Sharp increase of the effective mass near the critical density
in a metallic 2D electron system

A. A. Shashkin* and S. V. Kravchenko

Physics Department, Northeastern University, Boston, Massachusetts 02115

V. T. Dolgopolov

Institute of Solid State Physics, Chernogolovka, Moscow District 142432, Russia

T. M. Klapwijk

Department of Applied Physics, Delft University of Technology, 2628 CJ Delft, The Netherlands

We find that at intermediate temperatures, the metallic temperature dependence of the conductivity $\sigma(T)$ of 2D electrons in silicon is described well by a recent interaction-based theory of Zala et al. (Phys. Rev. B 64, 214204 (2001)). The tendency of the slope $\sigma^{-1} d\sigma/dT$ to diverge near the critical electron density is in agreement with the previously suggested ferromagnetic instability in this electron system. Unexpectedly, it is found to originate from the sharp enhancement of the effective mass, while the effective Landé $g$ factor remains nearly constant and close to its value in bulk silicon.

PACS numbers: 71.30.+h, 73.40.Qv

In dilute two-dimensional (2D) electron systems, the energy of electron-electron interactions dominates the kinetic energy making the system strongly correlated. The interaction strength is characterized by the Wigner-Seitz radius, $r_s$, which is equal in the single-valley case to the ratio of the Coulomb and the Fermi energies. According to the Fermi liquid theory (whose applicability to dilute 2D electron systems is discussed in Ref. [3]), the electron-electron interactions should give rise to a renormalization of the system parameters including the effective electron mass, $m$, and the effective $g$ factor. A sharp enhancement of the product $gm$ with decreasing electron density — possibly, a precursor of the ferromagnetic instability — has been observed in recent studies of the parallel field magnetoresistance of a metallic 2D electron system in high-mobility silicon metal-oxide-semiconductor field-effect transistors (MOSFETs) [4]. This agrees well with the $gm$ value obtained from the beating pattern of Shubnikov-de Haas oscillations in tilted magnetic fields [7-9].

Recently, temperature-dependent corrections to conductivity due to electron-electron interactions have been calculated by Zala et al. [9] based on the Fermi liquid approach. In contrast to pre-existing theories (see, e.g., Refs. [10,11]), the new theory incorporates strongly interacting 2D electron systems with electron densities, $n_s$, down to the vicinity of the metal-insulator transition (provided that the conductivity $\sigma \gg e^2/h$). For sufficiently strong interactions, it predicts a metallic temperature dependence of conductivity in the entire temperature range. At very low temperatures, in the “diffusive” regime ($T \ll h/k_B\tau$, where $\tau$ is the elastic relaxation time), this is Finkelstein’s weakly-metallic (logarithmic) conductivity [12,13]. At intermediate temperatures, in the “ballistic” regime ($T \gtrsim h/k_B\tau$; $T > 0.2 - 0.5$ K under the conditions of our experiments), the predicted $\sigma(T)$ is similar to the Gold-Dolgopolov linear dependence [10]:

$$\frac{\sigma(T)}{\sigma_0} = 1 - A k_B T,$$

where the slope, $A$, is determined by the interaction-related parameters: the Fermi liquid constants, $F_0^a$ and $F_1^s$. These parameters are responsible for the renormalization of the $g$ factor and the effective mass [3]

$$\frac{g}{g_0} = \frac{1}{1 + F_0^a}, \quad \frac{m}{m_b} = 1 + F_1^s$$

and can be determined experimentally [14]. The slope $A$ is predicted to rise as the interaction strength increases and the 2D electron system is driven toward the ferromagnetic instability. The last is expected to occur, in the simplest case, at $F_0^a = -1$, which corresponds to the diverging effective $g$ factor.

In this paper, we perform precision measurements of the temperature-dependent conductivity in a metallic 2D electron system in silicon over a wide range of electron densities above the critical electron density, $n_c$, for the metal-insulator transition. The theory of Zala et al. [9] is found to be successful in interpreting the experimental data in the ballistic regime. Knowing the product $gm$ from independent measurements, we determine both $g$ and $m$ as a function of $n_s$ from the slope of the temperature dependence of the conductivity. The tendency of the slope to diverge near the critical density is consistent with the suggested ferromagnetic instability in this electron system [15,16]. However, unlike in the simplest scenario for the ferromagnetic instability, it is the value of the effective mass that becomes strongly enhanced with
decreasing electron density, while the $g$ factor remains nearly constant, $g \approx g_0 = 2$ in bulk silicon.

Measurements were made in an Oxford dilution refrigerator with a base temperature of $\approx 30$ mK on high-mobility (100)-silicon samples similar to those previously used in Ref. [16]. The resistance was measured by a standard 4-terminal low-frequency technique. Excitation current was kept low enough to ensure that measurements were taken in the linear regime of response. Contact resistances in our samples were minimized by using a split-gate technique that allows one to maintain a high electron density in the vicinity of the contacts regardless of its value in the main part of the sample. In this paper we show results obtained on a sample with a peak mobility close to $3 \, \text{m}^2/\text{Vs}$ at 0.1 K.

Typical dependences of the normalized conductivity on temperature, $\sigma(T)/\sigma_0$, are displayed in Fig. 1 at different electron densities above the critical electron density for the metal-insulator transition which in this sample occurs at $n_c = 8 \times 10^{10}$ cm$^{-2}$ [17]; the value $\sigma_0$, which has been used to normalize $\sigma$, was obtained by extrapolating the linear interval of the $\sigma(T)$ dependence to $T = 0$. As long as the deviation $|\sigma/\sigma_0 - 1|$ is sufficiently small, the conductivity $\sigma$ increases linearly with decreasing $T$ in agreement with Eq. (1), until it saturates at the lowest temperatures [18].

In Fig. 2 we show the $n_s$ dependence of the inverse slope $1/A$ extracted from the $\sigma(T)$ data. Also shown for comparison is the magnetic energy, $\mu_B B_c = \pi h^2 n_s/gm$ (where $\mu_B$ is the Bohr magneton), corresponding to the onset of full spin polarization in this electron system, which is governed by the (enhanced) product $gm$ [4]. Over a wide range of electron densities, the values $1/A$ and $\mu_B B_c$ turn out to be close to each other. The low density data for $1/A$ are approximated well by a linear dependence which extrapolates to the critical electron density $n_c$ in a similar way to the behavior of the polarization field $B_c$ (note that the tendency of the polarization field to vanish near $n_c$ was also reported in Ref. [13]). We emphasize that it has been verified with the help of the weak-field low-temperature Hall effect measurements that the density of the delocalized electrons in the metallic phase is practically coincident with $n_s$.

As has already been mentioned, the coefficient $A$ in the linear-in-$T$ correction to conductivity of Eq. (1) is determined by the Fermi liquid parameters [3]:

$$A = -\frac{(1 + \alpha E_0^s) gm}{\pi h^2 n_s},$$

(3)

The factor $\alpha$ is equal to $8$ in our case where the temperature is small compared to the valley splitting [13]. This theoretical relation allows us to determine the many-body enhanced $g$ factor and mass $m$ separately using the data for the slope $A$ and the product $gm$ as a function of $n_s$ (the latter is known from independent measurements similar to those described in Ref. [13]).

In Fig. 3 we show the so-determined values $g/g_0$ and $m/m_b$ as a function of the electron density (the band mass $m_b$ is equal to 0.19$m_e$ where $m_e$ is the free electron mass). Note that in the range of $n_s$ studied here, the low-temperature conductivity $\sigma > 8e^2/h$. The behavior of $g$ and $m$ at electron densities below $n_s = 3 \times 10^{11}$ cm$^{-2}$ (corresponding to $r_s \approx 4.8$) turns out to be very different from that at electron densities above this value. In the high $n_s$ region (lower $r_s$), the enhancement of both $g$ and $m$ is relatively small, both values slightly increasing with decreasing electron density in agreement with earlier
FIG. 3. Renormalization of the effective mass (filled squares) and $g$ factor (dots) as a function of electron density. The data from Ref. [8] are shown by open symbols. The dashed lines are guides to the eye. The inset compares the theoretical dependence of the renormalization parameter $F_g$ (dots) calculated using Eq. (2) from our theory [9] to that calculated taking into account the renormalization of $m$ (squares). The dashed lines are guides to the eye.

data [20]. Also, the renormalization of the $g$ factor is dominant compared to that of the effective mass, which is consistent with theoretical studies [21]. The dependence $g(n_s)$ is described reasonably well by the theory: the inset of Fig. 3 compares the theoretical renormalization parameter $F_g^0 = -r_s/2(r_s + \sqrt{2})$ to that calculated using Eq. (3) and the data for $g(n_s)$.

In contrast, the renormalization in the low $n_s$ (critical) region, where $r_s \gg 1$, is much more striking. As the electron density is decreased, Fig. 3 shows that the renormalization of the effective mass overshoots abruptly while that of the $g$ factor remains relatively small, $g \approx g_0$, without tending to increase. Hence, the current analysis indicates that it is the effective mass that is responsible for the drastically enhanced $gm$ value near the metal-insulator transition.

The present results for the effective mass and $g$ factor in the critical region can be compared to the data of Ref. [8] obtained by analysis of the Shubnikov-de Haas oscillations in high-mobility Si MOSFETs. As seen from Fig. 3, data obtained from these two different methods are similar. Thus, the Fermi-liquid-based theory is adequate in describing the properties of dilute 2D electron systems.

It is important to discuss another consequence of the theory: the slope $A$ of the temperature dependence of the conductivity should increase as the ferromagnetic instability in a dilute 2D electron system is approached. Since renormalization parameters have not been theoretically calculated in the limit $r_s \gg 1$, the simplest scenario of the ferromagnetic instability is prompted by Eq. (2): $F_0^g \rightarrow -1$ causes the effective $g$ factor (and the slope $A$) to diverge. Experimentally, the slope $A$ tends to diverge near the critical electron density in a way similar to the behavior of the product $gm$ seen in Fig. 3. This is consistent with the conclusion of Ref. [4] about the possibility of ferromagnetic instability in this electron system. At the same time, the simplest scenario of a diverging $g$ factor is not the case; instead, it is the growing effective mass which controls the anomalous behavior of the dilute 2D electron system near the metal-insulator transition.

The effective mass enhancement was traditionally considered to be small, and, therefore, the value $m \approx m_b$ was used to calculate some of the important system parameters, e.g., the elastic relaxation time $\tau$ extracted from mobility. In Fig. 4, we compare the so-determined $\tau$ (circles) with that calculated taking into account the enhancement of the effective mass (squares). As seen from the figure, in the range of electron densities studied, the corrected $\tau$ keeps increasing down to the lowest $n_s$. Therefore, the mobility drop at low $n_s$ in high-mobility Si MOSFETs (see, e.g., Ref. [22]) turns out to come from the $m$ enhancement rather than from the decrease in $\tau$, although the value $\tau$ is still expected to vanish in the insulating phase. The observed behavior of $\tau$ is consistent with that of the temperature range corresponding to the ballistic regime (see Fig. 1), which gives additional confidence in our analysis down to the vicinity of the metal-insulator transition. Finally, values of $\tau$ much larger than those previously estimated yield appreciably smaller quantum level widths in perpendicular magnetic fields, which helps to understand why the Shubnikov-de Haas oscillations survive near the metal-insulator transition, as well as the origin of the oscillations of the metal-insulator phase boundary as a function of (perpendicular) magnetic field [23].
In summary, we have studied the temperature-dependent conductivity in a wide range of electron densities above the critical electron density for the metal-insulator transition. Using the recent theory of interaction-driven corrections to conductivity \[9\], we extract Fermi-liquid parameters from the experimental data and determine the many-body enhanced \(g\) factor and the effective mass. The tendency of the slope \(A\) of the temperature dependence of the conductivity to diverge near the critical density is in agreement with the suggested ferromagnetic instability in this electron system \[3\]. Unexpectedly, it is found to originate from the growing effective mass rather than the \(g\) factor. In addition, the mass enhancement is found to be responsible for the previously underestimated values of elastic scattering time near the metal-insulator transition.

We gratefully acknowledge discussions with I. L. Aleiner, P. T. Coleridge, L. I. Glazman, D. Heiman, J. P. Kotthaus, B. N. Narozhny, and A. Punnoose. This work was supported by NSF grants DMR-9803440 and DMR-9988283, RFBR grant 01-02-16424, Forschungspreis of A. von Humboldt Foundation, and the Sloan Foundation. TMK acknowledges support through NSF grant PHY99-07949.

---

* Permanent address: Institute of Solid State Physics, Chernogolovka, Moscow District 142432, Russia.

[1] E. Abrahams, S. V. Kravchenko, and M. P. Sarachik, Rev. Mod. Phys. 73, 251 (2001).
[2] L. D. Landau, Sov. Phys. JETP 3, 920 (1957).
[3] C. Chamon, E. R. Mucciolo, and A. H. Castro Neto, cond-mat/0107271.
[4] A. A. Shashkin, S. V. Kravchenko, V. T. Dolgopolov, and T. M. Klapwijk, Phys. Rev. Lett. 87, 086801 (2001).
[5] S. V. Kravchenko, A. A. Shashkin, D. A. Bloore, and T. M. Klapwijk, Solid State Commun. 116, 495 (2000).
[6] V. M. Pudalov, M. Gershenson, H. Kojima, N. Butch, E. M. Dizhur, G. Brunthaler, A. Prinz, and G. Bauer, cond-mat/0105081.
[7] S. V. Kravchenko, A. A. Shashkin, and V. T. Dolgopolov, cond-mat/0106056.
[8] V. M. Pudalov, M. E. Gershenson, and H. Kojima, cond-mat/0110160.
[9] G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B 64, 214204 (2001).
[10] A. Gold and V. T. Dolgopolov, Phys. Rev. B 33, 1076 (1986).
[11] S. Das Sarma and E. H. Hwang, Phys. Rev. Lett. 83, 164 (1999).
[12] A. M. Finkelstein, JETP Lett. 37, 517 (1983).
[13] A. Punnoose and A. M. Finkelstein, Phys. Rev. Lett. (in press); cond-mat/0105471.
[14] An attempt to determine the parameter \(F_0\) from the temperature-dependent conductivity of 2D holes in GaAs was made by Y. Y. Proskurnakov, A. K. Savchenko, S. S. Safonov, M. Pepper, M. Y. Simmons, and D. A. Ritchie, cond-mat/0109261.
[15] S. A. Vitkalov, H. Zheng, K. M. Mertes, M. P. Sarachik, and T. M. Klapwijk, Phys. Rev. Lett. 87, 086401 (2001).
[16] R. Heemskerk and T. M. Klapwijk, Phys. Rev. B 58, R1754 (1998).
[17] The metal-insulator transition point has been determined using both the criterion of a sign change of the temperature derivative of the resistance and that of vanishing activation energy and vanishing nonlinearity of current-voltage curves, see A. A. Shashkin, S. V. Kravchenko, and T. M. Klapwijk, Phys. Rev. Lett. (in press); cond-mat/0009181.
[18] An attempt to describe the temperature-dependent resistance in the metallic phase in a wide range of temperatures by activation law was made by V. M. Pudalov, JETP Lett. 66, 175 (1997).
[19] I. L. Aleiner, private communication. The valley splitting in the low magnetic field limit is equal to 1–2 K, dependent on a particular sample (V. M. Pudalov, A. Punnoose, G. Brunthaler, A. Prinz, and G. Bauer, cond-mat/0104317), which is consistent with the theoretical expectation (see Ref. [20]).
[20] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. 54, 437 (1982).
[21] N. Iwamoto, Phys. Rev. B 43, 2174 (1991); Y. Kwon, D. M. Ceperley, and R. M. Martin, Phys. Rev. B 50, 1684 (1994); G.-H. Chen and M. E. Raikh, Phys. Rev. B 60, 4826 (1999).
[22] S. V. Kravchenko, G. V. Kravchenko, J. E. Furneaux, V. M. Pudalov, and M. D’Iorio, Phys. Rev. B 50, 8039 (1994).
[23] A. A. Shashkin, G. V. Kravchenko, and V. T. Dolgopolov, JETP Lett. 58, 220 (1993).