Diagnostics based on continuous scanning LDV methods: numerical study

S Bruinsma, D Di Maio, T Tinga
Dynamics Based Maintenance, Department of Engineering Technology, Horst 2, 7522 LW, Enschede, The Netherlands

Abstract. This paper presents a research work on diagnostics using continuous scanning laser methods. Structural Health Monitoring based on laser vibrometry can exploit the ability of a scanning laser to measure vibration responses remotely both regarding temporal and spatial resolution. Therefore, it can be a powerful approach for rapid damage detection of components presenting an unhealthy dynamic behaviour. The work is focused on numerical analysis of a cantilever beam which is subjected to damage of different severity levels. The severity will be measured regarding response phase shift for a fixed excitation frequency. The simulated nodal responses of the surface being scanned by a laser beam are selected and used for simulating the spectral responses obtained from CSLDV methods. The main objective of this work is to determine the structural integrity based on an indicator obtained by referencing the relative spectral sidebands between a pristine and damage condition.

1. Introduction
In recent years, durability and extension of life of components has become an important topic of research. Structural Health Monitoring (SHM) focuses on detection of unhealthy dynamic behaviour as damage occurs and propagates in the structure. The early detection of damage is highly interesting for maintenance in the industry, which supported research for better diagnostic tools. These consist of destructive and non-destructive methods. Obviously, non-destructive methods are preferred, as these are most applicable in the industry for SHM. A conventional way of testing is to apply strain gauges to the component. However, these cannot be placed on rotating shafts, for example. Also, the fastening methods cannot always be applied due to high operating temperature. This can be omitted by using a non-destructive and non-contact measurement technique. Most of these techniques use either laser light, such as Holography, Speckle interferometry and Scanning Laser Doppler Vibrometry (SLDV), or a camera, like Digital Image Correlation (DIC). DIC uses a camera and image processing to obtain dynamic information [1]. DIC requires surface preparation and a lot of post processing to obtain results on the strain. Holography is an expensive method that allows for comprehensive measurements [3]. The downside for the industry is that the part needs to be isolated and placed in this large and finely tuned test set-up, increasing downtime of the system. Interferometry needs a similar sterile environment to do the measurements [9]. SLDV is a more robust approach that allows for the possibility of in-situ measurements [2]. Continuous SLDV strengthens this application by making it a very fast measurement method.

This paper focuses on Continuous SLDV, which main advantage is fast acquisition of both spacial and temporal information. CSLDV uses a laser that scans a vibrating structure. The time signal from a sinusoidal line scan can be modulated to a frequency spectrum [4], which can be used for SHM. The level of depth in damage detection is categorised in 4 levels, as explained in [5], the definition of the different levels are displayed in Table 1. Most of the current research with this technique is on locating the damage [6], which is quantified as SHM level 2. To exploit its main advantage of speed, the level of SHM is reduced to 1 in this study, where only the detection of damage is pursued.
### Table 1: Level of SHM

| Level  | Description                        |
|--------|------------------------------------|
| Level 1 | Determination that damage is present |
| Level 2 | Determination of the geometric location of the damage |
| Level 3 | Quantification of the severity of the damage |
| Level 4 | Prediction of the remaining service life |

A numerical study is undertaken, analysing a cantilever, aluminium beam. The prismatic beam is subjected to an oscillating force at the free end. The goal of the study is to determine the sensitivity of the CSLDV method to introduced damage. To accomplish this, different approaches to defining a damage criterion are taken.

### 2. Method

In this chapter the methodology of the research is presented. First the construction of the model is described. The model is checked and the sensitivity to the damage is analysed. This is then used to define damage indicators that are sensitive enough to show propagation of damage.

#### 2.1. Numerical Model

Researching the viability and sensitivity of this method is done on a simple geometry, a prismatic, cantilever beam. The dimensions of the beam are 0.4x0.04x0.01 m \((l, w, h)\) or \((x, z, y)\). The beam is modelled in Finite Element Program ANSYS Mechanical APDL 16.2. The beam is modelled with 200x20 \((x, y)\) rectangular SHELL181 elements. Only membrane bending is used, and full integration. All nodes are locked in \(z\)-direction, one end is fixed in \(y\)- and \(x\)-directions as well. At the other end, an excitation force is applied in \(y\)-direction. The harmonic response is simulated from 1 to 9000 Hz with a frequency step of 1 Hz. The model shows good correspondence to analytical eigen frequency calculations. The first six eigen frequencies are compared in Table 2.

### Table 2: Comparison Analytical and Numerical

|          | \(\omega_1\) | \(\omega_2\) | \(\omega_3\) | \(\omega_4\) | \(\omega_5\) | \(\omega_6\) |
|----------|--------------|--------------|--------------|--------------|--------------|--------------|
| Analytical (Hz) | 51.04 | 319.9 | 895.4 | 1755 | 2901 | 4334 |
| Numerical (Hz)  | 51 | 319 | 889 | 1731 | 2837 | 4196 |
| Error (%)       | <0.1 | 0.3 | 0.7 | 1.4 | 2.2 | 3.2 |

The prismatic beam has a consistent width (\(z\)-direction) of elements of 0.04 m. Damage is introduced to the model by reducing the width of the elements, which results in a stress concentration around the damaged elements. This way of modelling allows for a wide variety of elaborate damage cases, such as delamination and interlaminar cracks. In this study, a simple damage case is defined: a transverse cut, or saw cut. The transverse cut is placed at 0.04 m from the root, starting on the lowest row of elements, made in three severities. The most extreme case is the 50% damage case, where the cut is 0.005 m, thus half the thickness (in \(y\)-direction), which corresponds to 10 elements with reduced width. Furthermore, 25% of the thickness and 10% of the thickness are evaluated, corresponding to 5 and 2 elements respectively.

The ODS for all frequencies is extracted and a curve is fitted through the shape using the `polyfit`-function in MATLAB. The \(x\)-coordinates are transformed between \(-1\) and \(1\), to allow for simulation of the CSLDV time signal. This time signal is the result of equation 1, from [7].

\[
v(t) = \sum_{n=0}^{p} V_{Rn} \cos^n(\Omega t) \cos(\omega t)
\]  

(1)

Where \(p\) is the order of the polynomial used to describe the ODS, \(V_R\) is the real part of the polynomial coefficients, \(\Omega\) is the scan frequency of the CSLDV (in this paper 1.1 Hz.), \(\omega\) is the excitation frequency.
2.2. Sensitivity

To establish an indicator for the damage severity levels in relation to the vibration modes three FRFs were simulated for the damage cases. The crack is placed close to the root of the beam (10% the beam’s length, measured from the clamped end). The 50% transverse crack is 2 mm wide and 5 mm in length, which causes a frequency shift of less than 1% from the pristine natural frequency for almost all eigen frequencies, both FRFs from pristine and damage cases are presented in Figure 1.

![Figure 1: FRF of Pristine & D50T](image)

This behaviour is as expected, the applied damage to the structure reduces the stiffness in the beam [8]. The eigen frequency is proportional to $k/m$, where $k$ is stiffness and $m$ is mass. The severity of the damage is commonly measured in the frequency shift of the eigen frequencies. However, for a given structural damping, the response phase of a damaged structure shows much larger deviation from a pristine condition. As seen in Figure 1 this shows a higher sensitivity due to phase behaviour at a fixed frequency, such as the eigen frequency. Table 3 shows both the frequency shift and the phase shift with respect to the resonant frequency.

| Eigen | Frequency (Hz) | Phase shift at $\omega^P$ (deg) |
|-------|----------------|-------------------------------|
| $\omega_1$ | 51 | P | D10T | D25T | D50T | P | D10T | D25T | D50T |
| $\omega_2$ | 319 | 318 | 317 | 316 | -94 | -117 | -145 | -158 |
| $\omega_3$ | 889 | 889 | 888 | 887 | -92 | -97 | -107 | -117 |
| $\omega_4$ | 1731 | 1731 | 1731 | 1731 | -96 | -94 | -92 | -91 |
| $\omega_5$ | 2837 | 2837 | 2835 | 2833 | -94 | -97 | -105 | -115 |
| $\omega_6$ | 4196 | 4193 | 4184 | 4178 | -96 | -108 | -131 | -142 |
| $\omega_7$ | 5794 | 5785 | 5766 | 5750 | -100 | -120 | -148 | -157 |
| $\omega_8$ | 7616 | 7602 | 7571 | 7546 | -104 | -127 | -154 | -163 |

These differences are visualised in Figures 2 & 3. Where the found values are normalised to the pristine case and a relative shift is plotted.
Based on such indicator, the most sensitive modes to the damage can be highlighted and closely observed during the next phase of the research.

2.3. signal processing

All the ODSs simulated by the FEM are curve-fitted with a 15\textsuperscript{th} order polynomial. LDV output modulated signals are generated and processed by dFFT. The dFFT is discretised at the excitation frequency and subsequently at interval of the scan frequency, as in [10]. This results in a middle amplitude with sidebands at equal distances. The amount of sidebands is equal to the amount of polynomial coefficients. The magnitude of these sidebands carry the spatial information of the ODS. Meaning that the distribution of the amplitude of each sideband is characteristic for that system at that specific frequency.

When damage is introduced to the beam, this characteristic distribution of amplitudes over the sidebands differs, as seen in Figure 4. This is more clearly seen when a log scale on the y-axes is used.

As already introduced, 15 coefficients are used for the polynomial evaluation. However, the ODSs require a different amount of coefficients, less are needed if the shape is simple like the first bending mode. The minimal number of coefficients needed to adequately describe the ODS is investigated at the eigen frequencies of the beam. This is done by checking the MAC value of the fit for increasing number of polynomial coefficients.

| eigen freq | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------|---|---|---|---|---|---|---|---|
| Nr. of coeff. | 2 | 4 | 5 | 8 | 9 | 10 | 13 | 14 |
So for the second eigen frequency (319 Hz), only 4 coefficients are required to construct the ODS. This results in many near-zero coefficients, as can be seen in Figure 4. It is important to note that the amplitude of the sidebands does not simply shift up or down, this would mean that is is the same ODS but at lower or higher (excitation) amplitude. In the higher order sidebands, it is clear that this is not the case. However, as said before, these sidebands do not hold much meaning since the amplitudes are near-zero. So it becomes critical to analyse the difference of the first four sidebands as well.

The following two subsections will provide an attempt to quantify the deviation between a pristine and damaged case by developing an indicator to rank the damage severity with respect to mode shapes.

2.4. Fingerprint
A method to observe the difference between a pristine and damaged case is to create a so-called fingerprint. Every sideband is scaled by the one measured at the excitation frequency ($\omega$). This is done for each sideband ($i$) following equation 2.

$$FPA_i(\omega) = \frac{SBA_i(\omega + \Omega \cdot i)}{SBA(\omega)}$$

$FPA_i$ is the fingerprint (normalised) amplitude of sideband $i$, obtained by dividing sideband amplitude $SBA_i$ over the excitation or middle amplitude. Hence, both pristine and damaged case will present a typical fingerprint.

2.5. Sideband polynomial
As described before, the spectral sideband amplitudes are related to the polynomial coefficients of the wave shape of the ODS. The relation between the sideband amplitude and the polynomial coefficient is used to define a damage indicator. The difference in sideband amplitudes between the pristine case and damage case is related to the change in ODS, which is a polynomial function. The first sideband corresponds to the linear term, the second to the quadratic, and so on. This is integrated in the damage indicators defined below.

$$DIP_1 = \sum_{i=1}^{n} SBA_{Pi} - SBA_{Di}$$

$$DIP_2 = \sum_{i=1}^{n} SBA_{Pi} \cdot i - SBA_{Di} \cdot i$$

$$DIP_3 = \sum_{i=1}^{n} SBA_{Pi} \cdot i^{-1} - SBA_{Di} \cdot i^{-1}$$

$$DIP_4 = \sum_{i=1}^{n} (SBA_{Pi} \cdot i - SBA_{Di} \cdot i) \cdot \omega$$

Where $SBA_{Pi}$ is the $i^{th}$ sideband of the Pristine case. $\omega$ is the excitation frequency. This is done for a number of sidebands up to 15.

3. Results
In this section the results of the damage indicators described in the section above are shown.

3.1. Fingerprint
The fingerprint method yields a characteristic shape for the pristine case and the damaged case as seen in Figure 5. These shapes are referred to the fingerprint of the beam.
As can be seen in Figure 5, the higher order sidebands show most difference in relative amplitude. These are however most difficult to detect and in absolute numbers have very low value. The difference is visible in the first four sidebands when these are observed, see Figure 6. Detecting damage from this Figure can be done, but assigning a scalar value would be preferable. One possible damage criterion to obtain from this is the mean square error between the two fingerprints. The mean square error is 0.1 for the second eigen frequency. When this is evaluated over the entire frequency range, some spikes appear, as seen in Figure 7.

The reason for these spikes is investigated and can be explained with Figure 8.
From Figure 8, it can be concluded that a large difference in the middle amplitude results in unusually large Mean Square Errors.

3.2. Sideband polynomial
The result of the sideband polynomial methods are damage indicators that are highly dependent on the frequency and the method does not normalise the sideband amplitudes. Therefore the indicators only show meaningful difference around the eigen frequency, where the vibration amplitudes are generally high. Furthermore, at the anti-resonance a spike occurs, because vibration amplitudes are very small. The frequency dependency built in to equation 6 shows more consistency regarding the value indicated by the damage severity. However, at any frequency away from the eigen frequency, the damage is not indicated. The frequency dependent indicator is view in Figure 9.

4. Conclusions
In this study, the use of the CSLDV method for SHM purposes is explored. The method is fast in assessing the ODS of a beam by using the FFT frequency spectrum. This spectrum has a characteristic shape that can be used as a reference. The fingerprint method shows potential in detecting the difference between a damaged and pristine spectrum. However, this method shows inconsistent behaviour over the frequency range, making it an unsuitable approach for this application. The other methods suffer from the same problem, a very small window of frequencies is found where a useful value can be extracted. This makes it heavily model dependent and not much more useful than the current technique of measuring the shift in eigen frequency.
The problem with the fingerprint method is that the indicator is too dependent on the amplitude of one of
the sidebands used for the calculation and therefore makes the indicator unstable. The approach of relative contribution of the sidebands does hold. When fixed frequencies are observed, the overall amplitude changes when damage is introduced, as seen in the FRF. From an application point of view, this is also the case when not the exact same force is applied. So making the sidebands a relative, or normalised factor would make it comparable again.

Instead of normalising the sideband amplitudes to one sideband, it would be better to assign partial contributions to the sidebands in relation to all other sidebands. This is further investigated, mathematically and experimentally, in an upcoming paper.

References

[1] Stéphane Avril, Marc Bonnet, Anne-Sophie Bretelle, Michel Grédiac, François Hild, Patrick Ienny, Félix Latourte, Didier Lemosse, Stéphane Pagano, Emmanuel Pagnacco, and Fabrice Pierron. Overview of Identification Methods of Mechanical Parameters Based on Full-field Measurements. *Experimental Mechanics*, 48(4):381–402, 8 2008.

[2] P. Castellini, M. Martarelli, and E.P. Tomassini. Laser Doppler Vibrometry: Development of advanced solutions answering to technology’s needs. *Mechanical Systems and Signal Processing*, 20(6):1265–1285, 8 2006.

[3] I. Manuel De la Torre, María del Socorro Hernández Montes, J. Mauricio Flores-Moreno, and Fernando Mendoza Santoyo. Laser speckle based digital optical methods in structural mechanics: A review. *Optics and Lasers in Engineering*, 2015.

[4] Dario Di Maio. Use of continuous scanning LDV for diagnostics. In *Conference Proceedings of the Society for Experimental Mechanics Series*, 2016.

[5] Z A Jassim, N N Ali, F Mustapha, and N A Abdul Jalil. A review on the vibration analysis for a damage occurrence of a cantilever beam. *Engineering Failure Analysis*, 31:442–461, 2013.

[6] Jeong Tae Kim, Yeon Sun Ryu, Hyun Man Cho, and Norris Stubbs. Damage identification in beam-type structures: Frequency-based method vs mode-shape-based method. *Engineering Structures*, 2003.

[7] Milena Martarelli. Exploiting the Laser Scanning Facility for Vibration Measurements. 2001.

[8] W.M. Ostachowicz and M. Krawczuk. Analysis of the effect of cracks on the natural frequencies of a cantilever beam. *Journal of Sound and Vibration*, 150(2):191–201, 10 1991.

[9] Sridhar Reddy Guntaka, Manfred Hertwig, Torsten Flemming, and Ralf Usinger. Speckle interferometry for detection of subsurface damage in fibre-reinforced composites. *Meas. Sci. Technol.*, 5:100–104, 1994.

[10] A. B. Stanbridge and D. J. Ewins. Modal testing using a scanning laser Doppler vibrometer. *Mechanical Systems and Signal Processing*, 1999.