Chiral Symmetry and $N^*(1440) \to N\pi\pi$ Decay

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Abstract

The $N^*(1440) \to N\pi\pi$ decay is studied by making use of the chiral reduction formula. This formula suggests a scalar-isoscalar pion-baryon contact interaction which is absent in the recent study of Hernández et al. The contact interaction is introduced into their model, and is found to be necessary for the simultaneous description of $g_{RNN\pi}$ and the $\pi\pi$ and $\pi N$ invariant mass distributions.

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I. INTRODUCTION

In the theoretical understanding of baryon spectra, the Roper resonance $N^*(1440)$ is well known as the controversial object. Although the naive quark model assigns a radial excitation of constituent quarks to this resonance, its mass becomes so heavy that this interpretation does not work [1]. In connection with this problem there are some recent studies of baryon spectra based on, for example, the chiral algebra [2], the pentaquark (two-diquark and antiquark) model [3] and the chiral soliton model [4]. This is, however, still an open problem about the structure of $N^*(1440)$.

Besides being characteristic of the baryon spectra, the Roper resonance is important for the low energy hadron reactions. In particular, two pion production reactions [5, 6, 7, 8, 9] and some nuclear reaction [10] near threshold are sensitive to the $N^*(1440) \rightarrow N(\pi\pi)^{I=0}_{S\text{-wave}}$ decay, in which the Roper resonance goes into a nucleon accompanied by two pions with $S$-wave and isospin zero. The importance of this decay is emphasized by Manley et al. [11], which Particle Data Table [12] refers to. In their analysis, the two decay channels $N^*(1440) \rightarrow \Delta\pi$ and $N\epsilon$ are taken as the intermediate processes of the two-pion decay of $N^*(1440)$. Here the ‘$\epsilon$ meson’ describes the scalar-isoscalar $\pi\pi$ correlation which may correspond to the $\sigma$ meson.

Recently the $\pi\pi$ and $\pi N$ invariant mass distributions of $N^*(1440) \rightarrow N\pi\pi$ decay are examined by Hernández et al [13]. In their work the $\pi\pi$ re-scattering is explicitly considered to describe the scalar-isoscalar $\pi\pi$ correlation in $N^*(1440) \rightarrow N(\pi\pi)^{I=0}_{S\text{-wave}}$ decay instead of the $\epsilon$ propagation used in Manley’s analysis. Since there is no available data for the mass distributions, Hernández et al. considered Manley’s analysis as “experiment”, and employed its result for fixing their model parameters. Hernandez et al. well reproduced the “experimental results” of the invariant mass distributions.

Although their model is effective for the decay processes at the mass shell energy of $N^*(1440)$, it does not work in the $\pi\pi N$ threshold region. The coupling constant of the phenomenological Lagrangian $\mathcal{L}_{RN\pi\pi} = g_{RN\pi\pi} \Psi_N^\dagger \Psi_R \pi^a \pi^a$, extracted from the $\pi\pi N$ threshold amplitude, is much smaller than the one needed to explain the experimental data; $|g_{RN\pi\pi}^{\text{model}}/g_{RN\pi\pi}^{\text{expt}}| = 0.42$. They concluded that some extra contributions are necessary in order to solve this problem.

Chiral symmetry is one of the guiding principles for studying the low energy hadron
processes such as $N^*(1440) \rightarrow N\pi\pi$ decay. The systematic chiral expansion scheme is, however, difficult in the resonance region. As a result, phenomenological treatments have often been applied to the processes including baryon excitations, in which it is not clear whether chiral symmetry is appropriately reflected.

For the purpose of studying hadron reactions without missing chiral symmetry, the chiral reduction formula developed by Yamagishi and Zahed \cite{14} is a powerful method. This formula offers the Ward identity required by chiral symmetry for the scattering amplitudes of any hadron processes involving the pion. We can discuss the general framework of pion induced reactions separately from the detail of specific models.

In this paper, we discuss the general structure of $N^*(1440) \rightarrow N\pi\pi$ decay amplitude by making use of the Ward identity derived from the chiral reduction formula. Considering this general structure, we find that a scalar-isoscalar contact interaction for the pion-baryon vertex in the $N^*(1440) \rightarrow N(\pi\pi)^{I=0}_{S-wave}$ amplitude is absent in Ref. \cite{13}. We bring this contact interaction in the model of Ref. \cite{13}, and discuss its influence on the $N^*(1440) \rightarrow N\pi\pi$ decay. The scalar-isoscalar contact interaction has two aspects in its origin: one is due to the explicit chiral symmetry breaking and the other survives in the chiral limit. We will see that the former part plays an important role to solve the problem in Ref \cite{13}.

We stress here that the scalar-isoscalar contact interaction is not intuitively introduced in our discussion, but naturally appears in the general structure of $N^*(1440) \rightarrow N\pi\pi$ amplitude given by the Ward identity. Therefore we should include this interaction as long as there is no ad hoc reason to neglect it.

This paper is organized as follows. In Sec. II we explain the general structure of the $N^*(1440) \rightarrow N\pi\pi$ amplitude derived from the chiral reduction formula. After a brief review of Ref. \cite{13}, we introduce the scalar-isoscalar contact interaction in the decay amplitude. Taking account of this interaction, we calculate the $\pi\pi$ and $\pi N$ invariant mass distributions for the $N^*(1440) \rightarrow N\pi\pi$ decay. We show our results and discuss the important role of the contact interaction in Sec. III. Summary and conclusion are given in Sec. IV.

II. **THEORETICAL TREATMENT OF $N^*(1440) \rightarrow N\pi\pi$ DECAY**

A. General structure of amplitude

Using the chiral reduction formula \cite{14}, we decompose the $N^*(1440) \rightarrow N\pi\pi$ amplitude
FIG. 1: Kinematics of $N^*(1440) \rightarrow N \pi \pi$ decay. Here $p_R (p_N)$ is the four momentum of the Roper (nucleon), and $k_1, k_2$ and $a, b$ are the four momenta and isospin indices of pions.

$i\mathcal{T}$ as (see Fig. 1 for the kinematics of the $N^*(1440) \rightarrow N \pi \pi$ decay)

$$i\mathcal{T} = i\mathcal{T}_S + i\mathcal{T}_V + i\mathcal{T}_{AA}. \quad (1)$$

Each term on the right hand side is given by

$$i\mathcal{T}_S = -i\frac{m^2}{f_\pi} \delta^{ab} \langle N(p_N)|\hat{\sigma}(0)|N^*(p_R)\rangle, \quad (2)$$

$$i\mathcal{T}_V = -\frac{1}{2f_\pi^2} (k_1 - k_2)\epsilon^{abc} \langle N(p_N)|j_{V\mu}^c(0)|N^*(p_R)\rangle, \quad (3)$$

$$i\mathcal{T}_{AA} = +\frac{1}{f_\pi^2} k_1^\mu k_2^\nu \int d^4x e^{ik_1x} \langle N(p_N)|T^*(j_{A\mu}^a(x)j_{A\nu}^b(0))|N^*(p_R)\rangle, \quad (4)$$

where $\hat{\sigma}$ and $j_{V\mu}^c$ are the scalar density and the vector current, respectively, and $j_{A\mu}^a$ is the one-pion reduced axial current which is obtained by extracting the one-pion component from the ordinary axial current. The delta function $(2\pi)^4\delta^{(4)}(p_R - p_N - k_1 - k_2)$ for the momentum conservation is suppressed in these expressions.

$\mathcal{T}_S$ appears owing to the explicit chiral symmetry breaking, and vanishes in the chiral limit $m_\pi \rightarrow 0$. This term contributes only to the $N^*(1440) \rightarrow N(\pi\pi)^{I=0}_{S\text{-wave}}$ amplitude because of the scalar-isoscalar nature of $\hat{\sigma}$. $\mathcal{T}_{AA}$ also contributes to the $N^*(1440) \rightarrow N(\pi\pi)^{I=0}_{S\text{-wave}}$ amplitude. In contrast to $\mathcal{T}_S$, this term does not vanish in the chiral limit. The decay process in which baryon resonances appear in the intermediate steps, such as $N^*(1440) \rightarrow \Delta\pi$, is included in $\mathcal{T}_{AA}$. In the low energy region, $\mathcal{T}_V$ expresses the amplitude of $N^*(1440) \rightarrow N\rho$ decay process in which two pions are correlated in $P$-wave.
FIG. 2: The ‘open’ diagram (a), and the ‘closed’ diagram (b) for the $N^*(1440) \rightarrow N\pi\pi$ decay in Ref. [13]. The pion-baryon vertices are given by the standard axial derivative coupling. The blob in (b) expresses the $\pi\pi$ re-scattering.

Here we write explicit expressions of the $N^*(1440) \rightarrow N(\pi\pi)^{I=0}_{S\text{-wave}}$ amplitude. By introducing the form factors, we have

$$i\mathcal{T} \left[ N^*(1440) \rightarrow N(\pi\pi)^{I=0}_{S\text{-wave}} \right] = i\mathcal{T}_S + i\mathcal{T}_{AA}^C,$$

where $s_{\pi\pi} = (k_1 + k_2)^2$ is the invariant mass square of the emitted two pions and $\Phi_N$ [$\Phi_R$] and $\chi_N$ [$\chi_R$] are the isospin and spin Pauli spinors of the nucleon [Roper], respectively. $\mathcal{T}_{AA}^C$ is a part of $\mathcal{T}_{AA}$ contributing to the $N^*(1440) \rightarrow N(\pi\pi)^{I=0}_{S\text{-wave}}$ decay. The nonrelativistic approximation is taken for the baryon states in order to compare our results with those of Ref. [13]. The form factor $\sigma_{RN}(s_{\pi\pi})$ vanishes in $m_\pi \rightarrow 0$. $F_{AA}(s_{\pi\pi})$ does not include the single baryon poles owing to the notation of the $N^*(1440) \rightarrow N(\pi\pi)^{I=0}_{S\text{-wave}}$ decay.

B. Model of Hernández et al.
Now we proceed to a brief review of the model used in Ref. [13]. This model consists of the ‘open’ and ‘closed’ diagrams as shown in Fig. 2. The amplitudes are found in Ref. [13] for the propagation of $\Delta$ as an intermediate baryon. Similarly we can obtain the amplitudes also for the intermediate $N$ and $N^*(1440)$. In this model, the $N^*(1440) \rightarrow N(\pi\pi)^{I=0}_{S\text{-wave}}$ decay takes place only through the closed diagrams. The scalar-isoscalar correlation of the two pions is realized by the re-scattering mechanism based on the Lippmann-Schwinger equation as shown in Fig. 3, in which the tree level amplitude with $I = 0$ is provided by the lowest order chiral Lagrangian. The $I = 0$ $\pi\pi$ amplitude including the re-scattering effect is

$$t^{I=0}_{\pi\pi}(s_{\pi\pi}) = -\frac{6}{f_\pi^2} \frac{s_{\pi\pi} - m_\pi^2/2}{1 + (1/f_\pi^2)(s_{\pi\pi} - m_\pi^2/2)G(s_{\pi\pi})},$$

with the pion loop integral

$$G(s_{\pi\pi}) = i \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^2 - m_\pi^2 + i\varepsilon} \frac{1}{(P - l)^2 - m_\pi^2 + i\varepsilon},$$

where $P^2 = s_{\pi\pi}$. Using the dimensional regularization scheme with a renormalization scale $\mu = 1.2$ GeV, we obtain

$$G(s_{\pi\pi}) = \frac{1}{(4\pi)^4} \left(-1 + \ln \frac{m_\pi^2}{\mu^2} + \sigma \ln \frac{1 + \sigma}{1 - \sigma} - i\pi\sigma\right),$$

where $\sigma = \sqrt{1 - (4m_\pi^2/s_{\pi\pi})}$.

The amplitudes corresponding to the diagrams in Fig. 2 can be classified as $T_S$ and $T_{AA}$. The open diagram Fig. 2 (a) which expresses the decay processes through the baryon intermediate states is included in $T_{AA}$. As for the closed diagram Fig. 2 (b), we first note that the decay amplitudes are proportional to $t^{I=0}_{\pi\pi}(s_{\pi\pi})$ (see Eq. (8) in Ref. [13]). Rewriting $t^{I=0}_{\pi\pi}(s_{\pi\pi})$ as

$$t^{I=0}_{\pi\pi}(s_{\pi\pi}) = -\left(k_1 \cdot k_2 + \frac{3}{4}m_\pi^2\right) \frac{6}{f_\pi^2} \frac{2}{1 + (1/f_\pi^2)(s_{\pi\pi} - m_\pi^2/2)G(s_{\pi\pi})},$$

we can see that each part of the decay amplitude proportional to $k_1 \cdot k_2$ and $m_\pi^2$ belongs to $T_{AA}^C$ and $T_S$, respectively. The phenomenological amplitudes of Ref. [13] are consistent with the general structure based on the chiral reduction formula.

C. Scalar-isoscalar contact interaction

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1 See Ref. [15] for this solution on the basis of the on-shell tree amplitude.
In Ref. [13] it is only the $\pi\pi$ interaction that leads to the $N^*(1440) \to N(\pi\pi)^{I=0}_{S\text{-wave}}$ decay. The general expressions Eqs. (5)-(7) based on chiral symmetry, however, also allow the scalar-isoscalar contact interaction for the pion-baryon vertex to be the source of this decay.

The form factors in Eqs. (6) and (7) are constants at tree level, i.e. $\sigma_{RN}(s_{\pi\pi}), F_{AA}(s_{\pi\pi}) \to \sigma_{RN}, F_{AA}$, and we obtain

$$iT_S \to i\delta^{ab}\frac{\sigma_{RN}}{f_\pi}\Phi^\dagger_N \Phi_R \chi_R^\dagger \chi_R,$$

$$iT_{AA}^C \to -i\delta^{ab}(k_1 \cdot k_2)\frac{F_{AA}}{f_\pi^2}\Phi^\dagger_N \Phi_R \chi_R^\dagger \chi_R.$$ (12)

They are the scalar-isoscalar contact interactions for the pion-baryon vertex in the $N^*(1440) \to N(\pi\pi)^{I=0}_{S\text{-wave}}$ amplitude. It is natural to take account of these contact interactions in phenomenological treatments because there is no reason to discard these interactions. There is a difference in momentum dependence of Eqs. (12) and (13) arising from the origin of each amplitude in the chiral structure. Combining the contact interactions with the $\pi\pi$ re-scattering mechanism of Ref. [13], we obtain a new amplitude depicted in Fig. 4

$$iT_{new} = i\delta^{ab}\left(\sigma_{RN} - (k_1 \cdot k_2)F_{AA}\right) \left(1 + \frac{1}{6}G(s_{\pi\pi})t_{\pi\pi}^{I=0}(s_{\pi\pi})\right)\Phi^\dagger_N \Phi_R \chi_R^\dagger \chi_R.$$ (14)

We calculate the $\pi\pi$ and $\pi N$ invariant mass distributions of $N^*(1440) \to N\pi\pi$ decay including Eq. (14) in the phenomenological amplitude of Ref. [13].

Before ending this section, we note that $T_V$ is not considered in the following calculation because this term is irrelevant to the $N^*(1440) \to N(\pi\pi)^{I=0}_{S\text{-wave}}$ decay, which is also absent in Refs. [11, 13].
TABLE I: The value of constants in our calculation. The mass and total decay width of the Roper resonance is taken from Ref. [11]. As for the coupling constants, we use the same values as Ref. [13]. See Ref. [13] also for the explicit form of the interaction Lagrangian.

| Masses and Width | (MeV) | Constants |
|------------------|-------|-----------|
| $m_\pi$          | 139   | $f_{\pi NN}$ | 0.95 |
| $m_N$            | 939   | $f_{\pi N\Delta}$ | 2.07 |
| $m_\Delta$       | 1232  | $f_{\pi NR}$ | 0.40 |
| $m_R$            | 1462  | $f_{\pi RR}$ | 0.95 |
| $\Gamma_R$       | 391   | $f_\pi$ | 92.4 MeV |

III. RESULTS AND DISCUSSIONS

In this section, we show our results for the $\pi\pi$ and $\pi N$ invariant mass distributions of $N^*(1440) \to N\pi\pi$ decay calculated in the rest frame of the Roper resonance. We take $\Lambda$, $\sigma_{RN}$, and $F_{AA}$ as free parameters, which are the cutoff to regularize the closed diagram, and the form factors introduced in the last section, respectively. The values of these parameters are chosen so that we reproduce not only the mass distributions and the $N^*(1440) \to N\pi\pi$ decay width $\Gamma = 152$ MeV estimated by using Manley’s approach, but also $g_{RN\pi\pi}^{\text{expt}} = 1.6 \times 10^{-2}$ MeV$^{-1}$. As for the coupling constant of the $R\Delta\pi$ interaction $f_{R\Delta\pi}$, we use $f_{R\Delta\pi} = 1.1$ so that we have the same contribution of the open diagrams as Ref. [13]. In Table I, we summarize the values of masses and other constants which are fixed throughout this calculation.

A. Full results

Fig. 5 shows the mass distributions calculated by using $T_{\text{new}}$ and the amplitudes corresponding to the open and closed diagrams (the full calculation). The parameters are $\sigma_{RN} = 157$ MeV, $F_{AA} = 1.97 \times 10^{-3}$ MeV$^{-1}$, and $\Lambda = 700$ MeV. We also display the results of Ref. [13] for comparison. Both calculations are able to reproduce the “experimental results” of the $\pi\pi$ and $\pi N$ invariant mass distributions well. Note that our model succeeded in reproducing $g_{RN\pi\pi}^{\text{expt}}$ together with the mass distributions and the decay width, while $|g_{RN\pi\pi}^{\text{model}}/g_{RN\pi\pi}^{\text{expt}}| = 0.42$ in Ref. [13]. Because our $\Lambda$ and $f_{R\Delta\pi}$ take the same values as those of Ref. [13], the difference between these two calculations is purely due to the ampli-
FIG. 5: The (a) $\pi\pi$ and (b) $\pi N$ invariant mass distributions in our full calculation. The $M_{\pi\pi}$ and $M_{\pi N}$ are the $\pi\pi$ and $\pi N$ invariant masses, respectively. Our result is shown by the solid line. The “experimental result” (Manley’s approach) and the result of Ref. [13] are shown by the dashed and dotted line, respectively.

FIG. 6: Same as Fig. 5 but for the case of $F_{AA} = 0$.

tude Eq. [14]. Taking account of $T_{\text{new}}$, we obtain the consistent results in our calculation for the mass distributions and $g_{R_{N\pi\pi}}$.

**B. $\sigma_{RN}$ and $F_{AA}$**

We further examine the contribution of $T_{\text{new}}$ on the mass distributions. This amplitude consists of two terms proportional to $\sigma_{RN}$ and $F_{AA}$. The former is connected with the explicit breaking of chiral symmetry in the pion-baryon contact interaction [Eq. [12]], and the latter is derived from the scalar-isoscalar combination of two axial vector currents [Eq. [13]].
FIG. 7: Same as Fig. 5 but for the case without the closed diagrams.

Now we estimate each contribution of these terms separately. First, we set $F_{AA}$ to be zero and consider only the term proportional to $\sigma_{RN}$ with respect to $T_{\text{new}}$. We obtain $\sigma_{RN} = 160$ MeV and $\Lambda = 460$ MeV, and also use $f_{R\Delta\pi} = 1.1$. We find that this calculation reproduces the mass distributions (Fig. 6) and $g_{RN\pi\pi}$ as well as the full calculation. In this case $\Lambda$ becomes considerably small, while $\sigma_{RN}$ takes almost the same value as before.

Next we set $\sigma_{RN}$ to be zero and consider the term proportional to $F_{AA}$ in $T_{\text{new}}$. In this case we can not find acceptable values for the parameters $F_{AA}$ and $\Lambda$ reproducing the mass distributions and $g_{RN\pi\pi}$ even though we treat $f_{R\Delta\pi}$ as a free parameter. If we draw the mass distributions which is similar to Figs. 5 and 6 we obtain $g_{RN\pi\pi}$ far from $g_{RN\pi\pi}^{\text{expt}} = 1.6 \times 10^{-2}$ MeV$^{-1}$. This is also the case for Ref. [13] as explained in the introduction. These results imply that the term proportional to $\sigma_{RN}$ is necessary in reproducing the mass distributions and $g_{RN\pi\pi}$ simultaneously.

C. Phenomenological meaning of closed diagrams

We consider the phenomenological meaning of the closed diagram. There is no doubt that we should take the $\pi\pi$ correlation into account when we study the two-pion decay of the Roper resonance. We know that, however, there exist several ambiguities in the closed diagram; that is, this diagram depends highly on experimentally unknown cutoffs and coupling constants. Is it possible to avoid using the closed diagram for the phenomenological understanding of the $N^*(1440) \rightarrow N(\pi\pi)f_{S\text{-wave}}^{0}$ decay?

We try to discuss the case that includes all diagrams explained above but the closed diagrams. In this case we also obtain the parameter set so as to reproduce the mass
distributions (Fig. 7) and $g_{RN\pi\pi}$. The parameters become $\sigma_{RN} = 177$ MeV and $F_{AA} = -1.22 \times 10^{-3}$ MeV$^{-1}$. In comparison with the parameters in our full calculation, the change in the value of $\sigma_{RN}$ is small, while $F_{AA}$ changes its sign from positive to negative.

This result shows that $T_{new}$ effectively represents the contribution of the closed diagrams by tuning $\sigma_{RN}$ and, especially $F_{AA}$.

IV. CONCLUSIONS

We have shown that the amplitude proportional to $\sigma_{RN}$, which is connected with the explicit chiral symmetry breaking of pion-baryon interaction, is necessary for the simultaneous description of $g_{RN\pi\pi}$ and the $\pi\pi$ and $\pi N$ invariant mass distributions. Furthermore we have also shown that our amplitude derived from the contact interactions with $\sigma_{RN}$ and $F_{AA}$ are effectively substituted for the amplitude corresponding to the closed diagrams of Ref. [13].

A naive application of our experience at the low energy, in which the explicit breaking of chiral symmetry can be neglected for the phenomenological analysis, does not work for the present case at resonance energy. And, unless there is theoretical and/or experimental evidence, the closed diagram including baryon resonances is not prior to the contact interaction. In fact, we see that the closed diagram does not play an essential role for the phenomenological understanding of this decay. It will be desirable to study the $N^*(1440) \rightarrow N\pi\pi$ decay without using the closed diagram including some ambiguities for the baryon dynamics.

Motivated by the chiral reduction formula, we have introduced the contact interactions, and have obtained satisfactory results. The general discussion based on the chiral reduction formula tells us an effective way of the phenomenological treatment of hadron reactions.

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