The Rüchardt experiment revisited: using simple theory, accurate measurement and python based data analysis

Chris Shearwood and Peter A Sloan

Department of Physics, University of Bath, Bath, BA2 7AY, United Kingdom

E-mail: p.sloan@bath.ac.uk

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Abstract
This project uses the Rüchardt experiment to determine the ratio of specific heats and hence the number of degrees of freedom $f$ of different gases by measuring the frequency of damped simple harmonic motion where the gas provides the Hooke’s law like spring of a cylinder–piston system. This project links mechanics, electromagnetism, thermodynamics, statistical mechanics and quantum mechanics making it an excellent synoptic experiment for a year 2 undergraduate student and we detail our implementation of this team-based learning project. We present, for the Rüchardt experiment, simple derivations of the main relationships that govern the experiment, a detailed data analysis of the physics of the apparatus and the experimental data. We find $f(\text{He}) = 3.48 \pm 0.14$, $f(\text{N}_2) = 4.92 \pm 0.24$, $f(\text{Air}) = 4.96 \pm 0.25$, $f(\text{CO}_2) = 6.46 \pm 0.39$ at room temperature and atmospheric pressure. The results for CO$_2$ requires a statistical analysis of its vibrational modes. These results show that the expected results can be measured using fairly simple apparatus, coupled with careful analysis of large data sets.

Keywords: Ruchard’t’s experiment, adiabatic coefficient, piston, damped motion, dissipation

(Some figures may appear in colour only in the online journal)
1. Introduction

1.1. Teaching aims

The Rüchardt experiment [1] links quantum mechanics, Newtonian mechanics, statistical physics, thermodynamics, and due to our set-up electromagnetism. It is an ideal mid-year undergraduate physics experiment allowing physics and natural science students to use the wide variety of their taught physics, experimental techniques and computational analysis they have already been exposed to. This version of implementing and analysing the Rüchardt experiment therefore aims to bridge the gap between the more scripted and well-defined experiment they typically meet in their early years laboratory courses and the typically more open-ended, less defined projects, students tackle in their latter and final years.

Here we present our implementation of the Rüchardt experiment with particular emphasis on understanding the experimental apparatus and how it transforms the physical parameter in question, into a measurable quantity. We present detailed experimental results and explore the range over which they are valid before presenting the final results. We highlight places where basic student written computer simulation would underpin and strengthen the students’ understanding. We also highlight a few of the common difficulties that students encountered.

Critical to the success of this project, as a teaching activity, is that the students are already familiar with the physics concepts required for the project. It represents the realistic combination of, to the students, seemingly disparate aspects of their course into a coherent scientific narrative that is the ultimate learning outcome of this project.

2. Implementation of ‘team based learning’ project work

Here we will detail how the project is run in practice. A more full investigation of the teaching impact of this project is beyond the scope of this publication.

2.1. ‘Extended physics projects’ versus ‘physics experiments’

In a typical undergraduate physics year 1 or 2 experiments, students work in pairs and follow a script to use a particular item of apparatus to measure a physical property. However, in most scientific works there are usually a team of researchers working together on some common goal, for example, I typically work alongside another experimental physicist, a computational physicist, and a theoretical physicist. With one person taking on the project manager role to ensure, as smooth a possible process of investigation and reporting. One key point is that the roles we take are the roles we (in general) are good at and are interested in. By the time students are in the second half of their second year they too usually (rightly or wrongly) have an idea of whether they prefer the broad scope of experimental, computational or theoretical approaches to science.

We therefore re-wrote and re-branded a subset of our regular ‘Physics Experiments’ into what we call ‘Extended Physics Projects’ of which the Rüchardt experiment is one. Each such project has scope for an in-depth experimental investigation, for simple computational simulation or advanced data analysis, and for a theoretical underpinning. Each project is based on physics the students have already met so that their work will be focused on the scientific process and ‘doing’, not on learning some new aspect of physics (although that will naturally come depending on how far they get). The learning outcomes are that the students will have the ability to:

- Work efficiently as part of a team.
• Approach a scientific question from several directions and appreciate how each depends on the other and each adds to the whole.
• Present their individual work, both as a presentation and written report, setting it within the work of the whole project group.

Two other ‘Extended Physics Projects are the Haynes–Shockley experiment [2, 3] and variable star astronomical observations using small telescopes [4, 5].

The Haynes–Shockley experiment measures the diffusion, lifetime and drift of charge carriers in a semiconductor. Here students can measure the charge carriers using storage digital oscilloscopes. They can simulate a 1D random walk and mathematically derive a 1D diffusion equation to fit into the experimental and simulated data.

For the variable star astronomical observations, students determine which set of variable stars can be seen from our undergraduate Bath Physics Observatory (BPO) and which one is the optimum star on a particular night. They produce an observing plan, simulating exposure times, etc the number of exposure, repetition rates, and then take the photographs at the observatory over one or two nights observing. They also write a Python-based analysis pipeline to analyse existing data to extract the time-dependent apparent magnitude of a variable star and then apply it to their own data.

2.1.1. Project briefings. Each project has a brief outline of the overall goal of the project, approximate method of experimentation and a list of points that students can consider and follow if they wish. We have also devised a ‘problem sheet’ for each project with various show-that questions to help guide students at the beginning of their projects.

2.1.2. Choice of project and roles. From this briefing note, students submit a ranked order of preference of project choice and teams of 5 students are assigned to each project (with some project running with two or three separate teams) with the maximum number of first choices. Given the constraints of apparatus, staff and timetabling and the usually increasing year-on-year student numbers, in the future we may employ an automated Monte-Carlo approach to the Team assignments to maximise student ‘happiness’ given our constraints [6].

Once the teams were formed, students were then asked to choose two of the following roles, experimentalist, theoretician, computational physicist, experimental analyst and project manager. For each role, we provided a video recording of a short interview with a member of the department who was an expert in that role.

2.2. Weekly sessions and meeting

Students were expected to spend in total 64 h of time on the project, including the preparation of the presentation and report. 36 h were timetabled to be in the laboratory or the computer lab. The other time is for the students to work at home in whatever fashion they wished.

Projects ran for 8 weeks of the semester. Each week, each Team met their designated academic for 30 min supervision sessions. These meetings would ideally be run by the student project manager, but in practice were run by the academic. The technical lab staff were ever present in the main undergraduate laboratory and supervisors could of course meet their teams outside of these formal 30 min sessions. In fact, it was a pleasure to randomly drop into the laboratory or computer lab and find out what the Teams were up to.

2.3. Assessment

Assessment was at an individual student level and was based on only two outputs.
2.3.1. Presentations. Two sets of conferences were run, one at the beginning of the project, and one near the end. Each team member had to give one 7 min presentation (no group presentation). With three of the Team members presenting at an early conference, and two at a later conference. Teams decided who spoke when. Each conference session had presentation contributions from five Teams, and students of those Teams had to attend all the talks given in that sessions. Sessions were chaired by either an academic or a technician. All sessions were recorded and those recordings were made available for all the students to watch.

Marks were awarded for

- Attendance: 100% if they were there for the full session, 0% if not. Therefore a late student scored 0% for attendance.
- Giving feedback and asking a question. Each student was called upon at least twice in the session to give feedback to a speaker and ask a scientific question. If their feedback and question were entirely generic they scored 50%, if they were more insightful they gained 100%.
- If the presentations had a clear take-home-message the presenter was given 100%, if not then 50%.

This had the effect of having an attentive audience that the speaker was really speaking to—they did not just speak to the marker. That feedback was instant and from the students. The students were quite relaxed about the assessment and marking as most could easily score 100%. It was a delight to see so many of them enjoy the process and enjoy listening to their peers.

Due to strike action, one session was run by the students themselves. The Project Managers were instructed to chair the session and pass the microphone around for attendance and questions. Once back, the academic simply watched the recorded session and gave out the marks. It was instructive to see how something that was approached with dread by staff and students, oral presentation marking and feedback, was turned into an exciting, engaging, economic and useful activity for both staff and students.

2.3.2. Reports. To finish, each student submitted a Physical Review Letters style 7-page report, using a RevTex template, on the project focusing on their work and understanding. Students could ask for feedback on reports from their academic supervisors, each other and their regular physics tutors. The reports followed the exact same style and formatting students had used in the previous semester and in another assessment that focused on using LaTeX for report writing and python for data analysis and graphical presentation. This final report was therefore treated as an ‘exam’ of all the techniques they had learnt and gave them ample scope to produce written work of the same quality as their scientific work.

3. Introduction to the Rüchardt experiment

The central element of the Rüchardt experiment is using a gas as the spring in a piston–cylinder apparatus to produce simple-harmonic-motion (SHM) of the piston when displaced from equilibrium, see figure 1. The frequency of that SHM can then be related to the pressure of the gas and that in turn to the adiabatic ratio of the gas. Hence a macroscopic and easily measurable quantity can be traced back to the microscopic quantum mechanics of the gas particles.
From its initial reporting in 1929 [1] there has been a periodic updating of both the experiments [7–11] and its theoretical basis [12]. There has been a propensity to update the analysis and data-capture as more sophisticated data-capture and computer techniques become commonplace in undergraduate teaching laboratories, for example, with the advent of digital computing in teaching laboratories [13]. It is available to purchase through several laboratory supply companies, LabTrek and LD Didactic. We have used it at the University of Bath as both a year 1 and year 2 physics experiment. In the recent COVID instigated pivot to online laboratory courses, it was one of the experiments that turned into an ‘online’ experiment both at Bath and at Uppsala University in Sweden [14].

4. Theory

4.1. Damped SHM

The apparatus is in practice lightly damped so we actually measured damped SHM (dSHM). From most introductory dynamics or mechanic courses, we know that the solution to dSHM is

\[ z(t) = \Delta z \cos(\omega_d t - \alpha) \exp(-t/\tau), \tag{1} \]

where \( \Delta z \) is the starting displacement from the equilibrium position (See figure 1(B)), \( t \) is the time, and \( \alpha \) is some initial \( t = 0 \) phase offset. The piston release is usually taken as \( t = 0 \) so in general \( \alpha = 0 \). Finally, \( \tau \) is the decay-time, the time it takes for damping to reduce the amplitude to \( 1/e \) of the initial amplitude. The damped angular frequency \( \omega_d \) is related to the natural frequency \( \omega_0 \) of the undamped oscillations by

\[ \omega_d = \sqrt{\omega_0^2 - (1/\tau)^2} \tag{2} \]
with

$$\omega_0 = \sqrt{\frac{k}{M}},$$

(3)

where $M$ is the mass of the object doing the motion (the piston) and $k$ is the spring constant related to the SHM, which needs to be related to the properties of the gas.

### 4.2. Adiabatic process and degrees of freedom

An adiabatic process has no change in total energy, i.e. heat does not flow into or out of the system [15]. Here, to minimise the heat flow the cylinder and piston are made of glass, a poor thermal conductor. The experiments themselves only last a few seconds at most. As Fourier’s law of heat conduction states, any heat loss will be proportional to the temperature difference between the gas and the glass. Below we will propose that only in the most extreme experimental conditions (very small volume) would there be a large enough temperature difference to drive substantial heat transfer. There is some suggestion of this nonadiabatic nature in the volume dependence of the decay-time at small volumes.

If there is no heat transfer into or out of the gas system, then the work done by the piston on the gas (or vice versa) will be adiabatic. We therefore start with the well-known expression for an adiabatic process

$$pV^\gamma = \text{constant},$$

(4)

where $\gamma$ is heat capacity ratio,

$$\gamma = \frac{C_p}{C_V}$$

(5)

between the constant pressure heat capacity $C_p$ and constant volume heat capacity $C_V$. This then leads to the expression for $\gamma$ that depends on the identity of the gas particles through its number of degrees of freedom $f$

$$\gamma = 1 + \frac{2}{f},$$

(6)

We will discuss the significance of $f$ and its temperature and molecular dependence in section 7, but fundamentally it means that a measurement of the macroscopic and thermodynamic property $\gamma$ gives measurable insight into the structure and properties of the individual gas atoms or molecules. This is why this experiment is ideally suited to bridge the gap between scripted experiments and open-ended projects. It links easy-to-measure properties (the piston oscillating) to the microscopic, or quantum, properties of the individual atoms and molecules.

To estimate the accuracy required to determine significant difference between the four gases measured, table 1 presents the number of degrees of translational and rotational freedom each gas has. Vibrational degrees of freedom are discussed in section 7. Here we assume air is predominately nitrogen and oxygen, both diatomic molecules. This simple analysis suggests $\gamma$ will have to be measured to within an uncertainty of 10% and preferably closer to the 1% level.
4.3. From an adiabatic process to Hooke’s law

At equilibrium we have that

\[ p_0 V_0^\gamma = \text{constant}. \tag{7} \]

and the system is static because the pressure of the gas exerting an upwards force on the piston is equal to the downward force due to atmospheric pressure and the weight of the piston itself. We can therefore write that

\[ p_0 = p_{\text{atm}} + Mg/A, \tag{8} \]

where \( g \) is the gravitational acceleration and \( A \) the cross-sectional area of the piston. Atmospheric pressure was measured at the beginning of each experimental session, see table 2. Due to the weight of the piston being much lighter than the ‘weight’ of the atmosphere, the pressure correction is negligible, \( \sim 1 \text{kPa} \) compared with typical atmospheric pressure of \( 101 \text{kPa} \).

The net force \( F \) on the piston caused by a displacement, that is a change in gas-volume from \( V_0 \) to \( V \), is given by,

\[ F = p_0 \left( \frac{V}{V_0} \right)^\gamma A - p_0 A. \tag{9} \]

A computer simulation could employ this full expression, using for example the Verlet algorithm, at each piston position and so numerically find the time-dependence of the piston. However, here, to allow a match to SHM, we consider small volume displacements \( \Delta V \) giving

\[ V = V_0 + \Delta V. \tag{10} \]

A Taylor expansion of the force with such a small volume change \( \Delta V \) around the initial volume \( V_0 \) gives,
\[ F(V_0 + \Delta V) \approx F(V_0) + \Delta V \frac{dF}{dV} \bigg|_{V=V_0} + (\Delta V)^2 \frac{1}{2} \frac{d^2F}{dV^2} \bigg|_{V=V_0} + \text{small terms.} \]  

From equation (9), \( F(V_0) = 0 \) and

\[ \frac{dF}{dV} \bigg|_{V=V_0} = -\gamma \frac{p_0}{V_0} A. \]  

and

\[ \frac{d^2F}{dV^2} \bigg|_{V=V_0} = (\gamma^2 + \gamma) \frac{p_0}{V_0^2} A. \]  

Thus the net force acting on the piston caused by a small change in volume is

\[ F(\Delta V) \approx -\gamma p_0 \left( \frac{\Delta V}{V_0} \right) A + \frac{1}{2} (\gamma^2 + \gamma) \frac{p_0}{V_0^2} \left( \frac{\Delta V}{V_0} \right)^2 A. \]  

From this and taking only the 1st order, \( \Delta V \), term and typical values for helium, at 100 ml initial volume the maximum increase in pressure due to the oscillations will be \( \approx 4 \) kPs and at the smallest initial volume of 10 ml a pressure increase of \( \approx 40 \) kPa. Well within the safety limits of the glass apparatus.

For UK students the Taylor expansion is covered in the A-Level school course. At University it is generally included in year 1 maths for science courses and also touched on (at least in our institution) in at least two of their year 1 physics courses as well. But what they have not really done is apply it themselves to convert intractable equations into manageable equations that can then be solved and measured. This type of approximation is often key to understanding the physics of a system, but students normally do not see the Taylor expansion as a tool to aid them, but as a mathematical technique to be applied when asked. These projects are an opportunity to let students see these approximations in action, rather than just in exam questions.

Finally to mirror a Hooke’s law spring, we set \( \Delta V = A \Delta z \) and \( V_0 = z_0 A \) to give

\[ F(\Delta z) \approx -\gamma p_0 \left( \frac{\Delta z}{z_0} \right) A + \frac{1}{2} (\gamma^2 + \gamma) \frac{p_0}{z_0^2} \left( \frac{\Delta z}{z_0} \right)^2 A. \]  

If we assume \( \gamma \sim 1 \) for all gas particles, we can see that for say a 1 in 100 deviations from a linear response, e.g. where the linear term is 100 times larger than the quadratic term, we need that \( \Delta z/z_0 \sim 1/100 \). The normal experimental procedure was to displace by about 5 mm and release. The apparatus is made so that 1 mm of height corresponds to 1 ml of volume. At 100 ml volume, the cylinder is 100 mm high. Therefore for any low-volume measurements, well below 100 ml the response could significantly deviate from a linear response.

In the linear, or 1st order, regime we can identify that

\[ F = -k \Delta z \]  

with

\[ k = \gamma \frac{p_0 A^2}{V_0} \]
and \( \omega_0 = \sqrt{k/M} \), hence we have

\[
\omega_0 = \sqrt{\frac{p_0 A^2}{\gamma V_0 M}}.
\]

The pressure \( p_0 \) is a constant set by the atmospheric pressure, the mass of the piston is a constant, as is the area \( A \) of the piston. Therefore measuring the response \( \omega_0 \) over a range of (experimentally controlled) initial volumes \( V_0 \) will allow a measurement of the adiabatic ratio \( \gamma \).

Note, the same analysis for a purely diabatic process, that is one at constant temperature, gives the result \( \omega_0 = p_0 A^2 / V_0 M \), which means that if there is any heat flow from or to the apparatus it will naturally lead to a lowering of the measured value of \( \gamma \) towards 1.

From the ideal-gas law and the adiabatic relationship of equation (5) the gas temperature is given by

\[
T = T_0 \left( \frac{V_0}{V} \right)^{\gamma^{-1}}
\]

where \( T_0 \) is the initial temperature of the gas. For small volume changes around \( V_0 \) this can be Taylor expanded and so approximated as

\[
\Delta T \approx T_0 (1 - \gamma) \frac{\Delta V}{V_0}.
\]

The temperature change due to the oscillation will therefore be largest for gases with the largest \( \gamma \) value (e.g. helium) and at the smallest volumes. For helium it will be \( \Delta T \approx 9\,^\circ C \) at \( V_0 = 100\,\text{ml} \) and \( \Delta T \approx 90\,^\circ C \) at 10 ml, whereas for carbon dioxide both are lower: \( \Delta T \approx 4\,^\circ C \) at 100 ml and \( \Delta T \approx 41\,^\circ C \) at 10 ml. These results are broadly in line with, say, the same effect felt when pumping up tyres on a bike where the pump gets hot to touch. Therefore, any nonadiabaticity (driven by the larger temperature gradients from gas to external heat bath) will be more likely at the smaller volumes investigated.

Although we have not done this, it would be interesting and instructive to directly measure the time-dependence of the gas temperature and pressure, for example using a fast response thermocouple or thermistor and a pressure gauge. From this, students could try and use the displacement measurement to reconstruct the temperature and pressure and compare these with the measured values. That is, not only using the equations to compute other parameters but measuring all the parameters to verify that the equations hold.

4.4. Student misconceptions and difficulties

Most students struggled with the idea of a small-scale displacement and how that crucial step transforms a complicated relationship between here, pressure and volume, into a simple relationship with known dynamic outcome. This may have been the first time they were asked to derive a result like this themselves, rather than be given an equation that describes the piston dynamics.

5. Experimental

5.1. Apparatus and procedures

Our apparatus consists of a glass cylinder and a movable glass piston with a good gas-tight seal due to the exact fitting of the piston within the cylinder. This large glass-glass interface
only introduced minimal frictional damping. To measure the motion of the piston three small rare-earth magnets were glued to the top of the piston. As the piston/magnets moved this induced an electromotive force (emf) in a pick-up coil (50 ± 5) mm above the magnets and was measured and captured by a digital-storage oscilloscope. Figure 2 shows (A) a photograph of the whole apparatus and (B) a close-up photograph of the end of the piston and the gas-tight seal.

To ensure a smooth operation and to align the magnet and coil a laser light was passed from above through the coil onto the top of the stack of magnets. This allowed adjustment of the clamps and orientation of the coil to align it to the axis of the magnets as accurately as possible. Moreover, as the cylinder was filled with gas, so the starting point of the magnet would naturally rise. To keep a consistent average emf in the pick-up coil, the coil was moved to ensure a 50 mm gap between the top of the magnet stack and the base of the coil at each initial gas-volume measured. This level of accuracy was only important if a full understanding
of the magnitude of the emf signal was wanted and isn’t critical to the measurement of the adiabatic ratio $\gamma$.

A precise procedure for emptying, filling, purging and filling the cylinder with pure gas was followed. We estimate at most a 1% (ratio of volume of pipework open to air to volume of the maximum fill) contamination with air. This does not effect the air measurements, nor impact on the nitrogen measurements, but can influence on the helium (lowering $\gamma$) and carbon dioxide (raising $\gamma$) measurements, and we see some evidence of that.

To start the dSHM the piston was pushed down by hand by approximately 5 mm, reducing the volume by 5 ml, and released. A digital-storage oscilloscope was used to capture data onto a standard USB memory stick and transferred to a computer for analysis. The oscilloscope was set up to capture a single time-trace based on a trigger. Example time-traces are shown in figure 5. Each gas was measured over 10 initial volumes from 10 ml to 100 ml, see for example figure 6, with each volume measurement repeated 5 times. The volume measurements were taken by reading from the graduated scale of the cylinder the position of the black-seal ring. The uncertainty in all initial volume measurements was estimated to be $\pm 1$ ml. This is the main factor limiting the accuracy of the measurement of the adiabatic ratio $\gamma$.

A careful measurement of the actual volume of the cylinder–piston system to the graduated values found that there was an additional 6 ml of volume to the marked scale. This volume was attributed to the gas inlet tube system and slight curvature in the base of the piston. Therefore for a reported volume of, say 50 ml, the true, or corrected, volume was 56 ml. For ease of reading this correction is only made to the final fitting function of figure 8.

The piston and magnets had a combined mass of $M = (106.68 \pm 0.01)$ g, and the piston had a diameter of $(34.16 \pm 0.01)$ mm. The pick-up coil has a diameter of $(13 \pm 1)$ mm and $n = (1400 \pm 20)$ turns. Gases were of purity 99.8% or better. Local gravitational acceleration was calculated using the International Gravity Formula and using The University of Bath’s latitude and an elevation of the laboratory above sea level giving $g = 9.81134$ ms$^{-2}$. Such a precise value is not required, but was used to prompt students to be aware that not all assumed constants are constant. Showing how the uncertainty on this value propagates to the final answer would be in itself an interesting student challenge.

5.2. Damping and gas leakage

A weakness of our experimental method is the seal between the cylinder and piston. It must be sufficiently gas-tight to prevent leakage (and hence change of volume) of the gas, yet allow the nearly free movement of the piston within the cylinder. The apparatus here is simply sealed by the precise fit of the piston within the cylinder.

At volumes larger than 50 ml we find damping decay-times consistent at $\sim 1$ s (see figure 7) which when compared with the typical dSHM frequency of $\omega_d \sim 130$ rad s$^{-1}$ results in only a minor correction to the undamped frequency, in this example, of $+1 \times 10^{-3}$ rad s$^{-1}$. Below an initial volume of 50 ml there is a much larger damping effect possibly due to a rapid increase in the friction between piston and cylinder, or due to the increase in the temperature of the gas an increase in any nonadiabatic effects.

Figure 3 shows the volume of the apparatus filled with helium and with nitrogen gas naturally changes over time. This is due to the mass of the piston itself producing a pressure difference of $\Delta p = Mg/A = 1.14$ kPa which although much lower than the mean atmospheric pressure of 101.3 kPa is enough to drive gas from the piston through the seal. From this we determine the gas leakage rate as shown in figure 3(B). This is from neighbouring volume and
The leak rate shows a similar low-volume and high-volume regime as does the damping rate, this is unexpected. Given that the pressure difference between gas and atmosphere should be constant in this gas-leak experiment, and that the only change in the apparatus is the position of the seal, we expect a volume-independent leakage rate. Therefore, we speculate that there is an increase in the friction between seal and cylinder below the 50 ml mark. This naturally leads to an increased, and observed, damping with corresponding decrease in the decay-time. Furthermore, there will be an increase in the friction opposing the piston as it slowly descends in the

Figure 3. Gas leakage rates from the cylinder–piston apparatus. (A) The time-dependence of the volume of the system from an initial fill, of both Helium and of Nitrogen gas, to 100 ml as it naturally reduced due to the pressure imbalance due to the mass of the piston-magnet system. (B) Pair-wise simple differential of (A) to show the gas-leak rate.
cylinder leading to a reduction in the pressure difference between the gas and atmosphere and so a reduction in the leak rate.

What we can also see is during a typical oscillating experiment of say, 2 s, there will be at most a 0.1 ml loss of gas from the system. Therefore we can safely ignore such gas loss from the analysis of any one experimental time-trace measurement. In future experiment the volume before and after each dSHM run should be explicitly measured to quantify any gas leakage directly.

5.3. Magnetic field

There are of course many ways to measure the time-dependent movement of the piston. Here to ensure a non-invasive measurement, but based on introductory physics we make use of the electromotive force (emf) generated by a moving magnet. We use a set of small magnets glued to the top of the piston and a pick-up coil connected to an oscilloscope to measure the time-dependence of the induced electromotive force (emf) $\epsilon$. Through Faraday’s law of induction

$$\epsilon = -\frac{d\Phi_B}{dt},$$

where $\Phi_B$ is the magnetic flux through the area $A_c$ of an single turn of our pick-up coil multiplied by the number of turns $n$. If we take the far-field and on-axis description of the magnetic field of a dipole source we have the magnetic field strength

$$B(z) = \frac{2\mu_0}{4\pi z^3}m_B,$$

where $\mu_0$ is the vacuum permeability, $z$ is the distance away from the centre of the magnet and $m_B$ is the magnitude of the magnetic-dipole. Assuming the flux is the same for all the coils, in effect ignoring the height of the coil stack and the slight off-axis nature of the total area of the coils, we have

$$\Phi_B = A_c n \left(\frac{2\mu_0}{4\pi z^3}\right)m_B.$$  \hspace{1cm} (23)

Here a three-dimensional calculation using standard mathematical models the full dipole magnetic $B$-field could be done to convince a student that these approximations are valid in the far-field.

To find a value for $m_B$ the magnetic field strength was measured using a Hall probe at several distances from the magnet as shown in figure 4. On this log–log graph we would expect a linear dependence, which is evident at distances greater than 10 mm. As the experimental gas data used an initial $z_m = 60$ mm separation (approximate midpoint of coil height) with a $z_0 = 5$ mm initial SHM amplitude, we only consider this region (at lower distances the magnetic field measurement is probably sensitive to slight misalignment of the Hall probe both off-axis and off angle and will deviate away from the far-field approximate relationships used here). To ensure an equal weighting of the data within the usual least-square-fitting of SciPi’s ‘curvetfit’ module [16], we first took the logarithm of data to fit the expression $y = \log_{10}(\epsilon) - 3x$ and find $m_B = (0.54 \pm 0.11)$ Am$^{-2}$. That is, the fitted function of figure 4 is, in essence, $1/r^3$ across the data for distances greater than 10 mm.

The time-dependence of the measured signal emf $\epsilon$ is therefore introduced by the time-dependence of the position of the magnet as it oscillates relative to the pick-up coil with dSHM. This combines equations (1), (21) and (23). To solve this fully, the $B \propto 1/z^3$ dependence would need to be taken into account. (If that dependence is taken into account a
more sawtooth like emf waveform is computed from simple simulation and is consistent with some of our early measurements where we used too large an initial amplitude and at too close a position.) Here we make the further approximation that the displacement caused by the dSHM is small in comparison with the overall distance between magnet and coil. Taking a Taylor expansion to first order of equation (23) about a position $z_m$ gives

$$\Phi_m(z_m + \Delta z) = \left( \frac{m_B \mu_0 A_z n}{2\pi z_m^3} \right) - \Delta z \left( \frac{m_B^3 \mu_0 A_z n}{2\pi z_m^4} \right). \quad (24)$$

Now combining equations (1), (21) and (24) for the dSHM, Faraday’s law and the magnetic flux $z$-dependence gives

$$\epsilon(t) = \left( \frac{m_B^3 \mu_0 A_z n}{2\pi z_m^4} \right) \frac{d}{dt} z(t) \quad (25)$$

and so

$$\epsilon(t) = -\beta \left[ \omega_d \sin(\omega_d t + \phi) + \frac{1}{\tau} \cos(\omega_d t + \phi) \right] \exp(-t/\tau), \quad (26)$$

where we combine all the constants into one value $\beta = m_B^3 \mu_0 A_z n z_0^4 / 2\pi z_m^4$. The maximum emf will be just after at $t = 0$ where $\omega_d + \phi = \pi/2$. That would give a predicted maximum emf of $\sim 6$ mV which matches the measured values (see figure 5) of $\sim 6$ mV. Note that to measure the gas properties, as the relationship of equation (18) shows, the absolute magnitude of the emf is not required only the angular frequency.

5.4. Student misconceptions and difficulties

Students found using a digital oscilloscope for triggered acquisition difficult. Many reverted to a more user-friendly system they used in their year 1 laboratory units, but that had poor time and voltage resolution leading to quite severe discretisation.
Many students did not fix on an initial consistent piston displacement nor the height of pick-up coils, leading to data that did not fall within the dSHM approximation. Finally, as the emf signal looked like dSHM and not the slightly more nuanced expression of equation (26), most students struggled with the linkage of the dSHM through the magnet/coil system using Ampere’s law.

In the future, a sheet of problems or more direct hints may guide students to design a more systematic and consistent set of experiments. However, part of the learning outcome of this project is for students to discover that systematic and consistent experiments are fundamental to the scientific method.

6. Results and discussion

Figure 5 shows example emf time-traces for all four gases, (A) carbon dioxide, (B) nitrogen, (C) air and (D) helium, each with an initial volume of 60 ml. As they were all started with a \(~5\) mm compression they all have approximately the same initial conditions resulting in the emf time-traces having approximately the same initial emf value.
Figure 6. Series of typical emf time-traces for nitrogen gas over 10 cylinder volumes from 100 to 10 ml as indicated. To aid clarity each has been shifted vertically by 20 mV. The overlaid red line is, again, a line of best fit to equation (26) that was fitted over a time scale of $2\tau$ for each individual time-trace. The short black vertical line marks the $t = 2\tau$ positions.
All time-traces display the same main features: that there is an oscillation of approximately the same frequency, and an overall decay of that oscillation. To extract precise values for \( \omega_d \), \( \phi \) and \( \tau \) the SciPi [16] package curvefit was used with equation (26). Since each time-trace experiment was started by hand and therefore there will be a natural range of initial \( z_0 \) initial displacements, and since the absolute magnitude of the emf is not required to find the adiabatic gas parameters, all the amplitude terms were simply combined into the one fitting parameter \( \beta \). The (near perfectly) overlaid red curves in figure 5 best fits this model to the various data sets. The fitted values of \( \omega_d \) and \( \tau \) are displayed on the relevant plots.

To allow automated and unbiased fitting, the following procedure was coded into a python script. The maximum emf value was found and this point was set as \( t = 0 \), with all data points before that removed from the fitting data-set (Note, this is therefore a different definition of \( t = 0 \) than used in figure 1. In practice when fitting it is accounted for by a near \( \phi = \pi/\omega_d \) value). The lowest (most negative) data point was then found, and by comparing that time with the time of the maximum emf value an approximate value of \( \omega_d \) was determined. To find an approximate value for \( \tau \) a simple single exponential decay was fitted to the absolute value of the emf signal as a function of time. Finally using these parameters an initial value of \( \beta \) was determined to ensure the \( t = 0 \) value of equation (26) returned the correct \( t = 0 \) value of the measured signal. Using these initial inputs for the curve-fitting routine equation (26) was fitted to the data-set.

It was found (not shown here) that at very low amplitude, below 1 mV, there was a decrease in the damping lifetime. At these very low amplitudes it may be that there is an increase in the damping friction as the system begins to experience the effect of static friction. Therefore, finally, the same model was fitted again, but using the best-fit parameters found from that first fit and only fitting from \( t = 0 \) to \( t = 2\pi \). This can be seen in the extent of the red-line fit and vertical bar for each of the data sets shown in figure 5.

Figure 6 shows emf time-traces for nitrogen gas for all 10 measured initial volumes ranging from 100 to 10 ml in 10 ml steps. The (near perfectly) overlaid red line of the fitting function shows that the model developed culminating in equation (26) accurately describes the experiment across all gases measured and experimental parameters investigated.

Another analysis method that would naturally extract the frequency component would be Fourier transforms (FT) [10, 17] and their time-dependent counterpart of wavelet analysis [18, 19]. However, care would have to be taken with the interpretation of any FT since the presence of the magnet-coil system means the signal is not just dSHM. It would be an interesting, but quite advanced activity, to determine exactly how the full-width-at-half-maximum of an FT correlates with the physical parameters of interest. A time-dependent wavelet analysis could pick-up on the change in the decay-time at low amplitude vibrations which will lead to a change in the damped frequency. As with most physics experiments, there are many routes to interesting and robust analysis and this could provide a useful teaching resource on FT and wavelet analysis that can be compared and contrasted with the time-domain method used here.

Figure 7 presents the three fitting parameters, (A) amplitude of emf, (B) decay-time and (D) undamped angular frequency, for all four gases across all volumes. Each data point and associated error bar is the result of averaging across six time-traces for each set of experimental parameters. The emf amplitude \( \beta \), shown in panel (A) is proportional to the initial displacement \( z_0 \). Since each point is the mean amplitude taken from 6 individual time-traces and as each experiment was started by hand, there is a natural spread of initial displacements and hence amplitudes and associated error bar on each point representing the standard deviation of the six amplitudes. This is unlike the decay-time in (B) and angle frequency in (C) where the values are dependent on the adiabatic physics, and are independent (within our
approximations) from the piston starting position. We note that at very small volumes it becomes quite difficult to depress the piston by hand and this may lead to an inadvertent reduction of the amplitude at low ($V < 25$ ml) volumes.

What is striking is the volume dependence of the decay-time as shown in figure 7(B). It is reasonably constant and high for all gases for initial volumes in the range 100–50 ml. Below 50 ml, however, there is a stark reduction in the decay-time. This is the equivalent of an increase in damping friction. This friction-to-volume dependence, is similar to that proposed to explain the earlier gas leakage rate measurements. It is therefore likely that there is a significant difference in the friction between cylinder and piston within the apparatus for the piston below or above the 50 ml volume mark.

Figure 7. Gas and volume dependence of the parameters (A) emf amplitude $\beta$, (B) decay-time $\tau$ and (C) undamped angular frequency $\omega_0$. Each point and associated error bar is the average over 6 time-traces with parameters taken from the best fit of equation (26). Gases are: carbon dioxide, green solid line; nitrogen black dotted line; air, red dashed line; and helium, blue dash–dot line.
The gas and volume dependence of the undamped angular frequency is presented in figure 7(C). The rather long decay-times, relative to the typical period of oscillation, for volumes at or above 50 ml means there is little difference, typically less then \( +1 \text{ rad s}^{-1} \), between damped and the corrected undamped angular frequency, see equation (2). The frequency of oscillation was higher for helium at all volumes, nitrogen and air were closely matched, while carbon dioxide always had the lowest frequency. To restate for convenience equation (18)

\[
\omega_0 = \sqrt{\frac{\gamma p_0 A^2}{V_0 M}}
\]

where the dependence of the angular frequency on the identity of the gas must be due to the difference in the adiabatic ratio \( \gamma \) and so by extension the number of degrees of freedom of the individual gas. As expected, see table 1, helium should have the highest \( \gamma \) and so the highest \( \omega_0 \), nitrogen and air should be nearly equal and lower than helium.

In simple terms, the work done by the piston moving a certain distance is, for helium, all converted into kinetic energy of the atoms as there are no other degrees of freedom. This leads to the largest pressure increase which is the restoring force of the SHM-like spring system of this Rüchardt experiment. For nitrogen and air, that mechanical work is converted to both kinetic energy and rotational energy. Assuming all our gases behave as an ideal gas, that is no gas-gas interactions, the pressure is dependent on the kinetic energy, not rotational energy, therefore for those gases the same piston displacement will result in a smaller pressure response and hence appear as a softer-spring and associated reduced frequency of oscillation. This kinetic and rotational only analysis implies that carbon dioxide should be identical to nitrogen and air - not what is observed.

To quantitatively determine accurate values for \( \gamma \) for each gas figure 8(A) shows the same angular frequency data as figure 7(C), now on a log–log scale, and with the true initial volume that includes the extra 6 ml offset due to the gas tubing etc. It is clear that as the initial volume decreases, there is a deviation from the linear relationship found at larger volumes. This is plain in the fitted value of \( \gamma \) shown in figure 8(B) where the fitting was performed over a range of experimental results from 100 to 90 ml, to 100 to 20 ml (the 10 ml data was dramatically more of an outlier than the 20 ml data and so omitted from all analysis). Figure 8(C) shows the number of degrees of freedom \( f \) corresponding to each \( \gamma \) value. For the range of fitting up to and including the 50 ml value, there is a near invariant value for \( \gamma \) (and hence \( f \)). This range matches the range of volumes found earlier that do not suffer large damping effects, or possible large nonadiabatic effects. We therefore take our best values as those found from fitting from 100 to 50 ml data. Table 3 shows these values, and the values that would have been obtained with and without the 6 ml correction to the volume.

What is striking is the sensitivity to the initial volume that the number of degrees of freedom \( f \) has. As this is a systematic uncertainty, or offset, it is not accounted for by the normal uncertainty analysis of the various linear least-square fitting routines. To determine an error for this volume uncertainty, the same corrected-volume data was fitted again but with an additional correction volume of \(+1\) ml or \(-1\) ml (As the volume was measured by eye from the graduation marking, \(\pm 1\) ml is an estimation of the uncertainty in the volume measured in this way). The half difference of the final \( \gamma \) and \( f \) values from these fitting was used to determine an uncertainty in the original fitted values of \( \gamma \) and \( f \), see table 4.

The damping decay-time seen in figure 7 shows a clear trend, with carbon dioxide having the longest decay-time (least damping), air and nitrogen almost identical and helium having the shortest decay-time (most damping). For a dSHM system the quality \( Q \)-factor can be
related to the energy loss per cycle $\Delta E$ and the damping decay-time by

$$Q = 2\pi \frac{E}{\Delta E} = \frac{\omega_0 \tau}{2},$$

(28)

where $E$ is the total energy of the SHM system. If we assume the damping is mechanical in nature then we can approximate the energy loss per cycle $\Delta E$ to be the same for all the gases. What will be different is the total energy of the system. Given the equipartition of energy then the internal energy of a gas is given by

$$\text{Figure 8.} \text{ (A) The same data as figure 7(C) but here with a log–log scale to demonstrate the linear and nonlinear dependence of the frequency of oscillation and the true volume (an additional 6 ml). (B) The extracted fitting parameter } \gamma \text{ found from fitting to figure 7 across ranges of volumes: smallest range from 100 to 90 ml, largest range from 100 to 20 ml. (C) The number of degrees of freedom } f \text{ determined from (B). Note for ease of viewing, the data for Air has been slightly shifted in volume for plots (A, B, C) so as to not completely overlap with the nitrogen data. (D) } \tau \times \omega_0 \text{ values of the 100 ml experiment versus the number of degrees of freedom from the mean of the 100 ml to 50 ml values. The dashed line is a fit of the linear function } y = mx.$$
\[ E = f \frac{nRT}{2} \]  

(29)

where \( n \) is the number of moles of the gas. Therefore we could re-cast equation (28) as

\[ f \propto \omega_0 \tau. \]  

(30)

Therefore we should find a linear relationship between \( \omega_0 \tau \) and \( f \). From figure 8(D) we can see a linear relationship and that helium with the lowest \( f \) has the lowest \( \omega_0 \tau \), air and nitrogen are within experimental error of each other and carbon dioxide has the largest \( \omega_0 \tau \) value and largest number of degrees of freedom. The linear fit gives a slope of 102.6 ± 1.7.

Another analysis [20] suggests that the energy dissipation is dominated by heat transfer (nonadiabaticity) of the apparatus. This is somewhat in line with the thermal conductivity (at 300 K) of helium 156 mW m\(^{-1}\) K\(^{-1}\) being much greater than that of nitrogen 26 mW m\(^{-1}\) K\(^{-1}\), air 26.4 mW m\(^{-1}\) K\(^{-1}\) and carbon dioxide 17 mW m\(^{-1}\) K\(^{-1}\) [21]. Therefore any nonadiabatic effects would be more pronounced with helium. It would be interesting to try heavier noble gases, still with \( f = 3 \) but with thermal conductivities closer to the other gases studied, e.g. Argon 18 mW m\(^{-1}\) K\(^{-1}\). This may explain why helium’s measured \( \gamma \) is lower than expected as any nonadiabatic processes will skew the measured value of \( \gamma \) towards 1. Note that this can also be the case for any contamination by air etc.

### 7. Theoretical extension

Most reports of the Rüchardt experiment focus on the measurement of the adiabatic ratio \( \gamma \). Given the deviation of the value for carbon dioxide from an initial simple analysis, and as our implementation of the experiment had some focus on the underlying theoretical aspects, here we show how combining some statistical mechanics and quantum mechanics can lead to a greater insight of the measurement outcome. This is again with a view of demonstrating to
students that many aspects of their taught courses can be intertwined to give a deeper understanding of a scientific result.

Heat capacities, from the standard textbook ref [22], at 300 K of the gases are presented in table 5.

For the four studied gases, the measured experimental values from this work agree remarkably well with the known literature values. Nitrogen and Air are well within the experimental uncertainty. The measured value for carbon dioxide is lower but still within the uncertainty. Only Helium appears outside its expected range. As the heat capacity of helium has no temperature dependence (except near 4 K) we attribute this difference to a combination of air contamination, and perhaps a large nonadiabatic effect.

Heat capacity is dependent on the number of degrees of freedom a gas particle has. All gases have three translational degrees of freedom, each mode contributing \( \frac{R}{2} \) to the constant volume molar heat capacity \( C_{V,m} \), where \( R \) is the universal gas constant. Some have rotational degrees of freedom up to a maximum of 3, each contributing \( \frac{R}{2} \) to \( C_{V,m} \). Some have vibrational degrees of freedom with each vibration contributing \( R \) to \( C_{V,m} \). The constant volume molar heat capacity of a gas is therefore expressed as the sum

\[
C_{V,m} = \frac{1}{2}fR = \frac{1}{2}(3 + n_R + 2n_v)R,
\]

hence,

\[
f = 3 + n_R + 2n_v,
\]

where \( n_R \) is the number of rotational degrees of freedom and \( n_v \) is the number of vibrational degrees of freedom.

If the moment of inertia of a rotation is dependent on the size of the atomic nucleus (~5×10^{-4} Å), rather than the length of a chemical bond (~2 Å) then quantum mechanics dictates that that axis of rotation will be blocked at room temperature. That is, if the energy level spacing of the rotational energy states is much larger than the equipartition of energy \( k_BT/2 \) then that mode of rotation will be inaccessible to the molecule and so cannot effectively store energy—it is blocked—and does not contribute to the heat capacity. Helium has all 3-axes of rotation blocked, the linear molecules nitrogen and carbon dioxide will have one axis blocked and 2 free to rotate. As air is predominately nitrogen and oxygen, both linear diatomic molecules, on average Air will have only two modes of rotation. This analysis explains the measured values of \( f \) for helium, nitrogen and air, but carbon dioxide, a triatomic linear molecule therefore also with \( n_R = 2 \), is measured to be higher than 5 suggesting it has \( n_v = 0.9 \).

Like rotations, molecular vibrations only contribute to the heat capacity when there is enough thermal energy to excite them. Generally, at room temperature this energy is insufficient to allow the molecule to explore its full vibrational spectrum. The ‘classical’

| Gas     | \( \gamma \) | \( f \) |
|---------|--------------|--------|
| CO\(_2\) | 1.293        | 6.83   |
| N\(_2\) | 1.400        | 5.00   |
| Air     | 1.402        | 4.98   |
| He      | 1.667        | 3.00   |
| Ar      | 1.667        | 3.00   |
| C\(_3\)H\(_8\) | 1.127      | 15.78  |
The equipartition of energy assumption of \( k_B T / 2 \) will therefore not be valid, instead we must compute the partition function for each vibration in order to compute a vibration specific \( n_v \).

Carbon dioxide has four normal modes of vibrations, two energetically identical bending modes, a symmetric stretch and an asymmetric stretch, see table 6. For small vibrational amplitudes we can consider each an example of the quantum harmonic oscillator (another example of the Taylor expansion) with a characteristic vibrational energy ladder given by

\[
E_n = \left( n + \frac{1}{2} \right) \hbar \omega, \tag{33}
\]

where \( n \) is the quantum number of the vibration, \( \hbar \) is the reduced Planck’s constant and \( \omega \) is an angular frequency characteristic of the vibration. For a stiff vibration this will be high, resulting in more widely spaced vibrational states in energy, and for a softer vibration the states will be closer together. Therefore for a fixed temperature, here 300 K, it is the relative scale of that vibrational ladder to the equipartition of energy that will dictate if that vibration adds, and by how much, to the number of degrees of freedom and hence the adiabatic ratio.

The standard textbook of chemical information, the CRC Handbook of Chemistry and Physics, has tables of these parameters, but as it is written for a chemistry audience, physics students may struggle in converting from, usually a parameter in cm\(^{-1}\) to the more typical eV energy unit. From tables [23] and [24] of the CRC Handbook and taking \( \hbar \omega = \Delta E \) we can find the information required.

For each mode the partition function \( q^V \) is given by (see [25])

\[
q^V = \frac{1}{1 - \exp(-\Delta E / k_B T)} \tag{34}
\]

and the corresponding number of modes

\[
n_v = \left( \frac{\Delta E}{k_B T} \right)^2 \frac{\exp\left( \frac{-\Delta E}{k_B T} \right)}{\left[ 1 - \exp\left( \frac{-\Delta E}{k_B T} \right) \right]^2}. \tag{35}
\]

Table 6. Details of the vibrational modes of carbon dioxide and nitrogen gases. Schematic diagrams of vibrations, the vibrational excitation energy (\( \Delta E \)), their partitions \( q^V \) function at 300 K and the corresponding number of degrees of freedom \( n_v \) [23, 24].

| Gas | Vibration | Diagram | \( \Delta E / \text{eV} \) | \( q^V \) | \( n_v \) |
|-----|-----------|---------|----------------|--------|--------|
| CO\textsubscript{2} | Bend | | 0.0827 | 1.043 | 0.454 |
| CO\textsubscript{2} | Sym | | 0.165 | 1.002 | 0.069 |
| CO\textsubscript{2} | Asym | | 0.291 | 1.000 | 0.002 |
| N\textsubscript{2} | Sym | | 0.292 | 1.000 | 0.002 |
Finally

\[ f = 3 + 2 + 2 \sum \nu \nu \]  

(36)

Table 6 shows the computed \( \nu \nu \) values at 300 K. As expected the stretching vibration of nitrogen is not excited and so \( \nu \nu \approx 0 \) giving overall for nitrogen \( f = 5 \) in good agreement with our measured value and the literature values.

For carbon dioxide the lower energy bending mode is, however, quite excited, that is, it contributes to the heat capacity, whereas the two stretching modes contribute only a fractional amount. Figure 9 shows for CO2 the temperature dependence of the number of degrees of freedom for each of its three modes of vibrations and its overall total number of vibrational degrees of freedom. The difference in the stiffness of the three modes is evident in the different computed temperature onsets for each mode. At room temperature we can see that although the two identical bending modes are excited they are not fully excited and so do not make a complete (or integer) contribution to the overall number of vibrational degrees of freedom. Instead of carbon dioxide this analysis predicts \( \sum \nu \nu = 0.979 \) noting that there are two bending modes with the same energy. This, therefore means a predicted value of \( f = 6.958 \) which is in near agreement with the literature value of 6.83 and within the uncertainty of our measured value of 6.46 ± 0.39.

A similar analysis could be attempted for the rotational modes, but students may struggle to understand that quantisation, whereas the quantum harmonic oscillator is usually covered in most early years physics courses.

8. Proposed future work

As further work, the gases argon and propane are suggested as they lie at the extreme ends of the available \( \gamma \) values. Argon is a monotonic gas like helium but its larger mass means it had a much smaller thermal conductivity, more akin to carbon dioxide, and so it may be possible to
prevent, or measure, any heat transfer to or from the apparatus. Propane has a large number of degrees of freedom and from that it may be able to empirically determine a $\tau$ versus $f$ relationship to compare with theory. Finally attaching a small vacuum pump to the gas inlet system may reduce the chance of any contamination of the gases.

9. Conclusion

Through careful early years undergraduate level physics analysis of the experimental apparatus, the collected data and underlying theory, we show that the Rüchardt experiment is able to measure and differentiate between the adiabatic ratio, and hence degrees of freedom, of several small gases. We show that a complete description of the work requires mechanical dynamics, electromagnetism, thermodynamics, statistical mechanics and quantum mechanics. This makes it an ideal project to bridge the gap between scripted experiments and more advanced open-ended projects students typically tackle in their final years.

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Data availability statement

The data that support the findings of this study are openly available at the following URL/DOI: https://doi.org/10.15125/BATH-01199 [26].

Credit author statement

Chris Shearwood: Investigation, Resources, Methodology Peter Sloan Conceptualization, Software, Formal analysis, Data Curation, Writing, Visualization.

ORCID iDs

Peter A Sloan  https://orcid.org/0000-0002-0810-8468

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