State mapping and discontinuous entanglement transfer in a multipartite open system

Matteo Bina,1,2 Federico Casagrande,1,2 Marco G. Genoni,1,2 Alfredo Lulli,1,2 and Matteo G. A. Paris1,2,3

1Dipartimento di Fisica, Università di Milano, I-20133 Milano, Italy
2CNISM, UdR Milano, I-20133, Milano, Italy
3ISI Foundation, I-10133, Torino, Italy

(Dated: October 12, 2009)

We describe the transfer of quantum information and correlations from an entangled tripartite bosonic system to three separate qubits through their local environments also in the presence of various dissipative effects. Optimal state mapping and entanglement transfer are shown in the framework of optical cavity quantum electrodynamics involving qubit-like radiation states and two-level atoms via the mediation of cavity modes. For an input GHZ state mixed with white noise we show the occurrence of sudden death and birth of entanglement, that is discontinuously exchanged among the tripartite subsystems.

PACS numbers: 03.67.Mn,42.50.Pq

I. INTRODUCTION

As early as in 1935 Einstein, Podolsky, and Rosen [1] as well as Schrödinger [2] drew the attention on the correlations in quantum composite systems and the problems raised by their properties. Much later, theoretical [3] and experimental [4] cornerstones elucidated the issue of nonlocality. Entanglement is currently viewed as the key resource for quantum information (QI) processing [5], where it allowed a number of achievements such as teleportation [5], cryptography [6] and enhanced measurements [8]. The deep meaning of multipartite entanglement, its quantification and detection [9], the possible applications, are the object of massive investigation. As a matter of fact, optical systems have been a privileged framework for encoding and manipulating quantum information, since bipartite and multipartite entanglement may be effectively generated either in the discrete or continuous variable regime. On the other hand, the development of QI also requires localized registers, e.g. for the storage of entanglement in quantum memories.

Cavity quantum electrodynamics (CQED) [10] is a relevant scenario for this kind of investigations and has been addressed for the analysis of entanglement transfer of bipartite [11, 12, 13, 14, 15, 16, 17, 18, 19] and multipartite [20, 21, 22, 23] entanglement. In this framework we present a complete study on the entanglement dynamics of a nine parties open system whose implementation could be feasible in the optical regime of CQED [24, 25]. In particular we describe a system where three radiation modes, prepared in qubit-like entangled states, are coupled by optical fibers to three separate optical cavities each of them containing a trapped two-level atom. This paradigmatic example allows us investigating multipartite entanglement transfer and swapping in a more realistic way than in [20, 21], shedding light on fundamental processes related to quantum interfaces and memories in quantum networks [22, 23].

We demonstrate that a complete mapping of pure entangled states onto the tripartite atomic subsystem occurs when the external field has fed the cavities. If this field is then switched off, the quantum correlations can be periodically mapped onto the tripartite atomic and cavity mode subsystems according to a triple Jaynes-Cummings (JC) dynamics [26]. In the case of external radiation prepared in a mixed Werner state we deal with the recently observed phenomenon of entanglement sudden death (ESD) (and birth (ESB)) [27, 28], that in our case involves the abrupt vanishing (and raising) of quantum correlations in tripartite systems. Though this is in general a still open problem, nevertheless we can show the occurrence of discontinuous exchange of tripartite entanglement via ESD effects. We also describe the dissipative effects introduced by the presence of external environments, such as the decay of cavity modes, atomic excitations, and fiber modes. Some results are then reported in the case that the coupling between external and cavity mode fields is generalized from monomode to multimode.

In Sec. II we introduce the model of the physical system. In Sec. III we derive all main results concerning state mapping, entanglement transfer, and entanglement sudden death, also adding the presence of external environments. The case of multimode external-cavity field coupling is addressed in Sec. IV. Some conclusive remarks are reported in Sec. V.

II. MODEL OF THE PHYSICAL SYSTEM

We consider an entangled tripartite bosonic system \((f)\), prepared in general in a mixed state, interacting with three qubits \((a)\) through their local environments \((c)\) also in the presence of dissipative effects. In the interaction picture the system Hamiltonian has the form:

\[
\hat{H}^I = \hbar \sum_{J=A,B,C} \left[ g_J (\hat{c}_J \hat{\sigma}_J^T + \hat{c}_J^T \hat{\sigma}_J) \right] + \\
+ \sum_{J,K=A,B,C} \left[ \nu_{J,K}(t) (\hat{c}_J \hat{f}_K^T + \hat{c}_J^T \hat{f}_K) \right]
\]  

(1)
The operators $\hat{\gamma}_J, \hat{\sigma}_J^\dagger (\hat{f}_J, \hat{f}_J^\dagger)$ are the annihilation and creation operators for the cavity (external radiation) modes, while $\hat{\sigma}_J, \hat{\sigma}_J^\dagger$ are the raising and lowering operators for the atomic qubits in each subsystem ($J=A,B,C$). We consider real coupling constant $g_J$ for the atom-cavity mode interaction and $\nu_{J,K}(t)$ for the interaction of each cavity mode with the three modes of driving external radiation. We take time dependent constants in order to simulate the interaction switching-off at a suitable time $\tau_{\text{off}}$ for the external field. Now we take into account three processes of dissipation: the cavity losses at rate $\kappa_c$ due to interaction with a thermal bath with a mean photon number $\bar{n}$, the atomic spontaneous emission with a decay rate $\gamma_a$ for the upper level, and a loss of photons inside the fibers at a rate $\kappa_f$. All dissipative effects can be described under the Markovian limit by standard Liouville superoperators so that the time evolution of the whole system density operator $\hat{\rho}(t)$ can be described by the following ME written in the Lindblad form:

$$\dot{\hat{\rho}} = -\frac{i}{\hbar} [\hat{H}_c, \hat{\rho}] + \sum_{J=\text{A,B,C}} \left[ \hat{C}_{J,\rho} \hat{C}_{J,\rho}^\dagger + \hat{C}_{J,\rho}^\dagger \hat{C}_{J,\rho} + \hat{C}_{J,\rho} \hat{C}_{J,\rho}^\dagger \right]$$

where the non-Hermitian effective Hamiltonian is

$$\hat{H}_c = \hat{H}_c^\dagger - \frac{i}{2} \sum_{J} \left[ \hat{C}_{J,\rho}^\dagger \hat{C}_{J,\rho} + \hat{C}_{J,\rho} \hat{C}_{J,\rho}^\dagger \right].$$

The jump operators for the atoms are $\hat{\gamma}_J, \hat{\sigma}_J = \sqrt{\gamma_a} \hat{\sigma}_J$, for the fiber losses $\hat{C}_{J,\rho} = \sqrt{\kappa_c (\hat{n} + 1)} \hat{J}_J$ (loss of a photon) and $\hat{C}_{J,\rho}^\dagger = \sqrt{\kappa_c \kappa_f} \hat{J}_J$ (gain of a photon). The above ME can be solved numerically by the Monte Carlo Wave Function method [22]. From now on we consider dimensionless parameters, all scaled to the coupling constant $g_A$, and times $\tau = g_A t$.

As a significant example, the implementation of our scheme may be realized in the optical regime of CQED by choosing a continuous variable (CV) entangled field for the subsystem (f) and two-level atoms as qubits (a). Each qubit is trapped in a one-sided optical cavity (c), where the radiation modes can be coupled to the cavity modes via optical fibers as in [30]. In optical cavities thermal noise is negligible ($\bar{n} \approx 0$), spontaneous emission can be effectively suppressed, and single atoms can remain trapped even for several seconds [24].

Here we focus on external field prepared in a qubit-like entangled state $\hat{\rho}_f(0)$ because this is the condition for high entanglement transfer for CV field [21]. The generation of photon number multimode entangled radiation was recently demonstrated [31]. Under qubit-like behavior we can describe the entanglement of all three-qubit subsystems ($a, c, f$) by combining the information from tripartite negativity [32], entanglement witnesses [33] for the two inequivalent classes GHZ and W [34], and recently proposed criteria for separability [35]. In fact, the tripartite negativity $E^{(a)}(\tau)(a=\alpha, \beta, \gamma, f)$, defined as the geometric mean of the three bipartite negativities [36], is an entanglement measure providing only a sufficient condition for entanglement detection, though its positivity guarantees the GHZ-distillability that is an important feature in QF.

### III. STATE MAPPING AND TRIPARTITE ENTANGLEMENT TRANSFER FOR SINGLE MODE COUPLING

#### A. Hamiltonian regime for external field in qubit-like pure states

We first illustrate the Hamiltonian dynamics ($\{\hat{\kappa}_f, \hat{\kappa}_c, \gamma_a \} \ll 1$) for the external field prepared in a qubit-like entangled pure state $|\Psi(0)f\rangle_f$, atoms prepared in the lower state $|gg\rangle_a$, and cavities in the vacuum state $|000\rangle_c$. By choosing $\nu_{J,K}(\tau) = 0$ if $J \neq K$ and $\nu_{J,J} = g_A$ we describe single-mode fibers each one propagating a mode of the entangled radiation. We show that under optimal conditions for mode matching it is possible to map $|\Psi(0)f\rangle_f$ onto atomic and cavity mode states for suitable interaction times. Overall we are dealing with an interacting 9-qubit system, though the input field will be switched off at a time $\tau_{\text{off}}$ such that the atomic probability of excited state $p_\alpha(\tau)$ reaches the maximum. Injected field switch-off can be obtained, e.g., by rotating fiber polarization.

In Fig. [1] we show numerical results for the external field prepared in the GHZ state $|\Psi(0)f\rangle_f = (|000\rangle_f + |111\rangle_f)/\sqrt{2}$. In the time interval $0 < \tau < \tau_{\text{off}}$ (transient regime) each flying qubit transfers its excitation to the cavity which in turn passes it onto the atom (see Fig. [1]). The cavity mode, simultaneously coupled to the external field and to the atom, exchanges energy according to a Tavis-Cummings dynamics at an effective frequency $g/\sqrt{2}$ [10, 37]. During the transient up to time $\tau_{\text{off}} = \pi/\sqrt{2}$ the mean photon number $N^{(f)}(\tau) \equiv \langle \hat{c}^\dagger \hat{c}(\tau) \rangle$ in each cavity completes a cycle. In the same period the atomic excitation probability $p_\alpha(\tau)$ reaches its maximum value, while the input field has completely fed the cavity, i.e., its mean photon number $N^{(f)}(\tau) \equiv \langle \hat{f}^\dagger \hat{f}(\tau) \rangle$ vanishes. In Fig. [1], we show that in the transient the atomic tripartite negativity is always positive and $E^{(a)}(\tau_{\text{off}}) = 1$, that is the value of the injected GHZ state. Until $\tau_{\text{off}}$ the dynamics maps the whole initial state $|\Psi(0)f\rangle_f \otimes |000\rangle_c \otimes |gg\rangle_a$ onto the pure state $|000\rangle_f \otimes |000\rangle_c \otimes |\Psi(0)\rangle_a$, where $|\Psi(0)\rangle_a$ is obtained from $|\Psi(0)\rangle_f$ by the correspondence $|0\rangle_f \rightarrow |g\rangle_a$ and $|1\rangle_f \rightarrow |e\rangle_a$. This is confirmed in Fig. [1]: by the time evolution of the purity $\mu^{(a)}(\tau) = \text{Tr}_{a} [\rho_a^2(\tau)]$ and the fidelity $F^{(a)}(\tau) = a\langle \Psi(0) | \hat{\rho}_a(\tau) | \Psi(0) \rangle_a$, where $\hat{\rho}_a(\tau)$ is the atomic reduced density operator. As for the cavity mode dynamics we note that (see Fig. [1])
the local maximum of \( E^{(c)}(\tau_{\text{off}}/2) \) does not correspond to a pure state, i.e. the initial state \( |\Psi(0)\rangle \) cannot be mapped onto the cavity modes during the transient regime. The entanglement is only partially transferred to the cavity modes nevertheless allowing the building up of full atomic entanglement later on. This dynamics is quite different than in [21] where the entangled field was mapped onto the cavity modes before the interaction with the atoms.

At the end of the transient regime the external radiation is turned off and the subsequent dynamics is described by a triple JC rule by oscillations at the vacuum Rabi frequency \( 2g \), hence with a dimensionless period \( \pi \) as shown by cavity mean photon number and atomic probability in Fig. 1. The purities \( \mu^{(a,c)}(\tau) \) of \( \hat{\rho} \) in Figs. 1c,d oscillate at a double frequency between pure entangled (maximum negativity) and separable (zero negativity) states. In particular, at times \( \tau_m = \tau_{\text{off}} + m\pi \) (\( m = 0, 1, 2, \ldots \) ) the atoms are in the entangled states \( U_{\phi}^{(a)}|\Psi(0)\rangle_a \), where \( U_{\phi}^{(a)} = \bigotimes_j e^{-i\phi \sigma_j^a} \), is a local phase operator where \( \phi = 0 \) (\( \phi = \pi \) ) applies for even (odd) values of \( m \), that are the peaks of \( E^{(a)}(\tau) \) in Fig. 1b. At times \( \tau_n = \tau_{\text{off}} + (n + 1/2)\pi \) (\( n = 0, 1, 2, \ldots \) ) the cavity mode states are obtained by applying \( U_{\phi}^{(c)} = \bigotimes_j e^{-i\phi \sigma_j^c} \), where \( \phi = -\pi/2 \) (\( +\pi/2 \) ) for even (odd) values of \( n \), to the state \( |\Psi(0)\rangle_c \) derived from \( |\Psi(0)\rangle_f \) by the correspondence \( |0\rangle_f \leftrightarrow |0\rangle_c \) and \( |1\rangle_f \leftrightarrow |1\rangle_c \). By choosing to turn off the external field at times shorter than \( \tau_{\text{off}} \) we find a progressive reduction in the entanglement transfer to the atomic and cavity subsystems, simulating the effect of non perfect cavity mirror transmittance. Up to 10% changes in the value of \( \tau_{\text{off}} \), the fidelity \( F^{(a)}(\tau_{\text{off}}) \) remains above 99.9%. We remark that the state mapping process can be obtained for any \( |\Psi(0)\rangle_f \) written in a generalized Schmidt decomposition [34] as well as for mixed states as described below.

**B. Tripartite entanglement sudden death for external field in a Werner state**

For the injected field we consider the Werner state \( \hat{\rho}_{\text{f}}(0) = (1-p)|\text{GHZ}\rangle\langle \text{GHZ}| + \frac{p}{2} I \), \( 0 \leq p \leq 1 \), because it is relevant in QI and it is possible to fully classify its entanglement as a function of parameter \( p \). In fact, for \( 0 \leq p < \frac{4}{7} \) the state belongs to the GHZ class and to the W class up to \( p = \frac{4}{7} \). The tripartite negativity is positive up to \( 4/5 \), and due to the structure of \( \hat{\rho}_{\text{f}}(0) \), the state is clearly inseparable under all bipartitions (INS), i.e. it cannot be written as convex combination of biseparable states. For \( 4/5 < p \leq 1 \) it is known that the state is fully separable [35, 38].

The system dynamics can be divided into a transient and an oscillatory regime, and the state mapping of \( \hat{\rho}_{\text{f}}(0) \) onto atoms (cavity modes) still occurs at times \( \tau_m \) \( (\tau_n) \). Out of these times the density matrices of all subsystems lose the form of a GHZ state mixed with white noise but still preserve invariance under all permutations of the three qubits and present only one non vanishing coherence as in \( \hat{\rho}_{\text{f}}(0) \) . This greatly helps us in the entanglement classifications in the plane \( (\tau, p) \) shown in Fig. 2. In fact, in the regions where \( E^{(a)}(\tau) > 0 \) but out of W class we can exclude the biseparability. The fully separability criteria in [33] are violated only where \( E^{(a)}(\tau) > 0 \) so that if \( E^{(a)}(\tau) = 0 \) the state may be fully separable or biseparable. Nevertheless, in the latter case the state should be symmetric and biseparable under all bipartitions and hence it is fully separable [39]. For any fixed value of \( p \) in the range \( 0 < p < 4/5 \) we thus show the occurrence of entanglement sudden death and birth at the boundaries between fully separable and INS states. In particular, for \( 0 < p < 4/7 \) we find genuine tripartite ESD and ESB phenomena. Note that, for a fixed value of \( p \), the passage of atomic state during time evolution from W-class to GHZ-class (or viceversa) entangled states is permitted by the non-unitarity of the overall operation on the initial three qubits (not SLOCC). We also notice that for times \( \tau \geq \tau_{\text{off}} \) we can solve exactly the triple JC dynamics, confirming our numerical results and generalizing [40] to mixed states.

In Fig. 2 we see that, for increasing values of \( p \), there is an increase of both the slope of \( E^{(a)}(\tau) \) and the time interval of fully separability. In Fig. 2a we show in detail the transient dynamics of the tripartite negativities \( E^{(a)}(\tau,p) \) (\( \alpha = a, c, f \) ) in the crucial region around \( \tau_{\text{off}}/2 \). We consider some values of \( p \) where the atoms exhibit in times different classes of entanglement. We see that for \( p = 0.2 \), where the input state has GHZ class entanglement, the ESB of subsystems \((e, a)\) anticipates
the ESD of (f),(c), and there is an interval around \( \tau_{\text{off}}/2 \) where all three subsystems are entangled (of INS-type). As \( p \) grows, hence the initial state becomes more noisy, the effects of ESD occur earlier and those of ESB later. For \( p = 0.4 \), involving W-class entanglement, only at most two subsystems are simultaneously entangled (first (f),(c) and then (c),(a)). For \( p = 0.6 \), involving only entanglement of INS-type, the cavity modes do not entangle at all (see Fig. 2b). They physically mediate the discontinuous entanglement transfer from (f) to (a), where for \( p \rightarrow 4/5 \) the time interval without any entanglement increases while the entanglement level vanishes.

C. Effect of dissipation on state mapping

In the perspective of experimental implementation for QI purposes an important issue is the effect of dissipation on both state mapping and entanglement transfer. For external field prepared in a GHZ pure state we first evaluated the effect of cavity decay rates in the range \( 0 < \tilde{\kappa}_c \leq 0.5 \) for negligible values of all other decay rates. We consider as function of \( \tilde{\kappa}_c \) the behavior of the fidelities \( F^{(\alpha)}(\tau_{m,n}) \) and the tripartite negativities \( E^{(\alpha)}(\tau_{m,n}) \) (\( \alpha = c, f \)) at the first peaks (\( m = n = 0 \)). In Fig. 3 we see that the above functions of \( \tilde{\kappa}_c \) can be well fitted by exponential functions, whose decay rates for the atomic subsystem are \( \beta_{F}^{(c)} = 1.80, \beta_{E}^{(c)} = 2.94 \). As expected, quantum state mapping and entanglement transfer are by far more efficient onto atomic than cavity qubits. For instance, if \( \tilde{\kappa}_c = 0.1 \) we obtain a state mapping onto the atoms (cavity modes) with a fidelity of \( \approx 0.93 (\approx 0.83) \).

We can now add the further dissipative effect of atomic decay. For instance we find that, for an atomic decay rate \( \gamma_a = 0.03 \) and in the presence of cavity decay with a rate \( \tilde{\kappa}_c = 0.1 \), the fidelity of the atomic (cavity mode) subsystem reduces by 4.4% (8.9%).

Finally, we evaluate the effect of losses in the fibers used to inject the external field into each cavity. Clearly, this effect is relevant only up to the time \( \tau_{\text{off}} = 2.22 \). We evaluated the effects of fiber decay rates \( \tilde{\kappa}_f \) up to 1.0 for negligible values of atomic and cavity decay rates (\( \tilde{\kappa}_c << 1, \gamma_a << 1 \)) (see Fig. 4). We show the effect of parameter \( \tilde{\kappa}_f \) on cavity field mean photon number \( N^{(c)}(\tau_{\text{off}}/2) \) and atomic excitation probability \( p_c(\tau_{\text{off}}) \) and we see that the amount of energy transferred to the atoms and to the cavity modes decreases exponentially for increasing values of \( \tilde{\kappa}_f \): the decay rates are \( \approx 0.42 \) and \( \approx 0.82 \), respectively. Also the behavior of the tripartite negativity \( E^{(\alpha)}(\tau_0) \) and fidelity \( F^{(\alpha)}(\tau_0) \) at the first peak can be described versus \( \tilde{\kappa}_f \) by exponential functions whose decay rates are \( \approx 1.51 \) and \( \approx 1.95 \), respectively.

IV. STATE MAPPING AND TRIPARTITE ENTANGLEMENT TRANSFER FOR MULTI-MODE COUPLING

Finally, we consider multi-mode coupling of the external field to each cavity mode. For simplicity we choose equal coupling constants \( \tilde{\nu}_{J,K} \equiv \nu_{J,K}/g_A \neq 0 \) if \( K \neq J \) and we consider values in the range \( 0 - 1.4 \). In the transient regime the dynamics is sharply modified with respect to the case of single mode fiber shown in Fig. 1. By increasing the values of \( \tilde{\nu}_{J,K} \) the period of energy exchange decreases from \( 2\pi/\sqrt{2} \) to \( \approx 2.6 \). The maximum of cavity mode mean photon number grows.
Nevertheless, multimode coupling for larger values of transfer. We evaluate at time $\tau_{\text{off}}$ the atomic tripartite negativity $E^a$, the fidelity $F^a$ (dash-dot), and the atomic probability $p_a$ (dot), and at time $\tau_{\text{off}}/2$ the cavity mean photon number $N^c$. We consider an implementation in the optical bosonic system to three localized qubits through their tum information and entanglement from a tripartite system. In this paper we have addressed the transfer of quantum information and entanglement from a tripartite bosonic system to three localized qubits through their environments, also in the presence of external environments. We considered an implementation in the optical regime of CQED based on CV photon number entangled fields and atomic qubits trapped in one-sided optical cavities, where the radiation modes can couple to cavity modes by optical fibers.

In the nine-qubit transient regime the quantum state is mapped from tripartite entangled radiation to tripartite atomic system via the cavity modes. After the transient we switch off the external field. The subsequent triple JC dynamics, that we solved analytically (numerically) for
pure (mixed) input states, shows how the effect of mapping can further affect the atom-cavity six qubits system. Its relevance is in the possible manipulation for QI purposes of entanglement stored in separate qubits of atomic or bosonic nature. Hence the interest to put quantitative limits dictated by cavity, atomic and fiber mode decays, that we evaluated at the times where the transfer protocol is optimal.

In the case of a GHZ input state mixed with white noise, we provide a full characterization of the separability properties of the tripartite subsystems. We can then show the occurrence of entanglement sudden death effects at the tripartite level, deriving the conditions for the repeated occurrence of discontinuous exchange of quantum correlations among the tripartite subsystems. This is an issue of fundamental interest as well as worth investigating for all applications in quantum information processing, remarkably computing and error correction, where entanglement, which may be faster than decoherence, has to be carefully controlled.

An extension and comparison to other types of entangled qubit-like input fields and experimentally available CV fields \cite{41} will be presented elsewhere \cite{42}.

**Acknowledgments**

This work has been partially supported by the CNR-CNISM convention.

---

[1] A. Einstein, B. Podolski, and N. Rosen, Phys. Rev. A 47, 777 (1935).
[2] E. Schrödinger, Naturwissenschaften 23, 807 (1935).
[3] J. S. Bell, Physics (NY) 1, 195 (1964).
[4] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. 49, 1804 (1982).
[5] R. Jozsa and N. Linden, Proc. R. Soc. Lond. A 459, 2011 (2003).
[6] C. H. Bennett et al., Phys. Rev. Lett. 70, 1895 (1993).
[7] N. Gisin et al., Rev. Mod. Phys. 74, 1458 (2002).
[8] G. M. D’Ariano, P. Lo Presti, and M. G. A. Paris, Phys. Rev. Lett. 87, 270404 (2001).
[9] O. Gühne and G. Toth, Phys. Rep. 474, 1 (2009).
[10] S. Haroche and J. M. Raimond, *Exploring the Quantum* (Oxford University Press, 2006).
[11] M. Paternostro, W. Son, and M.S. Kim, Phys. Rev. Lett. 92, 197901 (2004).
[12] M. Paternostro, W. Son, M.S. Kim, G. Falci, and G.M. Palma, Phys. Rev. A 70, 022320 (2004).
[13] M. Paternostro, G. Falci, M. Kim, and G.M. Palma, Phys. Rev. B. 21, 4502 (2004).
[14] B. Julsgaard, J. Sherson, J.I. Cirac, J. Fiursek, and E.S. Polzik, Nature 432, 482 (2004).
[15] J. Hald, J.L. Sorensen, C. Shori, and E.S. Polzik, J. Mod. Opt. 47, 2599 (2000).
[16] W. Son, M. S. Kim, J. Lee, and D. Ahn, J. Mod. Opt. 49, 1739 (2002).
[17] J. Zou, G.L. Jun, S. Bin, L. Jian, and S.L. Qian, Phys. Rev. A 73, 042319 (2006).
[18] F. Casagrande, A. Lulli, M. G. A. Paris, Phys. Rev. A 75, 032336 (2007).
[19] F. Casagrande, A. Lulli, and M. G. A. Paris, Eur. Phys. J. ST 160, 71 (2008).
[20] M. Paternostro, W. Son, and S. M. Kim, Phys. Rev. Lett. 92, 107901 (2004); J. Lee et al., Phys. Rev. Lett. 96, 080501 (2006).
[21] F. Casagrande, A. Lulli, and M. G. A. Paris, Phys. Rev. A 79, 022307 (2009).
[22] J. I. Cirac et al., Phys. Rev. Lett. 78, 3221 (1996); A. D. Boozer et al., Phys. Rev. Lett. 98, 193601 (2007).
[23] A. Serafini et al., Phys. Rev. A 73, 022312 (2006).
[24] S. Nussmann et al., Nat. Phys. 1, 122 (2005); K. M. Fortier et al., Phys. Rev. Lett. 98, 233601 (2007); J. Ye, H. J. Kimble, and H. Satori, Science 320, 1734 (2008).
[25] A.B. Mundt et al., Phys. Rev. Lett. 89, 103001 (2002); M. Keller et al., Nature (London) 431, 1075 (2004).
[26] E. T. Jaynes and F. W. Cummings, Proc. IEEE 51, 89 (1963).
[27] K. Zyczkowski et al., Phys. Rev. A 65, 012101 (2001); C. Simon and J. Kempe, Phys. Rev. A 65, 052327 (2002); P.J. Dodd and J.J. Halliwell, Phys. Rev. A 69, 052105 (2004); T. Yu and J. H. Eberly, Phys. Rev. Lett. 93, 140404 (2004).
[28] M. P. Almeida et al., Science 316, 579 (2007); J. Laurat et al., Phys. Rev. Lett. 99, 180504 (2007); A. Salles et al., Phys. Rev. A 78, 022322 (2008).
[29] J. Dalibard, Y. Castin, and K. Molmer, Phys. Rev. Lett. 68, 580 (1992).
[30] A. Serafini, S. Mancini, and S. Bose, Phys. Rev. Lett. 96, 010523 (2006).
[31] M. Eibl et al., Phys. Rev. Lett. 92, 077901 (2004); N. Kiesel et al., Phys. Rev. Lett. 98, 063604 (2007); S.B. Papp et al., Nature 324, 764 (2009).
[32] C. Sabin and G. Garcia-Alcaine, Eur. Phys. J. D 48, 435 (2008).
[33] A. Acín et al., Phys. Rev. Lett. 87, 040401 (2001).
[34] W. Dürr, G. Vidal, and J. I. Cirac, Phys. Rev. A 62, 062314 (2000).
[35] O. Gühne and M. Seevinck, arXiv:0905.1349.
[36] G. Vidal and R. F. Werner, Phys. Rev. A 65, 032314 (2002).
[37] M. Tavis and F. W. Cummings, Phys. Rev. 188, 692 (1969).
[38] A. O. Pittenger and M.H. Rubin, Opt. Commun. 179, 447 (2000).
[39] B. Kraus, PhD thesis available on line as MPQ282.
[40] M. Ge, L.-F. Zhu, and L. Qiu, Commun. Theor. Phys. 49, 320 (2008).
[41] M. Boozer et al., Phys. Rev. Lett. 90, 167903 (2003); T. Aoki et al., Phys. Rev. Lett. 91, 080404 (2003); A. Allevi et al., Phys. Rev. A 78, 063801 (2008).
[42] M. Bina et al., in preparation.