CFD Study on Penetration of Sonic Boom Applying a Wavy-Water Model

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The penetration of sonic booms into the ocean and an estimation of the extent of their influence are important topics for supersonic and hypersonic aviation. Most prior studies were conducted assuming a flat-water surface. However, it is known that the effect of the surface waviness is not negligible in the case of real ocean environments. In the present study, the numerical method analyzes the two-dimensional flow field of sonic boom propagation across the air and water is proposed. When the flight Mach number of an aircraft is higher than 4.4, the underwater flow field becomes supersonic, and a sonic boom is generated there. The numerical results show that, in the underwater subsonic regime, the penetrating waves have the form of evanescent waves under either flat or wavy water surface, but the waviness-induced ripples with the smaller decay rate are responsible for deeper penetration. In contrast, in the supersonic underwater flow, the surface waviness hardly affects the penetration depth of underwater sonic booms, although it can slightly weaken the boom intensity. The effect of the amplitude of surface waviness on the underwater penetrating wave was also investigated.

Key Words: Sonic Boom Penetration, Underwater Overpressure, Wavy-Surface Interaction Effect, Computational Fluid Dynamics

Nomenclature

\[ \alpha: \text{decay rate} \]
\[ c: \text{speed of sound} \]
\[ c_s: \text{speed of surface wave} \]
\[ \delta: \text{amplitude of surface wave} \]
\[ H_N: \text{height of N-wave signature source} \]
\[ \eta: \text{elevation of water surface} \]
\[ I_p: \text{intensity ratio} \]
\[ L: \text{length of N-wave signature source} \]
\[ \lambda: \text{wavelength of surface wave} \]
\[ M: \text{Mach number} \]
\[ \mu: \text{Mach angle} \]
\[ p: \text{pressure} \]
\[ p^*: \text{overpressure} \]
\[ p_N^*: \text{peak overpressure of N-wave source} \]
\[ \rho: \text{density} \]
\[ t: \text{time} \]
\[ u, v: \text{velocity in } x- \text{ and } y-\text{directions} \]
\[ V: \text{speed of aircraft} \]
\[ x, y: \text{streamwise and vertical coordinates} \]

Subscripts

\[ a: \text{atmosphere} \]
\[ \text{max: maximum value} \]
\[ s: \text{water surface} \]
\[ w: \text{water} \]
\[ 0: \text{free water surface} \]
\[ \infty: \text{freestream} \]

1. Introduction

1.1. Background

During the last couple of decades, there has been interest in developing supersonic and hypersonic civil aviation in the world.$^1$ The potential benefits provided by high-speed transports not only for military demand but also for time-critical business trips and emergency response. However, sonic booms generated from aircrafts flying at speeds higher than Mach 1 are blamed for negative effects on humans, wildlife, and structures owing to their explosive sounds. (See Ref. 2) for the review on sonic booms.) In Manci’s report,$^3$ sound levels above 116 dB re 1 μPa (abbreviation of decibel referenced to pressure of 1 micropascal) probably lead to an adverse effect on mammals and results in several responses, including changing their habitation and proliferation, for example. Additionally, Cook, in his study, indicated that sonic booms, although possibly not triggering earthquakes directly, can potentially lead to avalanches or landslides, particularly in the regions previously stressed to instability.$^2$ Moreover, the lengthy and repeated impact on the distant airline of high-speed transports are also related with the public acceptance.

In view of these negative effects, supersonic flights typically are only allowed operating over oceans to minimize the influence on human. A major concern in turn was whether marine wildlife and environments would be disturbed by airborne sonic booms. Although the effect of noise on marine animals are not well known to date, there are a few typical effects, including behavioral change, masking (that animals difficulty perceive significant sounds for themselves because of the interference of ambient noises), hearing loss, and stranding.$^2$ These short-term responses to the noises may not only affect individual life functions but also be re-
sponsible for population-level consequences.\(^6\) Therefore, as one source of underwater noise, the penetration of sonic boom and its extent of influence are the important topics worth comprehensive evaluation.

### 1.2. Previous study on penetration of sonic booms

In 1968, Sawyers analytically derived a closed-form solution to the subsonic underwater overpressure induced by a sonic boom.\(^3\) In the analysis, Sawyers solved the pressure field using a linearized acoustic equation by directly specifying a N-wave signature on a flat-water surface. The penetrating pressure wave, according to Sawyers’ theory, evanescently propagates downward in water, and its intensity attenuated rapidly with water depth. Based on Sawyers’ analysis, some further research was sequentially conducted under different conditions and assumptions of analysis. For instance, the analysis was extended to the three-dimensional (3-D) underwater acoustic field by Moody,\(^8\) and was generalized into arbitrarily shaped boom signatures (instead of a N-wave) by Sparrow and Ferguson.\(^9\) These results presented slightly different waveforms from that in Sawyers’ analysis, but the evanescent decay as a significant characteristic was observed in the underwater acoustic field as well.

Additionally, there were several investigations on the effect of surface waviness for the realistic ocean feature. In Rochat and Sparrow’s numerical study with the finite difference procedure, the resultant overpressure showed the effect of focusing (amplifying) and defocusing (reducing) of the sonic boom arisen on troughs and crests of surface waviness, respectively.\(^10\) Meanwhile, Cheng and Lee proposed an analytical method that revealed the presence of downward-propagating waves induced by the interaction. The waves had deeper penetration than the sound waves in flat water, though they still complied with evanescent decay.\(^11,12\) Fincham and Maxworthy then conducted a laboratory-scaled experiment verifying the effect of wavy-surface interaction.\(^13\) According to their results, the wavelet frequencies of downward-propagating waves are consistent with Cheng and Lee’s analytical calculation.

Apart from the influence of the wavy-surface interaction, there is another important factor to cause deep penetration. For a level flight at \(M_o \geq 4.4\) (corresponding to the speed of sound in water, 1500 m/s), the penetrating waves converge into the shock waves in the supersonic regime (i.e., \(M_w \geq 1\)). Such phenomenon is referred to as underwater sonic boom, and \(M_w = 4.4\) is called “critical Mach number.” Unlike the evanescent waves, underwater sonic booms are capable of keeping their intensity and waveforms into very deep water, which implies that the sonic booms generated by hypersonic flights have an extensive-range effect than supersonic flights.

The preliminary study on the underwater sonic boom has been experimentally and analytically carried out by Intrieri and Malcolm\(^14\) and Wang et al.,\(^15\) respectively. The results showed that the pressure rise of sonic boom barely attenuate with water depth in 2-D uniform water. Furthermore, Chen and Suzuki studied the stratification effects on the underwater sonic boom in a 3-D space.\(^16\) In their numerical results, the vertical extent of the underwater sonic boom is limited by the sonic cutoff due to stratified water, and the lateral extent is constrained by interface cutoff due to the discontinuity of speed of sound on air-water interface.

### 1.3. Objective

As we have introduced above, Sawyers’ analysis using the flat-water model somewhat underestimated the influence on an actual ocean environment. The deeper penetration of a sonic boom was found in the wavy-surface model proposed by Cheng and Lee. However, in their published works, most efforts were concentrated on the underwater subsonic regime. Considering the advance of hypersonic transportation in the coming decades, the intensity and extent of underwater sonic booms generated by hypersonic flights will become more significant and require further investigation.

Additionally, in most previous studies, the analysis methods are not applicable to both subsonic and supersonic regimes. It is necessary to change the methods for the different flow regimes they dealt with. For instance, Sawyers’ and Cheng and Lee’s analytical solutions fail for the supersonic regime.\(^7,11,12\) Consequently, in the present research, the numerical method applicable to both subsonic and supersonic regimes will be established to simulate the penetration of sonic booms under wavy-water model. Based on the study, we not only have an insight into the features of penetrating sonic booms in the qualitative aspect, but also evaluate its intensity and decay rate in the quantitative aspect.

### 2. Methodology

#### 2.1. Problem setting

As can be seen in the schematic system shown in Fig. 1, the two-dimensional (2-D) simulation of a sonic boom traveling over a wavy body of water and its penetration are calculated for both air and water domains. The 3-D geometry of water surface and geometrical spreading effect are not considered in this paper. Additionally, we neglect fluid viscosity and gravity, and the flow is governed by the 2-D Euler equations, that is,
Note that, in the simulation model, the freestream velocity $V$ is set to \( \rho_w \) which are different from \( \rho_a \). On the wavy water surface, the elevation \( \eta \) is modeled by a sinusoidal wave train of amplitude $\delta$, wavelength $\lambda$, and wave speed $c_s$. The mathematical expression is given as

$$\eta(x, t) = \delta \sin \left[ 2\pi \left( x - (c_s + V)t \right) / \lambda \right].$$

Note that, in the simulation model, the freestream velocity $V$ is necessary to be added to the wave speed, so that points on the surface have relative velocity of $V + c_s$ with respect to N-wave source.

As a numerical study, the parameters in the system should be specified for computation. Here, the freestream pressure and density in the atmosphere are constantly set to $p_{a,\infty} = 1.013 \times 10^5 \text{Pa}$ and $\rho_{a,\infty} = 1.226 \text{kg/m}^3$, respectively. The velocity components are given as $u_{\infty} = c_{a,\infty} \times M_d$ and $v_{\infty} = 0$, where the speed of sound in the atmosphere is calculated to be $c_{a,\infty} = \sqrt{\gamma p_{a,\infty} / \rho_{a,\infty}} = 340.2 \text{m/s}$, and $M_d$ depends on the cases in simulation. In water domain, freestream values are also constant and identical to the value on the free surface, namely, $p_{w,\infty} = p_{w,0} = 1.013 \times 10^5 \text{Pa}$, and $\rho_{w,\infty} = \rho_{w,0} = 1000 \text{kg/m}^3$. For the N-wave signature source, its length and small peak overpressure are given as $L = 100 \text{m}$ and $p'_{N} = 0.001 \times p_{w,\infty} = 101.3 \text{Pa}$, referring to the typical sonic boom signature near the ocean surface.11] Meanwhile, the wavy interface is set at the realistic scale of an ocean environment. The wavelength of the surface wave is set to $\lambda = 1.5 \times L = 150 \text{m}$ with a wave speed of $c_s = L \times 0.1 \text{Hz} = 15 \text{m/s}$, which is much smaller than the freestream velocity in usual cases (i.e., $c_s \ll V$). Also, the amplitude is assumed to be the same order in Cheng and Lee’s study,11 which yields $\delta = 0.015 \times \lambda = 2.25 \text{m}$. Finally, decibel, the unit of sound pressure level (Lp), is a logarithmic scale of a relative quantity, which is evaluated from

$$L_p \text{ (dB)} = 20 \log_{10} \left( \frac{p'}{p_{\text{ref}}} \right)$$

where $p'$ is overpressure and $p_{\text{ref}}$ is reference pressure. Typically, in water the reference pressure is set to 1 $\mu$Pa, and the unit of sound level is abbreviated as dB re 1 $\mu$Pa (or only “dB” for short).

### 2.2. Computational grid

In the computational grid as shown in Fig. 2, the x-coordinate is in the streamwise direction originated at the leading-edge of N-wave signature, and the y-coordinate is in the vertical direction originated at the free water surface (a flat surface at rest without any external forces). Figure 3 presents the computational grids (shown in one-fourth of the actual grid nodes in each direction) for the case of
in the subsonic underwater cases are less than 5%, while it increases to 10.5% in the supersonic underwater case.

2.3. Boundary condition of air-water interface

The boundary condition on the interface is managed from two sides, namely, air side and water side. From a perspective of the air side, since a density ratio of water to air is considerably large (i.e., \( \rho_w \gg \rho_a \)), it is reasonable to treat the interface as a solid boundary when air flow implics on the water surface, which is given by

\[
p_{a,s} = p_{a,s-1}, \quad p_{a,s} = p_{a,\infty} \left( \frac{p_{a,s}}{p_{a,\infty}} \right)^{1/n},
\]

\[
u_{a,s} = (V_{a,s-1} \cdot \Omega) + V, \quad v_{a,s} = (V'_{a,s-1} \cdot \Omega),
\]

where \( V' = V - (V, 0) = (u - V, v) \) is perturbation velocity vector that removes the freestream from the flow velocity, \( t = (t_x, t_y) \) is unit tangential vector of water surface, and subscripts \( s \) and \( s-1 \) are the grid node on surface and its first internal one, respectively. From the water side, the transparent condition is specified for velocity components on the interface, the density is calculated using the Tait’s equation of state, and the pressure is given by the values on the air side, that is,

\[
p_{w,s} = p_{a,s}, \quad \rho_{w,s} = \rho_{w,\infty} \left( \frac{p_{w,s} + B}{p_{w,\infty} + B} \right)^{1/n},
\]

\[
u_{w,s} = u_{w,s-1}, \quad v_{w,s} = v_{w,s-1}.
\]

Now, the air and water domains are coupled together via the continuous pressure condition on the interface as shown in Eq. (10). Although it is noticed that the boom-induced velocity is not continuous from the two sides of interface, this discrepancy can be reasonably neglected in the simulation. Since the induced velocity is much smaller than the free-stream velocity under practical circumstances (the velocity ratio on an order of \( 10^{-3} \) or less), the effect of boom-induced velocity on interface is almost negligible in the system. Hence, we simply adopt this air-water interface condition in Eqs. (9) and (10) to couple the air and water domains in the simulation.

2.4. Simulation procedure

Figure 4 shows the flowchart about the complete procedure of simulation. The computational grid and primitive variables on nodes are initialized before the time evolution, the most time-consuming portion of the whole simulation. In the evolution, the fluxes were computed using Simple High-resolution Upwind Scheme (SHUS),\(^{18}\) which is one of the AUSM-family numerical schemes. Moreover, to achieve third-order spatial accuracy, the primitive variables are extrapolated using Monotone Upstream-centered Schemes for Conservation Law (MUSCL) with the van Albada slope limiter.\(^{19}\) For time integration, conserved quantity is solved using first-order explicit forward Euler method for the cases of unsteady wavy water.

Next, after updating the boundary values, the computational domains should be re-meshed to fit the air-water interface described by Eq. (7) at the next time level, which is followed by interpolation of the primitive variables from the old nodes to the new nodes in the next time level. To speed up
the calculation, especially in the time evolution, multi-core computation based on OpenMP (written in C++) is applied to the flow-field simulation. In this study, the simulations are conducted following the procedure shown in Fig. 4 with the five cases, including both the supersonic (i.e., $M_a = 1.2, 2, 3,$ and 4) and the hypersonic (i.e., $M_a = 5$).

3. Validation of Underwater Simulation

Before starting the discussion of numerical results, let us validate our CFD method by comparing with analytical results in Sawyers’ subsonic underwater study and Wang’s supersonic underwater study. The simulation model for validation is under the conditions of 2-D, uniform free-stream, and flat water surface with a given N-wave pressure source as the top boundary. First, for the underwater subsonic regime, the analytical acoustic field is derived by Sawyers with the method of Fourier transform, and the solution is presented as

$$p' = \frac{1 - 2\tilde{x}}{\pi} \left[\tan^{-1}\left(\frac{x}{m\tilde{y}}\right) - \tan^{-1}\left(\frac{\tilde{x} - 1}{m\tilde{y}}\right)\right] = \frac{m\tilde{y}}{\pi} \ln \left[\frac{(\tilde{x} - 1)^2 + m^2\tilde{y}^2}{\tilde{x}^2 + m^2\tilde{y}^2}\right],$$

where $\tilde{x}$ and $\tilde{y}$ are the x- and y-coordinates normalized by the signature length $L$, and $m = \sqrt{1 - M_w^2}$. Apparently, the solution holds only if $M_w < 1$ and thus $m \in \mathbb{R}$. On the other hand, Wang obtained his solution applicable to supersonic underwater by semi-similar transformation. The result shows an underwater sonic boom with unchanged pressure rise as follows:

$$p' = \frac{1 - 2(\tilde{x} - n\tilde{y})}{\pi} (0 \leq \tilde{x} - n\tilde{y} \leq 1),$$

$$0 \quad \text{(otherwise)},$$

Now, we present the simulation results solved by the numerical method introduced in Section 2 and the analytical solutions shown in Eqs. (11) and (12). For the subsonic regime at $M_w = 0.46$, Fig. 5(a) shows that the acoustic field near the N-wave signature source has the form of evanescent waves attenuating promptly. In Fig. 5(b), the numerical-based waveforms at three selected depths, namely, $y/L = 0.1, 0.5, 5$, are compared with Sawyers’ analytical solution. The results confirm the consistency between two methods, where the maximum difference is less than only 5%.

On the other hand, for the hypersonic flight at $M_a = 5$ (i.e., $M_w = 1.15$), underwater acoustic field is presented in Fig. 6(a). The pressure field resolved by the numerical approach converges into shock waves in the supersonic regime, which leads to the underwater sonic boom. In theory, 2-D
underwater sonic booms under the condition of flat and uniform water can propagate downward to an infinite depth with unchanged intensity and shape as shown in Wang’s analysis. Although a slight difference of magnitude is indicated in Fig. 6(b), the deviation from the analytical solution is only on the order of $10^{-4}$ that is small enough to be ignored. As a result, the examination confirms the feasibility using the present numerical approach.

4. Results and Discussion

4.1. Focusing and defocusing on the wavy-water surface

To begin with, let us investigate on the acoustic field around the contact point, a region where the sonic boom impinges on the water surface. The region includes the air domain near the surface and the shallow water. Figure 7 shows the snapshot of overpressure distribution around the contact point for the cases of wavy water with the flows at $Ma = 2$ and $Ma = 5$. The three kinds of waves in the acoustic field, including incident wave, reflected wave, and penetrating wave, have been resolved by the simulation. It can be seen that overpressure magnitude roughly jumps double when the incident wave approaches the water surface, which results from the overlap of incident and reflected shock waves. Unlike the constant value on the flat surface, the overpressure on the wavy surface varies with time owing to the interaction between the sonic boom and waviness. According to Rochat and Sparrow’s study, the phenomenon is induced by the focusing and defocusing on the wavy surface that periodically alters the pressure rise of sonic boom. In Table 2, we list the percent changes in the pressure rise on the wavy surface with respect to the pressure rise on the flat surface.

Table 2. Waviness-induced changes in the pressure rise on the wavy surface with respect to the pressure rise on the flat surface.

| $Ma$ | 1.2 | 2.0 | 3.0 | 4.0 | 5.0 |
|------|-----|-----|-----|-----|-----|
| Percent change in pressure rise (%) | 2.1 | 1.6 | 3.1 | 8.1 | 15.9 |

In addition to reflection, the incident wave also penetrates...
the water surface and leads to penetrating waves: the evanescent wave in the subsonic flow (as shown in Fig. 7(a)) and the underwater sonic boom in the supersonic flow (as shown in Fig. 7(b)). For the case of $M_d = 5$ in Fig. 7(b), the underwater sonic boom apparently has a periodic pattern on its shock waves, which is caused by the repeated exchange of focusing and defocusing of the airborne sonic boom on water surface. There are two reasons that the periodic pattern oppositely is difficult to be observed for the case of $M_d = 2$ in Fig. 7(a), even though the wavy-surface interaction also acts on the subsonic acoustic field. The major reason is that the penetrating wave as well as the influence of wavy-surface interaction are rapidly attenuated with water depth because of the diffusive behavior in a subsonic flow. Second, the interaction has a smaller effect in a lower Mach number as mentioned above. Despite inconspicuous observation in Fig. 7(a), wavy-surface interaction in deep water has remarkable influence on the acoustic field in the subsonic regime. In the next subsection, we shall present the results in the way of acoustic waveform, and discuss the influence of wavy-surface interaction in deep water.

4.2. Underwater acoustic waveforms

In Fig. 8 (shown in snapshots), overpressure waveforms are detected at different water depths for both $M_a = 2$ and $M_a = 5$. The detecting depths are selected as $y = 50$ m and 500 m representative of shallow and deep water depths, respectively. For the supersonic flight at $M_a = 2$, the Mach number in water is calculated to be $M_a = 0.46 < 1$, and the overpressure waveforms in flat water agree with Sawyers’ theory that underwater sound waves decay rapidly. In wavy water, the waveform in Fig. 8(a) presents the same order of magnitude in the shallow depth, while in deep water the ripples referred to the oscillating pattern in Fig. 8(b) are detected on one order of intensity higher than that in flat water. The Sawyers’ theory describes the acoustic diffusion from a N-wave signature source without other disturbance. Since these ripples are not generated from the original N-wave source but induced by the disturbance of surface waviness, the decay of ripples apparently does not comply with Sawyers’ theory. Although the acoustic field still has the diffusion characteristic, these interaction-induced ripples have slower decay and thus deeper penetration. Note that the reason for the slow decay of ripples has not been clarified yet in this paper and is left for the future works.

On the other hand, for the hypersonic flight at $M_a = 5$, the underwater flow transfers into the supersonic regime, namely, $M_a = 1.15$. The ripples are relatively unnoticeable in deep water as shown in Fig. 8(d). It is because penetration of the N-wave signature barely attenuates with water depth in the supersonic flow, and the overpressure peak still has five times magnitude of ripples at the deep depth of $y = 500$ m. That is, unlike the ripple-dominated acoustic field in subsonic deep water, the effect of ripples becomes trivial in the supersonic boom-dominated acoustic field. Although unremarkable effect of ripples, it is interesting to note that the pressure rise under the wavy surface at $y = 500$ m is 77.7% of the one in flat water. In theory, sonic boom propagates without decay in a 2-D flat water. However, the waviness-induced periodic pattern (referred to Fig. 7(b)) disturbs smoothness of the shock surface and thus leads to some energy dissipation. Consequently, the surface waviness is able to weaken the strength of the penetrating sonic boom in water. In the context of mitigation of underwater noises, the surface waviness seems to have the positive effect for hypersonic flights.

Remind that although the ripples are remarkable in subsonic deep water, there is more than one order difference of intensity from the sonic boom in supersonic water. For the given N-wave of $p_N' = 101.3$ Pa, the sound levels of underwater noises calculated by Eq. (8) are listed in Table 3. The ripples (136 dB) in subsonic water are 27 dB less than that of the penetrating sonic boom (163 dB) in supersonic water. From the perspective of influence on mammals as in-

![Fig. 8. Underwater acoustic waveforms in flat- and wavy-surface water. The corresponding sound levels are labeled on the right side of the figures.](image-url)
The curves under in dash-dotted lines) and wavy water (plotted in solid lines). The resultant decay rates with different Mach numbers and water surface conditions.

### 4.3. Decay rate of penetrating pressure waves

To quantify the decay in the penetrating waves, in Fig. 9 we depict the intensity ratios defined by $I_p$ (%) = $(|p|^2(y)|_{\text{max}}/|p|^2(y)|_{\text{max}}) \times 100\%$, which indicates the remaining intensity of the penetrating waves with water depth. These curves are classified into two categories: flat water (plotted in dash-dotted lines) and wavy water (plotted in solid lines). The curves under flat water are produced with the analytical solutions in Eqs. (11) and (12), and the ones in wavy water are yielded by the simulation in this study. Note also that the evaluation for wavy-surface is based on the snapshot of the flow field. We shall verify that evaluation are affected merely to a small extent with varying time in the end of the subsection.

Now, let us fit these curves with the power-law relation, that is,

$$I_p = \beta y^{-\alpha},$$  \hspace{1cm} (13)

where the constants $\alpha$ and $\beta$ for each case are listed in Table 4. Since the rapidness of decay in a power-law curve is determined by the constant $\alpha$, here the power $\alpha$ in the relationship is taken as the indicator to quantify the decay rate in this study. A smaller $\alpha$ implies a slower decay in penetrating waves. In Table 4, the average $\alpha$ of 0.60 in subsonic wavy water (which closely meets the analytical number, $\alpha = 1/2$, reported in Cheng and Lee’s analysis) is apparently smaller than average 1.80 in flat water. The result agrees with the previous observation in Section 4.2 that sound waves have deeper penetration owing to the waviness-induced ripples. It is also noticed that $\alpha$ slightly decreases with the increasing Mach number. As mentioned in the introduction, the evanescent wave would transfer into the sonic boom in water as $M_w \geq 4.4$. Accordingly, the decay rate $\alpha$ decreasing as $M_w \to 4.4$ (instead of the constant number in Cheng and Lee’s analysis) is more close to the actual phenomenon.

In addition to the cases listed in Section 2.1, the parametric study is also carried out to evaluate the relationship of the geometry of surface waviness and the decay rate in the penetrating wave. According to Ref. 10, the ratio of wave amplitude to length approximately keeps constant under usual ocean conditions. Here, we keep using $\delta/l = 0.015$ set in Section 2.1. The resultant decay rates with different amplitudes and corresponding wavelengths are depicted in Fig. 10. Expected, the decay rate increases with the smaller amplitude, because the effect of surface waviness reduces as the water surface becomes flat.

However, it is interesting to note that the decay rate decreases with the increasing amplitude only before $\delta = 3$ m and then starts to increase as $\delta > 3$ m. To satisfy $\delta/l = 0.015$, the longer wavelength accompanies with the increasing amplitude, which yields the lower frequency (or larger period) in the surface waviness. Since the interaction is substantially associated with the repeated exchange between focusing and defocusing on the wavy surface, a low frequency would reduce the response of sonic boom to the waviness. As a result, either too small wave amplitude or too large wavelength increases the decay rate of the penetrating wave, though the reason for extreme value at $\delta = 3$ m should be further investigated in the future works.

| Cases                | (a) | (b) | (c) | (d) |
|----------------------|-----|-----|-----|-----|
| Flat water           | 148 | 112 | 166 | 166 |
| Wavy water           | 152 | 136 | 168 | 163 |

Table 3. Sound levels (dB) measured from the four cases in Fig. 8.

| $M_w$ | $M_\alpha$ | $\alpha$ | $\beta$ |
|-------|------------|----------|---------|
| 1.2   | 0.28       | 1.86     | 0.03    |
| 2.0   | 0.46       | 1.85     | 0.04    |
| 3.0   | 0.69       | 1.82     | 0.07    |
| 4.0   | 0.92       | 1.69     | 0.22    |
| 5.0   | 1.15       | 0        | 100     |

Table 4. Curve-fitting of the intensity ratios with water depth in Fig. 9.
significantly based on either instantaneous or average results cannot changes in time-dependent decay rates. Hence, the evaluation is calculated to be 0.049, which is only 8.4% change from the average. The other simulation cases are evaluated in Fig. 12, where the results suggest the small effect of waviness-induced focusing and defocusing of sonic booms on the wavy-water surface. We extended the study in a 3-D space. In addition, the study on the wavy-surface interaction was for the special case that the aircraft and surface waviness as well. It increases and drop close to zero. Meanwhile, the decay rate is related with geometry of the surface waviness as well. It increases and thus leads to some energy dissipation. As a result, the intensity of penetrating sonic boom in wavy water is a little less than that in flat water.

In addition to the observation in the shallow depth, we also studied at the deep depth through the detection of waveforms in penetrating waves. The ripples referred to the oscillating pattern on the detected waveforms remains one order of magnitude larger than the wave in flat water. However, the presence of ripples oppositely becomes trivial in the supersonic underwater flow because of the boom-dominated acoustic field. From this aspect, the wavy-surface interaction has inconspicuous effect on the penetration depth of acoustic waves in the supersonic underwater flow.

Furthermore, the decay rate of penetrating waves was quantified with the power-law curve-fitting on intensity ratios. The results shows the average decay of $\gamma^{-0.60}$ in subsonic wavy water, and the power (without minus) slightly decreases with increasing Mach number, especially as $M_a \to 4.4$. As $M_a > 4.4$, the decay rate then has a sudden drop close to zero. Meanwhile, the decay rate is related with geometry of the surface waviness as well. It increases and gets close to the value in flat water when the surface waviness has either too small amplitude or too large wavelength.

In summary, the results can be classified into four categories as shown in Fig. 13. The conclusion drawn in this study was based on the 2-D model. Some effects induced in a 3-D space were not considered in the simulation. For instance, considering the decay due to the geometrical spreading, the decay rate would not be zero for the underwater sonic boom in a 3-D space. In addition, the study on the wavy-surface interaction was for the special case that the aircraft and surface waves moved in the same direction. Despite the special case, it is a significant preliminary study about the effect of surface waviness on the underwater acoustic field induced by the hypersonic flight. Based on this study, the extension into the 3-D model is expected in the future work.

5. Summary and Concluding Remarks

In this study, we numerically investigated the effects of surface waviness on the underwater acoustic field by the CFD method. The investigation was conducted from the three perspectives: the response of overpressure near the contact point, the underwater acoustic waveforms, and the decay rate of penetrating waves.

Around the contact point, the acoustic fields clarify the effect of waviness-induced focusing and defocusing of sonic booms on the wavy-water surface. We extended the study into the hypersonic flight, and the results still agree with that the effect of focusing and defocusing becomes more conspicuous with the increasing Mach number. Especially in the case of $M_a = 5$, the percent change in the pressure rise is almost ten times the change in the case of $M_a = 2$. In addition, the disturbance of focusing and defocusing propagating in the supersonic underwater flow results in the noticeable periodic pattern on the penetrating sonic boom. It is found that the disturbance destroys smoothness of the shock surface and thus leads to some energy dissipation. As a result, the intensity of penetrating sonic boom in wavy water is a little less than that in flat water.

Finally, the intensity ratios and decay rates in this paper are obtained from the snapshots of flow field. To clarify the influence due to the time-dependent change in flow field, we present the decay rate varying over time in Fig. 11. It is noticed that the decay rate periodically varies with the period of the surface wave, that is, $T = \lambda / (c_s + V) = 0.22$ s (for the case of $M_a = 2$), and it has an average of $\alpha = 0.58$ within a period. Furthermore, its maximum difference from the average is calculated to be 0.049, which is only 8.4% change from the average. The other simulation cases are evaluated as well in Fig. 12, where the results suggest the small changes in time-dependent decay rates. Hence, the evaluation based on either instantaneous or average results cannot significantly affect the discussion in our study.

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![Fig. 11. Periodic variation in the decay rate $\alpha$ with varying times for the case of $M_a = 2$ in wavy water. The red dashed line is the average within a period $T$.](image1)

![Fig. 12. Average decay rates and their maximum changes (presented in error bars) for the simulation cases of the five Mach numbers.](image2)

![Fig. 13. Summary of the underwater acoustic field in flat and wavy water.](image3)
References

1) Ueno, A. and Suzuki, K.: Preliminary Study on a Transportation Network of a Hypersonic Airliner, *J. Jpn. Soc. Aeronaut. Space Sci.*, 57 (2009), pp. 380–382. (in Japanese)
2) Maglieri, D. J., Bobbitt, P. J., Plotkin, K. J., Shepherd, K. P., Coen, P. G., and Richwine, D. M.: Sonic Boom: Six Decades of Research, Tech. Rept. NASA/SP-2014-622, 2014.
3) Manci, K. M., Gladwin, D. N., Villella, R., and Cavendish, M. G.: Effects of Aircraft Noise and Sonic Booms on Domestic Animals and Wildlife: A Literature Synthesis, Tech. Rept. NERC-88/29, 1988.
4) Cook, J. C., Goforth, T., and Cook, R. K.: Seismic and Underwater Responses to Sonic Boom, *J. Acoust. Soc. Am.*, 51 (1972), pp. 729–741.
5) National Research Council: *Ocean Noise and Marine Mammals*, National Academies Press, Washington (DC), 2003.
6) Wartzok, D. and Tyacke, P.: Elaboration of the NRC Population Consequences of Acoustic Disturbance (PCAD) Model, *Bioacoustics*, 17 (2008), pp. 286–288.
7) Sawyers, K. N.: Underwater Sound Pressure from Sonic Booms, *J. Acoust. Soc. Am.*, 44 (1968), pp. 523–524.
8) Moody, D. M.: Three-Dimensional Underwater Sound Pressure Field Due to Sonic Boom, *J. Acoust. Soc. Am.*, 119 (2006), pp. 1368–1372.
9) Sparrow, V. W. and Ferguson, T. J.: Penetration of Shaped Sonic Boom Noise into a Flat Ocean, AIAA Paper 97-0486, 1997.
10) Rochat, J. L. and Sparrow, V. W.: Two-Dimensional Focusing of Sonic Boom Noise Penetrating an Air-Water Interface, *AIAA J.*, 35 (1997), pp. 35–39.
11) Cheng, H. K. and Lee, C. J.: Sonic-Boom Noise Penetration under a Wavy Ocean: Theory, *J. Fluid Mech.*, 514 (2004), pp. 281–312.
12) Cheng, H. K. and Lee, C. J.: A Theory of Sonic Boom Noise Penetration under a Wavy Ocean, AIAA Paper 98-2958, 1998.
13) Fincham, A. and Maxworthy, T.: An Experimental Study of Sonic Boom Penetration under a Wavy Air-Water Interface, USC AME Rept. 09-11-2001, 2002.
14) Intrieri, P. F. and Malcolm, G. N.: Ballistic Range Investigation of Sonic-Boom Overpressures in Water, *AIAA J.*, 11 (1973), pp. 510–516.
15) Wang, J., Grifalce, C., Edwards, J., and Hashad, A.: Underwater Overpressure from Hypervelocity Sonic Booms, 14th AIAA/CEAS Aero. Conference, Canada, AIAA 2008-3036, 2008.
16) Chen, P.-H. and Suzuki, K.: Numerical Investigation on Underwater Sonic Boom Cutoff Phenomenon Induced by Hypersonic Aircraft, AIAA Scitech 2021 Forum, Virtual Event, AIAA 2021-1992, 2021.
17) Tamagawa, M. and Akamatsu, T.: Effects of Shock Waves on Living Tissues Using Shock Tubes for Bio-Tests, *Shock Waves @ Marseille III*, Springer, France, 1995, pp. 429–434.
18) Shima, E. and Joumouchi, T.: Role of CFD in Aeronautical Engineering (No. 14)–AUSM Type Upwind Schemes–, Proceedings the 14th NAL Symposium on Aircraft Computational Aerodynamics SP34, 1997, pp. 7–12.
19) Anderson, W. K., Thomas, J. L., and Van Leer, B.: Comparison of Finite Volume Flux Vector Splittings for the Euler Equations, *AIAA J.*, 24 (1986), pp. 1453–1460.

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