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Characteristics of EHF Wave Propagation in Hypersonic Plasma Sheaths Magnetized by Dipole Magnetic Fields

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Abstract: Communication blackout is always a serious threat to the flight tasks of modern hypersonic vehicles moving in near space. EHF communication is considered as a potential solution to the blackout problem. Nevertheless, EHF waves suffer from severe attenuation in hypersonic plasma sheaths. An external magnetic field could mitigate EHF wave attenuation in hypersonic plasma sheaths. Dipole magnetic fields, generated by coils, are feasible in realistic scenarios. In the present study, a model for EHF wave propagation in hypersonic plasma sheaths magnetized by dipole magnetic fields that are generated with coils is developed. The dissipation caused by the inhomogeneity of dipole magnetic fields and the magnetic field component of electromagnetic waves are compared with the dissipation yielded by the collision between electrons and neutral particles. The results show that collision is still the main dissipation mechanic for EHF waves. The study also found that, in the blunt-coned plasma sheath, the mitigation effect of a dipole magnetic field is weaker than that of a uniform magnetic field. The mechanics which yield the difference is analyzed. In addition, the relation between the characteristics of EHF waves and the coil parameters is investigated. Suggestions for the coil parameters and the operation frequencies of the EHF communication systems are made based on the investigation.

Keywords: hypersonic plasma sheath; communication blackout; EHF communication; EHF wave propagation

1. Introduction

Once a hypersonic vehicle moves in near space, most of the kinetic energy is converted into thermal energy and the neutral gas around it is heated and ionized. As a result, a plasma layer, which used to be called the plasma sheath, is formed. The peak electron density of the plasma sheath can be up to \(10^{20} \text{ m}^{-3}\). Normally, the communication signal cannot directly penetrate the plasma sheath; thus, the so-called “blackout” occurs. Communication blackouts can isolate hypersonic vehicles from ground-based or spaceborne base stations. Thus, they could lead to the failure of space missions [1–3].

In order to solve the black barrier problem, it is necessary to study the propagation characteristics of the signal in the plasma. To study the signal propagation characteristics in the plasma sheath, it is necessary to obtain the sheath parameters. There are usually four methods for this: using the plasma slab model [4], the simulated plasma environment generated by a ground test device [5,6], the public flight experiment data [7], and by solving the flow field distribution of the sheath based on the hydrodynamic model [8,9]. The
state-to-state model is one of the above-mentioned four models, focusing on the role of molecular vibration in the plasma’s evolution [10]. Previous studies have shown that EHF (extremely high frequency) communication could be a potential solution to the blackout problem [11,12]. Many models have been developed to study the propagation characteristics of EHF waves in hypersonic plasma sheaths. For example, in the frequency domain, EHF wave propagation in a plasma sheath was modeled with the scattering matrix method (SMM) [7,13] and the Wentzel–Kramers–Brillouin (WKB) method [14,15]. In the time domain, wave propagation used to be modeled with the finite difference time domain (FDTD) method [4,16]. Previous studies have shown that EHF waves propagating in plasma sheaths used to suffer from severe attenuation [4,12], which could make the wave very weak by the time it reaches the receiver. Such attenuation is led mainly by the high-density plasma and the collisions between free electrons and neutral particles [17]. On the other hand, the bit error rate (BER) for an EHF communication system is also influenced by the parameter of the plasma sheath [18]. In addition, the EHF signal propagations are influenced by many flight conditions, e.g., the atmospheric conditions surrounding the vehicle [19], the flight speed [20], the angle of attack (AOA) [21], and the shape of the vehicle [22]. In other words, the characteristics of EHF wave propagation in plasma sheaths is dynamic, which is always varying with time. In order to mitigate the communication blackout effectively, it is necessary to reduce the EHF wave attenuation in plasma sheaths. Some previous studies have shown that the EHF signal attenuation could be significantly reduced by introducing an external magnetic field near the onboard EHF antenna [16,23–25].

Nevertheless, previous studies that modeled EHF wave propagation in magnetized plasma sheaths were based on the linear electron motion equation. The magnetic field near the onboard antenna is more likely to be generated by a coil, which means it would be a dipole field. Whether there are other dissipation mechanisms for EHF waves in plasma sheaths magnetized by dipole fields has not yet been sufficiently studied. Moreover, the relation between the parameter of coils and the propagation of EHF waves is also, so far, still unclear.

In the present study, a propagation model for EHF waves in hypersonic plasma sheaths magnetized by dipole fields is developed based on the electron motion equation. It is found that the inhomogeneity of the dipole magnetic field and the magnetic field component of the electromagnetic wave forms a new dissipation mechanic. However, for EHF waves, the new dissipation mechanic has little effect compared to collision. On a blunt-coned vehicle, the effect of an ideal uniform magnetic field may not be achieved by applying a dipole magnetic field to reduce signal attenuation. The mechanics of those differences are analyzed. In addition, the present study investigates the relation between the coil parameter and the mitigation effect. Suggestions are made based on the present study.

2. The EHF Signal Propagation Model

As shown in Figure 1, the plasma sheath is magnetized. The external magnetic field is inhomogeneous and generated by a circular coil. Therefore, the magnetic field involved in the present study is a dipole field. The incident wave is meant to have the form of:

$$E = (\hat{E}_x e_x + \hat{E}_y e_y)e^{i(\omega_0 t - k_0 z)},$$

where $\omega_0$ is the angular frequency for the wave, and $k_0 = k_0 e_z$ is the wave vector propagating in free space.

High directive antennas used to be employed by EHF communication systems, i.e., the beam width would be very small. In the present study, the beam direction is supposed to be perpendicular to the coil in order to make the magnetic field direction in the center of the coil parallel to the wave vector. The wave–particle interactions occur in the coverage of antenna beam only. In such a case, the guiding center approximation is adopted, as it is in the present study. The displacement vector for an electron could be expressed as $r = r_0 + r_1$, $r_0$ is the cyclotron movement around the guiding center, and $r_1$ is the relatively
slow movement of the guiding center. The magnetic field is a weakly inhomogeneous, which satisfies the following relation:

$$| (r_0 \cdot \nabla) B_e |_0 \ll | B_0 |,$$

where $B_0$ is the magnetic field strength at the guiding center. The magnetic field generated by the coil in a cylindrical coordinate system could be expressed as:

$$B_e = B_z(r,z)e_z + B_r(r,z)e_r \approx B_0 + | (r_0 \cdot \nabla) B_e |_0 = B_0 + B_1,$$

where:

$$B_r = \frac{\mu_0 I}{4\pi} J_0^{2\pi} \frac{\cos \phi}{(r^2 + a^2 + z^2 - 2ar \cos \phi)^{3/2}} d\phi,$$

$$B_z = \frac{\mu_0 I}{4\pi} J_0^{2\pi} \frac{1}{(r^2 + a^2 + z^2 - 2ar \cos \phi)^{3/2}} d\phi,$$

$\mu_0$ is the magnetic permeability in the vacuum, $a$ is the radius of the coil, and $I$ is the current intensity.

**Figure 1.** The model for the wave propagation in an inhomogeneous plasma sheath. The external magnetic field is generated by a coil.

The electron motion equation is given below:

$$\frac{d^2 r}{dt^2} = \frac{q}{m} | E + \frac{dr}{dt} \times (B_0 + B_1) + \frac{dr}{dt} \times B | - \frac{dr}{dt},$$

where $q$ is the value of electron charge, $m$ is the mass of electron, $B$ is the magnetic field component of the incident wave, and $v$ is the collision frequency between electrons and neutral particles.

The electron motion equation in a homogeneous magnetic field is:

$$\frac{d^2 r_0}{dt^2} = \frac{q}{m} | E + \frac{dr_0}{dt} \times B_0 | - \frac{dr_0}{dt}. (6)$$

From (6), the dielectric constant tensor in the sheath can be solved:

$$\epsilon_r = \begin{pmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix},$$

$$\epsilon_1 = 1 + \frac{\omega_{pe}^2 (iv - \omega_0)}{\omega_0 (iv - \omega_0)^2 + (iv - \omega_0)^2},$$

where $\omega_{pe}$ is the plasma frequency, $\omega_0$ is the frequency of the incident wave, and $v$ is the collision frequency between electrons and neutral particles.
where $\omega_{pe}$ is the plasma frequency, which has the form of $\omega_{pe} = \sqrt{\frac{N_e q_e}{m}}$. $\varepsilon_0$ is the dielectric constant in vacuum, $N_e$ is the electron density, $\omega_c = -\frac{q B_0}{m}$ is the electron cyclotron frequency, and $i = \sqrt{-1}$.

The wave equation in magnetized plasma could be written in the form of:

$$\nabla(\nabla \cdot E) - \nabla^2 E = -\frac{1}{\varepsilon_0^2} \frac{\partial}{\partial t}\left(\frac{\partial E}{\partial r} \cdot E\right).$$

(10)

Considering that $k \cdot E = 0$, where $k$ is the wave vector in plasma, the wave equation in frequency domain can be written as:

$$k^2 E = k_0^2 \varepsilon_r \cdot E.$$

(11)

The wave number for the right-hand circular (RHC) and left-hand circular (LHC) modes are defined as $k_\pm = k_0 \sqrt{\varepsilon_\pm}$, where $+$ and $-$ correspond to the RHC and LHC modes, respectively.

The signal attenuation for the LHC mode in magnetized plasma is obviously lower than the RHC mode [16]. Hence, the present study will mainly concern the LHC mode.

For the LHC mode, $\hat{E}_x = -i \hat{E}_y$. By substituting that relation into (6), the following equation is obtained:

$$r_0 = \frac{q}{m} \frac{1}{\omega_0} \frac{1}{iv - \omega_c - \omega_0} E.$$

(12)

According to (12), it can be estimated that $|r_0| \approx 10^{-11} E$, which meets the weak inhomogeneity condition (2). $r_1$ can be regarded as a constant relative to a cyclotron period. By substituting $r_0$ into (5) and averaging it in each single cyclotron period, $\langle \frac{dr_1}{dt} \times B \rangle \approx \frac{dr_1}{dt} \times \langle B \rangle = 0$. Then, the electron motion equation in an inhomogeneous magnetic field can be written as:

$$\frac{d^2r_1}{dt^2} = \frac{q}{m} \left(\langle \frac{dr_0}{dt} \times (B_1 + B) \rangle + \langle \frac{dr_1}{dt} \times B_0 \rangle \right) - v \frac{dr_1}{dt},$$

(13)

where $q(\frac{dr_0}{dt} \times B_1)$ reflects the gradient drift.

$$q(\frac{dr_0}{dt} \times B_1) = q[\langle \omega_0 \cdot B_1 \rangle r_0 - \langle r_0 \cdot B_1 \rangle \omega_0]$$

$$= q\omega_0 |r_0| \left\{ 2 \frac{\partial B_1}{\partial r} |_{r_0} e_z, \right\}$$

(14)

where $\omega_0$ is a pseudo-vector whose value is equal to the frequency of the EM wave corresponding to $k_-$, which points to the negative direction of the z-axis.

The ponderomotive force $q(\frac{dr_0}{dt} \times B)$ cannot be obtained directly in the frequency domain. Thus, it is solved in the time domain as follows.

By substituting $k_- = \beta - ia$ into (11), the oscillating electric field after entering the plasma can be expressed as:

$$E = E_0 e^{-az} [e^{|B_0|} e^{|t \omega_0 - \beta z|} + e^{i(t_0 + \frac{x}{v})} e^{|t \omega_0 - \beta z|}] e^{i(\omega_0 t - \beta z)},$$

(15)

where $a$ is the attenuation coefficient and $\beta$ is the phase coefficient.

Faraday’s law in the frequency domain could be expressed as:

$$B = \frac{k_-}{\omega_0} (e_z \times E).$$

(16)
According to (12), (15) and (16), the following relation for the ponderomotive force could be obtained:

\[
q \langle \frac{d\mathbf{r}_0}{dt} \times \mathbf{B} \rangle = \frac{q^2 | \mathbf{E}_1 |}{mc} \frac{1}{|iv - \omega_e - \omega_0|} e z, \tag{17}
\]

where \( \theta = \theta_1 + \theta_2, \sqrt{\mathbf{E}_1 - \mathbf{E}_2} = | \mathbf{E}_1 - \mathbf{E}_2 | e^{i \theta_2}, \) and \( iv - \omega_e - \omega_0 = | iv - \omega_e - \omega_0 | e^{i \theta_1}. \)

According to (14) and (17), it can be seen that the gradient drift of electrons in the direction perpendicular to the wave vector is absent. In such a case, \( \mathbf{r}_1 = r_1 e_z, \) which indicates that \( \mathbf{r}_1 \) is always parallel to the z-axis.

In addition, it can be seen that the movement of electrons in an inhomogeneous magnetic field can still be regarded as a cyclotron movement around the guiding center, yet the guiding center is now moving along the magnetic field line at a speed of \( v \). In the coordinate system with the guidance center as the origin, the centrifugal force on an electron is:

\[
\mathbf{F}_{BC} = - (m \frac{d\mathbf{r}_1}{dt} \times \mathbf{B}) \cdot (| \frac{\nabla B_e}{B_e} | \nabla B_e | 0. \tag{18}
\]

Hence:

\[
| (| \frac{\nabla B_e}{B_e} | \nabla B_e | 0 = | \nabla B_e - \nabla B_e (| \frac{\nabla B_e}{B_e} | \nabla B_e) | | 0 = 0. \tag{19}
\]

Equation (19) indicates that it is not necessary to consider the curvature drift of the electrons.

Obviously, \( (q \frac{d\mathbf{r}_0}{dt} \times \mathbf{B}) \cdot \frac{d\mathbf{r}_1}{dt} \) originates from the incident wave. Thus:

\[
(q \frac{d\mathbf{r}_1}{dt} \times \mathbf{B}) \cdot \frac{d\mathbf{r}_0}{dt} + (q \frac{d\mathbf{r}_0}{dt} \times \mathbf{B}) \cdot \frac{d\mathbf{r}_1}{dt} = 0. \tag{20}
\]

The first term on the left side indicates that \( \mathbf{B}_e \) reduces \( \mathbf{r}_0 \), which results in the reduction of kinetic energy of the electron cyclotron motion, while the second term indicates that \( \mathbf{B}_e \) enlarges \( \mathbf{r}_1 \), i.e., it increases the kinetic energy of drift. The two terms are of the same magnitude with opposite signs. Therefore, \( \mathbf{B}_e \) makes an ignorable contribution to the total kinetic energy. The kinetic energy of electrons along the z-axis originates from the incident wave. The electrons are accelerated by the incident wave in the XY plane. Meanwhile, the inhomogeneous magnetic field transfers the kinetic energy of electrons in the XY plane into the Z direction.

\( \mathcal{F} \) contains the contributions of the dipole magnetic field inhomogeneities and the electromagnetic wave magnetic field components, and is defined as the following equation:

\[
\mathcal{F} = q (q \frac{d\mathbf{r}_0}{dt} \times \mathbf{B}) + q (q \frac{d\mathbf{r}_0}{dt} \times \mathbf{B}_1). \tag{21}
\]

Then, (13) can be written in the form of:

\[
\frac{d^2 \mathbf{r}_1}{dt^2} = \frac{\mathcal{F}}{m} - v \frac{d\mathbf{r}_1}{dt}. \tag{22}
\]

It can be solved such that \( \mathbf{r}_1 = \frac{\mathcal{F}}{mv} t. \) Therefore:

\[
\mathcal{F} \frac{dr_1}{dt} = mv \frac{dr_1}{dt} \frac{dr_1}{dt} + \frac{d}{dt} \frac{1}{2} (\mathbf{r}_1 \cdot \mathbf{r}_1) = \frac{\mathcal{F}^2}{mv}. \tag{23}
\]

This part of the electron movement yields dissipation of transmitting signals only, i.e., it will not transmit a receivable signal.

The total signal attenuation [20] is defined as:

\[
\tilde{A} t = \frac{|E_0|^2}{|E_l|^2} = e \tilde{E}_0 t_{max} z d t, \tag{24}
\]
where $L_{\text{max}}$ is the total length of wave propagation path within the plasma sheath. $|E_l|$ and $|E_0|$ are the amplitudes of the electric fields of the transmitted wave and incident wave, respectively. If the whole propagation path is divided into $N_L$ layers, the thickness of each layer is $L = L_{\text{max}}/N_L$. The propagation time of the EM wave in the layer of thickness $L$ is about $\tau = L \beta / \omega_0$. If $N_L$ is large enough, $\frac{\partial B}{\partial r} |_0$, $B_0$, $N$, $E$, and $\upsilon$ can be treated as constants in each single layer. Hence:

$$ A \tau \approx e^{\sum_{l=1}^{N_L} 2\alpha_l(l-1)/L}, \quad (25) $$

$$ \upsilon \approx W / \int_0^\tau m \frac{d\rho}{dt} \cdot \frac{d\rho}{dt} dt, \quad (26) $$

where $\alpha_l$ is the attenuation coefficient at $l = (n-1)L$, and $W$ is the energy dissipated by collision damping. According to (23) and (26), the corrected damping $\upsilon'$ is:

$$ \upsilon' = \frac{W'}{\int_0^\tau m \frac{d\rho}{dt} \cdot \frac{d\rho}{dt} dt} \approx \upsilon + \frac{\frac{\tau^3}{3\upsilon}}{m \frac{d\rho}{dt} \cdot \frac{d\rho}{dt}}. \quad (27) $$

Obviously, $\upsilon'$ consists of two parts, which are the damping $\upsilon$ caused by the collision and the damping $\upsilon' - \upsilon$ caused by $\mathcal{F}$, respectively.

### 3. Results and Analysis

The dipole magnetic field involved in the present study is supposed to be generated by a coil. According to (4), the magnetic field intensity along the wave propagation path is:

$$ B_\theta = \frac{\mu_0 I}{2a} \frac{a^3}{(a^2 + z^2)^{3/2}} e_z = B_e \frac{a^3}{(a^2 + z^2)^{3/2}} e_z, \quad (28) $$

where $I$ is the current in the coil, and $B_e$ is the magnetic field strength at the center of the coil.

In the present study, the plasma sheath parameters are obtained with a statistical model based on the RAM C-II experiment [16]. In particular, in the present study, the concerned plasma parameters are along the wave propagation path. Those parameters are shown in Figure 2a. The magnetic field intensity along the wave propagation path is shown in Figure 2b. The total length for the wave propagation path is $L_{\text{max}} = 0.23$ m.

Figure 2c,d shows an example of wave propagation in the given plasma layer. In that example, the incident wave frequency $f = 94$ GHz, $a = 0.01$ m, and $B_e = 1$ T.

In Figure 2c, $E$ denotes the amplitude of the electric field of the propagating wave by considering the dissipation yielded by $\upsilon$. $E'$ is the electric field by considering the dissipation yielded by $\upsilon'$. The red line indicates the difference between the two. It can be seen that the magnitudes of $E' - E$ are only $10^{-4} e_0$. In addition, according to the blue line, the magnitudes of the damping $\upsilon' - \upsilon$ are $10^0$, while, according to Figure 2a, the magnitudes of $\upsilon$ are $10^{10}$. In such a case, the collisions are still the dominant cause of signal attenuation in the magnetized plasma sheath. The drift caused by $\mathcal{F}$ makes little contribution to the signal attenuation. The dipole magnetic field mainly affects the attenuation via its spatial inhomogeneity, which is illustrated in detail in Figure 2d.

In Figure 2d, the attenuation coefficients in plasma sheaths magnetized by a uniform magnetic field and a dipole magnetic field are represented by green and blue lines, respectively. The attenuation coefficient in an unmagnetized plasma sheath is denoted by the red line, which almost coincides with the blue line. In other words, the dipole magnetic field makes an ignorable contribution to the weakening of the signal attenuation. On the other hand, the uniform magnetic field is obviously weakened by the signal attenuation in the plasma sheath. The reasons for such differences could be found from Figure 2.

According to Figure 2a,d, it can be seen that the spatial distributions of the attenuation coefficient and the electron densities are similar. Both the maxima of the attenuation
coefficient and the electron density along the wave propagation path is at \( L = 0.03 \) m, regardless of the external magnetic field. In other words, the energy loss of the propagating wave mainly occurs in a very small region around \( L = 0.03 \) m. On the other hand, according to Figure 2b, the dipole magnetic field strength decreases with \( L \) increasing. The maximum field strength is at \( L = 0 \), which is the wall of the vehicle. The strong magnetic field region is beyond the region of wave energy loss. In addition, the field strength around \( L = 0.03 \) m is too weak to effectively mitigate the wave energy loss. On the other hand, under the assumption of a uniform external magnetic field, the wave energy loss is always suppressed by the magnetic field over the whole propagation path. Therefore, the dipole magnetic field makes a poor contribution to the weakening of the wave attenuation, compared to a uniform magnetic field.

![Figure 2. Parameter distributions in the plasma sheath. (a) \( N_e \) and \( \nu \), (b) \( B_0 \), (c) \( E' - E \) and \( \nu' - \nu \), (d) \( \alpha \).](image)

Nevertheless, it should be noted that a uniform magnetic field is only an idealized assumption, although it was employed in some previous works and showed good performance for mitigating the wave attenuation. In a realistic scenario, a dipole magnetic field generated by the coil is more feasible. Thus, it is necessary to investigate how the wave attenuation varies with the parameters of the dipole field.

Figure 3a illustrates how the attenuation coefficient \( Att \) varies with the radius of the coil and the incident wave frequency under the condition of \( B_c = 1 \) T. As shown in the Figure, \( Att \) decreases as the wave frequency increases. Additionally, Figure 3a shows that the \( Att \) decreases as the coil radius increases. Yet, once the coil radius is greater than 0.06 m, the \( Att \) decreases with the coil radius increasing more slowly than that of the coil with a radius smaller than 0.06 m. In such a case, the coil radius is suggested to be around 0.06 m, according to the present study.
Figure 3. (a) $Att$ against $a$ and $f$, $B_c = 1$ T. (b) $Att$ against $B_c$ and $f$, $a = 0.06$ m. (c) $Att$ against $B_c$ and $a$, $f = 94$ GHz.

Figure 3b illustrates how the $Att$ varies with $B_c$ and wave frequency at a fixed coil radius of $a = 0.06$ m. As shown in the figure, $Att$ decreases as the wave frequency increases. In addition, $Att$ decreases as $B_c$ increases. It can be seen that the $Att$ of the 140-GHz and 225-GHz waves is less than 3 dB in the absence of a magnetic field. However, the
attenuation of the 225-GHz waves is relatively higher in a neutral atmosphere, which is not a good choice; the $Att$ of 94-GHz waves can reach 3 dB under a relatively weak external magnetic field, while 77-GHz waves require a strong external magnetic field. When the external magnetic field reaches 5 T, the $Att$ of 35-GHz waves is still greater than 3 dB. Therefore, the suggested carrier frequency is 94 GHz or 140 GHz, according to the present study.

Figure 3c illustrates how the $Att$ varies with $B_c$ and the radius of the coil when the wave frequency is $f = 94$ GHz. As shown in Figure 3c, the $Att$ decreases with the increase of the coil radius and the $B_c$. It can be seen that, while $B_c$ is small, the $Att$ is less affected by the coil radius. The 94-GHz waves obtain a 3-dB absorption coefficient, even if the coil size exceeds 0.1 m; $B_c$ needs to be close to 1 T. This is difficult to achieve for the current onboard magnetic field. Therefore, it is recommended to use 140 GHz as the carrier frequency.

4. Discussion

Previous studies have shown that an external magnetic field could help to mitigate the signal attenuation in hypersonic plasma sheaths. Nevertheless, most of previous work assumed that the external magnetic field is uniform. It should be realized that this is an idealized assumption. In realistic scenarios, strong magnetic fields are generated by the coil or a permanent magnet. On the other hand, only rarely could permanent magnets work stably in high-temperature environments. The temperature of the hypersonic plasma sheath could be up to thousands of Kelvins, which is not an appropriate working environment for a permanent magnet. Therefore, the magnetic field generated by the coil may be the only choice for hypersonic vehicles.

On the other hand, the present study has shown that, although the efficiency for the dipole magnetic field to weaken the attenuation is lower than that of a uniform magnetic field, it is still effective. Therefore, weakening the signal attenuation with a dipole magnetic field generated by the coil is a reasonable and possible approach to mitigate the communication blackout problem.

In addition, the present study has shown that a new dissipation yielded by the dipole magnetic field and the magnetic field component of an electromagnetic wave for high-frequency waves is ignorable, compared to the dissipation yielded by the collision between free electrons and neutral particles. Nevertheless, high-frequency communication is not the only possible approach to mitigate the “blackout”. Low-frequency waves, which could penetrate the plasma sheath via the skin depth effect, is also considered a potential solution to the blackout problem. Once the carrier wave is in the low-frequency (LF) band, the new dissipation could be significant. Moreover, the present study assumed that the wave vector is always parallel to the normal direction of the coil, which is not always true in realistic scenarios. Once the wave is propagating obliquely, the new dissipation may need to be taken into account.

5. Summary and Conclusions

The present study developed a wave propagation model based on the electron motion equation in a hypersonic plasma sheath magnetized by a dipole magnetic field. The dipole magnetic field is considered to be generated by coils. The mechanic for wave dissipation is analyzed based on the present model, and the effects of uniform and dipole magnetic fields on the mitigation of wave attenuation are compared. In addition, the impacts of the parameters of the coils are investigated. The conclusions are drawn below.

The wave dissipation in hypersonic plasma sheaths is mainly dominated by the absorption due to the collisions between free electrons and neutral particles. A dipole magnetic field generated by a coil is not as efficient as a uniform magnetic field for mitigating the wave attenuation for the blunt-coned plasma sheath. The reason is that the attenuation region in the plasma sheath is beyond the region of the maximum magnetic field. Nevertheless, a uniform magnetic field is not feasible enough in realistic scenarios. In such a case, the dipole magnetic field generated by coils could be a potential choice to mitigate high-
frequency wave attenuation in hypersonic plasma sheaths. The effects of dipole magnetic fields on wave attenuation mitigation dramatically increases with the coil radius while the radius is smaller than 0.06 m. The present study suggested that the communication system operating at 94 GHz or 140 GHz with a coil whose radius is 0.06 m could achieve good performance in terms of mitigating the communication blackout problem for hypersonic vehicles.

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