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LETTER

The role of the discount rate for emission pathways and negative emissions

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Abstract

The importance of the discount rate in cost-benefit analysis of long term problems, such as climate change, has been widely acknowledged. However, the choice of the discount rate is hardly discussed when translating policy targets—such as 1.5 °C and 2 °C—into emission reduction strategies with the possibility of overshoot. Integrated assessment models (IAMs) have quantified the sensitivity of low carbon pathways to a series of factors, including economic and population growth, national and international climate policies, and the availability of low carbon technologies, including negative emissions. In this paper we show how and to what extent emission pathways are also influenced by the discount rate. Using both an analytical and a numerical IAM, we demonstrate how discounting affects key mitigation indicators, such as the time when net global emissions reach zero, the amount of carbon budget overshoot, and the carbon price profile. To ensure inter-generational equity and be coherent with cost-benefit analysis normative choices, we suggest that IAMs should use lower discount rates than the ones currently adopted. For a 1000 GtCO2 carbon budget, reducing the discount rate from 5% to 2% would more than double today’s carbon price (from 21 to 55 $/tCO2) and more than halve the carbon budget overshoot (from 46% to 16%), corresponding to a reduction of about 300 GtCO2 of net negative emissions over the century.

1. Introduction

The social or consumption discount rate has been at the core of the discussion of climate change economics for many years, due to its importance for evaluating climate change impacts in the future against today’s costs of mitigating emissions (Weitzman 1994, 2001, Nordhaus 2007, Weitzman 2007). For example, in the DICE integrated assessment model (IAM) climate targets in line with 2 °C or less are typically attainable only for very low discount rates, like the one proposed in the Stern review (Stern 2006), or suggested in the intergovernmental working group report (Intergovernmental Working Group on Social Cost of Carbon, United States Government 2013). Similarly, analytical IAMs find the discount rate to be a crucial determinant of the Social Cost of Carbon (Golosov et al 2014, van den Bijsma et al 2016, van der Ploeg and Rezai 2019).
Yet, in practice, long term targets are often discussed and have been proposed (such as the well below 2 °C and the 1.5 °C objectives from the Paris Agreement), and detailed process models (DP-IAMs), characterized by a high level of disaggregation (Weyant 2017), have translated them into minimum cost emission pathways and their investment strategies (cost-effectiveness analysis). These assessments have focused on the role of key factors such as economic and population growth (Kriegler et al 2016, Marangoni et al 2017, Riahi et al 2017), national and international climate policies (Clarke et al 2009, Aldy et al 2016, Vrontisi et al 2018), fossil fuel availability (McJeon et al 2014, Kriegler et al 2016) and low carbon technologies (Kriegler et al 2014, Luderer et al 2018). Scenarios combining different assumptions about these drivers have been developed to quantify different mitigation challenges (Riahi et al 2017, van Vuuren et al 2017, Rogelj et al 2013, 2018). Recently, the role of emission pathways of negative emission options, such as afforestation and bioenergy with CCS (BECCS), has also been examined by IAMs (Edmonds et al 2013, Fuss et al 2013, Kriegler et al 2013, Tavoni and Socolow 2013) with a particular focus on stringent policy objectives, such as 2 °C and 1.5 °C (Rogelj et al 2018). However, the role of discounting and its interplay with negative emissions technologies in shaping emission pathways has been overlooked, while the importance of ethical considerations for these technologies has been acknowledged (Fuss et al 2016, Lenzi et al 2018). The only two contributions addressing this issue partly are Ermoliev et al (2008), who considers the role of the discount rate for management of catastrophic risks, and (Chen and Tavoni 2013) as a sensitivity analysis for the use of direct air capture (DAC).

The values of discount rates adopted in DP IAMs are around 5%–6% per year (IAMC 2018), in line with market interest rates. Moreover, across the world, discount rates adopted by national governments vary substantially in the range between 3.5% and 15% (see figure 1 in Emmerling (2018)). However, there are at least three reasons why lower, social discount rates should be considered when evaluating climate stabilization. First, economists suggest applying risk-free, public, and long-term interest rates when evaluating problems such as climate change (Weitzman 2001, Dasgupta 2008, Arrow et al 2013, Groom and Hepburn 2017). Expert elicitation indicates values around 2%–3% (Drupp et al 2018), and the U.S. Interagency Working Group on the Social Cost of Carbon uses a rate of 3% as central value (U.S. IAWG 2016). Second, cost-effective and cost-benefit analysis should be coherent: Nordhaus (2017) showed that the stringency of climate policy, as measured by the Social Cost of Carbon, is (exponentially) increasing as the discount rate is lowered, implying that very stringent climate targets are optimal only for low discount rates. Last not least, discounting has direct consequences for inter-generational equity: high values of the discount rate lower the mitigation effort of current generations at the expenses of future ones. This raises ethical problems, especially since future generations will be the ones bearing the majority of the impacts of climate change, which are typically not accounted for in low carbon mitigation pathways with the possibility of overshoot. Therefore, we suggest that lower, normative-based discounting is more appropriate when modeling the optimal timing of emission reductions, which is arguably the most important outcome of IAMs (Goulder and Williams 2012). And while some authors have argued to disentangle the market interest and social discount rates conceptually (Goulder and Williams 2012), in practice IAMs use a unique discount rate.

The aim of this paper is to show how the choice of the discount rate shapes emissions pathways constrained by carbon budgets compatible with 1.5 °C–2 °C, and to explore implications for the timing of abatement, carbon prices, and inter-generational burden sharing. To do so, we first develop a dynamic analytical model based on the Hotelling rule in resource economics whose simplicity and closed form solutions allow us to understand the basic dynamics. This model has been extensively applied in resource economics (Dasgupta and Heal 1974) to derive the optimal price path of an exhaustible resource over time. Given that a carbon budget is conceptually similar to an exhaustible resource, it has hence been applied to derive optimal emission prices and in general optimal policies with stock pollutants (Tahvonen 1997, Chakravorty et al 2006, Goulder and Williams 2012, Dietz and Venmans 2019) Then, we use a numerical DP-IAM with a richer process detail including different carbon dioxide removal (CDR) assumptions, which have been shown to be crucial for low temperature targets (Tavoni and Socolow 2013, IPCC 2018, van Vuuren et al 2018, Obersteiner et al 2018), allowing us to quantify the effects of discounting under alternative assumptions about negative emissions.

Analytical assessment of the discount rate and the timing of emission pathways. We develop a simple optimal control problem minimizing the discounted abatement costs of implementing a given carbon budget until the year 2100. The formal mathematical model is based on a standard intertemporal optimization model based on the Hotelling (Hotelling 1931) rule using a discount rate r. The model is fully described in the Methods section. From it, we obtain the closed form analytical expressions for three key indicators of the timing of mitigation: (i) the initial optimal carbon price, (ii) the net zero-emission year, and (iii) the carbon budget overshoot—defined as the total net negative emissions relative to the total carbon budget (see Methods, equations (4), (6) and (7)). Both the initial carbon price and the time of carbon neutrality are focal points in the climate policy discussions. Additionally, the budget overshoot provides information on how much negative emissions will be used: a value of 100% indicates total net negative emissions to be as large as the entire carbon budget. This is also an important mitigation indicator, given the prominent role that negative emissions play in low end scenarios,
as emphasized by the report about a global warming of 1.5 °C (IPCC 2018).

Figure 1 shows the results of the analytical solutions for carbon budgets from 400 up to 1600 GtCO₂, considering all CO₂ emissions over the horizon 2011–2100 (top panels). These budgets are in line with long term temperature targets of 1.5 °C to 2 °C (Rogelj et al 2018), and reflect the known uncertainties (Millar et al 2017). As it can be observed, a lower discount rate calls for higher initial carbon taxes and less overshoot of the carbon budget. As a result, less negative emissions are necessary, thus postponing the net zero-emissions year, see also figure A.3 in the supplementary information is available online at stacks.iop.org/ERL/14/104008/mmedia. The results on the budget overshoot (Panel c in the figure) are the most striking, especially for tighter budgets, as a one percentage point increase in the discount rate leads to up to a 50% increase in the overshoot. While these results are derived from one particular specification to ease representation, results are shown to hold for a more general analytical formulation, see section A.2.

How do these results compare with existing findings from scenarios generated by DP IAMs? We analyze data from the SSP database (Riahi et al 2017), a repository of results generated from this family of models, and in particular we consider scenarios that have carbon budgets around 1000 (between 900 and 1100) GtCO₂. On average across the five models, these mitigation scenarios reach net zero CO₂ emissions in 2075 and have a budget overshoot of around 14%. For a budget of around 400 GtCO₂ (200–600), in line with 1.5°, the values are 2055% and 91% respectively. These numbers are roughly in line with the output of the simple analytical model presented here, when considering a time discount rate of around 5%.

IAM analysis of the discount rate and the timing of emission pathways. The stylized analysis shows a potentially important role of the discount rate for cost-effective emission pathways. We move to a
complex DP-IAM in order to capture energy system and land-use change implications of different mitigation strategies in a more detailed fashion, which also include different assumptions about the availability of two key carbon dioxide removal technologies, namely biomass electricity generation plus CCS (BECCS) and Direct Air Capture (DAC). We perform the analysis with the WITCH model (Emmerling et al. 2016), which has been used extensively in the cost effective assessment of climate policies—including the scenarios analyzed in the recent IPCC reports.

We run the WITCH model for different carbon budgets (from 400 to 1600 GtCO₂ over the period 2010–2100) and vary the (global, averaged over the 21st century) discount rate between 1% and 8%. Moreover, we consider three scenarios based on different assumptions about the availability of CDR technologies:

- ‘no CDR’: a scenario without CO₂ removal options
- ‘only BECCS’: a scenario where the only negative emission technology available is bio-energy with carbon capture and storage (BECCS)
- ‘w/ BECCS + DAC’: a scenario where besides BECCS we also include direct air capture of CO₂ from ambient air (DAC).

This scenario design allows us to span the range of CDR assumptions considered in the literature. It yields $7 \times 8 \times 3 = 168$ scenarios, out of which four (the most extreme, i.e. the highest and lowest discount rates and only in the scenario with the more challenging technological options where DAC is not available) were found infeasible by the solver CONOPT. See the Methods section for assumptions and implementation details.

Results presented in figure 1 (lower panels) report the case where both negative emissions technologies are available (in figure A.5 we also report the implications of excluding one or both options). For a given carbon budget, a lower discount rate implies a higher initial carbon tax (see panel (d) in figure 1): moving from 5% to 2%, the carbon price which needs to be imposed today to meet a 1000 GtCO₂ budget increases from 21$/tCO₂ to 55$/tCO₂ when all negative emission technologies are available. The discount rate also affects the rate at which carbon prices grow, having a huge impact on their value in 2100: for the same scenario the 2100 price of carbon is 289$/tCO₂ under the 2% discount rate and 1093$/tCO₂ under a rate of 5% (see figure A.6).

Panel (e) in figure 1 shows the year in which net-zero carbon emissions are reached, which is delayed for lower discount rates. In accordance with the theoretical model, the behavior with respect to the discount rate is decreasing and convex, suggesting higher sensitivity for lower discount values. Moving from 5% to 2% for a 1000 GtCO₂ budget and full negative emissions delays the year of net negative emissions from 2072 to 2079.

Also in agreement with the analytical model, the carbon budget overshoot (Panel (f) in figure 1) shows a linearly increasing behavior with discounting. For the 1000 GtCO₂ budget, the overshoot decreases from 46% to 16% when lowering discounting from 5% to 2%. This represents a reduction of about 300 GtCO₂ of net negative emissions across the century.

How do the results change when one, or both, negative emission technologies are not available? figure A.5 reports the same results as in the bottom part of figure 1 but for all three negative emissions scenarios. Also for more pessimistic assumptions about negative emissions, the sensitivity patterns to discounting are confirmed for all three indicators, though their levels vary across scenarios. Lower negative emissions availability increases carbon prices and diminishes the carbon budget overshoot. Quantitatively, the possibility to absorb CO₂ from the atmosphere at large scale (meaning in the order of hundreds of GtCO₂) has a first order impact on low carbon pathways, especially on carbon prices. Nonetheless, varying time discounting over a sufficient range (e.g. from the currently assumed 5% to what we suggest are more appropriate values such as 2%) yields comparable changes for some indicators—like the timing of zero emissions and the budget overshoot.

Using all scenarios ($N = 164$), we estimate the effect of the discount rate, carbon budget, and CDR availability on the three policy indicators using OLS, see table 1. The results suggest that across all cases, a one percentage point increase in the discount rate increases the budget overshoot by 7 percentage points, and anticipates the net-zero year by more than two years. We also separately estimate the effects for the three different CDR cases, see the results in appendix A.3. The most important result is the strong impact of the discount rate on the budget overshoot, which is about 1.6% in the ‘no CDR’ case, increases to 5.7% in the ‘only BECCS’ scenario and 11.7% in the ‘w/ BECCS + DAC’ scenario for each percentage point increase in the discount rate.

In terms of energy and carbon intensity improvements, the three CDR scenarios show rather different pathways, which are themselves affected differently by changes in the discount rate. This is represented in figure 2, which compares the carbon and energy intensity in 2010 with projected values in 2050 and 2100 for different discount rates and CO₂ removal options. Without CO₂ removal, required energy intensity improvements are highest as carbon intensity is constrained to be non-negative (apart from afforestation). If DAC and BECCS are available, the required energy efficiency improvements are much lower, with an estimated value of 4 MJ/$ by the end of the century in comparison with less than 2.5 MJ/$ without CDR. Carbon intensity changes make up for this and the
More stringent budgets (from lower to higher panels), allowing less elbow room, reduce the potential for relative inequality across generations, though lower carbon budgets increase the absolute costs of reducing emissions (see the figure in the SI). Conversely, higher availability of negative emission technologies (from left to right panels), exacerbates the intergenerational conflict. For a discount rate of 2%–3% which we have claimed to be a more appropriate choice than what is currently assumed, the emission profile is such that the mitigation effort is equally distributed across generations, independently on the scenario and carbon budget considered. Finally, note that policy cost shares across regions also vary for different discount rate, even though to a minor extent, see figure A.10 in the SI.

**Conclusions.** The recently released IPCC special report on 1.5 °C (IPCC 2018) has reviewed a set of new stringent emission pathways, but has made almost no reference to the choice of time discounting, except saying that the impacts of varying discount rates on 1.5 °C (and 2 °C) mitigation strategies can be assessed to a limited degree. However, assumptions about discounting are embedded in the models which generate the scenarios used in these reports. For example, both in this and previous assessment report, a value of 5% has been applied to calculate the net present value of monetary flows such as carbon prices. Although this rate is in line with what is commonly used by detailed process IAMs, it is not necessarily the right value which should be used to calculate century-long emission reduction pathways.

We show that time discounting matters not just for cost benefit analysis of climate change, but also for the timing of emissions reductions (as found e.g. also in Gerlagh and van der Zwaan (2004)), the time when net zero emissions are reached, and the level of budget overshoot. In particular, moving from a market interest rate of 5%–6% to a social discount rate of 2%–3% significantly improves intergenerational equity. Future generations will have to bear the majority of the climate change impacts, whose costs are not typically accounted for in low carbon scenarios (Weyant 2017). Lowering the discount rate has important repercussions for the amount of negative emissions too: in our scenarios, moving from 5% to 2% reduces the budget overshoot (and hence negative emissions) by about one half. An appropriate choice of the discount rate would therefore automatically limit the role of these technologies, and suggest a different low carbon transition strategy which is more ambitious in its early stages and avoids deeply negative carbon intensities. We recommend more discussion and harmonization of time preferences for the IAMs involved in generating climate stabilization scenarios—including those which will inform the 6th assessment report of the

| Table 1. Influence of the discount rate, carbon budget, and CDR availability (no CDR is the reference category) on the three policy indicators. The statistical model regression uses the WITCH model results across all scenarios (for the net-zero year only scenarios where it occurs before 2100 are considered). |
| Net-zero year (1) | Budget overshoot (2) | log(Carbon price (2020)) (3) |
|------------------|---------------------|-----------------------------|
| dr               | −2.256***           | 0.066***                   |
|                  | (0.159)             | (0.099)                     |
| cb               | 0.021***            | −0.001***                  |
|                  | (0.001)             | (0.0001)                   |
| only BECCS       | −1.625***           | 0.151***                   |
|                  | (0.850)             | (0.052)                     |
| w/ BECCS + DAC   | −5.291***           | 0.544***                   |
|                  | (0.833)             | (0.051)                     |
| Constant         | 2069.181***         | 4.136***                   |
|                  | (1.207)             | (0.271)                     |
| Observations     | 138                 | 164                         |
| R²               | 0.833               | 0.665                       |
| Adjusted R²      | 0.828               | 0.657                       |
| Residual Std.    | 3.824               | 0.269                       |
| Error            | (df = 133)          | (df = 159)                  |
| F Statistic      | 163.798***          | 79.070***                   |

Note. *p < 0.1; **p < 0.05; ***p < 0.01.
IPCC. This can be done by multi-model ensembles—similarly to the several studies which have looked at other key drivers—as well as by aligning time discounting to the underlying socio-economic and policy assumptions.

2. Methods

2.1. Main analytical setting

We assume that a social planner is given the task to keep the cumulative global emissions below a certain carbon budget: this is a direct translation of the temperature goal agreed upon during the Paris agreement (UNFCCC 2015) for the time horizon of 2100. The time frame of the analytical model is thus from 2017 to 2100 and we denote by \( t = 0 \ldots T \) this time horizon. We denote by \( \int_{0}^{T} B(t) dt \) the cumulative baseline emissions at time \( T \). The carbon budget is then given by a fraction of the cumulative baseline emissions. We call \( \alpha \) this relative carbon budget \((0 < \alpha < 1)\), so that the carbon budget can be computed as \( \alpha \cdot \int_{0}^{T} B(t) dt \).

Mathematically, let MAC\((a)\) be the marginal abatement costs to reduce an amount \( a \geq 0 \), relative to a baseline trajectory of emissions \( B(t) \). That is, the MAC gives the relative marginal abatement costs: for example, MAC\((0.5)\) gives the costs of an instantaneous reduction of 50% compared to the baseline. Since we use the carbon price or tax \( p(t) \) as control variable, the instantaneous abatement level is given by the inverse of the MAC: \( a = \text{MAC}^{-1}(p) \). Then, the instantaneous emissions \( E(t) \) at time \( t \) are equal to:

\[
E(t) = B(t)[1 - \text{MAC}^{-1}(p(t))],
\]

where \( p(t) \) is the carbon price level at time \( t \). The cumulative emissions \( CE(t) \), which is the main quantity for the planning constraint, grows at the rate of the instantaneous emissions:

\[
CE(t) = E(t),
\]

with the constraint that \( CE(T) \leq \alpha \cdot \int_{0}^{T} B(t) dt \) or that the carbon budget is met.

The carbon tax is now calculated such that it minimizes the total, discounted, abatement costs, while still reaching the carbon budget goal. The instantaneous abatement costs are given by the integral of the marginal abatement costs from 0 to the amount abated at time \( t \). Since these are relative quantities, we have to multiply this relative abatement by the baseline emission level \( B(t) \) in order to obtain total abatement costs. By combining all the information provided in this setting, we obtain the following optimal control problem minimizing discounted (at rate \( r \)) total mitigation costs:

\[
\min_{p(t)} \int_{0}^{T} e^{-rt}B(t) \left( \int_{0}^{\text{MAC}^{-1}(p(t))} \text{MAC}(a) da \right) dt,
\]

s.t. \( CE(t) = E(t) \),

\[
CE(T) \leq \alpha \cdot \int_{0}^{T} B(t) dt.
\]

In the next section, we provide the assumptions and functional forms we used to obtain our results.

---

**Figure 2.** Influence of the discount rate on the world carbon intensity of energy and energy intensity of GDP over time for a carbon budget of 1000 GtCO₂, three scenarios of negative emissions and two periods of time (2050 and 2100). The carbon intensity is expressed in gCO₂eq per mega joule of primary energy and the energy intensity is expressed as mega joule per USD2005 of world GDP. The value in 2010 is shown in black.
2.1.1. Assumptions and functional forms

First, in order to simplify the analytical expressions, we assume a constant level of baseline emissions: \( B(t) = B \). In section A.2 of the supplementary information, we provide results with a more general linear or quadratic baseline. Second, we assume increasing marginal costs: \( \partial MAC(a)/\partial a > 0 \), which are moreover constant over time: this means that here we do not take into account price reductions through learning by doing or learning over time. Third, in order to incorporate this into our stylized model, we allow the MAC curve to be defined for higher than 100% abatement levels as well: the costs simply continue to grow for higher abatement. Finally, since it has been shown that the MAC curve becomes more convex for higher abatement, we assume a power-law functional form of the MAC, which has been found a good approximation of abatement costs curves empirically as in (Eyckmans et al 2000):

\[
MAC(a) = \gamma a^\beta, \quad \gamma, \beta > 0, \quad (3)
\]

where the convexity is parameterized by \( \beta \).

Now that we have defined the essential quantities for this analysis, we can solve the optimal control problem. This is detailed in supplementary information A.1. The main result from this is that we can find a closed form solution of the optimal carbon price.

\[
p(t) = p_0 e^{rt}.
\]

That is, in this model we obtain a carbon price similar to the Hotelling rule, where the initial carbon price \( p_0 \) is increasing at the discount rate, and computed as to achieve the carbon budget values for all scenarios. Note that we abstract from uncertainty here. Gollier (2018) shows that under uncertain growth and abatement costs, the Hotelling rule for the optimal carbon price is modified.

2.1.2. Analytical expression of the three key indicators

We can now compute analytically the three measures discussed in the main text. We start with the initial carbon tax \( p_0 \). The carbon tax at time 0 has to be calculated such that the cumulative emissions at time \( T \) are equal to the allowed carbon budget. Since we use a constant baseline, this carbon budget is equal to \( \alpha \cdot B \cdot T \). This means that we have to solve:

\[
\int_0^T B(1 - MAC^{-1}(p_0 e^{rt}))\,dt = \alpha \cdot B \cdot T
\]

for \( p_0 \). For the MAC curve defined in equation (3), the solution of this equation is:

---

**Figure 3.** The policy costs, expressed as % of baseline GDP, of the next 2 future generations (living in 2050–2080—in red—and 2080–2110—in blue) is compared to the policy cost of the generation living in 2020. A generation lasts for 30 years, e.g. the policy cost of the generation living in 2020 is the total undiscounted cost over the period 2020–2050.
This expression depends on the parameter $\gamma$, which (as it can be seen from 3) is equal to the price needed to abate 100%, $\text{MAC}(1) = \gamma$. For this reason, we can compute the initial carbon price relative to this maximum marginal abatement cost, i.e. $\frac{\gamma}{r}$, which is used to plot figure 1(a).

Since we assume that the baseline is constant, the emission path is always decreasing (see figure A.3 in the supplementary information). This means that once the instantaneous emission level becomes zero, all subsequent emission levels will be net negative. For this reason, we consider the moment of going to net negative emissions, $r^*$, a key indicator of the timing of emissions. By solving the equation $E(t^*) = 0$, where $E(t)$ is defined in equation (1), we obtain the following general expression for $r^*$:

$$
r^* = \frac{1}{r} \ln \left( \frac{\text{MAC}(1)}{p_0} \right),
$$

where $\text{MAC}(1)$ represents the marginal costs of abating 100% of baseline emissions. Substituting in the expression for $p_0$, the formula becomes:

$$
r^* = \frac{\beta}{r} \ln \left( \frac{\beta(e^{rT}/\beta - 1)}{(1 - \alpha)rT} \right).
$$

For large enough carbon budget $\alpha$, this expression can become larger than $T$: in this case, there is no moment in the considered time span at which the emissions are net negative. However, if $\alpha$ satisfies $\alpha < 1 + \frac{e^{rT}/\beta - 1}{rT}$, we can be sure that $r^*$ is indeed smaller than $T$. In this case, there will be a strictly positive overshoot, which is exactly equal to the total net negative emissions.

The carbon budget overshoot measure $OS$ can then be calculated as

$$
OS = \frac{\int_{0}^{T} E(t) \, dt}{\alpha \cdot B \cdot T}.
$$

By substituting in the expression for $E(t)$ and simplifying, we obtain a closed-form expression for the budget overshoot $OS$ as

$$
OS = \frac{1 - \alpha}{\alpha(e^{rT}/\beta - 1)} - 1 + \frac{\beta}{\alpha rT} \left( \frac{\beta(e^{rT}/\beta - 1)}{rT(1 - \alpha)} - 1 \right).
$$

Note that neither $p_0$, $r^*$, nor $OS$ depend on the baseline emission level $B$. However, all three parameters depend on the discount rate $r$. In particular, we find the following comparative statics results for the three measures:

$$
\frac{\partial p_0}{\partial r} = \frac{\beta \gamma (\beta + \alpha^2(rT - \beta))}{(1 - \alpha)^2 T^2} \left( \frac{\beta(e^{rT}/\beta - 1)}{(1 - \alpha)rT} \right)^{3/2} - \frac{1}{(1 - \alpha)rT} \beta(e^{rT}/\beta - 1) < 0
$$

$$
\frac{\partial r^*}{\partial \gamma} = -\frac{1}{r} \left( \beta + \beta \log \frac{\beta(e^{rT}/\beta - 1)}{(1 - \alpha)rT} \right)
- rT \left( \frac{1}{e^{rT} - 1} + 1 \right)
$$

$$
\frac{\partial OS}{\partial r} = \frac{\beta(\log((\alpha - 1)(-r)T) - \log(\beta(e^{rT}/\beta - 1)))}{\alpha^2 T^2}
+ \frac{e^{rT}(\alpha - 1) r T + \beta(e^{rT}/\beta - 1))}{\alpha(\beta(e^{rT}/\beta - 1))}.\n$$

While for $r^*$ and OS the effect of the discount rate is ambiguous, the initial carbon price always decreases in the discount rate. In the supplementary information (section A.3) we show that for a wide and reasonable range of parameter values, numerically, $r^*$ decreases while the overshoot $OS$ increases in $r$.

### 2.2. The numerical DP-IAM WITCH

In this section we describe the implementation in the IAM WITCH (Bosetti et al. 2006, Emmerling et al. 2016) to assess the quantitative magnitude of the effect of climate engineering on the optimal abatement path and a series of key variables of climate mitigation effort. WITCH has been used extensively in the literature of scenarios evaluating international climate policies, for example as a major contributor to scenarios reviewed by the IPCC in its fifth assessment report (https://tntcat.iiasa.ac.at/AR5DB). WITCH is a global model with 13 macro-regions. It is a long-term dynamic model based on a Ramsey optimal growth economic engine, and a hard linked energy system which provides a compact but exhaustive representation of the main abatement options both in the energy and non-energy sectors. The model is solved numerically in GAMS/CONOPT. A description of the model equations can be found on the model website at http://doc.witchmodel.org.

Here we solve the model using a non-cooperative solution method across world regions. At the non-cooperative solution, each region $n$ acts as one player maximizing its welfare. In this case, the set of players consists each of a single of the 13 macro-regions which constitute the model. For each region $n$ and time period $t$, inter-temporal utility is computed as discounted sum of utility based on a utility function of consumption $C_{t,n}$ per-capita over the whole population $l_{t,n}$ as

$$
W_n = \sum_t l_{t,n} \left( \frac{C_{t,n}}{\hat{c}} \right)^{1-\eta} - \frac{1}{1-\eta} e^{-\lambda t}.
$$

Each player takes at this optimization

$$
\max_{l_{t,n}} W_n
$$

the decisions of the other regions into account, which mainly consist in mitigation through investment in
different energy technologies \((I_{n,a})\), abatement of land-use emissions, and non-CO\(_2\) gases \((A_{n,a})\). In order to find the open-loop Nash equilibrium, we employ an iterative algorithm in which each region optimizes independently, and global values such as temperature are computed after each iteration. This algorithm has been shown to be robust to initial specifications (Bosetti et al. 2006).

WITCH is based on a Ramsey type inter-temporal optimal growth model. In such a model, the social discount rate at which economic financial flows are discounted is given by the Ramsey rule

\[
\eta_{n,a} = \delta + \eta g_{r,t,n},
\]

where \(\delta\) denotes the pure rate of time preference (PRTP), \(\eta\) the inverse of the elasticity of inter-temporal substitution, and \(g_{r,t,n}\) the consumption growth rate at time \(t\) in region \(n\). The WITCH model uses default values for both preference parameters of \(\delta = 1\%\) and \(\eta = 1.5\). Since the growth rate varies across regions and over time, the discount rate is in general higher in fast growing economies and typically declines over time as the growth rate dampens. On average, the global consumption growth rate over the 21st century is estimated at \(g = 1.6\%\) in the SSP2 baseline scenario, which implies that the average discount rate in WITCH equals \(r = 1\% + 1.5 \cdot 1.6\% = 3.4\%.\) We vary the two parameters \(\{\delta, \eta\}\) to generate 8 different average discount rates ranging from 1\% to 8\%. We re-calibrate and re-run the model for each discount rate, and in addition vary the stringency of the climate target and the availability of negative emissions. We explore seven carbon budgets \((400, 600, 800, 1000, 1200, 1400\) and \(1600\) GtCO\(_2\), for the period \(2011-2100\)) and three different technology scenarios of negative emissions, so that we ran a total of \(8 \times 7 \times 3 = 168\) scenarios (out of the 168 scenarios, 4 scenarios turned out to be infeasible: the cases with the lowest or highest discount rate in the scenarios without DAC).

The implementation of the direct air capture (DAC) technology in WITCH is based on a technology described in a report from the American Physical Society (Socolow et al. 2011). This technology is using water solutions containing hydroxide sorbents with high CO\(_2\) affinity to capture it from the atmosphere. The reference scale unit is able to capture 1 MtCO\(_2\) per year with a lifetime of 20 years. Such a unit has an initial estimate of investment cost of 1.6 billions USD, which can be translated into an annual capital cost of 185 millions USD per year, i.e. \(185\$\) per ton of CO\(_2\) (This cost estimate is in line with existing summaries of different cost estimates such as House et al (2011), even though higher potential costs have also been reported (Daggash et al 2019)). Operational costs are estimated to 90 USD per ton of CO\(_2\). The investment cost decreases of 1\% per year with an additional effect from learning by doing based on the cumulative capacity of CCS technologies (6\%). The energy requirements are \(1.1-1.9\) GJ of electricity per ton of CO\(_2\) and 6–10 GJ of natural gas per ton of CO\(_2\), varying over time to reflect efficiency improvements. The DAC expansion is limited to 1 GtCO\(_2\) per year of new DAC capacity at global level. For an in depth assessment of DAC with the WITCH IAM, see https://politi.polimi.it/handle/10589/135879. Bioenergy and carbon capture and sequestration (BECCS) competes with traditional fuel power plants when a sufficient carbon price is reached. Investment cost of BECCS is \(3700\$\) per kW and operation costs are \(10\$\) per MWh. Investment costs has a learning rate of 5\% per year with a floor cost of \(2000\$\) per kW. The BECCS expansion rate is limited to 7.5\% per year. The feed-stock for BECCS power plants is modeled through the soft-linking of WITCH with the land-use model GLOBIOM, which provides biomass prices and supply curves, and reacts to the demand for bio-energy and carbon prices. Carbon prices are implemented as first-best policy, that is, here we do not consider additional objectives or constraints such as on biodiversity or food security.

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Data availability statement

The data that support the findings of this study are openly available at https://doi.org/10.5281/zenodo.3338245.

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