Fluid–structure interaction modelling of the venous valve with elastic leaflets

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Abstract. The paper presents the results of a numerical study of the flow in a symmetric model of a healthy popliteal venous valve with different leaflet elasticity. The focus is on the analysis of the stagnant area under the valve leaflet. The stagnant area under the valve leaflet, as well as behind the valve, is of great practical interest to phlebologists in terms of the possibility of blood clots. The flow in the constructed simplified model of the venous valve gave good qualitative agreement with the clinical ultrasound data for a leaflet with Young modulus of 1.2 MPa.

1. Introduction
Bicuspid valves in veins prevent retrograde flow away from the heart [1]. They are able to regulate the flow and keep it unidirectional preventing the backflow when the pressure above the valves is greater than the pressure below them acting as a check valve in a pumping pipe system. Under load conditions, a healthy valve fully opens and closes on a periodic basis [2-4]. Under resting conditions, valve remains open and the amplitude of its leaflet’s oscillation is approximately 3-5% of the vein radius.

A venous valve eliminates any retrograde flow of blood in the vein circulation unless there are any disturbances such as clots [5]. Ageing makes venous valve leaflets more rigid preventing the normal functioning of the valve. Blood clots form in the stagnant zone, which is located behind the valve leaflets in the vein expansion behind the valve (sinus). The size of this zone is of great interest to phlebologists.

Research [6] provides useful insights for better diagnosis and understanding of the vein blockage induced by deep venous thrombosis and the occurrence of reverse flow in human veins, allowing for a proper detection of serious diseases related to deep venous insufficiency. The purpose of this simulation was to explore the effects of clot formation on the flow in a vein segment comprising several valves. Different periods of time were analyzed here according to the vein wall contraction and expansion, showing how each venous valve behaves. Previously, in our study [7] a two-dimensional computational model of the popliteal venous valve was constructed and the effect of the gap width between the valve leaflets (venous valve incompetence) on the valve reverse flow (reflux) was investigated. Paper [8] presents the results of a numerical study of the flow in this model with different leaflet elasticity. The focus is on the analysis of the velocity field and the stagnant region. Meanwhile,
the aforementioned and most of other studies consider the venous valve flow only under load conditions.

The purpose of the work is to study the effect of leaflet elasticity on the stagnant zone in the model of a geometrically healthy venous popliteal venous valve (figure 1a) under resting conditions, and also to compare the numerical results with the results of ultrasound measurements.

2. **Venous valve geometry**

In this study, a two-dimensional symmetric model of the venous valve was selected [4]. As the vein radius is $R = 5$ mm, this model corresponds to the popliteal vein valve (figure 1b).

![Image](image.png)

**Figure 1.** An ultrasound image (a) and the geometric model of the popliteal valve (b).

3. **A mathematical model of the flow in the venous valve**

A coupled numerical simulation of the fluid flow and leaflets motion was carried out using the fluid-structure interaction (FSI) technology in a generalized Lagrangian-Euler formulation. Hydrodynamic calculations were performed in Ansys Fluent, and mechanical calculations were performed in Ansys Mechanical. The combination of the two systems were coupled in System Coupling, a multidisplinare module of Ansys Workbench.

The equations of the fluid flow (Navier-Stokes equations) [4]:

$$\frac{\partial u_i}{\partial t} + (u_i - \bar{u}_i)\nabla u_j = -\frac{1}{\rho_f} \frac{\partial p}{\partial x_i} + \frac{1}{\rho_f} \nabla^2 \mu_f u_i, \quad i, j = 1, 2$$

$$\nabla u_i = 0,$$

where $u_i$ is the component of the fluid velocity at the nodes of the dynamic grid, $\bar{u}$ is the speed of the nodes of the computational grid at the point, $\rho_f$ and $\mu_f$ are the density and viscosity of the fluid: $\rho_f = 1040 \text{ kg/m}^3$, $\mu_f = 0.0035 \text{ Pa} \cdot \text{s}$. The Reynolds number, derived based on the diameter of the vein and the maximum velocity per cycle, is 350.

The equation of motion for leaflets (equation of elastodynamics) [4]:

$$\rho \frac{\partial^2 d_i}{\partial t^2} - (\nabla \times \zeta)_i = 0,$$

where $d$ is displacement, $\zeta = \int \sigma_s F^{\top}$ is the first Piola-Kirchhoff tensor, $\sigma_s = \frac{1}{J} F (\lambda (trE) I + 2\mu E) F^{\top}$ is the equation of state for the Saint-Venant-Kirchhoff model, $\lambda, \mu$ are Lamé constants (3). These parameters satisfy the following relations:

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)}$$

where $E$ is Young's modulus, and $\nu$ is Poisson's ratio. Mechanical properties of the valve leaflet: $E = 0.6; 1.2; 2; 8; 20$ MPa, $\nu = 0.3, \rho = 1200 \text{ kg/m}^3$. 


4. Boundary conditions
At the inlet boundary, the change in the average velocity rate in the cycle time was set by the formula

\[ V = A \sin (B(t - D)) + C, \quad A = 2.4 \text{ cm/s}, \quad B = 3.4 \text{ rad/s}, \quad C = 10 \text{ cm/s}, \quad D = 0.07 \text{ s}, \]

which was obtained by approximation (figure 2b) of the clinical spectral Doppler images (figure 2a). The cycle time is \( T = 1.9 \text{ s} \). A flat velocity profile is set at the inlet boundary. Constant pressure \( P = 0 \) is set at the outlet boundary. The symmetry condition is applied at the upper boundary, and the no-slip condition is applied at the bottom boundary. The condition of interface between solid and fluid regions was applied on the leaflet surface:

\[
\begin{align*}
    d_s^T &= d_f^T; \\
    u_s^T &= u_f^T; \\
    F_s^T &= -F_f^T,
\end{align*}
\]

where \( d \) is displacement, \( u \) is the velocity, and \( F \) is the force. The approximation of rigid walls was used due to the insignificance of their displacements for the popliteal vein.

![Figure 2](image)

**Figure 2.** Colour Doppler and spectral Doppler images of velocity in a venous valve of the popliteal vein (a), the maximum velocity curve digitized by ultrasound data, and the approximation curve (b).

5. Computation aspects
The hydrodynamic mesh consisted of triangular elements and was refined on the valve leaflet. A mesh independence study was conducted. The mesh with 18100 cells is enough to obtain a practically converged solution, the difference from the coarse mesh with 9050 cells is 15\%, and the maximum difference with the fine mesh (with 36200 cells) is 5\%. The mechanical mesh consisted of 800 rectangular elements.

A smoothing algorithm was used to set up a dynamic hydrodynamic meshes, it smoothes the mesh and avoids negative cell volumes in the calculation process, which may stop computation.

6. Results
Figure 3 shows axial velocity fields with streamlines for a leaflet with Young’s modulus of 1.2 MPa at different time instances of the cycle. It can be seen that a jet forms between the valve leaflets. A recirculation zone is formed behind the valve leaflet. A zone with low velocity (stagnant zone) is observed under the valve leaflet.
Figure 3. An axial velocity field and streamlines at four time instances of the cycle (E = 1.2 MPa).

Figure 4 shows the boundary of the stagnant zone (along the velocity isoline u = 0.2 mm/s) according to the calculation results with Young's modulus of 1.2 MPa, which has the best agreement with the clinical data on the stagnant region length. The length of the stagnant zone (l) averaged over the cycle is about 60% of the sinus length (L). The stagnant zone length is the minimum length from the base of the leaflet to the velocity isoline along which the boundary of the stagnant region is constructed.
Figure 4. The calculated position of the leaflet and the boundary of the stagnant zone at four time instances of the cycle (for E = 1.2 MPa) in comparison with the clinical ultrasound data.

When the leaflet elasticity decreases, the length of the stagnant zone increases (figure 5). This is due to the fact that with an increase in Young's modulus, the amplitude of oscillations (ΔR) of the leaflets decreases and the fluid below it moves at a lower velocity. With an increase in Young's modulus from 0.6 MPa to 20 MPa, the length of the stagnant zone increases from 0.4L to L.

Figure 5. The position of the leaflet and the boundary of the stagnant zone at four time instances of the cycle for three values of Young's modulus.

Figure 6 shows the dependence of the cycle-average stagnant zone relative length and the relative amplitude of the leaflets oscillation on Young’s modulus, in comparison with the clinical data. Note that the larger the amplitude of oscillations of the leaflets, the smaller the length of the stagnant zone. The best agreement with the clinical results is observed in calculations with $E = 1.2 - 2$ MPa.
Figure 6. The calculated values of the stagnant zone length and of the leaflet oscillation amplitude depending on Young's modulus in comparison with the clinical data.

7. Conclusions
The dependence was obtained between the stagnant zone length and the amplitude of the leaflet oscillations, and Young’s modulus. The length of the stagnant zone increases as the amplitude of leaflet oscillations decreases. The calculation results obtained in a symmetric two-dimensional model of the healthy venous valve of the popliteal vein gave good qualitative agreement with the clinical ultrasound data on the position and size of the stagnant zone behind the valve.

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