Optimum trade-off charts considering mass variation for the design of semi-active and passive shock absorbers for landing gear

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Abstract
In this study, we proposed two types of optimum trade-off charts considering mass variation for the design of a semi-active shock absorber and a passive shock absorber for landing gear. Each of these trade-off charts is formed from two curves indicating different types of data. Along one curve, the aircraft mass is constant, and shock absorbers having various dimensions are considered. Along the other curve, the aircraft mass varies from maximum to minimum values, and a specified shock absorber is considered. In order to generate optimum trade-off charts considering mass variation by means of a multi-objective optimization, we introduce a parameter related to the initial volume of gas inside the shock absorbers. We are able to establish and solve a multi-objective optimization problem and generate optimum trade-off charts considering mass variation. Using the optimum trade-off charts, we evaluate and compare the performance of semi-active and passive shock absorbers considering mass variation. It clearly demonstrates that the optimum trade-off charts are helpful in the design of landing gears for various aircrafts.

Key words: Landing gear, Multi-objective optimization, Shock absorber, Mass variation, Trade-off chart

1. Introduction

Shock absorbers are commonly used in applications such as aircraft landing gear (Chai, et al., 1996, Greenbank, et al., 1991), elevators, and coupling devices for railroad cars. Generally, the shock absorbers used in such applications are passive components. Passive shock absorbers can be optimally designed for specific conditions of use. However, passive shock absorbers cannot function optimally under conditions that differ from the specified conditions of use. For example, passive shock absorbers for elevators are problematic in that the resisting forces and the accelerations of impacting bodies become greater when the bodies impact under conditions such that the masses differ from the rated values (Maemori, et al., 1998). Furthermore, designing passive shock absorbers for aircraft landing gear that can optimally handle landing impact, as well as runway and taxiway unevenness, is difficult (Krüger, et al., 1997). In order to solve these problems, several studies have examined active shock absorbers and semi-active shock absorbers. In References (McGehee, et al., 1982) and (Howell, et al., 1990), active landing gear was experimentally compared to
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passive landing gear. In References (Ghiringhelli, et al., 2000) and (Ghiringhelli, et al., 2004), semi-active landing gear was numerically and experimentally investigated. Semi-active shock absorbers for landing gear (Berg, et al., 1998) and elevators, both of which were controlled electrorheologically, were investigated via simulations. In Reference (Choi, et al., 2003), a landing gear system featuring electrorheological/magnetorheological fluids was theoretically evaluated. In References (Krüger, et al., 2002), semi-active landing gear was designed using multi-objective optimization (Miettinen, et al., 2004), which has been applied in various studies (Altuazarra, et al., 2009). In Reference (Wang, et al., 1999), possible optimization strategies for semi-active landing gear at touchdown and during taxiing were discussed. In addition, problems concerning mass variation have been discussed (Kobayashi, et al., 2009, Kobayashi, et al., 2013, Maemori, et al., 2004 and Maemori, et al., 2003).

The most important factors in the design of shock absorbers are the maximum resisting force, the maximum displacement, and the efficiency of the shock absorbers. The maximum resisting force is a trade-off (Miettinen, et al., 2004) between the maximum displacement under a specified capacity and the efficiency of the shock absorber. In other words, the greater the maximum resisting force, the smaller the maximum displacement, and vice versa. Therefore, in designing semi-active shock absorbers that are subject to constraints on their maximum resisting forces and maximum displacements, it is important to clarify this trade-off, in which mass variation must be considered. However, this trade-off for semi-active shock absorbers has not yet been clarified. For landing gear that takes the above mass variation into account, the maximum acceleration of the aircraft mass is considered to be an important factor, because the acceleration of the mass becomes higher when the mass is smaller than the rated value. Therefore, it is important to clarify the trade-off between the maximum acceleration of the aircraft mass and the maximum displacement of the piston tube of the shock absorber in order to adequately design the shock absorber. However, this trade-off has not yet been clarified for semi-active shock absorbers.

We herein present optimum trade-off charts considering mass variation for a semi-active shock absorber with a bypass orifice and a passive shock absorber for landing gear. The present study is distinguished from previous studies in two respects. First, the optimum trade-off chart is formed from two kinds of lines. Along each line of one kind, the aircraft mass is constant, and shock absorbers having various dimensions are dealt with. On each line of the other kind, the aircraft mass varies from maximum to minimum values, and a specified shock absorber is dealt with. The goals of the present study are to evaluate and compare the performance of semi-active and passive shock absorbers considering mass variation and to determine whether a passive shock absorber can be appropriately replaced with a semi-active shock absorber. Second, a parameter with respect to the initial volume of gas inside the shock absorbers is introduced as a design variable. The goal here is to establish and solve a multi-objective optimization problem in order to generate the optimum trade-off charts.

The semi-active shock absorber considered in the present paper has a novel construction involving a hollow metering pin and a bypass orifice and a novel function in that the absorber is controlled for mass variation, as proposed in a previous paper (Maemori, et al., 2003). The proposed shock absorber has a greater possibility for practical application than other semi-active shock absorbers, such as those featuring electrorheological or magnetorheological fluids, because it is constructed from conventional components and materials. On the other hand, electrorheological and magnetorheological fluids are being developed for practical applications. In addition, the proposed shock absorber is expected to be more reliable than active shock absorbers, because, upon failure, it acts as a passive shock absorber.

In the present study, we deal with the mass of an aircraft in terms of discrete quantities, minimize the maximum vertical acceleration of each the discrete masses, and generate optimum trade-off charts for passive and semi-active shock absorbers. First, when the mass is a maximum, we minimize the maximum acceleration of the mass as well as the maximum displacement of the piston tube of the passive and semi-active shock absorbers as a multi-objective function, search the Pareto optimal set, and determine the optimum parameter with respect to the initial volume of gas inside the shock absorbers and the displacement of the piston tube, and the optimum orifice area depending on the displacement of the piston tube. Second, for each aircraft mass less than the maximum value, we minimize the maximum acceleration of the mass as a single objective function and determine the optimum bypass orifice area controlled by a stepping motor for each of the semi-active shock absorbers optimized in the above step. The total orifice area for each mass is the sum of the orifice area, depending on the displacement of the piston tube, and each bypass orifice area. The bypass orifice area is set as zero when the mass is maximum. The total orifice area and the relative velocity of the piston tube with respect to the cylinder determine the resisting force due to the dynamic pressure of the semi-active shock absorber. Third, an optimum trade-off curve for each of the discrete masses is generated from

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the maximum acceleration of the aircraft mass and the maximum displacement of the piston tube of each of the optimum semi-active shock absorbers. The optimum trade-off curve in the multi-optimum optimization is generated from the Pareto optimal set. When the acceleration of the mass is minimized, the total resisting force, which consists of the resisting forces due to the dynamic pressure and the gas pressure inside the shock absorber, is also minimized. Therefore, for each of the discrete masses, we can generate an optimum trade-off curve with respect to the maximum total resisting force and the maximum displacement of the piston tube of each of the optimum semi-active shock absorbers. For semi-active shock absorbers, we define an optimum trade-off chart for each maximum acceleration of the aircraft mass and resisting force of the shock absorber as a set of optimum trade-off curves. For passive shock absorbers, we define an optimum trade-off chart for each maximum acceleration of the aircraft mass and resisting force of the shock absorber as an optimum trade-off curve by multi-objective optimization when the mass is the maximum and trade-off curves obtained by simulation when the mass is less than the maximum value. Finally, we consider whether the method of multi-objective optimization, as well as the optimum trade-off charts considering mass variation proposed in the present paper, is effective in the design of semi-active and passive shock absorbers. Furthermore, we consider modifying this optimization problem and the optimum trade-off charts for application to another type of semi-active shock absorber for landing gear. In addition, we discuss the future development of these optimum trade-off charts. Table 1 lists the nomenclature used in the present paper.

2. Passive and Semi-active shock absorbers
2.1 Construction of passive and semi-active shock absorbers

Figure 1 shows the construction of a conventional passive shock absorber, and Fig. 2 shows the construction of a semi-active shock absorber with a bypass orifice, as proposed in a previous study (Maemori, et al., 2003), with orifice 1 and orifice 2 (bypass orifice) controlled for mass variation. Unlike the passive shock absorber, the semi-active shock absorber has a hollow metering pin, a control tube, holes 1, holes 2, orifice 2, and a stepping motor. At touchdown, the fluid in chamber A flows into chamber B through orifice 1 in the passive shock absorber (as shown in Fig. 1) and when mass $m_1$ is maximum in semi-active shock absorber (as shown in Fig. 2). When $m_1$ is less than maximum in the semi-active shock absorber, part of the fluid in chamber A flows into chamber B through orifice 1, and the remaining fluid flows into chamber B through orifice 2 (as shown in Fig. 2). The area of orifice 1 is determined by the circular hole and the metering pin and can be varied corresponding to only the relative displacement of the piston tube with respect to the cylinder. The area of orifice 2 is the area of the overlap formed by holes 1 and holes 2 (as shown in Fig. 2(b), the total number of holes, including holes 1 and 2 is four) and can be varied by the rotation of the control tube by the stepping motor, corresponding to the variation of mass $m_1$. The gas in chamber B is compressed by the total volume of fluid flowing through orifices 1 and 2. The pressure of the compressed gas produces the resisting force of the shock absorber spring. Dimensions $l_A$ and $l_B$ are the lengths of chambers A and B, respectively. The smaller these values, the smaller the shock absorber.

**Fig. 1** Schematic diagrams of passive shock absorber

**Fig. 2** Schematic diagrams of semi-active shock absorber
### 2.2 Resisting force of passive and semi-active shock absorbers and equations of motion

The resisting force $f_k$ of the shock absorber spring is expressed as

$$f_k = A_p p_0 [V_0 / (V_0 - A_h x_p)] - p_a A_p$$

where $p_0$ and $V_0$ are the initial pressure and volume of the gas in chamber B, and $x_p = x_1 - x_2$ (as shown in Fig. 3).

The resisting force $f_D$ due to the dynamic pressure is expressed as

$$f_D = (D_b / a^2) x_p^2 . D_n = \rho A_i / (2 C_p)$$

where $a$ is the total orifice area of orifices 1 and 2.

The total resisting force $f$ of the shock absorber is expressed as

$$f = f_k + f_D$$
Table 2 Optimization problems

| Type of shock absorber | Passive and semi-active shock absorbers | Semi-active shock absorber |
|------------------------|----------------------------------------|-----------------------------|
| Mass \( m_{i,n} \), step | \( n=1 \) | \( n=2,3,\ldots,N \) |
| Objective functions | \( \phi(D_i) = \max \left( \sum_{i=1}^{n} w_i \left| \dot{x}_{i,1} \right|, w_i x_{i,1,max} \right) \) (6) | \( \phi(D_\psi) = \max \left( \left| \dot{x}_{i,1} \right| \right) \) (13) |
| Total orifice area | \( a_i = a_m = (b_i - b_{i-1})(x_p - x_{p,i-1}) / (x_{p,i} - x_{p,i-1}) + b_{i-1} \) (7) | \( a_i = a_m^* + a_{\alpha,n} \) |
| Design variables | \( D_i = (b_1, b_2, \ldots, b_i, \ldots, b_{\text{max}}, \psi)^T \) (10) | \( D_i = a_{\alpha,n} \) (15) |
| Number of design variables | \( n_{pv} = l_{\text{max}} + 1 \) (11) | \( n_{pv} = 1 \) (16) |
| Constraints | \( b_i - a_{\psi,i} \leq 0, \quad a_{\psi,i} - b_i \leq 0 \) (\( i = 1,2,\ldots,l_{\text{max}} \)) | \( a_{n,i} - a_{n,I} \leq 0, \quad a_{n,I} - a_{n,i} \leq 0 \) |
| \( g_j (j = 1,2,\ldots,J_v) \) | \( x_{p,1,max} - x_{p,i+1} \leq 0, \quad \psi - \psi_i \leq 0, \quad \psi - \psi_{i-1} \leq 0 \) (12) | \( x_{p,1,max} - x_{p,i+1} \leq 0 \) (17) |

When the lift acting on masses \( m_1 \) and \( m_2 \) is equal to the gravitational force acting on these masses, the equation of motion for the two-mass system shown in Fig. 3 is expressed as

\[
\ddot{x}_i = -f / m_i, \quad \ddot{x}_2 = (f - kx_2) / m_2
\]

When \( 0 < x_p < x_{p,II} \), and as

\[
\ddot{x}_i = -kx_2 (m_1 + m_2), \quad \ddot{x}_2 = \ddot{x}_i
\]

When \( x_\psi = 0 \) or \( x_p = x_{p,II} \),

3 Method of Optimization

We explain the optimization problems using Table 2. In the optimization, we consider the mass \( m_1 \) of the aircraft as a discrete mass \( m_{1,n} (n = 1,2,\ldots,N) \). In the \( n = 1 \) case, namely, when \( m_1 \) is equal to the maximum mass \( m_{1,\text{max}} (= m_{1,3}) \), we optimize a parameter with respect to the displacement \( x_\psi \) of the piston tube and the initial volume of gas inside the shock absorber and optimize orifice area \( a_{\psi} \) depending on the displacement \( x_p \). In this step, the functions of the passive and semi-active shock absorbers are identical. In the \( n = 2,3,\ldots,N \) case, when mass \( m_1 \) is \( m_{1,n} (n = 2,3,\ldots,N) \), namely, when \( m_1 \) is less than the maximum mass \( m_{1,\text{max}} \) and greater than or equal to the minimum mass \( m_{1,\text{min}} (= m_{1,0}) \), we optimize the area of the bypass orifice controlled for the variation of \( m_1 \) in only the semi-active shock absorber. (The passive shock absorber does not have this bypass orifice.)

3.1 Optimization problems

3.1.1 Multi-objective optimization of passive and semi-active shock absorbers for the case of maximum mass: \( n = 1 \)
3.1.1.1 Objective function

Using the weighted minimax method, as shown by Eq. (6) in Table 2, we set the maximum vertical acceleration \( \dot{x}_{1,1,\text{max}} \) of aircraft mass \( m_{1,1} \) and the maximum displacement \( x_{p,1,\text{max}} \) of the piston tube of the passive and semi-active shock absorbers as a multi-objective function \( \psi(D_i) \) to be minimized, where \( w_1 \) and \( w_2 \) are weights. When the acceleration \( \dot{x}_{1,1,\text{max}} \) is minimized, the maximum total resisting force \( f_{1,\text{max}} \) of the passive and semi-active shock absorbers is also minimized, because, from Eq. (4), the maximum total resisting force \( f_{1,\text{max}} = m_{1,1} \dot{x}_{1,1,\text{max}} \).

3.1.1.2 Design variables

Passive shock absorbers are generally optimized for rated conditions. In other words, the orifice areas of the shock absorbers are optimized so that the total resisting force of the shock absorbers is nearly constant from beginning to end when both the impacting masses and impacting velocities have rated values. Similarly, in the design of the semi-active shock absorber, we optimize the orifice area \( a_{op} \) depending on displacement \( x_p \) during landing when mass \( m_1 = m_{1,\text{max}} \). The bypass orifice area is set as zero when the mass is maximum. Therefore, total orifice area \( a_i \) is equal to \( a_{op} \).

We express orifice area \( a_{op} \) in a piecewise linear approximation (as shown in Fig. 4 and Eq. (7) in Table 2) using the design variables as given by

\[
b = (b_1, b_2, \ldots, b_{i,\text{max}})^T
\]

which is optimized in this step only, where \( b_i \) \((i = 1, 2, \ldots, \text{Imax})\) (black circle, shown in Fig. 4) is the orifice area at position \( x_{p,i} \) of the piston tube. Orifice area \( a_{op} \) is a function of displacement \( x_p \), design variables \( b \), and specified value \( b_{i,\text{U}} \) (as determined by Fig. 4 and Eqs. (7) and (8)). Using the piecewise linear form, we can express various shapes of the curve and cope with allowable accuracy regarding the relationship between orifice area \( a_{op} \) and displacement \( x_p \). If lower accuracy of orifice area \( a_{op} \) is allowed, then we can reduce the number \( I_{\text{max}} \) of design variables \( b \), so that obtaining optimum design variables becomes easier (Maemori, et al., 2004).

We next consider whether we are able to adequately set this multi-objective optimization problem by setting design variables \( b \) only. If the initial volume and initial pressure of gas inside each shock absorber is identical, then each maximum displacement of the piston tube of the optimum shock absorbers, which is one of the objective functions, will be almost identical, because the maximum displacement of the piston tube is restricted by the resisting force due to the pressure of the gas inside the shock absorber (Kobayashi, et al., 2009), so that we will not be able to establish a multi-objective optimization problem.

Therefore, we propose to introduce a design variable with respect to the initial volume of gas inside the shock absorbers in this step as

\[
\psi = V_0 \big/ V_{0,\text{st}} = \left( x_p \big/ x_{p,\text{st}} \right) \left( x_{p,i} \big/ x_{p,i,\text{st}} \right) \quad (i = 1, 2, \ldots, \text{Imax}, IU)
\]

where the subscript \( st \) denotes the values of the standard passive and semi-active shock absorbers. Furthermore, we set the inside diameter \( d \) of gas chamber B (shown in Fig. 1) and the dimensions, such as \( A_D, A_I \) and \( p_0 \), of various shock absorbers so as to be identical to those of a standard shock absorber. In this case, Eq. (9) indicates that the upper limit of length \( l_A \) (as shown in Fig. 1) of chamber A of each of the various shock absorbers is different from that of the standard shock absorber, as is the upper limit of length \( l_B \) of the gas chamber of each of the various shock absorbers. Thus, we can deal with various shock absorbers that differ in total length, which is defined as \( l_A + l_B \) in this multi-objective optimization.

Adding this design variable \( \psi \), the design variables \( D_i \) and the number \( n_{DI} \) of components of \( D_i \) in the optimization step are expressed as Eqs. (10) and (11) in Table 2.

3.1.1.3 Constraints

In this step of the optimization, the design variables \( D_i \) are subject to the constraints of Eq. (12) in Table 2. In addition, the upper limit for the maximum displacement \( x_{p,1,\text{max}} \) is given as Eq. (12) in Table 2, so that the optimum orifice area \( a_{op} \) is independent of orifice area \( a_{U} \), as shown in Eq. (7).

3.1.1.4 Optimization problem
The optimization problem is generally expressed as a minimax problem, as shown in Table 2, where \( J_n \) is the number of constraints.

### 3.1.2 Single-objective optimization of the semi-active shock absorber for the case of an arbitrary mass that is less than the maximum: \( n = 2, 3, \ldots, N \)

#### 3.1.2.1 Objective function

For each of the optimum semi-active shock absorbers obtained in the above step \((n = 1)\), we set the maximum vertical acceleration \( \tilde{x}_{1,n}^{\text{max}} \) of the aircraft mass \( m_{1,n} \) as an objective function \( \phi(D_n) \) \((n = 2,3,\ldots,N)\), as shown by Eq. (13) in Table 2. If the number of optimum semi-active shock absorbers obtained in the \( n = 1 \) case is \( q \), then the number of objective functions \( \phi(D_n) \) to be minimized is \( q \) times \((N - 1)\), because each of \( q \) types of optimum shock absorbers is optimized for each of \((N - 1)\) types of masses. When the acceleration \( \tilde{x}_{1,n}^{\text{max}} \) is minimized, the maximum total resisting force \( f_{n,\text{max}} \) of the semi-active shock absorber is also minimized.

#### 3.1.2.2 Design variables

In the \( n = 2, 3, \ldots, N \) case, using the optimum orifice area \( a_{p,\text{opt}}^* \) obtained in the first step, we express the total orifice area \( a_n \) as Eq. (14) in Table 2 and optimize bypass orifice area \( a_{m,n} \) (shown in Fig. 4 and Eq. (15) in Table 2) as the design variable \( D_n \) for mass \( m_{1,n} \) \((m_{\text{min}} \leq m_{1,n} < m_{\text{max}})\). The number of design variables is given by Eq. (16) in Table 2.

#### 3.1.2.3 Constraints

The design variable is subject to the constraints of Eq. (17) in Table 2.

#### 3.1.2.4 Optimization problems

The optimization problems are generally expressed as shown in Table 2.

### 3.2 Optimization technique

#### 3.2.1 Multi-objective optimization of passive and semi-active shock absorbers for the case of maximum mass: \( n = 1 \)

The optimization problem in this step is solved Genetic algorithms (GA), in which all of the upper limits for the design variables must be identical, as must be the case for all of the lower limits for the design variables. Therefore, we transform the design variables \( \psi \) and \( D_i \) to the new design variables \( \overline{\psi} \) and \( \overline{D}_i \) as follows:

\[
\overline{\psi} = (a_{p,\text{U}} - a_{p,\text{L}})(\psi_U - \psi_L) / (\psi_U - \psi_L) + a_{p,\text{L}}, \quad \overline{D}_i = (b_1, b_2, \ldots, b_i, \ldots, b_{\text{max}}, \overline{\psi})
\]

where the constraints for \( \overline{\psi} \) are

\[
a_{p,\text{L}} \leq \overline{\psi} \leq a_{p,\text{U}}
\]

These design variables constraints are included in the genetic algorithm. The only remaining constraint in this step is as follows:

\[
x_{p,1,\text{max}} - x_{p,1,\text{max}}^* \leq 0
\]

By transforming the objective function \( \phi(\overline{D}_i) \) with the constraint shown in Eq. (20) using an exterior penalty function, we obtain the new unconstrained objective function to be minimized as

\[
P(\overline{D}_i) = \max(w_1|x_{\text{max}}^{\text{max}}|, w_2x_{p,1,\text{max}}^*) + r\max\left\{0, (x_{p,1,\text{max}}^* - x_{p,1,\text{max}})\right\}^2
\]

In addition, since genetic algorithms are basically optimization methods in which objective functions are maximized, we must transform the objective function \( P(\overline{D}_i) \) to be minimized to the function \( F(\overline{D}_i) \) to be maximized as
where \( P_T \) is a positive parameter.

### 3.2.2 Single-objective optimization of the semi-active shock absorber for the case of an arbitrary mass that is less than the maximum: \( n = 2, 3, \ldots, N \)

By transforming the objective function \( \phi(D_n) \) with the constraint shown in Eq. (17) using an exterior penalty function, we obtain the new unconstrained objective function to be minimized as

\[
P(D_n, r_k) = \sum_{i=1}^{N} \left[ \max \{0,(a_{n,i} - a_{u,i})\}^2 + \max \{0,(a_{n,i} - a_{w,i})\}^2 + \max \{0,(x_{p,n,\max} - x_{p,i})\}^2 \right] \tag{23}
\]

where \( r_k \) is a positive coefficient, and \( r_{K+1} > r_K \).

This optimization problem is solved using Powell’s conjugate direction method with quadratic interpolation.

### 4 Example of Trade-off Charts

#### 4.1 Example of generating optimum trade-off charts

The procedure for generating the optimum trade-off charts is as follows. First, from the multi-objective optimization of a semi-active shock absorber for the case of a maximum mass of 130t \((n=1)\), we determine the relationship between the maximum acceleration of the mass and the maximum displacement of the piston tube of the optimum shock absorber, as well as that between the maximum resisting force of the optimum shock absorber and the maximum displacement of its piston tube. These two relationships for the optimum semi-active shock absorber are identical to those for the optimum passive shock absorber. Second, from the single-objective optimization of the semi-active shock absorber for the case of a mass of 85t \((n=2)\) and for the case of a minimum mass of 40t \((n=3)\), we obtain two relationships. Third, for the passive shock absorber, we obtain two relationships through a simulation. Finally, we generate two types of charts from the three relationships among the cases of masses of 130, 85, and 40t for each of the optimum passive and optimum semi-active shock absorbers.

The parameters used in the calculations are \( A_p = 0.123 \text{m}^2 \), \( b_{st} = 0 \text{m}^2 \), \( D_D = 0.387 \text{kg} \cdot \text{m}^3 \), \( I_{\text{max}} = 5 \), \( I_{U} = 6 \), \( k = 4.3 \times 10^8 \text{N/m} \), \( m_2 = 1.8 \times 10^4 \text{kg} \) \( e = 1.1 \), \( p_0 = 10^5 \text{Pa} \), \( p_n = 1.01325 \times 10^5 \text{Pa} \), \( V_{0,n} = 7.3 \times 10^2 \text{m}^3 \), \( \dot{x}_{l,0} = 3 \text{m/s} \), \( x_{p,i,\text{st}} = 0, 0.13, 0.26, 0.39, 0.52 (= x_{p,\text{max,n}},) \), and 0.65 m (= \( x_{p,\text{U,n}} \)) (dotted line in Fig. 6).

In the \( n = 1 \) case, the parameters are \( a_{p,L} = 10^{-2} \text{m}^2 \), \( a_{p,U} = 7 \times 10^{-3} \text{m}^2 \), \( m_1 = m_{1,1} = 130t (= m_{\text{max}},) \), \( n_{\text{DV}} = 6 \), \( n_{\text{gen,\max}} = 4000, n_{\text{pop}} = 5, 6, 9, 10, P_T = 2, r = 10^4, w_1 = 1, w_2 = 0.1, 0.4, 0.5, 0.7, 0.9, 1.15, 2, \psi_L = 0.05, 0.1, \) and \( \psi_U = 2, 3 \). The values of \( P_T, r, \) and \( n_{\text{gen,\max}} \) were determined by trial and error in preparatory calculations. The starting points of the design variables \( \vec{D} \) were randomly determined in the range between \( a_{p,L} \) and \( a_{p,U} \). The initial random number seed is \( \text{idum} = -1000 \) and -10000.

In the \( n = 2 \) and \( n = 3 \) cases, the parameters are \( a_{n,L} = 10^{-2} \text{m}^2 \), \( a_{n,U} = 7 \times 10^{-3} \text{m}^2 \), \( m_1 = m_{1,2} = 85t. m_1 = m_{1,3} = 40t (= m_{\text{min}},) \), \( n_{\text{DV}} = 1, r_1 = 1, \) and \( r_{K+1}/r_K = 5 \). The values of \( r_1 \) and \( r_{K+1}/r_K \) were determined by trial and error and were used in our previous studies. For \( m_1 = 40t \) and 85t, the starting points were selected as \( a_{m,n} = 7 \times 10^{-3} \) and \( 10^{-5} \text{m}^2 \), respectively.

We obtained seven optimum semi-active shock absorbers and seven optimum passive shock absorbers. Furthermore, we added one optimum semi-active shock absorber and one optimum passive shock absorber obtained in the previous study for consideration herein (Maemori, et al., 2004). The results for these optimum shock absorbers are shown in Figs. 5 through 11. The optimum design variables are shown in Figs. 5, 6, and 7, and the optimum trade-off charts are shown in Figs. 10 and 11. Using Figs. 8 and 9, each of the eight solutions was determined to be optimum. In Figs. 5, 7, 10, and 11, approximate optimum straight lines were generated from the optimum points (eight figures) by linear interpolation.

#### 4.1.1 Optimum design variables

Figures 5 through 7 show the optimum design variables.
In the $n = 1$ case, we obtained seven optimum solutions ($\psi^* = 0.846, 1.221, 1.435, 1.689, 2.000, 2.438,$ and $3.000$), namely, seven optimum semi-active shock absorbers and seven passive shock absorbers. Furthermore, to the results obtained for the $n = 1$ case, we added one optimum solution ($\psi^* = 1$), obtained in a previous study (Maemori et al., 2004), as a standard shock absorber for $m_1 = 130t$. This optimum solution ($\psi^* = 1$), which was obtained in a single-objective optimization, would be able to obtain in the multi-objective optimization by determining adequately weights $w_1$ and $w_2$. Fig. 5, the eight white circles denoting the optimum points indicate the relationship between the optimum design variables $\psi^*$ and the maximum displacement $x_{p,1,\max}$ of the piston tube of the eight semi-active and eight passive shock absorbers for $m_1 = 130t$. In addition, seven straight lines generated from the white circles by interpolation indicate the relationship between the approximate optimum design variables $\psi^*$ and the maximum displacement $x_{p,1,\max}$. In Fig. 6, we show five of the eight sets of optimum design variables as examples, where the white symbols indicate optimum design variables $b_i^*(i = 1,2,...,5)$, the black symbols indicate $b_{i\psi}$, and the lines between $b_i^*(i = 1,2,...,5)$ obtained by interpolation indicate optimum orifice area $a_{i\psi}^*$. Using orifice area $a_{i\psi}^*$ shown in Fig. 6, we can decide the diameters of the metering pin of each of the five semi-active and five passive shock absorbers as examples. Generally, each of the design variables $b_i^*(i = 1,2,...,5)$ (white figures in Fig. 7) increases according to the increase of design variable $\psi^*$, i.e., according to the increase in the total length of the shock absorber. However, it is difficult to obtain a better approximate optimum curve from each of the optimum design variables $b_1, b_2,$ and $b_3$ by interpolation, because $b_1, b_2,$ and $b_3$ are slightly scattered, as shown in Fig. 7. Therefore, we must search more optimum points, for example, 20, 30, or 40 points, or more.

In the $n = 2$ case, we obtained seven optimum design variables $a_{m,2}^*$ ($= a_{m}^*$ for 85t) for the $n = 2$ case and $a_{m,3}^*$ ($= a_{m}^*$ for 40t) for the $n = 3$ case. Furthermore, to the results obtained for the $n = 2$ and 3 cases, we added one optimum design variable ($\psi^* = 1$), respectively, obtained in a previous study (Maemori, 2004), as a standard shock absorber. The optimum design variable $a_{m}^*$ increases according to the increase of design variable $\psi^*$ for both $m_1 = 85$ and 40t.

### 4.1.2 Resisting forces of passive and semi-active shock absorbers and time history of acceleration of aircraft mass

#### 4.1.2.1 Passive and semi-active shock absorbers: $n = 1$

We obtained the optimum total resisting force curves (solid line in Fig. 8) for the passive and semi-active shock absorbers for the case of a maximum mass of 130 t and the time histories of the acceleration of the aircraft mass (solid line in Fig. 9) for the optimum passive and semi-active shock absorbers. In Figs. 8 and 9, we show five optimum results
among the eight results (seven obtained for the \( n = 1 \) case and one obtained in a previous study) as examples. The effectiveness of the GA used in the present study was verified in the previous study (Maemori, 2004), in which the objective function, namely, the maximum acceleration of the aircraft mass obtained using the GA was almost identical to that obtained using Powell’s conjugate direction method. Each of the eight solutions was determined to be optimum when the top of the curve of the total resisting force (solid line in Fig. 8), as well as the acceleration (solid line in Fig. 9) of the aircraft mass, was approximately flat and there was no gap (as shown in Fig. 8(a)), where the term gap is the distance measured along a line parallel to the \( x_p \) axis between the end point of the total resisting force curve and the curve (dotted line in Fig. 8) of the resisting force of the shock absorber spring.

This is because the maximum value of the total resisting force curve, as well as the maximum value of the acceleration of the aircraft mass, would be minimized if the top of the curve were completely flat, under a specified energy absorbed by the shock absorber and a specified maximum displacement. In addition, the maximum displacement of the shock absorber with the gap is less than that without the gap. Therefore, under a specified energy absorbed by the shock absorber, the maximum total resisting force of the shock absorber with the gap is generally larger than that without the gap.

4.1.2.2 Semi-active shock absorber: \( n = 2, 3 \)

We obtained the optimum total resisting force curves (shown in Fig. 8) and the time histories of the acceleration of the aircraft mass (shown in Fig. 9) for the semi-active shock absorbers for the cases of masses of 85t (\( n = 2 \), thick dashed line) and 40t (\( n = 3 \), thick dash-dot-dashed line). In Figs. 8 and 9, we show five of the eight optimum results. None of these curves has a flat top, which differing from the curves for the \( n = 1 \) case. This difference is due to only one design variable. In contrast, in the \( n = 1 \) case, the number of design variables with respect to the orifice area is five. Each of the eight solutions was determined to be optimum when the value of the maximum peak of the total resisting...
force curve (thick dashed line and thick dash-dot-dashed line in Fig. 8) as well as that of the acceleration curve (thick dashed line and thick dash-dot-dashed line in Fig. 9) of the aircraft mass was identical to the value of one of the other peaks and there was no gap between the end point of the total resisting force curve (thick dashed line and thick dash-dot-dashed line in Fig. 8) and the curve (dotted line in Fig. 8) of the resisting force of the shock absorber spring.

### 4.1.2.3 Passive shock absorber: $n = 2, 3$

The resisting force curves and time histories obtained by simulation for the passive shock absorbers are shown as the thin dashed line and thin dash-dot-dashed line in Figs. 8 and 9. The curves for the passive shock absorbers are not optimum, because we can optimize passive shock absorbers for only a specified mass (130 t in this case).

### 4.1.3 Optimum trade-off charts

We now explain the optimum trade-off charts proposed in the present paper using Figs. 10 and 11. The dash-dot-dot-dashed lines in Figs. 10 and 11 and the thin solid line in Fig. 10 are lines of constant $\psi^*$. Each of these lines represents one of the eight optimum passive shock absorbers and one of the eight optimum semi-active shock absorbers examined in the present study.

We consider two types of trade-off. One is a relationship between the maximum acceleration $|x_{t,\text{max}}|$ (the value of the top of the curve shown in Fig. 9) of the aircraft mass and the maximum displacement $x_{p,\text{max}}$ (the value of the end point of the curve shown in Fig. 8) of the piston tube of the passive and semi-active shock absorbers in Fig. 10, and the other is a relationship between the maximum resisting force $f_{r,\text{max}}$ (the value of the top of the curve shown in Fig. 8) of the shock absorber and the maximum displacement $x_{p,\text{max}}$ in Fig. 11.

The eight black circles in Figs. 10 and 11 show a Pareto optimal set for the case of an aircraft mass of 130 t ($n = 1$). Here, $x_{p,\text{max}}$ is an objective function $x_{p,\text{t, max}}$. Furthermore, seven straight solid lines obtained from the black circles

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**Fig. 9** Time history of acceleration of aircraft mass

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through linear interpolation show approximate optimum solutions. We then define an optimum trade-off curve for the passive and semi-active shock absorbers as the Pareto optimal set and the straight solid lines generated from the set by the interpolation.

The eight triangles in Figs. 10(b) and 11(b) show an optimal set obtained by means of the single-objective optimization in the case of an aircraft mass of 85t \( n = 2 \), for each of eight optimum solutions \( \psi^* = 0.846, 1, 1.221, 1.435, 1.689, 2.000, 2.438, \) and 3.000), which is obtained for the \( n = 1 \) case. The eight inverted triangles in Figs. 10(b) and 11(b) show an optimal set obtained by means of the single-objective optimization for the case of an aircraft mass of 40t (\( n = 3 \)). The sets of these figures obtained for the cases of \( n = 2 \) and 3 are not Pareto optimal sets, because not all of the maximum displacements \( x_{p,\text{max}} (= x_{p,\text{s, max}}; n = 2, 3) \) are objective functions. Furthermore, we define an optimum trade-off curve for the semi-active shock absorber as the optimal set, which is obtained in the single-objective optimizations, and the lines (dashed lines for the case of 85 t and dash-dot-dashed lines for the case of 40t) generated from the set by linear interpolation.

For semi-active shock absorbers, we define an optimum trade-off chart as a set of the optimum trade-off curves for the cases of aircraft masses of \( m_{1,n} (n = 1,2, 
., N) \) (in the present study, \( N = 3 \), as shown in Fig. 10(b) for the maximum acceleration of the aircraft mass and in Fig. 11(b) for the resisting force of the shock absorber).

For the passive shock absorbers, we define an optimum trade-off chart as a set of the optimum trade-off curve for the case of an aircraft mass of \( m_{1,1} \) (in the present study, \( m_{1,1} = 130t \)) and the trade-off curves for the cases of masses \( m_{1,n} (n = 2,3, 
., N) \) (in the present study \( m_{1,2} = 85t \) and \( m_{1,3} = 40t \), as shown in Figs. 10(a) and 11(a)). The white symbols, the dashed lines, and the dash-dot-dashed lines in Figs. 10(a) and 11(a) show the trade-off curves for the passive shock absorber. However, these trade-off curves are not optimum for masses of 85t and 40t, because we can optimize passive shock absorbers for only a specified mass (130t in this case).

### 4.2 How to use the optimum trade-off charts

#### 4.2.1 Examination of the influence of mass variation on the maximum acceleration of the aircraft mass

Using the optimum trade-off charts shown in Figs. 10 and 11, we can examine the influence of the variation of aircraft mass on the maximum acceleration of the mass and the maximum resisting force of the shock absorber, for each of the shock absorbers of the optimum design variable \( \psi^* \), as shown in Fig. 5. For example, when \( \psi^* = 1.22 \), we obtain \( x_{p,1,\text{max}} = 0.606 \text{ m} \) using Fig. 5. As for the optimum passive shock absorber (as shown in Fig. 10(a)), when \( x_{p,\text{max}} = x_{p,1,\text{max}} = 0.606 \text{ m} \), we obtain \( |\dot{x}_{p,\text{max}}| / g = 0.698 (m_1 = 130t), 0.987 (m_1 = 85t), \) and \( 1.747 (m_1 = 40t) \) along the dash-dot-dotted line, which represents the optimum shock absorber for which \( \psi^* = 1.22 \). As for the optimum semi-active shock absorber (as shown in Fig. 10(b)), when \( x_{p,\text{max}} = x_{p,1,\text{max}} = 0.606 \text{ m} \), we obtain \( |\dot{x}_{p,\text{max}}| / g = 0.698 (m_1 = 130t), 0.789 (m_1 = 85t), \) and \( 1.01 (m_1 = 40t). \)

Furthermore, we can select the best shock absorber from a set of the optimum shock absorbers considering the mass variation. For example, when a required condition for a shock absorber is that \( x_{p,\text{U}} \leq 1.3 \text{ m} \), we obtain \( \psi^* \leq 2 \) using Eq. (9). Then, we can examine the influence of the variation of aircraft mass on the maximum acceleration of the mass, in the same manner as that for \( \psi^* = 1.22 \) using Fig. 10. As a result, we obtain the optimum semi-active shock absorber for which \( \psi^* = 2 \).

#### 4.2.2 Comparison of the optimum semi-active shock absorber with the optimum passive shock absorber

For each of the eight optimum passive shock absorbers, the maximum acceleration \( |\dot{x}_{p,\text{max}}| / g \) of mass \( m_1 \) increases according to the decrease (from 130t to 40t) in mass \( m_1 \), as shown in Fig. 10(a) (dash-dot-dotted lines). The reason for this increase is that the rate of the decrease of the maximum resisting force \( f_{\text{max}} \) is less than that of mass \( m_1 \), as shown in Fig. 11(a), where the maximum acceleration \( |\dot{x}_{p,\text{max}}| \) is obtained from the following equation: \( |\dot{x}_{p,\text{max}}| = f_{\text{max}} / m_1 \). For each of the eight optimum semi-active shock absorbers, the rate of increase in the maximum acceleration \( |\dot{x}_{p,\text{max}}| / g \) due to the decrease in mass \( m_1 \) is less than that for the case of the optimum passive shock absorber, as shown in Fig. 10(b) (dash-dot-dotted lines), because, in the semi-active shock absorber, not only is the resisting force \( f \) relatively constant, but each maximum displacement \( x_{p,\text{max}} \) is also greater than that in the passive shock absorber (Fig. 8).
4.2.3 Replacement of the optimum passive shock absorber with the optimum semi-active shock absorber

Using the optimum trade-off chart shown in Fig. 10, we consider replacing the optimum passive shock absorber with the optimum semi-active shock absorber. In the case of the optimum semi-active shock absorber, for example, when the optimum design variable is $\psi^* = 1$ (thin solid line), the maximum acceleration $\dot{\ddot{x}}_{\text{max}}/g$ is less than or equal to that for the case of the optimum passive shock absorber, as shown in Fig. 10. Namely, the ratio of the maximum acceleration in the optimum semi-active shock absorber to that in the optimum passive shock absorber is 1 when $m_1$ is 130 t, 0.79 when $m_1$ is 85 t, and 0.58 when $m_1$ is 40 t.

4.2.4 Replacement of the optimum semi-active shock absorber with the optimum passive shock absorber

We demonstrate how to use the optimum trade-off charts shown in Figs. 10 and 11 in order to determine $x_{p,1,\text{max}}$ and $\psi^*$ for the optimum passive shock absorber for the case in which $\dot{\ddot{x}}_{\text{max}}/g$ is equal to those of the semi-active shock absorber. For example, when $\psi^* = 1$ ($x_{p,1,\text{max}} = 0.506$ m, as shown in Fig. 5, for $m_1 = 130$ t) and $m_1 = 40$ t, for the case of the semi-active shock absorber, we obtain point A, at which $\dot{\ddot{x}}_{\text{max}}/g = 1.14$, which is determined using Fig.
10(b). For $m_1 = 40t$, in the case of the passive shock absorber, we can determine point B, at which $\frac{\dot{x}_1}{g} = 1.14$, by an interpolation using Fig. 10(a). We then determine point C along line BC, which indicates constant $\psi$, and, from Fig. 5, determine that $x_{p,max} = 1.17 m$ and $\psi' = 2.71$ (point A in Fig. 5) by interpolation. Therefore, we know that the sum of lengths $l_A$ and $l_B$ (shown in Fig. 1) in the optimum passive shock absorber must be increased by 174% compared to that (shown in Fig. 2) in the semi-active shock absorber, as calculated by Eq. (9), when the upper limit of length $l_B$ (shown in Fig. 1) is equal to $x_{p,U}$ (Choi, et al., 2003).

4.3 Optimization problems and optimum trade-off charts for another type of semi-active shock absorber

Next, we modify this optimization problem and the optimum trade-off charts proposed in the present paper for another type of semi-active shock absorber for landing gear, such as a shock absorber featuring magnetorheological fluid (Choi, et al., 2003 and Kobayashi, et al., 2009). If a semi-active shock absorber featuring magnetorheological fluid has four types of resisting forces, namely, forces due to dynamic pressure, fluid viscosity, gas pressure, and apparent viscosity of the fluid due to an applied electric current, and three types of resisting forces (all of the above, except for the apparent viscosity of the fluid) function when the aircraft mass is a minimum, and all of the resisting forces function when the mass is greater than the minimum value, we recommend the following optimization problems:

(1) when the mass is minimum, the optimization problem will be similar to the multi-objective optimization problem for the semi-active shock absorber with the bypass orifice, except for the mass. In this optimization step, the range of the displacement in the optimization of the orifice area depending on the displacement of the piston tube will be narrower than that in the case of the semi-active shock absorber with the bypass orifice, because the maximum displacement of the piston tube will be shorter than that for the semi-active shock absorber with the bypass orifice due to the lighter mass.
(2) when the mass is maximum, the objective function is the maximum acceleration of the mass, and the design variables are the amount of electric current and the remaining orifice area depending on the displacement of the piston tube.
(3) when the mass is between the maximum and minimum masses, each optimization problem will be similar to the single objective optimization problem for the semi-active shock absorber with the bypass orifice, except for the design variable. In this optimization step, the design variable is the amount of electric current.
(4) If these optimization problems are solved successfully, optimum trade-off charts will be generated in a manner similar to that for the semi-active shock absorber with the bypass orifice.

4.4 Future development of optimum trade-off charts

The optimum trade-off charts proposed in the present paper are intended as a prototype set and so should be refined for practical use. First, additional values will be selected for aircraft mass $m_1$. Second, although in the present paper the number of optimum points is eight for each mass $m_1$, additional optimum points will be searched in the future. In addition, the optimum trade-off charts will become more effective if parameters such as aircraft mass $m_1$, displacement $x_p$ of the shock absorber, and descent velocity $\dot{x}_{1,0}$ are non-dimensionalized.

5 Conclusions

We proposed two types of optimum trade-off charts considering mass variation for the design of a semi-active shock absorber with a bypass orifice and a passive shock absorber for landing gear. Each of the optimum trade-off charts is formed from two kinds of lines. On each line of one kind, the aircraft mass is constant, and on each line of the other kind, the aircraft mass varies from maximum to minimum values. In order to generate the optimum trade-off charts considering mass variation by means of a multi-objective optimization, we proposed the introduction of a parameter with respect to the initial volume of gas inside the shock absorbers, as a design variable. Using the optimum trade-off charts, the optimum semi-active shock absorber for landing gear was compared to the optimum passive shock absorber. Furthermore, we considered the modification of this optimization problem and the optimum trade-off charts
to those for another type of semi-active shock absorber for landing gear. Moreover, we discussed the future development of the optimum trade-off charts proposed in the present paper.

The conclusions are summarized as follows:

(1) The initial volume of gas inside the shock absorber was an effective parameter for setting and solving the multi-objective optimization problem and for generating the optimum trade-off charts considering mass variation.

(2) The optimum trade-off charts were effective for evaluating and comparing the performance of semi-active and passive shock absorbers considering mass variation and for appropriately replacing a passive shock absorber with a semi-active shock absorber.

(3) It appears that it is possible to modify the optimization problems and the optimum trade-off charts for the semi-active shock absorber with the bypass orifice to those for a shock absorber featuring a magnetorheological fluid.

(4) The optimum trade-off charts will become more effective if the charts are refined and parameters, such as aircraft mass and displacement of shock absorbers, are non-dimensionalized.

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