Abstract—For optimal operation of microgrids, energy management is indispensable to reduce the operation cost and the emission of conventional units. The goals can be impeded by several factors including uncertainties of market price, renewable generation, and loads. Real-time energy management system (EMS) can effectively address uncertainties due to the online information of market price, renewable generation, and loads. However, some issues arise in real-time EMS as battery-limited energy levels. In this paper, Lyapunov optimization is used to minimize the operation cost of the microgrid and the emission of conventional units. Therefore, the problem is multi-objective and a Pareto front is derived to compromise between the operation cost and the emission. With a modified IEEE 33-bus distribution system, general algebraic modeling system (GAMS) is utilized for implementing the proposed EMS on two case studies to verify its applicability.

Index Terms—Battery, energy management system (EMS), flexible and delay-tolerant load, Lyapunov optimization, microgrid, optimal power flow, renewable generation.

I. INTRODUCTION

MICROGRIDS are low- or medium-voltage power networks comprising various distributed energy resources [1]. Electrical demand in a grid-connected microgrid is provided by renewable energy sources (RESs), batteries, conventional generation (CG) units, and the main grid. Energy management is an essential part of microgrids and is implemented for various objectives such as cost minimization [2].

Day-ahead energy management methods are primarily introduced for the energy management of microgrids [3]. However, the prediction error of loads, RESs, and market prices introduce difficulties that must be addressed in day-ahead energy management methods. Therefore, the statistical information of data is used to address these uncertainties in stochastic energy management systems (EMSs). Several stochastic methods have been implemented to manage the uncertainties [3]. Scenario-based robust methods [4], [5], information-gap decision theory [6], [7], and risk management methods [8] are some of the approaches to address uncertainties in day-ahead EMSs. In [9], a day-ahead robust EMS is proposed, in which the uncertainties of loads and RESs are addressed using Taguchi’s orthogonal array method, and the uncertainty of market price by a robust optimization method.

Statistical information is used in stochastic EMS methods. Furthermore, scenario-based stochastic day-ahead EMS methods lead to large computation burdens as many scenarios are addressed.

Online information of RES, load, and market price is useful data for designing real-time EMS (RT-EMSs). The RT-EMS [10] is designed based on a model predictive control strategy that utilizes both the current state and the latest predicted information. Several real-time approaches have also been introduced [11]-[14].

RT-EMSs can be executed without any statistical information. In this case, errors arising from imperfect predictions are eliminated. Meanwhile, in some microgrids, electrical demand can be classified into two categories: basic usage and quality usage [15]. Basic usage leads to being provided immediately, whereas quality usage is flexible for the load. Quality usage is provided according to the requested quality of service (QoS) of the customer. In addition, according to demand response programs, loads can be delay-tolerant, which means that loads must be provided before the preferred deadline of the customer. It is challenging to maintain both the battery energy in the operation range and the satisfaction of the desired QoS in RT-EMSs. The proposed RT-EMS based on Lyapunov optimization can effectively address the abovementioned complexities without any statistical information. RT-EMS methods are used to schedule energy resources timely and with low computation burden.

In recent studies, Lyapunov optimization for real-time energy management has been investigated. In [16], two optimization problems for RT-EMSs are proposed and solved by Lyapunov optimization. The objectives include cost minimi-
zation of importing power from external sources and profit maximization with an additional decision variable of price setting to supply delay-tolerant loads by renewable sources and external units. However, the operation constraints of external units are not considered in [16]. In [17], an RT-EMS is formulated by Lyapunov optimization to supply delay-tolerant loads. Battery energy storage systems (BESSs), RESs, and electric utility provide the loads for the research in [17]. Although the minimization of energy supplied by CGs from the electric utility is intended in [17], the operation cost of battery is not considered. Distributed energy management of BESSs, which is implemented by storage aggregators using Lyapunov optimization, is proposed in [18].

BESS management using Lyapunov optimization in finite time horizons has been studied [19], [20]. Two types of degradation costs of BESS are considered. Loads are provided by RESs, BESSs, and the exchanged power with the main grid. Furthermore, delay-intolerant loads are supplied in [19], whereas delay-tolerant loads and a cost function for reducing the delay of load provision are considered [20]. However, some operation constraints are not considered, which are essential for the sound performance of RT-EMSs. A two-stage RT-EMS for a grid-connected microgrid with RESs, CGs, BESSs, and flexible loads has been proposed in [21].

With an objective function of operation cost minimization based on Lyapunov optimization, an RT-EMS in a microgrid considering operation limitations of the system using optimal power flow equations has been proposed in [22], which is not considered in [15]-[21]. The time-coupled constraints of BESS energy limitation and QoS provision for flexible loads are addressed with the Lyapunov optimization framework.

In this paper, the desired goals of the RT-EMS are to provide delay-tolerant loads before the deadline and maintain the load shedding for flexible loads lower than the customer requests. The customer of QoS satisfaction is also among the goals, which supplies delay-tolerant loads before the deadline and maintains BESSs in a desirable energy capacity range. The microgrid is operated in a connected mode and can exchange the power with the main grid. Furthermore, optimal power flow equations are considered, and voltages of buses are maintained in the desired range by additional constraints. The underlying network of the microgrid yields more realistic optimal values, which is similar to practical implementations.

Therefore, in contrast to the studies in [15]-[17] and [20]-[22], the objective of this paper is to supply both the flexible and delay-tolerant loads. Furthermore, the goal of this paper is to minimize both operation cost and emission function, which are not considered in [15]-[22]. Therefore, the RT-EMS is a multi-objective problem. The Pareto front is obtained to address RT-EMS multi-objective problem, which yields the best solutions for decision makers of microgrids. Therefore, decision makers can select desirable solutions according to their priorities. Moreover, [15]-[21] using Lyapunov optimization have addressed the problem without considering the underlying distribution network. It is a complicated problem when considering the underlying distribution network, line losses, and optimal power flow equations in EMS. Further nonlinear equations are added to the problem, which is complex in computation. However, addressing this problem in one time slot as a real-time problem reduces the computation burden. Optimal power flow equations enhance the accuracy results of EMS, and the bus voltage is maintained in the desired range and the line losses are addressed in advance. Therefore, less computation is required in lower layers of EMS.

This paper is an extended version of a conference paper [23]. Moreover, an additional case study, i.e., a modified IEEE 33-bus distribution system, is presented to verify the performance of the method, and the effect of different parameters on the RT-EMS is evaluated. The contributions of this paper are as follows:

1) An RT-EMS in a microgrid is implemented based on Lyapunov optimization for supplying flexible and delay-tolerant loads.

2) It has not been investigated that RT-EMS can minimize the operation costs and emission of CG units simultaneously. In this paper, the abovementioned problem is addressed with a Pareto front.

The rest of this paper is organized as follows. The Lyapunov optimization method is introduced in Section II. In Section III, the microgrid system is modeled and the formulation of RT-EMS is presented. Simulation results are presented in Section IV. Finally, conclusions are provided in Section V.

II. LYAPUNOV OPTIMIZATION METHOD

It is challenging to maintain both the battery energy in the defined range and QoS satisfaction according to the customer request in real-time management [15]. Lyapunov optimization can effectively address these complexities [22]. Compared with the optimal solution, results of the Lyapunov optimization are suboptimal, and it deviates from the global optimal value.

In Lyapunov optimization, the objective is to minimize the long-term time-average value of the expected cost function. Therefore, the optimization problem is modeled as (1) [24]. Formulas (2) and (3) are the equality and the inequality constraints, respectively.

\[
\min \lim_{t \to \infty} \sup (\bar{y}_i(t))
\]

s.t.

\[
\lim_{t \to \infty} \sup (\bar{y}_i(t)) \leq 0 \quad \forall i \in \{1, 2, ..., L\}
\]

\[
\lim_{t \to \infty} \sup (\bar{e}_j(t)) = 0 \quad \forall j \in \{1, 2, ..., J\}
\]

\[
a(t) \in A_{w_0}
\]

where \(A_{w_0}\) is the feasible region for the decision variables; \(\bar{y}_i(t)\) is the time average expectation value of the objective function in the optimization problem; \(a(t)\) is the decision variable; \(\bar{y}_i(t)\) and \(\bar{e}_j(t)\) are the time averages of expected values of the inequality constraint \(y_i(t)\) and the equality constraint \(e_j(t)\), respectively, which are defined as:

\[
\bar{y}_i(t) = \frac{1}{T} \sum_{t=0}^{T} \bar{y}_i(t)
\]

\[
\bar{e}_j(t) = \frac{1}{T} \sum_{t=0}^{T} \bar{e}_j(t)
\]

...
According to the Lyapunov optimization method, virtual queues are defined for time-average expected values of the equality and inequality constraints defined by (2) and (3), respectively [24].

Different queues can be defined [15], [19]. Virtual queues $Z_l(t), \forall l \in \{0, 1, ..., L\}$, and $H_j(t), \forall j \in \{0, 1, ..., J\}$ for (2) and (3) are defined as:

$$Z_l(t+1) = \max \{Z_l(t) + y_l(t), 0\}$$  \hspace{1cm} (7)

$$H_j(t+1) = H_j(t) + e_j(t)$$  \hspace{1cm} (8)

where $Z_l(t)$ and $H_j(t)$ are queues [22]. Then, the constraints in (2) and (3) are satisfied.

Definition: a process $Z_l(t)$ is mean rate stable if

$$\lim_{t \to \infty} \frac{E[|Q(t)|]}{t} = 0$$  \hspace{1cm} (9)

Note that $Z_l(t)$ and $H_j(t)$ are also called backlogs, which indicate the amount of work needed to perform [24]. In Lyapunov optimization, an objective is to push the backlogs toward a lower congestion state. Additionally, another objective is to minimize the penalty [24], which is defined as the operation cost and the emission in this paper. The drift-plus-penalty function is the weighted sum of the costs and backlogs. According to the Lyapunov optimization method, minimizing the upper bound of the drift-plus-penalty function will stabilize virtual queues and minimize the costs. In [24], it is shown that the optimum obtained by the Lyapunov optimization method is suboptimal for the actual stochastic time-average problem and provides a definite bound from the optimum solution.

Remark: virtual queues for satisfying the time-coupled constraints of batteries as well as flexible and delay-tolerant loads are defined in Section III-F. The stability of the defined virtual queues satisfies the expected time-average constraints [15], [17], [22].

III. SYSTEM MODEL

A typical model of microgrid is shown in Fig. 1. The RESs, BESSs, CGs, and main grid supply the electrical demand. A two-way information flow is considered in the microgrid to enable the EMS. The time slot is defined as a discrete time interval.

Emission reduction of the CG unit is another goal of this paper. Minimizing both the operation cost and the emission function are conflicting objectives. To find a compromise between the operation cost and the emission of the CG units, the Pareto front is applied. The weighted sum of the operation cost and emission function is minimized to obtain the Pareto front, which includes Pareto optimal solutions. The solutions on the Pareto front are the optimal solutions, and they are non-dominated according to the Pareto optimality definition. Other points outside the Pareto front are dominated and other solutions with better objective values exist.

Consequently, it is essential to consider the conflicting objectives and obtain the best solution for both of them. Furthermore, the goal of decision maker determines the importance of the objectives. In this paper, the weighted average of the operation and emission costs are considered as (10) using the weights $\lambda_{op}$ and $\lambda_{em}$ to attain the tradeoff between these two objectives. Therefore, the objective is to minimize $C(t)$, which is a combination of the operation cost $C_{op}(t)$ and the emission function $C_{em}(g, t)$:

$$C(t) = \lambda_{op} C_{op}(t) + \lambda_{em} C_{em}(g, t)$$  \hspace{1cm} (10)

$$\lambda_{op} + \lambda_{em} = 1$$  \hspace{1cm} (11)

where $C_{op}(t)$, $C_{em}(g, t)$, $C_{em}(g, t)$, and $C_0(t)$ are the operation costs of CG, battery, load shedding, and the exchanged power with the main grid, respectively.

A. CG Units

A CG unit is modeled by its fuel cost (13), the emission function (14), and operation constraints (15)-(17). The fuel cost is modeled by a quadratic function [22] as:

$$C_{op}(g, t) = a_{op}^2 P_{op}(g, t) + b_{op} P_{op}(g, t) + c_{op}$$  \hspace{1cm} (13)

where $P_{op}(g, t)$ is the active power of CG unit $g$; $\Delta t$ is the time duration of each slot; and $a_{op}$, $b_{op}$, and $c_{op}$ are the cost coefficients of CG fuel cost.

In addition to fuel cost, the emission function is modeled as a quadratic function:

$$C_{em}(g, t) = a_{em}^2 P_{em}(g, t) + b_{em} P_{em}(g, t) + c_{em}$$  \hspace{1cm} (14)

where $a_{em}$, $b_{em}$, and $c_{em}$ are the coefficients of CG emissions.

The output power is limited by the upper and lower bounds in (15) and the ramp rate limitations in (16):

$$P_{op}^{min}(g) \leq P_{op}(g, t) \leq P_{op}^{max}(g)$$  \hspace{1cm} (15)

$$-R_{op}(g) P_{op}^{max}(g) \leq P_{op}(g, t) - P_{op}(g, t-1) \leq R_{op}(g) P_{op}^{max}(g)$$  \hspace{1cm} (16)

where $P_{op}^{max}(g)$ and $P_{op}^{min}(g)$ are the upper and lower bound limitations, respectively; and $R_{op}(g)$ is the ramp limitation of CG unit $g$. Moreover, the output active power and reactive power of CG units are limited, considering the ratings of CGs as:

$$P_{op}^{2}(g, t) + Q_{op}^{2}(g, t) \leq S_{op}^{2}(g, t)$$  \hspace{1cm} (17)

where $Q_{op}(g, t)$ and $S_{op}(g, t)$ are the output reactive power and apparent power of CG unit $g$, respectively.
B. Battery

We consider the operation cost of the battery as (18) to penalize the fast charging and discharging of the battery, which would otherwise degrade the battery as:

$$C_b(b,t) = a_b^c (P_b(b,t)\Delta t) + c_b$$

where $P_b(b,t)$ is the output power; and $a_b$ and $c_b$ are the cost coefficients of battery $b$.

At each time slot, the battery energy is calculated based on the charging and discharging power of the battery as:

$$E(b,t+1) = E(b,t) + \eta_{ch} P_{h, ch}(b,t) \Delta t - \eta_{dv} P_{h, dv}(b,t) \Delta t$$

where $E(b,t)$ is the battery energy state at each time slot $t$; $\eta_{ch}$ and $\eta_{dv}$ are the charging and discharging efficiencies of the battery; and $P_{h, ch}(b,t)$ and $P_{h, dv}(b,t)$ are the charging and discharging power of battery $b$, respectively.

Note that the battery cannot be charged and discharged simultaneously. Therefore, $P_{h, dv}$ and $P_{h, ch}$ are zero when the battery is in the charging and discharging modes, respectively. Moreover, the battery has a limited power and energy capacity. Therefore, constraints (20) and (21) are used to satisfy the output power and battery energy limitations.

$$-P_b^{max}(b) \leq P_b(b,t) \leq P_b^{max}(b)$$

$$E^{min}(b) \leq E(b,t) \leq E^{max}(b)$$

where $P_b^{max}$ is the maximum output power; and $E^{min}(b)$ and $E^{max}(b)$ are the upper and lower bounds of energy level of the battery $b$, respectively.

$$P_b(b,t) = \eta_{ch} P_{h, ch}(b,t) - \eta_{dv} P_{h, dv}(b,t)$$

The limited capacity of battery inverter reduces the range of reactive power drawn from the battery as:

$$P_i^{max}(b,t) + Q_i^{max}(b,t) \leq S_i^{max}(b,t)$$

where $Q_i(b,t)$ and $S_i(b,t)$ are the output reactive power and apparent power of BESS $b$, respectively.

C. Flexible Electrical Loads

In modern microgrids, load flexibility is beneficial in many aspects [20]. Generally, loads can be classified into three categories, as depicted in Fig. 2.

![Classification of electrical load](image)

When the demand is high and costly generation units supply the demand, cost reduction can be facilitated by load flexibility. However, to provide reasonable power quality to customers, the load shedding cost is considered [22] as:

$$C_{fl}(fl,t) = \beta_{fl}(P_{fl}^{max}(fl,t)\Delta t - P_{fl}^{max}(fl,t)\Delta t)$$

where $P_{fl}(fl,t)$ and $P_{fl}^{max}(fl,t)$ are the provided and maximum requested flexible demands, respectively; and $\beta_{fl}$ is the consumer sensitivity to the load shedding.

A large $\beta_{fl}$ causes large costs and indicates that the consumer is more sensitive to load shedding. The long-term time-average expected value of the load shedding percentage is required to be lower than the tolerance control parameter $\alpha_{fl}$ defined in (25), which is set according to the customer request [22].

$$\lim_{t \to + \infty} \frac{1}{T} \sum_{t=0}^{T} E(A(P_{fl}(fl,t))) \leq \alpha_{fl}$$

where $P_{fl}^{max}(fl,t)$ is the minimum allowed load shedding. $\alpha_{fl}$ in (25) controls QoS, and a small $\alpha_{fl}$ implies less load shedding and higher satisfaction of customers. Assume that the load is bounded as:

$$P_{fl}^{min}(fl,t) \leq P_{fl}(fl,t) \leq P_{fl}^{max}(fl,t)$$

$$Q_{fl}^{min}(fl,t) \leq Q_{fl}(fl,t) \leq Q_{fl}^{max}(fl,t)$$

where $Q_{fl}(fl,t)$, $Q_{fl}^{max}(fl,t)$, and $Q_{fl}^{min}(fl,t)$ are the provided reactive power, upper bound limitation, and lower bound limitation of the flexible loads, respectively.

D. Delay-tolerant Electrical Loads

In addition to flexible loads, some delay-tolerant loads are considered. Consumers with delay-tolerant loads can tolerate their loads with delay. The provision of delay-tolerant loads before the maximum delay requires additional time-coupled constraints. In an RT-EMS, the time-coupled constraint of delay-tolerant load satisfaction is defined as a queue [17]. Delay-tolerant loads are stored in a queue, which is defined and updated as [16]:

$$Q_{DT}(DT,t+1) = \max \{Q_{DT}(DT,t) - P_{DT}(DT,t), 0 + P_{DT}^{max}(DT,t)\}$$

where $DT$ is the delay-tolerant load; and $P_{DT}(DT,t)$ and $P_{DT}^{max}(DT,t)$ are the provided and requested $DT$, respectively.

Furthermore, certain percentage of the delay-tolerant load is considered as a basic load. Therefore, constraint (30) is added to force certain percentage of supplied delay-tolerant loads. Furthermore, (31) is utilized to ensure that the load added to the queue defined by (29) is provided.

$$P_{DT}(DT,t) \geq P_{DT}^{min}(DT,t)$$

$$P_{DT}(DT,t) \leq P_{DT}^{max}(DT,t) + Q_{DT}(DT,t)$$

where $P_{DT}^{max}(DT,t)$ is the lower bound limitation of $DT$.

E. Model of Optimal Power Flow

The constraints of power distribution network are modeled [25] in this sub-section, which satisfies the operation constraints of the power grid and enables RT-EMS to maintain the voltage in the desired range.

$$P_{bus}(i) = P_{fl}(fl,t) + P_{DT}(DT,t) + P_{h}(h,t) - P_{g}(g,t) - P_{wt} - P_{pv}$$

where $P_{bus}(i)$, $P_{wt}$, and $P_{pv}$ are the active power, WT generation, and PV generation at bus $i$, respectively. Moreover, we can obtain:

$$Q_{bus}(i) = Q_{fl}(fl,t) + Q_{DT}(DT,t) + Q_{h}(h,t) - Q_{g}(g,t)$$

$$t \in \{0\}$$
where $Q_{bus_i}(t)$ and $Q_{qDT_i}$ are the reactive power and provided power of delay-tolerant loads at bus $i$, respectively.

The power $P_{bus_i}(t)$ is calculated as:

$$P_{bus_i}(t) = P_{lineij}(t) - r_j I_j(t) - \sum_k P_{linejk}(t)$$  \hspace{1cm} (34)

where $P_{lineij}(t)$, $I_j(t)$, and $r_j$ are the active power flow, square of current magnitude in p.u., current magnitude, and resistance of the line from bus $i$ to bus $j$, respectively.

The power $Q_{bus_i}(t)$ is calculated as:

$$Q_{bus_i}(t) = Q_{lineij}(t) - x_j I^2(t) - \sum_k Q_{linejk}(t)$$  \hspace{1cm} (35)

where $Q_{lineij}(t)$ and $x_j$ are the reactive power flow and resistance of the line from bus $i$ to bus $j$, respectively.

Moreover, we can obtain:

$$V_{bus_i}^2(t) = V_{lineij}^2(t)(2 r_{lineij} I_{lineij}(t) + x_{lineij} Q_{lineij}(t)) + (r_{lineij}^2 + x_{lineij}^2) I_{lineij}^2(t)$$  \hspace{1cm} (36)

where $V_{bus_i}(t)$ is the voltage at bus $i$.

$$I_{lineij}(t) = \frac{P_{lineij}(t) + Q_{lineij}(t)}{V_{bus_i}(t)}$$  \hspace{1cm} (37)

Instead of (37), inequality (38) is utilized to avoid non-convexity in the optimization problem [22], [25].

$$I_{lineij}(t) \geq \frac{P_{lineij}(t) + Q_{lineij}(t)}{v_{bus_i}(t)}$$  \hspace{1cm} (38)

where $v_{bus_i} = |V_{bus_i}(t)|$, $I_{lineij}(t) = |I_{lineij}(t)|$, and $I_{lineij}$ is the current from bus $i$ to $j$.

The constraint (38) is a rotated second-order cone constraint. The canonical form of a second-order cone and a rotated second-order cone constraint are defined as (39) and (40).

$$\begin{align*}
4x + b &\leq e^T x + d \\
x^T x &\leq y z \\
y \geq 0 \\
x \in \mathbb{R}^n \\
z \geq 0
\end{align*}$$  \hspace{1cm} (39)

$$\begin{align*}
x^T x &\leq y z \\
y \geq 0 \\
x \in \mathbb{R}^n, y, z \in \mathbb{R}
\end{align*}$$  \hspace{1cm} (40)

The constraint (38) is in the form of a rotated second-order constraint as:

$$P_{lineij}^2(t) + Q_{lineij}^2(t) \leq I_{lineij}(t)v_{bus_i}(t)$$  \hspace{1cm} (41)

Constraint (42) is used to restrain the bus voltages in the allowable range:

$$v_{bus_i}^\text{min} \leq v_{bus_i}(t) \leq v_{bus_i}^\text{max} \quad \forall i \in I \setminus \{0\}$$  \hspace{1cm} (42)

A microgrid exchanges power with the main grid in a grid-connected mode. It can import the power when the power of other units is insufficient to supply the demand. Furthermore, surplus electrical energy in a microgrid can be exported to the main grid to gain profits. The exchanged power with the main grid can be calculated according to (43) and (44).

$$P_{bus0}(t) = \sum_i P_{i0}(t)$$  \hspace{1cm} (43)

$$Q_{bus0}(t) = \sum_i Q_{i0}(t)$$  \hspace{1cm} (44)

where $P_{bus0}(t)$ and $Q_{bus0}(t)$ are the exchanged active and reactive power with the main grid, respectively.

The cost of exchanged power with the main grid can be obtained as:

$$C_q(t) = t(t) P_{bus0}(t) \Delta t$$  \hspace{1cm} (45)

where $t(t)$ is the market price; and $P_{bus0}(t)$ is positive (negative) when power is imported (exported) from (to) the main grid. The main grid can provide limited power to the microgrid owing to the transmission line. The limitations of main grid power sources is modeled as:

$$P_{\text{grid max}} \leq P_{bus}(t) \leq P_{\text{grid min}}$$  \hspace{1cm} (46)

where $P_{\text{grid max}}$ and $P_{\text{grid min}}$ are the maximum and minimum limitations of the power exchanged with the main grid, respectively.

**F. RT-EMS Problem**

In this paper, the RT-EMS method is used to schedule BESSs as well as flexible and delay-tolerant loads, i.e., components that complicate the RT-EMS problem. Constraint (19) shows that the energy level of BESS at a future time slot relies on the charging and discharging power at that time slot. Therefore, BESS is a time-coupled element that forces the time-coupled constraint to the problem. Furthermore, the limited energy capacity of BESS enforces constraint (21) to the problem. The real-time energy management of BESS is challenging as the BESS might be charged or discharged inappropriately [17]-[22]. Other time-coupled constraints in the problem are the QoS range of the flexible loads, i.e., constraint (25), and the provision of delay-tolerant loads. Constraint (25) is a time-coupled one because the overall QoS of the loads depends on the shedding load of the entire day. Furthermore, the delay-tolerant loads are time-coupled constraints in the problem, because unsupplied loads should be provided until the deadline is imposed by the customers. And the unsupplied delay-tolerant loads would cause the dissatisfaction of the customer.

Therefore, an appropriate method to address these complexities is required. In Lyapunov optimization, time-coupled constraints are addressed with queues and are satisfied by stabilizing the queues, which inhibits the full charging or discharging of BESS. Furthermore, load queues are stabilized to satisfy their requirements. In other RT-EMS methods such as the greedy algorithm, the problem is solved at each time slot without considering future data. Consequently, battery energy is used in the first time slot inefficiently, as will be described in Section IV. The Lyapunov optimization procedure is shown in Fig. 3.
In the Lyapunov optimization method, time-coupled constraints are converted to virtual queues, the objective of which is to maintain stable virtual queues and mean rates. And time-coupled constraints can be satisfied by the stable queues [24]. Therefore, time-coupled equality and inequality constraints are firstly defined by virtual queues. Next, the quadratic Lyapunov function is defined. Subsequently, the drift-plus-penalty function is introduced, which is the weighted sum of the drift function and the costs as a penalty. The drift-plus-penalty function is introduced, which is the weight of the remaining function is minimized subject to the operation constraints, excluding time-coupled limitations. 

RES, market price, and load are uncertainties in the microgrid. Minimizing the time-average expected value of the operation costs is the objective of this paper.

Problem I:

$$\begin{align*}
\text{min} & \quad \lim_{t \to \infty} \frac{1}{t} E(C(t)) \\
\text{s.t.} & \quad (11),(15)-(17),(19)-(23),(25)-(36),(38)-(44),(46)
\end{align*}$$

EMSs in microgrids are used for various objectives such as operation cost and emission minimization. The objective of the RT-EMS is to perform energy management at each time slot. However, the optimal solution for long-term management in Problem I, i.e., (47), is not obtained by cost minimization at each time slot, ignoring subsequent and previous states. Furthermore, time-coupled constraints including the limitation of battery energy in (21), provision of QoS for flexible loads in (25), and supply of delay-tolerant loads in (29) must be satisfied. The Lyapunov optimization proposes a suitable algorithm to address time-coupled constraints and achieves long-term management, as shown in [20]-[22]. In this method, time-coupled constraints are satisfied by maintaining stable mean rates of virtual queues.

1) Battery Virtual Queue

Battery energy is the time-coupled constraint as shown in (21), which is related to the previous charging and discharging states of the battery. Instead of retaining the battery energy rate in (21), the average battery charging and discharging is considered to be zero:

$$\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=t}^{t+1} E(P_b(b,\tau)\Delta t) = 0$$  \hspace{1cm} (48)$$

The virtual queue following Section II-A and (8) to satisfy constraint (21) is defined as:

$$B_b(b,t+1)=B_b(b,t)+P_b(b,t)\Delta t$$ \hspace{1cm} (49)$$

Summing virtual queue $B$ over time $\tau \in \{0,1,...,t-1\}$ and considering the expectation and infinite limits, we can obtain:

$$\lim_{t \to \infty} \frac{1}{t} \left( E(B_b(b,t)) - E(B_b(b,0)) \right) = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=t}^{t+1} E(P_b(b,\tau)\Delta t)$$ \hspace{1cm} (50)$$

It is assumed that $B_b(b,0)$ is bounded, thus we can obtain:

$$\lim_{t \to \infty} \frac{E(B_b(b,0))}{t} = 0$$ \hspace{1cm} (51)$$

If queue $H$ is mean rate stable, (51) is held. Subsequently, the right-hand side of (50) will be equal to zero. Consequent-

2) Virtual Queues of Flexible Loads

To satisfy constraint (25), a virtual queue is defined based on (7) as [21]:

$$Z_{dt}(fl,t+1) = \max \{Z_{dt}(fl,t)-\alpha_{dt},0\} + \alpha_{dt}(P_{dt}(fl,t))$$ \hspace{1cm} (52)$$

Applying the expectation, summing over the time $\tau \in \{0,1,...,t-1\}$, and applying the infinite limits, we can obtain:

$$\lim_{t \to \infty} \left( \frac{E(Z_{dt}(fl,t))}{t} - \frac{E(Z_{dt}(fl,0))}{t} \right) \geq -\alpha_{dt} + \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E(\alpha_{dt}(P_{dt}(fl,t)))$$ \hspace{1cm} (53)$$

Remark: if the virtual queue $Z_{dt}$ is mean rate stable, the left-hand side of (53) is zero, and constraint (25) is held [24].

3) Delay-aware Virtual Queue

For the modeling delay in the provision of delay-tolerant demands, the delay-aware virtual queue $H_{dt}(DT,t)$ is defined as:

$$H_{dt}(DT,t+1)=\max \{H_{dt}(DT,t) - P_{dt}(DT,t),0\} + \epsilon_{dt}\sum_{\tau=0}^{\infty} \alpha_{dt}(Q_{dt}(DT,t))$$ \hspace{1cm} (54)$$

where $\epsilon_{dt}$ is used to specify the requested service deadline of the customer. The virtual queue $H_{dt}(DT,t)$ is identified with the same serving rate as $Q_{dt}(DT,t)$. However, it has a different arrival rate, i.e., $1_{Q_{dt}(DT,t)>0}$ which is greater than zero when the load queue $Q_{dt}(DT,t)$ is not empty. Otherwise, the arrival rate is zero. The arrival rate ensures that the virtual queue $H_{dt}(DT,t)$ enlarges when the delay-tolerant load queue $Q_{dt}(DT,t)$ is not empty.

The time-coupled constraints of the battery and load, i.e., (21) and (25), respectively, are omitted in the optimization problem (47), whereas constraints (49) and (52) are added.

Problem II:

$$\begin{align*}
\text{min} & \quad \lim_{t \to \infty} \frac{1}{t} E(C(t)) \\
\text{s.t.} & \quad (11),(15)-(17),(19)-(23),(25)-(36),(38)-(44),(46),(49),(52),(54)
\end{align*}$$

However, constraint (21) is maintained to ensure that the battery energy is in the desired range.

4) Drift-plus-penalty Function

Minimizing the drift-plus-penalty will minimize and stabilize the queue backlogs. Meanwhile, it minimizes the cost function. We firstly define the Lyapunov function for the virtual queues as:

$$L(\Theta(t)) = \frac{1}{2} \left[ \beta_b \sum_b B_b^2(b,t) + \sum_s Z_b^2(s,fl,t) + \sum_{dt} (Q_{dt})^2 + H_{dt}(DT,t) \right]$$ \hspace{1cm} (56)$$

where $\Theta(t) = \{B,b_Z,dt,Q_{dt},H_{dt}\}$ and $\beta_b$ is the weight for the battery queue that yields a difference between the battery and load queues.

Then, we introduce the one-slot conditional Lyapunov drift as:
\[ \Delta(\Theta(t)) = E \left[ L(\Theta(t+1)) - L(\Theta(t)) \bigg| \Theta(t) \right] \] (57)

Minimizing the Lyapunov drift will minimize the queue backlogs which stabilize the queue mean rates and satisfy the described inequality constraints. However, only minimizing the Lyapunov drift would incur more costs. Hence, the operation cost and the emission function are added to the objective function. Furthermore, the goal of RT-EMS is to minimize the operation cost and emission simultaneously. The cost is penalized by a penalty factor \( V \). In this case, both time-coupled constraints and cost minimization are satisfied.

\[ \Delta(\Theta(t)) + VE \left( C(t) \bigg| \Theta(t) \right) \] (58)

The Lyapunov optimization method minimizes the upper bound of (58) instead of directly minimizing (58).

The conditional terms are eliminated because queues are known at each time slot. Squaring both sides of (49), the battery virtual queue is bounded as:

\[ B^2_s(b(t+1)) - B^2_s(b(t)) \leq 2B_s(b(t))P_s(b(t))\Delta t + \max \{(P^s_{\min})^2, (P^s_{\max})^2\}\Delta t^2 \] (59)

Lemma 1: for real positive variables \( a, b \), and \( q \), inequality (60) is held [17]:

\[ \max \{((q-b), 0 + a^2)\} \leq q^2 + a^2 + b^2 + 2q(a-b) \] (60)

Using (60), squaring both sides of (52), and considering \( A(P_{fl}, fl(t)) \leq 1 \), we can obtain:

\[ Z^2_s(b(t+1)) - Z^2_s(b(t)) \leq 2Z_s(b(t))A(P_s(b(t))) - \alpha^2 \] (61)

By squaring both sides of (29) and (54) and considering the inequality (60) in Lemma 1, we can obtain:

\[ Q_D^2(\text{DT, } t) - Q_D^2(\text{DT, } t) \leq 2(P_{\text{min}}^s(\text{DT}))^2 + \] \[ \sum_{fl} P_{\text{fl}}(\text{DT, } t)(P_{\text{min}}^s(\text{DT, } t) - P_{\text{fl}}(\text{DT, } t)) \] (62)

\[ H_D^2(\text{DT, } t) - H_D^2(\text{DT, } t) \leq \epsilon_D^2 + (P_{\text{min}}^s(\text{DT, } t))^2 + \] \[ \sum_{fl} H_{\text{fl}}(\text{DT, } t)(H_{\text{fl}}(\text{DT, } t) - H_{\text{fl}}(\text{DT, } t)) \] (63)

Substituting (59), (61), (62), and (63) into (58) and applying the conditional expectation, we can obtain:

\[ \Delta(\Theta(t)) + VE \left( C(t) \bigg| \Theta(t) \right) \leq B + \beta \sum_b B_s(b(t))E \left( P_s(b(t))\Delta t \bigg| \Theta(t) \right) + \] \[ \sum_{fl} Z_s(b(t))E \left( \max(0, \frac{P_{\text{fl}}(\text{DT, } t)}{P_{\text{fl}}(\text{min})(\text{DT, } t)}) \bigg| \Theta(t) \right) + \sum_{fl} (-H_{\text{fl}}(\text{DT, } t) - H_{\text{fl}}(\text{DT, } t)) \] \[ Q_{\text{fl}}(\text{DT, } t)E \left( P_{\text{fl}}(\text{DT, } t) \bigg| \Theta(t) \right) + VE \left( C(t) \bigg| \Theta(t) \right) \] \[ B = \frac{1}{2} \sum_{fl} \left[ (1 + \alpha^2_f) + (2P_{\text{min}}^s(\text{DT, } t))^2 + P_{\text{fl}}^2(\text{DT, } t) + \epsilon_D^2 \right] + \] \[ \frac{1}{2} \beta \max((P_{\text{min}}^s)^2, (P_{\text{min}}^s)^2) \Delta t^2 \] (64)

The Lyapunov optimization method minimizes the upper bound of (58) instead of directly minimizing (58), which involves calculating the upper bound of one-slot conditional Lyapunov drift function, considering the Lyapunov optimization method framework in [24]. This means that the term related to the cost function \( VE(C(t)\Theta(t)) \) is maintained.

In Lyapunov optimization, instead of directly minimizing the drift-plus-penalty function, the upper bound of (64) is minimized. At each time, the virtual queues \( \Theta(t) \) and system states including the power generated by RES, market price, and decision variables, i.e., \( P_{\text{fl}}(\text{fl}, t), P_{\text{g}}(\text{g}, t), P_{\text{fl}}(\text{fl}, t), P_{\text{fl}}(\text{DT, } t), P_{\text{fl}}(\text{DT, } t) \), are calculated by solving the following optimization problem.

Problem III:

\[ \min \left\{ V(C(t)) + \beta \sum_b B_s(b(t))P_s(b(t))\Delta t - \right. \] \[ \left. \sum_{fl} Z_s(b(t))P_{\text{fl}}(\text{min})(\text{DT, } t) - P_{\text{fl}}(\text{min})(\text{DT, } t) \right\} \]

s.t. (1), (15)–(17), (19)–(23), (27)–(36), (38)–(44), (46), (49), (52), (54)

IV. SIMULATION RESULTS

Two case studies are considered to evaluate the performance of the proposed method. The first case study is a microgrid obtained from [22], whereas the second one is a modified IEEE 33-bus distribution system [26]. The proposed RT-EMS method, i.e., Problem III, is evaluated in the general algebraic modeling system (GAMS) optimization tool. The LINDOGLOBAL nonlinear solver is used to address the nonlinear optimization of Problem III.

A. Case 1

Figure 4 shows the schematics of the microgrid test system used in case 1, in which simulations are performed in 5-min time slots. The electrical load is shown in Fig. 5.

The QoS control parameter is set to \( \alpha_e = 0.5 \). The cost coefficient of load shedding is set to \( \beta = 0.7 \). Wind and solar generations are shown in Fig. 6. The real-time price of exchanged power with the main grid is depicted in Fig. 7. As mentioned previously, no statistical information is required in the real-time energy management using the Lyapunov optimization method. CG and battery characteristics are listed in Tables I and II, respectively.
1) Case 1: Multi-objective RT-EMS

In the proposed RT-EMS, the objective is to minimize both the operation costs and the emission function of the units. As these objectives are conflicting, it is a multi-objective optimization problem. Hence, the Pareto front is obtained by varying the values of \( \lambda_{op} \) and \( \lambda_{em} \), which have already been defined in Section III. The Pareto-optimal solutions are shown on the Pareto-front line in Fig. 8.

The power production of CG unit is illustrated in Fig. 9 for two different values of \( \lambda_{em} \). Figure 9 shows that the increment of \( \lambda_{em} \) reduces the power of CG unit due to the emission of the CG units. Meanwhile, the output power of the BESS increases by the increment of \( \lambda_{em} \) to supply the remaining loads, as depicted in Fig. 10. The power of the RESs is used for charging the BESS and its amount is increased by the increment of \( \lambda_{em} \). The solution time for each time slot in case study I is calculated in 1.5 s, which is an appropriate duration for an RT-EMS with 5-min time slots.

2) Case 1: Effect of \( V \)

To analyze the effect of \( V \), the operation costs for \( V \) are obtained as shown in Fig. 12. As discussed in Section III,
the penalty, i.e., operation cost, is reduced when \( V \) is decreased due to the increment in the weight of operation costs.

Moreover, the battery energy levels are illustrated in Fig. 13 for \( V \). Increasing \( V \) is equivalent to decreasing the weight of the BESS virtual queue in the objective function, which is stabilized to maintain the energy level of BESS in an appropriate range. Therefore, the increment of \( V \) results in the reduction of the energy level of BESS over time at the end of the scheduled time.

![Fig. 12. Comparison of operation costs for \( V \) in case 1.](image)

The QoS values are depicted in Fig. 14 for \( V \). By increasing \( V \), the value of \( \Lambda(P_f(t)) \) decreases due to the increment in the weight of the operation costs and the weight reduction of the QoS virtual queue.

![Fig. 13. Battery energy level for \( V \) in case 1.](image)

3) Case 1: Effect of Battery Coefficient \( \beta_b \)

The effects of \( \beta_b \) on the operation and battery costs and battery energy are illustrated in Figs. 15 and 16, respectively.

The total cost increases with the increase of \( \beta_b \). Furthermore, the high value of \( \beta_b \) increases the battery power fluctuation due to the priority of the queue stability. Consequently, the fluctuations increase the battery and total costs. The battery energy shows that the increment of \( \beta_b \) increases the battery level.

![Fig. 14. QoS of loads for \( V \) in case 1.](image)

4) Case 1: Effect of \( \epsilon_{DT} \)

\( \epsilon_{DT} \) is used to control the provision time of the delay-tolerant loads. In this paper, it is set to be 2. However, to demonstrate the effect of this parameter on the provision time, the maximum deadline for different values of \( \epsilon_{DT} \) is calculated. In Table III, the maximum delay time and the objective function values for load 1 are presented. Table III shows that increasing \( \epsilon_{DT} \) will decrease the delay time. The value of the queues \( H_{DT}(DT,t) \) and \( Q_{DT}(DT,t) \) in the objective function (66) can be increased by decreasing the provision time and increasing \( \epsilon_{DT} \). However, \( \epsilon_{DT} \) should be selected so that \( \epsilon_{DT} \leq \epsilon_{DT}^{\text{max}} \). When \( \epsilon_{DT} \) is greater than \( \epsilon_{DT}^{\text{max}} \), it will cause the provision of the loads at the requested time. Therefore, the flexibility of the delay-tolerant loads for procrastination is no longer advantageous. \( \epsilon_{DT}^{\text{max}} \) is 10 in this case study, which is obtained by the simulation. \( \beta_b \) and \( V \) are set to be 50 and 10, respectively.

| \( \epsilon_{DT} \) | Maximum delay time (s) | Objective function value |
|-----------------|------------------------|--------------------------|
| 0               | 64                     | 4053                     |
| 2               | 59                     | 4068                     |
| 5               | 46                     | 4076                     |

5) Performance Comparison with Greedy Method

The greedy method is introduced in [16], [22] to demonstrate the efficiency of the proposed real-time method. The greedy method is modelled as Problem IV (67), in which the constraint (68) is used to ensure that the energy levels of BESS remain in the range. Constraint (69) is to guarantee that the flexible loads are provided according to the tolerance of the customer. Furthermore, to satisfy the delay-tolerant loads, constraint (70) is utilized to ensure that the delay-
tolerant loads are satisfied before or at the deadline. The constraint is used in [16], which indicates that the loads are satisfied and the remainder is satisfied at the deadline. Assume that the percentage of the delay-tolerant loads are provided at the requested time. Constraint (71) is applied for the supply of the delay-tolerant loads at the requested time.

Problem IV:

\[
\begin{align*}
\min & \quad C(t) \\
\text{s.t} & \quad (1), (15) - (17), (19) - (23), (27) - (34), (36) - (39), (41) \\
& \quad (E^{\text{min}}(b) - E(b, i))/\Delta t \leq P_s(b, t) \leq (E^{\text{max}}(b) - E(b, t))/\Delta t \\
& \quad A(P_{fl}, t, i) \leq \lambda \beta \\
& \quad P_{DT}^{\text{max}}(DT, i) - P_{DT}(DT, i) \leq P_{DT}(DT, t) \quad \forall i = DL \\
& \quad P_{DT}(DT, t) \geq \beta P_{DT}^{\text{max}}(DT, t)
\end{align*}
\]

Considering \( \lambda_{\text{em}} = 0.1 \) and \( \lambda_{\text{op}} = 0.9 \), the operation costs, emissions, and total costs of the simulation results for the proposed RT-EMS method and greedy method are given in Table IV, which indicate that the proposed RT-EMS method enforces less cost to the operator of the microgrid. Whereas the greedy method incurs more costs. Furthermore, the obtained energy level of the BESS is depicted in Fig. 17 for the proposed RT-EMS method and the greedy method. Figure 17 shows that the final energy level obtained by the proposed RT-EMS method reaches half of \( E^{\text{max}} \), and the energy level is not exhausted during the day. Additionally, Fig. 17 depicts that the energy level obtained by the greedy method reaches its minimum allowable level, i.e., \( E^{\text{min}} \). Therefore, BESS cannot be used at other times if it is required for the provision of the loads.

TABLE IV
PERFORMANCE OF PROPOSED RT-EMS METHOD COMPARED WITH GREEDY METHOD

| Performance | Operation cost ($) | Emission ($) | Total cost ($) |
|-------------|--------------------|-------------|---------------|
| Proposed RT-EMS method | 4241 | 418 | 3823 |
| Greedy method | 4446 | 410 | 4036 |

Fig. 17. Proposed RT-EMS method compared with the greedy method.

B. Case 2: Modified IEEE 33-bus Distribution System

The modified IEEE 33-bus distribution system, which is depicted in Fig. 18, is used in case 2 to further investigate the proposed RT-EMS method. The characteristics of this system are adopted from [27]. Note that in case 2, simulations are performed in 10-min time slots. In this paper, a CG unit is added to bus 22 of the modified IEEE 33-bus distribution system. The characteristics of the CG unit and BESSs in case 2 are shown in Tables V and VI, respectively. The load in bus 32 is shown in Fig. 19, and other loads exhibit the same pattern with the scaled amounts [27] proportional to bus 32, which are shown in Table VII. The loads in buses 2-17 are flexible, whereas the remaining loads are delay tolerant. The RES power generation is shown in Fig. 20. The initial BESS energy levels are considered as half of their energy capacity; \( \beta \) and \( \alpha \) are 500 and 0.5, respectively.
1) Case 2: Effect of $V$

To demonstrate the effect of $V$, $\beta_b$ is set to be 100. The effect of $V$ on the operation costs is depicted in Fig. 22. As shown in Fig. 22, the operation cost increases with the increase of $V$. The energy level of one of the BESSs for $V = 0.3$ is shown in Fig. 23.

By decreasing $V$, the energy fluctuation of BESS increases to satisfy the energy level of BESS in constraint (21), as the weight of the operation costs decreases. Additional costs are considered to be reduced. The calculated solution time for each time slot in case 2 is 2.2 s, which is an appropriate duration for RT-EMS with 5-min time slots.

2) Case 2: Impact of $\beta_b$

The effects of $\beta_b$ on the operation and BESS costs are shown in Fig. 24. To demonstrate the effect of $\beta_b$, $V$ is set to be 0.3. It can be concluded that the increment of $\beta_b$ incurs higher costs. The majority of the operation cost is related to the BESS costs, due to the increase in output power fluctuations of BESS. The energy levels of one BESS are shown in Fig. 25. The power fluctuations of BESS increase with $\beta_b$. It also leads to the increment in the value of constraint (21), which is satisfied by the drift-plus-penalty function. Note that constraint (21) is satisfied by stabilizing the defined virtual queue, which is achieved by minimizing the upper bound of drift-plus-penalty function in (66).

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**TABLE VII**

LOAD SCALE WITH RESPECT TO LOAD AT BUS 32

| Bus No. | Load scale with respect to load at bus 32 |
|---------|----------------------------------------|
| 11      | 0.21                                   |
| 5, 6, 9, 10 | 0.29                              |
| 3, 18, 19 | 0.43                                   |
| 2       | 0.48                                   |
| 31      | 0.71                                   |
| 32      | 1.00                                   |
| 20, 21, 22, 23 | 0.43                             |
| 12, 13, 15, 16, 17, 26, 27, 28, 33 | 0.29                     |
| 7, 8, 30 | 0.95                                   |
| 4, 14, 29 | 0.57                               |
| 24, 25  | 2.00                                   |

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**Fig. 21.** Real-time market price.

**Fig. 22.** Operation costs for $V$ in case 2.

**Fig. 23.** Energy level of BESS for $V=0.3$.

**Fig. 24.** Impact of $\beta_b$ on battery cost.

**Fig. 25.** Energy level of BESS for $\beta_b=50$.

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**V. CONCLUSION**

RT-EMS of a microgrid is employed in this paper based on the Lyapunov optimization method without any statistical information. The loads in the microgrid are classified into two categories: flexible and delay-tolerant demands. Lyapunov optimization is adopted to address the time-coupled constraints related to battery energy as well as the QoS of flexible and delay-tolerant loads. For each time-coupled constraint, a virtual queue is defined. In this paper, the objective of RT-EMS is to minimize the operation cost and the emission function simultaneously. Hence, the Pareto front is applied. In addition, the underlying operation limitations are
considered. Finally, the performance of RT-EMS is investigated in two case studies, including a modified IEEE 33-bus distribution system. The results based on different control parameters are derived, which indicate that the proposed online Lyapunov optimization method effectively utilizes the energy sources and BESSs. Furthermore, the desirable energy level in the BESSs is maintained and the QoS of flexible and delay-tolerant loads is provided.

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