Accelerated detectors in Dirac vacuum: the effects of horizon fluctuations

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Abstract
We consider an Unruh–DeWitt detector interacting with a massless Dirac field. Assuming that the detector is moving along a hyperbolic trajectory, we modeled the effects of fluctuations of the event horizon using a Dirac equation with random coefficients. First, we develop the perturbation theory for the fermionic field in the presence of randomness. Furthermore, we evaluate corrections due to the randomness in the response function associated with different model detectors.

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1. Introduction
The aim of our investigations is to take a pragmatic point of view toward seeking experimental consequences of horizon fluctuations in the Hawking effect [1]. The main motivations for such kinds of studies follow the ideas presented in [2]. In this work, in condensed matter physics, an analogue model for quantum gravity effects was proposed. See also [3–5]. Two general features of waves propagating in random fluids were considered. First, acoustic perturbations in a fluid define discontinuity surfaces that provide a causal structure with sound cones. Second, the propagation of acoustic excitations in random media are generally described by wave equations with random speed of sound [6, 7]. A quantum scalar field theory associated with acoustic waves was analyzed in a situation where the velocity of propagation of the acoustic wave and consequently the sound cone fluctuates.

Motivated by hydrodynamic models which reproduce features of black-hole physics, Unruh introduced the idea of analogous models [8]. In this scenario, suppose that we are able to construct a sonic black hole with a quantum Bose liquid. This system contains phonons as elementary quasi-particles. If the horizon radiates bosonic quasi-particles with the Hawking temperature, after introducing randomness, i.e. horizon fluctuations, the measurable spectral density must be modified. In a previous paper, the spectral density associated with a scalar field assuming event-horizon fluctuations was presented [9]. In particular, a uniformly accelerated
two-level system interacting with a massless scalar field was considered. Assuming the two-level system prepared in the ground state and the field in the Minkowski vacuum state, the main aim of this paper was to show how the transition rates are modified by event-horizon fluctuations. For modeling the event-horizon fluctuation, it was assumed that the scalar field satisfies a massless Klein–Gordon equation with a random coefficient. A modified response function with the same local temperature was found in the non-fluctuating case. The correction due to the event-horizon fluctuations has a Fermi–Dirac factor. A similar result was previously found by Takagi [10] in a quite different situation. For a careful discussion of the apparent inversion of statistics, see for example [11, 12]. In conclusion, the spectral density of bosonic quasi-particles in the presence of horizon fluctuations is given by the results presented in [9].

Another possibility that has been discussed in the literature is to generate black-hole analogues using a superfluid He^3 [13–15]. In this effective ‘black-hole-like’ spacetime, we also expect that the horizon radiates fermionic quasi-particles with the Hawking temperature. In this scenario, we are able to introduce randomness that simulates horizon fluctuations. How the spectral density of the fermions are modified due to the randomness is a fundamental question that must be answered. In this paper, we study how a particular model for the fluctuations of an event horizon can affect the transition rate of a two-level system interacting with a massless fermionic field [16, 17]. Since close to the horizon the black-hole metric takes the form of a Rindler line element, in order to capture the essential physical features of the horizon fluctuations, we consider a uniformly accelerated detector, with a proper acceleration α in Minkowski space. Our approach is the following: first we mimic horizon properties using accelerated detectors and we also assume that event-horizon fluctuations are modeled basically by centered, stationary and Gaussian random processes. Under these conditions, we obtain a random differential equation for the Dirac field that cannot be solve exactly. Therefore, we implement a perturbation theory, similar to the one developed in [18], in our case associated with a massless Dirac field.

The organization of the rest of the paper is as follows. In section 2, we discuss the perturbation theory for the fermionic field in a disordered medium. In section 3, we present the modified positive Wightman function due to the fluctuating horizon. In section 4, we study the modifications to transition probabilities of a Unruh–DeWitt detector due to event-horizon fluctuations. Finally, section 5 contains our conclusions. To simplify the calculations, we assume the units to be such that ℏ = c = k_B = 1.

2. Dirac field in a disordered medium

Our aim is to show how the transition rates of the Unruh–DeWitt detector interacting with a fermionic field are modified by event-horizon fluctuations. Since we implement horizon fluctuations introducing randomness in the Dirac equation, let us study a particular random Dirac equation given by

\[ i\gamma^0(1 + \mu(x))\frac{\partial}{\partial t} - i\gamma^i\nabla_i - m \Psi(t, x) = 0. \]  
\( (1) \)

Here, \( \Psi(t, x) \) represents a fermionic field, \( \gamma^0 \) and \( \gamma^i \) are the Dirac matrices and \( \mu(x) \) is a dimensionless random function of the spatial Cartesian coordinates. We consider a zero-mean random function

\[ \langle \mu(x) \rangle_{\mu} = 0. \]  
\( (2) \)

We also suppose that the noise is Gaussian distributed, i.e. the correlation of an odd number of noises is zero. For simplicity, we suppose white-noise correlations:

\[ \langle \mu(x)\mu(x') \rangle_{\mu} = \sigma^2\delta(x - x'). \]  
\( (3) \)
The symbol $\langle \cdots \rangle_\mu$ denotes the averaged ensemble of noise realizations and $\sigma$ represents the strength of the noise. The last statement is the pragmatic one. This is one of the simplest models one can choose which can exhibit light-cone fluctuations. To proceed, let us define the Fourier transforms to the Dirac field and also to the noise:

$$
\Psi_1(t, x) = \int \frac{d\omega}{2\pi} \frac{d^3k}{(2\pi)^3} e^{-i(\omega t - k \cdot x)} \Psi(\omega, k),
$$

$$
\mu(x) = \int \frac{d^3k}{(2\pi)^3} e^{ik \cdot x} \mu(k).
$$

Defining in equation (1) the free and perturbed part

$$
L_0 + L_1 /\Psi_1 = 0,
$$

where $L_0$ and $L_1$ are the matrices in the coordinates or momentum spaces, in the coordinate representation, we obtain

$$
L_0 = i\gamma^0 \partial_t - i\gamma^i \nabla_i - m
$$

and

$$
L_1 = i\gamma^0 \mu(x) \frac{\partial}{\partial t}.
$$

In the momentum space, $L_0$ and $L_1$ are written as

$$
L_0 = \left[ \gamma^0 \omega + \gamma^i k_i \right] \delta(k - k')
$$

and

$$
L_1 = \gamma^0 \omega \mu(k - k').
$$

Defining the full (operator valued) fermionic Green’s function,

$$
S = (L_0 + L_1)^{-1},
$$

and assuming a weak noise, a perturbative expansion for $S$ can be performed. We can write

$$
S = S_0 - S_0 L_1 S_0 + S_0 L_1 S_0 L_1 S_0 - \cdots,
$$

where $S_0$ is just the inverse of the free operator and it is Fourier transformed by

$$
S_0(t, x) = \frac{1}{(2\pi)^4} \int d^4k e^{-ik \cdot (x - x')} S_0(\omega, k).
$$

In the massless case, $S_0(\omega, k)$ is given by the well-known expression

$$
S_0(\omega, k) = -\frac{\gamma^0 \omega + \gamma^i k_i}{\omega^2 - k^2},
$$

where we omitted the prescription $i\epsilon$ in equation (12). After performing the average over the fluctuations, which can also be seen as an average in the ensemble of geometries in equation (10), we can write down the connected two-point Green’s function related to the Dirac field as a Dyson equation

$$
\langle S \rangle_\mu = S_0 - S_0 \Sigma(\langle S \rangle) S_0.
$$

where $\Sigma$ is the irreducible self-energy. The first contribution given by $\bar{S}^{(1)}(t, x)$ is of second order in the random function $\mu$. Using the Fourier representation, we have

$$
\bar{S}^{(1)}(t, x) = \frac{d^3k}{(2\pi)^3} \bar{S}^{(1)}(\omega, k) e^{-i(k \cdot x)},
$$

where

$$
\bar{S}^{(1)}(\omega, k) = \frac{1}{\omega^2 - k^2} \frac{(\gamma^0 \omega - \gamma^i k_i)}{(\omega^2 - k^2)}
$$

In the above equation, the self-energy $\Sigma(\omega, k)$ is given by

$$
\Sigma(\omega, k) = \int d^3k' \frac{(\gamma^0 \omega - \gamma^i k_i')}{\omega^2 - k'^2}.
$$
Only the first part of the integral in the above equation contributes to result $2i\pi^2\omega^2\gamma^0$. A straightforward calculation given in appendix A shows that the correction of the positive-frequency Wightman function is given by

$$
\tilde{S}^{(1)}(t, x) = \frac{\sigma^2}{2D}\gamma^0 I_1 + \frac{\sigma^2}{2D}\gamma^0 (\gamma^1 + \gamma^2 + \gamma^3) I_2,
$$

where $I_1$ and $I_2$ are the same as given by equations (A.11) and (A.12) and $D$ has been defined by $D \equiv (\Delta t - i\epsilon)^2 - |\Delta x|^2$. We are now able to apply these results to the calculation of the response function of the Unruh–DeWitt detector. This will be done in the next sections.

3. Wightman functions and Rindler noise

Our aim is to show how the transition rates are modified by event-horizon fluctuations. Before considering the Unruh–DeWitt detector, we need to understand how the Wightman function associated with the fermionic field changes if we take into account event-horizon fluctuations. To proceed, let us consider the line element of a four-dimensional Schwarzschild space-time describing a non-rotating uncharged black hole of mass $M$:

$$
d^2s^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2 - r^2d\Omega^2,
$$

where $d\Omega^2$ is the metric of a unit 2-sphere. Close to the horizon $r \approx 2M$, the line element given by equation (18) can be written as

$$
d^2s^2 = \left(\frac{\rho}{4M}\right)^2 dt^2 - d\rho^2 - 4M d\omega^2,
$$

where $\rho(r) = \sqrt{8M(r - 2M)}$ and the quantity $4M d\Omega^2$ describes the line element of a 2-sphere of radius $4M$. In turn, the other contribution can be identified with the line element of the two-dimensional Rindler edge by setting $t = 4M\alpha\tau$ and $\rho = \frac{\rho}{4M}$, for $0 < \rho \leq \infty$ and $-\infty < t < \infty$. Therefore, close to the horizon, the metric takes the form of a Rindler line element. In order to capture the essential physical features of the horizon fluctuations, we consider a uniformly accelerated detector, with a proper acceleration $\alpha$. We are interested in the situation where the detector moves in the plane $(t, x)$ ($y = 0, z = 0$). In this case, the relation to the Cartesian coordinates, adapted to inertial observers, is given by

$$
t = \alpha^{-1} \sinh \alpha \tau,
$$

$$
x = \alpha^{-1} \cosh \alpha \tau,
$$

where $\tau$ is the uniformly accelerated observer’s proper time. In the observer’s proper reference frame, it is necessary to Fermi–Walker transport the Dirac field due its non-stationarity [10]. We define a proper spinor in the following form:

$$
\Psi(\tau) = S_\tau \Psi(x(\tau)),
$$

where the corresponding matrix $S_\tau$ takes care of the Fermi–Walker transport. Here, the observer is, by definition, a Rindler observer; in a given time $\tau$ related to the laboratory frame, we define the boosts $\Lambda_\tau$, whose non-vanishing components are given by

$$
(\Lambda_\tau)_0^0 = (\Lambda_\tau)_1^1 = \cosh \alpha \tau,
$$

$$
(\Lambda_\tau)_0^1 = (\Lambda_\tau)_1^0 = -\sinh \alpha \tau,
$$

$$
(\Lambda_\tau)_2^2 = (\Lambda_\tau)_3^3 = 1.
$$

So the corresponding matrix elements of the spinor are given by

$$
S_\tau = e^{i\hat{e}_2^0} = \cosh(\alpha \tau/2) + \gamma_0\gamma_1 \sinh(\alpha \tau/2).
$$
The Wightman function will also be Fermi–Walker transported. The ‘proper’ Wightman function in the accelerated reference frame is called the Rindler noise for the Dirac field and is defined by

$$S(\tau, \tau') = S^+(x(\tau), x(\tau')) \mu S_{-\tau'}.$$

(24)

Changing the variables $\xi = \tau - \tau'$ and $\eta = \tau + \tau'$, the total Green’s function is given by

$$S(\xi, \eta) = S^{(0)}(\xi, \eta) + S^{(1)}(\xi, \eta).$$

(25)

The Rindler noise associated with the free case (non-random Dirac equations) is given by

$$S^{(0)}(\xi, \eta) = i \frac{2}{\pi^2} \frac{\alpha}{2} \left( \frac{\alpha \eta}{2} \right) \gamma^0 \cosh \left( \frac{\alpha \eta}{2} \right) + \gamma^1 \sinh \left( \frac{\alpha \eta}{2} \right) S_{-\eta} \sinh \left( \frac{\alpha \xi}{2} - i \epsilon_1 \right),$$

(26)

which is the result already known in the literature. Using equations (17), (A.11) and (A.12), we find that the Rindler noise, taking into account fluctuations of the event horizon, is given by

$$S^{(1)}(\xi, \eta) = - \left( \frac{\alpha}{2} \right)^6 \left\{ \sigma_1^2(\eta) \gamma^0 S_{-\eta} \frac{\sinh(\frac{\alpha \xi}{2})}{\sinh(\frac{\alpha \xi}{2} + \frac{\alpha \eta}{2} - i \epsilon_1)} + \sigma_2^2(\eta) \gamma^1 S_{-\eta} \frac{\sinh(\frac{\alpha \xi}{2})}{\sinh(\frac{\alpha \xi}{2} + \frac{\alpha \eta}{2} - i \epsilon_2)} \right\},$$

(27)

where some terms were absorbed by the $\epsilon_i$’s factor defined by

$$\epsilon_1 = \frac{\epsilon \cosh \left( \frac{\alpha \eta}{2} \right)}{7 \cosh \left( \frac{\alpha \xi}{2} \right)} \left( G_1(\eta) + 12 \right),$$

(28)

$$\epsilon_2 = \frac{\epsilon \cosh \left( \frac{\alpha \eta}{2} \right)}{7 \cosh \left( \frac{\alpha \xi}{2} \right)} \left( J_1(\eta) + 12 \right),$$

(29)

which are positive. The strength noises were rewritten by $\sigma_1^2(\eta) = -\sigma^2 G_1(\eta)$ and $\sigma_2^2(\eta) = -\sigma^2 J_1(\eta)$, which are also positive functions and grow with $\eta$, while the functions $F_1(\eta), G_1(\eta), H_1(\eta)$ and $J_1(\eta)$ are given by equations (B.2)–(B.5) in appendix B. This result shows how the fluctuating event horizon modifies the Rindler noise associated with the Dirac field. In the next section, we will apply them and the results presented in the previous section to find the response function associated with the Unruh–DeWitt detector.

### 4. Unruh–DeWitt detectors for Dirac particles

In this section, we use our results obtained with the Dirac field to study the response and transition probabilities associated with the Unruh–DeWitt detector; see [19, 20]. We assume a uniformly accelerated detector coupled to a massless Dirac field $\Psi$ with the following interacting Hamiltonian:

$$H_{\text{int}} = c_1 m(\tau) \bar{\Psi}(\tau) \dot{\Psi}(\tau).$$

(30)

In equation (30), $x^\mu(\tau)$ is the worldline of the two-level system parametrized by the proper time $\tau$, $m(\tau)$ is the monopole moment operator and $\bar{\Psi}$ denotes a Dirac conjugate to the field $\Psi$. Finally, $c_1$ is the coupling constant between the detector and the Dirac field.

We consider the detector as a point-like object with two energy levels given by $\omega_g < \omega_e$ and eigenstates $|g\rangle$ and $|e\rangle$. The gap energy between both states is $E = \omega_e - \omega_g$. Defining the initial state of the system at $\tau = 0$ as $|\tau_i\rangle = |g\rangle \otimes |\psi_i\rangle$ and a final state at time $\tau$ as $|\tau_f\rangle = |e\rangle \otimes |\psi_f\rangle$, we can use perturbation theory to compute the probability of transition of the Unruh–DeWitt detector; see [21, 22]. Let us assume that the Dirac field is prepared
in the Minkowski vacuum state \(|0, M\rangle\). In the first-order perturbation theory, we obtain the probability of transition \(P(\tau, 0)\) given by

\[
P(\tau, 0) = c_{\hat{g}}^2 |\langle e|m(0)\rangle|^2 R(E, \tau, 0),
\]

where the detector selectivity is given by \(c_{\hat{g}}^2 |\langle e|m(0)\rangle|^2\). Now, we will only concentrate on the response function which is given by

\[
R(E, \tau, 0) = \int_0^\tau d\tau' \int_0^\tau d\tau'' e^{-iE(\tau'' - \tau)} \langle 0, M| \overline{\Psi}(\tau') \hat{\Psi}(\tau') \overline{\Psi}(\tau'') \hat{\Psi}(\tau'') \rangle |0, M\rangle.
\]

Then, in this case, we have to calculate a four-point correlation function. Due to the Gaussian nature of the noise, this four-point correlation function can be computed in terms of a product of two-point correlation functions. In this case, one can find [22]

\[
\langle 0, M| \overline{\Psi}(\tau') \hat{\Psi}(\tau') \overline{\Psi}(\tau'') \hat{\Psi}(\tau'') \rangle |0, M\rangle = \text{Tr}[(S^+(\tau', \tau''))^2],
\]

where \(\text{Tr}\) is the trace and \(S^+(\tau', \tau'') = \langle 0, M| \overline{\Psi}(\tau') \overline{\Psi}(\tau'') \rangle |0, M\rangle\) is the positive-frequency Wightman function. Introducing again the variables \(\xi = \tau' - \tau''\) and \(\eta = \tau' + \tau''\), the response function may be written as [23, 24]

\[
R(E, \tau) = \frac{1}{2} \int_{-\tau}^{\tau} d\xi \int_{|\xi|}^{2\tau - |\xi|} d\eta e^{-iE\xi} \text{Tr}[(S^+)^2].
\]

After averaging over the fluctuations, we can put equation (25) into equation (34) to obtain the one-loop correction to the response function, hence, we expand the squared Wightman function in equation (33) resulting in four contributions to the response function, where omitting the arguments of the Wightman functions, which can be expressed by

\[
\text{Tr}[(S^+)^2] = \text{Tr}(S_0^2 + S_0S^{(1)} + S^{(1)}S_0 + (S^{(1)})^2).
\]

Substituting equation (35) into equation (34), we can write \(R(E, \tau) = R_0(E, \tau) + R_1(E, \tau)\), where

\[
R_0(E, \tau) = \frac{1}{2} \int_{-\tau}^{\tau} d\xi \int_{|\xi|}^{2\tau - |\xi|} d\eta e^{-iE\xi} \text{Tr}(S_0^2).
\]

and

\[
R_1(E, \tau) = \frac{1}{2} \int_{-\tau}^{\tau} d\xi \int_{|\xi|}^{2\tau - |\xi|} d\eta e^{-iE\xi} \text{Tr}(S_0S^{(1)} + S^{(1)}S_0).
\]

In the asymptotic limit, the first term \(R_0(E, \tau)\) gives the usual thermal contribution. The main aspect is that in the free case (non-random Dirac equation) the probability of excitation, i.e. \(E > 0\) (modulo the selectivity) of the Unruh–DeWitt detector, presents a Bose–Einstein factor [22]. We obtain \(\lim_{\tau \to \infty} R_0(E, \tau) = W_0(E, \tau)\), where

\[
W_0(E, \tau) = \frac{1}{64\pi} \frac{E\tau}{e^{2\pi E/m} - 1} (4\sigma^4 + 5E^2\sigma^2 + E^4).
\]

The second and third terms in equation (35) are the main corrections up to \(\sigma^2\) order; see equations (26) and (27). The last term is of the order \(\sigma^4\) and is discarded. Taking the limit \(\epsilon \to 0\), the leading contribution due to horizon event fluctuations is given by

\[
R_1(E, \tau) = -\frac{8i}{2\pi^2} \left(\frac{\alpha}{2}\right)^6 \int_{-\tau}^{\tau} d\xi e^{-iE\xi} \int_{|\xi|}^{2\tau - |\xi|} d\eta \left\{ -\sigma_1^2(\eta) + \sigma_2^2(\eta) \right\} \sin^2\left(\frac{\alpha\eta}{2}\right),
\]

where we defined \(\sigma_1^2(\eta) = \sigma_2^2(\eta) \cosh^2\left(\frac{\alpha\eta}{2}\right)\) and \(\sigma_2^2(\eta) = \sigma_2^2(\eta) \sinh^2\left(\frac{\alpha\eta}{2}\right)\). Let us write the \(\eta\) integrations in terms of two functions \(f_1(\tau, \xi)\) and \(f_2(\tau, \xi)\). Using \(f_1(\tau, \xi)\) and \(f_2(\tau, \xi)\), \(R_1(E, \tau)\) can be written as

\[
R_1(E, \tau) = -\frac{8i}{2\pi^2} \left(\frac{\alpha}{2}\right)^6 \int_{-\tau}^{\tau} d\xi e^{-iE\xi} \left\{ f_1(\tau, \xi) + f_2(\tau, \xi) \right\} \sin^2\left(\frac{\alpha\xi}{2}\right).
\]
To perform these integrals, it is convenient to replace $\chi = \alpha \xi$ and express each integral as

$$\frac{1}{\alpha} \int_{-\alpha \tau}^{\alpha \tau} d\chi' e^{-i \frac{E}{\alpha} \chi'} h(\tau, \chi') = \frac{1}{\alpha} \int_{-\infty}^{\infty} d\chi' e^{-i \frac{E}{\alpha} \chi'} h(\tau, \chi')$$

$$- \frac{1}{\alpha} \left[ \int_{-\infty}^{0} d\chi' e^{-i \frac{E}{\alpha} \chi'} h(\tau, \chi') + \int_{0}^{\infty} d\chi' e^{-i \frac{E}{\alpha} \chi'} h(\tau, \chi') \right],$$

(41)

where $h(\tau, \chi)$ are the terms similar to the ones in the curly brackets in equation (40). Replacing $\chi \rightarrow -\chi$ in the first integral in squared brackets, we can simplify all brackets to an integral given by

$$K(E, \tau) = -\frac{16}{\alpha} \left( \frac{\alpha}{2} \right)^9 \int_{\alpha \tau}^{\infty} d\chi \sin \left( \frac{E \chi}{\alpha} \right) h(\tau, \chi).$$

(42)

The nature of this term is associated with switching on and off of the interaction between the detector and the background field. It can be neglected if we assume large proper time intervals.

We now need to evaluate the infinity range integrals in equation (41). These integrals are given by

$$I_n(E, \tau) = \frac{1}{\alpha} \int_{-\infty}^{\infty} d\chi \frac{e^{-i \frac{E}{\alpha} \chi}}{\sinh^2(\chi/2)} h(\tau, \chi).$$

(43)

In these cases, $n = 9$. These integrals can be evaluated choosing the contours given in figure 1 and using the residue theorem proposed in [9, 21]. However, note that we have odd-order poles. In fact, these integrals will be associated with a nine-order residue of $h(\chi)$ at $\chi = 0$. Therefore, performing such an integral, the correction to the response function for the Unruh–DeWitt detector is given by

$$W_1(E, \tau) = \frac{\sigma^2(\tau)E^2}{e^{2\pi E/\alpha} + 1} F(E, \alpha),$$

(44)

where

$$F(E, \alpha) = \frac{(-298262\alpha^6 - 29302\alpha^4E^2 + 12817\alpha^2E^4 + 1842E^6)}{1260\pi \alpha}$$

(45)

and now $\sigma^2(\tau) \approx \sigma^2 \sinh(\alpha \tau) \approx \sigma^2 \alpha \tau$. 

Figure 1. Contours of integration of equation (43).
We note that the correction to the response function shows a ‘Fermi–Dirac-like’ factor with temperature given by $T = \frac{2}{\alpha}$, instead of a Bose–Einstein factor found in the free case. Since we assume a static random noise $\mu(x)$, the accelerated detector experiences event-horizon fluctuations of a non-stationary nature. Non-stationary means the time-dependent spectral density. The lack of stationarity does not necessarily mean that one cannot have a thermal-like response function with a local temperature. The temperature that appears in the response function correction is an averaged temperature over the fluctuations and the global temperature is related with the thermal equilibrium. With the above considerations, a possible interpretation of our result is that light-cone fluctuations in this model lead to the emergence of a local temperature as seen by accelerated observers.

So far we have evaluated the horizon fluctuating contribution to the response function for the Unruh–DeWitt detector coupled to a massless fermionic field. It is also possible to assume another interaction Hamiltonian and, consequently, another kind of detector. It is known that the interaction Hamiltonian for the scalar field coupled with the Unruh–DeWitt detector is a monopole moment. In other words, it is a two-level system operator coupled with a scalar field; hence, it is linear in the field. It gives a response function that depends on the two-point Green’s function instead of the four-point Green’s function found for the fermionic field case. Thus, instead of the Hamiltonian given in equation (30), we can find an analogue kind of monopole-moment detector similar to the scalar-case Hamiltonian. Despite the fact that this detector is less realistic, one can consider that it is equipped with a spinor $\Theta_1$ and coupled linearly to the field via the Hamiltonian

$$H_{int} = \bar{\Theta}(\tau) \Psi(x(\tau)) + \Psi(x(\tau)) \Theta(\tau),$$

where $x^\mu(\tau)$ is again the worldline of the two-level system parametrized by the proper time $\tau$, and $\Theta(\tau)$ is the fermionic monopole moment operator. To keep the Hamiltonian stationary to the observer’s proper reference frame, it is also necessary to Fermi–Walker transport the monopole operator and the Dirac field, in the same way as was done above using equations (21)–(23). One can also formulate a perturbation theory in the interaction picture where it is assumed that $\hat{\Theta} = S_\tau \Theta$ and $\bar{\hat{\Theta}} = \bar{\Theta} S_\tau$, rather than $\Theta$ and $\bar{\Theta}$, obey the time-evolution equations.

Following these steps, one can find the new response function:

$$F(E, \tau, 0) = \frac{1}{4} \text{Tr} \left[ \gamma^0 \int_0^\tau d\tau' \int_0^\tau d\tau'' e^{-iE(\tau - \tau'')} \langle 0 | \bar{\Psi}_\alpha(x(\tau')) \Psi_\beta(x(\tau'')) | 0 \rangle \right].$$

This result is similar to the one found in [28]. The response function is proportional to the two-point Wightman function instead of the four-point function found previously. The calculation is carried out in a finite time interval similarly as in equations (41) and (42), and up to first order in $\tau$, we have that the correction due to horizon fluctuations is

$$W_1(E, \tau) \approx \frac{\sigma^2(\tau)}{60\pi \alpha(e^{\frac{\alpha E}{\alpha}} - 1)} \left( -8059 \alpha^4 E - 55 \alpha^2 E^3 + 3684 E^5 \right),$$

where $\sigma^2(\tau)$ is the same as in equation (44). This integral can be evaluated using equation (43) with $n = 6$ considering the same contour given in figure 1. In the case without fluctuations, the two-point function is given by equation (26), and the response is

$$W_0(E, \tau) = \frac{\pi \tau [E^2 + (\alpha/2)^2]}{e^{\frac{\alpha E}{\alpha}} + 1}.$$

Comparing equations (48) and (49), we note that the event-horizon fluctuation modifies the Fermi–Dirac response found in the free case. A Bose–Einstein factor appears. As we discussed, it is also possible to define a local temperature, i.e. $T = \frac{2}{\alpha}$, in this non-equilibrium situation, and in a similar way, the effective noise strength is proportional to the proper time.
At this stage, we point out that there are different regularization schemes in the literature, which lies in the $\epsilon$ regulator in the Wightman function. Some authors (see, e.g., [22, 25–27]), based by causality arguments, proposed an alternative regularization procedure to the Wightman function. Basically, they assume $H_{int}$ as in equation (30) with a smeared form for the field operator (see, e.g., equation (81) from [22]) implying that the detector is no longer point like but now has a finite extension. These assumptions change the integrals present in equations (A.7), which are related to the Wightman function, by another integral given in equation (A.13).

To avoid possible confusion about our results, we briefly discuss the fact that a different choice of regularization does not make significant changes to our main results. Extending our analyses to this other regularization scheme, we need to modify equations (A.7)–(A.9) by the new function present in equations (A.13) and (A.14). Proceeding this way, and because the partial derivatives $(\partial/\partial t, \partial/\partial x)$ does not act on $(\dot{t}, \dot{x})$, we will find results similar to equation (17):

$$\bar{S}^{(1)}(t, x) = \frac{\sigma^2}{2D} \gamma^0 I_1 + \frac{\sigma^2}{2D} (\gamma^1 + \gamma^2 + \gamma^3) I_2,$$

where the prime denotes the finite extended detector functions, and $I_1'$ and $I_2'$ are the new functions defined by equations (A.15) and (A.16). The function $D' = (\Delta t - i\epsilon \delta \dot{t})^2 - (|\Delta x| - i\epsilon |\delta \dot{x}|)^2$.

Note that $I_1'$ and $I_2'$ are very similar to equations (A.11) and (A.12). In fact, the zeroth-order term in $\epsilon$ is the same as the zeroth-order term of $I_1$ and $I_2$. Changing the Cartesian coordinates to the Rindler coordinates, we find that the first-order Wightman function is similar to equation (27); in other words,

$$S^{(1)}(\xi, \eta) = -\left(\frac{\alpha^2}{2}\right)^6 \left\{ \sigma_1''(\eta) \gamma^0 S_{-\eta} - \frac{\sinh(\alpha \xi)}{\sinh(\frac{\alpha \xi}{2} - i\epsilon') \gamma^1} + \sigma_2''(\eta) \gamma^1 S_{-\eta} \frac{\sinh(\frac{\alpha \xi}{2})}{\sinh(\frac{\alpha \xi}{2} - i\epsilon')} \right\}.$$  

The difference lies in the definitions of the new functions $\epsilon''_{1,2}$, which now depend only on $\eta$:

$$\epsilon''_1 = \frac{\epsilon}{7} \left( \frac{G_2(\eta)}{F_2(\eta)} + 12 \right),$$

$$\epsilon''_2 = \frac{\epsilon}{7} \left( \frac{J_2(\eta)}{H_2(\eta)} + 12 \right),$$

which are positive functions, and $F_2(\eta), G_2(\eta), H_2(\eta)$ and $J_2(\eta)$ are the same as defined in equations (B.6)–(B.9). The strength noise ($\sigma(\eta)$) functions have also the same properties as discussed at the end of section 3 and are defined by $\sigma_1''(\eta) = -\sigma^2 F_2(\eta)$ and $\sigma_2''(\eta) = -\sigma^2 H_2(\eta)$. Then, after taking the limit $\epsilon \to 0$, one can use figure 1 again and apply the residue theorem to calculate the new integrals to the response function. After taking these calculations, one find equations (44) and (48) to both kinds of detectors considered before. So, our final expressions to the response function will be the same, leaving our conclusions unchanged.

5. Conclusions

The aim of our investigations was to take a pragmatic point of view toward seeking experimental consequences of horizon fluctuations in the Hawking effect. Recently, in condensed matter physics, an analogue model for quantum gravity effects was proposed. In the analogue model
scenario, suppose a sonic black hole with a quantum Bose liquid. This system contains phonons as elementary quasi-particles. If the horizon radiates bosonic quasi-particles with the Hawking temperature, after introducing randomness, i.e. horizon fluctuations, the measurable spectral density is modified. In a previous paper, the spectral density associated with a scalar field assuming event-horizon fluctuations was presented [9].

Another possibility that has been discussed in the literature is how to generate black-hole analogues using a superfluid He\(^3\). In this case, we expect that the horizon radiates fermionic quasi-particles with the Hawking temperature. In this scenario, if we are able to introduce randomness that simulates horizon fluctuations, the spectral density associated with the fermionic field is also modified.

In this paper, we have used these ideas to study the behavior of a massless fermionic field near a fluctuating event horizon using the techniques described before. First, we developed a perturbation theory associated with a massless fermionic field in the presence of random noise. After performing the calculations, we have applied these results to investigate the thermal radiation near the fluctuating horizon. We showed that the horizon fluctuation implies that the Unruh–DeWitt detector, which has a Bose–Einstein factor in the response function associated with the free case, now has a correction, which has a Fermi–Dirac factor. Since the static random medium implies light-cone fluctuations, the accelerated detector experiences event-horizon fluctuations of a non-stationary nature. Non-stationary means time-dependent spectral density. This is quite an interesting subject that has been discussed by many authors. See, for example, [29, 30]. The lack of stationarity does not necessarily mean that one cannot have a thermal-like response function with a local temperature, as we discussed. See [31–33]. From the above discussion, we see that it is possible to define a local temperature even in this non-equilibrium situation. In a less realistic detector, coupled linearly with a Dirac field, the correction has also an apparent change in the distribution measured by the response function with unchanged temperature. In this situation, the leading term has a Fermi–Dirac factor and in the correction a Bose–Einstein factor appears.

Finally, we discussed that the effects found here are originated by the fluctuations modeled by the random function present in equations (2) and (3) and not by the regularization procedure. In fact, when we use the regularization proposed in [25], we modify equation (A.9) with equation (A.14). This change gives a similar response to what was found in the standard regularization to both detectors considered in section 4. Note that in most of this paper we have considered the standard regularization. This choice is based on the comparison from our results to what is made in the standard literature. It seems that additional criteria are needed to choose one definition over another; however, as discussed, our conclusions do not depend on the type of regularization. Since the de Sitter horizon has also a thermal spectrum measured by the Unruh–DeWitt detector [34] for both bosons and fermions, a natural extension of our work is to study the effects of a fluctuating de Sitter horizon to such a kind of coupling. It is also natural to question the effects of introducing a time-dependent randomness [35]. These subjects are under investigation by the authors.

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Appendix A. One-loop correction to the positive-frequency Wightman function

Let us begin with equation (15) which gives the correction to the positive-frequency Wightman function. The self-energy is given in equation (16). By dimensional regularization, only the first term gives a contribution, which corresponds to $2\pi i\sigma^2\gamma^0$. Replacing this result in equation (15), this expression has a contribution to three different parts, i.e. $\gamma^{\mu 1}\gamma^f k_{\mu}k_{1} = \delta_{\mu}\omega^2 - 2\omega k_{\mu}\gamma^f \gamma^0 - \gamma^f k_{\mu}k_{1}$. Therefore, equation (14) can be written as

$$\tilde{S}^{(1)}(t, x) = 2\pi^2 i\sigma^2\gamma^0 \int \frac{d^3k}{(2\pi)^3} [I_6 + 2k_j\gamma^f\gamma^0 I_5 - \gamma^f k_jk_j I_4] e^{-ik\Delta x},$$  \hspace{1cm} (A.1)

where $I_n$ are the integrals defined by

$$I_n = \int \frac{d\omega}{(2\pi)^n} \frac{\omega^n}{(\omega^2 - k^2)^n} e^{-i\omega(t-\tau)},$$  \hspace{1cm} (A.2)

and $\Delta x = x - x'$. To calculate the positive-frequency Wightman function, we must evaluate equation (A.2) over the contour of integration for the variable $\omega$, as defined in [36]. In this case, only the pole $\omega = |k|$ is chosen and the residue theorem is used. This yields

$$I_n = \frac{i}{4} \omega^{n-3} [n - 1] - i|k|\Delta t \ e^{-i|k|\Delta t}. \hspace{1cm} (A.3)$$

The expression above is simplified and can be written in terms of the momenta as

$$\tilde{S}^{(1)}(t, x) = 2\pi^2 i\sigma^2\gamma^0 \int \frac{d^3k}{(2\pi)^3} \left[ \frac{i|k|^3}{4} [-i\Delta t|k| + 5] + 2ik_j\gamma_j\gamma^0 \frac{|k|^2}{4} [-i\Delta t|k| + 4] - i\gamma^f k_jk_j \frac{|k|^2}{4} [-i\Delta t|k| + 3] \right] e^{-i|k|\Delta t + ik\Delta x}. \hspace{1cm} (A.4)$$

The last two integrals can be simplified using the fact that $\int d^3k\gamma^f f(|k|) e^{-ik(t-x')} = \gamma^f \partial_t \int d^3k f(|k|) e^{-ik(t-x')}$. So our integrals are from the type

$$B^{(n)}_1 = \int \frac{d^3k}{(2\pi)^3} |k|^n e^{-i|k|\Delta x}. \hspace{1cm} (A.5)$$

The expression $\tilde{S}^{(1)}(t, x)$ can be written as

$$\tilde{S}^{(1)}(t, x) = 2\pi^2 i\sigma^2\gamma^0 \left( \frac{5i}{4} B^{(3)}_1 + \frac{1}{4} \Delta t B^{(4)}_1 - 2\gamma^f \partial_t B^{(2)}_1 \right) + i \gamma^f \gamma^0 \partial_t B^{(3)}_1 + \frac{3i}{4} \gamma^f \gamma^f \partial_t B^{(1)}_1 + \frac{\Delta t}{4} \gamma^f \gamma^f \partial_t B^{(2)}_1 \right). \hspace{1cm} (A.6)$$

After calculating the angular parts from the above integrals, it is useful to define a function $g_n(x, x')$ such that the function $B^{(n)}_1$ and its derivatives can be expressed in terms of $g_n(x, x')$ by the following expression:

$$g_n(x, x') = \int_0^\infty dk k^n (e^{-ik(\Delta t - i\epsilon - |\Delta x|}) - e^{-ik(\Delta t - i\epsilon + |\Delta x|)}). \hspace{1cm} (A.7)$$

In the last expression, $k$ is a real variable not a four-vector; thus, it can be put out from the integral through derivatives with respect to $\Delta t$ writing the last equation as

$$g_n(x, x') = \frac{1}{(i)^n} \frac{\partial^n}{\partial(\Delta t)^n} \int_0^\infty dk (e^{-ik(\Delta t - i\epsilon - |\Delta x|)} - e^{-ik(\Delta t - i\epsilon + |\Delta x|)}). \hspace{1cm} (A.8)$$
This expression is related to the Wightman positive function for the scalar field, in such a way that the \( g_n(x, x') \) function may be written as (see [36])

\[
g_n(x, x') = \frac{1}{(\Delta t)^n} \frac{\partial^n}{\partial (\Delta t)^n} \left( \frac{-2i|\Delta x|}{(\Delta t - i\epsilon)^2 - |\Delta x|^2} \right). \tag{A.9}
\]

The \( i\epsilon \) factor that was introduced in equation (A.7) regularizes the oscillatory integrand. The limit \( \epsilon \to 0 \) will be taken at the end of the calculations.

Thus, \( B_1^{(n)}\)'s integrals can be simplified by the expression

\[
B_1^{(n)} = \frac{1}{i(2\pi)^2|\Delta x|^2} g_{n+1}. \tag{A.10}
\]

Finally, substituting equations (A.9) and (A.10) into equation (A.6), we obtain the first contribution to the Wightman function which is given by equation (17), and the functions \( I_1 \) and \( I_2 \) are given by

\[
I_1 = -3684\Delta t^6 + 7539\Delta t^4|\Delta x|^2 - 8328\Delta t^2|\Delta x|^4 - 1005|\Delta x|^6
+ i\epsilon(18504\Delta t^2 - 24732\Delta t^2|\Delta x|^2 + 9354\Delta t|\Delta x|^4) \tag{A.11}
\]

and

\[
I_2 = -1660\Delta t^5|\Delta x| - 2024\Delta t^3|\Delta x|^3 - 156\Delta t|\Delta x|^5
+ i\epsilon(6620\Delta t|\Delta x| + 5056\Delta t^2|\Delta x|^3 + 12|\Delta x|^5). \tag{A.12}
\]

We have used for \( I_1 \) and \( I_2 \) a first-order expansion: \((x + a)^n \approx x^n + nx^{n-1}a\).

Note that in equation (A.7) we defined the regularization factor in the standard way \( t \to t - i\epsilon \). However, other regularization procedures are also possible. In order to show that our results are not a regularization artifact, we also consider the regularization proposed in [25]. In this case, the prescription factor is motivated by the finite extension of the detector and is given by \( e^{i\delta t(x)} \), where \( \delta t(x) = -\delta t + \delta x = \delta \sigma(x) = \dot{x}(\sigma) + \dot{x}(\sigma') \). The new \( g'_n(x, x') \) function is now

\[
g'_n(x, x') = \int_0^\infty dk k^n \left( e^{-i\delta|\Delta t + |\Delta x| - i\delta|\Delta x|} - e^{-i\delta|\Delta t + |\Delta x| - i\delta|\Delta x|} \right), \tag{A.13}
\]

and we obtain

\[
g'_n(x, x') = \frac{1}{(\Delta t)^n} \frac{\partial^n}{\partial (\Delta t)^n} \left( \frac{-2i|\Delta x|}{(\Delta t - i\delta)^2 - (|\Delta x| - i\delta|\Delta x|)^2} \right) \tag{A.14}
\]

which is given in terms of the Wightman function from [25] and the definition of the \( B_1 \) function is the same as in equation (A.10). Substituting equation (A.14) and (A.10) into equation (A.6), we obtain the first contribution to the Wightman function which is given by equation (50), and the functions \( I'_1 \) and \( I'_2 \) are given by

\[
I'_1 = -3684\Delta t^6 + 7539\Delta t^4|\Delta x|^2 - 8328\Delta t^2|\Delta x|^4 - 1005|\Delta x|^6
+ i\epsilon[(18504\Delta t^2 - 24732\Delta t^2|\Delta x|^2 + 9354\Delta t|\Delta x|^4)\delta t
+ (33312\Delta t^2|\Delta x|^3 - 15078\Delta t^2|\Delta x| + 6030|\Delta x|^5)\delta x)] \tag{A.15}
\]

and

\[
I'_2 = -1660\Delta t^5|\Delta x| - 2024\Delta t^3|\Delta x|^3 - 156\Delta t|\Delta x|^5
+ i\epsilon[(6620\Delta t|\Delta x| + 5056\Delta t^2|\Delta x|^3 + 12|\Delta x|^5)\delta t
+ (780\Delta t|\Delta x|^4 + 6072\Delta t^2|\Delta x|^2 + 1660\Delta t^3)|\delta x|]. \tag{A.16}
\]
Appendix B. One-loop correction to the Rindler noise

With the Rindler coordinates defined in equations (20), we are able to define the Rindler noise for the Dirac field by

\[ S(\tau, \tau') = S^\tau (x(\tau), x(\tau')) S_{-\tau}. \]

So using this definition and equations (17), (A.11) and (A.12), we find that in an accelerated reference frame

\[ S^\tau (\xi, \eta) = \frac{\sigma^2}{2D^2} \gamma^0 S_{-\eta} \left( \frac{2}{\alpha} \right)^5 \sinh^5 \frac{\alpha \xi}{2} \left[ \frac{2}{\alpha} \sinh \frac{\alpha \xi}{2} F_I(\eta) + i e G_I(\eta) \right] \]

\[ + \frac{\sigma^2}{2D^2} \gamma^1 S_{-\eta} \left( \frac{2}{\alpha} \right)^5 \sinh^5 \frac{\alpha \xi}{2} \left[ \frac{2}{\alpha} \sinh \frac{\alpha \xi}{2} H_I(\eta) + i e J_I(\eta) \right]. \] (B.1)

The functions \( F_I(\eta), G_I(\eta), H_I(\eta) \) and \( J_I(\eta) \) are defined as follows.

When \( i = 1, \)

\[ F_1(\eta) = -5478 \cosh^6 \frac{\alpha \eta}{2} + 12132 \cosh^4 \frac{\alpha \eta}{2} - 11343 \cosh^2 \frac{\alpha \eta}{2} + 1005, \] (B.2)

\[ G_1(\eta) = 6 \cosh \frac{\alpha \eta}{2} \left( 521 \cosh^4 \frac{\alpha \eta}{2} + 1004 \cosh^2 \frac{\alpha \eta}{2} + 1559 \right), \] (B.3)

\[ H_1(\eta) = -4 \cosh \frac{\alpha \eta}{2} \sinh \frac{\alpha \eta}{2} \left( 960 \cosh^4 \frac{\alpha \eta}{2} - 584 \cosh^2 \frac{\alpha \eta}{2} + 39 \right), \] (B.4)

\[ J_1(\eta) = 4 \sinh \frac{\alpha \eta}{2} \left( 2922 \cosh^4 \frac{\alpha \eta}{2} - 1270 \cosh^2 \frac{\alpha \eta}{2} + 3 \right). \] (B.5)

When \( i = 2, \)

\[ F_2(\eta) = F_1(\eta), \] (B.6)

\[ G_2(\eta) = -48 \left( 4565 \cosh^6 \frac{\alpha \eta}{2} - 10602 \cosh^4 \frac{\alpha \eta}{2} + 10126 \cosh^2 \frac{\alpha \eta}{2} - 1005 \right), \] (B.7)

\[ H_2(\eta) = H_1(\eta), \] (B.8)

\[ J_2(\eta) = -96 \sinh \frac{\alpha \eta}{2} \cosh \frac{\alpha \eta}{2} \left( 800 \cosh^4 \frac{\alpha \eta}{2} - 488 \cosh^2 \frac{\alpha \eta}{2} + 33 \right). \] (B.9)

Expanding the denominator in equation (B.1) and after some calculations, we find a simplified expression for the correction of the Rindler’s noise given in equation (27) when \( i = 1 \) and equation (51) when \( i = 2. \)

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