Abstract:

We study the dynamics of a confining vacuum in $N = 2 \, USp(4)$ gauge theory with $n_f = 4$. The vacuum appears to be a deformed conformal theory with nonabelian gauge symmetry. The low-energy degrees of freedom consist of four nonabelian magnetic monopole doublets of the effective $SU(2)$ colour group, two dyon doublets and one electric doublet. In this description the flavour quantum number is carried only by the monopoles. We argue that confinement is caused by the condensation of these monopoles, and involves strongly interacting nonabelian degrees of freedom.
1 Introduction

$N = 2$ gauge theories have been a consistent source of hints as to the nature of real-world QCD. Different types of confining vacua are realized in these models. For example some models exhibit confinement due to the condensation of monopoles charged under the maximally abelian subgroup, as in the $N = 1$ vacua surviving the adjoint mass perturbation in $N = 2$ SYM [1, 2].

However, this is not the typical situation in softly broken $N = 2$ theories with fundamental matter fields. In [3], [4], [5] the mechanism of confinement and dynamical symmetry breaking has been studied in some detail in these theories. In some vacua the low-energy degrees of freedom turn out to be nonabelian monopoles of the type studied by Goddard-Nuyts-Olive [6]; these objects (see [7], [8] for a semiclassical analysis) also carry a flavor quantum number and their condensation is responsible for confinement and dynamical flavor symmetry breaking.

The most interesting type of vacua found in [4] is based on the deformation of interacting superconformal field theories ([9], [10], [11]). The low-energy dynamics involves relatively non-local monopoles and dyons, as in the case first discovered by Argyres and Douglas [9]. In a previous paper [12] we have studied one of these confining vacua in $N = 2$ $SU(3)$ gauge theories with $N_f = 4$ quark hypermultiplets. Upon a small adjoint mass perturbation confinement occurs, as can be demonstrated through various considerations, such as the study of the large adjoint mass limit (see [4]). The low energy degrees of freedom consist of four $SU(2)$ gauge doublets monopoles, one dyon doublet and one electric doublet. These relatively nonlocal particles conspire to give a vanishing beta function, generalizing the abelian Argyres-Douglas mechanism [9] to a nonabelian effective theory. Giving the quark hypermultiplets different masses, the superconformal nonabelian vacuum split into six abelian local vacua; the fact that monopole condensates vanished in the equal mass limit suggested that confinement occurred in an essentially different way, involving strongly interacting nonabelian magnetic monopoles.

Confinement is described in this way in many vacua in $N = 2$ supersymmetric gauge theories. Given the importance of the problem and subtlety of the nonabelian
confinement mechanism, we believe that it is worthwhile to analyze as many explicit examples as possible, to deepen our insight into this phenomena further. In this paper we pursue the analysis for a vacuum in the \( USp(4) \) theory with \( N_f = 4 \). The degrees of freedom are found to be the same as in \([12]\), plus another dyon. This is a little puzzling because the \( \beta \)-function cancellation mechanism does not seem to work in the same way as in the previously studied \( SU(3) \) case.

Finally, we make a conjecture as to which degrees of freedom condense, by using the pattern of symmetry breaking known from the large adjoint mass analysis (see \[4]\); we conclude again that nonabelian monopoles play a fundamental role both in confinement and in dynamical symmetry breaking.

2 \( SU(2) \) Vacua in a \( USp(4) \), \( n_f = 4 \) Gauge Theory

The Seiberg-Witten curve of the \( USp(4) \) theory (\[13\]), with \( n_f \) quarks of different masses \( m_a \), is (setting \( \Lambda = 1 \))

\[
xy^2 = [x(x^2 - Ux - V) + 2 \prod_{a=1}^{n_f} m_a]^2 - 4 \prod_{a=1}^{n_f} (x + m_a^2),
\]

(2.1)

where

\[
U = \phi_1^2 + \phi_2^2, \quad V = -\phi_1^2\phi_2^2.
\]

(2.2)

\( \phi_1 \) and \( \phi_2 \) are the two components of the adjoint scalar field that breaks the gauge symmetry:

\[
\langle \phi \rangle = \begin{pmatrix}
\phi_1 & 0 & 0 & 0 \\
0 & \phi_2 & 0 & 0 \\
0 & 0 & -\phi_1 & 0 \\
0 & 0 & 0 & -\phi_2
\end{pmatrix}.
\]

(2.3)

In the case \( m_a = 0 \), the behavior of the curve is highly singular at the two points \( U = \pm 2, V = 0 \), where four of the five branchpoints coalesce. In these vacua we have \( \phi_1 = 0 \), and so the gauge symmetry is broken to \( USp(2) \times U(1) \simeq SU(2) \times U(1) \).

Giving equal masses to all of the quarks (\( m_a = m \)), these two singularities exhibit different behaviors: the \( U = 2 \) singularity splits into two separate singularities, each of which has two pairs of branchpoints colliding (\( U(1)^2 \) unbroken). The \( U = -2 \) singularity splits into three singularities: two of them have an unbroken \( U(1)^2 \), while the third has four colliding branchpoints (\[4\]).
Giving distinct masses to the quarks, each of two singularities near $U = 2$ splits into four, and the $SU(2)$ singularity near $U = -2$ splits into six; the former is a $4 + 4^*$ flavor representation of $U(4)$, the residual global symmetry, the latter is a $1 + 6 + 1$ representation. The $U = \pm 2, V = 0$ points are octet vacua of the original $SO(8)$ flavor symmetry.

We will analyze the $m_a = 0$ case, and try to understand the structure of the low-energy degrees of freedom and the mechanism of confinement.

### 3 Singularity structure in the massless case

In $m = 0$ case, the curve becomes
\[ y^2 = x(x^2 - Ux - V)^2 - 4x^3. \]  
(3.1)
The discriminant of the curve is
\[ \Delta_s \Delta_+ \Delta_, \]  
(3.2)
where
\[ \Delta_s = V^6; \]  
(3.3)
\[ \Delta_+ = (4 - 4U + U^2 + 4V); \]  
(3.4)
\[ \Delta_ - = (4 + 4U + U^2 + 4V). \]  
(3.5)
When two of these factors vanish, the singularity is maximal. $\Delta_s = \Delta_\pm = 0$ corresponds to the two superconformal points we are considering ($U = \pm 2, V = 0$).

Let’s consider one of these two points, $U = -2$. It is useful to redefine
\[ u = U + 2, \quad v = V, \]  
(3.6)
in order to have $u = 0, v = 0$ in the considered vacuum. At this point, $\Delta_s = \Delta_ - = 0$. Singularities near this point are located at
\[ \Delta_s = 0 \longrightarrow v = 0; \]
\[ \Delta_ - = 0 \longrightarrow v = -\frac{u^2}{4}. \]  
(3.7)
On the 3-sphere $|u|^2 + |v|^2 = R^2$ these equations describe two rings with linking number two (fig. 1).
Figure 1: Zero loci of the discriminant of the curve of the $\mathcal{N} = 2$, $USp(4)$, $n_f = 4$ theory at $m_a = 0$.

Imposing the vanishing of the first factor ($v = 0$), the resulting curve has 3 branch points colliding; the analysis of monodromies is difficult, because it is possible to get charges of the zero-mass particles only when there are pairs of branch points colliding ($U(1)^n$ symmetry); this will be clear in the following.

Problems become evident if one considers the monodromies around these two rings, as shown in figure 1. Choosing fundamental $\alpha$ and $\beta$ cycles as in figure 2, one can calculate the non-trivial monodromies of these around the singularity rings. The monodromy $M$ acts on the column vector of cycles

$$v = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \end{pmatrix}$$

(3.8)
as \( v \to M.v \). The result is the following:

\[
R_{m=0}^{1} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 2 & -1 & 0 \\
-2 & 0 & 0 & 1
\end{pmatrix}, \quad V_{m=0}^{1} = \begin{pmatrix}
2 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
R_{m=0}^{2} = \begin{pmatrix}
-1 & 2 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 2 & 1
\end{pmatrix}, \quad V_{m=0}^{2} = \begin{pmatrix}
2 & -2 & 1 & 0 \\
0 & 1 & 0 & 0 \\
-1 & 2 & 0 & 0 \\
2 & -4 & 2 & 1
\end{pmatrix}.
\]

One can easily extract magnetic and electric charges \((g_1, g_2; q_1, q_2)\) from the \(V_1, V_2\) monodromies using (13):

\[
M = \begin{pmatrix}
1 + q_1 g_1 & q_1 g_2 & q_1^2 & q_1 q_2 \\
q_2 g_1 & 1 + q_2 g_2 & q_1 q_2 & q_2^2 \\
-g_1^2 & -g_1 g_2 & 1 - q_1 g_1 & -q_2 g_1 \\
-g_1 g_2 & -g_2^2 & -q_1 g_2 & 1 - q_2 g_2
\end{pmatrix},
\]

and one obtains the following:

\[
V_1 = (1, 0; 1, 0), \quad V_2 = (1, -2; 1, 0).
\]

This is not the case for the \(R_1, R_2\) monodromies: they are not in the form (3.10). This is not surprising: at \( v = 0 \) three branch points coalesce, and there is not a \( U(1)^2 \) theory: one cannot expect that charges \( g_i, q_i \) make sense. Moreover, it is not a single
singularity. The composition of monodromies, when two or more singularities coalesce, respects (3.10) only if the particles are mutually local (for example, if they are in a flavor multiplet). In this case, we expect a flavor quartet (magnetic monopoles), plus other two particles; we will see that they are not relatively local with respect to the quartet.

4 Equal mass case and the massless limit

In order to understand what kinds of particles are present in the theory, we must give a little mass to quarks, in order to split the \( v = 0 \) ring into many simple rings.

We already know that by giving a common mass \( m_a = m \) to the quarks, the \( U = -2 \) singularity splits into three singularities. Making the stereographic projection, one must choose a spherical section of a radius sufficiently large to contain all three singularities: so he can go to the limit \( m \to 0 \) without crossing the sphere and altering the topological structure of the singularity rings. Monodromies obtained in such a way remain valid in the limiting case.

The curve becomes

\[
xy^2 = (x(x^2 - (u - 2)x - v) + 2m^4)^2 - 4(x + m^2)^4,
\]

and the discriminant has the three factors:

\[
\Delta_s = (m^4 + m^2(u - 4) - v)^4,
\]

\[
\Delta_1 = 16m^2 + u^2 + 4v,
\]

\[
\Delta_2 = 432m^8 + 32m^6(9u - 28) + 8m^2v(16 - 8u + u^2 - 6v) - \\
v^2(u^2 - 8u + 4(v + 4)) - 8m^2(2u^3 - 22u^2 - 12(8 + v) + u(9v + 80)).
\]

In the singularity, all three of these factors are null. In the \( m = 0 \) case \( \Delta_2 = v^2\Delta_+ \); this gives the \( v^6 \) factor.

The singularity structure on a 3-sphere of radius \( R = 1 \) has been analyzed numerically, with \( m = 0.1 \). The result are shown in figure 3: the \( R \) ring is a \( \Delta_s \) singularity, the \( V \) ring is a \( \Delta_1 \) singularity, the \( B \) and \( C \) rings are \( \Delta_2 \) singularities.
First of all, we note that the rings B and C can be continuously deformed to make them coincide with the R ring: this is what happens when $m \to 0$, and monodromies are not changed by this deformation. This means that this analysis gives correct information on the $m \to 0$ limit.

We computed non-trivial monodromies around the rings (see Appendix A). One can explicitly calculate only one monodromy for each ring, and obtain the others from these using the relations shown in Appendix A (A.3): the redundancy of these relations allows a non-trivial check, that works correctly. The charges obtained, with a global sign ambiguity, are:

- $R_1 = (1, 0; 0, 0)^4$; $R_2 = (0, 1; -1, 0)^4$;
- $R_3 = (-1, 1; 0, 0)^4$; $R_4 = (0, 0; 1, 0)^4$;
- $B_1 = (1, 1; -1, 0)$; $B_2 = (-3, 3; -1, 0)$;
\[ B_3 = (-1, 1; 1, 0); \quad B_4 = (3, -1; 1, 0); \]
\[ C_1 = (1, -1; 1, 0); \quad C_2 = (-1, 1; -1, 0); \]
\[ V_1 = V_6 = (1, 0; 1, 0); \]
\[ V_4 = V_3 = (-1, 2; -1, 0); \]
\[ V_2 = (3, 0; -1, 0); \quad V_5 = (-3, 2; 1, 0). \] (4.5)

As we expected, \( q_2 = 0 \) for all the charges, because \( \beta_2 \) cycle remains large approaching superconformal point. We expect also that particles relative to opposite arches of a singularity circle (ex. \( R_1 \) and \( R_3 \)) form doublets of the unbroken \( SU(2) \). So one can try to construct a transformation of cycles that give \((g_1, q_1)\) as the charges of \( U(1) \subset SU(2) \), and \((g_2, q_2)\) as the charges of \( U(1) \) orthogonal to \( SU(2) \); in this base, \((g_1, q_1)\) of the particles in the doublet must have opposite signs, and \((g_2, q_2)\) must have the same sign. It is possible to get this applying to charges the transformation

\[
T = \begin{pmatrix}
1 & 0 & 1 & 0 \\
1 & 2 & 1 & 0 \\
0 & 0 & 1 & -\frac{1}{2} \\
0 & 0 & 0 & \frac{1}{2}
\end{pmatrix}. \quad (4.6)
\]

Applying \( T \), one obtains a \( SU(2) \times U(1) \) basis in which the charges are correctly paired into doublets. They are

\[
(R_1, R_3) : (\pm 1, 1; 0, 0)^4; \\
(R_2, R_4) : (\pm 1, 1; \pm 1, 0)^4; \\
(B_1, B_3) : (0, 2; \pm 1, 0); \\
(B_2, B_4) : (\pm 4, 2; \pm 1, 0); \\
(C_1, C_2) : (\pm 2, 0; \pm 1, 0); \\
(V_1, V_4), (V_3, V_6) : (\pm 2, 2; \pm 1, 0); \\
(V_2, V_5) : (\pm 2, 2; \mp 1, 0). \quad (4.7)
\]

Following \[12\], we assume that if we want a correct interpretation of the theory only a section of fig. \[3\] must be chosen; other sections will give equivalent theories. In the \( m \to 0 \) limit the \( R, B, C \) rings coalesce. Observing figure \[4\] one can see that
Figure 4: Zero loci of the discriminant of the curve of the $\mathcal{N} = 2$, $USp(4)$, $n_f = 4$ theory, in the limit $m \to 0$. The four sections are the same as in fig. 1.

there are only two not equivalent sections ($s_1$ and $s_2$) in this limit. The $s_1$ and $s_2$ sections of figure 1 can be identified with the $\bar{s}_1$ and $\bar{s}_2$ of figure 4. The composition of surviving monodromies for the rings $R$, $B$, $C$ have to compose into those of (3.9). One can easily check that

$$R_{1m=0} = C_1 R_1 B_1;$$
$$R_{2m=0} = C_2 R_3 B_3,$$

(4.8)

and this shows that $C_1$, $R_1$, $B_1$ are the monodromies that compose into $R_{1m=0}$ of (3.9) in the limit $m \to 0$; the other monodromies ($R_2$, $R_4$, $B_2$, $B_4$, $V_2$, $V_3$, $V_5$ and $V_6$) have no role in the massless limit. Thus the sections to consider are $s_1$ and $s_2$ of figure 4. Corresponding charges relative to $U(1) \subset SU(2)$ are shown in table 1.

We conclude that the low energy degrees of freedom form $SU(2)$ doublets; in the
Table 1: Different sections of the singularities

\begin{array}{|c|c|}
\hline
\overline{s}_1 & \overline{s}_2 \\
\hline
R_1, R_3 : (\pm 1, 0)^4 & R_1, R_3 : (\pm 1, 0)^4 \\
B_1, B_3 : (0, \pm 1) & B_1, B_3 : (0, \pm 1) \\
C_1, C_2 : (\pm 2, \pm 1) & C_1, C_2 : (\pm 2, \pm 1) \\
V_1, V_4 : (\pm 2, \pm 1) & V_4, V_1 : (\mp 2, \mp 1) \\
\hline
\end{array}

SU(2) \times U(1) basis they are:

\( (R_1, R_3) : (\pm 1, 1; 0, 0)^4; \)
\( (B_1, B_3) : (0, 2; \pm 1, 0); \)
\( (C_1, C_2) : (\pm 2, 0; \pm 1, 0); \)
\( (V_1, V_4) : (\pm 2, 2; \pm 1, 0). \) (4.9)

5 Low-Energy Coupling Constant

Our aim is now to determine the form of the \( U(1)^2 \) low energy coupling constants near the singularity point. They are given by the symmetric \( \tau_{ij} \) matrix of the associated Riemann surface. Setting \( U = -2 + u, V = v \) the curve in the massless case assumes the following form:

\[ y^2 = x(x^2 - Ux - V)^2 - 4x^3 = x(x^2 - ux - v)(x^2 - (u - 4)x - v). \] (5.1)

The branch points in the \( x \) plane are the following:

\[ x_3 = 0, \quad x_{2,4} = \frac{u \pm \sqrt{u^2 + 4v}}{2}, \] (5.2)
\[ x_{1,5} = \frac{(u - 4) \pm \sqrt{(u - 4)^2 + 4v}}{2}. \] (5.3)

In the limit \( u, v \to 0 \) the points \( x_1, x_2, x_3, x_4 \) collide in \( x = 0 \); approximately,

\[ x_1 \simeq \frac{u^2 + 4v}{16}, \quad x_5 \simeq -4. \] (5.4)

Introducing the new variables

\[ \omega^2 = u^2 + 4v, \quad \alpha = \frac{u}{\omega}, \] (5.5)
one has:

\[ v = \omega^2(1 - \alpha^2); \quad u = \omega \alpha. \]  \hspace{1cm} (5.6)

The colliding branch points become:

\[ x_3 = 0, \quad x_{2,4} = \frac{(\alpha \pm 1)\omega}{2}, \quad x_1 = \frac{\omega^2}{16}. \]  \hspace{1cm} (5.7)

The matrix \( \tau_{ij} \) is a conformally invariant quantity; so one can multiply by \( \omega \) the positions of the branch points on the \( x \) plane without altering it. We have:

\[ x_3 = 0, \quad x_{2,4} = \frac{(\alpha \pm 1)}{2}, \quad x_1 = \frac{\omega}{16}, \quad x_5 = -\frac{4}{\omega}, \quad x_6 = \infty. \]  \hspace{1cm} (5.8)

In the limit \( \omega \to 0 \) one gets two couples of colliding branch points (at 0 and at \( \infty \)). One can now translate these points by the quantity \( a \) and then apply the transformation \( x \to \frac{1}{x} \). These transformations have no effect on the conformally invariant quantity \( \tau_{ij} \). The branch points become:

\[ x_3 = \frac{1}{a}, \quad x_1 = \frac{1}{a + \omega/16}, \quad x_{2,4} = \frac{2}{(2a + \alpha \pm 1)}, \quad x_5 = -\frac{\omega}{4 + \omega a}, \quad x_6 = 0. \]  \hspace{1cm} (5.9)

Fixing \( a = 1 \) and keeping only the lowest order terms in \( \omega \to 0 \):

\[ x_3 = 1, \quad x_1 = 1 - \frac{\omega}{16}, \quad x_{2,4} = \frac{2}{(2 + \alpha \pm 1)}, \quad x_5 = -\frac{\omega}{4}, \quad x_6 = 0. \]  \hspace{1cm} (5.10)

The couples of colliding branch points are now at 0 and at 1; there are no branch points at infinity and this makes the computation of the integrals around the \( \tilde{\alpha} \) and the \( \tilde{\beta} \) cycles easier in the basis shown in figure 5.

If one calculates the integral of the holomorphic forms \( \omega_1 = \frac{dx}{y} \), \( \omega_2 = \frac{xdx}{y} \) around the \( \tilde{\alpha} \) and \( \tilde{\beta} \) cycles:

\[ M_1 = \begin{pmatrix} f_{\tilde{\alpha}_1} \omega_1 & f_{\tilde{\alpha}_2} \omega_1 \\ f_{\tilde{\alpha}_1} \omega_2 & f_{\tilde{\alpha}_2} \omega_2 \end{pmatrix}, \quad M_2 = \begin{pmatrix} f_{\tilde{\beta}_1} \omega_1 & f_{\tilde{\beta}_2} \omega_1 \\ f_{\tilde{\beta}_1} \omega_2 & f_{\tilde{\beta}_2} \omega_2 \end{pmatrix}; \]  \hspace{1cm} (5.11)

then \( \tau_{ij} \) is given by:

\[ \tau_{ij} = M_2^{-1} M_1. \]  \hspace{1cm} (5.12)

Explicit calculations give:

\[ M_1 = \begin{pmatrix} \frac{2\pi i}{\sqrt{(1+\alpha)(3+\alpha)}} & \frac{2\pi i}{\sqrt{(1-\frac{1}{3\alpha})(1-\frac{1}{3\alpha})}} \\ 0 & \frac{2\pi i}{\sqrt{(1+\frac{1}{3\alpha})(1+\frac{1}{3\alpha})}} \end{pmatrix}. \]  \hspace{1cm} (5.13)
Figure 5: $\tilde{\alpha}$ and $\tilde{\beta}$ cycles used in the explicit calculation of $\tau_{ij}$.

\[
M_2 = \begin{pmatrix}
-\frac{-\log(\omega)}{\sqrt{(1+\alpha)(1+\alpha)}} & -\frac{-\log(\omega)}{\sqrt{(1-\frac{1}{1+\alpha})(1-\frac{1}{1+\alpha})}} \\
0 & \frac{-\log(\omega)}{\sqrt{(1-\frac{1}{1+\alpha})(1-\frac{1}{1+\alpha})}}
\end{pmatrix}
\]

and

\[
\tau_{ij} = \begin{pmatrix}
\frac{2\pi i}{-\log(\omega)} & 0 \\
0 & \frac{2\pi i}{-\log(\omega)}
\end{pmatrix}
\]

$M_1$ is exact, calculated using residues. In $M_2$ only the most divergent piece in the integral is kept; the 0 in $M_2$ is a finite non-divergent piece. So one has a non-diagonal correction to $\tau_{ij}$ of order $1/(\log(\omega)^2)$, which is difficult to compute.

Now it is easy to write $\tau_{ij}$ in our original basis. The matrix which transforms between the two bases is the following:

\[
\begin{pmatrix}
\tilde{\alpha}_1 \\
\tilde{\alpha}_2 \\
\tilde{\beta}_1 \\
\tilde{\beta}_2
\end{pmatrix} = \begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & -1 & 1 & 0 \\
-1 & 1 & 0 & 1 \\
-1 & 0 & 0 & 0
\end{pmatrix} \begin{pmatrix}
\alpha_1 \\
\alpha_2 \\
\beta_1 \\
\beta_2
\end{pmatrix} = S \begin{pmatrix}
\alpha_1 \\
\alpha_2 \\
\beta_1 \\
\beta_2
\end{pmatrix}.
\]

(5.16)

If the cycles transform with the symplectic matrix

\[
M = \begin{pmatrix}
A & B \\
C & D
\end{pmatrix},
\]

(5.17)
\( \tau_{ij} \) transforms as
\[
\tau \rightarrow (A \tau + B)(C \tau + D)^{-1}.
\] (5.18)

In the original basis one gets (using (5.18) with \( M = S^{-1} \)):
\[
\tau_{ij} = \begin{pmatrix}
-1 + \frac{4\pi i}{-\log(\omega)} & \frac{2\pi i}{\log(\omega)} \\
\frac{2\pi i}{\log(\omega)} & \frac{2\pi i}{-\log(\omega)}
\end{pmatrix},
\] (5.19)

and transforming into the \( SU(2) \times U(1) \) basis with \( M = (T^T)^{-1} \) (see \( T \) in (4.6)) one finds:
\[
\tau_{ij} = \begin{pmatrix}
-1/2 + \frac{i\pi}{-2\log(\omega)} & \frac{\pi}{-4\log(\omega)} \\
\frac{-4\log(\omega)}{\pi} & \frac{-2\log(\omega)}{\pi}
\end{pmatrix}.
\] (5.20)

At the end, in the \( \omega \to 0 \) limit, in the \( SU(2) \times U(1) \) basis it turns out that
\[
\tau_{ij} = \begin{pmatrix}
-1/2 & 0 \\
0 & 0
\end{pmatrix}.
\] (5.21)

The \( \beta \)-function cancellation mechanism found in \([9]\) and in \([12]\) does not generalized in an obvious way to this vacuum. We found above four \( SU(2) \) doublets of nonabelian monopoles, an electric doublet and two \((2,1)\) dyonic doublets. The monopoles cancel the contribution to the \( \beta \)-function of the nonabelian dual \( SU(2) \) gauge bosons. The one-loop contributions of the other degrees of freedom would cancel each other for \( \tau_{11} \) such that (see \([9],[12]\)):
\[
\sum_i (q_i + g_i \tau_{11})^2 = 2(1 + 2\tau_{11})^2 + 1 = 0,
\] (5.22)
i. e.,
\[
\tau_{11} = -\frac{1}{2} + \frac{\sqrt{2}}{4}i.
\] (5.23)
This condition is not satisfied.

We observe that neglecting the contribution of the electric doublet in (5.22) the condition becomes
\[
\tau_{11} = -\frac{1}{2},
\] (5.24)
and it would be satisfied by this vacuum. The \( \beta \)-function cancellation mechanism found in \([9]\) and \([12]\) can be generalized to this case only if the electric doublet decouples by some unknown reason from the low energy physics.

Evidently, further investigations are needed to understand this extremely subtle issue of nontrivial SCFT with nonabelian gauge symmetry.
6 Confinement and Chiral Symmetry Breaking Mechanism

The superconformal limit may be approached by first breaking chiral symmetry explicitly by introducing unequal bare quark masses. It is easy to see [4] that the non-local $SU(2)$ vacuum splits into 8 local $U(1)^2$ vacua. Adding an adjoint mass term $\mu \text{Tr} \Phi^2$ into each of these vacua the low energy degrees of freedom (monopoles, dyons) condense. However, in the superconformal limit all the condensates become zero (see the discussion of the $SU(3)$ case in [12] for more details).

The theory in the equal mass case has a $SU(N_f)$ chiral symmetry group, realized in the fundamental representation multiplets $Q, \tilde{Q}^*$. The massless case is particular: in this case the chiral symmetry is enhanced to $SO(2N_f)$ due to the presence of generators which mix $Q$ and $\tilde{Q}^*$.

The dynamical chiral symmetry breaking has been determined in [4] by studying the theory at large $\mu$ ($\gg \Lambda$). The result is that $SO(2N_f)$ is dynamically broken to $U(N_f)$. On the other hand, in our analysis we have found that the low energy degrees of freedom of our theory are a flavour quartet of monopoles and three flavour singlets dyons. It seems natural to assume that the chiral symmetry breaking is caused by the following flavour monopole condensation:

$$\langle M_i \tilde{M}^i \rangle = v. \quad (6.1)$$

This seems to be the only way to perform the breaking $SO(8) \to U(4)$ with our low energy degrees of freedom. This is also reasonable because our monopoles are strongly coupled.

As in the $SU(3)$ case [12], this suggests strongly that the mechanism of confinement and chiral symmetry breaking involves strongly coupled nonabelian monopoles and dyons.

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Appendix A Monodromies

Evaluating numerically the four monodromies $R_1$, $B_1$, $C_1$, $V_1$ one gets:

$$R_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -4 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & -1 & 2 & 0 \\ -1 & -1 & 1 & 1 \end{pmatrix}$$ \hspace{1cm} (A.1)

$$C_1 = \begin{pmatrix} 2 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 1 \end{pmatrix}, \quad V_1 = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \hspace{1cm} (A.2)$$

The topology of the four rings yields the relations

$$R_2 = B_1^{-1}R_1B_1, \quad R_3 = V_4^{-1}R_2V_4,$$

$$R_4 = B_3^{-1}R_3B_3, \quad R_1 = V_1^{-1}R_4V_1,$$

$$B_2 = V_4^{-1}B_1V_4, \quad B_3 = R_3^{-1}B_2R_3,$$

$$B_4 = V_1^{-1}B_3V_1, \quad B_1 = R_1^{-1}B_4R_1,$$

$$C_2 = V_4^{-1}C_1V_4, \quad C_1 = V_1^{-1}C_2V_1,$$

$$V_2 = R_1^{-1}V_1R_1, \quad V_3 = B_1^{-1}V_2B_1,$$

$$V_4 = C_1^{-1}V_3C_1, \quad V_5 = R_3^{-1}V_4R_3,$$

$$V_6 = B_3^{-1}V_5B_3, \quad V_1 = C_2^{-1}V_6C_2. \hspace{1cm} (A.3)$$
With these relations, one gets all of the monodromies, and re-obtain the initial ones with the last relation of each ring: this is a non trivial check of the monodromies. The check succeeds, and the monodromies are:

\[
R_2 = \begin{pmatrix}
1 & -4 & 4 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & -4 & 4 & 1
\end{pmatrix}, \quad R_3 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
-4 & 4 & 1 & 0 \\
4 & -4 & 0 & 1
\end{pmatrix},
\]

\[
R_4 = \begin{pmatrix}
1 & 0 & 4 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}, \quad B_2 = \begin{pmatrix}
4 & -3 & 1 & 0 \\
0 & 1 & 0 & 0 \\
-9 & 9 & -2 & 0 \\
9 & -9 & 3 & 1
\end{pmatrix},
\]

\[
B_3 = \begin{pmatrix}
0 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 \\
-1 & 1 & 2 & 0 \\
1 & -1 & -1 & 1
\end{pmatrix}, \quad B_4 = \begin{pmatrix}
4 & -1 & 1 & 0 \\
0 & 1 & 0 & 0 \\
-9 & 3 & -2 & 0 \\
3 & -1 & 1 & 1
\end{pmatrix},
\]

\[
V_2 = \begin{pmatrix}
-2 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
-9 & 0 & 4 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}, \quad V_5 = \begin{pmatrix}
-2 & 2 & 1 & 0 \\
0 & 1 & 0 & 0 \\
-9 & 6 & 4 & 0 \\
-6 & -4 & -2 & 1
\end{pmatrix},
\]

\[
V_3 = V_4 = \begin{pmatrix}
2 & -2 & 1 & 0 \\
0 & 1 & 0 & 0 \\
-1 & 2 & 0 & 0 \\
2 & -4 & 2 & 1
\end{pmatrix}, \quad C_2 = C_1, \ V_6 = V_1. \quad (A.4)
\]

Charges (4.5) can be calculated from these monodromies.