Outstanding Issues in Solar Dynamo Theory

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Summary. The magnetic activity of the Sun, as manifested in the sunspot cycle, originates deep within its convection zone through a dynamo mechanism which involves non-trivial interactions between the plasma and magnetic field in the solar interior. Recent advances in magnetohydrodynamic dynamo theory have led us closer towards a better understanding of the physics of the solar magnetic cycle. In conjunction, helioseismic observations of large-scale flows in the solar interior has now made it possible to constrain some of the parameters used in models of the solar cycle. In the first part of this review, I briefly describe this current state of understanding of the solar cycle. In the second part, I highlight some of the outstanding issues in solar dynamo theory related to the the nature of the dynamo $\alpha$-effect, magnetic buoyancy and the origin of Maunder-like minima in activity. I also discuss how poor constraints on key physical processes such as turbulent diffusion, meridional circulation and turbulent flux pumping confuse the relative roles of these vis-a-vis magnetic flux transport. I argue that unless some of these issues are addressed, no model of the solar cycle can claim to be “the standard model”, nor can any predictions from such models be trusted; in other words, we are still not there yet.

1 Introduction

Sunspots have been telescopically observed for centuries, starting with the pioneering observations of many, including Galileo Galilei in the early 17th century. Much later in the 20th century, Schwabe discovered that the number of sunspots on the solar surface vary cyclically, and Carrington discovered that the sunspots appear at lower and lower solar latitudes with the progress of the cycle. With Hale’s discovery of magnetic fields within sunspots in 1908, it became clear that the sunspot cycle is in fact a magnetic cycle (see Figure 1 for an overview of the magnetic butterfly diagram). Efforts to theoretically explain the origin of the solar cycle continued from then on and took a giant leap in 1955 when Parker outlined his theory of the solar cycle based on a magnetohydrodynamic (MHD) dynamo mechanism.
Fig. 1. The solar butterfly diagram depicting the latitude of sunspot appearance (think dark lines) with time. The background shows the weak and diffuse field outside of sunspots. Note that while the sunspot formation belt migrates equatorward, the weak field outside of it migrates poleward with the progress of the cycle, reversing the older polar field at the time of sunspot maximum.

In what follows, I briefly summarize the important concepts underlying the solar cycle that have been developed in the last half of the 20th century (Section 2). I then describe the current state of our understanding, concentrating on those ideas that are widely accepted as important for dynamo action (Section 3). Following this, I highlight the outstanding issues that need to be addressed towards developing a “standard model” of the solar cycle (Section 4). Finally, I end with some concluding remarks (Section 5).

Before we proceed, it is important here to state the scope of this review; this is neither meant to be a comprehensive review of all complementary ideas in solar dynamo theory and modeling, nor is it a reference source for important works in this field. Interested readers who desire these, are referred to the recent, and comprehensive review on the solar dynamo by Charbonneau (2005). This is a personalized account of the field as I perceive it to be.

2 Basic concepts

The interior of the Sun consists of highly ionized gas, i.e., plasma. The fundamental equation which governs the behavior (and generation) of magnetic fields in such a plasma system is the induction equation
\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}),
\] (1)

where \( \mathbf{B} \) is the magnetic field, \( \mathbf{v} \) the velocity field and \( \eta \) the effective magnetic diffusivity of the system. In astrophysical systems such as the Sun, the plasma has a very high characteristic magnetic Reynolds number (the ratio of the first to the second term on the R.H.S. of the above equation). In such a plasma the magnetic fields are frozen in the fluid and therefore the field and plasma movement are coupled. This allows the energy of convective flows in the solar convection zone (SCZ) to be drawn into producing and amplifying magnetic fields, which is the essence of the dynamo mechanism.

Under the approximation of spherical symmetry, applicable to a star such as the Sun, the magnetic and velocity fields can be expressed as

\[
\mathbf{B} = B_\phi \hat{e}_\phi + \nabla \times (A \hat{e}_\phi)
\] (2)

\[
\mathbf{v} = r \sin(\theta) \Omega \hat{e}_\phi + \mathbf{v}_p.
\] (3)

The first term on the R.H.S. of Equation 2 is the toroidal component (i.e., in the \( \phi \)-direction) and the second term is the poloidal component (i.e., in the \( r-\theta \) plane) of the magnetic field. In the case of the velocity field (Equation 3), these two terms correspond to the differential rotation \( \Omega \) and meridional circulation \( v_p \), respectively. The field of helioseismology has now constrained the profile of the solar differential rotation throughout the solar convection zone and it is therefore no longer a free parameter in models of the solar dynamo. The meridional circulation is observed in the surface and helioseismic inversions constrain it somewhat in the upper 10% of the Sun, however, the deeper counter-flow is not yet observed and is theoretically constructed by invoking mass conservation in conjunction with the solar density stratification.

Since the Sun rotates differentially, any pre-existing poloidal field would get stretched in the direction of rotation creating a toroidal component. Such horizontal toroidal flux tubes in the solar interior are subject to magnetic buoyancy (Parker 1955a) and therefore erupt out through the surface creating bipolar sunspot pairs. These bipolar sunspot pairs acquire a tilt due to the action of Coriolis force during their rise through the solar convection zone (SCZ), generating what is commonly known as the Joy’s law distribution of solar active region tilt angles. To complete the dynamo chain of events, the toroidal component of the magnetic field has to be converted back into the poloidal component. This necessitates the action of a non-axisymmetric mechanism, i.e., with non-zero vorticity. The first such proposed mechanism was due to Parker (1955b) who proposed that small-scale helical turbulence can twist rising toroidal flux tubes back into the \( r-\theta \) plane thereby recreating the poloidal field – a process which traditionally came to be known as the dynamo \( \alpha \)-effect; this became an essential ingredient in models of the solar dynamo.
3 Current state of our understanding

In the last two decades, simulations of the dynamics of buoyantly rising (thin) toroidal flux tubes showed that the initial strength of these flux tubes at the base of the SCZ had to be on the order of $10^5$ G, to match the morphological properties of active regions observed at the solar surface (D'Silva & Choudhuri 1993; Fan, Fisher & DeLuca 1994). However, the equipartition field strength of the magnetic field in the SCZ (i.e., the field strength at which magnetic and convective flow energies are in equipartition) is of the order of $10^4$ G. If the strength of the sunspot forming toroidal flux tubes are an order of magnitude greater than the equipartition field strength, then the helical convective flows would be unable to twist them as envisaged in the traditional dynamo $\alpha$-effect formalism.

This realization has now led the dynamo community to explore alternative mechanisms for the regeneration of the poloidal field. Amongst the various contenders, the so-called Babcock-Leighton (BL) mechanism is perceived to be the front-runner. In this mechanism, originally proposed by Babcock (1961) and Leighton (1969), the decay of tilted bipolar sunspot pairs, and the subsequent (net) poleward dispersal of their flux by surface processes such as diffusion, differential rotation and meridional circulation regenerates and reverses the solar poloidal field (Dikpati & Charbonneau 1999; Nandy & Choudhuri 2001). Although this process is now commonly referred to as the BL $\alpha$-effect, it may be noted that in spirit, this process is very different from the traditional $\alpha$-effect; in the latter formalism, averaging over small scale turbulence and the first-order-smoothing-approximation is required, whereas in the former, it is not. The BL mechanism for poloidal field generation is actually observed on the surface and has been substantiated with numerical surface-flux-transport simulations.

Since this poloidal field generation mechanism is primarily located in the near-surface layers, the generated poloidal field has to be transported back into the solar interior, where the toroidal field amplification and storage takes place (in the overshoot layer at the base of the SCZ). In most BL models of the solar cycle, this flux transport is achieved by meridional circulation, although turbulent diffusion is also expected to play a significant role. In such models, it is found that the meridional circulation governs the spatio-temporal distribution of sunspots on the solar surface (Nandy & Choudhuri 2002) and its speed determines the period of the solar cycle, even in regimes where the SCZ is diffusion dominated (Yeates, Nandy & Mackay 2008).

The amplitude of the sunspot cycle is found to be weakly correlated with the speed of the meridional circulation and the coefficient of turbulent diffusion. However, the threshold of magnetic buoyancy, i.e., the field strength at which stored toroidal flux tubes become magnetically buoyant and escape out of the overshoot layer, is a limiting factor on the amplitude of the solar cycle (Nandy 2002).
4 Outstanding issues

Although it seems that we have made much progress in the last decade or so in understanding many aspects of the solar cycle, this progress has also uncovered multiple aspects which pose a challenge to dynamo theory. I take this opportunity to discuss some of these outstanding issues.

4.1 Nature of the dynamo $\alpha$-effect

While the BL mechanism for poloidal field regeneration is observed on the solar surface, a few other $\alpha$-effect mechanisms have been proposed in recent times, which do not have any direct observational confirmation, but nevertheless may be functional in the solar interior. These $\alpha$-effects are driven by magnetic field instabilities or differential rotation instabilities and are spatially located around the base of the SCZ (for an overview of the various proposed dynamo $\alpha$-effects, see Charbonneau 2005).

What is unclear is the extent to which these proposed $\alpha$-effects may contribute to poloidal field regeneration in the Sun and the relative efficacy of these compared to the BL mechanism; this is connected to the following question (I believe first articulated by Manfred Schüssler): Is the observed BL mechanism a by-product of a dynamo mechanism that completely resides in the solar interior, or is it actually an integral part the dynamo mechanism? Although the success of dynamo models based on the BL mechanism argues for the latter scenario, in my view we still cannot rule out the possibility that other $\alpha$-effect mechanisms may contribute at least in parts to poloidal field generation. If this were to be the case it creates an interesting dilemma which is explained below.

For some time now many of us are content with the perception that we can observe one complete half of the solar dynamo mechanism, namely the poloidal field regeneration at the solar surface through active regions decay and dispersal. These surface observations have been used widely to constrain and fine tune dynamo models. Moreover, because the poloidal field of a given cycle feeds directly into producing the toroidal field of future cycle(s), these surface observations provide an useful tool to predict future cycle amplitudes. However, if some other, observationally unconstrained mechanism for poloidal field regeneration is actually more dominant than the BL mechanism, this would pose a serious challenge to our current perceptions of the solar cycle and would negatively impact attempts to predict future solar activity. Therefore, any evidence related to mechanisms of solar poloidal field generation has to be seriously evaluated to illuminate whether there are multiple mechanisms and if yes, what are their relative contributions to the overall dynamo.

4.2 Treatment of magnetic buoyancy

An issue coupled to the nature of the poloidal field regeneration mechanism is the treatment magnetic buoyancy and bipolar sunspot creation in models
of the solar cycle. It is believed that those strong toroidal flux tubes stored in the stably stratified region beneath the SCZ, which exceed a certain threshold (on the order of 100 kG), become magnetically buoyant when they emerge out in the SCZ and subsequently produce poloidal field. This whole process of buoyant eruption and poloidal field generation is treated in dynamo models through diverse implementations – almost all of which are not fully consistent with the philosophy of the BL mechanism.

The most popular approach, in the context of the BL dynamo, has been to approximate this process with a poloidal field source term that is located at near-surface layers (constrained by a prescribed spatially dependent function) and which is proportional to the toroidal field strength at the base of the SCZ. Although this approach typically has a upper quenching threshold which stops poloidal field creation when the toroidal field exceeds a certain threshold (in accordance with flux tube rise simulations which show that very strong toroidal flux tubes come out without any tilt), most modelers do not use a lower operational threshold. This causes even weak, sub-kG toroidal field to contribute to poloidal field creation through the BL mechanism, which goes against the spirit of the BL idea. An alternative approach, which has been used by some modelers, is to employ an explicit algorithm for magnetic buoyancy which searches for strong toroidal fields exceeding the buoyancy threshold and transporting this field to the surface layers, conserving flux in the process. However, this process too over-simplifies the surface-flux-dispersal process as it still uses a source term at the surface. Basically, the usage of this source-term preempts the surface-flux-transport process, which results in dynamo simulations giving results that are not consistent with surface-flux-transport simulations when a variable meridional flow is used (Schrijver & Liu 2008).

A more realistic, but rarely used approach is to buoyantly erupt spaced double rings of opposite radial field (akin to bipolar sunspot pairs) to the solar surface using a explicit buoyancy algorithm (for a comparative study of various buoyancy algorithms, including the double-ring approach, see Nandy & Choudhuri 2001). The source-term is discarded with in this approach. Surface differential rotation, meridional circulation and diffusion subsequently acts on these erupted double rings to generate the poloidal field in a truer representation of the BL philosophy. However, this approach is computationally intensive as it demands a very high grid-resolution, which is sufficiently close to solar AR spatial-scale. This being a impractical task, the over-simplified and somewhat questionable buoyancy prescriptions continue to be used in solar dynamo models. If an alternative, physically correct algorithm cannot be devised, it seems the brute-force solution to this problem is to implement more computationally efficient numerical algorithms for the solar dynamo that can handle very high grid-resolutions.
4.3 Origin of grand minima

Small, but significant variations in solar cycle amplitude is commonly observed from one cycle to another and models based on either stochastic fluctuations, or non-linear feedback, or time-delay dynamics exist to explain such variability in cycle amplitude (for overviews see, Charbonneau 2005; Wilmot-Smith et al. 2006). However, most models find it difficult to switch off the sunspot cycle completely for an extended period of time – such as that observed during the Maunder minimum – and subsequently recover back to normal activity.

Two important and unresolved questions in this context are what physical mechanism stops active region creation completely and how does the dynamo recover from this quiescent state. The first question is the more vexing one and still eludes a coherent and widely accepted explanation. The second question is less challenging in my opinion; the answer possibly lies in the continuing presence of another $\alpha$-effect (could be the traditional dynamo $\alpha$-effect suggested by Parker) which can work on weaker, sub-equipartition toroidal fields – to slowly build up the dynamo amplitude to eventually recover the sunspot cycle from a Maunder-like grand minima.

These are speculative ideas and one thing that can be said with confidence at this writing is that we are just scratching the surface as far as the physics of grand minima like episodes is concerned.

4.4 Parametrization of turbulent diffusivity

Typically, in many dynamo models published in the literature, the coefficient of turbulent diffusivity employed in much lower than that suggested by mixing-length theory (about $10^{13}$ cm$^2$/s; Christensen-Dalsgaard et al. 1996). This is done to ensure that the flux transport in the SCZ in advection dominated (i.e., meridional circulation is the primary flux transport process). There are many disadvantages to using a higher diffusivity value in these dynamo models. Usage of higher diffusivity values makes the flux transport process diffusion dominated, reducing the dynamo period to values somewhat lower than the observed solar cycle period. It also makes flux storage and amplification difficult and shortens cycle memory; the latter is the basis for solar cycle predictions. Nevertheless, this inconsistency between mixing-length theory and parametrization of turbulent diffusivity in dynamo models is, in my opinion, a vexing problem.

In the absence of any observational constraints on the depth-dependence of the diffusivity profile in the solar interior, this problem can only be addressed theoretically. One possible solution to resolving this inconsistency is by invoking magnetic quenching of the mixing-length theory suggested diffusivity profile. The idea is simple enough; since magnetic fields have an inhibiting effect on turbulent convection, strong magnetic fields should quench and thereby be subject to less diffusive mixing. The magnetic quenching of turbulent diffusivity is challenging to implement numerically, but seems to me to be the
best bet towards reconciling this inconsistency within the framework of the current modeling approach.

4.5 Role of downward flux pumping

An important physical mechanism for magnetic flux transport has been identified recently from full MHD simulations of the solar interior. This mechanism, often referred to as turbulent flux pumping, pumps magnetic field preferentially downwards, in the presence of rotating, stratified convection such as that in the SCZ (see e.g., Tobias et al. 2001). Typical estimates yield a downward pumping speed which can be as high as 10 m/s; this would make flux pumping the dominant downward flux transport mechanism in the SCZ, short-circuiting the transport by meridional circulation and turbulent diffusion. However, turbulent flux pumping is usually ignored in kinematic dynamo models of the solar cycle.

If indeed the downward pumping speed is as high as indicated, then turbulent flux pumping may influence the solar cycle period, crucially impact flux storage and amplification and also affect solar cycle memory. Therefore, turbulent flux pumping must be properly accounted for in kinematic dynamo models and its effects completely explored; this remains an issue to be addressed adequately.

5 Concluding remarks

Now let us elaborate on, and examine some of the consequences of the outstanding issues highlighted in the earlier section.

5.1 A story of communication timescales

To put a broader perspective on some of these issues facing dynamo theory, specifically in the context of the interplay between various flux-transport processes, it will be instructive here to consider the various timescales involved within the dynamo mechanism. Let us, for the sake of argument, consider that the BL mechanism is the predominant mechanism for poloidal field regeneration. Because this poloidal field generation happens at surface layers, but toroidal field is stored and amplified deeper down near the base of the SCZ, for the dynamo to work these two spatially segregated layers must communicate with each other. In this context, magnetic buoyancy plays an important role in transporting toroidal field from the base of the SCZ to the surface layers – where the poloidal field is produced. The timescale of buoyant transport is quite short, on the order of 0.1 year and this process dominates the upward transport of toroidal field.

Now, to complete the dynamo chain, the poloidal field must be brought back down to deeper layers of the SCZ where the toroidal field is produced and
There are multiple processes that compete for this downward transport, namely meridional circulation, diffusion and turbulent flux pumping.

Considering the typical meridional flow loop from mid-latitudes at the surface to mid-latitudes at the base of the SCZ, and a peak flow speed of 20 m/s, one gets a typical circulation timescale $\tau_v = 10$ years. Most modelers use low values of diffusivity on the order of $10^{11}$ cm$^2$/s, which makes the diffusivity timescale ($L_{SCZ}^2/\eta$, assuming vertical transport over the depth of the SCZ), $\tau_\eta = 140$ years; i.e., much more that $\tau_v$, therefore making the circulation dominate the flux transport. However, if one assumes diffusivity values close to that suggested by mixing length theory (say, $5 \times 10^{12}$ cm$^2$/s), then the diffusivity timescale becomes $\tau_\eta = 2.8$ years; i.e., shorter than the circulation timescale – making diffusive dispersal dominate the flux transport process.

If we now consider the usually ignored process of turbulent pumping, the situation changes again. Assuming a typical turbulent pumping speed on the order of 10 m/s over the depth of the SCZ gives a timescale $\tau_{\text{pumping}} = 0.67$ years, shorter than both the diffusion and meridional flow timescales. This would make turbulent pumping the most dominant flux transport mechanism for downward transport of poloidal field into the layers where the toroidal field is produced and stored.

5.2 Solar cycle predictions

As outlined in Yeates, Nandy & Mackay (2008), the length of solar cycle memory (defined as over how many cycles, the poloidal field of a given cycle would contribute to toroidal field generation) determines the input for predicting the strength of future solar cycles. The relative timescales of different flux transport mechanisms within the dynamo chain of events, and their interplay – based on which process (or processes) dominate, determine this memory. For example, if the dynamo is advection (circulation) dominated then the memory tends to be long, lasting over multiple cycles. However, if the dynamo is diffusion (or turbulent pumping) dominated, then this memory would be much shorter.

Now, within the scope of the current framework of dynamo models, I have argued that significant confusion exists regarding the role of various flux transport processes. So much so that we do not yet have a consensus on which of these processes dominate; therefore we do not have a so-called “standard-model” of the solar cycle yet. Should solar cycle predictions be trusted then?

Taking into account this uncertainty in the current state of our understanding of the solar dynamo mechanism, I believe that any solar cycle predictions – that does not adequately address these outstanding issues – should be carefully evaluated. In fact, under the circumstances, it is fair to say that if any solar cycle predictions match reality, it would be more fortuitous than a vindication of the model used for the prediction. This is not to say that modelers should not explore the physical processes that contribute to solar cycle predictability; indeed that is where most of our efforts should be. My concern
is that we do not yet understand all the physical processes that constitute the
dynamo mechanism and their interplay well enough to begin making predic-
tions. Prediction is the ultimate test of any model, but there are many issues
that need to be sorted out before the current day dynamo models are ready
for that ultimate test.

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