An electricity inventory model for power plant and transmission station system with in-house demand and carbon emission cost

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Abstract. This paper develops a mathematical model for optimizing energy storage size for a single power plant and a single transmission station. The electricity generated by the power plant is transmitted to the transmission and distribution station and then consumed by the end-customers. Besides providing electricity to end customers, a power plant also supplies electricity to their utility system to satisfy the in-house demand. The proposed model considers in-house demand, end-costumer’s demand and carbon emissions. The carbon emission is calculated from production activities. The objective of the model is to determine the supply size of in-house demand, shipment size of end-costumer’s demand, production lot size factor and emergency backup factor to minimize a total cost incurred by a power plant and transmission station. We utilize an inventory theory to develop our model and propose an algorithm to obtain the optimal solution for this problem. A numerical example is provided to illustrate the application of model.

Keywords: inventory model; electricity; power plant; transmission station; in-house demand; carbon emission

1. Introduction
The industrial sector is currently experiencing significant development. Electricity power plant has an important role in providing global electricity demand. The challenges faced by electricity power plant include strategic challenges and operational challenges, including fuel purchase decisions until the problems of determining electricity production lot size. Power plants expect an increase in profits. Power plant can manage the problem listed above using inventory model approach.

Inventory model research that discusses an electrical energy begins with Schneider, et al. [1] who developed a model to determine the optimal lot of energy storage using the inventory model approach. The object studied by Schneider et al. is EESS (Electrical Energy Storage System) in an apartment building. Demand and electricity supply in the system are assumed to be stochastic in nature. Furthermore, Wangsa and Wee [2] proposed an integrated distribution model of electricity in a supply chain consisting of power plant, transmission station, distribution station and end customers. Demand is assumed to be normally distributed and allows blackouts. Blackout is defined as the inability of a transmission station to meet the demand for electricity or in other words there is a power outage. The study aims to minimize the total combined costs incurred by power plants, transmission stations, distribution stations, and multi-consumers. The study of Wangsa and Wee was inspired by the previous scholars who investigated the joint economic-lot sizing problem (JELP). Previously, many scholars has studied JELP under various situations [3],[4],[5],[6],[7]. The main objective of JELP is to search the inventory decisions of the parties in supply chain system such that the joint total cost is minimized.
Most of research related to electricity distribution in a supply chain system didn’t consider environmental aspects, such as carbon emission. Research related to environmental aspects is increasingly becoming a concern for researchers today. Therefore, researchers began to pay attention to the quantity of carbon emissions produced in the inventory system. Research that considers the existence of carbon emissions begins with Hua, et al. [8]. They adopted the emission limitation into the classic economic ordering quantity (EOQ) model assuming that carbon emissions are linear with the demand. An EOQ is a simple model for controlling inventory in a single echelon system where the objective is to determine optimal ordering quantity such that the total cost is minimized. Wahab et al. [9] investigated carbon emissions in imperfect production system. Konur [10] proposed an EOQ model with heterogeneous truck types and carbon cap regulation. Furthermore, some researchers extended the previous model by considering more parties included in the inventory model. Jauhari et al. [11] investigated the impact of carbon emissions in a supply chain system considering an unequal delivery lot size. Wangsa [12] developed an inventory model that considers transportation costs, carbon emissions from the industrial and transportation sectors, and incentive policies and penalties from the government. Recently, Jauhari [13] examined the carbon emissions generated from production and delivery activities in a vendor-buyer model under adjusted production rate.

Based on the description above, it can be seen that there is no research on the electricity energy supply that incorporating the carbon emissions and the details of the production process. In an electricity production, the power plant must satisfy not only the demand from end customers, but also the demand from utility system (in-house demand). In this study, we consider a model for a single power plants and a single transmission station that considers carbon emission cost and fulfillment of two kinds of electricity demand, which are in-house demand and end customer’s demand. This paper is divided into six sections. Sections 1 is the introduction and the literature review of the study. Section 2 describes the overview of the problem. The model development is explained in section 3. Section 4 and section 5 shows a solution procedure and numerical example. Finally, conclusion and future directions are given in section 6.

2. Problem statement

In this research, we attempt to develop a mathematical model for a single power plant and a single transmission station, where the power plant fulfill two kinds of demand, i.e in-house electricity demand and regular demand from end customers. The carbon emission is considered in this model and calculated by considering the amount of carbon generated during production activities. Our objectives is to determine the optimal values of decision variables so the total cost incurred by power plant and transmission station can be minimized.

The notations used to develop the model are listed below:

2.1 Decision Variables

- \( Q \) electricity power consumption of transmission station (kW)
- \( q_u \) electricity power consumption of power plant (kW)
- \( k \) emergency backup factor of transmission station
- \( n \) distribution factor from power plant to transmission station

2.2 Input Parameters for Power Plant

- \( D_{in} \) in-house electrical demand rate in units per unit time (MW/year)
- \( P \) power supply rate in units per unit time (MW/year)
- \( t \) electricity time consumption (hour)
- \( h_p \) electricity holding cost per unit per time ($/kW/year)

2.3 Input Parameters for Transmission

- \( D \) regular electrical demand rate in units per unit time (MW/year)
- \( t \) electricity time consumption (hour)
- \( T_w \) setup and delivery time (year)
- \( \sigma \) standard deviation of demand per unit time (kW/year)
Here is the summary of the problem description:

- We consider a single power plant and a single transmission station.
- To meet the end customer’s demand \( D \), the transmission station orders a number of \( Q_t \) units to power plant. Furthermore, the power plant produces a number of \( nQ_t \) and delivers \( n \) times to transmission station.
- We assume that both the regular demand rate, in-house demand rate, and production rate are constant, \( P > D \) and \( P > D_u \).
- At first, power plant supplies electricity to fulfil the in-house demand and then continue to supply regular demand.

3. Model Development

A total of eight relevant costs is incorporated in this model. Power plants incurs three costs and transmission station incurs four costs. The description of each party cost is given below.
3.1 Power Plant’s Cost Formulation

Consider that the setup cost per setup is $S$ and the frequency of setup is $\frac{(D+d_u)}{(Q+q_u)tn}$. Thus, the setup cost per year charged by power plant is given by

$$C_s = S\left(\frac{(D+d_u)}{(Q+q_u)tn}\right)$$

(1)

Holding costs are costs incurred by the plant in the process of storing electrical energy before being sent to the transmission station. The average of inventory can be determined by calculating the area of the shaded part in Figure 1. Here, we use a basic inventory formulation from Jauhari [11]. The calculation of the average of electricity inventory is as follows:

$$I_{average} = \frac{1}{(Q+q_u)tn}\left[(Q+q_u)tn\left[\frac{(Q+q_u)tn(t-1)+\frac{q_u^2}{P}-\frac{(Q+q_u)tn}{2P}}{(Q+q_u)tn+\frac{q_u^2}{P}}\right]-\frac{q_t}{D}(1+2+\cdots+(n-1))\right]$$

(2)

$$I_{average} = \frac{(D+d_u)}{n(Q+q_u)tn}\left[\frac{-nQ(2Dq_u+(n+1)P(Q+q_u))}{2P}+nt(Q+q_u)\left(\frac{q_u^2}{P}+(n-1)(Q+q_u)t\cdot nt(Q+q_u))\right)}{2P}\right]$$

(3)

The electricity holding cost per unit time can be calculated as follows

$$C_{Hp} = h_p \left(\frac{(D+d_u)}{n(Q+q_u)tn}\left[\frac{-nQ(2Dq_u+(n+1)P(Q+q_u))}{2P}+nt(Q+q_u)\left(\frac{q_u^2}{P}+(n-1)(Q+q_u)t\cdot nt(Q+q_u))\right)}{2P}\right]$$

(4)

Carbon emission cost consists of two kinds of cost, namely indirect and direct costs (See Jauhari [11]). Here is the calculation of indirect emission cost.

$$IE_1 = \vartheta_{11}d_uEL_r$$

(5)

While direct emission cost is calculated by the following formula

$$IE_2 = \vartheta_{12}D$$

(6)

Thus, the total carbon emission cost per unit time is calculated by the equation below

$$C_e = (IE_1 + IE_2)C_{GHG} = C_{GHG}(\vartheta_{11}d_uEL_r + \vartheta_{12}D)$$

(7)

3.2 Transmission Station’s Cost Formulation

The ordering cost is the cost incurred by the transmission station to order electricity to the power plant. The ordering cost can be calculated by the following equation:

$$C_o = \frac{AD}{q_t}$$

(8)

The blackout cost is the cost incurred by the transmission station due to the loss of opportunity to fulfill the electricity demand from end consumers. The calculation of blackout costs is adopted from Wangsa and Wee [2]

$$C_{bs} = \pi \left(\frac{D}{q_t}\right) \sigma \left(\frac{q_t}{P} + T_W(f_\delta(k) - k[1 - F_\delta(k)])\right)$$

(9)

Holding costs are costs incurred by transmission stations in the process of storing electrical energy. The cost of storing electrical energy at the transmission station can be calculated by the equation (10). Please see Wangsa and Wee [2] for a detailed description.
\[ C_h = h_t \left( \frac{Q_t}{z} + k \sigma \sqrt{\frac{Q_t}{P} + T_W} \right) \]  

Where \( k \sigma \sqrt{\frac{Q_t}{P} + T_W} \) is emergency backup supply

Transmission costs are charged when the transmission station transmits electrical energy to end customers. Here is the calculation of transmission costs:

\[ C_f = F \frac{D}{Q_t} \]  

\[ JTC = \frac{AD}{Q_t} + \pi \left( \frac{D}{Q_t} \right) \sigma \sqrt{\frac{Q_t}{P} + T_W} (f_s(k) - k[1 - F_s(k)]) + h_t \left( \frac{Q_t}{z} + k \sigma \sqrt{\frac{Q_t}{P} + T_W} \right) + F \frac{D}{Q_t} + S \left( \frac{(D+d_u)}{(Q+q_ult)} \right) + h_p \left( \frac{(D+d_u)}{n(Q+q_ult)} \right) \]

\[ c_{GHT}(\partial_1 d_u E_L + \partial_2 D) \]  

### Solution Methodology

To find the solution of the proposed problem, we first take the first partial derivatives of \( JTC(Q, q_u, k, n) \) with respect to \( Q, q_u, \) and \( k \), respectively

\[ \frac{\partial JTC(Q,q_u,k,n)}{\partial Q} = \frac{AD}{Q_t^2} - \frac{DF}{Q_t^2} \frac{d}{dt} \frac{S(D+d_u)}{n(Q+q_u)^2} + \frac{h_p(D+d_u)}{(D+d_u)} \frac{n(1+n)Q_t^2 + n(F_1(k)+k(1-F_1(k)))Q_t + (D+d_u)}{2D} \]

\[ + \frac{n(F_1(k)+k(1-F_1(k)))Q_t + (D+d_u)}{2D} \frac{\n(Q+q_u)^2}{(D+d_u)} \frac{n(F_1(k)+k(1-F_1(k)))Q_t + (D+d_u)}{2D} \]

\[ + \frac{n(F_1(k)+k(1-F_1(k)))Q_t + (D+d_u)}{2D} \frac{n(Q+q_u)^2}{(D+d_u)} \frac{n(F_1(k)+k(1-F_1(k)))Q_t + (D+d_u)}{2D} \]

\[ + \frac{n(F_1(k)+k(1-F_1(k)))Q_t + (D+d_u)}{2D} \frac{n(Q+q_u)^2}{(D+d_u)} \frac{n(F_1(k)+k(1-F_1(k)))Q_t + (D+d_u)}{2D} \]

\[ \frac{\partial JTC(Q,q_u,k,n)}{\partial q_u} = \frac{AD+DF+q_u}{Q_t^2} + h_p(D+d_u) \frac{n(2D(n+1)p)(Q+q_u)\frac{(n-1)Q_t + nF_1(k)Q_t}{2D}}{n(Q+q_u)^2} \]

\[ \frac{AD+DF+q_u}{Q_t^2} + h_p(D+d_u) \frac{n(2D(n+1)p)(Q+q_u)\frac{(n-1)Q_t + nF_1(k)Q_t}{2D}}{n(Q+q_u)^2} \]

\[ \frac{\partial JTC(Q,q_u,k,n)}{\partial k} = \frac{AD+DF+w+h_p(D+d_u)Q_t^2}{n(Q+q_u)^2} + \frac{h_p(D+d_u)Q_t^2}{n(Q+q_u)^2} \]

Where, 

\[ Q = \sqrt{\frac{AD+DF+w+h_p(D+d_u)Q_t^2}{n(Q+q_u)^2} + y} \]

\[ \beta_1 + \beta_2 \]
In order to illustrate the above solution procedure, we provide data for parameters in Table 1.

5. Numerical Example

In order to illustrate the above solution procedure, we provide data for parameters in Table 1.
### Table 1. Parameter of Model

| Parameters                                      | Unit         | Values |
|------------------------------------------------|--------------|--------|
| Regular electrical demand rate \((D)\)          | MW/year      | 45     |
| In-house electrical demand rate \((D_u)\)       | MW/year      | 2.25   |
| Power supply rate \((P)\)                       | MW/year      | 650    |
| Average time consumption \((t)\)                | hour         | 24     |
| Setup and transmission time \((T_w)\)           | year         | 0.003  |
| Standard deviation of regular demand \((\sigma)\)| kWh/year     | 750    |
| Blackout cost \((\pi)\)                        | $/kWh        | 150    |
| Transmission station electricity holding cost \((h_t)\) | $/kW/year | 100    |
| Transmission cost \((S)\)                       | $/transmission | 500 |
| Power plant electricity holding cost \((h_p)\)   | $/kWh/year   | 10     |
| Setup cost \((S)\)                              | $/setup      | 5,600  |
| Indirect emission factor \((\theta_{I1})\)      | ton CO2/kWh  | 0.03   |
| Direct emission factor \((\theta_{I2})\)        | ton CO2/kWh  | 0.009  |
| Percentage of energy loss \((EL_r)\)            | %            | 10     |
| Emission carbon cost \((c_{GHG})\)              | $/ton CO2    | 30     |

In this paper, we use MATLAB R2014a software to run the proposed algorithm. Table 2 shows the computation results of the proposed algorithm. The optimal solutions are given as follows: electrical transmit frequency to in-house system and to transmission station \(n\) is 12 times, the amount of electrical supply for regular demand and in-house demand is 883.01 kW and 55.03 kW, respectively. Thus, the total expected cost is \$16,340.289/year.

### Table 2. The Computational Result

| \(n\) | \(Q\) (kW) | \(q_u\) (kW) | \(k\)  | \(JTC\) ($\) |
|-------|------------|--------------|-------|---------------|
| 1     | 2,110.25   | 51.63        | 3.170 | 20,023.452    |
| 2     | 1,483.66   | 51.67        | 3.270 | 18,272.428    |
| 3     | 1,241.14   | 51.31        | 3.319 | 17,520.837    |
| ...   | ...        | ...          | ...   | ...           |
| 10    | 895.07     | 54.53        | 3.407 | 16,351.997    |
| 11    | **883.01** | **55.03**    | **3.410** | **16,340.289** |
| 12    | 873.17     | 55.51        | 3.413 | 16,342.122    |

### Table 3. The Summary of Results

| Decision Variables                                      | Unit     | Values   |
|---------------------------------------------------------|----------|----------|
| Electrical energy consume by transmission station demand each cycle \((E_f = Qt)\) | kWh      | 21,192.24 |
| Electrical energy consume by in-house demand each cycle \((E_u = q_u t)\) | kWh      | 1,320.72  |
| Emergency backup \((B_s = k \sigma \sqrt{\frac{Q}{P} + T_w}})\) | kW       | 482.59   |
| Electrical energy produce by power plant per batch \((E_p = (Q + q_u) tn)\) | kWh      | 247,642.56 |

### 6. Conclusions

In this study, we formulated an electricity distribution model for a single power plant and a single transmission station. This paper contributes the current literature by incorporating fulfillment of two kinds of electricity demand and carbon emission cost. Minimum total cost is found by simultaneously optimizing supply frequency to in-house system and transmission station, amount of power supply to in-house system, amount of power supply to transmission station and emergency backup factor. For future research, it would be interesting to consider more parties in the electricity supply chain. As the number of parties involving in
the supply chain system is increased, the mathematical model would be more complex, hence it needs a comprehensive solution procedure. Investigating different carbon policies, such as carbon penalty, may also interesting to be done in an electricity distribution system.

References

[1] Schneider M, Beil K, Pfaller S, Schaede H, Rinderknecht S and Glock C 2016 Using inventory models for sizing energy storage system: An interdisciplinary approach J. Energy Storage 8 pp 339-348.

[2] Wangsa I D and Wee H M 2017 The Economical Modelling Of A Distribution System For Electricity Supply Chain J. Energy Syst. pp 1-21.

[3] Kurdhi N A, Lestari S M P and Susanti Y 2015 A fuzzy collaborative supply chain inventory model with controllable setup cost and service level constraint for imperfect items Int. J. Applied Manag. Sci. 7 p. 93–122.

[4] Kim M S and Sarkar B 2017 Multi-stage cleaner production process with quality improvement and lead time dependent ordering cost J. Cleaner Prod. 144 pp 572-590.

[5] Nagaraju D, Rao A R and Narayanan S 2016 Centralised and decentralised three echelon inventory model for optimal inventory decisions under price dependent demand Int. J. Log. Syst. Manag. 23 pp 147-170.

[6] Rad R H, Razmi J, Sangari M S and Ebrahimi Z F 2016 A note on the joint economic lot size model with stochastic demand and variable lead time Int. J. Log. Syst. Manag. 23 pp 76-97.

[7] Tahami H, Mirzazadeh A, Arshadi-Khamseh A and Gholakami Qadikolaei A 2016 A periodic review integrated inventory model for buyer’s unidentified protection interval demand distribution Cogent Eng. 3 1206689.

[8] Hua G, Cheng T C E and Wang S 2011 Managing Carbon Footprints in Inventory Management Int. J. Prod. Econ. 132 pp 178-185.

[9] Wahab M I M, Mamun, S M H and Ongkunaruk P 2011 EOQ models for a coordinated two-level international supply chain considering imperfect items and environmental impact Int. J. Prod. Econ. 134 pp 151-158.

[10] Konur D 2014 Carbon constrained integrated inventory control and truckload transportation with heterogeneous freight trucks Int. J. Prod. Econ. 153 pp 268-279.

[11] Jauhari W A 2018 A collaborative inventory model for vendor-buyer system with stochastic demand, defective items and carbon emission cost Int. J. Syst. Manag. 29 pp 241-269

[12] Wangsa I D 2017 Greenhouse Gas Penalty and Incentive Policies for a Joint Economic Lot Size Model with Industrial and Transport Emissions Int. J. Eng. Comp. 8 pp 453-480.

[13] Jauhari W A Pamuji A S and Rosyidi C N 2014 Cooperative inventory model for vendor-buyer system with unequal-sized shipment, defective items and carbon emission cost Int. J. Log. Syst. Manag. 19 pp 163-186