Photon and dilepton spectra from nonlinear QED effects in supercritical magnetic fields induced by heavy-ion collisions

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Abstract

We discuss properties of photons in extremely strong magnetic fields induced by the relativistic heavy-ion collisions. We investigate the vacuum birefringence, the real-photon decay, and the photon splitting which are all forbidden in the ordinary vacuum, but become possible in strong magnetic fields. These effects potentially give rise to anisotropies in photon and dilepton spectra.

Keywords: Supercritical magnetic field, Vacuum birefringence, Real-photon decay, Photon splitting

1. Introduction

Electromagnetic probes are expected to be penetrating probes of the matter created in the ultrarelativistic heavy-ion collisions. Especially, it is interesting to pursue the possibility if any aspect of the early-time dynamics can be probed. One of the ingredients recently bringing lots of excitments in the early-time dynamics is a strong magnetic field induced by the colliding nuclei \cite{1,2,3}. We would like to point out that the strong magnetic field gives rise to intriguing modifications of the photon properties which arise only in the presence of strong magnetic fields \cite{4,5,6,7}. Effects of the strong magnetic field, as well as the medium effects studied in, e.g., Refs. \cite{8,9}, will be important for the optics in the heavy-ion collisions.

2. Charged-fermion spectrum and resummation in supercritical magnetic fields

The important quantum effect of the magnetic field is the Landau-level discretization of the charged particle spectrum. The transverse energy level is discretized because of the periodic synchrotron motion, while the momentum longitudinal to the magnetic field is still continuous, so that the spectrum of charged fermion becomes anisotropic as $(B = (0, 0, B))$

$$E_n = \sqrt{m^2 + p^2_z + 2neB} \quad (n \geq 0).$$

(1)

This is particularly important to study the photon properties since the modification of the photon refractive index is entirely due to the quantum fluctuations of charged fermions in the Dirac sea responding to the propagating electromagnetic fields of photons.

A consequence of the anisotropic spectrum can be seen in the \textit{birefringence} shown in Fig. 1. Because of the anisotropic spectrum, the response of the charged particles depends on the polarization modes, i.e., the direction of the electric field, and thus the each polarization mode has a refractive index different from the other. As discussed below, an analogous effect called the \textit{vacuum birefringence} will be found in the presence of the magnetic field even without any medium.

To properly include effects of the Landau-level discretization, one needs to carry out the resummation with respect to the number of external-field insertions...
3. Vacuum birefringence and real-photon decay

Refractive indices can be obtained from the vacuum polarization diagram in Fig. 3. In the ordinary vacuum without external field, this quantum fluctuation does not give rise to any modification of the refractive index, because of the Lorentz and gauge symmetries. However, the external magnetic field breaks the Lorentz symmetry, and then the refractive indices get non-trivial modifications such as the polarization dependence discussed above due to the anisotropic fermion spectrum \( \xi \) from the Landau-level discretization.

3.1. Analytic computation of the polarization tensor

The general form of the polarization tensor is complicated due to the resummation. Nevertheless, one can get simple results in some particular limits. Figure 4 shows a summary of the relevant scales in the problem which are specified by the strength of the magnetic field and the photon momentum. For example, one can get a useful approximation in the strong field limit by including a contribution of only the lowest Landau level (LLL) \([4, 5, 13]\). However, the validity of the approximation also depends on the other scale, i.e., the photon momentum, and the LLL approximation will not suffice when the momentum scale becomes large. Actually, the regime of the large momentum and the strong field is relevant for the ultrarelativistic heavy-ion collisions.

The formal expression of the resummed polarization tensor has a gauge-invariant form

\[
\Pi^{\mu\nu}(q^2) = -(\chi_0 P^{\mu\nu} + \chi_1 P^{\mu\nu}_g + \chi_2 P^{\mu\nu}_\perp)
\]  

(2)

where the transverse projection operators \( P^{\mu\nu} = q^2 g^{\mu\nu} - q^\mu q^\nu \), \( P^{\mu\nu}_g = q^2 g^{\mu\nu}_g - q^\mu q^\nu_g \), and \( P^{\mu\nu}_\perp = q^2 g^{\mu\nu}_\perp - q^\mu q^\nu_\perp \) are defined by the metrics in the longitudinal and transverse subspaces \( g^{\mu\nu}_g = \text{diag}(1, 0, 0, -1) \) and \( g^{\mu\nu}_\perp = \text{diag}(0, -1, -1, 0) \), and the longitudinal and transverse momenta \( q^\mu_g = q^\mu g_{\mu\nu} \) and \( q^\mu_\perp = q^\mu g_{\mu\nu_\perp} \). Here, the magnetic field is applied in the third direction.

We could perform analytic computation of the coefficient functions \( \chi_{0,1,2} \) by using relations among special functions \([4]\). The analytic results are expressed by the wave functions of charged particles, namely the associated Laguerre polynomials, which naturally arise in the calculation, and by the summation with respect to the contributions of the Landau levels. The exact expressions of the resummed polarization tensor were missing in the last few decades (c.f. earlier attempts in Ref. \([14]\)). Our general result covers the whole parameter region in Fig. 4 and will be useful to study a wide variety of systems which contain quite different scales, e.g., heavy-ion collisions, neutron star/magnetars, early universe, high-intensity laser field, etc.
3.2. Refractive index in the LLL

By using the polarization tensor, we show the refractive index in the strong magnetic field [5]. To investigate the basic features, let us focus on the lowest Landau level which can be simply generalized by including the contribution of relevant Landau levels contained in the analytic result of the polarization tensor discussed above.

In the LLL approximation [4, 5, 13], two of the three coefficient functions are vanishing \( \chi_0 = \chi_2 = 0 \). This can be understood from the dimensional reduction in the LLL [10] where the charged fermions can fluctuate only in the direction of the external magnetic field (see Fig. 5). The nonvanishing component \( \chi_1 \) has a simple form

\[
\chi_{\text{LLL}} = \frac{e^2}{\pi} \cdot \frac{eB}{2\pi} - \frac{\omega^2}{q_{||}^2} (I(q_{||}^2) - 1),
\]

where the above form of \( I(q_{||}^2) \) is valid when \( 0 \leq q_{||}^2 < 4m^2 \) and can be analytically continued to the other regions \( q_{||}^2 < 0 \) and \( 4m^2 \leq q_{||}^2 \). A remarkable feature is that the \( \chi_1 \) acquires an imaginary part from the arctangent when the momentum goes beyond the invariant mass of the fermion and antifermion pair \( 4m^2 \leq q_{||}^2 \). This indicates decay of a real photon in the presence of external magnetic fields.

The dielectric constant is obtained from the solution of the Maxwell equation or equivalently from the pole position of the resummed photon propagator with insertions of the ring diagrams [4] as

\[
\epsilon = n^2 = \frac{1 + \chi_{\text{LLL}}}{1 + \chi_{\text{LLL}} \cos^2 \theta}
\]

When the dielectric constant has the imaginary part, \( \epsilon = \epsilon_{\text{real}} + i \epsilon_{\text{imag}} \), the refractive index also has both real and imaginary parts \( n = n_{\text{real}} + i n_{\text{imag}} \) which are related as

\[
n_{\text{real}} = \frac{1}{\sqrt{2}} \sqrt{|\epsilon| + \epsilon_{\text{real}}},
\]

\[
n_{\text{imag}} = \frac{1}{\sqrt{2}} \sqrt{|\epsilon| - \epsilon_{\text{real}}},
\]

with the absolute value \( |\epsilon| = \sqrt{\epsilon_{\text{real}}^2 + \epsilon_{\text{imag}}^2} \).

When photons are propagating in the supercritical magnetic fields, the real part of the refractive index can be, e.g., \( \sim 1.4 \) for \( B/B_c = 500 \) which is much larger than the refractive index of air (1.0003) and is comparable to that of water (1.333).

Figure 4 shows the imaginary part of the refractive index. The magnetic field is applied in the vertical direction, and an angle measured from the vertical axis corresponds to the angle between the direction of the magnetic field and the momentum of a photon. A distance from the origin to the red curve indicates the magnitude of the imaginary part. The angle dependence of the red curve indicates an anisotropy of the imaginary part. In this figure, the decay rate is large when the photons are propagating with angles \( \sim \pm \pi/4, \pm 3\pi/4 \), which will potentially give rise to the anisotropic spectrum of the direct photons in the ultrarelativistic heavy-ion collisions.

4. Photon splitting

We discuss another intriguing phenomenon called the photon splitting shown in Fig. 6. In the ordinary vacuum, the splitting of a photon into two photons is prohibited, because Furry’s theorem tells us that the two independent diagrams with clockwise and counterclockwise charge flows cancel for odd orders of the vector current correlators. However, with the help of external magnetic fields, the splitting process becomes possible because the diagrams could have in total an even number of external legs which are provided by an odd number of the external magnetic field and the three dynam-
cal photon lines. Although these diagrams are higher order in the naive order counting of the coupling constant, the strong magnetic field compensates the suppression. In the strong magnetic field limit, the dominant contribution would come from the triangle diagram composed of the three lowest Landau levels. However, we showed that this is not the case. As shown in Fig. 5, these 1+1 dimensional fluctuations can couple only to the polarization mode oscillating in the direction of the magnetic field. However, this splitting (parallel mode oscillating in the direction of the magnetic field) is independent of the magnetic field. However, this splitting (parallel mode oscillating in the direction of the magnetic field) is independent of the magnetic field. This may clarify earlier observations in the strong field limit \[16, 17\]. Details of our investigation will appear elsewhere \[18\].

5. Concluding remarks

We discussed the photon properties in the strong magnetic fields induced by the ultrarelativistic heavy-ion collisions. We would like to note that these phenomena have been discussed also in astrophysics \[13, 14\] and intense-laser physics \[20\]. Heavy-ion physics could be the first to observe such phenomena and get an impact on the interdisciplinary study.

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\[\text{Note the notations of } || \text{ and } \perp \text{ in the literature.}\]