Self-energy function of quantum-dot states and resonance fluorescence

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Abstract. We investigate the energy dependence of self-energy functions of quantum-dot states that describe the interaction of quantum dots with vacuum. It is shown that the variation of these self-energy functions can be significant and this can have a significant effect on processes of quantum interference from quantum dots.

1. Introduction
Resonance fluorescence from artificial atoms and quantum dots [1, 2] is now a major field of research. As was predicted by Mollow [3] and confirmed by a number of experiments [4–6], at relatively high laser intensities the fluorescence spectrum should consist of a symmetric triplet whose side components are separated from the central peak by distances equal to the generalized Rabi frequency. However, the recent experimental observations of the resonance fluorescence from the artificial atom and quantum dots [1, 2] showed an essential discrepancy with the predictions of quantum optics for two-level atomic systems. The observations of the resonance fluorescence from self-assembled quantum dots, being atom-like two-level emitters, showed [2] not only the asymmetry which for some detuning becomes very significant but also surprising sideband linewidths: there is a linear-linear zoom-in on Mollow sidebands at some laser powers. The discrepancy between the experiment and the theory could originate from the fact that the ordinary theory of the resonance fluorescence does not take into account the fact that the effect of the vacuum modes on the spectrum of resonance fluorescence may be significant. The fact that the interaction of emitters with the vacuum modes can be very significantly modified in nanomaterials has been demonstrated in [7]. In particular this should have a significant effect on the Lamb shift of the components of the Mollow triplet. In Refs. [8] it has been shown that under some conditions on the self-energy function of emitter states the contributions from the vacuum modes cannot be treated as small corrections. The aim of the present paper is to show that the above conditions can be realized in processes of resonance fluorescence from quantum dots.

2. Vacuum modes and resonance fluorescence from quantum dots
The strong interaction of an atom with the laser field resonant with an atomic transition is successfully treated within the quantum optics formalism [9,10]. When the laser mode frequency
\( \omega_L \) is close to the frequency \( \omega_R = \omega_e - \omega_g \) of the transition between two bound states \( |e \rangle \) and \( |g \rangle \) the atom-laser interaction is strong. In quantum optics this interaction is usually described by using the two-level model and the rotating wave approximation (RWA). In this approximation the laser-dressed states are the eigenstates of the system Hamiltonian \( H_{\text{RWA}} \) [10] and are given by

\[
| +, n \rangle = \cos \theta_n | e, n \rangle + \sin \theta_n | g, n + 1 \rangle; \quad | -, n \rangle = -\sin \theta_n | e, n \rangle + \cos \theta_n | g, n + 1 \rangle. \tag{1}
\]

Here \( | e, n \rangle \) (\( | g, n \rangle \)) denotes the state of the combined laser-atom system containing the atom in the bare state \( | e \rangle \) (\( | g \rangle \)) and \( n \) photons in the laser mode, and \( \theta_n \) is the mixing angle defined by \( \tan(2\theta_n) = -\Omega_n / \Delta \) with \( \Omega_n \) and \( \Delta \) being the Rabi frequency \( \Omega_n = 2g_L \sqrt{n + 1} \) and detuning, respectively. The energies of the dressed states are given by \( E_{\pm,n} = (n+1/2)\omega_L + (\omega_e - \omega_g)/2 \pm \Omega_R^{(n)}/2 \) with \( \Omega_R^{(n)} \) being the generalized Rabi frequency \( \Omega_R^{(n)} = \sqrt{\Omega_n^2 + \Delta^2} \). The states dressed by the interaction of the atom with the resonance laser field are not dressed by its interaction with the vacuum modes, and in our investigation of the problem we do not assume a priori that this interaction is weak in any case and use nonperturbative methods of its description based on the the generalized dynamical equation (GDE), which in [11] has been derived as a direct consequence of the first principles of quantum physics. Being equivalent to the Schrödinger equation in the case when the interaction in a quantum system is instantaneous, GDE allows one to extend quantum dynamics to the case of non-local-in-time interactions. This equation has been proven to provide a new insight of many problems in nuclear physics [12–15], atomic physics [16–20] and quantum optics [8,21]. Within this approach the QED bound state problem is solved by using the formalism of the self-energy function \( C_n(z) \) (see for details Refs. [22]), that describes the interaction of the particles in the state \( | n \rangle \) with the vacuum. The self-energy QED corrections to the energies \( E_n^{(0)} \) of the bare states are determined by the equation

\[
z - E_n^{(0)} - C_n(z) = 0 \quad \text{where} \quad C_n(z) \quad \text{in turn are determined by GDE. In the first approximation the solution of this equation is } E_n \equiv E_n^{(0)} + C_n^{(0)}(E_n^{(0)}) \equiv E_n^{(0)} + \Delta E^{(0)}_n - \frac{1}{2} \Gamma_n, \quad \text{where} \quad E_n \quad \text{is the energy of the real dressed state, } E_n^L \quad \text{is the Lamb shift, } \Gamma_n \quad \text{is the natural level width, and } C_n^{(0)}(z) \quad \text{is determined by the equation}
\]

\[
\frac{dC_n^{(0)}(z)}{dz} = -\langle n | H_I G^2_0(z) H_I | n \rangle, \tag{2}
\]

with \( H_I \) being the QED Hamiltonian. For this approximation to be valid, probability amplitudes of the nonradiative transitions between the states with the same total angular momentum \( J \), its projection \( J_z \) and parity must be much smaller than the energy distance between these states. In addition the variation of the function \( C_n(z) \) with the energy must be negligible on the energy interval of order of the value of the Lamb shift. This takes place in the case when the atom is in free space. In fact, in this case the energy distance between the states with the same \( J, J_z \) and parity are of order of \( \alpha^2 m_e c \), while the probability amplitudes of nonradiative transitions between them are of order of \( \alpha^3 m_e c \), and the self-energy function varies significantly with the energy only on energy intervals of order \( m_e c \). The situation is dramatically changed in the case of an emitter subject to resonance laser field because the energy distance between dressed states \( | +, n \rangle \) and \( | -, n \rangle \) having the same \( J, J_z \) and parity equals the Rabi frequency. As a result the nonradiative transitions between these states begin to play an important role. At leading order the probability amplitudes \( \langle \pm, n | \Sigma(z) | \mp, n \rangle \) of these transitions are given by the formula [8]

\[
\langle \pm, n | \Sigma(z) | \mp, n \rangle = \cos \theta \sin \theta (C_g^{(0)}(z) - C_e^{(0)}(z)), \quad \text{where} \quad C_g^{(0)}(z) \quad \text{and} \quad C_e^{(0)}(z) \quad \text{are the self-energy functions of the bare states } | g \rangle \quad \text{and} \quad | e \rangle \quad \text{respectively. At the same time, in the case of an atom being in free space the energy interval of order \( \Omega_R \) is too small for the variation with energy of } \langle \pm, n | \Sigma(z) | \mp, n \rangle \quad \text{to be noticeable. In this case the energy dependence of } \langle \pm, n | \Sigma(z) | \mp, n \rangle \quad \text{can be neglected, and they can be regarded as matrix elements of an additional term in the Hamiltonian}
\]
describing the interaction of the atom with resonance laser field. Adding such a term to the RWA Hamiltonian will lead only to a change in the mixing angle $\theta_0$, and, as a consequence, will not give rise to asymmetry of the Mollow spectrum. However, as we show below, in the case when a quantum dot acts as an emitter the variation of the self-energy functions $C_g(z)$ and $C_e(z)$ on an energy interval compared to the energy difference between the dressed states $|+, n\rangle$ and $|-, n\rangle$ can be very significant, and hence the vacuum modes are involved into the strong atom-laser interaction.

3. Self-energy functions of the quantum-dot states

Let us consider the model of a quantum dot interacting with two reservoirs in which the Hamiltonian of a system consisting of quantum dot and leads is described by $H = H_{QD} + H_T + H_{lead}$, where $H_{QD}$ is the Hamiltonian describing the quantum dot, $H_{lead}$ is the lead Hamiltonian, $H_T$ is the Hamiltonian which describes tunneling processes between dots and leads. The single-particle level spacing in the dot is assumed to be larger than any other energy scale (temperature, Coulomb interaction, and transport voltage) so that only one energy level needs to be taken into account. The quantum dot can be described by the single-level Anderson impurity model, in which $H_{QD}$ is of the form $H_{QD} = \sum_{\sigma = \uparrow, \downarrow} E d_\sigma^\dagger d_\sigma + U n_\uparrow n_\downarrow$, where the creation (annihilation) operator for an electron with spin $\sigma$ on the dot is given by $d_\sigma^\dagger$ and $d_\sigma$, and $n_\sigma = d_\sigma^\dagger d_\sigma$ is the corresponding number operator. The on-site repulsion $U$ describes the energy cost for double occupation (when we have two electrons in quantum dot) and stems from Coulomb interaction. The Hamiltonian $H_T$ is given by $H_T = \sum_{s,k,\sigma} V_s C_{s,k,\sigma}^\dagger d_\sigma + H.C.$, where $V_s$ is the momentum- and spin-independent tunnel matrix element, $C_{s,k,\sigma}^\dagger$ is creation (annihilation) operators for electrons with spin $\sigma$ and momentum $k$ in lead and $s = L, R$. The Hamiltonian $H_{lead}$ is given by $H_{lead} = \sum_{s,k,\sigma} E_{s,k} C_{s,k,\sigma}^\dagger C_{s,k,\sigma}$. The chemical potentials of the two leads differ by the applied bias $\mu_L - \mu_R = -eV$.

![Figure 1](image.png)

Figure 1. The derivative of the self-energy function $C(z)$ of a quantum-dot state for $R = 0.1$ eV, $U = 30R$, $\mu_L = \mu_R = 2$ eV and two different temperatures

We assume that the density of states $\rho_\sigma$ in the leads is constant for transport and determines the tunnel coupling strength $R_\sigma$ as $R_\sigma = 2\pi \rho_\sigma |V_s|^2$ and $R = R_L + R_R$. The Hilbert space for the quantum dot is spanned by the states $|0\rangle$ for an empty dot, $|\sigma\rangle$ for a singly occupied dot with spin $\sigma = \uparrow, \downarrow$ and $|d\rangle$ for a doubly occupied dot. The corresponding energies are $E_0$, $E_\sigma$, and $E_d$. By generalizing the equations derived in [23] for the self-energy function of quantum
dot we get

\[ C(z) = - \sum_s \frac{R_s}{2\pi} \int d\omega \left( \frac{f_s(\omega)}{\omega - z} + \frac{f_s(\omega)}{\omega + U - \omega} \right), \]

(3)

where \( f_s(\omega) = (1 + \exp(\frac{\omega - \mu_s}{k_B T}))^{-1} \) is the Fermi distribution function. As it follows from the results of calculations presented in Fig. 1 the variation of the self-energy function is very significant in the vicinity of the point \( z = 2 \text{ eV} \).

4. Conclusions

In this paper we have investigated the self-energy functions of quantum dots, in order to understand the effect of vacuum modes on the process of resonance fluorescence from quantum dots. We have shown that the variation of these functions on the energy interval compared to the energy difference between the dressed states \(|+, n\rangle\) and \(|-, n\rangle\) is very significant. This means that the vacuum modes have a significant effect on the resonance fluorescence from quantum dots, and the development of a theory that could take them into account in consistent way could remove the discrepancy between experiment and the theoretical predictions.

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