Comparison of Two Analysis Methods for Piezoelectric Laminated Plates

Liang Guo*, Ruishan Xing
Guangzhou Civil Aviation College, Aircraft Maintenance Engineering College, Guangzhou 510470, China

*Corresponding author e-mail: Guoliang@caac.net

Abstract. In this paper, the quasi-Shannon wavelet collocation precise integration method and the finite element method are used to solve the Hamilton regular equation of piezoelectric laminates. The cautions in the operation process of the two methods are analyzed, and the advantages and disadvantages of the two methods are compared. Numerical examples show that the wavelet collocation precise integration method which belong to the semi-analytical method is more accurate than the finite element method which is the numerical method.

Keywords: piezoelectric laminated plates; Hamilton canonical equation; quasi-shannon wavelet collocation precise integration method; finite element method.

1. Constitutive equations of piezoelectric materials

In engineering, piezoelectric laminates are used more and more widely, and many scholars have studied the structure of piezoelectric laminates. The electromechanical laws followed by the piezoelectric properties of piezoelectric crystals should be described by the piezoelectric constitutive equation which shows the relationship between electrical displacement $D$, electric field strength $E$, stress tensor $\sigma$ and strain tensor $\varepsilon$. Reference [1] gives four types of piezoelectric equations, of which the second type of piezoelectric equations, also called $e$-type equations, take strain and electric field strength as independent variables, and the matrix form is:

$$\begin{cases}
\sigma = c^E \varepsilon - e_\varepsilon E \\
D = e_\varepsilon \varepsilon + \lambda^E E
\end{cases} \quad (1)$$

In which $c^E$ is the short-circuit elastic stiffness constant tensor, $e$ is the piezoelectric constant tensor, $\lambda^E$ is clamping dielectric constant tensor.

For the constitutive equation, according to reference [2], Hamilton canonical equation for piezoelectric material is deduced as

$$\frac{\partial}{\partial z} \begin{bmatrix} P \\ Q \end{bmatrix} = D \begin{bmatrix} P \\ Q \end{bmatrix} - \begin{bmatrix} F \\ \theta \end{bmatrix}. \quad (2)$$
The basic governing equations used to describe piezoelectric materials include equilibrium equations, geometric equations, constitutive equations, and corresponding boundary conditions [3]. Among the three sets of basic equations, there are 22 equations and 22 unknowns, so they can be solved theoretically according to appropriate boundary conditions. Literature [4] roughly summarizes the solution method of piezoelectric laminates plates, and it can be seen that the solution method based on the equation of state and the three-dimensional theory is closer to the real solution. In this paper, based on the equation of state, the quasi-Shannon wavelet collocation precise integration method [5] and finite element method are used to solve the problem.

2. Program structure and precautions
The quasi-Shannon wavelet collocation precise integration method is a semi-analytical and semi-numerical method, While the finite element method is a numerical method. The collocation method program is written in mathematical software MATHEMATICA 8.0. The program flow chart is shown in Figure 1.

![Figure 1. Program Structure](image)

The dielectric constant refers to the absolute dielectric constant which is the clamping dielectric constant. Because the Hamilton matrix $\mathbf{D}$ contains both the order of the elastic constant $E$ and the order of $1/E$ [3], resulting in its strong singularity, it is easy to produce errors and even possible can not be calculated in the process of matrix product calculation or in the transfer of the matrix. In the calculation, it is necessary to carry out nondimensionalization processing[6] as follow
\[
(\vec{u}, \vec{v}, \vec{w}, \vec{D}_z) = \frac{1}{h} (u, v, w, D_z)
\]

\[
(\vec{\sigma}_{xz}, \vec{\sigma}_{yz}, \vec{\sigma}_{zz}, \vec{\phi}) = \frac{1}{E_x} (\sigma_{xz}, \sigma_{yz}, \sigma_{zz}, \phi)
\]

The elements in the dimensionless matrix are not much different in magnitude, and they can participate in the operation, but the final result should be returned to the dimension.

Commercial finite element software ANSYS16.0 is used for the finite element method, and the analysis procedure are shown in Figure 2. Element SOLID5 (Scalar Brick 5) is used for the piezoelectric layer, which is an eight-node hexahedral element with 6 degrees of freedom for each node. For the analysis of laminates, in order to ensure the continuity of the force and displacement between layers, the interlayer are bonded by the Glue method. The division accuracy of the grid in thickness direction is consistent with the plane direction.

The material characteristics required by ANSYS to analyze the piezoelectric model include relative dielectric constant, piezoelectric stress matrix or piezoelectric strain matrix, and elastic constant. Pay attention to conversion when inputting each data. The relative dielectric constant value should be calculated according to the constant strain value, that is, input according to the constant strain. Most of the published piezoelectric stress matrix \( \mathbf{e} \) are arranged in the order of \( x, y, z, yz, xz, xy \) based on the IEEE standard, while ANSYS input the matrix in the order of \( x, y, z, xy, yz, xz \), so the piezoelectric stress matrix \( \mathbf{e} \) when inputting into ANSYS needs to be exchanged as follows:

**Figure 2. ANSYS Analysis Procedure**

The material characteristics required by ANSYS to analyze the piezoelectric model include relative dielectric constant, piezoelectric stress matrix or piezoelectric strain matrix, and elastic constant. Pay attention to conversion when inputting each data. The relative dielectric constant value should be calculated according to the constant strain value, that is, input according to the constant strain. Most of the published piezoelectric stress matrix \( \mathbf{e} \) are arranged in the order of \( x, y, z, yz, xz, xy \) based on the IEEE standard, while ANSYS input the matrix in the order of \( x, y, z, xy, yz, xz \), so the piezoelectric stress matrix \( \mathbf{e} \) when inputting into ANSYS needs to be exchanged as follows:
Or when inputting the piezoelectric stress matrix in the piezoelectric equation of the second type into ANSYS, the following exchanges should be made:

\[
\begin{bmatrix}
  e_{11} & e_{12} & e_{13} \\
  e_{21} & e_{22} & e_{23} \\
  e_{31} & e_{32} & e_{33} \\
  y_{x} & e_{41} & e_{42} & e_{43} \\
  x_{z} & e_{51} & e_{52} & e_{53} \\
  x_{y} & e_{61} & e_{62} & e_{63}
\end{bmatrix} \rightarrow
\begin{bmatrix}
  e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} \\
  e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} \\
  e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36}
\end{bmatrix} (\text{ANSYS})
\] (4)

If the Young’s modulus, Poisson’s ratio and shear modulus are used when inputting the elastic constant matrix, the following conversion is required:

\[
S = C^{-1} =
\begin{bmatrix}
  C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
  C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\
  C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\
  0 & 0 & 0 & C_{44} & 0 & 0 \\
  0 & 0 & 0 & 0 & C_{55} & 0 \\
  0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}^{-1}
\] . (6)

\[
E_{x} = \frac{1}{S_{11}} , \quad E_{y} = \frac{1}{S_{22}} , \quad E_{z} = \frac{1}{S_{33}}
\]

\[
\mu_{xy} = \frac{S_{12}}{S_{11}} , \quad \mu_{xz} = \frac{S_{23}}{S_{22}} , \quad \mu_{yz} = \frac{S_{31}}{S_{33}}
\] . (7)

3. Numerical Examples

Consider a three-layer piezoelectric laminate made of two different materials with four sides clamped and electricity open-circuit on the upper and lower surfaces, as shown in Figure 3. The first layer and the third layer are made up of PZT-4 material, and the second layer is made of PVDF material. Properties of the two piezoelectric materials are presented in Table 1. The geometry parameters are: length and width \(a=b=1.0\)m, total thickness \(h=0.1\)m, thickness of each layer \(h_1=h_3=0.03\)m, \(h_2=0.04\)m. An uniform pressure \(q=1\)Pa is exerted on the top surface of the laminate\((z=0)\). The quasi-Shannon wavelet configuration fine integration method and finite element method were used to solve the problem, and the calculation results and error comparison between MATHEMATICA and ANSYS are shown in Table 2~5 .

\[
\begin{bmatrix}
  \frac{1}{S_{11}} , \quad \frac{1}{S_{22}} , \quad \frac{1}{S_{33}} \\
  \frac{S_{12}}{S_{11}} , \quad \frac{S_{23}}{S_{22}} , \quad \frac{S_{31}}{S_{33}} \\
  \frac{S_{45}}{S_{55}}
\end{bmatrix}
\]
**Figure 3.** Piezoelectric 3-layer Plates

**Table 1.** Material properties

| Elastic stiffness [Gpa] | $C_{11}$ | $C_{12}$ | $C_{13}$ | $C_{22}$ | $C_{23}$ | $C_{33}$ | $C_{44}$ | $C_{55}$ | $C_{66}$ |
|------------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| PZT-4                  | 132      | 71       | 73       | 132      | 73       | 115      | 26       | 26       | 30       |
| PVDF                   | 139      | 77.8     | 74.3     | 139      | 74.3     | 115      | 25.6     | 25.6     | 30.6     |

| Piezoelectric constant | $e_{31}$ | $e_{32}$ | $e_{33}$ | $e_{24}$ | $e_{15}$ |
|------------------------|----------|----------|----------|----------|----------|
| PZT-4                  | -4.1     | -4.1     | 14.1     | 10.5     | 10.5     |
| PVDF                   | -5.2     | -5.2     | 15.08    | 12.72    | 12.72    |

| Relative dielectric constant | $\epsilon_{11}/\epsilon_0$ | $\epsilon_{22}/\epsilon_0$ | $\epsilon_{33}/\epsilon_0$ |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| PZT-4                       | 804.6                       | 804.6                       | 659.7                       |
| PVDF                        | 1475                        | 1475                        | 1300                        |

**Table 2.** Maximum displacement value $w(a/2, h/2, h/2)$ and comparison (relative error)

| MATHEMATIC, $A_j=2$ | ANSYS, $4 \times 4$ | Error/% | MATHEMATIC, $A_j=3$ | ANSYS, $8 \times 8$ | Error/% | MATHEMATIC, $A_j=4$ | ANSYS, $16 \times 16$ | Error/% |
|---------------------|----------------------|---------|----------------------|---------------------|---------|---------------------|-----------------------|---------|
| 1.383E-10           | 1.415E-10            | 2.26    | 1.634E-10            | 1.604E-10           | 1.87    | 1.752E-10           | 1.645E-10             | 6.51    |

**Table 3.** Maximum electric potential value $\phi(a/2, h/2, h/2)$ and comparison (relative error)

| MATHEMATIC, $A_j=2$ | ANSYS, $4 \times 4$ | Error/% | MATHEMATIC, $A_j=3$ | ANSYS, $8 \times 8$ | Error/% | MATHEMATIC, $A_j=4$ | ANSYS, $16 \times 16$ | Error/% |
|---------------------|----------------------|---------|----------------------|---------------------|---------|---------------------|-----------------------|---------|
| 0.240E-1            | 0.275E-1             | 12.7    | 0.246E-1             | 0.253E-1            | 2.77    | 0.254E-1            | 0.250E-1              | 1.60    |
Table 4. Maximum displacement value $w(a/2,b/2)$ of each layer’s surface and relative error

| Surface Description               | MATHEMATICA, $j=3$ | ANSYS, 16×16 | Error/% |
|----------------------------------|--------------------|--------------|---------|
| Upper surface of the first layer $(0)$ | 1.612E-10         | 1.627E-10    | 0.92    |
| Upper surface of the second layer $(0.3h)$ | 1.631E-10         | 1.642E-10    | 0.67    |
| Lower surface of the second layer $(0.7h)$ | 1.629E-10         | 1.641E-10    | 0.73    |
| Lower surface of the third layer $(h)$ | 1.607E-10         | 1.624E-10    | 1.05    |

Table 5. Maximum electric potential value $\phi(a/2,b/2)$ of each layer’s surface and relative error

| Surface Description               | MATHEMATICA, $j=3$ | ANSYS, 16×16 | Error/% |
|----------------------------------|--------------------|--------------|---------|
| Upper surface of the first layer $(0)$ | 0.153E-1          | 0.175E-1     | 12.6    |
| Upper surface of the second layer $(0.3h)$ | 0.237E-1          | 0.243E-1     | 2.47    |
| Lower surface of the second layer $(0.7h)$ | 0.234E-1          | 0.241E-1     | 2.90    |
| Lower surface of the third layer $(h)$ | 0.143E-1          | 0.168E-1     | 14.9    |

Table 2 and Table 3 show that the maximum value increases with the increase of interpolation coefficient $j$. The maximum value of displacement through the collocation method is larger than the results obtained by ANSYS under the same pattern of meshing. It indicates that the flexibility of matrix generated by this algorithm is better, which demonstrates the configuration solution is closer to the real solution. Table 4 and Table 5 show that the results obtained by the collocation method coincide very well with the solutions of ANSYS, which proves that the approximate solution expressed by quasi-Shannon scale function is suitable for solving the fixed support boundary problem. In fact, if the piezoelectric constant of one of the piezoelectric materials is $e_{31}=e_{32}=e_{33}=e_{24}=e_{15}=0$, the laminated plate in the example becomes a laminated plate in which piezoelectric material and elastic material are superimposed.

4. Conclusion

In this paper, the quasi-Shannon wavelet collocation precision integration method and finite element method are used to solve the Hamilton canonical equations of piezoelectric laminates. The precautions in the operation of the two methods are analyzed. It needs to be dimensionless to solve the problem in the configuration method program, and the finite element method needs to convert various parameters to get the correct solution. The advantages and disadvantages of the two methods are compared. The example analysis shows that the wavelet collocation precise integration method is closer to the real solution than the finite element method, which is very suitable for solving the fixed-boundary problem, while the finite element method is more versatile.

References

[1] Fuxue Zhang, Likun Wang, Modern piezoelectrics. Science Press, Beijing 2001. (In Chinese)
[2] Yanhong Liu, Qingyuan Chen, Huiming Zhang, Analytical solution for laminated piezoelectric plates with clamped edges, Engineering Mechanics, 26 (2009) 6-11. (In Chinese)
[3] Liang Guo, Wavelet Collocation Precise Integration Method for Piezoelectric Laminated plates, Civil Aviation University of China, 2014. (In Chinese)
[4] Liang Guo, Research Methods on Mechanics Problems of Piezoelectric structure, Science
Mosaic, 05(2017):14-18. (In Chinese)

[5] Hong Wei Zhang, Yan Li Wang, Guang Hui Qing. A wavelet collocation precise integration method for laminated rectangular plates with four sides clamped[J]. Advanced Materials Research, 2013, 671-674: 1543-1551.

[6] Feng Liu, Duojie Jia, Guozhu Xi, Yonglin Ji, Method of normalization, Journal of Anshun University, 10(2008): 78-80. (In Chinese).