Renormalization of Hamiltonian QCD

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We study to one-loop order the renormalization of QCD in the Coulomb gauge using the Hamiltonian formalism. Divergences occur which might require counter-terms outside the Hamiltonian formalism, but they can be cancelled by a redefinition of the Yang–Mills electric field.

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1. Introduction

We study the renormalization of QCD in the Coulomb gauge Hamiltonian formalism. By Hamiltonian form, we mean that the Lagrangian contains only first order terms in time derivatives, and depends upon the conjugate momentum field $E_i^a$ as well as the (transverse) gluon field $A_i^a$ (here $a$ is the colour index and $i = 1, 2, 3$ is a 3-vector index). This form has a number of attractive features:

(i) As a Hamiltonian exists, the theory is explicitly unitary, without the necessity to cancel unphysical degrees of freedom with ghosts.

(ii) The Lagrangian form of the Coulomb gauge has “energy divergences” in some of its Feynman integrals, that is integrals of the form (we use $K$ for the spatial part of the 4-vector $k$)
\[ \int d^3 K dK_0 f(K, k_0) \]  

(1)

where \( f \) does not decrease as \( k_0 \rightarrow \infty \) (for fixed \( K \)). These divergences cancel between different Feynman graphs [1], but this cancellation has to be organized “by hand”. In the Hamiltonian form, each individual Feynman graph is free of such divergence. Formally ‘energy divergent’ integrals such as

\[
\int \frac{d^3 p}{(2\pi)^3} \int \frac{dp_0}{(2\pi)^3} \frac{p_0}{p_0^2 - p^2 + i\eta} \times \frac{1}{(P-K)^2}
\]

are assigned the value zero.

(iii) It has been argued [2] that the Coulomb gauge throws light on confinement. Certainly it is known [3] that, in the Coulomb gauge, the source of asymptotic freedom lies in the Coulomb potential.

In spite of (i) above, to 2-loop order, mild energy-divergences remain [4–6] which result in ambiguities which have to be resolved by a prescription. This is connected with questions of operator ordering [7].

For other applications of the Coulomb gauge, for example to lattice QCD, see [8,9].

The question addressed here is the following. Ultra-violet divergences exist which seem to require the existence of counter-terms containing second order terms in time derivatives, \( (\partial \tilde{A}^a_i/\partial t)^2 \). Do these take us out of the Hamiltonian form? We argue that this does not happen because the divergences concerned can be cancelled by a redefinition of the \( A^a_i \) field.

We do not use quite the strict Hamiltonian formalism. We retain the auxiliary field \( A^a_0 \), which contains no time derivatives and should be integrated out to give a nonlocal Coulomb potential term in the real Hamiltonian. It seems to be convenient, for the purposes of renormalization, to retain \( A^a_0 \) in the Lagrangian. Because of this, there is a ghost field, but it has an instantaneous propagator, and so is not relevant to unitarity. Its purpose is only to cancel out closed loops in the \( A^a_0 \) field.

2. The Feynman rules

The Lagrangian for the Coulomb gauge is

\[ \mathcal{L}' = \mathcal{L} - \frac{1}{2\alpha} (\partial_\mu A^a_\mu)^2 \]  

(3)

(where \( \alpha \) will eventually tend to zero to go to the Coulomb gauge),

\[
\mathcal{L} = -\frac{1}{4} F_{ij} \cdot F_{ij} - \frac{1}{2} (E_i)^2 + E_i \cdot F_0i + \partial_0 c^\dagger c + g (\partial_i c^\dagger c + g (A_i \wedge c)) + u_i \cdot (\partial_i c + g (A_i \wedge c))
\]

\[ + u_0 \cdot [\partial_0 c + g (A_0 \wedge c)] - \frac{1}{2} g K \cdot (c \wedge c) + g v_i \cdot (E_i \wedge c) \]

where we use a colour vector notation, and

\[ F_{ij}^a = \partial_i A^a_j - \partial_j A^a_i + g \epsilon^{abc} A^b_i A^c_j \]

and

\[ (A_i \wedge c)^a = \epsilon^{abc} A^b_i c^c \]  

(5)

Here \( c, c^\dagger \) are the ghost fields, and the sources \( u_i, v_n, \) and \( K \) are inserted for future use in formulating the BRST identities. The conjugate momentum (electric) field \( E_m \) could be integrated out to obtain the ordinary Lagrangian formalism, but for the Hamiltonian formalism it must be retained.

We will use indices \( m, n, \ldots = 1, 2, 3 \) to denote the (spatial) components of \( E \), so the seven fields are \( (A_0^a, A_0^0, E_0^a) \). We will use indices \( I, J, \ldots \) to denote the seven indices \( (i, 0, n) \). The bilinear part of the Lagrangian in momentum space is a \( 7 \times 7 \) matrix.
where

\[ T_{ij} \equiv \delta_{ij} - L_{ij}, \quad L_{ij} \equiv K_iK_j/K^2, \quad K^2 = k_0^2 - K^2. \]  

For the propagators, we need the inverse

\[ S_{ij}^{-1} \delta_{ab} = \begin{pmatrix} \frac{T_{ij}}{k_0^2 - \alpha L_{ij}/K^2} & \alpha k_0 K_i/(K^2)^2 & -ik_0 T_{in}/k^2 \\ \alpha k_0 K_j/(K^2)^2 & 1/K^2 + \alpha k_0^2/(K^2)^2 & ik_0 K_n/K^2 \\ ik_0 T_{mj}/k^2 & -ik_0 K_m/K^2 & T_{mn}K^2/k^2 \end{pmatrix}. \]  

We can now let \( \alpha \to 0 \), to obtain the Coulomb gauge. From this, and the interaction terms in the Lagrangian (4), we can read off the Feynman rules. We represent the \( A_i \) field by dashed lines, the \( E_m \) field by continuous lines, and the \( A_0 \) field by dotted lines. With this notation, we now list the rules (a factor of \( \frac{1}{(2\pi)^2} \) is to be included for each propagator, and a factor of \( (2\pi)^4 i \) for each vertex). If we choose the propagators in Fig. 1 to be the negative of the matrix (7), the extra factors of \( \frac{1}{(2\pi)^4} \) for the propagator and \( (2\pi)^4 i \) for the vertices cancel (see Figs. 1–3).

### 3. The ultra-violet divergences

The divergent graphs with 2 and with 3 external lines are shown in Figs. 4–31. Examples of the method of evaluation of divergent parts are given in Appendices A and B.

The ultra-violet divergent parts of these graphs are, in terms of the divergent constant (using dimensional regularization in \( 4 - \epsilon \) dimensions)

\[ i \quad j \quad \rightarrow \quad \frac{1}{k^2 + \eta} \left( \delta_{ij} - \frac{K_i K_j}{K^2} \right) \]

\[ E_m \quad A_j \quad \frac{i \kappa_0}{k^2 + \eta} \left( \delta_{mj} - \frac{K_m K_j}{K^2} \right) \]

\[ A_i \quad \rightarrow \quad \frac{i \kappa_0}{k^2 + \eta} \left( \delta_{im} - \frac{K_i K_m}{K^2} \right) \]

\[ m \quad \rightarrow \quad n \quad \frac{K^2}{k^2 + \eta} \left( \delta_{mn} - \frac{K_m K_n}{K^2} \right) \]

\[ E_m \quad A_0 \quad \frac{i K_m}{K^2} \]

\[ A_0 \quad \rightarrow \quad \frac{i K_n}{K^2} \]

Fig. 1. Feynman rules for the propagators in the Coulomb gauge.
Fig. 2. Feynman rules for the vertices in the Coulomb gauge. The arrows denote the directions of the momenta.

\[
ig f^{abc} \left[ \delta_{ij} (Q - P)_k + \delta_{jk} (R - Q)_i + \delta_{ki} (P - R)_j \right]
\]

\[
-a^2 f^{ace} f^{gold} (\delta_{ij} \delta_{kl} - \delta_{il} \delta_{kj})
\]

\[
-a^2 f^{ade} f^{bec} (\delta_{ik} \delta_{jl} - \delta_{il} \delta_{kj})
\]

\[
-a^2 f^{ade} f^{gce} (\delta_{ik} \delta_{jl} - \delta_{il} \delta_{kj})
\]

Fig. 3. Feynman rules for ghosts and sources in the Coulomb gauge. Doubled lines denote ghosts. The black arrows distinguish between ghosts and anti-ghosts. Momenta flow into the vertex.

\[
-\frac{1}{K^2}
\]

\[
-ig f^{mn} a Q_i
\]

\[
g f^{abc} \delta_{ij}
\]

\[
g f^{abc}
\]
\[
c = \frac{g^2}{16\pi^2} C_G \Gamma(\epsilon/2),
\]

(\text{where the superfix (4) and (5) etc. refers to the corresponding figure and } \Pi_{ij}, \Pi_{00} \ldots \Pi_{nn} \text{ denote self-energies, } V_{ijk}, V_{0i0} \ldots V_{00n} \text{ vertices and } \Lambda \text{ stands for diagrams with external ghost lines), are:}

\[
\Pi^{(4)}_{ij} = i c \left[ \frac{1}{3} k_0^2 \delta_{ij} + K^2 \delta_{ij} - K_i K_j \right] \delta_{ab}
\]

\[
\Pi^{(5)}_{00} = -\frac{1}{3} ic k_0 K_i \delta_{ab}
\]

\[
\Pi^{(6)}_{00} = \frac{1}{3} ic K^2 \delta_{ab}
\]

\[
\Pi^{(7)}_{mi} = 0
\]

\[
\Pi^{(8)}_{m0} = -\frac{4}{3} ic [i K_i \delta_{ab}]
\]

\[
\Pi^{(9)}_{mn} = -\frac{4}{3} ic \delta_{mn} \delta_{ab}
\]

\[
V^{(10)}_{ijk}(p, q, r) = -\frac{1}{3} c \delta_{abc} \left[ (Q - P)_k \delta_{ij} + (R - Q)_i \delta_{jk} + (P - R)_j \delta_{ik} \right]
\]

\[
V^{(11)}_{ijk}(p, q, r) = -\frac{5}{6} c \delta_{abc} \left[ (Q - P)_k \delta_{ij} + (R - Q)_i \delta_{jk} + (P - R)_j \delta_{ik} \right]
\]

\[
V^{(12)}_{ijk}(p, q, r) = -\frac{2}{3} c \delta_{abc} \left[ (Q - P)_k \delta_{ij} + (R - Q)_i \delta_{jk} + (P - R)_j \delta_{ik} \right]
\]

\[
V^{(13)}_{ijk}(p, q, r) = \frac{3}{2} c \delta_{abc} \left[ (Q - P)_k \delta_{ij} + (R - Q)_i \delta_{jk} + (P - R)_j \delta_{ik} \right]
\]

\[
V^{(14)}_{000}(p, q, r) = \frac{1}{4} c \delta_{abc} (R - Q)_i
\]

\[
V^{(15)}_{000}(p, q, r) = -\frac{1}{3} c \delta_{abc} (R - Q)_i
\]

\[
V^{(16)}_{000}(p, q, r) = \frac{1}{3} c \delta_{abc} (R - Q)_i
\]

\[
V^{(17)}_{000}(p, q, r) = \frac{1}{12} c \delta_{abc} (R - Q)_i
\]

\[
V^{(18)}_{0ij}(p, q, r) = 0
\]

\[
V^{(19)}_{0ij}(p, q, r) = \frac{2}{3} c \delta_{abc} (R - Q)_0
\]

\[
V^{(20)}_{0ij}(p, q, r) = -\frac{1}{3} c \delta_{abc} (R - Q)_0
\]

\[
V^{(21)}_{0ij}(p, q, r) = 0.
\]

\textbf{Graphs involving external } E_m \textbf{ line are}

\[
V^{(29)}_{mn0}(p, q, r) = \frac{1}{3} ic \delta_{mn}
\]

\[
V^{(30)}_{mn0}(p, q, r) = -\frac{1}{3} ic \delta_{mn}
\]

\[
V^{(31)}_{mn0}(p, q, r) = 0.
\]

\[\text{Fig. 4. The transverse gluon self-energy graphs.}\]
All other graphs involving external $E_m$-lines are convergent. The divergent parts of graphs with open ghost line are
Fig. 10. Graph contributing to the three-gluon vertex function.

Fig. 11. There are three graphs in this class with permutations of the vertices.

Fig. 12. Graph representing a class of six diagrams.
\[ \Lambda^{(22)ab}(q) = -\frac{4}{3}iCQ^2 \delta_{ab} \]  
\[ \Lambda^{(23)ab}(q) = -\frac{4}{3}CQ_1 \delta_{ab} \]  
\[ \Lambda^{(24)abc}(p, q) = 0 \]  
\[ \Lambda^{(25)abc}(p, q, r) = 0 \]  
\[ \Lambda^{(26)abc}(p, q) = 0 \]  
\[ \Lambda^{(27)abc}(p, q) = 0 \]  
\[ \Lambda^{(28)abc}(p, q) = 0. \]  

**Fig. 13.** There are three graphs in this class of diagrams.

**Fig. 14.** Graph with two external Coulomb lines (there are three diagrams in this class).

**Fig. 15.** There are two graphs in this class.
4. Counter-terms

Let

\[ \Gamma_0 = \int d^4x \mathcal{L}(x) \]

be the original action, \( \Gamma \) be the complete effective action, and let \( \Gamma_1 \) be the effective action to one-loop order. The complete BRST identities are
Fig. 19. Graph contributing to the \((A_i A_j A_0)\) three-point function which contains a three-gluon vertex.

Fig. 20. The \((A_i A_j A_0)\) graph with a three-gluon vertex.

Fig. 21. The \((A_i A_j A_0)\) graph with a four-gluon vertex.

Fig. 22. The ghost self-energy.
Fig. 23. Ghost and the $u_i$ source graph.

Fig. 24. The ghost vertex graph with a $K$ source.

Fig. 25. Graph with external $A_i$, ghost and anti-ghost lines.

Fig. 26. Graph with $u_0$ source, $E$, and $c$ lines.
So to one-loop order
\[ \Gamma_1 * \Gamma_0 + \Gamma_0 * \Gamma_1 \equiv \Delta \Gamma_1 = 0 \]  \hspace{1cm} (38)

So to one-loop order
\[ \Gamma_1 * \Gamma_0 + \Gamma_0 * \Gamma_1 \equiv \Delta \Gamma_1 = 0 \]  \hspace{1cm} (39)
where
\[
A = \frac{\partial \Gamma}{\partial \hat{A}_i} \cdot \frac{\partial}{\partial \hat{u}_i} + \frac{\partial \Gamma}{\partial \hat{u}_i} \cdot \frac{\partial}{\partial \hat{A}_i} + \frac{\partial \Gamma}{\partial \hat{A}_0} \cdot \frac{\partial}{\partial \hat{u}_o} + \frac{\partial \Gamma}{\partial \hat{u}_o} \cdot \frac{\partial}{\partial \hat{A}_0} + \frac{\partial \Gamma}{\partial \hat{c}} \cdot \frac{\partial}{\partial \hat{c}} + \frac{\partial \Gamma}{\partial \hat{K}} \cdot \frac{\partial}{\partial \hat{K}} + \frac{\partial \Gamma}{\partial \hat{v}_i} \cdot \frac{\partial}{\partial \hat{v}_i} + \frac{\partial \Gamma}{\partial \hat{E}_i} \cdot \frac{\partial}{\partial \hat{E}_i}
\]  
(40)
and
\[
A^2 = 0.
\]  
(41)

One class of solutions to this equation is of the form
\[
\Gamma_1^{(i)} = \Delta G,
\]  
(42)
where the allowed form of \( G \) is, in terms of constants \( a_5, \ldots, a_{11} \),
\[
G = a_5 A_i \cdot (u_i + \partial \hat{c}^i) + a_6 A_0 \cdot u_0 + a_7 \cdot K + a_8 E_i \cdot v_i + a_9 v_i \cdot \partial \hat{A}_0 + a_{10} v_i \cdot \partial \hat{A}_i + a_{11} v_i \cdot (A_0 \wedge A_i).
\]  
(43)
Other solutions of Eq. (39) are the explicitly gauge-invariant terms
\[
\Gamma_1^{(ii)} = a_1 (F_{ij})^2 + a_2 E_i \cdot F_{0i} + a_3 (F_{0i})^2 + a_4 (E_i)^2.
\]  
(44)

Finally, by differentiating (38) with respect to the coupling constant \( g \) and specialising to one-loop order, we see that
\[
\Delta \Gamma_1^{(iii)} = 0
\]  
(45)
where \((a_0\) being another divergent constant)
\[
\Gamma_1^{(iii)} = a_0 g \partial \Delta \Gamma_0 \frac{\partial}{\partial g}.
\]  
(46)
Combining these three contributions, we obtain
\[
\Gamma_1 = \Gamma_1^{(i)} + \Gamma_1^{(ii)} + \Gamma_1^{(iii)} = \int d^4x \mathcal{L}_1(x)
\]  
(47)
where
\[
\mathcal{L}_1 = a_1 (F_{ij})^2 + (a_2 + a_8 + a_9) E_i \cdot F_{0i} + (a_3 - a_9) (F_{0i})^2 + (a_4 - a_8) (E_i)^2 + a_5 F_{0i} \cdot \partial \hat{A}_i - (a_5 + \frac{1}{2} a_0) g F_{0i} \cdot (A_i \wedge A_j) - (a_0 + a_5 + a_9) g E_i \cdot (A_i \wedge A_0) + E_i \cdot (a_5 \partial \hat{A}_i - (a_5 \hat{A}_i - a_9 \hat{A}_0) - a_5 (u_i + \partial \hat{c}^i) \cdot \partial \hat{c} + a_0 g \partial \hat{c}^i \cdot (A_i \wedge c) - a_9 u_0 \cdot \partial \hat{c} + a_9 g u_0 \cdot (A_0 \wedge c) - a_7 (u_i + \partial \hat{c}^i) \cdot \{ \partial \hat{c} \}
\]
\[
+ g(A_i \wedge c) \} + a_0 g u_i \cdot (A_i \wedge c) - a_7 u_0 \cdot \{ \partial \hat{c} + g(A_0 \wedge c) \} + \frac{1}{2} g(a_7 - a_9) K \cdot (c \wedge c)
\]
\[
+ (a_0 - a_7) g v_i \cdot (E_i \wedge c).
\]  
(48)
The conditions coming from the vanishing ghost graphs Figs. 24–28 are particularly simple. They fix
\[ a_9 = -a_{10} \]
\[ a_{11} = -g a_9 \]
\[ a_0 = a_7 = -a_6. \] (49)

In order for the counter-terms to cancel the divergences in the other graphs, we require the conditions
\[ 4 a_1 - 2 a_5 = -c \]
\[ 4 a_1 - 3 a_5 - a_0 = \frac{1}{3} c \]
\[ a_3 - a_9 = -\frac{1}{6} c \]
\[ a_6 - a_5 = \frac{4}{3} c \]
\[ a_5 + a_7 = -\frac{4}{3} c \]
\[ a_4 - a_8 = \frac{2}{3} c \]
\[ a_2 + a_5 + a_8 + a_9 = 0. \] (50)

These equations do not fix the constants uniquely. We are free to make some choices. The term \((F_{0i})^2\) in \(\mathcal{L}^{(i)}\) Eq. (44) is not present in the original Hamiltonian form of the Lagrangian (4), so we choose
\[ a_3 = 0. \] (51)

We can also arrange for the combination
\[ -\frac{1}{2} (E_i)^2 + E_i \cdot F_{0i} \] (52)
to appear in \(\mathcal{L}^{(ii)}_1\) as it does in \(\mathcal{L}_0\). This requires (from (50))
\[ a_1 = -\frac{1}{4} c + \frac{1}{2} a_5 \]
\[ a_2 = c - 2 a_5 \]
\[ a_4 = -\frac{1}{2} c + a_5 \]
\[ a_6 = \frac{4}{3} c + a_5 \]
\[ a_7 = -\frac{4}{3} c - a_5 \]
\[ a_8 = -\frac{7}{6} c + a_5 \]
\[ a_9 = \frac{1}{6} c \]
\[ a_0 = -\frac{4}{3} c - a_5 \] (53)

and so
\[ \mathcal{L}^{(ii)}_1 = -4a_1 \left[ -\frac{1}{4} (F_{ij})^2 - \frac{1}{2} (E_i)^2 + E_i \cdot F_{0i} \right] \] (54)

proportional to the non-ghost part of the original Lagrangian (3).

Eq. (54) does not come from the BRST identities, it just emerges from the numerical values of the divergent integrals. It may be a consequence of some hidden Lorentz invariance.

The constants \(a_0, a_1, \ldots\) are still not uniquely fixed. There are two particularly simple choices.
(i) Choose \(a_0 = 0\) with \(a_5 = -\frac{4}{3}c\). Then we find
\[
\begin{align*}
& a_1 = -\frac{11}{12}c \\
& a_2 = \frac{11}{3}c \\
& a_4 = -\frac{11}{6}c \\
& a_6 = a_7 = 0 \\
& a_8 = -\frac{5}{2}c \\
& a_9 = \frac{1}{6}c.
\end{align*}
\] (55)

(ii) The second choice is \(a_1 = 0\) with \(a_5 = \frac{1}{2}c\). Then
\[
\begin{align*}
& a_0 = -\frac{11}{6}c \\
& a_2 = 0 \\
& a_4 = 0 \\
& a_6 = \frac{11}{6}c \\
& a_7 = -\frac{11}{6}c \\
& a_8 = -\frac{2}{3}c \\
& a_9 = \frac{1}{6}c.
\end{align*}
\] (56)

Note that \(a_0\) has the expected value for coupling constant renormalization.

The counter-terms in either case are
\[
\begin{align*}
L_1 &= -\frac{11}{12}c(F^2) - \frac{4}{3}cF_i \cdot \partial_i A_i + \frac{4}{3}cgF_i \cdot (A_i \wedge A_j) - \frac{1}{6}c(F_0)^2 + \frac{2}{3}c(E_i)^2 - \frac{4}{3}cgF_i \cdot \partial_i A_0 \\
&\quad + \frac{4}{3}c(u_i + \partial_i c^*) \cdot \partial_i c.
\end{align*}
\] (57)

The counter-terms in \(a_5, a_6, a_7, a_8\) and \(a_9\) are involved in a rescaling of the fields. Defining
\[
\begin{align*}
A_i' &= (1 + a_5)A_i \\
A_0' &= (1 + a_5)A_0 \\
E_m' &= (1 + a_6)E_m - a_5 F_{0m} \\
u_i' &= (1 - a_5)u_i \\
u_0' &= (1 - a_5)u_0 \\
c' &= (1 - a_7)c \\
K' &= (1 + a_7)K \\
g' &= (1 + a_6)g \\
c'^* &= (1 - a_5)c^* \\
v' &= (1 - a_8)v,
\end{align*}
\] (58)

we have from (48) that
\[
L_0 + L_1 = (1 - 4a_1)L_0(g', A_i', A_0', E', c', c'^*, u_i', u_0', K').
\] (59)
Note that $a_6$ which determines the renormalization of the Coulomb field $A_6^a$ has the same numerical value as $a_0$.

We have not calculated the divergences in graphs with four external lines. We assume they will be cancelled by the same counter-terms.

5. Comments

We conclude that there is no difficulty to one-loop order in renormalizing the Hamiltonian form of the Coulomb gauge. We guess that the renormalization would formally go through to higher orders, but then there is the problem mentioned in [4–6] of combining the renormalization of ultra-violet divergences with the resolution of energy-divergence ambiguities.

It is not quite obvious how the renormalization would be formulated if the $A_6^a$ field had been eliminated to give the non-local colour Coulomb potential (note the non-zero value of the $A_6^a$ field renormalization constant $a_6$ in (56)).

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Appendix A. Here we give as an example the evaluation of the ultra-violet divergent part of the graph in Fig. 20.

\[ V_{0k}^{abc}(q, -q, 0) = ig^3 C_{adj}^{abc} \int d^4p \frac{p_0p_k}{(p^2 + \eta)^2 (q + p)^2 + \eta} \times T_{ij}(P)T_{ji}(Q + P) \times \left[ (-2Q - P)_\nu \delta_{ij} + (Q - P)_\nu \delta_{jk} + (2P + Q)_\delta \right]. \]  (A.1)

Applying the integral

\[
\int dp_0 \frac{p_0}{(p^2 + \eta)^2 (q + p)^2 + \eta} = i\pi^2 \Gamma\left(\frac{5}{2}\right) q_0 \int_0^1 dy y(1 - y) \left\{ (P + yQ)^2 + y(1 - y)(-q^2 - \eta) \right\}^\frac{1}{2} \]  (A.2)

and power counting to (A.1)

\[
V_{0k}^{abc}(q, -q, 0) = -4g^3 C_{adj}^{abc} \sqrt{\pi} q_0 \Gamma\left(\frac{5}{2}\right) \int_0^1 dy y(1 - y) \times \int d^{3-\epsilon}pp_k \left\{ (P + yQ)^2 + y(1 - y)(-q^2 - \eta) \right\}^\frac{1}{2}, \]  (A.3)

leading to

\[
V_{0k}^{abc}(q, -q, 0) = -\frac{1}{3} C_{adj}^{abc} q_0 \delta_{jk}. \]  (A.4)

Appendix B. Example of self-energy evaluation $\Pi_{00}^{abc}$ in Eq. (11). Let $p, q$ be internal and $k$ external momentum, $p - q = k$. The sum of two graphs is

\[
(2\pi)^{-4} \frac{T_{ij}(P)T_{ji}(Q)}{p^2 q^2} \left[ \frac{1}{2} (P^2 + Q^2) - (i p_0)(i q_0) \right] \delta_{ab}. \]  (B.1)
where we have symmetrized the first term in $P, Q$. the minus sign in the second term comes from the opposite order of the $f^{abc}$ factors at the two vertices. Doing the $p_0$ integration by Cauchy, we get

$$\frac{(2\pi)^{-4}}{4} \frac{T_{ij}T_{ji}}{PQ} \frac{1}{(P + Q)^2 - k_0^2} (P + Q)[P^2 + Q^2 - 2PQ] \delta_{ab}. \quad (B.2)$$

The last factor $(P - Q)^2$ is approximately $(P \cdot K)^2 / P^2$. With this factor, the integral is only logarithmically divergent, and to get the divergent part we can put $Q = P$ everywhere. We use $T_{ij}(P)T_{ji}(P) = 2$. Then we get

$$\frac{(2\pi)^{-4}}{4} \frac{2\pi i}{4} K_i K_j \int d^3 x \frac{P_i P_j}{(P^2 + m^2)^{3/2}}. \quad (B.3)$$

So the divergent part is

$$\frac{1}{3} ic K^2 \delta_{ab}. \quad (B.4)$$

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1 Note that there was an error of sign in Eur. Phys. J. C37 (2004) 307–313 which however did not influence the final result.