A class of permanent magnetic lattices for ultracold atoms

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Abstract
We report on a class of configurations of permanent magnets on an atom chip for producing 1D and 2D periodic arrays of magnetic microtraps with non-zero potential minima and variable barrier height for trapping and manipulating ultracold atoms and quantum degenerate gases. We present analytical expressions for an infinite magnetic lattice for the relevant physical quantities and compare them with our numerical results. In one of the configurations of permanent magnets, we show how it is possible by changing the angle between the crossed periodic arrays of magnets to go from a 1D array of 2D microtraps to a 2D array of 3D microtraps and thus to continuously vary the barrier heights between the microtraps.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Magnetic lattices consisting of periodic arrays of current-carrying wires [1, 2] or permanent magnetic films [3, 4] have recently been proposed as an alternative to optical lattices [5, 6] for trapping and manipulating small clouds of ultracold atoms and quantum degenerate gases, including Bose–Einstein condensates. Magnetic lattices may be considered as complementary to optical lattices, in much the same way as magnetic traps are complementary to optical dipole traps: they do not require (intense and stable) laser beams and there is no decoherence or light scattering due to spontaneous emission; they can produce highly stable and reproducible potential wells leading to high trap frequencies; and only atoms in low magnetic field-seeking states are trapped, thus allowing the possibility of performing rf evaporative cooling in situ in the magnetic lattice and the study of very low temperature phenomena in a periodic lattice. Simple 1D magnetic lattices consisting of periodic arrays of traps or waveguides have been constructed using both current-carrying wires [7, 8] and permanent magnets [9–12] on atom chips.

In a recent paper, we proposed a 2D magnetic lattice consisting of two crossed periodic arrays of parallel rectangular magnets [4]. Here, we report on a new class of configurations of
permanent magnets comprising four periodic arrays of square magnets of different thickness for producing 1D and 2D arrays of magnetic microtraps with non-zero potential minima and variable barrier height. This class of magnetic lattices is more general and may prove easier to implement than the crossed periodic arrays of parallel rectangular magnets proposed previously [4]. Analytical expressions for an infinite magnetic lattice are presented for various physical quantities and compared with numerical calculations. In one of the configurations of magnets, we show that by varying the angle between the crossed arrays of magnets it is possible to go from a 1D array of 2D microtraps to a 2D array of 3D microtraps. In all of the configurations of permanent magnets in this paper, the magnetization is perpendicular to the surface.

2. Periodic arrays of square permanent magnets with bias magnetic fields

Figure 1 shows a configuration of four periodic arrays of square magnetic slabs with thicknesses \( t_1, t_2, t_3 \) and \( t_4 \). In section 2.1, we find the components of the magnetic field produced by a single array of square magnetic slabs (figure 2(a)) and then use these results to obtain the total magnetic field due to the four arrays of square magnetic slabs and an external bias field \( \mathbf{B}_1 = B_{1x}\hat{x} + B_{1y}\hat{y} \).
2.1. Single periodic array of square magnets

We first consider a single infinite periodic array of square magnetic slabs of thickness \( t \), at a distance \( s \) from the plane \( z = 0 \), with periodicity \( a \) along the \( x \) - and \( y \)-directions and perpendicular magnetization \( M_z \) (figure 2(a)). The sum of this configuration and a similar one displaced by \( a/2 \) along the \( x \)-axis gives an infinite periodic array of parallel rectangular magnets with periodicity \( a \) along the \( y \)-direction and the same magnetization \( M_z \) and thickness \( t \) (figure 2(b)). For distances from the surface which are large compared with \( a/2\pi \), the magnetic field \( \mathbf{B} \) produced by an infinite array of parallel rectangular magnets is \([4]\)

\[
(B_x, B_y, B_z) = (0, B_{0y} \cos(ky) e^{-kz}, B_{0y} \cos(ky) e^{-kz})
\]

where \( B_{0y} = B_0(\text{e}^{kt} - 1) \) \( \text{e}^{kx} \), \( k = 2\pi/a \) and \( B_0 = 4M_z \) (Gaussian units). Considering the corresponding surface currents around the magnetic slabs in figure 2(b), we find that the array of blue (grey in grey style) square magnets produces the same magnetic field along the \( y \)-axis as the array of red (black in grey style) magnets. Thus, the \( y \)-component of the magnetic field produced by the array of blue (grey in grey style) or red (black in grey style) magnets can be written as half of that given by (1). Furthermore, figure 2(a) is symmetrical with respect to \( x \) and \( y \), and therefore, for this infinite array of square magnets, the dependence of the \( x \)-component of the magnetic field on \( x \) is the same as the dependence of the \( y \)-component of the magnetic field on \( y \). According to Maxwell’s equations, \( \nabla \cdot \mathbf{B} = 0 \) which gives the \( z \)-component of the magnetic field produced by a single array of square magnetic slabs. Thus, the magnetic field \( \mathbf{B} \) for the configuration of magnets in figure 2(a) may be written as

\[
(B_x, B_y, B_z) = (B_{01y} \sin(ky) \text{e}^{-kz}, B_{01y} \sin(ky) \text{e}^{-kz}, B_{01y}[\cos(kx) + \cos(ky)]\text{e}^{-kz})
\]

where \( B_{01y} = B_0\left(\frac{\text{e}^{kt} - 1}{2}\right)\text{e}^{kx} \) and \( z \gg \frac{a}{2\pi} + s + t \).

2.2. Four periodic arrays of square magnets

Using equation (2), we can now write the components of the magnetic field due to the four arrays of square permanent magnetic slabs (figure 1) with external bias field \( \mathbf{B}_1 = B_{1x}\hat{x} + B_{1y}\hat{y} \) as

\[
B_x = B_{01x} \sin(kx) \text{e}^{-kz} + B_{1x}
\]

\[
B_y = B_{01y} \sin(ky) \text{e}^{-kz} + B_{1y}
\]

\[
B_z = [B_{01x} \cos(kx) + B_{01y} \cos(ky)] \text{e}^{-kz}
\]

where \( z \gg \frac{a}{2\pi} + \max(s_1 + t_1, s_2 + t_2, s_3 + t_3, s_4 + t_4) \) and

\[
B_{01x} = -B_{01} - B_{02} + B_{03} + B_{04}, \quad B_{01y} = B_{01} - B_{02} + B_{03} + B_{04}
\]

\[
B_{0i} = \frac{B_0}{2}(\text{e}^{ki} - 1)\text{e}^{ki} \quad (i = 1, 2, 3, 4).
\]

The magnitude of the magnetic field above the magnetic arrays is then

\[
B(x, y, z) = \left[ B_{1x}^2 + B_{1y}^2 + 2[B_{01x} B_{1x} \sin(kx) + B_{01y} B_{1y} \sin(ky)]e^{-kz} + \left[B_{01x}^2 + B_{01y}^2 + 2B_{01y} B_{01x} \cos(kx) \cos(ky)e^{-2kz} \right] \right]^\frac{1}{2}.
\]

This configuration of square magnets gives a 2D periodic lattice of 3D magnetic traps with non-zero potential minima given by

\[
B_{\text{min}} = \frac{|B_0 x B_{1y} - B_0 y B_{1x}|}{(B_{0x}^2 + B_{0y}^2)^\frac{1}{2}}
\]
which are located at
\[
(x_{\min}, y_{\min}) = \left( \left( n_x + \frac{1}{4} \right), \left( n_y + \frac{1}{4} \right) \right) a, \quad \text{where } n_x, n_y = 0, \pm 1, \pm 2, \ldots
\]  
\[
z_{\min} = \frac{a}{2\pi} \ln \left( \left( \frac{B^2_{0x} + B^2_{0y}}{-B_{0x}B_{1x} - B_{0y}B_{1y}} \right) \right).
\]

For a 2D lattice with non-zero magnetic field minima, (6) and (7b) impose the constraints
\[
B_{0x}B_{1y} \neq B_{0y}B_{1x}, \quad B^2_{0x} + B^2_{0y} > -B_{0x}B_{1x} - B_{0y}B_{1y} > 0
\]
on the geometrical parameters and the components of the bias magnetic field, where according to (4a) and (4b) \( B_{0x} \) and \( B_{0y} \) are independent of the magnetization and depend only on the geometrical constants \( a, t_1, t_2, t_3, s_1, s_2, s_3 \) and \( s_4 \).

The magnetic field barrier heights in the three directions are given by
\[
\Delta B_{1y} = \left[ \frac{4B^2_{0x}B^2_{1y} + (B_{0x}B_{1y} + B_{0y}B_{1x})^2}{B^2_{0x} + B^2_{0y}} \right]^{\frac{1}{2}} - B_{\min}, \quad j = 1, 2
\]
\[
\Delta B_{2} = \left( B^2_{1x} + B^2_{1y} \right)^{\frac{1}{2}} - B_{\min}
\]
where \( x_1 = x \) and \( x_2 = y \). Furthermore, the curvatures of the magnetic field at the centre of the traps can be written as
\[
\frac{\partial^2 B}{\partial x^2} = \frac{4\pi^2}{a^2} \frac{B_{0x}B_{1x}}{(B^2_{0x} + B^2_{0y})^2 |B_{0x}B_{1y} - B_{0y}B_{1x}|}, \quad j = 1, 2
\]
\[
\frac{\partial^2 B}{\partial z^2} = \frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2}
\]

Assuming magnetic fields for which the Zeeman splitting is small compared with the hyperfine splitting, i.e., \( B \ll \hbar \Delta v_{\text{hy}} / m \mu_n \) or \( B \ll 4.9 \text{ kG} \) for \(^{87}\text{Rb} \) atoms in hyperfine state \( F = 2 \) with magnetic quantum number \( m_F = \pm 2 \), the magnetic potential energy is \( U = m \mu_n B \). The potential energy minimum and the potential barrier heights in the \( x \)-, \( y \)-, and \( z \)-directions are \( U_{\min} = m \mu_n B_{\min}, \Delta U_{\perp} = m \mu_n \Delta B^z \), \( \Delta U_{\perp} = m \mu_n \Delta B^z \) and \( \Delta U_{\perp} = m \mu_n \Delta B^z \), respectively. Furthermore, the trap frequencies for an atom in hyperfine state \( F = 2 \) with magnetic quantum number \( m_F \) are given by \( \omega_x = \frac{2\pi \gamma}{a} \sqrt{\frac{\mu_n}{m}}, \quad \omega_y = \frac{2\pi \gamma}{a} \sqrt{\frac{\mu_n}{m}} \) and \( \omega_z = \sqrt{\omega_x^2 + \omega_y^2} \), where \( \gamma = \sqrt{m \mu_n \mu_n / m} \). \( \mu_n \) is the Landé g-factor, \( \mu_n \) is the Bohr magneton and \( m \) is the atomic mass.

2.2.1. Symmetrical 2D magnetic lattice. To create a 2D lattice which is symmetrical with respect to \( x \) and \( y \) we impose the constraint \( \Delta B^z = \Delta B^x \) from which we obtain \( B_{1y} = c_0 B_{1x} \), \( c_0 = B_{0x} / B_{0y}, B_{0x} \neq 0 \) and \( B_{0y} \neq 0 \). This is the same as the condition given in [4] for two crossed infinite periodic arrays of magnets with bias fields (figure 2 of [4]); so the analytical expressions for \( B_{\min}, x_{\min}, y_{\min}, z_{\min}, \omega_x, \omega_y, \omega_z, \Delta B^z, \Delta B^x, \Delta B^y, \Delta B^y, \Delta B^t, \Delta B^s \) and \( \Delta B^2 \) are the same as equations (18)–(19c) and (22)–(24b) in [4] but where the definitions of \( B_{0x} \) and \( B_{0y} \) are now given by (4a) and (4b). For a symmetrical 2D magnetic lattice with non-zero potential minima above the surface of the top array, we have constraints which are the same as equation (21) in [4].
2.3. Special cases of magnetic lattice

In this section, we consider special cases of the magnetic lattices introduced in section 2.2.

2.3.1. Single periodic array of rectangular magnets. When $s_3 = s_1 = 0, t_3 = t_1 = t$ and $t_4 = t_2 = 0$ (figure 2(b)), we obtain a 1D magnetic lattice consisting of a single array of rectangular magnets. From (4a) and (4b), we have

$$B_{0x} = 0, \quad B_{0y} = B_0(e^{kt} - 1). \quad (11)$$

Substituting (11) into (6)–(10b), we obtain expressions for $B_{\text{min}}, x_{\text{min}}, y_{\text{min}}, z_{\text{min}}, \Delta B^x, \Delta B^y, \Delta B^z, \frac{\partial^2 B}{\partial x^2}, \frac{\partial^2 B}{\partial y^2}$, and $\frac{\partial^2 B}{\partial z^2}$ which are the same as in [4].

2.3.2. Two crossed arrays of parallel rectangular magnets. When $t_3 = t_1, t_4 = t_2, s_2 = s_1 = 0$ and $s_4 = s_2 = s + t_1$ we obtain a 2D magnetic lattice consisting of two crossed arrays of rectangular magnets (figure 3(a)) as proposed previously in [4]. From (4a) and (4b), we have

$$B_{0x} = B_0(e^{kt_1} - 1), \quad B_{0y} = B_0(e^{kt_2} - 1) \quad (12)$$

Equation (12) is in agreement with the expressions given in [4] where this magnetic lattice has been studied in detail.

2.3.3. Three arrays of square magnets with different thickness. When $s_1 = s_2 = s_3 = t_4 = 0$ we obtain a 2D magnetic lattice consisting of three arrays of square magnets (figure 3(b)). This magnetic lattice was also introduced in [4] but it was studied only numerically. From (4a) and (4b), we have

$$B_{0x} = \frac{B_0}{2}(-e^{kt_1} + e^{kt_2} + e^{kt_3} - 1), \quad B_{0y} = \frac{B_0}{2}(e^{kt_1} - e^{kt_2} + e^{kt_3} - 1). \quad (13)$$

Using equations (6)–(10b) and (13) we can analytically determine the relevant quantities for this system. There is excellent agreement between the analytical results for an infinite lattice and the numerical results [4] in the central region of the lattice.

2.3.4. Chessboard configuration of square magnets. If we take $t_1 = t_2$ and $t_3 = 0$ in figure 3(b), we have a simple chessboard configuration of square magnetic slabs. In this case,
from (13), we obtain
\[ B_{0x} = B_{0y} = 0. \] (14)
Considering (21) in [4], we no longer have non-zero magnetic field minima.

3. Two crossed periodic arrays of parallel rectangular magnets at an arbitrary angle

3.1. General case

Here, we consider a generalization of the configuration of magnets in figure 3(a) in which the upper array of parallel rectangular magnets is at an arbitrary angle \( \theta \) about the \( z \)-axis normal to the surface (figure 4). The magnets in the lower array are parallel to the \( x \)-axis while those in the upper array are parallel to the \( x' \)-axis. The components of the total magnetic field due to both arrays of parallel magnets plus a bias field \( B_1 = B_{1x}\hat{x} + B_{1y}\hat{y} \) are

\[ B_x = -B_{0y'} \sin \theta \sin (k'y') e^{-kz} + B_{1x} \] (15a)
\[ B_y = [B_{0y'} \cos \theta \sin (k'y') + B_{0y} \sin (k'y)] e^{-kz} + B_{1y} \] (15b)
\[ B_z = [B_{0y'} \cos (k'y') + B_{0y} \cos (k'y)] e^{-kz} \] (15c)

where \( B_{0y'} = B_0(e^{kz_1} - 1) e^{kz_1}, B_{0y} = B_0(e^{kz_2} - 1) e^{kz_2}, y' = -x \sin \theta + y \cos \theta, z' = z \) and \( z \gg \frac{a}{2\pi} + s_2 + t_2 \). The magnitude of the magnetic field is then

\[
B(x, y, z) = \left\{ B_{1x}^2 + B_{1y}^2 + 2[B_{0y'}(-B_{1x} \sin \theta + B_{1y} \cos \theta) \sin (k'y') + B_{0y} B_{1y} \sin (k'y)] e^{-kz} + \left( B_{0y'}^2 + B_{0y}^2 - 2B_{0y'} B_{0y} [\cos (k'y') \cos (k'y) + \sin (k'y') \sin (k'y) \cos \theta] \right) e^{-2kz}\right\}^{\frac{1}{2}}.
\] (16)

If \( \theta > 0 \), these arrays of rectangular magnets can give a 2D periodic lattice of magnetic traps with non-zero potential minima given by

\[ B_{\text{min}} = \frac{|-B_{0y'} B_{1x} + B_{0y} B_{1y} \cos \theta + B_{0y'} B_{1y} \sin^2 \theta|}{\left( B_{0y'}^2 + B_{0y}^2 - 2B_{0y'} B_{0y} \cos \theta \right)^{\frac{1}{2}}} \] (17)
The potential minima are located at

\[
(x_{\text{min}}, x'_{\text{min}}) = \left( n_x + \frac{1}{4}, n_{x'} + \frac{1}{4} \right) \frac{a \sin \theta}{1 - \cos \theta}, \quad n_x, n_{x'} = 0, \pm 1, \pm 2, \ldots \tag{18a}
\]

\[
(y_{\text{min}}, y'_{\text{min}}) = \left( n_y + \frac{1}{4}, n_{y'} - \frac{1}{4} \right) a, \quad n_y, n_{y'} = 0, \pm 1, \pm 2, \ldots \tag{18b}
\]

\[
z_{\text{min}} = \frac{a}{2\pi} \ln \left( -\frac{B_{0y}^2 + B_{0y}^2 - 2B_{0y} B_{0y} \cos \theta}{B_{0y} B_{1x} \sin \theta + B_{0y} B_{1y} \cos \theta - B_{0y} B_{1y}} \right) \tag{18c}
\]

The period \( p \) of the magnetic potential minima, which are located along straight lines parallel to the \( x \) - and \( x' \) -axes (defined in figure 4), varies with \( \theta \) and is given by

\[
p = \left| \frac{a \sin \theta}{1 - \cos \theta} \right|. \tag{19}
\]

If \( \theta \) changes between zero and \( \pi/2 \), the period of the magnetic lattice varies between \( 2\pi \) and the minimum period \( a \), which is the period of the magnetic slabs along the \( y \) - and \( y' \) -axes. The constraints for a lattice with non-zero magnetic field minima are

\[
-B_{0y} B_{1x} + B_{0y} B_{1x} \cos \theta + B_{0y} B_{1y} \sin \theta \neq 0 \tag{20a}
\]

\[
B_{0y}^2 + B_{0y}^2 - 2B_{0y} B_{0y} \cos \theta > -B_{0y} B_{1x} \sin \theta + B_{0y} B_{1y} \cos \theta - B_{0y} B_{1y} > 0. \tag{20b}
\]

According to (17)–(18c) and the contour plots of the magnetic field for different values of \( \theta \) (figure 5), we have a 2D array of 3D microtraps with different periods in the \( x \) - and \( x' \) -directions, as \( \theta \) varies, except for the special case \( \theta = 0 \) where the magnetic lattice becomes a 1D lattice of 2D microtraps. Looking at figure 5 more carefully, we find that as \( \theta \) changes the distance between the magnetic traps along the interior bisector of the angle \( \theta \) between the positive \( y \) - and \( y' \) -axes also changes. The magnetic field barrier heights in the \( x \) - , \( x' \) - and \( z \) -directions are given by

\[
\Delta B^x = \left\{ B_{1x} - (-1)^j \frac{B_{0y} \sin \theta (B_{0y} B_{1y} - B_{0y} B_{1y} \cos \theta + B_{0y} B_{1y} \sin \theta)}{B_{0y}^2 + B_{0y}^2 - 2B_{0y} B_{0y} \cos \theta} \right\}^2 + \left\{ B_{1y} + (-1)^j \frac{(B_{0y} + B_{0y} \cos \theta)(B_{0y} B_{1y} - B_{0y} B_{1y} \cos \theta + B_{0y} B_{1y} \sin \theta)}{B_{0y}^2 + B_{0y}^2 - 2B_{0y} B_{0y} \cos \theta} \right\}^2 - B_{\text{min}}, \quad j = 1 \text{ or } 4 \tag{21a}
\]

\[
\Delta B^z = (B_{1x}^2 + B_{1y}^2)^{1/2} - B_{\text{min}} \tag{21b}
\]

where, \( x_1 = x, x_4 = x' \) and by definition we have

\[
\Delta B^x = B(x = x_{\text{max}}, y = y_{\text{min}}, z_{\text{min}}) - B_{\text{min}} \tag{22a}
\]

\[
\Delta B^y = B(x = x'_{\text{max}}, y = y'_{\text{min}}, z_{\text{min}}) - B_{\text{min}} \tag{22b}
\]

\[
\Delta B^z = B(x_{\text{min}}, y_{\text{min}}, z = \infty) - B_{\text{min}} \tag{22c}
\]

where

\[
(x_{\text{max}}, x'_{\text{max}}) = \left( n_x + \frac{3}{4}, n_{x'} + \frac{3}{4} \right) \frac{a \sin \theta}{1 - \cos \theta}, \quad n_x, n_{x'} = 0, \pm 1, \pm 2, \ldots \tag{23}
\]
We also have
\[\Delta B^\prime = B \left( y' = \frac{a}{4}, y = \frac{a}{4}, z_{\min} \right) - B_{\min} \quad (24a)\]
\[\Delta B^{\prime \prime} = B \left( y' = -\frac{3a}{4}, y = \frac{a}{4}, z_{\min} \right) - B_{\min}. \quad (24b)\]

The curvatures are given by

\[
\frac{\partial^2 B}{\partial x^2} = \frac{\pi^2}{a^2 F^2 G} B_{0y} \sin^2 \theta \left[ 2B_{0y} (B_{0y}^2 + B_{0x}^2) (B_{1x}^2 + B_{1y}^2) - B_{0y} B_{0x}^2 (B_{1x}^2 + 7B_{1y}^2) \cos \theta \right.
\]
\[\quad - 2B_{0y} (B_{0y}^2 + B_{0x}^2) (B_{1x}^2 - B_{1y}^2) \cos 2\theta + B_{0x} B_{0y}^2 (B_{1x}^2 - B_{1y}^2) \cos 3\theta \]
\[\left. + 2B_{0y} (3B_{0x}^2 + 2B_{0y}^2) B_{1x} B_{1y} \sin \theta - 4B_{0y} (B_{0y}^2 + B_{0x}^2) B_{1x} B_{1y} \sin 2\theta \right]
\[\quad + 2B_{0y} B_{0x}^2 B_{1x} B_{1y} \sin 3\theta \right] \quad (25a)\]

\[
\frac{\partial^2 B}{\partial y^2} = \frac{\pi^2}{a^2 F^2 G} B_{0y} \sin^2 \theta \left[ B_{0y} \left( -8B_{0y}^2 B_{1y}^2 + B_{0x}^2 (B_{1x}^2 - B_{1y}^2) \cos \theta \right. \right.
\]
\[\quad - B_{0y}^3 (B_{1x}^2 - B_{1y}^2) \cos 3\theta + 4B_{1y} (B_{0y}^2 + B_{0x}^2) B_{1y} \]
\[\quad + B_{0y} B_{1x} (B_{0y}^2 - B_{0x}^2 \cos 2\theta) \sin \theta \left. \right] \right] \quad (25b)\]

\[
\frac{\partial^2 B}{\partial z^2} = \frac{4\pi^2}{a^2 F^2 G} (B_{0y} B_{1y} - B_{0y} B_{1x} \cos \theta + B_{0y}^2 B_{1x} \sin \theta)^2 \quad (25c)\]

where \( F = \left| B_{0x}^2 + B_{0y}^2 - 2B_{0x} B_{0y} \cos \theta \right| \) and \( G = \left| B_{0y} B_{1x} + B_{0y} B_{1y} \cos \theta + B_{0y} B_{1y} \sin \theta \right| \).
3.2. Symmetrical lattice

If we have \( B_{1y} = \beta_0(\theta) B_{1x} \), where \( \beta_0(\theta) = B_{0y}/(B_{0x} \cos \theta + B_{0y}) \), then the barrier heights in the \( x \)- and \( x' \)-directions are equal. Thus, we have a symmetrical 2D lattice of 3D microtraps in the \( x \)- and \( x' \)-directions and the microtraps in the \( x \)- and \( x' \)-directions have the same period. Figure 6 shows the analytical results for an infinite lattice. These results are in excellent agreement with our numerical calculations in the central region of the lattice based on the software package Radia [13] interfaced to a Mathematica code. Here, the quantities of interest are

\[
B_{\min} = \beta_1(\theta)|B_{1x}|
\]

\[
(x_{\min}, x'_{\min}) = \left( n_x + \frac{1}{4}, n_x' + \frac{1}{4} \right) \frac{a \sin \theta}{1 - \cos \theta}, \quad n_x, n_x' = 0, \pm 1, \pm 2, \ldots
\]

\[
z_{\min} = \frac{a}{2\pi} \ln \frac{\beta_2(\theta)B_{0y}}{|B_{1x}|}
\]

\[
(\omega_x = \omega_{x'}, \omega_y = \omega_{y'}, \omega_z) = \frac{2\pi \gamma}{a} \sqrt{|B_{1x}|(\sqrt{\beta_3(\theta)}, \sqrt{\beta_4(\theta)}, \sqrt{\beta_5(\theta)})}
\]

\[
\Delta B^x = \Delta B^{x'} = \beta_6(\theta)|B_{1x}|, \quad \Delta B^z = \beta_7(\theta)|B_{1x}|
\]

where \( \gamma = \sqrt{m \gamma_p \mu_p / m} \) and \( \beta_1(\theta), \ldots, \beta_7(\theta) \) are dimensionless parameters which depend on \( c_0 = B_{0y}/B_{0x} \) and \( \theta \). Using the definitions \( c_1 = 1 - c_0^2, c_2 = 1 + c_0^2, h_1(\theta) = 1 + c_0 \cos \theta \) and \( h_2(\theta) = 1 + c_0^2 - 2c_0 \cos \theta \), we have

\[
\beta_1(\theta) = \frac{|c_1|}{|h_1(\theta)||h_2(\theta)|^2}
\]

\[
\beta_2(\theta) = \frac{h_1(\theta)h_2(\theta)}{2c_0 \sin \theta}
\]

\[
\beta_3(\theta) = \frac{2c_0 c_2 \sin^4 \theta}{|c_1 h_1(\theta)||h_2(\theta)|^2}
\]

\[
\beta_4(\theta) = \frac{c_0^2 [-8c_0 \cos \theta + c_2 (3 + \cos 2\theta)] \sin^2 \theta}{|c_1 h_1(\theta)||h_2(\theta)|^2}
\]

\[
\beta_5(\theta) = \frac{4c_0^2 \sin^2 \theta}{|c_1 h_1(\theta)||h_2(\theta)|^2}
\]
the central region of the lattice and $B_{1z} = -0.69(\mu G)$ was added to compensate for edge effects.

| Parameter                  | Definition                                      | Numerical results | Analytical results |
|----------------------------|-------------------------------------------------|-------------------|-------------------|
| $U_{\text{min}}/k_B$ (\mu K) | Potential energy minimum                         | 180               | 180               |
| $\Delta U_x/k_B$ (\mu K)    | Potential barrier height along $x$               | 559               | 559               |
| $\Delta U_y/k_B$ (\mu K)    | Potential barrier height along $y'\cdot$        | 558               | 559               |
| $\Delta U_z/k_B$ (\mu K)    | Potential barrier height along $z$               | 108               | 108               |
| $\omega_x/2\pi$ (kHz)       | Trap frequency along $x$                        | 131               | 131               |
| $\omega_y/2\pi$ (kHz)       | Trap frequency along $y$                        | 99                | 99                |
| $\omega_z/2\pi$ (kHz)       | Trap frequency along $z$                        | 164               | 164               |

\[
\beta_0(\theta) = \frac{\left[c_2^2 - 2c_1c_2^2 \cos \theta - 4c_1^2c_2 \cos 2\theta\right]^2}{|b_1(\theta)b_2(\theta)|} - \beta_1(\theta) \quad (27f)
\]

\[
\beta_2(\theta) = [1 + \beta_0(\theta)^2]^{1/2} - \beta_1(\theta). \quad (27g)
\]

Subject to $B_{1y} = \beta_0(\theta)B_{1z}$, the conditions for non-zero magnetic potential minima are

\[B_{0y} \neq B_{0y'}, \quad \beta_2(\theta)B_{0y} > |B_{1y}| > 0. \quad (28)\]

According to (26d), (26e) and (27e)–(27g), for a constant $c_0$, it is possible to change the trap frequencies and the magnetic field potential barriers by varying $\theta$ or the bias magnetic field $B_1$.

Table 1 shows that there is excellent agreement between the analytical results for an infinite lattice and the numerical results obtained in the central region of the lattice.

4. Three periodic arrays of parallel rectangular magnets

Here, we consider a special configuration involving three periodic arrays of parallel rectangular magnets. The bottom array (array 1) and the middle array (array 2) are at an angle $\theta$ with respect to each other, similar to the crossed arrays shown in figure 3, while the rectangular magnets in the top array (array 3) are parallel to those of the middle array and remain parallel as $\theta$ changes. The period $a_1$ is twice the periods $a_1$ and $a_2$. To obtain the components of the magnetic field due to these three arrays of magnets plus bias field $B_1 = B_{1x}\hat{x} + B_{1y}\hat{y}$, we add the magnetic field due to array 3 to that of arrays 1 and 2 (15a)–(15c):

\[B_x = -B_{0y'} \sin \theta \sin(ky') e^{-kz} - B_{0y} \sin \theta \sin(ky'/2) e^{-kz/2} + B_{1x} \quad (29a)\]

\[B_y = [B_{0y'} \cos \theta \sin(ky') + B_{0y} \sin(ky)] e^{-kz} + [B_{0y'} \cos \theta \sin(ky'/2)] e^{-kz/2} + B_{1y} \quad (29b)\]

\[B_z = [B_{0y'} \cos(ky') + B_{0y} \cos(ky)] e^{-kz} + B_{0y'} \cos(ky'/2) e^{-kz/2} \quad (29c)\]

where $z \gg a_1 + s_1 + t_2$ and $B_{0y} = B_0(e^{kz_1} - 1)e^{kz_1}, B_{0y'} = B_0(e^{kz_1} - 1)e^{kz_2}$ and $B_{0y''} = B_0(e^{kz_1} - 1)e^{kz_2}$. The magnetic field is shown for $\theta = \pi/6$ and $\theta = \pi/3$ in figures 6(a)–(c) and (d)–(f), respectively. The period $a_1$ is 1 \mu m, which is twice the periods $a_1$ and $a_2$. The thicknesses of the layers are $t_1 = 50$ nm, $t_2 = 40$ nm and $t_3 = 5$ nm and the lower surfaces of the layers are at distances $s_1 = 0, s_2 = 60$ nm and $s_3 = 105$ nm from the plane $z = 0$. The magnetization is as before and the components of the bias magnetic field
A class of permanent magnetic lattices for ultracold atoms

Figure 7. Magnetic field produced by three arrays of parallel rectangular magnets, as described in section 4. The parallel magnets in the middle and the top arrays have an angle with respect to the parallel magnets in the bottom array of \( \theta = \pi/6 \) in (a)–(c) and \( \theta = \pi/3 \) in (d)–(f). The curves show the analytical magnetic field along a line \((y = y_{\text{min}}, z = z_{\text{min}})\) parallel to the \(x\)-axis (a and d), along a line \((y' = y'_{\text{min}}, z = z_{\text{min}})\) parallel to the \(x'\)-axis (b and e) and along a line \((x = x_{\text{min}}, y = y_{\text{min}})\) parallel to the \(z\)-axis (c and f).

are \( B_{1x} = -4.08G, B_{1y} = -1.39G \) and \( B_{1z} = 0 \). According to figure 7, we can change the separation and also the barrier height between adjacent microtraps by varying \( \theta \). Figure 7 also shows the possibility of creating arrays of double well potentials.

5. Discussion and summary

We have introduced a new class of permanent magnetic lattices for ultracold atoms and quantum degenerate gases, including Bose–Einstein condensates and ultracold Fermi gases. The potential barriers between the adjacent microtraps can be altered by varying the bias magnetic field, as indicated by the relevant equations. We have also shown that for a configuration of two crossed arrays of rectangular magnets with variable angle \( \theta \) between the arrays, the magnetic lattice transforms from a 1D to a 2D lattice as \( \theta \) is changed from 0 to \( \pi/2 \). By varying the angle \( \theta \), it is also possible to continuously vary the depth of the traps and to change the period of the magnetic lattice. It is also possible to have two different periods \( a_y = a_1 \) and \( a'_y = a_2 \) in the \( y \)- and \( y' \)-directions, respectively, to produce a more versatile magnetic lattice.

The class of permanent magnetic lattices proposed here should be suitable for trapping and manipulating small clouds of ultracold atoms and quantum degenerate gases. Since only atoms in low field-seeking states are trapped in a magnetic lattice, it should be possible to perform evaporative cooling \textit{in situ} in the lattice thereby allowing the study of very low temperature phenomena in periodic lattices.

By using lattice periods of a few micrometres and controlling the barrier height between the microtraps it may be possible to use these configurations of magnetic lattice to study quantum tunnelling and to realize the BEC superfluid to Mott insulator quantum phase transition in a magnetic lattice.

Use of the BEC to Mott insulator transition for ultracold atoms located in 2D magnetic lattice sites may allow the preparation of single qubit atom on each site, which is important for scalable quantum information processing. An interesting possibility would be to use the
two crossed arrays of rectangular magnets with variable angle $\theta$ to study quantum tunnelling effects. This configuration may also be useful in principle for loading ultracold atoms into a 2D magnetic lattice, by initially loading the atoms into the 1D lattice and then changing the angle from $\theta = 0$ to $\theta = \pi/2$.

The experimental challenges involved in fabricating the proposed configurations of magnetic lattice include the ability to fabricate periodic arrays with sufficiently smooth magnetic potentials and equivalent magnetic lattice sites and to minimize the effects of the interaction of the ultracold atoms with the surface [14–20] in order to preserve quantum coherence of the atoms. For magnetic lattices with a period of a few micrometres, the potential minima are located a few micrometres from the surface of the magnetic lattice where the Casimir–Polder force [20] can be significant [18], leading to an attractive component that lowers the height of the barrier. It should be possible to minimize losses due to thermally induced spin flips caused by interaction with the ambient temperature surface [16–19] by using magnetic films whose thickness ($\leq 0.4 \mu m$ for the magnetic lattices considered here) is much less than the skin depth and by use of suitable substrates with low electrical conductivity [14]. The construction of a configuration involving two crossed periodic arrays of micrometre-thick rectangular magnets, with a variable angle $\theta$ between two arrays separated by about a micrometre, will also present considerable experimental challenges.

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