Lepton flavor violation in low-scale seesaw models: SUSY and non-SUSY contributions

A. Abada$^a$, M. E. Krauss$^b$, W. Porod$^b$, F. Staub$^c$, A. Vicente$^d$ and C. Weiland$^e$

$^a$ Laboratoire de Physique Théorique, CNRS – UMR 8627, Université de Paris-Sud 11, F-91405 Orsay Cedex, France

$^b$ Institut für Theoretische Physik und Astronomie, Universität Würzburg 97074 Würzburg, Germany

$^c$ Physikalisches Institut der Universität Bonn, 53115 Bonn, Germany

$^d$ IFPA, Dep. AGO, Université de Liège, Bat B5, Sart-Tilman B-4000 Liège 1, Belgium

$^e$ Departamento de Física Teórica and Instituto de Física Teórica, IFT-UAM/CSIC, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain

Abstract

Taking the supersymmetric inverse seesaw mechanism as the explanation for neutrino oscillation data, we investigate charged lepton flavor violation in radiative and 3-body lepton decays as well as in neutrinoless $\mu - e$ conversion in muonic atoms. In contrast to former studies, we take into account all possible contributions: supersymmetric as well as non-supersymmetric. We take CMSSM-like boundary conditions for the soft supersymmetry breaking parameters. We find several regions where cancellations between various contributions exist, reducing the lepton flavor violating rates by an order of magnitude compared to the case where only the dominant contribution is taken into account. This is in particular important for the correct interpretation of existing data as well as for estimating the reach of near future experiments where the sensitivity will be improved by one to two orders of magnitude. Moreover, we demonstrate that ratios like $\text{BR}(\tau \to 3\mu)/\text{BR}(\tau \to \mu e^+e^-)$ can be used to determine whether the supersymmetric contributions dominate over the $W^\pm$ and $H^\pm$ contributions or vice versa.


## Contents

1 Introduction ................................................. 4

2 Inverse seesaw model and its supersymmetric extension .......... 6

3 Low energy observables ....................................... 8
   3.1 Effective lagrangian ........................................ 8
   3.2 $\ell_\alpha \rightarrow \ell_\beta \gamma$ ......................... 8
   3.3 $\ell_\alpha^+ \rightarrow \ell_\beta^- \ell_\beta^- \ell_\beta^+$ .... 9
   3.4 $\ell_\alpha^- \rightarrow \ell_\beta^- \ell_\beta^- \ell_\beta^+$ .... 10
   3.5 $\ell_\alpha^+ \rightarrow \ell_\beta^- \ell_\gamma^- \ell_\gamma^+$ .... 10
   3.6 Coherent $\mu - e$ conversion in nuclei .................. 10

4 Results .................................................. 11
   4.1 Numerical setup ............................................ 11
   4.2 Numerical results .......................................... 13

5 Conclusions ................................................. 20

A Masses and vertices ......................................... 21
   A.1 Mass matrices .............................................. 21
   A.2 Vertices .................................................. 25
      A.2.1 Fermion-Scalar vertices ............................. 25
      A.2.2 Fermion-Vector vertices ............................ 28
      A.2.3 Scalar vertices ....................................... 29
      A.2.4 Scalar-Vector vertices .............................. 31
      A.2.5 Vector vertices ....................................... 32

B Renormalization Group Equations ................................ 32

C Loop Integrals ............................................ 36

D Photonic penguin contributions to LFV .......................... 38
   D.1 Feynman diagrams ......................................... 39
   D.2 Neutralino contributions .................................. 40
   D.3 Chargino contributions ................................... 40
   D.4 $W^+$ and $H^+$ contributions ............................. 40

E $Z$ and Higgs penguin contributions to LFV ....................... 41
   E.1 Feynman diagrams ......................................... 41
   E.2 Neutralino contributions .................................. 44
      E.2.1 Z-penguins .......................................... 44
      E.2.2 Scalar penguins ..................................... 45
   E.3 Chargino contributions ................................... 46
      E.3.1 Z-penguins .......................................... 46
      E.3.2 Scalar penguins ..................................... 47
   E.4 $W^+$ and $H^+$ contributions ............................. 50
      E.4.1 Z-penguins .......................................... 50
E.4.2 Scalar penguins .......................................................... 51

F  Box contributions to LFV .................................................. 54
   F.1 Four lepton boxes ....................................................... 54
      F.1.1 Feynman diagrams ............................................. 54
      F.1.2 Neutralino contributions ................................... 56
      F.1.3 Chargino contributions ...................................... 56
      F.1.4 $W^+$ and $H^+$ contributions ............................ 57
   F.2 Additional boxes for $\ell^-_\alpha \rightarrow \ell^-_\beta \ell^+_\gamma \ell^-_\gamma$ ........................................................................ 59
      F.2.1 Crossed neutralino contributions ......................... 59
      F.2.2 Crossed chargino contributions ........................... 60
      F.2.3 Crossed $W^+$ and $H^+$ contributions .................. 61
   F.3 Two-Lepton – Two-Quark boxes ................................... 64
      F.3.1 Feynman diagrams .............................................. 64
      F.3.2 Down quarks .................................................... 65
      F.3.3 Up quarks ..................................................... 67

G  Form factors of the 4-fermion operators ............................. 69
1 Introduction

The recent discovery of a bosonic state at the Large Hadron Collider (LHC) \cite{1,2} stands as a major breakthrough in particle physics. Although further confirmation is required, all data are compatible with the long-awaited Higgs boson, thus completing the Standard Model (SM) particle content. Furthermore, the properties and decay modes of this scalar are in good agreement with the SM expectations, making the SM picture more motivated than ever.

In this context, it is crucial to keep in mind that the SM cannot be the ultimate theory. In fact, and besides theoretical arguments such as the hierarchy problem, there are very good experimental reasons to go beyond the SM (BSM). The best of these motivations is the existence of non-zero neutrino masses and mixing angles, now firmly established by neutrino oscillation experiments \cite{3–5}. Since the SM lepton sector does not include them, one has to go beyond the SM.

A generic prediction in most of these neutrino mass models is lepton flavor violation (LFV), not only in the neutrino sector but also for the charged leptons. Depending of the exact realization of the neutrino mass model, the rates for the LFV processes can be very different. For instance, high-scale models typically predict small branching ratios, thus making LFV hard (if not impossible) to be discovered. In contrast, one expects measurable LFV rates if the scale of new physics is not far from the electroweak (EW) scale. These low-scale mechanisms generating neutrino masses are thus more attractive from a phenomenological point of view, since they offer a window to new physics thanks to their LFV promising perspectives. Moreover, they can be directly tested at the LHC via the production of new particles if these are light enough.

On the experimental side, the field of LFV physics will live an era of unprecedented developments in the near future, with dedicated experiments in different fronts\footnote{See [6] for a recent review.}. In the case of the muon radiative decay $\mu \to e\gamma$, the MEG collaboration has announced plans for future upgrades. These will allow for an improvement of the current bound, $\text{Br}(\mu \to e\gamma) < 5.7 \cdot 10^{-13}$ \cite{7}, reaching a sensitivity of about $6 \cdot 10^{-14}$ after 3 years of acquisition time \cite{8}. Limits on $\tau$ radiative decays are less stringent, but they are expected to be improved at Belle II \cite{9}. These will also search for lepton flavor violating $B$-meson decays. Moreover, the perspectives for the 3-body decays $\ell_\alpha \to 3 \ell_\beta$ are good as well. The decay $\mu \to 3 e$ was searched for long ago by the SINDRUM experiment \cite{10}, setting the limit $\text{Br}(\mu \to 3 e) < 1.0 \cdot 10^{-12}$. The future Mu3e experiment announces a sensitivity of $\sim 10^{-16}$ \cite{11}, which would imply a 4 orders of magnitude improvement. In the case of $\tau$ decays to three charged leptons, Belle II will again be the facility where improvements are expected \cite{12}, although recently the LHCb collaboration has reported first bounds on $\tau \to 3 \mu$ \cite{13}. The LFV process where the best developments are expected in the next few years is neutrinoless $\mu - e$ conversion in muonic atoms. In the near future, many different experiments will search for a positive signal. These include Mu2e \cite{14–16}, DeeMe \cite{17}, COMET \cite{18,19} and PRISM/PRIME \cite{20}. The expected sensitivities for the conversion rate range from a modest $10^{-14}$ in the near future to an impressive $10^{-18}$. Finally, one can also search for LFV in high-energy experiments, such as the LHC. A popular process in this case is the Higgs boson LFV decay to a pair of charged leptons, $h \to \ell_\alpha \ell_\beta$, with $\alpha \neq \beta$ \cite{21–30}. First bounds on $h \to \mu\tau$ have been reported by the CMS collaboration \cite{31}. For other possibilities to search for LFV at high-energy colliders, see \cite{32–48}. In table 1 we collect present bounds and expected near-future sensitivities for the most popular low-energy LFV observables.

With such a large variety of processes, a proper theoretical understanding of potential hierarchies or correlations in a given model becomes necessary. This goal requires detailed analytical
Table 1: Current experimental bounds and future sensitivities for some low-energy LFV observables.

| LFV Process | Present Bound | Future Sensitivity |
|-------------|---------------|--------------------|
| \( \mu \to e\gamma \) | \( 5.7 \times 10^{-13} \) \[7\] | \( 6 \times 10^{-14} \) \[8\] |
| \( \tau \to e\gamma \) | \( 3.3 \times 10^{-8} \) \[49\] | \( \sim 3 \times 10^{-9} \) \[9\] |
| \( \tau \to \mu\gamma \) | \( 4.4 \times 10^{-8} \) \[49\] | \( \sim 3 \times 10^{-9} \) \[9\] |
| \( \mu \to eee \) | \( 1.0 \times 10^{-12} \) \[10\] | \( \sim 10^{-16} \) \[11\] |
| \( \tau \to \mu\mu\mu \) | \( 2.1 \times 10^{-8} \) \[50\] | \( \sim 10^{-9} \) \[9\] |
| \( \tau^- \to e^-\mu^+\mu^- \) | \( 2.7 \times 10^{-8} \) \[50\] | \( \sim 10^{-9} \) \[9\] |
| \( \tau^- \to \mu^-e^+e^- \) | \( 1.8 \times 10^{-8} \) \[50\] | \( \sim 10^{-9} \) \[9\] |
| \( \tau \to eee \) | \( 2.7 \times 10^{-8} \) \[50\] | \( \sim 10^{-9} \) \[9\] |
| \( \mu^-, Ti \to e^-, Ti \) | \( 4.3 \times 10^{-12} \) \[51\] | \( \sim 10^{-18} \) \[20\] |
| \( \mu^-, Au \to e^-, Au \) | \( 7 \times 10^{-13} \) \[52\] | \( 10^{-15} - 10^{-18} \) |
| \( \mu^-, Al \to e^-, Al \) | | \( 10^{-14} \) \[53\] |
| \( \mu^-, SiC \to e^-, SiC \) | | |
experimental improvements, we aim in this work for a complete calculation of the various LFV observables taking into account all contributions at the same time. One of our results will be that there exist several regions in parameter space where cancellations between various contributions occur, changing the interpretation of existing and future experimental results. In order to do so we have made use of FlavorKit [75], a tool that combines the analytical power of SARAH [76–80] with the numerical routines of SPheno [81,82] to obtain predictions for flavor observables in a wide range of models. This setup makes use of FeynArts/FormCalc [83–88] to compute generic predictions for the form factors of the relevant operators and thus provides an automatic computation of the flavor observables. We use this setup to compute for the first time the Higgs penguin contributions to LFV in the inverse seesaw \(^2\). In addition, we improve previous studies in other aspects as well: (i) we make use of the full 2-loop renormalization group equations (RGEs) including all flavor effects in the SM and SUSY sectors to obtain the parameters entering the calculation, and (ii) we include for the first time the decays \(\tau^- \to \mu^+ e^-\) and \(\tau^- \to e^+ \mu^- \mu^-\).

The paper is organized as follows: in Section 2 we present the ISS model and its supersymmetric extension. The LFV observables induced by the extended particle content and the dominant contributions are discussed in Section 3, and in Section 4 we present our numerical results. In Section 5 we draw our conclusions. In the appendices we first introduce the formulae for the mass matrices and our convention for the loop integrals before presenting the additional contributions to the 1- and 2-loop RGEs compared to the MSSM case. More importantly, they contain the complete set of contributions to the LFV observables discussed in this paper.

2 Inverse seesaw model and its supersymmetric extension

In the inverse seesaw, the Standard Model field content is extended by \(n_R\) generations of right-handed neutrinos \(\nu_R\) and \(n_X\) generations of singlet fermions \(X\) (such that \(n_R + n_X = N_s\)), both with lepton number \(L = +1\) [55,59,90]. The corresponding Lagrangian before EWSB has the form

\[
\mathcal{L}_{\text{ISS}} = \mathcal{L}_{\text{SM}} - Y^i_{\nu} \nu_R^i \overline{H}_L L_i - M^i_{R} \nu_R^i X_j - \frac{1}{2} \mu_X^{\nu \bar{X}} X_i^{\nu} X_j + \text{h.c.},
\]

where a sum over \(i,j = 1, 2, 3\) is assumed\(^3\). \(\mathcal{L}_{\text{SM}}\) is the SM Lagrangian, \(Y_{\nu}\) are the neutrino Yukawa couplings and \(M_R\) is a complex mass matrix that generates a lepton number conserving mass term for the fermion singlets. The complex symmetric mass matrix \(\mu_X\) violates lepton number by two units and is naturally small, in the sense of 't Hooft [95], since in the limit \(\mu_X \to 0\) lepton number is restored. This Majorana mass term also leads to a small mass splitting in the heavy neutrino sector, which then become quasi-Dirac neutrinos.

After EWSB, in the basis \((\nu_L, \nu_R^C, X)\), the \(9 \times 9\) neutrino mass matrix is given by

\[
M_{\text{ISS}} = \begin{pmatrix}
0 & m_D^T & 0 \\
m_D & 0 & M_R \\
0 & M_R^T & \mu_X
\end{pmatrix},
\]

where \(m_D = \frac{1}{\sqrt{2}} Y_{\nu} v\) and \(v/\sqrt{2}\) is the vacuum expectation value (vev) of the Higgs boson.

\(^2\)The Higgs penguin contributions to LFV processes were first considered in the context of the inverse seesaw in [89]. However, our paper goes beyond this reference in two ways: by doing the computation in the mass basis and by taking into account all contributions to the Higgs penguins.

\(^3\)The ISS requires the introduction of at least two right-handed neutrinos in order to account for the active neutrino masses and mixings. The most minimal ISS realization [91–93] consists in the addition of two right-handed and two sterile neutrinos to the SM content. However, its minimal SUSY realization [94] requires only one pair of fermionic singlets.
Under the assumption $\mu_X \ll m_D \ll M_R$, the mass matrix $M_{\text{ISS}}$ can be block-diagonalized to give an effective mass matrix for the light neutrinos \cite{96}

$$M_{\text{light}} \simeq m_D^T M_R^{T-1} \mu_X M_R^{-1} m_D ,$$

whereas the heavy quasi-Dirac neutrinos have masses corresponding approximately to the entries of $M_R$.

As usual, one can easily obtain a supersymmetric version of the model by promoting the corresponding fields to superfields $\tilde{\psi}_i^C$ and $\tilde{X}_i$ ($i = 1, 2, 3$) and including the corresponding interactions in the superpotential. This reads

$$\begin{align*}
W &= W_{\text{MSSM}} + \varepsilon_{a b} Y_{\nu}^{a b} \tilde{\psi}_i^C \tilde{L}_i^a \tilde{H}_u^b + M_{R_{a i}} \tilde{\psi}_i^C \tilde{X}_j + \frac{1}{2} \mu_{X_{i j}} \tilde{X}_i \tilde{X}_j .
\end{align*}$$

$W_{\text{MSSM}}$ is the superpotential of the MSSM given by

$$W_{\text{MSSM}} = \varepsilon_{a b} Y_{\nu}^{a b} \tilde{U}_i^C \tilde{Q}_i^a \tilde{H}_u^b - \varepsilon_{a b} Y_{\nu}^{a b} \tilde{D}_i^C \tilde{Q}_i^a \tilde{H}_d^b - \varepsilon_{a b} Y_{\nu}^{a b} \tilde{E}_i^C \tilde{L}_i^a \tilde{H}_d^b + \varepsilon_{a b} \tilde{H}_u^a \tilde{H}_d^b \ ,$$

where we skipped the color indices. The corresponding soft SUSY breaking Lagrangian is given by

$$\begin{align*}
-L_{\text{soft}} &= -L_{\text{soft}}^{\text{MSSM}} + \tilde{\nu}_i^C m_{\tilde{\nu}_i} \tilde{\nu}_i^C + \tilde{X}_i \tilde{X}_j \tilde{X}_j + (T_{\nu}^{a b} \varepsilon_{a b} \tilde{\nu}_i^C \tilde{L}_i^a \tilde{H}_u^b + B_{M_{R}}^{ij} \tilde{Y}_{\nu}^{ij} \tilde{X}_i \tilde{X}_j + \frac{1}{2} \mu_{X_{i j}} \tilde{X}_i \tilde{X}_j + \tilde{X}_i \tilde{X}_j + \tilde{X}_i \tilde{X}_j + \text{h.c.} ) ,
\end{align*}$$

where $B_{M_{R}}^{ij}$ and $B_{\mu_X}^{ij}$ are the new parameters involving the scalar partners of the sterile neutrino states. Notice that while the former conserves lepton number, the latter violates lepton number by two units. Finally, $L_{\text{soft}}^{\text{MSSM}}$ collects the soft SUSY breaking terms of the MSSM.

$$\begin{align*}
-L_{\text{soft}}^{\text{MSSM}} &= \left( \varepsilon_{a b} Y_{\nu}^{a b} \tilde{U}_i^C \tilde{Q}_i^a \tilde{H}_u^b - \varepsilon_{a b} Y_{\nu}^{a b} \tilde{D}_i^C \tilde{Q}_i^a \tilde{H}_d^b - \varepsilon_{a b} Y_{\nu}^{a b} \tilde{E}_i^C \tilde{L}_i^a \tilde{H}_d^b + \varepsilon_{a b} \tilde{B}_{M_{R}}^{ij} \tilde{Y}_{\nu}^{ij} \tilde{X}_i \tilde{X}_j + \frac{1}{2} \mu_{X_{i j}} \tilde{X}_i \tilde{X}_j + \text{h.c.} \right) .
\end{align*}$$

The neutrino mass matrix has the same form as in Eq. (2), just replacing $v$ by $v_u$, the vev of the up-type Higgs boson. The mass matrices of this model are the same as in the MSSM apart from the sneutrino sector. Neglecting for the moment being the soft-breaking terms which lead to a splitting between the scalar and pseudoscalar parts, the corresponding mass matrix reads

$$m_{\nu_i}^2 = \begin{pmatrix}
m^2_D + \frac{1}{2} v^2 u^T Y_{\nu} Y_{\nu}^* + D_L & -\frac{1}{\sqrt{2}} (v_d \mu Y_{\nu} - v_u T_{\nu}) & \frac{1}{\sqrt{2}} v_u R (Y_{\nu}^T M_{\nu}^R) \\
-\frac{1}{\sqrt{2}} (v_d \mu Y_{\nu} - v_u T_{\nu}) & m_{\nu_C}^2 + M_R M_R^T + \frac{1}{2} v^2 u^T Y_{\nu} Y_{\nu}^* & -M_R \mu_S^T \\
\frac{1}{\sqrt{2}} v_u M_R Y_{\nu}^* & -M_R \mu_S^T & M_R^T M_R^* + m_S^2 + \mu_S \mu_S^* 
\end{pmatrix}$$

with

$$D_L = -m_D^2 \cos^2 \theta_W \cos 2\beta \mathbf{1} .$$

The complete mass matrices including the $B$-parameters as well as all other mass matrices can be found in Appendix A.1.
3 Low energy observables

The fact that the LHC has not yet seen any supersymmetric particles [97, 98] implies, at least in the specific SUSY model we consider in this work, that squarks and gluinos must be heavy. However, it could well be that sleptons, charginos and neutralinos are relatively light, thus having large contributions to LFV decays. Here we will consider the processes $\ell_\alpha \to \ell_\beta \gamma$, $\ell_\alpha \to \ell_\beta \ell_\gamma \ell_\delta$ and $\mu - e$ conversion in nuclei. In this section we will present the effective low-energy lagrangian and the basic formulae for the observables. This will also serve to fix our notation (we stay close to the conventions of Ref. [75]). The details for the calculations of the corresponding form factors can be found in appendices C–G.

3.1 Effective lagrangian

The interaction lagrangian relevant for LFV can be written as

$$\mathcal{L}_{\text{LFV}} = \mathcal{L}_{\ell\ell\gamma} + \mathcal{L}_{4\ell} + \mathcal{L}_{2\ell^2 d} + \mathcal{L}_{2\ell^2 u}.$$  

(10)

with

$$\mathcal{L}_{\ell\ell\gamma} = e \bar{\ell}_\beta \left[ \gamma^\mu \left( K_1^L P_L + K_1^R P_R \right) + im_{\ell_\alpha} \sigma^{\mu\nu} q_\nu \left( K_2^L P_L + K_2^R P_R \right) \right] \ell_\alpha A_\mu + \text{h.c.}$$  

(11)

$$\mathcal{L}_{4\ell} = \sum_{I=S,V,T} \sum_{X,Y=L,R} A_{XY}^I \bar{\ell}_\beta \Gamma_I P_X \ell_\alpha \bar{\ell}_\delta \Gamma_I P_Y \ell_\gamma + \text{h.c.}$$  

(12)

$$\mathcal{L}_{2\ell^2 d} = \sum_{I=S,V,T} \sum_{X,Y=L,R} B_{XY}^I \bar{\ell}_\beta \Gamma_I P_X \ell_\alpha \bar{d}_\gamma \Gamma_I P_Y d_\gamma + \text{h.c.}$$  

(13)

$$\mathcal{L}_{2\ell^2 u} = \mathcal{L}_{2\ell^2 d}|_{d \to u, B \to C}.$$  

(14)

Here $e$ is the electric charge, $q$ the 4-momenta of the photon, $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ are the usual chirality projectors and $\ell_\alpha$ and $d_\alpha$ denote the lepton and d-quark flavors, respectively. Furthermore, we have defined $\Gamma_S = 1$, $\Gamma_V = \gamma_\mu$ and $\Gamma_T = \sigma_{\mu\nu}$. We omit flavor indices in the form factors for the sake of simplicity. The underlying Feynman diagrams as well as the complete analytic results are given in appendices D–G.

Whenever possible, we have compared the explicit analytical formulae for the form factors with results already available in the literature. The supersymmetric contributions to boxes, Higgs penguins, photon penguins were found to perfectly agree with [74, 99], while the supersymmetric Z-penguins only differ from [74] via a constant term as pointed out in [73]. This constant term does not impact the result of [74] where a high-scale seesaw mechanism is considered but it can lead to non-physical results in low-scale seesaw models. We have also cross-checked our calculation of the non-SUSY boxes with [64], confirming their results. To the knowledge of the authors, this is the first calculation of the non-SUSY Higgs penguins in a two Higgs doublet Model, thus no comparison was possible.

3.2 $\ell_\alpha \to \ell_\beta \gamma$

In case of the radiative decay $\ell_\alpha \to \ell_\beta \gamma$, the corresponding decay width is given by [100]

$$\Gamma (\ell_\alpha \to \ell_\beta \gamma) = \frac{\alpha_{\text{em}} m_{\ell_\alpha}^5}{4} \left( |K_2^L|^2 + |K_2^R|^2 \right),$$  

(15)

where the dipole form factors $K_{2}^{L,R}$ are defined in Eq. (11), $\alpha_{\text{em}}$ being the fine structure constant.
Table 2: Relation between the form factors defined in this paper and the ones in [74]. Here
\( \hat{B}^{L,R} = B^{L,R} + B_{L,R,\text{Higgs}}^{L,R} \) and \( \hat{B}^{L,R}_3 = B^{L,R}_3 + B^{L,R}_{3,\text{Higgs}} \), and \( F_{XY} = \frac{F_x F_y}{e^2 m_2^2} \), with \( E^L \) and \( E^R \) the
tree-level Z-boson couplings to a pair of charged leptons (see appendix A.2).

3.3 \( \ell^-_\alpha \to \ell^-_\beta \ell^+_\beta \ell^+_\beta \)

Next, we consider the \( \ell^-_\alpha (p_1) \to \ell^-_\beta (p_2) \ell^+_\beta (p_3) \) 3-body decays. Using the operators in our LFV lagrangian, the decay width is given by

\[
\Gamma (\ell^-_\alpha \to 3 \ell^-_\beta) = \frac{m_{\ell^-_\alpha}}{512\pi^3} \left[ e^4 \left( |K^L_2|^2 + |K^R_2|^2 \right) \left( \frac{16}{3} \log \frac{m_{\ell^-_\alpha}}{m_{\ell^-_\beta}} - \frac{22}{3} \right) \right. \\
+ \frac{1}{24} \left( |A^S_{L\ell_L}|^2 + |A^S_{R\ell_R}|^2 \right) + \frac{1}{12} \left( |A^S_{L\ell_R}|^2 + |A^S_{R\ell_L}|^2 \right) \\
+ \frac{2}{3} \left( |\hat{A}^Y_{L\ell_L}|^2 + |\hat{A}^Y_{R\ell_R}|^2 \right) + \frac{1}{3} \left( |\hat{A}^Y_{L\ell_R}|^2 + |\hat{A}^Y_{R\ell_L}|^2 \right) + 6 \left( |A^T_{L\ell_L}|^2 + |A^T_{R\ell_R}|^2 \right) \\
+ \frac{e^2}{3} \left( K^L_2 A^{S\ast}_{RLL} + K^R_2 A^{S\ast}_{RRR} + \text{c.c.} \right) - \frac{2e^2}{3} \left( K^L_2 \hat{A}^{Y\ast}_{RLL} + K^R_2 \hat{A}^{Y\ast}_{RRL} + \text{c.c.} \right) \\
- \frac{4e^2}{3} \left( K^L_2 A^{V\ast}_{RLL} + K^R_2 A^{V\ast}_{RRL} + \text{c.c.} \right) \\
- \frac{1}{2} \left( A^{S\ast}_{L\ell_L} A^{T\ast}_{RLL} + A^{S\ast}_{R\ell_R} A^{T\ast}_{RRL} + \text{c.c.} \right) - \frac{1}{6} \left( A^{S\ast}_{L\ell_R} A^{Y\ast}_{RLL} + A^{S\ast}_{R\ell_L} A^{Y\ast}_{RRL} + \text{c.c.} \right) \bigg] .
\]

Here we have defined

\[
\hat{A}^Y_{XY} = A^Y_{XY} + e^2 K^X_1 \quad (X, Y = L, R)
\]

The mass of the leptons in the final state has been neglected in this formula, with the exception of
the numerical factors that multiply the \( K^L_2 \) contribution. Eq. (16) agrees with the one in
ref. [74], but includes in addition \( A^S_{L\ell_L} \) and \( A^S_{R\ell_R} \). In [74], these contributions were absorbed in the
corresponding vector form factors, \( A^V_{L\ell_L} \) and \( A^V_{R\ell_R} \), by means of a Fierz transformation [101]. The
relation between our coefficients and the ones of [74] is given in table 2.
3.4 $\ell^-_\alpha \rightarrow \ell^-_\beta \ell^-_\gamma \ell^+_\gamma$

We consider the $\ell^-_\alpha (p) \rightarrow \ell^-_\beta (p_1) \ell^-_\gamma (p_2) \ell^+_\gamma (p_3)$ 3-body decays, with $\beta \neq \gamma$. The decay width is given by

$$
\Gamma \left( \ell^-_\alpha \rightarrow \ell^-_\beta \ell^-_\gamma \ell^+_\gamma \right) = \frac{m^5_{\ell^-_\alpha}}{512 \pi^3} \left[ e^4 \left( |K^L_2|^2 + |K^R_2|^2 \right) \left( \frac{16}{3} \log \frac{m_{\ell^-_\alpha}}{m_{\ell^-_\gamma}} - 8 \right) + \frac{1}{12} \left( |A^S_{LL}|^2 + |A^S_{RR}|^2 \right) + \frac{1}{12} \left( |A^S_{LR}|^2 + |A^S_{RL}|^2 \right) \right.
$$

$$
+ \frac{1}{3} \left( \left| \hat{A}^V_{LL} \right|^2 + \left| \hat{A}^V_{RR} \right|^2 \right) + \frac{1}{3} \left( \left| \hat{A}^V_{LR} \right|^2 + \left| \hat{A}^V_{RL} \right|^2 \right) + 4 \left( |A^T_{LL}|^2 + |A^T_{RR}|^2 \right) - 2 e^2 \left( K^S_{LL} \hat{A}^V_{LR} + K^S_{RL} \hat{A}^V_{RR} + K^L_{LL} \hat{A}^V_{LR} \hat{A}^V_{RR} + c.c. \right) \right].
$$

(18)

Here we have used the same definition as in Eq. (17). Furthermore, as for $\ell^-_\alpha \rightarrow 3 \ell_\beta$, the mass of the leptons in the final state has been neglected in the decay width formula, with the exception of the dipole terms $K^L,R_{LL}$.

3.5 $\ell^-_\alpha \rightarrow \ell^+_\beta \ell^-_\gamma \ell^-_\gamma$

Finally, we consider the $\ell^-_\alpha (p) \rightarrow \ell^+_\beta (p_1) \ell^-_\gamma (p_2) \ell^-_\gamma (p_3)$ 3-body decays, with $\beta \neq \gamma$. The decay width is given by

$$
\Gamma \left( \ell^-_\alpha \rightarrow \ell^+_\beta \ell^-_\gamma \ell^-_\gamma \right) = \frac{m^5_{\ell^-_\alpha}}{512 \pi^3} \left[ \frac{1}{24} \left( |A^S_{LL}|^2 + |A^S_{RR}|^2 \right) + \frac{1}{12} \left( |A^S_{LR}|^2 + |A^S_{RL}|^2 \right) + \frac{2}{3} \left( \left| \hat{A}^V_{LL} \right|^2 + \left| \hat{A}^V_{RR} \right|^2 \right) + \frac{1}{3} \left( \left| \hat{A}^V_{LR} \right|^2 + \left| \hat{A}^V_{RL} \right|^2 \right) + 6 \left( |A^T_{LL}|^2 + |A^T_{RR}|^2 \right) - \frac{1}{2} \left( A^S_{LL} \hat{A}^T_{LR} + A^S_{RL} \hat{A}^T_{RR} + c.c. \right) - \frac{1}{6} \left( A^S_{LR} \hat{A}^V_{LR} + A^S_{RL} \hat{A}^V_{RL} + c.c. \right) \right].
$$

(19)

The same definitions and conventions as in the previous two observables have been used. Notice that this process does not receive contributions from penguin diagrams, but only from boxes.

3.6 Coherent $\mu - e$ conversion in nuclei

We now turn to the discussion of $\mu - e$ conversion in nuclei, which will follow the conventions and approximations described in Ref. [99, 102] (see also [103–105] for detailed works regarding the effective lagrangian at the nucleon level, [64, 106] for a calculation including the effects of the atomic electric field and [107] for recent improvements on the hadronic uncertainties). The conversion rate, relative to the the muon capture rate, can be expressed as

$$
\text{CR}(\mu - e, \text{Nucleus}) = \frac{p_e E_e m^3_e G_F^2 \alpha^3_{em} Z^4_{\text{eff}} F_p^2}{8 \pi^2 Z} \times \left\{ \left| (Z + N) \left( g^{(0)}_{LV} + g^{(0)}_{LS} \right) + (Z - N) \left( g^{(1)}_{LV} + g^{(1)}_{LS} \right) \right|^2 + \left| (Z + N) \left( g^{(0)}_{RV} + g^{(0)}_{RS} \right) + (Z - N) \left( g^{(1)}_{RV} + g^{(1)}_{RS} \right) \right|^2 \right\} \frac{1}{\Gamma_{\text{capt}}}.
$$

(20)

$Z$ and $N$ are the number of protons and neutrons in the nucleus and $Z_{\text{eff}}$ is the effective atomic charge [108]. Similarly, $G_F$ is the Fermi constant, $F_p$ is the nuclear matrix element and $\Gamma_{\text{capt}}$
represents the total muon capture rate. \( p_e \) and \( E_e \) (\( \approx m_\mu \) in our numerical evaluation) are the momentum and energy of the electron and \( m_\mu \) is the muon mass. In the above, \( g_{XK}^{(0)} \) and \( g_{XK}^{(1)} \) (with \( X = L, R \) and \( K = S, V \)) can be written in terms of effective couplings at the quark level as

\[
\begin{align*}
g_{XK}^{(0)} &= \frac{1}{2} \sum_{q=u,d,s} \left( g_{XK(q)} C_{K}^{(q,p)} + g_{XK(q)} C_{K}^{(q,n)} \right), \\
g_{XK}^{(1)} &= \frac{1}{2} \sum_{q=u,d,s} \left( g_{XK(q)} C_{K}^{(q,p)} - g_{XK(q)} C_{K}^{(q,n)} \right). \tag{21}
\end{align*}
\]

For coherent \( \mu - e \) conversion in nuclei, only scalar \( (S) \) and vector \( (V) \) couplings contribute \([102]\). Furthermore, sizable contributions are expected only from the \( u, d, s \) quark flavors. The numerical values of the relevant \( G_K \) factors are \([102, 109]\)

\[
\begin{align*}
G_{V}^{(a,p)} &= G_{V}^{(d,n)} = 2; \quad G_{V}^{(d,p)} = G_{V}^{(a,n)} = 1; \\
G_{S}^{(a,p)} &= G_{S}^{(d,n)} = 5.1; \quad G_{S}^{(d,p)} = G_{S}^{(a,n)} = 4.3; \\
G_{S}^{(s,p)} &= G_{S}^{(s,n)} = 2.5. \tag{22}
\end{align*}
\]

Finally, the \( g_{XK(q)} \) coefficients can be written in terms of the form factors in Eqs.(11), (13) and (14) as

\[
\begin{align*}
g_{LV(q)} &= \frac{\sqrt{2}}{G_F} \left[ e^2 Q_q (K_1^L - K_2^R) - \frac{1}{2} (C_{Lqq}^{LL} + C_{Lqq}^{LR}) \right], \\
g_{RV(q)} &= g_{LV(q)} \big|_{L \to R}, \\
g_{LS(q)} &= -\frac{\sqrt{2}}{G_F} \frac{1}{2} (C_{Lqq}^{SL} + C_{Lqq}^{SLR}), \\
g_{RS(q)} &= g_{LS(q)} \big|_{L \to R}. \tag{25}
\end{align*}
\]

Here \( Q_q \) is the quark electric charge \((Q_d = -1/3, Q_u = 2/3)\) and \( C_{Lqq}^{XX} = B_{XY}^{K} (C_{XY}^{K}) \) for \( d \)-quarks \((u \)-quarks\), with \( X = L, R \) and \( K = S, V \).

## 4 Results

### 4.1 Numerical setup

For the numerical examples we have implemented the model in the Mathematica package \textsc{SARAH} \([76–80]\), which creates the required modules for \textsc{SPheno} \([81, 82]\) to calculate the masses and mixing matrices including the complete 1-loop corrections. In the Higgs sector we include in addition the known 2-loop corrections to the Higgs mass from the MSSM \([110–115]\). However, this does not include 2-loop corrections stemming from the extended neutrino and sneutrino sectors, where we can have sizable Yukawa couplings. Moreover, \textsc{SARAH} calculates also the full 2-loop RGEs including the entire flavor structure for the model, which we have summarized in Appendix B. This will be of great importance in our numerical studies, as we use CMSSM-like boundary conditions, see below for their definition. In the flavor observables we include all possible contributions. These are calculated using the \textsc{FlavorKit} interface \([75]\). In the context of this project we have extended the lists of observables implemented in \textsc{FlavorKit} by \( \ell_e^{-} \to \ell_{\beta}^{-} \ell_{\beta}^{+} \ell_{\gamma}^{-} \) and \( \ell_{\alpha}^{-} \to \ell_{\beta}^{-} \ell_{\beta}^{+} \ell_{\gamma}^{+} \).

The numerical evaluation of each parameter point is performed as follows: the \( Y_{\nu} \) Yukawa couplings are calculated using a modified Casas-Ibarra parameterization \([116]\), properly adapted
conditions are applied

\[ \alpha^{-1}_{em} = 127.92783 \quad \alpha_S = 0.11720 \quad m_b(m_b) = 4.2 \text{ GeV} \quad m_e = 1.777 \text{ GeV} \quad G_B = 1.11639 \cdot 10^{-9} \text{GeV}^{-2} \quad M_Z = 91.18760 \text{ GeV} \quad m_t = 172.9 \text{ GeV} \]

Table 3: Input values for the SM parameters taken at \( M_Z \) unless otherwise specified.

for the inverse seesaw \[117, 118\] (and fixing \( M_R = 2 \text{ TeV}, \mu_X = 10^{-5} \text{ GeV} \) and the lightest neutrino mass \( m_{\nu_1} = 10^{-4} \text{ eV} \)):

\[ Y_\nu = \sqrt{v_u/V} D_{\sqrt{m_e}} D_{\sqrt{m_\nu}} U_{\text{PMNS}}. \]  

(27)

Here \( D_{\sqrt{m_e}} = \text{diag}(\sqrt{m_\nu_i}) \), \( D_{\sqrt{X}} = \text{diag}(\sqrt{X_i}) \), \( \hat{X}_i \) being the eigenvalues of \( X = M_R \mu_X^{-1} M_R^T \), and \( V \) is the matrix that diagonalizes \( X \) as \( V X V^T = \hat{X} \). Furthermore, we parameterize the complex orthogonal \( R \) matrix as

\[ R = \begin{pmatrix} 1 & 0 & 0 \\ \cos \theta_{13}^R & 0 & \sin \theta_{13}^R \\ 0 & 1 & 0 \\ \sin \theta_{13}^R & 0 & \cos \theta_{13}^R \end{pmatrix} \begin{pmatrix} \cos \theta_{12}^R & \sin \theta_{12}^R & 0 \\ 0 & 1 & 0 \\ -\sin \theta_{12}^R & \cos \theta_{12}^R & 0 \end{pmatrix}. \]  

(28)

Below we will set \( R \) to the unit matrix except when stated otherwise. We use the best-fit values for the neutrino oscillation parameters as given in \[119\]:

\[ \Delta m_{21}^2 = 7.60 \cdot 10^{-5} \text{ eV}^2, \quad \Delta m_{31}^2 = 2.48 \cdot 10^{-3} \text{ eV}^2, \]

\[ \sin^2 \theta_{12} = 0.323, \quad \sin^2 \theta_{23} = 0.467, \quad \sin^2 \theta_{13} = 0.0234. \]  

(29)

which are close to the ones obtained in \[3–5\]. We make use in our scans of the values

\[ Y_\nu = f \cdot 10^{-2} \cdot \begin{pmatrix} 0.0956 & -0.0589 & 0.0348 \\ 0.616 & 0.594 & -0.687 \\ 0.404 & 1.78 & 1.91 \end{pmatrix} \]  

(30)

fixed with \( f = 1 \) even if we vary \( M_R \). This is because one can always adjust \( \mu_X \) to fulfill neutrino oscillation data without affecting any of our observables.

**SPheno** derives the SM gauge and Yukawa couplings at \( M_Z \) where we take the masses and couplings given in table 3 as input. 2-loop RGEs for the dimensionless parameters are then used to evaluate these couplings at \( M_{\text{GUT}} \), defined by the requirement \( g_1 = g_2 \), where \( g_1 \) and \( g_2 \) are the couplings for the \( U(1)_Y \) and \( SU(2)_L \) gauge groups, respectively. At \( M_{\text{GUT}} \) the CMSSM boundary conditions are applied

\[ m_{\nu_C}^2 = m_X^2 = m_{\mu}^2 = m_{\tau}^2 = m_{\mu}^2 = m_{\tau}^2 = m_{\mu}^2 \equiv m_0^2 \]

\[ m_{H_u}^2 = m_{H_d}^2 \equiv m_0^2 \]

\[ M_1 = M_2 = M_3 \equiv M_{1/2} \]

\[ T_i \equiv A_0 Y_i \quad i = e, d, u, \nu \]

The mixed soft-term \( m_{X\nu_C}^2 \) is set to zero at the GUT scale and is not generated via RGE effects. Moreover, the phase of \( \mu \), which is an RGE invariant, is given as input. The ratio of the Higgs vevs, \( \tan \beta = \frac{v_u}{v_d} \), completes the list of input parameters. Then 2-loop RGEs are used to evolve these parameters to \( Q_{\text{EWSB}} = \sqrt{t_1 t_2} \). The numerical values for superpotential terms \( M_R \) and \( \mu_X \),
Figure 1: $\text{BR}(\mu \rightarrow e\gamma)$ as a function of $M_{\text{SUSY}}$ and $M_R$. The other parameters are given in the text. The gray area roughly corresponds to the parameter space excluded by the LHC experiments.

as well as for their corresponding soft terms $B_{M_R}$ and $B_{\mu X}$, are used as input at the SUSY scale. $B_\mu$ and $|\mu|$ are obtained as usual from the minimization conditions of the vacuum\footnote{In principle one could require that all $B$-parameters are proportional to each other, e.g. $B_\mu : B_{M_R} : B_{\mu X} = \mu : M_R : \mu_X$. However, as their actual value does not have any significant impact as long as this ratio is fulfilled up to a factor 2-3 we fix for simplicity $B_{M_R}$ and $B_{\mu X}$.}. At $M_{\text{SUSY}}$ the 1-loop corrected masses are calculated before the RGEs run down to $M_Z$ to re-calculate gauge and Yukawa couplings using the new SUSY corrections. These steps are iterated until the mass spectrum has converged with a numerical precision of $10^{-4}$. Afterwards, SPheno runs the RGEs to $Q = 160$ GeV for the calculation of the operators which contribute to quark flavor violating observables and to $Q = M_Z$ for the calculation of the operators needed for lepton flavor violating observables. These operators are then combined to compute the different observables using $\alpha(0)$, which includes to a large extent the effects from running the operators between $M_Z$ and the energy scale where the LFV processes take place (usually given by the mass of the decaying particle).

4.2 Numerical results

We will use the parameter values given in table 4 as starting point for our numerical computations unless stated otherwise. A variation of the soft SUSY parameters is denoted by a variation of $M_{\text{SUSY}}$, which actually implies a variation of three parameters at the same time $M_{\text{SUSY}} = m_0 = M_{1/2} = -A_0$. For completeness, we note that fixing the ratio $m_0/A_0$ usually gives a Higgs boson mass, $m_h$, that does not agree with the ATLAS and CMS measurements.
Nevertheless, we emphasize that (1) our results depend only weakly on the value of \( A_0 \), and (2) contributions mediated by \( h \) itself are subdominant. Therefore, the actual Higgs boson mass is of little importance for our investigations here.

We start with the discussion of \( \mu \) decays as the bounds are strongest in this case. In Fig. 1 we show the dependence of \( \text{BR}(\mu \rightarrow e\gamma) \) on \( M_R \) and \( M_{\text{SUSY}} \) as well as the individual dependence of the SUSY and non-SUSY contributions. The latter consist of \( \nu-W^\pm \) and the \( \nu-H^\pm \) contributions. There are two particular features: (i) if \( M_R = M_{\text{SUSY}} \) the SUSY contributions are more important than the non-SUSY ones and the relative importance of the SUSY contributions increases with the scale. The reason for the latter is that the mixing between light and heavy neutrinos decreases like \( \sim m_D/M_R \), whereas the mixing in the sneutrino sector decreases only logarithmically with the scale. (ii) The non-SUSY contributions can flip its sign. This is due to a sign-difference between the \( \nu-H^\pm \) and the \( \nu-W^\pm \) contributions to the coefficients \( K^{L,R}_2 \). This is in contrast to the analogous decay in the quark sector, \( b \rightarrow s\gamma \), where the \( W^\pm \) and \( H^\pm \)-contributions have always the same sign. The reason for this difference can be found in Eqs. (248)–(254), presented in appendix D, where the light neutrino masses appear instead of the mass of the heavy \( t \)-quark. We have checked explicitly, both numerically and analytically, that we recover the \( b \rightarrow s\gamma \) result if we replace the corresponding masses and Yukawa couplings. Finally, we stress that the scalar masses are functions of \( M_{\text{SUSY}} \), which explains why also the non-SUSY contribution actually depends on the SUSY scale. With our specific structure of the \( Y_{\nu} \) matrices we find that \( M_R \) has to be larger than \( M_{\text{SUSY}} \) for the sign flip to occur, which is also the reason why we do not observe it in case of \( M_R = M_{\text{SUSY}} \). The grey area corresponds to the part of the parameter space which is excluded in the CMSSM by the most recent ATLAS results [98]. However, we want to stress that even though the squark and gluino masses are essentially the same in our model as in the CMSSM, the cascade decays can be quite different due to (i) the enlarged sneutrino sector with additional light states and (ii) the different slepton masses. Thus, this is a rather conservative bound.

In Fig. 2 we display our results for the branching ratio \( \text{BR}(\mu \rightarrow 3e) \) as well as the various contributions to this decay. Here we find that for the case \( M_R = M_{\text{SUSY}} \) the non-SUSY boxes dominate. This fact was first noted in [63] and later confirmed by [64–66]. Note that this does not depend on the overall strength of the \( Y_{\nu} \) couplings, which we rescale as \( Y_{\nu} \rightarrow f Y_{\nu} \). This can be seen from the lower right plot: all contributions scale in the same way. However, the situation can change in principle if one allows for additional flavor violation in the soft SUSY breaking parameters. Note that the sign-flip induced by the \( H^\pm \) contributions is not as pronounced as in the case of \( \mu \rightarrow e\gamma \), where it led to a change of the overall sign, as the different contributions to the off-shell photon appear with different weights. However, it is the reason for the observed kink in the non-SUSY \( \gamma \)-penguin. We also observe that we have negative interference between non-SUSY \( Z \)-penguins and the corresponding box contributions. In particular, for larger values of \( M_R \) this can reduce \( \text{BR}(\mu \rightarrow 3e) \) by up to an order of magnitude. Since this is precisely the region which will be probed by future experiments, the possible appearance of these cancellations has to be

| \( m_0 \) | 1 TeV | \( M_{1/2} \) | 1 TeV |
| --- | --- | --- | --- |
| \( A_0 \) | -1.5 TeV | \( M_R \) | 2 TeV |
| \( B_{\mu_X} \) | 100 \( \mu_X \) | \( B_{M_R} \) | 100 \( M_R \) |
| \( \tan \beta \) | 10 | sign(\( \mu \)) | + |

Table 4: Standard values for the various parameters. \( M_R \) and \( \mu_X \) are taken proportional to the unit matrix.
The other parameters are given in the text. The gray area roughly corresponds to the parameter space excluded by the LHC experiments.

taken into account in order to interpret the experimental results properly.

Similar features appear in case of $\mu \to e$ conversion in nuclei, as exemplified for the case of an aluminium (Al) nucleus in Fig. 3. The main difference is that there is a large part of parameter space where a pronounced negative interference between the non-SUSY $Z$-penguin and the corresponding box contributions can occur. Note that with the expected sensitivity of $10^{-18}$ one can probe $Y_\nu$ couplings down to a few $\times 10^{-6}$ for $M_R = M_{SUSY} = 1$ TeV or, equivalently, to a mass scale of about 5 TeV in case of $Y_\nu$ as given in Eq. (30). As we found for the 3-body decays, for higher mass scales the non-SUSY $Z$-penguins can be as important as the corresponding box-diagrams. The overall features are essentially element independent as can be seen in Fig. 4 where we show all three observables discussed so far together and include also $\mu \to e$ conversion in titanium (Ti). In case $M_R \simeq M_{SUSY}$, we find that $\mu \to e$ conversion in nuclei is the most stringent LFV observable in our model.

Turning now to the LFV $\tau$ decays, we show in Fig. 5 several branching ratios for the scenario defined above. Unfortunately, they are too small to be observed in the near future. Below we will show alternative scenarios (in which the $R$ matrix is not assumed to be the unit matrix) where this is not the case. Nevertheless, they show an interesting feature which is quite generic in this model: $\text{BR}(\tau \to e\mu^+e^-) \simeq \text{BR}(\tau \to 3\mu)$ and $\text{BR}(\tau \to e\mu^+\mu^-) \simeq \text{BR}(\tau \to 3e)$. Particularly interesting is that these branching ratios are sensitive to the relative size of the non-SUSY contributions compared to the SUSY ones. We also stress that the various contributions contribute similarly

Figure 2: $\text{BR}(\mu \to 3e)$ as a function of $M_{SUSY}$, $M_R$ and an overall scaling parameter $f$ for $Y_\nu$. The other parameters are given in the text. The gray area roughly corresponds to the parameter space excluded by the LHC experiments.
as in case of $\mu \to 3e$. For completeness we note that $\text{BR}(\tau \to e\mu^+e^-)$ and $\text{BR}(\tau \to \mu^+\mu^-)$ are strongly suppressed, at least a factor of $10^{-6}$ with respect to the other 3-body decays, as they require at least one additional flavor violating vertex in the dominant contributions.

It is worth stressing that the fact that the $\mu$ observables are more constraining than the $\tau$ decays is correct in large parts of the parameter space. However, there is also a substantial part where the opposite is true, as exemplified in Fig. 6 where we tune the parameters such that both, $\mu$- and $\tau$-observables can be discovered in the next generation of experiments. For this we have adjusted the diagonal entries of $\mu_X$ as well as $\theta^R_{23}$ and calculated $Y_\nu$ using Eq. (27). Clearly, this part of the parameter space requires quite some hierarchy in $\mu_X$ to explain neutrino data correctly. Note that even in this part of parameter space the ratios $\text{BR}(\tau \to e\mu^+e^-) \simeq \text{BR}(\tau \to 3\mu)$ and $\text{BR}(\tau \to e\mu^+\mu^-) \simeq \text{BR}(\tau \to 3e)$ show the same dependence on the ratio $M_R/M_{\text{SUSY}}$ as in the previous case.

The impact of the $R$ matrix and the hierarchy in the $\mu_X$ entries is further illustrated for the decays $\ell_\alpha \to \ell_\beta\gamma$ in Fig. 7. Again, we have calculated $Y_\nu$ via Eq. (27), such that the results from neutrino oscillation experiments are explained correctly. One finds that, depending on the region in the parameter space, either the $\mu$ decay or the $\tau$ decays are more important. As in case of the 3-body decays, one finds fine-tuned combinations of the parameters where all decays can be observed in future experiments. Note that for fixed $\theta^R_{23}$ the branching ratios scale like $f_R^2/f_X$ where $f_R$ and $f_X$ denote an overall scaling of $M_R$ and $\mu_X$, respectively. Moreover, the branching ratios scale like $\tan^2\beta$ if the SUSY contributions dominate. In case the non-SUSY contributions

Figure 3: $\mu - e$ conversion on Al as a function of $M_{\text{SUSY}}$, $M_R$ and the scaling parameter $f$ for $Y_\nu$. The gray area roughly corresponds to the parameter space excluded by the LHC experiments.
dominate we find only a slight \( \tan \beta \) dependence for very large \( \tan \beta \) values.

Finally, let us comment on the Higgs penguin contributions to the different LFV observables. In all our numerical scans they have been found to be completely negligible and that is why we have decided not to include them in our figures. In principle, one could look for sizable Higgs penguin contributions by going to regions in parameter space with large \( \tan \beta \) and low pseudoscalar masses [89, 120]. This, however, would require dedicated parameter scans in order to overcome the constraints from flavor data, as these regions are already in strong tension after the LHCb measurement of the \( B_s \to \mu^+\mu^- \) branching ratio [121]. For this reason, we have not pursued this goal any further. Nevertheless, we have checked that the Higgs penguins contributions to \( \ell_\alpha \to \ell_\beta \ell_\gamma \ell_\delta \) and \( \mu-e \) conversion in nuclei have the expected decoupling behavior for large \( M_R \) and/or \( M_{SUSY} \) scales.

Figure 4: \( \text{BR}(\mu \to e\gamma) \), \( \text{BR}(\mu \to 3e) \), \( \mu-e \) conversion in Ti and Al as a function of \( M_R \) and \( M_{SUSY} \). The gray area roughly corresponds to the parameter space excluded by the LHC experiments.
Figure 5: Branching ratios for $\tau$ decays as a function of $M_R$ and $M_{SUSY}$. In the upper two plots the lines correspond to $\text{BR}(\mu \to 3e)$ (black solid), $\text{BR}(\tau \to 3e)$ (blue solid), $\text{BR}(\tau \to 3\mu)$ (red solid), $\text{BR}(\tau^{-} \to e^{-}\mu^{+}\mu^{-})$ (blue dashed) and $\text{BR}(\tau^{-} \to \mu^{+}e^{-}e^{-})$ (red dashed). The gray area roughly corresponds to the parameter space excluded by the LHC experiments.

Figure 6: $\mu$- and $\tau$-observables as a function of $\mu_X$. The underlying parameters are given in the plots. The lines correspond to $\text{BR}(\tau \to \mu\gamma)$ (full red), $\text{BR}(\tau \to 3\mu)$ (dashed red), $\text{BR}(\tau^{-} \to \mu^{-}e^{+}e^{-})$ (dotted red), $\text{BR}(\mu \to e\gamma)$ (full black) and $\text{BR}(\mu \to 3e)$ (dashed black). The light gray, red, yellow and blue bands show the expected future reach of the dedicated experiments to $\tau \to \mu\gamma$, $\tau \to 3\mu$, $\mu \to e\gamma$ and $\mu \to 3e$ as given in table 1.
Figure 7: Dependence of $\ell_\alpha \rightarrow \ell_\beta \gamma$ on $\mu_X^{33}$ and $\theta_{23}^R$. The lines correspond to BR($\mu \rightarrow e\gamma$) (black), BR($\tau \rightarrow e\gamma$) (blue dashed) and BR($\tau \rightarrow \mu\gamma$) (red dotted).
5 Conclusions

This paper represents the first complete computation of selected LFV observables in scenarios with light right-handed neutrinos. These include the radiative decays $\ell_\alpha \to \ell_\beta \gamma$, the 3-body decays $\ell_\alpha \to \ell_\beta \ell_\gamma \ell_\delta$ (in several variants) and neutrinoless $\mu - e$ conversion in nuclei. Our results are valid in the inverse seesaw and should also hold in low-scale type-I seesaw models with nearly conserved lepton number, the inverse seesaw being a specific realization of these models. Compared to previous studies, we have also included Higgs-penguins and considered non-supersymmetric as well as supersymmetric contributions to the corresponding LFV amplitudes simultaneously.

For the numerical examples we took a CMSSM inspired scenario where we also considered the limiting cases with either $M_R \gg M_{SUSY}$ and $M_{SUSY} \gg M_R$. Our main conclusions can be summarized as follows:

- The SUSY contributions dominate the induced photon penguins if both, $M_R$ and $M_{SUSY}$, are about the same size. For $M_R \lesssim M_{SUSY}/2$ the non-SUSY contributions start to dominate the radiative decays $\ell_\alpha \to \ell_\beta \gamma$.
- For low $M_R$ scales the LFV phenomenology is dominated by non-SUSY contributions. This holds in particular for the 3-body decays and $\mu - e$ conversion in nuclei. These are mainly given by boxes and $Z$-penguin diagrams containing right-handed neutrinos in the loop. In contrast to the usual high-scale seesaw models, in which their contributions to LFV processes are tiny, the right-handed neutrinos can play a major role in low-scale seesaw scenarios. In what concerns the non-SUSY box contributions, our results confirm previous claims in the literature [63–66]. Furthermore, we have highlighted the relevance of the non-SUSY $Z$-penguins, previously regarded as subdominant. They are particularly relevant for larger values of $M_R$, where we often find a negative interference between the $Z$-penguins and the box contributions. This will be particularly important when the next generation of experiments start to probe this mass region.
- The proper decoupling of the different contributions has been checked explicitly, e.g. we have checked that the SUSY-contributions, the $\nu^C \cdot X$ and the Higgs contributions decouple independently as expected.
- Currently, the radiative decay $\mu \to e\gamma$ is the most constraining LFV process. However, due to the promising experimental prospects in the near future, the situation will change. If the coming experiments perform as planned, $\mu \to 3e$ will be the most relevant LFV process in the mid term, whereas neutrinoless $\mu - e$ conversion in nuclei will set the strongest constraints in the long term.
- Ratios of $\tau$ LFV branching ratios can provide additional information about the dominant contributions. In particular, when the non-SUSY contributions dominate, one finds $\text{BR}(\tau^- \to \mu^- e^+ e^-)/\text{BR}(\tau \to 3\mu) \simeq \text{BR}(\tau^- \to e^- \mu^+ \mu^-)/\text{BR}(\tau \to 3e) \simeq 0.5 - 0.8$, whereas for a SUSY dominated scenario $\text{BR}(\tau^- \to \mu^- e^+ e^-)/\text{BR}(\tau \to 3\mu) \gg \text{BR}(\tau^- \to e^- \mu^+ \mu^-)/\text{BR}(\tau \to 3e)$. This can in turn be used to get a hint on the hierarchy between the seesaw and SUSY scales.

Acknowledgements

We thank Martin Hirsch and Ernesto Arganda for fruitful discussions. A.V. acknowledges partial support from the EXPL/FIS-NUC/0460/2013 project financed by the Portuguese FCT. M.E.K.
A Masses and vertices

We give first our conventions for the mass matrices as well as for the corresponding rotation matrices. These matrices are then used to express in appendix A.2 all the vertices needed to calculate the LFV observables.

A.1 Mass matrices

- **Mass matrix for Neutrinos**, Basis: \((\nu_L, \nu_R^C, X)\)

\[
m_\nu = \begin{pmatrix}
0 & \frac{1}{\sqrt{2}} v u Y^T & 0 \\
\frac{1}{\sqrt{2}} v u Y^T & 0 & M_R \\
0 & M_R^T & \mu_X 
\end{pmatrix}
\]  

This matrix is diagonalized by \(U V^*\):

\[
U V^* m_\nu U V^{\dagger} = m_\nu^{\text{dia}}
\]

- **Mass matrix for CP-odd Sneutrinos**, Basis: \((\sigma_L, \sigma_R, \sigma_X)\)

\[
m_{\nu i}^2 = \begin{pmatrix}
m_{\sigma_L \sigma_L} & m_{\sigma_L \sigma_R} & \frac{1}{\sqrt{2}} v_u \Re(Y^T M_R^*) \\
m_{\sigma_L \sigma_R} & m_{\sigma_R \sigma_R} & \Re(B_{M_R} - M_R \mu_X^*) \\
\frac{1}{\sqrt{2}} v_u \Re(M_R^T Y^T) & \Re(B_{M_R} - M_R \mu_X^*) & m_{\sigma_X \sigma_X}
\end{pmatrix}
\]

\[
m_{\sigma_L \sigma_L} = \frac{1}{2} v_u^2 \Re(Y^T Y^T) + \Re(m^2_\nu) + \frac{1}{8} (g_1^2 + g_2^2) (v_d^2 - v_u^2) \mathbf{1}
\]

\[
m_{\sigma_L \sigma_R} = -\frac{1}{\sqrt{2}} (v_d \Re(\mu Y^*) - v_u \Re(T^*_{\nu}))
\]

\[
m_{\sigma_R \sigma_R} = \Re(m^2_\nu) + \Re(M_R M_R^\dagger) + \frac{1}{2} v_u^2 \Re(Y^T Y^T)
\]

\[
m_{\sigma_X \sigma_X} = \Re(M_R^T M_R^*) + \Re(m^2_X) - \Re(B_{\mu_X}) + \Re(\mu_X \mu_X^*)
\]

This matrix is diagonalized by \(Z^i\):

\[
Z^i m_{\nu i}^2 Z^{i\dagger} = m_{\nu \nu}^{2\text{dia}}
\]
• Mass matrix for CP-even Sneutrinos, Basis: \((\phi_L, \phi_R, \phi_X)\)

\[
m_{\nu_R}^2 = \begin{pmatrix}
    m_{\phi_L\phi_L} & m_{\phi_L\phi_R}^T & \frac{1}{\sqrt{2}} v_u \mathfrak{R}\left(Y_{\nu}^T M_{R}^*\right) \\
    m_{\phi_L\phi_R} & m_{\phi_R\phi_R} & \mathfrak{R}\left(B_{M_R} + \mu_X M_{R}^*\right) \\
    \frac{1}{\sqrt{2}} v_u \mathfrak{R}\left(M_{R}^T Y_{\nu}\right) & \mathfrak{R}\left(B_{M_R} + \mu_X M_{R}^*\right) & m_{\phi_X\phi_X}
\end{pmatrix}
\] (39)

\[
m_{\phi_L\phi_L} = \frac{1}{2} v_u^2 \mathfrak{R}\left(Y_{\nu}^T Y_{\nu}\right) + \mathfrak{R}(m_t^2) + \frac{1}{8} \left(g_1^2 + g_2^2\right) \left(v_d^2 - v_u^2\right) \mathbf{1}
\] (40)

\[
m_{\phi_L\phi_R} = -\frac{1}{\sqrt{2}} \left( v_d \mathfrak{R}(\mu Y_{\nu}) - v_u \mathfrak{R}(T_{\nu}) \right)
\] (41)

\[
m_{\phi_R\phi_R} = \mathfrak{R}\left(m_t^2\right) + \mathfrak{R}\left(M_R M_{R}^*\right) + \frac{1}{2} v_u^2 \mathfrak{R}\left(Y_{\nu} Y_{\nu}^\dagger\right)
\] (42)

\[
m_{\phi_X\phi_X} = \mathfrak{R}\left(M_{R}^T M_{R}^*\right) + \mathfrak{R}\left(m_{\nu}^2\right) + \mathfrak{R}(B_{\mu_X}) + \mathfrak{R}(\mu_X M_{R}^*)
\] (43)

This matrix is diagonalized by \(Z^R\):

\[
Z^R m_{\nu_R}^2 Z_{R,\dagger} = m_{\nu_R}^{2,\text{dia}}
\] (44)

• Mass matrix for Down-Squarks, Basis: \((\tilde{d}_{L,\alpha_1}, \tilde{d}_{R,\alpha_2})\)

\[
m_{\tilde{d}}^2 = \begin{pmatrix}
    m_{\tilde{d}_L\tilde{d}_L}^2 & \frac{1}{\sqrt{2}} \left( v_d T_{d}^\dagger - v_u \mu Y_{d}^\dagger \right) \delta_{\alpha_1,\beta_2} \\
    \frac{1}{\sqrt{2}} \delta_{\alpha_2,\beta_1} \left( v_d T_{d} - v_u Y_{d} \mu^* \right) & m_{\tilde{d}_R\tilde{d}_R}^2
\end{pmatrix}
\] (45)

\[
m_{\tilde{d}_L\tilde{d}_L} = -\frac{1}{24} \left( 3g_2^2 + g_1^2 \right) \left( v_d^2 - v_u^2 \right) \delta_{\alpha_1,\beta_1} \mathbf{1} + \frac{1}{2} \delta_{\alpha_1,\beta_1} \left( 2m_{\tilde{d}}^2 + v_d^2 Y_{d} Y_{d}^\dagger \right)
\] (46)

\[
m_{\tilde{d}_R\tilde{d}_R} = \frac{1}{12} g_1^2 \left( v_d^2 - v_u^2 \right) \delta_{\alpha_2,\beta_2} \mathbf{1} + \frac{1}{2} \delta_{\alpha_2,\beta_2} \left( 2m_{\tilde{d}}^2 + v_d^2 Y_{d} Y_{d}^\dagger \right)
\] (47)

This matrix is diagonalized by \(Z^D\):

\[
Z^D m_{\tilde{d}}^2 Z_{D,\dagger} = m_{\tilde{d}}^{2,\text{dia}}
\] (48)

• Mass matrix for Up-Squarks, Basis: \((\tilde{u}_{L,\alpha_1}, \tilde{u}_{R,\alpha_2})\)

\[
m_{\tilde{u}}^2 = \begin{pmatrix}
    m_{\tilde{u}_L\tilde{u}_L}^2 & \frac{1}{\sqrt{2}} \left( -v_d \mu Y_{u}^\dagger + v_u T_{u}^\dagger \right) \delta_{\alpha_1,\beta_2} \\
    \frac{1}{\sqrt{2}} \delta_{\alpha_2,\beta_1} \left( -v_d \mu Y_{u} \mu^* + v_u T_{u} \right) & m_{\tilde{u}_R\tilde{u}_R}^2
\end{pmatrix}
\] (49)

\[
m_{\tilde{u}_L\tilde{u}_L} = -\frac{1}{24} \left( -3g_2^2 + g_1^2 \right) \left( v_d^2 - v_u^2 \right) \delta_{\alpha_1,\beta_1} \mathbf{1} + \frac{1}{2} \delta_{\alpha_1,\beta_1} \left( 2m_{\tilde{u}}^2 + v_d^2 Y_{u} Y_{u}^\dagger \right)
\] (50)

\[
m_{\tilde{u}_R\tilde{u}_R} = \frac{1}{2} g_1^2 \left( 2m_{\tilde{u}}^2 + v_d^2 Y_{u} Y_{u}^\dagger \right) + \frac{1}{6} \delta_{\alpha_2,\beta_2} \left( v_d^2 - v_u^2 \right) \delta_{\alpha_2,\beta_2} \mathbf{1}
\] (51)

This matrix is diagonalized by \(Z^U\):

\[
Z^U m_{\tilde{u}}^2 Z_{U,\dagger} = m_{\tilde{u}}^{2,\text{dia}}
\] (52)

22
• Mass matrix for Sleptons, Basis: \((\tilde{e}_L, \tilde{e}_R)\)

\[
m_{\tilde{e}}^2 = \begin{pmatrix}
    m_{\tilde{e}_L} e_L^* & \frac{1}{\sqrt{2}}(v_d T_e - v_u Y_e^*) \\
    \frac{1}{\sqrt{2}}(v_d T_e - v_u Y_e^*) & m_{\tilde{e}_R} e_R^*
\end{pmatrix}
\]

This matrix is diagonalized by \(Z^E\):

\[
Z^E m_{\tilde{e}}^2 Z^{E\dagger} = m_{\tilde{e}}^{2\text{dia}}
\]

• Mass matrix for CP-even Higgs, Basis: \((\phi_d, \phi_u)\)

\[
m_h^2 = \begin{pmatrix}
    \frac{1}{8}(g_1^2 + g_2^2)(3v_d^2 - v_u^2) + m_{H_d}^2 + |\mu|^2 & -\frac{1}{4}(g_1^2 + g_2^2)v_d v_u - \Re(B_\mu) \\
    -\frac{1}{8}(g_1^2 + g_2^2)v_d v_u - \Re(B_\mu) & \frac{1}{8}(g_1^2 + g_2^2)(v_d^2 - v_u^2) + m_{H_u}^2 + |\mu|^2
\end{pmatrix}
\]

This matrix is diagonalized by \(Z^H\):

\[
Z^H m_h^2 Z^{H\dagger} = m_h^{2\text{dia}}
\]

• Mass matrix for CP-odd Higgs, Basis: \((\sigma_d, \sigma_u)\)

\[
m_{A^0}^2 = \begin{pmatrix}
    \frac{1}{8}(g_1^2 + g_2^2)(v_d^2 - v_u^2) + m_{H_d}^2 + |\mu|^2 & \Re(B_\mu) \\
    \Re(B_\mu) & \frac{1}{8}(g_1^2 + g_2^2)(v_d^2 - v_u^2) + m_{H_u}^2 + |\mu|^2
\end{pmatrix}
\]

Gauge fixing contributions:

\[
m^2(Z) = \begin{pmatrix}
    \frac{1}{4}v_d^2 \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right)^2 & -\frac{1}{4}v_d v_u \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right)^2 \\
    -\frac{1}{4}v_d v_u \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right)^2 & \frac{1}{4}v_u^2 \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right)^2
\end{pmatrix}
\]

This matrix is diagonalized by \(Z^A\):

\[
Z^A m_{A^0}^2 Z^{A\dagger} = m_{A^0}^{2\text{dia}}
\]

• Mass matrix for Charged Higgs, Basis: \((H_d^-, H_u^{+*}), (H_d^{+*}, H_u^+)\)

\[
m_{H^-}^2 = \begin{pmatrix}
    m_{H_d}^2 & \frac{1}{4}g_2^2 v_d v_u + B_\mu \\
    \frac{1}{4}g_2^2 v_d v_u + B_\mu & m_{H_u^{+*}}^2
\end{pmatrix} + \xi_{W^-} m^2(W^-)
\]

\[
m_{H_{d}^{+*}} = \frac{1}{8}(g_2^2 (v_d^2 - v_u^2) + g_1^2 (v_d^2 + v_u^2)) + m_{H_d}^2 + |\mu|^2
\]
\[ m_{H_u^+ H_d^+} = \frac{1}{8} \left( g_1^2 \left( -v_d^2 + v_u^2 \right) + g_2^2 \left( v_d^2 + v_u^2 \right) \right) + m_{H_u}^2 + |\mu|^2 \]  

(64)

Gauge fixing contributions:

\[ m^2(W^-) = \begin{pmatrix} -\frac{1}{4} g_1^2 v_d^2 & -\frac{1}{4} g_2^2 v_d v_u \\ -\frac{1}{4} g_1^2 v_d v_u & -\frac{1}{4} g_2^2 v_u^2 \end{pmatrix} \]  

(65)

This matrix is diagonalized by \( Z^+ \):

\[ Z^+ m_{H^-}^2 Z^+ \dagger = m_{H^-}^{2,\text{dia}} \]  

(66)

- **Mass matrix for Neutralinos**, Basis: \((\tilde{\lambda}_B, \tilde{\nu}_d^0, \tilde{H}_d^0, \tilde{H}_u^0)\)

\[
m_{\tilde{\chi}_0^0} = \begin{pmatrix} M_1 & 0 & \frac{1}{2} g_1 v_d & \frac{1}{2} g_1 v_u \\ 0 & M_2 & \frac{1}{2} g_2 v_d & \frac{1}{2} g_2 v_u \\ -\frac{1}{2} g_1 v_d & \frac{1}{2} g_2 v_d & 0 & -\mu \\ \frac{1}{2} g_1 v_u & \frac{1}{2} g_2 v_u & -\mu & 0 \end{pmatrix} \]  

(67)

This matrix is diagonalized by \( N \):

\[ N^* m_{\tilde{\chi}_0^0} N = m_{\tilde{\chi}_0^0}^{\text{dia}} \]  

(68)

- **Mass matrix for Charginos**, Basis: \((\tilde{\chi}^-, \tilde{H}_d^-), (\tilde{\chi}^+, \tilde{H}_u^+)\)

\[
m_{\tilde{\chi}^-} = \begin{pmatrix} M_2 & \frac{1}{\sqrt{2}} g_2 v_u \\ \frac{1}{\sqrt{2}} g_2 v_u & \mu \end{pmatrix} \]  

(69)

This matrix is diagonalized by \( U \) and \( V \)

\[ U^* m_{\tilde{\chi}^-} V = m_{\tilde{\chi}^-}^{\text{dia}} \]  

(70)

- **Mass matrix for charged Leptons**, Basis: \((e_L), (e_R^*)\)

\[
m_{\tilde{e}} = \begin{pmatrix} \frac{1}{\sqrt{2}} v_d Y_e^T \end{pmatrix} \]  

(71)

This matrix is diagonalized by \( U_L^e \) and \( U_R^e \)

\[ U_L^e \tilde{e} U_R^{e\dagger} = m_{\tilde{e}}^{\text{dia}} \]  

(72)

- **Mass matrix for Down-Quarks**, Basis: \((d_{L,\alpha_1}), (d_{R,\beta_1}^*)\)

\[
m_{\tilde{d}} = \begin{pmatrix} \frac{1}{\sqrt{2}} v_d \delta_{\alpha_1 \beta_1} Y_d^T \end{pmatrix} \]  

(73)

This matrix is diagonalized by \( U_L^d \) and \( U_R^d \)

\[ U_L^d \tilde{d} U_R^{d\dagger} = m_{\tilde{d}}^{\text{dia}} \]  

(74)

- **Mass matrix for Up-Quarks**, Basis: \((u_{L,\alpha_1}), (u_{R,\beta_1}^*)\)

\[
m_{\tilde{u}} = \begin{pmatrix} \frac{1}{\sqrt{2}} v_u \delta_{\alpha_1 \beta_1} Y_u^T \end{pmatrix} \]  

(75)

This matrix is diagonalized by \( U_L^u \) and \( U_R^u \)

\[ U_L^u \tilde{u} U_R^{u\dagger} = m_{\tilde{u}}^{\text{dia}} \]  

(76)
A.2 Vertices

In this appendix we list all vertices relevant for our computations. Our conventions are as follows:

- Chiral vertices are parameterized as
  \[ \Gamma_{F_a F_b S_c}^L P_L + \Gamma_{F_a F_b S_c}^R P_R \]
  \[ \Gamma_{F_a V_c^\mu}^L \gamma_\mu P_L + \Gamma_{F_a V_c^\mu}^R \gamma_\mu P_R \]

- The momentum flow in vector and scalar-vector vertices is
  \[ \Gamma_{S_a S_b V_c^\mu} (p_{S_a}^\mu - p_{S_b}^\mu) \]
  \[ \Gamma_{V_c^\nu V_c^\rho} (g_{\rho\sigma}(-p_{V_c}^\sigma + p_{V_c}^\nu) + g_{\rho\sigma}(-p_{V_c}^\sigma + p_{V_c}^\nu) + g_{\rho\sigma}(-p_{V_c}^\sigma + p_{V_c}^\nu)) \]

Here we used polarization projectors \( P_{L,R} \), metric \( g_{\mu\nu} \) and momenta \( p \) of the external fields.

A.2.1 Fermion-Scalar vertices

\[ P_{ijk}^{c,L} = \Gamma_{\bar{\chi}^*_L \chi^*_R A_k^L}^L = - \frac{1}{\sqrt{2}} g_2 \left( U_{i1}^* V_{j2} Z_{k2}^A + U_{j2}^* V_{i1}^* Z_{k1}^A \right) \]  
(77)

\[ P_{ijk}^{c,R} = \Gamma_{\bar{\chi}^*_L \chi^*_R A_k^R}^R = \frac{1}{\sqrt{2}} g_2 \left( U_{i1} V_{j2} Z_{k2}^A + U_{j2} V_{i1}^* Z_{k1}^A \right) \]  
(78)

\[ P_{ijk}^{L} = \Gamma_{\bar{\chi}^*_L \chi^*_R A_k^L}^L = \frac{1}{2} \left( N_{i3}^* (g_{1} N_{j1}^* - g_{2} N_{j2}^* ) Z_{k1}^A - g_{2} N_{i2}^* N_{j2}^* Z_{k1}^A - g_{1} N_{i1}^* N_{j1}^* Z_{k2}^A + g_{2} N_{i2}^* N_{j1}^* Z_{k2}^A \right) \]
(79)

\[ P_{ijk}^{R} = \Gamma_{\bar{\chi}^*_L \chi^*_R A_k^R}^R = \frac{1}{2} \left( N_{i3}^* (g_{1} N_{i1} + g_{2} N_{i2}) N_{j3} + N_{i3} (g_{1} N_{j1} + g_{2} N_{j2}) \right) \]
(80)

\[ A_{ijk}^{d,L} = \Gamma_{\bar{d}_{ia} d_{ja} A_k^L}^L = - \frac{i}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^{3} U_{L,iab}^d \sum_{a=1}^{3} U_{R,ia}^d Y_{d,ab} Z_{k1}^A \]  
(81)

\[ A_{ijk}^{d,R} = \Gamma_{\bar{d}_{ia} d_{ja} A_k^R}^R = \frac{i}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^{3} \sum_{a=1}^{3} Y_{d,ab} U_{R,ja}^d U_{L,ib}^d Z_{k1}^A \]  
(82)

\[ A_{ijk}^{L} = \Gamma_{\bar{\ell}_{i j} A_k^L}^L = - \frac{i}{\sqrt{2}} \sum_{b=1}^{3} U_{L,jib}^e \sum_{a=1}^{3} U_{R,ia}^e Y_{e,ab} Z_{k1}^A \]  
(83)

\[ A_{ijk}^{R} = \Gamma_{\bar{\ell}_{i j} A_k^R}^R = \frac{i}{\sqrt{2}} \sum_{b=1}^{3} \sum_{a=1}^{3} Y_{e,ab} U_{R,ja}^e U_{L,ib}^e Z_{k1}^A \]  
(84)

\[ A_{ijk}^{u,L} = \Gamma_{\bar{u}_{ia} u_{ja} A_k^L}^L = - \frac{i}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^{3} U_{L,jib}^u \sum_{a=1}^{3} U_{R,ia}^u Y_{u,ab} Z_{k2}^A \]  
(85)

\[ A_{ijk}^{u,R} = \Gamma_{\bar{u}_{ia} u_{ja} A_k^R}^R = \frac{i}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^{3} \sum_{a=1}^{3} Y_{u,ab} U_{R,ja}^u U_{L,ib}^u Z_{k2}^A \]  
(86)

\[ A_{ijk}^{\nu,L} = \Gamma_{\bar{\nu}_{\nu j} A_k^L}^L = - \frac{i}{\sqrt{2}} \left( \sum_{b=1}^{3} U_{jib}^\nu \sum_{a=1}^{3} U_{\nu,3+a}^\nu Y_{\nu,ab} + \sum_{b=1}^{3} U_{ib}^\nu \sum_{a=1}^{3} U_{j3+a}^\nu Y_{\nu,ab} \right) Z_{k2}^A \]  
(87)
\[ A_{ijk}^{u,R} = \Gamma_{\nu,\nu_j}^R A_{ij}^\nu = \frac{i}{\sqrt{2}} \left( \sum_{b=1}^3 \sum_{a=1}^3 Y_{\nu,ab}^* U_{j3+a}^U U_{jb}^V + \sum_{b=1}^3 \sum_{a=1}^3 Y_{\nu,ab}^* U_{i3+a}^U U_{jb}^V \right) Z_{k2}^A \]  

(88)

\[ W_{ijk}^{u,L} = \Gamma_{\chi_i}^L u_{ij,b} = -\delta_{\beta \gamma} \left( g_2 U_{i1}^* \sum_{a=1}^3 U_{L,i_ja} Z_{ka}^D - U_{i2}^* \sum_{b=1}^3 U_{L,j_b}^* \sum_{a=1}^3 Y_{d,ab} Z_{k3+a}^D \right) \]  

(89)

\[ W_{ijk}^{u,R} = \Gamma_{\chi_i}^R u_{ij,b} = \delta_{\beta \gamma} \sum_{b=1}^3 \sum_{a=1}^3 Y_{u,ab} U_{R,i_ja} Z_{kb} V_{b2} \]  

(90)

\[ S_{ij}^{c,L} = \Gamma_{\chi_i}^L \chi_i^c h_k = -\frac{1}{\sqrt{2}} g_2 \left( U_{i1} V_{j_k L} Z_{k2}^H + U_{j_k L}^* V_{i1}^* Z_{k1}^H \right) \]  

(91)

\[ S_{ij}^{c,R} = \Gamma_{\chi_i}^R \chi_i^c h_k = -\frac{1}{\sqrt{2}} g_2 \left( U_{i1} V_{j_k L} Z_{k2}^H + U_{j_k L}^* V_{i1}^* Z_{k1}^H \right) \]  

(92)

\[ S_{ij}^L = \Gamma_{\chi_i}^L \chi_i^L h_k = \frac{1}{2} \left( N_{i3}^* \left( g_1 N_{i1} - g_2 N_{i2} \right) \right) Z_{k1}^H - g_2 N_{i2}^* N_{i3} N_{i1} Z_{k1}^H + \frac{1}{2} \left( N_{i3}^* \left( N_{j1}^* Z_{k1}^H - N_{j4}^* Z_{k2}^H \right) \right) \]  

(93)

\[ S_{ij}^R = \Gamma_{\chi_i}^R \chi_i^R h_k = \frac{1}{2} \left( Z_{k1}^H \left( -g_1 N_{i1} Z_{j3}^H + N_{i3} \left( g_1 N_{j1} Z_{j2}^H \right) \right) \right) \]  

(94)

\[ N_{ijk}^{d,L} = \Gamma_{\chi_i}^L \chi_i^L \delta_{b}^c = \delta_{\beta \gamma} \left( \frac{1}{\sqrt{2}} g_2 N_{i2}^* \sum_{b=1}^3 U_{L,i_ja}^d Z_{ka}^D - N_{i3}^* \sum_{b=1}^3 U_{L,j_b}^d \sum_{a=1}^3 Y_{d,ab}^* Z_{k3+a}^D - \frac{1}{3} g_1 N_{i1} \sum_{a=1}^3 U_{L,j_b}^d Z_{ka}^D \right) \]  

(95)

\[ N_{ijk}^{d,R} = \Gamma_{\chi_i}^R \chi_i^R \delta_{b}^c = -\delta_{\beta \gamma} \left( \sum_{b=1}^3 \sum_{a=1}^3 Y_{d,ab}^* U_{R,i_ja} Z_{kb}^D N_{i3} + \frac{\sqrt{2}}{3} g_1 \sum_{a=1}^3 Z_{k3+a}^D U_{R,i_ja} N_{i1}^* \right) \]  

(96)

\[ N_{ij}^L = \Gamma_{\chi_i}^L \delta_{ij} \delta_{b}^c = -N_{i3}^* \sum_{b=1}^3 U_{L,j_b}^d \sum_{a=1}^3 Y_{e,ab} Z_{k3+a}^E + \frac{1}{2} g_1 N_{i1} \sum_{a=1}^3 U_{L,j_b}^e \sum_{a=1}^3 Y_{e,ab} Z_{k3+a}^E + \frac{1}{2} g_2 N_{i2} \sum_{a=1}^3 U_{L,j_b}^e \sum_{a=1}^3 Y_{e,ab} Z_{k3+a}^E \]  

(97)

\[ N_{ij}^R = \Gamma_{\chi_i}^R \delta_{ij} \delta_{b}^c = -\left( \frac{1}{2} g_1 \sum_{a=1}^3 Z_{k3+b}^E U_{R,i_ja} N_{l1} + \sum_{b=1}^3 \sum_{a=1}^3 Y_{e,ab}^* U_{R,i_ja} Z_{k3+b}^E N_{i3} \right) \]  

(98)

\[ N_{i,j}^{u,L} = \Gamma_{\chi_i}^L \delta_{ij} \delta_{b}^c = -\delta_{\beta \gamma} \left( \frac{1}{\sqrt{2}} g_2 N_{i2}^* \sum_{a=1}^3 U_{L,i_ja}^u Z_{ka}^U + N_{i4}^* \sum_{b=1}^3 U_{L,j_b}^u \sum_{a=1}^3 Y_{u,ab} Z_{k3+a}^U + \frac{1}{3} g_2 \sum_{a=1}^3 U_{L,j_b}^u \sum_{a=1}^3 Y_{u,ab} Z_{k3+a}^U \right) \]  

(99)

\[ N_{i,j}^{u,R} = \Gamma_{\chi_i}^R \delta_{ij} \delta_{b}^c = \delta_{\beta \gamma} \left( \frac{2\sqrt{2}}{3} g_1 \sum_{a=1}^3 Z_{k3+b}^U U_{R,i_ja} N_{l1} - \sum_{b=1}^3 \sum_{a=1}^3 Y_{u,ab}^* U_{R,i_ja} Z_{k3+b}^U N_{i4} \right) \]  

(100)

\[ H_{i,j}^{d,L} = \Gamma_{\alpha}^L \delta_{ij} \delta_{b}^c = -\frac{1}{\sqrt{2}} \delta_{\alpha \beta} \sum_{b=1}^3 U_{L,j_b}^d \sum_{a=1}^3 U_{R,i_a}^d \sum_{b=1}^3 Y_{d,ab} Z_{k1}^H \]  

(101)

\[ H_{i,j}^{d,R} = \Gamma_{\alpha}^R \delta_{ij} \delta_{b}^c = -\frac{1}{\sqrt{2}} \delta_{\alpha \beta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{d,ab}^* U_{R,i_ja}^d U_{L,j_b}^d Z_{k1}^H \]  

(102)
\[ W_{ij}^{d,L} = \Gamma^L_{\chi^+ \delta^+ \tilde{\alpha} \bar{\kappa}_{\gamma}} = -\delta_{\beta \gamma} \left( g_2 V_{i1}^* \sum_{a=1}^{3} U_{L,ja}^* Z_{k3+a}^U - V_{i2}^* \sum_{b=1}^{3} U_{L,jb}^* \sum_{a=1}^{3} Y_{u,ab}^* Z_{k3+a}^U \right) \]  

\[ W_{ij}^{d,R} = \Gamma^R_{\chi^+ \delta^+ \tilde{\alpha} \bar{\kappa}_{\gamma}} = \delta_{\alpha \beta} \sum_{b=1}^{3} \sum_{a=1}^{3} Y_{d,ab}^* U_{R,ja}^d Z_{k3+a}^U U_{i2} \]  

\[ V_{ijk}^{u,L} = \Gamma^L_{u,\alpha \tilde{\alpha}} H_{k}^+ = \delta_{\alpha \beta} \sum_{b=1}^{3} \sum_{a=1}^{3} U_{L,jb}^e \sum_{b=1}^{3} U_{L,ia}^* Y_{u,ab}^* Z_{k2}^+ \]  

\[ V_{ijk}^{u,R} = \Gamma^R_{u,\alpha \tilde{\alpha}} H_{k}^+ = \delta_{\alpha \beta} \sum_{b=1}^{3} \sum_{a=1}^{3} Y_{d,ab}^* U_{R,ja}^* U_{L,ab}^* Z_{k1}^+ \]  

\[ V_{ijk}^{+L} = \Gamma^L_{\nu,\tilde{\nu}} \ell^i \bar{\epsilon}_j H_{k}^+ = \sum_{b=1}^{3} \sum_{a=1}^{3} U_{e,ab}^* U_{R,ja}^e \bar{U}_{e,ib}^* Z_{k1}^+ \]  

\[ V_{ijk}^{+R} = \Gamma^R_{\nu,\tilde{\nu}} \ell^i \bar{\epsilon}_j H_{k}^+ = \sum_{b=1}^{3} \sum_{a=1}^{3} Y_{e,ab}^* U_{R,ja}^e U_{L,ab}^* Z_{k1}^+ \]  

\[ H_{ijk}^{L} = \Gamma^L_{\ell^i \ell^j \ell^k} = -\frac{1}{\sqrt{2}} \left( - g_2 V_{i1}^* \sum_{a=1}^{3} U_{L,ja}^e Z_{k3+a}^i - V_{i2}^* \sum_{b=1}^{3} U_{L,jb}^e \sum_{a=1}^{3} Z_{k3+a}^i Y_{\nu,ab}^* \right) \]  

\[ H_{ijk}^{R} = \Gamma^R_{\ell^i \ell^j \ell^k} = -\frac{1}{\sqrt{2}} \left( g_2 V_{i1}^* \sum_{a=1}^{3} U_{L,ja}^e Z_{k3+a}^i + V_{i2}^* \sum_{b=1}^{3} U_{L,jb}^e \sum_{a=1}^{3} Z_{k3+a}^i Y_{\nu,ab}^* \right) \]  

\[ H_{ijk}^{u,L} = \Gamma^L_{u,\alpha \tilde{\alpha} \bar{\kappa}_{\gamma} h_{k}} = -\frac{1}{\sqrt{2}} \delta_{\alpha \beta} \sum_{b=1}^{3} \sum_{a=1}^{3} U_{L,ja}^* \sum_{b=1}^{3} Y_{u,ab}^* Z_{k2}^H \]  

\[ H_{ijk}^{u,R} = \Gamma^R_{u,\alpha \tilde{\alpha} \bar{\kappa}_{\gamma} h_{k}} = -\frac{1}{\sqrt{2}} \delta_{\alpha \beta} \sum_{b=1}^{3} \sum_{a=1}^{3} Y_{u,ab}^* U_{R,ja}^u U_{L,ib}^u Z_{k2}^H \]  

\[ H_{ijk}^{u,L} = \Gamma^L_{\nu,\tilde{\nu} h_{k}} = -\frac{1}{\sqrt{2}} \left( \sum_{b=1}^{3} U_{j,ja}^* \sum_{b=1}^{3} U_{j,ja}^* Y_{\nu,ab}^* \right) Z_{k2}^H \]  

\[ H_{ijk}^{u,R} = \Gamma^R_{\nu,\tilde{\nu} h_{k}} = -\frac{1}{\sqrt{2}} \left( \sum_{b=1}^{3} \sum_{a=1}^{3} U_{j,ja}^* U_{j,ja}^* Y_{\nu,ab}^* \right) Z_{k2}^H \]  

\[ F_{ijk}^{L} = \Gamma^L_{\chi^+ \nu \tilde{\nu} e_k} = V_{i2}^* \sum_{b=1}^{3} Z_{k3+a}^i \sum_{a=1}^{3} Y_{\nu,ab}^* \]  

\[ F_{ijk}^{R} = \Gamma^R_{\chi^+ \nu \tilde{\nu} e_k} = V_{i2}^* \sum_{b=1}^{3} Z_{k3+a}^i \sum_{a=1}^{3} Y_{\nu,ab}^* \]
\[ F_{\xi j}^{c,R} = \Gamma_{\xi_i^+ \nu_j^+ \gamma}^L = \Gamma_{\xi_i^- \nu_j^- \gamma}^R = e \delta_{ij} \] (126)

\[ C_{ij}^L = \Gamma_{\xi_i^+ \xi_j^- \gamma}^L = g_2 U_{j_1}^* \cos \Theta_W U_{i_1} + \frac{1}{2} U_{j_2}^* \left( -g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) U_{i_2} \] (127)

\[ C_{ij}^R = \Gamma_{\xi_i^+ \xi_j^- \gamma}^R = g_2 V_{i_1}^* \cos \Theta_W V_{j_1} + \frac{1}{2} V_{i_2}^* \left( -g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) V_{j_2} \] (128)

\[ M_{ij}^L = \Gamma_{\xi_i^0 \xi_j^0 \gamma}^L = -\frac{1}{2} \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \left( N_{j_3}^* N_{i_3} - N_{j_4}^* N_{i_4} \right) \] (129)

\[ M_{ij}^R = \Gamma_{\xi_i^0 \xi_j^0 \gamma}^R = \frac{1}{2} \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \left( N_{i_3}^* N_{j_3} - N_{i_4}^* N_{j_4} \right) \] (130)

\[ D_{ij}^L = \Gamma_{d_{a\alpha} d_{j\beta} \gamma}^L = \frac{1}{6} \delta_{\alpha\beta} \delta_{ij} \left( 3g_2 \cos \Theta_W + g_1 \sin \Theta_W \right) \] (131)

\[ D_{ij}^R = \Gamma_{d_{a\alpha} d_{j\beta} \gamma}^R = -\frac{1}{3} g_1 \delta_{\alpha\beta} \delta_{ij} \sin \Theta_W \] (132)

\[ \tilde{V}_{ij}^{u,L} = \Gamma_{\tilde{u}_{a\alpha} d_{j\beta} \gamma}^L = -\frac{1}{\sqrt{2}} g_2 \delta_{\alpha\beta} \sum_{a=1}^3 U_{L,ja}^* U_{L,ia}^u \] (133)

\[ \tilde{V}_{ij}^{u,R} = \Gamma_{\tilde{u}_{a\alpha} d_{j\beta} \gamma}^R = 0 \] (134)

\[ \tilde{V}_{ij}^{+,-} = \Gamma_{\tilde{e}_{a\alpha} \tilde{e}_{j\beta} \gamma}^L = -\frac{1}{\sqrt{2}} g_2 \sum_{a=1}^3 U_{L,ja}^* U_{L,ia}^V \] (135)

\[ \tilde{V}_{ij}^{+,-} = \Gamma_{\tilde{e}_{a\alpha} \tilde{e}_{j\beta} \gamma}^R = 0 \] (136)

\[ E_{ij}^L = \Gamma_{\tilde{e}_{a\alpha} \tilde{e}_{j\beta} \gamma}^L = \frac{1}{2} \delta_{ij} \left( -g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \] (137)

\[ E_{ij}^R = \Gamma_{\tilde{e}_{a\alpha} \tilde{e}_{j\beta} \gamma}^R = -g_1 \delta_{ij} \sin \Theta_W \] (138)

\[ U_{ij}^L = \Gamma_{\tilde{u}_{a\alpha} \tilde{u}_{j\beta} \gamma}^L = -\frac{1}{4} \delta_{\alpha\beta} \delta_{ij} \left( 3g_2 \cos \Theta_W - g_1 \sin \Theta_W \right) \] (139)

\[ U_{ij}^R = \Gamma_{\tilde{u}_{a\alpha} \tilde{u}_{j\beta} \gamma}^R = \frac{2}{3} g_1 \delta_{\alpha\beta} \delta_{ij} \sin \Theta_W \] (140)

\[ V_{ij}^L = \Gamma_{\tilde{e}_{a\alpha} \tilde{u}_{j\beta} \gamma}^L = -\frac{3}{4} \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \sum_{a=1}^3 U_{ja}^* U_{ia}^V \] (141)

A.2.2 Fermion-Vector vertices
\[ V_{ij}^R = \Gamma_{\nu ij}^R Z_\mu = \frac{1}{2} \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \sum_{a=1}^3 U_{ia}^* U_{ja}^V \] 

(142)

In addition, we introduce

\[ \hat{V}_{ij}^{d, L} = (\hat{V}_{ij}^{u, L})^* \quad \hat{V}_{ij}^{d, R} = (\hat{V}_{ij}^{u, R})^* \quad \hat{V}_{ij}^{+, L} = (\hat{V}_{ij}^{+, L})^* \quad \hat{V}_{ij}^{+, R} = (\hat{V}_{ij}^{+, R})^* \] 

(143)

### A.2.3 Scalar vertices

\[ A_{ij}^{h h} = \Gamma_{\nu ij}^h \Gamma_{\nu ij}^h = - \frac{i}{\sqrt{2} a} \left( 3 \sum_{j b} Z_{3}^{E \ast} 3 \sum_{k b} T_{e, a b} Z_{i 1}^{A} - 3 \sum_{j b} \sum_{k b} Z_{3}^{E \ast} T_{e, a b} Z_{i 1}^{A} \right) \] 

(144)

\[ \hat{A}_{ij}^{h h} = \Gamma_{\nu ij}^{h h} \] 

(145)

\[ \hat{A}_{ij}^{h h} = \Gamma_{\nu ij}^{h h} \] 

(146)

\[ \hat{A}_{ij}^{R R} = \Gamma_{\nu ij}^{R R} \] 

(147)

\[ H_{ij}^{h h} = \Gamma_{h h} H_{i j}^{h h} = - \frac{1}{4} \left( g_1^2 + g_2^2 \right) v_d Z_{i 1}^{A} + v_u Z_{i 1}^{A} \right) \sum_{a=1}^3 \left( - g_1^2 + g_2^2 \right) v_u Z_{k 1}^{A} + g_2^2 v_d Z_{k 2}^{A} \] 

(148)

\[ \hat{H}_{ij} = \Gamma_{h i j} \] 

(149)
\[
\bar{H}^{ij}_{\nu \nu} = \frac{1}{2} \left( -\frac{1}{4} \left( -\sqrt{2} \mu + \frac{3}{2} \sum_{c=0}^{b=1} \left( \sum_{a=1}^{3} Y_{\nu,ca} Z_{ka}^{*} \right) \right) \left( -\frac{1}{4} \left( -\sqrt{2} \mu + \frac{3}{2} \sum_{c=0}^{b=1} \left( \sum_{a=1}^{3} Y_{\nu,ca} Z_{ka}^{*} \right) \right) \right) \right)
\]
A.2.4 Scalar-Vector vertices

\[
A_{ij}^{hw} = \Gamma_{A_i^0} H_j^{-} W_{\mu}^+ = \frac{i}{2} g_2 \left( Z_{i}^{j_1} Z_{i}^{A} + Z_{j_2}^{+,+} Z_{i_1}^{+} \right) \tag{154}
\]

\[
H_{ij}^{hw} = \Gamma_{h_i^0} H_j^{-} W_{\mu}^+ = \frac{1}{2} g_2 \left( Z_{j_2}^{+,+} Z_{i_1}^{+} - Z_{j_1}^{+,+} Z_{i_1}^{+} \right) \tag{155}
\]

\[
F_{ij}^h = \Gamma_{H_i^{-}} H_j^{+,+} = -e \delta_{ij} \tag{156}
\]

\[
Z_{ij}^{bh} = \Gamma_{H_i^{-}} H_j^{+,+} = \frac{1}{2} g_1 \sin \Theta_W - g_2 \cos \Theta_W \right) \delta_{ij} \tag{157}
\]

\[
F_{ij}^e = \Gamma_\delta \delta_{ij} = -e \delta_{ij} \tag{158}
\]

\[
\bar{E}_{ij} = \Gamma_{e_i} e_j Z_{\mu} = g_1 \sin \Theta_W \sum_{a=1}^{3} Z_{i}^{E_a} Z_{j}^{E_a} + \frac{1}{2} \left( g_1 \sin \Theta_W - g_2 \cos \Theta_W \right) \sum_{a=1}^{3} Z_{i}^{E_a} Z_{j}^{E_a} \tag{159}
\]

\[
\bar{V}_{ij} = \Gamma_{\nu_i} \nu_j Z_{\mu} = \frac{i}{2} \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \sum_{a=1}^{3} Z_{i}^{\nu_{a}} Z_{j}^{\nu_{a}} \tag{160}
\]

\[
H_{ij}^{hw} = \Gamma_{h_i^0} W_{\mu}^+ W_{\mu}^- = -\frac{1}{2} g_2 \left( v_{d} Z_{i_1}^{H} + v_{u} Z_{i_2}^{H} \right) \tag{161}
\]

\[
F_{i}^{hw} = \Gamma_{H_i^{-}} W_{\alpha}^+ = -\frac{1}{2} \sin \Theta_W \cos^2 \left( v_{d} Z_{i_1}^{+,+} - v_{u} Z_{i_2}^{+,+} \right) \tag{162}
\]

\[
Z_{i}^{hw} = \Gamma_{H_i^{-}} W_{\alpha}^+ = \frac{1}{2} g_1 g_2 \left( v_{d} Z_{i_1}^{+,+} - v_{u} Z_{i_2}^{+,+} \right) \sin \Theta_W \tag{163}
\]

In addition, we introduce

\[
\tilde{A}_{ij}^{bw} = (A_{ij}^{hw})^* \quad \tilde{H}_{ij}^{bw} = (H_{ij}^{hw})^* \quad \tilde{F}_{i}^{bw} = (F_{i}^{bw})^* \quad \tilde{Z}_{i}^{bw} = (Z_{i}^{bw})^* \tag{165}
\]
A.2.5 Vector vertices

\[ F^w = \Gamma_{W^{\pm}e\gamma}W_\mu^\pm = g_2 \sin \Theta_W \]  \hspace{1cm} (166)

\[ Z^w = \Gamma_{W^{\pm}H}W_\mu^\pm Z_\mu = - g_2 \cos \Theta_W \]  \hspace{1cm} (167)

B Renormalization Group Equations

We give in the following the 2-loop RGEs for the considered model. For parameters present in the MSSM we show only the difference with respect to the MSSM RGEs. In general, the RGEs for a parameter \( X \) are defined by

\[ \frac{d}{dt} X = \frac{1}{16\pi^2} \beta_X^{(1)} + \frac{1}{(16\pi^2)^2} \beta_X^{(2)} \]  \hspace{1cm} (168)

Here, \( t = \log (Q/M) \), with \( Q \) the renormalization scale and \( M \) a reference scale.

Gauge Couplings

\[ \Delta \beta_{\alpha_1}^{(2)} = -\frac{6}{5} g_1^2 \text{Tr} \left( Y_\alpha Y_\alpha^\dagger \right) \]  \hspace{1cm} (169)

\[ \Delta \beta_{\alpha_2}^{(2)} = -2 g_2^2 \text{Tr} \left( Y_\alpha Y_\alpha^\dagger \right) \]  \hspace{1cm} (170)

Gaugino Mass Parameters

\[ \Delta \beta^{(2)}_{M_1} = -\frac{12}{5} g_1^2 \left( M_1 \text{Tr} \left( Y_\alpha Y_\alpha^\dagger \right) - \text{Tr} \left( Y_\alpha^\dagger T_\nu \right) \right) \]  \hspace{1cm} (171)

\[ \Delta \beta^{(2)}_{M_2} = 4 g_2^2 \left( - M_2 \text{Tr} \left( Y_\alpha Y_\alpha^\dagger \right) + \text{Tr} \left( Y_\alpha^\dagger T_\nu \right) \right) \]  \hspace{1cm} (172)

Trilinear Superpotential Parameters

\[ \Delta \beta_{Y_d}^{(2)} = -Y_d \text{Tr} \left( Y_d Y_d^\dagger Y_d^\dagger \right) - Y_d Y_u^\dagger Y_u \text{Tr} \left( Y_d Y_d^\dagger \right) \]  \hspace{1cm} (173)

\[ \Delta \beta_{Y_u}^{(2)} = Y_u Y_u^\dagger Y_u \]  \hspace{1cm} (174)

\[ \Delta \beta_{Y_e}^{(2)} = -2 Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e - 2 Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e - 3 Y_e Y_e^\dagger Y_e \text{Tr} \left( Y_u Y_u^\dagger \right) \]  
\[ - Y_e Y_e^\dagger Y_e \text{Tr} \left( Y_u Y_u^\dagger Y_e^\dagger \right) - Y_e \text{Tr} \left( Y_e Y_e^\dagger Y_e Y_e^\dagger \right) \]  \hspace{1cm} (175)

\[ \Delta \beta_{Y_u}^{(2)} = Y_u \text{Tr} \left( Y_u Y_u^\dagger \right) \]  \hspace{1cm} (176)

\[ \Delta \beta_{Y_e}^{(2)} = -3 Y_u \text{Tr} \left( Y_u Y_u^\dagger Y_u^\dagger \right) - 3 Y_u Y_d^\dagger Y_u \text{Tr} \left( Y_u Y_u^\dagger \right) - Y_u \text{Tr} \left( Y_e Y_e^\dagger Y_e Y_e^\dagger \right) \]  \hspace{1cm} (177)

\[ \beta_{Y_d}^{(1)} = 3 Y_d Y_d^\dagger Y_d + Y_d \left( - 3 g_2^2 + 3 \text{Tr} \left( Y_d Y_d^\dagger \right) - \frac{3}{5} g_1^2 + \text{Tr} \left( Y_d Y_d^\dagger \right) \right) + Y_d Y_d^\dagger Y_d \]  \hspace{1cm} (178)

\[ \beta_{Y_e}^{(2)} = +\frac{6}{5} g_2^2 Y_e Y_e^\dagger Y_e + 6 g_2^2 Y_e Y_e^\dagger Y_e \]  
\[ - 2 Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e - 2 Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e \]  
\[ - 4 Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e + Y_e Y_e^\dagger Y_e \left( - 3 \text{Tr} \left( Y_d Y_d^\dagger \right) + \frac{6}{5} g_1^2 - \text{Tr} \left( Y_d Y_d^\dagger \right) \right) - 9 Y_e Y_e^\dagger Y_e \text{Tr} \left( Y_u Y_u^\dagger \right) \]  
\[- 3 Y_e Y_e^\dagger Y_e \text{Tr} \left( Y_u Y_u^\dagger \right) \]
\[
\begin{align*}
+ Y_c \left( \frac{207}{50} g_1^4 + \frac{9}{5} g_1^2 g_2^2 + \frac{15}{2} g_1^4 + \frac{4}{5} \left( 20 g_2^2 + g_2^2 \right) \text{Tr} \left( Y_u Y_u^\dagger \right) - 3 \text{Tr} \left( Y_d Y_u^\dagger Y_u^\dagger \right) \\
- \text{Tr} \left( Y_e Y_e^\dagger Y_e^\dagger \right) - 9 \text{Tr} \left( Y_u Y_u^\dagger Y_u^\dagger \right) - 3 \text{Tr} \left( Y_e Y_e^\dagger Y_e^\dagger \right) \right) 
\end{align*}
\]

(179)

Bilinear Superpotential Parameters

\[
\Delta \beta_\mu^{(1)} = \mu \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) 
\]

(180)

\[
\Delta \beta_\nu^{(2)} = -\mu \left( 2 \text{Tr} \left( Y_e Y_e^\dagger Y_\nu Y_\nu^\dagger \right) + 3 \text{Tr} \left( Y_e Y_e^\dagger Y_\nu Y_\nu^\dagger \right) \right) 
\]

(181)

\[
\beta_\mu^{(1)} = \beta_x^{(2)} = 0 
\]

(182)

\[
\beta_{M_R}^{(1)} = 2 Y_\nu Y_\nu^\dagger M_R 
\]

(183)

\[
\beta_{M_R}^{(2)} = -2 \left( Y_\nu Y_\nu^\dagger Y_e Y_e^\dagger M_R + Y_\nu Y_\nu^\dagger Y_e Y_e^\dagger M_R \right) 
\]

(184)

Trilinear Soft-Breaking Parameters

\[
\Delta \beta_{T_d}^{(2)} = -2 Y_d Y_u^\dagger T_u \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) - T_d Y_u^\dagger Y_u \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) - 2 Y_d Y_u^\dagger Y_u \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) 
\]

\[
- T_d \text{Tr} \left( Y_e Y_e^\dagger Y_e^\dagger \right) - 2 Y_d \text{Tr} \left( Y_e Y_e^\dagger Y_e^\dagger \right) - 2 Y_d \text{Tr} \left( Y_e Y_e^\dagger Y_e^\dagger \right) 
\]

(185)

\[
\Delta \beta_{T_e}^{(2)} = 2 Y_e Y_e^\dagger T_e + T_e Y_e^\dagger Y_e 
\]

(186)

\[
\Delta \beta_{T_e}^{(2)} = -2 Y_e Y_e^\dagger Y_e Y_e^\dagger T_e - 4 Y_e Y_e^\dagger Y_e Y_e^\dagger T_e - 4 Y_d Y_d^\dagger T_e Y_e^\dagger Y_e - 4 Y_e Y_e^\dagger T_e Y_e^\dagger Y_e 
\]

\[
- 4 T_e Y_e^\dagger Y_e Y_e^\dagger Y_e - 2 T_e Y_e^\dagger Y_e Y_e^\dagger Y_e - 6 Y_e Y_e^\dagger T_e \text{Tr} \left( Y_u Y_u^\dagger \right) 
\]

\[
- 3 T_e Y_e^\dagger Y_e \text{Tr} \left( Y_e Y_u^\dagger \right) - 2 Y_e Y_e^\dagger T_e \text{Tr} \left( Y_e Y_u^\dagger \right) - T_e Y_e^\dagger Y_e \text{Tr} \left( Y_e Y_u^\dagger \right) 
\]

\[
- 6 Y_e Y_e^\dagger T_e \text{Tr} \left( Y_u^\dagger T_u \right) - 2 Y_e Y_e^\dagger T_e \text{Tr} \left( Y_u^\dagger T_u \right) - T_e \text{Tr} \left( Y_e Y_e^\dagger T_e Y_e^\dagger \right) 
\]

\[
- 2 Y_e \text{Tr} \left( Y_e Y_e^\dagger T_e Y_e^\dagger \right) - 2 Y_e \text{Tr} \left( Y_e Y_e^\dagger T_e Y_e^\dagger \right) 
\]

(187)

\[
\Delta \beta_{T_e}^{(2)} = 2 Y_e \text{Tr} \left( Y_e Y_e^\dagger T_e \right) + T_e \text{Tr} \left( Y_e Y_e^\dagger \right) 
\]

(188)

\[
\Delta \beta_{T_e}^{(2)} = -4 Y_u Y_u^\dagger T_u \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) - 5 T_u Y_u^\dagger Y_u \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) - 6 Y_u Y_u^\dagger Y_u \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) 
\]

\[
- T_u \text{Tr} \left( Y_e Y_e^\dagger Y_e^\dagger \right) - 2 Y_u \text{Tr} \left( Y_e Y_e^\dagger Y_e^\dagger \right) - 2 Y_u \text{Tr} \left( Y_e Y_e^\dagger Y_e^\dagger \right) 
\]

\[
- 3 T_u \text{Tr} \left( Y_e Y_e^\dagger Y_e^\dagger Y_e^\dagger \right) - 12 Y_u \text{Tr} \left( Y_e Y_e^\dagger Y_e^\dagger Y_e^\dagger \right) 
\]

(189)

\[
\beta_{T_e}^{(1)} = +2 Y_e Y_e^\dagger T_e + 4 Y_e Y_e^\dagger T_e + T_e Y_e^\dagger Y_e + 5 T_e Y_e^\dagger Y_e - \frac{3}{5} g_2^2 T_e - 3 g_2^2 T_e 
\]

\[
+ 3 T_e \text{Tr} \left( Y_e Y_e^\dagger \right) + T_e \text{Tr} \left( Y_e Y_e^\dagger \right) + Y_e \left( 2 \text{Tr} \left( Y_e^\dagger T_e \right) + 6 g_2^2 M_2 + 6 \text{Tr} \left( Y_e^\dagger T_e \right) + \frac{6}{5} g_2^2 M_1 \right) 
\]

(190)

\[
\beta_{T_e}^{(2)} = + \frac{12}{5} g_1^2 Y_e Y_e^\dagger T_e - \frac{12}{5} g_1^2 M_1 Y_e Y_e^\dagger Y_e - 12 g_2^2 M_2 Y_e Y_e^\dagger Y_e + \frac{6}{5} g_1^2 Y_e Y_e^\dagger T_e 
\]

\[
+ 6 g_2^2 Y_e Y_e^\dagger T_e + \frac{6}{5} g_1^2 T_e Y_e^\dagger Y_e + \frac{12}{5} g_1^2 T_e Y_e^\dagger Y_e + 12 g_2^2 T_e Y_e^\dagger Y_e 
\]
The RGEs of the soft SUSY breaking masses are usually written in terms of a set of traces (see [122]). In the model considered here, only one changes with respect to the MSSM:

\[
\Delta \sigma_{3,1} = \frac{1}{20} \frac{1}{\sqrt{5}} g_1 \left( -30 m_{H_u}^2 \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) + 30 \text{Tr} \left( Y_\nu m_i^2 Y_\nu^\dagger \right) \right)
\]
The resulting RGEs are:

\[ \Delta \beta_{m_q}^{(2)} = -2 T_u^\dagger T_u + 2 Y_u^\dagger m_2^2 Y_u + 4 m_H^2 u^\dagger u + m_q^2 Y_u^\dagger Y_u + Y_u^\dagger Y_u m_q^2 \] \(\text{Tr}(Y_u Y_u^\dagger)\) 
\[ - 2 \left( T_u^\dagger Y_u \text{Tr}(Y_u^\dagger T_u) + Y_u^\dagger T_u \text{Tr}(T_u^\dagger Y_u^\dagger) \right) + Y_u^\dagger Y_u \text{Tr}(m_q^2 Y_u Y_u^\dagger) \] 
\[ + Y_u^\dagger Y_u \text{Tr}(m_q^2 Y_u Y_u^\dagger) + Y_u^\dagger Y_u \text{Tr}(m_q^2 Y_u Y_u^\dagger) \] 
\[ (198) \]

\[ \Delta \beta_{m_i}^{(1)} = 2 m_H^2 Y_i^\dagger Y_i + 2 T_{\nu e}^\dagger T_{\nu e} + 2 Y_{\nu e}^\dagger m_2^2 Y_{\nu e} + Y_{\nu e}^\dagger Y_{\nu e} m_2^2 \]

\[ \Delta \beta_{m_i}^{(2)} = -3 (2 T_{\nu e}^\dagger T_{\nu e} + 2 Y_{\nu e}^\dagger m_2^2 Y_{\nu e} + 4 m_H^2 Y_{\nu e}^\dagger Y_{\nu e} + m_i^2 Y_{\nu e}^\dagger Y_{\nu e} + Y_{\nu e}^\dagger Y_{\nu e} m_i^2) \] \(\text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger)\) 
\[ - \left( 2 m_H^2 Y_{\nu e}^\dagger Y_{\nu e} + 2 m_H^2 Y_{\nu e}^\dagger Y_{\nu e} + m_i^2 Y_{\nu e}^\dagger Y_{\nu e} + Y_{\nu e}^\dagger Y_{\nu e} m_i^2 \right) \] 
\[ \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger) \] 
\[ - 2 \left( 4 m_H^2 Y_{\nu e}^\dagger Y_{\nu e} + 2 Y_{\nu e}^\dagger T_{\nu e}^\dagger T_{\nu e} + 2 Y_{\nu e}^\dagger T_{\nu e} T_{\nu e}^\dagger Y_{\nu e} + 2 T_{\nu e}^\dagger Y_{\nu e} Y_{\nu e} + 2 Y_{\nu e}^\dagger m_2^2 Y_{\nu e} + 2 Y_{\nu e}^\dagger m_2^2 Y_{\nu e} \right) \] 
\[ + 2 Y_{\nu e}^\dagger Y_{\nu e} Y_{\nu e}^\dagger Y_{\nu e}^\dagger + 3 T_{\nu e}^\dagger Y_{\nu e} \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger) + T_{\nu e}^\dagger Y_{\nu e} \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger) \] 
\[ + 3 Y_{\nu e}^\dagger Y_{\nu e} \text{Tr}(T_{\nu e}^\dagger T_{\nu e}) + Y_{\nu e}^\dagger Y_{\nu e} \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e}^\dagger) + 3 Y_{\nu e}^\dagger Y_{\nu e} \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e}^\dagger) \]
\[ + Y_{\nu e}^\dagger Y_{\nu e} \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e}^\dagger) \] 
\[ (200) \]

\[ \Delta \beta_{m_{\nu e}}^{(2)} = -2 \left( \left( m_H^2 + m_H^2 \right) \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger) + \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger Y_{\nu e}^\dagger) + \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger T_{\nu e}^\dagger) + \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger T_{\nu e} T_{\nu e}^\dagger) + \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger T_{\nu e} T_{\nu e}^\dagger) \right) \] 
\[ + \text{Tr}(Y_{\nu e} T_{\nu e}^\dagger T_{\nu e}^\dagger) + \text{Tr}(Y_{\nu e} T_{\nu e} T_{\nu e}^\dagger) + \text{Tr}(Y_{\nu e} T_{\nu e} T_{\nu e}^\dagger) \] 
\[ + \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e}^\dagger Y_{\nu e}^\dagger) + \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e}^\dagger Y_{\nu e}^\dagger) + \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e}^\dagger Y_{\nu e}^\dagger) + \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e}^\dagger Y_{\nu e}^\dagger) \] 
\[ (201) \]

\[ \Delta \beta_{m_{\nu e}}^{(1)} = 2 \left( m_H^2 \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger) + \text{Tr}(T_{\nu e}^\dagger T_{\nu e}^\dagger) + \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e}^\dagger) + \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e}^\dagger) \right) \]

\[ \Delta \beta_{m_{\nu e}}^{(2)} = -2 \left( \left( m_H^2 + m_H^2 \right) \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger) + \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger Y_{\nu e}^\dagger) + \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger T_{\nu e}^\dagger) + \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger T_{\nu e} Y_{\nu e}^\dagger) + \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger T_{\nu e} Y_{\nu e}^\dagger) \right) \]
\[ + 6 m_H^2 \text{Tr}(Y_{\nu e} Y_{\nu e}^\dagger Y_{\nu e} Y_{\nu e}^\dagger) + 6 Y_{\nu e} T_{\nu e} T_{\nu e}^\dagger Y_{\nu e}^\dagger + 6 Y_{\nu e} T_{\nu e} T_{\nu e}^\dagger Y_{\nu e} \] 
\[ + \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e} Y_{\nu e}^\dagger) + \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e} Y_{\nu e}^\dagger) + \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e} Y_{\nu e} Y_{\nu e} Y_{\nu e}^\dagger) + \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e} Y_{\nu e} Y_{\nu e} Y_{\nu e}^\dagger) \] 
\[ + \text{Tr}(m_i^2 Y_{\nu e} Y_{\nu e} Y_{\nu e} Y_{\nu e} Y_{\nu e}^\dagger) \] 
\[ (203) \]

\[ \Delta \beta_{m^2}^{(2)} = -2 \left( \left( 2 m_H^2 + 2 Y_H^\dagger m_2^2 Y_H + 4 m_H^2 Y_H^\dagger Y_H + 4 m_H^2 Y_H^\dagger Y_H + 4 m_H^2 Y_H^\dagger Y_H^\dagger + 4 m_H^2 Y_H^\dagger Y_H^\dagger + 4 m_H^2 Y_H^\dagger Y_H^\dagger + 4 m_H^2 Y_H^\dagger Y_H^\dagger + 4 m_H^2 Y_H^\dagger Y_H^\dagger m_2^2 \right) \] \(\text{Tr}(Y_H Y_H^\dagger)\) 
\[ + 2 \left( Y_H T_H^\dagger Y_H + Y_H T_H Y_H^\dagger + Y_H T_H^\dagger Y_H^\dagger + Y_H T_H Y_H^\dagger \right) \] 
\[ + Y_H Y_H^\dagger \text{Tr}(m_2^2 Y_H Y_H^\dagger) + Y_H Y_H^\dagger \text{Tr}(m_2^2 Y_H Y_H^\dagger) \] 
\[ (204) \]

\[ \Delta \beta_{m^2}^{(2)} = -2 \left( 3 m_H^2 Y_H^\dagger Y_H + 3 m_H^2 Y_H^\dagger Y_H + 3 m_H^2 Y_H^\dagger Y_H^\dagger + 3 m_H^2 Y_H^\dagger Y_H^\dagger + 3 m_H^2 Y_H^\dagger Y_H^\dagger + 3 m_H^2 Y_H^\dagger Y_H^\dagger m_2^2 \right) \] 
\[ (205) \]
\[
\beta_{m^2}^{(1)} = 2 \left( 2m^2_{H_u} Y_e Y_e^\dagger + 2T_v T_v^\dagger + 2Y_e m^2 T_Y + m^2_{H_d} v^\dagger v + Y_e T_y + m^2_{H_d} \right)
\]

\[
\beta_{m^2}^{(2)} = -\frac{2}{5} \left( 6g_1^2 m^2 T_Y Y_e^\dagger + 30g_2^2 m^2 T_Y T_Y^\dagger - 6g_1^2 T_T T_v^\dagger - 30g_2^2 T_T T_v^\dagger 
- 3g_1^2 m^2 T_Y Y_e^\dagger - 15g_2^2 m^2 T_Y T_Y^\dagger - 5g_2^2 Y_e m^2 T_Y^\dagger - 30g_2^2 Y_e m^2 T_Y^\dagger 
- 3g_1^2 Y_e T_T^\dagger m^2_{H_d} - 15g_2^2 Y_e T_T^\dagger m^2_{H_d} + 10m^2_{H_d} Y_e Y_e^\dagger Y_e^\dagger 
+ 10Y_e T_T^\dagger T_T^\dagger + 10Y_e T_T^\dagger T_T^\dagger + 10Y_e T_T^\dagger T_T^\dagger + 10T_T^\dagger T_T^\dagger T_T^\dagger + 10T_T^\dagger T_T^\dagger T_T^\dagger 
+ 10Y_e T_T^\dagger T_T^\dagger T_T^\dagger + 5m^2 T_T^\dagger T_T^\dagger + 5m^2 T_T^\dagger T_T^\dagger + 10Y_e T_T^\dagger T_T^\dagger T_T^\dagger + 10Y_e T_T^\dagger T_T^\dagger T_T^\dagger 
+ 5Y_e T_T^\dagger T_T^\dagger T_T^\dagger + 30T_v T_v^\dagger \text{Tr} \left( Y_u Y_u^\dagger \right) + 15m^2_{H_d} Y_e Y_e^\dagger \text{Tr} \left( Y_u Y_u^\dagger \right) 
+ 30Y_e m^2 T_T^\dagger \text{Tr} \left( Y_u Y_u^\dagger \right) + 15Y_e m^2 T_T^\dagger \text{Tr} \left( Y_u Y_u^\dagger \right) + 10Y_e m^2 T_T^\dagger \text{Tr} \left( Y_u Y_u^\dagger \right) + 5Y_e m^2 T_T^\dagger \text{Tr} \left( Y_u Y_u^\dagger \right) 
+ 2Y_e T_T^\dagger \left( 15g_2^2 M_2 + 15 \text{Tr} \left( Y_u T_u \right) + 3g_1^2 M_1 + 5 \text{Tr} \left( Y_u T_u \right) \right) + 30T_v T_v^\dagger \text{Tr} \left( T_y^a T_y^a \right) 
+ 10T_v T_v^\dagger \text{Tr} \left( T_y^a T_y^a \right) \right)
\]

\[
\frac{4}{5} Y_e T_T^\dagger \left( 3g_1^2 m^2_{H_u} + 15g_2^2 m^2_{H_u} + 6g_1^2 M_1 + 30g_2^2 M_2 \right) - 30m^2_{H_u} \text{Tr} \left( Y_u Y_u^\dagger \right) - 10m^2_{H_u} \text{Tr} \left( Y_v Y_v^\dagger \right) 
- 15 \text{Tr} \left( T_y^a T_y^a \right) - 5 \text{Tr} \left( T_y^a T_y^a \right) - 5 \text{Tr} \left( m^2 Y_T Y_T \right) - 15 \text{Tr} \left( m^2 Y_T Y_T \right) - 15 \text{Tr} \left( m^2 Y_T Y_T \right)
\]

\[
\beta_{m^2}^{(1)} = \beta_{m^2}^{(2)} = 0
\]

Vacuum expectation values

\[
\Delta \beta_{v_d}^{(2)} = v_d \text{Tr} \left( Y_e Y_e^\dagger Y_v Y_v^\dagger \right)
\]

\[
\Delta \beta_{v_u}^{(2)} = \frac{3}{10} \left( 5g_2^2 + g_1^2 \right) v_u \xi \text{Tr} \left( Y_v Y_v^\dagger \right) + v_u \text{Tr} \left( Y_v Y_v^\dagger Y_v Y_v^\dagger \right)
\]

C Loop Integrals

The \( B \)-functions with vanishing external momenta and the arguments \( (a, b) \) are given by

\[
B_0 = 1 - \log \left( \frac{b}{Q^2} \right) + \frac{1}{b - a} \left[ a \log \left( \frac{a}{b} \right) \right],
\]

\[
B_1 = \frac{1}{2} + \frac{1}{2} \log \left( \frac{b}{Q^2} \right) - \frac{1}{4(a - b)^2} \left[ a^2 - b^2 + 2a^2 \log \left( \frac{b}{a} \right) \right],
\]
The $C$-functions with vanishing external momenta and the arguments $(a, b, c)$ read

$$C_0 = -\frac{1}{(a-b)(a-c)(b-c)} \times \left[ b(c-a) \log \left(\frac{b}{a}\right) + c(a-b) \log \left(\frac{c}{a}\right) \right]$$

$$C_{00} = \frac{1}{8(a-b)(a-c)(b-c)} \times \left[ (c-a) \left( (a-b)(2 \log \left(\frac{a}{Q^2}\right) - 3)(b-c) - 2b^2 \log \left(\frac{b}{a}\right) \right) + 2c^2(b-a) \log \left(\frac{c}{a}\right) \right]$$

$$C_1 = -\frac{1}{2(a-b)^2(a-c)(b-c)^2} \times \left[ c^2(a-b)^2 \log \left(\frac{c}{a}\right) 
+ b(c-a) \left( (b-a)(b-c) - (a(b-2c) + bc) \log \left(\frac{b}{a}\right) \right) \right]$$

$$C_2 = -\frac{1}{2(a-b)(a-c)^2(b-c)^2} \times \left[ a^2(b-c)^2 \log \left(\frac{b}{a}\right) 
+ c(b-a) \left( (a-c)(b-c) + (c(a+b) - 2ab) \log \left(\frac{b}{c}\right) \right) \right]$$

$$C_{11} = -\frac{1}{6(a-b)^3(a-c)(b-c)^3} \times \left[ b(a-c)(-2a^2(b^2 - 3bc + 3c^2) + abc(b-3c) + b^2c^2) \log \left(\frac{b}{a}\right) 
- (b-a)(b-c) \left( -3ab + 5ac + b^2 - 3bc \right) + 2c^3(a-b)^3 \log \left(\frac{c}{a}\right) \right]$$

$$C_{12} = \frac{1}{6(a-b)^2(a-c)^2(b-c)^3} \times \left[ (a-b) \left( (a-c)(b-c)(a(b^2 + c^2) 
- bc(b+c)) + c^2(a-b)(3ab-c(a+2b)) \log \left(\frac{c}{a}\right) \right) 
+ b^2(a-c)^2(a(b-3c) + 2bc) \log \left(\frac{b}{a}\right) \right]$$

$$C_{22} = \frac{1}{6(a-b)(a-c)^3(b-c)^3} \times \left[ 2a^3(b-c)^3 \log \left(\frac{b}{a}\right) 
+ c(a-b) \left( 2 \left( c^2(a^2 + ab + b^2) + 3a^2b^2 - 3abc(a+b) \right) \log \left(\frac{b}{c}\right) 
- (a-c)(b-c)(-3c(a+b) + 5ab + c^2) \right) \right]$$

In the case of external photons, often the same combinations of $C$-functions appear. If the arguments are $(a, b, b)$, these can be expressed as

$$C_2 + C_{12} + C_{22} = C_1 + C_{12} + C_{11} = \frac{1}{12b(x-1)^3} \left( (x-1)(x(2x+5) - 1) - 6x^2 \log(x) \right)$$

$$C_0 + C_1 + C_2 = \frac{1}{2b(x-1)^3} \left( -x^2 + 2x \log(x) + 1 \right)$$

$$2C_{12} + 2C_{11} - C_2 = 2C_{12} + 2C_{22} - C_1 = \frac{1}{12b(x-1)^3} \left( 6(1 - 3x)x^2 \log(x) + (x-1)(x(31x - 26) + 7) \right)$$

$$C_1 + C_2 = \frac{1}{2b(x-1)^3} \left( 2x^2 \log(x) + (4 - 3x)x - 1 \right)$$

37
and for \((a, a, b)\) we get

\[
C_2 + C_{12} + C_{22} = \frac{1}{12b(x - 1)^4} \left( x((x - 6)x + 3) + 6x \log(x) + 2 \right) \\
C_0 + C_1 + C_2 = -\frac{1}{4b(x - 1)^4} \left( c^2 \log \frac{x}{a} + \frac{1}{4b(a - b)^4} \right)
\]

(227) (228)

In the previous expressions we used \(x = a/b\).

For the photonic monopole operators we define special loop functions

\[
M_{SFF}(a, b) = \frac{\left( a - b \right) (16a^2 - 29ab + 7b^2) + 6a^2(2a - 3b) \log \left( \frac{b}{a} \right)}{36(a - b)^4}
\]

(229)

\[
M_{FSS}(a, b) = \frac{6a^3 \log \left( \frac{b}{a} \right) + 11a^3 - 18a^2 b + 9ab^2 - 2b^3}{36(a - b)^4}
\]

(230)

\[
M_{FSV}(a, b) = \frac{\sqrt{a} (2a^3 + 3a^2 b + 6a^2 b \log \left( \frac{b}{a} \right) - 6ab^2 + b^3)}{12b(a - b)^4}
\]

(231)

\[
M_{FVV}(a, b) = \frac{\sqrt{a} (2a^3 + 3a^2 b + 6a^2 b \log \left( \frac{b}{a} \right) - 6ab^2 + b^3)}{12b(a - b)^4}
\]

(232)

\[
M_{FVS}(a, b) = \frac{6a^2(2a - 3b) \log \left( \frac{b}{a} \right) - (a - b) \left( 5a^2 - 22ab + 5b^2 \right)}{9(a - b)^4}
\]

(233)

The necessary box functions with the arguments \((a, b, c, d)\) read, in the limit of vanishing external momenta,

\[
D_0 = -\left[ \frac{b \log \frac{b}{a}}{(b - a)(b - c)(b - d)} + \frac{c \log \frac{c}{a}}{(c - a)(c - b)(c - d)} + \frac{d \log \frac{d}{a}}{(d - a)(d - b)(d - c)} \right]
\]

(234)

\[
D_{27} = -\frac{1}{4} \left[ \frac{b^2 \log \frac{b}{a}}{(b - a)(b - c)(b - d)} + \frac{c^2 \log \frac{c}{a}}{(c - a)(c - b)(c - d)} + \frac{d^2 \log \frac{d}{a}}{(d - a)(d - b)(d - c)} \right]
\]

(235)

In addition, we define

\[
I_{C_0D_0}(a, b, c, d) = C_0(a, b, c) + dD_0(a, b, c, d)
\]

(236)

D Photonic penguin contributions to LFV

In the following appendices we present our results for the form factors of the operators involved in our computation, done in the mass basis. The flavor of the external fermions will be denoted with Greek characters \((\alpha, \beta, \gamma, \delta)\), whereas the mass eigenstates of the particles in the loops will be denoted with Latin characters \((a, b, c, d)\). A sum over repeated indices will be assumed.
D.1 Feynman diagrams

We give in the following the contribution of each diagram to the different operators. We indicate the diagram by the corresponding index \((a_i)\) with \(i = 1, \ldots, 7\).
D.2 Neutralino contributions

\[ C_i = C_i(m_{\tilde{\chi}_0^2}^2, m_{\tilde{\nu}_e^0}^2, m_{\tilde{\nu}_\mu^0}^2) \]  

\[ A^{\text{2L}}_{(a_1)} = 2F_{c,b}^R(N_{a,a,b}^{a,R}(C_{12} + C_{22} + C_2)m_{\tilde{\nu}_e^0} + N_{a,b,c}^{a,R}(C_{11} + C_{12} + C_1)m_{\tilde{\nu}_\mu^0} - N_{a,b,c}^{a,R}(C_0 + C_1 + C_2)m_{\tilde{\chi}_0^2}) \]  

\[ A^{\text{1L}}_{(a_1)} = -N_{a,a,b}^{a,L}N_{a,b,c}^{a,L} F_{c,b}^R M_{\text{FSS}}(m_{\tilde{\chi}_0^2}^2, m_{\tilde{\nu}_e^0}^2) \]  

D.3 Chargino contributions

\[ C_i = C_i(m_{\tilde{\chi}_0^2}^2, m_{\tilde{\nu}_e^0}^2, m_{\tilde{\nu}_\mu^0}^2) \]  

\[ A^{\text{2L}}_{(a_2)} = -2 \left( \tilde{X}^{L}_{a,b,a} \chi^{c}_{c,\beta,a} (F_{b,c}^{c,L})^* C_{12} m_{\tilde{\nu}_e^0} - \tilde{X}^{R}_{a,b,a} (\chi^{L}_{c,\beta,a} (F_{b,c}^{c,R})^* (C_{12} + C_{22} + C_2)m_{\tilde{\nu}_e^0} + X^{R}_{c,\beta,a} (C_{11} + C_{12} + C_1)m_{\tilde{\nu}_e^0} - (F_{b,c}^{c,R})^* (C_0 + C_1 + C_2)m_{\tilde{\chi}_0^2}) \right) \]  

\[ A^{\text{1L}}_{(a_2)} = -\tilde{X}^{R}_{a,b,a} \chi^{L}_{c,\beta,a} (F_{b,c}^{c,R})^* M_{\text{FSS}}(m_{\tilde{\nu}_e^0}^2, m_{\tilde{\nu}_e^0}^2) \]  

D.4 W\(^+\) and H\(^+\) contributions

\[ C_i = C_i(m_{\tilde{\nu}_e^0}^2, m_{\tilde{\nu}_\mu^0}^2, m_{\tilde{H}_e^0}^2, m_{\tilde{H}_\mu^0}^2) \]  

\[ A^{\text{2L}}_{(a_4)} = 2F_{c,b}^R (V_{a,a,b}^{+,L} V_{a,a,b}^{+,R}(C_{12} + C_{22} + C_2)m_{\tilde{\nu}_e^0} + V_{a,a,b}^{+,R}(V_{a,a,b}^{+,L}(C_{11} + C_{12} + C_1)m_{\tilde{\nu}_e^0} - V_{a,a,b}^{+,R}(C_0 + C_1 + C_2)m_{\tilde{\nu}_e^0}) \]  

\[ A^{\text{1L}}_{(a_4)} = -V_{a,a,b}^{+,L} F_{c,b}^R M_{\text{FSS}}(m_{\tilde{\nu}_e^0}^2, m_{\tilde{\nu}_e^0}^2) \]  

\[ A^{\text{2L}}_{(a_5)} = 2(V_{a,a,b}^{+,L} V_{a,a,b}^{+,R} F_{c}^{hw} C_2)(m_{\tilde{\nu}_e^0}^2, m_{\tilde{\nu}_e^0}^2, m_{\tilde{W}_e^0}^2) \]  

\[ A^{\text{1L}}_{(a_5)} = (V_{a,a,b}^{+,L} V_{a,a,b}^{+,R} F_{c}^{hw} M_{\text{FVS}}(m_{\tilde{\nu}_e^0}^2, m_{\tilde{W}_e^0}^2) \]  

\[ A^{\text{2L}}_{(a_6)} = 2(V_{a,a,b}^{+,R}(V_{a,a,b}^{+,R} F_{c}^{hw} C_1)(m_{\tilde{\nu}_e^0}^2, m_{\tilde{W}_e^0}^2, m_{\tilde{H}_e^0}^2) \]  

\[ A^{\text{1L}}_{(a_6)} = (V_{a,a,b}^{+,L} V_{a,a,b}^{+,R} F_{c}^{hw} M_{\text{FSV}}(m_{\tilde{\nu}_e^0}^2, m_{\tilde{W}_e^0}^2) \]  

\[ C_i = C_i(m_{\tilde{\nu}_e^0}^2, m_{\tilde{W}_e^0}^2, m_{\tilde{H}_e^0}^2) \]  

\[ A^{\text{2L}}_{(a_7)} = -2F_{c}^{hw} (V_{a,a,b}^{+,L} (V_{a,a,b}^{-,R}(2C_{12} - C_1 + 2C_{22})m_{\tilde{\chi}_0^2}) \]
\[
A_{(a_7)}^{1L} = (\hat{V}_{a,a}^{+,L})^* (\hat{V}_{a,a}^{+,R})^* (2C_{11} + 2C_{12} - C_{2}) m_{\ell_\beta} + 3(\hat{V}_{a,\beta}^{+,R})^* (C_{1} + C_{2}) m_{\nu_a})
\]  
(254)

\[
A_{(a_7)}^{2L} = (\hat{V}_{a,a}^{+,L})^* (\hat{V}_{a,a}^{+,R})^* F^w M_{FV} (m_{\nu_a}^2, m_{W^-}^2)
\]  
(255)

These coefficients are related to the ones used in the calculation of the flavor observables by

\[
K_L^1 = \frac{1}{e} \sum_p A_{(a_p)}^{1L}
\]  
(256)

\[
K_L^2 = -\frac{1}{2e m_{\ell_\alpha}} \sum_p A_{(a_p)}^{2L}
\]  
(257)

E  \( Z \) and Higgs penguin contributions to LFV

E.1  Feynman diagrams

In the following \( B = Z, h_p, A_p^0 \) is used.

Neutralino diagrams

Self energy corrections

Vertex corrections
Chargino diagrams

Self energy corrections

\[ B_{\nu} i^a \tilde{\chi}^{-b} \]
\[ \tilde{\ell}^\beta \]
\[ \nu_{\bar{c}} \ell^c \]
\[ B \]

\[ c_1 \]

\[ B_{\bar{\nu}} R^a \tilde{\chi}^{-b} \]
\[ \tilde{\ell}^\beta \]
\[ \nu^R_{\bar{b}} \ell^c \]
\[ B \]

\[ c_2 \]

\[ c_3 \]

\[ c_4 \]

Vertex corrections

\[ \ell^\alpha \]
\[ \tilde{\chi}^- \]
\[ \nu_{\bar{c}} \]
\[ \ell^c \]
\[ B \]

\[ c_5 \]

\[ \ell^\alpha \]
\[ \tilde{\chi}^- \]
\[ \nu^R_{\bar{a}} \]
\[ \ell^c \]
\[ B \]

\[ c_6 \]

\[ \ell^\alpha \]
\[ \tilde{\chi}^+_a \]
\[ \nu^i_{\bar{c}} \]
\[ \nu^i_{\bar{c}} \]
\[ B \]

\[ c_7 \]

\[ \ell^\alpha \]
\[ \tilde{\chi}^+_a \]
\[ \nu^R_{\bar{b}} \]
\[ \nu^R_{\bar{c}} \]
\[ B \]

\[ c_8 \]
$W^+$ and $H^+$ diagrams

Self energy corrections
Vertex corrections

\[ I_1 = B_0(m_{\tilde{\chi}_a}^2, m_{\tilde{\ell}_b}^2) \]
\[ I_2 = B_1(m_{\tilde{\chi}_a}^2, m_{\tilde{\ell}_b}^2) \]
\[ V_{Z,(n_1,n_2)}^{LL} = \frac{(E_{\tilde{\chi}_a}^L N_{a,a,b} \tilde{N}_{a,b}^L I_2 m_{\tilde{\ell}_a}^2 - N_{a,a,b} \tilde{N}_{a,b}^R I_1 m_{\tilde{\ell}_a} m_{\tilde{\chi}_a}^0 + N_{a,a,b} \tilde{N}_{a,b}^L I_2 m_{\tilde{\ell}_a} m_{\tilde{\ell}_b})}{(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2)} + (\alpha \leftrightarrow \beta) \]  
\[ V_{Z,(n_1,n_2)}^{QR} = \frac{(E_{\tilde{\chi}_a}^R N_{a,a,b} \tilde{N}_{a,b}^L I_2 m_{\tilde{\ell}_a}^2 - N_{a,a,b} \tilde{N}_{a,b}^R I_1 m_{\tilde{\ell}_a} m_{\tilde{\chi}_a}^0 + N_{a,a,b} \tilde{N}_{a,b}^L I_2 m_{\tilde{\ell}_a} m_{\tilde{\ell}_b})}{(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2)} + (\alpha \leftrightarrow \beta) \]  

E.2 Neutralino contributions

E.2.1 Z-penguins

Self-energy corrections
Vertex corrections

\[ V_{Z,(n_3)}^{LL} = -2N_{a,a,b}^L N_{b,c,b}^R \bar{E}_{b,c} C_{00}(m_{\lambda_b}^2, m_{\chi_a'}^2, m_{\tilde{e}_b}^2) \]  
\[ V_{Z,(n_3)}^{LR} = -2N_{a,a,b}^L N_{b,c,b}^R \bar{E}_{b,c} C_{00}(m_{\lambda_b}^2, m_{\chi_a'}^2, m_{\tilde{e}_b}^2) \]  
\[ I_1 = B_0(m_{\chi_a}^2, m_{\tilde{e}_b}^2) \]  
\[ I_2 = C_0(m_{\lambda_b}^2, m_{\tilde{e}_a}^2) \]  
\[ I_3 = C_0(m_{\lambda_b}^2, m_{\tilde{e}_a}^2) \]  
\[ V_{Z,(n_4)}^{LL} = N_{b,a,a}^L N_{c,b,a}^R (\pm (M_{c,b}^L I_1 m_{\chi_b} m_{\tilde{e}_a}^2) + M_{c,b}^R (I_1 - 2I_2 + I_3 m_{\tilde{e}_a}^2)) \]  
\[ V_{Z,(n_4)}^{LR} = N_{b,a,a}^L N_{c,b,a}^R (\pm (M_{c,b}^L I_1 m_{\chi_b} m_{\tilde{e}_a}^2) + M_{c,b}^R (I_1 - 2I_2 + I_3 m_{\tilde{e}_a}^2)) \]

E.2.2 Scalar penguins

CP even scalars

Self-energy corrections

\[ I_1 = B_0(m_{\chi_a}^2, m_{\tilde{e}_b}^2) \]  
\[ I_2 = B_1(m_{\lambda_b}^2, m_{\tilde{e}_a}^2) \]  
\[ S_{h_{p,(n_1,n_2)}}^{LL}(H^L_{\beta,c,p}(-N_{a,a,b}^L N_{c,b,a}^R I_1 m_{\tilde{e}_c}^2) + N_{a,a,b}^R N_{c,b,a}^R I_1 m_{\tilde{e}_c}^2 - N_{a,a,b}^L N_{c,b,a}^R I_2 m_{\tilde{e}_c}^2)
\]  
\[ S_{h_{p,(n_1,n_2)}}^{LR}(H^L_{\beta,c,p}(-N_{a,a,b}^L N_{c,b,a}^R I_1 m_{\tilde{e}_c}^2) + N_{a,a,b}^R N_{c,b,a}^R I_1 m_{\tilde{e}_c}^2 - N_{a,a,b}^L N_{c,b,a}^R I_2 m_{\tilde{e}_c}^2)
\]

Vertex corrections

\[ S_{h_{p,(n_3)}}^{LL} = N_{a,a,b}^L N_{b,c,b}^R \bar{H}_{b,c} C_{00}(m_{\lambda_b}^2, m_{\tilde{e}_a}^2, m_{\chi_b}^2) \]  
\[ S_{h_{p,(n_3)}}^{LR} = N_{a,a,b}^L N_{b,c,b}^R \bar{H}_{b,c} C_{00}(m_{\lambda_b}^2, m_{\tilde{e}_a}^2, m_{\chi_b}^2) \]  
\[ I_1 = B_0(m_{\chi_b}^2, m_{\tilde{e}_b}^2) \]  
\[ I_2 = C_0(m_{\lambda_b}^2, m_{\tilde{e}_a}^2) \]  
\[ S_{h_{p,(n_4)}}^{LL} = N_{b,a,a}^L N_{c,b,a}^R (S_{c,b,p}^L I_1 m_{\chi_b} m_{\tilde{e}_a}^2 + S_{c,b,p}^R (I_1 + I_2 m_{\tilde{e}_a}^2)) \]  
\[ S_{h_{p,(n_4)}}^{LR} = N_{b,a,a}^L N_{c,b,a}^R (S_{c,b,p}^L I_1 m_{\chi_b} m_{\tilde{e}_a}^2 + S_{c,b,p}^R (I_1 + I_2 m_{\tilde{e}_a}^2)) \]

CP odd scalars

Self-energy corrections

\[ I_1 = B_0(m_{\chi_b}^2, m_{\tilde{e}_b}^2) \]  
\[ I_2 = B_1(m_{\lambda_b}^2, m_{\tilde{e}_a}^2) \]
\[ S_{A_{p}^{L}}^{LL}(n_{1},n_{2}) = (A_{\beta,c,p}^{L} - (N_{a,a,b}^{L} \tilde{N}_{a,a,b}^{R} I_{2} m_{\tilde{\ell}}^{2} + N_{a,a,b}^{R} \tilde{N}_{c,a,b}^{R} I_{1} m_{\tilde{\ell}} m_{\tilde{\ell}} - N_{a,a,b}^{R} \tilde{N}_{c,a,b}^{L} I_{2} m_{\tilde{\ell}} m_{\tilde{\ell}}) + (\alpha \leftrightarrow \beta) \]
\[ S_{A_{p}^{R}}^{LL}(n_{1},n_{2}) = (A_{\beta,c,p}^{R} - (N_{a,a,b}^{L} \tilde{N}_{a,a,b}^{R} I_{2} m_{\tilde{\ell}}^{2} + N_{a,a,b}^{R} \tilde{N}_{c,a,b}^{R} I_{1} m_{\tilde{\ell}} m_{\tilde{\ell}} - N_{a,a,b}^{R} \tilde{N}_{c,a,b}^{L} I_{2} m_{\tilde{\ell}} m_{\tilde{\ell}}) + (\alpha \leftrightarrow \beta) \]

Vertex corrections

\[ S_{A_{p}^{L}}^{LL}(n_{3}) = N_{a,a,b}^{L} \tilde{N}_{a,a,b}^{L} \tilde{A}_{p,b,c} C_{0} (m_{\tilde{\ell}}^{2}, m_{\tilde{\ell}}^{2}, m_{\tilde{\ell}}^{2}) m_{\tilde{\chi}_{a}}^{0} \]
\[ S_{A_{p}^{R}}^{LR}(n_{3}) = N_{a,a,b}^{L} \tilde{N}_{a,a,b}^{L} \tilde{A}_{p,b,c} C_{0} (m_{\tilde{\ell}}^{2}, m_{\tilde{\ell}}^{2}, m_{\tilde{\ell}}^{2}) m_{\tilde{\chi}_{a}}^{0} \]

**E.3 Chargino contributions**

**E.3.1 Z-penguins**

Self-energy corrections

\[ I_{1} = B_{0} (m_{\tilde{\chi}_{b}}^{2}, m_{\tilde{\chi}_{c}}^{2}) \]
\[ I_{2} = B_{1} (m_{\tilde{\chi}_{b}}^{2}, m_{\tilde{\chi}_{c}}^{2}) \]

\[ V_{Z_{(c_{1},c_{3})}}^{LL} = (E_{b,b,c}^{L} (W_{b,a,a}^{L} \tilde{X}_{b,a,a}^{R} I_{2} m_{\tilde{\ell}}^{2} - W_{b,b,a}^{R} \tilde{X}_{b,b,a}^{R} I_{1} m_{\tilde{\ell}} m_{\tilde{\ell}} - W_{b,b,a}^{R} \tilde{X}_{b,b,a}^{L} I_{2} m_{\tilde{\ell}} m_{\tilde{\ell}}) + (\alpha \leftrightarrow \beta) \]
\[ V_{Z_{(c_{1},c_{3})}}^{LR} = (E_{b,b,c}^{L} (W_{b,a,a}^{L} \tilde{X}_{b,a,a}^{R} I_{2} m_{\tilde{\ell}}^{2} - W_{b,b,a}^{R} \tilde{X}_{b,b,a}^{R} I_{1} m_{\tilde{\ell}} m_{\tilde{\ell}} - W_{b,b,a}^{R} \tilde{X}_{b,b,a}^{L} I_{2} m_{\tilde{\ell}} m_{\tilde{\ell}}) + (\alpha \leftrightarrow \beta) \]

\[ I_{1} = B_{0} (m_{\tilde{\chi}_{b}}^{2}, m_{\tilde{\chi}_{c}}^{2}) \]
\[ I_{2} = B_{1} (m_{\tilde{\chi}_{b}}^{2}, m_{\tilde{\chi}_{c}}^{2}) \]

\[ V_{Z_{(c_{2},c_{4})}}^{LL} = (E_{b,b,c}^{L} (\tilde{X}_{b,a,a}^{L} \tilde{X}_{b,a,a}^{R} I_{2} m_{\tilde{\ell}}^{2} - \tilde{X}_{b,a,a}^{R} \tilde{X}_{b,a,a}^{R} I_{1} m_{\tilde{\ell}} m_{\tilde{\ell}} - \tilde{X}_{b,a,a}^{R} \tilde{X}_{b,a,a}^{L} I_{2} m_{\tilde{\ell}} m_{\tilde{\ell}}) + (\alpha \leftrightarrow \beta) \]
\[ V_{Z_{(c_{2},c_{4})}}^{LR} = (E_{b,b,c}^{L} (\tilde{X}_{b,a,a}^{L} \tilde{X}_{b,a,a}^{R} I_{2} m_{\tilde{\ell}}^{2} - \tilde{X}_{b,a,a}^{R} \tilde{X}_{b,a,a}^{R} I_{1} m_{\tilde{\ell}} m_{\tilde{\ell}} - \tilde{X}_{b,a,a}^{R} \tilde{X}_{b,a,a}^{L} I_{2} m_{\tilde{\ell}} m_{\tilde{\ell}}) + (\alpha \leftrightarrow \beta) \]

Vertex corrections

\[ I_{1} = B_{0} (m_{\tilde{\chi}_{b}}^{2}, m_{\tilde{\chi}_{c}}^{2}) \]
\[ I_2 = C_0(m_{\chi_-}^2, m_{\chi_b}^2, m_{\nu_a}^2) \]  
\[ I_3 = C_0(m_{\chi_-}^2, m_{\chi_b}^2, m_{\nu_a}^2) \]  
\[ V_{LL}^{Z,(c)} = W_{b,a,a}^L \tilde{X}_{\beta,c,a}^R (-(C_{c,b}I_3m_{\chi_b}m_{\chi_-}) + C_{c,b}(I_1 - 2I_2 + I_3m_{\nu_a}^2)) \]  
\[ V_{LR}^{Z,(c)} = W_{b,a,a}^L \tilde{X}_{\beta,c,a}^R (-(C_{c,b}I_3m_{\chi_b}m_{\chi_-}) + C_{c,b}(I_1 - 2I_2 + I_3m_{\nu_a}^2)) \]  
\[ I_1 = B_0(m_{\chi_-}^2, m_{\chi_b}^2) \]  
\[ I_2 = C_0(m_{\chi_-}^2, m_{\chi_b}^2, m_{\nu_a}^2) \]  
\[ I_3 = C_0(m_{\chi_-}^2, m_{\chi_b}^2, m_{\nu_a}^2) \]  
\[ V_{LL}^{Z,(a)} = \tilde{X}_{b,a,a}^L \tilde{X}_{\beta,c,a}^R (-(C_{c,b}I_3m_{\chi_b}m_{\chi_-}) + C_{c,b}(I_1 - 2I_2 + I_3m_{\nu_a}^2)) \]  
\[ V_{LR}^{Z,(a)} = \tilde{X}_{b,a,a}^L \tilde{X}_{\beta,c,a}^R (-(C_{c,b}I_3m_{\chi_b}m_{\chi_-}) + C_{c,b}(I_1 - 2I_2 + I_3m_{\nu_a}^2)) \]  
\[ V_{LL}^{Z,(c)} = -2\tilde{X}_{a,a,b}^L \tilde{X}_{\beta,a,c}^R \tilde{V}_{b,c}^L C_0(0(m_{\chi_-}^2, m_{\nu_a}^2, m_{\nu_b}^2)) \]  
\[ V_{LR}^{Z,(c)} = -2\tilde{X}_{b,a,b}^L \tilde{X}_{\beta,b,c}^R \tilde{V}_{b,c}^L C_0(0(m_{\chi_-}^2, m_{\nu_a}^2, m_{\nu_b}^2)) \]  
\[ V_{LL}^{Z,(c)} = 2W_{b,a,a}^L \tilde{X}_{\beta,c,b}^R \tilde{V}_{b,c}^L C_0(0(m_{\chi_-}^2, m_{\nu_a}^2, m_{\nu_b}^2)) \]  
\[ V_{LR}^{Z,(c)} = 2W_{a,a,b}^L \tilde{X}_{\beta,b,c}^R \tilde{V}_{b,c}^L C_0(0(m_{\chi_-}^2, m_{\nu_a}^2, m_{\nu_b}^2)) \]  

E.3.2 Scalar penguins

CP even scalars

Self-energy corrections

\[ I_1 = B_0(m_{\chi_-}^2, m_{\chi_b}^2) \]  
\[ I_2 = B_1(m_{\chi_-}^2, m_{\nu_a}^2) \]  
\[ S_{b,a,(c)}^{LL} = (H_{\beta,c,a}^{L}(-(W_{b,a,a}^L \tilde{X}_{c,b,a}^R I_3m_{\chi_b}^2) + \tilde{X}_{b,a,a}^L \tilde{X}_{c,b,a}^R I_3m_{\chi_b}m_{\chi_-} - W_{b,a,a}^R \tilde{X}_{c,b,a}^L I_3m_{\chi_b}m_{\chi_-})/m_{\chi_b}^2 - m_{\chi_-}^2) + (\alpha \leftrightarrow \beta) \]  
\[ S_{b,a,(c)}^{LR} = (H_{\beta,c,a}^{L}(-(W_{b,a,a}^L \tilde{X}_{c,b,a}^R I_3m_{\chi_b}^2) + \tilde{X}_{b,a,a}^L \tilde{X}_{c,b,a}^R I_3m_{\chi_b}m_{\chi_-} - W_{b,a,a}^R \tilde{X}_{c,b,a}^L I_3m_{\chi_b}m_{\chi_-})/m_{\chi_b}^2 - m_{\chi_-}^2) + (\alpha \leftrightarrow \beta) \]  
\[ I_1 = B_0(m_{\chi_-}^2, m_{\nu_a}^2) \]  
\[ I_2 = B_1(m_{\chi_-}^2, m_{\nu_a}^2) \]  
\[ S_{b,a,(c)}^{LL} = (H_{\beta,c,a}^{L}(-(\tilde{X}_{b,a,a}^L \tilde{X}_{c,b,a}^R I_3m_{\chi_b}^2) + \tilde{X}_{b,a,a}^L \tilde{X}_{c,b,a}^R I_3m_{\chi_b}m_{\chi_-} - \tilde{X}_{b,a,a}^R \tilde{X}_{c,b,a}^L I_3m_{\chi_b}m_{\chi_-})/m_{\chi_b}^2 - m_{\chi_-}^2) + (\alpha \leftrightarrow \beta) \]  
\[ S_{b,a,(c)}^{LR} = (H_{\beta,c,a}^{L}(-(\tilde{X}_{b,a,a}^L \tilde{X}_{c,b,a}^R I_3m_{\chi_b}^2) + \tilde{X}_{b,a,a}^L \tilde{X}_{c,b,a}^R I_3m_{\chi_b}m_{\chi_-} - \tilde{X}_{b,a,a}^R \tilde{X}_{c,b,a}^L I_3m_{\chi_b}m_{\chi_-})/m_{\chi_b}^2 - m_{\chi_-}^2) + (\alpha \leftrightarrow \beta) \]
\[
+ \hat{X}^L_{b,a,a} \hat{X}^L_{c,b,a} I_1 m_{\xi_b} \bar{m}_{\ell_c}) / (m_{\ell_a}^2 - m_{\ell_c}^2) + (\alpha \leftrightarrow \beta)
\]

\[ (317) \]

**Vertex corrections**

\[
I_1 = B_0 (m_{\chi_b}^2, m_{\chi_c}^2)
\]
\[ (318) \]

\[
I_2 = C_0 (m_{\chi_b}^2, m_{\chi_c}^2, m_{\nu_a}^2)
\]
\[ (319) \]

\[
S_{h_{\nu,\nu}^L (c_5)}^{LL} = W_{b,a,a}^L \bar{X}_{\beta,\gamma,\delta}^L (S_{\nu,\nu}^L I_2 m_{\chi_b} m_{\chi_c}^2 + S_{\nu,\nu}^R (I_1 + I_2 m_{\nu_a}^2))
\]
\[ (320) \]

\[
S_{h_{\nu,\nu}^L (c_6)}^{LR} = W_{b,a,a}^L \bar{X}_{\beta,\gamma,\delta}^L (S_{\nu,\nu}^L I_2 m_{\chi_b} m_{\chi_c}^2 + S_{\nu,\nu}^R (I_1 + I_2 m_{\nu_a}^2))
\]
\[ (321) \]

\[
I_1 = B_0 (m_{\chi_b}^2, m_{\chi_c}^2)
\]
\[ (322) \]

\[
I_2 = C_0 (m_{\chi_b}^2, m_{\chi_c}^2, m_{\nu_a}^2)
\]
\[ (323) \]

\[
S_{h_{\nu,\nu}^L (c_5)}^{LL} = \hat{X}_{b,a,a}^L \hat{X}_{\beta,\gamma,\delta}^L (S_{\nu,\nu}^L I_2 m_{\chi_b} m_{\chi_c}^2 + S_{\nu,\nu}^R (I_1 + I_2 m_{\nu_a}^2))
\]
\[ (324) \]

\[
S_{h_{\nu,\nu}^L (c_6)}^{LR} = \hat{X}_{b,a,a}^L \hat{X}_{\beta,\gamma,\delta}^L (S_{\nu,\nu}^L I_2 m_{\chi_b} m_{\chi_c}^2 + S_{\nu,\nu}^R (I_1 + I_2 m_{\nu_a}^2))
\]
\[ (325) \]

\[
S_{h_{\nu,\nu}^L (c_0)}^{LL} = W_{a,a,b}^L \bar{X}_{\beta,\gamma,\delta}^L \tilde{H}_{p,q,r}^{ii} C_0 (m_{\chi_a}^2, m_{\nu_a}^2, m_{\nu_b}^2) m_{\chi_a}^2
\]
\[ (326) \]

\[
S_{h_{\nu,\nu}^L (c_0)}^{LR} = W_{a,a,b}^L \bar{X}_{\beta,\gamma,\delta}^L \tilde{H}_{p,q,r}^{ii} C_0 (m_{\chi_a}^2, m_{\nu_a}^2, m_{\nu_b}^2) m_{\chi_a}^2
\]
\[ (327) \]

\[
S_{h_{\nu,\nu}^L (c_7)}^{LL} = \hat{X}_{a,a,b}^L \hat{X}_{\beta,\gamma,\delta}^L \tilde{H}_{p,q,r}^{ii} C_0 (m_{\chi_a}^2, m_{\nu_a}^2, m_{\nu_b}^2) m_{\chi_a}^2
\]
\[ (328) \]

\[
S_{h_{\nu,\nu}^L (c_7)}^{LR} = \hat{X}_{a,a,b}^L \hat{X}_{\beta,\gamma,\delta}^L \tilde{H}_{p,q,r}^{ii} C_0 (m_{\chi_a}^2, m_{\nu_a}^2, m_{\nu_b}^2) m_{\chi_a}^2
\]
\[ (329) \]

\[
S_{h_{\nu,\nu}^L (c_0)}^{LL} = W_{a,a,b}^L \bar{X}_{\beta,\gamma,\delta}^L \tilde{H}_{p,q,r}^{ii} C_0 (m_{\chi_a}^2, m_{\nu_a}^2, m_{\nu_b}^2) m_{\chi_a}^2
\]
\[ (330) \]

\[
S_{h_{\nu,\nu}^L (c_0)}^{LR} = W_{a,a,b}^L \bar{X}_{\beta,\gamma,\delta}^L \tilde{H}_{p,q,r}^{ii} C_0 (m_{\chi_a}^2, m_{\nu_a}^2, m_{\nu_b}^2) m_{\chi_a}^2
\]
\[ (331) \]

CP odd scalars

**Self-energy corrections**

\[
I_1 = B_0 (m_{\chi_b}^2, m_{\chi_c}^2)
\]
\[ (335) \]

\[
I_2 = B_1 (m_{\chi_b}^2, m_{\chi_c}^2)
\]
\[ (336) \]

\[
S_{A_{\nu,\nu}^L (c_1, c_3)}^{LL} = \Lambda_{\beta,\gamma,\delta}^L (-W_{b,a,a}^L \bar{X}_{\gamma,\delta,\alpha}^L I_2 m_{\nu_a}^2) + W_{b,a,a}^R \bar{X}_{\gamma,\delta,\alpha}^R I_1 m_{\ell_a} m_{\chi_b} - W_{b,a,a}^R \bar{X}_{\gamma,\delta,\alpha}^R I_2 m_{\ell_a} m_{\ell_c}
\]
\[ (337) \]
\[ S_{L_{\mu}^{(c_1,c_2)}}^{LR} = (A_{\mu}^{L_c,c,c} \left( - (W_{b,a,c}^{L} \bar{B}_{b,c,a} I_2 m_{\ell_a}^2) + W_{b,a,c}^{R} \bar{B}_{c,b,a} I m_{\ell_a} m_{\ell_c} - W_{b,c,b}^{L} \bar{B}_{c,b,a} I_2 m_{\ell_a} m_{\ell_c} \right) + (\alpha \leftrightarrow \beta) \\
I_1 = B_0 (m_{\ell_a}^2, m_{\ell_c}^2) \\
I_2 = B_1 (m_{\ell_a}^2, m_{\ell_c}^2) \] (338)

\[ S_{L_{\mu}^{(c_2,c_1)}}^{LL} = (A_{\mu}^{L_c,c,c} \left( - (\hat{X}_{b,a,c}^{R} \bar{B}_{c,b,a} I_2 m_{\ell_a}^2) + \hat{X}_{b,a,c}^{R} \bar{B}_{c,b,a} I m_{\ell_a} m_{\ell_c} - \hat{X}_{b,c,b}^{L} \bar{B}_{c,b,a} I_2 m_{\ell_a} m_{\ell_c} \right) + (\alpha \leftrightarrow \beta) \\
S_{L_{\mu}^{(c_2,c_1)}}^{LR} = (A_{\mu}^{L_c,c,c} \left( - (\hat{X}_{b,a,c}^{R} \bar{B}_{c,b,a} I_2 m_{\ell_a}^2) + \hat{X}_{b,a,c}^{R} \bar{B}_{c,b,a} I m_{\ell_a} m_{\ell_c} - \hat{X}_{b,c,b}^{L} \bar{B}_{c,b,a} I_2 m_{\ell_a} m_{\ell_c} \right) + (\alpha \leftrightarrow \beta) \] (340)

Vertex corrections

\[ I_1 = B_0 (m_{\ell_a}^2, m_{\ell_c}^2) \\
I_2 = B_0 (m_{\ell_a}^2, m_{\ell_c}^2) \] (339)

\[ S_{L_{\mu}^{(c_1,c_1)}}^{LL} = W_{b,a,c}^{L} \bar{B}_{c,b,a} \left( P_{c,b,a}^{c,c} I_2 m_{\ell_a} - m_{\ell_c} + P_{c,b,a}^{c,c} (I_1 + I_2 m_{\ell_a}^2) \right) \] (344)

\[ S_{L_{\mu}^{(c_1,c_1)}}^{LR} = W_{b,a,c}^{L} \bar{B}_{c,b,a} \left( P_{c,b,a}^{c,c} I_2 m_{\ell_a} - m_{\ell_c} + P_{c,b,a}^{c,c} (I_1 + I_2 m_{\ell_a}^2) \right) \] (345)

\[ S_{L_{\mu}^{(c_2,c_2)}}^{LL} = \hat{X}_{b,a,c}^{L} \bar{B}_{c,b,a} \left( P_{c,b,a}^{c,c} I_2 m_{\ell_a} - m_{\ell_c} + P_{c,b,a}^{c,c} (I_1 + I_2 m_{\ell_a}^2) \right) \] (348)

\[ S_{L_{\mu}^{(c_2,c_2)}}^{LR} = \hat{X}_{b,a,c}^{L} \bar{B}_{c,b,a} \left( P_{c,b,a}^{c,c} I_2 m_{\ell_a} - m_{\ell_c} + P_{c,b,a}^{c,c} (I_1 + I_2 m_{\ell_a}^2) \right) \] (349)

\[ S_{L_{\mu}^{(c_1,c_0)}}^{LL} = W_{a,b,c}^{L} \bar{B}_{c,b,a} \hat{A}_{c,b,a}^{c,c} C_{0} (m_{\ell_a}^2, m_{\ell_c}^2, m_{\ell_b}^2) \] (350)

\[ S_{L_{\mu}^{(c_1,c_0)}}^{LR} = W_{a,b,c}^{L} \bar{B}_{c,b,a} \hat{A}_{c,b,a}^{c,c} C_{0} (m_{\ell_a}^2, m_{\ell_c}^2, m_{\ell_b}^2) \] (351)

\[ S_{L_{\mu}^{(c_2,c_0)}}^{LL} = \hat{X}_{a,b,c}^{L} \bar{B}_{c,b,a} \hat{A}_{c,b,a}^{c,c} C_{0} (m_{\ell_a}^2, m_{\ell_c}^2, m_{\ell_b}^2) \] (352)

\[ S_{L_{\mu}^{(c_2,c_0)}}^{LR} = \hat{X}_{a,b,c}^{L} \bar{B}_{c,b,a} \hat{A}_{c,b,a}^{c,c} C_{0} (m_{\ell_a}^2, m_{\ell_c}^2, m_{\ell_b}^2) \] (353)

\[ S_{L_{\mu}^{(c_0,c_0)}}^{LL} = W_{a,b,c}^{L} \bar{B}_{c,b,a} \hat{A}_{c,b,a}^{c,c} C_{0} (m_{\ell_a}^2, m_{\ell_c}^2, m_{\ell_b}^2) \] (354)

\[ S_{L_{\mu}^{(c_0,c_0)}}^{LR} = W_{a,b,c}^{L} \bar{B}_{c,b,a} \hat{A}_{c,b,a}^{c,c} C_{0} (m_{\ell_a}^2, m_{\ell_c}^2, m_{\ell_b}^2) \] (355)

\[ S_{L_{\mu}^{(c_0,c_0)}}^{LL} = \hat{X}_{a,b,c}^{L} \bar{B}_{c,b,a} \hat{A}_{c,b,a}^{c,c} C_{0} (m_{\ell_a}^2, m_{\ell_c}^2, m_{\ell_b}^2) \] (356)

\[ S_{L_{\mu}^{(c_0,c_0)}}^{LR} = \hat{X}_{a,b,c}^{L} \bar{B}_{c,b,a} \hat{A}_{c,b,a}^{c,c} C_{0} (m_{\ell_a}^2, m_{\ell_c}^2, m_{\ell_b}^2) \] (357)
E.4 W$^+$ and H$^+$ contributions

E.4.1 Z-penguins

Self-energy corrections

\[ I_1 = B_0(m_{\nu_a}^2, m_{H_u}^2) \quad (358) \]
\[ I_2 = B_1(m_{\nu_a}^2, m_{H_u}^2) \quad (359) \]

\[
V_{Z,(w_1,w_3)}^{LL} = (E_{\beta,c}^{L}(V_{a,a,b}^{+,L}V_{c,a,b}^{+,R}I_2m_{\ell_a}^2 - V_{a,a,b}^{+,R}V_{c,a,b}^{+,L}I_1m_{\ell_a}m_{\ell_c}) + V_{a,a,b}^{+,R}V_{c,a,b}^{+,L}I_2m_{\ell_a}m_{\ell_c})/(m_{\ell_a}^2 - m_{\ell_c}^2) + (\alpha \leftrightarrow \beta) \quad (360) 
\]

\[
V_{Z,(w_1,w_3)}^{LR} = (E_{\beta,c}^{L}(V_{a,a,b}^{+,L}V_{c,a,b}^{+,R}I_2m_{\ell_a}^2 - V_{a,a,b}^{+,R}V_{c,a,b}^{+,L}I_1m_{\ell_a}m_{\ell_c}) + V_{a,a,b}^{+,R}V_{c,a,b}^{+,L}I_2m_{\ell_a}m_{\ell_c})/(m_{\ell_a}^2 - m_{\ell_c}^2) + (\alpha \leftrightarrow \beta) \quad (361) 
\]

Vertex corrections

\[
V_{Z,(w_5)}^{LL} = -2V_{\alpha,a,b}^{+,L}V_{\beta,a,c}^{+,R}Z_{\beta,c}^{hh} C_0(m_{\nu_a}^2, m_{H_u}^2, m_{W_-}^2) \quad (365) 
\]
\[
V_{Z,(w_5)}^{LR} = -2V_{\alpha,a,b}^{+,L}V_{\beta,a,c}^{+,R}Z_{\beta,c}^{hh} C_0(m_{\nu_a}^2, m_{H_u}^2, m_{W_-}^2) \quad (366) 
\]

\[
V_{Z,(w_6)}^{LL} = \hat{V}_{a,i}^{+,L}\hat{V}_{\beta,a,c}^{+,R}Z_{\beta,c}^{hh} C_0(m_{\nu_a}^2, m_{H_u}^2, m_{W_-}^2) \quad (367) 
\]
\[
V_{Z,(w_6)}^{LR} = \hat{V}_{a,i}^{+,L}\hat{V}_{\beta,a,c}^{+,R}Z_{\beta,c}^{hh} C_0(m_{\nu_a}^2, m_{H_u}^2, m_{W_-}^2) \quad (368) 
\]

\[
V_{Z,(w_7)}^{LL} = V_{a,a,b}^{+,L}\hat{V}_{\beta,a}^{+,R}Z_{\beta}^{hh} C_0(m_{\nu_a}^2, m_{W_-}^2, m_{W_-}^2) \quad (369) 
\]
\[
V_{Z,(w_7)}^{LR} = V_{a,a,b}^{+,L}\hat{V}_{\beta,a}^{+,R}Z_{\beta}^{hh} C_0(m_{\nu_a}^2, m_{W_-}^2, m_{W_-}^2) \quad (370) 
\]

\[
I_1 = B_0(m_{W_-}^2, m_{W_-}^2) \quad (371) 
\]
\[
I_2 = C_0(m_{\nu_a}^2, m_{W_-}^2, m_{W_-}^2) \quad (372) 
\]
\[
I_3 = C_0(m_{\nu_a}^2, m_{W_-}^2, m_{W_-}^2) \quad (373) 
\]

\[
V_{Z,(w_9)}^{LL} = -\hat{V}_{a,i}^{+,L}\hat{V}_{\beta,a}^{+,R}Z_{\beta}^{hh} (-1 + 2I_1 + 2I_2 + I_3m_{\nu_a}^2) \quad (374) 
\]
\[
V_{Z,(w_9)}^{LR} = -\hat{V}_{a,i}^{+,L}\hat{V}_{\beta,a}^{+,R}Z_{\beta}^{hh} (-1 + 2I_1 + 2I_2 + I_3m_{\nu_a}^2) \quad (375) 
\]
\begin{align*}
I_1 &= B_0(m_{v_a}^2, m_{v_e}^2) \\
I_2 &= C_0(m_{v_e}^2, m_{H_e}^2) \\
I_3 &= C_0(m_{v_e}^2, m_{H_e}^2) \\
V_{Z,(w_0)}^{LL} &= V_{b,a,a}^{+}V_{c,a,b}^{+}(-((V_{c,b}^{L}I_3m_{v_a}m_{v_e}) + V_{c,b}^{-}(I_1 - 2I_2 + I_3m_{H_e}^2))) \\
V_{Z,(w_0)}^{LR} &= V_{b,a,a}^{+}V_{c,a,b}^{+}(-((V_{c,b}^{L}I_3m_{v_a}m_{v_e}) + V_{c,b}^{-}(I_1 - 2I_2 + I_3m_{H_e}^2)))
\end{align*}

E.4.2 Scalar penguins

CP even scalars

Self-energy corrections
\begin{align*}
I_1 &= B_0(m_{v_a}^2, m_{v_e}^2) \\
I_2 &= B_1(m_{v_a}^2, m_{v_e}^2) \\
S_{h_p,(w_1,w_3)}^{LL} &= (H_{\beta,c,p}^{L} - (V_{a,a,b}^{L}V_{a,b}^{+}I_2m_{\ell_a}^2) + V_{a,a,b}^{+}V_{a,b}^{+}I_1m_{\ell_a}m_{v_a} - V_{a,a,b}^{+}V_{a,b}^{+}I_2m_{\ell_a}m_{v_e} \\
&+ V_{a,a,b}^{+}V_{a,b}^{+}I_1m_{\ell_a}m_{v_e} + (\alpha \leftrightarrow \beta) \\
S_{h_p,(w_1,w_3)}^{LR} &= (H_{\beta,c,p}^{L} - (V_{a,a,b}^{L}V_{a,b}^{+}I_2m_{\ell_a}^2) + V_{a,a,b}^{+}V_{a,b}^{+}I_1m_{\ell_a}m_{v_a} - V_{a,a,b}^{+}V_{a,b}^{+}I_2m_{\ell_a}m_{v_e} \\
&+ V_{a,a,b}^{+}V_{a,b}^{+}I_1m_{\ell_a}m_{v_e} + (\alpha \leftrightarrow \beta) \\
I_1 &= B_0(m_{v_a}^2, m_{v_e}^2) \\
I_2 &= B_1(m_{v_a}^2, m_{v_e}^2) \\
S_{h_p,(w_2,w_4)}^{LL} &= - (H_{\beta,c,p}^{L} - (V_{a,a,b}^{L}V_{a,b}^{+}(-2\hat{V}_{c,a}^{+}L(1 - 2I_1)m_{v_a} + \hat{V}_{c,a}^{+}R(1 + 2I_2)m_{\ell_a}) + \hat{V}_{c,a}^{+}L(1 + 2I_2)m_{\ell_a} \\
&- 2\hat{V}_{c,a}^{+}R(1 - 2I_1)m_{\ell_a}m_{v_a})))/((m_{\ell_a}^2 - m_{v_a}^2)) + (\alpha \leftrightarrow \beta) \\
S_{h_p,(w_2,w_4)}^{LR} &= - (H_{\beta,c,p}^{L} - (V_{a,a,b}^{L}V_{a,b}^{+}(-2\hat{V}_{c,a}^{+}L(1 - 2I_1)m_{v_a} + \hat{V}_{c,a}^{+}R(1 + 2I_2)m_{\ell_a}) + \hat{V}_{c,a}^{+}L(1 + 2I_2)m_{\ell_a} \\
&- 2\hat{V}_{c,a}^{+}R(1 - 2I_1)m_{\ell_a}m_{v_a})))/((m_{\ell_a}^2 - m_{v_a}^2)) + (\alpha \leftrightarrow \beta)
\end{align*}

Vertex corrections
\begin{align*}
S_{h_p,(w_5)}^{LL} &= V_{a,a,b}^{+}V_{a,b}^{+}H_{\beta,c,p}^{L}C_0(m_{v_a}^2, m_{v_e}^2, m_{H_e}^2) \\
S_{h_p,(w_5)}^{LR} &= V_{a,a,b}^{+}V_{a,b}^{+}H_{\beta,c,p}^{L}C_0(m_{v_a}^2, m_{v_e}^2, m_{H_e}^2)
\end{align*}
\[ I_1 = B_0(m_{\nu_a}^2, m_{W^-}^2) \]
\[ I_2 = C_0(m_{\nu_a}^2, m_{H^-}^2, m_{W^-}^2) \]
\[ S_{h_p,(w_0)}^{LL} = -V_{a,i}^L \bar{V}_{+L}^{\beta,a,c} H_{p,c}^{h\nu} (I_1 + I_2 m_{\nu_a}^2) \]
\[ S_{h_p,(w_0)}^{LR} = -V_{a,i}^L \bar{V}_{+L}^{\beta,a,c} H_{p,c}^{h\nu} (I_1 + I_2 m_{\nu_a}^2) \]
\[ I_1 = B_0(m_{\nu_a}^2, m_{W^-}^2) \]
\[ I_2 = C_0(m_{\nu_a}^2, m_{W^-}^2, m_{H^-}^2) \]
\[ S_{h_p,(w_0)}^{LL} = V_{a,a,b}^L \bar{V}_{+L}^{\beta,a,c} H_{p,b}^{h\nu} (I_1 + I_2 m_{\nu_a}^2) \]
\[ S_{h_p,(w_0)}^{LR} = V_{a,a,b}^L \bar{V}_{+L}^{\beta,a,c} H_{p,b}^{h\nu} (I_1 + I_2 m_{\nu_a}^2) \]
\[ S_{h_p,(w_0)}^{LL} = 4 V_{a,i}^L \bar{V}_{+L}^{\beta,a} H_{p}^{h\nu} C_0(m_{\nu_a}^2, m_{W^-}^2, m_{W^-}^2) m_{\nu_a} \]
\[ S_{h_p,(w_0)}^{LR} = 4 V_{a,i}^L \bar{V}_{+L}^{\beta,a} H_{p}^{h\nu} C_0(m_{\nu_a}^2, m_{W^-}^2, m_{W^-}^2) m_{\nu_a} \]
\[ I_1 = B_0(m_{\nu_a}^2, m_{\nu_c}^2) \]
\[ I_2 = C_0(m_{\nu_a}^2, m_{\nu_c}^2, m_{H^-}^2) \]
\[ S_{h_p,(w_0)}^{LL} = V_{b,a,a}^L \bar{V}_{+L}^{\beta,c,a} (H_{c,b,p}^{h\nu} I_2 m_{\nu_b} m_{\nu_c} + H_{c,b,p}^{h\nu} (I_1 + I_2 m_{\nu_a}^2))) \]
\[ S_{h_p,(w_0)}^{LR} = V_{b,a,a}^L \bar{V}_{+L}^{\beta,c,a} (H_{c,b,p}^{h\nu} I_2 m_{\nu_b} m_{\nu_c} + H_{c,b,p}^{h\nu} (I_1 + I_2 m_{\nu_a}^2))) \]
\[ I_1 = B_0(m_{\nu_a}^2, m_{\nu_c}^2) \]
\[ I_2 = C_0(m_{\nu_a}^2, m_{\nu_c}^2, m_{W^-}^2) \]
\[ S_{h_p,(w_0)}^{LL} = 2 \bar{V}_{b,i}^L \bar{V}_{+L}^{\beta} (-2 H_{c,b,p}^{h\nu} I_2 m_{\nu_b} m_{\nu_c} + H_{c,b,p}^{h\nu} (1 - 2(I_1 + I_2 m_{\nu_a}^2)))) \]
\[ S_{h_p,(w_0)}^{LR} = 2 \bar{V}_{b,i}^L \bar{V}_{+L}^{\beta} (-2 H_{c,b,p}^{h\nu} I_2 m_{\nu_b} m_{\nu_c} + H_{c,b,p}^{h\nu} (1 - 2(I_1 + I_2 m_{\nu_a}^2)))) \]

### CP odd scalars

**Self-energy corrections**

\[ I_1 = B_0(m_{\nu_a}^2, m_{H^-}^2) \]
\[ I_2 = B_1(m_{\nu_a}^2, m_{H^-}^2) \]
\[ S_{\beta,(w_0)}^{LL} = (A_{\alpha,c}^{\nu} - (V_{a,a,b}^{+,L} \bar{V}_{+L}^{\nu,a,b} I_2 m_{\nu_a}^2) + V_{a,a,b}^{+,L} \bar{V}_{+L}^{\nu,a,b} I_1 m_{\nu_a} m_{\nu_a} - V_{a,a,b}^{+,L} \bar{V}_{+L}^{\nu,a,b} I_2 m_{\nu_a} m_{\nu_c}) \]
\[ + V_{a,a,b}^{+,L} \bar{V}_{+L}^{\nu,a,b} I_1 m_{\nu_a} m_{\nu_c})/m_{\nu_a}^2 + m_{\nu_c}^2 + (\alpha \leftrightarrow \beta) \]
\[ S_{\beta,(w_0)}^{LR} = (A_{\alpha,c}^{\nu} - (V_{a,a,b}^{+,L} \bar{V}_{+L}^{\nu,a,b} I_2 m_{\nu_a}^2) + V_{a,a,b}^{+,L} \bar{V}_{+L}^{\nu,a,b} I_1 m_{\nu_a} m_{\nu_a} - V_{a,a,b}^{+,L} \bar{V}_{+L}^{\nu,a,b} I_2 m_{\nu_a} m_{\nu_c}) \]
\[ + V_{a,a,b}^{+,L} \bar{V}_{+L}^{\nu,a,b} I_1 m_{\nu_a} m_{\nu_c})/m_{\nu_a}^2 + m_{\nu_c}^2 + (\alpha \leftrightarrow \beta) \]
\[ I_1 = B_0(m_{\nu_a}^2, m_{W^-}^2) \]
\[ I_2 = B_1(m_{\nu_a}^2, m_{W_-}^2) \] (418)

\[ S_{A_{\nu}^{\nu L}}^{LL}(w_2, w_4) = - (A_{\beta, c, p}^L (\hat{W}_{c,a}^{+ R} m_{\ell_{\alpha}} (\hat{W}_{c,a}^{+ L} (1 - 2 I_1) m_{\nu_a} + \hat{W}_{c,a}^{+ R} (1 + 2 I_2) m_{\ell_{\alpha}}) + \hat{W}_{c,a}^{+ L} (\hat{W}_{c,a}^{+ L} (1 + 2 I_2) m_{\nu_a}^2 - 2 \hat{W}_{c,a}^{+ R} (1 - 2 I_1) m_{\nu_a} m_{\ell_{\alpha}})))/(m_{\ell_{\alpha}}^2 - m_{\nu_a}^2) + (\alpha \leftrightarrow \beta) \] (419)

\[ S_{A_{\nu}^{\nu R}}^{LR}(w_2, w_4) = - (A_{\beta, c, p}^L (\hat{W}_{c,a}^{+ R} m_{\ell_{\alpha}} (\hat{W}_{c,a}^{+ L} (1 - 2 I_1) m_{\nu_a} + \hat{W}_{c,a}^{+ R} (1 + 2 I_2) m_{\ell_{\alpha}}) + \hat{W}_{c,a}^{+ L} (\hat{W}_{c,a}^{+ L} (1 + 2 I_2) m_{\nu_a}^2 - 2 \hat{W}_{c,a}^{+ R} (1 - 2 I_1) m_{\nu_a} m_{\ell_{\alpha}})))/(m_{\ell_{\alpha}}^2 - m_{\nu_a}^2) + (\alpha \leftrightarrow \beta) \] (420)

**Vertex corrections**

\[ S_{A_{\nu}^{\nu L}}^{LL}(w_2) = V_{a, a, b}^{R} \hat{V}_{a, a, c}^{\nu L} \bar{A}_{p, b}^{L} C_0(m_{\nu_a}^2, m_{H_c}^2, m_{H_b}^2) m_{\nu_a} \] (421)

\[ S_{A_{\nu}^{\nu R}}^{LR}(w_2) = V_{a, a, b}^{R} \hat{V}_{a, a, c}^{\nu L} \bar{A}_{p, b}^{L} C_0(m_{\nu_a}^2, m_{H_c}^2, m_{H_b}^2) m_{\nu_a} \] (422)

\[ I_1 = B_0(m_{H_c}^2, m_{W_-}^2) \] (423)

\[ I_2 = C_0(m_{\nu_a}^2, m_{H_c}^2, m_{H_b}^2) \] (424)

\[ S_{A_{\nu}^{\nu L}}^{LL}(w_2) = - \hat{V}_{a, a, b}^{R} \hat{V}_{a, a, c}^{\nu L} \bar{A}_{p, b}^{L} (I_1 + I_2 m_{\nu_a}^2) \] (425)

\[ S_{A_{\nu}^{\nu R}}^{LR}(w_2) = - \hat{V}_{a, a, b}^{R} \hat{V}_{a, a, c}^{\nu L} \bar{A}_{p, b}^{L} (I_1 + I_2 m_{\nu_a}^2) \] (426)

\[ I_1 = B_0(m_{H_b}^2, m_{W_-}^2) \] (427)

\[ I_2 = C_0(m_{\nu_a}^2, m_{H_b}^2, m_{H_b}^2) \] (428)

\[ S_{A_{\nu}^{\nu L}}^{LL}(w_2) = V_{a, a, b}^{R} \hat{V}_{a, a, c}^{\nu L} \bar{A}_{p, b}^{L} (I_1 + I_2 m_{\nu_a}^2) \] (429)

\[ S_{A_{\nu}^{\nu R}}^{LR}(w_2) = V_{a, a, b}^{R} \hat{V}_{a, a, c}^{\nu L} \bar{A}_{p, b}^{L} (I_1 + I_2 m_{\nu_a}^2) \] (430)

\[ I_1 = B_0(m_{\nu_c}^2, m_{\nu_c}^2) \] (431)

\[ I_2 = C_0(m_{\nu_c}^2, m_{\nu_c}^2, m_{H_a}^2) \] (432)

\[ S_{A_{\nu}^{\nu L}}^{LL}(w_2) = V_{a, a, b}^{R} \hat{V}_{a, a, c}^{\nu L} (A_{\nu, c, b}^{L} I_2 m_{\nu_a} m_{\nu_c} + A_{\nu, c, b}^{R} (I_1 + I_2 m_{\nu_a}^2)) \] (433)

\[ S_{A_{\nu}^{\nu R}}^{LR}(w_2) = V_{a, a, b}^{R} \hat{V}_{a, a, c}^{\nu L} (A_{\nu, c, b}^{L} I_2 m_{\nu_a} m_{\nu_c} + A_{\nu, c, b}^{R} (I_1 + I_2 m_{\nu_a}^2)) \] (434)

\[ I_1 = B_0(m_{\nu_c}^2, m_{\nu_c}^2) \] (435)

\[ I_2 = C_0(m_{\nu_c}^2, m_{\nu_c}^2, m_{W_-}^2) \] (436)

\[ S_{A_{\nu}^{\nu L}}^{LL}(w_{10}) = 2 \hat{V}_{b, a, c}^{R} \hat{V}_{b, c, a}^{\nu L} (- 2 A_{\nu, c, b}^{R} I_2 m_{\nu_a} m_{\nu_c} + A_{\nu, c, b}^{L} (1 - 2 (I_1 + I_2 m_{\nu_a}^2))) \] (437)

\[ S_{A_{\nu}^{\nu R}}^{LR}(w_{10}) = 2 \hat{V}_{b, a, c}^{R} \hat{V}_{b, c, a}^{\nu L} (- 2 A_{\nu, c, b}^{R} I_2 m_{\nu_a} m_{\nu_c} + A_{\nu, c, b}^{L} (1 - 2 (I_1 + I_2 m_{\nu_a}^2))) \] (438)

\[ S_{A_{\nu}^{\nu L}}^{LL}(w_{10}) = 2 \hat{V}_{b, a, c}^{R} \hat{V}_{b, c, a}^{\nu L} (- 2 A_{\nu, c, b}^{R} I_2 m_{\nu_a} m_{\nu_c} + A_{\nu, c, b}^{L} (1 - 2 (I_1 + I_2 m_{\nu_a}^2))) \] (439)
F  Box contributions to LFV

F.1  Four lepton boxes

F.1.1  Feynman diagrams

Neutralino diagrams

Chargino diagrams
\[ \ell_\alpha \rightarrow \bar{\ell}_\beta \]
\[ \nu^R_d \rightarrow \tilde{\chi}^-_a \]
\[ \tilde{\nu}^l_b \rightarrow \tilde{\chi}^+_{c_0} \]
\[ \ell_\gamma \]

(c_7)

\[ \tilde{\nu}^l_c \rightarrow \tilde{\chi}^+_{c_0} \]
\[ \ell_\gamma \]

(c_8)

\( W^+ \) and \( H^+ \) diagrams

\[ \ell_\alpha \rightarrow \bar{\ell}_\beta \]
\[ \nu_a \rightarrow \tilde{\ell}_\delta \]
\[ H^+_d \rightarrow H^+_b \]
\[ \ell_\gamma \]

(\( w_1^l \))

\[ \ell_\alpha \rightarrow \bar{\ell}_\beta \]
\[ \nu_a \rightarrow \tilde{\ell}_\delta \]
\[ W^+ \rightarrow H^+_b \]
\[ \ell_\gamma \]

(\( w_2^l \))

\[ \ell_\alpha \rightarrow \bar{\ell}_\beta \]
\[ \nu_a \rightarrow \tilde{\ell}_\delta \]
\[ W^+ \rightarrow W^+ \]
\[ \ell_\gamma \]

(\( w_3^l \))

\[ \ell_\alpha \rightarrow \bar{\ell}_\beta \]
\[ \nu_a \rightarrow \tilde{\ell}_\delta \]
\[ H^+_d \rightarrow H^-_b \]
\[ \ell_\gamma \]

(\( w_5^l \))

\[ \ell_\alpha \rightarrow \bar{\ell}_\beta \]
\[ \nu_a \rightarrow \tilde{\ell}_\delta \]
\[ H^+_d \rightarrow W^- \]
\[ \ell_\gamma \]

(\( w_6^l \))

\[ \ell_\alpha \rightarrow \bar{\ell}_\beta \]
\[ \nu_a \rightarrow \tilde{\ell}_\delta \]
\[ W^+ \rightarrow W^- \]
\[ \ell_\gamma \]

(\( w_7^l \))
F.1.2 Neutralino contributions

\[ S_{(n1)}^{LL} = \frac{1}{2} N_{a,a,d}^N \tilde{N}_{a,a,d}^L \tilde{N}_{a,a,d}^L \delta_{c,d} m_{\tilde{\chi}_0^0} m_{\tilde{\chi}_e^0} D_0 (m_{\tilde{\chi}_a^0}^2, m_{\tilde{\chi}_c}^2, m_{\tilde{\chi}_d}^2, m_{\tilde{\chi}_e}^2) \]  
\[ S_{(n1)}^{LR} = -2 N_{a,a,d}^N \tilde{N}_{a,a,d}^R \tilde{N}_{a,a,d}^L \delta_{c,d} D_0 (m_{\tilde{\chi}_a^0}^2, m_{\tilde{\chi}_c}^2, m_{\tilde{\chi}_d}^2, m_{\tilde{\chi}_e}^2) \]  
\[ V_{(n1)}^{LL} = -N_{a,a,d}^L \tilde{N}_{a,a,d}^R \tilde{N}_{a,a,d}^L \delta_{c,d} D_27 (m_{\tilde{\chi}_0^0}^2, m_{\tilde{\chi}_e}^2, m_{\tilde{\chi}_d}^2, m_{\tilde{\chi}_e}^2) \]  
\[ V_{(n1)}^{LR} = -N_{a,a,d}^L \tilde{N}_{a,a,d}^R \tilde{N}_{a,a,d}^R \delta_{c,d} D_27 (m_{\tilde{\chi}_0^0}^2, m_{\tilde{\chi}_e}^2, m_{\tilde{\chi}_d}^2, m_{\tilde{\chi}_e}^2) \]  
\[ T_{(n1)}^{LL} = \frac{1}{8} N_{a,a,d}^N \tilde{N}_{a,a,d}^L \tilde{N}_{a,a,d}^L \delta_{c,d} m_{\tilde{\chi}_0^0} m_{\tilde{\chi}_c}^2 D_0 (m_{\tilde{\chi}_a^0}^2, m_{\tilde{\chi}_c}^2, m_{\tilde{\chi}_d}^2, m_{\tilde{\chi}_e}^2) \]  

F.1.3 Chargino contributions

\[ S_{(c1)}^{LL} = -W_{a,a,d}^L \tilde{X}_{a,a,b}^L \tilde{W}_{a,a,b}^L \delta_{c,d} m_{\tilde{\chi}_a^0} m_{\tilde{\chi}_c} D_0 (m_{\tilde{\chi}_a^0}^2, m_{\tilde{\chi}_c}^2, m_{\tilde{\chi}_d}^2, m_{\tilde{\chi}_e}^2) \]  
\[ S_{(c1)}^{LR} = -W_{a,a,d}^L \tilde{X}_{a,a,b}^R \tilde{W}_{a,a,b}^L \delta_{c,d} D_0 (m_{\tilde{\chi}_a^0}^2, m_{\tilde{\chi}_c}^2, m_{\tilde{\chi}_d}^2, m_{\tilde{\chi}_e}^2) \]  
\[ V_{(c1)}^{LL} = -W_{a,a,d}^L \tilde{R}_{a,a,b}^R \tilde{X}_{a,a,b}^L \delta_{c,d} D_27 (m_{\tilde{\chi}_0^0}^2, m_{\tilde{\chi}_e}^2, m_{\tilde{\chi}_d}^2, m_{\tilde{\chi}_e}^2) \]  
\[ V_{(c1)}^{LR} = -W_{a,a,d}^L \tilde{R}_{a,a,b}^R \tilde{X}_{a,a,b}^R \delta_{c,d} D_27 (m_{\tilde{\chi}_0^0}^2, m_{\tilde{\chi}_e}^2, m_{\tilde{\chi}_d}^2, m_{\tilde{\chi}_e}^2) \]  
\[ T_{(c1)}^{LL} = \frac{1}{8} N_{a,a,d}^N \tilde{N}_{a,a,d}^L \tilde{N}_{a,a,d}^L \delta_{c,d} m_{\tilde{\chi}_0^0} m_{\tilde{\chi}_c}^2 D_0 (m_{\tilde{\chi}_a^0}^2, m_{\tilde{\chi}_c}^2, m_{\tilde{\chi}_d}^2, m_{\tilde{\chi}_e}^2) \]
\[ V_{(c_1)}^{LL} = - \hat{X}_{a,a,d}^{L} \hat{X}_{\beta,a,b}^{R} \hat{X}_{c,\gamma,b}^{L} \hat{X}_{d,c,d}^{L} D_{27}(m_{\chi_a}^2, m_{\chi_c}^2, m_{\nu_d}^2, m_{\nu_b}^2) \]  
\[ V_{(c_1)}^{LR} = - \hat{X}_{a,a,d}^{L} \hat{X}_{\beta,a,b}^{R} \hat{X}_{c,\gamma,b}^{L} \hat{X}_{d,c,d}^{R} D_{27}(m_{\chi_a}^2, m_{\chi_c}^2, m_{\nu_d}^2, m_{\nu_b}^2) \]  
\[ S_{(c_1)}^{LL} = - W_{a,a,d}^{L} \hat{X}_{\beta,a,b}^{R} \hat{X}_{c,\gamma,b}^{L} W_{d,c,d}^{L} D_{27}(m_{\chi_a}^2, m_{\chi_c}^2, m_{\nu_d}^2, m_{\nu_b}^2) \]  
\[ S_{(c_1)}^{LR} = - W_{a,a,d}^{L} \hat{X}_{\beta,a,b}^{R} \hat{X}_{c,\gamma,b}^{L} W_{d,c,d}^{R} D_{27}(m_{\chi_a}^2, m_{\chi_c}^2, m_{\nu_d}^2, m_{\nu_b}^2) \]

F.1.4 \( W^+ \) and \( H^+ \) contributions

\[ S_{(w_1)}^{LL} = - V_{a,a,d}^{+L} V_{\beta,a,b}^{+L} V_{c,\gamma,b}^{+L} V_{d,c,d}^{+L} m_{\nu_a} m_{\nu_c} D_0(m_{\nu_a}^2, m_{\nu_c}^2, m_{H_d}^2, m_{H_b}^2) \]  
\[ S_{(w_1)}^{LR} = - V_{a,a,d}^{+L} V_{\beta,a,b}^{+L} V_{c,\gamma,b}^{+L} V_{d,c,d}^{+R} m_{\nu_a} m_{\nu_c} D_0(m_{\nu_a}^2, m_{\nu_c}^2, m_{H_d}^2, m_{H_b}^2) \]

\[ V_{(w_1)}^{LL} = - V_{a,a,d}^{+L} V_{\beta,a,b}^{+L} V_{c,\gamma,b}^{+L} V_{d,c,d}^{+R} D_{27}(m_{\nu_a}^2, m_{\nu_c}^2, m_{H_d}^2, m_{H_b}^2) \]  
\[ V_{(w_1)}^{LR} = - V_{a,a,d}^{+L} V_{\beta,a,b}^{+L} V_{c,\gamma,b}^{+L} V_{d,c,d}^{+R} D_{27}(m_{\nu_a}^2, m_{\nu_c}^2, m_{H_d}^2, m_{H_b}^2) \]

\[ S_{(w_2)}^{LL} = 2 V_{a,a,d}^{+L} \hat{V}_{\beta,a}^{+, L} \hat{V}_{c,\gamma,b}^{+, L} (J_{c,\alpha} D_0(m_{\nu_c}^2, m_{W}^2, m_{H_d}^2, m_{H_b}^2) - 2D_{27}(m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{H_d}^2)) \]  
\[ \text{(485)} \]
\[ S_{LR}^{(w_2)} = 2V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+R} \hat{V}_{\delta,c,d}^{+L} (I_{C_0}D_0) (m_{\nu_a}^2, m_{\nu_c}^2, m_{H_d}^2, m_{H_d}^2) - 2D_27 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (486)

\[ V_{LL}^{(w_3)} = V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} m_{\nu_a} m_{\nu_c} D_0 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (487)

\[ V_{LR}^{(w_2)} = V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+R} \hat{V}_{\delta,c,d}^{+L} m_{\nu_a} m_{\nu_c} D_0 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (488)

\[ S_{LL}^{(w_3)} = 2V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} (I_{C_0}D_0) (m_{\nu_a}^2, m_{H_b}^2, m_{W}^2, m_{W}^2) - 2D_27 (m_{\nu_a}^2, m_{\nu_c}^2, m_{H_b}^2, m_{H_b}^2) \] (489)

\[ S_{LR}^{(w_3)} = 2V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} (I_{C_0}D_0) (m_{\nu_a}^2, m_{H_b}^2, m_{W}^2, m_{W}^2) - 2D_27 (m_{\nu_a}^2, m_{\nu_c}^2, m_{H_b}^2, m_{H_b}^2) \] (490)

\[ V_{LL}^{(w_3)} = V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} m_{\nu_a} m_{\nu_c} D_0 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (491)

\[ V_{LR}^{(w_2)} = V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} m_{\nu_a} m_{\nu_c} D_0 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (492)

\[ T_{LL}^{(w_3)} = -V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} D_27 (m_{\nu_a}^2, m_{\nu_c}^2, m_{H_b}^2, m_{W}^2) \] (493)

\[ S_{LL}^{(w_3)} = -4V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} m_{\nu_a} m_{\nu_c} D_0 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (494)

\[ S_{LR}^{(w_3)} = -4V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} m_{\nu_a} m_{\nu_c} D_0 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (495)

\[ V_{LL}^{(w_3)} = -4V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} (I_{C_0}D_0) (m_{\nu_a}^2, m_{W}^2, m_{W}^2, m_{W}^2) - 3D_27 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (496)

\[ V_{LR}^{(w_2)} = -4V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} I_{C_0}D_0 (m_{\nu_a}^2, m_{W}^2, m_{W}^2, m_{W}^2) \] (497)

\[ T_{LL}^{(w_3)} = V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} m_{\nu_a} m_{\nu_c} D_0 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (498)

\[ S_{LL}^{(w_2)} = \frac{1}{2} V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} m_{\nu_a} m_{\nu_c} D_0 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (499)

\[ S_{LR}^{(w_2)} = -2V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} m_{\nu_a} m_{\nu_c} D_0 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (500)

\[ V_{LL}^{(w_3)} = -\frac{1}{2} V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} D_27 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (501)

\[ V_{LR}^{(w_2)} = -V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} D_27 (m_{\nu_a}^2, m_{\nu_c}^2, m_{H_b}^2, m_{H_b}^2) \] (502)

\[ T_{LL}^{(w_2)} = V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} m_{\nu_a} m_{\nu_c} D_0 (m_{\nu_a}^2, m_{\nu_c}^2, m_{H_b}^2, m_{H_b}^2) \] (503)

\[ S_{LL}^{(w_3)} = -V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} (I_{C_0}D_0) (m_{\nu_a}^2, m_{W}^2, m_{W}^2, m_{W}^2) - 8D_27 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (504)

\[ S_{LR}^{(w_3)} = 2V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} m_{\nu_a} m_{\nu_c} D_0 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (505)

\[ V_{LL}^{(w_3)} = V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} (I_{C_0}D_0) (m_{\nu_a}^2, m_{W}^2, m_{W}^2, m_{W}^2) - 2D_27 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (506)

\[ V_{LR}^{(w_3)} = V_{a,a,d}^{+L} \hat{V}_{c,\gamma}^{+L} \hat{V}_{\delta,c,d}^{+L} m_{\nu_a} m_{\nu_c} D_0 (m_{\nu_a}^2, m_{\nu_c}^2, m_{W}^2, m_{W}^2) \] (507)
F.2.1 Crossed neutralino contributions

In the case of $F.2$ Additional boxes for $\ell_s$ it is necessary to calculate the crossed diagrams with exchanged indices $\beta \leftrightarrow \gamma$ explicitly.

F.2.2 Crossed neutralino contributions

$$T_{LL}^{(w)} = - \frac{1}{4} V_{\alpha a,\alpha} \tilde{V}_{\beta c,\beta} I_{C_0 D_0} \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right)$$  \hspace{1cm} (508)

$$S_{LL}^{(w)} = - V_{\alpha a,\alpha} \tilde{V}_{\beta c,\beta} I_{C_0 D_0} \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right) - 8D_2 \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right)$$  \hspace{1cm} (509)

$$S_{LR}^{(w)} = 2 V_{\alpha a,\alpha} \tilde{V}_{\beta c,\beta} I_{C_0 D_0} \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right)$$  \hspace{1cm} (510)

$$V_{LL}^{(w)} = V_{\alpha a,\alpha} \tilde{V}_{\beta c,\beta} I_{C_0 D_0} \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right) - 2D_2 \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right)$$  \hspace{1cm} (511)

$$V_{LR}^{(w)} = V_{\alpha a,\alpha} \tilde{V}_{\beta c,\beta} I_{C_0 D_0} \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right)$$  \hspace{1cm} (512)

$$T_{LL}^{(w)} = - \frac{1}{4} V_{\alpha a,\alpha} \tilde{V}_{\beta c,\beta} I_{C_0 D_0} \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right)$$  \hspace{1cm} (513)

$$S_{LL}^{(w)} = - 4 V_{\alpha a,\alpha} \tilde{V}_{\beta c,\beta} I_{C_0 D_0} \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right)$$  \hspace{1cm} (514)

$$S_{LR}^{(w)} = - 8 V_{\alpha a,\alpha} \tilde{V}_{\beta c,\beta} I_{C_0 D_0} \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right) - 3D_2 \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right)$$  \hspace{1cm} (515)

$$V_{LL}^{(w)} = - 2 V_{\alpha a,\alpha} \tilde{V}_{\beta c,\beta} I_{C_0 D_0} \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right)$$  \hspace{1cm} (516)

$$V_{LR}^{(w)} = - 4 V_{\alpha a,\alpha} \tilde{V}_{\beta c,\beta} I_{C_0 D_0} \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right)$$  \hspace{1cm} (517)

$$T_{LL}^{(w)} = V_{\alpha a,\alpha} \tilde{V}_{\beta c,\beta} I_{C_0 D_0} \left( m_{\psi_a}^2, m_{\psi_c}^2, m_{H_d}^2, m_{\nu_a}^2 \right)$$  \hspace{1cm} (518)
\[ T_{LL}^{(c_1')} = \frac{1}{8} N_{d,\alpha,a}^L N_{\beta,\beta,b}^L N_{d,\beta,c}^L \frac{m_{\tilde{\chi}_d^0}}{m_{\tilde{\chi}_b^0}} D_0(m_{\tilde{\nu}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(528)

\[ T_{LL}^{(c_2')} = \frac{1}{8} N_{d,\alpha,a}^L N_{\beta,\beta,b}^L N_{d,\beta,c}^L \frac{m_{\tilde{\chi}_d^0}}{m_{\tilde{\chi}_b^0}} D_0(m_{\tilde{\nu}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(529)

### F.2.2 Crossed chargino contributions

\[ S_{LL}^{(c_1')} = \frac{1}{2} X_{d,a,a}^{L} X_{b,c,c}^{L} m_{\tilde{\chi}_d^0} D_0(m_{\tilde{\nu}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(530)

\[ S_{LR}^{(c_1')} = -2 X_{d,a,a}^{R} X_{b,c,c}^{R} D_27(m_{\tilde{\chi}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(531)

\[ V_{LL}^{(c_1')} = X_{d,a,a}^{L} X_{b,c,c}^{R} D_27(m_{\tilde{\chi}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(532)

\[ V_{LR}^{(c_1')} = \frac{1}{2} X_{d,a,a}^{L} X_{b,c,c}^{R} D_27(m_{\tilde{\chi}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(533)

\[ T_{LL}^{(c_1')} = -\frac{1}{8} X_{d,a,a}^{L} X_{b,c,c}^{L} m_{\tilde{\chi}_d^0} D_0(m_{\tilde{\nu}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(534)

\[ S_{LL}^{(c_2')} = \frac{1}{2} X_{d,a,a}^{L} X_{b,c,c}^{L} m_{\tilde{\chi}_d^0} D_0(m_{\tilde{\nu}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(535)

\[ S_{LR}^{(c_2')} = 2 X_{d,a,a}^{L} X_{b,c,c}^{L} D_27(m_{\tilde{\chi}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(536)

\[ V_{LL}^{(c_2')} = -X_{d,a,a}^{L} X_{b,c,c}^{L} D_27(m_{\tilde{\chi}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(537)

\[ V_{LR}^{(c_2')} = \frac{1}{2} X_{d,a,a}^{L} X_{b,c,c}^{L} D_27(m_{\tilde{\chi}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(538)

\[ T_{LL}^{(c_2')} = -\frac{1}{8} X_{d,a,a}^{L} X_{b,c,c}^{L} m_{\tilde{\chi}_d^0} D_0(m_{\tilde{\nu}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(539)

\[ S_{LL}^{(c_3')} = \frac{1}{2} X_{d,a,a}^{L} X_{b,c,c}^{L} m_{\tilde{\chi}_d^0} D_0(m_{\tilde{\nu}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(540)

\[ S_{LR}^{(c_3')} = 2 X_{d,a,a}^{L} X_{b,c,c}^{R} D_27(m_{\tilde{\chi}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(541)

\[ V_{LL}^{(c_3')} = -X_{d,a,a}^{L} X_{b,c,c}^{L} D_27(m_{\tilde{\chi}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(542)

\[ V_{LR}^{(c_3')} = \frac{1}{2} X_{d,a,a}^{L} X_{b,c,c}^{R} D_27(m_{\tilde{\chi}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(543)

\[ T_{LL}^{(c_3')} = -\frac{1}{8} X_{d,a,a}^{L} X_{b,c,c}^{L} m_{\tilde{\chi}_d^0} D_0(m_{\tilde{\nu}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(544)

\[ S_{LL}^{(c_4')} = \frac{1}{2} X_{d,a,a}^{L} X_{b,c,c}^{L} m_{\tilde{\chi}_d^0} D_0(m_{\tilde{\nu}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(545)

\[ S_{LR}^{(c_4')} = 2 X_{d,a,a}^{L} X_{b,c,c}^{L} D_27(m_{\tilde{\chi}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(546)

\[ V_{LL}^{(c_4')} = -X_{d,a,a}^{L} X_{b,c,c}^{R} D_27(m_{\tilde{\chi}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(547)

\[ V_{LR}^{(c_4')} = \frac{1}{2} X_{d,a,a}^{L} X_{b,c,c}^{R} D_27(m_{\tilde{\chi}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(548)

\[ T_{LL}^{(c_4')} = -\frac{1}{8} X_{d,a,a}^{L} X_{b,c,c}^{L} m_{\tilde{\chi}_d^0} D_0(m_{\tilde{\nu}_d^0}, m_{\tilde{\nu}_b^0}, m_{\tilde{\nu}_c^0}) \]  
(549)
\[ S_{L\ell}^{LL}(c) = \frac{1}{2} X_{\nu d,a} X_{\nu d,c} X_{\nu b,a} X_{\nu b,c} m_\chi D_0(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (550)
\[ S_{L\ell}^{LR}(c) = -2 X_{\nu d,a} X_{\nu b,c} X_{\nu b,a} X_{\nu b,c} D_{27}(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (551)
\[ V_{LL}^{LL}(c) = X_{\nu d,a} X_{\nu d,c} X_{\nu b,a} X_{\nu b,c} D_{27}(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (552)
\[ V_{LL}^{LR}(c) = \frac{1}{2} X_{\nu d,a} X_{\nu d,c} X_{\nu b,a} X_{\nu b,c} m_\chi D_0(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (553)
\[ T_{LL}^{LL}(c) = -\frac{1}{8} X_{\nu d,a} X_{\nu d,c} X_{\nu b,a} X_{\nu b,c} m_\chi D_0(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (554)

\[ S_{L\ell}^{LL}(c) = \frac{1}{2} X_{\nu d,a} X_{\nu d,c} X_{\nu b,a} X_{\nu b,c} m_\chi D_0(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (555)
\[ S_{L\ell}^{LR}(c) = -2 X_{\nu d,a} X_{\nu d,c} X_{\nu b,a} X_{\nu b,c} D_{27}(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (556)
\[ V_{LL}^{LL}(c) = X_{\nu d,a} X_{\nu d,c} X_{\nu b,a} X_{\nu b,c} D_{27}(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (557)
\[ V_{LL}^{LR}(c) = \frac{1}{2} X_{\nu d,a} X_{\nu d,c} X_{\nu b,a} X_{\nu b,c} m_\chi D_0(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (558)
\[ T_{LL}^{LL}(c) = -\frac{1}{8} X_{\nu d,a} X_{\nu d,c} X_{\nu b,a} X_{\nu b,c} m_\chi D_0(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (559)

\[ S_{L\ell}^{LL}(c) = \frac{1}{2} \hat{X}_{\nu d,a} \hat{X}_{\nu d,c} \hat{X}_{\nu b,a} \hat{X}_{\nu b,c} m_\chi D_0(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (560)
\[ S_{L\ell}^{LR}(c) = -2 \hat{X}_{\nu d,a} \hat{X}_{\nu d,c} \hat{X}_{\nu b,a} \hat{X}_{\nu b,c} D_{27}(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (561)
\[ V_{LL}^{LL}(c) = \hat{X}_{\nu d,a} \hat{X}_{\nu d,c} \hat{X}_{\nu b,a} \hat{X}_{\nu b,c} D_{27}(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (562)
\[ V_{LL}^{LR}(c) = \frac{1}{2} \hat{X}_{\nu d,a} \hat{X}_{\nu d,c} \hat{X}_{\nu b,a} \hat{X}_{\nu b,c} m_\chi D_0(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (563)
\[ T_{LL}^{LL}(c) = -\frac{1}{8} \hat{X}_{\nu d,a} \hat{X}_{\nu d,c} \hat{X}_{\nu b,a} \hat{X}_{\nu b,c} m_\chi D_0(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (564)

\[ S_{L\ell}^{LL}(c) = \frac{1}{2} \hat{X}_{\nu d,a} \hat{X}_{\nu d,c} \hat{X}_{\nu b,a} \hat{X}_{\nu b,c} m_\chi D_0(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (565)
\[ S_{L\ell}^{LR}(c) = -2 \hat{X}_{\nu d,a} \hat{X}_{\nu d,c} \hat{X}_{\nu b,a} \hat{X}_{\nu b,c} D_{27}(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (566)
\[ V_{LL}^{LL}(c) = \hat{X}_{\nu d,a} \hat{X}_{\nu d,c} \hat{X}_{\nu b,a} \hat{X}_{\nu b,c} D_{27}(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (567)
\[ V_{LL}^{LR}(c) = \frac{1}{2} \hat{X}_{\nu d,a} \hat{X}_{\nu d,c} \hat{X}_{\nu b,a} \hat{X}_{\nu b,c} m_\chi D_0(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (568)
\[ T_{LL}^{LL}(c) = -\frac{1}{8} \hat{X}_{\nu d,a} \hat{X}_{\nu d,c} \hat{X}_{\nu b,a} \hat{X}_{\nu b,c} m_\chi D_0(m_{\chi_d}^2, m_{\chi_b}^2, m_{\nu_d}^2, m_{\nu_b}^2) \] (569)

F.2.3 Crossed W⁺ and H⁺ contributions

\[ S_{(w^+)}^{LL} = \frac{1}{2} V_{\nu d,a} V_{\nu b,a} V_{\nu b,c} V_{\nu b,c} D_0(m_{\nu_d}^2, m_{\nu_b}^2, m_{H_a}^2, m_{H_c}^2) \] (570)
\[ S_{(w^+)}^{LR} = 2 V_{\nu d,a} V_{\nu b,c} V_{\nu b,a} V_{\nu b,c} D_{27}(m_{\nu_d}^2, m_{\nu_b}^2, m_{H_a}^2, m_{H_c}^2) \] (571)
\[ V_{(w^+)}^{LL} = -V_{\nu d,a} V_{\nu b,a} V_{\nu b,c} V_{\nu b,c} D_{27}(m_{\nu_d}^2, m_{\nu_b}^2, m_{H_a}^2, m_{H_c}^2) \] (572)
\[ V_{\text{LR}}(w') = \frac{1}{2} V_{d,a,a} \beta, b, a \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{H_a}^2, m_{H_c}^2) \]  
\[ T_{\text{LL}}(w') = -\frac{1}{8} V_{d,a,a} \beta, b, a \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{H_a}^2, m_{H_c}^2) \]  
\[ I_1 = I_{C_0} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ I_2 = D_{27} (m_{\nu_d}^2, m_{\nu_b}^2, m_{H_a}^2, m_{W^-}^2) \]  
\[ I_3 = m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{H_a}^2, m_{W^-}^2) \]  
\[ S_{\text{LL}}(w') = \frac{1}{4} (-3 V_{d,a,a} \beta, b, a \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{H_a}^2, m_{W^-}^2) \]  
\[ S_{\text{LR}}(w') = \frac{3}{4} (-V_{d,a,a} \beta, b, a \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{H_a}^2, m_{W^-}^2) \]  
\[ S_{\text{LL}}' = \left( V_{d,a,a} \beta, b, a \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{H_a}^2, m_{W^-}^2) \right) \]
\[ T_{\text{LL}}(w') = \frac{1}{8} (V_{d,a,a} \beta, b, a \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{H_a}^2, m_{W^-}^2) \]  
\[ I_1 = I_{C_0} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ I_2 = D_{27} (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ I_3 = m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ S_{\text{LL}}(w') = \frac{1}{4} (-3 V_{d,a,a} \beta, b, a \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ S_{\text{LR}}(w') = \frac{3}{4} (-V_{d,a,a} \beta, b, a \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ V_{\text{LL}}'(w') = V_{d,a,a} \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ V_{\text{LR}}'(w') = V_{d,a,a} \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ T_{\text{LL}}(w') = \frac{1}{8} (V_{d,a,a} \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ S_{\text{LL}}(w') = 8 V_{d,a,a} \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ S_{\text{LR}}(w') = 8 V_{d,a,a} \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ V_{\text{LL}}(w') = -4 V_{d,a,a} \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ V_{\text{LR}}(w') = 2 V_{d,a,a} \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ S_{\text{LL}}'(w') = \frac{1}{2} V_{d,a,a} \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]  
\[ S_{\text{LR}}'(w') = 2 V_{d,a,a} \beta, b, a \delta, d, c m_{\nu_b} m_{\nu_d} D_0 (m_{\nu_d}^2, m_{\nu_b}^2, m_{W^-}^2, m_{H_a}^2) \]
\[ V^{LL}_{(u'_a)} = -\frac{1}{2} V^{+L}_{d,\alpha,a} \hat{V}^{+L}_{\beta,b,a} D_0(m_{\nu_b}^2, m_{\nu_d}^2, m_{H_a}^2, m_{H_c}^2) \] (597)

\[ V^{LR}_{(u'_a)} = V^{+L}_{d,\alpha,a} \hat{V}^{+L}_{\beta,b,a} D_27(m_{\nu_b}^2, m_{\nu_d}^2, m_{H_a}^2, m_{H_c}^2) \] (598)

\[ T^{LL}_{(u'_a)} = \frac{1}{8} V^{+L}_{d,\alpha,a} \hat{V}^{+L}_{\beta,b,a} D_0(m_{\nu_b}^2, m_{\nu_d}^2, m_{H_a}^2, m_{H_c}^2) \] (599)

\[ S^{LL}_{(u'_a)} = -8 V^{+L}_{d,\alpha,a} \hat{V}^{+L}_{\beta,b,a} D_27(m_{\nu_b}^2, m_{\nu_d}^2, m_{H_a}^2, m_{W^2}) \] (600)

\[ S^{LR}_{(u'_a)} = -8 V^{+L}_{d,\alpha,a} \hat{V}^{+L}_{\beta,b,a} D_27(m_{\nu_b}^2, m_{\nu_d}^2, m_{H_a}^2, m_{W^2}) \] (601)

\[ V^{LL}_{(u'_a)} = 2 V^{+L}_{d,\gamma,a} \hat{V}^{+L}_{\beta,b,a} D_27(m_{\nu_b}^2, m_{\nu_d}^2, m_{H_a}^2, m_{H_c}^2) \] (602)

\[ V^{LR}_{(u'_a)} = -2 V^{+L}_{d,\gamma,a} \hat{V}^{+L}_{\beta,b,a} D_27(m_{\nu_b}^2, m_{\nu_d}^2, m_{H_a}^2, m_{W^2}) \] (603)

\[ S^{LL}_{(u'_a)} = -8 V^{+L}_{d,\alpha,a} \hat{V}^{+L}_{\beta,b,a} D_27(m_{\nu_b}^2, m_{\nu_d}^2, m_{H_a}^2, m_{H_c}^2) \] (604)

\[ S^{LR}_{(u'_a)} = -8 V^{+L}_{d,\alpha,a} \hat{V}^{+L}_{\beta,b,a} D_27(m_{\nu_b}^2, m_{\nu_d}^2, m_{H_a}^2, m_{H_c}^2) \] (605)

\[ V^{LL}_{(u'_a)} = 2 V^{+L}_{d,\gamma,a} \hat{V}^{+L}_{\beta,b,a} D_27(m_{\nu_b}^2, m_{\nu_d}^2, m_{H_a}^2, m_{H_c}^2) \] (606)

\[ V^{LR}_{(u'_a)} = -2 V^{+L}_{d,\gamma,a} \hat{V}^{+L}_{\beta,b,a} D_27(m_{\nu_b}^2, m_{\nu_d}^2, m_{H_a}^2, m_{H_c}^2) \] (607)

\[ S^{LL}_{(u'_a)} = 8 V^{+L}_{d,\alpha,a} \hat{V}^{+L}_{\beta,b,a} D_27(m_{\nu_b}^2, m_{\nu_d}^2, m_{H_a}^2, m_{H_c}^2) \] (608)

\[ S^{LR}_{(u'_a)} = 32 V^{+L}_{d,\alpha,a} \hat{V}^{+L}_{\beta,b,a} D_27(m_{\nu_b}^2, m_{\nu_d}^2, m_{W^2}) \] (609)

\[ V^{LL}_{(u'_a)} = -2 V^{+L}_{d,\alpha,a} \hat{V}^{+L}_{\beta,b,a} D_27(m_{\nu_b}^2, m_{\nu_d}^2, m_{W^2}) \] (610)

\[ V^{LR}_{(u'_a)} = 4 V^{+L}_{d,\alpha,a} \hat{V}^{+L}_{\beta,b,a} D_27(m_{\nu_b}^2, m_{\nu_d}^2, m_{W^2}) \] (611)
F.3 Two-Lepton – Two-Quark boxes

F.3.1 Feynman diagrams

Neutralino diagrams

\[ (n_1^d) \]

\[ \ell_\alpha \rightarrow \tilde{\nu}_a \rightarrow \tilde{\chi}_d^0 \rightarrow \tilde{\ell}_\beta \]
\[ \tilde{d}_\delta \rightarrow \tilde{d}_c \rightarrow d_\gamma \]

\[ (n_1^u) \]

\[ \ell_\alpha \rightarrow \tilde{\nu}_a \rightarrow \tilde{\chi}_d^0 \rightarrow \tilde{\ell}_\beta \]
\[ \bar{u}_\delta \rightarrow \bar{u}_c \rightarrow u_\gamma \]

Chargino diagrams

\[ (c_1^d) \]

\[ \ell_\alpha \rightarrow \tilde{\nu}_a \rightarrow \tilde{\chi}_d^+ \rightarrow \tilde{\ell}_\beta \]
\[ \tilde{d}_\delta \rightarrow \tilde{d}_c \rightarrow d_\gamma \]

\[ (c_1^u) \]

\[ \ell_\alpha \rightarrow \tilde{\nu}_a \rightarrow \tilde{\chi}_d^+ \rightarrow \tilde{\ell}_\beta \]
\[ \bar{u}_\delta \rightarrow \bar{d}_c \rightarrow u_\gamma \]
$W^+$ and $H^+$ diagrams

\[ (w_1^d) \]
\[ (w_2^d) \]
\[ (w_3^d) \]
\[ (w_4^d) \]

$w_1^u$
$w_2^u$
$w_3^u$
$w_4^u$

F.3.2 Down quarks

Neutralino

\[ S_{LL(n_4)}^{L} = \frac{1}{2} N_{d,a,a} N_{b,a} N_{\bar{b},b,a} N_{\bar{c},c} \tilde{\chi}_{0}^{d,L} \tilde{\chi}_{0}^{d,L} \sum_{g} m_{\chi_{g}} m_{\tilde{\chi}_{0}} D_{0}(m_{\chi_{g}}^{2}, m_{\tilde{\chi}_{0}}^{2}, m_{\tilde{\chi}_{0}}^{2}, m_{\tilde{\chi}_{0}}^{2}) \] (612)

\[ S_{LR(n_4)}^{L} = 2 N_{d,a,a} N_{b,a} N_{\bar{b},b,a} N_{\bar{c},c} \tilde{\chi}_{0}^{d,R} \tilde{\chi}_{0}^{d,R} D_{27}(m_{\chi_{g}}^{2}, m_{\tilde{\chi}_{0}}^{2}, m_{\tilde{\chi}_{0}}^{2}, m_{\tilde{\chi}_{0}}^{2}) \] (613)

\[ V_{LL(n_4)}^{L} = - N_{d,a,a} N_{b,a} N_{\bar{b},b,a} N_{\bar{c},c} \tilde{\chi}_{0}^{d,L} \tilde{\chi}_{0}^{d,L} D_{27}(m_{\chi_{g}}^{2}, m_{\tilde{\chi}_{0}}^{2}, m_{\tilde{\chi}_{0}}^{2}, m_{\tilde{\chi}_{0}}^{2}) \] (614)
\[ V_{LR}^{(m_1^2)} = \frac{1}{2} N_{d,a,a} N_{\beta,b,a} N_{d,c} \chi_{d} m_{\chi_{d}} D_0(m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2) \] (615)

\[ T_{LL}^{(m_1^2)} = -\frac{1}{8} N_{d,a,a} N_{\beta,b,a} N_{d,c} \chi_{d} m_{\chi_{d}} D_0(m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2) \] (616)

\[ S_{LL}^{(m_2^2)} = \frac{1}{2} N_{d,a,a} N_{\beta,b,a} N_{d,c} \chi_{d} m_{\chi_{d}} D_0(m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2) \] (617)

\[ S_{LR}^{(m_2^2)} = 2 N_{d,a,a} N_{\beta,b,a} N_{d,c} \chi_{d} m_{\chi_{d}} D_0(m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2) \] (618)

\[ V_{LL}^{(m_2^2)} = -\frac{1}{2} N_{d,a,a} N_{\beta,b,a} N_{d,c} \chi_{d} m_{\chi_{d}} D_0(m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2) \] (619)

\[ V_{LR}^{(m_2^2)} = N_{d,a,a} N_{\beta,b,a} N_{d,c} \chi_{d} m_{\chi_{d}} D_0(m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2) \] (620)

\[ T_{LL}^{(m_2^2)} = -\frac{1}{8} N_{d,a,a} N_{\beta,b,a} N_{d,c} \chi_{d} m_{\chi_{d}} D_0(m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2, m_{\chi_{d}}^2) \] (621)

Chargino

\[ S_{LL}^{(c_2^2)} = \frac{1}{2} X_{d,a,a} \chi_{d,b,a} W_{d,c}^{d,d} \chi_{b,c} m_{\chi_{b}} D_0(m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2) \] (622)

\[ S_{LR}^{(c_2^2)} = 2 X_{d,a,a} \chi_{d,b,a} W_{d,c}^{d,d} \chi_{b,c} m_{\chi_{b}} D_0(m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2) \] (623)

\[ V_{LL}^{(c_2^2)} = -X_{d,a,a} \chi_{d,b,a} W_{d,c}^{d,d} \chi_{b,c} m_{\chi_{b}} D_0(m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2) \] (624)

\[ V_{LR}^{(c_2^2)} = 2 X_{d,a,a} \chi_{d,b,a} W_{d,c}^{d,d} \chi_{b,c} m_{\chi_{b}} D_0(m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2) \] (625)

\[ T_{LL}^{(c_2^2)} = -\frac{1}{8} X_{d,a,a} \chi_{d,b,a} W_{d,c}^{d,d} \chi_{b,c} m_{\chi_{b}} D_0(m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2) \] (626)

\[ S_{LL}^{(c_2^2)} = \frac{1}{2} X_{d,a,a} \chi_{d,b,a} W_{d,c}^{d,d} \chi_{b,c} m_{\chi_{b}} D_0(m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2) \] (627)

\[ S_{LR}^{(c_2^2)} = 2 X_{d,a,a} \chi_{d,b,a} W_{d,c}^{d,d} \chi_{b,c} m_{\chi_{b}} D_0(m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2) \] (628)

\[ V_{LL}^{(c_2^2)} = -X_{d,a,a} \chi_{d,b,a} W_{d,c}^{d,d} \chi_{b,c} m_{\chi_{b}} D_0(m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2) \] (629)

\[ V_{LR}^{(c_2^2)} = 2 X_{d,a,a} \chi_{d,b,a} W_{d,c}^{d,d} \chi_{b,c} m_{\chi_{b}} D_0(m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2) \] (630)

\[ T_{LL}^{(c_2^2)} = -\frac{1}{8} X_{d,a,a} \chi_{d,b,a} W_{d,c}^{d,d} \chi_{b,c} m_{\chi_{b}} D_0(m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2, m_{\chi_{b}}^2) \] (631)

\[ W^+ \text{ and } H^+ \]

\[ S_{LL}^{(w_1^2)} = -V_{a,a,d} \chi_{a,b,a} W_{\gamma,b,c} \delta_{d,c} m_{\nu_{a}} m_{\nu_{c}} D_0(m_{\nu_{a}}^2, m_{\nu_{c}}^2, m_{\nu_{a}}^2, m_{\nu_{c}}^2) \] (632)

\[ S_{LR}^{(w_1^2)} = -V_{a,a,d} \chi_{a,b,a} W_{\gamma,b,c} \delta_{d,c} m_{\nu_{a}} m_{\nu_{c}} D_0(m_{\nu_{a}}^2, m_{\nu_{c}}^2, m_{\nu_{a}}^2, m_{\nu_{c}}^2) \] (633)

\[ V_{LL}^{(w_1^2)} = -V_{a,a,d} \chi_{a,b,a} W_{\gamma,b,c} \delta_{d,c} D_0(m_{\nu_{a}}^2, m_{\nu_{c}}^2, m_{\nu_{a}}^2, m_{\nu_{c}}^2) \] (634)

\[ V_{LR}^{(w_1^2)} = -V_{a,a,d} \chi_{a,b,a} W_{\gamma,b,c} \delta_{d,c} D_0(m_{\nu_{a}}^2, m_{\nu_{c}}^2, m_{\nu_{a}}^2, m_{\nu_{c}}^2) \] (635)

\[ S_{LL}^{(w_2^2)} = 2 V_{a,a,d} \chi_{a,b,a} W_{\gamma,b,c} \delta_{d,c} D_0(m_{\nu_{a}}^2, m_{\nu_{c}}^2, m_{\nu_{a}}^2, m_{\nu_{c}}^2) \] (636)
\[
S_{LR}^{(w_2)} = 2 V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} (I_{C_0} D_0 (m_{v_a}^2, m_{W^+}^2, m_{H^+_d}^2) - 2 D_{27}(m_{W^+}^2, m_{v_a}^2, m_{v_a}^2))
\]

\[
V_{LL}^{(w_2)} = V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} m_{v_a} m_{a_c} D_0 (m_{v_a}^2, m_{v_a}^2, m_{W^+}^2)
\]

\[
V_{LR}^{(w_2)} = V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} m_{v_a} m_{a_c} D_0 (m_{v_a}^2, m_{v_a}^2, m_{W^+}^2)
\]

\[
T_{LL}^{(w_2)} = - V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} D_{27}(m_{v_a}^2, m_{v_a}^2, m_{W^+}^2)
\]

\[
S_{LL}^{(w_2)} = 2 V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} (I_{C_0} D_0 (m_{v_a}^2, m_{H^+_d}^2, m_{W^+}^2)) - 2 D_{27}(m_{v_a}^2, m_{v_a}^2, m_{S22}^2, m_{W^+}^2)
\]

\[
S_{LR}^{(w_2)} = 2 V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} (I_{C_0} D_0 (m_{v_a}^2, m_{H^+_d}^2, m_{W^+}^2)) - 2 D_{27}(m_{v_a}^2, m_{v_a}^2, m_{S22}^2, m_{W^+}^2)
\]

\[
V_{LL}^{(w_2)} = V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} m_{v_a} m_{a_c} D_0 (m_{v_a}^2, m_{v_a}^2, m_{H^+_d}^2)
\]

\[
V_{LR}^{(w_2)} = V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} m_{v_a} m_{a_c} D_0 (m_{v_a}^2, m_{v_a}^2, m_{H^+_d}^2)
\]

\[
T_{LL}^{(w_2)} = V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} D_{27}(m_{v_a}^2, m_{v_a}^2, m_{H^+_d}^2, m_{W^+}^2)
\]

\[
S_{LL}^{(w_2)} = - 4 V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} m_{v_a} m_{a_c} D_0 (m_{v_a}^2, m_{v_a}^2, m_{W^+}^2)
\]

\[
S_{LR}^{(w_2)} = - 4 V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} m_{v_a} m_{a_c} D_0 (m_{v_a}^2, m_{v_a}^2, m_{W^+}^2)
\]

\[
V_{LL}^{(w_2)} = - 4 V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} m_{v_a} m_{a_c} D_0 (m_{v_a}^2, m_{v_a}^2, m_{W^+}^2)
\]

\[
V_{LR}^{(w_2)} = - 4 V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} m_{v_a} m_{a_c} D_0 (m_{v_a}^2, m_{v_a}^2, m_{W^+}^2)
\]

\[
T_{LL}^{(w_2)} = - 4 V_{u,a,d}^+, L \hat{V}^+_{\beta,a} \hat{V}^+_{\gamma,c} \bar{V}^{d,R}_{\delta,c} m_{v_a} m_{a_c} D_0 (m_{v_a}^2, m_{v_a}^2, m_{W^+}^2)
\]

\[\text{F.3.3 Up quarks}\]

Neutralino

\[
S_{LL}^{(u_1)} = \frac{1}{2} N_{d,a,a}^L \bar{N}_{\beta,b,a}^L N_{b,a,c}^L \bar{N}_{\delta,c,d}^L D_27(m_{\chi_d}^2, m_{\chi_d}^2, m_{\varepsilon_a}^2, m_{\varepsilon_a}^2)
\]

\[
S_{LR}^{(u_1)} = \frac{1}{2} N_{d,a,a}^L \bar{N}_{\beta,b,a}^L N_{b,a,c}^L \bar{N}_{\delta,c,d}^L m_{\chi_d} m_{\chi_d} D_0 (m_{\chi_d}^2, m_{\chi_d}^2, m_{\varepsilon_a}^2, m_{\varepsilon_a}^2)
\]

\[
V_{LL}^{(u_1)} = - N_{d,a,a}^L \bar{N}_{\beta,b,a}^L N_{b,a,c}^L \bar{N}_{\delta,c,d}^L D_27(m_{\chi_d}^2, m_{\chi_d}^2, m_{\varepsilon_a}^2, m_{\varepsilon_a}^2)
\]

\[
V_{LR}^{(u_1)} = \frac{1}{2} N_{d,a,a}^L \bar{N}_{\beta,b,a}^L N_{b,a,c}^L \bar{N}_{\delta,c,d}^L D_27(m_{\chi_d}^2, m_{\chi_d}^2, m_{\varepsilon_a}^2, m_{\varepsilon_a}^2)
\]

\[\text{F.3.3 Up quarks}\]

Neutralino

\[
S_{LL}^{(u_2)} = \frac{1}{2} N_{d,a,a}^L \bar{N}_{\beta,b,a}^L N_{b,a,c}^L \bar{N}_{\delta,c,d}^L m_{\chi_0} m_{\chi_0} D_0 (m_{\chi_0}^2, m_{\chi_0}^2, m_{\varepsilon_a}^2, m_{\varepsilon_a}^2)
\]

\[
S_{LR}^{(u_2)} = \frac{1}{2} N_{d,a,a}^L \bar{N}_{\beta,b,a}^L N_{b,a,c}^L \bar{N}_{\delta,c,d}^L m_{\chi_0} m_{\chi_0} D_0 (m_{\chi_0}^2, m_{\chi_0}^2, m_{\varepsilon_a}^2, m_{\varepsilon_a}^2)
\]

\[
V_{LL}^{(u_2)} = \frac{1}{2} N_{d,a,a}^L \bar{N}_{\beta,b,a}^L N_{b,a,c}^L \bar{N}_{\delta,c,d}^L m_{\chi_0} m_{\chi_0} D_0 (m_{\chi_0}^2, m_{\chi_0}^2, m_{\varepsilon_a}^2, m_{\varepsilon_a}^2)
\]

67
\[ V_{(u_2)}^{LR} = N_{d,a,a}^{L} \bar{N}_{b,b,a}^{R} \bar{N}_{d,G,c}^{u,L} N_{d,c}^{a,R} D_2 \left( m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2 \right) \] (658)

\[ T_{(u_2)}^{LL} = \frac{1}{8} N_{d,a,a}^{L} \bar{N}_{b,b,a}^{R} \bar{N}_{d,G,c}^{u,L} N_{d,c}^{a,R} D_0 \left( m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2 \right) \] (659)

Chargino

\[ S_{LL}^{(c_1)} = \frac{1}{2} X_{d,a,a}^{L} \bar{X}_{b,b,a}^{L} W_{d,c}^{u,L} W_{d,c}^{u,L} m_{c_{1a}}^2 \bar{m}_{c_{1a}}^2 D_0 \left( m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2 \right) \] (660)

\[ S_{LR}^{(c_1)} = 2 X_{d,a,a}^{L} \bar{X}_{b,b,a}^{L} W_{d,c}^{u,R} W_{d,c}^{u,R} D_2 \left( m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2 \right) \] (661)

\[ V_{LL}^{(c_1)} = - \frac{1}{2} V_{d,a,a}^{L} \bar{V}_{b,b,a}^{L} W_{d,c}^{u,R} W_{d,c}^{u,R} m_{c_{1a}}^2 \bar{m}_{c_{1a}}^2 D_0 \left( m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2 \right) \] (662)

\[ V_{LR}^{(c_1)} = V_{d,a,a}^{L} \bar{V}_{b,b,a}^{L} W_{d,c}^{u,R} W_{d,c}^{u,R} D_2 \left( m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2 \right) \] (663)

\[ T_{LL}^{(c_1)} = \frac{1}{8} X_{d,a,a}^{L} \bar{X}_{b,b,a}^{L} W_{d,c}^{u,L} W_{d,c}^{u,L} m_{c_{1a}}^2 \bar{m}_{c_{1a}}^2 D_0 \left( m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2 \right) \] (664)

\[ S_{LL}^{(c_2)} = \frac{1}{2} \tilde{X}_{d,a,a}^{L} \tilde{X}_{b,b,a}^{L} W_{d,c}^{u,L} W_{d,c}^{u,L} m_{c_{1a}}^2 \bar{m}_{c_{1a}}^2 D_0 \left( m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2 \right) \] (665)

\[ S_{LR}^{(c_2)} = 2 \tilde{X}_{d,a,a}^{L} \tilde{X}_{b,b,a}^{L} W_{d,c}^{u,R} W_{d,c}^{u,R} D_2 \left( m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2 \right) \] (666)

\[ V_{LL}^{(c_2)} = - \frac{1}{2} V_{d,a,a}^{L} \tilde{V}_{b,b,a}^{L} W_{d,c}^{u,R} W_{d,c}^{u,R} m_{c_{1a}}^2 \bar{m}_{c_{1a}}^2 D_0 \left( m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2 \right) \] (667)

\[ V_{LR}^{(c_2)} = \tilde{V}_{d,a,a}^{L} \tilde{V}_{b,b,a}^{L} W_{d,c}^{u,R} W_{d,c}^{u,R} D_2 \left( m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2 \right) \] (668)

\[ T_{LL}^{(c_2)} = \frac{1}{8} \tilde{X}_{d,a,a}^{L} \tilde{X}_{b,b,a}^{L} W_{d,c}^{u,L} W_{d,c}^{u,L} m_{c_{1a}}^2 \bar{m}_{c_{1a}}^2 D_0 \left( m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2, m_{c_{1a}}^2 \right) \] (669)

\[ W^+ \ and \ H^+ \]

\[ S_{LL}^{(w_2)} = - V_{a,a,d}^{+}, V_{b,a,b}^{+}, V_{c,a,d}^{+}, V_{d,a,d}^{+}, m_{a,d}^2 D_0 \left( m_{a,d}^2, m_{a,d}^2, m_{a,d}^2, m_{a,d}^2 \right) \] (670)

\[ S_{LR}^{(w_2)} = - V_{a,a,d}^{+}, V_{b,a,b}^{+}, V_{c,a,d}^{+}, V_{d,a,d}^{+}, D_2 \left( m_{a,d}^2, m_{a,d}^2, m_{a,d}^2, m_{a,d}^2 \right) \] (671)

\[ V_{LL}^{(w_2)} = V_{a,a,d}^{+}, V_{b,a,b}^{+}, m_{a,d}^2 D_2 \left( m_{a,d}^2, m_{a,d}^2, m_{a,d}^2, m_{a,d}^2 \right) \] (672)

\[ V_{LR}^{(w_2)} = V_{a,a,d}^{+}, V_{b,a,b}^{+}, m_{a,d}^2 D_2 \left( m_{a,d}^2, m_{a,d}^2, m_{a,d}^2, m_{a,d}^2 \right) \] (673)

\[ S_{LL}^{(w_2)} = - 4V_{a,a,d}^{+}, V_{b,a,b}^{+}, V_{c,a,d}^{+}, V_{d,a,d}^{+}, D_2 \left( m_{a,d}^2, m_{a,d}^2, m_{a,d}^2, m_{a,d}^2 \right) \] (674)

\[ S_{LR}^{(w_2)} = - 4V_{a,a,d}^{+}, V_{b,a,b}^{+}, V_{c,a,d}^{+}, V_{d,a,d}^{+}, D_2 \left( m_{a,d}^2, m_{a,d}^2, m_{a,d}^2, m_{a,d}^2 \right) \] (675)

\[ V_{LL}^{(w_2)} = V_{a,a,d}^{+}, V_{b,a,b}^{+}, V_{c,a,d}^{+}, m_{a,d}^2 D_2 \left( m_{a,d}^2, m_{a,d}^2, m_{a,d}^2, m_{a,d}^2 \right) \] (676)

\[ V_{LR}^{(w_2)} = V_{a,a,d}^{+}, V_{b,a,b}^{+}, V_{c,a,d}^{+}, m_{a,d}^2 D_2 \left( m_{a,d}^2, m_{a,d}^2, m_{a,d}^2, m_{a,d}^2 \right) \] (677)

\[ T_{LL}^{(w_2)} = - V_{a,a,d}^{+}, V_{b,a,b}^{+}, V_{c,a,d}^{+}, m_{a,d}^2 D_2 \left( m_{a,d}^2, m_{a,d}^2, m_{a,d}^2, m_{a,d}^2 \right) \] (678)

\[ S_{LL}^{(w_3)} = - 4V_{a,a,i}^{+}, V_{b,a,b}^{+}, V_{c,a,d}^{+}, D_2 \left( m_{a,d}^2, m_{a,d}^2, m_{a,d}^2, m_{a,d}^2 \right) \] (679)
G  Form factors of the 4-fermion operators

We define the sum over all penguin diagrams as

\[ V_{Z,\text{sum}}^{LL} = \frac{1}{16\pi^2} \left( \sum_a V_{Z,(\eta a)}^{LL} + \sum_a V_{Z,(\zeta a)}^{LL} + \sum_a V_{Z,(\omega a)}^{LL} \right) \]  
\[ V_{Z,\text{sum}}^{LR} = \frac{1}{16\pi^2} \left( \sum_a V_{Z,(\eta a)}^{LR} + \sum_a V_{Z,(\zeta a)}^{LR} + \sum_a V_{Z,(\omega a)}^{LR} \right) \]  
\[ S_{h_p,\text{sum}}^{LL} = \frac{1}{16\pi^2} \left( \sum_a S_{h_p,(\eta a)}^{LL} + \sum_a S_{h_p,(\zeta a)}^{LL} + \sum_a S_{h_p,(\omega a)}^{LL} \right) \]  
\[ S_{h_p,\text{sum}}^{LR} = \frac{1}{16\pi^2} \left( \sum_a S_{h_p,(\eta a)}^{LR} + \sum_a S_{h_p,(\zeta a)}^{LR} + \sum_a S_{h_p,(\omega a)}^{LR} \right) \]  
\[ S_{A_p,\text{sum}}^{LL} = \frac{1}{16\pi^2} \left( \sum_a S_{A_p,(\eta a)}^{LL} + \sum_a S_{A_p,(\zeta a)}^{LL} + \sum_a S_{A_p,(\omega a)}^{LL} \right) \]  
\[ S_{A_p,\text{sum}}^{LR} = \frac{1}{16\pi^2} \left( \sum_a S_{A_p,(\eta a)}^{LR} + \sum_a S_{A_p,(\zeta a)}^{LR} + \sum_a S_{A_p,(\omega a)}^{LR} \right) \]  

and the sum over all boxes as

\[ X_{(\text{sum})}^{LL} = \frac{1}{16\pi^2} \left( \sum_a X_{(\eta a)}^{LL} + \sum_a X_{(\zeta a)}^{LL} + \sum_a X_{(\omega a)}^{LL} \right) \]  
\[ X_{(\text{sum})}^{LR} = \frac{1}{16\pi^2} \left( \sum_a X_{(\eta a)}^{LR} + \sum_a X_{(\zeta a)}^{LR} + \sum_a X_{(\omega a)}^{LR} \right) \]  

with \( X = V, S, T \) and \( x = \ell, d, u \). With these, we can finally obtain the form factors of the 4-lepton operators as follows:

\[ A_{LL}^V = V_{Z,\text{sum}}^{LL} \frac{1}{m_Z^2} + V_{(\text{sum})}^{LL} \]
\[ A_{LR}^{V} = V_{Z, \text{sum}}^{LR} E_{\gamma, \delta}^{R} \frac{1}{m_{Z}^{2}} + V_{(\text{sum})}^{LR} \]  
\[ A_{LL}^{S} = \sum_{p} S_{h_{p}, \text{sum}}^{LL} H_{\gamma, \delta, \mu_{p}}^{L} \frac{1}{m_{h_{p}}^{2}} + \sum_{p} S_{A_{p}^{0}, \text{sum}}^{LL} A_{\gamma, \delta, \mu_{p}}^{L} \frac{1}{m_{A_{p}^{0}}^{2}} + S_{(\text{sum})}^{LL} \]  
\[ A_{LR}^{S} = \sum_{p} S_{h_{p}, \text{sum}}^{LR} H_{\gamma, \delta, \mu_{p}}^{R} \frac{1}{m_{h_{p}}^{2}} + \sum_{p} S_{A_{p}^{0}, \text{sum}}^{LR} A_{\gamma, \delta, \mu_{p}}^{R} \frac{1}{m_{A_{p}^{0}}^{2}} + S_{(\text{sum})}^{LR} \]  
\[ A_{LL}^{T} = T_{(\text{sum})}^{LL} \]  
\[ A_{LR}^{T} = T_{(\text{sum})}^{LR} \]  
\[ B_{LL}^{V} = V_{Z, \text{sum}}^{LL} D_{\gamma, \delta}^{L} \frac{1}{m_{Z}^{2}} + V_{(\text{sum})}^{LL} \]  
\[ B_{LR}^{V} = V_{Z, \text{sum}}^{LR} D_{\gamma, \delta}^{R} \frac{1}{m_{Z}^{2}} + V_{(\text{sum})}^{LR} \]  
\[ B_{LL}^{S} = \sum_{p} S_{h_{p}, \text{sum}}^{LL} H_{\gamma, \delta, \mu_{p}}^{LL} \frac{1}{m_{h_{p}}^{2}} + \sum_{p} S_{A_{p}^{0}, \text{sum}}^{LL} A_{\gamma, \delta, \mu_{p}}^{LL} \frac{1}{m_{A_{p}^{0}}^{2}} + S_{(\text{sum})}^{LL} \]  
\[ B_{LR}^{S} = \sum_{p} S_{h_{p}, \text{sum}}^{LR} H_{\gamma, \delta, \mu_{p}}^{LR} \frac{1}{m_{h_{p}}^{2}} + \sum_{p} S_{A_{p}^{0}, \text{sum}}^{LR} A_{\gamma, \delta, \mu_{p}}^{LR} \frac{1}{m_{A_{p}^{0}}^{2}} + S_{(\text{sum})}^{LR} \]  
\[ B_{LL}^{T} = T_{(\text{sum})}^{LL} \]  
\[ B_{LR}^{T} = T_{(\text{sum})}^{LR} \]  
\[ C_{LL}^{V} = V_{Z, \text{sum}}^{LL} U_{\gamma, \delta}^{L} \frac{1}{m_{Z}^{2}} + V_{(\text{sum})}^{LL} \]  
\[ C_{LR}^{V} = V_{Z, \text{sum}}^{LR} U_{\gamma, \delta}^{R} \frac{1}{m_{Z}^{2}} + V_{(\text{sum})}^{LR} \]  
\[ C_{LL}^{S} = \sum_{p} S_{h_{p}, \text{sum}}^{LL} H_{\gamma, \delta, \mu_{p}}^{U, L} \frac{1}{m_{h_{p}}^{2}} + \sum_{p} S_{A_{p}^{0}, \text{sum}}^{LL} A_{\gamma, \delta, \mu_{p}}^{U, L} \frac{1}{m_{A_{p}^{0}}^{2}} + S_{(\text{sum})}^{LL} \]  
\[ C_{LR}^{S} = \sum_{p} S_{h_{p}, \text{sum}}^{LR} H_{\gamma, \delta, \mu_{p}}^{U, R} \frac{1}{m_{h_{p}}^{2}} + \sum_{p} S_{A_{p}^{0}, \text{sum}}^{LR} A_{\gamma, \delta, \mu_{p}}^{U, R} \frac{1}{m_{A_{p}^{0}}^{2}} + S_{(\text{sum})}^{LR} \]  
\[ C_{LL}^{T} = T_{(\text{sum})}^{LL} \]  
\[ C_{LR}^{T} = T_{(\text{sum})}^{LR} \]  

and the other chiralities are given by \( X_{RL}^{W} = X_{LR}^{W}(R \leftrightarrow L) \) and \( X_{RR}^{W} = X_{LL}^{W}(R \leftrightarrow L) \) (\( X = A, B, C; W = S, T, V \)).

References

[1] ATLAS Collaboration, G. Aad et al., Phys.Lett. B716, 1 (2012), arXiv:1207.7214.
[2] CMS Collaboration, S. Chatrchyan et al., Phys.Lett. B716, 30 (2012), arXiv:1207.7235.
[3] D. Forero, M. Tortola, and J. Valle, Phys.Rev. D86, 073012 (2012), arXiv:1205.4018.
[4] M. Gonzalez-Garcia, M. Maltoni, J. Salvado, and T. Schwetz, JHEP 1212, 123 (2012), arXiv:1209.3023.

[5] F. Capozzi et al., Phys.Rev. D89, 093018 (2014), arXiv:1312.2878.

[6] S. Mihara, J. Miller, P. Paradisi, and G. Piredda, Ann.Rev.Nucl.Part.Sci. 63, 531 (2013).

[7] MEG Collaboration, J. Adam et al., Phys.Rev.Lett. 110, 201801 (2013), arXiv:1303.0754.

[8] A. Baldini et al., (2013), arXiv:1301.7225.

[9] T. Aushev et al., (2010), arXiv:1002.5012.

[10] SINDRUM Collaboration, U. Bellgardt et al., Nucl.Phys. B299, 1 (1988).

[11] A. Blondel et al., (2013), arXiv:1301.6113.

[12] A. Bevan et al., (2014), arXiv:1406.6311.

[13] LHCb collaboration, R. Aaij et al., Phys.Lett. B724, 36 (2013), arXiv:1304.4518.

[14] Mu2e Collaboration, R. Carey et al., (2008).

[15] Mu2e Collaboration, D. Glenzinski, AIP Conf.Proc. 1222, 383 (2010).

[16] Mu2e Collaboration, R. Abrams et al., (2012), arXiv:1211.7019.

[17] DeeMe Collaboration, M. Aoki, PoS ICHEP2010, 279 (2010).

[18] COMET Collaboration, Y. Cui et al., (2009).

[19] COMET Collaboration, Y. Kuno, PTEP 2013, 022C01 (2013).

[20] T. P. working group, Search for the $\mu \to e$ Conversion Process at an Ultimate Sensitivity of the Order of $10^{-18}$ with PRISM, http://j-parc.jp/researcher/Hadron/en/pac_0606/pdf/p20-Kuno.pdf.

[21] G. Blankenburg, J. Ellis, and G. Isidori, Phys.Lett. B712, 386 (2012), arXiv:1202.5704.

[22] R. Harnik, J. Kopp, and J. Zupan, JHEP 1303, 026 (2013), arXiv:1209.1397.

[23] S. Davidson and P. Verdier, Phys.Rev. D86, 111701 (2012), arXiv:1211.1248.

[24] A. Arhrib, Y. Cheng, and O. C. Kong, Europhys.Lett. 101, 31003 (2013), arXiv:1208.4669.

[25] A. Arhrib, Y. Cheng, and O. C. Kong, Phys.Rev. D87, 015025 (2013), arXiv:1210.8241.

[26] P. Bhupal Dev, R. Franceschini, and R. Mohapatra, Phys.Rev. D86, 093010 (2012), arXiv:1207.2756.

[27] M. Arana-Catania, E. Arganda, and M. Herrero, JHEP 1309, 160 (2013), arXiv:1304.3371.

[28] A. Falkowski, D. M. Straub, and A. Vicente, JHEP 1405, 092 (2014), arXiv:1312.5329.

[29] E. Arganda, M. Herrero, X. Marcano, and C. Weiland, (2014), arXiv:1405.4300.

[30] J. Kopp and M. Nardecchia, (2014), arXiv:1406.5303.
[31] CMS Collaboration, CMS Collaboration, CMS-PAS-HIG-14-005 (2014).

[32] W. Porod and W. Majerotto, Phys.Rev. D66, 015003 (2002), arXiv:hep-ph/0201284.

[33] A. Bartl et al., Eur.Phys.J. C46, 783 (2006), arXiv:hep-ph/0510074.

[34] M. Hirsch, J. Valle, W. Porod, J. Romao, and A. Villanova del Moral, Phys.Rev. D78, 013006 (2008), arXiv:0804.4072.

[35] M. Hirsch, S. Kaneko, and W. Porod, Phys.Rev. D78, 093004 (2008), arXiv:0806.3361.

[36] S. Kaneko, J. Sato, T. Shimomura, O. Vives, and M. Yamanaka, Phys.Rev. D78, 116013 (2008), arXiv:0811.0703.

[37] F. del Aguila and J. Aguilar-Saavedra, Nucl.Phys. B813, 22 (2009), arXiv:0808.2468.

[38] A. Atre, T. Han, S. Pascoli, and B. Zhang, JHEP 0905, 030 (2009), arXiv:0901.3589.

[39] A. Abada, A. Figueiredo, J. Romao, and A. Teixeira, JHEP 1010, 104 (2010), arXiv:1007.4833.

[40] J. Esteves et al., JHEP 1012, 077 (2010), arXiv:1011.0348.

[41] J. Esteves, J. Romao, M. Hirsch, F. Staub, and W. Porod, Phys.Rev. D83, 013003 (2011), arXiv:1010.6000.

[42] A. Abada, A. Figueiredo, J. Romao, and A. Teixeira, JHEP 1108, 099 (2011), arXiv:1104.3962.

[43] A. Abada, A. Figueiredo, J. Romao, and A. Teixeira, JHEP 1208, 138 (2012), arXiv:1206.2306.

[44] P. Bandyopadhyay, E. J. Chun, H. Okada, and J.-C. Park, JHEP 1301, 079 (2013), arXiv:1209.4803.

[45] S. Mondal, S. Biswas, P. Ghosh, and S. Roy, JHEP 1205, 134 (2012), arXiv:1201.1556.

[46] A. Das and N. Okada, Phys.Rev. D88, 113001 (2013), arXiv:1207.3734.

[47] A. Teixeira, A. Abada, A. Figueiredo, and J. Romao, (2014), arXiv:1402.1426.

[48] P. S. Bhupal Dev, S. Mondal, B. Mukhopadhyaya and S. Roy, JHEP 1209, 110 (2012), arXiv:1207.6542.

[49] BaBar Collaboration, B. Aubert et al., Phys.Rev.Lett. 104, 021802 (2010), arXiv:0908.2381.

[50] K. Hayasaka et al., Phys.Lett. B687, 139 (2010), arXiv:1001.3221.

[51] SINDRUM II Collaboration., C. Dohmen et al., Phys.Lett. B317, 631 (1993).

[52] SINDRUM II Collaboration, W. H. Bertl et al., Eur.Phys.J. C47, 337 (2006).

[53] DeeMe Collaboration, M. Aoki, AIP Conf.Proc. 1441, 599 (2012).

[54] A. J. Buras, B. Duling, T. Feldmann, T. Heidsieck, and C. Promberger, JHEP 1009, 104 (2010), arXiv:1006.5356.
[55] R. Mohapatra and J. Valle, Phys.Rev. D34, 1642 (1986).
[56] A. Elsayed, S. Khalil, and S. Moretti, Phys.Lett. B715, 208 (2012), arXiv:1106.2130.
[57] M. Hirsch, M. Malinsky, W. Porod, L. Reichert, and F. Staub, JHEP 1202, 084 (2012), arXiv:1110.3037.
[58] E. J. Chun, V. S. Mummidi, and S. K. Vempati, (2014), arXiv:1405.5478.
[59] J. Bernabeu, A. Santamaria, J. Vidal, A. Mendez, and J. Valle, Phys.Lett. B187, 303 (1987).
[60] A. Ilakovac and A. Pilaftsis, Nucl.Phys. B437, 491 (1995), arXiv:hep-ph/9403398.
[61] F. Deppisch and J. Valle, Phys.Rev. D72, 036001 (2005), arXiv:hep-ph/0406040.
[62] F. Deppisch, T. Kosmas, and J. Valle, Nucl.Phys. B752, 80 (2006), arXiv:hep-ph/0512360.
[63] A. Ilakovac and A. Pilaftsis, Phys.Rev. D80, 091902 (2009), arXiv:0904.2381.
[64] R. Alonso, M. Dhen, M. Gavela, and T. Hambye, JHEP 1301, 118 (2013), arXiv:1209.2679.
[65] D. Dinh, A. Ibarra, E. Molinaro, and S. Petcov, JHEP 1208, 125 (2012), arXiv:1205.4671.
[66] A. Ilakovac, A. Pilaftsis, and L. Popov, Phys.Rev. D87, 053014 (2013), arXiv:1212.5939.
[67] C.-H. Lee, P. Bhupal Dev, and R. Mohapatra, Phys.Rev. D88, 093010 (2013), arXiv:1309.0774.
[68] M. Hirsch, F. Staub, and A. Vicente, Phys.Rev. D85, 113013 (2012), arXiv:1202.1825.
[69] H. Dreiner, K. Nickel, F. Staub, and A. Vicente, Phys.Rev. D86, 015003 (2012), arXiv:1204.5925.
[70] M. Hirsch, W. Porod, L. Reichert, and F. Staub, Phys.Rev. D86, 093018 (2012), arXiv:1206.3516.
[71] A. Abada, D. Das, A. Vicente, and C. Weiland, JHEP 1209, 015 (2012), arXiv:1206.6497.
[72] M. E. Krauss, W. Porod, and F. Staub, Phys.Rev. D88, 015014 (2013), arXiv:1304.0769.
[73] M. E. Krauss et al., Phys.Rev. D90, 013008 (2014), arXiv:1312.5318.
[74] E. Arganda and M. J. Herrero, Phys.Rev. D73, 055003 (2006), arXiv:hep-ph/0510405.
[75] W. Porod, F. Staub, and A. Vicente, (2014), arXiv:1405.1434.
[76] F. Staub, (2008), arXiv:0806.0538.
[77] F. Staub, Comput.Phys.Commun. 181, 1077 (2010), arXiv:0909.2863.
[78] F. Staub, Comput.Phys.Commun. 182, 808 (2011), arXiv:1002.0840.
[79] F. Staub, Computer Physics Communications 184, pp. 1792 (2013), arXiv:1207.0906.
[80] F. Staub, Comput.Phys.Commun. 185, 1773 (2014), arXiv:1309.7223.
[81] W. Porod, Comput.Phys.Commun. 153, 275 (2003), arXiv:hep-ph/0301101.
[82] W. Porod and F. Staub, Comput.Phys.Commun. 183, 2458 (2012), arXiv:1104.1573.

[83] T. Hahn and M. Perez-Victoria, Comput.Phys.Commun. 118, 153 (1999), arXiv:hep-ph/9807565.

[84] T. Hahn, Comput.Phys.Commun. 140, 418 (2001), arXiv:hep-ph/0012260.

[85] T. Hahn, Nucl.Phys.Proc.Suppl. 89, 231 (2000), arXiv:hep-ph/0005029.

[86] T. Hahn, Nucl.Phys.Proc.Suppl. 135, 333 (2004), arXiv:hep-ph/0406288.

[87] T. Hahn, eConf C050318, 0604 (2005), arXiv:hep-ph/0506201.

[88] B. Chokoufe Nejad, T. Hahn, J.-N. Lang, and E. Mirabella, (2013), arXiv:1310.0274.

[89] A. Abada, D. Das, and C. Weiland, JHEP 1203, 100 (2012), arXiv:1111.5836.

[90] R. Mohapatra, Phys.Rev.Lett. 56, 561 (1986).

[91] M. Malinsky, T. Ohlsson, Z.-z. Xing, and H. Zhang, Phys.Lett. B679, 242 (2009), arXiv:0905.2889.

[92] M. Gavela, T. Hambye, D. Hernandez, and P. Hernandez, JHEP 0909, 038 (2009), arXiv:0906.1461.

[93] A. Abada and M. Lucente, Nucl.Phys. B885, 651 (2014), arXiv:1401.1507.

[94] M. Hirsch, T. Kernreiter, J. Romao, and A. Villanova del Moral, JHEP 1001, 103 (2010), arXiv:0910.2435.

[95] G. ’t Hooft, NATO Adv.Study Inst.Ser.B Phys. 59, 135 (1980).

[96] M. Gonzalez-Garcia and J. Valle, Phys.Lett. B216, 360 (1989).

[97] CMS Collaboration, S. Chatrchyan et al., JHEP 1406, 055 (2014), arXiv:1402.4770.

[98] ATLAS Collaboration, G. Aad et al., (2014), arXiv:1405.7875.

[99] E. Arganda, M. Herrero, and A. Teixeira, JHEP 0710, 104 (2007), arXiv:0707.2955.

[100] J. Hisano, T. Moroi, K. Tobe, and M. Yamaguchi, Phys.Rev. D53, 2442 (1996), arXiv:hep-ph/9510309.

[101] E. Arganda, private communication.

[102] Y. Kuno and Y. Okada, Rev.Mod.Phys. 73, 151 (2001), arXiv:hep-ph/9909265.

[103] J. Vergados, Phys.Rept. 133, 1 (1986).

[104] J. Bernabeu, E. Nardi, and D. Tommasini, Nucl.Phys. B409, 69 (1993), arXiv:hep-ph/9306251.

[105] A. Faessler, T. Kosmas, S. Kovalenko, and J. Vergados, (1999), arXiv:hep-ph/9904335.

[106] R. Kitano, M. Koike, and Y. Okada, Phys.Rev. D66, 096002 (2002), arXiv:hep-ph/0203110.
[107] A. Crivellin, M. Hoferichter and M. Procura, Phys.Rev. D89, 093024 (2014), arXiv:1404.7134.

[108] H. Chiang, E. Oset, T. Kosmas, A. Faessler, and J. Vergados, Nucl.Phys. A559, 526 (1993).

[109] T. Kosmas, S. Kovalenko, and I. Schmidt, Phys.Lett. B511, 203 (2001), arXiv:hep-ph/0102101.

[110] A. Brignole, G. Degrassi, P. Slavich, and F. Zwirner, Nucl.Phys. B631, 195 (2002), arXiv:hep-ph/0112177.

[111] G. Degrassi, P. Slavich, and F. Zwirner, Nucl.Phys. B611, 403 (2001), arXiv:hep-ph/0105096.

[112] A. Brignole, G. Degrassi, P. Slavich, and F. Zwirner, Nucl.Phys. B643, 79 (2002), arXiv:hep-ph/0206101.

[113] A. Dedes and P. Slavich, Nucl.Phys. B657, 333 (2003), arXiv:hep-ph/0212132.

[114] A. Dedes, G. Degrassi, and P. Slavich, Nucl.Phys. B672, 144 (2003), arXiv:hep-ph/0305127.

[115] B. Allanach, A. Djouadi, J. Kneur, W. Porod, and P. Slavich, JHEP 0409, 044 (2004), arXiv:hep-ph/0406166.

[116] J. Casas and A. Ibarra, Nucl.Phys. B618, 171 (2001), arXiv:hep-ph/0103065.

[117] L. Basso et al., Comput.Phys.Commun. 184, 698 (2013), arXiv:1206.4563.

[118] A. Abada, D. Das, A. Teixeira, A. Vicente, and C. Weiland, JHEP 1302, 048 (2013), arXiv:1211.3052.

[119] D. Forero, M. Tortola, and J. Valle, (2014), arXiv:1405.7540.

[120] K. Babu and C. Kolda, Phys.Rev.Lett. 89, 241802 (2002), arXiv:hep-ph/0206310.

[121] LHCb Collaboration, R. Aaij et al., Phys.Rev.Lett. 110, 021801 (2013), arXiv:1211.2674.

[122] S. P. Martin and M. T. Vaughn, Phys.Rev. D50, 2282 (1994), arXiv:hep-ph/9311340.