Direct measurement of single fluxoid dynamics in superconducting rings

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We have measured the dynamics of individual magnetic fluxoids entering and leaving photolithographically patterned thin film rings of underdoped high-temperature superconductor Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$, using a variable sample temperature scanning SQUID microscope. These measurements can be understood within a phenomenological model in which the fluxoid number changes by thermal activation of a Pearl vortex in the ring wall. We place upper limits on the “vison” binding energy in these samples from these measurements.

Although there is a vast literature on vortex dynamics in superconductors [1], with a few notable exceptions [2,3] this work has involved indirect measurements of collective motions of vortices through, to cite a few recent examples, transport [4–7], voltage noise [8], magnetization loops [9], persistent currents in rings [10,11], or microwave impedance [12]. In this Letter, we present direct measurements of the dynamics of individual magnetic fluxoids leaving and entering superconducting rings with a well defined geometry. These measurements represent a new tool for studying vortex dynamics.

Our measurements were made on 300 nm thick films of the high-temperature superconductor Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (BSCCO), epitaxially grown on (100) SrTiO$_3$ substrates using magnetron sputtering. The oxygen concentration in these films was varied by annealing in oxygen or argon at 400-450 °C. The films were photolithographically patterned into circular rings using ion etching. The rings had outside diameters of 40, 60, and 80 µm, with inside diameters half the outside diameters. The film for the current measurements had a broad resistive transition (90% of the extrapolated normal state resistance at T=79 K, 10% at T=46 K) before patterning. After patterning the rings had T$_c$’s from 25 K to 33 K, as indicated by inductive measurements. The rings were cooled in a magnetic field sufficient to trap one vortex in each ring. The dots in Figure 1(b) are a cross-section through the center of one ring. The curve in Figure 1(b) is modeled as follows:

Consider a thin film ring of thickness d ≪ λ_L (the London penetration depth) with radii a < b in the plane z = 0. The London equations for the film interior read

$$j = -\frac{c\Phi_0}{8\pi^2\lambda_L^2} \left( \nabla \theta + \frac{2\pi}{\Phi_0} A \right),$$

where j is the supercurrent density, $\Phi_0 = hc/2e$ is the superconducting flux quantum, $\theta$ is the order parameter phase, and $A$ is the vector potential. Since the current in the ring must be single valued, $\theta = -N \varphi$, where $\varphi$ is the azimuth and the integer $N$ is the winding number (vorticity) of the state. Integrating j over the film thickness d, we obtain:
\[ g_{\varphi} = g(r) = \frac{c\Phi_0}{4\pi^2\lambda} \left( \frac{N_A}{r} - \frac{2\pi}{\Phi_0} A_{\varphi} \right), \]

where \( g(r) \) is the sheet current density directed along the azimuth \( \varphi \), and \( \Lambda = 2\lambda^2/d \) is Pearl’s film penetration depth \([13]\). The vector potential \( A_{\varphi} \) can be written as

\[ A_{\varphi}(r) = \int_{a}^{b} dp \, g(p) a_{\varphi}(p; r, 0) + \frac{r}{2} H, \]

where the last term represents a uniform applied field \( H \) in the \( z \) direction and \( a_{\varphi}(p; r, z) \) is the vector potential of the field created by a circular unit current of a radius \( p \): \([13]\)

\[ a_{\varphi}(p; r, z) = \frac{4}{ck} \sqrt{\frac{\rho}{r}} \left[ \left( 1 - \frac{k^2}{2}\right) K(k) - E(k) \right], \]

\[ k^2 = \frac{4\rho r}{(\rho + r)^2 + z^2}. \]

Here, \( K(k) \) and \( E(k) \) are the complete elliptic integrals in the notation of Ref. \([16]\).

Substituting Eq. (3) and (4) into (2), we obtain an integral equation for \( g(r) \):

\[ \frac{4\pi^2\Lambda}{c} \int_{-\rho}^{\rho} dp \, g(p) \left[ \frac{\rho^2 + r^2}{\rho + r} K(k_0) - (\rho + r) E(k_0) \right], \]

where \( k_0^2 = 4\rho/(\rho + r)^2 \). This equation is solved by iteration for a given integer \( N \) and field \( H \) to produce current distributions which we label as \( g_N(H, r) \).

After \( g_N(H, r) \) is found, the field outside the ring can be calculated using Eq. (3):

\[ h_z(N; r, z) = \frac{2}{c} \int_{a}^{b} dp \, g_N(H, \rho) \left[ K(k) \right. \left. + \frac{\rho^2 - r^2 - z^2}{(\rho - r)^2 + z^2} \right] E(k) + H. \]

The flux through the SQUID is obtained numerically by integrating Eq. (3) over the pickup loop area. The line in Fig. 1b is a two parameter fit of this integration of Eq. (3) to the data, resulting in \( z = 3.5 \mu m \), and \( \Lambda = 9 \mu m \) (corresponding to \( \lambda_L = 1.1 \mu m \)).

The fluxoid number \( \Phi_a \) of a ring could be changed by varying an externally applied flux \( \Phi_a = A_{eff} H \), \( A_{eff} \approx \pi(a^2 + b^2)/2 \) \([13]\), and monitored by positioning the SQUID pickup loop directly over it \([3]\). We always observed single fluxoid switching events, as determined by the agreement (to within 10%) of the measured spacing in applied flux between vortex switching events with our calculations for \( |\Delta N| = 1 \), in the experiments reported here. Switching distributions \( P(\Phi_a,i) \) were obtained by repeatedly sweeping the applied field, in analogy with experiments on Josephson junctions \([20]\). The transition rates \( \nu \) of the fluxoid states were determined from this data using

\[ \nu(\Phi_{a,m}) = \frac{d\Phi_a/dt}{\Delta\Phi_a} \ln \left\{ \frac{\sum_{j=1}^{m} P(\Phi_{a,j})}{\sum_{i=1}^{m-1} P(\Phi_{a,i})} \right\}, \]

where \( m = 1 \) labels the largest \( \Phi_a \) in a given switching histogram peak \([21]\), and \( \Delta\Phi_a \) is the flux interval between data points.

\[ \Phi_a/\Phi_0 \]

\[ N = -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \]

\[ \nu \quad (sec^{-1}) \]

\[ E_r(N) = \frac{\Phi_a^2}{2L}(N - \phi_a)^2, \]

where \( \phi_a = \Phi_a/\Phi_0 \), and \( L \) is the inductance of the ring. We take the ring maximum energy during the transition \( N \rightarrow N' \) to be

\[ E_{\nu}(N, N') = E_{\nu} - \alpha(N + N')/4 + (E_r(N) + E_r(N'))/2, \]

where the first two terms on the right hand side represent the energy required to nucleate a vortex in the ring wall. The maximum vortex energy in a straight thin film superconducting strip of width \( W \ll \Lambda \) (carrying no transport supercurrent) is \([22]\)

\[ E_{\nu} = \frac{\Phi_a^2}{8\pi^2\Lambda} \ln \left\{ \frac{2W}{\pi \xi} \right\}. \]
Since \( \lambda_L \propto 1/\sqrt{1-t^4} \) (\( t = T/T_c \)), we take \( E_V = E_{V0}(1-t^4) \). \( \alpha \) is a fitting parameter which describes the reduction of the vortex nucleation energy with increasing \( N \). For thermal activation, the \( N \to N' \) transition rate is \( \nu = \nu_0 e^{-E_{n}(N,N')/k_B T} \), where \( \nu_0 \) is an attempt frequency and the activation energy \( E_{n}(N,N') = E_l(N,N') - E_l(N) \). For this ring (from fits to the data of Figs. 2, 3 and 4) we take \( E_{V0}=6 \times 10^{-13} \) erg, \( \alpha=1.7 \times 10^{-16} \) erg, and \( \Phi_0^2/2L=4.6 \times 10^{-14} \) erg. Taking \( \Lambda = 200 \mu m \) at \( T=31.5 \) K from fits of Eq. (1) to SSM data, \( \xi = 3.2/\sqrt{1-t} \) nm, and \( W = 20 \mu m \) we find \( E_{V0} = 1.55 \times 10^{-12} \) erg, a factor of 2.6 larger than the \( 6 \times 10^{-13} \) erg from our fits. It has been proposed that surface defects could reduce the barrier to entry of vortices in type-II superconductors. If we calculate an effective inductance for the ring as \( L^* = \Phi_0/I_s \), where \( I_s = \int_0^d d^3r \rho_N(\mathbf{H} \cdot \mathbf{r}) \), and use the solution of Eq. (2) for \( N = 1 \), \( H = 0 \), and \( \Lambda = 200 \mu m \), we find \( \Phi_0^2/2L^*=1.8 \times 10^{-14} \) erg, a factor of 2.6 smaller than the \( 4.6 \times 10^{-14} \) erg from our fit. For ring temperatures close to \( T_c \), telegraph noise (Figure 3a) from thermally activated switching \( N \leftrightarrow N' \) occurs. This noise peaks when \( \Phi_a = (N+1/2)\Phi_0 \), \( N \) an integer. Figure 3b shows the telegraph noise frequency (number of steps up per sec) as a function of applied field for the ring of Figure 2 at a temperature \( T=31.6 \) K. The line is the prediction of the model described above, using the same fitting parameters, and writing the telegraph noise frequency as \[ \nu = \frac{2\nu_0 e^{-E_{in}/k_B T} e^{-E_{out}/k_B T}}{e^{-E_{in}/k_B T} + e^{-E_{out}/k_B T}}, \] where \( E_{in} = E_n(N,N+1) \) and \( E_{out} = E_n(N+1,N) \). Finally, the symbols in Figure 4 show the telegraph noise frequency for the same ring as in Figures 2 and 3, as a function of temperature with \( \Phi_a = \Phi_0/2 \). The line is the prediction of our simple model, with the same fitting parameters as above.

The data shown in Figures 2, 3, and 4 are all consistent with an attempt frequency \( \nu_0 \approx 3 \times 10^8 \) sec\(^{-1}\). Glazman and Fogel \[26\], in a treatment of quantum tunneling of vortices, write \( \nu_0 = \sqrt{\Phi_0 B_{c2}/4\pi \Delta m} \), where the vortex mass \( m = \hbar \eta/\Delta \), and the damping parameter \( \eta = \Phi_0 B_{c2} d/\rho n c^2 \). Taking the second critical field \( B_{c2} \) equal to 1T for \( T_c-T=1 \)K, the normal state sheet resistivity \( \rho_n=1200 \mu \)Ohm-cm, and \( \Delta = 5k_B T_c = 2.24 \times 10^{-14} \) erg, we find \( \nu_0 = 9.7 \times 10^8 \) sec\(^{-1}\), within a factor of three of our measurements. A second estimate, in a treatment of thermal activation of vortices \[27\], is \( \nu_0 = 6.96(D/a^2)\sqrt{E_V/k_B T} \), where the diffusion constant \( D = k_B T/\eta \). Using \( a=40 \mu m \), and the other parameters the same as before, this expression gives \( \nu_0 = 1.5 \times 10^8 \) sec\(^{-1}\). The disparity between these two estimates provides a measure of the uncertainties involved.

FIG. 3. a) Telegraph noise signal vs. time for the ring of Figure 2 at \( T=31.4 \) K, \( \Phi_a = \Phi_0/2 \). b) Telegraph noise frequency vs externally applied field \( \Phi_a/\Phi_0 \) for telegraph noise of this ring at \( T=31.6 \) K.

FIG. 4. Telegraph noise frequency as a function of temperature, with \( \Phi_a = \Phi_0/2 \) for the ring of Figures 2 and 3 (dots). The line is the prediction of the model discussed in the text.
in calculating attempt frequencies.

Senthil and Fisher \cite{29} and Sachdev \cite{30} have proposed tests of the idea that the electron is fractionalized in the high-\(T_c\) cuprate superconductors. The Senthil-Fisher proposal is to look for persistence of vorticity in underdoped cuprate cylinders as they are cycled through the superconducting transition temperature. This experiment has been performed by Bonn et al. \cite{30} on single crystals of YBa\(_2\)Cu\(_3\)O\(_{6.3}\), and by us on the present ring samples, with no evidence to date for this persistent vorticity. The predicted effect depends on the existence of a gapped topological excitation, dubbed a “vison”, that is associated with a conventional vortex below the superconducting transition temperature, but which persists up to the pseudogap temperature. The present experiments provide another test of these ideas. It has been proposed that the vison should have a binding energy above the superconducting transition temperature of order \(E_{\text{vison}} \sim k_B T^*\) \cite{29}. The pseudogap temperature \(T^*\) is estimated to be approximately 300 K \cite{31} for BSCCO with a \(T_c\) of 30 K. If we assume that the vortex thermal activation energy at \(T_c\) is equal to the vison binding energy \(E_{\text{vison}}\) (since the vortex core and supercurrent contributions to the vortex energy have gone to zero), then the telegraph noise frequency extrapolated to \(T_c\) should be given by \(\nu(T = T_c) = \nu_0 e^{-E_{\text{vison}}/k_B T_c}\), or \(E_{\text{vison}} = k_B T_c \ln(\nu_0/\nu(T = T_c))\). The larger of our two estimates for the attempt frequency \(\nu_0\) would lead to \(E_{\text{vison}}/k_B \sim 60 K\). The smaller estimate would indicate a smaller binding energy. A conservative estimate for an upper limit of the attempt frequency would be \(\nu_0 \sim c/b = 4 \times 10^{12} \text{ sec}^{-1}\), where \(c\) is the speed of light, and \(b\) is the outer radius of the ring, which would lead to an upper limit for \(E_{\text{vison}}/k_B \sim 300 K\).

In conclusion, we have demonstrated a technique for measuring the dynamics of single vortices on a relatively short time scale, limited by the modulation frequency (100KHz) of our SQUID electronics (modulation frequencies 10\(^5\) times faster have been demonstrated). These measurements were made possible by using cuprate superconductors that were highly underdoped, so that the penetration depths were longer than the ring wall lengths, and the vortex activation energies were comparable to the temperature, over an appreciable temperature range below \(T_c\). Although these measurements were apparently made in a regime where the fluxoid transitions were mediated by thermally activated Pearl vortices, it may be possible to use similar techniques to study single vortex tunneling \cite{29}.

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