Supplementary Materials for

Negative reflection of nanoscale-confined polaritons in a low-loss natural medium

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S1. Specular reflection in isotropic media

![Schematics of reflection of polaritons in isotropic media. (A) Reflection of polaritons upon momentum conservation at the boundary ($\Delta k_\parallel = k_\parallel^r - k_\parallel^i = 0$). The black solid line represents the polaritonic IFC, where $k_{i,r}$ and $S_{i,r}$ are always parallel. (B) Real-space illustration of reflection of isotropic polaritons. The angle of reflection $\theta_r$ is always equal to that of incidence $\theta_i$ (specular reflection).](image)

S2. Reflection of highly-confined polaritons in natural elliptic media

![Schematics of reflection of polaritons in elliptic media. (A) Reflection of polaritons upon momentum conservation at the boundary ($\Delta k_\parallel = k_\parallel^r - k_\parallel^i = 0$). The black solid line represents the polaritonic IFC, where $k_{i,r}$ and $S_{i,r}$ are generally non-collinear parallel. (B) Real-space illustration of reflection of elliptic polaritons. The angle of reflection $\theta_r$ is different to that of incidence $\theta_i$.](image)
**Fig. S3.** Visualization of reflection of nanoscale-confined polaritons in a natural elliptic medium. (A) Illustration of back-reflection of elliptic PhPs in k-space and real space for in-plane elliptical media. (B) Full-wave numerical simulation $E_z(x,y)$ of back-reflected polaritons excited by a point dipole on $\alpha$-MoO$_3$ as they back-reflect on a mirror tilted at an angle $\varphi = 38^\circ$. (C) Experimental s-SNOM amplitude image $s_3(x,y)$ of elliptic PhPs back-reflecting on a mirror tilted at an angle $\varphi = 38^\circ$ fabricated on $\alpha$-MoO$_3$. $\mathbf{k}_{i,r}$ and $\mathbf{S}_{i,r}$ indicate the wavefronts and Poynting vectors of the incident/reflected elliptic PhPs, respectively.

**S3. Schematics of negative reflection upon subwavelength mirrors**

**Fig. S4** Schematics of negative reflection of HPhPs upon extended and subwavelength mirrors. (A) HPhPs are launched by the tip along very different spatial directions (blue arrows). Upon reflection on an extended mirror, the outcoming HPhPs propagate along very different directions (red arrows), thereby generating complex interferences and obstructing clear visualization of the extreme case of negative reflection. (B) In contrast, a subwavelength mirror allows us to select waves propagating along one specific spatial direction (red arrow), enabling a clear visualization of negative reflection along a given direction by avoiding interferences with the rest of incoming waves (blue lines). Moreover, the subwavelength mirror guarantees a narrow beam of the reflected HPhPs, which is necessary to determine the direction of the polaritonic energy flux (i.e. the direction of Poynting vector, $\mathbf{S}$) in the near-field images (see Figs. 2,3 in the main text and Fig. S5).
Fig. S5. Extraction of the direction of the Poynting vector from experimental near-field images. (A) Experimental s-SNOM amplitude image $s_3(x,y)$ of HPhPs back-reflecting on a mirror tilted at an angle $\phi = 38^\circ$ fabricated on $\alpha$-MoO$_3$. We take several profiles (denoted as 1-4) along different in-plane directions. (B) By drawing the envelope to the line profiles, we find the direction of propagation of the Poynting vector as the direction along which the profile has the longest envelope (line profile-4).

S4. Experimental s-SNOM near-field amplitude and phase images of HPhPs at different incident frequencies

Figure S6. Near-field amplitude of negative reflection of nanoscale-confined polaritons in a natural medium as a function of the incident frequency. Experimental s-SNOM near-field amplitude images $s_3(x,y)$ of HPhPs back-reflecting on mirrors tilted at an angle $\phi = 38^\circ$ fabricated on $\alpha$-MoO$_3$ at different frequencies $\omega_0$. 
Figure S7. Near-field phase of negative reflection of nanoscale-confined polaritons in a natural medium as a function of the incident frequency. Experimental s-SNOM near-field phase images $\phi_3(x, y)$ of HPhPs back-reflecting on mirrors tilted at an angle $\varphi = 38^\circ$ fabricated on $\alpha$-MoO$_3$ at different frequencies $\omega_0$.

S5. Experimental s-SNOM near-field amplitude and phase images of HPhPs for different angles of the mirror $\varphi$

Figure S8. Near-field amplitude of negative reflection of nanoscale-confined polaritons in a natural medium as a function of the angle of the mirror. Experimental s-SNOM near-field amplitude images $s_3(x, y)$ of HPhPs back-reflecting on mirrors tilted fabricated on $\alpha$-MoO$_3$ at different angles $\varphi$ for a fixed incident frequency $\omega_0 = 889$ cm$^{-1}$.
Figure S9. Near-field phase of negative reflection of nanoscale-confined polaritons in a natural medium as a function of the angle of the mirror. Experimental s-SNOM near-field phase images $\phi_3(x,y)$ of HPhPs back-reflecting on mirrors tilted fabricated on $\alpha$-MoO$_3$ at different angles $\phi$ for a fixed incident frequency $\omega_0 = 889$ cm$^{-1}$.

S6. Analytical shape of a total hyperbolic retroreflector

In this section, we consider a plane wave propagating inside a biaxial half-space (that, without loss of generality, we will assume infinite along the $z$ direction). We will assume that the dielectric permittivity tensor of the medium has the following diagonal form:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}.$$ 

We will also consider that the plane wave, polarized in the $xy$-plane and with wavevector $k$ and Poynting vector $S$, is incident on the surface $f(x,y) = 0$ (see Figure S10). The dispersion of an electromagnetic wave propagating in the medium is given by Fresnel’s equation, which gives rise to the following IFC:

$$F \equiv \frac{k_x^2}{\varepsilon_y} + \frac{k_y^2}{\varepsilon_x} - k_0^2 = 0,$$  \hspace{1cm} (S1)
where \( k_0 = \omega / c \), \( \omega \) and \( c \) is the frequency of the wave and the speed of light, respectively. In the lossless case, i.e., when the dielectric permittivities are completely real, the Poynting vector is parallel to the group velocity. Therefore, \( S \) is given by \( S \sim \nabla F (k_x, k_y) \), or, more explicitly:

\[
S_x \approx \frac{k_x}{\varepsilon_y}, \quad S_y \approx \frac{k_y}{\varepsilon_x}.
\]

When considering the optical losses of the medium, the direction of the Poynting vector will be slightly tilted with respect to the one predicted by the previous equation, but we will neglect this effect here for the sake for simplicity, i.e. we will restrict ourselves to the lossless case.

**Figure S10. Optical paths of HPhPs back-reflecting on a hyperbolic retroreflector.**

Schematics of a retroreflector in (A) real space and (B) momentum space. \( S \) and \( k \) illustrate the directions of the Poynting and wave vectors, respectively. In (B), the waves back-reflecting at the points 1 and 2 follow different optical paths.

To fulfill momentum conservation at the boundary, any wave whose wave vector is perpendicular to the boundary surface will be reflected along the same direction but in the opposite direction. Therefore, the shape of the total reflector must be perpendicular to the IFC given by (S1).

\[
\frac{x^2}{\varepsilon_x} - \frac{y^2}{\varepsilon_y} = C. \quad (S2)
\]

In order to determine \( C \), we consider the phase of the wave. For the electric field to be maximized at the location of the source, the wave must propagate to the reflector and reflect back with constructive interference at the source. Let us consider now two different optical paths to the point (0,0): one along the x-axis and the other starting from an arbitrary point of the lens surface (labeled as “2” in Figure S10). The phase of the waves propagating along these paths are the following:

\[
\begin{align*}
\phi_1 &= 2k_x(0)d, \\
\phi_2 &= 2(k_y(\phi)y + k_x(\phi)x), \quad (S3)
\end{align*}
\]
where $d$ is the distance from the origin to point 1; $k_x(\phi)$ and $k_y(\phi)$ are the wave vector components in the direction marked by the angle $\phi$. On the other hand, the angle $\phi$ can be calculated geometrically as $\tan \phi = \frac{\varepsilon_x k_x}{\varepsilon_y k_y}$. These values are determined by the IFC of the medium in which the wave propagates by eq. Fresnel. At $\phi = 0$, $k_y = 0$ and hence $k_x(0) = \sqrt{\varepsilon_y k_0}$.

Substituting these two expressions into Eq. (S3) we get:

\[
\phi_1 = 2\sqrt{\varepsilon_y k_0 d},
\]

\[
\phi_2 = 2 \frac{\sqrt{\varepsilon_x k_0}}{1 + \frac{\varepsilon_x x^2}{\varepsilon_y y^2}} \left( y + \frac{\varepsilon_x x^2}{\varepsilon_y y^2} \right) = 2 \sqrt{\varepsilon_x y^2 + \varepsilon_y x^2 k_0},
\]

For these waves to focus on the point $(0, 0)$, and, consequently, for the curve $f(x, y) = 0$ to be a total retroreflector, both phases must be equal, i.e. $\phi_1 = \phi_2$. Under this condition, the phases of all the waves coming through the lens to the point $(0, 0)$ will be equal, as the point “2” is arbitrary.

The shape of the lens in real space is thus defined by:

\[
\frac{x^2}{\varepsilon_x} - \frac{y^2}{\varepsilon_y} = -\left( \frac{d}{\sqrt{\varepsilon_x}} \right)^2.
\]

Note that the parameter $d/\varepsilon_x$ corresponds to the distance from the origin to the vertex of the hyperbola. For the interference to be constructive, the condition $\phi_1 = \phi_2 = 2\sqrt{\varepsilon_y k_0 d} = 2\pi m$, with $m \in \mathbb{Z}$, must be fulfilled:

\[
d = \frac{\pi m}{\sqrt{\varepsilon_y k_0}}.
\]

In all the simulations performed in the work we have set $m = 1$, i.e., the first interference order. With the help of the derived hyperbola, we can perform full-wave simulations, as shown in Fig. 4 of the main text.

We note that this derivation assumes that the shape of the IFC is given by Eq. (1), which does not consider the finite thickness of the slab. A more realistic, accurate derivation, should consider such finite thickness, e.g. as described by the dispersion of polaritons in biaxial slabs given in ref. (35).

**S7. Nanoresonator for HPhPs under far-field illumination**

In this section we conduct a simulation of PhPs focusing inside a hyperbolic nanoresonator under far-field illumination. As shown in Fig. S11, we find that the phases of PhP waves launched by the edges of each reflector element are opposite, which is analogous to the phase difference of the polariton waves launched by resonant antennas (40, 41). As such, there is a destructive interference in the center of the structure. The shape of the total hyperbolic retroreflector, which is derived by assuming that all the waves arrive with the same phase does not account for constructive
interference in that case. We note that the focusing effect of antenna-launching PhPs arises from the interference between their high-momentum components, which has been studied by our group (28) and others (29, 30), however, it is out of the scope of the present work.

In any case, the combination of edge launching and negative reflection effects undoubtedly provides an interesting avenue for future research.

**Figure S11. Nanoresonator for HPhPs under far-field illumination.** Simulated near-field image Re\[E_z(x,y)\] (left) and |E_z(x,y)| of HPhPs launched by the edges of a hyperbolic nanoresonator consisting of two retroreflectors, on a 200-nm-thick \(\alpha\)-MoO_3 slab at an incident laser frequency of \(\omega_0 = 920 \text{ cm}^{-1}\).

**S8. Absolute value of the z-component of the electric field created by a point dipole on the center of a hyperbolic nanoresonator**

**Figure S12. Nanoresonator for HPhPs.** Simulated near-field image |E_z(x,y)| of HPhPs reflecting on a hyperbolic nanoresonator consisting of two retroreflectors on a 200-nm-thick \(\alpha\)-MoO_3 slab at an incident laser frequency of \(\omega_0 = 920 \text{ cm}^{-1}\) (the same structure as the one shown in Fig. 4C). The source is a point dipole located at a distance equal to the HPhP wavelength \(\lambda_p = 1.7 \mu\text{m}\) from the edge.
Figure S13. *Retrorreflector for HPhPs.* Simulated near-field amplitude $|E_z(x,y)|$ of HPhPs reflecting on a hyperbolic retroreflector on a 100-nm-thick $\alpha$-MoO$_3$ layer at $\omega_0 = 900$ cm$^{-1}$. 

$2\lambda_p = 2.6 \mu m$ for this wavelength.