagna, O. Nicrosini, F. Piccinini and G. Passarino, Comput. Phys. Commun. 117 (1999) 278.
[25] D. Bardin, P. Christova, M. Jack, L. Kalinovskaya, A. Olchevski, S. Riemann and T. Riemann, Comput. Phys. Commun. 133 (2001) 229.
[26] P. A. Raczka and A. Szymacha, Phys. Rev. D 54 (1996) 3073.
[27] F. Le Diberder and A. Pich, Phys. Lett. B 289 (1992) 165.
[28] K. Ackerstaff et al., OPAL Collaboration, Eur. Phys. J. C 7 (1999) 571.
[29] P. Ball, M. Beneke and V. M. Braun, Nucl. Phys. B 452 (1995) 563.
[30] K.G. Chetyrkin, J.H. Kühn, and M. Steinhauser, Phys. Lett. B351 (1995) 331.
[31] K. G. Chetyrkin, J. H. Kühn and M. Steinhauser, Phys. Rev. Lett. 75 (1995) 3394.
[32] E. Tournefier, “Extraction of $\alpha_s$ and constraint on the Higgs mass from electroweak fits at the Z resonance,” hep-ex/9810042.
[33] K. Ackerstaff, et al., OPAL Collaboration, Eur. Phys. J. C 7 (1999) 571.
[34] OPAL Collaboration, “Precision Tau and Electron Neutral Current Asymmetry Measurements from the Tau Polarization at OPAL”, OPAL Physics Note PN438.
[35] P. Abreu et al., DELPHI Collaboration, Eur. Phys. J. C 14 (2000) 585.
[36] Private communication, T. Abe.
[37] K. Abe et al., SLD Collaboration, hep-ex/0009063.
K. Abe et al., SLD Collaboration, SLAC-PUB-8542 Talk given at the 30th International Conference on High-Energy Physics (ICHEP 2000), Osaka, Japan, 27 Jul - 2 Aug 2000.
K. Abe et al., SLD Collaboration, Phys. Rev. D 63 (2001) 032005.
[38] D. Abbaneo et al., LEP Heavy Flavor Working Group Collaboration, Eur. Phys. J. C 4 (1998) 185.
[39] DELPHI Collaboration, “Measurement of the Forward-Backward Asymmetries of $e^+e^- \rightarrow Zb\bar{b}$ and $c\bar{c}$ using prompt leptons”, DELPHI 2000-101 CONF 400.

[40] ALEPH collaboration, “Inclusive semileptonic branching ratios of $b$ hadrons produced in $Z$ decays”, ALEPH 2000-069, CONF-2000-047, Contributed paper to ICHEP2000 #133.

[41] S. Eidelman and F. Jegerlehner, Z. Phys. C 67 (1995) 585.

[42] H. Burkhardt and B. Pietrzyk, Phys. Lett. B 356 (1995) 398.

[43] R. Alemany, M. Davier and A. Hocker, Eur. Phys. J. C 2, 123 (1998).

[44] M. Davier and A. Hocker, Phys. Lett. B 419 (1998) 419.

[45] J. H. Kuhn and M. Steinhauser, Phys. Lett. B 437 (1998) 425.

[46] S. Groote, J. G. Korner, K. Schilcher and N. F. Nasrallah, Phys. Lett. B 440 (1998) 375.

[47] J. Erler, Phys. Rev. D 59 (1999) 054008.

[48] F. Jegerlehner, “Hadronic effects in $(g - 2)(\mu)$ and $\alpha_{\text{QED}}(m_Z)$: Status and perspectives,” hep-ph/9901386.

[49] A. D. Martin, J. Outhwaite and M. G. Ryskin, Phys. Lett. B 492 (2000) 69.

[50] B. Pietrzyk, “The global fit to electroweak data”, talk presented at ICHEP 2000, Osaka, July 27- August 2, 2000. LAP-EXP-2000-06. H. Burkhardt and B. Pietrzyk, LAPP-EXP-2001-03.

[51] Z.G. Zhao, BES Collaboration, “New R values in 2-5 GeV from BES” talk presented at ICHEP2000, OSKA, July 27- August 2, 2000.