Pion to Photon Transition Form Factors with Basis Light-Front Quantization

Chandan Mondal

Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, China

in collaboration with

Sreeraj Nair (IMPCAS), Shaoyang Jia (ANL), Xingbo Zhao (IMPCAS) and James P. Vary (ISU)

December 1, 2021
Overview

BLFQ-NJL Model and Applications

Pion DA & $\pi \rightarrow \gamma^* \gamma$ Transition Form Factor

$\pi \rightarrow \gamma^* \gamma^*$ Transition Form Factor

Conclusions

CM, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)

• Sreeraj Nair : Tuesday, Nov. 30 at 14:30, Parallel Session 1-B

• Jiangshan Lan : Thursday, Dec. 2 at 9:05, McCartor Award Session
Basis Light-Front Quantization (BLFQ)

A computational framework for solving relativistic many-body bound state problems in quantum field theories\(^1\)

\[ P^- P^+ |\Psi\rangle = M^2 |\Psi\rangle \]

- **\(P^-\)**: light-front Hamiltonian
- **\(P^+\)**: longitudinal momentum
- **\(|\Psi\rangle\)**: mass eigenstate
- **\(M^2\)**: mass squared eigenvalue for eigenstate **\(|\Psi\rangle\)**
- First-principles / effective Hamiltonian as input
- Evaluate observables

\[ O \sim \langle \Psi | \hat{O} | \Psi \rangle \]

- Direct access to light-front wavefunction of bound states

**GOAL**

Light-front wave-functions

3D imaging

Proton spin

FFs

GPDs

TMDs...

---

\(^1\) Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).
Effective Hamiltonian: BLFQ-NJL Model

\[
| \pi \rangle_{\text{phys}} = a \ | q\bar{q} \rangle + b \ | q\bar{q}g \rangle + c \ | q\bar{q}q\bar{q} \rangle + \ldots
\]

\[
H_{\text{eff}} = \frac{\vec{k}^2 + m_q^2}{x} + \frac{\vec{k}^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x) r^2 + \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x (x(1-x) \partial_x) + H_{\text{NJL}}
\]

kinetic energy

transverse confining potential [2]

longitudinal confining potential [3]

Nambu–Jona-Lasinio (NJL) interaction [4]

---

1. Jia and Vary, Phys. Rev. C 99, 035206 (2019)
2. Brodsky, Teramond, Dosch and Erlich, Phys. Rep. 584, 1 (2015).
3. Li, Maris, Zhao and Vary, Phys. Lett. B 758, 118 (2016)
4. Klimt, Lutz, Vogl and Weise, Nucl. Phys. A 516, 429-468 (1990).
Meson Light-Front Wave Functions (LFWFs)

- Valence LFWFs in orthonormal bases

\[ \psi_{rs}(x, \vec{\kappa}_\perp) = \sum_{n,m,l} \langle n, m, l, r, s | \psi \rangle \times \phi_{nm}(\vec{\kappa}_\perp) \chi_l(x) \]

- Transverse direction (2D-HO)

\[ \phi_{nm}(\vec{\kappa}_\perp) \sim (|\vec{\kappa}_\perp|)^{|m|} \times \exp(-\vec{\kappa}_\perp^2) L_n^{|m|}(\vec{\kappa}_\perp^2); \quad 0 \leq n \leq N_{\text{max}} \]

- Longitudinal direction (Jacobi polynomial basis)

\[ \chi_l(x) \sim x^{\beta/2}(1 - x)^{\alpha/2} P_l^{(\alpha, \beta)}(2x - 1); \quad 0 \leq l \leq L_{\text{max}} \]

- Coefficients \( \langle n, m, l, r, s | \psi \rangle \) : eigenvector in BLFQ basis representation.

\[ ^2 \text{Li, Maris, and Vary, Phys. Rev. D 96, 016022 (2017)} \]
BLFQ-NJL model parameters

• Parameters are fixed to
  • reproduce ground state masses
  • experimental charge radii of $\pi^+$ and the $K^+$ \(^1\)

• Successfully applied to
  • compute the PDAs and the EMFFs \(^1\)
  • PDFs for the pion and the kaon and pion-nucleus induced Drell-Yan cross sections \(^2\)\(^3\)
  • GPDs \(^4\)

• Summary of the model parameters

| Valence flavor | $N_{\text{max}}$ | $L_{\text{max}}$ | $\kappa$(MeV) | $m_q$(MeV) | $m_{\bar{q}}$(MeV) |
|----------------|----------------|----------------|---------------|------------|-------------------|
| $ud$           | 8              | 8–32           | 227           | 337        | 337               |
| $u\bar{s}$     | 8              | 8–32           | 276           | 308        | 445               |

\(^1\) Jia and Vary, Phys. Rev. C 99, 035206 (2019)
\(^2\) Lan, CM, Jia, Zhao, Vary, Phys. Rev. Lett. 122 172001 (2019)
\(^3\) Lan, CM, Jia, Zhao, Vary, Phys. Rev. D 101, 034024 (2020)
\(^4\) Adhikari, CM, Nair, Xu, Jia, Zhao and Vary, [arXiv:2106.04954] accepted by Phys. Rev. D
Applications: Light Meson PDFs

Light-front effective Hamiltonian, $H_{\text{eff}}$: ($\mu_{0\pi}^2 = 0.240 \pm 0.024 \text{ GeV}^2$)

Diagonalizing $H_{\text{eff}} \Rightarrow$ LF wavefunction $\Rightarrow$ Initial PDFs $\Rightarrow$ Scale evolution $^1$.

$$\psi_{rs}(x, \vec{k}^\perp) = \sum_{n,m,l} \langle n,m,l,r,s|\psi \rangle \times \phi_{nm}(\vec{k}^\perp) \chi_l(x)$$

- 2D-HO $\phi_{nm}(\vec{k}^\perp)$ in the transverse plane.
- Jacobi polynomial basis $\chi_l(x)$ in the longitudinal direction.

$^1$Lan, CM, Jia, Zhao, Vary, Phys. Rev. Lett. 122 172001 (2019)
Moments of Pion PDF

Moments of the valence quark PDF

\[ \langle x^n \rangle = \int_0^1 dx \, x^n f_v(x, \mu^2), \quad n = 1, 2, 3, 4. \]

Consistent with global fit, lattice QCD, and phenomenological models.

\[ ^1 \text{Lan, CM, Jia, Zhao, Vary, Phys. Rev. D 101, 034024 (2020)} \]
GPDs → Transverse Densities

- Moments of GPDs:
  \[
  \int_0^1 dx \ x^{n-1} H_\pi(x, b_\perp^2) = A_{n0}(b_\perp^2), \\
  \int_0^1 dx \ x^{n-1} E_{\pi T}(x, b_\perp^2) = B_{Tn0}(b_\perp^2).
  \]

- Define density
  \[
  \rho_n(b_\perp, s_\perp) = \frac{1}{2} \left[ A_{n0}(b_\perp^2) - \frac{s_\perp \epsilon^{ij} b_j^\perp}{m_\pi} B_{Tn0}(b_\perp^2) \right],
  \]

- Reasonable agreement with Lattice QCD
Distribution Amplitudes

DAs of pseudoscalar states

\[ \phi(x, \mu_0) \sim \frac{1}{\sqrt{x(1-x)}} \int \frac{d^2 \vec{k}_\perp}{2(2\pi)^3} \frac{(\psi_{\uparrow\downarrow} - \psi_{\downarrow\uparrow})}{\sqrt{2}} \]

- DA evolution: ERBL equations (Gegenbauer basis)
  
  Ruiz, et. al. PRD 66, (2002)

- Oscillations \( \rightarrow \) Basis artifacts

- With increasing \( L_{\text{max}} \) the DA tends toward a smooth function

- Evolved DA (10 GeV\(^2\)) : Asymptotic DA

Decay constant \( f_\pi \):

BLFQ (Basis [8, 32]): 145.3 MeV

Experimental data: 130.2 \( \pm \) 1.7 MeV

\[ L_{\text{max}} = 8 \]
\[ L_{\text{max}} = 32 \]
\[ L_{\text{max}} \rightarrow \infty \]

\[ \phi(x, \mu) \]

\[ x \]

Asymptotic

Initial DA

E791 data

Evolved DA

\[ x \]

Asymptotic

Initial DA

E791 data

Evolved DA

- Consistent with the FNAL-E-791

\(^1\) Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)
Moments of Pion DA

\[ \langle z_p \rangle = \int_0^1 dx \ z^p \ \phi(x, \mu), \]

\[ z \equiv (2x - 1) \text{ when } p \geq 1 \text{ and } z \equiv x \text{ for } p = -1. \]

|                  | \( \mu(\text{GeV}) \) | \( \langle z_2 \rangle \)     | \( \langle z_4 \rangle \)     | \( \langle z_6 \rangle \)     | \( \langle x^{-1} \rangle \) |
|------------------|----------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| BLFQ-NJL (this work) | 1, 2               | 0.221, 0.217              | 0.099, 0.097              | 0.057, 0.055              | 3.21, 3.17               |
| Playkurtic [90]  | 2                   | 0.220^{+0.009}_{-0.006}  | 0.098^{+0.008}_{-0.005}  | ...                      | 3.13^{+0.14}_{-0.10}    |
| NLC sum rules [93]| 2                   | 0.248^{+0.016}_{-0.015}  | 0.108^{+0.005}_{-0.003}  | ...                      | 3.16(9)                  |
| LF quark model [44]| \sim 1             | 0.24(22)                  | 0.11(9)                   | 0.07(5)                  | ...                      |
| Sum rules [91]    | 1                   | 0.24                      | 0.11                      | ...                      | ...                      |
| AdS/QCD [55]      | \sim 1             | 0.25                      | 0.125                     | 0.078                    | 3.98                     |
| LF holographic \((B = 0)\) [46]| 1, 2              | 0.180, 0.185              | 0.067, 0.071              | ...                      | 2.81, 2.85               |
| LF holographic \((B \gg 1)\) [46]| 1, 2              | 0.200, 0.200              | 0.085, 0.085              | ...                      | 2.93, 2.95               |
| Renormalon model [98] | 1              | 0.28                      | 0.13                      | ...                      | ...                      |
| Instanton vacuum \((MIA 1)\) [92]| 1, 2             | 0.237, 0.218              | 0.112, 0.094              | 0.066, 0.052              | ...                      |
| Instanton vacuum \((MIA 2)\) [92]| 1, 2             | 0.239, 0.220              | 0.113, 0.096              | 0.067, 0.053              | ...                      |
| Sum rules [2]     | 2                   | 0.343                     | 0.181                     | ...                      | 4.25                     |
| Dyson-Schwinger [RL, DB] [99]| 2              | 0.280, 0.251              | 0.151, 0.128              | ...                      | 5.5, 4.6                 |
| QCD background field theory sum rule [47]| 1            | 0.271(13)                  | 0.138(10)                 | 0.087(6)                 | 3.95                     |
| QCD background field theory sum rule [47]| 2            | 0.254(10)                  | 0.125(7)                  | 0.077(6)                 | 3.33                     |
| Lattice QCD [100] | 2                   | 0.28(1)(2)                | ...                      | ...                      | ...                      |
| Lattice QCD [94]  | 2                   | 0.2361(41)(39)            | ...                      | ...                      | ...                      |
| Lattice QCD [101] | 2                   | 0.27(4)                   | ...                      | ...                      | ...                      |
| Lattice QCD [95]  | 2                   | 0.2077(43)                | ...                      | ...                      | ...                      |
| Lattice QCD [96]  | 2                   | 0.234(6)(6)               | ...                      | ...                      | ...                      |
| Lattice QCD [97]  | 2                   | 0.244(30)                 | ...                      | ...                      | ...                      |
| Asymptotic QCD    | \infty             | 0.200                     | 0.086                     | 0.048                    | 3.00                     |

\[ ^1 \text{Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)} \]
Radiative Decay Width

• Alder-Bell-Jackiw (ABJ) anomaly\(^{12}\) relations

\[ F_{\pi \gamma}^{\text{ABJ}}(0) = \frac{1}{2\sqrt{2}\pi^2 f_{\pi^0}} \]

• The radiative decay width

\[ \Gamma_{\pi \to \gamma \gamma} = \frac{\pi}{4} \alpha_{\text{EM}}^2 M_{\pi}^3 |F_{\pi \gamma}(0)|^2 \]

| \([N_{\text{max}}, L_{\text{max}}]\) | \([8, 8]\) | \([8, 16]\) | \([8, 32]\) | Experimental data \([102]\) |
|---------------------------------|----------|----------|----------|------------------|
| \(f_{\pi}\) (MeV)              | 142.8    | 144.8    | 145.3    | 130.2            |
| \(\Gamma_{\pi \to 2\gamma}\) (keV) | \(7.22 \times 10^{-3}\) | \(7.03 \times 10^{-3}\) | \(6.98 \times 10^{-3}\) | \((7.82 \pm 0.22) \times 10^{-3}\) |

• Demonstrate a good convergence trend.

---

\(^{1}\) Adler, Phys. Rev. 177, 2426 (1969)  
\(^{2}\) Bell and Jackiw, Nuovo Cimento A 60, 47 (1969)  
\(^{3}\) Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)
\[ \pi \to \gamma^* \gamma \text{ Transition Form Factor} \]

\[ \langle \gamma (P - q) | J^\mu | P (P) \rangle = -i e^2 F_{\pi \gamma} (Q^2) \epsilon^\mu \nu \rho \sigma P_\nu \epsilon_\rho q_\sigma, \]

- Evaluated from convolution of a hard scattering amplitude (HSA) with DA.

- Results for \( \{ N_{\text{max}}, L_{\text{max}} \} = \{ 8, 8 \}, \{ 8, 16 \}, \) and \( \{ 8, 32 \} \) (upper panel)

- Good convergence trend.

- Consistent with Brodsky-Lepage limit \(^1\): \( Q^2 F_{\pi \gamma} (Q^2 \to \infty) = \text{const.} \)

- ERBL evolution effects \& \( \alpha_s \) order correction considered (lower panel).

---

\(^1\) Lepage and Brodsky, Phys. Rev. D 22, 2157 (1980)

\(^2\) Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)
\[ Q^2 F_{\pi\gamma}(Q^2) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 dx \, T_H(x, Q^2) \phi(x, \bar{x}Q), \]

- The hard scattering amplitude (HSA)\(^{12}\)
  \[ T_H(x, Q^2) = \frac{1}{1 - x} + O(\alpha_s) + \ldots \]

- Assumption \( \phi(x, (1 - x)Q) \simeq \phi(x, Q) \): reasonable at \( Q^2 \to \infty \)^2.

- NOT well justified below the asymptotic region.

- Need to take into account the ERBL evolution effects.

---

\(^{1}\) Braaten, Phys. Rev. D 28, 524 (1983)
\(^{2}\) Brodsky, Cao, and de Teramond, Phys. Rev. D 84, 033001 (2011)
\(^{3}\) Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)
\[ F_{\pi\gamma^*\gamma^*}(Q_1^2, Q_2^2) = \frac{\sqrt{2}}{3} f_{\pi} \int_0^1 dx \left. T_{H}^{\gamma^*\gamma^* \rightarrow \pi^0} (x, Q_1^2, Q_2^2) \right|_{\phi(x, \bar{Q})} \]

\[ T_{H}^{\gamma^*\gamma^* \rightarrow \pi^0} = \frac{1}{(1 - x)Q_1^2 + xQ_2^2} + \mathcal{O}(\alpha_s) + \ldots \]

- \( F_{\pi\gamma^*\gamma^*}(Q_1^2, Q_2^2) \sim 1/(Q_1^2 + Q_2^2) \) when \((Q_1^2, Q_2^2) \rightarrow \infty\).

- Consistent with PQCD prediction\(^1\).

- Vector meson dominance (VMD) model: \( F_{\pi\gamma^*\gamma^*}^{VMD}(Q_1^2, Q_2^2) \sim 1/(Q_1^2 Q_2^2) \).

- Qualitative behavior \( \rightarrow \) consistent with the LFQM results.

- Singly virtual TFF \( \rightarrow \) by setting one of the momentum transfers to zero.

---

\(^1\) Lepage and Brodsky, Phys. Rev. D 22, 2157 (1980)

\(^2\) Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)
\[ \pi \rightarrow \gamma^* \gamma^* \] Transition Form Factor

\[ F_{\pi\gamma^*}(Q_1^2, Q_2^2) = \frac{\sqrt{2}}{3} f_{\pi} \int_0^1 dx \, T_{H\gamma^*\gamma^* \rightarrow \pi^0}(x, Q_1^2, Q_2^2) \phi(x, \bar{Q}) \]

| \((Q_1^2, Q_2^2)\) | \(F_{\pi\gamma^*}^{LO}\), this work | \(F_{\pi\gamma^*}^{NLO}\), this work | LO | NLO | LFQM [54] | VMD [54] |
|---------------------|-----------------|-----------------|-----|-----|-----------|-----------|
| (6.48, 6.48)        | 10.39–10.56     | 9.59–9.75       | 9.52| 8.78| 9.08      | 1.957 ± 0.022 |
| (16.85, 16.85)      | 3.99–4.06       | 3.73–3.79       | 3.66| 2.69| 3.58      | 0.322 ± 0.004 |
| (14.83, 4.27)       | 7.55–7.72       | 7.00–7.14       | 6.91| 6.39| 6.76      | 1.301 ± 0.014 |
| (38.11, 14.95)      | 2.65–2.69       | 2.48–2.52       | 2.42| 2.27| 2.40      | 0.163 ± 0.002 |
| (45.63, 45.63)      | 1.47–1.50       | 1.39–1.41       | 1.35| 1.33| 1.33      | 0.046 ± 0.001 |

1 Choi, Ryu and Ji, Phys. Rev. D 99, 076012 (2019)
1 Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)
Effective Hamiltonian with One Dynamical Gluon

\[ | \pi \rangle_{\text{phys}} = a |q\bar{q}\rangle + b |q\bar{q}g\rangle + c |q\bar{q}q\bar{q}\rangle + \ldots \]

\[ H_{\text{eff}} = \frac{\vec{k}^\perp 2 + m_q^2}{x} + \frac{\vec{k}^\perp 2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}^\perp 2 \]

\[ + \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x x(1-x)\partial_x \]

\[ + H_{\text{vertex}} + H_{\text{inst}} \]

1. Lan, Fu, CM, Zhao and Vary, arXiv:2106.04954 [hep-ph].
2. Brodsky, Teramond, Dosch and Erlich, Phys. Rep. 584, 1 (2015).
3. Li, Maris, Zhao and Vary, Phys. Lett. B (2016).
4. Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998).
DA & Transition Form Factors

\[ | \pi \rangle_{\text{phys}} = a | q\bar{q} \rangle + b | q\bar{q}g \rangle + c | q\bar{q}q\bar{q} \rangle + \ldots \]

\( \{N_{\text{max}}, L_{\text{max}}\} = \{14, 15\} \)

\[ (Q_1^2 + Q_2^2) F_{\gamma \pi}^{1O}(Q^2) \text{ [GeV]} \]

Decay constant \( f_{\pi} \):

BLFQ : 138 MeV

Experimental data: 130.2 \( \pm \) 1.7 MeV

\(^1\) Work in progress; Jiangshan Lan: Thursday, Dec. 2 at 9:05, McCartor Award Session
Conclusions

- Pion structure from the eigenstates of light-front effective Hamiltonians

- \(|q\bar{q}\rangle (\text{BLFQ-NJL model})\) and \(|q\bar{q}\rangle + |q\bar{q}g\rangle\) (with QCD interactions).

- LF Hamiltonian \(\Rightarrow\) Wavefunctions \(\Rightarrow\) Observables.

- \(\pi\rightarrow\gamma^*\gamma\) TFF: Consistent with Belle data; deviates from BaBaR data.

- \(F_{\pi\gamma^*}(Q_1^2, Q_2^2) \sim 1/(Q_1^2 + Q_2^2)\) when \((Q_1^2, Q_2^2)\to\) large.

Thank You