Leptogenesis from Split Fermions

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We present a new type of leptogenesis mechanism based on a two-scalar split-fermions framework. At high temperatures the bulk scalar vacuum expectation values (VEVs) vanish and lepton number is strongly violated. Below some temperature, $T_c$, the scalars develop extra dimension dependent VEVs. This transition is assumed to proceed via a first order phase transition. In the broken phase the fermions are localized and lepton number violation is negligible. The lepton-bulk scalar Yukawa couplings contain sizable CP phases which induce lepton production near the interface between the two phases. We provide a qualitative estimation of the resultant baryon asymmetry which agrees with current observation. The neutrino flavor parameters are accounted for by the above model with an additional approximate U(1) symmetry.

Our model also accounts for the neutrino flavor parameters using an additional approximate U(1) lepton symmetry as described below.

The model requires a flat extra dimension, $|x_5| \leq \pi R$, compactified on an orbifold, $S_1/Z_2$, where the bulk scalars are odd with respect to the $Z_2$. The work in \textsuperscript{1} is extended to account for the neutrino flavor parameters which are induced by a variant of the minimal seesaw model \textsuperscript{3, 4}. Our model consists of $L^1$, SU(2) lepton doublets, $\ell^1$, charged leptons, where $i$ is a flavor index, $N_{1,2}$ SM singlet neutrinos (in principle an additional singlet neutrino can be added provided that its couplings to the SM fields are sufficiently suppressed) and, $\Phi_{1,2}$, the SM singlets bulk scalars. The relevant part of the Lagrangian is given by:

$$
\mathcal{L} = \mathcal{L}_Y + \mathcal{L}_L + V_{\Phi,T} ,
$$

where $\mathcal{L}_Y$ contains the interactions between the leptons and the bulk scalars. $\mathcal{L}_L$ contains the lepton violating interactions and the Yukawa couplings to the SM Higgs. $V_{\Phi,T}$ is the bulk scalar temperature dependent potential which drives the first order PT.

The actual mechanism of creating the asymmetry is similar to the SM electroweak baryogenesis case. At $T_c \sim R^{-1}$ a bubble is formed. Almost immediately it fills the whole compact extra dimension and expands in the 4D direction. Incoming leptons from the unbroken phase hit the bubble wall. Reflection asymmetries and lepton violating interactions induce an excess of leptons which is then overtaken by the bubble.

We first focus on $\mathcal{L}_Y$ and derive a qualitative estimate for the excess of leptons near the bubble wall. This is first done under the assumption that the rate for lepton violation, $\Gamma_L$, is infinite (zero) in the unbroken (broken) phase. Then we consider $\mathcal{L}_L$ which accounts for the observed lepton flavor parameters. We calculate $\Gamma_L^{-1}$ and find that it is much longer than other dynamical time scales relevant to our model. This induces a further suppression in the resultant asymmetry. We then derive a qualitative estimation of $n_B/s$ which agrees with the observed value.
We also briefly comment on the requirement from $V_{\Phi,T}$ as to yield a first order PT.

**Interaction with the bubble.** The interaction between fermions of each representation of the SM, $\Psi = L, \ell, N$, and the bubble wall are given by

\[
\mathcal{L}_Y = \frac{1}{\sqrt{M}} \left( f_{ij1}^* \Phi_1 + f_{ij2}^* \Phi_2 \right) \bar{\Psi}^i \Psi^j, \tag{2}
\]

where $f_{ij1}^*$ are representation dependent hermitian matrices and $M$ is the fundamental scale of the above 5D effective theory. By a unitary rotation $f_{ij1}^*$ can be brought to a real diagonal form which preserves a $U(1)^3$ symmetry. A-priori, $f_{ij2}^*$ contains three phases. Two phases can be eliminated using part of the above $U(1)^3$ symmetry [the whole expression in (2) is invariant under a residual $U(1)$ flavor symmetry] and thus a single physical CPV phase, $\varphi$, is present. Generically, all the elements of $f_{ij2}^*$ are of order one. Consequently, the asymmetry between a process and its CP conjugate induced by $f_{ij1}^*$ is expected to be sizable. This is in clear contrast with the SM baryogenesis case, where CP asymmetries induced by the Higgs bubble are tiny due to the smallness of the Jarlskog determinant [7].

For our mechanism to work, $T_c$ should be of the order of $R^{-1}$ as follows: At low temperatures, $T \ll R^{-1}$, the theory is essentially 4D, in particular the Yukawa interactions with the bulk scalars are absent. Thus, $T_c \ll R^{-1}$ is unacceptable. On the other hand, at $T \gg R^{-1}$, thermal processes will induce lepton violating interaction even in the broken phase which is unacceptable. Thus we must have $T_c \sim R^{-1}$.\(^1\) This implies that, on-shell, high KK modes are irrelevant since their statistical weights are exponentially suppressed. Thus, for our purpose, it is enough to consider only the first few KK modes. In practice, we shall consider only the zero and the first KK modes.

Since we are only interested in having a qualitative estimation of the BAU, we shall not provide a complete finite temperature analysis of the dynamics near the bubble wall. We shall apply the naive thin wall approximation. This approximation\(^1\) can overestimate the resultant asymmetry. One should therefore view our final result as an upper bound. Note, however, that when using results from the 4D case\(^2\) for the values of the mean free path, $l_{T1}^{-1} \sim g_2^2 T_c$, the velocity of the wall, $v_w \sim 0.1$ and the wall width, $\delta_w \sim T_c^{-1}$, then the required conditions for the approximation are met.

As explained above, in this part we consider the case where there is no suppression due to inefficient lepton violating interaction. The resultant excess of lepton number, $\tilde{n}_L$, is given by\(^7\)\(^12\):

\[
\tilde{n}_L = - \sum_{n,m=0,1,2} \int \frac{dp_3}{2\pi} n^u(E(n,p_3)) \Delta(E), \tag{3}
\]

where $n^u(E(n,p_3))$ stands for Fermi-Dirac distribution function for an $n$ KK state with 4D momentum $p_3$ in the unbroken phase in the bubble wall rest frame. \(\Delta(E) = \text{Tr} \left( R^i_{n \bar{m}} R_{n m} - R^i_{\bar{m} \bar{n}} R_{m \bar{n}} \right)\), where $R_{nm}$ stands for the reflection coefficients which are matrices in flavor space. For example, $R^i_{nm}$ is related to an incoming $n = 0, 1, 2$ KK lepton state ($n \pm$ are the two helicities) of a flavor $i$ which is reflected into an $m = 0, 1, 2$ KK state (conserving angular momentum) of flavor $j$. $R_{nm}$ corresponds to the CP-conjugate processes. Using CPT the expression for $\tilde{n}_L$\(^13\) is further simplified\(^7\)\(^13\)\(^2\):

\[
\tilde{n}_L \approx - \int \frac{dp_3}{2\pi} \left\{ n^u(E(0,p_3)) - n^u(E(1^+, p_3)) \right\} \Delta(E),
\]

\[
\Delta(E) \approx \text{Tr} \left( R^u_{01,01^+} - R^u_{1^+,01} \right). \tag{4}
\]

To first order $v_w$ we get\(^13\):

\[
\tilde{n}_L = - \frac{v_w}{T} \int \frac{dE}{2\pi} \left( E - \sqrt{E^2 - R^2} \right) \times \frac{n_0(E) [1 - n_0(E)]}{E} \Delta(E), \tag{5}
\]

where $n_0(E)$ is a Fermi-Dirac distribution function in an unboosted frame and the integral is taken over the region which yields the dominant contribution. From\(^13\) we learn that our problem is reduced into finding $\Delta(E)$. In principle, the calculation of $\Delta(E)$ within the thin wall approximation is straightforward. One should solve the Dirac eq. for the fermions in the broken and unbroken phase regions. Then by matching the WFs at the bubble wall one can extract the reflection matrices. In practice the required analysis, even numerically, is very hard. Thus, we shall apply the following approximation: We compute the reflection coefficients in a single generation framework. We expect that the value of the reflection coefficients will not change significantly while promoting the model into a three generation one. The essential difference is that in the latter case unsuppressed CPV sources are present. Consequently, $R^{u}_{01,01^+}$ will pick up an order one phase relative to its CP conjugate. Qualitatively, we therefore expect the following:

\[
\Delta(E) \sim |R_{01^+}|^2 \sin \varphi \sim |R_{01^+}|^2, \tag{6}
\]

where $R_{01^+}$ is calculated in a single flavor model. Even in this case the effective bulk scalar background\(^2\) has a

\(^1\) Constraints on the reheating temperature in this case are not very stringent and may be evaded\(^1\)\(^4\).

\(^2\) We do not apply the quasi-particle treatment\(^6\)\(^1\)\(^4\). Unlike the SM case, the asymmetry is created even in the absence of these thermal effects.
complicated structure and the Dirac eq. in the bubble cannot be solved analytically. As a rough approximation for the effective VEV we thus take it to be of a step function shape, \( \langle \Phi \rangle = \frac{1}{\sqrt{M}} \left[ \theta(x_3) - \theta(-x_3) \right] \).

In this case the fermion WFs can be found analytically in the whole region. They are characterized by a spatial 4D momentum, \( k_3 \) and a KK index \( n \). As angular momentum perpendicular to the wall is preserved, it is enough to consider only a single (negative) helicity state. Here we give only the \( x_3 \) dependent part of the WFs. In the unbroken phase the WF is of the form:

\[
\Psi_n^\pm (k_3, n) = \left( N_1^\pm \cos \left( \frac{n \pi}{2} \right), N_2^\pm \sin \left( \frac{n \pi}{2} \right) \right), \tag{7}
\]

where \( E^2 = \frac{n^2}{2} + k_3^2 \) and \( N_1^\pm \) are normalization constants. Due to the orbifold transformation assignment the upper [lower] component in (7) which corresponds to the upper [lower] component in (7) which corresponds to a left [right] handed field is described by an even [odd] function. Including also virtual (off shell) modes for the reflected WF, the generic solution is a linear combination of the above functions. In the broken phase, the zero mode WF is given by:

\[
\Psi_{01}^b (k_3, 0) = \left( N_0^+ e^{f \sqrt{2} \xi x_3}, 0 \right), \tag{8}
\]

and the other KK states are described by:

\[
\Psi_n^b (k_3, n) = \left( N_n^+ \cos \left( \frac{n \pi}{2} \right) + \frac{L}{M} \sin \left| \frac{n \pi}{2} \right|, N_n^- \sin \left( \frac{n \pi}{2} \right) \right), \tag{9}
\]

where \( E^2 = f^2 v_\xi^2 + \frac{\eta^2}{2} + k_3^2 \). In split fermion models one typically finds \( f v_\xi \gtrsim 10 \). In the broken phase, therefore, only the zero mode can be produced on shell.

\( R_{01}^+ \) is found by matching the wave function of an incoming zero mode with momentum \( k_3 \) onto generic WFs of reflected and transmitted fermions. To match the above WFs, inclusion of the off-shell modes is required and the actual matching was done numerically. To test our calculation we verified that we get a zero reflection coefficient for \( ER < 1 \). In addition we checked that unitarity and CPT are satisfied by considering also the case of an incoming first KK mode. For this, we also computed \( R_{1-1} \) and the corresponding transmission coefficients (a more detailed analysis, including the generalization to three generations, will be presented elsewhere).

The results of our analysis are shown in fig. where we plot \( |R_{01}^+|^2 \) for energies in the relevant range, \( 1 < ER < 2 \). The figure shows that a sizable reflection is found over most of the energy range. Using the result for \( \Delta (E) \) we numerically performed the integral and find that \( \tilde{n}_L \sim 10^{-2} \times v_w \). \( \tag{10} \)

**Lepton flavor sector.** To naturally realize the minimal seesaw scenario we imposed an additional U(1) lepton symmetry. It is assumed to be broken by a small parameter, \( \varepsilon_L \), which controls the amount of lepton violation at all temperatures. Consider the following charge assignment for the leptons under the U(1)_L, \( Q(L) = Q(\ell) = 3, \quad Q(N) = 1 \). The relevant part of the 5D Lagrangian is given by

\[
\mathcal{L}_L = \frac{\varepsilon^6_L}{M} LLHH + \frac{\varepsilon^2_L}{\sqrt{M}} N^1 LH + \varepsilon^2_L MNN, \tag{11}
\]

where we suppressed the flavor indices and dimensionless coefficients.

The model yields a hierarchical pattern for the neutrino masses (see e.g. [16]):

\[
m_{atm} \sim 5 \times 10^{-2} \text{ eV}^2, \quad m_{sol} \sim 8 \times 10^{-3} \text{ eV}^2, \tag{12}
\]

and the third neutrino mass is smaller. We require that the masses induced by the bare term \( (L^2 H^2) \) are small, say below \( m_{sol}/5 \). This implies an upper bound on \( \varepsilon_L \):

\[
\varepsilon_L \sim 0.055 \left( \frac{R^{-1}}{5000 \text{ TeV}} \right)^{\frac{3}{2}}, \tag{13}
\]

with \( MR \sim 100 \) and \( R^{-1} \gtrsim 5000 \text{ TeV} \).

Given (11) and (13) we can compute the strength of lepton flavor violation at \( T_c \) in the unbroken phase. An example for lepton violating process is \( NL \rightarrow L \ell \) which is mediated by a Higgs, \( t \)-channel, exchange between \( L \) and \( N \). The amplitude for this a process is \( A_{LL} \propto \varepsilon_L^2 \left( \varepsilon_L^2 M/T_c \right)^2 \), where we used the fact that at \( T_c \) the heavy neutrinos are dynamical, \( \varepsilon_L^2 M \sim M_N \gtrsim \varepsilon_L^2 M \gtrsim T_c \). The typical inverse time scale for lepton coherent production near the wall is \( \tau_w^{-1} \approx v_w/l_w \sim v_w/T_c \). Thus, the lepton violating rate is much smaller than \( \tau_w^{-1} \):

\[
\Gamma_L \sim |A_{LL}|^2 T_c \tau_w \sim \frac{\varepsilon_L^4}{v_w^4} \left( \frac{M}{T_c} \right)^4 \sim 10^{-8}. \tag{14}
\]

In this particular model, therefore, lepton violating interactions are inefficient in converting the excess of lepton into anti-lepton, near the wall, before it is overtaken by
the bubble. Thus, the result in (10) overestimated the resultant excess of leptons.

Let us briefly describe how the neutrino flavor parameters are accounted for in our model. We denote the required suppression of the 4D neutrino Dirac masses due to the small overlaps, as \( \epsilon_{\text{sol, atm}} \). \( \epsilon_{\text{sol}} \) and \( \epsilon_{\text{atm}} \) characterize the overlap between \( N_1 \) (or 2) and \( L_{1,2} \) while the other overlaps are smaller. Using the experimental data we find: \( \epsilon_{\text{atm}} , \epsilon_{\text{sol}} \sim (0.02, 0.007) \). This pattern is yielded by the following bulk scalar-lepton Yukawas, using the notation of [1] (assuming for simplicity that the Yukawa are flavor diagonal),

\[
J^{L_{1,2}, 3} = 12, -28, 15, \quad X^{L_{1,2}, 3} = -1, 0.7, 1, \\
J^{N_{1,2}} = \pm 50, \quad X^{N_{1,2}} = 0.55, 0.55, \quad a_3 = 1. \quad (15)
\]

**Final results.** For our crude estimation of \( n_L \) we use the relation \( n_L \sim \Delta n^2 \Gamma_{L \tau_w} \) where \( \Delta n^2 = 2 \) is the number of leptons produced by a single lepton violating process. Using (10) we find

\[
\frac{n_L}{s} \sim \Delta n^2 \Gamma_{L \tau_w} \frac{n_L}{s} \sim 10^{-10},
\]

where we used the relation \( s \approx 2\pi^2 g^4 / 45 \approx 45 \). This is in agreement with the observed value, \( n_B/s = O \left( 10^{-10} \right) \).

Let us also briefly list the requirements from \( V_{\Phi, T} \) as to have a first order PT (in order to check whether the requirement below are realistic a finite temperature analysis is needed which is left for a future work). We assume that the dominant part of \( V_{\Phi, T} \) is given by:

\[
V_{\Phi, T} = \frac{a^2}{4M^4} \phi^6 + \frac{b}{M} \phi^4 + cM^2 \phi^2, \quad (17)
\]

where \( a, b, c \) are real and \( c \) is strongly temperature dependent. A first order PT implies that at \( T_c \), the true and the false vacua are separated by a barrier. This requires that \( b < 0 \) and that \( c \) is a monotonic increasing function of \( T \) with \( c(T_c) \sim \frac{b^2}{2a} \).

**Conclusions.** In this work we presented a new class of baryogenesis models within the split fermions framework. We focused on the lepton sector and demonstrated how our mechanism works. Note that the final value we got for the BAU, \( O \left( 10^{-10} \right) \), is much smaller than our naive expectation \( O \left( 10^{-2} \right) \). The extra suppression is model dependent and comes from our particular realization of the minimal see-saw model using the approximate U(1) lepton symmetry, which is not inherent to the above framework. Furthermore, we believe that the above mechanism is quite general and can be also applied to other split fermion models related to the quark sector [12].

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