Quantum Measurement Operations with Nanomagnets and SQUIDs

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The low-energy behaviour of 2 coupled nanomagnets or 2 coupled SQUIDs, interacting with their environment, can be described by the model of a “Pair or Interacting Spins Coupled to an Environmental Sea” (PISCES). These physical systems can then be used for a measurement operation in which system, apparatus and environment are all treated quantum mechanically. We design a “Bell/Coleman-Hepp” measuring system, and show that in principle one may design a situation in which quantum interference between system and apparatus can upset the usual measurement operation.

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1. INTRODUCTION

The question of whether quantum mechanics applies at the mesoscopic or macroscopic levels has generated an enormous amount of work in recent years. As emphasised by Leggett, theoretical discussions of this topic should include realistic physical models, with the ultimate goal of testing the predictions of quantum mechanics against experimental results. In particular, it is crucial to include all couplings between the quantum systems and their environment (which according to Quantum theory, is itself also quantum-mechanical).

In this context, it is surprising that no discussion of a Quantum Measurement Operation ever considered both system and apparatus, both coupled to some external environment, with all three described in a fully quan-
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tum mechanical way. Apart from the practical interest of such a discussion (see below), it is also essential if one is to examine the cornerstone of the Copenhagen interpretation, viz., that quantum states can only be defined with reference to some measuring system.

We consider here a fully quantum-mechanical description of a measurement scheme using systems that have already shown evidence of macroscopic quantum behaviour, i.e., SQUIDs and nanomagnets. The low-energy dynamics can be reduced to the “PISCES” model, the dynamics of which was recently elucidated.

2. Models of Quantum Measurements

A quantum measurement typically involves some “measuring coordinate” (usually macroscopic), and a quantum system which is being measured. Both are coupled to their environment. The simplest example is where the quantum “system” is stable in one of 2 interesting quantum states, and the apparatus likewise. In this case the apparatus works as a measuring device if its final state is correlated with the initial state of the system.

One common such model, the “Bell/Coleman-Hepp” model, has Hamiltonian

\[ H^{(A)}(\vec{\tau}) + H^{(s)}(\vec{\sigma}) + \frac{1}{2}K \hat{\tau}_x (1 - \hat{\sigma}_z) \] (1)

where \( \vec{\tau} \) is a Pauli vector describing the apparatus, and \( \vec{\sigma} \) a Pauli vector for the system. The coupling \( K = \pi \), so that if \( \vec{\sigma} \) is in initial state \( | \uparrow > \), then \( \vec{\tau} \) is unaffected, whereas if \( \vec{\sigma} \) is initially \( | \downarrow > \), then \( \vec{\tau} \) flips (with no change in \( \vec{\sigma} \)). We thus have the classic “ideal measurement” scheme, for which

\[ | \uparrow > | \uparrow > \rightarrow | \uparrow > | \uparrow > \]
\[ | \uparrow > | \downarrow > \rightarrow | \downarrow > | \downarrow > \] (2)

ie., the final state of the apparatus is uniquely correlated with the initial state of the system.

Such a scheme can in principle be obtained starting with a large variety of initial “microscopic” high-energy Hamiltonians. Consider, eg., a Hamiltonian

\[ H = \frac{1}{2} (M_Aq_A^2 + M_sq_s^2) + V_A(q_A) + V_s(q_s) - \xi_Aq_A \]

\[ -K_oq_sq_A + \frac{1}{2} \sum_{k=1}^{N} m_k(\dot{x}_k^2 + \omega_k^2x_k^2) + \sum_{k=1}^{N} (c_k^{(s)} q_s + c_k^{(A)} q_A)x_k \] (3)

where \( q_A \) and \( q_S \) are the “coordinate” of the apparatus and system respectively, \( V_s(q_s) \) and \( V_A(q_A) \) describe symmetric 2-well systems (with minima
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at $q_s = \pm 1, q_A = \pm 1$) and the coupling $-K_o q_A q_s$ is produced by expanding the interaction in powers of $q_A$ and $q_s$. A “bias field” $\xi_A$ also acts on the apparatus. The environment is composed of delocalised harmonic oscillators $x_k$ modes, with linear coupling to the macroscopic systems.

Below a “UV cut-off” energy $\Omega_0$, only the 2 lowest levels of each system will be important; thus if $kT, K_0, \xi_A \ll \Omega_0$, it is straightforward to truncate Eq. (3) to the low-energy Hamiltonian

$$H = H_0(\tau, \sigma) + H_{osc}(\{x_k\}) + H_{int}(\tau, \sigma; \{x_k\}) \quad (4)$$

where $H_{osc}$ and $H_{int}$ are the same as in Eq. (3) (with $q_A$ and $q_s$ replaced by $\tau_z$ and $\sigma_z$, and with the restriction that all $\omega_k \ll \Omega_0$); and

$$H_0 = \Delta_A \tau_x + \Delta_s \sigma_x - \xi_A \hat{\tau}_z - K \tau_z \sigma_z \quad (5)$$

Here $\Delta_A$ and $\Delta_s$ are calculable tunneling matrix elements and $K$ is a renormalised interaction, produced by integrating out high energy environmental modes ($\omega_k > \Omega_0$).

We refer to Hamiltonians like (4) as PISCES Hamiltonians (PISCES being an acronym for “Pair or Interacting Spins Coupled to an Environmental Sea”). We have recently solved the dynamics of this model [2]. Here we shall (a) describe under what conditions it behaves like the ideal measuring scheme (2), and (b) discuss how microscopic theory for SQUIDs or nanomagnets, operating in the quantum regime, shows they are described by (4). This then allows us to see how one may experimentally probe the quantum measurement operation.

3. Nanomagnets and SQUIDs

Consider as an example 2 conducting nanomagnets imbedded in a metallic matrix- they interact via dipolar interactions and also via the conduction electron bath. For this problem (of great importance in disordered magnets, metallic glasses, and “colossal magnetoresistance” systems), the interaction with nuclear spins is not important (unlike for insulating nanomagnets!). In the quantum regime, each nanomagnet behaves like a “spin-boson” system [5] with an Ohmic coupling to the bath of dimensionless strength $\alpha_\beta \sim g^2 S^4/3$, where $\beta = 1, 2$ labels each nanomagnet, $g = JN(0)$ is the dimensionless exchange coupling at each electronic spin site in the nanomagnet, and $S$ is the spin quantum number of the “giant spin” produced when the electronic spins lock together at low $T$. Since typically $g \sim 0.05 - 0.2$ for metals, we get very strong coupling ($\alpha \gg 1$), even for very small metallic nanomagnets; however one can also vary $N(0)$, going even to semimetals, to produce very
small $\alpha$. In isolation, if $\alpha \gg 1$, these “giant Kondo” spins are frozen by the electron bath until either $kT, \xi > \Omega_0$, where $\xi = g\mu_B SH_0$ is the bias in an external field along the easy axis, and $\Omega_0$ is roughly the single ion anisotropy ($\sim 1K$), and independent of $S$. Truncation of the pair of nanomagnets to the quantum regime produces a PISCES Hamiltonian like eq. (4), with a renormalised interaction $K_\tau^z \sigma^z = [V_{\text{dip}}(R) + J(R)]\tau^z \sigma^z$, where $R$ is the radius vector between the nanomagnet centres, and $V_{\text{dip}}$ is the dipolar interaction; transverse spin-spin couplings are irrelevant. The electron-mediated interaction $J(R)$ depends on the shape and surface properties of each nanomagnet (for analytic calculations see [6]), but quite generally

$$J(R) \sim \frac{\sqrt{\alpha_1 \alpha_2}}{(2k_F R)^3} \varepsilon_F \equiv \alpha_{12}(R) \varepsilon_F$$

where $\varepsilon_F$ is the Fermi energy (ie., the bandwidth, in this simple calculation), and $k_F$ the Fermi wave-vector.

As a quite different example consider 2 interacting SQUIDs. The results are intuitively obvious; we get a PISCES model, with coupling described by a mutual inductance and capacitance, and possibly a dissipative coupling through the circuitry (depending on the experimental set-up); we give details elsewhere. One can also consider a SQUID ring coupled to a nanomagnet, with one or other behaving as a measuring device.

4. Dynamics of the PISCES Model

Let us briefly review the dynamics of the PISCES model- controlled by the direct interaction $K$, the temperature $T$, two renormalised tunneling matrix elements $\Delta_\beta$, and 3 dissipative couplings, which we will assume here to be Ohmic, with values $\alpha_\beta$ and $\alpha_{12}(R)$. There are 4 main dynamical regimes (as shown in Fig. (1) for the symmetric case $\alpha_1 = \alpha_2 = \alpha$, and $\Delta_1 = \Delta_2 = \Delta$, viz:

(i) The “Locked Phase” ($K \gg T, \Delta_\beta/\alpha_\beta$): The 2 spins lock together, in either ferro- or antiferromagnetic states depending on the sign of $K$, and behave like a single spin-boson system, with tunneling matrix element $\Delta_c = \Delta_1 \Delta_2/|K|$ and coupling $\alpha_c(R) = \alpha_1 + \alpha_2 \pm \alpha_{12}(R)$ to the oscillator bath.

(ii) The “Mutual Coherence” phase ($\Delta_\beta/\alpha_\beta \gg T \gg K; K \gg \Delta_\beta$): The thermal energy $\gg K(R)$, but if the dissipative couplings $\alpha_\beta$’s are small ($\alpha_\beta \ll 1$), the energy scale $\Delta_\beta^*/\alpha_\beta$ can dominate even if $\Delta_\beta < K$. The 2 systems are then in a superposition of quantum states, and the dynamics is composed of damped “beat”-like oscillations, at 3 different frequencies.

(iii) The “Correlated Relaxation” or High-T phase ($T \gg \Delta_\beta/\alpha_\beta, K$). Each spin relaxes incoherently, but in the time-dependent bias generated
by the other- there are 3 characteristic relaxation times. These oscillations simply reflect the fact that the system is in a superposition of quantum states.

(iv) The rather boring “perturbative regime” ($K \ll \Delta_{1}$), in which the total coupling only weakly affects the single spin-boson behaviour

An important feature of the PISCES model is that as soon as $K > \Delta_{1}$, the single spin-boson model results are invalid, and decoherence effects are massively amplified. Coherent oscillations can only exist in the Mutual Coherence regime.

5. The Measurement Operation

We assume our 2 systems are coupled by a renormalised “ferromagnetic” interaction $K(R)$ and also that (i) $\Delta_{A} \gg \Delta_{s}$ and (ii) $K > \xi_{A} \gg \Delta_{1}, kT$. Requirement (i) means the apparatus reacts quickly to any change in the
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system state. Requirement (ii) means that the bias $\xi_A$ holds the apparatus in state $|\uparrow\rangle$ when there is no coupling; when $K$ acts, it must overcome this bias. The resulting net bias between initial and final states of the apparatus must be $\gg \Delta_A, kT$, so that apparatus transitions are irreversible.

We immediately see that this system will behave according to eq. (2). The combined system-apparatus starts off either in $|\uparrow\uparrow\rangle$ (and stays there) or in $|\uparrow\downarrow\rangle$ (and then tunnels inelastically to $|\downarrow\downarrow\rangle$). Typically the apparatus-bath coupling is strong, so the apparatus relaxes quickly. Notice that the quantum dynamics of the system is frozen as soon as the apparatus-system coupling is switched on, since $K \gg \Delta_s$ the measurement suppresses the quantum dynamics of the measured system. This is a general feature of quantum measurements. Notice also the dissipative effect of the remaining oscillators (with $\omega_k < \Omega_o$) on the system is quite different once $K$ is switched on – dissipative effects are stronger when a system is biased. However the most interesting result comes from the mutual coherence regime- if accessible (eg., by varying $R$), we can set up a situation where quantum interference between system and apparatus can mess up the usual “classical” behaviour of the apparatus. In such a situation, all the concepts of measurement theory, based on a classical apparatus, would be incorrect. This result, if seen experimentally, would be intriguing at the very least (as well as further testing quantum mechanics at the macroscopic scale).

Thus models of this kind, including apparatus and system (as well as the environment), have interesting light to shed on measurement theory. The PISCES model appears to be the first attempt to discuss all 3 partners in the measurement operation on an equal and fully quantum-mechanical level.

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