The Dark Force: Astrophysical Repulsion from Dark Energy

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(Dated: January 27, 2015)

Dark energy (i.e., a cosmological constant) leads, in the Newtonian approximation, to a repulsive force which grows linearly with distance. We discuss possible astrophysical effects of this “dark” force. For example, the dark force overcomes the gravitational attraction from an object (e.g., dwarf galaxy) of mass $10^7 M_\odot$ at a distance of 23 kpc. It seems possible that observable velocities of bound satellites (rotation curves) could be significantly affected, and therefore used to measure the dark energy density.

I. INTRODUCTION

The discovery of dark energy, which accounts for the majority of the energy in the universe, is one of the most significant of the last 20 years. While the repulsive properties of dark energy are well known in the cosmological context, they have not been as thoroughly understood on shorter, astrophysical, length scales. Previous work has constrained the cosmological constant on solar-system scales [1], but its effects are obviously too small to be directly observed. We discuss the repulsive “dark” force, and its astrophysical effects on galactic scales.

II. NEWTONIAN GRAVITY AND COSMOLOGICAL CONSTANT

The Einstein equation with cosmological constant $\Lambda$ is

$$ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8 \pi G T_{\mu\nu} + g_{\mu\nu} \Lambda. \hspace{1cm} (1) $$

Contracting both sides with $g^{\mu\nu}$, one gets

$$ R_{\mu\nu} = 8 \pi G \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) - g_{\mu\nu} \Lambda. \hspace{1cm} (2) $$

In the Newtonian limit, one can decompose the metric tensor as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$. Specifically, we are interested in the 00-component of the Einstein equation. We parameterize the 00th-component of the metric tensor as

$$ g_{00} = 1 + 2 \Phi, \hspace{1cm} (3) $$

where $\Phi$ is the Newtonian gravitational potential. To leading order, one can show that [2]

$$ R_{00} \approx \frac{1}{2 \nabla^2 g_{00}} = \nabla^2 \Phi. \hspace{1cm} (4) $$

In the inertial frame of a perfect fluid, its 4-velocity is given by $u_\mu = (1, 0)$ and we have

$$ T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu} = \text{diag}(\rho, p), \hspace{1cm} (5) $$

where $\rho$ is the energy density and $p$ is the pressure. For a Newtonian (non-relativistic) fluid, the pressure is negligible compared to the energy density, and hence $T \approx T_{00} = \rho$. As a result, in the Newtonian limit, the 00-component of the Einstein equation reduces to

$$ \nabla^2 \Phi = 4 \pi G \rho - \Lambda, \hspace{1cm} (6) $$

which is just the modified Poisson equation for Newtonian gravity, including cosmological constant. This equation can also be derived from the Poisson equation of Newtonian gravity, $\nabla^2 \Phi = 4 \pi G (\rho + 3p)$, with source terms from matter and dark energy; $p \approx 0$ for non-relativistic matter, and $p = -\rho$ for a cosmological constant.

Assuming spherical symmetry, we have $\nabla^2 \Phi = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Phi}{\partial r} \right)$ and the Poisson equation is easily solved to obtain

$$ \Phi = - \frac{GM}{r} - \frac{\Lambda}{6} r^2, \hspace{1cm} (7) $$

where $M$ is the total mass enclosed by the volume $\frac{4}{3} \pi r^3$. The corresponding gravitational field strength is given by

$$ g = - \nabla \Phi = \left( - \frac{GM}{r^2} + \frac{\Lambda}{3} r \right) \hat{r}. \hspace{1cm} (8) $$

Therefore, the cosmological constant leads to a repulsive force whose strength grows linearly with $r$.

One can also derive $g$ by starting with the de Sitter-Schwarzschild metric [3]

$$ ds^2 = \left( 1 - \frac{2GM}{r} - \frac{\Lambda}{3} r^2 \right) dt^2 - \left( 1 - \frac{2GM}{r} - \frac{\Lambda}{3} r^2 \right)^{-1} dr^2 - r^2 d\Omega^2 \hspace{1cm} (9) $$

which describes the spacetime outside a spherically symmetric mass distribution $M$ in the presence of a cosmological constant $\Lambda$. One then obtains Eq. (7) and hence Eq. (8) by identifying Eq. (3) with the 00-component of the de Sitter-Schwarzschild metric.

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III. GALAXIES

The results obtained in previous section are relevant for galaxies. For instance, in the presence of the cosmological constant $\Lambda$, Eq. (8) describes the Newtonian gravitational field strength outside a galaxy with a spherically symmetric mass distribution $M$. From Eq. (5), it is clear that when $r$ is sufficiently large, the repulsive dark force will dominate over the gravitational attraction. The critical value of $r$ beyond which this happens is given by

$$r_c = \left( \frac{3 GM}{\Lambda} \right)^{1/3} = \left( \frac{3 M}{8 \pi \rho_\Lambda} \right)^{1/3},$$  

where $\rho_\Lambda = \frac{\Lambda}{8 \pi G} \approx (2.3 \times 10^{-3} \text{eV})^4$ is the observed energy density of the cosmological constant. Table I displays galactic masses in units of solar mass $M_\odot$ and the corresponding $r_c$.

Typical galaxies, including our Milky Way, have total mass (including dark matter) $\gtrsim 10^{11-12} M_\odot$ and sizes $\sim 50\text{kpc}$. According to Table I, $r_c \gtrsim 500\text{kpc}$ for these galaxies and so the dark force is not likely to affect internal dynamics, but may impact galaxy-galaxy interactions, and limit the size of galaxy clusters ($\sim 10^{14} M_\odot$, size $\sim \text{Mpc}$).

Some dwarf galaxies have total mass (including dark matter) $\lesssim 10^7 M_\odot$. These include Ursa Major II, Coma Berenices, Leo IV, Leo T, Canes Venatici I, Canes Venatici II, and Hercules (analyzed by [4]), and also Leo II [5] and Leo V [6]. The irregular galaxy Leo A also has a mass around $10^7 M_\odot$. For galaxies with mass $\lesssim 10^7 M_\odot$, we have $r_c \lesssim 23\text{kpc}$. Thus, their galactic rotation curves could be affected by the dark force: rotational velocities of stars or gas clouds bound to these galaxies should be smaller than that predicted by ordinary Newtonian gravity. This in turn could provide a novel way to measure the cosmological constant in the near future. Rotation curves for many galaxies have been measured to radii of $\sim 30\text{kpc}$ or more, and for some dwarf galaxies to $\sim 10\text{kpc}$ [8].

Low surface brightness (LSB) galaxies may also be worthy of investigation [9]. Some LSBs with total mass $\sim 10^{10} M_\odot$ have disks as large as 100 kpc.

In the event that dark energy is dynamical, as opposed to a rigid cosmological constant, it might form inhomogeneous clumps on galactic length scales. This behavior could, in principle, be detectable through the effects discussed here: A from astrophysical measurements would be larger than the known cosmological value.

IV. MISSING SATELLITE PROBLEM

Observations indicate fewer satellite galaxies than predicted by numerical simulations involving cold dark matter. This is known as the missing satellite problem [10]. For instance, simulations predict a few hundred satellite galaxies within a Mpc radius of the Local Group, but we have observed at least five times fewer.

For galaxies of mass $\sim 10^{11-12} M_\odot$, we have $r_c \gtrsim 500\text{kpc}$, suggesting that the dark force may play a limiting role in the binding of satellite galaxies.

Acknowledgements. We thank Robert Scherrer, James Schombert, Brian O’Shea and Jay Strader for useful conversations. This work was supported by the Office of the Vice-President for Research and Graduate Studies at Michigan State University.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Galaxy Mass & $r_c$ \\
\hline
$10^9 M_\odot$ & 11 kpc \\
$10^9 M_\odot$ & 23 kpc \\
$10^9 M_\odot$ & 50 kpc \\
$10^{10} M_\odot$ & 107 kpc \\
$10^{11} M_\odot$ & 234 kpc \\
$10^{12} M_\odot$ & 498 kpc \\
$10^{13} M_\odot$ & 1.07 Mpc \\
$10^{14} M_\odot$ & 2.31 Mpc \\
$10^{15} M_\odot$ & 4.98 Mpc \\
\hline
\end{tabular}
\caption{Galaxy masses (units of solar mass $M_\odot$) and the corresponding $r_c$.}
\end{table}

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