Influence of external perturbations on the interaction between grains in plasma

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Abstract. An experimental study of dust grain dynamics in plasma was carried out for cluster systems relaxing to their equilibrium state after the action of a laser beam. The influence of this external perturbation on the potential between interacting dust grains was analyzed. We found that the spatial asymptotes of the pair potential for perturbed and equilibrium dust systems differ. The possible reasons for this phenomenon are discussed.

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1. Introduction

A study of the inter-grain interactions for dusty plasma (consisting of electrons, ions, neutral gases and dust grains of micron size) is of great interest. Information on the interaction potential profile is necessary for an analysis of various physical characteristics (pressure, internal energy, viscosity, etc) and is essential for understanding the nature of phase transitions in dust subsystems of complex plasma [1–3]. Most of the analytical models, proposed for the conditions of laboratory dusty plasma, are based on consistent solution of a Poisson equation and a kinetic equation for the ion component [3–7]. In this case practically all of the diversity of the proposed electrostatic potentials $\phi(l)$ may be presented in the form of a simple approximation for the pair interaction energy $U(l) = eZ\phi(l)$ [3–7]:

$$U(l) = \sum_{i=1}^{2} \{ a_i l^{-i} + b_i l^{-i} \exp \left( -l/\lambda_i \right) \}. \tag{1}$$

Here $eZ$ is the dust charge; $l$ is the distance between interacting grains; $a_i$, $b_i$ and $\lambda_i$ are the coefficients depending on the parameters of dusty plasma.

For determining the pair potential and the charge for dust grains in plasma, various external perturbations are often used with subsequent numerical analysis of the dynamic response of the investigated system [5, 8–12]. For the passive diagnostics of pair interaction between dust particles, approximate solutions to integral equations of the statistical theory are also used [13–16]. The method developed for the analysis of equilibrium systems [17, 18], which is based on solving the inverse problem for an overdetermined system of motion equations of interacting particles, has a number of advantages. This technique allows one to recover a spatial distribution of pair interaction forces (and, accordingly, the pair potential) and parameters of an external confining potential. Unlike the methods mentioned previously, this technique does not perturb the analyzed systems, does not require any preliminary information (on acting forces) except information on the displacements of particles, and can be applied as well to strongly correlated as to fluid dust systems. Nevertheless, at the present time, the problem of the form of interaction potential between dust in plasma (including the influence of external perturbations) requires further investigations.

In this paper, we present the results of an experimental study of dust particle dynamics in the cluster systems relaxing to its equilibrium state after the external action. The physical characteristics of dust clouds (including the pair interaction between dust particles) for its equilibrium and perturbed states were analyzed.

2. Experiment and results

Experiments were carried out for two-dimensional (2D) cluster systems, consisting of 27 ($N_p = 27$) monodisperse particles (material density $\rho \approx 1.5 \text{ g cm}^{-3}$, radius $a = 3.92 \mu\text{m}$), in a weakly ionized argon plasma of a capacitive RF discharge at pressures $P = 0.04–0.15$ Torr and discharge power $W \sim 2–6$ W. The simplified scheme of the experimental setup is shown in figure 1. To form a radial trap for dust clouds, a metallic ring of diameter 3 cm and height 0.2 cm was mounted on the lower electrode. To visualize a process, we illuminated the dust clusters by a horizontally spread plane laser sheet of vertical thickness $\sim 300 \mu\text{m}$ provided by a 660 nm diode laser with a radiation power $\sim 100$ mW. Dust trajectories in the horizontal plane were registered by a high-definition (HD) camera with a frame rate of $\sim 300–400 \text{s}^{-1}$. To evaluate
the vertical displacements of the particles, an additional video camera was installed on the side. Video data were processed by special software, which allowed us to identify the positions of individual dust particles.

As a source of disturbance of the dust subsystem, we used a beam of an Ar$^+$ laser with a radiation power of $\sim 4$ W. Due to the laser exposure the dust cluster displaced along the beam and its ordered structure became disordered, see figure 2(a). Pulse duration was 1–2 s, which was enough for a transition to a new excited state. Magnifying laser power has led to a more significant shift of the cloud of particles and to an increase in their initial kinetic energy after removal of the laser influence, but this increasing did not lead to a significant difference of the results. After removal of the external influence, the dust cloud relaxed to its equilibrium (stationary) state. Illustrations of the particle’s positions in the dust cluster depending on time $t$ after switching off the Ar$^+$ laser are presented in figures 2(a)–(d). Here we assume that the characteristic relaxation time $t_r$ is a time of kinetic energy damping in $e^2$ times after removal of the external influence. The experimental pair correlation functions $g(l)$ for an excited dust cloud (averaged for $t < t_r$) and for a stationary structure (at $t \gg t_r$) are presented in figure 2(e).

Figures 3(a) and (b) illustrate the particle kinetic energy reduction during their relaxation to an equilibrium state as the normalized values $K^* = MV^2/(2T)$ versus time; here $V$ is the grain velocity in the horizontal plane and $T$ is the kinetic temperature of grains related to their stochastic (thermal) motions in the equilibrium state. The thick color lines in these figures mark an approximation of $K^*$ by the function $f(t) = A_1 + A_2 \exp(-\chi_r t)$ (here $A_1 = 1$ is the $K^*$ value in a stationary state, and the sum of $(A_1 + A_2)$ is determined by the maximum value of $K^*$) for various time intervals: blue curve: from 0 to $t \gg t_r$; red curve: from $0.5t_r$ to $t \gg t_r$. The $\chi_r$ values obtained for various time intervals are close to constant within 10% errors for both experiments presented in figures 3(a) and (b). Thus, the damping is proportional to $\propto \exp(-\chi_r t)$, where $2/\chi_r = t_r$. A similar asymptotic behavior of the energy relaxation was found in the investigations of a plasma crystal perturbation by applying a short electric pulse to wires located close to the particles [19] and by introducing a modulation to the powered lower electrode [20].

For diagnostics of the thermal velocity of dust particles $V_T = (2T/M)^{1/2}$, their friction coefficient $\nu_{fr}$ and the effective coupling parameter $\Gamma^* = 1.5\nu_{pn}U''/2T$ [21], we used the
Figure 2. Dispositions of dust particles versus time \( t \) after the laser’s perturbation (a)–(d) and pair correlation functions (e) averaged for \( t < t_r \) and for stationary structure (\( t \gg t_r \)) in the experiments at \( W = 4 \) W and \( P = 0.04 \) Torr. Panel (a) shows an excited dust cloud due to laser exposure (\( t = 0 \) corresponds to the moment of switching off the laser).

Figure 3. Normalized grain kinetic energies \( K^* \) (thin lines) during relaxation of dust clouds in the experiments at \( W = 4 \) W and \( P \): (a) 0.135 Torr and (b) 0.04 Torr. The thick color lines mark an approximation of \( K^* \) by the function \( f(t) = A_1 + A_2 \exp(-\chi_r t) \) for different time intervals: blue curve: from 0 to \( t \gg t_r \); red curve: from \( t_r \) to \( t \gg t_r \).

This technique is based on an analysis of mass-transfer processes for a short observation time [22]; here \( M \) and \( T \) are the particle mass and temperature and \( U^{''} \equiv U^''(l_{pm}) \) is the second derivative of a pair potential energy at the point of the most probable interparticle distances \( l_{pm} \), which corresponds to the first maximum of the pair correlation function (see figure 2(d)). This technique is based on the analogies of particle dynamics (namely, the velocity autocorrelation function).
functions \( \langle V(0)V(t) \rangle \) and the time evolution of mean square displacements \( \langle x^2(t) \rangle, \langle y^2(t) \rangle \) in liquid and crystalline structures and allows one to obtain the grain temperature \( T \), the friction coefficient \( v_t \) and the value of \( U''(l_{pm}) \) by the best fit of the measured functions \( \langle V(0)V(t) \rangle, \langle x^2(t) \rangle, \langle y^2(t) \rangle \) and the corresponding analytical solutions for the harmonic oscillator at short observation times. The obtained dust parameters together with the values of \( l_{pm} \) are given in tables 1 and 2 for some experiments considered here for illustrating our results.

In order to determine the characteristic frequencies of the trap \( \omega = (eZ\alpha/M)^{1/2} \) (here \( \alpha \) is the gradient of the trap field) and profiles of the pair interaction force \( F_{int}(l) \equiv -\partial U/\partial l \) for perturbed and equilibrium structures, we used the method based on solving an overdetermined set of motion equations of dust particles [17, 18]

\[
\frac{d^2\vec{l}_k}{dt^2} = -v_t \frac{d\vec{l}_k}{dt} + \frac{1}{M} \left[ \sum_j F_{int}(\vec{l}_k - \vec{l}_j) + F_{ext}(\alpha, \vec{l}_k) \right]
\]

(2)

consisting of \( \sim N_p \times (\Delta t/dt) \) equations; here \( N_p \) is the number of dust particles in an analyzed system, \( \Delta t \) is the duration of particle motion, \( dt \) is the time step related to the videoregistration frame rate, \( \vec{l}_k(t) \) is the experimentally measured trajectories of each of all \( N_p \) dust particles, and the unknown interaction and external confining forces \( (F_{int}, F_{ext}) \) are approximated by splines or various combinations of power-law and exponential functions [17]. Applying this method to a perturbed dust structure (by analyzing the motion of dust particles after an external influence) has several advantages over its use for equilibrium systems. The main advantages are [23]: many fewer experimental data for solving the inverse problem (shorter duration of the experiment); lower requirements for temporal and spatial resolution of particle motion; a possibility of recovering the interaction potential profile at distances considerably smaller than the mean interparticle distance.

| Table 1. The dust structure parameters \((T, l_{pm}, Z, \lambda, \Gamma^*)\) for the experiments at different pressures \(P\) and an RF discharge power \(W = 4\) W. |
|---|---|---|---|---|
| \(P\) (Torr) | \(T\) (eV) | \(l_{pm}\) (mm) | \(Z/1000\) | \(\lambda\) (mm) | \(\Gamma^*\) |
| 0.135 | 0.7 ± 0.1 | 0.775 | 9.8 ± 0.50 | 1.45 ± 0.07 | 345 ± 30 |
| 0.10 | 0.8 ± 0.1 | 0.815 | 10.2 ± 0.50 | 1.35 ± 0.07 | 315 ± 30 |
| 0.07 | 0.9 ± 0.12 | 0.875 | 10.7 ± 0.6 | 1.3 ± 0.06 | 290 ± 30 |
| 0.04 | 1 ± 0.13 | 0.925 | 11.5 ± 0.6 | 1.25 ± 0.06 | 270 ± 30 |

| Table 2. The friction coefficient \(v_t\) and the characteristic frequency of the trap \(\omega_{1..3} = (eZ\alpha/M)^{1/2}\), recovered by various approximations (see the text) for the experiments at different pressures \(P\) and an RF discharge power \(W = 4\) W. |
|---|---|---|---|---|
| \(P\) (Torr) | \(v_t\) (s\(^{-1}\)) | \(\omega_1\) (s\(^{-1}\)) | \(\omega_2\) (s\(^{-1}\)) | \(\omega_3\) (s\(^{-1}\)) |
| 0.135 | 24 ± 1.5 | 9.5 ± 0.3 | 11.5 ± 0.3 | 10.5 ± 0.5 |
| 0.10 | 19 ± 1.2 | 9.3 ± 0.3 | 11.0 ± 0.3 | 10.5 ± 0.5 |
| 0.07 | 14 ± 1.2 | 9.1 ± 0.3 | 10.5 ± 0.3 | 10.0 ± 0.5 |
| 0.04 | 8.0 ± 0.5 | 9.0 ± 0.3 | 10.0 ± 0.3 | 9.5 ± 0.5 |
Estimates of the characteristic frequency of the trap $\omega_{1,3} = (eZ\alpha/M)^{1/2}$, experimentally obtained for the perturbed and equilibrium systems ($\omega_1$ and $\omega_2$, respectively) as well as analytically calculated ($\omega_2$), are shown in Table 2. The theoretical estimate $\omega_3$ has been obtained on the basis of approximation for the trap gradient $\alpha$ in the case of an equilibrium cluster of particles interacting via a Yukawa-type screened potential [24]:

$$\alpha \approx \frac{2\pi e Z_p}{(N + 1)^3} \left[ 1 - \left( 1 + N\kappa \right) \exp \left( -(N + 1)\kappa \right) \right] + \frac{\kappa \exp \left( -\kappa \right) \left[ 1 - \exp \left( -N\kappa \right) \right]}{\left[ 1 - \exp \left( -\kappa \right) \right]^2} - 1. \tag{3a}$$

Here $N = R/l_{pm} \propto (N_p/\pi)^{1/2}$ is the number of the characteristic distances within the radius $R$ of the dust clusters, and $\kappa = l_{pm}/\lambda$, where $\lambda$ is the screening length. For a system of particles interacting via the Coulomb potential ($\kappa \rightarrow 0$) or for the case of $N\kappa < 1$, equation (3a) is reduced to

$$\alpha \approx \frac{2\pi e Z_p}{(N + 1)^3} l_{pm}^2. \tag{3b}$$

An additional estimate of the characteristic trap frequency $\omega_4 = (eZ\alpha/M)^{1/2}$ has been obtained from our experimental data by using numerical studies of the rate of a finite cloud relaxation to its equilibrium state after switching off the external influence on the system [23]:

$$\chi_t = \nu_t - 2\sqrt{\frac{v_t^2}{4} - \omega_4^2} \quad \text{for } \omega_4 < \nu_t/2, \tag{4a}$$

$$\chi_t = \nu_t \quad \text{for } \omega_4 > \nu_t/2. \tag{4b}$$

Since the characteristic frequency of the trap can be obtained by (4) only for $\omega_4 < \nu_t/2$, we have made such an estimate for the plasma trap at pressure $P = 0.135$ Torr ($v_t = 24 \pm 1.5$ s$^{-1}$). The obtained value $\omega_4 = 10.0 \pm 1.0$ s$^{-1}$ coincides with other estimates for this pressure, see $\omega_{1,3}$ in Table 2. It should also be noted that when $\omega_4 > \nu_t/2$ measuring the relaxation rate $\chi_t$ can provide an additional independent way of estimating the friction coefficient $\nu_t$.

Pair interaction forces $F_{\text{int}}(l)$ obtained by solving the inverse problem (2) are plotted in figures 4(a) and (b) for the experiments at RF discharge power $W = 4$ W and various pressures. Changing the discharge power in the range from 2 to 6 W did not have a noticeable effect on the results. Recovery of the force profiles has been carried out for various time intervals $\Delta t$: from 0 to 0.5$t_2$, from 0.5$t_2$ to $t_r$, and for $t \gg t_r$. Approximations of the experimental data by the Coulomb $f(l) = (eZ/l)^2$ and Debye $f(l) = (eZ/l)^2 \exp(-l/\lambda)(1 + l/\lambda)$ forces are shown in figure 5. The estimations of dust charges $Z$ and screening length $\lambda$, obtained by these fits, are given in Table 1.

3. Discussion and conclusion

Table 1 shows that as the pressure increases, the value of $Z$ slightly decreases and the value of $\lambda$ grows. Meanwhile, under the conditions of a steady-state (unperturbed) dust cloud, the temperature $T$ of dust particles and the average distance $l_{pm}$ between them are decreasing, and the coupling parameter $\Gamma^*$ is increasing. Note that the temperature of dust particles $T$ is 20–30 times as high as the typical room temperature. Such an anomalously high kinetic temperature of the particles is observed in most of the experiments with gas discharge plasma [3, 21]. Most
Figure 4. Profiles of $F_{\text{int}}(l)/M$ in the experiments at $W = 4$ W and $P = 0.135$ Torr (a) and $P = 0.04$ Torr (b) recovered for various time intervals: 1: $t \gg t_r$; 2: from $0.5t_r$ to $t_r$; 3: from 0 to $0.5t_r$.

Figure 5. Recovered profiles of $F_{\text{int}}(l)/M$ (symbols) for: 1: $t \gg t_r$, $P = 0.04$ Torr; 2: $t \gg t_r$, $P = 0.135$ Torr; 3: $0 > t > 0.5t_r$, $P = 0.04$ Torr; 4: $0 > t > 0.5t_r$, $P = 0.135$ Torr. The solid lines are the fits by: 1 and 2: Coulomb functions; 3: Debye force with $\lambda = 0.125$ cm; 4: Debye force with $\lambda = 0.145$ cm.

probably, this effect is caused by fluctuations of the electric field due to spatial and temporal variations of the particle charge in plasma [3, 21, 25–28].

Let us now consider the measured $\omega_1$, $\omega_2$ and $\omega_3$ for different pressures (see table 2). It can be easily seen that as the pressure increases, all the values of the characteristic trap frequency are slightly increasing (within 5–10%). Since the observed changes are small and comparable with the experimental errors, we cannot conclude that they are caused by specific physical processes. Nevertheless, it should be noted that gradients of electric fields in RF-discharge plasma usually
increase with increasing pressure [29]. As for the differences in the frequencies $\omega_1$, $\omega_2$ and $\omega_3$ obtained for a fixed gas pressure, they are also small (5–10%) and comparable with the experimental errors.

It can be easily seen that in figure 4 the shape of $F_{\text{int}}(l)$ (and of the pair potential, respectively) recovered for equilibrium and perturbed systems has a different spatial asymptotics. The simplest explanation of the observed phenomenon could be related to limitations of the 2D diagnostics for real three-dimensional systems. For example, if there are unaccounted for particle displacements in a direction perpendicular to the plane of the dust cloud, the measured interparticle distance may be lower than the real one. Then the force, determined at these ‘undervalued’ distances, seems to be less than its real value, and its profile may be distorted and seem to be Debye-like. However, the observations from the side-view camera showed that the variance of the vertical displacements of particles $\delta z$ under the laser excitation did not exceed 9% of their typical interparticle distance $l_{pm}$. In this case, an error in determining the interparticle distance, related to vertical displacements of particles, can be estimated as $\delta l/l_{pm} \approx 0.5(\delta z/l_{pm})^2 \approx 0.005$, which is comparable with the spatial resolution of the HD camera used for the registration of dust positions in the horizontal plane. Thus, the reason for the observed phenomenon is probably deeper and fundamental.

Since the parameters of the trap recovered for perturbed and stationary dust clouds do not change significantly (see $\omega_1$ and $\omega_2$ in table 2) we can assume that the change in the potential profile is not related to a significant difference in the parameters of the surrounding plasma. Such changes in the interaction potential may be caused by qualitative differences in the statistical state between the perturbed and equilibrium systems (see the pair correlation functions in figure 2(d)). So, for example, the ‘Debye’ asymptotics of the pair potential was observed in the experiments for measuring the radial pair interaction in the perturbed systems of two particles in plasma of capacitive RF [8] and dc [9] discharges. A similar asymptotic behavior was found in studies of the interaction between ‘large’ probe particles and smaller particles of dust cloud (perturbed by the probe one) levitating in plasma of inductive RF discharge [10]. Finally, the Debye potential was observed in the experimental investigations of weakly correlated dust structures presented in [13–16]. In contrast to the mentioned results, the measurements of spatial distribution of the interaction potential in equilibrium strongly correlated dust systems revealed a characteristic Coulomb asymptotic behavior of the potential profile at distances greater than one or two mean interparticle distances [17, 18]. As for a theory, the asymptotic behavior of a spatial distribution of electrostatic potential around a particle in a collisional isotropic plasma has been studied previously [6, 30–34]. In particular, the models proposed in [6, 34] predict the unscreened Coulomb asymptotic behavior of the potential profile at large distances (exceeding the ion mean free path) from the particle in a weakly collisional isotropic plasma. However, the mentioned models were proposed for an isolated particle and do not account for the surrounding dust particles.

In summary, the results of the experimental study of the dust particle’s dynamics are presented for cluster systems, relaxing to their equilibrium state after an external influence on the system. The interaction forces between dust particles have been measured. It is found that the shape of the pair potential recovered for equilibrium and perturbed systems has a different spatial asymptotics. This may be related to the fact that the same dust ensemble in a particular plasma environment exhibits both types of interaction behavior depending on whether it is in an equilibrium state or a non-equilibrium state. This assumption may explain why several previous experimental investigations reported either one or the other interaction behavior.
Another possible reason for this phenomenon may be the qualitatively different statistical state of the investigated systems, because the distribution of plasma ions/electrons around a charged dust particle may depend on the correlation of surrounding dust particles having about its own distribution of the plasma particles. Note, for example, that the power law $F_{\text{int}}(l) \propto l^{-2}$, obtained for the ordered unperturbed dust subsystem, can provide a proof of validity of the Wigner–Seitz-cell model in the strongly ordered systems [35, 36].

As the phenomenon described in this paper is observed for the first time, for its consistent and correct analysis we need a numerical simulation of the problem taking into account all charged plasma particles: dust, electrons and ions. (The present analytical models and their numerical verification apply only to the case of an isolated dust particle.) The final clarification of the issue is a topic for future theoretical, numerical and experimental studies.

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