Thermal Error Compensation Method Based on Floyd Algorithm

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Abstract. In this paper, a new compensation method of thermal error is proposed based on Floyd algorithm and homogeneous transformation matrices (HTMs). With the topological structure and measured data, the thermal error modelling is established. Then, according to the improved Floyd minimum-distance method and ant colony algorithm-based back propagation neural network (ACO-BPN), the error compensation models were established respectively. A comparison between the Floyd algorithm compensation method and a typical method ACO-BPN is performed, and the results shows that the Floyd algorithm has advantages of higher compensation accuracy, more effective and shorter time than ACO-BPN. Under variable temperatures conditions, the Floyd compensation method also has higher compensation accuracy.

Introduction

As for machine tool, the machining accuracy is an important index to evaluate its performance and characteristics [1], and it is influenced by many factors. Among them, thermal errors account for about 40% of the total machining errors [2].

In the past, errors were minimized by improving thermal properties of the machine tool through mechanical design and manufacturing technology [3]. However, there are many physical limitations that cannot be overcome solely by design techniques during the practical implementation. Therefore, the studies on error compensation technique used to improve machine tool accuracy have become more and more significant in this field [4].

A robust and accurate thermal error modeling is the most key step to correct and compensate thermal errors [5]. Thermal error model must be built accurately based on experimental measurements. Yang et al. established a relationship between temperature and thermal error of a spindle using artificial neural networks (ANNS) [6]. El Ouafi et al. constructed an artificial neural network model for spindle thermal errors with the temperature drawing on statistical methodology, which effectively improves the machining accuracy [7].
The scholars Lin et al. and Zhao et al. established a spindle thermal error model based on least square support vector machine theory [8]. In this paper, HTMs will be adopted to establish the thermal error model.

Among the existing compensation methods, the ACO-BPN are the most commonly used methods. Yang et al. used the ACO-BPN to establish thermal errors model and provided a compensation method [9]. In this paper, a new compensation method of thermal error is proposed based on Floyd algorithm.

Error Modeling of NC Machine Tool

Experiment and Measurement

A 5-axis vertical machining center, as shown in Fig.1, is selected to study the influence mechanism of thermal error on machining accuracy. The site of error measurement is shown in Fig. 2.

The machine reference origin was set to be the starting point for this measurement, and the positioning error was measured every 50 mm in the whole stroke range of 600 mm. To detect the thermal error, the positioning error was measured under different temperature conditions. Firstly, the initial geometric error was measured at normal temperature of 20 °C when the machine tool was initially switched on [10]. And then, the machine tool was warmed up by moving the X-axis slide along its stroke range with a feed rate of 10m/min. Meanwhile, the temperature measurement was taken at same intervals until the parts reached a thermal equilibrium state.

Figure 1. 5-axis Vertical Machining Center.

Figure 2. Error Measurement.
Error Modeling of NC Machine Tool

In this research, error modeling is based on homogeneous transformation matrices mothod. The homogenous transformation matrices are shown in Table 1, and the five-axis machine tool can be abstracted into a topology framework as shown in Fig. 3.

Suppose that the tool forming point coordinate in the tool coordinate system is:

\[ P_t = \begin{pmatrix} P_{tx} & P_{ty} & P_{tz} & 1 \end{pmatrix}^T \] (1)

The work-piece forming point coordinate in the work-piece coordinate system is:

\[ P_w = \begin{pmatrix} P_{wx} & P_{wy} & P_{wz} & 1 \end{pmatrix}^T \] (2)

When the machine tool moves in ideal form, the tool forming point and work-piece forming point will overlap together, and \( M_{o,T} P_t = M_{o,W} P_{ideal} \) can be obtained, where \( M_{o,T} \) means the homogenous transformation matrix of the tool branch and \( M_{o,W} \) means the homogenous transformation matrix of the workpiece branch, and then:

\[
M_{o,X}^P M_{o,Y}^P M_{X,Y}^S M_{Y,Z}^P M_{Z,B}^P M_{B,T}^P P_t = M_{o,Y}^P M_{o,Y}^S M_{Y,A}^P M_{Y,A}^S M_{A,W}^P M_{A,W}^S P_{ideal}
\] (3)

In Eq.(3), \( P \) and \( S \) mean static and motion respectively, and so \( M_{i-1,j}^P \) refers to the ideal static homogenous transformation matrix of the adjacent body and \( M_{i-1,j}^S \) refers to the ideal motion homogenous transformation matrix of the adjacent body as shown in Table 1.

From Eq.(3), we can obtain the ideal tool forming point in the workpiece coordinate system as follows:

\[
P_{ideal} = \left( M_{o,Y}^P M_{o,Y}^S M_{Y,A}^P M_{Y,A}^S M_{A,W}^P M_{A,W}^S \right)^{-1} M_{o,X}^P M_{o,Y}^S M_{X,Z}^P M_{X,Z}^S M_{Z,B}^P M_{B,T}^P P_t \] (4)

In the actual machining process, the actual position of tool forming point will inevitably deviate from the ideal position of tool forming point. As a result, the actual tool forming point in the workpiece coordinate system is given as:

\[
P_{actual} = \left( E M_{o,W} \right)^{-1} E M_{o,T} P_t \] (5)

In Eq. (5), the error homogenous transformation matrix of the workpiece branch \( E M_{o,W} \) and the error homogenous transformation matrix of the tool branch \( E M_{o,T} \) can be described as follows:

\[
E M_{o,W} = M_{o,X}^P \times E M_{o,X}^P \times M_{o,Y}^P \times E M_{o,Y}^P \times M_{o,Y}^S \times E M_{o,Y}^S \times M_{Y,A}^P \times E M_{Y,A}^P \times E M_{Y,A}^S \times E M_{Y,A}^S
\times E M_{Y,A}^P \times M_{Y,A}^P \times E M_{Y,A}^S \times E M_{Y,A}^S \times E M_{A,W}^P \times E M_{A,W}^P
\times E M_{A,W}^S \times E M_{A,W}^S
\]

\[
E M_{o,T} = M_{o,X}^P \times E M_{o,X}^P \times M_{o,X}^S \times E M_{o,X}^S \times M_{X,Z}^P \times E M_{X,Z}^P \times M_{X,Z}^S \times E M_{X,Z}^S
\times M_{Z,B}^P \times E M_{Z,B}^P \times M_{Z,B}^S \times E M_{Z,B}^S \times E M_{B,T}^P \times E M_{B,T}^P \times E M_{B,T}^S \times E M_{B,T}^S
\] (6)
In Eq.(6), $^E M_{i-1,i}^P$ refers to the static error homogenous transformation matrix of the adjacent body and $^E M_{i-1,i}^S$ refers to the motion error homogenous transformation matrix of the adjacent body as shown in Table 1. And then, error of five-axis machine tool caused by the gap between actual forming point and ideal forming point can be obtained by:

$$E = ^E M_{0,w}^P \cdot \text{world} - ^E M_{0,t}^P$$

(7)

Where the values of the expressions in the Eq.(3)-Eq.(6) can be obtained from the deformation of Table 1, and $E$ represents geometric errors of this machine tool and it contains three parts $E_x$, $E_y$, and $E_z$, and then:

$$E = [E_x, E_y, E_z, 0]^T$$

(8)

Table 1. Characteristic Matrices of the 5-axis Machine Tool.

| adjacent body | Body ideal static, motion HTMs | Body static, motion error HTMs |
|---------------|--------------------------------|-------------------------------|
| 0—1 X-axis    | $M_{O,x}^P = I_{4x4}$          | $^E M_{O,x}^P = I_{4x4}$       |
|               | $M_{S,x}^P = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $^E M_{S,x}^P = \begin{bmatrix} 1 & -\Delta \gamma_x & \Delta \beta_x & \Delta \alpha_x \\ \Delta \gamma_x & 1 & -\Delta \alpha_x & \Delta \gamma_x \\ -\Delta \beta_x & \Delta \alpha_x & 1 & \Delta \alpha_x \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| 1—2 Z-axis    | $M_{X,z}^P = I_{4x4}$          | $^E M_{X,z}^P = I_{4x4}$       |
|               | $M_{S,x,z}^P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $^E M_{S,x,z}^P = \begin{bmatrix} 1 & -\Delta \gamma_z & \Delta \beta_z & \Delta \alpha_z \\ \Delta \gamma_z & 1 & -\Delta \alpha_z & \Delta \gamma_z \\ -\Delta \beta_z & \Delta \alpha_z & 1 & \Delta \alpha_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| 2—3 B-axis    | $M_{Z,B}^P = I_{4x4}$          | $^E M_{Z,B}^P = I_{4x4}$       |
|               | $M_{S,z,b}^P = \begin{bmatrix} \cos B & 0 & \sin B & 0 \\ 0 & 1 & 0 & 0 \\ -\sin B & 0 & \cos B & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $^E M_{S,z,b}^P = \begin{bmatrix} 1 & -\Delta \gamma_B & \Delta \beta_B & \Delta \alpha_B \\ \Delta \gamma_B & 1 & -\Delta \alpha_B & \Delta \gamma_B \\ -\Delta \beta_B & \Delta \alpha_B & 1 & \Delta \alpha_B \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| 3—4 Tool      | $M_{B,T}^P = \begin{bmatrix} 1 & 0 & 0 \times_{td} & x_{td} \\ 0 & 1 & 0 & L + y_{td} \\ 0 & 0 & 1 & z_{td} \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $^E M_{B,t}^P = \begin{bmatrix} 1 & -\Delta \gamma_{td} & \Delta \beta_{td} & \Delta \alpha_{td} \\ \Delta \gamma_{td} & 1 & -\Delta \alpha_{td} & \Delta \gamma_{td} \\ -\Delta \beta_{td} & \Delta \alpha_{td} & 1 & \Delta \alpha_{td} \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| 0—5 Y-axis | 6—7 workpiece |
|------------|---------------|
| $M_{y}^{P} = I_{4x4}$ | $M_{y}^{w} = I_{4x4}$ |
| $M_{y}^{S} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $M_{y}^{w} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y_{w} \\ 0 & 0 & 1 & z_{w} \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| $\varepsilon M_{y}^{P} = \begin{bmatrix} 1 & -\Delta \gamma_{y} & 0 & 0 \\ \Delta \gamma_{y} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $\varepsilon M_{y}^{w} = \begin{bmatrix} 1 & -\Delta \gamma_{w} & \Delta \beta_{w} & \Delta \zeta_{w} \\ \Delta \gamma_{w} & 1 & -\Delta \alpha_{w} & \Delta \zeta_{w} \\ -\Delta \beta_{w} & \Delta \alpha_{w} & 1 & \Delta \zeta_{w} \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |

Figure 3. Topology Framework of the Machine Tool.

**Thermal Error Compensation Method Based on ACO-BPN and Floyd Algorithm**

In recent years, the compensation method NC tool-path planning modified became a popular approach. Especially, the ACO-BPN algorithm is used in the shortest path planning frequently. However, the computing process of this algorithm is time consuming. Floyd algorithm is a new path planning method that has never been used in error compensation in early days even though it is easy to understand and design. This chapter mainly introduces compensation principle of the two algorithms.
ACO-BPN Compensation Operating Principle

The main idea of the neural network model is: hypothesis neural networks have $m$ parameters which will be optimize, the parameters are arranged in order, into the $p_1, p_2, \ldots, p_m$. For any of these parameters, Initialized to $N$ arbitrary nonzero value constitute a collection of $I_p$. The number of ants is $S$, all the ants from the first set of randomly selected elements in search of food, and find food after return to nest collecting information, repeat the steps when all ants collected the same route, the network of the optimal solution is found. When all ants converge to the same path, the number of iterations is completed, and the optimal solution is obtained. Process flow diagram is shown in Fig.4.

![Figure 4. ACO-BPN Algorithm Flow Chart.](image)

Floyd Algorithm Compensation Operating Principle

The Floyd algorithm is implemented by calculation of weight matrix. Adjacency matrix $D^{(0)}$ stands for the distance between every two nodes, such as $w_i$ and $w_j$. First, $D^{(1)}$ is calculated. One of all possible paths is figured out between $w_i$ and $w_j$. By comparison, the best path can be identified. The new iteration adjacency matrix $D^{(i)}$ is replaced the initial one. The elements in $D^{(i)}$ represent the better path every two points by one iteration. And by adding one node between $w_i$ and $w_j$, the path changes to be better or shorter distance directly. On the other hand, to improve reliability, an iterative matrix $D^{(k)} = (d^{(k)}_{ij})$ is established. A new node $w_r$ is set between $w_i$ and $w_j$, and then the path length is compared with the previous path. If $d^{(k-1)}_{ir} \leq d^{(k-1)}_{ij}$ or $d^{(k-1)}_{jr} \geq d^{(k-1)}_{ij}$ is certain, the new path length will not be shorter than that containing the previous node $w_r$. The actual point is not close to target due to manufacturing defect, and the compensation method function makes actual target point extended to target. One of all possible paths is figured out between $w_i$ and $w_j$. With comparison, the better path can be figured out. The above-described process is the key of Floyd algorithm compensation operating principle. Fig.5 shows the Floyd algorithm compensation flow chart.
Discussion about Floyd Algorithm Compensation

The most striking similarity to ACO-BPN and Floyd algorithm compensation method is that they compare weights to search the best one. However, the ACO-BPN is over relying on the BPN, and its computation is more time-consuming than Floyd algorithm compensation. Moreover, Floyd algorithm does not require additions and subtractions at some particular location as the ACO-BPN do.

In this paper, the ACO-BPN is compared to Floyd algorithm compensation. Numerical simulation data (Fig.6) showed that the Floyd algorithm compensation is more effective than ACO-BPN compensation method. Floyd algorithm compensation reduced average errors 68 μm to 4 μm, decreasing it by 94.2%. However, average ACO-BPN compensation reduced errors from 68 μm to 6 μm decreasing it by 91.2%. Therefore, the compensation effect of Floyd algorithm is better than ACO-BPN.

Furthermore, Floyd algorithm compensation requires a lower number of simulations than ACO-BPN. Table 2 shows the iterations and statement frequency of the comparison between Floyd algorithm and ACO-BPN.

Figure 5. Shows the Floyd Algorithm Compensation Flow Chart.

Figure 6. Numerical Simulation Data Comparison.
Table 2. Comparison between Floyd Algorithm and ACO-BPN.

| Iterations (n) | Statement frequency | Proportion Floyd/ACO-BPN |
|---------------|---------------------|-------------------------|
|               | ACO-BP | Floyd | Floyd/ACO-BPN |
| 3             | 27     | 4     | 0.148        |
| 4             | 64     | 18    | 0.281        |
| 5             | 125    | 48    | 0.384        |
| 6             | 216    | 100   | 0.463        |
| 7             | 343    | 180   | 0.525        |
| 8             | 512    | 294   | 0.574        |

The comparison between Floyd algorithm and ACO-BPN highlights that when iterations number \( n \) increases, the statement frequencies will both increase. However, the Floyd’s statement frequency is lower than that of ACO-BPN. As the comparison demonstrated, the Floyd algorithm is more effective in compensating machine errors.

Compensation Effect of Floyd Algorithm under Variable Temperatures Conditions

Even though environment temperature keeps constant, during machining the effects of temperature on machine tool mechanical components cannot be neglected. Therefore, Section 4.4 introduces compensation effect of Floyd algorithm to variable temperatures, on thermal error and its relationship with the temperature of components. Consider the tool coming to a certain position. Three sensors installed on driving part, follower of key components and surroundings, respectively, of each moving axis detect temperatures. Taking X-axis for example, sensors are installed on feed screw nut, flanged bearing unit WBK and on machine bed. Acquisition time of temperature data is set to 100 minutes. In this paper, positioning error datas was selected synchronously after the machine tool had been warmed up for 10, 20, 35, 50, and 100 min, respectively. Table 3 lists part of the measured data.

Table 3. Part of the Measured Data.

| time (min) | Temperature °C | Machine bed |
|-----------|---------------|-------------|
|           | feed screw nut | WBK         |
| 10        | 18.88         | 17.69       | 17.58       |
| 20        | 19.18         | 17.92       | 17.69       |
| 35        | 20.54         | 18.56       | 17.95       |
| 50        | 21.12         | 18.74       | 18.05       |
| 100       | 22.08         | 18.95       | 18.32       |

Fig. 7 shows the thermal errors on X-axis and the numerical simulation of Floyd algorithm compensation effect under variable temperatures.
Fig. 7 shows that Floyd algorithm compensation method keeps the errors within the range [-6μm, 7μm]. Furthermore, the compensation method provides excellent results under variable temperatures conditions.

**Conclusion**

This paper proposes a new thermal compensation method for machine tool based on Floyd algorithm and HTMs. By compensating the thermal errors that affect the tool path, the proposed compensation model has advantages of higher compensation precision, more effective, shorter time than ACO-BPN. Under variable temperatures conditions, the Floyd compensation method also had good closed-loop robustness and the results assess the effectiveness of the proposed method.

Despite the progress in the proposed method, how to compensate or reduce the uncertain error terms by Floyd algorithm is unconsidered in this paper. However, it is more and more important to analyze the cutting results with consideration of uncertain error terms, especially in high accuracy machining field.

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