Dephasing by two-level systems at zero temperature by unitary evolution

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November 2, 2018

Abstract

We analyze the unitary time evolution of a conduction electron, described by a two-level system, interacting with two-level systems (spins) through a spin-spin interaction and prove that coherent spin states of the conduction electron are obtained in the strong coupling regime, when the number of the spins is taken to be, formally, infinitely large (thermodynamic limit). This model describes a spin interacting with a spin-bath in a strong coupling regime and gives a dephasing time at zero temperature that agrees with the recent experimental results for quantum dots. Dephasing is proved to occur as the conduction electron oscillates between the two states with frequency going to infinity in the thermodynamic limit, that is, increasing the number of spins. Then, it is shown that the only meaning that can be attached to such oscillations with infinite frequency is by an average in time, eliminating the off-diagonal terms of the density matrix. This model is in agreement with a recent proposal of appearance of classical states in quantum mechanics due to the large number of components of a quantum system, if properly prepared for the states of each component part. The strong coupling study of the model is accomplished through the principle of duality as recently introduced in perturbation theory [M.Frasca, Phys. Rev. A 58, 3439 (1998)].
1 Introduction

Recent experiments have shown that decoherence at zero temperature can happen in mesoscopic devices as quantum dots and nanowires [1, 2]. Indeed, standard Landau’s theory of Fermi liquids disagrees with such findings and rather agrees with the point of view that, missing any possible exchange of energy between a system and a reservoir, no decoherence should appear at $T = 0$ and any dephasing time $\tau_\phi$ should go to infinity as the temperature goes to zero. Experimentally, it is observed that such a time saturates at well defined values rather than to go to infinity as the temperature decreases. Besides, Ferry et al. [2] have shown that, for quantum dots, the dephasing time saturates going like the inverse of the number of the electrons in the dots.

Several attempts to explain such a disagreement between theory and experiment have been put forward. An approach discussed in Ref. [3] explains the dephasing as an effect of zero-point fluctuations being these the only residual of a quantum system at zero temperature but this approach has undergone severe criticism in Ref. [4]. Vacuum fluctuation effects on dephasing was also studied in Ref. [5]. In Ref. [6, 7] has been proposed to consider the effect of two-level tunneling systems (TLS), representing the impurities in the system, as the cause of dephasing: Recent computation by Mohanty et al. and Altshuler et al. [8] have shown that the density of impurities needed to have the right order of magnitude for the dephasing time is larger than what is found in mesoscopic devices. In the same line of considering TLS as the origin of dephasing, it has been proposed that a two-channel Kondo interaction [7] can be the reason of the saturation. A critical point in this proposal is the value of the Kondo temperature. An evaluation of the Kondo temperature in mesoscopic systems has been given in [9].

The aim of this paper is to give an analysis of a model in the strong coupling regime for TLSs coupled to a conduction electron, and prove that the dephasing is just a property of the unitary evolution of the model in the “thermodynamic limit” where the number of the TLS is formally taken to be increasingly large. This is in agreement with a recent proposal for decoherence as originating from unitary evolution of N quantum systems with N becoming increasingly large [11]. The main point of this approach is that it depends on the way the system is prepared. When quantum fluctuations can be neglected with respect to the mean values then, by the Ehrenfest theorem the system follows
the classical equations of motion without no significant deviation.

The model we analyze has been also discussed, in a different regime and with some minor differences by Hänggi and Shao [1], but here we apply the strong coupling analysis and a different decoherence mechanism to achieve agreement with experiments on quantum dots. Our decoherence mechanism is non dissipative, being derived by the unitary evolution, taking the thermodynamic limit. A similar kind of effect has been recently uncovered in a generalization of the model we consider here [12], but considering a coupling for all the spin components. Decoherence is produced dynamically and no arbitrary division between bath and system has to be done.

We will show that, modelling the conduction electron by a two-level system, with a spin-spin term in the strong coupling regime for the interaction with TLSs, the above scheme for decoherence applies and the correlators for the conduction electron go to zero in the thermodynamic limit, that is, increasing the number of spins without bound. It is interesting to point out that we are considering the case of strong coupling where small perturbation theory does not apply contrarily at the analysis of Mohanty et al. [8]. What we will prove is that the conduction electron evolves in time as a coherent spin state (CSS) [13, 14], that is, a state having mean values following the classical equations of motion and the uncertainty product at the minimum. These states are SU(2) analogous to the coherent states describing a laser field that derives from bosonic operators. What we will show is that the conduction electron, being in the ground state, evolves in time with a coherent spin state when the number of interacting spins is taken to become very large [14]. The electron oscillates between the two states it can access with frequency going to infinity in this limit (thermodynamic limit). A system having a time-scale going to zero or, that is the same, an infinite frequency of oscillations can have physical meaning only if it is averaged in time as we will prove.

The dephasing time we obtain goes like the inverse of the number of spins in the bath and this agrees with the experimental results given in [2] where the spins involved are those of the 2D electron gas in the dot.

In order to study this model in the strong coupling regime we apply the dual Dyson series [15]. This approach has been pioneered by Bender et al. [16] in quantum field theory. These results can be framed in a general formulation given in Ref. [15] that permits to prove the main assert of this paper.
The paper is so structured. In Sec. 2 we introduce the model of spin-spin interaction. In Sec. 3 we give a description of the method of strong coupling perturbation theory or dual Dyson series. In Sec. 4 the perturbative solution of the model in the strong coupling regime is given. In Sec. 5 we give the conclusions with some discussion of the theoretical results we obtain. Finally, in two appendices we discuss about the model of non dissipative decoherence we use and the coherent spin states.

2 A model for the conduction electron in a mesoscopic device

With small modifications with respect to the model of Ref. [11] we take for the conduction electron (here and in the following $\hbar = 1$)

$$H_{el} = \frac{\Omega_0}{2} \sigma_z$$

(1)

being $\Omega_0$ the energy separation of the ground and the excited states. A two-level approximation proves to be a good one also for the study of Kondo models [9, 17]. Besides, devices for representing qubit have also been devised in Ref. [18]. To fix the ideas we assume that we are treating the electron spin but, any other property that can be characterized by two eigenvalues can be used for the following analysis.

In the same way one can write the Hamiltonian of TLSs representing the spin bath

$$H_{TLS} = \frac{1}{2} \sum_{i=1}^{N} \left( \Delta_{xi} \tau_{xi} + \Delta_{zi} \tau_{zi} \right),$$

(2)

being $\tau_{xi}, \tau_{zi}$ the Pauli matrices for the $i$-th TLS, $\Delta_{xi}$ and $\Delta_{zi}$ the parameters of the spins belonging to the bath.

Finally, we hypothesize that the TLSs and a conduction electron interact through an spin-spin term, that is [11]

$$H_K = -J \sigma_x \cdot \sum_{i=1}^{N} \tau_{xi},$$

(3)

being $J$ the strength of the coupling. This kind of approximation is used e.g. in Josephson-junction devices [18, 19] where tunneling is the
main effect. Dephasing by TLSs has been suggested also in this case \[20\]. So, finally, we write the Hamiltonian that we want to analyze as

\[ H = H_{el} + H_{TLS} + H_K = \frac{\Omega_0}{2}\sigma_z + \frac{1}{2}\sum_{i=1}^{N}(\Delta_x \tau_{xi} + \Delta_z \tau_{zi}) - J\sigma_x \cdot \sum_{i=1}^{N} \tau_{zi}. \] (4)

We will assume that the interaction and the electronic terms will prevail on the TLS Hamiltonian. Besides, the initial state of TLSs can be cast in the form

\[ |\chi_0\rangle = \prod_{i=1}^{N} |\lambda_i\rangle \] (5)

with \(\tau_{xi}|\lambda_i\rangle = \lambda_i|\lambda_i\rangle\), being \(\lambda_i = \pm 1\) and also

\[ |\lambda_i\rangle = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}_i + \lambda_i \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}_i \right]. \] (6)

Our aim will be the study of this model in the strong coupling regime, that is, when the interaction term is larger than the TLS Hamiltonian as is the term of the conduction electron. This is the crucial approximation we take in this paper.

### 3 Duality in perturbation theory

The concept of duality in perturbation theory has been introduced in Ref.\[13\]. This idea embodies the pioneering work done by Bender at al.\[16\] in quantum field theory where the kinetic term is taken as a perturbation and use is made of path integral formalism.

This approach can be easily formalized by considering the Schrödinger equation

\[ (H_0 + \lambda V)\psi(t) = i\frac{\partial}{\partial t}\psi(t) \] (7)

being \(\lambda\) a parameter, and \(H_0\) and \(V\) indicate two parts by which the original Hamiltonian has been split. Our aim is to use the freedom in the choice of the two parts to obtain two different perturbation series. Indeed, if \(\lambda \to 0\) one obtains the Dyson series

\[ |\psi(t)\rangle = e^{-iH_0t}Te^{-i\lambda \int_0^t dt'V_I(t')}|\psi(0)\rangle \] (8)

being \(T\) the time ordering operator and

\[ V_I(t) = e^{iH_0t}Ve^{-iH_0t}, \] (9)
having assumed both $H_0$ and $V$ time independent.

For $\lambda \to \infty$, we rescale time as $\tau = \lambda t$ so that eq.(7) becomes

$$\left(\frac{1}{\lambda}H_0 + V\right)\psi(\tau) = i\frac{\partial}{\partial \tau}\psi(\tau)$$

(10)

and so, by this rescaling, we have reverted the role of $H_0$ and $V$ giving a Dyson series with the development parameter $\frac{1}{\lambda}$

$$\psi(t) = e^{-iV\tau}Te^{-\frac{i}{\lambda}\int_0^\tau d\tau' H_0F(\tau')}\psi(0)$$

(11)

being now

$$H_{0F}(\tau) = e^{iV\tau}H_0e^{-iV\tau}.$$  

(12)

This perturbation series is dual to the series eq.(8) being its development parameter $\frac{1}{\lambda}$ that is the inverse of the one of the series eq.(8). But this has been obtained by a symmetry of the original Hamiltonian where one part or the other can be chosen arbitrarily: This is duality in perturbation theory. Indeed, a series can be obtained from the other by the interchange $H_0 \leftrightarrow V$, setting $\lambda = 1$. It is interesting to note that, if the initial condition is e.g. an eigenstate of $H_0$, the Dyson series gives a trivial leading order yielding the same eigenstate multiplied by a phase factor while the dual Dyson series gives a non trivial one being the exponential factor $e^{-iV\tau}$ multiplied by the eigenstate of $H_0$. The argument can be reversed but this simmetry in the perturbation series can be broken by the way a quantum system is initially prepared. In quantum mechanics, also for strong perturbations, it is always taken as initial state the one of the unperturbed system.

It is quite easy to recover the result of Bender et al.\cite{16} by taking for $V$ the kinetic term of a $\lambda \phi^4$ theory.

It is interesting to point out here that some condition on the boundedness of the domains of the operators $H_0$ and $V$ should be extended from the Dyson series to its dual as some of the most important operators in quantum mechanics are unbounded. In the following this problem will be of no concern as our operators act on a finite dimensional Hilbert space.

4 Solution in the strong coupling regime

Our aim is to show how non dissipative decoherence, as described in Appendix \ref{app1}, appears by the dynamics of model (4). We will see
that coherent spin states are the solution at the leading order (see Appendix B for a description of spin coherent states).

From the Hamiltonian (4) we can easily get the strong coupling expansion by taking

\[ H_0 = \frac{1}{2} \sum_{i=1}^{N} (\Delta_{xi} \tau_{xi} + \Delta_{zi} \tau_{zi}) \]  

\[ V = \frac{\Omega_0}{2} \sigma_z - J \sigma_x \cdot \sum_{i=1}^{N} \tau_{xi} \]  

and the dual Dyson series is given by

\[ U_D = U_0(t) \left[ 1 - i \int_0^t H_{0F}(t') dt' - \int_0^t dt' \int_0^{t'} dt'' H_{0F}(t') H_{0F}(t'') + \cdots \right] \]  

where

\[ U_0(t) = \exp \left[ -it \left( \frac{\Omega_0}{2} \sigma_z - J \sigma_x \cdot \sum_{i=1}^{N} \tau_{xi} \right) \right] \]  

and

\[ H_{0F}(t) = U_0(t) \left[ \frac{1}{2} \sum_{i=1}^{N} (\Delta_{xi} \tau_{xi} + \Delta_{zi} \tau_{zi}) \right] U_0^\dagger(t) \]  

that is, a really non trivial result already at the leading order if the initial state is an eigenstate of \( H_0 \) giving in this case \( U_0(t) \) multiplied by this same state.

Firstly, let us evaluate the leading order result. By the disentanglement formula (77) of Appendix B, we obtain immediately

\[ U_0(t) = \exp \left( -i \hat{\Lambda}(t) \sigma_+ \right) \exp \left( -\ln \hat{\Sigma}(t) \sigma_3 \right) \exp \left( -i \hat{\Omega}(t) \sigma_- \right) \]  

being the operators

\[ \hat{\Lambda}(t) = \frac{-J \sum_{i=1}^{N} \tau_{xi} \sin(\hat{\Omega} t)}{\Omega \hat{\Sigma}(t)} \]  

\[ \hat{\Sigma}(t) = \cos(\hat{\Omega} t) + i \frac{\Omega_0}{2\hat{\Omega}} \sin(\hat{\Omega} t) \]  

\[ \hat{\Omega}(t) = \sqrt{\frac{\Omega_0^2}{4} + J^2 \left( \sum_{i=1}^{N} \tau_{xi} \right)^2} \]  

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We note that these operators are diagonal on the environment state and so we have to compute\cite{21}

\[ \bar{U}_D(t) = \langle \chi_0 | U_D(t) | \chi_0 \rangle \] (22)

that is, assuming e.g. \( \sum_{i=1}^{N} \tau_{x_i} | \chi_0 \rangle = -N | \chi_0 \rangle \), at the leading order

\[ \bar{U}_0(t) = \exp (-i \Lambda(t) \sigma_) \exp (-\ln \Sigma(t) \sigma_3) \exp (-i \Lambda(t) \sigma_-) \] (23)

and

\[ \Lambda(t) = \frac{NJ \sin(\Omega t)}{\Omega} \Sigma(t) \] (24)
\[ \Sigma(t) = \cos(\Omega t) + i \frac{\Omega_0}{2\Omega} \sin(\Omega t) \] (25)
\[ \Omega(t) = \sqrt{\frac{\Omega_0^2}{4} + (NJ)^2}. \] (26)

Instead, the first order term is given by

\[ -i \bar{U}_0(t) \int_0^t \langle \chi_0 | H_0 | \chi_0 \rangle dt' \] (27)

that is,

\[ i \bar{U}_0(t) \frac{N}{2} \Delta_x t \] (28)

a secular term with \( \Delta_x = \frac{1}{N} \sum_{i=1}^{N} \Delta_{x_i} \), unbounded in the limit \( t \to \infty \). This terms can be resummed (see Ref.\cite{22}) to give an exponential correction to the leading order \( \exp \left( i \frac{N}{2} \Delta_x t \right) \).

At the second order one has

\[ -\bar{U}_0(t) \int_0^t dt' \int_0^{t'} dt'' \bar{U}_0^\dagger(t') \langle \chi_0 | H_0 U_0(t') U_0^\dagger(t'') H_0 | \chi_0 \rangle \bar{U}_0(t'') \] (29)

To compute this term we note that

\[ \sum_{i=1}^{N} \Delta_{x_i} \tau_{z_i} | \chi_0 \rangle = \sum_{i=1}^{N} \Delta_{x_i} | \chi_0 \rangle \] (30)

where

\[ | \chi_0 \rangle = |-1 \rangle_1 |-1 \rangle_2 \cdots |-1 \rangle_i \cdots |-1 \rangle_N \] (31)
that is, (30) is a non normalized state orthogonal to $|\chi_0\rangle$. Then, after some algebra one gets
\begin{equation}
-\frac{1}{2}\frac{N^2}{4}\Delta_x^2 t^2 \hat{U}_0(t)
- \frac{1}{4}\sum_{i=1}^{N} \Delta_x^2 \hat{U}_0(t) \int_{0}^{t} dt' \int_{0}^{t'} dt'' e^{i t'} \left[ \frac{\Omega_0}{2} \sigma_z \right] e^{-i t'} \left[ \frac{\Omega_0}{2} \sigma_z + (N-1) J \sigma_x \right] (32)
\end{equation}
\begin{equation}
e^{i t''} \left[ \frac{\Omega_0}{2} \sigma_z + (N-1) J \sigma_x \right] e^{-i t''} \left[ \frac{\Omega_0}{2} \sigma_z + NJ \sigma_x \right]. (33)
\end{equation}
The first term is just the second order term of the Taylor series that enters into the resummation of $\exp \left( i \frac{N}{2} \Delta_x t \right)$ and can be eliminated.

If $N \gg 1$ the second term can be evaluated. It gives
\begin{equation}
- \frac{1}{4} \sum_{i=1}^{N} \frac{\Delta_x^2}{J^2} \hat{U}_0(t) \left( 1 - e^{i J t \sigma_z} + i J t \sigma_x \right) (34)
\end{equation}
and again we obtain a secularity. This can be resummed as done for the other one but we do not pursue this aim here. Rather, we note that for consistency reasons we need to have $\sum_{i=1}^{N} \frac{\Delta_x^2}{J^2} \ll 1$, otherwise we miss convergence. This condition should be kept in the thermodynamic limit $N \to \infty$ and realizes the feature of this strong coupling expansion at this order. It is interesting to note that the contribution $\Omega_0$ of the conduction electron plays no role in the thermodynamic limit and this is in agreement with the experimental results given in [2]. Indeed, in such a case we have
\begin{equation}
\hat{U}_0(t) \approx \exp \left( -i t N J \sigma_x \right) (35)
\end{equation}
that using the relation $\sigma_x = \sigma_+ + \sigma_-$ proves to be the operator generating a coherent spin state as given in eq.(57) being now $\zeta = -i t N J$.

If the conduction electron is in the extremal state $\left( \begin{array}{c} 0 \\ 1 \end{array} \right)$, we obtain
\begin{equation}
Rabi oscillations with frequency $\Omega_R = 2 N J$.
\end{equation}

Our aim is to see if, in the limit $N \to \infty$, a meaning can be attached to the above evolution operator. We are requested to give a sense to the limits
\begin{equation}
\lim_{N \to \infty} \cos(Nx) (36)
\end{equation}
\begin{equation}
\lim_{N \to \infty} \sin(Nx) (37)
\end{equation}
and this can be done if we rewrite the above as the values of divergent series. Indeed, one has
\begin{equation}
\cos(Nx) = 1 - \int_{0}^{Nx} \sin(y)dy (38)
\end{equation}
\[
\sin(Nx) = \int_0^{Nx} \cos(y) dy \tag{39}
\]

that we can reinterpret for our aims through the Abel summation to divergent integrals as

\[
\lim_{\epsilon \to 0^+} \lim_{N \to \infty} \int_0^{N_x} e^{-\epsilon y} \cos(y) dy \tag{40}
\]

\[
\lim_{\epsilon \to 0^+} \lim_{N \to \infty} \int_0^{N_x} e^{-\epsilon y} \sin(y) dy, \tag{41}
\]

where the order with which the limits are taken is important. These are proper constructions for the thermodynamic limit in this case and give 0 and 1 respectively so that, the evolution operator in the thermodynamic limit can be taken to be zero. This is the correct meaning to be attached physically to functions having time scales of variation going to 0 (instantaneous variation). Indeed, whatever physical apparatus one uses to make measurements on such a system, unavoidably an average in time will be made that, in this case, gives zero.

This conclusion is fundamental for the density matrix that, in this way, turns out to have the off-diagonal terms averaged to zero and the diagonal ones to \( \frac{1}{2} \). This is decoherence, as promised, in the thermodynamic limit. For the sake of completeness, we report here the case of the density matrix. We have for the conduction electron in the ground state

\[
\rho(t) = \exp \left( -itNJ\sigma_x \right) \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \exp \left( itNJ\sigma_x \right) \tag{42}
\]

that yields

\[
\rho_{1,1}(t) = \frac{1 - \cos(2NJt)}{2} \tag{43}
\]

\[
\rho_{1,-1}(t) = -i \frac{1}{2} \sin 2NJt \tag{44}
\]

\[
\rho_{-1,1}(t) = \frac{1}{2} \sin 2NJt \tag{45}
\]

\[
\rho_{-1,-1}(t) = \frac{1 + \cos(2NJt)}{2}. \tag{46}
\]

Applying the above argument in the limit \( N \to \infty \) gives the required result. From the above matrix elements we find a natural time scale for the model for \( N \gg 1 \), that is \( \tau_0 = \frac{\pi}{NJ} \), that, as already said, goes to zero in the thermodynamic limit.
The differences with the case of quantum dissipative systems\cite{24} can be straightforwardly understood. A quantum dissipative system is generally characterized by substituting sums with integrals and doing hypothesis on the spectral function for the energy of the bath. In this way dissipation is recovered in a form of a Langevin equation. In our case, decoherence appears without dissipation as a dynamical process by the unitary evolution, solving directly the Schrödinger equation for the many-body problem and checking directly the solution. In this way, no ad hoc hypothesis is needed on the bath that now is taken together with the system as a single system whose evolution has to be studied. Decoherence at zero temperature is then obtained.

An analog situation for the bosonic case is discussed in Ref.\cite{10}.

5 Discussion and Conclusions

The above analysis shows that, in a strong coupling regime, if the number of spins belonging to the bath becomes increasingly large, one has firstly that the motion of the conduction electron enters into a coherent spin state oscillating between the two states we have supposed it can access. Secondly, if the thermodynamic limit, as defined through the resummation technique of Abel, is taken, then, the conduction electron is localized in one of the two states with probability $\frac{1}{2}$. So, the TLSs act like a classical measurement apparatus erasing the interference terms of the density matrix that, in this way, takes a mixed form.

Such an approach can give an explanation to dephasing at zero temperature recently observed in quantum dots\cite{2}. Indeed, we have derived a time scale for the oscillations of the conduction electron, that is, the period $\tau_0 = \frac{\pi \hbar}{N J}$ (having restated the Planck constant). But this time should be the same of the saturated values experimentally observed, i.e. a few nanoseconds. This, in turn, means that $N J \approx \frac{k_B T_0}{0.1 K}$ the saturation temperature and so, the order of magnitude of energy is a few of $\mu eV$. This result is in agreement with the saturation time for quantum dots with $N$ being the number of spins (electrons) found in the dot.

The most important conclusion is that in this paper we have given a consistent set of methods to approach the study of a model in the strong coupling regime where, in the thermodynamic limit, effects of decoherence may appear in the zero temperature case. The model
agrees with recent experiment on quantum dots.

Appendices

A Classical states by unitary evolution

The main result we present here, based on the results of Ref.[10], is that, $N$ TLSs, in the thermodynamic limit, understood as the formal limit $N \to \infty$, behaves, if their states are properly disordered, as a classical system and so, a two-level system interacting with such a bath undergoes decoherence[25]. The critical point is that the classical behavior of such N-TLSs system emerges or not depending on the way the system is prepared. The question pointed out in Ref.[10, 25] is that the most realistic situation is the one having the N-TLSs in the state

$$|\chi_0\rangle = \prod_{i=1}^{N} (\alpha_i | -1 \rangle_i + \beta_i | 1 \rangle_i)$$

(47)

being

$$|\alpha_i|^2 + |\beta_i|^2 = 1$$

(48)

and

$$\tau_{zi} | \pm 1 \rangle_i = \pm | \pm 1 \rangle_i$$

(49)

being $\tau_{zi}$ the Pauli matrix for the i-th component of the N-TLSs. The TLS Hamiltonian by itself, without interactions, can be diagonalized and, for the sake of simplicity, we assume all the parameters to be not dependent on the number $i$ of the given TLS, as, in this way we cannot expect the result to change too much. So, the Hamiltonian of the TLSs can be cast in the form

$$H = \lambda \sum_{i=1}^{N} \tau_{zi}.$$  

(50)

having $\lambda = \sqrt{\Delta^2_x + \Delta^2_z}$. Then, the system evolves in time as

$$|\chi_0(t)\rangle = \prod_{i=1}^{N} (\alpha_i e^{i\lambda t} | -1 \rangle_i + \beta_i e^{-i\lambda t} | 1 \rangle_i)$$

(51)
It can be proven that

\[ \frac{\Delta H}{\langle H \rangle} \propto \frac{1}{\sqrt{N}} \tag{52} \]

being \((\Delta H)^2\) the variance computed on the state \(|\chi_0(t)\rangle\), or, that is the same, on the state \(|\chi_0(0)\rangle\) and \(\langle H \rangle\) the mean computed on the same state. So, quantum fluctuations are negligible in the thermodynamic limit. The same can be said for the spin components \(\Sigma_x = \sum_{i=1}^{N} \tau_{xi}\) and \(\Sigma_y = \sum_{i=1}^{N} \tau_{yi}\) that obeys the classical equations of motion by the Ehrenfest theorem

\[
\langle \dot{\Sigma}_x \rangle = -2\lambda \langle \Sigma_y \rangle \tag{53}
\]

\[
\langle \dot{\Sigma}_y \rangle = 2\lambda \langle \Sigma_x \rangle \tag{54}
\]

without any significant deviation due to quantum fluctuations. This is exactly the behavior of a coherent spin state\([14]\): quantum and classical dynamics coincide when the Hamiltonian is a linear combination of the generators of the symmetry group from which the coherent states originate and the system is found in the extremal state, to be defined later, or in a coherent state. In both these cases, the evolution of the quantum system is described by a coherent state. It is important to emphasize that we have described an unitary evolution and no dissipative effect is really involved. Anyhow, it should be emphasized that taking the thermodynamic limit is a fundamental step in our approach.

**B Coherent spin states**

We limit our analysis to the case of a SU(2) group for a spin \(1/2\) particle whose generators can be set through the three Pauli matrices \(\sigma_{+}, \sigma_{-}\) and \(\sigma_{z}\) having the algebra

\[
[\sigma_{+}, \sigma_{-}] = \sigma_{z} \tag{55}
\]

\[
[\sigma_{z}, \sigma_{\pm}] = \pm 2\sigma_{\pm}. \tag{56}
\]

Then, a coherent spin state can be defined as \([13, 14]\)

\[
|\frac{1}{2}, \zeta\rangle = e^{\zeta \sigma_{+}} |\frac{1}{2}, -\frac{1}{2}\rangle \tag{57}
\]

being \(\sigma_{z} |\frac{1}{2}, -\frac{1}{2}\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle\) the lower eigenstate said an extremal or ground state, and \(\zeta\) a complex number. Analogously, one has
If the Hamiltonian is built in such a way to be a linear combination of the generators of the SU(2) group, the time evolution of a coherent state gives again a coherent state. If the initial state is an extremal one, then the state evolves into a coherent state. In both these cases, there is no difference between classical and quantum dynamics. What really we have done is just to rotate the extremal state on the plane x-y by an angle $\theta$ around the axis $(\sin \phi, -\cos \phi, 0)$. This can be realized by setting $\zeta = \frac{\theta}{2} e^{-i\phi}$ into eq.(57). In terms of the eigenstates $|\frac{1}{2}, \frac{1}{2}\rangle$ and $|\frac{1}{2}, -\frac{1}{2}\rangle$ one can write as for bosonic coherent states

$$|\frac{1}{2}, \zeta\rangle = \frac{1}{(1 + \tau \tau^*)^2} \exp(\tau \sigma_x)|\frac{1}{2}, -\frac{1}{2}\rangle = \frac{1}{(1 + \tau \tau^*)^2} \left(|\frac{1}{2}, -\frac{1}{2}\rangle + \tau |\frac{1}{2}, \frac{1}{2}\rangle\right)$$

being

$$\tau = \tan \frac{\theta}{2} e^{-i\phi} = \frac{\zeta \sin |\zeta|}{|\zeta| \cos |\zeta|}$$

(58)

(59)

This description of a spin $\frac{1}{2}$ through a rotated state is a particular case of a more general concept that coherent spin states, to be similar to a bosonic coherent state of N particles, describe an assembly of N spin $\frac{1}{2}$ particles.

A simple way to generate a coherent spin state is given by a spin $\frac{1}{2}$ particle in a constant magnetic field with Hamiltonian given by

$$H = -\frac{1}{2} g \mu_B B_x \sigma_x - \frac{1}{2} g \mu_B B_y \sigma_y$$

(60)

that has a time evolution operator

$$U = \cos (\Omega t) + i \frac{B_x \sigma_x + B_y \sigma_y}{\sqrt{B_x^2 + B_y^2}} \sin (\Omega t)$$

(61)

being $\Omega = \frac{1}{2} g \mu_B \sqrt{B_x^2 + B_y^2}$. Then, if we apply the above operator on the extremal state $|\frac{1}{2}, -\frac{1}{2}\rangle$ we get

$$|\frac{1}{2}, \zeta\rangle = \cos (\Omega t) |\frac{1}{2}, -\frac{1}{2}\rangle + i \frac{B_x - i B_y}{\sqrt{B_x^2 + B_y^2}} \sin (\Omega t) |\frac{1}{2}, \frac{1}{2}\rangle.$$ 

(62)

So, choosing

$$\tau = i \frac{B_x - i B_y}{\sqrt{B_x^2 + B_y^2}} \tan (\Omega t)$$

(63)
we realize that we have built a coherent spin state. It is important to note that in this case the extremal state is not an eigenstate of the Hamiltonian (60). Besides, these are minimum uncertainty states for the rotated components by an angle $\phi$. That is, if we define

$$\langle \sigma_{\eta} \rangle = \langle \sigma_x \rangle \cos \phi + \langle \sigma_y \rangle \sin \phi$$

$$\langle \sigma_{\xi} \rangle = -\langle \sigma_x \rangle \sin \phi + \langle \sigma_y \rangle \cos \phi$$

and being

$$\langle \sigma_x \rangle = \cos \phi \sin(2\theta)$$

$$\langle \sigma_y \rangle = \sin \phi \sin(2\theta)$$

$$\langle \sigma_z \rangle = -\cos(2\theta)$$

then

$$\langle \sigma_{\eta} \rangle = \sin(2\theta)$$

$$\langle \sigma_{\xi} \rangle = 0$$

and finally

$$(\Delta \sigma_{\eta})^2 = \cos^2(2\theta)$$

$$(\Delta \sigma_{\xi})^2 = 1$$

$$(\Delta \sigma_z)^2 = \sin^2(2\theta).$$

This means that the uncertainty relations become

$$\Delta \sigma_{\eta} \Delta \sigma_{\xi} = \frac{1}{2} |\langle [\sigma_{\eta}, \sigma_{\xi}] \rangle| = |\langle \sigma_z \rangle|$$

$$\Delta \sigma_{\eta} \Delta \sigma_z = \frac{1}{2} |\langle [\sigma_{\eta}, \sigma_z] \rangle| \geq |\langle \sigma_{\xi} \rangle| = 0$$

$$\Delta \sigma_{\xi} \Delta \sigma_z = \frac{1}{2} |\langle [\sigma_{\xi}, \sigma_z] \rangle| = |\langle \sigma_{\eta} \rangle|.$$

For the special case of $\sigma_{\eta}$ and $\sigma_z$ these can be seen as classical commuting variables. To complete this section we give some disentanglement formulas. The following relation is true

$$\exp \left( \lambda_+ \sigma_+ + \lambda_- \sigma_- + \frac{\lambda_3}{2} \sigma_3 \right) = \exp(\Lambda_+ \sigma_+) \exp \left[ -\frac{1}{2} (\ln \Lambda_3) \sigma_3 \right] \exp(\Lambda_- \sigma_-)$$
where

\[ \Lambda_3 = \left( \cosh \alpha - \frac{\lambda_3}{2\alpha} \sinh \alpha \right)^{-2} \] (78)

\[ \Lambda_{\pm} = \frac{2\lambda_{\pm} \sinh \alpha}{2\alpha \cosh \alpha - \lambda_3 \sinh \alpha} \] (79)

\[ \alpha^2 = \frac{1}{4} \lambda_3^2 + \lambda_+ \lambda_- \] (80)

For the particular case of \[ \lambda_{\pm} = i\theta \] and \[ \lambda_3 = 0 \] one gets

\[ \exp[i\theta(\sigma_+ + \sigma_-)] = \exp[i(\tan \theta)\sigma_+] \exp\left[ -\frac{1}{2} \ln(\cos^2 \theta) \sigma_3 \right] \exp[i(\tan \theta)\sigma_-]. \] (81)

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