I. INTRODUCTION

The quality of recording media depends crucially on the intrinsic (microscopic) switching-field distribution (SFD) of the media particles, which determines both magnetic stability and attainable recording density. It is straightforward to obtain the SFD of a diluted magnetic system by taking the derivative of the DC demagnetization (DCD) curve. In the case of high-density magnetic recording media, where magnetic interactions between the media particles are not negligible, the problem of extracting the SFD from bulk magnetization curves cannot be solved rigorously any more and the shape (and to a lesser extent the location) of the extracted SFD will depend on certain model assumptions.

Two conceptually different algorithms have been suggested recently to extract the intrinsic $D(H_s)$ of perpendicular recording media (PRM) from macroscopic magnetization curves using a set of recoil loops. The analysis technique by van de Veerdonk et al. assumes a constant effective demagnetization factor $N_{\text{eff}}$ for deshearing recoil loops, from which the DCD curve is extracted. Extraction of the DCD curve and deshearing the recoil loops are performed simultaneously in order to arrive at a self-consistent solution. Although the algorithm converges after a couple of iterations, it is not clear how reliable the self-consistent solution may be under the assumption of an effective demagnetization factor independent of the magnetization. Another drawback of the method is that it requires assumptions about the shape of the SFD, which of course is not known.

The second algorithm, referred to as $\Delta H(M, \Delta M)$-method, is a generalization of the $\Delta H$ method originally proposed by Tagawa and Nakamura. The $\Delta H(M, \Delta M)$-method approximates interactions on the mean-field level in a sense that all microscopic magnetization configurations $\mathbf{M}(r)$ representing the same macroscopic magnetization value $M$ produce the same volume-averaged internal field $H_i(M)$. An implicit assumption underlying the $\Delta H(M, \Delta M)$-method is that each particle acts as a square hysteron. Then, the lower branch of the major loop $M^- (H_a)$ can be represented as:

$$M^- (H_a) = -M_{\text{sat}} + 2 \int_{-\infty}^{H_{\text{sat}}(H_a,M)} D(H_s) dH_s,$$  \hspace{1cm} (1)

where $D(H_s)$ is the SFD. Any recoil loop $M^+(H_r, H_a > H_r)$ originating from the upper branch $M^+$ at $H_r$ can be written as

$$M^+(H_r, H_a) = M^+(H_r) + 2 \int_{-\infty}^{H_{\text{sat}}(H_a,M)} D(H_s) dH_s.$$  \hspace{1cm} (2)

In order for Eq. 2 to hold, two conditions must be met: firstly, $M^+(H_r, H_a > H_r)$ has to saturate at fields $|H_a| \leq |H_r|$ and secondly, recoil loops must not cross each other. Then the coercivity distribution can be assumed to consist of disjunct segments $D(H_s)dH_s$ (non-interacting hysterons). The inverse $I^{-1}$ of the cumulative distribution

$$I(H) = \int_{-\infty}^{H} D(H_s) dH_s$$  \hspace{1cm} (3)

for a given magnetization value $M$ can then be obtained by taking the difference in field position, $\Delta H(M)$, between $M^- (H_a)$ and recoil loop $M^+(H_r, H_a > H_r)$,

$$\Delta H(M, \Delta M) = H_a(M^-) - H_a(M^+) = I^{-1} \left[ \frac{M_{\text{sat}} + M}{2} \right] - I^{-1} \left[ \frac{M_{\text{sat}} + (M - \Delta M)}{2} \right],$$  \hspace{1cm} (4)

where $\Delta M = M^+(H_r) - (-M_{\text{sat}})$. By fitting the $\Delta H(M, \Delta M)$ curves against the inverse of a certain parameterized distribution function, the key features of the
SFD can be extracted. In case I is a normal distribution centered at \( H_c \), Eq. 4 writes to

\[
\Delta H(M, \Delta M) = \sqrt{2\sigma} \left[ \text{erf}^{-1}(m) - \text{erf}^{-1}(m - \Delta m) \right],
\]

(5)

where lower case m’s are the magnetization values relative to the saturation magnetization \( M_{sat} \).

The third method in our comparative analysis is based on the first-order-reversal curve (FORC) distribution, defined as

\[
\rho(H_r, H_a) = -\frac{1}{2} \frac{\partial^2 M(H_r, H_a)}{\partial H_r \partial H_a}.
\]

(6)

Although there is no principal difference between a FORC and a recoil loop, the numerical determination of \( \rho \) from a set of FORCs requires recoil loops measured on a grid equidistant in \( H_r \) and \( H_a \).

For an assemblage of square hysterons, the FORC distribution is identical to their Preisach distribution and we can obtain their SFD as

\[
D_{\text{FORC}}(H_c) = \int_{-\infty}^{\infty} \rho(H_c, H_b) dH_b,
\]

(7)

where \( H_c = (H_a - H_r)/2 \) and \( H_b = (H_a + H_r)/2 \) are the coordinates of the Preisach plane, defined by the microscopic switching field \( H_c \) and the bias field \( H_b \). As with the \( \Delta H(M, \Delta M) \) method, the FORC method is reliable as long as the media particles can be reasonably well described by square hysterons. As opposed to the two algorithms above, however, the method based on Eq. 6 relies neither on a mean-field approximation nor on a certain model form for \( D(H_a) \) and therefore imposes the fewest constraints on the data.

II. COMPUTATIONAL DETAILS

The three methods presented above are best tested on a system with known SFD. For this purpose, we micromagnetically computed sets of FORCs for a PRM, using the OOMMF code (v. 1.1b2). The simulated medium typically consisted of \( N \sim 2 \cdot 10^3 \) particles, arranged on a regular grid (mesh size \( s = 5 \) nm), inscribed in a circle of 250 nm diameter. We chose a circular boundary to minimize the effects of corners. The easy axes of the particles are all oriented the same way, roughly perpendicular to the surface (\( \theta_K = 89.42 \) deg) so as to avoid numerical problems that may arise when the applied magnetic field \( H \) (\( \theta_H = 90 \) deg) is exactly collinear with the easy direction. We modified the module maganis.cc to produce normal distributed values of the uniaxial magnetocrystalline anisotropy constant, \( K_u \) such that \( \sigma_{H_K} = 0.1 \langle H_K \rangle \), with \( H_K = 2 K_u/\mu_0 M_{sat} \). It has been shown that a small amount of intergranular exchange helps to reduce the increase in the recording transition parameter due to a small distribution in \( H_K \). We therefore set the nondimensional intergranular exchange coupling constant \( h_{ex} = A/(K_u s^2) \) to \( h_{ex} = 0.046 \) in all our simulations. In order to systematically explore the effects of dipolar interactions, we vary the dipolar interaction strength by using a scaling factor \( \phi < 1 \) for \( M_{sat} \), where \( \phi \) represents the volume fraction of magnetic material in each cell. The uniaxial magnetocrystalline anisotropy constant, \( K_u \), is scaled simultaneously to keep the value of the microscopic coercivity \( H_K \) constant as \( \phi \) varies. We normalize all magnetic fields by \( H_K \) and use a mean-field type equation to define the material-independent dipolar interaction strength as \( \phi_Q = \phi/Q \), i.e.,

\[
h_{eff} = h_a + \alpha \frac{\phi}{Q} \frac{M}{M_{sat}} \equiv h_a + \alpha \phi Q m,
\]

(8)

where \( \alpha \) is the mean-field parameter and \( Q \) denotes the quality factor \( Q = 2K_u/\mu_0 M_{sat}^2 \). For hcp Cobalt at room temperature, \( Q = 0.422 \).

III. RESULTS AND DISCUSSION

Figure 1 shows the estimated width \( \sigma^* \) of the SFD as extracted with the three algorithms tested here. In the hypothetical limit case of a PRM controlled solely by exchange-coupling (\( \phi_Q \leq 0 \)), the \( \Delta H(M, \Delta M) \) method is able to capture the intrinsic width of the SFD with great accuracy. For PRM largely dominated by dipolar interactions, however, the \( \Delta H(M, \Delta M) \) method is consistently less accurate than the FORC method, which renders the least deviation of \( \sigma^* \) from \( \sigma \) for \( \phi_Q > 0.15 \). In contrast, the \( \sigma^* \) extracted with the \( N_{eff} \) method always show the largest deviations from \( \sigma \). Importantly, the \( N_{eff} \)
The intrinsic SFD cut-off at a very good degree despite the shift in location (Fig. 2).

All the three methods are primarily concerned with finding the right scale parameter (i.e., \( \sigma^* \) or FWHM) of the SFD distribution, while the location parameter is considered to be invariant. As can be seen in Figure 1 the observed values of \( H_c \) are progressively shifted to lower values with increasing \( \phi_Q \), which is due to dipolar interactions between the media particles, deflection of the effective field away from the easy axes. According to the Stoner-Wohlfarth relationship,

\[
H_\phi(\psi) = H_K \left(\cos^2 + \sin^2 \psi\right)^{-3/2}
\]

observed coercive forces of \( \sim 0.8H_K \) suggest that a large fraction of grains experience a local effective field that deviates by some 5% from the applied field direction, giving rise to curvilinear hysterons, in other words, reversible magnetization processes. In the FORC diagram (Fig. 2), curvilinear hysterons manifest themselves in the form of the small negative region centered at \((H_c, H_b) = (1, -0.05)H_K\). This way, one can determine directly from the FORC diagram if and to what degree the model assumptions are not strictly met.

Since all methods tested here rely on square hysterons, their underlying assumption starts breaking down with increasing dipolar coupling \( \phi_Q \), albeit at different values of \( \phi_Q \). More importantly, the fact that the methods do not all start to fail at the same point also shows that the presence of reversible magnetization processes is not the most crucial limitation to a method’s applicability, which rather is restricted by imposing constraints on the internal field distribution. Compared to the \( N_{eff} \) method, the \( \Delta H(M, \Delta M) \) method puts a less tight constraint on the expected magnetization curves and so has a larger range of applicability. The very absence of any such approximations makes the method based on the FORC distribution the most robust algorithm and therefore most suitable for characterizing ultra-high-density PRM.

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Micromagnetically computed FORCs for $\phi_Q = 0.35$.

Comparison between expected $\Delta H(M, \Delta M)$ curves (solid lines) for $\sigma^* = 0.133$ and $\Delta H(M, \Delta M)$ dataset (dots). Not all $\Delta H(M, \Delta M)$ curves are shown to keep the picture tidy.

Average deviation of expected $\Delta H(M, \Delta M)$ curves (according to Eq. 5) from $\Delta H(M, \Delta M)$ dataset as a function of $\sigma$. The best fit is obtained for $\sigma_{FIT} = 0.133$, which defines $\sigma^*$. 