Transport Properties near the z=2 Insulator-Superconductor Transition

Denis Dalidovich and Philip Phillips
Loomis Laboratory of Physics
University of Illinois at Urbana-Champaign
1100 W. Green St., Urbana, IL, 61801-3080

We consider here the fluctuation conductivity near the point of the insulator-superconductor transition in a system of Josephson junction arrays in the presence of particle-hole asymmetry and Ohmic dissipation. The transition is characterized by the dynamical critical exponent \( z = 2 \), opening the possibility of the perturbative renormalization-group (RG) treatment. The coupling to the Ohmic heat bath, giving the finite quasiparticle life-time, leads to the non-monotonic behavior of the dc conductivity as a function of temperature in the leading logarithmic approximation.

### A. Introduction

In granular superconducting thin films and fabricated 2D Josephson junction arrays (JJA), phase coherence is destroyed at zero temperature, whenever the intergran Josephson tunneling of Cooper pairs becomes smaller than the Coulomb interaction between grains.\([1][2]\). The occurrence of this insulator-superconductor quantum phase transition (IST) can be best seen from conductivity measurements. At the transition point, the resistivity is observed to be temperature-independent and close in value to the quantum of resistance for charge 2\(e\) particles \( R_Q = \sigma_Q^{-1} = h/4e^2 \). There are many theoretical works devoted to the explanation of this remarkable result\([3][4]\). Cha et al., for instance, calculated the zero-temperature collisionless conductivity at the transition point within the effective 3D Ginzburg-Landau (GL) action for an \( M \)-component bosonic field\([5]\), that can be derived from the commensurate 2D Bose-Hubbard model. The influence of the quartic term in this approach, considered up to the first order in \( 1/M \), was shown to reduce the mean-field value of the conductivity \( (\pi/8)\sigma_Q = \pi e^2/2h \) by 36%. Fazio and Zappala calculated this conductivity applying the dimensional regularization in \( \epsilon = 3 - d \)\([6]\). However, as was shown later by Damle and Sachdev\([7]\), the collisionless zero-temperature conductivity is not the conductivity that is measured experimentally at a finite temperature. This conclusion stems from the non-commutativity of the limits \( \omega \rightarrow 0, T \rightarrow 0 \) and \( \omega = 0, T \rightarrow 0 \) in the general expression for \( \sigma(\omega, T) \) at the transition point. The former limit describes collisionless transport, while it is the latter limit that must be taken to assess the experimentally-relevant collision-dominated conductivity. With collisions neglected, the finite temperature dc conductivity is singular on the insulating side, so the inclusion of quasiparticle damping is necessary for its regularization. Collisions are properly accounted for utilizing the quantum kinetic equation which represents, in general, a complicated non-linear integral equation for the distribution function of thermally excited quasiparticles. The kernel of this equation is determined by the quartic non-linearity in the GL action responsible for collisions between quasiparticles. The approximate methods, such as \( 1/M \) or \( \epsilon = 3 - d \) expansion, should be used to solve the kinetic equation close to the transition point\([11][12]\). Another way of regularization of the singular Drude conductivity is the inclusion of the phenomenological Ohmic dissipation. Experimentally, the latter plays a central role, because in combination with disorder it can lead to the temperature-independent resistivity as \( T \rightarrow 0 \)\([13]\). From the theoretical point of view Ohmic dissipation is always a relevant perturbation, affecting thus the functional form of universal quantities near the IST point\([14][14]\). The mentioned sources of dissipation lead to different temperature dependences of the conductivity, and, generally speaking, should be treated simultaneously\([14]\).

Thus far, most of the approaches have used as a starting point the particle-hole symmetric GL action that is space-time isotropic at \( T = 0 \) and belongs to the universality class with the mean-field dynamical exponent \( z = 1 \). However, the behavior in the critical region can be considerably affected by a term that breaks the symmetry between particles and holes. The physical picture leading to the modified action is realized in a system of JJA in the presence of uniformly-distributed frustrating offset charges \( q_x \neq 2e \) that cannot be eliminated by Cooper pair tunneling\([15][16]\). The microscopic quantum phase model describing this situation is also known to be equivalent to the clean incommensurate Bose-Hubbard model\([17]\). The IST in this case is characterized by the \( z = 2 \) dynamical critical exponent. Thus, the quartic term in the GL action is marginally irrelevant, and there exists the range of parameters \( T \) and \( \delta \) (the deviation from the zero-temperature critical point) that permits all interesting physical quantities to be calculated in the perturbative renormalization-group (RG) treatment.\([18][19]\). In this region the static quantities can be regarded in the leading approximation as some universal functions of \( \delta \) and \( T \), independent of the quartic interaction strength, \( u \). This region persists as long as \( \delta \) and \( T \) are logarithmically or even double logarithmically small.
In this paper we use this approach to calculate the dc conductivity close to the point of the IST, governed by $z = 2$, within leading logarithmic accuracy. It is shown, using the Kubo formula, that in the absence of any dissipative mechanism, the real part of finite temperature conductivity is singular and non-universal on the insulating side. Its regularization by inclusion of the coupling to the Ohmic heat bath leads to the logarithmic temperature dependence in the quantum critical (QC) regime. The conductivity increases in the leading approximation as $\ln \ln (1/T)$ with $T \to 0$ right above the transition, and decreases monotonically in the quantum disordered (QD) regime. Consequently, we claim then that crossing over from the QD to the QC regime with decreasing temperature is accompanied by a somewhat re-entrant behavior of the dc conductivity. The regularization that results from the finite lifetime of quasiparticles, is possible only once the Umklapp scattering is included. This is a consequence of the Galilean invariance of the action, which explicitly describes the system of interacting charges of one sign. These processes ensure that the inverse lifetime of quasiparticles, $1/\tau_U \sim e^{-A/T}$, leads, in the absence of other sources of dissipation, to an exponentially large fluctuation conductivity on the insulating side. We discuss the applicability of this approach to the description of the finite-T charge transport near the 2D metal-superconductor transitions.

B. Renormalization Group Analysis

The general form of the GL functional that models the behavior near the IST point in the presence of uniformly-distributed offset charges is derived to be \[ F[\psi] = \int d^2r \int d\tau \left\{ \left[ \left( \nabla + \frac{i e^*}{\hbar} A(r, \tau) \right) \psi^*(r, \tau) \right] \left[ \left( \nabla - \frac{i e^*}{\hbar} A(r, \tau) \right) \psi(r, \tau) \right] + \lambda \psi^*(r, \tau) \partial_\tau \psi(r, \tau) \right. \\
+ \kappa^2 |\partial_\tau \psi(r, \tau)|^2 + \delta |\psi(r, \tau)|^2 + \frac{u}{2} |\psi(r, \tau)|^4 \left\} + F_{\text{dis}} \right. \]

(1)

where $A(r, \tau)$ is the vector potential, due to applied electric field, $e^* = 2e$, and $\delta$ is proportional to the inverse correlation length. In Fourier space, the dissipation term, $F_{\text{dis}} = \eta \sum_\mathbf{k}, \omega_n |\omega_n| |\psi(\mathbf{k}, \omega_n)|^2$ corresponds to the Ohmic model of Caldeira and Leggett. Parameters $\kappa$ and $\lambda$ (linear in the value of offset charges) measure the strength of quantum fluctuations. We will regard $\kappa/\lambda = O(1)$. The term with $\kappa$ is clearly irrelevant in the region of interest, so we can neglect it and measure all quantities, having the dimensionality of energy, in units of $\lambda$. The key idea behind the calculation of the general, frequency-dependent conductivity in the critical region is that in 2D at finite temperature, the conductivity obeys \[ \sigma_{\alpha\beta}(\delta, T, \omega, u) = \sigma_{\alpha\beta}(T(l^*), \omega(l^*), u(l^*)). \]

(2)

In the momentum-shell RG, $l^*$ denotes the scale at which the effective size of the Kadanoff cell is on the order of the correlation length. We assume without loss of generality that the scaling stops when $\delta(l^*) = 1$. The frequency in the above equation scales trivially, $\omega(l) = \omega e^{2T}$, although we will be interested only in the dc conductivity, most commonly measured in experiments. The dissipation term with $\eta$, preserving the $z = 2$ universality class, should be also incorporated in the RG equations. However, because any non-zero $\eta$ profoundly changes the analytic structure of a quasiparticle propagator, it is convenient to consider the cases of zero and non-zero $\eta$ separately.

$\eta = 0$: The one-loop finite-$T$ RG equations for this problem in 2D are of the form \[ \frac{dT(l)}{dl} = z T(l) \]

(3)

\[ \frac{d\delta(l)}{dl} = 2\delta(l) + \frac{2K_2 u(l)}{\exp[(1 + \delta(l))/T(l)]} \]

(4)

\[ \frac{du(l)}{dl} = (2 - z)u(l) - C[\delta(l), T(l)]u(l)^2, \]

(5)

where the coefficient \[ C[\delta(l), T(l)] \approx \begin{cases} \frac{K_2}{2(1 + \delta(l))} & T(l) \ll 1 \\ \frac{5K_2 T(l)}{(1 + \delta(l))^2} & T(l) \gg 1 \end{cases} \]

(6)

Here $K_2 = (1/2\pi)$, and in the one-loop approximation we can assume $z = 2$. The solution of the above system is qualitatively different in the quantum disordered and quantum critical regimes.

Quantum disordered regime: We are in the QD regime, if the inequality $\delta \gg T$ between the bare parameters holds. With sufficient accuracy the term with $u(l)$ in the righthand side of Eq. (4) can be neglected, and we obtain in this regime, that $T^* = \frac{T}{\delta}$, giving \[ T^* = T, \quad u^* \approx \frac{8\pi}{\ln \frac{T}{u^*}}. \]

(7)

The asterisk denotes that the parameters under scaling must be taken at $l = l^*$. The result for $u^*$ is obtained with logarithmic accuracy, i.e we consider $l^*$ to be large. This condition ensures the validity of the perturbation theory.

Quantum critical regime: This region is characterized by the opposite condition, $\delta \ll T$. The integration of
the RG equations in this case consists of two steps. In the first step, we integrate over $l$ from 0 to $\tilde{l}$, such that $T(\tilde{l}) = 1$. We have then $\delta(\tilde{l}) = (\delta/T + O(1/\ln T^{-1})) \ll 1$. So, the scaling must continue into the second step in which $T(l) \gg 1$. In this, classical region, the strength of interactions is measured not by $u(l)$, but by $v(l) = u(l)T(l)$ \[15\], and for $l > \tilde{l}$ we must consider the equations

$$\frac{dv(l)}{dl} = 2v(l) - \frac{5K_2v(l)^2}{(1 + \delta(l))^2}$$  \hspace{1cm} (8)

$$\frac{d\delta(l)}{dl} = 2\delta(l) + \frac{2K_2v(l)}{1 + \delta(l)}$$  \hspace{1cm} (9)

Assuming, that throughout the scaling $v(l) \ll 1$, that is we are working in the weakly classical region, we obtain, using $v(\tilde{l}) = 4\pi/\tilde{l}$, that $v(\tilde{l}) \approx (4\pi/\tilde{l}) e^{2(l - \tilde{l})}$. Substitution of this result into Eq. \[8\], and subsequent integration gives us with the logarithmic accuracy the equation for $l^*$ in the QC regime:

$$\delta(l^*) = \frac{4}{l}(l^* - \tilde{l})e^{2(l^* - \tilde{l})} = 1.$$  \hspace{1cm} (10)

Solving this within double logarithmic accuracy, we find that

$$l^* = \frac{1}{2} \ln \left[ \frac{1}{T} \ln \ln \left( \frac{1}{T} \right) \right],$$  \hspace{1cm} (11)

$$T^* = \ln \frac{1}{T}, \quad v^* = \frac{2\pi}{\ln \ln \frac{1}{T}}.$$  \hspace{1cm} (12)

The validity of the above solution requires not only the condition $\ln \ln (1/T) \gg 1$, but also $\ln \ln (1/T) \gg 1$.

$\eta \neq 0$: The presence of the term $\eta|\omega_n|$ in the RG equation for $\delta$ leads to the divergency in the sum over frequencies. So the momentum-shell RG with the upper frequency cutoff $\Gamma$ should be used \[19\], and the obtained one-loop equations are of the form

$$\frac{d\delta(l)}{dl} = 2\delta(l) + f^{(2)}(\delta(l), T(l))u(l)$$  \hspace{1cm} (13)

$$\frac{du(l)}{dl} = -f^{(4)}(\delta(l), T(l))u(l)^2,$$  \hspace{1cm} (14)

where $f^{(2)}$ and $f^{(4)}$ are some rather robust functions of $\delta(l)$, $T(l)$ and $\eta$, that we don’t write out here. It is essential, that the results for $T^*$ and $l^*$ remain the same in both the QC and QD regimes, if we are interested only in the leading logarithmic approximation. However, the subdominant temperature dependent corrections appear to be different. $u^*$ and $v^*$ are also logarithmically small, though have the cutoff dependent numerical prefactors.

C. Transport Properties

We see, that upon scaling, for small enough $T$ and $\delta$, the parameters $u^*$ and $v^*$ remain small, and in the first approximation one can calculate the conductivity with the help of the Kubo formula,

$$\sigma_{\alpha\beta}(i\omega_n) = -\frac{\hbar}{\omega_n} \int d^2r \int d\tau \frac{\delta^2 \ln Z}{\delta A_\alpha(\tau, r) \delta A_\beta(0)} e^{i\omega_n \tau},$$

applied to the Gaussian part of Eq. \[1\]. The standard calculations for the longitudinal conductivity, which we denote simply as $\sigma$, lead to the result

$$\sigma(i\omega_n^*) = \frac{2(e^*)^2 T^*}{\hbar \omega_n^*} \sum_{\omega_n^*} \int d^2k \frac{2k_2^2 G(k, \omega_n^*)}{(\pi^2)^2} G(\bar{k}, \omega_n^*) - G(k, \omega_n^* + \omega_n^*),$$  \hspace{1cm} (15)

where $G(k, \omega_n^*) = (i\omega_n^* + \epsilon_n^*)^{-1}$ is the usual Matsubara Green function. The rescaled temperature $T^*$ and the energy of quasiparticles $\epsilon_n^* = 1 + k^2$ are employed in the righthand side in accordance with Eq. \[2\] ($\omega_n^* = 2\pi m T^*$). We must perform then the analytical continuation to real frequencies after doing the summation over frequencies $\omega_n^*$.

If the absence of any dissipative mechanism, we obtain:

$$\sigma(\omega^*) = \left[ \frac{i}{\omega^*} + \pi \delta(\omega^*) \right] \frac{(e^*)^2}{2\hbar T^*} \cos^2 \left( \frac{1 + k^2}{2T^*} \right).$$  \hspace{1cm} (16)

Though the integral over $k$ is calculable exactly, we state here only the results in the limiting cases of the QD and QC regimes:

$$\sigma(\omega) = \left( \frac{i}{\omega} + \pi \delta(\omega) \right) \frac{(e^*)^2}{\hbar T} \cos^2 \left( \frac{1 + k^2}{2T^*} \right) \frac{e^{-\delta/T}}{\ln \ln \frac{1}{T}} \quad \text{QC};$$  \hspace{1cm} (17)

The expression in the QC regime is written with triple logarithmic accuracy. We see that on the insulating side, the real part of the collisionless conductivity does not have any regular contribution, as $\omega \rightarrow 0$. The singular part disappears as well at $T = 0$. Moreover, the form of the frequency dependence in Eq. \[17\] is characteristic for the conductivity of a superconducting phase. This unphysical result suggests that the finite life-time of quasiparticles should be accounted for in the correct determination of transport properties.

The inclusion of the non-zero Ohmic dissipation provided by the last term in Eq. \[4\], changes the analytical properties of Matsubara sums, and the analytical continuation in Eq. \[13\] should be performed by contour integration

$$\sigma(\omega^*) = \frac{(e^*)^2}{\hbar \omega^*} \int_0^\infty k^3dk \int_{-\infty}^{\infty} \coth \frac{z}{2T^*} dz \left[ (G^R(z) - G^A(z)) (G^R(z) + G^A(z) - G^R(z + \omega^*) - G^A(z - \omega^*)) \right],$$  \hspace{1cm} (18)
where the retarded and advanced Green functions $G^{R/A}(z) = (z + i\epsilon_k^* \mp i\eta z)^{-1}$ have been introduced. In the limit $\omega \to 0$, expansion of the corresponding Green functions and subsequent integration by parts yields the dc conductivity:

$$\sigma = \frac{e^2}{2\pi h} \int_0^\infty k^3 dk \int_\infty^{-\infty} dx \frac{dx}{\sinh^2 x} \times \frac{8\eta T^* x^2}{(e_k^2 + 2xT^*)^2 + (2\eta T^* x)^2}. \quad (19)$$

In the QD regime $T^* = T/\delta \ll 1$, and, as long as $\eta \ll T/\delta$, the integral over $x$ is determined by the two competing contributions: from the region near the minimum of denominator at $x_0 = -\epsilon_k^* / 2T^*$ and from the vicinity of $x = 0$. Evaluating those contributions and integrating subsequently over momentum $k$, we receive the total conductivity:

$$\sigma = \frac{2e^2}{h} \left[ \frac{T}{\delta \eta} e^{-\delta/T} + \frac{2\pi}{9} \left( \frac{\eta T^*}{\delta} \right)^2 \right]. \quad (20)$$

As can be seen, the first term is dominant for very small values of dissipation, $\eta < (\delta/T)^{3/2} e^{-\delta/3T}$, while for larger $\eta$ only the second contribution should be retained. In any of those cases the conductivity monotonically decreases as $T \to 0$.

In the QC regime the double logarithmic accuracy implies that $T^* = \ln(1/T) \gg 1$. With the same accuracy, the main contribution to the integral over $x$ comes from $x < 1$, allowing us to set $x^2 / \sinh^2 x \approx 1$. We obtain then after simple integrations, assuming $\eta \leq 1$, that

$$\sigma = \frac{e^2}{h} \frac{1 + \eta^2}{\eta} \ln \frac{1}{T}. \quad (21)$$

This suggests, that the conductivity is not universal in the QC regime and increases upon lowering the temperature, albeit very slowly, as a double logarithm of $1/T$. One can conclude that, because of the crossover from the QC to the QD region upon lowering temperature for any non-zero $\delta$, it may be possible to observe a non-monotonic behavior of the resistivity as a function of temperature with a dip when $T \sim \delta$. Though the derived results are strictly applicable in rather tiny regimes, where $\delta$ and $T$ are logarithmically small, the results found here may be observable beyond those boundaries.

Now consider the role of mutual scattering of quasiparticles as the other possible source of dissipation. Eq. (1) suggests that our action is Galilean invariant, describing the system of charge carriers of one definite sign. In the Hamiltonian formalism, our system is identical to that of weakly-interacting Bose particles in 2D [24]. The linearization of the corresponding collision integral in the kinetic equation in a small correction to the equilibrium distribution, proportional to $(k|E)$,

$$\left( \frac{\partial}{\partial t} + e^*E(t) \frac{\partial}{\partial k} \right) f(k, t) = I[f(k, t)], \quad (22)$$

reveals, that $I[f(k, t)]$ turns to zero as a consequence of momentum conservation. This situation is different from the $z = 1$ case, in which the system consists of colliding particles and holes having the same energy [1]. This does not mean, however, that the mutual collisions of quasiparticles in the $z = 2$ case can not lead to the finite lifetime. The appropriate way to see this is to go beyond the continuum limit and recall that the real experimental systems, such as JJA, have a lattice periodicity. This suggests that in case of a square lattice, the quasiparticle energies $\epsilon_k = 4 - 2(\cos k_x + \cos k_y)$ are periodic functions of the quasimomentum $k$, and the Umklapp processes need to be taken into account. The corresponding inverse scattering time $1/\tau_U$ can be estimated from [24]

$$\frac{1}{\tau_U} \sim u^2 \int n(\epsilon_{k_1})(1 + n(\epsilon_{k_2})) \delta(\epsilon_{k_1} + \epsilon_{k_3} - \epsilon_{k_2} - \epsilon_{k_3}) \cdot (2\pi)^2 \delta(k + k_1 - k_2 - k_3 - b) \frac{d^2 k_1 d^2 k_2 d^2 k_3}{(2\pi)^2 (2\pi)^2 (2\pi)^2}, \quad (23)$$

where the integration runs over the first Brillouin zone, and $b$ is the reciprocal lattice vector. The non-zero $b$ implies that the conditions imposed by $\delta$-functions on quasimomenta and energies of colliding quasiparticles can both be satisfied only if one of the incoming and one of the outgoing quasimomenta come from the interior of the first Brillouin zone. Because the main contribution, as can be seen from the Kubo formula, comes from small $k$, $k_2$ must be large. Physically, this means that the damping of the critical fluctuations of the order parameter arises from the collisions with the non-critical modes. One can easily estimate that $1/\tau_U \sim e^{-A/T}$ where the prefactor varies slowly $\delta$ and $T$, and $A$ is some numerical constant determined by the concrete form of the first Brillouin zone and the quasiparticle dispersion law. This inverse scattering rate, valid in both QC and QD regimes, is very small near the quantum phase transition point. This implies that in the absence of any Ohmic dissipation $\eta$ the conductivity $\sigma \propto \tau_U \propto e^{-A/T}$, and hence exponentially increases with temperature. The latter conclusion can be verified by making the substitution $\eta x T \to (1/2\tau_U)$ in Eq. (19). Such behavior suggests that Ohmic dissipation plays the dominant role near the 2D IST point in the presence of charge frustration and reinstates the insulating behavior. For JJA, fabricated from $s$-wave superconductors, weak non-magnetic disorder is non-pair breaking. Hence, as long as the pairs remain intact, it can not lead to dissipative terms in our action [3]. Ohmic dissipation arises if the superconducting grains are embedded in a conducting environment [23] in which case Eqs. (20) and (21) give the conductivity due to fluctuations of the superconducting order parameter.

Conventional disorder leads, however, to dissipation in a $d$-wave superconductor because in this case it is a
pair-breaking perturbation \[27\]. The finite temperature fluctuation conductivity near this disorder-driven metal-superconductor quantum phase transition can be investigated using the same action \[1\], provided we are not too close to the transition point where the disorder affects the critical properties. Depending on parameters of the microscopic Hamiltonian, \( \eta \) can be both small and large, compared to unity \[27\]. If \( \eta \leq 1 \), the conductivity in the QC regime is given by Eq. \[21\], and we should use the last term of Eq. \[20\] for the QD regime. If \( \eta \gg 1 \), the relevant conductivity was discussed using \( N = \infty \) approach \[14\]. This work was funded by the DMR98-12422 of the NSF research fund.

\[1\] For a review see A. M. Goldman and N. Markovic (1998).
\[2\] H. M. Jaeger, D. B. Haviland, B. G. Orr, and A. M. Goldman, Phys. Rev. B \textbf{40}, 182 (1989).
\[3\] H.S.J. van der Zant, et. al. Phys. Rev. B \textbf{54}, 10081, (1996).
\[4\] S. Doniach, Phys. Rev. B \textbf{24}, 5063, (1981).
\[5\] A. van Otterlo, K.-H Wagenblast, R. Fazio, and G. Schönh, Phys. Rev. B \textbf{48}, 3316 (1993).
\[6\] M.P.A. Fisher and G. Grinstein, Phys. Rev. Lett. \textbf{60}, 208, (1988).
\[7\] X.G. Wen and A. Zee, Intl. J. Mod. Phys. B \textbf{4}, 437, (1990).
\[8\] Min-Chul Cha, et. al. Phys. Rev. B \textbf{44}, 6883, (1991).
\[9\] R. Fazio and D. Zappala, Phys. Rev. B \textbf{53}, R8883, (1996)
\[10\] I.F. Herbut, Phys. Rev. Lett. \textbf{81}, 3916, (1998).
\[11\] Kedar Damle and Subir Sachdev, Phys. Rev. B \textbf{56}, 8714, (1997)
\[12\] Subir Sachdev. Phys. Rev. B \textbf{57}, 7157, (1998).
\[13\] N. Mason and A. Kupitelnik, Phys. Rev. Lett. \textbf{82}, 5341, (1999).
\[14\] D. Dalidovich and P. Phillips, Phys. Rev. Lett. \textbf{84}, 737, (2000).
\[15\] D. Dalidovich and P. Phillips, cond-mat/0005119.
\[16\] G. Grignani, et. al., Phys. Rev. B \textbf{61}, 11676, (2000).
\[17\] Matthew P. A. Fisher, et.al., Phys. Rev. B \textbf{40}, 546, (1989).
\[18\] Daniel S. Fisher and P.C. Hohenberg, Phys. Rev. B \textbf{37}, 4936, (1988).
\[19\] A.J. Millis, Phys. Rev. B \textbf{48}, 7183, (1993).
\[20\] Subir Sachdev, T. Senthil and R. Shankar, Phys. Rev. B \textbf{50}, 258, (1994).
\[21\] A. O Caldeira and A. J. Leggett, Phys. Rev. Lett. \textbf{46}, 211 (1981).
\[22\] S.L. Sondhi, et. al., Rev. Mod. Phys. \textbf{69}, 315, (1997).
\[23\] Ziqiang Wang, et. al., Phys. Rev. B \textbf{61}, 8326, (2000).
\[24\] E. M. Lifshitz and L. P. Pitaevskii, Physical Kinetics, Pergamon Press, 1984.
\[25\] P. W. Anderson, J. Phys. Chem. Solids \textbf{11}, 26 (1959).
\[26\] K. Wagenblast, A. van Otterlo, G. Schönh and G. Zimanyi, Phys. Rev. Lett. \textbf{79}, 2730, (1997).