Regular superconducting solution for interior of the Kerr-Newman (KN) spinning black-hole has \( g = 2 \) as that of the Dirac electron and paid attention as a classical model of electron coupled with gravity \( 1, 2, 3, 4, 5, 6, 7, 8 \). For parameters of electron (in the units \( G = C = \hbar = 1 \)) \( J = 1/2, m \sim 10^{-22}, e^2 \sim 137^{-1} \) the black-hole horizons disappear and the Kerr singular ring of the Compton radius \( a = h/2m \sim 10^{22} \) turns out to be naked. This ring forms the gate to a negative sheet of the Kerr geometry, significance of which is the old trouble for interpretation of the source of Kerr geometry. The attempts by Brill and Cohen to match the Kerr exterior with a rotating spherical shell and flat interior \( 10, 11, 12 \) have not lead to a consistent solution, and Israel suggested to truncate the negative sheet, replacing it by a thin (rotating) disk spanned by the Kerr ring \( 3 \). This model led to a consistent source which was build from a very exotic superluminal matter, besides the Kerr singular ring remained naked at the edge of the disk. López \( 4 \) transformed this model to the model of an oblate rotating thin shell which covers the Kerr ring. By especial choice of the boundary of the bubble, \( r = r_0 = e^2/2m \) (\( r \) is the Kerr oblate spheroidal coordinate), he matched continuously the Kerr exterior with the flat interior, and therefore he obtained a consistent KN source without the KN singularity. However, like the other models, the López model was not able to explain the origin of Poincaré stress, and a negative pressure was introduces by López phenomenologically. Meanwhile, the necessary tangential stress appears naturally in the domain walls field models, in particular, in the models based on Higgs \( 6 \). It suggests that a consistent field descriptions of this problem should contain, along with the KN Einstein-Maxwell sector, the sector of Higgs fields and sector of interaction between the em field (coupled to gravity) and Higgs fields corresponding to superconducting properties of the KN source. Up to our knowledge, despite several attempts and partial results, no one has been able so far to obtain a consistent field model of the KN source related with Higgs field, and in this paper we present apparently first solution of this sort which is consistent in the limit of thin domain wall boundary of the bubble. The considered KN source represents a generalization of the López model and incorporate the results obtained earlier in \( 9, 10, 11, 12 \). Although we consider here only the case of thin domain wall, generalization to the case of a finite thickness is also possible by methods described earlier in \( 10 \). The described in \( 6, 10 \) KN source was a bag formed by a potential \( V(r) \) interpolating between the external ‘true’ vacuum \( V^{(ext)} = 0 \) and a ‘false’ (superconducting) vacuum \( V^{(in)} = 0 \) inside of the bag. It was shown that corresponding Higgs sector may be described by \( U(1) \times U(1) \) Witten field model with the given by Morris in \( 18 \) super-potential. It was shown also in \( 6, 10, 13 \) that consistency of the Einstein-Maxwell sector with such a phase transition may be perfectly performed in the KS formalism with use of the Gürses and Gürsey ansatz \( 19 \). However, the problem with consistency appeared in \( 6 \) by the treatment of interaction between the em and Higgs field. In this short note we improve this deficiency which allows us to present consistent solution.

2. Phase transition in gravitational sector. Following \( 6, 7, 12 \), for external region we use the exact KN solution in the Kerr-Schild (KS) form of metric

\[
g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_{\mu}k_{\nu}, \tag{1}
\]

where \( \eta^{\mu\nu} \) is metric of the auxiliary Minkowski background in Cartesian coordinates \( x^{\mu} = (t, x, y, z) \). Electromagnetic (em) KN field is given by vector potential

\[
A_{\mu KN} = Re\frac{e}{r + ia \cos \theta}k^{\mu}, \tag{2}
\]

where \( k^{\mu}(x^{\mu}) \) is the null vector field which is tangent to a vortex field of null geodesic lines, the Kerr principal null congruence (PNC). For the KN solution function \( H \) has the form

\[
H = \frac{m r - e^2/2}{r^2 + a^2 \cos^2 \theta}. \tag{3}
\]
The used by Kerr especial spherical oblate coordinates $r, \theta, \phi_K$, are related with the Cartesian coordinates as follows:

$$x + iy = (r + ia)e^{i\phi_K} \sin \theta, \quad z = r \cos \theta.$$  

Vector field $k^\mu$ is represented in the form

$$k_\mu dx^\mu = dr - dt - a \sin^2 \theta d\phi_K. \quad (4)$$

For the metric inside of the oblate bubble we use the KS ansatz $f(r)$ in the form suggested by Gürses and Gürsey $[14]$ with function

$$H = \frac{f(r)}{r^2 + a^2 \cos^2 \theta}.$$  

If we set for interior $f(r) = f_{int} = \alpha r^4$, the Kerr singularity will be suppressed.

For exterior, $r > r_0$, we use $f(r) = f_{KN} = mr - e^2/2$ corresponding to KN solution. Therefore, $f(r)$ describes a phase transition of the KS metric from ‘true’ to ‘false’ vacuum (see fig.1).

![FIG. 1: Matching of the metric for regular bubble interior with metric of external KN field.](image)

We assume that the zone of phase transition, $r \approx r_0$, is very thin and metric is continuous there, so the point of intersection, $f_{int}(r_0) = f_{KN}(r_0)$, determines position of domain wall $r_0$ graphically and yields the ‘balance matter equation’ $[9, 12, 13, 14]$,

$$m = m_{em}(r_0) + m_{mat}(r_0), \quad m_{em}(r_0) = \frac{e^2}{2r_0}, \quad (5)$$

which determines $r_0$ analytically. Interior has constant curvature, $\alpha = 8\pi\Lambda/6$. For parameters of electron $\alpha = J/m = 1/2m >> r_0 = e^2/2m$ and the axis ratio of the ellipsoidal bubble is $r_0/\alpha = e^2 \approx 137^{-1}$, so the bubble has the form of highly oblated disk.

3. Brief summary of the Higgs sector $[9, 14]$. The corresponding phase transition is provided by Higgs model with two complex Higgs field $\Phi$ and $\Sigma$, two related gauge fields $A^\mu$ and $B^\mu$, and one auxiliary real field $Z$. This is a given by Morris $[18]$ generalization of the $U(1) \times U(1)$ field model used by Witten for superconducting strings $[20]$.

The potential $V(r) = \sum_i |\partial W|^2$, where $\partial_1 = \partial_{\Phi}$, $\partial_2 = \partial_Z$, $\partial_3 = \partial_{\bar{\Phi}}$, is determined by superpotential

$$W = \lambda Z(\Sigma - \eta^2) + (cZ + m)\Phi\bar{\Phi},$$

where $c, m, \eta, \lambda$ are real constants. It forms a domain wall interpolating between the internal (‘false’) and external (‘true’) vacua.

The vacuum states obey the conditions $\partial_i W = 0$ which yield $V = 0$

1) for ‘false’ vacuum ($r < r_0$): $Z = -m/c; \Sigma = 0; |\Phi| = \eta\sqrt{\lambda/c}$, as well as

2) for ‘true’ vacuum ($r > r_0$): $Z = 0; \Phi = 0; \Sigma = \eta$.

4. Interaction between the KN and Higgs fields. We set $B^\mu = 0$ and use only the gauge fields $A^\mu$ which interacts with the Higgs field $\Phi(x) = |\Phi(x)|e^{i\chi(x)}$ having a nonzero vev inside of the bubble, $|\Phi(x) |_r < r_0 = \Phi_0$, and vanishing outside. The related part of the field model is a copy of the Nielsen-Olesen (NO) field model for a vortex string in superconducting media $[21]$, however, there is principal difference in topology, since we describe superconducting interior of the bubble, contrary to superconducting external media of the NO model. The Lagrangian $L_I = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} (\partial_\mu \Phi)(\partial^\mu \Phi)^*, \quad (6)$

where $D_\mu = \nabla_\mu + ic A_\mu$, and $\Phi(x)$ is determined by superpotential $i\chi(\mu + c A_\mu)$. In external region $|\Phi| = 0$ and the em field has to correspond to exact KN solution, $A^\mu = A^\mu_{KN}$. Following López, we fix the boundary of bubble at $r_0 = r_e = e^2/2m$ (one half of the classical radius of electron), which yields flat interior, $\alpha = 0$, and allows us to use flat d’Alembertian and set $D_\mu = \partial_\mu + ic A_\mu$ for $r < r_0$. We assume that current has to be expelled from interior of the bubble to its boundary (perfect superconductor), and set in interior $I_\mu = 0$, which yields

$$\Box A_\mu = 0 = e|\Phi|^2 (\chi + e A^{(in)}_\mu) .$$

The KN gauge field $A_\mu$ is given by $[2]$ and $[11]$. Inside of the bubble the specific Kerr angular coordinate $\phi_K$ turns out to be inconsistent with the simple angular coordinate of the Higgs field, $\phi = -i \ln((x + iy)/\rho), \rho = (x^2 + y^2)^{1/2}$. Using the relation between corresponding differentials $d\phi_K = \phi + \frac{adr}{r^2 + a^2}$, we transform vector potential on the boundary and inside of the bubble to the form

$$A_\mu dx^\mu = -\frac{er}{r^2 + a^2 \cos^2 \theta} [dt + a \sin^2 \theta d\phi] + \frac{2er dr}{r^2 + a^2}. \quad (7)$$

which shows that radial component $A_r$ is full differential. At the external side of the bubble $r = r_e + 0 = e^2/2m$, the value of potential is $A_\mu dx^\mu |_{r = r_0 + 0} = (-\frac{er}{r^2 + a^2 \cos^2 \theta} [dt + a \sin^2 \theta d\phi] + \frac{2er dr}{r^2 + a^2})$. The lines of KN vector-potential in equatorial plane are tangent to the Kerr singular ring, and approaching the string-like edge of bubble by $\cos \theta = 0$ the potential takes the form

$$A^{(str)}_\mu dx^\mu = -\frac{2m}{e} [dt + ad\phi] + \frac{2er dr}{(r^2 + a^2)}. \quad (8)$$
In particular, the tangent component at the edge is $A_{\nu}^{(\text{str})} = -2ma/e$. Since $J = ma$ we have $A_{\nu}^{(\text{str})} = -2J/e$. Setting $J = \frac{m}{\pi}, n = 1, 2, \ldots$ we find out that vector potential forms on the edge of bubble a closed quantized Wilson loop

$$S = \oint eA_{\phi}^{(\text{str})} d\phi = -2\pi n,$$

which has to be matched with angular periodicity of the Higgs field $\Phi = \Phi_0 \exp(i\chi)$ and fixes its $\phi$-dependence, $\Phi \sim \exp\{i\alpha \chi\}$. For the time-like component of $A_{\mu}$ inside of the bubble, the r.h.s. of (6) states $\chi_0 = -eA_{\nu}^{(\text{str})}(r)$, which determines the $\chi_0$ to be a constant corresponding to frequency of oscillations of the Higgs field, $\chi_0 = \omega = -eA_{\nu}^{(\text{str})} = 2m$. Radial component of the KN field is a full differential, and being extended inside the bubble, it is compensated by Higgs field in agreement with the r.h.s. of (6). Therefore, the Higgs field acquires the form

$$\Phi(x) = \Phi_0 \exp\{i\omega t - i \ln(r^2 + a^2) + i\alpha \phi\}. \quad (9)$$

For exclusion of the region of string-like loop at equator, the time and $\phi$ components of the gauge field have a chock crossing the boundary of bubble, which determine a distribution of circular currents over the bubble boundary.

5. Consistency. We find out that inside of the bubble $D_{\mu}\Phi = i\partial_{\mu}\chi + eA_{\nu}^{(\text{in})} \equiv 0$. Together with the result that $V^{(\text{in})} = 0$, it leads to vanishing of the stress-energy tensor of matter inside of the bubble, $T^{(\text{in})}_{\mu\nu} = (D_{\mu}\Phi)[(D_{\nu}\Phi) - \frac{1}{2}g_{\mu\nu}][D_{\lambda}\Phi][D^{\lambda}\Phi]$, and provides flatness of the interior in agreement with our assumptions.

Therefore, the obtained superconducting solution turns out to be consistent in the limit of the infinitely thin domain wall. It should be emphasized two important distinctions of this model from the typical soliton-like field models:

i) oscillations of the Higgs field with frequency $\omega = 2m$, and ii) the appearance of quantum Wilson loop on the edge of KN disk.

The model can also be generalized for the domain walls of a finite thickness $\delta$, if the parameter $r_0$ is much greater then $\delta$. In this case, as it was considered in [6], one can use the flat domain wall approximation, and the exact field equations corresponding to finite region of phase transition may be integrated, at least numerically.

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