Different models of connection flexibility in dynamic analysis of plane frame systems

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Abstract: Nowadays, in Vietnamese structural design standards for steel frames and semi-precast reinforced concrete structural frame systems, the connections of beams and columns are commonly considered as perfectly hinged or rigidly fixed. This is adopted because of the simplicity in design and analysis procedure but it is not consistent with the condition taking into account actual behaviour in practice of structural systems. The connections between columns and beams have partial restraint depending on the type of used connections. The presence of connection flexibility tends to reduce frame stiffness in comparison with the case of fully fixed connection and hence increases the frame’s natural period of vibration. However, it is also more time required because of complexity of analysis process with consideration of connection flexibility in the case of lack of suitable analysis method. The paper presents the dynamic analysis of plane frame systems with different models of semi-rigid connections using finite element method. The used models in dynamic analysis are rotational spring and rotational-lateral spring. The analysis procedure was implemented using MathCad programming software and the results of this study will obtained by conducting of numerical examples. From there, the conclusions and recommendation of dynamic analysis for steel and semi-precast concrete frames with models of connection flexibility and the differences between used models will be proposed.

1. Introduction
Semi-rigid frames are considered frames, in which the beam-column connections are neither perfectly pinned nor rigid. In practical engineering structures, all frames are semi-rigid in nature because truly pinned or perfectly rigid connections do not exist. The design instructions for these types of structures have been mentioned in many foreign construction standards with specific calculation recommendations considering the connection flexibility. However, in Vietnam, there is still no specific domestic calculation instructions in design of these structures considering the actual behavior of the connections. When designing these structures, it is either used foreign standards or considered the assumption that the connections between beams and columns are perfectly hinged or rigidly fixed. This assumption does not reflect the actual behavior of practical structures. The calculation with consideration of connection flexibility is more constituent with the behavior of the work of the practical structures. Thereof, if the calculation takes into account the connection flexibility, the analysis results will be more accurate and closer to the actual behavior of the real structures. If the influence of flexibility of semi-rigid connections is underestimated, and they are treated in the design as pinned, it has a negative impact on cost of a structure. Thus, in the analysis and design of steel frames the connection between beams and columns should be modeled as semi-rigid connections.

The connection flexibility of steel and semi-precast concrete frame structures have been concerned by many scientists from over the worlds and many studies on this issue are published with different
problems [1-8]. In most of these studies the connections between beams and columns of semi-rigid frames were modeled by rotational springs. In fact, frame elements have semi-rigid behavior through the axial direction of themselves. In [9], the behaviors of beam-column connections are designed through either rotational or lateral effects. The additional effect of lateral springs already discussed in [9] on statically analysis. In this paper, the dynamic analysis is further discussed with the whole range of the values rotational and lateral springs which correspond with the end-fixity factor. The end-fixity factor is analyzed from a value greater than 0 to near 1 (0 is the value of end-fixity factor that corresponds with the case of pinned connection and 1-the case of rigidly fixed).

2. Dynamic analysis of plane frame systems with consideration of different models of semi-rigid connection

The flexibility of beam-column connections of frames is modelled by rotational springs and lateral springs. Each model has its characteristics. In this section, it is introduced about their properties and element stiffness matrix in finite element analysis.

2.1. Rotational spring model

In rotational spring model, the connection is represented by rotational springs near to beam-column joints. The presence of rotational springs will restrain the rotations $\theta_{rA}$ and $\theta_{rB}$ at the ends of the beam AB [9].

![Figure 1. Beam element with semi-rigid connections.](image)

The unit of stiffness of rotational spring is kN m rad. In the analysis of semi-rigid plane frames this model is illustrated in figure 2.

![Figure 2. The rotational spring model for beam-column connections.](image)

The important step of analysis using finite element method is determination of global stiffness matrix of structure system. The element stiffness matrices for beams and columns can be written as following.

The beam stiffness matrix [10]
In which

\[ s_{ij}^* = \left( 4 + \frac{12E \cdot I_b}{l \cdot R_{ABA}} \right) \left( 1 + \frac{4E \cdot I_b}{l \cdot R_{ABA}} \right) \left( 1 + \frac{4E \cdot I_b}{l \cdot R_{ABB}} \right) \left( E \cdot I_b \right)^2 \left( \frac{4}{R_{ABA} \cdot R_{ABB}} \right) \]

\[ s_{ji}^* = \left( 4 + \frac{12E \cdot I_b}{l \cdot R_{ABA}} \right) \left( 1 + \frac{4E \cdot I_b}{l \cdot R_{ABA}} \right) \left( 1 + \frac{4E \cdot I_b}{l \cdot R_{ABB}} \right) \left( E \cdot I_b \right)^2 \left( \frac{4}{R_{ABA} \cdot R_{ABB}} \right) \]

\[ s_{ij} = \frac{s_{ji}^*}{2} \]

\[ \left( 1 + \frac{4E \cdot I_b}{l \cdot R_{ABA}} \right) \left( 1 + \frac{4E \cdot I_b}{l \cdot R_{ABB}} \right) \left( E \cdot I_b \right)^2 \left( \frac{4}{R_{ABA} \cdot R_{ABB}} \right) \]

\[ \left( 1 + \frac{4E \cdot I_b}{l \cdot R_{ABA}} \right) \left( 1 + \frac{4E \cdot I_b}{l \cdot R_{ABB}} \right) \left( E \cdot I_b \right)^2 \left( \frac{4}{R_{ABA} \cdot R_{ABB}} \right) \]

\[ s_{ij}^* = \left( \frac{4 + 12E \cdot I_b}{l \cdot R_{ABA}} \right) \left( 1 + \frac{4E \cdot I_b}{l \cdot R_{ABA}} \right) \left( 1 + \frac{4E \cdot I_b}{l \cdot R_{ABB}} \right) \left( E \cdot I_b \right)^2 \left( \frac{4}{R_{ABA} \cdot R_{ABB}} \right) \]

\[ R_{ABA}, R_{ABB} \] – initial stiffness rotational spring (in this study it is used the symbol \( C_\theta = R_{ABA}, C_\theta = R_{ABB} \))

\( E, I_b, A_b, l \) are respectively elastic modulus, moment of inertia, area of cross section, length of the beams.

The column stiffness matrix [10]

\[ \mathbf{K}_{b \rightarrow c} = \frac{E \cdot I_b}{l} \]

\[ \left[ \begin{array}{cccc} \frac{12}{l^2} \gamma_1 & 0 & 0 & \frac{12}{l^2} \gamma_2 \\ 0 & \frac{6}{l_e} \gamma_1 & 0 & \frac{6}{l_e} \gamma_2 \\ 0 & 0 & \frac{A}{I_c} & 0 \\ 4\gamma_3 & -\frac{6}{l_e} \gamma_2 & 0 & 2\gamma_4 \\ \frac{12}{l^2} \gamma_1 & 0 & \frac{6}{l_e} \gamma_2 & \frac{A}{I_e} \end{array} \right] \]

In which the coefficients are determined by following formula
\[ \begin{align*}
\gamma_1 &= \frac{(kl_r)^3 \cdot \sinh (kl_r)}{12(2 - 2\cosh (kl_r) \pm kl_r \cdot \sinh (kl_r))}; \\
\gamma_2 &= \frac{(kl_r)^2 \cdot (\cosh (kl_r) - 1)}{6(2 - 2\cosh (kl_r) \pm kl_r \cdot \sinh (kl_r))}; \\
\gamma_3 &= \frac{(kl_r) \cdot (kl_r \cdot \cosh (kl_r) - \sinh (kl_r))}{4(2 - 2\cosh (kl_r) \pm kl_r \cdot \sinh (kl_r))}; \\
\gamma_4 &= \frac{(kl_r) \cdot (\sinh (kl_r) - kl_r)}{(2 - 2\cosh (kl_r) \pm kl_r \cdot \sinh (kl_r))}.
\end{align*} \] (4)

Where \( k = \left( \frac{P}{EI} \right)^{1/2} \), “+” sign corresponds with tensile axial force and “-” sign – compressive axial force. In the case of that the axial force is small, \( \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 1 \).

### 2.2. Rotational-lateral spring model

This model is represented by rotational and lateral springs near beam-column connections. The presence of these spring not only restrain the rotations \( \theta_i \) and \( \theta_k \), but also the lateral displacements \( \delta_i \) and \( \delta_k \) at the ends of beams. This model is illustrated in figure 3.

![Figure 3. The rotational-lateral spring model in semi-rigid frame structures.](image)

The beam stiffness matrix can be written as followings:

\[ \begin{pmatrix}
\frac{A_b}{I_b} + R_{bi} & 0 & 0 & -\frac{A_b}{I_b} - R_{bi} & 0 & 0 \\
\frac{\left(s'_u + 2s'_y + s''_y\right)}{l^2} & \frac{\left(s'_u + s'_y\right)}{l} & 0 & -\frac{\left(s'_u + 2s'_y + s''_y\right)}{l^2} & \frac{\left(s'_u + s'_y\right)}{l} \\
\frac{\left(s'_u + s'_y\right)}{l} & 0 & -\frac{\left(s'_u + s'_y\right)}{l} & \frac{s'_y}{l} & 0 \\
\frac{A_b}{I_b} + R_{bi} & 0 & 0 & \frac{A_b}{I_b} + R_{bi} & 0 \\
\frac{\left(s'_u + 2s'_y + s''_y\right)}{l^2} & \frac{\left(s'_u + s'_y\right)}{l} & 0 & -\frac{\left(s'_u + s'_y\right)}{l} & \frac{s'_y}{l} \\
\end{pmatrix} \] (5)

in which


\[
R_{bi} = \frac{l}{I_b} \cdot \frac{A_b \cdot (k_i \cdot k_j)}{l \cdot (k_i \cdot k_j) + E \cdot A_b \cdot (k_i \cdot k_j)}
\]  

(6)

where, \(k_i, k_j\) – lateral stiffness of the second model. The unit of lateral stiffness is kN/m;

2.3. Dynamic analysis of plane frames with consideration of two models of connection flexibility using finite element method

This study is to investigate the dynamic characteristics of semi-rigid frames and the influences of connection flexibility on the dynamic characteristics. The equation of motion for a semi-rigid frames in the case of free vibration is written as followings:

\[
M \cdot \ddot{\mathbf{v}} + \mathbf{K} \cdot \mathbf{v} = 0
\]  

(7)

Where \(\ddot{\mathbf{v}}\) and \(\mathbf{v}\) are acceleration and displacement vectors of a structure respectively.

\(M\) and \(\mathbf{K}\) are respectively mass and element stiffness matrices of semi-rigid plane frames.

In this study, the dynamic characteristics of semi-rigid frames are determined by modal analysis. The frequency \(\omega_i\) and period \(T_i\) of a vibration will be investigated. The influences of connection flexibility will be discussed. It is solved the eigenvalue problem, that means the determinant of the system has to be equal to zero. From equation (8) it is found corresponding frequency \(\omega_i\).

\[
\begin{vmatrix}
\mathbf{K} - \omega^2 \cdot \mathbf{M}
\end{vmatrix} = 0
\]  

(8)

The effects of connection flexibility are modeled using rotational springs having stiffness \(C_{\alpha,i,j}\) and rotational-lateral springs having stiffness \(k_{i,j}\) at the ends of the beams. To reflect the relative stiffness of the beams and the rotational end-spring connections, an end-fixity factor is used [10]. It is defined as:

\[
\eta = \frac{\alpha}{\phi} = \frac{1}{1 + \frac{3}{R_i}}
\]  

(9)

Where \(\alpha\) is the member end rotation, \(\phi\) is the combined rotation of the member and the connection due to a unit end moment, \(R_i = (C_{\alpha,i,j} \cdot I_b \cdot (EI_b))^{-1}\). Note that \(\eta\) falls between zero and one, it defines the stiffness of each end connection relative to the adjoining beam (\(\eta = 0\) corresponds with the case of perfectly pinned connection, \(\eta = 1\) – fully fixed connection).

3. Numerical example

In this paper, two-storey semi-rigid frames with two above mentioned models as shown in figure 4 and figure 5. All frames have same geometrical dimensions, cross-section and material properties for comparison of influences of connection flexibility on dynamic characteristics. Similarly, in this study it is clarifies the different effects of used different models of flexibility connection on dynamic characteristics.

Given material properties and geometrical sizes as followings: elastic modulus \(E=20 \cdot 10^7\) kN/m\(^2\), cross-section of beams C150×70, cross-section of columns 1160×81×5.0, \(l=4\)m, \(h=3.5\)m.
**Figure 4.** Two-storey semi-rigid frame with rotational spring model in dynamic analysis.

**Figure 5.** Two-storey semi-rigid frame with rotational -lateral spring model in dynamic analysis.

**Table 1.** Results of dynamic analysis with rotational spring model

| End-fixity factor $\eta$ | Frequency $\omega_i$ (rad/s) | Period $T_i$ (s) |
|--------------------------|-----------------------------|-----------------|
|                          | 1-st mode | 2-nd mode | 3-rd mode | 1-st mode | 2-nd mode | 3-rd mode |
| 1 (rigid)                | 6.532     | 6.795     | 34.52     | 0.9617    | 0.9245    | 0.182     |
| 0.8                      | 6.532     | 6.795     | 30.03     | 0.96175   | 0.9246    | 0.209     |
| 0.6                      | 6.532     | 6.795     | 26.60     | 0.9618    | 0.9246    | 0.236     |
| 0.4                      | 6.5317    | 6.794     | 23.48     | 0.96185   | 0.92465   | 0.267     |
| 0.3                      | 6.5315    | 6.793     | 21.82     | 0.96185   | 0.92457   | 0.287     |
| 0.1                      | 6.5317    | 6.792     | 17.66     | 0.9619    | 0.9245    | 0.355     |
| 0 (pinned)               | 6.531     | 6.663     | 14.44     | 0.962     | 0.943     | 0.435     |

**Table 2.** Results of dynamic analysis with rotational-lateral spring model

| End-fixity factor $\eta$ | Lateral spring stiffness $k_i$ (kN/m) | Frequency $\omega_i$ (rad/s) | Period $T_i$ (s) |
|--------------------------|---------------------------------------|-----------------------------|-----------------|
|                          | 1-st mode | 2-nd mode | 3-rd mode | 1-st mode | 2-nd mode | 3-rd mode |
| 1 (rigid)                | $10^7$    | 6.532     | 6.795     | 34.52     | 0.9617    | 0.9245    | 0.182     |
| 0.8                      | $10^6$    | 6.5317    | 6.795     | 30.03     | 0.9618    | 0.9246    | 0.209     |
| 0.6                      | $10^5$    | 0.65314   | 6.794     | 26.60     | 0.96184   | 0.9246    | 0.2362    |
| 0.4                      | $10^4$    | 0.6531    | 6.794     | 23.40     | 0.96191   | 0.92466   | 0.268     |
| 0.3                      | $10^3$    | 6.531     | 6.79     | 21.60     | 0.96198   | 0.9248    | 0.289     |
| 0.1                      | $10^2$    | 0.65308   | 6.78     | 17.55     | 0.96199   | 0.9249    | 0.358     |
| 0 (pinned)               | 0         | 6.5305    | 6.662     | 14.43     | 0.962     | 0.943     | 0.435     |

From the results of calculation, it can be shown that the different models of semi-rigid connection effects differently on the dynamic analysis.
4. Conclusions
The flexibility of beam-column connections is presented by rotational and rotational-lateral springs models that partially reflect actual behaviour of real structures. The stiffness matrices are introduced for dynamical analysis by finite element method. In this study, the subroutine is written for dynamical analysis using finite element method by MathCad programming software that helps to perform any geometrically dimensions of structures easily.

The effects of connection flexibility are investigated. The different types of models of connection flexibility effect differently on dynamic behaviour of structures. The study results show that the connection flexibility tends to increase periods, whereas it tends to decrease the frequency. The first and the second frequencies tend to increase with very little changes, but the third frequency changes significantly corresponding with the change of the value of fixity factor. The frequencies of plane frames with rotational spring model are greater in comparison with the case with rotational-lateral spring models, whereas the periods of the same analysis are reversed.

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