The recent star formation history of the *Hipparcos* solar neighbourhood

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ABSTRACT

We use data from the *Hipparcos* catalogue to construct colour–magnitude diagrams for the solar neighbourhood, which are then treated using advanced Bayesian analysis techniques to derive the star formation rate history, \( SFR(t) \), of this region over the last 3 Gyr. The method we use allows the recovery of the underlying \( SFR(t) \) without the need of assuming any a priori structure or condition on \( SFR(t) \), and hence yields a highly objective result. The remarkable accuracy of the data permits the reconstruction of the local \( SFR(t) \) with an unprecedented time resolution of \( \approx 50 \) Myr. An \( SFR(t) \) that has an oscillatory component of period \( \approx 0.5 \) Gyr is found, superimposed on a small level of constant star formation activity. Problems arising from the non-uniform selection function of the *Hipparcos* satellite are discussed and treated. Detailed statistical tests are then performed on the results, which confirm the inferred \( SFR(t) \) to be compatible with the observed distribution of stars.

Key words: methods: statistical – stars: formation – solar neighbourhood.

1 INTRODUCTION

The problem of deducing the star formation rate history, \( SFR(t) \), of the Milky Way has generally been attempted in terms of indirect inferences, mostly through chemical evolution models. The validity of these methods relies on the soundness of the assignation of a ‘chemical age’ to each of the studied stars. Generally a metallicity indicator is chosen and used to measure the metal content of a number of stars, which are then binned into age groups through the use of an age–metallicity relation derived from a chemical evolution model. For example, Rocha-Pinto & Maciel (1997) take a variety of age–metallicity relations (AMRs) from the literature and use a closed box chemical evolution model to translate the AMRs into \( SFR(t) \), allowing for an intrinsic Gaussian spread in the AMR assumed to be constant in time.

The advantages of these methods are that large samples of stars, both in the solar neighbourhood and further away within the disc of the Galaxy, can be studied. An inferred \( SFR(t) \) can be constructed over an ample time range and spatial extent within the Galaxy, which is consistent with the metallicities of the sample studied and the chemical evolution model proposed. However, the validity of the AMR assumption can not be checked independently of the proposed chemical evolution model, and is necessarily dependent on the choice of the mixing physics of the interstellar medium (ISM), the possible infall of primordial non-enriched gas and the still largely unknown Galactic formation scenario in general. This last problem also affects attempts at inferring the \( SFR(t) \) from stellar kinematics (e.g. Gomez et al. 1990, Marsakov et al. 1990).

With the recent availability of the *Hipparcos* satellite catalogue (ESA 1997) we are now in a position to attempt recovery of the local \( SFR(t) \) directly, without the need of any model dependent assumptions. Previous direct approaches have been undertaken through the binning of observed stars into age groups according to the degree of chromospheric activity as measured through selected emission-line features, with conflicting results depending on the assumed age–activity relation (e.g. Barry 1988 and Soderblom et al. 1991). Using this technique, Rocha-Pinto et al. (2000a) have derived a star formation history from the chromospheric activity–age distribution of a larger sample comprising 552 stars, and have found evidence for intermittency in the \( SFR(t) \) over 14 Gyr. The *Hipparcos* catalogue offers high-quality photometric data for a large number of stars in the solar neighbourhood, which can be used to construct a colour–magnitude diagram (CMD) for this region. Once a CMD is available, it is in principle possible to recover the \( SFR(t) \) that gave rise to the observed distribution of stars, assuming only a stellar evolutionary model in terms of a set of stellar tracks. In practice, the most common approach to inverting CMDs has been to propose a certain parametrization for the \( SFR(t) \), used to construct synthetic CMDs, which are statistically compared with the observed ones in order to select the values of the parameters that result in a best match CMD. Examples of the above are found in Chiosi et al. (1989), Aparicio et al. (1990) and Mould, Han & Stetson (1997), using Magellanic and local star clusters, and Mighell & Butler (1992), Smecker-Hane et al. (1994), Aparicio & Gallart (1995), Tolstoy & Saha (1996) and Mighell (1997), using local dwarf spheroidals (dSphs).
We have extended these methods in Hernandez, Valls-Gabaud & Gilmore (1999, henceforth Paper I) by combining a rigorous maximum likelihood statistical approach, analogous to that introduced by Tolstoy & Saha (1996), with a variational calculus treatment. This allows a totally non-parametric solution of the problem, where no a priori assumptions are introduced. This method was applied by Hernandez, Gilmore & Valls-Gabaud (2000, henceforth Paper II) to a set of Hubble Space Telescope CMDs of local dSph galaxies in order to infer the SFR(t) of these interesting systems. The result differs from what can be obtained from a chemical evolution model, in that a direct answer is available, with a time resolution which depends only on the accuracy of the observations.

Limitations on the applicability of our method to the Hipparcos data appear in connection to the selection function of the catalogue. The need to work only with complete volume-limited samples limits the age range over which we can recover the SFR. In Section 2 our conclusions.

In Section 2 we give a summarized review of the method introduced in Paper I, the sample selection and results are discussed in Section 3. Section 4 presents a careful statistical testing of our results, and Section 5 our conclusions.

2 THE METHOD

In this section we give a summary description of our Hertzsprung–Russell (HR) diagram inversion method, which was described extensively in our Paper I. In contrast with other statistical methods, we do not need to construct synthetic CMDs for each of the possible star formation histories being considered. Rather, we use a direct approach that solves for the best SFR(t) compatible with the stellar evolutionary models assumed and the observations used.

The evolutionary model consists of an isochrone library and an initial mass function (IMF). Our results are largely insensitive to the details of the latter, for which we use

\[
\rho(m) \propto \begin{cases} 
  m^{-1.3}, & 0.08 M_\odot < m \leq 0.5 M_\odot, \\
  m^{-2.2}, & 0.5 M_\odot < m \leq 1.0 M_\odot, \\
  m^{-2.7}, & 1.0 M_\odot < m.
\end{cases}
\]

(1)

The above fit was derived by Kroupa, Tout & Gilmore (1993) for a large sample towards both Galactic poles and all the solar neighbourhood, and therefore applies to the Hipparcos data.

As we shall be treating here only data from the solar neighbourhood derived by the Hipparcos satellite, we shall assume for the observed stars a fixed metallicity of [Fe/H] = 0. This assumption is valid as we will only be treating stars within a short distance from the Sun with a small spread in age. Once the metallicity is fixed we use the latest Padova isochrones (Fagotto et al. 1994; Girardi et al. 1996), together with a detailed constant phase interpolation scheme using only stars at constant evolutionary phase, to construct an isochrone library with a chosen temporal resolution.

To transform the isochrones from the theoretical HR diagram to the observed CMDs we used the transformations provided by the calibrations of Lejeune, Cuisinier & Buser (1997), which are appropriate for the solar metallicities considered here. Using the updated calibrations given by Bessell, Castelli & Plez (1998) does not change the transformations significantly in the regime used here, unlike the case for giant and asymptotic giant branch stars (see Weiss & Salaris 1999).

In this case we implement the method with a formal resolution of 15 Myr, compatible with the high resolution of the Hipparcos observations. It is one of the advantages of the method that this resolution can be increased arbitrarily (up to the stellar model resolution) with computation times scaling only linearly with the resolution.

Our only other inputs are the positions of, say, n observed stars in the HR diagram, each with a colour and luminosity ci and li, respectively. Starting from a full likelihood model, we first construct the probability that the n observed stars resulted from a certain SFR(t). This will be given by

\[
\mathcal{L} = \prod_{i=1}^{n} \left[ \int_{t_{0}}^{t_{1}} SFR(t)G_i(t) \, dt \right].
\]

(2)

where

\[
G_i(t) = \int_{m_{0}}^{m_{1}} \frac{\rho(m; t)}{2\pi\sigma(i)\sigma(c_i)} \exp \left[ -\frac{D(l_i; t, m)^2}{2\sigma(l_i)} \right] \times \exp \left[ -\frac{D(c_i; t, m)^2}{2\sigma(c_i)} \right] \, dm.
\]

In the above expression ρ(m;t) is the density of points along the isochrone of age t around the star of mass m, and is determined by the assumed IMF together with the duration of the differential phase around the star of mass m. The ages t0 and t1 are minimum and a maximum ages that need to be considered, as m0 and m1 are a minimum and a maximum mass considered along each isochrone, e.g. 0.6 and 20 M_\odot, σ(i) and σ(c_i) are the amplitudes of the observational errors in the luminosity and colour of the i-th star. These values are supplied by the particular observational sample one is analysing. Note that for the sample we have selected (Section 3), there is no correlation between these errors. Finally, D(l_i; t, m) and D(c_i; t, m) are the differences in luminosity and colour, respectively, between the i-th observed star and a general star of age and mass (m, t). We shall refer to G_i(t) as the likelihood matrix, since each element represents the probability that a given star, i, was actually formed at time t with any mass.

A similar version was introduced in Paper I but was restricted to the case of observational errors only in one variable, which was adequate for the problem of studying the SFR(t) of local dSph galaxies treated in Paper II. Equation (2) is essentially the extension from the case of a discretized SFR(t), used by Tolstoy & Saha (1996), to the case of a continuous function in the construction of the likelihood. The challenge now is to find the optimum SFR(t) without evaluating equation (2), i.e. without introducing a fixed set of test SFR(t) cases from which one is selected.

The condition that ∆L(SFR) = 0 has an extremal can be written as

\[
\delta L(SFR) = 0.
\]

(3)

and a variational calculus treatment of the problem applied. First we develop the product over i using the chain rule for the variational derivative, and divide the resulting sum by L to obtain

\[
\sum_{i=1}^{n} \left[ \frac{\delta}{\delta SFR(t)} \int_{t_{0}}^{t_{1}} SFR(t)G_i(t) \, dt \right] = 0.
\]

(3)
Introducing the new variable \( Y(t) \) defined as
\[
Y(t) = \int \sqrt{\text{SFR}(t)} \, dt \Rightarrow \text{SFR}(t) = \left[ \frac{dY(t)}{dt} \right]^{2},
\]
and introducing the above expression into equation (3) we can develop the Euler equation to yield
\[
\frac{d^2 Y(t)}{dt^2} \sum_{i=1}^{n} \frac{G_i(t)}{I_i(t)} = -\frac{dY(t)}{dt} \sum_{i=1}^{n} \frac{dG_i/dt}{I_i(t)},
\]
where
\[
I_i(t) = \int_{t_0}^{t} \text{SFR}(t)G_i(t) \, dt.
\]

This in effect has transformed the problem from one of searching for a function that maximizes a product of integrals (equation 2) to one of solving an integro-differential equation (equation 4). We solve this equation iteratively with the boundary condition \( \text{SFR}(t_1) = 0 \).

Details of the numerical procedure required to ensure convergence to the maximum likelihood \( \text{SFR}(t) \) can be found in our Paper I, where a more complete development of the method is also found. In Paper I the method was tested extensively using synthetic HR diagrams, obtaining very satisfactory results. Equation (4) will be satisfied by any stationary point in the likelihood, not just the global maximum, it is therefore important to check that the numerical algorithm implemented converges to the true \( \text{SFR}(t) \). This was shown in our Paper I using synthetic HR diagrams extensively, and testing the robustness of the method to changes in the initialization condition of the algorithm. An independent test of the validity of the results was also implemented, and is described in Section 4.

The main advantages of our method over other maximum likelihood schemes are: (1) the totally non-parametric approach the variational calculus treatment allows, and (2) the efficient computational procedure, where no time consuming repeated comparisons between synthetic and observational CMDs are necessary, as the optimal \( \text{SFR}(t) \) is solved for directly.

2.1 Two tests

We now present two examples of the performance of the method in cases similar to the CMDs of the Hipparcos sample. The left-hand panel of Fig. 1 shows a synthetic CMD produced from the first input \( \text{SFR}(t) \), resulting in a number of stars similar to that which the Hipparcos samples yield for small errors in \( V - I \) (<0.12 mag) and \( M_V \) (<0.02 mag). The positions of the simulated stars are then used to construct the likelihood matrix, which is used to recover the inferred \( \text{SFR}(t) \) through an iterative numerical procedure (see Paper I). The right-hand panel of Fig. 1 shows the last three iterations of the method (solid curves) and the input \( \text{SFR}(t) \) (a three-burst \( \text{SFR}(t) \) – dotted curve). It can be seen that the main features of the input \( \text{SFR}(t) \) are accurately recovered. The age, duration and shape of the input \( \text{SFR}(t) \) are clearly well represented by the final inferred \( \text{SFR}(t) \). As the difference between successive isochrones diminishes with age, and since the errors remain constant, the accuracy of the recovery procedure diminishes with the age of the stellar populations being treated. This is seen in that the first burst is very accurately recovered, whilst the last one appears somewhat spread out.

The last example is shown in Fig. 2, which is analogous to Fig. 1. Here an \( \text{SFR}(t) \) that is constant over a large period is treated. On first inspection, the HR diagram of this case appears to be almost identical to that of the previous case, however, given the extremely small errors assumed (typical of the Hipparcos data) the method is capable of distinguishing and accurately recovering the input \( \text{SFR}(t) \) of these two cases. The small number of stars (~450) results in a degree of shot noise, which has to be artificially suppressed using a smoothing procedure, the result of which is seen in the residual short-period oscillations of the inferred \( \text{SFR}(t) \). This smoothing procedure reduces the effective resolution of the method to 50 Myr. Note that as in the previous example, the inversion method successfully recovers the main features of the

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**Figure 1.** Left-hand panel: simulated colour–magnitude diagram resulting from the first test \( \text{SFR}(t) \). Right-hand panel: first test \( \text{SFR}(t) \) – dashed line; the solid curves are the last three iterations of the method, showing an accurate reconstruction of the input \( \text{SFR}(t) \).
input SFR(t). In these two tests only stars blueward of \( V - I = 0.7 \) were considered in the inversion procedure (see below).

3 SAMPLE SELECTION AND RESULTS

In order to apply the method described in the preceding section to the Hipparcos data we would like to construct a volume-limited sample, where no biases appear between stars of different ages. Furthermore, such a sample should contain a sufficient number of stars from all age groups being considered, i.e. it must go down in magnitude to the turn-off point corresponding to the oldest age being considered. Although the Hipparcos satellite produced a catalogue with very well understood errors and highly accurate magnitude and colour determinations for a large number of stars, the sample has to be reduced through several cuts before it complies with the restrictions required by our method.

The Hipparcos catalogue provides an almost complete sample of stars in the solar neighbourhood. The limiting magnitude depends both on spectral type and Galactic latitude (ESA 1997). For the types earlier than G5 that we consider here, the limiting \( V \) magnitude is given by \( V_{\text{lim}} = 7.9 + 1.1 \sin|b| \). To avoid unnecessary complications, we consider cuts of the type \( V = \) constant at all latitudes with \( V > 7.9 \).

Fig. 3 shows a graph of \( M_V \) versus distance for all stars in the Hipparcos catalogue with distance errors smaller than 20 per cent and an apparent magnitude \( m_V < 7.9 \). The solid lines show the inclusion criteria for one possible volume-limited sample, complete to \( M_V < 3.15 \) and \( m < 7.25 \). As it can be seen, the maximum age that can be considered will not be very large, as the number of stars in a volume-limited sample complete to absolute magnitudes greater than four rapidly dwindles. After experimenting with synthetic CMDs of known SFR\((t)\), produced using our isochrone grid and constructed to have the same numbers of stars as a function of lower magnitude limit as in Fig. 3, and recovering the SFR\((t)\) using our method, we identified 3 Gyr as the maximum age that we can accurately treat with the data at hand. This fixes \( t_0 = 0 \) and \( t_1 = 3 \) Gyr as the temporal limits in equation (2), where the use of 200 isochrones establishes the formal resolution of the method to be 15 Myr.

Although the absolute magnitude errors correlate tightly with the distance, the colour errors correlate more strongly with the apparent magnitude and can actually represent the dominant error in inverting the CMDs. The solid curve shows the \( m_V < 7.25 \) completion limit, which implies errors similar to those used in the simulations of Figs 1 and 2. It will be with complete volume-limited samples with this apparent magnitude limit that we will be dealing.

As the limit in \( M_V \) is moved to dimmer stars, the structure of the SFR\((t)\) at older ages is better recovered by the inversion method, but the number of younger stars diminishes (see Fig. 1) and the SFR\((t)\) of the younger period is under-represented in the recovered SFR\((t)\). We constructed a variety of somewhat independent

\[ \text{Figure 2. Left-hand panel: simulated CMD resulting from the second test SFR}(t). \text{ Right-hand panel: second test SFR}(t) \pm \text{dashed line; the solid curves are the last three iterations of the method, showing an accurate reconstruction of the input SFR}(t). \]

\[ \text{Figure 3. Stars in the Hipparcos catalogue with distance errors <20 per cent. The sharp lower envelope shows the completeness limit of } m < 7.9. \text{ The straight lines show one volume-limited sample at } M_V < 3.15. \text{ The curve shows the completeness limit of } m < 7.25; \text{ which corresponds to errors similar to those used in the simulations of Figs 1 and 2.} \]

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Hipparcos CMDs for different absolute magnitude limits in the range $3.0 < M_V < 3.5$, and obtained highly compatible answers.

Also, all our samples include a certain number of contaminating stars with ages greater than the 3 Gyr limit considered by the inversion method, mostly in the red giant branch (RGB) region, as their turn-off points appear at magnitudes dimmer than the minimum ones considered. To avoid these stars, the inversion method considers only stars bluerward of $V-I = 0.7$, as carried out in the synthetic examples of Figs 1 and 2.

As the fraction of stars produced by the SFR($t$) which live into the observed CMD diminishes with age, in inverting a well-populated CMD the older regions of the SFR($t$) are underestimated by the recovery method. This is compensated by a correction factor given by the assumed IMF and the mass at the tip of the RGB as a function of time, as discussed in Paper I.

Once the IMF, metallicity, positions of the observed stars in the CMD and observational errors in both coordinates (also supplied by the Hipparcos catalogue) are given, they are used to construct the likelihood matrix $G_t(t)$, which is the only input given to the inversion method. The small number of stars present in any volume-limited sample (~450) leaves us insensitive to the existence of small features in the SFR($t$) that produce only a few stars. The limited numbers of stars also reduces the resolution of the method, as a smoothing function has to be applied to suppress instabilities in solving the integro-differential equation of the problem. This final smoothing reduces the effective temporal resolution of the method to 50 Myr, still much higher than that of any indirect chemical evolution inference of SFR($t$).

### 3.1 Kinematic and geometric corrections

Once the apparent and absolute magnitudes of the sample have been chosen, the set of stars to be studied is fully specified. The positions of which in the CMD are compared with the assumed isochrones to construct the likelihood matrix, which is the only input required by the numerical implementation of the method as described above. The resulting SFR($t$) will be representative of the stars that were used in the construction of the likelihood matrix.

To normalize the various inferred SFR($t$) from samples with different $M_V$ limits, and hence complete out to different distances, we apply the following kinematic and geometric corrections.

Let $F(v,h)$ be the fraction of the time a star with vertical velocity $v$ at the disc plane spends between heights $-h$ and $+h$. Then, for a cylindrical sample complete to height $h$ above and below the disc plane,

$$N(t) = \frac{N_s(t)}{\sqrt{2\pi\sigma^2(t)}} \int_{-\infty}^{\infty} \frac{\exp[-v^2/2\sigma^2(t)]}{F(v,h)} dv,$$

where $N(t)$ is the number of stars a stellar population of age $t$ contains, $N_s(t)$ the number of stars of age $t$ observed and $\sigma(t)$ the time dependent velocity dispersion of the several populations. As volume-limited samples are generally spherical around the Sun, a further geometric factor is required, giving

$$N(t) = \frac{3N_s(t)}{\sqrt{2\pi\sigma(t)^2}} \int_{-\infty}^{\infty} \frac{\exp[-v^2/2\sigma^2(t)]}{F(v,h)} dv,$$

where $R$ is the radius of the observed spherical volume-limited sample, $r$ is a radial coordinate and $h^2 = R^2 - r^2$. To estimate $F(v,h)$, one requires the vertical force law of the Galactic disc at the solar neighbourhood. The best direct estimate of this function remains that of Kuijken & Gilmore (1993), who show this function to deviate from that of a harmonic potential to a large degree. We integrate numerically this detailed force law to obtain $F(v,h)$. We use $\sigma(t) = 20 \text{ km s}^{-1}$, which is appropriate for the metallicity and age ranges that we are studying (Edvardsson et al. 1993; Wyse & Gilmore 1995). Note also that the scatter in metallicity within 80 pc is rather small (Garnett & Kobulnicky 2000) and will not significantly change our results.

In this way, assuming a Gaussian distribution for the vertical velocities of the stars and a given $\sigma(t)$, an observed $N_s(t)$ can be transformed into a total $N(t)$, which is equal to the total projected disc quantity.

In our case the SFR($t$) given by the method takes the place of $N_s(t)$, and equation (6) is used to obtain a final star formation history that accounts for the kinematic and geometric factors described. This function is then normalized, through the total number of stars in the relevant sample, to give the deduced SFR($t$) in units of $M_\odot \text{Myr}^{-1} \text{kpc}^{-2}$.

### 3.2 Results

Fig. 4 shows the CMD corresponding to a volume-limited sample complete to $M_V < 3.15$ for stars in the Hipparcos catalogue with errors in parallax of less than 20 per cent and $m_V < 7.25$ (left-hand panel). The right-hand panel of this figure shows the result of applying our inversion method to this CMD (solid curve). The dotted envelope encloses several alternative reconstructions arising from different $M_s$ cuts and gives an estimate of the errors likely to be present in our results, which can be seen to increase with time. The reconstruction based on the $(M_V, B - V)$ diagram gives essentially the same results.

A certain level of constant star formation activity can be seen, superimposed on to a strong, quasi-periodic component with a period close to 0.5 Gyr, as encoded in the positions of the observed stars in the CMD. The sharp feature seen towards $t = 3$ Gyr could be the beginning of a fifth cycle, truncated by the boundary condition SFR($3$) = 0. We have performed tests with synthetic CMDs using the same numbers of stars and magnitude limits as in Fig. 4 with a variety of SFR($t$). The method efficiently discriminates between constant and periodic input SFR($t$), and correctly recovers features such as those found in the inferred SFR($t$) of Fig. 4. We conclude that as far as the Padova isochrones at solar metallicity are representative of the observational properties of the stars in the CMDs, the SFR($t$) of the solar neighbourhood over the last 3 Gyr has been that shown in Fig. 4. The unprecedented time resolution of our SFR reconstruction makes it difficult to compare with the results derived from chromospheric activity studies (Rocha-Pinto et al. 2000a), although qualitatively we do find the same activity at both 0.5 and 2 Gyr, but not the decrease between 1 and 2 Gyr.

One possible interpretation of a cyclic component in the SFR($t$) of the solar neighbourhood can be found in the density wave hypothesis (Lin & Shu 1964) for the presence of spiral arms in late-type galaxies. As the pattern speed and the circular velocity are in general different, a given region of the disc (e.g. the solar neighbourhood) periodically crosses an arm region where the increased local gravitational potential might possibly trigger an episode of star formation. In the simplest version of this scenario, we can take the pattern angular frequency $\Omega_p$ equal to twice the circular frequency $\Omega$ at the position of the Sun (Binney & Tremaine 1987), valid within a flat rotation curve region. The time interval $\Delta t$ between encounters with an arm at the solar
neighbourhood will be given in general by

\[ \Delta t = \frac{2\pi}{m[\Omega - \Omega_p]}, \]

that is

\[ \Delta t = \frac{0.22 \text{ Gyr}}{m} \left( \frac{\Omega}{29 \text{ km s}^{-1} \text{kpc}^{-1}} \right)^{-1} \left( \frac{\Omega_p}{\Omega} - 1 \right)^{-1}, \]

where \( m \) is the number of arms in the spiral pattern. The classical value of the pattern speed, \( \Omega_p = 0.5 \Omega \approx 14.5 \text{ km s}^{-1} \text{kpc}^{-1} \) would imply that the interaction with a single arm (\( m = 1 \)) should be enough to account for the observed regularity in the recent star formation rate history. However, more recent determinations tend to point to much larger values (e.g. Mishurukov, Pavloskaya & Suchkov 1979; Avedisova 1989; Amaral & Lépine 1997) closer to \( \Omega_p \approx 23-24 \text{ km s}^{-1} \text{kpc}^{-1} \), which would then imply that the regularity present in the reconstructed \( SFR(t) \) would be consistent with a scenario where the interaction of the solar neighbourhood with a two-armed spiral pattern would have induced the star formation episodes we detect. In fact, given the uncertainties in the pattern speed, mean circular velocity and solar radius, both the \( m = 2 \) and \( m = 4 \) cases could explain the 0.5 Gyr periodicity we detect. These arms are clearly detected in, for instance, the distribution of free electrons in the Galactic plane (Taylor & Cordes 1993). This is reminiscent of the explanations put forward to account for the inhomogeneities observed in the velocity distribution function, where well-defined branches associated with moving groups of different ages (Asiain, Figueras & Torra 1999; Chereul, Crézé & Bienaymé 1999; Skuljan, Hearnswood & Cottrell 1999) could perhaps be also associated with an interaction with spiral arm(s), although in this case the time-scales are much smaller.

Alternatively, if the solar neighbourhood is closer to the corotation radius, the Galactic bar could have triggered star formation in the solar neighbourhood with episodes separated by about 0.5 Gyr if the pattern speed of the bar is larger than about 40 km s\(^{-1}\)kpc\(^{-1}\) (Dehnen 1999).

Of course, other explanations are possible, for example the cloud formation, collision and stellar feedback models of Vazquez & Scalo (1989) predict a phase of oscillatory \( SFR(t) \) behaviour as a result of a self-regulated star formation regime. Close encounters with the Magellanic Clouds have also been suggested to explain the intermittent nature of the star formation rate on longer time-scales (Rocha-Pinto et al. 2000b).

We have tested the ability of our method to accurately distinguish oscillatory components in the \( SFR(t) \) with tests such as those shown in Figs 1 and 2. The oscillatory component in the case shown in Fig. 1 is successfully recovered by the method. In the case shown in Fig. 2 however, although the main features are also accurately recovered, a level of small amplitude fluctuations appear spuriously. This last feature is of such a low level that if a CMD were produced from the method of Fig. 1 with the same total number of stars, each small fluctuation would produce a number of stars of the order of 1.

Our answer shown in Fig. 4 shows not only a large-scale oscillatory component, but superimposed onto this, a certain level of small-amplitude fluctuations. Given the total number of stars present in our sample, we can not rule out the possibility (quite possibly the case, in fact) that these small fluctuations are numerical, as they are of amplitude similar to the ones discussed and appearing in Fig. 2, and are actually buried within the error envelope. The main oscillatory component with a period of 0.5 Gyr, however, involves a sufficiently large number of stars to be objectively identified.

A larger and independent data set from which to derive \( SFR(t) \) would be necessary in order to extend our results to a broader age range and a more extensive region of the Galactic disc. Increasing the number of stars available for the HR inversion procedure would also allow us to recover finer features and reduce the small numerical fluctuations discussed above.

### 4 TESTING THE RESULTS

In our Paper I we tested this method using synthetic CMDs produced from known star formation histories, with which we could assess the accuracy of the result of the inversion procedure as shown in Figs 1 and 2. In working with real data, we require the introduction of an independent method of comparing our final
result with the starting CMD, in order to check that the answer our inversion procedure gives is a good answer. From our Paper I we know that when the stars used in the inversion procedure are indeed produced from the isochrones and metallicity used to construct the likelihood matrix, the inversion method gives accurate results. The introduction of an independent comparison between our answer and the data is hence a way of checking the accuracy of the input physics used in the inversion procedure, i.e. the IMF, metallicity and observational parameters.

The most common procedure of comparing a certain SFR(t) with an observed CMD is to use the SFR(t) to generate a synthetic CMD, and compare this with the observations using a statistical test to determine the degree of similarity between the two.

The disadvantage, however, is that one is not comparing the SFR(t) with the data, but rather a particular realization of the SFR(t) with the data. The distinction becomes arbitrary when large numbers of stars are found in all regions of the CMD, which is generally not the case. Following a Bayesian approach, we prefer to adopt the W statistic presented by Saha (1998), essentially

\[ W = \prod_{i=1}^{B} \frac{(m_i + s_i)!}{m_i!s_i!}, \]

where B is the number of cells into which the CMD is split, and \( m_i \) and \( s_i \) are the numbers of points in the two distributions being compared. This asks for the probability that two distinct data sets are random realizations of the same underlying distribution. In implementing this test we first produce a large number (500) of random realizations of our inferred SFR(t), and compute the W statistic between pairs in this sample of CMDs. This gives a distribution that is used to determine a range of values of \( W \) expected to arise in random realizations of the SFR(t) being tested. Next, the W statistic is computed between the observed data set and a new large number of random realizations of the SFR(t) (also 500), this gives a new distribution of W that can be objectively compared with the one arising from the model–model comparison to assess whether both data and modelled CMDs are compatible with a unique underlying distribution.

![Figure 5](https://academic.oup.com/mnras/article-abstract/316/3/605/970653/reuse/fig5)

**Figure 5.** Left-hand panel: simulated CMD from the inferred SFR(t) down to \( M_V < 3.15 \), containing a similar number of stars to the volume-limited sample complete to the same limit. Right-hand panel: histogram of W statistics for 500 model–model comparisons – solid curve. The dashed histogram gives the W statistics of 500 data–model comparisons, showing the inferred SFR(t) to be compatible with this volume-limited sample.

Fig. 5 shows a synthetic CMD produced from our inferred SFR(t) for the solar neighbourhood, down to \( M_V = 3.15 \). This can be compared with the Hipparcos CMD complete to the same \( M_V \) limit of Fig. 4. A visual inspection reveals that there are approximately equal numbers of stars in each of the distinct regions of the diagram, a more rigorous statistical comparison is also included. The right-hand panel of Fig. 5 shows a histogram of the values of the W statistic for 500 random realizations of our inferred SFR(t) in a model–model comparison. This gives the range of values of the W statistic likely to appear in comparisons of two CMD diagrams arising from the same underlying SFR(t), our inferred SFR(t). The dashed histogram shows the results of 500 synthetic CMDs versus the observed Hipparcos data set. The compatibility of both sets of W values is clear. In this way the observed stars in the volume-limited sample complete to \( M_V = 3.15 \) and the isochrones, IMF and metallicity used in estimating the inferred SFR(t), shown in Fig. 4, are shown to be compatible with each other. Similar tests were also performed changing the limiting \( M_V \) in the range 3.0–3.5 and comparing against the corresponding Hipparcos sample. This produces alternative CMDs that contain different stars (see Fig. 3), which were compared with synthetic CMDs always from the same central inferred SFR(t). The results were always equal or better than that shown in Fig. 5, model–model and data–model distributions of W having mean values well within 1σ of each other, where σ refers to either the model–model or the data–model W distributions.

5 CONCLUSION

We have applied the method developed in our Paper I to the data of the Hipparcos catalogue. An objective solution for the SFR(t) of the solar neighbourhood over the last 3 Gyr was found, which can be shown to be consistent with the complete volume-limited Hipparcos samples relevant to this age range. A structured SFR(t) is obtained showing a cyclic pattern with a period of about 0.5 Gyr superimposed on some degree of underlying star formation activity, which increases slightly with age. No random bursting behaviour was found at the time resolution of 0.05 Gyr of our
method. A first-order density wave model for the repeated encounter of Galactic arms could explain the observed regularity.

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