How Phase-Breaking Affects Quantum Transport Through Chaotic Cavities

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We investigate the effects of phase-breaking events on electronic transport through ballistic chaotic cavities. We simulate phase-breaking by a fictitious lead connecting the cavity to a phase-randomizing reservoir and introduce a statistical description for the total scattering matrix, including the additional lead. For strong phase-breaking, the average and variance of the conductance are calculated analytically. Combining these results with those in the absence of phase-breaking, we propose an interpolation formula, show that it is an excellent description of random-matrix numerical calculations, and obtain good agreement with several recent experiments.

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Recently there has been great interest in the effects of quantum-mechanical interference on electronic transport through ballistic quantum dots. In these microstructures both the phase-coherence length and the elastic mean free path exceed the system dimensions. Thus the leads into the dot can be thought of as electron waveguides and the dot itself as a resonant cavity.

Experimentally, one observes random (but reproducible) fluctuations in the conductance as the magnetic field, the Fermi energy, or the shape of the cavity is changed. The sensitivity to small changes in these parameters shows that these fluctuations are caused by quantum interference. In addition to conductance fluctuations, several interference effects which survive averaging over many microstructures are observed. In particular, an increase in the average resistance at zero magnetic field called the weak-localization correction (WLC). The two main features of these interference effects are their shape—the characteristic field or energy to which they are sensitive, or more precisely the functional dependence on these parameters—and their magnitude—simply how big these quantum corrections are compared to the classical conductance. Here we concentrate on the magnitude and merely note that a theory for the shape has been extensively developed.

A theory of the magnitude of quantum interference effects in chaotic cavities has recently been developed by making a statistical guess for the S-matrix describing the scattering. In Refs. 3 and 4, developed a random S-matrix theory by assigning to S an “equal a priori distribution” once the symmetry requirements were imposed. The results for the average, variance, and probability density of the conductance were in good agreement with numerical calculations for a chaotic cavity connected to two waveguides. However, the random-matrix predictions for both the weak-localization correction and the variance are larger than the experimental results. In addition, the measured probability density is close to a Gaussian distribution when there are two propagating modes per lead (N = 2), while random-matrix theory predicts a Gaussian distribution only for N ≥ 3.

Inherent in the model of Refs. 3 and 4 is the assumption (among others) that one can neglect phase-breaking processes which destroy the coherence of the wave function. In this paper we show that this assumption is largely responsible for the discrepancy between theory and experiment mentioned above. We make specific predictions for the dependence of the quantum transport corrections on the degree of phase-breaking which may be tested by future experiments.

To simulate the effects of phase-breaking events we adopt a model suggested by M. Büttiker in addition to the physical leads 1, 2 attached to reservoirs at chemical potentials µ1, µ2, a lead 3 connects the cavity to a phase-randomizing reservoir at µ3. This model has been discussed extensively for disordered materials and, more recently, for ballistic quantum dots by Marcus, et al. A similar model has been used for absorption of microwaves in chaotic-scattering from cavities. Requiring the current in lead 3 to vanish determines µ3: the two-terminal dimensionless conductance is then found to be

\[ g \equiv G/(e^2/h) = 2 \left[ T_{21} + \frac{T_{23}T_{31}}{T_{32} + T_{31}} \right], \tag{1} \]

where \( T_{ij} \) is the transmission coefficient for “spinless electrons” from lead j to lead i. The factor of 2 accounts for spin explicitly. We call N the number of channels in leads 1 and 2, \( N_\phi \) that in lead 3, and \( N_T = 2N + N_\phi \).

We now make the fundamental assumption of an “equal a priori distribution” for the total \( N_T \times N_T \) scattering matrix S, once the symmetry requirements have been imposed. S is, of course, unitary, and is symmetric in the absence of a magnetic field because of time-reversal symmetry. We assume that the statistics of the total S-matrix are given by the Circular Ensembles of random matrix theory. For \( B = 0 \) the orthogonal ensemble (COE, denoted \( \beta = 1 \)) is appropriate while for
nonzero $B$ we use the unitary ensemble (CUE, $\beta = 2$). In contrast to previous studies of the eigenphases, we treat the statistics of $g$ given in Eq. (1). Here we have $\langle T_{ij} \rangle = N_i N_j / (N_T + \delta_{31})$, a result closely related to the Hauser-Feshbach formula in nuclear physics. The WLC is then given by

$$\delta g = -N/N_\phi + O(1/N_\phi^2).$$  \hspace{1cm} (7)$$

For the variance one finds from Eq. (5)

$$\text{varg} = \left( \frac{N}{N_\phi} \right)^2 \left[ 1 + \frac{2 - \beta}{2N} \right] + \cdots$$  \hspace{1cm} (8)$$

and, for the ratio of the variances for $\beta = 1, 2$,

$$\left( \frac{\text{varg}}{\delta g} \right)^{(\beta=1)} = 2 \left( 1 + \frac{1}{2N} \right) + \cdots. \hspace{1cm} (9)$$

Of course the magnitude of the quantum corrections decreases as the number of phase-breaking channels increases. Note that the power controlling the decay of the variance is twice that of the average conductance. The deviation of the ratio of the variances from 2 is highly unusual; for $N = 1$ the ratio can be as high as 3.

We present in Figs. 1, 2 and 3 results of simulations of the random-matrix model in order to check the asymptotic behavior and suggest improvements. The points are obtained by generating random $N_T \times N_T$ unitary or orthogonal matrices and computing $g$ from Eq. (1). In Fig. 1, the value $N = 2$ was selected for comparison with the experiment of Ref. [3]. The log-log inset in Fig. 1 show that the convergence of the numerical to the asymptotic results [Eqs. (6), (8), dotted lines] is rather slow. The curves in the main parts of Fig. 1 are interpolation formulae suggested by the results for $N_\phi = 0$ and $N_\phi \gg 1$. For the WLC, Eqs. (2c) and (3) suggest the relation

$$\delta g \sim -N/(2N + N_\phi). \hspace{1cm} (10)$$

For the fluctuations of the conductance, we combine square roots of variances,

$$(\text{varg})^{1/2} = \left[ (\text{varg})^{1/2}_{N_\phi=0} + (\text{varg})^{1/2}_{N_\phi\gg 1} \right]^{-1}, \hspace{1cm} (11)$$

because in the numerical simulations the change in varg is clearly linear in $N_\phi$, not quadratic. So, for $\beta = 2$

$$\text{varg} \sim N^2 / [(4N^2 - 1)^{1/2} + N_\phi^2]. \hspace{1cm} (12)$$

For $\beta = 1$ a similar, but more complicated, expression holds. These interpolation formulae agree very well with the numerical results; $\delta g$ and varg are nearly independent of $N$ in the perfectly coherent limit—a result known as “universality”—phase-breaking channels cause the magnitudes to vary. Thus the universality can only be seen if $N_\phi \ll N$; otherwise, the behavior is approximately linear, as in some
experiments. Clearly, phase-breaking must be included in interpreting the experiments.

In addition to the mean and variance, the probability density of the conductance, \( w(g) \), is experimentally measurable. The numerical results in Fig. 3 show that as \( N_\phi \) increases \( w(g) \) tends towards a Gaussian and is therefore fully characterized by the mean and variance given above. For \( N \geq 3 \) the distribution is essentially Gaussian even for \( N_\phi = 0 \), so that \( N_\phi \) affects \( w(g) \), apart from changing the mean and variance, only for \( N < 3 \). For \( N = 1 \), the highly non-Gaussian distribution at \( N_\phi = 0 \) become approximately Gaussian at \( N_\phi = 3 \); the intermediate distributions, \( N_\phi = 1,2 \), are shown in Fig. 3. For \( N = 2 \), the deviation of \( w(g) \) for \( N_\phi = 0 \) from a Gaussian is smaller than for \( N = 1 \), and hence fewer phase-breaking channels produce a Gaussian distribution. We show numerical results only for \( N_\phi = 1 \).

Finally, we use our model to interpret the experiments of Refs. 4, 8 and 9. First, the solid circles in Fig. 1 show the measured \( N_\phi, \delta g, \) and var\( g \) of Ref. 4 in the \( N = 2 \) case. Before comparison the experimental variance must be corrected for thermal averaging: convolution over the derivative of the Fermi function produces a reduction factor of \( \sim 0.22 - 0.38 \) for an electron temperature of \( 50 - 100 \) mK. We have increased the measured var\( g \) by the inverse of this reduction factor; the WLC is not affected by thermal averaging since it is already an average effect. The error bars shown result from both the uncertainty in temperature in the case of the variance and the experimental fluctuations at small \( B \) for the variance and WLC. (For the moment, we do not assign further physical significance to the oscillations seen in the experiment.) Note that in Fig. 1 we have not fit the theory to the data and yet the agreement is very good. Second, in Fig. 2 we show as solid symbols the data of all three Refs. 4, 8 and 9. The value of \( N_\phi \) deduced from comparing to our calculations is in good agreement with the value estimated independently in the experiments: \( N_\phi = 7 - 9 \) for Ref. 4, \( N_\phi = 2 \) for Ref. 8 and \( N_\phi = 4 - 8 \) for Ref. 9. Finally, the probability density of the conductance is found to be Gaussian in Ref. 4. For the estimated \( N_\phi = 4 - 8 \), we obtain a Gaussian distribution and so are consistent. Observation of the interesting non-Gaussian distribution of \( g \) obtained in the theory for \( N = 1,2 \) and \( N_\phi = 0 \) requires greatly reduced phase-breaking.

In conclusion, we have presented a random-matrix model that simulates the effects of phase-breaking events on electronic transport through ballisitic chaotic cavities. The analysis of recent experiments indicates that one can find a value of \( N_\phi \), the number of phase-breaking channels, that allows a consistent description of the data. Further experiments are needed to test quantitatively the dependence on \( N \) and \( N_\phi \) that we predict.

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Note added— While preparing this paper for publication, we received a preprint by P. W. Brouwer and C. W. J. Beenakker with some overlapping material for \( N = 1 \).
FIG. 1. Magnitude of quantum transport effects as a function of the number of phase-breaking channels, $N_\phi$, on linear (main panels) and log-log (insets) scales. (a) The weak-localization correction ($N = 2$). (b) The variance in both the orthogonal (squares) and unitary (triangles) cases ($N = 2$). (c) The ratio of the variance in the orthogonal case to that in the unitary for $N = 1, 2, $ and 6; the arrows mark the $N_\phi \to \infty$ limit for $N = 1, 2$. Open symbols are numerical results (20,000 matrices used, statistical error the size of the symbol). Solid lines are interpolation formulae. Dotted lines are asymptotic results. Solid circles are experimental results of Ref. 9 corrected for thermal averaging. The interpolation formulae are excellent except for $N = 1$ and small $N_\phi$ [panel (c)].

FIG. 2. The magnitude of quantum transport effects as a function of the number of channels in the leads, $N$, for fixed $N_\phi = 0, 2, 4, $ and 8. (a) The weak-localization correction. (b) The variance for the orthogonal case ($B = 0$). (c) The variance for the unitary case (nonzero $B$). Open symbols are numerical results (as in Fig. 1). Solid lines are interpolation formulae. Solid symbols are experimental results of Refs. 4 (triangles), 8 (squares), and 9 (circles) corrected for thermal averaging. The introduction of phase-breaking decreases the “universality” of the results but leads to good agreement with experiment.

FIG. 3. The probability density of the conductance in the orthogonal (first column) and unitary (second column) cases for $N = 1$ (first row) and $N = 2$ (second row). Increasing the phase-breaking from zero (dashed lines, analytic) to $N_\phi = 1$ (plus symbols, numerical) to $N_\phi = 2$ (squares, numerical) moves the distribution towards a Gaussian.
Figure 1.
Figure 2.
Figure 3.