Three-body analysis of incoherent $\eta$-photoproduction on the deuteron in the near threshold region

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A three-body calculation of the reaction $\gamma d \rightarrow \eta np$ in the energy region from threshold up to $30$ MeV above has been performed. The primary goal of this study is to assess the importance of the three-body aspects in the hadronic sector of this reaction. Results are presented for the $\eta$-meson spectrum as well as for the total cross section. The three-body results differ significantly from those predicted by a simple rescattering model in which only first-order $\eta N$- and $NN$-interactions in the final state are considered. The major features of the experimental data are well reproduced although right at threshold the rather large total cross section could not be explained.

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I. INTRODUCTION

In this letter we would like to present theoretical results for total and differential cross sections of the reaction $\gamma d \rightarrow \eta np$ close to the threshold region. The need for an improved theory for describing the low-energy $\eta$-production on the lightest nuclei stems from the recent precise measurements of such processes on light nuclei using hadronic and electromagnetic probes. The experimental results are substantially larger than a mere phase-space calculation near threshold \cite{1,2}. It is reasonable to associate this effect with a rather strong attractive force which governs the dynamical features of the $\eta NN$-system in the low-energy regime. In a previous paper \cite{3} we found that the $\eta NN$-dynamics allows the existence of virtual s-wave three-body states near zero energy. The natural extension of these findings would be to investigate to what degree such states may influence the processes considering the $\eta NN$-system in different spin-isospin channels. Of particular interest are the reactions of coherent ($\gamma d \rightarrow \eta d$) and incoherent ($\gamma d \rightarrow \eta np$) $\eta$-photoproduction on the deuteron, which recently have become the subject of attention of different experimental collaborations \cite{2,4,5}.

As for the coherent process, the investigation of this channel within a three-body approach has been performed in \cite{6}. Unfortunately, due to the smallness of the coherent $\eta$-photoproduction cross section resulting in extremely small $\eta$-meson yields, only qualitative statements about the size of the experimental cross section of the coherent reaction is available at present.

In this work, we focus our attention exclusively on the incoherent $\eta$-photoproduction where the deuteron breaks up. Recent precision measurements of the break-up channel \cite{2} require a more sophisticated theoretical analysis. Indeed, this reaction has already been considered in a previous paper \cite{7} within the so-called rescattering model as a first step beyond the simple impulse approximation (IA). In this model, final state interaction is taken into account by including the single rescatterings between two of the final particles as most important correction to the IA. As was shown in \cite{7}, without such rescattering effects a strong suppression of $\gamma d \rightarrow \eta np$ cross section was found because the IA requires a large spectator nucleon momenta for producing a low-energy $\eta$-meson on the deuteron. For this reason, the inclusion of first order $\eta N$- and $NN$-interactions provides a mechanism to distribute the high momentum between two nucleons and thus avoids this suppression. In view of the importance of the first-order rescattering it is natural to ask about the role of higher order rescattering terms. Indeed, in a system of three low-energy particles with rather strong attractive forces, one can expect a considerable overlap of their wave functions. Therefore, there is no guarantee that the contributions from higher order $\eta N$- and $NN$-rescatterings will be small. This fact distinguishes the present reaction from those with pions where the smallness of the pion-nucleon scattering length allows one to take into account only the lowest order $NN$-interaction in the final state just above threshold \cite{8}. Thus, the rescattering approximation can hardly be considered as a satisfactory method for describing low-energy $\eta$-photoproduction on light nuclei. It must also be noted that the successive inclusion of the next higher order terms of the multiple scattering

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expansion is not well advised, because the corresponding Neumann series is expected to converge very slowly due to the proximity of the virtual three-body poles mentioned above. As a consequence, the inclusion of terms of quite high order will be required rendering such an approach inadequate. These considerations point to the necessity of a three-body approach, where the final state interaction is included to all orders.

In the present paper, the reaction $\gamma d \to \eta np$ is considered as a three-body problem in the hadronic sector. The formalism is based on a separable s-wave approximation for the two-body potentials. This approach may be justified by the fact that in the energy region under consideration the $\eta N$-resonance $S_{11}(1535)$ as well as the $^1S_0$ and $^3S_1$ poles of the $NN$ scattering matrix determine essentially the driving $\eta N$- and $NN$-forces. Our primary goal is to investigate the main properties of the incoherent $\eta$-photoproduction on the deuteron near threshold. We also compare our results with those predicted by the first order rescattering approximation in order to assess the shortcomings of this method.

In the Sect. II the three-body equations for the $\gamma d \to \eta np$ transition amplitude are derived. In Sect. III we calculate the meson spectrum for a fixed photon energy and compare our results for the total cross section with available experimental data.

II. GENERAL FORMALISM

Here we will briefly describe the main points of the formal part, and comment on the most important approximations involved in our calculation. A more detailed and rigorous development of the formalism as well as the method for the numerical solution of the three-particle integral equations will be presented elsewhere. Firstly, we note that the three-body concept of the $\eta$-photoproduction on a deuteron possesses in principle a straightforward solution of the three-body multichannel problem where the coupling between the channels $\gamma NN$ and $\eta NN$ is taken into account. However, since we will restrict ourselves to the first order only in the electromagnetic interaction, it is more appropriate to obtain the dynamical equations of our model starting from the Faddeev formalism for the pure hadronic $\eta NN$ system. Our approach with respect to the hadronic part is essentially based on the separable approximation scheme of Alt, Grassberger and Sandhas with an isobar ansatz for the $\eta N$ system. As is mentioned above, this ansatz should represent a fair approximation of the real dynamics in the energy region examined here. We use the following notations for the basic three-body channels:

\begin{align}
N^* & \quad \text{for the } \eta N\text{-isobar plus spectator nucleon,} \\
d & \quad \text{for the } NN\text{-isobar plus spectator meson.} 
\end{align}

and analogous notations for the two-body subsystems. Then the separable input

$$t_i = |f_i|\tau_i|f_i| \quad (i = N^*, d)$$

allows one to obtain two coupled operator equations for the rearrangement amplitudes

\begin{align}
X_d &= 2Z_{dN^*}\tau_{N^*}X_{N^*}, \\
X_{N^*} &= Z_{N^*d} + Z_{N^*d}\tau_d X_d + Z_{N^*N^*}\tau_{N^*}X_{N^*}. 
\end{align}

The amplitudes $X_d$ and $X_{N^*}$ determine the transitions from the initial on–energy-shell state $\eta d$ to the quasi-two-body final states $\eta d$ and $NN^*$, respectively. The potentials $Z_{ij}$ have a conventional meaning, namely

$$Z_{ij} = \langle f_i|G_0|f_j\rangle,$$

with $G_0$ being the free three-body Greens function.

Turning now to the photoproduction process $\gamma d \to \eta np$, we treat the electromagnetic interaction perturbatively, keeping only the terms up to first order in the e.m. coupling. Therefore, in order to introduce the electromagnetic part, one just has to replace in (3) the $\eta$-meson by the photon in the entrance channel. This yields then the coupled equations which are formally similar to (3)

\begin{align}
X_d &= 2Z_{dN^*}\tau_{N^*}X_{N^*}, \\
X_{N^*} &= B_{N^*} + Z_{N^*d}\tau_d X_d + Z_{N^*N^*}\tau_{N^*}X_{N^*}. 
\end{align}

Here the “born term” $B_{N^*}$ is determined as

$$B_{N^*} = \langle f_{N^*}^{(\gamma)}|G_0^{(\gamma)}|f_d\rangle.$$


where $G_0^{(\gamma)}$ is the free propagator in the $\gamma NN$-sector and $|f^*_{N^*}\rangle$ is the electromagnetic vertex function for the transition $\gamma N \to N^*$. Equations (3) are represented diagrammatically in Fig. 1. Clearly, the impulse approximation is equivalent to the replacements $X_d \to 0$ and $X_{N^*} \to 0$ in the right-hand sides of (3), i.e.,

\begin{align}
X^1_d &= 0, \\
X^1_{N^*} &= B_{N^*}.
\end{align}

(7a) (7b)

In order to obtain the first-order rescattering approximation, in which only the single $\eta N^*$- and $NN$-scattering terms are retained, one has just to set $X_d = 0$ and $X_{N^*} = B_{N^*}$. Then one has

\begin{align}
X^{\text{resc}}_d &= 2Z_{dN^*} \tau_{N^*} B_{N^*}, \\
X^{\text{resc}}_{N^*} &= B_{N^*} + Z_{N^*} \tau_{N^*} B_{N^*}.
\end{align}

(8a) (8b)

After projecting (3) and correspondingly (4) and (5) onto the $L = 0$ states, we end up with a system of one-dimensional integral equations in momentum space, which can be solved by matrix inversion. The well-known problem of logarithmic singularities by the rearrangement terms $Z_{N^*} \tau_{N^*}$ and $Z_{N^*} \tau_{d}$ was avoided using the contour deformation method of [10].

The transition matrix element for the break-up process $\gamma d \to \eta np$ may be expressed in terms of the rearrangement amplitudes (3) as follows

\begin{align}
\langle \vec{p}_1, \vec{p}_2, \vec{q}|T(W)|\hat{\vec{k}} \rangle &= \langle \vec{p}_1|f_{N^*}\rangle \tau_{N^*}(W - E_{p_2} - \frac{p_2^2}{2M_{N^*}}) \langle \vec{p}_2|X_{N^*}(W)|\hat{\vec{k}} \rangle \\
&+ \langle \vec{p}_2|f_{N^*}\rangle \tau_{N^*}(W - E_{p_1} - \frac{p_1^2}{2M_{N^*}}) \langle \vec{p}_1|X_{N^*}(W)|\hat{\vec{k}} \rangle \\
&+ \langle \vec{p}_2|f_d\rangle \tau_d(W - \omega_{\eta} - \frac{q^2}{4M_N}) \langle \vec{q}|X_d(W)|\hat{\vec{k}} \rangle.
\end{align}

(9)

Here, $W$ denotes the total three-body c.m. energy and $\hat{\vec{k}}$ the photon momentum. Furthermore, the three-momenta of the final nucleons and $\eta$-meson are denoted by $\vec{p}_1$, $\vec{p}_2$ and $\vec{q}$, and their total energies by $E_{p_1}$, $E_{p_2}$ and $\omega_{\eta}$, respectively. The propagators $\tau_{N^*}(E_{N^*})$ and $\tau_d(E_d)$ depend on the invariant energies of the corresponding two-body subsystems. Their explicit form is given in (3). The arguments of the vertex functions $\langle \vec{p}|f_i\rangle$ are the respective two-body relative momenta, whose nonrelativistic expressions read ($i = 1, 2$)

\begin{align}
\vec{p}_m &= \frac{qM_N - \vec{p}_m m_\eta}{M_N + m_\eta}, \\
\vec{p}_i &= \frac{\vec{p}_1 - \vec{p}_2}{2}.
\end{align}

(10)

In the actual calculations, only the s-wave configuration ($J^\pi; T$) = (0$^-$; 1) of the $\eta NN$-system was taken into account. As was discussed in [3], due to the spin-isospin selection rules, this state gives the major contribution to the $\gamma d \to \eta np$ cross section. We also use the nonrelativistic kinetic energies for all three final particles, since we restrict ourselves to the near-threshold region where this approximation is well justified. The contribution from pion exchange in the rearrangement potential $Z_{N^*} \tau_{N^*}$ was ignored in view of its insignificance as was shown in [3]. The parametrization of the hadronic separable t-matrices (3) is given in detail in [3]. For the s-wave singlet and triplet $NN$-states we use the fit of Yamaguchi [1]. The $N^*$ parameters were chosen in such a way that the main hadronic width of the $S_{11}(1535)$ resonance, $\Gamma_{N^* \to \pi N} \approx \Gamma_{N^* \to \eta N} \approx 75$ MeV [3], is reproduced. As for the electromagnetic vertex $|f^*_{N^*}\rangle$, it is natural to associate it with the $\gamma N \to S_{11}(1535)$ helicity amplitude $A_{1/2}^{(N)}$ which is of great experimental importance. Thus one has

\begin{align}
\langle \vec{p}|f^*_{N^*}\rangle &= A_{1/2}^{(N)} \sqrt{\frac{M_{N^*}}{M_{N^*} + M_N}}.
\end{align}

(11)

Since we restrict our consideration to the $\eta NN$-state with isospin $T = 1$, only the dominant isovector part of the $\gamma N \to S_{11}(1535)$ transition has been taken into account. The corresponding amplitude was derived as follows. First we have determined the helicity amplitude in the proton channel $A_{1/2}^{(p)}$ by fitting the experimental cross section data for the elementary process $\gamma p \to \eta p$ of [13]. Then the isovector part $A_{1/2}^{(v)}$ was extracted as $A_{1/2}^{(v)} = 0.9A_{1/2}^{(p)}$ which is compatible with different analyses [3, 13, 14].

Finally, we present here the formula for the exclusive c.m. cross section used in the present calculation
\[
\frac{d\sigma^{c.m.}}{d\omega_\eta} = \frac{1}{(2\pi)^3} \frac{E_d \omega_\eta E_{p_1} E_{p_2}}{W} \frac{1}{2} \left| \left\langle \tilde{p}_1, \tilde{p}_2, \tilde{q} | T | \tilde{k} \right\rangle \right|^2 d\Omega_\eta d\phi_{\eta 1} dE_{p_1},
\]

where \( E_d \) is the deuteron energy and \( \phi_{\eta 1} \) is the azimuthal angle of the nucleon momentum \( \tilde{p}_1 \) counted from the \( \tilde{k} - \tilde{q} \) plane in the frame with the z-axis directed along \( \tilde{q} \). The factor \( \frac{1}{2} \) appears after summing and averaging over the spin projections. The semi-inclusive cross section \( d\sigma^{c.m.} / d\omega_\eta \) and the total cross section \( \sigma_{\text{tot}} \) considered below are obtained from (12) by appropriate integrations.

### III. RESULTS AND DISCUSSION

We would like to begin our discussion with the \( \eta \)-meson spectrum \( d\sigma^{c.m.} / d\omega_\eta \) as function of the \( \eta \) c.m. energy, shown in Fig. 3 for two different photon energies. Its specific form was discussed already in 2. A strong s-wave attraction between the two nucleons at very low relative energy shifts visibly a major part of the spectrum towards higher energies, resulting in a rather pronounced peak near the boundary of the phase space at the maximum of the available meson energy, where the relative energy of the two nucleons is close to zero. It is obviously a manifestation of the large \( NN \) singlet scattering length, i.e., of the antibound \( ^1S_0 \)-state. As our calculation shows, the first-order rescattering approximation underestimates substantially the complete three-body result just above threshold, but overestimates it at higher energies. Although the form of the spectrum is not changed qualitatively, the peak is much more pronounced in the complete calculation.

The difference between the results of the first-order rescattering and the complete three-body calculation is most apparent in the energy dependence of the total cross section shown in Fig. 2, where we compare it also with the inclusive \( \gamma d \rightarrow \eta X \) data from 4. As shown in 4 and noted in the introduction, rescattering gives a very important contribution in particular just above threshold, where it leads to a strong enhancement of the meson yield. However, truncating the multiple scattering expansion after the first order gives only a qualitative description of this effect. Inclusion of rescattering to all orders within the three-body approach results in a considerably stronger effect in the near threshold region so that the energy dependence of the cross section becomes slightly concave (solid curve “1” in Fig. 3), which is not characteristic for photoproduction reactions with more than two particles in the final state. We associate this effect with the virtual \( \eta NN \)-pole found in 3 in the \( (J^P; T) = (0^-; 1) \)-state which pulls down the essential part of the meson yield to the near-threshold region.

When comparing our results with the experimental data we see, first of all, that the three-body calculation constitutes a very important improvement of the theory which reproduces fairly well the qualitative features of the data in the near threshold region up to about 650 MeV photon energies, in particular with respect to the energy dependence. On the other hand, one notes right at threshold still a substantial underestimation of the data by the theory. One reason for this disagreement might be, of course, a contribution of the coherent channel \( \gamma d \rightarrow \eta d \) to the experimental inclusive yield. However, to ascribes the whole difference to the coherent process alone would mean to assign a rather large cross section of about 100 nb at \( E_\gamma = 635 \) MeV to the coherent process. While such a value is not in conflict with the recent measurements of 4 it nevertheless overestimates substantially the corresponding PWIA-calculations for the case \( A_{1/2}^{(v)} = 0.9A_{1/2}^{(p)} \) 7. In order to reach a more definite conclusion one needs, of course, more precise data for the total cross section of the coherent reaction. Finally, with respect to the underestimation of the data at energies above 650 MeV, one should keep in mind that we have included in the calculation only the \( \eta NN \)-state with total orbital momentum \( L=0 \). Thus at higher photon energies, where the higher partial waves are needed to fill the available phase space, our calculation cannot be considered to be realistic.

In order to complete our discussion, we have investigated the sensitivity of the results to the strength of the interaction, i.e., to a variation of the \( N^* \)-parameters. As is shown in 4 the \( \eta NN^* \) and \( \pi NN^* \) coupling constants influence the position of the \( \eta NN \) S-matrix poles rather strongly and thus may also affect the cross section value. Indeed, this is proven by a calculation using different sets of the \( \eta NN^* \)- and \( \pi NN^* \)-couplings, the results of which are also shown in Fig. 3. In order to illustrate the size of the variation of the interaction strength by varying the model parameters, we quote also the corresponding values of the \( \eta N \)-scattering length \( a_{\eta N} \) associated with the choice of the coupling constants. This parameter characterizes the low-energy \( \eta N \)-interaction and is quoted in most analyses of the \( \eta N \)-interaction (for a review of the \( a_{\eta N} \)-values see, e.g. 5). We conclude from Fig 4 that the results are not very sensitive to the choice of \( a_{\eta N} \), so that even with a rather large value of \( a_{\eta N} = (0.91 + i0.25) \) fm (solid curve “2” in Fig. 3), we cannot describe the data sufficiently well.
IV. CONCLUSION

We have presented theoretical results for the cross section of the reaction $\gamma d \rightarrow \eta np$ in the energy region up to 30 MeV above threshold. The calculation is performed within a three-body formalism with s-wave separable $\eta N$- and $NN$-potentials. We found that the first-order rescattering approximation is insufficient, since the full three-body treatment, including the full hadronic final state interaction, enhances significantly at lower energies the $\eta$-meson spectrum while at the higher energies a less pronounced but still sizeable decrease appears. These results demonstrate clearly that any "minimal" realistic model for the $\eta$-production near threshold must be based on a three-body description of the $\eta NN$-dynamics.

Our calculation explains at least qualitatively the anomalous behaviour of the experimental $\eta$-meson yield observed just above threshold. On the other hand, a quantitative agreement with the experimental results is not achieved, even with a rather large $\eta N$-scattering length $a_{\eta N}=(0.91+i0.25)$ lying on the boundary of values allowed by modern analyses. Whether the discrepancy could be explained by the contribution from the coherent channel $\gamma d \rightarrow \eta d$ is at present an open question.

In view of our results on the $\eta$-meson spectrum as presented in Fig. 2, we would like to emphasize the fact that very useful and interesting information on the mechanism of the low-energy $\eta$-photoproduction may be obtained from a measurement of this spectrum. The characteristic feature of the meson spectrum with its sharp, pronounced maximum close to the high energy kinematic limit should make it possible to identify the final state interaction as a major source of an anomalously large cross section of the reaction $\gamma d \rightarrow \eta np$ close to the threshold. On the contrary, if there is no simple explanation for the noted shift between the data and the model predictions, then one has to invoke some additional unexplored mechanism in $\eta$-photoproduction.

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\end{itemize}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{Diagrammatic representation of the three-body equations for $\eta$-photoproduction on the deuteron.}
\end{figure}
FIG. 2. The $\eta$-meson spectrum for the reaction $\gamma d \rightarrow \eta np$ versus the total $\eta$ c.m. energy $\omega_\eta$ for two lab photon energies. Notation of the curves: dotted: impulse approximation (IA), dashed: inclusion of first-order $\eta N$- and $NN$-rescattering, solid: full three-body calculation.

FIG. 3. Total cross section for the reaction $\gamma d \rightarrow \eta np$. Notation of the curves as in Fig. 2. The results obtained within the three-body approach with different sets of $\eta NN^*$ and $\pi NN^*$ couplings are presented as the solid curves “1” and “2”. The corresponding values of the $\eta N$-scattering length $a_{\eta N}$ are given in the legend. The inclusive $\gamma d \rightarrow \eta X$ data are taken from [2].