Noise spectroscopy of a single spin with spin polarized STM

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We show how the noise in a spin polarized STM tunneling current gives valuable spectroscopic information on the temporal susceptibility of a single magnetic atom residing on a non-magnetic surface.

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I. INTRODUCTION

No fundamental principle precludes the measurement of a single spin, and therefore the capability to make such a measurement depends on our ability to develop a detection method of sufficient spatial and temporal resolution. The standard electron spin detection technique—electron spin resonance—is limited to a macroscopic number ($\geq 10^{10}$) of electron spins [1]. Recent experiments employing spin polarized STM [2] open new possibilities of investigation of magnetic systems at spatial resolutions of the Angstrom scale. The bulk of the experiments performed to date have been on magnetically ordered states, such as antiferromagnets.

Here we propose to use a spin polarized STM tunnel current to gain spectroscopic information on a single magnetic atom on a non-magnetic surface. A scheme is depicted in Fig.(1). We start with a magnetic atom of spin $S$ placed on an otherwise non-magnetic substrate. As we will explain in the text, the noise in the current flowing from the STM tip will allow us, in certain instances, to directly measure the single spin time dependent susceptibility. This is yet another case where the spectroscopy of current noise allows us (where other methods often fail) to directly probe the highly disordered quantum states of a microscopic system (in this case a single spin).

In earlier works, one of us and others examined similar issues at finite magnetic fields. This work is crucially different from its predecessors in that we consider the situation at a vanishing external field, and within the context of a spin polarized tunnel current. Our work also differs in the former context from [3], and further offers a transparent and accessible account of the underlying physics. Throughout the paper we propose and analyze aspects of a new experimental technique for probing single spin dynamics.

The outline of this article is as follows: In Section(II), we derive the general relation between the noise $\langle |\delta I(\omega)|^2 \rangle$ in the current and the single spin susceptibility $\chi(\omega)$. Under quite general circumstances, the noise in the current is directly proportional to the single spin susceptibility, so that measuring the noise in the current immediately gives us valuable information about the single spin susceptibility. In Section(III), we will elaborate on the origin of the spin dependent tunneling matrix elements that form much of the backbone of our proposal. In Section(IV), we will discuss the decoherence resulting from backaction effects. Here, we will also relate the spin scattering relaxation rate and DC charge transport and estimate how large the various quantities might be in realistic setups. We will conclude, in Section(V), by evaluating the typical signal to noise ratios for our experimental proposal.
II. HOW TO PROBE THE SPIN SUSCEPTIBILITY VIA A MEASUREMENT OF THE NOISE

There is a major difference between probing a single spin (as discussed here) and probing a macroscopic magnetic system. In the case of a single atomic spin $S(t)$, we perform a measurement on a microscopic state that is quantum and highly fluctuating due to interactions with its environment. The tunneling experiment discussed here is a specific example of noise spectroscopy, wherein we extract spectroscopic information, such as the single spin relaxation time $T^*$ from the measurement of the noise in the current. The main idea of noise spectroscopy in STM is that the time dependence of the tunneling current will have a characteristic relaxation time that is directly related to spin relaxation time $T^*$. Here $T^*$ does not originate from a directly applied external magnetic field (there is none) but rather from substrate excitations and other effects.

The time varying tunneling current $I(t)$ will have a fluctuating component, apart from its average DC value $I_0$. Current noise is given by the Fourier transform of the time-dependent fluctuations of the electrical current leading to the power spectra $\langle |I(\omega)|^2 \rangle$ within the frequency domain. Here, we will focus on the power spectrum of the tunneling current and argue that it is proportional to the local spin susceptibility,

$$\chi(\omega) = \langle S_\omega \cdot S_{-\omega} \rangle = \frac{1}{\omega^2 + 1/T^*},$$  \hspace{1cm} (1)

of the single atom on the substrate. In the above, $T^*$ denotes the relaxation time to the polarization axis of the single local spin (for a macroscopic collection of spins, this would be none other than the standard $T_1$ of NMR gauging the relaxation time to the polarization axis). We reiterate that in the present discussion there is a direct externally applied magnetic field $T^*$, and albeit the similarity the origin of the relaxation time here and in NMR of a completely different origin.

A long time exponential relaxation for $\chi(t) = \langle S(t)S(0) \rangle \sim \exp[-t/T^*]$, has as its Fourier transform the Lorentzian of Eqn.(1). In the Lorentzian form of Eqn.(1), the inverse relaxation time $(T^*)^{-1}$ trivially appears as the linewidth in $\chi(\omega)$. If additional short time spin modulations occur, these will augment $\chi(\omega)$ by additional high frequency terms.

In what follows, most of our focus will be on the long time (short frequency) behavior of the current noise. In thermal noise, the spectral density $\langle |I(\omega)|^2 \rangle$ is independent of frequency (e.g. being $4k_B T / R$ at a temperature $T$ for current flowing through a resistor of magnitude $R$). Shot noise, on the other hand, results from the discrete pulse-like character of electrical current. Its magnitude exhibits a white noise (frequency independent) spectrum that is proportional to the average DC current, $\langle |I(\omega)|^2 \rangle \sim a I_0$ (where the numerical constant $a = 2$ for a simple conductor), up to a certain cut-off frequency which is related to the time required for an electron to travel through the conductor.

Examples of noise spectroscopy include the noise NQR measurements [5], noise in Faraday rotation [6] and, recently, noise spectroscopy of a local spin dynamics in STM [7]. The central feature present in all of these examples is that the quantum system is not driven by external fields. Rather, the noise in the signal itself (e.g. thermal, shot, ...) sans any applied polarizing field allows us to extract spectroscopic information. Here we consider the excess noise produced by a single spin whose time-dependent quantum state we wish to probe. We suggest that over a certain parameter range, the excess noise generated by the single spin will be the dominant noise source in a single spin tunnel junction.

To make matters concrete, consider the tunneling between two contacts in the presence of a localized spin $S$. The Hamiltonian of this system assumes the form

$$\hat{H} = \sum_{n\sigma} \epsilon_n c_{Ln\sigma} c_{Ln\sigma}^\dagger + \sum_{nn'\alpha\beta} c_{Ln\alpha}^\dagger t_{0\alpha} + t_1 \hat{S} \cdot \sigma \langle c_{Ln'\beta} \rangle + (L \rightarrow R).$$ \hspace{1cm} (2)

In the above, $\hat{S}$ is the Pauli matrix vector with matrix indices $\alpha$ and $\beta$, the fermionic $c_{Ln\sigma} ; c_{Ln\sigma}^\dagger$ are the annihilation and creation operators of electrons in the $n$-th eigenstate of the lead $\lambda = L, R$ with $\sigma = \pm 1$ the (up/down) spin polarization label. The left lead (L) is the STM tip, and the right lead (R) refers to the surface. For infinite “free” leads, the eigenstates $|n\rangle$ simply correspond to the various plane wave states $\{k\}$. In real systems, all hoppings matrices $t$ will carry additional $n, n'$ labels. However, to make the notation more compact we will dispense with these indices. The wavefunctions of our system are superpositions of the direct product states

$$|\psi_L\rangle \otimes |\psi_S\rangle \otimes |\psi_R\rangle$$ \hspace{1cm} (3)

- the direct product of the state of the left contact, the impurity spin, and the right contact. The tunneling matrix $t$ present in the second term of Eqn.(2) couples all of these different states. It has two contributions: the term proportional to $t_0$ describes the spin independent tunneling while the term proportional to $t_1$ depicts the spin dependent contributions arising from the exchange interaction for electrons tunneling to the magnetic atom. Only the second term in Eqn.(2) will give rise to net current flow from the left to the right contact. In section(III), we will explain the origin and elaborate on the magnitude of the spin independent and dependent terms. A chemical potential shift (voltage drop) between the two bands $\{\epsilon_n\}$ and $\{\epsilon_{nR}\}$ will lead to a DC current within the steady state.
Henceforth, we will assume that the tunneling electrons are partially spin polarized. There are several situations where such interactions may materialize. In a ferromagnetically coated tip a chemical potential difference $2(\delta \mu_a)$ separates the two different spin polarizations: $\epsilon_{Ln\sigma} = \epsilon_{Ln} + \sigma \delta \mu_a$. Ferromagnetically ordered tips have proven to be very successful in the study of magnetic structures [2].

Very recently, an alternative approach using antiferromagnetically coated tips with no ferromagnetic order has been used to produce spin polarized current [4]. These tips might have potential benefits as compared to ferromagnetic tips. A ferromagnetic tip may produce a field of $O(1)$ Teslas at a separation of few angstroms from the surface. Ferromagnetic tips may therefore lead huge precession frequencies that are difficult to measure. In the case of an antiferromagnetic tip, there is a vanishing dipolar field.

Both of these techniques may be used for the measurements proposed here. In what follows, we are not interested in a specific model for how the spin polarized current is generated. We define a parameter that is related to the spin polarized current to the net tunneling current. This parameter will be determined by a particular microscopic model of the tip. Hereafter, we will treat $A$ as a phenomenological parameter.

As may be seen by examining the spin dependent contribution to $I$, the tunneling electrons exert torques on the localized spin $S$ which lead to corrections to the spin dynamics [8]. To lowest order in $t_1$, however, such effects are not present. Similarly, in what follows, we ignore the spin-flip interactions between the local moment and electrons in the substrate (i.e. we assume that the experimental temperature is higher than any relevant Kondo temperature ($T > T_K$)). Below the Kondo temperature, we may not ignore the spin-flip interactions of the localized spin with the substrate electrons- the local spin susceptibility will be heavily influenced by these interactions. Here we will only consider $T > T_K$ (a free impurity spin) for the sake of simplicity. For temperatures lower than the Kondo scale, the Kondo effect might manifest itself through interesting changes in the observed current noise that we discuss here for the free spin case.

To make our expressions slightly more concise, we will employ the Heisenberg representation and absorb the time dependence in all operators $\hat{O}_i$, i.e. $\hat{O}(t) = \exp[\imath H t] \hat{O}_S \exp[-\imath H t]$, with $\hat{O}_S = \hat{O}(t = 0)$ the operator in the Schrödinger representation which we now forsake. All finite time expectation values $\langle \hat{O}(t) \rangle$ that appear will represent $\sum_{\psi_i(t = 0)} p_i \langle \psi_i(0) \mid \hat{O}(t) \mid \psi_i(0) \rangle$ with $\psi_i(t = 0)$ the zero time wavefunction of the Schrödinger representation which does not evolve within the Heisenberg formulation and $p_i$ its probability within the density matrix formulation.

Let us now give a simple qualitative description of the effect that we address here. By directly computing $dN_L/dt$ we find that the charge current [9]

$$\dot{I}(t) = -\imath e \sum_{nn'\alpha\beta} c^\dagger_{Ln\alpha}(t)[t_0 \delta_{\alpha\beta} + t_1 \hat{S}(t) \cdot \hat{\sigma}_{\alpha\beta}]c_{Rn'\beta}(t) + h.c.,$$

with $e$ the electronic charge.

Thus, we see that the tunneling current has a part that depends on the localized spin via a scalar product,

$$\delta I(t) = e t_1 \hat{S}(t) \cdot \hat{\sigma}_{spin}(t),$$

where

$$\hat{\sigma}_{spin}(t) = -\imath \sum_{nn'\alpha\beta} c^\dagger_{Ln\alpha}(t)\hat{\sigma}_{\alpha\beta}c_{Rn'\beta}(t) + h.c.,$$

is the spin polarization dependent contribution to the electronic current. This expression has a very transparent meaning. Its z-component, $I^z_{spin}(t) = -\imath (c^\dagger_{L,z} A_{CR,z} - c^\dagger_{L,z} A_{CR,z}^\dagger) + h.c.$, is the net flow of up spin minus the flow of down spin. A DC current of polarized electrons flowing from the tip to the surface trivially leads to a uniform spin polarized current $I_{spin}$. We assume that there is a non-vanishing steady spin polarized current component tunneling from the tip to the surface, assumed to be aligned along (or defining) the z axis: $(\langle \hat{I}_{spin}(t) \rangle = \delta_{t,z} \hat{A}_{\uparrow} + \text{time dependent fluctuations}$, with a finite $A \neq 0$. An application of the diagrammatic analysis introduced in [10], reveals that four contributions result each of which is, at most, of the order of the contribution that we discuss below. To lowest order in the hopping amplitude $t_1$, the electronic current-current correlation function originating from the spin dependent part that we wish to probe,

$$\langle \{ \delta I(t), \delta I(t') \} \rangle = e^2 t_1^2 \langle \hat{S}^z(t) \hat{S}^z(t') \rangle \langle \hat{I}_{spin}^z(t) \hat{I}_{spin}^z(t') \rangle + \langle t \leftrightarrow t' \rangle,$$

where $\{ \}$ denotes a symmetrized correlator, $i, j = x, y, z$ denote the spin components. To lowest non-trivial order in $t_1$, we need to treat the two temporal correlation functions $\chi(t - t') = \langle \hat{S}^z(t) \hat{S}^z(t') \rangle$ and $C(t - t') = \langle \hat{I}_{spin}^z(t) \hat{I}_{spin}^z(t') \rangle \rightarrow \langle \hat{I}_{spin}^z(t) \rangle \langle \hat{I}_{spin}^z(t') \rangle = \delta_{t,z} \delta_{t,z} A$ (for $|t - t'| \rightarrow \infty$) independently. To this order, the wavefunctions with respect to which we compute the expectation values are the products of pieces describing the decoupled evolution of both the spin and of the left and right contacts separately. To make connection with the main proposal in this paper, we note that, when Fourier transformed, the symmetrized correlator $\langle \{ \delta I(t), \delta I(t') \} \rangle$ is none other than the current noise spectrum at various frequencies originating from the local spin. For small $(t_1/t_0) \ll 1$ (the experimental situation), $\mathcal{O}(\langle \{ \delta I(t), \delta I(t') \} \rangle) = (t_1/t_0)^2 t_0^2$. In evaluating the spin current correlator $C(t)$, we ignore the fluctuating contributions present for short times (large frequencies). The
spin current correlator $C(t)$ has a finite, asymptotic, long time value, $A$, that reflects the spin polarized DC current emanating from the STM tip. In Fourier space, the current power spectrum is given by a convolution of the two power spectra associated with $S$ (i.e. $\chi(\omega)$) and $\sigma$ (the spin current correlator $C(\omega)$):

$$
|\langle \delta I(\omega) \rangle|^2 = e^2 t_1^4 \int \frac{d\omega_1}{2\pi} \chi(\omega_1)C(\omega - \omega_1) + (\omega \to -\omega).
$$

(8)

At low frequencies, $C(\omega) \simeq 2\pi A\delta(\omega)$, and, consequently,

$$
|\langle \delta I(\omega) \rangle|^2 = 2Ae^2 t_1^4 \chi(\omega) + ...
$$

(9)

The ellipsis in Eqn.(9) refer to the contribution to the convolution of Eqn.(8) from the finite frequency (short time) contributions to $C(t)$. Assuming such finite frequency contributions in $C(\omega)$ have low spectral weight, the effect of these contributions will be low. This low order result is augmented by higher order corrections in $t_1$ as well as contributions arising from the connected component of the current current correlator which amounts to a shot noise like contribution.

As noted, in Eqn.(9) we neglected the effect of finite frequency components of $C(\omega)$ in the convolution of Eqn.(8). We believe the large finite frequency components of the spin current $C(\omega)$ to be small as these correspond to transient fluctuations about an assumed steady state. This assumption is not necessary, however. More generally, we may deal with the full convolution in Eqn.(8) directly without invoking any assumptions. We may potentially do this by first performing a measurement on a reference state. For instance, if we initially replace the single spin by a large cluster of a large fixed spin, we may then experimentally measure the resulting spin current $C(\omega)$. Armed with the knowledge of the spin current (from this earlier measurement on the magnetic cluster), once the noise spectrum $|\langle I(\omega) \rangle|^2$ is measured for the single spin, we can directly deconvolve Eqn.(8) to obtain the spin susceptibility [11].

Eqs.(8, 9) are our central result. They vividly illustrate how the spectroscopy of the noise in the tunneling current $|\langle \delta I(\omega) \rangle|^2$ allows us to directly probe the spectrum of spin fluctuations encapsulated in $\chi(\omega)$. The spin polarized tunneling current provides a reference frame with respect to which we may measure the fluctuations of the localized spin $S(t)$. We note that these results are generally applicable to systems displaying strong correlations (such as impurities in the Kondo regime).

III. THE ORIGIN OF SPIN DEPENDENT TUNNELING

We now elaborate on the origin and magnitude of the spin dependent tunneling matrix elements of Eqn.(2). The spin dependence of the tunneling originates from the direct exchange dependence of the tunneling barrier [7]. The overlap of the electronic wave functions of the tip and surface, separated by a distance $d$ is exponentially small and is given by a spin dependent tunneling matrix element,

$$
\hat{t} = \gamma \exp[-\sqrt{\frac{\Phi - JS(t) \cdot \sigma}{\Phi_0}}],
$$

(10)

where we explicitly include the direct exchange between tunneling electron spin $\sigma$ and the local spin $S$. Here, $J$ is the exchange interaction between the electrons tunneling from the tip and the local precessing spin $S$. In the above, $\hat{t}$ is to be understood as a matrix in the internal spin indices, and $\Phi$ is the tunneling barrier height. Typically, $\Phi$ is a few $eV$. As a canonical value we may assume $\Phi = 4eV$, $\Phi_0 = \frac{\hbar^2}{md^2}$ is related to the distance $d$ between the tip and the surface [12]. As the exchange term in the exponent is small compared to the barrier height, we may expand the exponent in $JS$. Explicitly, $\hat{t}$ may be written as

$$
\hat{t} = t_0 + t_1 \hat{\sigma} \cdot S(t),
$$

(11)

where,

$$
t_0 = \gamma \exp((-\Phi/\Phi_0)^{1/2}) \cosh[JS/(2\Phi) \sqrt{\Phi/\Phi_0}],
$$

(12)

describes spin independent tunneling. The spin dependent amplitude,

$$
t_1 = \gamma \exp((-\Phi/\Phi_0)^{1/2}) \sinh[JS/(2\Phi) \sqrt{\Phi/\Phi_0}],
$$

(13)

For estimates we may employ the typical rule of thumb $t_1/t_0 \simeq JS/2\Phi \ll 1$.

IV. BACKACTION EFFECT OF THE TUNNELING CURRENT ON THE SPIN

We may use the tunneling Hamiltonian of Eqn.(2) to estimate the decay rate of the localized spin state resulting from the spin scattering interaction associated with $t_1$. To second order this calculation is equivalent to a simple application Fermi’s golden rule leading to an up-down spin flip rate $\frac{1}{\tau_s} = \pi t_1^2 N_L N_R eV$, with $V$ is the voltage applied between the left and right electrodes. Similarly, the DC tunneling current $I_0$ is given by the tunneling rate of conduction electrons $\frac{1}{\tau_s} = \pi t_0^2 N_L N_R eV$, where $N_{L,R}$ denotes the density of states at the Fermi level of the tip and surface respectively [13].

Diagrammatically, both the spin scattering and electronic scattering rates arise from the simple bubble diagram whose real-time propagators are $G_L(t)$ and
$G_R(-t)$, where $G_{L,R}$ correspond to the left and right Green’s functions for the conduction electrons respectively. The sole difference between the two (spin dependent/independent) scattering rates is encapsulated in the prefactors. In the spin dependent scattering case ($\tau_s^{-1}$) the raw value of the single loop integral appearing in the bubble diagram needs to be scaled by $t_2^2$. The spin independent scattering rate ($\tau_e^{-1}$) is the much same albeit a scaling by the spin independent scattering amplitude squared ($t_2^2$).

Comparing these simple results, we find

$$\frac{1}{\tau_s} = \left(\frac{t_1}{t_0}\right)^2 \frac{1}{\tau_e} \sim \left(\frac{JS}{2F}\right)^2 \frac{1}{\tau_e}. \quad (14)$$

The important outcome of this analysis is that the current induced broadening predicts a spin relaxation rate $\frac{1}{\tau_s} \propto I_0$ which may be experimentally tested. This result has a very simple interpretation: the impinging foreign electron tunneling rate for a DC current of magnitude $I_0 = 1nA$ is given by $\frac{1}{\tau_s} \sim 10^{10} Hz$. By contrast, the probability to produce a spin flip, sparked by the tunneling electrons, is proportional to $t_2^2$, which leads to Eqn.(14) for the linewidth. The full intrinsic line width is further enhanced by the coupling of the spin to the environment (e.g. the interaction between the spin and various substrate excitations) which may indeed further increase the spin flip rate. The net observed linewidth,

$$(T^*)^{-1} \simeq \tau_s^{-1} + \tau_{env}^{-1}, \quad (15)$$

includes both backaction contributions ($\tau_s^{-1}$) and the aforementioned linewidth broadening due to coupling to the environment ($\tau_{env}^{-1}$). The inverse backaction relaxation time scale sets a lower bound on the net relaxation linewidth of the single impurity spin.

Given the typical values of the parameters in Eqn.(14), we estimate $\frac{1}{\tau_s} \sim 5 \times 10^{9} Hz$ for our DC current of $I_0 \sim 1nA$, $J \sim 1$ eV, $\Phi \sim 4$ eV, and $S = 1/2$. Future experiments will help to clarify the linewidth dependence on the various parameters.

V. A SIZABLE SIGNAL TO NOISE RATIO

We now demonstrate that the (signal to noise) ratio of $|\delta I(\omega \rightarrow 0)|^2$ (the excess noise induced by the impurity spin) to $|I_{shot}(\omega \rightarrow 0)|^2$ (the shot noise already present in the absence of the impurity spin) can be of quite significant (of order unity).

The finite frequency ratio $|\delta I(\omega)|^2/|I_{shot}(\omega)|^2$ was computed in a multitude of systems and shown to be bounded by 4 or other numbers of order unity. By contrast, the low or zero frequency signal to noise ratio $|\delta I(\omega \rightarrow 0)|^2/|I_{shot}(\omega \rightarrow 0)|^2$ was found to be unbounded in many instances. For one calculation amongst many others demonstrating this explicitly, see e.g. [14]. Nevertheless, as we highlight below, the low frequency signal to noise ratio, $|\delta I(\omega \rightarrow 0)|^2/|I_{shot}(\omega \rightarrow 0)|^2$, in our system is stringently bounded from above by a number of order unity, just as it is in many finite frequency situations. By considering typical parameter values, we will show that this ratio may, potentially, saturate this upper bound and be markedly large. The spin dynamics as manifested through the current noise may be very easily discernible.

To estimate this ratio, we note that the average spin dependent contribution to the current,

$$\langle \delta I(t) \rangle = c \tau (S'(t)\hat{I}_{\text{spin}}(t)) \quad (16)$$

leading to $O(c \tau A^{1/2} (S^2(t)))$. Here, as throughout, $\hbar = 1$, and the spin is dimensionless.

To obtain estimates of orders of magnitude let us inspect Eqs.(4, 6). The net electronic current $I_e$ of Eqn.(4) is of the order of $I_0$. Insofar as orders of magnitude are concerned, the spin current ($I_{\text{spin}}$) defined in Eqn.(6) satisfies

$$O(A^{1/2}) = O(I_{\text{spin}}) = O\left(\frac{I_0}{e \tau_0}\right) = O\left(\frac{I_0}{e \tau_0}\right). \quad (17)$$

The shot noise, $\langle I_{\text{shot}}^2 \rangle = a c I_0$, where the numerical constant $a = O(1)$. Noting that $I_0 = \frac{\Phi}{e}$, and making use of Eqn.(9), the signal to noise ratio is found to be

$$\frac{\langle |\delta I(\omega \rightarrow 0)|^2 \rangle}{\langle |\delta I_{\text{shot}}(\omega \rightarrow 0)|^2 \rangle} \sim \frac{t_2^2}{t_0} \chi(\omega) \frac{1}{\tau_e}. \quad (18)$$

The ratio ($t_1/t_0$) $\sim O\left(\frac{1}{\tau_e}\right)$. Inserting the Lorentzian form of $\chi(\omega)$ from Eqn.(1), we obtain

$$\frac{\langle |\delta I(\omega \rightarrow 0)|^2 \rangle}{\langle |\delta I_{\text{shot}}(\omega \rightarrow 0)|^2 \rangle} \sim \frac{T^*}{\tau_e} \left(\frac{t_1}{t_0}\right)^2 \sim \left(\frac{J}{2F}\right)^2 T^* \frac{1}{\tau_e}. \quad (19)$$

Inserting typical values, $J \simeq 0.1 - 1$ eV, $\Phi \simeq 4$ eV, $T^* \sim 10^{-8} - 10^{-6}$ seconds, and $\tau_e \sim 10^{-10}$ seconds (1nA), we find the above signal to noise ratio is, naively, 1-100. As promised, we now demonstrate, within this allowed empirical regime, the signal to noise ratio will typically veer towards the lower end of the spectrum (i.e. may be of order unity at most). The bottleneck in the signal to noise ratio is set by the backaction effect of the single spin on the tunneling current. More explicitly, fusing Eqs.(14,15) together,

$$(T^*)^{-1} \simeq \tau_s^{-1} + \tau_{env}^{-1} \sim \left(\frac{JS}{2F}\right)^2 \frac{1}{\tau_e} + \tau_{env}^{-1}, \quad (20)$$

leading us to conclude that $T^* \lesssim \left(\frac{2F}{JS}\right)^2 \tau_e$.

Inserting this back in the last line of Eqn.(19),

$$\frac{\langle |\delta I(\omega \rightarrow 0)|^2 \rangle}{\langle |\delta I_{\text{shot}}(\omega \rightarrow 0)|^2 \rangle} \lesssim \left(\frac{J}{2F}\right)^2 T^* \frac{1}{\tau_e} \lesssim O(1). \quad (21)$$
As the generic signal to noise ratio is bounded both from below and above by a number of order unity, the signal may indeed be quite sizable (of the order of the shot noise) and may be detected.

This sizable ratio offers promise for such single spin detection and related small system applications.

Note added. While finishing this paper we became aware of work by Bulaevskii, Hrushka and Ortiz [3], where a similar problem was considered for a specific case of a ferromagnetic STM tip. Our results are qualitatively similar to the results obtained by Bulaevskii et al.

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\[ e \frac{dN_L}{dt} = -ie[N_L,H] = -ie[c_L^\dagger c_L, H] \]

\[ \rightarrow -ie[c_L^\dagger c_L, c_R^\dagger ic_R + h.c.] \] (22)

Taking note of the trivial \([c_L^\dagger c_L, c_R^\dagger ic_R + h.c.] = -c\), the charge current

\[ e \frac{dN_L}{dt} = -ie[c_L^\dagger ic_R] + h.c. \] (23)

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