On the possibility of the generation of an artificial wormhole

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We model spacetime foam by a gas of virtual wormholes. Then applying an external field one may change the density of virtual wormholes and try to organize a wormhole-like structure in space. The relation between an additional distribution of virtual wormholes and the external field is considered for the homogeneous case. We show that the external fields suppress the density of virtual wormholes which sets an obstruction for creating an actual wormhole in a straightforward fashion. We also present a rough idea of a more complicated model for the artificial creation of a wormhole.

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I. INTRODUCTION

It is well known that physics of wormholes involves the two important features. The first one is that wormholes violate the null energy condition (NEC) [1] which requires the presence of an exotic (which does not exist in lab experiments) matter. The second important feature is the fact that topology changes are rigorously forbidden in classical general relativity, e.g., see the theorem of Geroch in Ref. [2]. Nevertheless, there exists a widely spread misunderstanding that it is enough to prepare an exotic NEC-violating matter and this may allow to create a wormhole artificially. This is surely not true, for it does not matter which sort of the stress energy tensor we may use as a source in classical general relativity, topology will remain the same. An exotic energy produces only an exotic dynamics but not any topology change. In particular, the possibility to create a new universe (or baby universe) by a man-made process discussed first in Refs. [3] and recently in connection to NEC-violating theories in Ref. [4] does not assume any topology change but rather an inflating dynamics for a small portion of space.

The first rigorous model which may allow to describe the creation of an artificial wormhole was suggested recently in Ref. [5]. This model is based on quantum gravity effects, namely, on the spacetime foam picture [6, 7]. It is assumed that at very small (Planckian) scales spacetime is filled with a gas of virtual wormholes. The virtual wormhole represents a quantum topology fluctuation (tunnelling between different topologies) which takes place at very small (Planckian) scales and lasts for a very short (Planckian) period of time. It does not obey to the Einstein equations and, therefore, violates readily NEC. Virtual wormholes are described by Euclidean wormhole configurations which were first suggested in Refs. [8]. Our model is based on the fact that a coherent set of virtual wormholes may work as an actual wormhole [9, 10] (in somewhat different context analogous idea was discussed in Ref. [11]). Thus, by applying an external classical field one may govern the intensity of such topology fluctuations (i.e., the density of virtual wormholes) and try to organize an artificial wormhole. In this case a topology change does not occur at all and this is not in the contradiction with the Geroch theorem.

We in Ref. [5] missed however the sign of the Green function and assumed the delta-correlation for the bias function (i.e., for the additional sources generated on throats of wormholes) which, in general, is not correct. In the present paper we consider a more general but still a homogeneous case. We show how an external field relates to topology fluctuations (i.e., to the perturbations in the number density of virtual wormholes). The naive expectations, e.g., that a homogeneous external electric field may redirect virtual wormholes along the direction of the electric lines, seem to be not working. Indeed, if it were the case, such a phenomenon would be observed long ago in laboratory experiments. Instead, it turns out that external fields somewhat suppress the number density of wormholes. Such a behaviour becomes clear if we recall that virtual wormholes diminish the energy density of zero-point fluctuations (i.e., the vacuum energy density) [12]. The external classical field carries always the positive energy density and, therefore, in the presence of any external classical field the vacuum energy density becomes somewhat higher. Which means that the number density of virtual wormholes should be somewhat smaller.

At first glance the fact that an external classical field suppresses the density of virtual wormholes means that the straightforward creation of an actual wormhole meets some obstruction (which seems to agree with Ref. [4] where an obstruction for creating a universe in the laboratory was reported). However it is clearly not the case. One may

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consider a more complicated models, e.g., in the case of a metastable vacuum state (which probably took place in the very early Universe) or (more realistically) one may use a dual (in some sense) field which suppresses a some number density of wormholes save a coherent set [19]. We see that such a dual fields involve more energy density and have the more complicated configurations. This make explain why such a phenomenon (creating of a wormhole-like structure) was not observed so far. We should also reserve the possibility that inhomogeneous external fields (which are still out of our consideration) may produce a consistent model for creating an artificial wormhole which requires the further study.

The negative perturbation in the number density of wormholes in the presence of external fields may however lead to a positive shift in the value of the cosmological constant [12] (we assume that in the absence of external fields the cosmological constant should be exactly zero). Whether it is possible to use such a phenomenon to explain the observed acceleration of the Universe (or to implement the ideas of Ref. [4]) or not requires the further study.

II. GREEN FUNCTION IN A GAS OF VIRTUAL WORMHOLES

Consider now the simplest massless scalar field and construct the Green function in the presence of a gas of wormholes. The Green function obeys the Laplace equation

\[-\Delta G(x, x') = \delta(x - x')\]

with proper boundary conditions at throats (we require \(G\) and \(\partial G/\partial n\) to be continual at throats). The Green function for the Euclidean space is merely

\[G_0(x, x') = \frac{1}{4\pi^2(x - x')^2}\]

(and \(G_0(k) = 1/k^2\) for the Fourier transform). In the presence of a single wormhole which connects two Euclidean spaces this equation admits the exact solution e.g., see details in Ref. [10, 14]. In this case the wormhole is described by the metric \((\alpha = 1, 2, 3, 4)\)

\[dx^2 = h^2(r) \delta_{\alpha\beta} dx^\alpha dx^\beta,\]

where

\[h(r) = 1 + \theta(a - r) \left(\frac{a^2}{r^2} - 1\right)\]

and \(\theta(x)\) is the step function. Such a wormhole has vanishing throat length. Indeed, in the region \(r > a\), \(h = 1\) and the metric is flat, while the region \(r < a\), with the obvious transformation \(y^\alpha = \frac{a^2}{r^2} x^\alpha\), is also flat for \(y > a\). Therefore, the regions \(r > a\) and \(r < a\) represent two Euclidean spaces glued at the surface of a sphere \(S^3\) with the center at the origin \(r = 0\) and radius \(r = a\).

For the outer region of the throat \(S^3\) the source \(\delta(x - x')\) generates a set of multipoles placed in the center of sphere which gives the corrections to the Green function \(G_0\) in the form (we suppose the center of the sphere at the origin)

\[\delta G = -\frac{1}{4\pi^2 x^2} \sum_{n=1}^{\infty} \frac{1}{n + 1} \left(\frac{a}{x}\right)^{2n} \left(\frac{x'}{x}\right)^{n-1} Q_n,\]

where \(Q_n = \frac{4\pi^2}{2n} \sum_{l=0}^{n-1} \sum_{m=-l}^{l} Q_{nlm}^* Q_{nlm}\) and \(Q_{nlm}(\Omega)\) are four-dimensional spherical harmonics e.g., see [15]. And analogous expression exists for the inner region of the sphere \(S^3\) [10, 14]. In the present paper we shall consider a dilute gas approximation and, therefore, it is sufficient to retain the lowest (monopole) term only. We point out that the monopole terms coincide for the inner and outer regions of the throat \(S^3\). A single wormhole which connects two regions in the same space is a couple of conjugated spheres \(S^3_a\) of the radius \(a\) with a distance \(\bar{X} = \bar{R}_+ - \bar{R}_-\) between centers of spheres. So the parameters of the wormhole are \(\xi = (a, \bar{R}_+, \bar{R}_-).\) The interior of the spheres is removed and surfaces are glued together. Then the proper boundary conditions (the actual topology) can be accounted for by adding the bias of the source

\[\delta(x - x') \rightarrow N(x, x') = \delta(x - x') + b(x, x').\]
In the approximation $a/X \ll 1$ (e.g., see also [16]) the bias takes the form

$$b_1 (x, x', \xi) = \frac{a^2}{2} \left( \frac{1}{(R_- - x')^2} - \frac{1}{(R_+ - x')^2} \right) \times$$

$$\times \left[ \delta^4 (x - R_+) - \delta^4 (x - R_-) \right]$$

(5)

We expect that virtual wormholes have throats $a \sim \ell_{pl}$ of the Planckian size, while in the present paper we are interested in much larger scales. Therefore, the form (5) is sufficient for our aims. However this form is not acceptable in considering the short-wave behavior and vacuum polarization effects (e.g., the stress energy tensor). In the last case one should account for the finite value of the throat size and replace in (5) the point-like source with the surface density (induced on the throat) e.g., see for details [12], $\delta^4 (x - R_{\pm}) \rightarrow \frac{1}{2\pi^2 a^3} \delta (|x - R_{\pm}| - a)$.

In the rarefied gas approximation the bias function for the gas of wormholes is additive, i.e.,

$$b (x, x') = \sum b_1 (x, x', \xi_i) = \int b_1 (x, x', \xi) F(\xi) d\xi,$$

(6)

where $F(\xi)$ is the number density of wormholes in the configuration space which is given by

$$F(\xi) = \sum_{i=1}^{N} \delta (\xi - \xi_i).$$

(7)

In the vacuum case this function has a homogeneous distribution $\rho(\xi) = \langle 0 | F(\xi) | 0 \rangle = \rho (a, X)$, then for the mean bias $\bar{b} = \langle 0 | b | 0 \rangle$ we find

$$\bar{b} (x' - x') = \int a^2 \left( \frac{1}{R_-^2} - \frac{1}{R_+^2} \right) \delta^4 (x - x' - R_+) \rho (a, X) d\xi$$

(8)

Consider the Fourier transform $\rho (a, X) = \int \rho (a, k) e^{-ikx} \frac{d^4k}{(2\pi)^4}$, then we find for $b (k) = \int b (x) e^{ikx} d^4x$ the expression

$$b (k) = \frac{4\pi^2}{k^2} \int a^2 (\rho (a, k) - \rho (a, 0)) da,$$

(9)

which forms the background cutoff function $\overline{N} (k) = 1 + \overline{b} (k)$, so that the regularized vacuum Green function $G_{\text{reg}} (k)$ has the form

$$G_{\text{reg}} (k) = \frac{1}{k^2} \overline{N} (k).$$

(10)

General properties of the cutoff is that $\overline{N} (k) \rightarrow 0$ as $k \gg k_{pl}$ and $\overline{N} (k) \rightarrow \text{const} \leq 1$ in the low energy limit as $k \ll k_{pl}$. The next our aim is to find the change of the bias in the presence of an external field. To this end we consider the structure of the partition function.

### III. STRUCTURE OF THE GENERATING FUNCTIONAL

To relate the additional distribution of virtual wormholes and an external current we consider now the generating functional (the partition function) which is used to generate all possible correlation functions in quantum field theory (and the perturbation scheme when we include interactions)

$$Z_{\text{total}} (J) = \sum_\tau \sum_\varphi e^{-S_E}$$

(11)

where the sum is taken over field configurations $\varphi$ and topologies $\tau$ (wormholes), the Euclidean action is

$$S_E = -\frac{1}{2} (\varphi \Delta \varphi) - (J \varphi),$$

(12)
and we use the notions \((J, \varphi) = \int J(x) \varphi(x) \, d^3x\). Here \(J\) denotes an external current. The sum over field configurations \(\varphi\) can be replaced by the integral \(Z^* (J) = \int [D\varphi] e^{-S_E} \) which gives

\[
Z^* = Z_0(G) e^{J G J},
\]

where \(Z_0(G) = \int [D\varphi] e^{\frac{i}{\hbar} (\varphi \Delta \varphi)}\) is the standard expression and \(G = G(\xi_1, \ldots, \xi_N)\) is the Green function for a fixed topology, i.e., for a fixed set of wormholes \(\xi_1, \ldots, \xi_N\).

Consider now the sum over topologies \(\tau\). To this end we restrict with the sum over the number of wormholes and integrals over parameters of wormholes:

\[
\sum_{\tau} \sum_{N} \int \prod_{i=1}^{N} d\xi_i = \int [DF]
\]

where \(F\) is given by \((7)\). We point out that in general the integration over parameters is not free (e.g., it obeys the obvious restriction \(|R_i^+ - R_i^-| \geq 2a_i\)). This defines the generating function as

\[
Z_{\text{total}} (J) = \int [DF] Z_0(G) e^{J G J},
\]

In this expression \(J\) may serve as an external current which generates an external classical background field \(\varphi_{\text{ext}} = GJ\).

In the vacuum case virtual wormholes have a homogeneous distribution. Therefore, if we remain within homogeneous distributions \(F\) (which sets also additional restrictions on possible external fields), then the structure of the partition function \(Z_{\text{total}} (J)\) can be investigated a little bit further. Indeed, for homogeneous states in the Fourier representation the bias \(N(x, x', \xi) \rightarrow N(k, k', \xi)\) takes the form

\[
N(k, k') = N(k, \xi) \delta(k - k'),
\]

then the true Green function takes the form (compare with \((10)\)) \(G(k) = G_0(k) N(k, \xi)\) and for the total partition function we find

\[
Z_{\text{total}} (J_{\text{ext}}) = \int [DN(k)] e^{-I(N(k))} e^{\frac{i}{\hbar} \sum_{k} N(k) |J_k|^2},
\]

where \(\sum_{k} = \int \frac{d^4k}{(2\pi)^4}\) and \([DN] = \prod_{k} dN_k\). The functional \(I(N)\) comes from the integration measure (which includes the Jacobian of transformation from \(F(\xi)\) to \(N(k)\))

\[
e^{-I(N)} = \int [DF] Z_0(N(k, \xi)) \delta(N(k) - N(k, \xi))
\]

and has the sense of the action for the bias function \(N(k)\). In the true vacuum case \(J = 0\) and by means of using the expression \((16)\) we find the two-point Green function in the form

\[
G(k) = \frac{\overline{N}(k)}{k^2}
\]

where \(\overline{N}(k)\) is the cutoff function (the mean bias) which is given by

\[
\overline{N}(k) = \langle 0 | N(k) | 0 \rangle = \frac{1}{Z_{\text{total}} (0)} \int [DN] e^{-I(N)} N(k).
\]

The action \(I(N)\) can be expanded as \((20)\)

\[
I(N) = I(\overline{N}) + \frac{1}{2} \sum_{kp} M_{k,p} \Delta N(k) \Delta N^*(p) + ...\]

where \(\Delta N(k) = N(k) - \overline{N}(k)\) and the kernel \(M_{k,p} = 1/\sigma_{k,p}^2\) can be expressed via the dispersion of vacuum topology fluctuations as

\[
\sigma_{k,p}^2 = \overline{\Delta N(k) \Delta N^*(p)} = \frac{1}{Z_{\text{total}} (0)} \int [DN] e^{-I(N)} \Delta N(k) \Delta N^*(p).
\]
IV. TOPOLOGY FLUCTUATIONS IN THE PRESENCE OF EXTERNAL FIELDS

Consider now topology fluctuations in the presence of an external current. In the presence of an external current $J^{ext}$ the intensity of topology fluctuations changes. Indeed using (10), (18) we find

$$I(N, J^{ext}) = I(N) + \frac{1}{2} \sum_{k,p} M_{k,p} \Delta N(k) \Delta N(p) - \frac{1}{2} \sum_k \frac{1}{k^2} |J_k^{ext}|^2 N(k)$$

where $\delta N(k) = N(k) - \overline{N}(k)$,

$$I(N, J^{ext}) = I(N) - \frac{1}{8} \sum_{k,p} \sigma_{k,p}^2 \left( \frac{1}{k^2} |J_k^{ext}|^2 \right) \left( \frac{1}{p^2} |J_p^{ext}|^2 \right),$$

and $\overline{N}(k, J^{ext}) = \overline{N}(k) + \delta b_k (J)$ with

$$\delta b_k (J) = \frac{1}{2} \sum_p \sigma_{k,p}^2 \frac{1}{p^2} |J_p^{ext}|^2.$$

We recall that this expression works only in the case when the external current does not destroy the homogeneity of vacuum state but still it is exact and contains yet unknown kernel $\sigma_{k,p}^2$. It is possible to get an analogous expression by perturbation method without the restriction to homogeneity of the vacuum state (i.e., in the presence of an arbitrary external field) but it has (as well as the exact form of the kernel $\sigma_{k,p}^2$) too complicated structure and we present it elsewhere [17]. Here we only briefly describe the way to get its exact expression. First, one has to notice that the bias has the structure $N(x, x') = \delta(x - x') + b(x, x')$ where $b$ depends on wormhole parameters $\xi$ and is given by (10). Therefore, the dispersion $\sigma$ reduces to $\sigma_{1,2}^2 = \int \Delta b_1 \Delta b_2 |0 \rangle >$, while the former may be expressed via the two-point correlation function for the wormhole number density in the configuration space (7) as $\sigma_{1,2}^2 = \int b_1(\xi)b_2(\xi')\rho(\xi, \xi')d\xi d\xi'$, where $\rho(\xi, \xi') = \int \Delta F(\xi) \Delta F(\xi') |0 \rangle >$. In a gas of wormholes $\rho$ has the structure $\rho(\xi, \xi') = \rho(\xi) \delta(\xi - \xi') + \nu(\xi, \xi')$, where $\rho(\xi) = \rho(a, X) = \int \Delta F(\xi) |0 \rangle >$ is the mean vacuum density which we used in (8) and $\nu$ describes correlations. In the rarefied gas approximation $\nu$ may be neglected to the leading order, while in general the dependence on $\xi$ can be found directly from (13) as $\rho(\xi_1, \xi_2) \sim Z_0(\xi_1, \xi_2)$.

V. CONCLUSION

The expression (22) shows that $\delta b_k (J) > 0$ contrary to the naive expectations. In particular, for the background bias (6), (13) the value $\overline{N}(k)$ is always less than unity and $\overline{b}(k) < 0$. This means that external fields always suppress the density of virtual wormholes. Indeed let us assume the homogeneous perturbation in the presence of $J \neq 0$

$$\delta \rho(a, X) = \delta n \delta (a - a_0) \frac{1}{2} (\delta^4 (X - r_0) + \delta^4 (X + r_0)),$$

where $\delta n = \delta N/V$ is the change in the density of wormholes. We point out that in the vacuum case the background density of wormholes is always positive $n \geq 0$, while the value $\delta n$ admits both signs. The above distribution corresponds to a set of wormholes with the throat size $a_0$, oriented along the same direction $r_0$ and with the distance between throats $r_0 = |R_+ - R_-|$. Then $\delta \rho(a, k)$ reduces to $\delta \rho(a, k) = \delta n \delta (a - a_0) \cos (kr_0)$, where $(kr_0) = k_\mu r_\mu^0$. Thus from (6) we find $\delta b_k (J) = -\delta n a_0^2 \frac{4\pi}{3} (1 - \cos (kr_0))$ and substituting this into (22) we may define the external field $\varphi = GJ$ which generates such a perturbation $\delta \rho$ in the vacuum distribution of wormholes. The property $\delta b_k (J) > 0$ means that the perturbation in the wormhole number density is always negative $\delta n(J) < 0$.

According to discussions in Ref. [11] there may exist a situation when the speed of light remains isotropic (in an appropriate external field) but slightly exceeds the vacuum value. Such a phenomenon can be in principle observed in an experiment analogous to that by the OPERA Collaboration [13].

The negative value $\delta n(J) < 0$ means that the possibility to create a homogeneous wormhole-like structure in space in a straightforward fashion meets some obstructions. The way out of this difficulty is to consider a metastable
vacuum state which may appear in the presence of an inhomogeneous external field (which require the further study). Still there is a more simple situation when the external field suppresses the dual distribution of wormholes, e.g., the distribution in the form

$$\delta \rho_{\text{dual}}(a, X) = \delta n - \delta \rho(a, X),$$  \hspace{1cm} (24)

where $\delta \rho(a, X)$ is given by [23]. Then the negative value $\delta n < 0$ will mean the presence of a some excess of virtual wormholes with the distribution [23], i.e., the formation of the wormhole-like structure discussed in Ref. [10]. We recall that to suppress all virtual wormholes simply impossible since it would require an infinite energy.

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[20] In this expansion the mean cutoff $\overline{N}(k)$ is merely the solution of $\frac{d \overline{N}(k)}{dk} = 0$. 