Charge transport through a SET with a mechanically oscillating island

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We consider a single-electron transistor (SET) whose central island is a nanomechanical oscillator. The gate capacitance of the SET depends on the mechanical displacement, thus, the vibrations of the island may strongly influence the current-voltage characteristics, current noise, and higher cumulants of the current. Harmonic oscillations of the island and oscillations with random amplitude (e.g., due to the thermal activation) change the transport characteristics in a different way. The noise spectrum has a peak at the frequency of the island oscillations; when the island oscillates harmonically, the peak reduces to a δ-peak. We show that knowledge of the SET transport properties helps to determine in what way the island oscillates, to estimate the amplitude, and the frequency of the oscillations.

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The interplay of electric currents through nanostructures with their mechanical degrees of freedom has attracted a lot of interest recently, both from the experimental and theoretical side.123456789 One of the central questions of this new field of nanophysics is how the vibrations of the oscillating part of a nanodevice influence its transport properties and vice versa. A number of nanomechanical devices were investigated in the last years, e.g., so-called single-electron shuttles456. On the theoretical side, it was shown recently that electric currents passing through a dirty nanowire can stimulate its vibrations10. Indications for thermal vibrations of suspended single-wall nanotubes doubly clamped between two contacts were observed11.

Recently, the nanomechanical properties of single-electron transistors (SETs) have started to attract attention. SETs built from carbon nanotubes are a natural candidate for this type of question. For instance, it was shown that the equilibrium shape of a suspended nanotube studied as a function of a gate voltage shows features related to single-electron electronics, e.g., Coulomb “quantization” of the nanotube displacement7.

In this Letter, we discuss how vibrations of the central island of the SET change the current and the noise. We show that the transport characteristics of the SET differ for islands oscillating thermally or harmonically. The current noise spectrum has a peak at the frequency of the island vibrations that reduces to a δ-peak when the island oscillates harmonically. Therefore, knowing the transport properties of the SET can help to determine in what way the island oscillates, and to find the amplitude and frequency of the oscillations.

The system that we want to study – a SET with a mechanically oscillating island – is sketched in Fig. 1. We assume that the island is coupled to the left (L) and right (R) leads by tunnel junctions, and its capacitance to the gate \( C_g(z) \) depends on the coordinate \( z \) that measures the deviation of the island from its equilibrium position. When the island oscillates, the gate capacitance changes with the position of the island center \( z \), and therefore the transport properties of the SET change.

FIG. 1: (a) Sketch of a single-electron transistor (SET) with an oscillating island. The island is coupled to the left (L) and right (R) leads by tunnel junctions, and its capacitance to the gate \( C_g(z) \) depends on the coordinate \( z \) that measures the deviation of the island from its equilibrium position. (b) Equivalent circuit of the device.

We assume that electronic transport through the SET can be described by sequential tunneling. In this case, it is governed by four tunneling rates121314: the rate for electrons to tunnel onto the central region from the left \( \Gamma_{n+1 \rightarrow n} \) and right \( \Gamma_{n \rightarrow n+1} \) and the rates for electrons to tunnel off the central region. The rates can be calculated via Fermi’s golden rule. The energy change
corresponding to the first tunnel process is
\[ \Delta E_{n\rightarrow n+1}^L = e(V_g - V_L) + \varepsilon_n, \] (1)
\[ \varepsilon_n = \frac{e^2}{C_S} \left[ \frac{1}{2}n + \frac{C_R(V_R - V_g) + C_L(V_L - V_g)}{e} \right], \]
where \( C_S = C_L + C_R + C_g(z) \). Defining \( \gamma(z) = -e/(1 - \exp(\beta\varepsilon)) \), the rates can be written as
\[ \Gamma_{n\rightarrow n+1}^L(z) = \frac{1}{e^2R_L} \gamma(\Delta E_{n\rightarrow n+1}^L(z)), \] (2)
where \( R_L \) is the resistance of the left junction. The other rates can be written similarly. The charge state of the SET is characterized by the probability \( \rho_n(t) \) to find \( n \) excess electrons on the island. The time evolution of \( \rho_n(t) \) is governed by the master equation [12, 13, 14, 15]:
\[ \frac{\partial \rho_n}{\partial t} = \sum_{a=\pm 1} \left( \Gamma_{i\rightarrow a} - \Gamma_{i\rightarrow -a} \right) \rho_i, \] (3)
where \( \Gamma_{i\rightarrow j} = \Gamma_{i\rightarrow j}^L + \Gamma_{i\rightarrow j}^R \). The current and all its cumulants can be expressed through \( \Gamma \) and \( \rho \) [17].

Equation 3 has to be supplemented with the equation describing the oscillations of the island. The electrostatic force that acts on the island is \( f(z) = \partial z C_g(V_g - \varphi)^2/2 \), where \( \varphi = (C_L V_L + C_R V_R + C_LV_G + q)/C_S \) is the potential of the island, and \( q = \sum \rho_i n_e \) the average charge. Then the motion of the island can be described by a Langevin equation,
\[ \ddot{z} + \eta \dot{z} + \omega_0^2 z = y + [f(z) - f(0)]/m, \] (4)
where \( m \) is the island mass; \( \eta \approx \omega_0/Q \) where \( Q \) is the quality factor; \( y \) is a random force simulating the interaction with the thermal bath, \( \langle y(t) y(t') \rangle = 2\pi Q \delta(t - t') \) [18]. Typically \( Q \approx 10^5 - 10^4 \) [1] and \( \eta \ll \omega_0 \). If the island is a nanotube, \( C_g \) is a functional of the deviation \( z(x) \) where \( x \) is the coordinate along the nanotube [7]. The rates depend only on integral quantities like \( \int z(x) dx/L \) (\( L \) is the length of the nanotube). Their dynamics can be described by Eq. 4 if the amplitude of the nanotube oscillations does not exceed its diameter by several orders of magnitude.

The current going through the left junction is
\[ I(t) = e \sum_{j} \left[ \Gamma_{j\rightarrow j+1}^L(t) - \Gamma_{j\rightarrow j-1}^L(t) \right] \rho_j(t). \] (5)
We are interested in the current averaged over a time interval \( \tau \) much larger than the characteristic period \( T_0 \) of the island oscillations: \( \bar{I} = \int_0^\tau I(t, z(t)) dt/2\pi, \tau \rightarrow \infty. \)

The typical frequency of micromechanical oscillations is \( \omega_0 \approx 100\text{MHz} \). If electrons tunnel through the SET with a similar frequency, the current will be of the order of \( I \sim e\omega_0 \approx 10^{-11}\text{A} \). However, the current going through a typical SET is usually several orders of magnitude bigger. Hence, during the oscillation period \( T_0 \) many electrons go through the SET and thus it is possible to neglect the time derivative in the master equation 3. Then all the methods used for current calculation in “usual” SETs are applicable to the case with the oscillating island [14]. Averaging the current over time can be replaced by averaging over \( P(z) \), the density of the probability distribution for the deviation \( z \), i.e., \( \bar{I} = \int P(z) I(z) dz \). If the island oscillations are thermally activated, \( P(z) \propto \exp(-z^2/2\omega_0^2) \), where \( \langle z^2 \rangle = k_BT/m\omega_0^2 \). If the island oscillates harmonically, \( z(t) = z_0 \sin(\omega t) \), then \( P(z) = 1/\pi \sqrt{1-(z/z_0)^2} \).

In these expressions, the driving terms \( \sim f \) in Eq. 4 that couple the current in the SET with its mechanical degrees of freedom were neglected. This term is usually much smaller than \( \omega_0^2 z \) on the left-hand side of Eq. 4 (e.g., for the SET parameters in recent experiments, see Ref. 5); the small parameter is \( z_0 \max \partial_t \ln C_i \). The driving terms may become important, e.g., when an a.c. bias near the resonance frequency \( \omega_0 \) is applied to the terminal(s) of the SET. Then the a.c. bias drives the island oscillations and the \( f \)-terms in Eq. 4 are important to determine the amplitude \( z_0 \) of the oscillations [19].

In general, the current and noise in a SET cannot be calculated analytically for arbitrary transport voltages even when the island is static. Analytical progress can be made if we restrict ourselves to the case of small driving voltages near the onset voltage, and temperatures much below the charging energy \( \epsilon^2/C_S \), i.e., \( \gamma(z) \approx \theta(z) \) in Eq. 3. In the case of a static island, the performance of the SET as a transistor and electrometer reaches an optimum in this regime [20]. In this region, the transport characteristics of the SET are also most sensitive to mechanical oscillations of the island, so this regime is the most interesting. Only two states of the island have to be taken into account; the probability \( \rho \) has only two nonzero values \( \rho_{00}, \rho_{11} \). If \( V_L < V_R \) an electron enters the island with the rate \( \Gamma_{0\rightarrow 1}^L \) from the left lead and goes away with the rate \( \Gamma_{1\rightarrow 0}^R \) into the right lead. The average current will be
\[ \bar{I} = \int dz P(z) \frac{e\Gamma_{n\rightarrow n+1}^L(z)\Gamma_{n\rightarrow n+1}^R(z)}{\Gamma_{n\rightarrow n+1}^L(z) + \Gamma_{n\rightarrow n+1}^R(z)}, \] (6)
where \( n = [C_g^{(0)} V_g/e], C_g^{(0)} \equiv C_g(z = 0) \), and \([\ldots]\) means the integer part. Assuming that the capacitances depend only weakly on \( z \), Eq. 6 can be expanded with respect to \( z \). To proceed, we define
\[ J(z) = \frac{1}{eR_L\Delta E_{n\rightarrow n+1}^L + R_L\Delta E_{n\rightarrow n+1}^R}. \] (7)
Using \( \Delta E_{n\rightarrow n+1}^R = e(V_R - V_L) - \Delta E_{n\rightarrow n+1}^L \) and defining \( z_1, z_2 \) to be the roots of the equations \( \Delta E_{n\rightarrow n+1}^L = 0 \) and
FIG. 2: Current gate-charge characteristics for symmetric SET ($R_L = R_R$) with $C_g \gg C_{L,R}$; $V_L = -V_R = V/2$, $Q_0 = -C_{g0}V_g$. (a) $V = 0.2$, ($|V|/2 < V_{osc}$); (b) $V = 0.5$, ($|V|/2 \approx V_{osc}$), (c) $V = 1$, ($|V|/2 > V_{osc}$). The voltage is measured in units of $|e|/2C_\Sigma(z = 0)$, the current in units of $I_0 = (e/C_{g0}^0)/(R_L + R_R)$. The dashed curve corresponds to a static island, the solid curve to a harmonically oscillating island, $z = z_0 \sin(\omega_0 t)$; $z_0 (\partial_z C_g)/C_g = 5.6 \cdot 10^{-3}$ (this is typical for SETs where the island is a nanotube [64]); then $z_0 \approx 5r$, where $r$ is a typical nanotube diameter). The dotted curve in Fig. 11 illustrates what happens if $V_{osc}/(|e|/2C_{g0}^0) = 5 > 1$ and Eq. 5 is not valid. The dash-dotted curves correspond to the case of thermal motion; the thermal average $\langle z^2 \rangle_T \equiv k_B T/m \omega_0^2$ is chosen to be equal $\langle z_0 \sin(\omega_0 t) \rangle_T^2 = z_0^2/2$. The integer part of $Q_0/e$ is the number of electrons on the island in the static regime when $V_L = V_R = 0$. The curves are periodic in the static case, but not if the island oscillates. The areas under the peaks in the static and in the dynamical cases are the same.

$$\Delta E_{n+1 \rightarrow n}^R = 0,$$
we get from Eq. 6

$$I \approx \int_{\min(z_1, z_2)}^{\max(z_1, z_2)} dz P(z)[J(0) + z(\partial_z J(z)|_{z \rightarrow 0}) + z^2 \frac{1}{2}(\partial_z^2 J(z)|_{z \rightarrow 0})]. \quad (8)$$

This formula is valid also for $V_L > V_R$. Using Eq. 11 we find $z_2 - z_1 \approx e(V_R - V_L)/\partial_z \varepsilon_n|_{z \rightarrow 0}$. Thus, if $z_0$ is a characteristic amplitude of island oscillations then it is natural to define the voltage scale

$$V_{osc} = \frac{z_0}{e}(\partial_z \varepsilon_n|_{z \rightarrow 0}). \quad (9)$$

In the limiting case $C_L, C_R \ll C_{g0}^0$,

$$V_{osc} = \frac{z_0}{e}(\partial_z C_g)|_{z \rightarrow 0}^0 e(n + 1/2). \quad (10)$$

Eq. 8 is valid if $V_{osc} < e/C_\Sigma$. If the driving voltages applied to the SET terminals are much larger than $V_{osc}$, the integration limits in Eq. 8 can be extended to infinity because they far exceed $z_0$, the scale of decay of $P(z)$. The second term in Eq. 8 vanishes and

$$I = I(z = 0) + \frac{1}{2} \langle z^2 \rangle_T \frac{\partial^2}{\partial z^2} I(z)|_{z \rightarrow 0}, \quad (11)$$

where $\langle z^2 \rangle_T = \int P(z)z^2 dz$. The first term in Eq. 11 is the current for a static island. If the driving voltages applied to SET terminals are smaller than $V_{osc}$ then the second term (linear in $z$) in Eq. 11 does not vanish; in this regime the current-voltage characteristics is largely influenced by island oscillations. The small parameter in the expansion Eq. 8 is $z_0 \partial_z C_g$. The second term in Eq. 11 is of second order in this parameter. If we investigate the $I - V$ characteristics, island oscillations will smear the Coulomb gap within a voltage band of width of order $V_{osc}$.

The $I - V_g$ characteristics of a symmetric SET ($R_L = R_R$, $C_L = C_R \ll C_{g0}$) with an oscillating island is shown in Fig. 2. The dashed curves correspond to the case of the static island. The solid and dash-dotted curves show the case of a harmonically oscillating island or an island subject to thermal equilibrium fluctuations. Figures 2(a-c) also show how the $I - V_g$ characteristics change when the driving voltage $V$ is smaller, of the order of, or larger than $V_{osc}$. Within each peak of the curves, $n$ is constant,
therefore $V_{osc}$ is also constant within the peak. The most interesting case is shown in the first panel of Fig. 2(a). For the harmonically oscillating island, the peaks split and their width becomes larger with the characteristic scale $V_{osc}$; when the island moves due to thermal activation the peaks do not split but their width also becomes larger with $V_{osc}$. Thus, the type of motion of the island leaves a characteristic trace in the $I - V_g$ plot. Equation (3) well describes $I - V_g$ characteristics when the peaks do not overlap, like in Figs. 2a and b. It follows from Eq. (3) that the areas under the peaks the in static and dynamical cases are equal. The $I - V_g$ characteristics is periodic in the static case, but not periodic for an oscillating island because $V_{osc}$ changes from peak to peak, see, e.g., Eq. (10). For this graph, we used the parameters of the nanotube model of the island (see Ref. [2]), we chose, e.g, $C_y(z) = L/2 \ln(2(R-z)/r)$, where $r = 0.65\text{nm}$ and $L = 500\text{nm}$ are the nanotube radius and length, and $R = 100\text{nm}$ is the distance to the gate. If $C_L, C_R$ and $C_y$ are of the same order, the $I - V_g$ characteristics is qualitatively similar to what is shown in Fig. 2. If the junction is asymmetric, $R_R \neq R_L$, the peaks in Fig. 2 become nonsymmetric. Figure 3 shows the $I - V$ characteristics for $R_R \gg R_L$. The oscillations of the island smear the Coulomb gap within a voltage range of the order of $V_{osc}$.

We will now discuss the current noise in a SET with a moving island. When the island oscillates, the irreducible current-current correlator $S(\tau, \Theta) = \langle (I_L(\Theta + \tau/2)I_L(\Theta - \tau/2)) \rangle$ depends on both $\tau$ and $\Theta$ (rather than only on $\tau$ as in the case of the static island). However, since the charging events in the SET are correlated on time scales much shorter than the period of the island oscillations, the dependence of $S$ on $\tau$ is much stronger than on $\Theta$. In other words, the low-frequency noise can be calculated at a given position of the island (e.g., as it is done in Ref. [14]), and then averaging over time as it was already done for the current above. The result of this procedure is presented in Fig. 4, which shows the dependence of the Fano factor [21] on $V_g$. Here we assumed that the driving voltage is smaller than $V_{osc}$, i.e., the system is in the region in which the influence of the oscillations of the island on the transport properties of the SET is maximal. The Fano factor dips get split due to harmonic oscillations of the island; the scale of the splitting is $V_{osc}$. In contrast to that, the dips are washed out by thermal equilibrium oscillations of the island. Here, we assumed that the noise frequency is much below the frequency of the island oscillations, $\omega_0$.

We now consider the noise spectral density, i.e., the Fourier transform $S(\omega, \Theta)$ of $S(\tau, \Theta)$. If $\omega$ approaches $\omega_0$, the correction to the noise from the motion of the island is $\sim \overline{(\delta I)^2}\langle \zeta^2 \rangle$, where $\langle \zeta^2 \rangle$ is the irreducible correlator of the deviation $z$ at frequency $\omega$. It has a $\delta$-peak at $\omega_0$ if the island oscillates harmonically. In contrast, if the island moves due to thermal activation, $\langle \zeta^2 \rangle = 2\eta k_B T / m (\omega^2 - \omega_0^2)^2 + \omega^2 \eta^2$, the noise peak has a width of the order of the oscillation damping factor $\eta$, see Eq. (4). A similar result was obtained numerically in Ref. [22]. Thus, measuring the noise spectrum allows to find the frequency of island oscillations and gives information on the nature of the oscillations.

In conclusion, we have discussed how vibrations of the island in a SET change its transport properties. The transport characteristics of the SET can be used to determine the nature of island motion, in particular, to estimate the amplitude and frequency of its oscillations.

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[1] A. N. Cleland, Foundations of Nanomechanics (Springer, Heidelberg, 2002).
[2] A. N. Cleland and M. L. Roukes, Nature 392, 160 (1998).
[3] L. Y. Gurel, A. Isacsson, M. V. Vonova, B. Kasemo, R. I. Shkletter, and M. Jonson, Phys. Rev. Lett. 80, 4526 (1998).
[4] A. Erbe, R. H. Blick, A. Tilke, A. Kriede, and J. P. Kotthaus, Appl. Phys. Lett. 73, 3751 (1998).
[5] C. Weiss and W. Zwerger, Europhys. Lett. 47, 97 (1999).
[6] J. Cao, Q. Wang, D. Wang, and H. Dai, cond-mat/0312239.
[7] S. Sapmaz, Ya. M. Blanter, L. Gurevich, and H. S. J. van der Zant, Phys. Rev. B 67, 235414 (2003).
[8] A. D. Armour, M. P. Blencowe, and Y. Zhang, cond-mat/0307528.
[9] A. Mitra, I. Aleiner, and A. J. Millis, cond-mat/0311503.
[10] A. V. Shytov, L. S. Levitov, and C. W. J. Beenakker, Phys. Rev. Lett. 88, 228303 (2002); M. Kindermann and
C. W. J. Beenakker, Phys. Rev. B 66, 224106 (2002).
[11] B. Babic, J. Furer, S. Sahoo, Sh. Farhangfar, and C. Schönenberger, Nano Letters 3, 1577 (2003).
[12] C. W. J. Beenakker, Phys. Rev. B 44, 1646 (1991).
[13] D. V. Averin, A. N. Korotkov, and K. K. Likharev, Phys. Rev. B 44, 6199 (1991).
[14] S. Hershfield, J. H. Davies, P. Hyldgaard, C. J. Stanton, and J. W. Wilkins, Phys. Rev. B 47, 1967 (1993).
[15] C. Bruder and H. Schoeller, Phys. Rev. Lett. 72, 1076 (1994).
[16] D. A. Bagrets and Yu. V. Nazarov, Phys. Rev. B 67, 085316 (2003).
[17] L. D. Landau and E. M. Lifshits, in Course in Theoretical Physics, Vol. 8 (Pergamon Press, Oxford, 1984).
[18] L.D. Landau and E.M. Lifshits, in Course in Theoretical Physics, Vol. 5 (Pergamon Press, Oxford, 1996).
[19] O. Usmani and Ya. M. Blanter, in preparation.
[20] G. L. Ingold and Yu. V. Nazarov, in Single Charge Tunneling, edited by H. Grabert and M. H. Devoret, NATO ASI Series B Vol. 294 (Plenum Press, New York, 1992).
[21] Ya. M. Blanter and M. Büttiker, Phys. Rep. 336, 1 (2000).
[22] A. D. Armour, cond-mat/0401387.