Field induced conducting state in Mott insulator

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Electron–electron repulsion, on the one hand, can result in bound pair, which has heavy effective mass. On the other hand, it is also the cause of Mott insulator. We study the effect of a staggered magnetic field on a Hubbard model. We find that a bound pair with large energy bandwidth can be formed under the resonant staggered field, being the half of Hubbard repulsion strength. Accordingly, the system exhibits following dynamical behaviors: (i) When an electric field is applied, fast bound pair Bloch oscillation occurs, while a single electron is frozen. (ii) When a quenching resonant field is applied to an initial antiferromagnetic Mott insulating state, the final state becomes doublon conducting state manifested by the non-zero $\eta$ correlator and large charge fluctuation. Our finding indicates that the cooperation of electron-electron correlation and modulated external field can induce novel quench dynamics.

I. INTRODUCTION

The Hubbard model represents a minimal starting point for modeling strongly correlated systems\cite{1–5}. It has relevance for a broad range of correlation phenomena, including high-temperature superconductivity\cite{6}, metal-insulator transition\cite{7} quantum criticality\cite{8} to interacting topological states of matter\cite{9}. Over the past decades, the Hubbard model lies at the heart of investigating the cold atom quantum simulations since these strongly correlated systems are faithful representations of the bosonic or fermionic Hubbard models\cite{9–11}. Recent developments in quantum simulations of the Hubbard model suggest that these cold atom systems are particularly suitable for studying quantum many-body dynamics\cite{12}, which is less refereed in the content of condensed matter systems. A fascinating subject is quantum quench dynamics in the strongly correlated system. It studies how a many-body system is prepared initially in a certain initial state, and evolves unitarily in time following the sudden change of the parameters to a final Hamiltonian\cite{13}. Dynamics induced by a quantum quench has become an active topic of research because it poses many fundamental questions that can also be studied by current generation experiments.

A typical feature of quench dynamics is that the considered many-body systems are usually far from equilibrium, which can induce many intriguing phenomena in condensed-matter materials\cite{14,15}. One striking example of recent experimental observations is the light-induced superconducting-like properties in some high-$T_c$ cuprates\cite{15–21} and alkali-doped fullerenes\cite{22–23}, which have stimulated many theoretical investigations\cite{23,24}. In addition, the pump-probe experiments realized in cold-atom setups have been also triggered an intensive theoretical investigation of out-of-equilibrium dynamics in quantum many-body systems (see e.g. the reviews\cite{25,26}). They have been successfully used for understanding the mechanisms of relaxation\cite{25,26} showing that local-in-space observables relax at large times. Such novel dynamic phenomena are ultimately attributed to the cooperation between the interaction and the external field.

This paper aims to study the effect of the staggered external field on a Fermi-Hubbard model in the deep Mott insulating phase. We show that the quench resonant field can build a bridge for the transport of the particles demonstrated by the large charge fluctuation and enhancement of the doublon-doublon correlation. Specifically, an initial anti-ferromagnetic insulating state becomes a conducting state under the action of the post-quench Hamiltonian. The underlying mechanism, which can be elucidated by the two-particle system, is that: The combination between the on-site interaction and the presence of resonant staggered field culminates in the formation of a bound pair (BP) with large energy bandwidth. However, the bandwidth of a single particle is shrunk. Correspondingly, the system exhibit two distinct dynamic behaviors when the dc electric field is applied. For the scattering states, the particles are frozen in the initial place. As for the BPs, they show breathing behavior populating a large interval that is determined by the bandwidth of BP. This is in stark difference with the case of the staggered field free, wherein the BP is frozen at the starting point. Hence, the large-scale migration of BPs induced by the external field provides an insight into the dynamics transition of the system from the Mott insulating to conducting phase. It is hoped that these results can motivate further studies of both fundamental aspects and potential applications of quench interacting many-body systems. The rest of the paper is structured as follows: In Sec. \textsection II, we introduce the Hubbard model subjected to a staggered field. The exact two-particle equivalent Hamiltonian and effective Hamiltonian about the BP solution are investigated based on the symmetry of the system. Sec. \textsection III is devoted to investigate the dynamics transition of the system from the Mott insulating to conducting phase. It is hoped that these results can motivate further studies of both fundamental aspects and potential applications of quench interacting many-body systems. The rest of the paper is structured as follows: In Sec. \textsection IV, the transition from the Mott insulating to doublon conducting phase is investigated through quench dynamics by examining the charge fluctuation and $\eta$ correlator. We conclude and discuss our results in Sec. \textsection V.
In the absence of the staggered magnetic field $V = 0$, the system Hamiltonian $H$ has two sets of SU(2) symmetry. The first of these relates to spinful particles which can be characterized by the generators

$$s^z = (s^z)^\dagger = \sum_{j \in A \cup B} s_j^z,$$  \hspace{1cm} (3)

$$s^\pm = \sum_{j \in A \cup B} s_j^\pm,$$ \hspace{1cm} (4)

where the local operators $s_j^+ = c_{j,\uparrow}^\dagger c_{j,\downarrow}$ and $s_j^- = (n_{j,\uparrow} - n_{j,\downarrow})/2$ obey the Lie algebra, i.e., $[s_j^+, s_j^-] = 2s_j^z$, and $[s_j^+, s_j^+] = \pm s_j^z$. The second, often referred to as $\eta$ symmetry, relates to spinless quasiparticles. The corresponding generators can be given as

$$\eta^+ = (\eta^-)^\dagger = \sum_{j \in A \cup B} \eta_j^+,$$ \hspace{1cm} (5)

$$\eta^- = \sum_{j \in A \cup B} \eta_j^-,$$ \hspace{1cm} (6)

with $\eta_j^+ = \lambda c_{j,\uparrow}^\dagger c_{j,\downarrow}^\dagger$ and $\eta_j^- = (n_{j,\uparrow} + n_{j,\downarrow} - 1)/2$ satisfying commutation relation, i.e., $[\eta_j^+, \eta_j^-] = 2\eta_j^z$, and $[\eta_j^+, \eta_j^+] = \pm \eta_j^z$. Here we assume a bipartite lattice and $\lambda = 1$ for $j \in \{A\}$ and $-1$ for $j \in \{B\}$. Evidently, the presence of the staggered field spoils such SU(2) symmetries but remains $s^z$ symmetry, that is $[s^z, H] = 0$. Hence, the system can be divided into different invariant subspaces in terms of the quantum number of $s^z$.

**II. MODEL**

**A. the Hubbard model with staggered potential**

We consider a many-body quantum system $H = H_0 + H_s$ and show the basic property determined by the interplay between the interaction and external field. The system Hamiltonian given by the Hubbard model with bipartite lattice

$$H_0 = -\sum_{i \in \mathcal{A}, j \in \mathcal{B}} J_{ij} c_{i,\sigma}^\dagger c_{j,\sigma} + H.c. + U \sum_{j \in \mathcal{A} \cup \mathcal{B}} n_{j,\uparrow} n_{j,\downarrow},$$ \hspace{1cm} (1)

is subjected to a staggered external field

$$H_s = \frac{V}{2} \sum_{j \in \mathcal{A}} (n_{j,\uparrow} - n_{j,\downarrow}) - \frac{V}{2} \sum_{j \in \mathcal{B}} (n_{j,\uparrow} - n_{j,\downarrow}).$$ \hspace{1cm} (2)

where the operator $c_{j,\sigma}$ is the annihilation operator of a spin-$\sigma$ fermion at site $j$, satisfying the usual fermion anticommutation relations $\{c_{i,\sigma}^\dagger, c_{j,\sigma'}\} = \delta_{i,j} \delta_{\sigma,\sigma'}$ and $\{c_{i,\sigma}, c_{j,\sigma'}\} = 0$; the system can be divided into two disjoint sets $\mathcal{A}$ and $\mathcal{B}$ and the hopping matrix elements $J_{ij}$ are required to be real and satisfy $J_{ij} = J_{ji}$; the number operators are $n_{j,\sigma} = c_{j,\sigma}^\dagger c_{j,\sigma}$, while $U$ is the on-site energy denoting the repulsive particle-particle interaction; the term $V$ describes spin-dependent staggered potentials. For convenience and clarity, the number of the sublattice sites and the filled-particles are denoted by $N$ and $M$, respectively. The considered system is illustrated in Fig. 1.

Here, we want to stress that the staggered magnetic field can alter significantly the mobility of the particles. A simple example is employed to elucidate the underlying mechanism in Fig. 2. In the absence of the staggered field $V = 0$, the system Hamiltonian $H$ has two sets of SU(2) symmetry. The first of these relates to spinful particles which can be characterized by the generators

$$s^z = (s^z)^\dagger = \sum_{j \in A \cup B} s_j^z,$$ \hspace{1cm} (3)

$$s^\pm = \sum_{j \in A \cup B} s_j^\pm,$$ \hspace{1cm} (4)

where the local operators $s_j^+ = c_{j,\uparrow}^\dagger c_{j,\downarrow}$ and $s_j^- = (n_{j,\uparrow} - n_{j,\downarrow})/2$ obey the Lie algebra, i.e., $[s_j^+, s_j^-] = 2s_j^z$, and $[s_j^+, s_j^+] = \pm s_j^z$. The second, often referred to as $\eta$ symmetry, relates to spinless quasiparticles. The corresponding generators can be given as

$$\eta^+ = (\eta^-)^\dagger = \sum_{j \in A \cup B} \eta_j^+,$$ \hspace{1cm} (5)

$$\eta^- = \sum_{j \in A \cup B} \eta_j^-,$$ \hspace{1cm} (6)

with $\eta_j^+ = \lambda c_{j,\uparrow}^\dagger c_{j,\downarrow}^\dagger$ and $\eta_j^- = (n_{j,\uparrow} + n_{j,\downarrow} - 1)/2$ satisfying commutation relation, i.e., $[\eta_j^+, \eta_j^-] = 2\eta_j^z$, and $[\eta_j^+, \eta_j^+] = \pm \eta_j^z$. Here we assume a bipartite lattice and $\lambda = 1$ for $j \in \{A\}$ and $-1$ for $j \in \{B\}$. Evidently, the presence of the staggered field spoils such SU(2) symmetries but remains $s^z$ symmetry, that is $[s^z, H] = 0$. Hence, the system can be divided into different invariant subspaces in terms of the quantum number of $s^z$.

**B. bound pair solution**

To gain physical insight into subsequent many-body dynamics, we first investigate the two-particle invariant subspaces. When $V = 0$, the 1D homogeneous system can harbor the bound pair (BP) solution with the form of $\epsilon_{bp} = \sqrt{U^2 + 16J^2 \cos^2 \frac{2\beta}{10}}$. The corresponding bandwidth is $8J^2/U$ as $J \ll U$. This becomes very complicated when staggered field presents. For clarity and simplicity, we suppose that the Hamiltonian $H$ still describes a 1D homogeneous ring system. The translation symmetry combing with $s^z$ symmetry allows us to construct the basis of invariant subspace $s^z = 0$ as follow

\[
\begin{pmatrix}
|j, 1, k\rangle \\
|j, 2, k\rangle \\
|j, 3, k\rangle \\
|j, 4, k\rangle \\
\end{pmatrix} = \sum_l \frac{1}{\sqrt{N}} e^{ik(l+j)} \begin{pmatrix}
\alpha_{l+j,1,\uparrow}^\dagger c_{l+j,\downarrow}^\dagger |\text{Vac}\rangle \\
\alpha_{l+j,2,\uparrow}^\dagger c_{l+j,\downarrow}^\dagger |\text{Vac}\rangle \\
\beta_{l+j,3,\uparrow}^\dagger c_{l+j,\downarrow}^\dagger |\text{Vac}\rangle \\
\beta_{l+j,4,\uparrow}^\dagger c_{l+j,\downarrow}^\dagger |\text{Vac}\rangle \\
\end{pmatrix},
\]

with the notation $\alpha_{l,\sigma}^\dagger = c_{l,\sigma}^\dagger$ ($l \in \{A\}$) and $\beta_{l,\sigma}^\dagger = c_{l,\sigma}^\dagger$ ($l \in \{B\}$). Here $k = 2n\pi/N$ is the momentum vector indexing the subspace and $j = 0...N - 1$ describes the relative distance between the two particles. The other two invariant subspaces $s^z = \pm 1$ are not considered since the bound states only reside in the subspace with $s^z = 0$. 

**Fig. 1:** Schematic illustration of the structure of the Hubbard model concerned in this work, which supports the transition from insulating to conducting states induced by the competition between on-site repulsion and staggered magnetic field. It consists of two sub-lattices $A$ (empty) and $B$ (grey). The electrons are subjected by opposite magnetic fields which are denoted by orange and green shaded regions, respectively. We present a possible electron filling configuration to illustrate the underlying mechanism of the transition. The corresponding on-site energy is denoted under each sits. (i) In the case of $V = 0$ and large $U$, the hopping processes denoted by red arrows are allowed, while green is forbidden. The half-filled ground state is Mott insulating state even at half filling. (ii) When the resonant magnetic field is switched on, the hopping processes denoted by green arrows are allowed, while red is forbidden, supporting conducting state even at half filling.
In such a two-particle Hilbert space, the Hamiltonian $H$ can be written as

$$H_k = \sum_{j=0}^{N-1} h_{j,k} - J \sum_{j=0}^{N-1} \sum_{m} |j, m, k\rangle \langle j, m+1, k| + \text{H.c.}$$

$$+ \sum_{j=0}^{N-1} \sum_{m} \mu_m |j, m, k\rangle \langle j, m, k| + U(\langle 0, 1, k| \langle 0, 1, k| + \langle 0, 3, k| \langle 0, 3, k|),$$

and

$$h_{j,k} = \lambda_1 |j, 1, k\rangle \langle j + 1, 4, k| + \lambda_2 |j, 2, k\rangle \langle j + 1, 1, k|$$

$$+ \lambda_3 |j, 2, k\rangle \langle j + 3, k|$$

$$+ \lambda_4 |j, 3, k\rangle \langle j + 1, 4, k| + \text{H.c.}$$

with $\lambda_{1,2,3,4} = -J (1, e^{ik}, e^{ik})$ and $\mu_{1,2,3,4} = (0, V, 0, -V)$. Note that $|N, m, k\rangle = |0, m, k\rangle$. The expression of $H_k$ in Eq. (8) has a clear physical meaning: $j$, $m$, $k$ denotes the site state for the $j$-th site on the $m$-leg of a 4-leg ladder system with the modulated inter-cell coupling. The structure of $H_k$ is schematically illustrated in Fig. 2(a). The BP state mainly piles up at the 0-th site due to the near resonance condition of $|\Delta|/J \ll 1$, where $\Delta = U - V$. Hence, the corresponding approximate solution can be obtained within such unitcell. The involved four kets are denoted by purple box in Fig. 2(b). In the basis of $\{|0, 1, k\rangle, |0, 2, k\rangle, |0, 3, k\rangle, |N - 1, 2, k\rangle\}$, the matrix form of effective BP Hamiltonian is given as

$$H_{\text{eff}} = -J \left( \begin{array}{cccc} -\Delta/J & 1 & 0 & e^{-ik} \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -\Delta/J & 1 \\ e^{ik} & 0 & 1 & 0 \end{array} \right) + V I_4,$$  \hspace{1cm} (10)

Taking the linear transformation,

$$\left( \begin{array}{c} |1, k\rangle_{\text{bp}} \\ |2, k\rangle_{\text{bp}} \\ |3, k\rangle_{\text{bp}} \\ |4, k\rangle_{\text{bp}} \end{array} \right) = \frac{1}{\sqrt{2}} \left( \begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array} \right) \left( \begin{array}{c} |0, 1, k\rangle \\ |0, 2, k\rangle \\ |0, 3, k\rangle \\ |N - 1, 2, k\rangle \end{array} \right),$$  \hspace{1cm} (11)

the effective BP Hamiltonian can be rewritten as

$$H_{\text{eff}} = \left( \begin{array}{cc} h_{1,\text{eff}} & 0 \\ 0 & h_{2,\text{eff}} \end{array} \right) + V I_4,$$  \hspace{1cm} (12)

with

$$h_{1,\text{eff}} = -J \left( \begin{array}{cc} \Delta/J & 2 i \sin (k/4) \\ -2i \sin (k/4) & 0 \end{array} \right),$$  \hspace{1cm} (13)

$$h_{2,\text{eff}} = -J \left( \begin{array}{cc} \Delta/J & -2 \cos (k/4) \\ 2 \cos (k/4) & 0 \end{array} \right).$$  \hspace{1cm} (14)

Evidently, the effective $H_{\text{eff}}$ can be divided into two invariant subspaces, each of which is described by a $k$-dependent two-level system. The corresponding eigenenergies are

$$\varepsilon_{1,\sigma}^k = \frac{\Delta}{2} + \sigma \sqrt{\left(\frac{\Delta}{2}\right)^2 + 4J^2 \sin^2 \frac{k}{4}},$$  \hspace{1cm} (15)

$$\varepsilon_{2,\sigma}^k = \frac{\Delta}{2} + \sigma \sqrt{\left(\frac{\Delta}{2}\right)^2 + 4J^2 \cos^2 \frac{k}{4}},$$  \hspace{1cm} (16)

with $\sigma = \pm$. For each subsystem, there exists an energy gap $\Delta$ between the two bands. When the resonance condition ($\Delta = 0$) is achieved, not only the upper and lower energy bands are connected with each other ($\varepsilon_{j=1}^0 = \varepsilon_{j=2}^0$, $j = 1, 2$), but also four bands form a single BP band with the width $4J$. This evidence suggests that the bound pair velocity is altered from the order of $J^2/U$ to $J$. To check this point, we compare the result obtained by Eqs. (15)-(16) with that of the exact diagonalization method in Fig. 3. It demonstrates that two kinds of results accord with each other in the large $U$ limit. This characteristic plays the key role to under the following Bloch oscillation (BO) dynamics.

### III. DYNAMICS OF TWO-PARTICLE SYSTEM

It is well known that a particle in a periodic potential with an additional constant force performs BO, which can be explained by the formation of a Wannier-Stark ladder energy spectrum. BO is an ubiquitous phenomenon of coherent wave transport that can be observed in a wide variety of physical system and reflects the spectrum.
property of the system in the absence of the weak dc electric field. In addition, a cluster of bound particles can also present the BO which shares the same mechanism of the single-particle case. In this section, we use the BO to investigate the influence of a staggered field on the transport properties of the system.

**A. single-electron solution: Rice-Mele model**

We first study the 1D bipartite Hamiltonian in the single-particle invariant subspace, wherein the interaction $U$ has no effect. On the basis of

$$
|\lambda, j, +\rangle = c_{j,\uparrow}^\dagger |\text{Vac}\rangle,
$$

$$
|\lambda, j, -\rangle = c_{j,\downarrow}^\dagger |\text{Vac}\rangle,
$$

where $|\text{Vac}\rangle$ is the vacuum state of fermion $c_{j,\sigma}$, and $\lambda = A (B)$ when $j \in A (B)$, the corresponding Hamiltonian can be given as

$$
H = H_0 + H_s + H_e,
$$

with

$$
H_0 = -J \sum_{j, \sigma} |\langle A, j, \sigma | B, j, \sigma \rangle| + |B, j, \sigma \rangle \langle A, j + 1, \sigma | + \text{H.c.},
$$

$$
H_s = -\frac{V}{2} \sum_{j, \sigma} |\sigma \langle A, j, \sigma | A, j, \sigma \rangle| - |B, j, \sigma \rangle \langle B, j, \sigma |,
$$

$$
H_e = F \sum_{j, \sigma} (2j + 1) |\langle A, j, \sigma | A, j, \sigma \rangle| + 2j |B, j, \sigma \rangle \langle B, j, \sigma |.
$$

Here $H_e$ describes the dc electric field. In the absence of $H_s$, the Hamiltonian characterizes a Rice-Mele model with homogeneous hopping. Performing the Fourier transformation,

$$
|\lambda, q, \sigma \rangle = \frac{1}{\sqrt{N}} \sum_j e^{iqj} |\lambda, j, \sigma \rangle,
$$

where $q = 2l \pi / N$, $l \in [0, N - 1]$, the Hamiltonian $H$ can be block diagonalized due to its translational symmetry, i.e., $H = \sum_{q, \sigma} H_{q, \sigma}$. In this new basis, the matrix form of $H_q$ is

$$
H_{q, \sigma} = -J \begin{pmatrix}
\sigma V / 2J & 1 + e^{iK}
1 & -\sigma V / 2J
\end{pmatrix},
$$

which admits the eigen spectrum as

$$
\epsilon_{q, \pm}^{(1)} = \pm \sqrt{(V/2)^2 + 4J^2 \cos^2(q/2)}.
$$

Evidently, the two-particle spectrum consists of scattering and bound bands. The scattering band can be theoretically constructed through the Eq. 25. In particular, the upper scattering band of Figs. 4(a1)-(a2) can be obtained by the combination of two individual particles, i.e.,

$$
\epsilon_{k,m}^{(2)} = \epsilon_{q_1,\pm}^{(1)} + \epsilon_{q_2,\pm}^{(1)}.
$$

Analogously, the middle and lower scattering bands can be given as $\epsilon_{k,m}^{(2)} = \epsilon_{q_1,\pm}^{(1)} + \epsilon_{q_2,\pm}^{(1)}$, and $\epsilon_{k,\mp}^{(2)} = \epsilon_{q_1,\mp}^{(1)} + \epsilon_{q_2,\pm}^{(1)}$, respectively. When $J << V$, the single-particle bandwidth is in proportion to $J^2 / U$ which is similar to that of the bound pair spectrum as $V = 0$. This indicates that the presence of the staggered field suppresses the group velocity of the single particle but enhances the bound pair group velocity. It is conceivable that when $V$ is large enough, the existence of dc electric field will not induce the large-scale oscillation of scattering particles in coordinate space. In the following, we examine the dynamics of the system in the single-particle and two-particle Hilbert spaces, respectively.

**B. Bloch oscillation**

In the preceding section, the effect of the staggered field on the band structure is discussed. For simplicity, we...
The presence of $V$ results in the two-band structure for the single-particle subspace so that the scattering bands of (a1) consists of three parts with different energy scale. Such the scattering band can be understood by the superposition of the energies of two-free particles. The resonant staggered field expands the BP band but suppresses the scattering band. The different band structures determine the system dynamics in the presence of a dc electric field. The panels (b1)-(b2) describes the BP dynamics, while the (c1)-(c2) exhibit the time evolution of the two scattering-particle state $|\Phi_1(0)\rangle$. It clearly shows that the scattering particles are frozen at their initial locations and the BP can widen and shrink periodically populating an interval $4J/F$ when dc electric field is applied. This indicates that the BP band induced by staggered field is $4J$, which again confirms our analysis.

only consider the dynamics when the resonant field is applied. The Hamiltonian can be given as $H_{BO} = H + H_e$, where the dc electric field is $H_e = F \sum_{\sigma, j \in A, B} j n_{j, \sigma}$. To show the difference between the dynamics of the single-particle and two-particle cases, the initial states are chosen as

$$|\Phi_1(0)\rangle = \frac{1}{\sqrt{2}} c_{N/2, \uparrow}^\dagger c_{3N/2, \downarrow}^\dagger |\text{Vac}\rangle,$$

$$|\Phi_2(0)\rangle = \frac{1}{\sqrt{2}} c_{N, \uparrow}^\dagger c_{3N, \downarrow}^\dagger |\text{Vac}\rangle.$$ (26)

These are two types of wave packets, which are initially located at two separate sites and a single site, respectively. Evidently, the existence of interaction $U$ only affects $|\Phi_2(0)\rangle$ but does no effect on $|\Phi_1(0)\rangle$. For the initial state $|\Phi_1(0)\rangle$, the two-particle dynamics can be understood as the superposition of the motions of two independent particles regardless of whether the staggered external field is switched on. For the second type of initial state $|\Phi_2(0)\rangle$, the interaction $U$ binds the two particles together so that the two particles behave like a single composite particle. The dc electric field forces the spectrum of the system to consists of Wannier–Stark ladders$^{61,62}$ and hence drives the two initial states to undergo the BO, respectively. A typical breathing behavior can be observed in Figs. 4(b1) and (c2). It shows that the width of the wave packet oscillates strongly, while its position remains constant. In general, the dynamics is confined to a spatial interval, the width of which is determined by the ratio of the bandwidth of the considered system with $H_e = 0$ to the strength $F$. Specifically, the BP can be moved on the lattice with effective hopping $2J^2/U$ when $V = 0$ and large $U$ limit are supposed, whereas a single particle’s hopping is $J$. As such, the breathing mode of two scattering particles $|\Phi_1(t)\rangle$ extends over an interval $4J/F$, and $|\Phi_2(t)\rangle$ is frozen at the initial site. Note that the period of BO for BP is $\pi/2F$ rather than $\pi/F$ since the external field felt by the BP is $2F$ when it migrates. In contrast, the existence of the resonant field shrinks the bandwidth of the scattering band to the order of $J^2/U$. On the other hand, it broadens the width of the bound band to $4J$ instead of $J^2/U$ with frequency $F$. As a consequence, the dynamics of the BP and scattering particles are interchanged as shown in Figs. 4(c1)-(c2). So far, we have demonstrated how the external resonance field can improve the mobility of BP in the matter by changing the structure of the energy spectrum through the example of two particles. This mechanism could provide some physical insights to understand the subsequent many-body dynamics.
c particles since the basis of \{staggered field\} provides a bridge for the mobility of the particles. With the spirit of exhibiting a vanishing particle charge fluctuation \(\Delta n_j\), which is 6-site Hubbard model at half filling and \(S_z = 0\). Evidently, the resonant field can induce the oscillation behaviors of both quantities. It can be served as a dynamic signature to identify the transition from the Mott insulating to conducting state.

**IV. QUENCH DYNAMICS AT HALF FILLING**

In this section, we turn to investigate the many-body quench dynamics. Before quench, the Hamiltonian of the considered system is \(H_0\), which is a standard Hubbard Hamiltonian. When \(J \ll U\), the system is in the Mott insulating phase such that there exists an excitation gap in the spectrum. In the following, we focus on the system at half filling. In this condition, each site contains only one particle. The on-site interaction prohibits the movement of the particles. With the spirit of the two-particle case, one can imagine that the resonant staggered field provides a bridge for the mobility of the particles since the basis of \(\{c_{j,\uparrow}^\dagger c_{j,\downarrow}^\dagger \text{ Vac}, c_{j,\uparrow}^\dagger c_{j+1,\downarrow}^\dagger \text{ Vac}, c_{j+1,\uparrow}^\dagger c_{j,\downarrow+1,\downarrow}^\dagger \text{ Vac}, \ldots\}\) stays at the same energy shell \((U)\). Therefore, the conductivity of the system is enhanced.

**A. Transition from Mott insulating to conducting state**

To see how does the staggered field affect the conductivity, we propose a quench scheme in the following. The scheme is that: The system is initialized in the ground state of \(H_0\) and then evolved with a post-quench Hamiltonian \(H = H_0 + H_s\). To measure the conductivity of the system, two physical quantities are introduced. The first one is the doublon-doublon correlation

\[
C_{i,j} = \langle \eta_i^+ \eta_j^\dagger \rangle \forall i,j, i \neq j,
\]

the constant number of which denotes that the corresponding state has off-diagonal long-range order (ODLRO). It also provides a possible definition of superconductivity, as a finite value of this quantity can be shown to imply both the Meissner effect and flux quantization. In general, the \(\eta\)-pairing state of \(H_0\) possesses ODLRO. Unfortunately, such state is the excited state rather than the ground state of \(H_0\). In addition, the presence of \(H_s\) spoils the \(\eta\) symmetry so that such superconducting properties of the system may not exist. However, the correlator \(C_{i,j}\) can reflect the conductivity of the system to some extent. The other quantity used to examine the conductivity is the charge fluctuation \(\Delta n_j\) defined by

\[
\Delta n_j = \sqrt{(n_j)^2 - \langle n_j \rangle^2}.
\]

It is well known that when the on-site repulsive interaction between fermions is large enough in the Mott phase, the number fluctuation would become energetically unfavorable, forcing the system into a number state and exhibiting a vanishing particle charge fluctuation \(\Delta n_j\). Beyond the Mott insulator regime, the fermions are delocalized with the nonvanishing charge fluctuation. For the half-filled free fermion system, straightforward algebra show that \(\Delta n_j = \sqrt{2}/2\), which can be served as a benchmark to examine the following result.

Again, we focus on 1D lattice realizations due to their numerical tractability. We perform numerical calculation where a 6-site Hubbard model at half filling is considered. Taking the ground state of \(H_0\) as the initial state \(|\Phi(0)\rangle\), one can observe the variations of the correlator \(C_r(t) = \langle \Phi(t) | n_i^+ n_{i+r}^\dagger | \Phi(t) \rangle\) and \(\Delta n_j(t) = \sqrt{\langle \Phi(t) | n_j^2 | \Phi(t) \rangle - \langle \Phi(t) | n_j | \Phi(t) \rangle^2}\) as time, where \(|\Phi(t)\rangle = e^{-iHt} |\Phi(0)\rangle\). Note that the ground state \(|\Phi(0)\rangle\) of 1D Hubbard model is the Mott insulator with strong antiferromagnetic correlations. Fig. shows that the value of \(V\) which we quench to determines the center value of two such oscillated quantities. When the resonant field is applied, i.e. \(V = U\), the maximum center values of \(C_r(t)\) and \(\Delta n_j(t)\) can be reached, which is in agreement with our previous analysis. To determine the effect of \(V\), we introduce the average correlator and charge fluctuation in the time interval \([0, t_f]\), defined as follows

\[
\overline{C}_r = \frac{1}{t_f} \int_0^{t_f} C_r(t) \, dt,
\]

\[
\overline{\Delta n}_j = \frac{1}{t_f} \int_0^{t_f} \Delta n_j(t) \, dt,
\]

**FIG. 5:** Time evolutions of correlator \(C_r(t)\) and charge fluctuation \(\Delta n_j(t)\) for different staggered fields \(V\) (units of \(J\)) with a given on-site interaction \(U/J = 50\). The system is initialized in the ground state of \(H_0\), which is a 6-site Hubbard model at half filling and \(S_z = 0\). Evidently, the resonant field can induce the oscillation behaviors of both quantities. It can be served as a dynamic signature to identify the transition from the Mott insulating to conducting state.
where $t_f$ represents the cutoff quench time. Average $Cr$ and $\Delta n_j$ as functions of parameter $V$ for different $r$ values are plotted in Fig. 6. It can be shown that the peaks appear in both $Cr$ and $\Delta n_j$ when resonant filed is taken. Comparing to the free fermion case, the increase of such two quantities suggests that the system experiences a transition from Mott insulating to conducting phase. This scheme presents an alternative approach to control the conductivity of the system from the dynamical prospective.

V. SUMMARY

In summary, we investigate how does the interplay between the external field and on-site interaction $U$ alter the conductivity of the system. When the resonant field is applied, the two-particle case shows that there can exist an energy shell of $U$, which allows the movement of BP with the hopping energy scale of $J$ rather than $J^2/U$. This phenomenon can be extended to the dilute system. However, for the case of a single particle, the movement of the particle is limited due to the existence of the staggered field, which is reflected by the fact that the bandwidth of the single-particle spectrum is suppressed from $J$ to $J^2/U$ order. These facts can be examined by the BO. Specifically, when two particles are bound together, they show large-scale BO in coordinate space. On the contrary, when the initial two particles are far away, the dc electric field suppresses the two particles at the initial position. This unique two-particle property paves the way to understand the many-body quench dynamics. For the system at half filling, the cooperation of resonant staggered field and on-site interaction forces the initial antiferromagnetic Mott insulating state to become a steady conducting state. Our finding provides a new perspective to understand the influence of external fields on strongly correlated systems and suggests a new dynamical mechanism to control the conductivity.

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