Lightweight and Secure Two-Party Range Queries over Outsourced Encrypted Databases

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Abstract

With the many benefits of cloud computing, an entity may want to outsource its data and their related analytics tasks to a cloud. When data are sensitive, it is in the interest of the entity to outsource encrypted data to the cloud; however, this limits the types of operations that can be performed on the cloud side. Especially, evaluating queries over the encrypted data stored on the cloud without the entity performing any computation and without ever decrypting the data become a very challenging problem. In this paper, we propose solutions to conduct range queries over outsourced encrypted data. The existing methods leak valuable information to the cloud which can violate the security guarantee of the underlying encryption schemes. In general, the main security primitive used to evaluate range queries is secure comparison (SC) of encrypted integers. However, we observe that the existing SC protocols are not very efficient.

To this end, we first propose a novel SC scheme that takes encrypted integers and outputs encrypted comparison result. We empirically show its practical advantage over the current state-of-the-art. We then utilize the proposed SC scheme to construct two new secure range query protocols. Our protocols protect data confidentiality, privacy of user’s query, and also preserve the semantic security of the encrypted data; therefore, they are more secure than the existing protocols. Furthermore, our second protocol is lightweight at the user end, and it can allow an authorized user to use any device with limited storage and computing capability to perform the range queries over outsourced encrypted data.

Keywords: Secure Comparison, Range Query, Encryption, Cloud Computing

1 Introduction

For many companies, especially in the case of small and medium size businesses, maintaining their own data can be a challenging issue due to large capital expenditures and high day-to-day operational costs. Therefore, data owners may be more interested in outsourcing their data and operations related to the data. Along this direction, cloud computing \([3,16,43]\) offers a promising solution due to various advantages such as cost-efficiency and flexibility. Due to various privacy reasons \([40, 41, 45, 49]\) and as the cloud may not be fully trusted, users encrypt their data at first place and then outsource them to the cloud. However, this places limitations on the range of operations that can be performed over encrypted data in the cloud. In recent years, query processing over encrypted data stored in the cloud has gained significant importance as it is a common feature in many outsourced service-oriented databases.

In this paper, we focus on processing range queries over encrypted data in the cloud. A range query, where records are retrieved if the values of a specific field lie in the range \((\alpha, \beta)\), is one among the highly desirable queries. For example, consider the situation where a hospital outsources its patients’ medical data to a cloud after the data were properly encrypted. If at some future time, suppose a researcher wants to access this hospital’s data for analyzing the disease patterns of all the young patients whose ages lie between 18 and 25. For privacy reasons, the input query by the researcher should not be revealed to the cloud. In addition, due to efficiency and privacy reasons, the entire patients’ medical data should not be revealed to the researcher. That is, on one hand, we claim that a trivial solution where an authorized user can download the whole data from the cloud and decrypt them to perform range query locally is not practical from user’s computation perspective. On the other hand, for privacy reasons, only the disease information of a patient whose age is in \((18, 25)\) should be revealed to the researcher. We refer to such a process as privacy-preserving range query (PPRQ) over encrypted data. At a high level, the PPRQ protocol should securely compare the user’s search input (i.e., \(\alpha\) and \(\beta\)) with the encrypted field values (stored in the cloud) upon which the user wants to filter the data records.

Based on the above discussions, it is clear that the underlying basic security primitive required to solve the PPRQ problem is secure comparison of encrypted integers. Secure comparison (SC) is an important building block in many distributed and privacy-preserving applications such as secure electronic voting (e.g., \([17]\)), private auctioning and
bidding (e.g., [8, 13]), and privacy-preserving data mining (e.g., [2, 55]). First, we observe that the existing custom-designed SC protocols (e.g., [6, 19, 22]) require encryptions of individual bits of inputs rather than simple encrypted integers; therefore, making them less efficient. Secondly, the traditional two-party computation methods based on Yao’s garbled-circuit technique seem to a better choice to solve the SC problem. Indeed, some recent implementations, such as FastGC [31], demonstrate that such generic approaches can outperform the custom-designed protocols. Nevertheless, we show that the SC protocol constructed using Yao’s garbled-circuit on FastGC is still less efficient (see Section 4 for details). Along this direction, we first propose a novel SC protocol that is more efficient than the methods based on the above two approaches.

Apart from ensuring data confidentiality, which is commonly achieved by encrypting the data before outsourcing, two important privacy issues related to the PPRQ problem are: 1) preserving the privacy or confidentiality of an input query and 2) preventing the cloud from learning the data access patterns. While the privacy of user’s input query can be protected by the security of the underlying encryption schemes, hiding the data access patterns from the cloud is a challenging task. This is because of the fact that the encrypted data resides in the cloud (which acts as a third party). As mentioned in [21, 52], by monitoring the data access patterns, the cloud can reconstruct the correspondence between the plaintext data and the encrypted data based on the access pattern frequencies to each piece of data. These access patterns can actually violate the security guarantee of the underlying encryption schemes used to encrypt the outsourced data.

1.1 Access Patterns and Semantic Security

By data access patterns, we mean the relationships among the encrypted data that can be observed by the cloud during query processing. For example, suppose there are five records \( t_1, \ldots, t_5 \) in a database \( D \) and let \( E_{pk}(\cdot) \) denote a semantically secure encryption function, such as Paillier cryptosystem [38]. Assume that these records are encrypted (i.e., \( E_{pk}(t_1), \ldots, E_{pk}(t_5) \)) and stored on a cloud, denoted by \( C \). After processing a user’s range query, let \( E_{pk}(t_2) \) and \( E_{pk}(t_3) \) be the output returned to the user. More details on how this is achieved are given in Section 5.

In cryptography, it is a common belief that an encryption scheme needs to be at least secure against chosen-plaintext attack (i.e., semantic security). In other words, the ciphertexts should be indistinguishable from an (computationally bounded) adversary’s perspective. From the previous example, before processing the user query, \( C \) cannot distinguish \( E_{pk}(t_1), \ldots, E_{pk}(t_5) \) because \( E_{pk}(\cdot) \) is semantically secure. However, after processing the user query, \( C \) learns that the encrypted data can be partitioned into two groups: \( \{E_{pk}(1), E_{pk}(3), E_{pk}(4)\} \) and \( \{E_{pk}(2), E_{pk}(5)\} \). More specifically, \( \{E_{pk}(1), E_{pk}(3), E_{pk}(4)\} \) is distinguishable from \( \{E_{pk}(2), E_{pk}(5)\} \). This breaks the semantic security of \( E_{pk}(\cdot) \). Thus, to protect the confidentiality or semantic security of the outsourced data, access patterns should be hidden from the cloud who stores and processes the data.

A naïve approach to hide the access patterns is to encrypt the database with symmetric key encryption schemes (e.g., AES) and then outsourcing them to the cloud. However, during the query processing step, the cloud cannot perform any algebraic operations over the encrypted data. Thus, the entire encrypted database has to be downloaded by the authorized user which is not practical especially for mobile users and large databases. On the other hand, to avoid downloading the entire encrypted database from the cloud and to hide access patterns, a user can adopt Oblivious RAM (ORAM) techniques [48, 52].

The main goal of Oblivious RAM is to hide which data record has been accessed by the user. To utilize ORAM, the user needs to know where to retrieve the record on the cloud through certain indexing structure. Since a dataset can contain hundreds of attributes and a range query can be performed on any of these attributes, one indexing structure to utilize ORAM is clearly insufficient. How to efficiently utilize multiple indexes on multiple attribute of the data is still an open problem with ORAM techniques. In addition, since the current computations required for ORAM techniques cannot be parallelized, we cannot take full advantage of the large-scale parallel processing capability of a cloud.

To provide better security guarantee and shift the entire computation to the cloud, this paper proposes two novel PPRQ protocols by utilizing our new SC scheme as the building block. Our protocols protect data confidentiality and privacy of user’s input query. At the same time, they hide data access patterns from the cloud service providers. Also, our second protocol is very efficient from the end-user perspective.
1.2 Our Contributions

We propose efficient protocols for secure comparison and PPRQ problems over encrypted data. More specifically, the main contributions of this paper are two-fold:

(i). Secure Comparison. As mentioned earlier, the basic security primitive required to solve the PPRQ problem is secure comparison (SC) of encrypted integers. Since the existing SC methods are not that efficient, we first propose an efficient and probabilistic SC scheme. Because the proposed SC scheme is probabilistic in nature, we theoretically analyze its correctness and provide a formal security proof based on the standard simulation paradigm [25]. We stress that, our SC scheme returns the correct output for all practical applications.

(ii). Privacy-Preserving Range Query. We construct two novel PPRQ protocols using our new SC scheme as the building block. Our protocols achieve the desired security objectives of PPRQ (see Section 2 for details), and the computation cost on the end-user is very low since most computations are shifted to the cloud. Also, the computations performed by the cloud can be easily parallelized to drastically improve the response time.

The rest of the paper is organized as follows. In Section 2, we formally discuss our problem statement along with the threat model adopted in this paper. A brief survey of the existing related work is presented in Section 3. Our new secure comparison scheme is presented in Section 4 along with a running example. Also, apart from providing a formal security proof, we theoretically analyze the accuracy guarantee of our SC scheme. Additionally, in this section, we empirically compare the performance of our SC scheme with the existing methods and demonstrate its practical applicability. The proposed PPRQ protocols, which are constructed on the top of our SC scheme, are presented in Section 5. Finally, we conclude the paper with possible future work in Section 6.

2 Problem Settings and Threat Model

2.1 Architecture and Desired Security Properties

Consider a data owner Alice holding the database $D$ with $n$ records denoted by $\langle t_1, \ldots, t_n \rangle$. Let $t_{i,j}$ denote the $j^{th}$ attribute value of tuple $t_i$, for $1 \leq i \leq n$ and $1 \leq j \leq w$, where $w$ denotes the number of attributes. We assume that Alice encrypts her database $D$ attribute-wise with an additive homomorphic encryption scheme that is semantically secure, such as Paillier cryptosystem [38], and outsources the encrypted database to the cloud. Without loss of generality, let $T$ denote the encrypted database. Besides the data, Alice outsources the future query processing services to the cloud. Now, consider a user Bob who is authorized by Alice to access $T$ in the cloud. Suppose, if at some future time, Bob wants to execute a range query $Q = \{k, \alpha, \beta\}$ over encrypted data in the cloud, where $k$ is the attribute index upon which he wants to filter the records with $\alpha$ and $\beta$ as the lower and upper bound values, respectively. Briefly, the goal of the PPRQ protocol is to securely retrieve the set of records, denoted by $S$, such that the following property holds.

$$\forall \ t' \in S, \ \alpha \leq t'_k \leq \beta$$

where $t'_k$ denotes the $k^{th}$ attribute value of data record $t'$. More formally, we define the PPRQ protocol as follows:

$$\text{PPRQ}(T, Q) \rightarrow S$$

For any given PPRQ protocol, we stress that the following privacy requirements should be met:

(a) Bob’s input query $Q$ should not be revealed to the cloud.

(b) During any of the query processing steps, contents of $D$ should not be disclosed to the cloud.

(c) The data access patterns should not be revealed to the cloud. That is, for any given input query $Q$, the cloud should not know which data records in $D$ belong to the corresponding output set $S$. Also, access patterns related to any intermediate computations should not be revealed to the cloud. In other words, the semantic security of the encrypted data needs to be preserved.

(d) $D - S$ (i.e., the set of records not satisfying $Q$) should not be disclosed to Bob.

(e) At the end of the PPRQ protocol, $S$ should be revealed only to Bob and no information is revealed to the cloud.
2.2 Threat Model

In this paper, privacy/security is closely related to the amount of information disclosed during the execution of a protocol. Proving the security of a distributed protocol is very different from that of an encryption scheme. In the proposed protocols, our goal is to ensure no information leakage to the participating parties other than what they can deduce from their own inputs and outputs. To maximize security guarantee, we adopt the commonly accepted security definitions and proof techniques in the literature of secure multiparty computation (SMC) to analyze the security of the proposed protocols. SMC was first introduced by Yao’s Millionaires’ (two-party) problem [54, 55], and it was extended by Goldreich et al. [27] to the multi-party case. It was proved in [27] that any computation which can be done in polynomial time by a single party can also be done securely by multiple parties.

There are three common adversarial models under SMC: semi-honest, covert and malicious. An adversarial model generally specifies what an adversary or attacker is allowed to do during an execution of a secure protocol. In the semi-honest model, an attacker (i.e., one of the participating parties) is expected to follow the prescribed steps of a protocol. However, the attacker can compute any additional information based on his or her private input, output and messages received during an execution of the secure protocol. As a result, whatever can be inferred from the private input and output of an attacker is not considered as a privacy violation. An adversary in the semi-honest model can be treated as a passive attacker whereas an adversary in the malicious model can be treated as an active attacker who can arbitrarily diverge from the normal execution of a protocol. On the other hand, the covert adversary model [4] lies between the semi-honest and malicious model. More specifically, an adversary under the covert model may deviate arbitrarily from the rules of a protocol, however, in the case of cheating, the honest party is guaranteed to detect this cheating with good probability.

In this paper, to develop secure and efficient protocols, we assume that parties are semi-honest for two reasons. First, as mentioned in [31], developing protocols under the semi-honest setting is an important first step towards constructing protocols with stronger security guarantees. Second, it is worth pointing out that most practical SMC protocols proposed in the literature (e.g., [28, 30, 31, 37]) are implemented only under the semi-honest model. By semi-honest model, we implicitly assume that the cloud service providers (or other participating users) utilized in our protocols do not collude. Since current known cloud service providers are well established IT companies, it is hard to see the possibility for two companies, e.g., Google and Amazon, to collude to damage their reputations and consequently place negative impact on their revenues. Thus, in our problem domain, assuming the participating parties are semi-honest is very realistic.

However, in Section 4.2(2), we discuss strategies to extend the proposed SC protocol to be secure under the malicious and the covert models. Since SC is the main component of the proposal PPRQ protocols, we believe that the same strategies can be used to make the PPRQ protocols secure under the malicious and covert models. Due to space limitations, we will not provide detailed discussions on how to modify the proposed PPRQ protocols to be secure under other adversarial models and leave it as part of our future work.

Formally, the following definition captures the security of a protocol under the semi-honest model [25].

**Definition 1**  Let $a_i$ be the input of party $P_i$, $\Pi_i(\pi)$ be $P_i$’s execution image of the protocol $\pi$ and $b_i$ be its output computed from $\pi$. Then, $\pi$ is secure if $\Pi_i(\pi)$ can be simulated from $\langle a_i, b_i \rangle$ and distribution of the simulated image is computationally indistinguishable from $\Pi_i(\pi)$.

In the above definition, an execution image generally includes the input, the output and the messages communicated during an execution of a protocol. To prove a protocol is secure under the semi-honest model, we generally need to show that the execution image of a protocol does not leak any information regarding the private inputs of participating parties [25].

3 Related Work

In this section, we first briefly review upon the existing work related to our problem domain. Then, we refer to the additive homomorphic properties and the corresponding encryption scheme adopted in this paper. Finally, we discuss the secure comparison problem and point out two (different) best-known solutions to solve this problem.
3.1 Keyword Search on Encrypted Data

A different but closely related work to querying on encrypted data is “keyword search on encrypted data”. The main goal of this problem is to retrieve the set of encrypted files stored on a remote server (such as the cloud) that match the user’s input keywords. Along this direction, much work has been published based on searchable encryption schemes (e.g., [7,15,50,53]). However, these works mostly concentrate on protecting data confidentiality and they do not protect data access patterns. Though some recent works addressed the issue of protecting access patterns while searching for keywords [23,35], at this point, it is not clear how their work can be mapped to range queries which is an entirely different and complex problem than simple exact matching.

3.2 Existing PPRQ Methods

The PPRQ problem has been investigated under different security models such as order-preserving encryption [1,9] and searchable public key encryption schemes [10,47]. Range queries over encrypted data was first addressed by Agrawal et al. [1]. They have developed an order-preserving encryption (OPE) scheme for numeric data that can support indexing to efficiently access the encrypted data stored on an untrusted server. The basic idea behind the OPE scheme is to map plaintexts into ciphertexts by preserving their relative order. That is, for any given ciphertexts \( c_1 \) and \( c_2 \) corresponding to plaintexts \( p_1 \) and \( p_2 \), if \( p_1 \geq p_2 \) then it is guaranteed that \( c_1 \geq c_2 \). Such a guarantee allows the untrusted server (i.e., the cloud in our case) to easily process range queries even if the data are encrypted. As an improvement, Boldyreva et al. [9] provided a formal security analysis and an efficient version of the OPE scheme. Nevertheless, the main disadvantage of OPE schemes is that they are not secure against chosen-plaintext attacks (CPA). This is because of the fact that OPE schemes are deterministic (i.e., different encryptions of a given plaintext will result in the same ciphertext) and they reveal relative ordering among plaintexts. Due to the above reasons, the ciphertexts are distinguishable from the server’s perspective; therefore, OPE schemes are not IND-CPA secure.

As an alternative, in the past few years, researchers have been focusing on searchable public key encryption schemes by leveraging cryptographic techniques. Along this direction, in particular to range queries, some earlier works [10,47] were partly successful in addressing the PPRQ problem. However, as mentioned in [29], these methods are susceptible to value-localization problem; therefore, they are not secure. In addition, they leak data access patterns to the server. Recently, Hore et al. [29] developed a new multi-dimensional PPRQ protocol by securely generating index tags for the data using bucketization techniques. However, their method is susceptible to access pattern attacks (this issue was also mentioned as a drawback in [29]) and false positives in the returned set of records. More specifically, the final set of records has to be weeded by the client to remove false positives which incurs computational overhead on the client side. In addition, since the bucket labels are revealed to the server, we believe that their method may lead to unwanted information leakage.

Vimercati et al. [20] proposed a new technique for protecting confidentiality as well as access patterns to the data in outsourced environments. Their technique is based on constructing shuffled index structures using B+-trees. In order to hide the access patterns, their method introduces fake searches in conjunction with the actual index value to be searched. We emphasize that their work solves a different problem - mainly how to securely outsource the index and then obliviously search over this data structure. Their technique has a straight-forward application to keyword search over encrypted data since it deals with exact matching. However, at this point, it is not clear how their work can be extended for range queries that require implicit comparison operations to be performed in a secure manner.

We may ask if we can use fully homomorphic cryptosystems (e.g., [23]) which can perform arbitrary computations over encrypted data without ever decrypting them. However, such techniques are very expensive and their usage in practical applications have yet to be explored. For example, it was shown in [24] that even for weak security parameters one “bootstrapping” operation of the homomorphic operation would take at least 30 seconds on a high performance machine.

As an independent work, Bajaj et al. [5] developed a new prototype to execute SQL queries by leveraging server-hosted tamper-proof trusted hardware in critical query processing stages. However, their work still reveals data access patterns to the server. Recently, Samanthula et al. [44] proposed a new PPRQ protocol by utilizing the secure comparison (SC) protocol in [6] as the building block. Perhaps, their method is the most closely related work to the protocols proposed in this paper. However, the SC protocol in [6] operates on encrypted bits rather than on encrypted integers; therefore, the overall throughput in their protocol is less. In addition, their protocol leaks data access patterns to the
cloud service provider.

Hence, in order to provide better security and improve efficiency, this paper proposes two novel PPRQ protocols that protect data confidentiality, privacy of user’s input query and hide data access patterns.

### 3.3 Additive Homomorphic Encryption Scheme

In the proposed protocols, we utilize an additive homomorphic encryption scheme (denoted by \( \text{HEnc}^+ \)) that is probabilistic in nature. Without loss of generality, let \( E_{\text{pk}} \) and \( D_{sk} \) be the encryption and decryption functions of an \( \text{HEnc}^+ \) system, where \( \text{pk} \) and \( \text{sk} \) are the public and secret keys, respectively. Given a ciphertext and \( \text{pk} \), it is impossible for an (computationally bounded) adversary to retrieve the corresponding plaintext in polynomial time. Let \( N \) denote the RSA modulus (or part of public key \( \text{pk} \)). In general, the \( \text{HEnc}^+ \) system exhibits the following properties:

- Given two ciphertexts \( E_{\text{pk}}(a) \) and \( E_{\text{pk}}(b) \), where \( a, b \in \mathbb{Z}_N \), we can compute the ciphertext corresponding to \( a + b \) by performing homomorphic addition (denoted by \(+_h\)) on the two ciphertexts:
  \[
  D_{sk}(E_{\text{pk}}(a) +_h E_{\text{pk}}(b)) = a + b;
  \]

- Using the above property, for any given constant \( u \in \mathbb{Z}_N \), the homomorphic multiplication property is given by:
  \[
  D_{sk}(E_{\text{pk}}(a)^u) = a \cdot u;
  \]

- The encryption scheme is semantically secure \([26] \), that is indistinguishability under chosen-plaintext attack (IND-CPA) holds.

Any \( \text{HEnc}^+ \) system can be used to implement the proposed protocols; however, this paper uses the Paillier cryptosystem \([38] \) due to its efficiency.

### 3.4 Secure Comparison (SC)

Let us consider a party \( P_1 \) holding two Paillier encrypted values \( (E_{\text{pk}}(x), E_{\text{pk}}(y)) \) and a party \( P_2 \) holding the secret key \( \text{sk} \) such that \( (x, y) \) is unknown to both parties. The goal of the secure comparison (SC) protocol is for \( P_1 \) and \( P_2 \) to securely evaluate the functionality \( x \geq y \). The comparison result, denoted by \( c \), is 1 if \( x \geq y \), and 0 otherwise. At the end of the SC protocol, the output \( E_{\text{pk}}(c) \) should be known only to \( P_1 \). During this process, no other information regarding \( x \), \( y \), and \( c \) is revealed to \( P_1 \) and \( P_2 \).

We emphasize that other variations of SC include \( (x, E_{\text{pk}}(y)), (E_{\text{pk}}(x), y), \) or shares of \( x \) and \( y \) as private inputs. On one hand, the existing SC methods based on Yao’s garbled-circuit technique (e.g., \([34] \)) assume that \( x \) and \( y \) are known to \( P_1 \) and \( P_2 \) respectively. However, such techniques can be easily modified to handle the above input cases with minimal cost. For completeness, here we briefly explain how to construct a secure comparison circuit (denoted by \( \text{SC}_x \)) using \( E_{\text{pk}}(a) \) and \( E_{\text{pk}}(b) \) as \( P_1 \)’s input. Since it would be complex (and costly) to include encryption and decryption operations as a part of the circuit, we discuss a simple method to compute the random shares of \( x \) and \( y \) from \( (E_{\text{pk}}(x), E_{\text{pk}}(y)) \) using homomorphic properties. Initially, \( P_1 \) masks the encrypted inputs by computing \( E_{\text{pk}}(x + r_1) \) and \( E_{\text{pk}}(y + r_2) \), where \( r_1 \) and \( r_2 \) are random numbers (known only to \( P_1 \)) in \( \mathbb{Z}_N \), and sends them to \( P_2 \). Upon decryption, \( P_2 \) gets his/her random shares as \( x + r_1 \mod N \) and \( y + r_2 \mod N \). Also, \( P_1 \) sets his/her corresponding random shares as \( N - r_1 \) and \( N - r_2 \). After this, \( P_1 \) can construct a garbled-circuit, where \( P_2 \) acts as the circuit evaluator, based on the following steps:

1. Add the random shares (as a part of the circuit) to get \( x \) and \( y \).
2. Compute the comparison result on \( x \) and \( y \) using \([34] \). We stress that the comparison result \( c \) is not known to either of the parties since the result is encoded as a part of the garbled-circuit.
3. Add a random value (known only to \( P_1 \)) to the comparison result. The masked value is the final output of the circuit which will be known only to \( P_2 \).
Next, $P_2$ encrypts the masked comparison result and sends it to $P_1$. Finally $P_1$ removes the masking factor to compute $E_{pk}(c)$ using homomorphic properties. We emphasize that the addition operations in the above circuit should be followed by an implicit modulo $N$ operation to compute the correct result. At a high level, the above circuit seems to be simple and efficient. Nevertheless, as we show in Section 4.3 such traditional techniques are much less efficient than our proposed SC scheme.

In this paper, we do not consider the existing secure comparison protocols that are secure under the information theoretic setting. This is because, the existing secure comparison protocols under the information theoretic setting are commonly based on linear secret sharing schemes, such as Shamir’s [46], which require at least three parties. We emphasize that our problem setting is entirely different than those methods since the data in our case are encrypted and our protocols require only two parties. Our protocols, which are based on additive homomorphic encryption schemes, are orthogonal to the secret sharing based SC schemes. Nevertheless, developing a PPRQ protocol by using the secret sharing based SC methods that protect the data access patterns is still an open problem; therefore, it can be treated as an interesting future work.

On the other hand, there exist a large number of custom-designed SC protocols (e.g., [6, 19, 22]) that directly operate on encrypted inputs. Since the goal of this paper is not to investigate all the existing SC protocols, we simply refer to the most efficient known implementation of SC (here we consider methods based on Paillier cryptosystem to have a fair comparison with our scheme) that was proposed by Blake et al. [6]. We emphasize that the SC protocol given in [6] requires the encryptions of individual bits of $x$ and $y$ as the input rather than $(E_{pk}(x), E_{pk}(y))$. Though their protocol is efficient than the above garbled-circuit based SC method (i.e., $SC_p$) for smaller input domain sizes, we show that our SC scheme outperforms both the methods for all practical values of input domain sizes (see Section 4.3 for details). Also, it is worth pointing out that the protocol in [6] leaks the comparison result $c$ to at least one of the involved parties. However, by using the techniques in [19], we can easily modify (at the expense of extra cost) the protocol of [6] to generate $E_{pk}(c)$ as the output without revealing $c$ to both parties.

4 The Proposed SC Scheme

As mentioned above, which we also show empirically in the later part of this section, the existing well-known SC methods in [6, 19, 22] are not that efficient. Therefore, to improve efficiency without compromising security, we propose a novel secure scheme, denoted by $SC_p$, for efficient comparison of encrypted integers. In our $SC_p$ protocol, the output is $E_{pk}(c)$ and is revealed only to $P_1$. That is, the comparison result $c$ is not revealed to $P_1$ and $P_2$. We stress that $SC_p$ can be easily modified to generate shared output. Therefore, depending on the application requirements, our $SC_p$ protocol can be used as a building block in larger privacy-preserving tasks.

The overall steps involved in the proposed $SC_p$ protocol are given in Algorithm 1. The basic idea of $SC_p$ is for $P_1$ to randomly choose the functionality $F$ (by flipping a coin), where $F$ is either $x \geq y$ or $y \geq x + 1$, and to obliviously execute $F$ with $P_2$. Briefly, depending on $F$, $P_1$ initially computes the encryption of difference between $x$ and $y$, say $d$. Then, $P_1$ and $P_2$ collaboratively decide the output based on whether $d$ lies in $[0, 2^m)$ or $[N - 2^m, N)$. Since $F$ is randomly chosen and known only to $P_1$, the output of functionality $F$ remains oblivious to $P_2$. Before explaining the steps of $SC_p$ in detail, we first discuss the basic ideas underlying our scheme which follow from Observations 1, 2, and 3.

**Observation 1** For any given $x$ and $y$ such that $0 \leq x, y < 2^m$, we know that $0 \leq d < 2^m$ if $x \geq y$ and $N - 2^m \leq d < N$ otherwise, where $d = x - y$. Note that “$N - y$” is equivalent to “$-y$” under $\mathbb{Z}_N$. Then, we observe that $d - d' = 0$ only if $x \geq y$, where $d'$ denotes the integer corresponding to the $m$ least significant bits of $d$. On the other hand, if $x < y$, then we have $d - d' > 0$.

The above observation is clear from the fact that $d'$ always lies in $[0, 2^m)$. On one hand, when $x \geq y$, we have $d' = d$ since $d \in [0, 2^m)$. On the other hand, if $x < y$, we have $d \in [N - 2^m, N)$; therefore, $d > d'$.

**Observation 2** For any given $x$, let $x' = x + r \mod N$, where $r$ is a random number in $\mathbb{Z}_N$ (denoted by $r \in_R \mathbb{Z}_N$). Here the relation between $x'$ and $r$ depends on whether $x + r \mod N$ leads to an overflow (i.e., $x + r$ is greater than $N$) or not. We observe that $x'$ is always greater than $r$ if there is no overflow. In the case of overflow, $x'$ is always less than $r$. 

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Algorithm 1 \( \text{SC}_p(E_{pk}(x), E_{pk}(y)) \rightarrow E_{pk}(c) \)

**Require:** \( P_1 \) has Paillier encrypted values \( (E_{pk}(x), E_{pk}(y)) \), where \( (x, y) \) is not known to both parties and \( 0 \leq x, y < 2^m \); (Note: The public key \( (g, N) \) is known to both parties whereas the secret key \( sk \) is known only to \( P_2 \))

1: \( P_1 \):
   (a). \( l \leftarrow 2^{-1} \mod N \) and \( d' \leftarrow 0 \)
   (b). Randomly choose the functionality \( F \)
   (c). if \( F : x \geq y \) then \( E_{pk}(d) \leftarrow E_{pk}(x - y) \)
      if \( F : y \geq x + 1 \) then \( E_{pk}(d) \leftarrow E_{pk}(y - x - 1) \)
   (d). \( \delta \leftarrow E_{pk}(d) \)

2: for \( i = 1 \) to \( m \) do:
   (a). \( P_1 \):
      - \( \tau_i \leftarrow \delta \ast E_{pk}(r_i) \), where \( r_i \in_R \mathbb{Z}_N \)
      - Send \( \tau_i \) to \( P_2 \)
   (b). \( P_2 \):
      - \( \tau'_i \leftarrow D_{sk}(\tau_i) \)
      - if \( \tau'_i \) is even then \( s_i \leftarrow E_{pk}(0) \)
        else \( s_i \leftarrow E_{pk}(1) \)
      - Send \( s_i \) to \( P_1 \)
   (c). \( P_1 \):
      - if \( r_i \) is even then \( E_{pk}(d_i) \leftarrow s_i \)
        else \( E_{pk}(d_i) \leftarrow E_{pk}(1) \ast s_i^{N-1} \)
      - \( E_{pk}(d') \leftarrow E_{pk}(d') \ast E_{pk}(d_i)^2^{i-1} \)
        \{update \( \delta \} \)
      - \( \Phi \leftarrow \delta \ast E_{pk}(d_i)^N \) and \( \delta \leftarrow \Phi^l \)
      - if \( i = m \) then
        - \( G \leftarrow E_{pk}(d) \ast E_{pk}(d')^{N-1} \)
        - \( G' \leftarrow G^r \), where \( r \in_R \mathbb{Z}_N \)
        - Send \( G' \) to \( P_2 \)

3: \( P_2 \):
   (a). Receive \( G' \) from \( P_1 \)
   (b). if \( D_{sk}(G') = 0 \) then \( c' \leftarrow 1 \)
      else \( c' \leftarrow 0 \)
   (c). Send \( E_{pk}(c') \) to \( P_1 \)

4: \( P_1 \):
   (a). Receive \( E_{pk}(c') \) from \( P_2 \)
   (b). if \( F : x \geq y \) then \( E_{pk}(c) \leftarrow E_{pk}(c') \)
      if \( F : y \geq x + 1 \) then \( E_{pk}(c) \leftarrow E_{pk}(1) \ast E_{pk}(c')^{N-1} \)
Observation 3 For any given \( x' = x + r \mod N \), where \( N \) is odd, the following property regarding the least significant bit of \( x \) (denoted by \( x_0 \)) always hold:
\[
x_0 = \begin{cases} 
\lambda_1 \oplus \lambda_2 & \text{if } r \text{ is even} \\
1 - (\lambda_1 \oplus \lambda_2) & \text{otherwise}
\end{cases}
\]

Here \( \lambda_1 \) denotes whether an overflow occurs or not, and \( \lambda_2 \) denotes whether \( x' \) is odd or not. That is, \( \lambda_1 = 1 \) if \( r > x' \) (i.e., overflow), and 0 otherwise. Similarly, \( \lambda_2 = 1 \) if \( x' \) is odd, and 0 otherwise. Observe that \( 1 - (\lambda_1 \oplus \lambda_2) \) denotes the negation of bit \( \lambda_1 \oplus \lambda_2 \). Also note that the RSA modulus \( N \), which is a product of two large prime numbers, is always odd in the Paillier cryptosystem \[33\].

By utilizing the above observations, the proposed SCp protocol aims to securely compute \( E_{pk}(d - d') \) and check whether \( d - d' = 0 \) or not. To start with, \( P_1 \) initially computes the multiplicative inverse of 2 under \( \mathbb{Z}_N \) and assigns it to \( l \). In addition, he/she sets \( d' \) to 0. Then, \( P_1 \) chooses the functionality \( F \) as either \( x \geq y \) or \( y \geq x + 1 \) randomly. Depending on \( F \), \( P_1 \) computes the encryption of difference between \( x \) and \( y \) using homomorphic properties\[4\] as below:

- If \( F : x \geq y \)
  \[
  E_{pk}(d) = E_{pk}(x) \ast E_{pk}(y)^{N-1} = E_{pk}(x - y)
  \]
- If \( F : y \geq x + 1 \)
  \[
  E_{pk}(d) = E_{pk}(y) \ast E_{pk}(x + 1)^{N-1} = E_{pk}(y - x - 1)
  \]
- Observe that if \( F : x \geq y \), then \( d - d' = 0 \) only if \( x \geq y \). Similarly, if \( F : y \geq x + 1 \), then \( d - d' = 0 \) only if \( y \geq x + 1 \).
- Assign \( E_{pk}(d) \) to \( \delta \).

After this, \( P_1 \) and \( P_2 \) jointly compute \( E_{pk}(d') \) in an iterative fashion. More specifically, at the end of iteration \( i \), \( P_1 \) knows the encryption of \( i^{th} \) least significant bit as well as the encryption of integer corresponding to the \( i \) least significant bits of \( d \), for \( 1 \leq i \leq m \). Without loss of generality, let \( d_i \) denote the \( i^{th} \) least significant bit of \( d \). Then, we have \( d' = \sum_{i=1}^{m} d_i \ast 2^{i-1} \). In the first iteration, \( P_1 \) randomizes \( \delta = E_{pk}(d) \) by computing \( \tau_1 = \delta \ast E_{pk}(r_1) \) and sends it to \( P_2 \), where \( r_1 \) is a random number in \( \mathbb{Z}_N \). Upon receiving \( \tau_1 \), \( P_2 \) decrypts it to get \( \tau_1' = D_{sk}(\tau_1) \) and checks its value. Note that \( \tau_1' = d + r_1 \mod N \). Following from Observation\[3\] if \( \tau_1' \) is odd, \( P_2 \) computes \( s_1 = E_{pk}(1) \), else he/she computes \( s_1 = E_{pk}(0) \), and sends it to \( P_1 \). Observe that \( s_1 = E_{pk}(\lambda_2) \). Also, to compute \( \lambda_1 \), we need to perform secure comparison between \( r_1 \) and \( \tau_1 \). However, in this paper, we assume that \( \lambda_1 \) is always zero (i.e., no overflow). We emphasize that though we assume no overflow, \( \tau_1' = d + r_1 \mod N \) can still have overflow which depends on the actual values of \( d \) and \( r_1 \). Nevertheless, in the later parts of this section, we show that for many practical applications, the above probabilistic assumption is very reasonable.

Once \( P_1 \) receives \( s_1 \) from \( P_2 \), he/she computes \( E_{pk}(d_1) \), encryption of the least significant bit of \( d \), depending on whether \( r_1 \) is even or odd as below:

- If \( r_1 \) is even, then \( E_{pk}(d_1) = s_1 = E_{pk}(\lambda_2) \).
- Else \( E_{pk}(d_1) = E_{pk}(1) \ast s_1^{N-1} \mod N^2 = E_{pk}(1 - \lambda_2) \)

\[1\] In Paillier cryptosystem, ciphertext multiplications are followed by modulo \( N^2 \) operation so that the resulting ciphertext is still in \( \mathbb{Z}_{N^2} \). However, to avoid cluttering the presentation, we simply omit the modulo operations.
Since $\lambda_1$ is assumed to be 0, following from Observation $[3]$ we have $\lambda_1 \oplus \lambda_2 = \lambda_2$. Also, note that “$N - 1$” is equivalent to “-1” under $\mathbb{Z}_N$. Then, $P_1$ updates $E_{pk}(d')$ to $E_{pk}(d_1)$. After this, $P_1$ updates $\delta$ to $E_{pk}\left(\left\lfloor \frac{d}{2} \right\rfloor \right)$, encryption of quotient when $d$ is divided by 2, by performing following homomorphic additions:

- $\Phi = \delta \cdot E_{pk}(d_1)^{N-1} = E_{pk}(d - d_1)$
- $\delta = \Phi^j = E_{pk}((d - d_1) * 2^{-1}) = E_{pk}\left(\left\lfloor \frac{d}{2} \right\rfloor \right)$

The main observation is that $d - d_1$ is always a multiple of 2; therefore, $(d - d_1) * 2^{-1}$ always gives the correct quotient under $\mathbb{Z}_N$. The above process is continued iteratively such that in iteration $i$, $P_1$ knows $E_{pk}(d') = E_{pk}(\sum_{j=1}^{i} d_j * 2^{-1})$ and updates $\delta$ accordingly, for $1 \leq i \leq m$.

In the last iteration, $P_1$ computes the encryption of difference between $d$ and $d'$ as $G = E_{pk}(d) * E_{pk}(d')^{N-1} = E_{pk}(d - d')$. Then, he/she randomizes $G$ by computing $G' = G^r$ and sends it to $P_2$, where $r$ is a random number in $\mathbb{Z}_N$. After this, $P_2$ decrypts $G'$ and sets $c' = 1$ if $D_{sk}(G') = 0$, and $c' = 0$ otherwise. Also, $P_2$ sends $E_{pk}(c')$ to $P_1$. Finally, depending on $F$, $P_1$ computes the output $E_{pk}(c)$ as below:

- If $F : x \geq y$, then set $E_{pk}(c)$ to $E_{pk}(c')$.
- Otherwise, compute the negation of $E_{pk}(c')$ and assign it to $E_{pk}(c)$. That is, $E_{pk}(c) = E_{pk}(1) * E_{pk}(c')^{N-1} = E_{pk}(1 - c')$.

**Example 1** Suppose $x = 1$, $y = 5$, and $m = 3$. Let us assume that $P_1$ holds $(E_{pk}(1), E_{pk}(5))$. Under this case, we show various intermediate results during the execution of the proposed SC protocol. Without loss of generality, we assume that $P_1$ chooses the functionality $F : y \geq x + 1$. Initially, $P_1$ computes $E_{pk}(d) = E_{pk}(3)$ and sets it to $\delta$.

Since $m = 3$, $SC_{P}$ computes $E_{pk}(d')$ in three iterations. For simplicity, we assume that $r_i$'s are even and there is no overflow. Note that, however, $r_i$ is different in each iteration.

**Iteration 1:**

- $\tau_1' = 3 + r_1 \mod N = \text{an odd integer}$
- $E_{pk}(d_1) = s_1 = E_{pk}(1)$
- $E_{pk}(d') = E_{pk}(d_1) = E_{pk}(1)$
- $\delta = E_{pk}\left(3 - 1 \cdot 2^{-1}\right) = E_{pk}(1)$

**Iteration 2:**

- $\tau_2' = 1 + r_2 \mod N = \text{an odd integer}$
- $E_{pk}(d_2) = s_2 = E_{pk}(1)$
- $E_{pk}(d') = E_{pk}(d') * E_{pk}(d_2)^2 = E_{pk}(3)$
- $\delta = E_{pk}\left(1 - 1 \cdot 2^{-1}\right) = E_{pk}(0)$

**Iteration 3:**

- $\tau_3' = r_3 \mod N = \text{an even integer}$
- $E_{pk}(d_3) = s_3 = E_{pk}(0)$
- $E_{pk}(d') = E_{pk}(d') * E_{pk}(d_3)^4 = E_{pk}(3)$
- $\delta = E_{pk}(0)$

At the end of the 3rd iteration, $P_1$ has $E_{pk}(d') = E_{pk}(3) = E_{pk}(d)$. After this, $P_1$ computes $G' = E_{pk}(r * (d - d')) = E_{pk}(0)$ and sends it to $P_2$. Upon receiving, $P_2$ decrypts $G'$ to get 0, sets $c'$ to 1, and sends $E_{pk}(c')$ to $P_1$. Finally, $P_1$ computes $E_{pk}(c) = E_{pk}(1 - c') = E_{pk}(0)$. It is clear that, since $x < y$, we have $c = 0$. 

\[\square\]
4.1 Correctness Analysis

In this sub-section, we theoretically prove that our SC_p scheme generates the correct result with very high probability. First, we emphasize that the correctness of SC_p depends on how accurately can P_1 and P_2 compute E_{pk}(d'). Since d' = \sum_{i=1}^{m} d_i \cdot 2^{m-1} is computed in an iterative fashion, this further implies that the correctness depends on the accuracy of the least m significant bits computed from d.

In each iteration, r_i can take any value in \mathbb{Z}_N. We observe that if r_i \in [N - 2^m, N), only then the corresponding computed encrypted bit of d_i, i.e., E_{pk}(d_i) can be wrong (due to overflow). That is, the number of possible values of r_i that can give rise to error are 2^m. Since we have N number of possible values for r_i, the probability for producing wrong bit is \frac{2^m}{2N} \approx \frac{1}{2^{K-m}}, where K is the encryption key size in bits. Therefore, the probability for computing the encryption of d_i correctly is approximately 1 - \frac{1}{2^{K-m}}. This probability remains the same for all the bits since r_i is chosen independently in each iteration. Hence, the probability for SC_p to compute the correct value of E_{pk}(d') is given by:

\[ \left(1 - \frac{1}{2^{K-m}}\right)^m \approx e^{-\frac{m}{2^{K-m}}} \]

In general, for many real-world applications, m can be at most 100 (since 0 \leq x, y < 2^{100} is sufficiently large enough to suit most applications). Therefore, for 1024-bit key size, the probability for SC_p to produce the correct output is approximately e^{-\frac{m}{2^{100}}} \approx 1. Hence, for practical domain values of x and y, with a probability of almost 1, the SC_p protocol gives the correct output E_{pk}(d'). We emphasize that even in the extreme case, such as m = 950, the probability for SC_p to produce correct E_{pk}(d') is e^{-\frac{m}{2^{100}}} \approx 1.

Additionally, following from Observation 1, the value of d - d' is equal to 0 iff the corresponding functionality under F is true. In practice, as mentioned above, we have K > m. When F is true, the property 0 \leq d, d' < 2^m holds and the integer corresponding to the m least significant bits of d is always equivalent to d'. Therefore, the decryption of G' = E_{pk}(r \cdot (d - d')) by P_2 will result in 0 iff F is true. In particular, when F : y \geq x + 1, the negation operation by P_1 makes sure that the final output is equal to E_{pk}(c). Hence, based on the above discussions, it is clear that the proposed SC_p scheme produces correct result with very high probability.

4.2 Security Analysis

4.2.1 Proof of Security under the Semi-honest Model

The security goal of SC_p is to prevent P_1 and P_2 from knowing x and y. In addition, the comparison result should be protected from both P_1 and P_2. Informally speaking, since d is the only value related to x and y, either d or part of d is always hidden by a random number; therefore, P_2 does not know anything about d. As a result, P_2 knows nothing about x and y. On the other hand, since P_1 does not have the decryption key and the comparison result is encrypted, P_1 does not know x \geq y or y \geq x + 1. Moreover, because P_1 randomly selects which functionality between x \geq y and y \geq x + 1 to compute, P_2 does not know the comparison result either. However, we may ask why to randomly select a functionality. If we do not, P_2 will know, for example, the first value is bigger than the second value. This seemingly useless information can actually allow P_2 to learn data access patterns which in turn breaks the semantic security of the underlying encryption scheme. Next we provide a formal proof of security for SC_p under the semi-honest model.

As stated in Section 2, to prove the security of the proposed protocol under the semi-honest setting, we adopt the well-known security definitions and techniques in the literature of secure multiparty computation. To formally prove SC_p is secure 25, we need to show that the simulated execution image of SC_p is computationally indistinguishable from the actual execution image of SC_p. An execution image generally includes the messages exchanged and the information computed from these messages. Therefore, according to Algorithm 1, the execution image of P_2 can be denoted by \Pi_{P_2}, where

\[ \Pi_{P_2} = \{ \langle E_{pk}(\delta + r_i), \delta + r_i \mod N, \langle G', b \rangle \rangle \text{ for } 1 \leq i \leq m \} \]

Note that \delta + r_i \mod N is derived from E_{pk}(\delta + r_i), where the modulo operator is implicit in the decryption function. P_2 receives G' at the last iteration and b denotes the decryption result of G'. Let the simulated image of P_2 be \Pi^{S}_{P_2}, where

\[ \Pi^{S}_{P_2} = \{ \langle s_i, s_i' \rangle, \langle s, b' \rangle \rangle \text{ for } 1 \leq i \leq m \} \]
Both $s_1^i$ and $s$ are randomly generated from $\mathbb{Z}_{N^2}$, and $s_2^i$ is randomly generated from $\mathbb{Z}_N$. Since $E_{pk}$ is a semantically secure encryption scheme with resulting ciphertext size less than $N^2$, $E_{pk}(\delta + r_i)$ and $G'$ are computationally indistinguishable from $s_1^i$ and $s$, respectively. Also, as $r_i$ is randomly generated, $\delta + r_i \mod N$ is computationally indistinguishable from $s_2^i$. Furthermore, because the functionality is randomly chosen by $P_1$ (at step 1(b) of Algorithm 1), $b$ is either 0 or 1 with equal probability. Thus, $b$ is computationally indistinguishable from $b'$. Combining all these results together, we can conclude that $\Pi_{P_2}$ is computationally indistinguishable from $\Pi_{P_1}^S$. This implies that during the execution of $SC_p$, $P_2$ does not learn anything about $x$ and $y$. Intuitively speaking, the information $P_2$ has during an execution of $SC_p$ is either random or pseudo-random, so this information does not disclose anything regarding $x$ and $y$.

Similarly, the execution image of $P_1$ can be denoted by $\Pi_{P_1}$, where
\[
\Pi_{P_1} = \{s_i, E_{pk}(c') | 1 \leq i \leq m\}
\]
Let the simulated image of $P_1$ be $\Pi_{P_1}^S$, where
\[
\Pi_{P_1}^S = \{s_i', s | 1 \leq i \leq m\}
\]
Both $s_i$ and $s$ are randomly generated from $\mathbb{Z}_{N^2}$. Since $E_{pk}$ is a semantically secure encryption scheme with resulting ciphertext size less than $N^2$, $s_i$ and $E_{pk}(c')$ are computationally indistinguishable from $s_i'$ and $s$, respectively. Therefore, $\Pi_{P_1}$ is computationally indistinguishable from $\Pi_{P_1}^S$. This implies that $P_1$ does not learn anything about the comparison result. Combining with previous analysis, we can say $SC_p$ is secure under the semi-honest model.

### 4.2.2 Security against Malicious Adversary

After proving that $SC_p$ is secure under the semi-honest model, the next step is to extend it to a secure protocol against malicious adversaries. Under the malicious model, an adversary (i.e., either $P_1$ or $P_2$) can arbitrarily deviate from the protocol to gain some advantage (e.g., learning additional information about inputs) over the other party. The deviations include, as an example, for $P_1$ (acting as a malicious adversary) to instantiate the $SC_p$ protocol with modified inputs ($E_{pk}(x'), E_{pk}(y')$) and to abort the protocol after gaining partial information. However, in $SC_p$, it is worth pointing out that neither $P_1$ nor $P_2$ knows the comparison result. In addition, all the intermediate results are either random or pseudo-random values. Thus, even when an adversary modifies the intermediate computations he/she cannot gain any information regarding $x$, $y$, and $c$. Nevertheless, as mentioned above, the adversary can change the intermediate data or perform computations incorrectly before sending them to the honest party which may eventually result in the wrong output. Therefore, we need to ensure that all the computations performed and messages sent by each party are correct. We now discuss two different approaches from the literature to extend the $SC_p$ protocol and make it secure under the malicious model.

The standard way of preventing the malicious party from misbehaving is to let the honest party validate the other party’s work using zero-knowledge proofs [14]. First of all, we stress that input modification in any secure protocol cannot be prevented [25]; therefore, we proceed as follows. On one hand, if $P_1$ is a malicious adversary and the input of $SC_p$ is generated as a part of an intermediate step, then the honest party (i.e., $P_2$) can validate it correctness using zero knowledge proofs. On the other hand, where $E_{pk}(x)$ and $E_{pk}(y)$ are not part of an intermediate step, we assume that the input is committed (e.g., explicitly certified by the data owner). Under this case, the honest party can validate the intermediate computations of $P_1$ based on the committed input. Also, we assume that there exist no collusion between $P_1$ and $P_2$ (i.e., at most one party is malicious). Note that such an assumption is necessary to construct secure protocols under the malicious model.

Recently, Nikolaenko et al. [37] discussed a mechanism for the honest party to validate the data sent by the adversary under (asymmetric) two-party setting. Their approach utilizes Pedersen commitments [42] along with the zero-knowledge proofs to prove modular arithmetic relations between the committed values. However, checking the validity of computations at each step of $SC_p$ can significantly increase the overall cost.

An alternative approach, as proposed in [32], is to instantiate two independent executions of the $SC_p$ protocol by swapping the roles of the two parties in each execution. At the end of the individual executions, each party receives the output in encrypted form. This is followed by an equality test on their outputs. More specifically, suppose $E_{pk_1}(c_1)$ and $E_{pk_2}(c_2)$ be the outputs received by $P_1$ and $P_2$ respectively, where $pk_1$ and $pk_2$ are their respective public keys.
A simple equality test on $c_1$ and $c_2$, which produces an output value of 1 if $c_1 = c_2$ and a random number otherwise, is sufficient to catch the malicious adversary. That is, the malicious party, which will be caught in the case of cheating, acts as a covert adversary [4]. Under the covert adversary model, the parties can shift the verification step until the end and then directly compare the final outputs. We emphasize that the equality test based on the additive homomorphic encryption properties which was used in [32] is not applicable to our problem. This is because, the outputs in our case are in encrypted format and the corresponding ciphertexts (resulted from the two executions) are under two different public key domains. Nevertheless, $P_1$ and $P_2$ can perform the equality test by constructing a garbled-circuit based on the similar steps as mentioned in the SC$_g$ protocol.

4.3 Performance Comparison of SC$_p$ with Existing Work

In this sub-section, we empirically compare the computation costs of SC$_p$ with those of SC$_g$. As discussed in Section 3.4, the SC$_g$ protocol is based on the Yao’s garbled-circuit technique. Besides SC$_g$, another well-known solution to the secure comparison of Paillier encrypted integers was proposed by Blake et al. [6]. However, as mentioned earlier, their protocol requires the encryptions of bits rather than pure integers as the inputs. Nevertheless, one could combine their protocol with the existing secure bit-decomposition (SBD) methods to solve the SC problem. Recently, Samanthula et al. [44] proposed a new SBD method and combined it with [6] to solve the SC problem. We denote such a construction by SC$_b$. To the best of our knowledge, SC$_b$ is the most efficient custom-designed method (under Paillier cryptosystem) to perform secure comparison over encrypted integers.

To better understand the efficiency gains of SC$_p$, we need to compare its computation costs with both SC$_g$ and SC$_b$. For this purpose, we implemented all the three protocols in C using Paillier’s scheme [38] and conducted experiments on a Intel® Xeon® Six-Core™ 3.07GHz PC with 12GB memory running Ubuntu 10.04 LTS. In particular to SC$_g$, we constructed and evaluated the circuit using FastGC [51] framework on the same machine. Since SC$_b$ is secure under the semi-honest model, in our implementation we assume the semi-honest setting for a fair comparison among the three protocols. That is, we did not implement the extensions to SC$_p$ that are secure under the malicious setting.

For encryption key size $K = 1024$ bits (a commonly accepted key size which also offers the same security guarantee as in FastGC [51]), the comparison results are as shown in Figure 1. Following from Figure 1 it is clear that the computation costs of both SC$_p$ and SC$_b$ grow linearly with the domain size $m$ (in bits) whereas the computation cost of SC$_g$ remains constant at 2.01 seconds. This is because, the SC$_g$ protocol uses random shares as input instead of encryptions of $x$ and $y$. On one hand, the computation costs of SC$_b$ varies from 1.58 to 7.96 seconds when $m$ is changed from 20 to 100. On the other hand, the computation costs of SC$_p$ increases from 0.23 to 1.16 seconds when $m$ is changed from 20 to 100. It is evident that SC$_p$ outperforms both the protocols irrespective of the value of $m$. Also, for all values of $m$, we observe that SC$_p$ is at least 6 times more efficient than SC$_b$. In addition, when $m = 20$, our SC$_p$ protocol is around 8 times more efficient than the circuit-based SC$_p$ protocol.

From a privacy perspective, it is important to note that SC$_p$ and SC$_g$ guarantee the same level of security by not revealing the input values as well as the comparison result to $P_1$ and $P_2$. Although SC$_b$ leaks the comparison result...
to at least one of the participating parties, as mentioned in Section 3.4, it can be easily modified to our setting at the expense of additional cost. Since SC_p provides similar security guarantee, but more efficient than SC_p and SC_b, we claim that SC_p can be used as a building block in larger privacy-preserving applications, such as secure clustering, to boost the overall throughput by a significant factor. Furthermore, we emphasize that our SC_p scheme is more reliable than SC_b in terms of round complexity. More specifically, SC_p require m+1 number of communication rounds whereas SC_b require 2m+1 number of communication rounds between P_1 and P_2. On the other hand, SC_g requires constant number of rounds. Nevertheless, we would like to point out that the round complexity of our SC_p scheme can be reduced to (small) constant number of rounds by using the Carry-Lookahead Adder with similar computation costs. Since the constant-round SC_p protocol is much more complex to present and due to space limitations, in this paper, we presented the SC_p protocol whose round complexity is bounded by O(m). Also, we emphasize that the efficiency of SC_p (and other SC protocols) can be improved further by using alternative HEnc^+ systems (e.g., [19]) which provide faster encryption than a Paillier encryption.

5 The Proposed PPRQ Protocols

In this section, we propose two novel PPRQ protocols over encrypted data in the cloud computing environment. Our protocols utilize the above-mentioned SC_p scheme and secure multiplication (SMP) as the building blocks. In addition, we analyze the security guarantees and complexities of the proposed protocols in detail. The two protocols act as a trade-off between efficiency and flexibility. In particular, our second protocol incurs negligible computation cost on the end-user.

Both protocols consider two cloud service providers denoted by C_1 (referred to as primary cloud) and C_2 (referred to as secondary cloud) which together form a federated cloud [12]. As justified in Section 2.2 for the rest of this paper, we assume that the probability of collusion between C_1 and C_2 is negligible (which is reasonable in practice). We emphasize that such an assumption has been commonly used in the related problem domains (e.g., [11]). The main intuition behind this assumption is as follows. Suppose the two servers can be implemented by two cloud service providers, such as Google and Amazon. Then it is hard to imagine why Google and Amazon want to collude to damage their reputation which could cost billions to repair.

Under the above cloud setting, Alice initially generates a Paillier public-secret key pair (pk, sk) and sends the secret key sk to C_2 through a secure channel whereas pk is treated as public information. Additionally, we explicitly make the following practical assumptions in our problem setting:

- Alice encrypts her database D attribute-wise using her public key pk. More specifically, she computes T_{i,j} = E_{pk}(t_{i,j}), where t_{i,j} denotes the j\textsuperscript{th} attribute value of data record t_i, for 1 ≤ i ≤ n and 1 ≤ j ≤ w. After this, she outsources the encrypted database T to C_1. It is important to note that the cost (both computation and communication) incurred on Alice during this step is a one-time cost. In the proposed protocols, after outsourcing T to C_1, Alice can remain offline since the entire query processing task is performed by C_1 and C_2.

- The attribute values lie in [0, 2\textsuperscript{m}], where m is the domain size of the attributes (in bits). In general, m may vary for each attribute. However, for security reasons, we assume that m is the same for all attributes. One way of selecting m is to take the maximum out of all attribute domain sizes. For the rest of this paper, we assume m is published.

- We assume that the number of data records (i.e., n) and attributes (i.e., w) can be revealed to the clouds. We emphasize that Alice can include some dummy records to D (to hide n) and dummy attributes to each record (to hide w). However, for simplicity, we assume that Alice does not add any dummy records and attributes to D. The values of n and w are treated as public.

- All parties are assumed to be semi-honest and there is no collusion between different parties. However, we stress that by combining the malicious SC_p protocol with zero-knowledge proofs, we can easily extend our protocols to secure protocols under the malicious model. Also, we assume that there exist secure communication channels

\[\text{Footnote 2: For a better security, the data owner Alice can mask m by adding a small random number m'} \text{ (where both m and m' are known only to Alice) to it. Under this case, the value of m + m' can be treated as public information.}\]
between each pair of parties involved in our protocols. Note that the existing secure mechanisms, such as SSL, can be utilized for this purpose.

- We assume that the set of authorized users (decided by Alice) who can access $D$ is known to $C_1$ and $C_2$. This is a practical assumption as it will also be useful for them to verify users’ identity during authentication \[16\]. We emphasize that the above assumptions are commonly made in the literature of related problem domains, and we do not make any abnormal assumptions.

### 5.1 Protocol 1

In the proposed first protocol, referred to as PPRQ$_1$, we assume that each authorized user generates a public-secret key pair. In particular, we denote Bob’s public-secret key pair by $(pk_b, sk_b)$.

After outsourcing the attribute-wise encrypted database of $D$ (i.e., $T$) by Alice to $C_1$, if at some future time, suppose Bob wants to perform a range query on the encrypted data in the cloud. Let $k$ be the attribute index upon which he wants to filter the records. During the query request step, he first computes the additive random shares of lower and upper bound values in his query. That is, he computes random shares $\{\alpha_1, \alpha_2\}$ and $\{\beta_1, \beta_2\}$ such that $\alpha = \alpha_1 + \alpha_2 \mod N$ and $\beta = \beta_1 + \beta_2 \mod N$, where $\alpha$ and $\beta$ are the lower and upper bound values of his range query. Note that $0 \leq \alpha, \beta < 2^n$. The goal here is for Bob to securely retrieve the data record $t_i$ only if $\alpha \leq t_{i,k} \leq \beta$, for $1 \leq i \leq n$. We emphasize that $\alpha$ and $\beta$ are private information of Bob; therefore, they should not be revealed to Alice, $C_1$ and $C_2$.

The overall steps involved in the proposed PPRQ$_1$ protocol are shown in Algorithm 2. To start with, Bob initially sends $\{k, \alpha_1, \beta_1\}$ and $\{\alpha_2, \beta_2\}$ to $C_1$ and $C_2$, respectively. Upon receiving $\{\alpha_1, \alpha_2\}$ from Bob\[3\], $C_2$ computes $\{E_{pk}(\alpha_1), E_{pk}(\alpha_2)\}$ and sends it to $C_1$. Then, $C_1$ computes the encrypted values of $\alpha$ and $\beta$ locally using additive homomorphic properties. That is, $C_1$ computes $E_{pk}(\alpha)$ as $E_{pk}(\alpha_1) \times E_{pk}(\alpha_2)$ and $E_{pk}(\beta)$ as $E_{pk}(\beta_1) \times E_{pk}(\beta_2)$. After this, $C_1$ and $C_2$ jointly involve in the following set of operations, for $1 \leq i \leq n$:

- Securely compare $T_{i,k}$, i.e., the encryption of $k^{th}$ attribute value of data record $t_i$ in $D$, with $E_{pk}(\alpha)$ and $E_{pk}(\beta)$ using the SC$_p$ protocol (in parallel). Without loss of generality, suppose $L_i = SC_p(T_{i,k}, E_{pk}(\alpha))$ and $M_i = SC_p(E_{pk}(\beta), T_{i,k})$. At the end of this step, the outputs $L_i$ and $M_i$, which are in encrypted format, are known only to $C_1$.

- Securely multiply $L_i$ and $M_i$ using the secure multiplication (SMP) protocol. The SMP protocol is one of the basic building blocks in the field of secure multiparty computation \[23\]. Briefly, given a party $P_1$ holding $(E_{pk}(a), E_{pk}(b))$ and a party $P_2$ with $sk$, the SMP protocol returns $E_{pk}(a \times b)$ to $P_1$. During this process, no information regarding $a$ and $b$ is revealed to $P_1$ and $P_2$. An efficient implementation of SMP is given in the Appendix. Let $O_i$ denote the output of SMP($L_i, M_i$). The observation here is $O_i = E_{pk}(1)$ only if $L_i = M_i = E_{pk}(1)$. This further implies that $t_{i,k} \geq \alpha$ and $\beta \geq t_{i,k}$. Otherwise, $O_i = E_{pk}(0)$. The output $O_i$ is known only to $C_1$. Since $O_i$ is an encrypted value, neither $C_1$ nor $C_2$ know whether the corresponding record $t_i$ matches the query condition $\alpha \leq t_{i,k} \leq \beta$.

- Generate a dataset $T'$ such that $T_{i,j}' = SMP(T_{i,j}, O_i)$, for $1 \leq j \leq w$. We emphasize that $T_{i,j}' = T_{i,j}$ iff $O_i$ is an encryption of 1. That is, if index $i$ satisfies the property $\alpha \leq t_{i,k} \leq \beta$, then $T_{i,j}' = T_{i,j}$. Otherwise, all the entries in $T_{i,j}'$ are encryptions of 0’s. At the end, the output $T'$ is known only to $C_1$.

After this, $C_1$ locally involves in the following set of operations, for $1 \leq i \leq n$ and $1 \leq j \leq w$:

- Randomize $T_{i,j}'$ using additive homomorphic property to get $U_{i,j} = T_{i,j}' \times E_{pk}(r_{i,j})$, where $r_{i,j}$ is a random number in $\mathbb{Z}_N$. Also, encrypt the random number $r_{i,j}$ using Bob’s public key $pk_b$ to get $V_{i,j} = E_{pk}(r_{i,j})$.

- Perform a row-wise permutation on $U$ and $V$ to get $X = \pi(U)$ and $Y = \pi(V)$. Here $\pi$ is a random permutation function known only to $C_1$. Also, $C_1$ randomly permutes the vector $O$, i.e. he/she computes $Z = \pi(O)$. Then, $C_1$ sends $X, Y$, and $Z$ to $C_2$.

\[3\] Note that if Bob is not an authorized user (which is usually decided by Alice), then $C_2$ simply dumps the query request of Bob.
Algorithm 2 PPRQ₁(T, Q) → S

Require: sk is known only to Alice and C₂; sk₀ is known only to Bob; whereas pk and pk₀ are public; π is known only to C₁; Q = \{k, \alpha, \beta\} is private to Bob

{Step 1 - Query Request}
1: Bob:
   (a). \( \alpha_1 + \alpha_2 \mod N \leftarrow \alpha \) and \( \beta_1 + \beta_2 \mod N \leftarrow \beta \)
   (b). Send \{k, \alpha_1, \beta_1\} to C₁ and \{\alpha_2, \beta_2\} to C₂

{Steps 2 to 5 - Data Processing}
2: C₂ sends \{E_{pk}(\alpha_2), E_{pk}(\beta_2)\} to C₁
3: C₁:
   (a). \( E_{pk}(\alpha) \leftarrow E_{pk}(\alpha_1) \cdot E_{pk}(\alpha_2) \) and \( E_{pk}(\beta) \leftarrow E_{pk}(\beta_1) \cdot E_{pk}(\beta_2) \)
4: C₁ and C₂, for \( 1 \leq i \leq n \) do:
   (a). \( L_i \leftarrow SC_p(T_{i,k}, E_{pk}(\alpha)) \), here only C₁ receives \( L_i \)
   (b). \( M_i \leftarrow SC_p(E_{pk}(\beta), T_{i,k}) \), here only C₁ receives \( M_i \)
   (c). \( O_i \leftarrow \text{SMP}(L_i, M_i) \), here \( O_i \) is known only to C₁
   (d). \( T'_{i,j} \leftarrow \text{SMP}(T_{i,j}, O_i) \), for \( 1 \leq j \leq w \)
5: C₁:
   (a). for \( 1 \leq i \leq n \) and \( 1 \leq j \leq w \) do:
      • \( U_{i,j} \leftarrow T'_{i,j} \cdot E_{pk}(r_{i,j}) \), where \( r_{i,j} \in R \mathbb{Z}_N \)
      • \( V_{i,j} \leftarrow E_{pk_0}(r_{i,j}) \)
   (b). Row-wise permutation: \( X \leftarrow \pi(U) \) and \( Y \leftarrow \pi(V) \)
   (c). \( Z \leftarrow \pi(O) \)
   (d). Send \( X, Y \) and \( Z \) to C₂

{Step 6 - Query Response}
6: C₂, for \( 1 \leq i \leq n \) do:
   (a). if \( D_{sk}(Z_i) = 0 \) then:
      • \( x_{i,j} \leftarrow D_{sk}(X_{i,j}) \), for \( 1 \leq j \leq w \)
      • Send \( (x_i, Y_i) \) to Bob
   else Ignore \( X_i \) and \( Y_i \)

{Step 7 - Data Decryption}
7: Bob:
   (a). \( S \leftarrow \emptyset \)
   (b). foreach entry \( (x_i, Y_i) \) received from C₂ do:
      • \( \gamma_{i,j} \leftarrow D_{sk_0}(Y_{i,j}) \), for \( 1 \leq j \leq w \)
      • \( t'_j \leftarrow x_{i,j} - \gamma_{i,j} \mod N \), for \( 1 \leq j \leq w \)
      • \( S \leftarrow S \cup t'_j \)
Upon receiving, $C_2$ filters the entries of $X$ and $Y$ using $Z$ as follows. We observe that if $D_{sk}(Z_i) = 1$, i.e., $O_i = L_i = M_i = E_{pk}(1)$, then the $k^{th}$ column value of $t_{z^{-1}(i)}$ satisfies the input range query condition, for $1 \leq i \leq n$. This is because, when $L_i = E_{pk}(1)$, we have $t_{z^{-1}(i),k} \geq \alpha$. On the other hand, when $M_i = E_{pk}(1)$, we have $t_{z^{-1}(i),k} \leq \beta$. Therefore, when $O_i = L_i = M_i = E_{pk}(1)$, the desired condition $\alpha \leq t_{z^{-1}(i),k} \leq \beta$ always holds. Hence, under this case, $C_2$ decrypts $X_i$ attribute-wise to get $x_{i,j} = D_{sk}(X_{i,j})$, for $1 \leq j \leq w$, and sends the entry $(x_{i,j}, Y_i)$ to Bob. Observe that $x_{i,j}$ is a random number in $Z_N$. On the other hand, if $D_{sk}(Z_i) = 0$, we have $L_i = E_{pk}(0)$ or $M_i = E_{pk}(0)$; therefore, the corresponding $k^{th}$ column value does not lie in $(\alpha, \beta)$. Hence, under this case, $C_2$ simply ignores $X_i$ and $Y_i$. Note that since $Z$ is a randomly permuted vector of $O$ and as $\pi$ is known only to $C_1$, $C_2$ cannot trace back which data record in $D$ corresponds to $Z_i$.

After receiving the entries (if there exist any) from $C_2$, Bob initially sets the output set $S$ to $\emptyset$. Then, he proceeds as follows for each received entry $(x_{i,j}, Y_i)$ and $1 \leq j \leq w$:

- By using his secret key $sk_b$, decrypt $Y_i$ attribute-wise to get $\gamma_{i,j} = D_{sk_b}(Y_{i,j})$.
- Remove randomness from $x_{i,j}$ to get $t'_j = x_{i,j} - \gamma_{i,j}$. Based on the above discussions, it is clear that $t'$ will be a data record in $D$ that satisfies the input range query $Q$, i.e., $\alpha \leq t'_k \leq \beta$ always holds.
- Finally, Bob adds the data record $t'$ to his output set: $S = S \cup t'$.

### 5.1.1 Security Analysis

Informally speaking, during the query request step of Bob, only the additive random shares of the boundary values (i.e., $\alpha$ and $\beta$) are sent to $C_1$ and $C_2$. That is, $\alpha$ and $\beta$ are never revealed to Alice, $C_1$ and $C_2$. However, the attribute index $k$ upon which he wants to execute the range query is revealed to $C_1$ for efficiency reasons. Also, since $C_1$ does not have the decryption key and as all the values it receives are in encrypted form, $C_1$ cannot learn anything about the original data. In addition, the information $C_2$ has is randomized by adding randomly chosen numbers. Thus, $C_2$ does not learn anything about the original data either. Because each data record is encrypted attribute-wise, the index $k$, the number of attributes, and the size of the database do not violate semantic security of the encryption scheme. Therefore, the privacy of Bob is always preserved.

To formally prove the security of PPRQ1 under the semi-honest model, we need to use the Composition Theorem given in [25]. The theorem says that if a protocol consists of sub-protocols, the protocol is secure as long as the sub-protocols are secure plus all the intermediate results are random or pseudo-random. Using the same proof strategies presented in Section 4.2, we can easily show that the messages seen by $C_1$ and $C_2$ during steps 2, 3, 5 and 6 of Algorithm 2 are pseudo-random values. In addition, as proved earlier, the SC$_p$ scheme is secure, and the SMP protocol given in the Appendix is secure since all the intermediate values are computationally indistinguishable from random values. Using the Composition Theorem, we can claim PPRQ1 is secure under the semi-honest model. In a similar fashion, by utilizing the SC$_p$ and SMP protocols that are secure against malicious adversaries, we can construct a PPRQ1 protocol that is secure under the malicious model.

In the PPRQ1 protocol, the data access patterns are protected from both $C_1$ and $C_2$. First, although the outputs of SC$_p$ and SMP are revealed to $C_1$, they are in encrypted format. Therefore, the data access patterns are protected from $C_1$. In addition, even though the vector $Z$ is revealed to $C_2$, it cannot trace back to the corresponding data records due to the random permutation of $O$ by $C_1$. Thus, the data access patterns are further protected from $C_2$. Also, due to randomization by $C_1$, contents of $D$ are never disclosed to $C_2$. However, we emphasize that the value of $k$ (part of $Q$) is revealed to $C_1$ for efficiency reasons. Also, $C_2$ will know the size of the output set $|S|$, i.e., the number of data records satisfying the input range query $Q$. At this point, we believe that $|S|$ can be treated as minimal information as it will not be helpful for $C_2$ to deduce any information regarding $\alpha, \beta$, and contents of $D$. Hence, we claim that the PPRQ1 protocol preserves the semantic security of the underlying encryption scheme.

### 5.1.2 Computation Complexity

In the proposed PPRQ1 protocol, for each record $t_i$, $C_1$ and $C_2$ jointly execute SC$_p$ and SMP as sub-routines twice and $w + 1$ times, respectively. Also, $C_1$ has to randomize the attribute values of each record (which requires $w$ encryptions). In addition, he/she has to encrypt the corresponding random values using Bob’s public key. This requires $w$ encryptions per record. Furthermore, $C_2$ has to perform $w$ decryptions for each output record. Therefore, for $n$ records, the
computation cost of the federated cloud (i.e., the combined cost of $C_1$ and $C_2$) is bounded by $O(n)$ instantiations of SC$_p$, $O(w \ast n)$ instantiations of SMP, and $O(w \ast n)$ encryptions (assuming that the encryption and decryption times are almost the same under Pailler’s scheme).

On the other hand, Bob’s computation cost mainly depends on the data decryption step in PPRQ in which he has to perform $w$ decryptions for each record in $S$. Hence, Bob’s total computation cost in PPRQ$_1$ is bounded by $O(w \ast |S|)$ encryptions (under the assumption that time for encryption and decryption are the same under Pailler’s scheme). Plus, assuming the constant-round SC$_p$ protocol, we claim that PPRQ$_1$ is also bounded by a constant number of rounds. For large values of $|S|$ (which depends on the query $Q$ and database $D$), Bob’s computational cost can be high. Therefore, with the goal of improving Bob’s efficiency, we present an alternate PPRQ protocol in the next sub-section.

5.2 Protocol 2

Similar to PPRQ$_1$, the proposed second protocol (referred to as PPRQ$_2$) consists of two cloud providers $C_1$ and $C_2$ where Alice outsources her encrypted database to $C_1$. However, unlike PPRQ$_1$, there is no need for Bob to generate a public-secret key pair in PPRQ$_2$. Instead, we assume that Alice shares her secret key $sk$ between $C_1$ and $C_2$ using threshold-based (Paillier) cryptosystem [18]. More specifically, let $sk_1$ and $sk_2$ be the shares of $sk$ such that Alice sends $sk_1$ and $sk_2$ to $C_1$ and $C_2$, respectively. By doing so, PPRQ$_2$ aims at shifting the total expensive operations obliviously between the two clouds; thereby, improving the efficiency of Bob in comparison to that of in PPRQ$_1$. That is, the user Bob in PPRQ$_2$ can take full advantage of cloud computing at the expense of additional cost on the federated cloud. Note that, under the above threshold cryptosystem [18], a decryption operation requires the participation of both parties.

We emphasize that the building blocks utilized in this paper, i.e., SC$_p$ and SMP, can be easily extended to the threshold-based setting with the same security guarantee and outputs. Without loss generality, let TSC$_p$ and TSMP denote the corresponding protocols constructed for SC$_p$ and SMP under the threshold-based setting.

The main steps involved in the proposed PPRQ$_2$ protocol are highlighted in Algorithm 5. To start with, upon receiving Bob’s query request, $C_1$ and $C_2$ in the TSC$_p$ and TSMP protocols to compute $O$ and $T’$. This process is similar to steps 1 to 4 of PPRQ$_1$. Note that, at the end of this step, only $C_1$ knows $O$ and $T’$. After this, $C_1$ randomizes the entries of $T’$ attribute-wise and also encrypts the corresponding random factors using the public key $pk$. That is, he/she computes $U_{i,j} = T’_{i,j} \ast E_{pk}(r_{i,j})$ and $H_{i,j} = E_{pk}(r_{i,j})$, for $1 \leq i \leq n$ and $1 \leq j \leq w$, where $r_{i,j}$ is a random number in $Z_N$. Also, $C_1$ partially decrypts $O$ component-wise using his/her secret key share $sk_1$ to get $O’_i = D_{sk_1}(O_i)$, for $1 \leq i \leq n$. Then, $C_1$ performs a row-wise permutation on $U$ and $H$ to get $X = \pi(U)$ and $W = \pi(H)$, respectively. Here $\pi$ is a random permutation function known only to $C_1$. In addition, $C_1$ randomly permutes the vector $O’$ to get $Z = \pi(O’)$. Then, $C_1$ sends $X, W$, and $Z$ to $C_2$.

Upon receiving, $C_2$ filters the entries of $(X, W)$ using $Z$ and proceeds as follows:

- Decrypt each entry in $Z$ using his/her secret key share $sk_2$ and check whether it is 0 or 1. Similar to PPRQ$_1$, if $D_{sk_2}(Z_i) = 1$, then we observe that the corresponding data record $X_i$ satisfies the range query condition. Under this case, $C_2$ randomizes both $X_i$ and $W_i$ attribute-wise using Alice’s public key $pk$. More specifically, he/she computes $X’_{i,j} = X_{i,j} \ast E_{pk}(r’_{i,j})$ and $Y’_{i,j} = W_{i,j} \ast E_{pk}(r’_{i,j})$, where $r’_{i,j}$ is a random number in $Z_N$ known only to $C_2$. Then, $C_2$ partially decrypts $Y’_{i,j}$ to get $W’_{i,j} \leftarrow D_{sk_2}(Y’_{i,j})$ and sends $(X’, W’)$ to $C_1$.

- On the other hand, if $D_{sk_2}(Z_i) = 0$, then the corresponding data record $X_i$ do not satisfy the query condition. Therefore, $C_2$ simply ignores $(X_i, W_i)$.

Now, for each received entry $(X’, W’)$, $C_1$ performs the following set of operations to compute the encrypted versions of data records that satisfy the query condition locally:

- Decrypt $W’$ attribute-wise using his/her secret key share $sk_1$. That is, compute $h_{i,j} = D_{sk_1}(W’_{i,j})$. Observe that $h_{i,j} = r_{i,j} + r’_{i,j}$ mod $N$, where $1 \leq j \leq w$ and $r’_{i,j}$ is known only to $C_2$.

- Remove the random factors (within the encryption) from $X’$ attribute-wise by computing $H_{i,j} = X’_{i,j} \ast E_{pk}(N - h_{i,j})$. Note that $N - h_{i,j}$ is equivalent to $-h_{i,j}$ under $Z_N$. By the end of this step, $C_1$ has encrypted data records $H_i$ that satisfy Bob’s range query.
Algorithm 3 PPRQ$_2(T, Q) \rightarrow S$

**Require:** $sk$ is private to Alice; $sk_1$ and $\pi$ are private to $C_1$; $sk_2$ is private to $C_2$; $Q = \{k, \alpha, \beta\}$ is private to Bob

Steps 1 to 4 are the same as in PPRQ$_1$

5: $C_1$: 

(a). **for** $1 \leq i \leq n$ and $1 \leq j \leq w$ **do**:

- $U_{i,j} \leftarrow T'_{i,j} \ast E_{pk}(r_{i,j})$, where $r_{i,j} \in R \mathbb{Z}_N$
- $H_{i,j} \leftarrow E_{pk}(r_{i,j})$

(b). $O'_i \leftarrow D_{sk_1}(O_i)$, for $1 \leq i \leq n$

(c). Row-wise permutation: $X \leftarrow \pi(U)$ and $W \leftarrow \pi(H)$

(d). $Z \leftarrow \pi(O')$; send $X, W$ and $Z$ to $C_2$

6: $C_2$, **for** $1 \leq i \leq n$ **do**:

(a). if $D_{sk_2}(Z_i) = 1$ then:

- **for** $1 \leq j \leq w$ **do**:
  - $X'_{i,j} \leftarrow X_{i,j} \ast E_{pk}(r'_{i,j})$, where $r'_{i,j} \in R \mathbb{Z}_N$
  - $Y'_{i,j} \leftarrow W_{i,j} \ast E_{pk}(r'_{i,j})$
  - $W'_{i,j} \leftarrow D_{sk_2}(Y'_{i,j})$

- Send $(X'_{i}, W'_{i})$ to $C_1$

else Ignore $(X_{i}, W_{i})$

7: $C_1$, **foreach** received entry $(X'_{i}, W'_{i})$ from $C_2$ **do**:

(a). **for** $1 \leq j \leq w$ **do**:

- $h_{i,j} \leftarrow D_{sk_2}(W'_{i,j})$
- $H_{i,j} \leftarrow X'_{i,j} \ast E_{pk}(N - h_{i,j})$
- $H'_{i,j} \leftarrow H_{i,j} \ast E_{pk}(\hat{r}_{i,j})$, where $\hat{r}_{i,j} \in R \mathbb{Z}_N$
- $\Phi_{i,j} \leftarrow D_{sk_1}(H'_{i,j})$; send $\Phi_{i,j}$ to $C_2$ and $\hat{r}_{i,j}$ to Bob

8: $C_2$, **foreach** received entry $\Phi_i$ from $C_1$ **do**:

(a). **for** $1 \leq j \leq w$ **do**:

- $\Gamma_{i,j} \leftarrow D_{sk_2}(\Phi_{i,j})$; send $\Gamma_{i,j}$ to Bob

9: Bob:

(a). $S \leftarrow \emptyset$

(b). **foreach** received entry $(\Gamma_{i,j}, \hat{r}_{i,j})$ **do**:

- $t'_j \leftarrow \Gamma_{i,j} - \hat{r}_{i,j} \mod N$, for $1 \leq j \leq w$
- $S \leftarrow S \cup t'_j$

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Now, $C_1$ randomizes $H_i$ attribute-wise to get $H'_{i,j} = H_{i,j} \ast E_{pk}(r_{i,j})$, for $1 \leq j \leq w$. Here $r_{i,j}$ is a random number in $\mathbb{Z}_N$ known only to $C_1$. Also, $C_1$ partially decrypts $H'_i$ attribute-wise to get $\Phi_{i,j} = D_{sk_1}(H'_{i,j})$, sends $\Phi_{i,j}$ to $C_2$ and $r_{i,j}$ to Bob, for $1 \leq j \leq w$.

In addition, for each received entry $\Phi_{i,j}$, $C_2$ decrypts it attribute-wise to get $\Gamma_{i,j} = D_{sk_2}(\Phi_{i,j})$ and sends the results to Bob. Note that, due to randomization by $C_1$, $\Gamma_{i,j}$ is always a random number in $\mathbb{Z}_N$.

Finally, for each received entry pair $(\Gamma_{i,j}, r_{i,j})$, Bob retrieves the corresponding output record and proceeds as below:

- Remove randomness from $\Gamma_i$ attribute-wise to get $t'_i = \Gamma_{i,j} - r_{i,j} \mod N$, for $1 \leq j \leq w$. We observe that $t'_i \in D$ and the property $\alpha \leq t'_i \leq \beta$ always holds.
- Include data record $t'_i$ to the output set: $S = S \cup t'_i$.

### 5.2.1 Security Analysis

The security proof of PPRQ$_2$ is similar to that of PPRQ$_1$. Briefly, due to random permutation of $O'$ by $C_1$; $C_2$ cannot trace back to the data records satisfying the query condition. In addition, as the comparison results (in encrypted form) are known only to $C_1$ who does not have access to the secret key $sk$, the data access patterns are protected from $C_1$. Therefore, we claim that the data access patterns are protected from both $C_1$ and $C_2$. Furthermore, no other information regarding the contents of $D$ is revealed to the cloud service providers since the intermediate decrypted values are random in $\mathbb{Z}_N$. However, in PPRQ$_2$, $k$ (part of $Q$) is revealed to $C_1$ whereas $|S|$ is revealed to $C_1$ and $C_2$. As mentioned earlier in the security analysis of PPRQ$_1$, this is treated as a minimal information leakage since it cannot be used to break the semantic security of the encryption scheme.

### 5.2.2 Computation Complexity

The computation cost of the federated cloud (i.e., the combined cost of $C_1$ and $C_2$) in PPRQ$_2$ is bounded by $O(n)$ instantiations of TSC, $O(w+n)$ instantiations of TSMP and $O(w*(n+|S|))$ encryptions and decryptions. In general, assuming the decryption time under threshold cryptosystem is (at most) two times more than an encryption operation, the computation cost of the federated cloud in PPRQ$_2$ is (at most) twice to that of PPRQ$_1$. However, unlike PPRQ$_1$, during the data retrieval step of PPRQ$_2$, Bob does not perform any decryption operations. Thus, the computation cost of Bob in PPRQ$_2$ is negligible compared that of in PPRQ$_1$. Remember that Bob's computation cost in the PPRQ$_1$ protocol is bounded by $O(w*|S|)$ decryptions.

At first, it seems that the proposed PPRQ protocols are costly and may not scale well for large databases. However, we stress that the computations involved on each data record are fully independent of others. In particular, the execution of sub-routines SC and SMP (similarly, TSC and TSMP) on a data record does not depend on the operations of other data records. Therefore, in the cloud computing environment where high performance parallel processing can be easily achieved using multiple cores, we believe that the scalability issue in the proposed PPRQ protocols can be eliminated or mitigated. Furthermore, by using the existing MapReduce techniques (such as Hadoop) in the cloud, the performance of the proposed PPRQ protocols can be improved drastically. We leave the above low-level implementation details for future work. Nevertheless, the main advantages of the proposed PPRQ protocols are that they protect data confidentiality and privacy of user’s input query. In addition, they protect data access patterns and in particular PPRQ$_2$ incurs negligible computation cost on the end-user.

### 6 Conclusions

Query processing in distributed databases has been well-studied in the literature. In this paper, we focus on the privacy-preserving range query (PPRQ) problem over encrypted data in the cloud. We observed that most of the existing PPRQ methods reveal valuable information, such as data access patterns, to the cloud provider; thus, they are not secure from both data owner and query issuer’s perspective.

In general, the basic security primitive that is required to solve the PPRQ problem is the secure comparison (SC) of encrypted integers. Since the existing SC methods (both custom-designed and garbled-circuit approaches) are not efficient, we first proposed a new probabilistic SC scheme that is more efficient than the current state-of-the-art SC
protocols. Then, we proposed two novel PPRQ protocols by using our SC scheme as the building block under the cloud computing environment. Besides ensuring data confidentiality, the proposed PPRQ protocols protect query privacy and data access patterns from the cloud service providers. In addition, from end-user’s perspective, our second protocol is significantly more efficient than our first protocol.

In this work, we proposed the SC scheme whose round complexity is bounded by \( O(m) \). Therefore, developing a constant round SC protocol using Carry-Lookahead Adders will be the primary focus of our future work. Another interesting direction is to extend our PPRQ protocols to multi-dimensional range queries and analyze their trade-offs between security and efficiency. We will also investigate alternative methods and extend our work to other complex conjunctive queries.

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Possible Implementation of SMP. Consider a party $P_1$ with private input $(E_{pk}(a), E_{pk}(b))$ and a party $P_2$ with the secret key $sk$. The goal of the secure multiplication (SMP) protocol is to return the encryption of $a * b$, i.e., $E_{pk}(a * b)$ as the output to $P_1$. During this protocol, no information regarding $a$ and $b$ should be revealed to $P_1$ and $P_2$. First, we emphasize that one can construct a SMP protocol by using the garbled-circuit technique. However, we observe that our custom-designed SMP protocol (as explained below) is more efficient than the circuit-based method. The basic idea of our SMP protocol is based on the following property which holds for any given $a, b \in \mathbb{Z}_N$:

$$a * b = (a + r_a) * (b + r_b) - a * r_b - b * r_a - r_a * r_b$$

(1)

where all the arithmetic operations are performed under $\mathbb{Z}_N$. The overall steps involved in the proposed SMP protocol are shown in Algorithm 4. Briefly, $P_1$ initially randomizes $a$ and $b$ by computing $a' = E_{pk}(a) * E_{pk}(r_a)$ and $b' = E_{pk}(b) * E_{pk}(r_b)$, and sends them to $P_2$. Here $r_a$ and $r_b$ are random numbers in $\mathbb{Z}_N$ known only to $P_1$. Upon receiving, $P_2$ decrypts and multiplies them to get $h = (a + r_a) * (b + r_b) \mod N$. Then, $P_2$ encrypts $h$ and sends it to $P_1$. After this, $P_1$ removes extra random factors from $h' = E_{pk}((a + r_a) * (b + r_b))$ based on Equation 1 to get $E_{pk}(a * b)$. Note that, under Paillier cryptosystem, “$N - x$” is equivalent to “$-x$” in $\mathbb{Z}_N$.

**Algorithm 4 SMP$(E_{pk}(a), E_{pk}(b)) \rightarrow E_{pk}(a * b)$**

**Require:** $P_1$ has $E_{pk}(a)$ and $E_{pk}(b)$; $P_2$ has $sk$

1. $P_1$:
   - (a). Pick two random numbers $r_a, r_b \in \mathbb{Z}_N$
   - (b). $a' \leftarrow E_{pk}(a) * E_{pk}(r_a)$
   - (c). $b' \leftarrow E_{pk}(b) * E_{pk}(r_b)$; send $a', b'$ to $P_2$

2. $P_2$:
   - (a). $h_a \leftarrow D_{sk}(a')$; $h_b \leftarrow D_{sk}(b')$
   - (b). $h \leftarrow h_a * h_b \mod N$
   - (c). $h' \leftarrow E_{pk}(h)$; send $h'$ to $P_1$

3. $P_1$:
   - (a). $s \leftarrow h' * E_{pk}(a)^{N-r_b}$
   - (b). $s' \leftarrow s * E_{pk}(b)^{N-r_a}$
   - (c). $E_{pk}(a * b) \leftarrow s' * E_{pk}(r_a * r_b)^{N-1}$