Abstract

We assume that nucleon antinucleon annihilation is a fast process leading to a classical coherent pion pulse. We develop the quantum description of such pion waves based on the method of coherent states. We study the consequences of such a description for averages of charge types and moments of distributions of pion momenta with iso-spin and four-momentum conservation taken into account. We briefly discuss the applicability of our method to annihilation at rest, where we find agreement with experiment, and suggest other avenues for its use.
I. INTRODUCTION

Low energy nucleon-antinucleon annihilation is a fertile area for studying hadron dynamics and particularly pionization. Experiments give information on pion numbers, spectra, and correlations and seem to show little dependence on initial energy from annihilation at rest to kinetic energies of a few hundred MeV. Results do depend on the iso-spin and spin quantum numbers of the annihilating pair. Understanding these results is a serious challenge to theory.

In this paper we develop a description of nucleon-antinucleon annihilation into pions based on the creation of a coherent classical pion wave in the annihilation process and the quantization of this wave using the method of coherent states. About 40% of annihilations go through meson resonances, mostly rho and omega mesons, and then to pions [1]. These channels do not lead to prompt pions and are not part of our treatment. Our first studies show our picture accounts for the major phenomenological features of the pions seen in low energy annihilation. Our picture divides the process into two steps. The first is the dynamics of the nucleon and antinucleon. We have little to say about this except to assume that once these two “touch” and annihilation begins it proceeds very rapidly. This rapid annihilation leads to a pion pulse or pion wave that forms the basis of our coherent state. It is the description and consequences of that state and the way in which its features reflect the spatial and temporal evolution of the annihilation region as well as its quantum numbers that is our principal concern [2].

Our description of annihilation as occurring by the rapid radiation of pions, draws its inspiration from recent studies of annihilation in the Skyrme model. Sommermann et al [3] have studied numerically the collision of two Skyrmions, one of baryon number one and the other of baryon number minus one. They find that once there is significant overlap of the two, they disappear into pion radiation at nearly the causal limit. Shao et al [4] studied this rapid decay in a schematic model. They considered an initial well localized “blob” of Skyrmionic matter with baryon number zero and total energy about twice the single Skyrmion energy, and studied its time evolution. They found that it decays into a pion wave nearly at the causal limit. That pion wave is a propagating, but well localized pulse. This picture of the annihilation process in terms of a rapid coherent radiation pulse is as far from the thermodynamic one as a broadcasting radio antenna is from a black body.

The Skyrmionic description is classical, and is meant as purely suggestive. Does a classical starting point make sense for annihilation? Let us assume that the rapid conversion to pion radiation seen in the Skyrme treatments is a feature of the real annihilation. Then the pions are radiated from this “bright flash” as a coherent pulse, or coherent pion quantum wave. Since the mean number of pions radiated is not small, and the total energy to be radiated is large compared with the pion mass, $\mu$, we can envisage this wave to be, in first approximation, a classical coherent wave. The remaining problem is then to quantize this wave. This we do with the well known method of coherent states. Our picture, then, has two central assumptions. First we assume that rapid annihilation leads to a classical pion wave pulse. This is a consequence of the rapid time scale of the annihilation and the large energy (compared to the pion mass) released. Second we assume that this classical wave can be quantized using coherent states. This is basically the first term in a description with expansion parameter $1/\hat{N}$, where $\hat{N}$ is the mean number of pions. The error in our
second assumption can therefore be estimated.

It should be noted that our starting point arises in classical QCD, which is the non-perturbative domain of the theory. This seems to us the correct starting point for the study of annihilation. It may be that other strongly interacting systems are also best approached from this direction. It has recently been shown that that is true for the nuclear force [5]. In such treatments it is the reintroduction of quantum aspects that is the problem. For radiation fields, coherent states are constructed to solve that problem.

In the next Section, we develop the coherent state formalism for the pions. We begin with a description of classical pion waves from a source, introduce coherent states, discuss the constraints of iso-spin and of energy and momentum conservation and give some statistical features of the coherent state. In Section 3 we briefly investigate the consequences of our description for the annihilation data. We only touch on what could be done, but we do see that our approach generally agrees with experiment. Finally Section 4 discusses some directions for further work and presents a brief summary.

II. COHERENT STATE FORMALISM FOR PIONS

A. Classical Pion Waves

We imagine nucleon antinucleon annihilation as occurring very rapidly from a small region of high energy density. In first approximation, we take this region to radiate a classical coherent pion wave. If we further neglect $\pi - \pi$ interactions and the back reaction of the radiated pions on the source, we can write the classical wave equation for the pion wave $\Phi(r, t)$, as

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2} - \mu^2\right)\Phi(r, t) = S(r, t).$$

Here $S(r, t)$ is the source of the pion field, $\Phi$. This source will be assumed to be sharply localized in space and time. For the moment we neglect the isospin degree of freedom and will return to it later. We also leave out the factor of $-4\pi$ in front of the source that is familiar from electrodynamics, since it adds nothing here. Eqn (1) gives the form of the pion wave as a classical wave radiated by the source $S$. The details of the annihilation mechanism are contained in this source function.

The equation for $\Phi$ is easily solved in terms of $S$ by taking Fourier transforms. We define the Fourier transform of $\Phi$ by

$$\Phi(r, t) = \int \frac{d^3k \, d\omega}{(2\pi)^2} e^{ikr} e^{-i\omega t} \phi(k, \omega),$$

and a corresponding function, $s(k, \omega)$ as the Fourier transform of $S(r, t)$. The pion field is then given by

$$\Phi(r, t) = \int \frac{d^3k \, d\omega}{(2\pi)^2} e^{ikr} e^{-i\omega t} \frac{s(k, \omega)}{\omega^2 - k^2 - \mu^2}.$$
so and using the pole contributions only to evaluate the $\omega$ integral in (3), we obtain the approximate expression for the pion field, for $t > 0$,

$$\Phi(r, t) = -i \int \frac{d^3k}{(4\pi \omega_k)} s(k, \omega_k) e^{i k \cdot r - i \omega_k t}. \quad (4)$$

where $\omega_k = \sqrt{k^2 + \mu^2}$. This form for the pion field is exact in the radiation region, but neglects non-radiating parts of the field coming from the $\omega$ dependence of $s$. In the radiation zone there is no source and $\Phi$ given in (4) satisfies the source free wave equation. It is this form for $\Phi$ that we will take to represent the radiated pions of nucleon antinucleon annihilation.

**B. Coherent States**

It is not the classical pion wave that is detected in annihilation experiments but rather the quanta of that field, the pions. We must therefore extract the quantum content of this classical wave. This is exactly what the coherent state method achieves. Pioneered by Glauber [6] for optics in situations where the photon granularity is significant but not overwhelming, the coherent state formalism allows one to construct a quantum state that corresponds, in the large number of field quanta limit, to the classical wave field. Coherent states for pions were studied by Horn and Silver [7], and many of the arguments given here were considered by them. They did not, however, study annihilation.

Because the pion field satisfies the wave equation without sources, it can be decomposed into a linear superposition of free waves with wave number $k$ and corresponding energy $\pm \omega_k$. That is the content of Eqn (4). We can bring in quantum mechanics for this wave by introducing the standard creation and annihilation operators for the modes $k$, $a_k$ and $a_k^\dagger$. These satisfy the usual harmonic oscillator commutation relations,

$$[a_k, a_{k'}^\dagger] = \delta(k - k'), \quad (5)$$

with all other commutators zero. The quantum field operator corresponding to the pion field is then

$$\Phi_{qm}(r, t) = \int \frac{d^3k}{(2\pi)^{3/2}} (a_k e^{i k \cdot r} e^{-i \omega_k t} + a_k^\dagger e^{-i k \cdot r} e^{i \omega_k t}) \quad (6)$$

The coherent state associated with a given classical state, $\Phi(r, t)$, is the quantum state that is an eigenstate of the positive frequency part of $\Phi_{qm}$, $\Phi_{qm}^+$, with the corresponding positive energy part of the classical wave as its eigenvalue. Recall that the positive frequency part of the state has the $e^{-i \omega_k t}$ dependence. In the quantum operator for the field, this is the part that goes with the annihilation operator. Since the modes for each $k$ are independent, finding the eigenstate is equivalent to finding the eigenstate of one single annihilation operator. The full state can then be built up by superposing the independent modes.

The normalized eigenstate, $|\lambda\rangle$, with eigenvalue, $\lambda$, of a single mode annihilation operator, $a$, ($a |\lambda\rangle = \lambda |\lambda\rangle$), is given by

$$|\lambda\rangle = e^{-\frac{\lambda}{2}} e^{\lambda a^\dagger} |0\rangle \quad (7)$$
where \( |0\rangle \) is the vacuum state. This equation for the eigenstate of \( a \) is the key to the entire coherent state formalism. 

Consider now the normalized quantum state \( |f\rangle \) defined by

\[
|f\rangle = \exp(-\frac{1}{2} \int d^3k f^*(k) f(k) + \int d^3k f(k) a_k^\dagger) |0\rangle.
\]  

(8)

This is just a coherent state in which each mode \( k \) carries weight \( f(k)d^3k \). It is clear, for the positive frequency part of the field, that

\[
\Phi^+_{qm}(r,t)|f\rangle = \varphi(r,t)|f\rangle,
\]  

(9)

with the eigenfunction \( \varphi \) given by

\[
\varphi(r,t) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{ik\cdot r} e^{-i\omega_k t} f(k).
\]  

(10)

This \( \varphi \) satisfies the source free wave equation for any function \( f \). We then take \( |f\rangle \) to be the normalized quantum state corresponding to the classical wave \( \varphi \). Note that \( |f\rangle \) does not contain a fixed number of quanta, but is rather a coherent superposition of states of different numbers of quanta.

The meaning of a coherent state arising from a classical source can be clarified by relating it to the standard S-matrix description of a field generated by a classical external source. The classical source, \( S \), coupled to the field, \( \Phi \), creates from the vacuum the quantum state

\[
\exp(-i \int d^3r dt \Phi_{qm}(r,t) S(r,t)) |0\rangle.
\]  

(11)

Upon four dimensional Fourier transformation and integration over \( dk_o \), this reduces, up to a normalization, to Eqn (8). The exponent in (11) is just the S-matrix in the interaction representation, and makes clear both the connection of the S-matrix and coherent state approaches and why the resulting state has an indefinite number of quanta.

In the electromagnetic case it seems natural to define states with indefinite numbers of photons, but even then if the wave has a fixed total energy only a fixed number of photons above some frequency can be detected. With pions the description in terms of a coherent mixture of states with different numbers of pions seems less comfortable, particularly in view of the finite pion mass. No one would suggest representing a pion wave with total energy near the pion rest mass as a classical object. In annihilation, however, the total energy released is about 13 times the pion rest mass, and the approximation of representing it by a classical coherent wave seems at least plausible. The purpose of this paper is to show that that description is not only plausible but useful and gives dynamical insight into the

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1 We are talking here of field coherent states. These are the coherent states appropriate to classical fields. They are different from coherent states expressed as polynomials in the creation operators and used in descriptions of material objects such as atoms, molecules, deformed nuclei and Skyrmions. (For an introduction to these states see [3])
annihilation process and may have application in a number of other processes in which many pions are created.

To make the pion coherent state correspond to the radiating solution \( \Phi(\mathbf{r}, t) \) of Eqn (4), that is to make \( \varphi \) in (9) or (10) correspond to the positive frequency part of \( \Phi \), we must take in (8)

\[
f(\mathbf{k}) = -i \frac{s(\mathbf{k}, \omega_k)}{2\omega_k} \sqrt{\frac{2\pi}{4\pi}}.
\]

(12)

With this choice, our quantum coherent state represents the classical radiating pion wave generated by the source \( S(\mathbf{r}, t) \).

C. Iso-spin

The pion is an iso-vector and comes in three charge states. In principle each charge state can have its independent source, corresponding classical wave, and coherent state. This is most easily formulated by making \( \Phi(\mathbf{r}, t) \) and \( S(\mathbf{r}, t) \) of Eqn (1) each iso-vectors. Then each charge state could have a source density that varies arbitrarily in space and time. In fact nucleon antinucleon annihilation occurs in a state of fixed total iso-spin, 0 or 1. The full pion wave must have the same total iso-spin. This condition of total iso-spin conservation imposes a global condition on the source and on the coherent state. Since the different \( \mathbf{k} \) modes of the wave are independent, they must each combine to the correct total iso-spin, which means that the iso-spin source density cannot depend on \( \mathbf{r} \) or \( t \). In the coherent state \( |f\rangle \) of (8) we expect in general that \( f(\mathbf{k}) \) is an iso-vector dotted into the now iso-vector creation operator. But if we are to construct states of fixed iso-spin for any \( \mathbf{k} \) configuration, the iso-vector dependence of \( f \) must be independent of \( \mathbf{k} \). In that case \( f \) is a constant vector in iso-spin space. The direction of that iso-vector depends on the source.

For the moment let us consider a coherent state specified by a fixed unit vector in iso-spin space \( \hat{T} \) and an iso-scalar function \( f(\mathbf{k}) \). Call that state \( |f, \hat{T}\rangle \). In place of (8) we write

\[
|f, \hat{T}\rangle = \exp\left(-\frac{1}{2} \int d^3k f(\mathbf{k}) f^*(\mathbf{k}) + \int d^3k \hat{T} \cdot a^\dagger_{\mathbf{k}} f(\mathbf{k})\right) |0\rangle.
\]

(13)

That is a coherent state constructed from a particular superposition of pions, that superposition made up of pions all pointing in the iso-direction of \( \hat{T} \). This is not yet a state of fixed iso-spin. In fact it is easy to see that it contains states of all iso-spin. A state of fixed iso-spin must be projected from it. Call the projected state of total iso-spin \( I \) with z-component \( I_z \), \( |f, I, I_z\rangle \). The projection is given by

\[
|f, I, I_z\rangle = \nu \int \frac{d\hat{T}}{\sqrt{4\pi}} |f, \hat{T}\rangle Y_{I, I_z}^*(\hat{T})
\]

(14)

where \( Y_{I, I_z}(\hat{T}) \) is the usual spherical harmonic, and \( \nu \) is a normalization constant. The state defined in (14) is orthogonal to states of different total iso-spin or z-component projection. To see this and to calculate the normalization let us study the inner product of two projected states. This is given by
\[
\langle f, I, I_z|f, I', I'_z \rangle = \nu^2 \int \frac{d\hat{T}d\hat{T}'}{4\pi} \langle f, \hat{T}|f, \hat{T}'\rangle Y_{I,I_z}(\hat{T})Y_{I',I'_z}(\hat{T}')
\] (15)

It is easy to show that
\[
\langle f, \hat{T}|f, \hat{T}'\rangle = \exp \left( -\frac{(\hat{T} - \hat{T}')^2}{2} \int d^3k f(k)f^*(k) \right)
\] (16)

For a strong field, large \( f \), this inner product is very sharply peaked around \( \hat{T} = \hat{T}' \). We will show in the next section that
\[
\int d^3k f(k)f^*(k) = \hat{N}
\] (17)

where \( \hat{N} \) is the mean number of pions emitted in the annihilation. We will use this notation here both to save writing and to remind us that this integral, or \( \hat{N} \), is large. To show the orthogonality in (16) write
\[
e^{\hat{T}\cdot\hat{T}'\hat{N}} = 4\pi \sum_{l,m} i^l j_l(-i\hat{N})Y^*_{l,m}(\hat{T})Y_{l,m}(\hat{T}')
\] (18)

From this it is easy to show that
\[
\langle f, I, I_z|f, I', I'_z \rangle = \nu^2 e^{-\hat{N}} i^l j_l(-i\hat{N})\delta_{I,I'}\delta_{I_z,I'_z}
\] (19)

from which the normalization can be read off. In particular for large \( \hat{N} \), we can use the asymptotic form of the Bessel function to yield for the normalization
\[
\nu = \sqrt{2\hat{N}}.
\] (20)

We should note that our iso-spin projection has the interesting property that states of even iso-spin contain only even numbers of pions and states of odd iso-spin only odd numbers. To escape this restriction, as the data does, we need to include the channels for annihilation into meson resonances.

**D. Probing the Coherent State: No Iso-spin**

We would now like to examine the properties of the coherent pion state generated by the assumed initial source \( S(\mathbf{r}, t) \). We begin with a discussion of the case without iso-spin. This coherent state is given by (8) with \( f(|k|) \) given in terms of the Fourier transform of \( S \) as in (12). Although the state does not contain a fixed number of pions, we can ask what is the average number, \( \hat{N} \). This is just the expectation value of the total pion number operator in the coherent state. We need
\[
\hat{N} = \langle f| \int d^3k a_k^\dagger a_k|f \rangle.
\] (21)

(Note that we are working in the Heisenberg representation so that the states are time independent while the operators in general carry a time dependence. However, the number
operator is time independent.) This expectation value is easily evaluated using the special properties of the coherent states to give (17). Thus we see that the integral that enters to normalize the coherent state in (8), is just \( \hat{N} \). It is large \( \hat{N} \) that corresponds to large field (\( f \) or \( S \) large) and thus to a good approximate correspondence between the classical and quantum descriptions. It is also clear from this that the average single pion momentum distribution is given by

\[
\frac{dN(k)}{d^3k} = f^*(k)f(k). \tag{22}
\]

in terms of the square of the Fourier transform of the source density.

In the same way one obtains for the total energy released, the expectation of the Hamiltonian, or

\[
E = \int d^3k \omega_k f^*(k)f(k). \tag{23}
\]

In a quantum process, this energy is sharp and should be put equal to the total energy released in the annihilation. This fact serves as a convenient way to normalize \( f \). The sharpness of the state can be seen by calculating the dispersion in the number of pions emitted. One finds

\[
\sigma^2 = \langle f|N^2 - (\hat{N})^2|f \rangle = \hat{N}, \tag{24}
\]

which shows that the fractional dispersion in \( N \) is \( 1/\sqrt{\hat{N}} \). This result also follows from the fact that the probability of finding a state of \( m \) pions is given by a Poisson distribution, as we shall show below.

One can use the coherent state to calculate the amplitude for finding some particular quantum configuration. For example the amplitude for finding \( m \) pions of momenta \( p_1 \ldots p_m \) is given by

\[
\langle p_1, \ldots p_m | f \rangle = \frac{1}{\sqrt{m!}} f(p_1) \ldots f(p_m)e^{-\hat{N}/2} \tag{25}
\]

The probability of finding \( m \) pions of any momentum is then

\[
P_m = \frac{1}{m!} e^{-\hat{N}} (\int |f(p)|^2 d^3p)^m \tag{26}
\]

which is the Poisson distribution. It is clear that the coherent state does not “know” not to emit more pions than energy conservation permits. We will turn to these constraints of energy and momentum conservation below. Our discussion here serves to emphasize that the coherent state approach is better at giving average information over the ensemble of annihilations than it is for particular quantum states.
E. Probing the Coherent State: With Iso-spin

If we include iso-spin projection as in Section 2.3, we can also probe questions about iso-spin populations. For example let us study the average number of pions of a particular type in a projected coherent state with total iso-spin $I$ and $z$-component $I_z$. This is given by

$$\hat{N}_\mu = \langle I, I_z, f | \int d^3k a^\dagger_{k,\mu} a_{k,\mu} | I, I_z, f \rangle,$$

where $\mu$ is the pion type ($+, -, 0$) and there is no sum over $\mu$ implied. Using the forms of the projected states, this becomes

$$\hat{N}_\mu = \nu^2 \int \frac{d\hat{T} d\hat{T}'}{4\pi} Y^*_{I,I_z}(\hat{T}) \hat{T}^*_\mu \hat{T}'_{\mu} Y_{I,I_z}(\hat{T}') \exp(\hat{N}(\hat{T} \cdot \hat{T}' - 1)) \int d^3k f^*(k)f(k)$$

Using (18) and expressing the $\hat{T}$ in terms of spherical harmonics, the integrals can be calculated in terms of standard identities. We continue to use (17) to express the integral over $f$ in terms of $\hat{N}$. We now use (17) to define $\hat{N}$ but as we shall see it is still the average number of pions summed over all pion types. We then find

$$\hat{N}_\mu = \hat{N} \sum_{l,m} \frac{2l+1}{2l+1} \langle l0, 10|l, I0 \rangle \langle lm, 1\mu|l, II_z \rangle^2$$

in terms of a sum over Clebsch-Gordan coefficients. In obtaining (29) we have used the asymptotic forms of the Bessel functions (large $\hat{N}$) so that there is no $l$ dependence in the normalization.

For nucleon-antinucleon annihilation there are only two choices for $I$, 0 or 1. For the case of $I = 0$ (29) reduces to

$$\hat{N}_\mu = \hat{N}/3,$$

for any $\mu$. As we expect for a state of iso-spin zero the average number of pions is type independent, and each is one third of $\hat{N}$, making $\hat{N}$ the true average number of pions summed over types, as promised. For $I = 1$ there are three cases. For $I_z = 0$ one finds

$$\hat{N}_+ = \hat{N}/5,$$
$$\hat{N}_- = \hat{N}/5,$$
$$\hat{N}_0 = 3\hat{N}/5.$$  

We see that again the sum of averages over types is $\hat{N}$. Note the dominance of $\pi^0$, an important feature of the data. For $I_z = 1$ one finds

$$\hat{N}_+ = 2\hat{N}/5,$$
$$\hat{N}_- = 2\hat{N}/5,$$
$$\hat{N}_0 = \hat{N}/5.$$  

with the same result for $I_z = -1$. It is no surprise that for the $I = 0$ case or for the $I = 1$, $I_z = 0$ case the average number of $\pi^+$ and $\pi^-$ is the same. It is a bit more puzzling for $I = 1,$
\( I_z = \pm 1 \). What happened to charge conservation? The initial \( I_z = \pm 1 \) state has charge \( \pm 1 \). However, \( \hat{N}_\pm \) is a number of order \( \hat{N} \). To that order all three \( I_z \) values do correspond to states of average charge zero. Charge conservation is a \( 1/\hat{N} \) effect, and we are neglecting terms of that order. We will return to this approximation below.

The same charge ratios can be obtained in (28) by making the overlap of (16) proportional to \( \delta(\hat{T} - \hat{T}') \). We have already commented that this overlap is strongly peaked around \( \hat{T} = \hat{T}' \) for large \( \hat{N} \). This delta function approximation is correct in the large \( \hat{N} \) limit and it makes calculations of the average pion number by type and of higher moments of the distribution very simple.

We illustrate this delta function method in the calculation of isospin number correlations. For example the joint average of \( \mu \) type and \( \nu \) type pions in a state of total isospin \( I \) with \( z \)-component \( I_z \) is

\[
\hat{N}^2_{\mu,\nu} = \langle I, I_z, f | \int d^3k a_{k,\mu}^\dagger a_{k,\mu} \int d^3p a_{p,\nu}^\dagger a_{p,\nu} | I, I_z, f \rangle.
\]

In the large \( \hat{N} \) limit where the overlap may be replaced by a delta function, we obtain,

\[
\hat{N}^2_{\mu,\nu} = \hat{N}^2 \int d\hat{T} Y^*_{I,I_z}(\hat{T}) \hat{T}_\mu^* \hat{T}_\nu^* \hat{T}_\mu \hat{T}_\nu Y_{I,I_z}(\hat{T}).
\]

In this expression the components of \( \hat{T} \) are, in the usual spherical coordinates,

\[
\hat{T}_0 = \cos \theta \\
\hat{T}_+ = -\frac{1}{\sqrt{2}} \sin \theta e^{i\phi} \\
\hat{T}_- = \frac{1}{\sqrt{2}} \sin \theta e^{-i\phi}.
\]

Higher order forms are obtained from the generalization of (33) and (34). They come from higher powers of the number operator for the \( \rho \)th type in the expectation value with the coherent state replacement

\[
\int d^3k a_{k,\rho}^\dagger a_{k,\rho} = \hat{N} \hat{T}_\rho^* \hat{T}_\rho.
\]

The integrals that appear in (34) are elementary, and hence these higher moments are easily calculated.

Greiner, Gong and Muller [18] have recently introduced polynomial coherent states in a discussion of pion condensates in heavy ion collisions. They consider only \( I = 0 \) states and calculate the iso-spin pair correlations defined as

\[
C_{\mu,\nu} = \frac{\hat{N}^2_{\mu,\nu}}{N_{\mu} N_{\nu}} - 1.
\]

We obtain the same values as they do for \( C_{\mu,\nu} \) for large \( \hat{N} \), but our method is simpler. We can also use the method to calculate the pair correlations in other states. For example we find for \( I = 1, I_z = 0, C_{0,0} = 4/21 \) and for \( I = 1, I_z = 1, C_{0,0} = 8/7 \). Other moments
and other amplitudes are also easily calculated and agree with those of Greiner, Gong and Muller where calculated by them.

Thus we see that the coherent state approach makes definite predictions about average charge ratios in annihilations and can also be used to calculate higher moments of those distributions. It also predicts that the single pion momentum distribution should be independent of pion type.

F. Energy and Momentum Conservation

So far the coherent state we have defined does not have definite total energy or momentum. For example it contains contributions from states with only one pion or with arbitrary numbers of pions, neither of which is allowed by conservation of total energy and momentum. This fault is not serious for large $\hat{N}$, but is significant for the intermediate $\hat{N}$ of annihilation. Energy and momentum conservation can be imposed on the coherent state, as was shown by Horn and Silver [7], at the expense of some further complication of the formalism.

Let us begin by imposing energy and momentum conservation on the coherent state without iso-spin. The method for imposing energy and momentum on the coherent state is formally the same as the projection method for imposing fixed iso-spin. The projections commute, but taking them together complicates the physics. For iso-spin one first constructs a state with all the pions pointing in the same direction in iso-spin and then averages over that direction with the appropriate $SU(2)$ eigenfunction weight. To construct a state of fixed energy and momentum (or four-momentum) one first constructs a state of all the pions at a fixed place in space and time and averages over those places and times with the appropriate eigenfunctions of definite energy and momentum. To do this define the operator $F(r, t)$ by

$$F(r, t) = \int d^3 p f(p) a_p^\dagger \exp(i p \cdot r - i \omega_p t).$$  \hspace{1cm} (38)

Then the (un-normalized) coherent state at $r$ and $t$ is

$$|f, r, t\rangle = e^{F(r, t)} |0\rangle.$$  \hspace{1cm} (39)

It is more concise as well as more physical to use four dimensional notation. We call $x$ the four-vector of position with space part $r$ and time part $t$. Thus $|f, r, t\rangle$ becomes $|f, x\rangle$. Then the state of fixed total four-momentum, $K$ is given by

$$|f, K\rangle = \int \frac{d^4 x}{(2\pi)^4} e^{-i K \cdot x} |f, x\rangle.$$  \hspace{1cm} (40)

To see how this works and to calculate the normalization, consider the overlap of two states of different four-momentum. One easily finds

$$\langle f, K'|f, K\rangle = \int \frac{d^4 x d^4 x'}{(2\pi)^8} \exp(-i K \cdot x + i K' \cdot x') e^{\rho(x-x')}$$  \hspace{1cm} (41)

where

$$\rho(x) = \int d^3 p |f(p)|^2 e^{-i p \cdot x}$$  \hspace{1cm} (42)
and where the fourth component of $p$ under the integral is $\omega_p = \sqrt{p^2 + \mu^2}$. A simple change of variable reduces the overlap to

$$\langle f, K' | f, K \rangle = \delta^4(K - K') \int \frac{d^4x}{(2\pi)^4} e^{iK \cdot x} e^{\rho(x)}$$

(43)

This clearly shows that states of different four-momentum are orthogonal, but the remaining integral is singular due to the undamped large $x$ behavior of the integrand. Horn and Silver also pointed out this problem. It arises from the first term in the expansion of the exponential. This term, a “one”, corresponds to the no pion contribution to the annihilation process. Since there is in fact no such contribution that conserves energy and momentum, we can subtract it out. Similarly there is no one pion state that can contribute to annihilation and conserve four-momentum, thus we may also subtract out the one meson state and define a coherent state of total four-momentum $K$ that starts with two pions. This is given in terms of

$$|f, x, 2\rangle = (e^{F(x)} - F(x) - 1)|0\rangle$$

(44)

by

$$|f, K, 2\rangle = \int \frac{d^4x}{(2\pi)^4} e^{-iK \cdot x} |f, x, 2\rangle.$$  

(45)

We then find

$$\langle f, K', 2 | f, K, 2 \rangle = \delta^4(K - K') \int \frac{d^4x}{(2\pi)^4} e^{iK \cdot x} (e^{\rho(x)} - \rho(x) - 1).$$

(46)

The integral now is convergent and we have lost nothing by the subtractions since we have only removed states that cannot contribute physically. However the integral in (46) is still quite delicate and difficult to evaluate numerically because it contains delta functions coming from the $d^4x$ integral. We can get around this problem and shed more physical light on the expression by expanding the exponential and interchanging the order of integration. Let us call the integral in (46) $\mathcal{I}(K)$. We can write

$$\mathcal{I}(K) = \int \frac{d^4x}{(2\pi)^4} e^{iK \cdot x} \sum_{m=2} \frac{\rho^m(x)}{m!},$$

(47)

$$= \sum_{m=2} \mathcal{I}_m(K)/m!.$$  

Using the definition of $\rho$ and interchanging integrations one finds

$$\mathcal{I}_m(K) = \int \delta^4(K - p_1 - p_2 \ldots - p_m) \prod_{i=1}^m d^3p_i |f(p_i)|^2,$$

(48)

where as before we take the fourth component of $p_i$ to be $\sqrt{p_i^2 + \mu^2}$. Integrals of this form are easily done by Monte Carlo methods following a program given by Barger and Phillips. [11]
The individual terms in the $m$ sum in (47) represent the relative contributions to annihilation into $m$ pions. For fixed total energy the sum must terminate. For example, nucleon-antinucleon annihilation at rest cannot go into more than 13 pions, and in that case $I_m$ must be zero for $m > 13$. In fact $I_m/m! I$ is just the fraction of annihilations that go into $m$ pions. Thus the mean pion number is given by

$$\hat{\mathcal{N}} = \sum_{m=2}^{13} \frac{mI_m(K)}{m! I(K)}$$

(49)

This is equal to the expression that one arrives at by taking the expectation value of the number operator in the state $|f, K, 2\rangle$ and normalizing. This expression can also be written

$$\hat{\mathcal{N}} = \int \frac{d^4x}{(2\pi)^4} e^{iK \cdot x} \rho(x) e^{\rho(x)} / \int \frac{d^4x}{(2\pi)^4} e^{iK \cdot x} e^{\rho(x)}$$

(50)

Note that $\rho(0) = \hat{\mathcal{N}}$ in terms of the old expression for $\hat{\mathcal{N}}$, (17). For $\hat{\mathcal{N}}$ large, $\rho(x)$ is large, and the integrands in (50) are dominated by the factors of $e^{\rho}$ and $e^{-iK \cdot x}$. The expression (50) can then be thought of in terms of a ratio of improper integrals

$$\hat{\mathcal{N}} \approx \int \frac{d^4x}{(2\pi)^4} e^{iK \cdot x} \rho(x) e^{\rho(x)} / \int \frac{d^4x}{(2\pi)^4} e^{iK \cdot x} e^{\rho(x)}$$

(51)

These integrals can be approximated by the method of stationary phase. For annihilation at rest, the total energy released, $K_0 = E$, is large, and then the stationary phase point comes at $x \approx 0$. Thus the $\rho(x)$ in the numerator of (51), but not in the exponent, can be evaluated at $x = 0$ to give that the new $\hat{\mathcal{N}}$ is approximately equal to the old one. Corrections are of order $1/\hat{\mathcal{N}}$. We have verified this in a number of numerical examples.

We see that four-momentum conservation can indeed be implemented for the coherent state, but for average quantities, and for $\hat{\mathcal{N}}$ large, that constraint is not important. As we will see below, it is important for the probability distribution and for higher moments of the distribution. It is clear that one can combine the method outlined above for projecting onto states of good four-momentum with the method of the previous section for projecting onto states of good iso-spin. We now turn to a discussion of how to do that.

**G. Probing the Coherent State: With Iso-spin and Four-momentum**

Combining the methods of previous sections, we can construct a state of fixed total four-momentum $K$ and fixed iso-spin $I$ with z-component $I_z$. We first generalize $F$ of (38) to give

$$F(x, T) = \int d^3p f(p) e^{-ip \cdot x} a_{\mu, \nu}^\dagger T_{\mu
u}$$

(52)

Then the appropriate generalization of (44) to include iso-spin is

$$|f, x, T, 2\rangle = (e^{F(x, T)} - F(x, T) - 1)|0\rangle$$

(53)

giving the state (not normalized)
\[ |f, K, I, I_z, 2\rangle = \int \frac{d^4x}{(2\pi)^4} \frac{d\hat{T}}{\sqrt{4\pi}} e^{iK \cdot x} |f, x, T, 2\rangle Y_{I_z}^*(T). \] (54)

The inner product of two such states belonging to different four-momentum and iso-spin gives a four dimensional delta function on four-momentum and Kroncker deltas on iso-spin and its z-component times the following normalization integral

\[ \mathcal{I} = \int \frac{d^4x}{(2\pi)^4} \frac{d\hat{T}d\hat{T}'}{4\pi} e^{iK \cdot x} Y_{I_z}^*(\hat{T}) Y_{I_z}^*(\hat{T}') (e^{\rho(x)\hat{T} \cdot \hat{T}'} - \rho(x)\hat{T} \cdot \hat{T}' - 1), \] (55)

This integral can be calculated by the same expansion methods used in (46) and (47). In terms of the \( I_m(K) \) defined in (47) the normalization integral of (55) is given by

\[ \mathcal{I} = \sum_{m=2}^{\infty} \frac{I_m(K)}{m!} F(m, I), \] (56)

where

\[ F(m, I) = \int \frac{d\hat{T}d\hat{T}'}{4\pi} Y_{I_z}^*(\hat{T}) Y_{I_z}^*(\hat{T}') (\hat{T} \cdot \hat{T}')^m \] (57)

This integral can be done using the identity in (18) by differentiating (18) with respect to \( \hat{N}_m \) times and then setting \( \hat{N} \) to zero. One finds

\[ F(m, I) = \begin{cases} 0 & I > m \text{ and } I - m \text{ is odd} \\ \frac{m!}{(m-1)!!(I+m+1)!!} & I \leq m \text{ and } I - m \text{ is even} \end{cases} \] (58)

It should be recalled that sums over \( m \) as in (56) have finite upper limits because of the constraint of energy conservation.

The expected number of pions of type \( \mu \) corresponding to (27) is now

\[ \hat{N}_\mu = \frac{1}{\mathcal{I}} \int \frac{d^4x}{(2\pi)^4} \frac{d\hat{T}d\hat{T}'}{4\pi} e^{iK \cdot x} Y_{I_z}^*(\hat{T}) Y_{I_z}^*(\hat{T}') \hat{T}_\mu \hat{T}^*_\mu \rho(x)(e^{\rho(x)\hat{T} \cdot \hat{T}'} - 1), \] (59)

The joint average of \( \mu \) type and \( \nu \) type pions as in (33) is given by

\[ \hat{N}_{\mu\nu}^2 = \frac{1}{\mathcal{I}} \int \frac{d^4x}{(2\pi)^4} \frac{d\hat{T}d\hat{T}'}{4\pi} e^{iK \cdot x} Y_{I_z}^*(\hat{T}) Y_{I_z}^*(\hat{T}') \hat{T}_\mu \hat{T}^*_\mu \hat{T}_\nu \hat{T}^*_\nu \rho^2(x)e^{\rho(x)\hat{T} \cdot \hat{T}'} \] (60)

In obtaining (60), we have neglected a term from the commutator that is down by \( 1/\hat{N} \). In these expressions, \( \rho \) is still given as in (42). These expectation values can be evaluated, as before, by expanding. Once again they are expressed in terms of the \( I_m \). We have

\[ \hat{N}_\mu = \frac{1}{\mathcal{I}} \sum_{m=2}^{\infty} \frac{I_m(K)}{(m-1)!} G_\mu(m-1, I, I_z), \] (61)

and

\[ \hat{N}_{\mu\nu}^2 = \frac{1}{\mathcal{I}} \sum_{m=2}^{\infty} \frac{I_m(K)}{(m-2)!} H_{\mu\nu}(m-2, I, I_z), \] (62)
where

\[ G_\mu(m, I, I_z) = \int \frac{d\hat{T}d\hat{T}'}{4\pi} Y_{I\mu}^*(\hat{T})Y_{I\nu}^*(\hat{T}')\hat{T}_\mu\hat{T}'_\nu(\hat{T} \cdot \hat{T}')^m \]  

(63)

\[ = \sum_{l_n} F(m, l) \frac{2l + 1}{2l + 1} (\langle l0, 10|l0, 1\mu\langle I_z\rangle \rangle)^2 \]

and

\[ H_{\mu\nu}(m, I, I_z) = \int \frac{d\hat{T}d\hat{T}'}{4\pi} Y_{I\mu}^*(\hat{T})Y_{I\nu}^*(\hat{T}')\hat{T}_\mu\hat{T}'_\nu(\hat{T} \cdot \hat{T}')^m \]  

(64)

\[ = F(m, I) (\langle 10, 10|00\rangle \langle 1\mu, 1\nu|00\rangle^2 + 2F(m, I) \langle 10, 10|00\rangle \langle 1\mu, 1\nu|00\rangle \langle 10, 10|20\rangle \langle 00, 20\rangle \langle 1\mu, 1\nu|2\mu + \nu\rangle \langle II_z, 2\mu + \nu|II_z\rangle \rangle + \sum_{l_n} F(m, l) \frac{2l + 1}{2l + 1} (\langle 10, 10|20\rangle \langle 00, 20\rangle \langle 1\mu, 1\nu|2\mu + \nu\rangle \langle II_z, 2\mu + \nu|II_z\rangle \rangle)^2 \]

Let us examine these forms in the cases of physical interest. For \( I = 0 \), it is easy to show that

\[ G_0(m, 0, 0) = G_1(m, 0, 0) = G_{-1}(m, 0, 0) = F(m, 1)/3 \]  

(65)

Then from (56) and (61) we get

\[ \hat{N}_+ = \hat{N}_- = \hat{N}_0 = \hat{N} \]  

(66)

as we expect. For \( I = 1, I_z = 0 \), we find

\[ G_0(m, 1, 0) = F(m, 0)/3 + 4F(m, 2)/15 \]

\[ G_1(m, 1, 0) = F(m, 2)/5 \]

\[ G_{-1}(m, 1, 0) = F(m, 2)/5. \]  

(67)

These do not seem to give the same charge ratios we obtained before in this state, but those were calculated in the large \( \hat{N} \) limit. In that limit the dominant terms in the sums of (56) and (61) come from terms with \( m \sim \hat{N} \). Thus the dominant terms in \( F(m, l) \) that matter have \( m >> l \) (recall that \( l \) is of order \( I \) which is in turn of order 1). In this limit it is easy to show that \( F(m, l)/F(m, l + 2) \sim 1 \) with corrections of order \( 1/m \). In that limit (67) yields the results of (31). Similarly for \( I = 1, I_z = 1 \), we find

\[ G_0(m, 1, 1) = F(m, 2)/5 \]

\[ G_1(m, 1, 1) = F(m, 0)/3 + F(m, 2)/15 \]

\[ G_{-1}(m, 1, 1) = 2F(m, 2)/5. \]  

(68)

which yields (32) in the large \( \hat{N} \) limit, and shows how charge conservation enters for finite \( \hat{N} \). One can calculate the two pion averages in the same way. The factors that enter become for \( I = 1, I_z = 0 \)

\[ H_{00} = 9F(m, 1)/25 + 12F(m, 3)/175 \]  

(69)
and for $I = 1, I_z = 1$

$$H_{00} = F(m, 1)/25 + 8F(m, 3)/175 \quad (70)$$

which yield our previous results for the iso-spin pair correlations $C_{\mu,\nu}$ defined in Sect. 2.5 in the large $\hat{N}$ limit.

The evaluation of the averages now depend on the functional form of $f(k)$. We turn to it in the next section.

III. PHENOMENOLOGY

As a first orientation into the phenomenological content of the coherent pion wave description of annihilation, let us study the relation between the average pion number $\hat{N}$ of (17) and the energy released, (23), for a very simple model of $f$. We do not take $f$ directly from the experimental pion spectrum since the pions from the meson resonance channels confuse the extraction. Rather suppose we assume that the pion field source turns on at $t = 0$ and then decays exponentially in time, and that it has a spherically symmetric Yukawa shape. (Spherical symmetry is appropriate for annihilation at rest.) That is we take for $S(r, t)$ of (1),

$$S(r, t) = 0, \quad t < 0$$
$$= S_0 \frac{e^{-ar}}{r} t e^{-\gamma t}, \quad t > 0 \quad (71)$$

where $S_0$ is the source strength and where we have put in a factor of $t$ to make the time dependence continuous at $t = 0$. Using (12) this leads to

$$|f(k)|^2 = \frac{C_0 k^2}{(k^2 + \alpha^2)^2(\omega_k^2 + \gamma^2)^2 \omega_k^2} \quad (72)$$

where $C_0$ is a strength and where we have multiplied $f$ by $k$ to model the p-wave nature of pion emission. With this form, the integrals for the mean number of pions and for the mean energy (17) and (23) cannot both be done analytically, but they can be easily evaluated numerically. This requires a choice of the parameters $\alpha, \gamma$ and $C_0$. The last can be fixed by requiring that the average energy be the energy released in annihilation. In units of the pion mass ($\mu = 1$), and for annihilation at rest, this energy is 13.87. For the range parameters, we take $\alpha = \gamma = 2$. This corresponds to an annihilation region with a time and distance scale of half a pion Compton wave length - a reasonable size. It is not the purpose of this paper to make a careful study of the source function and to fit it to data. Rather we simply want to demonstrate that a quite reasonable choice for functional form and size yields a correspondingly reasonable account of the data. Thus we have not made an exhaustive parameter search. With our choice for parameters we find that (17) gives for the average pion number $\hat{N} = 6$. With the same choice of parameters, the average pion number calculated with four-momentum conservation imposed as in (50) gives $\hat{N} = 6.4$. This is certainly within $1/\hat{N}$ of the unconstrained value, as promised. Also $\hat{N}$ about 6 agrees with the data, for reviews see [12], [13], particularly when it is realized that we are only modeling
that part of annihilation that goes into uncorrelated pions, and not the part that goes into other mesons that subsequently decay into pions. Since these typically give few pions, the average number without them will be somewhat higher than the total average number.

Although the average number from the Poisson distribution and the distribution constrained by four-momentum conservation come out very close, the actual probability distributions in pion number are rather different. In Fig. 1 we show the probability of finding \( m \) pions as a function of \( m \) for the Poisson and for the “m-sum” cases (as defined in (47) and (48)). We see that the constrained case has a far narrower distribution than the Poisson. For the Poisson distribution the \( \sigma (\sigma^2 \text{is defined in (24)}) \) is \( \sigma = \sqrt{\hat{N}} = 2.45 \). For the distribution constrained by four-momentum conservation we find \( \sigma = 0.88 \), which is quite close to the value deduced [14] from experiment, \( \sigma = 1.02 \). This value has a statistical error of at least five percent and an additional unknown error from modeling the distribution of neutrals. In fact the entire pion multiplicity distribution from the constrained calculation shown in Fig. 1 is quite close to the experimental distribution [14], [13]. Our constrained distribution is also indistinguishable (on the scale of the figure) from a Gaussian distribution with the same average number and same variance. That is what a statistical model would give.

One can also use the combination of iso-spin projection and four momentum projection of Sect. 2.7 to calculate the pion number distributions. They are shown in Fig. 2a and 2b for \( I = 0 \) and \( I = 1 \), \( I_z = 0 \) respectively. Recall that for \( I = 0 \) only an even number of pions is possible while for \( I = 1 \) there is only an odd number. The data is an appropriate average over these two, and again strongly resembles our figures [14], [13]. Annihilation into vector mesons must be added to evade this odd-even effect.

Let us now turn to a comparison of our picture with experiment for the average number of pions by iso-spin type. We compare with the ratios reported in Sedlak and Simak [12]. To do this we need to estimate the relative population of \( I = 0 \) and \( I = 1 \) in proton antiproton annihilation at rest. We do so following the work of Locher and Zou [17]. Then using the charge ratios in states of good iso-spin we find \( \hat{N}_0/\hat{N}_+ = 1.53 \) with an error of about ten percent to be compared with the experimental number reported as \( 1.27\pm .14 \) [14], [12]. This is remarkable agreement. Note that these ratios do not depend at all on the details of our parameters, but do depend on the structure of the coherent state picture. In particular, it should be noted that the excess of \( \pi_0 \)'s found experimentally comes out naturally in our treatment. Given \( |f(k)|^2 \), we can calculate charge averages including four-momentum conservation as in Sect. 2.7. Using the same parameters, we can find the average pion number by charge type and the variance in the various iso-spin states. These are shown in Table 1. The \( I = 1, I_z = 1 \) case shows the effect of charge conservation, for \( I_z = 1 \), the average number of \( \pi^+ \) is one larger than the average number of \( \pi^- \). We also see that with four-momentum conservation the ratio of \( \pi^0 \) to \( \pi^+ \) in the \( I = 1, I_z = 0 \) case is somewhat larger than in the unrestricted case.

To make our formalism a real theory of the annihilation process we need to model annihilation in space and time as a classical source of pions. We have presented a very simple example as an orientation. More sophisticated models of the pion source could be made based on a dynamical theory of the annihilation process, for example the Skyrme model calculation [3], or on nucleon-antinucleon potential model calculations. These could include the spatial asymmetry seen in these annihilation calculations [3]. This asymmetry would
lead to a different momentum distribution for the pions. Alternately one could use (22) to extract the form of \( f(k) \) from the single pion momentum distribution data. We must also model the substantial part of annihilation that goes through meson resonances. We plan to return to these ideas in later work.

Given a form for \( f(k) \), one can calculate not just single pion averages, but also correlations \([15], [16]\) and joint momentum distributions for multiple pions. The great advantage of the pion coherent state approach is that it makes detailed, specific and rather restrictive predictions for these observables and thus is easily verifiable, or perhaps falsifiable.

**IV. FUTURE WORK AND SUMMARY**

Clearly much work remains to be done to develop and test the coherent state approach developed here for pions radiated from nucleon antinucleon annihilation. Predictions for higher moments of pion distributions from nucleon antinucleon states of sharp iso-spin have to be confronted with data. Correlations have to be studied. Theories of nucleon antinucleon interaction and of the development of the annihilation region have to be studied and interpreted in terms of a source of the pion radiation. It may be possible, for example, to model annihilation classically in the Skyrme model including the non-linear interactions of the pions, and the effects of the annihilation back on the source. Such a description would lead, far enough away from the annihilation region, to a free, non-interacting pion wave. This wave could then be the source of the coherent state. This approach is far simpler than a full quantum theory of the annihilation. Beyond this study of the annihilation process, we must study the effect of other quantum numbers such as G-parity and angular momentum on annihilation in the coherent state description. Finally a better understanding of the relation between annihilation into pion radiation and into other mesons is needed. We are exploring all these questions. Should these detailed descriptions fail, it would be interesting to go beyond the coherent state picture by including squeezed states.

One can also imagine other applications of the general methods advocated here. The basic idea is to look for any fast coherent processes in which the total energy radiated is large compared with the mass of the radiated quanta and in which the interaction among those quanta or by the quanta back on the source can be neglected. Then the radiation process is described classically, and the subsequent wave quantized using coherent states. Conserved quantum numbers have to be imposed on these states as we have imposed iso-spin. One can then explore averages, moments, momentum distributions and their relation to theradiating source, all as we did for annihilation. Greiner, Gong and Muller \([18]\) have discussed one such possible application in heavy ion collisions, there are many more. There may be similar opportunities in the description of hadronization in QCD jets \([19]\). Other examples will occur to the reader. It may even be possible to lift the restriction of no self interaction of the field or of the field back on the source so long as one can treat these classically and then use coherent states to quantize only in the radiation zone. Many QCD processes, like annihilation, may best be treated by starting in the classical non-perturbative domain of QCD. It may also be possible to extend these ideas to heavy ion collisions in general \([20]\), where one could take into account the statistical nature of the process by introducing a density matrix and averaging over an appropriate ensemble of coherent states. Such methods are well known in quantum optics \([3]\), and may be an attractive alternative to the
thermodynamic approach to heavy ion collisions.

In summary we have seen that the large energy released and the possibility of very short reaction time suggest that a classical, coherent pion wave pulse is radiated in nucleon antinucleon annihilation. This wave can be quantized using the method of coherent states. From this description averages and higher moments of pion distributions in annihilation can be calculated. In particular by imposing iso-spin and four-momentum conservation, interesting correlations among pion charge types are easily calculated. Momentum distributions contain information on the spatial and temporal distribution of the pion wave source. Preliminary calculations show that all these features obtained from the coherent state picture agree with the data and therefore suggest that the coherent classical pion pulse picture of annihilation is valid for that part of annihilation that goes directly into pions. We also speculate on other applications of classical meson waves and their subsequent quantization.

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Figure Captions

Figure 1. The probability distribution, $P_m$, for finding $m$ pions from nucleon antinucleon annihilation at rest. The open circles are the Poisson distribution and the solid squares are the distribution with the constraint of four-momentum conservation imposed. That distribution cannot be distinguished (on the scale of the figure) from a Gaussian of the same mean and variance.

Figure 2. The probability distribution, $P_m$, for finding $m$ pions from nucleon antinucleon annihilation at rest with iso-spin projection. In all cases the constraint of four-momentum conservation has been imposed. Figure 2a is for $I = 0$ and Figure 2b for $I = 1, I_z = 0$. The open circles are the probabilities with iso-spin projected and the solid squares without iso-spin projection. The solid squares are the same as the solid squares in Figure 1. Note that in our model, for $I = 0$ only an even number of pions is possible and for $I = 1$ only an odd number. This makes the iso-spin projected distributions appear peaked.
Table 1: The mean pion number and variance, and the mean pion number by charge type from nucleon antinucleon annihilation at rest in each of the three iso-spin states with the constraint of four-momentum conservation included.
| Iso-spin state | $\hat{N}_+$ | $\hat{N}_-$ | $\hat{N}_0$ | $\bar{N}$ | $\sigma$ |
|---------------|------------|------------|------------|--------|--------|
| $I = 0, I_z = 0$ | 2.07 | 2.07 | 2.07 | 6.21 | 0.82 |
| $I = 1, I_z = 0$ | 1.08 | 1.08 | 4.24 | 6.4 | 0.95 |
| $I = 1, I_z = 1$ | 3.15 | 2.15 | 1.08 | 6.38 | 0.95 |
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