Global Analysis of Stage Structure Two Predators Two Prey Systems Under Harvesting Effect for Mature Predators

R A Pratama, M F V Ruslau and Nurhayati

Department of Mathematics Education, Faculty of Teacher Training and Education, Musamus University, Jl. Kamizaun Mopah Lama Merauke 99611, Indonesia.

E-mail: pratama@unmus.ac.id

Abstract. This article examines a prey-predator population model system with structural stages. Development of a mathematical model of a sustainable population of a population of living things. Structure stages are formed in predator populations, namely immature and mature. The predation function that corresponds to the characteristics in the ecosystem is the predation process of Holling I. The interaction in the population model that is carried out analysis is the equilibrium value formed from the population model. There are eight equilibrium values that arise from simple simulations. The equilibrium is \( E_1(0,0,0,0) \), \( E_2(0,k,0,0) \), \( E_3(k,0,0,0) \), \( E_4(k,k,0,0) \), \( E_5(0,0,0,A_1) \), \( E_6(A_2,0,A_3,A_4) \), \( E_7(0,A_5,A_6,A_7) \) and \( E_8(A_8,A_9,A_{10},A_{11}) \). However, only one equilibrium value is analyzed to obtain stability. Stability is seen by requiring four eigenvalues with the Jacobian matrix. As well as the chosen value is used to find the amount of harvest carried out. The linearization of differential equations is an alternative way in this study to obtain equilibrium values. Each equilibrium value has the characteristics and terms of its stability. The Routh-Hurwitz criterion becomes a condition of its stability characteristics. Meanwhile, exploitation efforts in the population are carried out to see the changes that occur. Harvesting carried out obtained harvesting business \( W = 0.01313666667 \). For the maximum benefit obtained \( \pi = 4.997259008 \). This advantage is the stability and sustainability of the ecosystem.

1. Introduction

There are many species in the living ecosystem on earth that are conducting interaction[1]. Interactions that occur are influenced by many factors in the ecosystem. Some interact because they defend themselves, natural disasters, and the exploitation of humans[2]. Therefore the balance and sustainability of species in the ecosystem must remain sustainable.

One form of interaction that occurs is predator-prey population dynamics, namely interactions between prey and predator populations[3]. After the first inventors of the population dynamic system namely Lotka and Volterra, the dynamic system experienced developments made in the predator-population population dynamic system[4]. In the most recent scientific studies using the influence of stage-structure, size-structure, developmental stage, harvesting, refuge and etc[5].

Because it has extensive studies and has an impact on an ecosystem concept many researchers have developed a predator-prey model with stage-structure. It is the improvement of the population model that approaches the real context of life in the ecosystem that will have a good impact on being able to formulate policies or decisions[6]. This policy or decision can be adopted by the stakeholders, in this case, the relevant government[7].
In the predator-prey interaction model itself, there are many interactions that make up the composition therein[8]. Intraspecific interactions in predator populations, which are interactions between individuals themselves in a prey or predator population. In addition, there is also a functional response that the researchers approach to develop it[9]. Predation interactions that use predator response functions in a predation interaction. Functional response is a function of prey density which states the amount of consumption per predator in prey. This response function can also describe changes in predation rates at varying prey densities[10].

In theoretical ecology, there are some very well-known functional responses in a dynamic system of predator-prey interactions referred to as Holling Type-I (linear), Holling Type-II (cyrtoid), Holling Type-III (sigmoid), Holling Type-IV, functional Hassel-Verley, and Beddington-e-response response type functional responses, etc[11][12]. Much of the world's research has discussed and highlighted some predator-prey relationships with stage-structured with various functional responses. In the development of a broader functional Holling response became a discussion of population dynamics ideas that were widely discussed in the development of the branch of mathematics in ecology.

Many predator-prey population dynamics models are developed to see the relationship with stage-structure. This stage-structure separates the life span of predators, which are divided into immature and mature[13]. Immature predator and mature predator population are the focus of development in this study. This age separation is because there is a very productive age range and has the potential for exploitation[14].

In the life of a certain ecological ecosystem, exploitation activities can be carried out. The exploitation referred to in this study is a harvesting[15]. Harvesting is done to meet market demand. So that harvesting is not only to obtain abundant yields but to obtain profits. Therefore, the benefits of harvesting efforts must be sustainable in the long term. In harvesting efforts, the researcher will involve a profit function for the sustainability of the exploited population.

2. Predator-Prey Population Model

The study of this study will focus on population dynamics consisting of two prey and two predators. The growth rate of each population is logistic. For the two prey populations each adopted perfect logistical growth. While growth for predator populations is closely related to interactions between prey one populations \((x_1)\) and prey two \((x_2)\). Predator populations are divided into immature \((y)\) and mature \((z)\). Interaction between mature predator populations with prey one and prey two, using the Holling I functional response.

This is because the interactions reflect the characteristics of the ecosystem. The population of prey one and prey two live exist in an environment of capacity. The survival of predator immature is very dependent on the care of food sources from mature predators. Intense interaction of getting food is done by mature to both prey populations, while immature do not do direct interaction with prey[2]. The speed of immature age transfer to mature becomes very important in this study. The speed will be the coefficient that can be controlled on an ecosystem balance. Besides mature having interactions with each prey, mature also has intraspecific interactions involving its own population.

The model is explained as follows

\[
\frac{dx_1}{dt} = x_1 \rho_1 \left(1 - \frac{x_1}{k}\right) - \beta x_1 z \\
\frac{dx_2}{dt} = x_2 \rho_2 \left(1 - \frac{x_2}{k}\right) - ax_2 z \\
\frac{dy}{dt} = c\beta x_1 z + dax_2 z - (\tau + \delta_1) y \\
\frac{dz}{dt} = \tau y - (\nu z + \delta_2) z
\]

(1)

with initial values

\(x_1(0) > 0; x_2(0) \geq 0; y(0) \geq 0; z(0) \geq 0\)
with $\frac{dx_1}{dt}$, $\frac{dx_2}{dt}$, $\frac{dy}{dt}$ and $\frac{dz}{dt}$ is a population growth of prey one, prey two, immature predator and mature predator. Variables $\rho_1$ and $\rho_2$ respectively are intrinsic growth rates in the prey population. The variable $k$ is the carrying capacity for the population in the model. The functional form of mature response with prey one is $\beta x_1 P$ and the functional response of mature with prey two is $\alpha x_2 P$, each using the Holling I functional response. For $\beta$ and $\alpha$ is the rate of predation of predatory mature predation on prey. While $c$ and $d$ are the rates change resulting from predation interactions that occur. The rate of change in immature to mature which is conducive is symbolized by $y$ and the coefficient of introspective mature predator interaction is symbolized by $v$. For $\delta_1$ and $\delta_2$ is the rate of natural mortality by immature and mature predators.

For exploitation or harvesting efforts carried out in this model, only in mature populations. One of the main objectives of this study is to look at the strong effects of harvesting. This is done to consider population growth that exists for mature populations. Therefore the population dynamics model (1) changes because of the harvesting effort.

$$\frac{dx_1}{dt} = x_1 \rho_1 \left( 1 - \frac{x_1}{k} \right) - \beta x_1 z$$
$$\frac{dx_2}{dt} = x_2 \rho_2 \left( 1 - \frac{x_2}{k} \right) - \alpha x_2 z$$
$$\frac{dy}{dt} = c \beta x_1 z + d \alpha x_2 z - (\tau + \delta_1) y$$
$$\frac{dz}{dt} = \tau y - (vz + \delta_2) z - q_1 W z$$

(2)

For $q_1$ is the rate of capture at mature predators that are harvested. While $W$ is a variable of harvesting business. In the population dynamics model (2) operational differential equations will be carried out, to obtain a balance with the characteristics of each population. The operation of the differential equation takes into account the dimensions of the model.

| Table 1. Definition of variables in the model (2). |
| Variable | Definition | Dimension |
|----------|------------|-----------|
| $x_1$    | Population prey one | [N] |
| $x_2$    | Population prey two | [N] |
| $y$      | The population of predators (Immature) | [N] |
| $z$      | The population of predators (mature) | [N] |

| Table 2. The parameter values in the model (2). |
| Parameter | Definition | Dimensions |
|-----------|------------|-------------|
| $\rho_1$  | The intrinsic growth rate of the first prey | [T]$^{-1}$ |
| $\rho_2$  | The intrinsic growth rate of the second prey | [T]$^{-1}$ |
| $k$       | Carrying capacity | [N] |
| $\beta$   | Predation rate from predator to first prey | [N]$^{-1}$ [T]$^{-1}$ |
| $\alpha$  | Predation rate from predator to second prey | [N]$^{-1}$ [T]$^{-1}$ |
| $c$       | Change efficiency from first predation | - |
| $d$       | Change efficiency from second predation | - |
| $\tau$    | Rate of change in immature to mature | [T]$^{-1}$ |
| $v$       | The coefficient of introspective mature | [N]$^{-1}$ [T]$^{-1}$ |
3. Results and Discussion

Equilibrium points that meet for analysis of the model (2) are 8 equilibrium points. Each equilibrium point that meets is $(x_1, x_2, y_1, y_2, z_1, z_2, p_1, p_2)$. Where $A_0 = k$

$$A_1 = \frac{E_1 q_1 + \delta_2}{v}, A_2 = \frac{(N_1 E_1 + N_2)}{M_1}, A_3 = \frac{(N_3 E_2^2 - N_4 E_1 - N_5)}{M_1}, A_4 = \frac{(N_6 - N_7 E_1)}{M_1}, A_5 = \frac{(N_8 E_1 + N_9)}{M_2}, A_6 = \frac{(N_1 E_1^2 - N_11 E_1 - N_12)}{(M_2)}, A_7 = \frac{(N_13 - N_14 E_1)}{\alpha^2 \sec + \tau_0 \rho_2 + \nu_0 \rho_2}, A_8 = \frac{(N_15 + N_16 E_1)}{M_3}, A_9 = \frac{(N_18 - N_16 E_1)}{M_3}, A_{10} = \frac{(N_19 E_2^2 - N_20 E_1 - N_21)}{(M_2)}, A_{11} = \frac{(N_22 - N_22 E_1)}{M_3},$$

where $N_1 = (\beta t \delta_1 q_1 + \beta \delta_1 q_1), N_2 = (\beta t \delta_1 + \beta \delta_1 \delta_2 + \tau \rho_1 + \nu \delta_1 \rho_1), M_1 = (\beta^2 c t k + \tau v_1 + \nu \delta_1 \rho_1), N_3 = (-\beta^2 c t k \rho_1 q_1^2 - \beta^2 c \delta_1 k \rho_1 q_1^2), N_4 = (\beta^3 c^2 t k_2 \rho_1 q_1 - 2\beta^2 c \delta_2 \delta_1 k_1 q_1 - 2\beta^2 c \delta_1 \delta_2 k_1 q_1 - \beta \delta_1 t k_1 q_1^2 - \beta \delta_1 t k_1 q_1^2), N_5 = (\beta^3 c^2 \delta_2 k_2^2 q_1 + \beta^2 c^2 \tau k_2^2 q_1^2 - \beta^2 c \tau k_2^2 \delta_1 q_1 - \beta \delta_1 c t k_1 q_1), N_6 = (\beta \delta_1 t k_1 - \tau \rho_1 \delta_2 - \nu \delta_1 \rho_1), N_7 = (\tau \rho_1 q_1 + \delta_1 q_1), N_8 = (\alpha \delta_2 t k + \nu \delta_2 + \nu \delta_1 \rho_2), M_2 = (\alpha \delta_2 t k + \nu \delta_2 + \nu \delta_1 \rho_2), N_9 = (-\alpha \delta_1 t k q_1 q_1^2 + \alpha^2 d k \delta_1 \rho_2 q_1^2), N_{10} = (\alpha^2 d t k_2^2 q_1^2 + 2\alpha^2 c^2 d t k_2^2 q_1^2 - \alpha^2 d t \delta_2^2 k_2^2 + \alpha^2 d^2 \delta_2^2 k_2^2 + \alpha^2 d^2 t k_2^2 q_2^2 - \alpha^2 d^2 \delta_2^2 k_2^2 q_2^2 - \alpha^2 d^2 t k_2^2 q_2^2 - \alpha^2 d \delta_1 \rho_1 q_1^2), N_{11} = (\alpha^2 d^3 t k^2 q_1^2 + 2\alpha^2 c^2 d t k^2 q_1^2 q_1^2 - \alpha^2 d^3 t k^2 q_1^2 q_1^2 + \alpha^2 c^2 t k^2 q_1^2 q_1^2 - \alpha^2 c^2 t \delta_2^2 k_2^2 q_1^2 q_1^2 - \alpha^2 d \delta_1 \rho_1 q_1^2), N_{12} = (\alpha^2 d^3 t k^2 q_1^2 + 2\alpha^2 c^2 d t k^2 q_1^2 q_1^2 - \alpha^2 d^3 t k^2 q_1^2 q_1^2 - \alpha^2 d^2 t k_2^2 q_1^2 q_1^2 - \alpha^2 d^2 \delta_2^2 k_2^2 q_1^2 q_1^2 - \alpha^2 d \delta_1 \rho_1 q_1^2, N_{13} = (\alpha \delta_2 t k_2 - \tau \rho_2 - \nu \delta_1 \rho_2), N_{14} = (\tau \rho_2 q_1 + \nu \delta_1 \rho_2), N_{15} = (\alpha^2 d t k_1^2 q_1^2 + \alpha \beta \delta_1 t k_1^2 q_1^2 + \alpha \beta \delta_1 k_1^2 \rho_1^2 + \alpha^2 d^2 \delta_1 \rho_1 q_1^2, N_{16} = (\beta \delta_1 t k_1 q_1 + \beta \delta_1 \delta_2 q_1^2, N_{17} = (\alpha \delta_2 t k_2^2 q_1^2 + \alpha \beta \delta_2 t k_2^2 q_1^2 q_1^2 + \alpha \beta \delta_2 k_2^2 \rho_1^2 + \alpha^2 d^2 \delta_2^2 k_2^2 q_1^2 q_1^2 + \alpha \beta \delta_2 k_2^2 q_1^2 q_1^2 + \alpha^2 d \delta_1 \rho_1 q_1^2), N_{18} = (\alpha \delta_2 t k_2 + \beta \delta_2 t k_2 + \beta \delta_1 \rho_1 q_1^2 + \nu \delta_1 \rho_1 q_1^2, N_{19} = (-\alpha \delta_1 t k q_1^2 q_1^2 q_1^2 + \alpha \delta_1 t k q_1^2 q_1^2 q_1^2 - \alpha \delta_1 t k q_1^2 q_1^2 q_1^2 - \alpha \delta_1 t k q_1^2 q_1^2 q_1^2 - \alpha \delta_1 t k q_1^2 q_1^2 q_1^2).$

With positive equilibrium for operating the system of differential equations model (2) it is $E_0(x_1^*, x_2^*, y_1^*, y_2^*, z_1^*, z_2^*, p_1^*, p_2^*)$. By taking the equilibrium point the Jacobian matrix is obtained as follows:

$$J = \begin{bmatrix} J_{11} & 0 & 0 & J_{14} \\ 0 & J_{22} & 0 & J_{24} \\ J_{31} & J_{32} & J_{33} & J_{34} \\ 0 & 0 & J_{43} & J_{44} \end{bmatrix}.$$
where
\[ J_{11} = \rho_1 - \frac{2 \rho_1 x_1'}{k} - \beta z' \]
\[ J_{14} = -\beta x_2' \]
\[ J_{22} = \rho_2 - \frac{2 \rho_2 x_2'}{k} - \alpha z' \]
\[ J_{24} = \alpha x_2' \]
\[ J_{31} = \beta cz' \]
\[ J_{32} = \alpha dz' \]
\[ J_{33} = -\tau - \delta_2 \]
\[ J_{34} = \alpha dx_2' + \beta cx_1' \]
\[ J_{43} = \tau \]
\[ J_{44} = -2\nu z' - Wq_1 - \delta_1 \]

Equation of the Jacobian characteristics of the matrix \( J(E_0) \) is
\[ \lambda^4 + A_1 \lambda^3 + A_2 \lambda^2 + A_3 \lambda + A_4 = 0, \]
where
\[ A_1 = -(J_{11} + J_{22} + J_{33} + J_{44}), \]
\[ A_2 = J_{11}J_{22} + J_{11}J_{33} + J_{11}J_{44} + J_{22}J_{33} + J_{22}J_{44} + J_{33}J_{44} - J_{12}J_{21} - J_{13}J_{31} - J_{14}J_{41} - J_{34}J_{43}, \]
\[ A_3 = J_{11}J_{44} + J_{12}J_{23} + J_{12}J_{32} + J_{13}J_{23} + J_{13}J_{32} + J_{14}J_{43} + J_{14}J_{34} + J_{14}J_{34} - J_{12}J_{23} - J_{13}J_{32} - J_{14}J_{43} - J_{22}J_{33} - J_{23}J_{32} - J_{34}J_{43}, \]
\[ A_4 = J_{11}J_{22}J_{33}J_{44} + J_{12}J_{23}J_{34}J_{44} + J_{13}J_{23}J_{34}J_{44} + J_{14}J_{23}J_{34}J_{44} + J_{12}J_{23}J_{34}J_{44} - J_{12}J_{23}J_{34}J_{44} - J_{12}J_{23}J_{34}J_{44} - J_{12}J_{23}J_{34}J_{44}, \]

Using the Routh Hurwitz stability criteria, the polynomial model (2) has eigenvalues whose real part is negative if they meet conditions \( A_1 > 0, A_2 > 0, A_3 > 0 \) d an \( A_1A_2A_3 - A_3^2 - A_2^2A_4 > 0. \)

4. **Benefits of harvesting**

The polynomial equilibrium point model (2) produces stability that indicates ecosystem balance. At this asymptotic equilibrium, point harvesting is done to see population movements in the ecosystem[16]. Harvesting is done on adult predators using harvesting parameter coefficients. For harvesting business actions taken also consider the cost of harvesting and income. Therefore, the total cost we form in \( TC = c_1E_1 \) and the total of all income is formulated in \( TR = p_1E_1 \). Respectively \( c_1 \) and \( p_1 \) are the coefficients of the cost of harvesting in mature and the coefficient of income for the mature population.

A profit function for harvesting business in mature will be seen as follows
\[ \pi = \frac{p_1z'E_1 - c_1E_1}{H_1} \]
\[ \pi = \frac{(H_1E_1^2 - H_2E_1)}{H_3} - c_1E_1 \]

where:
\[ H_1 = (-p_1\tau q_1\rho_1\rho_2 - p_1 \delta_1 \rho_1\rho_2), H_2 = (adp_1\tau k\rho_1\rho_2 + \beta cp_1\tau k\rho_1\rho_2 - p_1\tau \delta_2 \rho_1\rho_2 - p_1 \delta_1 \rho_2\rho_1) \]
\[ H_3 = (a^2/secp_1 + \beta^2\tau k\rho_2 + p_1 \rho_2 + \delta_1 \rho_1). \]

5. **Numerical Simulation**

The parameter values used in the simulation are selected using relevant basic assumptions. The parameter values in this numerical simulation are \( \rho_1 = 1.5, \rho_2 = 1.8, k = 100, \beta = 0.02, \alpha = 0.01, c = 0.02, d = 0.04, \delta_1 = 0.04, \delta_2 = 0.02, \tau = 0.06, \nu = 0.003. \) From the simulation parameters, the equilibrium values are obtained \( E_0(x_1',x_2',y',z'). \) is (89.18918919, 95.49549550, 5.989773557, 8.108108108).
Furthermore for the Jacobi matrix obtained from \(E_0\) is

\[
J(E_0) = \begin{bmatrix}
-1.337837838 & 0 & 0 & -1.783783784 \\
0 & -1.718918919 & 0 & -0.954954955 \\
0.003243243243 & 0.003243243243 & -0.1 & 0.07387387388 \\
0 & 0 & 0.06 & -0.06864864865
\end{bmatrix}
\]

The characteristic polynomial of \(J(E_0)\) is

\[
\lambda^4 + 3.225405406 \lambda^3 + 2.817585099 \lambda^2 + 0.3957985942 \lambda + 0.006438977356 = 0.
\]

From the above polynomial, the eigenvalue is

\[
\lambda_1 = -1.33805926002379, \lambda_2 = -0.149651220431219, \lambda_3 = -0.0187063169423948, \lambda_4 = -1.7189886082526.
\]

6. **Numerical Simulation with Harvesting**

The parameter number on the numerical simulation is \(\rho_1 = 1.5, \rho_2 = 1.8, k = 100, \beta = 100, \beta = 0.02, \alpha = 0.01, c = 0.02, d = 0.04, \delta_1 = 0.04, \delta_2 = 0.02, \tau = 0.06, v = 0.003, q_1 = 1, p_1 = 100, c_1 = 50.\) Equilibrium point at \(E_0\) with \(W > 0\) so \(E_0(W) = (x_1', x_2', y', z')\) becomes

\[
x_1' = 89.18918919 + 386.1003861W, \\
x_2' = 95.49549550 + 160.8751609W, \\
y' = -633.5624096W^2 - 196.1807367W + 5.989773557, \\
z' = -289.5752896W + 8.108108108.
\]

So that the profit function obtained is

\[
\pi = -28957.52896W^2 + 760.8108108W
\]

The profit function in equation (4) is obtained by harvesting stationary namely \((W) = (0.01313666667).\) Obtaining the value of \((W)\) will be a parameter in the equation model (2). Substitute the stationary point \((W)\) into the profit function (4), then gain \(\pi = 4.997259008.\) We will discuss a harvesting effect that is carried out in each population. After this, we will discuss changes in population due to harvesting. Here we really expect a good curve for a sustainable population.

7. **Population Dynamics from The Mature Harvesting Effect**

This simulation process is used to see population changes in time \((t)\), which is a reference in returning decisions or policies. Observations were made on prey population \((x_1)\) and \((x_2)\) population immature \((y)\) and mature population \((z)\). The dynamics that occur in each population are presented in the following population growth curve.

![Figure 1. Trajectories for prey \((x_1)\) with and without harvesting.](image-url)
It is clear that Figure 1, Figure 2, Figure 3 and Figure 4 interpret characteristic trajectories. Harvesting effects carried out on mature populations greatly affect population growth in the model (2). Figures 1 and 2 show relatively stable changes from before and after harvesting. The population of prey \((x_1)\) and \((x_2)\) has increased in population, this is because the prey population is heading for stable growth.
because the predator population is harvested. While for the immature population \((y)\) to decrease, this is because mature predators that reproduce are also reduced. While for the mature predator population \((z)\), it has decreased towards stability. In the trajectories shown, it is clear that harvesting is very influential on population growth in the ecosystem model (2). Therefore, we need a policy for sustainable sustainability but it can still provide benefits for human economic life.

8. Conclusion
The predator-population population model in the ecosystem model (2), shows a balanced dynamic in the ecosystem. The optimal stages of structure and harvest are a number of highly significant influences. Equilibrium found in the dynamics of this population there are 8 points. The equilibrium is \(E_1(0, 0, 0, 0)\), \(E_2(k, 0, 0, 0)\), \(E_3(k, 0, 0, 0)\), \(E_4(k, k, 0, 0)\), \(E_5(0, 0, 0, A_1)\), \(E_6(A_2, 0, A_3, A_4)\), \(E_7(0, A_5, A_6, A_7)\) and \(E_8(A_8, A_9, A_{10}, A_{11})\). Of the eight equilibrium, only one value fulfills stability. The chosen value is used to find the amount of harvest carried out. The equilibrium value taken is \(E_0(x_1, x_2, y, z)\). Harvesting carried out obtained harvesting business \(W = 0.01313666667\). For the maximum benefit gained \(\pi = 4.997259008\). This advantage is the stability and sustainability of the ecosystem. Population extinction in this ecosystem system is very much avoided. It is from the value of this harvesting business and profits that affect population growth in the ecosystem.

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