Inventory-Production System Subject to Production Constraints with Non-Zero Target Inventory

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Abstract: Stock levels of inventory-production systems has to be optimized such that inventory holding cost reduce meanwhile customer satisfaction increase. But not all companies has such capability due to different types of limitations that exist in the real world. One of these limitations is production constraints which forces manufacturing line to operate under specific upper and lower boundaries. A control theory model of this phenomena has been proposed for zero target inventory resulting in stock out during peak market demand. In this paper we aim to extend it to non-zero target inventory conditions to analyze safety stock levels. Therefore we will be able to find a certain target inventory for each demand frequency to prevent stock out and increase customer satisfaction.

Key Words: NIOBPCS, Inventory, Production, Stock out, Customer service level

1. Introduction

An efficient production and inventory planning system controls both material and information flows throughout the supply chain. And since supply chain lead time is often higher than promised delivery time, a specific safety stock level need to be stored to guarantee steady flows and satisfy specific customer service level (Li and Jiang, 2012). Some researchers believe that safety stock could be implemented on unfinished items too (Whybark and Williams, 1976), while others argue that it is not necessary to apply it to the whole chain because a safety stock level for finished product automatically increase the stock levels at upstream (Nahmias, 2009). Regardless of this controversial argument, it is generally accepted that safety stock act as a buffer against different demand fluctuations and uncertainties to maintain a predefined service level (Bonney, 1994).

Safety stock is emphasized when the manufacturing line cannot afford high fluctuations and thus cannot follow highly fluctuated demands. In this case manufacturing line are forced to operate smoothly to prevent capacity idleness (Parsanejad and Matsukawa, 2014). Although capacity utilization is an important target in inventory-production systems but customer satisfaction should not be neglected. To overcome this conflict we need to install extra stock of finished items to respond customer needs. In turn having extra inventory levels increase the cost in different formats such as holding, maintenance or opportunity costs. These contradictory aspects of production and inventory systems show an inevitable trade-off between production quantities, inventory level and customer satisfaction (Graves 1988; Zinn and Marmorstein, 1990), forcing supply chain managers to install a control system in order to solve the problem such that all stockholders of the chain being satisfied.

In this paper we aim to analyze a Nonlinear Inventory Order-Based Production Control System (NIOBPCS) for non-zero target inventory levels to discover the optimum level of safety stock for different demand frequencies. NIOBPCS is an extend version of Inventory Order-Based Production Control System (IOBPCS) subject to upper and lower production constraints implying a production smoothing phenomena. In this nonlinear inventory-production system the production signal is not allowed to fluctuated more than a specific level so that production line operate smoothly. Although an smooth production line has capacity utilization benefits but customer will not be satisfied due to late delivery or stock out. Therefore we need to introduce a comprehensive model to cover all of the above-mentioned issues.

In the following steps we briefly explain NIOBPCS proposed by Parsanejad and Matsukawa (2014) and then implement non-zero target inventory policy to investigate behavior of the system subject to production smoothing constraints and then analyze relationship among production quantity, inventory levels, stock out and customer satisfaction. And since we use control engineering methodology to model and solve the problem, first we review the related literature.

2. Literature review

Control theory was first applied to production and inventory control problems by Simon (1952) on continuous systems and then extended to discrete systems by Vassian (1955). Towill (1982) proposed IOBPCS where the system has production process with lead time of Tp and two control parameters, i.e., Ta , which represents demand averaging, and Ti, which represents filling the gap between the on-hand inventory level and the target inventory level. The Laplace transform block diagram of IOBPCS is shown in figure 1 where D represents the external demand with amplitude of M, frequency of w and average value of d, P represents production, I represents inventory, and TINV represents target inventory.
IOBPCS was extended into variable IOBPCS by setting a variable target inventory level instead of a constant target inventory level (Edgill and Towill, 1990). John et al. (1994) and Disney et al. (2000) extended the IOBPCS model into APIOBCS by considering work-in-process inventory. There are also other extensions of IOBPCS, for example, using different forecasting mechanisms (Riddalls and Benett, 2002; Dejonckheere et al., 2002), introducing discrete time (Disney and Towill, 2003; Dejonckheere et al., 2003), applying state space (Lalwani et al., 2006) and taking into account remanufacturing in the supply chain (Zhou et al., 2006).

3. NIOBPCS

One of the obstacles that hinders further development of control theory in the context of inventory-production system is linear assumptions of the studies (Ortega and Lin, 2004). In practice inventory-production systems usually behave as nonlinear due to waste, vulnerability, uncertainty, congestion, bullwhip, economy of scale, and self-interest (Blanco et al., 2011). Owing to the complexity of nonlinear analysis, there are a few studies published in this field. In our knowledge, there are only two research categories related to our research in this paper. The first category is research considering the capacity constraint (Ishii and Imori, 1996; Grubbstrom and Wang, 2003; Haksever and Moussourakis, 2005; Grubbstrom and Huynh, 2006; Wang et al., 2009; Rinaldi and Zhang, 2012; Jia, 2013), and the second category is the research considering the order constraint in the system (Wang et al., 2012; Wang et al., 2014).

Production smoothing is one of the most important supply chain phenomena that has nonlinear property. Parsanejad and Matsukawa (2014) propose a nonlinear model using control engineering methodology to analyze production smoothing by extending IOBPCS.

In IOBPCS if manufacturing line is allowed to produce items without any limitation the production signal can limitlessly fluctuate to satisfy the market demand. This system must be prepared for peak production to track the demand. This condition is illustrated in figure 2a, where system A sometimes works with high capacity, but often portion of production capacity is idle. But unlimited production system has higher capacity and therefore could follow the demand pattern rapidly, but it suffer from higher production cost.

In practice companies have capacity constraints that force manufacturing lines to operate under constraints. In this case, production quantity could not be higher than the capacity constraint, as shown in figure 2b. Consequently, part of the demand will not be satisfied. This unsatisfied demand could be produced during the lower load period, supporting the introduction of lower production bound, as shown in figure 2c. This postponed production can also be justified as pre-production for the next peak market demand.

System C has three main advantages compared with systems A and B. First, it operates with less capacity compared with system A. Second, it utilizes a higher portion of its capacity compared with system B. Finally, the production signal in system C is smoother than both system A and B. Alongside these advantages, production constraints of system C will cause some negative effects, such as inventory amplification, delivery delay, and stock out.

To model system C we need to add an upper and lower amplitude limiter after production component. This nonlinearity can be defined as a production smoother as shown in figure 3. Using this component, the amplitude of production is cut at upper and lower bounds, where X represents input and Y represents the output of the nonlinear component. If we apply this production constraints on IOBPCS, a nonlinear model i.e. NIOBPCS is created as shown in figure 4.

Parsanejad and Matsukawa (2014) formalize NIOBPCS assuming the demand of production system composed of constant and sinusoidal signals representing average and variable demand respectively. This type of demand has been analyzed in a number of papers especially in the field of linear control theory. Edgill and Towill (1990) analyze weekly, monthly and seasonal sinusoidal demand using IOBPCS and VIOBPCS and compare the results. Towill and del Vecchino (1994) use IOBPCS to analyze seasonality in a three echelon supply chain and discover demand amplification or attenuation. Dejonckheere et al., (2002), (2003), and (2004) analyze sinusoidal demand for APIOBPCS, Order up to policy, and APVIOBPCS respectively. All of these studies investigate linear models but since NIOBPCS is nonlinear, the linear approach is no longer useful for modeling and solving the problem.

Parsanejad and Matsukawa (2014) applied a nonlinear methodology to discover frequency response of NIOBPCS as shown in figure 5 and 6, where the horizontal axis is demand frequency and vertical axis is amplitude ratio of production and inventory signal. figure 5 and 6 show that for different demand frequencies what is the amplitude of production and inventory signals.

Frequency response of the system for M=0.6 is equivalent to frequency response of IOBPCS and illustrates that the amplitude of production and inventory signals is only a function frequency of market demand. But frequency response the system for higher M represent NIOBPCS where amplitude of production and inventory signals are not only a function of frequency of demand but also a function of amplitude of demand. We also observe that for higher demand amplitudes, amplitude ratio of production signal of NIOBPCS becomes less than IOBPCS, showing more production smoothing compared with less demand amplitudes. And also amplitude ratio of inventory signal of NIOBPCS which is representation of inventory swings, for $M \leq 1$ is more similar to NIOBPCS due to less production smoothing, but for $1 < M$ is descending when frequency of demand increase.
4. NIOBPCS WITH NON-ZERO TINV

All aforementioned results of NIOBPCS is for zero target inventory levels. Having zero target inventory the system experiences stock outs during market demand peaks. This situation can be affordable for companies that the inventory holding cost is extremely high and low customer service levels is accepted. On the other hand the total cost of inventory management can be defined as the sum of inventory holding and shortage costs (Persona et al., 2007). So we need to take into account both holding associated cost and shortage cost in planning and control of an inventory-production system.

In practice there are different situations for supply chains where different cost elements of the system would be differ. Due to the importance of customer satisfaction, the cost of a system with stock outs might be higher than inventory holding cost. And also the positive relationship between customer service level and inventory holding levels is straightforward. Since the safety stock levels are exponentially related to the desired level of customer service (Zinn and Marmorstein 1990), we need to consider which level of customer service is appropriate for the company then allocate the safety stock to satisfy it. We cannot implement safety stock for NIOBPCS with zero target inventories, thus we need to consider a non-zero target inventory policy. In this case the problem would be finding a level of target inventory to achieve a certain customer service level.

Target inventory is one of the inputs of NIOBPCS. The other input is demand of the market. If we put target inventory equal to zero, it means that we are eliminating its effects. Switching from zero to non-zero target inventory result in some changes in the system behavior. To analyze its behavior we need to analyze the system response for each input separately then combine the results. Considering the only input of non-zero target inventory the inventory level at the steady state conditions would be exactly equal to non-zero target inventory. It means that the only effect of increasing target inventory is increasing the mean value of inventory signal. Hence we could set the target inventory such that the inventory signal always become a positive number.

Since negative inventory indicates stock outs, we need to increase target inventory levels up to the amplitude ratio of inventory for each frequency as illustrated in figure 6. If we set target inventory equal to amplitude ratio of inventory signal the safety stock will be zero and stock out does not occur. For lower amounts of target inventory levels there would be a specific amount of stock out in the system that implies a specific customer dissatisfaction.

We implement a simulation to prove our assertion. The result of simulation for production lead time of $T_p=4$, time to adjust demand of $T_a=8$ and time to adjust inventory of $T_i=4$, demand signal with frequency of $w=0.2$, amplitude of $M=1$ and average value of $d=5$, are shown in the figure 7 for $TINV=0$ and $TINV=6.57$ separately.

In figure 7, the black sinusoid curve is demand signal, the
red line is limited production signal which is subject to upper and lower constraints, the green curve is inventory signal when TINV=0 and the green dash line is inventory signal when TINV=6.57. We found TINV=6.57 from figure 6 where the amplitude of inventory signal for w=0.2, M=1 is 6.57. And if we set TINV=6.57 it acts as average inventory levels in output and elevates the inventory signal to above zero and thus the system will not have any stock out. The safety stock in this target inventory level is zero but any uncertain fluctuation in demand may result in stock out and to be more safer an inventory-production system need to increase target inventory level to maintain enough confidence levels.

5. DISCUSSION

The value of inventory amplitude for each frequency which could be found from figure 6, is the border of having and not having stock out. The target inventories more than this value result in less probability of stock out in case of uncertainty, and the target inventories less than this value result in stock out. This finding is consistent with the theoretical relationship between safety stock and service level. The amount of stock out would be the area under inventory signal where it became negative. Needless to mention that the area under inventory signal where the inventory signal is positive is equal to real inventory holding in the stock. For more clarification we simulate the system behavior for the above mentioned specifications for an arbitrary target inventory i.e. TINV≺2 and the results are shown in figures 8. In figure 8 the stock out for each period is shown by black area under negative inventory signal. It shows that for target inventories less than amplitude ratio of inventory signal shown at figure 6, (i.e. TINV = 2 ≺ 6.57), the stock out still exists because part of inventory signal in this target inventory falls below zero and causes backlog orders.

On the other hand figure 8 also shows that there are inventory holding where the inventory signal is positive. In this case the amount of inventory holding of the system cannot compensate stock outs. Indeed the average inventory is not enough to prevent stock out and that is why we argue that the system require at least the amount of target inventory equal to inventory amplitude shown at figure 6.

For demand amplitudes less than production constraints M ≤ 1 there is no significant difference between IOBPCS and NIOBPCS because the cutting production amplitude by smoothing constraints is small. But for demand amplitudes more than production constraints i.e. 1 < M, there are meaningful differences between IOBPCS and NIOBPCS. For 1 < M and in low frequencies the need for safety stock is considerably higher than other frequencies due to large amount of stock out in low frequencies. It means that inventory production managers or supply chain controllers could find necessary safety stock levels from figure 6 and apply it in one stage or the whole chain. In practice they could use this safety stock to buffer demand fluctuation and get rid of stock outs and increase customer service levels.

6. CONCLUSION

In this paper we extended the NIOBPCS to non-zero target inventory levels. The target inventory influences on safety stock levels that acts as a buffer in front of uncertainties and demand fluctuations. The results of study indicate that increasing the target inventory increases safety stock. The target inventory of zero, leads to significant stock outs and low customer service levels. We proved that the minimum target inventory level must be at least equal to amplitude of inventory, in order to prevent stock out. The more the target inventory, the more safety factors and more confidence interval in terms of less probability of stock out. A stationary target inventory may not suffice rapid changes of the market and therefore future extension of this study can be analyzing Variable Inventory Order-Based Production Control System (VIOPBCS) where the target inventory is a dynamic function of demand.

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