Optical mapping of oscillatory stresses in transparent media

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Abstract. The phase-modulated optical signal produced by the light propagation through the stressed transparent media is detected using the non-steady-state photoelectromotive force technique. The mechanical system including the glass plate and piezoelectric transducer demonstrates the resonant behavior in the vicinity of 100 kHz. The measured distribution of the optical phase is a bell-shaped surface for these frequencies. The simulation of the piezooptic response shows the qualitative coincidence with the experimental data and provides the maps of the stress field. The characterization of the $\beta$-Ga$_2$O$_3$ adaptive detector is performed for the light wavelength $\lambda = 532$ nm.

1. Introduction
The detection of phase-modulated optical signals using adaptive detectors is usually carried out using the vibrometric scheme, where the light phase incursion is created by the reflection from an oscillating object [1]. Such an additional phase can obviously appear in the light beam propagating in a stressed transparent material as well. In this work we apply the non-steady-state photo-EMF technique to the detection of phase-modulated optical signals produced by the oscillating stresses in the transparent material.

2. Experimental procedure and results
The experiments with the detection of oscillatory stresses are carried out with the arrangement shown in Fig. 1. One of the beams passes through the transparent glass plate PM-15. The plate is made of glass K8 (BK7 analog) and has the diameter of 30.3 mm and thickness of 15.87 mm. The front and back surfaces have the optical quality. The lateral (cylindric) surface has two diametrically arranged 6.6 mm flats, which were used for clamping. The oscillatory stresses in this plate are created by the piezoelectric transducer. The adaptive detector is made of monoclinic $\beta$-Ga$_2$O$_3$ crystal and has dimensions 2.00×2.15×1.35 mm along crystallographic directions [100], [010] and direction perpendicular to plane (001).

First, we have calibrated our adaptive detector by measuring the dependencies of the signal amplitude versus temporal and spatial frequencies. The frequency transfer function looks like that of differentiating RC-circuit with the cut-off frequency of 13 Hz (for the light intensity $I_0=0.4$ W/cm$^2$). The spatial frequency $K=4.0$ $\mu$m$^{-1}$ chosen for the further experiments with the glass plate is very close to its optimum value maximizing the signal.

The photo-EMF signal produced by the stress oscillations in the glass plate has the frequency dependence with resonant peaks at 93 and 119 kHz. The resonant behavior is associated with the mechanical resonances of the system including the glass plate, piezoelectric transducer and clamp.
The glass plate has the rather complicated geometry, and one can expect that the mechanical stresses, deformations and resulting amplitude of phase modulation non-uniformly distribute across the plate surface. We have measured this distribution (Fig. 1) by moving the clamp with the glass plate and transducer in the directions perpendicular to the light beam. The measurements are performed at 93.5 kHz, i.e. at the first resonant frequency. As seen, the distribution has the nearly radial symmetry with the maximum at the center of the plate.

3. Simulation

The second part of the work contains the simulation of the light phase distribution in the stressed glass plate. The analytic approach for such a problem requires solution of motion equation for elastic medium. Since the cylinder has the two flats, and its height is finite, the analytic approach seems too difficult. Besides, the obtained solution would be valid only for this geometry, which was chosen rather arbitrarily. So, the numerical solution is much more reasonable, and we use the structural mechanics module of COMSOL Multiphysics software to cope with this problem. The geometry of the problem is the same as that used in the experiment. The bottom flat of the cylinder is fixed, the top flat oscillates with amplitude 1.5 nm and frequency 104 kHz. Figure 2 presents the diagonal components of the stress tensor $\sigma$ calculated for the cross section at $z=d/2$, i.e. at half height of the cylinder. The analogous distributions of these values at different $z$ including cylinder faces were calculated. They slightly differ from those presented in Fig. 2, so in the further analysis we can consider stresses $\sigma_{11}$, $\sigma_{22}$ and $\sigma_{33}$ as functions of coordinates $x$ and $y$ but not $z$.

Then we proceed with the calculation of the light phase increment arising via piezooptic effect. These calculations are nearly the same as those for the electrooptic effect, except the tensor of...
Dielectric impermeability is defined in a different way: \( \varepsilon_{ij} = \pi_{ijkl} \sigma_{kl} \), where \( \pi_{ijkl} \) is the tensor of piezooptic coefficients. If the stresses and corresponding phase increments are rather small, then the desired amplitude of phase modulation in the light wave polarized along \( y \)-axis is expressed as

\[
\Delta(x, y) = -\frac{2\pi}{\lambda} n_0 d \left[ \left( \frac{n_0^2 \pi_{12}}{2} + \frac{\nu}{E} \right) \sigma_1(x, y) + \left( \frac{n_0^2 \pi_{11}}{2} + \frac{\nu}{E} \right) \sigma_2(x, y) + \left( \frac{n_0^2 \pi_{12}}{2} - \frac{\nu}{E} \right) \sigma_3(x, y) \right].
\]

where \( n_0 \) is refractive index, \( E \) is the Young's modulus, \( \nu \) is the Poisson's ratio, and reduced notation of tensors is used. After substitution of the calculated stress tensor distributions (Fig. 2) we obtain the map of the light phase amplitude [2]. The calculated distribution of the phase modulation amplitude is a bell-shaped surface, its maximum is located in the vicinity of the plate's center, as it was observed in the experiment (Fig. 1).

Let us estimate the sensitivity of the developed technique. The minimal detectable amplitude of phase modulation is of \( \Delta_{\text{min}} = 1.6 \times 10^{-3} \) rad for the power of the signal beam \( P_s = 1 \) mW, detection bandwidth \( \delta f = 1 \) Hz and load resistor \( R_L = 300 \) k\( \Omega \). Since the piezooptic coefficients \( \pi_{ij} \) are of the same order of magnitude and the Poisson's ratio \( \nu \) is of order of unity, we can roughly estimate the minimal detectable amplitude of the stress oscillation:

\[
\sigma_{\text{min}} \sim \left[ \frac{2\pi}{\lambda} n_0 d \left( \frac{n_0^2 \pi_{ij}}{2} + \frac{\nu}{E} \right) \right]^{-1} \Delta_{\text{min}} = (1 - 2) \times 10^3 \text{ Pa.}
\]

The conventional holographic methods employ the counting of the interference fringes in the pattern. The phase step between adjacent bright and dark fringes is of \( \pi \), and this value is much higher than minimal detectable amplitude of our sensor. The better sensitivity of the adaptive detector mainly results from the narrow detection bandwidth, which is of order of 1 Hz in our setup and can easily be decreased to 0.01 Hz and lower using modern lock-in voltmeters.

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**References**

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