Elliptic flow of resonances at RHIC: probing final state interactions and the structure of resonances

C. Nonaka and B. Müller
Department of Physics, Duke University, Durham, NC 27708, USA

M. Asakawa
Department of Physics, Kyoto University, Kyoto 606-8502, Japan

S. A. Bass
Department of Physics, Duke University, Durham, NC 27708, USA and
RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA

R. J. Fries
School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA
(Dated: March 30, 2022)

We propose the measurement of the elliptic flow of hadron resonances at the Relativistic Heavy Ion Collider as a tool to probe the amount of hadronic final state interactions for resonances at intermediate and large transverse momenta. This can be achieved by looking at systematic deviations of the measured flow coefficient \( v_2 \) from the scaling law given by the quark recombination formalism. Our method can be generalized to explore the structure of exotic particles, such as the recently found pentaquark \( \Theta^+ \)(1540).

Recently, progress has been made in our understanding of the hadronization of a quark gluon plasma created in high energy heavy ion collisions. Both the yields and the anisotropic flow \( v_2 \) of hadrons at transverse momenta above 2 GeV, observed at the Relativistic Heavy Ion Collider (RHIC), are described well by a combination of two mechanisms: (i) the statistical recombination of thermal constituent quarks from a quark gluon plasma phase during the phase transition, (ii) fragmentation of jets which experienced energy loss due to the surrounding medium.

The elliptic flow \( v_2 \) is defined as the second Fourier coefficient of the azimuthal hadron spectrum

\[
\frac{d^2N}{P_T d\Phi d\eta} = \frac{dN}{2\pi P_T dP_T} \left[ 1 + 2v_2(P_T) \cos 2\Phi + \ldots \right]. \tag{1}
\]

It measures the elliptic anisotropy in the spectrum since

\[
v_2(P_T) = \langle \cos 2\Phi \rangle = \frac{\int d\Phi \cos 2\Phi \frac{d^2N}{P_T dP_T d\Phi}}{\int d\Phi \frac{d^2N}{P_T dP_T d\Phi}}. \tag{2}
\]

Such an asymmetry naturally arises in heavy ion collisions with non-vanishing impact parameter \( b > 0 \).

It has been shown that in the region where recombination of partons dominates the hadronization process — i.e., for \( P_T < 4 \) (6) GeV for mesons (baryons) — and mass effects are suppressed — i.e., \( P_T > 2 \) GeV — \( v_2 \) obeys the simple scaling law,

\[
v_2(P_T) = n v_2^q(P_T/n), \tag{3}
\]

where \( n \) is the number of valence quarks of the hadron. \( v_2 \) is the measured flow coefficient. We will assume throughout the paper that strange quarks and light quarks have the same elliptic flow \( v_2^q \). Measurements of strange hadrons support this assumption. \( v_2 \) at higher transverse momentum, \( P_T > 6 \) GeV, where fragmentation of partons dominates, we expect the \( v_2 \) of all hadrons to lie on a universal curve since the effect of fragmentation functions largely cancels in Eq. (2). We refer the reader to Refs. \( \text{[2]} \) \( \text{[3]} \) for a more detailed discussion.

The scaling law \( \text{[3]} \) has been impressively confirmed by measurements at RHIC. Pions and kaons \( (n = 2) \), protons, \( \Lambda \) and \( \Xi \) \( (n = 3) \) all fall on one universal curve, if \( v_2 \) and \( P_T \) are divided by the number \( n \) of valence quarks \( \text{[3]} \) \( \text{[10]} \). Slight deviations can be observed and are largest for pions. This can be due to its nature as a Goldstone boson — its mass is much smaller than the sum of its constituent quark masses — or because a large fraction of all pions are not created at hadronization but by the subsequent decay of hadron resonances \( \text{[11]} \). This poses the interesting question how the existence of a hadronic phase generally affects hadron production at intermediate and large momenta. We suggest that the amount of hadronic rescattering can be determined by measuring \( v_2 \) for several hadronic resonances.

Hadrons can experience rescattering after hadronization, until kinetic freeze-out takes place. Experimentally, it has been found that the kinetic freeze-out temperature at RHIC \( \text{[12]} \) is much lower than the expected QCD phase transition temperature \( \text{[13]} \). Resonances produced from a hadronizing quark gluon plasma decay in the hadronic medium. If their decay products rescatter, the signal is lost in the experimental measurement, since the original resonance cannot be reconstructed from a correlation analysis of its daughter hadrons. Likewise, hadrons from the medium may scatter into a resonance state and thus...
contribute to the final measured yield. The total measured resonance yield will therefore have two contributions:

1. **primordial resonances**: resonances produced from a hadronizing quark gluon plasma whose decay products have not rescattered (QGP mechanism).

2. **secondary resonances**: resonances produced in the hadronic final state via hadron-hadron rescattering (HG mechanism).

In this paper, we will show that $v_2$ differs between the two cases. Key to our analysis is the observation that the elliptic flow $v_2$ will be *additive* for any type of composite particle with respect to the $v_2$ of its constituents. Note that this feature is mimicked in a hydrodynamic model when mass effects are neglected. Then the elliptic flow of all particles follows the same line through the origin, $v_2(P_T) \propto P_T$. The scaling law simply maps this line into itself. Only the deviation from the ideal hydro behavior with increasing $P_T$, manifest in the saturation of $v_2$ at intermediate $P_T$, makes the scaling law non-trivial. The $v_2$ of a $K_0^*$ meson in the recombination domain will therefore be the sum of the $v_2$ contributions of the $d$ and $\bar{s}$ quarks if it has been formed from a hadronizing QGP, or the sum of the contributions of the $K^+$ and $\pi^-$ if it has been formed through the coalescence of a kaon and a pion in the hadron phase. Our discussion of deriving the formalism for the coalescence of partons can be extended to the coalescence of hadrons in a straightforward way \[1, 2, 3\]. Therefore we can here assume the same additive feature for hadron-hadron coalescence as well. Naively, if the kaon and pion themselves are formed by quark recombination, we expect a scaling of the $K_0^*$ with $n = 4$ in the HG case, while it scales with $n = 2$, like the stable mesons, if it is created at the phase transition. The scaling will be altered by the possibility that one or both of the coalescing hadrons could come from fragmentation, which breaks the scaling law. The final result will be a mixture of both scenarios.

We assume that the $v_2$ of stable hadrons (stable with regard to strong interactions) is not affected by hadronic interactions for $P_T$ above 2 GeV. This is a sound assumption in light of the experimental facts, as discussed above. Let $r(P_T)$ be the fraction of resonances that have escaped from hadronization without rescattering of the decay products vs. the total measured yield. Given the knowledge of $v_2$ in the two limiting cases, the QGP mechanism and the HG mechanism, the fraction $r(P_T)$ can be determined by measuring $v_2$ at intermediate $P_T$, between 2 and 6 GeV via

$$ v_2^{\text{measured}} = r(P_T)v_2^{\text{QGP}} + (1 - r(P_T))v_2^{\text{HG}}. \quad (4) $$

One may be able to deduce the widths and cross sections of resonances in the hadronic medium by measuring $r(P_T)$ for several resonances. Our scheme can be generalized to explore the structure of exotic particles, e.g. a large molecular state, as discussed in the case of the recently discovered pentaquark $\Theta^+(1540)$. We will discuss this in more detail below.

![FIG. 1: (Color online) Elliptic flow $v_2$ as a function of $P_T$ for the $\Sigma^*(1385)$ (top) and the $K_0^*(892)$ (bottom). Dashed line: resonance contribution from hadronization (QGP); dotted line: resonance contribution from coalescence in a hadron gas (HG); solid line: calculation for a 30% contribution of hadronic resonance formation.](image-url)
be extracted from data.

For quark recombination we used the scaling law \( v_2 \) together with the parton \( v_2^q \) extracted and discussed in \( v_2 \). Since the fragmentation functions for the resonances are unknown, we utilize the universal curve for the \( v_2 \) contribution from fragmentation as obtained in \( v_2 \).

Due to the transition from the recombination to the fragmentation regime, the maxima of the HG and QGP curves do not scale like 5/3 and 4/2 for the \( \Sigma^* \) and \( K^0 \) respectively. Instead, the HG curve is brought down by contributions from pions, kaons or \( \Lambda \)s from fragmentation, violating the scaling law. Still, the difference between the two limiting cases can be as large as 30% between 2 and 6 GeV.

The fraction of the total resonance yield being produced by re-generation in the hadronic phase cannot be inferred through the final measured resonance yield since the amount of signal loss in resonance reconstruction due to rescattering of the resonance decay products cannot be experimentally determined. The scheme outlined above to determine \( r(P_T) \) therefore provides complimentary information to the resonance yield measurements via invariant mass reconstruction. The determination of \( r(P_T) \) allows for an estimate of the gain term of resonance production in the hadronic phase - together with the yield information this allows for a far better estimate of the actual QGP production of resonances and for an improved estimate of the signal loss due to rescattering of the decay products. These estimates are of high relevance for the interpretation of statistical model fits with respect to resonances yields.

We can now utilize the scaling law for the elliptic flow - and its transition to a universal value for \( v_2 \) in the fragmentation domain - to probe the structure of exotic particles, such as the \( \Theta^+ (1540) \) pentaquark state. The discovery of this novel hadronic state with five valence quarks (\( uudds \)) has recently been reported in \( v_2 \). Its existence was predicted in 1997 by Diakonov et al. in the chiral quark soliton model (CQSM) \( v_2 \). Several other groups have by now confirmed this important discovery \( v_2 \). \( v_2 \) is the first unambiguous five quark state, since other candidates for five quark states, such as the \( \Lambda (1405) \), have quark compositions that can mix with three quark states like \( uudds \leftrightarrow uds \). Recently, NA49 reported the observation of two more states that could be the two missing pentaquarks in the antidecuplet \( v_2 \). The yield of the \( \Theta^+ \) in heavy ion collisions has been estimated in \( v_2 \). \( v_2 \).

While the discovery of the \( \Theta^+ \) has undoubtedly established the existence of pentaquarks, its structure, and even its spin and parity, have not yet been experimentally verified. It is still an open question whether pentaquarks like the \( \Theta^+ \) have a compact structure with all five valence quarks being closely confined, just as the three quarks in the nucleon, or whether they are molecular bound states of a baryon and a meson. A bound state of two diquarks and an antiquark was recently suggested as well \( v_2 \).

Loosely bound states with large geometric cross section whose binding energies are much less than the temperature, like the deuteron, are dissociated in the hadron phase even if they were produced at hadronization. Such a medium induced breakup reaction does not allow for the reconstruction of the state via two-particle mass spectra, since the initial momentum of the scattering partner is unknown. These bound states are unobservable. Thus, the loosely bound states and molecular states observed in heavy ion collisions are produced close to the kinetic freeze-out hypersurface \( v_2 \), in the case of the pentaquark by \( K^+ + n \to \Theta^+ \) and \( K^0 + p \to \Theta^+ \) (summarily denoted as \( KN \)). The hadronic phase can be viewed as a filter that tends to let the \( \Theta^+ \) from hadronization only survive if it is a compact five quark state.

With respect to the azimuthal anisotropy of pentaquarks produced in heavy-ion collisions at RHIC, we expect it to fulfill the scaling law \( v_2 \) with \( n = 5 \). This would be an impressive manifestation of its exotic five quark character. On the other hand, we expect modifications to hold, as we discussed them for conventional resonances. However, we want to point out, that in the case of conventional baryon resonances the scaling law holds for \( n = 3 \) in the QGP scenario, while it is roughly \( n = 5 \) in the HG case. For a pentaquark it would be \( n = 5 \) in both cases. The difference would merely come from the possibility that for the reaction \( N + K \to \Theta^+ \) one or both of the initial hadrons could come from fragmentation. The same is true for a deuteron, that scales approximately with \( n = 6 \) in both cases.

If the \( \Theta^+ \) is a molecular state, the momentum fraction \( x_K \) of the kaon in a lightcone frame is expected to be approximately given by \( r_K = M_K/(M_K + M_N) \sim 0.35 \), where \( M_K \) and \( M_N \) are the masses of the kaon and nucleon, respectively. We therefore use the wave functions \( v_2 \).

\[
|\phi_\theta(x_K, x_N)|^2 = \delta(x_K - 0.35),
\]

\[
|\phi_\Delta(x_p, x_n)|^2 = \delta(x_p - 0.5).
\]

In Fig. \( v_2 \) we show \( v_2 \) for the different scenarios discussed above. Note that the scenario with all pentaquarks from hadronization (QGP) is now providing the upper limit, while it was the lower limit in the case of conventional resonances. We include the deuteron in the study to give an example of a well known molecular state. Above \( P_T = 3 \) GeV we find a considerable difference, up to 20%, between the scenario where the pentaquark is created at the phase transition — and is therefore a compact five quark state — and the scenario where it originates from \( KN \) coalescence. If a \( KN \) coalescence takes place to form a \( \Theta^+ \) between 4 and 8 GeV, it is likely that either the kaon or neutron (or both) stems from fragmentation which lowers the resulting \( v_2 \). At very high \( P_T \), both the kaon and the nucleon have to come from fragmentation. This results in a power law spectrum that falls off like \( P_T^{-2b} \), approximately twice as rapidly as a directly fragmenting particle with \( \sim P_T^{-b} \). Therefore, direct fragmentation will dominate at very large \( P_T \) and will eventually lead \( v_2 \) down to the universal curve.
For the deuteron as a shallow molecular bound state we expect pn recombination at the kinetic freeze-out to dominate. The deuteron is therefore ideal as a benchmark measurement. First results for the $v_2$ of the deuteron have been announced by the PHENIX collaborations.[23]. As expected, this particle seems to follow the scaling law with $n = 6$ within the large error bars. With the upcoming high statistics Au+Au run, error bars and $P_T$ coverage should be greatly improved, allowing for a gauging of the formalism.

Let us comment on the third possible scenario for the $\Theta^+$, that was advocated by Jaffe and Wilczek[22]. If it is a collection of two tightly bound ($ud$) diquarks and an $\bar{s}$ quark, recombination would predict the $P_T$ dependence of $v_2$ to be very similar to that of a “democratic” 5 quark state. This, however, is only true if the diquarks are also formed at hadronization and the individual quarks have not been correlated early on with enough time to have thermalized diquark states. In that case one would expect a scaling with $n = 3$.

In summary, we propose the measurement of the elliptic flow of hadronic resonances at the Relativistic Heavy Ion Collider. After recombination scaling has been confirmed for the $v_2$ of stable particles, deviations from this scaling for resonances can directly measure the fraction of resonances that escape from hadronization without rescattering in the hadronic phase at intermediate and large transverse momentum. This allows conclusions to be drawn about the importance of the hadronic phase in this region of phase space.

In turn, this method can be applied to explore the structure of exotic particles, such as the recently found $\Theta^+(1540)$. The amount of rescattering in the hadronic phase is sensitive to the size of the system, eventually distinguishing between a compact hadron and a molecular state. The deuteron, with a known molecular structure, can serve as a precise benchmark for these measurements. The same method is also applicable to determine the structure of other exotic candidates with a possible molecular structure such as $\Lambda(1405)$, $a_0(980)$, $f_0(980)$, and so on.[24, 25, 26].

Acknowledgments

We thank J. Kapusta, V. Greco and C.M. Ko for stimulating discussions. This work was in part supported in part by RIKEN, Brookhaven National Laboratory, Grant-in-Aid by the Japanese Ministry of Education No. 14540255, and DOE grants DE-FG02-96ER40945 and DE-AC02-98CH10886. S.A.B. acknowledges support from an Outstanding Junior Investigator Award (DOE grant DE-FG02-03ER41239) and R.J.F. has been supported by DOE grant DE-FG02-87ER40328.

[1] R. J. Fries, B. Müller, C. Nonaka, and S. A. Bass, Phys. Rev. Lett. 90, 202303 (2003).
[2] R. J. Fries, B. Müller, C. Nonaka, and S. A. Bass, Phys. Rev. C 68, 044902 (2003).
[3] C. Nonaka, R. J. Fries, and S. A. Bass, Phys. Lett. B 583, 73 (2003).
[4] V. Greco, C. M. Ko, and P. Lévai, Phys. Rev. Lett. 90, 202302 (2003); V. Greco, C. M. Ko, and P. Lévai, Phys. Rev. C 67, 034902 (2003).
[5] R. C. Hwa and C. B. Yang, Phys. Rev. C 67, 034902 (2003).
[6] J. Y. Ollitrault, Phys. Rev. D 46, 229 (1992).
[7] S. A. Voloshin, Nucl. Phys. A 715, 379 (2003).
[8] D. Molnár and S. A. Voloshin, Phys. Rev. Lett. 91,
[9] J. Castillo [STAR Collaboration], talk at QM2004, Oakland, USA, January 11-17, 2004.
[10] P. Sorensen [for the STAR Collaboration], J. Phys. G 30, S217 (2004).
[11] V. Greco and C. M. Ko, nucl-th/0402020.
[12] J. Adams et al. [STAR Collaboration], nucl-ex/0310004.
[13] C. Bernard et al., Nucl. Phys. B (Proc. Suppl.) 119, 523 (2003).
[14] T. Nakano et al. [Leps Collaboration], Phys. Rev. Lett. 91, 012002 (2003).
[15] D. Diakonov, V. Petrov, and M. V. Polyakov, Z. Phys. A 359, 305 (1997).
[16] V. V. Barmin, Phys. Atom. Nucl. 66, 1715 (2003) [Yad. Fiz. 66, 1763 (2003)] (hep-ex/0304040).
[17] S. Stepanyan et. al. [CLAS Collaboration], Phys. Rev. Lett. 91, 252001 (2003).
[18] J. Barth et al. [SAPHIR Collaboration], Phys. Lett. B 572, 127 (2003).
[19] C. Alt et al. [NA49 Collaboration], hep-ex/0310014.
[20] J. Randrup, Phys. Rev. C 68, 031903 (2003); J. Letessier, G. Torrieri, S. Steinke and J. Rafelski, hep-ph/0310188.
[21] L. W. Chen, V. Greco, C. M. Ko, S. H. Lee, and W. Liu, nucl-th/0308006.
[22] R. L. Jaffe and F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003).
[23] M. Kaneta [PHENIX Collaboration], talk at QM2004, Oakland, USA, January 11-17, 2004.
[24] R. L. Jaffe, Phys. Rev. D 15, 267 (1977); ibid. 15, 281 (1977).
[25] M. Alford and R. L. Jaffe, Nucl. Phys. B 578, 367 (2000).
[26] F. E. Close and N. A. Törnqvist, J. Phys. G 28, R249 (2002).