Do the Age of the Universe and the Hubble Constant Depend on What Scale One Observes Them?

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Abstract

The apparent cosmological conflict between the age of the Universe, predicted in the standard Friedman cosmology by using the recent measurement of the larger Hubble constant from a direct calibration of the distance to the Virgo galaxy cluster, and the ages of the oldest stars and globular clusters is resolved by invoking the scale dependence of cosmological quantities, including the age of the Universe. The distance dependence or the running of cosmological quantities is motivated by the asymptotically-free higher-derivative quantum gravity. The running can also be derived by “properly” modifying the Friedman equations. This property can also provide partial explanation of the apparent disagreement between the two recent measurements of the Hubble constant using NGC 4571 at 15 Mpc and NGC 5253 at 4 Mpc.

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Recent measurements of the Hubble constant, $H_0$ using NGC 4571 (distance $= 14.9 \pm 1.2$ Mpc) in Virgo cluster $^1$ and NGC 5253 (distance $= 4.1$ Mpc) in a galaxy closer than Virgo cluster $^2$, and their implied ages of the Universe have become a subject of heated controversy. In addition to the disagreement between the two measurements, the major problem is that the age of the Universe predicted by the use of the larger value of $H_0$ from NGC 4571 ($H_0 = 87 \pm 7$ Km/sec Mpc) and the standard Friedman cosmology with flat space becomes about half the measured ages of 14 to 18 Gyr for the oldest stars and globular clusters $^3$. In this article, we show that the application of the concept of the running of the physical quantities according to the Renormalization Group Equations (RGE) $^4$ to cosmology can solve the age puzzle and explain the difference between the two $H_0$ measurements, at least partially. It is well-known that masses as well as coupling constants are running, i.e. their values depend on what energy or momentum scales one measures or calculates them. If the gravitational constant $G$ is asymptotically free as suggested by recent works $^5$, “modified” Friedman-like cosmological equations are shown to imply that $G$, $H_0$, and $\Omega_0 (= (\rho_0/c^2))$ are all increasing functions of scale or distance, even at present epoch, whereas the age of the Universe, $t_0$, is a decreasing function of distance at which one measures or calculates it.

There are several ways to modify the Friedman cosmology. The best is, of course, to start with “the ultimate theory” of quantum gravity. Since this has not been realized as yet, one can approach in the way described in Refs. $^6$ and $^7$ by incorporating the running gravitational constant into the Friedman equations. One can also modify the Robertson-Walker metric by relaxing the Cosmological Principle, which states that the Universe has symmetric homogeneous space-time, with ”homogeneous” replaced by ”only globally homogeneous”. This leads to the running of $H_0$, $R_0$ and $\Omega_0$ as functions of scale. To accomplish this, it is also necessary to modify the Einstein equation itself by replacing its right-hand side by $8\pi G(d)T_{\mu\nu}$ with the constraint that the covariant derivative of $G(d)T_{\mu\nu}$ be zero where $d$ denotes distance. This, of course, modifies the equation of state and mimics the running gravitational constant as a function of distance. Details on this approach will be discussed elsewhere $^8$.

To set the stage for later cosmological argument, let us consider the following metric in order to illustrate the case of simply changing the metric,

$$dr^2 = dt^2 - R^2(t, r)(dr^2 + r^2d\Omega^2). \tag{1}$$

When $R(t, r) = R(t)$, Eq.(1) reduces to the Robertson-Walker metric for $k = 0$, which is a consequence of the Cosmological Principle. Equation (1) describes the Universe which has spherically symmetric ( and only globally homogeneous) space-time. Physical rational for this relaxing is as follows. When one looks at a large scale object such as, say, a cluster, then due to the increasing amount of dark matter and/or larger gravitational constant, it appears that $\Omega_0$ has increased from that of a galaxy scale. Since the scale factor is governed by $(G\rho)$ which now depends on scale, it must depend on the scale $r$ as well as $t$ ( see $^8$ for detail).

For an illustrative purpose, let us consider the case when

$$R(t, r) = a(t) [1 + \alpha (ra(t))^n]; \alpha > 0, n > 0, \tag{2}$$
which is clearly an increasing function of \( r \). That is, the local \((r = 0)\) scale factor at any given time is always the smallest, and the scale factor at any fixed time increases as distance (from us) increases. In this case the metric is obviously not that of the Robertson-Walker. (The case, in which \( R(t, r) \) is factored out as \( a(t)S(r) \), reduces to the Robertson-Walker metric with redefinition of \( r \).) The the expansion rate becomes

\[
H(t, r) = \frac{\dot{R}}{R} = \left[ \frac{\dot{a}}{a} \right] \frac{1 + (n + 1)\alpha r^n [a(t)]^n}{1 + \alpha r^n [a(t)]^n}
\]

which is now an increasing function of \( r \). Strictly speaking, the expansion rate must be defined by \((\dot{D}/D)\), where \( D(t, r) = \int R(t, r)dr \), but for scales up to, say, the Virgo cluster which has \( z \sim 0.004 \) and \( r \sim 4 \times 10^{-3} \) (assuming the horizon has \( r \sim 1 \)), the definition used in Eq.(3) is, for our present purpose, a good approximation to the more rigorous one. In general, however, one needs caution in defining the expansion rate in the modified Friedman cosmology, especially for dealing with much more distant or larger objects.

The present value, \( H_0(r) \), in Eq.(3), is no longer universal, but instead depends on what scale one uses to observe or calculate \( H_0 \). If \( H_0^2(r) \) is proportional to the gravitational constant \( G \) as in the case of the Friedman cosmology, \( G \) increases as \( r \) increases as well. Also, since the age of the Universe, when it is calculated using the cosmological equations, is inversely proportional to \( H_0(r) \), the age of the Universe becomes older when estimated using a smaller scale, whereas it decreases as the scale becomes larger. This does not mean, however, that the age depends on where one calculates. In fact, every observer who uses the same scale \( r \) at \( t_0 \) would obtain the same age. Although the above discussion has nothing to do with the RGE, physics involved is precisely that of the running of masses and coupling constants as functions of energy or momentum scale based on the RGE. As in the case of running masses and coupling constants, direct comparison of cosmological quantities makes sense only when they were observed or calculated at the same scale. The same quantity can take different values at different scales. Therefore, the measured ages of the oldest stars and globular clusters in our neighborhood (less than 100 Kpc, say) have to be compared with the age of the Universe calculated at the same distance scale. Now the question is “How can this scenario be realized?” Here, we follow the approach used in Refs. [6] and [7].

In the asymptotically-free higher-derivative quantum gravity [5], an inflationary period eventually settles into a standard Friedman epoch that behaves as if the Universe is matter-dominated, but with a very important modification. That is, the gravitational constant \( G \) that appears in the Friedman equations is replaced by the one which is asymptotically free, i.e., \( G \) increases as a function of distance scale according to the RGE [5,6]. This \( G \) takes the value of the Newton’s value \( G_N \) at short distances but slowly starts to rise as distance increases. For this asymptotically-free behavior, see, for example, figure 2 of Ref. [6] and figure 1 of [7]. In addition to the above modification, there are other consequences.

During the inflation, space is flattened so that the \( k = 0 \) case (flat) is realized. Therefore, the present value of \( \Omega_0 \equiv (\rho_0/\rho_c) \) is unity, where \( \rho_0 \) is the present density and \( \rho_c \) is the present density in the case of a flat Universe. However, there is a very important difference between the standard cosmology and the present one. Since \( G \) is distance-dependent (or running), \( \Omega_0 \) also becomes distance-dependent. That is, \( \Omega_0 \) does not have to be unity everywhere. \( \Omega_0 = 1 \) may be realized at a very large distance, maybe near the horizon or at some point.
further out than the horizon. Therefore, in our local neighborhood $\Omega_0$ is less than unity and locally open even though the Universe as a whole has no curvature. (This point has also been noticed by others in different context [9].) In short, some cosmological quantities may run as functions of distance scale. The running of $H_0$ and $\Omega_0$ and others can, in principle, be calculated based on their RGE, but unfortunately it is not feasible at present because of the lack of a satisfactory quantum version of gravity. Therefore, we adapt the following phenomenological approach.

The modified Friedman-like equations with $k = -1$ (this is true in our local neighborhood since the local $\Omega_0$ is less than unity), that have been suggested by the asymptotically-free higher-derivative quantum gravity, are [6,7]

$$\dot{R}^2(t, r) = \frac{8\pi}{3}\rho G(d)R^2(t, r) + 1; \quad G \cdot \rho = G \cdot \rho_0 \left[\frac{R_0}{R}\right]^3,$$

(4)

$$\Omega_0(d_0) = \frac{G \cdot \rho_0}{G \cdot \rho_{c,0}} = \frac{8\pi G(d_0)\rho_0}{3H_0^2(d_0)},$$

(5)

where the subscript zero denotes the present values, and $G(d)$ is the value of $G$ at (proper) distance $d$ , if one interprets $d$ as an inverse of momentum, or simply $r$ if one uses the comoving distance parameter. But, at present epoch, this distinction is irrelevant. The validity of the above equations is, admittedly, not on a firm ground as one would like it to be. But, in this article, we shall assume them as phenomenological and treat them as such. With this cautionary remark, we now investigate some consequences of Eqs.(4) and (5). The behavior of $G(d)$ given in Ref. [6] and [7] can be best fitted by the following expression:

$$G(d) = G_N(1 + 0.3 d^{0.15}) \equiv G_N[1 + \delta_G(d)],$$

(6)

where $d$ is expressed in units of Kpc [6]. Since $d$ is a proper distance, we can rewrite

$$d(t, r) = \left[\frac{R(t, r)}{R(t_0, r)}\right]^{10^7r} 10^7,$$

(7)

where $r = 1$ corresponds roughly to the size of the horizon and $r = 3 \cdot 10^{-7}$ gives 1 Kpc at present. In fact, in Eq.(7) $d$ does not have to depend on $t$ as mentioned already. Such a model is presented in Ref. [6]. It is to be emphasized here that an analytic solution for $R(t, r)$ cannot be obtained from Eq.(4) but since $d$ is a function of $t$ and $r$, $R$ obviously has to be a function of $t$ and $r$. This behavior is precisely the same as that of the example discussed in Eq.(2). As a consequence, the expansion rate $H = (\dot{R}(t, r)/R(t, r))$ (recall that this approximation is valid for $z \ll 1$) is also a function of $r$ or distance. As mentioned already, $\Omega_0$ also depends on the distance scale. That is, both $H_0$ and $\Omega_0$ run as functions of distance.

An interpretation of Eq.(5) as an indication of growing $\Omega_0(d_0)$ with distance was discussed in Ref. [6], leading to the necessity of less or no dark matter in the Universe. In arriving at this conclusion, it was assumed in Ref. [6] that $H_0$ has no distance dependence. It was also shown in Ref. [7] that Eq.(7) leads to an increased power of the two point correlation.
function at large distances without help of dark matter with \( \Omega_0 = 1 \). In Refs. \[7\] and \[9\], the possibility of growing \( H_0 \) with scale was also mentioned with no further elaboration.

Equations (4) and (5) lead to the following phenomenological expressions for the running behavior of \( H_0 \) and \( \Omega_0 \):

\[
H_0(d_0) \simeq H_0 \sqrt{1 + \Omega_0 \delta_G(d_0)},
\]

\[
\Omega_0(d_0) \simeq \Omega_0 \frac{1 + \delta_G(d_0)}{1 + \Omega_0 \delta_G(d_0)},
\]

where bars denote local values at, say, \( r \simeq 0 \) or \( d_0 \simeq 0 \) and \( \delta_G(d_0) \) is given by Eq.(6). Both \( H_0(d_0) \) and \( \Omega_0(d_0) \) are increasing functions of \( d_0 \). However, the scale dependence of \( H_0(d_0) \) is much slower than that of \( \Omega_0(d_0) \). In deriving Eqs.(8) and (9), we have assumed that the \( d_0 \) dependence of \( H_0^2 \) is mainly due to that of \( G \) so that the \( d_0 \) dependence of \( R(t_0, d_0) \) is, in general, much slower than those of \( H_0(d_0) \) and \( G(d_0) \) (i.e., \( R(t_0, d_0) \simeq R(t_0, d_0 \simeq 0) \) for up to the distance of our interest, 15 Mpc. This assumption is explicitly justified in a model considered in Ref.[8]. We must also caution that Eqs.(8) and (9) did not take into account a further increase of \( \Omega_0 \) due to the possibility that the dark matter content increases as distance increases. (This effectively makes \( \delta_G(d_0) \) grows faster than Eq.(6) if one keeps \( \rho_0 \) as constant.) Equation (8) states that \( H_0 \) obtained at distance of 15 Mpc has to be somewhat larger than that of 4 Mpc since \( H_0 \) is slowly increasing as distance increases, which explains the discrepancy mentioned above, at least qualitatively. Our estimate based on Eq.(8) without dark matter indicates that the growth of \( G(d) \) is not sufficiently fast enough to explain the discrepancy between the recent two measurements of \( H_0 \), although up to about 20% increase of \( H_0 \) from 4 Mpc to 15 Mpc can be accounted for. However, within one and half standard deviations, the two measurements can be made consistent in our picture of growing \( H_0 \) without dark matter. The increase of \( \Omega_0(d_0) \) is shown to be consistent with observations (see also Ref.[6]) with much less dark matter than \( \Omega_0 = 1 \) requires. Results of detailed calculations with (and without) the contributions from dark matter will be given elsewhere [8].

We now proceed to discuss the age problem. Combination of Eqs. (4) and (5) yields, with the approximation mentioned already,

\[
\frac{1}{R_0^2(d_0)} \simeq \frac{1}{R_0^2(d_0 \simeq 0)} \simeq [1 - \Omega_0(d_0)] H_0^2(d_0).
\]

Note that in our approximation the distance dependences of \( \Omega_0(d_0) \) and \( H_0(d_0) \) on the right-hand side are supposed to be cancelled with each other.

Using Eqs. (4), (8), (9), and (10), we find the age equation

\[
\int_0^1 \frac{\sqrt{x} dx}{x [1 - \Omega_0(d_0)] + \Omega_0(d_0) [1 + \delta_G(x, r)]} \simeq \bar{H}_0 t_0(d_0),
\]

where
\[ \delta G(x, r) = 0.3 \left[ \frac{10^7 r x}{3} \right]^{0.15}, \] 

(12)

and

\[ x = \frac{R(t, r)}{R(t_0, r)}. \] 

(13)

An identical expression to Eq.(11), with exception that \( \delta G \) has no \( x \) dependence, can also be derived classically by modifying the Robertson-Walker metric and the Einstein equation [8]. The following comments are in order:

1. When \( \Omega_0 \) and \( H_0 \) are replaced by the standard \( \Omega_0 \) and \( H_0 \), and \( \delta G(x, r) \) is set equal to 0, Eq.(11) reduces to the age equation for \( k = -1 \) in the Friedman cosmology.

2. Equation (11) is valid up to the scale where our approximate definition of \( H \) is still valid.

3. The age of the Universe \( t_0(d_0) \) depends on the value of \( d_0 \) or \( r \), i.e., \( t_0(d_0) \) is also running as a function of \( d_0 \) so that the age depends on which distance or scale we are interested in. As can be seen from Eqs.(11) and (12), the smaller the value of \( r \), the older the age becomes and vice versa. This is our main result.

We now present some results of numerical calculations based on Eq.(11). Interestingly, it turns out that as long as \( d_0 \lesssim 100 \) Kpc, \( t_0(d_0) \) in our neighborhood is insensitive to \( d_0 \). It only depends mildly on the choice of \( \Omega_0(d_0) \). We find, for \( d_0 \lesssim 100 \) Kpc,

\[ t_0 \simeq \frac{0.9}{H_0} \quad \text{for} \quad \Omega_0 = 0.1 \]
\[ t_0 \simeq \frac{0.85}{H_0} \quad \text{for} \quad \Omega_0 = 0.2, \] 

(14)

where \( H_0 \) and \( \Omega_0 \) denote our “local” values, as defined earlier. We believe that \( \Omega_0 = 0.1 \) case is more appropriate because this value is consistent with the dark matter content in our galactic halo as well as with the bound from nucleosynthesis. The choice of the local value of \( H_0 \) is somewhat ambiguous due to our inability to measure it locally. Therefore, we take, for definiteness, the most commonly used canonical lower value \( H_0 = 50 \) Km/sec Mpc. The resulting age is

\[ t_0 \simeq 18 \quad \text{Gyr}, \] 

(15)

which is comfortably long enough to accommodate the known ages of oldest stars and globular clusters, also obtained in our neighborhood. We emphasize that this comparison is justified because both are measured or calculated at the same scale.

The age at the distance of 15 Mpc becomes

\[ t_0(d_0 = 15\text{Mpc}) \simeq \frac{0.81}{H_0} = 16 \text{Gyr}, \] 

(16)
which is shorter than that of Eq.(15). This, however, is anticipated, as mentioned already, because of the running (decreasing as scale increases) of $t_0(d_0)$.

Another age of interest is the one at the distance of 4 Mpc. The result is $t_0(d_0) \simeq 16.4$ Gyr. We mention here that because of the approximation we used in defining $H$, we cannot use Eq.(11) to calculate the age at scales much beyond the Virgo cluster. For comparison, the standard Friedman cosmology with $\Omega_0 = 1$ gives the age

$$t_0 = \frac{0.66}{H_0},$$

where $H_0$ is a fixed value, independent of scale. Therefore, $t_0$ in Eq.(17) does not run. This causes problems when different observed values of $H_0$ are used in Eq.(17). It is worth mentioning that in order to be self-consistent, one has to take into account the effective growth of $\delta G(x,r)$ in Eq.(11) due to dark matter in addition to that given in Eq. (6). This is because we have used effective $\Omega_0(d_0)$ values such as 0.1 for $d_0 \lesssim 1$ Kpc and 0.4 for $d_0 = 15$ Mpc which include the possible dark matter contributions. In order to see the sensitivity of the age on the content of dark matter, we have increased the coefficient 0.3 in Eq. (12) by 50% which requires a large amount of dark matter. The resulting ages turn out to decrease only by 2% and 8%, respectively, for $d_0 \lesssim 100$ Kpc and $d_0 = 15$ Mpc.

The final, but important comment on the age calculation concerns the interpretation of various running ages. First, as mentioned already, direct measurements of the ages of local stars and globular clusters do not depend on the use of $H_0$. Therefore, their ages should be compared with the age of the Universe estimated at the same scale, i.e., with the so-called local age of the Universe given in Eq. (15). The agreement is excellent. Then, what does the age of, for example, 16 Gyr at the distance of 15 Mpc mean? It is simply the age for that scale which cannot be compared directly with the ages obtained at different scales, as the fine structure constant $\alpha$ measured at low energies is different from the one measured at LEP energy.

In summary, we have demonstrated that in a modified Friedman-like cosmology based on the asymptotically-free quantum gravity, admittedly phenomenological in nature, even the present values of $\Omega_0$, $H_0$ and $R_0$ all increase as the scale increases. We can make the two recently measured values of $H_0$ consistent with each other with this growing $H_0$ (with scale) within one and half standard deviations, only because $H_0$ does not increase fast enough. We have also shown that when one calculates the age of the Universe, the result depends on what scale one uses. In fact, the age, $t_0$, is a decreasing function of scale. The calculated age of the Universe in our local neighborhood is about 18 Gyr for the assumed local values $\Omega_0 = 0.1$ and $H_0 = 50$ Km/sec Mpc, consistent with the ages of the oldest stars and globular clusters, 14 to 18 Gyr. Our neighborhood is locally open with $\Omega_0 = 0.1$, although the Universe as a whole is flat.

In contrast, the calculated ages of the Universe at distances of 4 Mpc and 15 Mpc from us are 16.4 and 16 Gyr, respectively. We argue that these ages are simply not our local ages and thus should not, in principle, be compared with the ages of the oldest stars or globular clusters in our neighborhood, although they agree within errors. Instead, they should be used in relating quantities observed or calculated at the same scales.

How does the running of $G$ affect the early Universe? Let us take the nucleosynthesis as an example. The relevant $G$ calculated at the horizon scale within which microphysics
operates at the time of nucleosynthesis is the same as $G_N$ since $d$ in Eq.(7) and thus $\delta_G(d)$ in Eq.(6) are practically zero. Therefore, the nucleosynthesis proceeds in the standard manner.

It goes without saying that it is important to continue further study of gravity beyond that of Einstein’s because cosmological implications are far-reaching. Only when we have a satisfactory understanding of quantum gravity on hand, the validity of crucial equations such as those in Eqs.(4) and (5) can be critically checked and the RGE’s for various cosmological quantities can directly be derived. An entirely different, but classical approach based on the modification of the Robertson-Walker metric by relaxing the Cosmological Principle and the Einstein equation so that $G$ runs with scale, and further details on how $H_0(d)$ and $\Omega_0(d)$ behave as functions of scale, as well as their cosmological implications are given elsewhere.

We close with the following remark. In our modified cosmology, the standard formulas that relate distance, redshift, $H_0$ and so on are modified to accommodate the scale dependence of $R(t, r)$. Although our results are valid up to the scales where the approximate definition of $H$ is still valid (i.e., for small $z$), the expected modifications could be significant for larger redshifts when the correct definition of $H$ is used. Therefore, previously obtained values of, for example, $H_0$ from very distant objects with large $z$ based on the standard cosmology have to be reevaluated in the modified Friedman cosmology. This will be discussed elsewhere.

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REFERENCES

[1] M.J. Pierce, et al, Nature 371, 385 (1994).
[2] A. Sandage, et al, Astrophys. J. 423, L13 (1994).
[3] A. Sandage, Astrophys. J. 331, 583 (1988); A. Sandage, Astrophys. J. 350, 631 (1990); Y.-W. Lee, Astrophys. J. 350 (1990); Y.-W. Lee, Astronom. J. 104, 1780 (1992).
[4] M. Gell-Mann and F. Low, Phys. Rev. 95, 1300 (1954); D. Gross and F. Wilczek, Phys. Rev. Letters, 26, 1343 (1973); H.D. Politzer, Phys. Rev. Letters, 26, 1346 (1973).
[5] J. Julve and M. Tonin, Nuovo Cimento, 46B, 137 (1978); E.S. Fradkin and A.A. Tseytlin, Nucl. Phys. B201 469 (1982); E.G. Avramidy and A.O. Barvinsky, Phys. Lett. B159, 269 (1985).
[6] T. Goldman, J. Pérez-Mercader, F. Cooper and M. Martin Nieto, Phys. Lett. B281, 219 (1992).
[7] O. Bertolami, J.M. Mourão and J. Pérez-Mercader, Phys. Lett. B311, 27 (1993).
[8] C.W. Kim, A. Sinibaldi and J. Song, to be published.
[9] A. Linde, “Lectures on Inflationary Cosmology”, SU-ITP-94-36, hep-th/9410082.