RATE OF GROWTH OF DISTRIBUTIONALLY CHAOTIC FUNCTIONS

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Abstract. We investigate the permissible growth rates of functions that are distributionally chaotic with respect to differentiation operators. We improve on the known growth estimates for $D$-distributionally chaotic entire functions, where growth is in terms of average $L^p$-norms on spheres of radius $r > 0$ as $r \to \infty$, for $1 \leq p \leq \infty$. We compute growth estimates of $\partial / \partial x_k$-distributionally chaotic harmonic functions in terms of the average $L^2$-norm on spheres of radius $r > 0$ as $r \to \infty$. We also calculate sup-norm growth estimates of distributionally chaotic harmonic functions in the case of the partial differentiation operators $D^\alpha$.

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