Spatiotemporal Characterization of Nonlinear Interactions between Selectively Excited Radially Symmetric Modes of a Few-Mode Fiber

Sai Kanth Dacha\textsuperscript{1,2,*}, Thomas E. Murphy\textsuperscript{1,3}

\textsuperscript{1}Institute for Research in Electronics and Applied Physics (IREAP), University of Maryland, College Park, MD 20740, USA
\textsuperscript{2}Department of Physics, University of Maryland, College Park, MD 20740, USA
\textsuperscript{3}Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20740, USA
\textsuperscript{*sdacha@umd.edu}

Abstract: In order to build a more complete understanding of recently observed spatiotemporal nonlinear phenomena in few- and multi-mode optical fibers (FMFs/MMFs), it is of interest to not only measure the constituent fundamental nonlinear interactions between the individual fiber modes, but also to resolve the output in space and time. FMFs are ideal for exciting a small number of modes, and are chosen for this study. In this Article, we report complete spatiotemporal measurements of two nonlinearly interacting LP\textsubscript{0n} modes of a step-index FMF. Modes are selectively excited using a novel implementation of a phase mask, namely end-face patterning using ion-beam milling. The output is resolved in space and time by raster-scanning, in the near field, a scanning optical microscope tip that is coupled to high-speed detection.

1. Introduction

The rapid growth in demand for long-haul fiber-optic communication systems since the 1970s drove an active interest in understanding nonlinear optics in single-mode fibers (SMFs), as nonlinear impairments had to be understood in order to successfully transmit information over long lengths of fiber. Despite being developed before the SMF, and despite the relative ease in manufacturing them, multimode fibers (MMFs), on the other hand, have largely remained under the radar. This is owing largely to significant intermodal dispersion leading to pulse broadening. Present-day fiber optic communication systems are fast approaching the Shannon capacity limit for single-mode fibers in the face of ever-increasing capacity demands. Space-division multiplexing (SDM) is the next frontier to improve network capacity further [1]. Implementing SDM systems in FMFs and MMFs requires a good understanding of the nonlinear effects over long lengths of fiber [2]. This interest has been one of the driving forces for the up and coming research area that multimode fiber nonlinear optics is. Over the past few years, the availability of high-power transform-limited pulsed lasers has made it possible to probe the nonlinear domain of operation in FMFs and MMFs like never before, by making it possible to achieve high nonlinear phase shifts without the need for long lengths of fiber (which would inevitably be limited by intermodal dispersion, especially for short pulses).

Multimode nonlinear optics is inherently more complex than single-mode nonlinear optics thanks to the availability of an additional degree of freedom: space. The vast number (~100s) of spatial modes supported in commercially available MMFs make possible a wide variety and number of intramodal and intermodal nonlinear interactions. This in opens the door for a plethora of spatiotemporal nonlinear phenomena, some of which have been uncovered in recent experiments, including Kerr-induced beam cleanup in graded-index MMFs [3] [4], multimode solitons [5], geometric parametric instability [6], multi-octave spanning supercontinuum generation [7] and spatiotemporal modulation instability [8]. Such phenomena are of great interest not only from a fundamental science perspective, but also in practical applications ranging from high-power beam
Notwithstanding the importance of the aforementioned seminal results, in this Article, we seek to address two key shortcomings in the way multimode nonlinearity is measured and modeled, respectively. The first relates to experimental measurement techniques. Traditional measurement techniques, including spectral and temporal measurements of the entire beam and spatial imaging using CCD/CMOS cameras, all average over two out the three measurement axes (space, time and spectrum). In the most common type of measurements (spectral measurements coupled with spatial imaging of the output), both the spectrum analyzer as well as the CCD/CMOS camera average over many pulses. As a result, many interesting dynamics that happen within one pulse duration are missed, and the spatiotemporal nature of multimode nonlinearity is not efficiently captured. We tackle this problem by introducing here a novel method for measuring the output MMFs and FMFs in both space and time. Specifically, raster scan a near-field scanning electron microscope (NSOM) tip in the near-field of the MMF/FMF output end-face, while collecting a time trace at each spatial location. By stitching together the measured spatially-resolved time traces, we demonstrate the temporal evolution of the instantaneous intensity profile within one pulse duration, with a temporal resolution of 40 ps and spatial resolution of 300 nm.

Spatiotemporal measurements of multimode fiber nonlinearity have been reported before. In [10], Krupa et al. report camera images, time-traces and spectra of the output for the phenomenon of supercontinuum generation in graded-index (GRIN) MMFs, although both the temporal as well as spectral measurements were spatially averaged across the fiber core area. In [11], Krupa et al. show the spatiotemporal nature of Kerr beam cleanup by measuring the time-traces of light at different spatial locations on the output end-face of a graded-index MMF. This was done by translating a photodiode with a small active area placed in the path of the collimated output beam. In this Article, we attempt to go one step further and report complete spatiotemporal measurements of the entire output beam, for the first time to our knowledge.

The second shortcoming we seek to address in this Article relates to the understanding of the nature of nonlinearity itself. Currently, there exist two complementary models of nonlinear propagation of optical pulses in MMFs: the (3+1)D nonlinear Schrödinger equation for the complex field envelope (also known as the Gross-Pitaevskii equation), and the generalized multimode nonlinear Schrödinger equations (GMM-NLSE) [12]. The nonlinear wave equation is most efficient for numerically simulating the nonlinear propagation of pulses when a large (~100s) number of modes are excited [6]. It describes nonlinearity as a spatially local effect, similar to case of bulk media, thereby sidestepping the modal picture. The GMM-NLSE on the other hand is best suited for studying and numerically simulating pulse propagation when the number of excited modes is small (~10s). The GMM-NLSE treats optical nonlinearity, notably different from the case of bulk media, as acting at the modal level (in other word, spatially non-local). The validity of the latter picture of nonlinearity is of fundamental importance not only in establishing a more complete understanding of multimode nonlinear effects broadly, but also specifically in FMF-based SDM applications where the number of co-propagating modes is small (~10s). As a result, in order to study this problem, we choose a step-index FMF as our platform. We further restrict the already small number of allowed modes in FMFs by etching a phase mask directly on the FMF input end-face by means of focused ion-beam (FIB) milling. The phase mask restricts the number of excited modes to the smallest non-trivial number possible: two. We then measure the output of this system in space and time simultaneously using the technique described above and compare our results with the predictions of the GMM-NLSE and its modal treatment of nonlinearity.
2. Selective Mode Excitation

Under the weakly guiding approximation, the linearly polarized (LP) spatial modes of a step-index FMF are given by Bessel functions of the first kind:

\[
\psi_{lm}(r, \phi) = N_{lm} J_l \left( \frac{U_{lm} r}{a} \right) \frac{\cos(q_0 \phi)}{\sin(q_0 \phi)}
\]

where \( r \) and \( \phi \) are the radial and azimuthal coordinates, \( N_{lm} \) is the normalization constant, \( l \) and \( m \) are the radial and azimuthal mode orders, \( a \) is the core radius, and \( U_{lm} = a(k_0^2 n_1^2 - \beta_{lm}^2)^{1/2} \) with \( k_0 \), \( n_1 \) and \( \beta \) denoting the vacuum wave number, core refractive index and the modal propagation constant. The normalization constant \( N_{lm} \) is chosen such that the modal profiles obey the orthonormality relation \( \int \psi_p \psi_q^* r dr d\phi = \delta_{pq} \). For the work that we report in this paper, we are only interested in radially-symmetric input excitations, and therefore we focus our attention on the radially-symmetric \( LP_{0m} \) modes alone. The fiber that we work with is a step-index FMF with a core diameter \( 2a = 20 \mu m \) and numerical aperture \( NA = 0.14 \). For such a fiber, at our laser wavelength \( \lambda_0 = 1064 \text{ nm} \), three radially symmetric modes exist: \( LP_{01}, LP_{02} \) and \( LP_{03} \).

A Gaussian laser beam focused onto the input end-face of the FMF can be broken down into the three radially symmetric guided modes:

\[
E_x(r, \phi, z = 0, t) = \sum_{p=1}^{3} \zeta \psi_p(r, \phi) A_p(z = 0, t)
\]

where in (2), \( \zeta = (\frac{2\pi c \epsilon_0}{\lambda_0})^{-1/2} \), \( c \) and \( \epsilon_0 \) are speed of light in vacuum and vacuum permittivity respectively, and \( A_p \) is the modal amplitude. Note that we have switched from the \( \psi_{lm} \) notation to the \( \psi_p \) notation given that the modes of interest all have the radial mode order \( l = 0 \). By exploiting the orthonormality \( \int \psi_p \psi_q^* r dr d\phi = \delta_{pq} \) of the spatial modes, we then write down the expressions for the modal amplitudes in terms of the electric field of the input beam:

\[
A_p(z = 0, t) = \frac{1}{\zeta} \int E_x(r, \phi, z = 0, t) \psi_p^*(r, \phi) r dr d\phi
\]

The modal amplitudes as in (3) are such that \( |A_p|^2 \) is the power in mode \( p \) in Watts. We then define the modal coupling efficiency here as the fraction of input power in mode \( p \): \( \eta_p = |A_p|^2 / P_{in} \). In Figure 1, we plot numerically calculated modal coupling efficiencies (\( \eta_p \)) for the three modes as a function of input beam radius. Two key inferences that can be drawn from the plot are as follows: i) it is not possible to excite a higher order \( LP_{0m} \) mode selectively, and more importantly ii) it is not possible to selectively excite a desired combination of two radially symmetric modes with comparable powers without significant power in the third mode. In order to study the nonlinear interactions between individual modes, it is helpful to be able to selectively excite two modes with comparable powers in them.

Selective excitation of modes has been typically achieved using spatial light modulators (SLMs) [13] [14]. However, SLM based systems can be bulky, difficult to align, and are prone to damage under high fluence illumination that is required to observe nonlinear optical effects. Selective excitation of OAM modes has also been achieved using fork gratings on the fiber end-face [15]. We have employed a method based on the discussion of thin-film deposition in [16], although instead of depositing thin films onto the FMF input end-face, we directly etch a phase mask onto the input end-face of the FMF using Focused Ion Beam (FIB) milling. An accelerated beam of \( \text{Ga}^+ \) ions is focused onto the FMF input end-face to a spot size of about \( 90 \text{ nm} \). (The FMF end-face is first coated with a \( 20 \text{ nm} \) layer of Au:Pd alloy in order to make the sample conducting in order to help mitigate charging effects during the milling process.)
The ion beam is steered so as to mill away a disc pattern of radius \( r_m \) and depth \( d_m \) as shown in Figure 2. The removal of SiO\(_2\) in the disc region creates a phase difference between the light that passes through the disc and the light that doesn’t pass through it, thereby creating a phase mask.

In the presence of a phase mask described by the function \( \Theta(r, \phi) \), the modal amplitudes are calculated by rewriting (3) as follows:

\[
A_p(z = 0, t) = \frac{1}{\xi} \iint e^{i\Theta(r, \phi)} E_x(r, \phi, z = 0, t)\psi_p^*(r, \phi) r dr d\phi
\]  

\( (4) \)

Figure 3 shows the result of a numerical sweep over the parameter space of \( (d_m, r_m) \), for a
fixed input beam radius of 8.4 \( \mu m \). The radius of the mask is varied from 0 to 10 \( \mu m \), while the depth of mask is varied from 0 to 1 \( \mu m \) (i.e. roughly one wavelength). The coupling efficiency \( \eta_p \) to each of the modes at each coordinate on the parameter grid is calculated and plotted as a color map. Regions of our interest on this color map include those that have negligible power in one mode and comparable power in the other two. The chosen operating point is marked by * (in green) in Figure 3, at which the \( LP_{03} \) color map shows very low coupling efficiency, while \( LP_{01} \) and \( LP_{02} \) color maps show comparable efficiencies. The calculated modal coupling efficiencies at * are \( \eta_1 = 0.47 \), \( \eta_2 = 0.31 \) and \( \eta_3 = 0.0008 \) for \( LP_{01} \), \( LP_{02} \) and \( LP_{03} \) modes respectively. The final result of this FIB milling process is shown in the scanning electron microscopy (SEM) image of the FMF input end-face shown in Figure 4.

Fig. 3. Numerical calculation of modal coupling efficiencies as a function of phase mask radius (\( r_m \)) and depth (\( d_m \)). The chosen operating point marked by * in green is \((r_m, d_m) = (5.28 \mu m, 0.53 \mu m)\).

Fig. 4. Scanning Electron Microscopy (SEM) image of FMF input end-face after FIB milling process: The darker disc at the center indicates the area where milling was performed.

3. Nonlinear Optics in FMFs: Theory and Modeling

In SMFs, propagation of a pulse through the fiber in both linear and nonlinear regimes is described by the nonlinear Schrödinger equation (NLSE). In MMFs and FMFs, because of the existence of multiple spatial modes, pulse propagation is instead described by a system of coupled nonlinear partial differential equations. The generalized multimode nonlinear Schrödinger equations (GMM-NLSE) were first described in 2008 in [12]. Upon neglecting the Raman and shock terms
and orders of dispersion above 2, the GMM-NLSE can be written down, in the reference frame of the fundamental mode, as follows:

\[
\frac{\partial A_p(z,t)}{\partial z} = i(\beta_0^{(p)} - \beta_1^{(p)})A_p - (\beta_1^{(p)} - \beta_1^{(1)}) \frac{\partial A_p}{\partial t} - i \frac{\beta_2^{(p)}}{2} \frac{\partial^2 A_p}{\partial t^2} + i \sum_{l,m,n} \gamma_{lmp} A_l A_m A_n^* \tag{5}
\]

where \(A_p(z,t)\) is the temporal pulse envelope corresponding to mode \(p\), \(\beta_n^{(p)}\) is the \(n^{th}\) term in the Taylor expansion of the wavenumber for mode \(p\), \(\gamma_{lmp} = \frac{n_{lmp}}{c A_{lmp}^{eff}}\) is the nonlinear coefficient for the \(A_l A_m A_n^*\) interaction, \(n_2\) is the nonlinear intensity-dependent refractive index, and \(A_{lmp}^{eff}\) is the effective area of interaction for modes \(l, m, n\) and \(p\). \(A_{lmp}^{eff}\) is given by the following equation, where and \(dS = rdrd\phi\) is the area element:

\[
A_{lmp}^{eff} = \sqrt{\int |\psi|^2 dS \int |\psi_m|^2 dS \int |\psi_n|^2 dS \int |\psi_p|^2 dS} / \int \psi^\dagger \psi_m \psi_n \psi_p dS \tag{6}
\]

The setup for our experiments consists of an FMF (20 \(\mu\)m core diameter; step-index) that is 1.24 m long, and we work with laser pulses that are 720 ps wide. Because of the broad pulse that we use, it becomes possible to neglect chromatic dispersion and simply set \(\beta_2 = 0\). Further, the two excited \(LP_{100}\) modes have different propagation constants. The beat length corresponding to this difference 1/|\(\beta_0^{(1)} - \beta_0^{(2)}| = 0.557 \text{ mm} \ll L = 1.24 \text{ m}, the FMF length. As a result, the coherent terms in the right-hand-side of (5) average out to zero [17]. The simplified GMM-NLSE for two co-propagating \(LP_{100}\) modes in a step-index FMF can therefore be written as:

\[
\frac{\partial A_1(z,t)}{\partial z} = i(\gamma_{1111}|A_1|^2 + 2\gamma_{1122}|A_2|^2)A_1 \tag{7}
\]

and

\[
\frac{\partial A_2(z,t)}{\partial z} = i(\beta_0^{(2)} - \beta_0^{(1)})A_2 + (\beta_1^{(2)} - \beta_1^{(1)}) \frac{\partial A_2}{\partial t} + i(\gamma_{2222}|A_2|^2 + 2\gamma_{2211}|A_1|^2)A_2 \tag{8}
\]

where the nonlinear terms on the right-hand-side represent self-phase modulation (SPM) and cross-phase modulation (XPM) of the two modes. For ease of theoretical analysis, we make yet another assumption here: the two modes propagate with the same group velocity (i.e. \(\beta_1^{(1)} = \beta_1^{(2)}\)). Equation 7 remains the same, while 8 is modified as:

\[
\frac{\partial A_2(z,t)}{\partial z} = i(\beta_0^{(2)} - \beta_0^{(1)})A_2 + i(\gamma_{2222}|A_2|^2 + 2\gamma_{2211}|A_1|^2)A_2 \tag{9}
\]

Because the coherent terms on the right-hand-side averaged to zero, the equations do not allow for permanent intermodal power transfer. As a result, \(|A_1|^2\) and \(|A_2|^2\) remain constant as a function of \(z\). The analytical solutions to (7) and (9) can therefore be written down as \(A_1(z,t) = A_1(0,t) e^{\Gamma_1(t)}\) and \(A_2(z,t) = A_2(0,t) e^{i(\beta_0^{(2)} - \beta_0^{(1)})z + \beta_1^{(2)} e^{i\Gamma_2(t)}}\) where \(\Gamma_1(t) = (\gamma_{1111}|A_1|^2 + 2\gamma_{1122}|A_2|^2)\) and \(\Gamma_2(t) = (\gamma_{2222}|A_2|^2 + 2\gamma_{2211}|A_1|^2)\)

What this means is that the two \(LP_{100}\) modes propagate through the fiber to acquire different nonlinear chirps arising from the differences in \(\gamma_{lmp}\) values and from the differences in powers in the two modes. The net measurable intensity at the FMF output, as a function of space and time, is then given by:

\[
I(x, y, z = L, t) = |\psi_1(x, y)A_1(z = L, t) + \psi_2(x, y)A_2(z = L, t)|^2 \tag{10}
\]
If we launched a Gaussian pulse into two $LP_{0n}$ modes, (10) predicts that at some location $(x,y)$ on the FMF output end-face, we would observe the interference of two Gaussian pulses that have acquired different nonlinear chirps, i.e. a non-Gaussian temporal pulse shape. By the same token, (10) also predicts that the temporal dependence of instantaneous intensity as the pulse rises and falls. These broad predictions of the analytical model (based on certain simplifying assumptions) form a good starting point with which to compare the results of our numerical simulations and experimental results. In the following sections, we present our experimental setup, measurements, and compare them numerical Split-Step Fourier Method (SSFM) simulations of (5).

4. Experiment

The experimental setup consists of a YAG microchip laser ($\lambda_0 = 1064$ nm) that gives out 720 ps pulses at a 1 kHz repetition rate. The laser pulses have an energy of about 135 $\mu$J, and the energy of the pulses going into the laser is controlled by a series combination of a half-wave plate (HWP) followed by a polarizing beam splitter (PBS) such that the input peak power is 15 kW. Using a plano-convex lens of focal length ($f = 25.4$ mm), the laser beam is focused down to a spot with radius of 8.4 $\mu$m on the patterned input end-face of an FMF. The patterning has been done in order to selectively excite the $LP_{01}$ and $LP_{02}$ modes with comparable amplitudes, as described in Section 2.

The FMF is a step-index fiber that has a core diameter of 20 $\mu$m (numerical aperture $NA = 0.14$) and a length of 1.24 m. At the output end-face of the FMF, we employ a near-field scanning optical microscope (NSOM) tip that is brought in close proximity ($\ll 1$ $\mu$m) to the FMF end-face. The NSOM tip tapers into a single-mode fiber segment that is connected to a high-speed detector consisting of a 10 GHz photo-receiver and a high-speed real-time oscilloscope.

The NSOM tip has an aperture of 250 $\pm$ 50 nm, and is mounted on a piezo-controlled translation stage that can be used to raster scan the tip across the FMF output end-face. By using this setup, we record the temporal output along a grid of spatial locations on the FMF output end-face at a resolution of a few 100s of nm. Further, having recorded the output time-traces at each pixel on the output spatial grid, we reconstruct a temporal evolution of the 2-D intensity profile exiting the FMF end-face. In other words, we resolve the output in space and time simultaneously, thereby capturing the spatiotemporal nature of intermodal nonlinear interactions.

Because this technique involves synchronous measurements of the time-domain waveform at multiple pixels on the FMF output end-face, it is imperative that the laser produce stable, repeatable pulses. As a result, prior to taking any measurements, a statistical analysis was performed on the laser output power to ensure reasonable pulse-to-pulse repeatability. Further, a clean trigger signal for the oscilloscope is key to be able to make repeatable and reliable measurements. Such a trigger signal is extracted from the laser beam by steering a 1% reflection of the beam onto a second high-speed PIN photodiode. We achieve a spatial resolution of 300 nm over the core diameter of 20 $\mu$m and a temporal resolution of 40 ps.

5. Results and Discussion

Using the scanning NSOM-tip method described above, the time-domain output is recorded at various spatial locations on the FMF output end-face. Figure 6(a) shows, for reference, the radial profiles of the two selectively excited modes: $LP_{01}$ and $LP_{02}$. Figures 6(b) through 6(d) show the time-domain output recorded at 3 selected spatial locations: $r = 0$ (on-axis), $r = 4.4$ $\mu$m, and $r = 7.2$ $\mu$m respectively. The radial symmetry of the modes that we chose to excite allows us to,
As our simplified theoretical model (10) above predicted, we observe interference fringes in the time-domain arising from the overlap, in time and space, of two modes that have acquired different nonlinear chirps. Further, at the three selected values of \( r \), the two modes have different magnitudes and phases, leading to a different time-domain pattern at each value of \( r \). As the plots in Figure 6 show, there is reasonable agreement between the experimentally observed output and the results of numerically simulating (5). The observation of significant temporal fringes at \( r = 4.4 \mu m \), where neither of the two modal intensities have their maxima, supports the validity of the modal picture of nonlinearity.

The temporal data collected at various \( r \) values at various angles of azimuth are then used to reconstruct the 2D spatial intensity pattern at the FMF output for time-instances within one pulse duration. In other words, the spatially-resolved temporal data can be used to reconstruct what one would see at the output with a ps-scale ultrafast video camera, except without the need for such a camera.

Figure 7 shows the output of such a reconstruction. The Figure shows the input Gaussian pulse for reference. Reconstruction of the output intensity profile at three time instances (\( t = -0.66 \) ns, \( t = 0 \) and \( t = 0.86 \) ns) brings out a very interesting pattern. At both (\( t = -0.66 \) ns as well as (\( t = 0.86 \) ns, when the instantaneous input power and therefore the nonlinearity is low, the output intensity pattern looks like a Gaussian that is centered on-axis. However, as the pulse reaches its peak at (\( t = 0 \)), the two modes acquire a nonlinear phase difference. Using the aforementioned expressions for the nonlinear time-dependent nonlinear phases the two modes acquire (\( \Gamma_1(t) \) and \( \Gamma_2(t) \)), the peak nonlinear phase difference between the two modes comes out to be roughly one \( \pi \), for our experimental parameters. As a result, at the pulse peak (\( t = 0 \)), the two modes interfere with an additional \( \pi \) phase difference. This converts on-axis peak to an on-axis local minimum, and the Gaussian-like intensity pattern to a donut shape. Further, the results of numerically simulating (5) are shown as insets in Figure 7. One can therefore see that the experimental measurements are in close agreement with the predictions of the
simplified theoretical model and also with numerical simulations. (Note: See supplementary videos 1 and 2 for illustrations based on experimental data and numerical simulations respectively.)

5.1. Validity of Our Model

Our model, based on the GMM-NLSE, explains these findings as follows: as the pulse rises towards its peak, the instantaneous phase difference between the two modes varies with time as they acquire different instantaneous nonlinear phase shifts via SPM and XPM. As a result, the instantaneous spatial pattern formed by their interference also varies with time. Measured at one spatial location, this manifests as fringes in the time domain. The time-domain interference fringes and the time-varying output intensity pattern are two sides of the same coin, and are caused simply by a change in relative phase (with time) between the two modes at the output end-face of the FMF.

A simple test to verify the validity of this model would be to find a different mechanism – different from optical nonlinearity – to vary the relative phase between the two modes as they reach the output end-face of the FMF, and compare the output intensity patterns from such a test to what our spatiotemporal measurements show. We achieved this by heating a section of the fiber on a hot plate. The rise in temperature causes two effects: i) thermal expansion of the length of the fiber, and ii) change in refractive index of the fiber core, which causes a change in modal propagation constants. However, the thermal effect on the modal propagation constants would be the same for both the modes. Since the relevant quantity for interference of the two modes is the difference in their propagation constants, the second of the effects listed above can be neglected for the purposes of these measurements.
In the absence of any nonlinearity, the difference in linear phases acquired by the two modes at the output is given by $\theta_{12} = \delta \beta L$. In the presence of heat, $L$ changes slightly, resulting in a change in the phase difference with which the two modes overlap at the output, thereby resulting in a temperature-dependence of the interference pattern. This change in the output intensity pattern is easily observed on a CMOS/CCD camera. Figure 8 shows the FMF output recorded on a CMOS camera at 3 different temperatures (corresponding to 3 different values of relative phase between the two modes) at low input power. As one can see, the output intensity profile varies from a Gaussian-like shape to an annulus and back; exactly as in our spatiotemporal measurements of nonlinearity. (Note that when the fiber is stretched by heating, it introduces a relative phase difference between the two modes that remains constant through the duration of a pulse. Additionally, as the input power is kept low for these measurements to keep out any nonlinear effects, there is no time-dependent phase introduced. As a result, it is sufficient to use a CMOS camera (which averages over many pulses) to capture the output intensity profile.)

The agreement between the predictions of our simplified theoretical model, numerical simulations, experimental measurements as well as these thermal measurements indicates not only the validity of the novel measurement technique that we introduced that we introduced in this Article, but also reaffirms the validity of the modal picture of nonlinearity.

6. Conclusion

In order to probe the nonlinear interaction between two individual spatial modes, it is desirable to preferentially excite only the modes of interest. We have achieved this using a novel implementation of a phase mask, that involves etching the mask directly onto the fiber input end-face by means of FIB milling. While hard-writing a mask onto the input end-face has the disadvantage of
Fig. 8. FMF output recorded on a CMOS camera at low input power at 3 different temperatures. As the temperature of a 20 cm long FMF section is increased, the length of the core increases on the micron-scale due to thermal expansion, leading to a slightly different modal overlap at each temperature. As the temperature is swept from 50°C to 150°C, the output intensity profile switches between a Gaussian-like shape and an annulus, just as it did within one pulse duration in the presence of nonlinearity.

being less flexible as compared to an SLM setup, it has some key advantages such as compactness and ease of integration into chip-scale photonic circuits. Such a mask is also not prone to damage under the influence of high laser power, and is a power efficient way of exciting a desired mode combination.

Having excited the desired combination of modes, we measured the output in both space and time. Our spatiotemporal measurement technique that employs a raster-scanned NSOM-tip brings out the rich spatiotemporal nonlinear dynamics that happen within the duration of a single pulse that are not possible to observe using traditional CCD/CMOS cameras and optical spectrum analyzers. For the case of two LP_{01} modes excited in a step-index FMF, our measurements demonstrate the existence of interference fringes in the time-domain output, as predicted by the GMM-NLSE. Further, upon using the raster-scanned measurements to reconstruct the temporal evolution of the instantaneous intensity profile at the FMF output end-face, we see that the instantaneous intensity profile transforms from a Gaussian-like shape to an annulus, and back, as the pulse rises, peaks and falls. These form the first complete spatiotemporal measurements of multimode nonlinearity to our knowledge.

In order to confirm that the phenomenon that we observe is a result of a time-varying relative phase difference between two overlapping spatial modes, we varied the relative phase difference by using a mechanism (heating a section of the fiber) different from optical nonlinearity and observed the change in output intensity pattern. The change in output intensity pattern with temperature was seen to be consistent with spatial inference of two modes that have acquired different time-dependent nonlinear phases. Further, the match in theoretical, numerical, experimental as well as the thermal measurements demonstrate the validity of the modal picture of nonlinearity.

The methods presented here could find applications in further study of spatiotemporal nonlinear phenomena reported previously, such as Kerr-induced beam cleanup and supercontinuum generation in GRIN MMFs, to help shed more light on the mechanisms behind them. Future directions of work could also include resolving the output not just in space and time but also in polarization. Spatiotemporal measurements of the output could also be very useful in the development of multimode spatiotemporally mode-locked lasers, where both the output mode
quality and the output pulse shape are of interest. The effects of optical nonlinearity in MMFs and FMFs are fundamentally spatiotemporal in nature. In order to best understand the physics of these systems and the nonlinear dynamics that arise in them, a full (2+1)D diagnostic that can measure in both space and time (or frequency) is required [18]. The near-field measurement technique that we presented here serves as a promising tool with which to better understand nonlinear optics in MMFs and FMFs.

Acknowledgments

We acknowledge the support of the Maryland NanoCenter and its AIMLab.

References

1. Richardson, D., Fini, J. & Nelson, L. Space-division multiplexing in optical fibres. Nature Photon 7, 354â€“362 (2013).

2. R.-J. Essiambre, R. W. Tkach, and R. Ryf. “Fiber nonlinearity and capacity: Single-mode and multimode fibers,” in Optical Fiber Telecommunications, Optics and Photonics, 6th ed., edited by I. P. Kaminow, T. Li, and A. E. Willner (Academic Press, Boston, 2013), Chap. 1, pp. 1â€“43.

3. Zhanwei Liu, Logan G. Wright, Demetrios N. Christodoulides, and Frank W. Wise, “Kerr self-cleaning of femtosecond-pulsed beams in graded-index multimode fiber,” Opt. Lett. 41, 3675-3678 (2016).

4. Krupa, K., Tonello, A., Shalaby, B. et al. Spatial beam self-cleaning in multimode fibres. Nature Photon 11, 237â€“241 (2017). https://doi.org/10.1038/nphoton.2017.32.

5. W. H. Renninger and F. W. Wise, “Optical solitons in graded-index multimode fibres,” Nat. Commun. 4, 1719 (2012).

6. Krupa, Katarzyna & Tonello, Alessandro & BarthÃľlÃľmy, Alain & Couderc, Vincent & Shalaby, B. & Bendahmane, Abdelkrim & Millot, G. & Wabnitz, Stefan. (2016). Observation of Geometric Parametric Instability Induced by the Periodic Spatial Self-Imaging of Multimode Waves. Physical Review Letters. 116. 10.1103/PhysRevLett.116.183901.

7. M. A. Eftekhar, L. G. Wright, M. S. Mills, M. Kolesik, R. Amezcua Correa, F. W. Wise, and D. N. Christodoulides, “Versatile supercontinuum generation in parabolic multimode optical fibers,” Opt. Express 25, 9075-9087 (2016).

8. Wright, L., Liu, Z., Nolan, D. et al. Self-organized instability in graded-index multimode fibres. Nature Photon 10, 771â€“776 (2016). https://doi.org/10.1038/nphoton.2016.227.

9. Picozzi, Antonio & Millot, G. & Wabnitz, Stefan. (2015) Nonlinear optics: Nonlinear virtues of multimode fibre. Nature Photonics. 9. 289-291. 10.1038/nphoton.2015.67.

10. K. Krupa, C. Louot, V. Couderc, M. Fabert, R. Guenard, B. M. Shalaby, A. Tonello, D. Pagnoux, P. Leproux, A. Bendahmane, R. Dupiol, G. Millot, and S. Wabnitz, “Spatiotemporal characterization of supercontinuum extending from the visible to the mid-infrared in a multimode graded-index optical fiber,” Opt. Lett. 41, 5785-5788 (2016).

11. Krupa, Katarzyna & Tonello, Alessandro & Couderc, Vincent & BarthÃľlÃľmy, Alain & Millot, G. & Modotto, Daniele & Wabnitz, Stefan. (2018). Spatiotemporal light-beam compression from nonlinear mode coupling. Physical Review A. 97. 10.1103/PhysRevA.97.043836.

12. Francesco Poletti and Peter Horak, "Description of ultrashort pulse propagation in multimode optical fibers,” J. Opt. Soc. Am. B 25, 1645-1654 (2008).

13. L. RishÃÿj, B. Tai, P. Kristensen, and S. Ramachandran, "Soliton self-mode conversion: revisiting Raman scattering of ultrashort pulses,” Optica 6, 304-308 (2019).

14. Z. Zhu, L. G. Wright, J. Carpenter, D. Nolan, M. Li, D. N. Christodoulides, and F. W. Wise, "Mode-Resolved Control and Measurement of Nonlinear Pulse Propagation in Multimode Fibers,” in Conference on Lasers and Electro-Optics, OSA Technical Digest (online) (Optical Society of America, 2018), paper FTh4E.7.

15. Zhenwei Xie, Shecheng Gao, Ting Lei, Shengfei Feng, Yan Zhang, Fan Li, Jianbo Zhang, Zhaohui Li, and Xiaocong Yuan, “Integrated (de)multiplexer for orbital angular momentum fiber communication,” Photon. Res. 6, 743-749 (2018).

16. Chin-Lin Chen, "Excitation of higher order modes in optical fibers with parabolic index profile,” Appl. Opt. 27, 2553-2556 (1988).

17. Horak, Peter & Poletti, Francesco. (2012). Multimode Nonlinear Fibre Optics: Theory and Applications. 10.5772/27489.

18. Krupa, Katarzyna & Tonello, Alessandro & BarthÃľlÃľmy, Alain & Mansuryan, Tigran & Couderc, Vincent & Millot, G. & Daniele, Modotto & Babin, Sergey & Stefan, Wabnitz. (2019). Multimode Nonlinear Fibre Optics, a spatiotemporal avenue. APL Photonics. 4. 10.1063/1.5119434.