Equation of State, Flow, Fluctuations and $J/\psi$ suppression

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Radial flow observed at AGS/SPS energies is very strong, with collective velocities of matter reaching about 0.5c for central collisions of the heaviest ions. The lattice-based Equation of State (EOS) is however rather soft, due to the QCD phase transition. We show that both statements are consistent only if proper kinetic-based treatment of the freeze-out is made. In fact chemical and thermal freeze-out happen at quite different conditions, especially at SPS. Event-by-event fluctuations can shed new light on this problem. We also propose new model of “anomalous” $J/\psi$ suppression found for PbPb collisions, related it to prolonged lifetime of dense matter due the “softest point” of the EOS.

1. THE RADIAL FLOW: AN INTRODUCTION

One of the major goals of heavy ion physics is to learn the EOS of hot/dense hadronic matter. Although for central collisions of heavy ions at the SPS the energy density of the order of few $GeV/fm^3$ is reached, it remains unknown when and how the matter becomes (locally) equilibrated. We also do not know whether new phase of matter - Quark-Gluon Plasma (QGP) - is actually produced. One well-known strategy addressing these issues relies on (very rare) processes happening at earlier stages, the e/m probes \textsuperscript{[18]} or $J/\psi$ suppression (see section 5). Both lead to exciting experimental findings, much discussed at this conference.

Another approach (to be mostly discussed in this talk) is based on hadronic observables, well measured now in high-statistics experiments. Although re-scatterings tend to erase most traces of the dense stage, some of them are preserved and accumulated during the expansion: collective flow is one of them. Existing data strongly suggest that the hadronic system does indeed behave as a truly macroscopic one. Rather detailed phenomenology of the so called directed flow was covered here by Ollitrault, so I focus on radial (axially symmetric) flow\textsuperscript{[2]} for central collisions.

Its very existence was widely debated for years, but (although at QM97 we have still witnessed remnants of this debate) it seems to be proven now “beyond a reasonable doubt”. The HBT data, Coulomb effects (or any reasonable event generator) show that transverse size at freeze-out significantly exceed that of parent nuclei. An excellent test for existence of the flow is provided by deuterons. The shape of their spectrum, its slope

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\textsuperscript{2} The longitudinal flow was discussed in multiple hydro-based works. However, due to uncertainty in initial conditions, predictive power of hydrodynamics is rather limited.
$T_d$ and even yield are all very sensitive to it. The flow implies a correlation between position and momentum, which helps to produce deuterons. If this correlation is removed (see \[7\], where in RQMD output the nucleon’s positions or momenta were interchanged) deuteron spectra dramatically change.

Phenomenology of the $m_t$ slopes $T_r$ is qualitatively explained by flow. However, looking at it quantitatively one finds that their understanding should be improved. True, one may get a very good fit with some velocity profiles and fixed decoupling temperature $T_f$. But the spectra allow for multiple fits, with wide trade-offs between $< v_t >$ and $T_f$. Furthermore, those fits are based on grossly oversimplified picture: (i) the ad hoc profiles do not correspond to hydro with a realistic EOS; (ii) the main assumption - for all secondaries one should expect the same $v_t$ and $T_f$ - obviously contradicts to elementary kinetics. Different secondaries do have very different cross sections, and therefore decouple from flow at different times.

As one can see from experimental talks at this conference, by now we have rich experimental systematics of $m_t$ slopes. First of all (as shown by NA44 at QM96), the slopes have very strong A dependence. While the pp data show no trace of the radial flow (as noted long ago \[8\]), it is seen for nuclear collisions. Furthermore, $< v_t >$ is increased by about factor two from SS to PbPb. Such trend qualitatively contradicts to predictions of most of the hydro models in literature, with A-independent freeze-out.

Then there is significant dependence on particle species: while $\pi, K, N, d$ slopes show about linear increase with the particle mass. New data for strange hadrons $\phi, \Lambda, \Xi, \Omega$ deviate from this line. (This does not mean that they do not flow together with others for some time - they may be just decoupled earlier, due to smaller cross sections.)

The next point: data show strong rapidity dependence of the radial flow. The strongest effect (and its A-dependence) comes preferentially from mid-rapidities.

Finally: dependence on the collision energy $E/A$. For Bevalac/SIS energies $< v_t >$ steadily grows with $E/A$, and new low energy runs at AGS show that it is also the case in the whole AGS domain. At 11 GeV/A flow velocity is about the same as at SPS (160-200 GeV/A). It is very important to know what happens in between: most probably there exist a maximum at some energy $E$.\[4\]

2. THE EQUATION OF STATE

We use rather standard EOS of hadronic matter, a resonance gas for hadronic phase \[15\] and a simple bag-type quark-gluon plasma, with a Bag constant fitted to $T_c = 160 MeV$. In Fig.(a) we show its phase boundary \[15\] and the paths the matter elements follow during adiabatic expansion. As both baryon number and entropy is conserved, the lines are marked by their ratio. Those for $n_b/s = 0.02, 0.1$ correspond approximately to SPS (160-200 GeV/A).

\[3\]The slopes are not temperatures, as they also include effect of the flow to be discussed, and resonance decays.

\[4\]Unless a saturation occurs exactly 10 GeV/c. But as EOS is changing drastically for corresponding energy densities (both due to changing meson/baryon ratio and to the phase transition), exact cancellation of its influence on flow is unlikely.

\[5\]This model is known to have wrong behavior of the phase boundary at large densities, and even with excluded volume for baryons (which we use) one finds some problems. Significant progress in understanding of high-density matter was made recently, see talks of T.Schaefer and K.Rajagopal.
Figure 1. (a) Paths in the $T - \mu$ plane for different baryon admixture, for resonance gas plus the QGP; (b) the ratio of pressure to energy density $p/\epsilon$ versus $\epsilon$, for different baryon admixture.

GeV A) and AGS (11 GeV A) heavy ion collisions, respectively. (Note a non-trivial zigzag shape, with slight re-heating in the mixed phase.) The paths end in hadronic phase by freeze-out: as we discuss below, it is non-universal for different collisions and even volume elements. Two points in Fig.1(a) correspond to chemical freeze-out, extracted from thermal fit to particle composition observed at AGS and SPS (from [3]). Both (inside error bars) coincide with the left corners of the zigzag path.

The final velocity of the observed collective “flow” is time integral of the acceleration, which is proportional to $p/\epsilon$ ratio plotted in Fig.1(b). Note that the QCD resonance gas in fact has a very simple EOS $p/\epsilon \approx const$, while in the “mixed phase” is softer indeed. Furthermore, there is a deep minimum known as the “softest point” [5]. The contrast between “softness” of matter at dense stages and relative “stiffness” at the dilute ones is much stronger for the SPS case, because there are less baryons.

Typical solution for 11.6A GeV Au+Au is shown in Fig.2 while for 160A GeV Pb+Pb it is shown in Fig.3. First of all, they are qualitatively different. At AGS the longitudinal and transverse expansion are not qualitatively different, while at SPS the longitudinal flow has already distinct ultra-relativistic features, most isotherms being close to Bjorken hyperbola $\tau = \sqrt{t^2 - z^2}$. Less obvious observation (resulting from particular EOS with the QCD phase transition) is a dramatic difference in the transverse expansion. The AGS case can be described as “burning in”, the lines of constant energy density moves inward with some small constant speed. At SPS the mixed phase matter burns into the low density hadron gas at a cylinder (known also as a “burning log”), which has nearly time-independent transverse radius 6-8 fm.
Figure 2. Hydrodynamical solutions for central (a) 11.6A GeV Au+Au and (b) 160A GeV Pb+Pb. The solid contours are energy density contours, with the bold contour being the boundary between the mixed and hadronic phase ($e = 0.35\text{GeV}/\text{fm}^3$). The dotted contours are the longitudinal (left) and radial (right) velocity contours, with values starting from the left of 0.01, 0.05, 0.1, 0.2, ...

3. KINETICS OF THE FREEZE-OUT

We have already remarked that in order to explain flow (and many other things) quantitatively, the hydro solution should be complimented with (well known!) kinetics of the dilute hadron matter at the end. So the practical objective of [2] was to create a next generation Hydro-Kinetic model for heavy ion collisions, HKM for short. It incorporates basic elements of the macroscopic approach – (i) thermodynamics of hadronic matter, (ii) hydrodynamics of its expansion, and (iii) realistic hadronic kinetics at the freeze-out, plus practically important (iv) resonance decays. Most elements of the model have in fact been worked out in literature, some are new, but they are practically taken together for the first time.

Before we go into specifics, let us make brief comments on literature. Many hydro-based works have parameterized the initial conditions in a way which results in rough reproduction of the $y, p_t$ spectra of different species, but none so far addressed the details of the radial flow systematics discussed above. Probably the closest in spirit to our work
The cascade “event generators” (Fritjof, Venus, RQMD, ARC etc) are widely used, and (at least) RQMD provides a reasonable radial flow. (They are much more beloved by experimentalists than theorists, who of course know that all of these models know nothing about QGP, and thus directly contradict to QCD theory/lattice.) Hydrodynamics and cascades are not physics alternatives, but just complementary tools, most useful for dense and dilute conditions, respectively. These days, with approaching RHIC/LHC, one has to work with thousands of secondaries. Direct simulation of all re-scatterings are neither practical not necessary: as soon as the system becomes much larger than the correlation length, it can be cut into independent “cells”. The most reasonable strategy for RHIC/LHC is, to my mind, a combination (parton cascade)-(hydro)-(hadronic cascade).

One should make clear distinction between chemical and thermal freeze-outs. Reactions changing particle composition have cross sections very different from those for elastic collisions. In most hydro-based papers published so far it was however ignored, and the expansion was simply cut off at fixed $T$, usually about 140 MeV. It is clear now that at SPS chemical freeze-out happens at $T_{ch} \approx 160$ MeV, while thermal one occurs (for heavy ions, in their center) to temperatures as low as $T_{th} = 100 - 120$ MeV. It does not look like a huge difference, but in terms of density, space-time picture and flow it is is crucial. Extra few fm/c time available at the end for heavy ion collisions (PbPb relative to SS) leads to “extra push” by stiff hadronic gas, especially for nucleons. (This is where the twice stronger collective flow for heavy ions comes from.)

Apart of flow, there are of course other means to verify the difference between chemical and thermal freeze-outs. If they do not coincide, the chemical potentials cannot stay at zero. The effect should in fact be directly observable in $p_t$ distribution of pions as extra low $p_t$ enhancement (on top of resonance decays). We have calculated that for central PbPb at SPS it should reach $\mu_\pi = 60 - 80$ MeV. Taking the ratio of NA44 data for PbPb/SS we indeed found extra low-$p_t$ enhancement compatible with it.

HKM uses “local” kinetic condition: the (relevant) collision rate $1/\tau_{coll}$ becomes comparable to the expansion rate $1/\tau_{expansion} = \partial_t u_\mu$. Using it in a form $\tau_{expansion}/\tau_{coll} = 1/2$ we determine the freeze-out (3-d) surface. Representative case is shown in Figs. as sections by time t - longitudinal coordinate z plane (at transverse coordinate r=0) and the t-r plane (z=0). Note significant dependence on the particle kind. The larger is the system, the lower is $T_{th}$ on this surface. Furthermore, the shape of the freeze-out surface is very different from that of isotherms. It means that there is a significant variation of this temperature over the surface itself: in order to find the coolest pion gas, one should look at the very center of central collisions of heaviest nuclei at highest energy!

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6 This paper also uses local freeze-out conditions. Unfortunately, their method (referred to as “global” hydrodynamics) include transverse plane averaging, which is unnecessary and significantly obscures the results obtained. However, to the extent flow is concerned, our findings agree.

7 It was originally pointed out by G.Baym, see quantitative discussion in [9].

8 For clarity: those potentials are conjugated to total number of particles, so say for pions they enter distributions of $\pi^+, \pi^-, \pi^0$ with the same sign.
4. THE RADIAL FLOW: SOME RESULTS

The improved freeze-out condition leads to huge difference for the flow. Although it does not significantly prolong total lifetime, it significantly increases the lifetime of hadronic phase at which the matter is most “stiff”.

Distribution over transverse velocities calculated at the freeze-out surfaces is shown in Figs. 4. These distributions have sharp peak at their right end, more pronounced at SPS. Its position depends significantly on the particle type, reaching $v_t = 0.6$ for N at SPS. (However velocity distribution for the isotherms $T = .14 GeV$ peak is at much smaller $v_t \approx .17$. This difference is smaller for medium ions, not shown.) In Figs. 5 we show how this translates into the observable quantity, the $m_t$ slopes $\tilde{T}(y)$ plotted as a function of rapidity $y$. For AuAu data at AGS Fig. 5(a) one can see, that our results are slightly below RQMD ones: it is because this version of RQMD has a (specially tuned) repulsive baryon-induced potential, on the top of the pure cascade. Approximate agreement with RQMD at SPS energies is due to “softness” of RQMD EOS: at early times the energy is stored not in resonances, but in (longitudinally stretched) classical strings, producing no pressure in the transverse direction.

Let us stress that we have not attempted any fine tuning of the parameters. The main ingredients, EOS and freeze-out condition were fixed at an early stage and not modified when hadron spectra/slopes were calculated. Our main objective was to test mostly the systematics of the flow discussed in the introduction.

If we would plot just $p_t$ spectra, one would see no difference.
Figure 4. Transverse velocity distribution over the various freeze-out surfaces for 11.6A GeV Au+Au collision (a) and 160A GeV Pb+Pb (b).

5. THE “SOFTEST POINT” AND THE $J/\psi$ SUPPRESSION

Recent studies by the NA50 collaboration \cite{16} of the $J/\psi$ suppression in Pb(158 GeV-A)-Pb collisions show dramatic differences with the extrapolations of known trends, from lighter beams and pA collisions. These data, if correct, imply existence of a new suppression mechanism, which sets in \cite{16} for PbPb for impact parameters $b < 8\, fm$.) They may be a long-awaited manifestations of collective effects, related to dramatic changes in properties of hot/dense hadronic matter related to the QCD phase transition. In general, “anomalous” $J/\Psi$ suppression can be caused either by (i) increased absorption rate or (ii) by increased time the $J/\Psi$ spends in a dense/hot matter. The available literature is mostly devoted to the former possibility, while we study the latter one.

The oldest (and the simplest idea \cite{18}) is that if Quark-Gluon Plasma (QGP) is formed, its free gluons can rather easily “ionize” charmonium states by photo-effect-type reaction $gJ/\psi \rightarrow \bar{c}c \cite{23,13,24}$. Another idea \cite{21,22} is that successive charmonium levels $\psi', \chi, J/\psi$ should “melt” (fail to exist) at increasing density due to Debye screening in the Quark-Gluon Plasma (QGP).

There is a significant difference between these two mechanisms. “Melting” scenario implies existence of certain thresholds as a function of energy density, and one can indeed expect rapid onset of the suppression. Furthermore, it naturally explains why different charmonium states get suppressed at different $\epsilon$ ($b, E_t$). Although the realistic model of this type is yet to be worked out, this scenario may be potentially successful.

In view of NA50 data, scenario based on the gluonic “photo-effect” has problems. Like any other probabilistic process (“co-mover” absorption) it cannot create discontinuous behavior by itself. Therefore an additional hypothesis was made \cite{20} - it is the formation of QGP which is discontinuous, it happens suddenly after a certain critical energy density.

\footnote{The surprising and much debated feature of the data is that this new regime seems to set in extremely sharply, see fig.\ref{fig:5}. Let me therefore stress again, that we try to describe trend in the whole region, not just its onset.}
\( \epsilon_c \) is reached. But thermodynamics tells us that in a first-order phase transition there is discontinuity in temperature but not in energy density: the mixed phase interpolates linearly between the energy densities of the two phases, \( \epsilon_{\text{min}} \) and \( \epsilon_{\text{max}} \). Relying on non-equilibrium fluctuations require too much fine tuning. Furthermore, this scenario has a phenomenological problem: any sudden transition into QGP phase at some \( \epsilon_c \) results in a jump of the total entropy produced, while the data on \( <n_{\text{ch}}(E_t)> \) does not seem to have a jump at \( E_t \approx 50 \text{GeV} \).

We have mentioned above that QCD phase transition leads to a very non-trivial EOS, with a deep minimum in \( p/\epsilon \) at the “softest point”, \( \epsilon_{\text{max}} \sim 1-2 \text{GeV/fm}^3 \). It was pointed out in [5] (see also later study [6]) that hydrodynamical expansion with such EOS has a non-monotonous dependence on initial conditions. When initial energy density is close to the “softest point”, the expansion is more slow. As a result, the QGP lifetime has a peak as a function of collision energy (or impact parameter \( b \)), increasing by significant factor (2-3), as compared to that at central collisions at SPS.

In [17] we study whether the non-trivial features of \( J/\psi \) suppression can be explained by such increase in the QGP lifetime. Before we go into specifics, let us point out qualitative differences between this scenario and the one based on \( J/\psi \) “melting”. The main difference is that in the latter case the complete suppression of corresponding charmonium states are assumed for all \( \epsilon > \epsilon_c \), while in our case the anomaly is concentrated around \( \epsilon \approx \epsilon_{\text{max}} \) and at larger \( \epsilon \) the survival probability increases back.

Estimating initial dependence of energy density (e.g. by “wounded nucleon model”), one can see that indeed the impact parameter \( b \approx 8 \text{fm} \) in PbPb collision roughly corresponds to the “softest point”. The hydro expansion for non-central collisions is complicated by the so called directed flow. Fortunately for \( J/\psi \) suppression one may ignore it: expansion is predominantly longitudinal during the first few fermi/c.

So, assuming that absorption rates are unchanged, we study whether changes in QGP lifetime can affect \( J/\psi \) suppression and reproduce NA50 data. The main problem here is “leakage” of \( J/\psi \) from the system, which limits the sensitivity of \( J/\psi \) yield to the
Figure 6. (a) Three variants of QGP lifetime as a function of impact parameter $b$ lead to $J/\psi$ suppression (relative to Glauber theory) shown in (b). The points are preliminary NA50 data. (c) The $J/\psi$ mean $p_T^2$ versus transverse energy $E_T$. “This work” means variant B in figs (a,b).

QGP lifetime. We have reproduce known observables (distribution in transverse energy $E_T$ and its correlation with $b$) by a small Monte Carlo model. For Pb(158-GeV)-Pb collisions at given impact parameter it generates charmonium states, which are filtered through nuclear absorption, and then put in a hot de-confined medium. The survival probability in it is $e^{-\Gamma t_p}$ where $t_p$ is the time spent in this medium (see fig.6(a)) and $\Gamma$ is the “ionization” rate. $\Gamma$ is chosen to match the observed suppression for central events. To calculate the time inside plasma we need to distribute these events in coordinate and momentum space. The distribution in the transverse plane is given by Glauber theory, like in [27] and the distribution in the longitudinal direction is presumed uniform.

The question is then whether this “semi-realistic” model provides good description of NA50 data at all $b$, especially near the discontinuity. The result of the calculation is shown in in Fig.6(b). Because the usual Glauber-type absorption on nucleons describes well the p-A and light lighter ion data (not shown) as well as peripheral PbPb data, we take it out of the picture, plotting only deviations from it. As one can see from Fig.6(b), the increased QGP lifetime leads to additional suppression for intermediate $E_T = 50 - 100 GeV$. “Leakage” and other smearing effects do not allow to reproduce a jump seen in data, as expected. However, extra QGP lifetime does improve agreement with most of the data points for $E_T > 60 GeV$. In particularly, it reproduces vanishingly small slope of the suppression in this region.

Potentially important way to separate different suppression mechanisms is related with $p_T$-dependence of the suppression. We have therefore calculated $<p_T^2>_{J/\psi}$ vs. $E_T$, see Fig.6(c). The usual initial state parton re-scattering leads to $<p_T^2>_{J/\psi}$ growing with $E_T$. However, if $J/\psi$ produced in the central region are destroyed and only peripheral ones

\footnote{The origin of a (non-statistical?) fluctuation at $E_T \approx 100 GeV$ remains unanswered experimentally.}
survive, then (as suggested by Kharzeev et al) this dependence becomes inverted, with a
decrease at most central collisions. Our scenario leads to intermediate flat dependence,
because some $J/\psi$ from the center may still “leak” out. It was encouraging to see that
NA50 data (first shown at this conference after my talk) show exactly such dependence!

Finally, in [17] we have also discussed the conversion of $J/\psi$ into its spin partner $\eta_c$.
This channel is one more potential “sink” for $J/\psi$ and to our knowledge has not been
investigated previously: however our estimates of the rate of conversion show that it
hardly can be essential.

6. EVENT-BY-EVENT FLUCTUATIONS AND THERMODYNAMICS

Event-by-event fluctuations are deviations of mean value of some observable calculated
in an event, relative to that averaged over the whole ensemble of events. In general, those
can be of dynamical or of statistical nature, and I focus on the latter ones (which are
guaranteed to be there). Their sensitivity to some non-trivial details of the dynamics were
first pointed out in [13]. It was noticed that dispersion of the $<p_t>_{\text{event}}$ distribution
strongly depends on a cascade model used. In particular, those based on “initial rescat-
terings” or superposition of independent pp events predicted much larger fluctuations,
compared to models with multiple re-scattering of secondaries. This paper has triggered
experimental studies (reported at QM97 by Roland): the fluctuations observed by the
NA49 experiment were found to be perfect Gaussian for several orders of magnitude,
without any unusual tails. Its width is very small, which clearly rules out any model of
the former type.

The essence of it is that the total entropy generated in the collisions shows up in the
magnitude of fluctuations. In general, any statistical fluctuations can be derived from
the famous Boltzmann expression relating entropy $S$ and probability $P$, written in the
Einstein form $P \sim \exp(S)$.

Before we turn to specifics, few general comments are appropriate. Applying thermo-
dynamical theory of fluctuations we rely on statistical independence of different volume
elements. The fluctuations of temperature or particle composition should be independent
on whether all elements of the excited system have their freeze-out at the same time or
not, whether they freeze-out at rest in the same coordinate frame or are all moving with
different velocities. The relevant thing is how the global entropy of the system $S$ depends
on the particular observable under consideration.

In general, the fluctuations accurately predicted by this expression neither should be
small, nor the discussed system should have huge number of degrees of freedom. The
requirement is that the system is equilibrated, equally populating all its available phase
space.

The first example we consider [11,12] is the fluctuations in apparent freeze-out tem-
perature $\tilde{T}$. Standard thermodynamics tells us that temperature fluctuations are given

\[^{12}\text{As an example of such approach to multi-hadron production reaction, let me refer to two (my own old) papers [14] where such approach was used for the }K/\pi\text{ ratio dependence on pion multiplicity, as well as probabilities of exclusive channels in low energy }\bar{p}p\text{ annihilation. Remarkably, simple ideal gas formulae for the entropy correctly predicted them, starting from reactions with only 4 secondaries!}\]

\[^{13}\text{As discussed above, the }m_t\text{ slopes are affected by collective flow: but estimates show that the corresponding correction fluctuate less and mostly cancels in ratios, especially for pions.}\]
by $P \sim \exp[-\frac{C_v(T)}{2}(\Delta T/T)^2]$ where $C_v(T)$ is the heat capacity of a hadronic matter. It is an extensive quantity $C_v = T \frac{\partial S}{\partial T}|_{T,V}$, proportional to the total volume (or the number of particles $N$) of the system, so the relative fluctuations are $O(1/N^{1/2})$ as expected.

The key point [12] is that $C_v(T)$ has strong $T$-dependence, which can be used as a “thermometer”. In the vicinity of the QCD phase transition $C_v(T)$ has a peak. So, if the observed hadrons are emitted directly from the QGP clusters/mixed phase (as suggest by some people based on chemical freeze-out), the fluctuations of $\tilde{T}$ should be very small. However, if chemical and thermal freeze-outs are different (as we strongly advocated above) and the observed fluctuations happen in cool gas, its $C_v(T)$ is smaller. We argued above that larger systems cool to lower $T_f$. Therefore we end up with a (counter-intuitive) prediction: (properly scaled) fluctuations for (central) heavy ion collisions should show stronger fluctuations compared to medium ions or peripheral collisions.

The second example [12] deals with fluctuations of occupation of particular bins in the histograms. Deviations from trivial Poisson statistics are induced by quantum statistics. For the ideal Bose-Einstein (Fermi-Dirac) gas one gets $\langle \Delta n_k^2 \rangle = n_k (1 \pm n_k)$. So, by measuring fluctuations in occupation number of different bins one can measure the quantum degeneracy $n_k$ of the gas at freeze-out. Let us briefly mention the magnitude of the effect. Pions in a chemically equilibrated gas ($\mu_\pi = 0$) with zero momenta should have fluctuations enhanced by a factor 1.5. If pions have a non-zero $\mu_\pi$ (as advocated above), the effect becomes stronger: at $p_t = 0$ Bose enhancement reaches a factor of 2. Finally, if there are dynamical low momentum excess pions (e.g. due to DCC) one should observe even larger fluctuations in the corresponding bins.

7. Summary and outlook

The main result of our studies of the radial flow in central collisions can be understood in hydro framework only if the special care is taken about the freeze-out. When it is properly done, all puzzles are gone and the “standard” lattice-based EOS including softness due to the QCD phase transition approximately describe the data, for all ions, both at AGS and SPS. Further work, in combination with elliptic flow data, is however needed to tell if this description is unique.

One significant conclusion is existence of a rather large gap between “chemical” and “thermal” freeze-outs. For central PbPb collisions at SPS the former corresponds to $T_{ch} \approx 160 MeV$ while the latter (in the very center) cools down to $T_{ch} \approx 100 - 120 MeV$. If so, the non-zero chemical potential for pions should appear: there are first indication for reality of this effect in NA44 data. Completely new set of ideas has recently emerged, relating event-by-event fluctuations and thermodynamics. Potentially it is very useful way to learn more about the freeze-out conditions. It is also relatively simple tool for large-acceptance detectors like NA49 or STARS.

Finally, the non-trivial EOS leads to non-monotonous energy/impact parameter dependence of the dense matter lifetime, with the maximum corresponding to the “softest point” [5]. We have speculated above that this effect contributes to $J/\psi$ suppression, and suggested a particular model to be compared with data. We hope that all experiments

[14] The same quantity is in essence measured in the usual HBT: the difference is that we propose to detect Bose-Einstein effect of all pions in a bin, rather than a particular pair.
will now study the $E_t$ dependence of their observables, or run at another (lower) $E/A$.

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