CP asymmetries in dileptonic decays of B-meson in MSSM

S. Rai Choudhury and Naveen Gaur

Department of Physics & Astrophysics,
University of Delhi,
Delhi - 110 007, India.
E-mail: src@ducos.ernet.in, naveen@physics.du.ac.in

Abstract: Scalar interactions (in effective Hamiltonian) can give significant variation of various experimental observables, as compared to their respective Standard Model values, in dileptonic decays of B-meson. Also the quark level transition \(b \rightarrow d\ell^+\ell^-\) can be useful to test CP violation. Here we will do comparative study of CP violation in two independent processes, which have the same quark level transition \((b \rightarrow d\ell^+\ell^-)\), \(B \rightarrow X_d\ell^+\ell^-\) (the inclusive decay mode) and the exclusive channel \(B_d \rightarrow \ell^+\ell^-\gamma\) (radiative dileptonic decay mode). We will mainly focus on the comparative study of scalar interactions on the CP asymmetries in these two different channels.

Keywords: Supersymmetric Standard Model, B-physics, Higgs Physics

URL: http://physics.du.ac.in/~naveen
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1. Introduction

The Flavor changing neutral current (FCNC) $b \to s(d)$ transition can be a very useful probe of the weak interaction sector of SM because this transition is forbidden in the tree approximation and goes through a loop which is second order in weak interaction. In SM this transition occurs through a intermediate $t, c$ or $u$ quark. Among the processes having quark level $b \to s(d)$ transition, the ones having leptons in final state are more interesting because they are relative clean. The pure leptonic and semi-leptonic decays can also be useful because they, over and above the branching ratio, can give us many other experimentally measurable observable associated with pair of final state leptons like lepton pair forward backward asymmetry (FB asymmetry) and the three polarization asymmetries $^1$. These decays thus can be very useful in testing the structure of effective Hamiltonian and can also be used to test new physics beyond SM. One can also look at CP violation in these transitions. If we

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$^1$the three polarization asymmetries are longitudinal, transverse and normal, in pure leptonic mode, like $B_s \to \ell^+\ell^-$ there can only be one polarization asymmetry which is longitudinal because the kinematics of this mode allows only one independent momenta
look at $b \to s\ell^+\ell^-$, the transitions involving intermediate $t$, $c$ and $u$ quarks enter with CKM factors $V_{tb}V_{ts}^*$, $V_{cb}V_{cs}^*$ and $V_{ub}V_{us}^*$ respectively. Using the Wolfenstein’s parameterization of CKM matrix [1] we can see that: $V_{tb}V_{ts}^* \sim \lambda^2$, $V_{cb}V_{cs}^* \sim \lambda^2$ and $V_{ub}V_{us}^* \sim \lambda^4$ where $\lambda = \sin \theta_C \approx 0.22$. So we can see that $V_{ub}V_{us}^*$ can be neglected as compared to the other two. The unitarity relation for CKM factors hence reduces to $V_{tb}V_{ts}^* + V_{cb}V_{cs}^* \approx 0$. So effectively we can remove one in favor of other and hence we are left with only one overall CKM factor. Its phase will not show up in the transition rate and hence CP-violation would not show up.

But the situation for $b \to d$ transition is different. Here the contributions of intermediate $t$, $c$ and $u$ quarks are respectively $V_{tb}V_{td}^*$, $V_{cb}V_{cd}^*$ and $V_{ub}V_{ud}^*$ and all of these are of order $\lambda^3$ and in general all three of them can have different phase and hence the $b \to d\ell^+\ell^-$ transition rate would be sensitive to CP-violating phases. This was studied in the case of inclusive [2, 4] channel and exclusive channel [3] within SM. Lately the scalar (and pseudoscalar) interactions (in effective Hamiltonian) have attracted lot of interest in various purely leptonic [5, 7] and semi-leptonic decays like $B \to \pi\ell^+\ell^-$, $B \to \rho\ell^+\ell^-$ [6, 11, 12, 20] $B \to K(K^*)\ell^+\ell^-$ [18], $B_d \to \ell^+\ell^-$ [9, 13]. The effects of the scalars on CP asymmetries in the exclusive decays $B \to \pi\ell^+\ell^-$ and $B \to \rho\ell^+\ell^-$ was discussed in our earlier work [10]. But as emphasized in some works [9, 13] the radiative dileptonic decay mode $B_s \to \ell^+\ell^-\gamma$ is also very sensitive to the scalar interactions. This present work is a comparative study of the CP asymmetries in inclusive dileptonic decay $B \to X_d\ell^+\ell^-$ and exclusive decay $B_d \to \ell^+\ell^-\gamma$. We will mainly focus on the effects of the scalar interactions on the CP asymmetries of these two channels.

The simplest and one of the most favourite extension of SM has been Minimal Supersymmetric extention of the SM (MSSM). In MSSM there are five scalars (Higgs) as compared to one in SM. The importance of these scalars also called as Neutral Higgs Bosons (NHBs) have been extensively discussed [5–7, 9–14] in literature and we will use MSSM for our comparative study of CP asymmetries. As known that in MSSM we have to include some additional operators, over and above the usual SM operators in effective Hamiltonian. These operators arise in MSSM because of the NHBs and the coefficients (Wilson coefficients) for with these operators are proportional to $m_d m_b \tan^2 \beta$, for large $\tan \beta$ which means that the $\tau$ lepton processes would be affected most with a much lesser effect for the ones with $\mu$. Here in our work we will be going to take the final state leptons to be $\tau$. Although in SM the Branching ratios of both $B \to X_d\tau^+\tau^-$ ($\sim 10^{-8}$) and $B_d \to \tau^+\tau^-\gamma$ ($\sim 10^{-10}$) is very low but it still might be possible to observe it in future e.g. in LHC-B where more than $10^{11}$, $B_d$ mesons are expected to be produced. Also in MSSM these branching ratios can be enhanced by an order in certain allowed region of MSSM parameter space $^2$.

$^2$in fact for radiative dileptonic decay there can be a enhancement by two orders as we have
The paper is organized as follows: In section \[2\] we will discuss the effective Hamiltonian for \( b \rightarrow d \ell^+ \ell^- \). In section \[3\] we will discuss CP violation in the inclusive decay mode \( B \rightarrow X_d \ell^+ \ell^- \). In section \[4\] we will discuss the exclusive dileptonic decay mode \( B_d \rightarrow \ell^+ \ell^- \gamma \) and finally in section \[5\] we will discuss our results and conclusions.

2. The Effective Hamiltonian

The effective Hamiltonian for the decay \( b \rightarrow d \ell^+ \ell^- \) can be written as \[6\]:

\[
H_{\mathrm{eff}} = \frac{4G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{td}^* \left[ \sum_{i=1}^{10} C_i O_i + \sum_{i=1}^{10} C_Q Q_i - \lambda_u \{ C_1 [O^u_1 - O_1] + C_2 [O^u_2 - O_2] \} \right] \tag{2.1}
\]

where we have used the unitarity of the CKM matrix \( V_{tb} V_{td}^* + V_{ub} V_{ud}^* \approx -V_{cb} V_{cd}^* \), and \( \lambda_u = V_{ub} V_{ud}^*/V_{tb} V_{td}^* \). Here \( O_1 \) and \( O_2 \) are the current current operators, \( O_3, \ldots, O_6 \) are called QCD penguin operators and \( O_9 \) and \( O_{10} \) are semileptonic electroweak penguin operators \[8\] \[^3\]. The new operators \( Q_i (i = 1, \ldots, 10) \) arises due to NHB exchange diagrams \[5, 6\]. In this work we will use the Wolfenstein parameterisation \[1\] of CKM matrix with four real parameters \( \lambda, A, \rho \) and \( \eta \) where \( \eta \) is the measure of CP violation. In terms of these parameters we can write \( \lambda_u \) as:

\[
\lambda_u = \frac{\rho(1-\rho) - \eta^2}{(1-\rho)^2 + \eta^2} - i \frac{\eta}{(1-\rho)^2 + \eta^2} + O(\lambda^2) \tag{2.2}
\]

For inclusive decay we will also make use of:

\[
\frac{|V_{tb} V_{td}^*|^2}{|V_{cb}|^2} = \lambda^2 [(1-\rho)^2 + \eta^2] + O(\lambda^4) \tag{2.3}
\]

The additional operators \( O^u_{1,2} \) are:

\[
O^u_1 = (\bar{d}_\alpha \gamma_\mu P_L u_\beta) (\bar{u}_\beta \gamma_\mu P_R b_\alpha)
\]
\[
O^u_2 = (\bar{d}_\alpha \gamma_\mu P_L u_\alpha) (\bar{u}_\beta \gamma_\mu P_R b_\beta)
\tag{2.4}
\]

The resulting QCD corrected matrix element relevant to us can be written as:

\[
\mathcal{M} = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{td} \left\{ -2 C^{eff}_7 \frac{m_b}{q^2} (\bar{d} \sigma_{\mu \nu} q^\nu P_R b) (\bar{\ell} \gamma_\mu \ell) + C^{eff}_9 (\bar{d} \gamma_\mu P_L b) (\bar{\ell} \gamma_\mu \ell) + C_{10} (\bar{d} \gamma_\mu P_R b) (\bar{\ell} \gamma_\mu \ell) + C_{Q_1} (\bar{d} P_R b) (\bar{\ell} \ell) + C_{Q_2} (\bar{d} P_L b) (\bar{\ell} \gamma_5 \ell) \right\} \tag{2.5}
\]

where \( q \) is the momentum transfer and \( P_{L,R} = (1 \mp \gamma_5)/2 \) and where we have neglected mass of d quark. The Wilson coefficients \( C^{eff}_7 \) and \( C_{10} \) are given in many shown earlier for \( B_s \rightarrow \ell^+ \ell^- \gamma \) \[13\].

[^3]: the only difference being that s quark is replaced by d quark
works [6, 16, 21] and the other Wilsons $C_{Q_1}$ and $C_{Q_2}$ are given in [6, 7]. The definition of $C_{eff}^9$ is [2, 4]:

$$C_{eff}^9 = \xi_1 + \lambda_u \xi_2$$

(2.6)

with

$$\xi_1 = C_9 + g(\hat{m}_c, \hat{s})(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) - \frac{1}{2} g(\hat{m}_d, \hat{s})(C_3 + 3C_4) - \frac{1}{2} g(\hat{m}_b, \hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) + \frac{2}{9} (3C_3 + C_4 + 3C_5 + C_6)$$

(2.7)

$$\xi_2 = [g(\hat{m}_c, \hat{s}) - g(\hat{m}_c, \hat{s})](3C_1 + C_2)$$

(2.8)

where

$$g(\hat{m}_i, \hat{s}) = -\frac{8}{9} \ln(\hat{m}_i) + \frac{8}{27} + \frac{4}{9} y_i - \frac{2}{9} (2 + y_i) \sqrt{1 - y_i}$$

$$\times \left\{ \Theta(1 - y_i) \left( \ln \left( \frac{1 + \sqrt{1 - y_i}}{1 - \sqrt{1 - y_i}} \right) - i\pi \right) + \Theta(1 - y_i) \frac{1}{2} \arctan \frac{1}{\sqrt{1 - y_i}} \right\}$$

with $y_i \equiv 4\hat{m}_i^2/\hat{s}$.

We will incorporate the long-distance contributions due to charm quark resonances, i.e. $c\bar{c}$ intermediate states, by using the substitution [3, 6, 12, 19, 22]:

$$g(\hat{m}_c, \hat{s}) \rightarrow g(\hat{m}_c, \hat{s}) - \frac{3\pi}{\alpha^2} \sum_{V = J/\psi, \psi'} M_V Br(V \rightarrow l^+l^-) \Gamma_V^{total} \frac{M_V}{(s - M_V^2) + i\Gamma_V^{total} M_V}$$

(2.10)

we are now equipped with the effective Hamiltonian and the matrix element and we proceed to calculate the CP asymmetries in next two sections.

3. Inclusive decay mode $B \rightarrow X_d \ell^+\ell^-$

3.1 Decay rate and FB asymmetry

The decay width as a function of invariant mass of lepton pair is given by [6]:

$$\frac{d\Gamma}{d\hat{s}} = \frac{G_F^2 m_b^5}{768\pi^5} \alpha^2 |V_{tb} V_{td}^*|^2 (1 - \hat{s})^2 \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \Sigma_{B \rightarrow X_d \ell^+\ell^-}$$

(3.1)

where

$$\Sigma_{B \rightarrow X_d \ell^+\ell^-} = 4|C_7^{eff}|^2 \left( 1 + \frac{2\hat{m}_\ell^2}{\hat{s}} \right) \left( 1 + \frac{2}{\hat{s}} \right) + |C_9^{eff}|^2 \left( 1 + \frac{2\hat{m}_\ell^2}{\hat{s}} \right) (1 + 2\hat{s})$$

$$+ |C_{10}|^2 \left( 1 - 8\hat{m}_\ell^2 + 2\hat{s} + \frac{2\hat{m}_\ell^2}{\hat{s}} \right) + 12Re (C_7^{eff} C_9^{eff} \left( 1 + \frac{2\hat{m}_\ell^2}{\hat{s}} \right))$$

$$+ \frac{3}{2} |C_{Q_1}|^2 (\hat{s} - 4\hat{m}_\ell^2) + \frac{3}{2} |C_{Q_2}|^2 \hat{s} + 6Re (C_{10} C_{Q_2}) \hat{m}_\ell$$

(3.2)
To remove the uncertainties in the value of $m_b$ we normalize the above decay rate to the charged current decay rate:

$$\Gamma(B \rightarrow X_c \ell \nu) = \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 f(\hat{m}_c) k(\hat{m}_c)$$  \tag{3.3}$$

where $f(\hat{m}_c)$ is the phase space factor and $k(\hat{m}_c)$ is the QCD corrections to the semi-leptonic decay rate, these factors are given in appendix. The differential branching ratio hence becomes:

$$\frac{dBr(B \rightarrow X_d \ell^+ \ell^-)}{d\hat{s}} = \frac{\alpha^2 |V_{tb} V^*_{td}|^2}{4\pi^2} \frac{Br(B \rightarrow X_c e \bar{\nu}_e)}{|V_{cb}|^2} f(\hat{m}_c) k(\hat{m}_c) \sum_{B \rightarrow X_d \ell^+ \ell^-}$$  \tag{3.4}$$

Figure 1: Branching ratio of $B \rightarrow X_d \tau^+ \tau^-$ with invariant mass of dileptons. All the parameters of mSUGRA and SUGRA are given in appendix A.

As has been earlier on also mentioned that FB asymmetry is also very sensitive to the new physics. For completeness we give the expression of FB asymmetry also. The definition of the FB asymmetry is:

$$A_{FB} = \frac{\int_0^1 d\cos \theta \frac{d\tau}{d\cos \theta} d\cos \theta - \int_{-1}^0 d\cos \theta \frac{d\tau}{d\cos \theta} d\cos \theta}{\int_0^1 d\cos \theta \frac{d\tau}{d\cos \theta} d\cos \theta + \int_{-1}^0 d\cos \theta \frac{d\tau}{d\cos \theta} d\cos \theta}$$  \tag{3.5}$$
where $\theta$ is the angle between the momentum of B-meson and $\ell^-$ in the CM frame of dileptons. The analytical expression of the FB asymmetry is:

$$A_{FB}(\hat{s}) = \frac{6}{\Sigma_{B\to X_d\ell^+\ell^-}} Re \left[ 2C^{eff}_7C_{10} + C^{eff}_9 C_{10} \hat{s} + 2C^{eff}_7 C_{Q_2} \hat{m}_\ell + C^{eff}_9 C_{Q_1} \hat{m}_\ell \right] (3.6)$$

### 3.2 CP asymmetries

Next we define the CP violating partial width asymmetry as:

$$A_{CP}(\hat{s}) = \frac{d\Gamma}{d\hat{s}} - \frac{d\bar{\Gamma}}{d\hat{s}} = \frac{d\Gamma(\bar{b} \to \bar{d}\ell^+\ell^-)}{d\hat{s}} + \frac{d\bar{\Gamma}(\bar{b} \to \bar{d}\ell^+\ell^-)}{d\hat{s}} (3.7)$$

where

$$\frac{d\Gamma}{d\hat{s}} = \frac{d\Gamma(b \to d\ell^+\ell^-)}{d\hat{s}} , \quad \frac{d\bar{\Gamma}}{d\hat{s}} = \frac{d\bar{\Gamma}(\bar{b} \to \bar{d}\ell^+\ell^-)}{d\hat{s}} (3.8)$$

In going from $\Gamma$ to $\bar{\Gamma}$ the only change would be in the term having $C^{eff}_9$ in the matrix element. The definition of $C^{eff}_9$ is given in eqn.(2.6). Now to find $\bar{\Gamma}$ the definition of $C^{eff}_9$ changes to:

$$C^{eff}_9 = \xi_1 + \lambda_0^a \xi_2 (3.9)$$

one can easily calculate the expression of CP-violating partial width asymmetry from the expression of decay width eqn.(3.1), the expression of CP-violating partial width

![Figure 2: FB asymmetry of $B \to X_d\tau^+\tau^-$ with invariant mass of dileptons](image-url)
asymmetry is:

\[ A_{CP}(\hat{s}) = \frac{-2Im\lambda_u\Delta_{B\to X_d\ell^+\ell^-}}{\Sigma_{B\to X_d\ell^+\ell^-} + 2Im\lambda_u\Delta_{B\to X_d\ell^+\ell^-}} \]  

(3.10)

where \( \Sigma_{B\to X_d\ell^+\ell^-} \) is given in eqn.(3.2) and \( \Delta_{B\to X_d\ell^+\ell^-} \) is:

\[ \Delta_{B\to X_d\ell^+\ell^-} = Im(\xi_1^*\xi_2)\left(1 + \frac{2m^2_\tau}{s}\right)\left(1 + 2\hat{s}\right) + 6Im(C_{7}^{eff}\xi_2)\left(1 + \frac{2m^2_\tau}{s}\right) \]  

(3.11)

Figure 3: CP violating asymmetry \( A_{CP} \) in \( B \to X_d\tau^+\tau^- \) with invariant mass of dileptons.

As argued in many earlier works [2, 3, 10] that by measuring the FB asymmetries of \( B \) and \( \bar{B} \) also one can observe the CP violating phase of the CKM matrix.

While discussing the CP violation through the FB asymmetries it is important to fix up the sign convention. The reason for this is that there are generally two conventions available in literature regarding this sign. One is followed by Krüger and Sehgal [3] where the difference of FB asymmetries of \( B \) and \( \bar{B} \) was taken as the measure of CP violation. The other convention is where the sum of FB asymmetries of \( B \) and \( \bar{B} \) is taken to be the extent of CP violation [2]. Actually both these conventions are same, the reason for this is that sign of FB asymmetry for \( B \) and \( \bar{B} \) are different. In fact in the limit of strict CP conservation:

\[ A_{FB}(\bar{B}) = - A_{FB}(B) \]  

(3.12)
We can easily understand this because CP conjugation not only requires exchange 
b \leftrightarrow \bar{b} but also \ell^- \leftrightarrow \ell^+. Since the two dileptons are emitted back to back in 
dilepton CM frame, the asymmetry defined in terms of direction of \ell^- (for both \(B\) and \(\bar{B}\)) changes sign under CP transformation \(^5\). Any deviation from eqn. (3.12) will 
give us another measure of CP violation. We for this define a CP violating parameter 
in FB asymmetry as:

\[ \delta_{FB} = A_{FB}(B) + A_{FB}(\bar{B}) \]  

(3.13)

Using the expression of the FB asymmetry eqn.(3.6) we can get:

\[ \delta_{FB} = \frac{2Im\lambda_u \left[ -Im\xi_2(C_{10}\hat{s} + C_{Q_1}\hat{\tilde{m}_\ell})\Sigma_{B\rightarrow X_d\ell^+\ell^-} + 2\Delta_{B\rightarrow X_d\ell^+\ell^-} N_1 \right]}{\Sigma_{B\rightarrow X_d\ell^+\ell^-} \left( \Sigma_{B\rightarrow X_d\ell^+\ell^-} + 4Im\lambda_u \Delta_{B\rightarrow X_d\ell^+\ell^-} \right)} \]  

(3.14)

with

\[ N_1 = 2C_{10}^{eff}C_{10} + (Re\xi_1 + Re\lambda_u Re\xi_2 - Im\lambda_u Im\xi_2)(C_{10}\hat{s} + C_{Q_1}\hat{\tilde{m}_\ell}) + 2C_{Q_2}^{eff}C_{Q_2}\hat{\tilde{m}_\ell} \]  

(3.15)

and \(\Sigma_{B\rightarrow X_d\ell^+\ell^-}\) is given in eqn.(3.2)

\(^5\)Krüger & Schgal [3] haven’t considered this sign change or in other words for \(B\) they calculate 
FB asymmetry wrt \(\ell^-\) but for \(\bar{B}\) they calculate FB asymmetry wrt \(\ell^+\)
4. Exclusive decay mode $B_d \rightarrow \ell^+\ell^−\gamma$

4.1 Decay rate and FB asymmetry

The procedure for calculation of the decay rate of $B_d \rightarrow \ell^+\ell^−\gamma$ is exactly same as that of $B_s \rightarrow \ell^+\ell^−\gamma$ [9, 13] with the replacement $s \rightarrow d$. As explained earlier [9, 13] the exclusive $B_d \rightarrow \ell^+\ell^−\gamma$ decay is induced by the inclusive $b \rightarrow d\ell^+\ell^−$ one. So, we have to start with QCD corrected effective Hamiltonian for related quark level process $b \rightarrow d\ell^+\ell^−$ given in eqn.(2.1).

In order to obtain the matrix element for $B_d \rightarrow \ell^+\ell^−\gamma$ decay, a photon line should be hooked to any of the charged internal or external lines. As has been pointed out before [24], contributions coming from hooking a photon line from any charged internal line will be suppressed by a factor of $m_b/M_W^2$, and hence we neglect them in our further analysis. When photon is attached to the initial quark lines the corresponding matrix element is the so called structure dependent (SD) part of the amplitude which can be written as:

$$M_{SD} = \frac{\alpha^3 G_F}{\sqrt{2\pi}} V_{tb}V_{td}^* \left\{ A \varepsilon_{\mu\nu\rho\sigma} \epsilon^{*\mu}p^{\sigma}q^{\nu} + iB \left( \epsilon^{*\mu}(pq) - (\epsilon^*p)q_{\mu} \right) \bar{\ell}\gamma^\mu\ell \right. $$

$$+ \left[ C \varepsilon_{\mu\nu\rho\sigma} \epsilon^{*\mu}p^{\sigma}q^{\nu} + iD \left( \epsilon^{*\mu}(pq) - (\epsilon^*p)q_{\mu} \right) \bar{\ell}\gamma^\mu\gamma^5\ell \right\} \quad (4.1)$$

where definition of form factors and A, B, C and D are given in appendix (B). In the definitions of A and B (given in eqn.(B.1) the value of $C_9^{eff}$ is given by eqn.(2.6).

We can see from eqn.(4.1) that neutral scalar exchange parts do not contribute to the structure dependent part.

When the photon is attached to the lepton lines using the eqns.(B.6,B.7,B.8) and the conservation of vector current we can get the contribution to the Bremsstrahlung part (called internal Bremsstrahlung IB) part as:

$$M_{IB} = \frac{\alpha^3 G_F}{\sqrt{2\pi}} V_{tb}V_{td}^* i2 m_\ell f_{B_d} \left\{ \left( C_{10} + \frac{m_{B_d}^2}{2m_\ell m_b} C_{Q_2} \right) \bar{\ell} \left[ \frac{\not k \not P_{B_d}}{2p_+q} - \frac{P_{B_d}^2 \not k}{2p_-q} \right] \gamma^5 \ell \right. $$

$$+ \left. \frac{m_{B_d}^2}{2m_\ell m_b} C_{Q_1} \left[ 2m_\ell \left( \frac{1}{2p_-q} + \frac{1}{2p_+q} \right) \bar{\ell} \not k \ell \right. \right. $$

$$+ \left. \left. \bar{\ell} \left( \frac{\not k \not P_{B_d}}{2p_+q} - \frac{P_{B_d}^2 \not k}{2p_-q} \right) \ell \right] \right\}. \quad (4.2)$$

where $P_{B_d}$ and $f_{B_d}$ are the momentum and decay constant of the $B_d$ meson. $p_-$ and $p_+$ are the four momenta of $\ell^−$ and $\ell^+$ respectively.

The total matrix element for $B_d \rightarrow \ell^+\ell^−\gamma$ is obtained as a sum of $M_{SD}$ and $M_{IB}$ terms:

$$M = M_{SD} + M_{IB} \quad (4.3)$$
From above matrix element we can get the square of the matrix element as, (with photon polarizations summed over)

\[
\sum_{\text{photon pol}} |\mathcal{M}|^2 = |\mathcal{M}_{SD}|^2 + |\mathcal{M}_{IB}|^2 + 2\text{Re}(\mathcal{M}_{SD}\mathcal{M}_{IB}^*) \tag{4.4}
\]

with

\[
|\mathcal{M}_{SD}|^2 = 4 \left| \frac{\alpha^{3/2}G_{F}^{3/2}}{\sqrt{2\pi}} V_{tb} V_{td}^{*} \right| f_{B_{d}} m_{\ell}^{2} \left[ C_{10} + \frac{m_{B_{s}}^{2}}{2m_{c}m_{b}} C_{Q_{2}} \right] \left\{ 8 + \frac{1}{(p-q)^{2}} \right\}
\]

\[
\times (-2m_{B_{d}}^{2}m_{\ell}^{2} - m_{B_{d}}^{2}p^{2} + p^{4} + 2p^{2}(p+q)) + \frac{1}{(p-q)^{2}}(6p^{2} + 4(p+q))
\]

\[
+ \frac{1}{(p-q)^{2}} \left( -4m_{B_{d}}^{2}m_{\ell}^{2} + 2p^{4} \right) \left\{ 8 + \frac{1}{(p-q)^{2}} \right\}
\]

\[
\times (6m_{B_{d}}^{2}m_{\ell}^{2} + 8m_{c}^{4} - m_{B_{d}}^{2}p^{2} - 8m_{\ell}^{2}p^{2} + p^{4} - 8m_{\ell}^{2}(p+q) + 2p^{2}(p+q))
\]

\[
+ \frac{1}{(p-q)^{2}}(-4m_{\ell}^{2} + 6p^{2} + 4(p+q)) + \frac{1}{(p+q)^{2}}(6m_{B_{d}}^{2}m_{\ell}^{2} + 8m_{c}^{4} - m_{B_{d}}^{2}p^{2} - 8m_{\ell}^{2}(p+q) + 2p^{2}(p+q))
\]

\[
+ \frac{1}{(p+q)^{2}}(-4m_{\ell}^{2} + 6p^{2} + 4(p+q)) + \frac{1}{(p+q)^{2}}(4m_{B_{d}}^{2}m_{\ell}^{2} + 16m_{\ell}^{2} - 16m_{\ell}^{2}p^{2} + 2p^{4}) \right\} \tag{4.6}
\]

\[
2\text{Re}(\mathcal{M}_{SD}\mathcal{M}_{IB}^*) = 16 \left| \frac{\alpha^{3/2}G_{F}^{3/2}}{\sqrt{2\pi}} V_{tb} V_{td}^{*} \right| f_{B_{d}} m_{\ell}^{2} \left[ C_{10} + \frac{m_{B_{s}}^{2}}{2m_{c}m_{b}} C_{Q_{2}} \right] \left\{ - \text{Re}(A) \right\}
\]

\[
\times \left( \frac{(p-q+p+q)}{(p-q)(p+q)} \right)^{3} + \text{Re}(D) \left( \frac{(p-q)^{2}(p-q-p)}{(p-q)(p+q)} \right)
\]

\[
+ \left( \frac{m_{B_{s}}^{2}}{2m_{c}m_{b}} C_{Q_{1}} \right) \left\{ \frac{\text{Re}(B)}{(p-q)(p+q)} \left( -(pq)^{3} - 2(p+p_{+})(p+q)^{2} \right)
\right.
\]

\[
\left. -2(p-p_{+})(p+q)^{2} + 4m_{\ell}^{2}(p+q)(p-q) \right) + \text{Re}(C)
\]

\[
\times \left( \frac{(pq)^{2}(p-q-p)}{(p-q)(p+q)} \right) \tag{4.7}
\]

The differential decay rate of $B_{d} \rightarrow \ell^{+}\ell^{-}\gamma$ as a function of invariant mass of lepton pair is given by:

\[
\frac{d\Gamma}{ds} = \left| \frac{\alpha^{3/2}G_{F}^{3/2}}{2\sqrt{2\pi}} V_{tb} V_{td}^{*} \right| \frac{m_{B_{d}}^{5}}{16(2\pi)^{3}} (1 - s) \sqrt{1 - \frac{4m_{\ell}^{2}}{s}} \Sigma_{B_{d} \rightarrow \ell^{+}\ell^{-}\gamma} \tag{4.8}
\]
with $\Sigma_{B_d \to \ell+\ell-\gamma}$ defined as

$$\Sigma_{B_d \to \ell+\ell-\gamma} = \frac{4}{3} m_{B_d}^2 (1 - \hat{s})^2 \left[ (|A|^2 + |B|^2) (2\hat{m}_\ell^2 + \hat{s}) + (|C|^2 + |D|^2)(-4\hat{m}_\ell^2 + \hat{s}) \right]$$

$$+ \frac{64 f_{B_d}^2 \hat{m}_\ell^2}{m_{B_d}^2} \left( C_{10} + \frac{m_{B_d}^2}{2m_\ell m_b} C_{Q_1} \right)^2 \left[ (1 - 4\hat{m}_\ell^2 + \hat{s}) \ln(\hat{z}) - 2\hat{s} \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \right] \left( 1 - \hat{s} \right)^2 \frac{1}{\sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}}}$$

$$- \frac{64 f_{B_d}^2 \hat{m}_\ell^2}{m_{B_d}^2} \left( \frac{m_{B_d}^2}{2m_\ell m_b} C_{Q_1} \right)^2 \left\{ \frac{(-1 + 12\hat{m}_\ell^2 - 16\hat{m}_\ell^4 - \hat{s}^2) \ln(\hat{z})}{(1 - \hat{s})^2 \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}}} \right\} + 32 f_{B_d}^2 \hat{m}_\ell^2 \left( C_{10} + \frac{m_{B_d}^2}{2m_\ell m_b} C_{Q_2} \right) Re(A) \times \frac{(-1 + \hat{s}) \ln(\hat{z})}{\sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}}} - 32 f_{B_d}^2 \hat{m}_\ell^2 \left( \frac{m_{B_d}^2}{2m_\ell m_b} C_{Q_1} \right) Re(B) \times \frac{(-1 + \hat{s}) \ln(\hat{z})}{\sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}}}$$

$$\times \frac{\left[ (1 - 4\hat{m}_\ell^2 + \hat{s}) \ln(\hat{z}) - 2\hat{s} \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \right]}{\sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}}}$$

Equation (4.9)

where $\hat{s} = p^2/m_{B_d}^2$, $\hat{m}_\ell^2 = m_\ell^2/m_{B_d}^2$, $\hat{z} = \frac{1 + \sqrt{1 - \frac{4m_\ell^2}{\hat{s}}}}{1 - \sqrt{1 - \frac{4m_\ell^2}{\hat{s}}}}$ are dimensionless quantities.

We can also calculate the FB asymmetry from use of eqn.(4.5). The analytical expression of FB asymmetry is:

$$A_{FB} = \left[ -2 \frac{m_{B_d}^2}{m_\ell^2} Re(A^* D + B^* C) (1 - \hat{s})^2 \hat{s} \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \right]$$

$$+ \frac{32 f_{B_d}}{m_\ell^2} \frac{(-1 + \hat{s}) \ln(\hat{z})}{\sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}}} \left\{ \left( C_{10} + \frac{m_{B_d}^2}{2m_\ell m_b} C_{Q_2} \right) Re(D) \right\}$$

$$+ \left( \frac{m_{B_d}^2}{2m_\ell m_b} C_{Q_1} \right) Re(C) \right] \right] / \Sigma_{B_d \to \ell+\ell-\gamma}$$

Equation (4.10)

4.2 CP asymmetries

One can also calculate the CP asymmetries as defined in eqn.(3.11) and eqn.(3.13). The expression of CP violating partial width asymmetry is:

$$A_{CP} = \frac{-2Im \lambda u \Delta_{B_d \to \ell+\ell-\gamma}}{\Sigma_{B_d \to \ell+\ell-\gamma} + 2Im \lambda u \Delta_{B_d \to \ell+\ell-\gamma}}$$

Equation (4.11)

with $\Sigma_{B_d \to \ell+\ell-\gamma}$ given in eqn.(4.9) and expression of $\Delta_{B_d \to \ell+\ell-\gamma}$ is:

$$\Delta_{B_d \to \ell+\ell-\gamma} = \left\{ G_1(p^2) + F_1(p^2) \right\} Im(\xi_1^* \xi_2)$$
Figure 5: Branching ratio of $B_d \to \tau^+ \tau^- \gamma$ with invariant mass of dileptons

\[ \frac{2m_b}{p^2} \left\{ G_1(p^2)G_2(p^2) + F_1(p^2)F_2(p^2) \right\} C_{10} \text{Im}(\xi_2) \times T_1(\hat{s}, \hat{m}_\ell) \]
\[ + \left( C_{10} + \frac{m_{B_s}^2}{2m_\ell m_b}C_{Q_2} \right) G_1(p^2)T_2(\hat{s}, \hat{m}_\ell) \times \text{Im}\xi_2 \]
\[ + \left( \frac{m_{B_s}^2}{2m_\ell m_b}C_{Q_1} \right) F_1(p^2)T_3(\hat{s}, \hat{m}_\ell) \times \text{Im}\xi_2 \]  

(4.12)

with

\[ T_1(\hat{s}, \hat{m}_\ell) = \frac{1}{m_{B_d}^2} \frac{4(1 - \hat{s})^2(2\hat{m}_\ell^2 + \hat{s})}{3} \]  

(4.13)

\[ T_2(\hat{s}, \hat{m}_\ell) = 16f_{B_d} \frac{\hat{m}_\ell^2 (-1 + \hat{s})\ln(\hat{z})}{m_{B_d}^2} \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \]  

(4.14)

\[ T_3(\hat{s}, \hat{m}_\ell) = -16f_{B_d} \frac{(1 - 4\hat{m}_\ell^2 + \hat{s})\ln(\hat{z}) - 2\hat{s} \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}}}{m_{B_d}^2} \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \]  

(4.15)

Similarly we can calculate the second CP violating parameter $\delta_{FB}$ as defined in eqn.(3.13). The expression of $\delta_{FB}$ is:

\[ \delta_{FB} = \frac{2Im\lambda_u \times \left[ -\Sigma_{B_d \to \ell^+ \ell^- \gamma}L_1 + 2 \Sigma_{B_d \to \ell^+ \ell^- \gamma}L_2 \right]}{\Sigma_{B_d \to \ell^+ \ell^- \gamma}(\Sigma_{B_d \to \ell^+ \ell^- \gamma} + 4Im\lambda_u \Sigma_{B_d \to \ell^+ \ell^- \gamma})} \]  

(4.16)
with $\Sigma_{B_d \to \ell^+ \ell^- \gamma}$ and $\Delta_{B_d \to \ell^+ \ell^- \gamma}$ are given in eqns. (4.3) and (4.12) respectively. $L_2$ is just the numerator of the expression of FB asymmetry in eqn. (4.10) and $L_1$ is given as:

$$L_1 = -2(1 - s^2) s \sqrt{1 - \frac{4m_T^2}{s}} \left[ DG_1(p^2) Im(\xi_2) + CF_1(p^2) Im(\xi_2) \right]$$ \hspace{1cm} (4.17)

5. Results and discussion

We have performed the numerical analysis of all the asymmetries, branching ratios and FB asymmetries whose analytical expressions are given in previous sections.

The MSSM that we are working with is the simplest (and having the least number of parameters) SUSY model, but even this still has too many of parameters to do any meaningful phenomenology with it. There are many choices available to restrict this large parameter space. We have opted for Supergravity (SUGRA) model for our analysis. In this model the universality of all the scalar masses and coupling constants at the unification scale is assumed. So in minimal SUGRA (mSUGRA) model we only have five parameters (in addition to SM parameters) namely: $m$ the unified mass of all the scalars at GUT scale , $M$ the unified gaugino mass at GUT scale, $A$ the universal trilinear coupling at unification scale , $\tan\beta$ the ratio of vacuum expectation values of the two Higgs doublets and finally $sgn(\mu)$. We have
also considered another model where we have relaxed the condition of the universality of the scalar masses at GUT scale. This sort of model lately has been advocated in many works [7, 10, 12, 13, 15] In this model we have taken the squark sector and Higgs sector to have different unified masses at GUT scale. So here we have another parameter which we have taken to be the pseudoscalar Higgs mass \(^6\). About the sign of convention of \(\mu\), we are following the convention where \(\mu\) enters the chargino mass matrix with +ve sign. In all of our numerical analysis we have taken a 95% CL bound \([23]\)

\[
2 \times 10^{-4} < Br(B \rightarrow X_s\gamma) < 4.5 \times 10^{-4}
\]

which is in agreement with CLEO and ALEPH results. Our results are given in Figs.(1 - 8).

From our numerical analysis we can conclude :

1. **Branching ratios** : As we can see from Figure(1) for inclusive mode \((B \rightarrow X_d\ell^+\ell^-)\) that there can be significant increase in the branching ratio of this decay mode both in mSUGRA and SUGRA model as compared to SM . This has been stated earlier on also \([6]\) in context of \(B \rightarrow X_s\ell^+\ell^-\). As we can see from Figure(5), this pattern (that branching ratio shows significant increase from SM results) repeats for exclusive mode \((B_d \rightarrow \ell^+\ell^-\gamma)\) again this has earlier on stressed in earlier works \([9, 13]\).

\(^6\)our choice of parameters is given in Appendix A
2. **FB asymmetries**: As we can see from Figures(2) and (6) that FB asymmetries also shows fairly large deviations from SM results both in mSUGRA and SUGRA. Again this point has been stressed in many earlier works [6, 9, 11, 13].

3. **CP violating partial width asymmetry**: The effect scalars on CP violating asymmetries in exclusive decay modes $B \to \pi \ell^+ \ell^-$ and $B \to \rho \ell^+ \ell^-$ as already been discussed in our earlier work [10]. There it was shown that the CP violating partial width asymmetries for both the exclusive modes decrease with the introduction of scalars in the theory (Higgs here). Here as we can see from Figure(3) that the same trend is present for the inclusive decay mode $B \to X_d \ell^+ \ell^-$, but contrastingly, as we can see from Figure(7) the exclusive decay mode $B_d \to \ell^+ \ell^- \gamma$ doesn’t show up this trend. In fact in this decay mode the CP violating partial width asymmetry increases with switching on the scalar effects.

4. **CP violation via FB asymmetries**: For estimating this effect we have introduced $\delta_{FB}$. As we can see from Figure(4) that this parameter follows the trend followed by $B \to \pi \ell^+ \ell^-$ and $B \to \rho \ell^+ \ell^-$ (noted in [10]) , which is that $\delta_{FB}$ increases with switching on of the scalar effects as compared to the SM values. But here again as we can see from Figure(8) the trend for $B_d \to \ell^+ \ell^- \gamma$ is opposite, here in mSUGRA and SUGRA $\delta_{FB}$ reduces as compared to the SM value.
Although the branching ratios of both $B \to X_d \tau^+ \tau^-$ and $B_d \to \tau^+ \tau^- \gamma$ are very low but with upcoming B-factories like LHC-b where more than $10^{11} B_d$ will be produced, one can hope of observing these modes. In semi-leptonic decays as far as the branching ratios and FB asymmetries are concerned, branching ratio tends to increase, and FB asymmetry tends to decrease with increasing the scalar effects. This has been noted in many different decay modes like: $B_s \to \ell^+ \ell^- [9, 13]$, $B \to \pi \ell^+ \ell^- [10, 14]$, $B_s \to \ell^+ \ell^- [5, 11, 20]$, $B \to X_s \ell^+ \ell^- [6, 7, 11]$, $B \to (K, K^*) \ell^+ \ell^-$ [14, 18]. But as we can see the CP asymmetries doesn’t follow the same trend. For some channels they decrease and for other they increase. So in brief the measurement of CP asymmetries although a challenging task, could be very useful for more information about scalar effects and hence any new physics.

**Acknowledgments**

This work was supported under SERC scheme of Department of Science and Technology, India

**A. Input parameters and constants**

\[
\begin{align*}
    f(\hat{m}_c) &= 1 - 8\hat{m}_c^2 + 8\hat{m}_c^4 - \hat{m}_c^8 - 24\hat{m}_c^4 \ln(\hat{m}_c) \\
    k(\hat{m}_c) &= 1 - \frac{2\alpha_s(m_b)}{3\pi} \left[ \left( \frac{\pi^2}{4} - \frac{31}{4} \right)(1 - \hat{m}_c^2) + \frac{3}{2} \right]
\end{align*}
\]  

(A.1) \hspace{1cm} (A.2)

The branching ratio of charged current semi-leptonic decay mode $B \to X_c e\bar{\nu}_e$ we are taking to be:

\[ Br(B \to X_c e\bar{\nu}_e) = 10.4 \% \]

The parameters we have used for our numerical analysis are:

\[
\begin{align*}
    m_\tau &= 1.77 \text{ GeV} \\
    m_b &= 4.8 \text{ GeV} , m_c = 1.4 \text{ GeV} , m_t = 176 \text{ GeV} , m_{B_d} = 5.26 \text{ GeV} \\
    f_{B_d} &= 1.8 , \alpha = \frac{1}{129} , \tau_B = 1.5 \times 10^{-12} \text{ s}
\end{align*}
\]

Wolfenstein parameters:

\[
\begin{align*}
    \rho &= -0.07 , \eta = 0.34 , \lambda = 0.22 , A = 0.84
\end{align*}
\]

For mSUGRA the parameters we have taken as:

\[
\begin{align*}
    m &= 200 \text{ GeV} , M = 500 \text{ GeV} , A = 0 , \tan\beta = 45 , \text{sgn}(\mu) = +ve
\end{align*}
\]

The additional parameter for SUGRA, the pseudoscalar Higgs mass is taken to be $m_A = 281 \text{ GeV}$
B. Form factors

\[ A = \frac{1}{m_{B_d}^2} [C^{eff}_G G_1(p^2) - 2C^{eff}_7 \frac{m_B}{p^2} G_2(p^2)], \]
\[ B = \frac{1}{m_{B_d}^2} [C^{eff}_F F_1(p^2) - 2C^{eff}_7 \frac{m_B}{p^2} F_2(p^2)], \]
\[ C = \frac{C_10}{m_{B_d}^2} G_1(p^2), \]
\[ D = \frac{C_10}{m_{B_d}^2} F_1(p^2). \] \hspace{1cm} (B.1)

In getting above eqns we have used following definitions of the form f actors \cite{25}
\[ \langle \gamma | \bar{d} \gamma \mu (1 \pm \gamma_5) b | B_d \rangle = \frac{e}{m_{B_d}} \varepsilon_{\mu \alpha \beta \sigma} \epsilon^*_\alpha p_\beta q_\sigma G_1(p^2) \mp i[(\epsilon^*_\mu pq) - (\epsilon^* p)q_\mu)] F_1(p^2) \] \hspace{1cm} (B.2)
\[ \langle \gamma | \bar{d} i \sigma_{\mu \nu} (1 \pm \gamma_5) b | B_d \rangle = \frac{e}{m_{B_d}} \varepsilon_{\mu \alpha \beta \sigma} \epsilon^*_\alpha p_\beta q_\sigma G_2(p^2) \pm i[(\epsilon^*_\mu pq) - (\epsilon^* p)q_\mu)] F_2(p^2) \] \hspace{1cm} (B.3)

another relation we can get by multiplying \( p_\mu \) on both the sides of eqn.(B.3) :
\[ \langle \gamma | \bar{d} (1 \pm \gamma_5) b | B_d \rangle = 0 \] \hspace{1cm} (B.4)

Here \( \epsilon_\mu \) and \( q_\mu \) are the four vector polarization and momentum of photon respectively.

The definition of the form factors used in above eqns for our numerical analysis are \cite{25} :
\[ G_1(p^2) = \frac{1}{1 - p^2/5.6^2} \text{GEV}, \ G_2(p^2) = \frac{3.74}{1 - p^2/40.5} \text{GEV}^2, \]
\[ F_1(p^2) = \frac{0.8}{1 - p^2/6.5^2} \text{GEV}, \ F_2(p^2) = \frac{0.68}{1 - p^2/30} \text{GEV}^2. \] \hspace{1cm} (B.5)

when photon is emitted from lepton lines we use following definitions :
\[ \langle 0 | \bar{d} b \frac{1}{B_d} \rangle = 0 \] \hspace{1cm} (B.6)
\[ \langle 0 | \bar{d} \sigma_{\mu \nu} (1 + \gamma_5) b \frac{1}{B_d} \rangle = 0 \] \hspace{1cm} (B.7)
\[ \langle 0 | \bar{d} \gamma \gamma b \frac{1}{B_d} \rangle = - if_{B_d} P_{B_d \mu} \] \hspace{1cm} (B.8)

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