IMPLICATIONS OF RUNNING NEUTRINO PARAMETERS 
FOR LEPTOGENESIS AND FOR 
TESTING MODEL PREDICTIONS

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The running of neutrino parameters in see-saw models and its implications for leptogenesis and for testing predictions of mass models with future precision experiments are discussed using analytical approximations as well as numerical results.

1. Introduction

Fermion mass models and the leptogenesis\(^1\) mechanism for explaining the baryon asymmetry of our universe typically operate at high energies close to the unification scale \(M_U\) or at the see-saw scales, i.e. the masses the heavy right-handed neutrinos. Our knowledge about the neutrino masses and mixings on the other hand mainly stems from experiments on neutrino oscillations, performed at low energy. In order to compare the high-energy predictions with the low energy experimental data, the renormalization group (RG) running of the relevant quantities has to be taken into account. In see-saw models for neutrino masses (type I and type II), this requires solving the RGEs\(^2\)\(^-\)\(^7\) for the effective neutrino mass matrix for the various effective theories which arise from successively integrating out the heavy degrees of freedom, in particular the heavy right-handed neutrinos.

2. Implications for Leptogenesis

For the leptogenesis mechanism, the relevant scale is the mass \(M_1\) of the lightest right-handed neutrino, or, in the type II case, possibly also the mass scale \(M_\Delta\) of the lightest SU(2)\(_L\)-triplet. In the energy range between the leptogenesis scale and the electroweak scale \(M_{EW}\), we can consider the running of the effective neutrino mass operator, which is produced from integrating out the heavy right-handed neutrinos and/or triplets.
2.1. Enhancement of the Decay Asymmetries

The decay asymmetries\(^8,10,9\) for type I leptogenesis as well as for type II leptogenesis (via the lightest right-handed neutrino) can be written as\(\varepsilon_1 \sim -\frac{M_1}{v_{\text{EW}}} \langle m^\text{BAU}_{\text{eff}} \rangle\), where \(\langle m^\text{BAU}_{\text{eff}} \rangle \equiv \frac{1}{(Y^\nu)^2} \sum_i \text{Im} \left[(Y^\nu)^*_{if}(Y^\nu)^*_{ig}(m_{\nu})_{fg}\right]\) is an effective mass for leptogenesis. \(Y^\nu\) is the neutrino Yukawa matrix and we have considered the case of hierarchical right-handed neutrino masses and \(M_\Delta \gg M_1\). In the SM or for a moderate tan\(\beta\) in the MSSM, the RG running from high to low energy leads mainly to a scaling of the neutrino mass matrix \(m^\nu\) (see Fig. 1). Including the RG effects thus leads to an enhancement of the decay asymmetry for leptogenesis\(^{11,12}\) by a factor of roughly 20% in the MSSM and 30% - 50% in the SM.\(^{12}\)

![Figure 1. Scaling of the neutrino mass eigenvalues \(m_i\) from the RG evolution of the effective neutrino mass matrix in the MSSM (Fig. 1(a)) and in the SM (Fig. 1(b)).\(^{12}\)

In the MSSM with large tan\(\beta\), we have shown the evolution of \(m_3\) for a normal mass ordering and with CP phases set to 0. For the plots, \(m_t = 178\ \text{GeV}\) has been used.

2.2. Correction to the Bound on the Neutrino Mass Scale

From the requirement of successful thermal type I leptogenesis, a bound on the absolute neutrino mass scale can be derived.\(^{13}\) Among the significant corrections to this bound, included in recent calculations, are the effects from RG running between low energy and \(M_1\). With an increased decay asymmetry \(\varepsilon_1\) as discussed in section 2.1, the produced baryon asymmetry increases as well. However, scattering processes which tend to wash out the baryon asymmetry are enhanced as well, which typically over-compensates the correction to the bound from the enhanced decay asymmetry and makes the bound on the neutrino mass scale more restrictive.\(^{12,14,15}\)
2.3. Implications for Resonant Leptogenesis

Resonant leptogenesis\textsuperscript{16} relies on a small splitting between the masses of the lightest right-handed neutrinos, $M_1$ and $M_2$, of the order of their decay widths. Given a model for neutrino masses with such a small mass splitting defined at $M_U$, it can be affected significantly by the RG evolution of the mass matrix of the heavy right-handed neutrinos from $M_U$ to $M_1 \approx M_2$. On the other hand, one can also have exactly degenerate masses $M_1 = M_2$ at high energy and generate the required mass splitting radiatively\textsuperscript{17}.

3. Implications for Testing Model Predictions by Future Precision Experiments

Future reactor and long-baseline experiments have the potential to measure the neutrino mixing angles to a high precision. For testing the predictions of mass models using such precision measurements, the RG corrections will be important, in particular, if the observed mixing angles turn out to be close to theoretically especially interesting values. For conservative estimates of the RG effects, analytical formulae\textsuperscript{19,20,12} for the running of the neutrino parameters below the see-saw scales can be used. For an accurate determination of the RG effects in specific models, the model-dependent running above and between the see-saw scales can contribute significantly and often even dominates the RG effects\textsuperscript{18,6}. Formulae which allow an analytic understanding of the running above the see-saw scales are in preparation\textsuperscript{21}.

3.1. Radiative Generation of $\theta_{13}$

One important parameter is the mixing angle $\theta_{13}$. The knowledge of its value will allow to discriminate between many fermion mass models and furthermore, only if $\theta_{13}$ is not too small, future experiments on neutrino oscillations have the potential to measure leptonic CP violation. Do we expect $\theta_{13}$ very close to zero at low energy? Even if $\theta_{13} = 0$ is predicted by some model at high energy, RG running will in general generate $\theta_{13} \neq 0$ at low energy. From a conservative estimate using the analytical formulae below the see-saw scales, it has been shown\textsuperscript{12} that the RG corrections are often comparable to, or even exceed the expected sensitivity of future experiments. Note that for $\theta_{13}$, small values of CP phases (as predicted e.g. by certain type II see-saw models\textsuperscript{22}) can protect against large RG corrections\textsuperscript{12}. Radiative generation of $\theta_{13}$ from running above the see-saw scales has been analyzed in Ref. 23.
3.2. Modification of Complementarity Relations for $\theta_{12}$

With the present neutrino data, complementarity relations$^{24-27}$ such as $\theta_{12} + \theta_C = \pi/4$ (with $\theta_C$ being the Cabibbo angle) are allowed and will be tested by future experiments to a high accuracy. RG corrections can lead to significant modifications of such relations for $\theta_{12}$.$^{26}$

3.3. Corrections to Maximal Mixing $\theta_{23}$

The present best-fit value for $\theta_{23}$ is close to maximal. Typically, mass models predict a deviation of $\theta_{23}$ from maximality, which is within reach of future long baseline experiments.$^{28}$ If $\theta_{23}$ turns out to be close to maximal, this would point towards a symmetry which fixes maximal mixing at high energy $M_U$. However, even if $\theta_{23} = \pi/4$ is predicted by some model at high energy, RG corrections from the running between $M_U$ and low energy generate a deviation of the low energy value for $\theta_{23}$ from maximality.$^{12,28}$ In many cases, even for hierarchical neutrino masses, this deviation is comparable to, or exceeds the sensitivity of future experiments (see Fig. 2).

![Figure 2](image-url)

(a) Experimental sensitivity

(b) RG corrections

Figure 2. Fig. 2(a) shows the expected sensitivity of future long-baseline experiments (combined MINOS, ICARUS, OPERA, JPARC-SK and MuMI; see Ref. 28 for details) for excluding maximal mixing $\sin^2 \theta_{23} = 0.5$ in 10 years at $1\sigma$, $2\sigma$ and $3\sigma$ (from light to dark shading). The dashed line shows the currently allowed region for $\theta_{23}$ at $3\sigma$. Fig. 2(b) (from Ref. 28; see also Ref. 12 for details) shows a conservative estimate (ignoring $Y_\nu$-effects) for the RG corrections to maximal $\theta_{23}$ from the running between $M_U$ and $M_{EW}$ in the MSSM. The contour lines correspond to $\Delta \sin^2 \theta_{23} = 0.02, 0.05, 0.08$ and 0.1.
4. Summary and Conclusions

Facing the high expected sensitivities of future experiments on the neutrino parameters, RG corrections are increasingly relevant for testing predictions of mass models. They are particularly important, if the neutrino mass spectrum turns out to be non-hierarchical or if experiments find the lepton mixing angles close to specific values such as 0 for $\theta_{13}$, $\pi/4$ for $\theta_{23}$ or compatible with complementarity relations such as $\theta_{12} + \theta_C = \pi/4$ (with $\theta_C$ being the Cabibbo angle) for $\theta_{12}$. For leptogenesis, the scaling of the neutrino masses by RG effects enhances the decay asymmetries for type I/II leptogenesis, effects washout parameters and finally lowers the bound on the absolute neutrino mass scale from the requirement of successful thermal type I leptogenesis.

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