Tuneable frequency up-conversion based on biased asymmetric coupled quantum well structure

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Abstract. The behaviours of the optical nonlinear susceptibility $\chi^{(3)}$ responsible for the phase-conjugate beam of frequency conversion in non-degenerate four-wave mixing (NDFWM) are studied for a biased asymmetric coupled quantum well (ACQW) structure. It is shown that the frequency up-conversion peak position determined by $\chi^{(3)}$ is very sensitive to the external inverse electric field strength applied among the grown direction of quantum well but its value is insensitive to that. In other words, the frequency up-conversion peak has a large shift but its value maintains a constant when the electric field strength increases in a small bias range. The characteristics of the tuneable-frequency and the power balance of the ACQW structure may provide the high-efficient wavelength conversion in the optical communication system.

1. Introduction

Asymmetric coupled quantum well (ACQW) structures [1, 2] have been an interesting issue for using optoelectronic devices because the coupling effect between the two wells can not only provide a large quantum-confined Stark shift of the transition energy [3], a large refractive index change and a large second-order optical nonlinearity [4], but also allow for the nonlinear resonances beyond terahertz detuning rates in non-degenerate four-wave mixing (NDFWM) [5].

In general, the NDFWM mechanism is physically Bragg scattering from the index gratings results from the beating between the input pump- and probe-beams for bulk semiconductor materials [6]. Such gratings are associated with the modulation of both carrier density occupation probability in each (sub)-band at the pump-probe detuning rate $\delta = |\nu_p - \nu_i|$, where $\nu_p$ and $\nu_i$ are the frequencies of pump- and probe-beams, respectively, carrier-density pulsations [7] provide the most efficient scattering mechanism; however, they are unfortunately limited by the recombination lifetime $\tau$ [8], which is typically on the order of 100ps, so that the effectiveness of a Bragg reflection signal of large detuning rate via resulting gratings rapidly decreases at detuning rates in excess of several tens gigahertz [9]. In this sense, the ACQW structure is propitious to use as transparent all-optical-wavelength converters [10] and phase-conjugate generators [11]. In this paper, it is discussed that the third-order optical nonlinear susceptibility $\chi^{(3)}$ responsible for the phase-conjugate beam when the external inverse electric field strength along the grown direction of quantum well changes. It is shown that the resonant peak position of $\chi^{(3)}$ in the frequency domain is not only controlled by the inverse electric field but also its peak value may maintain a constant in a

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small bias range. From a view of application, the ACQW structure as a high-efficient wavelength converter in the optical communication system can realize tuneable-frequency and power balance for converted signal.

2. Coupled quantum well structure and wave functions

ACQW structure considered consists of a wide well (WW), a narrow well (NW) and a thin barrier, as shown in figure 1(a). The widths of WW, NW are \( L_w \) and \( L_n \), respectively, and the thickness of the barrier layer between the two wells and \( b \). If the \( b \) is thin enough, so that the significant overlap of electronic wave functions in the two wells may take place at bias, the lowest electronic states \(|1\rangle\) and \(|2\rangle\) are respectively the transition angular frequencies between the first heavy-hole sublevel in the NW and that in the WW, and \( \omega_1 \) and \( \omega_2 \) are the transition angular frequencies from \( |0\rangle \) to \(+\rangle\) and \( -\rangle\), respectively, and \( \omega_0=\frac{\omega_1+\omega_2}{2} \) is the central angular frequency of the upper two delocalized states \(+\rangle\) and \(+\rangle\) relative to a lower localized state \( |0\rangle \).

![Figure 1](image_url)

Figure 1. Level diagrams of coupled quantum well structure: (a) is in uncoupled case, a double well structure, where \( \omega_1 \) and \( \omega_2 \) are respectively the transition angular frequencies between the first heavy-hole sublevel in the NW and that in the WW, and (b) is in coupled case, a ACQW, as an inverse static electric field is applied along the grown direction of quantum well, where \( \omega_1 \) and \( \omega_2 \) are the transition angular frequencies from \( |0\rangle \) to \(+\rangle\) and \( -\rangle\), respectively, and \( \omega_0=\frac{\omega_1+\omega_2}{2} \) is the central angular frequency of the upper two delocalized states \(+\rangle\) and \(+\rangle\) relative to a lower localized state \( |0\rangle \).

and the thickness of the barrier layer between the two wells and \( L_b \). If the \( L_b \) is thin enough, so that the significant overlap of electronic wave functions in the two wells may take place at bias, the lowest electronic states \(|1\rangle\) and \(|2\rangle\) are mixed into the new stationary states \(+\rangle\) and \(-\rangle\), and the heavy-hole may be viewed as a ground state due to large effective mass, as shown in figure 1(b). As a good approximation, the two delocalized states \(+\rangle\) and \(+\rangle\) in the conduction band are written, respectively, as [12]

\[
|+\rangle = \frac{1}{\sqrt{2}} (|1\rangle+|2\rangle) ,
|\rangle = \frac{1}{\sqrt{2}} (|1\rangle-|2\rangle)
\]

with energy eigenvalues as \( E_- \) and \( E_+ \). The corresponding angular frequency splitting \( \omega_\times = (E_- - E_+)/\hbar \) may be written as

\[
\omega_\times = (\hbar \kappa / m_L) \exp(-\kappa L)
\]

(2)

where \( \kappa = 2\pi / \lambda \), \( \lambda = 2\pi \hbar / \sqrt{2m(U-\bar{E})} \) is the de Broglie wavelength of the electron tunneling, \( \bar{E} = (E_\text{ew} + E_\text{en})/2 \) is the average quantum-confined energy of the electronic ground state in the two wells at zero bias and \( U \) is the barrier height depending on the biased electric field strength \( F \) along the growth direction of quantum well. The well depth at bias may be approximately written as \( U = \Delta E_{\text{CB}} + eFL_b \), where \( \Delta E_{\text{CB}} \) denotes the well depth at zero bias.
and $eFL_n$ is a triangular potential, as a result of the external electric field applied along the growth direction of quantum well.

In the single particle state approximation, the lowest one-dimensionless quantum-confined wave function $\varphi_{\alpha}(z)$ for electron ($\alpha = e$) or heavy-hole ($\alpha = h$) in the two isolated wells satisfies the stationary Schrödinger equation [13]

$$\left\{-\frac{\hbar^2}{2m_\alpha} \frac{\partial^2}{\partial z^2} + \left[ V(z) \pm eFz \right] \right\} \varphi_{\alpha}(z) = E_\alpha \varphi_{\alpha}(z),$$

where $+$, $-$ correspond to electron and heavy hole, respectively, and $V(z)$ is the potential of quantum well. One assumes the electronic or heavy-hole effective mass in the well layer is the same that in the barrier layer and $V(z) = 0$. Solving Eq.(3), $\varphi_{\alpha}(z)$ is given by linear combinations of the Airy function $\text{Ai}(\ldots)$ and $\text{Bi}(\ldots)$

$$\varphi_{\alpha}(z) = c_\alpha \text{Ai}(\eta_\alpha) + \tilde{c}_\alpha \text{Bi}(\eta_\alpha)$$

with the argument as $\eta_\alpha$ linearly depending on the energy eigenvalue $E_\alpha$ and the position coordinate $z$ at the quantum central position as zero:

$$\eta_\alpha = -\left[ \frac{2m_\alpha}{(\varepsilon hF)^2} \right]^{1/3} \left( E_\alpha \mp eFz \right).$$

$c_\alpha$ and $\tilde{c}_\alpha$ are determined by the boundary condition of the single quantum well as shown in Figure 1.

3. Third-Order Nonlinear Optical Susceptibility

In the basis of both the density matrix approach and the rotting-wave approximation [14], the third-order optical nonlinear susceptibility $\chi^{(3)}$ responsible for the phase-conjugate beam via NDFWM is

$$\chi^{(3)}(\omega_i; -\omega_1, -\omega_2, \omega_3) = -\frac{i|\mu_{\alpha}|^4}{\varepsilon_0 \hbar^4 V^3 \gamma_{-\alpha}} \frac{1}{\gamma_{-\alpha} - i(\omega_0 + \omega_3 / 2 - \omega_2)}$$

$$\times \left[ \frac{1}{\gamma_{-\alpha} - i(\omega_0 + \omega_3 / 2 - \omega_2)} \gamma_{0\alpha} + i(\omega_3 - \omega_2) - \frac{1}{\gamma_{-\alpha} - i(\omega_0 - \omega_3 / 2 - \omega_2) \gamma_{+\alpha} + i(\omega_3 - \omega_2)} \right]$$

$$+ \frac{1}{\gamma_{+\alpha} + i(\omega_0 + \omega_3 / 2 - \omega_2)} \gamma_{0\alpha} + i(\omega_3 - \omega_2) - \frac{1}{\gamma_{+\alpha} + i(\omega_0 + \omega_3 / 2 + \omega_2) \gamma_{+\alpha} + i(\omega_3 - \omega_2)}.$$
Here \( \omega_1 \) and \( \omega_2 \) are the angular frequencies of pump- and probe-beam, respectively; \( \varepsilon_0 \) is the vacuum dielectric constant, \( V \) is the effective volume of well and barrier layers, \( \gamma_i \) is the energy relaxation rate of the \( i \)-th sub-band and \( \gamma_{ij} \) is the dephasing rate between the sub-levels \( |i\rangle \) and \( |j\rangle \). \( \mu_{0-} \) and \( \mu_{0+} \) are the optical dipole matrix elements from the heavy-hole state \( |0\rangle \) to the two delocalized electronic states \( |\rightarrow\rangle \) and \( |\rightarrow+\rangle \), respectively; the terms associated with \( |\mu_{0-}|^4 \) and \( |\mu_{0+}|^4 \) are the nonlinear optical absorption and dispersion arising from the transitions \( |0\rangle \rightarrow |\rightarrow\rangle \) and \( |0\rangle \rightarrow |\rightarrow+\rangle \), respectively, and the terms with \( |\mu_{0+}|^2 \) and \( |\mu_{0+}|^2 \) make an extra contribution to the optical nonlinearity because of the coupling between the two wells.

Making use of representation transformation, the optical dipole transition elements \( \mu_{0-} \) and \( \mu_{0+} \) in the representation \( \{ |0\rangle, |+\rangle, |\rightarrow\rangle \} \) is transformed as \( \mu_{10} \) and \( \mu_{20} \) in the single quantum well representation \( \{ |0\rangle, |1\rangle, |2\rangle \} \). It is

\[
\mu_{10} = \frac{1}{\sqrt{2}} (\mu_{10} + \mu_{20}), \quad \mu_{0-} = \frac{1}{\sqrt{2}} (\mu_{10} - \mu_{20}),
\]

where \( \mu_{10} \) and \( \mu_{20} \) represent the optical dipole matrix element of the transitions \( |0\rangle \rightarrow |1\rangle \) and \( |0\rangle \rightarrow |2\rangle \), respectively. Thus \( \mu_{0-} \) and \( \mu_{0+} \) are attributed to the calculation of \( \mu_{10} \) and \( \mu_{20} \) in the single quantum well representation.

3. Analysis of results

From expression (4), it is known that \( \chi^{(3)} \) depends not only on \( \gamma_i \) and \( \gamma_{ij} \) but also depends on \( \omega_1 \), \( \mu_{0-} \) and \( \mu_{0+} \). In general, the values of \( \gamma_i \) and \( \gamma_{ij} \) are treated by the screening of the Coulomb and Fröhlich interactions in the fully dynamic random-phase approximation [15], which is related to the temperature of the sample, the impurities, surface roughness and concentration fluctuation. One assumes \( \gamma_i \approx 2.0 \text{ps}^{-1} \) and \( \gamma_{ij} \approx 2.4 \text{ps}^{-1} \) for \( \text{In}_{1-x}\text{Ga}_x\text{P} - \text{Ga}_x\text{In}_{1-x}\text{As} \) ACQW structure in the condition of low temperature. By using the band-gap formulae

\[
E_{g_{\text{In}_{1-x}\text{Ga}_x\text{P}}} = 1.35 + 0.668x + 0.758x^2 \quad \text{and} \quad E_{g_{\text{In}_{1-x}\text{Ga}_x\text{As}}} = 0.35 + 0.629x + 0.436x^2,
\]

we have

\[
E_{g_{\text{In}_{1-x}\text{Ga}_x\text{P}}} = 1.829 \text{eV} \quad \text{and} \quad E_{g_{\text{Ga}_x\text{In}_{1-x}\text{As}}} = 0.74 \text{eV}.
\]

Correspondingly, the effective electronic and heavy-hole masses are \( m_e \approx 0.08 m_0 \) and \( m_{hh} = 0.609 m_0 \), respectively; the spin-orbit splitting is \( \Delta = 89 \text{meV} \). The band-gap offset satisfies \( \Delta E_g = E_{g_{\text{Ga}_x\text{In}_{1-x}\text{As}}} - E_{g_{\text{In}_{1-x}\text{Ga}_x\text{P}}} \).
\[ \Delta E_{cb} + \Delta E_{vb} \]. If the ratio of conduction band offset to valence one is 6:4, the \( \Delta E_{cb} \approx 635\text{meV} \) and the \( \Delta E_{vb} \approx 436\text{meV} \). The lowest quantum-confined energies of electron are \( E_e^W \approx 21\text{meV} \) for the WW with \( L^W = 15\text{ nm} \) and \( E_e^N \approx 47\text{meV} \) for the NW with \( L^N = 10\text{ nm} \), respectively. Similarly, the energies of heavy-hole are \( E_h^W \approx 2.8\text{meV} \) for the WW and \( E_h^N \approx 3.7\text{meV} \) for the NW. Thus, the energy difference between the electronic states in coupling and the heavy-hole state is obtained as \( E_0 \approx (E_{1e}^W + E_{1h}^N)/2 + (E_{1h}^W + E_{1h}^N)/2 \approx 0.78\text{eV} \), corresponding to the transition frequency \( \nu_0 = \omega_0 / 2\pi = 188.3\text{THz} \).

Our interest is that the frequency up-conversion peak changes with external electric field strength. Figure 2(a) denotes the dots of the frequency up-conversion peak determined by \( |\chi^{(3)}| \) with respect to electric field strength and Figure 2(b) is the peak value changes with electric field strength. As seen easily, the peak intensity of the large detuning signal via NDFWM maintains approximately a constant for a small bias range from 0 to -15keV/cm. Apparently, the ACQW structure as a tuneable up-frequency converter has the characteristic of output power balance for the phase-conjugate beam.

4. Conclusions

The third-order optical nonlinear susceptibility responsible for the phase-conjugate beam in NDFWM is analyzed for a biased ACQW structure. It is shown that the detuning of the frequency up-conversion may be in excess of 5THz based on the In\textsubscript{0.532}Ga\textsubscript{0.468}P-Ga\textsubscript{0.468}In\textsubscript{0.532} As ACQW structure consisting of a wide well of 15nm width, a narrow well of 10nm width and a barrier of 4nm width. Also, the resonance peak of the frequency up-conversion maintains approximately a constant and goes with the tuneable frequency shift in excess of gigahertz when the inverse electric field strength changes from 0 to 15keV/cm. This means that the ACQW structure as a tuneable all-optical wavelength converter of output power balance has a potential application in the optical communication system.
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