Rethinking: Deep-learning-based Demodulation and Decoding

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Abstract—Over the past few decades, the information theory community has worked to develop modulation and encoding that achieve the Shannon capacity in the constraint of the low implementation complexity. In this paper, we focus on the demodulation/decoding of the complex modulations/codes that approach the Shannon capacity. Theoretically, the maximum likelihood (ML) algorithm can achieve the optimal error performance whereas it has $O(2^k)$ demodulation/decoding complexity with $k$ denoting the number of information bits. Recent progress in deep learning provides a new direction to tackle the demodulation and the decoding. The purpose of this paper is to analyze the feasibility of the neural network to demodulate/decode the complex modulations/codes close to the Shannon capacity and characterize the error performance and the complexity of the neural network. Regarding the neural network demodulator, we use the golden angle modulation (GAM), a promising modulation format that can offer the Shannon capacity approaching performance, to evaluate the demodulator. It is observed that the neural network demodulator can get a close performance to the ML-based method while it suffers from the lower complexity order in the low-order GAM. Regarding the neural network decoder, we use the Gaussian codebook, achieving the Shannon capacity, to evaluate the decoder. We also observe that the neural network decoder achieves the error performance close to the ML decoder with a much lower complexity order in the small Gaussian codebook. Limited by the current training resources, we cannot evaluate the performance of the high-order modulation and the long codeword. But, based on the results of the low-order GAM and the small Gaussian codebook, we boldly give our conjecture: the neural network demodulator/decoder is a strong candidate approach for demodulating/decoding the complex modulations/codes close to the Shannon capacity owing to the error performance of the near-ML algorithm and the lower complexity.

Index Terms—Deep learning, demodulation, decoding, Shannon capacity

I. INTRODUCTION

In 1948, Shannon has provided the mathematical theory of communication [1]. Guided by the theory of Shannon, the researchers have tried their best to develop communication techniques that meet the Shannon limit in the following decades.

Various modulation formats have been developed and applied, such as pulse amplitude modulation (PAM), phase shift keying (PSK), quadrature amplitude modulation (QAM), amplitude-PSK (APSK), star-QAM, etc. Among them, QAM is the most well-known modulation scheme and is deployed in various communication systems. However, there is an asymptotic shaping-loss of $\pi e/6 \approx 1.53$ dB in additive Gaussian noise channel according to Shannon-Hartley theorem [2]. To overcome the shaping-loss, probabilistic-shaping and geometric-shaping techniques are studied to design novel modulation schemes. They aim at optimizing the location and probability of occurrence of constellation points to achieve complex Gaussian distribution. Examples of some early works are trellis shaping [3], nonuniform-QAM [4], and asymmetric constellations [5]. The recent work in [6] proposes the golden angle modulation (GAM) that asymptotically approaches the Shannon capacity as the number of signal constellation points grows. Meanwhile, the biggest advance in coding theory is the discovery of low-density parity-check (LDPC) codes [7], turbo codes [8] and polar codes [9], where the polar codes asymptotically achieve the Shannon limit.

Regarding the demodulation and the decoding, the maximum likelihood (ML) algorithm is universal and optimal for the demodulation and the decoding in theory. However, the ML-based demodulation/decoding is impractical with the exponential complexity. Thus, it is attractive for researchers to design a new demodulation/decoding algorithm that approaches the error performance of the ML algorithm with the lower complexity.

In demodulation, a generalized bit level demodulation scheme for M-ary QAM systems is proposed in [10], which significantly reduces the complexity and has almost the same bit error rate (BER) performance as the ML algorithm. The authors in [11] propose the deep convolutional neural network to demodulate the Rayleigh-faded signal and the results show the deep convolutional neural network has a lower bit error probability compared to other demodulators such as the support vector machine. In [12], the authors propose the deep-learning-based demodulator in short-range multipath channels, where the deep belief network and the stacked autoencoder are applied to their demodulation system. In [13], the mixed neural network using the convolutional neural network and recurrent neural network is proposed to demodulate the received signal. The authors in [14] propose a demodulator based on the convolutional neural network with variable input and output length. The hard bit information is used to train the convolutional neural network and the log probability ratio based on the output layer of the trained network is proposed to realize the soft demodulation.

In decoding, Gallager proposes the belief propagation for the LDPC code decoding, which is an iterative soft-decoding algorithm [15]. In [16], the authors propose an iterative turbo decoder including two soft-input soft-output decoders. In [9], the successive cancellation algorithm is designed for the polar code decoding. In addition to the above decoding algorithms for specific codes, the universal decoding algorithms also have been widely studied. In [17], the authors propose a universal and near-ML decoder based on the reordering of the received
symbols according to their reliability measure. However, the decoder involves the Gauss-Jordan elimination and thus suffers from the high computation complexity. In [18], the authors introduce a universal decoder that rank-orders noise sequences from most likely to least likely. Further, the decoder in [18] can realize the ML decoding for arbitrary codebooks in discrete channels with or without memory. Owing to the powerful fitting ability of the deep neural network, it shows a good performance on the decoding. The authors in [19] propose the deep learning method for improving the belief propagation (BP) algorithm by assigning weights to the edges of the Tanner graph, which can achieve an error performance close to the traditional BP decoders with fewer iterations. In [20], the recurrent neural decoder architecture based on the method of successive relaxation is proposed for improving the error performance and reducing the computational complexity. The authors in [21] view the decoding as the classification problem and train the weights of the neural network decoder using the generated dataset that contains a large number of codewords. It is observed in [21] that structured codes such as the polar codes are easier to learn than the random codes and the neural networks are difficult to train for long codes.

Despite a variety of schemes on the deep-learning-based demodulation and decoding, they never analyze the feasibility of using the neural network to demodulate/decode the complex modulations/codes close to the Shannon capacity. In fact, modulations/codes that are closer to the Shannon capacity generally have the higher demodulation/decoding complexity. In this paper, we use the GAM and the Gaussian codebook to evaluate the neural network demodulator and decoder, respectively, in terms of the error performance and the complexity. The GAM can overcome the asymptotic shaping loss seen in QAM and offers the Shannon capacity approaching performance [6]. Gaussian codebooks have often been used to prove direct coding theorems for the Gaussian channel [22], [23]. We boldly give our conjecture: the neural network demodulator/decoder is a strong candidate approach for demodulating/decoding the complex modulations/codes close to the Shannon capacity owing to the error performance of the near-ML algorithm and the lower complexity order. The conjecture is based on the following fact/vision: 1) For the low-order GAM and the small Gaussian codebook, the neural network demodulator/decoder obtains the good performance, i.e., the error performance close to the ML algorithm and the lower complexity order; 2) The rapid development of computing resources makes it possible for the deep neural network to train the high-order modulation and the long codeword.

II. SYSTEM MODEL

A. Modulation-Demodulation Framework

Consider the modulation-demodulation model, in which the transmitted signal is subjected to additive white Gaussian noise. The received symbol is formulated as

\[ y_1 = x_1 + \nu_1 \]  

where \( y_1 \in \mathbb{C}^{n_1} \) is the received symbol; \( \nu_1 \in \mathbb{C}^{n_1} \) is the noise, which follows independent identically distributed zero-mean circularly symmetric complex Gaussian (CSCG) distribution with variance \( \sigma_1^2 \); \( x_1 \in \mathbb{C}^{n_1} \) is the transmitted symbol obtained by modulating the information bit \( b_1 \in \{0,1\}^{k_1} \). In this paper, we assume that the GAM is used for the transmitter modulator. The \( m \)th constellation point of GAM can be denoted as

\[ s_m = a_m e^{2 \pi i \theta_m}, \quad m \in I_M \]  

where \( M = 2^{k_1} \) is the modulation order; \( a_m \) is the radius of \( m \)th constellation symbol; \( \theta = 1 - (\sqrt{5} - 1)/2; 2 \pi \theta \approx 137.5^\circ \) is the golden angle in radians; \( I_M = \{1,2,\ldots,M\} \) is defined as a shorthand of the index set. It’s noted that \( a_{m+1} > a_m \) in golden angle modulation and the later constellation point turns a fixed angle relative to the previous constellation point. Hence, the shape of the constellation is like an increasing spiral. Every constellation point has a unique index and the number of constellation points of GAM can be increased arbitrarily, which makes the design of GAM constellation more flexible than QAM. In this paper, we use disc-GAM and its distribution of constellations point is disc-shape (see [6] in detail). The amplitude of the \( m \)th constellation point can be expressed as

\[ a_m = \sqrt{2 P_1 m / (M + 1)}, \quad m \in I_M \]  

where \( P_1 \) is the average power constraint. An example of disc-GAM is illustrated in Fig. 1.

The receiver gets the \( y_1 \) and demodulates it as \( \hat{b}_1 \) using the demodulator, i.e.,

\[ \hat{b}_1 = \mathcal{G}_1(y_1) \]  

where \( \mathcal{G}_1 \) denotes the demodulator at the receiver.

B. Coding-Decoding Framework

We suppose that the information bit \( b_2 \in \{0,1\}^{k_2} \) is mapped into the code sequence \( x_2 \in \mathbb{R}^{n_2} \), where each element is selected at random independent identically distributed according to \( \mathcal{N}(0,P_2) \). Here, \( P_2 \) is the average power constraint.
and the generated Gaussian codebook is known by both the
transmitter and the receiver. Then, the code sequence \( \mathbf{x}_2 \) is
transmitted over a channel that is subjected to additive white
Gaussian noise. The received signal can be written as
\[
y_2 = \mathbf{x}_2 + \mathbf{v}_2
\]
where \( y_2 \) is the channel output and \( \mathbf{v}_2 \) is the zero-mean
Gaussian noise with variance \( \sigma_2^2 \), i.e., \( \mathbf{v}_2 \sim \mathcal{N}(0, \sigma_2^2 \mathbf{I}_{n_2}) \).

The receiver gets the \( y_2 \) and decodes it as \( \hat{b}_2 \) using the
decoder, i.e.,
\[
\hat{b}_2 = \mathcal{G}_2(y_2)
\]
where \( \mathcal{G}_2 \) denotes the decoder at the receiver.

III. DEEP-LEARNING-BASED DECODER

In this section, we firstly introduce the ML algorithm, which is
used for the baseline of evaluating the neural network demodulator/decoder.
Then, the neural network demodulator/decoder is introduced in detail.
Essentially, the demodulator and the decoder have the same mathematical properties,
and recovering the received signal to bits. For convenience,
we use \( y \) to represent \( \mathbf{y}_1 \) and \( y_2 \), and similar operations also
are applied to \( x_1, x_2, b_1, b_2, b_1, b_2, k_1, k_2, n_1 \) and \( n_2 \).

A. ML Demodulator/Decoder

An optimal and universal demodulation/decoding algorithm
is named the ML algorithm, which is formulated as
\[
\hat{b} = \arg \max_{b \in \mathcal{B}} p(y | b)
\]
where \( \mathcal{B} \) is the set of all possible cases. As shown in Section
II, the \( M \)-order GAM consists of \( 2^k \) possible symbols, in
which \( \mathcal{B} \) denotes the set of all possible symbols. Meanwhile,
the generated codebook contains \( 2^{k_2} \) codewords, in which \( \mathcal{B} \)
denotes the set of all possible codewords. The ML algorithm
compared the received noisy signal \( y \) with each of the set \( \mathcal{B} \)
and picks the one closest to \( y \). Although the ML algorithm is
the optimal, obviously, it suffers from the \( \mathcal{O}(2^k) \) complexity.
Thus, the ML algorithm is impractical in the communication system especially when the length of the information \( b \) is large.

B. Neural Network Demodulator/Decoder

In this paper, we design a simple and universal full-connected feedforward neural network for demodulation/decoding, which consists of \( L \) hidden layers. In a more mathematical way, the neural network demodulator/decoder can be represented as
\[
\hat{b} = f(y; \theta) = f^{(L)} \left( \left[ f^{(L-1)} \left( \left[ f^{(L-2)} \left( \cdots \left( f^{(1)}(y) \right) \right] \right. \right] \right) \right)
\]
where \( \theta \) is the optimal parameters of the neural network and
the mapping function of the layer \( i \) with the weight \( w_i \in \mathbb{R}^{n_i \times n_{i-1}} \) and the bias \( \mathbf{e}_i \in \mathbb{R}^{n_i} \) is denoted as \( f^{(i)} : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i} \), i.e.,
\[
f^{(i)}(\mathbf{d}_i) = g(\mathbf{w}_i \mathbf{d}_i + \mathbf{e}_i)
\]
where \( \mathbf{d}_i \in \mathbb{R}^{n_i} \) is the input of the \( i \)-th layer and \( g(\cdot) \) is the
activation function. The number of the neuron of the input layer
and the output layer are depended on the length of the channel output \( y \) and the information bit \( b \), respectively. Theoretically,
the neural network can approximate any continuous function
owing to the nonlinear activation functions when the number
of neurons is large enough.

To obtain the optimal \( \theta \), the neural network needs to be
properly trained. The training set contains the received signal
sample \( y \) and the information bit \( b \), which can be denoted as
\[
\mathcal{D} = \{(y, b) \}_i^J
\]
where \( J \) denotes the number of samples in the training set.
The training of the neural network uses the mean squared error
(MSE) as the loss function, which is defined as
\[
\mathcal{L} = \mathbb{E}(\mathbf{b} - \hat{\mathbf{b}})^2
\]
where \( \hat{\mathbf{b}} \) is the output of the neural network and \( \mathbb{E}(\cdot) \) is the operator
denoting the mean. Once the optimal \( \theta \) is obtained,
the neural network demodulator/decoder can used for the
demodulation/decoding of the received noisy signal in the
testing phase by using (8).

IV. PERFORMANCE ANALYSIS

In this section, we evaluate the performance of the neural
demodulator/decoder including the error performance and the
complexity. For the convenience of representation, the ‘NN’
and ‘ML’ are used to represent the neural network demodulator/decoder and the maximum likelihood demodulator/decoder,
respectively. Furthermore, the signal to noise ratio (SNR)
is measured by the \( E_b/N_0 \), where \( E_b \) and \( N_0 \) denote the
energy of a bit and the power spectral density of the noise,
respectively.

A. Parameter Setting

For the demodulation, the 4GAM, 16GAM, 64GAM, and
256GAM are used for the performance evaluation, i.e., \( k_1 = 2, 4, 6, 8 \). The oversampling factor is set as 10, i.e., a symbol
contains \( n_1 = 10 \) samples. The transmitted signal is normalized,
i.e., the average power \( P_1 = 1 \). For the decoding, the
codeword with the size \( k_2 = 2, 4, 6, 8 \) are used for performance evaluation. The code rate is 0.5, i.e., \( k_2/n_2 = 0.5 \). The
code sequence is normalized, i.e., the average power \( P_2 = 1 \).

We use a very large neural network as the architecture of
the neural network demodulator/decoder that consists of four
hidden layers and the number of the neuron corresponding to
each layer are 1024, 512, 256, 128, respectively. The size of
the training and the testing for each codeword/symbol index are \( 2^{18} \) and \( 2^{17} \), respectively. The batch size is 256, the ReLu
is used for the activation function and we use the Adam as the
optimizer.

B. Numerical Results

Observation 1: The neural network demodulator achieves the
error performance close to the ML demodulator when the
modulation order is low. (cf. Fig. 2)
Conjecture: The neural network demodulator/decoder is a strong candidate approach for demodulating/decoding the complex modulations/codes close to the Shannon capacity owing to the error performance of the near-ML algorithm and the lower complexity order.

In the observation 1, 2, 3, and 4, we can conclude that the NN demodulator/decoder has much lower complexity order while achieving error performance close to the ML algorithm in the case of the low-order modulation and the NN demodulator does not grow so fast. The trend of time computation is consistent with the complexity evaluation. Thus, we conclude that the NN demodulator has a lower complexity order compared with the ML demodulator.

Observation 3: The neural network decoder achieves the error performance close to the ML decoder when the codeword is short. (cf. Figs. 3 and 4)

In Fig. 3, we plot the BER curve of the NN and the ML decoder for $k_2 = 2, 4, 6, 8$. It is observed that the NN decoder can achieve the near-ML performance especially when the codeword is short. Furthermore, we give the BER of the NN and ML decoder when $k_2 = 8$ and the size of the training samples is $2^{10}, 2^{14}, 2^{18}$. It can be seen that the large training size can improve the error performance of the NN decoder and the near-ML performance is achieved when the training size is $2^{18}$. Thus, we conclude that the NN decoder trained with a sufficiently large number of samples can achieve the near-ML performance for the given large-dimensional neural network.

Observation 4: The neural network decoder has a lower complexity order compared with the ML decoder. (cf. Table II)

Table II illustrates the computational complexity of the decoding for the NN and ML decoders with different codeword sizes. Obviously, the computational complexity order of the NN decoder is significantly less than that of the ML decoder when decoding the same codeword. Furthermore, we also see that as the length of the codeword grows, the time consumption of the ML decoder grows much faster than the NN decoder. Thus, we can conclude that the NN decoder has a lower complexity order compared with the ML decoder.

In Fig. 2 the BER performance of the NN demodulator and the ML demodulator are evaluated for the 4GAM, 16GAM, 64GAM, and 256GAM. Obviously, the performance of the NN demodulator is close to the ML demodulator, especially for low-order modulation. This is because that the set $B$ for the low-order modulation contains fewer possible symbols than the high-order modulation. As a result, the mapping function of the GAM for the low-order modulation can be learned more easily by the neural network.

Observation 2: The neural network demodulator has a lower complexity order compared with the ML demodulator. (cf. Table II)

In Table II the computational complexity of the NN demodulator and the ML demodulator are evaluated for the 4GAM, 16GAM, 64GAM, and 256GAM. We can see that the complexity order of the NN demodulator is lower than the ML demodulator. Furthermore, Table II also explicitly provides numerical results of time consumption of each implementation. It is observed that the computational time of the ML demodulator increases rapidly with $k_1$ while that of the NN demodulator is close to the ML demodulator, especially for low-order modulation.

Observation 4: The neural network decoder has a lower complexity order compared with the ML decoder. (cf. Table II)

Table II illustrates the computational complexity of the decoding for the NN and ML decoders with different codeword sizes. Obviously, the computational complexity order of the NN decoder is significantly less than that of the ML decoder when decoding the same codeword. Furthermore, we also see that as the length of the codeword grows, the time consumption of the ML decoder grows much faster than the NN decoder. Thus, we can conclude that the NN decoder has a lower complexity order compared with the ML decoder.
short codeword. Limited by the current training resources, we cannot evaluate the performance of the high-order modulation and the long codeword such as $k_1 = 10000$ and $k_2 = 10000$. But, with the development of offline resources such as the graphical processing units (GPUs), the performance evaluation of the high-order modulation and the long codeword will be implemented. In fact, the Shannon capacity can be obtained when the constellation point or the code sequence is Gaussian distributed and the modulation order or the length of the codeword is infinite. Here, based on the results of the low-order GAM and the small Gaussian codebook, we boldly give our conjecture: the neural network demodulator/decoder is a strong candidate approach for demodulating/decoding the complex modulations/codes close to the Shannon capacity owing to the error performance of the near-ML algorithm and the lower complexity order.

V. CONCLUSION
In this paper, the feasibility of using the neural network to demodulate/decode the complex modulations/codes close to the Shannon capacity is analyzed. We evaluate the performance of the neural network demodulator/decoder in the case of the low-order GAM and the small Gaussian codebook. The results show that the NN demodulator/decoder has much lower complexity order while achieving error performance close to the ML algorithm. Based on the promising results of the low-order GAM and the small Gaussian codebook, we also give our conjecture: the neural network demodulator/decoder is a strong candidate approach for demodulating/decoding the complex modulations/codes close to the Shannon capacity owing to the error performance of the near-ML algorithm and the lower complexity order.

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