1. Introduction

The cleanliness of liquid steel is becoming more and more important with the increasing demands for high quality steel year by year. Nonmetallic inclusions in liquid steel can greatly influence the properties of steel such as workability, surface quality, and fatigue strength adversely. So inclusion removal is one of the main objectives in steelmaking industry. However, inclusion removal in the metallurgical reactor is a complicated process, which involves in fluid flow, inclusion collision and coalescence, floatation, adhesion to the wall and gas bubbles, so it is very difficult to carry out an excellent simulation including all above involved phenomena.

It should be noted that the agglomeration among inclusions leads to the generation of larger inclusions which is harmful for steel products, but the newly generated larger inclusions also have more chances to be removed by floatation. So the clean steel can be produced by controlling the inclusion agglomeration process consequently. Nowadays, the particle trajectory model can trace individual inclusion movement,1–4) and the mass conservation model can give the spatial distribution of inclusions with same size, 5,6) but both of them can not give a satisfied description for the inclusion agglomeration process. Thus, by dividing all inclusions into several groups according to their size, the population balance model (PBM)7–9) based on the Smoluchowski equation, has been developed to simulate the inclusion collision and agglomeration behavior, but the population balance model can not give the spatial distribution of the inclusions in the metallurgical reactors. In addition, the Smoluchowski equation is applied as the source term of the inclusion number density transport equation to simulate the fluid flow and the inclusion agglomeration process simultaneously,10–13) but such a complicated model also needs more powerful computer.

Meanwhile, some industrial trial results showed that the fractional inclusion number density had an exponential relationship with the inclusion radius,7,9,14,15) some new models have been developed on the assumption of exponential size distribution of inclusions.16–19) By solving the transport equations of the inclusion characteristic number density and concentration, the number and mass conservation model can consider the effect of fluid flow on inclusion agglomeration process. Although the number and mass conservation model has been applied to the inclusion removal in the case of gas injection by some researchers, but the current model did not take the effect of injected gas bubbles on the inclusion removal into account,17,18) and ignored the Stokes collisions among inclusions which has been proved to be significant if the difference in inclusion radius is large.7,13,20,21)

Thus, the purpose of the present work is to develop an inclusion number and mass conservation model in which the Stokes collision and bubble adhesion are taken into account. On the other hand, the secondary refining in ladle stirred by bottom gas injection plays a significant role in the steelmaking industry. And some researches have been carried out to investigate the effect of different multi-plug configuration on the mixing phenomenon in gas-stirred ladle.22,23) So the newly developed model will be employed to investigate the inclusion removal process when the argon...
gas is injected through one plug placed centrally, one plug placed eccentrically at half radius, two and three plugs placed at half radius, as shown in Fig. 1. And the position of main plane in the following figures is also shown in Fig. 1.

2. Mathematical Model

The mathematical model for fluid flow and inclusion removal is based on the following assumptions:

(a) The fluids in both the gas and liquid phases are Newtonian, viscous and incompressible, and the fluid flow is at the steady state.

(b) The effect of top slag on fluid flow is neglected and the free surface is thought to be flat.

(c) The gas bubbles are spherical and the interaction among bubbles is not considered.

(d) The fluid flow in the ladle is assumed to be an isothermal process.

(e) The effect of inclusion movement on fluid flow is neglected.

(f) Inclusions are spherical and each inclusion moves independently before the collision occurs.

(g) The fractional inclusion number density has an exponential relationship with the inclusion radius and can be expressed as \( r^3 \exp(-B r) \):

Thus, the inclusion characteristic number density, concentration and radius can be expressed as:

\[
N^* = \int_0^\infty f(r)dr = \frac{A}{B}, \quad C^* = \int_0^\infty \frac{4}{3} \pi r^3 f(r)dr = 8\pi \frac{4}{B^2}
\]

and \( r^3 \sqrt{6/B} \), respectively. Furthermore, \( C^* \) can also be expressed as the function of \( N^* \) and \( r^3 \):

\[
C^* = N^* \cdot (4/3) \pi r^3
\]

In order to give a basic idea of the mathematical model, a schematic of the model has been shown in Fig. 2. First of all, an Eulerian–Eulerian model was employed to simulate the two-phase flow in the ladle. Then the transient transport equations for inclusion characteristic number density and concentration, which consider different inclusion removal approaches and collision mechanisms, have been solved to investigate the variations of inclusion characteristic parameters with space and time. In the calculation, new values of \( N^* \) and \( C^* \) at each grid point during the whole process are obtained by each iteration and the values of \( A \) and \( B \) can also be derived, which indicate new inclusion size distribution of inclusions at each grid point.

2.1. Theory of Multiphase Flow

The multiphase flow in gas-stirred ladle is simulated by an Eulerian–Eulerian model. The continuity and momentum conservation equations for the \( k \)-th phase are shown as follows:

Continuity equation:

\[
\nabla \cdot (\rho_k \mathbf{u}_k) = 0 \quad \text{(1)}
\]

Momentum conservation equation:

\[
\nabla \cdot (\rho_k \mathbf{u}_k \mathbf{u}_k) = -\nabla p + \mathbf{F}_{\text{drag},k} + \alpha_k \rho_k g + \nabla \left[ \alpha_k \mu_{\text{eff}} (\nabla \mathbf{u}_k + (\nabla \mathbf{u}_k)^T) \right] \quad \text{(2)}
\]

where \( k=1,2 \) denotes the gas and liquid phases, respectively. \( \rho_k \) is the density of the \( k \)-th phase, \( \mathbf{u}_k \) is the velocity vector of the \( k \)-th phase, \( p \) is the pressure, \( \mu_{\text{eff}} \) is the effective viscosity and can be determined by the standard \( k-\varepsilon \) turbulence model. Due to the whole space domain is shared by the two phases, the sum of their volume fractions should be equal to one, i.e. \( \alpha_1 + \alpha_2 = 1 \). \( \mathbf{F}_{\text{drag},k} \) is the drag force between two phases and can be expressed as:

\[
\mathbf{F}_{\text{drag},k} = -\mathbf{F}_{\text{drag},g} = \frac{3}{4} \frac{C_D}{d_i} \alpha_k \rho_k |\mathbf{u}_g - \mathbf{u}_i| \left( \mathbf{u}_g - \mathbf{u}_i \right) \quad \text{(3)}
\]

where \( C_D \) is the drag coefficient:

\[
C_D = \max \left( \frac{24}{Re_b} \left( 1 + 0.15 Re_b^{0.64} \right), 0.44 \right) \quad \text{(4)}
\]
Here, the bubble Reynolds number $Re_b$ is the function of the bubble diameter $26$ and can be expressed as:

$$Re_b = \frac{\rho_d u_{b} - \bar{u}_d}{\mu}$$

with

$$d_b = 0.091\left(\frac{\sigma}{\rho_d}\right)^{0.5}\left(\frac{4Q_g}{\pi d_b^5}\right)^{0.44}$$

where $\sigma$ is the surface tension, $Q_g$ is the gas flow rate, and $d_b$ is the diameter of the nozzle exit.

### 2.2. Inclusion Removal Model

#### 2.2.1. Inclusion Transport Equations

The inclusion transport equations which describe collision and aggregation among inclusions can be expressed as follows $16-18$:

$$\frac{\partial}{\partial t}(\rho N^*) + \nabla \cdot (\rho \bar{u}_N N^*) = \nabla \cdot (D_{eff} \nabla N^*) + S_{N^*}$$

$$\frac{\partial}{\partial t}(\rho C^*) + \nabla \cdot (\rho \bar{u}_C C^*) = \nabla \cdot (D_{eff} \nabla C^*) + S_{C^*}$$

#### 2.2.2. Collision Mechanism

The collisions have a constant floatation velocity in vertical direction since the density of liquid steel is larger than that of the inclusion. So the convection velocities in Eq. (7-1) and Eq. (7-2) are the sum of the liquid steel velocity and the inclusion characteristic floatation velocities $16-18$. Here, $\bar{u}_N$ and $\bar{u}_C$ are the inclusion characteristic floatation velocities and can be derived as follows:

$$N^* \bar{u}_N = \int_0^\infty \bar{u}_{Stokes,f}(r)dr$$

$$C^* \bar{u}_C = \int_0^\infty \bar{u}_{Stokes,fr}(r)dr$$

Here, $\bar{u}_{Stokes,fr} = 2g(p_c - p_v)r^3/9\rho_v r_v$ is the inclusion floatation velocity $31$. Therefore, the inclusion characteristic floatation velocities can be expressed as:

$$\bar{u}_N = \frac{2}{\sqrt[3]{36}} \frac{2g}{9\rho_v r_v} (p_c - p_v) r^2$$

$$\bar{u}_C = \frac{20}{\sqrt[3]{36}} \frac{2g}{9\rho_v r_v} (p_c - p_v) r^2$$

Here, the source term $S_{N^*}$ accounts for the effects of bubble adhesion and coalescence among inclusions on the inclusion number density, while the source term $S_{C^*}$ accounts for the effect of bubble adhesion on the inclusion concentration.

$$S_{N^*} = W(r_r, r_l) + S_{N^*(bubble)}$$

$$S_{C^*} = S_{C^*(bubble)}$$

#### 2.2.3. Inclusion Removal by Bubble Adhesion

The force field model developed by Kwon $30,31$ was employed to compute the possibility of bubble-inclusion attachment:

$$P_{bp} = \frac{3}{2} \left( \frac{1}{94} \right)^{1/2} \left( \frac{3}{2} Re_b \right) \left( \frac{r_b}{r_l} \right)^{1/2}$$

Thus, the decrease of inclusion number density and concentration due to bubble adhesion can be expressed as:

$$S_{N^*(bubble)} = -\int_0^\infty P_{bp} \mu_b N_p \pi (r_b + r)^2 A e^{-Br} dr$$

$$S_{C^*(bubble)} = -\pi P_{bp} N_p \frac{2}{\sqrt[3]{36}} N^* r^2 + \frac{2}{\sqrt[3]{6}} N^* r^2 + N^* r^2$$

Thus, both the turbulent collisions and Stokes collisions have been taken into account.

The turbulent collision rate can be expressed as $27,28$:

$$\beta_T(r_r, r_l) = 0.952 \sqrt[3]{\pi} \left( \frac{5}{6\pi\mu_r r^3 (4/15\pi)^{0.5}} \right)^{0.242} \left( \frac{\rho}{\rho_l} \right)^{0.5} (r_r + r_l)^3$$

Here, $\nu_l$ is the kinematical viscosity of the liquid steel, the value of Hamaker constant $A^*$ is $2.3 \times 10^{-20}$ J for alumina inclusion, $28$, and $r$ is the initial size of the monomer particle.

### References

$7,13,20,21$
where $N_b$ is the number density of argon bubbles and is given by:

$$N_b = \frac{\alpha_b}{\frac{4}{3} \pi r_b^3}$$

(18)

### 2.2.4. Inclusion Removal by Top Slag and Refractory Wall

At the top slag, it is assumed that 80% of the inclusions which reach top slag are removed while the remaining 20% of the inclusions are entrained into the liquid steel.14,16) The inclusions adhesion to the refractory wall can be thought as the mass diffusion of boundary layer.7,9,14,16) Moreover, at the ladle bottom, the reverse effect of inclusion flotation velocity on the inclusion adhesion has also been taken into account. Thus, the boundary fluxes for number density and concentration are listed in Table 1.

### 2.3. Boundary Conditions and Solution Method

The no-slip wall boundary conditions were applied at the refractory walls. For the free surfaces in ladle, the symmetry boundary condition was imposed. Furthermore, the gas bubbles reaching the free surface were assumed to escape at their flotation velocity. The finite volume method32) was used to solve these partial differential equations. Consequently, the computational fluid dynamics package, CFX, was employed to perform the calculation. And the grids consist of about 900 000 control volumes. The convergence criteria is that the value of the root mean square normalized residual for variables was less than $1 \times 10^{-5}$ and the global imbalances, which means the ratios of the difference between the total input mass flux and the total output mass flux to the total input mass flux was less than 0.1%. The dimensions of ladle and other parameters are shown in Table 2. And the initial distribution of inclusion size is indicated as $f(r) = 1.925 \times 10^{16} e^{-0.4552 \times 106 r^2}$. Therefore, the initial inclusion number density is $N^* = 4.23 \times 10^{12}$/m$^3$.

### 3. Results and Discussion

#### 3.1. Flow Field

Figure 3 shows the computed flow patterns at the main vertical plane in the ladle when argon gas was injected through the porous plugs at different positions. In Fig. 3(a), only one porous plug was placed at the bottom centre (Type A). As a result, the upstream jet due to gas injection from the porous plug forms a plume and expands as it rises. Then the plume splits into two main streams and both streams flow down along the side wall. And the velocity of liquid steel along with the symmetric axis is greater than that at other places. In Fig. 3(b), the argon gas was injected through the porous plug placed eccentrically (Type B). Unlike central bottom blowing, the plume bends toward the side wall because of the returning flow, and a large recirculation zone in the ladle is formed. Figure 3(c) shows the predicted flow patterns for dual-plug injection placed oppositely at half radius (Type C). And there are two types of recirculation zones in the ladle. One is middle recirculation between two plumes, while another is sidewall recirculation between the plume and the side wall. Figure 3(d) shows that there is a large recirculation pattern in the ladle when three
porous plugs were placed at half radius (Type D). Such a large recirculation pattern is similar to that in the ladle of Type B, but the velocity at the free surface in the case of Type D is more uniform than that in the case of Type B, and the center of recirculation zone in the case of Type D is very close to the center of the ladle. Figures 3(a)–3(d) shows that the porous plug configurations have a great effect on the flow field in the ladle.

Figure 4 shows the maximum of turbulence kinetic energy is mainly near the free surface. The turbulence kinetic
energy is greater at the plumes, but the greater turbulence kinetic energy gradient exists near the plumes. Moreover, the turbulence kinetic energy is weaker near the junction of the vertical side wall and the base of ladle, so it is difficult for inclusions to collide with each other.

Figure 4 also shows that the turbulence kinetic energy at the region except the plume is weaker when only one porous plug was adopted (Type A and B). And the turbulence kinetic energy becomes more uniform when two porous plugs were adopted. It can be seen that the turbulence kinetic energy is weak only near the junction of the vertical side wall and near the bottom. Moreover, the turbulence kinetic energy and the mean velocity of liquid steel are very uniform in the case of Type D. On the other hand, Fig. 4 also shows that the maximum of turbulence kinetic energy decreases with the increasing porous plugs because the overall argon gas flow rate is 400 NL/min for different types of ladle.

Figure 5 shows that the distribution of the turbulent energy dissipation rate is very similar to that of the turbulence kinetic energy. Since increasing turbulent energy dissipation rate can increase the turbulent collision rate among inclusions, the number density of inclusions at the region with large turbulent energy dissipation rate should be relatively smaller than other regions in the ladle.

3.2. Inclusion Removal Process

Figure 6 shows the predicted distribution of the inclusion number density under different porous plug configurations after 400 s. It can be seen that the inclusion number density in the plume zones is less than that in the sidewall recirculation zones. Three factors lead to this phenomenon. Firstly, inclusions attached to gas bubbles can be removed directly when bubbles float up to the top slag. Secondly, the turbulence kinetic energy is very strong in plume zones and the stronger turbulent energy dissipation rate can promote the collision and coalescence among inclusions effectively. Thirdly, the upward velocity of the liquid steel is very large in plume zones, which is also in favor of inclusion removal by floatation. In this way, inclusions in the plume zone have more chances to reach the free surface.

Furthermore, the porous plug configuration has a great effect on the distribution of inclusion number density. As shown in Fig. 6(a), the distribution of inclusion number density is symmetrical in the ladle of Type A. And the inclusion number density is large near the bottom except the plume zone and decreases upwards. The key reason is that the turbulence kinetic energy increases upwards and stronger turbulent flow can promote the collision among inclusions. Figure 6(b) shows that the inclusion number density at the recirculation center is great when the porous plug placed eccentrically. Two key factors lead to this phenomenon. Firstly, the weak turbulent flow limits the collision and aggregation among inclusions. Secondly, a number of smaller inclusions accumulate at the recirculation center by the transportation of liquid steel. Figure 6(c) shows the distribution of inclusion number density in the case of dual plugs. The inclusion number density at the sidewall recirculation is much larger than other regions. The key factor is that the turbulence kinetic energy is too small to promote the collision among inclusions. Besides, the circulation zones near the free surface also have an unfavorable effect on the floatation of inclusions. Figure 6(d) shows the distribution of inclusion number density is more uniform in the ladle of Type D than that in the ladle of other types. Except in the corner near the two-phase domain, there is no obvious accumulation of inclusions in the ladle of Type D. This is because the gas bubbles can disperse very well on the
condition of three pours plugs, which is in favor of inclusion attachment to gas bubbles. Figure 7 shows that the variation of inclusion number density for different sizes ranges with time under different porous plug configurations. Figure 7(a) shows that the number density of 0–5 \( \mu m \) inclusions decreases drastically due to the collision and coalescence among inclusions. Figure 7(b) shows that the number density of 5–10 \( \mu m \) inclusions decreases slowly. Figure 7(c) shows that the number density of 10–15 \( \mu m \) inclusions increases firstly and then decreases when one or two porous plugs were adopted (Types A, B and C), while the number density of 10–15 \( \mu m \) inclusions decreases all the time when three porous plugs were adopted (Type D). Figure 7(d) shows that the number density of 15–20 \( \mu m \) increases firstly and then decreases. It should be noted that the terminal number density becomes even larger than the initial value when one porous plug was placed centrally. Furthermore, Fig. 7(e) shows that the number density of inclusions larger than 20 \( \mu m \) increases firstly and then decreases, and the terminal inclusion number density is larger than the initial value on the condition of one porous plug configurations (Type A and B). Collision and aggregation among inclusions lead to such interesting phenomena. At the beginning of the process, the number of new bigger inclusions after aggregation is much more than that of the removed inclusions, so the number of smaller inclusions decreases while the total number of bigger inclusions increases. The decrease of the number of small inclusions results in the decrease of the number of big inclusion after aggregation. Once the number of big inclusions after aggregation is less than that of big inclusions removed, the total number of big inclusions decreases.

Table 3 shows that the terminal inclusion concentration is quite different under different porous plug configurations after 1 000 s. As can be seen, the terminal inclusion concentration decreases from 127.8 to 82.3 ppm when one porous plug moves from the center to the eccentric position. The main reason is that when one porous plug was placed eccentrically, the gas bubbles dispersed more widely which can promote the efficiency of inclusion attachment to gas bubbles. Moreover, when the only one porous plug was placed eccentrically, the horizontal velocity component becomes more dominant than that in the ladle under one porous plug placed centrally. Consequently, inclusions have more residence time at the free surface which is good for the top slag to trap the inclusions. Thus, the mass ratio of removed inclusions by the top slag increases from 86.1 to 87.2% while the porous plug configuration changes from Type A to B.

On the other hand, the inclusion removal rate increases with the increasing number of porous plugs. The terminal inclusion concentration falls down to 56.1 ppm when two porous plugs are placed at half radius oppositely (Type C), while the terminal inclusion concentration falls down to 32.7 ppm when three porous plugs are placed at half radius (Type D). The main reason is that the gas bubbles dispersed more widely with the increasing number of porous plugs and it is easy for the inclusions to be removed by adhesion to bubbles. Therefore, the mass ratio of removed inclusion by top slag in the ladle of Type C and D also rises up to 94.2% and 95.9% respectively.

Table 3 also shows that on the condition of different porous plug configurations, more than 85% of inclusions are removed by the top slag, about 4–13% of inclusions are trapped by the side wall and the inclusions trapped by the bottom are negligible. Moreover, it should be noted that the increasing number of porous plugs is also in favor of increasing the flow velocity along the side wall. However, it is difficult for the inclusions near the wall to be removed when the velocity along the side wall is greater. Thus, the relative removed inclusion mass ratio by the side wall decreases with the increasing number of porous plugs.
4. Conclusions

(1) A mathematical model, which considers the interaction between bubbles and inclusions, Stokes collisions and turbulent collisions among inclusions, has been developed to predict the variation of inclusion spatial distribution with time in the ladle.

(2) The porous plug configurations have a profound effect on the inclusion removal rate in ladle. The eccentric position is better than the central position for inclusion removal on the condition of one porous plug configuration, and the terminal inclusion concentration decreases with the increasing number of porous plugs.

(3) For inclusion removal, the inclusion trapped by the
top slag is the main manner, inclusion adhesion to the side wall is the minor manner, and inclusion adhesion to the bottom wall is negligible. After 1000 s, the inclusions larger than 15 μm increases in the ladle of one plug configuration and decreases in the ladle of two or more plug configurations. Therefore, on the condition of one plug configuration, it is necessary to prolong the treating time in order to avoid newly generated larger inclusions detained in liquid steel.

Acknowledgements

This work was supported by the National High-tech R&D Program of China (No. 2009AA03Z530), National Natural Science Foundation of China and Shanghai Baosteel (No. 50834010), the Key Project of Chinese Ministry of Education (No. 108036), SRF for ROCS, SEM (No. 20071108-2) and NEU, 111 Project (No. B07015), China Postdoctoral Science Foundation (No. 20070421065), and Science and Technology Planning Project of Liaoning Province (No. 2009221007).

Nomenclature

\[ A \]: Constant
\[ A^* \]: Hamaker constant
\[ B \]: Constant
\[ C^* \]: Inclusion characteristic volume concentration
\[ C_D \]: Drag coefficient
\[ d_b \]: Bubble diameter
\[ d_n \]: Nozzle diameter
\[ D \]: Diffusion coefficient
\[ f \]: Fractional inclusion number density
\[ F \]: Force vector
\[ g \]: Gravitational acceleration vector
\[ k \]: Turbulent energy
\[ N^* \]: Inclusion characteristic number density
\[ N_b \]: Bubble number density
\[ \rho \]: Density
\[ \rho_p \]: Possibility of bubble-inclusion attachment
\[ Q \]: Flow rate
\[ r, r^* \]: Inclusion radius and characteristic radius
\[ Re_b \]: Bubble Reynolds number
\[ S \]: Source term
\[ u \]: Velocity vector
\[ u_{\text{eff}} \]: Inclusion characteristic number density flotation velocity
\[ u_{\text{c}} \]: Inclusion characteristic volume fraction flotation velocity
\[ v \]: Kinematic viscosity
\[ \beta_s \]: Stokes collision rate
\[ \beta_t \]: Turbulent collision rate
\[ \mu \]: Viscosity

Subscripts

\[ g, l \]: Gas, liquid
\[ b \]: Bubble
\[ \text{eff} \]: Effective
\[ p \]: Inclusion

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