Universality in solar flare and earthquake occurrence

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Earthquakes and solar flares are phenomena involving huge and rapid releases of energy characterized by complex temporal occurrence. By analysing available experimental catalogs, we show that the stochastic processes underlying these apparently different phenomena have universal properties. Namely both problems exhibit the same distributions of sizes, inter-occurrence times and the same temporal clustering: we find afterflare sequences with power law temporal correlations as the Omori law for seismic sequences. The observed universality suggests a common approach to the interpretation of both phenomena in terms of the same driving physical mechanism.

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Solar flares are highly energetic explosions\textsuperscript{1} from active regions of the Sun in the form of electromagnetic radiation, particle and plasma flows powered by strong and twisted magnetic fields. Since they cause disturbances on radio-signals, satellites and electric-power on the Earth, much interest has been devoted in the last years to space weather forecasts\textsuperscript{2,3}. Recent studies have shown that solar flares also affects the Sun’s interior, generating seismic waves similar to earthquakes\textsuperscript{4}. Actually, despite having different causes, solar flares are similar to earthquakes in many respects, for example in the impulsive localised release of energy and momentum and their huge fluctuations\textsuperscript{5}. The analogy with earthquake occurrence is also supported by the observation of power laws\textsuperscript{6,7,8} in the distribution of flare sizes, $P(s)$, related to the Gutenberg-Richter law for the earthquake magnitude distribution. Various interpretations have been proposed for these power law distributions ranging from Magneto-Hydro-Dynamics\textsuperscript{1,9} to turbulence\textsuperscript{10} up to Self Organized Criticality\textsuperscript{11,12,13}. A better understanding of solar flares and coronal mass ejections from the Sun requires knowledge of the structural details of these events and their occurrence in time. This could greatly improve the prediction of violent space weather and the understanding of the physical processes behind solar events.

Here we present evidence that the same empirical laws widely accepted in seismology characterize, surprisingly, also the size and time occurrence of solar flares. In particular the same temporal clustering holds both for earthquake, where it is known as the Omori law, and solar flare catalogues: a mainflare triggers a sequence of afterflares. The evidence of a universal statistical behaviour suggests the possibility of a common approach to long term forecasting and rises as well deep questions concerning the nature of the common basic mechanism.

A statistical approach to earthquake catalogues has revealed a scale invariant feature of the phenomenon, as indicated by power law distributions for relevant physical observables\textsuperscript{14,15}, such as the seismic moment distribution of earthquakes, $P(s) \sim s^{-\alpha}$, where the exponent $\alpha \in [1.6,1.7]$ is essentially the same in different areas of the world\textsuperscript{16}. This relation corresponds to the Gutenberg-Richter law for the distribution of the earthquake magnitude $M$ via the relation $M = 2/3 \log(s) - K$, where $K$ is a constant\textsuperscript{17}. It is also observed that earthquakes tend to occur in clusters temporally located after large events: the Omori law states that the number of aftershocks at time $t$ after a main event, $N_A(t)$, decays as a power law $N_A(t) \sim t^{-p}$ with $p \approx 1$\textsuperscript{18}. Finally, the distribution of intertimes between consecutive earthquakes, $P(\Delta t)$, is not a simple power law, but has a non trivial functional form which, like the other quantities mentioned before, is essentially independent of the geographical region or the magnitude range considered\textsuperscript{19}. These observations suggest that $P(\Delta t)$, $N_A(t)$ and $P(s)$ are distinctive features of earthquakes and, thus, fundamental quantities for a probabilistic analysis of the phenomenon characterizing its amplitude and time scales.

In this letter we analyse several catalogues of solar flares and compare them with the Southern California catalogue for earthquakes. Since emissions at different wavelengths are related to different radiation mechanisms, we present a comparison among solar data from X-ray observations in three different energy ranges and different periods of solar cycle, by using on-line available flare catalogues. More specifically, Soft X-ray data in the (1.5-2.4) keV and (3.1-24.8) keV ranges are obtained from the Geostationary Operational Environmental Satellite (GOES) systems\textsuperscript{20}. We consider $N_e = 21567$ events from January 1992 to December 2002 covering the entire 11-years solar cycle. Solar flares in the Hard X-ray range...
FIG. 1: (Color online) The number distribution, $n(\Delta t)$, of intertimes, $\Delta t$, between consecutive events in solar flare and earthquake catalogues. Solar data refer to X-ray observations in three different energy ranges covering different periods of solar cycle: soft X-ray data in the (1.5-2.4) keV and (3.1-24.8) keV ranges from the GOES catalogue ( ■ symbols); hard X-ray (>25 keV) from the BATSE catalogue ( □ symbols); intermediate X-ray (10-30 keV) from the WATCH catalogue ( ○ symbols). Earthquake intertimes data are from the California catalogue for events with magnitude $M \geq 2$ ( ● symbols).

(>25 keV) are obtained from the Burst and Transient Source Experiment (BATSE) that gives $N_e = 6658$ flares from April 1991 to May 2000 [21]. Finally $N_e = 1551$ events from January 1990 to July 1992 in the Intermediate X-ray (10-30 keV) range are analysed from the WATCH experiment [22]. Many earthquake catalogues exist and the universality of their statistical features has already been established [16, 19]. Thus, for clarity, we only consider here the Southern California earthquake catalogue [23] which has $N_e = 88470$ events with magnitude $M \geq 2$ in the years from 1967 to 2002.

The intertime distribution has been already investigated both for earthquakes [19] and solar flares [24]. The intertime, $\Delta t$, is the time between the start of a flare (or an earthquake) and the start of the next one as reported in the above catalogues. Here, for a catalogue with $N_e$ data, we count the number of events $n(\Delta t)$ having intertime between $\Delta t$ and $\Delta t + \lambda/N_e$, where $\lambda$ is a constant setting the binning of raw data. This is the statistically relevant quantity to consider [21], since $n(\Delta t)/\lambda \to P(\Delta t)$ in the limit $N_e \to \infty$, and, thus, in the following we refer to $n(\Delta t)$. Here we choose $\lambda$ such that $\lambda/N_e = 75$ sec for the California catalogue and use the same value for all catalogues. Fig.1 shows the intertime distributions, $n(\Delta t)$, for the three different solar

flares data sets and for the California earthquake catalogue. Solar flares data scale one on top of the other to a very good approximation and, interestingly, they all appear to collapse, within statistical errors, on the same non trivial distribution function of earthquake intertimes. In particular, this data collapse is obtained without rescaling $\Delta t$ by any suitable factor: the intertime duration, $\Delta t$, is expressed in the same units (seconds) for all data sets. Thus, Fig.1 shows that the same intertime distribution function and, surprisingly, even the same time range characterize these apparently different physical processes in the magnitude range of the above catalogues.

The scaling behaviour of $n(\Delta t)$ for different solar phases is a widely debated subject [24, 25, 27] and a dependence on the solar phase [22] has been observed also in the case of Coronal Mass Ejection [27]. The result for flares has been obtained by taking into account only events with a peak flux larger than $10^{-6}Wm^{-2}$ (class C1). We have then considered separately data from the GOES catalog corresponding to maximum and minimum solar activity and used the same binning procedure as Fig.1. In order to take into account the different level of background X-ray flux, we have set different thresholds for different phases: events greater of class C1 in the maximum and class B1 ($10^{-7}Wm^{-2}$) in the minimum phase [24]. Fig.2 shows that data from different phases fall on the same universal curve.

The other crucial quantity to be investigated is the distribution of flares sizes, $P(s)$, i.e., the distribution of the
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In terms of the magnitude, the data from flare catalogs is compared with the earthquake seismic moments distribution, $M$. The universal distribution is well fitted by a power law with exponent $\alpha = 1.65 \pm 0.1$ (shown as a solid line in the picture). $s_0 = 10^{-7} \text{W/m}^2$ for GOES, $s_0 = 600 \text{cmnts/(sec 2000cm}^2\)$ for BATSE, $s_0 = 3000 \text{cnt/bin for WATCH, } s_0 = 30 \cdot 10^{-6} \text{Nm for earthquakes.}$

flare peak intensity, $s$, from the above catalogues. This is compared with the earthquake seismic moments distribution, $P(s)$, from the Southern California catalogue. In order to normalize the different units and experimental ranges used in each catalogue, here we scale the values, $s$, of each data set by a given constant amount $s_0$. Then, we calculate the number of events $n(s/s_0)$ with sizes between $s/s_0$ and $s/s_0 + \lambda/N_e$. Excellent data collapse is observed in Fig. 3 with all data fitted over almost three decades by a power law $n(s/s_0) \sim (s/s_0)^{-\alpha}$ with an exponent $\alpha = 1.65 \pm 0.1$, in agreement with previous results on solar flares and earthquakes. Therefore, in analogy to earthquakes, from the above observations it is possible to introduce a Richter scale for flares where their “magnitude”, $M$, is defined via the relation: $M(s) = 2/3\log(s) - K_F$, where $K_F$ is a constant.

In terms of the magnitude the data from flares catalogues are therefore found to follow the Gutenberg Richter law introduced for earthquakes.

Further evidence of structural similarities in the statistics of the two phenomena is given by the analysis of correlations between events within each of these catalogues. It would be interesting to compare the time correlation between main-events and the sequence of their after-events, as in the Omori law. We define a “main-event” as an event with magnitude $M > M_{\text{main}}$; its “after-events” are the following events with $M_{\text{cut}} < M < M_{\text{main}}$, where $M_{\text{cut}}$ is a cutoff for small background events. The basic difference with the usual definition used in seismology is that an event with $M < M_{\text{main}}$ considered as “aftershock” may instead be an independent event totally unrelated to the preceding “mainshock”. Furthermore, an event with $M > M_{\text{main}}$ considered as “mainshock” may have been triggered by a previous larger event. Despite these differences, our definition can be straightforwardly applied to flare catalogues too and tends to the standard one for large enough $M_{\text{main}}$ and $M_{\text{cut}}$: here we fix $M_{\text{cut}} = M_{\text{main}} - 2.5$. In Fig. 4 we show the number of “after-event”, $n_A(t)$, at time $t$ after a “main-event”, for all the mentioned data sources. Interestingly, the time correlation function, $n_A(t)$, has the same functional form in all catalogues. A power law $n_A(t) \sim 1/t$ (straight line in the picture), as the Omori law, fits the data. The results are quite robust with respect to changes of the parameter $M_{\text{main}}$, provided that $M_{\text{main}}$ is large enough as previously explained. We apply the same procedure to analyse the rate of occurrence of events leading up to a main event and observe that also “fore-flares” follow the same power law behaviour (Omori law) as foreshocks, even if more symmetrical behaviour is observed in the flare case.

Fig.s 1, 3 and 4 indicate that the statistical properties of size and time scales of solar flares (independently of the energy range and the temporal location in the solar cycle of the X-ray radiation) and earthquakes are essentially the same within current statistical accuracy. It is tempting to look at the observed universality in the perspective of the theory of critical phenomena. In the past analogy between the two phenomena was proposed on the basis of the same theoretical model. Here we follow a completely different approach: we directly compare experimental catalogues, we observe universal behaviour and therefore we propose the presence of a common driving physical mechanism. Most earthquakes occur where the elastic energy builds up owing to relative motion of tectonic plates. Schematically, as the friction locks the sliding margins of the plates, energy load increases. When elastic stress overcomes the threshold of frictional resistance, it is relaxed causing the occurrence of an earthquake. This “stick-slip” behaviour redistributes the stress-energy field in the crust generating new earthquakes where and when the local slipping threshold is exceeded. A quantitative prediction on the aftershocks decay cannot be derived by simple stress transfer but can be obtained in terms of a state variable constitutive formulation, where the rate of earthquakes results from the applied stressing history. This formulation gives account for long-range correlations between earthquakes affecting the shape of the whole intertime distribution.
FIG. 4: (Color online) The correlation function, $n_A(t)$, i.e., the number of "after-event" at time $t$ after a "main-event" for the same catalogues of Fig.1. To find the best collapse, data from different catalogues are rescaled by a given amount $t_0$ ($t_0 = 700$ sec for GOES, $t_0 = 60$ sec for BATSE, $t_0 = 1$ sec WATCH and $t_0 = 43$ sec for California earthquakes). As for Fig.1 and Fig.3, the "aftershock" rate of occurrence for earthquakes and solar flares scale very well. For comparison, we also plot an Omori power law $n_A(t/t_0) \sim t_0/t$ (solid line).

The universal scaling of Figures 1-4 suggests a similar physical mechanism at the basis of solar flares occurrence. Flares and X-ray jets arise in active solar regions where magnetic flux emerges from the solar interior and interacts with ambient magnetic field. These interactions are thought to occur in electric current sheets separating regions of opposite magnetic polarity. The dynamics and energetics of these sheets are governed by a complex magnetic field structure. Opposite fluxes lead to rearrangement of field lines building up magnetic stress up to a breaking point where magnetic energy is released in a flare via magnetic reconnections. The observed temporal clustering of Fig. 4 shows that the rate of flare occurrence decreases in time as a power law after a "mainflare". Since the same behaviour is found for seismic sequences, here we propose that the mechanism at the basis of seismic energy redistribution can be responsible for "afterflare" occurrence. In particular magnetic stress transfer in Solar Corona plays the role of elastic stress redistribution on the Earth crust. As a consequence the state-rate formulation can be generalized to solar flares, namely the flare triggering depends on the entire history of magnetic stresses. Beyond issues of fundamental science, the present results can also have very practical consequences as, for instance, to improve the prediction of violent space weather by applying established methods of seismic forecasting.

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