Mass gap effects and higher order electroweak Sudakov logarithms

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Abstract

The infrared structure of spontaneously broken gauge theories is phenomenologically very important and theoretically a challenging problem. Various attempts have been made to calculate the higher order behavior of large double-logarithmic (DL) corrections originating from the exchange of electroweak gauge bosons resulting in contradictory claims. We present results from two loop electroweak corrections for the process $g \rightarrow f_R f_L$ to DL accuracy. This process is ideally suited as a theoretical model reaction to study the effect of the mass gap of the neutral electroweak gauge bosons at the two loop level. Contrary to recent claims in the literature, we find that the calculation performed with the physical Standard Model fields is in perfect agreement with the results from the infrared evolution equation method. In particular, we can confirm the exponentiation of the electroweak Sudakov logarithms through two loops.

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1 Introduction

Recently there have been various attempts to calculate higher order corrections in the electroweak theory with differing outcomes [1, 2, 3, 4, 5]. At future collider experiments in the TeV regime, large double logarithmic (DL) corrections originating from the exchange of the Standard Model (SM) gauge bosons need to be taken into account for precision measurements designed to disentangle new physics effects. Even two loop effects are expected to change typical cross sections by a few percent. The reason for these large effects is simply that cross sections in the electroweak theory depend on the infrared cutoff, the gauge boson mass $M$, and don’t cancel out in semi-inclusive, or even in fully inclusive cross sections [6].

Thus it is of considerable importance and of theoretical interest to study the higher order behavior of these Sudakov logarithms in general scattering amplitudes. In Ref. [1] a general method of resumming these large DL corrections has been presented based on the gauge invariant infrared evolution equation method [7]. In Ref. [5] the approach was extended to the subleading level and shown to agree with explicit one loop calculations. In both cases these corrections were found to exponentiate also in the case of broken gauge theories. Recently, however, an explicit two loop calculation [8] in the Coulomb gauge found non-exponentiating contributions for external fermion lines originating from a mass gap between the photon and Z-boson.

It is the purpose of this paper to refute these claims and also to demonstrate with a calculation in terms of the physical fields, that exponentiation of electroweak Sudakov logarithms holds at the two loop level. For this purpose we study the reaction $g \rightarrow f_R f_L$, which is ideally suited to study the mass gap effects from the exchange of the neutral electroweak gauge bosons.

In the next section we briefly review the calculation of the one loop DL corrections in the Feynman gauge. This gauge leads to a particularly simple way of calculating DL corrections and only QED-like topologies have to be considered in our example. The important difference to QED is of course the fact that the two neutral gauge bosons have different masses. These effects are then investigated at the two loop level using the physical SM fields. We only consider virtual corrections with the understanding that for physical cross sections also the real soft emission contributions have to be included. We then compare our result with the literature and summarize our findings briefly.

2 Two loop results

We begin by briefly recalling the Sudakov method [1] of extracting DL corrections from Feynman diagrams[1]. In the Feynman gauge, the propagator of the exchanged boson

\footnote{More details, including the effect of fermion masses, are given in Ref. [10].}
must connect two external lines in order to generate the double pole structure needed to obtain DL corrections. The large argument in the logarithms is \( s = 2p_1p_2 \), where we denote with \( p_1 \) the four momentum of \( f_R \) and with \( p_2 \) the four momentum of \( f_L \).

The Sukakov decomposition along these external momenta is given by

\[
k = vp_2 + up_1 + k_\perp
\]

For the boson propagator we use the identity

\[
\frac{i}{suw - m^2 - k_\perp^2 + i\varepsilon} = \mathcal{P} \frac{i}{suw - m^2 - k_\perp^2} + \pi\delta(suw - m^2 - k_\perp^2)
\]

writing it in form of the real and imaginary parts (the principle value is indicated by \( \mathcal{P} \)). The latter does not contribute to the DL asymptotics and at higher orders gives subsubleading contributions. Rewriting the measure as \( d^4k = d^2k_\perp d^2k_{\parallel} \) with

\[
d^2k_\perp = |k_\perp|d|k_\perp|d\phi = \frac{1}{2}dk_\perp^2d\phi = \pi dk_\perp^2
\]

\[
d^2k_{\parallel} = |\partial(k^0, k^\perp)/\partial(u, v)|dudv \approx \frac{s}{2}dudv
\]

where we turn the coordinate system such that the \( p_1, p_2 \) plane corresponds to 0, \( x \) and the \( y, z \) coordinates to the \( k_\perp \) direction so that it is purely spacelike. The last equation follows from \( p_{1u}^2 = 0 \), i.e. \( p_{1u}^2 \approx p_{2u}^2 \) and

\[
(p_{1x}p_{2x} - p_{2x}p_{1x})^2 \approx (p_{1x}p_{20} - p_{2x}p_{10})^2 = (p_{1x}p_{20})^2 = s/2
\]

At the one loop level, the electroweak corrections are depicted in Fig. 2. We use the Feynman gauge and thus the Feynman rules depicted in Fig. 1. The fermion masses are neglected for simplicity. They can, however, be added without changing the nature of the higher order corrections. For right handed fermions we only need to consider...
Figure 2: The one loop electroweak SM Feynman diagrams leading to DL corrections in the Feynman gauge for $g \to f_R f_L$. Only the vertex corrections from the neutral Z-boson (zigzag-lines) and the photon propagators contribute. At higher orders only corrections to these two diagrams need to be considered in the DL approximation. The photonic corrections are regulated by a fictitious mass terms $\lambda$. In physical cross sections, the $\lambda$-dependence is canceled by the effect of the emission of soft and collinear bremsstrahlung photons.

The neutral electroweak gauge bosons, i.e. we are concerned with an $U(1)_R \times U(1)_Y$ gauge theory which is spontaneously broken to yield the Z-boson and photon fields. The DL-contribution of a particular Feynman diagram is thus given by

$$M_k = M_{\text{Born}} \mathcal{F}_k$$

where the $\mathcal{F}_k$ are given by integrals over the remaining Sudakov parameters at the $n$-loop level:

$$\mathcal{F}_k = \left(\frac{\alpha_f}{2\pi}\right)^n \prod_{i=1}^n \int_0^1 \int_0^1 \frac{du_i}{u_i} \frac{dv_i}{v_i} \Theta_k$$

The $\Theta_k$ describe the regions of integration which lead to DL corrections. At one loop, the diagrams of Fig. 2 lead to

$$M_{\text{DL}}^{R(1)} = M_{\text{Born}}^{R(1)} \left(1 - \frac{\alpha_f}{2\pi} \int_0^1 \frac{du}{u} \int_0^1 \frac{dv}{v} \left[\Theta(suv - \lambda^2) + \frac{s^2_w}{c_w^2} \Theta(suv - M^2)\right]\right)$$

$$= M_{\text{Born}}^{R(1)} \left(1 - \frac{\alpha_f}{4\pi} \left[\log^2 \frac{s}{\lambda^2} + \frac{s^2_w}{c_w^2} \log^2 \frac{s}{M^2}\right]\right)$$

which is the well known result from QED plus the same term with a rescaled coupling (see Fig. 1) and infrared cutoff. The restriction to right handed fermions allows us to focus solely on the mass gap of the neutral electroweak gauge bosons. The $W^\pm$ only couples to left handed doublets. At the two loop level we have to consider more diagrams than in the QED case. The relevant Feynman graphs that give DL corrections in the Feynman gauge are depicted in Fig. 3. Only these corrections can yield four logarithms at the two loop level in the Feynman gauge. Otherwise one cannot obtain the required pole terms (as is well known in QED [10]). They contain diagrams where the exchanged gauge bosons enter with differing on-shell regions, i.e. differing integration regions which give large DL corrections. It is instructive to revisit the case
Figure 3: The two loop electroweak SM Feynman diagrams leading to DL corrections in the Feynman gauge for $g \rightarrow f_R f_L$. The neutral Z-boson (zigzag-lines) and the photon propagators possess different on-shell regions due to the mass gap.
of pure QED corrections, since the topology of the graphs yielding DL contributions in the Feynman gauge is unchanged. In QED at the two loop level, the scalar integrals corresponding to the first row of Fig. 3 are given by:

\[ S_1^{\text{QED}} \equiv \int_{u_0}^{1} \frac{du_1}{u_1} \int_{v_0}^{1} \frac{dv_1}{v_1} \int_{u_2}^{1} \frac{du_2}{u_2} \int_{v_2}^{1} \frac{dv_2}{v_2} \theta(su_1 v_1 - \lambda^2) \theta(su_2 v_2 - \lambda^2) \theta(u_1 - u_2) \theta(v_1 - v_2) \quad (9) \]

\[ S_2^{\text{QED}} \equiv \int_{u_0}^{1} \frac{du_1}{u_1} \int_{v_0}^{1} \frac{dv_1}{v_1} \int_{u_2}^{1} \frac{du_2}{u_2} \int_{v_2}^{1} \frac{dv_2}{v_2} \theta(su_1 v_1 - \lambda^2) \theta(su_2 v_2 - \lambda^2) \theta(u_1 - u_2) \theta(v_2 - v_1) \quad (10) \]

Thus, in QED we find the familiar result

\[ S_1^{\text{QED}} + S_2^{\text{QED}} = \int_{u_0}^{1} \frac{du_1}{u_1} \int_{v_0}^{1} \frac{dv_1}{v_1} \int_{u_2}^{1} \frac{du_2}{u_2} \int_{v_2}^{1} \frac{dv_2}{v_2} \theta(su_1 v_1 - \lambda^2) \theta(su_2 v_2 - \lambda^2) \theta(u_1 - u_2) \]

\[ = \int_{u_0}^{1} \frac{du_1}{u_1} \int_{v_0}^{1} \frac{dv_1}{v_1} \int_{u_2}^{1} \frac{du_2}{u_2} \int_{v_2}^{1} \frac{dv_2}{v_2} \theta(su_1 v_1 - \lambda^2) \theta(su_2 v_2 - \lambda^2) \theta(u_1 - u_2) \]

\[ = \frac{1}{2} \left( \frac{1}{2} \log^2 \frac{s_1}{\lambda^2} \right)^2 \quad (11) \]

which yields the second term of the exponentiated one loop result in Eq. (8) for \( s_w \rightarrow 0 \). In the electroweak theory, we also need to consider the remaining diagrams of Fig. 3. The only differences occur because of the rescaled coupling according to the Feynman rules in Fig. 1 and the fact that the propagators have a different mass. Thus the second row of Fig. 3 leads to

\[ S_{1,2}^{M,\lambda} = \frac{s_w}{c_w^2} \int_{u_0}^{1} \frac{du_1}{u_1} \int_{v_0}^{1} \frac{dv_1}{v_1} \int_{u_2}^{1} \frac{du_2}{u_2} \int_{v_2}^{1} \frac{dv_2}{v_2} \theta(su_1 v_1 - M^2) \theta(su_2 v_2 - \lambda^2) \theta(u_1 - u_2) \]

\[ = \frac{s_w}{c_w^2} \int_{u_0}^{1} \frac{du_1}{u_1} \int_{v_0}^{1} \frac{dv_1}{v_1} \int_{u_2}^{1} \frac{du_2}{u_2} \int_{v_2}^{1} \frac{dv_2}{v_2} \theta(su_1 v_1 - M^2) \theta(su_2 v_2 - \lambda^2) \theta(u_1 - u_2) \]

\[ = \frac{s_w}{c_w^2} \left[ \frac{1}{8} \log^4 \frac{s}{\lambda^2} - \frac{1}{6} \log^3 \frac{s}{\lambda^2} \log \frac{M^2}{\lambda^2} + \frac{1}{24} \log^4 \frac{M^2}{\lambda^2} \right] \quad (12) \]

where we indicate the gauge boson masses of the two propagators in the scalar functions. Analogously we find for the remaining two rows

\[ S_{1,2}^{M,M} = \frac{s_w}{c_w^2} \int_{u_0}^{1} \frac{du_1}{u_1} \int_{v_0}^{1} \frac{dv_1}{v_1} \int_{u_2}^{1} \frac{du_2}{u_2} \int_{v_2}^{1} \frac{dv_2}{v_2} \theta(su_1 v_1 - \lambda^2) \theta(su_2 v_2 - M^2) \theta(u_1 - u_2) \]

\[ = \frac{s_w}{c_w^2} \int_{u_0}^{1} \frac{du_1}{u_1} \int_{v_0}^{1} \frac{dv_1}{v_1} \int_{u_2}^{1} \frac{du_2}{u_2} \int_{v_2}^{1} \frac{dv_2}{v_2} \theta(su_1 v_1 - \lambda^2) \theta(su_2 v_2 - M^2) \theta(u_1 - u_2) \]

\[ = \frac{s_w}{c_w^2} \left[ \frac{1}{4} \log^2 \frac{s}{M^2} \log \frac{s}{\lambda^2} - \left( S_{1,2}^{M,\lambda} \right) \right] \quad (13) \]

and

\[ S_{1,2}^{M,M} = \frac{1}{2} \frac{s_w^4}{c_w^4} \left( \frac{1}{2} \log^2 \frac{s}{M^2} \right)^2 \quad (14) \]
Thus, we find for the full two loop electroweak DL-corrections

\[ M_{DL}^{R(2)} \equiv M_{Born}^{R} \left( 1 + \delta_{R}^{(1)} + \delta_{R}^{(2)} \right) \]  

(15)

with

\[ \delta_{R}^{(1)} = -\frac{\alpha f}{4\pi} \left( \log^2 \frac{s}{\lambda^2} + \frac{s_w^2}{c_w^2} \log^2 \frac{s}{M^2} \right) \]  

(16)

and

\[ \delta_{R}^{(2)} = \left( \frac{\alpha f}{2\pi} \right)^2 \left[ S_{QED}^1 + S_{QED}^2 + S_{M,\lambda}^1 + S_{M,\lambda}^2 + S_{\lambda,M}^1 + S_{\lambda,M}^2 + S_{M,M}^1 + S_{M,M}^2 \right] \]

\[ = \left( \frac{\alpha f}{2\pi} \right)^2 \left[ \frac{1}{8} \log^4 \frac{s}{\lambda^2} + \frac{s_w^2}{4c_w^2} \log^2 \frac{s}{M^2} \log \frac{s}{\lambda^2} + \frac{s_w^4}{8c_w^4} \log^4 \frac{s}{M^2} \right] \]

\[ = \frac{1}{2} \left[ -\frac{\alpha f}{4\pi} \left( \log^2 \frac{s}{\lambda^2} + \frac{s_w^2}{c_w^2} \log^2 \frac{s}{M^2} \right) \right]^2 \]  

(17)

which is precisely the second term of the exponentiated one loop result in the process of \( g \rightarrow f_R \bar{f}_L \). In the next section we review briefly the infrared evolution equation method and compare Eq. (15) with other results in the literature.

### 3 Comparison with other results

In Ref. [4] a general method for calculating DL corrections in the electroweak theory was presented. It is based on calculating the DL corrections first in the high energy regime \( s \gg \mu^2 \geq M^2 \), where \( \mu^2 \) is an infrared cutoff on the \( k^2 \) of the exchanged gauge bosons. The corrections are then obtained by solving the infrared evolution equation, based on a non-Abelian generalization of Gribov’s bremsstrahlung theorem [11]. The solution in this regime is then taken as the initial condition for the general case where \( M^2 \leq \mu^2 \). In this way the mass gap between the neutral electroweak gauge bosons is incorporated in a natural way by considering the effective theories in each domain and requiring equality at the weak scale \( M \). In Ref. [3] an alternative derivation was presented based on the virtual contributions to the Altarelli-Parisi splitting functions. In this approach it is evident that all Sudakov logarithms, for fermions and transversely polarized gauge bosons even on the subleading level, are related to external lines and exponentiate. For longitudinally polarized gauge bosons all DL corrections can be resummed by utilizing the Goldstone boson equivalence theorem [3].

In order to compare the result of Refs. [4] with Eq. (15) we specialize on the case with an external right handed fermion and left handed anti-fermion pair. In terms of the quantum numbers of the unbroken theory, we obtain from Refs. [4] \[ M_{DL}^{R} = M_{Born}^{R} \exp \left( -\frac{1}{2} \sum_{k=1}^{2} \left[ \frac{g^2}{(4\pi)^2} T_k (T_k + 1) + \frac{g^f}{(4\pi)^2} \frac{T_k}{4} \right] \log^2 \frac{s}{M^2} \right) \]
\[ \times \exp \left( -\frac{1}{2} \sum_{k=1}^{2} \frac{e_f^2}{(4\pi)^2} 2 \log \frac{s}{\mu M} \log \frac{M^2}{\mu^2} \right) \]

\[ = \mathcal{M}_{\text{Born}}^{R} \exp \left( -\frac{e_f^2}{(4\pi)^2 c_w^2} \log^2 \frac{s}{M^2} \right) \times \exp \left( -\frac{e_f^2}{(4\pi)^2} \left[ \log^2 \frac{s}{\mu^2} - \log^2 \frac{s}{M^2} \right] \right) \]

\[ = \mathcal{M}_{\text{Born}}^{R} \exp \left( -\frac{e_f^2}{(4\pi)^2} \left[ \log^2 \frac{s}{\mu^2} + \frac{s_w^2}{c_w^2} \log^2 \frac{s}{M^2} \right] \right) \]

(18)

where we use the fact that for right handed fermions \( T = 0, Y = 2Q_f \) and \( g' = \frac{e}{c_w} \).

Eq. (18) thus reproduces the two loop result in Eq. (15) where we assume that for small values of the infrared cutoff \( \mu \sim \lambda \).

In Ref. [4], the same result is obtained for the DL-vertex corrections to all orders based on a generalization of QCD-results for the electromagnetic form factor. Also the subleading Sudakov logarithms are in agreement with Ref. [5] to all orders.

Ref. [2] is also in agreement with the above results for right handed particles. It disagrees, however, for left handed particles with the result obtained from the infrared evolution method.

The only paper which contains a real two loop calculation in terms of the physical fields is presented in Ref. [8]. It uses, other than in the present work, a new formulation of the Coulomb gauge also for the massive gauge bosons. The authors of Ref. [8] find non-exponentiating DL terms at the two loop level for both right and left handed fermions and trace these terms back to the fact that there is a mass gap between the neutral gauge bosons “since \( \frac{m_f}{\lambda} \gg \sqrt{s} \gg 1 \).”

There are several comments in order. Firstly, our explicit calculation for the right handed case in the Feynman gauge contradicts their claims, the overall result of course being gauge independent. Since for right handed fermions the Z-boson coupling is proportional to \( \frac{s_w}{c_w} \), one can simply switch off all Z-boson effects by taking the limit \( \frac{s_w}{c_w} \rightarrow 0 \) as is evident by our result in Eq. (13). If the result of Ref. [8] is a consequence of the aforementioned reasons, the non-exponentiating terms should also vanish in that limit (since in the Feynman gauge all DL-terms from Z-bosons must couple \( \sim s_w \)). They, however, do not. The authors of Ref. [8] claim that there are new uncanceled contributions from so called frog-diagrams \( \overrightarrow{2} \) where the \( s_w \) terms are indeed canceled by the \( W^\pm Z \)-vertex) because of the massless photon and the massive Z-boson. It should be noted, however, that those diagrams are infrared finite because the virtualities are cut off by \( k_{\perp}^2 \geq 4M^2 \) in order to produce the \( W^\pm \) loop. The limit \( \lambda \rightarrow 0 \) exists, and in particular, \( \lambda \) does not enter as an argument of a logarithm. Thus, the region

\[ \overrightarrow{2} \]

These are two-loop two point functions where the external fermions are coupled to neutral gauge bosons with an induced \( W^\pm \)-loop in the Coulomb gauge. In this gauge, all DL corrections are contained in two point functions unlike the situation for covariant gauges.
producing large logarithms for both neutral gauge bosons is in fact identical. If the mass gap was the origin for the new terms and would vanish if \( \lambda = M \), it should reveal itself through a term which vanishes in that limit. While the authors claim that there is exponentiation in that case, their result shows no sign of a mass gap. Thirdly, the argument should evidently be independent of the fermion masses. Thus for massless fermions, i.e. \( m_f \sim \lambda \), the abovementioned reasoning fails since \( \mathcal{O}(1) \sim \frac{m_f}{\lambda} \ll \frac{\sqrt{s}}{M} \) and one expects the effective theory to be that of the unbroken phase.

In addition the author would also like to point out that for the left handed case, the authors of Ref. \cite{8} obtain the same spurious contribution as in the right handed case plus a term which is non-analytic in the coupling \( \sim |e_f| \). While non-analytic contributions occur in bound state problems from restrictions of the effective region of integration, these terms are disallowed in general scattering amplitudes by the analyticity properties of S-matrix elements. The results of Ref. \cite{8} should thus be rechecked, with a special focus on the subtleties of the new Coulomb gauge formulation for massive gauge bosons.

4 Conclusions

In this paper we have calculated the two loop DL corrections to the process \( g \rightarrow f_R f_L \) using the SM fields. It was shown that the result is in agreement with the calculation based on the infrared evolution equation method. In particular, the exponentiation found in Refs. \cite{1,5} is confirmed for this process, and it is shown that the mass gap between the photon and Z-boson does not spoil the higher order Sudakov resummation. Our result contradicts explicitly recent claims \cite{8} about the breakdown of exponentiation through mass gap effects. Rather it demonstrates in a straightforward way that the matching conditions of the infrared evolution equation method naturally incorporate the different masses of the neutral electroweak gauge bosons.

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