Study of the $\Upsilon(1S) \to DP$ decays

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Abstract

Inspired by the potential prospects of high-luminosity dedicated colliders and the high enthu-
siasms in searching for new physics in the flavor sector at the intensity frontier, the $\Upsilon(1S) \to D^-\pi^+,$ $\bar{D}^0\pi^0$ and $D_s^-K^+$ weak decays are studied with the perturbative QCD approach. It is
found within the standard model that the branching ratios for the concerned processes are tiny, about $O(10^{-18}),$ and far beyond the detective ability of current experiments unless there exists some significant enhancements from a novel interaction.
Searching for possible new physics (NP) beyond the standard model (SM) of particles from precise measurements with huge statistical data is a popular fashion nowadays for experimentalists and theorists. The $b$ quark weak decays are ideal places to explore NP effects, because at the intensity frontier, there will be more than $10^{14}\ b\bar{b}$ pairs with $300\ ab^{-1}$ dataset at LHCb [1, 2] and about $5\times10^{10}\ b\bar{b}$ pairs with $50\ ab^{-1}$ dataset at Belle-II [2, 3] in the near future. The $b$ rare decays usually have tiny branching ratios within SM, and are often used to look for NP, because an obvious deviations from SM predictions might be a smoking gun of NP. Of course, the precondition is a detailed and comprehensive investigations into specific processes within SM. According to the future experimental prospects, in this paper, we will study the $\Upsilon(1S) \rightarrow DP$ decays (here $P = \pi$ and $K$) within SM in order to offer a ready reference for future analysis.

The $\Upsilon(1S)$ meson is one of the $b\bar{b}$ bound states (bottomonium). The mass of $\Upsilon(1S)$ meson, $m_{\Upsilon(1S)} = 9460.30(26)\ MeV$ [4], is less than the open flavor threshold. The predominant decay is through the annihilation of the $b\bar{b}$ pairs into three gluons, with branching ratio $Br(\Upsilon \rightarrow ggg) = 81.7(7)\%$ [4]. However, the hadronic decays are hindered by the phenomenological Okubo-Zweig-Iizuka (OZI) rule [5–7], and result in the extremely narrow decay width, $\Gamma_{\Upsilon} = 54.02(1.25)\ keV$ [4]. So far, the sum of measured branching ratio of 100 exclusive hadronic modes is only about 1.2% [4]. Besides the strong and electromagnetic transitions, the $\Upsilon(1S)$ meson can decay through the weak interactions, for example, the flavor non-conservation processes of $\Upsilon(1S) \rightarrow DP$ decays. It is estimated that the branching ratios of $\Upsilon(1S)$ weak decays should usually be very small, about $2/\tau_{\Upsilon}\Gamma_{\Upsilon} \sim \mathcal{O}(10^{-8})$. Within SM, the $\Upsilon(1S) \rightarrow DP$ decays are induced by the $W^\pm$ exchange, see Fig. 1.

![Feynman diagram for the $\Upsilon(1S) \rightarrow D^-\pi^+$ decay within SM.](image)

Some $10^8\ \Upsilon(1S)$ mesons have been collected at resonances by the Belle detector [8]. A much more number of $\Upsilon(1S)$ with great precision is expected at the running SuperKEKB and upgraded LHC accelerators. Besides the direct production via $e^+e^- \rightarrow \Upsilon(1S)$, the $\Upsilon(1S)$ meson can also be produced via the $\Upsilon(nS) \rightarrow \pi\pi\Upsilon(1S)$ and $\eta\Upsilon(1S)$ transitions (where $n \geq$
2) and initial state radiation processes $e^+e^- \rightarrow \pi\pi\Upsilon(1S)$. The huge amount of data make the study of $\Upsilon(1S)$ weak decay interesting and worthwhile, although very challenging.

With the help of the high performance of Belle-II and LHCb detectors, and the assistance of sophisticated analysis technology and methods, events of the $\Upsilon(1S) \rightarrow DP$ decays should in principle be easily selected. On the one hand, the final states carry definite energies and momenta in the rest frame of the $\Upsilon(1S)$ meson; on the other hand, the identification of a single charmed meson is free from inefficiently double tagging, and provides a conclusive evidence of the $\Upsilon(1S)$ weak decay. And what’s more, the phenomenon of an abnormally large production rate of a single charmed mesons would be a hint of NP.

As far as we know, the $\Upsilon(1S) \rightarrow DP$ decays have not be studied seriously yet. From the experimental point of view, inadequate data samples and tiny branching ratios might be the main considerations. From the theoretical point of view, one of the principal problems is how to properly calculate the hadron transition matrix elements due to our limited informations about the hadronization mechanisms, the long-distance contributions, and so on.

From Fig. 1, it is clearly seen that there are simultaneously many scales involved in the theoretical calculation of the $\Upsilon(1S) \rightarrow DP$ decays, such as the mass of $W$ gauge boson $m_W$, the $b$ quark mass $m_b$ and the QCD characteristic scale $\Lambda_{\text{QCD}}$. In general, different dynamics correspond to different scales. Here, we will adopt the commonly acknowledged treatment by using the effective theory. The effective Hamiltonian in charge of the $\Upsilon(1S) \rightarrow DP$ decays is written as [9],

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cb}^* \sum_{i=1}^{10} f_i C_i(\mu) O_i(\mu) + \text{H.c.},$$

where $G_F \simeq 1.166 \times 10^{-5} \text{ GeV}^{-2}$ [4] is the Fermi coupling constant. The Cabibbo-Kobayashi-Maskawa (CKM) factor $|V_{ub} V_{cb}^*| = 1.463(54) \times 10^{-4}$ [4]. The factor $f_i = +1$ for tree operators $O_{1,2}$ and $-1$ for penguin operators $O_{3-10}$, respectively. The Wilson coefficients $C_i$ are calculable with the renormalization group improved perturbation theory at the scale of $m_W$, and then evolved to the scale of $\mu$. The operators describing the local interactions among four quarks are defined as follows.

$$O_1 = \left[ \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha \right] \left[ \bar{b}_\beta \gamma_\mu (1 - \gamma_5) c_\beta \right],$$

$$O_2 = \left[ \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \right] \left[ \bar{\bar{b}}_\beta \gamma_\mu (1 - \gamma_5) c_\alpha \right],$$

$$O_3 = \left[ \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) c_\alpha \right] \sum_q \left[ \bar{q}_\beta \gamma_\mu (1 - \gamma_5) q_\beta \right].$$
\[ O_4 = \left[ \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) c_\beta \right] \sum_q \left[ \bar{q}_\beta \gamma^\mu (1 - \gamma_5) q_\alpha \right], \quad (5) \]

\[ O_5 = \left[ \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) c_\alpha \right] \sum_q \left[ \bar{q}_\beta \gamma^\mu (1 + \gamma_5) q_\beta \right], \quad (6) \]

\[ O_6 = \left[ \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) c_\beta \right] \sum_q \left[ \bar{q}_\beta \gamma^\mu (1 + \gamma_5) q_\alpha \right], \quad (7) \]

\[ O_7 = \left[ \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) c_\alpha \right] \sum_q \frac{3}{2} Q_q \left[ \bar{q}_\beta \gamma^\mu (1 + \gamma_5) q_\beta \right], \quad (8) \]

\[ O_8 = \left[ \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) c_\beta \right] \sum_q \frac{3}{2} Q_q \left[ \bar{q}_\beta \gamma^\mu (1 + \gamma_5) q_\alpha \right], \quad (9) \]

\[ O_9 = \left[ \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) c_\alpha \right] \sum_q \frac{3}{2} Q_q \left[ \bar{q}_\beta \gamma^\mu (1 - \gamma_5) q_\beta \right], \quad (10) \]

\[ O_{10} = \left[ \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) c_\beta \right] \sum_q \frac{3}{2} Q_q \left[ \bar{q}_\beta \gamma^\mu (1 - \gamma_5) q_\alpha \right], \quad (11) \]

where \( \alpha \) and \( \beta \) are color indices and the sum over repeated indices is understood. \( Q_q \) is the electric charge of quark \( q \) in the unit of \( |e| \), and \( q \in \{u, d, c, s, b\} \).

With the interaction Hamiltonian of Eq.(1), the decay amplitudes for the \( \Upsilon(1S) \rightarrow DP \) decays can be written as,

\[ \mathcal{A}(\Upsilon \rightarrow DP) = \langle DP | \mathcal{H}_{\text{eff}} | \Upsilon \rangle = \frac{G_F}{\sqrt{2}} V_{ub} V_{cb}^* \sum_{i=1}^{10} f_i C_i(\mu) \langle DP | O_i(\mu) | \Upsilon \rangle. \quad (12) \]

It is seen that the decay amplitudes of Eq.(12) are clearly factorized into four parts: the couplings of weak interactions \( G_F \), the CKM factors \( V_{ub} V_{cb}^* \), the Wilson coefficients \( C_i \) summarizing the physical contributions above the scale of \( \mu \), and the hadronic matrix elements (HMEs) \( \langle O_i \rangle = \langle DP | O_i(\mu) | \Upsilon \rangle \) containing the physical contributions below the scale of \( \mu \). The product of the first three parts, \( G_F, V_{ub} V_{cb}^* \) and \( C_i \), can be regarded as the effective coupling of operators \( O_i \), and has been well known. The HMEs \( \langle O_i \rangle \) describing the transitions from quarks to participating hadrons are the core and difficulty of theoretical calculations. In addition, the QCD radiative corrections to HMEs should be included in order to obtain a physical amplitude by cancelling the scale \( \mu \) dependence of the Wilson coefficients.

Recently, some QCD-inspired phenomenological models, such as the QCD factorization (QCDF) approach \[10–15\] based on the collinear approximation, and the perturbative QCD (pQCD) approach \[16–22\] based on \( k_T \) factorization, have been successfully applied to exclusive nonleptonic \( B \) meson decay processes. Using these models, HMEs have a simple structure. They are generally expressed as the convolution of scattering sub-amplitudes arising
from hard gluon exchanges among quarks and the wave functions (WFs) reflecting the non-perturbative contributions. The scattering sub-amplitudes are in principle calculable order by order with the perturbative theory. WFs are universal and process-independent, and could be obtained by nonperturbative methods or from data. So the theoretical calculation of HMEs becomes reasonably practical. The $W^\pm$ exchange topology of Fig. 1 corresponds to annihilation topologies (see Fig. 2) within the effective theory of Eq. (1). Another two unknown parameters or more will be introduced to deal with the endpoint divergences of the annihilation amplitudes with the QCDF approach [23–31]. While the transverse momentum effects and Sudakov factors are considered to settle the endpoint contributions of quark scattering amplitudes and hadronic WFs with the pQCD approach [16–22]. In this paper, we will investigate the $\Upsilon(1S) \to DP$ decays with the pQCD approach. The master pQCD formula for decay amplitudes could be factorized into three parts: the hard contributions above the scale of $\mu$ incorporated into the Wilson coefficients $C_i$, the perturbatively calculable quark scattering amplitudes $H$ near the scale of $\mu$, and the long-distribution contributions below the scale of $\mu$ incorporated into hadronic WFs $\Phi$.

$$A_i = \int dx_1 dx_2 dx_3 db_1 db_2 db_3 C_i(t_i) H_i(x_1, x_2, x_3, b_1, b_2, b_3) \Phi_T(x_1, b_1) e^{-S_T} \Phi_D(x_2, b_2) e^{-S_D} \Phi_P(x_3, b_3) e^{-S_P},$$

where $x_i$ is the longitudinal momentum fraction of the valence quark, $b_i$ is the conjugate variable of the transverse momentum, and $e^{-S_i}$ is the Sudakov factor.

With the convention of Refs. [32–36], the relevant mesonic WFs and distribution amplitudes (DAs) are defined as follows.

$$\langle 0 | \bar{b}_\alpha(0) b_\beta(z) | \Upsilon(p_1, e_\parallel) \rangle = \frac{f_T}{4} \int d^4 k_1 e^{-i k_1 \cdot z} \{ f_\Upsilon^\parallel [m_T \phi_T^v - \not{p}_T \phi_T^t] \} \beta_\alpha,$$

$$\langle \bar{D}(p_2) | \bar{q}_\alpha(z) c_\beta(0) | 0 \rangle = -i \frac{f_D}{4} \int d^4 k_2 e^{-i k_2 \cdot z} \{ \gamma_5 (\not{p}_2 + m_D) \phi_D \} \beta_\alpha,$$

$$\langle P(p_3) | \bar{q}_\alpha(z) u_\beta(0) | 0 \rangle = -i \frac{f_P}{4} \int d^4 k_3 e^{-i k_3 \cdot z} \{ \gamma_5 [\not{p}_3 \phi_P^a + \mu_P \phi_P^p - \mu_P (\not{n}_- \not{n}_+ - 1) \phi_P^t] \} \beta_\alpha,$$

where $f_T$, $f_D$ and $f_P$ are decay constants. $\mu_P = 1.6 \pm 0.2$ GeV [34] is the chiral mass. $n_+ = (1, 0, 0)$ and $n_- = (0, 1, 0)$ are the light cone vectors, and satisfy the relations of $n_+^2 = 0$.
and \( n_+ n_- = 1 \). In the rest frame of the \( \Upsilon(1S) \) meson, the kinematic variables are defined as follows.

\[
P_{\Upsilon} = p_1 = \frac{m_{\Upsilon}}{\sqrt{2}} (1, 1, 0),
\]

\[
P_D = p_2 = \frac{m_{\Upsilon}}{\sqrt{2}} (1, r_D^2, 0),
\]

\[
P_P = p_3 = \frac{m_{\Upsilon}}{\sqrt{2}} (0, 1 - r_D^2, 0),
\]

\[
k_1 = x_1 p_1 = \frac{m_{\Upsilon}}{\sqrt{2}} (x_1, x_1, \vec{k}_1 T),
\]

\[
k_2 = x_2 p_2^+ = \frac{m_{\Upsilon}}{\sqrt{2}} (x_2, 0, \vec{k}_2 T),
\]

\[
k_3 = x_3 p_3^- = \frac{m_{\Upsilon}}{\sqrt{2}} (0, x_3 (1 - r_D^2), \vec{k}_3 T),
\]

\[
\epsilon_{\Upsilon}^\parallel = \frac{1}{\sqrt{2}} (1, -1, 0),
\]

where \( k_i, x_i \) and \( \vec{k}_{iT} \) are respectively the momentum, longitudinal momentum fraction and transverse momentum, as shown in Fig. 2(a). The mass ratio \( r_D = \frac{m_D}{m_{\Upsilon}} \). \( \epsilon_{\Upsilon}^\parallel \) is the longitudinal polarization vector. The explicit DA expressions [32–36] are as follows.

\[
\phi^v_{\Upsilon}(x) = A x \bar{x} \exp\left\{ - \frac{m_b^2}{8 \beta_1^2 x \bar{x}} \right\},
\]

\[
\phi^l_{\Upsilon}(x) = B \xi^2 \exp\left\{ - \frac{m_b^2}{8 \beta_1^2 x \bar{x}} \right\},
\]

\[
\phi_D(x) = C x \bar{x} \exp\left\{ - \frac{1}{8 \beta_2^2} \left( \frac{m_q^2}{x} + \frac{m_e^2}{\bar{x}} \right) \right\},
\]

\[
\phi_D(x, b) = 6 x \bar{x} \left\{ 1 - C_D \xi \right\} \exp\left\{ - \frac{1}{2} \omega_D^2 b^2 \right\},
\]

\[
\phi^a_P(x) = 6 x \bar{x} \left\{ 1 + a_1^P C_1^{3/2}(\xi) + a_2^P C_2^{3/2}(\xi) \right\},
\]

\[
\phi_P^P(x) = 1 + 3 \rho_+^P - 9 \rho_-^P a_1^P + 18 \rho_+^P a_2^P
+ \frac{3}{2} (\rho_+^P + \rho_-^P) (1 - 3 a_1^P + 6 a_2^P) \ln(x)
+ \frac{3}{2} (\rho_+^P - \rho_-^P) (1 + 3 a_1^P + 6 a_2^P) \ln(\bar{x})
- \left( \frac{3}{2} \rho_+^P - \frac{27}{2} \rho_-^P a_1^P + \frac{27}{2} \rho_-^P a_2^P \right) C_1^{1/2}(\xi)
+ (30 \eta_P - 3 \rho_+^P a_1^P + 15 \rho_+^P a_2^P) C_2^{1/2}(\xi),
\]

\[
\phi_P^P(x) = \frac{3}{2} (\rho_-^P - 3 \rho_-^P a_1^P + 6 \rho_-^P a_2^P)
\]
\[
-C_1^{1/2}(\xi) \{ 1 + 3 \rho_+^P - 12 \rho_+^P a_1^P + 24 \rho_+^P a_2^P \\
+ \frac{3}{2} (\rho_+^P + \rho_-^P) (1 - 3 a_1^P + 6 a_2^P) \ln(x) \\
+ \frac{3}{2} (\rho_+^P - \rho_-^P) (1 + 3 a_1^P + 6 a_2^P) \ln(\bar{x}) \} \\
- 3 (\rho_+^P a_1^P - \frac{15}{2} \rho_-^P a_2^P) C_2^{1/2}(\xi),
\]

where \( \bar{x} = 1 - x \) and \( \xi = x - \bar{x} = 2x - 1 \). \( \beta_1 = m_b \alpha_s(m_b) \) and \( \beta_2 = m_D \alpha_s(m_D) \) are the shape parameters of DAs for the \( \Upsilon(1S) \) and \( D \) mesons. \( a_i^P \) and \( C_n^m(\xi) \) are the Gegenbauer moment and Gegenbauer polynomials. The other shape parameters of pseudoscalar DAs are \( \rho_+^P = \frac{m_\pi^2}{\mu_\pi^2}, \rho_0^P \simeq \frac{m_s}{\mu_K}, \rho_-^P = 0, \) and \( \eta_P = \frac{f_3 P}{f_P \mu_P} \). The parameters \( A, B \) and \( C \) in Eq.(24), Eq.(25) and Eq.(26) can be determined by the normalization conditions.

\[
\int_0^1 dx \phi_{\Upsilon}^{v,t}(x) = 1,
\]

\[
\int_0^1 dx \phi_{D_0}(x) = 1.
\]

It should be pointed out there are many models for DAs of the \( D \) mesons, for example, Eq.(30) of Ref.[36]. In this paper, we take the typical models of Eq.(26) and Eq.(27) for examples to illustrate the model dependence of results. For the scenario I of Eq.(26), the mass of light quark is \( m_{u,d} = 310 \) MeV and \( m_s = 510 \) MeV [37]. For the scenario II of Eq.(27), the shape parameters \( C_D = 0.5 \) and \( \omega_D = 0.1 \) GeV for the \( D_{u,d} \) meson and \( C_D = 0.4 \) and \( \omega_D = 0.2 \) GeV for the \( D_s \) meson [36].

![Feynman Diagrams](image)

**FIG. 2:** The Feynman diagram for the \( \Upsilon(1S) \to D^-\pi^+ \) decay with the pQCD approach, where (a,b) are factorizable diagrams, and (c,d) are nonfactorizable diagrams. The dots denote appropriate interactions, and the dashed circles denote quark scattering amplitudes.

The lowest order Feynman diagrams for the \( \Upsilon(1S) \to D\pi \) decay with the pQCD approach is shown in Fig. 2. After a series of calculation with the pQCD formula of Eq.(13), the
expressions of the decay amplitude and branching ratio are written as follows.

\[
A(\Upsilon \to DP) = F \frac{G_F}{\sqrt{2}} V_{ub} V_{cb}^* \left\{ \left( a_2 - a_3 - a_4 + \frac{1}{2} a_7 + \frac{1}{2} a_9 \right) (A_{c}^{LL} + A_{b}^{LL}) + \left( C_1 - C_4 + \frac{1}{2} C_{10} \right) (A_{c}^{LL} + A_{b}^{LL}) - \left( C_6 - \frac{1}{2} C_{8} \right) (A_{c}^{LR} + A_{b}^{LR}) \right\},
\]

\[
Br = \frac{p_{cm}}{2 \pi m_{\Upsilon}^2 \Gamma_{\Upsilon}} |A(\Upsilon \to DP)|^2,
\]

where the factor \( F = \frac{1}{\sqrt{2}} \) for \( P = \pi^0 \), and \( F = +1 \) for \( P = \pi^+ \) and \( K^+ \). \( p_{cm} \) is the center-of-mass momentum of final states in the rest frame of the \( \Upsilon(1S) \) meson. The building blocks of amplitudes \( A_j^i \) are listed in Appendix A. With the input parameters in Table I, the numerical results of the branching ratios for the \( \Upsilon(1S) \to DP \) decays are shown in Table II.

TABLE I: The values of the input parameters, where their central values will be regarded as the default inputs unless otherwise specified. The numbers in parentheses are errors.

| mass and decay constants of the particles [4] |
|---------------------------------------------|
| \( m_{\pi^0} = 134.98 \text{ MeV}, \) |
| \( m_{D^0} = 1864.84(5) \text{ MeV}, \) |
| \( f_\pi = 130.2(1.2) \text{ MeV}, \) |
| \( m_{\pi^\pm} = 139.57 \text{ MeV}, \) |
| \( m_{D^\pm} = 1869.5(4) \text{ MeV}, \) |
| \( f_K = 155.7(3) \text{ MeV}, \) |
| \( m_{K^\pm} = 493.68 \text{ MeV}, \) |
| \( m_{D_s^\pm} = 1969.0(1.4) \text{ MeV}, \) |
| \( f_{3\pi} = 0.45(15) \times 10^{-2} \text{ GeV}^2 [34], \) |
| \( m_b = 4.78(6) \text{ GeV}, \) |
| \( f_D = 212.6(7) \text{ MeV}, \) |
| \( f_{3K} = 0.45(15) \times 10^{-2} \text{ GeV}^2 [34], \) |
| \( m_c = 1.67(7) \text{ GeV}, \) |
| \( f_{D_s} = 249.9(5) \text{ MeV}, \) |
| \( f_{\Upsilon(1S)} = 676.4(10.7) \text{ MeV} [38], \) |

| Gegenbauer moments at the scale of \( \mu = 1 \text{ GeV} [34] \) |
|---------------------------------------------|
| \( a_1^\pi = 0, \) |
| \( a_2^\pi = 0.25(15), \) |
| \( a_1^K = 0.06(3), \) |
| \( a_2^K = 0.25(15) \) |

TABLE II: Branching ratios for the \( \Upsilon(1S) \to DP \) decays in the unit of \( 10^{-18} \). The uncertainties come from the DAs of \( \phi^{s,t} \) due to variation of \( m_b, \phi_D \) due to variation of \( m_c \) for scenario I and \( C_D \pm 0.2 \) and \( \omega_D \pm 0.04 \text{ GeV} \) for scenario II, \( \phi^{a,p,t}_P \) due to variation of \( \mu_P \) and \( a_2^P \), respectively.

| mode | \( D^- \pi^+ \) | \( \overline{D}^0 \pi^0 \) | \( D_s^- K^+ \) |
|------|----------------|----------------|----------------|
| scenario I | \( 1.157_{-0.012}^{+0.013} \times 10^{-5}, \) | \( 0.579_{-0.006}^{+0.006} \times 10^{-5} \) | \( 1.775_{-0.017}^{+0.018} \times 10^{-18} \) |
| scenario II | \( 0.774_{-0.009}^{+0.009} \times 10^{-5} \) | \( 0.387_{-0.004}^{+0.004} \times 10^{-5} \) | \( 1.294_{-0.013}^{+0.014} \times 10^{-18} \) |

(1) For one specific process, the branching ratios of scenario I is larger than those of scenario II. The branching ratios are sensitive to the DA models for the \( D \) mesons.
(2) Because of the relations among decay constants, \( i.e., f_{D_s} > f_D \) and \( f_K > f_\pi \), there is a clear hierarchical pattern among branching ratios.

\[
Br(\Upsilon(1S)\rightarrow D_sK) > Br(\Upsilon(1S)\rightarrow D_d\pi) > Br(\Upsilon(1S)\rightarrow D_u\pi).
\]  

(35)

In addition, there is a relation, \( Br(\Upsilon(1S)\rightarrow D_d\pi) \approx 2 Br(\Upsilon(1S)\rightarrow D_u\pi) \), because of the quark compositions of electrically neutral pion, \( i.e., |\pi_0^0\rangle = \frac{|u\bar{u}\rangle - |d\bar{d}\rangle}{\sqrt{2}} \).

(3) Besides the uncertainties listed in Table II, another 7\% and 3\% uncertainties will come from the CKM factor \( |V_{ub}V_{cb}^*| \) and decay constants, respectively.

(4) There are many possible reasons for the small branching ratios. Some of them are selected and listed as follows. (a) Almost all of the decay width of the \( \Upsilon(1S) \) meson come from the strong and electromagnetic interactions. The \( \Upsilon(1S) \) weak decays are strongly suppressed by the Fermi coupling constant \( G_F \sim 10^{-5} \), compared with the couplings of \( \alpha_s \sim 10^{-1} \) and \( \alpha_{em} \sim 10^{-2} \). (b) These annihilation processes are dynamically suppressed by helicity. (c) These processes are highly suppressed by the CKM factor of \( |V_{ub}V_{cb}^*| \sim 10^{-4} \). (d) According to the conservation law of angular momentum, decay amplitudes are only the \( P \)-wave contributions. (e) These decays are suppressed by color due to the \( W \) exchange between quarks of different final states.

(5) The branching ratios for the \( \Upsilon(1S) \rightarrow DP \) decays within SM are the order of \( 10^{-18} \), which are too small to be measurable in the near future. Of course, it is possible that some extraordinary effects from NP may significantly enhance these branching ratios, and produce an observable phenomena. This is the very thing we are looking for in future.

In summary, considering the developmental opportunities and important challenges at the high-luminosity dedicated heavy-flavor factories in the future, the exclusive two-body nonleptonic \( \Upsilon(1S) \) decays through the weak interactions into final states including only one charmed meson, \( \Upsilon(1S) \rightarrow DP \), are studied for the first time with the pQCD approach within SM. Our results show that (1) the \( \Upsilon(1S) \rightarrow D_sK \) decay has relatively large occurrence probability among the concerned processes; (2) the branching ratios for the \( \Upsilon(1S) \rightarrow DP \) decay are tiny, \( \mathcal{O}(10^{-18}) \), and impossible to measure at Belle-II and LHCb during the next decades. One experimental signal of the \( \Upsilon(1S) \rightarrow D_sK \) and/or \( D\pi \) decays will be an obvious deviation from the SM prediction and an omen of NP.
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Appendix A: Building blocks of decay amplitudes

For the sake of convenience in writing, some shorthands are used. There should always be a Sudakov factor corresponding to each WF with the pQCD approach. So the shorthands are $\phi_T^{s,t}(x_1) e^{-S_T}$, $\phi_D = \phi_D(x_2) e^{-S_D}$, $\phi_p^a = \phi_p^a(x_3) e^{-S_p}$ and $\phi_p^{s,t} = \frac{\mu_p}{m_T} \phi_p^{s,t}(x_3) e^{-S_p}$.

\[ a_i = \begin{cases} 
C_i + \frac{1}{N_c} C_{i+1}, & \text{for odd } i; \\
C_i + \frac{1}{N_c} C_{i-1}, & \text{for even } i. 
\end{cases} \tag{A1} \]

According to the pQCD formula of Eq. (13), the amplitude building block $A_i^j$ should be a function of the Wilson coefficients $C_i$. That is to say, the expression $C_k A_i^j$ in Eq. (33) should actually be $A_i^j C_k$. As to the amplitude building block $A_i^j$, the subscript $i$ corresponds to the indices of Fig. 2, and the superscript $j$ refers to the two possible Dirac structures $\Gamma_1 \otimes \Gamma_2$ of the operator $(\bar{q}_1 \Gamma_1 q_2)(\bar{q}_3 \Gamma_2 q_4)$, namely $j = LL$ for $\gamma^\mu (1 - \gamma_5) \otimes \gamma_\mu (1 - \gamma_5)$ and $j = LR$ for $\gamma^\mu (1 - \gamma_5) \otimes \gamma_\mu (1 + \gamma_5)$. The expressions of $A_i^j$ are written as follows.

\[ \mathcal{C} = m_T^4 f_T f_D f_P \frac{\pi C_F}{N_c}, \tag{A2} \]

\[ A_{a}^{LL} = \mathcal{C} \int_0^1 dx_2 dx_3 \int_0^\infty db_2 db_3 \alpha_s(t_a) H_{ab}(\alpha_g \beta_a, b_2, b_3) C_i(t_a) S_i(x_2) \phi_D \left\{ \phi_p^a (1 - r_{D}) x_2 + 2 r_D \phi_p^a [x_2 - (1 - r_{D})] \right\}, \tag{A3} \]

\[ A_{b}^{LL} = \mathcal{C} \int_0^1 dx_2 dx_3 \int_0^\infty db_2 db_3 \alpha_s(t_b) H_{ab}(\alpha_g \beta_b, b_3, b_3) C_i(t_b) S_i(x_3) \phi_D \left\{ \phi_p^a (1 - r_{D}) [x_3 (1 - r_{D}) - r_{D}^2] + r_D (1 - r_{D}^2) (x_3 - \bar{x}_3) [\phi_p^b + \phi_p^f] + 2 r_D \phi_p^f \right\}, \tag{A4} \]

\[ A_{c}^{LL} = \frac{\mathcal{C}}{N_c} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty db_1 db_2 \alpha_s(t_c) H_{cd}(\alpha_g \beta_c, b_1, b_2) C_i(t_c) \phi_D \left\{ \phi_T^c [\phi_p^a (1 - r_{D}) (x_1 (1 + r_{D}) - x_2 r_{D}^2 - x_3 (1 - r_{D}^2)) + \phi_p^b (1 - r_{D}) (x_1 (1 + r_{D}) - x_2 r_{D}^2 - x_3 (1 - r_{D}^2)) + \phi_p^f (1 - r_{D}) (x_1 (1 + r_{D}) - x_2 r_{D}^2 - x_3 (1 - r_{D}^2)) \right\} \frac{\pi C_F}{N_c}. \]
\[ H_{ab}(\alpha, \beta, b_i, b_j) = -\frac{\pi^2}{4} b_i b_j \{ J_0(b_j \sqrt{\alpha}) + i Y_0(b_j \sqrt{\alpha}) \} \]

\[ A_c^{LR} = \frac{C}{N_c} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty db_1 db_2 \alpha_s(t_c) H_{cd}(\alpha_g \beta_c, b_1, b_2) C_i(t_c) \]

\[ \phi_D \{ \phi_P^t (1 - r_D^2)^2 (x_2 - x_1) + r_D \phi_P^t (x_2 - x_3 (1 - r_D^2)) \]

\[ -r_D \phi_P^t (2 x_1 - x_2 - x_3 (1 - r_D^2)) \]

\[ + r_D \phi_P^t (x_2 - x_3 (1 - r_D^2)) + r_D \phi_P^t (2 x_1 - x_2 - x_3 (1 - r_D^2)) \]

\[ -\frac{1}{2} \phi_P^t [\phi_P^a (1 - r_D^2) + 4 r_D \phi_P^t] \}

\[ A_c^{LR} = \frac{C}{N_c} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty db_1 db_2 \alpha_s(t_c) H_{cd}(\alpha_g \beta_c, b_1, b_2) C_i(t_c) \]

\[ \phi_D \{ \phi_P^t (1 - r_D^2)^2 (x_2 - x_1) + r_D \phi_P^t (x_2 - x_3 (1 - r_D^2)) \]

\[ -r_D \phi_P^t (2 x_1 - x_2 - x_3 (1 - r_D^2)) \]

\[ + \frac{1}{2} \phi_P^t [\phi_P^a (1 - r_D^2) + 4 r_D \phi_P^t] \}

\[ A_c^{LL} = - A_c^{LR} (x_1 \rightarrow \bar{x}_1, t_c \rightarrow t_d, \beta_c \rightarrow \beta_d) \]
where \( I_0, J_0, K_0 \) and \( Y_0 \) are Bessel functions. The expression of \( s(x, Q, b) \) can be found in Ref.[18]. \( \gamma_q = -\frac{\alpha_s}{\pi} \) is the quark anomalous dimension.

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