Properties of the Resonance $\Lambda(1520)$ as seen in the Forward Electroproduction at JLab Hall A

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Abstract. High-resolution spectrometer measurements of the reaction $H(e,e'K^+)X$ at small $Q^2$ are used to extract the mass and width of the $\Lambda(1520)$. We investigate dependence of the resonance parameters on different parametrizations of the background and the resonance peak itself. Our final values for the Breit-Wigner parameters are $M = 1520.4 \pm 0.6$ (stat.) $\pm 1.5$ (syst.) MeV and $\Gamma = 18.6 \pm 1.9$ (stat.) $\pm 1$ (syst.) MeV. The width appears to be more sensitive to the assumptions than the mass. We also estimate, for the first time, the pole position for this resonance and find that both the pole mass and width seem to be smaller than their Breit-Wigner values.

I. INTRODUCTION

The $\Lambda(1520)$ is considered to be one of the best known baryon resonances. It is an excited hyperon state of the highest rank (4 stars) in the Review of Particle Physics (RPP) [1]. But its properties are still not exceedingly well understood. For example, its rather intensive decay to the $\pi\pi\Lambda$ channel [1] gives a hint at an unexpectedly strong coupling to the kinematically suppressed subchannel $\pi\Sigma(1385)$, stronger than to the main decay channels $\pi\Sigma$ and $KN$. This could mean that the $\Lambda(1520)$ has a molecular nature (see, e.g., Ref. [2] and references therein). On the other hand, evidence has been suggested [3] that a new, previously unobserved, resonance $\Sigma(1380)$ with $J^P = 1/2^-$ and decay to $\Lambda\pi$ might influence the mode $\Lambda(1520) \rightarrow \pi\pi\Lambda$. These examples show that more detailed studies of the $\Lambda(1520)$, which may clarify both its nature and the hyperon spectroscopy, are needed.

As a first step, we investigate degree of precision available for the basic resonance parameters of the $\Lambda(1520)$ [4]. Surprisingly, all the experimental inputs shown in RPP [1] for its mass and width are dated before 1980. Many new experiments performed since then have not been accounted for. Meanwhile, they provided much more data with the $\Lambda(1520)$ observed either as the main goal or as a byproduct (even more data are expected to appear in near future).

Most of the earlier high-precision mass and width values, in particular those used for averaging in RPP [1], had come either directly, from production measurements with bubble chambers, or indirectly, from partial wave analyses of scattering data. High-energy spectrometers did not have good enough resolution for such measurements, and their results could not compete with bubble chamber data, even despite much higher statistics. Now, the High Resolution Spectrometers (HRS’s) [5] in the JLab Hall A experiment [6] provided an exciting accuracy ($\sigma = 1.5$ MeV), comparable to the best previous resolutions. That is why their measurements are competitive with any previous ones.

Note also that experiments used for averaging in RPP, applied very different assumptions to extract the resonance parameters. However, influence of those assumptions on the parameter values has never been analyzed. We investigate this problem [4] on the base of the Hall A data set [6].

II. JLAB HALL A EXPERIMENT

High-resolution measurements of the missing mass (MM) spectra were developed at near-forward production angles in the reactions $ep \rightarrow e'K^+X$ and $e'\pi^+X$. The experiment [6] took place in Hall A at the Jefferson Lab using a 5.09 GeV electron beam incident on a 15 cm liquid hydrogen target. Scattered electrons were detected in one of the HRS’s in coincidence with electroproduced hadrons in the second HRS. Each spectrometer was positioned at 12.5° relative to the beamline, but the use of additional septum magnets [7] allowed to reach smaller production...
angles, down to \( \sim 6^\circ \). In this configuration, the spectrometers have an effective acceptance of approximately 4 msr in solid angle and \( \pm 4.5\% \) in momentum, while still maintaining their nominal \( 10^{-4} \) full-width half-max momentum resolution \( \sigma \). To obtain the desired MM coverage, the central momentum of the electron HRS was varied between 1.85 and 2.00 GeV, while the central momentum of the hadron HRS was changed between 1.89 and 2.10 GeV. In such configuration, the average momentum transfer of the virtual photon was \( \langle Q^2 \rangle \approx 0.1 \text{ (GeV/c)}^2 \), and the average cm photon energy was \( \langle E_{\gamma}^{\text{cm}} \rangle = 1.1 \text{ GeV} \) which means that \( \langle W \rangle = 2.53 \text{ GeV} \). For the kaon kinematics, the cm scattering angle was \( 5.6^\circ \pm 1.85 \text{ and } 2.00 \text{ GeV, while the central momentum of the hadron HRS was changed between 1.89 and 2.10 GeV. In such configuration, the average momentum transfer of the virtual photon was } \langle Q^2 \rangle \approx 0.1 \text{ (GeV/c)}^2 , \text{ and the average cm photon energy was } \langle E_{\gamma}^{\text{cm}} \rangle = 1.1 \text{ GeV which means that } \langle W \rangle = 2.53 \text{ GeV. For the kaon kinematics, the cm scattering angle was } 5.6^\circ \pm 4.5\% \text{ in momentum, while still maintaining their nominal } 10^{-4} \text{ full-width half-max momentum resolution } \sigma . \text{ Calibration of HRS’s was based on precise measurements of the known MM peaks for the neutron (in } \pi^+ X \text{) and for the hyperons } \Lambda(1116), \Sigma^0(1193) \text{ (both in } K^+ X \text{). These three baryons decay through weak or electromagnetic interactions, and the proper widths of their peaks are negligible. Therefore, the observed widths of the peaks directly determine the MM resolutions at the corresponding momenta of the registered meson (} \pi^+ \text{ or } K^+ \text{). Then, the resolution of HRS for the } \Lambda(1520) \text{ may be determined by extrapolation of those resolution values to the MM region of } 1520 \text{ MeV. Such a procedure gave the resolution } \sigma = 1.5 \text{ MeV } \sigma . \text{ We apply this resolution when fitting the data. Further experimental details can be found in Refs. [4, 6, 8].}

III. PRESENT ANALYSIS OF THE \( \Lambda(1520) \)

The main initial goal of the Hall A experiment [6] was to search for possible very narrow resonances with \( \Gamma \sim 1 \text{ MeV} \). Therefore, the MM spectra were scanned with 1 MeV steps. This is not adequate for the much broader resonances. To avoid strong fluctuations and excessively large statistical errors for every experimental point, we use now the 4 MeV bin size to analyze the data for \( \Lambda(1520) \). Such a size is adequate for the resolution with \( \sigma = 1.5 \text{ MeV} \), it provides smaller statistical errors and fluctuations.

Generally, we describe the MM spectra in the form

\[
\text{Fit} = BW + BG ,
\]

where \( BW \) is the Breit-Wigner (BW) contribution for the resonance \( \Lambda(1520) \), and the term \( BG \) combines all other contributions (including other possible resonances), which provide a background for the \( \Lambda(1520) \).

The BW contribution may be written as

\[
BW = A_{BW} \Gamma(M_X) D(M_X) .
\]

In the non-relativistic form, we have

\[
D_{\text{non}}^{-1}(M_X) = (M_X - M_0)^2 + \Gamma^2(M_X)/4 .
\]

In the relativistic form, this denominator should look like \( |M_X^2 - [M_0 - i \Gamma(M_X)/2]|^2 \), but usually \( \Gamma^2 \ll M_0^2 \), and \( D^{-1} \) may be approximately written in the form

\[
D_{\text{rel}}^{-1}(M_X) = (M_X^2 - M_0^2)^2 + M_0^2 \Gamma^2(M_X) .
\]

With the energy-independent width, we just define \( \Gamma(M_X) \equiv \Gamma_0 \). Parameters \( M_0 \) and \( \Gamma_0 \) are the conventional BW mass and width of the \( \Lambda(1520) \).

The energy-dependent width has a more complicated structure. Let us recall that the total width is the sum of partial ones for all decay modes, \( \Gamma(M_X) = \sum_i \Gamma_i(M_X) \). Here we emphasize that every partial width should have its own energy dependence, corresponding to the threshold and kinematical properties of the particular decay channel.

Decays of the \( \Lambda(1520) \) are dominated by only three channels, \( K^+ N, \pi^+ \Sigma, \text{ and } \pi^+ \Lambda \). Branching ratios for two other modes, \( \pi^+ \Sigma \text{ and } \Lambda \gamma \), are not more than 1% each [1], and we neglect them here. Also neglected is the channel \( \Sigma^0 \gamma \), with an even lower branching ratio [1]. Further, we define \( \Gamma_0 = \Gamma(M_0) \). Here we also consider \( M_0 \) and \( \Gamma_0 \) as the BW mass and width of the \( \Lambda(1520) \). In all cases \( M_0 \) and \( \Gamma_0 \) are fitting parameters.

Now, for extracting the mass and width of the \( \Lambda(1520) \), we restrict ourselves to the mass interval from 1.45 to 1.65 GeV (this corresponds to 13,070 detected events). To the peak, we apply the BW term [2], while the background is described by various combinations. Further, for the best-fit procedure, we use two not quite equivalent methods, least-squares (min-\( \chi^2 \)) and log-likelihood (LL) ones. In such a way, we obtain several sets of numerical values for pairs \( (M_0, \Gamma_0) \) and can trace their dependence on the assumptions used.
IV. RESULTS AND THEIR DISCUSSIONS

We begin with considering changes of the BW mass $M_0$. They appear to be rather small, though not always negligible. The non-relativistic expression (3) for the BW term provides $M_0$ values lower than the relativistic Eq.(1), at the level of $\sim 0.05$ MeV. Similarly, the LL procedure gives lower $M_0$ than min-$\chi^2$; the difference may be as small as 0.01 MeV, but may reach $\sim 0.15$ MeV. Structure of the background can also shift $M_0$; the difference for various variants which we use is, again, not more than 0.15 MeV.

A more essential effect comes from energy dependence of the width. Description with the energy-dependent width results in $M_0$ about 0.6 MeV lower than for the energy-independent width. Interestingly, it is at the same level as our statistical uncertainty for $M_0$, which is $\sim 0.6$ MeV in all the studied cases.

Changes of the BW width $\Gamma_0$ also show some regularities. Shifts of results for using the (non-)relativistic Eqs.(6) or (1) are not more than 0.06 MeV. The LL fitting gives a lower width than min-$\chi^2$; the difference may be 0.2 MeV, but may reach $\sim 0.9$ MeV (for comparison, our statistical uncertainty for the width is $\sim 2$ MeV).

More complicated is the influence of the background description. Most influential in fitting our data is contribution of the resonance $\Lambda(1405)$. Its exclusion diminishes $\Gamma_0$; the difference may be up to $\sim 1$ MeV (the corresponding shift of $M_0$ is much smaller, not more than 0.1 MeV).

Now we are able to formulate some conclusions, which may have a more general character.

- The width of $\Lambda(1520)$ is sufficiently small, so the relativistic and non-relativistic forms give practically the same values of $M_0$ and $\Gamma_0$ (at the present level of accuracy).
- By definition, the LL fitting always provides a larger value of $\chi^2$, than the min-$\chi^2$ fitting. However, formally they should be equivalent at asymptotically high statistics. In this sense, the present statistics is not asymptotical yet. Both $M_0$ and $\Gamma_0$ are different in the two fittings, the differences are comparable to the statistical uncertainties of those BW parameters. In terms of $\chi^2$ per degree of freedom, $\chi^2$/dof, which is typically $\sim 1.5$ in our studies here, the LL fitting is up to 0.05 higher than min-$\chi^2$.
- The $M_0$ has a smaller statistical uncertainty and is less affected by any change of fitting procedure than the $\Gamma_0$. The width is especially sensitive to the background form.

For extracting our resulting BW parameters, we use the BW relativistic expression (2), (4) with energy-dependent width (both points are theoretically motivated to be more reasonable). The corresponding least-squares (min-$\chi^2$) fit in the mass interval 1.45 - 1.65 GeV is shown in Fig. 1 together with all separate contributions. This fit corresponds to $\chi^2$/dof=1.46 and gives

$$M_0 = 1520.4 \pm 0.6 \text{ MeV} , \quad \Gamma_0 = 18.6 \pm 1.9 \text{ MeV} ,$$

with pure statistical uncertainties. Each of the two BW parameters has also a systematic uncertainty. It is about 1.5 MeV, mainly due to the mass scale uncertainty, for $M_0$, and about 1 MeV, mainly due to the fitting procedure ambiguity, for $\Gamma_0$.

For comparison, the same description with the LL fit gives slightly lower values

$$M_0 = 1520.3 \pm 0.6 \text{ MeV} , \quad \Gamma_0 = 17.8 \pm 1.9 \text{ MeV} ,$$

and the higher value $\chi^2$/dof=1.50.

Even without accounting for the systematic uncertainties, both $M_0$ and $\Gamma_0$ are in reasonable agreement with previous experimental values and with their RPP average [1]. It is worth to note, however, that the uncertainties in the later works used in RPP are larger than those in the earlier works. This may hint that the uncertainties stated in the earlier works (and, therefore, in the average) are too optimistic.

Now we can discuss position of the $S$-matrix pole corresponding to the $\Lambda(1520)$, that has never been discussed for hyperons. It is determined by vanishing of the denominator (4), i.e., by a solution of the equation

$$(W_p^2 - M_0^2)^2 + M_0^2 \Gamma^2(W_p) = 0 .$$

This non-linear equation may have non-unique solutions. Physically reasonable is only one of them, close to $(M_0 - i \Gamma_0/2)$. It is convenient, therefore, to rewrite Eq.(7) in the form

$$W_p = M_0 [1 - i \Gamma(W_p)/M_0]^{1/2} .$$

Its complex solution gives the pole mass $M_p = \text{Re} W_p$ and the pole width $\Gamma_p = -2 \text{Im} W_p$. 

FIG. 1: (Color on line) Fit to the experimental MM distribution is shown by the solid line. The short-dashed line is the BW contribution for the $\Lambda(1520)$ (note an asymmetric form, due to the energy-dependent width). The total background (dot-dashed line) is the sum of the linear binomial (long-dashed line) and the BW tail of the $\Lambda(1405)$ (dotted line).

Numerical solution with values $^5$ for $M_0$ and $\Gamma_0$ gives

$$M_p = 1518.8 \text{ MeV}, \quad \Gamma_p = 17.2 \text{ MeV}.$$  \hfill (9)

Note that $M_p < M_{BW}$, $\Gamma_p < \Gamma_{BW}$, with the mass difference exceeding the statistical uncertainty. Such relation for masses may be rather general, as suggested by comparison with the mass pairs (BW and pole) shown for $\pi N$ resonances in Listings of RPP $^1$. This assumption is supported by the analysis of Eq.\( ^5 \) presented in Ref. $^4$.

In summary, we have found the mass and width of the $\Lambda(1520)$ in near-forward electroproduction. The extracted BW parameters of the resonance are shown to depend not only on experimental data, but also on the way of their treatment. The extracted width is more sensitive to the various treatments than the mass. For the $\Lambda(1520)$, the non-resonance background should be accurately studied and understood if one intends to extract the mass and, especially, width with uncertainty of order 1 - 2 MeV. Having the BW mass and width $^5$, we also give the first estimate $^9$ of the pole parameters for the $\Lambda(1520)$. The pole values for both mass and width tend to be lower than the BW values.

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