Statistical process control for AR(1) or non-Gaussian processes using wavelets coefficients

A Cohen, T Tiplica and A Kobi
L’UNAM, LARIS Systems Engineering Research Laboratory, ISTIA Engineering School, 62 Avenue Notre Dame du Lac 49000 Angers, France
E-mail: achraf.cohen@univ-angers.fr

Abstract. Autocorrelation and non-normality of process characteristic variables are two main difficulties that industrial engineers must face when they should implement control charting techniques. This paper presents new issues regarding the probability distribution of wavelets coefficients. Firstly, we highlight that wavelets coefficients have capacities to strongly decrease autocorrelation degree of original data and are normally-like distributed, especially in the case of Haar wavelet. We used AR(1) model with positive autoregressive parameters to simulate autocorrelated data. Illustrative examples are presented to show wavelets coefficients properties. Secondly, the distributional parameters of wavelets coefficients are derived, it shows that wavelets coefficients reflect an interesting statistical properties for SPC purposes.

1. Introduction
Statistical process control (SPC) gathers several statistical techniques in order to monitor manufacturing processes and service operations. SPC mainly involves the implementation of control charts. They are among the most widely techniques used to detect changes in Gaussian models parameters. Based on statistics and exploiting the data collected from processes, control charts are particularly designed to detect assignable causes. Various procedures and control schemes have been proposed for detecting mean and/or variance change. The Xbar-R and Xbar-S are the firsts control schemes proposed [1]. Afterwards, EWMA [2] and CUSUM [3] are introduced, which are more effective than Xbar chart, especially for small shifts. Nowadays, ideas to vary control charts parameters (sample size, sampling interval, control limit coefficient) have been developed considerably by proposing several charts, called adaptive control charts. For more details, see [4–6]. Recently, a comparison study was done to evaluate the detection performance of nine control charts, for monitoring process mean [7]. It shows that CUSUM/EWMA are the best charts in terms of FSSI (Fixed Sample Size and Sampling Intervals) control charts.

In the last decade the multi-scale SPC techniques, which combining SPC classical models (X̄, EWMA, CUSUM, PCA, etc.) and multi-scale decomposition using wavelets analysis [8–11], are taking a big part in the SPC literature, especially in the multivariate context [12–20]. Most of the published papers that investigated this combination focused on the use of wavelets as data preprocessing tool, such as de-noising [21–23], data reduction [24; 25], or feature extraction [26; 27], in order to improve the detection performance. These research works are mainly motivated by the ability of wavelets analysis to extract signals time-frequency/scale components. For
example to identify the corrosion intensity the time-frequency plan was used to localize the corrosion [28; 29].

Independence is one of the two hypotheses that most SPC procedures suppose valid. However, data collected from processes, mostly via sensors, are generally autocorrelated. This aspect can also exist in quality characteristic data, especially when short sampling interval are used. The main effect of autocorrelation in control charts is the inflation of the false alarms rate, consequently the Average Run Length (ARL) is improperly calculated. To face this issue two approaches exist: 1) adjust control limits to take into account the autocorrelation, it’s done by estimating the true variance of the process and 2) construct a times series model that fits the process data, then monitor the residues using classical SPC tools. A good literature review regarding SPC procedures for monitoring autocorrelated processes is done here [30].

On the other hand, violation of normality hypothesis can also generate dramatic consequences in terms false alarms rate, for more details see [31–33]. Three solutions are proposed to deal with non-normality: 1) use data transformation techniques in order to make data normal distribution-like, such as Box-Cox [34] and Jonhson [35] transformations; 2) adjust control limits by taking into account data statistical characteristic (kurtosis and skewness) and 3) create control chart that is adapted to the distribution of the data. For instance, CUSUM for observations exponentially distributed are proposed [36].

In these references [13; 26] authors noted that wavelets analysis has the ability to decrease strongly autocorrelation degree, also wavelets coefficients are normally distributed even if original data are not Gaussian. Nevertheless, there is no study in the literature regarding the evaluation of autocorrelation and normality of wavelets coefficients. We note also that there is a lack of case studies in order to highlight the benefit of using wavelets coefficients to face autocorrelation and normality issues for SPC procedures.

The goal of this paper is to propose a result regarding wavelets coefficients probability distribution, using orthogonal wavelets families. Therefore we shed light on the use of wavelets coefficients for SPC purpose. We present briefly the result regarding the evaluation of normality and autocorrelation of wavelets coefficients.

This paper is organized as follows: the second section introduces the wavelets coefficients and their distributional characteristics. We present the findings related to study of autocorrelation and normality of wavelets coefficients. In the third section we are presenting the results and discussions for SPC issues. Finally, in the last section conclusions and perspectives are presented.

2. Statistical properties of wavelets coefficients

MultiResolution Analysis (MRA) [37] provides multiscale decomposition using orthogonal wavelets families across filter banks. MRA has led to create Fast Wavelet Transform (FWT) and then multiply applications of wavelets, especially in image processing. For more details see [8; 10; 37]. Wavelets coefficients, approximations $a_j(k)$ and details $d_j(k)$, are given as follows:

$$a_j(k) = \sum_{i=0}^{l} h[i]a_{j-1}[2k - i]$$

$$d_j(k) = \sum_{i=0}^{l} g[i]a_{j-1}[2k - i]$$

Where $a_0 = x$ the original signal, $j$ represents the decomposition scale; $k \in Z$; $l$ is the filter length; $h$ and $g$ are scaling and wavelets filters respectively.

2.1. Autocorrelation & wavelets coefficients

This part aims to evaluate the wavelets coefficients Autocorrelation Function (AF). We used an autoregressive model AR(1) with positive autocorrelation in order to simulate autocorrelated

$$A(k) = \sum_{i=0}^{l} h[i]a_{j-1}[2k - i]$$

$$D(k) = \sum_{i=0}^{l} g[i]a_{j-1}[2k - i]$$

Where $A_0 = x$ the original signal, $j$ represents the decomposition scale; $k \in Z$; $l$ is the filter length; $h$ and $g$ are scaling and wavelets filters respectively.
data. This model is used by several authors to evaluate control charts performance in presence of autocorrelation [38–40]. AR(1) model is defined as follows:

\[ X_t = c + \varepsilon_t + \varphi_1 X_{t-1} \] (3)

Where \( \varepsilon_t \sim N(0, \sigma^2_0) \) and we used a stationary model \( (c = 0 \text{ and } |\varphi_1| < 1) \). Simulation study was done for different window size \( N \in \{4, 8, 16, 32, 64, 128\} \), \( \varphi_1 \in \{0.1, 0.2, ..., 0.7, 0.8, 0.9, 0.95\} \) and wavelets families (Haar, Daubechies, Symlets, Coiflets). Results for \( \varphi_1 \in \{0.7, 0.8, 0.9, 0.95\} \) are presented in tables 1 and 2. One can remark that globally wavelets analysis decreases strongly the autocorrelation present in the original data. Moreover, in the case of Haar and db2 wavelets one can remark that serial correlation \( (\text{AF}(T = 2)) \) is equal to zero. For instance, when \( \varphi_1 = 0.95 \) and using \( N = 64 \) we obtained for Haar a serial correlation that equal to \(-0.12 (sd = 0.21) \) (sd: standard-deviation) and for db2 we got \( 0.18(0.28) \). But for db3 one can see from table 2 that autocorrelation is evaluated as \( 0.52(0.22) \). Regarding results in tables 1 and 2, we have concluded that the autocorrelation can be removed using Haar, db2 especially for large signals \( N > 32 \), and by using others wavelets autocorrelation can be strongly debased.

2.1.1. An illustrative example

In this example we show the autocorrelation functions behavior of wavelets coefficients using db2 wavelet and \( N = 128 \). The signal has an autoregressive parameter equal to \( \varphi_1 = 0.95 \), see figure 1(a)-(b). The figure 1(c) shows the autocorrelation function of approximation coefficients (6 coefficients in this case), one can remark that there is no serial correlation \( (0.03(0.30), \text{see the table 2}) \), the same conclusion can be achieved in the case of wavelets coefficients (App. + Det.). By the same way, the figure 1(d) shows the autocorrelation function of details coefficients. It is clear that details coefficients are sightly autocorrelated \( (-0.31(0.16), \text{see the table 2}) \), but it could be very acceptable compared to the autocorrelation of original data \( \varphi_1 = 0.95 \). For more details see tables 1 and 2.

![Figure 1](image-url)

**Figure 1.** Wavelets coefficients autocorrelation functions; (a): original data X with \( \varphi_1 = 0.95 \); (b): autocorrelation function of X; (c): autocorrelation function of approximation coefficients of X using db2; (d): autocorrelation function of details coefficients of X using db2
Table 1. Evaluation of wavelets coefficients autocorrelation (App. : approximation, Det. : detail)
| $N$ | Coefficients | Haar | db2 | db3 | db4 | db5 | sym2 | sym3 | sym4 | sym5 | coif1 | coif2 | coif3 | coif4 | coif5 |
|-----|--------------|------|-----|-----|-----|-----|------|------|------|------|-------|-------|-------|-------|-------|
| 4   | App.+Det.    | -0.23(0.24) | —   | —   | —   | —   | —    | —    | —    | —    | —      | —      | —      | —      | —      |
|     | App.         | —     | —   | —   | —   | —   | —    | —    | —    | —    | —      | —      | —      | —      | —      |
|     | Det.         | -0.32(0.24) | —   | —   | —   | —   | —    | —    | —    | —    | —      | —      | —      | —      | —      |
| 8   | App.+Det.    | -0.20(0.23) | 0.26(0.26) | —   | —   | —   | —   | —    | —    | —    | —      | —      | —      | —      | —      |
|     | App.         | —     | —   | —   | —   | —   | —    | —    | —    | —    | —      | —      | —      | —      | —      |
|     | Det.         | -0.13(0.30) | -0.32(0.27) | —   | —   | —   | —   | —    | —    | —    | —      | —      | —      | —      | —      |
| 16  | App.+Det.    | -0.15(0.21) | 0.38(0.26) | 0.50(0.23) | 0.51(0.22) | —   | —   | —    | —    | —    | —      | —      | —      | —      | —      |
|     | App.         | —     | —   | —   | —   | —   | —    | —    | —    | —    | —      | —      | —      | —      | —      |
|     | Det.         | -0.09(0.27) | -0.25(0.22) | -0.19(0.27) | -0.14(0.27) | —   | —   | —    | —    | —    | —      | —      | —      | —      | —      |
| 32  | App.+Det.    | -0.10(0.25) | 0.32(0.28) | 0.52(0.22) | 0.54(0.23) | 0.68(0.15) | 0.32(0.27) | 0.53(0.23) | 0.60(0.22) | 0.56(0.19) | 0.58(0.19) | 0.70(0.16) | —      | —      | —      |
|     | App.         | —     | —   | —   | —   | —   | —    | —    | —    | —    | —      | —      | —      | —      | —      |
|     | Det.         | -0.01(0.21) | 0.18(0.28) | 0.43(0.25) | 0.47(0.26) | 0.63(0.18) | 0.18(0.26) | 0.44(0.25) | 0.53(0.24) | 0.53(0.19) | 0.53(0.19) | 0.66(0.17) | —      | —      | —      |
| 64  | App.+Det.    | -0.12(0.21) | 0.18(0.28) | 0.48(0.22) | 0.51(0.21) | 0.61(0.18) | 0.11(0.29) | 0.48(0.21) | 0.54(0.23) | 0.49(0.22) | 0.49(0.22) | 0.67(0.16) | 0.78(0.11) | 0.78(0.11) | 0.78(0.11) |
|     | App.         | —     | —   | —   | —   | —   | —    | —    | —    | —    | —      | —      | —      | —      | —      |
|     | Det.         | -0.11(0.23) | -0.30(0.17) | -0.26(0.16) | -0.25(0.16) | -0.23(0.15) | -0.30(0.17) | -0.28(0.15) | -0.24(0.14) | -0.25(0.14) | -0.25(0.14) | -0.25(0.14) | -0.30(0.13) | -0.20(0.16) | -0.19(0.17) | -0.18(0.17) |
| 128 | App.+Det.    | -0.11(0.20) | 0.03(0.25) | 0.31(0.22) | 0.33(0.22) | 0.54(0.17) | 0.06(0.25) | 0.30(0.23) | 0.44(0.22) | 0.37(0.21) | 0.37(0.22) | 0.59(0.16) | 0.73(0.11) | 0.73(0.11) | 0.72(0.12) |
|     | App.         | —     | —   | —   | —   | —   | —    | —    | —    | —    | —      | —      | —      | —      | —      |
|     | Det.         | -0.12(0.21) | -0.31(0.16) | -0.28(0.14) | -0.26(0.14) | -0.30(0.13) | -0.32(0.16) | -0.28(0.14) | -0.27(0.12) | -0.27(0.13) | -0.31(0.12) | -0.25(0.10) | -0.26(0.11) | -0.24(0.11) | —      |

**Table 2.** Evaluation of wavelets coefficients autocorrelation (App. : approximation, Det. : detail)
2.2. Normality & wavelets coefficients

The evaluation of the normality of wavelets coefficients was carried out by using 5 normality tests (Jarque-Bera [41], Shapiro-Wilk [42; 43], Anderson-Darling [44], Lilliefors [45] and Kolmogorov-Smirnov [46; 47]). Simulations show that wavelets details coefficients are normally-like distributed as demonstrated in the figure 3. On the other hand, wavelets approximations are normally distributed except in some cases (the complete study is not presented here, because of limited number of pages). The decision was taken in each simulation by majority strategy (for example if there are 3 tests that do not reject normality then the normality is decided). The table 3 presents summary results regarding the most used distributions (Exponential, Uniform, Weibull, Rayleigh, Gamma). It shows the parameters formula of Gaussian distribution of wavelets coefficients (approximation and details), if normality is decided.

| Distribution  | Wavelets coefficients | Normality | Gaussian parameters |
|---------------|-----------------------|-----------|---------------------|
| Exponential ($\lambda$) App. $a_j$ Det. $d_j$ | $\otimes$ | $N(\mu = 2^{1/2} \lambda; \sigma^2 = \lambda^2)$ |
| Uniform (a,b) App. $a_j$ Det. $d_j$ | $\otimes$ | $N(\mu = \frac{a+b}{2}; \sigma^2 = \frac{(b-a)^2}{12})$ |
| Weibull ($\alpha,\beta$) App. $a_j$ Det. $d_j$ | $\otimes$ | $N(\mu = 2^{1/2} \beta \Gamma(1 + \frac{1}{\alpha}); \sigma^2 = \beta^2 \Gamma(1 + \frac{2}{\alpha}) - \mu^2)$ |
| Rayleigh ($\sigma_R$) App. $a_j$ Det. $d_j$ | $\otimes$ | $N(\mu = 2^{1/2} \sigma_R \sqrt{\frac{\pi}{2}}; \sigma^2 = \frac{4\pi}{7} \sigma_R^2)$ |
| Gamma ($\alpha,\beta$) App. $a_j$ Det. $d_j$ | $\otimes$ | $N(\mu = 2^{1/2} \alpha \beta; \sigma^2 = \alpha \beta^2)$ |

Table 3. Distributional parameters of wavelets coefficients when they are normally distributed

2.2.1. An illustrative example

The aim of this example is to display the distribution of wavelets coefficients when original data are exponentially distributed. In the figure 2, data was generated from exponential distribution with $\lambda = 5$. One can see from the figure 3 that wavelets coefficients have the characteristic symmetric "bell curve" shape.

Figure 2. Exponentially distributed data with $\lambda = 5$
Figure 3. Wavelets coefficients histograms using Haar wavelet and $N = 8$; Coef.1 corresponds to the approximation and Coef.2-8 correspond to the details.

3. Wavelets coefficients and SPC purposes

SPC aims to monitor process characteristics in order to detect changes in the mean and/or variance of the processes. Based on statistics (indicators) control charts are designed and control limits are derived from the statistic probability distribution. In the following theorem, we present a result regarding the probability distribution of wavelets coefficients (details and approximations).

**Theorem 1.** Assume $X = [x_1, x_2, \ldots, x_n]$ is a signal, where $x_i$ are identically normally distributed random variables ($x_i \sim \mathcal{N}(\mu, \sigma^2)$). Consider Orthonormal compactly supported wavelets (Haar, Daubechies, Symlets, Coiflets). The multiresolution analysis of $X$ provides wavelets coefficients as follows:

$$a_j(k) \sim \mathcal{N}(2^j \mu, \sigma^2)$$
$$d_j(k) \sim \mathcal{N}(0, \sigma^2)$$

Which are normally distributed random variables ($\mu$ and $\sigma^2$ are defined in the table 3 for non-Gaussian distributions). Moreover, if the Haar wavelet is used then they are independent (even if $x_i$ are autocorrelated), else autocorrelation is strongly debased (see tables 1 et 2).

One can remark, from the proof of the theorem 1 (see appendix A), that wavelets coefficients are summation of normally distributed variables consequently they follow the normal distribution.
From the theorem 1, we show that wavelets coefficients present some interesting distributional characteristics that reveal the original data features. One can remark that the approximation coefficients amplify the mean of the original data. Regarding details coefficients, they have a mean equal to zero (see window2 in Fig. 4b). Therefore, this characteristic could be useful to detect change in variance, because the mean change does not affect the details coefficients, as we shown in previous work [20].

Figure 4. (a): Observations with mean change (positive shift) at 40 ; (b): Wavelets coefficients (approximations $A_i$ and details $d_i$) using db2 wavelet

In the figure 4, we display the wavelets coefficients behavior when change in the mean is occurring. We consider a window size of length $L=8$ observations/subgroups, as the smallest one that can be used rationally with the db2 wavelet. Consequently we get eight wavelets coefficients at the scale one: four approximations coefficients and four details coefficients. The first 39 observations plotted in the figure 4(a) consist of observations randomly generated from a normal distribution $N(0,1)$ (an in-control process), and the last 41 observations, consist of observations randomly generated from a normal distribution $N(20,1)$ (out-of-control process with mean change). One can conclude that wavelets coefficients reflect clearly the mean change.

- Note, from the theorem 1, that $a_j(k)$ coefficients have the same distribution behavior as the average of the data $\bar{X}$ (Xbar). In fact, if $X \sim N(\mu_0, \sigma_0^2)$ then $\bar{X} \sim N(\mu_0, \frac{\sigma_0^2}{n})$. A mean change $\delta$ is defined as : $\mu_1 = \mu_0 + \delta \bar{\sigma}_0$ and can be detected in $\bar{X}$ chart as $\delta \sqrt{n}$ shift. When the sample size is $n = 2^J$, then the shift is equal to $\delta 2^{J/2}$. By the same way $\delta$ can be detected in wavelets approximations coefficients $A_J$ (statistic based on wavelets approximations coefficients, in the case of the Haar wavelet the $A_J$ is the approximation coefficient), at the highest scale $J$, as $\delta 2^{J/2}$ (see theorem 1). Furthermore, we note that $\bar{X}$ statistic reduces the variance of the data and keeps the mean equal to the mean of the data. However, in terms of detectability the $A_J$ and $Xbar$ charts are totally equivalent (see the figure 5(b)). Regarding wavelets details coefficients $d_j(k)$, they follow normal distribution with mean equal to zero and variance reflecting the variance of the data.

- DeWave chart are proposed based on wavelet details in order to detect variance change. It shows that DeWave performs better than S-chart when large samples are used [20].
In practice, DeWave and $A_J$ control charts can be implemented by doing a Discrete Wavelet Transform. Using the Haar wavelet and a sample of size $n = 2^J$; we obtain one approximation coefficient that will be used to monitor the process mean, and $n-1$ details coefficients that will be used to control the process variability. A special case where $n=2$, the wavelets details coefficients are corresponding to the Range (R) chart to monitor the process variability.

**Figure 5.** (a): Observations with mean change at 10th sub-group of size 4 ; (b): Xbar control chart and $A_J$ statistic (approximation coefficient) using Haar wavelet.

4. Conclusion

In this paper we expose the multiresolution analysis benefits in terms of handling two challenges that face SPC procedures. Firstly we propose a result regarding distributional parameters of wavelets coefficients. Secondly, we demonstrate that wavelets coefficients could be normally-like distributed even if original data have non-Gaussian distributions and we drive their distribution parameters. Moreover, we show the ability of wavelets analysis to debase the autocorrelation degree. Our study on AR(1) processes shows that the Haar wavelet is very recommended in order to decorrelate data. Illustrative examples and discussions are also proposed. Next research will concern the publication of the complete studies with examples of industrial applications. Future works should concern more applications of wavelets coefficients to monitor processes that are autocorrelated or characterised by non-Gaussian data. More studies are also needed to highlight the assets of wavelets coefficients to improve SPC performances.

Appendix A

Wavelets and scaling filters must satisfy some conditions [8; 27], in order to provide a multi-resolution analysis. We introduce the following conditions, which will be useful for the proof.
We consider $x_i \sim \mathcal{N}(\mu, \sigma^2)$:

$$
\sum_n h_n = \sqrt{2} \quad (4)
$$

$$
\sum_n g_n = 0 \quad (5)
$$

**Proof.** The approximation wavelets coefficients, at the first scale:

$$
a_1(k) = \sum_{i=1}^l h[i]x[2k - i]
$$

The expectation and the variance give that:

$$
E(a_1) = E(\sum_{i=1}^l h[i]x[2k - i]) = \sum_{i=1}^l h_i \ast E(x) = 2^{1/2} \ast \mu
$$

$$
V(a_1) = V(\sum_{i=1}^l h[i]x[2k - i]) = \sum_{i=1}^l h_i^2 \ast V(x) = \sigma^2
$$

At the second scale:

$$
a_2(k) = \sum_{i=1}^l h[i]a_1[2k - i]
$$

The expectation and the variance give that:

$$
E(a_2(k)) = E(\sum_{i=1}^l h[i]a_1[2k - i]) = \sum_{i=1}^l h_i \ast E(a_1(i)) = 2^{2/2} \ast \mu
$$

$$
V(a_2(k)) = \sigma^2
$$

By the same way, at the higher scales, we conclude at scale $j$:

$$
E(a_j(k)) = 2^{j/2} \ast \mu
$$

$$
V(a_j(k)) = \sigma^2
$$

Proceeding by the same previous proof, one can show that details wavelets coefficients, at scale $j$:

$$
E(d_j(k)) = 0
$$

$$
V(d_j(k)) = \sigma^2
$$
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