Self-calibration Applied in Converting Simulation Surveying

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Abstract  In the field of converting simulation surveying and traditional close range photogrammetry, it has been developed so far to survey objects by commercial digital camera and this technique is applied widely in every part of production. In order to get three-dimensional information of objects, commercial digital camera must be examined. For a long time, digital camera has been examined by DLT. Then there must be a high-precision control field. For realizing surveying without control points, a method for self-calibration is proposed.

Keywords  absolute conic; absolute quadric; self-calibration

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Introduction

In close-range photography and converting simulation surveying, it is usually commercial digital camera being used in measuring and surveying target objects, whose elements of interior orientation, namely \((x_0,y_0)\), \(f\) are not known. So, before measuring and surveying target objects, it must be done to examine elements of interior orientation. For a long time, digital camera has been examined by DLT with which there must be a high-precision control field. As a result, the scope of applying commercial digital camera in surveying is limited. In this paper, a method for self-calibration is discussed and realized.

1 Calibration of digital camera

1.1 Self-calibration of digital camera

A method for self-calibration affords feasibility in realizing real-time controlling and simulation, which does not again depend on the control points. In pinhole imaging, the relation between the object point \(M[x,y,z]^T\) and the image point \(m[u,v]^T\) can be expressed as follows:

\[
s[u \ v \ z]^T = P \cdot [x \ y \ z \ l]^T
\]

where \(s\) is a random scale coefficient; \(P\) is a \(3\times4\) perspective projection matrix, which is also expressed as the result of multiplying the elements of interior orientation by the elements of exterior orientation, namely:

\[
P = K \cdot [R \ t]
\]

where

\[
K = \begin{bmatrix}
-fk_x & fk_z \cos \theta & u_0 \\
0 & -\frac{fk_y}{\sin \theta} & v_0 \\
0 & 0 & 1
\end{bmatrix}
\]

where \(f\) is principal distance of camera; \(k_x, k_z\) are horizontal scale coefficient and vertical scale coefficient respectively; \(u_0, v_0\) are coordinates of principal point of photograph; \(\theta\) is the angle between image coordinate axes; \(R, t\) are rotation matrix and displacement matrix respectively from object coordinate
system to image coordinate system\cite{1}.

Make \( f_x = -f_k u, s = f_k \cos \theta, f_v = -f_k / \sin \theta \), the elements of interior orientation which are needed to be examined are \((f_x, f_v, s, u_0, v_0)\).

### 1.2 Basic notion

1) Homogeneous space. In Euclidean space, a point may be expressed as \( x = (X, Y, Z)^T \). This formula can be expanded as \( x = [X, Y, Z, t]^T \), in the same time, the Euclidean space is turned into Homogeneous space, where \( t = 0 \) means a point which lies in an infinite far plane.

2) Absolute conic. An absolute conic is an imaginary conic which lies in an infinite far plane and can be expressed as: \( X^2 + Y^2 + Z^2 = 0 \), \( t = 0 \). Here \( x = (X, Y, Z, 0)^T \) is an optional point on it.

An absolute conic defines a circle on an infinite far projection plane whose radius is \( i = \sqrt{-1} \). It is provided with invariability under the transformation of translation, rotation and scaling. Based on the property, it is concluded that the perspective projection image \( w \) of absolute conic is still an absolute conic on which the coordinates of any point do not depend on the position of digital camera. When taking pictures in different situation, absolute conic and its image are same. So, the coordinates of the image \( w \) relate only to the elements of the interior orientation and do not relate to the elements of the exterior orientation. As a result, the digital camera can be calibrated with absolute conic whose perspective projection image is expressed as follows:

\[
D = K^{-T}K^{-1}
\]  

(3)

With the method of self-calibration, it is first to deduce \( w \) or \( w' \), then to calculate \( K^{-T}K^{-1} \), to get inverse matrix of \( K^{-T}K^{-1} \), namely \( KK^{-T} \), then to decompose \( KK^{-T} \) with Cholesky. Finally the matrix \( K \) about the elements of interior orientation is gotten.

3) Absolute conicoid. An absolute conicoid \( \Omega' \) is a special imaginary conicoid in the Homogeneous space which equals to allelomorph of absolute conic \( \Omega \). From the point of algebra, \( \Omega' \) is expressed as follows in Homogeneous space:

\[
\Omega' = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}
\]

(4)

Let \( P \) is projection matrix, then \( \Omega' \) should be satisfied with condition \( w' = P\Omega' P^T \).

4) Homography matrix. In Homogeneous space, the transformation between two different reference frames can be described by Homography matrix.

5) Two views geometry. In two views, the corresponding points in images follow the epipolar geometry. In Fig. 1, the images are taken at two different positions \( C, C' \). The images of the ground point \( x \) in left and right view are \( m, m' \) respectively. Based on the epipolar geometry, \( m \) must lie in epipolar line \( l_m \) and \( m' \) must lie in epipolar line \( l'_m \). The restrict relation is the basic geometry relation between two views which is named epipolar geometry customarily. From the view point of algebra, it can be expressed as:

\[
m^T F m = 0
\]

(5)

where the matrix \( F \) is a \( 3 \times 3 \) matrix whose rank is 2 and is customarily named fundamental matrix. Because the fundamental matrix \( F \) includes all geometry information between two views, it is very important to exactly calculate \( F \) for calibration and three-dimensional reconstruction. The method of eight points is used to calculate \( F \) based on Eq.(5) which is as follows: let \( m = [u,v,1]^T, m' = [u',v',1]^T \), \( F = [F_{11}, F_{12}, F_{13}, F_{21}, F_{22}, F_{23}, F_{31}, F_{32}, F_{33}] \), a pair of corresponding points \( (m, m') \) can be expressed as \([uu', vu', uu', vv', u, v, 1]f = 0\); construct \( n(n \geq 8) \) equations and then build up the linear equations \( Af = 0 \); calculate \( Af = 0 \) with the method of SUV.

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![Epipolar geometry](image-url)
camera stations; \( k \) is the line to joint one station with the other one; \( e, e' \) are epipolar points respectively in two views. Let \( II \) is a plane that the line \( k \) is passing by. The plane \( II \) intersects two image planes at line \( l, l' \) respectively. Let \( p, p' \) be projection matrixes corresponding to two views respectively. The epipolar transformation defines a homography matrix between two lines of \( l, l' \).

If plane \( II \) is tangent to absolute conic \( \Omega \), line \( l \) is tangent to the image \( w \) of absolute conic and line \( l' \) is tangent to \( w' \). The absolute conic is not depended on the camera station. So, it is right that \( w = w' \), that is \( \omega = w' \). With the epipolar transformation, the line \( l_i \) is corresponding to \( l_i' \) and \( l_2 \) is corresponding to \( l_2' \) [1].

Provided \( D \) is a matrix expression of absolute conic \( w \). Based on the definition of \( D \), epipolar line \(< e, m > \) which is from the epipolar point \( e \) to the image point \( m \) is tangent to \( w \) and the point \( m \) must be in the absolute conic \( w \), namely:

\[
(p \times m)^T D (p \times m) = 0 \tag{6}
\]

where the matrix \( D \) is composed of 6 parameters of which there are 5 parameters independence and this is accordant to the elements of camera interior orientation.

Suppose the coordinate of point \( m \) is \((x, y, 0)\). The Eq.(6) comes into existence respectively on two views. As a result, we can conclude as follows:

\[
\begin{align*}
A_{11}x^2 + 2A_{12}xy + A_{22}y^2 &= 0 \\
A'_{11}x'^2 + 2A'_{12}x'y' + A'_{22}y'^2 &= 0
\end{align*}
\tag{7}
\]

where \( A_{ij} \) and \( A'_{ij} \) are decided by matrix \( D \) and epipolar points \( e, e' \).

Let \( m = (x, y, 0), m' = (x', y', 0) \). When \( m' \) is equals to \( Nm \),

\[
< e, m > = H < e', m' > \tag{8}
\]

where \( H \) is homography matrix. The transformation \( N \) is equals to as follows:

\[
T = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{9}
\]

where parameters \( a, b, c, d \) can be calculated by epipolar points \( e, e' \) and the corresponding image points \( q_i - q_i' \). From the relation between stereoscopic pair, we can conclude as follows:

\[
T = \frac{p_i q_{i2} - p_i q_{i3}}{p_3 q_{i1} - p_i q_{i3}}, \quad T' = \frac{p_i' q_{i2} - p_i' q_{i3}}{p_3 q_{i1} - p_i' q_{i3}} \tag{10}
\]

Based on Eqs.(9) and (10), parameters \( a, b, c, d \) can be calculated with least squares adjustment. Then Eq.(7) can be rewritten as:

\[
\begin{align*}
A_{11} + 2A_{12}T + A_{22}T^2 &= 0 \\
A'_{11}(bT + c) + 2A'_{12}(bT + c)(T + a) + A'_{22}(T + a)^2 &= 0
\end{align*}
\tag{11}
\]

In Eq.(11), every equation is a conic about \( T \). Dividing coefficients of one equation by corresponding coefficients of the other equation, let the ratio is equal, we can conclude as follows:

\[
\begin{align*}
A_{12}(A'_{22}a^2 + A'_{22}c^2 + 2A'_{12}ac) - (A_{12}c + A'_{12}a + A'_{22}bc + A'_{22}ab)A_{11} &= 0 \\
A_{22}(A'_{22}a^2 + A'_{22}c^2 + 2A'_{12}ac) - (2A_{12}b + A'_{22}a' + A'_{11}b^2)A_{11} &= 0
\end{align*}
\tag{12}
\]

Then Eq.(12) is Kruppa equation.

Based on the upper referred deduction, 4 equations can be built up based on a stereoscopic pair. But, the number of the elements of interior orientation is 5. So, it is necessary to take pictures form 3 different camera stations for calculating the elements of interior orientation.

The method of self-calibration based on the equation Kruppa does not reconstruct projective transformations between all images but only establish equation between two images. In the situation where there are a lot of difficulties to uniform all images into a coordinate system, the method is superior to hierarchic self-calibration [3]. But, when calibrating camera based on the equation Kruppa, there is instability in final results with the number of images increasing. As a result, the application of this method is limited.

1.3.2 Hierarchic self-calibration

With the method of hierarchic self-calibration, the projection transformation between the sequences of images must be rebuilt up firstly, then the affine parameters and the elements of interior orientation are calculated under a restriction of absolute conic.

An absolute conicoid \( \Omega \) is corresponding to the
dual \( w' \) of the absolute conic on the image, namely:

\[
\begin{align*}
    w' = KK^T = \lambda P\Omega^T P^T
\end{align*}
\]  

(13)

The formula is a ligament by which the restriction of the elements of interior orientation can be transferred to the restriction on \( \Omega^T \).

Supposed \( K_i \) is the matrix about the elements of interior orientation corresponding to the \( i \)th image. Based on Eq.(13), it can be concluded as follows:

\[
K_iK_i^T = P_i\Omega^T P_i^T
\]  

(14)

Because the matrix \( \Omega^* \) is symmetrical and its rank is 3, the \( \Omega^* \) in Eq.(14) can be written in detail as follows:

\[
\begin{align*}
    \lambda^T P_i \begin{bmatrix} KK^T & K^T a_n & 0 \\ a_n^T K & a_n^T a_n & 0 \\ \vdots & \vdots & \vdots \end{bmatrix} P_i^T = KK^T
\end{align*}
\]  

(15)

And then all unknown parameters can be calculated with optimizing the following formula:

\[
\begin{align*}
    c_r(K_i, K, a_n) = \sum \| P(K_iK_i^T) - F(P_i\Omega^T P_i^T) \|_2
\end{align*}
\]  

(16)

In order to guarantee the convergence of Eq.(16), a good initial value of \( K_i \) must be given. Here, suppose slope element \( s \) be zero and the coordinates of principal point of photograph are known. So, Eq.(14) can be turned into linear equation. Let the initial value of Eq.(16) be the results of the referred linear equation as referred above, the convergence of Eq.(16) can be guaranteed\(^{2, 4, 5}\).

### 1.4 Steps of solution

1) Calculate the fundamental matrix \( F \) with the method of 8 points. Find 8 corresponding image points from the left and right image respectively, the fundamental matrix can be obtained with the method of SUV decomposing.

2) Calculate the coordinates of the epipolar point on the right image. At first, any of two epipolar lines are gotten with coplanar equation. Then the epipolar point can be acquired by calculating the point of intersection.

3) Construct the projection matrix. The projection matrix can be built up with the following formula:

\[
\begin{align*}
    P' &= \begin{bmatrix} e' \end{bmatrix} F + e'v^T \lambda e'
\end{align*}
\]

\[
\begin{align*}
    P = [I | 0]
\end{align*}
\]

where \( I \) is unit matrix; \( e' \) is the coordinate of epipolar point on the right image; \( F \) is the fundamental matrix; \( \lambda \) is random value; \( v \) is \( 3 \times 1 \) matrix.

4) Calculate the unknown parameters based on the hierarchic formulas as referred above.

### 2 Results and analyses

In this experiment, an active control frame is used on which 40 control points are set. Images are taken in different positions. In order to give an impersonal and exact evaluation about self-calibration, firstly, one result is acquired with the DLT and then the other result is gotten with self-calibration. In this experiment, the camera has been calibrated for three times under 20, 30 and 40 control points respectively. The results are shown in Table 1.

From Table 1, we can conclude that the result with self-calibration accords with the results of DLT. In other words, it satisfies the requirement of measurement to calibrate the camera with self-calibration.

### Table 1 Results of DLT and self-calibration

| Control points number | DLT | Self-calibration |
|-----------------------|-----|-----------------|
|                       | \( x_0 \)/pixel | \( y_0 \)/pixel | \( f \) | \( x_0 \)/pixel | \( y_0 \)/pixel | \( f \) |
| 20                    | 1 104.81 | 734.15 | 1 728.50 | 1 103.87 | 733.90 | 1 727.91 | 1 728.84 |
| 30                    | 1 104.81 | 734.25 | 1 728.53 | 1 104.54 | 734.21 | 1 727.82 | 1 728.65 |
| 40                    | 1 104.81 | 734.28 | 1 728.56 | 1 103.97 | 734.36 | 1 727.83 | 1 729.66 |

### References

[1] Hartley R, Zisserman A(2000) Multiple view geometry in computer vision[M]. Cambridge: The Press Syndicate of the University of Cambridge

[2] Meng Xiaoqiao, Hu Zhanyi(2003)Recent progress in camera self-calibration[J]. Acta Automatica Sinica, 29(1):110-124 (in Chinese)

[3] Mendonca P, Cipolla R(1999)A simple technique for self-calibration[C]. IEEE Conference on Computer Vision and Pattern Recognition, Fort Collins, Colorado

[4] Faugeras O, Luong Q T(2001)The geometry of multiple images[M]. Cambridge: The MIT Press

[5] Zeller C, Faugeras O(1996)Camera self-calibration from video sequences: the Kruppa equations revisited, research report 2793[R]. INRIA Sophia-Antipolis, France