An introduction to the general boundary formulation of quantum field theory

Daniele Colosi
Escuela Nacional de Estudios Superiores, Unidad Morelia, Universidad Nacional Autónoma de México, Campus Morelia, C.P. 58190, Morelia, Michoacán, México
E-mail: dcolosi@enesmorelia.unam.mx

Abstract. We give a brief introduction to the so-called general boundary formulation (GBF) of quantum theory. This new axiomatic formulation provides a description of the quantum dynamics which is manifestly local and does not rely on a metric background structure for its definition. We present the basic ingredients of the GBF, in particular we review the core axioms that assign algebraic structures to geometric ones, the two quantisation schemes so far developed for the GBF and the probability interpretation which generalizes the standard Born rule. Finally we briefly discuss some of the results obtained studying specific quantum field theories within the GBF.

1. Introduction
One of the most relevant obstacle to the realisation of the synthesis of general relativity and quantum mechanics, in the yet-to-be-found theory of quantum gravity, stems from the different ways in which the two theories are formulated. In particular, general relativity describes the geometry of space-time as a dynamical field that evolves according to equations of motion, the Einstein’s field equations. Such dynamics is not described with respect to a non-dynamical reference system. This property is sometimes called background independence. In contrast, a fixed, non-dynamical space-time metric (flat or curved) is an essential ingredient in the standard formulation of quantum theory from the point of view of the mathematical and conceptual foundations of the theory. In particular, in the framework of quantum field theory, quantum fields evolve in agreement with the causal structure determined by the space-time on which they are defined. For example, they are required to satisfy the so-called ‘microcausality relation’ expressed as the commutation of fields evaluated at space like separated points.

One possibility in order to construct a quantum theory of gravity, is to learn how to incorporate in quantum field theory the background independence of general relativity, namely how to describe the dynamics of quantum fields without reference to a fixed background space-time metric. A first step in this direction is represented by the development since the late 1980s of the so-called ‘Topological Quantum Field Theory’ (TQFT) [1] which has provided the first examples of background independent quantum theories in the sense that the quantum evolution (involving also topological change) is defined on manifolds not carrying a metric structure. In particular, TQFT has been successful in constructing quantum gravity models in three space-time dimensions where the classical theory has only global but not local degrees of freedom,
namely it reduces to a topological theory\(^1\).

In this note we present an introduction to a new axiomatic formulation of quantum theory called the general boundary formulation (GBF) [3, 4] which is based on two main ingredients: The mathematical framework of TQFT and a generalisation of the Born rule that guarantees a consistent physical interpretation. In particular the axioms of the GBF provide a consistent assignment of algebraic structures to geometric ones. State spaces are associated with hypersurfaces, namely oriented \((n-1)\)-dimensional manifolds, and amplitude maps are associated with regions, namely oriented \(n\)-dimensional manifolds. Hypersurfaces arise as boundaries of regions. The physical interpretation of these structures is obtained by generalising the ordinary Born rule of quantum mechanics [5]: Probabilities and expectation values of observables are extracted from maps associated to regions. A remarkable aspect of these structures is that no background space-time metric is required for their implementation.

When considering quantum fields defined on manifold carrying a metric structure, the GBF does not impose any special choice for the boundaries of regions where dynamics takes place. For example, in the standard formulation of quantum field theory in Minkowski space-time, states are defined on equal time hyperplanes. Their evolution from an initial hyperplane at time \(t_i\) to a final one at time \(t_f\) is described by a temporal evolution operator, as in the formula \(\langle \psi_f | U(t_i, t_f) | \psi_i \rangle\) representing the transition amplitude from the initial state \(\psi_i\) to the final state \(\psi_f\). So, the space-time region under consideration is the time-interval region that we denote as \([t_i, t_f]\). In the case of quantum fields on curved spaces, hyperplanes are replaced by Cauchy surfaces, but the structure of transition amplitudes (in terms of initial and final states and an evolution operator) is unchanged. The standard \(S\)-matrix is then constructed by taking the temporal asymptotic limit, namely \(t_i \to -\infty, \ t_f \to \infty\). As has been mentioned, the GBF is not bounded to a specific region even in Minkowski space-time. Its versatility allows to consider more general regions, e.g., regions whose boundary is timelike, or connected or even compact are allowed. The implementation of the standard formulation of the quantum field theory fails in these situations. Concrete examples of such kind of boundaries have been considered in the GBF and will be discussed later.

The new way to describe the dynamics of quantum fields provided by the GBF appears to be interesting in many respects: First of all, the GBF can be viewed as an extension and a generalisation of quantum theory, offering new perspectives on aspects of the theory, as will be shown when discussing the quantization of scalar field in Minkowski space-time in non-time-interval regions. Moreover, the GBF offers useful tools for implementing theoretical treatments more close to the experimental setups. Indeed, experiments often involve an apparatus which occupies a certain volume in space and is only active during a temporal interval. Therefore, the boundary of space-time region concerned by real experiments is limited in space and in time, namely is a compact region in four dimensions; precisely such region is of the kind the GBF can deal with no conceptual difficulties (although the technical ones could be non-trivial).

Secondly, since general space-time regions are considered in the GBF, it may solve the difficulties that arise from restricting the notion of dynamics to the evolution of initial state to a final one, i.e. from the restriction to time-interval regions. In Anti-de Sitter space, no asymptotic temporal regions exist, and consequently the construction of the standard \(S\)-matrix appears to be unfeasible due to the lack of temporal asymptotic free states. The peculiar geometry of Anti-de Sitter space does not represent an obstacle for a general boundary quantum field theory: Indeed, an amplitude for spatially, instead of temporally, asymptotic states can be implemented (and has been implemented, as discussed later).

Thirdly, some of the conceptual problems of background independent theory can be solve in the GBF. We briefly mention the problem of locality, understood as the problem of isolating the

\(^1\) One of the most studied TQFT model of quantum gravity in three dimensions is the Turaev-Viro model [2].
physical system of interest from the rest of the universe. In standard Minkowski-based quantum field theory, the $S$-matrix of two spacelike separated systems factorizes in the product of two $S$-matrices, one for each system. This is the so called cluster decomposition of the $S$-matrix and it relies on the existence of a fixed causal structure, provided by the Minkowski metric. But in a background independent theory no fixed metric or causal structure is available. Then, how can we separate the system from the rest of the universe? Given a region and its boundary, the GBF treatment rests only on the dynamics unfolding in the interior of that region and on the configurations of the fields on the boundary hypersurface(s). No additional data are needed, in particular data referring to the exterior of the region. In other words, locality is naturally implemented within the GBF.

Finally, the GBF may contribute in the resolution of the problem of quantum gravity, which has been originally the main motivation for its development. In that respect it is interesting to notice that the GBF is certainly compatible with quantum gravity in three dimensions, where the theory is topological and the GBF is precisely based on the mathematical framework of TQFT. Moreover, current approaches to quantum gravity like Spin Foam\textsuperscript{2} \cite{6} models and Group Field Theory \cite{7} fits into the GBF.

2. Basic axioms, state spaces and amplitude maps

We present in this section a reduced and simplified version of the core axioms of the GBF and refer the reader to \cite{8} for the complete list.

Let denote with $M$ an oriented topological manifold of dimension $n$ that we call `region’. We indicate with $\Sigma$ an oriented manifold without boundary of dimension $(n-1)$, namely a `hypersurface’. We can think of hypersurfaces as boundaries of regions (and the orientations of the regions induce those of the boundaries). Moreover, regions can be glued along (components of) their common boundaries to produce other regions.

To each hypersurface $\Sigma$ is associated a Hilbert space $H_\Sigma$ with inner product $\langle \cdot, \cdot \rangle_{\Sigma}$. The dual Hilbert space $H_\Sigma^*$ is then associated with the same manifold but with opposite orientation, which is denoted by $\Sigma^*$, namely $H_{\Sigma^*} = H_\Sigma^*$. The following composition rule is satisfied by the Hilbert spaces: If the hypersurface $\Sigma$ decomposes into a disjoint union of hypersurfaces as $\Sigma = \Sigma_1 \cup \cdots \cup \Sigma_m$ then the Hilbert space $H_\Sigma$ results to be given by the tensor product of Hilbert spaces: $H_{\Sigma_1} \otimes \cdots \otimes H_{\Sigma_m}$.

To each region $M$ is associated an amplitude map $\rho_M : H_{\partial M} \to \mathbb{C}$, from the Hilbert space $H_{\partial M}$ defined on boundary $\partial M$ of the region $M$. According to the gluing rule, if two regions $M_1$ and $M_2$ share a common boundary $\Sigma$ and are glued along it, the amplitudes maps $\rho_{M_1}$ and $\rho_{M_2}$ can be composited into the map $\rho_{M_1 \cup M_2}$ for the union region, $\rho_{M_1 \cup M_2} = \rho_{M_1} \circ \rho_{M_2}$. The symbol $\circ$ represents composition of maps, which involves a complete sum over an orthonormal basis of the Hilbert space $H_\Sigma$ associated with the common hypersurface $\Sigma$,

$$\rho_{M_1 \cup M_2}(\psi_1 \otimes \psi_2) = \sum_{i \in I} \rho_{M_1}(\psi_1 \otimes \xi_i) \rho_{M_2}(\xi_i^* \otimes \psi_2), \tag{1}$$

where $\psi_1 \in H_{\partial M_1 \setminus \Sigma}$, $\psi_2 \in H_{\partial M_2 \setminus \Sigma}$ and $\{\xi_i\}_{i \in I}$ is an orthonormal basis of $H_\Sigma$. In contrast, if the two regions are disjoint, the amplitude map of the union region is equal to the product of the amplitude maps of the two regions, namely $\rho_{M_1 \cup M_2}(\psi_1 \otimes \psi_2) = \rho_{M_1}(\psi_1) \rho_{M_2}(\psi_2)$. Finally, if the region $M$ reduces to the empty region $\tilde{\Sigma}$, the boundary of which is given by the union of a hypersurface and a copy of this same hypersurface but with opposite orientation, i.e. $\partial \tilde{\Sigma} = \Sigma \cup \Sigma^*$, ($\Sigma^*$ being a copy of $\Sigma$) then the amplitude map reduces to the inner product of the Hilbert space associated to the hypersurface: $\rho_{\tilde{\Sigma}}(\psi_1 \otimes \psi_2) = \langle \psi_1, \psi_2 \rangle_{\Sigma}$.

\textsuperscript{2} See Oeckl’s talk at the Mexilazos Conference, held in Morelia, 10 November 2012, available at http://matmor.unam.mx/eventos/mexilazos2012/files/Oeckl-ML2012.pdf.
The amplitude map $\rho_M$ can be seen as a generalisation of the standard transition amplitude, which is recovered in the standard setting, namely when the space-time region $M$ reduces to the standard time-interval region $[t_i, t_f]$. In this case the boundary hypersurface $\partial M$ is given by the disjoint union of flat equal time hyperplanes $\Sigma_t$ and $\Sigma_{t'}$ (with opposite orientation) labeled by the time instants $t_i$ and $t_f$. According to the composition rule of Hilbert spaces, we have $\mathcal{H}_{\partial M} = \mathcal{H}_{t_i} \otimes \mathcal{H}_{t_f}$. Then, the amplitude map for the boundary state $\psi_i \otimes \psi_f \in \mathcal{H}_{t_i} \otimes \mathcal{H}_{t_f}$ results to be: $\rho_M(\psi_i \otimes \psi_f) = \langle \psi_f | U(t_i, t_f) | \psi_i \rangle$, which is the expression of the standard transition amplitude for the evolution from the state $\psi_i$ to the state $\psi_f$ determined by the temporal evolution operator $U(t_i, t_f)$.

In the GBF quantum observables are associated with regions $M$ and implemented as maps, called observable maps, from the Hilbert space $\mathcal{H}_{\partial M}$ to the complex number, $[9]$. A set of axioms prescribes how different observables (associated with different regions) compose in analogy with the composition of amplitude maps. Indeed, amplitude maps can be considered as special cases of observable maps.

The implementation of all these algebraic structures for a specific theory can be realized through the application of a quantization procedure to a classical field theory. So far, two quantization schemes have been developed within the GBF and we shall present them in the next section.

3. Quantization schemes
We consider a scalar field in an $N$-dimensional Lorentzian manifold with metric $g$ described by the linear action $S_M(\phi) = \int_M d^N x L(\phi, \partial_x \phi)$ in the space-time region $M$. We do not require the boundary hypersurface $\partial M$ to be a Cauchy or a spacelike hypersurface. The classical theory equips the boundary $\partial M$ with a symplectic vector space $(L_{\partial M}, \omega_{\partial M})$, $L_{\partial M}$ being the space of (germs of) solutions of the Euler-Lagrange equations in a neighbourhood of $\partial M$, and $\omega_{\partial M}$ is the symplectic structure given by the anti-symmetric bilinear form $\omega_{\partial M}(\xi, \eta) = \frac{1}{2}[\xi, \eta]_{\partial M} - \frac{1}{2}|\eta, \xi|_{\partial M}$ with

$$\{\xi, \eta\}_{\partial M} = \int_{\partial M} d^{N-1} \sigma \eta(x(\sigma)) \left( n^\mu \frac{\delta L}{\delta \partial_\mu \xi} \right) (x(\sigma)), \quad (2)$$

where $n^\mu$ is the unit normal vector to the hypersurface $\partial M$. An additional ingredient is needed to quantize the theory: A compatible complex structure $J_{\partial M}$ defined on $L_{\partial M}$ such that $J_{\partial M}^2 = -id_{\partial M}$ and is compatible with the symplectic structure, i.e. $\omega_{\partial M}(J_{\partial M} \xi, J_{\partial M} \eta) = \omega_{\partial M}(\xi, \eta)$.

Based on the above ingredients two quantization schemes have been developed so far for the GBF: The holomorphic quantization, inspired by geometric quantisation, and the Schrödinger-Feynman one where the path integral quantization is implemented.

The symplectic and complex structures define a real and complex inner product on the space $L_{\partial M}$ respectively as

$$g_{\partial M}(\xi, \eta) = 2\omega_{\partial M}(\xi, J_{\partial M} \eta), \quad \{\xi, \eta\}_{\partial M} = g_{\partial M}(\xi, \eta) + 2i\omega_{\partial M}(\xi, \eta). \quad (3)$$

The complex inner product $\langle \cdot, \cdot \rangle_{\partial M}$, upon completion, makes $L_{\partial M}$ a complex Hilbert space.

In the holomorphic representation, the Hilbert space $\mathcal{H}^{h}_{\partial M}$ is given by the set of square integrable holomorphic functions on $L_{\partial M}$. The amplitude map for a coherent state $\psi \in \mathcal{H}^{h}_{\partial M}$, defined by $\xi \in L_{\partial M}$, is

$$\rho_M(\psi) = \exp \left( \frac{1}{4} g_{\partial M}(\xi, \bar{\xi}) \right), \quad (4)$$

where $\bar{\xi}$ is a complex solution of the equations of motion of the form $\xi = \xi^R - i\xi^I$. The elements $\xi^R$ and $\xi^I$ are global solutions in the region $M$ mapped to the corresponding (germs of) solutions in $L_{\partial M}$.

We are neglecting some mathematical technicalities; the rigorous construction can be found in Ref. [8].
Within the Schrödinger-Feynman quantization scheme, quantum states are in the Schrödinger representation, namely the Hilbert space $\mathcal{H}_{\partial M}^S$ is the set of wave functionals defined on the space of field configurations. The amplitude map for a state $\psi \in \mathcal{H}_{\partial M}^S$ is formally given by

$$\rho_M(\psi) = \int \mathcal{D}\phi \psi(\phi) \int_{\phi|\partial M = \varphi} \mathcal{D}\phi \ e^{iS_M(\phi)},$$

(5)

where the inner integral is extended over the space-time field configurations $\phi$ that reduce to $\varphi$ on the boundary hypersurface $\partial M$ and the outer integral is over all the configurations $\varphi$.

The two representations turn out to be equivalent, in the sense that the Hilbert spaces $\mathcal{H}_{\partial M}^S$ and $\mathcal{H}_{\partial M}^A$ are related by an isomorphism [10]. Moreover, under such isomorphism the amplitude maps (4) and (5) coincide.

The two quantization prescriptions have been designed in order to obtain a general boundary quantum field theory. Indeed, it can be shown that the Hilbert spaces and the amplitude maps, in both representations, satisfy the axioms of the GBF.

4. Probability interpretation

Probabilities are extracted from the amplitude map by selecting appropriate subspaces of the Hilbert space associated to the boundary hypersurface, $\mathcal{H}_{\partial M}$, in agreement with the following idea. Imagine we perform a measurement on a quantum system. First we prepare the system in an initial state, i.e. the initial state encodes the information of the system prior to the measurement process. Then, after the measurement, we ask the question about the probability to observe the system on a final state, i.e. the final state encodes the information relative to observation. The result is a standard quantum mechanical probability which is a conditional one: The probability of observing the final state given that the system was prepared in the initial state.

In the GBF, initial and final states are elements of $\mathcal{H}_{\partial M}$, and information about preparation and observation is consequently encoded on subspaces of $\mathcal{H}_{\partial M}$. We call $S \subset \mathcal{H}_{\partial M}$ the closed subspace of the boundary Hilbert space that represents ‘preparation of the system’, in the sense that $S$ encodes the information concerning what can be considered as resulting form the preparation of the system. The closed subspace $A \subset \mathcal{H}_{\partial M}$ represents ‘observation of the system’, i.e. this subspace encodes the information on what is interpreted as the result of the observation of the system. The conditional probability $P(A|S)$ for observing $A$ given that $S$ has been prepared is defined as

$$P(A|S) = \frac{\langle \rho_M \circ P_S, \rho_M \circ P_A \rangle_{\partial M}}{\langle \rho_M \circ P_S, \rho_M \circ P_S \rangle_{\partial M}},$$

(6)

where $\langle \cdot, \cdot \rangle_{\partial M}$ is the inner product in $\mathcal{H}_{\partial M}$, $P_S$ and $P_A$ are the orthogonal projectors onto the subspaces $S$ and $A$ respectively and $\circ$ denotes composition of maps. This expression makes sense if the maps $\rho_M \circ P_S$ and $\rho_M \circ P_A$ are linear continuous maps from $\mathcal{H}_{\partial M}$ to $\mathbb{C}$ and also $\rho_M \circ P_S$ does not vanish.

The conditional probability $P(A|S)$ is a generalisation of the standard Born rule of quantum theory and reduces to it in the standard setting, namely when the space-time $M$ is the time-interval region $[t_i, t_f]$. In this case, as in Sec. 2, we have $\mathcal{H}_{\partial M} = \mathcal{H}_{t_i} \otimes \mathcal{H}_{t_f}$. We now choose the subspaces $S$ and $A$ as follows, $S = \psi_i \otimes \mathcal{H}_{t_f}$ and $A = \mathcal{H}_{t_i} \otimes \psi_f$. The inner products of the composition of maps in (6) result to be: $\langle \rho_M \circ P_S, \rho_M \circ P_A \rangle_{\partial M} = |\rho_M(\psi_i \otimes \psi_f)|^2$ and $\langle \rho_M \circ P_S, \rho_M \circ P_S \rangle_{\partial M} = \sum_{k \in J} |\rho_M(\psi_i \otimes \xi_k)|^2 = 1$ where $\{\xi_k\}_{k \in J}$ is an orthonormal basis of the space $\mathcal{H}_{t_f}$. Finally, the conditional probability (6) reads $P(A|S) = |\rho_M(\psi_i \otimes \psi_f)|^2 = |\langle \psi_f | U(t_i, t_f) | \psi_f \rangle|^2$, which is the standard Born rule.
5. Results from the GBF

This section presents a partial list of results obtained by implementing the GBF for specific quantum field theories in the space-time and for different regions.

(i) A minimal requirement any new formulation of quantum theory is the ability to reproduce known and well established results. In [11, 12] we apply the GBF for a general interacting real massive scalar field in Minkowski space-time in two different settings: the standard time-interval region \([t_i, t_f]\) and the hypercylinder region, namely a ball of radius \(R\) centred at the origin of space and extended over all of time. In the region \([t_i, t_f]\) standard results as the Feynman propagator, the perturbative treatment, the \(S\)-matrix were recovered. In particular the \(S\)-matrix arises as the temporal asymptotic limit of the amplitude map \(\rho_{[t_i, t_f]}\). The boundary of the hypercylinder region is timelike and connected, two aspects that render the standard formulation of quantum field theory of difficult applicability. In contrast, a mathematically well defined quantization of the field was realised within the GBF. Notice that the connectedness of the boundary prevents to talk about transition. Nevertheless the GBF provides a consistent description of dynamics. The standard \(S\)-matrix is obtained as the spatial asymptotic limit \(R \to \infty\) of the amplitude map \(\rho_R\) for the hypercylinder region. An isomorphism exists between the Hilbert spaces \(\mathcal{H}_{[t_i, t_f]}\) and \(\mathcal{H}_R\) associated with the boundary of the two regions. Consequently the tensor product of an \(n\)-incoming particles state and an \(m\)-outgoing particles state in \(\mathcal{H}_{[t_i, t_f]}\) is mapped into an \((m + n)\)-particles state in \(\mathcal{H}_R\). It is worth noticing that an \(n\)-particles state on the hypercylinder is neither ‘incoming’ nor ‘outgoing’. Consequently the crossing symmetry of the \(S\)-matrix is automatically implemented by the amplitude map \(\rho_R\). In particular, crossing symmetry could be viewed as a prediction (which is verified) of the dynamics described in the spatially asymptotic hypercylinder region. An additional result from the quantum theory in the hypercylinder is the manifest quantization of the so-called evanescent modes, which are modes of the field exponentially decaying in space, and which are completely hidden in the time-interval region.

(ii) The results obtained in Minkowski space-time have been extended in de Sitter space in [13], where the quantum theory of a real massive scalar field in the presence of a general interaction was studied perturbatively in the time-interval region and the hypercylinder region. In particular the amplitude map for the hypercylinder region leads, in the spatially asymptotic limit, to a new representation of the \(S\)-matrix and the Feynman propagator. Moreover, a new type of propagator proposed by Polyakov in [14] can be derived in a simple way from the time-interval region by sending one of the boundary to infinity\(^4\).

(iii) As mentioned in Sec. 1, the standard formulation of the \(S\)-matrix in Anti-de Sitter space is obstructed by the lack of temporally asymptotic regions, and consequently temporally asymptotic free states. However, spatially asymptotic free states of a quantum scalar field theory can be defined within the GBF. In this respect, the hypercylinder region turns out to be a useful tool, as realised in [15] where the asymptotic amplitude map defined for the hypercylinder region was proposed as definition of \(S\)-matrix in Anti-de Sitter space.

(iv) The spacetime regions of main interest in the GBF are those with a compact boundary since in such regions the GBF will provide a manifestly local description of quantum dynamics. The applications mentioned in the previous points involve infinitely extended regions. Instead, in [16] a real scalar field was considered in a two dimensional Riemannian space-time, both in the time-interval region and in a disk region. The disk region has a circle of radius \(R\) centred at origin as its boundary hypersurface. Analogous results to those obtained in Minkowski were recovered. In particular, Hilbert spaces were defined on the boundary of the disk region and amplitude maps for the regions.

\(^4\) Colosi D, in preparation.
The dynamics of a real scalar field in two dimensional Rindler space was studied in [17]. The GBF was successfully implemented in the usual time-interval and then in a region bounded by a timelike connected hypersurface determined by a constant value of the Rindler spatial coordinate, namely an hyperbola. The quantum theories in the two regions were compared and shown to be equivalent in the sense discussed in the first point of this list.

Considerable work has been devoted to the understanding of the so-called Unruh effect (see the review [18]). In [19] this effect has been interpreted as the coincidence of expectation values of a class of observables computed on two different states: On the one hand on the vacuum state of a scalar field in Minkowski space-time and on the other hand on a mixed state in Rindler space. In particular these expectation values are obtained from the observable maps associated with two time-interval regions (in Minkowski and in Rindler spaces). The observables considered are the Weyl observables, which are linear functionals of the field $\phi$ of the form $F(\phi) = \int dx \mu(x)\phi(x)$ where $\mu(x)$ has compact support in the right wedge of Minkowski.

The formal development as well as the examples of its application show that the GBF provides a powerful and versatile formulation of quantum theory. The GBF is still a work in progress and many lines of further research may be pursued in the future, among which we mention: The relation between the quantization schemes presented in Sec. 3 and the canonical quantization prescription; the consideration of more general regions, in particular of regions with compact boundaries in Lorentzian space-time; the definition of KMS states and the treatment of the Hawking effect; a proposal for a new candidate theory of quantum gravity.

Acknowledgments
This work was supported in part by UNAM-DGAPA-PAPIIT through project grant IA-102314.

References
This work was supported in part by UNAM-DGAPA-PAPIIT through project grant IA-102314.

[1] Atiyah M 1988 Topological quantum field theories Inst. Hautes Études Sci. Publ. Math. 68 175-86
[2] Turaev V 1994 Quantum invariants of knots and 3-manifolds (New York: De Gruyter)
[3] Oeckl R 2003 A ‘general boundary’ formulation for quantum mechanics and quantum gravity Phys. Lett. B 575 318-24 (Preprint hep-th/0306025)
[4] Oeckl R 2008 General boundary quantum field theory: Foundations and probability interpretation Adv. Theor. Math. Phys. 12 319-52 (Preprint hep-th/0509122)
[5] Oeckl R 2007 Probabilities in the general boundary formulation J. Phys. Conf. Ser. 67 012049 (Preprint hep-th/0612076)
[6] Perez A 2013 The Spin Foam Approach to Quantum Gravity Living Rev. Rel. 16 3 (Preprint arXiv:1205.2019 [gr-qc])
[7] Freidel L 2005 Group field theory: An overview Int. J. Theor. Phys. 44 1769-83 (Preprint arXiv:hep-th/0505016)
[8] Oeckl R 2012 Holomorphic quantization of linear field theory in the general boundary formulation SIGMA 8 050 (Preprint arXiv:1009.5615 [hep-th])
[9] Oeckl R 2012 Observables in the General Boundary Formulation Quantum Field Theory and Gravity (Regensburg, 2010) (Basel: Birkhauser) pp. 137-156 (Preprint arXiv:1101.0367 [hep-th])
[10] Oeckl R 2012 The Schrödinger representation and its relation to the holomorphic representation in linear and affine field theory J. Math. Phys. 53 072301 (Preprint arXiv:1109.5215 [math-ph])
[11] Colosi D and Oeckl R 2008 Spatially asymptotic S-matrix at spatial infinity Phys. Lett. B 665 310-13 (Preprint arXiv:0710.5203 [hep-th])
[12] Colosi D and Oeckl R 2008 Spatially asymptotic S-matrix from general boundary formulation Phys. Rev. D 78 025020 (Preprint arXiv:0802.2274 [hep-th])
[13] Colosi D 2009 General boundary quantum field theory in de Sitter spacetime (Preprint arXiv:1010.1209 [hep-th])
[14] Polyakov A M 2008 De Sitter space and eternity Nucl. Phys. B 797 199-217 (Preprint arXiv:0709.2899 [hep-th])
[15] Colosi D, Dohse M and Oeckl R 2012 S-Matrix for AdS from General Boundary QFT J. Phys. Conf. Ser. 360 012012 (Preprint arXiv:1112.2225 [hep-th])
[16] Colosi D and Oeckl R 2009 States and amplitudes for finite regions in a two-dimensional Euclidean quantum field theory *J. Geom. Phys.* 59 764-80 (*Preprint* arXiv:0811.4166 [hep-th])

[17] Colosi D and Rätzel D 2013 Quantum field theory on timelike hypersurfaces in Rindler space *Phys. Rev. D* 87 12, 125001 (*Preprint* arXiv:1303.5873 [hep-th])

[18] Crispino L C B, Higuchi A and Matsas G E A 2008 The Unruh effect and its applications *Rev. Modern Phys.* 80 787-838 (*Preprint* arXiv:0710.5373)

[19] Colosi D and Rätzel 2013 The Unruh effect in general boundary quantum field theory *SIGMA* 9 019 (*Preprint* arXiv:1204.6268 [hep-th])