Applications of Markov Chain in Forecast

XIA YUTONG
Ul ink College, Guangzhou, Guangdong, 511458, China
*Corresponding author’s e-mail: yuxia2186@ulinkcollege.com

Abstract. The article is going to introduce Markov chain and its application into business. Two main daily life examples are applied for the explanation of this theory. In the Markov process, the probability of one state only depends on the next previous state. A mathematical model of market forecast and weather forecast is able to build through constructing transition probability matrix, analysing and computing with Markov chain. According to the data and calculation verification, we can see Markov Chain is a scientific, effective and convenient method for prediction and can be used for solving daily issues.

1. Introduction

In the production, operation, and management of enterprises, the evolution of Markov Chain does not depend on the past and only affects by the situation in current. Markov Chain process is widely used in communication, biology, society, science and other academic areas, which is especially used for sales conditions prediction, interest rates analysis, education evaluation, natural disasters and epidemic forecasts, etc.

The following approaches set up the mathematical mode of Markov chain to do forecast about a business’ future market share and to a weather forecast in a specific region. The data were used for the first approach is hypothetical while there are high validity numerical data be used for the weather forecast one. The two aspects use of application for Markov chain is more likely to reflect the benefits of that. The significance of weather forecast in our daily life is prominent. For example, diseases could be prevent to some extent because people are able to choice increase or decrease their clothes according to the weather condition. Besides, many of the labourers such as farmers are also need to know the forecast of weather since they need to make sure their plants are grown under correct temperature and humidity. Without the assist of prediction, numerous things in our life can not be forecast in advance, therefore some troubles may occur because people didn’t prepare.

2. Markov Chain

Markov chain is a stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event[1]. Frankly speaking, which is a process the forecast only regarding to the future outcomes on the foundation of current situation. The basic concepts of that were introduced by Andrei Markov in 1907, since then which was developed by a number of leading mathematicians, such as A. Kolmogorov, W. Feller etc. [2]

The definition can be expressed in mathematical terms as follow:
A stochastic process X = {Xn, n ε N} in a countable space S is a discrete-time Markov chain if:
For all n ≥ 0, Xn ε S
For all n ≥ 1 and for all i0, ...i n−1, in εS, we have:
P{Xn = in |Xn−1 = in−1, ..., X0 =i0 } =P{Xn =in|Xn−1 =in−1 } [3]
There are two main types of Markov Chain which determine by the various of state space. The first type of Markov Chain defined as the discrete-time Markov chain, which is on a countable and measurable state space, the Harris chain is an example for that. The Continuous-time Markov Chain is another type of that. The state space for that is continuity and the Wiener process is an example of that.

3. Weather Forecast

Markov Chains are used to calculate for the probabilities of events express in the form of transition matrix. For example, according to the weather information of London, we can make a transition diagram as below [In fig. transition matrix of weather in London]:

![Transition matrix of weather in London](image)

Figure 1. Transition matrix of weather in London

If today is a sunny day, an average of 85% that the next day is another sunny day and only 15% that the next day is a cloudy day. Besides, if it's a rainy day today, then 55% chance that the next day will be another rainy day and 45% chance that it will be a sunny day.

In our weather forecast example, we define $S = [\text{Sunny} \hspace{1em} \text{Rainy}]$. Therefore, assumed that today is a sunny day, we define $S_0 = [1 \hspace{1em} 0]$ because there is hundred percent of a sunny day and zero chance of a rainy day.

A transition matrix is defined as

$$P = \begin{bmatrix} S & R \\ \hline S & 0.85 & 0.15 \\ R & 0.45 & 0.55 \end{bmatrix}$$

Therefore, the next state, $S_1$, we can calculate the matrix product $S_1 = S_0P$.

$$\begin{bmatrix} 1 \hspace{1em} 0 \end{bmatrix} \cdot \begin{bmatrix} 0.85 & 0.15 \\ 0.45 & 0.55 \end{bmatrix} = \begin{bmatrix} 1 \cdot 0.85 + 0.15 \cdot 0 \\ 0.45 \cdot 1 + 0.55 \cdot 0 \end{bmatrix} = \begin{bmatrix} 0.85 \\ 0.15 \end{bmatrix}$$

Therefore $P(R|S)=0.15$, $P(S|S)=0.85$

Since the formula for computing successive state is $S_n = S_0P^n$, the general formula for probability of a process ending up in a certain state is: $S_n = S_0P^n$ [4]

Under the situation of today is a sunny day, the weather of third day after today is computing by:

3 days after today: $S_3 = S_0P^3 = \begin{bmatrix} 1 \hspace{1em} 0 \end{bmatrix} \cdot \begin{bmatrix} 0.85 & 0.15 \\ 0.45 & 0.55 \end{bmatrix}^3 = \begin{bmatrix} 0.766 \\ 0.236 \end{bmatrix}$

5 days after today: $S_5 = S_0P^5 = \begin{bmatrix} 1 \hspace{1em} 0 \end{bmatrix} \cdot \begin{bmatrix} 0.85 & 0.15 \\ 0.45 & 0.55 \end{bmatrix}^5 = \begin{bmatrix} 0.751 \\ 0.2489 \end{bmatrix}$

50 days after today: $S_{50} = S_0P^{50} = \begin{bmatrix} 1 \hspace{1em} 0 \end{bmatrix} \cdot \begin{bmatrix} 0.85 & 0.15 \\ 0.45 & 0.55 \end{bmatrix}^{50} = \begin{bmatrix} 0.750 \\ 0.250 \end{bmatrix}$

100 days after today: $S_{100} = S_0P^{100} = \begin{bmatrix} 1 \hspace{1em} 0 \end{bmatrix} \cdot \begin{bmatrix} 0.85 & 0.15 \\ 0.45 & 0.55 \end{bmatrix}^{100} = \begin{bmatrix} 0.750 \\ 0.250 \end{bmatrix}$

From this we can conclude that when $n \to \infty$, the probabilities will converge to a steady state, indicating that in the long-term, 75% of weather will be sunny day and 25% of weather will be rainy.
What we can see that the steady-state probabilities of this Markov chain do not depend upon the initial state[4].

4. Market Share Forecast
In this case, the Markov chain is going to use for predict the future market share for a multinational garments company K. Which could be used as a reference for K’s future decision despite the fact that which is risky. It’s able to set up a Markov model base on the problem and according to the transition probability to computing out the steady state probability after a time period. There are two mainly type of manufactures for K, the domestic one (in China) and overseas, there are some differences between the products produced in each factories. The frequency of customers buy the garments on an quarterly basis in average. According to the market research, we can see that 60% of consumers who buy K’ domestic product are willing to purchase it again within a quarter, while 45% of those who buy K’ international product are willing to turn to K’s domestic product within the quarter period. By compiling these probabilities into a table, we get the transition matrix M as following:

| Last quarter | Domestic E1 | Oversea E2 |
|--------------|------------|-----------|
| Domestic E1  | 0.6        | 0.4       |
| Oversea E2   | 0.45       | 0.55      |

\[ M = \begin{bmatrix} 0.6 & 0.4 \\ 0.45 & 0.55 \end{bmatrix} \]

Figure 2. Market trend for K’s product

In our market share forecast example, we define \( S = [\text{Domestic Oversea}] \). Thus, if the market share for K’s product in the market is 65% domestic and 35% oversea in last quarter, we define

\[
S_0 = \begin{bmatrix} 0.65 \\ 0.35 \end{bmatrix}.
\]

\[
M = \begin{bmatrix} 0.6 & 0.4 \\ 0.45 & 0.55 \end{bmatrix}
\]

The market share \( S_1 \) of K’s products in the market is:

\[
S_1 = S_0 \cdot M = \begin{bmatrix} 0.65 & 0.35 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.45 & 0.55 \end{bmatrix} = \begin{bmatrix} 0.5475 \\ 0.4525 \end{bmatrix}
\]

Thus, the next quarter’s market share for K’s product in the market is 54.8% in domestic and 45.3% oversea. Use the same theory, K’s market share for next few quarter is able to be computed:

1 quarters from now:

\[
S_1 = S_0 \cdot M^1 = \begin{bmatrix} 0.65 & 0.35 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.45 & 0.55 \end{bmatrix}^1 = \begin{bmatrix} 0.5475 \\ 0.4525 \end{bmatrix}
\]

3 quarters from now:

\[
S_3 = S_0 \cdot M^3 = \begin{bmatrix} 0.65 & 0.35 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.45 & 0.55 \end{bmatrix}^3 = \begin{bmatrix} 0.5298 \\ 0.4701 \end{bmatrix}
\]

5 quarters from now:

\[
S_5 = S_0 \cdot M^5 = \begin{bmatrix} 0.65 & 0.35 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.45 & 0.55 \end{bmatrix}^5 = \begin{bmatrix} 0.5294 \\ 0.4705 \end{bmatrix}
\]
20 quarters from now:

\[ S_{20} = S_0 \cdot M^{20} = \begin{bmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{bmatrix}^{20} = \begin{bmatrix} 0.5294 & 0.4705 \end{bmatrix} \] (11)

In a nutshell, the market share for K’s product for a very long term will in a steady state as 53% in domestic market and 47% in overseas market.

5. Conclusion

The theory of Markov Chain is an important concept in computing stochastic processes and which is also a successful combination of Linear Algebra and Probability theory. Accurate probability can always get through the use of Markov Chain whatever for long-term or short-term prediction. It also reduces the need for large amounts of data taken into account because of the property of Markov Chain-- the future probability only depends on the current state not the history one. However, not every forecast have high validity. Put market share forecast into the cases, the data used for prediction may not consist with the actual data and errors occurred. On top of that, the change of transmission matrix is one of the most prominent factors. To be specific, we assumed that the transmission matrix is steady during the time period. But in fact, many reasons such as change in fashion trend, income level increase, attractive promotion campaign and impressive innovative product all cause the unstable feature of transmission matrix, which led to the expansion of the error forecast. Thus, the prediction of one product’s market share after a long-term can not be used as the determining factor of decision-making, only act the role of a reference.

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