Quantum Limited displacement sensing in 3D cavity optomechanics

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Ultra-high sensitivity detection of quantum-scale displacements in cavity optomechanics ideally optimises the combined errors from measurement back-action and those from imprecision from laser quantum shot noise. This sets the well-known Standard Quantum Limit (SQL). Normally quantum cavity optomechanics deals with the cooling and detection of a single degree of freedom, typically along the cavity axis. However, recent cavity optomechanics experiments with optically trapped, levitated nanoparticles, are uniquely 3D in character and can exhibit very strong optomechanical coupling between an optical mode of the cavity and centre of mass motion in every direction. We investigate this scenario here and show that the sensing analysis is typically far from the straightforward addition of independent contributions along the \( \hat{x} \), \( \hat{y} \) and \( \hat{z} \) axes. We identify additional 3D contributions that transfer energy and measurement back-action between the degrees of freedom. We show it is possible to effectively suppress 3D terms at parameters where there is near complete destructive interference between different coupling pathways, in order to approach and even surpass the SQL.

The coupling of mechanical motion to the optical mode of a cavity permits not only strong cooling but also ultra-sensitive detection displacement and has led to breakthroughs ranging from ground state cooling of a mechanical oscillator [1][2] to the detection of gravitational waves [3] by LIGO. The parallel field of optomechanics employing levitated dielectric particles began later but has also experienced rapid development [4][5]. The unique potential of levitated cavity optomechanics in terms of decoupling from environmental heating and decoherence, coupled with the sensitivity of displacement sensing offered by optical cavities was already recognised in 2010 [6][8]. Actual experimental realisations were hindered by formidable technical challenges: the levitated nanoparticles must be cooled from room temperatures, initially millions of quanta above the quantum ground state. Most initial proposals were for self-trapping set-ups [7][9][10], with trapping and cooling both provided by the cavity modes [11], but this failed to overcome the key challenge to trap stably at high vacuum [10][12].

In order to overcome this roadblock, hybrid set-ups combining for instance a tweezer and cavity traps [6][13]; or a hybrid electro-optical trap [13][15], or a tweezer and near-field of a photonic crystal [16], allowed some progress towards the ultimate goal of quantum ground state cooling. Some of these hybrid set-ups allow for fully 3D cavity mechanics dynamics with optomechanical coupling constants along every axis \( g_x, g_y, g_z \): in effect a displacement of the nanoparticle, even if perpendicular to the intercavity axis can correspond to an apparent change in the “length” of the cavity or rather effect a change in its resonant frequency. Typically, non-axial coupling is weak.

However this year a very significant breakthrough was provided by the realisation that the light coherently scattered (CS) into an undriven cavity has major advantages [17][19]: the resulting optomechanical couplings are extremely strong even for comparatively low mean cavity photon numbers, minimising the deleterious effects of photon scattering. [20][22]. As a result, strong 3D coupling and thus nanoparticle cooling along every direction to, or near, quantum regimes is now in view. This motivates our study of 3D displacement sensing, focussed on 3D optical trapped system in particular for the strong couplings offered by CS-cavity levitation. In particular, in this work we demonstrate the importance of considering the complete linearised Hamiltonian, with previously overlooked direct coupling terms \( g_{xy}, g_{xz}, g_{yz} \) without which the cooling and sensing dynamics cannot be fully understood.

Displacement sensing. — For a cavity mode \( \hat{a} \), standard optomechanical displacement sensing will involve a measurement of some quadrature of the optical field \( \hat{Q}_q = e^{-i\phi} \hat{a} + e^{i\phi} \hat{a}^\dagger \), with coupling to a mechanical displacement \( \hat{q} \), usually set by the cavity axis, with coupling strength \( g \) described by the well known equation of linearised optomechanics:

\[
\dot{\hat{Q}}_q(\omega) = i\eta_q(\omega)\hat{q}(\omega) + \sqrt{\kappa}\hat{Q}_q^{in}(\omega)
\]

where \( \dot{\hat{Q}}_q^{in} \) represent measurement imprecision, typically from incoming quantum photon shot-noise fluctuations, while \( \kappa \) is the cavity linewidth. The optical susceptibility, describing the shape of the cavity resonance, is \( \eta_q = e^{-i\phi}\chi(\omega,\Delta) - e^{i\phi}\chi^*(-\omega,\Delta) \) where \( \chi(\omega,\Delta) = -i(\omega + \Delta) + \frac{\kappa}{2} \) and \( \Delta \) is the detuning of the light from the cavity resonance. Understanding the SQL of displacement sensing in optomechanics usually proceeds via analysis of errors in Eq.[1] or related form.

In the 3D case, the measured optical quadrature in general now couples to displacements \( \hat{q}_j \) along all directions \( j = x, y, z \):

\[
\dot{\hat{Q}}_q(\omega) = i\sum_j \eta_{q_j} g_j \hat{q}_j(\omega) + \sqrt{\kappa}\hat{Q}_q^{in}(\omega)
\]
where $\Phi_j \equiv \Phi$ for the normal optomechanical case where displacement couples to amplitude of the light, but $\Phi_j \equiv \Phi - \pi/2$ for the new scenario in the 3D levitated experiments \cite{20,22} where it can couple to the optical phase quadrature.

With a simple adjustment to relate the intracavity field to the cavity output field via input-output relations, the corresponding PSD of the measured signal is used to estimate a displacement spectrum $S_{\Delta q\Delta q} \simeq \gamma^2 \nu g_j^2 S^1D_j$ in the 1D case. A key question is whether one might straightforwardly extend to the 3D displacement spectra by simply considering the sum of the independent PSD contributions $S_{\Delta q\Delta q} \simeq \sum_{j=x,y,z} \gamma^2 \nu g_j^2 S^1D_j$. We show here that this is not the case. We find that the most significant differences are new 3D back-action terms that redistribute energy between mechanical modes and add additional optical back-action; attaining the 3D SQL may require suppression of these terms. We show also that even if the $\tilde{q}_j$ were independent the separate back-action contributions are correlated and interfere.

In the standard 1D quantum theory of cavity optomechanics, the displacement spectra are calculated from cavity amplified noise fluctuations $\tilde{D}_j^{1D}$, comprising thermal fluctuations of the mechanical modes plus, in addition, fluctuations representing the back-action effect of the incoming photon shot-noise. Neglecting certain normalisation terms (see \cite{23} for full-details) we have:

$$\tilde{q}_j(\omega) \equiv \tilde{D}_j^{1D} \simeq \sqrt{\Gamma} \tilde{Q}_j^{\text{therm}}(\omega) + i \sqrt{\nu} g_j \mu_j(\omega) \tilde{Q}_j^{\text{in}}(\Phi=0)$$

where $\Gamma$ is a mechanical damping. The $\mu_j(\omega, \omega_j) = \chi(\omega, \omega_j) - \chi^*(\omega, \omega_j)$ is a mechanical susceptibility function that determines the back-action spectrum generated by incoming quantum shot noise $\tilde{Q}_j^{\text{in}}(\omega)$. In the above 1D equations, the $\tilde{Q}_j^{\text{therm}}(\omega)$ might represent the true signal we wish to measure, while the imprecision and measurement back-action contributions in Eqs\[1\] and \[3\] represent measurement errors. Minimising their combined effect yields the well-known SQL \cite{12,2}.\footnote{FIG. 1. (a.) Schematic of 3D cooling set-up in levitated optomechanics: a nanoparticle held by a tweezer trap within a cavity. The cavity is undriven, but is populated by photons coherently scattered from the tweezer. The nanoparticle is placed at a point $\phi \simeq k x_0$ from the anti-node of the cavity field. Cooling and detection of the centre of mass displacement in 3D along $x, y, z$ is possible. (b.) The pattern of coherent photon scattering (taken from \cite{21}) into the cavity depends on the tilt $\theta$ of the tweezer polarization axis. (c.) Compares displacement PSDs using analytical expressions from Quantum Linear Theory (dashed lines) and with numerical solutions of the Langevin equations of motion (solid lines) using the tweezer and cavity potentials for the $x$ (black), $y$ (red) and $z$ (blue) motion. The latter does not assume any values for the optomechanical coupling strengths or equilibrium positions, and includes nonlinearities. Agreement between analytcs and numerics is excellent. The analysis shows that the $x, y$ modes are strongly hybridised and show a double-peaked structure (even for $\Delta \gg \omega_j$ for (i) low $\phi$ (top panel), because of direct coupling $g_{xy}$ and (ii) large $\phi \sim 0.5\pi$ (bottom panel) because of indirect cavity mediated coupling $g_{xy}$. Near $\phi \approx \pi/4$ (middle panels) in contrast, destructive interference between the direct and indirect pathways approximately decouples the modes. Parameters similar to the experiment in \cite{21}: input power $P_{in} = 0.17W$, $\Delta = -300$ kHz, however sphere radius $R_0 = 100nm$ and finesse $\mathcal{F} = 150,000$ are slightly larger, with gas pressure $P = 10^{-6}$ mbar. $\theta = 0.2\pi$.}

3D Cavity optomechanics. – As a first approximation to a 3D system, one might simply replace, in Eq\[2\] $g_j \tilde{q}_j(\omega) \equiv g_j \tilde{D}_j$ and directly obtain the PSD for the homodyne spectrum, in other words replace the displacement noises by their 1D equivalents. We note that even in this straightforward case, the error analysis does not simply yield a sum of the 1D PSDs $S_{\Delta q\Delta q}^{1D}$. The reason is that, while the thermal contributions are uncorrelated $\langle \tilde{Q}_j^{\text{therm}}(\omega) \tilde{Q}_j^{\text{therm}}(\omega') \rangle = \delta_{jj} \delta(\omega + \omega')$ and thus contribute independently to the PSDs, the separate back-actions are all correlated with each other and with the imprecision noises. They must interfere correctly $\sum_{j} g_j \chi^{BA}_j(\omega) \langle \tilde{Q}_j^{\text{in}}(\Phi=0) \tilde{Q}_{k,j}(\omega) \rangle$. This can be physically significant in the important quantum optomechanics scenario where the quantum back-action spectra dominate the dynamics. This leads to well-known quantum signatures such as sideband asymmetries and optical (ponderomotive) squeezing. Correlations between optical back action and imprecision noise play an important role in LIGO displacement sensing \cite{24}. However, one of our key findings is that we find additional, genuinely 3D, contributions and we can write the displacement noise spectrum in the form:

$$\tilde{q}_j(\omega) \simeq \tilde{D}_j^{1D} + \sum_{k \neq j} g_{jk}^3(\omega) \tilde{D}_k^{1D}$$

from which we can obtain all 3D PSDs analytically: i.e each displacement, in addition to the usual 1D noises terms, receives contributions from the 1D noises of the other two degrees of freedom, determined by a 3D coupling function $g_{jk}^{3D}(\omega)$ which we can give in closed form (the exact solution in fact includes higher order correc-


tion terms, see [23] which are essential for accuracy, but for clarity we discuss only the lowest order here) and which quantifies the deviation from 1D behaviour.

$\text{FIG. 2. (a.) Comparison between full 3D QLT (solid lines) with the equivalent 1D QLT (dotted lines) for the PSDs of the x, y, z displacements. While 3D QLT includes all optomechanical couplings $g_{xy}, g_{xz}, g_{yz}$ for the full coupled problem, 1D QLT obtains 3 independent PSDs $S_{iD}$, with all couplings set to zero except $g_j$. Parameters are similar to Fig.1 with $\Delta \gg \omega_j$, but pressure is set to $P = 10^{-7}$ mbar for phonon occupancies near the quantum regime $n \sim 1 - 3$. While in general the $x, y$ 3D PSDs are strongly perturbed (have a double-peaked structure) we see that for $\phi \approx \pi/4$ (middle panels) they are very close to their 1D forms. The $z$ mode contributes only weakly as it is well separated in frequency. At lower (higher) $\phi$ there are large differences between 3D and 1D due to the direct (indirect) pathways as seen in the top (bottom) panels. However, when there is destructive interference between direct and indirect pathways, there 1D and 3D forms nearly coincide. (b.) For the optical output spectra (corresponding to homodyne detection of the amplitude quadrature of the cavity output, violet lines) the very large squeezing by the $z$ mode at $\phi \approx 0$ lowers the precision floor for the $x, y$ PSDs. We compare also with the measurement back-action (BA) spectra (green curves) obtained for $P \to 0$ if we neglect photon scattering losses. Dotted lines are the 1D BA equivalent and once again, at $\phi = \pi/4$ these are also very close to the 3D equivalent.

To understand $\mathcal{G}_{jk}^{1D}(\omega)$, we revisit the quadratic Hamiltonians of linearised optomechanics which consider small displacements from an equilibrium point $x_0, y_0, z_0, \vec{a}$ where the mean photon number in the cavity is $n_0 = |\vec{a}|^2$. Usually we write $\hat{H} = \sum_j \hat{h}_j^{(0)} + \hat{V}_j^{\text{int}}$ where $\hat{h}_j^{(0)} = -\Delta \hat{a}^\dagger \hat{a} + \frac{\omega_j}{2} (\hat{q}_j^2 + \hat{p}_j^2)$ and $\hat{V}_j^{\text{int}} = g_j (\hat{a}^\dagger \hat{a} + \hat{a}^\dagger) \hat{q}_j$. We take $\omega_0 \approx \omega_\perp = \Delta < 0$ for a red-detuned cavity. However for consistency, the full Hamiltonian to quadratic order should be

$$\hat{H} = \sum_j [\hat{h}_j^{(0)} + \hat{V}_j^{\text{int}}] + \sum_{k \neq j} g_{jk} \hat{q}_j \hat{q}_k$$

and can also include additional direct coupling terms of strength $g_{jk}$. These are distinct from nonlinear, position squared coupling terms $g_j (\hat{a}^\dagger \hat{a} + \hat{a}^\dagger) \hat{q}_j^2$, which lead to observed sidebands at $2\omega_j$ in optically trapped systems at higher temperatures [15, 21].

For this case, we can show that

$$\mathcal{G}_{jk}^{1D}(\omega) = \frac{\mu_j \eta_0 \eta_0}{M_j} |\mu_j(\omega)g_j \eta_0 - g_j|$$

The prefactor, $\mu_j(\omega)$, is the mechanical susceptibility, with $M_j = 1 + g_j^2 \kappa \eta_0$, is a function peaked around $\omega \approx \pm \omega_j$, one of the mechanical frequencies. However, it is the terms in the square brackets that are of most interest. One can see they describe the interference between a direct $\propto g_j$, and a cavity mediated, indirect $\propto g_j \eta_0$ coupling between any two displacements. In other words, suppressing or conversely, enhancing 3D dynamics will involve either suppressing or correspondingly enhancing the 3D coupling via destructive or constructive interference of direct and indirect pathways near $\omega \approx \omega_j$.

Tweezer-cavity setup.— The above is quite generic to an arbitrary 3D optomechanics set-up. Here we apply this to the new experiments pioneered in [20, 21] which involve levitating a dielectric nanoparticle in a tweezer within a cavity. The tweezer axis and cavity axis (taken as $x$ direction) are tilted at an angle of $\theta$. The cavity in these set-ups is undriven but is populated entirely by light coherently scattered from the tweezer field and the particle moves under the combined effect of the tweezer trapping field and the coherently scattered light as explained in [20, 21]. We give the full potential in [22], but to a good approximation, the tweezer represents a trapping Hamiltonian equivalent to $\sum_j \hat{h}_j^{(0)}$, while the interaction with the cavity mode yields an interaction potential:

$$\frac{V_{\text{int}}}{h} = E_d \cos(\phi + \frac{\kappa}{2} (x \cos \theta + y \sin \theta))(ae^{-i\beta(z)} + a^\dagger e^{i\beta(z)})$$

where $\phi \sim kx_0, \beta(z) = kz - \arctan(z/z_R)$ and $z_R$ is the Rayleigh range. As shown in Fig.1, $x_0$ is the displacement between the tweezer focus and an antinode of the cavity. $E_d$ is an amplitude determined by the particle polarisability and input power to the tweezer. Expanding $V_{\text{int}}$ to quadratic order provides the light-matter couplings $g_j$, the matter-matter couplings $g_{kjk}$ as well as corrections to the mechanical frequencies and the equilibrium points (see [23] for details).

The direct coupling has not previously been considered in the experimental analysis [20, 22] but we find they can be of great importance; one can show that $g_{xy} = g_z g_\perp 2Re(\bar{a}) \cos^2 \phi \approx g_z g_\perp \bar{a}^2 \cos^2 \phi$ for $j = x, y$. Thus depending on the positioning, $\Delta$ or $\kappa$, the direct couplings contribution can be similar or exceed the cavity mediated coupling.
To test our analysis, we compare analytical, closed form PSDs obtained with 3D QLT and Eq.4 with direct solutions of the nonlinear equations of motion, using the tweezer and cavity potential functions. We emphasize that in the full nonlinear equations, the $g_j$ and $g_{jk}$ are not input parameters but are emergent properties of the combined potentials, and appear in the limit of low-amplitude displacements in the mechanical and optical degrees of freedom (the linear regime). Fig.1b compares the symmetrised analytical quantum spectra with the nonlinear Langevin numerics showing excellent agreement. We obtain excellent agreement in both thermal (high gas pressure, and quantum regimes where phonon occupancies are a few quanta, except that the numerics show effects of nonlinearities (additional peaks in the optical spectra) and the unsymmetrised quantum spectra show sideband asymmetries. Nevertheless the comparisons underline the importance of the direct coupling terms $g_{jk}$ at low $\phi$, leading to double peaked structures in Fig.1c) for $\phi \approx 0$ where $g_x g_y \approx 0$; conversely, as $\phi \rightarrow \pi/2$, where $g_{xy} \rightarrow 0$, cavity mediated coupling from $g_x g_y$ once again hybridise the $x, y$ peaks.

We can show that $i\eta_0(\omega) \rightarrow \frac{\Delta}{2(\kappa/2)^2+\Delta^2}$ if $\Delta \gg \omega$ (and we are interested primarily in the region $\omega \sim \omega_j$). Thus as $\Delta \rightarrow \infty$,

$$G_{xy}^{3D}(\omega) = G_{xy}^{3D} \rightarrow \alpha \left( g_x g_y \Delta(1 - 2 \cot^2 \phi) \right)$$

and the $x, y$ coupling $G_{xy}^{3D}(\omega)$ thus vanishes for $\phi \approx \pi/4$, thus near $x_{so} = \lambda/8$. This occurs since $g_{xy} \propto \text{Re}(\alpha)$ where $\alpha$ is the mean cavity field, which follows the cavity resonance.

The situation for the $G_{zz}^{3D}(\omega)$ couplings is more involved. A peculiarity of the coherently scattered tweezer light in the cavity is that the optomechanical coupling is of the form $g_z i(\hat{a}^\dagger - \hat{a})\hat{z}$, i.e., the displacement couples to the momentum quadrature of the cavity. In this case, $G_{zz}^{3D}(\omega) = \frac{i\mu_j}{M_j} \left[ \frac{g_j g_z (i\eta_{zj}/2(\omega)) - \frac{\kappa}{(\kappa/2)^2+\Delta^2}}{\kappa} \right]$, but $G_{zz}^{2D}(\omega) = \frac{i\mu_j}{M_j} \left[ g_j g_z (\eta_{zj}/2(\omega) - \frac{\kappa}{(\kappa/2)^2+\Delta^2}) \right]$. In other words, $G_{zz}^{2D} \neq G_{zz}^{3D}$ and both couplings cannot be suppressed simultaneously. In any case, for $\Delta \rightarrow \infty$, $\eta_{zj} \rightarrow \frac{\kappa\omega_j}{(\kappa/2)^2+\Delta^2}$, thus even where there is destructive interference, only the real part is fully cancelled. Nevertheless, the 3D coupling is attenuated for $\Delta \gg \omega_j$.

The near cancellation the direct and indirect coupling terms means that the 3D couplings $G_{xy}^{3D}(\omega)$ vanish; as the $z$ peaks are of much lower frequency, mixing with those is less important. This indicates that we have approximately 1D dynamics. To test this we explicitly compare with 1D QLT (all $G_{jk}^k = 0$ in Fig.3 (left panels) and verify that for $\phi \approx \pi/4$ the 3D PSDs approximate closely their 1D form. At these points one would expect the sensing analysis to approximate the standard 1D SQL. In Fig.2 we look also at the effect of ponderomotive squeezing both in quantum regimes with a thermal contribution (pressure $P = 10^{-7}$ mbar as well as the back-action limit where we allow only the noise from quantum shot noise fluctuations. Here we can see that as the $x, y, z$ contributions interfere, the strong squeezing by one mode ($z$) can lower the noise imprecision floor for detection of the $x, y$ modes (upper panel).

In Fig.3 we show that although lower equilibrium phonon occupancies are possible in the usual red-sideband cooling resonant regimes $\Delta \approx -\omega_j$, the effective 1D dynamics which may be advantageous for quantum sensing is a feature of the $-\Delta \gg \omega_j$ regimes, so that the regime which is optimal for sensing may be somewhat different from regime optimal for cooling.

**Conclusions** We have shown that 3D optomechanical displacement sensing can be far from a trivial sum of PSDs from three orthogonal $x, y, z$ coordinates. Although our work focusses on recent experiments on 3D centre of mass cooling of levitated nanospheres, some of the conclusions are generic. We show one may be able switch on and switch off some of the additional 3D effects and that these can give advantages in terms of exceeding usual quantum back action limited occupancies for a given coordinate. 3D optomechanics opens the way to new forms of force and displacement sensing, including sensing the direction as well as magnitude.
[1] Warwick P. Bowen and Gerard J. Milburn. *Quantum Optomechanics*. CRC Press, 2015.

[2] Markus Aspelmeyer, Tobias J. Kippenberg, and Florian Marquardt. Cavity optomechanics. *Rev. Mod. Phys.*, 86:1391–1452, Dec 2014.

[3] B. P. Abbott et al. Observation of gravitational waves from a binary black hole merger. *Phys. Rev. Lett.*, 116:061102, Feb 2016.

[4] James Millen, Tania S Monteiro, Robert Pettit, and Nick A. Vamivakas. Optomechanics with levitated particles. *arXiv:1907.08198*, 2019.

[5] Zhang-Qi Yin, Andrew Geraci, and Tongcang Li. Optomechanics of levitated dielectric particles. *Int. J. Mod. Phys. B*, 27:1330018, 2013.

[6] O. Romero-Isart, M.L. Juan, R. Quidant, and J.I. Cirac. Toward quantum superposition of living organisms. *New J. Phys.*, 12:033015, 2010.

[7] D. E. Chang, C. A. Regal, S. B. Papp, D. J. Wilson, J. Ye, O. Painter, H. J. Kimble, and P. Zoller. Cavity opto-mechanics using an optically levitated nanosphere. *Proceedings of the National Academy of Sciences*, 107(3):1005–1010, 2010.

[8] P. F. Barker and M. N. Shneider. Cavity cooling of an optically trapped nanoparticle. *Phys. Rev. A*, 81:023826, Feb 2010.

[9] G. A. T. Pender, P. F. Barker, Florian Marquardt, J. Millen, and T. S. Monteiro. Optomechanical cooling of levitated spheres with doubly resonant fields. *Phys. Rev. A*, 85:021802, Feb 2012.

[10] T. S. Monteiro, J. Millen, G. A T Pender, Florian Marquardt, D. Chang, and P. F. Barker. Dynamics of levitated nanoshpere: Towards the strong coupling regime. *New Journal of Physics*, 15, 2013.

[11] Nikolai Kiesel, Florian Blaser, Uroš Delić, David Grass, Rainer Kaltenbaek, and Markus Aspelmeyer. Cavity cooling of an optically levitated submicron particle. *Proceedings of the National Academy of Sciences*, 110(35):14180–14185, 2013.

[12] Peter Asenbaum, Stefan Kuhn, Stefan Nimmrichter, Ugur Sezer, and Markus Arndt. Cavity cooling of free silicon nanoparticles in high vacuum. *Nature Communications*, 4:2743 EP –, Nov 2013. Article.

[13] Pau Mestres, Johann Berthelot, Marko Spasenović, Jan Gieseler, Lukas Novotny, and Romain Quidant. Cooling and manipulation of a levitated nanoparticle with an optical fiber trap. *Applied Physics Letters*, 107(15):151102, 2015.

[14] J. Millen, P. Z. G. Fonseca, T. Mavrogordatos, T. S. Monteiro, and P. F. Barker. Cavity cooling a single charged levitated nanosphere. *Phys. Rev. Lett.*, 114:123602, Mar 2015.

[15] P. Z. G. Fonseca, E. B. Aranas, J. Millen, T. S. Monteiro, and P. F. Barker. Nonlinear dynamics and strong cavity cooling of levitated nanoparticles. *Phys. Rev. Lett.*, 117:173602, Oct 2016.

[16] Lorenzo Magrini, Richard A. Norte, Ralf Riedinger, Igor Marinković, David Grass, Uroš Delić, Simon Gröblacher, Sungkun Hong, and Markus Aspelmeyer. Near-field coupling of a levitated nanoparticle to a photonic crystal cavity. *Optica*, 5(12):1597–1602, Dec 2018.