Research and modeling of wrinkles and control of rectangular membrane structures with high-class modeling in on-orbit conditions

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Abstract. In the previous paper, the authors described a mathematical model of the process, this paper is devoted to the issues of modeling and analysis. The effects of different parameters such as temperature gradients, membrane’s aspect ratio, pre-stress, thickness, anchor points, and elastic properties of the membrane and the multilayer on the characteristic wrinkling behavior (wavelength, size, critical load and location) have been examined. It is found that the critical buckling condition of the membrane is dependent on the compressibility, anchor points, aspect ratio and the thickness of the membrane, and it’s out of plane displacement depends on the magnitude of the applied in-plane tension. The number of wrinkles decreased with increasing the number of anchor points of the membrane. An effective active flatness control method is used to minimize the surface error (RMS error). Also, it is seen that the multilayer geometry helps in reducing the surface error of the membrane structure.

1. Introduction
The large space structures are required for the deep space exploration, high speed communication, and accurate earth observation. But the weight of the payload increases when the size of structures increase in the conventional space structures and probability of the successes decreases. Also, the operation cost required to transport mass to the destination is high as well as the volume of payload is limited. Hence, achieving minimal launch mass and volume of space structures is always an important task for the space mission. [1]–[5]. Therefore, ultra-lightweight and ultra-large membrane structures are the key element of inflatable reflector, solar sail, solar collector, sun-shields, deployable mirrors surface for the next generation of spacecraft. Many such membrane based space structures have been tested like IN-Step Inflatable antenna experiment [6], [7]. IKAROS [8], CubeSat [9], [10].

Thin membranes are often subjected to stresses and strains when an external force applied. Membrane structure is very flexible and has strong geometric nonlinearity and can leads to an out of plane displacement due to their negligible flexural rigidity and presence of in-plane compressive stresses [2], [11]–[14]. Hence it is essential and desirable for a better understanding of the effects of wrinkles on the structural performance and its stability. Surface wrinkling may also be utilized to develop techniques for probing the surface characteristics of materials. [13], [15], [16].
Many authors have discussed the various analytical, numerical and experimental methods to study the wrinkling behavior of thin membrane structures [16]–[25]. Reissner (1938) derived the tension field theory without initial tension under in-plane torsion to and analyzed the circular membrane [26]. Stein and Hedgepeth (1961) to analyze partly wrinkled membranes using Stein’s theory [27]. Using the Rayleigh-Ritz method, the circular stretched membrane is analyzed without bending rigidity, but geometrical stiffness considered by [28]. Damil et al [29] have discussed a wrinkling phenomenon using a new multiscale approach taking one-dimensional beam model. Blandino et al [30] have discussed the wrinkling phenomena at the isothermal condition and prove that number of wrinkles increase with the tensile loads. Also, the amplitude of wrinkles increases with increase in thermal gradient. The full shell methods and the reduced resolution methods can be used to solve the wrinkle problems. For the thermo-mechanical load, Kodjo et al [31] have modified Roddamen’s model [18] to considering thermal effect in the model.

In this paper, thermal effect on flat membrane antenna has been discussed at orbital conditions. Also, authors discussed the wrinkling analysis of the membrane structures under different loading conditions, the effect of prestress, thickness variation, elastic properties, anchor points. Thermal stresses have been introduced in the classical equation of within the framework of the elastic shell theory. Here Asymptotic Numerical Method is used to solve the nonlinear equations and effective active flatness control method is used to minimize the RMS error of the surface wrinkles of the membrane structure. Also, the analysis of bi-layer structure has been performed to minimize the wrinkled area.

2. Wrinkles Details
The deformed shape under these unsymmetrical loading is shown in Fig. 1, and it is significantly different from symmetric loading a continuous, large diagonal wrinkle goes between the two more heavily loaded corners. In addition to this diagonal wrinkle, fans of small wrinkles can still be seen near the other two corners. The out-of-plane displacements in this plot have been amplified 100 times, for clarity of wrinkle details. The comparison of out of plane displacement for all $T_1/T_2$ ratios are shown in Fig. 2 at distance 0.280 m from the corner.

3. RMS Error Calculation
The RMS error of wrinkle amplitude has been calculated by.

$$\text{RMS Error} = \sqrt{\frac{\sum(x-x)^2}{n}}$$

Where $\bar{x}$ the mean value of amplitude and $n$ is the total number of nodes on the membrane structure. First RMS error has been calculated for all cases ($T_1/T_2=1$ to 5) and plotted in with respect to $T_1/T_2$ ratios as shown in Fig. 3. Graph clearly shows that the RMS error increases with increases in the load ratios ($T_1/T_2$).

Figure 1. Wrinkles on square membrane with symmetric loading $T_1/T_2=1$ & asymmetric loading ($T_1/T_2\neq1$)
Figure 2. Out of plane displacement with respect to different $T_1/T_2$ ratio of 0.5 m x 0.5 m square membrane.

Figure 3. Comparison of RMS Error with respect to $T_1/T_2$ ratios.

Figure 4. Comparison of Analytical, Numerical and Experimental Results of the 0.5 m x 0.5 m membrane.
4. Comparison of Numerical and Experimental Results of Square Membrane

In this sub-section, numerical and experimental are compared with under various loading conditions the wrinkles details. Fig. 4 shows the comparison of wrinkles amplitude (out–of-plane displacement) of 0.5 m x 0.5 m Aluminized Kapton membrane under corner tensile loading at 170 mm rom the corner. Experimental plot confirms the validation of numerical and analytical model. The RMS error of same is listed in Table 1. Similarly, Out-of-plane displacement of the 1.0 m x 1.0 m square membrane is compared at 250 mm from the corner for $T_1/T_2=1$ as shown in Fig 5. At Fig. 6 shows the comparison of winkle’s amplitude along midway of the membrane side for $T_1/T_2=5$. The comparison of experiment and numerical value of amplitude of wrinkles at 170 mm from the corner of the Kapton membrane with size of 0.5 m x 0.5 m under different $T_1/T_2$ ratios are shown in Fig. 7. The RMS error of different size of membrane with various $T_1/T_2$ ratios are compared in the Table 2. Previously, it confirmed that RMS error increase with increase in $T_1/T_2$ ratios. RMS error increase with increase in size of membrane.

| Table 1. RMS error comparison of 0.5 mx 0.5 m membrane |
|-----------------------------------------------|
|                  | RMS Error (mm) |
| Analytical       | 0.052          |
| Numerical        | 0.053          |
| Experimental     | 0.056          |

Figure 5. Comparison of experimental and numerical results of out of plane displacement at a distance 250 mm from corner of the 1.0 mx 1.0 m square membrane with symmetric loading ($T_1/T_2=1$)

Figure 6. Comparison of experimental and numerical results of out of plane displacement at a mid-edge of the 1.0 mx 1.0 m square membrane with asymmetric loading ($T_1/T_2=5$)
Figure 7. The comparison of experiment and numerical value of amplitude of wrinkles at 170 mm from the corner of the membrane (size of 0.5 m x 0.5 m)

Table 2. RMS error of different size of Square Membrane with various T1/T2 ratio

| T1/T2 | RMS error (mm) |
|-------|----------------|
|       | 0.2 m² | 0.25 m² | 0.5 m² | 0.75 m² | 1.0 m² | 1.2 m² |
| 1     | 0.035 | 0.038 | 0.053 | 0.079 | 0.091 | 0.156 |
| 1.5   | 0.058 | 0.063 | 0.071 | 0.113 | 0.143 | 0.298 |
| 2     | 0.072 | 0.076 | 0.099 | 0.178 | 0.253 | 0.589 |
| 2.5   | 0.095 | 0.113 | 0.126 | 0.257 | 0.509 | 0.883 |
| 5     | 0.253 | 0.293 | 0.357 | 0.659 | 0.996 | 1.256 |

5. Effect of Pre-Stretching and Thickness
This sub-section considers the variation of the wrinkle details with the magnitude of the applied loads. Fig. 8 compares the out of plane displacement at cross-section at a distance of 170 mm from point corner at different pre-stretched loading. It is seen that wrinkle amplitudes increase, but wrinkle wavelength decreases when applied load is increased. Also note that the small downwards wrinkle almost disappears, leaving an almost antisymmetric down-up wrinkle. At Fig. 9 shows the effect of pre-stretching on wrinkle formation at different location of membrane. It is observed that there is no effect on the wrinkle’s amplitude at edge but all other places its increased and after greater stretching it convergence to constant.

Figure 8. Effect of pre-stretching on wrinkle formation
6. Conclusion
In this paper, different structural and thermomechanical condition has been discussed to analyses wrinkling behavior of membrane structures. The following conclusions have been summarized.

RMS error has been reduced by inducing the more symmetric anchor/tension points by using effective active flatness control design and double layer design. The optimum number of anchor points and respective load value can be calculated by genetic algorithms.

The scaling analysis has been performed on edge stretched membrane structures. It is found that increase in length to width aspect ratio ($\beta$) that amplitude of wrinkles also increased. The wrinkles wavelength decreases with increase in the tensile strain. But the wrinkle amplitude first increases then decreases due to increase in tensile strain. Also, it is confirmed that the wrinkle’s amplitude and surface error increases with increase in length to width aspect ratio ($\beta$).

In the future, these studies will be continued in order to simulate the conditions of orbital motion.

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8. References
[1] C. H. M. Jenkins, “Recent Advances in Gossamer Spacecraft,” Progress in astronautics and aeronautics, vol. 212. American Institute of Aeronautics and Astronautics, 2006.
[2] C. H. M. Jenkins, Gossamer Spacecraft: Membrane and inflatable structures technology for space applications, vol. 191. AIAA, 2001.
[3] E. J. Ruggiero and D. J. Inman, “Gossamer Spacecraft: Recent Trends in Design, Analysis, Experimentation, and Control,” J. Spacecr. Rockets, vol. 43, no. 1, pp. 10–24, 2006.
[4] J. Santiago-Prowald and H. Baier, “Advances in deployable strcutures and surfaces for large apertures in space,” CEAS Sp. J., vol. 5, no. 3–4, pp. 89–115, 2013.
[5] L. Scialino, A. Ihle, M. Migliorelli, N. Gatti, L. Datashvili, K. van’t Klooster, and J. Santiago Prowald, “Large deployable reflectors for telecom and earth observation applications,” CEAS Sp. J., vol. 5, no. 3–4, pp. 125–146, 2013.
[6] R. E. Freeland and G. Bilyeu, “In-Step Inflatable Antenna Experiment,” Acta Astronaut., vol. 30, pp. 29–40, 1993.
[7] R. E. Freeland and G. Bilyeu, “In-Step Inflatable Antenna Experiment,” Int. Astronaut. Fed., 1992.
[8] O. Mori, H. Sawada, R. Funase, M. Morimoto, T. Endo, T. Yamamoto, Y. Tsuda, Y. Kawakatsu, J. Kawaguchi, and Y. Miyazaki, “IKAROS Demonstration Team, and Solar Sail Working Group, ‘First Solar Power Sail Demonstration by IKAROS.,’” Trans. Japan Soc. Aeronaut. Sp. Sci. Aerosp. Technol. Japan, vol. 8, 2009.

[9] A. Babuscia, T. Choi, J. Sauder, A. Chandra, and J. Thangavelautham, “Inflatable antenna for CubeSats: Development of the X-band prototype,” in IEEE Aerospace Conference Proceedings, vol. 2016 – June, pp. 1–11.

[10] N. Chahat, S. Member, R. E. Hodges, S. Member, J. Sauder, M. Thomson, E. Peral, and Y. Rahmat-samii, “CubeSat Deployable Ka-Band Mesh Reflector Antenna Development for Earth Science Missions,” IEEE Trans. Antennas Propag., vol. 64, no. 6, pp. 2083–2093, 2016.

[11] Y. Xu, F. Guan, H. Huang, and Q. Ye, “Thermal Distortion Analysis of Inflatable Antenna Structures Considering Inflation Gas,” J. Aerosp. Technol. Manag., vol. 8, no. 4, pp. 475–482, 2016.

[12] E. Cerda and L. Mahadevan, “Geometry and physics of wrinkling,” Phys. Rev. Lett., vol. 90, no. 7, p. 74302, 2003.

[13] E. Cerda, K. Ravi-Chandar, and L. Mahadevan, “Thin films: Wrinkling of an elastic sheet under tension,” Nature, vol. 419, no. 6907, p. 579, 2002.

[14] M. Liu, J. Huang, and Y. Wang, “Analysis of wrinkled membrane structures based on a wrinkle-wave model,” AIP Adv., vol. 7, no. 1, 2017.

[15] K. Efimenko, M. Rackaitis, E. Manias, A. Vaziri, L. Mahadevan, and J. Genzer, “Nested self-similar wrinkling patterns in skins,” Nat. Mater., vol. 4, no. 4, p. 293, 2005.

[16] H. Soni, R. A. Pelcovits, and T. R. Powers, “Wrinkling of a thin film on a nematic liquid-crystal elastomer,” Phys. Rev. E, vol. 94, no. 1, 2016.

[17] C. W. J. Oomens and J. D. Janssen, “The wrinkling of thin membranes: Part I—theory,” J. Appl. Mech., vol. 54, p. 885, 1987.

[18] D. G. Roddemann, J. Drukker, C. W. J. Oomens, and J. D. Janssen, “The wrinkling of thin membranes: Part II—numerical analysis,” J. Appl. Mech., vol. 54, no. 4, pp. 888–892, 1987.

[19] W. Wong and S. Pellegrino, “Wrinkled Membranes III: Numerical Simulations,” J. Mech. Mater. Struct., vol. 1, no. 1, pp. 63–95, 2006.

[20] Q. Tao, C. Wang, Z. Xue, Z. Xie, and H. Tan, “Wrinkling and collapse of mesh reinforced membrane inflated beam under bending,” Acta Astronaut., vol. 128, pp. 551–559, 2016.

[21] R. Zhao and D. Wei, “Wrinkling Analysis of Stiffness Equivalence in Membrane Structures,” in Advances in Structures, Pts I–5, vol. 163–167, L. J. Li, Ed. 2011, pp. 1976–1979.

[22] C. G. Wang, H. F. Tan, X. W. Du, and Z. M. Wan, “Wrinkling prediction of rectangular shell-membrane under transverse in-plane displacement,” Int. J. Solids Struct., vol. 44, no. 20, pp. 6507–6516, 2007.

[23] Y. W. Wong and S. Pellegrino, “WRINKLED MEMBRANES PART III : NUMERICAL SIMULATIONS,” Mech. Mater. Struct., vol. 1, no. 1, 2006.

[24] Y. W. Wong and S. Pellegrino, “WRINKLED MEMBRANES PART II : ANALYTICAL MODELS,” Mech. Mater. Struct., vol. 1, no. 1, 2006.

[25] Y. W. Wong and S. Pellegrino, “WRINKLED MEMBRANES PART I : EXPERIMENTS,” J. Mech. Mater. Struct., vol. 1, no. January, pp. 61–93, 2006.

[26] R. K. Miller and J. M. Hedgepeth, “An algorithm for finite element analysis of partly wrinkled membranes,” AIAA J., vol. 20, no. 12, pp. 1761–1763, 1982.

[27] M. Stein and J. M. Hedgepeth, “Analysis of partly wrinkled membranes,” National Aeronautics and Space Administration Washington, 1961.

[28] H. Furuya, Y. Miyazaki, and Y. Akutsu, “Experiments on Static Shape Control of One-Dimensional Creased Membrane By Piezoelectric Films,” in 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, 2002, no. April, pp. 1–4.

[29] N. Damil, M. Potier-Ferry, and H. Hu, “Membrane wrinkling revisited from a multi-scale point
of view,” Adv. Model. Simul. Eng. Sci., vol. 1, no. 1, p. 6, 2014.

[30] J. R. Blandino, J. D. Johnston, and U. K. Dharamsi, “Corner wrinkling of a square membrane due to symmetric mechanical loads,” J. Spacecr. Rockets, vol. 39, no. 5, pp. 717–724, 2002.

[31] K. Attipou, H. Hu, F. Mohri, M. Potier-Ferry, and S. Belouettar, “Thermal wrinkling of thin membranes using a Fourier-related double scale approach,” Thin-Walled Struct., vol. 94, pp. 532–544, 2015.