A Rate-Dependent Effective-Temperature Model of Shear Band Formation During Flow

Adam R. Hinkle† and Michael L. Falk†‡§∗

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Recent shear experiments of carbopol gels have revealed the formation of a transient shear band before reaching the steady state, which is characterized by homogeneous flow. Analysis of this phenomenon using a time-evolving effective temperature in the shear transformation zone (STZ) theory predicts that the observed fluidization proceeds via two distinct processes. A shear band nucleates and gradually broadens via disordering at the interface of the band. Simultaneously, spatially homogeneous fluidization is induced outside of the shear band where the disorder of the gel grows uniformly. Experimental data are used to determine the physical parameters of the theory, and direct, quantitative comparison is made to measurements of the structural evolution of the gel.

1 Introduction

Yield stress fluids (YSFs) are ubiquitous in everyday life, and their special properties have merited intense research1–9. Examples include gels, clay suspensions, foams, concentrated emulsions, and colloids. These seemingly distinct substances exhibit a similar mechanical response when subjected to shear deformation: Below a critical (yield) stress these materials remain elastic, behaving as solids, but above this critical stress they are able to deform and flow as viscous liquids. This characteristic ability makes them extremely sought-after for many applications. Recent experiments have reported the formation of regions of highly localized strain, known as shear bands, in simple YSFs11–14, where “simple” implies that memory and aging effects are thought not to play a significant role in the mechanical behavior of these materials. Simple YSFs have been generally believed to transition from the solid to the liquid state uniformly as a homogeneous system2,3. Recent Couette-cell experiments of shear in a carbopol gel have instead revealed the formation of transient shear bands before the gel reaches a steady state characterized by a linear velocity profile and Herschel-Bulkley rheology11.

The onset of plastic flow in YSFs and other similar materials has been investigated by a number of theoretical approaches, but direct, quantitative comparison has been lacking with regard to shear banding and the process of fluidization. Among these theoretical frameworks are soft glassy rheology (SGR)15 and mode coupling theory (MCT)16–18. The SGR model in particular has been successful in qualitatively describing the general steady-state power-law behaviors seen in experiments, and criteria have been proposed to characterize the onset of shear banding19. MCT has been limited to very simple theoretical investigations of systems involving monodispersed colloidal glasses, and has not been able to predict aging, which is well captured by SGR. One drawback of SGR is that it assumes the existence of a “noise temperature” that controls activation rates of plastic processes. The physical basis for this noise temperature is however not clear, and its dynamics have not been derived from fundamental or mesoscale principles.

A minimal theoretical model of a Newtonian fluid using the Krieger-Dougherty constitutive relation20 has been used to qualitatively model transient shear banding in a Couette-cell geometry. The shortcomings of this approach are that it does not describe a yield stress or solid-state elastic response, nor does it quantitatively capture key experimental observations of shear banding in YSFs, i.e. the correct spatial and temporal evolution of the shear band and fluidization times of the system. More importantly, this model attributes transient shear banding to the nature of the Couette-cell apparatus itself. Specifically it suggests that the gradient in the stress field, which exists simply due to a cylindrical geometry, leads to a higher shear rate and unjamming of material near the inner cylinder of the Couette cell.

In this paper we propose an alternative description of the phenomenon of transient shear banding based on the microscopic structure of the gel. We adapt the effective-temperature hypothesis of the shear transformation zone (STZ) theory as a rheological model for shear localization in a simple YSF.
The Eulerian rate-of-deformation tensor is equal to the Lie time-derivative of the form, which competes with the STZ-transition rates, and where we STZ dynamics. For a monotonically loaded, athermal system of-deformation tensor plastic-strain rate as a matter of convention.

The STZ theory is a general framework for characterizing plasticity in amorphous materials, and provides a continuum-level mean-field approximation for flow based on an assumption of local rearrangements of a material’s structure via the activation of STZs. STZs are orientational point defects that mediate plastic flow by accommodating rearrangement. In this paper we use data from the aforementioned experiments to determine the physical parameters of the theory and make precise experimental connections. A distinguishing feature of the STZ theory is that it is based on a specific model of molecular rearrangements, which have been observed directly in numerical simulations and analog experiments. The present formulation of the STZ theory has been extended from the original work of Argon and his proposal of “shear transformations” to explain plastic deformation in metallic glasses, as well as from the free-volume and flow-defect theories of Turnbull, Cohen, Spaepen, and others. The STZs themselves have been postulated to have internal degrees-of-freedom. The STZs not only transform from one orientation to another; they are created and annihilated during configurational fluctuations at a rate that is proportional to the rate of energy-dissipation per STZ.

A flow rule for the plastic component of the Eulerian rate-of-deformation tensor $D_{ij}^{pl}$, which we will subsequently call the plastic-strain rate as a matter of convention, follows from the STZ dynamics. For a monotonically loaded, athermal system where there are no rate-dependent processes such as aging, which compete with the STZ-transition rates, and where we assume there to be a low STZ density, the flow rule can take the form,

$$D_{ij}^{pl} = f_{ij}(x)e^{-1/\chi},$$

where $f_{ij}$ is a monotonic tensor-function of the deviatoric Cauchy stress $s_{ij}$. One critical way the STZ theory differs from its predecessors (and indeed other theories) is through the introduction of the quantity $\chi$ and its relationship to an “effective temperature” $T_{eff}$ that is defined as

$$T_{eff} = \frac{\partial U_c}{\partial S_c},$$

where $U_c$ and $S_c$ are the material’s potential energy and entropy respectively of only the structural degrees-of-freedom. This is to be distinguished from the usual thermodynamic temperature $T$ which accounts for equilibrium systems in which the degrees-of-freedom relax on timescales short compared to the observation time. The typical definition of $T$ is applicable to the fast, i.e. vibrational degrees-of-freedom, but the structural degrees-of-freedom may be out of equilibrium. In the effective-temperature STZ formalism, the dimensionless scalar field $\chi$ is defined as $\chi = k_B T_{eff}/E_z$, where $k_B$ is the Boltzmann factor. Here $E_z$ is a typical energy required to create an STZ. In the athermal limit the dynamical equation for the dimensionless effective temperature $\chi$ takes the form

$$\dot{\chi} = \frac{s_{ij}D_{ij}^{pl}}{c_{eff}} (\chi - \chi_e) + D_{\chi} \frac{\partial^2 \chi}{\partial x_i \partial x_j},$$

using Cartesian tensors and the Einstein summation convention.

The first term on the RHS in Eq. (3) represents a source of plastic work per unit time that does mechanical work on the structural degrees-of-freedom. The parameter $c_{eff}$ is the volumetric effective-heat capacity with dimensions of energy per unit volume, determining the energy input per unit increment of dimensionless effective temperature. In flowing regions $\chi$ converges to a limiting value $\chi_e$, which represents the steady-state dimensionless effective temperature where the work done to shear the structure no longer causes an increase in disorder. Simulations and theoretical investigations have indicated that in the most general case, $\chi_e$ is rate dependent.

As we show later this will allow regions of the material to evolve towards different steady-state effective temperatures depending upon the applied strain-rate. The final term in Eq. (3) describes the diffusion of the dimensionless effective temperature through an effective thermal diffusivity $D_{\chi} = k_{eff}/c_{eff}$ with dimensions length-squared per unit time, where $k_{eff}$ is the effective thermal conductivity, which is only non-zero in the presence of shear. In its rate-dependent form, $D_{\chi}$ is typically taken to have the explicit form $D_{\chi} = l^2 \sqrt{D_{mn}^\text{pl} D_{mn}^\text{pl}}$ where $l$ introduces a lengthscale that is approximately the size of an STZ (on the order of a molecular lengthscale).

The effective-temperature theory for Couette-cell shear is established in Sec. 2. Section 3 presents the results of this effective-temperature model and makes direct and quantitative comparisons with recent experimental results. We conclude in Sec. 4 with a discussion of these results and their implications for understanding transients during flow in YSFs, as well as for direction for further investigation using the effective-temperature hypothesis as a rheological model.
2 Basic Theory

In this section we adapt the effective-temperature description in the STZ theory for a Couette-cell system, whose geometry is described in Fig. 1. A similar application for an idealized infinite strip has been used, while the general, tensorial framework of the STZ theory has also been presented. The symmetry of the Couette cell allows us to treat the problem as an effectively one-dimensional Cartesian system, and we further assume that the gap is sufficiently small enough that the variation in the stress along the radial direction is negligible, which is consistent with experimental findings. A key experimental observation of the rheology of simple YSFs is that the steady-state behavior of the system is described by the well-known Herschel-Bulkley (HB) relation, and this will be used to determine the steady-state flow rule for $D_{ij}^{pl}$.

2.1 Plastic-Strain Rate

The one-dimensional plastic-strain rate $\dot{\varepsilon}^{pl}$ from the STZ theory takes the form

$$\dot{\varepsilon}^{pl} = f(s) e^{-1/\chi}.$$ \hfill (4)

We next assume that the steady-state behavior of the gel is given by the HB relation, namely $s = s_c + A (\varepsilon^{pl})^n$ for some parameters $n$ and $A$ which characterize a specific system that begins to unjam and flow plastically when the deviatoric stress $s$ reaches $s_c$, the critical stress. Here the HB coefficient $A$ has units of Pa·$s^n$, and sets the timescale for the fluidization of the gel. The plastic-strain rate is related to the measure of disordering by a function $f = f(s)$. We require that Eq. 4, the form of the constitutive law in the STZ theory, reduce to the HB relation when $\chi = \chi_\infty$. Therefore $\dot{\varepsilon}^{pl}$ in the steady state $\varepsilon_\infty^{pl}$ must be

$$\varepsilon_\infty^{pl} = \left( \frac{s - s_c}{A} \right)^{1/n}.$$ \hfill (5)

This allows $f$ to be determined so that

$$\varepsilon_\infty^{pl} = f(s) e^{-1/\chi_\infty} = \left( \frac{s - s_c}{A} \right)^{1/n}$$ \hfill (6)

and we find

$$f(s) = \left( \frac{s - s_c}{A} \right)^{1/n} e^{1/\chi_\infty}. \hfill (7)$$

Upon substituting the expression for $f$ into Eq. 4, we arrive at the expression for the plastic-strain rate, namely

$$\dot{\varepsilon}^{pl} = \left( \frac{s - s_c}{A} \right)^{1/n} e^{1/\chi - 1/\chi}.$$ \hfill (8)

This result provides a continuum constitutive-law relating the plastic-strain rate to the deviatoric stress within the framework of an effective temperature. Note that the HB relation for steady-state flow is recovered when the system has evolved to a fully disordered state, i.e. $\chi = \chi_\infty$.

2.2 Stress Rate

To derive the differential equation for the deviatoric-stress rate we express the strain rate as the sum of the elastic and plastic components, namely

$$\dot{\varepsilon} = \dot{\varepsilon}^{el} + \dot{\varepsilon}^{pl}. \hfill (9)$$

The velocity $v = v(y)$ as a function along the Couette-cell radius $y$ in Fig. 1 is then given by

$$v(y) = \int \dot{\varepsilon} dy = \int \dot{\varepsilon}^{el} dy + \int \dot{\varepsilon}^{pl} dy \hfill (10)$$

and when integrated across the full gap-width, one finds

$$v_o = \frac{\dot{w}}{\mu} + \left( \frac{s - s_c}{A} \right)^{1/n} \int_{R_1}^{R_o} e^{1/\chi} \, dy \hfill (11)$$

where $\mu$ is the elastic shear modulus (with dimensions of stress), and $\bar{\gamma} = v_o/w$ is the average strain-rate across the gap found by imposing a velocity $v_o$ at the inner cylinder. We have assumed that the elastic response is linear and that all nonlinear behavior results from the plastic response. Rearranging we find that

$$\dot{s} = \mu \bar{\gamma} - \mu \left( \frac{s - s_c}{A} \right)^{1/n} \int_{R_1}^{R_o} e^{1/\chi} \, dy. \hfill (12)$$

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Fig. 1 The edge of the inner Couette-cell cylinder ($R_1 = 23.9$ mm) and outer cylinder edge ($R_o$) with gel (white) where the gap width $w = R_o - R_1 = 1.1$ mm. A constant strain-rate $\dot{\gamma} = 0.7$ s$^{-1}$ is applied to the inner cylinder causing the nucleation of a shear band at $R_1$ which broadens outward in the $y$-direction.
2.3 Rate-Dependent Effective Temperature

Central to the STZ theory is the notion that the local plastic-strain rate depends on the changing state of the amorphous structure, which is described by \( \dot{\chi} = \chi(y, t) \). At any time, \( \dot{\chi} \) quantifies the disordering of the material as it is strained or stirred slowly. In the athermal limit, and if \( D_\chi \) is a function of \( \dot{\varepsilon}^{pl} \), the time-evolution \( \dot{\chi} \) will be nonzero only if a state of stress greater than a critical stress is present. Stresses above the critical stress induce STZ rearrangements which cause \( \chi \) to evolve towards its steady-state value.

In our present model the steady state is determined by the two terms on the RHS of Eq. 3, a source term \( s_i D_{ij}^{pl} (\chi_\infty - \chi) / c_{eff} \) and a Laplacian term \( D_\chi \partial^2 \chi / \partial x_i \partial x_j \), which acts to diminish gradients in \( \chi \). As such, the steady state simply occurs when \( \dot{\chi} = \chi_\infty \), since the Laplacian of \( \chi \) becomes negligible in the steady state.

In the simplest case we are free to choose \( \chi_\infty \) to be a constant. It was shown however that regions of the material sheared at higher rates are driven to higher steady-state values of the effective temperature. Therefore we will treat \( \chi_\infty \) as a strain-rate dependent parameter \( \chi_\infty = \chi_\infty (\dot{\varepsilon}^{pl}) \) with the form given by

\[
\chi_\infty = \chi_\infty^0 - \frac{\alpha_o}{\log \left( \frac{\dot{\varepsilon}^{pl}}{\dot{\varepsilon}_o} \right)}
\]

(13)

which is an approximation to an earlier proposed form where \( \alpha_o \) is a dimensionless parameter which uniformly increases or decreases the value of \( \chi_\infty \). Here \( \dot{\varepsilon}_o^{pl} \) is the asymptotic (maximum) plastic-strain rate. The constant \( \chi_\infty^0 \) is the value of \( \chi_\infty \) when \( \dot{\varepsilon}^{pl} \rightarrow 0 \). Equation (13) is plotted in Fig. 2 where its effect on lower rates becomes apparent. We note that

with this form the value of \( \chi_\infty \) will evolve with time, but as the steady state is reached the plastic-strain rate converges to that given by the HB relation. We can then write a “steady-state” value for \( \chi_\infty \) in Eq. 13 as

\[
\chi_\infty = \chi_\infty^0 - \frac{\alpha_o}{\log \left( \frac{s - s_c}{A} \right)^{1/n}}.
\]

(14)

The stress function \( f \) which appears in the expression for the plastic strain-rate in Eq. 8 now has the explicit form

\[
f(s) = \left( \frac{s - s_c}{A} \right)^{1/n} \exp \left( \frac{\alpha_o}{\chi_\infty^0 - \frac{s - s_c}{A}} \right)^{-1}.
\]

(15)

The rate-of-change of \( \chi \) is given by Eq. 3 and after substituting in Eq. 8 the expression for \( \dot{\chi} \) becomes

\[
\dot{\chi} = \frac{2s}{c_{eff}} \left( \frac{s - s_c}{A} \right)^{1/n} e^{1/\chi_\infty - 1/\chi} (\chi_\infty - \chi) + D_\chi \frac{\partial^2 \chi}{\partial y^2}.
\]

(16)

As discussed in the Introduction the diffusivity \( D_\chi = l^2 |\dot{\varepsilon}^{pl}| \). The parameter \( l \) sets a lengthscale for the model that should be on the order of the size of an STZ. This choice was first introduced for an STZ-theory analysis of metallic glasses.

3 Model Simulations

The equations of motion (EOM) of the effective-temperature model that we have presented in Sec. 2 are

\[
\dot{\varepsilon}^{pl} = \left( \frac{s - s_c}{A} \right)^{1/n} e^{1/\chi_\infty - e^{-1/\chi}}
\]

(17)

\[
\dot{s} = \mu \dot{\gamma} - \mu \left( \frac{s - s_c}{A} \right)^{1/n} e^{1/\chi_\infty - \frac{1}{w} \int_0^w e^{-1/\chi} dy}
\]

(18)

\[
\dot{\chi} = \frac{2s}{c_{eff}} \left( \frac{s - s_c}{A} \right)^{1/n} e^{1/\chi_\infty - 1/\chi} (\chi_\infty - \chi) + D_\chi \frac{\partial^2 \chi}{\partial y^2}
\]

(19)

where \( \chi_\infty \) is a function of the local plastic-strain rate given by Eq. 13.

The three nonlinear PDEs take the form of coupled, partial integro-differential equations which we integrate in time. At time \( t = 0, s = 0 \) so that the fluid is initially unstrained in our analysis. Boundary conditions for \( \chi \) must also be placed at the rotor and the outer wall of the Couette cell. We impose a no-conduction boundary condition \( \partial \chi / \partial y = 0 \), since \( \chi \) is restricted to the gel itself.

Table 1 contains the values of the parameters which appear in the model’s EOM. It includes those parameters which
Table 1 The parameters characterizing the gel for both the numerical model (NUM) and the experiment (EXP). The ‘-’ indicates the was not experimentally measured or the parameter is dimensionless.

| PARAMETERS | UNIT | NUM | EXP |
|------------|------|-----|-----|
| Herschel-Bulkley exponent | n  | 0.55 | 0.55 |
| Critical (yield) stress | $s_c$ | Pa | 29.8 | 26.9 |
| Herschel-Bulkley coefficient | $A$ | Pa $s^n$ | 0.88 | 9.8 |
| Couette-cell gap width | w | mm | 1.1 | 1.1 |
| Diffusivity lengthscale | $l^2$ | mm$^2$ | $10^{-8}$ | - |
| Elastic shear modulus | $\mu$ | Pa | 3.9 | - |
| Volumetric effective-heat capacity | $c_{\text{eff}}$ | Pa | 3500 | - |
| $\chi_{\text{eff}}$ in small $\dot{\epsilon}^\text{pl}$ limit | $\chi_o$ | - | 0.0075 | - |
| Steady-state dimensionless constant | $\alpha_\text{pl}$ | - | 0.90 | - |
| Asymptotic plastic-strain rate | $\dot{\epsilon}_{\text{pl}}^\infty$ | s$^{-1}$ | $8 \times 10^5$ | - |

The parameters characterizing the gel for both the numerical model (NUM) and the experiment (EXP). The ‘-’ indicates the parameter was not experimentally measured or the parameter is dimensionless.

arise from the STZ phenomenology as well as those reported from experiments. The central parameter in the effective-temperature hypothesis is the volumetric effective-heat capacity $c_{\text{eff}}$ which appears in the the equation for $\chi$. Moreover, the quantity $c_{\text{eff}}$ has the physical significance of being the amount of plastic work per unit volume required to cause a fractional relaxation of $\chi$ to its steady state. We find that the values in Tab. 1 provide the best match of the model with the experimental data. The values of the STZ parameters were initially chosen to be within the ranges reported in prior work, and then adjusted to match the behavior of the experiment. Experimentally reported parameters, such as $A, n,$ and $s_c$ which describe the HB steady-state behavior, were adjusted after initially using their measured values, if doing so improved the fitting. The HB coefficient $A$ e.g. needed to be decreased by approximately an order of magnitude to obtain the best fit to the experiment. The exponent $n$ in the HB relation was not adjusted from its measured value, as it is known to be more robust than the other HB-relation parameters which are more significantly affected by the gel’s particular preparation and treatment.

The initial dimensionless effective-temperature field $\chi_o$ was given the form of a spatially uniform background with a small perturbation.

The gel was initially prepared to remove memory effects using the same protocol before each experiment: A pre-shear lasting 60 s at $10^3$ s$^{-1}$ was applied in the clockwise direction. Then 60 s at $10^3$ s$^{-1}$ was applied in the counterclockwise direction. The shear was then instantaneously stopped and the gel was allowed to rebuild for 120 s. The experiments then involved shearing at a constant rate for $10^4$ s while the stress was measured. The experiment was repeated at different applied strain-rates, and for each rate a shear band was observed to nucleate near the rotating inner wall of the cell. After nucleation the band widened until the onset of fluidization, at which point the entire gel transitioned to a homogeneously flowing state, revealing a distinct process of unjamming. During the transient characterized by the broadening of the shear band, the shear rate and velocity profile of the gel outside the band were found to be non-zero.

Fig. 3 The dimensionless effective temperature $\chi$ at various times during the shear-induced deformation. The initial field $\chi(y,0) = \chi_o$ (heavy black dashes) grows into a shear band as time evolves. The increasing value of $\chi$ shows the band broadening across the gap (y-direction) beginning at the rotor ($y = 0$), before reaching a uniform steady-state.

Fig. 4 Plastic-strain rates at different times (each color) during the simulation. As plastic strain develops in the gel, $\dot{\epsilon}_{\text{pl}}^\text{pl}$ takes on a large value near the inner rotor ($y = 0$). As a shear band forms and traverses the gap the system fully fluidizes, until steady state is reached and everywhere $\dot{\epsilon}_{\text{pl}}^\text{pl} = 0.7$ (solid black line), the value of the applied shear-rate. The inset highlights the evolution to steady state near the interface of the shear band.
The gel’s restructuring under shear is captured by the evolution of $\chi$ during the numerical calculations. In Fig. 3 $\chi$ is plotted at different times during the deformation. The bottommost, dashed black curve in Fig. 3 illustrates $\chi(y, 0) = \chi_0$, the state before shearing begins. The time-evolution of the plastic-strain rate is described in Fig. 4 and reflects these changes in $\chi$. As was observed experimentally, the simulations reveal the formation of a shear band, which nucleates from the perturbation near the inner cylinder and begins to grow, broadening across the gap. In the simulations this is followed by an abrupt transition during which the gel reaches a fully unjammed state and completely fluidizes. The stress-time curve of the simulation also agrees with the experimental measurements as seen in Fig. 6, where in the model the gel undergoes a very brief, linearly elastic regime followed by plastic flow until the steady-state stress given by the HB relation is reached. The rate dependence of $\chi_\infty$ as described by Eq. 13 has the effect of lowering its value for lower strain-rates. Figure 2 plots Eq. 13 for different strain rates and shows $\chi_\infty = 0.04636$ in the steady state for an applied strain-rate $\dot{\gamma} = 0.7$ s$^{-1}$. This is consistent with the value $\chi$ evolves towards in Fig. 3.

The sudden fluidization of the gel after the formation of the shear band is also evident in the velocity profiles shown in Fig. 5. The portion of the velocity curves with the significantly steeper slope near the inner cylinder is indicative of a shear band, while the lower slope near the outer cylinder suggests that the gel outside the shear band is also gradually flowing as it fluidizes. Experimental measurements confirm the non-zero velocity for the gel in front of the shear band. Direct, quantitative comparisons of the velocity profiles of the simulations and the published experimental data are difficult because of the significant wall slip found in the experiment especially at early times, effects which are not included in the model. In Fig. 5 we have compared two instances from the simulations against the experimental data where the wall slip was minimal. We see the presence of the two distinct fluidization processes in the model as a result of the effective-temperature dynamics and the particular form of the constitutive law from the STZ theory that we are using. The equation of motion for $\chi$ takes the form of a second-order parabolic PDE (e.g., the heat equation) with source and Laplacian terms as seen in Eq. 3. In the case of spatially homogeneous solutions, like those found in the steady state, Eq. 3 reduces to $\ddot{\chi} = g(\chi)$, where $g$ is a nonlinear, inhomogeneous term in $\chi$. Here, the function $g$ is similar in form (although asymptotically different) to the logistic reaction term found in the F-KPP equation that describes solidification and reaction-diffusion fronts. In the case of the F-KPP equation, a consideration of the stability of equilibria reveals that there are two fixed points, one corresponding to a metastable state and the other to a stable state into which material is transforming. Similarly, $g$ also has a stable fixed-point which corresponds to the shear-banding region, but instead of a metastable state outside the shear band, $g \rightarrow 0$ exponentially, as $\chi \rightarrow 0$. Consequently the gel outside the shear band is extremely sluggish, although it remains sub-
ject to gradual fluidization in the presence of a shear stress, even far from the shear-banding region.

From Fig. 6 we see the simulation reaches the steady-state stress after approximately 2,900 s. This is also observed from the velocity profile across the gap shown in Fig. 5. The experimentally reported fluidization time for an applied shear-rate of \( \dot{\gamma} = 0.7 \text{ s}^{-1} \) was approximately 2500 s. The fluidization times of the simulation at other \( \dot{\gamma} \) begin to deviate from experimentally measured times the further \( \dot{\gamma} \) deviates from 0.7 \text{ s}^{-1}. However, the exponent in the power law for the fluidization time reported in the experiment (\( \alpha = 2.3 \)) can be recovered if \( c_{\text{eff}} \) is a function of the applied strain-rate \( c_{\text{eff}} = c_{\text{eff}}(\dot{\gamma}) \). The form \( c_{\text{eff}} = 288 \dot{\gamma}^{-2.0} \) reasonably matches the fluidization times for the applied strain-rates in the experiments. This would imply that the fraction of plastic work that goes into disordering the gel increases as the strain rate increases, and the power law of the fluidization time \( \tau_f \sim \dot{\gamma}^{-\alpha} \) arises from this aspect of the physics. This could reflect rate effects in the gel’s mechanical response which cause the gel to accommodate deformation through structural disordering at high rates that it could accommodate through relaxation processes at low rates. It is not clear that this is the only possible means of rectifying this issue regarding the fluidization timescale, and this issue is worthy of further study.

4 Conclusions

We have presented a phenomenological effective-temperature model of Couette-cell shear experiments of a carbopol gel where transient shear banding is observed. This theoretical model is based on the STZ description, and results in two distinct fluidization processes: the shear band formation and the simultaneously competing homogeneous fluidization. We have made direct, quantitative comparisons with experiments which demonstrate reasonable agreement with the stress-time behavior of the gel.

In this theory, consistent with the interpretation of Divoux, et al., stress gradients are not the primary cause of the strain localization or the fluidization. This is in contrast to other theoretical descriptions in which the strain localization is directly associated with the fluidization and comes solely from the non-linear response of the fluid coupled to the stress inhomogeneity that arises from the geometry of the Couette-cell. One possible experimental check would be to see if such shear banding and fluidization would take place if the experiments were repeated for a different shear geometry, such as shearing the gel between two parallel plates. This model does confirm that normal stresses appear to play no role in shear-band formation, as found by the \( \lambda \)-based rheological model.

The effective-temperature theory also improves several aspects of existing visco-plastic models which lack any notion of a yield stress, a defining physical feature of YSFs. The mechanical response of the gel is quantitatively well captured by the simulations, as evidenced by the stress-time curve. The simulations in this paper reveal that the two-stage fluidization seen in experiments likely arises from the equation of motion for the effective temperature and the particular form of the constitutive law postulated by the effective-temperature theory. The parameters of the theory provide a physical connection to the thermodynamics of the gel’s structural state under shear. One open question is the origin of the strain-rate dependence of the fluidization time. A possible explanation is that the volumetric effective-heat capacity, describing the fraction of plastic work that disorders the gel and emerges phenomenologically in the effective-temperature dynamics, changes as a power law of the applied strain-rate.

The effective-temperature approach that we have proposed here would still benefit from further development. A number of consequential phenomena exist which we have largely ignored in the present model, e.g. aging and wall slip, both of which are known to play at least some role in the recent shear experiments of carbopol. A generalization of this model and incorporation of additional physics is a matter of our on-going research.

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References

1 A. Bonn and M. Denn, Science, 2009, 324, 1401.
2 A. R. et al., Phys. Rev. E, 2007, 76, 051408.
3 P. M. et al., Phil. Trans. R. Soc. A, 2009, 367, 5139.
4 P. Moller, J. Mewis and D. Bonn, Soft Matter, 2006, 2, 274.
5 P. Coussot, Soft Matter, 2007, 3, 528.
6 H. A. Barnes, J. Non-Newtonian Fluid Mech., 1997, 70, 1.
7 G. Ovarlez, S. Rodts, X. Chateau and P. Coussot, Rheol. Acta, 2009, 48, 831.
8 P. Schall and M. van Hecke, Annu. Rev. Fluid Mech., 2010, 42, 67.
9 R. Bird, D. Gance and B. J. Yarusso, Rev. Chem. Eng., 1982, 1, 1.
10 P. Coussot, Rheometry of pastes, suspensions, and granular materials, Wiley New York, 2005.
11 T. Divoux, Phys. Rev. Lett., 2010, 104, 208301.
12 T. Divoux, V. Grenard and S. Manneville, Phys. Rev. Lett., 2013, 110, 018304.
13 T. Divoux, C. Barentin and S. Manneville, Soft Matter, 2011, 7, 8409.
14 T. Divoux, D. Tamarri, C. Barentin, S. Teitel and S. Manneville, Soft Matter, 2012, 8, 4151.
15 S. M. Fielding, M. E. Catesb and P. Sollich, Soft Matter, 2009, 5, 2378.
16 M. Fuchs and M. Cates, Phys. Rev. Lett., 2002, 89, 248304.
17. J. M. Brader, T. Voigtmann, M. E. Cates and M. Fuchs, *Phys. Rev. Lett.*, 2007, 98, 058301.
18. M. Fuchs and M. E. Cates, *Faraday Disc.*, 2003, 123, 267.
19. R. L. Moorcroft and S. M. Fielding, *Phys. Rev. Lett.*, 2013, 110, 086001.
20. X. Ilia, A. Puisto, A. Lehtinen, M. Mohtaschemi and M. Alava, *Phys. Rev. E*, 2013, 87, 022307.
21. E. Bouchbinder and J. Langer, *Phys. Rev. E*, 2009, 80, 031133.
22. M. Falk and J. Langer, *Phys. Rev. E*, 1998, 57, 7192.
23. M. Falk and J. Langer, *MRS Bull.*, 2000, 25, 40.
24. L. Pechenik, *Phys. Rev. E*, 2005, 72, 021507.
25. L. Pechenik and J. Langer, *Phys. Rev. E*, 2003, 68, 061507.
26. E. Bouchbinder and J. Langer, *Phys. Rev. E*, 2009, 80, 031131.
27. E. Bouchbinder and J. Langer, *Phys. Rev. E*, 2009, 80, 031132.
28. E. Bouchbinder, *Phys. Rev. E*, 2008, 77, 051505.
29. E. Bouchbinder and J. S. Langer, *Phys. Rev. E*, 2011, 83, 061503.
30. J. Langer, *Phys. Rev. E*, 2004, 70, 041502.
31. M. Falk and J. Langer, *Annu. Rev. Condens. Matter Phys.*, 2011, 1, 062910.
32. M. Manning, J. Langer and J. Carlson, *Phys. Rev. E*, 2007, 76, 056106.
33. M. Manning, E. Daub, J. Langer and J. Carlson, *Phys. Rev. E*, 2009, 79, 016110.
34. J. Langer and M. Manning, *Phys. Rev. E*, 2007, 76, 056107.
35. E. Bouchbinder, J. Langer and I. Procaccia, *arXiv*, 2006, cond-mat/0611026.
36. A. Mehta and S. Edwards, *Physica A*, 1989, 157, 1091.
37. L. Cugliandolo, J. Kurchan and L. Peliti, *Phys. Rev. E*, 1997, 55, 3898.
38. M. Lundberg, K. Krishna, N. Xu, C. O’Hern and M. Dennin, *Phys. Rev. E*, 2008, 77, 041505.
39. E. Daub, D. Klaumünzer and J. Löffler, *Phys. Rev. E*, 2014, 90, 062405.
40. A. Argon and H. Kuo, *Materials Science and Eng.*, 1979, 39, 101.
41. A. Argon, *Acta Metall.*, 1979, 27, 47.
42. D. Turnbull and M. Cohen, *J. Chem. Phys.*, 1970, 52, 3038.
43. F. Spaepen, *Acta Metall.*, 1979, 27, 47.
44. T. Haxton and A. Liu, *Phys. Rev. Lett.*, 2007, 99, 195701.
45. R. A. Fisher, *Ann. Eugenics*, 1937, 7, 355.
46. A. Kolmogorov, I. Petrovsky and N. Piskunov, *Bull. Univ. Moscow, Ser. Intern., Sec. A*, 1937, 1, 1.
47. W. van Saarloos, *Physics Reports*, 2003, 386, 29.
