Complex field as inflaton and quintessence

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Abstract
We investigate cosmology with complex scalar field. It is shown that such models can describe two stages of inflation and the oscillatory regime. Thus we don’t need both quintessence and Λ-term. It is enough to have single complex field to obtain both inflation and present accelerated universe. If the present accelerated expansion was created by the complex field then universe must escape eternal acceleration. Besides, we will show that cosmological equations with complex field admit stationary solution without any cosmological constant.

1 Introduction.

Recently, Steinhardt and Turok suggested a new cosmological scenario called Cyclic Universe [1]. This scenario is the development of their Ekpyrotic Universe [2]. They write "The discovery of dark energy is a complete surprise from the point-of-view of big bang and inflationary cosmology" [3]. Dark energy is the reason of expansion of the universe to accelerate.

Steinhardt-Turok point-of-view is too flat but there is some thing. It is easy to formulate usual inflationary theory to obtain recently discovered accelerated expansion of the universe. To do it one can add a small constant term $V_0 > 0$ to the potential $V(\phi)$ [4] . If cosmological constant $V_0 \sim 10^{-120}$ (in Planck units) then one get the present acceleration [5], but is not terrifically because it is not clear why should $V_0$ be so small? This approach return us to the old mystery of vacuum energy [6].

Another way to solve the problem of accelerated expansion is a new kind of matter called Λ-field or quintessence [7]-[8] . It is clear that a new kind of matter lead to new problems. It will be better to obtain present inflation using the same field which was cause of inflation in early universe. It is not clear how one can do it by-passing the problem of Λ-term in the case of real field (vide supra) so in this work we examine the question: can complex field support this role?

The main goal of this work is to show that complex field can (as a matter of principle) lead to both inflation in early universe and present acceleration of universe. In other words, we don’t need two different kind of matter to understand inflation and present acceleration and we don’t need wrapped in mystery Λ-term.

We deal with qualitative investigation and we demonstrate three separate bits of mosaic: the first inflation, the oscillatory regime and the second inflation. Whether the exact

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1Cosmology with negative potentials were considered in [5].
2Interesting alternative to quintessence is the models like Chaplygin gas [9].
solution (analytical or numerical) exist where these bits are glued in complete mosaic? We will not discuss it in this paper.

2 Complex field.

Complex fields in quantum and classical cosmology were introduced at first time in [10, 11]. As we have shown in [12], the complex field give us an inflationary cosmology with natural exit as distinct from models with a real inflaton. In the other hand, the complex field in exact soluble models (which was studied in [12]) leads to very small inflation and we supposed that a satisfactory model may contain two scalar fields: one real inflaton and one complex "anti-inflaton" i.e. the field that triggers the end of inflation. As we shall see in the next section, this conclusion is not valid in the case of chaotic inflation.

Let us consider Fridman-Robertson-Walker universe containing a minimally coupled complex scalar field $\Phi(t)$ with potential $V(|\Phi|) = m^2 \Phi \Phi^*$. The field equations can be derived by the minimizing the action (in units with $8\pi G = c = 1$):

$$S = \int dt \sqrt{-g} \left( \frac{1}{2} R + g^{ik} \Phi_i^* \Phi, - V(|\Phi|) \right).$$

We choose $\Phi = \phi(t) \exp (i\theta(t)) / \sqrt{2}$ with the cyclic variable $\theta(t)$ therefore

$$M \equiv \phi^2 \dot{\theta} = \text{const.}.$$ 

We call the conserved quantity $M$ the "charge". Finally, the Einstein equations reduced to the set

$$H^2 + \frac{k}{a^2} = \frac{1}{6} \left( \dot{\phi}^2 + m^2 \phi^2 + \frac{M^2}{\phi^2} \right), \quad \ddot{\phi} + 3H \dot{\phi} + m^2 \phi = \frac{M^2}{\phi^2} \left( \frac{1}{\phi} - \frac{3H}{\dot{\phi}} \right), \quad (1)$$

where $a = a(t)$ is the expansion scale factor and $H = \dot{a}/a$ is the Hubble expansion parameter. If $M = 0$ then centrifugal terms are disappear and one get usual equations for the real field $\phi$.

The centrifugal terms leads to the important difference between real and complex fields. For instance, the equations (1) admit stationary solution without any cosmological constant ($k = +1$):

$$a = a_0 = \sqrt{\frac{3}{mM}}, \quad \phi = \phi_0 = \sqrt{\frac{M}{m}}, \quad H = H_0 = 0. \quad (2)$$

(2) is the simplest solution of the system (1). In the case of general position one can’t solve these equation exactly. Some examples of exact soluble potentials are contains in [12]. For example, one of these potentials has the form

$$V = \frac{m^2 \phi^2}{2} - V_0, \quad V_0 = \frac{m^2}{3} + \frac{3M^2}{4},$$

so we have model where the potential may be become negative for $\phi < \sqrt{2V_0}/m$. In the case of real field, cosmologies with negative potentials were investigated in [5]. One of major conclusions is that such models enter a stage of contraction. For the complex field we have also the case. The great number of confirmatory models take place in the [12]. One can show this using the simple argumentation: let consider the cosmological equation
(1) for the arbitrary potential \( V(\phi(t)) \equiv V(t) \) and \( k = 0 \). With the exception of quantity \( \dot{\phi}^2 + M^2/\phi^2 \) one get the Riccati equation

\[
\dot{H} + 3H^2 = V,
\]

which can be linearized by the substitution \( H = (\log \psi)/3 \). As a result, one get the Schrödinger equation for the \( \psi(t) = a^3(t) \),

\[
\ddot{\psi} = 3V(t)\psi.
\]  

Thus, to find the scale factor one need solve the Schrödinger equation (3) with zero eigen-value. Let us assume that \( V \to 0 \) as \( t \to \infty \). If \( V > 0 \) then the solution \( \psi(t) \) is not normalizable function. In other words, \( a(t) = t^{\psi/3} \to \infty \) as \( t \to \infty \). It is not true when \( V < 0 \) therefore negative potential \( V \) can lead to the contraction though we deal with the open model \( (k = 0) \).

3 Chaotic inflation.

One start with Linde chaotic inflation for the real field. Let \( M = 0 \), then the equations (1) reduces to the well known system

\[
H^2 = \frac{1}{6}\left(\dot{\phi}^2 + m^2\phi^2\right), \quad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0. \tag{4}
\]

The inflation is appear when energy density is accumulated in potential, i.e.

\[
\dot{\phi}^2 \ll m^2\phi^2, \quad |\dot{\phi}| \ll m^2 \cdot |\phi| . \tag{5}
\]

Using (4), (5) one get \( H^2 = m^2\phi^2/6 \) and \( 3H\dot{\phi} = -m^2\phi \) therefore

\[
\phi(t) = \phi_0 - \sqrt{\frac{2}{3}}mt. \tag{6}
\]

The conditions (5) are valid if \( \phi_0 \gg 1 \) and \( t < \sqrt{3/2\phi_0/m} \equiv t_e \), where \( t_e \) is the time of exit from inflation. During inflation \( (t < t_e) \) one may suppose that \( \phi \sim \phi_0 \) so \( H = m\phi_0/\sqrt{6} = \text{const} \). In other words, we get the de Sitter-like regime. The number of e-foldings at time \( t_e \) is \( \alpha(t_e) = \phi_0^2/2 \) therefore the condition \( \phi_0 \gg 1 \) lead to good inflation. For example, if \( \phi_0 = 10 \) then \( \alpha(t_e) \sim 50 \). We will hold \( \phi_e = \phi(t_e) = 1 \).

All that is well known Linde chaotic inflation for the real inflaton. Let consider the equation for complex field (1). These equation are differ from the (4) because the system (1) has additional "centrifugal terms". The Linde inflation will be valid if "centrifugal terms" will be small, i.e.

\[
\frac{M}{m} \ll \phi^2, \quad \frac{M^2}{m^2} \ll \frac{|\dot{\phi}| \phi^3}{3H}.
\]

The first condition will be true if \( M \ll m \). It is strong inequality guaranteeing that \( M^2/\phi^3 \ll m^2\phi \) right up to the end of inflation. The second condition can be written as \( M/m \ll \sqrt{2}\phi_0/\sqrt{3} \), i.e. it is coincide with the first, practically.

Thus, usual Linde chaotic inflation can take place for the complex field if \( M/m \ll \phi_0 \). Nevertheless, we should use more strong condition \( M/m \ll 1 \) which we’ll need later. In this regime one can neglect "centrifugal terms" and complex field is indistinguishable from the real one.
4 The oscillatory regime.

One should continue the analogy between real and complex field for the oscillatory regime. We start from the real field. If the \( \phi \) oscillates near \( \phi = 0 \) with frequency \( m \gg H \) then the second equation of the system (4) can be written as

\[
\ddot{\phi} + m^2 \phi = 0. \tag{7}
\]

One find the solution of (7) in the form \( \phi = \Phi(t) \sin mt, \) where \( \Phi(t) \) is the slow variable function. Neglecting \( \dot{\Phi} \) one calculate the pressure:

\[
p = \frac{1}{2} \dot{\phi}^2 - \frac{m^2 \phi^2}{2} = \frac{m^2 \Phi^2}{2} \cos 2mt,
\]
so taking an average over many oscillations one get \( < p > = 0. \) Thus we have the dust equation of state. In this case the solution of Einstein equations is well known: \( a \sim t^{2/3} \) and \( H = 2/3t. \) Substituting \( H \) into the first equation of (4) one get

\[
\phi = \frac{2\sqrt{2}}{3mt} \sin mt. \tag{8}
\]

It’s clear that (8) can’t describe oscillations of complex field (even approximately). This because \( \phi = 0 \) at \( t_N = \pi N/m \) (and \( \dot{\phi} = 0 \) at \( t = t_N + \pi/2m \)) therefore centrifugal terms are not small close to these points. Maybe we can use this regime during the time when \( \phi \neq 0 \) and \( \dot{\phi} \neq 0? \) It is not right because during the time the number of oscillations less than one. Thus one need another oscillatory regime. Fortunately, it is easy to find one.

Now let us assume that the real field \( \phi \) oscillates near \( \phi = A(t) \neq 0: \)

\[
\phi = A(t) + \Phi(t) \sin mt. \tag{9}
\]
Calculating the pressure one get

\[
p = \frac{1}{2} \left( \dot{A}^2 - m^2 A^2 + (m\Phi + 2A)m\Phi \cos mt - 2m^2 A\Phi \sin mt \right),
\]
therefore

\[
< p > = \frac{1}{2} \left( \dot{A}^2 - m^2 A^2 \right).
\]
If one need to obtain \( a \sim t^{2/3} \) then \( < p > = 0 \) so \( A(t) = A_0 e^{kmt} \) with \( k = \pm 1. \) Substituting the expression for the \( A(t) \) into the (9) and (9) into the first equation of (4) we have

\[
\phi = A_0 e^{kmt} + \sqrt{\frac{8}{9m^2t^2} - 2A_0^2 e^{2kmt} \sin mt}. \tag{10}
\]

(10) is generalization of (8). If \( A_0 = 0 \) then (10)=(8).\(^3\)

Let \( z = mt, \theta = A_0 e^{kz}. \) There are two condition which lead to good behavior of (10): the radicand must be positive and \( \phi \neq 0. \) Both of them can be written as compact inequality,

\[
\frac{2\sqrt{2}}{3} < A_0 e^{kz} < \frac{2}{\sqrt{3}}. \tag{11}
\]

\(^3\)Note, the condition \( < p > = 0 \) is not necessary. One can use some another condition.
It is clear that the oscillatory regime work during finite time whereupon one of inequalities (11) will be break. There are two different cases when \( k = 1 \) (i) and \( k = -1 \) (ii). If \( k = -1 \) then one get two cases (iia) and (iib). The case (iia) take place when \( 2\sqrt{3} > A_0/e > 2\sqrt{3}/3 \) while (iib) take place when \( A_0/e > 2\sqrt{3} \). The time of this regime can be sufficiently large to create enough number of pairs.

Now let us consider the same regime for the complex field. We need that "centrifugal terms" will be small during regime of oscillations (10). It is possible if the reason of exit from this regime is the violation of second member of two-sided inequality (11). In this case \( \phi \) and \( \dot{\phi} \) will be nonvanishing function right up to intersection of curves \( \theta \) and \( 2/\sqrt{3} \) therefore the "centrifugal terms" will be finite and we can choose \( M \) so small to suppress these terms. Suitable situations are (i) and (iib).

We need just these cases to obtain small "centrifugal terms" both during and just on completion of the oscillatory regime. If \( \phi \) is the decreasing function of \( t \) then we have the point of time \( t_* \) such that "centrifugal terms" are the first order of smallness. Taking into account these terms via the perturbation theory we have a new dynamics which is characteristic precisely of the complex field. As we shall see, that will do to obtain new inflation.

## 5 Second inflation.

Let us consider the system (1). We suppose that all energy density is dominated by the effective potential \( U = U(\phi) \),

\[
\frac{1}{2} \ddot{\phi}^2 \ll U, \quad |\dddot{\phi}| \ll |U'|, \quad U \equiv \frac{m^2 \phi^2}{2} + \frac{M^2}{2} \phi^2. \tag{12}
\]

So \( H = \sqrt{U/3} \). After simple calculations we get two possible expressions for the \( \dot{\phi} \):

\[
\dot{\phi}_\pm = \frac{1}{2\phi\sqrt{U}} \left( -U' \phi \pm \sqrt{(U'\phi)^2 - 12M^2U} \right).
\]

Let \( \phi_0 \) is the value of \( \phi \) at which inflation begins. Besides, we’ll use the condition \( \phi \gg \sqrt{M/m} \) which was obtained above. Using Taylor’s series (\( M \) is the small parameter) we get

\[
\dot{\phi}_+ \sim -\sqrt{\frac{3}{2}} \frac{M^2}{m\phi^2}, \quad \dot{\phi}_- \sim -\sqrt{\frac{2}{3}} \frac{m}{\phi^2}.
\]

The solution \( \phi_- \) is the usual Linde inflation (6) whereas the \( \phi_+ \) is new solution which is characteristic precisely of the complex field. After integration we have

\[
\phi_+ = \left( \phi_{0,+}^3 - \sqrt{\frac{3}{2}} \frac{3M^2t}{m} \right)^{1/3} \tag{13}
\]

The condition (12) can be written as

\[
\phi \gg \left( \sqrt{\frac{3}{2}} \frac{M^2}{m^2} \right)^{1/3}
\]
and it is jointly satisfiable with $\phi \gg \sqrt{M/m}$. The number of e-foldings during the new inflation ($\alpha_+(t)$) and during Linde inflation ($\alpha_-(t)$) are

$$\alpha_+(t) = \frac{m^2}{12M^2} \left( \phi_{0,+}^4 - \phi^4(t) \right), \quad \alpha_-(t) = \frac{1}{4} \left( \phi_{0,-}^2 - \phi^2(t) \right).$$

Thus, to the end of inflation the number of e-foldings will be

$$\alpha_+(t^{(+)}_e) \sim \frac{m^2\phi_{0,+}^4}{12M^2}, \quad t^{(+)}_e = \frac{\sqrt{2m\phi_{0,+}^3}}{\sqrt{27M^2}}.$$

while for the Linde inflation we have $t^{(-)}_e = \sqrt{3}\phi_{0,-}/2m$.

During the time $t \ll t^{(+)}_e$ we have $\phi \sim \phi_{0,+}$ and $H = \sqrt{U(\phi_{0,+})/3} = \text{const}$, so we have de Sitter-like regime. Thus, if the $M$ is small the equations (1) admits two inflations. The first (6) is usual Linde chaotic inflation which took place in the early universe. The second (13) is new and take place only for the universe with complex scalar field. We get the designing question: it is possible to equate this regime with recently discovered accelerated expansion of the universe? It will be very nicely to obtain the positive answer. This because such explanation has two advantages:

1. Both the inflation in the early universe and the present accelerated expansion of universe are sequent of existence and dynamics of single object - complex scalar field. We don’t need "quintessence".
2. We don’t need cosmological constant so we have not problems with it’s anomalous small value.

The time of the first inflation is $t^{(-)}_e \sim 10^{-35}$ s. The time $t^{(+)}_e$ is counts off from the time when the second inflation began. We see that this time is finite, i.e. if the present accelerated expansion was created by the complex field then universe must escape eternal acceleration. The time of the second inflation $t^{(+)}_e$ must be much more long than $t^{(-)}_e$ if the second inflation is the present acceleration. We choose $\phi_{0,+} = 1$, therefore $t^{(+)}_e/t^{(-)}_e = 4m^2/9\phi_{0,-}M^2$. This ratio will be larger then unit if

$$\frac{M}{m} \ll \frac{2}{3\sqrt{\phi_{0,-}}}.$$ (14)

(14) is strong inequality and it lay on $M$ very hard restriction (in comparison with the inequality $M/m \ll 1$). In units with $\hbar = c = 1$ we have $M \ll 0.42 \times 10^{27}$ GeV$^3$ and if $t^{(+)}_e \sim 10^{10}$ years then $M \sim 1900\phi_{0,+}$ GeV$^3$ if $\phi_{0,-} = 10^{25}$ GeV and $m = 10^{12}$ GeV.

6 Conclusion.

Thus, complex field can be both inflaton and quintessence. The aim of this work is to show that a new way to solve well known cosmological problems is exist in principle. The next step is the search of exact solution (numerical or analytical) where these bits are glued in complete mosaic:

the first inflation $\rightarrow$ the oscillatory regime $\rightarrow$ the second inflation.
It is possible that such solutions are not exist. That would be a pity because this way to solve both problems of the inflation (with exit) and the present accelerated expansion of universe is looking very fine.

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