Defrosting in an Emergent Galileon Cosmology

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We study the transition from an Emergent Galileon condensate phase of the early universe to a later expanding radiation phase. This “defrosting” or “preheating” transition is a consequence of the excitation of matter fluctuations by the coherent Galileon condensate, in analogy to how preheating in inflationary cosmology occurs via the excitation of matter fluctuations through coupling of matter with the coherent inflaton condensate. We show that the “minimal” coupling of matter (modeled as a massless scalar field) to the Galileon field introduced by Creminelli, Nicolis and Trincherini in order to generate a scale-invariant spectrum of matter fluctuations is sufficient to lead to efficient defrosting, provided that the effects of the non-vanishing expansion rate of the universe are taken into account. If we neglect the effects of expansion, an additional coupling of matter to the Galileon condensate is required. We study the efficiency of the defrosting mechanism in both cases.

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I. INTRODUCTION

Over the past decades, more and more observational and theoretical discoveries and puzzles have hinted that maybe our current understanding of gravity does not encompass the whole picture. As such they motivated the study of modifications of the theory of general relativity, either through the introduction of new types of matter having unprecedented properties, or by modifying the way gravity itself propagates and couples to matter.

On one hand, the current observed accelerated expansion of the universe has caused a first clash with theoretical predictions \cite{1,2}. While the anthropic argument might be used to explain both the existence and the smallness of a cosmological constant \cite{3}, the discovery of accelerated cosmological expansion has also engendered interest to explore theoretical possibilities for a modification of General Relativity (GR) on cosmological scales, i.e. an infrared modification of GR. Along that line, there has been significant progress in developing screening mechanisms \cite{4}, such as the Chameleon mechanism \cite{5–10}, the Symmetron mechanism \cite{11–13}, the Vainshtein mechanism \cite{14–16} (which encompasses massive gravity theories \cite{17–19}, degravitation \cite{20–23}, brane induced gravity models \cite{24–29}, and Galileon theories \cite{30–42}). These mechanisms are based on the assumption of extra scalar degree(s) of freedom, coupling gravitationally to both the baryonic and the dark sector, in such a way that the evolution of cosmological scales is affected to match the observed accelerated expansion, without being detectable through local experiments such as solar system tests of gravity \cite{1}.

On the other hand, modified approaches to gravity might ameliorate some problems which are encountered in the earliest stages of the evolution of the universe. The Null Energy Condition (NEC), which states that $T_{\mu\nu}k^\mu k^\nu \geq 0$ for every null vector $k^\mu$, implies for a Friedmann-Robertson-Walker (FRW) universe that $\dot{H} \leq 0$. Going back in time, this leads to the initial cosmological singularity. If the NEC is always satisfied, this might lead one to ask why is the universe expanding so rapidly in the first place. However, such a question is entangled with the question of the ultraviolet (UV) completion of gravity, because as one goes backward in time, $H$ and the energy density increase, space contracts, until $H \sim m_{pl}$ and it becomes necessary to appeal to quantum gravity to understand the origin of the expansion of the universe. The objective of finding an alternative history of the universe in which quantum gravity effects do not become important has been a motivation to consider modifications of General Relativity by introducing new fields that would violate the NEC. On this basis many alternative cosmological scenarios have been proposed, such as string gas cosmology \cite{46–51}, the pre-Big-Bang scenario \cite{52–54}, models of Ekpyrotic, cyclic \cite{53–64}, and bouncing cosmology (see \cite{63} for a review of older work on bouncing cosmology and \cite{66} for a review of more recent approaches), and even higher dimensional inflation \cite{65}.

A general class of such a violation of the NEC was studied formally for the first time in the context of ghost condensation \cite{68}. Instead of being doomed to contain disastrous instabilities as they were initially thought to \cite{69}, NEC-violating ghost condensates provided a general stable framework and opened many avenues for novel cosmological

\footnote{Note that there are also suggestions that an instability to infrared (IR) fluctuations might lead to a dynamical relaxation mechanism for the cosmological constant \cite{43–44}.}
The second class of such models are obtained by making use of Galileons. Initially introduced in the context of the DGP model \cite{24,25} and later proposed formally as a generic local infrared modification of General Relativity in order to explain the late-time accelerated expansion, Galileons are defined by imposing an extra scalar degree of freedom, $\pi$, kinetically mixed with GR. \cite{30}. Demanding that they must obey the \textit{Galilean symmetry},

$$
\pi(x) \rightarrow \pi(x) + c + b_\mu x^\mu,
$$

imposing that this symmetry also be a symmetry of the Lagrangian, and that the $\pi$ equation of motion (EoM) be exactly second-order in derivatives leads one to the conclusion that in four dimensions, only five possible interaction terms are allowed in the Lagrangian, one per order of interaction. The fact that the EoM for $\pi$ is still only second order (regardless of the presence of higher order interaction terms in the Lagrangian) ensures that no ghost degree of freedom will stain the theory. Under these assumptions, Galileons provide a natural realization of the Vainshtein screening mechanism, with a self-accelerating solution at cosmological scales that decouples from short scales. However, Galileons also give rise to interesting cosmological scenarios when promoting the symmetry group of the Galileon Lagrangian from the Galilean symmetry to the conformal group $\text{SO}(4,2)$. Among the three possible maximally symmetric solutions for $\pi$, choosing the time dependent one that breaks $\text{SO}(4,2)$ to the isometry group of four-dimensional de Sitter space $\text{SO}(4,1)$, one obtains a stable strong violation of the NEC in which the universe is expanding at late times, even in the case when it is not initially doing so. This scenario, dubbed the “Galilean Genesis” \cite{74}, renders the usual assumption of an initial large and positive $H$ completely unnecessary.

More precisely, the “Galilean Genesis” scenario postulates a universe that is asymptotically Minkowski ($M_4$) in the past. \cite{74} Choosing the de Sitter solution $\pi_{dS}$ of the Galileon (which has zero energy density) in that limit will cause the scale factor to grow exponentially as we approach $t \to 0^-$. Perturbations around this background were previously shown to be stable, but they do not give rise to significant cosmological perturbations and do not produce significant squeezing on large scales. The necessity for any cosmological scenario to produce the observed spectrum of scale-invariant primordial cosmological perturbations makes the introduction of a second scalar matter field, $\sigma$, necessary. Conformal symmetry requires that any other field coupling to the Galileon does so treating the Galileon as a dilaton, i.e. through an effective metric

$$
g_{\mu
u}^f = e^{2\sigma} g_{\mu
u}.
$$

If $g_{\mu\nu}$ can be approximated as $\eta_{\mu\nu}$, as is the case here asymptotically in the past, then $\sigma$ will behave as in a “fake” de Sitter space and its dynamics will be the same as if space was undergoing inflation. In particular, the spectrum of perturbations in the matter field will undergo squeezing and will be scale invariant. In the original paper proposing this model \cite{74}, it was assumed that $\pi$ departs from its de Sitter solution (due to its coupling to gravity) and as energy is transferred to the matter field, $\rho_{\text{tot}}$ increases until the system exits the regime of validity of the Galileon effective field theory. At that point, $(M_4, \pi_{dS})$ ceases to be a valid background to expand around, and it was assumed that the energy density would be transferred to the standard adiabatic mode of regular matter, so that the universe would then proceed to a standard radiation-dominated FRW phase, through a sort of “defrosting” similar to reheating in standard inflation.

However, even if Galileon Genesis provides a successful implementation of an effective inflationary phase, the importance of the defrosting stage should not be underestimated. It might be the case that the qualitative argument presented above conceals a graceful exit problem, for example if the Galileon fails to transfer sufficient energy density to the matter field $\sigma$ to allow for a transition toward a radiation-dominated epoch. Hence one must ensure that a preheating stage transfers most of the energy density to the regular matter field. Another source of concern is how the NEC-violating Galileon will react to a coupling with standard matter. It might be the case that, due to non-conventional kinetic properties, a rapid increase of the energy density in $\sigma$ will back-react on the Galileon by \textit{accelerating} it instead of slowing it down as one would intuitively expect. In that case, again, the standard matter field will never come to dominate the evolution of $H$, and an evolution toward a radiation-dominated phase will not be possible.

Another point that makes the study of preheating/defrosting crucial is that it is this process that determines the amplitude of the adiabatic primordial cosmological perturbations produced. A viable cosmological model must produce an amplitude compatible with the $\delta \rho/\rho \sim 5 \times 10^{-5}$ COBE normalization, and the amount of fine-tuning required to attain such an amplitude (if possible) in a given model gives information about the naturalness of the

\footnote{In this aspect the Galileon Genesis scenario is a realization of the “Emergent Universe Scenario” of \cite{72}.}
model. Moreover, the precise preheating mechanism describes how isocurvature perturbations, if they are produced either before or during reheating, will influence the observable adiabatic spectrum of perturbations.

In the case at hand, the fundamental growing fluctuations are produced in the matter field, while no sizeable perturbations are produced in the \( \pi \) field. Since at the classical level in the background, the Galileon represents the adiabatic field, all sizeable scale-invariant perturbations are produced in the form of entropy fluctuations. In order to produce the observed adiabatic spectrum, these entropy modes must therefore be transferred to an additional adiabatic degree of freedom. However, the precise calculation of the amplitude of the \( \delta \rho/\rho \) spectrum will not be the main focus of the current paper. This is because this calculation is made more complicated by the absence of non-vanishing classical background, which renders \( \delta \rho/\rho \) second order in the perturbations of \( \sigma \) and which makes the spectrum of energy density perturbations in \( \sigma \) qualitatively very different from the spectrum of field perturbations in \( \sigma \) \cite{76}.

In this paper, we make the idea of preheating and defrosting more precise in the context of Galileon Genesis and ensure that the model does not suffer from a graceful exit problem. In Section III we review the Galileon Genesis formalism, first in the case of decoupling from gravity, where \( \pi \) is identically in the de Sitter configuration, and then when coupling to gravity is re-established, in which case the de Sitter configuration in Minkowski space is only the limiting solution as \( t \to -\infty \). In this more realistic model, \( H \sim -1/t^3 \) and the energy density in \( \pi \) is created suddenly as \( t \) approaches the singularity, which sets the right conditions for an efficient preheating. In Section III we review the preheating formalism. In Section IV we study how preheating proceeds in Galileon Genesis in the limit where gravity is decoupled from the evolution of the fields. That is, we assume the de Sitter configuration for \( \pi \) and \( H = 0 \) as the background for the evolution of \( \sigma \). It is found that, for minimal coupling between the Galileon and the matter field, the energy transfer to \( \sigma \) is not sufficient to overcome the growth of the Galileon as it evolves toward the singularity at \( t = 0 \). To solve this problem, we explore the consequences of the introduction of further couplings between \( \pi \) and \( \sigma \), in the form of a potential term for \( \sigma \). Such terms are chosen in such a way that they do not spoil the near scale-invariance of the spectrum of perturbations, but make it slightly red-tilted. Moreover, during the fictitious de Sitter phase, the amplitude of fluctuation modes remain time-independent after their freeze-out, which hints that this solution is as attractor, until the time of preheating, at which point they start growing. Upon the inclusion of such couplings, the energy transfer to \( \sigma \) is made efficient enough for preheating to proceed. Finally, it is found that in the case of both minimal and non-minimal coupling, the back-reaction of \( \sigma \) on \( \pi \) slows down the Galileon and making it evolve towards another maximally symmetric solution: \( \pi = 0 \).

Finally, in Section V we reintroduce the coupling of the background to gravity. The introduction of a growing \( H \) and a Galileon departing from its de Sitter configuration has the surprising effect of accelerating the growth of the energy density in the matter field close to the singularity, making it fast enough to render obsolete the need for extra couplings between \( \pi \) and \( \sigma \). However, such an acceleration does not spoil the scale invariance of the spectrum at earlier times. The scale-invariant part of the spectrum remains slightly red-tilted, while a trough at scales corresponding to the scales freezing out at the beginning of preheating allows the UV end of the spectrum to be heavily blue-tilted. Hence the smallest scale modes freezing out during preheating dominate the energy density in \( \sigma \) and permit efficient defrosting. Moreover, the re-introduction of gravity does not spoil the back-reaction of \( \sigma \), which is still found to slow down the Galileon.

II. REVIEW OF GALILEON GENESIS

We will work with the simplest version of the Galileon minimally coupled to gravity. In this case, the action of the Galileon scalar field \( \pi \) is given by

\[
S_{\pi} = \int d^4x \sqrt{-g} \left[ f^2 e^{2\pi} (\partial \pi)^2 + \frac{f^3}{\Lambda^3} (\partial \pi)^2 + \frac{f^3}{2\Lambda^3} (\partial \pi)^4 \right],
\]

(3)

where \( f \) sets the mass scale of the Galileon field (which is taken to be dimensionless) and \( \Lambda \) is a second mass scale which sets the energy at which the higher derivative terms in the action become important. Lorentz indices are contracted with the metric \( g_{\mu\nu} \) and the “box” operator is built out of metric covariant derivatives.

In the absence of coupling to gravity, there is a “de-Sitter” solution \( \pi_{dS} \) of the equations which follow from (3):

\[
e^{\pi_{dS}} = -\frac{1}{H_0 t}
\]

(4)

valid in the time range \(-\infty < t < 0 \). In the above, the constant \( H_0 \) is given by

\[
H_0^2 = \frac{2\Lambda^3}{3f}.
\]

(5)
As we shall discuss at the beginning of Section IV, scalar matter fields (like regular matter fields) which are minimally coupled to the effective metric evolve as if they were minimally coupled to a de Sitter metric. The energy-momentum tensor of the Galileon field can be derived in the standard way and is given in [74]. It can be verified that the solution (4) has vanishing energy density and pressure which scales as \(-t^{-4}\). Thus, it has an equation of state which violates the NEC. Since its pressure and the energy density vanish as \(t \to -\infty\) the solution (4) can be taken to be the asymptotic solution in the far past even in the presence of gravity. Thus, it corresponds to an emergent Universe which approaches Minkowski space-time as \(t \to -\infty\). The NEC violation allows for a transition to an expanding phase. In fact, solving the Friedmann equations to leading order in Newton’s constant \(G\) yields a background solution \(\pi_0\) which scales as

\[
\pi_0 = \pi_{dS} - \frac{1}{2} \frac{f^2}{m^2_{pl}} H_0^2 t^2 \quad t \to -\infty
\]

with an associated Hubble constant which increases as

\[
H \approx -\frac{1}{3} \frac{f^2}{m^2_{pl}} \frac{1}{H_0^2 t^3}.
\]

The Hubble constant and the correction term in \(\pi_0\) compared to the de Sitter solution \(\pi_{dS}\) increase without bound as \(t \to 0\). Hence the perturbative expansion in \(G\) will break down at some \(t \sim -H_0^{-1} f/m_{pl}\). The equations in fact lead to a divergence in \(H\) at some time \(t_0\). As shown in [74], the asymptotic behavior as \(t \to t_0\) is given by

\[
e^{\pi_0} \approx \frac{8}{\sqrt{3}} \frac{f}{m_{pl}} \frac{1}{(t_0 - t)^2} \quad t \to t_0,
\]

with

\[
H \approx \frac{16}{3} \frac{f^2}{m^2_{pl}} \frac{1}{H_0^2 (t_0 - t)^3}.
\]

The cosmological scale factor \(a(t)\) then scales as

\[
a(t) \approx \exp\left[\frac{8 f^2}{3 H_0^2 M^2_{pl} (t_0 - t)^2}\right].
\]

The above solution describes a universe which emerges from a flat Minkowski gravitational vacuum in the limit \(t \to -\infty\) and then begins to expand more and more rapidly (which is possible because the Galileon violates the NEC). Eventually, \(\pi\) becomes strongly coupled and the effective field theory description of the Galileon breaks down. As the Galileon field grows in strength, its coupling to regular matter fields becomes important, and it is to the study of the effects of these couplings which we now turn.

III. PREHEATING: SETUP AND BASIC EQUATIONS

If the Galileon genesis scenario is to successfully connect to late-time cosmology, there needs to be a mechanism which drains energy-momentum from the Galileon field and creates regular matter. This challenge is analogous to that faced in inflationary universe cosmology. In an inflationary model [77], the energy density at the end of the period of inflation is contained in the spatially homogeneous condensate of the inflaton field, the scalar field responsible for generating inflation - in the same way that at the end of the Galileon genesis phase the stress-energy is contained in the spatially homogeneous Galileon field condensate. In the same way that couplings between the inflaton field and regular matter need to be introduced to describe the energy transfer at the end of inflation - a process called “reheating” - coupling terms between the Galileon field and regular matter need to be introduced in Galileon cosmology. As is usually done in studies of reheating in inflationary cosmology (see [78] for a recent review), we will model regular matter as another scalar field.

In the case of inflationary cosmology, reheating was first studied perturbatively [79, 80]. However, it was realized [81] that the perturbative analysis misses out on the coherent nature of the inflaton condensate and in fact gives completely wrong results for the duration of time the energy transfer takes. It was shown [81] (see also [82]) that parametric resonance effects during the oscillation of the inflaton condensate lead to a rapid energy transfer and produce an out-of-equilibrium state of matter particles. This initial phase of energy transfer was later [83] denoted “preheating”. The process in an expanding cosmological background was then studied in more detail in [84, 85].
As in the case of inflationary preheating, we expect coherence effects of the Galileon condensate to be crucial when studying the energy transfer between the Galileon background and matter, a process which we will call “defrosting” of the Galileon condensate state. Hence, we will employ the same formalism as is used in inflationary preheating, namely a semiclassical analysis in which the linear matter field fluctuations are quantized in the classical background given by the Galileon condensate.

Let us denote the scalar field representing matter by $\chi$ (in the application to the Galileon genesis scenario this field will be the $\sigma$ field mentioned earlier). We will treat $\chi$ as a free scalar field. The non-trivial dynamics comes from the coupling of $\chi$ to gravity and (in our case) to the Galileon. The first step in the semi-classical analysis is to determine the canonically normalized matter field $\bar{\chi}$. For a standard kinetic term of $\chi$ (i.e. in particular in the absence of coupling of $\chi$ to the Galileon), and in the case of minimal coupling of $\chi$ to gravity, the canonical field is

$$\bar{\chi} = a\chi.$$

The action then takes canonical form if we use conformal time $\tau$ related to the physical time $t$ via

$$dt = a(t)d\tau.$$

We expand $\chi$ in terms of creation and annihilation operators $\hat{a}_k$ and $\hat{a}_k^\dagger$ as

$$\bar{\chi}(x, t) = \frac{V^{1/2}}{(2\pi)^3} \int d^3k (\bar{\chi}_k(t)^* \hat{a}_k e^{i\mathbf{k}\cdot\mathbf{x}} + \chi_k(t) \hat{a}_k^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}},$$

where $V$ is the spatial cutoff volume and $\mathbf{k}$ is the comoving momentum vector. The creation and annihilation operators obey the usual canonical commutation relations. In the case of a free scalar field $\chi$ minimally coupled to gravity and not coupled to the Galileon, the mode functions $\bar{\chi}_k$ satisfy the equation

$$\bar{\chi}_k'' + \omega_k^2 \bar{\chi}_k = 0,$$

with

$$\omega_k^2 = k^2 + m_\chi^2 a^2 - \frac{a''}{a}$$

where a prime indicates the derivative with respect to $\tau$ and $m_\chi$ is the mass of $\chi$.

Note that the effective square frequency $\omega_k^2$ can be negative if $a''/a$ is positive and if $k$ is sufficiently small. This is the case for fluctuations with wavelengths larger than the Hubble radius. On these long wavelength scales the fluctuation amplitude increases while the microphysical oscillations freeze out. This is the squeezing of fluctuations on super-Hubble scales which is responsible for the growth and classicalization of quantum vacuum perturbations in inflationary cosmology (see [86, 87] for reviews of the theory of cosmological fluctuations and [88, 89] specifically for the question of classicalization). We will see in the next section that due to the coupling with the Galileon condensate field, matter perturbations evolve as regular matter fluctuations would in an effective time-dependent metric given by (2). Thus, long wavelength fluctuations are excited, leading to a transfer of pressure from the Galileon to regular matter. Note that in inflationary cosmology it is the coupling in the interaction potential between the inflaton field and the matter field which leads to the parametric excitation of long wavelength matter fluctuations.

In the semi-classical analysis we will assume that the $\bar{\chi}$ field starts out (mode by mode) in its vacuum state. With the field normalization chosen, taking expectation values of $\chi$ correlation functions in an initial vacuum state corresponds to calculating classical averages of these correlation functions using as initial values of $\bar{\chi}_k$ their harmonic oscillator ground state values

$$\bar{\chi}_k(t_i) = \frac{1}{\sqrt{2k}}.$$  

In the following, we will show that the same scalar field $\sigma$ which was introduced in [74] with applications for generating cosmological perturbations in mind can provide a good model for the matter into which the initial Galileon stress-energy flows.

We briefly recall why the field $\sigma$ was introduced in [74]. The starting point is the observation that the initial spectrum of curvature fluctuations induced by the Galileon field is blue if it stems from initial vacuum fluctuations.

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3 Gravitational waves in an expanding universe undergo a similar squeezing process [90].
(which is the obvious initial state for fluctuations in the emergent Galileon cosmology). It is a vacuum spectrum with a spectral index $n_s = 3$ (scale-invariance corresponds to $n_s = 1$). Since the curvature fluctuations are constant on super-Hubble scales, its spectrum remains blue. Hence, a different mechanism is required to generate a scale-invariant spectrum. In [74] it was pointed out that a massless scalar field which couples to the Galileon only through the kinetic part of the Lagrangian (minimally coupled to the effective metric (2)) acquires a scale-invariant spectrum since the matter field evolves as if it were in de Sitter space (we will review this result in the following section). Regular matter (modeled as a massless scalar field) must couple to the Galileon in exactly the same way 4. In the following section we will show that the induced growth of fluctuations is strong enough to efficiently drain energy-momentum from the Galileon, thus leading to successful “defrosting” of the cosmological Galileon condensate.

IV. PREHEATING: ANALYTICAL ANALYSIS

A. MinimalCoupling

We start by considering the minimal interaction of the Galileon with the massless scalar matter field $\sigma$ representing regular matter. The part of the action involving $\sigma$ is:

$$S_I = \int d^4x \sqrt{-g} \mathcal{L}_I(\pi, \sigma) = \int d^4x \sqrt{-g} \left(-e^{2\pi} \partial_\mu \sigma \partial^\mu \sigma\right).$$

Here, $\sigma(x, t)$ has units of energy, and the minus sign is required for $\sigma$ to have positive energy density, and hence to behave like a regular matter field. We first study the case in which gravity is decoupled, so that the indices are contracted with the Minkowski metric, $\eta_{\mu\nu}$. Moreover, $\pi$ is chosen to start out at $t \to -\infty$ and to follow the de Sitter background solution, so that $\sigma$ behaves as if it was in a “fake” de Sitter background. The coupling is minimal in the sense that conformal invariance requires any coupling of $\sigma$ with $\pi$ to be through the “fake” metric $g_{\mu\nu}^f = e^{2\pi} \eta_{\mu\nu}$.

The equation of motion (EoM) for $\sigma$, upon the field rescaling

$$\sigma \rightarrow u^{-1}(t) \tilde{\sigma} \equiv e^{-\pi} \tilde{\sigma}$$

(18)

to obtain the canonically-normalized variable, and upon performing the Fourier transform

$$\tilde{\sigma}(x, t) = \int \frac{d^3k}{(2\pi)^3} e^{i kx} \sigma_k(t) V^{1/2},$$

(19)

(where $V$ is the cutoff volume coming from putting the theory in a finite box), is then:

$$\ddot{\sigma}_k + \left(k^2 - \frac{2}{t^2}\right) \dot{\sigma}_k = 0.$$ (20)

As discussed in Section [11] we want to match this with the usual equation for the canonically normalized massless scalar matter field $\chi$ in a fixed background $a(\tau)$:

$$\chi''_k + \left(k^2 - \frac{a''}{a}\right) \chi_k = 0,$$ (21)

where the primes denote derivatives with respect to the conformal time $\tau$. In the case of a de Sitter background, we have

$$\frac{a''}{a} = \frac{2}{\tau^2}.$$ (22)

Thus, the analogy between the two EoMs is clear. In our case, however, the real metric is flat Minkowski, which means $a = 1$ and $\tau = t$. However, the function $a(t)$ from the inflationary case matches to an expression $a^f(t)$, the “fake” scale factor, in [20]. This way, the standard analysis of inflationary cosmology can be applied to canonically

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4 Treating regular matter as a massless field is a good approximation since the mass of Standard Model matter fields is many orders of magnitude smaller than typical mass scales relevant in the very early universe.
quantize the matter field and get the power spectrum of perturbations in $\sigma_k$ at the time when they freeze out. Note however that, in contrast to the case of massless inflation, the field rescaling $u(t)$ used earlier in (18) and $a^f(t)$ defined here need not be equal. Even though they are for the simple coupling with $\pi$ considered here, as soon as a potential term for $\sigma$ is introduced (as will be done in the following subsection), they cease to be.

For the de Sitter Galileon background $\pi_{dS}$, the fake scale factor satisfies the equation $\ddot{a}^f/a^f = 2/t^2$. The solutions for $a^f$ are hence

$$a^f = c_1 t^2 + c_2 t,$$

(23)

where $c_1$ and $c_2$ are constant coefficients. As in our case $t \to 0$, the growing mode is selected and we can use $a^f = 1/H_0 t$. We choose $a^f$ to be unitless for simplicity, but note that the overall normalization and units of $a^f$ bear no physical significance, and are therefore irrelevant. All of the physical information is enclosed in the scaling $u(t)$, whose normalization is fixed by the requirement of transforming the kinetic term of $\sigma$ in (17) into a canonical kinetic term in flat space for $\tilde{\sigma}$.

Before turning to our analysis of defrosting of the Galileon background via production of $\sigma$ excitations, we will review why a scale-invariant spectrum of fluctuations of $\sigma$ emerges [74]. For a given $k$, a mode of $\tilde{\sigma}_k$ will oscillate with constant amplitude as long as the corresponding length scale stays within the fake Hubble radius, i.e. for

$$k^2 > k^2_{frz} \equiv \frac{\ddot{a}^f(a^f(t_{frz}))}{a^f(t_{frz})^2} = \frac{2}{t^2_{frz}},$$

(24)

and it will freeze out at $k = k_{frz}$. For $k < k_{frz}$, there is a mode of $\tilde{\sigma}_k$ which grows as $a^f$. Hence, the spectrum of the field $\sigma$ will be given by:

$$\langle \sigma_k \sigma_{k'} \rangle = (2\pi)^3 \delta(k + k') |\sigma_k|^2 = (2\pi)^3 \delta(k + k') |u^{-1}(t)\tilde{\sigma}_k(t)|^2$$

$$= (2\pi)^3 \delta(k + k') (u(t))^{-2} \frac{(a^f(t_{frz}))^{-2}}{(a^f(t))^2} |\tilde{\sigma}_k(t_{frz})|^2$$

$$= (2\pi)^3 \delta(k + k')(H_0 t_{frz})^2 \frac{1}{\sqrt{2k}} \frac{1}{k^3}$$

(25)

Hence the spectrum of perturbations is scale invariant.

We are now interested to know, first, whether the energy density transferred to $\sigma$ from $\pi$ is sufficient for the matter field to overcome the Galileon energy-wise. Second, we want to ensure that this process of energy transfer will, as one would intuitively think, indeed back-react on $\pi$ to slow it down and make it evolve toward the $\pi = 0$ Lorentz invariant vacuum solution. The fulfilment of these two conditions will ensure that the “fake” de Sitter phase eventually comes to an end, and, as the $\pi$ field is driven to zero by the growth of $\sigma$, that it will be followed by a radiation-dominated epoch.

The energy density in $\sigma$ can be computed from the stress-energy tensor of the part of the action involving $\sigma$, namely $S_I$:

$$T_{\mu\nu}(\sigma) = [2\partial_\mu \sigma \partial_\nu \sigma - g_{\mu\nu}(\partial \sigma)^2] e^{2\pi}.$$

(26)

Note that this expression involves the $\sigma$ field in position space, not the rescaled field $\tilde{\sigma}$). The energy density and pressure in $\sigma$ for the chosen $\pi$ background are thus given by:

$$\rho_\sigma(x, t) = \frac{1}{(H_0 t)^2} \left( \dot{\sigma}^2 + \frac{(\nabla \sigma)^2}{a^2} \right) \Rightarrow \bar{\rho}_\sigma = \frac{1}{V} \int d^3 x \rho_\sigma(x, t)$$

$$= \int \frac{d^3 k}{(2\pi)^3 (k H_0)^2} \left( \dot{\sigma}_k^2 + \frac{k^2 \sigma_k^2}{a^2} \right)$$

$$= \int \frac{d^3 k}{(2\pi)^3} \rho_\sigma(k, t)$$

(27)
\[ p_{\sigma}(x, t) = \frac{1}{(H_0 t)^2} \left( \hat{a}^2 - \frac{1}{3} \left( \nabla \sigma \right)^2 \right) \]  \quad \Rightarrow \quad \bar{\rho}_p = \int \frac{d^3k}{(2\pi)^3} \frac{1}{(tH_0)^2} \left( \hat{a}^2 - \frac{1}{3} \frac{k^2 \sigma^2}{a^2} \right) 
abla \sigma \]  
\[ = \int \frac{d^3k}{(2\pi)^3} \rho_{\sigma}(k, t) \]  \quad (28)

where by $\bar{\rho}_{\sigma}$ and $\bar{\rho}_p$ we mean the space averages of these quantities over the spatial volume $V$, and we have then used the Fourier modes of $\sigma$ to re-write the expressions. We perform such a spatial average for the sake of comparison with $\rho_{\pi}$, which is a homogeneous quantity. Moreover, in the last step by defining $\rho_{\sigma}(k, t)$ and $\rho_{\pi}(k, t)$ we mean the energy density and pressure in each $k$-modes contributing to the spatial averages $\bar{\rho}_{\sigma}$ and $\bar{\rho}_p$ respectively, not the Fourier transform of $\rho_{\sigma}(x, t)$ and $\rho_{\pi}(x, t)$.

To know the amount of energy density transferred to the $\sigma$ field, we need to integrate the energy density over all $k$-modes. Since we started out in Minkowski space at $t = -\infty$, $H$ is initially zero, so $H_0$ and $k$ are physical (as opposed to comoving). Hence the scales of interest to us are the ones with $\lambda \sim 1 \text{mm}$, or $k_i = 10^{-31} \text{m}^{-1}$, which give the wavelength of modes corresponding to the current large scale structure at the time of reheating (if the scale of reheating is taken to be comparable to the scale of particle physics Grand Unification). We set the IR cutoff to be a bit larger than that scale.

Also, we note that modes below the scale $k_{frz}$ are still in their stage of quantum vacuum oscillation, and hence they do not contribute to the renormalized energy density (obtained by subtracting the vacuum contribution). The UV cutoff in the integral over momenta at time $t$ is therefore set by the smallest scale for which the mode functions have frozen out at the time at the time $t$. This value of $k$ is given by $k_{frz}$, and it corresponds to the “fake” Hubble radius at time $t$.

We use the solution

\[ \sigma_k(t) = u(t)^{-1} \left( a'_{frz}(t) \right)^{-1} \left( a'(t) \right)^{-1} \bar{\sigma}_k(t_i) \]  \quad (29)

from above to express the growing mode of $\sigma_k$ that will contribute the most to $\rho_{\sigma}(k, t)$. We can then express the energy density in each $k$-mode satisfying $k \ll k_{pre}$, that is, every mode that is frozen out, as:

\[ \rho_{\sigma}(k, t) = \frac{1}{k^2 t^2}. \]  \quad (30)

Hence

\[ \bar{\rho}_{\sigma} = \int_{k_i}^{k_{frz}} \frac{d^3k}{(2\pi)^3} \rho_{\sigma}(k, t) = \frac{1}{2\pi^2 t^2} \left( \frac{1}{t_i^2} - \frac{1}{t^2} \right), \]  \quad (31)

where we have used $t_i = \sqrt{2}/k_i$, and $k_{frz} = \sqrt{2}/t_i$. We see that energy density transfer from $\pi$ to the matter field $\sigma$ will only be sufficient for $\rho_{\sigma}$ to grow as $\sim 1/t^4$ close to the time of defrosting/preheating as $t \to 0$.

Note that the energy density in $\sigma$ scales with the same power of $t^{-1}$ as the background pressure of the Galileon condensate (which is Minkowski spacetime with $\pi = \pi_{dS}$ and $\rho_{\pi} = 0$, $\pi = \pi_{dS}$). Since the energy density of $\pi_{dS}$ vanishes, we see that $\rho_{\pi}$ will immediately dominate the total energy density. Naively, this might lead us to expect that the universe should defrost/preheat quickly (we shall come back to this point soon). But it also makes clear that the back-reaction of $\sigma$ on $\pi$ could be very important. Hence, we now turn to the analysis of this back-reaction, with the goal to ensure that it will slow down the Galileon from its de Sitter solution toward the solution $\pi = 0$.

Including the variational derivative of $S_I$, the equation of motion for $\pi$ gives:

\[ \pi \left[ 1 - \frac{2}{H_0^2} e^{-2\pi \hat{\pi}^2} + \frac{4}{3H_0^2} e^{-2\pi \nabla^2 \sigma} \right] \]  \quad \Rightarrow \quad \pi \left[ -\hat{\nabla}^2 + \frac{2}{H_0^2} e^{-2\pi \nabla^2 \sigma} \nabla^2 \sigma \right] 
\[ = -\hat{\pi}^2 + (\nabla \pi)^2 + \nabla^2 \pi + \frac{2}{H_0^2} e^{-2\pi} \nabla^2 \nabla^2 \pi \]  
\[ + \frac{2}{3H_0^2} e^{-2\pi} \left[ 2(\nabla \hat{\pi})^2 - \hat{\pi}^2 (\nabla \pi)^2 - \hat{\pi} \nabla \hat{\pi} \nabla \pi - \hat{\pi} \nabla^2 \sigma \right] 
\[ - \frac{1}{f^2} \left[ \hat{\sigma}^2 - (\nabla \sigma)^2 \right]. \]  \quad (32)

We are interested in studying the back-reaction of the linear fluctuations of $\sigma$ (computed above) on the background of $\pi$. We use the following expansion of $\pi$ in a fixed Minkowski background:

\[ \pi = \pi_{dS} + \delta \pi + \delta^{(2)} \pi \]  \quad (33)
\[ \sigma = \delta \sigma + \delta^{(2)} \sigma, \]  \quad (34)
where the background is homogeneous, but higher order perturbations are allowed not to be. The expansion parameter is the amplitude of the linear fluctuations. The term $\delta \sigma$ is the linear fluctuations in $\sigma$ which we have just studied. The term $\delta \pi$ corresponds to the linear fluctuations in $\pi$. However, we already know from section [31] that inhomogenous adiabatic linear perturbations in $\pi$ will be cosmologically irrelevant. Hence, when expanding to linear order, we will find no significant growing solutions because all contributions from $\sigma$ will be second order and $\rho_{dS}$ is an attractor. We can directly go to second order and neglect the contributions from $\delta \pi$. We obtain

$$\delta^{(2)} \pi - \frac{2}{t} \delta^{(2)} \dot{\pi} - \frac{4}{t^2} \delta^{(2)} \pi - \frac{1}{3} \nabla^2 (\delta^{(2)} \pi) = \frac{1}{f^2} \left[ \delta \pi^2 - (\nabla \delta \sigma)^2 \right]. \tag{35}$$

In order to have a better intuition of the way the $\sigma$ source terms will drive the Galileon backreaction, we go to Fourier space. We are mainly interested in the homogeneous back-reaction $\delta^{(2)} \pi_{k=0}$. In the equation for that zero mode, the $\sigma$ terms will turn into an integral over $k$-space where each $\sigma_k$ mode couples with the corresponding $\pi_{-k}$ mode. To perform that integral we use, as before, the rescaled field $\tilde{\sigma}_k$ and the fake scale factor $a^f$ to express $\sigma_k$ for $k < k_{frz}$ as $\sigma_k = H_0/k^{3/2}$. Finally, the modes $k > k_{frz}$ are oscillating with constant amplitude, so that they do not contribute to the renormalized value of the integral. Once the integral is performed, we are left with the following equation for the back-reaction on the Galileon background:

$$\delta^{(2)} \pi_{k=0} - \frac{2}{t} \delta^{(2)} \dot{\pi}_{k=0} - \frac{4}{t^2} \delta^{(2)} \pi_{k=0} = -\frac{H_0^2}{2\pi^2 f^2} \left( 1 - \frac{1}{t^2} \right). \tag{36}$$

Since the source term is negative (recall that $t > t_i$), we thus conclude that the growth of $\sigma$ with time will indeed slow down the Galileon background, and this way will make it move from the de Sitter solution toward the $\pi = 0$ solution. From this analysis, we can hope that the defrosting/preheating indeed proceeds and ends the fake de Sitter phase, leading to a radiation dominated expanding phase $^5$.

However, if we now go back to the comparison of the growth of $\rho_\sigma$ relative to $\rho_\pi$, and, for the purpose of comparison, we look at a more realistic background in which the coupling to gravity has been re-introduced close to the time of preheating, we see that, due to the singularity in the solution, $\rho_\pi$ grows as $\sim 1/t^6$ as $t \to 0$. This means that the growth of $\rho_\sigma$ will not be fast enough to overcome $\pi$ in a more realistic setup if we neglect the effects of a non-zero real $H$ on $\sigma$.

There are few avenues to overcome this apparent difficulty. A first one is to introduce further interaction terms in $S_I$ that would make the coupling between $\pi$ and $\sigma$ stronger only close to the singularity. This way, the scale invariance of the spectrum of $\sigma$ perturbations would be preserved, but the growth of $\rho_\sigma$ close to the singularity could be made much faster. This is the avenue we explore next.

### B. Non-Minimal Coupling

Our goal here is to add coupling terms between the Galileon and the matter field in $S_I$ that make $\rho_\sigma$ grow faster than $\rho_\pi$ as $t \to 0$, in such a way that the scale invariance of the spectrum of $\sigma$ perturbations is preserved. That is to say, the rescaling $u(t)$, and therefore the kinetic term of $\sigma$, must remain unchanged. A potential term $V(\sigma)$ including $\pi$ or its derivative must therefore be included. Moreover it should induce a correction to $a^f$ that is higher order than $1/t^2$, so that far from $t = 0$ (while the matter field is inflating due to the effect of fake de Sitter) the $\sigma_k$ modes that are frozen out do not vary with time. The higher order correction in $1/t^2$ to $a^f$ will ensure that such an effect only arises during preheating.

We chose to consider a term of the form $e^{2n_\pi} \partial_\mu \pi \partial^\mu \pi \sigma^2$ with $n \geq 2$, but a term $e^{n_\pi} \sigma^2$ with $n \geq 2$ would have the same effect. Even though such terms explicitly break the Galileon symmetry, it is in a very mild way as $t \to -\infty$ since $\sigma$ starts out at zero. Moreover, we expect any coupling of the Galileon to an extra degree of freedom to break the Galileon symmetry, and so if we aim to use the Galileon to build a cosmological model, we must expect having to break this symmetry in one way or another.

We therefore consider the following action for the $\sigma$ field interacting with $\pi$:

$$S_I = \int d^4 x \sqrt{-g} \left( -e^{2n_\pi} \partial_\mu \pi \partial^\mu \sigma - e^{n_\pi} \partial_\mu \pi \partial^\mu \pi \sigma^2 \right). \tag{37}$$

$^5$ Note, in particular, that there is no instability in the system - one might have feared that the excitation of $\sigma$ would lead to an increase in the amplitude of $\pi$. 


The field rescaling $u(t)$ remains as before. The equation of motion for the rescaled field $\tilde{\sigma}_k(t)$ in Fourier space becomes:

$$\ddot{\tilde{\sigma}}_k + \left( k^2 - \frac{2}{t^2} - H_0^2 \left( \frac{1}{H_0 t} \right)^{2n} \right) \tilde{\sigma}_k = 0. \tag{38}$$

With the new coupling, the definition of the fake scale factor $a^f$ becomes

$$\frac{\dot{a}^f}{a^f} = \frac{\dot{u}(t)}{u(t)} + V_{\sigma} = \frac{2}{t^2} + \frac{H_0^2}{(H_0 t)^{2n}}. \tag{39}$$

We now see that it will no longer be equal to the field rescaling $\sigma$, which explains the choice of different notations for these two variables.

In what follows, we fix $n = 2$, the minimum required value. $a^f$ has an analytical solution which can be approximated in the limits $t \to -\infty$ during the fake de Sitter regime, which we call the IR regime (since it corresponds to the solution valid when the IR end of the $\tilde{\sigma}_k$ spectrum freezes out), and $t \to 0$ during the preheating phase, which we call the UV regime (since it corresponds to an approximation valid when the UV end of the $\tilde{\sigma}_k$ spectrum freezes out). The initial conditions are chosen such that $a^f, \dot{a}^f \to 0$ as $t \to -\infty$:

$$a^f(t) = 3H_0 t \left[ H_0 \sinh \left( \frac{1}{H_0 t} \right) - \cosh \left( \frac{1}{H_0 t} \right) \right]$$

$$= -3 \sum_{n=0}^{\infty} \frac{1/(H_0 t)^{2n+1}}{(2n+1)!(2n+3)}$$

$$\rightarrow_{t \to -\infty} a^f_{IR}(t) = -\frac{1}{H_0}$$

and

$$\rightarrow_{t \to 0} a^f_{UV}(t) = -3H_0 t e^{-1/(H_0 t)} \left( \frac{H_0 t + 1}{2} \right). \tag{41}$$

The overall normalization of $a^f$ is chosen such that the IR limit of the spectrum matches the corresponding expression in the case of minimal coupling. The approximate solution in the IR remains valid up to $t \sim -H_0^{-1}$, at which point the extra term in the differential equation for $a^f$ starts to dominate. It can then be interpolated with the asymptotic solution in the UV, which starts being a good approximation at $t \gtrsim -H_0^{-1}/2$. The time of freeze-out of a mode $k$ will now be given by:

$$t_{frz} = -\sqrt{\frac{1}{k^2} + \frac{1}{k^2} \sqrt{1 + k^2/H_0^2}}$$

$$\rightarrow_{k \to 0} t_{IR}^{frz} = -\frac{\sqrt{2}}{k}$$

and

$$\rightarrow_{k \to \infty} t_{UV}^{frz} = -\frac{1}{\sqrt{H_0 k}}. \tag{43}$$

The solution for $\sigma_k$ in the regime $k < k_{frz}$ is therefore:

$$\sigma_k(t) = e^{-\pi a_k^f(t)} a^f(t_{frz}) \frac{1}{\sqrt{2k}}$$

$$\rightarrow_{k \to 0} \sigma_k^{IR} \approx \begin{cases} \frac{H_0 k^{3/2}}{H_0 t^2}, & |t| \gtrsim H_0^{-1} \\ \frac{3H_0^2 k^{3/2} e^{-1/(H_0 t)}(H_0 t + 1/a_k t)}{2}, & 0 < |t| \lesssim H_0^{-1}/10 \end{cases} \tag{44}$$

and

$$\rightarrow_{k \to \infty} \sigma_k^{UV} \approx \frac{H_0^2 t^{3/2} e^{-3/2} H_0 t + 1}{\sqrt{2} \sqrt{H_0 k}}. \tag{45}$$

Figure 1 shows the $\sigma_k$ power spectrum as a function of time. We see that, upon the addition of the extra term in the action, the IR end of the $\sigma_k$ spectrum stays scale invariant and remains constant after freeze-out (which is the condition for the solution to be an attractor) until the time of preheating, which happens at $|t| \lesssim H_0^{-1}$. Modes freezing out very close to reheating will not be characterized by a scale-invariant spectrum, but rather by a red spectrum, as can be seen from 45. However these modes are outside of the observable range since they re-enter the oscillatory regime just after preheating and will not have any observable consequences to leading order. However it might be interesting to see whether these small scales can couple together to have effects on larger scales that are observable today, in the form of the introduction of non-Gaussianity, for example.
FIG. 1: Evolution of the power spectrum of the $\sigma_k$ perturbations after freeze-out. The level curves show constant amplitudes of perturbations. On the colour map, blue means smaller amplitude and red means larger amplitude. Modes below the dotted line are frozen out, while modes with paler colour are still oscillating. If we choose for a chosen value of $H_0 = 5 \times 10^{-4}$, the green region corresponds to an amplitude of perturbation of $\sim 10^{-5}$. The horizontal axis shows the time-evolution of the perturbations for a given $k$ mode, while the vertical axis shows the $k$ spectrum at a given time. In the IR end (i.e. for $k < H_0$), the spectrum is scale invariant and constant until the onset of preheating after $t = -H_0^{-1}$, at which point the amplitude of the fluctuations starts to grow. On the other side, the UV end of the spectrum is slightly red and its amplitude increases slightly with time.

Repeating the analysis presented above to get the energy density in $\rho_\sigma$, it is easy to see that one obtains an extra term in the stress-energy for $\sigma$ that will add an extra $\frac{1}{2} \frac{1}{tH_0^2} \sigma^2$ to the energy density in each $k$ mode. Integrating as above, we obtain (being careful to separate properly the regions of $k$ space and the temporal regions of validity of our approximate solutions)

$$\bar{\rho}_\sigma \simeq \int_{k_i}^{H_0} \frac{d^3k}{(2\pi)^3} \left( \frac{1}{kt^2} + \frac{1}{t^6 H_0^2 k^3} \right) = \frac{1}{2\pi^2 t^2} \left( \frac{1}{t^2} - \frac{1}{H_0^2} \right) + \frac{1}{2\pi^2 \beta H_0^2 \ln \left[ \frac{t}{t_i} \right]} \quad \text{for } |t| > H_0^{-1} \text{ and } k < H_0, \quad (46)$$

during the IR regime, or fake de Sitter epoch. At $t \sim -H_0^{-1}$, we enter the UV regime: the behaviour of the modes that froze out during the IR regime changes and the modes that freeze out have a different time evolution. It is at this time that the defrosting/preheating starts, and $\bar{\rho}_\sigma$ will now be given by:

$$\bar{\rho}_\sigma \simeq \int_{k_i}^{H_0} \frac{d^3k}{(2\pi)^3} \rho^{IR}(k,t) + \int_{H_0}^{k} \frac{d^3k}{(2\pi)^3} \rho^{UV}(k,t)$$

$$= \frac{H_0^2}{2\pi^2} e^{-2/H_0 t} \left[ \left( \frac{H_0^2}{2} - \frac{1}{t_i^2} \right) O(H_0^4 t^4) + \ln \left( \frac{H_0 t_i}{\sqrt{2}} \right) O \left( \frac{1}{t^2} \right) \right] + \frac{1}{2\pi^2} e^{-1/H_0 t} \left[ O \left( \frac{1}{H_0^{12} t^{16}} \right) \right] \quad (47)$$

for $|t| < H_0^{-1}$ and $k > H_0$, where we have only included the lowest order terms in $t$ in the result for clarity. With the new coupling, $\bar{\rho}_\sigma$ grows sufficiently fast when $|t| < H_0^{-1}$ to overcome even a more realistic estimate of the growth rate of $\rho_\sigma$ which diverges as $\sim 1/t^6$ as $t \to 0$. Hence the growth of the energy density in the matter field will be fast enough to allow for initiation of preheating.

It only remains to check whether the introduction of the new coupling term back-reacts on the $\pi$ background in a way as to slow it down, or if it spoils the relationship we had previously obtained. However, it is straightforward to
check that the only modification is that equation \(35\) acquires the extra source term on the r.h.s:

\[
\frac{2t\delta\sigma\dot{\delta}\sigma - 3\delta\sigma^2}{f^2H_0^2t^4}.
\] (48)

Since we know from above (and as can easily be seen from Figure \(1\) that \(\delta\sigma\) and \(\dot{\delta}\sigma\) are always positive, and since \(t < 0\), the contribution of this extra term is always **negative**. Hence the source term back-reacting on the homogeneous mode of the \(\pi\) background becomes more negative compared to the minimal coupling case, which means that the Galileon will be even more efficiently driven toward the \(\pi = 0\) solution and preheating will be completed on a more rapid time scale.

Qualitatively, as the Galileon rolls toward \(t = 0\), it excites fluctuations in the matter field. From the point of view of the matter field, it gets excited in exactly the same manner as if it was immersed in de Sitter space. However, instead of getting the energy for particle production from the metric, it is the Galileon that transfers it through its coupling to matter. Hence as fluctuations in the matter field get amplified, the Galileon is slowed down. By adding an extra coupling term, the energy transfer toward the matter field was made much more efficient starting at \(|t| < H_0^{-1}\), hence the \(\sigma\) growth was accelerated and \(\pi\) was accordingly decelerated, making preheating possible.

If, instead, we had chosen to add a coupling term of the form \(e^{2n}\sigma^2\) with \(n = 2\), the effect on \(\sigma\) would have been qualitatively the same and the only difference on the \(\pi\) back-reaction is that the extra source term would have been

\[-\frac{4\delta\sigma^2}{f^2H_0^2t^2}.
\] (49)

Hence the Galileon would still have been slowed down, but in a less efficient way than with the coupling considered above. If we had chosen \(n > 2\), the energy transfer to \(\sigma\) and the resulting deceleration back-reaction on \(\pi\) would have been even faster.

## V. REINTRODUCING THE COUPLING TO GRAVITY

In the above section, it was discussed how an additional degree of freedom, which we took to be a matter scalar field, behaves when coupled both minimally and non-minimally to the Galileon in its de Sitter solution, when fields are decoupled from gravity. It was realized that in the case of minimal kinematic coupling between \(\pi\) and \(\sigma\), the energy density in \(\pi\) grows much faster compared to the minimal coupling case, which means that the Galileon will be even more efficiently driven toward the \(\pi = 0\) solution and preheating will be completed on a more rapid time scale.

Qualitatively, as the Galileon rolls toward \(t = 0\), it excites fluctuations in the matter field. From the point of view of the matter field, it gets excited in exactly the same manner as if it was immersed in de Sitter space. However, instead of getting the energy for particle production from the metric, it is the Galileon that transfers it through its coupling to matter. Hence as fluctuations in the matter field get amplified, the Galileon is slowed down. By adding an extra coupling term, the energy transfer toward the matter field was made much more efficient starting at \(|t| < H_0^{-1}\), hence the \(\sigma\) growth was accelerated and \(\pi\) was accordingly decelerated, making preheating possible.

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Hence the Galileon would still have been slowed down, but in a less efficient way than with the coupling considered above. If we had chosen \(n > 2\), the energy transfer to \(\sigma\) and the resulting deceleration back-reaction on \(\pi\) would have been even faster.
Since we know the background solution in terms of physical time, it is easier to work in terms of $t$ instead of $\tau$.

In order to find the explicit form of the spectrum of perturbations, a solution for $t_{frz}$ is required for both asymptotic background regimes: $\tau \to -\infty$ and $\tau \to t_0$. Starting by analysing the first regime, we write the explicit form of the time-dependent mass term in (52) in terms of the physical time as follows:

$$\frac{(a^f)''}{a^f} \to \frac{1}{H_0^2} \left[ \begin{array}{c} \pi + \pi^2 + \pi H + 2H^2 + \dot{H} \\ \frac{1}{6} \frac{m_{pl}^2}{\sigma_{IR}} \frac{k^2}{t^2} \left[ \frac{2}{t^2} + \frac{2}{3} \frac{f^2}{m_{pl}^2} \frac{1}{H_0^2 t^4} + 2H^2 + \frac{8}{9} \frac{f^4}{m_{pl}^4} \frac{1}{H_0^4 t^6} \right] \end{array} \right].$$ (54)

Evaluating (54) at $t_{frz}$ and setting the whole equation equal to $k^2$ yields the relationship between the time of freezeout and the wavenumber of a mode which we need.

The region where the background (7) is valid extends up to $t \sim -H_0^{-1} f/m_{pl}$, and hence matches what we earlier called the IR region. Since $a$ starts out equal to 1 as $t \to -\infty$, equation (54) is dominated by the $2/t^2$ over the whole region, so that $t_{frz} \approx -\sqrt{2}/k$ for all times when $a \approx 1$, just as in the previous section. This ensures that this region the spectrum is quasi-scale invariant:

$$\sigma_{IR}^k = \frac{H_0}{k_{IR}} e^{\frac{1}{6} \frac{m_{pl}^2}{\sigma_{IR}} \frac{k^2}{t^2}} \quad k < k_{frz} \lesssim H_0 m_{pl}/f, \quad -H_0^{-1} f/m_{pl} \gtrsim t > t_{frz}. \tag{55}$$

We now look at the growth of energy density in the matter field. From (29), the energy density in each frozen $k$ mode is given by

$$\rho_{IR}^k(t, k) \approx \frac{3}{4\pi^2} \frac{H_0^2 m_{pl}^2}{f^2} \left[ \begin{array}{c} \frac{2}{3} \frac{f^2}{m_{pl}^2} \frac{1}{H_0^2 t^4} \end{array} \right] \quad k < k_{frz} \lesssim H_0 m_{pl}/f, \quad -H_0^{-1} f/m_{pl} \gtrsim t > t_{frz}. \tag{56}$$

The energy density $\tilde{\rho}_\sigma$ during the IR regime will therefore be given by:

$$\tilde{\rho}_{IR} = \int_{k_{frz}}^k d^3k (2\pi)^3 \rho_{IR}^k (t, k) \approx \frac{3}{4\pi^2} \frac{H_0^2 m_{pl}^2}{f^2} \left[ \begin{array}{c} \exp \left[ -\frac{2}{3} \frac{f^2}{m_{pl}^2} \frac{1}{H_0^2 t^4} \right] - \exp \left[ \frac{2}{3} \frac{f^2}{m_{pl}^2} \frac{1}{H_0^2} \left( \frac{1}{t_1^4} - \frac{1}{t^4} \right) \right] \end{array} \right] \quad t \lesssim -H_0^{-1} f/m_{pl}. \tag{57}$$

As expected, Taylor expanding the exponential, we recover the gravity-decoupled result (51) in the limit when $t \to -\infty$.

As $t \approx -H^{-1}$, the background solution shifts smoothly from (7) to (11), and $\sigma$ enters a new regime of evolution that we call the UV regime. Now, the time-dependent mass is given by

$$\frac{(a^f)''}{a^f} \to \frac{1}{H_0^2} \left[ \begin{array}{c} \frac{6}{(t_0 - t)^2} + \frac{80}{3} \frac{f^2}{m_{pl}^2} \frac{1}{H_0^2 (t_0 - t)^4} + \frac{512}{9} \frac{f^4}{m_{pl}^4} \frac{1}{H_0^4 (t_0 - t)^6} \end{array} \right]. \tag{58}$$

In the last step, we have used the fact that the term $\frac{6}{(t_0 - t)^2}$ has already stopped being dominant at the time when the background enters the UV regime, and we have also neglected the $\sim \frac{1}{H_0^2 (t_0 - t)^2}$ term since we are interested in the behaviour of $\sigma$ as the background gets close to the singularity at $t_0$.

Working in this approximation, the freeze-out time in the UV regime is given by a Lambert W function $W_0$ of the comoving wavenumber, which cannot be expressed in terms of elementary functions and whose divergence at infinity is slower than the divergence of a logarithmic function:

$$(t_0 - t_{frz}) \approx \frac{4}{3H_0} \left( \frac{1}{W_0 \left( \frac{2}{3 \sqrt{3}} \left( \frac{k_{IR}}{H_0} \right)^{2/3} \right)} \right), \quad t \to t_0. \tag{59}$$
FIG. 2: Power spectrum of $\sigma_k$ fluctuations after freezeout when the full background with gravitational coupling is considered, obtained by solving numerically the differential equation for the background $\pi$ and $H$ and the time-dependent mass in (58). The colour map is logarithmic in the amplitude of the power spectrum, and increases from blue to red. The horizontal axis shows the time evolution of the perturbations for a given $k$ mode, while the vertical axis shows the logarithm of the $k$ spectrum at a given time. Modes below the dotted line are frozen out with the indicated amplitude, while modes with paler colour above the dotted line are still oscillating. The first thing to note is that the spectrum is now constant for all modes that are frozen out. At the IR end (i.e. for $k \lesssim H_0$), the spectrum is almost scale-invariant and slightly red-tilted until the end of that regime after $t = -H_0^{-1}$. On the other side, the UV end of the spectrum is very tilted towards the blue, and the onset of this UV regime marks the onset of preheating. In between the two regimes, the graph shows an interesting feature in the power spectrum: a small trough between $k = 10^{-4}$ and $k = 10^{-3}$. This feature allows for a red-tilted power spectrum during the fake de Sitter phase, or IR regime, where the spectrum is almost scale invariant, at the same time as an efficient preheating with a blue spectrum in the UV end, so that the power spectrum becomes highly dominated by UV modes as defrosting proceeds.

Inserting this result into (59), we obtain the approximate solution for $\sigma_k$ in the UV regime:

$$k_{frz} > k \gtrsim H_0 m_{pl}/f, \quad t > t_{frz} \gtrsim -H_0^{-1} f/m_{pl}, \quad \sigma_k^{UV} \approx \frac{m_{pl} \sqrt{2\epsilon} W_0 \left( \sqrt{\sqrt{\frac{2}{f}} m_{pl} H_0} \right)^{2/3}}{\frac{2}{3} \pi^{2/3} \left( \frac{k}{H} \right)^{4/3}} \left( \frac{\pi^{2/3}}{m_{pl}} \right)^{1/2} \left( \left( \frac{k}{H} \right)^{-2/3} \right) \left( \frac{m_{pl} H_0 \sqrt{3}}{f} \right) \left( \frac{2}{3} \right)^{2/3} \left( \frac{k}{H} \right)^{4/3}$$

(60)

Hence we see that as the matter field enters the UV regime, the power spectrum of $\sigma_k$ perturbations starts to deviate from scale independence and is multiplied by an additional divergent part that grows as the square of a Lambert W function as the background approaches the singularity. This means that in the UV regime the spectrum gets more and more tilted toward the blue as $t \to t_0$. Therefore, not only does the re-introduction of the coupling to gravity in the background push the singularity forward in time from $t = 0$ to $t = t_0 \sim \frac{2}{3} m_{pl} H_0^{-1}$, but, quite surprisingly, as $\sigma$ approaches the singularity, its perturbations grow faster than when we fix $H = 0$.

If we find the full numerical solution for the background Galileon and Hubble rate, as well as for the time-dependent mass (58) and insert them into (59), we obtain solutions for $\sigma_k$ for every $k$ after their freeze-out and can compute the full power spectrum of $\sigma_k$ perturbations. The result is shown in Figure 2 for $f = m_{pl}$ and $H_0 = 10^{-4}$. One interesting feature of the power spectrum is the small trough between $k = 10^{-4}$ and $k = 10^{-3}$. Because of this feature, the power spectrum in the scale invariant region (on scales larger than $k^{-1} = 10^5$) is slightly red tilted up to scales where it reaches the bottom of the trough. For larger values of $k$ it then becomes very heavily blue tilted.
Turning now to the evaluation of the growth of the energy density during the UV regime, we write the energy density in every frozen $k$-mode as:

$$\rho^{IR}(k,t) \simeq \frac{2^6 f^2}{3 \pi^2 m_{pl}^2 H_0^3(t_0 - t)^4} \left[ \frac{H_0^2}{k} e^{\frac{4}{3} \frac{m_{pl}^2}{H_0^2(t_0 - t)^2}} \right] \left( \frac{3 \pi^2}{m_{pl}^2} \right)^2 \left( k \frac{H_0}{m_{pl}} \right)^2$$

$k < k_{frz} \lesssim H_0 \frac{m_{pl}}{f}, \ t_{frz} < -H_0^{-1} f \frac{m_{pl}}{f} \lesssim t$;

$$\rho^{UV}_\sigma(k,t) \simeq \frac{2^4 f}{\pi^2 m_{pl}^2 H_0^3(t_0 - t)^4} \left[ W_0 \left( \frac{2\pi \sigma}{3} \frac{m_{pl}}{H_0} \right) \right] \left( \frac{3 \pi^2}{m_{pl}^2} \right)^{2/3}$$

$k > k_{frz} \gtrsim H_0 \frac{m_{pl}}{f}, -H_0^{-1} \frac{f}{m_{pl}} \lesssim t_{frz} < t$.

The total averaged energy density $\bar{\rho}_\sigma$ during the UV regime is now:

$$\bar{\rho}^{UV}_\sigma = \int_{k_{frz}}^{k_{frz}^f = -H_0^{-1}} \frac{d^3 k}{(2\pi)^3} \rho^{IR}(k,t) + \int_{k_{frz}}^{k_{frz}^f = -H_0^{-1}} \frac{d^3 k}{(2\pi)^3} \rho^{UV}(k,t)$$

$$\simeq \frac{2^4 f}{\pi^2 m_{pl}^2 H_0^3(t_0 - t)^4} \left[ \frac{2 f^2}{m_{pl}^2} \right] - \exp \left( 2 \frac{f^2}{3 m_{pl}^2} H_0^2 t_0^2 \right) + \frac{3^3}{\pi^2} \frac{f^2}{m_{pl}^2 H_0^3(t_0 - t)^4} \int dw (1 + w) \bar{W}_0 \left( \frac{2 f^2}{m_{pl}^2} \right)$$

where we have used the change of variables $we^w = x$ and $W_0(x) = w$ with $x = \frac{2 f^2}{3 \pi^2} \left( \frac{k}{H_0} \right)^{2/3}$. The limits of integration are still from $w$ evaluated at the scale freezing out at the time when the background just enters the UV regime, up to $w$ evaluated to the scale freezing out at the time when we want to know the energy density. Considering that a realistic value of $f$ implies $f \sim m_{pl}$, it is realistic to fix $f = m_{pl}$. Making use of that assumption, the remaining integral can be evaluated, yielding a leading diverging behaviour close to the singularity at $t_0$:

$$\bar{\rho}^{UV}_\sigma \sim \frac{2^{11}}{32 \pi^2} \frac{W_0 \left( \frac{2 f^2}{m_{pl}^2} \right)}{H_0^6 (t_0 - t)^{10}}, \ t \rightarrow t_0.$$

We have used (61) to relate $k$-modes to their corresponding time of freeze-out. We now see that when the coupling to gravity is re-introduced, the leading contribution to the average energy density in $\sigma$, $\bar{\rho}_\sigma$, in the UV regime grows faster than $\sim 1/(t_0 - t)^{10}$, while, as discussed before, the energy density in the Galileon, $\rho_\pi$, grows as $1/(t_0 - t)^6$. This means that when a full treatment of the background including couplings to gravity is considered, then as the Galileon rolls close to the singularity at $t_0$, the energy density in the matter field with minimal kinetic coupling will eventually come to dominate the Galileon, allowing for the onset of preheating.

The only thing left now for us to check in order to ensure that the Galilean Genesis model does not suffer from a graceful exit problem is the back-reaction of $\sigma$ on $\pi$. That is, we need to verify that, once coupling to gravity is re-introduced, the growth of $\sigma$ will still slow down the Galileon and not accelerate it, and make it evolve from the de Sitter configuration toward its $\pi = $ constant solution. This, however, is very straightforward to check, since although the EoM for $\pi$ (still working in physical time) becomes significantly more complex (see (61)), the source term coming from $\sigma$ is still the same as before and does not involve any additional terms in $H$. That is, the source term from $\sigma$ is still:

$$- \frac{1}{f^2} \left[ \dot{\sigma}^2 - \frac{(\nabla \sigma)^2}{a^2} \right].$$

When computing the back-reaction EoM for $\pi$ in Fourier space, we again assume a fixed background metric. Indeed, it is reasonable to assume that the time scale for the instability in $\sigma$ to develop and overcome the evolution of the background is much shorter than the characteristic time scale for any metric perturbations to become important. It is therefore possible to write the contribution of $\sigma$ to the background Galileon back-reaction equation $\delta^{(2)} \pi_{k=0}$ as a source term on the r.h.s. of the form:

$$\Delta \text{EoM}_{\delta^{(2)} \pi_{k=0}} = - \frac{1}{f^2} e^{-2\sigma_0} \bar{\rho}_\sigma.$$

Since $\sigma$ is a regular matter field $\rho_\sigma$ is obviously always a positive quantity. Therefore, the additional source term that $\sigma$ contributes to the back-reaction to the Galileon background always has the effect of slowing down $\pi$. A more
explicit approximate expression of this source term can be obtained in each of the two regimes considered above for π₀ and ρ_σ. In the IR regime, we obtain:

\[ \Delta^{IR} EoM_{\delta^{(2)}\pi=0} \approx -\frac{3H_0^4}{4\pi^2} m_{pl}^2 \left[ e^{\frac{t^2}{2m_{pl}^2}} \left( \frac{d^2}{dt^2} - \frac{1}{t} \right) \right] , \quad t \simeq -H_0^{-1} f/m_{pl} , \quad (67) \]

while in the UV regime, setting \( m_{pl} = f \), we obtain that the leading contribution to the source term is:

\[ \Delta^{UV} EoM_{\delta^{(2)}\pi=0} \approx -\frac{2}{3\pi^2} f^2 W_0 \left( \frac{2^4 \pi^4}{3^2 \pi_0^2 H_0^2 (t_0-t)^2} \right) H_0^2 (t_0-t)^6 , \quad t \to t_0 . \quad (68) \]

We can therefore conclude that, as fluctuations in the matter field get amplified because of their immersion in fake de Sitter space, and as the total energy density in σ grows accordingly, the Galileon from which σ gets its energy slows down. This therefore ensures that the Galileon Genesis scenario does not suffer from a graceful exit problem, and that the system will proceed to a FRW phase dominated by σ, in which the NEC is re-established with \( \dot{H} < 0 \).

VI. CONCLUSIONS AND DISCUSSION

In this paper we have studied the transition between an Emergent Galileon background phase and the radiation phase of an expanding universe. We have shown that, at least when including the effects of non-vanishing expansion on the background fields, the same coupling of the Galileon condensate to regular scalar field matter introduced in [74] is sufficient to ensure a rapid energy transfer to the matter field which then via its back-reaction on the Galileon background leads to a slowing down of the Galileon condensate.

There are similarities and differences between the defrosting transition of the Galileon background studied here and preheating in inflationary cosmology. In both cases, it is the coherent dynamics of the background matter field which drives the production of regular matter. Here it is the dynamics of the Galileon background, in inflationary cosmology it is the coherent oscillations of the inflaton condensate at the end of the period of inflation. However, here it is the same squeezing of fluctuations which leads to scale-invariant matter fluctuations which leads to defrosting/preheating, whereas in inflationary cosmology the generation of scale-invariant fluctuations and the reheating instability are separate processes.

In inflationary cosmology, it is mostly long wavelength modes which are excited during preheating. On the other hand, we have shown here that efficient Galileon defrosting is based on a sharp blue tilt of the spectrum in the UV. In light of these similarities and differences it would be of great interest to study reheating in Galileon-based inflation models [91].

We wish to end with a comment on the generation of curvature fluctuations in the Emergent Galileon scenario: since the background matter has vanishing background value, a scale-invariant spectrum of the matter fields does not lead to scale-invariant spectrum of curvature fluctuations (see e.g. [74]) since the curvature fluctuations are quadratic in the matter perturbations. In addition, if the matter fluctuations have Gaussian statistics, the curvature perturbations with not be Gaussian. This is an issue which merits further study.

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