Infrared Regularization of Yang–Mills Theories

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ABSTRACT

We introduce an infrared regulator in Yang–Mills theories under the form of a mass term for the nonabelian fields. We show that the resulting action, built in a covariant linear gauge, is multiplicatively renormalizable by proving the validity at all orders of the Slavnov identity defining the theory.
1 Introduction

It is a commonly accepted statement that the Higgs mechanism constitutes the only way to give masses to the gauge bosons in the framework of a local, renormalizable and unitary theory. On the other hand, the Higgs boson, i.e. the particle whose existence is entailed by the Yang-Mills (YM) theory, has not been discovered yet. This negative experimental fact renders meaningful all efforts towards alternative theories describing massive YM fields. The task is formidable, and despite the many trials attempted in the last thirty years, no alternative theory to the Yang–Mills–Higgs model has been proposed in order to give mass to the gauge bosons and to fulfill the three necessary requirements of

1. locality
2. renormalizability
3. unitarity.

In particular, the issue of unitarity for massive YM theory can be satisfied only with a spontaneous symmetry breaking mechanism. This old and somehow dormant subject is now knowing a kind of second youth. For instance, much discussion arose again on the old Curci-Ferrari (CF) model [1], due to some very recent contributions [2, 3]. In this note, we first recall the basic features of the CF theory and we explain the reasons why it cannot be considered as alternative to the Yang-Mills-Higgs model. Then, we propose a very simple way to give masses to the YM fields by means of a local and renormalizable model, without addressing the problem of its unitarity, our aim being that of giving a BRS invariant infrared regularization for the nonabelian fields. In fact a completely satisfactory regularization for YM theories at low momenta is not currently available; it is indeed known that giving masses to the YM vector bosons through the Higgs mechanism, may destroy the asymptotic freedom. This is basically due to the fact that the contribution of the scalars tends to make positive the slope of the beta function in YM theories. Together with the non–discovery of the Higgs particle, this fact constitutes another good reason for avoiding scalars altogether.

2 The Curci-Ferrari model

The most interesting local and renormalizable massive non-abelian gauge model not involving scalars is still represented by the CF model [1], whose classical action contains the mass term

\[ S_{\text{mass}}^{CF} = \int d^4x \left( \frac{m^2}{2} A^2 + \alpha m^2 \bar{\phi} \phi \right), \]  

(2.1)
where \((\bar{c}c)\) is the (anti)ghost and \(\alpha\) is the gauge parameter. Once the gauge is fixed, the propagator for the YM fields reads

\[
\Delta_{\mu\nu}^{ab} = \frac{\delta_{ab}}{k^2 - m^2} \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) + \frac{\alpha k_\mu k_\nu}{k^2} \frac{\delta_{ab}}{k^2 - \alpha m^2}.
\]  
(2.2)

An unphysical pole appears at \(k^2 = \alpha m^2\). The fact that, due to the particular mass term \((2.1)\), an identical pole is present in the ghost propagator, justifies the hope that the CF model could be unitary.

The gauge-fixing term of the CF action is

\[
S_{\text{CF}}^{GF} = \int d^4x \left( b^a \partial A^a + \bar{c}^a \partial^\mu (D_\mu c)^a + \frac{1}{2} \alpha b^a - \frac{1}{2} \alpha f^{abc} b^a c^b c^c - \frac{1}{8} \alpha f^{abc} c^b c^c f^{emn} c^m c^n \right),
\]  
(2.3)

where \(b(x)\) is the Lagrange multiplier usually introduced to enforce the gauge condition. The total CF action

\[
S^{\text{CF}} = -\frac{1}{4g^2} \int d^4x F^2 + S^{\text{CF}}_{gf} + S^{\text{CF}}_{\text{mass}}
\]  
(2.4)

is invariant under the set of field transformations

\[
\begin{align*}
\delta S^{\text{CF}} A^a_\mu &= - (D_\mu c)^a \\
\delta S^{\text{CF}} c^a &= \frac{1}{2} f^{abc} b^a c^b c^c \\
\delta S^{\text{CF}} \bar{c}^a &= b^a \\
\delta S^{\text{CF}} b^a &= -m^2 c^a.
\end{align*}
\]  
(2.5)

The CF model is affected by a few problems, here we would like to stress the most serious ones:

**Non-linearity of the gauge condition** The equation of motion of the Lagrange multiplier is nonlinear

\[
\frac{\delta S^{\text{CF}}}{\delta b^a} = \partial A^a + \alpha b^a - \frac{1}{2} \alpha f^{abc} c^b c^c
\]  
(2.6)

and therefore the gauge condition defined by \((2.6)\) holds only classically, since it cannot be implemented at the quantum level, contrary to what happens in the ordinary, massless, case. As a consequence, the multiplier cannot be eliminated from the quantum action, and an additional symmetry must be used to guarantee the renormalizability of the model \([4]\)

\[
\begin{align*}
\partial A^a_\mu &= \partial c^a \\
\partial b^a &= \frac{1}{2} f^{abc} c^b c^c \\
\partial c^a &= \delta c^a = 0
\end{align*}
\]  
(2.7)

However, the most unpleasant implication of the non linear gauge-fixing condition \((2.4)\) is that the hypersurface crossing the gauge orbits in order to choose a representative for each of them, is not defined for the quantum theory. This fact, which is related to the presence in the CF action of a quartic term in the ghosts, leads to a weakness of the very concept of gauge fixing for nonlinear choices like \((2.6)\).
Non-unitarity The CF model is invariant under the transformations (2.3), which, being non-nilpotent
\[(s^{CF})^2 = -m^2 \delta ,\]
cannot even be classified as of the BRS type. The lack of nilpotency of the pseudo-BRS operator (2.3) did not prevent from showing that the model is renormalizable by means of five multiplicative renormalization constants [4]. In general, the physical states coincide with the cohomology classes of the nilpotent BRS operator defining the theory. The definition of “physical space” for a theory described by a non-nilpotent BRS operator is not clear, and it is not at all surprising that the non-nilpotency of the symmetry describing the theory is the central point of the proofs on non-unitarity of the CF model [4, 5, 6, 7, 8, 2], the first of which was given by Curci and Ferrari themselves [5]. Later, Ojima [6] explicitly found a state with negative norm between the “would be” physical states of the CF model, thus undoubtedly concluding about the lack of unitarity. Ojima’s proof was very recently improved in [2], where a whole class of “physical states” having negative norm has been found.

Mass dependence Finally, whatever the observables of the CF model are, the control of their dependence on the parameter \(m^2\), considered as an infrared regulator and not as a physical mass, is difficult, and it has not been accomplished.

3 Infrared Regularized Theory

In this section we introduce a mass for the vector bosons, relaxing the condition of unitarity. In other words, here the mass must be considered as an infrared regulator, which eventually will be set equal to zero. The objective is to write a local action for massive YM fields, which is invariant under a set of nilpotent BRS transformations, whose cohomology classes – and hence the physical operators – are the same as in the massless case.

We start from the massless gauge-fixed YM action
\[S = -\frac{1}{4g^2} \int d^4x \, F^2 + s \int d^4x \left( \bar{c}^{a} \partial A^{a} + \frac{\alpha}{2} \bar{c}^{a} b^{a} \right),\]
which is invariant under the usual nilpotent BRS field transformations
\[
\begin{align*}
    sA^{a}_{\mu} &= -(D_{\mu}c)^{a} \\
    sc^{a} &= \frac{1}{2} f^{abc} c^{b} c^{c} \\
    s\bar{c}^{a} &= \bar{b}^{a} \\
    sb^{a} &= 0 .
\end{align*}
\]
Then, we introduce two external sources \(\{\sigma(x), \tau(x)\}\), organized in a BRS doublet
\[
\begin{align*}
    s\sigma &= \tau \\
    s\tau &= 0 ,
\end{align*}
\]
and we add to the action (3.1) the cocycle
\[ s \int d^4x \sigma \frac{A^2}{2}. \]  
(3.4)

Performing a shift in the source \( \tau(x) \)
\[ \tau(x) \rightarrow \hat{\tau}(x) \equiv \tau(x) + m^2, \]  
(3.5)

the classical action
\[ S_{mass} = -\frac{1}{4g^2} \int d^4x F^2 + s \int d^4x \left( \epsilon^a \partial A^a + \frac{\alpha}{2} \epsilon^a b^a + \sigma \frac{A^2}{2} \right) \]
\[ = \int d^4x \left( -\frac{1}{4g^2} F^2 + b^a \partial A^a + \epsilon^a \partial^\mu (D_\mu c)^a + \frac{\alpha}{2} b^2 + (\tau + m^2) \frac{A^2}{2} + \sigma A^a_\mu \partial^\mu c^a \right) \]  
(3.6)

acquires an infrared regulator under the form of a mass term for the YM fields.

The action \( S_{mass} \) is invariant under the nilpotent BRS transformations (3.2), enlarged by the doublet (3.3). Moreover, as in the massless case, the gauge condition is implemented by the following \textit{linear} field equation of the Lagrange multiplier
\[ \frac{\delta S_{mass}}{\delta b^a} = \partial A^a + \alpha b^a. \]  
(3.7)

In the Landau gauge, \textit{i.e.} for \( \alpha = 0 \), the action \( S_{mass} \) satisfies two additional constraints:
the ghost equation of the Landau gauge [9]
\[ \mathcal{F}^a S_{mass} = \int d^4x \left( \frac{\delta}{\delta c^a} + f^{abc} \frac{\delta}{\delta b^c} \right) S_{mass} = 0, \]  
(3.8)

and the following identity
\[ W S_{mass} = \int d^4x \left( \frac{\delta}{\delta \sigma} + c^a \frac{\delta}{\delta b^a} \right) S_{mass} = 0. \]  
(3.9)

The Slavnov identity describing the theory is
\[ S(\Sigma_m) = \int d^4x \left( \frac{\delta \Sigma_m}{\delta \Omega^a_\mu} \frac{\delta \Sigma_m}{\delta A^a_\mu} + \frac{\delta \Sigma_m}{\delta L^a} \frac{\delta \Sigma_m}{\delta c^a} + \epsilon^a \frac{\delta \Sigma_m}{\delta b^a} + \hat{\tau} \frac{\delta \Sigma_m}{\delta \sigma} \right) = 0. \]  
(3.10)

In (3.10), \( \Sigma_m \) is the classical action \( S_{mass} \), increased by a source term
\[ \Sigma_m = S_{mass} + S_{ext}, \]  
(3.11)

introduced to define the composite operators constituted by the nonlinear BRS transformations of the quantum fields
\[ S_{ext} = \int d^4x \left( \Omega^a_\mu s A^a_\mu + L^a s c^a \right), \]  
(3.12)

by means of two external sources \( \Omega(x) \) and \( L(x) \).
The proof of the renormalizability of the theory develops according to straightforward lines [10]. Because of the presence of the massive parameter $m^2$, one must distinguish between ultraviolet and infrared dimensions of the fields, according to the large- and small- momentum behaviour of the propagators. The result is listed in the table, together with the Faddeev-Popov assignments.

| $d_{uv}$ | $d_{ir}$ | $\Phi$ | $A^a_\mu$ | $\epsilon^a$ | $\bar{c}^a$ | $b^a$ | $\sigma$ | $\tau$ | $\Omega^{a\mu}$ | $L^a$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 2 | 2 | 2 | 2 | 3 | 4 | 2 | 2 | 3 | 4 |

Table Ultraviolet, infrared dimensions and Faddeev–Popov charges.

The stability of the theory can be checked by perturbing the classical action $\Sigma_m$ with an integrated functional $\Sigma_c$, which, according to the quantum action principle [11], is the most general one having $d_{uv} \leq 4$ and $d_{ir} \geq 4$. After imposing the identities defining the theory on the perturbed action $\Sigma_m + \varepsilon \Sigma_c$, we will show that the perturbation $\Sigma_c$ can be reabsorbed in $\Sigma_m$ by a number of redefinitions of the fields and parameters, which counts the renormalization constants.

As in the massless case [10], because of the linearity of the gauge condition (3.7), the functional $\Sigma_c$ does not depend on the multiplier $b(x)$. Moreover, it contains the antighost $\bar{c}(x)$ and the source $\Omega(x)$ only through the combination

$$\hat{\Omega}^{a\mu} = \Omega^{a\mu} + \partial^\mu \bar{c}^a.$$  \hspace{1cm} (3.13)

In the Landau gauge, the two additional symmetries of the classical action (3.8) and (3.9) imply that the ghost $c(x)$ and the source $\sigma(x)$ appear only differentiated. In addition, since the mass to the YM fields is provided by the spontaneous symmetry breaking in the direction of the external field $\tau(x)$, the set of constraints on $\Sigma_c$ is completed by

$$\left. \frac{\delta \Sigma_c}{\delta \tau(x)} \right|_{\psi=0} = 0 \ , \ \psi(x) = \text{all fields} \ ,$$  \hspace{1cm} (3.14)

and by the shift equation controlling the dependence of the classical action $\Sigma_m$ upon the massive parameter $m^2$

$$\left( \frac{\partial}{\partial m^2} - \int d^4x \frac{\delta}{\delta \tau(x)} \right) \Sigma_m = 0 \ .$$  \hspace{1cm} (3.15)

Finally, the Slavnov identity (3.10) imposed on the perturbed action, at first order in the infinitesimal parameter $\varepsilon$ translates into the following Slavnov condition

$$B_{\Sigma_m} \Sigma_c = 0 \ ,$$  \hspace{1cm} (3.16)

where $B_{\Sigma_m}$ is the linearized Slavnov operator

$$B_{\Sigma_m} = \int d^4x \left( \frac{\delta \Sigma_m}{\delta \hat{\Omega}^{a\mu}} \frac{\delta}{\delta A^a_\mu} + \frac{\delta \Sigma_m}{\delta A^a_\mu} \frac{\delta}{\delta \hat{\Omega}^{a\mu}} + \frac{\delta \Sigma_m}{\delta \Omega_a} \frac{\delta}{\delta \bar{c}^a} + \frac{\delta \Sigma_m}{\delta \sigma} \frac{\delta}{\delta \tau} + \hat{\tau} \frac{\delta}{\delta \sigma} \right) \ .$$  \hspace{1cm} (3.17)
which, by effect of (3.10), is nilpotent

\[(B_{\Sigma_m})^2 = 0 .\]  

(3.18)

The Slavnov condition (3.16) is easily solved once we remark that the only difference with respect to the massless case is the presence of the external sources \(\sigma\) and \(\hat{\tau}\), which appear in the Slavnov operator (3.17) as a BRS doublet, and consequently non altering the cohomological structure of the theory [10]. Therefore, the solution of the condition (3.16) can depend on the new fields \(\sigma(x)\) and \(\hat{\tau}(x)\) only through a trivial cocycle.

The most general solution, satisfying the whole set of constraints, is

\[\Sigma_c = Z_g \int d^4x \ F^2 + B_{\Sigma_m} \int d^4x \left( Z_A \hat{\Omega}^{a\mu} A^a_{\mu} + Z_c L^a \epsilon^a + Z_m \sigma A^2 \right) , \]  

(3.19)

where the four constants \((Z_g, Z_A, Z_c, Z_m)\) correspond respectively to renormalizations of the gauge coupling constant, the YM field, the ghost field and the parameter \(m^2\). We remark that in the Landau gauge, as a consequence of the symmetries (3.8) and (3.9), the ghost field and the mass parameter do not renormalize

\[Z_c = Z_m = 0 \quad \text{(Landau gauge)}. \]  

(3.20)

For what concerns the presence of anomalies, again the fact that the external fields \(\sigma(x)\) and \(\hat{\tau}(x)\) are BRS doublets, insures that, algebraically, the only anomaly is the ABJ one, which has a vanishing coefficient, since all the fields are in the adjoint representation of the gauge group. One may easily verify also the absence of infrared anomalies, \textit{i.e.} of counterterms having infrared dimension \(d_{\text{ir}} < 4\).

In the CF model, the correlation functions of physical observables depend upon the gauge parameter \(\alpha\), and it has been argued that the \(\alpha\)--independence could be recovered when \(m^2 = 0\) [3, 5]. In our case, the introduction of the mass parameter does not change the dependence of the classical action on the gauge parameter \(\alpha\), nor the fact that it doesn’t renormalize. Hence, the usual proof [12] of gauge independence of the theory based on the “extended” BRS symmetry, holds true. The argument of [12] consists in extending the classical identity

\[\frac{\partial \Sigma_m}{\partial \alpha} = B_{\Sigma_m} \int d^4x \frac{1}{2} \epsilon^a b^a \]  

(3.21)

to all orders of perturbation theory

\[\frac{\partial \Gamma}{\partial \alpha} = B_\Gamma (\Delta \cdot \Gamma) , \]  

(3.22)

where \(\Gamma\) is the quantum vertex functional and \(\Delta \cdot \Gamma\) is a quantum insertion, by exploiting the non-renormalization properties of the parameter \(\alpha\), or, in other words, the linearity of the gauge condition (3.7). The quantum relation (3.22) states the non-physical character of the gauge parameter, from which follows that the correlators of the physical observables
are independent from $\alpha$. The nature of the parameter $m^2$ is different, since even classically it does not satisfy an identity analogous to (3.21). We can only say that the physical observables do not depend from the shifted external field $\hat{\tau}(x)$, since the cohomology of the linearized Slavnov operator is independent from it. Nevertheless a $m^2$ dependence of the Green functions of the physical operators cannot be excluded; indeed Eq. (3.15), which is to become the Callan-Symanzik equation in the quantized theory, controls the mass dependence of amplitudes as due to radiative corrections.

4 Conclusions

We provided the YM fields of a mass $m^2$, through the spontaneous symmetry breaking in the direction of an external source, in the framework of a local and renormalizable theory. In absence of unitarity, the mass we introduced must be considered an infrared regulator, and not a physical mass. Unlike what happens in the CF model, we are able to write a true Slavnov identity, with a linear gauge choice. The physical observables do not depend on the gauge parameter $\alpha$. Moreover, in the Landau gauge the mass $m^2$ does not renormalize. A precise analysis needs a choice of a renormalization procedure; we can only comment that in the dimensional scheme the counterterms are polynomials in any dimensional parameter and therefore the effective Lagrangian possesses a smooth zero mass limit. For this reason, and since the external field whose shift gives mass to the gauge fields does not appear in the cohomology space, we expect that the zero mass limit of the massive model (modulo gauge fixing delicacies) should go smoothly to the usual massless Yang-Mills theory [13].

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