Rapid thermalization by adaptive flow reorientation

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Abstract. Aim of this paper is the enhancement of scalar transport (heat, chemical species) in flow systems with reorientations of a laminar base flow. Conventional heating/mixing protocols comprise of temporal or spatial periodic reorientations of these base flows to promote fluid mixing. However, thermal homogenisation rates of scalar fields are not necessarily accelerated with these approaches due to the substantial effect of diffusion and/or chemical reactions on heat/chemical transport. In the present study we numerically study heat transport with an adaptive approach for an entire parameter space of fluid and flow properties. Key to the approach is real-time control of the fluid flow based on the scalar field due to an efficient numerical model. Results show that the adaptive approach can significantly enhance heat transport over the conventional periodic heating/mixing approach designed for efficient mixing.

1. Introduction
Efficient transport of scalar quantities (heat, chemical species) in laminar flows is key to a wide range of industrial applications including compact equipment for process intensification and micro-fluidic devices as well as enhanced subsurface flows for resource extraction or groundwater remediation. Applications in these fields would greatly benefit from enhanced scalar transport. In traditional approaches, spatial or temporal periodic reorientations of flow fields are designed to accelerate scalar field homogenisation. These periodic approaches redistribute heat/chemicals throughout a fluid through protocols designed for effective fluid mixing [1]. However, boundary heating requires sufficiently large temperature gradients at the circumference to facilitate rapid heating. Furthermore, fluid mixing may not be the best approach to achieve heating, in particular in case of significant diffusion/chemical reactions. Therefore, recent efforts have focussed on mechanisms to produce aperiodic or controlled reorientation schemes to accelerate scalar homogenisation [2]. In [3] we presented an adaptive approach for “optimal” selection of the flow field reorientation such that boundary heat-transfer was enhanced. However, we investigated improvements in heating rate only for a single case of fluid and flow properties. This work extends the analysis of the adaptive approach by studying heat transport for a range of practically relevant fluid and flow properties. In Sec. 2, we briefly describe the model of a Rotated Arc Mixer (RAM) that illustrates the system of interest for studying heat transport. This model is then used to study boundary heating enhancement with the adaptive approach over the conventional periodic approach in Sec. 3. Finally, conclusions and recommendations for future research are presented in Sec. 4.
2. Methodology

The Rotated Arc Mixer (RAM) following [3] is used to study the effect of system parameters on fluid heating. The RAM consists of a circular domain $D = \{(r, \theta) \in \mathbb{R}^2 | r \leq R, -\pi \leq \theta \leq \pi\}$ of radius $R$ enclosed by circumference $\Gamma$ which contains a fluid. Heat-transfer from the hot boundary $\Gamma$ is governed by the energy conservation law, which for the flow conditions in the RAM reduces to the dimensionless advection-diffusion equation of the form [3]

$$\frac{\partial \tilde{T}}{\partial \bar{t}} = -\mathbf{v}(\tilde{x}) \cdot \nabla \tilde{T} + \frac{1}{P \varepsilon} \nabla^2 \tilde{T}, \quad T(\tilde{x}, 0) = T_0, \quad \text{and} \quad T(\tilde{x}, \bar{t})|_\Gamma = T_\infty > T_0,$$  

(1)

with $\tilde{T} = T_\infty - T$ the transient temperature field where $T_0 = 0$ and $T_\infty = 1$ such that $0 \leq T \leq 1$, $P \varepsilon = UR/\alpha$ the Péclet number, $U$ the characteristic velocity and $\alpha$ the thermal diffusivity. The studied flow field consists of Stokes flows that differ by only a reorientation according to

$$\mathbf{v}(\tilde{x}) (r, \theta) = \mathbf{v}_1 (r, \theta + (\tilde{t} - 1) \Theta) = \mathbf{v}_1 \left( \mathcal{R}(\tilde{t}, \Theta) \right),$$  

(2)

where $u(\tilde{t})$ denotes the “orientation scheme” that activates aperture $u_n \in \mathcal{U} = \{u_0, u_1, \ldots\}$, which induces fluid motion during time interval $n\tau \leq \tilde{t} \leq (n + 1)\tau$. Aperture activations are considered that induce clock-wise circulating Stokes flow through viscous drag at $\Gamma$. In conventional periodic protocols, the set $\mathcal{U}$ consists of sequential activations that are optimized towards mixing by adjusting the apertures’ activation time ($\tau$) [5]. However, an adaptive protocol that optimizes the aperture activation sequence based on the future transient field $\tilde{T}$ results in schemes that can significantly improve heating rates [3]. Key enabler for this adaptive protocol is a spectral decomposition of (1) for a steady Stokes flow $k$ following

$$\tilde{T}(\tilde{x}, \tilde{t}) = \sum_{m=0}^{M} \chi^{(k)}(k) \Phi_m^{(k)}(\tilde{x}) e^{\lambda_m \tilde{t}} = \sum_{m=0}^{M} \chi^{(k)}(k) \Phi_m^{(1)}(\mathcal{R}_k(\tilde{x})) e^{\lambda_m \tilde{t}}, \quad \tilde{T}(\tilde{x}, 0) = \sum_{m=0}^{M} \chi^{(k)}(k) \Phi_m^{(k)}(\tilde{x}).$$  

(3)

with $(\Phi_m(\tilde{x}), \lambda_m)$ an eigenfunction-eigenvalue pair of (1). Reorientation of aperture-wise Stokes flows following (2) carries over to eigenfunctions $\Phi_m^{(k)}(\tilde{x})$ and thus admits a spectral decomposition for aperture $k$ from that of the base flow (aperture $k = 1$) [3]. This enables sufficiently fast predictions of the temperature evolution for each aperture $k$ to allow real-time optimization of $\int_D \tilde{T}^2(\tilde{x}, \tilde{t}) d^2\tilde{x} \approx \|\tilde{T}(\tilde{x}, \tilde{t})\|$ in closed loop as shown in Fig. 1. Note that the fluid is only heated by a hot boundary.

![Figure 1: The closed loop for adaptive flow reorientation in the Rotated Arc Mixer (RAM).](image)

3. Performance investigation

Here, fluid heating with each protocol is investigated for a range of Péclet numbers where both advection and diffusion significantly impact heat transport. The effectiveness in producing homogeneity is defined by the non-dimensional time steps ($t_e$) required for the transient field to be within a certain threshold $\varepsilon$, i.e. $\|\tilde{T}(\tilde{x}, t_e)\| \leq \varepsilon$. Homogenisation rates for each protocol are investigated by comparing the transient times of orientation schemes to reach $\varepsilon$ as

$$\chi(P \varepsilon, \tau) = \frac{t_{e,a}(P \varepsilon, \tau)}{t_{e,a}(P \varepsilon, \tau)},$$  

(4)
Figure 2: (a) Homogenisation times ($t_{\varepsilon,a}$) for the closed loop Fig. 1, (b) homogenisation ratios ($\chi$) and (c) a zoom of homogenisation ratios for various Péclet numbers and activation times for both the current parameters (circle) and the parameters (cross) from [3].

where subscripts “a” and “p” indicate time till homogenisation with either the adaptive or the periodic protocol, respectively. Ratios (4) exceeding unity ($\chi > 1$) denote accelerated thermal homogenisation rates for the adaptive protocol that are larger than the conventional periodic one. The homogenisation rates and protocol performance for Péclet numbers ($500 \leq Pe \leq 4000$) and activation times ($10^0 \leq \tau \leq 10^{1.5}$) of interest are shown in Fig. 2. Heating is significantly promoted with schemes from the adaptive protocol compared to those from the periodic protocol as activation time decreases in Fig. 2b. Clearly visible in Fig. 2b is a substantial region where the adaptive protocol significantly outperforms the periodic protocol and can reach up to $\chi > 3.2$ for high Péclet numbers and small aperture activation times. A 'transition zone' can be observed in Fig. 2b beyond which performance with the adaptive protocol shows a drastic improvement in heating rates compared to the periodic protocol. Note that heating rates for the periodic scheme match results presented in [5].

Figure 3: Comparison of the transient field in advection dominated transport for orientation schemes obtained with a periodic and an adaptive protocol for $Pe = 2249$ and $\tau = 3.8$.

insufficient to promote effective heat transport by advection with a periodic scheme. Mixing studies on the RAM with $\Theta = 2\pi/3$ show the transition in Fig. 2b marks a region of effective mixing with schemes from the periodic approach as $\tau$ increases [4]. Therefore, the region after $\tau \approx 3.2$ ($\log_{10}(3.2) \approx 0.5$) in Fig. 2b is magnified in Fig. 2c. A close examination of the zoom in Fig. 2c reveals that heating rates are significantly promoted around the zone with the adaptive protocol. Fig. 3 shows, for example, that heating rate is improved by 50% ($\chi = 2$) for $Pe = 2249$.
at $\tau = 3.8$ and by 24% ($\chi = 1.32$) for $Pe = 10^3$ at $\tau = 5$, respectively. We investigate these points (see Fig. 2c) further for any associated characteristics by temporal snapshots in Fig. 4. Snapshots for the periodic scheme (top row) in Fig. 4 reveal a "pattern" that reorients with the activated aperture. Heat transport seems to be confined at the circumference. Snapshots for the adaptive scheme (middle-bottom row) show improved heat dispersion throughout the domain such that heat transfer from the boundary is promoted.

4. Conclusions

Heat transfer into an initially cold fluid from a hot boundary is influenced by both fluid and flow parameters. Previous work showed that flow field reorientations obtained with an adaptive approach can significantly improve this heat transfer compared to a periodic approach for a single parameter case. This work shows that the adaptive approach yields a systematic improvement over a large span of these parameters. Future efforts are directed towards experimental investigations of the adaptive approach, determination of the spectral decomposition from CFD/experimental data using e.g. Dynamic Mode Decomposition (DMD), dedicated observer design for transient field estimation in experiments and control synthesis for further enhancement of scalar transport.

References

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