Mid-infrared Period–Luminosity Relations for Miras in the Large Magellanic Cloud

Patryk Iwanek®, Igor Soszyński®, and Szymon Kozłowski®
Astronomical Observatory, University of Warsaw, Al. Ujazdowskie 4, 00-478 Warsaw, Poland; piwanek@astrouw.edu.pl
Received 2020 December 21; revised 2021 June 22; accepted 2021 June 25; published 2021 September 29

Abstract

We present the mid-infrared (mid-IR) period–luminosity relations (PLRs) using over 1000 Mira variables in the Large Magellanic Cloud, for the four Wide-field Infrared Survey Explorer and the four Spitzer bands. These PLRs cover a mid-IR wavelength range from 3.4–22 μm and are presented separately for the oxygen-rich (O-rich) and carbon-rich (C-rich) Miras. These relations can be used to measure distances to individual O-rich and/or C-rich Mira stars with an accuracy of 5% and 12%, respectively. They are the most accurate Mira PLRs in the mid-IR to date.

Unified Astronomy Thesaurus concepts: Mira variable stars (1066); Long period variable stars (935); Pulsating variable stars (1307); Large Magellanic Cloud (903); Distance indicators (394); Stellar distance (1595); Asymptotic giant branch stars (2100); Carbon stars (199); Late-type giant stars (908)

Supporting material: machine-readable table

1. Introduction

Mira variables are fundamental-mode pulsators with periods ranging from ~100 to over 1000 days. They belong to a class of late-type, low- or intermediate-mass pulsating red giants also known as long-period variables (LPVs). Miras are asymptotic giant branch (AGB) tracers of old- and intermediate-age populations (Iben & Renzini 1983; Whitelock 2012). Like other AGB stars, they can be divided into oxygen-rich (O-rich) and carbon-rich (C-rich) giants, depending on their surface composition (Riebel et al. 2010). Miras, thanks to their large brightness variations, can be easily found in the Milky Way and other Local Group galaxies, and they are an extensively studied subclass of LPVs.

Since 1912, radially pulsating stars have been widely used as Galactic and extragalactic distance indicators thanks to the discovery of the period–luminosity relation (PLR; Leavitt & Pickering 1912) of classical Cepheids. The history of research on PLRs for Miras dates back to the 1920s and began when Gerasimović (1928) noticed that Miras with longer periods are on average fainter at optical wavelengths. This result has been confirmed by, e.g., Wilson & Merrill (1942).

Glass & Lloyd Evans (1981) were the first to report the existence of a PLR for Miras at near-infrared (NIR) wavelengths. Their research was based on 11 Miras from the Large Magellanic Cloud (LMC), and it appeared that the PLR for Miras in the NIR was much tighter than in the optical passbands. This NIR PLR was redefined by Feast et al. (1989) and Hughes & Wood (1990).

The availability of long-term photometric data for millions of stars from large-scale sky surveys such as MACHO (Alcock et al. 2000), EROS (Expérience pour la Recherche d’Objets Sombres; Ansari 1996), and OGLE (Optical Gravitational Lensing Experiment; Udalski et al. 2015) allowed an in-depth study of the PLRs of LPVs. Based on MACHO data, Wood et al. (1999) and then Wood (2000) showed five distinct, parallel sequences in the period–luminosity (PL) plane. It turned out that one of these sequences was composed of Miras (sequence C in Figure 1 in Wood 2000).

The research on pulsating red giants conducted by OGLE team members has significantly expanded our knowledge about PLRs for such stars. For instance, Soszyński et al. (2004) showed that OGLE small-amplitude red giants (OSARGs) form a completely different set of PL sequences than Miras and semiregular variables (SRVs). In turn, Soszyński et al. (2005) proposed a photometric method to distinguish between O-rich and C-rich AGB stellar populations.

Many modern studies on Mira PLRs (e.g., Whitelock et al. 2006, 2008; Riebel et al. 2010; Ita & Matsunaga 2011; Riebel et al. 2015; Whitelock et al. 2017; Yuan et al. 2017a, 2017b, 2018; Bhardwaj et al. 2019) have shown that these stars can be used as an excellent extension of the cosmic distance ladder. Due to their well-defined PLRs and high luminosity in the IR passbands, which are less affected by interstellar extinction, Miras can be used as an attractive distance indicator (e.g., Matsunaga et al. 2009; Huang et al. 2018; Qin et al. 2018; Molina et al. 2019; Huang et al. 2020; Urago et al. 2020).

Recently, Iwanek et al. (2021) analyzed Mira variability over a wide range of wavelengths, covering 0.1–40 μm. The authors used densely covered, accurate OGLE J-band light curves to create templates and fit them to NIR and mid-infrared (mid-IR) data from multiple sky surveys, extending the study to longer wavelengths (up to 40 μm) by fitting spectral energy distributions (SEDs). As a result, they determined the variability amplitude ratios and phase lags for a range of wavelengths. Additionally, the authors presented synthetic PLRs in the NIR and mid-IR for existing and future sky surveys.

The aim of this paper is to derive mid-IR PLRs for LMC Miras discovered in OGLE-III data (Soszyński et al. 2009), using Wide-field Infrared Survey Explorer (WISE; Wright et al. 2010) and Spitzer (Werner et al. 2004) data, the most accurate distance to the LMC (Pietrzyński et al. 2019), and a reddening map of the LMC (Skowron et al. 2021).

Despite many previous works on Mira PLRs, this subject has not been comprehensively studied in the mid-IR. Using the reddening map of the LMC and the distance to the LMC accurate to 1% we provide the most accurate Mira PLRs to date. Additionally, for the first time, we present PLRs separately for O-rich and C-rich Miras, in the WISE W1, W2, W3, and W4 bands.
2. Data

2.1. Sample of Miras

OGLE is one of the largest and the longest lasting time-domain variability sky surveys worldwide. Since 1992, the Galactic bulge (BLG), and later also the Magellanic Clouds (MCs) and the Galactic disk (GD) have been regularly monitored to search for variability. These observations have enabled many scientific discoveries, catalogs of variable stars, and two-band, multi-decade-long time-series photometry.

One of the products of the third phase of the OGLE project (OGLE-III; Udalski 2003) is the catalog of LPVs in the LMC (Soszyński et al. 2009). The authors discovered almost 100,000 LPVs, including OSARGs, SRVs, and Mira-type stars. The two latter subclasses were separated using the $I$-band pulsation amplitude (for Miras, $\Delta I > 0.8$ mag). Using color–color ($(V-I)$ versus $(J-K)$) and Wesenheit ($W_I$ versus $W_{JK}$) diagrams, Soszyński et al. (2009) divided Miras into O-rich and C-rich stars. This division was evaluated and confirmed by Ita & Matsunaga (2011). In the original catalog, Soszyński et al. (2009) published two-band ($I$ and $V$ in the Johnson–Cousins photometric system), 13 yr long light curves covering the time span 1996–2009. The full catalog can be found in online databases through the OGLE webpage.

2.2. Optical Data

In this paper, we used 1663 published LMC Miras (Soszyński et al. 2009) with provided coordinates and surface chemistry classifications (1194 C-rich and 469 O-rich). The OGLE-III phase ended in 2009, and at the beginning of 2010 the OGLE project restarted its observations as the fourth phase (OGLE-IV). A detailed description of the OGLE-IV phase, instrumentation, calibration, data reduction, observational cadence, and the sky coverage can be found in Udalski et al. (2015).

We supplemented publicly available Mira light curves with the non-public OGLE-IV data collected over an additional 10 yr since 2010. We revised their pulsation periods using two-decade-long light curves. To find periods, we used the TATRY code (Schwarzenberg-Czerny 1996). The code employs periodic orthogonal polynomials to fit the data and the analysis of variance (ANOVA) statistic to evaluate the quality of the fit. We visually inspected each light curve, phase folded it with its new pulsation period, and manually removed obvious outlying points. The updated OGLE light curves have on average $\sim$1300 and $\sim$100 data points in the $I$ band and $V$ band, respectively, obtained between 1996 December 29 and 2020 March 15.

2.3. Mid-IR Data

The role of IR observations in studying PLRs and measuring cosmic distances is invaluable because the influence of interstellar dust on stellar light decreases with increasing wavelength. Moreover, Mira variables are known to undergo the mass-loss phenomenon (e.g., Perrin et al. 2020), which may be the cause of the formation of circumstellar dust shells around these stars. The impact of circumstellar shells on the observed stellar radiation, as for interstellar dust, is smaller at longer wavelengths.

We crossmatched our catalog of Mira stars with two space-mission databases containing mid-IR observations: WISE (Wright et al. 2010) and Spitzer (Werner et al. 2004). WISE is a 40 cm diameter IR space telescope that has monitored the entire sky in four bands: W1 (3.4 $\mu$m), W2 (4.6 $\mu$m), W3 (12 $\mu$m), and W4 (22 $\mu$m). In the standard mode, the WISE telescope has observed each sky location every six months, collecting several independent exposures during one flyby. The situation was slightly different in the case of the LMC area, which was observed much more frequently due to the polar trajectory of the telescope. As a result, the light curves of stars located in the LMC were covered with much denser sampling. The main mission of the WISE telescope ended in 2010; however, observations in W1 and W2 have continued since 2011 as part of the Near Earth Object WISE Reactivation Mission (NEOWISE-R; Mainzer et al. 2011, 2014). For this analysis, we retrieved the data from the AllWISE MultiePOCH Photometry Table, and NEOWISE-R Single Exposure (L1b) Source Table using the NASA/IPAC Infrared Science Archive.

The Spitzer Space Telescope is an 85 cm diameter telescope with three IR instruments on board: Infrared Array Camera (IRAC), Infrared Spectrograph (IRS), and Multiband Imaging Photometry for Spitzer (MIPS). The LMC was observed by Spitzer in the Surveying the Agents of Galaxy’s Evolution (SAGE; Meixner et al. 2006) survey using IRAC in four bands: 3.6, 4.5, 5.8, and 8.0 $\mu$m, also called [3.6], [4.5], [5.8], and [8.0], or $I_1$, $I_2$, $I_3$, and $I_4$, respectively. We downloaded all available measurements from the SAGE IRAC Epoch 1 and Epoch 2 Catalog using the NASA/IPAC Infrared Science Archive. Light curves from these databases contain mostly two epochs.

The above-mentioned databases were searched for counterparts within $1''$ around each Mira’s coordinates. For further analysis, we used Spitzer light curves containing two epochs in each IRAC band only. This left us with 1470 light curves (out of 1663) collected from 2005 October to November. The WISE data had to be cleared of significant outliers. After a two-step cleaning procedure, we removed light curves with fewer than 100 data points in the W1 band and W2 band and fewer than three epochs in the W3 band and W4 band from further analyses. Finally, this left us with 1311 Miras observed by the WISE telescope. The median number of data points per light curve was 645, 703, 63, and 22 in the W1, W2, W3, and W4 bands, respectively. The WISE data were collected between 2010 February 8 and 2010 June 17 (AllWISE data), and between 2010 December 13 and 2019 December 1 (NEOWISE-R data). A detailed description of the data extraction and cleaning procedure can be found in Iwanek et al. (2021). In Figure 1, we present the transmission curves of the WISE and Spitzer bands.

The analysis of the SEDs of the LMC Miras performed by Iwanek et al. (2021) showed that photometric measurements in the W4 band (22 $\mu$m) are typically significantly biased. Meanwhile, the OGLE, VISTA survey of the Magellanic Cloud system (VMC), Spitzer, WISE (<20 $\mu$m), and MIPS (i.e., one of the Spitzer instruments that observed the sky at 24 $\mu$m) measurements were well described by either a single (in the case of O-rich Miras) or a double (in the case of C-rich Miras) Planck function; the W4 measurements clearly deviated from the fits. We conducted an analysis of the PLR in the W4

1 http://ogle.astrow.edu.pl

2 https://wise2.ipac.caltech.edu/docs/release/allwise/

3 https://irsa.ipac.caltech.edu/applications/Gator/
band for completeness purposes only, but they should not be used in future analyses.

3. Methods

3.1. Corrections for Interstellar Extinction

Interstellar extinction has a strong influence on stellar light and should be taken into account when measuring accurate PLRs. Although this effect is smaller at longer wavelengths and the amount of dust toward the LMC is much smaller than, for example, toward the BLG, it should not be neglected.

Recently, Skowron et al. (2021) published the most accurate optical reddening maps of the Magellanic Clouds using Red Clump (RC) stars. The reddening \( E(V - I) \) and the upper \( \sigma_{+E(V-I)} \) and lower \( \sigma_{-E(V-I)} \) uncertainties can be obtained for a given right ascension (R.A.) and declination (Decl.) via an online form on the OGLE webpage.\(^4\) Following the authors’ remarks, the extinction \( A_I \) could be calculated as

\[
A_I \approx 1.5 \times E(V - I),
\]

where the coefficient can vary between 1.1 and 1.7, depending on the dust characteristics. Therefore, we used a coefficient equal to 1.5. The analysis of the SEDs of the LMC with an uncertainty equal to 0.2, which is consistent with the coefficient 1.505 for \( A_I \) derived for \( RV = AV/E(B - V) = 3.1 \) by Schlafly & Finkbeiner (2011). Knowing that

\[
E(V - I) = AV - A_I,
\]

the extinction in the \( V \) band is represented by the relation

\[
AV = (2.5 \pm 0.2) \times E(V - I),
\]

with the uncertainty

\[
\sigma_{AV} = 2.5 \times E(V - I) \times \sqrt{\frac{0.2^2}{2.5} + \left( \frac{\sigma_{+E(V-I)} + \sigma_{-E(V-I)}}{2 \times E(V - I)} \right)^2}.
\]

The \( V \)-band extinction could be transformed to extinctions in the WISE and Spitzer bands using the extinction laws derived by Chen et al. (2018) and Wang & Chen (2019). We summarize these transformations in Table 1 for the WISE W1, W2, and W3, and Spitzer [3.6] \( \mu \)m, [4.5] \( \mu \)m, [5.8] \( \mu \)m, and [8.0] \( \mu \)m bands. Considering the uncertainty \( \sigma_{AV} \) (see Equation (4)) and the transformation uncertainties \( \sigma_{AV/AV} \) presented in Table 1, the extinction uncertainties \( \sigma_{A_{\lambda}} \) in a given WISE or Spitzer band can be calculated as

\[
\sigma_{A_{\lambda}} = A_{\lambda} \times AV \times \sqrt{\left( \frac{\sigma_{AV/AV}}{A_{\lambda}/AV} \right)^2 + \left( \frac{\sigma_{AV}}{AV} \right)^2}. \tag{5}
\]

As there is a lack of information in the literature on the extinction transformation from the \( V \) band to \( W4 \) band, we simply assumed that \( A_{W4} \) is equal 0. Iwanek et al. (2021) showed that the median \( A_{[8.0]} \) is equal to 0.007, and is one order of magnitude smaller than extinction in the \( J \) band (1.25 \( \mu \)m). Knowing that the influence of interstellar dust on stellar light decreases with increasing wavelength (see, e.g., Cardelli et al. 1989), the true extinction in the \( W4 \) band toward the LMC must be close to 0.

Moreover, Iwanek et al. (2021) compared the extinction obtained with the method described above with the classical reddening law derived by Cardelli et al. (1989). The authors concluded that the difference between these two methods is less than 10\%, while extinction in the mid-IR is, in general, comparable to a typical photometric uncertainty of a given survey.

The mean values of the extinction uncertainties in the WISE (with an exception of the \( W4 \) band) and Spitzer bands are presented in Table 2. Throughout the paper, all stellar brightnesses and colors are extinction corrected.

3.2. Template Light Curves and Mean Magnitudes

To measure mean magnitudes from sparsely sampled variable mid-IR light curves, we first modeled the high-cadence OGLE I-band data with a semiparametric Gaussian process regression (GPR) model (He et al. 2016). In short, the model consists of a number of independent parts that include the mean magnitude, a low-frequency trend across cycles, a periodic term that models the

![Figure 1. Transmission curves for the WISE and Spitzer bands.](image-url)
main variability, and finally a stochastic term that absorbs any short-term deviations. We then fitted the GPR templates by modifying their amplitudes, and shifting them in time and magnitude to match the mid-IR light curves. Having the best-fitting parameters for the variability amplitude ratios, phase-shifts, and magnitude shifts, we transformed the light-curve templates to the mid-IR data sets of interest. A detailed description of making templates and fitting them to mid-IR data can be found in Iwanek et al. (2021).

We used the transformed templates to measure the mean magnitudes in the WISE and Spitzer bands. Each template light curve was transformed to flux scale, fitted with a third-order truncated Fourier series, integrated to determine the mean brightness, and finally transformed back to the magnitude scale. The uncertainties of the mean magnitudes were determined from a $\chi^2$ surface. In Table 3, we present the basic parameters for each Mira star from our sample (ID, coordinates, type, pulsation period) and measured mean brightnesses with uncertainties in each WISE and Spitzer band.

### Table 3

| Number | R.A. (h: m: s) | Decl. (°: m: s) | type | $P$ (d) | $m_{W1}$ (mag) | $\sigma_{m_{W1}}$ (mag) | ... | $m_{[8.0]}$ (mag) | $\sigma_{m_{[8.0]}}$ (mag) |
|--------|----------------|-----------------|------|---------|----------------|-------------------|-----|----------------|-------------------|
| 00055  | 04:29:49.87    | −70:19:00.7     | C    | 289.78  | 9.315          | 0.018             | ... | −9.999         | −9.999             |
| 00082  | 04:30:44.96    | −69:50:41.0     | O    | 162.33  | −9.999         | −9.999             | ... | 10.959         | 0.061              |
| 00094  | 04:30:54.95    | −69:28:35.5     | C    | 335.87  | 10.048         | 0.017             | ... | 9.585          | 0.019              |
| 00096  | 04:30:59.53    | −69:57:16.1     | C    | 385.50  | 9.495          | 0.022             | ... | 7.884          | 0.035              |
| 00098  | 04:31:03.28    | −69:34:15.3     | C    | 323.23  | 10.064         | 0.009             | ... | 8.606          | 0.023              |
| 00115  | 04:31:27.40    | −70:40:41.4     | C    | 176.04  | −9.999         | −9.999             | ... | −9.999         | −9.999             |
| 00144  | 04:31:54.49    | −68:42:26.3     | C    | 369.67  | 9.879          | 0.014             | ... | 7.974          | 0.144              |
| 00225  | 04:33:17.24    | −68:09:28.8     | C    | 487.44  | 9.539          | 0.063             | ... | 6.908          | 0.061              |
| 00265  | 04:33:43.68    | −70:09:50.5     | C    | 444.04  | 9.885          | 0.041             | ... | 7.198          | 0.044              |
| 91928  | 06:16:49.54    | −70:43:03.7     | C    | 370.72  | 9.892          | 0.018             | ... | −9.999         | −9.999             |

Note. We provide the coordinates, surface chemistry classification, pulsation periods, and mean magnitudes with uncertainties for all WISE and Spitzer bands. Mean magnitudes presented in this table are corrected for interstellar extinction. The full star ID is as in the original catalog (Soszyński et al. 2009) and is composed of OGLE-LMC-LPV-, followed by the number. The table rows are sorted by the star ID. A −9.999 value in the columns with the mean magnitudes means that the star was not found in the mid-IR database, or the star was rejected from the analysis due to reasons described in Section 2. This table is available in its entirety in a machine-readable form in the online journal. A portion is shown here for guidance regarding its form and content.

Figure 2. CMDs (upper row) and period–color diagrams (lower row) for the LMC Miras. CMDs are colored by the pulsation period, while the period–color diagrams are divided into C-rich (red) and O-rich (blue) Miras. The left plots present the NIR color ($J-K$), while the middle and right plots present mid-IR colors, ($W1-W2$) and ([3.6]–[5.8]), respectively.

4. Mid-IR PL and Period–Luminosity–Color Relations for Miras

PLRs for Miras are tighter and better defined at IR wavelengths than in optical bands, as Mira PLRs are generally
flat in the optical (see e.g., Iwanek et al. 2021 for an explanation of this effect) and also blurred by the stronger effects of interstellar and circumstellar extinction, hence making them harder to utilize. Use of the Wesenheit index improves PLRs in optical bands; however, they are still quite broad. Adding a color-term to the relations in optical bands makes them harder to utilize. Use of the Wesenheit index improves the distance to the LMC.

The authors showed that in the optical bands this kink is at \( \log P = 2.3 \), while in the NIR it is at \( \log P = 2.6 \). Bhardwaj et al. (2019) found that this kink shifts to longer periods with increasing wavelength. The authors showed that in the optical bands this kink is at \( \log P = 2.48 \), while in the NIR it is at \( \log P = 2.54 \). These results are consistent with those of Ita & Matsunaga (2011), confirming a kink at even longer periods in the mid-IR bands.

Another approach to using O-rich Miras assumed the construction of PLRs using a quadratic fit. For example, this approach was used by Yuan et al. (2017b) to determine PLRs for fewer than 180 O-rich Miras from the central region of the LMC, using time-series observations in the \( JHK \) bands. The same method was used to construct PLRs for Miras discovered in the M33 galaxy (Yuan et al. 2017a, 2018), and to construct LMC Miras PLRs based on Gaia DR2 data (Gaia Collaboration et al. 2018; Bhardwaj et al. 2019). Other authors proposed a linear fit to both O- and C-rich Miras separately (e.g., Riebel et al. 2010, 2015), or even a single linear fit to both groups of Miras in the LMC (e.g., Urago et al. 2020).

Due to the large scatter of C-rich Mira PLRs, Ita & Matsunaga (2011) decided to analyze O-rich Miras only. They introduced PLRs in the Spitzer mid-IR bands as a combination of two linear fits, with a kink at \( \log P = 2.6 \). Bhardwaj et al. (2019) found that this kink shifts to longer periods with increasing wavelength. The authors showed that in the optical bands this kink is at \( \log P = 2.48 \), while in the NIR it is at \( \log P = 2.54 \). These results are consistent with those of Ita & Matsunaga (2011), confirming a kink at even longer periods in the mid-IR bands.

Another approach to using O-rich Miras assumed the construction of PLRs using a quadratic fit. For example, this approach was used by Yuan et al. (2017b) to determine PLRs for fewer than 180 O-rich Miras from the central region of the LMC, using time-series observations in the \( JHK \) bands. The same method was used to construct PLRs for Miras discovered in the M33 galaxy (Yuan et al. 2017a, 2018), and to construct LMC Miras PLRs based on Gaia DR2 data (Gaia Collaboration et al. 2018; Bhardwaj et al. 2019). Other authors proposed a linear fit to both O- and C-rich Miras separately (e.g., Riebel et al. 2010, 2015), or even a single linear fit to both groups of Miras in the LMC (e.g., Urago et al. 2020).

### Table 4

Mid-IR PLRs for C-rich and O-rich Miras in the WISE and Spitzer Bands

| λ     | \( a_{0,λ} \) | \( a_{0,λ,ab} \) | \( a_{1,λ} \) | \( a_{2,λ} \) | \( σ_λ \) | \( χ^2/dof^a \) | \( N_{in} \) | \( N_{fit} \) | \( N_{red} \) |
|-------|-------------|----------------|-------------|-------------|--------|---------------|-----------|-----------|-----------|
| WISE W1 (Figure 3(a)) | 11.006 ± 0.023 | -7.471 ± 0.033 | -4.198 ± 0.074 | 0.000 | 0.258 | 0.97 | 999 | 982 | 17 |
| WISE W2 (Figure 3(b)) | 10.916 ± 0.030 | -7.561 ± 0.038 | -6.445 ± 0.094 | 0.000 | 0.326 | 0.97 | 998 | 986 | 12 |
| WISE W3 (Figure 3(c)) | 10.545 ± 0.052 | -7.932 ± 0.058 | -9.040 ± 0.164 | 0.000 | 0.584 | 0.99 | 1018 | 1012 | 6 |
| Spitzer [3.6] (Figure 6(a)) | 10.734 ± 0.028 | -7.743 ± 0.037 | -4.147 ± 0.091 | 0.000 | 0.344 | 0.89 | 1072 | 1064 | 8 |
| Spitzer [4.5] (Figure 6(b)) | 10.775 ± 0.033 | -7.702 ± 0.041 | -5.788 ± 0.106 | 0.000 | 0.406 | 0.93 | 1072 | 1066 | 6 |
| Spitzer [5.8] (Figure 6(c)) | 10.789 ± 0.040 | -7.689 ± 0.047 | -7.185 ± 0.130 | 0.000 | 0.494 | 0.76 | 1072 | 1066 | 6 |
| Spitzer [8.0] (Figure 6(d)) | 10.691 ± 0.044 | -7.786 ± 0.050 | -8.329 ± 0.143 | 0.000 | 0.541 | 0.96 | 1072 | 1065 | 7 |

Notes.

a Linear model in a form: \( M_{bol} = a_{0,λ} + a_{1,λ} × (\log P - 2.3) \).

b Quadratic model in a form: \( M_{bol} = a_{0,λ} + a_{1,λ} × (\log P - 2.3) + a_{2,λ} × (\log P - 2.3)^2 \).

c Scatter in the PLR.

d \( χ^2 \) calculated using Equation (9) divided by the degrees of freedom.

e Absolute zero-point with uncertainty calculated using a distance modulus of \( μ = 18.477 \) mag (Pietrzyński et al. 2019), and systematic and statistic uncertainties of the distance to the LMC.

---

The Astrophysical Journal, 919:99 (12pp), 2021 October 1

Iwanek, Osypiński, & Koziolowski
4.1. Fitting Procedure

We used the observed mean magnitudes and pulsating periods presented in Table 3 to create PLRs. Our goal was to fit C-rich and O-rich Miras separately; therefore, we adopted the same division as provided by (Soszyński et al. 2009, which we also present in Table 3). The PLRs for the C-rich Miras were fitted with the linear function

\[ m_{\lambda,\text{fit}} = a_{0,\lambda} + a_{1,\lambda} \times (\log P - 2.3), \]

where \( \lambda \) is any of the WISE and Spitzer bands.

Since the PLRs for O-rich Miras appear to be non-linear, we made both linear and quadratic fits. The quadratic fit was made for the entire range of pulsation periods, and had the form

\[ m_{\lambda,\text{fit}} = a_{0,\lambda} + a_{1,\lambda} \times (\log P - 2.3) + a_{2,\lambda} \times (\log P - 2.3)^2. \]

The O-rich Miras with pulsation periods shorter than \( \sim 400 \) days (i.e., \( \log P = 2.6 \)) follow the linear PLRs (e.g., Ita & Matsunaga 2011). Therefore, we also performed fits in the form presented by Equation (6) for Miras with such pulsation periods.

We fitted models to all O-rich and C-rich Miras using the weighted least squares method, using sigma-clipping procedures, with weights that are represented by the covariance matrix \( \mathcal{C}_{kl} \), and its inverse \( \mathcal{B}_{kl} \equiv C_{kl}^{-1} \) following the notation of Gould (2003)

\[ C_{kl} = \delta_{kl} \times (\sigma_{m,\lambda,k}^2 + \sigma_{m,\lambda,l}^2), \]

\[ \chi^2 = \sum_{k=1}^{N} \sum_{l=1}^{N} (m_{\lambda,k} - m_{\lambda,\text{fit},k}) B_{kl} (m_{\lambda,l} - m_{\lambda,\text{fit},l}). \]

where \( \delta_{kl} \) is the Kronecker delta (\( \delta_{kl} = 1 \) when \( k = l \), or \( \delta_{kl} = 0 \), when \( k \neq l \)). The diagonal components of the \( \mathcal{C}_{kl} \) matrix

![Figure 3. PLRs for C-rich Miras from the LMC in mid-IR WISE bands: (a) W1, (b) W2, (c) W3, and (d) W4. In each case, the left (right) y-axis of the top panel presents the observed (absolute) mean magnitude in a given band \( m_{\lambda} \) as a function of the logarithm of the pulsation period \( P \) (in days), while the bottom panel presents residuals calculated as the difference between the measured and predicted absolute magnitudes from the fit. In the top panels, we present the best fit with the solid line, while dashed lines present the range of \( \pm 3\sigma \), where \( \sigma \) means the dispersion along the fit. The model is given in each plot (with the exception of panel (d) for the W4 band due to the photometric issues in this band) in the form \( m_{\lambda,\text{fit}} = a_{0,\lambda} + a_{1,\lambda} \times (\log P - 2.3) \). The final samples of C-rich Miras are plotted with red points. The errorbars of \( \log P \) are smaller than the point size. Stars rejected during the 3\( \sigma \) clipping procedure are plotted in gray. The final 3\( \sigma \) range, above which there are no outliers, is plotted as a dashed line in the bottom plots. Panel (d) for the W4 band is shown here for completeness.](image-url)
Equation (8) contain the sum of the squares of the uncertainty of the mean magnitude $\sigma_{m,\lambda}$ and the intrinsic PLR scatter $\sigma_{\lambda}$, which is updated iteratively during the sigma-clipping procedure. The off-diagonal elements of the matrix are equal to 0. For each fit, we also calculated $\chi^2$ in the form presented by Equation (9).

The PLRs need to be calibrated using objects with known and well-measured distances. The distance to the LMC was measured with 1\% accuracy by Pietrzyński et al. (2019), who found $d = 49.59 \pm 0.09$ (statistical) $\pm 0.54$ (systematic) kpc (distance modulus $\mu = 18.477$). We calculated the absolute zero-point $a_{0,\lambda,\text{abs}}$ as the difference $a_{0,\lambda} - \mu$, while the uncertainty of the absolute zero-point is the sum of the zero-point $a_{0,\lambda}$ uncertainty resulting from the fit, and both statistical and systematic uncertainties of the distance, all combined in quadrature.

4.2. PLC Relations for C-rich Miras

A strong correlation between colors and pulsation periods of Mira variables exists in the optical and NIR bands. Therefore, PLC relations usually have much lower scatter than PLRs, especially for C-rich Miras (see e.g., Bhardwaj et al. 2019). The PLC relation can be described in the form of the following equation

$$m_{\lambda,\text{fit}} = a_{0,\lambda} + a_{1,\lambda} \times (\log P - 2.3) + c \times (m_{\lambda} - m_{\lambda,\text{abs}}),$$

where $c \times (m_{\lambda} - m_{\lambda,\text{abs}})$ is the color-term. In the case of optical and NIR PLC relations, the coefficient $c$ is statistically significant with a relatively small uncertainty (see Table 2 in Bhardwaj et al. 2019).
We fitted PLC relations (Equation (10)) to the C-rich Miras in the WISE bands using the \((W1−W2)\) color index, while in the Spitzer bands we tried fits with two color indices, i.e., \([3.6]−[4.5]\) and \([3.6]−[5.8]\). In each case, the fit including the color-term resulted in a statistically insignificant \(c\) coefficient—\(c\) was close to zero with a comparable uncertainty. This led us to the conclusion that in the mid-IR, the dependence between the pulsation period and color does not exist, or is very small.

To investigate this topic further, we used magnitudes in NIR bands \((J\text{ and } K)\) extracted from the 2MASS All-Sky Catalog of Point Sources (Cutri et al. 2003). We searched for objects within 1″ radius around each LMC Mira’s coordinates. We found 1656 (out of 1663) Mira counterparts. We corrected the \(J\) and \(K\) magnitudes for interstellar extinction using the same method as described in Section 3.1, using transformations between bands derived by Chen et al. (2018) and Wang & Chen (2019).

In Figure 2, we present color–magnitude diagrams (CMDs, upper row) for all Miras plotted with NIR and mid-IR data mentioned above. The color map in each case presents the pulsation periods of the Miras. In the lower row of Figure 2, we present period–color diagrams using the same color-term as in the CMDs. In the period–color diagrams, we plotted C-rich (red) and O-rich (blue) Miras, separately.

From the CMDs it is clear why the color-term must have an influence on the PLR scatter in the optical and NIR bands and why it is negligible in the mid-IR. For the \(K\) band, we can find very bright and blue Miras with the exact same periods as much fainter and red Miras. This is not the case for the mid-IR data, where the period seems to depend on the brightness only.

In the left plot of the lower panel in Figure 2, we present period–color diagram \((J−K)\) versus \(\log P\), which is commonly used to divide AGB stars into C- and O-rich (Soszyński et al. 2009; Ita & Matsunaga 2011). It is clear that the Miras form
two distinct groups. A similar division is also seen for the mid-IR colors (middle and right plot), but it seems that the overlap of both groups is slightly larger than that in NIR. However, the mid-IR period–color diagram could also be used for the division of AGB stars into C-rich and O-rich groups in other galaxies.

5. Results

Using the PLR models and fitting procedures, described in Section 4.1, we performed fits to the data sets using the weighted least squares method with a $3\sigma$ clipping procedure. We iteratively fitted a linear model in the form of Equation (6) to the O-rich and C-rich Miras, and a quadratic model in the form of Equation (7) to the O-rich Miras only. In each iteration, we rejected points deviating by more than $3\sigma$ from the model, until no outliers were left. The final parameters of the fitted models (mid-IR PLRs), the absolute zero-point $a_0,\lambda_{abs}$ with its uncertainty, are presented in Table 4, along with dispersions $\sigma_3$, $\chi^2$/dof, the number of initial data points $N_{init}$, the final number of data points after rejecting outliers $N_{fin}$, and the number of outliers rejected during the $3\sigma$ clipping procedure $N_{out}$. The PLRs for each band are presented in Figures 3–8 (WISE, C-rich Miras), 4 (WISE, O-rich Miras, linear fit), 5 (WISE, O-rich Miras, quadratic fit), 6 (Spitzer, C-rich Miras), 7 (Spitzer, O-rich Miras, linear fit), and 8 (Spitzer, O-rich Miras, quadratic fit).

As mentioned in Section 2.3, the SEDs fitting showed that photometric measurements in the W4 band are significantly biased. Therefore, in Figures 3–5, we present the PLRs in the W4 band for completeness purposes only. We do not provide the best-fitting parameters in Table 4 and in Figures 3–5 because PLRs based on biased photometric magnitudes lead to biased PLRs, and should not be used in future studies.

The PLRs for the C-rich Miras, both in the WISE and Spitzer bands, have significantly larger dispersions $\sigma_3$ than the ones obtained for the O-rich Mira PLRs. One of the reasons for this is the presence of gas and dust ejecta from these stars that can significantly alter their luminosity (by introducing long-term trends or variations), which in turn can lead to larger dispersions. As a rule of thumb, changes of brightness (the mean and amplitude) of O-rich Miras are reasonably stable over time, so that scatter along PLRs is smaller than for the C-rich Miras.

Figure 6. Same as in Figure 3, but for the Spitzer bands: (a) [3.6] $\mu$m, (b) [4.5] $\mu$m, (c) [5.8] $\mu$m, and (d) [8.0] $\mu$m.
5.1. Comparison of the Mean Magnitude Measuring Methods

One of the basic parameters that characterizes a star is its mean magnitude. The mean magnitude should not depend on the measurement method; however, various methods come with their own advantages and disadvantages, the latter leading to biases. Since the mean magnitude is extremely important for PLRs analyses, we investigated the influence of several methods on the zero-point \( a_0, \), slope \( a_1, \), and dispersion \( \sigma_\lambda \), of the PLRs.

The PLRs in this paper (Figures 3–8 and Table 4) show the mean mid-IR magnitudes obtained by integrating third-order Fourier series on the fitted OGLE templates to mid-IR data (see Section 3.2 in this paper, and Iwanek et al. 2021 for more details). In principle, in some cases this method could fail, as the Mira brightness in addition to the typical periodic variability may exhibit aperiodic variations of the mean magnitude and amplitude. We compared these PLRs (in each Spitzer and WISE band) with PLRs constructed using mean magnitudes measured in two different ways. The PLR fitting method is the same as described in Section 4.1.

In contrast to more sophisticated methods (i.e., the integrated Fourier series fit), we also measured the mean magnitudes using two simple methods: (1) the regular flux-based mean of the scaled OGLE templates to the mid-IR data, and (2) the sum of maximum and minimum fluxes divided by two and converted to magnitude. The magnitudes have been corrected for interstellar extinction as described in Section 3.1. We then constructed PLRs separately for the C-rich Miras (in the full range of pulsation periods) and for the O-rich Miras (with linear fit to periods \( P \leq 2.6 \)).

Let us compare the differences between the PLR parameters for the mean magnitudes obtained with the different methods to the corresponding uncertainties of our base models (presented in Table 4). In both cases (base-method (1) and base-method (2)), the smallest differences from our base models were obtained for the mean magnitudes calculated as a regular mean (method (1)). For the C-rich Miras, the differences in zero-points are at the level of 0.5\( \sigma \), while for the O-rich Miras these differences are less than 0.5\( \sigma \), for both Spitzer and WISE data. The slope of PLRs varies around 3\( \sigma \) for our base models of C-rich Miras, while this differences for the O-rich Miras are
smaller than 0.5σ. Dispersions are the smallest, almost in all cases, for the mean magnitudes obtained from the integration of the Fourier series and are used in the PLRs presented in Table 4 and in Figures 3–8. Method (2) significantly underestimates the mean magnitudes and causes much poorer results than the base model.

6. Conclusions

In this paper, we used the catalog of Mira-type variable stars in the LMC from the OGLE survey (Soszyński et al. 2009) to construct the most accurate mid-IR PLRs to date. We crossmatched the known LMC Miras with the WISE (Wright et al. 2010) and Spitzer databases (Werner et al. 2004) to obtain mid-IR light curves. We used two-decade-long OGLE light curves to create templates using the GPR model (He et al. 2016), and we fitted these templates to the mid-IR data (Iwanek et al. 2021). We measured the mean magnitudes in the mid-IR WISE and Spitzer bands by fitting a third-order truncated Fourier series, and integrated them. The mean magnitudes were corrected for interstellar extinction using the reddening map of the LMC (Skowron et al. 2021), and extinction curves in the mid-IR bands (Chen et al. 2018; Wang & Chen 2019). We calibrated the Mira PLRs using the most accurate distance measurement to the LMC (Pietrzyński et al. 2019).

The PLRs presented in Table 4 can be used to measure distances to Mira-type stars in the Milky Way and nearby galaxies. The distance to an individual Mira star can be measured with an accuracy at the level of 5% and 12% for O-rich and C-rich Mira, respectively.

In future work, we are going to measure the mid-IR distances to the tens of thousands of Mira-type stars in the Milky Way discovered in OGLE data. It will allow us to study the threedimensional structure of the Milky Way.

We are deeply grateful to the anonymous referee for his/her detailed report. The referee raised several points that turned out to be of significant importance. The suggestions given by the referee allowed us to greatly improve this manuscript, but more importantly, these comments motivated us to explore the subject of Mira variability in depth. As a result, the referee’s remarks led to the creation of a second paper titled “Multi-wavelength Properties of Miras” (Iwanek et al. 2021).

We thank Drs. Jan Skowron, Przemek Mróz, Radek Poleski, and Mariusz Gromadzki, and Marcin Wrona for comments that...
This publication makes use of data products from WISE, which is a joint project of the University of California, Los Angeles, and the Jet Propulsion Laboratory/California Institute of Technology, funded by the National Aeronautics and Space Administration (NASA). This work is based in part on archival data obtained with the Spitzer Space Telescope, which was operated by the Jet Propulsion Laboratory, California Institute of Technology under a contract with NASA.

**ORCID iDs**

Patryk Iwanek @ https://orcid.org/0000-0002-6212-7221
Igor Soszyński @ https://orcid.org/0000-0002-7777-0842
Szymon Kozłowski @ https://orcid.org/0000-0003-4084-880X

**References**

Alcock, C., Allsman, R. A., Alves, D. R., et al. 2000, ApJ, 542, 281
Ansari, R. 1996, VA, 40, 519
Bhardwaj, A., Kanbur, S., He, S., et al. 2019, ApJ, 884, 20
Cardelli, J. A., Clayton, G. C., & Mathis, J. S. 1989, ApJ, 345, 245
Chen, X., Wang, S., Deng, L., & de Grijs, R. 2018, ApJ, 859, 137
Cutri, R. M., Skrutskie, M. F., van Dyk, S., et al. 2003, 2MASS All Sky Catalog of Point Sources
Feast, M. W., Glass, I. S., Whitelock, P. A., & Catchpole, R. M. 1989, MNRAS, 241, 375
Gaia Collaboration, Brown, A. G. A., Vallenari, A., et al. 2018, A&A, 616, A1
Gerasimović, B. P. 1928, PNAS, 14, 963
Glass, I. S., & Lloyd Evans, T. 1981, Natur, 291, 303
Gould, A. 2003, arXiv:astro-ph/0310577
He, S., Yuan, W., Huang, J. Z., Long, J., & Macri, L. M. 2016, AJ, 152, 164
Huang, C. D., Riess, A. G., Hoffmann, S. L., et al. 2018, ApJ, 857, 67
Huang, C. D., Riess, A. G., Yuan, W., et al. 2020, ApJ, 899, 5
Hughes, S. M. G., & Wood, P. R. 1990, AJ, 99, 784
Iben, I. J., & Renzini, A. 1983, ARA&A, 21, 271
Iita, Y., & Matsunaga, N. 2011, MNRAS, 412, 2345
Iwanek, P., Kozłowski, S., Gromadzki, M., et al. 2021, ApJS, in press (arXiv:2107.03397)
Leavitt, H. S., & Pickering, E. C. 1912, HarCi, 173, 1
Mainzer, A., Bauer, J., Cutri, R. M., et al. 2014, ApJ, 792, 30
Mainzer, A., Bauer, J., Grav, T., et al. 2011, ApJ, 731, 53
Matsunaga, N., Kawadu, T., Nishiyama, S., et al. 2009, MNRAS, 399, 1709
Meixner, M., Gordon, K. D., Indebetouw, R., et al. 2006, AJ, 132, 2268
Molina, C. N., Borrisova, J., Catelan, M., et al. 2019, MNRAS, 482, 5567
Perrin, G., Rdigway, S. T., Lacour, S., et al. 2020, A&A, 642, 882
Pietrzyński, G., Graczyk, D., Gallenne, A., et al. 2019, Natur, 567, 200
Qin, W., Nataf, D. M., Zakamska, N., Wood, P. R., & Casagrande, L. 2018, ApJ, 865, 47
Riebel, D., Boyer, M. L., Sinivasan, S., et al. 2015, ApJ, 807, 11
Riebel, D., Meixner, M., Fraser, O., et al. 2010, ApJ, 723, 1195
Schlafly, E. F., & Finkbeiner, D. P. 2011, ApJ, 737, 103
Schwarzenberg-Czerny, A. 1996, ApJL, 460, L107
Skowron, D. M., Skowron, J., Udalski, A., et al. 2021, ApJS, 252, 23
Soszyński, I., Udalski, A., Kubiak, M., et al. 2004, AcA, 54, 129
Soszyński, I., Udalski, A., Kubiak, M., et al. 2005, AcA, 55, 351
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2009, AcA, 59, 239
Udalski, A. 2003, AcA, 53, 291
Udalski, A., Szymański, M. K., & Szymański, G. 2015, AcA, 65, 1
Urango, R., Omudaka, T., Nagayama, T., et al. 2020, ApJ, 891, 50
Wang, S., & Chen, X. 2019, ApJ, 877, 116
Werner, M. W., Alcock, C., Allsman, R. A., et al. 1999, in IAU Symp. 191, Wide-Field Variability Surveys: A 21st Century Perspective – 22nd Los Alamos Stellar Pulsation – Conf. Series Meeting (EPJ Web of Conferences 152) ed. M. Catelan & W. Gieren (Les Ulis: EDP Sciences), 01009
Wilson, R. E., & Merrill, P. W. 1942, ApJ, 95, 248
Wood, P. R. 2000, PASA, 17, 18
Wood, P. R., Alcock, C., Allsman, R. A., et al. 1999, in IAU Symp. 191, Asymptotic Giant Branch Stars, ed. T. Le Bertre, A. Lebre, & C. Waelkens (Cambridge: Cambridge Univ. Press), 151
Wright, E. L., Eisenhardt, P. R. M., Mainzer, A. K., et al. 2010, AJ, 140, 1868
Yuan, W., He, S., Macri, L. M., Long, J., & Huang, J. Z. 2017a, AJ, 153, 170
Yuan, W., Macri, L. M., He, S., et al. 2017b, AJ, 154, 149
Yuan, W., Macri, L. M., Javadi, A., Lin, Z., & Huang, J. Z. 2018, AJ, 156, 112

...