Suppressing decoherence by preparing the environment

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Abstract. To protect a quantum system from decoherence due to interaction with its environment, we investigate the existence of initial states of the environment allowing for decoherence-free evolution of the system. For a class of models in which a two-state system and a dynamical environment interact through a Hamiltonian restricted to be a tensor product, we prove that such states exist if and only if the interaction and self-evolution Hamiltonians of the environment share an eigenstate. If decoherence by state preparation is not possible, we show that initial states minimizing decoherence result from a delicate compromise between the environment and interaction dynamics.

1. Introduction
Decoherence provides an elegant framework to explain why an open quantum system coupled to its environment will exhibit a set of preferred states, usually ruling out a coherent superposition of arbitrary states. In [1], Zurek showed that even if the superposition principle treats all quantum states equally, the interaction between the quantum system and its environment would select a restricted number of “pointer” states (einselection) and destroy the phase coherence of superpositions of those pointer states (decoherence). This phenomenon presents a formidable challenge for such applications as quantum computation. In this simplest model of decoherence, it was readily realized that initial states of the environment exist that allow for decoherence-free unitary evolution of the quantum system. These peculiar states were usually neglected on the basis that “in realistic cases, such highly ordered initial states and symmetrical system-environment couplings are unlikely to be relevant” [2] or because the environment self-evolution would preclude such a unitary evolution [3].

In this paper, we investigate the conditions under which such initial states of the environment do exist in a framework where the quantum system interacts with its environment through a tensor product of Hamiltonians and the environment also evolves by itself. The results obtained underline the crucial role of the environment’s self-evolution. The ability to identify and prepare such special initial states could be used in order to store quantum states. Indeed, even if the environment dynamics cannot be controlled, it might be possible to prepare it in a specific initial state. The interest of preparing the environment has already been briefly studied from

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a theoretical point of view in [4] for a qubit coupled to a bosonic bath. In quantum dots, longer decoherence times can be obtained by narrowing the distribution of nuclear spin states. Propositions have been made for such a narrowing [5, 6, 7] and it has been demonstrated experimentally [8, 9, 10, 11]. However, our results restrict what can be expected from such a technique.

More precisely, we obtain a mathematical condition for the existence of an initial state allowing decoherence-free evolution in the presence of an interaction between a two-state system and its environment, with in addition a self-evolution of the environment. The interaction is restricted to a tensor product of Hermitian operators acting respectively on the system and on the environment. The result is stated in terms of the spectra of the two Hamiltonians acting on the environment (written $\tilde{H}$ and $H_E$ below). Next, we analyze in detail a particular model, which is an extension of the model introduced by Zurek in [1]. Next, we assess the impact of imperfect state preparation. Finally, we discuss how to choose an initial state that minimizes decoherence when decoherence-free evolution via environment preparation is not in fact possible. Certain mathematical proofs are given in the appendices.

While our original motivation stemmed from the peculiar behavior of models of decoherence for certain initial states of the environment, our work can be related to the substantial body of work on decoherence suppression. Passive strategies to fight against decoherence began with the concept of decoherence-free subspaces introduced in the seminal paper of Lidar, Chuang and Whaley [12], following ideas introduced in [13, 14]. This breakthrough was later generalized to noiseless subsystems where protection arises from symmetry in the noise [15, 16, 17]. Active strategies are generically referred to as quantum error correction [18, 19, 20, 21]. A framework accounting for both noiseless subsystems and quantum error correction called operator quantum error correction (OQEC) was devised in [22, 23]. OQEC led to a large body of work in which conditions are derived for decoherence suppression that are independent of the state of the environment. However, recent work [24, 25] deriving the conditions for OQEC in the context of continuous dynamics provides a criterion for arbitrary system-environment Hamiltonians conditioned on the state of the environment. This criterion, although very elegant and general, is difficult to apply operationally. Although it provides a method for testing whether or not a system can be corrected by a given unitary operator, it does not suggest a correction operator that is suitable. On the contrary, our simple, yet less general, model provides a criterion which can be computed efficiently.

2. The model

We consider a quantum system $S$ and its environment $E$ with dynamics described by a Hamiltonian of the form

$$H = S \otimes \tilde{H} + 1 \otimes H_E.$$  \hfill (1)

The Hermitian operator $S$ acts in the Hilbert space of $S$, which we take to be two-dimensional. The quantum system is thus taken to be a quantum bit (qubit) [26]. The Hermitian operators $\tilde{H}$ and $H_E$ act in the Hilbert space of the environment.

The total Hamiltonian (1) induces pure dephasing and is typical of a coupling between the system and the environment that commutes with the self-evolution of the system [27].

Without loss of generality, we can assume $S = \sigma^z$. Its eigenstates, of eigenvalues $\pm 1$, are written $|0\rangle$ and $|1\rangle$, respectively.

Suppose that the global system is in a product state at $t = 0$:

$$|\Psi(0)\rangle = |\psi(0)\rangle \otimes |I\rangle$$

where $|\psi(0)\rangle = a |0\rangle + b |1\rangle$ is an arbitrary normalized pure qubit state. At a later time $t$, the state evolves to

$$|\Psi(t)\rangle = a |0\rangle \otimes |\xi_0(t)\rangle + b |1\rangle \otimes |\xi_1(t)\rangle$$
where
\[ i \frac{d}{dt} |\epsilon_k(t)\rangle = H_k |\epsilon_k(t)\rangle, \quad k = 0, 1, \] (2)
and where we have defined \( H_0 \equiv H_E + \tilde{H} \) and \( H_1 \equiv H_E - \tilde{H} \) with initial condition \( |\epsilon_0(0)\rangle = |\epsilon_1(0)\rangle = |1\rangle \).

Since \( |\Psi(t)\rangle \) is no longer a product state in general, the reduced density matrix \( \rho(t) \) of the quantum system \( S \) no longer describes a pure state: the system has decohered. To quantify this, the off-diagonal elements of \( \rho(t) \) in the basis \( \{|0\rangle, |1\rangle\} \) are reduced by a factor
\[ r(t) = \langle \epsilon_0(t)|\epsilon_1(t)\rangle. \]

Essentially, \( r(t) \) quantifies how distinguishable the states \( |\epsilon_0(t)\rangle \) and \( |\epsilon_1(t)\rangle \) are. If they differ only by a phase, i.e. \( |r(t)| = 1 \), they are indistinguishable and the system \( S \) remains in (or has returned to) a pure state. If \( r(t) = 0 \), they can be distinguished with certainty, decoherence is complete, and the state of the quantum system reduces to a statistical mixture \( |a|^2|0\rangle\langle 0| + |b|^2|1\rangle\langle 1| \). In general, the system has only undergone partial decoherence and a straightforward calculation shows that its purity is
\[ P(t) = \text{Tr} \rho^2(t) = 1 - 2|a|^2|b|^2(1 - |r(t)|^2) \]
where \( |r(t)|^2 \) is the Loschmidt echo [28].

Thus, a qubit prepared in an arbitrary coherent superposition of pointer states will remain pure if and only if the environment can be prepared in a state \( |I\rangle \) for which
\[ |r(t)| = 1 \quad \text{for all} \ t \] (3)
i.e., for which \( |\epsilon_0(t)\rangle \) and \( |\epsilon_1(t)\rangle \) are the same state up to a time-dependent phase at all time.

3. Condition for coherent evolution
Given that the state evolves according to (2), what can be said of the Hamiltonians \( \tilde{H} \) and \( H_E \) in order that for some initial state \( |I\rangle \), \( |\epsilon_0(t)\rangle \) and \( |\epsilon_1(t)\rangle \) satisfy the coherence criterion (3)? The factor \( r(t) \) is given by
\[ r(t) = \langle I| e^{iH_0 t} e^{-iH_1 t} |I\rangle. \]

Clearly, if \( |I\rangle \) is an eigenstate of both \( H_0 \) and \( H_1 \), then \( r(t) \) is a phase and (3) is satisfied, so the existence of a common eigenstate of these Hamiltonians (or, equivalently, of \( \tilde{H} \) and \( H_E \)) is a sufficient condition. It is also a necessary condition, although the state \( |I\rangle \) itself need not be a common eigenstate. To understand this, write \( H_0 \) in terms of its spectral decomposition, with eigenvalues \( \{\lambda_j^{(0)}\} \) and associated projection operators \( \{\Pi_j^{(0)}\} \) (so that \( H_0 = \sum_j \lambda_j^{(0)} \Pi_j^{(0)} \)), and similarly for \( \tilde{H}_1 \). Then
\[ r(t) = \sum_{j,k} e^{i(\lambda_j^{(0)} - \lambda_k^{(1)}) t} \langle I| \Pi_j^{(0)} \Pi_k^{(1)} |I\rangle. \] (4)

In general, \( r(t) \) contains terms of different frequencies. However, in order for it to remain of unit magnitude for all \( t \), only one frequency can appear; all others must be associated with vanishing coefficients. The easiest way for this to occur is if \( |I\rangle \) is a common eigenstate of \( H_0 \) and \( H_1 \), in which case the double sum collapses to a single term. A more general possibility is if \( |I\rangle \) is a linear combination of common eigenstates of \( H_0 \) and \( H_1 \) corresponding to the same energy difference. In terms of the original Hamiltonians \( \tilde{H} \) and \( H_E \), \( |I\rangle \) must be a linear combination of eigenstates of \( H_E \) and each of these must also be a degenerate eigenstate of \( \tilde{H} \). Stated otherwise, we must be able to find a basis in which \( \tilde{H} = C_M \oplus \tilde{H}' \) and \( H_E = D_M \oplus H'_E \), where \( C_M \) is an
$M$-dimensional constant matrix (proportional to the identity) and $D_M$ is an $M$-dimensional diagonal matrix; $|I\rangle$ can be any vector in the first $M$ dimensions. Thus, preparing an initial state of the environment perfectly maintaining the coherence of the qubit at all times can only be done if both the interaction Hamiltonian and the environment self-evolution Hamiltonian exhibit a specific structure, namely they share an eigenstate. Obviously, this is not likely to occur generically, an observation that can easily be made quantitative.

Notice that if the environment self-evolution Hamiltonian $H_E$ is zero, any eigenstate of the interaction Hamiltonian is a suitable initial state. Thus, it is the dynamics of the environment that usually prevents the existence of such an initial state. In the following section, we will make this situation explicit by computing the loss of coherence induced by adding dynamics to an otherwise-static environment.

4. Environment evolution as a perturbation

In this section, we consider a solvable model due to Zurek [1] to which we add a new term $I \otimes H_E$ to provide dynamics to an otherwise-static environment. We consider the case in which the environment is initially prepared in an eigenstate of $\tilde{H}$ and show that the self-evolution of the environment will destroy the coherence of the system. In this model, the environment consists of $n$ spin-$1/2$ particles. The dimension of the environment Hilbert space is $N = 2^n$. Every particle of the environment interacts with the system through a $\sigma_z \sigma_z$ interaction. The eigenstates of the Pauli matrix $\sigma_z^k$ acting on the $k^{th}$ spin are $\{|0\rangle_k, |1\rangle_k\}$. The strength of the interaction between the system and the $k^{th}$ spin of the environment is measured by a coupling constant $g_k \in \mathbb{R}$. Thus, the Hamiltonian is

$$H = \sigma_z^S \otimes \sum_{k=1}^n g_k \sigma_z^k. \quad (5)$$

The eigenstates of $\tilde{H} = \sum_{k=1}^n g_k \sigma_z^k$ are the states in which the $k^{th}$ spin of the environment is in an eigenstate of $\sigma_z^k$, i.e., the states $|x\rangle = \bigotimes_{k=1}^n |x_k\rangle_k$ where all $x_k \in \{0,1\}$. A convenient way to order these states is to consider that $x$ is a number between 0 and $N - 1$ of which the binary representation is $x = x_1x_2 \ldots x_n$. Thus,

$$\forall x \in \{0,1\}^n \quad \tilde{H} |x\rangle = \omega_x |x\rangle$$

where $\omega_x = \sum_{k=1}^n (-1)^{x_k} g_k$. Throughout this section, we will assume that the coupling constants $\{g_k\}$ are chosen so that the eigenvalues $\omega_x$ are distinct.

Figure 1. Decoherence model with dynamical environment

So far, the spins of the environment do not interact with each other; thus, preparing the environment in any state $|x\rangle$, being an eigenstate of $\tilde{H}$, will give rise to coherent evolution of the system. We will now add a self-evolution of the environment of the form

$$H_{\mathcal{E}} = \lambda \sum_{x,y \in \{0,1\}^n} |x\rangle \langle y|$$
where \( \lambda \in \mathbb{R} \) is a perturbation parameter. This Hamiltonian is proportional to the projector on the state \( |\Phi\rangle = \frac{1}{\sqrt{N}} \sum_{x \in \{0,1\}^n} |x\rangle = \bigotimes_{k=1}^{n} |+\rangle_k \), where \( |+\rangle = \frac{1}{\sqrt{2}} (|0\rangle_k + |1\rangle_k) \):

\[
H_\varepsilon \propto \lambda N |\Phi\rangle \langle \Phi|.
\]

Clearly, the interaction and the environment Hamiltonians do not share a common eigenstate because all eigenstates of \( \hat{H} \) have a small overlap with \( |\Phi\rangle \).

Suppose that the environment is prepared in the state

\[
|0\rangle = \bigotimes_{k=1}^{n} |0\rangle_k
\]

which is an eigenstate of \( \hat{H} \) with eigenvalue \( \omega_0 \).

Let us compute the decoherence factor \( r(t) \) for this initial state of the environment. Standard perturbation theory for \( \lambda \ll \min_{x,y \in \{0,1\}^n} |\omega_x - \omega_y| \) shows that

\[
|r(t)|^2 = 1 - 16\lambda^2 \sum_{x \neq 0} \sin^4 \left( \frac{\omega_x - \omega_0}{2} t \right) \frac{\omega_x - \omega_0}{(\omega_0 - \omega_x)^2}.
\]

The time average of this quantity is given by

\[
\overline{|r(t)|^2} = 1 - 6\lambda^2 \sum_{x \neq 0} \frac{1}{(\omega_0 - \omega_x)^2}.
\]

Therefore, the time average is reduced by a factor proportional to \( \sum_{x \neq 0} \frac{1}{(\omega_0 - \omega_x)^2} \), which characterizes the density of energy levels near the unperturbed energy level.

**5. Imperfect preparation of the environment**

Suppose now that the condition for coherent evolution in the original model (1) is satisfied, so that there is at least one state for which the time evolution preserves the coherence of the system \( S \). We have assumed implicitly our ability to prepare perfectly the initial state of the environment. Notice that this strong assumption is nonetheless weaker than requiring control over the environment dynamics at all times. However, preparing the environment in a given initial state is a very difficult task which might only be achieved partially.

Let us give a simple example in the case of Zurek’s model (5). When trying to prepare the state (6), the goal is to prepare all spins in the state \( \alpha_k |0\rangle_k + \beta_k |1\rangle_k \) with \( \alpha_k = 1 \) and \( \beta_k = 0 \). Suppose that we are only able to ensure that \( |\beta_k|^2 \leq \varepsilon \ll 1 \) for all \( k \), i.e., all spins of the environment are prepared with a small error. In that case, the average value \( \overline{|r(t)|^2} \) is bounded by

\[
\overline{|r(t)|^2} \geq \left( (1 - \varepsilon)^2 + \varepsilon^2 \right) \frac{\varepsilon \ll 1}{1 - 2n\varepsilon}
\]

which is attained if \( |\beta_k|^2 = \varepsilon \) for all \( k \). Thus, for small independent errors on each of the \( n \) spin of the environment, the coherence loss is proportional to \( n \). This particular example indicates that even if an initial state allowing for decoherence-free evolution exists, the difficulty in preparing it will grow with the size of the environment, as one would expect intuitively.

A simple calculation shows the robustness of our method under imperfect preparation, an important feature in view of potential practical implementation. If the environment is prepared in the desired state \( |I\rangle \), the system remains pure

\[
|\Psi^{SE}(t)\rangle = U(t) \left( |\psi^S\rangle \otimes |I\rangle \right) = R(t) |\psi^S\rangle \otimes V(t) |I\rangle,
\]
whereas for imperfect preparation, say state $|\hat{I}\rangle$, the evolution leads to $|\tilde{\Psi}^{SE}(t)\rangle$. The evolution being unitary, the fidelity [26] remains constant, i.e.

$$\mathcal{F}\left(|\tilde{\Psi}^{SE}(t)\rangle, |\tilde{\Psi}^{SE}(t)\rangle\right) = |\langle I|\tilde{I}\rangle|.$$  

To compare the trace over the environment of both states, we use Uhlmann’s characterization of fidelity [29] as the maximal overlap over all purifications, which implies

$$\mathcal{F}\left(\text{Tr}_E |\tilde{\Psi}^{SE}\rangle\langle \tilde{\Psi}^{SE}|, \text{Tr}_E |\tilde{\Psi}^{SE}\rangle\langle \tilde{\Psi}^{SE}|\right) \geq |\langle \psi^{SE} | \tilde{\Psi}^{SE}\rangle|.$$  

Hence, the fidelity between the states of the system evolving under perfect (on the one hand) and imperfect (on the other) initialization of the environment is bounded:

$$\mathcal{F}\left(\mathcal{R}(t) |\psi^{S}\rangle, \text{Tr}_E |\tilde{\Psi}^{SE}\rangle\langle \tilde{\Psi}^{SE}|\right) \geq |\langle I|\tilde{I}\rangle|.$$  

(7)

This calculation yields the same result as the much more general work of [30, 25] on the robustness of OQEC.

6. Imperfect control of the environment

We now address a situation which is in a sense opposite to the one just considered. Rather than having perfect control over the dynamics (so that a common eigenstate of $\hat{H}$ and $H_E$ can be made to exist) but an imperfect ability to prepare the initial state, suppose that we can prepare perfectly any state we wish but that no such common eigenstate exists. Given that the coherence of $S$ cannot be preserved, is there an optimal choice of initial state, that is, one for which the ensuing decoherence is in some sense minimized? To address this question, we must first specify what we mean by “optimal.” Do we wish to minimize average decoherence, in which case we would not “see” brief but significant drops in the coherence? Alternatively, do we wish to minimize the maximum decoherence, in which case a significant drop in coherence will make a state appear to be a bad choice even though it may be good on average? In the following we will adopt this latter criterion.

It is somewhat easier to use the combinations $H_{0,1}$ rather than $\hat{H}$ and $H_E$. We wish to find the state $|I\rangle$ for which the minimum value of $|r(t)|$ is maximal, in the case where several frequencies are present in the sum (4). The general case appears difficult to analyze, but one might expect that the best choice of $|I\rangle$ is an eigenstate of one of the two Hamiltonians and a combination of two eigenstates of the other Hamiltonian. For instance, if $\Pi_1^{(0)}|I\rangle = (\Pi_1^{(1)} + \Pi_2^{(1)})|I\rangle = |I\rangle$, then (4) becomes

$$r(t) = e^{i\lambda_1^1 t} \left(e^{-i\lambda_2^2 t} \langle I|\Pi_1^{(1)}|I\rangle + e^{-i\lambda_2^2 t} \langle I|\Pi_2^{(1)}|I\rangle\right)$$

and

$$|r_{\text{min}}| = |\langle I|\Pi_1^{(1)}|I\rangle - \langle I|\Pi_2^{(1)}|I\rangle|.$$  

This expectation turns out not to be the best choice, in general. To see this, we examine the simplest example in which no common eigenstate exists, namely, an environment consisting of a single qubit with $H_{0,1}$ describing its interaction with non-parallel magnetic fields of the same intensity. Let the precession frequency be $\omega$ and let the direction of the magnetic fields corresponding to $H_0$ and $H_1$ be $\hat{m}_0 = (\sin \alpha, 0, \cos \alpha)$ and $\hat{m}_1 = (-\sin \alpha, 0, \cos \alpha)$, with $0 < \alpha < \pi/2$, respectively. The initial state can be taken to be a spin aligned along a third direction, $\hat{v} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$, say. Its evolution according to each of the Hamiltonians is simple: the spin precesses around $\hat{m}_i$ with frequency $\omega$ so $|\psi_i(t)\rangle = |\tilde{\psi}_i(t)\rangle$ where $\tilde{\psi}_i(t)$ is $\hat{v}$ rotated about $\hat{m}_i$ by angle $\omega t$. Then $r(t) = |\langle \psi_0(t)|\psi_1(t)\rangle|$ and $|r(t)| = \cos(\gamma(t)/2)$, where
\( \gamma(t) \) is the angle between \( \hat{v}_0(t) \) and \( \hat{v}_1(t) \). Thus we would like to find the vector \( \hat{v} \) for which the maximum angle between \( \hat{v}_0(t) \) and \( \hat{v}_1(t) \) as they precess is minimized. If, as was conjectured above, we choose \( \hat{v} = \hat{m}_0 \), then it is easy to see that the maximum angle is the lesser of \( 4\alpha \) and \( 2\pi - 4\alpha \), reached after half a precession. Although it is surprisingly difficult to find this maximum angle for an arbitrary \( \hat{v} \), the optimal choice turns out to depend on the angle \( \alpha \) as indicated in Figure 2.

Figure 2. Numerical determination of \( |r_{\min}| \) as a function of \( \alpha \)

If \( \alpha \leq \pi/3 \), it is best to choose \( \hat{v} = \hat{y} \), perpendicular to \( \hat{m}_0 \) and \( \hat{m}_1 \), resulting in \( \gamma_{\max} = 2\alpha \) (attained after a quarter-rotation) and

\[
|r_{\min}| = \cos \alpha.
\]

If \( \alpha \geq \pi/3 \), an optimal choice is \( \hat{v} \) collinear with \( \hat{m}_0 \) (or \( \hat{m}_1 \)), resulting in \( \gamma_{\max} = 2\pi - 4\alpha \) (attained after a half-rotation) and

\[
|r_{\min}| = \cos(\pi - 2\alpha).
\]

Thus, the optimal value of \( |r_{\min}| \) is close to unity if the directions of the two magnetic fields are almost parallel. More surprising is the fact that the best choice of initial state is far from what one would naively have guessed. In the case in which \( \alpha \leq \pi/3 \), the optimal choice is \( \hat{v} = \hat{y} \), which does not correspond to an eigenstate of either Hamiltonian.

To sum up, two regimes emerge. On the one hand, for \( \alpha > \pi/3 \), the system-environment interaction prevails whereas the self-evolution of the environment is only a perturbation. In this regime, the system and the environment play symmetric roles. Reducing decoherence in this case boils down to minimizing the entanglement. The optimal initial states are thus the pointer states of the environment, i.e. the eigenstates of \( \hat{H} \). On the other hand, for \( \alpha < \pi/3 \), the self-evolution Hamiltonian \( H_E \) dominates over the interaction Hamiltonian \( \hat{H} \). However, the evolution of the quantum system relies essentially on the interaction. Thus, for the evolution of the system, \( \hat{H} \), no matter how small, cannot be considered a perturbation with respect to \( H_E \). In that case, the evolution of the environment is dominated by its own dynamics but the impact on the quantum system is mediated by the interaction Hamiltonian. Hence, finding an analytical criterion characterizing an initial state of the environment that optimally limits the subsequent decoherence of the system remains an unresolved challenge.

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References

[1] Zurek W H 1982 Phys. Rev. D 26 1862–1880
[2] Schlosshauer M 2007 Decoherence and the Quantum to Classical Transition (Springer-Verlag Berlin)
[3] Paz J P and Zurek W H 2001 Coherent Matter Waves, Lectures from the 72nd Les Houches Summer School, 1999 (Springer-Verlag, Berlin) pp 533–614
[4] Dajka J, Mierzejewski M, Luczka J and Haenggi P 2009 Physica E In Press ISSN 1386-9477
[5] Klauser D, Coish W A and Loss D 2006 Phys. Rev. B 73 205302
[6] Ribeiro H and Burkard G 2009 Phys. Rev. Lett. 102 216802
[7] Coish W A 2009 Nat. Phys. 5 710–711
[8] Greilich A, Shabanev A, Yakovlev D R, Efros A L, Yugova I A, Reuter D, Wieck A D and Bayer M 2007 Science 317 1896–1899
[9] Reilly D, Taylor J, Petta J, Marcus C, Hanson M and Gossard A 2008 Science 321 817
[10] Xu X, Yao W, Sun B, Steel D G, Bracker A S, Gammon D and Sham L J 2009 Nature 459 1105–1109
[11] Latta C, Hoge A, Zhao Y, Vamivakas A N, Maletinsky P, Greilich A, Reuter D, Wieck A D and Bayer M 2007 Science 317 1896–1899
[12] Lidar D A, Chuang I L and Whaley K B 1998 Phys. Rev. Lett. 81 2594–2597
[13] Duan L M and Guo G C 1998 Phys. Rev. A 57 737–741
[14] Zanardi P and Rasetti M 1997 Phys. Rev. Lett. 79 3306–3309
[15] knill E, Laflamme R and Viola L 2000 Phys. Rev. Lett. 84 2525–2528
[16] De Filippis S 2000 Phys. Rev. A 62 052307
[17] Yang C P and Gea-Banacloche J 2001 Phys. Rev. A 64 032309
[18] Bennett C H, DiVincenzo D P, Smolin J A and Wootters W K 1996 Phys. Rev. A 54 3824–3851
[19] Knill E and Laflamme R 1997 Phys. Rev. A 55 900–911
[20] Shor P W 1995 Phys. Rev. A 52 R2493–R2496
[21] Steane A M 1996 Phys. Rev. Lett. 77 793–797
[22] Kribs D, Laflamme R and Poulin D 2005 Phys. Rev. Lett. 94 180501
[23] Kribs D, Laflamme R, Poulin D and Lesosky M 2006 Quantum Information & Computation 6 382–399
[24] Oreshkov O, Lidar D A and Brun T A 2008 Phys. Rev. A 78 022333
[25] Oreshkov O 2008 Topics in quantum information and the theory of open quantum systems Ph.D. thesis University of Southern California
[26] Nielsen M A and Chuang I L 2000 Quantum Information and Quantum Information (Cambridge University Press
[27] Hornberger K 2009 Entanglement and Decoherence (Lecture Notes in Physics vol 768) (Springer Berlin / Heidelberg) pp 221–276
[28] Cucchietti F M, Paz J P and Zurek W H 2005 Phys. Rev. A 72 052113
[29] Uhlmann A 1976 Rep. Math. Phys. 9 273–279
[30] Oreshkov O 2008 Phys. Rev. A 77 032333
[31] Katô T 1966 Perturbation theory for linear operators (Springer)