Astrophysical Solutions are Incompatible
with the Solar Neutrino Data

S. Bludman, N. Hata, and P. Langacker

Department of Physics, University of Pennsylvania, Philadelphia, PA 19104
(June 1, 1993, UPR-0572T)

Abstract

We consider the most general solar model, using the neutrino fluxes as free parameters constrained only by the solar luminosity, and show that the combined solar neutrino data exclude any astrophysical solution at 98\% C.L. Our best fit to the $^7$Be and $^8$B fluxes is respectively $<7\%$ and $37\pm4\%$ of the standard solar model prediction, but only with a large $\chi^2$ (5.6 for 1 d.f.). This best fit to the fluxes contradicts explicit nonstandard solar models, which generally reduce the $^8$B flux more than the $^7$Be. Those models are well parameterized by a single parameter, the central temperature.
Each of the solar neutrino experiments \cite{1-5} show deficits of the solar neutrino flux compared to the standard solar model (SSM) predictions \cite{6,7} as summarized in Table I. Numerous astrophysical solutions have been proposed to explain the discrepancy between theory and experiments \cite{8-12}. One category of such proposals changes the input parameters of the solar models, assuming that the uncertainties of those quantities might be significantly underestimated in the SSMs. For example, the neutrino fluxes are known to be sensitive to the opacity, and explicit models have been constructed with smaller values of the opacity or smaller values of the heavy element abundance. The nuclear reaction cross sections, which are extrapolated from laboratory conditions, are another potential source of uncertainties: there might be some mechanism affecting the low energy cross sections and reducing the neutrino production, and such effects might even be correlated among the different reactions. A second category of proposals attributes the neutrino deficit to mechanisms such as rotation, magnetic fields, turbulent diffusion, mixing of elements, or hypothetical weakly interacting particles (WIMPs) that are not included in the SSMs. Both kinds of theoretical proposals usually reduce the expected flux of high- and medium-energy neutrino production by lowering the temperature in the core region where the nuclear fusion takes place, and can be parameterized by a lower central temperature ($T_C$). In previous studies using the power law dependence of the neutrino fluxes on $T_C$ \cite{13,14,5}, it was shown quantitatively that such cooler sun models are incompatible with the experimental data, especially because the higher Kamiokande observed rate relative to the Homestake rate cannot be explained so long as cool sun models reduce the expected $^8$B flux more than the $^7$Be flux. This failure of astrophysical resolutions of the solar neutrino deficit suggests particle physics solutions, such as the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism \cite{15}, which fits all observations and is taken as a strong hint of neutrino mass and mixings \cite{14,16-20}.

In this paper, we remove the assumption of a power law dependence and examine arbitrary solar models by allowing the four relevant neutrino fluxes $\phi(pp)$, $\phi(Be)$, $\phi(B)$, and $\phi(CNO)$ to change freely. We do not advocate such models or claim that they are consistent with other solar observations, but only show that they are incompatible with the solar
neutrino data. In our most general solar model, we assume: (1) The Sun is in quasi-static equilibrium and generates energy by nuclear fusion in the \(pp\) and the CNO chains; (2) Astrophysical mechanisms may change the magnitude of each neutrino flux component, but do not significantly distort the energy spectra of the individual components \[21\]. (Particle physics solutions such as the MSW effect often depend on neutrino energy and therefore do distort the neutrino spectrum.); (3) All reported experimental results are correct, as well as the calculations of radio-chemical detector cross sections. Because the Kamiokande and Homestake results are crucial to our conclusions, we will also consider the possibilities that their uncertainties have been underestimated.

By our first assumption, the well-measured solar luminosity imposes the constraint

\[
\phi(pp) + 0.979 \phi(Be) + 0.955 \phi(CNO) = 6.57 \times 10^{10} \text{ cm}^{-2}\text{s}^{-1},
\]  

(1)

among the \(pp\), \(^7\)Be, and CNO fluxes, when the different energies carried off by neutrinos are taken into account.

In Figs. 1–4 we present the results of all solar neutrino experiments in the \(\phi(Be) - \phi(B)\) plane in units of the Bahcall-Pinsonneault predicted fluxes. Essentially all astrophysical solutions, including insensible models, are represented in the plane, from \(\phi(Be)/\phi(Be)_{SSM} = \phi(B)/\phi(B)_{SSM} = 1\) for the SSM to \(\phi(Be)/\phi(Be)_{SSM} = \phi(B)/\phi(B)_{SSM} = 0\) for the minimum rate model \[22\]. Fig. 1 shows the constraints from each experiment obtained by minimizing the \(\chi^2\) with respect to \(\phi(pp)\) and \(\phi(CNO)\) at each point subject to the luminosity constraint. Our \(\chi^2\) fit includes experimental uncertainties as well as detector cross section uncertainties. The uncertainties of minor fluxes (\(pep, hep, \) and \(^{17}\)F) are included, but contribute negligibly. The Kamiokande result constrains only the \(^8\)B flux, while the Homestake data and combined SAGE-GALLEX data constrain both the \(^7\)Be and \(^8\)B fluxes. At 90% confidence level (C.L.), none of the experiments are consistent with the Bahcall-Pinsonneault SSM. Indeed, the combined experiments together allow only a small parameter space around \(\phi(Be)/\phi(Be)_{SSM} \sim 0\) and \(\phi(B)/\phi(B)_{SSM} \sim 0.4\), but with a large \(\chi^2\) value: 5.6 for 0 degrees of freedom (3 data – (4 parameters – 1 constraint)). No general statistical interpretation exists in such a case,
other than to conclude that this model is excluded. If one considers the $^7$Be flux to be fixed at 0, then the fit has 1 degree of freedom and this possibility is excluded at the 98% C.L. This shows that any astrophysical solution in which the spectral shape of the individual neutrino fluxes is unchanged is incompatible with observations.

Fig. 4 shows the confidence levels of the combined fit in the two dimensional $\phi(\text{Be}) - \phi(\text{B})$ subspace. The contours are determined by $\chi^2 = \chi^2_{\text{min}} + \Delta \chi^2$, where $\chi^2_{\text{min}}$ is obtained allowing an unphysical negative $^7$Be flux. We could alternately have taken $\chi^2_{\text{min}}$ in the physical region by restricting the probability distribution to the physical region ($\phi(\text{Be}) \geq 0$). This procedure would have ignored the fact that the best fit is very poor, and would grossly overestimates the allowed region. We therefore present our results as a qualitative display of the confidence levels.

We also show in Fig. 2 the Bahcall-Pinsonneault SSM with 90% C.L. uncertainties, the 1000 Monte-Carlo SSMs of Bahcall and Ulrich [8], the central value of the Turck-Chièze-Lopes (TC) SSM [4], and various explicit nonstandard solar models constructed to solve the solar neutrino problem: the low Z model in which the heavy element abundance is reduced by 90% from the standard value [8]; the low opacity models with 10 and 20% reduced opacity [4]; the solar models with increased $pp$ cross sections ($S_{11}$) by 30, 50, 80, 100, and 150% from the SSM value [10]; and the solar model with WIMPs [12,8]. Also the power laws for the core temperature and $S_{11}$ obtained from the Monte-Carlo SSMs are extrapolated from the SSM region and displayed. The uncertainty due to the $p+^7$Be cross section (9.3%) is shown as error bars.

Because the decay of $^8$B follows the reaction $p+^7$Be $\rightarrow ^8$B $+$ $\gamma$, all explicit nonstandard models predict more reduction of the $^8$B flux than the $^7$Be flux (i.e., $\phi(\text{Be})/\phi(\text{Be})_{\text{SSM}} > \phi(\text{B})/\phi(\text{B})_{\text{SSM}}$). Any reduction of the $^7$Be production rate affects both the $^8$B and $^7$Be flux equally. Other uncertainties in the $p+^7$Be rate affect only the $^8$B flux. Therefore, unless there is some independent mechanism to suppress only the $^7$Be neutrino emission, all realistic nonstandard solar models are in serious contradiction to the solar neutrino data, which constrain the two fluxes to $\phi(\text{Be})/\phi(\text{Be})_{\text{SSM}} < 0.07$ and $\phi(\text{B})/\phi(\text{B})_{\text{SSM}} =$
0.37 ± 0.04 (1σ). This emphasizes that there are two solar neutrino problems: (I) The neutrino fluxes observed in every experiment are significantly below SSM predictions at 90% C.L. (II) Kamiokande and Homestake together allow only the very implausible fit \( \phi(\text{Be}) / \phi(\text{Be})_{\text{SSM}} \ll \phi(\text{B}) / \phi(\text{B})_{\text{SSM}} \).

The two curves in Figs. 2–4 assume that the \(^7\text{Be}\) and \(^8\text{B}\) neutrino fluxes each depend simply on powers of the central temperature \( T_c \) (solid curve) or of the pp nuclear cross section factor \( S_{11} \) (dot-dashed curve), while the pp and CNO neutrino fluxes are adjusted to obey the solar luminosity constraint (Eq. 1). For the solid curve we assumed \( \phi(\text{Be}) \sim T_c^8, \phi(\text{B}) \sim T_C^{18} \) so that \( \phi(\text{B}) \propto \phi(\text{Be})^{2.25} \). For the dot-dashed curve we assumed \( \phi(\text{Be}) \sim S_{11}^{-0.97}, \phi(\text{B}) \sim S_{11}^{-2.59} \) so that \( \phi(\text{B}) \propto \phi(\text{Be})^{2.67} \). Those exponents were obtained by Bahcall and Ulrich from 1000 SSMs with input parameters randomly distributed near the most probable values [8].

The nonstandard solar models (the low opacity models, the low Z model, and the models with large \( S_{11} \)) illustrated in Fig. 2 include physically unreasonable models with \( S_{11} \) as large as 2.5 times and \( T_C \) as small as 0.97 times their most probable values. Within their theoretical uncertainties, all model predict \( \phi(\text{B}) \propto \phi(\text{Be})^n \) with \( n = 2.25 - 2.67 \). Most extremely nonstandard solar models still lead to \(^7\text{Be}\) and \(^8\text{B}\) neutrino fluxes that are adequately parameterized by simple power laws, e.g., in the central temperature or \( S_{11} \). (Exceptions include the maximum rate model [8], the WIMP model, and the model with \( S_{34} = 0 \) [8].) This happens because, although the Sun as whole is not self-homologous (polytropic), over the range of temperatures and densities in the Sun’s inner core \( (r < 0.2R_\odot) \), 91% of the neutrino and energy production derives from the single pp reaction, and all the nuclear reactions and opacities in the present Sun can be approximated by power laws. Consequently, when the luminosity is held constant, large changes in input parameters lead only to nearly homologous changes in core temperatures, mass, and radius.

So far, we have shown that the Kamiokande and Homestake results together, if correct, essentially exclude any astrophysical solutions. What if either experiment were wrong? In Fig. 3 we show the enlarged allowed region of the combined fit when Homestake’s quoted experimental error is tripled. The data still strongly disfavor the nonstandard solar models:
the cooler sun with $T_C$ reduced by 5% is only allowed at $\sim 1\%$ C.L. Expressed otherwise, the best fit with $\phi(\text{Be}) = 0$ corresponds to a Homestake rate of 2.9 SNU; the best cool sun fit when the Homestake error is tripled is 3.3 SNU, compared with the value of $2.23 \pm 0.23$ in Table I. We have also carried out a calculation with the cross section uncertainties for the chlorine and the gallium detectors increased by factors of three, and obtained a similar result (Fig. 1). Of course, if one entirely disregards either of these two experiments, a large class of nonstandard models become possible.

In summary we have considered the most general solar model with minimal constraints using the neutrino fluxes as free parameters, and shown that the fit is excluded by the solar neutrino data at 98% C.L., i.e., essentially any astrophysical solution is incompatible with the quoted data. Furthermore, this very improbable best fit point requires $\phi(\text{Be})/\phi(\text{Be})_{SSM} < 0.07$ and $\phi(\text{B})/\phi(\text{B})_{SSM} = 0.37 \pm 0.04$ (1$\sigma$), which is inconsistent with virtually all explicit nonstandard solar models, which predict a larger reduction of the $^8\text{B}$ flux than the $^7\text{Be}$ flux. Increasing the Homestake experimental error or the detector cross section errors by factors three does not justify the nonstandard solar models.

We conclude that at least one of our original assumptions are wrong, either (1) Some mechanism other than the $pp$ and CNO chains generates the solar luminosity, or the Sun is not in quasi-static equilibrium; (2) The neutrino energy spectrum is distorted by some mechanism such as the MSW effect; (3) Either the Kamiokande or Homestake result is grossly wrong.

We also noted that almost all explicit nonstandard models fall on a narrow band in the $\phi(\text{Be}) - \phi(\text{B})$ plane, and can be characterized by a single effective parameter, the core temperature [13,14].

It is pleasure to thank Eugene Beier for useful discussions. This work is supported by the Department of Energy Contract DE-AC02-76-ERO-3071.
REFERENCES

[1] R. Davis, Jr., et al., in Proceedings of the 21th International Cosmic Ray Conference, Vol. 12, edited by R. J. Protheroe (University of Adelaide Press, Adelaide, 1990), p. 143; R. Davis, Jr., in International Symposium on Neutrino Astrophysics, Takayama, Kamioka, 1992 (unpublished).

[2] K. S. Hirata et al., Phys. Rev. Lett. 65, 1297 (1990); 65, 1301(1990); 66, 9 (1991); Phys. Rev. D 44, 2241 (1991).

[3] Y. Suzuki, in International Symposium on Neutrino Astrophysics, Takayama, Kamioka, 1992 (unpublished).

[4] A. I. Abazov, et al., Phys. Rev. Lett. 67, 3332 (1991); V. N. Gavlin, in XXVI International Conference on High Energy Physics, Dallas, 1992, edited by J. Sanford (AIP, New York, 1993).

[5] GALLEX Collaboration, P. Anselmann et al., Phys. Lett. B 285, 376 (1992); 285, 390 (1992).

[6] J. N. Bahcall and M. H. Pinsonneault, Rev. Mod. Phys. 64, 885 (1992).

[7] S. Turck-Chièze and I. Lopes, Astrophys. J. 408, 347 (1993).

S. Turck-Chièze, S. Cahen, M. Cassé, and C. Doom, Astrophys. J. 335, 415 (1988).

[8] J. N. Bahcall and R. N. Ulrich, Rev. Mod. Phys. 60, 297 (1988); J. N. Bahcall, Neutrino Astrophysics, (Cambridge University Press, Cambridge, England, 1989).

[9] D. Dearborn, private communications.

[10] V. Castelliani, S. Degl’Innocenti, and G. Fiorentini, Phys. Lett. B 303, 68 (1993).

[11] A. Bertin et al., Phys. Lett. B 303, 81 (1993).

[12] J. Faulkner and R. L. Gilliland, Ap. J. 299, 994 (1985); R. L. Gilliland, J. Faulkner,
W. H. Press, and D. N. Spergel, Ap. J. 306, 703 (1986).

[13] S. A. Bludman, D. C. Kennedy, and P. G. Langacker, Phys. Rev. D 45, 1810 (1992); Nucl. Phys. B373, 498 (1992);

[14] S. A. Bludman, N. Hata, D. C. Kennedy, and P. G. Langacker, Phys. Rev. D 47, 2220 (1993).

[15] L. Wolfenstein, Phys. Rev. D 17, 2369 (1978); 20, 2634 (1979);

S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. 42, 1441 (1985); Nuo. Cim. 9C, 17 (1986).

[16] N. Hata and P. Langacker, University of Pennsylvania preprint UPR-0570T (1993).

[17] X. Shi, D. N. Schramm, and J. N. Bahcall, Phys. Rev. Lett. 69, 717 (1992); X. Shi and D. N. Schramm, Phys. Lett. B 283, 305 (1992); Fermilab preprint 92/322-A.

[18] J. M. Gelb, W. Kwong, and S. P. Rosen, Phys. Rev. Lett. 69, 1864 (1992).

[19] P. I. Krastev and S. T. Petcov, Phys. Lett. B 299, 99 (1993).

[20] L. Krauss, E. Gates, and M. White, Phys. Lett. B 299, 94 (1993).

[21] J. N. Bahcall, Phys. Rev. D 44, 1644 (1991).

[22] J. N. Bahcall, B. T. Cleveland, R. Davis, Jr., and J. K. Rowley, Astrophys. J. 292, L79 (1985).
TABLES

TABLE I. The standard solar model predictions of Bahcall and Pinsonneault [6] and of Turck-Chièze and Lopes [7], along with the results of the solar neutrino experiments.

|                  | BP SSM       | TCL SSM     | Experiments            |
|------------------|--------------|-------------|------------------------|
| Kamiokande       | 1 ± 0.14     | 0.77±0.19   | 0.50±0.07 BP-SSM       |
| Homestake (Cl)   | 8±1 SNU      | 6.4±1.4 SNU | 2.23±0.23 SNU (0.28±0.03 BP-SSM) |
| SAGE & GALLEX (Ga) | 131.5^{+7}_{6} SNU | 122.5±7 SNU | 71±15 SNU (0.54±0.11 BP-SSM) |
FIGURES

FIG. 1. Each experiment is fit to the \( pp, ^7\text{Be}, ^8\text{B}, \) and CNO fluxes, imposing only the luminosity constraint. The fit neutrino fluxes are plotted in the \(^7\text{Be}-^8\text{B}\) plane in units of the Bahcall-Pinsonneault predicted fluxes \([6]\). This parameter space represents all possible astrophysical solutions consistent with our (minimal) assumptions. The 90\% C.L. uncertainties of the Bahcall-Pinsonneault SSM are shown in the upper-right corner.

FIG. 2. The allowed region from the combined fit of the Kamiokande, Homestake, and gallium results at 90, 95, and 99\% C.L. allowing (unphysical) negative values for \( \phi(\text{Be}) \). For \( \phi(\text{Be}) \geq 0 \), \( \chi^2 = 5.6 \) for the best fit, and the model is excluded at 98\% C.L. for 1 degree of freedom. Therefore any astrophysical solution is excluded at \( \geq 98\% \) C.L. Also shown are various nonstandard solar models and the power laws of the central temperature (\( T_C \)) and the \( pp \) cross section (\( S_{11} \)) obtained from the Bahcall-Ulrich SSMs \([8]\), and extrapolated from the SSM region. All nonstandard models other than WIMPs can be approximately parameterized by \( T_C \) or \( S_{11} \). The error bars show the uncertainty of \( \phi(\text{B}) \) due to the \( p+^7\text{Be} \) cross section \([6]\).

FIG. 3. The combined fit when the Homestake experimental error is tripled.

FIG. 4. The combined fit when the detector cross section uncertainties are tripled from 3.3\% (Homestake) and 4\% (gallium).
$\frac{\phi(^8B)}{\phi(7Be)} / \frac{\phi(7Be)}{\phi(7Be)_{SSM}}$

\text{Ga}

SSM

$90\% \text{ C.L.}$

Kamiokande

Cl
\[ \frac{\phi(8\text{B})}{\phi(7\text{Be})_{SSM}} \]

- Monte Carlo SSMs
- TC SSM
- Low Z
- Low Opacity
- WIMPs
- Large S\textsubscript{11}

\[ \phi(7\text{Be}) / \phi(7\text{Be})_{SSM} \]

Combined Fit

\[ 90\% \text{ C.L.} \]

\[ 95\% \text{ C.L.} \]

\[ 99\% \text{ C.L.} \]

- S\textsubscript{11} Power Law
- T\textsubscript{C} Power Law

\[ \text{90\% C.L.} \]

\[ \text{95\% C.L.} \]

\[ \text{99\% C.L.} \]
