Hadron masses from a Kaluza-Klein like Model

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Abstract

The purpose of this paper is to calculate the masses of the hadrons. More precisely the masses of the scalar and vector mesons as well as the baryons having $\frac{3}{2}$ and $\frac{1}{2}$ spin are calculated. The standard model of particle physics, delivering excellent results in many aspects of particle physics, as the particles decay rates or the anomalous magnetic moment, fails at calculating particle masses. Therefore, for this paper a Kaluza-Klein like model was developed based on the structure of the Standard Model. The model consists of 10 dimensions which are: one time, three usual macroscopic space and six compactified dimensions. Excitations - disturbances traveling with the speed of light on the 10D spacetime - are introduced. An excitation on a compactified dimension can induce a mass in 4D spacetime; it is accompanied by an integer, the excitation number, and has a well-defined spin. The model’s free parameters are computed using the measured masses of the charged leptons, including the upper bound of the mass difference between electron and positron, the mesons $\pi^0, \pi^+, \rho^0, \phi, \psi, \Upsilon$, the top-quark mass, and the anomalous magnetic moment of the electron. The most important of the derived parameters are the compactification radius $\rho$, the weak coupling $\alpha_w$, the strong coupling $\alpha_s$ and the anti-neutrino to neutrino density ratio $\delta$. The formulas for calculating hadron masses are given and applied to approximately one hundred composite particles, which are compiled within four separate tables. The comparison between the measured masses of 64 hadrons and the calculations shows relative errors below 0.05 for 50% and below 0.1 in 85% of all cases. Pearson’s correlation coefficient of masses measured versus calculated is $r = 0.99$ with a significance of $p < 10^{-56}$. The calculation results of another 36 hadron masses, not yet experimentally determined, are stated.

Keywords: hadron masses, Kaluza-Klein theory, geodesic equation, Standard Model, antineutrino/neutrino density, neutrino mass

1 Introduction

The Standard Model of particle physics [1-7] is one of the most successful theories of physics. The strength of the model is its ability to explain the forces
between particles as the exchange of bosons [8] whereby the photon in case of electromagnetism, the 8 gluons for the strong and the \( W^+, W^- \) and the \( Z \) for the weak interaction operate as mediators. One of the model's greatest successes is the calculation of the anomalous magnetic dipole moment of the electron through quantum mechanical corrections, as first introduced by Julian Schwinger in 1948 [9] with the first order correction on the predictions of the Dirac equation. Later, higher order corrections established the agreement between theoretical and experimental results of up to 12 significant digits [10].

There are however weaknesses such as the well-known failure to provide any information on dark matter and dark energy [11]; the inability to describe gravity; and the inability to calculate the hadron masses. Thus far, it appears that there is no room within the Standard Model for providing information with regards to dark matter and dark energy. For the missing gravitational interaction, some extensions in the form of quantum gravity [12] with the spin2 graviton that serves as an exchange boson, have been suggested. An additional shortcoming is the model's inability to calculate meson and baryon masses, the particles representing ordinary matter, and the unstable particles created in high-energy particle smashing experiments.

The Standard Model scenario involves the acting of various fields within the flat 4-dimensional spacetime. General relativity [13], on the other hand, uses curved spacetime to describe gravity on a larger scale - the main theory of cosmology [14]. But what if instead of looking for a graviton as the force creating boson of quantum gravity, the alternative could be a multidimensional spacetime to describe the strong, the electromagnetic and the weak interactions? The first attempt dates back to the year 1921, when Theodor Kaluza [15, 16] presented a 5-dimensional model to include electromagnetism. The quantum aspect was introduced in 1926 by Oskar Klein [17, 18]. A summary on the temporary development was published in 1987 by Appelquist, Chodos and Freund titled "Modern Kaluza-Klein Theories" [19]. In 1999 another summary [20] was published with a similar title by Paul Wesson. The update came in 2019 [21].

One of the focal points of many papers dealing with Kaluza-Klein theories concentrate on the inclusion of electromagnetism in 5-dimensional spacetime. These developments frequently involve the construction of a 5-dimensional line element and the composition of the respective Einstein tensor. Higher dimensional approaches come with string theory [22-24]. While string theory introduces a rich mathematical methodology, it is unable to deliver results that can be compared with data gathered from experiments. The Standard Model comes with the quantum structure of elementary particles which is capable of calculating the decay probabilities of particles [25]. However, a general method for calculating hadron masses is still lacking [26, 27]. In addition, in order to describe the features and capabilities, a method should be found within the framework of a multi-dimensional spacetime in which to portray particle physics.

Is it reasonable to exchange the robust Standard Model for something that so far has been able to describe gravitation very well while at the same time be unable to contribute to the very small, particle physics? And is it possible that the Standard Model and a multidimensional relativity both be meaningful
and able to produce results? More precisely, what if both approaches represent two sides of the same coin, such as in the sense of dualism? The famous double slit experiment [28] that displays wave and particle nature at the same time points towards such a possibility. Therefore, within this article a model is introduced which describes the outlines of the Standard Model in terms of a "Kaluza-Klein like theory": an approach for looking at the Standard Model from a different perspective, and not as a contradictory method. The following development consists of ten deformable dimensions: one time, three macroscopic space and another 6 compactified space dimensions. Within all dimensions, an excitation, a temporary deformation of this 10-dimensional canvas, can always move with the speed of light. In the usual 4D space, an excitation is identified as a gravitational wave. In a compactified dimension, a stable excitation has a well-defined length, depending on the dimension’s radius. The actual radii of the six compactified dimensions are able to change dynamically depending on the specific excitations, which are identified as particles. During the development of the model, the explicit spacetime structure is given. The next step is an explanation of the three interactions and the corresponding formulas for calculating particle masses. The particles are identified in terms of the 6 compactified dimensions. Afterwards, the model’s free constants are determined; the most important are the compactification radius $\rho$, the weak coupling $\alpha_{\text{w}}$, the strong coupling $\alpha_{\text{s}}$ and the neutrino $\nu$ to anti-neutrino $\bar{\nu}$ distribution ratio $\delta$. For this purpose, the measured masses of the leptons, the mesons $\pi^0, \pi^+, \rho^0, \phi, \psi, \Upsilon$, the top-quark mass (as given by the Particle Data Group [29]) and the magnetic moment of the electron are used. From here onwards, there are no more free parameters available. Then, the calculation rules for the scalar and vector mesons; and for the spin $3/2$ and $1/2$ baryons are stated. The masses of approximately 100 hadrons will be calculated and compared with the experimental data, if available. The relative error between the measured and the calculated mass is $< 0.05$ for most of the comparable hadrons.

2 Model

2.1 Embedding and container space

The start point is a 10-dimensional space consisting of 1 time and 9 space dimensions. The time and three space dimensions constitute the usual spacetime. The other six space dimensions are compact dimensions. For calculational reasons, this 10-dimensional space is embedded within a 20-dimensional container space. The connections between the physical 10D space $x^i$ and the container
space \( y^i \) is given as:

\[
\begin{align*}
  y^0 &= ct \\
  y^1 &= x^1 \\
  y^2 &= x^2 \\
  y^3 &= x^3 \\
  y^i &= r^i \cdot \cos (\varphi_i) = r^i \cdot \cos \left( \frac{x^i}{r^i} \right) \\
  y^{10} &= 0 \\
  y^{11} &= 0 \\
  y^{12} &= 0 \\
  y^{13} &= 0 \\
  y^{i+10} &= r^i \cdot \sin (\varphi_i) = r^i \cdot \sin \left( \frac{x^i}{r^i} \right)
\end{align*}
\]

As such, \( r^i \) is the radius of the compact dimension with indices \( i = 4, 5, 6, 7, 8, 9 \).

The container space has a simple flat metric of the form

\[
\eta_{ab} = \text{diag}(1, -1, -1, 1, 1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1, -1, -1, -1, -1)
\]

with \( a \) and \( b \) from 0 to 19.

The connection between the 10-dimensional physical space and the 20-dimensional container space is given as

\[
g_{\mu\nu} = \frac{\partial y^a}{\partial x^\mu} \frac{\partial y^b}{\partial x^\nu} \eta_{ab}
\]

This results in a spacetime signature of \((1 \, -1 \, -1 \, 1 \, 1 \, 1 \, 1 \, 1 \, -1 \, -1 \, 1 \, 1 \, 1 \, -1 \, -1 \, -1 \, -1)\) - see appendix 8.9.

The 6 additional compactified dimensions are subdivided into sections for the purpose of: the strong interaction - dimensions 4 to 6; the electric interaction - dimension 7; and the weak interaction - dimensions 8 and 9.

### 2.2 The angular momentum of a stable excitation on a compactified dimension

A particle is a combination of associated excitations, which waves around some of the six compact tubes forming one unit. The signal velocity around a tube is always \( c \), the speed of light. A full revolution around a tube (double value) is

\[
4\pi r^i = n^i \lambda^i
\]

with \( n^i \) being an integer. The wavelength is connected to the speed of light and the frequency \( v \) in the usual way.

\[
c = v \cdot \lambda
\]
The momentum of an excitation is
\[ p^i = \hbar k^i \] (6)
whereby \( k^i = 2\pi/\lambda^i \) is the wave vector. With these definitions, the spin (angular momentum) of an excitation on one tube is
\[ s^i = r^i \times p^i = r^i \times \frac{2\pi\hbar}{\lambda^i} = r^i \times \frac{n^i\hbar}{2r^i} = n^i\frac{\hbar}{2} \] (7)
, independent of the tube radius.

2.3 The electric interaction - dimension 7

The start of the development is a point-particle current (as seen in the usual 4D space) of four-velocity
\[ J^\mu_e = \frac{e}{3} \cdot o \cdot \delta^3(\vec{x} - \vec{y}) \cdot w^\mu \] (8)
and the covariant Liénard-Wiechert potential \[30\] caused by a point-particle of four-velocity \( v^\mu \).
\[ A^\mu_e = \frac{\frac{e}{3} \cdot n}{4\pi\varepsilon_0 c^2} \cdot \frac{v^\mu}{|\vec{y} - \vec{x}_0|} \] (9)
The potential energy between these two particles is:
\[ U_{no}(|\vec{x} - \vec{x}_0|) = \iiint_{-\infty}^{\infty} J^\mu_e A^\mu_e dy_1 dy_2 dy_3 \] (10)
\[ = \frac{e^2no}{8\pi\varepsilon_0 c^2g} \cdot \frac{u^\mu \nu_n}{|\vec{x} - \vec{x}_0|} = \frac{\alpha_e chno_{no}(z)}{9\rho z} \]
Here \(|\vec{x} - \vec{x}_0|\) is abbreviated as \( \rho \cdot z \), with a constant \( \rho \) of dimension length, and a dimensionless parameter \( z \). The invariant expression is condensed as:
\[ w^\alpha v_\alpha = g_{\alpha\beta} w^\alpha v^\beta = c^2 g_{\alpha\beta} \beta^\alpha \beta^\beta = c^2 f_{no}(z) \] (11)
with
\[ \beta^\mu_o = \begin{bmatrix} \gamma_o & \gamma_o \beta_o & 0 & 0 & sign(o_4) & sign(o_5) & sign(o_6) & sign(o_7) & sign(o_8) & sign(o_9) \end{bmatrix} \]
and
\[ \beta^\mu_n = \begin{bmatrix} \gamma_n & \gamma_n \beta_n & 0 & 0 & sign(n_4) & sign(n_5) & sign(n_6) & sign(n_7) & sign(n_8) & sign(n_9) \end{bmatrix} \]
(see appendix 8.7). As such \( f_{no}(z) \) contains all \( z \)-depended parts of the expression. In particular, this includes the metric tensor \( g_{\mu\nu}(z) \), and the normalized velocity \( \beta = \frac{v}{c} \), as well as \( \gamma = \frac{1}{\sqrt{1-\beta^2}} \). The force on such a particle is
\[ \vec{F}_e(z) = -\frac{1}{\rho} \cdot \frac{\partial}{\partial z} \left( \frac{\alpha_e chno_{no}(z)}{9\rho z} \right) = \frac{\alpha_e chno_{no}(z)}{9\rho z^2} - \frac{\alpha_e chno \frac{\partial f_{no}(z)}{\partial z}}{9\rho^2 z} \] (12)
The rest and kinetic part of the electric energy of the excitation \( o \) is anticipated as

\[
E^e_o(z) = \left( \hbar \omega (r_o) + \frac{A\alpha_e c}{9 \rho} \right) \cdot \gamma_o(z) = \left( \frac{\alpha c}{r_o(z)} + \frac{A\alpha_e c}{9 \rho} \right) \cdot \gamma_o(z) \tag{13}
\]

in which the second term represents a constant, multiplied by the \( \gamma_o \)-factor. The total electric energy is the sum of (13) and (10), which is conserved and hence it follows

\[
0 = \frac{\partial}{\partial z} \left( E^e_o(z) + U^e_o(z) \right) \tag{14}
\]

\[
0 = -\frac{\alpha c}{r(z)} \gamma_o(z) + \left( \frac{\alpha c}{r(z)} + \frac{A\alpha_e c}{\rho} \right) \left( \frac{d}{dz} \gamma_o(z) \right) - \frac{\alpha c \gamma_o(z)}{\rho} \left( f_{no}(z) - z \left( \frac{d}{dz} f_{no}(z) \right) \right)
\]

This differential equation allows to calculate the tube radius of dimension 7 (with \( o \) and \( n \) being the excitation numbers), which are written with the respective index from here on.

\[
r_o(z) = \frac{9\gamma_o(z)\rho \zeta_o}{-A\alpha_e \gamma_o(\zeta) + \alpha c \zeta_o f_{no}(z) + 9\rho \zeta_o} \tag{15}
\]

The limit of the undisturbed radius at infinity is (see "The undisturbed tube radii" below):

\[
\lim_{z \to \infty} r_o(\zeta) = \frac{9\rho \zeta_o(\infty)}{-A\alpha_e \gamma_o(\infty) + 9\rho \zeta_o} = \frac{9\rho}{\alpha_e \text{signum} (\zeta_o)} \tag{16}
\]

hence allowing to fix the free constant \( C \) as:

\[
C = \frac{\alpha_e (A + |\zeta_o|) \gamma_o(\infty)}{9\zeta_o \rho} \tag{17}
\]

Here \( \rho \) is identified as the compactification radius, which will be calculated later.

The radius of equation (15) becomes

\[
r_o(\zeta) = \frac{9\gamma_o(z)\rho \zeta_o}{\alpha_e \cdot \left( \gamma_o(\infty) \cdot (A + |\zeta_o|) - \frac{n_\tau f_{no}(z)}{z} \right) - \gamma_o(z)A} \tag{18}
\]

The energy generated via the electric interaction of the particle is achieved by inserting (18) into equation (13).

\[
E^e_o(z) = \left( \frac{\alpha c}{r_o(z)} + \frac{A\alpha_e c}{9 \rho} \right) \cdot \gamma_o(z) = \frac{\alpha c c}{9 \rho} \cdot \left( \gamma_o(\infty) \cdot (A + |\zeta_o|) - \frac{n_\tau f_{no}(z)}{z} \right) \tag{19}
\]
Equation (19) describes the electric rest and kinetic energy. To calculate the mass, the potential energy (10) must be added to (19) and following is the division by $c^2$. The observable mass becomes

$$m_o = \frac{E_o(z) + U_{no}(z)}{c^2} = \frac{\alpha_e \hbar \cdot (A + |\sigma|) \gamma_o(\infty)}{9c\rho} \tag{20}$$

, which is not dependent on $z$.

### 2.4 The strong interaction - dimensions 4, 5, and 6

At this point the color representation is introduced as:

$$c_{\text{red}} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad c_{\text{green}} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad c_{\text{blue}} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \tag{21}$$

For an anti-color each component is multiplied by $-1$. The calculations of the tube radii are done analog to the electric case. Again, starting with a point-particle current of four-velocity $w^\mu$ for each dimension the current is

$$J^\mu = e_s \cdot \delta^3(\vec{x} - \vec{y}) \cdot w^\mu n_i \tag{22}$$

and the vector potential (simplified ansatz form [31-34]) depending linearly on the distance between the two-color charges,

$$A^\mu_s = \frac{e_s}{4\pi\varepsilon_s c^2} \cdot |\vec{y} - \vec{x}_0| v^\mu o_i = \frac{e_s}{4\pi\varepsilon_s c^2} \cdot \frac{z}{\rho} v^\mu o_i \cdot (\vec{y} - \vec{x}_0)$$

the potential energy between these two particles is calculated in the usual way.

$$U^s_{no}(z) = \iiint_{-\infty}^{\infty} J^\mu_s A^\mu_s dy_1 dy_2 dy_3 \tag{24}$$

$$= \frac{e^2 n_o}{4\pi\varepsilon_s c^2} \cdot w^\mu v^\mu i \cdot n_i o_i \cdot \frac{z}{\rho}$$

$$= \frac{\alpha_s c \hbar n_i o_i}{\rho} \cdot f_{no}(z) \cdot z$$

The force becomes:

$$F^s = -\frac{1}{\rho} \cdot \frac{\partial}{\partial z} U^s_{no}(z) \tag{25}$$

$$= -\frac{\alpha_s c \hbar n_i o_i}{\rho^2} \left( \frac{df_{no}(z)}{dz} \cdot z + f_{no}(z) \right)$$

The rest and kinetic color energy of a tube’s excitation is assumed to be of the same format as in the electric case:

$$E^s_o(z) = \left( \frac{\alpha_s c \hbar}{r_o(z)} + \frac{\alpha_s c A}{\rho} \right) \cdot \gamma_o(z) \tag{26}$$
and as before it follows:

\[
0 = \frac{\partial}{\partial z}(E^*_s(z) + U^*_\text{no}(z))
\]

\[
0 = \left(\frac{\alpha_s c h}{r_i(z)} + \frac{\alpha_s c h A}{\rho} \right) \cdot \frac{\partial \gamma_o(z)}{\partial z} - \frac{\alpha_s c h \gamma_o(z)}{r_i^2(z)} \cdot \frac{\partial r_o}{\partial z} + \frac{\alpha_s c h n_i o_i}{\rho} \cdot \left( \frac{d U_{\text{no}}(z)}{dz} \cdot z + f_{\text{no}}(z) \right)
\]

The radii of the color tubes \((i = 4, 5, 6)\) become:

\[
r_o_i(z) = \frac{\rho o_i \gamma_o(z)}{-\alpha_s A \gamma_o(z) - \alpha_s n_i o_i f_{\text{no}} z + C \rho o_i}
\]

Fixing the free constant is archived:

\[
\lim_{z \to 0} r_o_i = \frac{o_i \rho \gamma_o(0)}{-\alpha_s A \gamma_o(0) + o_i \rho C} = \frac{\rho}{\alpha_s \text{signum} (o_i)}
\]

with the result:

\[
C = \frac{\alpha_s (A + |o_i|) \gamma_o(0)}{o_i \rho}
\]

The tube radius becomes:

\[
r_o_i(z) = \frac{\rho o_i \gamma_o(z)}{\alpha_s (\gamma_o(0) \cdot (A + |o_i|) - o_i n_i f_{\text{no}} z - \gamma_o(z) A)}
\]

At this point, without the potential energy, the color energy becomes

\[
E^S_o(z) = \frac{\alpha_s c h}{\rho} \left( (A + |o_i|) \gamma_o(0) - n_i o_i \cdot f_{\text{no}} \cdot z \right)
\]

Finally, the color mass is

\[
m_o_i = \frac{E^S_o(z) + U^*_\text{no}(z)}{c^2} = \frac{\alpha_s h}{c \rho} \cdot (A + |o_i|) \gamma_o(0)
\]

### 2.5 The weak interaction - dimensions 8 and 9

The point-particle currents of the weak interaction are

\[
J_{w}^{\mu} = e_F \cdot \delta^3(\vec{x} - \vec{y}) \cdot w^{\mu} n_i
\]

In the electric case, the vector potential shows a \(1/z\) dependence. For the strong interaction the vector potential has a \(z\) dependency. The electric interaction needs one dimension while the strong interaction occupies three dimensions. The weak interaction uses two dimensions. Therefore, the conjecture for the
weak potential is a logarithmic dependence since this appears to be the plausible functional behavior for this number of dimensions.

\[
A_w^\alpha = \frac{e_F}{4\pi\varepsilon_w c^2} \frac{\nu^\mu}{\rho} \alpha_i n_i \cdot \ln(z) \tag{35}
\]

The associated potential energy as previously demonstrated

\[
U_{no}^w(z) = \int\int\int_{\infty}^{\infty} J_w^\alpha A_{\mu}^w dy_1 dy_2 dy_3 \tag{36}
\]

\[
= \frac{e_F^2}{4\pi\varepsilon_w c^2} \cdot \frac{\nu^\mu \nu_\mu}{\rho} \alpha_i n_i \cdot \ln(z)
\]

\[
= \alpha_w c \chi \alpha_i n_i \cdot \ln(z) f_{no}(z) / \rho
\]

with the force

\[
\vec{F}_w = -\frac{1}{\rho} \cdot \frac{\partial}{\partial z} U_{no}^w(z) \tag{37}
\]

\[
= -\frac{\alpha_w c \chi \alpha_i n_i}{\rho^2} \cdot \left( \ln(z) \cdot \frac{\partial f_{no}(z)}{\partial z} + \frac{f_{no}(z)}{z} \right)
\]

With the same consideration as above the rest and kinetic energy is written as

\[
E^w_{\omega}(z) = \left( \frac{\alpha_i c \hbar}{r_o(z)} + \frac{\alpha_w c \chi A}{\rho} \right) \cdot \gamma_o(z) \tag{38}
\]

As before the total energy, the sum of (38) and (36) is conserved, which results in:

\[
0 = \frac{\partial}{\partial z} (E^w_{\omega}(z) + U_{no}^w(z)) \tag{39}
\]

\[
0 = \left( \frac{\alpha_i c \hbar}{r_o(z)} + \frac{\alpha_w c \chi A}{\rho} \right) \cdot \frac{\partial \gamma_o(z)}{\partial z} - \frac{\alpha_i c \hbar}{r_o^2(z)} \cdot \frac{\partial r_o}{\partial z}
\]

\[
+ \frac{\alpha_w c \chi \alpha_i n_i}{\rho} \cdot \left( \ln(z) \cdot \frac{\partial f_{no}(z)}{\partial z} + \frac{f_{no}(z)}{z} \right)
\]

Once again, (39) allows for calculating the tubes’ radii with the result

\[
r_o^i(z) = \frac{\alpha_i \rho^i \gamma_o(z)}{-A \alpha_w \gamma_o(z) - \alpha_w \alpha_i n_i \ln(z) f_{no}(z) + C \rho o_i} \tag{40}
\]

for \( i = 8, 9 \).

Not mentioned so far, is a screening effect playing a role here, too. Every particle reacting to the weak interaction is also affected by the nearby neutrinos and anti-neutrinos. This means that besides the interaction with a specific second particle, there is an additional contribution of nearby neutrinos and anti-neutrinos. Be \( \delta \) the average distance \( z_v \) of the nearest neutrino compared...
to the average distance \( z_{\bar{\nu}} \) of the nearest anti-neutrino. \( \delta \) is also equal to the cubic root of the anti-neutrino density \( \rho_{\bar{\nu}} \) divided by the neutrino density \( \rho_{\nu} \). For a homogeneous density, this connection is for any given volume \( V = n_{\nu} z_{\nu}^3 \rho^3 = n_{\bar{\nu}} z_{\bar{\nu}}^3 \rho^3 \)

\[
\frac{\rho_{\bar{\nu}}}{\rho_{\nu}} = \frac{n_{\bar{\nu}}}{n_{\nu}} \cdot \frac{z_{\nu}}{z_{\bar{\nu}}} = \frac{n_{\bar{\nu}} n_{\nu} z_{\nu}^3 \rho^3}{n_{\nu} n_{\bar{\nu}} z_{\bar{\nu}}^3 \rho^3} = \frac{z_{\nu}^3}{z_{\bar{\nu}}^3} = \delta^3
\]

This establishes the neutrino \( z_{\nu} \) to anti-neutrino \( z_{\bar{\nu}} \) rate

\[
\delta = \frac{z_{\nu}}{z_{\bar{\nu}}} = \left( \frac{\rho_{\bar{\nu}}}{\rho_{\nu}} \right)^{\frac{1}{3}} \tag{41}
\]

\( \delta \) alters the undisturbed tube radii of the weak interaction. Therefore, the fixing of the free constant becomes

\[
\lim_{z \to 1} r_{\alpha} = \frac{\alpha_i \rho \gamma_o(1)}{C \rho \alpha_i - \alpha_w A \gamma_o(1)} = \frac{\rho}{\alpha_w \cdot (\text{signum} (\alpha_i) - \ln(\delta))} \tag{43}
\]

, which results in

\[
r_{\alpha}(z) = \frac{\rho \alpha_i \gamma_o(z)}{\alpha_w (\gamma_o(1) \cdot [(1 - \text{signum} (\alpha_i) \ln(\delta)) \cdot |\alpha_i| + A] - \ln(z) f_{n\alpha}(z) \cdot n_i \alpha_i - A \gamma_o(z))} \tag{44}
\]

The overall distribution of neutrinos and anti-neutrinos expressed though \( \delta \), provides a very small contribution to a particle’s energy

\[
E_{\alpha}(z) = \frac{\alpha_w c \hbar}{\rho} \cdot (\text{signum} (\alpha_i) \ln(\delta)) \cdot |\alpha_i| + A \cdot \gamma_o(1) - f_{\alpha}(z) \ln(z) n_i \alpha_i)
\]

(45)

Once again, the mass becomes

\[
m_{\alpha} = \frac{E_{\alpha}(z) + U_{\alpha}^w(z)}{c^2} = \frac{\alpha_w c \hbar \gamma_o(1)}{\epsilon \rho} \cdot [(1 - \text{signum} (\alpha_i) \ln(\delta)) \cdot |\alpha_i| + A \gamma_o(1) - f_{\alpha}(z) \ln(z) n_i \alpha_i)]
\]

(46)

2.6 The undisturbed tube radii

The undisturbed radius of a tube is the radius not altered by any potential energy depending on \( z \). The remaining energy is

\[
E_{\text{undisturbed}} = \frac{\alpha c \hbar \gamma_o}{r(z = \infty, 0, 1)} = \frac{\alpha c \hbar \gamma_o}{\rho} \cdot F(o, n, \ldots)
\]

(47)

, and must be non-zero, not to violate the energy conservation. The solution for \( r \) yields

\[
r = \frac{\rho}{\alpha \cdot F(o, n, \ldots)}
\]

(48)
The ansatz for the interactions is

\[ F(o_7,n_7) = \text{signum}(o_7) \cdot \text{signum}(|n_7|) \Rightarrow r = \frac{\rho}{\alpha_s \cdot \text{signum}(o_7)} \quad (49) \]

\[ F(o_{4,5,6},n_{4,5,6}) = \text{signum}(o_{4,5,6}) \cdot \text{signum}(|n_{4,5,6}|) \Rightarrow r = \frac{\rho}{\alpha_s \cdot \text{signum}(o_{4,5,6})} \]

\[ F(o_{8,9},n_{8,9}) = \text{signum}(o_{8,9}) \cdot \text{signum}(|n_{8,9}|) - \ln(\delta) \]

\[ \Rightarrow \begin{cases} r = \frac{\rho}{\alpha_w \cdot \text{signum}(o_{8,9})} - \ln(\delta) & \text{if entangled with another particle} \\ r = \frac{\rho}{\alpha_w \cdot \ln(\delta)} & \text{if not entangled} \end{cases} \]

The additional term \( \ln(\delta) \) is a consequence of the formula of the potential energy \( (36) \) of the weak interaction in which the mean distances to the neutrinos and anti-neutrinos play a role.

\[ U_w = \frac{\alpha_w c \hbar \gamma_0}{\rho} \cdot (\ln(z_{ve}) + \ln(z_{\bar{v}e})) = -\frac{\alpha_w c \hbar \gamma_0}{\rho} \cdot \ln(\delta) \quad (50) \]

This "potential" is independent of \( z \) and does not give rise to any force.

### 2.7 The particle mass of strong, electric, and weak interaction

The Lorentz factors for all three interactions at their specific fix points (see appendix 8.8) at which their radii are equal to the undisturbed radii are

\[ \gamma_o(\infty) = 1 \quad \gamma_o(0) = 1 \quad \gamma_o(1) = 1 \quad (51) \]

For all the additional dimensions - the three interactions strong, electric, and weak - the total mass \( m_o \), without the contribution of magnetism, of a particle is

\[ m_o = \frac{\hbar}{c \rho} \cdot \left[ \alpha_s |o_1| + |o_5| + |o_6| + \frac{\alpha_e |o_7|}{9} \right] \]

\[ + \alpha_w \sum_{i=8}^{9} ([1 - \text{signum}(o_i) \ln(\delta)] \cdot |o_i|) \]

\[ + \frac{\hbar A}{c \rho} \cdot \left[ 3\alpha_s + \frac{\alpha_e}{9} + 2\alpha_w \right] \quad (52) \]

### 2.8 Masses depending solely on the weak sector

The situation differs somewhat for particles that are affected exclusively by the weak interaction. Only nearby neutrinos and anti-neutrinos effectively alter the value of the weak tubs' radii. Equation (36) reduces to

\[ U_w^w(z) = 0 \quad (53) \]
Consequently, there is no force
\[ F_w = -\frac{1}{\rho} \cdot \frac{\partial}{\partial z} U_{\alpha}^w(z) = 0 \]  \hspace{1cm} (54)

The energy of the excitation in the rest frame has the identical structure as for the general weak interaction (38) but without the dependency on \( z \):
\[ E_{\alpha}^w = \frac{\alpha_i c \hbar}{r_{o_i}} + \frac{\alpha_w c \hbar A}{\rho} \]  \hspace{1cm} (55)

The non-varying tube radius is
\[ r_{o_i} = -\frac{\rho}{\alpha_w \ln(\delta)} \]  \hspace{1cm} (56)

which then results in the excitation energy
\[ E_{\alpha}(z) = \frac{\alpha_w c \hbar}{\rho} \cdot (-\alpha_i \ln(\delta) + A) \]  \hspace{1cm} (57)

The mass becomes \((i = 8,9)\)
\[ m_{\alpha} = \frac{E_{\alpha}(z) + U_{\alpha}^w(z)}{c^2} = \frac{\alpha_w c \hbar}{c \rho} (-\alpha_i \ln(\delta) + A) \]  \hspace{1cm} (58)

### 2.9 Calculation of particle velocity, acceleration, and force

In the usual four-dimensional spacetime with \( u^\alpha = \gamma^\beta \alpha c \), the expression
\[ u^\alpha u_\alpha = c^2 \text{ with } \alpha = 0, 1, 2, 3 \]  \hspace{1cm} (59)

is conserved. The 10-dimensional version, conserved as well, changes with the number of active compact dimensions. It has the form of
\[ u^\mu u_\mu = (1 + n_{abcd})c^2 = c^2 \left\{ \begin{array}{cl} 0 & \text{leptons} \\ 2 & \text{quarks} \end{array} \right. \mu = 0, 1, 2, 3, \ldots, 9 \]  \hspace{1cm} (60)

with \( n_{abcd} \) being the result of adding active compact dimensions with a positive and subtracting those with a negative signature. The additional dimensions are indexed 4 to 9. For a coordinate system, with the two particles located on the \( z = z^1 \)-axis and \( z^2 = z^3 = 0 \), the velocity can be calculated as a function of the distance between the particles. By means of the metric tensor of equation (3), the calculation of the Christoffel symbols can be performed in the usual way with repeated indices summed from 0 to 9.
\[ \Gamma_{\nu \xi}^\mu = \frac{1}{2} g^{\mu \kappa} (\partial_\nu g_{\kappa \xi} + \partial_\xi g_{\kappa \nu} - \partial_\kappa g_{\nu \xi}) \]  \hspace{1cm} (61)

The acceleration is calculated by means of the geodesic equation.
\[ a^\mu = -\Gamma_{\nu \xi}^\mu u^\nu u^\xi \]  \hspace{1cm} (62)

The force is established as:
\[ F^\mu(z) = \frac{E_{\alpha}(z)}{c^2} a^\mu \]  \hspace{1cm} (63)
2.10 Coulomb force between two slow moving charges

The electric force between two charges \( q_1 = \frac{e}{7} \cdot \sigma \) and \( q_2 = \frac{e}{7} \cdot n \) of first-generation particles \(-A = 0\), which will be shown below - is calculated for slow movement \( \beta \approx 0 \) and \( f_{no} = 1 \) (see appendix 8.7.). The radius (18) reduces to:

\[
r_{\sigma}(z) = \frac{9\rho \text{signum} (\sigma)}{\alpha_e \cdot \left(1 - \frac{n\pi \text{signum}(\sigma)}{z}\right)}
\]

(64)

The mass \( E_{\sigma}(z)/c^2 \) associated with equation (19) is used. While the full solution of (63) is rather long, the first terms of the series in \( \rho \) are manageable.

\[
F^1(R) = \frac{\alpha_e c h n \pi \sigma}{9R^2} - \frac{4c h n \pi^2 |\sigma| \pi \rho^2}{R^4} + O (\rho^3)
\]

(65)

\[
= -E_{\sigma}(z)/c^2 \cdot \Gamma_{77} \cdot c^2 + \cdots = \frac{\alpha_e c h n \pi \sigma}{9R^2} + \cdots
\]

\( R = \rho \cdot z \) is the distance in meters. \( \rho \) is of the order of femtometer as shown below. For distances above \( 10^{-13} \) m, the first term of (65) is dominant, and represents the well-known formula of the Coulomb force of electrostatics.

3 Calculation of the model’s constants

3.1 The compactification radius \( \rho \)

The undisturbed radius of the electric tube is \( 9\rho/\alpha_e \). The time for an electron excitation (of length \( \lambda = 4\pi r/3 \) and velocity \( c \)) to revolve around the tube is:

\[
\Delta T = \frac{3\lambda}{c} = \frac{4\pi r}{c} = \frac{36\pi \rho}{\alpha_e c}
\]

(66)

With this, the electric current calculates as:

\[
I = -\frac{e}{\Delta T} = -\frac{e\alpha_e c}{36\pi \rho}
\]

(67)

The classic magnetic dipole moment is defined ( \( n = 2 \) being the winding number, \( A \) is the area the current flows around) as:

\[
\mu = nIA = -2 \cdot \frac{e\alpha_e c}{36\pi \rho} \cdot \pi \cdot \frac{81\rho^2}{\alpha_e^2} = -\frac{9ec\rho}{2\alpha_e}
\]

(68)

The magnetic dipole moment of an electron - electric dimension’s spin \( S_e = 3h/2 \) - is:

\[
\mu_S = -\frac{g_\mu e S_e}{h} = -\frac{3gech}{4m_e}
\]

(69)

Comparing these two expressions gives the value of the compactification radius \( \rho \) by using the measured magnetic moment \( g_e = 2.00231930436182(52) \) as:

\[
\rho = \frac{ge\hbar \alpha_e}{6m_e c} = 9.4040252(14) \cdot 10^{-16} \text{ m}
\]

(70)
3.2 The weak coupling $\alpha_w$ and the neutrino distribution ratio $\delta$

The mass of the three leptons $e, \mu, \tau$ are calculated according to equation (52):

$$m_{e,\mu,\tau} = \frac{\hbar}{c \rho} \left[ \frac{\alpha_e}{3} + 2\alpha_w (1 - \ln(\delta)) \right] + \frac{\hbar A_{e,\mu,\tau}}{c \rho} \left[ 3\alpha_s + \frac{\alpha_e}{9} + 2\alpha_w \right]$$  \hspace{1cm} (71)

The couplings for the strong $\alpha_s$, the weak interaction $\alpha_w$, and the constant $\delta$ are at this point not yet known. But solving the equation (71) for $A_e$ and setting in the numbers of the known values, results in:

$$A_e = \frac{0.00002538274 - 18 \alpha_w \cdot (1 - \ln(\delta))}{27 \alpha_s + 0.007297352536 + 18 \alpha_w}$$  \hspace{1cm} (72)

It is known that $\alpha_s[35]$ is in the order of one and $\alpha_w[36]$ is about $10^{-6}$, while $\delta$ is almost one, which results in $\ln(\delta)$ being close to zero. It shows $A_e < 10^{-6}$ as being very small. So, the first term of (71) is approximately three orders of magnitude larger than term two. This suggests that $A_e$ is exactly zero, and in the wake of it all, $A_s$ of the first generation are presumed to be zero. There is a second argument supporting the accuracy. Later in this article the higher dimensional structure of the photon will be deduced. For the photon to be massless $A_{\text{first generation}} = 0$ must hold true. From here onward the calculations will be done accordingly: $A_e = A_{\mu e} = A_d = A_u = 0$.

The mass of an electron becomes (see equation (52)) for the electric mass contribution and the weak mass part:

$$m_e^- = \frac{\hbar}{c \rho} \left[ \frac{\alpha_e}{3} + 2\alpha_w \cdot (1 - \ln(\delta)) \right]$$  \hspace{1cm} (73)

The positron is

$$m_e^+ = \frac{\hbar}{c \rho} \left[ \frac{\alpha_e}{3} + 2\alpha_w \cdot (1 + \ln(\delta)) \right]$$  \hspace{1cm} (74)

The upper limit for the mass difference [29,37] between a positron and an electron is $|m_e^+ - m_e^-| < 8 \cdot 10^{-9}$. With the electron mass and the small mass difference, it is possible to calculate the weak coupling $\alpha_w$ and estimate $\delta$.

$$\alpha_w = \frac{3c \rho m_{\text{average}} - \hbar \alpha_e}{6\hbar} = 1.41040(26) \cdot 10^{-6}$$  \hspace{1cm} (75)

$$= \exp \left( \frac{c \rho m_{\text{average}} \cdot 8 \cdot 10^{-9}}{4\hbar \alpha_e} \right) = \left\{ \begin{array}{l}
1.0000034530(64) \text{ if } m_{e+} > m_{e-} \\
0.9999965467(64) \text{ if } m_{e+} < m_{e-}
\end{array} \right.$$  \hspace{1cm} (76)

To complete the list of constants in terms of the leptons with electric charge, the measured masses of the muon and the tau plus equation (71) are used to calculate the lepton constants (Table 1).

Table 1: Values of the lepton constants

| $A_e$  | $A_{\mu e}$ | $A_{\tau}$ |
|---|---|---|
| 0  | 0.33561(26) | 5.6698(44) |
| $A_{\nu e}$ | unknown | unknown |
3.3 The strong coupling $\alpha_s$ and the magnetic self-interaction within the mesons

By means of equation (52), the strong, electric, and weak mass ratios of the quarks, can be stated.

\[
m_{d,s,b} = \hbar c \rho \left[ 2\alpha_s + \frac{1}{9}\alpha_e + 2\alpha_w \cdot (1 - \ln(\delta)) \right] + \frac{h A_{d,s,b}}{c \rho} \cdot \left[ \frac{3\alpha_s + \frac{\alpha_e}{9} + 2\alpha_w}{9} \right] + \hbar A_{d,s,b} c \rho \cdot \left[ 3\alpha_s + \frac{\alpha_e}{9} + 2\alpha_w \right] 
\]

\[(77)\]

\[
m_{\bar{d},\bar{s},\bar{b}} = \hbar c \rho \left[ 2\alpha_s + \frac{1}{9}\alpha_e + 2\alpha_w \cdot (1 + \ln(\delta)) \right] + \frac{h A_{d,s,b}}{c \rho} \cdot \left[ \frac{3\alpha_s + \frac{\alpha_e}{9} + 2\alpha_w}{9} \right] + \hbar A_{d,s,b} c \rho \cdot \left[ 3\alpha_s + \frac{\alpha_e}{9} + 2\alpha_w \right] 
\]

\[(78)\]

\[
m_{u,c,t} = \hbar c \rho \left[ 2\alpha_s + \frac{2}{9}\alpha_e + \alpha_w \cdot (1 + \ln(\delta)) \right] + \frac{h A_{u,c,t}}{c \rho} \cdot \left[ \frac{3\alpha_s + \frac{\alpha_e}{9} + 2\alpha_w}{9} \right] + \hbar A_{u,c,t} c \rho \cdot \left[ 3\alpha_s + \frac{\alpha_e}{9} + 2\alpha_w \right] 
\]

\[(79)\]

\[
m_{\bar{u},\bar{c},\bar{t}} = \hbar c \rho \left[ 2\alpha_s + \frac{2}{9}\alpha_e + \alpha_w \cdot (1 - \ln(\delta)) \right] + \frac{h A_{u,c,t}}{c \rho} \cdot \left[ \frac{3\alpha_s + \frac{\alpha_e}{9} + 2\alpha_w}{9} \right] + \hbar A_{u,c,t} c \rho \cdot \left[ 3\alpha_s + \frac{\alpha_e}{9} + 2\alpha_w \right] 
\]

\[(80)\]

The differences of the quark masses are not caused exclusively by the strength of the interaction itself (first term), rather it is strongly induced by the constant represented by the second term. As previously mentioned, all constants of the first generation are treated as zero $A_e = A_v = A_d = A_u = 0$. Not including the magnetic self-interaction, the first-generation meson masses $\tilde{m}$ become:

\[
\tilde{m}_\pi^0 = \tilde{m}_{d\bar{d}} + \tilde{m}_{u\bar{u}} = m_d + m_{\bar{d}} = \tilde{m}_{\rho^0} 
\]

\[(81)\]

\[
\tilde{m}_\pi^+ = \tilde{m}_{u\bar{d}} = m_u + m_{\bar{d}} = \tilde{m}_{\rho^+} 
\]

Each of the quarks and anti-quarks represents a dipole and possesses a magnetic field as well. The energy of a dipole $\mu$ within a magnetic field is written as

\[
U_{\text{dipole}} = -\mu \cdot B = -\frac{m \cdot c^2 \cdot \alpha_T \cdot n_7}{m} \quad \text{with} \quad \mu \sim \frac{\alpha_T}{m} \quad \text{and} \quad B \sim n_7 
\]

\[(82)\]

which is rewritten as $c^2$ times a constant $M$ multiplied by the electric excitation numbers of the two quarks and divided by the quark mass. The energy of one quark’s magnetic field within a given volume is $\frac{\mu^2}{2\mu_0}$ and following is the energy

\[
U_B = \int \frac{B^2}{2\mu_0} dV = N c^2 n_7^2 
\]

\[(83)\]

which is written as the constant $N$ multiplied by $c^2$ and the excitation number squared. The details can be found in appendix (8.4.2). In combination with the other quarks, the magnetic fields must first be combined in order to calculate the magnetic energy of a meson or baryon. The meaning of the symbols is
\( \vec{B} \): magnetic field; \( o \): excitation number of quark one; \( n \): excitation number of quark two; and \( m \): particle mass as given in equations (77) to (80). The mass contributions due to magnetism for mesons and baryons differ. For mesons, it can be stated as

\[
m_{q\bar{q}} = m_q + m_{\bar{q}} + m_{q\bar{q}}^{\text{dipole}} + m_{\bar{q}q}^{\text{dipole}} \tag{84}
\]

The dipole masses for scalar mesons are all negative, which reduces their mass. For vector mesons dipole masses are positive with a mass increasing effect. In addition, there are differences in calculating magnetic contributions for scalar or vector mesons. The calculations for mesons is - starting with parallel spin orientation - a spin flip causes a quark or anti-quark excitation number to be multiplied by \(-1\). Hence, the calculation rule for a scalar meson is:

\[
m_{q\bar{q}}^{\vec{B}} = N \cdot (o_7 - n_7)^2 \quad m_{q\bar{q}}^{\text{dipole}} = -\frac{M}{m_q} \cdot |o_7 \cdot n_7| \quad m_{\bar{q}q}^{\text{dipole}} = -\frac{M}{m_{\bar{q}}} \cdot |n_7 \cdot o_7| \tag{85}
\]

while for a vector meson, the calculation is:

\[
m_{q\bar{q}}^{\vec{B}} = N \cdot (o_7 + n_7)^2 \quad m_{q\bar{q}}^{\text{dipole}} = \frac{M}{m_q} \cdot |o_7 \cdot n_7| \quad m_{\bar{q}q}^{\text{dipole}} = \frac{M}{m_{\bar{q}}} \cdot |n_7 \cdot o_7| \tag{86}
\]

Now it is possible to state the mass formulas for the first-generation mesons.

\[
m_{\pi^0} = \frac{m_d + m_{\bar{d}} + m_u + m_{\bar{u}}}{2} + 10N - \frac{1}{2} \left( \frac{M}{m_d} + \frac{M}{m_{\bar{d}}} + \frac{4M}{m_u} + \frac{4M}{m_{\bar{u}}} \right) \tag{87}
\]

\[
m_{\pi^+} = m_u + m_{\bar{d}} + N - 2\frac{M}{m_u} \quad m_{\pi^-}^{\vec{B}} = \frac{2M}{m_{\bar{d}}} \tag{88}
\]

\[
m_{\rho^0} = \frac{m_d + m_{\bar{d}} + m_u + m_{\bar{u}}}{2} + \frac{1}{2} \left( \frac{M}{m_d} + \frac{M}{m_{\bar{d}}} + \frac{4M}{m_u} + \frac{4M}{m_{\bar{u}}} \right) \tag{89}
\]

\[
m_{\rho^+} = m_u + m_{\bar{d}} + 2\frac{M}{m_u} + 2\frac{M}{m_{\bar{d}}} \tag{90}
\]

By using the measured masses of the mesons \( \pi^0, \pi^+ \) and \( \rho^0 \) along with the respective mass formulas of (87), it is possible to calculate the numerical values of the magnetic multipliers and the strong coupling (Table 2).

Table 2: The magnetic multiplier values and the strong coupling

| \( M \) | 4.74681(57) \cdot 10^{-29} \text{ kg}^2 | \( N \) | 1.323(10) \cdot 10^{-29} \text{ kg} | \( \alpha_s \) | 0.49743(39) |

4 Particle identification

The identification of the fermions is straightforward. Color and electric charge are easily set. A quark comes with one of the colors: red, green, or blue, and the respective excitation numbers. The electric excitation number is minus one
for a down quark, two for an up quark and minus three for an electron. Each excitation number adds up a spin $\hbar/2$ times excitation number. To get an overall spin $\hbar/2$, the weak excitations adjust respectively. A neutrino has weak excitations, only. This particle comes as a superposition of the weak excitations. Hence, the excitation number formally written as $\frac{1}{2}$, is the same as for the weak contribution of a u-quark (Table 3).

Table 3: Fermions of the first generation

| Particle      | $T_s$ strong | $T_s$ strong | $T_s$ strong | $T_s$ electric | $T_s$ weak | $T_s$ weak | Mass formula of electric, strong, and weak interaction without the contribution of magnetism |
|---------------|-------------|-------------|-------------|---------------|------------|------------|---------------------------------------------------------------------------------|
| electron      | 0           | 0           | 0           | -3            | 1          | 1          | $m_e = \frac{\hbar}{c}\left(\frac{a_e}{3} + 2a_w(1 - \ln(\delta))\right)$ |
| e neutrino    | 0           | 0           | 0           | 0             | -1/2       | -1/2       | $m_{\nu e} = \frac{\hbar}{c}\cdot a_w \ln(\delta)$ |}

For the higher generation particles, the excitation settings are identical to the one from the first generation. The difference comes solely with the additional mass constants. The respective anti-particles get all the excitation numbers multiplied by $-1$. The Standard Model gauge bosons’ excitation numbers can be deduced from known decay processes. In the process of electron-positron annihilation [38] two photons are produced (Figure 1).

$$e^+ + e^- \rightarrow 2\gamma$$

\[
\begin{pmatrix}
0 \\
0 \\
0 \\
3 \\
-1
\end{pmatrix}
+ \begin{pmatrix}
0 \\
0 \\
0 \\
-3 \\
1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
1
\end{pmatrix}
+ \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
-1
\end{pmatrix}
\]

Figure 1: Electron-positron annihilation into two photons
Two real measurable particles are created. The photons, as can be seen using equation (58), are massless. However, how can the other gauge bosons be accounted for? Do they exist as autonomous particles similar to the photon? The \( \eta_b \) meson has one decay mode evolving into a three-gluon event (Figure 2).

\[
\begin{align*}
\eta_b & \rightarrow 3g \\
& = \left( \begin{array}{c}
1 \\
-1 \\
0 \\
-1 \\
1 \\
1
\end{array} \right) \left( \begin{array}{c}
-1 \\
1 \\
0 \\
1 \\
-1 \\
0
\end{array} \right) \rightarrow \left( \begin{array}{c}
1 \\
-2 \\
1 \\
0 \\
0 \\
0
\end{array} \right) + \left( \begin{array}{c}
-2 \\
1 \\
0 \\
1 \\
0 \\
0
\end{array} \right)
\end{align*}
\]

Figure 2: \( \eta_b \) decay into three gluons

A direct measurement is, however, not possible. The questions that arise: Is the gluon a specific configuration in 10D-space in the decay process of the original particles and the decay product? Or does a real intermediate state, a gluon, exist independently in the process? The same questions can be directed at the other gauge bosons. In the neutron decay into a proton, an electron, and an anti-electron neutrino, a \( W^- \) has been declared as an intermediate state (Figure 3).

\[
W^- \rightarrow e^- + \bar{\nu}_e \\
= \left( \begin{array}{c}
0 \\
0 \\
0 \\
-3 \\
3/2 \\
3/2
\end{array} \right) \rightarrow \left( \begin{array}{c}
0 \\
0 \\
0 \\
-3 \\
1 \\
1
\end{array} \right) + \left( \begin{array}{c}
0 \\
0 \\
0 \\
0 \\
1/2 \\
1/2
\end{array} \right)
\]

Figure 3: A \( W^- \) boson decays into an electron and an anti-electron neutrino

It is important to note that: \( 1/2 \) must be deciphered as a superposition with excitation numbers one and zero. \( 3/2 \) stands for a superposition of 1 and 2.

The last intermediate boson is the \( Z \), which can decay via various channels e.g., into an electron-positron pair.

\[
Z \rightarrow e^- + e^+ \\
= \left( \begin{array}{c}
0 \\
0 \\
0 \\
0 \\
SP(1/-1) \\
SP(1/-1)
\end{array} \right) \rightarrow \left( \begin{array}{c}
0 \\
0 \\
0 \\
-1 \\
1 \\
1
\end{array} \right) + \left( \begin{array}{c}
0 \\
0 \\
0 \\
1 \\
-1 \\
-1
\end{array} \right)
\]

Figure 4: \( Z \) decay into a electron-positron pair. \( SP(1/-1) \) stands for a superposition of the states 1 and -1.

The photon and the other intermediate bosons are presented in Table 4.
Masses are created as excitations of the compactified dimensions; therefore, a Higgs-like boson is not a necessity within this development. An excitation always travels with the speed of light, while manifesting as a mass in ordinary spacetime when induced through the higher dimensions.

5 Hadron masses

The magnetic multipliers as well as the coupling constants are not dependent on particle generation. That allows to calculate the hadron masses. First the meson masses scalar and vector - and afterwards the baryon masses - spin $3\hbar/2$ and $\hbar/2$ - are calculated. The search for lists of particles ended by using those of Wikipedia [39, 40]. For each table a hypothetical combination with a top quark is added to demonstrate the expected masses. The measured particle masses shown are used in the comparison to the calculated values and to estimate the relative error. All other calculations, however, rest on data of the latest PDG publications [29]. All lists include the particle name, the symbol, quark content, measured and calculated masses in MeV/$c^2$ and kg, together with the standard deviations, the relative error, and the mass formula. The measured masses in kg are not stated with standard deviation, since these numbers are no longer used in any further calculation.

5.1 Meson masses

The spin of scalar mesons is anti-parallel, hence resulting in a net spin of $S = 0$ (Table 5).

| Particle | $\tau_4$ strong | $\tau_5$ strong | $\tau_6$ strong | $\tau_7$ electric | $\tau_8$ weak | $\tau_9$ weak | Mass formula |
|----------|------------------|------------------|------------------|-------------------|----------------|----------------|--------------|
| Photon $\gamma$ | 0 | 0 | 0 | 0 | 1 | -1 | $m_\gamma = -\frac{\alpha_e \hbar}{c\rho} \cdot [1 - 1 \cdot \ln(\delta)]$ massless from (58) |
| 8 Gluons as superposition constructed out of quantities as this color red to green changing example $g_{\gamma\delta}$ | 1 | -2 | 1 | 0 | 0 | 0 | $m_{\text{gluon}} = \frac{4 \alpha_e \hbar}{c\rho} \approx 7.44 \cdot 10^{-29} \text{kg}$ $\approx 417 \frac{\text{MeV}}{c^2}$ If gluons are real intermediate particles massive |
| $W^+$ | 0 | 0 | 0 | 3 | -3/2 | -3/2 | massive |
| $W^-$ | 0 | 0 | 0 | -3 | 3/2 | 3/2 | massive |
| $Z$ | 0 | 0 | 0 | 0 | SP 1/1 | SP 1/1 | massive |
The vector mesons have a net spin of \( S = \hbar \). The quark and the anti-quark spins point in the same direction (Table 6).

Table 6: Masses of vector mesons

| Name       | Symbol | Quark content | Measured mass | Calculated mass | Relative error | Mass formula |
|------------|--------|---------------|---------------|-----------------|----------------|--------------|
| Particle   | \( \pi^0 \) | ud             | 3.0001(80)    | 3.0000(80)      | 0              | \( m_{\pi^+} = m_q + M_{\text{dipole}} \) |
| Particle   | \( \pi^+ \) | uud            | 3.0001(80)    | 3.0001(80)      | 0              | \( m_{\pi^+} = m_q + M_{\text{dipole}} \) |
| K meson    | \( K^0 \) | c\bar{d}       | 1.770(2)(4)   | 1.7702(6)       | 0.0006         | \( m_{K^0} = m_c + m_{\text{dipole}} \) |
| K meson    | \( \bar{K}^0 \) | c\bar{d}       | 1.7702(6)     | 1.7702(6)       | 0              | \( m_{\bar{K}^0} = m_c + m_{\text{dipole}} \) |
| Upsilon    | \( \Upsilon \) | bb             | 3.996(10)     | 3.9960(10)      | 0.000          | \( m_{\Upsilon} = m_b + m_b \) |

5.2 Baryon masses

Baryons consist of 3 quarks. They come as \( 3h/2 \) and \( h/2 \) spin versions. As with the mesons, the calculations of the magnetic masses differ depending on the spin status. The general mass statement is identical for both versions, but the specific calculation of the various contributions differs. The baryon mass is

\[
m_{q_1 q_2 q_3} = m_{q_1} + m_{q_2} + m_{q_3} + m_{\text{dipole}}
\]

(88)
The spin $3h/2$ baryons’ magnetic masses are

$$m_{q_0}^{B} = N \cdot |o_7 + n_7 + p_7|^2 \quad m_{q_0}^{\text{dipole}} = \frac{M}{m_{q_0}} \cdot |o_7 \cdot (n_7 + p_7)|$$

$$(89) \quad m_{q_n}^{\text{dipole}} = \frac{M}{m_{q_n}} \cdot |n_7 \cdot (p_7 + o_7)| \quad m_{q_p}^{\text{dipole}} = \frac{M}{m_{q_p}} \cdot |p_7 \cdot (o_7 + n_7)|$$

Each quark combines with their two neighboring quarks; therefore, the order of the quarks is of no concern (Table 7).

Table 7: Masses of Baryons with spin $3h/2$

| Name  | Symbol | Quark content | Measured mass $M_{\text{mass}}$ (GeV) | Calculated mass $M_{\text{calc}}$ (GeV) | Relative error $\frac{\text{Mass Error}}{M_{\text{mass}}}$ (MeV) | Mass formula |
|-------|--------|---------------|---------------------------------------|------------------------------------------|------------------------------------------------|--------------|
| Delta | $\Delta^+$ | ssn          | 1332(2)                               | 2.1963 $\pm 10^{-3}$                     | 4.489(14) $\times 10^{-3}$                         | $\Delta^+$ = $s$ + $n$ + $p$ |
| Delta | $\Delta^0$ | ssn          | 1332(2)                               | 2.1963 $\pm 10^{-3}$                     | 2.050(39) $\times 10^{-3}$                         | $\Delta^0$ = $s$ + $n$ + $p$ |
| Delta | $\Delta^-$ | ssn          | 1332(2)                               | 2.1963 $\pm 10^{-3}$                     | 1.069(14) $\times 10^{-3}$                         | $\Delta^-$ = $s$ + $n$ + $p$ |
| Sigma  | $\Sigma^+$ | ssn          | 1389(3)                               | 2.0607 $\pm 10^{-3}$                     | 2.445(55) $\times 10^{-3}$                         | $\Sigma^+$ = $s$ + $n$ + $p$ |
| Sigma  | $\Sigma^0$ | ssn          | 1388(3)                               | 2.0607 $\pm 10^{-3}$                     | 2.599(73) $\times 10^{-3}$                         | $\Sigma^0$ = $s$ + $n$ + $p$ |
| Sigma  | $\Sigma^-$ | ssn          | 1387(3)                               | 2.0607 $\pm 10^{-3}$                     | 2.599(73) $\times 10^{-3}$                         | $\Sigma^-$ = $s$ + $n$ + $p$ |
| Charmed Sigma  | $\Sigma^+$ | csn          | 2516.4(20)                             | 4.405 $\pm 10^{-3}$                      | 4.69(2) $\times 10^{-3}$                           | $\Sigma^+$ = $c$ + $n$ + $p$ |
| Charmed Sigma  | $\Sigma^0$ | csn          | 2516.4(20)                             | 4.405 $\pm 10^{-3}$                      | 2.599(73) $\times 10^{-3}$                         | $\Sigma^0$ = $c$ + $n$ + $p$ |
| Bottom  | $\Xi^-$ | ssn          | 5410.3(27)                             | 1.059 $\pm 10^{-3}$                      | 1.810(92) $\times 10^{-3}$                         | $\Xi^-$ = $s$ + $n$ + $p$ |
| Bottom  | $\Xi^0$ | ssn          | 5410.3(27)                             | 1.059 $\pm 10^{-3}$                      | 1.031(91) $\times 10^{-3}$                         | $\Xi^0$ = $s$ + $n$ + $p$ |
| Bottom  | $\Xi^+$ | ssn          | 1355.9(52)                             | 2.796 $\pm 10^{-3}$                      | 2.699(92) $\times 10^{-3}$                         | $\Xi^+$ = $s$ + $n$ + $p$ |
| Bottom  | $\Xi^+$ | csn          | 2445.5(30)                             | 4.735 $\pm 10^{-3}$                      | 4.547(50) $\times 10^{-3}$                         | $\Xi^+$ = $c$ + $n$ + $p$ |
| Bottom  | $\Xi^0$ | csn          | 2445.5(30)                             | 4.735 $\pm 10^{-3}$                      | 4.547(50) $\times 10^{-3}$                         | $\Xi^0$ = $c$ + $n$ + $p$ |
| Bottom  | $\Xi^-$ | csn          | 2445.5(30)                             | 4.735 $\pm 10^{-3}$                      | 4.547(50) $\times 10^{-3}$                         | $\Xi^-$ = $c$ + $n$ + $p$ |
| Bottom  | $\Xi^-$ | unknown      | 7.3(71)                               | 7.3(71)                                 | 7.3(71)                                         | $\Xi^-$ = $c$ + $n$ + $p$ |
| Bottom  | $\Xi^0$ | unknown      | 7.3(71)                               | 7.3(71)                                 | 7.3(71)                                         | $\Xi^0$ = $c$ + $n$ + $p$ |
| Bottom  | $\Xi^0$ | unknown      | 7.3(71)                               | 7.3(71)                                 | 7.3(71)                                         | $\Xi^0$ = $c$ + $n$ + $p$ |
For the spin $\frac{1}{2}$ baryons, the situation is different. Here, one of the quarks performed a spin flip, therefore breaking the symmetry. Consequently, different configurations result in different masses, while the quark content remains the same. An example is the different masses of the $\Lambda^0$ and the $\Sigma^0$ baryons, both of which consist of one $d$, one $u$ and one $s$ quark. This symmetry breaking is flipped - together with the selection rules (details see appendix 8.4.3) then causes changes with the calculation of the magnetic masses to

$$m^B_{qoqnp} = N \cdot \left| (\sigma_7 + n_7)^2 - p_7^2 \right|$$

$$m_{qo}^{\text{dipole}} = \frac{M}{m_{qo}} \cdot |\sigma_7 \cdot n_7| \quad m_{qn}^{\text{dipole}} = \frac{M}{m_{qn}} \cdot |n_7 \cdot \sigma_7| \quad m_{qp}^{\text{dipole}} = 0$$

with the results to be found in Table 8.

Table 8: Masses of Baryons with spin $\frac{1}{2}$
5.3 Correlation between measured and calculated hadron masses

The correlations between measured and calculated hadron masses display the suitability of the mass calculation (Table 9 and Figure 5).

Table 9: Pearson’s correlation between measured and calculated hadron masses

| Name        | Symbol | Quantum content | Measured mass | Calculated mass | Relative error | Mass formula |
|-------------|--------|-----------------|---------------|-----------------|---------------|--------------|
| Poton       | P²⁺  | uς, uς         | 5.1823(5)     | 5.1823(5)       | 0.104          | mₚ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Neutron     | N⁰   | nu              | 1.6742(58)    | 1.6742(58)      | 0.003          | mₙ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Lambda      | Λ⁰   | u, u            | 1.1158(6)     | 1.1158(6)       | 0.073          | m₈ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Charmed Lambda | A⁰     | d, b            | 2.286.8(4)    | 2.286.8(4)      | 0.095          | m₈ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Bottom Lambda | B⁰     | u, b            | 5.019.0(14)   | 5.019.0(14)     | 0.008          | m₈ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Sigma plus   | Σ⁺⁺   | us               | 1.189.57(10)  | 1.2262(10)      | 0.059          | m₈ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Sigma zero   | Σ⁰    | sb               | 1.192.04(24)  | 1.2265(24)      | 0.059          | m₈ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Sigma minus  | Σ⁻⁻   | ss               | 1.197.44(30)  | 1.2846(30)      | 0.059          | m₈ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Charmed Sigma | Σ⁺⁺⁺   | us               | 2.412.9(11)   | 2.412.9(11)     | 0.059          | m₈ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Bottom Sigma | Σ⁻⁻⁻   | sb               | 2.412.7(11)   | 2.412.7(11)     | 0.059          | m₈ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Bottom Sigma | Σ⁻⁻⁻   | ss               | 2.412.7(11)   | 2.412.7(11)     | 0.059          | m₈ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Β⁺         | B⁺    | u²               | 1.313(60)     | 1.313(60)       | 0.048          | m₈ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Β⁻         | B⁻    | d²               | 1.313(60)     | 1.313(60)       | 0.048          | m₈ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Charmed Κ   | Κ⁺⁺   | us               | 2.407(60)     | 2.407(60)       | 0.048          | m₈ = mₑ + mₑ + 2M + 3H + M³ + H² |
| Charmed Κ   | Κ⁻⁻   | do               | 2.407(60)     | 2.407(60)       | 0.048          | m₈ = mₑ + mₑ + 2M + 3H + M³ + H² |

5.3 Correlation between measured and calculated hadron masses

The correlations between measured and calculated hadron masses display the suitability of the mass calculation (Table 9 and Figure 5).

Table 9: Pearson’s correlation between measured and calculated hadron masses
and the associated significance \( p \).

| Number of particles | Particle group  | Pearson's r | \( p \)     |
|---------------------|----------------|-------------|------------|
| 11                  | Scalar Mesons | 0.997       | 1.52E-11   |
| 9                   | Vector Mesons | 1           | 5.34E-12   |
| 20                  | Spin 3/2 Baryons | 0.974       | 4.93E-13   |
| 24                  | Spin 1/2 Baryons | 0.998       | 9.52E-28   |
| 64                  | Hadrons        | 0.992       | 3.56E-57   |

Figure 5: Correlation between measured and calculated masses with the two outliers \( uuu \) and \( uuc \). The heavy scalar meson consisting of one b- and one anti-b-quark with mass (measured/calculated) 9398/10041 MeV/\( c^2 \) is not included within this figure as well as those scalar and vector mesons used for determining the model’s constants.

6 Discussion

This Kaluza-Klein like model enables the calculation of meson and baryon masses. In 50% of all comparisons the relative error between measured and calculated masses is smaller than 0.05 and 85% show deviations below 0.1. The model is based on the structure of the standard model of particle physics with the three interactions strong, electromagnetic, and weak but it does use the mathematical methods of general relativity. Consequently, gravity is an integral part of this 10-dimensional construct. Beside the usual 4-dimensional spacetime there are 6 compactified dimensions responsible for the strong (dimensions 4 to 6), the electric (dimension 7), and weak (dimensions 8 & 9) interactions. Excitations in 10 dimensions always travel with the speed of light. Stable waves around the tubes evolve naturally within the framework of the 10-dimensional spacetime and are responsible for the main fraction of mass. A substantial contribution to the particle mass, at least for the light quarks, results from magnetism. However, there appear to be subtle mass contributing effects, which are not rendered perfectly. E.g., the radii as given in (94) are expected to be identical. Within
the equations (18), (31), and (44), however, the flavor constants might not completely cancel for fast particles. When calculating the magnetic constants $N, M$ and the strong coupling $\alpha_s$, three out of four first generation meson masses and their respective equations (87), are needed. Depending on the selection of the mesons, the numerical values of the constants and the strong coupling vary slightly. Nevertheless, calculating the masses of the composite particles mesons and baryons (Tables 5 to 8) produces sensible results. A detailed knowledge of the metric tensor and the $\gamma$-factors of the quarks, relative to the hadron’s center of mass system is not necessary for the calculation. By combining the rest energy, the kinetic energy, and the potential energy, the terms containing those expressions cancel. Differences between measured and calculated masses are small. However, there are two major exceptions. An abnormally large difference is found with the $\Delta^{++}$ baryon, with spin $3\hbar/2$. It has a calculated mass of approximately 112% higher than the measured value. This could be caused by the mass measurement, which does not discriminate between the delta mesons with a different electric charge. Nonetheless, it seems that the extremely high magnetic field strength of this triple up quark arrangement combines differently than other baryons with a lower field strength. Something similar can be observed with the charmed sigma double plus $\Sigma^{++}_c$, which has an identical magnetic field strength. The quark content is uuc, hence the relative influence of its magnetic field on the mass is smaller and has a deviation of 36%. It can be expected to find the mass calculation of the so far not yet measured baryons ucc and ccc being overstated as well. Spin $\hbar/2$ baryons consisting of three different quark flavors do exist with 2 slightly different masses. This model is capable of calculating the different masses. For example, when comparing the $\Lambda^0$ to $\Sigma^0$, both have the same quark content of dsu, but with different mass. During the developmental process of the model, it was not only possible but also necessary to calculate some important constants: the compactification radius $\rho$, the weak $\alpha_w$ and the strong $\alpha_s$ coupling and $\delta$ the neutrino distribution ratio. $\delta$ evolves from the two compactified dimensions responsible for the weak interaction with a logarithmic dependency on the mean distance to the neutrino/anti-neutrino. In this context $\delta$ is responsible for the extremely low neutrino mass. Therefore, the estimate for the neutrino’s upper mass border from the maximal possible mass difference between positron and electron, is $m_{\nu_e} = \frac{2\hbar}{g} \alpha_w \ln(\delta) \leq 0.001 \text{eV}$, for the lightest neutrino. It is not clear what the mass of the second or third generation neutrino might be. By taking into account the number received for $m_{\nu_e}$ is the upper bound, the value appears to be in agreement with the upper limits of $1.1 \text{eV}$ reported by the KATRINA collaboration [41], or the $0.33 \text{eV}$ of the astrophysical Chandra observations [42]. However, additional experimental verification is needed. Could there be areas in space with a different neutrino to anti-neutrino distribution? And can this difference be measured? The particle mass, e.g., of the electron, would change slightly, as would the magnetic moment. In the calculation above, using the mass data from PDG [29] for $e^-$ and $e^+$, the bigger $\delta$ value was used, which means the number of anti-neutrinos is bigger than the one of neutrinos. As a consequence, the neutrino mass is positive and the anti-neutrino mass negative. The overall combined mass of neutrinos and
anti-neutrinos is negative. In the event of a smaller number of anti-neutrinos compared to neutrinos, the neutrino mass would be negative and the mass of anti-neutrinos positive, yielding a positive total mass.

In the context of this paper, the couplings play the roles of constants, since mesons and baryons are described in its 'stable' situation and not in the process of decay. This does not mean that quarks and anti-quarks exist in a static configuration. Quite the opposite is true. The hadrons' masses arise as "solid" due to the averaging of the kinetic energies connected to strong, electric, and weak interaction.

The tensor calculus of general relativity is used, but it is not necessary to expect the 10D spacetime to be a smooth canvas. The averaging processes might mimic smoothness, which for small time intervals \( \ll 10^{-24} \) s and small distances \( \ll 10^{-15} \) m might not hold true. The full relativistic expression of the invariant term \( f_{no}(z) \) is not known. However, for small, normalized velocities \( \beta \ll 1 \), the values are \( f_{no}^{\text{leptons}} = 1 \) and \( f_{no}^{\text{quarks}} = -1 \) (see appendix 8.7). The radii of slow-moving particles are explicitly computable, and via equation (3) the metric tensors are calculable too. Equation (60) permits the calculation of velocity albeit keeping in mind \( f_{no} \) is approximated. For quarks within a meson this results in the velocities depicted in Figure 6. A further approximation, caused by the computational difficulties, took place. The weak interaction, which might significantly influence the outcome for \( z < 0.1 \), is not included within the calculation.

\[
\begin{align*}
u \bar{u} & \quad u \bar{d} & \quad d \bar{d}
\end{align*}
\]

Figure 6: Normalized velocity \( \beta = \frac{v}{c} \) as a function of the normalized distance \( z = \frac{R}{\rho} \) between two quarks as results of equation (60).

All accelerations are calculated using the geodesic equation of the 10-dimensional spacetime. In such sense all forces within this model are exclusively caused by the curvature of spacetime. A further analysis of the hadrons' internal accelerations and forces between quarks seems possible, which at this point is outside the scope of this paper.
7 Conclusion

Hadrons, scalar and vector mesons, as well as baryons with spin $3\hbar/2$ and $\hbar/2$ were calculated. 50% of comparisons between measurement and calculation have a relative error below 0.05 and 85% below 0.1. This, together with the Pearson correlation coefficient of $r = 99$ demonstrates the model’s predictive power, a model consisting of a 10-dimensional construct, with the usual 4-dimensional spacetime and 6 compactified dimensions. Excitations, temporary deformations always travel with the speed of light. Excitations on the compactified dimensions can form stable waves with a wavelength $\lambda = \frac{4\pi r_o}{\sigma}$, and with $r_o$ : radius of the compact dimension, and $\sigma$ : an integer, the excitation number. The energy of such a construct is in accordance with Louis de Broglie’s interpretation of wave mechanics \[ E = \hbar \cdot \omega = \frac{c\hbar}{r_o}. \] This energy and the contribution from magnetism is the reason for the matter measured in 4D spacetime. In general, within this model, all energy/matter and dynamics are manifestations of the excitations acting on the 10D structure. The undisturbed radii of the compactified dimensions are: 

- $\rho_\alpha$ : for the strong; 
- $\rho_e$ : for the electric; 
- $\rho_w$ : for the weak interaction with $\rho$ being the compactification radius. Within the model, the constants, the most important to mention are $\rho, \alpha_w, \alpha_s, \delta$, were derived by means of the measured masses of the charged leptons, the particles $\pi^0, \pi^+, \rho^0, \phi, \psi, \Upsilon$, the top-quark mass $m_t$ and the anomalous magnetic dipole moment $g$. Once this was completed, the hadron masses were calculated. This data can be found in the Tables 5 to 8. The masses induced by the excitations of the weak interaction are influenced by the proximity of neutrinos and antineutrinos. On the other hand, the mixing ratio directly and minimally effects the particle masses of hadrons and charged leptons, while dominating the neutrino masses. The particle masses do provide some information on the neutrino/anti-neutrino distribution. Further, it seems the model has the potential for analyzing the creation and decay processes of particles without the use of Feynman diagrams [44].

8 Appendix

8.1 Statistical error analysis

The newly established system constants are $\rho$ : compactification radius, $\alpha_w$ : weak coupling, $\alpha_s$ : strong coupling, $\delta$ : neutrino distribution ratio, $M&N$ : magnetic constants, $A_{d,u,s,c,b,t}$ : flavor constants, $A_{e,\mu,\tau,\nu_e,\nu_\mu,\nu_\tau}$ : lepton constants. To calculate these constants the measured masses (Table 11) of the charged leptons and following mesons are used $\pi^0, \pi^+, \rho^0, \phi, \psi, \Upsilon$, the top-quark mass $m_t$ and the anomalous magnetic dipole moment $g$ (Table 13). The statistical deviations of the calculated constants, quark and hadron masses were
obtained using Gauss error propagation (91).

\[ s_Z = \sqrt{\left( \frac{\partial Z}{\partial a} \right)^2 \cdot s_a^2 + \left( \frac{\partial Z}{\partial b} \right)^2 \cdot s_b^2 + \cdots} \] (91)

Equation (91) is used to compute the standard deviation of a quantity \( Z \), which is calculated from parameters given as the mean values \( a, b, \ldots \) and standard deviations \( s_a, s_b, \ldots \) for the numerical result of equations (70), (75), (76), Tables 1, 2, 5, 6, 7, 8, 10, and 11.

### 8.2 Parameters calculated within the model

#### Table 10: List of derived system constants

| Name                      | Symbol | Value                              | Input parameters                  |
|---------------------------|--------|------------------------------------|-----------------------------------|
| Compactification radius   | \( \rho \) | 9.4040252(14) \cdot 10^{-14}m      | \( g, \alpha, m_e^- \)           |
| Weak coupling             | \( \alpha_\mu \) | 1.41040(26) \cdot 10^{-6}         | \( m_e^+, m_e^-, \rho, \alpha_e \) |
| Neutrino distance ratio   | \( \delta \) | \( 1.0000034530(64) \) if \( m_e^+ < m_e^- \) 0.9999965467(64) \( m_e^- > m_e^+ \) | \( m_e^+, m_e^-, \rho, \alpha_e \) |
| Strong coupling           | \( a_s \) | 0.49743(39)                        | \( m_{e^+}, m_{e^-}, m_{\mu^0}, \alpha_e, \alpha_{\mu^0}, \delta \) |
| Magnetic moment’s constant| \( M \) | 4.74681(57) \cdot 10^{-28}kg^2    | \( m_{e^+}, m_{e^-}, m_{\mu^0}, \alpha_e, \alpha_{\mu^0}, \delta \) |
| Magnetic field’s constant | \( N \) | 1.323(10) \cdot 10^{-28}kg         | \( m_{e^+}, m_{e^-}, m_{\mu^0}, \alpha_e, \alpha_{\mu^0}, \delta \) |
| Electron constant         | \( A_e \) | 0                                  |                                   |
| Electron neutrino constant| \( A_{\nu_e} \) | 0                                  |                                   |
| Muon constant             | \( A_\mu \) | 0.33561(26)                        | \( m_\mu \)                       |
| Muon neutrino constant    | \( A_{\nu_\mu} \) | unknown                            |                                   |
| Tau constant              | \( A_\tau \) | 5.6698(44)                         | \( m_\tau \)                      |
| Tau neutrino constant     | \( A_{\nu_\tau} \) | unknown                            |                                   |
| d-flavor constant         | \( A_d \) | 0                                  |                                   |
| u-flavor constant         | \( A_u \) | 0                                  |                                   |
| s-flavor constant         | \( A_s \) | 0.86050(84)                        | \( m_s \)                         |
| c-flavor constant         | \( A_c \) | 4.14863(52)                        | \( m_c \)                         |
| b-flavor constant         | \( A_b \) | 15.320(12)                         | \( m_b \)                         |
| t-flavor constant         | \( A_t \) | 550.8(1.0)                         | Calculated using \( m_t \) of [29] |

#### Table 11: List of quark masses (without contributions of magnetism)
### 8.3 Parameters from literature used in the calculations

Table 12: List of measured particle masses used in the calculations [29]

| Measured particle masses used in the calculation | Mass [kg] | Mass [MeV/c²] |
|------------------------------------------------|-----------|---------------|
| Electron/Positron                              | 9.109384033(55) · 10⁻³¹ | 0.5109989461(31) |
| Muon                                           | 1.883531692(43) · 10⁻²⁸ | 105.6583745(24) |
| Tau                                            | 3.16754(21) · 10⁻²⁴ | 177.86(12) |
| Pion⁻                                          | 2.4061801(89) · 10⁻²⁸ | 134.9768(5) |
| Pion⁺                                          | 2.4880683(32) · 10⁻²⁸ | 139.57039(18) |
| Rho⁻                                           | 1.38203(41) · 10⁻²⁷ | 775.26(23) |
| Phi                                            | 1.817354(29) · 10⁻²⁴ | 1019.461(16) |
| J/Ψ                                            | 5.520726(11) · 10⁻²⁴ | 3096.900(6) |
| Upsilon                                        | 1.786808(55) · 10⁻²⁶ | 10023.26(31) |

Table 13: Constants from literature used in the calculations [29]

| Measurement                          | Symbol | Value                                     |
|--------------------------------------|--------|-------------------------------------------|
| Speed of light                       | c      | 299792458 m/s                             |
| Reduced Planck constant              | h = h/π² | 1.054571817 · 10⁻³⁴ Js                  |
| Electron’s anomalous magnetic dipole | g_e    | 2.00231930436182(52)                      |
| Electric coupling                    | α_e    | 7.2973525693(11) · 10⁻³                   |

### 8.4 Magnetic mass contributions

The currents, the electrical charges divided by the time an excitation needs for a full turn around on a tube, are the sources for the magnetic fields and the magnetic dipoles.

\[
I_oγ = \frac{σ_{γEC}}{12π|r_oγ|}, \quad I_νγ = \frac{ν_{γEC}}{12π|r_νγ|}, \quad I_ργ = \frac{ρ_{γEC}}{12π|ρ_γ|} \tag{92}
\]

The unknowns here are the radii. Equation (18) is the formula for the calculation but indeterminate parameters prevent an actual calculation. Fortunately, there
exist two expressions for calculating the dipole moment of a first generation particles namely

\[ \mu = 2IA = \frac{\omega e|\tau|c}{6} \quad \text{and} \quad \mu_s = \frac{-\omega g_e e \hbar}{4m_o c}. \]

Rearranging the terms leads to

\[ |r_o7| = \frac{3g_o7\hbar}{2m_o7c}. \quad (93) \]

In contrast to the radius calculation of the electron, with the anomalous dipole moment \( g_e \) precisely known, \( g_o7 \) for quarks is not determined. Here two assumptions take place:

1. \( |g_e - g_o7| < 5 \)
2. \( g_u \approx g_d \)

In addition, the masses of u- and d-quarks without the contribution of magnetism are treated as equal. Following the radii of the quarks are set identical

\[ |r_d| = |r_u| = |r_s| = |r_c| = |r_b| = |r_t| \quad (94) \]

because quarks of second and third generation differ from those of the first generation in the second part - the constant - of equation (52) only. The distance between quarks in mesons or baryons are not static. Therefore, the radii of (94) are understood as the average values of the dynamic systems.

The magnetic field’s energy of a meson - a quark anti-quark pair - with the electric charges \( o7 \) and \( n7 \) is:

\[ U_B = \int \frac{(B_o7 + B_n7)^2}{2\mu_0} dV \quad (95) \]

Here are \( B_o7 = \mu_0 I_o7 F(z) \) and \( B_n7 = \mu_0 I_n7 F(z) \) the two magnetic fields consisting of \( \mu_0 \), the permeability of vacuum, \( I \) the electric current, and \( F(z) \) a factor depending on the specific location in space. Following, the energy of the electric field can be stated as

\[ U_B = \alpha_e c \hbar N' \left( \frac{o7}{|r_o7|} + \frac{n7}{|r_n7|} \right)^2 \quad (96) \]

where \( N' \) contains the contribution of the integration. The substitution \( \alpha_e = \frac{\mu_o e^2 c}{4\pi \hbar} \) was used. In the last step all constants are united in the magnetic field constant \( N \).

\[ m_{qq}^B = \frac{U_B}{c^2} = (o7 + n7)^2 N \quad (97) \]
The contribution of each magnetic dipole within the magnetic field of the partner quark/anti-quark is

$$U_{oM} = |\mu_7 B_{n7}| = \frac{\alpha_e |\sigma n_7| g h^2 M'}{6m_q |r_{n7}|}$$

$$U_{nM} = |\mu_{n7} B_{o7}| = \frac{\alpha_e |n_7 \sigma| g h^2 c M'}{6m_\bar{q} |r_{o7}|}$$

As in the case of the magnetic field energy, all constants are subsumed within the magnetic moment constant $M$. The additional masses caused by the dipole moments are

$$m_{dipole}^q = \frac{|\sigma n_7| M}{m_q} \quad m_{dipole}^{\bar{q}} = \frac{|\sigma n_7| M}{m_\bar{q}}$$

Apostil 1: In the calculations of the compactification radius $\rho$, the weak coupling constant $\alpha_w$, and the neutrino distance ratio $\delta$, the magnetic field energy of the electron was neglected. The electron’s electric radius is $r_e = \frac{9\rho}{\alpha_e} = \frac{3g_h}{2m_e c} \approx 1.12 \times 10^{-12} m$ compared to the mean quark radius of $r_q = \frac{3g_h}{2m_q c} \approx 2.83 \times 10^{-15} m$.

This causes the electron’s $B$-field energy being weaker by a factor $r_e^2 < 6 \times 10^{-6}$ compared to the quark field strength.

Apostil 2: There is no "color magnetism" because each color consists of a positive and a negative excitation number cancelling each other.

Apostil 3: "Weak magnetism" is suppressed by a factor $\frac{2\alpha_w}{\alpha_e} < 2 \times 10^{-4}$ relative to the electromagnetic contribution.

### 8.4.1 Vector meson contribution

Equations (86) are those derived as above - (97) and (99). The contributions of (99) are always positive for vector mesons.

### 8.4.2 Scalar meson contribution

Equations (85) are again derived from above equations (97) and (99) with the $n$-quark flipped exchanging $n_7$ with $-n_7$. The contributions of (99) are always negative for scalar mesons.

### 8.4.3 Spin $\frac{3}{2}$ baryon contribution

The difference to the vector meson case of equation (86) is an additional $B$-field/magnetic dipole only, which directly leads to the equations (89).

### 8.4.4 Spin $\frac{1}{2}$ baryon contribution

The obvious scheme for calculating the magnetic mass seems to alter equation (89) by exchanging $p_7$ with $-p_7$ in analogy to the procedure of the scalar mesons (85). This, however, does not lead to meaningful mass results. A different
mechanism must be active. Clearly, the symmetry of (89) is broken in a way to allow having different mass values for same quark content with different quark flavors flipped. This is achieved with the equations (90) in which the magnetic field and the dipole moment of the flipped quark appear to be oriented perpendicular to the fields and dipole moments of the other quarks. In addition the following selection rules take place:

1. For baryons consisting of a mixture of u-like (u, c, t quarks) and d-like quarks (d, s, b quarks), the non-flipped quarks always represent a u-d-like combination.

2. In case of a baryon consisting of a u-d-mixture but build of three different quark species, there always exist two combinations with a different quark flipped. These are the spin \( \frac{1}{2} \) baryons with identical quark content but dissimilar mass.

3. For baryons with charge "++" (three u-like quarks) or charge "," (three d-like quarks) the quark of the biggest mass is flipped.

8.5 Coordinates and velocities within the ten dimensions

The coordinate origin is chosen to be at the entangled partner-particle location. So, \( z^1 = z \) is the normalized distance between two entangled particles. The normalized distance between the two particles is of interest only. Therefore, the coordinate systems \( z^1 \)-axis is selected to point in the direction of the particle and the two other spatial coordinates are zero. Within the compact dimensions a stable excitation is spread out onto the complete cycle \( 4\pi r_i \). For some calculations, e.g. of the Coulomb force, it is necessary to use one single coordinate, the average of the compact dimension’s range.

\[
z^1 = z \quad z^2 = z^3 = 0 \quad z^i = \frac{2\pi r_i}{\rho} \quad i = 4, ..., 9 \tag{100}
\]

The velocities of the 3-space is as usual smaller \( c \) for massive particle (induced by the higher dimensions) and equal to the speed of light for massless particles and excitations (gravitational waves). Within the higher dimensions velocity is always the speed of light.

\[
u^0 = \gamma c \quad v^1 = \gamma \beta c \quad v^2 = v^3 = 0 \tag{101}
\]

\[
u^i = signum(\alpha_i) \quad c \quad i = 4, ..., 9
\]

8.6 The metric tensor

The components of the metric tensor are influenced by the distance to the next particle and marginally by the average distances to the next neutrino and anti-
neutrino. The metric tensor has the form
\[ g_{\mu\nu}(z) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & g_{11}(z) & 0 & 0 & g_{14}(z) & g_{15}(z) & 0 & g_{17}(z) & g_{18}(z) & g_{19}(z) \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & g_{41}(z) & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & g_{51}(z) & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & g_{71}(z) & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & g_{81}(z) & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & g_{91}(z) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
\end{pmatrix} \] (102)

For \( 1 \ll z \) all off-diagonal elements approaching zero and \( g_{11}(1 \ll z) = -1 \). The components can be calculated directly using equation (3). A simplification of \( g_{11} \) can be archived by calculating the metric tensors separately for each interaction: s: strong, e: electric, and w: weak and combining the components to
\[ g_{11}(z) = g_{11}^s(z) + g_{11}^e(z) + g_{11}^w(z) + 2 \] (103)

8.7 The invariant expression \( f_{no}(z) \)

The relativistic expression of \( f_{no}(z) \) is not known. For charged leptons at rest, however, the expression is
\[ f_{no}(z, \beta^i_o = 0, \beta^i_n = 0) = 1 \] (104)

\[ = g_{00} - g_{77} \pm \frac{1}{2} (g_{88} + g_{99}) \]

The upper signs are for two particles having a plus/minus charge combination, the lower sign combination otherwise. For quarks within a meson or baryon
\[ f_{no}(z, \beta^i_o = 0, \beta^i_n = 0) = -1 \] (105)

\[ = g_{00} + g_{ii} + g_{jj} \mp g_{77} \pm \frac{1}{2} (g_{88} + g_{99}) \]

, depends on the active color - \( i, j = 4, 5, 6 \) with \( i \neq j \) - and the contribution of the superposition of dimensions 8 and 9.

8.8 Lorentz factors at the interactions’ fix point locations

Any velocity below the speed of light is consistent with the equations (18), (31), and (44) at the location of the fix points. When applying equation (60) - \( w^\mu u_\mu = (1+n_{acd})c^2 \) with \( n_{acd} \): sum of active compact dimensions and taking the signature into consideration - for each interaction one of the possible solutions is \( \beta = 0 \) resulting in \( \gamma = 1 \). These settings are determined for each interaction separately. In combination of two or three interactions active, the velocity at the fix point can take different values.
8.9 The logic of the spacetime signature

The spacetime signature does influence the results of the strong, electric, and weak forces. The setting must be compatible with the following demands:

1. 4D spacetime signature is chosen as \((1 - 1 - 1 - 1)\).

2. For the Coulomb force \(f_{\text{no}}(z) = 1\) is proven, from which follows a positive signature for the electric dimension and a negative signature for the weak dimensions.

3. Color forces take effect at a distance between two quarks at about one femtometer. In this case too, a positive signature for the color dimensions follows necessarily.

With these demands respected the spacetime signature becomes \((1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1)\).

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