Abstract. We design a device employing a couple of star-gearing mechanisms attached to both sides of a wheelchair to climb and descend stairs mainly but also other obstacles, giving independence and mobility to the user. The star system proportions high maneuverability and stability under several functionality conditions. The kinematical analyses use Lagrange formulation, employed to study the star-system performance over a rough plane and ascending and descending stairs. We illustrate the viability of the design performing the stability studies during the ascent or descent of stair.

1. Introduction

Physical disability is a problem that when affected people are in their current economic edge, represent a significant disadvantage concerning their independent activities. According to the INEGI, in Mexico, almost 5% of the population has some disability [1]. Between the causes of disability are inherent illnesses, old age, injuries from accidents and in minor proportion, problems at birth. Between those people with disabilities, 58% requires for their independence the use of a wheelchair. This data means that a wheelchair is a fundamental need for their mobility and personal development in the society. Most beyond the rejection that the user faces to rehabilitate and participate into the cultural environment and society is the acceptation by part of the society itself, which gives not an opportunity to develop their capabilities in front of the failure of the medicine to cure people with this injury. Until the medical technological advances improve, a suitable device is essential to living independently and realizes their life’s daily activities.

Nowadays, the wheelchairs are specialized devices, interested in how to perform the control of the motion without establishes a burden on the rider and not only for moving recovering or aged patients use. They are tools that proportionate independence into the mobility and autonomy, not for stay confined in their homes or for rehabilitation institutions compelling. Such improvement requires designing evolitional wheelchairs that give independence and autonomy, converting them in an extension of disabled people to develop their activities of the quotidiant life. The innovation in the design of wheelchairs involves the interplay between cultural needs, technology, and adaptability to the environment [2]. The conventional wheelchairs have mobility and excellent performance on many
surfaces or even terrain, but they do not have the capacity for mobility in stairs or with some small obstacles. At this respect, some systems have been developed, with the capacity of stair climbing; a few more are under development to construct a robot capable to climbing and descending steps slopes and stairs [3, 4].

The actual devices use trajectory tracks control or implement a wheel-track locomotive system [5]. Other uses a sophisticated high-cost hybrid locomotion system (like IBOT4). Nevertheless, the several systems used until now, in Mexico, we have no one to save such barriers and obstacles and remain a great problem for wheelchair users [6]. In our proposal, our system consists of a frame with two star-gearing mechanisms attached, in which there are two servomotors commanded by a simple electronic device permitting an automated control of tridimensional gyroscopes coupled to the axis of the chair. In Brazil, a similar device has been already constructed independently [7]. With the analyses we lay, the foundations to establish dimensional and power features to develop the system for loading and control that fulfill all norms and regulations, more accessible and economical to build up, than other sophisticated and expensive systems. We include all variables and degrees of freedom to manipulate the system through the use of Lagrangian analyses. Was developed a conventional wheelchair 3D computational model to simulate the free down-hill fall in a straight ladder. A 3D star-system also was simulated to the same situation. In both cases, is studied the rider’s GC. Also was developed the stability analyses during the ascent of the stairs employing the theorem of Krasovskii, minimizing the constructed Lyapunov function.

2. Mathematical model
Is know that sharp changes of acceleration during the management of a wheelchair represent a high risk for the patient, is duly considered, for this reason, a path control as essential. The main details of the kinematical model can found in references [8, 9]. The employed model is illustrated in figure 1, is furnished with two star-gearing mechanisms attached on both sides, and for running stabilizing the system uses two casters as front wheels. We use the midpoint \( P_0(x_0, y_0) \) in the device as the reference control point, where is fixed the reference axis system \( \hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}} \), with respect the inertial reference system \( \hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}} \).

The radius of the wheels is called \( a \), \((\hat{V}_w, \hat{\omega}_w)\) are the linear velocities and angular velocities of the wheels, \( L_1 \) the width of the device as is shown in figure 2. As a first consideration is that no tire slippage happens, then the regular rigid body kinematics study can be done. After performing the corresponding Lagrange analyzes, we found the control rules for the rough plane and ascend motion (the complete calculation can found in references [8, 9]), which is a relation between the torques applied to the wheels and the equivalent inertial matrix. The solution of the differential equations is performed by Runge-Kutta method for the position in any instant of time \( (x, y, \theta) \). It is shown the relevant results in the simulations that appear in the figures 3 and 4. The control laws give us the plausible safe movement of the device, time for the advocate for the stability in the ascent of stairs. The control can be achieved using constraints or forces.

3. Stability
We are going to use servo-constraints using the control satellite’s technology or spacecraft. The use of gyrostats is crucial because it does not modify the distribution of mass in the device. We employed the technique of Lyapunov for stabilizer our device studying the equilibrium position of the rider’s CG. The idea to control the motion is to choose moments of internal forces. The complete analyzes can found in references [9, 10]. To study the system, we use two reference systems. One attached to the static inertial frame \( \{\hat{\mathbf{r}}\} = (x_1, x_2, x_3) \) and the other called \( \{\hat{\mathbf{b}}\} = (\xi, \eta, \zeta) \) used to describe the moving system coincident with the main principal inertial axis of the system. The relation between both is a direction cosines matrix \( \alpha_{ij}, i, j = 1, \ldots, 3 \), which will be used to equilibrate the rider’s GC

\[
\begin{pmatrix}
\xi \\
\eta \\
\zeta
\end{pmatrix} =
\begin{pmatrix}
\alpha_{11} & \alpha_{12} & \alpha_{13} \\
\alpha_{21} & \alpha_{22} & \alpha_{23} \\
\alpha_{31} & \alpha_{32} & \alpha_{33}
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix}.
\]
**Figure 1.** Squeem of the star-gearing device.

**Figure 2.** The two reference systems \((\vec{K}, \vec{J}, \vec{R})\), attached to the device and the inertial one \((\vec{I}, \vec{J}, \vec{R})\).

**Figure 3.** Schematic representations of the GC and velocity evolution down-hill stair descend of the regular wheelchair.

**Figure 4.** Depicted the behavior of star system under the same conditions to compare.
Figure 5. Schematic of the star device ascending a step of the straight stair.

Figure 6. Example of the results obtained after the optimization of the parameters. From left to right on top, we depict the behavior of the trend $U_i$ control momentums of the gyro rotors systems versus $\tau$. In the following two squares appears the parameters $\eta_i(\tau)$ vs. $\tau$. In last appear $\xi_i(\tau)$ vs. $\tau$. The following lines give us the cyclic limit for the behavior of $\eta_i$ vs. $U_1$, $\xi_i$ vs. $U_1$ (of reference [10]).
The gyroscopes are employed to control our device, assuming one for each principal axis of inertia generating attitude control moments upon the vertical position. Their rotation axes are coincident also with the device principal axes. \( (I_i, \theta_i) \) represent the moments of inertia of the gyrostat and their angular velocity respectively, whose relative motion give rise to the control moments

\[
I_1(\dot{\omega}_1 + \dot{\varphi}_1) = u_1 - k_1\dot{\varphi}_1  \\
I_2(\dot{\omega}_2 + \dot{\varphi}_2) = u_2 - k_2\dot{\varphi}_2  \\
I_3(\dot{\omega}_3 + \dot{\varphi}_3) = u_3 - k_3\dot{\varphi}_3.
\]

The rotational dynamics come from \( \{\dot{\theta}\} \), is expressed by the Euler Equations and are characterized by \( (A_i, \omega_i) \) the principal moments of inertia and angular velocity of the device

\[
A_1(\dot{\omega}_1) + (A_1\omega_2 + I_3\dot{\varphi}_3)\omega_2 - (A_2\omega_2 + I_2\dot{\varphi}_2)\omega_3 + I_1\dot{\varphi}_1 = M_1  \\
A_2(\dot{\omega}_2) + (A_2\omega_1 + I_1\dot{\varphi}_1)\omega_3 - (A_3\omega_3 + I_3\dot{\varphi}_3)\omega_1 + I_2\dot{\varphi}_2 = M_2  \\
A_3(\dot{\omega}_3) + (A_3\omega_2 + I_2\dot{\varphi}_2)\omega_1 - (A_1\omega_1 + I_1\dot{\varphi}_1)\omega_2 + I_3\dot{\varphi}_3 = M_3.
\]

The optimal stabilization function includes all changing parameters (see figure 5), requiring define an efficiency indicator to build the Lyapunov function and minimize it through the values of control momentums employing the Krasovskii theorem (the complete calculations can found in references [9, 10]). It is shown some results in figure 6.

\[
Ef = \frac{1}{4}\int_0^\infty \left( 4A_1\xi_2^2 + \frac{1}{A_1} \left[ \sigma_3\eta_{32} - \sigma_2\eta_{23} + v_1 \left( \sum_{i=1}^3 h_i\eta_{i1} - A_1\xi_1 \right) - U_1 \right]^2 + 4A_2\xi_2^2 \\
+ \frac{1}{A_2} \left[ \sigma_1\eta_{13} - \sigma_3\eta_{31} + v_2 \left( \sum_{i=1}^3 h_i\eta_{i2} - A_2\xi_2 \right) - U_2 \right]^2 + 4A_3\xi_3^2 \\
+ \frac{1}{A_3} \left[ \sigma_2\eta_{21} - \sigma_1\eta_{12} + v_3 \left( \sum_{i=1}^3 h_i\eta_{i3} - A_3\xi_3 \right) - U_3 \right]^2 \right) dt.
\]

4. Results and discussions

In the simulations realized using the Visual Nostran program it was layout a straight stair with a high of 1.6 m, consisting of eight 20 cm steps. Concerning the study of the velocities in the corresponding three axes of symmetry appears surprising result. For example, the X component of the velocity results in an immediately crashing cause for the descent of stairs, precipitating it tower front, with sudden rotation respect the axes of the steps, then instantaneously rotate respect the Y and Z axes, provoking a complete not control in the descent. The depicted pick in the figures 3 establishes the moments in which the wheelchair impact with the steps, changing their orientation. Otherwise, in figure 4 the star system is maintained almost constant during their evolution in the descent since the X rotation coincides with the star device’s rotation. The descent is stable without sudden changes in the Y and Z rotation axis during the three seconds of their trajectory afterward; also the X displacements are nulls. To balance the system, we take advantage of the servo-moments impelled by the gyrostats, incorporating a stabilizer impetus to maintain the vertical equilibrium position of the raider’s GC, enabling changes in the direction of motion of the device. Our strategy was deriving the stabilization control laws under the Lagrangian methods; this means that we can manage stabilization concerning energy. Such control laws should remain in Lagrangian form establishing a closed-loop dynamics for our device, as a guide. Also, the Lyapunov-based method to analyze stability will permit us to achieve the equilibrium in an asymptotically stable way when are included the constraints forces of the gyroscopes as dissipative controls. The Lyapunov function also gives us information concern how to choose the controls gains to become a closed-loop of stability. Thus, is performed the conservation of
energy into the closed loop between the energy of the device and the control forces. So far, with fixed gains, the input controls not need to be so large to acquire the stabilization. In this way, the new terms appear in the equation of motion just as desired control directions. For achieve reliability, is used the theory of optimal control and the Krasovskii’s theorem to choose optimally the tread control momentums showed in figure 6, where are depicted examples of numerical experiments for few seconds of reaction, although a fraction of seconds is available for the gyrostats. Thus, is ensured the stabilization with minimum cost in energy. Also, is checked that the steady-states rotation is asymptotically stable according to with Lyapunov method under the closed-loop. A few numerical simulations were done to check the validity of the calculations. Is shown the controlled orientation vectors in figure 6, where the behavior of the different components of director cosines that determines the deviation between both reference systems is displayed. Also, are depicted the cycle limit corresponding to the respective involving angles concerning the momentum \( U_1 \).

5. Conclusions

Thanks to the stabilization control laws under the Lagrangian methods and the numerical simulations, the star-gearing device was assessed during the transit through straight ladders as viable and their control as feasible. Using the control theory and the Lyapunov functions, we developed a closed-loop of stabilization based on gyroscopes and evaluated them with numerical experiments (was done an example for 3 seconds, but can be resolved for every appropriate time needed). Is obtained an optimal stability state, in which the patient’s safety during their transit upon stairs was the priority.

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