Determination of Cost-Effective Sand Mixing Ratio for Improvement of High Liquid Limit Soil Based on Fuzzy Comprehensive Evaluation

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Abstract. In order to save soil resources and increase the utilization range of high liquid limit soil, the high liquid limit soil was improved by mixing sand, and the cost-effective sand mixing ratio was determined with the help of fuzzy comprehensive evaluation method. It is shown that the sand-mixing improvement method can significantly reduce the liquid limit and plasticity index of high liquid limit soil, and increase its value of California bearing ratio to meet the property requirements for embankment filling. The fuzzy comprehensive evaluation results show that the improvement effect and cost of sand-mixed high liquid limit soil reach a comprehensive optimal state when the sand mixing ratio is 20%.

1. Introduction

High liquid limit soil is a kind of weak soil with high natural water content, high liquid limit and high fine particle content, and it is widely distributed in tropical and subtropical countries [1]. The Chinese standard stipulates that the soil with a fine particle content not less than 50% and a liquid limit higher than 50% is high liquid limit soil [2]. High liquid limit soil cannot be used as embankment filling for its low performance in engineering. To save soil resources and cost, it is common to improve high liquid limit soil by adding admixtures [3].

Sand-mixing improvement method for weak soil is low in cost, environmentally friendly, and easy to apply in engineering. Research efforts have been devoted to sand-mixing improvement of weak soil. For example, Park et al. [4] found that as sand content in bentonite increased, the angle of internal friction and the water permeability of bentonite increased while the cohesion decreased. Choo et al. [5] investigated the compressibility and small strain stiffness of kaolin clay mixed with different amounts of sand. Zhuang et al. [6] studied the dynamic characteristics of sand-mixed expansive soil. The above studies indicate that sand can improve the gradation of soil, improve the water permeability and the compressive strength of weak soil, therefore it is feasible to improve high liquid limit soil by mixing sand.

Fuzzy comprehensive evaluation method is a comprehensive evaluation method based on fuzzy mathematics, which can solve the problem that is difficult to quantify. Fuzzy comprehensive evaluation
method is widely used in engineering practice [7,8]. In this paper, sand is used as the admixture to improve the engineering performance of high liquid limit soil. Fuzzy comprehensive evaluation method is used to determine cost-effective sand mixing ratio to make the improvement effect and cost of improved soil reach a comprehensive optimal state.

2. Materials and Methods

2.1. Materials

The high liquid limit soil used in this paper is taken from Yunluo expressway, Guangdong Province, China. The physical and mechanical properties of high liquid limit soil in natural state are shown in table 1, and the particle grading curve is shown in figure 1.

| Moisture content (%) | Liquid limit (%) | Plasticity index | Density (g/cm³) | Void ratio | Compression coefficient (MPa-1) | Quick direct shear test | Cohesion (kPa) | Angle of internal friction (°) |
|----------------------|-----------------|-----------------|-----------------|------------|-------------------------------|------------------------|----------------|-----------------------------|
| 27.0                 | 59.1            | 32.3            | 1.75            | 0.96       | 0.44                          | 34.3                   | 15.6           |                             |

Figure 1. Particle grading curves of high liquid limit soil and river sand

The sand used in this paper is river sand from the Pearl River Basin in China, as shown in figure 2. The physical and mechanical properties of river sand are shown in table 2, and the particle grading curve is shown in figure 1.

| Moisture content (%) | Apparent density (g/cm³) | Mud content (%) | Quick direct shear test | Cohesion (kPa) |
|----------------------|--------------------------|-----------------|-------------------------|----------------|
| 0                    | 2.58                     | 0.03            | 0                       | 16.2           |

Figure 2. The river sand from the Pearl River Basin in China
2.2. Improvement Test of Sand-Mixed High Liquid Limit Soil
The sand mixing ratio \( \eta \) is the ratio of the mass of dry sand to the mass of dry soil, and the improved high liquid limit soils with different sand mixing ratios \( \eta = 15\%, 20\%, 25\% \) and \( 30\% \) were made. According to the Chinese test code [9], geotechnical tests were carried out on the improved soils \( \eta = 15\%, 20\%, 25\% \) and \( 30\% \) and untreated soil \( \eta = 0\% \).

2.3. Evaluation Model of Cost-Effective Sand Mixing Ratio
In this paper, the improvement effect and cost of sand-mixed high liquid limit soil were taken as the evaluation factors with the help of fuzzy comprehensive evaluation method.

2.3.1. Establishment of Factor Set. Factor set \( U \) is a set of factors that affect the evaluated object. In order to determine cost-effective sand mixing ratio, the factors in this paper were improvement effect and cost, and the factor set \( U \) was established as,

\[
U = \{u_1, u_2\} = \{\text{Improvement effect, Improvement cost}\} \quad (1)
\]

Since it is difficult to evaluate the improvement effect of improved soil by a single factor, the improvement effect \( u_1 \) is a set of subfactor related to the liquid limit, strength and compressibility of the improved soil. In this paper, the subfactor set \( u_1 \) included five subfactors: liquid limit \( W_L \) , free expansion rate \( F_s \) , California bearing ratio value \( CBR \) , unconfined compressive strength \( q_u \) and compression coefficient \( a_r \). The subfactor set \( u_1 \) was established as,

\[
u_1 = \{u_{11}, u_{12}, u_{13}, u_{14}, u_{15}\} = \{w_L, F_s, CBR, q_u, a_r\} \quad (2)
\]

2.3.2. Establishment of Comment Set. Comment set \( V \) is a set of comments of the evaluated object. The single-factor evaluation for each subfactor in subfactor set \( u_1 \) was made, thereby obtaining the membership degree \( r_i (0 \leq r_i \leq 1, i = 1, 2, ..., 5) \) of each subfactor \( u_i \) with respect to comment set \( V \). In same way, the membership degree \( r_j (0 \leq r_j \leq 1) \) of factor \( u_2 \) with respect to comment set \( V \) can be obtained.

2.3.3. Determining Weight Coefficient by Analytic Hierarchy Process. In this paper, analytic hierarchy process was used to determine the weight coefficient \( W_u \) of factor set \( U \) and the weight coefficient \( W_{u_1} \) of subfactor set \( u_1 \). Analytic hierarchy process can make the weight coefficient more objective and practical, easy to express quantitatively, thereby improving the accuracy of fuzzy comprehensive evaluation results.

2.3.4. Fuzzy Comprehensive Evaluation. The fuzzy comprehensive evaluation matrix \( R_1 \) of subfactor set \( u_1 \) was established as,

\[
R_1 = [r_{i1}, r_{i2}, r_{i3}, r_{i4}, r_{i5}] \quad (3)
\]

where \( r_{ij} (0 \leq r_{ij} \leq 1, j = 1, 2, ..., 5) \) is the membership degree of each subfactor \( u_{1j} \) with respect to comment set \( V \).
The comprehensive evaluation result $B_1$ of subfactor set $u_1$ can be determined as,

$$B_1 = W_{u_1} \cdot R_1^T$$  \hspace{1cm} (4)

where $R_1^T$ is the transpose matrix of $R_1$, $W_{u_1}$ is the weight coefficient of subfactor set $u_1$.

The final result $B$ of the fuzzy comprehensive evaluation can be determined as,

$$B = W_u \cdot [r_1 \ r_2]^T$$  \hspace{1cm} (5)

where $r_1$ is the norm of matrix $B_1$, $r_2$ is the membership degree of factor $u_2$ with respect to comment set $V$, $W_u$ is the weight coefficient of factor set $U$.

The larger the value of $|B|$ is, the closer the evaluated sand mixing ratio is to the most cost-effective sand mixing ratio.

3. Results and discussions

3.1. Physical and Mechanical Properties of Improved Soil

The particle size distribution of untreated soil ($\eta = 0\%$) and improved soils ($\eta = 15\%, 20\%, 25\%$ and $30\%$) are shown in table 3. The coarse particle content of high liquid limit soil increased obviously after mixing with sand, the gradation was improved. The increased coarse particles were mainly distributed in the particle size range of 0.5mm $\sim$ 1.0mm.

| Sand mixing ratio | Content less than the particle size / % |
|-------------------|-----------------------------------------|
|                   | <0.075mm | <0.25mm | <0.5mm | <1mm | <2mm | <5mm | <10mm | <20mm |
| 0%                | 89.81    | 92.57   | 92.95  | 93.43 | 93.68 | 95.98 | 98.83 | 100   |
| 15%               | 78.34    | 83.33   | 86.09  | 91.82 | 93.02 | 96.45 | 98.96 | 100   |
| 20%               | 75.15    | 80.76   | 84.19  | 91.38 | 92.83 | 96.57 | 98.99 | 100   |
| 25%               | 72.22    | 78.41   | 82.44  | 90.97 | 92.66 | 96.68 | 99.02 | 100   |
| 30%               | 69.52    | 76.23   | 80.82  | 90.60 | 92.51 | 96.80 | 99.06 | 100   |

The basic properties of untreated soil ($\eta = 0\%$) and improved soils ($\eta = 15\%, 20\%, 25\%$ and $30\%$) are shown in table 4. The free expansion rate $F_3$ decreased with the increase of sand mixing ratio $\eta$. As sand mixing ratio $\eta$ increased, the liquid limit $\omega_l$ and plasticity index $I_p$ of the improved soils decreased. The liquid limits of improved soils were all less than 50 $\%$, and the plasticity indexes were all less than 22. Chinese technical regulation [10] stipulates that soils with a liquid limit greater than 50$\%$ and a plasticity index greater than 26 cannot be used as embankment filling. Therefore, sand-mixing improvement method can enable the high liquid limit soil to be used as embankment filling.

| Sand mixing ratio | Free expansion rate (%) | Liquid limit (%) | Plasticity index |
|-------------------|-------------------------|-----------------|-----------------|
| 0%                | 21.8                    | 59.1            | 32.3            |
| 15%               | 8.0                     | 49.9            | 21.5            |
| 20%               | 6.2                     | 46.8            | 17.6            |
| 25%               | 4.6                     | 45.8            | 16.2            |
| 30%               | 2.9                     | 45.5            | 15.4            |
The strength test results of untreated soil ($\eta = 0\%$) and improved soils ($\eta = 15\%, 20\%, 25\%$ and $30\%$) are shown in table 5. The unconfined compressive strength $q_u$ of high liquid limit soil was greatly improved after mixing with sand. The unconfined compressive strength $q_u$ increased first and then decreased with the sand mixing ratio $\eta$, and reached the maximum when the sand mixing ratio $\eta$ was 20%. The values of California bearing ratio $CBR$ of improved soils were all higher than 3, which met the requirement for California bearing ratio $CBR$ of embankment filling in Chinese technical regulation [10]. Compared with untreated soil, the compression coefficient $\eta_c$ of improved soil decreased slightly.

### Table 5. The strength test results of untreated soil and improved soils

| Sand mixing ratio | Unconfined compressive strength (MPa) | California bearing ratio value (%) | Compression coefficient ($\eta_c$, MPa-1) |
|-------------------|---------------------------------------|----------------------------------|------------------------------------------|
| 0%                | 0.07                                  | 2.8                              | 0.25                                     |
| 15%               | 0.48                                  | 4.5                              | 0.22                                     |
| 20%               | 0.54                                  | 5.1                              | 0.21                                     |
| 25%               | 0.42                                  | 5.7                              | 0.20                                     |
| 30%               | 0.33                                  | 6.1                              | 0.18                                     |

3.2. Results of Fuzzy Comprehensive Evaluation

3.2.1. Variation of Factor and Subfactors with Sand Mixing Ratio. When the high liquid limit soil was improved by mixing sand, the improvement cost can be approximately determined by the mixing amount of sand. The cost was represented by 1.00 when the sand mixing ratio $\eta$ was 15%, therefore based on the mixing amount of sand, the cost was represented by 1.33 when the sand mixing ratio $\eta$ was 20%. In the same way, the cost was represented by 1.67 when the sand mixing ratio $\eta$ was 25%, and the cost was represented by 2.00 when the sand mixing ratio $\eta$ was 30%. The variations of factor and subfactors with sand mixing ratio $\eta$ are shown in table 6.

### Table 6. Variation of factor and subfactors with sand mixing ratio

| Sand mixing ratio $\eta$ | Improvement effect $u_i$ | Liquid limit $W_i$ (%) | Free expansion rate $F_i$ (%) | California bearing ratio $CBR$ (%) | Unconfined compressive strength $q_u$ (MPa) | Compression coefficient $\eta_c$ (MPa-1) |
|--------------------------|---------------------------|------------------------|-------------------------------|--------------------------------------|------------------------------------------|------------------------------------------|
| 15%                      | $u_{11}$                  | 49.9                   | 8.0                           | 4.5                                  | 0.48                                     | 0.22                                     |
| 20%                      | $u_{12}$                  | 46.8                   | 6.2                           | 5.1                                  | 0.54                                     | 0.21                                     |
| 25%                      | $u_{13}$                  | 45.8                   | 4.6                           | 5.7                                  | 0.42                                     | 0.20                                     |
| 30%                      | $u_{14}$                  | 45.5                   | 2.9                           | 6.1                                  | 0.33                                     | 0.18                                     |

3.2.2. Calculation of Membership Degree. For subfactors including liquid limit $W_i$, free expansion rate $F_i$, and compression coefficient $\eta_c$, the lower the value is, the better the improvement effect is, therefore reduced semi trapezoid distribution function was selected as their membership function. For the subfactors including California bearing ratio value $CBR$ and unconfined compressive strength $q_u$, the greater the value is, the better the improvement effect is, therefore ascended semi trapezoid distribution function was selected as their membership function. For improvement cost $u_2$, reduced semi trapezoid distribution function was selected as its membership function. Based on the value of each factor and subfactor in table 6 and the requirements for embankment filling in Chinese technical regulation [10], the membership function of each factor and subfactor is shown in table 7.
Table 7. Membership functions of factor and subfactors

| Factor or subfactor | Membership function |
|---------------------|---------------------|
| \( u_1 \) Liquid limit \( w_L \) | \( r_{11} = \begin{cases} 1 & , \ x < 30 \\ \frac{50-x}{20} & , \ 30 \leq x \leq 50 \\ 0 & , \ x > 50 \end{cases} \) |
| \( u_2 \) Free expansion rate \( F_s \) | \( r_{12} = \begin{cases} 1 & , \ x < 30 \\ \frac{40-x}{40} & , \ 0 \leq x \leq 40 \\ 0 & , \ x > 40 \end{cases} \) |
| \( u_3 \) California bearing ratio \( CBR \) | \( r_{13} = \begin{cases} 0 & , \ x < 2 \\ \frac{x-2}{6} & , \ 2 \leq x \leq 8 \\ 1 & , \ x > 8 \end{cases} \) |
| \( u_4 \) Unconfined compressive strength \( q_u \) | \( r_{14} = \begin{cases} 0 & , \ x < 0.2 \\ \frac{x-0.2}{0.4} & , \ 0.2 \leq x \leq 0.6 \\ 1 & , \ x > 0.6 \end{cases} \) |
| \( u_5 \) Compression coefficient \( a_v \) | \( r_{15} = \begin{cases} 0 & , \ x < 0.1 \\ \frac{0.3-x}{0.2} & , \ 0.1 \leq x \leq 0.3 \\ 1 & , \ x > 0.3 \end{cases} \) |
| \( u_2 \) Improvement cost | \( r_2 = \begin{cases} 1 & , \ x < 1 \\ \frac{2-x}{1} & , \ 1 \leq x \leq 2 \\ 0 & , \ x > 2 \end{cases} \) |

3.2.3. Calculation of Weight Coefficient. According to the engineering experience and the comparison result table of the factor importance (shown in table 8), the relative importance of each factor and subfactor was compared. In the factor set \( U \), the improvement effect \( u_1 \) was significantly more important than the improvement cost \( u_2 \). In the subfactor set \( u_t \), the liquid limit \( w_L \) was slightly more important than the free expansion rate \( F_s \) and the unconfined compressive strength \( q_u \), the California bearing ratio \( CBR \) was strongly more important than the liquid limit \( w_L \), and the compression coefficient \( a_v \) was significantly more important than the liquid limit \( w_L \).

The judgment matrix \( P_U \) of factor set \( U \) was established as,

\[
P_U = \begin{bmatrix} 1 & 5 \\ 1/5 & 1 \end{bmatrix}
\]  \hspace{1cm} (6)

The judgment matrix \( P_{u_t} \) of subfactor set \( u_t \) was established as,
The eigenvector \( \mathbf{A}_U \) of the judgment matrix \( \mathbf{P}_U \) is the weight coefficient \( \mathbf{W}_U \) of factor set \( U \), and \( \mathbf{W}_U \) was as follows,

\[
\mathbf{W}_U = \mathbf{A}_U = \begin{bmatrix} 0.83 & 0.167 \end{bmatrix}
\]

(8)

The eigenvector \( \mathbf{A}_{u_1} \) of the judgment matrix \( \mathbf{P}_{u_1} \) is the weight coefficient \( \mathbf{W}_{u_1} \) of subfactor set \( u_1 \), and \( \mathbf{W}_{u_1} \) was as follows,

\[
\mathbf{W}_{u_1} = \mathbf{A}_{u_1} = \begin{bmatrix} 0.099 & 0.051 & 0.523 & 0.051 & 0.275 \end{bmatrix}
\]

(9)

In order to verify whether the weight distribution is reasonable, the consistency of the judgment matrix \( \mathbf{P}_U \) and \( \mathbf{P}_{u_1} \) was checked. The judgment matrix \( \mathbf{P}_U \) is a second-order matrix, and the second-order matrix is a consistent matrix. The judgment matrix \( \mathbf{P}_{u_1} \) is a fifth-order matrix, and the method for consistency check is as follows:

(a) The consistency indicator \( C.I. \) is determined as,

\[
C.I. = \frac{\lambda_{\text{max}} - n}{n - 1}
\]

(10)

where \( n \) is the order of the judgment matrix, \( \lambda_{\text{max}} \) is the maximum eigenvalue of the judgment matrix.

(b) The consistency ratio \( C.R. \) is determined as,

\[
C.R. = \frac{C.I.}{R.I.}
\]

(11)

where \( R.I. \) is the random consistency indicator and the value of \( R.I. \) is 1.12 when the order of the judgment matrix is 5.

**Table 8.** Comparison result of factor importance

| Factor importance                                      | Explanation                                      | \( f(x, y) \) | \( f(y, x) \) |
|--------------------------------------------------------|--------------------------------------------------|---------------|---------------|
| \( x \) is as important as \( y \)                    | \( x \) and \( y \) have the same contribution   | 1             | 1             |
| \( x \) is slightly more important than \( y \)       | The contribution of \( x \) is slightly greater  | 3             | 1/3           |
|                                                        | than that of \( y \)                              |               |               |
| \( x \) is significantly more important than \( y \)  | The contribution of \( x \) is significantly     | 5             | 1/5           |
|                                                        | greater than that of \( y \)                     |               |               |
| \( x \) is strongly more important than \( y \)       | The contribution of \( x \) is strongly greater   | 7             | 1/7           |
|                                                        | than that of \( y \)                              |               |               |
| \( x \) is more significant than \( y \)              | The contribution of \( x \) is more significant   | 9             | 1/9           |
|                                                        | than that of \( y \)                              |               |               |
| \( x \) and \( y \) are between the above two adjacent judgments | The median of the above two adjacent judgments | 2,4,6,8       | 1/2,1/4,1/6,1/8 |


The consistency ratio $C.R.$ of judgment matrix $P_{u1}$ was 0.06, less than 0.10. Therefore, the judgment matrix $P_{u1}$ had good consistency, which proved that the weight distribution was reasonable.

3.2.4. The determination of Cost-Effective Sand Mixing Ratio. Table 9 shows the fuzzy comprehensive evaluation results of each sand mixing ratio $\eta_i$. It is noticed that when the sand mixing ratio $\eta$ is 20%, the value of $[B]$ is the largest, which is 0.526. Therefore, sand mixing ratio $\eta$ of 20% is the cost-effective sand mixing ratio.

Table 9. Fuzzy comprehensive evaluation results of different sand mixing ratios

| Sand mixing ratio $\eta$ | 15% | 20% | 25% | 30% |
|--------------------------|-----|-----|-----|-----|
| The value of $[B]$       | 0.505 | 0.526 | 0.517 | 0.501 |

4. Conclusions
In this paper, the high liquid limit soil is improved with different sand mixing ratio, and the cost-effective sand mixing ratio is determined by the help of fuzzy comprehensive evaluation method. The main conclusions are as follows:

1) Compared with untreated soil, the physical and mechanical properties of improved soil are improved greatly, which enables the improved soil to be used as embankment filling. The liquid limit of the improved soil is less than 50%, and the plasticity index is less than 22.

2) Taking improvement cost and improvement effect as evaluation factors, based on the results of fuzzy comprehensive evaluation, the improvement effect and cost of sand-mixed high liquid limit soil reach a comprehensive optimal state when the sand mixing ratio is 20%.

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