Cross Subaperture Averaging Generalized Sidelobe Canceler Beamforming Applied to Medical Ultrasound Imaging

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Abstract: For adaptive ultrasound imaging, a reliable estimation of the covariance matrix has a decisive influence on the performance of beamformers. In this paper, we propose a new cross subaperture averaging generalized sidelobe canceler approach (GSC-CROSS) for medical ultrasound imaging, which uses the cross-covariance matrix instead of the traditional covariance matrix estimation. By using the more stable and accurate estimation of the covariance matrix, GSC-CROSS performs well in both lateral resolution and contrast. Experiments are conducted based on the simulated echo data of scattering points and a cyst target. Beamforming responses of scattering points show that GSC-CROSS can improve the lateral resolution by 76.9%, 68.8%, and 17.1% compared with delay-and-sum (DS), synthetic aperture (SA), and the traditional generalized sidelobe canceler (GSC), respectively. Also, imaging of the cyst target shows that compared with DS, SA, and GSC, the contrast increases by 101%, 32.6%, and 63.5%, respectively. Finally, the actual echo data collected from a medical ultrasonic imaging system is applied to reconstruct the image. Results show that the proposed method has a good performance on lateral resolution and contrast. Both the simulated and experimental data demonstrate the effectiveness of the proposed method.

Keywords: GSC-CROSS; adaptive beamforming; minimum variance; covariance matrix

1. Introduction

Medical ultrasound imaging has the characteristics of high transmission capacity and low harm to the human body, which plays an important role in medical diagnosis technologies [1–3]. The key to the quality of echo images is beamforming algorithms. Nowadays, the Delay-and-Sum (DS) is widely used due to its low complexity [4]. However, the DS suffers wide main lobe and high sidelobe, which makes the lateral resolution and contrast of the image poor [5]. To improve the beamforming response, Jørgen Arendt Jensen [6] proposed synthetic aperture (SA) beamforming which achieves focusing in both transmission and reception process for every point in the field. Though SA can improve the lateral resolution and contrast, the sidelobe is still at a high level [7].

Recently, adaptive beamforming has been widely used in radar, sonar, communication, and other fields which use array signal processing. Huang et al. proposed robust adaptive beamforming (RAB) for acoustic imaging [8,9]. The adaptive beamforming aims to calculate a weight vector based on the echo data. The weight vector is equivalent to a spatial filter that can maintain the desired signal and suppress the interference and noise signal of the echo data [10]. Because the weight vector varies with the echo data, the beamforming responses improve significantly. For medical ultrasound imaging, adaptive beamforming was first proposed by Capon [11]. Johan-Fredrik Synnevåg et al. studied the minimum variance beamforming (MV) [12,13]. However, the MV algorithm has the problem of signal cancellation [14]. To improve the beamforming response, Li et al. studied
generalized sidelobe canceler beamforming for medical ultrasound imaging systems [15]. By separating the linear constraint space, GSC converts the constrained problem of the adaptive beamforming algorithm into an unconstrained problem, thus overcoming the signal cancellation problem of MV [16,17].

For adaptive beamforming, due to the high correlation of medical ultrasonic echo signals [18], some processing algorithms must be introduced to eliminate the correlation. The commonly used method is the subaperture averaging proposed by J. E Evans [19]. However, obtaining the covariance matrix with the subaperture averaging, the performance of GSC will approach the non-adaptive beamformer (SA) as the number of subarrays increase. Furthermore, although the lateral resolution improved, the contrast did not improve correspondingly. Therefore, how to obtain a reliable estimate of the covariance matrix that can simultaneously improve the lateral resolution and contrast has become a research hotspot.

In this paper, we put forward a new cross subaperture averaging generalized sidelobe canceler algorithm (GSC-CROSS) for medical ultrasound imaging, which uses echo signals from different subarrays to calculate a cross-covariance matrix. By using the more stable and accurate cross-covariance matrix instead of the traditional subaperture averaging one, the quality of the echo image improves significantly. Simulated and experimental results show that this method can improve both lateral resolution and contrast of the image and the performance is stable under different numbers of subarrays.

The outline of this paper is as follows: In Section 2, we introduce the mathematical model of ultrasound imaging and the basic principles and application methods of the GSC. In Section 3, we explain the basic principle and realization of GSC-CROSS in detail. Moreover, we show the experimental results based on simulated and actual echo data in Section 4. In Section 5, we discuss the advantages of the proposed method and compare it with other beamformers. Finally, conclusions of the study are provided in Section 6.

2. Background

2.1. Sensor Signal Model

For medical ultrasound imaging, we use adaptive beamformers to process ultrasonic data that has achieved transmitted and received focusing by SA. For a linear array consisting of M transducer elements, the output of the beamformer can be expressed as [20]:

\[ y(k) = \mathbf{w}^H(k) \mathbf{X}(k) = \sum_{i=1}^{M} \mathbf{w}_i^*(k)x_i(k) \]  

(1)

where \( k \) is the time index. \( \mathbf{X}(k) \) consists of M time-delayed echo data, \( \mathbf{X}(k) = [x_1(k), x_2(k), \cdots, x_M(k)]^T \), and \( \mathbf{w}(k) = [\mathbf{w}_1(k), \mathbf{w}_2(k), \cdots, \mathbf{w}_M(k)]^T \) is the weight vector of the beamforming, \( * \) represents complex conjugate, \( (\cdot)^T \) and \( (\cdot)^H \) represent matrix transpose and matrix conjugate transpose, respectively. For adaptive beamformers, we artificially divide \( \mathbf{X}(k) \) into three parts as follow:

\[ \mathbf{X}(k) = \mathbf{S}(k) + \mathbf{I}(k) + \mathbf{N}(k) \]  

(2)

where \( \mathbf{S}(k) \) represents the desired signal, which is reflected by the detection point, \( \mathbf{I}(k) \) is the echo signal introduced from the sidelobe direction called interference signal, and \( \mathbf{N}(k) \) represents the noise signal, including thermal noise and scattering noise.

For adaptive beamforming, the weight vector \( \mathbf{w}(k) \) is equivalent to a spatial filter, which can suppress interference and noise signals while retaining the desired signal. Since the weight vector \( \mathbf{w}(k) \) is calculated from the echo data, it can vary with echo data adaptively. Thus, it can improve the quality of echo images.

Similarly, Equation (1) can also represent non-adaptive beamforming algorithms if the vector \( \mathbf{w}(k) \) is preset. For example, we can use the window functions, such as Hamming, Hanning, and Blackman, etc., which is called the apodization technique [21]. Especially, the output of the beamformer will be the result of the synthetic aperture (SA)
when \( \omega(k) = [1, 1, \ldots, 1]^T \). It is equivalent to adding a rectangular window to the beam. In this situation, the \( \omega(k) \) can also be expressed as [22]:

\[
\omega = \frac{I^{-1}a}{a^H I^{-1}a}
\]

(3)

where \( I \) is an identity matrix and \( a \) denotes a steering vector.

### 2.2. Generalized Sidelobe Canceler (GSC)

The GSC algorithm was originally proposed by Griffiths [17]. For adaptive algorithms, the beamformers should follow the principle of suppressing the interference and noise signal while maintaining the desired signal. This principle can be expressed as a mathematical constraint as follows [23]:

\[
\min \omega^H R \omega, \text{ subject to } \omega^H a = 1
\]

(4)

where \( R \) is the covariance matrix of the interference and noise signal, and \( a \) denotes a steering vector. In the present study, \( a \) becomes a vector of ones because the echo data has been delayed and focused. Equation (4) can be solved by utilizing the Lagrange method, and the optimal weight vector is given by [24]

\[
\omega = \frac{R^{-1}a}{a^H R^{-1}a}
\]

(5)

Equation (5) is usually used as the adaptive weight of MV beamformer, but GSC will decompose it further. Figure 1 shows the schematic diagram of the GSC. It can separate the linear constraint with an adaptive filter, thereby converting the constrained optimization problem of Equation (4) into the unconstrained optimization problem. The weight vector \( \omega \) is decomposed into an adaptive weight vector \( \omega_q \) and a non-adaptive weight vector \( \omega_a \). Moreover, the \( \omega_q \) locates in the constrained subspace while the \( \omega_a \) is orthogonal to the constrained subspace. This structure can overcome the problem of signal cancellation.

![Figure 1. Structure of generalized sidelobe canceler.](image)

As we can see in Figure 1, the upper part is the non-adaptive branch where all the signals can pass and determine the non-adaptive weight \( \omega_q \). Moreover, we set \( \omega_q \) parallel to the desired signal \( S(k) \) to ensure that the desired signal can pass through the branch without loss. Meanwhile, the bottom half is the adaptive branch where blocking matrix \( B \) is provided to block the desired signal [25]. Thus, only interference and noise signals can pass and determine the adaptive weight \( \omega_a \). Finally, by subtracting the adaptive branch output \( Y_a(k) \) from the non-adaptive branch output \( Y_q(k) \), we can remove the interference and noise signals from the non-adaptive branch while retaining the desired signal. The weight vector of GSC can be expressed as:

\[
\omega = \omega_q - B \omega_a
\]

(6)

Then, we can redefine the constraints of Equation (4) using Equation (6) as

\[
\omega_a = \arg \min_{\omega_a} \left( (\omega_q - B \omega_a)^H R (\omega_q - B \omega_a) \right)
\]

subject to \( a^H \omega_q = 1 \)

(7)
By traditional subaperture averaging, the main diagonal components of the sub-covariance ultrasound data. However, it will cause the degradation of the sample covariance matrix.

### Proposed Method

#### Figure 2.

Schematic diagram of subaperture averaging.

**2.3. Estimation of Covariance Matrix**

Observing (6)–(9), we can see that it is necessary to acquire the interference and noise covariance matrix to calculate the weight vector \( \omega \). However, in practice, \( R \) is usually unavailable and estimated with the sample covariance matrix \( \hat{R} \), defined as [28]:

\[
\hat{R} = X(k)X(k)^H
\]

For medical ultrasound systems, echo data are broadband and coherent signals, so that formula (12) cannot be directly used in the data processing. To acquire a reliable estimation of the covariance matrix \( \hat{R} \), we should eliminate the correlation first. Usually, subaperture averaging is applied. As shown in Figure 2, subaperture averaging divides the array into \( P \) overlapping subarrays with length \( L \), and the covariance matrix is obtained by averaging the covariance matrices from all subarrays [29]. The estimation of the sample covariance matrix \( \hat{R} \) can be expressed as:

\[
\hat{R} = \frac{1}{P} \sum_{p=1}^{P} G_p(k) G_p^H(k)
\]

where \( P \) denotes the number of subarrays and \( P = M - L + 1 \), and \( L \) denotes the length of subarrays. The larger \( L \) is, the smaller \( P \) is, the better beamformer is, however, the worse the stability of the beamformer will be [30]. Usually, \( L \) is less than \( M/2 \).

**Figure 2.** Schematic diagram of subaperture averaging.

### 3. Proposed Method

With the help of subaperture averaging, GSC can be applied to process medical ultrasound data. However, it will cause the degradation of the sample covariance matrix \( \hat{R} \). By traditional subaperture averaging, the main diagonal components of the sub-covariance matrix \( G_p(k) G_p(k)^H \) correspond to the square of the echo signal \( |x_i(k)|^2 \) received by the subarrays. The larger \( L \) is, the smaller \( P \) is, the better beamformer is, however, the worse the stability of the beamformer will be [30]. Usually, \( L \) is less than \( M/2 \).
same element, which is always positive, while the rest of the sub-covariance matrix consists of $x_i(k)x_j(k)$, which are positive or negative randomly. Therefore, after averaging according to Equation (13), the elements on the diagonal of the covariance matrix are much larger than those in other positions, which makes the sample covariance matrix $R$ approach the identity matrix $I$. Observing Equations (3) and (5), GSC gradually degenerates into SA, as the number of subarrays increases.

To improve the accuracy and stability of the estimation of the covariance matrix, we put forward a new cross subaperture averaging generalized sidelobe canceler approach (GSC-CROSS) for medical ultrasound systems. This method uses the echo data from different subarrays to estimate the covariance matrix, and the new matrix is called the cross-covariance matrix. With the more stable and accurate cross-covariance matrix, rather than the traditional covariance matrix obtained from subaperture averaging, the beamformer response improves significantly.

The cross subaperture averaging considers not only the covariance matrix of the same subarray but also the covariance matrix between different subarrays. The expression of the cross-covariance matrix is:

$$\hat{R} = \frac{1}{P^2} \sum_{i=1}^{P} \sum_{j=1}^{P} G_i(k)G_j^H(k)$$

(14)

where $i, j$ denote subarray numbers. By introducing the cross-covariance matrix, the main diagonal components of the sub-covariance matrix may not always be positive, so the increase of diagonal components in $\hat{R}$ will be avoided, which makes the estimation of the covariance matrix more stable and reliable.

Observing Equation (14), the cross-covariance matrix requires $P^2$ matrix multiplications while the conventional covariance matrix requires $P$ times. This will lead to a huge increase in the computational cost to obtain images. Here, we propose to calculate the mean-subarray $\bar{G}$ first, then the cross-covariance matrix can be express as:

$$\bar{G} = \frac{1}{P} \sum_{i=1}^{P} G_i(k)$$

(15)

$$\hat{R} = \bar{G}\bar{G}^H$$

(16)

According to Equations (15) and (16), we can obtain the cross-covariance matrix with a onetime matrix multiplication. Therefore, the computational cost is reduced significantly, which always means the faster imaging speed.

In the present study, diagonal loading is applied to ensure that the matrix can be inverted correctly. It will add a tiny noise signal $\varepsilon$ into the sample covariance matrix $\hat{R}$ [31]. The $\varepsilon$ can be defined as:

$$\varepsilon = \frac{1}{\Delta \times L} \text{tr} (\hat{R})$$

(17)

$$\hat{R} = \hat{R} + \varepsilon I$$

(18)

where $\text{tr}(\cdot)$ is the trace of the matrix and $\Delta$ is a constant whose value ranges from 10 to 100. In general, smaller $\varepsilon$ provide the image better lateral resolution, but the computation may fail due to numerical instability [8,9].

4. Results

In this section, we present several results including simulated and experimental data to compare the difference between the proposed method with DS, SA, and GSC in terms of lateral resolution, contrast, and sidelobe level. The simulated data is obtained by Field II [32,33]. It is a simulation tool widely used in the medical ultrasonic imaging field to
verify the performance of beamforming algorithms. In the present study, we simulate a 3.33 MHz, 64-element linear array with a 0.24 mm inter-element spacing along with 7.1 MHz sampling frequency and 1500 m/s sound velocity. Scattering points and a circular cyst are introduced as detection targets in the simulation experiment. Moreover, the experimental data is collected from a medical ultrasound imaging system with the same parameters as the simulated one.

4.1. Simulated Point Targets

There are 14 scattering points distributed from depth 25–60 mm and lateral direction −7 mm to 7 mm. Moreover, the echo images of point targets processed by various algorithms are shown in Figure 3. Figure 4 shows the lateral variation of the point at the depth of 35 mm and 45 mm. According to Figure 4, we can analyze the full width at half maximum (FWHM) and the peak sidelobe level (PSL) of each algorithm, which represents the lateral resolution and sidelobe energy, respectively. Moreover, Table 1 lists the calculation results of FWHM and PSL of each algorithm at the depth z = 40 mm. Observing the FWHM, GSC-CROSS can increase the FWHM by 76.9%, 68.8%, and 17.1% compared with DS, SA, and GSC. As for ultrasound imaging, the lower PSL is, the better the echo image will be. Compared with DS, SA, and GSC, PSL with GSC-CROSS is reduced by 75.64, 61.96, and 43.44 dB, respectively. Therefore, the sidelobe noise in the images processed by DS, SA, and GSC is obvious, while it is basically not observed in GSC-CROSS. The point targets simulation experiment shows that the GSC-CROSS algorithm can improve the lateral resolution of the echo image while suppressing sidelobe energy.

![Figure 3](image_url)

**Figure 3.** Beamforming responses of simulated point targets with different algorithms. (a) DS; (b) SA; (c) GSC (L = 32, Δ = 10); (d) GSC-CROSS (L = 32, Δ = 10). The dynamic range of image is 80 dB.

![Figure 4](image_url)

**Figure 4.** Lateral variation of point at (a) z = 35 mm; (b) z = 40 mm.

| Algorithm   | FWHM (mm) | PSL (dB) |
|-------------|-----------|----------|
| DS          | 1.47      | -13.08   |
| SA          | 1.09      | -26.76   |
| GSC         | 0.41      | -45.28   |
| GSC-CROSS   | 0.34      | -88.72   |

**Table 1.** FWHM and PSL of point targets at depth z = 40 mm.
4.2. Simulated Cyst Targets

To analyze the performance of each algorithm on image contrast, we simulate a cyst phantom in a high speckle noise environment by Field II. The circular cyst with a radius of 5 mm is located at (x, z) = (0, 40) mm. Since the cyst mimics the lesions which are anechoic inside, the interior of the cyst has no scattering points while several scattering points with different coefficients are placed outside the cyst randomly to increase the speckle noise. Figure 5 shows the echo images of various algorithms. The lateral variation at the center of the circle is shown in Figure 6. Table 2 lists the contrast and generalized contrast-to-noise ratio (gCNR) of each image in detail. Compared with DS, SA, and GSC, contrast with GSC-CROSS is improved by 101%, 32.6%, and 63.5%, respectively. Because the interior of the cyst is anechoic, the lower the mean intensity inside the cyst in the echo image is, the better the sidelobe suppression is. Compared with DS, SA, and GSC, GSC-CROSS can reduce the mean intensity inside the cyst by 47.83, 33.45, 34.65 dB, respectively. Moreover, observing the edge area of the cyst, the boundary between the cyst and background in the images of DS, SA, and GSC is blurred because of the interference, while GSC-CROSS results in a distinguishable edge. Observation index gCNR [34] is robust against dynamic range alterations. It is usually used to judge whether the algorithm can improve the contrast indeed, rather than the effect of dynamic range changes. The gCNR describes the success rate for an ideal observer to detect the lesion. The closer gCNR is to 1, the higher probability of lesion detection of the algorithm. In Table 2, the GCC-CROSS has the highest gCNR, which means that this method is easier for the detection of lesions and can improve the contrast. The cyst targets simulation experiment shows that GSC-CROSS can effectively suppress the interference and improve the contrast.

| Algorithm | FWHM (mm) | PSL (dB) |
|-----------|-----------|----------|
| DS        | 1.73      | 9.45     |
| SA        | 1.09      | 7.56     |
| GSC (L = 32, \(\Delta = 20\)) | 0.34 | 2.78 |
| GSC-CROSS (L = 32, \(\Delta = 20\)) | 0.26 | 2.12 |

Figure 5. Contrast of beamforming responses for different beamforming algorithms. (a) DS; (b) SA; (c) GSC (L = 32, \(\Delta = 20\)); (d) GSC-CROSS (L = 32, \(\Delta = 20\)). The dynamic range of image is 80 dB.

Figure 4. Lateral variation of point at (a) \(z = 35\) mm; (b) \(z = 40\) mm.
Figure 6. Lateral variation at z = 40 mm of Figure 5.

Table 2. Contrast and gCNR of beamforming responses for different beamforming algorithms.

| Algorithm | Mean Intensity in the Cyst Region (dB) | Mean Intensity in the Background (dB) | Contrast (dB) | gCNR   |
|-----------|----------------------------------------|---------------------------------------|--------------|--------|
| DS        | −45.71                                 | −20.02                                | 25.69        | 0.968  |
| SA        | −60.09                                 | −21.19                                | 38.90        | 0.998  |
| GSC       | −58.89                                 | −27.34                                | 31.55        | 0.996  |
| GSC-CROSS | −93.54                                 | −41.95                                | 51.59        | 1.000  |

Contrast = mean intensity in the background–mean intensity in the cyst region.

4.3. Actual Data Experiment

To verify the effect of the algorithms, we used the echo data collected from an actual ultrasound imaging system for an experiment. Compared with the simulated echo data, the actual echo data suffers from low Signal-to-Noise Ratio and focus error due to the sound velocity error and energy attenuation in complex tissue, as well as the inaccurate system parameters. Thus, the actual echo image is weaker than the simulation result. The echo images processed by each algorithm are shown in Figures 7 and 8 shows the lateral variation at the depth of 75.6 mm and 88 mm. Table 3 lists the lateral resolution, contrast and gCNR of images with different beamformers. Since it is difficult to distinguish the scattering point and cyst in the image with DS, we only analyze the images with SA, GSC, and GSC-CROSS in the following text. Comparing the gCNR, the value of GSC-CROSS is higher than that of GSC and SA. Thus, the lesion is easier to detect by GSC-CROSS and the contrast is improved indeed. Compared with SA and GSC, the lateral resolution with GSC-CROSS is improved by 56.9% and 54.8%, and the contrast with GSC-CROSS is improved by 65.3% and 97.1%. Moreover, observing the beamforming responses of GSC and SA, we can see that their performance is closed. This is because we set a small value of L, so there are more subarrays used to estimate the covariance matrix, which degrades the GSC beamformer. This experiment shows that even in the situation of high noise, GSC-CROSS can still obtain better lateral resolution and contrast. Moreover, as the subarrays increase, GSC-CROSS can also maintain a good performance which means that it can maintain good performance and stability together. The actual data experiment results prove the effectiveness of the proposed algorithm.

Table 3. FWHM, contrast and gCNR of different beamforming algorithms.

| Algorithm  | FWHM (mm) | Contrast (dB) | gCNR |
|------------|-----------|---------------|------|
| SA         | 2.53      | 8.56          | 0.684|
| GSC        | 2.41      | 7.18          | 0.523|
| GSC-CROSS  | 1.09      | 14.15         | 0.952|
Figure 7. Images of actual echo data with different algorithms. (a) DS; (b) SA; (c) GSC (L = 16, Δ = 20); (d) GSC-CROSS (L = 16, Δ = 20). The dynamic range of image is 80 dB.

Figure 8. Lateral variation of point at (a) z = 75.6 mm; (b) z = 88 mm.

4.4. Computational Cost

In the process of computing, the matrix multiplication takes lots of computational cost. Considering a transducer array with P subarrays, the conventional covariance matrix requires P times of the matrix multiplication, and the cross-covariance matrix requires P^2 times. Thus, calculating the cross-covariance matrix as Equation (14) directly will greatly increase the amount of computation. However, in our GSC-CROSS method, we can obtain the cross-covariance matrix with only one-time multiplication. Table 4 shows the imaging speed of traditional GSC, GSC with conventional cross-covariance matrix using Equation (14) (GCS-cross) and GSC-CROSS to obtain the images in Figure 7. The computation time is measured on a platform with an Intel® Core i5-7400 CPU (Intel, USA) of 3.00GHZ with a RAM of 32G. This experiment shows that GSC-CROSS can reduce the computational cost of calculating the cross-covariance matrix, and it provides a faster imaging speed than the GSC beamformer.

Table 4. Imaging speed with covariance matrix and cross-covariance matrix.

| Algorithm     | Computation Time (s) |
|---------------|----------------------|
| GSC           | 325                  |
| GSC-cross     | 8395                 |
| GSC-CROSS     | 146                  |

5. Discussion

According to the simulation experiment of point targets and cyst target, as well as the real data experiment, we prove that due to the more reliable covariance matrix estimation, GSC-CROSS can not only improve the lateral resolution but also lead to better contrast. In the simulated point targets experiment shown in Figure 3; Figure 4, we can see that GSC-CROSS has the narrowest main lobe width compared with DS, SA, and GSC, indicating that GSC-CROSS slightly improves the lateral resolution of the medical ultrasound system. Meanwhile, GSC-CROSS also has outstanding performance in suppressing sidelobe energy.
It can inhibit the PSL to $-88.7$ dB, which is much lower than the other three algorithms. Thus, when we set the dynamic range at 80 dB, the artifact caused by the sidelobe can hardly be observed in Figure 3d, while it is obvious in Figure 3a–c.

The cyst target simulation experiment shows GSC-CROSS also performs well in improving contrast. Compared with DS, SA, and GSC, the contrast with GSC-CROSS is improved by about 20 dB. Observing the beamforming response processed by GSC and SA, we can see that though the lateral resolution is improved, the contrast is not improved correspondingly. It is because the filtering effect of GSC is associated with the degree of freedom of the system, and it will have good performance with the condition that the number of interference sources is less than L-2. However, the degree of freedom after subaperture averaging is L-1, so the beamformer will be weakened [7]. Therefore, GSC is usually combined with processing methods to improve the performance of contrast [30,35]. These processing methods will increase the complexity and computational demand for GSC. However, GSC-CROSS uses the echo signals from different subarrays to estimate the cross-covariance matrix, which is more reliable than the traditional covariance matrix. Furthermore, the number of sub-matrices involved in averaging the cross-covariance matrix is much larger than that in the traditional covariance matrix, which makes the interference and noise suppressed better. Thus, compared with GSC, GSC-CROSS dramatically improves the contrast as well as the lateral resolution for the medical ultrasound system.

The cross-covariance matrix introduces more matrix multiplications, which will slow down the imaging speed. In a previous study [22], only partial subarrays are extracted by sampling to estimate the cross-covariance matrix, limiting the performance of the beamformer. In our method, by calculating the mean-subarray first, the cross-covariance matrix can be obtained with a one-time matrix multiplication. It allows us to use all the subarrays to estimate the matrix and takes less time. Thus, our method can improve the imaging speed and it is even faster than the traditional GSC beamformer.

For adaptive beamforming, some parameters of the processing algorithms have a great influence on the performance of the beamformer. Figure 9 shows the echo images with GSC-CROSS using different parameters, and the contrast of each image is listed in Table 5. Parameter L determines the number of subarrays P. For traditional GSC, the smaller the L is, the more subarrays P there are, the worse beamformer will be. However, because of a more stable covariance matrix estimation, GSC-CROSS shows high robustness to the change of L(P). In Table 5, we can see that when L changes, the contrast hardly changes. The parameter $\Delta$ determines the energy of white noise $\epsilon$ introduced into the sample covariance matrix. The larger the $\Delta$, the smaller the noise energy $\epsilon$ that will be added. Comparing Figure 9a,c, when $\Delta$ increases, the speckle signal of background area is not well preserved in Figure 9c, which indicates the stability of the covariance matrix will be reduced.

![Figure 9](image-url)  
**Figure 9.** Beamforming responses of simulated cyst target with different parameters. (a) GSC-CROSS ($L = 24$, $\Delta = 20$); (b) GSC-CROSS ($L = 16$, $\Delta = 20$); (c) GSC-CROSS ($L = 24$, $\Delta = 100$). The dynamic range of image is 80 dB.
Table 5. Contrast of beamforming responses of GSC-CROSS with different parameters.

| Algorithm   | Parameters | Contrast (dB) |
|-------------|------------|---------------|
| GSC-CROSS   | L = 24, Δ = 20 | 50.77         |
| GSC-CROSS   | L = 16, Δ = 20 | 50.15         |
| GSC-CROSS   | L = 24, Δ = 100 | 53.60        |

6. Conclusions

In this paper, we use cross subaperture averaging to estimate the cross-covariance matrix which is more accurate and stable than the traditional covariance matrix. Through simulation and actual data experiments, the proposed method has significant improvement in lateral resolution and contrast of the medical ultrasonic system and it also provides faster imaging speed. As an optimization for covariance matrix, this method can be applied to a variety of beamformers based on GSC, which can further improve the quality of medical ultrasound images.

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