On Extracting Heavy Quark Parameters from Moments with Cuts

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I.I. Bigi\textsuperscript{a} and N. Uraltsev\textsuperscript{b}\
\textsuperscript{a} Department of Physics, University of Notre Dame du Lac\
Notre Dame, IN 46556, USA\
\textsuperscript{b} INFN, Sezione di Milano, Milan, Italy

Abstract

We point out that applying the photon energy cut significantly modifies the moments of energy spectrum in $B \rightarrow X_s + \gamma$ decays, with a certain class of effects not accounted for in the mostly used OPE expressions. This leads to a systematic bias in the extracted values of the $b$ quark mass and other heavy quark parameters. The apparent $b$ quark mass increases typically by 70 MeV or more, together with an even more dramatic downward shift in the kinetic expectation value. Accounting for these cut-related shifts brings different measurements into a good agreement, when the OPE-based theory employs the robust approach. These nonperturbative effects are exponential in the effective hardness severely lowered by high cuts, and do not signify a breakdown of the $1/m_b$ expansion itself. Similar effects in semileptonic $b \rightarrow c$ decays are briefly addressed. We stress the utility of the second moment of $E_\gamma$ once these effects are incorporated.

\textsuperscript{*}On leave of absence from Department of Physics, University of Notre Dame, Notre Dame, IN 46556, USA and St. Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188300, Russia
Treating effects of strong interactions in decays of beauty particles is of primary importance nowadays when many precision experimental studies are being carried out. Inclusive distributions in radiative and semileptonic decays are the portal to accurately determining the nonperturbative heavy quark parameters controlling short-distance observables in \( B \) decays. Their utility rests on an consistent expansion in \( 1/m_b \), the inverse b quark mass [1]. Recent experimental data generally show a nontrivial agreement between quite different – and a priori unrelated – measurements at the nonperturbative level, on one hand, and consistency with the QCD-based OPE theory.

In order to enjoy in full the potential of a small expansion parameter provided by the heavy quark mass, the observable in question must be sufficiently inclusive. However experimental cuts imposed for practical reasons – to suppress backgrounds etc. – often essentially degrade the effective hardness \( Q \) of the process. This brings in another expansion parameter \( 1/Q \) effectively replacing \( 1/m_b \) in certain QCD effects. The reliability of the expansion greatly deteriorates for \( Q \ll m_b \). This phenomenon is particularly important in \( b \to s + \gamma \) decays where experiments so far have imposed cut \( E_\gamma > 2 \text{ GeV} \) or even higher.

The theoretical aspects of such limitations have been discussed during the last couple of years [2, 3, 4]. In particular, the effective ‘hardness’ of the inclusive \( b \to s + \gamma \) decays with \( E_\gamma > 2 \text{ GeV} \) amounts to only about 1.25 GeV, which casts doubts on the precision of the routinely used expressions incorporated into the fits of heavy quark parameters.

In the present letter we point out that these effects are numerically significant, lead to a systematic bias that often exceeds naive error estimates and therefore cannot be ignored. Evaluating them in the most straightforward (although somewhat simplified) way we find, for instance for \( b \to s + \gamma \) decays

\[
\tilde{m}_b \simeq m_b + 70 \text{ MeV} \\
\tilde{\mu}_\pi^2 \simeq \mu_\pi^2 - (0.15 \div 0.2) \text{ GeV}^2 \tag{1}
\]

where \( \tilde{m}_b \) and \( \tilde{\mu}_\pi^2 \) are the apparent values of the \( b \) quark mass and of the kinetic expectation value, respectively, as extracted from the \( b \to s + \gamma \) spectrum with \( E_\gamma > 2 \text{ GeV} \) in a usual way. Correcting for these effects eliminates alleged problems for the OPE in describing different data and rather leads to a too good agreement between the data on different types of inclusive decays.

Moreover, this resolves the controversy noted previously: while the values of \( \bar{\Lambda} \) and \( \mu_\pi^2 \) reportedly extracted from the CLEO \( b \to s + \gamma \) spectrum were found to be significantly below the theoretical expectations, the theoretically obtained spectrum itself turned out to yield a good description of the observed spectrum when we evaluated it based on these theoretically preferred values of parameters [2].

The bias in Eqs. (1) depends on the position of the cut (more precisely, on the gap \( \frac{m_b}{2} - E_{\text{cut}} \)) and the actual values of other heavy quark parameters. The quoted estimates assume moderate values, \( m_b(1\text{ GeV}) \simeq 4.6 \text{ GeV} \) and \( \mu_\pi^2(1\text{ GeV}) \simeq 0.43 \text{ GeV}^2 \). If \( m_b \) becomes lower and/or the true \( \mu_\pi^2 \) increases as may be indicated by the most recent data, the bias increases further.
**OPE and cuts** The origin of these effects and why they are missed in the standard application of the OPE and in estimates of the theoretical accuracy, have been discussed elsewhere [4]. In brief, considering a constrained fraction of the $B \to X_s + \gamma$ events

$$1 - \Phi_\gamma(E) = \frac{1}{\Gamma_{bs\gamma}} \int^M_B E \, dE \frac{d\Gamma_{bs\gamma}}{dE_\gamma}. \quad (2)$$

(or similarly truncated photon energy moments), the simple-minded approach routinely expands the spectrum in powers of $1/m_b$. Ignoring perturbative bremsstrahlung one obtains a $\delta$-like spectrum peculiar for two-body decays, and the expansion around the free-quark kinematics does not change this – it only generates higher derivatives of $\delta(E_\gamma - m_b/2)$:

$$\frac{1}{\Gamma^0_{bs\gamma}} \frac{d\Gamma_{bs\gamma}}{dE_\gamma} = a \delta(E_\gamma - m_b/2) + b \delta'(E_\gamma - m_b/2) + c \delta''(E_\gamma - m_b/2) + \ldots \quad (3)$$

where $a$, $b$, ... are given by the $B$ meson expectation values of local $b$-quark operators. Naively computing $1 - \Phi_\gamma(E)$, or spectral moments over the restricted domain in this way would yield unity in Eq. (2) for any $E > m_b/2$ – a result clearly meaningless on physical grounds. The actual behavior of the spectrum and the moments is described by the heavy quark distribution function. Its tail is indeed exponentially suppressed by a typical factor $e^{-c Q/\mu_{hadr}}$ at $Q(E_\gamma) \gg \mu_{hadr}$. However, for $Q(E_\gamma) \sim \mu_{hadr}$ there is little suppression of the missed tail contribution; the error becomes of order one.

This obvious point is missed in the naive application of the OPE and in the way to gauge the theoretical uncertainty based on it. Conceptually this is related to the limited range of convergence of the OPE for the width, determined in this case by the support of the heavy quark distribution function [4].

Here we rather concentrate on the numerical consequences for $B \to X_s + \gamma$ decays. To this end we first turn off perturbative effects altogether. The spectrum then is described by the nonperturbative light-cone distribution function $F(k_+)$:

$$\frac{1}{\Gamma} \frac{d\Gamma}{dE_\gamma} = 2F(2E_\gamma - m_b). \quad (4)$$

Although not necessary for our purpose, one can imagine a theoretical heavy quark limit with fixed hardness $Q$:

$$Q \equiv m_b - 2E_{cut} = \text{fixed}, \quad m_b \to \infty. \quad (5)$$

The fully integrated moments of $F(x)$ and therefore of the spectrum give then directly the underlying heavy quark parameters:

$$\int_0^\infty k F(\Lambda - k) \, dk = \Lambda, \quad \int_0^\infty (k - \Lambda)^2 F(\Lambda - k) \, dk = \frac{\mu^2}{3}, \text{ etc.} \quad (6)$$

As mentioned above, in the standard practical-OPE–based formulae these relations remain the same for the moments with the cut as well (provided $E_{cut} < m_b/2$ which is always
assumed) – yet not in reality. Paralleling the routinely used way we therefore introduce
\[
\bar{\Lambda}(E_{\text{cut}}) = \frac{\int_{E_{\text{cut}}} (M_B - 2E_\gamma) \frac{d\Gamma}{dE_\gamma} dE_\gamma}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dE_\gamma} dE_\gamma}, \quad \tilde{\mu}_\pi^2(E_{\text{cut}}) = 3 \left[ \frac{\int_{E_{\text{cut}}} (M_B - 2E_\gamma)^2 \frac{d\Gamma}{dE_\gamma} dE_\gamma}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dE_\gamma} dE_\gamma} - \bar{\Lambda}^2(E_{\text{cut}}) \right].
\]

Clearly the apparent value \(\bar{\Lambda}(E_{\text{cut}})\) is always below the actual \(\bar{\Lambda}\). This is illustrated by Fig. 1a where we plot the cut-related ‘bias’ – the difference between \(\bar{\Lambda}\) and \(\bar{\Lambda}(E_{\text{cut}})\) as a function of energy \(E_{\text{cut}}\). It turns out to be quite significant.

\[\text{Figure 1:} \quad \text{The shifts } \delta m_b = m_b - \bar{m}_b \text{ in the quark mass (a) and } \delta \mu_\pi^2 = \mu_\pi^2 - \tilde{\mu}_\pi^2 \text{ in the kinetic operator (b) introduced by imposing a lower cut in the photon energy in } B \to X_s + \gamma. \text{ Blue and maroon curves correspond to two different ansätze for the heavy quark distribution function, } F_1 \text{ and } F_2, \text{ respectively.}
\]

The naive extraction of the kinetic expectation value through the variance of the truncated distribution undercounts it even more dramatically as Fig. 1b illustrates, since higher moments are more sensitive to the tail of the distribution.

To gauge the sensitivity to the choice of the heavy quark distribution function we follow Ref. [2] evaluating the effect for two ansätze – one exponential in \(k_+\), and the other in \(k_+^2\) yielding an even faster decreasing tail:
\[
F_1(k_+) = N_1 (\bar{\Lambda} - k_+)^\alpha e^{ck_+} \theta(\bar{\Lambda} - k_+), \quad F_2(k_+) = N_2 (\bar{\Lambda} - k_+)^\beta e^{-d(\bar{\Lambda} - k_+)^2} \theta(\bar{\Lambda} - k_+); \quad (8)
\]

the parameters are adjusted in such a way as to yield the same actual \(m_b\) (\(\bar{\Lambda}\)) and \(\mu_\pi^2\). (For \(m_b = 4.6\) GeV and \(\mu_\pi^2 = 0.43\) GeV\(^2\) we have \(\alpha = 2.22\) and \(\beta = 0.773\).) Curiously, the ‘deficit’ \(\bar{\Lambda} - \bar{\Lambda}(E_{\text{cut}})\) practically does not depend on the choice at \(E_{\text{cut}}\) around 2 GeV, and even the deficit in \(\mu_\pi^2\) is reasonably stable.

Why are the effects of the cut so significant? They are exponential in the inverse hadronic scale \(\mu_{\text{hadr}}\), but are governed by the hardness \(Q \approx m_b - 2E_{\text{cut}}\) rather than by \(m_b\):
\[
\bar{\Lambda} - \bar{\Lambda}(E_{\text{cut}}) \propto \mu_{\text{hadr}} e^{-\frac{Q}{\mu_{\text{hadr}}}}, \quad \mu_\pi^2 - \tilde{\mu}_\pi^2 \propto \mu_{\text{hadr}}^2 e^{-\frac{Q}{\mu_{\text{hadr}}}}; \quad (9)
\]
(the exponent may have a form of a power of \( Q/\mu_{\text{hadr}} \)). Even at \( m_b \to \infty \) these effects survive unless \( Q \) is made large as well!

As explained in Ref. [4] the bias terms (9) are associated from a theoretical viewpoint with the limited (in fact, zero) convergence radius of the OPE. This becomes practically relevant due to the presence of a subseries in powers of \( 1/Q \) rather than \( 1/m_b \). The limitations on convergence appear due to a factorial growth of the power coefficients, a rather universal property of the OPE. In this respect one may associate this effect with quark-hadron duality [5]. Yet it has no features peculiar to local quark hadron duality violation intrinsic to inclusive decay widths in actual Minkowski world. (For a discussion of the notorious subtleties in the notion of quark-hadron duality, see reviews [6, 5]). For instance, these effects are truly exponential and do not oscillate.1

The validity of the routinely applied expressions for the moments with cuts for actual decays is additionally complicated by perturbative corrections. Incorporated into fits are naive sums of pure perturbative and pure nonperturbative terms:

\[
M_n^{\text{np}} \to M_n^{\text{np}} + M_n^{\text{pert}}(\alpha_s, m_b, E_{\text{cut}}) \tag{10}
\]

where nonperturbative corrections to the moments \( M_n \) still do not depend on \( E_{\text{cut}} \). This is not true in general, but would hold if the actual spectrum were exactly a convolution of the perturbative and nonperturbative spectra,

\[
\frac{d\Gamma}{dE_\gamma} = \int dk \frac{d\Gamma^{\text{pert}}(E_\gamma - k)}{dE} \frac{d\Gamma^{\text{np}}(m_b^2 + k)}{dE} \tag{11}
\]

provided no cut is introduced (or its effect on the pure nonperturbative distribution is negligible). We hasten to add, though that the effects we consider are unrelated to this complication and rather represent an independent phenomenon – they are significant even in the complete absence of perturbative corrections.

Since the perturbative effects can potentially modify the effect, we have evaluated the cut-induced deficit in \( \bar{\Lambda} \) and \( \bar{\mu}^2_\pi \) including short-distance corrections. Namely, we considered \( \bar{\Lambda} \), \( \bar{\mu}^2_\pi \) in Eqs. (7) for the complete spectrum obtained by the convolution (11) of the perturbative and primordial (nonperturbative) ones, and compared them to the naive sum Eq. (10) which would indeed hold for a sufficiently low cut. Including the perturbative spectrum as detailed in Ref. [2] we found no appreciable change in \( \bar{\Lambda} - \bar{\Lambda} \) or \( \mu^2_\pi - \bar{\mu}^2_\pi \) at realistic cuts (a small increase emerged only at \( E_{\text{cut}} \approx 1 \text{ GeV} \)).

Based on these results we conclude:

\begin{itemize}
  \item the value of \( m_b \) as routinely extracted from the \( b \to s + \gamma \) spectrum is to be decreased by an amount of order 70 MeV;
  \item relative corrections to \( \mu^2_\pi \) are even more significant and can naturally constitute a shift of 0.2 GeV². This arises on top of other potential effects.
\end{itemize}

**Practical implications** Accepting the above shifts at face value and using the rather arbitrary choice \( m_b = 4.595 \text{ GeV} \), \( m_c = 1.15 \text{ GeV} \), \( \mu^2_\pi = 0.45 \text{ GeV}^2 \), \( \bar{\rho}_D^3 = 0.06 \text{ GeV}^3 \) and

---

1Peculiarities of real local duality would appear here only at the next-to-leading order in \( 1/m_b \) and, therefore are not of much interest.
\( \rho_{LS}^3 = -0.15 \text{ GeV}^3 \) adjusted to accommodate \( \langle M_X^2 \rangle_{E_\ell > 1 \text{ GeV}} \), we obtain

\[
\langle M_X^2 \rangle \simeq 4.434 \text{ GeV}^2 \quad [\text{cf. (4.538 \pm 0.093) GeV (New DELPHI)}]
\]

\[
\langle M_X^2 \rangle_{E_\ell > 1.5 \text{ GeV}} \simeq 4.177 \text{ GeV}^2 \quad [\text{cf. 4.180 GeV}^2 \text{ (BaBar), 4.189 GeV}^2 \text{ (CLEO)}]
\]

\[
\langle E_\ell \rangle \simeq 1.389 \text{ GeV} \quad [\text{cf. (1.383 \pm 0.015) GeV (DELPHI)}]
\]

\[
\langle E_\gamma \rangle_{E_\gamma > 2 \text{ GeV}} \simeq 2.329 \text{ GeV} \quad [\text{cf. (2.346 \pm 0.034) GeV (CLEO)}]
\]

\[
\langle E_\gamma^2 - \bar{E}_\gamma^2 \rangle_{E_\gamma > 2 \text{ GeV}} \simeq \frac{0.0202}{0.0233} \text{ GeV}^2 \quad [\text{cf. (0.0226 \pm 0.0066 \pm 0.0020) GeV}^2 \text{ (CLEO)}] \quad (12)
\]

(experimental data are from Refs. [7, 8, 9, 10]). \( E_\ell^\text{cut} \)-dependence of \( \langle M_X^2 \rangle \) is also reproduced, see Fig. 2. The counterpart of the above exponential cut-related effects for the semileptonic decays has not been incorporated here, however.

Figure 2: Experimental values of the average hadronic mass square \( \langle M_X^2 \rangle \) in \( B \to X_c \ell \nu \) at different lower cuts in \( E_\ell \) and the literal OPE prediction (blue curve) for the stated heavy quark parameters. The DELPHI point assumes no cut on \( E_\ell \).

It should be noted that the cut-induced shifts \( \tilde{m}_b - m_b \), \( \tilde{\mu}_s^2 - \mu_s^2 \) are not unambiguously determined in terms of a few known HQ parameters, but rather depend on the actual shape of the heavy quark distribution function; in particular it is driven by the asymptotics of certain high-dimension expectation values. The evaluation presented above relies on the

\(^2\text{The two values for the second } E_\gamma \text{-moment correspond to the two ansätze; they are obtained discarding higher-order power corrections to the light-cone distribution function.}\)
most natural assumptions about the function. Strictly speaking, they can vary for more contrived ansätze. Therefore, the estimates in Eqs. (1) can be conservatively viewed as the minimal possible theoretical inaccuracy of the usual naive evaluations. The safest way to tackle possible ambiguity is to resort to more inclusive moments without too severe cuts.

**Cuts in semileptonic moments.** As argued in Ref. [4] similar cut-related ‘exponential’ biases missed in the naive OPE applications affect the truncated moments in the semileptonic decays as well. Their description, even simplified is less transparent and would be more involved, though. In particular, the light-cone distribution functions for $b \to s + \gamma$ is replaced by a different function, the form of which actually is not universal. Yet the qualitative trend is expected the same – it should smoothly interpolate the case of literal OPE expression at low $E_\ell$ cuts and the values at a high cut dictated simply by the actual hadrons kinematics. This would replace step- or $\delta$-like behavior in the formal OPE expressions.

Numerical aspects are less certain. Keeping in mind that for $B \to X_c \ell \nu$ the effective hardness

$$Q_{sl} \simeq m_b - E_{cut} - \sqrt{E_{cut}^2 + m_c^2}$$

at $E_{cut} = 1.5$ GeV is about 1.25 GeV [3], nearly the same as for $B \to X_s + \gamma$ with $E_\gamma \gtrsim 2$ GeV, we may expect quite significant effects. To get a rough idea of the possible magnitude of the bias we can use a simplified rule of thumb – assume that $m_b$ in the semileptonic decay can be just replaced by an effective larger value $\tilde{m}_b$ apparent in $b \to s + \gamma$ at the commensurate cut yielding the same hardness. In other words, we mimic the effect of decreasing hardness by an additional effective nonperturbative running of the heavy quark mass at low scales. As illustrated above, this is to increase $m_b$ (or, equivalently decrease $\Lambda$) by about 70 MeV, a significant change.

Yet the semileptonic decay characteristics strongly depend on both $m_b$ and $m_c$. To stay on the conservative side we assume the apparent shift in $m_c$ as high as in $m_b$ (which would reflect just the heavy flavor symmetry):

$$m_c \to \tilde{m}_c \approx m_c + (\tilde{m}_b - m_b).$$

(14)

In actuality the corrections to $m_c$ are typically somewhat smaller due to $1/m_c^k$ terms.

As discussed elsewhere [11], for actual $B$ decays both lepton moments and $\langle M_X^2 \rangle$ depend on more or less the same combination of masses $m_b - 0.65m_c$. This means that the literal ansatz (14) would suppress the effect by a factor of 3 to 4, yet $1/m_c$ corrections may be thought to softening this suppression. We then expect the exponential terms in semileptonic decays with $E_{cut} \simeq 1.5$ GeV to introduce effects on the same scale as shifting $m_b$ upward by up to 25 to 30 MeV (assuming fixed $m_c$ and other heavy quark parameters). This rule of thumb is useful to get an idea of the ultimate theoretical accuracy one can count on.

For example, the CLEO’s cut moment $R_1 = \langle E_\ell \rangle_{E_\ell > 1.5 \text{GeV}}$ is approximately given by [3]

$$R_1 = 1.776 \text{ GeV} + 0.27(m_b - 4.595 \text{ GeV}) - 0.17(m_c - 1.15 \text{ GeV})$$

at $|V_{ub}/V_{cb}| = 0.08$ (15)
(the above mentioned values of the nonperturbative parameters are assumed). An increase in $m_b$ by only 20 MeV would then change

$$R_1 \rightarrow R_1 + 0.0055 \, \text{GeV} \quad (16)$$

and would perfectly fit the central CLEO’s value 1.7810 GeV. It is worth noting that Eqs. (15)-(16) make it explicit that the imposed cut on $E_\ell$ degrades the theoretical calculability of $R_1$ far beyond its experimental error bars, the fact repeatedly emphasized over the last year. Unfortunately, this was not reflected in the fits of parameters which placed much weight on the values of $R_0 - R_2$ just owing to their small experimental uncertainties, whilst paying less attention to actual theoretical errors.\(^3\)

Similar reservations apply to the quality of theory for CLEO’s $R_2$ representing the second moment with the cut, with the only difference that the effective hardness only deteriorates for higher moments.

The ratio $R_0$ is the normalized decay rate with the cut on $E_\ell$ as high as 1.7 GeV, and for it hardness $Q$ is below 1 GeV. A precision – beyond just semiquantitative – treatment of nonperturbative effects is then questionable, and far more significant corrections should be allowed for.

We note that there is a good agreement of most data referring to sufficiently ‘hard’ decay distributions with the theory based on the OPE in QCD, in the ‘robust’ OPE approach. The latter was put forward \([3]\) to get rid of vulnerable and unnecessary assumptions of the usually employed fits of the data. This consistency likewise refers to the absolute values of the heavy quark parameters necessary to accommodate the data. Theory anticipates, however that the expansion becomes deceptive with increase of the experimental cuts. Here we have addressed the most obvious effects, those from the variety of ‘exponential’ terms in the effective hardness. While presently not amenable to precise theoretical treatment, they can be estimated using quite natural assumptions and are found to be far too significant for $E_\text{cut}^\ell \gtrsim 1.5$ GeV and $E_\text{cut}^\gamma \gtrsim 2$ GeV often employed in experiment. Taking these corrections at face value and incorporating in our predictions, we get a good, more than qualitative agreement with “less short-distance” inclusive decays as well.

To summarize, the cuts essentially decreasing the hardness in $B$ decays introduce terms some of which are exponentially suppressed though only in the effective hardness, but not suppressed by powers of the heavy quark mass. They significantly change the extracted values of the heavy quark parameters leading to an apparent suppression of the magnitude of the nonperturbative ones in $B$ mesons. Accounting for such effects appears necessary in $B \rightarrow X_s + \gamma$ decays unless the cut on photon energy is pushed well below present 2 GeV. This brings the practical predictions for various inclusive characteristics into a good agreement. Moreover, it seems that the second moment of the photon spectrum discarded for the fits so far, does impose informative constraints\(^3\)

\(^3\)N.U. acknowledges that a similar in spirit criticism of the theory error treatment in \([12]\) was expressed by D. Hitlin at the BaBar Workshop, SLAC December 2002.
provided these effects are properly incorporated.

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