Similarity of the Wall Jet Resulting from Planar Underexpanded Impinging Jets

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ABSTRACT

Wall shear stress is mapped across nozzle parameters (stand-off height, jet hydraulic diameter, and nozzle pressure ratio) for two dimensional planar underexpanded impinging jets. Computational fluid dynamics is used to calculate the flow field resulting from impinging jets with height-to-diameter ratios of 15-30 and nozzle pressure ratios of 1.2-3.0. The wall jet resulting from underexpanded impinging jets is found to be self-similar in the same triple-layer structure as traditional wall jets. The effects of compressibility on the wall jet were found to be insignificant, and adjustment for mean density changes are not necessary for wall jets with Mach numbers of less than 0.8. Power laws with source dependent coefficient are assumed for local wall jet variables. It is found that normalizing by momentum, rather than characteristic length or source velocity, is beneficial for achieving similarity. Power laws for normalized maximum velocity, friction velocity, and maximum and half-maximum velocity wall distances versus momentum normalized streamwise location are developed. Source dependent coefficients are determined as a function of nozzle parameters using the conjugate gradient method. These power laws allow for complete mapping of wall shear stress on the impingement surface for a range of nozzle parameters.

I. INTRODUCTION

Impinging jets have been studied extensively; their characterization is useful in biological, chemical, and engineering applications. These studies tend to focus on heat and mass transfer [1-4]. The goal of this work is to analyze the properties of the wall jet originated from the impingement of underexpanded planar jets, with application to aerodynamic micro-particle sampling. Previous studies of underexpanded jets have generally been motivated by the flow dynamics and acoustics of short takeoff and vertical landing aircraft [5-7]; thus, the jets of interest were axisymmetric, and the wall jet region has not been the primary focus. The main advantages of using underexpanded planar jets for the removal of micro-particles from a surface are: (i) planar jets provide an advantage over circular jets as the velocity in the wall jet sustains further from impingement, (ii) planar jets cover a larger area where the particle is removed, (iii) supersonic jets produce high wall shear stress in the wall jet region, (iv) isentropic nozzle relations allow for straightforward calculations of fluid properties at the jet exit, convenient for numerical and experimental studies.

For characterization of aerodynamic particle resuspension, it is useful to characterize the wall shear stress resulted from jet impingement. Measurements of wall shear stress are challenging; for example, Young et al. [8] used oil-film interferometry to measure the shear stress from an impinging supersonic jet. Their experiment has shown promise, but oil-film interferometry is limited in its precision. Tu & Wood [9] conducted a comprehensive study of wall shear stress developed from a subsonic impinging jet using Preston and Stanton tube measurements, but their results were affected by the measurement apparatus, and their conclusions cannot be extrapolated for compressible jets. Smedley et al. [10] and Phares et al. [11] investigated the removal of microspheres from impinging jets and used theoretical shear stress profiles, adhesion forces, and particle removal rates to infer shear stress along the plate. Their results find shear stress to be directly related to particle forces, but do not account for compressibility and turbulent effects.

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These conclusions were produced for subsonic and nearly sonic jets and are not applicable to underexpanded jets. Velocity measurements near the wall can be used to elucidate the values of the shear stress. Loureiro [12] demonstrated Laser Doppler Anemometry (LDA) could be used to measure velocity within 50 micrometers of the wall accurately; but, in boundary layers with the necessary wall shear stress to remove trace explosive materials, the viscous sublayer may be only 20 micrometers thick [13]. Keedy et al. [14], using Birch’s [15] model for the virtual origin of underexpanded jets, also illustrated that explosive particles would only be removed with high-pressure jets at low standoff distances. In general, there is a scarcity of reliable wall shear stress data in the scientific literature, especially for compressible and planar impinging jets.

The planar wall jet has been studied extensively, but all of these studies are based on a wall jet developing from a jet attached to a wall, and not the wall jet resulting from flow impingement. Thus, it is unclear if the previous results related to wall jet similarity formulations would hold for the impinging jet scenario. The wall jet resulting from axisymmetric impinging jets has been studied experimentally and examined for similarity by Yadav [16] and Yao [17]; these results, along with the work of Loureiro to parametrize impinging jets, demonstrated that the wall jet developed downstream of impingement do demonstrate self-similar behavior. This provides motivation for this work, which is to examine the similarity in the wall jet resulting from normal planar jet impingement. We present a parametric study that characterizes the velocity profile and wall shear stress of the wall jet resulting from planar, underexpanded impinging jets.

A. Wall Jet Theory

For mapping of wall shear stress, it is useful to examine the wall jet portion of the flow from a similarity perspective. The planar turbulent wall jet has consistently been shown to have incomplete similarity, which is that a single solution cannot describe the velocity profile of the wall jet. Thus, one must separate the wall jet into three regions; a self-similar wall layer where viscous forces are dominant, a self-
similar outer layer which behaves analogously to a free jet, and an overlap layer where the velocity is closest to the maximum. A triple layered incomplete similarity is achieved by matching the self-similar outer and wall regions while considering source dependence. This source dependence has been studied for true wall jets but is not defined for the wall jets resulting from impinging jets. The equation of motion for the wall jet is defined as:

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \left[ v \frac{\partial U}{\partial y} - \frac{\tau}{\rho} \right]
\]

\[u \to 0 \text{ as } y \to \infty; \quad u = v = 0 \text{ at } y = 0.
\]

As first proposed by Glauert [18], the equations of motion are assumed to be solved by outer and inner self-similar equations. The outer region becomes:

\[u = u_m(x)f_o^m(\eta_o)\]  
\[\eta = \frac{y}{y_{0.5}(x)}\]

George [19] demonstrated that the classical “law of the wall” coordinates for turbulent boundary layers can be used for turbulent wall jets.

\[u = u_r(x)f_i(y^+)\]  
\[y^+ = \frac{yu_r(x)}{v}\]

The inner and outer regions must then be rectified, in what has traditionally been called the overlap region. George [19] concluded that the overlap velocity profile could be accurately described in both inner and outer similarity coordinates, but Gertsen [20] demonstrated the velocity in this overlap region can be more accurately described in the form of a defect law.

\[u = u_m(x) - u_r(x)f^i(\eta)\]  
\[\eta_m = \frac{y}{y_m(x)}\]

The solutions to these similarity equations have been determined separately by George [19] and Gertsen [20]. For this work, we examine the x-dependent variables, which when determined can be used to describe the rest of the flow field. Thus, we are interested in developing relations for \(y_{0.5}, y_m, u_m,\) and \(u_r\). For each of these variables, we will assume a power law relation [21] with source dependence as incomplete similarity is expected.

\[y_{0.5} \sim \beta_1 x^{\alpha_1}, \quad y_m \sim \beta_2 x^{\alpha_2}, \quad u_m \sim y_{0.5}^{\alpha_3}, \quad u_r \sim \beta_4 x^{\alpha_4}\]

In order to determine the power law exponents, one must determine proper scaling through dimensional analysis.

**B. Dimensional Analysis**

In the description of the planar impinging jet, we consider seven parameters: \(x \sim L\), the streamwise distance from the impingement point; \(y \sim L\), the distance from the impingement surface; \(h \sim L\), the standoff height of the jet; \(d \sim L\), the jet hydraulic diameter; \(\rho \sim ML^{-3}\), the fluid density; \(\nu \sim L^2 T^{-1}\), the kinematic viscosity; and \(U_0 \sim L^1 T^{-1}\), the velocity at the jet exit, where L, M, and T are the units of length, mass and time, respectively. Using these variables for dimensional analysis yields the following non-dimensional groups:

\[
\Pi_1 = \frac{h}{d}, \Pi_2 = \frac{x}{h}, \Pi_3 = \frac{y}{h}, \Pi_4 = \frac{U_0 d}{v}
\]
Narashima [22] demonstrated that scaling $x$ and $y$ by the momentum flux of the source is effective when writing power laws for the velocity in wall jets. While George [19] defines the momentum flux as $U_0^2 d/2$ for underexpanded jets, one must consider the changes in density by defining the momentum flux as $J = \rho_0 U_0^2 d/2$. This normalization yields the following non-dimensional versions of $x$, $y$, $u_\tau$, and $u_m$:

\[
X = \frac{Jx}{\rho_\infty \mu_\infty}, \quad Y = \frac{Jy}{\rho_\infty \mu_\infty}, \quad U_\tau = \frac{u_\tau \mu_\infty}{J}, \quad U_m = \frac{u_m \mu_\infty}{J}
\]

This procedure does not account for all the source dependence, however. In order to fully characterize source dependence for underexpanded impinging jets, one must consider a Reynolds number associated with the jet width and the nozzle pressure ratio (NPR) as well as the standoff height to hydraulic diameter ratio.

\[
Re_n = \frac{U_0 d}{v_\infty}, \quad NPR = \frac{P_0}{P_\infty}
\]

Wygnanski [23] determined that $Y_{0.5}$, $Y_m$, $U_\tau$, and $U_m$ can be expressed as power laws of the form:

\[
Y_{0.5} = \beta_1 X^{\alpha_1} \quad (5)
\]
\[
Y_m = \beta_2 X^{\alpha_2} \quad (6)
\]
\[
U_m = \beta_3 Y_{0.5}^{\alpha_3} \quad (7)
\]
\[
U_\tau = \beta_4 X^{\alpha_4} \quad (8)
\]

If the exponents and source dependent coefficients of the expressions above can be determined, one can define the entire wall jet flow-field resulting from underexpanded jet impingement as a function of the nozzle Reynolds number, height to jet width ratio, and NPR.

### C. A Note on Compressibility in the Wall Jet Region

While underexpanded impinging jets provide high wall shear stress, which is ideal for aerodynamic particle sampling, flow in the wall jet region is compressible, potentially introducing complications in similarity formulations. The effects of density fluctuations on turbulence have been shown by Morkovin [24] to be negligible for compressible jets for $Ma < 1.5$. The range of cases in this work is limited to subsonic wall jets ($Ma < 0.8$), so the turbulent properties will not be affected by compressibility. Mean density effects may still be important, however. Ahlman [25] found through DNS study that mean density effects were only significant in the wall normal direction by comparing Reynolds and Favre averaged velocity profiles for the outer layer and comparing traditional wall coordinates with semi-local [26] and Van Driest [27] scaling. When examining velocity profiles in this work, we also found that mean density effects are minimal. Plotting profiles in Van Driest and semi-local scaling yielded no noticeable improvement in similarity analysis (see SI 1). For that reason, considering the effects of compressibility on wall jet similarity is not necessary for the range of Mach numbers presented in this work. It is important to note that this assumption is not likely to be valid for transonic and supersonic wall jets.

### II. COMPUTATIONAL STUDY

Numerical simulations for this work were performed using ANSYS FLUENT 17.2. The pressure-velocity coupled algorithm (QUICK scheme) solved the steady-state Favre-Averaged Navier-Stokes equations.
\[
\begin{align*}
\frac{\partial (\rho \vec{u}_i)}{\partial x_i} &= 0 \\
\frac{\partial (\rho \vec{u}_i \vec{u}_j)}{\partial x_i} &= -\frac{\partial \rho}{\partial x_i} + \frac{\partial \bar{\tau}_{ij}}{\partial x_i} - \frac{\partial (\rho u_i' u_j'')}{\partial x_j} \\
\frac{\partial}{\partial x_j} \left( \rho \bar{u}_j \left( \bar{h} + \frac{1}{2} \bar{u}_i \bar{u}_j \right) + \bar{u}_j \rho u_i'' u_j'' \right) &= \frac{\partial}{\partial x_j} \left( \bar{u}_i \left( \bar{\tau}_{ij} - \rho u_i'' u_j'' \right) - \bar{q} - \rho u_j'' h'' + \tau_{ij} u_i'' - \frac{1}{2} \rho u_j'' u_i'' u_j'' \right)
\end{align*}
\]

As demonstrated by Alvi [28] and discussed by Fillingham [29], the \( k - \omega \) shear stress transport (SST) turbulence model was found to be appropriate for compressible impinging jets for resolving the boundary layer. The SST model uses \( k - \epsilon \) away from the wall in the free stream and free jet portions of the flow while using \( k - \omega \) near the wall where it is more accurate. Figure 2 shows the computational domain. The inlet boundary condition is defined as the exit of an isentropic nozzle where the flow is choked; thus, it is defined by a total pressure and a static pressure where the total pressure is necessarily (for an ideal diatomic gas) 1.893 times the static pressure. The walls are defined as isothermal, no-slip boundaries. The outlets are defined as atmospheric pressure outlets.

![FIG. 2. Schematic of CFD domain and boundary conditions.](image)

The mesh contains ~600,000 elements, with the first node placed at one micrometer away from the wall, ensuring that the viscous sublayer is fully resolved for all cases. Mesh independence was ensured as the mesh was refined for a single case by a factor of 1.5 with no change in results. The matrix of conditions for which computations were conducted is given in Table I. The chosen cases encompass the range condition resulting in sufficient wall shear stress for microparticle resuspension [13,14] without transitioning supersonic flow in the wall jet region. If the flow in the wall jet is supersonic, a separate characterization is necessary.

| \( h/d \) | \( d \) (mm) | \( NPR \) |
|---------|------------|-------|
| 15, 17.5, 25, 30 | 0.5, 1 | 1.2, 1.6, 2.0, 2.4, 2.8 |

The CFD model used in this work has been previously used to study axisymmetric impinging jets and has shown good correlation with the experimental observations using Schlieren photography [29]. For planar
jets, pressure sensitive paint was used for validation of the normal pressure profile on the impingement surface. 3D simulations were conducted to ensure 2D the models are accurate. For more information, see SI section 2.

III. RESULTS AND DISCUSSION

A. Wall Jet Velocity Profiles

Velocity profiles from the CFD simulations are examined to determine if the flow in the wall jet region is self-similar. Traditionally, the planar wall jet has been considered self-similar in the coordinates presented in Eq. 2. Here, normalization by $y_{0.5}$ and $u_m$ appears to yield similarity, as was observed by Wygnanski [23], but truly only applies to the outer region ($y > y_{0.5}$) of the jet as demonstrated by George [19] and Gertsen [20]. The velocity profile for the outer region is identical to that of a free jet and thus can be described by:

$$f_o' = 1 - (\tanh k\eta)^2$$

$$k = \text{atanh} \sqrt{\frac{1}{2}}$$

Figure 3 illustrates the self-similarity in the outer region for three different geometries and NPRs, comparing the CFD simulations to the analytical solution, Eq. 12. The self-similarity develops downstream of the impingement point for $x/h > 0.2$.

In order to examine similarity in the overlap layer, the coordinates described by Eq. 4 are used in a defect relation given by Eq. 13. Gertsen developed an analytical expression for the velocity profile:

$$f' = \frac{1}{0.41} \left( -\ln \eta_m - \frac{5}{6} + \frac{3}{2} \eta_m^2 - \frac{2}{3} \eta_m^3 \right)$$
In Figure 4, velocity profiles are plotted in defect coordinates for two geometries and two NPRs alongside Eq. 13. The overlap layer similarity takes longer to develop ($x/h > 0.4$) than inner and outer layers.

FIG. 4. Velocity profiles plotted in defect coordinates for four different cases against the theoretical profile (Eq.13). Profiles demonstrate similarity independent of the geometry and nozzle pressure ratio.

It appears similarity is obtained in the wall layer as well, but the analytical expression derived from the equations of motion does not apply. In order to obtain similarity in the viscous wall layer, the velocity profiles are plotted in the traditional law of the wall coordinates. Figure 5 plots the wall layer for the same cases as Figures 3 and 4.

FIG. 5. Velocity profiles plotted in wall coordinates for four different cases against $y^+ = u^+$. Profiles demonstrate similarity independent of the geometry and nozzle pressure ratio.
Figures 4 and 5 demonstrate that there is no analytical expression that accurately characterizes the “buffer” region (between the linear and log law portions of the wall layer) of the velocity profiles as is the case with all turbulent boundary layers. Plotting data from this work in the established similarity coordinates for wall jets has established confidence in the assumptions that compressible effects are negligible and that impinging jets produce wall jets in the same triple layer similarity structure established in the existing literature [30].

B. Power Laws

After confirming the similarity of the wall jet region, we analyze the dependence of the similarity variables in the form of source-dependent power laws on $x$. The characteristic length of wall jet velocity profiles has been debated [18-20], generally $y_{0.5}$, the distance from the wall in the outer region where the velocity is half of the maximum, is used. George argued that this choice is arbitrary, but $y_{0.5}$ has repeatedly [22,23,31] proven to be useful in characterizing the similarity of wall jet velocity profiles. George also demonstrated that momentum normalized $y_{0.5}$ can be accurately described by a source dependent power law in the $x$-direction with a virtual origin. Figure 6 plots $Y_{0.5}$ against $X$ for all geometries with 1 mm jet hydraulic diameter and all NPRs. One can clearly see a virtual origin is necessary for similarity. This is consistent with the previous reports [19,23]. While for traditional wall jets there is not an obvious physical choice for a virtual origin, with impinging jets the standoff height is the logical choice. Here, we define the virtual origin location as $X_0 = -\frac{jh}{\rho\infty\mu\infty}$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{Momentum normalized half velocity wall distance plotted against momentum normalized x-location for all height-to-diameter ratios colored by the nozzle pressure ratio.}
\end{figure}

Figure 7a demonstrates the effectiveness of using standoff height as a virtual origin. The similarity is nearly obtained, but an adjustment for source dependence based on nozzle pressure ratio improves the fit; $\beta_1 \sim NPR^{0.15}$ yields a linear relationship as shown in Figure 7b.
George [19] proposed that the dependence of $y_{0.5}$ on $x$ is necessarily non-linear by the Asymptotic Invariance Principle (he finds the exponent to be 0.97). Gertsen [20] suggests that a linear relationship is expected. In the least squares sense, $\alpha_1$ was found to be 0.99, which for all intents and purposes can be taken to be unity. This leads to a final expression for $Y_\frac{1}{2}$

$$Y_\frac{1}{2} = \beta_1 (X - X_0) \tag{14}$$

$$\beta_1 = 0.083 \times NPR^{0.15}$$

When characterizing maximum velocity in the wall jet, George [19] demonstrated that using a power law based on a local length scale is more accurate than using a power law in $x$. Intuitively, one would take $y_m(x)$ as the length scale for characterizing $u_m$, but, as stated earlier, $y_{0.5}(x)$ has lower source dependency; it is also easier to measure, as shown experimentally by Eriksson [31] and with DNS by Naqavi [32]. George [19] proposes that the decay exponent for $u_m$ as a function of $y_{0.5}$ is universal for wall jets. Figure 8 plots momentum normalized maximum velocity against $y_{0.5}$ with and without pressure source adjustment. The NPR is the only source adjustment, which is consistent with the finding that a power law for maximum velocity based on the local length scale is universal.
FIG. 8. Momentum normalized max velocity plotted against momentum normalized half max velocity location: (a) without nozzle pressure source dependence adjustment; (b) with nozzle pressure source dependent adjustment and fit (Eq. 15).

After adjusting for nozzle pressure ratio, the power law for maximum velocity becomes:

\[ U_m = \beta_3 X^{-0.51} \]

\[ \beta_3 = 0.0051 \times NPR^{0.15} \]

The exponent for the decay of maximum velocity is -0.51, which is slightly lower than the exponent determined by George [19] of -0.527. The relationship between local length scale and maximum velocity is the most characteristic of traditional wall jets, and the fact that the relationship developed in this work is consistent with those in the literature provides confidence in the assumption that the wall jet developed for impinging jets exhibits the same length scale dependence as simple wall jets.

In order to use defect law coordinates, one must characterize the maximum velocity location, \( y_m \), as a function of \( x \). Similarity of wall jets generally assumes the ratio, \( r = y_m/y_{0.5} \), to be constant, but this is only strictly true as \( x \to \infty \) [19,20]. For impinging jets, we are interested in characterizing the flow near the impingement point, so this approach is not adequate. Thus, a separate power law for \( y_m \) is proposed.

Figure 9a plots momentum normalized maximum velocity location against momentum normalized \( x \). It is evident that a virtual origin is not necessary. The dependence on a virtual origin correction for \( y_{0.5} \), but not for \( y_m \), may be explained by tracing streamlines. The streamlines that pass through \( y_m \) decelerate to near zero velocity in the stagnation region at impingement while the streamlines associated with \( y_{0.5} \) do not experience rapid deceleration. The source dependence as determined by least squares fitting is plotted in Figure 7b. The final expression for \( y_m \) is:

\[ Y_m = \beta_2 X^{0.49} \]

\[ \beta_2 = 0.00027 \times NPR^{0.33} \times \frac{\frac{\hat{h}}{d}}{Re_n^{0.85}} \]
The exponent for $y_m$ in this work of 0.49 is significantly lower than reported in the literature. Tang [33] found the exponent to be 0.717 using LDA while Naqavi [32] found the exponent to be 0.743 based on the DNS calculations. Historically, $y_m$ has been difficult to characterize and formulations in the literature are less consistent than for $y_{0.5}$, primarily because the velocity varies slowly in the region near the maximum and thus the precise location of the maximum velocity is difficult to accurately measure. This makes it difficult to determine the reason for the disparity in the exponents. It is possible that it is a property of a wall jet resulting from impinging jets. It is also possible that the max velocity region takes longer to develop, and the use of a larger domain would yield a different decay exponent. Compressibility did not have an appreciable effect on the decay exponent, as fitted for cases $Ma\sim0.3$ and $Ma\sim0.8$ had exponents of -.50 and -.48, respectively, which is an insignificant change relative to the difference between the exponents found in this work and previous literature on wall jets.

Friction laws in the existing literature are generally expressed as a friction coefficient, which is a function of a local Reynolds. When examining the data in this work, the friction coefficient power law is only accurate in the region of $\frac{h}{h} > 1.0$. Using downstream data, a friction law has the best fit:

$$c_f = \left( \frac{u_f}{u_m} \right)^2 = 0.0029Re_l^{-0.19}$$

(17)

$$Re_l = \frac{u_m y_{0.5}}{v_{0.5}}$$

This formulation agrees with the existing friction laws from the literature [19,31,34]. These friction laws, however, are inconsistent across experimental and DNS data and are highly dependent on the momentum source; thus, for this work, we will characterize friction velocity directly as we have for maximum velocity.

Figure 10a demonstrates the need for the source dependent adjustment as shown in Figure 10b. Momentum normalized friction velocity can be expressed as:
\[ U_c = \beta_4 X^{-0.3} \]  
(18)

\[ \beta_4 = 0.021 \times \frac{h}{d} \times NPR^{-0.07} \times Re_n^{-0.5} \]

Note that it is difficult to obtain physical interpretations from the source dependent exponents, as there is an insufficient analytical or experimental investigation into planar impinging jets. Further investigation is needed to interpret the source dependent exponents. With the characterization of friction velocity, all regions of wall jet have been expressed in the form of a power law.

![Graph showing momentum normalized friction velocity plotted against momentum normalized x-location.](image)

**FIG. 10.** Momentum normalized friction velocity plotted against momentum normalized x-location: (a) without nozzle pressure source dependence adjustment; (b) with nozzle pressure source dependent adjustment and fit (Eq. 18).

### C. Wall Shear Stress

While compressibility does not have a significant effect on the power laws or similarity, the change in mean density is not negligible. For this reason, we cannot determine wall shear stress directly from friction velocity. Here, we formulate a power law for momentum normalized wall shear stress, \( \tau^* \).

\[ \tau^* = \frac{\tau}{\rho_\infty \left( \frac{\mu_\infty}{J} \right)^2} \]

Figure 11 plots momentum normalized wall shear stress against momentum normalized x with and without source dependence, demonstrating a power law is appropriate for wall shear stress.
The source dependent power law developed for momentum normalized wall shear stress is:

\[
\tau^* = \beta_5 \cdot X^{-61}
\]

\[
\beta_5 = 0.00059 \cdot \frac{h^{-45}}{d} \cdot NPR^{-18} \cdot Re_n^{-1.0}
\]

The power law developed in this work demonstrates a slower decay of wall shear stress than those in the literature for traditional wall jets. Wygnanski [23] found the decay exponent to be \(-1.07\), while Naqavi [32] found an exponent of \(-0.967\) via DNS. As there is no existing DNS or accurate experimental study on the wall jet from planar impinging jets, it is difficult to determine where this discrepancy comes from. Further investigation is necessary to determine if this decay is characteristic of wall jets resulting from underexpanded impinging jets.

**IV. CONCLUSIONS**

We have conducted a parametric computational study of planar underexpanded impinging jets and determined wall jet velocity profiles and wall shear stress. The 2D numerical simulations examine underexpanded impinging jets over a range of jet parameters (jet standoff distance, jet hydraulic diameter, and jet nozzle pressure ratio) and establish similarity variables. The wall jet developed from planar jet impingement does indeed have the same triple-layered structure as the classical wall jets. This provides confidence in using the same x-dependent length scales and velocities \((y_{0.5}, y_m, u_r, \text{and } u_m)\) as have been used in the literature when examining wall jets. We found that compressibility effects are insignificant when considering similarity; that is, density adjusted similarity coordinates do not yield improvement over traditional coordinates for wall jets with \(Ma < 0.8\). After ensuring similarity in the wall jet, power laws for the x-dependent variables were developed. Normalization by momentum, as opposed to length scales, was effective in reducing nozzle pressure source dependence in the power laws. Jet nozzle parameters (h, d, NPR) have a significant effect on the coefficients of the power laws, while power law exponents are...
independent of nozzle parameters. This allows for the development of broadly applicable power law relations, with source dependent coefficients, for the characteristic variables of wall jets. While the mean density effects are insignificant for power law formulation, the change in mean density is non-negligible, thus wall shear stress must be characterized separately from friction velocity. A power law was developed for normalized wall shear stress in the same manner as for the wall jet variables, allowing for the prediction of wall shear stress within a maximum error of 8%, as a function of only jet hydraulic diameter, standoff height, NPR, and x-coordinate.

Supersonic impinging jets have primarily been studied for applications to vertical takeoff vehicles, and thus the downstream wall jet region has not been examined. The underexpanded jets introduce complicated shock structures and compressibility effects that hindered their analytical studies. The high-speed nature of the flow makes accurate experimental examination challenging. This study of the wall jet originating from underexpanded planar impinging jets provides a simple, direct calculation of the wall shear stress resulting from these jets.

V. ACKNOWLEDGMENTS
This work was supported by the DHS Science and Technology Directorate, Homeland Security Advanced Research Projects Agency, Explosives Division, and UK Home Office [grant no. HSHQDC-15-C-B0033] and was facilitated using advanced computational, storage, and the networking infrastructure provided by the Hyak supercomputer system at the University of Washington.
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