Analysis of Time and Frequency Response for Single/Multi-Tone Stimulus Harmonic Phase-Reference Based on Prime Number Algorithm

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Received: 14 September 2020; Accepted: 30 October 2020; Published: 3 November 2020

Abstract: We present a spectrally-dense phase-reference and its calibration for nonlinear vector network analyzers (NVNAs) using a step recovery diode (SRD) comb-generator with a multi-tone stimulus. Frequency selection for multi-tone stimulus based on prime number algorithms was used with the Digital Real-Time Oscilloscope (DRTO) to avoid the sub-Nyquist spurs components and to increase the effective sampling rate so that the waveform can be observed in greater detail. The measured results were calibrated to minimize drift and jitter and achieved excellent agreement between the prime number and the exact frequency strategies except at the sub-Nyquist frequencies. The analysis indicates that the prime number selected frequencies show significantly improved performance by avoiding the DRTO distortion components. We have verified the validity of the method described in this paper by experimental measurement results.

Keywords: digital real-time oscilloscope (DRTO); harmonic phase-reference; multi-tone; prime number algorithm; sub-Nyquist spurs; nonlinear vector network analyzer (NVNA)

1. Introduction

The high peak-to-average-power-ratio (PAPR) of modern communication systems require more sophisticated amplifier designs to achieve higher efficiency [1–3]. When a power amplifier is working under large-signal conditions, the amplifier distortion will cause some of the energy to be spread to sidebands. Therefore, pre-distortion algorithms are used to add complementary components to cancel these unwanted terms. The RF power amplifier under the actual working state is time-invariant, and the observed hysteresis is due to long-term memory effects. Besides, the intermodulation components are asymmetric, and their phases are varied.

S-parameters theory describes only the linear characteristics of microwave devices and does not reflect the true response under high power stimulus. S-parameters can only provide information about linear characteristics, and a vector network analyzer (VNA) provides phase-locking to the 10 MHz reference, but the relative phases for each frequency are arbitrary. The Large Signal Network Analyzer (LSNA) and NVNA can measure the nonlinear behavior of RF devices or systems. The frequency grid spacing for the NVNA and the LSNA is typically limited to 10 MHz and 600 MHz [4], respectively. The nonlinear harmonic relationships can be characterized by X-parameters, describing the complex responses of the microwave devices under large-signal stimulus [5–9]. The nonlinear signal components can be used to measure the intermodulation components and the harmonics of the multi-tone stimulus.
signals. An external phase-reference standard is used as part of the NVNA nonlinear calibration to determine the eight complex-valued calibration coefficients of the NVNA at all the frequencies of interest.

The dense spectrum of the phase reference can be acquired by stimulating step recovery diode (SRD) with a pulsed-RF signal of over 99% duty cycle [10]. SRD modifies the local current flow in response to the field stimulus and is sensitive to the feedback that changes the voltage across the nonlinear device. Therefore, the pulse signal was unstable, and the waveform displayed drifts and jitters on the screen. Besides, the harmonic phase reference of such stimulus signals has only the harmonics due to the adjacent channel power ratio (ACPR) [11]. In order to break the limit of harmonic-only phase reference limitation, it is necessary to pursue a multi-tone stimulus with similar characteristics as designed waveforms, which proves to be stable than the high duty cycle signal.

There are two main advantages of using the multi-tone stimulus for NVNA phase reference [12]. Firstly, as the stimulus tone frequency increases, the power spectrum lines at the harmonics will be flattened out, and the fading effect will be weakened. Secondly, it optimizes the setup configuration in view of real measurement and research requirements. The multi-tone determines the available intermodulation bandwidth around each harmonic frequency.

The sampling oscilloscopes are designed for the periodic signal measurement, such as the Pseudo-Random Binary Sequence (PRBS) signals and the pulse signals [13]. Besides, the channel mismatch calibration [14] and time-base distortion (TBD) [15] in the sampling oscilloscope need to be corrected by the complex algorithms. Therefore, the DRTO is adopted to measure the multi-tone stimulus signals response for the NVNA phase reference. In order to avoid the error introduced by the sub-Nyquist spur components of noise, we have adopted the prime number algorithms to avoid noise interference falling on the targeted frequency [16]. This paper proposed the multi-tone prime number frequency selection algorithms to improve the time and frequency response of SRD comb generator harmonic phase reference for NVNA.

In this paper, multi-tone stimulus phase reference based on prime number algorithms for NVNA is proposed. The structure of the paper is as follows. Section 1 briefly surveys the methods for NVNA phase reference based on SRD comb generator, whereas Section 2 looks at the methodologies of single-tone and multi-tone stimulus and outlines the merits for the prime number algorithms. Details on the raw data processing based on the alignment principle are illustrated in Section 3. In the end, discussions and conclusions are summarized in Section 4. This paper extends the work presented at the CPEM 2018 conference [17].

2. Methodology

In this section, the methodology of prime number algorithms will be proposed and illustrated. The significant effect of prime number algorithms embodies in two aspects. On the one hand, the ADCs in the DRTO will create the sub-Nyquist terms that modulate the stimulus signal. The modulated terms will be superimposed on the measured results, and there will be distortion in the waveform. The prime number algorithm selects frequencies for the stimulus signal and avoids these errors. On the other hand, it increases the effective sampling rate to a great extent, and some details in the waveform can be observed clearly.

2.1. Prime Number Algorithm

The prime number algorithm is divided into two conditions, one is a single tone strategy, and the other is multi-tone strategy. The multi-tone strategy of the prime number algorithm is based on the single-tone strategy frequency selection rules.

2.1.1. Single-Tone Strategy

Using the same notation as [18] to select the frequency by prime number rules, P is the number of time samples in a block and R is the cycles of the targeted frequency, which are used to provide statistical information about the waveform. P and R can be decomposed into several prime numbers,
respectively, as have been defined in Equations (5)–(7) in [18]. The key point in the selection algorithm is that P or R should avoid large prime number factors, which will rule out lots of choices for the selection. The value for P is arbitrary and can be chosen to give smaller prime factors or a closer frequency to the target. Due to the fact that SRD is not a fully time-invariant system and the waveform will alter as the frequency changes, it is worth picking values of P and R that are both below and above the frequency target so that the magnitude and phase can be interpolated. P and R can be mapped onto a single cycle to provide the statistical information of detailed oversampling. The SRD time-domain output comparison of prime number frequency and exact frequency is depicted in Figure 1, which are represented by the red dot and the blue dot, respectively. The actual frequency SRD output will be overlapped, and the sharp pulse is not precise, while due to the slight deviation in the stimulating frequency, the overall waveform of prime number frequency can be delineated in great detail, and it is obvious to tell the pulse formation of the SRD.

The SRD time domain output comparison of two frequencies. Red dots indicate the prime number frequency SRD output and blue dots indicate the exact frequency SRD output.

2.1.2. Multi-Tone Strategy

There are many ADCs with an 8-bit resolution in DRTO to achieve the high-speed sampling rate. The minor fluctuation in the DC level and response in single ADC will generate sub-Nyquist spur components [19]. Based on the rules of the single-tone frequency selection algorithm, we proposed the algorithm to select the frequency of multi-tone modulated signal. One significant aspect of the prime number algorithm for multi-tone stimulus strategy is to steer away from the sub-Nyquist components:

\[ f_{spur} = S_{DRTO}/m_{ADC} \]

where \( S_{DRTO} \) denotes the sampling rate of DRTO, and \( m_{ADC} \) is the number of ADC in DRTO. The noise response of DRTO is performed to determine the number of ADC. By applying no signal to DRTO, ten groups of raw noise data were acquired. The averaged noise response was depicted in Figure 2, and the parasitic components above the noise floor are the sub-Nyquist spurs. As can be concluded from the results, the minimum sub-Nyquist spur will be 0.25 GHz, and the total number of the ADCs in DRTO is 320, which can be used to select frequencies excluded from the sub-Nyquist spur and the harmonics. What is different from the single-tone algorithm is that the granularity \( \Delta f \) of multi-tone stimulus will be set by the DAC sample rate in the vector signal generator (VSG) rather than the DRTO sample rate. The granularity \( \Delta f \) also corresponds to the modulation period of the multi-tone stimulus, thus providing additional criteria for the frequency selection process to satisfy. If the number of ADC is small, then it is possible that the lowest sub-Nyquist term will correspond to the harmonic value
rather than the fundamental. It is notable that re-modulation will occur around the sub-Nyquist tones. Therefore, the harmonic values should also comply with these criteria. The implication is that the overall period may be slightly longer or shorter than the targeted granularity. The “added or reduced samples” is for the selection of nondegenerate combinations of $P$ samples and $R$ cycles in the block. The granularity of the multi-tone stimulus will be set by the DAC sampling space, so the cycle number of DAC should also be taken into consideration. $k$ represents the $k^{th}$ stimulus of the multi-tone signal or the DAC.

$$R_k = \prod_{j=1}^{m_k} r_{jk} \quad (R_k = R_{DRTO} \ or \ R_{DAC})$$

(2)

Figure 2. Noise and sub-Nyquist spur response of oscilloscope. The total number of the ADC is 320, calculated from the spur components.

To clearly clarify the multi-tone stimulus prime number algorithm, the following steps are performed to select the frequency for the multi-tone modulated signal:

1. Determine the oversampling ratio (also the DAC cycles) and the targeted frequency interval $\Delta f_t$, then estimate the added or reduced DAC samples according to the oversampling ratio and DAC sampling rate, see (9).
2. Calculate the period of the DAC ($T_{DAC}$) using the slightly modified DAC samples and then give the modulation frequency $\Delta f_{DAC}$.
3. The appropriate number for the DRTO samples will be determined by (12), which is the round value of the added or reduced results of the product of the DAC period and DRTO sampling rate.
4. The cycle number of the carrier frequency ($R_{DRTOc}$) and the $M^{th}$ upper band frequency ($R_{MDROTo}$) or the $M^{th}$ lower band ($R_{MDRTOo}$) frequency are settled by (5)-(7).
5. Evaluate the frequency of the carrier and sideband frequencies based on (8), and they should avoid the sub-Nyquist spurs.
6. Ensure the DRTO samples and the cycle numbers do not contain common factors.
7. If the calculated multi-tone frequencies or the prime number factors contradict with the above standards, then the added or reduced samples should be modified to meet the selection criteria.

For the single-tone case, both the $P$ and the $R$ terms are included in the figure of merit, but on reflection, only the $P$ terms are important for the Fourier Transform criteria. For the multi-tone case, all the multi-tones are the harmonics of comb frequency spacing, and it is the $R$ values that
change. Therefore, a new figure of merit should be defined to avoid too many common prime numbers. One possibility is to include the degenerate “r” terms from each value of R.

\[
FoM_{mt} = \frac{1}{\sum_{i=1}^{n_g} p_i + \sum_{k=1}^{\text{ntone}} \sum_{j=1}^{n_k} r'_{jk}}
\]

(3)

where \( r'_{jk} = r_{jk} \cap p_i \).

The prime number factor sets of the two parameters are excluded from each other, which ensures no degeneracy and maximizes the number of ADC levels excised. In a multi-tone signal, it is difficult to achieve \( P \) and \( R \) factors rule of (5) in [18] for all multi-tone frequencies, but the number of coincidences should be minimized. For example, the samples \( P \) with factors (2, 3, 5, 11) and \( R_1 \) with factors (2, 13), \( R_2 \) with factors (3, 7) is feasible, but \( P \) with (2, 3, 5, 11) and \( R_1 \) with (2, 13), \( R_2 \) with (2, 7) it is not because factor 2 is common to all.

In a multi-tone signal, a series of relationships \( R_1 \& f_1, R_2 \& f_2 \ldots \) should be determined, and in each case, we will get different sets of prime factors. The carrier frequency cycle number of the multi-tone stimulus can be calculated from (4), and the \( R_{DRTO} \) represents the round number of the product term:

\[
R_{DRTO} = \text{round}(\frac{f_{\text{Target}}}{S_{DRTO}} \times P_{DRTO})
\]

(4)

where the \( f_{\text{Target}} \) represents the targeted carrier frequency. Then the selected frequency can be acquired:

\[
f_{\text{Prime}} = \frac{R_{DRTO} S_{DRTO}}{P_{DRTO}}
\]

(5)

As for multi-tone upper band frequency selection, the \( M^{th} \) cycle number can be calculated from:

\[
R_{MDRTO_p} = \text{round}(\frac{f_{\text{Prime}} + M \Delta f_{DAC}}{S_{DRTO}} \times P_{DRTO})
\]

(6)

where the \( \Delta f_{DAC} \) represents the frequency resolution of the multi-tone signal, and \( M \) represents the order of the upper or lower sideband tone. Similarly, as for multi-tone lower band frequency selection, the \( M^{th} \) cycle number will be:

\[
R_{MDRTO_n} = \text{round}(\frac{f_{\text{Prime}} - M \Delta f_{DAC}}{S_{DRTO}} \times P_{DRTO})
\]

(7)

Once the cycle number is settled, the corresponding stimulus prime number frequency can be described as follow:

\[
f_{MPrime} = \frac{R_X S_{DRTO}}{P_{DRTO}} \quad \text{(where } R_X = R_{MDRTO_p} \text{ or } R_{MDRTO_n} \text{)}
\]

(8)

2.1.3. Example Calculations

Taking the multi-tone stimulus as an example, the oscilloscope used in this experiment is Agilent MSOV334A, which has a sampling rate of 80 G Sa/s and bandwidth of 33 GHz. The clock rate in the VSG E8267D can reach 100 MHz according to the User’s Manual. The example is targeted at a three-tone signal with a carrier frequency of 1 GHz (\( f_{\text{Target}} \)) and granularity of 1 MHz (\( \Delta f \)). To give an epoch of two modulation periods, it can achieve an oversampling ratio of 2 (\( R_{DAC} \)) to give 160k points per cycle. This will give an acquisition period of 4 \( \mu \)s, and this can be extended as required to provide additional statistical data. In the following calculation process, add one \( (k_{DRTO} = -1) \) of the DRTO samples and reduce one \( (k_{DAC} = 1) \) of the DAC samples to ensure nondegenerate combinations.
of $P$ and $R$ values. The non-zero value for $k$ means that there will be a phase shift of $\pi/80$ for each successive repeat in the sequence.

$$P_{DAC} = \frac{R_{DAC}S_{DAC}}{\Delta f_t} - k_{DAC}$$

$$T_{DAC} = \frac{P_{DAC}}{S_{DAC}}$$

The actual modulation frequency and the actual sampling numbers can be extracted by (11) and (12):

$$\Delta f_{DAC} = \frac{R_{DAC}}{T_{DAC}}$$

$$P_{DRTO} = T_{DAC}S_{DRTO} - k_{DRTO}$$

The specific values for the nine-tone stimulus with a targeted carrier frequency of 1 GHz and targeted frequency granularity of 1 MHz are listed in Table 1. The optimum choice of $P_{DAC}$ and $P_{DRTO}$ values corresponds to 199 and 159,201, respectively.

Table 1. Example of nine-tone stimulus.

| Item   | Value | Factor | Item   | Value                           |
|--------|-------|--------|--------|---------------------------------|
| $P_{DAC}$ | 199   | 199    | $T_{DAC}$ | 1990 ns                        |
| $P_{DRTO}$ | 159,201 | 3, 7, 19 | $\Delta f_{DAC}$ | 1,005,025,126 Hz              |
| $R_{DRTO}$ | 1998 | 2, 3, 37 | $f_{pprime}$ | 1,004,013,793.883 Hz        |
| $R_{DRTO}$ | 1996 | 2, 499 | $f_{pprime}$ | 1,003,008,775.071 Hz        |
| $R_{DRTO}$ | 1994 | 2, 987 | $f_{pprime}$ | 1,002,003,756.258 Hz        |
| $R_{DRTO}$ | 1992 | 2, 3, 83 | $f_{pprime}$ | 1,000,998,737.445 Hz        |
| $R_{DRTO}$ | 1990 | 2, 5, 199 | $f_{pprime}$ | 999,993,718.632 Hz         |
| $R_{DRTO}$ | 1988 | 2, 7, 71 | $f_{pprime}$ | 998,988,699.820 Hz         |
| $R_{DRTO}$ | 1986 | 2, 3, 331 | $f_{pprime}$ | 997,983,681.007 Hz         |
| $R_{DRTO}$ | 1984 | 2, 31 | $f_{pprime}$ | 996,978,662.194 Hz         |
| $R_{DRTO}$ | 1982 | 2, 991 | $f_{pprime}$ | 995,973,643.382 Hz         |

2.2. Implementation

The measurement schematic diagram of the multi-tone stimulus phase reference is shown in Figure 3. The DRTO adopted the phase-locked mode via 10 MHz external clocks, as demonstrated in [20]. The VSG and the pulse signal generator (PSG) Agilent 81160A were connected to the external clock to ensure the synchronization of the three instruments. The PSG provides a trigger signal for the DRTO, and the VSG generates the modulated multi-tone signal to drive the SRD comb generator. During the measurement, the data acquired from the DRTO is imported into MATLAB and transformed into the frequency domain (FFT) for analysis of complex frequency characteristics of multi-tone NVNA phase reference. The objective is to observe the dense spectrum of the intermodulation components around the carrier and harmonics. In the experiment, the SRD pulse generator (Herotek GC1026RC) was driven at an RMS power level of 18 dBm. The measured epoch gives a frequency resolution of 500 kHz, and the nine-tone stimulus has a minimum frequency spacing of 1 MHz, providing additional frequency points to validate time base fidelity.
The VSG, the PSG, and the DRTO are all locked to an external Rubidium atomic clock to ensure the synchronization and frequency traceability of the three instruments (Figure 4). The SRD comb generator is stimulated by the RF signal generated from the VSG, and this waveform is recorded by Channel 1 of the DRTO. Channel 3 of the DRTO is used as an event trigger using the pulse signal from the PSG. The bandwidth of the SRD comb generator and the DRTO are 20 GHz and 33 GHz, respectively, which is adequate to capture the full SRD spectrum.

3. Data Processing

The relative phase arrangement of multi-tone frequencies will generate amplitude modulation components and phase modulation components. If the signal is only amplitude modulated, then the peak must be set to protect the SRD, which also means that over much of the SRD epoch is under-driven and produces fewer harmonics. Phase-modulation will increase the harmonic content as frequency increases, which prevents the spectral spreading effect. Therefore, it is important to balance the AM and PM components to control the growth of the sidebands at higher harmonics.

The power level will affect fundamental and harmonic components in the frequency domain. The wider bandwidths and higher power levels enhanced the quality of the phase reference signal for NVNA. Besides, SRD can be stimulated at a higher power level of Schroeder phase multi-sine signal because the energy can be spread to harmonic bands; thus, the pulse formation phenomenon appears...
to be more obvious than zero phases. As for the $N$-tone stimulus signal, the Schroeder phases $\phi_k$ can be obtained by:

$$\phi_k = \frac{-k(k - 1)\pi}{N}$$  \hspace{1cm} (13)

The subscript $k$ denotes $k^{th}$ phase value of stimulus signals. The nine-tone stimulus of Schroeder phases is shown in Table 2. The radians have been transformed into angle values. In the third row, $\Phi_k$ denotes phase-unwrapping results of the Schroeder phase, which are centro-symmetric about the carrier phase. The time-domain response of multi-tone stimulus and the uncertainty is depicted in Figure 5. There does not appear to be any structure to the uncertainty of time-aligned results determined from the standard deviation of time response (100 times). The lack of significant structure suggests that there is little phase error in the corrected result. As the stimulus increased, unlike the first inset of varied amplitude, the second inset with sharp pulse formation provides a better phase standard for NVNA.

**Table 2.** Schroeder phases of nine-tone stimulus.

| $k$ | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\phi_k$ | 0   | -40 | -120| -240| -400| -600| -840| -1120| -1440|
| $\Phi_k$ | 0   | 320 | 240 | 120 | 320 | 120 | 240 | 320 | 0   |

**Figure 5.** Time response and uncertainty of multi-tone signal with Schroeder phase.

The comb results for the two strategies at 3 GHz are shown in Figure 6 and Table 3. The prime-number algorithms reduce the effect of sub-Nyquist spurs in the spectrum (the difference is $2.8^\circ$ at 3 GHz).

**Figure 6.** Nine-tone frequency response at 3 GHz. Error Vector Magnitude (EVM) includes both the magnitude and phase components. The arrows indicate the relevant vertical axis. The mean EVM is $-37$ dBc, excluding the point at 3 GHz.
Take the 1 MHz resolution as an example. There are 100 groups of raw data acquired from the DRTO, and for each group, the record length is 160,000 points of the sampled waveform. Two periods of time-domain wave information are contained in each measured epoch. However, the points in each group are not synchronous with other groups because of the time delay in the DRTO data acquisition of each sampling. In order to minimize the influence of noise signal or other interference in time-domain and acquire the optimal waveform, 99 groups of time-domain input amplitude measured values should be processed by the cross-correlation algorithm to be aligned to the standard group.

The alignment principle is based on matrix convolution theory. The first row of the matrix is chosen as the standard group of waveforms, and the remainder of the matrix rows will be shifted in order according to first row data. The time-domain translation position is decided by the maximum value of convolution. Every time the position is shifted backward or forwards, the data in the shifted row of the matrix is convoluted by the first row until the maximum correlation result is acquired and convolution results are stored in another matrix. The maximum correlation is the position that the corresponding data group needs to shift.

If the maximum value in the convolution matrix is the last group of data, there is no need for this group to shift. After the overall cycle of the entire points, the best position correlated with the first row is itself. When the 100th group points have been shifted to the optimum position according to the convolution results, the aligned data matrix will have the maximum correlation with each other, and the interference of noise signal on the original signal will be cut down. Under this circumstance, we can acquire the aligned data for the further processing of the characteristics of the multi-tone signals.

The complex waveform results for the raw and aligned waveforms are shown in Figure 7 at fundamental, fifth, and eleventh harmonics. After alignment, the results still show scatter with a circular distribution, consistent with noise. Phase-noise would distribute the residual signals in an arc.
In this part, the frequency domain data processing results will be analyzed, and the two strategies will be compared to verify the validity of prime number algorithms. The amplitude response around fundamental, second, and third harmonics for the nine-tone stimulus waveform is shown in Figure 8. In the exact frequency case, the results show higher values for the inter-comb frequencies because all of the comb/ADC intermodulation products will add coherently. In the prime number case, there is a phase shift, which explains the noise reduction feature of the latter strategy. To compare the two strategies, the components points were shown in Figure 8. The results show good agreement between the two strategies (see Figure 9). It is worth noting that the difference of the two strategies at the sub-Nyquist spurs will be higher than other points, which means the prime number algorithm will offset the influence of the sub-Nyquist spurs on the noise floor and at the same time, the other frequency point responses are almost same as the exact frequency strategy. To sum up, the prime number strategy is equivalent to the exact frequency strategy with improvement of the frequency response at the sub-Nyquist spurs.

Figure 8. Amplitude response under nine-tone stimulus for Exact (a,c,e) and Prime number (b,d,f) strategies at 1 GHz, 2 GHz, and 3 GHz, respectively.
Figure 9. Magnitude and phase frequency response differences between the two strategies around 1 GHz (a,b), 2 GHz (c,d), and 3 GHz (e,f) for a nine-tone stimulus.

4. Discussion and Conclusions

The residual spur level at the fundamental is 48 dBC, corresponding to a phase uncertainty of 0.23°, calculated as the arctan of the residual signals (Figure 10). Also, the results in Figure 8 indicate larger residual components in the results falling between the harmonic components for the exact case. The spurs from the time-interleaved ADCs in the DRTO may introduce systematic errors that will degrade the measurement of the instrument impedance match if this interferes with the CW source within the VNA. The objective is to improve the time and frequency performance of dense comb SRD-based phase-reference use with an NVNA. The multi-tone prime-number frequency-selection algorithm was applied to minimize the effect of sub-Nyquist spurs in the result. Because the RF power is concentrated close to the SRD harmonic frequency, the available power is higher at comb harmonics.
This is relevant to amplifiers used for RF communications where the carrier frequency may lie between 1–4 GHz, but the modulation range will only be 5–50 MHz with a comb spacing of 1 MHz or lower. A full comb with this frequency spacing would provide very little power, particularly at high comb frequencies. The results indicate the improvement of time and frequency response for single-tone or multi-tone stimulus harmonic phase reference based on a prime number algorithm, which makes it possible to extend NVNA measurement ability of the modulated signal. Future efforts will be employed to minimize the phase noise and prime number algorithm optimization for nonlinear characterization of multi-tone stimulus. Due to the merits of this prime number algorithm, it is a promising candidate in applications of the NVNA calibration systems.

Author Contributions: Conceptualization, H.Y.; methodology, H.Y., M.L. and F.M.; data measurement, H.Y. and M.L.; writing—original draft preparation, H.Y.; writing—review and editing, D.A.H., L.A. and C.W.; funding acquisition, D.A.H. and C.W. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the General Financial Grant from the China Postdoctoral Science Foundation (2017M611367), Heilongjiang Postdoctoral Fund (LBH-Z17056), Zhejiang Lab (2019MC0AB03, 2019MC0AD01), and by the UK Department for Business, Energy, and Industrial Strategy. He Yu is thankful for the financial support from the China Scholarship Council (CSC, No. 201906120114) during her research.

Conflicts of Interest: The authors declare no competing financial interest.

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