On the probability of the collision of a Mars-sized planet with the Earth to form the Moon

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ABSTRACT
The problem of the formation of the Moon is still not explained satisfactorily. While it is a generally accepted scenario that the last giant impact on Earth between some 50 to 100 million years after the starting of the formation of the terrestrial planets formed our natural satellite, there are still many open questions like the isotopic composition which is identical for these two bodies. In our investigation we will not deal with these problems of chemical composition but rather undertake a purely dynamical study to find out the probability of a Mars-sized body to collide with the Earth shortly after the formation of the Earth-like planets. For that we assume an additional massive body between Venus and Earth, respectively Earth and Mars which formed there at the same time as the other terrestrial planets. We have undertaken massive n-body integrations of such a planetary system with 4 inner planets (we excluded Mercury but assumed one additional body as mentioned before) for up to tens of millions of years. Our results led to a statistical estimation of the collision velocities as well as the collision angles which will then serve as the basis of further investigation with detailed SPH computations. We find a most probable origin of the Earth impactor at a semi-major axis of approx. 1.16 AU.

Key words: celestial mechanics – planets and satellites: general - Moon

1 INTRODUCTION
An assumed giant impact of an additional Mars-sized object (Theia) onto the Earth could have led to the formation of the Moon after the planets already had their actual mass and no more gas was present in the Solar System. Many recent publications deal with this topic, e.g., Asphaug (2014), Nakajima & Stevenson (2015), Quarles & Lissauer (2015), Jacobson et al (2014), Jacobson & Morbidelli (2014) [=JM], Izidoro et al (2014), since the first ideas developed by Hartmann & Davis (1975), Cameron & Ward (1976), and later Canup & Asphaug (2001). Detailed collision scenaria were studied e.g., by Cameron (1997), Canup (2004), Canup (2008), and Canup (2013) where the collision was modelled with the aid of sophisticated SPH codes. In a most recent article Kaib & Cowan (2013) [=KC] the authors concentrate on the feeding zone of the planet to form with respect to the planet’s volatile inventory and isotopic composition. Because of the highly random outcome they ask the question of how deterministic the outcome of the planetary formation is. In fact the correspondence of the results of the different model computations of n-body codes is very small. Most of these modelisations have been undertaken to understand the architecture of our Solar System, which results only from a subset of the chosen initial conditions. In KC the authors estimated the likelihood that the mass of Theia could be equal to the mass of the Earth, but the probability is rather low. Their results coincide well with JM who estimated the collision probability of bodies with comparable masses as being low. According to these results we have fixed the mass of the additional planet (the ‘projectile planet’) to $m_{\text{Mars}}$ for our computations. Other investigations aimed for high velocity encounters e.g. the one by Cuk et al (2012) who assumed high velocity collisions for smaller masses of Theia $(0.025m_{\text{Earth}} < m_{\text{Theia}} < 0.05m_{\text{Earth}})$, but the results of KC show that such an event may not be very probable because a spin rate of the Earth of the order of 2 hours can only be achieved by big impactors – and these events are rare. Because of the same reason the scenario proposed by Reufer et al (2013) where they look for a steeper collision angle is not very probable. KC undertake 150 different simulations with 3 different underlying models: a first model with Jupiter and Saturn on circular orbits, a second one with initially small eccentricities of Jupiter and Saturn, and a third one according to the model of Hansen (2009). Whereas in the first two models 100 self interacting bodies (distributed between 0.5 and 4 AU with small eccentricities) were integrated which then end up as planets, the last one starts with 400 embryos in an annulus between 0.7 and 1 AU and – according to the authors – represent more or less the outcome of the Grand Tack model (Walsh et al 2012). It is therefore appropriate to make such a study – which is orientated versus the collision of a Mars-sized object with the Earth – on the basis of these results.
1.1 A possible formation of a ‘Theia’ in the early planetary system

It is well known that the outcome of computations of the development of the early Solar System depends highly on the initial conditions after the gas in the disk disappeared. Several different approaches lead to different ‘planetary systems’ although all these attempts have been undertaken to understand the architecture of our system (e.g. [Hansen 2009, Izidoro et al 2014]). Our assumptions is based on the outcome of the Grand Tack [Walsh et al 2012] where after the inward migration of Jupiter and Saturn a later outward migration triggered the formation of the terrestrial planets. In our numerical integrations we started with these two gas giants in their actual position and 100 planetesimals randomly distributed between 0.4 AU < a_planetesimal < 2.7 AU with masses in the order of the Moon. In Fig. 1 we plotted the results of one out of 16 simulations where the architecture of this simulated planetary system turned out to be close to the one of our Solar System. Nevertheless there are two important differences: an additional planet (about the size of Mars) between the ‘Earth’ (here with only 60 % of its actual mass) and a ‘Mars’ (with more than the double of its actual mass); the planet at 0.5 AU can be seen as a Venus equivalent. While this is just one example of resulting configurations we use it to motivate our choice of initial conditions described in the section below: a Mars-sized planet between Earth and Mars. The example chosen is not fully artificial when we look at Fig. 2 where all the simulations are combined into one graph which shows the accumulation of planets around 1 AU.

2 DYNAMICAL MODEL AND NUMERICAL SETUP

Our dynamical models for the early Solar System where chosen in the following way:

- **Model 1 [M1]**, consisting only of the terrestrial planets excluding Mercury and an additional planet in between the orbits of Earth and Venus.
- **Model 2 [M2]**, consisting of the terrestrial planets excluding Mercury, the gas giants Jupiter and Saturn, and an additional planet in between the orbits of Earth and Mars.

Because our tests have shown that for the additional planet between Venus and Earth the influence of Jupiter and Saturn can be neglected we just used a 5-body problem for M1. For both models we adopted as initial conditions the current osculating elements of the planets and added a Mars-sized object, Theia, with semi-major axes 0.8 AU < a_Theia < 0.94 AU (M1) and 1.06 AU < a_Theia < 1.4 AU (M2), respectively. Between scenarios we varied Theia’s semi-major axes with δa_Theia = 0.005 AU. The reason of taking the orbital elements of the planets as they are today is the following: we know that for the billions of years into the past the orbits were the same (e.g. [Laskar 1996]) or only slightly different. The additional planet – the possible impactor – should then be in a quasi stable orbit after the formation of the planetary system. To find such an orbit we did not vary the eccentricity nor the inclination and set them to e_Theia = 0.075 and i = 2°. For the other orbital elements of Theia we used randomly chosen values between 0° and 360°. For every fixed semi-major axis 25 such osculating elements were computed as initial conditions. For achieving the highest possible precision with respect to the collision angle and velocity the integration method was the one we have used for many years for most of our computations ([Dvorak et al 2003, 2015, Gliuzzi et al 2013]). The Lie-integration has an automatic step-size control and is well adapted for such kind of computations ([Hanslmeier & Dvorak 1984, Eggl & Dvorak 2010, Lichtenegger 1984, Delva 1985]). With regard to the formation process of the terrestrial planets and the estimated time of collision of Theia with the Earth of 95 ± 32 Myr ([Jacobson & Morbidelli 2014]) the integration time was up to 50 Myrs.

3 EVOLUTION OF TWO SELECTED SAMPLES

To demonstrate the sensitivity of the dynamical evolution in this regions we show a stable seeming orbit and another one very rapidly becoming unstable. Both had initial conditions outside the Earth orbit with only slightly different semi-major axis, and the same initial conditions for the other orbital elements.

In Fig. 3 and Fig. 5 we show two examples of the development of the semi-major axes of a fictitious Theia and the terrestrial planets, one for a stable orbit and another one which is chaotic very soon. The very regular variation of the eccentricity of this stable orbit of Theia is depicted in Fig. 3 (0 < e_Theia < 0.04) which ends in a sudden increase up to e = 0.15, a close approach and even a collision with the Earth after 17 Myrs. In the other example (Fig. 5) we show a chaotic orbit suffering from multiple close encounters with the Earth visible through the chaotic signal of Theia’s semi-major axis already after 1 Myr. Caused by a sequence of very close encounters between Earth and Theia the orbit of the fictitious planet
Collision of a Mars-sized planet to form the Moon

jumps inside the orbit of the Earth after 2.2 Myrs. Also the Earth is suffering from these repeatedly close encounters and moves a little outside which can be explained by the conservation of the momentum of this planet pair. After a capture close to the 6:5 MMR between them (2.2 Myrs) a final collision with the Earth ends the lifetime of Theia. Mars and Venus are not affected at all in this example.

Figure 3. Time development of the semi-major axes of the terrestrial planets and additional planet Theia leading to a collision with the Earth.

Figure 4. Time development of the eccentricities of the Earth, Venus, and in addition the one of Theia (in red) from the example in Fig. 3. Please see text for more.

Figure 5. Time development of the semi-major axes of a highly chaotic orbit leading to an escape after 3 Myrs.

Figure 6. Time development of the eccentricity (y-axis) of a highly chaotic orbit of Theia from Fig. 5 with a subsequent escape after a close encounter with the Earth; the eccentricity of Mars is the regular curve close to \( e = 0.1 \).

Figure 7. Schematic view of the collision parameters impact angle and impact velocity for an Earth-colliding small planet.

4 IMPACT PARAMETERS

The goal of these investigations is to determine the collision probability for an additional Mars-sized planet Theia which could have formed during the early formation (e.g. Grand Tack scenario) between the Earth and Mars or between Earth and Venus. In addition to this we study the impact velocity as well as the impact angle (Fig. 7 – expressed by the impact parameter in different studies [Leinhardt & Stewart 2012; Maindl & Dvorak 2014]).

Together with the mass and composition of the two colliding bodies these two quantities determine the outcome of such a cosmic catastrophic event. In Fig. 8 we show the different impact parameters according to our simulations, where one can see that most of the encounter velocities are just above the escape velocity of the two planets \( (v_{\text{esc}} \sim 9 \text{ km/s}) \) and only one impact happens with a 15 percent higher velocity. The distribution of the impact angles is asymmetric with slightly more head-on collisions; we can explain it by the fact that close to the Earth Theia suffers from a strong acceleration toward the center of the larger planet (we remember the mass ratio \( \mu = 0.1 \)).

The actual collision outcome in terms of fragmentation strongly depends on the impact velocities, impact angles, and the mass ratio \( \mu \) [Leinhardt & Stewart 2012; Maindl et al. 2014]. The number of fragments varies with the impact angle such that even for relatively high impact velocities strongly inclined collisions may lead to two major survivors (hit-and-run) whereas head-on impacts may destroy the involved bodies. In our scenarios the low impact...
velocities suggest surviving bodies and probably a Moon forming from a debris ring.

5 THE OVERALL PICTURE

For the integrated several thousands of orbits and the chosen grid of semi-major axes a fixed eccentricity and a fixed value of the inclination was chosen; the initial mean longitude was varied randomly (see Sect. 2). In Fig. 8 we show the combined results for all computed orbits by plotting the initial difference in mean longitudes (ΔTheia − ΔEarth) versus semi-major axes and color coding the ‘stability character’ of the orbit which was classified as follows: red to light blue circles according to the escape time. These color points stand for an unstable orbit either due a close encounter with Venus (inner part) or due to a close encounter with Mars or even with the Earth. We note such an orbit as one typical example is depicted in Fig. 5. Dark blue circles stand for stable orbits for the whole integration time; the black dots stand for a collisions with the Earth.

According to the chosen initial conditions in semi-major axis we divided the whole domain where the terrestrial planets move in 5 regions: (a) aTheia < 0.875 AU, (b) 0.875 AU < aTheia < 0.94 AU, (c) the region around the Earth, (d) 1.06 AU < aTheia < 1.165 AU, and (e) aTheia > 1.165 AU. In regions b and d we plotted the number of collisions with the Earth for each of the 25 initial mean longitudes per initial aTheia (Fig. 10). We observe that the maximum number of collisions with the Earth is about 36% (9 out of 25 initial conditions), which occurs once for regions b (aTheia = 0.95 AU) and d (aTheia = 1.075 AU), respectively.

In regions a and e many mean motion resonances (MMR) with the Earth act to destabilize the orbits of a hypothetical Theia, see Fig. 11 which also shows the most important MMRs together with the longest duration of stability of the collision orbits in regions b and d. One can see that the number of Earth collisions increases with the decreasing distance to the Earth. We did not make computations close to the Earth (inside and outside its orbit) – region e – with the exceptions of some sample orbits, which have shown that almost all are destabilized very soon after a close encounter or even a collision. Because our interest was to find collisional orbits which were stable for several tens of millions of years in between Earth and Venus and also between Earth and Mars we do not show these results. It is easy to understand that only then an additional planet may have formed in the early formation stages of our planetary system; we showed even an example from our own computations of the formation of the planets in Fig. 11 where a Theia like planet was formed outside the Earths orbit.

It is obvious that close to the inner region e the stability time is short because the Earth is relatively close by, whereas we find times up to 30 Myrs of stability for an orbit before an escape in region d.

Table 1. Collisions in region b, see text for details.

| aTheia | CT-mean | CT-max | CT-min | n  |
|--------|---------|--------|--------|----|
| 0.875  | 3.41    | 6.648  | 1.555  | 3  |
| 0.880  | 6.13    | 6.129  | 6.129  | 1  |
| 0.885  | 12.87   | 18.542 | 2.892  | 3  |
| 0.890  | 8.52    | 12.691 | 4.342  | 2  |
| 0.895  | 7.02    | 13.973 | 3.332  | 4  |
| 0.900  | 8.21    | 23.669 | 0.313  | 3  |
| 0.905  | 1.83    | 4.528  | 0.114  | 3  |
| 0.910  | 2.06    | 2.904  | 1.216  | 2  |
| 0.915  | 1.92    | 3.747  | 0.090  | 2  |
| 0.920  | 4.22    | 6.168  | 1.628  | 4  |
| 0.925  | 9.76    | 15.685 | 0.057  | 3  |
| 0.930  | 0.26    | 0.626  | 0.051  | 3  |
| 0.935  | 0.02    | 0.024  | 0.024  | 1  |
| 0.940  | 0.97    | 4.872  | 0.002  | 9  |
Collision of a Mars-sized planet to form the Moon

5

The longest time to collision [Myr]

| $\Delta r_{\text{Theia}}$ | CT-mean | CT-max | CT-min | n |
|------------------------|----------|--------|--------|---|
| 1.065                  | 0.17     | 0.235  | 0.013  | 2 |
| 1.070                  | 0.52     | 1.255  | 0.008  | 4 |
| 1.075                  | 0.86     | 4.518  | 0.010  | 9 |
| 1.080                  | 3.60     | 10.471 | 0.017  | 8 |
| 1.085                  | 11.10    | 0.314  | 0.104  | 4 |
| 1.090                  | 3.22     | 18.218 | 0.023  | 7 |
| 1.095                  | 8.21     | 14.190 | 0.205  | 6 |
| 1.100                  | 15.20    | 8.838  | 0.117  | 4 |
| 1.105                  | 2.30     | 17.066 | 0.426  | 4 |
| 1.110                  | 2.62     | 6.225  | 0.392  | 3 |
| 1.115                  | 2.59     | 3.580  | 1.259  | 3 |
| 1.120                  | 5.28     | 15.63  | 1.529  | 5 |
| 1.125                  | 23.43    | 23.435 | 23.435 | 1 |
| 1.130                  | -        | -      | -      | - |
| 1.135                  | 10.19    | 14.778 | 1.299  | 3 |
| 1.140                  | 10.12    | 9.691  | 5.27   | 2 |
| 1.145                  | 7.97     | 18.064 | 3.525  | 7 |
| 1.150                  | -        | -      | -      | - |
| 1.155                  | 9.44     | 9.443  | 9.443  | 1 |
| 1.160                  | 31.19    | 31.193 | 31.193 | 1 |
| 1.165                  | 20.26    | 30.895 | 9.617  | 2 |

Table 2. Collisions in region d

| region | $\Delta r_{\text{Theia}}$ in AU | stable | eject | collision |
|--------|---------------------------------|--------|-------|-----------|
| a      | 0.750-0.875                     | 50.50  | 47.25 | 2.25      |
| b      | 0.875-0.940                     | 47.08  | 40.61 | 12.31     |
| c      | 0.940-1.060                     | -      | -     | -         |
| d      | 1.060-1.165                     | 44.73  | 29.27 | 26.00     |
| e      | 1.165-1.350                     | 52.16  | 45.12 | 2.72      |

Table 3. Statistics of collisions (units percents)

The results for the mean time before a collision occurs are shown in Fig. 12. It is obvious, that close to the inner region c this time is short because the Earth is relatively close by, whereas this mean collision time increases in regions b and d.

6 CONCLUSIONS

Thousands of orbits of a hypothetical additional planet Theia in the early phase of the Solar System were integrated and classified according their dynamics. The aim was to find an orbit stable for sufficiently long time (tens of Myrs) inside or outside the orbit of the Earth which then could lead to a collision building a companion of the Earth. The impactor was assumed having the mass of Mars which then – after a collision – could lead to an additional body like our Moon.

Tabs. 1 and 2 summarize our results with respect to the mean, maximum and minimum collision times for the inner region b and the outer region d, respectively. The last columns show the number of Earth collisions of Theia, which is always smaller than 40 percent of the 25 integrated orbits for one fixed semi-major axis. Finally, Tab. 3 shows the overall statistics. Approximately 50 % of all integrated orbits (in total about 2000, the different tests in regions a and e included) turned out to be stable up to the chosen integration time of up to 50 Myrs. The same almost 50 % in these two regions escape due to close encounters either with Venus or Mars (we did not count the number of Venus and Mars collisions) and only 2 % suffered from impacts on Earth. In regions b again approx. 50 % turned out to be stable, but the number of Earth colliders was increased to 12 %. Region d is the best candidate for a collision of Theia with the Earth after millions of years of stability: 26 % of such colliders were observed here and especially around 1.16 AU (compare Fig. 11) the chance is high that the Moon-producing planet Theia was formed here together with the other terrestrial planets (compare Fig. 1). We plan to make more of these numerical experiments on a finer grid and for longer integration times and furthermore we will combine these results with detailed computations of the collisions (SPH codes); all this will bring us a step further in the knowledge of the formation of our Moon.

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