Locomotion Control of Snake-like Robot utilizing Friction Forces: Stability Verification of Model Following Servo Controller

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Abstract. The purpose of this paper is to verify the effectiveness of Model Following Servo Control (MFSC) as a stabilizing control measure for systems with uncontrollable disturbances. Both the gravity compensation control system of the 1-link manipulator and the head-position control system of the two-wheeled robot is designed. In gravitational compensation control, gravitational acceleration is defined as an uncontrollable state. In the head-position control, the frictional force in the axial direction of the skidding wheel is defined as an uncontrollable state. The effectiveness of the MFSC as a stabilizing control system for systems containing uncontrollable states is verified via numerical simulations.

Keywords: Snake-like Robot, Model Following Servo Controller, Gravity Compensation, Head Position Control

1. Introduction

The snake is propelled by the characteristic that it is slippery in the propulsive direction and non-slippery in the normal direction. In the process of designing a control system, this dynamic feature is often assumed to have an infinite difference in frictional force. This assumption is treated as a constraint that the body of the robot will not slip in the normal direction. Based on this constraint, motion control systems have been designed and implemented [1, 2, 3, 4].

However, in the real environment, the friction force varies from place to place. Under these conditions, the snake-like robot has the potential to experience body skidding. Some studies based on this assumption have been reported [5, 6, 7, 8, 9]. For example, in [9], body skidding is represented by a hybrid model to realize a propulsion control system. The hybrid model method requires accurate model switching for the control system to properly determine the control input. To accurately switch models, it is necessary to accurately measure
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and estimate a large number of state quantities, including velocity and force dimensions. To do this, a large number of sensors must be installed or the state quantities must be estimated with high accuracy. However, to accurately estimate the state quantity, an accurate model is required. In other words, there is a co-dependency between the accuracy of the model and the state estimation system. Therefore, stability can easily be broken.

The snake locomotes actively generating frictional force in the normal to its body through its twisting motion. This means that the snake receives forces from the environment to locomote itself. This force is considered to be a kind of disturbance because it has a different magnitude depending on the location due to the change of friction coefficient. In this study, we consider a snake-like robot as a robot that locomotes receiving an unknown disturbance passively from the environment. Then, the propulsive control problem of the snake-like robot considering skidding can be attributed to the unknown disturbance estimation problem and the control system design problem using the disturbance.

In this paper, we focus on the control system design problem using the disturbance. This disturbance is difficult to model because it is a frictional force that varies from place to place. Therefore, this disturbance is treated as an uncontrollable state. Model Following Servo Control (MFSC) [10] is one of the control measures for servo control that includes the uncontrollable states. It can converge a plant model to a stably designed uncontrolled reference trajectory. However, the effectiveness of MFSC as a stabilizing control measure for systems with uncontrollable disturbances that are not guaranteed to be stable has not yet been verified.

The purpose of this paper is to verify the effectiveness of MFSC as a stabilizing control measure for systems with uncontrollable disturbances as state quantities. In this study, the effectiveness is verified in two stages: the first stage is the gravity compensation control of a 1-link manipulator, and the second stage is the leading position control of a two-wheel vehicle robot. In the gravitational compensation control of a single-link manipulator, the gravitational acceleration is defined as an uncontrollable state quantity, and it is verified whether the link angle can be stabilized to the horizontal point. The gravitational acceleration is naturally uncontrollable because it always keeps a constant value. When considering the influence of gravity, servo control or gravity compensation control is generally used. In this study, we consider the gravitational acceleration as an uncontrollable state quantity and design a control system using MFSC. In this study, the control system to be designed is a stabilizing control system rather than a servo control system, and it is verified that MFSC is effective for stabilizing control of systems with gravity as a disturbance. In the head position control of a two-wheeled vehicle robot, we control a skidding two-wheeled vehicle robot that is subjected to frictional forces in the axial direction. The two-wheeled vehicle robots, like snake-like robots, are non-holonomic systems, and thus are suitable for verification as simple models of snake-like robots. In the plant model, the LuGre Friction Model [11] is used as the friction force to represent the skidding. In the control model, the frictional force is treated as an uncontrollable state as a disturbance, however its exact magnitude is assumed to be known. A head position control system for a two-wheeled vehicle robot using MFSC is designed, and its effectiveness is verified through numerical simulations.

This study is organized as follows: Section 2. deals with the gravity compensation control of a 1-link manipulator. The equations of motion of the 1-link manipulator are derived
with the horizontal origin, and the MFSC is designed based on the derived equations of motion. The gravitational acceleration is treated as an uncontrollable state quantity. Section 3. deals with the head position control of a two-wheeled robot. The equations of motion for a skidding two-wheeled robot subjected to a frictional force in the axle direction are derived. The friction force is then modeled based on the LuGre Friction Model. A head position control system is designed using MFSC, in which the friction force in the axle direction is an uncontrollable state quantity, and its effectiveness is verified by numerical simulations. Section 4. concludes the paper.

2. Gravity Compensation Control of 1-link Manipulator

In this section, we design a gravity compensation control system for a 1-link manipulator. The first step is to derive the equations of motion of the 1-link manipulator. We define a coordinate system with the horizontal position as the origin and derive the equations of motion. Next, we design a stabilizing control system using the MFSC. Since the convergence point is horizontal, the system will not converge properly if the control system does not properly compensate for the effect of gravity. Finally, we show by numerical simulation that MFSC can converge the system properly.

2.1. Modeling of 1-link Manipulator

Figure 1 shows the schematic figure of the 1-link manipulator, and Table 1 shows the list of physical parameters. The $\theta$[rad] is the angle of the manipulator, horizontal, that is, the angle from the $x$-axis. Therefore, for the state quantity of the manipulator to converge to the zero point, it is necessary to take into account the effect of gravitational acceleration to maintain the state. The equation of motion of the 1-link manipulator is formulated as follows:

$$(J + mr^2)\ddot{\theta} + c\dot{\theta} + rmg \cos \theta = u.$$  \hspace{1cm} (1)

![Figure 1: The schematic figure of the 1-link manipulator](image)
Table 1: Physical Parameters of the 1-link Manipulator

| Parameters                  | Notation | Value |
|-----------------------------|----------|-------|
| Mass [kg]                   | $m$      | 1.0   |
| Inertia moment [kg·m²]      | $J$      | 1.0   |
| Viscous friction coefficient [Nm·s/rad] | $c$ | 1.0 |
| Length of link [m]          | $r$      | 1.0   |
| Gravity [m/s²]              | $g$      | 9.81  |

2.2. Control System Design for 1-link Manipulator

We design a control system based on MFSC using (1). We define the state of the system as follows:

$$x = \begin{bmatrix} \dot{\theta} & \dot{\theta}^2 & g \end{bmatrix}^T.$$

The state-space representation is formulated as follows:

$$\dot{x} = Ax + bu,$$

$$A = \begin{bmatrix} 0 & -\frac{c}{J + mr^2} & -\frac{mr \cos \theta}{J + mr^2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix}.$$ 

The input $u$ that stabilize the system can be obtained as follows:

$$u = -\frac{1}{R} b_1^T \begin{bmatrix} P_1 & p_2 \end{bmatrix} x.$$

Where, $P_1$ and $p_2$ are the positive definite symmetric solution of the Riccati equation and the solution of the matrix algebraic equation shown below, respectively.

$$A_1^T P_1 + P_1 A_1 + Q_1 - \frac{1}{R} P_1 b_1 b_1^T P_1 = 0,$n

$$A_2^T P_2 + P_1 a_2 - \frac{1}{R} P_1 b_1 b_1^T P_2 = 0.$$n

where, $Q_1$ and $R$ are weight matrices, chosen to be semi-positive and positive definite, respectively.

2.3. Numerical Simulation of Gravity Compensation Control

The effectiveness of the designed MFSC for gravity compensation control of a 1-link manipulator is verified by numerical simulation. MaTX (Visual C++ 2012) version 5.3.45 [12] is performed for simulation. The simulation parameters are shown in Table 1. The simulation time was 10 s, the sampling interval was 0.01 s, and the algorithm for solving the
ordinary differential equation was the *rkf45* function, which is a solver function using the Runge-Kutta-Fehlberg method. The initial angle and the initial angular velocity of the 1-link manipulator was set to 1 rad and 0 rad/s respectively. The weight matrices are set as follows:

\[
Q_1 = \text{diag}(10, 1), \quad R = 1.
\]

The simulation results are shown in Figs. 2-4.

Figure 2 and Fig. 3 show the time variation of the angle and angular velocity of the manipulator, respectively: Figure 2 and Fig. 3 show that the angle of the manipulator converges to the horizontal position despite the influence of gravity. Figure 4 shows that the MFSC is applying the necessary torque to the system to make the angle converge to the horizontal. Therefore, it is shown that the MFSC can realize effective gravity compensation control by using gravity as an uncontrollable state quantity.

![Figure 2: Time variation of the manipulator angle.](image)

![Figure 3: Time variation of the manipulator angular velocity.](image)
3. Head Position Control of a Two-Wheeled Robot

In this section, we design a head position control system for a two-wheeled vehicle robot. First, we derive the equations of motion of the two-wheeled robot. The equations of motion are based on a skidding model in which a friction force acts in the direction of the robot axial. The friction force is modeled by the LuGre Friction Model for the plant model. Next, we design a head position control system using the MFSC. The MFSC is designed based on the equation of state in which the frictional force acting in the axial direction is an uncontrollable state quantity. Finally, it is shown by numerical simulation that the MFSC can converge the system properly.

3.1. Modeling

We derive the dynamics model of the two-wheeled vehicle robot. The schematic model of the robot is shown in Fig. 5, and its parameters are listed in Table 2. The robot skids with a friction force in the axial direction. The LuGre Friction Model is used as the friction force in the axial direction and is known to be able to handle static friction force, Coulomb friction force, and viscous friction force in a single model.

From Fig. 5, the dynamics model of the two-wheeled vehicle robot skidding with frictional force in the axial direction is derived as (4).

\[
M \dot{\mathbf{v}} + \mathbf{N} \mathbf{v} = \mathbf{E} \mathbf{u} - \mathbf{F} \mathbf{f},
\]

\[
\mathbf{v} = \begin{bmatrix} \omega & v_x & v_y \end{bmatrix}^T, \quad \mathbf{u} = \begin{bmatrix} f_x & f_y \end{bmatrix}^T, \quad M = \text{diag}(J, m, m),
\]

\[
\mathbf{N} = \begin{bmatrix} c & 0 & 0 \\ 0 & 0 & -\omega \\ 0 & \omega & 0 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 0 & d \\ 1 & 0 \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.
\]
Figure 5: The schematic figure of the two wheeled robot.

Table 2: Physical Parameters

| Parameters                          | Notation | Value |
|-------------------------------------|----------|-------|
| Mass [kg]                           | $m$      | 1.0   |
| Inertia moment [kg-m$^2$]           | $J$      | 1.0   |
| Viscous friction coefficient [Nm-s/rad] | $c$     | 1.0   |
| Length from center of gravity to head [m] | $d$     | 0.5   |
| Viscous friction coefficient of floor [N-s/m] | $\sigma_2$ | 1.0   |
| Coulomb friction parameter          | $l_c$    | 0.003 |
| Static friction parameter           | $l_s$    | $2l_c$|
| Striebeck velocity [m/s]            | $v_s$    | 0.1   |
| Friction coefficient                | $\mu$    | 0.3   |
| Gravity [m/s$^2$]                   | $g$      | 9.81  |

In (4), $f$ is formulated by the LuGre Friction Model as follows:

$$f = (\sigma_1 + \sigma_2)v_y + (\sigma_0 - \sigma_1 \frac{|v_y|}{g(v_y)})z,$$

$$\dot{z} = v_y - \frac{v_y}{g(v_y)}z,$$

$$g(x) = l_c + (l_s - l_c)e^{-x/v_s},$$

$$\sigma_0 = \frac{\mu mg}{l_c},$$

$$\sigma_1 = 2m \sqrt{\frac{\mu g}{l_c}}.$$
3.2. Control System Design

We design the head position control system using friction force based on MFSC. We define the state of the system as follows:

\[ x = \begin{bmatrix} x_h & y_h & \omega & v_x & v_y & f \end{bmatrix}^T, \]

including the head position of the robot \((x_h, y_h)\). Note that only \(f\) is an uncontrollable state quantity, while the others are controllable. Also, \(f\) is the frictional force and its exact value is unknown. Therefore, this value must be estimated. This estimation problem is not addressed in this paper but is assumed to be accurately estimated by Unscented Kalman Filter or other estimation methods. From Fig. 5, \((x_h, y_h)\) can be formulated as follows:

\[ \begin{bmatrix} x_h \\ y_h \end{bmatrix} = \begin{bmatrix} x + d \cos \theta \\ y + d \sin \theta \end{bmatrix}, \]

The time derivative is derived as follows:

\[ \begin{bmatrix} \dot{x}_h \\ \dot{y}_h \end{bmatrix} = \begin{bmatrix} \dot{x} - d \sin \theta \dot{\theta} \\ \dot{y} + d \cos \theta \dot{\theta} \end{bmatrix} = S\nu, \]

\[ S = \begin{bmatrix} -d \sin \theta & \cos \theta & -\sin \theta \\ d \cos \theta & \sin \theta & \cos \theta \end{bmatrix}. \]

The state-space representation of the system is formulated as (5).

\[ \dot{x} = Ax + Bu, \quad (5) \]

\[ A = \begin{bmatrix} 0 & S & 0 & 0 \\ 0 & -M^{-1}N & M^{-1}F & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ 0 & A_r \end{bmatrix}, \]

\[ B = \begin{bmatrix} 0 \\ M^{-1}E \\ 0 \end{bmatrix} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}. \]

For the system, consider a cost function defined as (6).

\[ J = \int_{-\infty}^{\infty} x^T Q x + u^T R u \, dt, \quad (6) \]

The input \(u\) that minimizes (6) can be obtained as follows:

\[ u = -R^{-1}B^T_1 \begin{bmatrix} P_1 & P_2 \end{bmatrix} x. \quad (7) \]

Where, \(P_1\) and \(P_2\) are the positive definite symmetric solution of the Riccati equation and the solution of the matrix algebraic equation shown below, respectively.

\[ A_1^T P_1 + P_1 A_1 + Q_1 - P_1 B_1 R^{-1} B_1^T P_1 = 0, \]

\[ A_2^T P_2 + P_1 A_2 - P_1 B_1 R^{-1} B_1^T P_2 = 0, \]

where, \(Q_1\) and \(R\) are weight matrices, chosen to be semi-positive and positive definite, respectively.
3.3. Numerical Simulations

We verify that the MFSC can control the head position of the two-wheeled vehicle robot that includes the uncontrollable friction force in its state. MaTX (Visual C++ 2012) version 5.3.45 [12] is performed for simulation. The simulation parameters are shown in Table 2. The simulation time was 10 s, the sampling interval was 0.01 s, and the algorithm for solving the ordinary differential equation was the \( \text{rkf45} \) function, which is a solver function using the Runge-Kutta-Fehlberg method. The weight matrices were

\[
Q_1 = \text{diag}(10, 10, 1, 1, 100), \\
R = \text{diag}(1, 1).
\]

In this paper, the two simulations with different initial positions are performed. In the first simulation, the initial head position of the robot was set to (-10, 10) m, and the initial pose was 0 rad. The simulation results are shown in Figs. 6-11. Figures 6-8 shows that the head position of the robot is converged to the origin by the designed control system. Figures 9-11 show that the input varies appropriately according to the frictional force acting in the axial direction and the variation in speed.

In the second simulation, the initial head position of the robot was set to (10, -3) m, and the initial pose was 0 rad. The simulation results are shown in Figs. 12-17. Figures 12-14 shows that the head position of the robot is converged to the origin by the designed control system. Figures 15-17 show that the input varies appropriately according to the frictional force acting in the axial direction and the variation in speed. The system is stabilized even when the uncontrollable friction force is included in the state, which confirms that MFSC is effective for stabilization control problems involving uncontrollable states.

![Figure 6: The time variation of the head position. (The initial head position: (-10, 10) m)](chart.png)
4. Conclusion

In this paper, the effectiveness of MFSC as the stabilizing control measure for systems with uncontrollable disturbances has been verified through simulation. The effectiveness of the control was verified in two stages: the first stage was the gravity compensation control of a 1-link manipulator, and the second stage was the leading position control of a two-wheeled vehicle robot. In the gravity compensation control of the 1-link manipulator, we defined the acceleration of gravity as an uncontrollable state quantity and verified whether the link angle can be stabilized to the horizontal point. For this purpose, the equations of motion of the single-link manipulator were derived, and a control system was designed using the MFSC with gravitational acceleration as an uncontrollable state quantity. Numerical simulations confirmed that the MFSC can stabilize the link angle to the horizontal point as gravity compensation control. We defined the frictional force in the axial direction of the skidding
wheel as an uncontrollable state quantity and verified the possibility of controlling the head position of the skidding two-wheeled robot that receives the frictional force in the axial direction. The dynamics model of the two-wheeled vehicle robot subjected to the frictional force based on the LuGre Friction Model in the axial direction has been derived, and the state equation has been formulated with the frictional force as the unknown disturbance. Based on the state equation, the stabilizing control system using MFSC has been designed and its effectiveness has been confirmed by numerical simulations.

In future work, we will work on the head position control of the snake-like robot. We consider the frictional force in the axial direction of the snake robot as an uncontrollable state quantity and design the MFSC. We also address the problem of estimating the friction force. We will apply state estimation methods such as the Extended Kalman filter and the Unscented Kalman filter to the two-wheeled vehicle robot subjected to frictional forces in the axial direction, and verify the accuracy of the frictional force estimation.
Figure 11: The time variation of the friction force. (The initial head position: (-10, 10) m)

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Figure 12: The time variation of the head position. (The initial head position: (10, -3) m)
Figure 13: Time variation of the robot’s posture. (The initial head position: (10, -3) m)

Figure 14: Trajectory of head position and center of gravity position. (The initial head position: (10, -3) m)

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Figure 15: The time variation of the input. (The initial head position: (10, -3) m)

Figure 16: The time variation of the velocities. (The initial head position: (10, -3) m)

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Figure 17: The time variation of the friction force. (The initial head position: (10, -3) m)

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