Stability and spring constant investigation for micromachined inductive suspensions: theoretical analysis vs. experimental results

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Abstract. We present a linear analytical model coupled with experimental analysis to discuss stability of a levitated proof mass (PM) in a micromachined inductive suspension (MIS), which has been previously introduced and characterized. The model is a function of the MIS geometry, describes the dynamics of a levitated disk-shaped PM near the equilibrium point, and predicts conditions for stable levitation. The experimental setup directly measures the lateral component of the Lorentz force, which has a stabilization role in the MIS structure, as well as the vertical levitation force. The experimental setup is further used to derive mechanical parameters such as stiffness values relative to lateral, vertical and angular displacements, proven to be in excellent agreement with the values predicted by the analytical model.

1. Introduction
Downscaling of micromachined inductive suspensions (MIS) has been enabled by the advances in the field of micro-electro-mechanical systems (MEMS) technologies. Due to their non-contact operation based on electromagnetic induction, MIS constitute the building blocks of a wide range of systems operating in a frictionless, dust-free environment, such as gyroscopes, micromotors, micro-positioning systems.

The first achievements in the field of MIS have been reported by Shearwood et al. in 1995, the technology being essentially based on surface micromachining, while the main application proposed was a rotating micro-gyroscope [1]. Shearwood et al. [2] reported 1000 rpm rotation speed and showed that this limit is attributed to a slight wobble during rotation that excited a lateral resonant mode. The performance was improved by Zhang et al. [3] by separating the so-called rotation and levitation coils, reporting a maximum speed of 3000 rpm. However, there was still plenty of room for improvement since the maximum theoretical rotation speed in air is of the order of 100000 rpm [2].

Most of the MIS structures reported so far, including the examples mentioned above, extensively make use of planar microfabrication techniques in order to define the coil structures required to achieve various functions such as levitation, rotation and stabilization of the proof mass (PM) to be levitated.³

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While defining the metal track for these coils is extremely straightforward for one single metal layer, any addition for multi-layered/multi-windings coils tremendously increases the fabrication complexity, at the same time decreasing the yield. Each additional metal layer requires additional masks corresponding to metallization, etching, insulation, vias. Moreover, the current carrying capabilities of metal layers obtained via planar microfabrication techniques (evaporation/sputtering, electroplating) are rather limited. In order to circumvent these problems we have developed a coil winding technology based on an automatic wirebonder [4] for the fabrication of 3D microcoils with a high degree of freedom in terms of shape, height, winding pitch, and number of windings. Using this technology, we reported a 3D micromachined inductive suspension (3D MIS), which was preliminarily characterized [5, 6] demonstrating a dramatically reduced threshold current necessary to achieve levitation, along with increased levitation height compared to Shearwood et al.[2], due to the superior number of ampere-turns in the case of 3D solenoidal microcoils as opposed to the coils obtained through planar technology.

The purpose of this paper is to give a brief theoretical background as a basis to describe the stability and to calculate characteristic parameters of the system such as stiffness values. At the same time, this is the first direct verification of the theoretical model through direct experimental measurements of the forces acting on the proof mass.

2. Operating principle and fabrication

The basics of the 3D MIS proposed by our group have been previously introduced and the fabrication process has been presented in detail [6]. The device consists of two coaxial wirebonded microcoils that are excited with AC signals having 180° phase-shift with respect to each other. The inner coil is responsible for the levitation of the conductive disk (aluminium, 25 μm thick), while the outer coil is responsible for the stabilization of the proof mass. By applying an AC signal to the coils, a time-variable magnetic flux is created, thus inducing eddy currents in the PM. These eddy currents also generate a magnetic field that interacts with the excitation field producing the levitation and the stabilization effects respectively. By biasing the outer coil with an AC signal having 180° phase-shift with respect to the inner coil, a restoring force is introduced that pulls the PM back towards the equilibrium position, thus shaping a potential well which defines the stability of the system.

The fabrication process of the 3D MIS starts with the metallization of a Pyrex wafer in order to define the contact pads for microcoil connection. A CrAu layer is evaporated, followed by UV patterning and electroplating up to a thickness of 10 μm. After dissolving the photoresist mold used for electroplating and etching away the CrAu seed layer, tall SU-8 2150 structures are defined by UV photolithography to serve as support for the subsequent coil winding process. The height of these support structures is approximately 700 μm. The final step of the fabrication is the coil winding process, which is performed using an automatic coil-winding machine.

The PM to be levitated is laser-cut to a diameter of 3200 μm from a 25 μm thick Al foil.

![Figure 1. Schematic of the 3D MIS.](image1)

![Figure 2. Photo of the 3D MIS without PM.](image2)
Table 1. Parameters of the 3D MIS prototype.

| Parameter                                | Value          |
|------------------------------------------|----------------|
| Radius of the levitation coil, \( r_l \) | 1000 \( \mu m \) |
| Radius of the stabilization coil, \( r_s \) | 1900 \( \mu m \) |
| The coils pitch of winding, \( p \)      | 25 \( \mu m \)  |
| Number of windings for stabilization coil, \( N \) | 12 |
| Number of windings for levitation coil, \( M \) | 20 |
| Radius of PM, \( r_{PM} \)               | 1600 \( \mu m \) |
| Thickness of PM                           | 25 \( \mu m \)  |

3. Linear analytical model for MIS

3.1. Theoretical background

In the present paper, we adapt and apply the linear analytical model previously developed in [7, 8] to the MIS structure based on 3D wirebonded microcoils. The linear theoretical model can be seen as the analogy for the 3D MIS of the harmonic oscillator model, allowing us to predict the important parameters of the structure such as different stiffness components and, as a consequence, the corresponding resonant frequencies. The model starts from the dynamical equations written for the three degrees of freedom required to completely characterize the system: vertical displacement (\( l \)), lateral displacement (\( s \)), and angular displacement (\( \varphi \)).

\[
\begin{align*}
ml'' + \mu_l l' + kl' &= F_l; \\
ms'' + \mu_s s' + ks' &= F_s; \\
J\varphi'' + \mu \varphi' + k_s s' + k_{\varphi} \varphi &= M_{\varphi};
\end{align*}
\]

where \( m \) is the mass of PM, \( J \) is the moment of inertia of the PM about the axis lying on its equatorial plane, \( \mu_l, \mu_s, \) and \( \mu_{\varphi} \), and are the damping coefficients of the PM relative to the appropriate velocities of the generalized coordinates \( l, s \) and \( \varphi \), respectively; \( F_l, F_s, \) and \( M_{\varphi} \) are the generalized forces and torque acting on the PM relative to the generalized coordinates; \( k_l, k_s, k_{\varphi} \) and \( k_{sp} \) are the stiffness coefficients corresponding to the generalized coordinates.

As we have previously demonstrated [7, 8], the stable levitation condition requires that the following inequalities are fulfilled simultaneously:

\[
\begin{align*}
&k_l > 0; \quad (2a) \\
&k_s > 0; \quad (2b) \\
&k_{\varphi} > 0; \quad (2c) \\
&k_s \cdot k_{\varphi} - k_{sp}^2 > 0. \quad (2d)
\end{align*}
\]

3.2. Direct experimental measurement of stiffness coefficients

The strategy to measure the stiffness coefficients corresponding to vertical, lateral and angular displacement is to apply a force in a controlled manner and to measure the displacement determined by this force. In order to perform these measurements, we employed a microforce sensing probe (FT-S100, FEMTO-TOOLS AG Switzerland) and a mechanical probe (FT-FS1000, FEMTO-TOOLS AG Switzerland), simultaneously recording the applied force and the vertical and lateral displacement, respectively. The force resolution of the sensing probe was 0.005 \( \mu N \) and the displacement resolution 5 nm.

Figure 3 shows the schematic of the experimental setup. Each coil was fed with a square wave AC current provided by a current amplifier (LCF A093R). The amplitude and the frequency of the current...
in each coil were controlled by a function generator (Arb studio 1104D) via a computer. In this experiment the levitation height, $h$, of the PM was 104 $\mu$m, measured with a laser distance sensor (LK-G32). Root mean square currents in the stabilization and levitation coils were to be $I_S = 0.106$ A and $I_L = 0.11$ A, respectively, at a frequency of 12 MHz. For force and displacement measurements the microforce sensing probe was moved towards the PM until mechanical contact, as shown in the inset of figure 4. Once in contact, the force applied to the PM and the linear displacement along the direction of action were recorded.

To evaluate the lateral stiffness, the tip of the sensing probe was tilted to an angle of 15° with respect to the PM plane due to topological restrictions. The lateral displacement measurement results are shown in figure 4, with a lateral stiffness coefficient, $k_s$, calculated to be $3.0 \times 10^{-3}$ N·m$^{-1}$. Vertical and angular stiffness coefficients, $k_v$ and $k_\phi$, respectively, have been evaluated indirectly by applying the sensing probe vertically at two different points located at $r_{p1}=300$ $\mu$m and $r_{p2}=1450$ $\mu$m from the PM center (insets of figure 5 and 6). Force versus displacement measurements are shown in figure 5 and 6. The corresponding stiffness values are: $k_{p1}=4.5 \times 10^{-2}$ N·m$^{-1}$ and $k_{p2}=1.2 \times 10^{-2}$ N·m$^{-1}$. Substituting $k_{p1}, k_{p2}, r_{p1}$ and $r_{p2}$ into the set below:

$$
\begin{align*}
    k_v &= \frac{(r_{p2}^2 - r_{p1}^2)}{r_{p1}} k_{p1}, \\
    k_v &= \frac{k_{p1}}{k_{p2}} - 1, \\
    k_{p1} &= \frac{k_{p2}}{1 - r_{p2}^2 (k_{p2} / k_v)},
\end{align*}
$$

(3)

the desired stiffness coefficients can be calculated. Hence we have the vertical stiffness $k_v$ relative to the generalized coordinated, $l$: $k_l = 5.3 \times 10^{-2}$ N·m$^{-1}$, and the angular stiffness $k_\phi$, relative to the generalized coordinate, $\phi$: $k_\phi = 3.3 \times 10^{-8}$ N·m·rad$^{-1}$. On the other hand, we have also calculated the same stiffness coefficients using the linear analytical model. A comparison between the values measured experimentally and the calculated ones are shown in Table 2, showing a very good agreement between the experiment and the
model. We also plot the set of inequalities (2), therefore providing a map of stability for the PM as shown in figure 7, where the variable $d$ is the difference between the PM radius and the levitation coil radius, $d = r_{PM} - r_l$.

| Stiffness coefficients | Measured values | Calculated values |
|-------------------------|-----------------|------------------|
| $k_s$, [N·m$^{-1}$]    | $3.0 \times 10^{-3}$ | $3.0 \times 10^{-3}$ |
| $k_t$, [N·m$^{-1}$]    | $5.3 \times 10^{-2}$ | $4.0 \times 10^{-2}$ |
| $k_p$, [N·m·rad$^{-1}$] | $3.3 \times 10^{-8}$ | $0.8 \times 10^{-8}$ |

Figure 7. Stability map of the levitating proof mass as derived from the set of inequalities (2). The arrows show the sign change when the boundary is crossed in the indicated direction.

4. Conclusions and outlook

The present work provides a theoretical insight in the 3D MIS reported previously [6], as well a comparison between the important parameters calculated by means of the theoretical model and the values obtained from direct experiment. This is the first time when the levitation and stabilization forces have been directly measured in such a structure. Moreover, this paper provides the means to describe the dynamics of the 3D MIS structure and clearly formulates the condition for stable levitation. The ability to theoretically predict the stability and the spring constants of the MIS enables future designs with improved dynamics for applications such as gyroscopes or high speed rotating micro-motors.

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