Description and Evaluation Approach for Uncertainty of RS Images Classification

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Abstract With the development of researches on the classification quality of remote sensing images, researchers thought that uncertainty is the main factor that influences classification quality. This study puts forward an approach to uncertainty representation, which is developed from two aspects: formalized description and comprehensive evaluation. First, we complete the classification using fuzzy surveillance approach, taking it as a formalized description of classification uncertainty. Then we introduce a hybrid entropy model for classification uncertainty evaluation, which can meet the requirement of comprehensive reflection of several uncertainties, while constructing the evaluation index from pixel scale with the full consideration of the different contribution to the error rate of each pixel. Finally, an application example will be studied to examine the new method. The result shows that the evaluation results fully reflect the classification quality, when compared with the conventional evaluation method which constructs models from unitary uncertainty and category scale.

Keywords classification uncertainty; formalized description; comprehensive evaluation; hybrid entropy model

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Introduction

Remote sensing images are widely used in many aspects of society today, such as in urban planning, land use survey, environmental improvement, public safety, etc. In the late 1970s, the scientists began to research on the problem of uncertainty in RS images classification, and they gradually thought that, uncertainty is the main factor that influences the classification quality, and only uncertainty itself is determinate. Therefore, the methods for evaluating uncertainty of images classification have drawn extensive attention.

The evaluation methods of RS images classification accuracy have developed from quantitative to qualitative and from non-position to position[1]. In the process, many evaluation methodologies, such as variance analysis technique, multivariable Bayesian estimation technique, and the smallest accuracy analytical methods[2-8], have been designed. Especially, error matrix is the prevailing one because of its strictest statistical characteristics. However, it is also not a good expression of the integrity quality information. In 1992 Foody presented that, for the overestimated to accidental consistency in the Kappa coefficient calculation, the overall classification accuracy is underestimated[9]. Koukoulas and Blackburn (2001) even thought that, Kappa coefficient was not suitable to be a precision in-
dicator as non-probability measurement\cite{10}.

Obviously, the primary limitation of traditional methods comes from their basic assumption: each pixel can be unambiguously classified to a single category, so they neglect the uncertain information. Moreover, the previous researches have been mainly developed from the unitary aspect of uncertainty. As a result, the evaluation index cannot express the comprehensive effect from several aspect of uncertainty, which is ubiquitous and indivisible. In addition, error matrix is based on category scale, which represents the combined effects of all the pixels in a certain category, without taking into account different contributions to the error rate of each pixel. For example, the error rate of built-up land and agricultural land has lower contribution than that of farmland and grassland in classification result.

We focus on the uncertainty modeling to solve the previous statement problem. The RS image of Huangshi district is taken as the study data. Based on a framework of fuzzy set theory and entropy theory, the study developed from two aspects, formalized description and comprehensive evaluation. First, the classification is carried out by using fuzzy surveillance approach, taking the result as formalized description of classification uncertainty. Then hybrid entropy model is introduced for classification uncertainty evaluation, which can meet the requirement of comprehensive reflection of several uncertainties, while evaluation index is generated from pixel scale with the full consideration of different contribution to the error rate of each pixel. This experimental result demonstrates that the proposed method is feasible and reasonable for description and evaluation of classification uncertainty.

1 Formalized description of classification uncertainty

1.1 Uncertainty analysis of RS data

Uncertainty is the degree to which the spatial process and features are hardly decided and the inherent property of the different spatial processes and features in nature\cite{11}. According to the meanings of uncertainty, analyzing the data sources and methods of RS images classification, we can get that the formation of the uncertainty of RS images classification includes two aspects.

One is the inherent random uncertainty in the imaging process, which is caused by the instability of natural phenomena. The earth’s surface is a complicated geographic synthesis with macroscopical order and microcosmic mess, so in the real world, even in the same type of geographic objects, the characteristic spectrum of different individuals can be only similar, but cannot be completely equal. This property in characteristic spectrum of geographic objects reflects as the uncertainty of grey image features, and it put up the phenomena of same object different images and same image different objects.

The second one is the peculiar fuzzy uncertainty in the classifying process. Because the RS data structure is raster pixel with a certain scale, no matter how much the RS satellite resolution can achieve, the geographic features and phenomena covered by a pixel may not be completely consistent. The characteristics of image pixels are the interactional results by various geographic objects, which reflect as mix element.

1.2 Fuzzy surveillance image classification supporting the feasibility

At present, maximum likelihood classification is the major method of RS supervised classification. According to its theory, we can find out that using probability vector to classify pixels into a certain kind of class is not completely affirmative, thus, it is a decision-making course with uncertainty. However, traditional classification methods only consider maximal probability vector. Correspondingly, many useful information of uncertainty are lost, and they are called “hard classifier” classification method. Based on this, in 1992, Goodchild pointed out that, comparable to traditional “hard classification” technology, the uncertainty of RS classification could be expressed by the entire probability vector\cite{12}.

Fuzzy mathematics reflects its superiority in managing this kind of uncertainty conception. To describe the uncertainty which is introduced into maximum likelihood classification, we can make the probability
vector produced in classification as a fuzzy set to deal with. This method greatly expresses the useful uncertainty information which is resulted from the course of classification.

1.3 Formalized description based on Fuzzy Surveillance Image Classification

The essential principle of maximum-likelihood classification is to generate the conditional probability \( p(\omega_x \mid x) \), i.e., the value of each pixel \( x \) belongs to each category \( \omega \), so it can be taken as the membership \( \mu_{\omega} (x) \) in fuzzy set. In this way, the posterior conditional probability can be described:

\[
\{ p(\omega_1 \mid x), p(\omega_2 \mid x), \ldots, p(\omega_i \mid x), \ldots, p(\omega_n \mid x) \} \quad (1)
\]

In this equation, \( p(\omega_i \mid x) \) equals to the membership \( \mu_{\omega_i} (x) \). Therefore, the fuzzy objects can be represented by a planar fuzzy matrix. That is to say, fuzzy matrix can be taken as a formalized description for uncertainty of RS images classification.

![Uncertain polygon](a) Uncertain polygon

![Uncertain line](b) Uncertain line

Fig.1 Uncertainty description of image classification

2 Comprehensive evaluation of classification uncertainty

2.1 Hybrid entropy model supporting the feasibility

Entropy is a useful concept that shows special predominance in measuring the uncertainty of special data. Actually, many scholars have already recognized the value of entropy theory, and have tried to build precision evaluating models based on it such as information entropy, fuzzy entropy, cross-entropy, divergence degree, correlative coefficient, scalar product and Euclidian distance\(^{[13]}\).

From the analysis in section 1.1, we know that there are two kinds of pixel uncertainties in a grid image. One is random uncertainty and the other is fuzzy uncertainty. At present, evaluating models for classification uncertainty are just for the unitary aspect of uncertainty. Therefore, considering the general uncertainties simultaneously generated by random and fuzzy characteristics, we build up a comprehensive evaluation model for the two kinds of uncertainties based on hybrid entropy theory, which can measure the connections of various uncertainties.

2.2 Hybrid entropy model

In practical issues, for measuring the general uncertainties simultaneously aroused by random and fuzzy characteristics in a system, a hybrid entropy model for evaluating uncertainty of special data\(^{[14,15]}\) was created. The model is a combined product space \( R \times F \) determined by random space \( R \) and fuzzy space \( F \). The general distribution functions that represent the two kinds of uncertainties are mappings as \( f : R \times F \rightarrow \{0,1\} \).

(1) First, random space \( R \) is represented by information entropy \( H_r(X) \) in order to measure the average uncertainties of information sources, i.e., the discrete information sources is depicted as: \( \{X \cdot P\} \) \( \{X : a_1, a_2, \ldots, a_n; P(X) : p_1, p_2, \ldots, p_n\} \), \( p_i \) equals the probability of incident \( a_i \), and \( p_i \geq 0 \), \( \sum_{i=1}^{n} p_i = 1 \), then the equation of Shannon information entropy is:

\[
H_r(X) = E[-\log p_i] = -\sum_{i=1}^{n} p_i \log p_i \quad (2)
\]

(2) Second, fuzzy space \( F \) is represented by fuzzy entropy \( H_f(A) \). Assuming there is a discussed area \( U = \{x_1, x_2, \ldots, x_j\} \) and that \( A \) is a fuzzy subset on \( U \), then for \( \forall x \in U \), there is a corresponding
value $\mu_a(x) \in [0, 1]$, and $\mu_a(x)$ is just the membership of fuzzy subset A. Deluca and Termini (1972) brought forward a fuzzy entropy model without regarding the distribution functions of probability:

$$H_f(A) = -k \sum_{i=1}^{n} [\mu_a(x_i) \log \mu_a(x_i) + (1 - \mu_a(x_i)) \log (1 - \mu_a(x_i))]$$

(3) Finally, general uncertainties $R \times F$ are represented by mixed entropy $H_h(R, F)$. The postulates of the process are: the model degenerates to a statistic entropy such as one where there is no fuzzy, or it degenerates to a fuzzy entropy such as one where there is no random. Taking the above postulates into account and combining Shannon information entropy plus Deluca-Termini fuzzy entropy model, we can introduce discrete hybrid entropy $H_h(R, F)$ as:

$$H_h(R, F) = -k \sum_{i=1}^{n} [p_i \mu_i \log p_i + p_i (1 - \mu_i) \log (1 - \mu_i)] + \sum_{i=1}^{n} (1 - p_i) [\mu_i \log \mu_i + (1 - \mu_i) \log (1 - \mu_i)]$$

(4)

The above formulation can meet the demand of postulates. We can conclude the final formulation by decomposing from it:

$$H_h(R, F) = H_s + H_f - H_{cf}$$

(5)

Obviously, in the right part of the equation above, $H_s, H_f$ respectively represent statistic entropy and fuzzy entropy defined in Eq.2 and Eq.3, and $H_{cf}$ represents crossed-entropy which can be regarded as intercrossed effects of random and fuzzy. Accordingly, in a combined space made up by random space and fuzzy space, the general uncertainties, i.e., hybrid entropy equals to the sum of information entropy and fuzzy entropy subtracting crossed-entropy.

### 2.3 Comprehensive evaluation based on hybrid entropy model

(1) In RS images processing, information entropy is used to measure the dispersed degree and uniformity of brightness values of pixels. That is to say, information entropy can be the measurement of random uncertainty in the classification, i.e., it can estimate uncertainty aroused by the phenomena of same object different images and “different objects same image”.

From Eq.2, we can infer that statistic entropy mainly depends on probability density distribution functions of characteristic space. Because vegetation spectra are distributed randomly, spatial eigenvector is a variable normally distributed. The probability density distribution function is:

$$p(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ -\frac{(x - E)^2}{2\sigma^2} \right\}$$

(6)

Combining Eq.6 with Eq.2, we can see that the information entropy lies on distribution variance $\sigma$ and expectations $E$.

(2) In addition, the fuzzy uncertainty of images classification can be measured by fuzzy entropy, which was aroused by the phenomena of mix element.

Eq.3 tells us that the membership function determines the fuzzy entropy mostly. Based on the image classification results by using maximum likelihood method, the posterior probability $p(x_i | \omega)$, i.e., the degree of each pixel belonging to each category, is generated to be the membership $\mu_{\omega}(x)$ in fuzzy sets. In this way, the fuzzy entropy of pixel $x_i$ can be gained from Eq.3 as:

$$H_f(x) = -k \sum_{i=1}^{n} [\mu_{\omega}(x_i) \log \mu_{\omega}(x_i) + (1 - \mu_{\omega}(x_i)) \log (1 - \mu_{\omega}(x_i))]$$

(7)

Here $k= 1$ and $n$ equals to the number of classes.

(3) In general, the comprehensive uncertainty of RS image classification is comprised by random uncertainty and fuzzy uncertainty, which can be evaluated together according to hybrid entropy in Eq.5. In this equation, the parameters to calculate crossed-entropy are the same as that to calculate information entropy and fuzzy entropy. Therefore, the hybrid entropy can be reckoned by adding statistic entropy and crossed-entropy together.

### 3 Result and analysis

#### 3.1 Study data

The study area is the Xisai district of Huangshi City, which is a typical central plain rural area. The RS data
used in this study is SPOT-5 in 2005. Its spatial resolution is 2.5 m×2.5 m and the study area is as big as 2,957×2,401. Also, the image was processed by geometric and radiation correction and orthorectified. Other relevant supporting data include the land-use map in the same year.

3.2 Classification result analysis

The study area is a typical rural land, and is mostly composed of such land types as river, pond, resident, road, farm land and woodland. For convenience, we classify the image into three categories: water (river and pond), construction land (resident and road), and agriculture land (farm land and woodland).

By using fuzzy surveillance classification method, the land-use classification map and the uncertainty description result can be obtained. PCI software is used for this stage. Fig.1 shows the classification result and Fig.2 shows the uncertainty description of image classification. Meanwhile, the values of class correlation matrix, class covariance matrix, and probability are recorded in PCI reports to obtain modeling parameters.

First, CSR Signature Report was used to obtain the statistical information of feature space, including class correlation matrix and class covariance matrix, where we can generate the mean value and the deviation value of the feature space of each category. These values are shown in Table 1.

![The study area in Huangshi City of Hubei Province, China](image)

![The SPOT image of study area in Huangshi City, 2005](image)

**Fig.2 Location of study area**

| Feature space | Mean/m | Deviation (dev) |
|--------------|--------|-----------------|
| Wave band1   | 27.76  | 228.91          |
| Wave band2   | 37.84  | 212.49          |
| Wave band3   | 83.94  | 219.06          |

3.3 Evaluation result analysis

From the fuzzy surveillance classification results, the parameters for evaluation modeling can be obtained. Then the value of hybrid entropy should be calculated according to Eq.5. Fig.3 shows the hybrid entropy values of each pixel.

![Table 1 The information of feature space of each category](image)

A region with the largest variety of hybrid entropy is chosen for discussion and the hybrid entropy of this region is shown in Fig.4.
In Fig. 4, there are two regions $A$ and $B$ whose values of the hybrid entropy are smallest; region $C$ has the largest hybrid entropy value. Therefore, the relationship of hybrid entropy to the uncertainties of pixels can be captured by analyzing the variance among these regions.

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- **Fig. 3 Hybrid entropy values of each pixel**
- **Fig. 4 Hybrid entropy value of a certain region**

(1) If the minimum value in region $A$ and $B$ is zero, we can conclude that the pixels within these regions entirely belong to one category. In these regions, the phenomena of same object different images and same image different objects are almost absent and the number of the mix pixels is near to zero; accordingly, the comprehensive uncertainty is minimal.

(2) Region $C$ has the largest hybrid entropy value. It corresponds to the boundary lines in classification results, where the most pixels are mixed. Moreover, due to the effects of different object spectra, the comprehensive uncertainty is maximal.

(3) In the areas surrounding region $A$ and region $B$, the hybrid entropy values increase as the pixels deviate. In the areas surrounding region $C$, the situation is opposite. It tells that the uncertainty of images classification tends to increase gradually from the region $A$ and region $B$ to outside, and it arrives at peak value when it meets the boundary of categories.

According to the analysis above, some conclusions can be inferred: the more the value of hybrid entropy $H_s(R,F)$ approaches zero, the smaller the pixels uncertainty; contrarily, the more the value of hybrid entropy deviates from zero, the bigger the pixels uncertainty. The variety trend of comprehensive uncertainty is shown in Fig. 5.
4 Conclusion

(1) According to RS images classification based on fuzzy MLC, we can get the classification results, and by reading the classification results file, obtain the expected value, variance, the relevant fuzzy membership file, as well as the useful uncertainty information of all the pixels, taking it as a formalized description of classification uncertainty.

(2) According to the experiment, it is feasible to introduce the hybrid entropy model into comprehensive evaluation on the uncertainty of RS images classification in pixel scale. The index evaluates the random uncertainty caused by same object different images, and same image different objects, and the fuzzy uncertainty resulted from mix element. The experiment results show that hybrid entropy is rational and clear. Also, this evaluation method makes up the deficiency of the traditional evaluation method based on error matrix. It reflects the comprehensive uncertainty of each pixel, which is more according to the reality of different pixels resulting in different error contribution.

(3) Admittedly, the method is far from being perfect. It is noticed that, the study describes the uncertainty of RS images by fuzzy MLC, which is based on probability and statistics, and the hypothesis image features is normal probability distribution. However, in high-dimensional feature space, this hypothesis is untrue. Therefore, the problem, which is concerned with applying the proposed method to a more general situation with feature space of non-normal probability distribution, will be the research orientation in the future.

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