WHY WE NEED DARK ENERGY

D. PAVÓN*

Departamento de Física, Universidad Autónoma de Barcelona,
Bellaterra, 08193, Spain
*E-mail: diego.pavon@uab.es

N. Radicella*

Dipartimento di Fisica “E.R. Caianiello”, Università di Salerno, I-84084 Fisciano, Italy

It is argued that dark energy - or something dynamically equivalent at the background level - is necessary if the expanding universe is to behave as an ordinary macroscopic system; that is, if it is to tend to some thermodynamic equilibrium state in the long run.

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1. Introduction

The standard cosmological cold dark matter model was in good health until around the last decade of the previous century when it was realized that the fractional density of matter falls well below unity. Its full dismissal came shortly after, about the close of the century, with the discovery of the current cosmic acceleration. However, to account for the acceleration in homogeneous and isotropic models one must either introduce some exotic energy component with a huge negative pressure (called dark energy) or, more radically, devise some theory of gravity more general than Einstein’s. Thus, both solutions look somewhat forced and not very aesthetical. Here we very briefly argue that dark energy (or something equivalent) is demanded on thermodynamic grounds. That is to say, we provide what we believe is a sound thermodynamic motivation for the existence of dark energy. Details can be found in Ref. [1].

2. Simple models

Isolated, ordinary, macroscopic systems spontaneously tend to thermodynamic equilibrium. This constitutes the empirical basis of the second law of thermodynamics. Succinctly, the latter asserts that the entropy, $S$, of such systems can never decrease, $S' \geq 0$, and that it must be concave, $S'' < 0$, at least during the last stage of approaching equilibrium. It seems worthwhile to explore the consequences of this law when applied to spatially flat Friedmann-Robertson-Walker universes.

The entropy is contributed by two terms, the entropy of the apparent horizon, $S_A = k_B A/(4\pi P_{by}) = k_B \pi/H^2$ (where $A$ denotes the horizon area), and the entropy of matter and fields, $S_f$, enclosed by it. Let $\rho$ and $w$ be the energy density and equation of state parameter (assumed constant) of the latter. Then, $S'_A \propto (1 + w)/(a\rho)$ and $S''_A \propto (1 + w)(2 + 3w)/(a^2 \rho)$ (the prime means derivative with respect to the scale factor). Likewise, $S'_f \propto (1 + w)(1 + 3w)/(aH)$ and $S''_f = (1 + w)(1 + 3w)(1 + w)a^{(9w-1)/2}$. These expressions reveal that radiation dominated universes
cannot tend to thermodynamic equilibrium in the long run. The same is true for universes dominated either by matter or phantom energy. However, for dark energy dominated universes with $-1 \leq w < -2/3$, one has $S'_A + S'_f > 0$ at all times, and $S''_A + S''_f < 0$ at least when $a \to \infty$. This suggests the necessity of non-phantom dark energy, or some modified gravity scenario able to lead to a suitable acceleration at late times, for the Universe to behave as an ordinary macroscopic system.

3. More general models

Thus far we have restricted ourselves to cosmological models with $w = \text{constant}$ and governed by Einstein gravity. This section very briefly reviews a handful of more general models that show overall compatibility with the observational constraints.

(i) For the model of Barboza and Alcaniz, in which the universe is dominated by pressureless matter and dark energy with equation of state $w(z) = w_0 + w_1 z(1 + z)(1 + z^2)^{-1}$, the second law is fulfilled either if $w_1 < 2/3$ and $-1 < w_0 < -2/3$, or if $2/3 \leq w_1 < 1$ and $-1 < w_0 < -w_1$.

(ii) In the original Chaplygin gas model (p = $-A/\rho$, $\rho = \sqrt{A + (B/a^n)}$), that unifies matter and dark energy, $S'_f < 0$ and $S''_f > 0$ when $a \to \infty$; nevertheless, $S'_f > 0$ and $S''_f < 0$ in the same limit.

(iii) The holographic dark energy model of Ref. 5 in which the dark energy density varies as the area of the apparent horizon and decays into matter in such a way that the ratio of both densities is a constant (thus solving the coincidence problem), is also seen to tend to thermodynamic equilibrium at late times.

(iv) In the modified gravity model of Dvali et al. our four dimensional Universe is considered a brane embedded in a five dimensional bulk with flat Minkowski metric. There is no dark energy, just matter and radiation. The condition for having $S'_f > 0$ and $S''_f < 0$ as $a \to \infty$ reduces to a bound on the current number of dust particles, which is fulfilled by a huge margin.

(v) The Cardassian model, proposed by Freese and Lewis, also dispenses altogether with dark energy but introduces an extra term, proportional to $\rho^\alpha$, in the usual Friedmann’s equation. Because it can be mapped to a dark energy model with $w = \alpha - 1$, it presents a healthy thermodynamic behavior provided $\alpha$ lies in the range $(0, 2/3)$.

(vi) Lastly, torsion gravity replaces the scalar curvature in Einstein relativity by the torsion $\tau$, the action being

$$I = \frac{1}{16\pi G} \int d^4 x \sqrt{-g}(\tau + f(\tau)) + I_{\text{matter}}.$$

In the of Bengochea and Ferraro, $f(\tau) = -\alpha(-\tau)^{-n}$. Using the best fit values for $\alpha$ and $n$, it is found that $S'_f < 0$ and that $S''_f > 0$ in long run, thereby the universe does not tend to thermodynamic equilibrium.
4. Conclusions
Altogether, (i) neither a radiation dominated nor a cold dark matter dominated universe can tend to thermodynamic equilibrium in the long run. However, dark energy dominated universes with constant $w$ in the range $-1 \leq w < -2/3$ and the $\Lambda$CDM model can. (ii) Phantom models with $w = \text{constant}$, cannot. (iii) Dark energy models with $w \neq \text{constant}$ deserve a separate analysis. In particular, the Chaplygin gas model can, as well as the model of Barboza and Alcaniz for an substantial range of its parameters, and some holographic models. (iv) Several modified gravity models, such as DGP and Cardassian models, can, but the model of Bengochea and Ferraro cannot. (v) Models in which the present accelerated stage of expansion is just transitory conflict with the second law. (vi) The entropy of the Universe seems to tend to some maximum value (of about $H^{-2}$ when $a \to \infty$), but in order to reach a firmer conclusion, accurate measurements regarding $H(z)$ are called for. (vii) Finally, the existence of dark energy or -equivalently- some modified gravity theory could have been expected on thermodynamic grounds.

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References
1. N. Radicella and D. Pavón, *Gen. Relativ. Grav.* **44**, 685 (2012).
2. H.B. Callen, *Thermodynamics*, (J. Wiley, New York, 1960).
3. E.M.Jr. Barboza and J.S. Alcaniz, *Phys. Lett.* B **666**, 415 (2008).
4. A.Y. Kamenschchik, U. Moschella, and V. Pasquier, *Phys. Lett.* B **511**, 265 (2001).
5. W. Zimdahl and D. Pavón, *Class. Quantum Grav.* **24**, 5461 (2007).
6. G. Dvali, G. Gabadadze, and M. Porrati, *Phys. Lett.* B **485**, 208 (2000).
7. K. Freese and M. Lewis, *Phys. Lett.* B **540**, 1 (2002).
8. G.R. Bengochea and R. Ferraro, *Phys. Rev.* D **79**, 124019 (2009).