Light neutralino dark matter in $U(1)_X$SSM

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Abstract

The $U(1)_X$ extension of the minimal supersymmetric standard model (MSSM) is called as $U(1)_X$SSM with the local gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$. $U(1)_X$SSM has three singlet Higgs superfields beyond MSSM. In $U(1)_X$SSM, the mass matrix of neutralino is $8 \times 8$, whose lightest mass eigenstate possesses cold dark matter characteristic. Supposing the lightest neutralino as dark matter candidate, we study the relic density. For dark matter scattering off nucleus, the cross sections including spin-independent and spin-dependent are both researched. In our numerical results, some parameter space can satisfy the constraints from the relic density and the experiments of dark matter direct detection.

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I. INTRODUCTION

There are several existences of dark matter in the universe, and dark matter contribution is more important than the visible matter. The earliest and the most compelling evidences for dark matter are the luminous objects that move faster than one expects \[1\]. The other evidences for the dark matter can be found in Refs. \[2, 3\]. Besides the gravitational interaction, dark matter can take part in weak interaction \[4\]. To keep the relic density of dark matter, dark matter should be stable and live long. People have paid much attention to dark matter for many years, but they have not known its mass and interaction property. The non-baryonic matter density is \(\Omega h^2 = 0.1186 \pm 0.0020\) \[5\], and the standard model (SM) can not explain this problem. It implies that there must be new physics beyond the SM. From the present researches, axions, sterile neutrinos, weakly interacting massive particles (WIMPs) \[4, 6\] etc. are dark matter candidates.

Considering the shortcoming of SM, physicists extend it and obtain a lot of extended models. In these new models, the minimal supersymmetric standard model (MSSM) \[7\] is the favorite one, where the lightest neutralino can be dark matter candidate \[8\]. Furthermore, MSSM is also extended by people, and its U(1) extensions are interesting \[9, 10\]. In this work, we extend the MSSM with the \(U(1)_X\) gauge group \[11\]. On the base of MSSM, we add three right-handed neutrinos and three singlet Higgs superfields \(\hat{\eta}, \hat{\bar{\eta}}, \hat{S}\). The right-handed neutrinos not only can give tiny masses to light neutrinos but also produce the lightest scalar neutrino possessing dark matter character. Our \(U(1)_X\) extension of MSSM is called as \(U(1)_X\) SSM \[12, 13\], which relieves the so called little hierarchy problem that is in the MSSM. The baryon number violating operators are avoided because the \(U(1)_X\) gauge symmetry breaks spontaneously. So, the proton is stable.

In \(U(1)_X\) SSM, there are the terms \(\mu \hat{H}_u \hat{H}_d\) and \(\lambda_H \hat{S} \hat{H}_u \hat{H}_d\). \(\hat{S}\) is the singlet Higgs superfield and possesses a non-zero VEV \(v_S/\sqrt{2}\). Therefore, \(U(1)_X\) SSM has an effective \(\mu_{\text{eff}} = \mu + \lambda_H v_S/\sqrt{2}\), which will probably relieve the \(\mu\) problem. As discussed in Ref. \[14\], one can particularly put the \(\mu\) term to zero by a redefinition \(\frac{v_S}{\sqrt{2}} \rightarrow \frac{v_S}{\sqrt{2}} - \frac{\mu}{\lambda_H}\). The singlet \(S\) can improve the lightest CP-even Higgs mass at tree level. Then large loop corrections to the 125 GeV Higgs mass are not necessary. \(S\) can also make the second light neutral CP-even Higgs heavy at TeV order. This can easily satisfy the constraints for heavy Higgs from experiments (such as LHC). If we delete \(S\), the second light neutral CP-even Higgs is light,
whose mass varies from 150 GeV to 400 GeV. Considering the limit for the heavy neutral CP-even Higgs, we should make the second light neutral CP-even Higgs heavier. The singlet $S$ can produce this effect.

In our previous work [13], the lightest CP-even scalar neutrino is supposed as dark matter candidate in the framework of $U(1)_{X}$SSM. Its relic density and the cross section scattering from nucleus have been researched in detail. Some works of scalar neutrino dark matter can be found in Refs. [15–18]. Here, we study the lightest neutralino as dark matter candidate [19].

In the base of the neutralino ($\tilde{\chi}$), wino ($\tilde{W}^0$) and higgsinos ($\tilde{H}^0_d$, $\tilde{H}^0_u$), the neutralino mass eigenstates in the MSSM has the parameters $\tan \beta$, $M_1$, $M_2$ and $\mu$. The lower limit on the lightest neutralino $\chi^0_1$ mass is about 46 GeV, which can be derived from the Large Electron Positron (LEP) chargino mass limit [20]. While, this limit increases to well above 100 GeV, in the constrained MSSM (cMSSM) [21]. In pMSSM, the authors research the lightest neutralino below 50 GeV satisfying the constraints from LHC and XENON100 [22].

Many people have studied the phenomenology of lightest neutralino in MSSM [8] and there are a lot of works of neutralino dark matter in several models. They enrich the dark matter research and give light to the direct research of dark matter.

After this introduction, some content of $U(1)_{X}$SSM is introduced in section II. In section III, we suppose the lightest neutralino as a dark matter candidate and we study its relic density. The direct detection of the lightest neutralino scattering off nuclei is researched in section IV, which includes both the spin-independent cross section and spin-dependent cross section. The numerical results of the relic density and cross sections of dark matter scattering are all calculated in section V. We give our discussion and conclusion in section VI. Some formulae are collected in the appendix.

II. THE $U(1)_X$SSM

Extending the local gauge group from $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ and adding three Higgs singlets $\hat{\eta}$, $\hat{\bar{\eta}}$, $\hat{S}$, right-handed neutrinos $\hat{\nu}$ to MSSM, one can obtain $U(1)_X$SSM [13]. The introduction of right-handed neutrinos can explain the neutrino experiments. The mass squared matrix of CP-even Higgs is $5 \times 5$, because the CP-even parts of $\eta$, $\hat{\eta}$, $S$ mix with the neutral CP-even parts of $H_u$, $H_d$. We take into account the one loop corrections for the lightest CP-even Higgs with 125 GeV. The condition is
similar for the CP-odd Higgs, whose mass squared matrix is also 5 $\times$ 5. The sneutrinos are departed into CP-even sneutrinos and CP-odd sneutrinos, whose mass squared matrixes are both 6 $\times$ 6. Here, we show the $U(1)_X$ charges of the MSSM superfields: $\hat{Q}_i(0)$, $\hat{u}_i^c(-\frac{1}{2})$, $\hat{d}_i^c(\frac{1}{2})$, $\hat{L}_i(0)$, $\hat{e}_i^c(\frac{1}{2})$, $\hat{H}_u(\frac{1}{2})$, $\hat{H}_d(-\frac{1}{2})$. In table I, the superfields beyond MSSM are collected in detail.

The concrete forms of the Higgs superfields are

$$H_u = \left( \begin{array}{c} H_u^+ \\ \frac{1}{\sqrt{2}}(v_u + H_u^0 + iP_u^0) \end{array} \right), \quad H_d = \left( \begin{array}{c} \frac{1}{\sqrt{2}}(v_d + H_d^0 + iP_d^0) \\ H_d^- \end{array} \right),$$

$$\eta = \frac{1}{\sqrt{2}}(v_\eta + \phi_\eta^0 + iP_\eta^0), \quad \bar{\eta} = \frac{1}{\sqrt{2}}(v_\bar{\eta} + \phi_{\bar{\eta}}^0 + iP_{\bar{\eta}}^0),$$

$$S = \frac{1}{\sqrt{2}}(v_S + \phi_S^0 + iP_S^0).$$ (1)

$v_u$ and $v_d$ are the VEVs of the Higgs doublets $H_u$ and $H_d$. While, $v_\eta$, $v_{\bar{\eta}}$ and $v_S$ are the VEVs of the Higgs singlets $\eta$, $\bar{\eta}$ and $S$. The angles $\beta$ and $\beta_\eta$ are defined as $\tan \beta = v_u/v_d$ and $\tan \beta_\eta = v_\eta/v_{\bar{\eta}}$.

The sneutrino fields $\tilde{\nu}_L$ and $\tilde{\nu}_R$ read as

$$\tilde{\nu}_L = \frac{1}{\sqrt{2}}(\phi_L + i\sigma_L), \quad \tilde{\nu}_R = \frac{1}{\sqrt{2}}(\phi_R + i\sigma_R).$$ (2)

We show the superpotential and the soft breaking terms in $U(1)_X$SSM

$$W = l_W^T \hat{S} + \mu \hat{H}_u \hat{H}_d + M_S \hat{S} \hat{S} - Y_d \hat{d} \hat{q} \hat{H}_d - Y_e \hat{e} \hat{l} \hat{H}_d + \lambda_H \hat{S} \hat{H}_u \hat{H}_d$$

$$+ \lambda_C \hat{S} \hat{\eta} \hat{\bar{\eta}} + \frac{\kappa}{3} \hat{S} \hat{S} \hat{S} + Y_a \hat{u} \hat{q} \hat{H}_u + Y_X \tilde{\nu} \tilde{\nu} + Y_\nu \tilde{\nu} \hat{H}_u + \mu_\eta \tilde{\eta} \tilde{\bar{\eta}}.$$

$$L_{soft} = L_{soft}^{\text{MSSM}} - B_S S^2 - L_S S - \frac{T_X}{3} S^3 - T_{\lambda_C} S \eta \bar{\eta} + \epsilon_{ij} T_{\lambda_H} S H_{ij}^\dagger H_{ij}$$

$$- T_{\lambda_C} \tilde{\eta} \tilde{\nu} \tilde{\nu} + \epsilon_{ij} T_{\lambda'_{ij}} H_{ij}^\dagger \tilde{\nu} + \epsilon_{ij} T_{\lambda'_{ij}} H_{ij}^\dagger \tilde{\nu} + m_{\eta}^2 |\tilde{\eta}|^2 - m_{\bar{\eta}}^2 |\tilde{\bar{\eta}}|^2 - m_S^2 S^2$$

$$- (m_{\tilde{\nu}_R}^2)^{T_{\lambda'_{ij}}} \tilde{\nu}^\dagger \tilde{\nu} - \frac{1}{2} \left( M_X \lambda_X^2 + 2 M_{BB'} \lambda_B \lambda_{\tilde{\nu}} \right) + h.c.$$ (3)
Here, $\mathcal{L}_{\text{soft}}^{\text{MSSM}}$ represents the soft breaking terms of MSSM. Obviously, the $U(1)_X$SSM is more complicated than the MSSM. In our previous work, $Y^Y$ represents the $U(1)_Y$ charge and $Y^X$ denotes the $U(1)_X$ charge. We have proven that $U(1)_X$SSM is anomaly free, and the details can be found in Ref. [13]. In $U(1)_X$SSM, there are two Abelian groups $U(1)_Y$ and $U(1)_X$, which cause the gauge kinetic mixing. This effect is the characteristic beyond MSSM and it can also be induced through RGEs.

We write the covariant derivatives of $U(1)_X$SSM in the general form

$$D_\mu = \partial_\mu - i \left( Y^Y, Y^X \right) \left( g_Y, g'_{YX} \right) \left( \begin{array}{c} A_{\mu}^Y \\ A_{\mu}^X \end{array} \right),$$

(4)

with $A_{\mu}^Y$ ($A_{\mu}^X$) denotes the gauge field of $U(1)_Y$ ($U(1)_X$). Considering the fact that the two Abelian gauge groups are unbroken, we change the basis through a correct matrix $R$ and redefine the $U(1)$ gauge fields

$$\left( \begin{array}{c} g_Y, g'_{YX} \\ g'_{XY}, g'_X \end{array} \right) R^T = \left( \begin{array}{c} g_1, g_{YX} \\ 0, g_X \end{array} \right), \quad R \left( \begin{array}{c} A_{\mu}^Y \\ A_{\mu}^X \end{array} \right) = \left( \begin{array}{c} A_{\mu}^Y \\ A_{\mu}^X \end{array} \right).$$

(5)

Different from MSSM, the $U(1)_X$SSM gauge bosons $A_{\mu}^X$, $A_{\mu}^Y$ and $V^3_\mu$ mix together at the tree level. In the basis $(A_{\mu}^Y, V^3_\mu, A_{\mu}^X)$, the corresponding mass matrix reads as

$$\left( \begin{array}{ccc} 1 & 1 & \frac{1}{8}g_1g_{YX}v^2 \\ 1 & 1 & \frac{1}{8}g_1g_{YX}v^2 \\ \frac{1}{8}g_1g_{YX}v^2 & \frac{1}{8}g_2g_{YX}v^2 & \frac{1}{8}g_2g_{YX}v^2 \end{array} \right),$$

(6)

with $v^2 = v_u^2 + v_d^2$ and $\xi^2 = v_\eta^2 + v_\bar{\eta}^2$. One can diagonalize the above mass matrix by an unitary matrix including two mixing angles $\theta_W$ and $\theta'_W$. $\theta_W$ is Weinberg angle and $\theta'_W$ is defined as

$$\sin^2 \theta'_W = \frac{1}{2} - \frac{(g_{YX}^2 - g_1^2 - g_2^2)v^2 + 4g_2^4\xi^2}{2\sqrt{(g_{YX}^2 + g_1^2 + g_2^2)v^4 + 8g_2^4(g_{YX}^2 - g_1^2 - g_2^2)v^2\xi^2 + 16g_2^4\xi^4}}.$$  

(7)

The lightest neutralino is supposed as dark matter candidate, and we obtain the mass matrix of neutralino in the basis $(\lambda_{\tilde{B}}, \tilde{W}^0, \tilde{H}_d, \tilde{H}_u, \lambda_{\tilde{X}}, \tilde{\eta}, \tilde{\bar{\eta}}, \tilde{s})$. This is caused by the super partners of the added three Higgs singlets and new gauge boson, which mix with the MSSM neutralino superfields.

$$M_{\chi^0} = \begin{pmatrix} A & B \\ B^T & C \end{pmatrix}.$$ 

(8)
The concrete forms of $A$, $B$ and $C$ are

$$A = \begin{pmatrix} M_1 & 0 & -\frac{g_1}{2}v_d & \frac{g_1}{2}v_u \\ 0 & M_2 & \frac{1}{2}g_2v_d & -\frac{1}{2}g_2v_u \\ -\frac{g_1}{2}v_d & \frac{1}{2}g_2v_d & 0 & m\tilde{H}_d\tilde{H}_u^0 \\ \frac{g_1}{2}v_u & -\frac{1}{2}g_2v_u & m\tilde{H}_u\tilde{H}_d^0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} M_{BB'} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ m\lambda S\tilde{H}_d^0 & 0 & 0 & -\frac{\lambda_\mu v_u}{\sqrt{2}} \\ m\lambda S\tilde{H}_u^0 & 0 & 0 & -\frac{\lambda_\mu v_d}{\sqrt{2}} \end{pmatrix}, \quad (9)$$

$$C = \begin{pmatrix} M_{BL} & -g_Xv_\eta & g_Xv_\eta & 0 \\ -g_Xv_\eta & 0 & \frac{1}{\sqrt{2}}\lambda_Cv_S + \mu_\eta & \frac{1}{\sqrt{2}}\lambda_Cv_\eta \\ g_Xv_\eta & \frac{1}{\sqrt{2}}\lambda_Cv_S + \mu_\eta & 0 & \frac{1}{\sqrt{2}}\lambda_Cv_\eta \\ 0 & \frac{1}{\sqrt{2}}\lambda_Cv_S + \mu_\eta & \frac{1}{\sqrt{2}}\lambda_Cv_\eta & m_{\tilde{s}\tilde{s}} \end{pmatrix}, \quad (10)$$

$$m\tilde{H}_d\tilde{H}_u^0 = -\frac{1}{\sqrt{2}}\lambda_Hv_u - \mu, \quad m\tilde{H}_u\lambda_S^0 = -\frac{1}{2}(g_YX + g_X)v_d,$$

$$m\tilde{H}_u\lambda_S^0 = \frac{1}{2}(g_YX + g_X)v_u, \quad m_{\tilde{s}\tilde{s}} = 2M_S + \sqrt{2}\kappa v_S. \quad (11)$$

This matrix is diagonalized by $N$

$$N^*M_{\tilde{\chi}^0}N^T = M_{\tilde{\chi}^0}^{diag}. \quad (12)$$

It is too difficult to obtain exactly the analytic forms of the eigenvalues, eigenvectors and $N$ for $M_{\tilde{\chi}^0}$. With some supposition, we can deduce the lightest neutralino mass and eigenvector approximately. Comparing with $A$ and $C$, the matrix $B$ is very small. In this condition, we can use $Z_N^T$ to simplify $M_{\tilde{\chi}^0}$ with matrix $\zeta$, whose elements are all small parameters of the order $B/A$.

$$Z_N^T = \begin{pmatrix} 1 - \frac{1}{2}\zeta^T\zeta & -\zeta^T \\ \zeta & 1 - \frac{1}{2}\zeta^T\zeta \end{pmatrix}, \quad Z_N^T M_{\tilde{\chi}^0} Z_N = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix},$$

$$\kappa_1 = A - \frac{1}{2}(\zeta^TA + A\zeta^T\zeta) - \zeta^TB^T - B\zeta + \zeta^TC\zeta,$$

$$\kappa_2 = \zeta^TA^T + B^T\zeta^T + \zeta^B + C - \frac{1}{2}(\zeta^TC + C\zeta^T). \quad (13)$$

$\zeta^T$ can be calculated from the equation $A\zeta^T + B - \zeta^TC = 0$. If we take the simplest approximation, it is

$$Z_N^T M_{\tilde{\chi}^0} Z_N \sim \begin{pmatrix} A & 0 \\ 0 & C \end{pmatrix}. \quad (14)$$
In this work, we suppose the lightest neutralino $m_{\chi_0^0}$ is different from the MSSM condition, that is to say $m_{\chi_0^0}$ dominantly comes from the matrix $\mathcal{C}$ and MSSM neutralinos in the matrix $\mathcal{A}$ are heavy. Therefore, we calculate the mass eigenstates of $\mathcal{C}$, which is tedious to solve the common quartic equation with one unknown quantity. Considering the constraint that $\tan \beta_\eta$ is near 1, we use $s_m = v_\eta - v_\eta$ with the relation $v_\eta \gg s_m$. So, $\mathcal{C}$ turns to

$$
\mathcal{C} = \begin{pmatrix}
M_{BL} & -g_X v_\eta & g_X v_\eta + g_X s_m & 0 \\
-g_X v_\eta & 0 & \lambda_C v_S + \mu_\eta & \lambda_C v_\eta + \lambda_C s_m \\
g_X v_\eta + g_X s_m & \lambda_C v_S + \mu_\eta & 0 & \lambda_C v_\eta \\
0 & \lambda_C v_\eta + \lambda_C s_m & \lambda_C v_\eta & m_{\tilde{\tau}}
\end{pmatrix},
$$

with $\lambda_C = \lambda_C / \sqrt{2}$.

The eigenvalues of $\mathcal{C}$ are deduced to the leading order according to the small parameter $s_m$.

$$
m_{\chi_0^0}^{(0)} = \frac{1}{2} \left( M_{BL} - \lambda_C v_S - \mu_\eta + \sqrt{8g_X^2 v_\eta^2 + (\lambda_C v_S + \mu_\eta + M_{BL})^2} \right),
$$

$$
m_{\chi_0^0}^{(0)} = \frac{1}{2} \left( -M_{BL} + \lambda_C v_S + \mu_\eta + \sqrt{8g_X^2 v_\eta^2 + (\lambda_C v_S + \mu_\eta + M_{BL})^2} \right),
$$

$$
m_{\chi_0^0}^{(0)} = \frac{1}{2} \left( \lambda_C v_S + \mu_\eta + m_{\tilde{\tau}} + \sqrt{8(\lambda_C^2 v_S^2 + (m_{\tilde{\tau}} - \lambda_C v_S - \mu_\eta)^2) \right),
$$

$$
m_{\chi_0^0}^{(0)} = \frac{1}{2} \left( \lambda_C v_S + \mu_\eta + m_{\tilde{\tau}} - \sqrt{8(\lambda_C^2 v_S^2 + (m_{\tilde{\tau}} - \lambda_C v_S - \mu_\eta)^2) \right).$$

We take $M_{BL}$ and $m_{\tilde{\tau}}$ are both positive parameters. To satisfy the constraint from $m_{Z'}$, $v_\eta$ is large and bigger than $v_S$. So it is easy to see that $m_{\chi_0^0}$ and $m_{\chi_0^0}$ are large values. Using $m_{\tilde{\tau}} \gg M_{BL}$ and $\mu_\eta$, $m_{\chi_0^0}$ is smaller than $m_{\chi_0^0}$, so $m_{\chi_0^0}$ is the lightest neutralino mass $m_{\chi_0^0}$.

For the lightest neutralino mass, we consider the correction at the order $s_m$.

$$
m_{\chi_0^0}^{(1)} = -\frac{2(\lambda_C^2 s_m v_\eta)}{\sqrt{8(\lambda_C^2 v_S^2 + (m_{\tilde{\tau}} - \lambda_C v_S - \mu_\eta)^2}}.
$$

At the leading order, the eigenvector of $m_{\chi_0^0}$ is

$$
V_{\chi_0^0}^{(0)} = \frac{1}{\sqrt{2 + a_0^2}} (0, 1, 1, a_0) \sim \left(0, \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right),
$$

$$
a_0 = \frac{4(\lambda_C^2 v_\eta)}{\lambda_C v_S + \mu_\eta - m_{\tilde{\tau}} - \sqrt{8(\lambda_C^2 v_S^2 + (m_{\tilde{\tau}} - \lambda_C v_S - \mu_\eta)^2}}.
$$

At the leading order, the eigenvector of $m_{\chi_0^0}$ is

$$
V_{\chi_0^0}^{(0)} = \frac{1}{\sqrt{2 + a_0^2}} (0, 1, 1, a_0) \sim \left(0, \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right),
$$

$$
a_0 = \frac{4(\lambda_C^2 v_\eta)}{\lambda_C v_S + \mu_\eta - m_{\tilde{\tau}} - \sqrt{8(\lambda_C^2 v_S^2 + (m_{\tilde{\tau}} - \lambda_C v_S - \mu_\eta)^2}}.
$$

At the leading order, the eigenvector of $m_{\chi_0^0}$ is

$$
V_{\chi_0^0}^{(0)} = \frac{1}{\sqrt{2 + a_0^2}} (0, 1, 1, a_0) \sim \left(0, \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right),
$$

$$
a_0 = \frac{4(\lambda_C^2 v_\eta)}{\lambda_C v_S + \mu_\eta - m_{\tilde{\tau}} - \sqrt{8(\lambda_C^2 v_S^2 + (m_{\tilde{\tau}} - \lambda_C v_S - \mu_\eta)^2}}.
$$
Here, $a_0$ is a small parameter. The $s_m$ correction to eigenvector $V^{(0)}_{\chi_d^0}$ is $V^{(1)}_{\chi_d^0}$
\[
V^{(1)}_{\chi_d^0} = \frac{s_m}{v_\eta} \sqrt{b_1^2 + c_1^2} \left(0, 0, b_1, c_1\right),
\]
\[
b_1 = \sqrt{8(\lambda_C')^2v_\eta^2 + m_{\tilde{g}}^2 + m_{\tilde{d}}^2}, \quad c_1 = \frac{1}{2} \left(\frac{m_{\tilde{d}}}{\sqrt{8(\lambda_C')^2v_\eta^2 + m_{\tilde{d}}^2}} - 1\right). \quad (19)
\]
$c_1$ is much smaller than $b_1$, then $V^{(0)}_{\chi_d^0}$ can be simplified as
\[
V^{(1)}_{\chi_d^0} \sim \frac{s_m}{v_\eta} \left(0, 0, 1, 0\right). \quad (20)
\]
In the whole, the eigenvector of the lightest neutralino is dominantly composed by the linear combination of $\tilde{\eta}$ and $\tilde{\bar{\eta}}$.

In the MSSM, the lightest CP-even Higgs mass at tree level is no more than 90 GeV, and the loop corrections to the lightest CP-even Higgs mass can be large. Including the leading-log radiative corrections from stop and top particles, we write the mass of the lightest CP-even Higgs boson in the following form
\[
m_h = \sqrt{(m_{h_1}^0)^2 + \Delta m_h^2}. \quad (21)
\]
Here, $m_{h_1}^0$ represents the lightest Higgs boson mass at tree level and $\Delta m_h^2$ is shown analytically
\[
\Delta m_h^2 = \frac{3m_t^4}{16\pi^2} \left[\tilde{t} + \frac{1}{2} + \tilde{X}_t\right] + \frac{1}{16\pi^2} \left(\frac{3m_t^2}{2v^2} - 32\pi\alpha_3\right) \left(t^2 + \tilde{X}_t\tilde{t}\right),
\]
\[
\tilde{t} = \log \frac{M_{\tilde{T}}^2}{m_t^2}, \quad \tilde{X}_t = \frac{2\tilde{A}_t^2}{M_{\tilde{T}}^2} \left(1 - \frac{\tilde{A}_t^2}{12M_{\tilde{T}}^2}\right).
\]
with $\alpha_3$ denoting the strong coupling constant. The parameter $\tilde{A}_t$ is $\tilde{A}_t = A_t - \mu \cot \beta$ with $A_t$ representing the trilinear Higgs stop coupling. $M_{\tilde{T}} = \sqrt{m_{\tilde{t}1}m_{\tilde{t}2}}$ and $m_{\tilde{t}1,2}$ are the stop masses. To save space in the text, other used couplings are collected in the appendix.

### III. RELIC DENSITY

Supposing the lightest neutralino($\chi_1^0$) as dark matter candidate, we calculate the relic density. The constraint of dark matter relic density is severe, and the concrete value is $\Omega_D h^2 = 0.1186 \pm 0.0020$ [5]. The $\chi_1^0$ number density $n_{\chi_1^0}$ should satisfy the Boltzmann equation
\[
\frac{dn_{\chi_1^0}}{dt} = -3H n_{\chi_1^0} - \langle \sigma v \rangle_{SA} (n_{\chi_1^0}^2 - n_{\chi_1^0 eq}^2) - \langle \sigma v \rangle_{CA} (n_{\chi_1^0 n_{\phi} - n_{\chi_1^0 eq} n_{\phi eq}}). \quad (23)
\]
For $\chi_1^0$, we take into account self-annihilation and co-annihilation with another particle $\phi$. At the temperature $T_F$, the annihilation rate of $\chi_1^0$ is approximately equal to the Hubble expansion rate, and the lightest neutralino freezes out. We suppose $\chi_1^0$ is the lightest SUSY particle and $m_\phi$ is larger than $m_{\chi_1^0}$. The relevant formulae are

$$
\langle \sigma v \rangle_{SA} n_{\chi_1^0} + \langle \sigma v \rangle_{CA} n_\phi \sim H(T_F),
$$

$$
n_\phi = \left( \frac{m_\phi}{m_{\chi_1^0}} \right)^{3/2} \text{Exp} \left( \frac{m_{\chi_1^0} - m_\phi}{T} \right) n_{\chi_1^0}
$$

$$
\left[ \langle \sigma v \rangle_{SA} + \langle \sigma v \rangle_{CA} \left( \frac{m_\phi}{m_{\chi_1^0}} \right)^{3/2} \text{Exp} \left( \frac{m_{\chi_1^0} - m_\phi}{T} \right) \right] n_{\chi_1^0} \sim H(T_F). \tag{24}
$$

After we study the self-annihilation cross section $\sigma(\chi_1^0 \chi_1^0 \rightarrow \text{anything})$ and co-annihilation cross section $\sigma(\chi_1^0 \phi \rightarrow \text{anything})$, $\langle \sigma v \rangle_{SA}$ and $\langle \sigma v \rangle_{CA}$ are gotten. The annihilation results can be written as $\sigma v_{rel} = a + b v_{rel}^2$ in the mass center frame. Here, $v_{rel}$ is the relative velocity of the two particles in the initial states. Using the following formula, we can approximately calculate the freeze-out temperature ($T_F$)

$$
x_F = \frac{m_{\chi_1^0}}{T_F} \simeq \ln \left[ \frac{0.076 M_{Pl} m_{\chi_1^0} (a + 6b/x_F)}{\sqrt{g_* x_F}} \right], \tag{25}
$$

with $M_{Pl}$ denoting the Planck mass.

The relativistic degrees of freedom with mass less than $T_F$ is represented by $g_*$. The cold non-baryonic dark matter density is simplified in the following form

$$
\Omega_D h^2 \simeq \frac{1.07 \times 10^9 x_F}{\sqrt{g_* M_{Pl} (a + 3b/x_F)}} \text{ GeV}. \tag{26}
$$

It is well known that, the self-annihilation processes are dominant in general condition. We show the researched concrete self-annihilation processes: $\chi_1^0 + \chi_1^0 \rightarrow A + B$, A and B represent final states $(Z, h)$, $(W, W)$, $(Z, Z)$, $(h, h)$, $(\bar{u}, u)$, $(\bar{d}, d)$, $(\bar{l}, l)$, $(\bar{\nu}, \nu)$. Here $i = 1, 2, 3$ and $h$ represents the lightest CP-even Higgs. The neutrinos in final state are just three light neutrinos not including heavy neutrinos.

For co-annihilation processes, if the mass of another particle is almost equal to the mass of $\chi_1^0$, they give considerable contributions to the annihilation cross section.

a. The lightest neutralino $\chi_1^0$ annihilates with heavier neutralinos $\chi_1^0 (k = 2 \ldots 8)$, whose final states are same as those produced by self-annihilation processes.

b. $\chi_1^0 + \chi^- \rightarrow \{ (\nu, l^-), (\bar{u}, d), (W^-, Z), (W^-, \gamma), (W^-, h^0) \}$. The corresponding processes are obtained by the charge conjugate transformation.
c. $\chi_1^0 + \bar{L}^- \to \{(\gamma, l^-), (Z, l^-), (h^0, l^-)\}$. Similar as the condition b, condition c also has charge conjugate processes.

d. $\chi_1^0 + \bar{\nu}^R(\bar{\nu}^I) \to \{(\nu, Z), (l^-, W^+), (l^+, W^-)\}$.

The co-annihilation between $\chi_0^1$ and scalar quarks($\bar{U}, \bar{D}$) are neglected, because scalar quark masses are very heavy and much larger than $m_{\chi_0^1}$.

IV. DIRECT DETECTION

The experiment constraints for the direct detection of dark matter become strict more and more. The lightest neutralino scatters off nucleus, and the process is $\chi_1^0 + q \to \chi_1^0 + q$. The exchanged particles can be CP-even Higgs $H_0^j$, CP-odd Higgs $A_0^j$, gauge bosons $Z, Z'$. For the CP-odd Higgs $A_0^j$ contribution, there are two suppression factors: 1. The Yukawa coupling $Y_q$ of light quark; 2. The operators $\bar{\chi}_0^1 \chi_0^1 \bar{q}q$ and $\bar{\chi}_0^1 \gamma^\mu \gamma^5 \chi_0^1 \bar{q}q$ are suppressed by the factors $q^2$ and $q^4$ respectively[29]. Therefore, we neglect the CP-odd Higgs contribution. Because neutralino is Majorana particle, the operator $\bar{\chi}_0^1 \gamma^\mu \gamma^5 \chi_0^1 \bar{q}q$ disappears. The dominant operators at quark level are $\bar{\chi}_0^1 \chi_0^1 \bar{q}q$ and $\bar{\chi}_0^1 \gamma^\mu \gamma^5 \chi_0^1 \bar{q}q$ obtained from CP-even Higgs and vector bosons $Z, Z'$ contributions[29].

The quark level operators should be converted to the effective nucleus operators. To convert the operator $\bar{\chi}_0^1 \chi_0^1 \bar{q}q$, we use the following formulae[29]

$$a_q m_q \bar{q}q \to f_N m_N \bar{N}N,$$

$$f_N = \sum_{q=u,d,s} f_T^{(N)}(q) a_q + \frac{2}{27} f_T^{(G)} \sum_{q=c,b,t} a_q, \quad f_T^{(N)} = 1 - \sum_{q=u,d,s} f_T^{(N)}(q). \quad (27)$$

Integrating out heavy quark loops, the coupling to gluons is induced, which is included in $f_N$. We show the concrete values of the parameters $f_T^{(N)}$[30],

$$f_T^{(p)} = 0.0153, \quad f_T^{(n)} = 0.0191, \quad f_T^{(n)} = 0.0447,$$

$$f_T^{(p)} = 0.0110, \quad f_T^{(n)} = 0.0273, \quad f_T^{(n)} = 0.0447. \quad (28)$$

$\bar{\chi}_0^1 \gamma^\mu \gamma^5 \chi_0^1 \bar{q}q$ is a spin-dependent operator, which is converted to the effective nucleus operator with the following formulae.

$$d_q \bar{q} \gamma^\mu \gamma^5 q \to a_N \bar{N} s_{\mu}^{(N)} N, \quad \langle N | \bar{q} \gamma^\mu \gamma^5 q | N \rangle = s_{\mu}^{(N)} \Delta q^{(N)}, \quad a_N = \sum_{u,d,s} d_q \Delta q^{(N)}. \quad (29)$$
with $s^{(N)}_\mu$ denoting the spin of nucleus. In the numerical calculation, we use the parameters of DarkSUSY

$$\Delta u^{(p)} = \Delta d^{(n)} = 0.77, \quad \Delta d^{(p)} = \Delta u^{(n)} = -0.47, \quad \Delta s^{(p)} = \Delta s^{(p)} = -0.15.$$  

(30)

For the spin-independent operator $\bar{\chi}_0^0 \chi_0^0 \bar{q}q$, the scattering cross section reads as

$$\sigma = \frac{1}{\pi} \hat{\mu}^2 [Z_p f_p + (A - Z_p) f_n]^2,$$  

(31)

with $Z_p$ denoting the number of proton, and $A$ representing the number of atom.

The scattering cross section for the spin-dependent operator $\bar{\chi}_1^0 \gamma_\mu \gamma_5 \chi_0^0 \bar{q}q \gamma^\mu \gamma_5 q$ is shown as

$$\sigma = \frac{16}{\pi} \hat{\mu}^2 a^2 N J_N (J_N + 1),$$  

(32)

with $J_N$ is the number of angular momentum for the nucleus. The corresponding formula for one nucleon is

$$\sigma = \frac{12}{\pi} \hat{\mu}^2 a^2 N.$$  

(33)

V. NUMERICAL RESULTS

To study the numerical results, we should take into account the experimental constraints. One strict constraint from experiment is the mass(125 GeV) of the lightest CP-even Higgs. $Z'$ boson mass constraint is also important. The mass bounds for $M_{Z'}$ from LHC are more severe than the limits from the low energy data. In the Sequential Standard Model, the lower mass limit of $Z'_{SSM}$ is 4.5 TeV at 95% confidence level(CL). The Lower mass limits of the $Z'$ boson in the left-right symmetric model and the (B-L) model are respectively 4.1 TeV and 4.2 TeV. The upper bound on the ratio between $M_{Z'}$ and its gauge coupling is $M_{Z'}/g_X \geq 6$ TeV at 99% CL. Considering the LHC experimental data, tan $\beta_\eta$ should be smaller than 1.5. We take into account the above constraints and choose the parameters to satisfy the relation $M_{Z'} > 4.5$ TeV.

Here, we also add other experiment limits. The considered mass limits for the particles beyond SM are: 1 the mass limits for heavy neutral Higgs ($H^0, A^0$) and charged Higgs ($H^\pm$); 2 the mass limits for neutralino, chargino, sneutrino, scalar charged lepton, squark. The decays of the lightest CP even Higgs ($m_{h^0} = 125$ GeV) such as $h^0 \rightarrow \gamma + \gamma, h^0 \rightarrow Z + Z$...
and $h^0 \to W + W$ are considered. The constraint from $B \to X_s + \gamma$ is also taken into account. With new experiment data of muon g-2 from the Fermion National Accelerator Laboratory (FNAL) [36], the deviation between experiment and SM prediction is $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 251(59) \times 10^{-11}$ and increases to $4.2\sigma$. We study muon g-2 in $U(1)_X SSM$ in the previous work [37], and consider this limit here.

Therefore, we use the following parameters

$$M_S = 2.7 \text{ TeV}, \ T_\kappa = 1.6 \text{ TeV}, \ g_{YX} = 0.2, \ g_X = 0.3, \ \lambda_C = -0.08, \ \lambda_H = 0.1,$$

$$v_\eta = 15.5 \times \cos \beta_\eta \text{ TeV}, \ v_\bar{\eta} = 15.5 \times \sin \beta_\eta \text{ TeV}, \ Y_{X11} = Y_{X22} = 0.5, \ Y_{X33} = 0.4,$$

$$T_{\lambda_H} = 0.3 \text{ TeV}, \ T_{X11} = T_{X22} = T_{X33} = -1 \text{ TeV}, \ T_{e11} = T_{e22} = T_{e33} = -3 \text{ TeV},$$

$$T_{\lambda_C} = -0.1 \text{ TeV}, \mu_\eta = 10 \text{ GeV}, \ M_{U11}^2 = M_{U22}^2 = 10 \text{ TeV}^2, \ M_{Q33}^2 = M_{U33}^2 = 3.5 \text{ TeV}^2,$$

$$l_W = 4 \text{ TeV}^2, \ M_{e11}^2 = M_{e22}^2 = M_{e33}^2 = 0.5 \text{ TeV}^2, \ T_{u11} = T_{u22} = T_{u33} = -2 \text{ TeV}, \ \kappa = 1,$$

$$T_{d12} = T_{d21} = 0.2 \text{ TeV}, \ M_{L11}^2 = M_{L22}^2 = M_{L33}^2 = 1.9 \text{ TeV}^2, \ B_\mu = B_S = m_S^2 = 1 \text{ TeV}^2,$$

$$M_{E11}^2 = M_{E22}^2 = M_{E33}^2 = 3.54 \text{ TeV}^2, \ \tan \beta_\eta = 0.8, \ T_{\nu11} = T_{\nu22} = T_{\nu33} = 0.5 \text{ TeV}. \hspace{1cm} (34)$$

To simplify the numerical discussion, most of the parameters $T_{\nu}, \ T_{X}, \ T_u$ etc. are supposed as diagonal matrices and we use the supposition

$$M_{D11}^2 = M_{D22}^2 = M_{D33}^2 = M_D^2, \hspace{1cm} T_{d11} = T_{d22} = T_{d33} = T_d. \hspace{1cm} (35)$$

A. The relic density of neutralino dark matter

With the supposition that the lightest neutralino $\chi^0_1$ is the lightest SUSY particle (LSP), we research the relic density of $\chi^0_1$. In this subsection, we adopt the parameters as $M_{Q11}^2 = M_{Q22}^2 = M_D^2 = 10 \text{ TeV}^2$ and $T_d = 1 \text{ TeV}$.

$M_1$ is the mass of $U(1)_X$ gaugino and appears in the neutralino mass matrix. Therefore, $M_1$ can affect neutralino masses and mixing to some extent. In the Figure [40] we plot the relic density in the banded gray area with $\pm 3\sigma$ sensitivity. The relic density versus $M_1$ is represented by solid line ($M_2 = 1 \text{ TeV}$) and dotted line ($M_2 = 2 \text{ TeV}$) with the parameters $M_{BL} = 1 \text{ TeV}$, $\tan \beta = 9$, $\mu = 0.5 \text{ TeV}$ and $M_{BB} = 0.4 \text{ TeV}$. It is obvious that both the solid line and the dotted line versus $M_1$ vary weakly in the region $[400, 1800] \text{ GeV}$. The both lines are in the $\pm 3\sigma$ band. At the point $M_1 = 1200 \text{ GeV}$, the relic density is very near its central value, which can well satisfy the experiment constraint. Generally speaking,
the two lines are very near. With the used parameters, the mass of the lightest neutralino $\chi_1^0$ is around 302 GeV, and the other SUSY particles are all much heavier than $\chi_1^0$. So, the self-annihilation processes are dominant. That is to say, the contributions from the co-annihilation processes are tiny. Because the masses of exchanged virtual particles are not near $2 \cdot m_{\chi_1^0}$, the resonance annihilation affecting the relic density strongly can not take place.

In this parameter space, the masses of some SUSY particles that can co-annihilate with the lightest neutralino are collected here: the second light neutralino mass $m_{\chi_2^0} \sim 800$ GeV, the lightest scalar neutrino mass (CP-even and CP-odd) $m_\nu \sim 1600$ GeV, the lightest scalar lepton mass $m_{L_1} \sim 880$ GeV, the lightest chargino mass $m_{\chi_1^+} \sim 780$ GeV, the lightest scalar quark mass $m_{\tilde{q}_1} \sim 1800$ GeV.

![Graph](image)

**FIG. 1:** The relic density $\Omega_D h^2$ versus $M_1$ is plotted by the solid line (dotted line) with $M_2 = 1(2)$ TeV.

$M_{BB'}$ is the mass of the $U(1)_Y$ and $U(1)_X$ gaugino mixing and presents in the mass matrix of neutralino, which can affect the relic density through the mixing matrix. Here, we use the parameters as $M_1 = 1.2$ TeV, $M_2 = 1$ TeV, $M_{BL} = 1$ TeV, $\tan \beta = 9$. In the Fig 2, the numerical results of the relic density versus $M_{BB'}$ are shown by the solid line ($\mu = 0.5$ TeV) and dotted line ($\mu = 0.4$ TeV) respectively. The solid line is above the dotted line. When $M_{BB'}$ is near zero, $\Omega_D h^2$ can not satisfy the experimental constraint. The numerical results of $\Omega_D h^2$ corresponding to $M_{BB'}$ regions [-500, -300] GeV and [300, 500] GeV are better.

$M_{BL}$ is the mass of the new gaugino, and it has influence on the mass matrix of neutralino. Therefore, $M_{BL}$ can have considerable effect on the relic density. With the parameters
$M_1 = 1.2\text{TeV}, \ M_2 = 1\text{TeV}, \ \mu = 0.5\text{TeV}, \ M_{BB'} = 0.4\text{TeV}$, we plot the relic density versus $M_{BL}$ in the Fig. 3. The solid line corresponds to $\tan \beta = 9$, and the dotted line corresponds to $\tan \beta = 5$. Both the solid line and the dotted line become small with the increasing $M_{BL}$, and they possess similar behavior. For the both lines, the best point is around $M_{BL} = 1000$ GeV. As $M_{BL} < 800$ GeV or $M_{BL} > 1150$ GeV, the obtained numerical results of $\Omega_{Dh^2}$ exceed the experimental data.

In order to scan the parameters more efficiently, we plot the parameters in $3\sigma$ sensitivity of the relic density with two variables. With the parameters $M_1 = 1.2\text{TeV}, \ M_2 = 1\text{TeV}, \ M_{BL} = 1\text{TeV}, \ M_{BB'} = 0.4\text{TeV}$, we plot the allowed results in the plane of $\tan \beta$ and $\mu$, which is shown in the Fig. 4. $\tan \beta$ appears in almost all the mass matrixes of Fermions,
scalars and Majoranas, and it must be a sensitive parameter. From the Fig. 4 one can find that $\tan \beta$ should be in the region from 2.5 to 23. The corresponding values of $\mu$ are approximately in the scope ($-1100 \sim -600$) GeV and ($0 \sim 1100$) GeV. The allowed region of $\mu > 0$ is larger than that of $\mu < 0$.

![Fig. 4: The allowed parameters in the plane of $\tan \beta$ and $\mu$.](image)

$M_{BB'}$, $M_2$, $M_{BL}$ and $M_1$ are all in the mass matrix of neutralino. So we study their effects and allowed regions. In the Fig. 5 we plot the results versus $M_{BB'}$ and $M_2$ with $\tan \beta = 9, \mu = 0.5\text{TeV}, M_1 = 1.2\text{TeV}, M_{BL} = 1\text{TeV}$. $M_2$ smaller than 3500 GeV is acceptable. $M_{BB'}$’s region is almost symmetric relative to $M_{BB'} = 0$, whose value should be in the region ($-700 \sim -200$) GeV and ($100 \sim 600$) GeV. The region near the point (0, 0) is excluded. In the plane of $M_{BL}$ and $M_1$, the numerical results of the relic density are researched as $\tan \beta = 9, \mu = 0.5\text{TeV}, M_2 = 1\text{TeV}, M_{BB'} = 0.4\text{TeV}$. It is obvious that the allowed region in the Fig. 5 is smaller than that in the Fig. 4 and Fig. 5. The points gather around the narrow band near $M_{BL} = 1000$ GeV.

To find large parameter space satisfying the relic density, we plot the relic density versus the lightest neutralino mass $M_{\chi_1^0}$ with the parameters: $2 \leq \tan \beta \leq 50$, and $-2\text{TeV} \leq \mu \leq 2\text{TeV}, -2\text{TeV} \leq M_{BL} \leq 2\text{TeV}, -2\text{TeV} \leq M_{BB'} \leq 2\text{TeV}, -2\text{TeV} \leq M_1 \leq 2\text{TeV} -2\text{TeV} \leq M_2 \leq 2\text{TeV}$. The results are shown in the Fig. 7 where the gray band represents the relic density in three $\sigma$ sensitivity. One can easily see that large reasonable parameter space in near $M_{\chi_1^0} \sim 300\text{GeV}$. In the $M_{\chi_1^0}$ region (120 GeV to 280 GeV), there are also reasonable parameter space for $\Omega_D h^2$, but these parameter space are much smaller than the reasonable parameter space for $M_{\chi_1^0} \sim 300\text{GeV}$.
B. The cross section of neutralino scattering off nucleus

In this subsection, the cross section of the lightest neutralino scattering off nucleus is numerically researched with the parameters $M_1 = 1.2\text{TeV}$, $M_2 = 1\text{TeV}$, $M_{BL} = 1\text{TeV}$, $\tan \beta = 9$, $\mu = 0.5\text{TeV}$, $M_{BB'} = 0.4\text{TeV}$. Both the spin-independent cross section and spin-dependent cross section are studied here. The constraint from the relic density is taken into account. In our used parameter space, the mass of the lightest neutralino is about 300 GeV. For dark matter mass $\sim 300$ GeV, the corresponding experimental limit on spin-independent direct detection is about $2.3 \times 10^{-46}$ cm$^2$ for Xenon in 1σ sensitivity. While, it is about twice as large for PandaX [38, 39]. The experimental constraint on spin-independent cross section is much more severe than that on spin-dependent cross section. The direct detection experimental limit on spin-dependent cross section is about $4.0 \times 10^{-41}$ cm$^2$ for
FIG. 7: The relic density $\Omega D h^2$ versus the lightest neutralino mass $M_{\chi^0_1}$

Xenon1T experiment. The corresponding constraint is around $1.4 \times 10^{-40}$ cm$^2$ for PandaX-II$^{30, 38}$.

To simplify the discussion, we suppose $M_{Q11}^2 = M_{Q22}^2 = M_Q^2$. At first, the spin-independent cross section is researched with the parameters $M_Q^2 = 10$ TeV$^2$. $T_d$ is in the non-diagonal element of the mass squared matrix for scalar down type quarks. Therefore, $T_d$ should influence the scattering cross section. In the Fig. 8, the numerical results of the spin-independent cross section versus $T_d$ are plotted by the solid line ($M_D^2 = 6$ TeV$^2$) and dotted line ($M_D^2 = 5$ TeV$^2$). Generally speaking, the both lines are at the order of $10^{-47}$ cm$^2$, which are about one order smaller than the experimental bound.

Secondly, we calculate the spin-dependent cross section as $T_d = 1$ TeV. $M_Q^2$ are the important diagonal elements in the mass squared matrixes of scalar quarks, and they can strongly affect the masses of scalar quarks. In the Fig. 9, the numerical results of spin-dependent cross section versus $M_Q^2$ are represented by the solid line (dotted line) with $M_D^2 = 10$ TeV$^2$ (5 TeV$^2$). The dotted line is above the solid line. With the same $M_Q^2$ in the region ($3.0 \times 10^6$ GeV$^2 \sim 3.0 \times 10^7$ GeV$^2$), the values of the dotted line are about $2 \times 10^{-44}$ cm$^2$ larger than the values of the solid line. Generally speaking, they are about three orders smaller than the experimental bounds.
FIG. 8: The spin-independent cross section versus $T_d$, the solid line corresponds to $M_D^2=6\text{TeV}^2$ and the dotted line corresponds to $M_D^2=5\text{TeV}^2$.

FIG. 9: The spin-dependent cross section versus $M_Q^2$ is plotted by the solid line (dotted line) with $M_D^2 = 10(5)\text{TeV}^2$.

VI. DISCUSSION AND CONCLUSION

We extend MSSM with the $U(1)_X$ local gauge group and obtain the so called $U(1)_X$ SSM. In the $U(1)_X$ SSM, there are several superfields beyond MSSM, such as right-handed neutrinos, three singlet Higgs superfields $\hat{\eta}$, $\hat{\eta}$, $\hat{S}$. As discussed in MSSM, the lightest neutralino is studied in detail as dark matter candidate. While, both the lightest sneutrino and the lightest neutralino can be dark matter candidates in $U(1)_X$ SSM. Supposing the lightest CP-even sneutrino as LSP and dark matter candidate, we research its relic density and the scattering cross section off nucleus in our previous work [13]. $U(1)_X$ SSM has richer phenomenology than MSSM. To compare the scalar neutrino condition, we research the lightest neutralino
as dark matter candidate in this work.

To calculate the relic density of $\chi^0_1$, we consider the self-annihilation and co-annihilation processes. In our used parameter space, the masses of the SUSY particles except $\chi^0_1$ are all heavier enough than the mass of $\chi^0_1$. Therefore, self-annihilation processes are dominant and co-annihilation processes are suppressed by the exponential function. In the whole, this is the general condition. The resonance annihilation does not take place, because the masses of the exchanged virtual particles are not near $2m_{\chi^0_1}$. From our numerical results, we find that $M_{BB'}$ and $M_{BL}$ in the neutralino mass matrix are sensitive parameters for the relic density. The reason is that both $M_{BB'}$ and $M_{BL}$ affect neutralino mixing. Large reasonable parameter space supports $m_{\chi^0_1} \sim 300$ GeV, though $m_{\chi^0_1}$ can be smaller with reasonable parameter space. The obtained numerical results can well satisfy the experimental constraints from the relic density of dark matter. The cross section of $\chi^0_1$ scattering off nucleus are also calculated in this work. The spin-independent and spin-dependent cross sections are at least one order smaller than their experimental constraints. This work makes up for the dark matter research[13], where just the lightest CP-even scalar neutrino is supposed as dark matter.

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Appendix A: The coupling

The couplings of neutralino and gauge bosons are $\chi^0 - \chi^0 - Z$, $\chi^0 - \chi^0 - Z'$ and $\chi^0 - \chi^0 - W^\pm - W^\pm$. Their concrete forms are shown as

$$\mathcal{L}_{\chi^0 \chi^0 Z} = \bar{\chi}^0_i \left\{ \frac{i}{2} \left[ \left( g_1 \cos \theta' W \sin \theta_W + g_2 \cos \theta_W \cos \theta' W - (g_{YX} + g_X) \sin \theta' W \right) \times (N_{j3}^* N_{i3} - N_{j4}^* N_{i4}) - 2 g_X \sin \theta' W (N_{i6}^* N_{j6} - N_{i7}^* N_{j7}) \right] \gamma_\mu P_L \right\} \chi^0_j Z^\mu,$$

$$\mathcal{L}_{\chi^0 \chi^0 Z'} = \bar{\chi}^0_i \left\{ \frac{i}{2} \left[ \left( (g_1 \sin \theta_W + g_2 \cos \theta_W) \sin \theta' W - (g_{YX} + g_X) \cos \theta' W \right) \times (N_{i3}^* N_{j3} - N_{i4}^* N_{j4}) - 2 g_X \sin \theta' W (N_{i6}^* N_{j6} - N_{i7}^* N_{j7}) \right] \gamma_\mu P_R \right\} \chi^0_j Z'^\mu,$$

for $i, j = 1, 2, 3$.
The concrete form of neutralino-chargino-charged Higgs coupling $(\chi^0 - \chi^0 - Z$ and $\chi^0 - \chi^0 - Z^\prime$ contribute to the self-annihilation, and the coupling $\chi^0 - \chi^\pm - W^\pm$ gives correction to co-annihilation. We deduce the coupling of neutralino-lepton-slepton$(\chi^0 - l - \tilde{L})$.

$$\mathcal{L}_{\chi^0 l\tilde{L}} = \chi^0 \left\{ i \left( \frac{1}{\sqrt{2}} (g_1 N^*_i + g_2 N^*_i + g_{YX} N^*_i) Z^E_{k_3} - i \lambda (g_1 N^*_i + g_2 N^*_i + (g_{YX} + g_X) N^*_i) P_L - g_2 (2 g_{YX} N^*_i + (g_{YX} + g_X) N^*_i) P_L \right) e_j \tilde{L}_k \right\}.$$ (A4)

$Z^E$ is used to diagonalize the mass squared matrix of slepton.

The coupling of neutralino-neutralino-CP-even Higgs $(\chi^0 - \chi^0 - H^0)$ is

$$\mathcal{L}_{\chi^0\chi^0 H} = \chi^0 \left\{ i \left( \frac{1}{2} (2 g_{YX} (N^*_i + N^*_i + N^*_i N^*_j) - \sqrt{2} \lambda \chi_2 (N^*_i + N^*_i + N^*_i N^*_j)) \right) Z^H_{k_3} + \lambda (2 g_{YX} (N^*_i + N^*_i + N^*_i) P_L - (2 g_{YX} (N^*_i + N^*_i + N^*_i) P_L \right) e_j \tilde{L}_k \right\}.$$ (A5)

The concrete form of neutralino-chargino-charged Higgs coupling $(\chi^0 - \chi^\pm - H^\pm)$ is

$$\mathcal{L}_{\chi^0 \chi^\pm H^\pm} = \chi^\pm \left\{ i \left[ - g_{2 \chi^0} (2 \lambda H N^*_j Z^+_{k_3}) + \sqrt{2} (g_1 + g_2 N^*_j + (g_{YX} + g_X) N^*_j) Z^+_{k_3} \right) \right\}.$$
Neutralinos interact with neutrinos and sneutrinos in the following form

\[
\mathcal{L}_{\chi^{0} \nu_{i} \bar{\nu}_{j}} = \chi_{i}^{0} \frac{1}{2} \sum_{a=1}^{3} \left( - \sqrt{2} N_{i7}^{*} \sum_{b=1}^{3} Y_{x,ab} (Z_{k3+b}^{I,*, V} U_{j3+a}^{V,*, I} + U_{j3+a}^{V,*, I} Z_{k3+b}^{I,*, V}) + (g_{YX} N_{i5} - g_{2N_{i2}} + N_{i1} g_{1}) U_{j3+a}^{V,*, I} Y_{x,ab}^{*} N_{i7} + \frac{1}{2} \left( \sum_{b=1}^{3} \left( Z_{k3+b}^{I,*, V} U_{j3+a}^{V,*, I} + Z_{k3+b}^{I,*, V} U_{j3+a}^{V,*, I} \right) Y_{x,ab}^{*} N_{i7} + Z_{k3+a}^{I,*, V} U_{j3+a}^{V,*, I} (g_{1} N_{i1} - g_{2N_{i2}} + g_{YX} N_{i5}) \right) P_{R} \right) \nu_{j} \tilde{\nu}_{k},
\]

(A7)

There are also neutralino-quark-squark couplings

\[
\mathcal{L}_{\chi^{0} d \bar{d}} = -\frac{i}{6} \chi_{i}^{0} \left\{ \left( \sqrt{2} (g_{1} N_{i1} - 3 g_{2N_{i2}} + g_{YX} N_{i5}) Z_{k3+j}^{D} + 6 N_{i3} Y_{d,j} Z_{k3+j}^{D} \right) P_{L} + \left( 6 Y_{d,j} Z_{k3+j}^{D} N_{i3} + \sqrt{2} Z_{k3+j}^{D} \right) \left[ 2 g_{1} N_{i1} + (2 g_{YX} + 3 g_{X}) N_{i5} \right] P_{R} \right\} d_{j} \bar{d}_{k},
\]

(A8)

\[
\mathcal{L}_{\chi^{0} u \bar{U}} = -\frac{i}{6} \chi_{i}^{0} \left\{ \left( \sqrt{2} (g_{1} N_{i1} - 3 g_{2N_{i2}} + g_{YX} N_{i5}) Z_{k3+j}^{U} + 6 N_{i4} Y_{u,j} Z_{k3+j}^{U} \right) P_{L} - \left( \sqrt{2} Z_{k3+j}^{U} \left( (3 g_{X} + 4 g_{YX}) N_{i5} + 4 g_{1} N_{i1} - 6 N_{i4} Y_{u,j} Z_{k3+j}^{U} \right) \right) P_{R} \right\} u_{j} \bar{U}_{k}^{*}.
\]

(A9)

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