Plasma-enhanced metamaterials using microwave radiative power transfer

To cite this article: Hyunjun Kim and Jeffrey Hopwood 2018 Plasma Sources Sci. Technol. 27 095007

View the article online for updates and enhancements.
Plasma-enhanced metamaterials using microwave radiative power transfer

Hyunjun Kim and Jeffrey Hopwood

Electrical and Computer Engineering, Tufts University, 161 College Avenue, Medford, MA 02155, United States of America

E-mail: hopwood@ece.tufts.edu

Received 4 June 2018, revised 14 August 2018
Accepted for publication 20 August 2018
Published 14 September 2018

Abstract

Plasmas are created within a metamaterial structure using low pressure argon and xenon gases. The metamaterial creates an effective negative permeability by using 3D arrays of split ring resonators (SRR) fabricated on thin substrates. Microwave energy incident on this material excites one set of SRRs at 1.5 GHz, which breaks-down the low pressure gas and creates a steady-state plasma. Because this plasma effectively short-circuits the SRRs, the negative permeability is lost. Therefore, a second set of SRRs is included within the metamaterial. These SRRs are isolated from the plasma and shown to be immune to plasma interference. We show that this set of isolated SRRs can maintain negative permeability properties at the design frequency of 1.9 GHz, even with plasma present. Having demonstrated negative permeability, the electromagnetic properties and wave transmission of this metamaterial with periodic SRR-sustained xenon and argon plasmas are investigated. The permittivity of plasma is calculated from collisional Langmuir probe measurements of the plasma density. Spatially resolved measurements show that the central region of each SRR-sustained plasma has negative plasma permittivity with measured plasma density in the range of \(0.5\times10^{11}\) cm\(^{-3}\). The spatial average of the plasma density, however, is too low to create negative permittivity throughout the material.

Keywords: metamaterials, microwave plasma, plasma diagnostics

1. Introduction

Metamaterials are artificial composites which may have extraordinary properties such as negative relative permeability \((\mu_r)\) and permittivity \((\varepsilon_r)\) within a narrow frequency band. For some decades the concept of a double-negative metamaterial has been known. For this class of material, overlapping electric and magnetic resonances give \(\varepsilon_r < 0\) and \(\mu_r < 0\) simultaneously [1–3]. This unusual situation causes a negative refractive index and a negative propagation constant. Double-negative materials are potentially attractive for a wide range of applications such as microwave absorbers, cloaking devices, optical detectors and imagers, perfect lenses, and advanced antenna systems [4–8]. These properties are primarily dependent on total size and volume of the metamaterial and a sufficiently large cross-section in three dimensions is most beneficial. However, there is a challenge for the fabrication process to make large scale metamaterials in three dimensions [9–11].

The split ring resonator (SRR) is one example of a unit cell used in the practical implementation of metamaterials because it is known to provide a band of negative permeability at its resonant frequency. Many researchers have reported unique electromagnetic properties in various SRR structures [12–17]. Recently, we have introduced a three dimensional SRR array which is capable of self-igniting argon microplasmas within the gaps of the SRRs [18]. The presence of a high pressure argon plasma was found to decrease the energy coupling among adjacent resonators in the metamaterial. This decoupling resulted in wave transmission spectra that changed by orders of magnitude in the presence of plasma. The high-pressure microplasmas created in that work, however, were known to have \(\varepsilon_r > 0\) due to high collisionality, and therefore the plasmas were found to...
simply modulate the effective permeability of the SRR array. In this work, we investigate low pressure plasmas sustained within 3D SRR arrays in order to reduce the collisional nature of the plasma and attempt to obtain $\varepsilon_p < 0$.

As reported in [18], plasma can be useful in metamaterials since a plasma exhibits a variable electromagnetic response. If the plasma frequency is greater than the excitation frequency, the collisionless plasma has the desired negative permittivity property needed for double negative behavior. Specifically, the complex plasma permittivity is readily controlled through the electron density and gas pressure. Therefore, combining the dynamic electrical properties of plasma with the novel characteristics of metamaterials is promising for applications requiring reconfigurable metamaterials [19–23]. Once plasmas have formed inside the metamaterial, the properties of the material can be adjusted by the applied power. Reconfigurable double-negative metamaterials, however, will likely require a spatially-averaged negative permittivity throughout the metamaterial, so a large volume of overdense plasma is required. Additionally, the plasma pressure must be low such that the electron collision frequency is less than the electromagnetic wave frequency ($\nu < \omega$); this is typically the order of 1 Torr, or less, for microwave excitation. As an example, Sakai et al have reported double-negative behavior by creating a diffuse plasma within periodic metamaterials inside a waveguide [24]. They report that a high density plasma ($>1 \times 10^{17} \text{m}^{-3}$) in the negative $\mu$ spaces of metamaterials exhibits double-negative behavior in the macroscopic view, as the appearance of plasma allowed wave propagation through the metamaterial that is normally cut off. Similarly, Nakamura et al have employed pulsed microwaves in a metamaterial-immersed waveguide system [25]. They measured negative $\mu$ without plasma and negative plasma permittivity was suggested with the assumption that the negative $\mu$ space is also maintained when plasma is present.

In this paper, we report the existence of an overlooked plasma-loading effect in the SRR configuration by directly measuring wave transmission spectra through such metamaterials in the presence of plasma and extracting the effective permeability. Since low-pressure plasma diffuses over the surface of the SRRs, it quenches the quality factor of the resonator such that the desired property of $\mu_r < 0$ is lost. To overcome this problem, we have configured another metamaterial structure with two sets of SRRs. The first set creates plasmas by resonating with high power microwave radiation from a horn antenna at 1.5 GHz. The second set of SRRs is tuned to have a different resonant frequency (1.9 GHz) and is intentionally isolated from contact with the plasma. The second set provides the required negative $\mu$ even when the first set loses effectiveness due to plasma loading. In addition to confirming the existence of negative $\mu$ in this new metamaterial structure, we determine the plasma permittivity within the negative $\mu$ region by measuring the ion density using a collisional Langmuir probe system. Then the spatial profile of plasma density and permittivity are used to estimate the average permittivity of the SRR-sustained plasmas. Although regions of negative plasma permittivity exist, the average permittivity is found to be positive.

2. Experimental method

Figure 1 shows the detailed geometry of the SRR unit cell, an individual 2D substrate, and the full 3D configuration of the SRR array made of 9 substrates. The unit cell consists of two SRRs, one fabricated on each side of the substrate (Rogers™ TMM3, $\varepsilon_r = 3.27$, thickness of 1.5 mm). We designate the first and the second set of SRRs by their unique function: plasma creation ($p$-SRR) and negative permeability ($\mu$-SRR). The $p$-SRR has a square shape with an outer and inner length of 17.5 and 15.3 mm, and a discharge gap of 150 $\mu$m where plasma formation first occurs. The $\mu$-SRR is fabricated on the opposite side of the TMM substrate such that it is isolated from the plasma and may maintain negative permeability in the presence of plasma on the $p$-SRR. The second SRR has a rectangular shape with outer lengths of 17.5 and 11 mm and the ends are separated by a 4 mm gap. This shape avoids plasma formation as well as limits any electric field interference and parasitic coupling with the $p$-SRR. Therefore, the two sets of SRRs function independently. The unit cells are fabricated into 3 × 3 arrays on TMM substrates. In the complete 3D structure, the spacing between 2D arrays along the y-direction is determined by a Teflon™ mounting structure (not shown). As indicated in figure 1, the electromagnetic waves propagate along the z-direction ($k_z$), and $E$ and $H$ fields are polarized in the $x$ and $y$-direction, respectively. With this incoming wave, a high amplitude $E$ field is formed within the discharge gaps of the $p$-SRRs which enables a self-initiated discharge in the pressure range of 1–260 Torr using radiated microwave power of 80 W.

Figure 2 shows the 3D structure within an anechoic vacuum chamber as previously described in [18]. Microwave power is supplied to a broadband antenna located outside the chamber. The microwave radiation from the antenna passes through a vacuum window into the chamber and excites the 3D array. For all data we report the amplifier output power, $P_0$, but the net power delivered from the wideband horn antenna is approximately 0.8 $P_0$ due to reflection, impedance mismatching, and transmission losses in this system.

Xenon and argon gases are fed into the chamber through independent gas lines. The chamber is evacuated by a mechanical pump with a resulting base pressure inside of the chamber of 50 mTorr and a measured leak rate of 0.2 mTorr min$^{-1}$. Experiments using argon benefit from improved gas purity by flowing and pumping gas simultaneously, but xenon experiments use a static gas fill to conserve Xe usage.

The schematic for measurements of the normalized reflection coefficients ($S_{11}$) and the normalized transmission coefficients ($S_{21}$) is provided in figure 2. For transmission measurements, the output port of the vector network analyzer (VNA) was combined with the fixed-frequency oscillator (1.5 GHz), amplified, and transmitted from the broadband antenna. The plasma-sustaining power at 1.5 GHz is typically 20–150 W; the $S_{21}$ probing power from the amplified VNA signal (1.2–2.4 GHz) is always less than 3 W and is well below the threshold for plasma ignition. The transmitted signal was then received by the VNA through a monopole antenna located in the chamber on the back side of the metamaterial. In order to
measure $S_{11}$ the receiving port of the VNA was manually
switched to the reflection port of the directional coupler.

The calibration curves for both measurements were
taken without metamaterials, thus defining the 0 dB level
before placing the metamaterials in the chamber. Note that
$S$-parameters greater than 0 dB do not represent gain,
but simply indicate that more power was transmitted
to the monopole antenna when the metamaterial was in the
chamber compared to the transmission through the empty
chamber.

For diagnostics of the plasma, the monopole antenna
was removed from the chamber and replaced by a movable
Langmuir probe as shown in the lower frame of figure 2.

For measurements of the spatial plasma density profile,
the size of the SRR structure was reduced to $3 \times 3 \times 5$ and
measurements were taken along the center line of this smaller
3D array. This downsizing was necessary because when the
Langmuir probe was deeply inserted into the metamaterial,
the probe body interfered with the electromagnetic coupling
among the SRRs and disturbed the uniform ignition of the
plasmas; the smaller array was less sensitive to the intrusion
of the probe body.

The probe consisted of a rigid coaxial line which had
a metal tip with a length $L_w = 2.3$ mm and a radius $R_w = 0.25$ mm. It is recommended to use a probe tip for which
$R_w \ln(L_w/2R_w)$ is smaller than the electron mean-free path $\lambda_e$. In
this case the probe’s size parameter is $\approx 0.4$ mm, which is less
than the electron mean-free path of $0.7 < \lambda_e < 1.6$ mm in the
operating pressure range of $0.5$–$2$ Torr [26]. Because the
plasma system lacks any grounded surfaces, the probe is
essentially functioning as an asymmetric double Langmuir
probe with the body of the coax serving as the second elec-
trode. This made the collection of the electron saturation cur-
rent impossible as the electron current to the tip was limited by
the ion current to the probe’s body. Ion saturation currents were
used to obtain the plasma density as described below.

The probe was attached to an electrostatic plasma probe
controller (Hiden, ESP004) which captured the current versus
voltage curve ($i$–$v$). The plasma density $n_0$ was estimated
from the measured ion saturation current $i_{is}$ to the small probe
tip using

$$i_{is} = 0.61 e n_0 u_s A_p,$$

where $e$ is the electron charge, $u_s$ is the collisional ion velo-

city at the sheath edge, and $A_p$ is the effective collection area
of the probe tip including sheath expansion. For the colli-
sional ion sheath, $u_s$ and $A_p$ were obtained by [27]

$$u_s = \frac{u_B}{\sqrt{1 + (\pi \lambda_D e/2 \lambda_e)},}$$

$$A_p = \pi (R_w + s)[2(L_w + s) + (R_w + s)],$$

where $u_B = \sqrt{eT_e/M}$ is the Bohm velocity for ion mass $M$,
$\lambda_D e = \sqrt{\varepsilon_0 T_e/0.61 e n_0}$ is the Debye length for electrons, $\varepsilon_0$
is the permittivity in vacuum, $\lambda_e$ is the ion mean free path, and
$s$ is the collisional sheath width. Equations (1)–(3) were
solved iteratively. The electron temperature $T_e$ was
estimated from the slope of the logarithm of the measured
current at the asymptotic crossing point [28]. To help corrobora-
te the probe measurements, we installed a 35 GHz
interferometer to compare the relative electron density
between argon and xenon plasmas. Two small horns are

![Figure 1. Geometry of the dual SRR structure. A unit cell of the SRR structure consists of two SRRs: the $p$-SRR for producing plasma and the $\mu$-SRR for creating negative permeability; these are fabricated on opposite sides of the substrate. The full $3 \times 3 \times 9$ SRR metamaterial structure is shown on the left with only the $p$-SRRs visible.](image-url)
placed outside glass windows at both sides of the vacuum chamber (figure 2, bottom). The phase shift of the 35 GHz microwave signal through the metamaterial was nulled and the additional phase shift through the plasma along the y-direction determines the line integrated electron density \[29, 30\]. Based on the measured electron density of argon and xenon plasma, the plasma permittivity for various discharge conditions is obtained as discussed in section 3.2.

Figure 2. Schematic of measurements for the S-parameters (\(S_{11}\) and \(S_{21}\)), the plasma density \(n_0\) by the Langmuir probe, and the line integrated electron density by interferometer.

3. Results and discussion

Figure 3 shows the 3D array of SRR supporting a total of 81 argon plasmas at 0.6 Torr. Note that all of the plasmas were created on the side of each substrate where the \(p\)-SRRs were fabricated and that the plasmas extend outside the discharge gap and partially cover the \(p\)-SRRs. The spaces between 2D arrays were partially filled by plasmas, as shown in figure 3, but the backside of each TMM substrate where the \(\mu\)-SRRs
reside is mostly free of visible plasma as shown in the lower left corner of the top view. In the following section, the transmission spectra will show that the resonance and negative permeability of the $p$-SRR is lost upon plasma formation, but the $μ$-SRR continues to function properly.

Figure 4 shows measured transmission spectra ($S_{21}$) through the metamaterial structure as a function of the applied power. The spectra are complex, so we begin the description by focusing on the SRR array without plasma as indicated by the dashed curve in each plot. There are two groups of frequencies for which the transmission is greatly attenuated. The first group occurs near 1.4 GHz and is due to the array of $p$-SRRs. The multiple resonances are due to electromagnetic coupling and resonance splitting among the 81 identical $p$-SRRs [17, 31–35]. One notes that increasing the separation between SRR planes from $d_y = 6.4$ mm to $d_y = 10$ mm
changes the amount of coupling and alters the transmission properties of the material according to coupled mode theory [35]. The second group of non-transmissive frequencies occurs near 1.9 GHz and is due to the coupled resonances of the 81 μ-SRRs. As we will show in the following section, the loss of transmission through the material is the result of SRR resonance, but it can also be interpreted as a frequency band for which \( \mu < 0 \). This causes the wave vector to become imaginary such that the wave does not propagate and transmission minima occur.

Next we examine the transmission through the array in the presence of a weak Xe plasma. For the lowest possible power of 20 W, the metamaterial becomes transmissive at 1.3 GHz. One might incorrectly interpret this as a double-negative metamaterial for which the propagation constant becomes real due to negative plasma permittivity and negative SRR permeability. However, we note that (1) the resonances due to the μ-SRRs remain non-transmissive indicating \( \mu < 0 \) and \( \varepsilon > 0 \), and (2) the measurements of electron density presented later show that \( \varepsilon > 0 \) on average. In addition, it is well-known that low pressure plasmas in contact with SRRs will act as lossy volumes and quench the resonance [36]. Therefore, the appearance of weak Xe plasma simply removes the negative permeability characteristic near 1.3 GHz and the wave passes in the normal manner (\( \mu > 0 \), \( \mu > 0 \)). In the case of weak argon plasma (\( P_{\alpha} = 40 \) W), we see that the S-SRR plasma is not completely eliminated because the argon plasma is smaller in size than Xe. But increasing the amplifier power to 90 W creates a larger and more dense argon plasma that also eliminates the p-SRR resonance. As an aside, we also observe this transmissive behavior in a simple array consisting solely of p-SRRs. This undesirable finding suggested the addition of the μ-SRR to recover negative \( \mu \). The addition of the μ-SRR did not affect the p-SRR response, however, due to the non-overlapping resonance frequency and geometric offset as shown in figure 1.

Switching our attention to the upper frequencies, we note that the appearance of plasma on the p-SRR causes an upward shift in the resonance frequency of the μ-SRR near 1.9–2.1 GHz. Increasing power causes the plasma size to increase and the plasma begins to encroach on the μ-SRR resulting in some loss of sharpness to the peaks due to plasma damping. However, even at the highest power tested, the plasmas do not expand sufficiently toward the μ-SRRs to completely eliminate the upper resonances. Extraction of the effective permeability in the next section will show that the μ-SRR permeability remains negative in the presence of plasma.

3.1. Negative permeability of the μ-SRRs with plasma present

Techniques for the extraction of the permittivity and permeability from measured S-parameters have been widely used to evaluate metamaterial structures [37–41]. Through the measured values of \( S_{11} \) and \( S_{21} \), the material’s reflection coefficient \( \Gamma \) and the propagation factor \( z \) can be obtained from

\[
\Gamma = \frac{S_{11} + S_{22} - \Gamma}{1 - \Gamma(S_{11} + S_{22})},
\]

where \( X = (S_{11}^2 - S_{22}^2 + 1)/2S_{21} \). \( \alpha \) is the attenuation constant, \( \beta \) is the phase constant, and \( d \) is the sample thickness. The sign for \( \Gamma \) is typically chosen to satisfy \( \Gamma < 1 \), but numerical challenges are still associated by the complex square root and the limitation of phase ambiguity in the frequency domain (\( 0 < \beta < 2\pi \)). Moreover, the method assumes well-defined boundary conditions of the material which are difficult to reproduce in our experiment due to the constraints imposed by the vacuum chamber.

We used a common approximation to avoid these ambiguity issues. Assuming that \( \varepsilon \approx 1 - (\alpha + j\beta d) \), the wave propagation vector \( (k = \alpha + j\beta) \) and the relative permeability can be written as [37]

\[
k \approx \frac{1}{d} \left[ 1 - (S_{11} + S_{22})(1 + \Gamma) \right],
\]

\[
\mu_r \approx \frac{2}{k_0 d} \frac{1 - (S_{11} + S_{22})}{1 + (S_{11} + S_{22})},
\]

where \( k_0 \) is the wave propagation number in vacuum. These assumptions can provide a unique wave vector and reasonable \( \mu_r \) without phase and sign discontinuities. The approximation requires that the sample be smaller than the wavelength. The SRR array thickness \( d \) in the present configuration (72 mm) is comparable to the wavelength within the substrate (TMM3, \( \varepsilon_r = 3.27 \)) at 2 GHz, so this is marginally valid. Without the approximation, however, the measurement noise and phase discontinuities result in unrealistic extraction of permeability values [42, 43]. The final issue with the extraction method is the accurate measurement of \( S_{11} \). The reflection cross-section of the SRR array is similar to the antenna aperture and the anechoic lining of the vacuum chamber absorbs much of the backscattered wave. Therefore, the normalized \( S_{11} \) parameter measured from the experiment is not sufficiently precise.

To overcome this problem, we note that the reflection coefficient near the SRR resonances is typically close to unity and therefore use \( S_{11} \approx 0 \) dB. With this approximation the method is no longer able to extract the permittivity, and therefore we defer that measurement to the final section of the paper using another method.

Figure 5 shows the magnitude and phase of the transmission through the SRR array as well as the extraction of the effective permeability of the array with and without plasma present. The frequency range displayed in figure 5 is restricted to the band near the μ-SRR array because, as previously discussed, once plasma forms the permeability is \( \mu_r \approx 1 \) in the lower frequency band of the p-SRRs. Without plasma present, we find that the relative permeability of the array is less than –10 near the resonance of 1.94 GHz. In the presence of plasma, we note that the permeability remains negative as power is increased, although the plasma shifts the region of negative \( \mu \) upward in frequency toward 2 GHz. There is a limit to the effectiveness of this particular dual resonator design, however, as shown by the data for \( P_{\alpha} = 150 \) W. In the case of argon gas, the plasma is smaller in volume and the permeability remains negative. The xenon plasma, however, is larger in volume and we observe that it
extends outward to contact the \( \mu \)-SRRs at the highest measured power. In this case the negative permeability property is lost, thus confirming the hypothesis that SRR metamaterials containing plasmas must be designed to isolate the loading effect of the plasma.

Figure 6 shows the extracted \( \mu_r \) at xenon plasma pressures of 0.6, 1, and 2 Torr for the same 3 \( \times \) 3 \( \times \) 9 metamaterial configuration. The minimum discharge pressure was limited to 0.6 Torr since parasitic plasmas started to form at the edges between the metamaterial and the Teflon mounting device. The measurements show that negative \( \mu_r \) was easily lost at lower pressure where the volume of plasmas was visibly increased. Here we see that the negative \( \mu_r \) is adversely affected by the plasma volume when plasma comes in contact with the surface of the \( \mu \)-SRRs. Since the plasma size is small compared to the SRR spacing \((d_y = 6.4 \text{ mm})\) at 2 Torr, the desired negative \( \mu_r \) is still observed, even at the maximum power of 150 W. In the case of argon (not shown), the strong resonance and negative \( \mu_r \) were observed at the maximum power of 150 W, due to the smaller size of the argon plasma. Finally, with greater separation between the SRRs \((d_y = 10 \text{ mm})\), the negative \( \mu_r \) can be observed with both xenon and argon plasmas at the maximum power of 150 W (figure 4). In summary, the isolated \( \mu \)-SRR can produce a negative permeability in the presence of plasma as long as the adjacent \( p \)-SRR’s plasma does not encroach close enough to damp its oscillations.

### 3.2. Plasma density and permittivity

In this section we extract the permittivity of the plasma by measuring the spatial variation of the plasma density. Table 1 lists the peak plasma density \( n_0 \), the electron temperature \( T_e \), the calculated bulk plasma permittivity \( \varepsilon_p \), and the collision frequency for momentum transfer \( v_m \) in argon and xenon for pressures of 0.6, 1, and 2 Torr. In these experiments, we also wish to create negative \( \mu_r \) at higher plasma power (i.e., larger electron density), therefore \( d_y \) was chosen to be 10 mm to limit the plasma’s interaction with the \( \mu \)-SRRs as noted above. Since plasma formation within the full 3 \( \times \) 3 \( \times \) 9 array is very sensitive to interference by the Langmuir probe, the plasma density was measured in the center of a structure with 3 \( \times \) 3 \( \times \) 5 SRRs.
The local plasma density, however, was found to be invariant to the number of 2D arrays as each SRR is responsive to the local $E$-field in this radiative power system [18].

The peak electron density in the argon plasma is approximately $10^{11}$ cm$^{-3}$ which provides a marginally negative permittivity at the resonance frequency of the $\mu$-SRRs. The real part of the plasma’s relative permittivity was estimated by the Drude dispersion relation using

$$\varepsilon_p = 1 - \frac{e^2 n_0}{\varepsilon_0 m (\omega^2 + \nu_m^2)} = 1 - \frac{\omega_p^2}{(\omega^2 + \nu_m^2)},$$

where $\varepsilon_0$ is the permittivity of vacuum, $m$ is the electron mass, and $\omega_p = \sqrt{e^2 n_0 / \varepsilon_0 m}$ is the plasma electron frequency. Assuming a Maxwellian electron energy distribution, the collision frequency is found from

$$\nu_m = n_e \langle \sigma_m(v)v \rangle = n_e K_m,$$

where $n_e$ is the gas density. $K_m$ is the normalized collisional rate constant which is determined by averaging the elastic electron-neutral collision cross-section $\sigma_m$ and electron speed $v$ [44]. The lowest plasma permittivity ($\varepsilon_p \approx -2.4$) is observed at an argon pressure of 1 Torr. Although higher pressure can increase the electron density, the increased pressure results in a higher collision frequency and a permittivity that trends toward the positive range according to (10). An attempt to increase the electron density by using Xe gas is also reported in table 1. The surprising result shows that the readily-ionized Xe atoms actually produce a lower plasma density. Typically, Xe has a higher electron density compared to argon as reported at pressures of a few mTorr [45]. However, we note that the electron temperature is significantly reduced by higher pressures of Xe, and this results in the lower xenon plasma density measured here. To confirm this result, we also measured the line integrated electron density by microwave interferometer. The result for argon plasma was $5 \times 10^{10}$ cm$^{-2}$, which is higher than that of the xenon plasma ($3 \times 10^{10}$ cm$^{-2}$) in spite of the larger visual appearance of the Xe plasma.

It is possible to examine this result using a simple plasma model. The plasma density in both xenon and argon plasma can be written as

$$n_0 = \frac{P_{abs}}{P_{loss}} = \frac{P_{abs}}{e n_0 A_{eff} \varepsilon_T},$$

where $P_{abs}$ is the power absorbed by the plasma, $A_{eff}$ is the effective area for energy loss, and $\varepsilon_T$ is the total energy loss per electron–ion pair [30]. Mistoco et al have derived the collisional energy loss $\varepsilon_c$ (per electron–ion pair generated) for argon and xenon plasma, and show that $\varepsilon_c$ dramatically increases for lower electron temperature [30, 44]. Using measured electron temperatures we find that Xe plasma has $\varepsilon_c \approx 500$ eV at 1 eV, but $\varepsilon_c < 100$ eV in the case of argon plasma. Given that the ion sheath potential is small, we assume $\varepsilon_T \approx \varepsilon_c$ and find that $P_{loss}(Xe) / P_{loss}(Ar) \approx 3$. For a comparable absorbed power, this shows that Ar will have three times the electron density which is consistent with table 1 and the interferometer measurements. Similarly, Dagang et al [46] have reported on xenon plasma characteristics in the pressure range of 0.3–1 Torr, and find that ionization and collisional processes were decreased by higher pressures corresponding to an electron temperature range of 1–4 eV. Therefore, higher collisional loss with low electron temperature reduces steady-state ionization processes in Xe, which results in lower xenon plasma density in the vicinity of 1 Torr. The peak permittivity of Xe plasmas is near zero in table 1 and we conclude that Xe is not a viable path to negative permittivity in SRR-produced plasmas in this pressure regime.

### Table 1. Plasma properties as a function of pressure at fixed power of 120 W.

| Pressure (Torr) | $n_0$ ($10^{11}$ cm$^{-3}$) | $T_e$ (eV) | $\varepsilon_p$ | $\nu_m$ ($10^9$ s$^{-1}$) | $n_0$ ($10^{11}$ cm$^{-3}$) | $T_e$ (eV) | $\varepsilon_p$ | $\nu_m$ ($10^9$ s$^{-1}$) |
|---------------|-----------------|---------|--------|-----------------|-----------------|---------|--------|-----------------|
| 0.6           | 0.8             | 2.2     | -1.8   | 1.9             | 0.3             | 1.3     | -0.1  | 3.4             |
| 1             | 1.0             | 1.8     | -2.4   | 2.2             | 0.5             | 1.2     | -0.4  | 5.1             |
| 2             | 1.0             | 1.7     | -2.0   | 4.1             | 0.6             | 1.1     | -0.1  | 9.2             |

Figure 6. Extracted $\mu_\perp$ at xenon pressure of 0.6, 1, and 2 Torr through a metamaterial structure of $3 \times 3 \times 9$ ($d_y = 6.4$ mm) with existence of xenon plasmas.
The plasma is most intense near the discharge gap of each $p$-SRR as shown in figure 3, and the plasma parameters at these locations are reported in table 1. It is desirable to understand the distribution of plasma density within the array and, therefore, the spatial variation of the plasma permittivity. The spatial average of the permittivity is estimated by measuring the plasma profile in the $yz$-plane by scanning the Langmuir probe through the array in the $z$-direction as shown in figure 2. Each spatially-resolved scan of the ion saturation current was repeated for three positions in the $y$-direction. Figure 7 shows the spatial permittivity profile for argon and xenon gas pressures of 0.6 and 1 Torr with a constant microwave power ($1.5$ GHz, $120$ W). In this case, the electron collisions are minimal ($\omega > \nu_{el}$) and $\varepsilon_p \approx 1 - \omega_p^2 / \omega^2$ was used for the contour plot. Note that the dashed-line indicates the boundary for which $\varepsilon_p = 0$ ($n_0 \approx 0.3 \times 10^{11}$ cm$^{-3}$). A larger volume of the xenon plasma than argon plasma was observed, but the volume of the weak Xe plasma exhibiting negative $\varepsilon_p$ was very small. The estimated averaged permittivity in the volume between 2D SRR arrays is 0.37 and 0.42 for argon pressures of 0.6 and 1 Torr, respectively. In the case of xenon, however, the average permittivity was only 0.7 due to the low plasma density. Therefore, while the metamaterial exhibited $\mu_r < 0$ due to the isolated $\mu$-SRR array and local regions where $\varepsilon_p < 0$, the average permittivity was greater than zero and double negative behavior was not observed.

This experiment inherently requires a steady-state plasma in order to allow the acquisition of the frequency-domain transmission spectra and subsequent extraction of the effective permeability. Previous experimental studies of pulsed resonator microplasmas [47] using time-gated imaging spectroscopy, however, suggest that the expansion time of an argon microplasma is on the order of 1 ms. Therefore, the volume-average permittivity is naturally closer to unity if one employs a short, pulsed-plasma metamaterial for which the plasma does not reach its full steady-state volume. Regardless, if the plasma has sufficient size and density to create negative permittivity throughout the metamaterial volume, then that plasma may not contact the SRR or negative permeability will be lost.

4. Conclusion

Experimental investigation of plasma-enhanced 3D metamaterials shows that low-pressure plasma loading is highly likely
to remove the resonance within the metamaterial, resulting in the loss of negative permeability $\mu$. An improved design of the metamaterial consists of two sets of SRRs, one group self-initiates and sustains the plasma ($\mu$-SRRs) and the other group provides negative permeability ($\mu$-SRRs). This arrangement can maintain negative $\mu$ in the presence of plasmas as long as plasma does not quench the $\mu$-SRRs. Periodic plasmas within the metamaterial structure are dimensionally scalable and are shown to be easily extended to at least $3 \times 3 \times 9$.

Using microwave transmission spectra through the metamaterial, we extracted the permeability in the presence of xenon and argon plasmas. The experiments show negative permeability due to the strong unquenched resonance of the $\mu$-SRRs. Although some plasma loading effect on the $\mu$-SRRs is also observed, it is mainly due to the primary plasmas contacting the surfaces of the $\mu$-SRRs at the highest radiated powers. Further improvements to the structure will focus on better isolating the structures responsible for negative $\mu$.

With maximum plasma density within the plasma volume of $\sim 1 \times 10^{11} \text{ cm}^{-3}$ for argon and $\sim 0.6 \times 10^{11} \text{ cm}^{-3}$ for xenon plasmas, we report that regions of negative permittivity exist. However, the spatial average permittivity is $\sim 0.4$ and $\sim 0.7$ for argon and xenon plasma, respectively. Spatial average permittivity is positive in both gases because the region of plasma having negative permittivity is much smaller than the free space volume within the metamaterial. As a result we could not observe double-negative behavior through this plasma-enhanced metamaterial structure.

In future experiments, we will continue to investigate double-negative behavior in an improved metamaterial structure where the spatial average permittivity becomes negative by creating a larger plasma volume and a higher plasma density within the negative permeability spaces.

Acknowledgments

This work was supported by the Air Force Office of Scientific Research (AFOSR) under Award No. FA9550-14-10317 through a Multi-University Research Initiative (MURI) Grant titled ‘Plasma-Based Reconfigurable Photonic Crystals and Metamaterials’ with Dr Mitat Birkan as the program manager.

ORCID iDs

Jeffrey Hopwood © https://orcid.org/0000-0002-0265-7095

References

[1] Veselago V G 1968 The electrodynamics of substances with simultaneously negative values of $\varepsilon$ and $\mu$ Sov. Phys.—Usp. 10 509–14
[2] Pendry J B 2000 Negative refraction makes a perfect lens Phys. Rev. Lett. 85 3966–9
[3] Shelby R A, Smith D R and Schultz S 2001 Experimental verification of a negative index of refraction Science 292 77–9
[4] Pendry J B, Schurig D and Smith D R 2006 Controlling electromagnetic fields Science 312 1780–2
[5] Schurig D, Mock J J, Justice B J, Cummer S A, Pendry J B, Starr A F and Smith D R 2006 Metamaterial electromagnetic cloak at microwave frequencies Science 314 977–80
[6] Landy N I, Sajuyigbe S, Mock J J, Smith D R and Padilla W J 2008 Perfect metamaterial absorber Phys. Rev. Lett. 100 207402
[7] Singh P K, Ameri S K, Chao L, Afsar M N and Sonkusale S 2013 Broadband millimeterwave metamaterial absorber based on embedding of dual resonators Prog. Electromagn. Res. 142 625–38
[8] Rout S, Shkreinemaher D, Sonkusale S and Padilla W 2010 Embedded HEMT/metamaterial composite devices for active terahertz modulation 2010 23rd Annual Meeting IEEE Photon. Soc. 437–8
[9] Chen W T, Chen C J, Wu P C, Sun S, Zhou L, Guo G Y, Hsiao C T, Yang K Y, Zheludev N I and Tsai D P 2011 Optical magnetic response in three-dimensional metamaterial of upright plasmonic meta-molecules Opt. Express 19 12837–42
[10] Soukoulis C M and Wegener M 2010 Optical metamaterials—more bulky and less lossy Science 330 1633–4
[11] Burckel D B, Wendt J R, Ter Eyck G A, Ginn J C, Ellis A R, Brenner I and Sinclair M B 2010 Micrometer–cubic unit cell 3D metamaterial layers Adv. Mater. 22 5053–7
[12] Zaherta S, Yalcinkaya A D and Torun H 2015 Rectangular split-ring resonators with single-split and two-splits under different excitations at microwave frequencies AIP Adv. 5 117220
[13] Okamoto T, Otsuka T, Sato S, Fukuta T and Haraguchi M 2012 Dependence of LC resonance wavelength on size of silver split-ring resonator fabricated by nanosphere lithography Opt. Express 20 24059–67
[14] Seo J-H, Kim T-H, Kang C, Kee C-S and Lee J-S 2016 Distribution of THz electric field in the split-ring resonator metamaterials based on the thin film geometry Curr. Appl. Phys. 16 329–34
[15] Aydin K and Ozbay E 2006 Identifying magnetic response of split-ring resonators at microwave frequencies Opto-Electron. Rev. 14 193–9
[16] Dextre R A and Xu K G 2017 Effect of the split-ring resonator width on the microwave microplasma properties IEEE Trans. Plasma Sci. 45 215–22
[17] Gay-Balmaz and Martin O J P 2002 Electromagnetic resonances in individual and coupled split-ring resonators J. Appl. Phys. 92 2929
[18] Kim H, Parsons S and Hopwood J 2018 Spatially adjustable microplasma generation in proto-metamaterials using microwave radiative power transfer Plasma Sources Sci. Technol. 27 015010
[19] Kourtzanidis K, Pederson D M and Raja L L 2016 Optical magnetic response in three-dimensional metamaterials based on embedding of dual resonators AIP Adv. 6 045004
[20] Parsons S, Gregorioso J and Hopwood J 2017 Microwave plasma formation within a 2D photonic crystal Plasma Sources Sci. Technol. 26 055002
[21] Varault S, Gabard B, Sokoloff J and Bolioli S 2011 Plasma-based localized defect for switchable coupling applications Appl. Phys. Lett. 98 134103
[22] Parsons S and Hopwood J 2017 Millimeter wave plasma formation within a 2D photonic crystal IEEE Electron Device Lett. 38 1602–5
[23] Tamayama Y and Sakai O 2017 Microplasma generation by slow microwave in an electronically induced transparency-like metasurface J. Appl. Phys. 121 073303
[24] Sakai O, Iio S and Nakamura Y 2013 Overdense microwave plasma generation in a negative-permeability space J. Plasma Fusion Res. 8 1406167
[25] Nakamura Y and Sakai O 2014 High-density plasma source using negative-permeability metamaterial with tuned wave attenuation Japan. J. Appl. Phys. 53 03DB04-1
[26] Godyak V A, Piejak R B and Alexandrovich B M 2002 Electron energy distribution function measurements and plasma parameters in inductively coupled argon plasma Plasma Sources Sci. Technol. 11 525–43
[27] Minayeva O B and Hopwood J 2003 Langmuir probe diagnostics of a microfabricated inductively coupled plasma on a chip J. Appl. Phys. 94 2821
[28] Godyak V A and Alexandrovich B M 2015 Comparative analyses of plasma probe diagnostics techniques J. Appl. Phys. 118 233302
[29] Dickson M, Qian F and Hopwood J 1996 Quenching of electron temperature and electron density in ionized physical vapor deposition J. Vac. Sci. Technol. A 15 340
[30] Lieberman M A and Lichtenberg A J 1994 Principle of Plasma Discharges and Material Processing (New York: Wiley)
[31] Kurs A, Karalis A, Moffatt R, Joannopoulos J D, Fisher P and Solijačić M 2007 Wireless power transfer via strongly coupled magnetic resonances Science 317 83–6
[32] Wu C, Hoskinson A R and Hopwood J 2011 Stable linear plasma array at atmospheric pressure Plasma Sources Sci. Technol. 20 045022
[33] Singh P K, Hopwood J and Sonkusale S 2014 Metamaterials for remote generation of spatially controllable two dimensional array of microplasma Sci. Rep. 4 5964
[34] Zhang Z-B and Hopwood J 2009 Linear arrays of stable atmospheric pressure microplasmas Appl. Phys. Lett. 95 161502
[35] Haas H A and Huang W 1991 Coupled-mode theory Proc. IEEE 79 1505–18
[36] Iza F and Hopwood J 2005 Split-ring resonator microplasma: microwave model, plasma impedance and power efficiency Plasma Sources Sci. Technol. 14 397–406
[37] Ziolkowski R W 2003 Design, fabrication, and testing of double negative metamaterials IEEE Antennas Propag. 51 1516–29
[38] Ziolkowski R W and Heyman E 2001 Wave propagation in media having negative permittivity and permeability Phys. Rev. E 64 056625
[39] Nicolson A M and Ross G F 1970 Measurement of the intrinsic properties of materials by time-domain techniques IEEE Trans. Instrum. Meas. 19 377–82
[40] Arslanagić S, Hansen T V, Mortensen N A, Gregerson A H, Sigmund O, Ziolkowski R W and Breinbjerg O 2013 A review of the scattering parameter extraction method with clarification of ambiguity issues in relation to metamaterial homogenization IEEE Antennas Propag. 55 91–106
[41] Smith D R, Vier D C, Koschny T and Soukoulis C M 2005 Electromagnetic parameter retrieval from inhomogeneous metamaterials Phys. Rev. E 71 036617
[42] Woodley J and Mojahedi M 2010 On the signs of the imaginary parts of the effective permittivity and permeability in metamaterials J. Opt. Soc. Am. B 27 1016–21
[43] Qi J, Keitunen H, Wallén H and Sihvola A 2010 Compensation of Fabry–Pérot resonances in homogenization of dielectric composites IEEE Antennas Wireless Propag. Lett. 9 1057–60
[44] Mistoco V F and Bilén S G 2008 Numerical modeling of a miniature radio-frequency ion thruster 44th AIAA/ASME/SAE/ASEE Joint Propulsion Conf. and Exhibit (Harford, CT) AIAA 2008-5194 (https://doi.org/10.2514/6.2008-5194)
[45] Schwabedissen A, Benck E C and Roberts J R 1997 Langmuir probe measurements in an inductively coupled plasma source Phys. Rev. E 55 3450
[46] Dagang A N, Kondo A, Motomura H and Jinno M 2009 Mercury-free electrodeless discharge lamp: effect of xenon pressure and plasma parameters on luminance J. Phys. D: Appl. Phys. 42 095202
[47] Hoskinson A R, Yared A and Hopwood J 2015 Gas heating and plasma expansion in pulsed microwave-excited microplasmas Plasma Sources Sci. Technol. 24 055002