PERTURBATIVE AND NON-PERTURBATIVE ISSUES IN HEAVY QUARK FRAGMENTATION

Matteo CACCIARI
Dipartimento di Fisica, Università di Parma, Italy, and INFN, Sezione di Milano, Gruppo Collegato di Parma

We review the state-of-the-art of our understanding of heavy quark fragmentation. Recent $e^+ e^-$ data for $B$ mesons are compared to the most up-to-date theoretical predictions, and the need for inclusion of a non-perturbative component is discussed. Experimental analyses in moments space are suggested, and it is pointed out how perturbative and non-perturbative contributions are to be properly matched. Failure to do so can result in large phenomenological discrepancies. An example is given for $B^+$ hadroproduction at the Tevatron.

1 Introduction

This talk will be devoted to a review of our present understanding of heavy quark fragmentation, i.e. of the processes where a heavy quark is produced in a hard collision and then hadronizes before decaying. For the quark to be “heavy”, its mass has to be larger than the QCD scale $\Lambda_{QCD} \simeq 200 – 300$ MeV. The top quark, however, is too heavy and decays weakly before hadronizing. Throughout our discussion “heavy quark” will therefore have to be understood as a charm or a bottom quark.

Let us start by considering, for the sake of definiteness, an experiment observing a $B$ meson. Since we know it to contain a $b$ quark, whatever the initial state of the process we can envision its production to proceed as roughly sketched in Fig. 1: A so-called hard process, described by perturbative QCD (pQCD), produces a bottom quark. Subsequently, soft strong interactions (the non-perturbative ‘np’ blob) bind this heavy quark to light ones into a hadronic state.

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resulting into the observed $B$ meson with only a fraction $z$ of the original momentum of the $b$ quark.

From such a simplified description alone, it appears clear that any separation between “hard” and “soft” processes, which is to say between perturbative and non-perturbative QCD, is at best arbitrary. We shall focus more on this issue in the following.

People knowledgeable with light quark fragmentation only might not be immediately familiar with the following fact: In the heavy quark case the pQCD term does not need collinear factorization to be performed. It is the large mass $m$ of the heavy quark that takes care of regulating the collinear singularities, acting as an infrared cutoff. Hence, pQCD alone can yield a finite result and be predictive at the ‘leading twist’ level. Further discrepancies with the experimental results (aside, of course, unknown higher order perturbative contributions) can then be attributed to a non-perturbative part, to be studied in the form of power corrections.

Many analyses, and also intuitive arguments, predict for the leading term of such power corrections a $\frac{\Lambda}{m}$ behaviour, $\Lambda$ being a hadronic scale of the order of a few hundred MeV. These corrections are therefore parametrically small, $m$ being larger than $\Lambda$, but can be numerically large. For instance, they are definitely larger than typical power corrections in event shapes studies at LEP, where the suppressing scale is $\sqrt{s} \simeq 90$ GeV rather than the heavy quark mass. Heavy quark fragmentation is therefore an ideal place to observe and analyze such corrections.

2 Perturbative Issues

Let us consider the production of the heavy quark $Q$ in $e^+e^-$ collisions, $e^+e^- \to \gamma/Z \to Q + X$. An experimentally observable variable usually looked at is the energy fraction (with respect to the beam energy) of a heavy hadron $H$ containing the heavy quark, $x_E \equiv E_H/E_{\text{beam}}$.

Fixed order perturbative calculations are available up to order $\alpha_s^2$. These results do however contain large logarithmic terms which need to be resummed to all orders:

- log($s/m^2$) terms are large when the centre-of-mass energy $\sqrt{s}$ is much larger than the heavy quark mass $m$. Such a situation is easily met at LEP energies for both charm and bottom production;
- $1/(1-x_E)$ and log($1-x_E$)/($1-x_E$) terms are large when the energy of the observed particle is close to the maximum allowed one, and are due to gluon radiation being inhibited close to the phase space boundaries.

Mele and Nason first considered the resummation of log($s/m^2$) terms up to next-to-leading logarithmic (NLL) level. They achieved it by rewriting the $e^+e^- \to \gamma/Z \to Q + X$ differential
cross section in a factorized form

\[ \sigma_N(\sqrt{s}, m) = C_N(\sqrt{s}, \mu_F)D_N^{\text{ini}}(\mu_F, m) + \mathcal{O}(m/\sqrt{s})^0. \]  

(1)

The factorization scale \( \mu_F \) separates the two functions \( C \) and \( D^{\text{ini}} \), and Altarelli-Parisi evolution can be used to resum the collinear logarithms in \( D^{\text{ini}}(\mu_F, m) \) by evolving from an initial scale \( \mu_0 \approx m \) up to the hard scale \( \mu_F \approx \sqrt{s} \), so that \( D_N^{\text{ini}}(\mu_F, m) = E_N(\mu_F, \mu_0)D_N^{\text{ini}}(\mu_0, m) \).

It is to be noted that all the information concerning the hard scattering process is now contained in the coefficient function \( C_N(\sqrt{s}, \mu_F) \). The initial condition \( D_N^{\text{ini}}(\mu_0, m) \), on the other hand, only contains information about physics taking place around the heavy quark mass scale \( m \). This process independence of the initial condition function has been recently established in a more formal setting. It is worth reminding that, while process-independent, \( D_N^{\text{ini}}(\mu_0, m) \) is of course still factorization scheme dependent, like the coefficient function and the Altarelli-Parisi evolution factor. Both Ref. and Ref. make use of the \( \overline{\text{MS}} \) scheme.

After performing the resummation of the \( \log(s/m^2) \) terms one still has to take care of the potentially terms related to suppression of radiation close to the phase space borders. Such terms show up as large logarithms in the large-\( N \) limit of the Mellin transforms. For instance, in this limit the initial condition reads:

\[
D_N^{\text{ini}}(\alpha_s(\mu_0^2); \mu_0^2, \mu_0^2, m^2) = 1 + \frac{\alpha_s(\mu_0^2)}{\pi} C_F \left[ -\ln^2 N + \left( \ln \frac{m^2}{\mu_0^2} - 2\gamma_E + 1 \right) \ln N + 1 - \frac{\pi^2}{6} + \gamma_E - \gamma_E^2 + \left( \gamma_E - \frac{3}{4} \right) \ln \frac{m^2}{\mu_0^2} + \mathcal{O}(\frac{1}{N}) \right] + \mathcal{O}(\alpha_s^2).
\]

Resummation for these so-called Sudakov logarithms was performed at the leading log (LL) level and at NLL level, but in a process dependent way. In Ref., on the other hand, NLL resummation has been revisited in a fully process independent way, exploiting the factorization properties of \( D_N^{\text{ini}}(\mu_0, m) \) and providing NLL resummed expressions for both the \( e^+e^- \) coefficient function and the process independent initial condition.

The resummation of all the large logarithms present in the perturbative calculation for \( D_N(\sqrt{s}, m) \) allows for a prediction which is at the same time reliable and accurate. The dependence on the unphysical factorization scales \( \mu_F \) and \( \mu_0 \), for instance, is greatly reduced and under control after the Sudakov logarithms are resummed with NLL accuracy.

3 Non-Perturbative Issues

Despite having achieved a reliable perturbative prediction, inclusion of non-perturbative effects is still mandatory for a meaningful comparison with the experimental data, for instance the ones recently published by the ALEPH and SLD Collaborations. After calculating a cross section for, say, \( b \) quark production, the one for \( B \) meson is obtained by convoluting it with a non-perturbative component:

\[ D_N^B = D_N^b D_N^{\text{np}}. \]

Pretty much at odds with the available literature, I shall consider comparisons between theory and experiments in \( N \)-moments space rather than with differential distributions in \( x_E \) space.

The most obvious reason for doing so is that only in moments space a smooth departure of the experimental results from the perturbative prediction can be seen: low-\( N \) moments are

\[ \text{From now on we shall make use of Mellin moments, } D_N \equiv \int_0^1 x^{N-1} D(x) \, d(x), \text{ which turn convolutions into products.} \]
Figure 2: ALEPH experimental data compared to theoretical results. The dashed line is the perturbative prediction. The solid line is a combination of the former with a non-perturbative component fitted to the $D_2 = \langle x_E \rangle$ point.

closest, and the gap widens as $N$ increases. Indeed, the non-perturbative contribution can be predicted to behave like

$$D_N^{np} = 1 - (N - 1) \frac{\Lambda}{m} + O\left(\frac{\Lambda^2}{m^2}\right).$$

(4)

In $x_E$ space, on the other hand, one usually tries to compare a whole curve, and the shape of the perturbative prediction has clearly little to do with the experimental one, especially in the $x_E \approx 1$ region, where the peak is. This is due to the fact that all-order power corrections become important here, and prompt one to include from the very start a “large" non-perturbative term, for instance in the form of some smearing function like the Peterson et al. one, in order to fit the data. Doing so, however, immediately obscures the fact that non-perturbative contributions can be seen as a small $\frac{\Lambda}{m}$ correction to the perturbative result.

Figure 3 clearly shows this point. The purely perturbative result fails in describing even the lowest $N$ moment (see left panel), hence showing that inclusion of a non-perturbative component is mandatory. However, its job looks much worse in $x_E$ space (right panel), particularly in the region around and beyond the peak. But it suffices to fit a 1-parameter non-perturbative form $D_N^{np}(x; \alpha)$ to the $D_2 \equiv \langle x_E \rangle$ point in $N$ space, to produce a curve which describes very well the low-$N$ region, and even $x_E$ data much better (albeit not perfectly). We wish to stress that no effort has been made in this case to achieve a particularly good description of the whole $x_E$ distribution, the emphasis being rather on showing that a “reasonable" functional form, fitted to a single point in moments space in terms of a numerically small $\frac{\Lambda}{m} \approx 0.1$ correction, can already produce a decent agreement. Good fits of the low-$N$ moments, on the other hand, will be important for the issue we are going to discuss in the next Section.

4 Phenomenology

The theoretical machinery of fragmentation functions for heavy quark can be used to make predictions for processes other than $e^+e^-$ production, exploiting the property of process inde-

In this case a $D^{np}(x; \alpha) = (\alpha + 1)(\alpha + 2)x^\alpha(1 - x)$ form (or, rather, its Mellin transform) was used, and the fit returned $\alpha = 27.45$. It can easily be seen that this form is compatible with the leading power correction upon replacing $\alpha$ with $2m/\Lambda$. This equality suggests $\Lambda \approx 350$ MeV, in line with our intuitive expectations.
Figure 3: On the left, the experimental data from CDF on $B^+ \rightarrow X$ production in $p\bar{p}$ collisions at the Tevatron, compared to theoretical predictions. On the right, moments of ALEPH data on $B$ fragmentation in $e^+e^-$ collisions compared to a purely perturbative theoretical prediction and to various models/fits for non-perturbative contributions. Plots from Ref. [10]. No Sudakov resummation was included in this case.

dependence of the initial condition $D^{ini}$. For instance, bottom cross sections at large transverse momentum $p_T$ in hadronic collisions can be calculated, providing a resummation of large $\log(p_T/m)$ terms \[\text{References: 13, 14}\]. At the same time, inclusion of the non-perturbative component determined from $e^+e^-$ fits allows to make predictions for the production of $B$ mesons, which are the particles which can be directly observed.

Obtaining accurate theoretical predictions requires however great care in at least two instances:

- a “non-perturbative term” is not an observable quantity. It cannot be determined in absolute terms, but only relatively to how one defines the perturbative part and its parameters. Therefore, when fitting, say, $e^+e^-$ data one extracts a non-perturbative function which should then be used only together with a perturbative description of the same kind (leading, next-to-leading, resummed, etc.) as the one it has been determined with, and with the same parameters ($\Lambda_{QCD}$, $m$, ...);

- different processes may depend on different details of the non-perturbative contribution. For instance, the peak in $x_F$ space is clearly the most prominent feature in $e^+e^-$ processes, but not the most important one when calculating $B$ meson production in $p\bar{p}$ collisions, where instead a good determination of the moments around $N \simeq 4$ (and taking care of employing the same kind of perturbative description in $e^+e^-$ and $p\bar{p}$ processes) increases the prediction, bringing it in agreement with the data within the errors \[\text{References: 14}\].

That these issues are not merely of academic interest is shown by the case of $B$ production at the Tevatron. \[\text{References: 13}\]. Figure 3 shows that the “standard” procedure, used by the experimental Collaboration, of evaluating the cross section by convoluting the perturbative calculation for $b$ quark with a Peterson et al. function with $\epsilon = 0.006$ underestimates the data by almost a factor of three. On the other hand, employing a non-perturbative contribution fitted to $e^+e^-$ data in moments space (the “$N=2$ fit”), so as to get a good description of moments around $N \simeq 4$ (and taking care of employing the same kind of perturbative description in $e^+e^-$ and $p\bar{p}$ processes) increases the prediction, bringing it in agreement with the data within the errors.
5 Conclusions

Heavy quark fragmentation is now a mature subject. Fixed order perturbative calculations have been performed up to order $\alpha_s^2$, collinear and Sudakov resummations are known up to next-to-leading logarithmic accuracy.

Comparisons to experimental data however need the addition of a non-perturbative component. The fairly large contributions expected in this process make it a good one to study power corrections. Performing the fits and the comparisons in moments space helps disentangling the leading power correction from the higher order ones.

Factorization of terms related to the hard scattering from terms related to the heavy quark mass scale suggests some form of universality for such non-perturbative terms, which can therefore be fitted in one process and used to give predictions in another.

When transporting this information care must however be taken to properly match the perturbative and non-perturbative terms, which are not independently measurable quantities, and to carefully consider what details of the non-perturbative function need to be well known. Failure to do so may result in predictions with a far larger degree of uncertainty than one might expect.

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